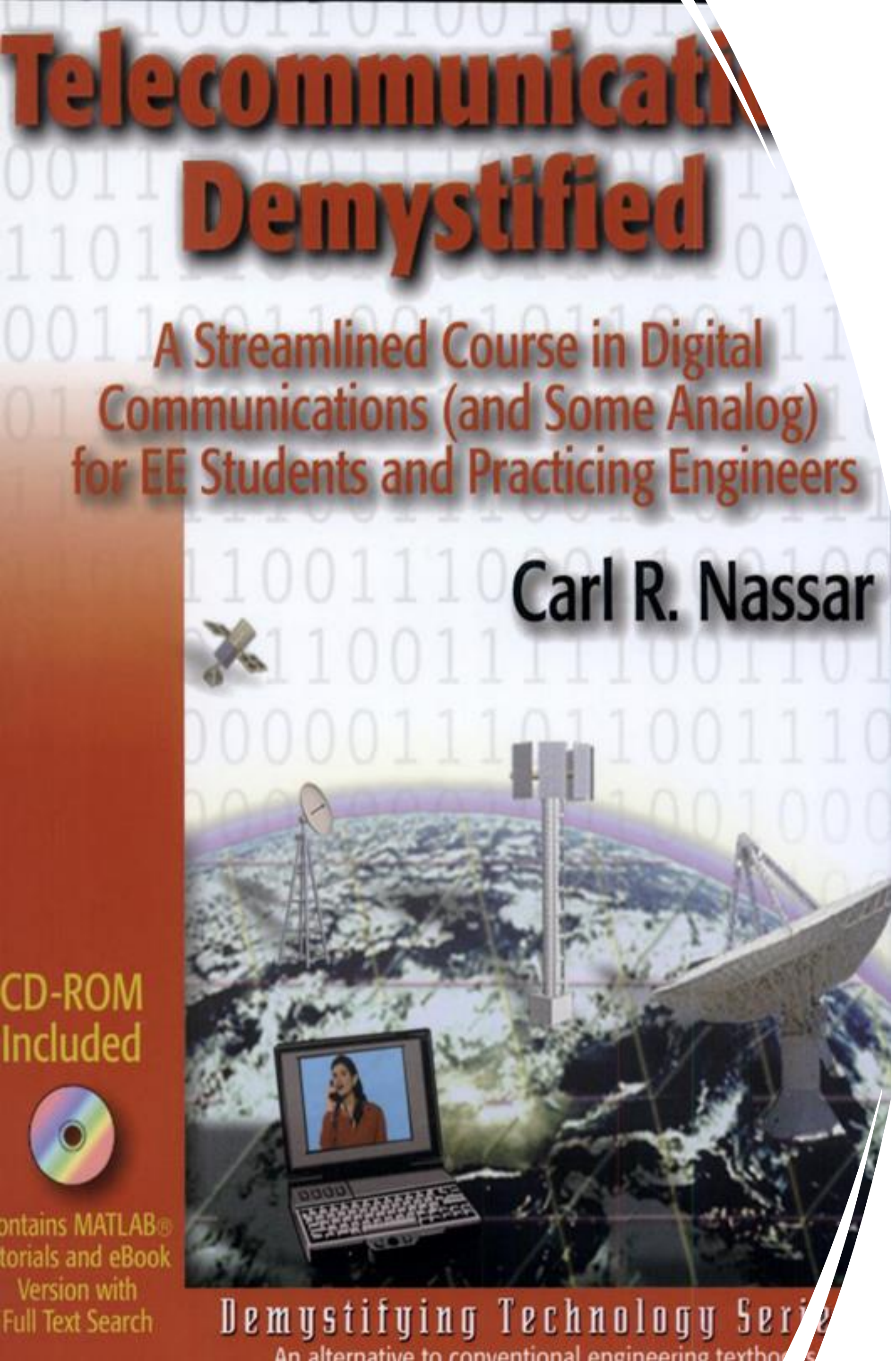


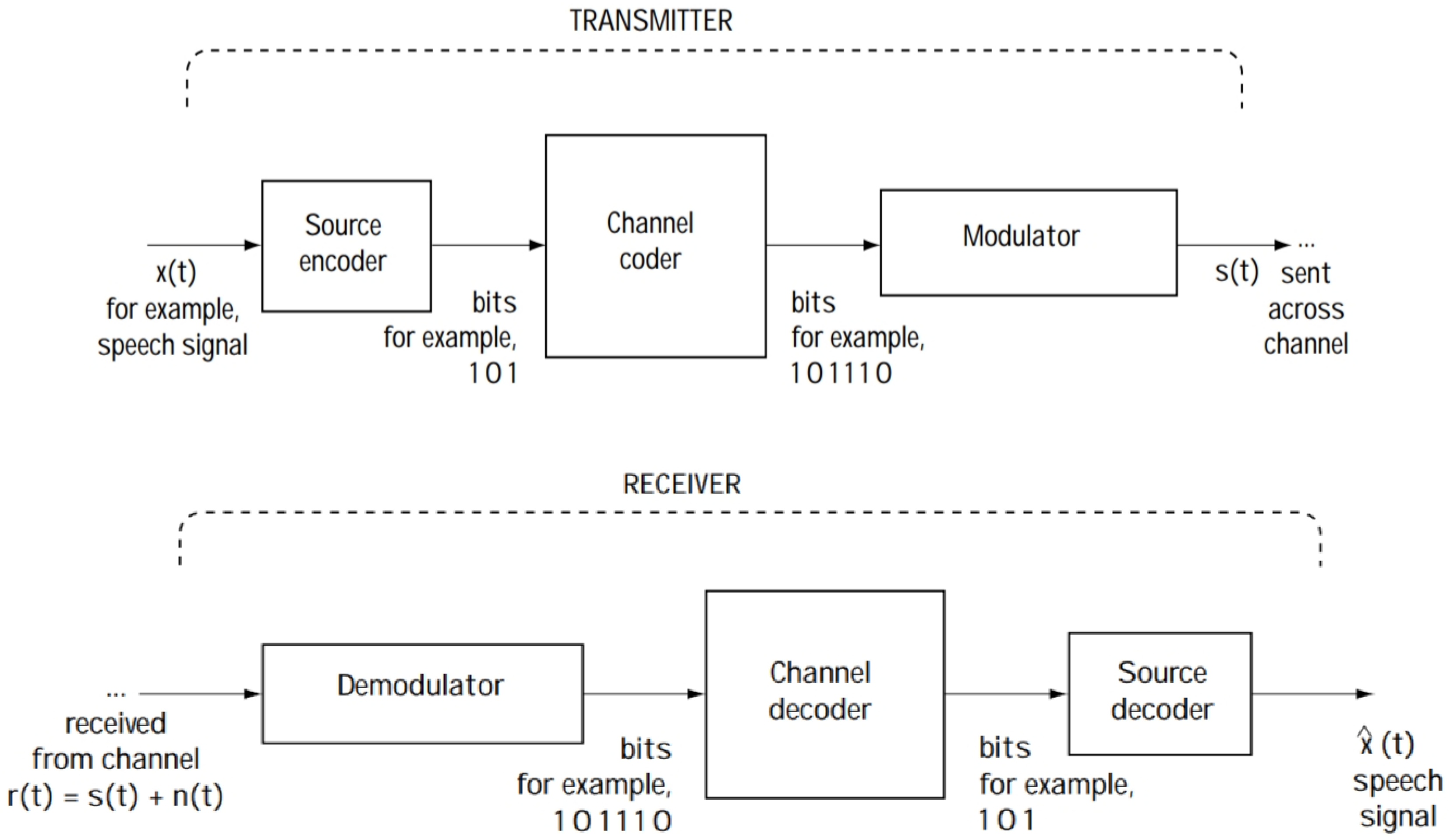
5. Block Coding and Decoding

5.1 Simple Block Coding

5.2 Linear block codes

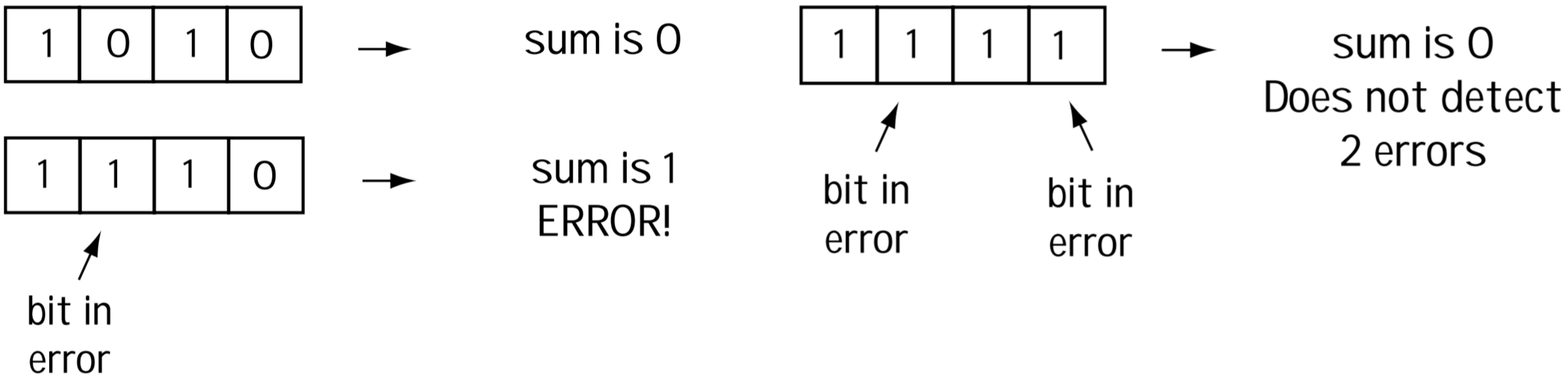
5.3 Performance of the Block Coders

5. Block Coding and Decoding



5.1 Simple Block Coding

- The Single Parity Check Bit Coder - Channel coding where one bit is added to create a total sum of 0 is called *even parity*. You can instead add one more bit so that the total when adding all bits is 1, and this is called *odd parity*.

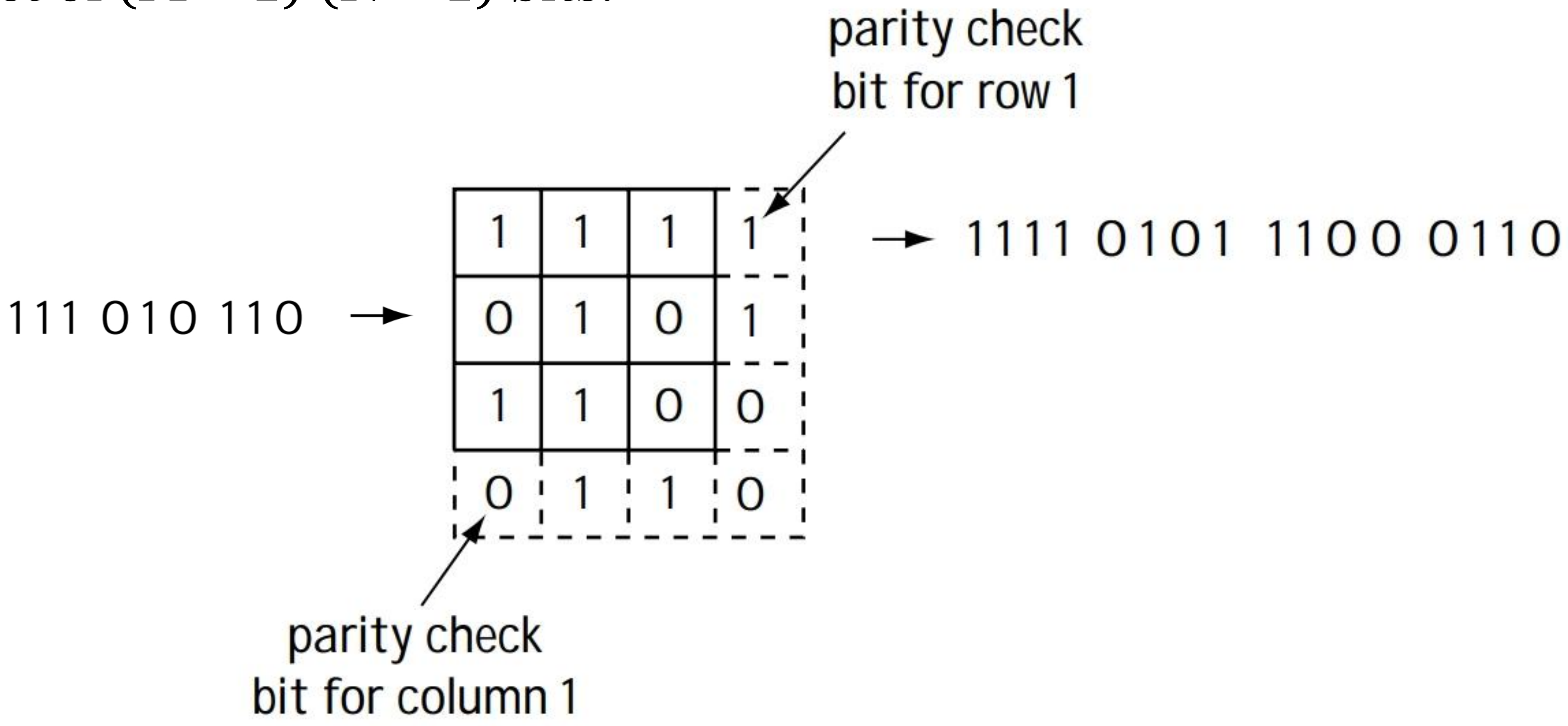


5.1 Simple Block Coding

- Consider a block code that maps each incoming set of k bits to an outgoing set of n bits:
  1. First, in shorthand notation, this block code will be called an (n,k) code.
  2. This block code is said to have a code rate of k/n.
  3. This block code is also said to have a redundancy of (n–k)/k.
  4. And finally, this code is said to have (n–k) redundant bits (that is, check bits or parity bits), which refer to the added (n–k) bits.

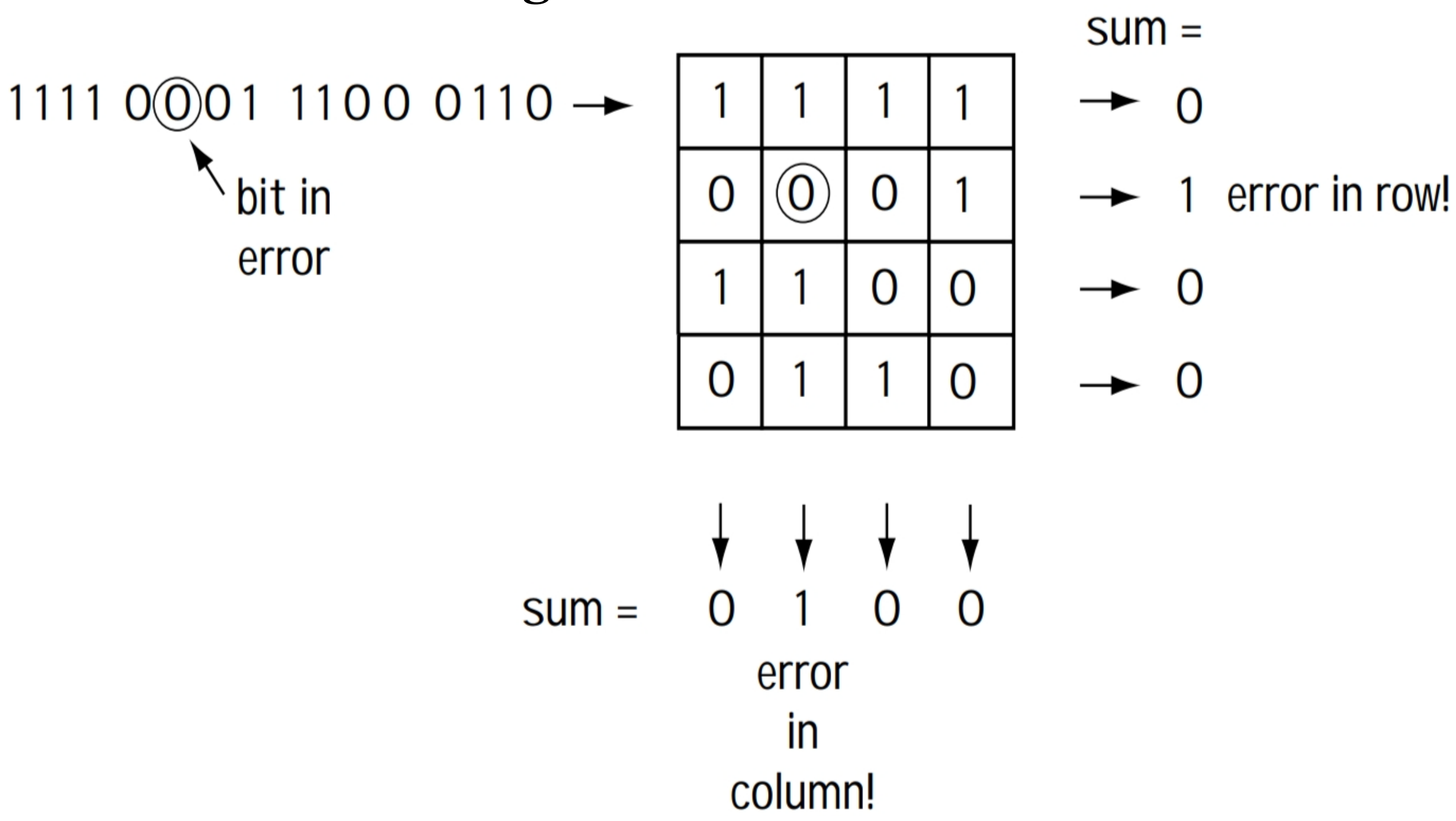
5.1 Simple Block Coding

- Rectangular Codes - In rectangular codes, each set of M·N bits are mapped to a set of (M + 1)·(N + 1) bits.



5.1 Simple Block Coding

- Channel Decoders for Rectangular Codes



5.2 Linear block codes

- Linear block coders are a group of block coders that follow a special set of rules when choosing which set of outputs to use. The rules are as follows, using a (6,3) code for illustrative purposes:

  V_n = the set of all possible 64 6-bit sequences

  U = the set of eight 6-bit sequences output at the channel coder

- Using this notation, the rule is this:

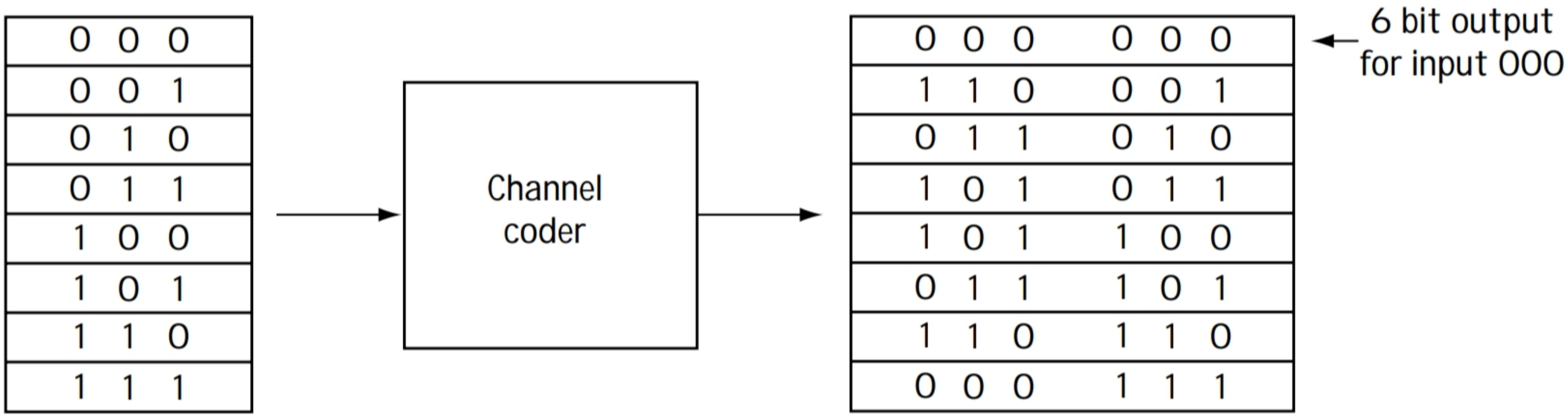
  U must be a subspace of V_n.

- This means two very simple things:

  1. U must contain {000000}

  2. Adding (modulo 2) any two elements in U must create another element in U.

5.2 Linear block codes



5.2 Linear block codes

input bits		output bits			
0	0	0	0	0	0
0	1	0	1	0	1
1	0	1	0	1	0
1	1	1	1	1	1

- A linear block code?

5.2 Linear block codes

1 1 0 → **Channel coder : uses look-up table** → 1 1 0 1 1 0

Input	Output
0 0 0	0 0 0 0 0 0
0 0 1	1 1 0 0 0 1
0 1 0	0 1 1 0 1 0
0 1 1	1 0 1 0 1 1
1 0 0	1 0 1 1 0 0
1 0 1	0 1 1 1 0 1
1 1 0	1 1 0 1 1 0
1 1 1	0 0 0 1 1 1

- vector $\mathbf{m}$ (1 0 1)
- vector $\mathbf{u}$ = (0 1 1 1 0 1)
- $\mathbf{u} = \mathbf{m}\,G$

5.2 Linear block codes

Channel coder : uses look-up table

1 1 0 → 1 1 0 1 1 0

Input	Output
0 0 0	0 0 0 0 0 0
0 0 1	1 1 0 0 0 1
0 1 0	0 1 1 0 1 0
0 1 1	1 0 1 0 1 1
1 0 0	1 0 1 1 0 0
1 0 1	0 1 1 1 0 1
1 1 0	1 1 0 1 1 0
1 1 1	0 0 0 1 1 1

$$G = \left(P_{3\times3} \vdots I_{3\times3}\right)$$

$$= \begin{pmatrix} P_{11} & P_{12} & P_{13} & \vdots & 1 & 0 & 0 \\ P_{21} & P_{22} & P_{23} & \vdots & 0 & 1 & 0 \\ P_{31} & P_{32} & P_{33} & \vdots & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{u} = \mathbf{m}G$$

$$= \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

5.2 Linear block codes

input bits	output bits
0 0	0 0 0 0
0 1	0 1 0 1
1 0	1 0 1 0
1 1	1 1 1 1

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{u}_{m=(0\ \ 0)} = \mathbf{m}G = \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{u}_{m=(0\ \ 1)} = \mathbf{m}G =$$

$$\mathbf{u}_{m=(1\ \ 0)} = \mathbf{m}G =$$

$$\mathbf{u}_{m=(1\ \ 1)} = \mathbf{m}G =$$

5.2 Linear block codes

- The Decoding

$$GH = \mathbf{0}$$

$$G = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \qquad H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

5.2 Linear block codes

- The Decoding

$$\mathbf{v}H = \mathbf{u}H = \mathbf{m}GH = \mathbf{m}\mathbf{0} = \mathbf{0}$$

$$\begin{aligned}
\mathbf{v} &= \mathbf{u} + \mathbf{e} \\
\mathbf{v}H &= \mathbf{u}H + \mathbf{e}H \\
&= \mathbf{m}GH + \mathbf{e}H \\
&= \mathbf{m}\mathbf{0} + \mathbf{e}H \\
&= \mathbf{0} + \mathbf{e}H \\
&= \mathbf{e}H
\end{aligned}$$

5.2 Linear block codes

- Mapping errors e to syndromes **S**

e	**S** = **e** H
0 0 0 0 0 0	0 0 0
0 0 0 0 0 1	1 0 1
0 0 0 0 1 0	0 1 1
0 0 0 1 0 0	1 1 0
0 0 1 0 0 0	0 0 1
0 1 0 0 0 0	0 1 0
1 0 0 0 0 0	1 0 0
0 1 0 0 0 1	1 1 1

5.3 Performance of the Block Coders

- Performances of single parity check bit coders/decoders

$$P_m = \sum_{\substack{j=2 \\ j \in even}}^{n} P(j,n) \qquad P(j,n) = \binom{n}{j} p^j (1-p)^{n-j}$$

- Performance of rectangular codes

$$P = \sum_{j=2}^{n} P(j,n)$$

- Performance of linear block codes

$$t = \left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor \qquad P = \sum_{j=t+1}^{n} P(j,n)$$

5.3 Performance of the Block Coders

- Benefits and Costs of Block Coders