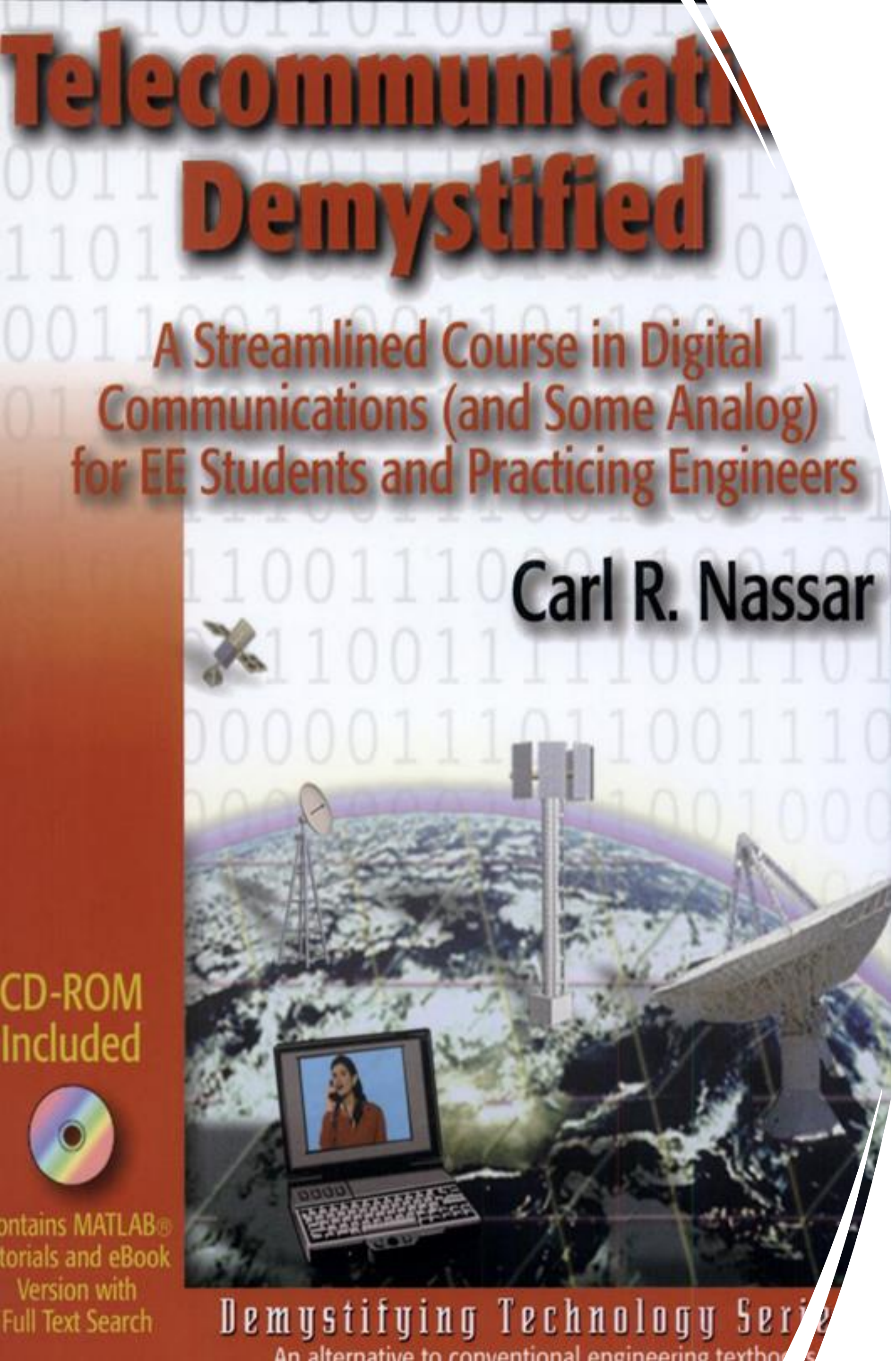


2. A Review of Some Important Math

2.1 Random Variables

2.2 Random Processes

2.3 Signals and Systems

2.1 Random Variables

- A *random event*, A, refers simply to an event with an unknown outcome.
- An example of a random event is tomorrow's weather.
- A *random variable*, x, is a number whose value is determined by a random event, A.
- For example, it may be tomorrow's outdoor temperature.

2.1 Random Variables

- One way to fully characterize our random variable x is by a function called the *probability distribution function*, $F_x(X)$.
- The function $F_x(X)$ is defined in words as follows: $F_x(X)$ is the likelihood that the random variable x is less than the number X.
- In a nice, neat equation, $F_x(X)$ is defined as

$$F_x(X) = P(x \leq X)$$

- where P(___) is shorthand for the words "probability that ___ happens."
- The probability distribution function is also known as the *cumulative distribution function (CDF)*.

2.1 Random Variables

- Example 1: Suppose there are 6 balls in a bag. The random variable X is the weight of a ball (in kg) selected at random. Balls 1, 2, and 3 weighs 0.5 kg; Balls 4 and 5 weighs 0.25 kg; and Ball 6 weighs 0.3 kg. Write the Probability for X.

weight of a ball (*X*)	Probability *P(x)*
0.25	2/6
0.30	
0.50	

2.1 Random Variables

- Example 2: Suppose we toss two dice. Make a table of the probabilities for the sum of the dice.

X	1	2	3	4	5	6	7	8	9	10	11	12	13
$P(x)$													
$F_x(X)$													

2.1 Random Variables

- Here are four simple properties of $F_x(X)$:

(1) $0 < F_x(X) < 1$: that is, since $F_x(X)$ represents the probability that $x < X$, it, like all probabilities, must be between 0 (never happens) and 1 (always happens).

(2) $F_x(-\infty) = 0$: that is, $F_x(-\infty) = \mathrm{P}(x < -\infty) =$ (the probability that x is less than $-\infty$) $= 0$ (since no number can be smaller than $-\infty$).

(3) $F_x(\infty) = 1$: that is, $F_x(\infty) = \mathrm{P}(x < \infty) =$ (the probability that x is less than ∞) $= 1$ (since every value must be smaller than ∞).

(4) $F_x(\mathrm{x}_1) \geq F_x(\mathrm{x}_2)$ if $\mathrm{x}_1 > \mathrm{x}_2$: that is, for example, the probability that x is less than 20 is at least as big as the probability that x is less than 10 .

2.1 Random Variables

- Example 3: The number of old people living in houses on a randomly selected city block is described by the following probability distribution.

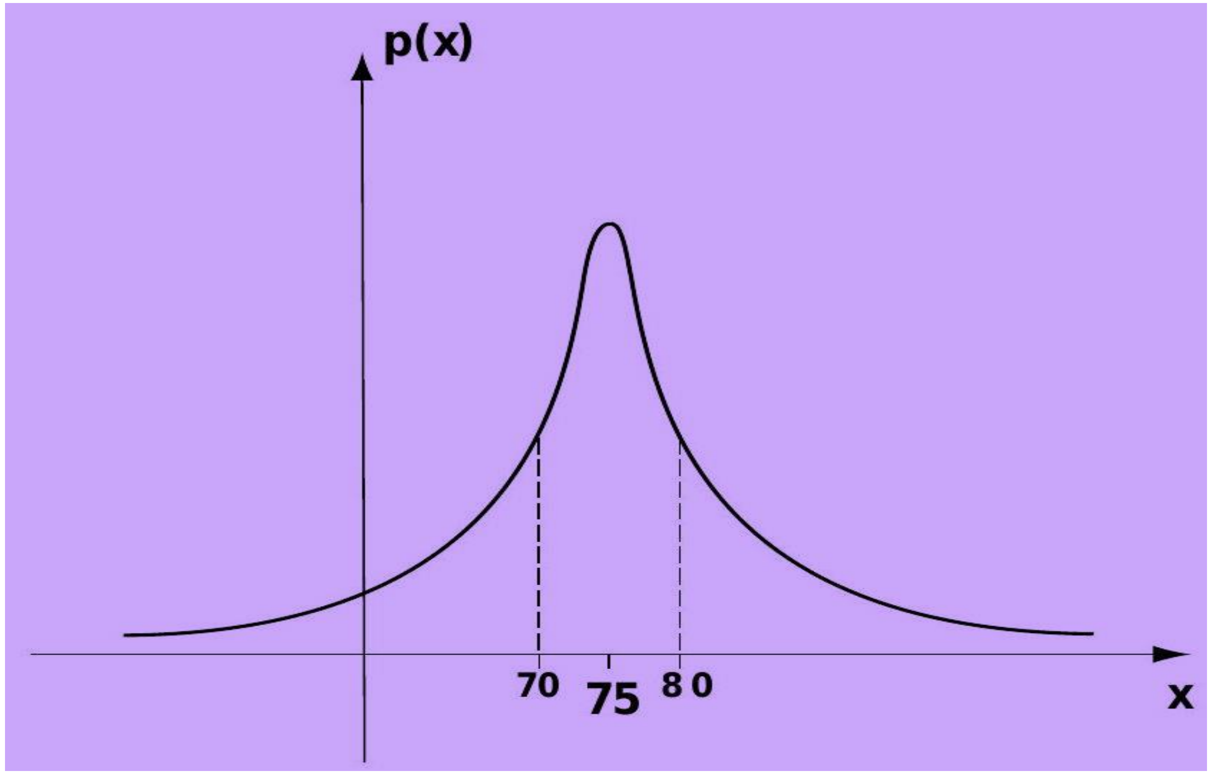
Number of adults (X)	Probability $P(x)$	$F_x(X)$
3	0.50	
4	0.25	
5	0.10	
6		

2.1 Random Variables

- A second way to describe our random variable x is to use a different function called the *probability density function* (*pdf* for short).
- The *pdf* for this variable is denoted $p_x(x)$ or $p(x)$

$$P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} p(x)dx$$

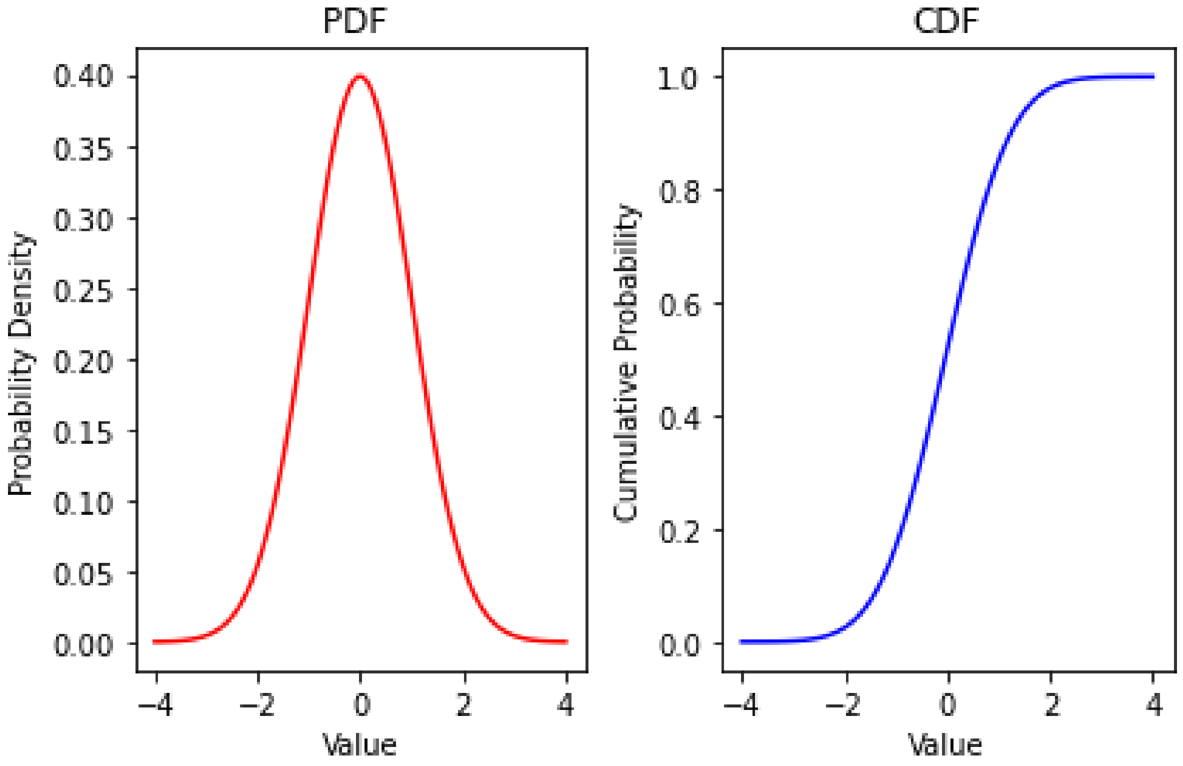
2.1 Random Variables



- if you want to know how likely it is that tomorrow's temperature x will be between 70 degrees and 80 degrees, all you have to do is integrate $p(x)$ over the range of 70 to 80 .
- $p(x)$ at x=70 gives you an idea how likely it is that tomorrow's temperature will be about 70 degrees.

2.1 Random Variables

- CDF is the integral of the PDF, and the PDF is the derivative of the CDF.



2.1 Random Variables

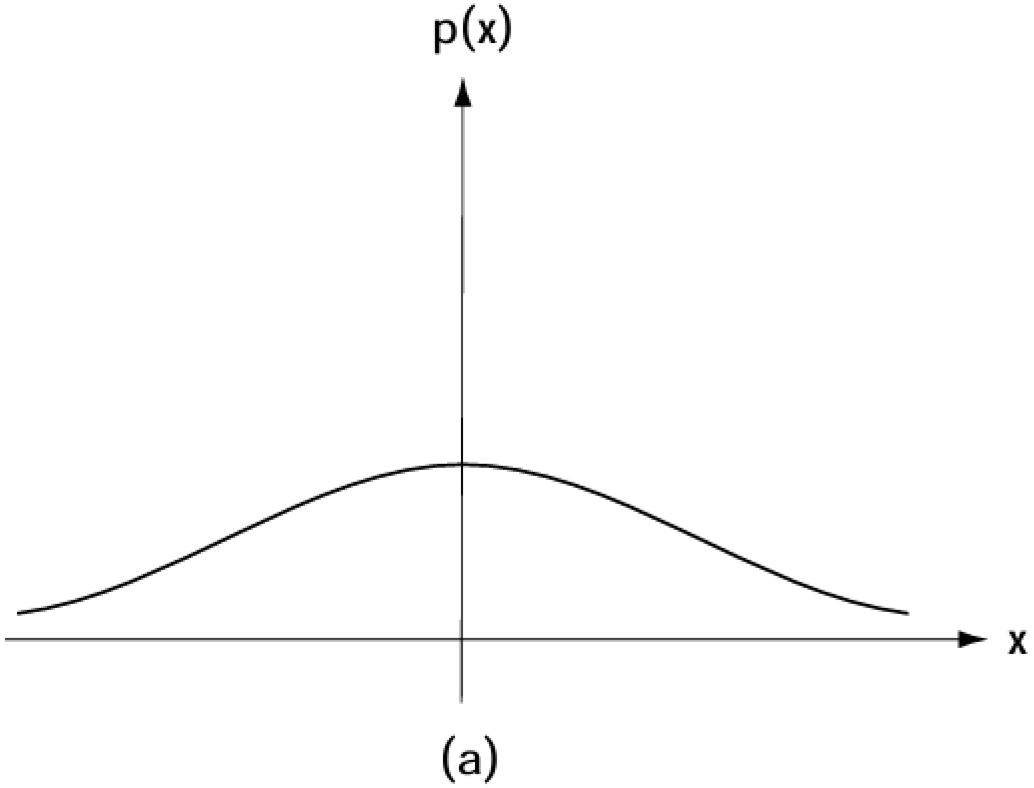
- The mean, x_m (also known as E(x)) : One thing you can tell me is the average (or mean) value of x

$$x_m = \int_{-\infty}^{\infty} x p(x) dx$$

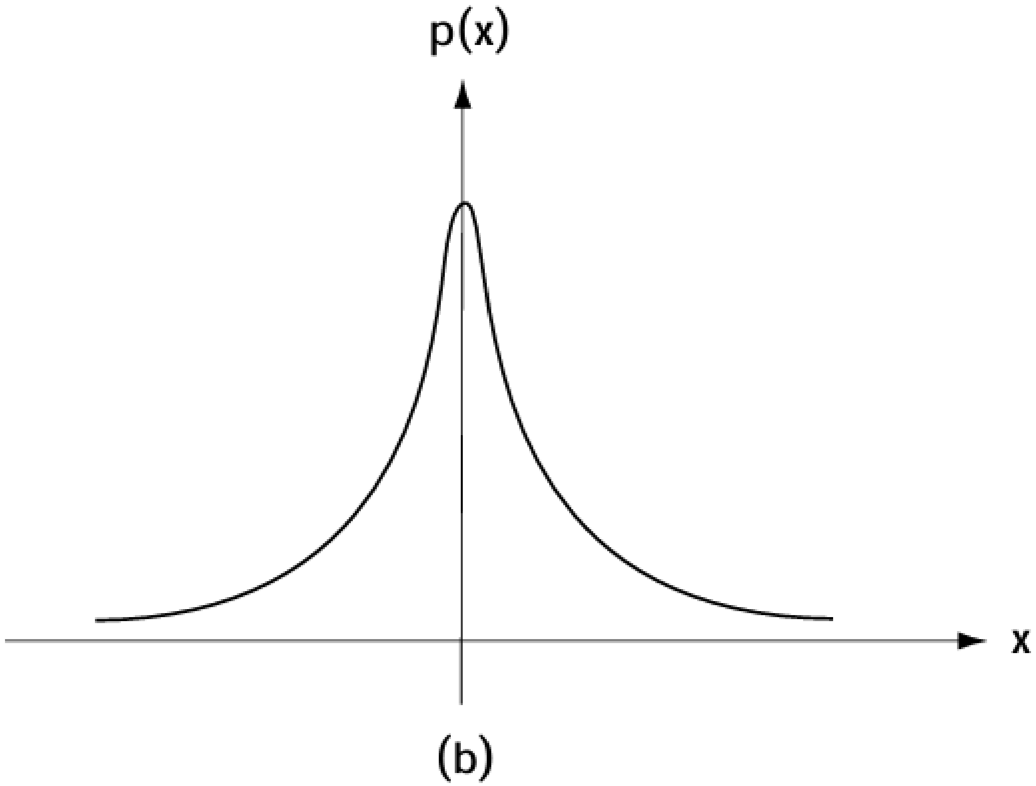
- The variance, σ_n^2 : Another important piece of information about the random variable x is how much x varies.

$$\sigma_n^2 = \int_{-\infty}^{\infty} (x - x_m)^2 p(x) dx$$

2.1 Random Variables



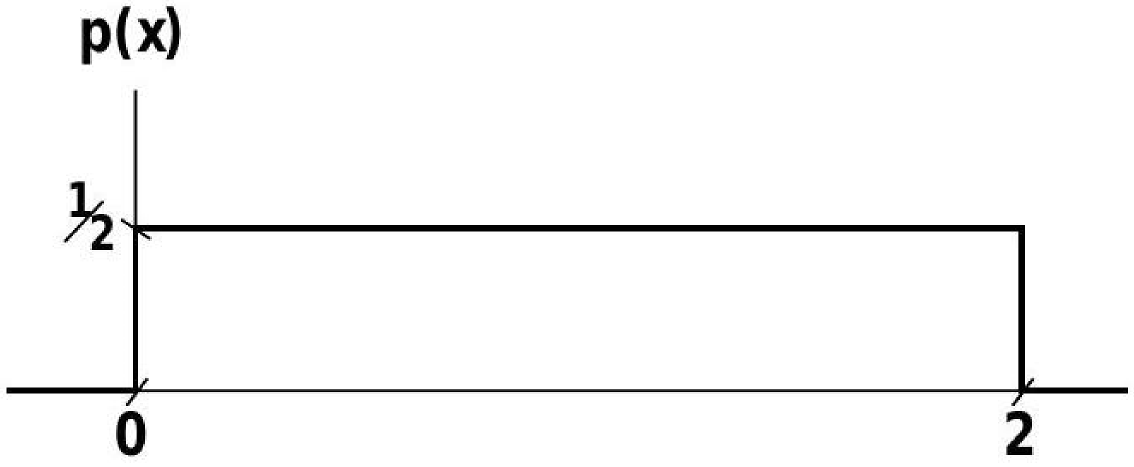
Random variable x with large variance



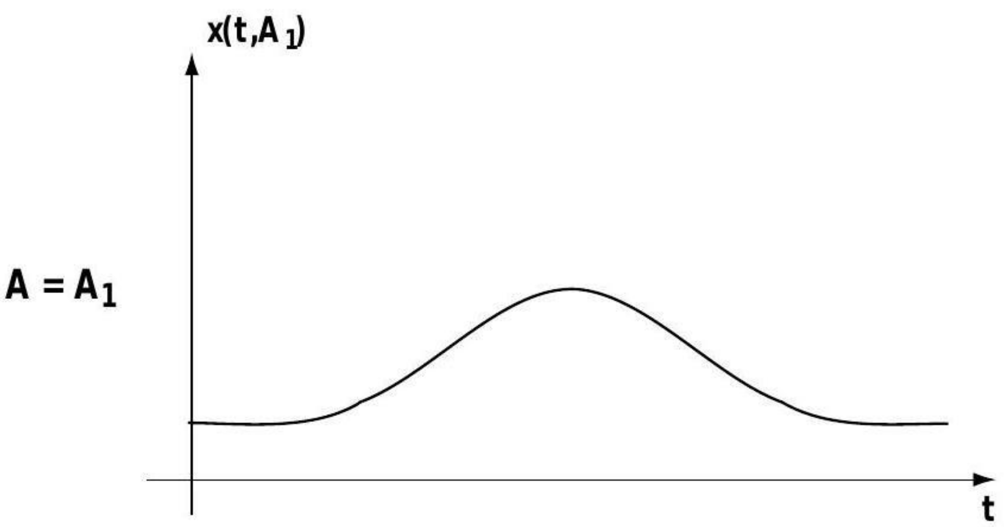
Random variable x with small variance

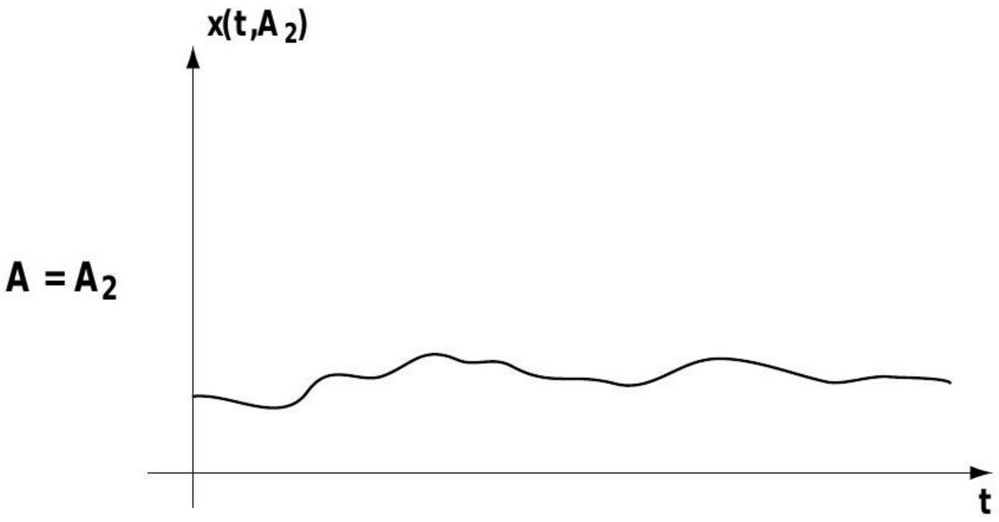
2.1 Random Variables

- Example 4: Given a random variable x and told that x has a probability distribution function $p(x)$, determine its mean and its variance.



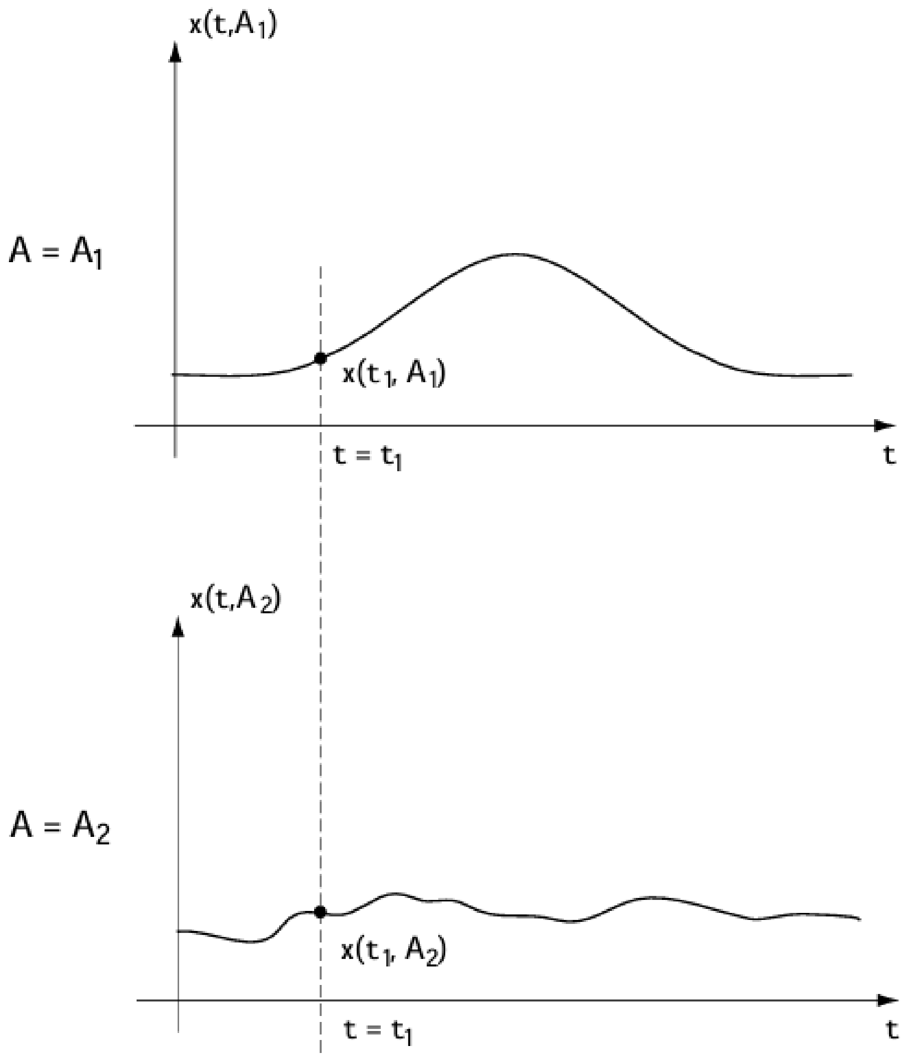
2.2 Random Processes





- A random process, $x(t)$, is a function of time t, where the exact function that occurs depends on a random event A.
- For example, let $x(t)$ be tomorrow's temperature as it changes with time over the day; the values of $x(t)$ will depend on A (whether it's sunny or not).
- So, we can write $x(t) = x(t, A)$ to indicate that the time function depends on the random event A. Here, $x(t, A)$ is a random process.

2.2 Random Processes



- There's one very important thing to note about a random process $x(t, A)$.
- At time $t = t_1$, we have $x(t_1, A)$, which is a number whose exact value depends on A.
- That's just a random variable! So, the sample of a $A = A_1$ random process $x(t, A)$ at $t = t_1$ is a random variable.
- We'll call it $x_1 = x_1(A)$.

2.2 Random Processes

- the mean of $x(t_1, A) = x_1$: this number (which may be different at different times t_1) tells you the average value of $x(t, A)$ at $t = t_1$. This value can be generated using the equation

$$x_m(t_1) = \int_{-\infty}^{\infty} x_1 p(x_1) dx_1$$

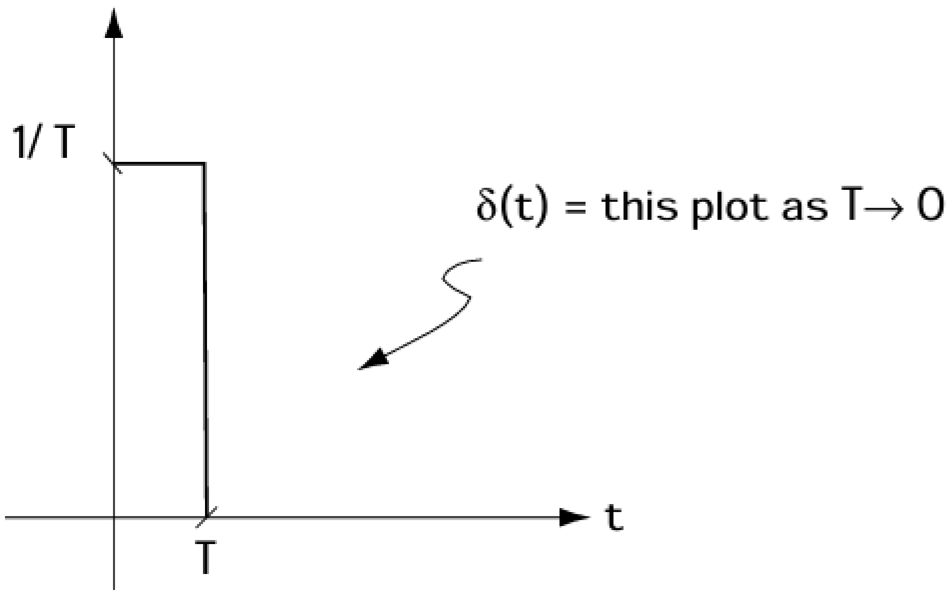
2.2 Random Processes

- the autocovariance, $\mathrm{R_x(t_1, t_2)}$: this number (which may be different for different t_1 and t_2 values) describes the relationship between the random variable $x(t_1, A) = x_1$ and the random variable $x(t_2, A) = x_2$.

$$R_x(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(x_1 - x_m(t_1)\right)\left(x_2 - x_m(t_2)\right) p(x_1, x_2) dx_1 dx_2$$

- The larger this number, the more closely related $x(t_1, A)$ is to $x(t_2, A)$. This value can be generated mathematically through the equation

2.3 Signals and Systems



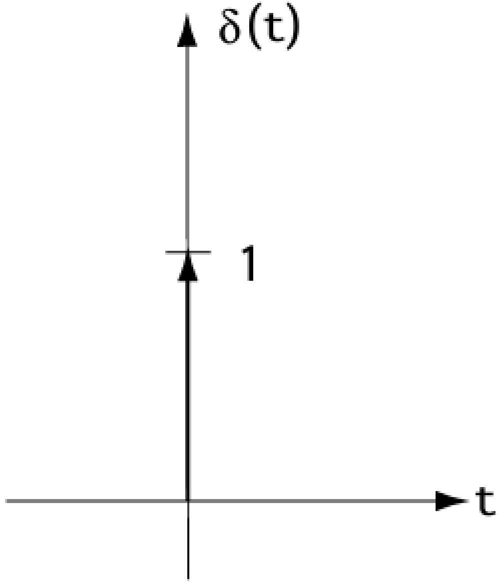
- $\delta(t)$, which is called the impulse function (or delta function, or just impulse)
- $\delta(t)$ is infinitely tall;
- $\delta(t)$ is infinitely skinny; and
- the area under the $\delta(t)$ function is 1

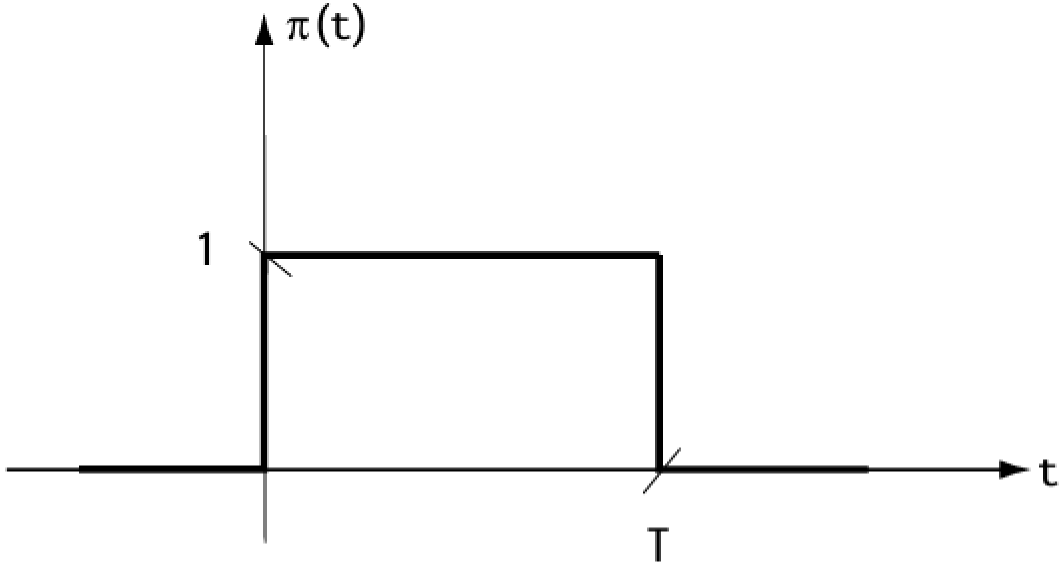
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

2.3 Signals and Systems

- A square wave function, $\pi(t)$.
- This function is of height 1 and duration T.

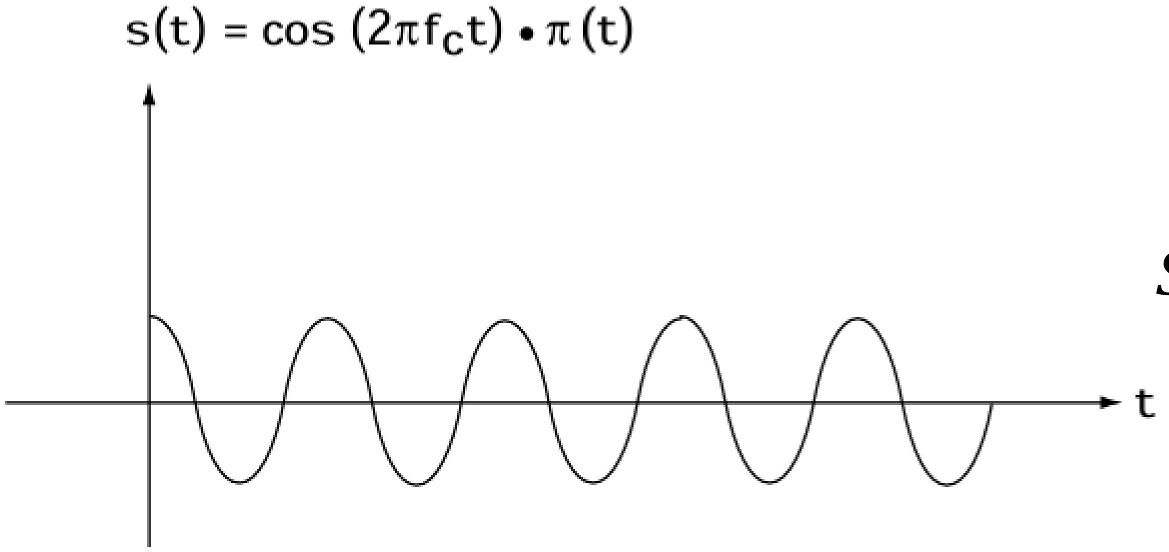
$$\pi(t)=\begin{cases}1, & 0\le t\le \mathrm{T}\\ 0, & \text{else}\end{cases}$$





2.3 Signals and Systems

- To describe signals, discovered by a fellow named Fourier.
- Any time signal $s(t)$ can be created by adding together cosine and sine waveforms with different frequencies.
- You can describe $s(t)$ by indicating how much of each frequency f you need to put together to make your signal $s(t)$.

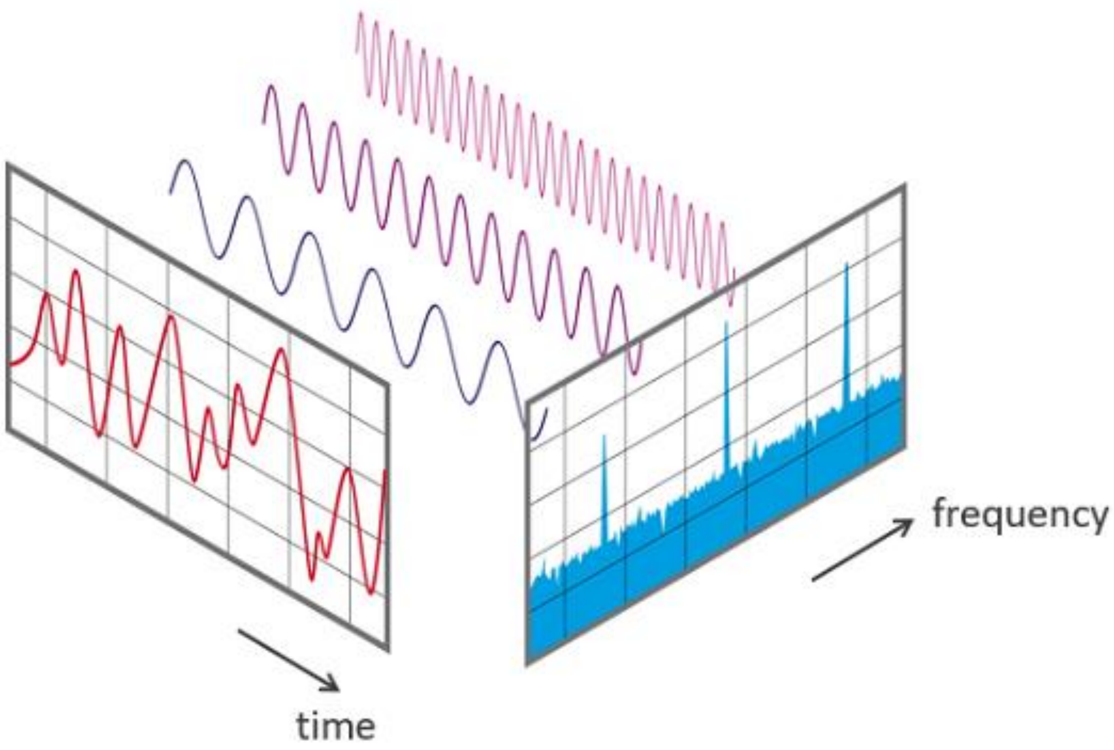


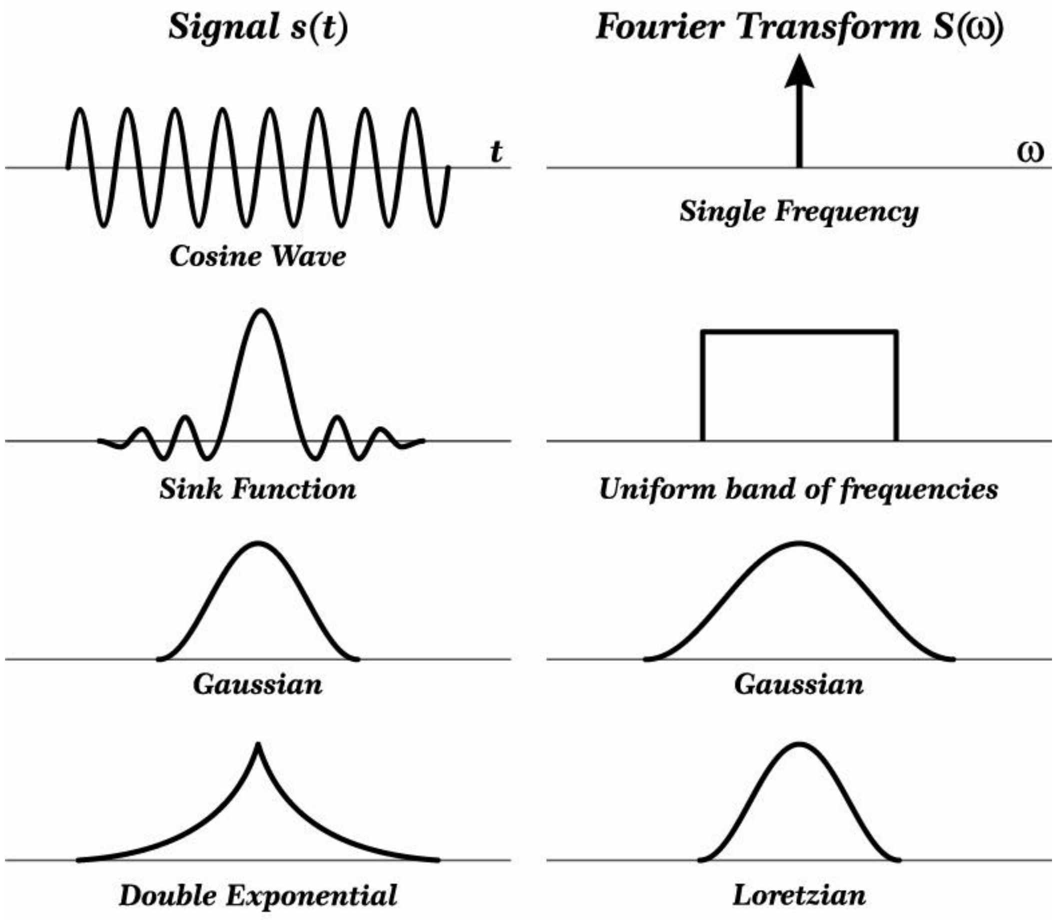
$$s(t) = \cos(2\pi f_c t) \cdot \pi(t)$$

$$S(f) = F\{s(t)\} = \int_{-\infty}^{\infty} s(t) e^{-j2\pi t} dt$$

2.3 Signals and Systems

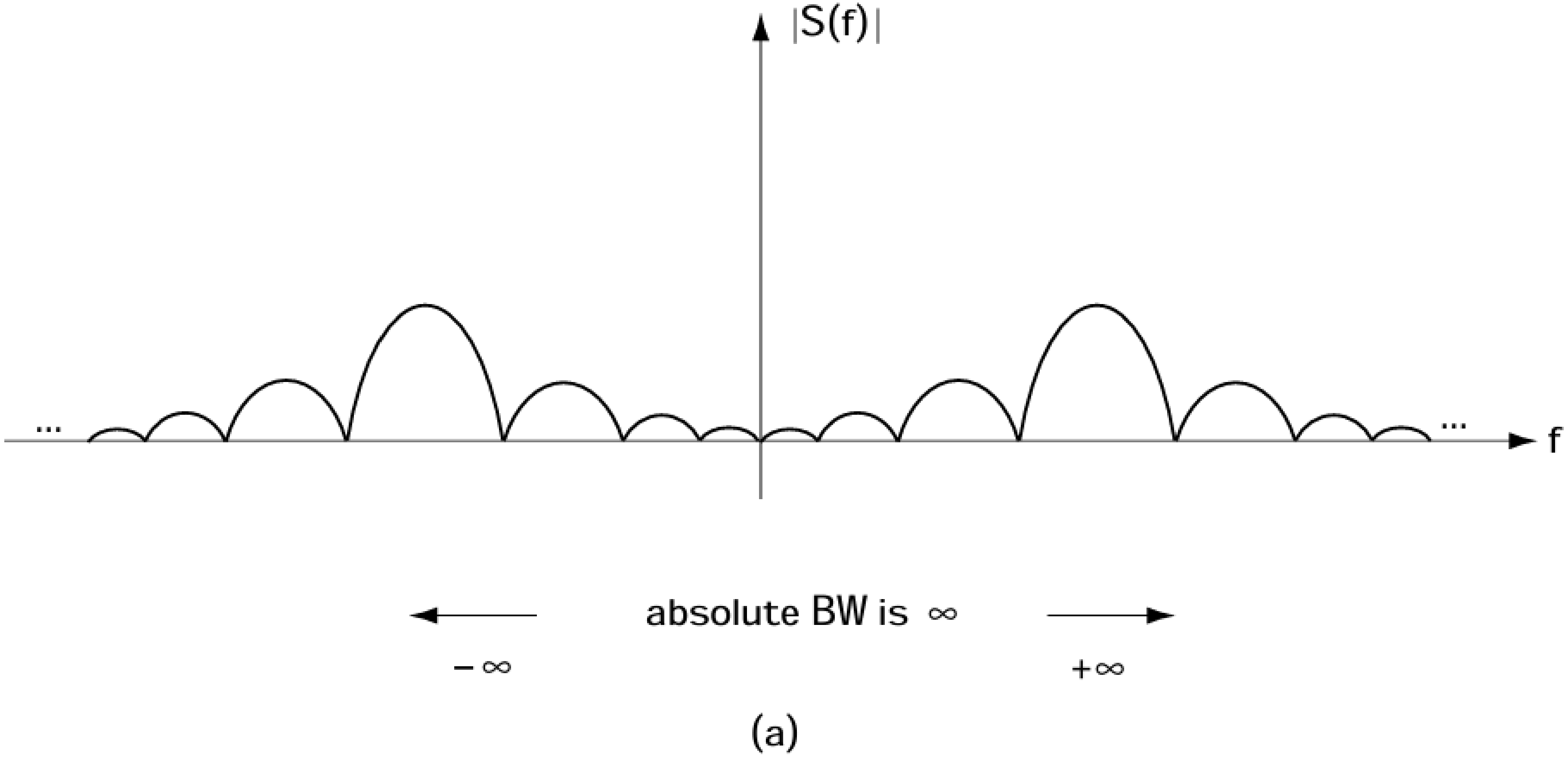
- Fourier Transform





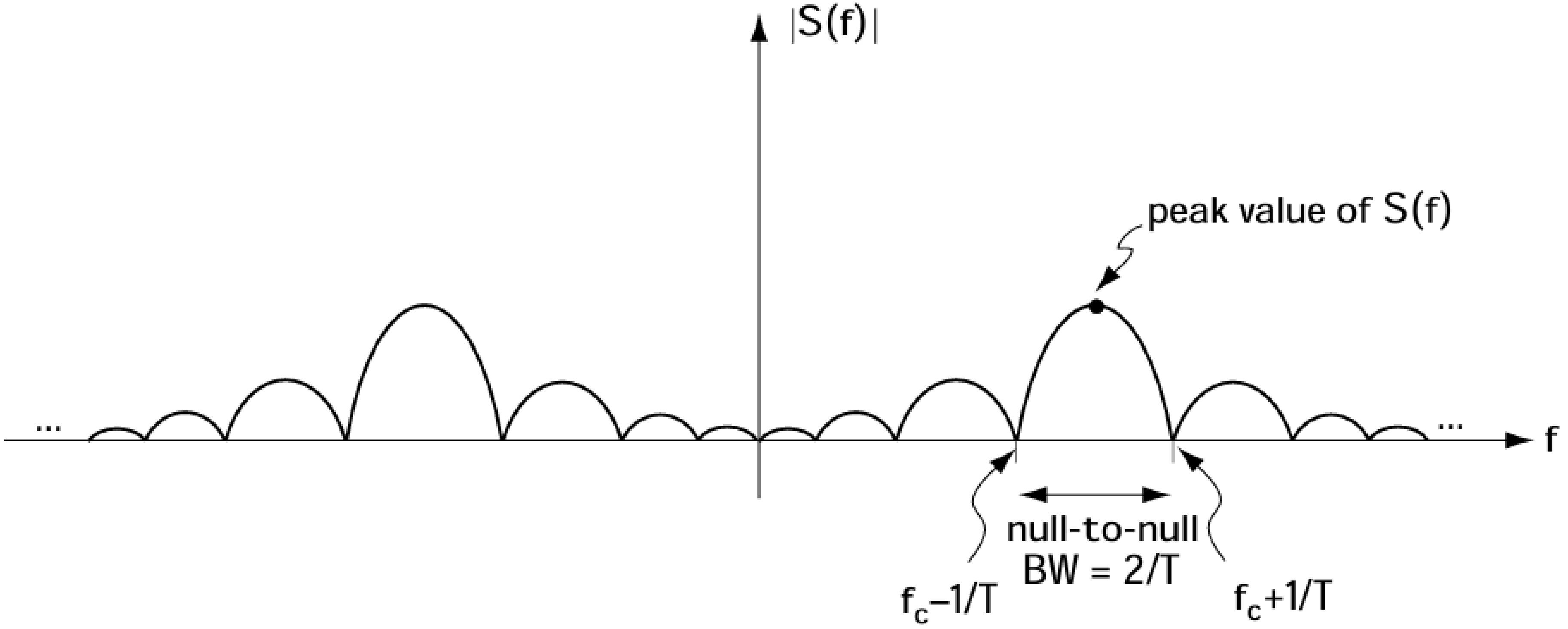
2.3 Signals and Systems

- Bandwidth



2.3 Signals and Systems

- Bandwidth



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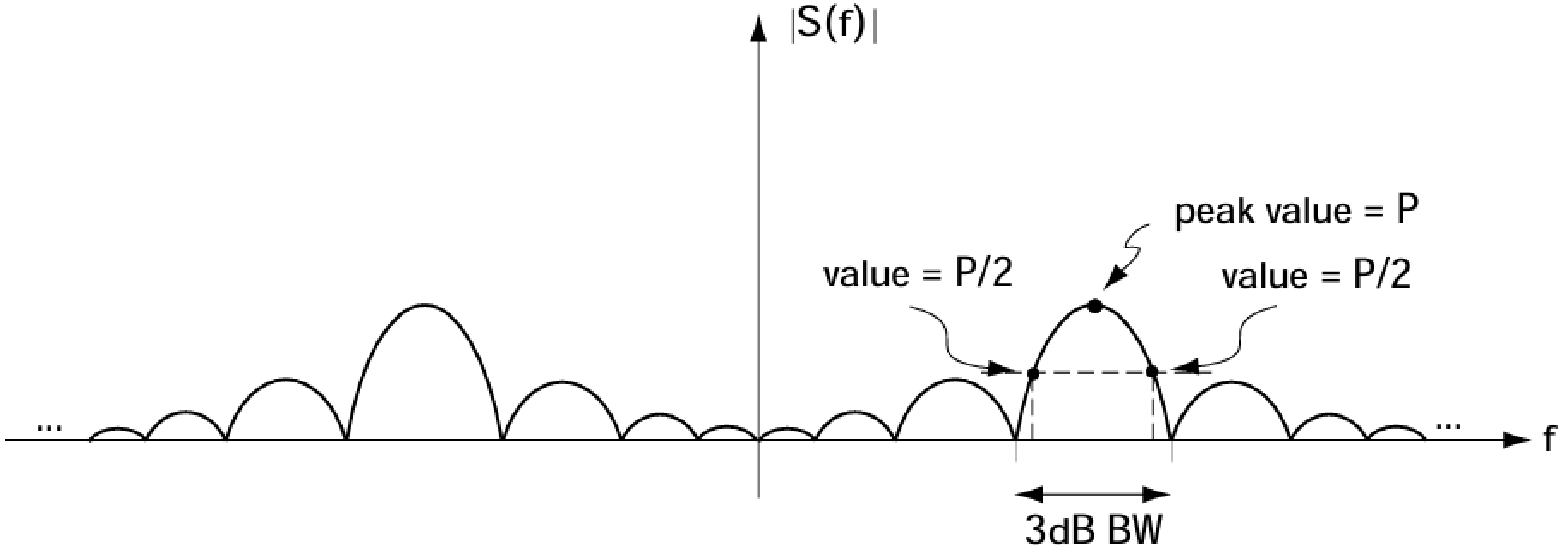
2.3 Signals and Systems

- Bandwidth



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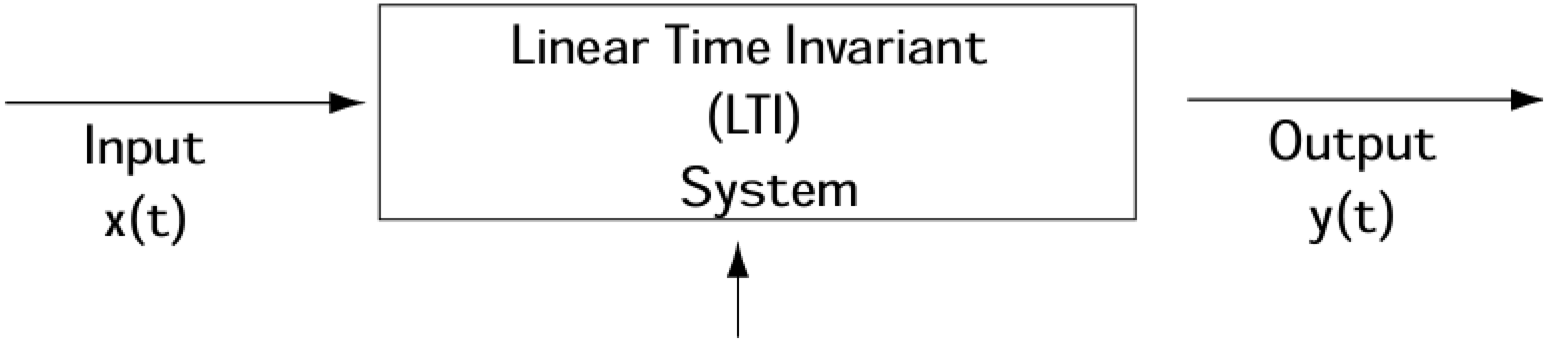
2.3 Signals and Systems

- Linear Time Invariant (LTI) System

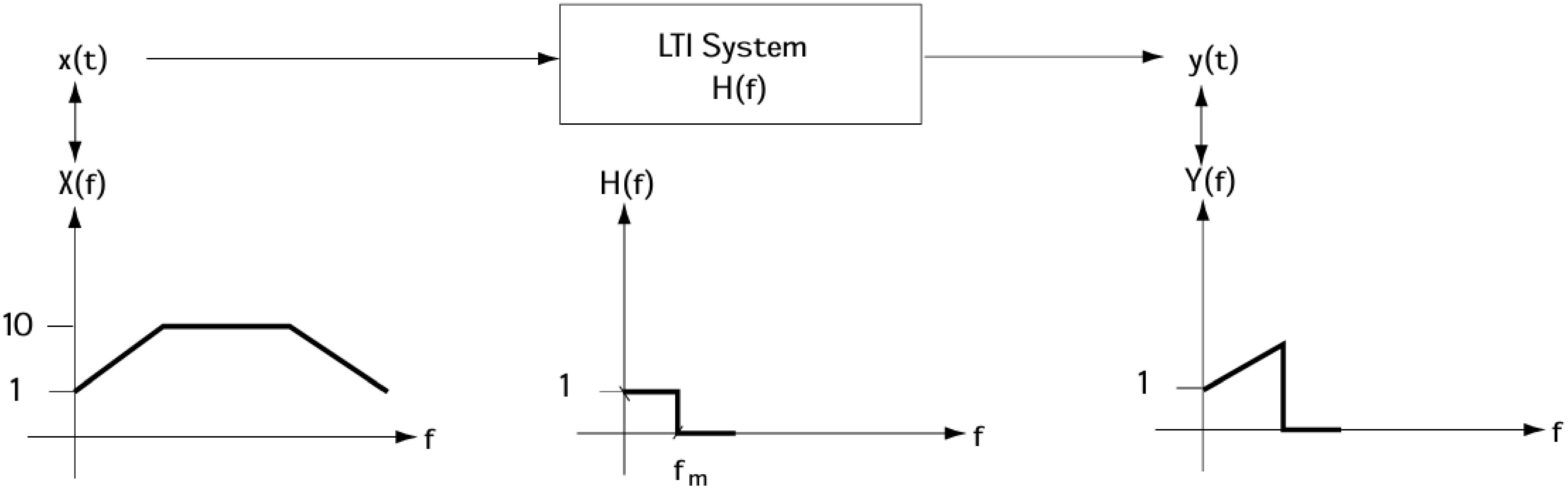


$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$Y(f) = H(f)X(f)$$

2.3 Signals and Systems

- Linear Time Invariant (LTI) System



2.3 Signals and Systems

- Linear Time Invariant (LTI) System