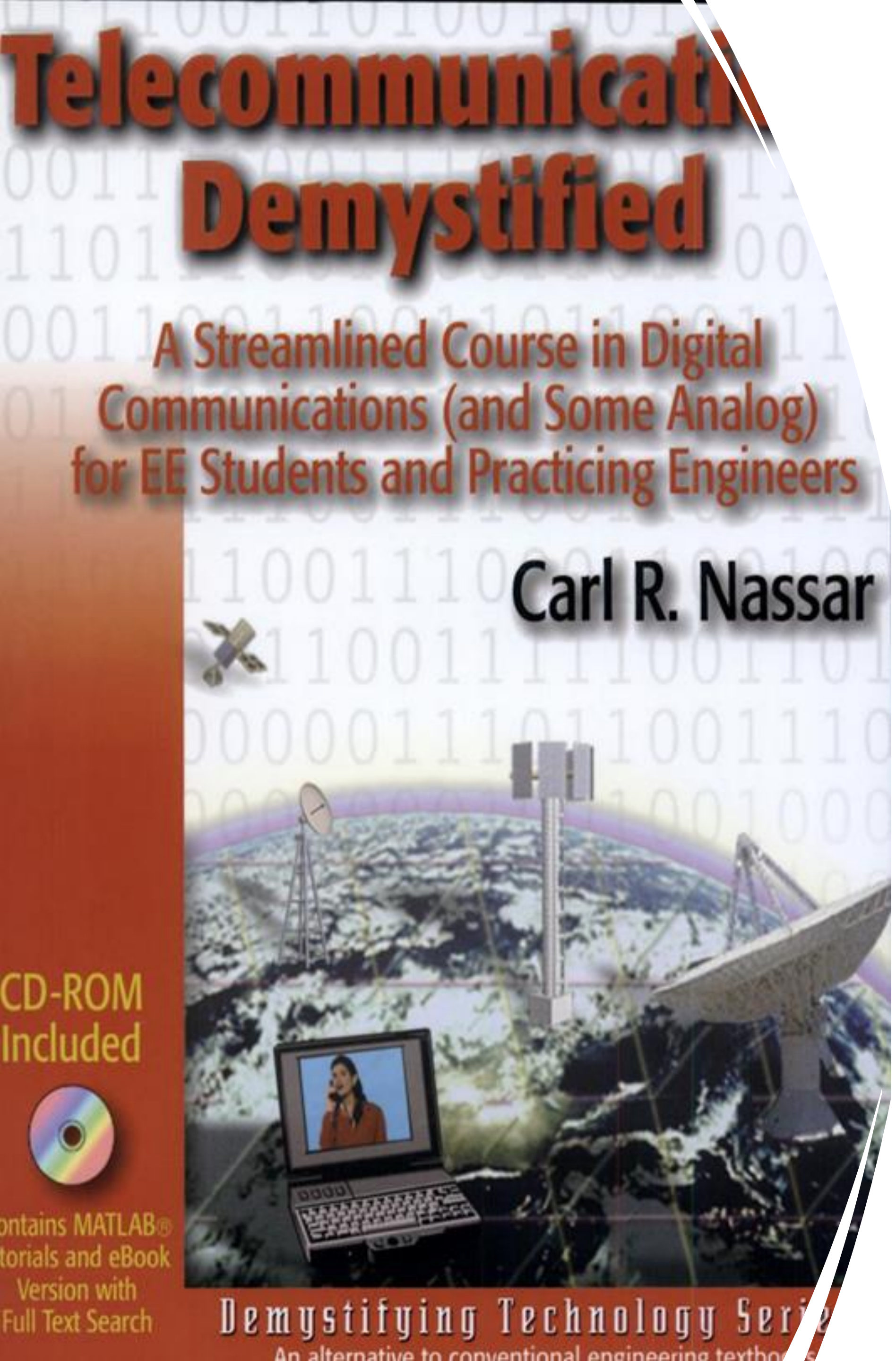


4. Modulators and Demodulators

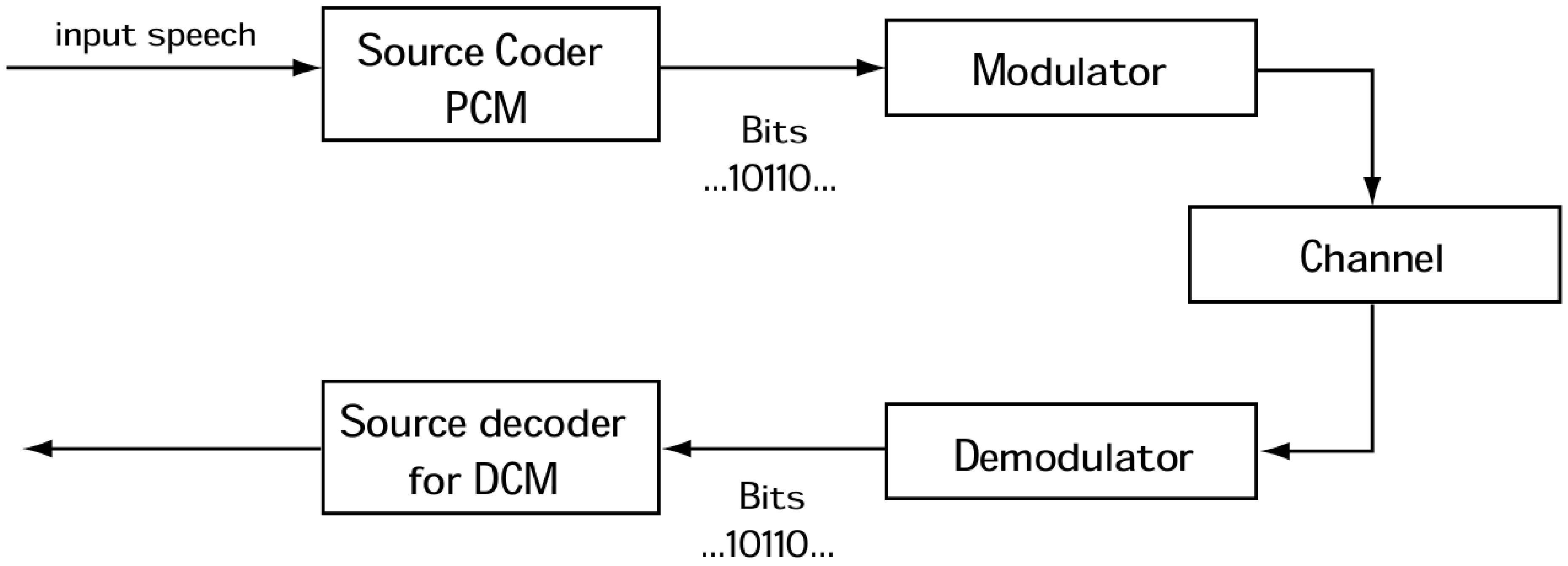
4.1 Baseband Modulators

4.2 Bandpass Modulators

4.3 Modulator Signal

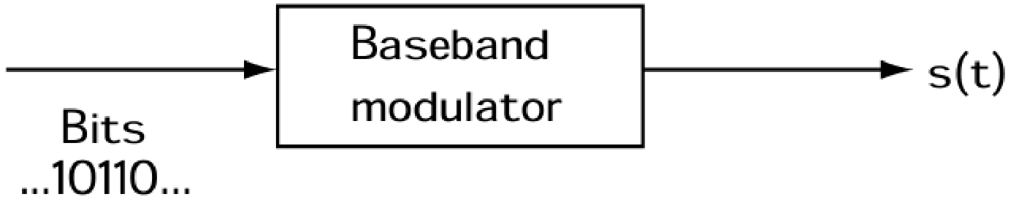
4.4 Demodulator

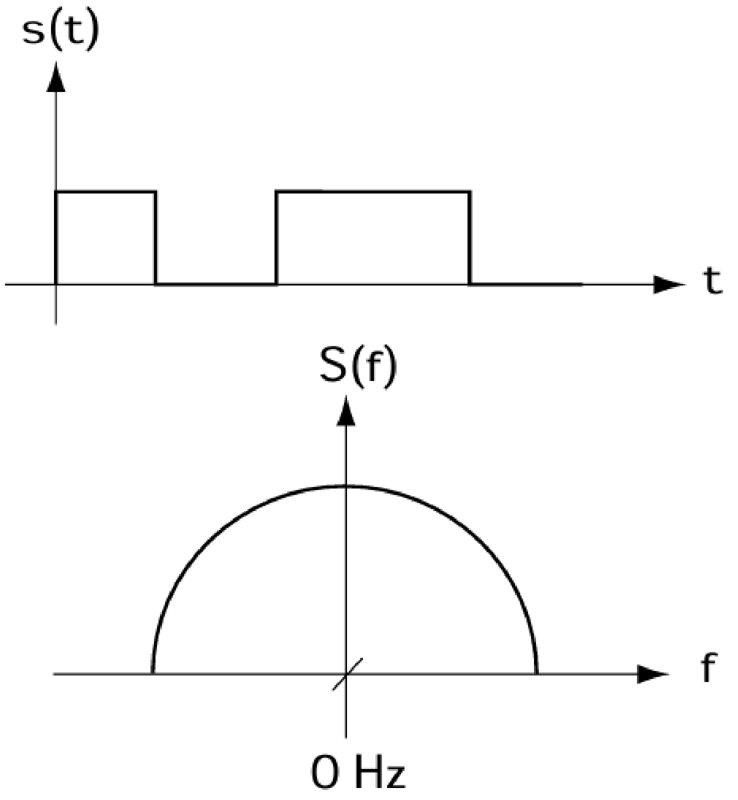
4. Modulators and Demodulators



4.1 Baseband Modulator

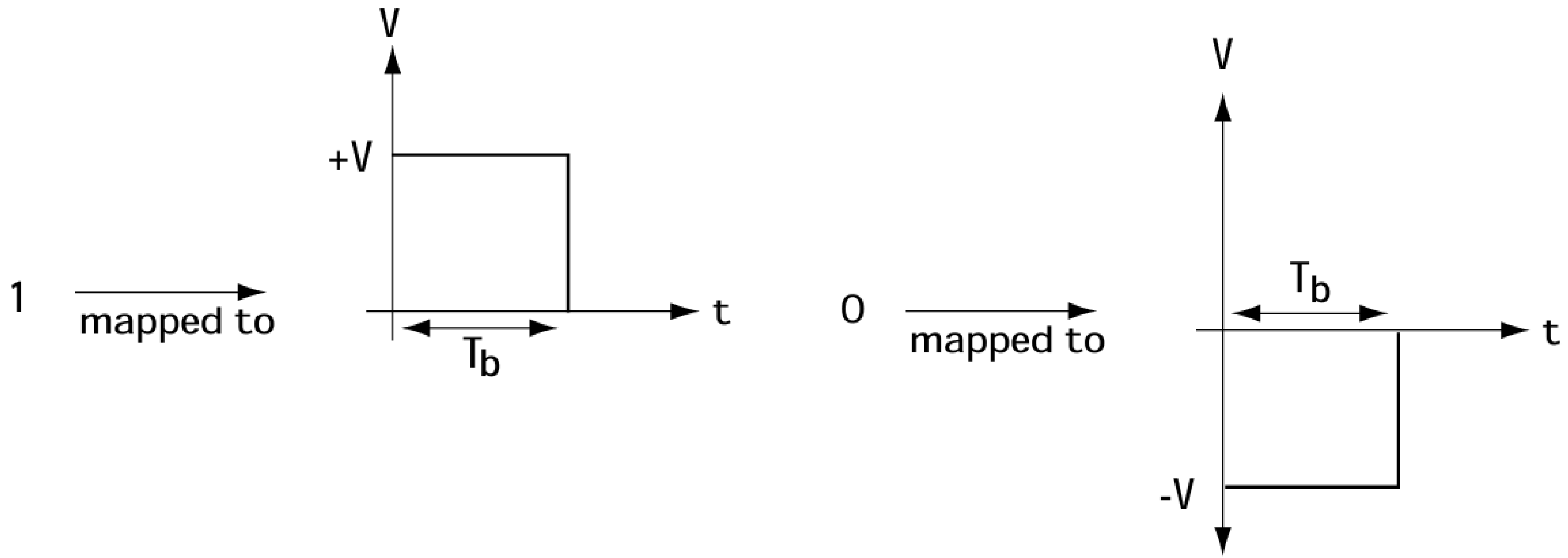
- *Baseband modulators* are devices that turn your bit stream into a waveform centered around 0 Hz





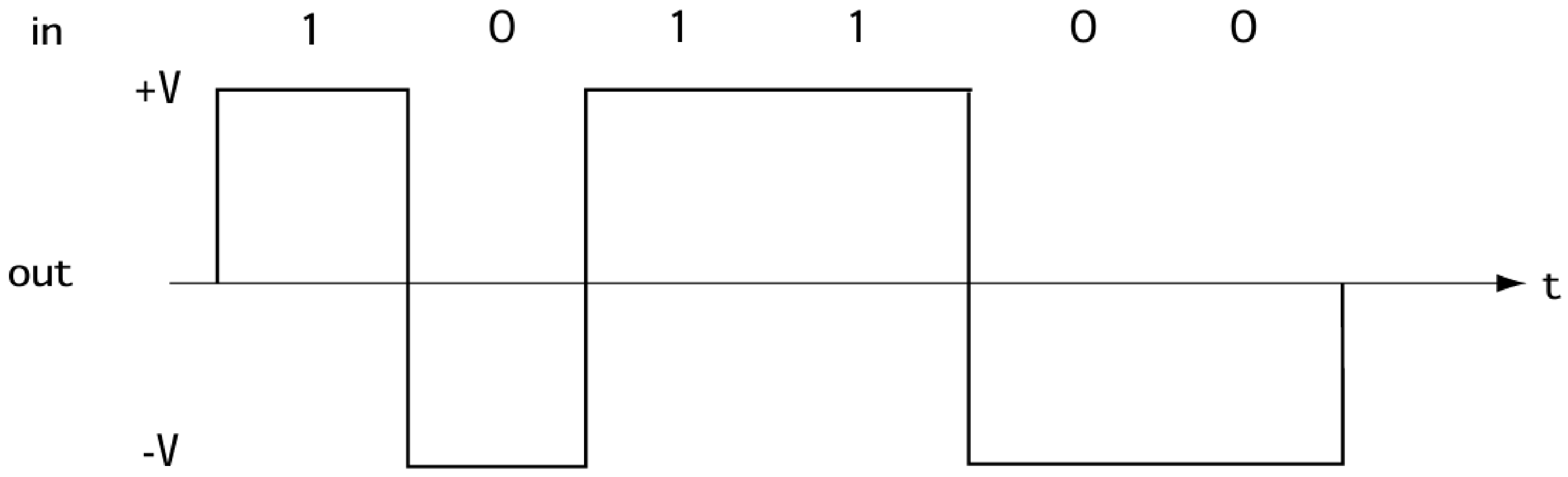
4.1 Baseband Modulator

- NRZ modulator (NRZ-L)



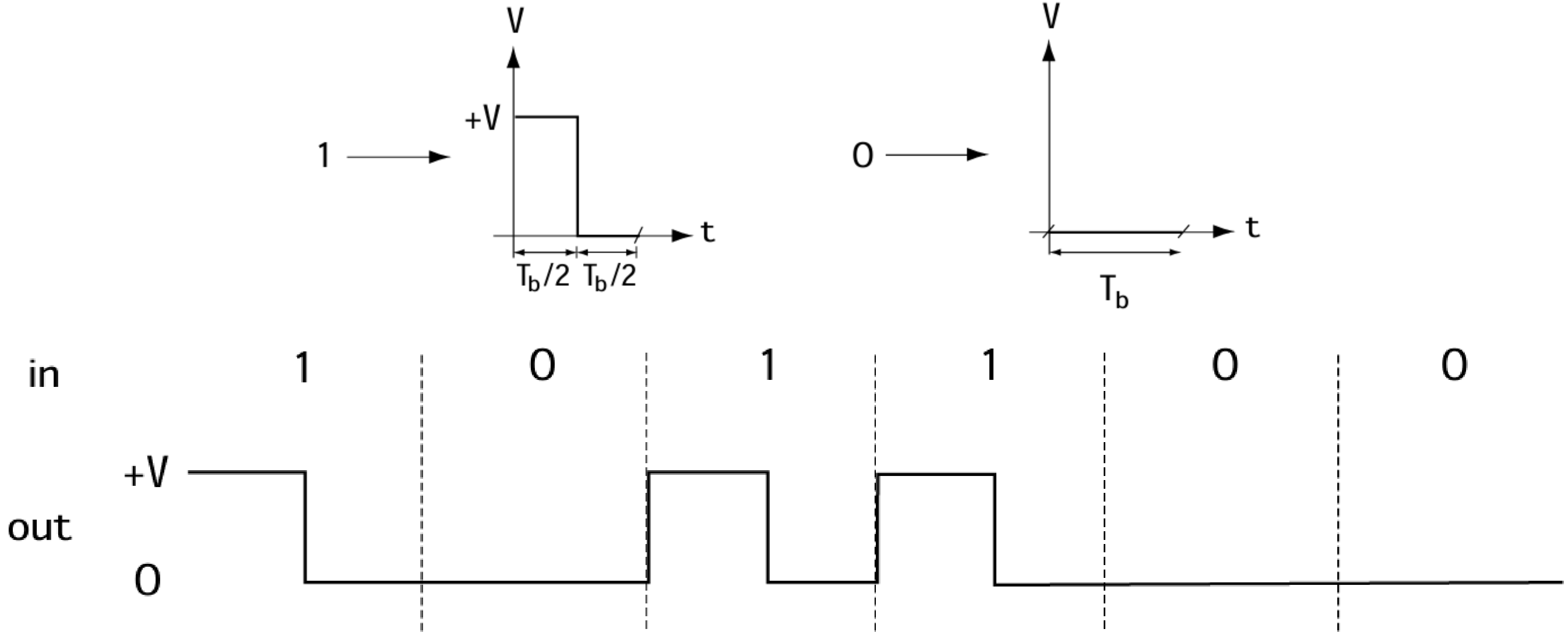
4.1 Baseband Modulator

- NRZ modulator (NRZ-L)



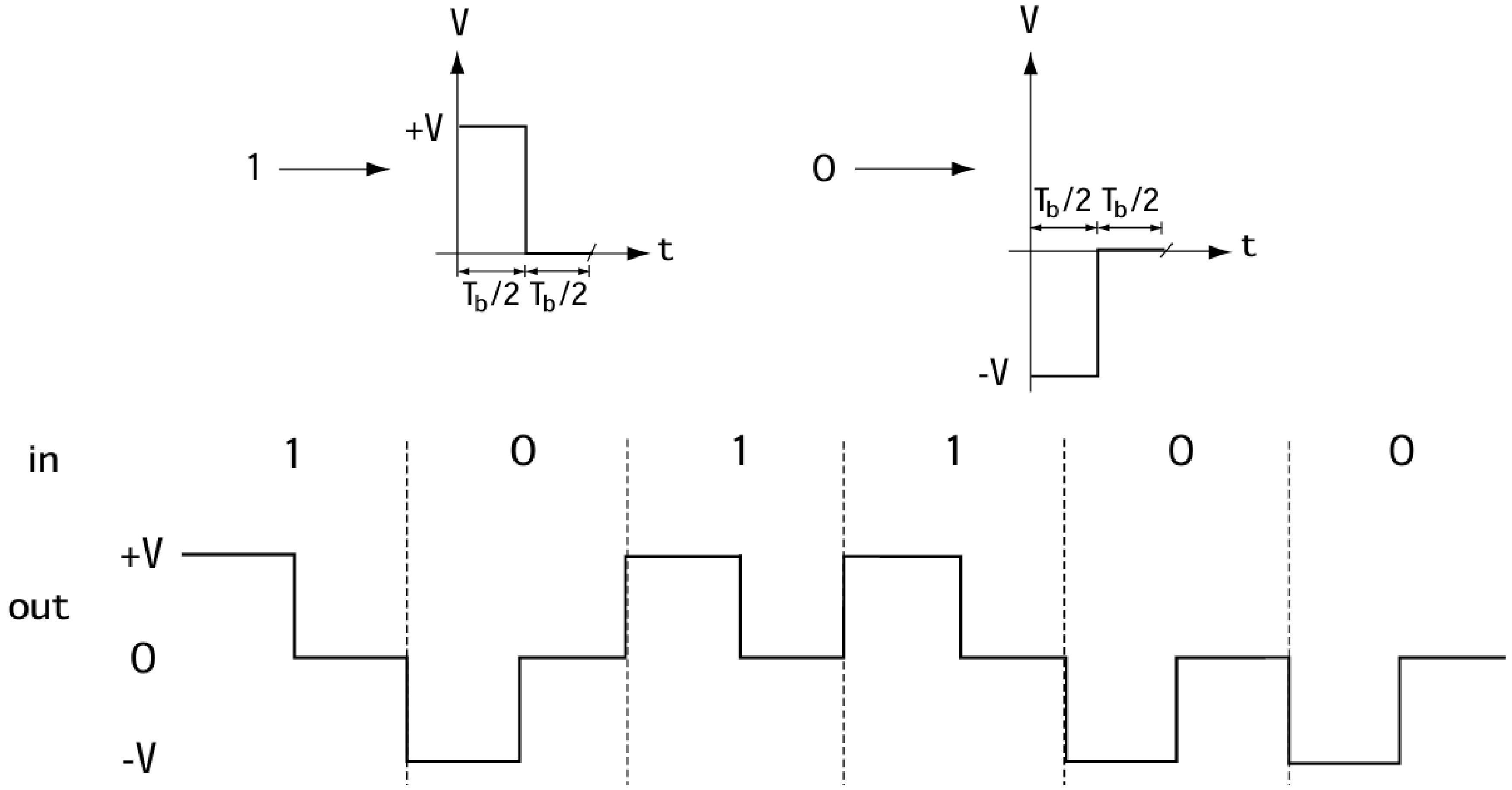
4.1 Baseband Modulator

- RZ Modulators (Unipolar RZ)



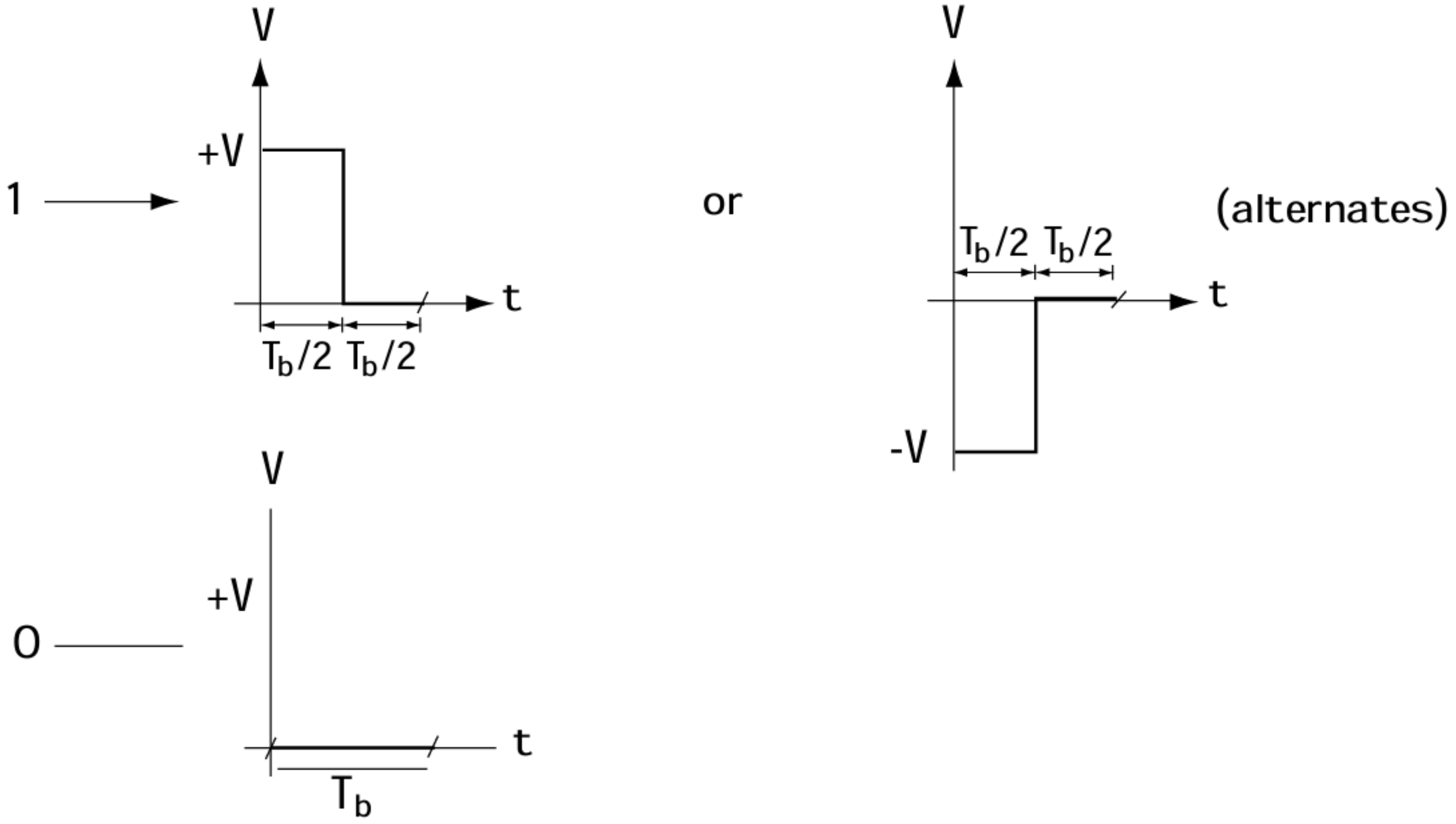
4.1 Baseband Modulator

- RZ Modulators (Bipolar RZ)



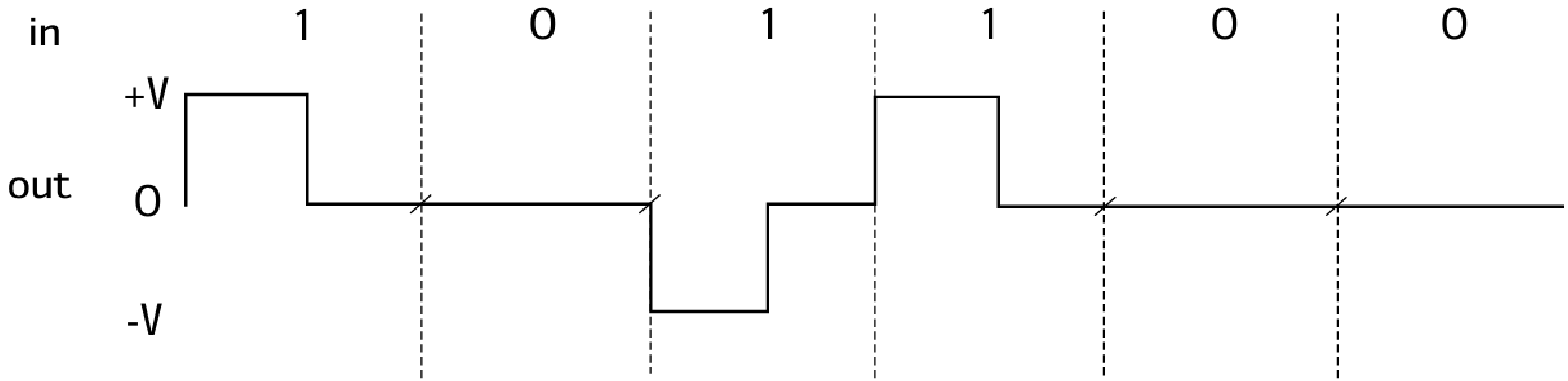
4.1 Baseband Modulator

- RZ Modulators (RZ-AMI)



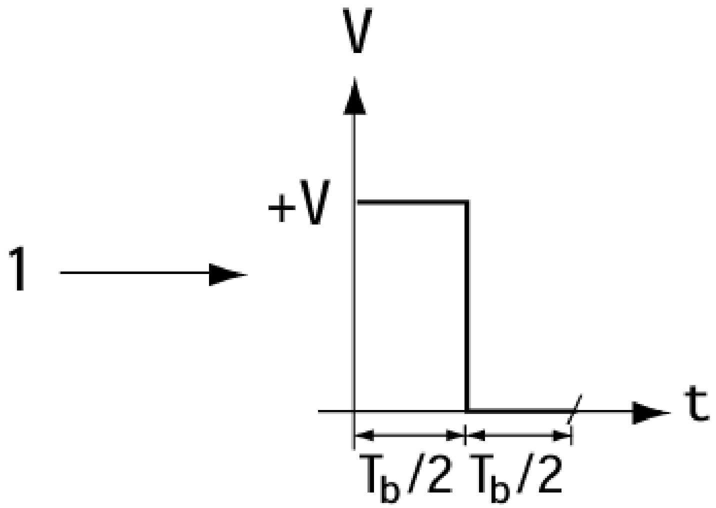
4.1 Baseband Modulator

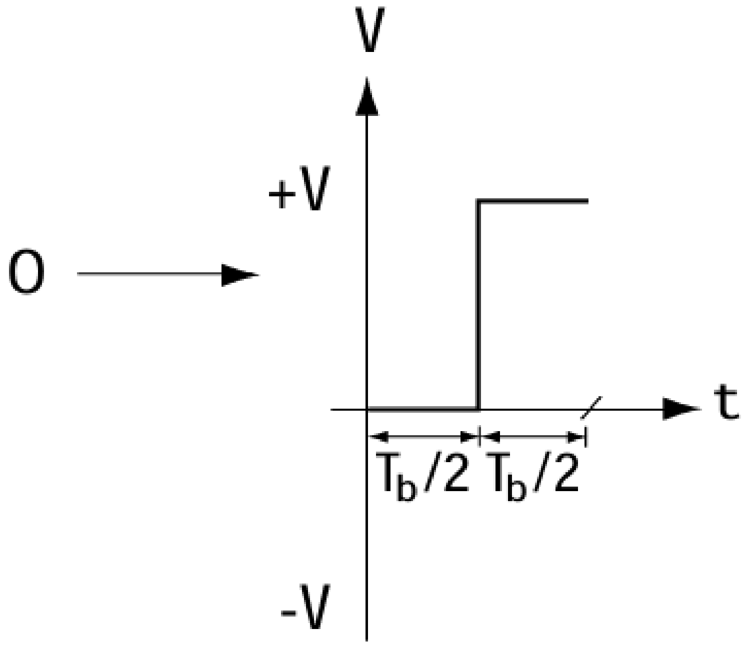
- RZ Modulators (RZ-AMI)



4.1 Baseband Modulator

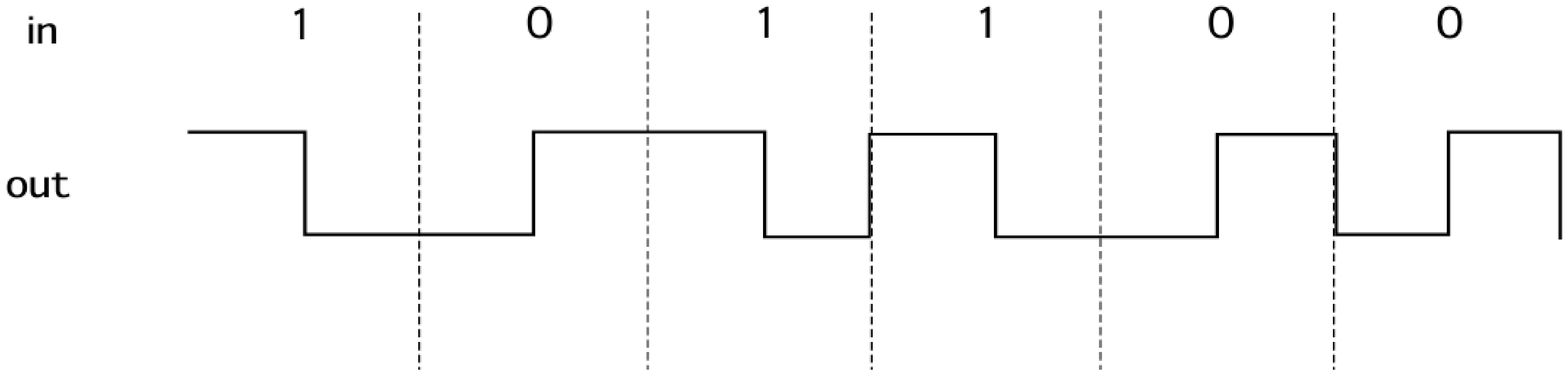
- Manchester Coding





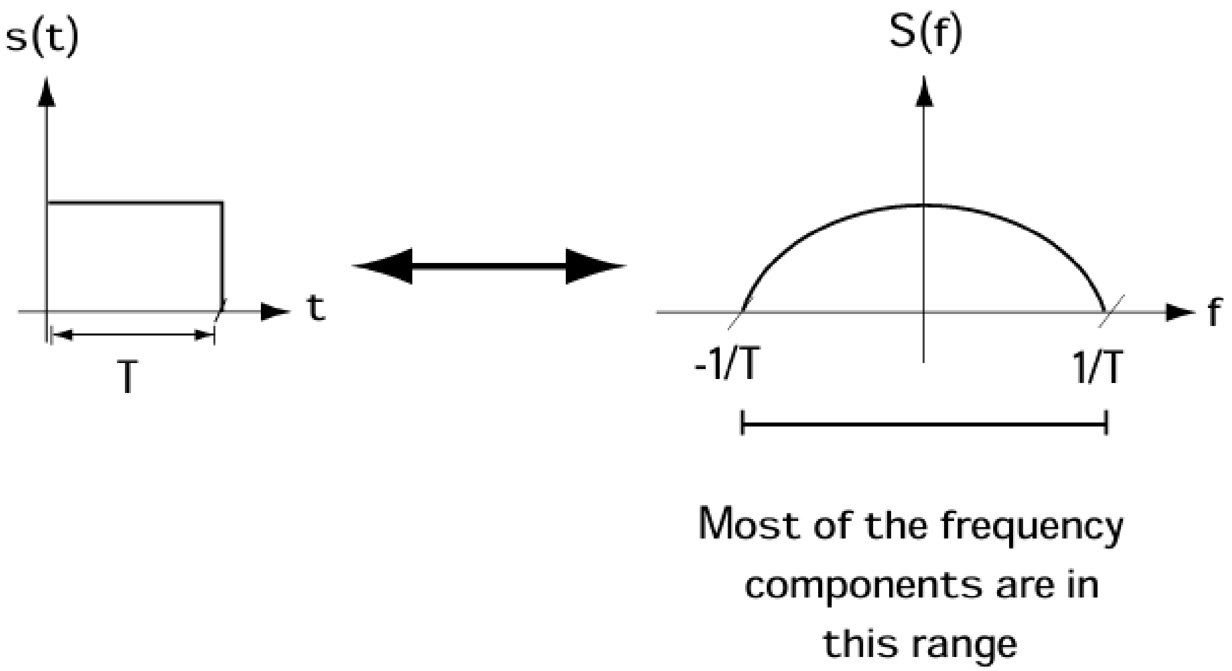
4.1 Baseband Modulator

- Manchester Coding



4.1 Baseband Modulator

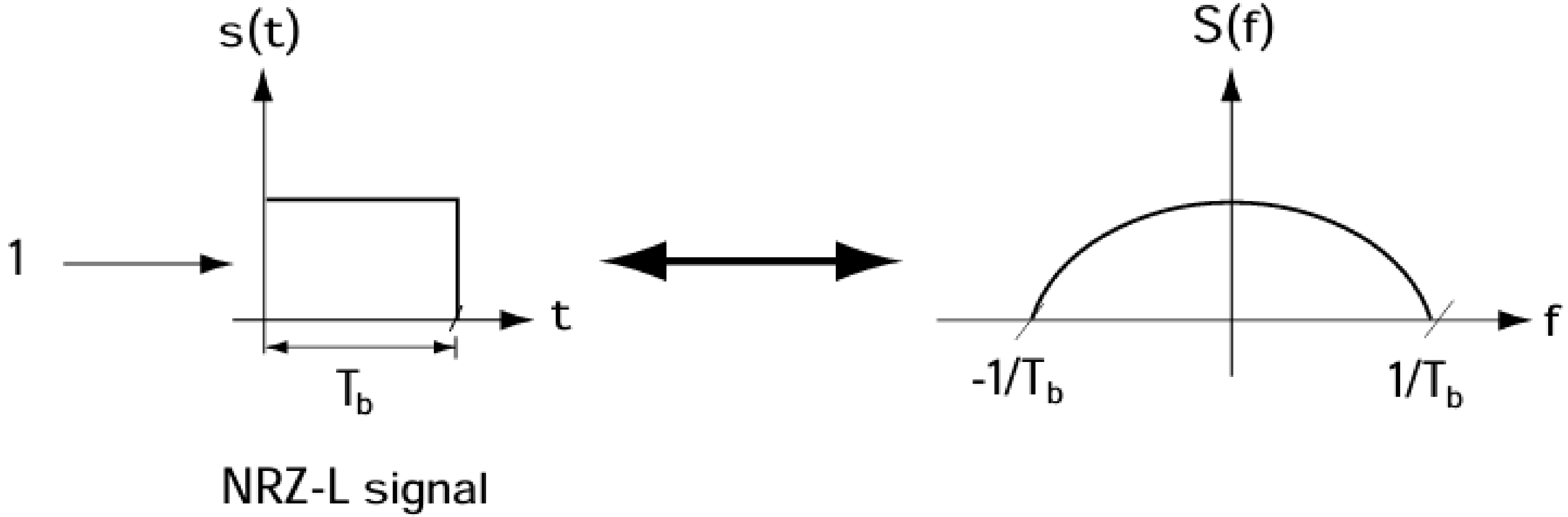
- Bandwidth considerations



(a)

4.1 Baseband Modulator

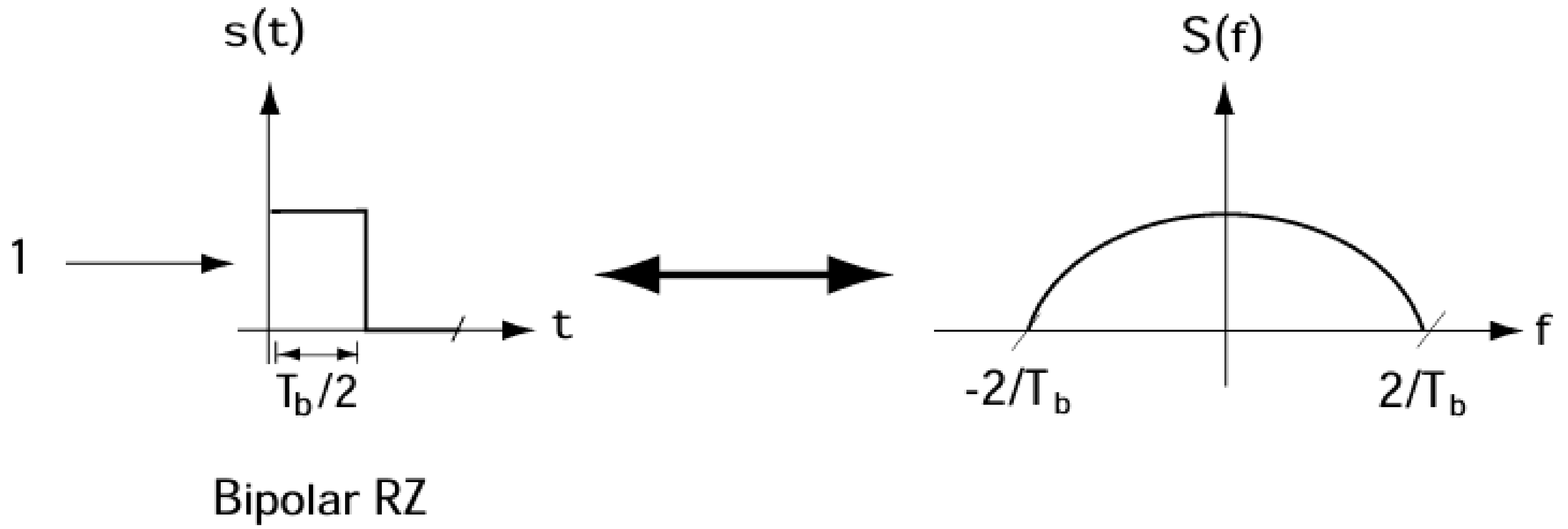
- Bandwidth considerations



(b)

4.1 Baseband Modulator

- Bandwidth considerations



(c)

4.2 Bandpass Modulator

- A bandpass modulator takes incoming bits and outputs a waveform centered around frequency ω_c
- This is the modulator you want to use when your communication channel will provide safe passage to frequencies around ω_c

$$s(t) = A\cos(\omega t + \theta)$$

4.2 Bandpass Modulator

- Amplitude Shift-Keying modulators (ASK)
- This refers to the modulators that, given the input bits, create the waveform

$$s(t)=A\cos(\omega t+\theta)$$

where the input bits are stuffed in the amplitude (A).

- We'll start with the simplest of the ASK modulators, called *Binary ASK* or *B-ASK* for short.

4.2 Bandpass Modulator

- 4-ASK

Input bits	Output waveform	Output waveform (shorthand form)
00	$s_0(t) = -3A \cos\omega_c t,\ iT \le t < (i+1)T$	$-3A \cos \omega_c t \cdot \pi\ (t-iT)$
01	$s_1(t) = -A \cos \omega_c t,\ iT \le t < (i+1)T$	$-A \cos \omega_c t \cdot \pi\ (t-iT)$
10	$s_2(t) = A \cos \omega_c t,\ iT \le t < (i+1)T$	$A \cos \omega_c t \cdot \pi\ (t-iT)$
11	$s_3(t) = 3A \cos \omega_c t,\ iT \le t < (i+1)T$	$3A \cos \omega_c t \cdot \pi\ (t-iT)$

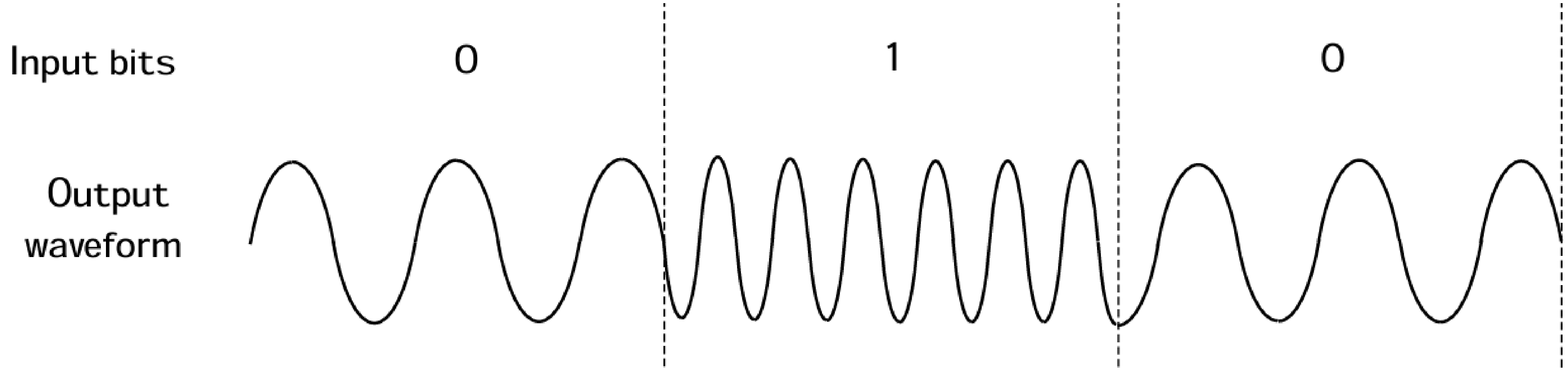
4.2 Bandpass Modulator

- 8-ASK

Input bits	Output waveform	Output waveform (shorthand form)
000	$s_0(t) = -7A \cos \omega_c t,\ iT \le t < (i+1)T$	$-7A \cos \omega_c t \cdot \pi\ (t-iT)$
001	$s_1(t) = -5A \cos \omega_c t,\ iT \le t < (i+1)T$	$-5A \cos \omega_c t \cdot \pi\ (t-iT)$
010	$s_2(t) = -3A \cos \omega_c t,\ iT \le t < (i+1)T$	$-3A \cos \omega_c t \cdot \pi\ (t-iT)$
011	$s_3(t) = -A \cos \omega_c t,\ iT \le t < (i+1)T$	$-A \cos \omega_c t \cdot \pi\ (t-iT)$
100	$s_4(t) = A \cos \omega_c t,\ iT \le t < (i+1)T$	$A \cos \omega_c t \cdot \pi\ (t-iT)$
101	$s_5(t) = 3A \cos \omega_c t,\ iT \le t < (i+1)T$	$3A \cos \omega_c t \cdot \pi\ (t-iT)$
110	$s_6(t) = 5A \cos \omega_c t,\ iT \le t < (i+1)T$	$5A \cos \omega_c t \cdot \pi\ (t-iT)$
111	$s_7(t) = 7A \cos \omega_c t,\ iT \le t < (i+1)T$	$7A \cos \omega_c t \cdot \pi\ (t-iT)$

4.2 Bandpass Modulator

- Frequency Shift-Keying modulators (FSK)
- As the name suggests, here we stuff the information bits into the frequency.
- We'll look at the simplest first, which is *BFSK* (*binary FSK*).



4.2 Bandpass Modulator

- Frequency Shift-Keying modulators (FSK)

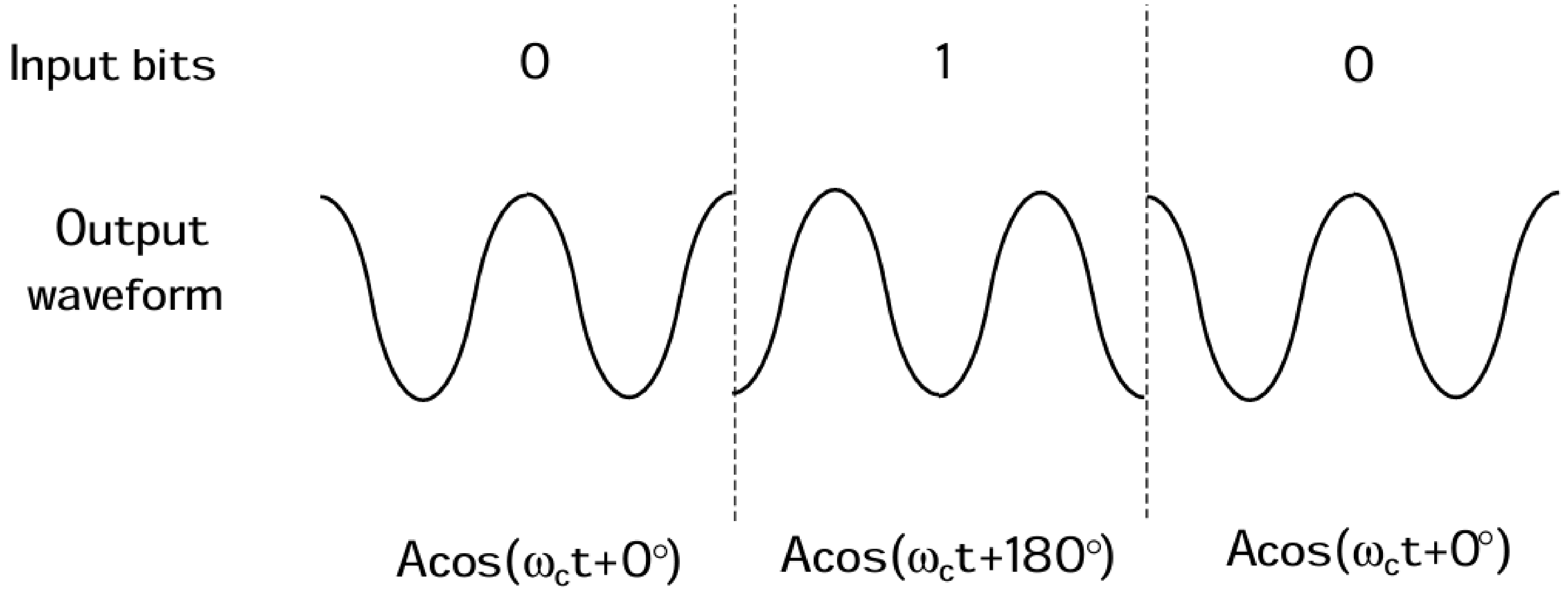
	Input bits	Output waveform	Output waveform (shorthand)
BFSK	0	$s_0(t) = A\cos((\omega_c+\Delta\omega_0)t),\ iT \le t < (i+1)T$	$A\cos((\omega_c+\Delta\omega_0)t) \cdot \pi(t-iT)$
	1	$s_1(t) = A\cos((\omega_c+\Delta\omega_1)t),\ iT \le t < (i+1)T$	$A\cos((\omega_c+\Delta\omega_1)t) \cdot \pi(t-iT)$
4-FSK	00	$s_0(t) = A\cos((\omega_c+\Delta\omega_0)t),\ iT \le t < (i+1)T$	$A\cos((\omega_c+\Delta\omega_0)t) \cdot \pi(t-iT)$
	01	$s_1(t) = A\cos((\omega_c+\Delta\omega_1)t),\ iT \le t < (i+1)T$	$A\cos((\omega_c+\Delta\omega_1)t) \cdot \pi(t-iT)$
	10	$s_2(t) = A\cos((\omega_c+\Delta\omega_2)t),\ iT \le t < (i+1)T$	$A\cos((\omega_c+\Delta\omega_2)t) \cdot \pi(t-iT)$
	11	$s_3(t) = A\cos((\omega_c+\Delta\omega_3)t),\ iT \le t < (i+1)T$	$A\cos((\omega_c+\Delta\omega_3)t) \cdot \pi(t-iT)$

4.2 Bandpass Modulator

- Phase Shift-Keying modulators (PSK)
- With these, input bits are mapped into output waveforms of the form

$$s(t)=A\cos(\omega t+\theta)$$

and the information bits are stuffed in the phase θ.



4.2 Bandpass Modulator

- Phase Shift-Keying modulators (PSK)

	Input bits	Output waveform	Output waveform (shorthand form)
BPSK	0	$s_0(t) = A\cos(\omega_c t+0°),\ iT \le t < (i+1)T$	$A\cos(\omega_c t+0°) \cdot \pi(t-iT)$
	1	$s_1(t) = A\cos(\omega_c t+180°),\ iT \le t < (i+1)T$	$A\cos(\omega_c t+180°) \cdot \pi(t-iT)$
4-PSK	00	$s_0(t) = A\cos(\omega_c t+0°),\ iT \le t < (i+1)T$	$A\cos(\omega_c t+0°) \cdot \pi(t-iT)$
	01	$s_1(t) = A\cos(\omega_c t+90°),\ iT \le t < (i+1)T$	$A\cos(\omega_c t+90°) \cdot \pi(t-iT)$
	10	$s_2(t) = A\cos(\omega_c t+180°),\ iT \le t < (i+1)T$	$A\cos(\omega_c t+180°) \cdot \pi(t-iT)$
	11	$s_3(t) = A\cos(\omega_c t+270°),\ iT \le t < (i+1)T$	$A\cos(\omega_c t+270°) \cdot \pi(t-iT)$

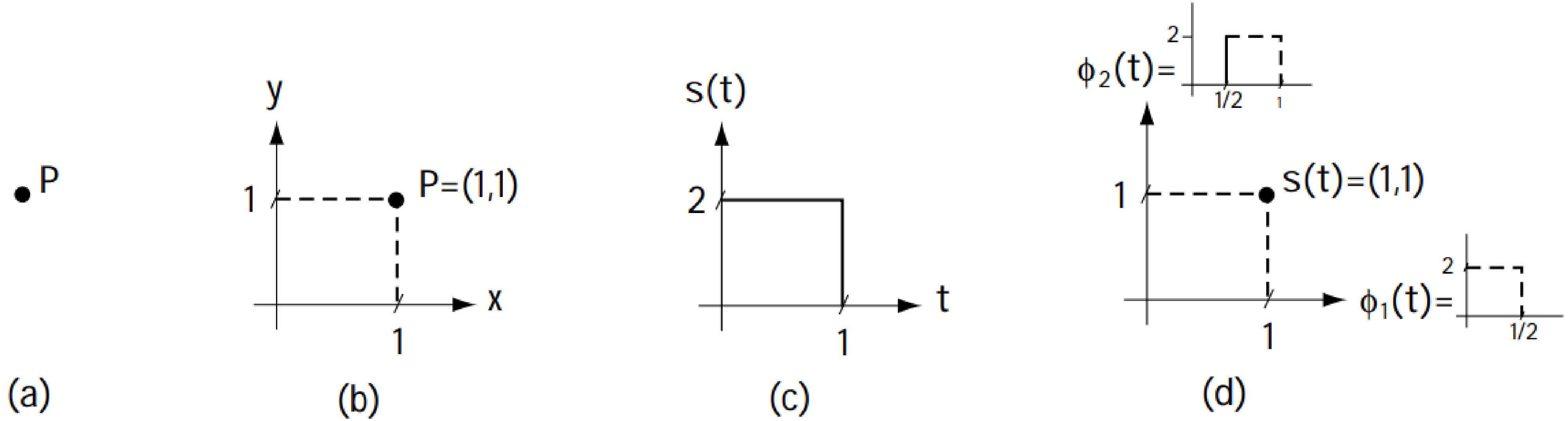
4.2 Bandpass Modulator

- Phase Shift-Keying modulators (PSK)

	Input bits	Output waveform	Output waveform (shorthand form)
8-PSK	000	$s_0(t) = A\cos(\omega_c t+0°),\ iT \le t < (i+1)T$	
	001	$s_1(t) = A\cos(\omega_c t+45°),\ iT \le t < (i+1)T$	
	010	$s_2(t) = A\cos(\omega_c t+90°),\ iT \le t < (i+1)T$	
	011	$s_3(t) = A\cos(\omega_c t+135°),\ iT \le t < (i+1)T$	
	100	$s_4(t) = A\cos(\omega_c t+180°),\ iT \le t < (i+1)T$	
	101	$s_5(t) = A\cos(\omega_c t+225°),\ iT \le t < (i+1)T$	
	110	$s_6(t) = A\cos(\omega_c t+270°),\ iT \le t < (i+1)T$	
	111	$s_7(t) = A\cos(\omega_c t+315°),\ iT \le t < (i+1)T$	

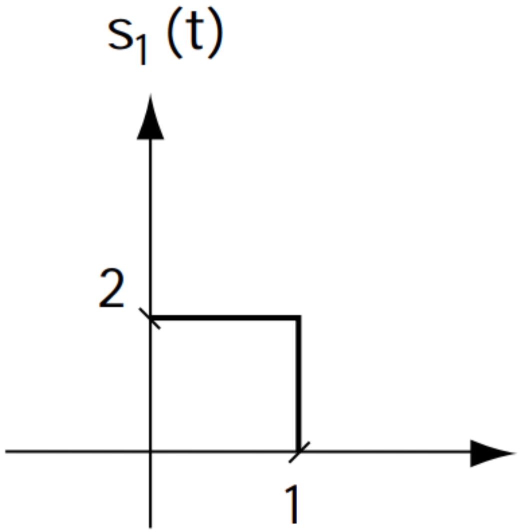
4.3 Modulator Signal

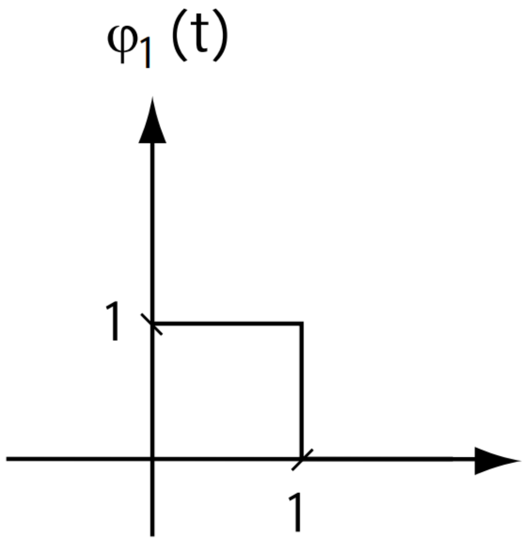
- Representing points and signals

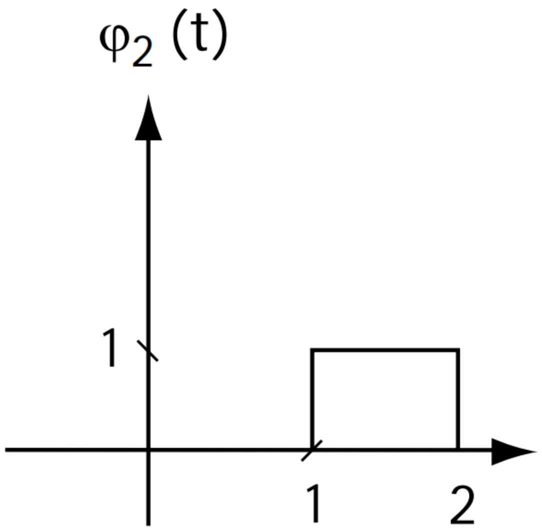


4.3 Modulator Signal

- Representing points and signals

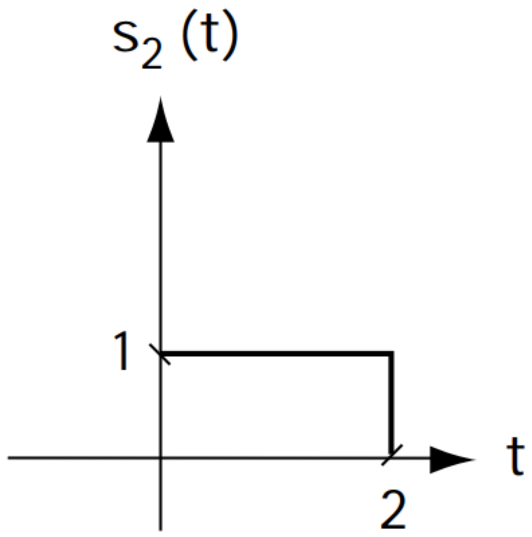


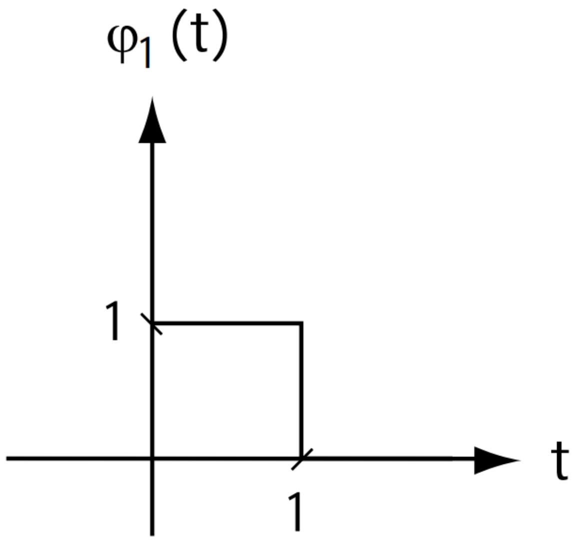


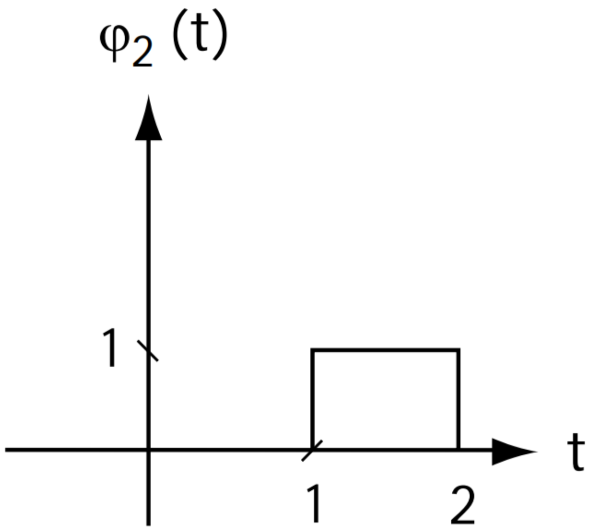


4.3 Modulator Signal

- Representing points and signals







4.3 Modulator Signal

- orthonormal basis

$$s_1(t) = s_{11}\varphi_1(t) + s_{12}\varphi_2(t) + \ldots + s_{1N}\varphi_N(t)$$

$$\ldots$$

$$s_M(t) = s_{M1}\varphi_1(t) + s_{M2}\varphi_2(t) + \ldots + s_{MN}\varphi_N(t)$$

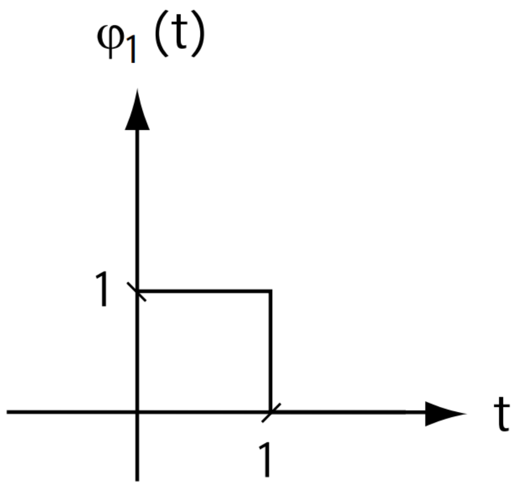
$$\int_{-\infty}^{\infty} \varphi_i(t)\varphi_j(t)\,dt = 0, i \neq j$$

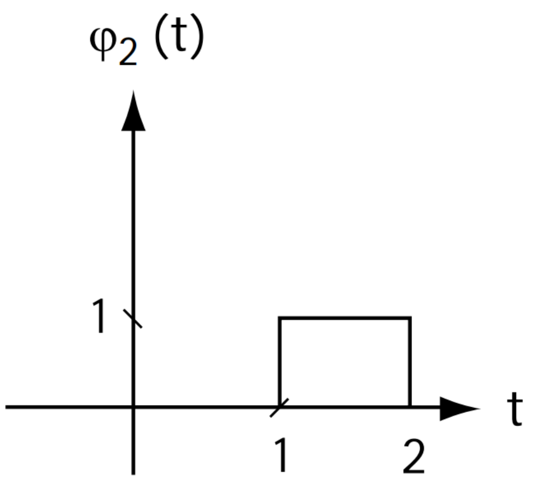
$$\int_{-\infty}^{\infty} \varphi_j(t)\varphi_j(t)\,dt = 1$$

$$s_{ij} = \int_{-\infty}^{\infty} s_i(t)\varphi_j(t)\,dt$$

4.3 Modulator Signal

- orthonormal basis



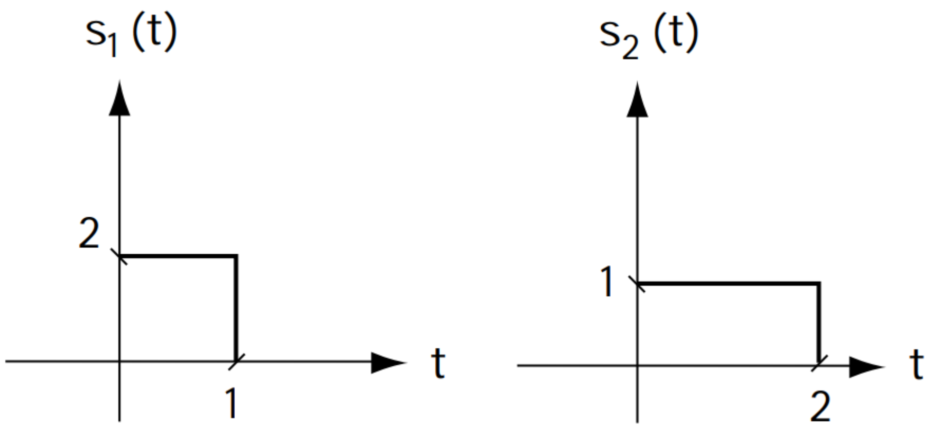


$$\int_{-\infty}^{\infty} \varphi_i(t)\varphi_j(t)\,dt = 0, i \neq j$$

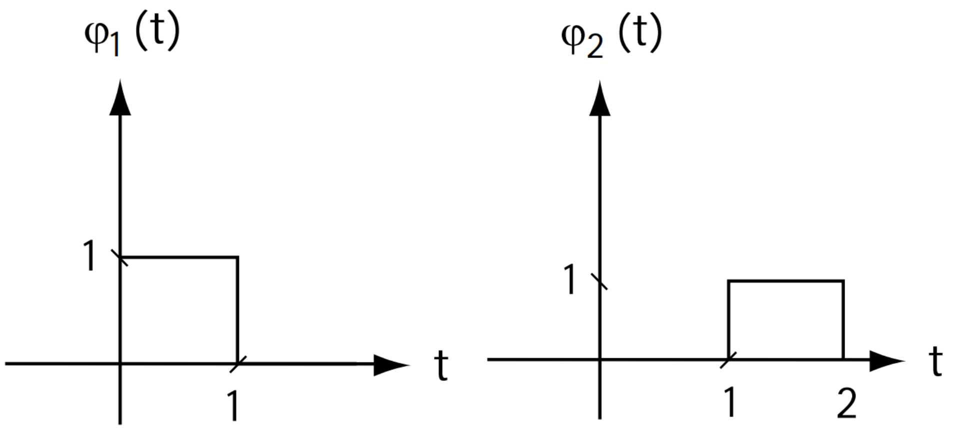
$$\int_{-\infty}^{\infty} \varphi_j(t)\varphi_j(t)\,dt = 1$$

4.3 Modulator Signal

- orthonormal basis



signals

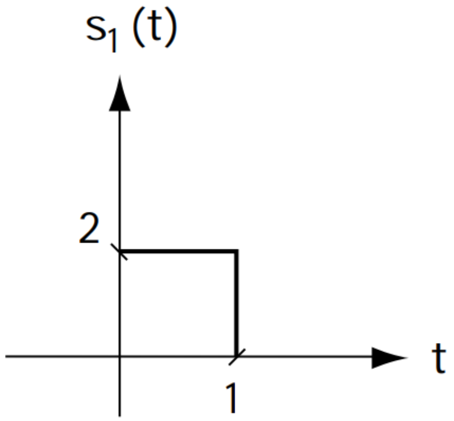


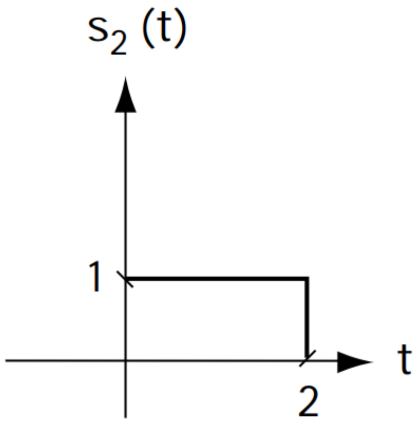
orthonormal basis

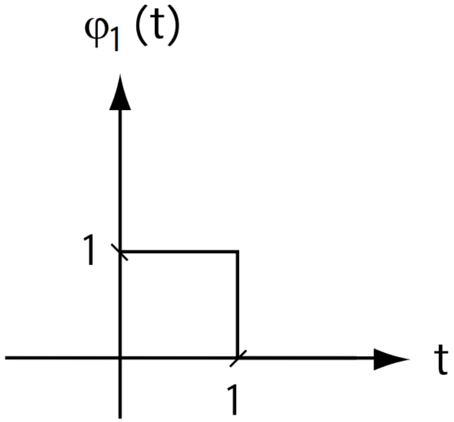
$$\left.\begin{aligned} s_1(t) &= s_{11}\varphi_1(t) + s_{12}\varphi_2(t) + \ldots + s_{1N}\varphi_N(t) \\ &\ldots \\ s_M(t) &= s_{M1}\varphi_1(t) + s_{M2}\varphi_2(t) + \ldots + s_{MN}\varphi_N(t) \end{aligned}\right\} \quad s_{ij} = \int_{-\infty}^{\infty} s_i(t)\varphi_j(t)\,dt$$

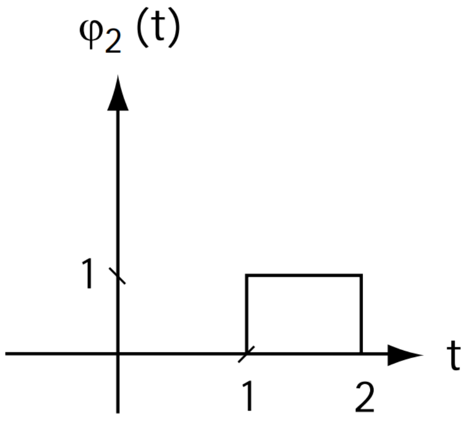
4.3 Modulator Signal

- orthonormal basis





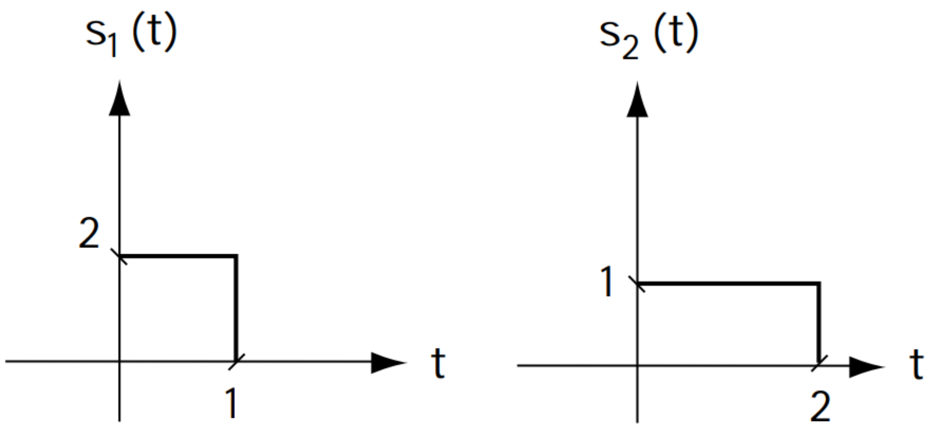


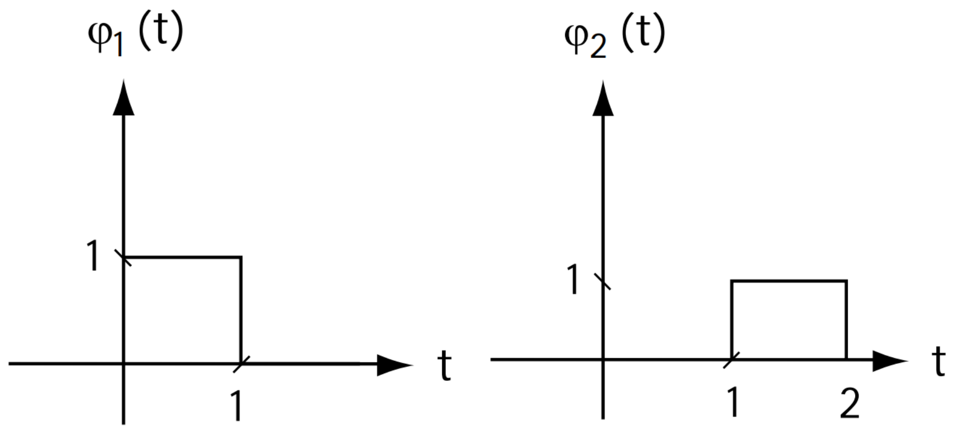


$$s_1(t) = 2\phi_1(t) + 0\phi_2(t) \quad \begin{cases} s_{11} = \int_{-\infty}^{\infty} s_1(t)\phi_1(t)dt = 2 \\ s_{12} = \int_{-\infty}^{\infty} s_1(t)\phi_2(t)dt = 0 \end{cases}$$

4.3 Modulator Signal

- orthonormal basis





$$s_2(t) = 1(t) + 1\phi_2(t) \quad \begin{cases} s_{21} = \int_{-\infty}^{\infty} s_2(t)\phi_1(t)dt = 1 \\ \\ s_{22} = \int_{-\infty}^{\infty} s_2(t)\phi_2(t)dt = 1 \end{cases}$$

4.3 Modulator Signal

- Gram-Schmidt orthogonalization procedure

(1) To get $\phi_1(t)$, just compute

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

where $E_1 = \int_{-\infty}^{\infty} s_1(t)s_1(t)dt$.

(2) To get $\phi_2(t)$, compute

$$\phi_2(t) = \frac{\theta_2(t)}{\sqrt{E_{\theta 2}}}$$

where $\theta_2(t) = s_2(t) - s_{21}\phi_1(t)$ and $E_{\theta 2} = \int_{-\infty}^{\infty} \theta_2(t)\theta_2(t)dt$.

4.3 Modulator Signal

- Gram-Schmidt orthogonalization procedure

(3) To get $\phi_3(t)$, just compute

$$\phi_3(t) = \frac{\theta_3(t)}{\sqrt{E_{\theta 3}}}$$

where $\theta_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)$

and $E_{\theta 3} = \int_{-\infty}^{\infty} \theta_3(t)\theta_3(t)dt$.

(4) Keep going, up to $\phi_M(t)$, and if you get $\phi_k(t) = 0$

along the way, just throw that one out, because you don't need it.

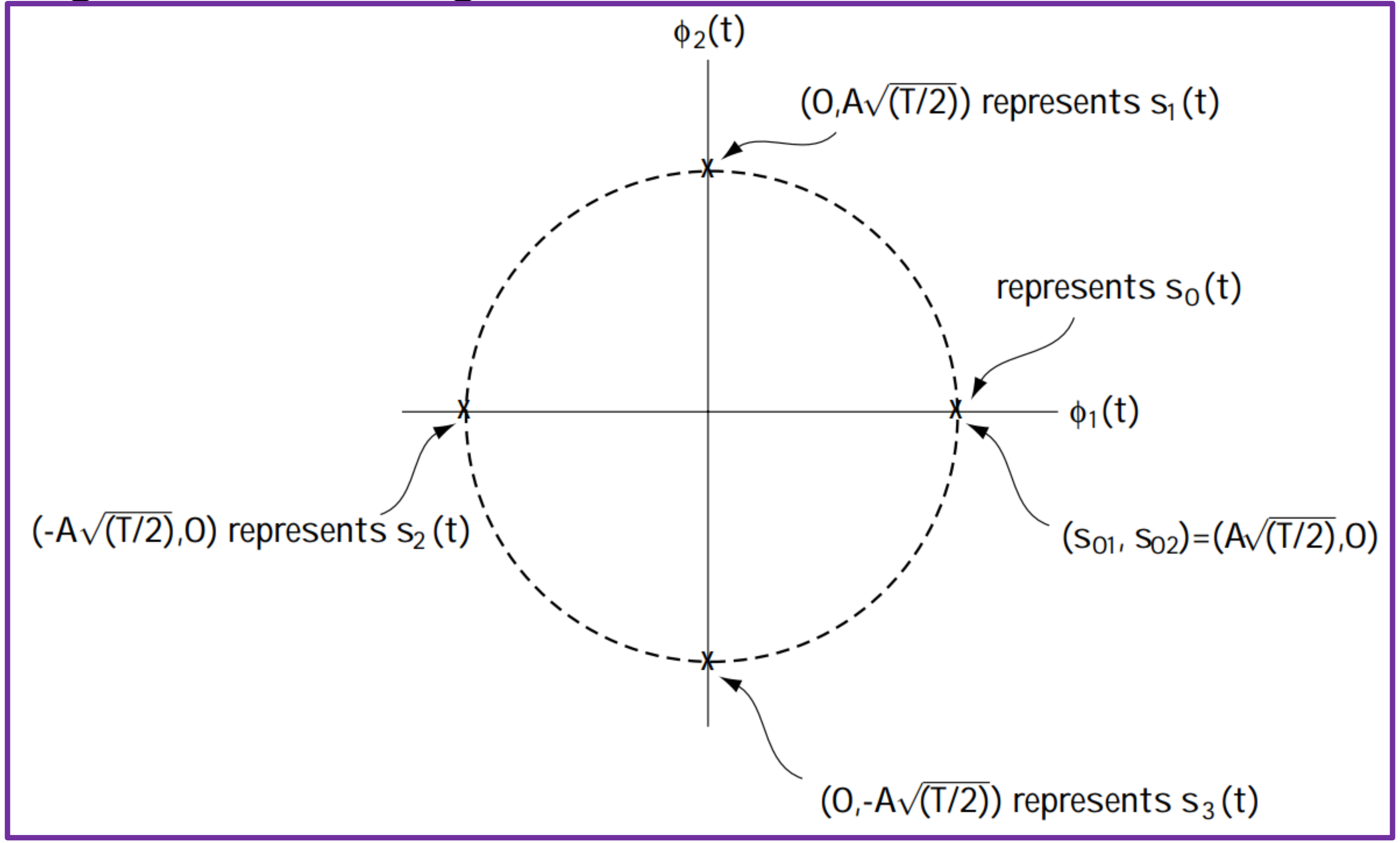
4.3 Modulator Signal

- Representing Modulated Signals : 4 - PSK

	Input bits	Output waveforms				
4-PSK	00	$s_0(t) = A\cos(\omega_c t + 0°)\pi(t-iT)$	=	$A\cos(\omega_c t)\cdot\pi(t\text{-}iT)$	+	0
	01	$s_1(t) = A\cos(\omega_c t + 90°)\cdot\pi(t-iT)$	=	0		$-A\sin(\omega_c t)\cdot\pi(t\text{-}iT)$
	10	$s_2(t) = A\cos(\omega_c t + 180°)\cdot\pi(t-iT)$	=	$-A\cos(\omega_c t)\cdot\pi(t\text{-}iT)$	+	0
	11	$s_3(t) = A\cos(\omega_c t + 270°)\cdot\pi(t-iT)$	=	0		$+A\sin(\omega_c t)\cdot\pi(t\text{-}iT)$

4.3 Modulator Signal

- Representing Modulated Signals : 4 - PSK



4.3 Modulator Signal

- Representing Modulated Signals : 8 - PSK

	Input bits	Output waveforms				
8-PSK	000	$s_0(t) = A\cos(\omega_c t + 0^\circ)\cdot\pi(t-iT)$	$=$	$A\cos(\omega_c t)\cdot\pi(t\text{-}iT)$	$+$	0
	001	$s_1(t) = A\cos(\omega_c t + 45^\circ)\cdot\pi(t-iT)$	$=$	$\frac{A}{\sqrt{2}}\cos(\omega_c t)\cdot\pi(t\text{-}iT)$	$-$	$\frac{A}{\sqrt{2}}\sin(\omega_c t)\cdot\pi(t\text{-}iT)$
	010	$s_2(t) = A\cos(\omega_c t + 90^\circ)\cdot\pi(t-iT)$	$=$	0		$-A\sin(\omega_c t)\cdot\pi(t-iT)$
	011	$s_3(t) = A\cos(\omega_c t + 135^\circ)\cdot\pi(t-iT)$	$=$	$\frac{-A}{\sqrt{2}}\cos(\omega_c t)\cdot\pi(t-iT)$	$-$	$\frac{A}{\sqrt{2}}\sin(\omega_c t)\cdot\pi(t-iT)$

4.3 Modulator Signal

- Representing Modulated Signals : 8 - PSK

100	$s_4(t) = A\cos(\omega_c t+180^\circ)\cdot\pi(t-iT)$	=	$-A\cos(\omega_c t)\cdot\pi(t-iT)$	$+\ 0$
101	$s_5(t) = A\cos(\omega_c t+225^\circ)\cdot\pi(t-iT)$	=	$\frac{-A}{\sqrt{2}}\cos(\omega_c t)\cdot\pi(t-iT)$	$+\ \frac{A}{\sqrt{2}}\sin(\omega_c t)\cdot\pi(t-iT)$
110	$s_6(t) = A\cos(\omega_c t+270^\circ)\cdot\pi(t-iT)$	=	0	$+\ A\sin(\omega_c t)\cdot\pi(t-iT)$
111	$s_7(t) = A\cos(\omega_c t+315^\circ)\cdot\pi(t-iT)$	=	$\frac{A}{\sqrt{2}}\cos(\omega_c t)\cdot\pi(t-iT)$	$+\ \frac{A}{\sqrt{2}}\sin(\omega_c t)\cdot\pi(t-iT)$

4.3 Modulator Signal

- Representing Modulated Signals : ASK

	Output waveform	Output waveform represented on orthonormal basis
BASK	$s_0(t)$	$\mathbf{s}_0 = s_{01} = -A\sqrt{T/2}$
	$s_1(t)$	$\mathbf{s}_1 = s_{11} = A\sqrt{T/2}$
4-ASK	$s_0(t)$	$\mathbf{s}_0 = s_{01} = -3A\sqrt{T/2}$
	$s_1(t)$	$\mathbf{s}_1 = s_{11} = -A\sqrt{T/2}$
	$s_2(t)$	$\mathbf{s}_2 = s_{21} = A\sqrt{T/2}$
	$s_3(t)$	$\mathbf{s}_3 = s_{31} = 3A\sqrt{T/2}$

4.3 Modulator Signal

- Representing Modulated Signals : ASK

8-ASK	$s_0(t)$	$\mathbf{s}_0 = s_{01} = -7A\sqrt{T/2}$
	$s_1(t)$	$\mathbf{s}_1 = s_{11} = -5A\sqrt{T/2}$
	$s_2(t)$	$\mathbf{s}_2 = s_{21} = -3A\sqrt{T/2}$
	$s_3(t)$	$\mathbf{s}_3 = s_{31} = -A\sqrt{T/2}$
	$s_4(t)$	$\mathbf{s}_4 = s_{41} = A\sqrt{T/2}$
	$s_5(t)$	$\mathbf{s}_5 = s_{51} = 3A\sqrt{T/2}$
	$s_6(t)$	$\mathbf{s}_6 = s_{61} = 5A\sqrt{T/2}$
	$s_7(t)$	$\mathbf{s}_7 = s_{71} = 7A\sqrt{T/2}$

4.3 Modulator Signal

- Representing Modulated Signals : QAM
- The information bits are stuffed into both the phase (θ) and the amplitude (A) of the cosine waveform. That is, a typical output waveform for QAM looks like this

$$s_j(t) = A_j \cos\left(\omega_c t + \theta_j\right) \cdot \pi(t - iT)$$

- we can rewrite this as

$$s_j(t) = A_j \cos\left(\theta_j\right) \cos(\omega_c t) \cdot \pi(t - iT) - A_j \sin\left(\theta_j\right) \sin(\omega_c t) \cdot \pi(t - iT)$$

4.3 Modulator Signal

- Representing Modulated Signals : QAM
- Using the orthonormal basis as

$$s_j(t) = s_{j1}\phi_1(t) + s_{j2}\phi_2(t)$$

where

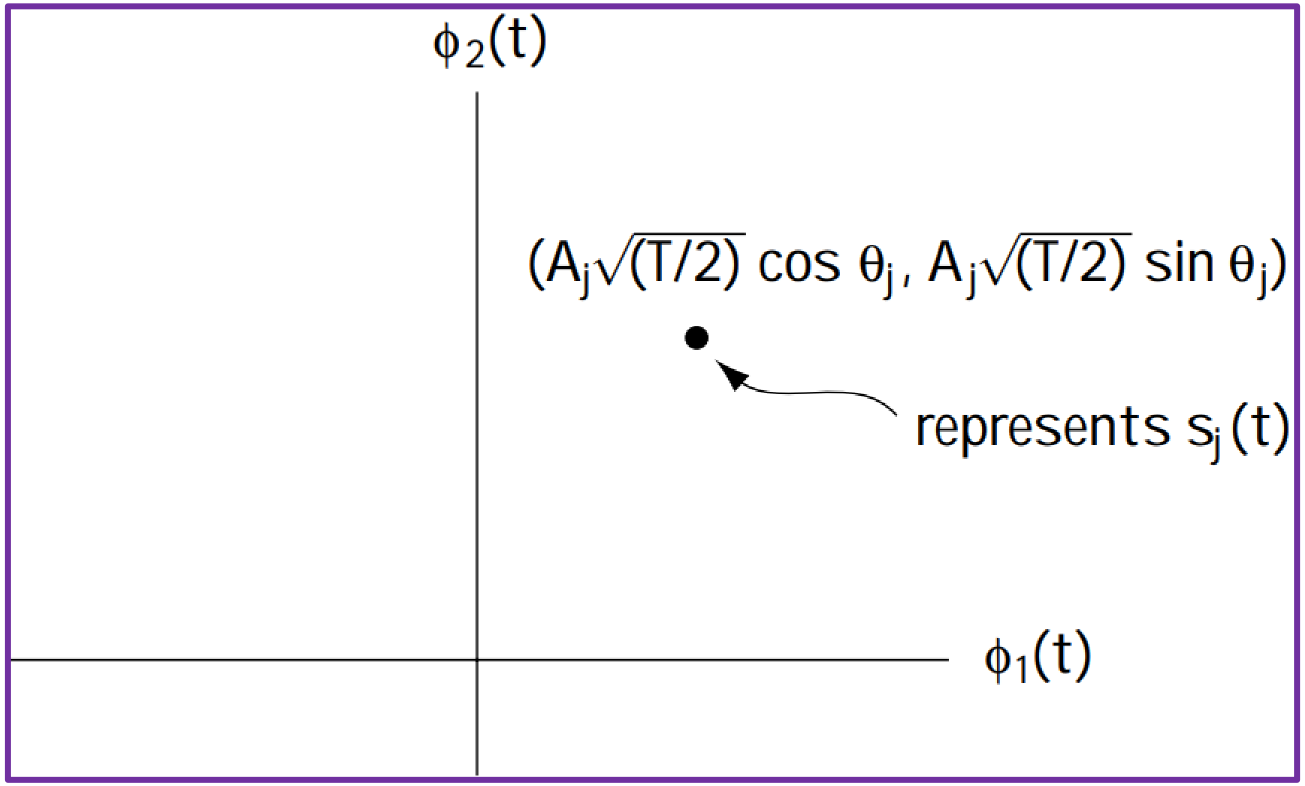
$$\phi_1(t) = +\sqrt{2/T}\cos(\omega_c t) \cdot \pi(t - iT)$$

$$\phi_2(t) = -\sqrt{2/T}\sin(\omega_c t) \cdot \pi(t - iT)$$

4.3 Modulator Signal

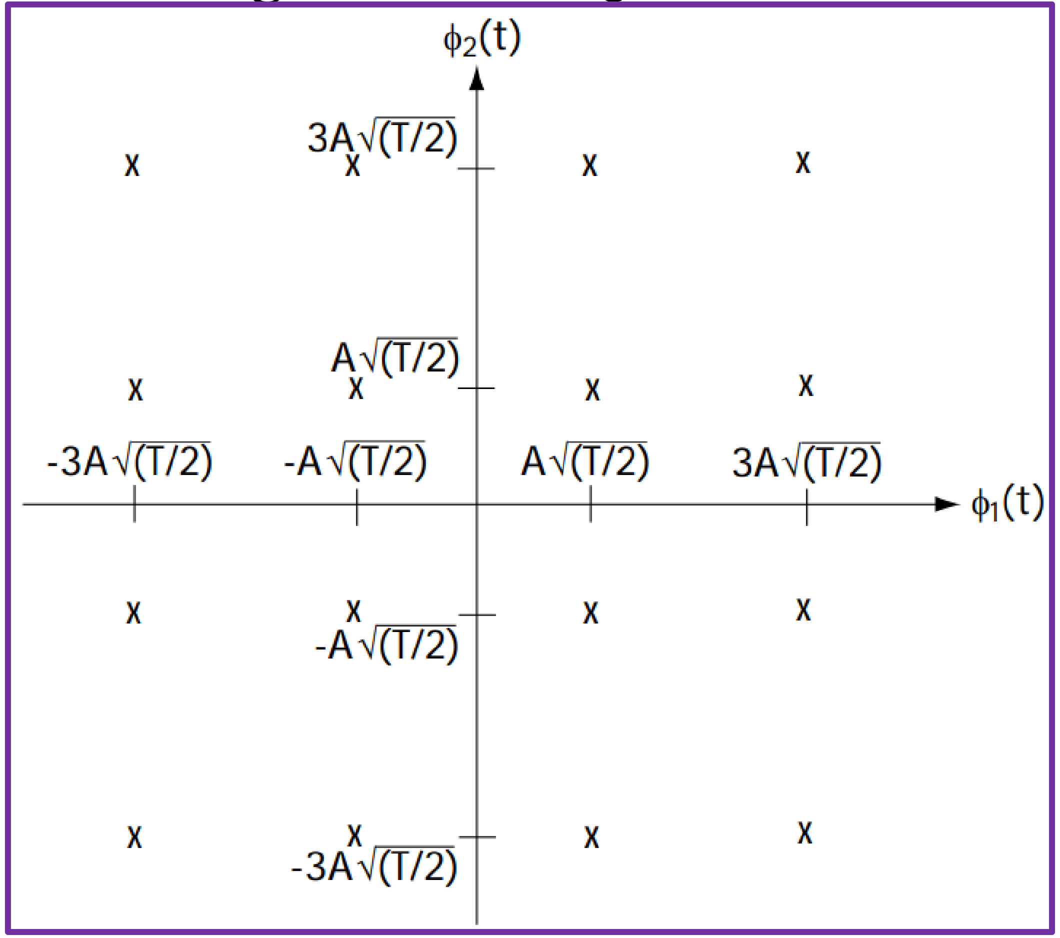
- Representing Modulated Signals : QAM

$$s_j(t) \leftrightarrow \underline{s}_j = (s_{j1}, s_{j2}) = \left(A_j\sqrt{T/2}\cos\theta_j, A_j\sqrt{T/2}\sin\theta_j\right)$$



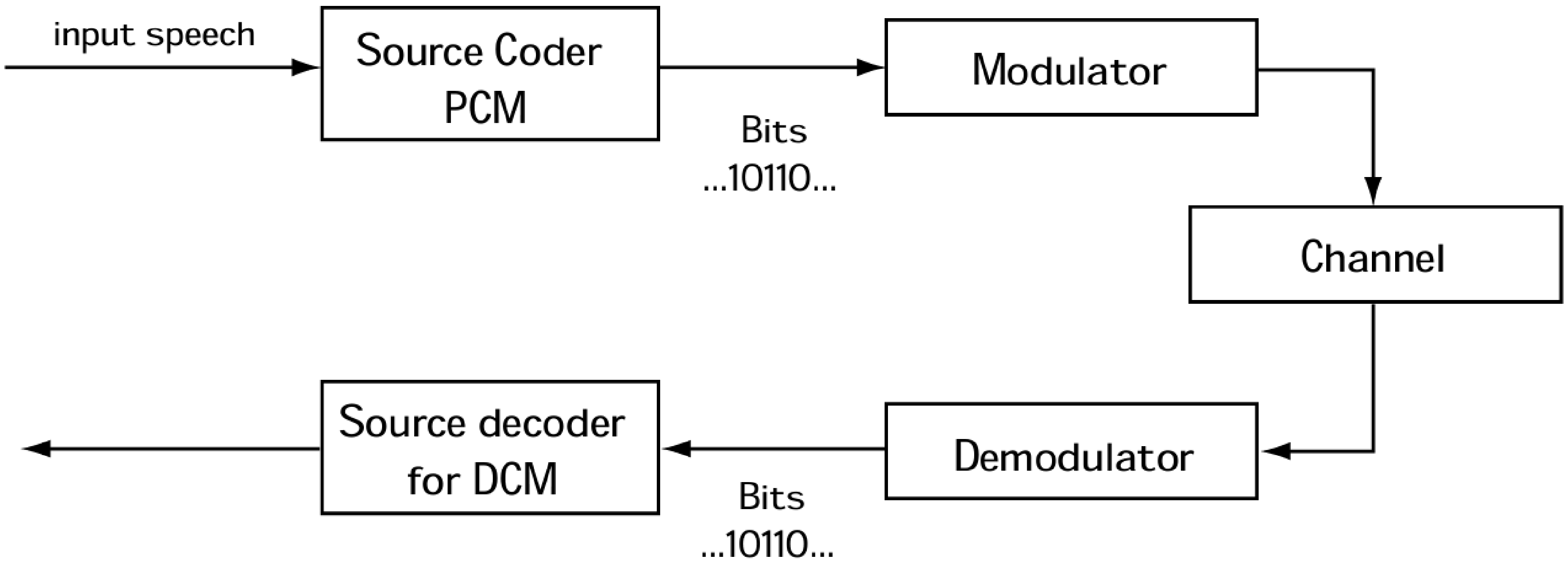
4.3 Modulator Signal

- Representing Modulated Signals : 16 - QAM

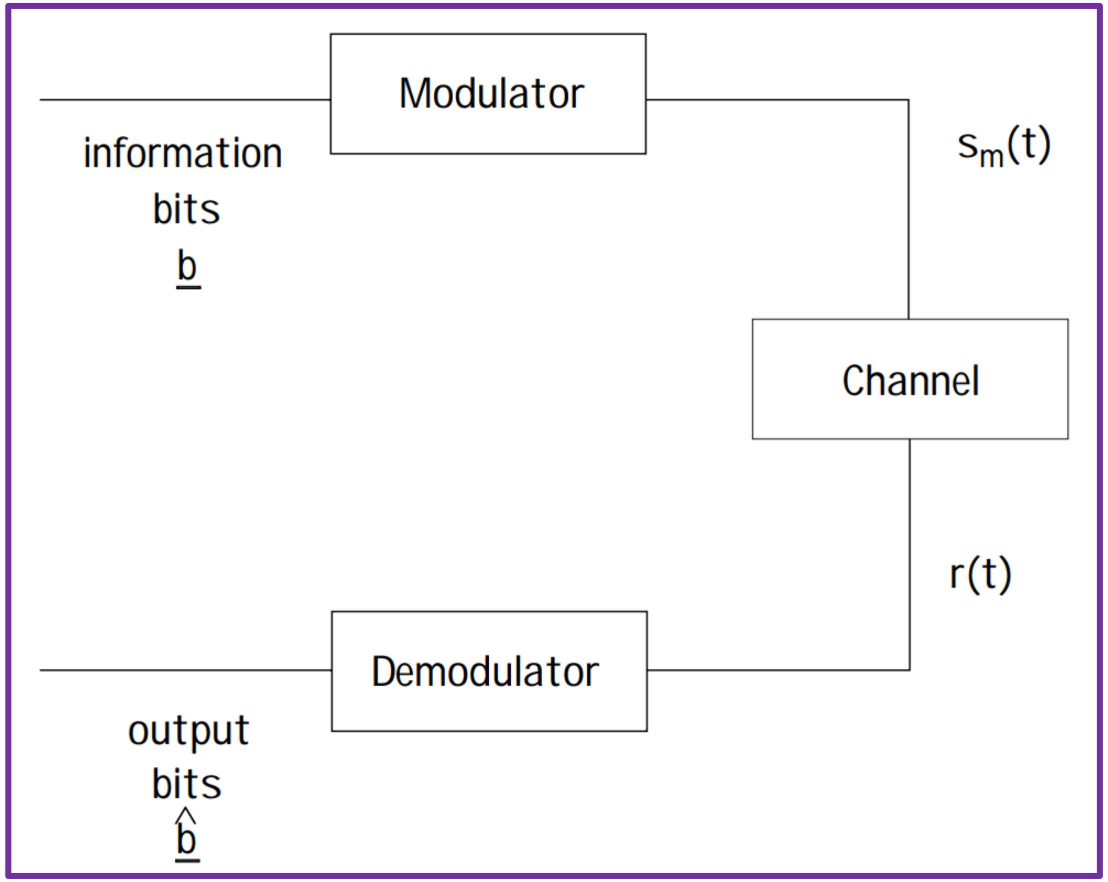


4.4 Demodulator

- The demodulator is a device that gets the signal sent across the channel and turns it back into bits.

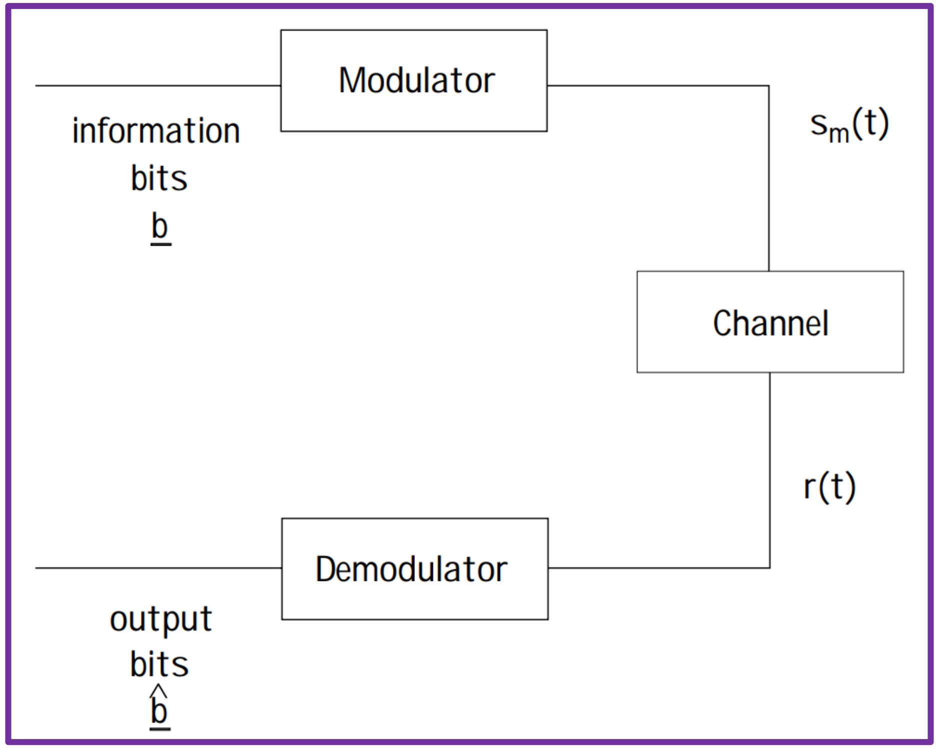


4.4 Demodulator



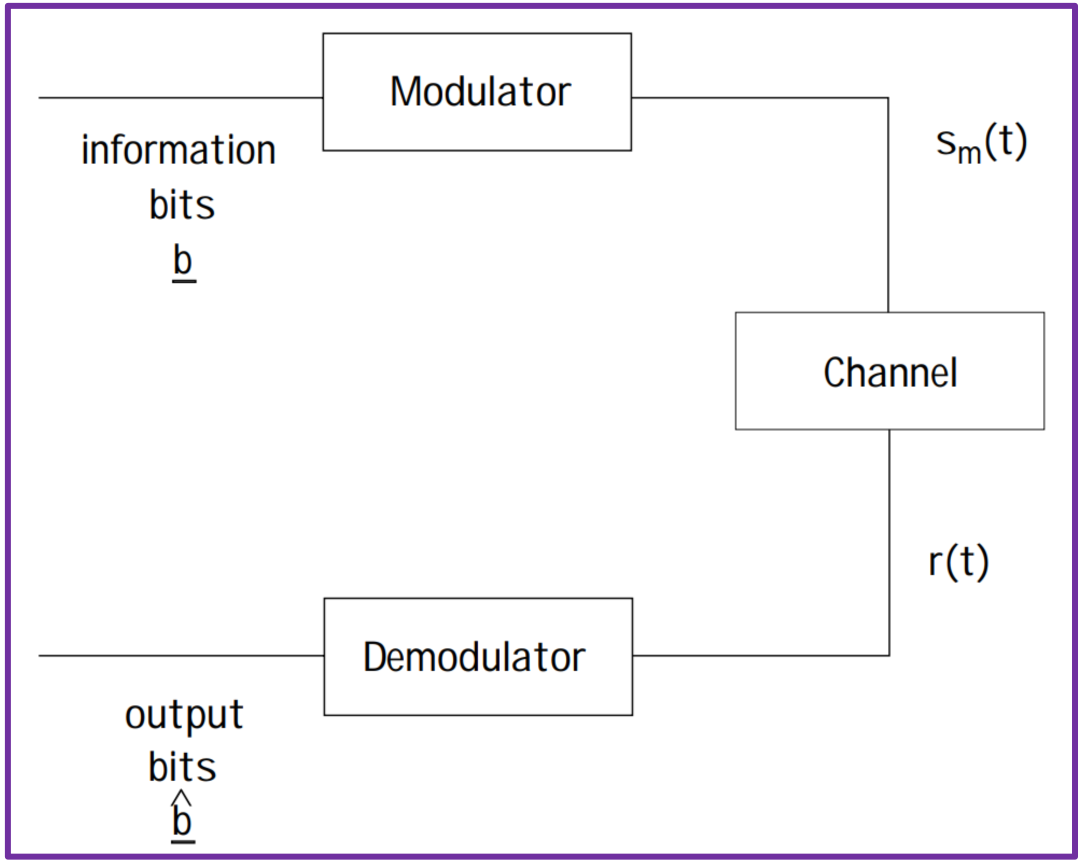
- The key to building a good demodulator is to minimize the effects of noise and give the highest probability of guessing the correct sent signal.
- This will make sure that the bits that leave the demodulator (receiver side) are as close as possible to the bits that come into the modulator (transmitter side).

4.4 Demodulator



- $r(t) = s_m(t) + \eta(t)$
- $r(t) = r_1\phi_1(t) + r_2\phi_2(t) + \cdots + r_N\phi_N(t)$

4.4 Demodulator



$$r_1 = \int r(t)\varphi_1(t)dt$$

$$r_1 = \int (s_m(t)+\eta(t))\varphi_1(t)dt$$

$$r_1 = \int s_m(t)\varphi_1(t)dt + \int \eta(t)\varphi_1(t)dt$$

$$r_1 = s_{m1} + \eta_1$$

4.4 Demodulator

$$r_2 = s_{m2} + \eta_2 \qquad r_3 = \int r(t)\varphi_3(t)dt$$

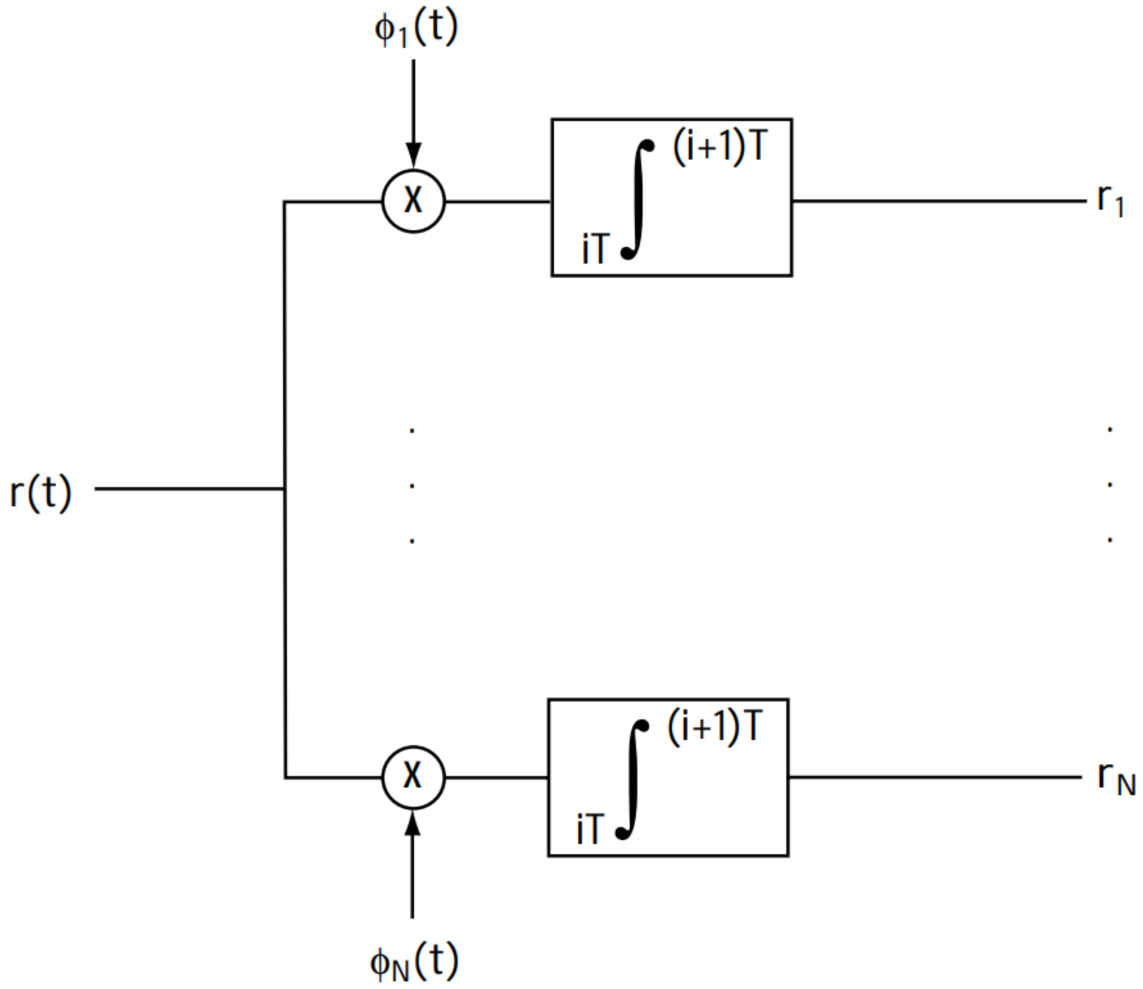
$$r_3 = \int \left(s_m(t) + \eta(t)\right)\varphi_3(t)dt$$

$$r_3 = \int s_m(t)\varphi_3(t)dt + \int \eta(t)\varphi_3(t)dt$$

$$r_3 = \int \left(s_{m1}\varphi_1(t) + s_{m2}\varphi_2(t)\right)\varphi_3(t)dt + \int \eta(t)\varphi_3(t)dt$$

$$r_3 = s_{m1}\int \varphi_1(t)\varphi_3(t)dt + s_{m2}\int \varphi_2(t)\varphi_3(t)dt + \int \eta(t)\varphi_3(t)dt$$

4.4 Demodulator



Correlator receiver front end