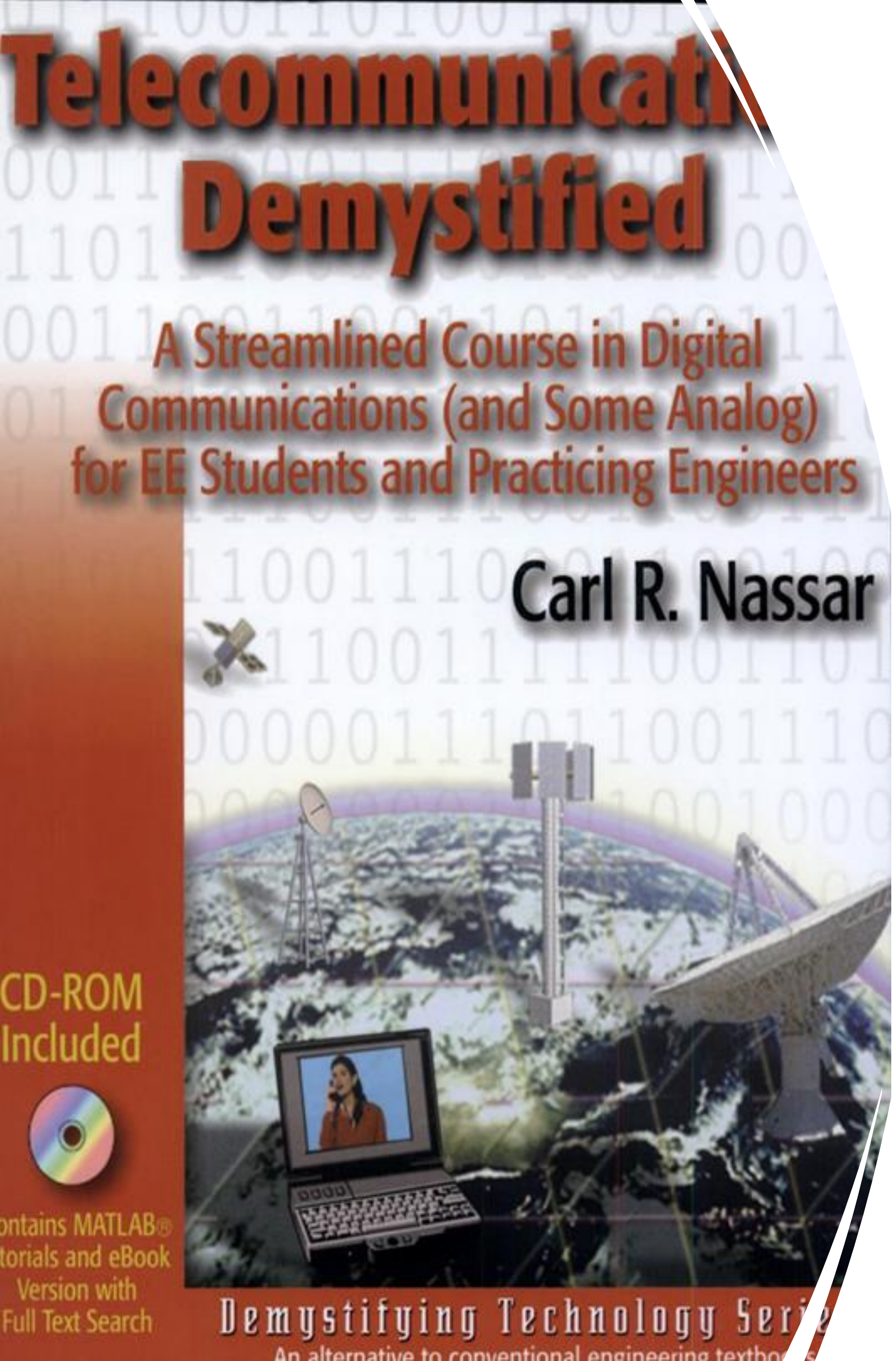


3. Source Coding and Decoding

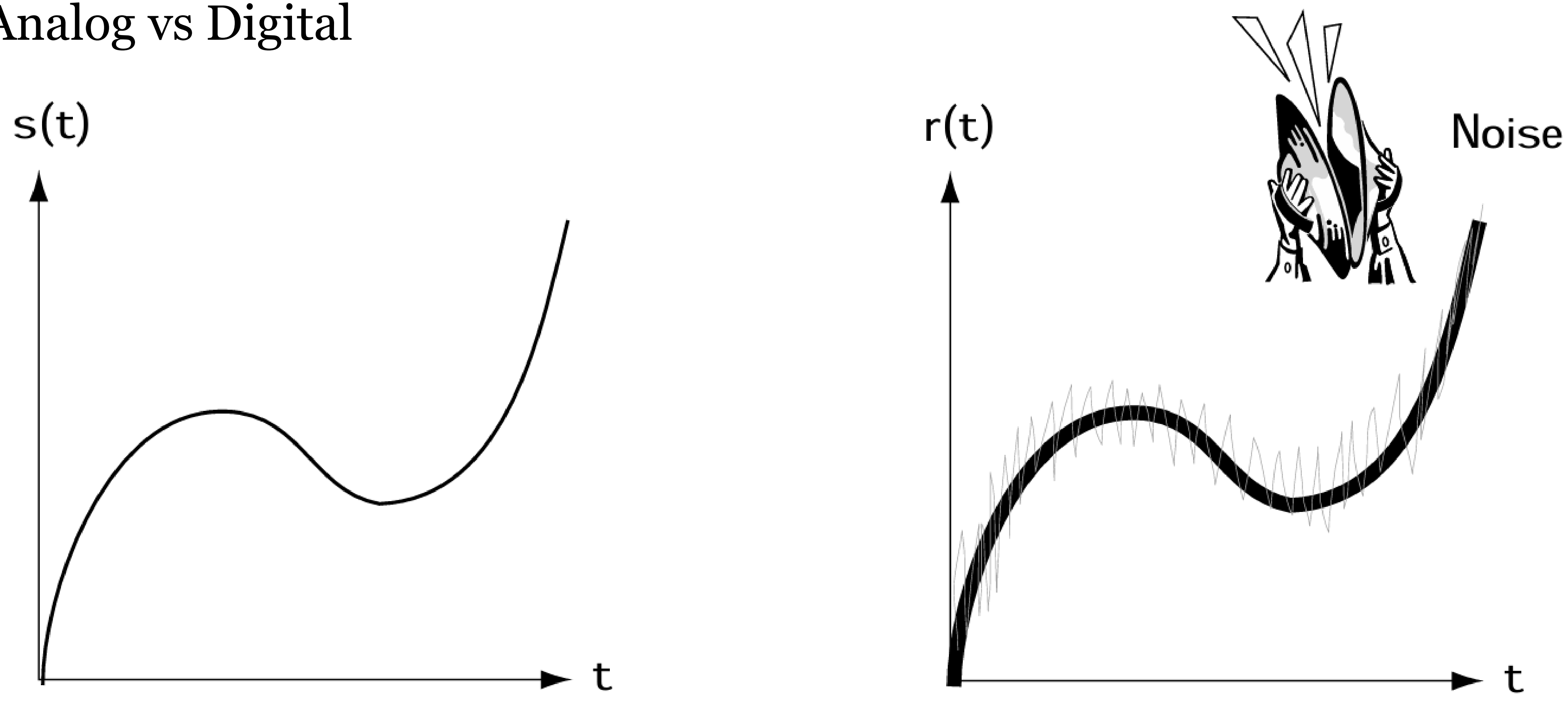
3.1 Sampling

3.2 Quantization

3.3 Source Coding

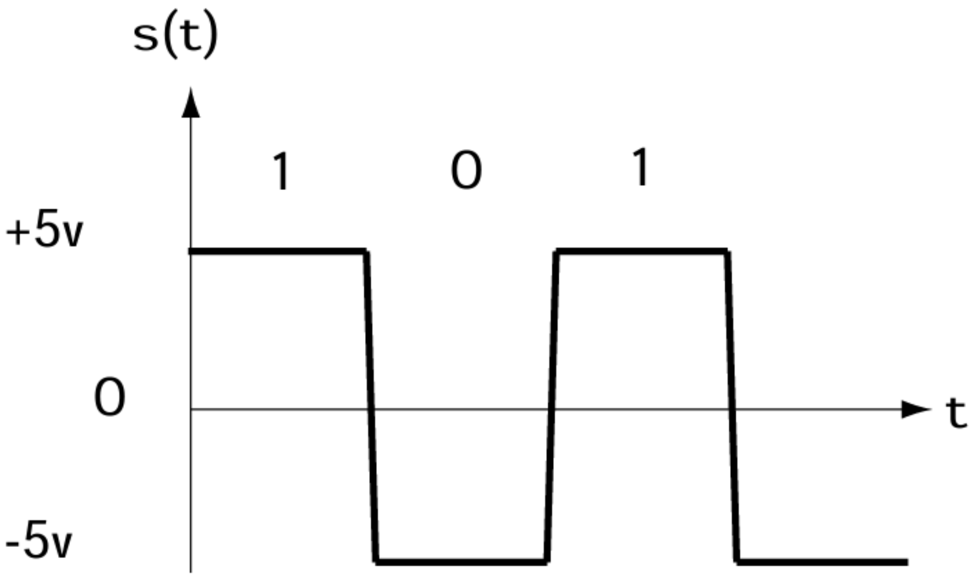
3. Source Coding and Decoding

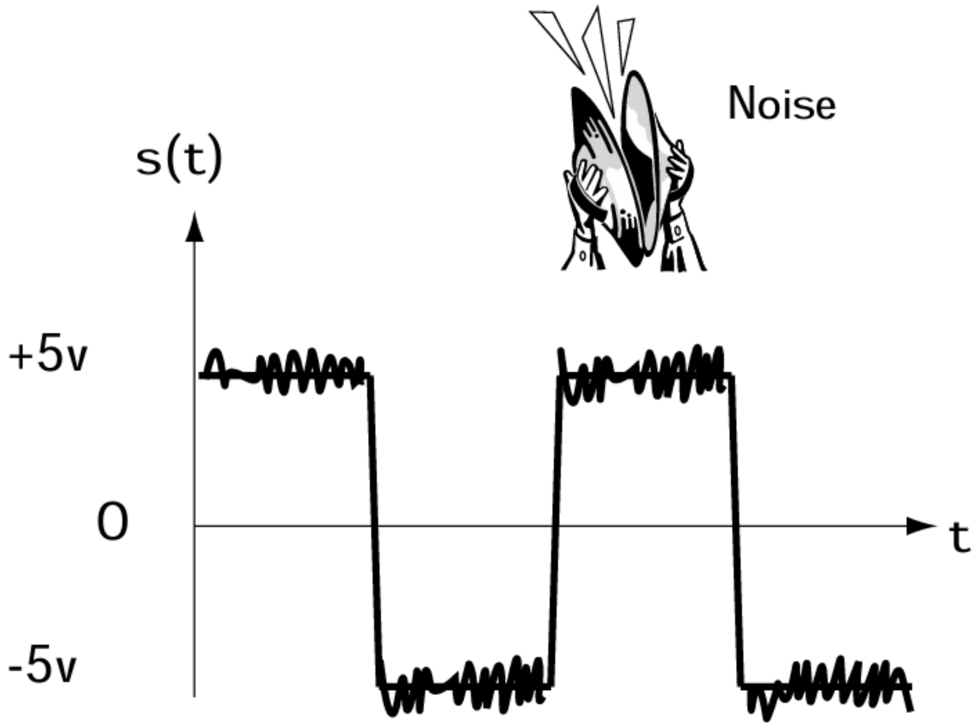
- Analog vs Digital



3. Source Coding and Decoding

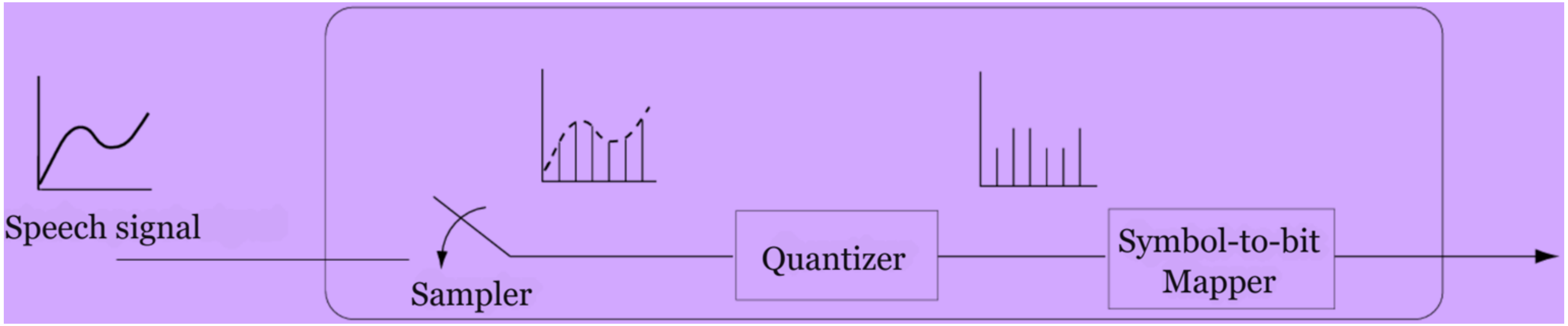
- Analog vs Digital





3. Source Coding and Decoding

- This chapter talks about how to turn an analog signal into a digital one, a process called source coding.
- Before going on, just a brief reminder about why we want to turn analog signals to digital. So many naturally occurring sources of information are analog (human speech, for example), and we want to make them digital signals so we can use a digital communication system.



3.1 Sampling

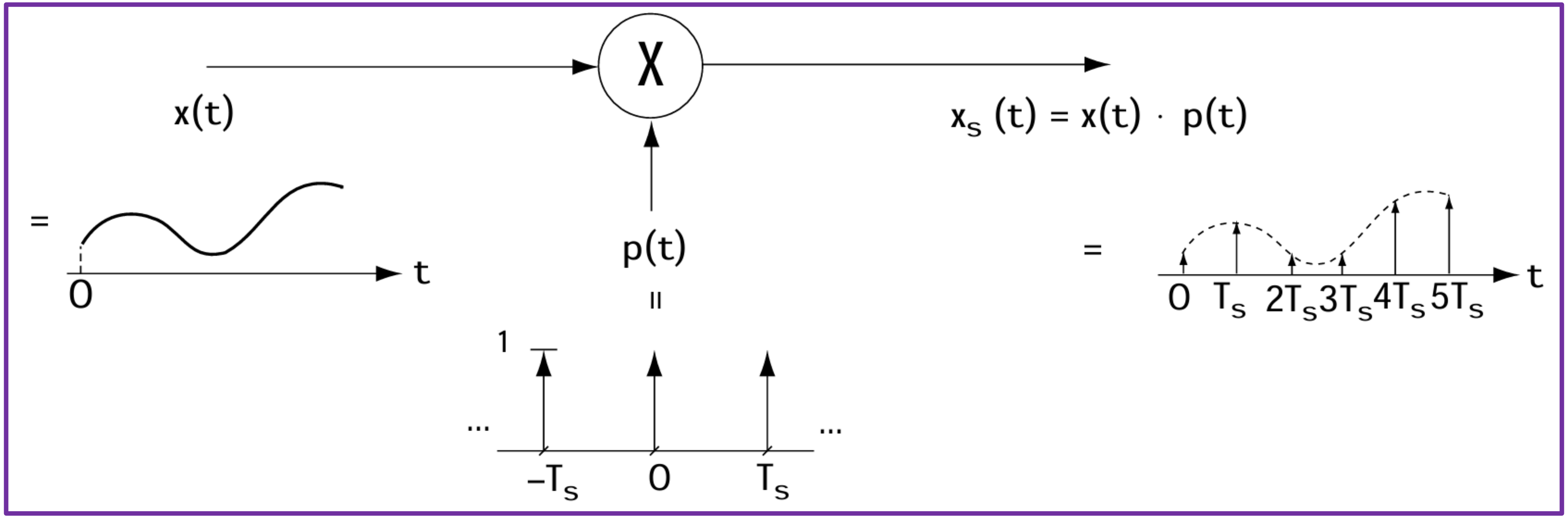
- The key first step in turning any analog signal to a digital one is called sampling. *Sampling* is the changing of an analog signal to samples (or pieces) of itself.
- There are three methods of sampling that we'll look at together, Ideal Sampling, Zero-order Hold Sampling, Natural Sampling

3.1 Sampling

- Ideal Sampling

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

$$x_s(t) = x(t) \cdot p(t) = x(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$



3.1 Sampling

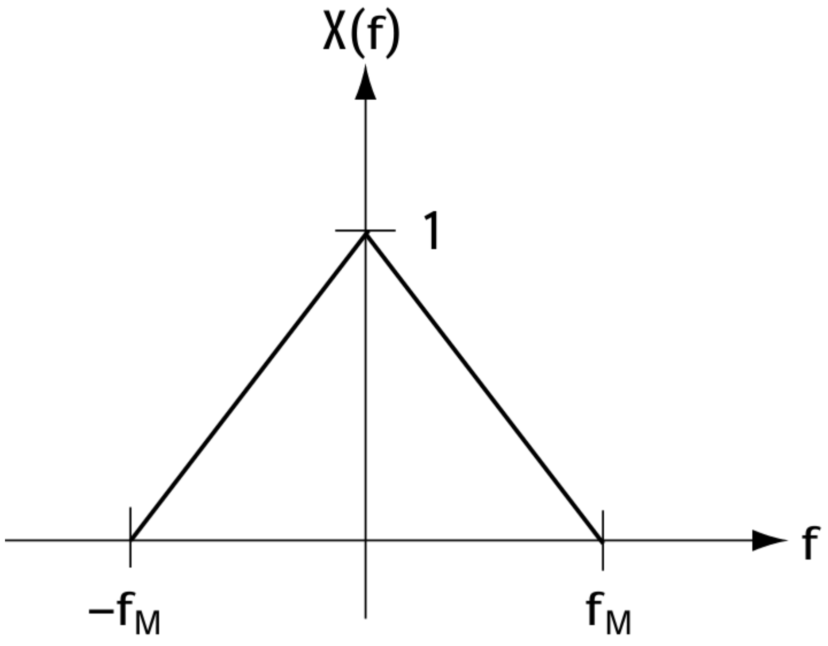
- Ideal Sampling

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s) \xrightarrow[\text{transform}]{\text{Fourier}} P(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - k f_s)$$

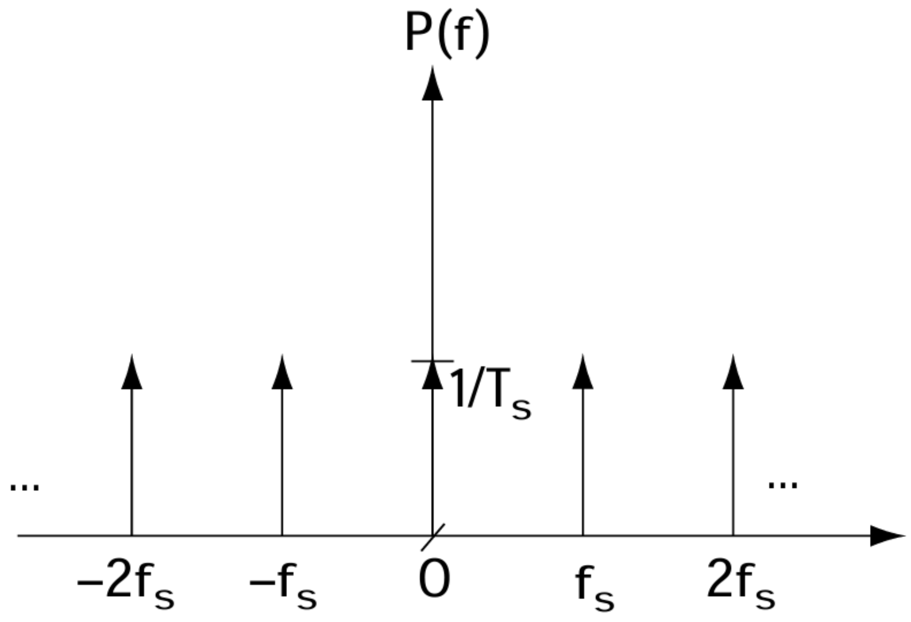
- where $f_s = 1/T_s$ and f_s is called the *sampling rate*.

3.1 Sampling

- Ideal Sampling



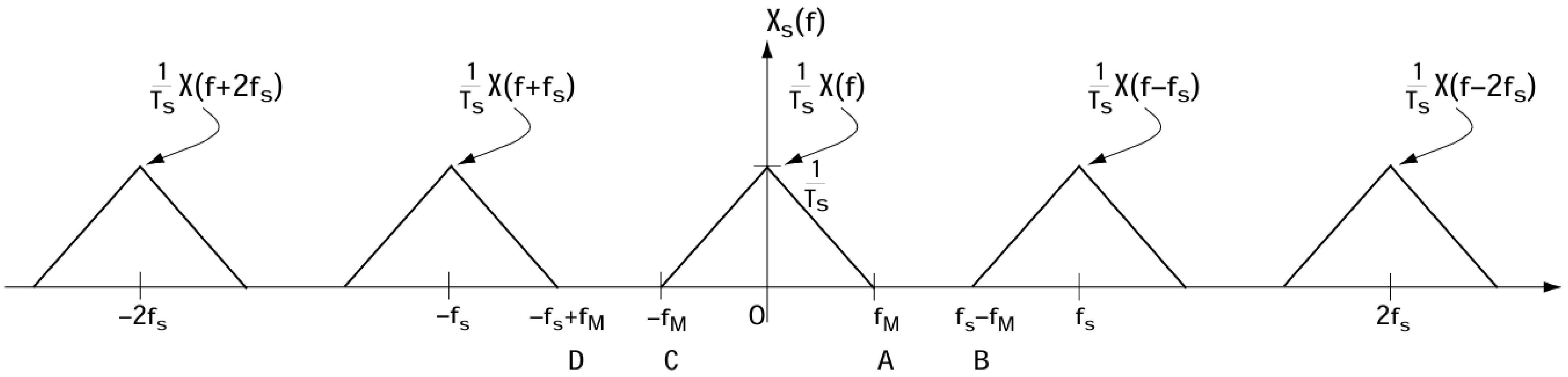
Fourier transform of the input signal x(t)



Fourier transform of the impulse train p(t)

3.1 Sampling

- Ideal Sampling

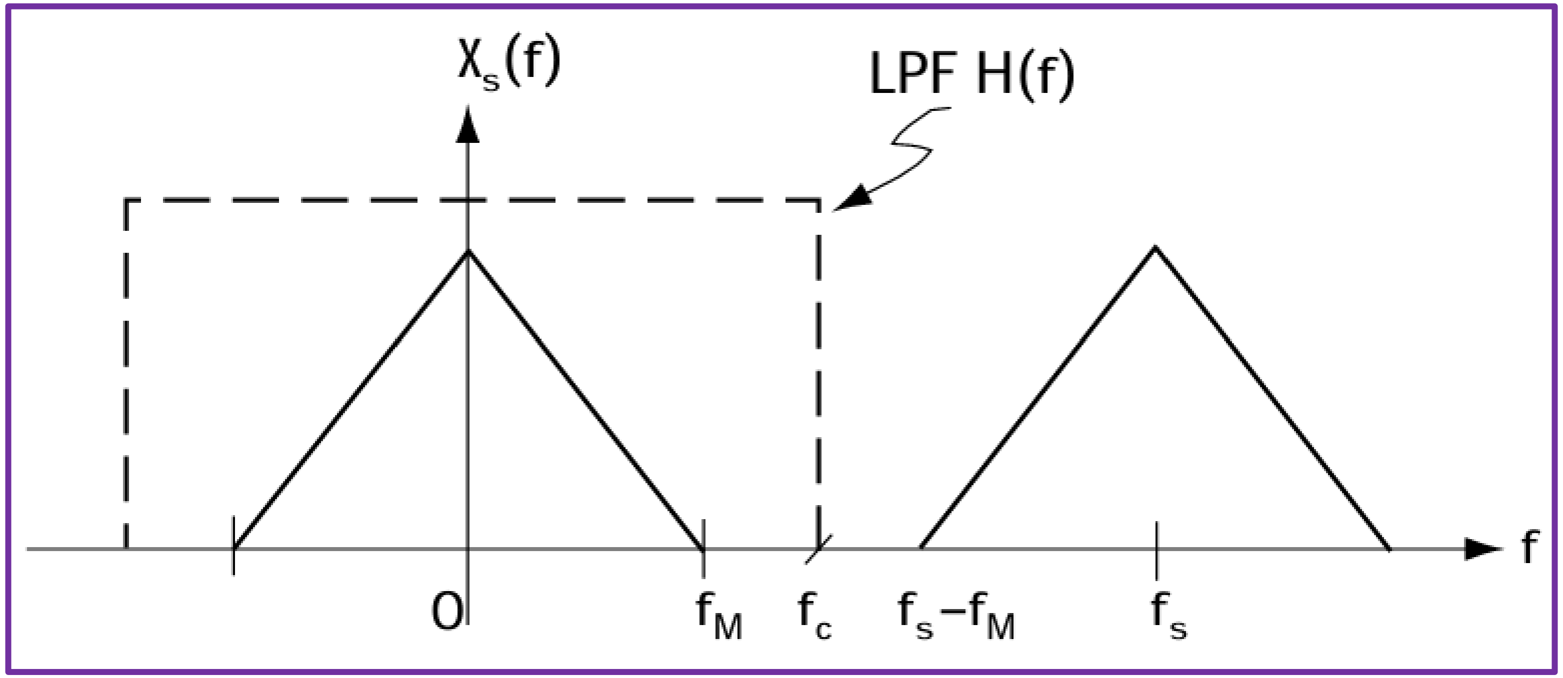


Fourier transform of the ideal sampling output $x_s(t)$

- The sampling theorem simply states that a signal can be recovered from its samples as long as it is sampled at $f_s > 2f_m$.

3.1 Sampling

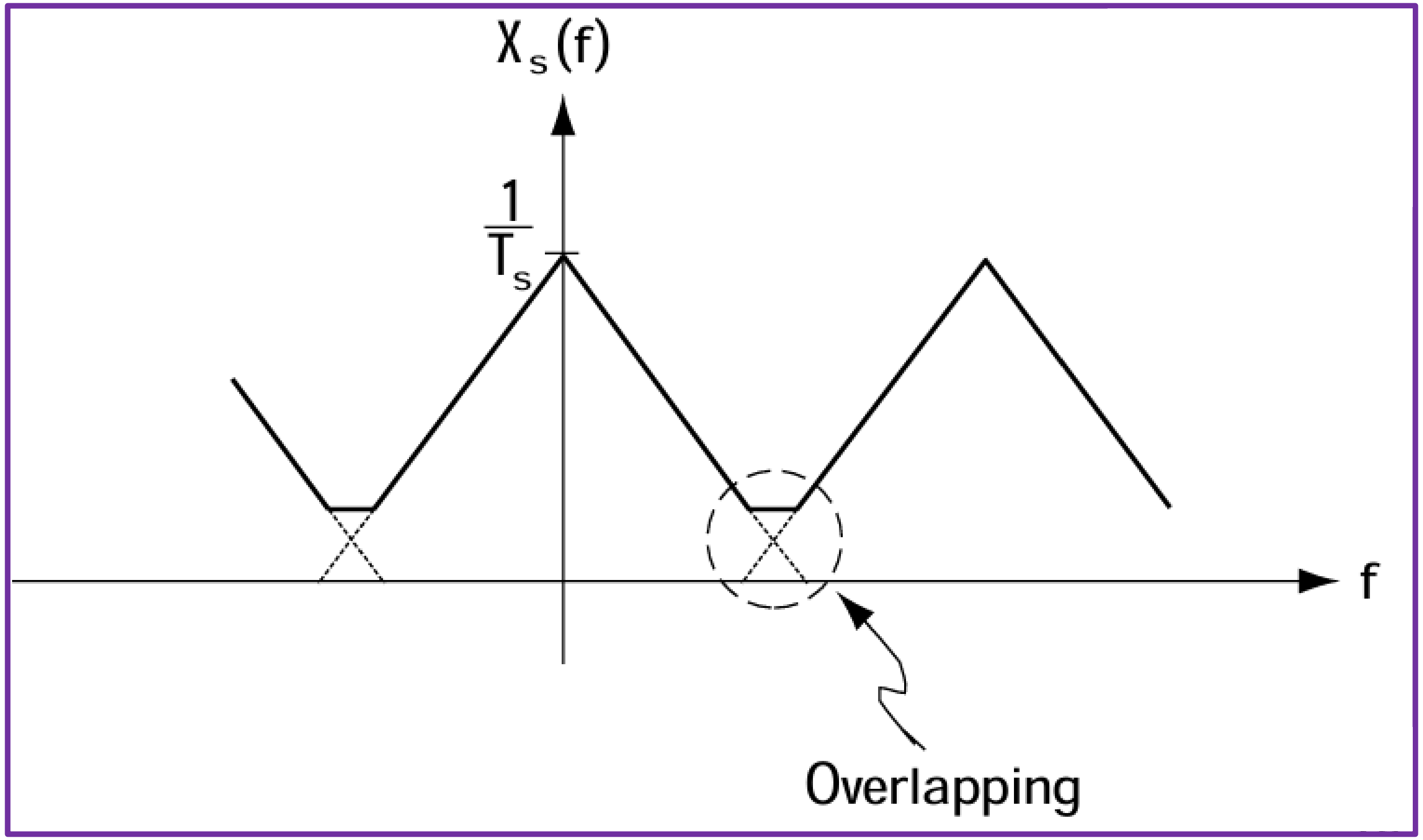
- Ideal Sampling



- The *Nyquist rate* is the smallest *sampling rate* f_s that can be used if you want to recover the original signal from its samples. From what we just saw, we know that $f_N = 2f_M$.

3.1 Sampling

- Ideal Sampling



3.1 Sampling

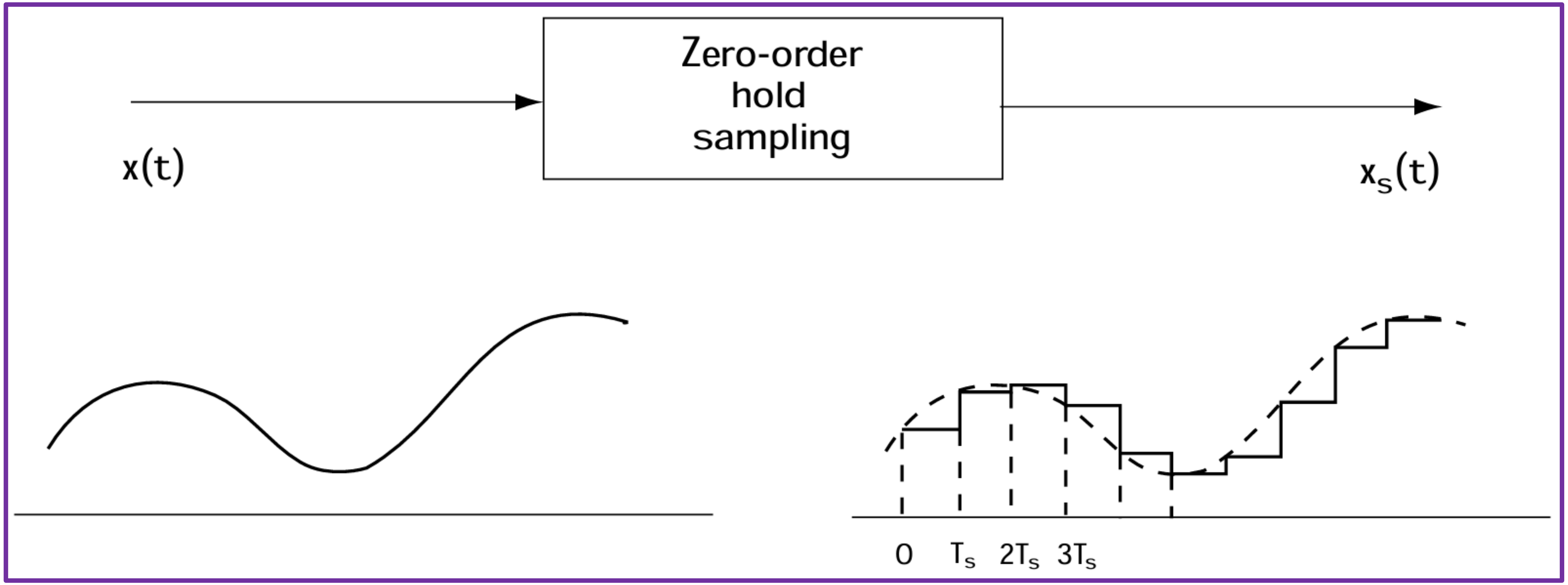
- Example 1: Determine the Nyquist sampling rate for the following signals.

(a) $$x(t)=\frac{\sin(4000\pi\, t)}{\pi\, t}$$

(b) $$x(t)=\frac{\sin^2(4000\pi\, t)}{\pi^2\, t^2}$$

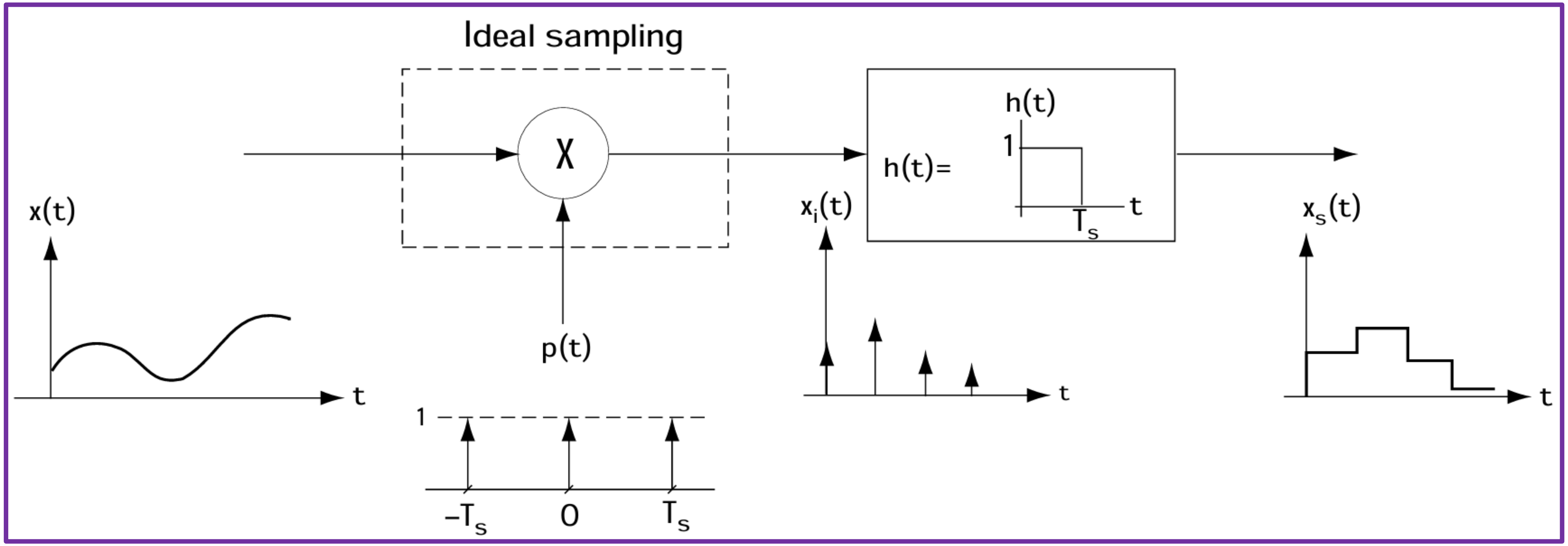
3.1 Sampling

- Zero-order Hold Sampling



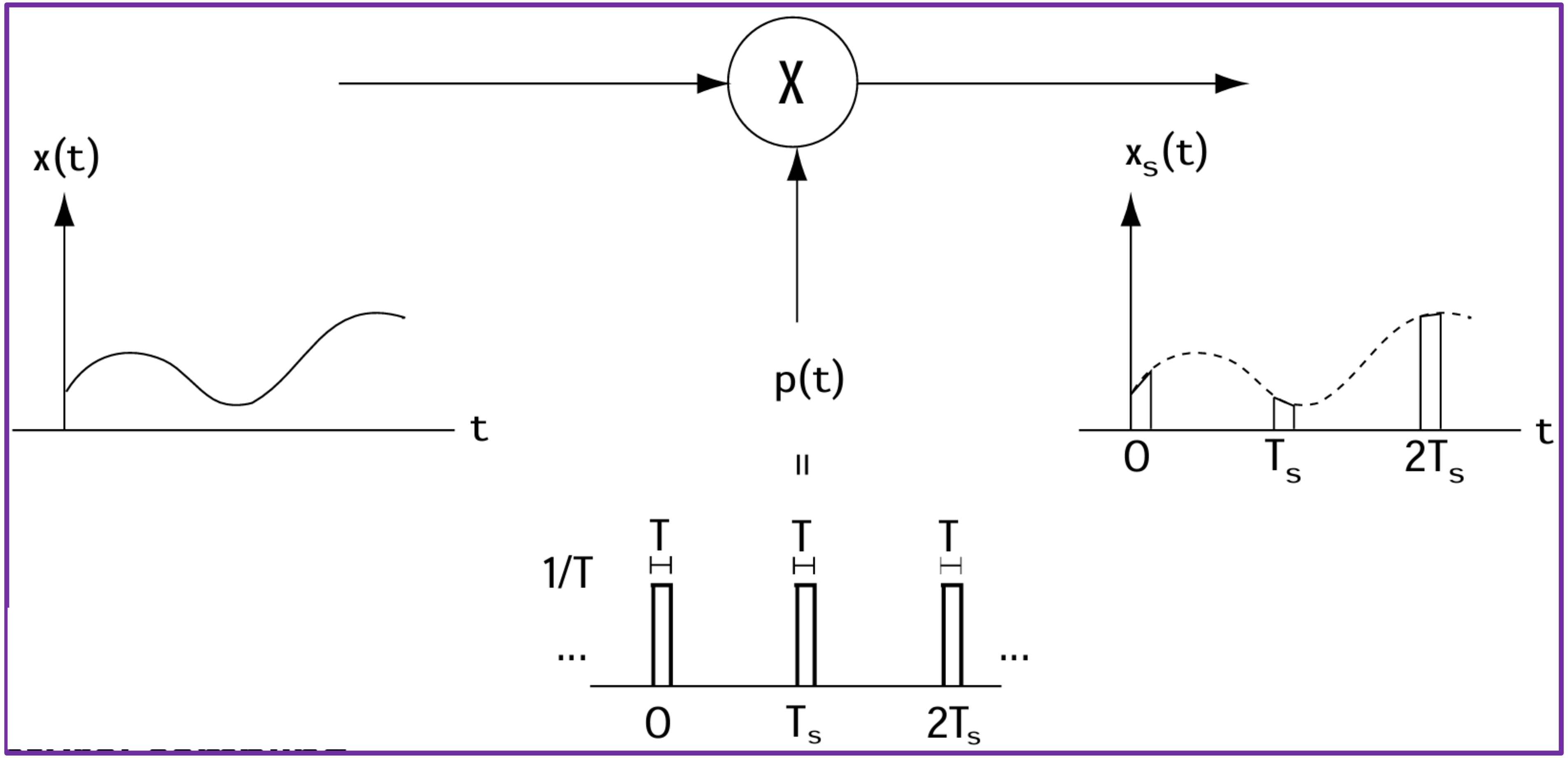
3.1 Sampling

- Zero-order Hold Sampling

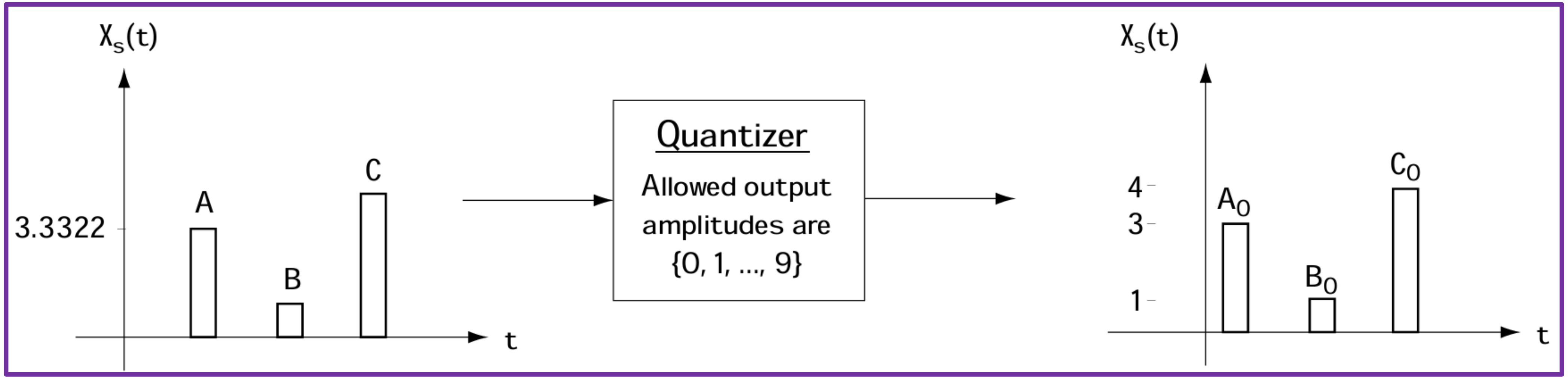


3.1 Sampling

- Natural Sampling



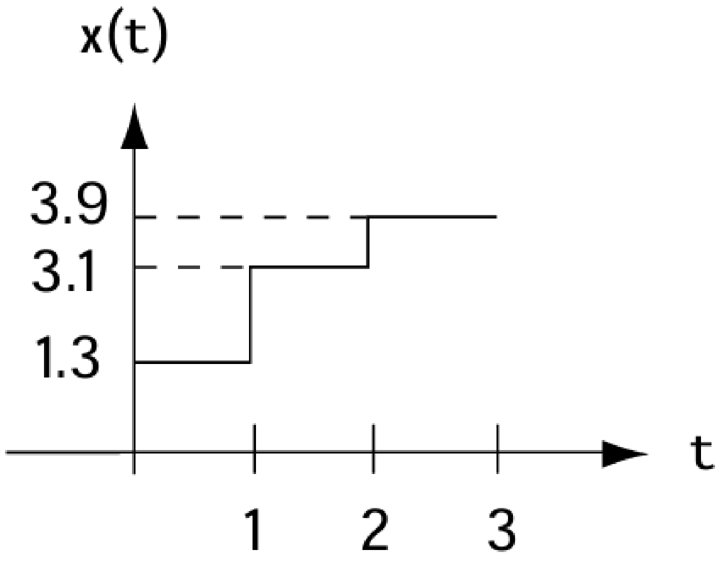
3.2 Quantization



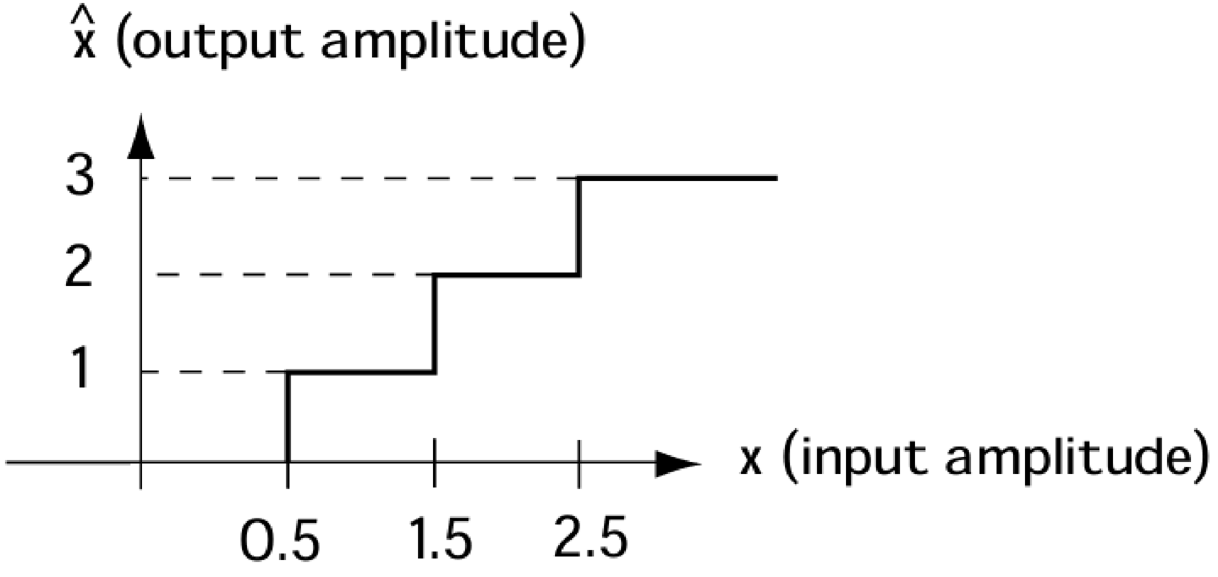
- The operation carried out in source coding is called *quantization*, and the device which does it is called a *quantizer*.
- It is actually just an "*amplitude changer*"

3.2 Quantization

- Example 2: Consider the quantizer with the input shown in Figure (a) and with an input amplitude–output amplitude relationship drawn in Figure (b). Draw a plot of its output.

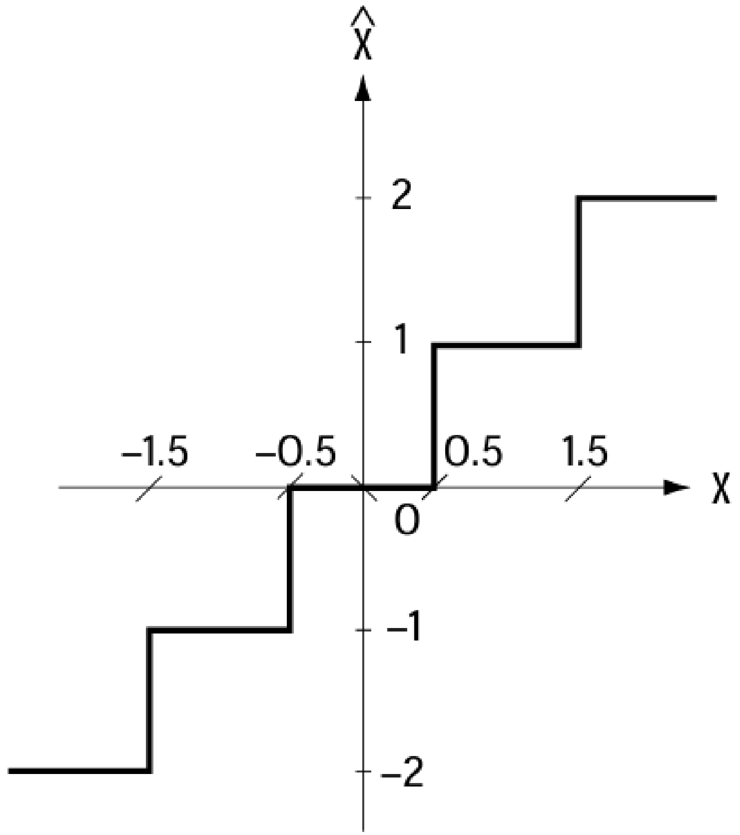


(a)

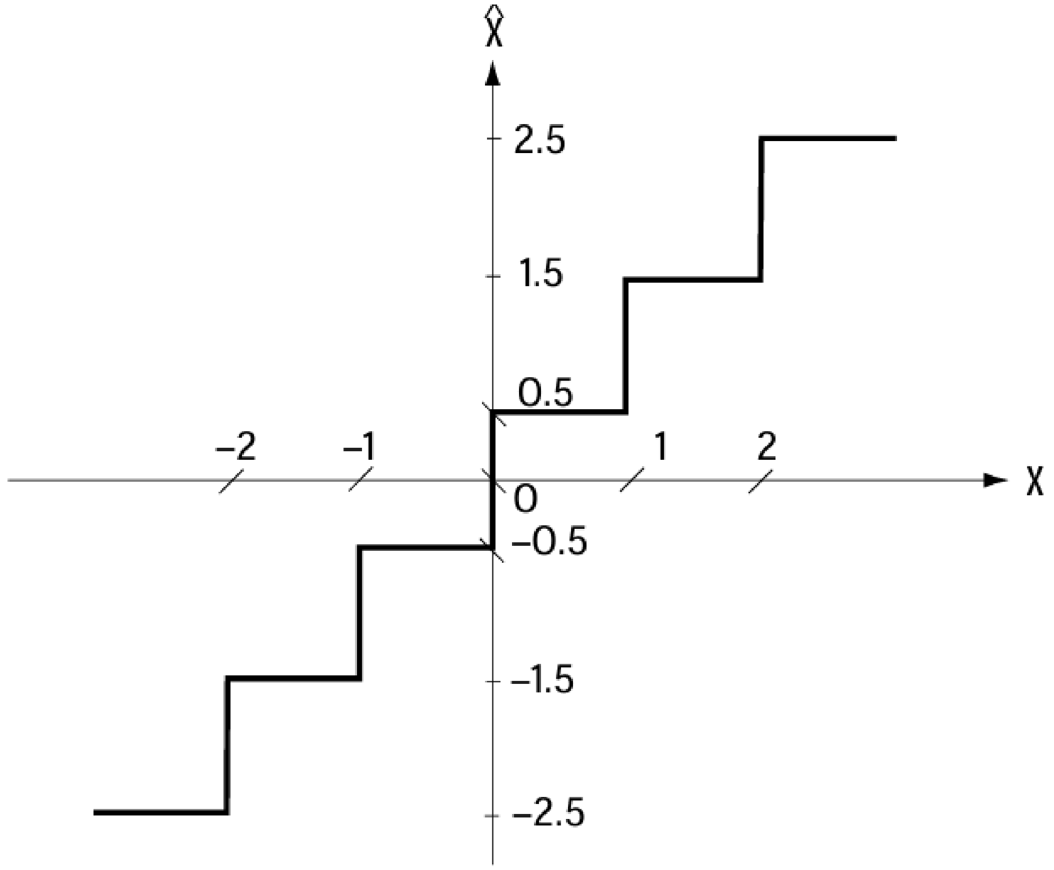


(b)

3.2 Quantization

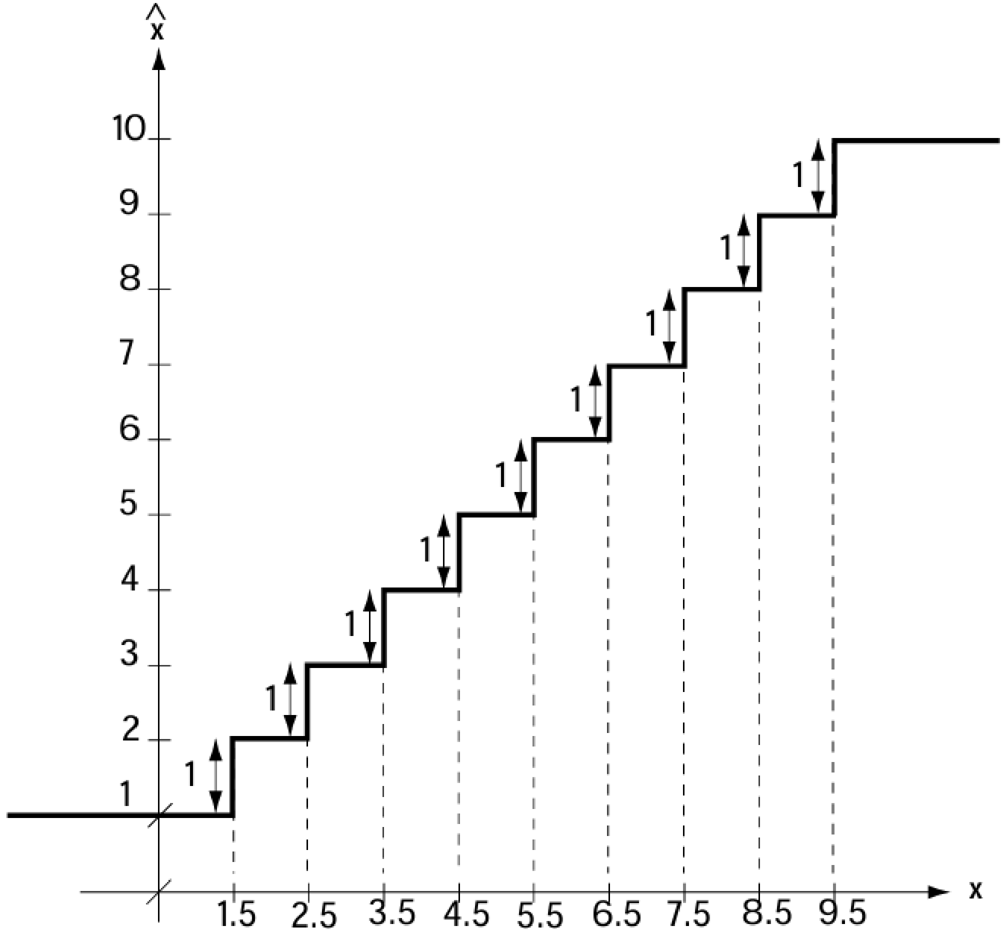


mid-tread quantizer

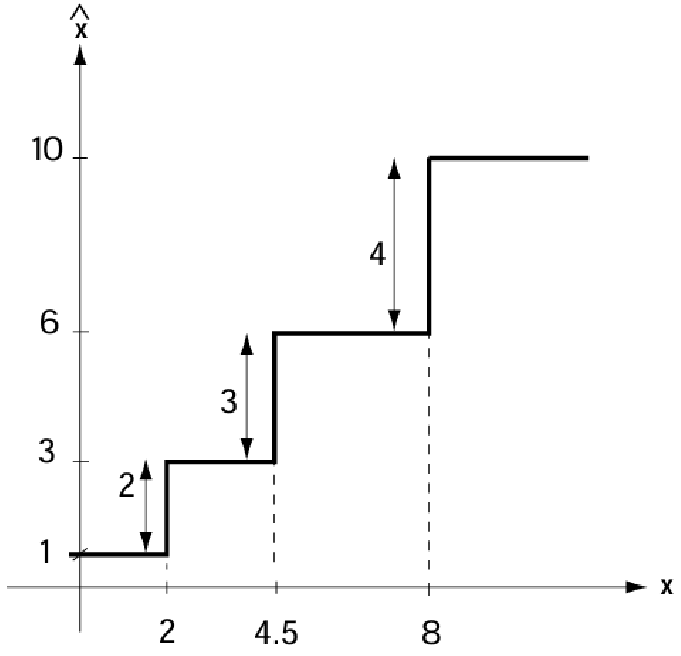


mid-riser quantizer

3.2 Quantization



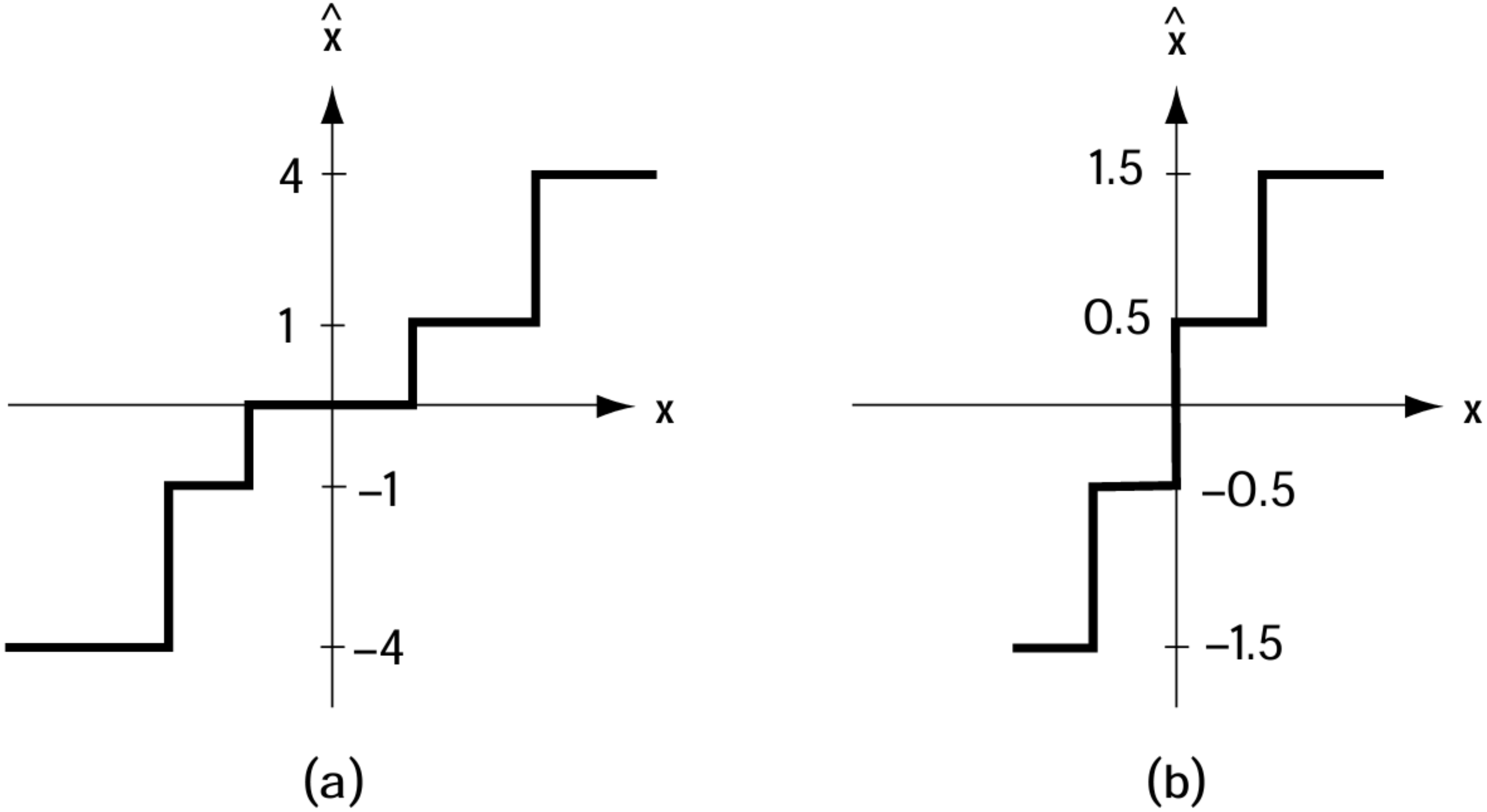
uniform quantizer



non-uniform quantizer

3.2 Quantization

- Example 3: Looking at the quantizers in Figure, determine if they are mid-tread or mid rise and if they are uniform or non-uniform.



3.2 Quantization

- Measures of Performance

$$e(x) = |\hat{x} - x|$$

$$mse = E\left[(x - \hat{x})^2\right] = \int_{-\infty}^{\infty} (x - \hat{x})^2 \, p_x(x)\,dx$$

$$SQNR = \frac{P_s}{P_e} = \frac{\int_{-\infty}^{\infty} (x - x_m)^2 \, p_x(x)\,dx}{mse}$$

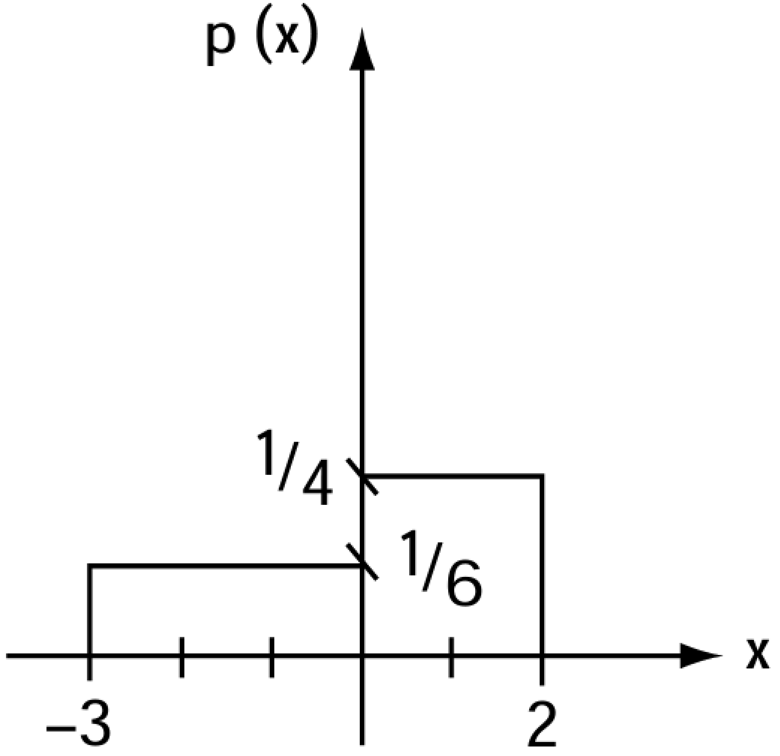
3.2 Quantization

- Example 4: Consider a quantizer with an input described in Figure

(a) Draw a quantizer with 7 levels. Make it mid-tread, let it have –3 as its smallest output value, and make sure that the step size is 1.

(b) Evaluate the *mse* of your quantizer given the input.

(c) Evaluate the SQNR.



3.3 Source Coding

- In this section we'll put samplers and quantizers together to build a source coder.
- Because there are other ways to build source coders, as we'll see later.
- This source coder is given a very particular name—the *pulse code modulator (PCM)*.

$$\text{bit rate} = \text{symbol rate} \times {}^{\#\ \text{of bits}}\!\big/_{\text{symbol}}$$

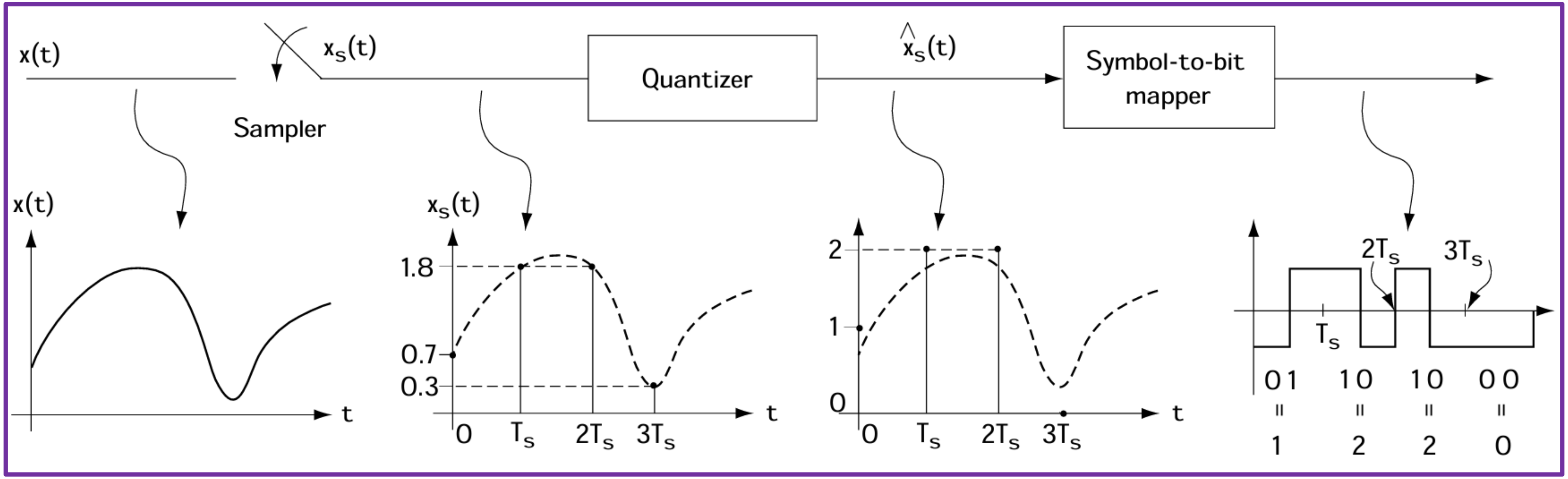
3.3 Source Coding

- Example 5: A computer sends:
  - 100 letters every 4 seconds
  - 8 bits to represent each letter
  - the bits enter a special coding device that takes in a set of bits and puts out one of 32 possible symbols.

What is the bit rate and what is the symbol rate out of the special coding device?

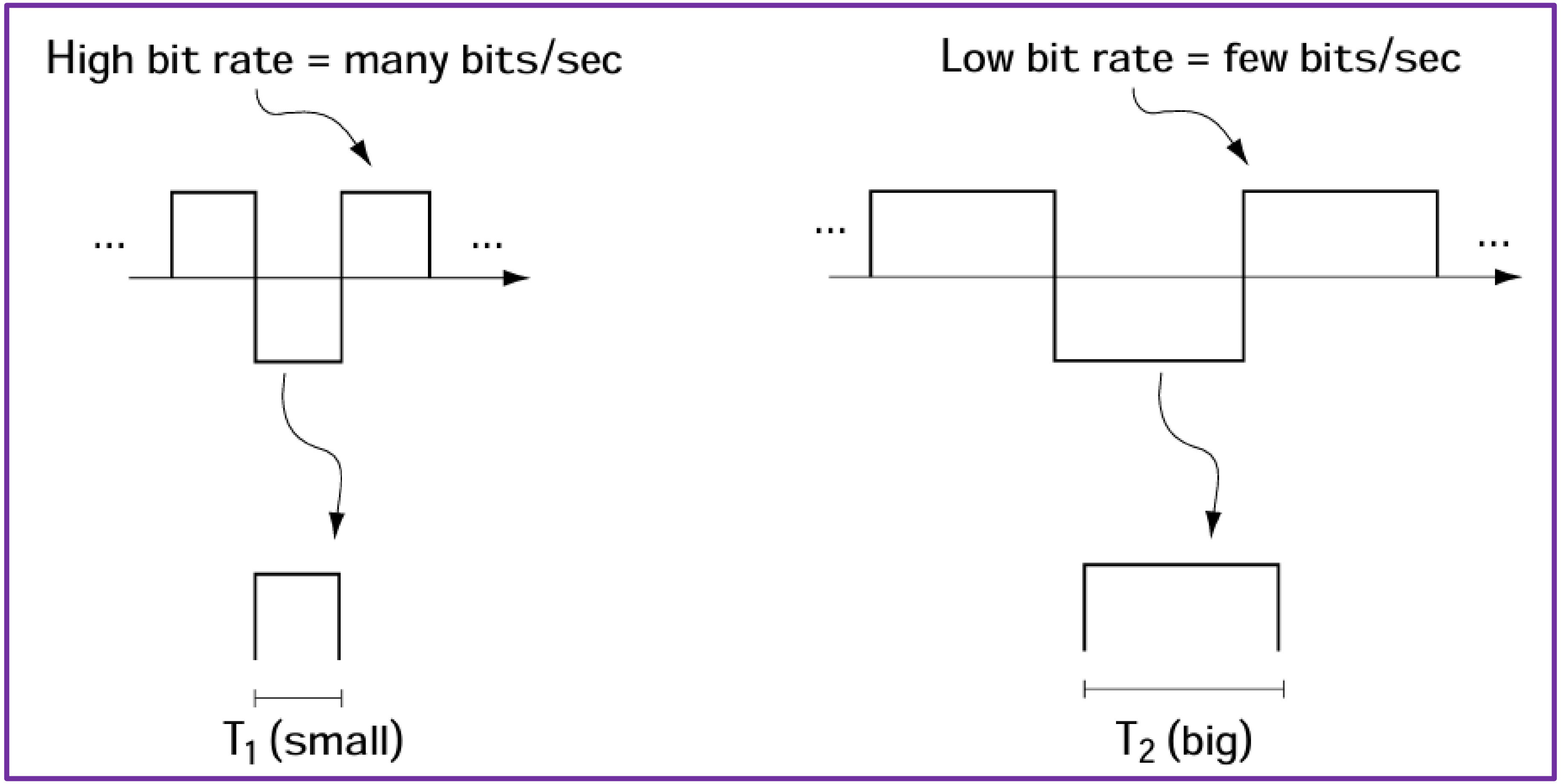
3.3 Source Coding

- Pulse Code Modulator (PCM).



3.3 Source Coding

- Pulse Code Modulator (PCM).



3.3 Source Coding

- Pulse Code Modulator (PCM).