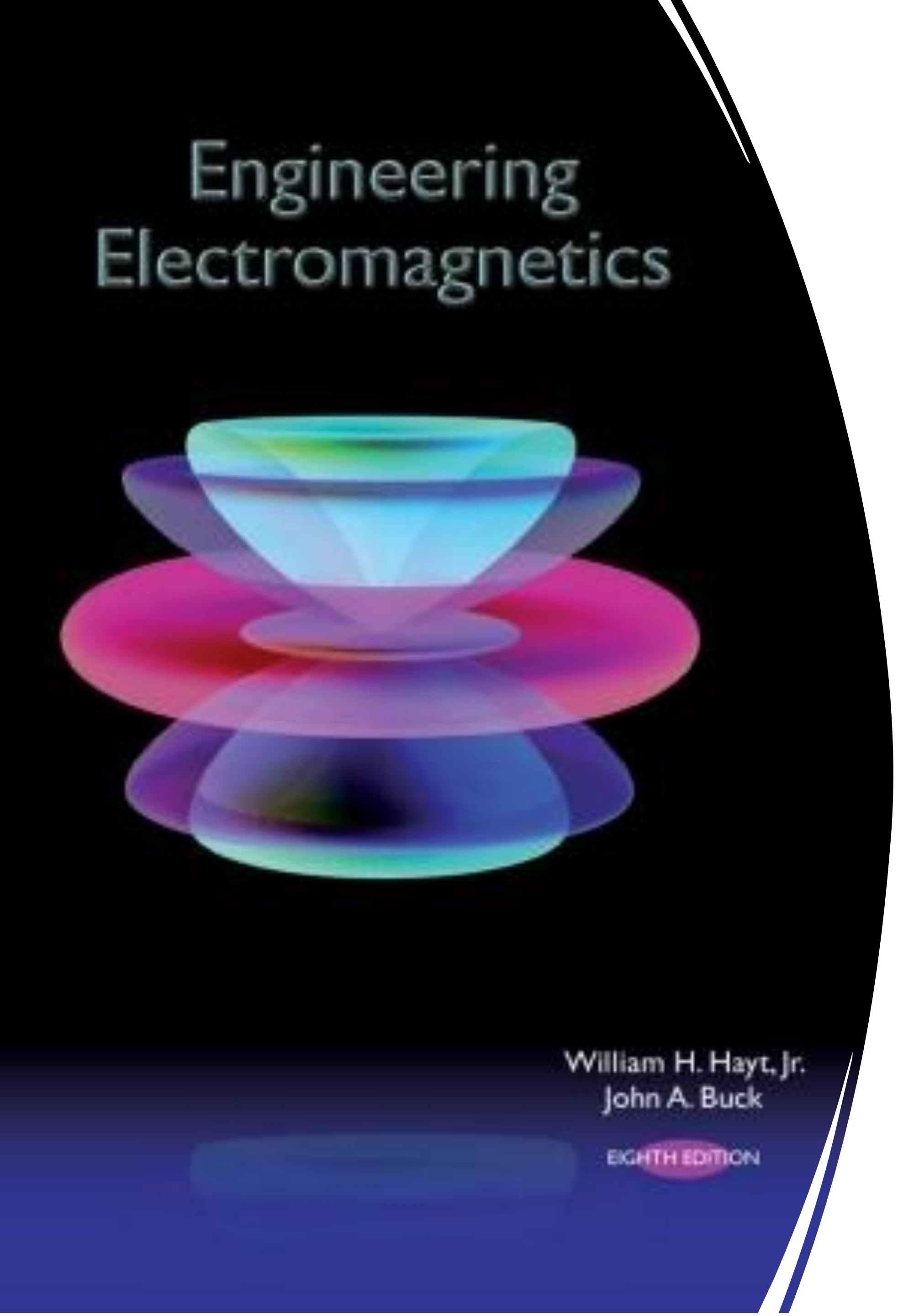


8. The Uniform Plane Wave

8.1 Wave Propagation in Free Space

- When considering electromagnetic waves in free space, we note that the medium is sourceless ($\rho_v = \mathbf{J} = 0$). Under these conditions, Maxwell's equations may be written in terms of **E** and **H** only as

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

8.1 Wave Propagation in Free Space

- Transverse electromagnetic (TEM)

$\mathbf{E} = E_x \mathbf{a}_x$, or that the electric field is polarized in the x direction. The curl of $\mathbf{E}$ reduces to a single term:

$$\nabla \times \mathbf{E} = \frac{\partial E_x}{\partial z} \mathbf{a}_y = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} = -\mu_0 \frac{\partial H_y}{\partial t} \mathbf{a}_y$$

The directions of $\mathbf{E}$ and $\mathbf{H}$ and the direction of travel are mutually orthogonal. Using the y-directed magnetic field, and the fact that it varies only in z

$$\nabla \times \mathbf{H} = -\frac{\partial H_y}{\partial z} \mathbf{a}_x = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \epsilon_0 \frac{\partial E_x}{\partial t} \mathbf{a}_x$$

8.1 Wave Propagation in Free Space

- Transverse electromagnetic (TEM)

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu_0 \frac{\partial^2 H_y}{\partial t \partial z} \qquad \frac{\partial^2 H_y}{\partial z \partial t} = -\epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \qquad \frac{\partial^2 H_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2}$$

8.1 Wave Propagation in Free Space

- We further identify the propagation velocity:

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s} = c$$

- The intrinsic impedance of free space is defined as

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \doteq 120\pi \qquad \Omega$$

8.1 Wave Propagation in Free Space

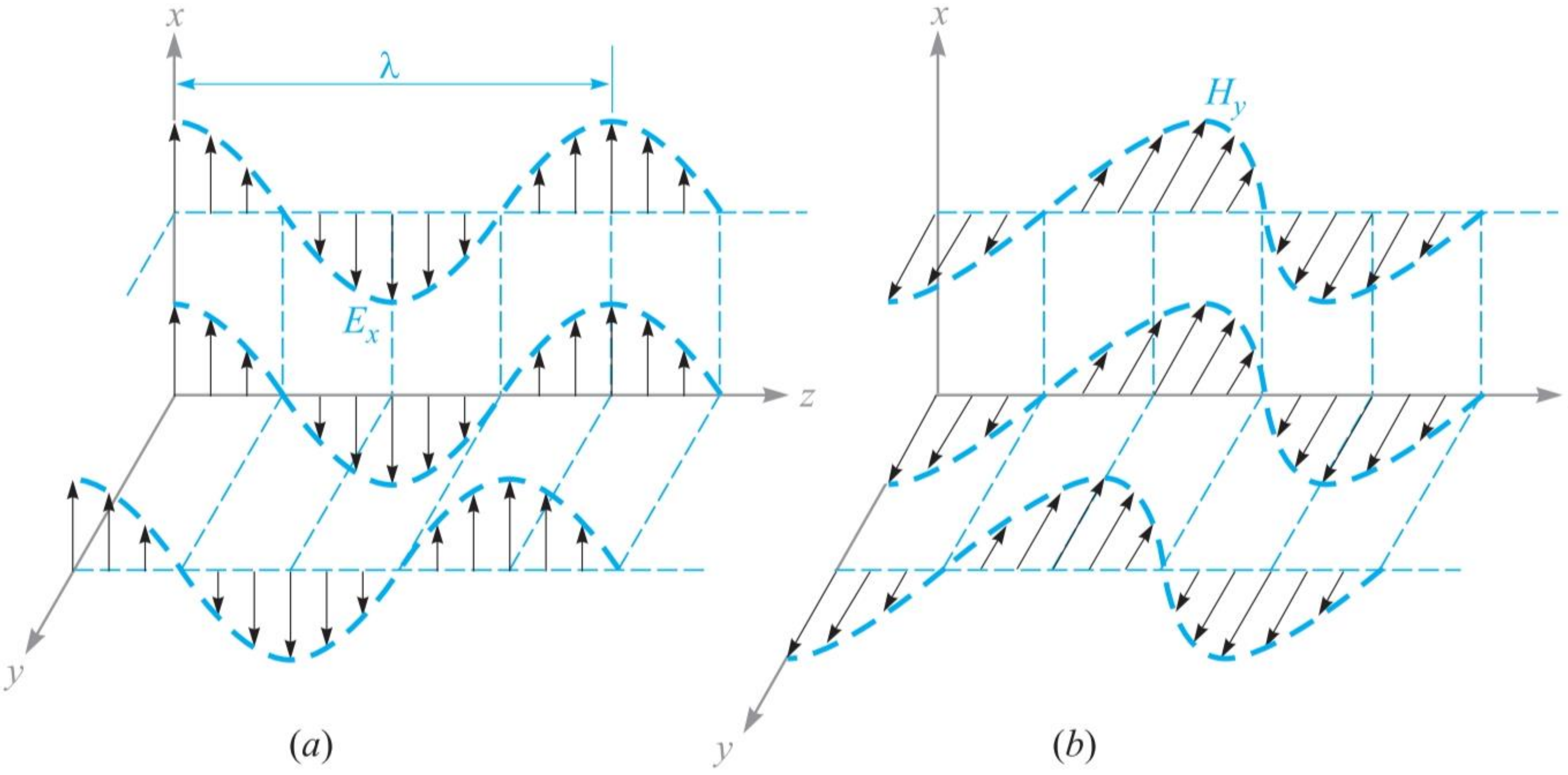
- The equations of the forward- and backward-propagating:

$$E_x(z,t) = f_1(t - z/v) + f_2(t + z/v)$$

where again f_1 and f_2 can be any function whose argument is of the form $t \pm z/v$.

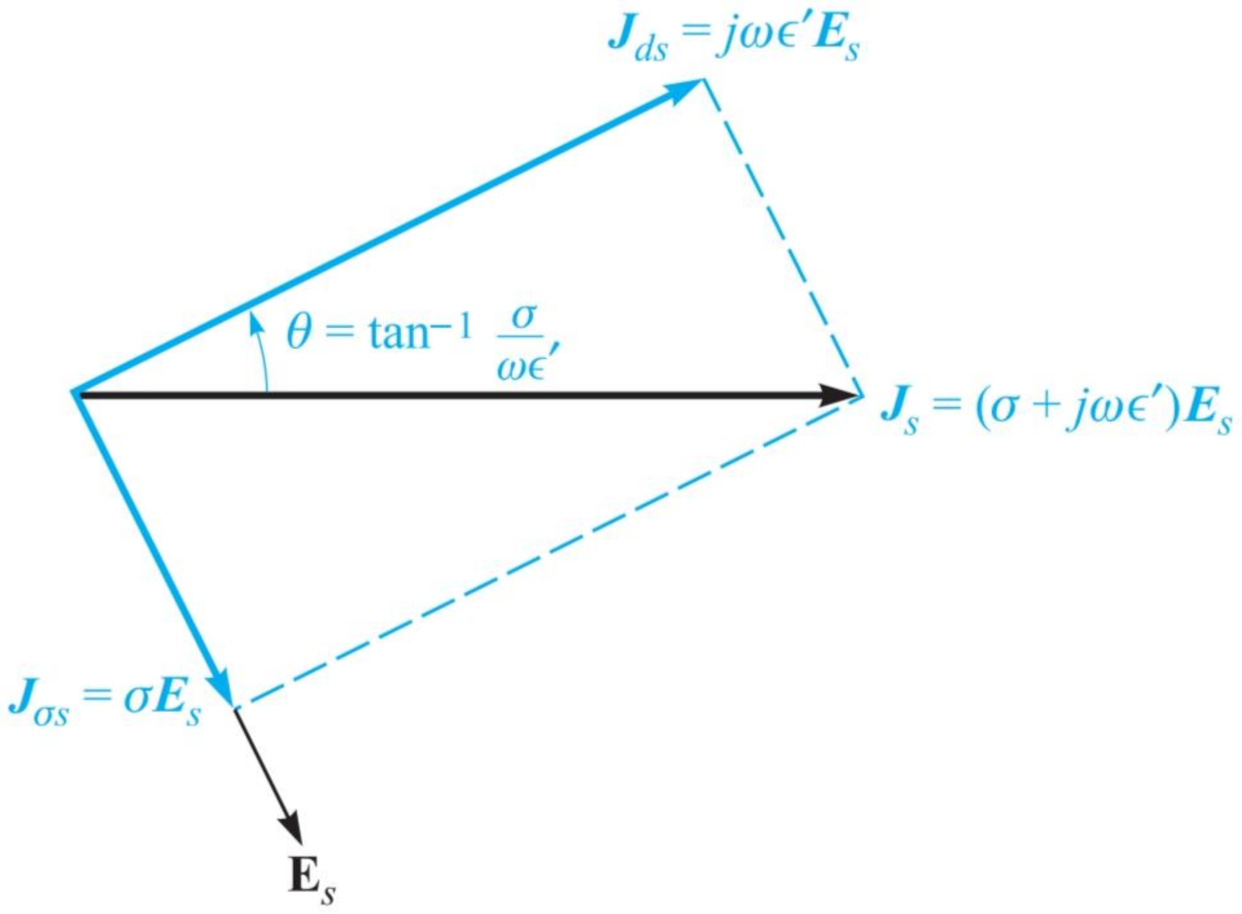
$$\begin{aligned} E_x(z,t) &= \mathcal{E}_x(z,t) + \mathcal{E}'_x(z,t) \\ &= |E_{x0}| \cos[\omega(t - z/v_p) + \phi_1] + |E'_{x0}| \cos[\omega(t + z/v_p) + \phi_2] \\ &= \underbrace{|E_{x0}| \cos[\omega t - k_0 z + \phi_1]}_{\text{forward } z \text{ travel}} + \underbrace{|E'_{x0}| \cos[\omega t + k_0 z + \phi_2]}_{\text{backward } z \text{ travel}} \end{aligned}$$

8.1 Wave Propagation in Free Space



8.2 Wave Propagation in Dielectrics

- The time-phase relationship between J_{ds}, $J_{\sigma s}$, J_s, and E_s. The tangent of θ is equal to $\sigma/\omega\epsilon'$, and $90° - \theta$ is the common power-factor angle, or the angle by which J_s leads E_s.



8.3 Poynting's Theorem and Wave Power

- The total power flowing out of the volume is

$$\oint_{\text{area}} (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} \qquad \text{W}$$

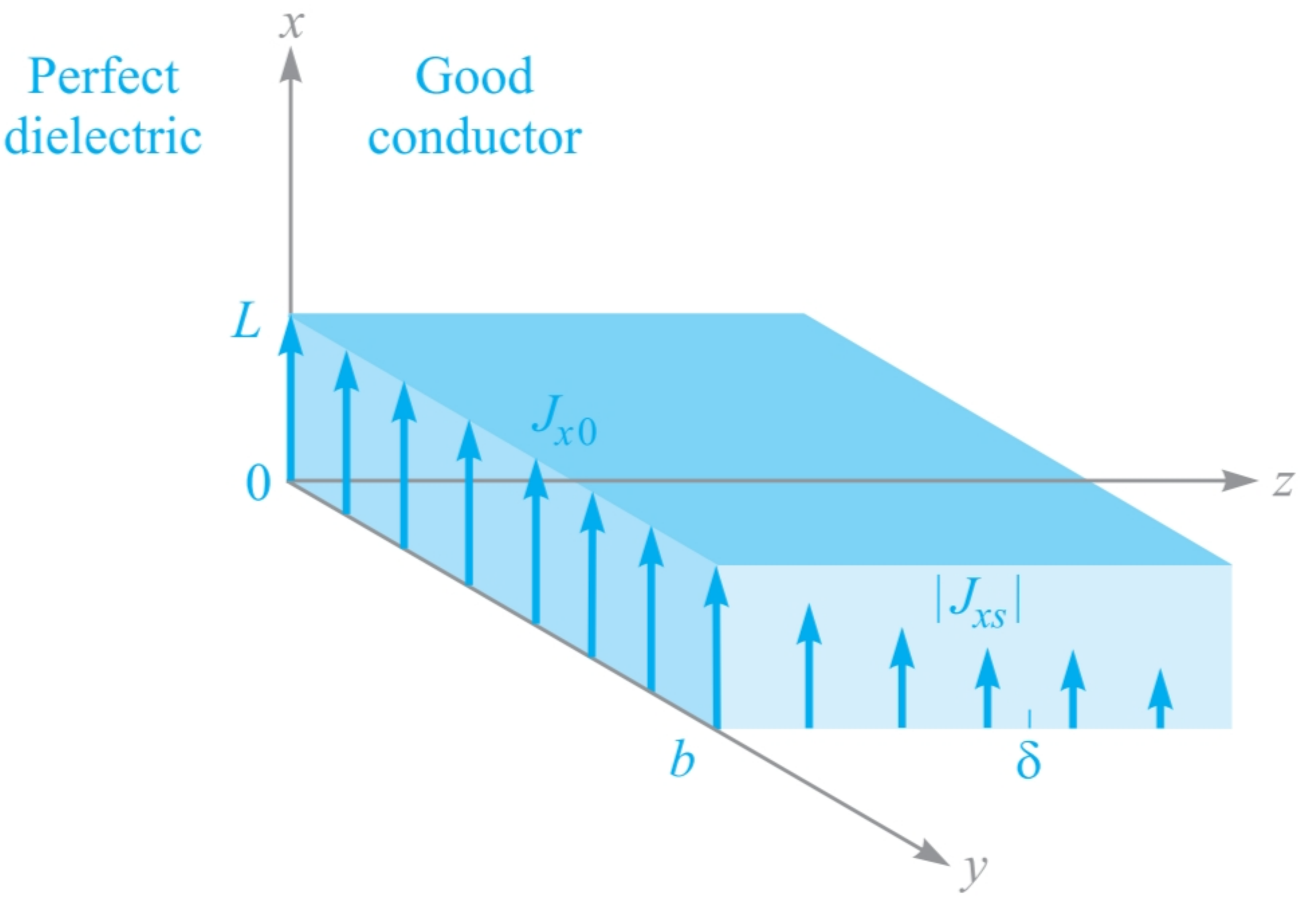
where the integral is over the closed surface surrounding the volume. The cross product $\mathbf{E} \times \mathbf{H}$ is known as the Poynting vector, $\mathbf{S}$,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \qquad \text{W/m}^2$$

which is interpreted as an instantaneous power density, measured in watts per square meter (W/m^2). The direction of the vector $\mathbf{S}$ indicates the direction of the instantaneous power flow at a point, and many of us think of the Poynting vector as a "pointing" vector.

8.4 Propagation in Good Conductors: Skin Effect

- The current density $J_x = J_{x0} e^{-z/\delta} e^{-jz/\delta}$ decreases in magnitude as the wave propagates into the conductor. The average power loss in the region $0 < x < L, 0 < y < b, z > 0$, is $\delta b L J_{x0}^2 / 4\sigma$ watts.



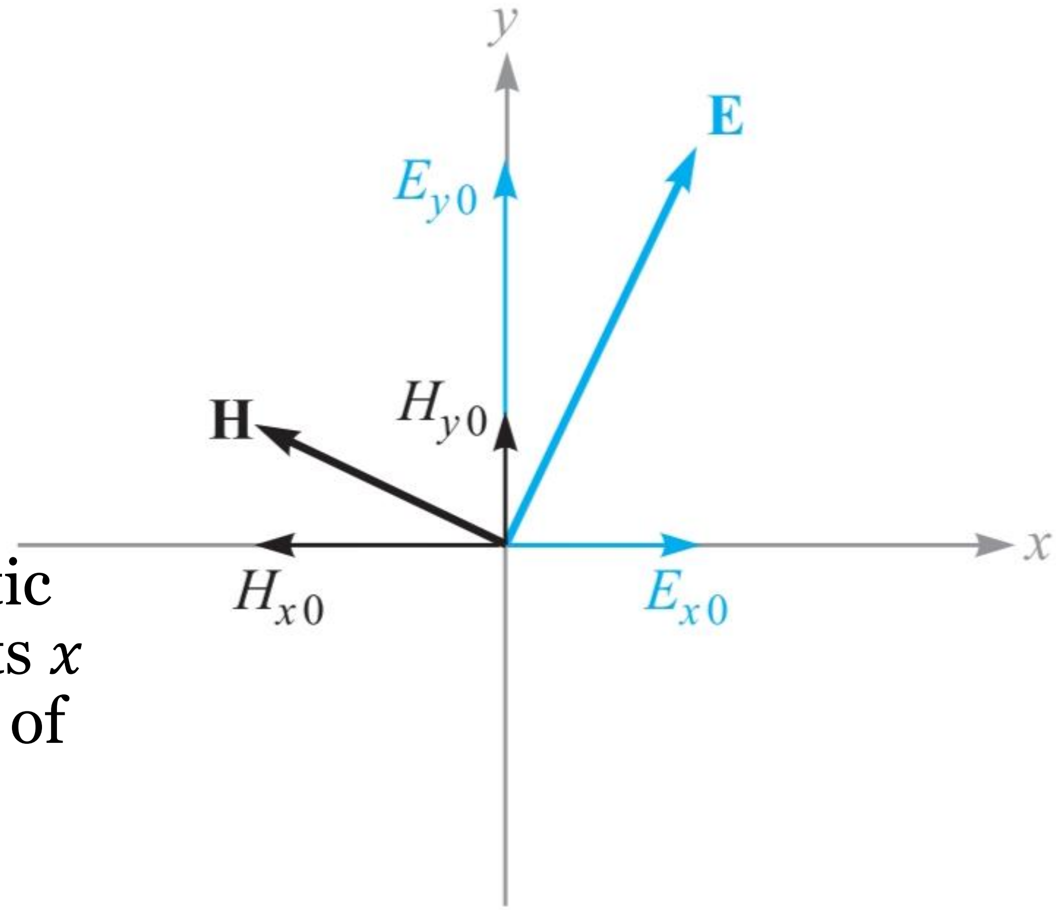
8.5 Wave Polarization

- Electric and magnetic field configuration for a general linearly polarized plane wave propagating in the forward z direction

$$\mathbf{E}_s = \left(E_{x0}\mathbf{a}_x + E_{y0}\mathbf{a}_y\right)e^{-\alpha z}e^{-j\beta z}$$

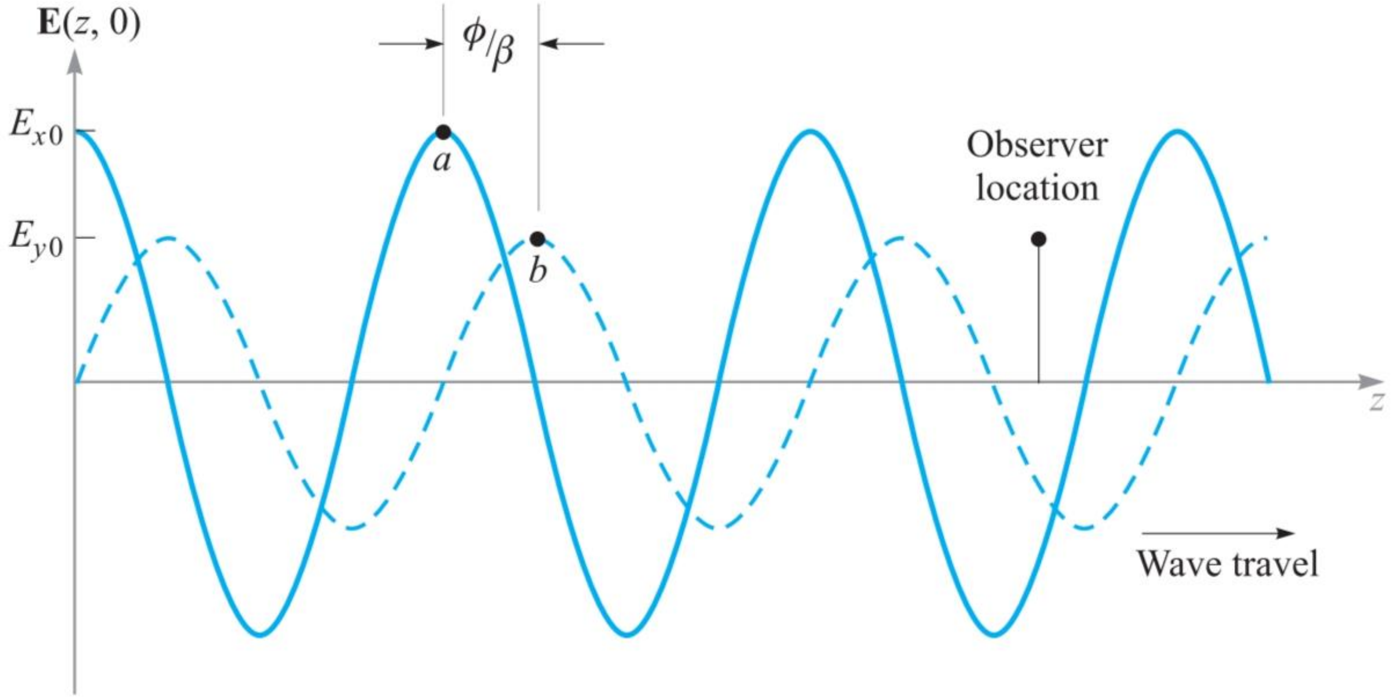
where E_{x0} and E_{y0} are constant amplitudes along x and y. The magnetic field is readily found by determining its x and y components directly from those of $\mathbf{E}_s$. Specifically, $\mathbf{H}_s$ for the wave is

$$\mathbf{H}_s = \left[H_{x0}\mathbf{a}_x + H_{y0}\mathbf{a}_y\right]e^{-\alpha z}e^{-j\beta z}$$



8.5 Wave Polarization

- Plots of the electric field component magnitudes as functions of z. Note that the y component lags behind the x component in z. As time increases from zero, both waves travel to the right. Thus, to an observer at a fixed location, the y component leads in time.



8.5 Wave Polarization

- Representation of a right circularly polarized wave. The electric field vector (in white) will rotate toward the *y* axis as the entire wave moves through the *xy* plane in the direction of k. This counterclockwise rotation (when looking toward the wave source) satisfies the temporal right-handed rotation convention as described in the text. The wave, however, appears as a left-handed screw, and for this reason it is called left circular polarization in the other convention.