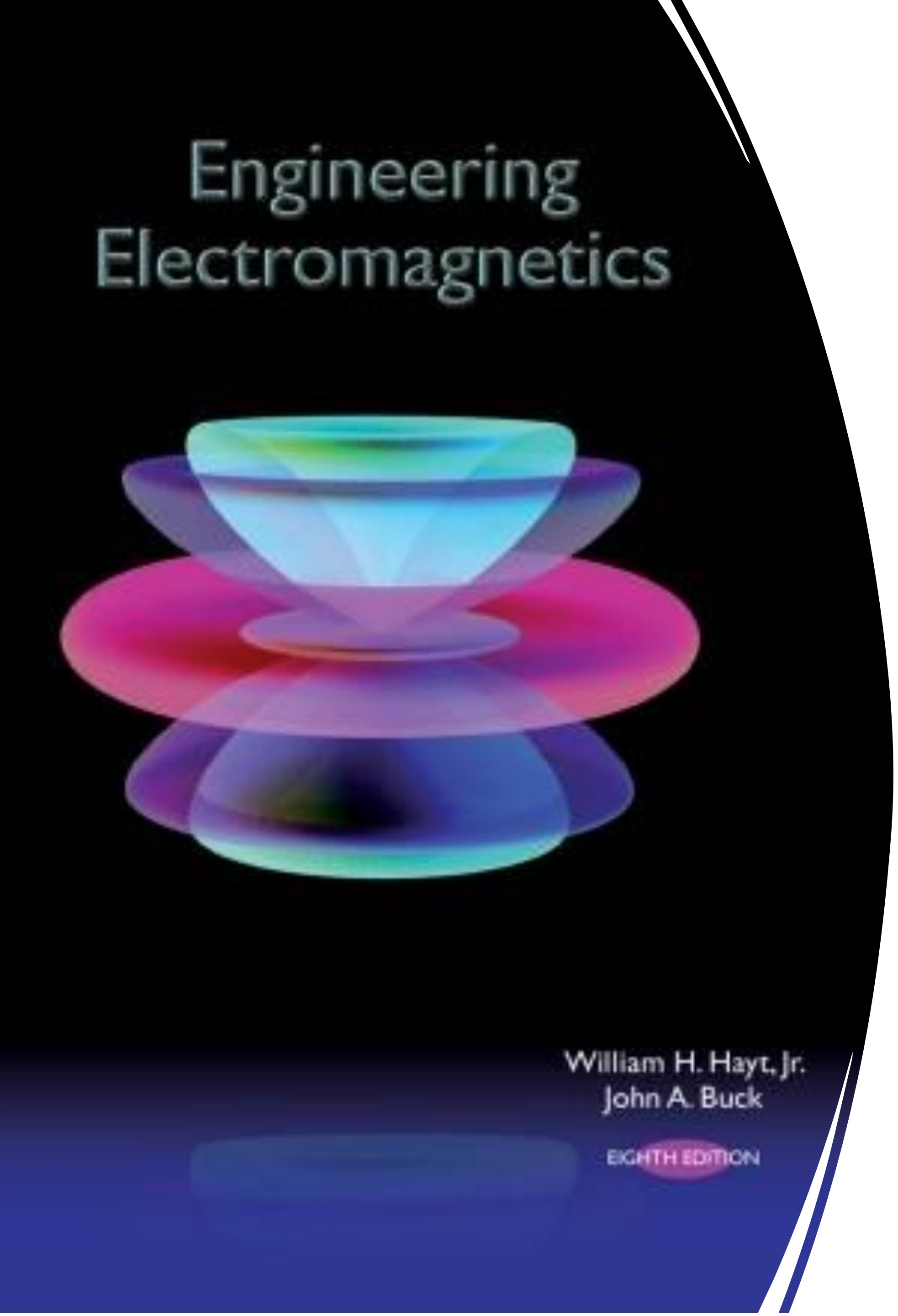


2. Coulomb's Law and Electric Field Intensity

2.1 The Experimental Law of Coulomb

- Coulomb stated that the force between two very small objects separated in a vacuum or free space by a distance

$$F = k\frac{Q_1 Q_2}{R^2} \qquad k = \frac{1}{4\pi\epsilon_0} \qquad \epsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi} 10^{-9}\ \mathrm{F/m}$$

- The coulomb is an extremely large unit of charge, for the smallest known quantity of charge is that of the electron (negative) or proton (positive), given in SI units as 1.602×10^{-19}C; hence a negative charge of one coulomb represents about 6×10^{18} electrons.
- Coulomb's law shows that the force between two charges of one coulomb each, separated by one meter, is 9×10^{9} N, or about one million tons.

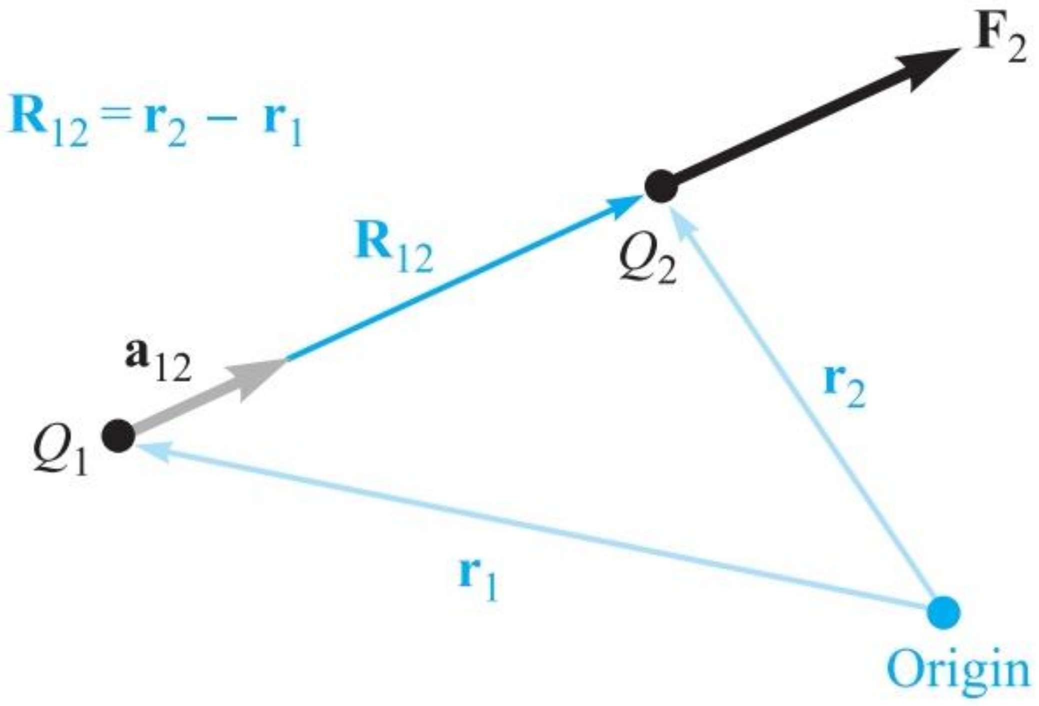
2.1 The Experimental Law of Coulomb

- The force acts along the line joining the two charges and is repulsive if the charges are alike in sign or attractive if they are of opposite sign.
- Let the vector $\mathbf{r}_1$ locate Q_1, whereas $\mathbf{r}_2$ locates Q_2. Then the vector $\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1$ represents the directed line segment from Q_1 to Q_2.
- The vector $\mathbf{F}_2$ is the force on Q_2 and is shown for the case where Q_1 and Q_2 have the same sign. The vector form of Coulomb's law is

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}$$

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

2.1 The Experimental Law of Coulomb



- If Q_1 and Q_2 have like signs, the vector force F_2 on Q_2 is in the same direction as the vector R_{12}.

2.1 The Experimental Law of Coulomb

A charge $Q_A = -20\mu\text{C}$ is located at $A(-6,4,7)$, and a charge $Q_B = 50\mu\text{C}$ is at $B(5,8,-2)$ in free space. If distances are given in meters, find: (a) $\mathbf{R}_{AB}$; (b) R_{AB}. Determine the vector force exerted on Q_A by Q_B if $\epsilon_0 =$ (c) $10^{-9}/(36\pi)\text{F/m}$; $(d) 8.854 \times 10^{-12}$ F/m.

Ans. $11\mathbf{a}_x + 4\mathbf{a}_y - 9\mathbf{a}_z$ m; 14.76 m; $30.76\mathbf{a}_x + 11.184\mathbf{a}_y - 25.16\mathbf{a}_z$mN; $30.72\mathbf{a}_x + 11.169\mathbf{a}_y - 25.13\mathbf{a}_z$mN

2.2 Electric Field Intensity

- A force field that is associated with charge, Q_1. Call this second charge a test charge Q_t. The force on it is given by Coulomb's law

$$\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

- Writing this force as a force per unit charge gives the electric field intensity, $\mathbf{E}_1$ arising from Q_1 :

$$\mathbf{E}_1 = \frac{\mathbf{F}_t}{Q_1} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

- The electric field of a single point charge becomes:

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2}\mathbf{a}_R$$

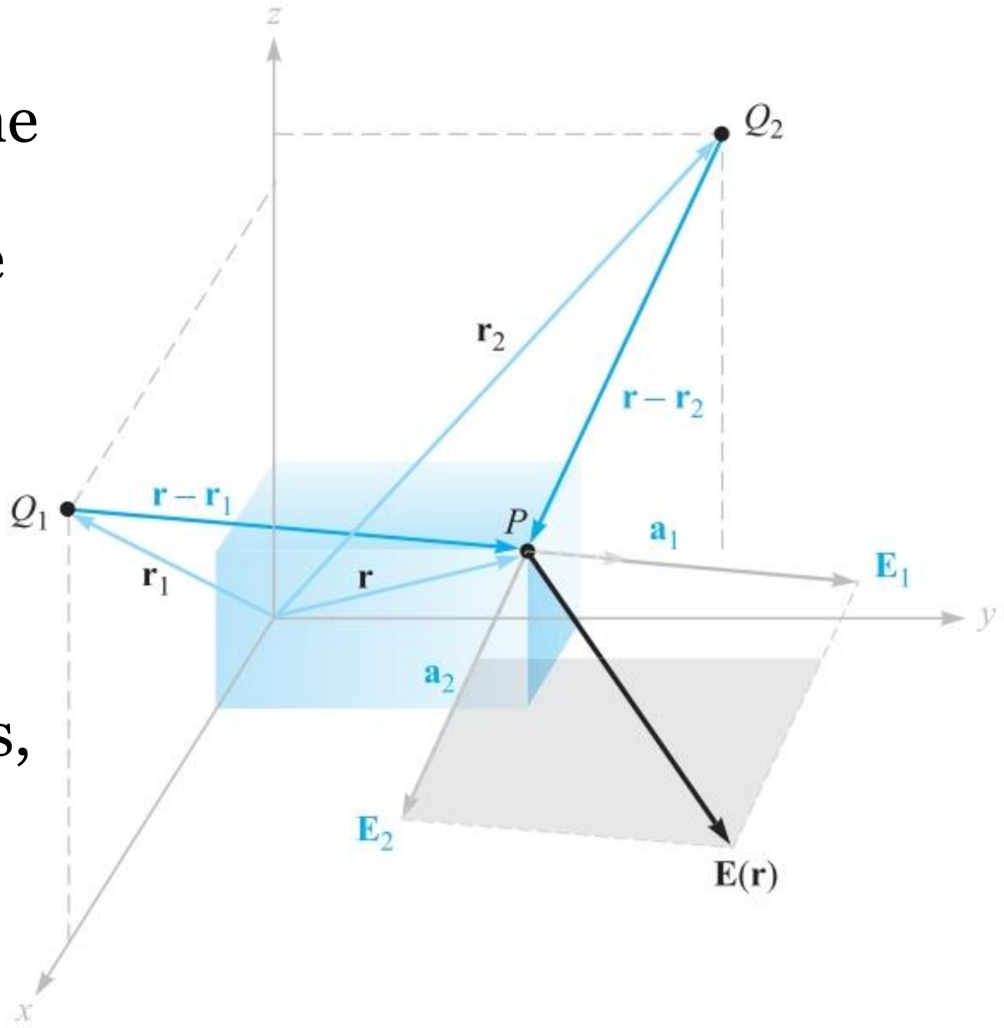
2.2 Electric Field Intensity

- Because the coulomb forces are linear, the electric field intensity arising from two point charges, Q_1 at $\mathbf{r}_1$ and Q_2 at $\mathbf{r}_2$, is the sum of the forces on Q_t caused by Q_1 and Q_2 acting alone

$$\mathbf{E}(\mathbf{r}) = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$

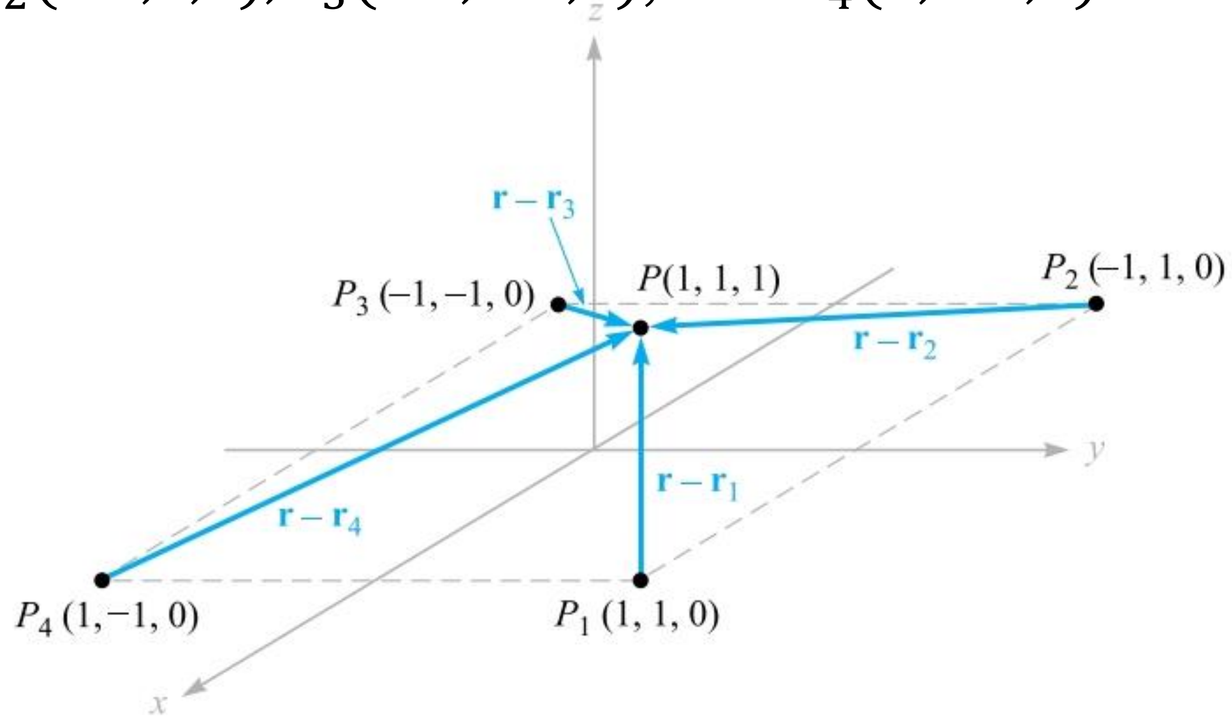
- If we add more charges at other positions, the field due to n point charges is

$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_m|^2}\mathbf{a}_m$$



2.2 Electric Field Intensity

Find $\mathbf{E}$ at $P(1,1,1)$ caused by four identical 3-nC charges located at $P_1(1,1,0), P_2(-1,1,0), P_3(-1,-1,0)$, and $P_4(1,-1,0)$



Ans. $\mathbf{E} = 6.82\mathbf{a}_x + 6.82\mathbf{a}_y + 32.8\mathbf{a}_z$ V/m

2.3 Field Arising from a Continuous Volume Charge Distribution

- We denote volume charge density by ρ_ν, having the units of coulombs per cubic meter $(\mathrm{C/m^3})$.The small amount of charge ΔQ in a small volume Δv is

$$\Delta Q = \rho_\nu \Delta v$$

- and we may define ρ_ν mathematically by using a limiting process,

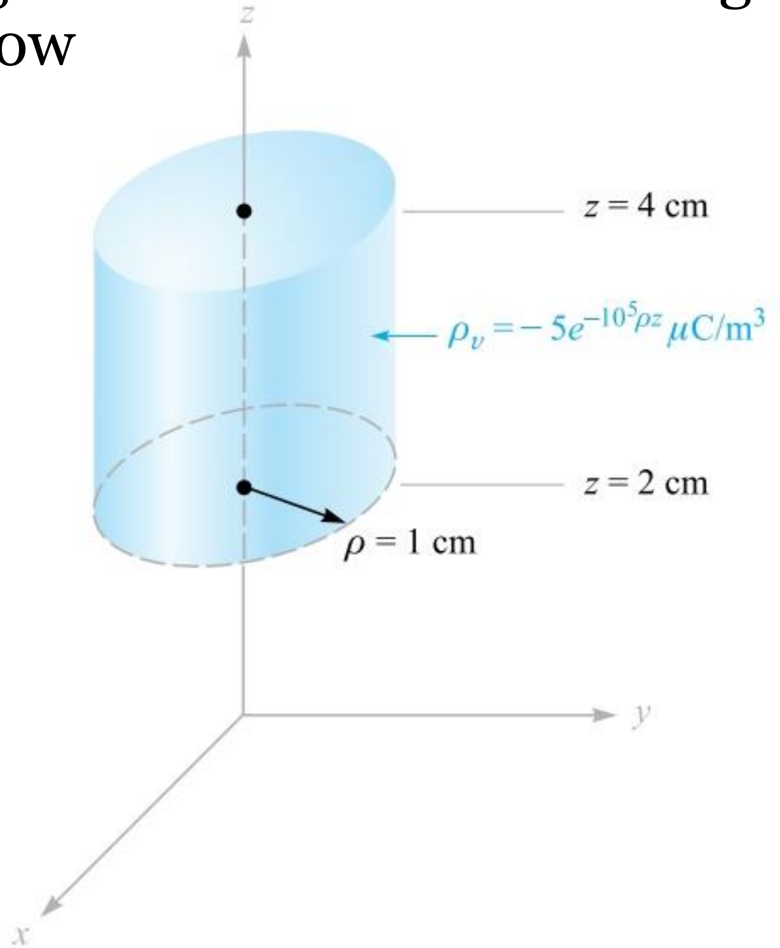
$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

- The total charge within some finite volume is obtained by integrating throughout that volume,

$$Q = \int_{\mathrm{vol}} \rho_\nu dv$$

2.3 Field Arising from a Continuous Volume Charge Distribution

Find the total charge contained in a 2-cm length of the electron beam shown in figure below



Ans. 0.0785pC

2.3 Field Arising from a Continuous Volume Charge Distribution

Calculate the total charge within each of the indicated volumes:

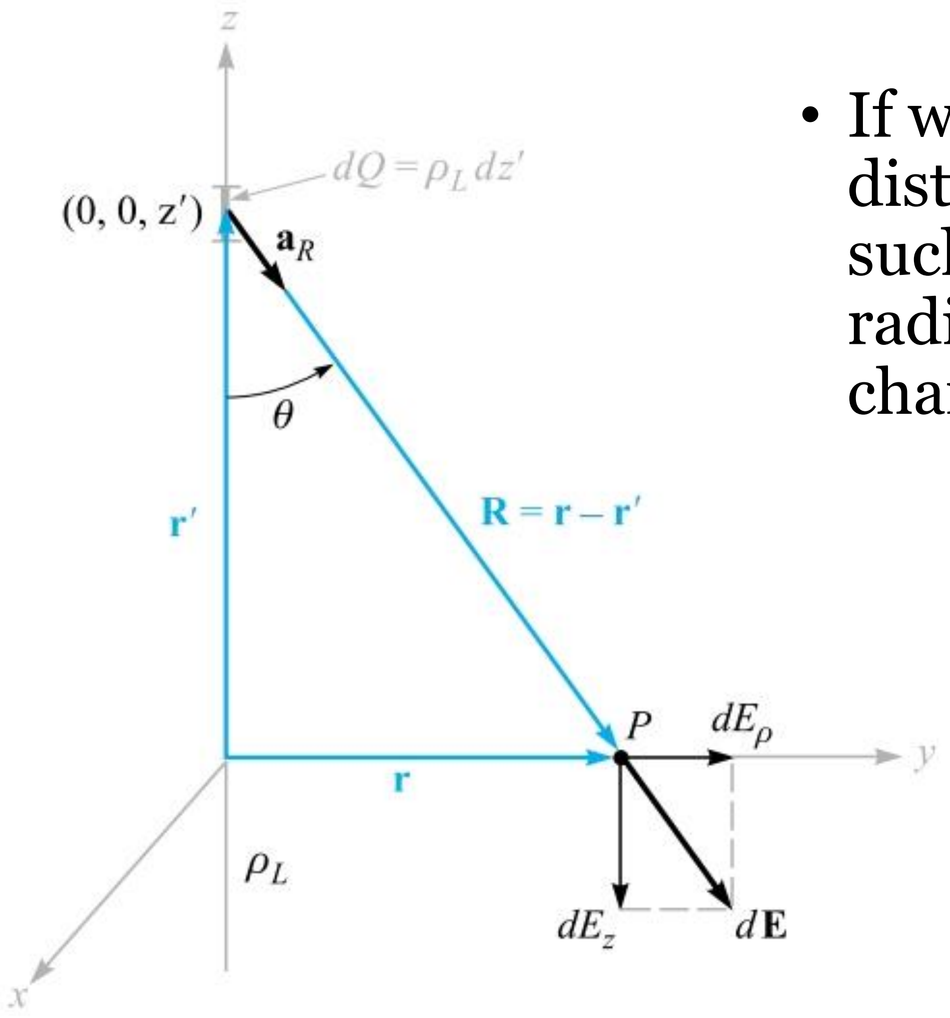
$(a)\ 0.1 \le |x|, |y|, |z| \le 0.2$: $\rho_v = \dfrac{1}{x^3 y^3 z^3}$;

$(b)\ 0 \le \rho \le 0.1, 0 \le \phi \le \pi, 2 \le z \le 4$; $\rho_v = \rho^2 z^2 \sin 0.6\phi$;

(c) universe: $\rho_v = e^{-2r}/r^2$.

Ans. 0; 1.018mC; 6.28C

2.4 Field of a Line Charge



- If we now consider a filamentlike distribution of volume charge density, such as a charged conductor of very small radius, we find it convenient to treat the charge as a line charge of density ρ_L C/m.

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\mathbf{a}_\rho$$

2.4 Field of a Line Charge

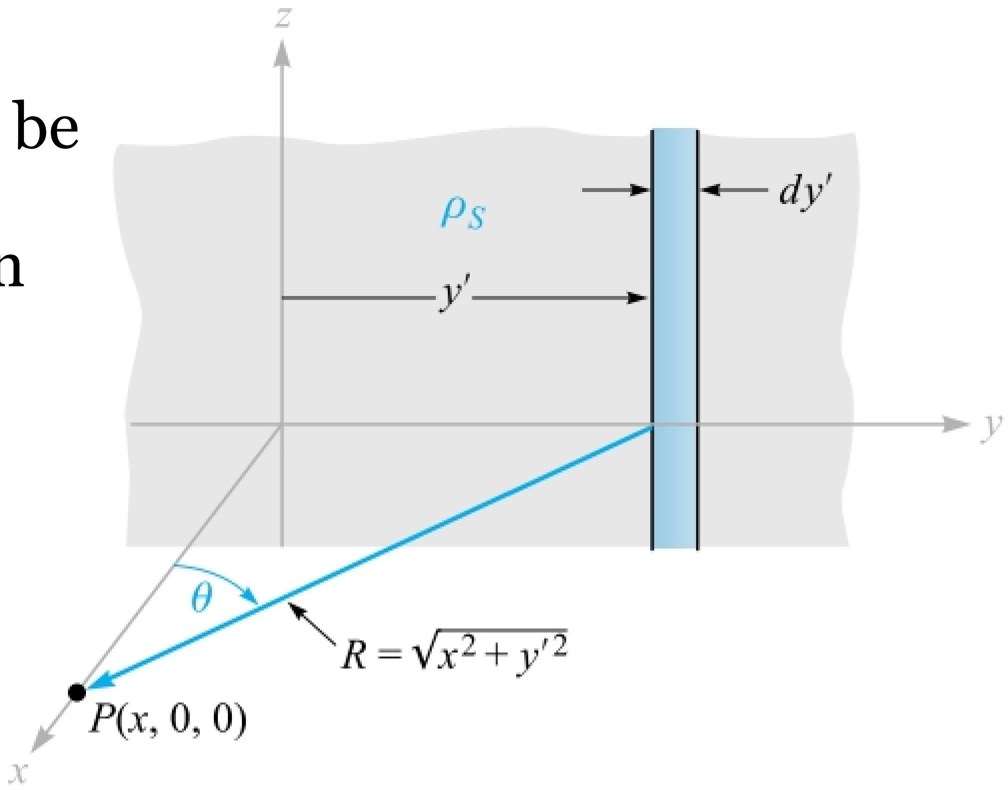
Infinite uniform line charges of 5nC/m lie along the (positive and negative) x and y axes in free space. Find $\mathbf{E}$ at: $(a)P_A(0,0,4)$; $(b)P_B(0,3,4)$.

Ans. $45\mathbf{a}_z$ V/m; $10.8\mathbf{a}_y + 36.9\mathbf{a}_z$ V/m

2.5 Field of a Sheet of Charge

- Another basic charge configuration is the infinite sheet of charge having a uniform density of ρ_S C/m^2.
- Such a charge distribution may often be used to approximate that found on the conductors of a strip transmission line or a parallel-plate capacitor.

$$\mathbf{E} = \frac{\rho_S}{2\epsilon_0}\mathbf{a}_N$$



2.5 Field of a Sheet of Charge

Three infinite uniform sheets of charge are located in free space as follows: 3nC/m^2 at $z = -4, 6\text{nC/m}^2$ at $z = 1$, and -8nC/m^2 at $z = 4$. Find **E** at the point: $(a) P_A(2,5,-5)$; (b) $P_B(4,2,-3)$; (c) $P_C(-1,-5,2)$; (d) $P_D(-2,4,5)$.

Ans. $45\mathbf{a}_z$ V/m; $10.8\mathbf{a}_y + 36.9\mathbf{a}_z$ V/m

Problems

1. Point charges of 50nC each are located at $A(1,0,0), B(-1,0,0), C(0,1,0)$, and $D(0,-1,0)$ in free space. Find the total force on the charge at A.
2. Let a point charge $Q_1 = 25\text{nC}$ be located at $P_1(4,-2,7)$ and a charge $Q_2 = 60\text{nC}$ be at $P_2(-3,4,-2)$. (a) If $\epsilon = \epsilon_0$, find **E** at $P_3(1,2,3)$. (b) At what point on the y axis is $E_x = 0$?
3. Find **E** at the origin if the following charge distributions are present in free space: point charge, 12nC, at $P(2,0,6)$; uniform line charge density, 3nC/m, at $x = -2, y = 3$; uniform surface charge density, 0.2nC/m^2 at $x = 2$.