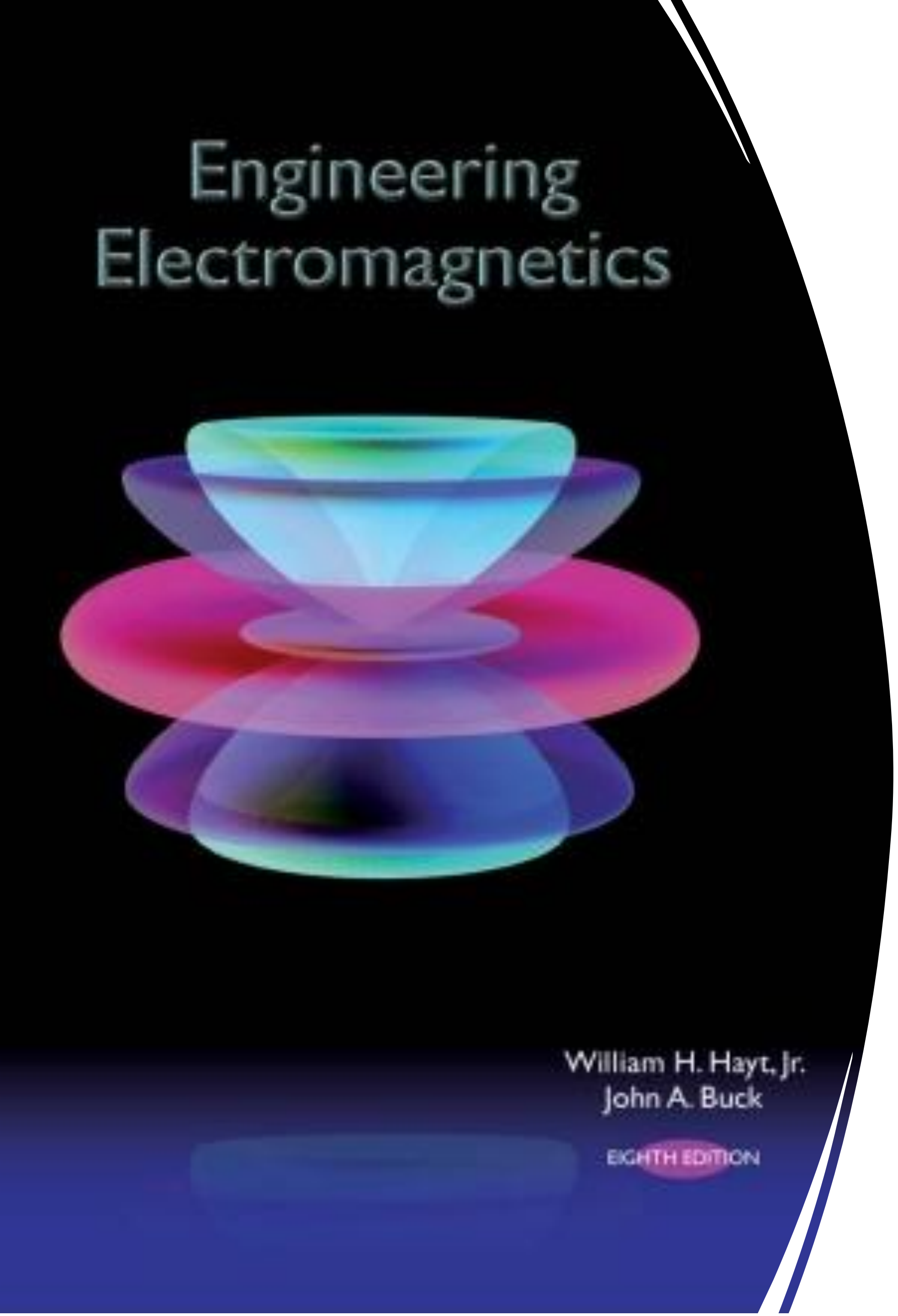


5. The Steady Magnetic Field

5.1 Biot-Savart Law

5.2 Ampere's Circuital Law

5.3 Curl

5.4 Stokes' Theorem

5.5 Magnetic Flux and Magnetic Flux Density

5.1 Biot-Savart Law

- The Biot-Savart law may be written concisely using vector notation as

$$d\mathbf{H} = \frac{I d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \frac{I d\mathbf{L} \times \mathbf{R}}{4\pi R^3}$$

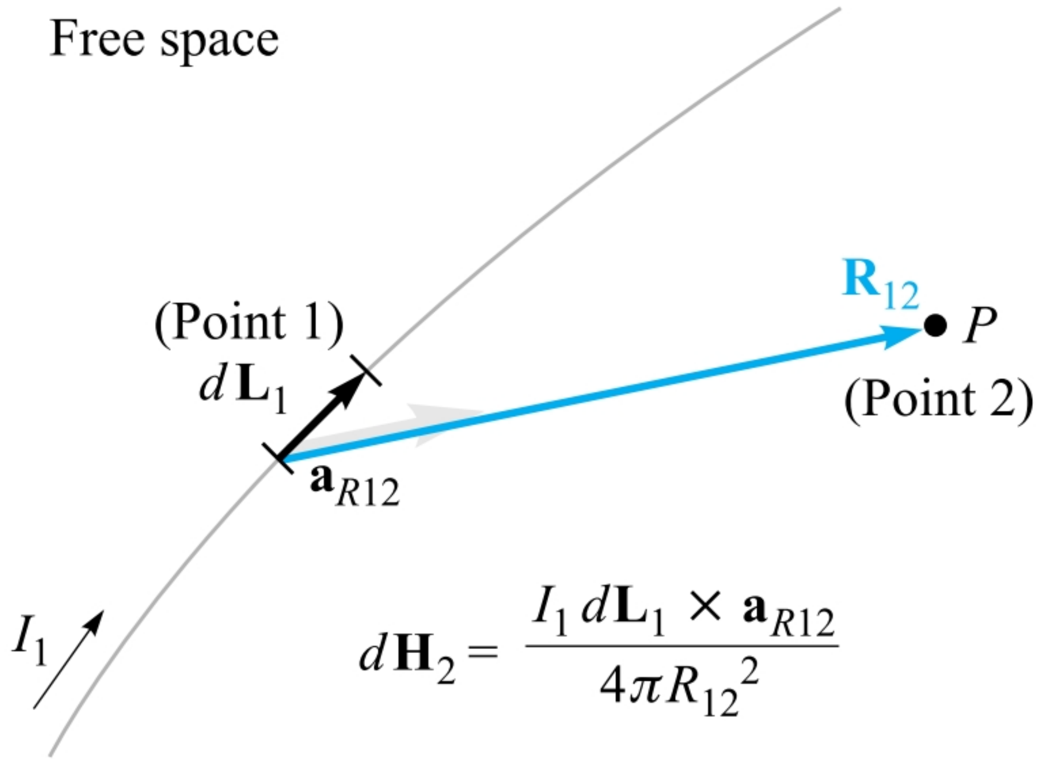
The units of the magnetic field intensity **H** are evidently amperes per meter (A/m).

- If we locate the current element at point 1 and describe the point P at which the field is to be determined as point 2, then

$$d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}$$

5.1 Biot-Savart Law

- The law of Biot-Savart expresses the magnetic field intensity $d\text{H}_2$ produced by a differential current element $I_1 d\ \text{L}_1$. The direction of $d\text{H}_2$ is into the page.



$$d\mathbf{H}_2 = \frac{I_1\, d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^{\ 2}}$$

5.1 Biot-Savart Law

- It follows that only the integral form of the Biot-Savart law can be verified experimentally,

$$\mathbf{H} = \oint \frac{I d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$$

5.2 Ampere's Circuital Law

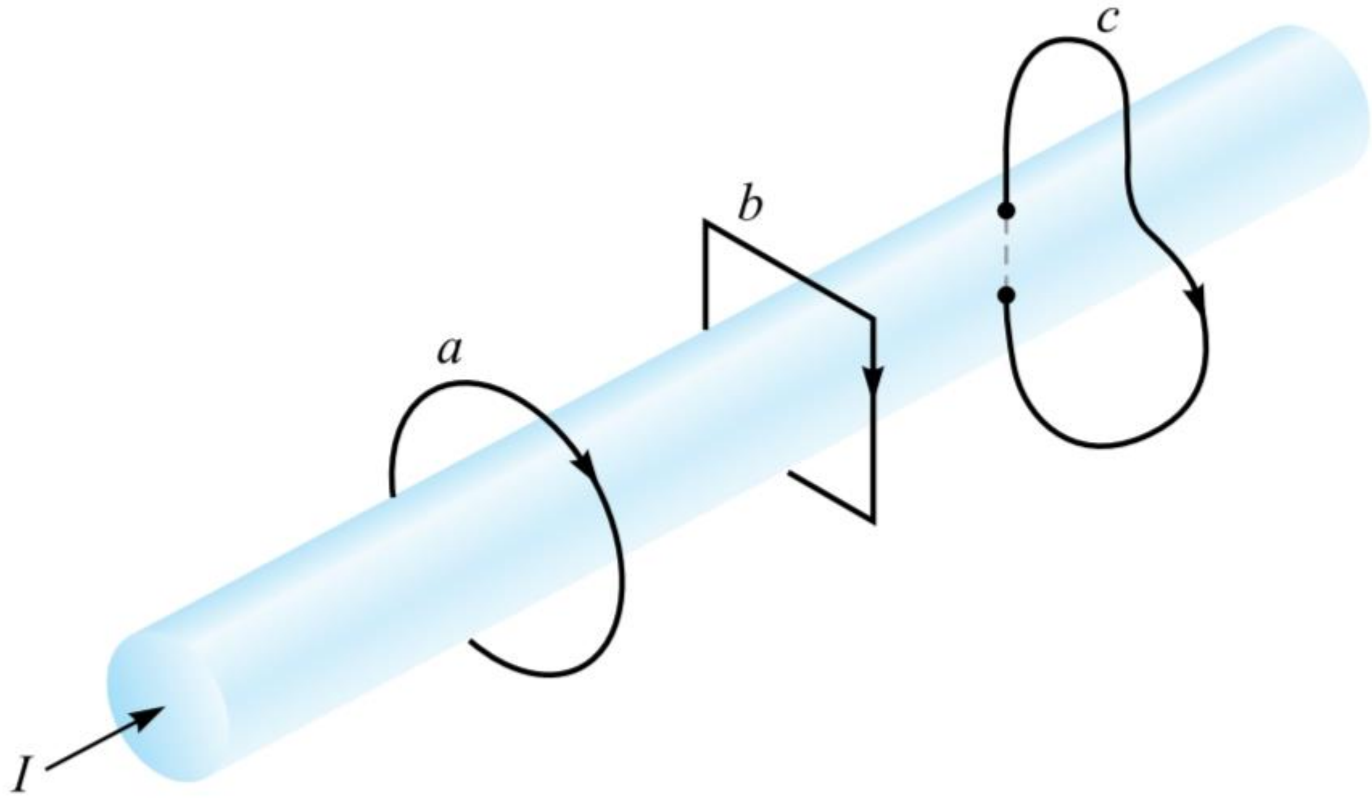
- Ampère's circuital law states that the line integral of **H** about any closed path is exactly equal to the direct current enclosed by that path,

$$\oint \mathbf{H} \cdot d\mathbf{L} = I$$

- We define positive current as flowing in the direction of advance of a right-handed screw turned in the direction in which the closed path is traversed.

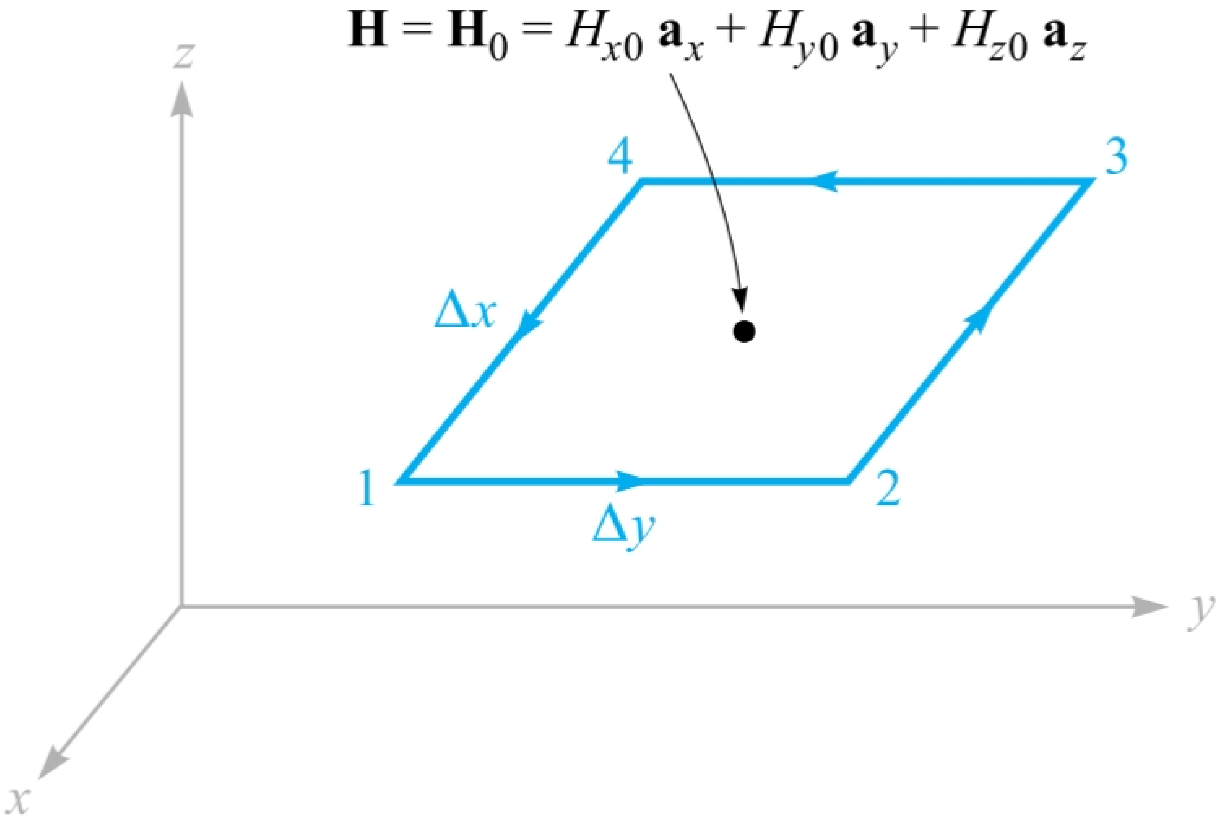
5.2 Ampere's Circuital Law

- A conductor has a total current I. The line integral of H about the closed paths a and b is equal to I, and the integral around path c is less than I, since the entire current is not enclosed by the path.



5.2 Ampere's Circuital Law

- An incremental closed path in rectangular coordinates is selected for the application of Ampère's circuital law to determine the spatial rate of change of H.



5.2 Ampere's Circuital Law

Evaluate total current I for the field $\mathbf{H} = 6xy\mathbf{a}_x - 3y^2\mathbf{a}_y$ A/m and the rectangular path around the region, $2 \le x \le 5, -1 \le y \le 1, z = 0$.

Ans. -126 A

5.3 Curl

- After beginning with Ampere’s circuital law equating the closed line integral of H to the current enclosed.
- We see that a component of the current density is given by the limit of the quotient of the closed line integral of H about a small path in a plane normal to that component and of the area enclosed as the path shrinks to zero.

$$(\text{curl}\ \mathbf{H})_N = \lim_{\Delta S_N \to 0} \frac{\oint \mathbf{H} \cdot d\mathbf{L}}{\Delta S_N}$$

5.3 Curl

$$\text{curl } \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z$$

This result may be written in the form of a determinant,

$$\text{curl } \mathbf{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

- and may also be written in terms of the vector operator,

$$\text{curl } \mathbf{H} = \nabla \times \mathbf{H}$$

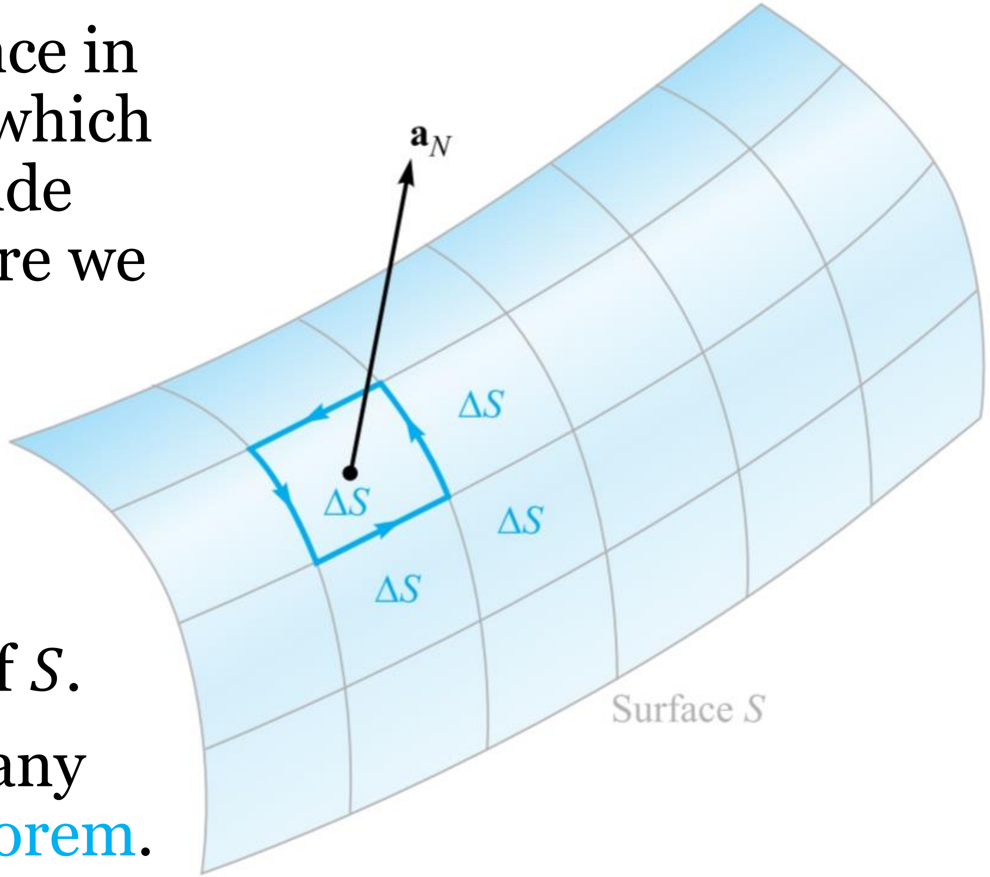
5.4 Stokes' Theorem

- because every interior wall is covered once in each direction. The only boundaries on which cancellation cannot occur form the outside boundary, the path enclosing S. Therefore we have

$$\oint \mathbf{H} \cdot d\mathbf{L} \equiv \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$$

where $d\mathbf{L}$ is taken only on the perimeter of S.

- The equation is an identity, holding for any vector field, and is known as Stokes' theorem.



5.4 Stokes' Theorem

Evaluate total current I for the field $\mathbf{H} = 6xy\mathbf{a}_x - 3y^2\mathbf{a}_y$ A/m and the rectangular path around the region, $2 \leq x \leq 5, -1 \leq y \leq 1, z = 0$.

Ans. -126 A

5.5 Magnetic Flux and Magnetic Flux Density

- In free space, let us define the magnetic flux density **B** as

$$\mathbf{B} = \mu_0 \mathbf{H} \quad \text{(free space only)}$$

- where **B** is measured in webers per square meter ($\mathrm{Wb/m^2}$)

- In a newer unit adopted in the International System of Units, tesla (T). An older unit that is often used for magnetic flux density is the gauss (G), where 1 T or 1 $\mathrm{Wb/m^2}$ is the same as 10,000G. The constant μ_0 is not dimensionless and has the defined value for free space, in henrys per meter (H/m), of

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$$

- The name given to μ_0 is the permeability of free space.

5.5 Magnetic Flux and Magnetic Flux Density

- Let us represent magnetic flux by Φ and define Φ as the flux passing through any designated area,

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

- In the example of the infinitely long straight filament carrying a direct current I, the **H** field formed concentric circles about the filament. Because $\mathbf{B} = \mu_0 \mathbf{H}$, the **B** field is of the same form. The magnetic flux lines are closed and do not terminate on a "magnetic charge".
- For this reason, Gauss's law for the magnetic field is

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

Problems

1. The magnetic field intensity is given in a certain region of space as $\mathbf{H} = [(x + 2y)/z^2]\mathbf{a}_y + (2/z)\mathbf{a}_z$ A/m. (a) Find $\nabla \times \mathbf{H}$. (b) Find $\mathbf{J}$. (c) Use $\mathbf{J}$ to find the total current passing through the surface $z = 4, 1 \leq x \leq 2, 3 \leq z \leq 5$, in the $\mathbf{a}_z$ direction. (d) Show that the same result is obtained using the other side of Stokes' theorem.
2. Let $\mathbf{A} = (3y - z)\mathbf{a}_x + 2xz\mathbf{a}_y$ Wb/m in a certain region of free space. (a) Show that $\nabla \cdot \mathbf{A} = 0$. (b) At $P(2, -1, 3)$, find $\mathbf{A}, \mathbf{B}, \mathbf{H}$, and $\mathbf{J}$.