Sigma Notation: Introduction to Summation

Video companion

1 Sigma notation (Σ)

Examples that will be seen in this video:

$$\sum_{i=1}^{4} i^2 = 30$$

$$\sum_{i=1}^{5} (2i + 3) = 45$$

$$\sum_{j=3}^{7} \frac{j}{2} = \frac{25}{2}$$

2 First example

Example:

$$\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2$$

$$= 30$$

$i = 1$ on bottom tells us to *start* with $i = 1$.
4 on top tells us to *finish* with $i = 4$.
Implicitly know that you increment by 1.

For each number i that you count,

$$i = 1 : i^2 = 1^2$$
$$i = 2 : i^2 = 2^2$$
$$i = 3 : i^2 = 3^2$$
$$i = 4 : i^2 = 4^2$$

then the Σ tells you to *sum* the results.

3 Second example

Example:

$$\sum_{i=1}^{5}(2i+3) = (2(1)+3) + (2(2)+3) + (2(3)+3) + (2(4)+3) + (2(5)+3)$$
$$= 45$$

Work for problem:

$$i = 1 : 2i + 3 = 2(1) + 3$$
$$i = 2 : 2i + 3 = 2(2) + 3$$
$$i = 3 : 2i + 3 = 2(3) + 3$$
$$i = 4 : 2i + 3 = 2(4) + 3$$
$$i = 5 : 2i + 3 = 2(5) + 3$$

4 Third example

Example:

$$\sum_{j=3}^{7}\frac{j}{2} = \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \frac{6}{2} + \frac{7}{2} = \frac{25}{2}$$
$$\sum_{r=3}^{7}\frac{r}{2} = \frac{25}{2}$$

j and r are "dummy indices," symbols for counters.

$$\sum_{\smiley=3}^{7}\frac{\smiley}{2} = \frac{25}{2}$$

Common choices for indices:
$i,\ j,\ k,\ l,\ r,\ m,\ n$