

# Numbers: Algebra with Inequalities

Video companion

## 1 Introduction

- Review algebra with equalities ( $=$ )
  - how?
  - why?
- Learn algebra with inequalities ( $<$ ,  $>$ ,  $\leq$ ,  $\geq$ )
  - what works
  - A BIG WARNING

## 2 Algebra with equalities

$$\begin{aligned}4 &= 4 \\4 + 3 &= 4 + 3 \\7 &= 7 \quad \checkmark\end{aligned}$$

**Rule:**

If  $a = b$ , then  $a + c = b + c$ .

Example:

$$\begin{aligned}x + 3 &= 10 \\(x + 3) - 3 &= 10 - 3 \\x &= 7\end{aligned}$$

Similarly with multiplication,

$$\begin{aligned}4 &= 4 \\2 \cdot 4 &= 2 \cdot 4 \\8 &= 8 \quad \checkmark\end{aligned}$$

$$\begin{aligned} 4 &= 4 \\ (-3) \cdot 4 &= (-3) \cdot 4 \\ -12 &= -12 \quad \checkmark \end{aligned}$$

**Rule:**

If  $a$ ,  $b$ , and  $c$  are numbers, and  $c \neq 0$ , and  $a = b$ , then  $c \cdot a = c \cdot b$ .

Example:

$$\begin{aligned} -5x &= 15 \\ \left(-\frac{1}{5}\right) \cdot (-5x) &= \left(-\frac{1}{5}\right) \cdot 15 \\ x &= -3 \end{aligned}$$

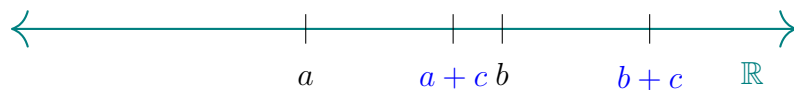
### 3 Algebra with inequalities

$$\begin{aligned} 4 &< 7 \\ 4 + 2 &\stackrel{?}{<} 7 + 2 \\ 6 &\stackrel{?}{<} 9 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 4 &< 7 \\ 4 - 1 &\stackrel{?}{<} 7 - 1 \\ 3 &\stackrel{?}{<} 6 \quad \checkmark \end{aligned}$$

**Rule:**

If  $a < b$ , then  $a + c < b + c$ .



Example:

$$\begin{aligned}x + 3 &< 10 \\(x + 3) - 3 &< 10 - 3 \\x &< 7\end{aligned}$$

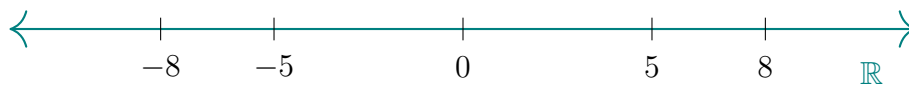


$$x \in (-\infty, 7)$$

Test cases with multiplication:

$$\begin{aligned}5 &< 8 \\3 \cdot 5 &\stackrel{?}{<} 3 \cdot 8 \\15 &\stackrel{?}{<} 40 \quad \checkmark\end{aligned}$$

$$\begin{aligned}5 &< 8 \\(-1) \cdot 5 &\stackrel{?}{<} (-1) \cdot 8 \\-5 &\stackrel{?}{<} -8 \quad \times \\-5 &> -8 \quad !\end{aligned}$$



**Rule:**

Suppose  $a < b$ .

If  $c > 0$ , then  $a \cdot c < b \cdot c$ .

If  $c < 0$ , then  $a \cdot c > b \cdot c$ .

Example:

$$\begin{aligned} -2x &< 10 \\ \left(-\frac{1}{2}\right) \cdot (-2x) &> \left(-\frac{1}{2}\right) \cdot 10 \\ x &> -5 \end{aligned}$$

