Data Science Math Skills
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Functions: Composition and Inverse

Video companion

1 Introduction

- Composing two functions
  - Basic identity
  - A warning
- Inverse functions
  - Basic identity
  - A neat picture
  - A warning

2 Composing functions

Definition: Given functions f and g, $(g \circ f)(x) = g(f(x))$, and $(f \circ g)(x) = f(g(x))$

Example:

$$\begin{aligned} f(x) &= x^2 \\ g(x) &= x + 5 \end{aligned}$$

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g(x^2) = x^2 + 5 \\ g(f(2)) &= g(2^2) = 2^2 + 5 = 9 \\ (f \circ g)(x) &= f(g(x)) \\ &= f(x+5) = (x+5)^2 \neq x^2 + 5 \end{aligned}$$

3 Inverse functions

Example:

$$f(x) = 2x$$
$$g(x) = \frac{1}{2}x$$

$$(g \circ f)(x) = g(f(x)) = g(2x) = \frac{1}{2}(2x) = x$$

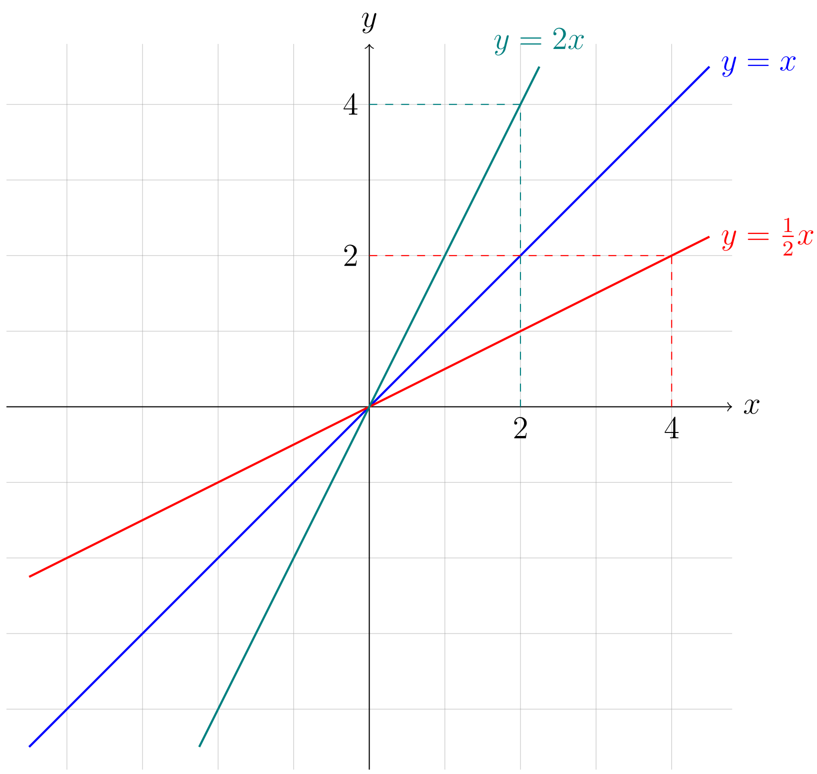
Notice: true for all x

$$(g \circ f)(3) = g(f(3)) = g(2 * 3) = \frac{1}{2}(2 * 3) = 3$$
$$(g \circ f)(\pi) = g(f(\pi)) = g(2 * \pi) = \frac{1}{2}(2 * \pi) = \pi$$

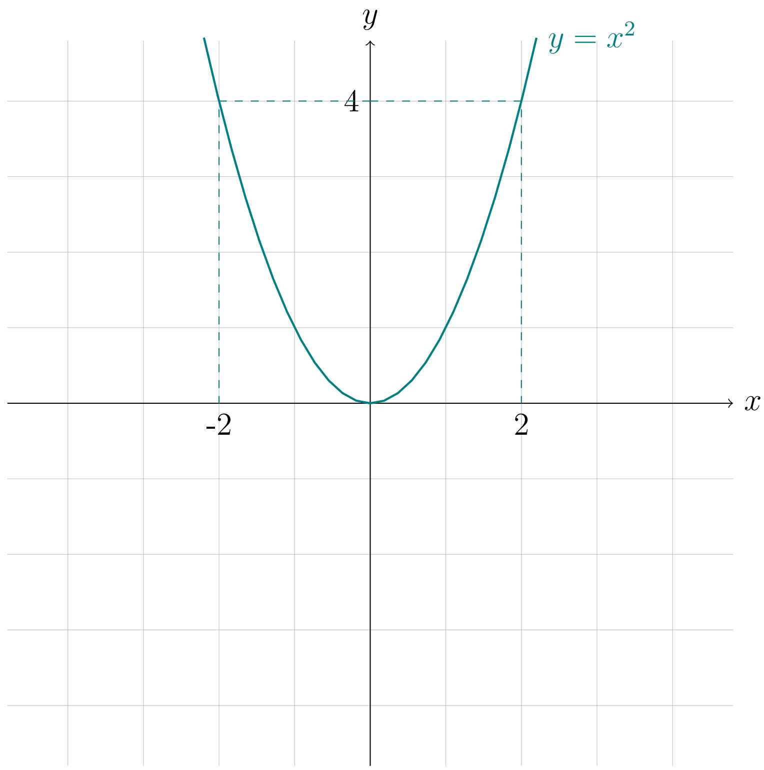
f and g are *inverses* of each other, i.e. f undoes what g does.

$$g = f^{-1}$$

4 Graphical depiction



Warning: not every function $f : \mathbb{R} \to \mathbb{R}$ has an inverse.



Warning: if the graph of f fails the horizontal line test, then f has no inverse. The only invertible functions are those that are either strictly increasing or strictly decreasing.