

Cartesian Plane: Distance Formula

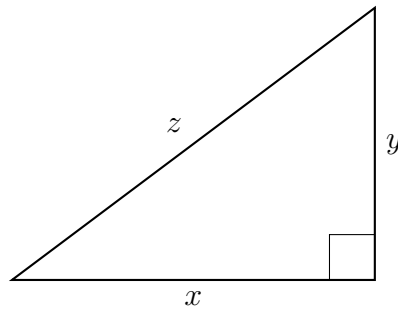
Video companion

1 Introduction

In this video:

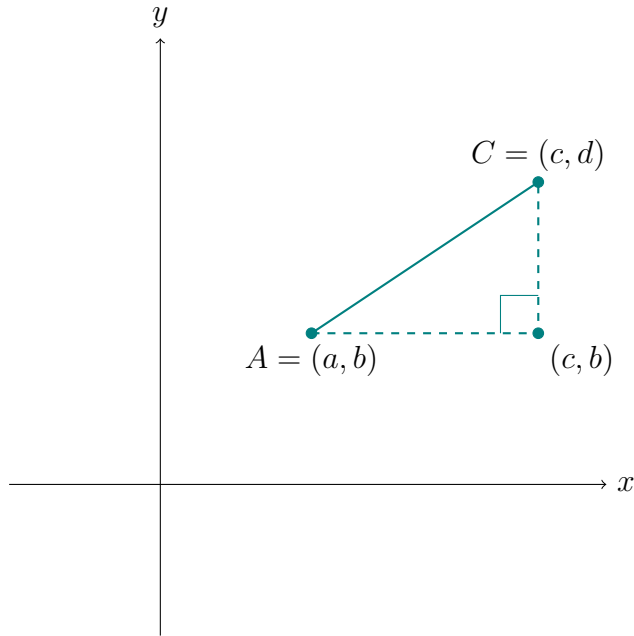
- The distance formula
- Nearest neighbors
- Clustering

2 Pythagorean theorem



$$z^2 = x^2 + y^2$$
$$z = \sqrt{x^2 + y^2}$$

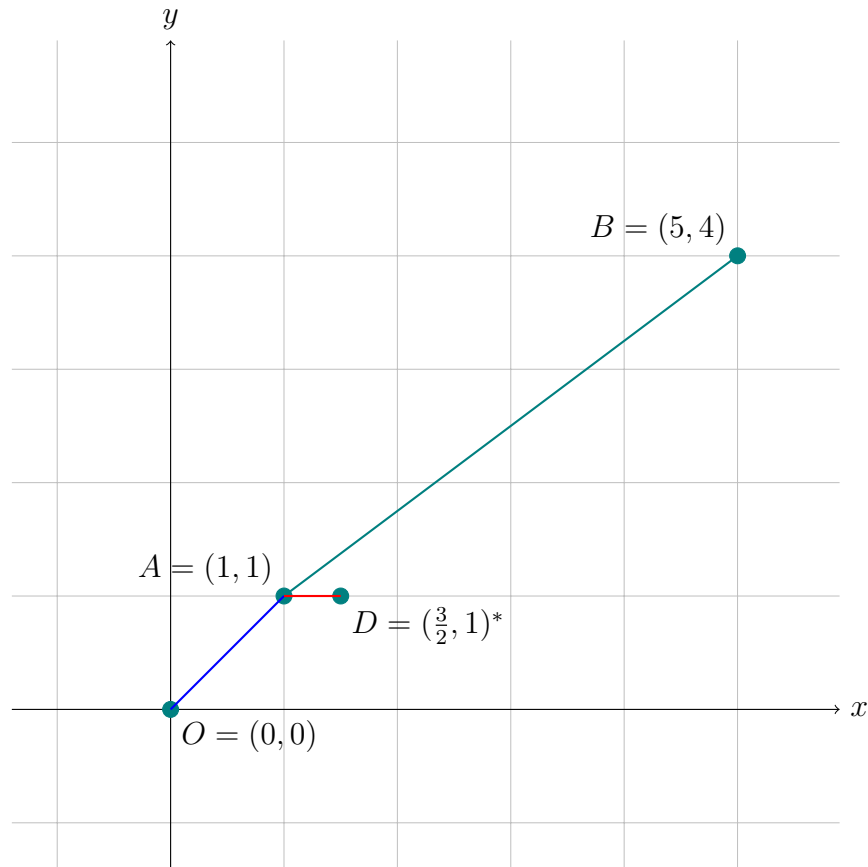
3 Graph of distance formula



Distance formula:

$$\text{dist}(A, C) = \sqrt{(c - a)^2 + (d - b)^2}$$

4 Example and nearest neighbors



$$\begin{aligned}\text{dist}(A, B) &= \sqrt{(5 - 1)^2 + (4 - 1)^2} \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{dist}(A, O) &= \sqrt{(1 - 0)^2 + (1 - 0)^2} \\ &= \sqrt{2} \approx 1.4\end{aligned}$$

$$\begin{aligned}\text{dist}(A, D) &= \sqrt{\left(\frac{3}{2} - 1\right)^2 + (1 - 1)^2} \\ &= \frac{1}{2}\end{aligned}$$

*Note that the x and y values of point D are reversed in the video, but it does not matter in calculating the distance from A .

Consider set S :

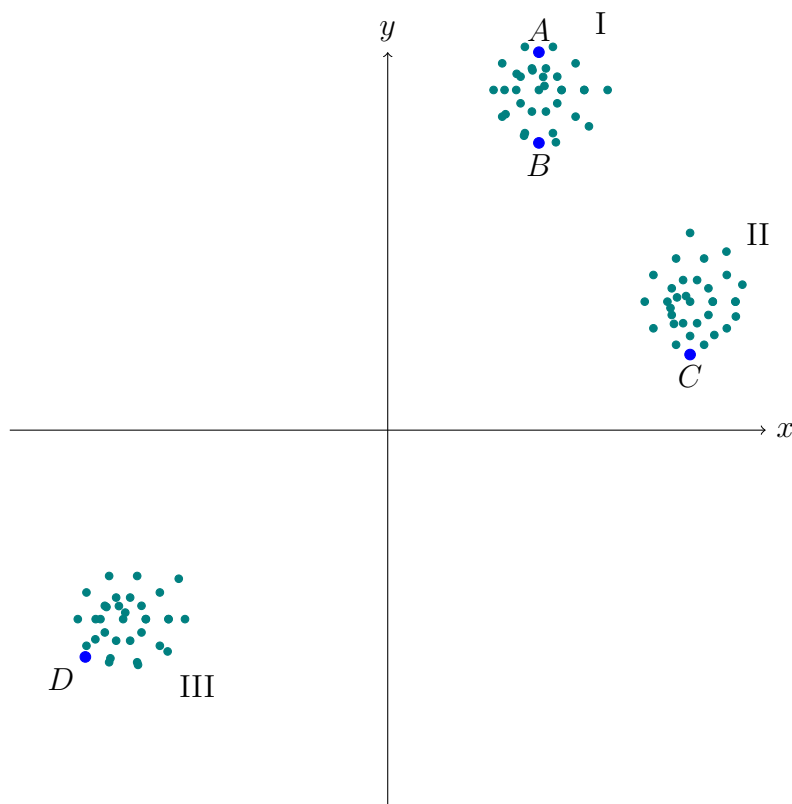
$$S = \{O, B, D\}$$

The *nearest neighbor* of A in S is D .

The second nearest neighbor of A in S is O .

The third nearest neighbor of A in S is B .

5 Clustering



Three clusters: I, II, and III

If A and B are in cluster I,
and C is in cluster II,
and D is in cluster III,

Then $\text{dist}(A, B) \ll \text{dist}(A, C),$
 $\ll \text{dist}(A, D)$