A3 - CS 480

Q1) $K(x,x') = e^{-x^Tx/2\sigma^2} e^{x^Tx'/\sigma^2} e^{-x'^Tx'/2\sigma^2}$

Let $c(x) = e^{-x^Tx/2\sigma^2}$.

Then, $K(x,x') = c(x) \cdot e^{x^Tx'/\sigma^2} \cdot c(x')$

Consider the Taylor series expansion for $f(x) = e^x$ about $x=0$:

① $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

By ①, $$K(x,x') = c(x) \cdot \left[\sum_{n=0}^{\infty} \frac{\left(\frac{x^Tx'}{\sigma^2}\right)^n}{n!} \right] \cdot c(x')$$

$$= c(x) \cdot \left[\sum_{n=0}^{\infty} \frac{K_{poly(n)}(x,x')}{\sigma^{2n} \cdot n!} \right] \cdot c(x')$$

Thus, the Gaussian kernel is an infinite sum of all polynomial kernels.

As shown in class, each polynomial kernel, $K_{poly(n)}(x,x')$, can be broken down into some basis functions, say Φ_n.

In the infinite dimensional space corresponding to the Gaussian kernel, each $\phi(x) \in \Phi_n$, is multiplied by $\frac{c(x)}{\sqrt{\sigma^{2n} \cdot n!}}$

By taking the dot product of 2 vectors in this basis space, we arrive at the kernel above.