TWO-WHEELED SELF BALANCING BOT

CHAPTER 1:
INTRODUCTION

Balance bot is a two wheeled robot which can balance itself without any extra support. The balance bot can be considered as an advancement of the classic inverted pendulum problem. Inverted pendulum applications are plenty; for example the human body is an inverted pendulum balancing the upper body around our ankle joints in every step. In the recent decade Segways, making use of bodily movements for steering, have emerged on the market. In common, these applications share that their centre of mass is located above their pivot points, thus requiring active control in order to balance.

In order to balance a two-wheeled inverted pendulum robot it is necessary to have accurate information of the current tilt angle from using a measurement unit. Furthermore a controller needs to be implemented to compensate for said tilt. In order to balance the bot, the magnitude and direction in which the bot is tilting is measured using accelerometer and gyroscope. The signals obtained are then used to move the wheels in the direction in which the bot is falling. Complementary filter is used to obtain better results from the accelerometer and gyroscope. Initially the Linear Quadratic Regulator (LQR) controller was used to control the Bot, but there were some complications with respect to the physical limitations of the Bot. Then we switched to Cascaded Proportional Derivative (PD) controller. Here, Cascaded Proportional Derivative (PD) is used to control the motor speed.

CHAPTER 2:

LITERATURE SURVEY

The applications based on inverted pendulum theory are becoming very common with the passage of time. The uniqueness and wide applications of this unstable system has gained popularity among researchers and robotics enthusiasts around the world. This section provides an insight and literature review to the current technology available to construct a two-wheel self-balancing robot. It also highlights various advancements in this technology with time and also encompasses various methods used by researchers on this topic.

Segway (Dean Kamen, 2001) is a commercially available two wheel balancing robot which was produced by Segway Inc. of New Hampshire, USA [2]. It was invented by David Kamen in 2001. It is a very successful personal transporter most commonly used by security guards in mega shopping malls to cover a vast area. Its advertising suggests the robot is ideal for adventure, law enforcement and transportation in general. It has capability to achieve a speed of 20km/h.

iBot is another application of two wheel robots developed by David Kamen [3]. In 2006, David Kamen introduced a mobile wheelchair. It was capable of balancing on two wheels. It can climb stairs and it increases its height when it is balanced on two wheels. The purpose of increasing height is to establish line of sight between disable people and other people.

CHAPTER 3:

THEORY

A **Simple pendulum** is one which can be considered to be a point mass suspended from a string or rod of negligible mass. When the Bob in the pendulum is left from the maxima point, the Bob oscillates to and forth and eventually comes to rest in the Equilibrium position.

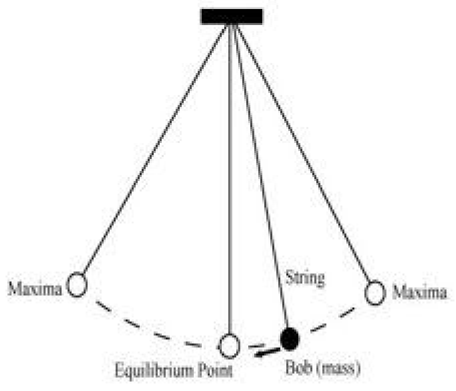


Fig 3.1 Simple Pendulum

An **Inverted pendulum** is a pendulum that has its center of mass above its pivot point. It is **unstable** and without additional help will fall over. It can be suspended stably in this inverted position by using a control system.

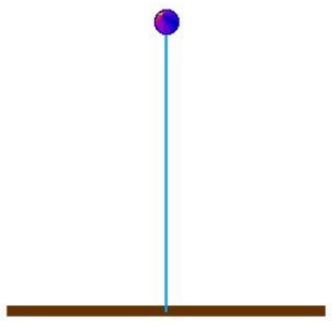


Fig 3.2 Inverted Pendulum

Stable Equilibrium - If a system always returns to the equilibrium point after small perturbations. (Simple Pendulum)

Unstable Equilibrium - If a system moves away from equilibrium point after small perturbations (Inverted Pendulum)

3.1 Cart Pendulum Model

The aim is to balance a pendulum upside down on a cart that is allowed to move about. A sensor is used to detect the position of the pendulum which sends the information over to a computer which performs certain computations and instructs actuators to move the cart in a way to make the pendulum vertical again.

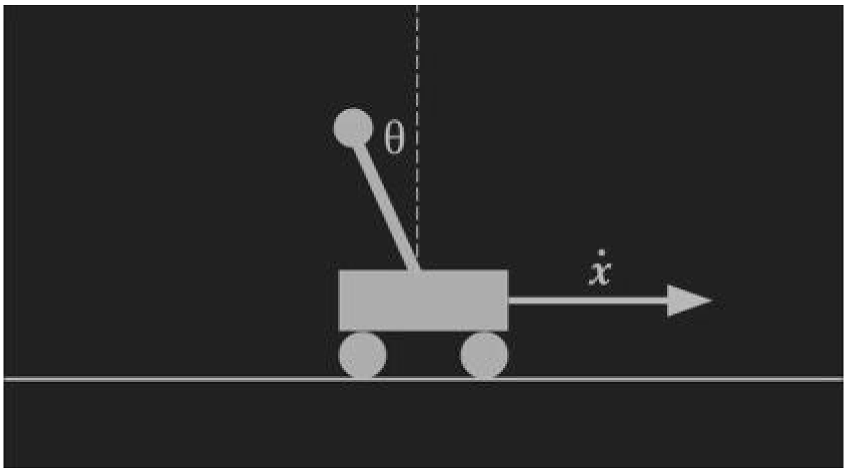


Fig 3.3 Cart Pendulum model

The Goal of the Cart Pendulum model is to keep **θ = 0** by moving the cart in the direction of the falling mass. Our Bot is modelled with the same principle.

3.2 Mathematical Model of the Bot

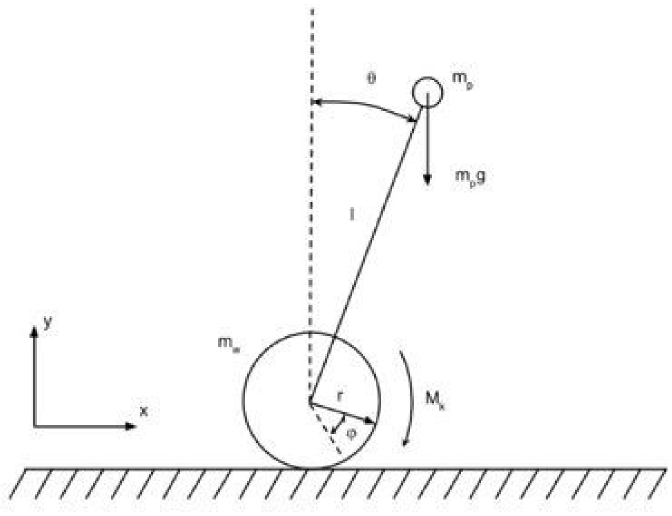


Fig 3.4 Diagram of our Bot

Consider the inverted pendulum system on the wheel, pictured below. Assume that the system moves without friction. The pendulum has a mass m p ,attached to a weightless rod of length l to the wheel. The wheel is considered as a ring of radius r and mass m w . The engine torque M k acts on the wheel

m p : pendulum mass
m w : wheel mass
l: pendulum length
r: wheel radius
θ: angle between the pendulum and the vertical line
φ: angle of rotation of the wheel relative to its initial position
M k : motor torque

To study the system, we use the Euler-Lagrange equations. (1.1)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}(q,\dot{q})\right) - \frac{\partial L}{\partial q}(q,\dot{q}) = \tau$$

Express the position of the center of the wheel through the angle of rotation:

$$x = r\phi \ . \ (1.2)$$

Note that the coordinates of the center of mass of the pendulum are found by the following relationships:

$$x_g = x + l sin(\theta)_{(1.3)}$$
$$y_g = l cos(\theta)_{(1.4)}$$

First, we calculate the representation for the kinetic energy of the system.
Kinetic pendulum energy is equal to

$$T_p = \tfrac{1}{2}[m_p \dot{y}_g^2(t) + m_p \dot{x}_g^2(t)] \ . \ (1.5)$$

The kinetic energy of the wheel is

$$T_w = \tfrac{1}{2}[m_w \dot{x}^2(t) + m_w r^2 \dot{\phi}^2(t)]$$

Substituting the above equations, Total Kinetic Energy is

$$T = \tfrac{1}{2} m_p l^2 \dot{\theta} sin(\theta)^2 + \tfrac{1}{2} m_p r^2 \dot{\phi}^2 + \tfrac{1}{2} m_p l^2 \dot{\theta}^2 cos(\theta)^2 + m_w r^2 \dot{\phi}^2 + \tfrac{1}{2} m_p l^2 \dot{\theta}^2$$

Potential Energy is

$$\Pi = m_p g l cos(\theta)$$

The Lagrange function is given by the formula

$$L = T - \Pi$$

As a result,

$$L = \tfrac{1}{2} m_p l^2 \dot{\theta} sin(\theta)^2 + \tfrac{1}{2} m_p r^2 \dot{\phi}^2 + \tfrac{1}{2} m_p l^2 \dot{\theta}^2 cos(\theta)^2 + m_w r^2 \dot{\phi}^2 +$$
$$\tfrac{1}{2} m_p l^2 \dot{\theta}^2 - m_p g l cos(\theta)$$

We derive the equations of motion using the Lagrange equations of the second kind (1.1). As generalized coordinates, we take the angles of rotation of the wheel and the pendulum. Then the vector of generalized coordinates is represented as vector of generalized forces is as follows:

$$q = \begin{pmatrix} \phi \\ \theta \end{pmatrix}$$

$$\tau = \begin{pmatrix} M_k \\ 0 \end{pmatrix}$$

Substituting the derivatives into the Lagrange equations (1.1), we derive the equations of movement :

$$r\cos(\theta) l m_p \ddot{\theta} + r^2(m_p + 2m_w)\ddot{\phi} - r\sin(\theta)\theta^2 l m_p = M_k$$
$$\ddot{\phi}\cos(\theta) l m_p r - m_p g l \sin(\theta) + 2m_p l^2 \ddot{\theta} = 0$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$
$$q = \begin{pmatrix} \phi \\ \theta \end{pmatrix}$$
$$M(q) = \begin{bmatrix} r^2(m_p + 2m_w) & r\cos(\theta) l m_p \\ r\cos(\theta) l m_p & 2m_p l^2 \end{bmatrix}$$
$$C(q,\dot{q}) = \begin{bmatrix} 0 & -r\dot{\theta}\sin(\theta) l m_p \\ 0 & 0 \end{bmatrix}$$
$$G(q) = \begin{pmatrix} 0 \\ -m_p g l \sin(\theta) \end{pmatrix}$$
$$\tau = \begin{pmatrix} M_k \\ 0 \end{pmatrix}$$

To control the resulting system, we construct a linear-quadratic controller. To do this, we linearize the obtained equations of motion in neighborhood of the zero position of the pendulum using Jacobians:

$$\frac{d}{dt}\begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \end{bmatrix} = AX + BM_k = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-m_p g}{r(m_p+4m_w)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g(m_p+2m_w)}{l(m_p+4m_w)} & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2}{r^2(m_p+4m_w)} \\ 0 \\ \frac{1}{lr(m_p+4m_w)} \end{bmatrix} M_k$$

A – state matrix,

X – state vector

B – input matrix

Mk – input vector

CHAPTER 4:

DESIGN ANALYSIS

4.1 HARDWARE DESIGN

Hardware used:

1. Arduino Mega 2560
2. MPU6050 Sensor
3. 300 RPM DC Gear Motors with Encoders
4. L298N Dual H-Bridge Motor Controller
5. XBee Module
6. 3 cell Li-Po battery

4.1.1 Arduino Mega 2560

The Arduino Mega 2560 is a microcontroller board based on the Atmega 2560.

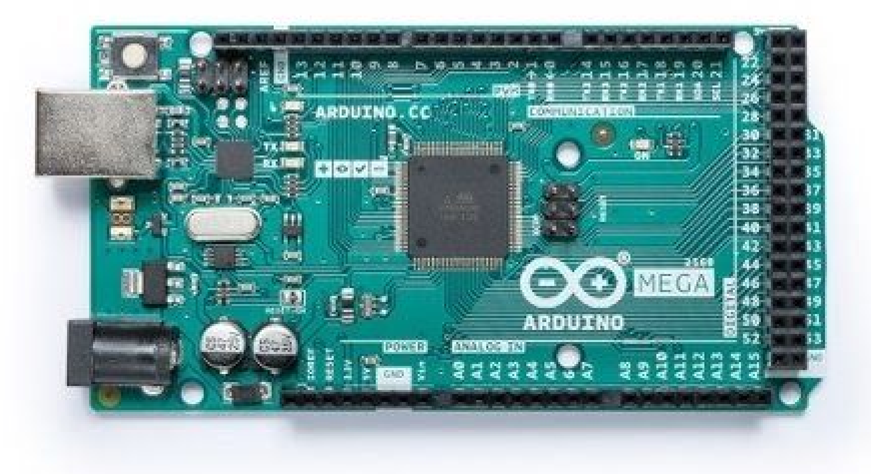


Fig 4.1 Arduino Mega 2560 Board

Features:

- 54 digital input/output pins (of which 15 can be used as PWM outputs)
- 16 Analog input pins
- 4 UARTs (hardware serial ports)
- 16 MHz crystal oscillator
- 256 KB of Flash Memory
- 6 external interrupts
- 5 V Operating Voltage
- 37 g in weight

4.1.2 MPU6050 Sensor

MPU-6050 is an 8 pin 6 axis gyro and accelerometer in a single chip. This module works on **I2C** serial communication. For I2C this has **SDA** and **SCL** lines.

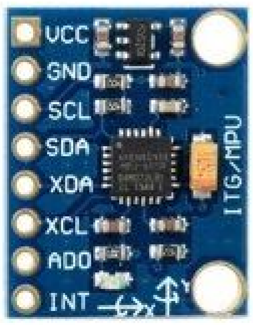


Fig 4.2 MPU6050 Sensor

3-axis Gyroscope

The MPU-6050 consist of a 3 axis gyroscope which can detect rotational velocity along the x,y,z axis with micro electro mechanical system technology (MEMS).

- When the sensor is rotated along any axis a vibration is produced due to Coriolis effect which is detected by the MEMS.

- 16-bit ADC is used to digitize voltage to sample each axis (32767 to - 32768).
- **+/- 250**, +/- 500, +/- 1000, +/- 2000 are the full scale range of output.
- Angular velocity is measured along each axis in **degree per second** unit.

3-axis Accelerometer

The MPU-6050 consist of a 3 axis accelerometer which can detect angle of tilt or inclination along the x,y,z axis with micro electro mechanical system technology (MEMS).

- Acceleration along the axes deflects the moving mass which in turn unbalances the differential capacitor which is detected with electro mechanical system technology (MEMS). The Output amplitude is proportional to acceleration.
- 16-bit ADC is used to digitize the values (32767 to - 32768).
- Acceleration is measured along each axis in terms of **g (9.8 m/s2)**
- **+/- 2g**, +/- 4g, +/- 8g, +/- 16g are the full scale range of output.
- In a normal position that is when the device is placed on a flat surface the values are 0 g on x axis, 0 g on y axis and +1 on z axis.

4.1.3 300 RPM DC Gear Motors with Encoders

A Gear motor is a specific type of electrical motor that is designed to produce high torque while maintaining a low horsepower, or low speed, motor output. Gear motors are primarily used to reduce speed in a series of gears, which in turn creates more torque. This is accomplished by an integrated series of gears or a gearbox being attached to the main motor rotor and shaft via a second reduction shaft. The second shaft is then connected to the series of gears or gearbox to create what is known as a series of reduction gears. Generally speaking, the longer the train of reduction gears, the lower the output of the end, or final, gear will be.

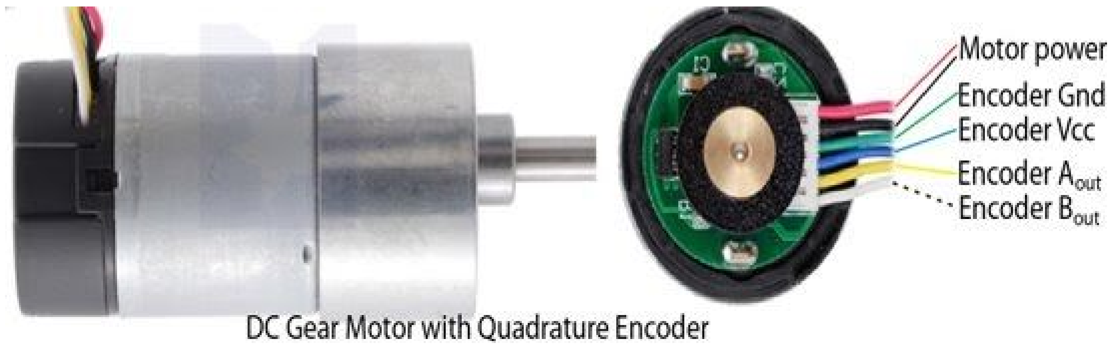


Fig 4.3 DC Gear Motor with Quadrature Encoder

Motor Specifications:

Stall Current = 2.3-2.5 Amps

Stall Torque = 8.5 Kg-cm

Base RPM = 9000 rpm

Shaft RPM after Gear Box = 300 rpm

Gear Box Ratio = 1:30

Quadrature Encoders are handy sensors that lets us measure the speed and direction of a rotating shaft and keep track of how far we have moved. A quadrature encoder has two outputs - Channel A and B - each of which will produce digital pulses when the thing they are measuring is in motion. We connect channel A of both the Motors as Interrupts in Arduino. We trigger the interrupts in Rising Edge. We then check the value of channel B at that instant. If channel B is 0, then the motor is moving in clockwise direction else if channel B is 1, then the motor is moving in anti-clockwise direction.

Encoder Specification:

270 Pulses per revolution per channel.

4.1.4 L298N Dual H-Bridge Motor Controller

H-Bridge is used in controlling motors speed and direction. The speed is controlled by Pulse Width Modulation (PWM).

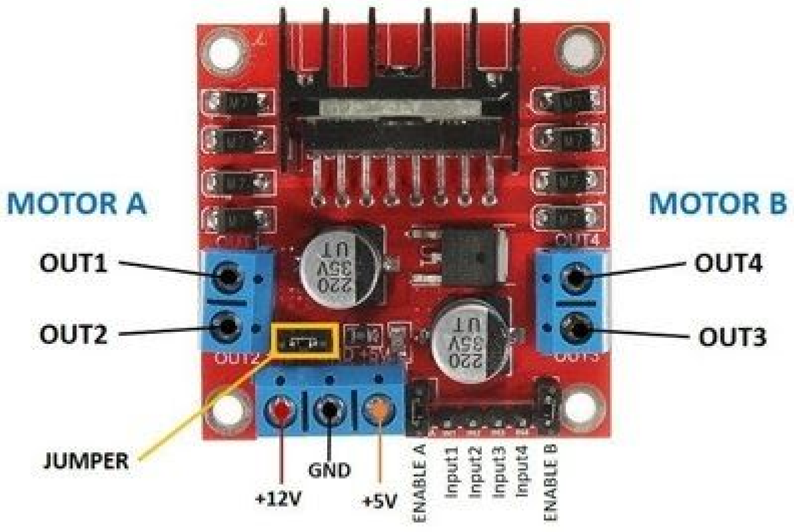


Fig 4.4 Dual H-Bridge Motor Controller

Specifications:

- Double H bridge Drive Chip: L298N
- Logical voltage: 5V
- Drive voltage: 5V-35V
- Logical current: 0-36mA
- Drive current: 2A (MAX single bridge)
- Max power: 25W
- Dimensions: 43 x 43 x 26mm
- Weight: 26g

4.1.5 XBee Module

XBee S2C is a RF module designed for wireless communication or data exchange and it works on ZigBee mesh communication protocols that sit on top of IEEE 802.15.4 PHY. The module provides wireless connectivity to end-point devices in any ZigBee mesh networks.

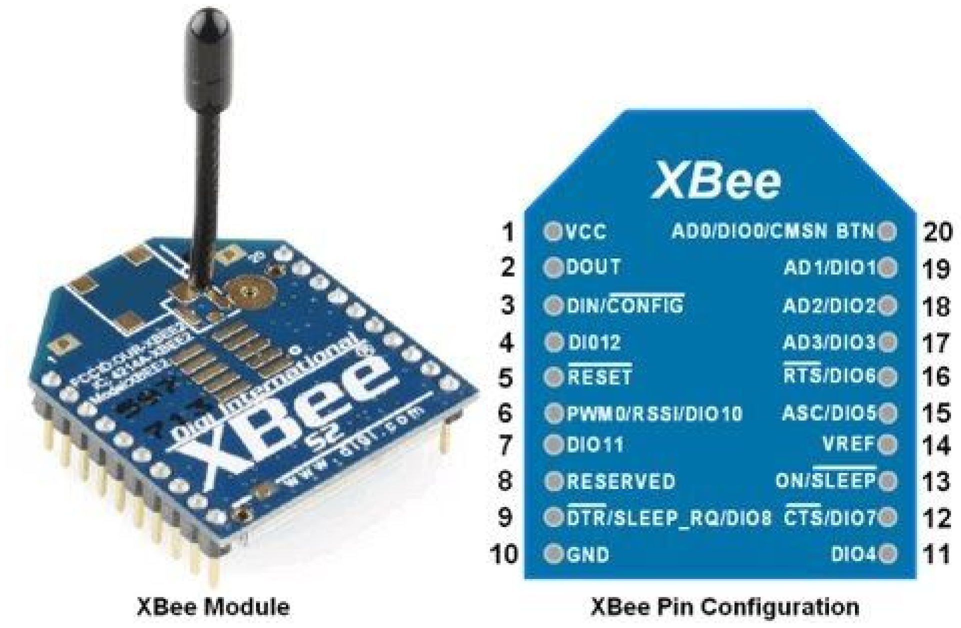


Fig 4.5 XBee Module

Features and Electrical Characteristics

- Transmission Frequency: 2.4GHz to 2.5GHz
- Indoor/Urban Range: 200ft
- Outdoor RF line-of-sight Range: up to 4000ft
- RF Data Rate: 250,000 bps
- Supply Voltage Range: +2.1V to +3.6V
- Operating Current: 33mA (at3.3V, for Normal mode) , 45mA (at 3.3V,for Boost mode) Maximum output current on all pins together: 40mA

4.1.6 3 cell Li-Po Battery (Lithium Polymer)

Fig 4.6 Li-Po Battery

Specifications:

Voltage / Cell Count – 11.1 V / 3S
A LiPo cell has a nominal voltage of 3.7V. For the 11.1V battery, that means that there are three cells in series (which means the voltage gets added together). A "3S" battery pack means that there are 3 cells in Series. So a three-cell (3S) pack is 11.1V, and so on.

Capacity – 2200 mAh
The capacity of a battery is basically a measure of how much power the battery can hold. The unit of measure here is milliamp hours (mAh). This is saying how much drain can be put on the battery to discharge it in one hour.

Discharge Rating ("C" Rating) – 35C
The C Rating is simply a measure of how fast the battery can be discharged safely and without harming the battery. 35C = 35 x Capacity (in Amps) Calculating the C-Rating of our example battery: 35 x 2.2 = 77A The resulting number is the maximum sustained load you can safely put on the battery. Going higher than that will result in, at best, the degradation of the battery at a faster than normal pace. At worst, it could burst into flames. So our battery can handle a maximum continuous load of **77A**.

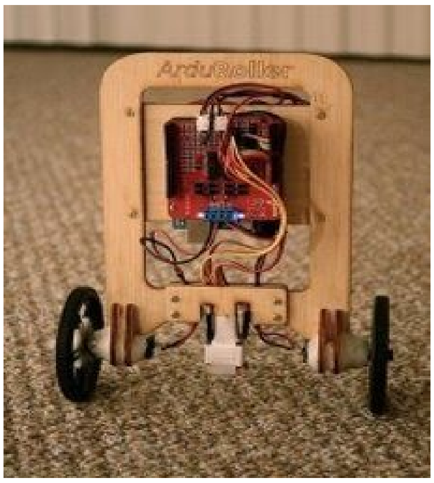
4.2 CHASSIS DESIGN

Two types of Chassis design are available:

1. Vertical design
2. Stackable design

Stability and Response – The moment of inertia of the robot must be high so that it tilts more slowly when external torque is applied. The mass in the vertical design is distributed evenly, whereas it is distributed as different tiers in the stacked design. Hence, the different tiers can be moved up and down in order to shift the center of gravity as required. The taller the robot, more the stable it is.

Construction – The stacked design is more feasible to construct compared to the vertical design. Various components can be easily mounted and removed, without having to worry much about the CG shift as the tiers can easily be shifted later as required. Another major advantage is that, the stacked design is scalable. For example, if we want the robot to carry some weight, this can be easily done in the stacked design by adding a new tier. The design can be constantly updated and new features can be incorporated with ease. On the other hand, creating vertical stacks (along the depth) is not very feasible.



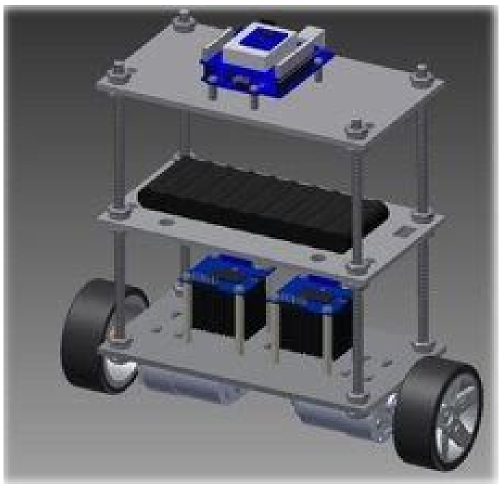


Fig 4.7 Vertical Design (left) and Stacked Design (right)

4.3 SIGNAL PROCESSING

Signal processing is a field of engineering that focuses on analysing, modifying and synthesizing signals such as sound, images, biological measurements etc. In signal processing, a “Filter” is a device or process that removes some unwanted components or features from a signal. Filtering is a class of signal processing, the defining feature of filters being the complete or partial suppression of some aspect of the signal. Most often, this means removing some frequencies or frequency bands. Frequency filter circuits (such as low-pass, high-pass, band-pass, and band-reject) shape the frequency content of signals by allowing only certain frequencies to pass through.

We have used 3 types of filters:

1. Low pass filter (Used to filter Accelerometer data)
2. High pass filter (Used to filter Gyroscope data)
3. Complementary filter (Used to combine Accelerometer and Gyroscope data)

4.3.1 Low Pass Filter

The low-pass filter has a gain response with a frequency range from zero frequency (DC) to cutoff frequency (ωc), which means that any input that has a frequency below the ωc gets a pass, and anything above it gets attenuated or rejected. The gain approaches zero as frequency increases to infinity

The input signal of the filter shown here has equal amplitudes at frequencies ω1 and ω2. After passing through the low-pass filter, the output amplitude at ω1 is unaffected because it’s below the cutoff frequency ωc. However, at ω2, the signal amplitude is significantly decreased because it’s above ωc.

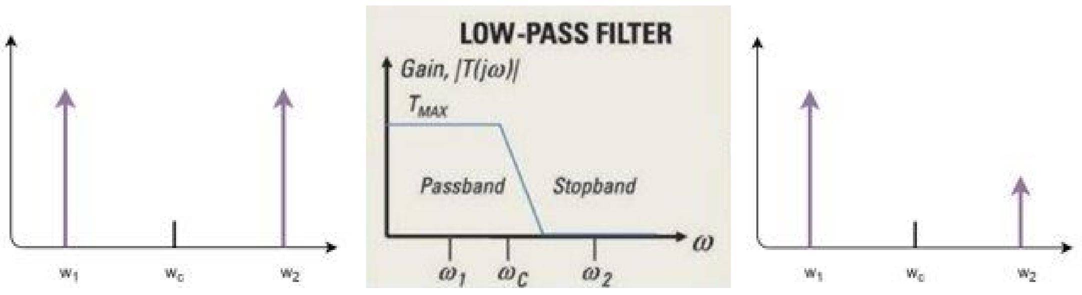


Fig 4.8 low Pass Filter

Equation for low-pass filter:

y[n] = (1-alpha).x[n] + alpha.y[n-1]

x[n] is the unfiltered input signal

y[n-1] is the previous filtered output signal

y[n] is the filtered final output signal

alpha = (tau)/(tau + dt) where tau is the desired time constant (how fast you want the readings to respond) and dt = 1/fs where fs is your sampling frequency.

Calculation of the Pitch angle using Accelerometer is done using the formula

acc = arcTan(Ax / Az)

First we pass the signals Ax and Az through Low pass filters, then we find the angle acc.

4.3.2 High Pass Filter

Opposite to the Low-pass filter, the high-pass filter has a gain response with a frequency range from the cutoff frequency (ωc) to infinity. Any input having a frequency below ωc gets attenuated or rejected. Anything above ωc passes through unaffected.

The input signal of the filter shown here has equal amplitude at frequencies ω1 and ω2. After passing through the high-pass filter, the output amplitude at ω1 is significantly decreased because it's below ωc, and at ω2, the signal amplitude passes through unaffected because it's above ωc.

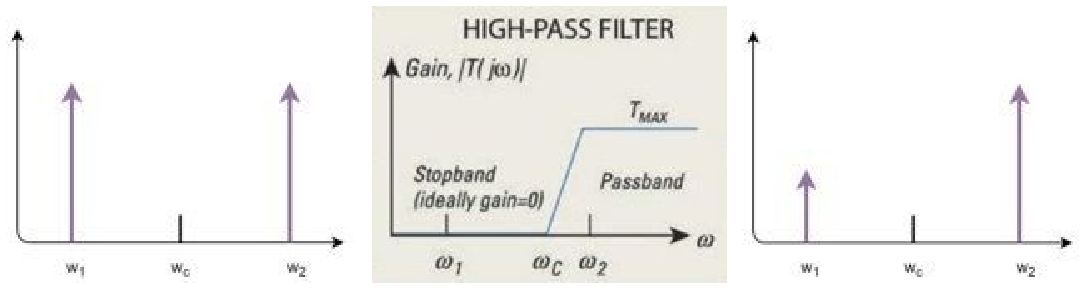


Fig 4.9 High Pass Filter

Equation for high-pass filter:

y[n] = (1-alpha).y[n-1] + (1-alpha)(x[n]-x[n-1])

x[n] is the unfiltered input signal

x[n-1] is the previous input signal

y[n-1] is the previous filtered output signal

y[n] is the filtered final output signal

alpha = (tau)/(tau + dt) where tau is the desired time constant (how fast you want the readings to respond) and dt = 1/fs where fs is your sampling frequency.

The gyroscope data **Gy** is taken

4.3.3 Complementary Filter

Complementary filter is a simple estimation technique to combine measurements. The basic complementary filter is shown in the figure below where x and y are noisy measurements of signal z; Ẑ is the estimate of z produced by the filter. Assume that the noise in y is mostly high frequency, and the noise in x is mostly low frequency. Then G(s) can be made a low pass filter to filter out the high-frequency noise in y. If G(s) is low-pass, [1-G(s)] is the compliment, i.e., a high pass filter which filters out the low-frequency noise in x.

The calculated tilt angle from the accelerometer data has slow response time, while the integrated tilt angle from the gyro data is subjected to drift over a period of time. In other words, we can say that the accelerometer data is useful for long term while the gyro data is useful for short term. Idea behind complementary filter is to take slow moving signals from accelerometer and fast moving signals from a gyroscope and combine them. Complementary filter is designed in such a way that the strength of one sensor will be used to overcome the weakness of the other sensor which is complementary to each other.

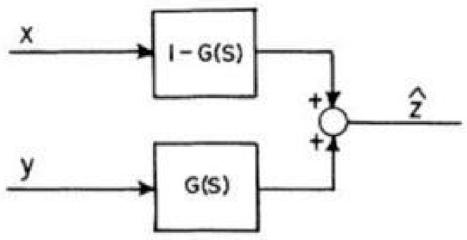


Fig 4.10 Complementary Filter

Equation for Complementary filter:

angle = (1- alpha)(angle + gyro * dt) + (alpha)(acc)

First reading is the angle as obtained from gyroscope integration. Second reading is the one from accelerometer.

4.4 CONTROLLER DESIGN

A controller is a system, which provides the desired response by controlling the output. In our case, we have worked on two controllers

1. Linear Quadratic Regulator (LQR) Controller
2. Cascaded Proportional Derivative (P-D) Controller

4.4.1 Linear Quadratic Regulator (LQR)

Linear Quadratic Regulator is a powerful tool which helps us choose the K matrix according to our desired response.

Here we use a cost function:

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt$$

where Q and R are positive semi-definite diagonal matrices (positive semi-definite matrices are those matrices whose all the eigenvalues are greater than or equal to zero). Also to remind you that for a diagonal matrix, the diagonal entries are its eigenvalues. x and u are the state vector and input vector respectively. Let us say that you have your system with four states and one input.

Let us say that you have your system with four states and one input. Then input = u and

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad Q = \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & Q_4 \end{bmatrix}$$

Then,

$$x^T Q x + u^T R u = Q_1 x_1^2 + Q_2 x_2^2 + Q_3 x_3^2 + Q_4 x_4^2 + R u^2$$

The controller is of form **u = −Kx** which is a Linear controller and the underlying cost function is Quadratic in nature and hence the name Linear Quadratic Regulator. With a careful look at the integrand of the cost function J, we may observe that each Qi are the weights for the respective states xi . So, the trick is to choose weights Qi for each state xi so that the desired performance criteria is achieved. Greater the state objective is, greater will be the value of Q corresponding to the said state variable. We can choose **R = 1** for single input system. In case of inverted pendulums, we know that angle θ with the vertical and the angular velocity ˙θ is very important and hence the weights corresponding to them should be more as compared to linear position x and linear velocity $\dot{x}$. LQR minimizes this cost function J based on the chosen matrices Q and R. Its a bit complicated to find out matrix K which minimizes this cost function. What is required to be done is to choose the Q and R matrix appropriately to get the desired performance. Mathematically its done with Algrebriac Ricatti Equation (ARE) but here we are using Octave's inbuilt lqr function.

4.4.2 Cascaded P-D Controller

e(t) = Error is the difference between the desired position (Set Point) and the current position (Tilt Angle).

Proportional Term

The response of the proportional term is simply a constant (Kp) times the current error. A very low gain (Kp) will make the robot very unresponsive to disturbances. Whereas, a high gain will make the robot over responsive. The controller will not only correct the present error but will also introduce its own error, and in turn try the correct this self-induced error again. Hence, it induces oscillations about the desired position. As oscillations become larger and larger, the robot will become unstable and finally fall down.

Response of the proportional term is given as:

proportional_term = kp * error

Derivative Term

The derivative term is proportional to rate of change of error. It is not dependent on the current magnitude of error and is incapable of minimizing the error on its own. The purpose of the derivative term is to anticipate the future behavior of the error. Thus, it helps in achieving the desired position much faster. However, a high magnitude of derivative gain (Kd) can make the robot very jittery.

Derivate response is given by:

derivative_term = kd * (current_error – previuos_error)/sampleTime

Finally, the desired output response can be achieved by tuning these PD gains properly to control their individual contributions in the output. The output of the PD control algorithm is given as: **PD_output = proportional_term + derivative_term**

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt}$$

4.4.2.1 Cascaded P-D controller Architecture

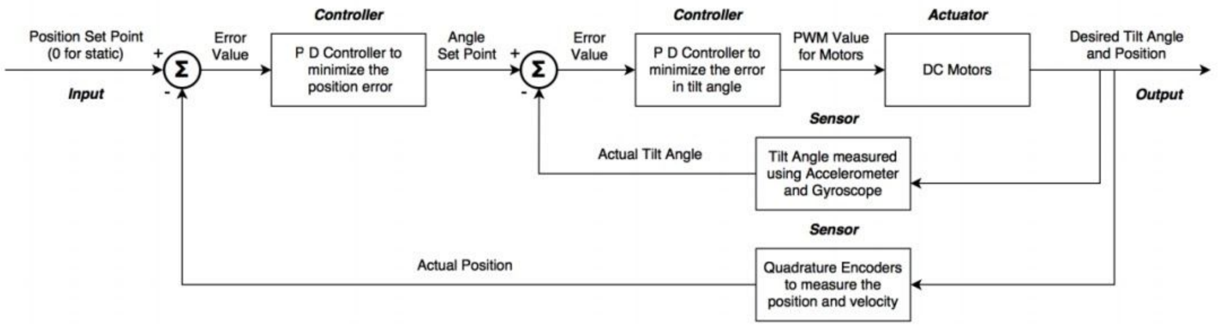

Position Set Point (0 for static)
Input
+
-
Error Value
Controller
P D Controller to minimize the position error
Angle Set Point
+
-
Error Value
Controller
P D Controller to minimize the error in tilt angle
PWM Value for Motors
Actuator
DC Motors
Desired Tilt Angle and Position
Output
Sensor
Actual Tilt Angle
Tilt Angle measured using Accelerometer and Gyroscope
Sensor
Actual Position
Quadrature Encoders to measure the position and velocity

Fig 4.11 Cascaded PD controller architecture

The inner PD loop controls the **tilt angle** and the outer PD loop controls the **position** of the robot.

The tilt angle of the balance bot is measured using the accelerometer and the gyroscope fused using a complementary filter. A PD controller is designed to minimize the error between current tilt angle and the desired set point. The output of the PD controller is directly used to set the PWM of the motors to control its speed and the direction (forward or backward) based on the direction of tilt.

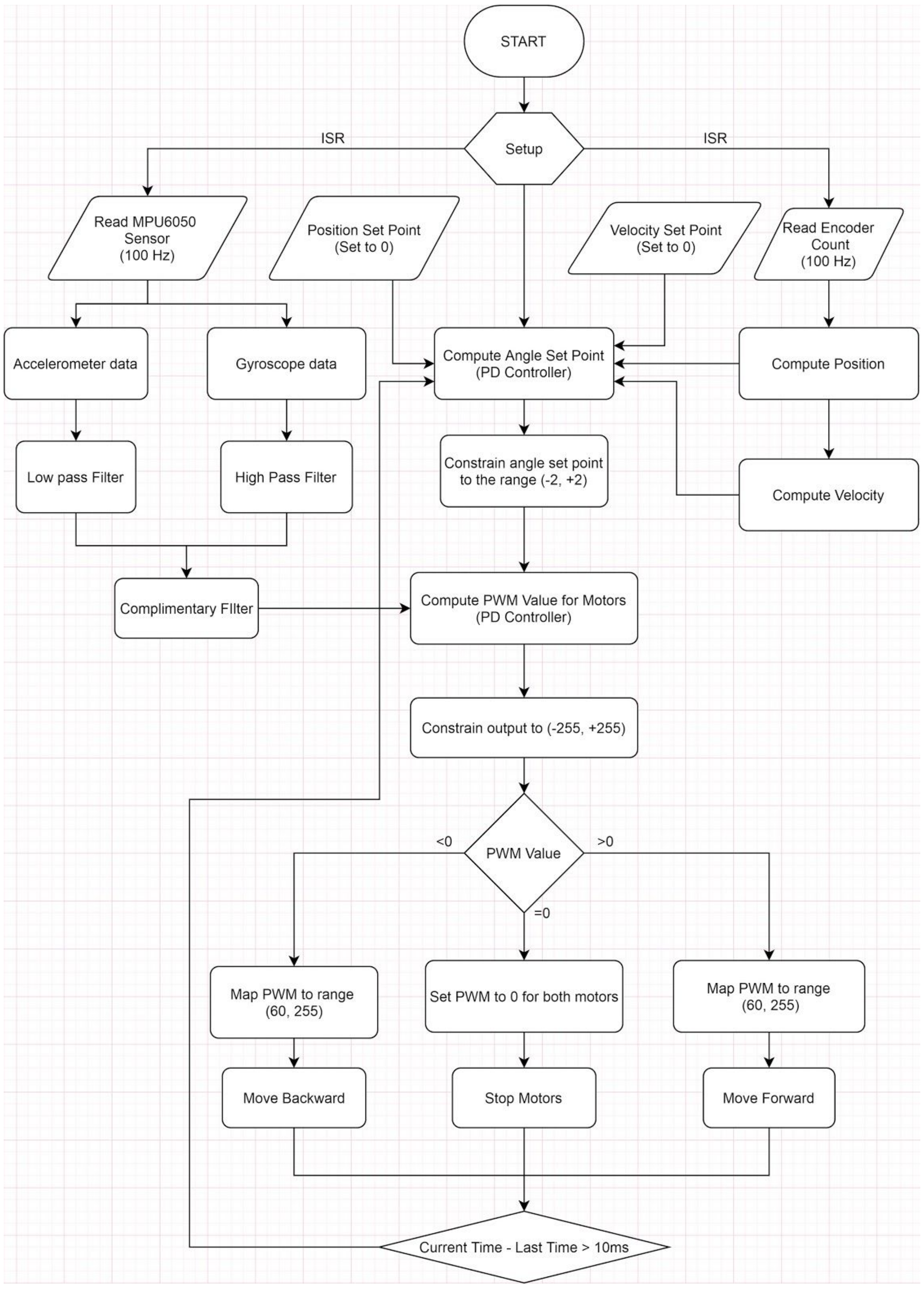
Another PD controller is designed which minimizes the error between the current position and the desired position (zero for static). The output of this PD controller changes the Angle Set Point. Initially consider the angle set point to be zero. If the robot moves linearly for a given distance, the position PD controller will change the angle set point in the opposite direction (say +2 deg). This in turn makes the robot now balance itself at +2 deg instead of 0 degrees (upright). As the position error approaches to zero, the angle set point also approaches zero and thus the motor will automatically slow down and position error is corrected.

4.5 SOFTWARE DESIGN

The tilt angle from the IMU sensor is computed in parallel every 10ms with the help Timer 3 ISR. The encoders' channel A is connected to interrupt pins and encoder count is incremented/decremented based on the state of channel B inputs. Timer 1 ISR is used to compute the RPM of the motors every 100ms. PD computation task is scheduled for every 10ms.

First, the encoder PD output is computed to determine the angle set point. Process variables of this PD loop are the position and velocity (RPM of motors) of the robot. Velocity is measured in order to prevent the robot from accelerating too much and falling down.

Once the angle set point is computed, the angle PD loop is run to compute the PWM values for the motors. This output is limited to the range (-255, 255). If the output is positive, the motors are set to rotate in the forward direction and for negative output, motors are set to rotate in the backward direction in order to balance the robot as required. The PWM values for the motors are constrained to the range (60, 255) so as to not operate in the dead zone of the motors. Finally, the entire process runs in a while loop.


START
Setup
ISR
ISR
Read MPU6050 Sensor (100 Hz)
Position Set Point (Set to 0)
Velocity Set Point (Set to 0)
Read Encoder Count (100 Hz)
Accelerometer data
Gyroscope data
Compute Angle Set Point (PD Controller)
Compute Position
Low pass Filter
High Pass Filter
Constrain angle set point to the range (-2, +2)
Compute Velocity
Complimentary FIlter
Compute PWM Value for Motors (PD Controller)
Constrain output to (-255, +255)
PWM Value
<0
>0
=0
Map PWM to range (60, 255)
Set PWM to 0 for both motors
Map PWM to range (60, 255)
Move Backward
Stop Motors
Move Forward
Current Time - Last Time > 10ms

CHAPTER 5:

RESULTS

5.1 CHASSIS

We selected **Stackable design** for our Bot

5.1.1 Tier design

- Material selected: Acrylic sheet
- Thickness: 3mm
- Designed using AutoCAD software
- Fabricated with Laser cutter

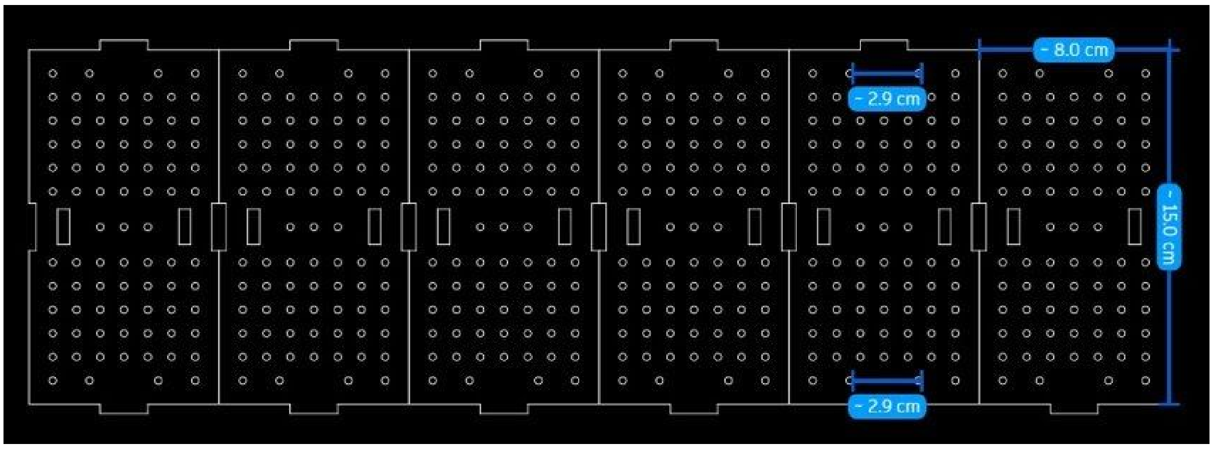


Fig 5.1 2D CAD drawing of the Chassis

5.1.2 Placement of Components

A total of 4 tiers are used, starting from Tier 0 (lowest tier) to Tier 3 (upper most tier)

Tier 0:

Lower side – Motors and MPU6050 Sensor

Upper side – Li-Po Battery

Tier 1:

Upper side – Power distribution circuit and Motor driver

Tier 2:

Upper side – Arduino Mega 2560 and XBee module

Tier 3:

No components are placed on this Tier. This tier is used for support and to increase the height of the Bot

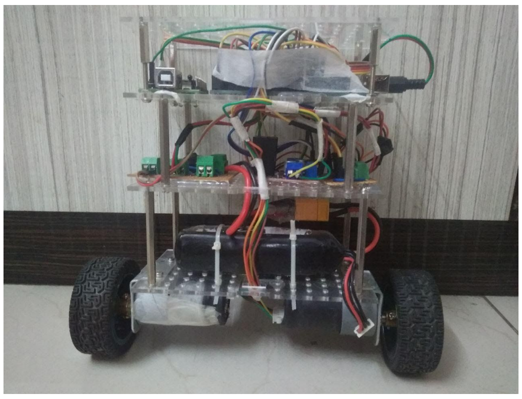


Fig 5.2 Front view of the Bot

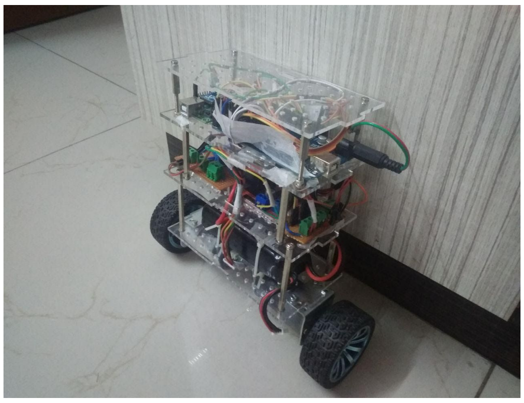


Fig 5.3 Diagonal view of the Bot

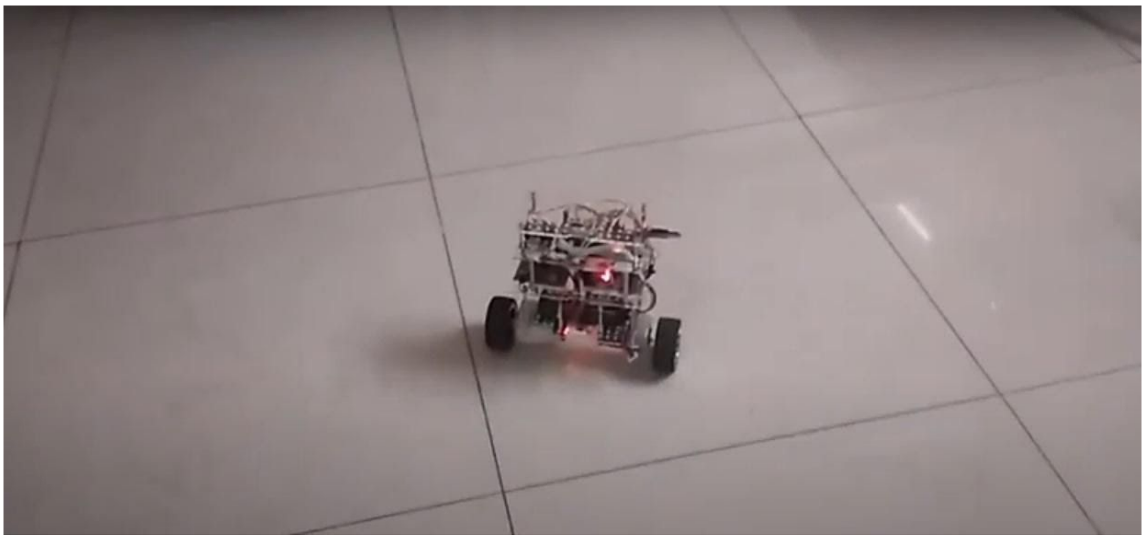


Fig 5.4 The Bot balancing in 2 wheels

5.2 SIGNAL PROCESSING

5.2.1 Accelerometer

Calculation of the Pitch angle using Accelerometer is done using the formula

acc = arcTan(Ax / Az)

First we pass the signals Ax and Az through Low pass filters, then we find the angle acc **f_cut= 5, dt = 0.01, alpha = 0.76**

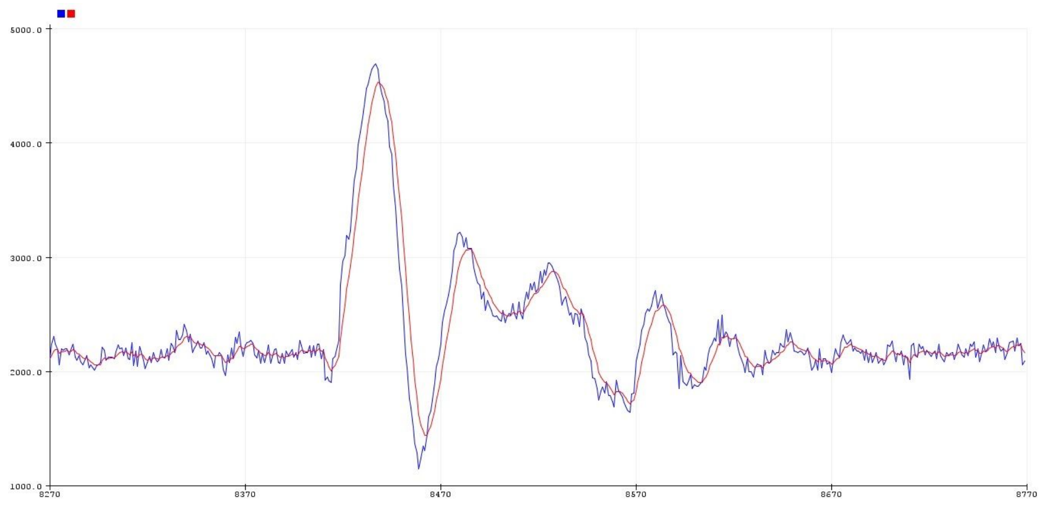


Fig 5.5 ***Ax*** *signal -- Blue - Raw signal , Red - filtered signal*

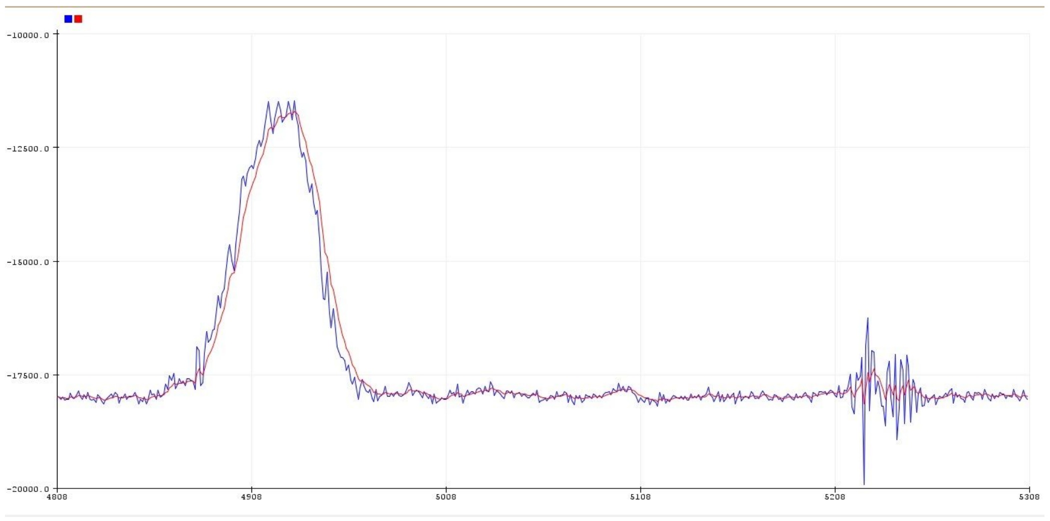


Fig 5.6 ***Az*** *signal -- Blue - Raw signal , Red - filtered signal*

5.2.2 Gyroscope

The gyroscope data **Gy** is taken

f_cut= 5, dt = 0.01, alpha = 0.76

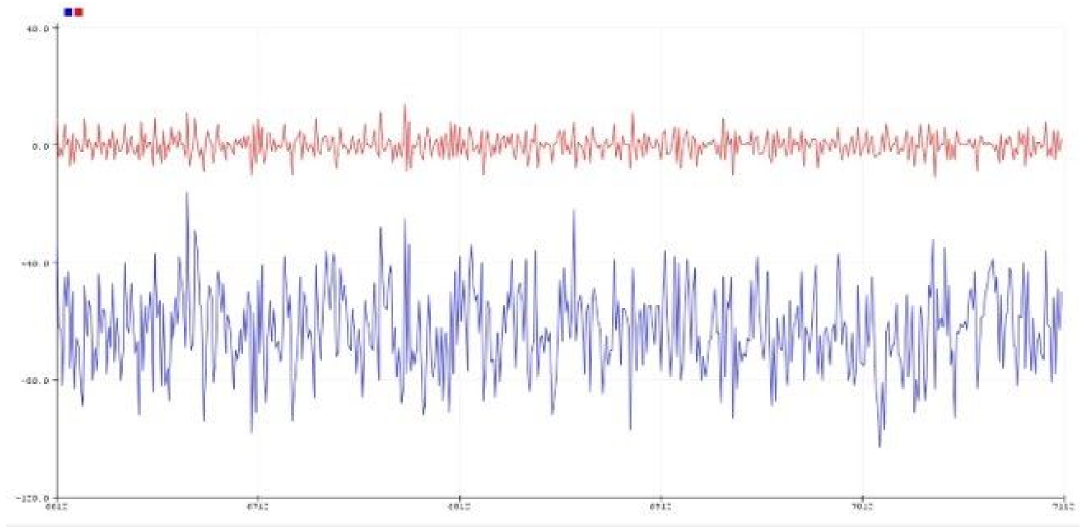


Fig 5.7 ***Gy*** *signal -- Blue - Raw signal , Red - filtered signal*

5.2.3 Pitch Angle

angle = (1- alpha)(angle + gyro * dt) + (alpha)(acc)

First reading is the angle as obtained from gyroscope integration. Second reading is the one from accelerometer.

alpha = 0.15

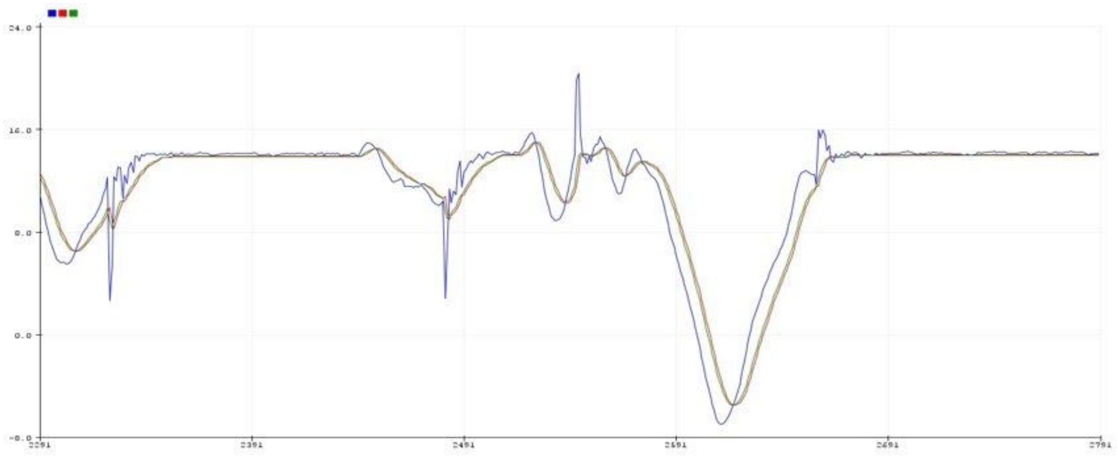


Fig 5.8 Blue – acc, Red – gyro, Green – pitch angle

5.3 CONTROLLER

5.3.1 Linear Quadratic Regulator (LQR)

5.3.1.1 Simulation

```
mp = 0.985
 r = 0.032
mw = 0.04
g = 9.804
l = 0.067
A=[ 0    1                    0                          0
       0    0          (-1*mp*g)/(r*(mp+4*mw))           0
       0    0                    0                          1
       0    0      (g*(mp+2*mw))/(l*(mp+4*mw))       0 ]

B=[                0
          2/(r^2*(mp+4*mw))
                   0
          -1/(l*r*(mp+4*mw))   ]

Q=[ 2  0   0   0
      0   1   0   0
      0   0  10  0
      0   0   0   5  ]
R=1
State vector = [ phi    phi_dot    theta    theta_dot ]
Initial condition= [0  0  3.14/4  0]           final condition =[0  0  0  0]
```

5.3.1.2 Simulation Results

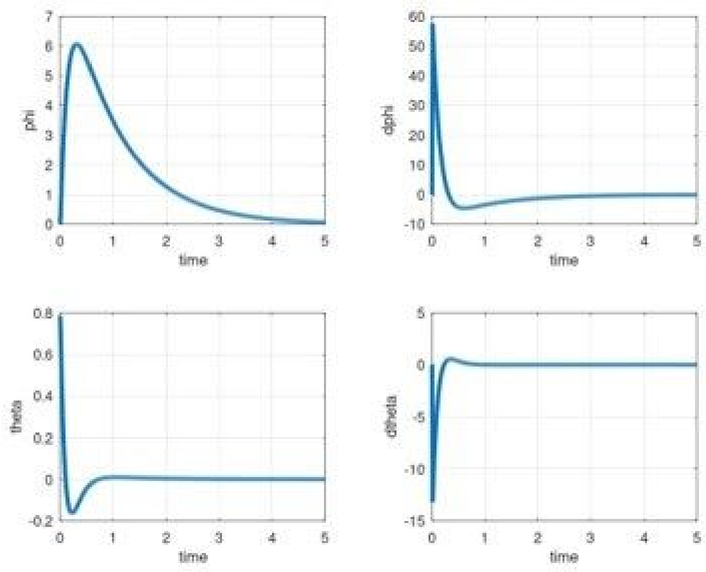


Fig 5.9 State variables behaviour

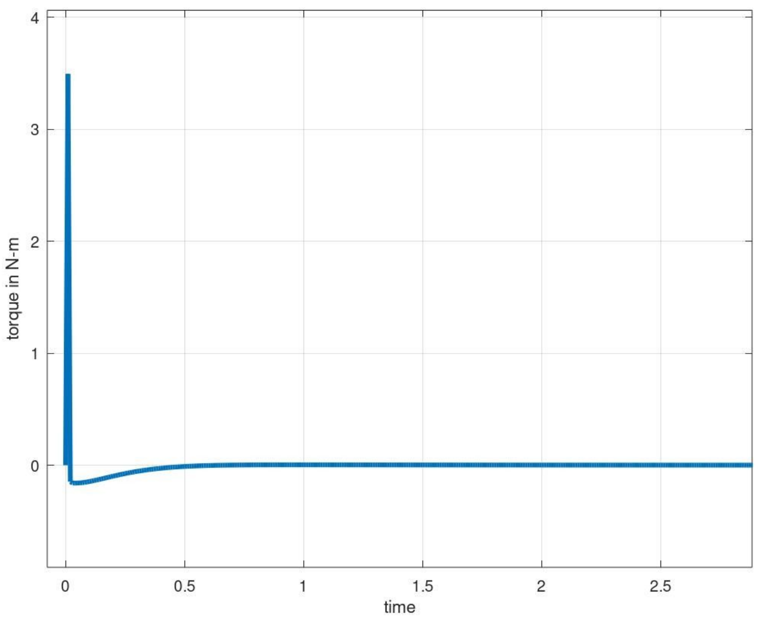


Fig 5.10 Torque produced by the controller

K matrix obtained:

K = [-0.047409 -0.059344 -4.450723 -0.513819]

Simulation Peak Torque = **3.494 Nm**

Practical Peak Torque = 0.833*2 = **1.66 Nm**

As we can see that the practical peak torque that our motors can generate is lesser than half of the peak torque that is required in the simulation. Due to this physical constraint of the motors used, we didn't get good results with the LQR controller.

5.3.2 Cascaded P-D Controller

5.3.2.1 Controller Tuning Results

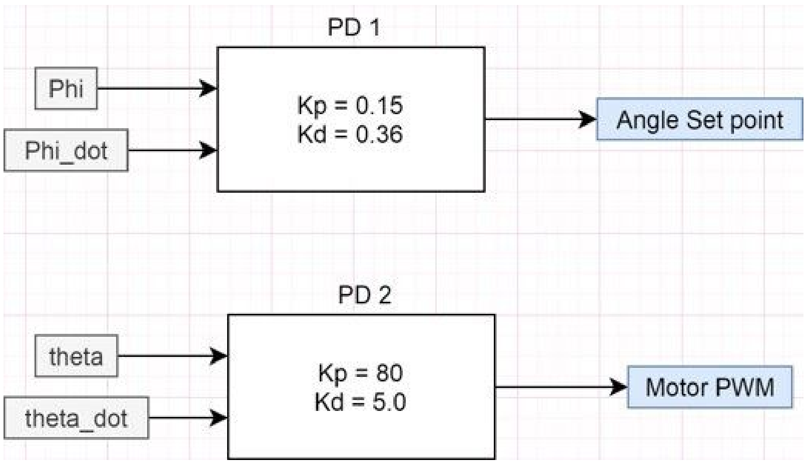


Fig 5.11 PD controller blocks

After a lot of tuning, we could achieve stability of the bot in terms of tilt angle.

The K values were found to be:

PD1 :

Kp = 0.15, Kd = 0.36

PD2:

Kp = 80, Kd = 5.0

While tuning the controller in the Bot, we used the XBee module. XBee module helped us to wirelessly change the values of Kp and Kd and made the tuning process faster. It is highly recommended to use a wireless communication module during controller tuning.

CHAPTER 6:

CONCLUSION

1. The stacked design proved to be efficient in terms of stability because of the increased y-axis depth.
2. Placing the battery at the lowest tier improved the response time of the Bot and increased the maximum recoverable tilt angle.
3. However while implementing the LQR controller, we noticed that the maximum torque produced by our Bot's motors did not meet the required criteria in the simulation.
4. We switched to cascaded PD design and noticed that the Bot is able to achieve Tilt Angle Control but not Position control. Hence the Bot is able to Balance itself on two wheels using the cascaded PD controller.

To improve the Bot further,

1. We can reduce the overall weight of the Bot by 3D printing few components such as studs
2. We can use higher torque motor

CHAPTER 7:

REFERENCES

[1] Changkai Xu , Ming Li and Fangyu Pan “The System Design and LQR control of a Two-wheels Self-balancing Mobile Robot “

[2] Fengxin Sun, Zhen Yu and Haijiao Yang “A Design for Two-Wheeled Self-Balancing Robot Based on Kalman Filter and LQR”

[3] Wei An and Yangmin Li “Simulation and Control of a Two-wheeled Self-balancing Robot“

[4] Osama Jamil, Mohsin Jamil,Yasar Ayaz and Khubab Ahmad “Modeling, Control of a Two-Wheeled Self-Balancing Robot”

[5] Hau-Shiue Juang and Kai-Yew Lum “Design and Control of a Two-Wheel Self-Balancing Robot using the Arduino Microcontroller Boar