The System Design and LQR control of a Two-wheels Self-balancing Mobile Robot

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***Abstract*—This paper describes the control method and mathematical model of a two-wheels self-balancing mobile robot. The mobile robot designed by author uses two parallel wheels to keep balance and uses a three axes gyroscope as position, velocity and accelerated speed detection sensor. This paper presents the mathematical modeling of two wheels inverted pendulum mobile robot. The dynamic model of the system has been established to design and analyze the control system. The control program and the controller LQR are designed in LabVIEW module.**

Keywords-component: two-wheels self-balancing mobile robot, mathematical model, LQR control, LabVIEW

I. INTRODUCTION

Compared with traditional multiwheel mobile robot, the two-wheels self-balancing mobile robot is characterized by the ability of balancing on its coaxial two wheels and the zero radius rotation. In recent years, the number of research projects, journal papers, and doctoral thesis about two-wheel mobile robot has rapidly increased [1-7]. The kinematic model of a two-wheeled self-balancing mobile robot is built by the reference [1]. The reference [2] proposes a novel second order sliding mode controller using twisting algorithm, disturbance observer and dynamics of double inverted pendulum. The reference [3] presents the linear modeling of two wheels inverted pendulum mobile robot and the robustness of sliding mode control. PID control system is implemented to pilot the motors so as to keep the system in equilibrium in the reference [4]. The reference [5] presents the design of two-wheel self-balanced electric vehicle based on MEMS. The reference [6] uses the LQR control strategy to help the robot to keep balance. A commercially available system, SEGWAY [7], which is a man-machine vehicle, has been invented by Dean Kamen.

This paper is successful in overcoming the current researches' insufficiency and achieving its aims to balance a two-wheel autonomous self-balancing robot based on the inverted pendulum model. The design of the control routine employs LQR control strategy to help the robot to keep balance.

II. MODELING

The simplified model of our two-wheels self-balancing mobile robot is a removable first order inverted pendulum. It is a mobile platform, which can achieve balance by its self-regulation. For cognizing the mobile robot more exactly, we need to research its mathematical model. The establishment of more exact model is the premise of designing control system and control algorithm.

A. Mechatronics system design:

The robotic systems include mechanical system, control system and sensing system.

National Instruments embedded hardware platforms is selected for our control system. A compactRIO embedded system is an industrial controlling and acquisition system. It contains one embedded processor (cRIO-9014 real-time PowerPC embedded controller for CompactRIO, 2 GB disc on chip non-volatile storage, 128 MB DRAM), one eight-slot reconfigurable chassis (cRIO-9104 8-slot, 3 M gate reconfigurable chassis for CompactRIO) containing a user-programmable FPGA, and hot-swappable industrial I/O modules (NI 9205, NI 9401, NI 9263, NI 9411).

The three axes gyroscope 3DM-GX1 from MicroStrain corp. is built into the mobile robot for our sensing system. 3DM-GX1 gyroscope is a miniature system that detects angle, angular velocity, velocity and acceleration of the mobile robot. The sensor signal of 3DM-GX1 contains complete temperature compensation, so that it ensures the signal's accuracy in any temperature.

The dynamical system contains two disc type electric machines (Maxon EC 90 flat), two drivers (Accelnet Micro Panel) and batteries (48AH). The Maxon EC 90 flat is a kind of DC brushless servo motor with high torque. The Accelnet Micro Panel is a compact, DC powered digital servo amplifier for position, velocity, torque control of DC brush or brushless motors.

The photo of CAD design and a real mobile robot is shown in fig. 1. The platform structured by aluminum bars contains three layers. In the first layer, there are motors, reducers, batteries, two wheels of vehicle and two small safety wheels in front and back. In the middle layer, there are control module CompactRIO, gyroscope 3DM-GX1 and drivers Accelnet Micro Panel. We can place other components on the top layer afterwards, such as robotic arms.

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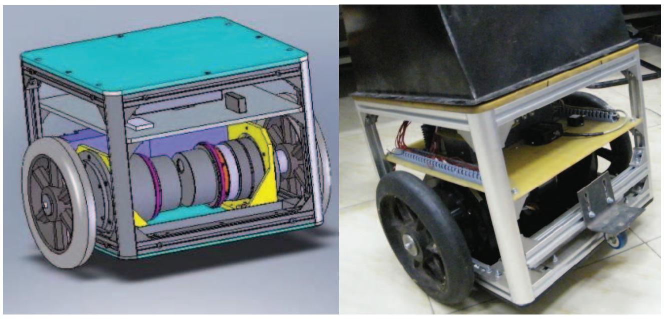

Figure 1. Photo of CAD design and a real mobile robot.

B. Mathematical modeling:

The methods of the two-wheels mobile robot's mathematical modeling are divided into Newtonian mechanics kinematics modeling and Lagrange's equation kinetic modeling. This paper uses the Lagrange's equation to model system kinetic equation through the way of system capacity.

The mobile robot can be divided into two wheels and a body. Their simplified model is shown in fig. 2.

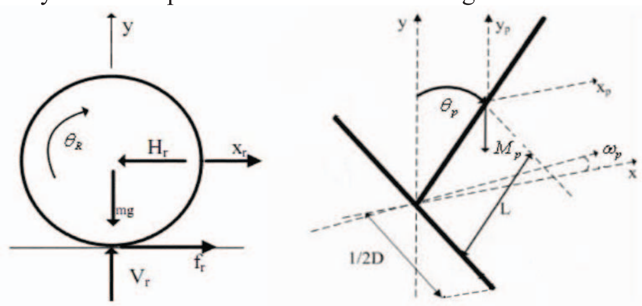

Figure 2. Simplified model of mobile robot system

In figure 2,

$M_R = M_{RL} = M_{RR}$: mass of the two wheels;

M_p : mass of the body;

R : radius of the wheel;

L : distance between body's centre of mass and z axis;

D : distance between two wheels;

θ_p : inclined angle of body;

ω_p : angular velocity of body round z axis;

δ : rotation angle of body round y axis;

$\dot{\delta}$: angular velocity of body round y axis;

θ_{RL}, θ_{RR} : rotation angle of two wheels;

X_{RL}, X_{RR} : displacement of two wheels;

$\dot{X}_{RM}$: speed of body;

The mathematical expression of Lagrange's equation is shown in (1).

$$\frac{d}{d_t}\left(\frac{\partial T}{\partial_{qk}}\right) - \frac{\partial T}{\partial_{qk}} = Q_k, (k = 1,2,3...) \quad (1)$$

In equation (1), T is the total kinetic energy of system, q_k is the generalized coordinate of system, Q_k is the generalized moment of system。

The translation kinetic energy of two wheels is shown in (2).

$$T_1 = \frac{1}{2} M_{RL} X_{RL}^2 + \frac{1}{2} M_{RR} X_{RR}^2 \quad (2)$$

The rotation kinetic energy of two wheels is shown in (3).

$$T_2 = \frac{1}{2} J_{RL} \theta_{RL}^2 + \frac{1}{2} J_{RR} \theta_{RR}^2 \quad (3)$$

The translation kinetic energy of body's centre of mass is shown in (4).

$$T_3 = \frac{1}{2} M_p ((\theta_p L \cos\theta_p + \dot{X}_{RM})^2 + (-\theta_p L \sin\theta_p)^2) \quad (4)$$

The rotation kinetic energy of body round the axis which is through centre of mass and parallel to z axis is shown in (5).

$$T_4 = \frac{1}{2} J_{p\theta} \theta_p^2 \quad (5)$$

The rotation kinetic energy of body round y axis is shown in (6).

$$T_5 = \frac{1}{2} J_{p\theta} \delta^2 \quad (6)$$

As a result, the total kinetic energy of mobile robot system is calculated in (7).

$$T = T_1 + T_2 + T_3 + T_4 + T_5 = \frac{1}{2} M_{RL} X_{RL}^2 + \frac{1}{2} M_{RR} X_{RR}^2 + \frac{1}{2} J_{RL} \theta_{RL}^2 + \frac{1}{2} J_{RR} \theta_{RR}^2 + \frac{1}{2} M_p ((\theta_p L \cos\theta_p + \dot{X}_{RM})^2 + (-\theta_p L \sin\theta_p)^2 + \frac{1}{2} J_{p\theta} \theta_p^2 + \frac{1}{2} J_{p\theta} \delta^2 \quad (7)$$

We select $q_1 = \theta_{RL}$, $q_2 = \theta_{RR}$ and $q_3 = \theta_p$ as variables of motion state of system $(\theta_{RL}, \theta_{RR}, \theta_p)$. Obviously, the system described by these three variables with holonomic constraints can determine position of mass point uniquely. Therefore we select $\theta_{RL}, \theta_{RR}, \theta_p$ as generalized coordinate of the system. The corresponding generalized force contains: the torque of left wheel C_L, the torque of right wheel C_R, the torque of vehicle body ($M_p gL \sin\theta_p - C_L - C_R$).

The mathematical model of two wheels and body system is obtained by substituting the total kinetic energy, generalized coordinate and generalized force into the Lagrange's equation. The three equations are shown in (8, 9, and 10)

$$(M_{RL}R^2 + J_{RL} + \frac{1}{4} M_p R^2 + \frac{R^2}{D^2} J_{p\delta})\ddot{\theta}_{RL} + (\frac{1}{4} M_p R^2 - \frac{R^2}{D^2} J_{p\delta})\ddot{\theta}_{RR} + \frac{1}{2} M_p RL \ddot{\theta}_p = C_L \quad (8)$$

$$(M_{RR}R^2 + J_{RR} + \frac{1}{4} M_p R^2 + \frac{R^2}{D^2} J_{p\delta})\ddot{\theta}_{RR} + (\frac{1}{4} M_p R^2 - \frac{R^2}{D^2} J_{p\delta})\ddot{\theta}_{RL} + \frac{1}{2} M_p RL \ddot{\theta}_p = C_R \quad (9)$$

$$\frac{1}{2} M_p RL \ddot{\theta}_{RL} + \frac{1}{2} M_p RL \ddot{\theta}_{RR} + (J_{p\theta} + M_p L^2)\ddot{\theta}_p = M_p gL\theta_p - (C_L + C_R) \quad (10)$$

According to these equations, assumed $\sin\theta_p \approx \theta_p$,

$\cos\theta_p \approx 1$, ignored high order term of θ_p, we can get the linearization equation in (11)

$$\begin{pmatrix} \dot{X}_{RM} \\ \dot{V}_{RM} \\ \dot{\theta}_p \\ \dot{\omega}_p \\ \dot{\delta} \\ \ddot{\delta} \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} X_{RM} \\ V_{RM} \\ \theta_p \\ \omega_p \\ \delta \\ \dot{\delta} \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ B_2 & B_2 \\ 0 & 0 \\ B_4 & B_4 \\ 0 & 0 \\ B_6 & -B_6 \end{bmatrix} \begin{pmatrix} C_L \\ C_R \end{pmatrix} \quad (11)$$

In matrix (11),

$$A_{23} = \frac{-M_p^2 L^2 g}{M_p J_{p\delta} + 2(J_{p\theta} + M_p L^2)(M_R + J_R / R^2)}$$

$$A_{43} = \frac{M_p^2 gL + 2M_p gL(M_R + \frac{J_R}{R^2})}{M_p J_{p\theta} + 2(J_{p\theta} + M_p L^2)(M_R + \frac{J_R}{R^2})}$$

$$B_2 = \frac{\frac{J_{p\theta} + M_p L^2}{R + M_p L}}{M_p J_{p\theta} + 2(J_{p\theta} + M_p L^2)(M_R + \frac{J_R}{R^2})}$$

$$B_4 = \frac{-\frac{R+L}{R} M_p - 2(M_R + \frac{J_R}{R^2})}{M_p J_{p\theta} + 2(J_{p\theta} + M_p L^2)(M_R + \frac{J_R}{R^2})}$$

$$B_6 = \frac{\frac{D}{2R}}{J_{p\delta} + \frac{D^2}{2R}(M_R R + \frac{J_R}{R})}$$

C. *System Decoupling*

Characteristic equation of DC brushless motor is shown in (12).

$$C_m = K_m (U - K_e \cdot \dot{\theta}_p) / R \qquad (12)$$

The whole system model of the two-wheels self-balancing mobile robot is obtained by substituting the equation (12) into the matrix (11). Then we can decouple the system; divide the matrix into two independent equations: one describes the state of self-balancing and another describes the state of turn round. The system decoupling equations are shown in (13, 14, and 15).

$$\begin{pmatrix} \dot{X}_{RM} \\ \dot{V}_{RM} \\ \dot{\theta}_p \\ \dot{\omega}_p \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-2K_m K_e B_2}{R} & A_{23} & 2K_m K_e B_2 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2K_m K_e B_4}{R} & A_{43} & 2K_m K_e B_4 \end{bmatrix} \begin{pmatrix} X_{RM} \\ V_{RM} \\ \theta_p \\ \omega_p \end{pmatrix} + \begin{bmatrix} 0 \\ K_m B_2 \\ 0 \\ K_m B_4 \end{bmatrix} U_\theta \quad (13)$$

$$\begin{pmatrix} \dot{\delta} \\ \ddot{\delta} \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-DK_m B_6}{R} \end{bmatrix} \begin{pmatrix} \delta \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 \\ K_m B_6 \end{pmatrix} U_\delta \quad (14)$$

$$\begin{pmatrix} U_L \\ U_R \end{pmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} \begin{pmatrix} U_\theta \\ U_\delta \end{pmatrix} \quad (15)$$

We can calculate the control parameters of the independent two subsystems, and then obtain the output torques U_L and U_R of the two system motors.

III. CONTROLLER AND SIMULATION

The target of LQR (Linear Quadratic Regulator) is to keep the mobile robot balance, when the system receipt disturb signals, and minimize the dynamic error and energy consumption.

$$J = \int_0^\infty (x^T Q x + u^T R u) d_t \qquad (16)$$

The equation (16) is function matrix of LQR. We select $Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, R=1, as the initial value. The system simulation model is shown in fig. 3.

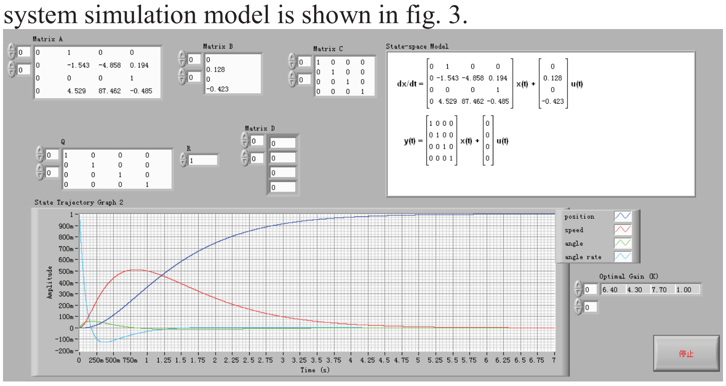

Figure 3. System simulation interface with the initial Q, R

The system achieves state of balance for 5.5 seconds. So we regulate the proportion between Q_{11} and Q_{33} for many times. The convergence time of the system has shortened into two seconds. We finally select $Q = \begin{bmatrix} 300 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, R=1. The system simulation model at this time is shown in fig. 4.

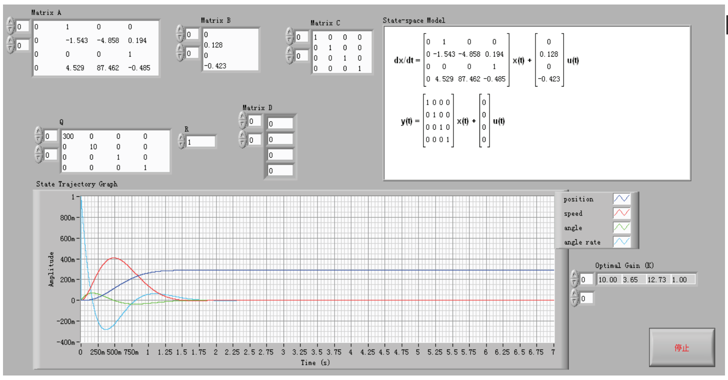

Figure 4. System simulation interface with the final Q,R

The system feedback matrix is $K_2 = [10 \quad 3.65 \quad 12.73 \quad 1]$ at this time.

IV. CONCLUSION

The technologies developed of the two-wheels self-balancing mobile robot consist of the robot structure design, system mechanical model build, LQR motion controller design, and simulation of the system states.

Future work will be focused on higher speed motion and higher degree of accuracy motion control in space. More and more useful function will be developed.

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