Why is Compression Needed?

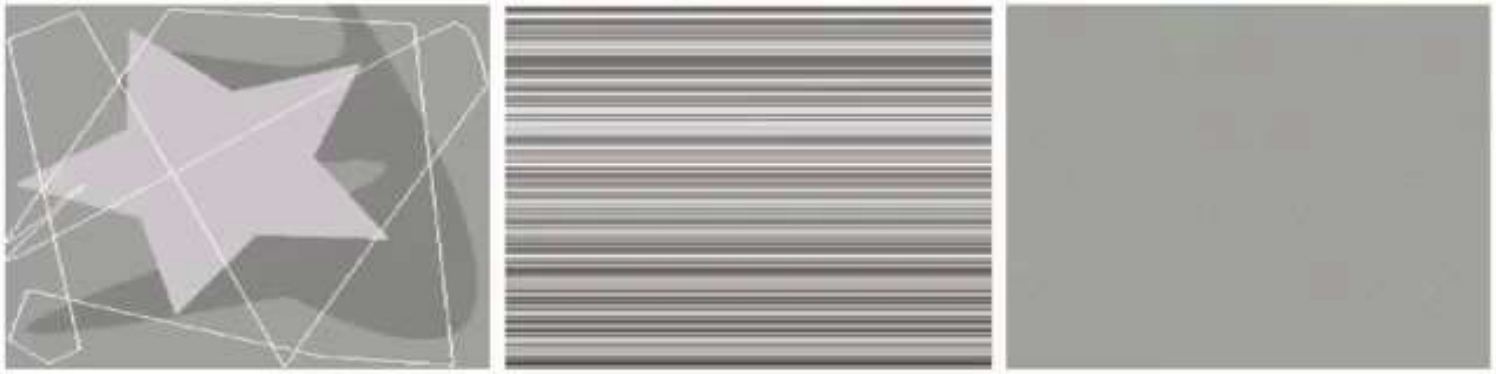
$1000 \times 1000 \times 24 \times 30 \times 60 \times 120 =$

very large number

Chapter 8

Image Compression

The reasons we can compress:



a b c

FIGURE 8.1 Computer generated 256 × 256 × 8 bit images with (a) coding redundancy, (b) spatial redundancy, and (c) irrelevant information. (Each was designed to demonstrate one principal redundancy but may exhibit others as well.)

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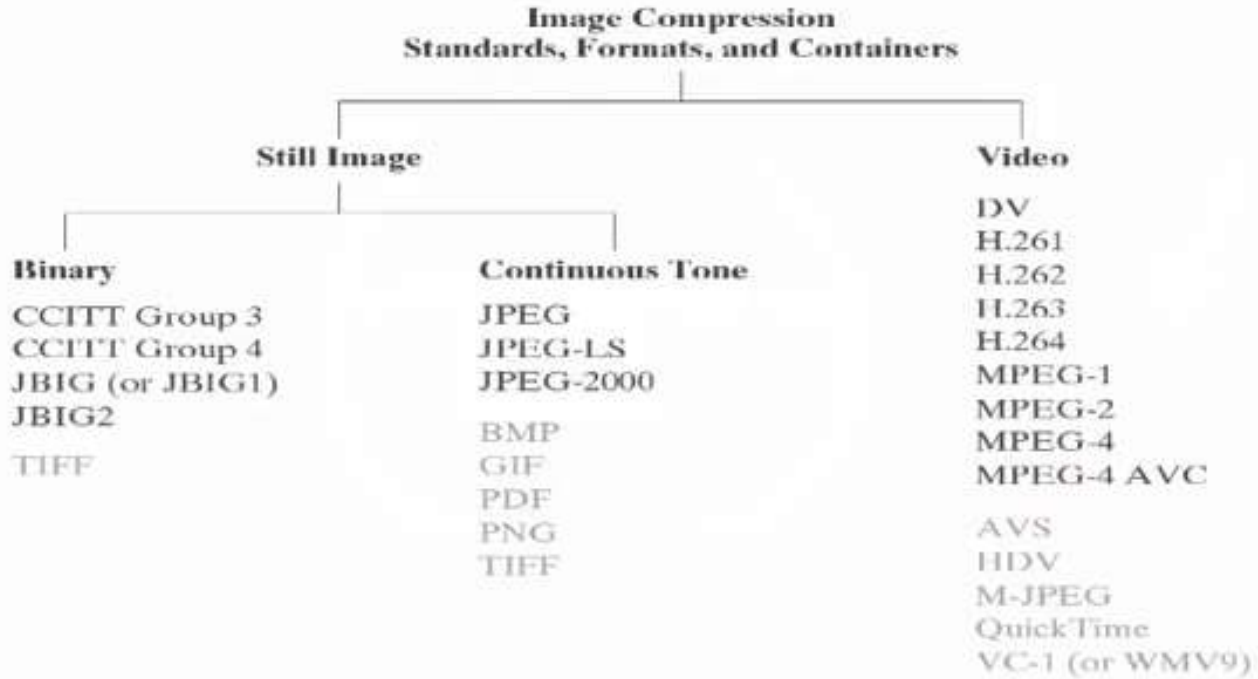
Image Compression



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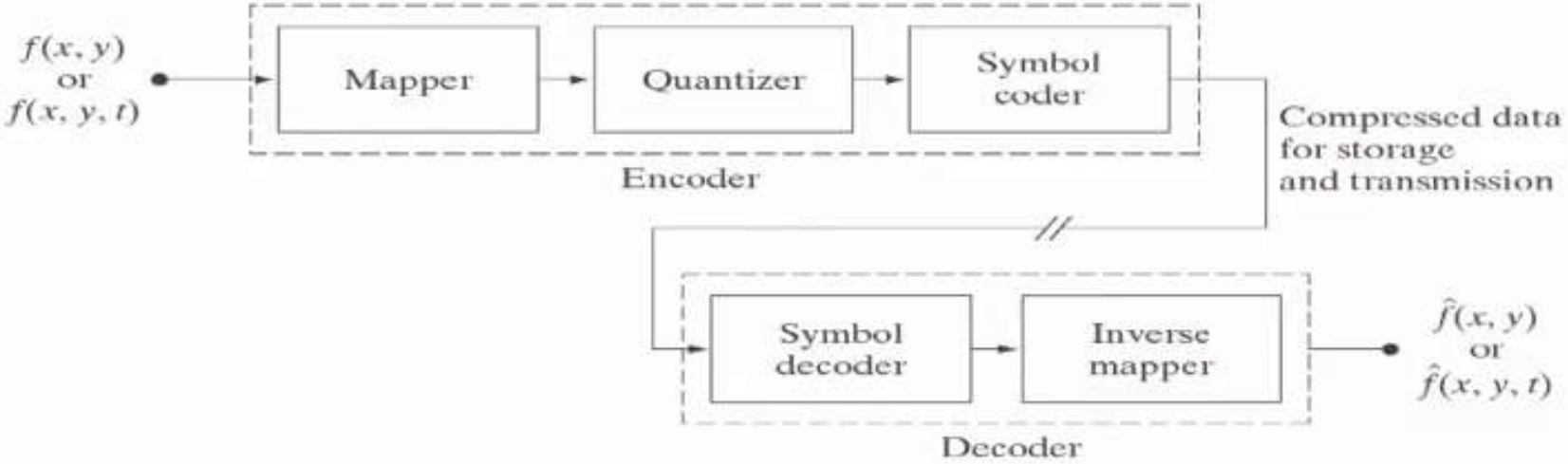
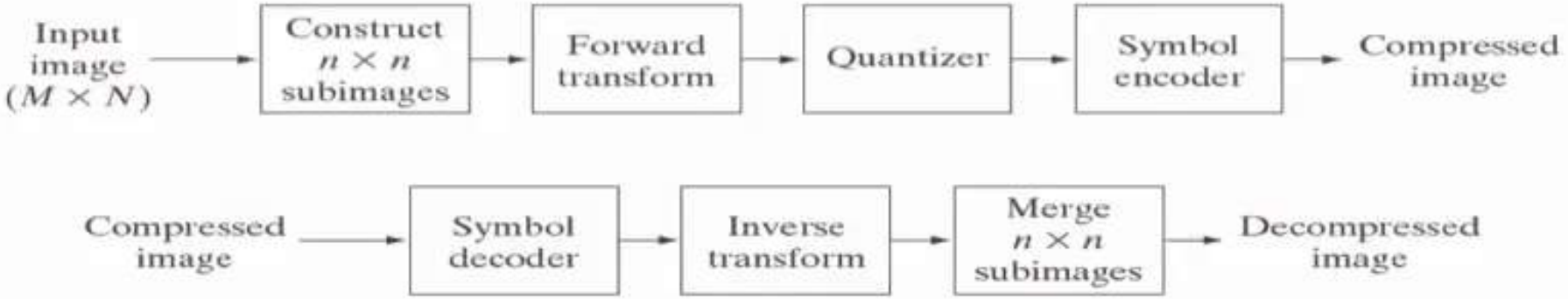


Chapter 8

Image Compression

JPEG



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JPEG

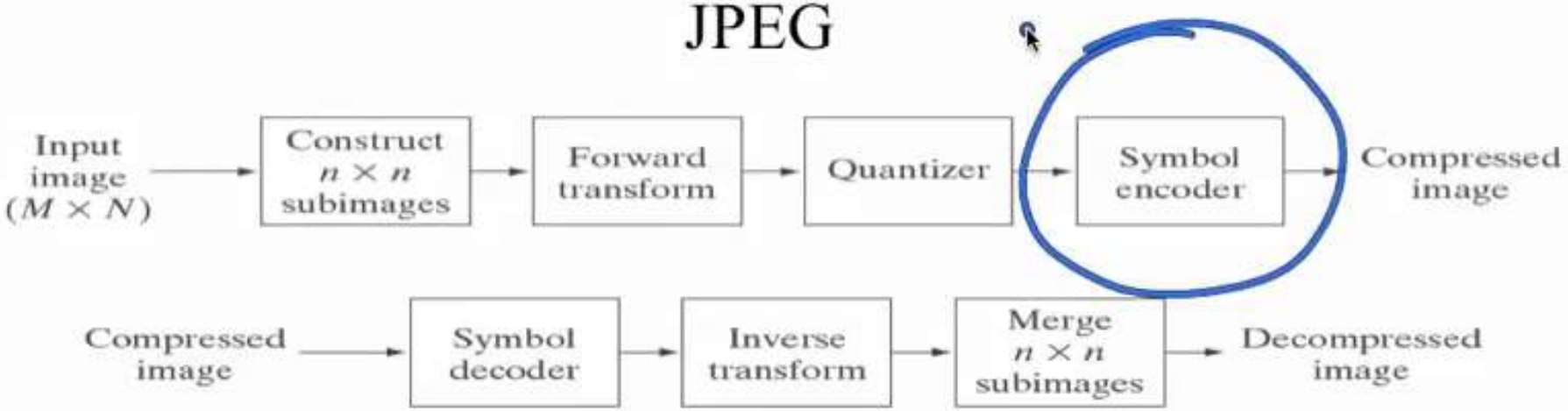

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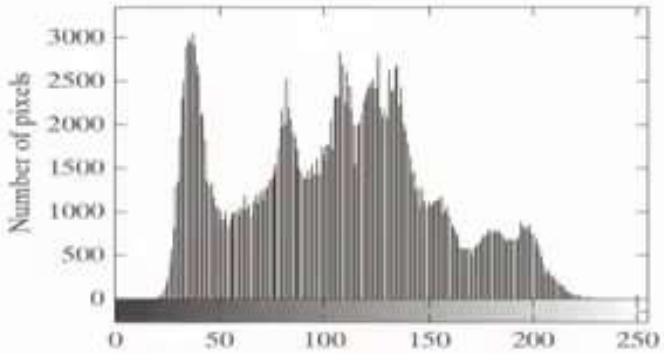
Image Compression

Are all pixels equal?



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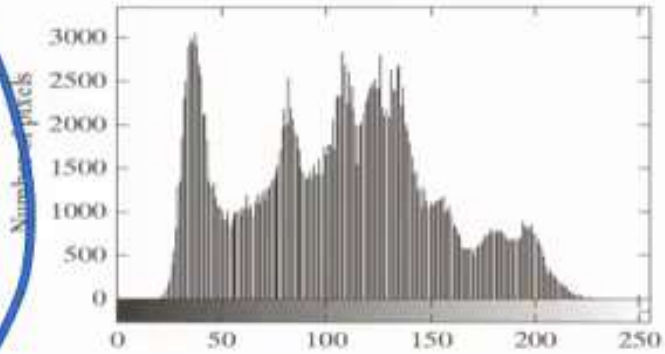
Are all pixels equal?



Lena

Histogram

Chapter 8

Image Compression

Are all pixels/symbols equal?

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

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Image Compression

Are all pixels/symbols equal?

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

$0.25 \times 2 + 0.47 \times 1 + 0.25 \times 3 + 0.03 \times 3 =$

1.81

Huffman Coding

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

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Image Compression

Original source			Source reduction							
Symbol	Probability	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

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Image Compression

Original source			Source reduction							
Symbol	Probability	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

$$\text{Entropy} = H = -\sum_{\text{symbols}} p(s) \log_2 p(s)$$

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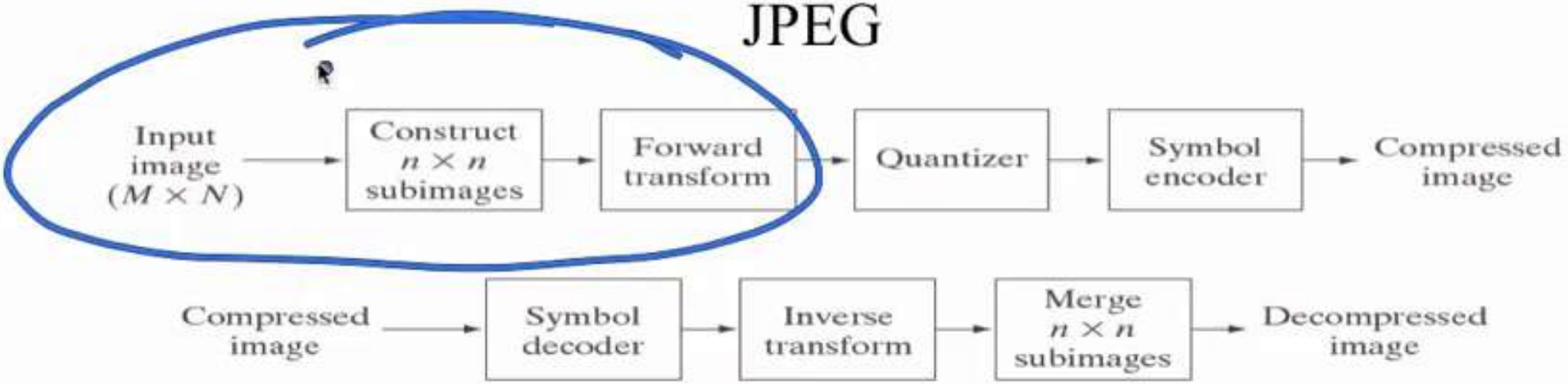
Image Compression

JPEG



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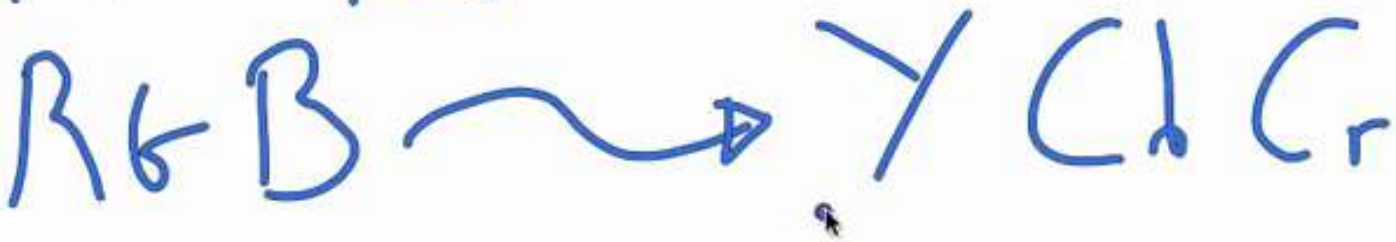

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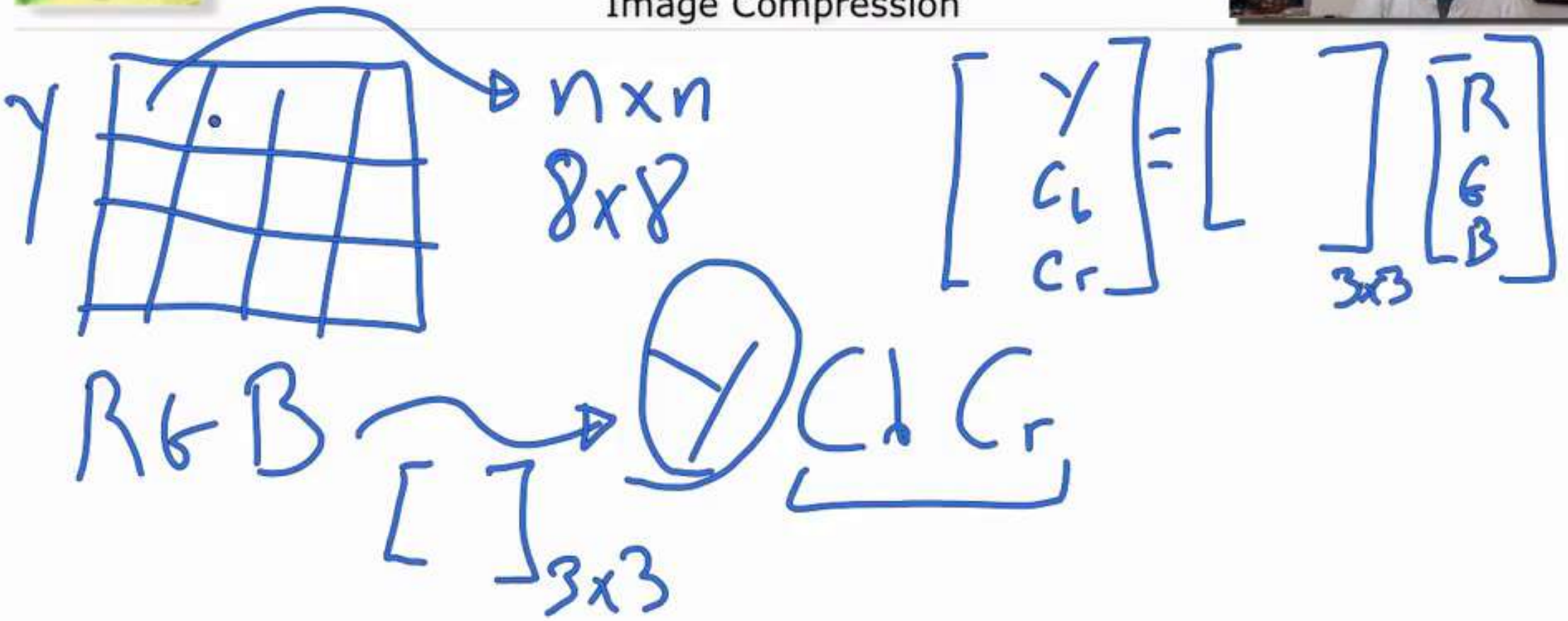

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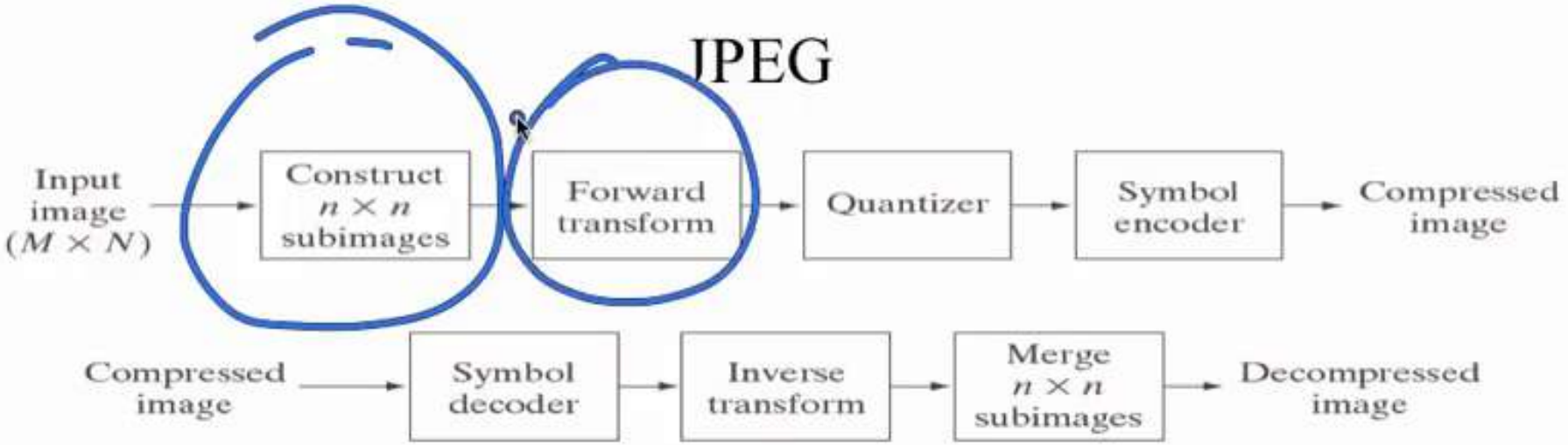

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Image Compression

MSE = Mean Square Error

$$MSE = \left[\frac{1}{\text{\# of pixels}} \sum_{\text{pixels}} (\hat{f} - f)^2 \right]^{1/2}$$

n x n Kahunen-Loève

KLT

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Image Compression

$$T(u,v) = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} f(x,y)\, r(x,y,u,v)$$

$n \times n$

$$f(x,y) = \sum_{u=0}^{n-1} \sum_{v=0}^{n-1} T(u,v)\, s(x,y,u,v)$$

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$$r(x,y,u,v) = s(x,y,u,v)$$

$$= \alpha(u)\alpha(v)\cos\left[\frac{(2x+1)u\pi}{2n}\right] \cdot$$

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{n}} & u = 0 \\ \sqrt{\frac{2}{n}} & u \neq 0 \end{cases} \qquad \cos\left[\frac{(2y+1)v\pi}{2n}\right]$$

DCT

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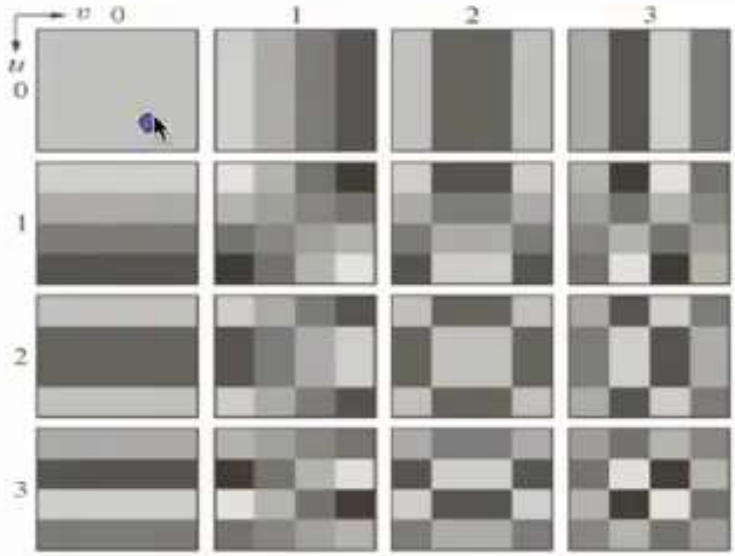
Image Compression

Discrete Cosine Transform

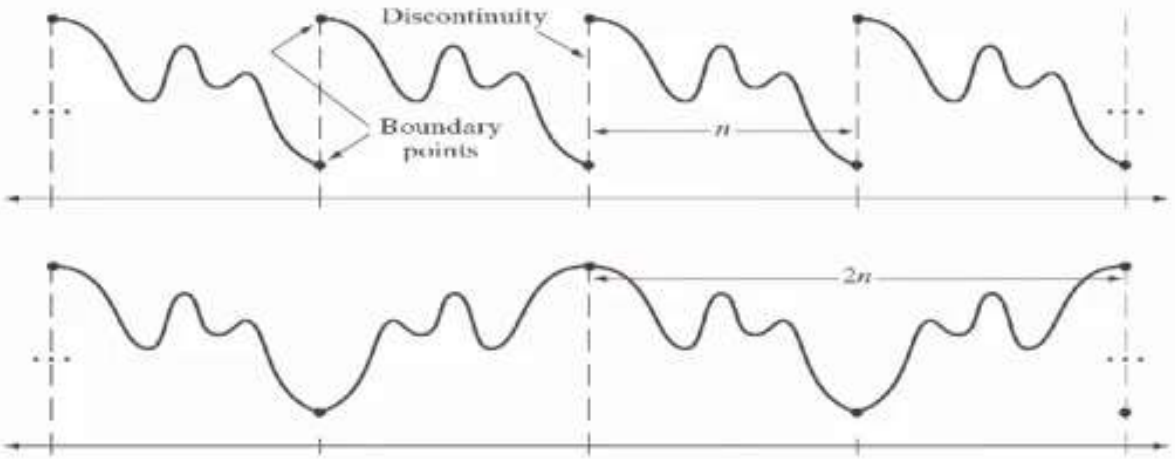


$n = 4$

Chapter 8

Image Compression

Why DCT?



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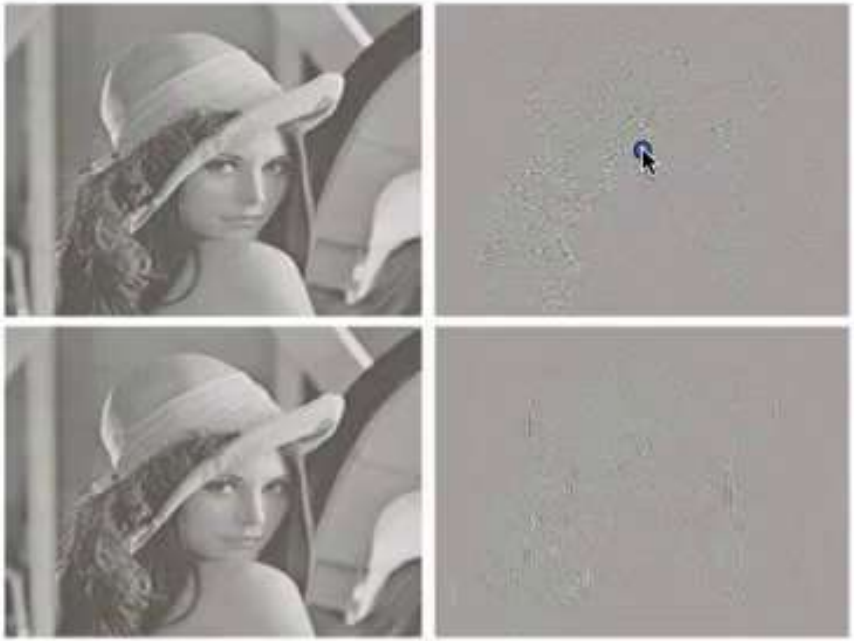

a b
c d

FIGURE 8.28 Approximations of Fig. 8.9(a) using 12.5% of the 8×8 DCT coefficients: (a)—(b) threshold coding results; (c)—(d) zonal coding results. The difference images are scaled by 4.

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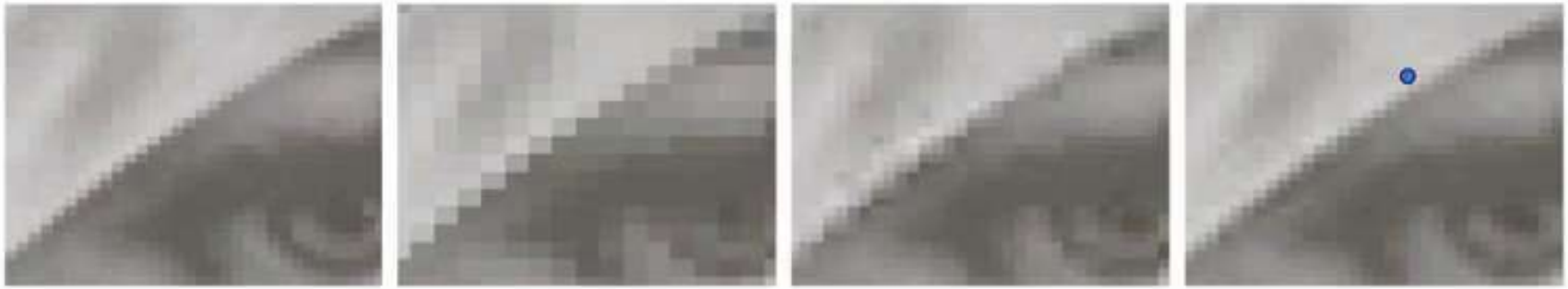

a b c d

FIGURE 8.27 Approximations of Fig. 8.27(a) using 25% of the DCT coefficients and (b) 2×2 subimages, (c) 4×4 subimages, and (d) 8×8 subimages. The original image in (a) is a zoomed section of Fig. 8.9(a).

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Image Compression

JPEG

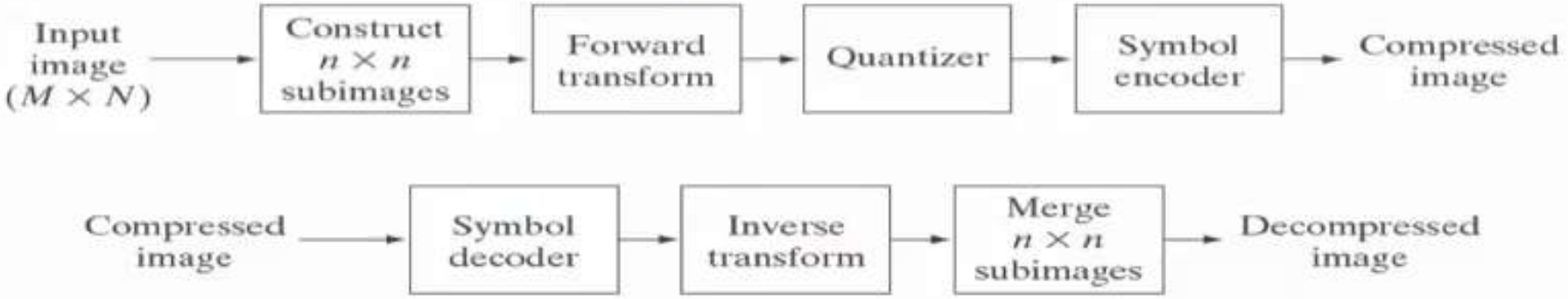

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Image Compression

1	1	1	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

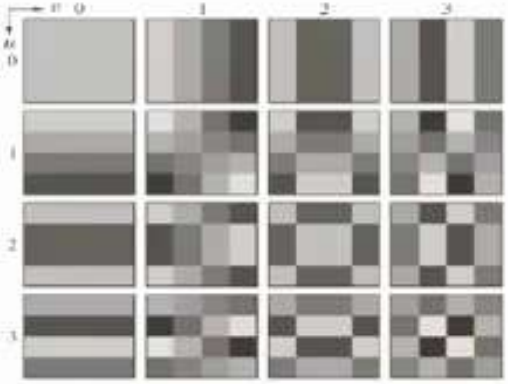
8	7	6	4	3	2	1	0
7	6	5	4	3	2	1	0
6	5	4	3	3	1	1	0
4	4	3	3	2	1	0	0
3	3	3	2	1	1	0	0
2	2	1	1	1	0	0	0
1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0

1	1	0	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

0	1	5	6	14	15	27	28
2	4	7	13	16	26	29	42
3	8	12	17	25	30	41	43
9	11	18	24	31	40	44	53
10	19	23	32	39	45	52	54
20	22	33	38	46	51	55	60
21	34	37	47	50	56	59	61
35	36	48	49	57	58	62	63

a b
c d

FIGURE 8.29
A typical (a) zonal mask, (b) zonal bit allocation, (c) threshold mask, and (d) thresholded coefficient ordering sequence. Shading highlights the coefficients that are retained.



$$T(u,v) = \sum_x \sum_y f(x,y)\, r(x,y,u,v)$$

$$\hat{f}(x,y) = \sum_u \sum_v \hat{T}(u,v)\, r(x,y,u,v)$$

$\hat{T}(0,0)$ $\hat{T}(0,1)$ $\hat{T}(1,0)$

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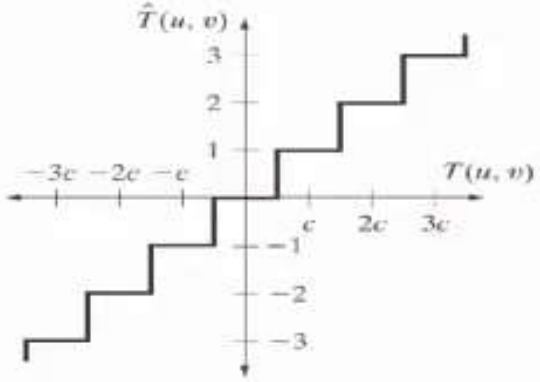

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

a b

FIGURE 8.30 (a) A threshold coding quantization curve [see Eq. (8.2-29)]. (b) A typical normalization matrix.

Students: Good place to take a break if you need it.

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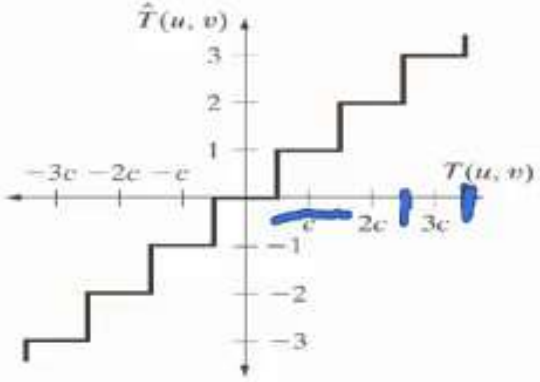

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

a b

FIGURE 8.30 (a) A threshold coding quantization curve [see Eq. (8.2-29)]. (b) A typical normalization matrix.

$$\left[\frac{T(0,10)}{16}\right] \times 16 \simeq \hat{T}(0,10)$$

$$0..15 \simeq 0$$

$$16..31 \simeq 1$$

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Image Compression

52	55	61	66	70	61	64	73
63	59	66	90	109	85	69	72
62	59	68	113	144	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	63	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

EXAMPLE 8.17: JPEG baseline coding and decoding.

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FIGURE 8.31 Approximations of Fig. 8.9(a) using the DCT and normalization array of Fig. 8.30(b): (a) $\mathbf{Z}$, (b) $2\mathbf{Z}$, (c) $4\mathbf{Z}$, (d) $8\mathbf{Z}$, (e) $16\mathbf{Z}$, and (f) $32\mathbf{Z}$.

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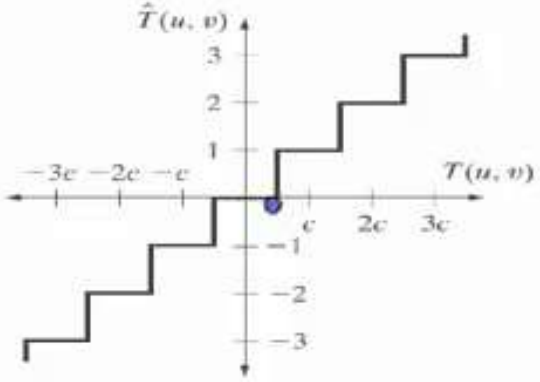
Image Compression



16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

a b

FIGURE 8.30 (a) A threshold coding quantization curve [see Eq. (8.2-29)]. (b) A typical normalization matrix.

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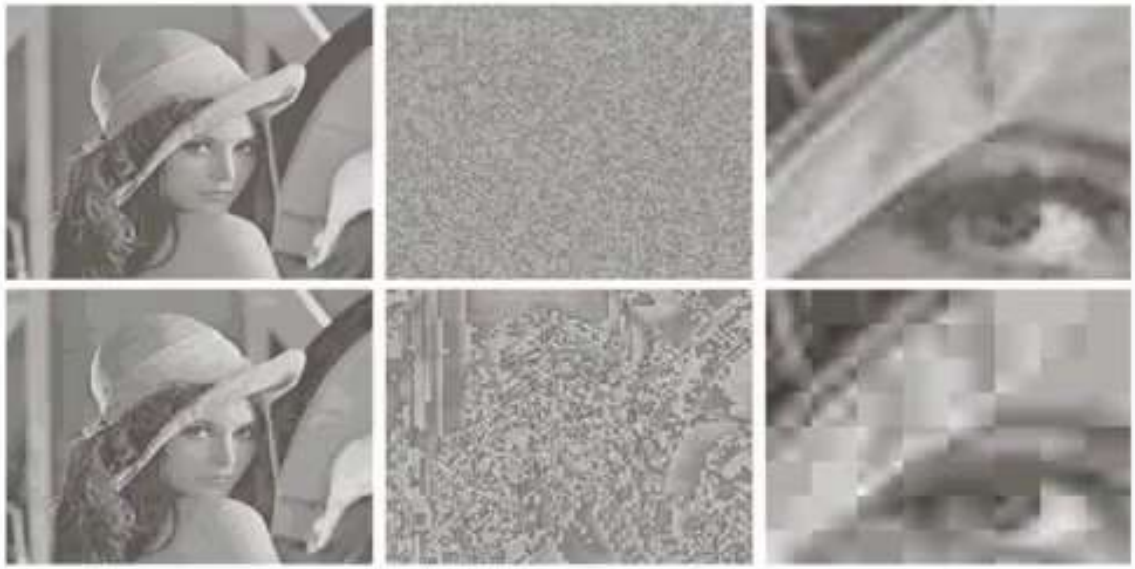


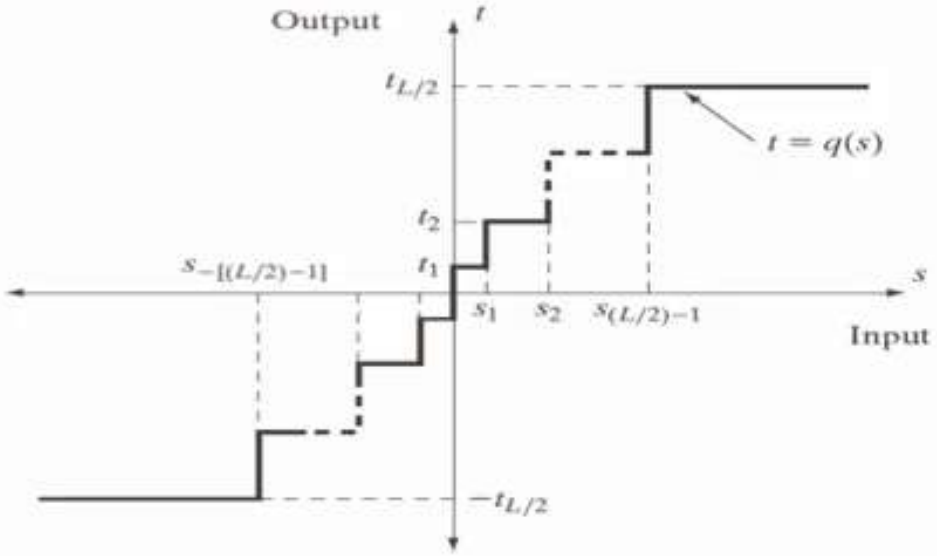
a b c
d e f

FIGURE 8.32 Two JPEG approximations of Fig. 8.9(a). Each row contains a result after compression and reconstruction, the scaled difference between the result and the original image, and a zoomed portion of the reconstructed image.

Chapter 8

Image Compression

Max-Lloyd Optimal Quantizer



Error

$(s_i - t_i)^2$

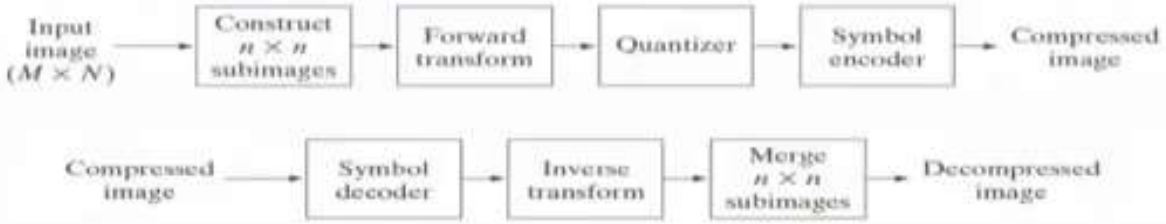
$$\int_{s_{i-1}}^{s_i} (s_i - t_i)\, p(s)\, ds$$

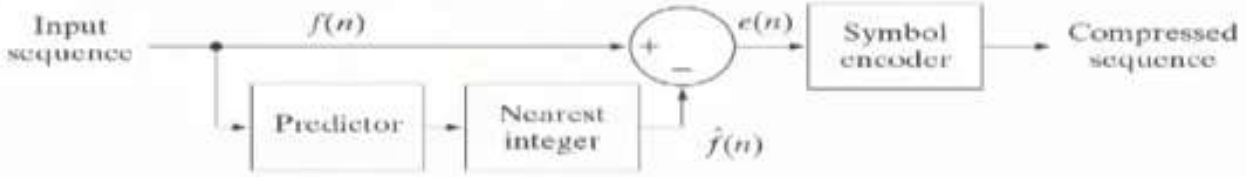
s_i – average t_i, t_{i+1}

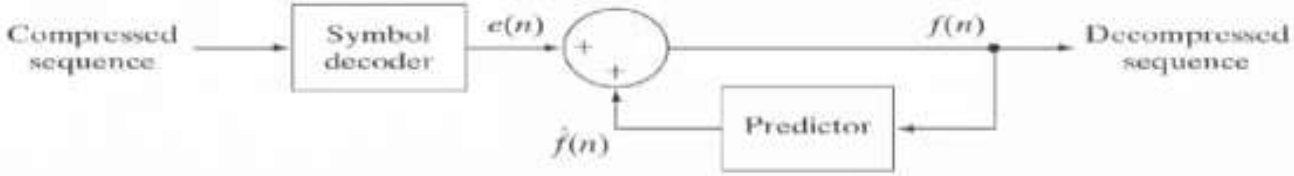
Chapter 8

Image Compression

Predictive lossless compression







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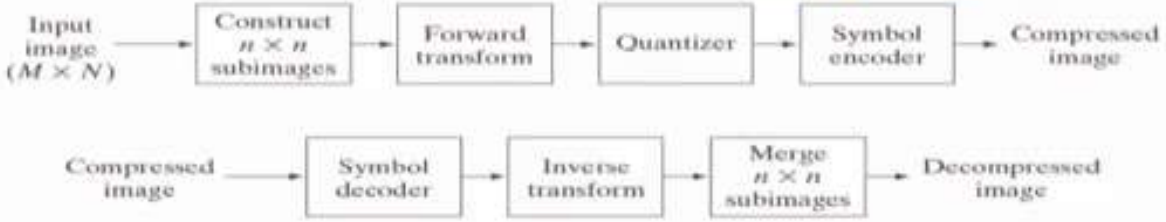
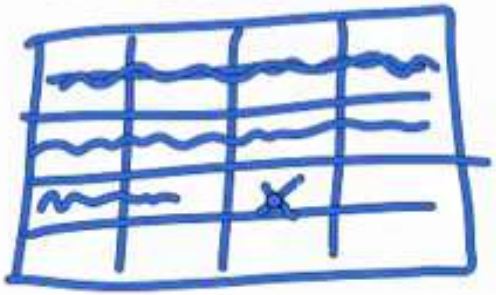
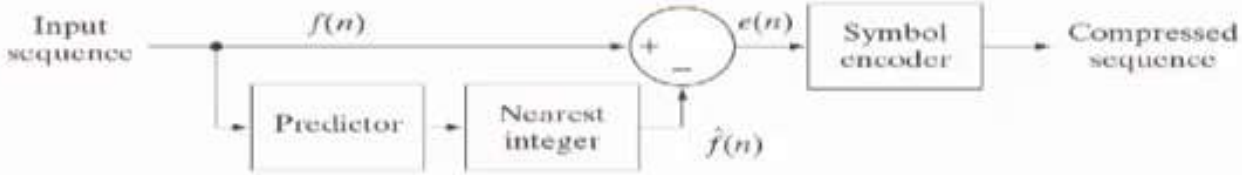
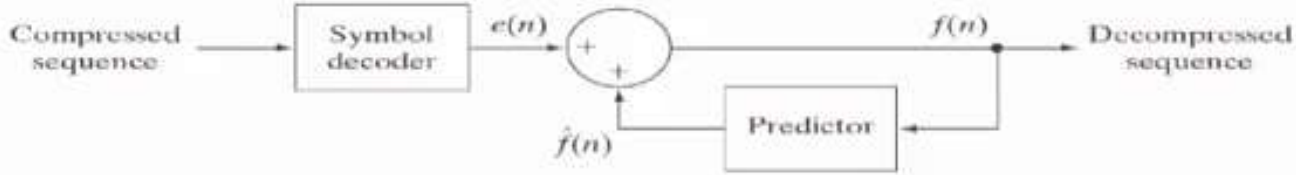
Predictive lossless compression

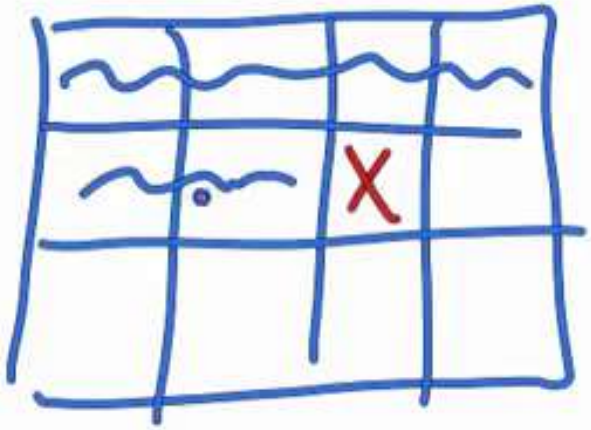








Examples of predictors and JPEG-LS



$f(x-1,y)$

$e = f(x-1,y) - f(x,y)$

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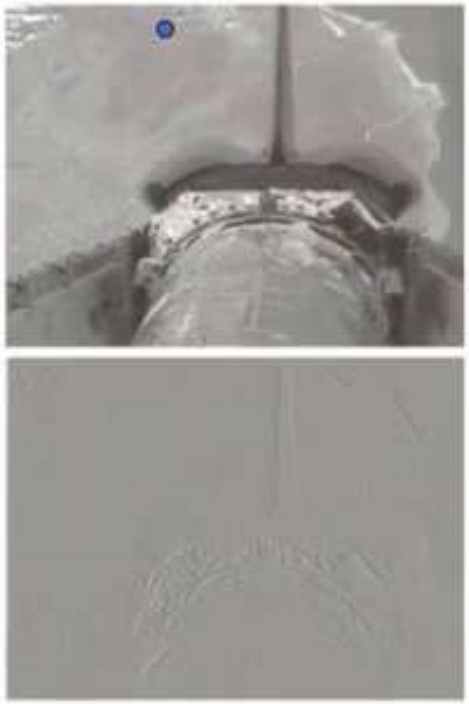
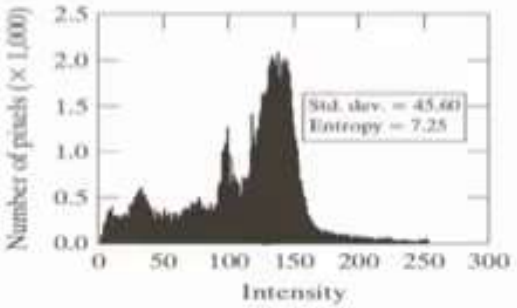
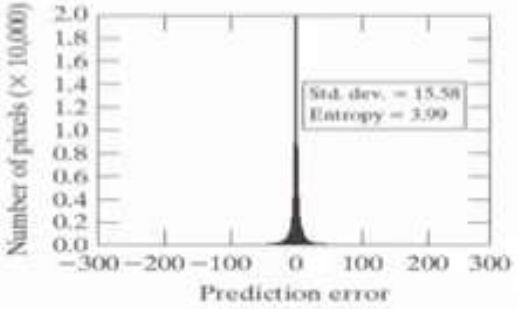
Chapter 8

Image Compression







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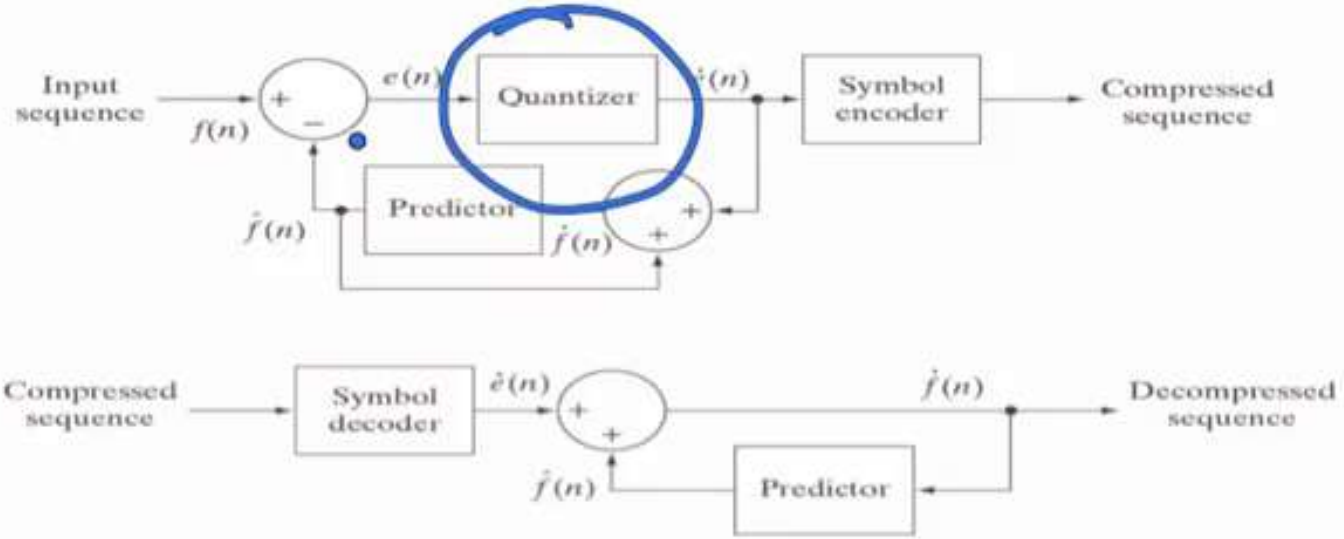

a
b

FIGURE 8.41
A lossless predictive coding model: (a) encoder; (b) decoder.

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Image Compression

Temporal prediction (MPEG)

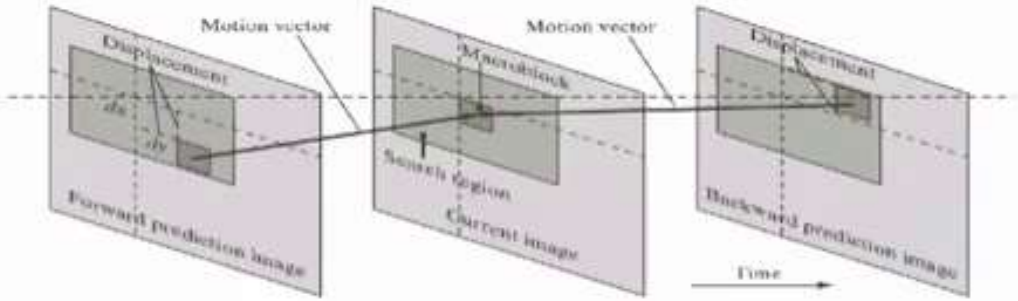

FIGURE 8.36 Macroblock motion specification.

Students: Good place to take a break if you need it.

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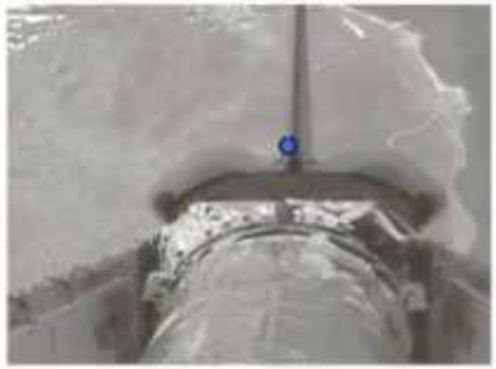

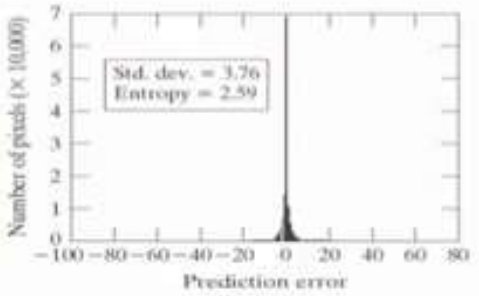

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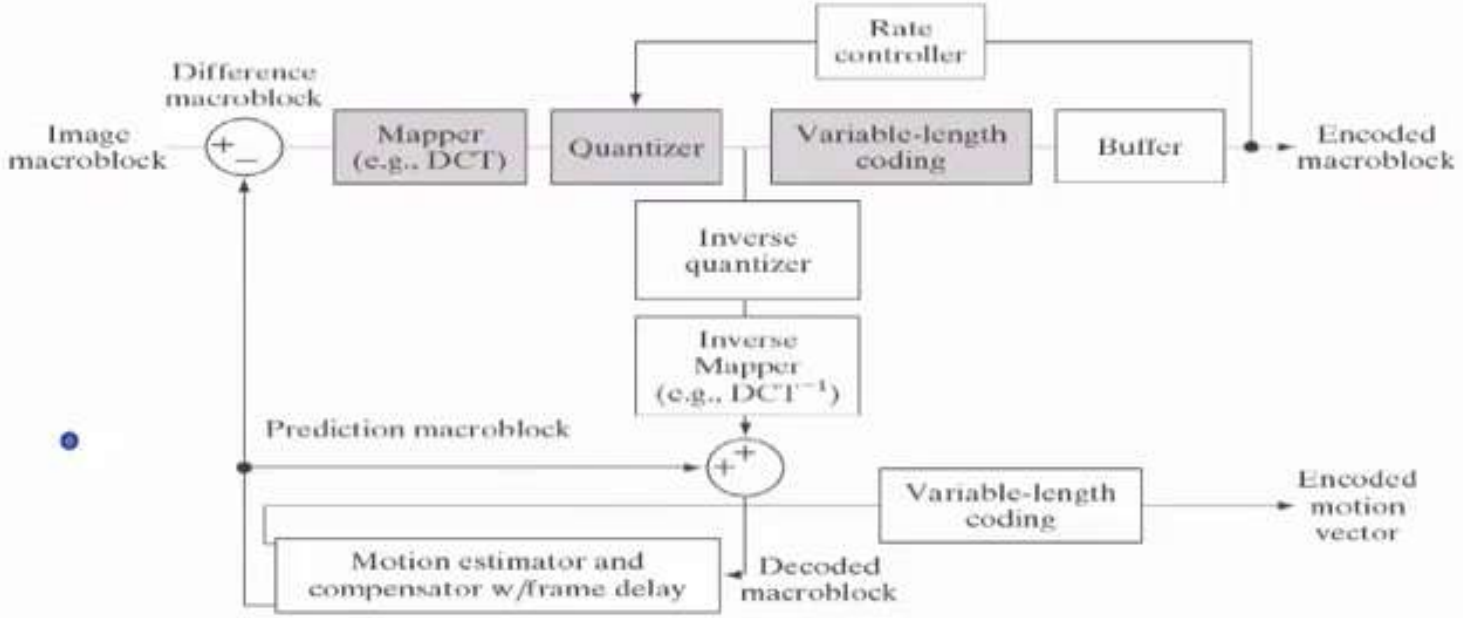
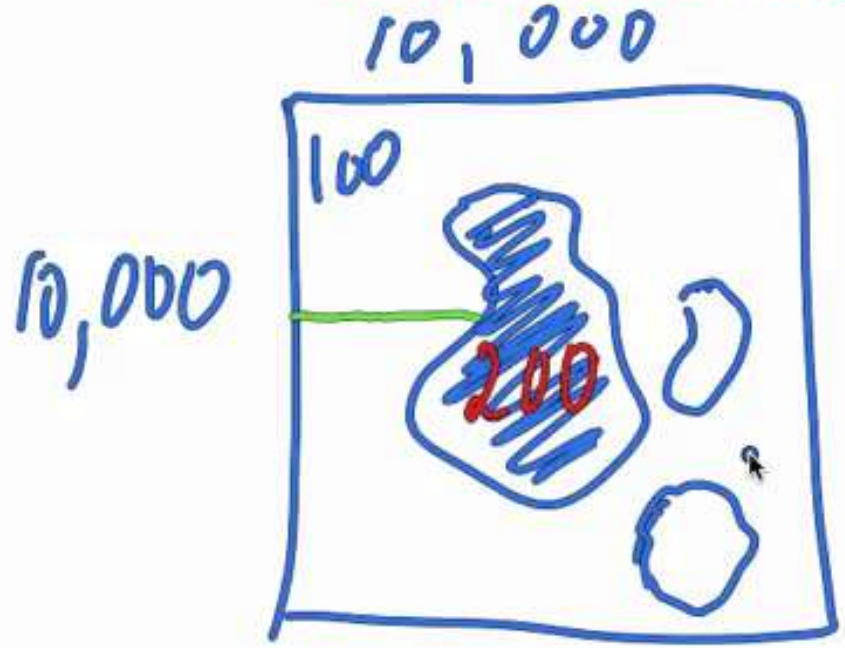


Bonus: Run-length Coding



3000, 1000