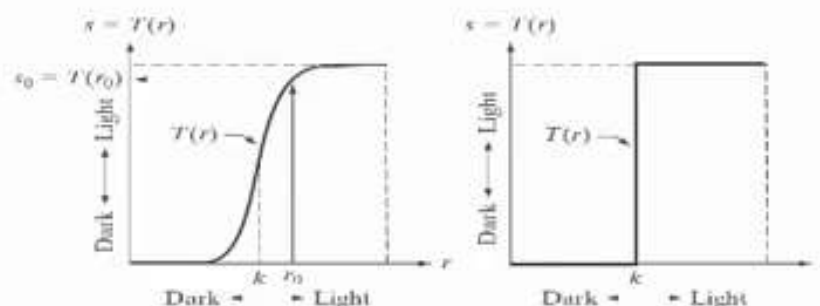




# Digital Image Processing, 3rd ed. Gonzalez & Woods

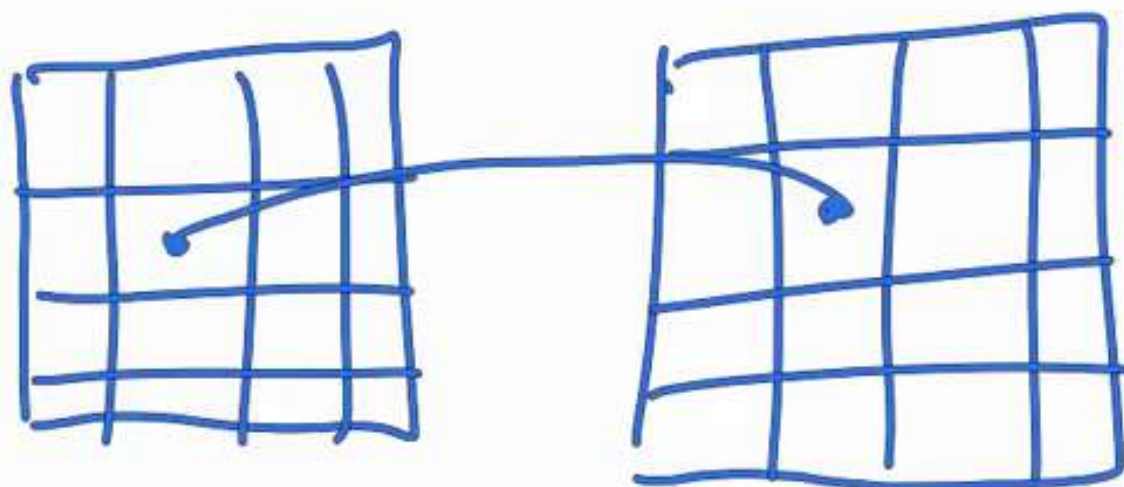
www.ImageProcessingPlace.com

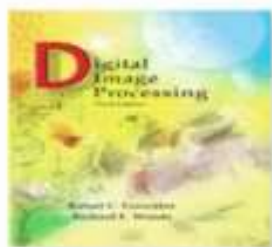
## Chapter 3 Intensity Transformations & Spatial Filtering



a b

**FIGURE 3.2**  
Intensity transformation functions.  
(a) Contrast-stretching function.  
(b) Thresholding function.

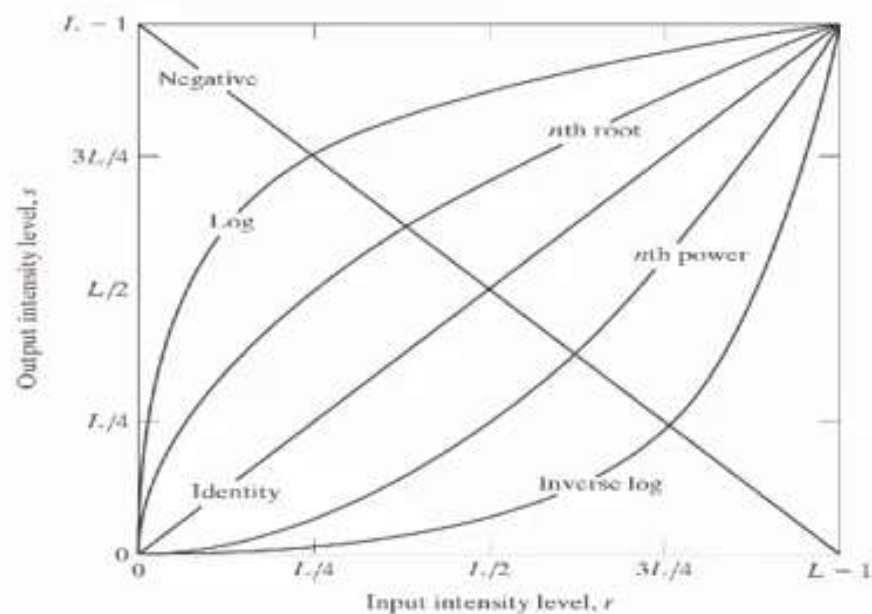




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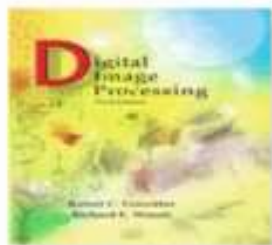
$$S = T$$

$$S = C \log(r+1)$$

$$L' = \frac{L}{2}$$

$$S = F(r)$$

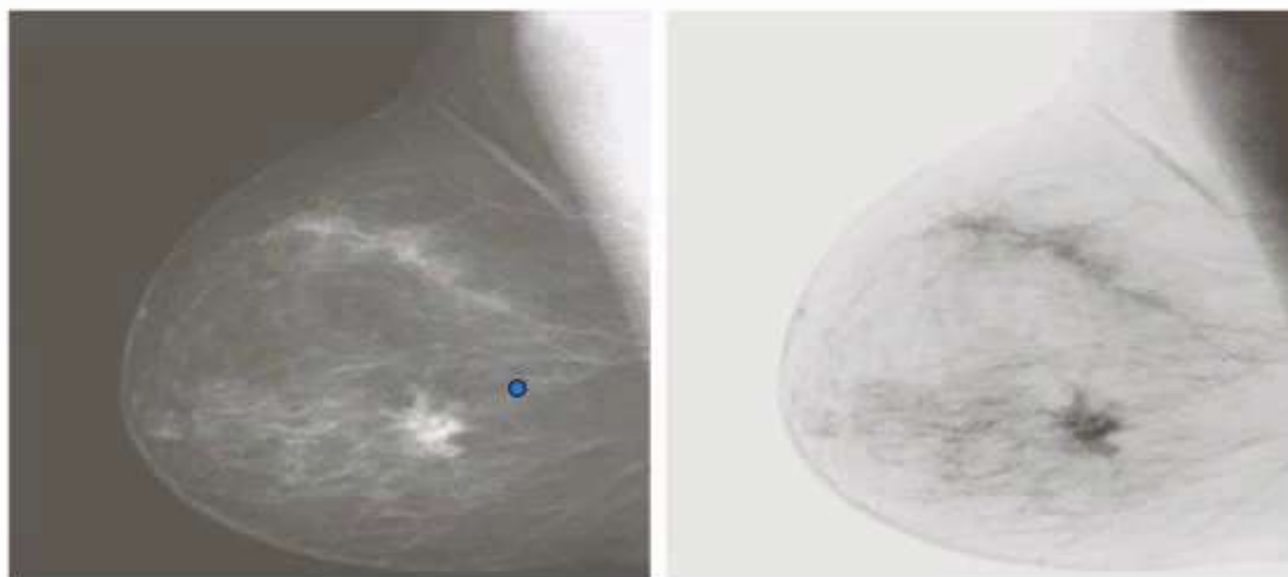
$$S = L - r$$



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a b

**FIGURE 3.4**  
(a) Original digital mammogram.  
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).  
(Courtesy of G.E. Medical Systems.)

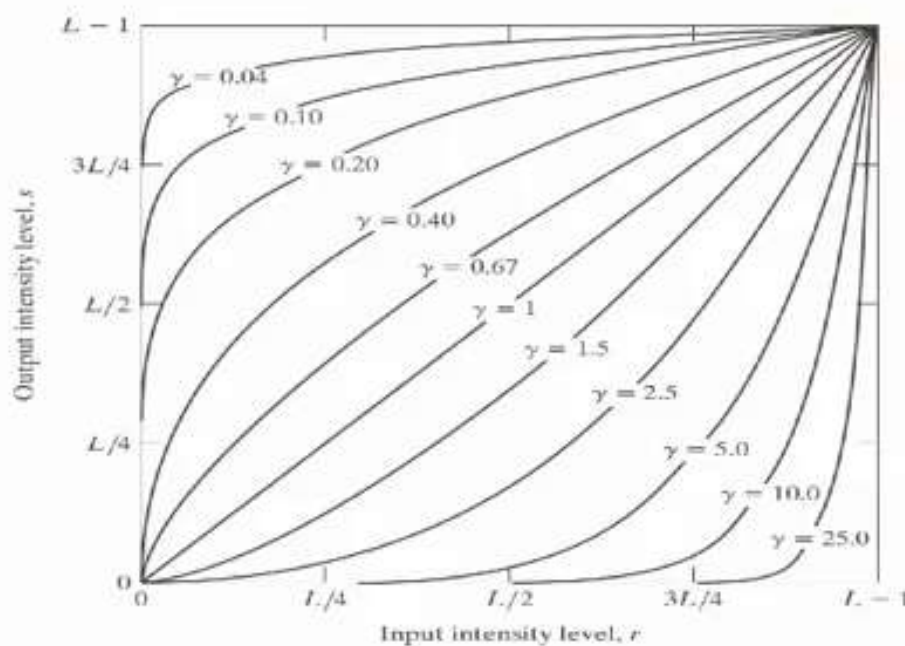
$$S = L - r$$



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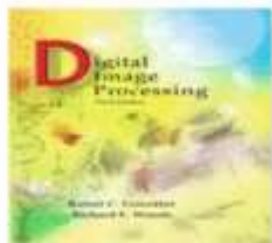
## Chapter 3 Intensity Transformations & Spatial Filtering



**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases). All curves were scaled to fit in the range shown.

$$S = C r^\gamma$$

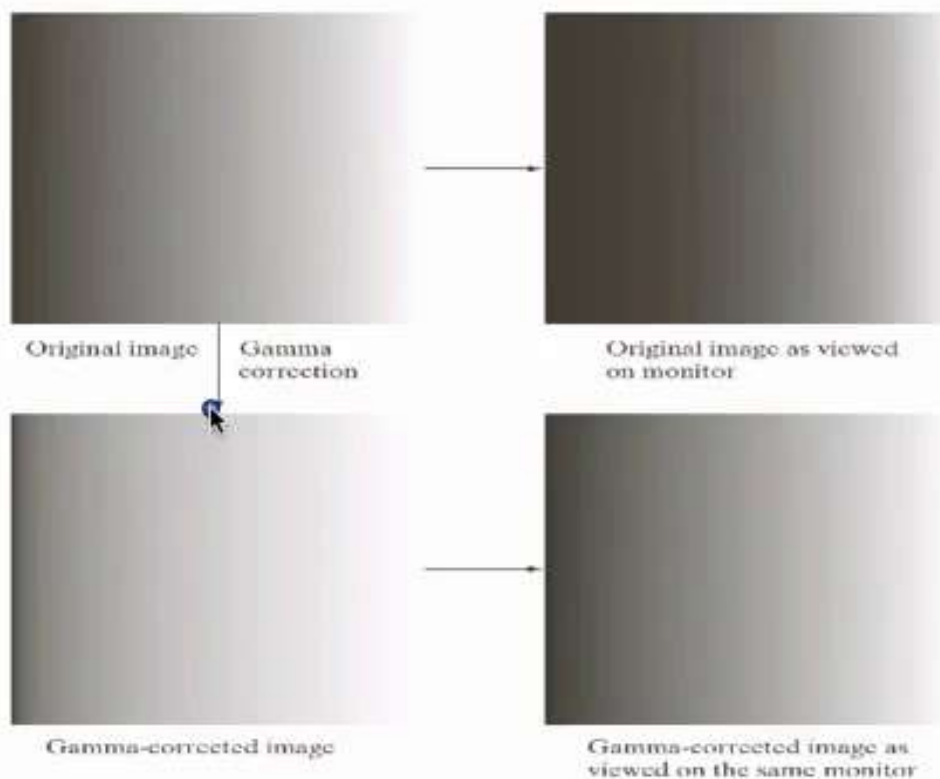




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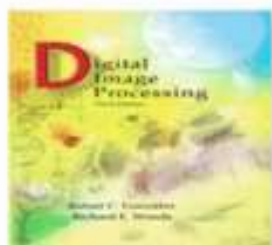
www.ImageProcessingPlace.com

## Chapter 3 Intensity Transformations & Spatial Filtering



a b  
c d

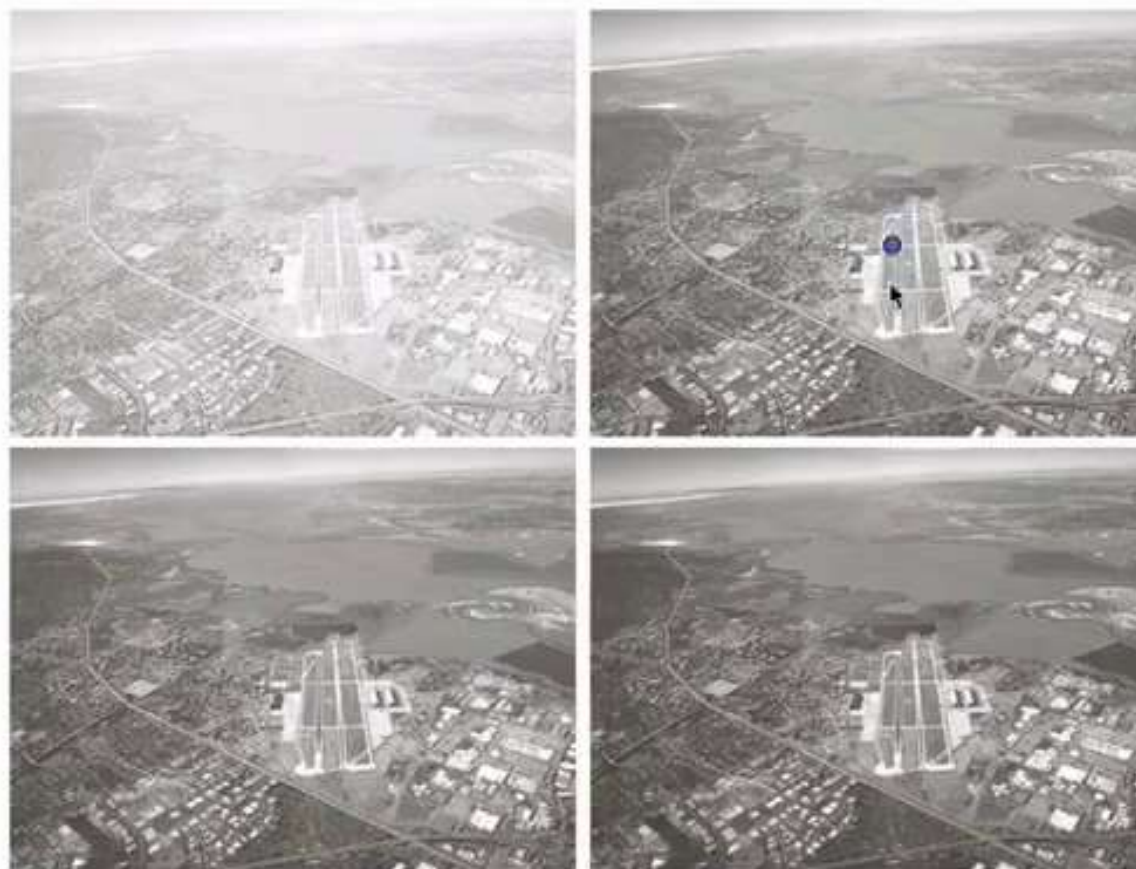
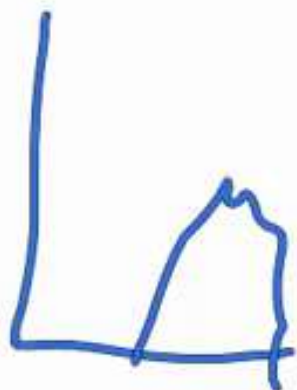
**FIGURE 3.7**  
(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).



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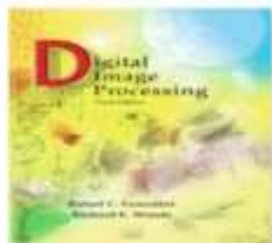
## Chapter 3 Intensity Transformations & Spatial Filtering



a b  
c d

**FIGURE 3.9**

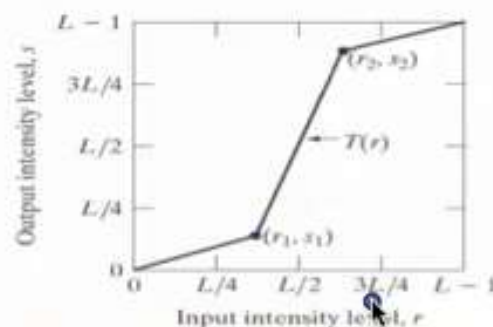
(a) Aerial image.  
(b)–(d) Results of  
applying the  
transformation in  
Eq. (3.2-3) with  
 $c = 1$  and  
 $\gamma = 3.0, 4.0,$  and  
 $5.0$ , respectively.  
(Original image  
for this example  
courtesy of  
NASA.)



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a b  
c d

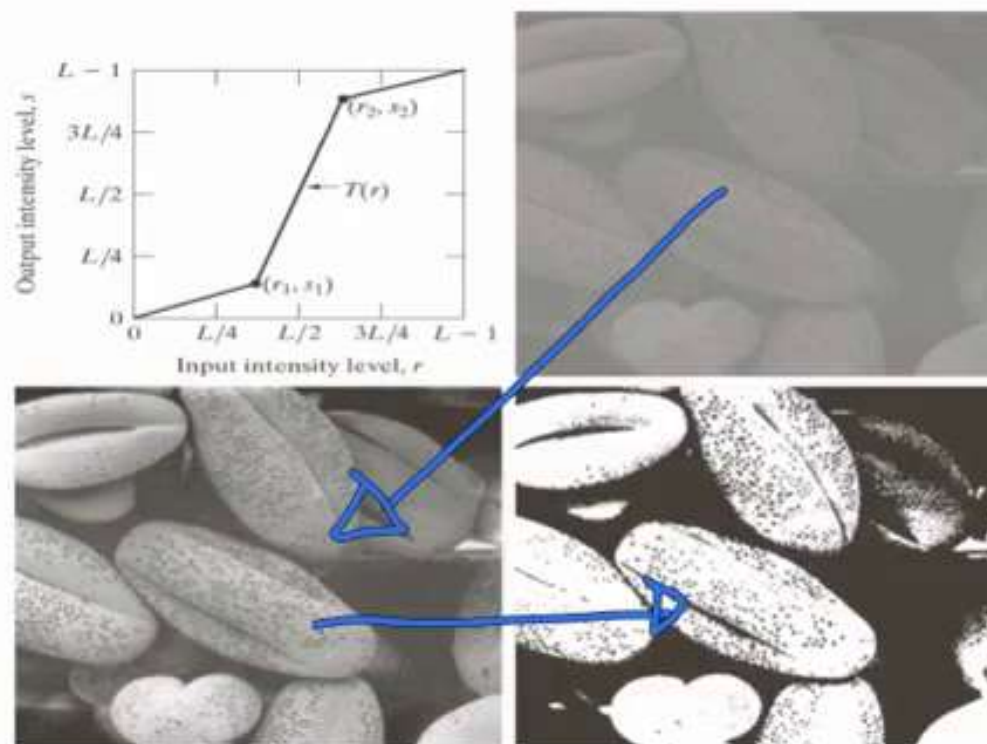
**FIGURE 3.10**  
Contrast stretching.  
(a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



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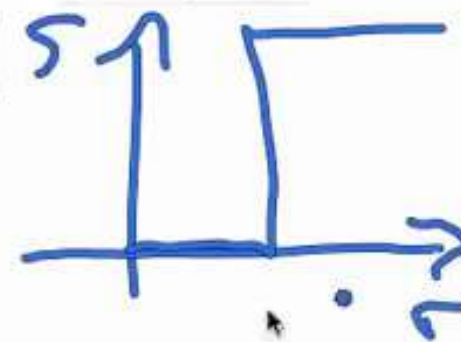
www.ImageProcessingPlace.com

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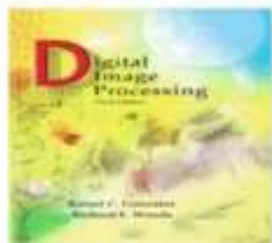


a b  
c d

**FIGURE 3.10**  
Contrast stretching.  
(a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)







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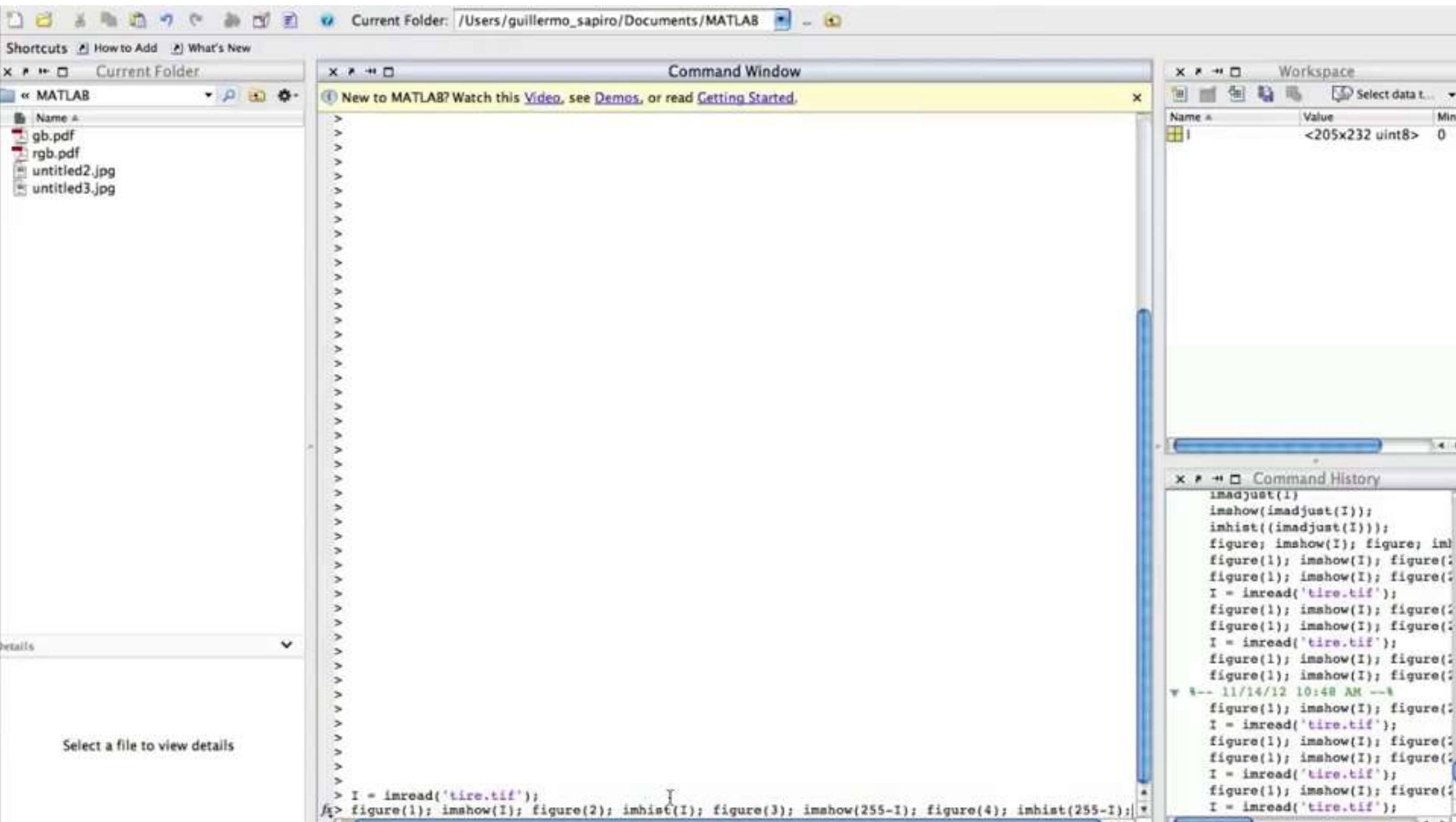
www.ImageProcessingPlace.com

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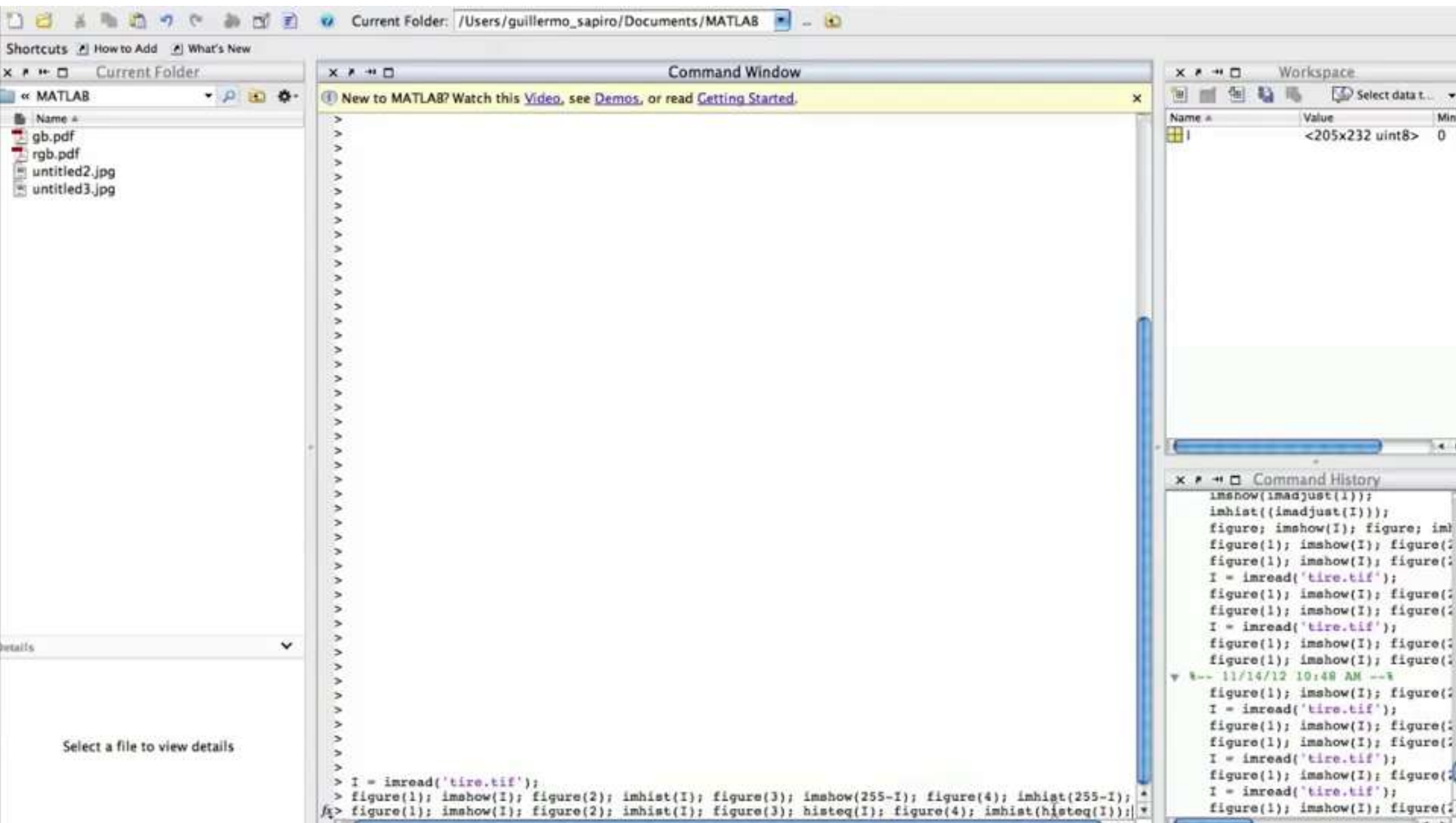


a b c

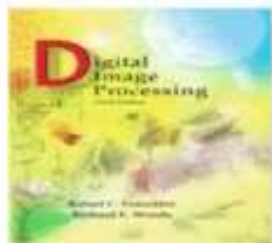
**FIGURE 3.12** (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)









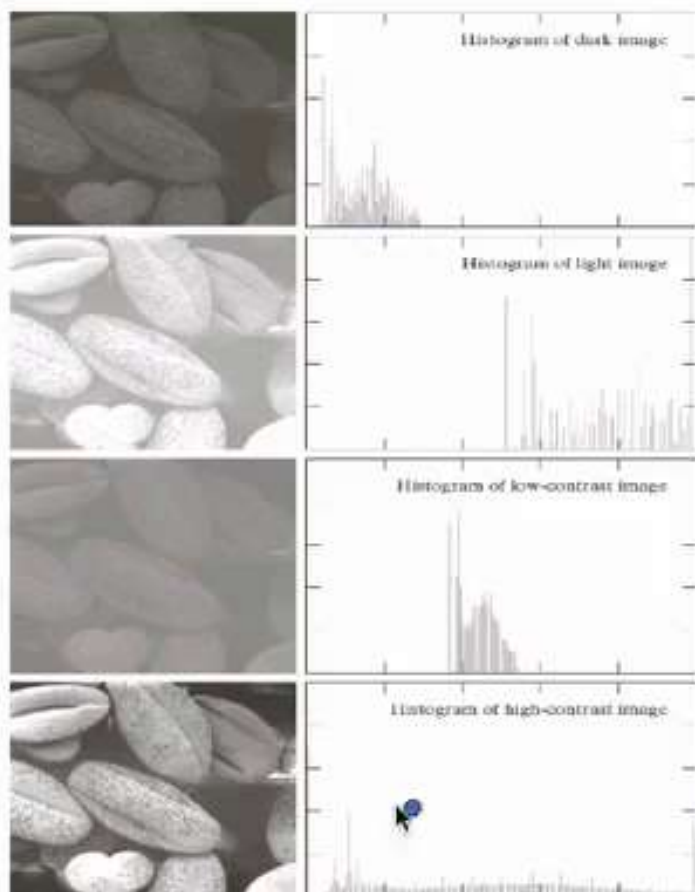


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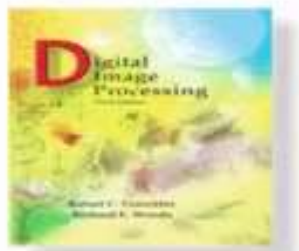
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### Intensity Transformations & Spatial Filtering



**FIGURE 3.16** Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.



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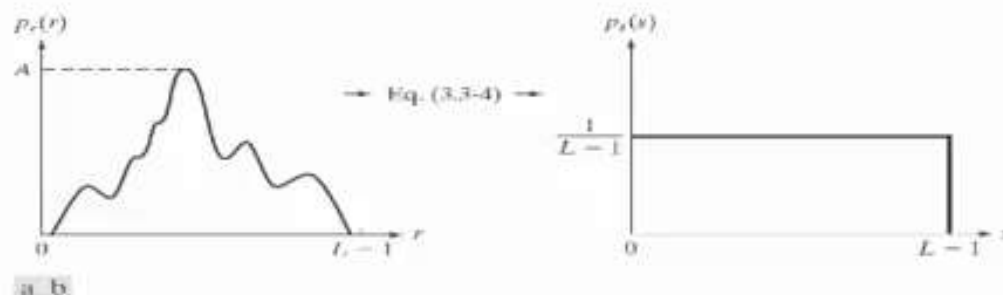
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## Histogram Equalization



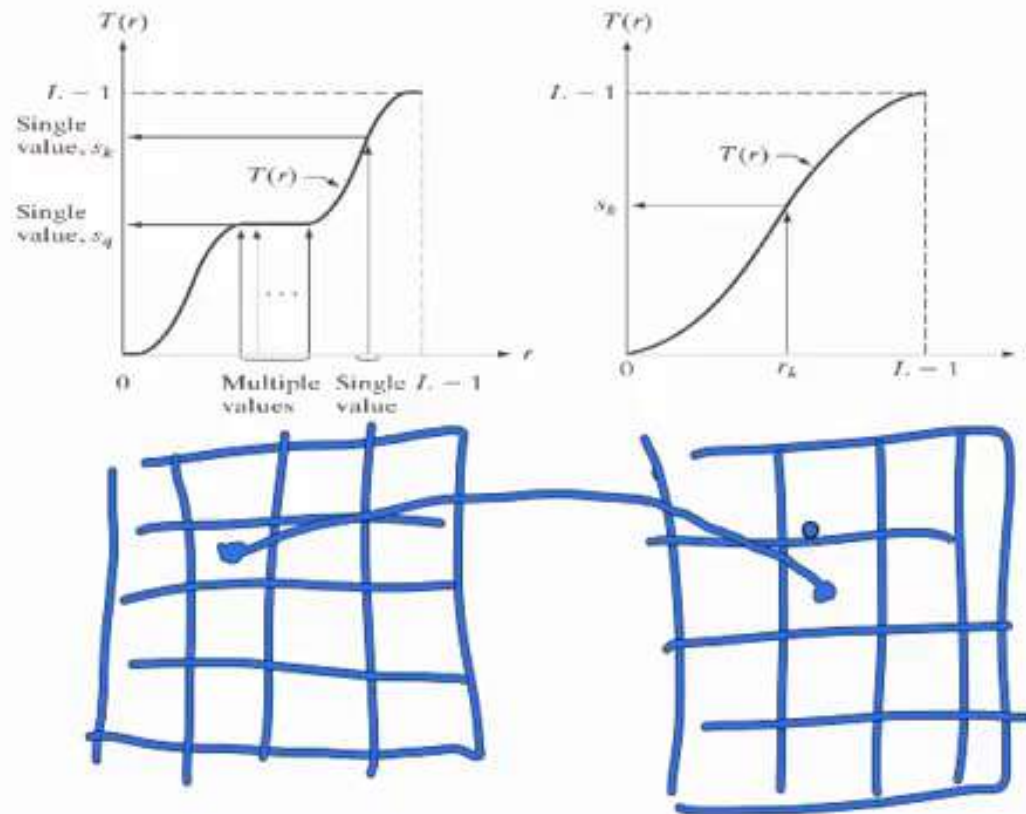
**FIGURE 3.18** (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels,  $r$ . The resulting intensities,  $s$ , have a uniform PDF, independently of the form of the PDF of the  $r$ 's.



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**FIGURE 3.17**  
(a) Monotonically increasing function, showing how multiple values can map to a single value.  
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.



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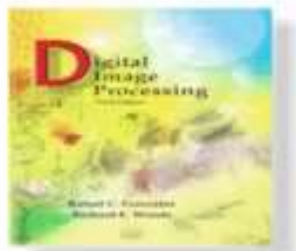
## Chapter 3

### Intensity Transformations & Spatial Filtering



$$s = T(r) \quad P_s(s) = P_r(r) \left| \frac{dr}{ds} \right|$$
$$s = T(r) = (L-1) \int_0^r P_r(w) dw$$
$$\frac{ds}{dr} = \frac{dT(r)}{dr} = \frac{d(L-1) \int_0^r P_r(w) dw}{dr}$$





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$$P_S(s) = P_r(r) \left| \frac{dr}{ds} \right| =$$

$$= P_r(r) \left| \frac{1}{(L-1)p_r(r)} \right| = \frac{1}{L-1}$$

$$S = T(r) = (L-1) \int_0^r P_r(w) dw$$



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$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

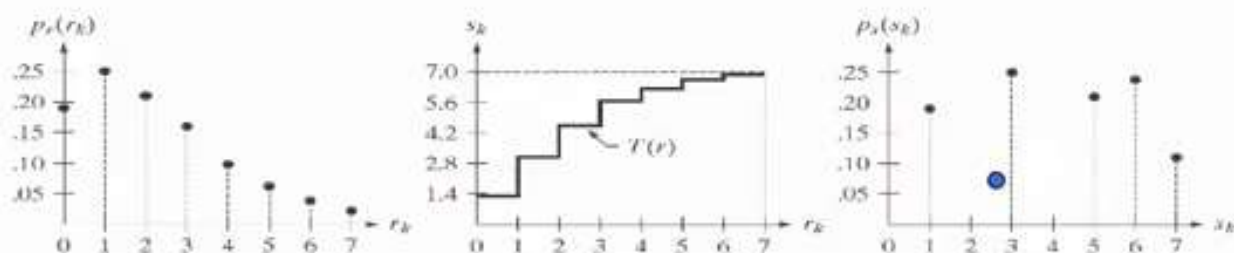
**TABLE 3.1**  
Intensity  
distribution and  
histogram values  
for a 3-bit,  
 $64 \times 64$  digital  
image.



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a b c

**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

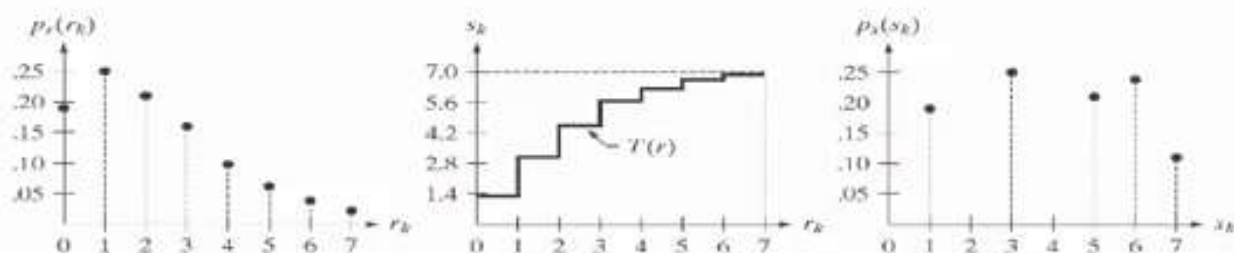
$$\int_0^r p_r(u) du$$



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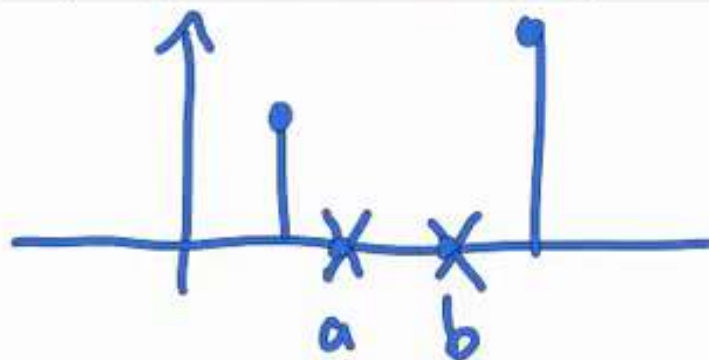
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a b c

**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.



$$T(a) \stackrel{?}{=} T(b)$$

$$T(a) = T(b)$$

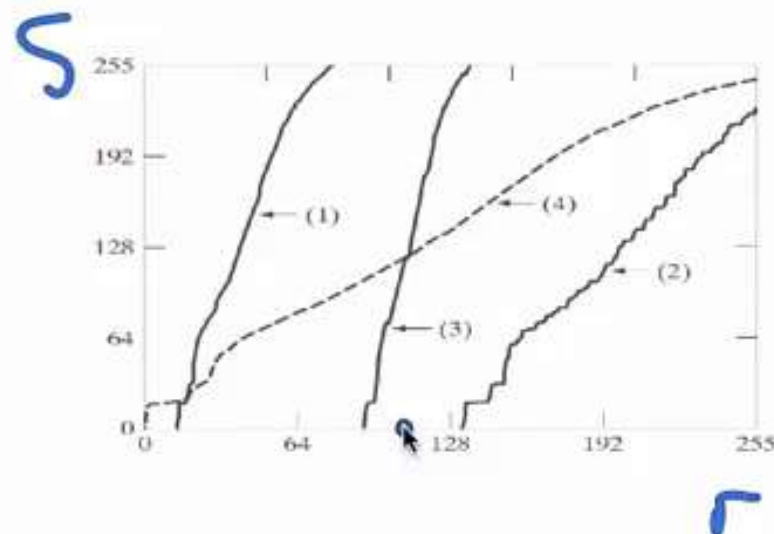
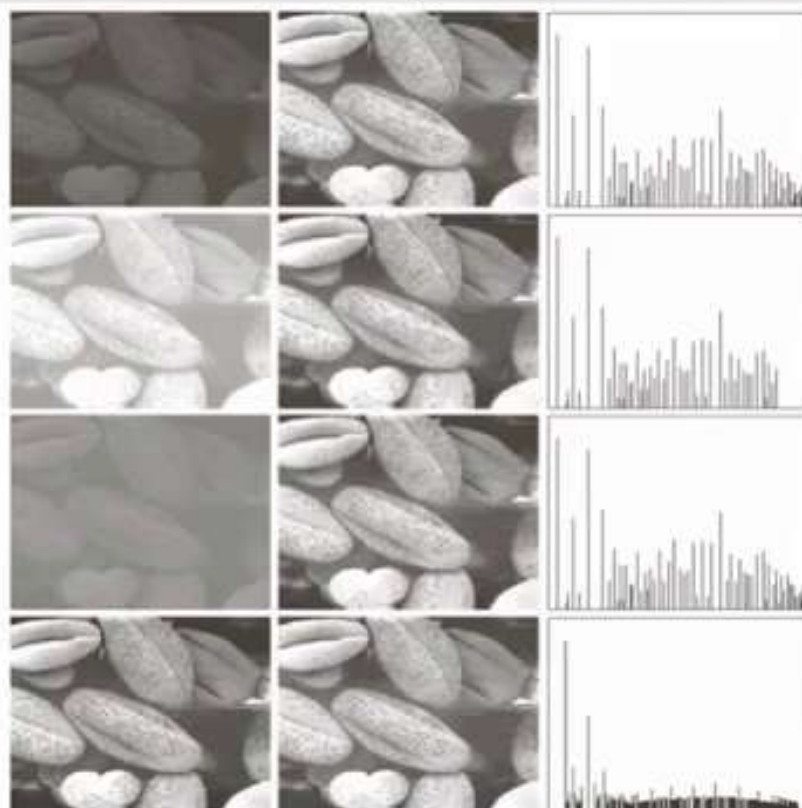




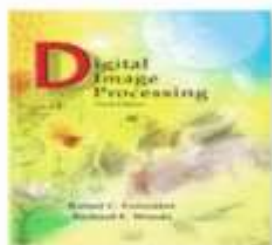
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**FIGURE 3.20** Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.



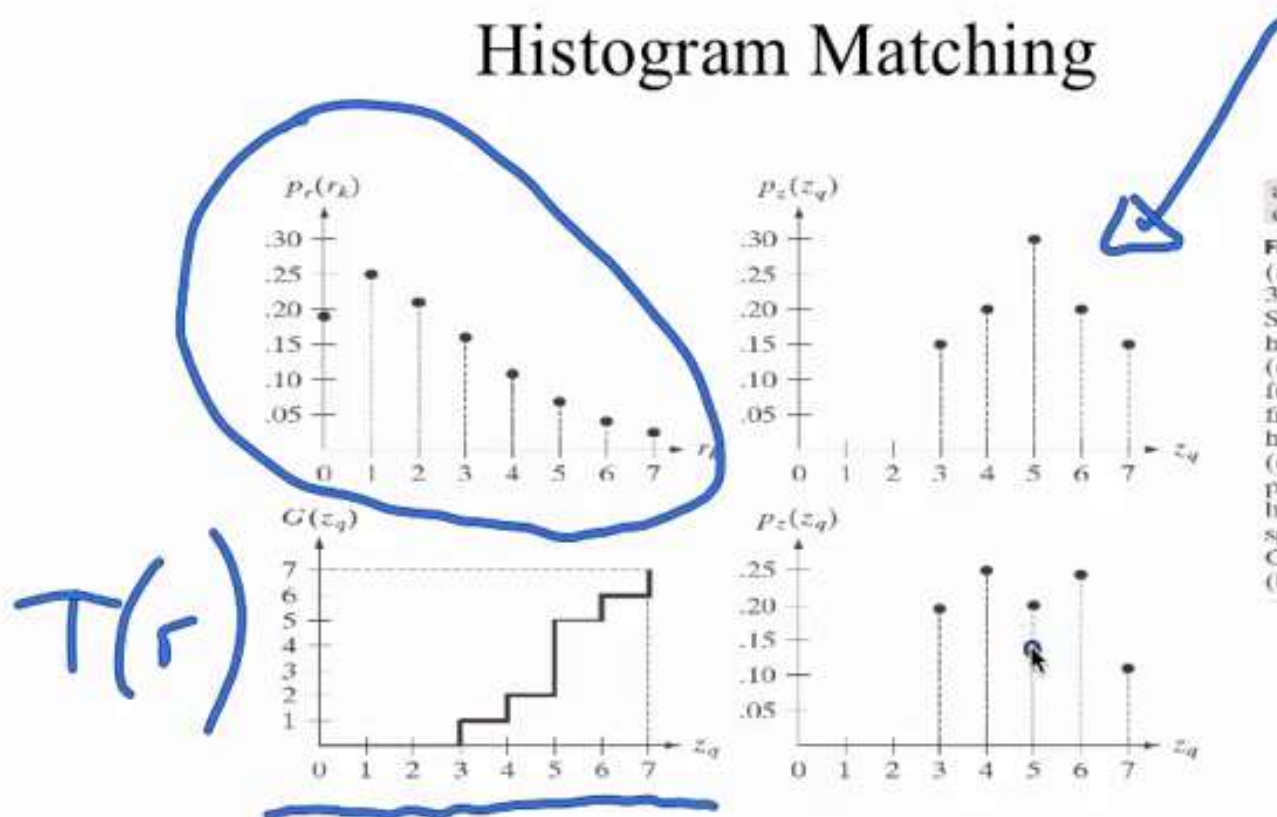
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### Histogram Matching



a b  
c d

**FIGURE 3.22**  
(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).



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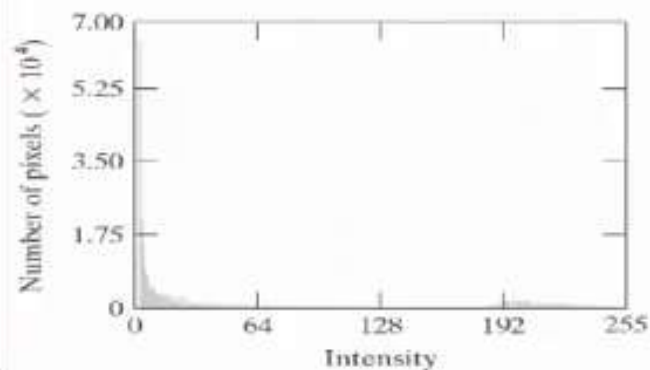




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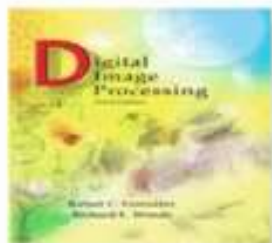
## Chapter 3 Intensity Transformations & Spatial Filtering



a b

**FIGURE 3.23**  
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

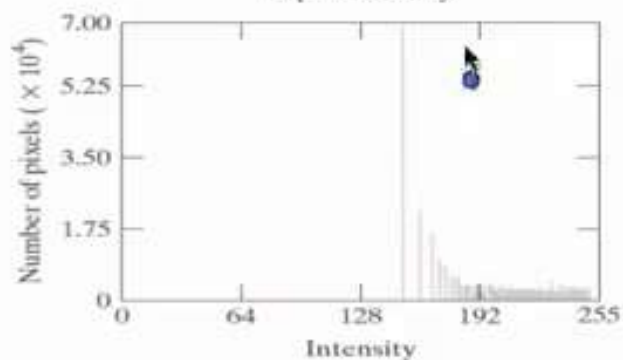
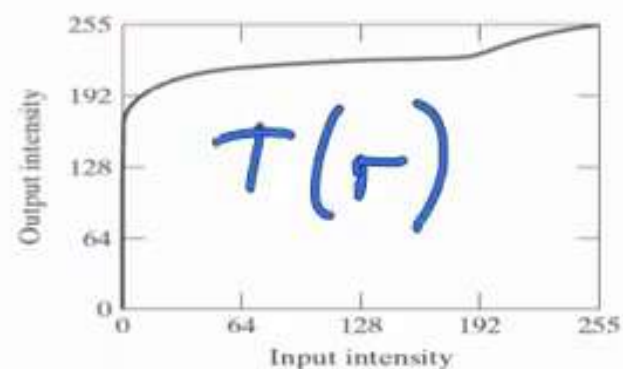


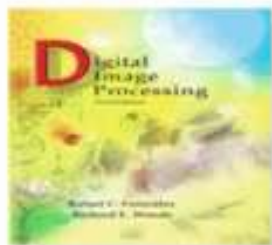


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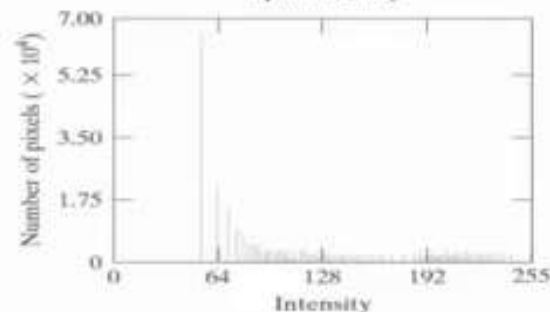
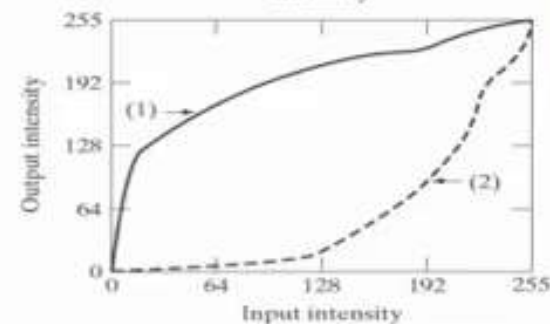
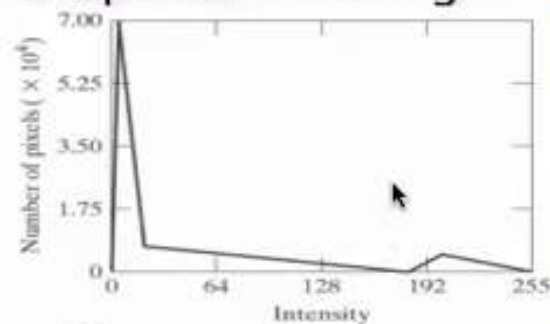
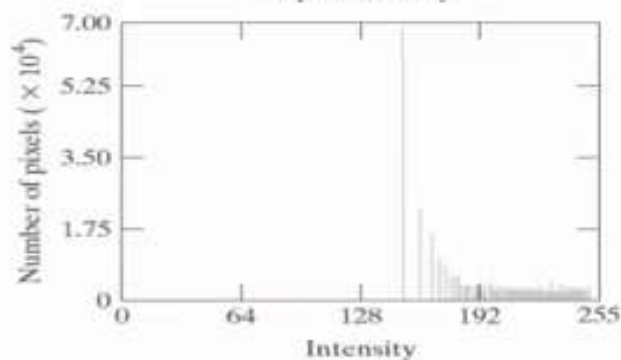
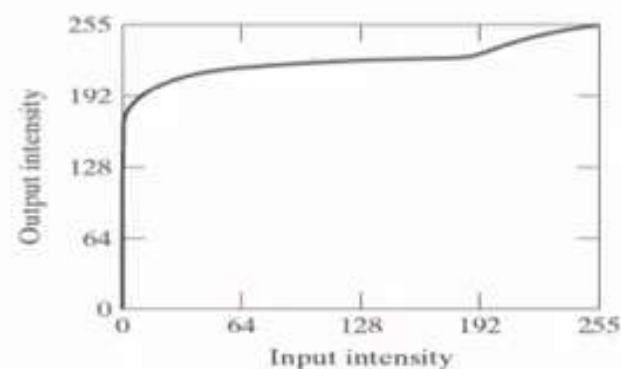




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**a b c**

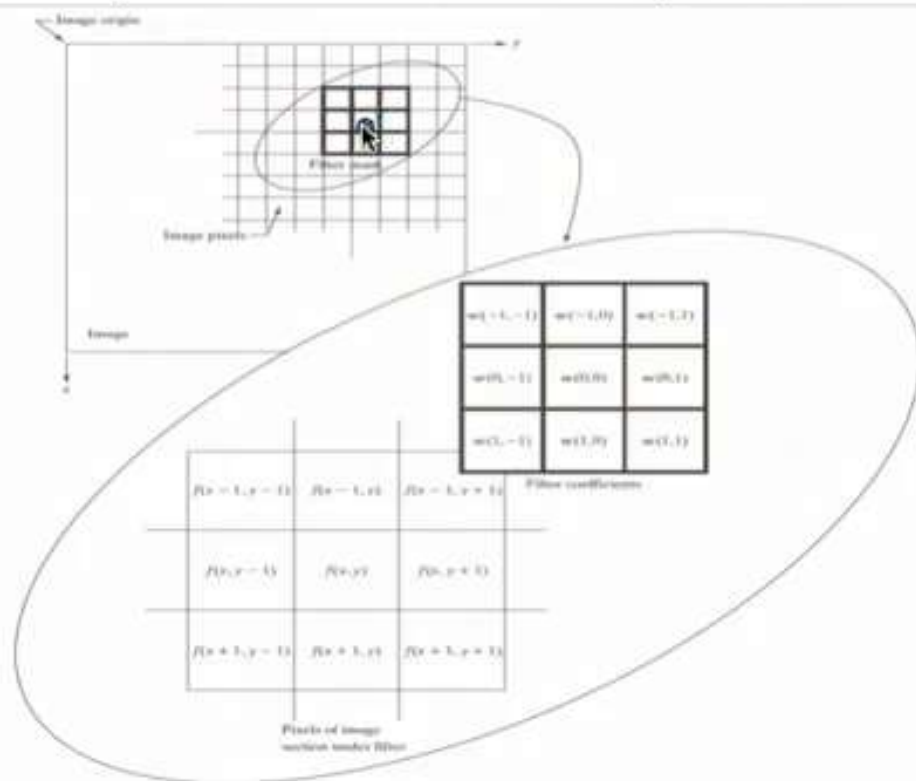
**FIGURE 3.27** (a) SEM image of a tungsten filament magnified approximately 130 $\times$ . (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)



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$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$

**FIGURE 3.28** The mechanics of linear spatial filtering using a  $3 \times 3$  filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.





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$$\frac{1}{9} \times$$

1	1	1
1	1	1
1	1	1

$$\frac{1}{16} \times$$

1	2	1
2	4	2
1	2	1

a b

**FIGURE 3.32** Two  $3 \times 3$  smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.



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**FIGURE 3.33** (a) Original image, of size  $500 \times 500$  pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes  $m = 3, 5, 9, 15$ , and  $35$ , respectively. The black squares at the top are of sizes  $3, 5, 9, 15, 25, 35, 45$ , and  $55$  pixels, respectively; their borders are  $25$  pixels apart. The letters at the bottom range in size from  $10$  to  $24$  points, in increments of  $2$  points; the large letter at the top is  $60$  points. The vertical bars are  $5$  pixels wide and  $100$  pixels high; their separation is  $20$  pixels. The diameter of the circles is  $25$  pixels, and their borders are  $15$  pixels apart; their intensity levels range from  $0\%$  to  $100\%$  black in increments of  $20\%$ . The background of the image is  $10\%$  black. The noisy rectangles are of size  $50 \times 120$  pixels.





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a b c

**FIGURE 3.34** (a) Image of size  $528 \times 485$  pixels from the Hubble Space Telescope. (b) Image filtered with a  $15 \times 15$  averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

## Averaging, Gaussian Filtering, Heat Flow

$$1, 2, 3 \quad a$$
$$(a-1)^2 + (a-2)^2 + (a-3)^2$$

No

Yes - 0

Yes - 2

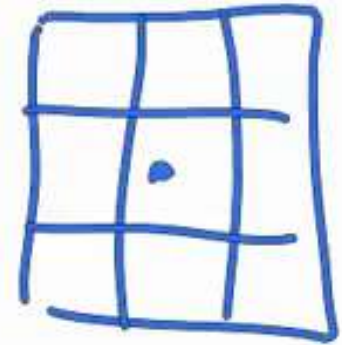
Yes - 4



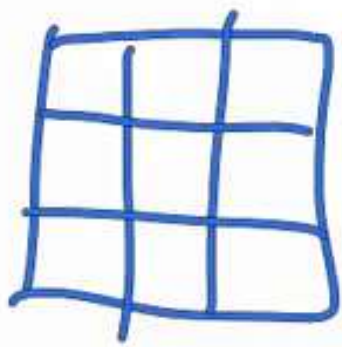


## Averaging, Gaussian Filtering, Heat Flow

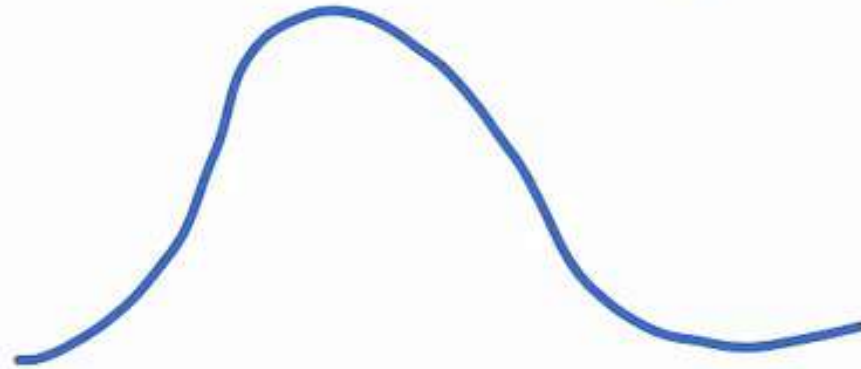
$$N = \sum_i (a - a_i)^2$$
$$a = \overline{a_i}$$



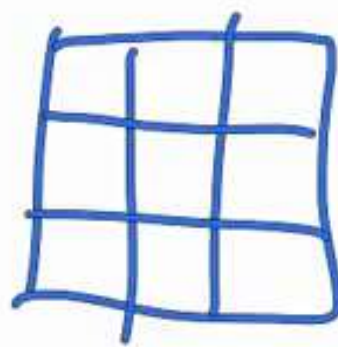
## Averaging, Gaussian Filtering, Heat Flow

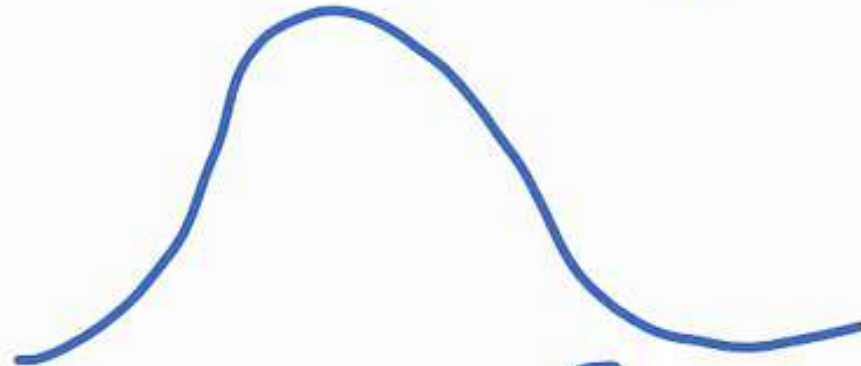


$$f(x, y) = f(x, y) * G(0, \sigma)$$



## Averaging, Gaussian Filtering, Heat Flow

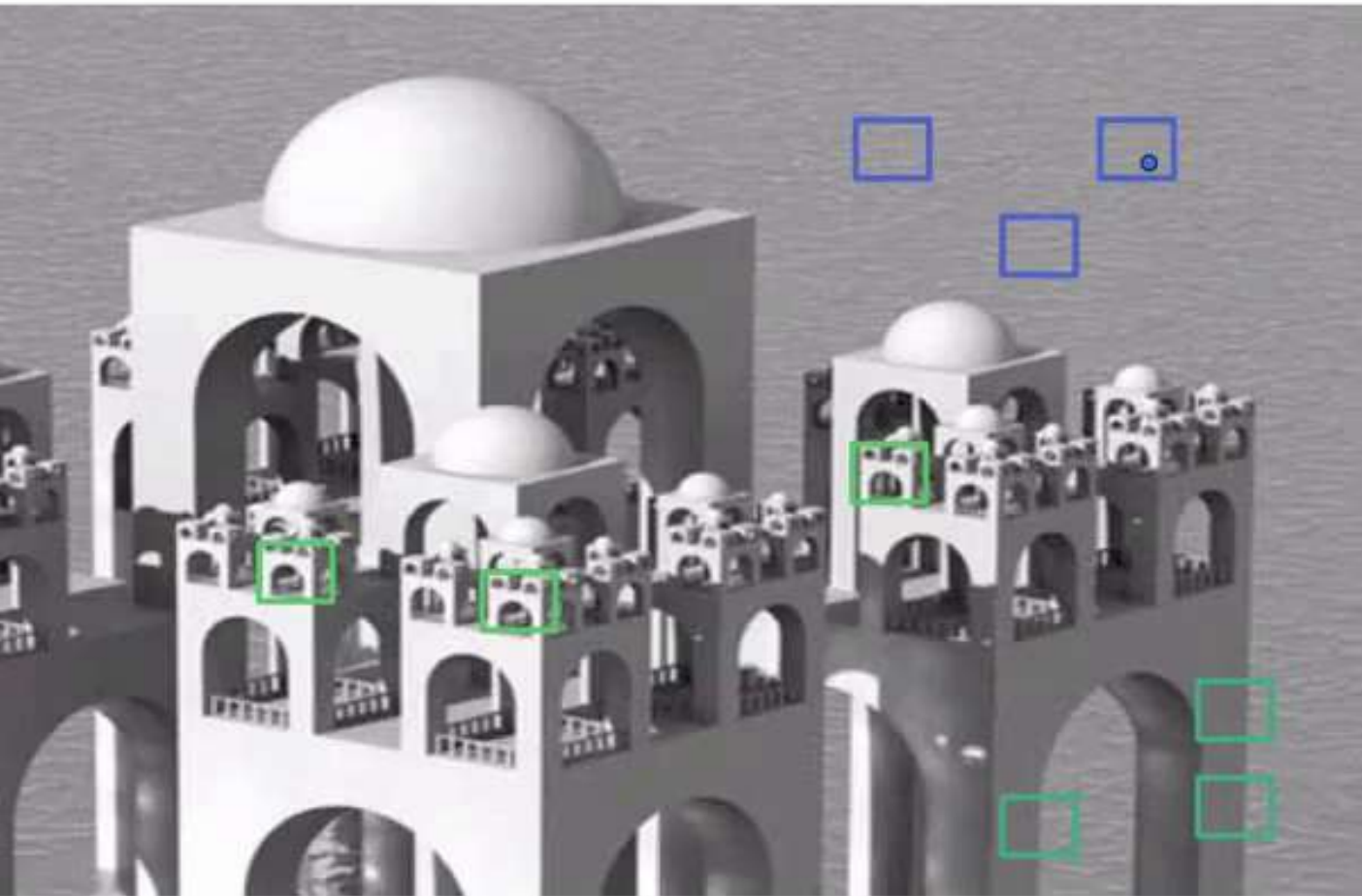

$$f(x, y, t) = f(x, y) * G(0, \sigma)$$



$$\frac{df}{dt} = \Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



## Non Local Means/Average



Credits: Glasner et al. – Buades et al.





demo.ipol.im/demo/bcm\_non\_local\_means\_denoising/result?key=D7B0F397B350FA1730C849EF38F46FA7

The algorithm result is displayed hereafter. It ran in 5.82s.  
You can run again this algorithm with new data.

Run again?:

Results (sigma: 25)

- Noisy ☒
- Denoised ☐
- Original ☐
- Difference ☐




demo.ipol.im/demo/bcm\_non\_local\_means\_denoising/result?key=D7B0F397B350FA1730C849EF38F46FA7

The algorithm result is displayed hereafter. It ran in 5.82s.  
You can run again this algorithm with new data.

Run again?:

Results (sigma: 25)

Noisy

Denoised 

Original

Difference





demo.ipol.im/demo/bcm\_non\_local\_means\_denoising/result?key=D7B0F397B350FA1730C849EF38F46FA7

The algorithm result is displayed hereafter. It ran in 5.82s.  
You can run again this algorithm with new data.

Run again?:

Results (sigma: 25)

Noisy

Denoised

Original ☒

Difference



demo.ipol.im/demo/bcm\_non\_local\_means\_denoising/result?key=D7B0F397B350FA1730C849EF38F46FA7

The algorithm result is displayed hereafter. It ran in 5.82s.  
You can run again this algorithm with new data.

Run again?:

Results (sigma: 25)

Noisy

Denoised

Original

Difference





demo.ipol.im/demo/bcm\_non\_local\_means\_denoising/result?key=5ABA2A4448E8980B01B51B37F6486422

article demo archive

Please cite this article if you publish results obtained with this online demo.

The algorithm result is displayed hereafter. It ran in 16.33s.  
You can run again this algorithm with new data.

Run again?:

Results (sigma: 35)

- Noisy ☒
- Denoised
- Original
- Difference



Please cite this article if you publish results obtained with this online demo.

The algorithm result is displayed hereafter. It ran in 16.33s.  
You can run again this algorithm with new data.

Run again?: [new input](#) [different parameter or subimage](#)

Results (sigma: 35)

- Noisy
- Denoised**
- Original
- Difference



Please cite this article if you publish results obtained with this online demo.

The algorithm result is displayed hereafter. It ran in 16.33s.  
You can run again this algorithm with new data.

Run again?:

Results (sigma: 35)

- Noisy
- Denoised
- Original ☒
- Difference



demo.ipol.im/demo/bcm\_non\_local\_means\_denoising/result?key=5ABA2A4448E8980B01B51B37F6486422

article demo archive

Please cite this article if you publish results obtained with this online demo.

The algorithm result is displayed hereafter. It ran in 16.33s.  
You can run again this algorithm with new data.

Run again?:

Results (sigma: 35)

Noisy  
Denoised  
Original  
Difference



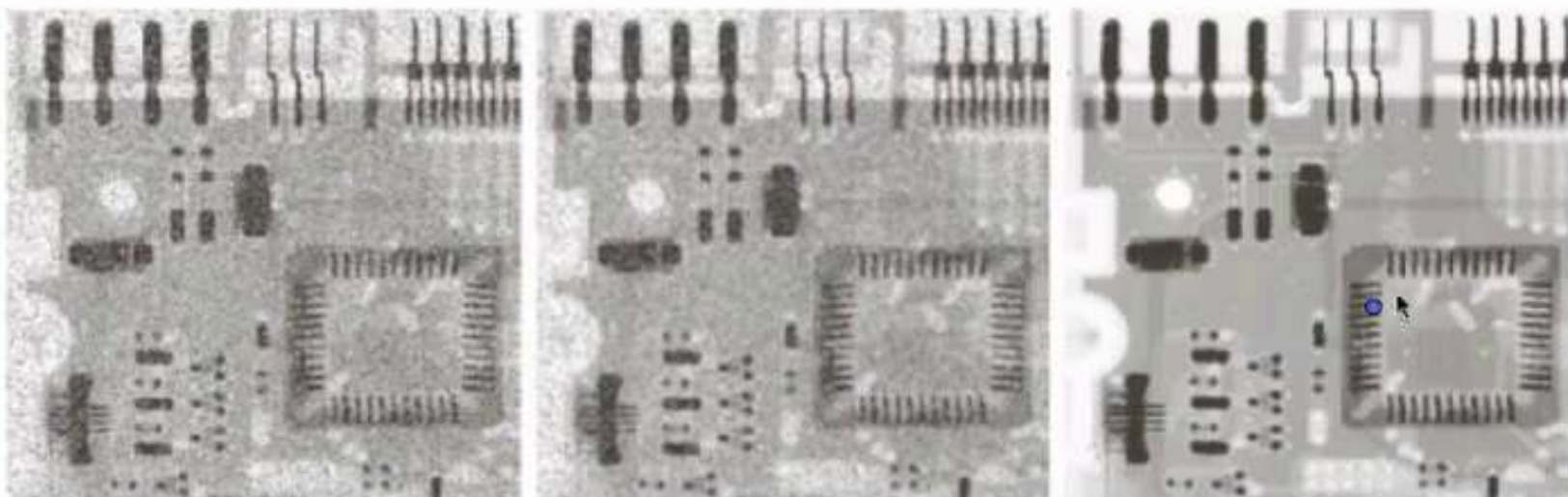




## *Digital Image Processing, 3rd ed.* Gonzalez & Woods

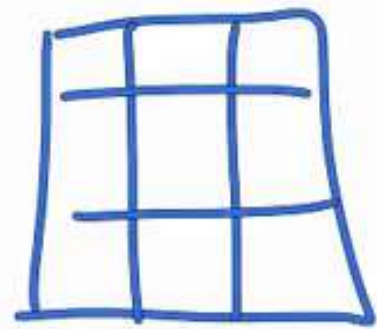
[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

### Chapter 3 Intensity Transformations & Spatial Filtering



a b c

**FIGURE 3.35** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a  $3 \times 3$  averaging mask. (c) Noise reduction with a  $3 \times 3$  median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



$$\sum_i (a - a_i)^2$$

Mean

$$\sum f(a - a_i)$$

$$\sum |a - a_i|$$



## Guillermo Sapiro

Duke  
UNIVERSITY

Select a file to view details.

Watch this [Video](#), see [Demos](#), or read [Getting Started](#).

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fx>> I = imread('eight.tif');  
>> J = imnoise(I,'salt & pepper',0.09);  
>> K = medfilt2(J);  
fx>> figure, imshow(I); figure, imshow(J), figure, imshow(K)
```



Name *	Value	Min
CX	<205x232 doubl...	-85
CY	<205x232 doubl...	-11
I	<242x308 uint8>	56
J	<242x308 uint8>	0
K	<242x308 uint8>	0

```
help images
I = imread('eight.tif');
J = imnoise(I, 'salt & pepper',
figure, imshow(I), figure, im
J = imnoise(I, 'salt & pepper',
figure, imshow(I), figure, im
help medfilt2
I = imread('eight.tif');
J = imnoise(I, 'salt & pepper',
K = medfilt2(J);
figure, imshow(J), figure, im
J = imnoise(I, 'salt & pepper',
figure, imshow(I); figure, im
I = imread('eight.tif');
J = imnoise(I, 'salt & pepper',
K = medfilt2(J);
fig
I = imread('eight.tif');
J = imnoise(I, 'salt & pepper',
K = medfilt2(J);
```





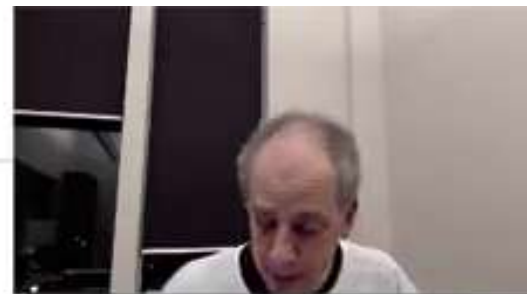


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## Chapter 3

### Intensity Transformations & Spatial Filtering

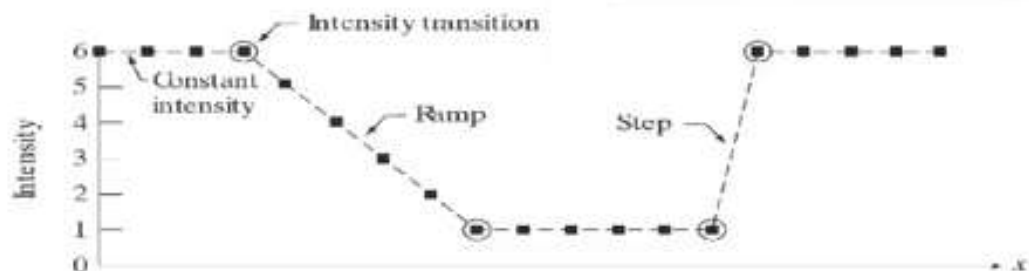


$$\frac{\partial F}{\partial x} \approx f(x+1) - f(x)$$

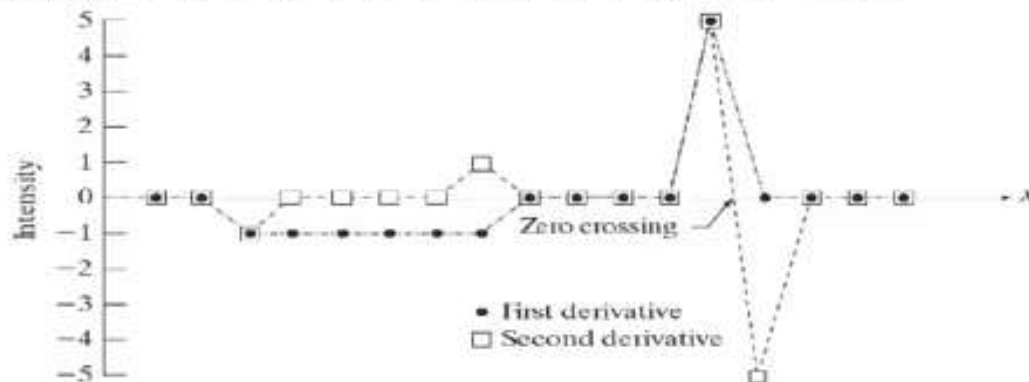
0	0	0
0	-1	1
0	0	0

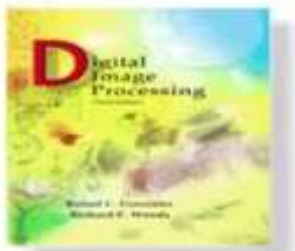
$$\frac{\partial^2 F}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

0	0	0
1	-2	1
0	0	0



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative	0	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	0	5	0	0	0	0
2nd derivative	0	0	0	-1	0	0	0	0	1	0	0	0	0	0	5	-5	0	0	0



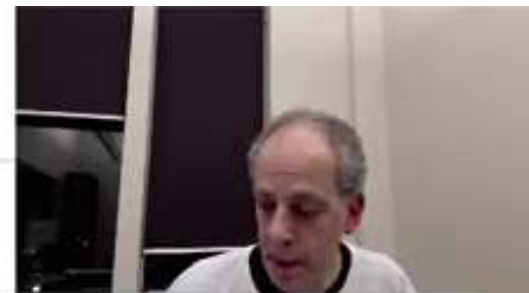


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## Chapter 3

### Intensity Transformations & Spatial Filtering



$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = \Delta F$$

$$= f(x+1) + f(x-1) - 2f(x) + f(y+1) + f(y-1) - 2f(y)$$

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1



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## Chapter 3 Intensity Transformations & Spatial Filtering



0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1



a  
b c  
d e

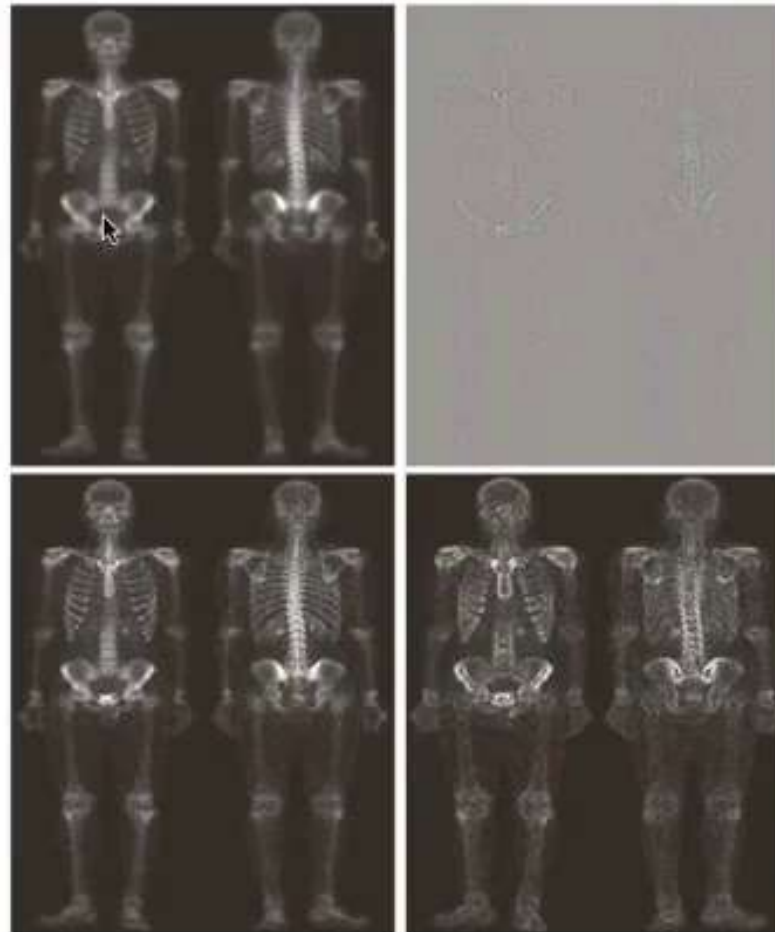
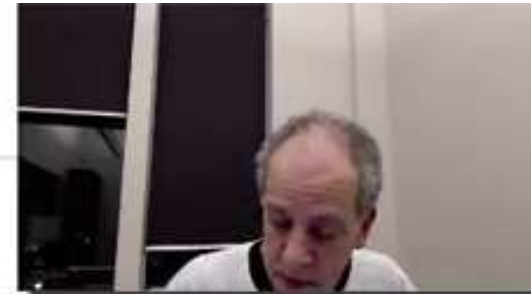
**FIGURE 3.38**  
(a) Blurred image of the North Pole of the moon.  
(b) Laplacian without scaling.  
(c) Laplacian with scaling.  
(d) Image sharpened using the mask in Fig. 3.37(a).  
(e) Result of using the mask in Fig. 3.37(b).  
(Original image courtesy of NASA.)



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## Chapter 3 Intensity Transformations & Spatial Filtering



a b  
c d  
**FIGURE 3.43**  
(a) Image of whole body bone scan.  
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).  
(d) Sobel gradient of (a).





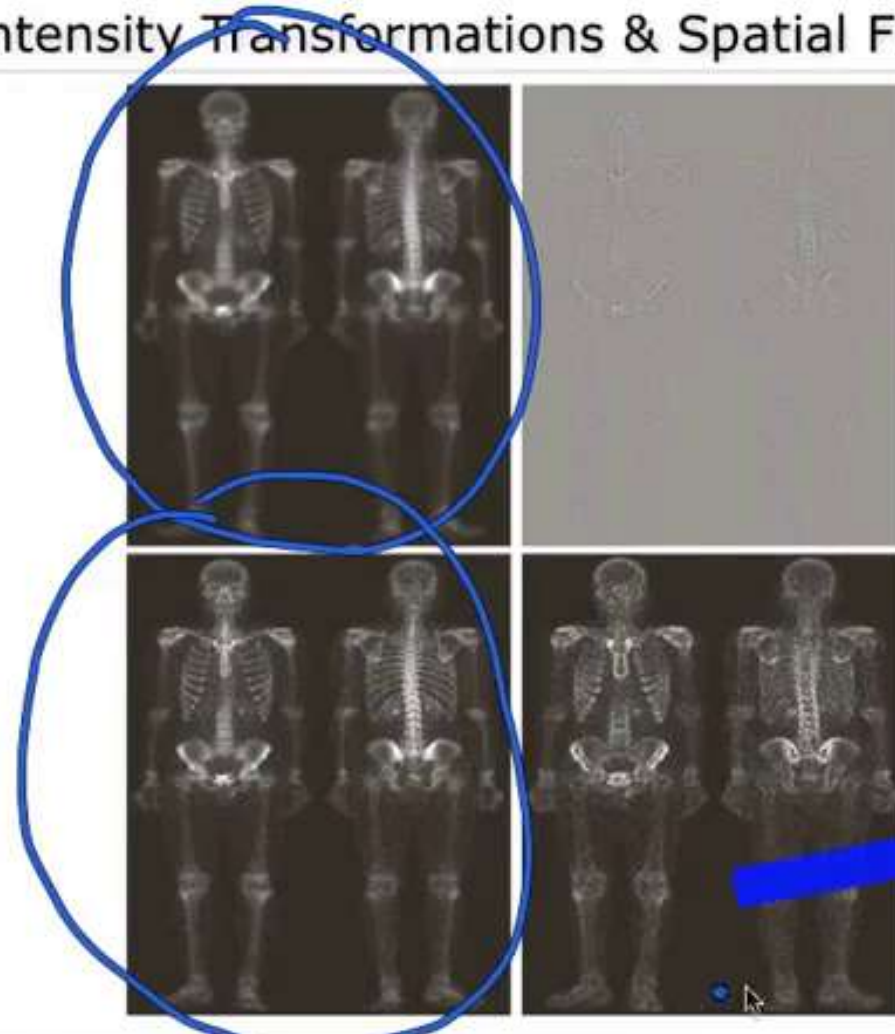


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## Chapter 3

### Intensity Transformations & Spatial Filtering



a b  
c d

**FIGURE 3.43**

(a) Image of whole body bone scan.  
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).  
(d) Sobel gradient of (a).

Sobel  
Edges



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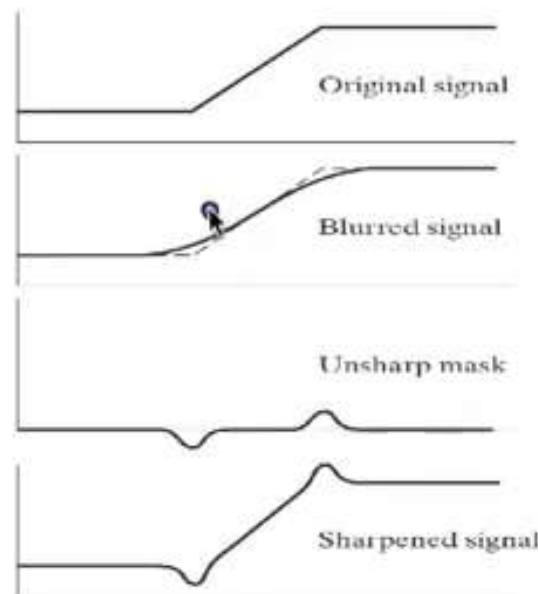
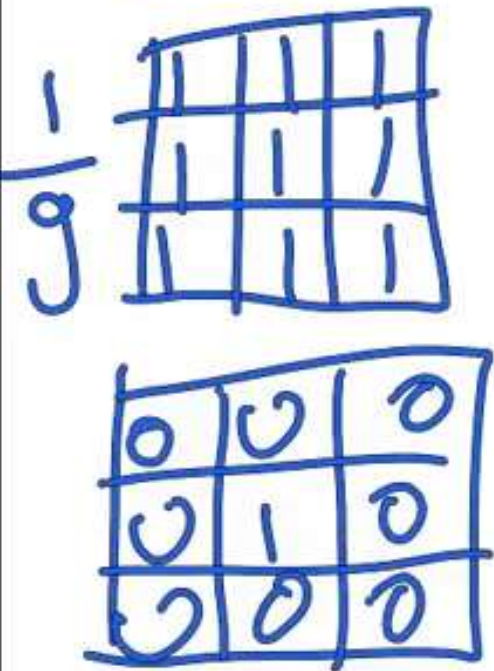
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## Chapter 3

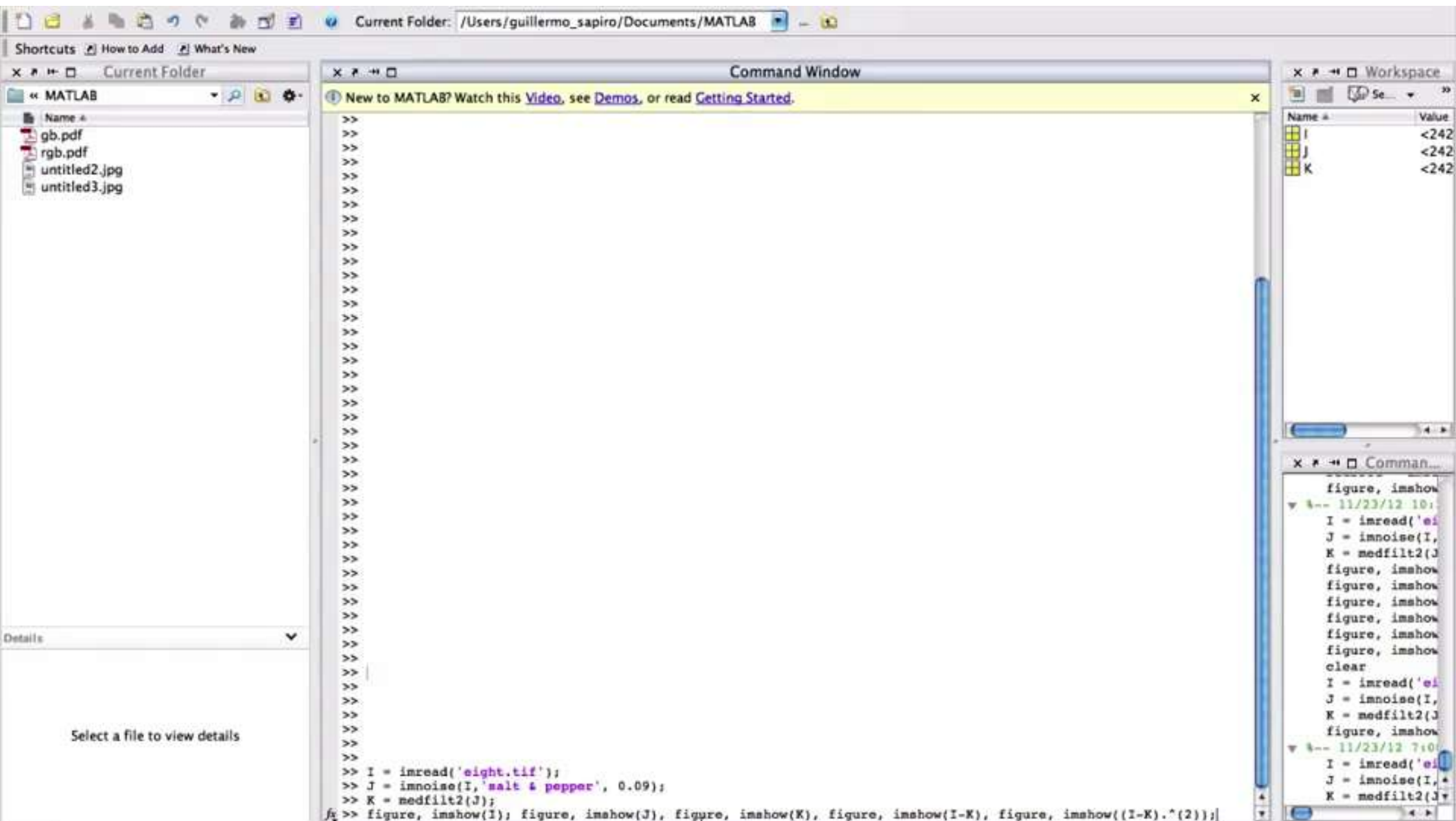
### Intensity Transformations & Spatial Filtering



Unsharp



**FIGURE 3.39** 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

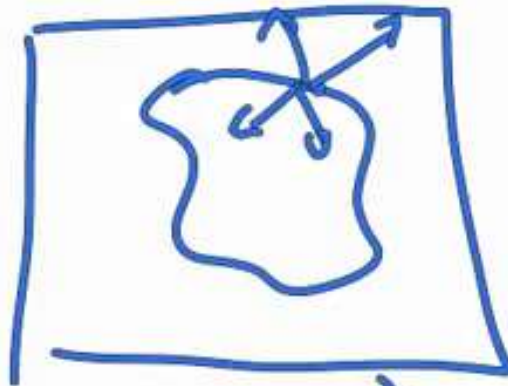






## Edge Detection and Color Edge Detection

$$\nabla f(x, y) = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$



$(R, G, B)$

$$|\nabla f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

