

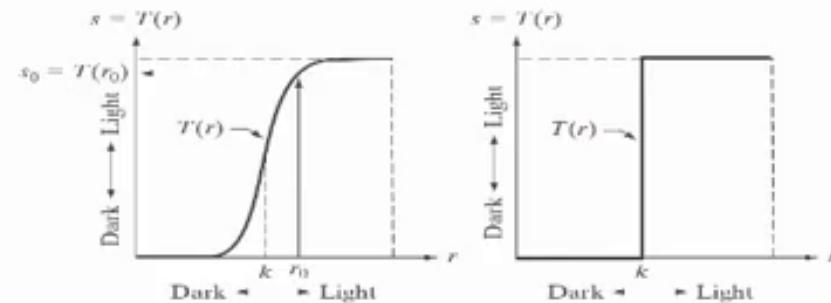
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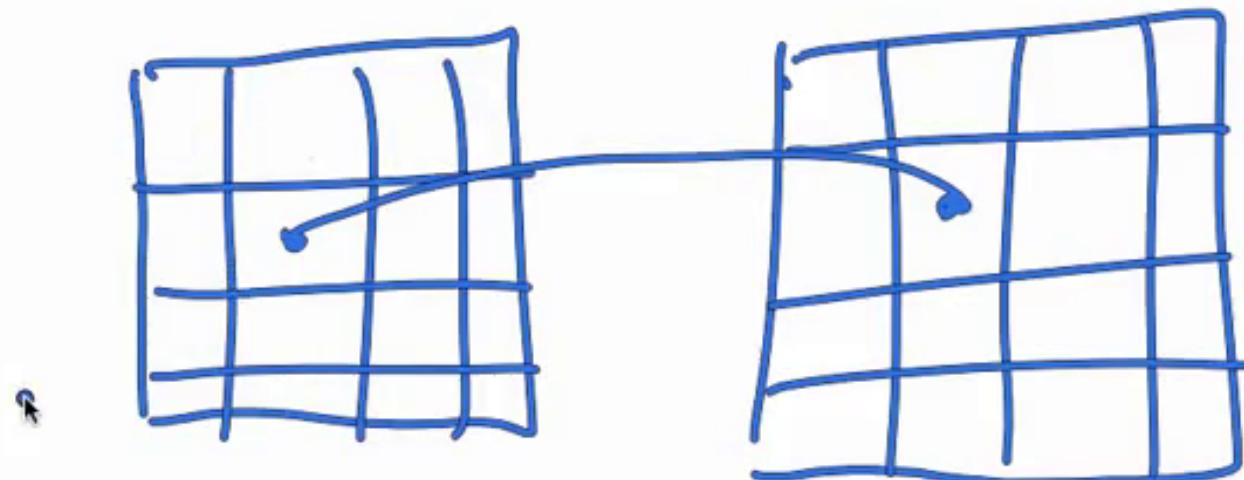
Chapter 3

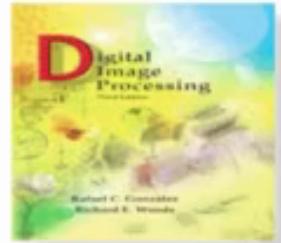
Intensity Transformations & Spatial Filtering



a b

FIGURE 3.2
Intensity
transformation
functions.
(a) Contrast-
stretching
function.
(b) Thresholding
function.





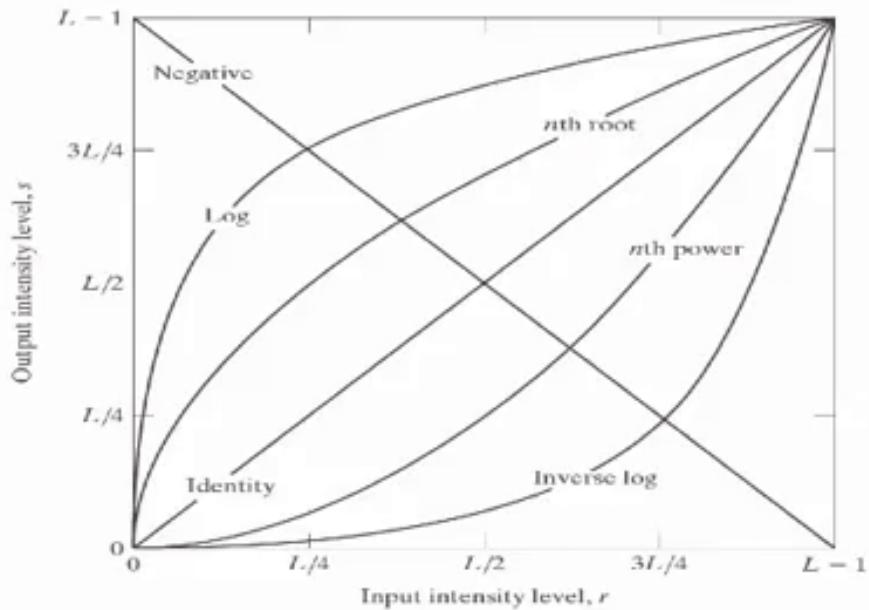
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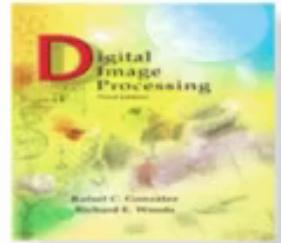
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$$\begin{aligned}S &= \Gamma \\S &= C \log(r+1) \\[10pt] S &= F(\Gamma) \\S &= L - \Gamma\end{aligned}$$



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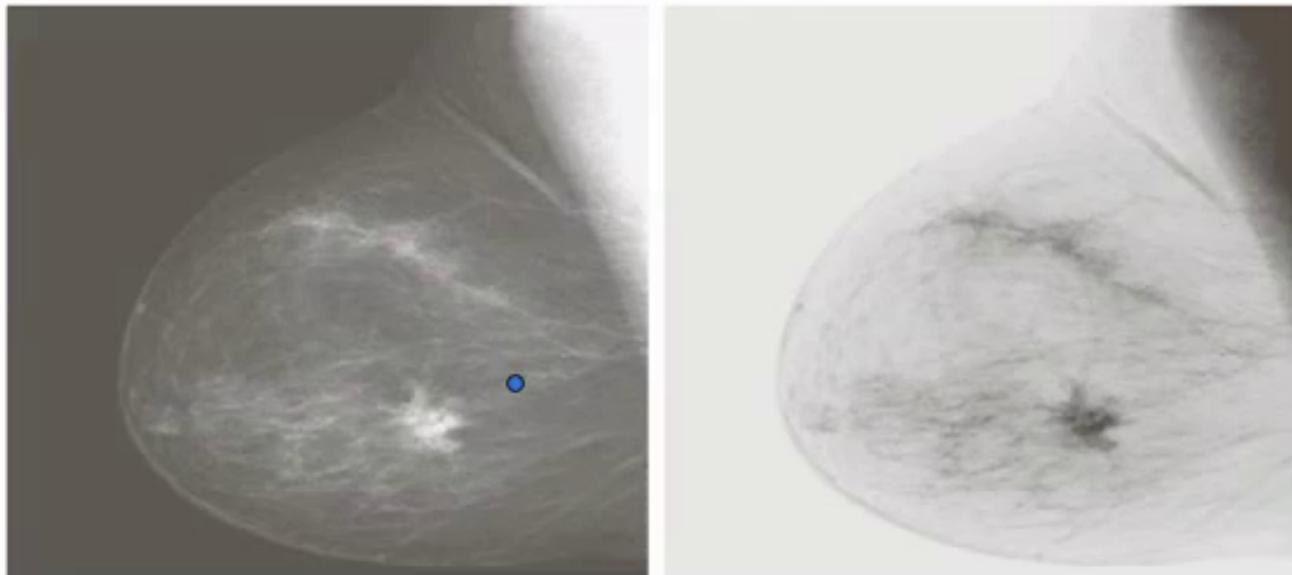
Chapter 3

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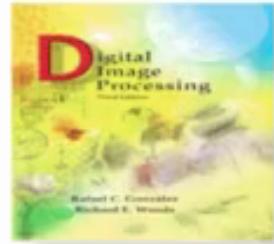


a b

FIGURE 3.4
(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)



$$S = L - r$$



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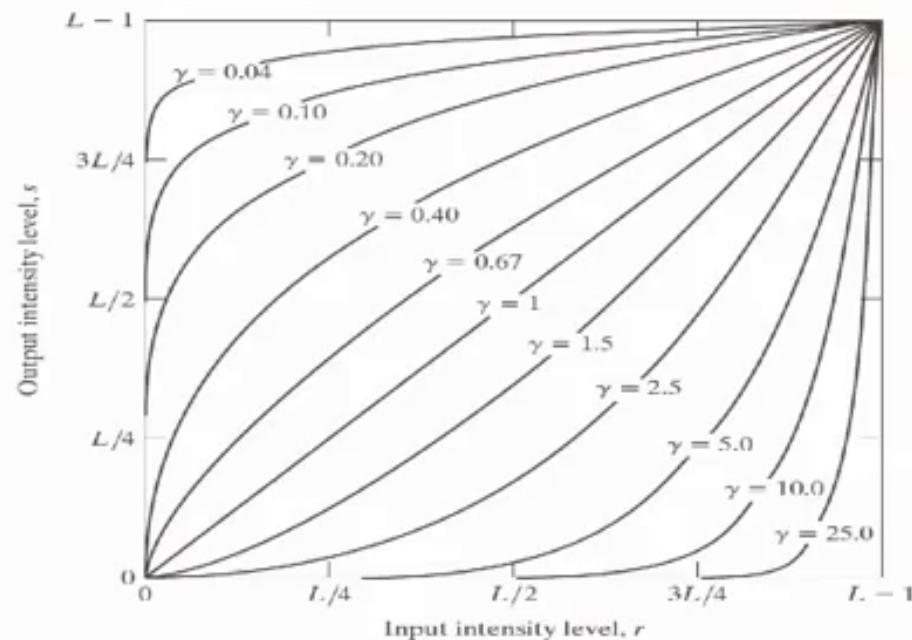
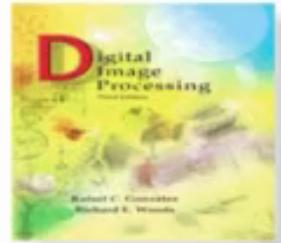


FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

A handwritten blue equation $S = C R^\gamma$ is written on the right side of the figure. A blue mouse cursor arrow points to the γ symbol in the equation.



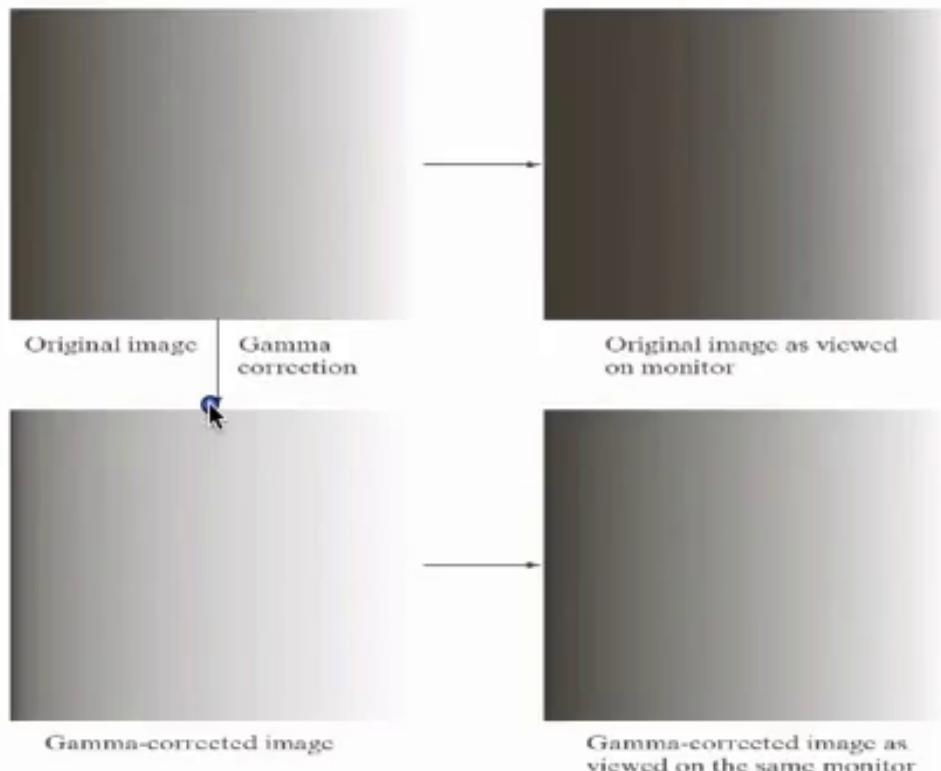
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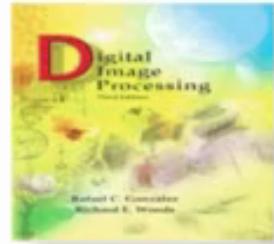
Chapter 3

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a b
c d

FIGURE 3.7
(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).



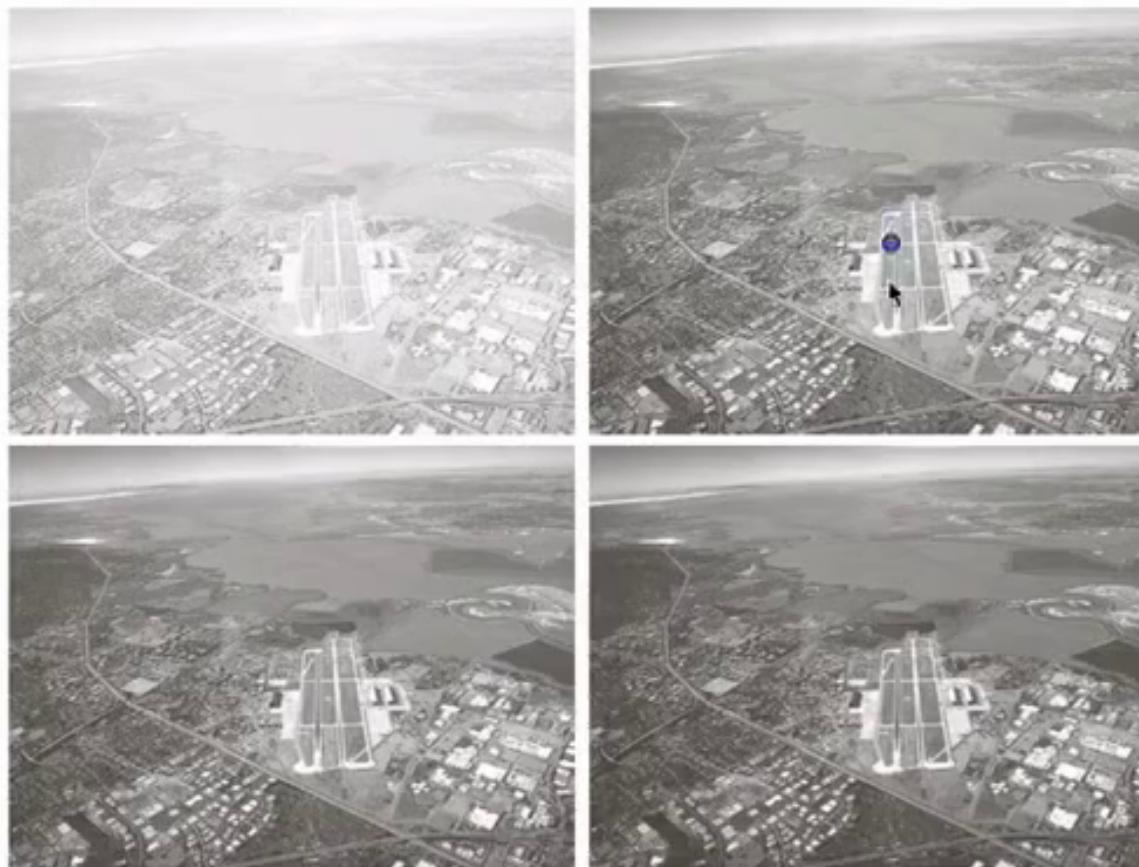
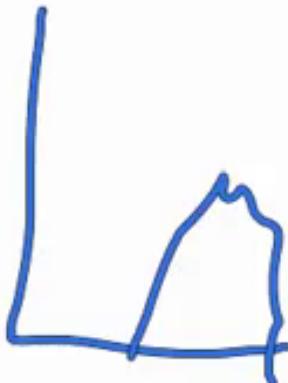
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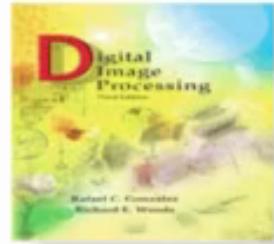
Chapter 3

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a
b
c
d

FIGURE 3.9
(a) Aerial image.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and $5.0,$ respectively.
(Original image for this example courtesy of NASA.)

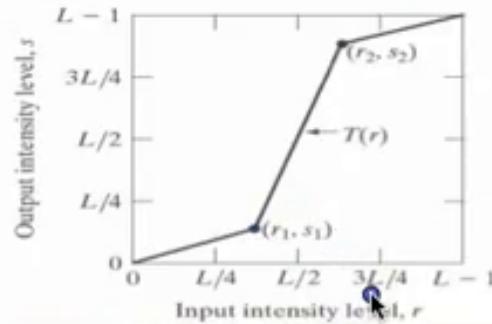


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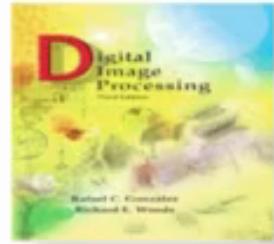
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a
b
c
d

FIGURE 3.10
Contrast stretching.
(a) Form of
transformation
function. (b) A
low-contrast image.
(c) Result of
contrast stretching.
(d) Result of
thresholding.
(Original image
courtesy of Dr.
Roger Heady,
Research School of
Biological Sciences,
Australian National
University,
Canberra,
Australia.)

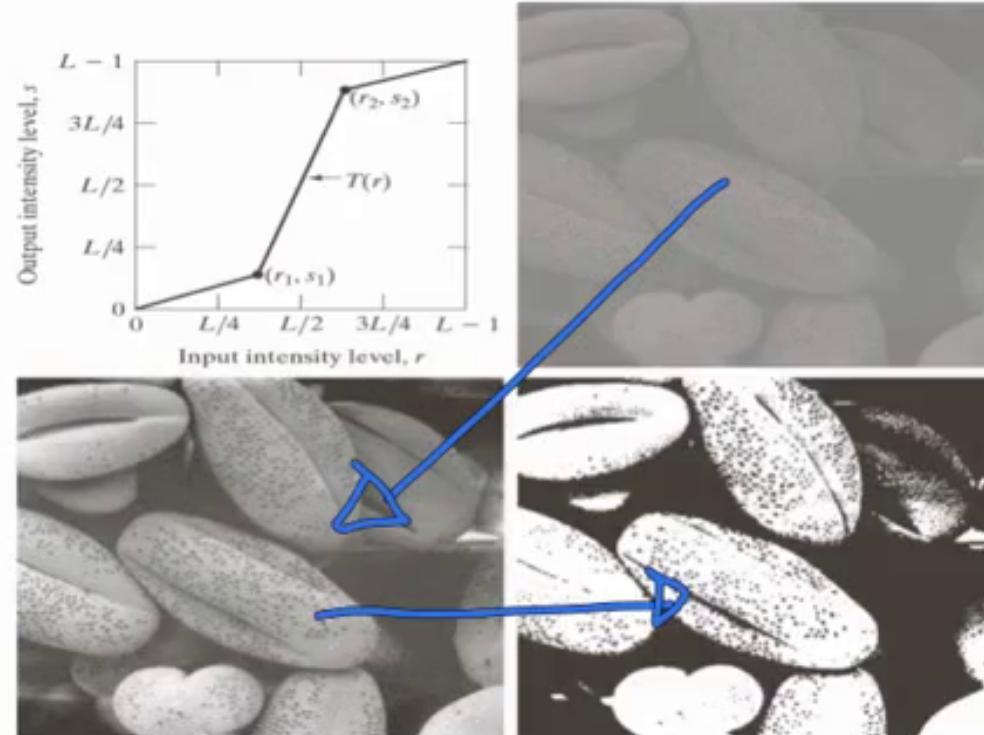


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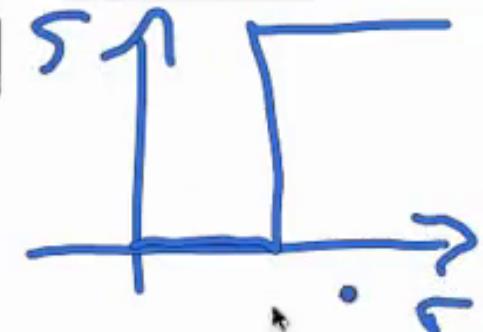
www.ImageProcessingPlace.com

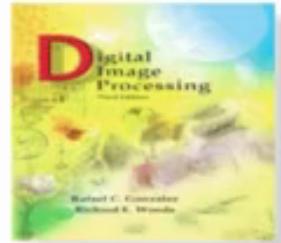
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a
b
c
d

FIGURE 3.10
Contrast stretching.
(a) Form of
transformation
function. (b) A
low-contrast image.
(c) Result of
contrast stretching.
(d) Result of
thresholding.
(Original image
courtesy of Dr.
Roger Heady,
Research School of
Biological Sciences,
Australian National
University,
Canberra,
Australia.)





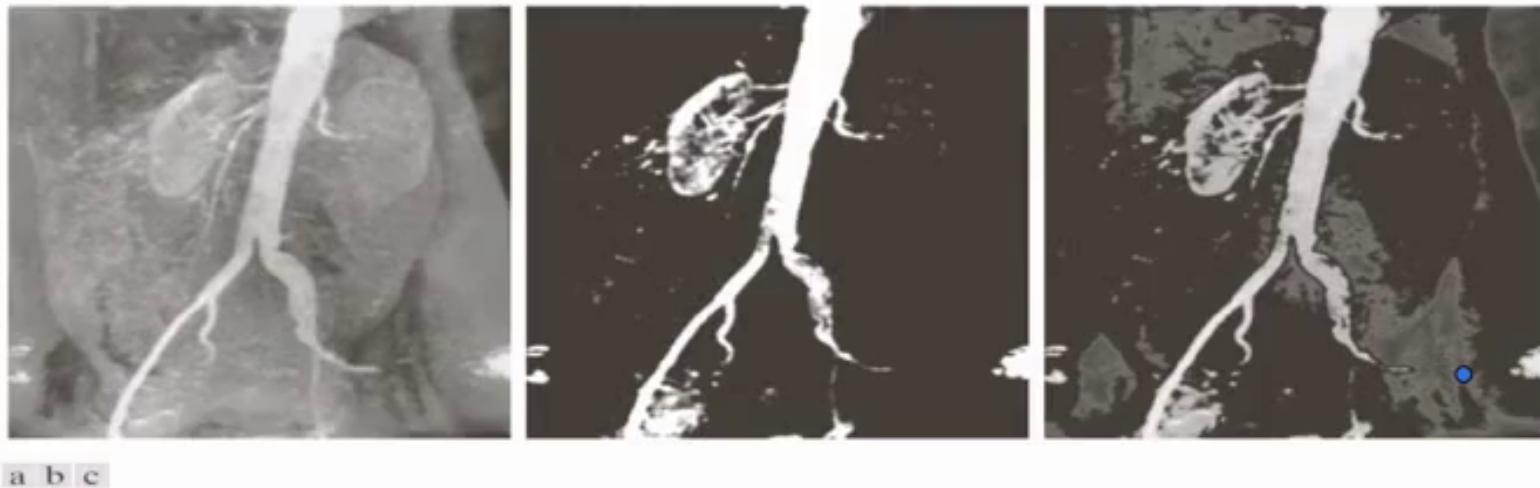
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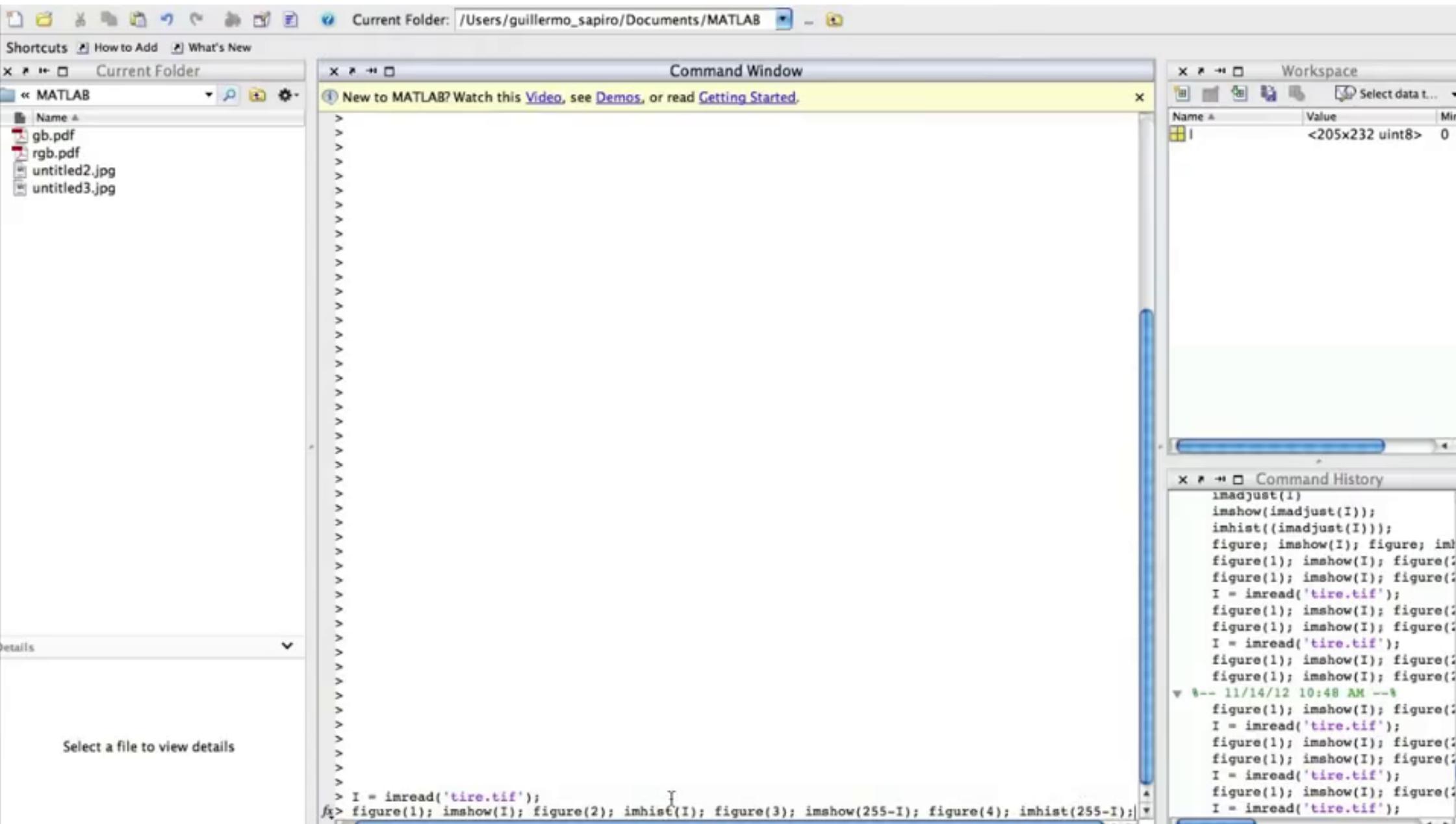
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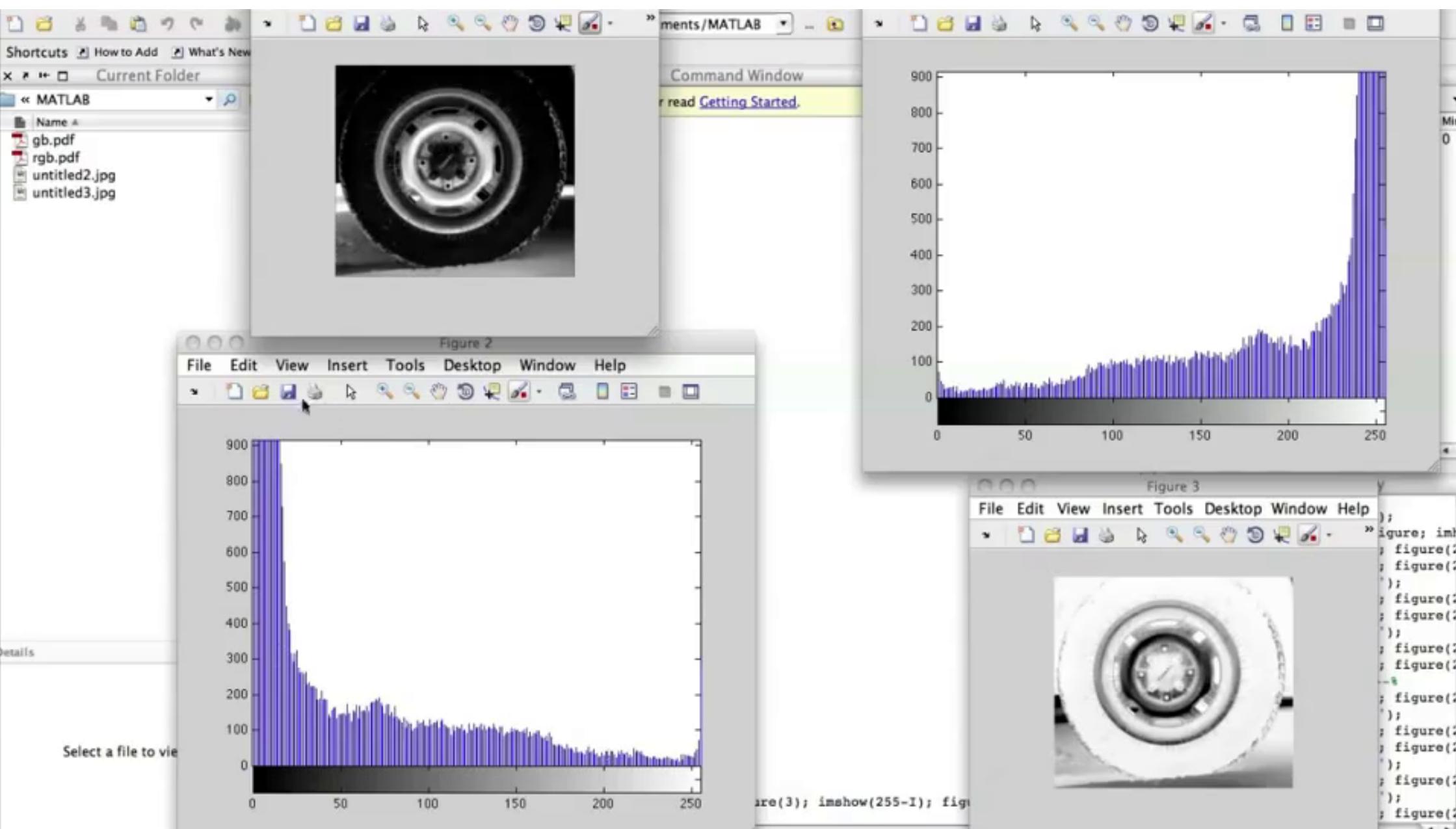
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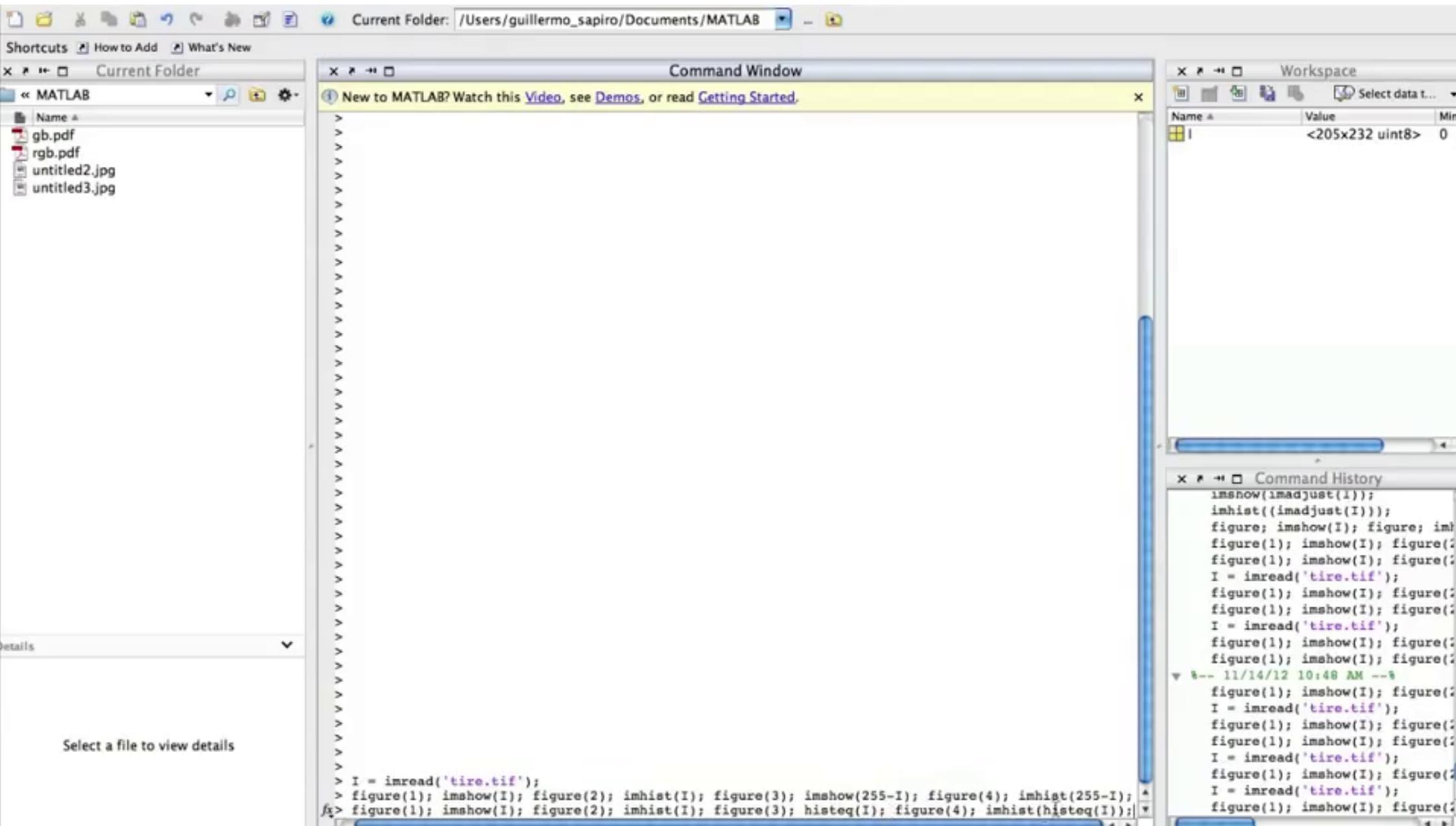


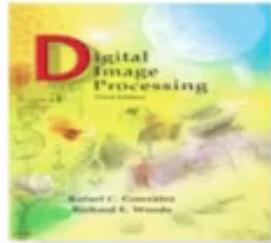
a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)









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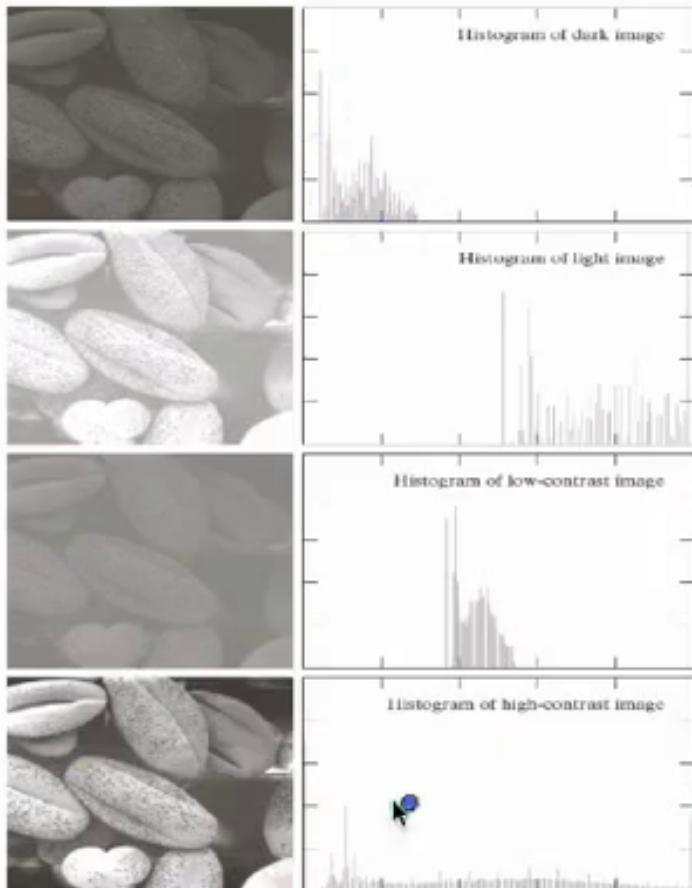
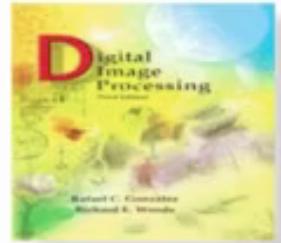


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.



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Histogram Equalization

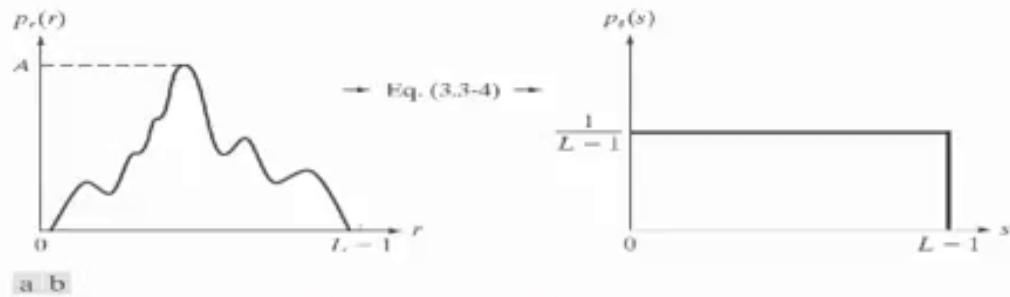
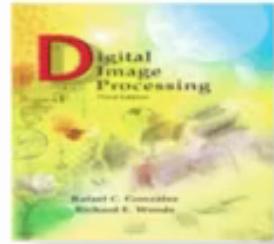


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r . The resulting intensities, s , have a uniform PDF, independently of the form of the PDF of the r 's.



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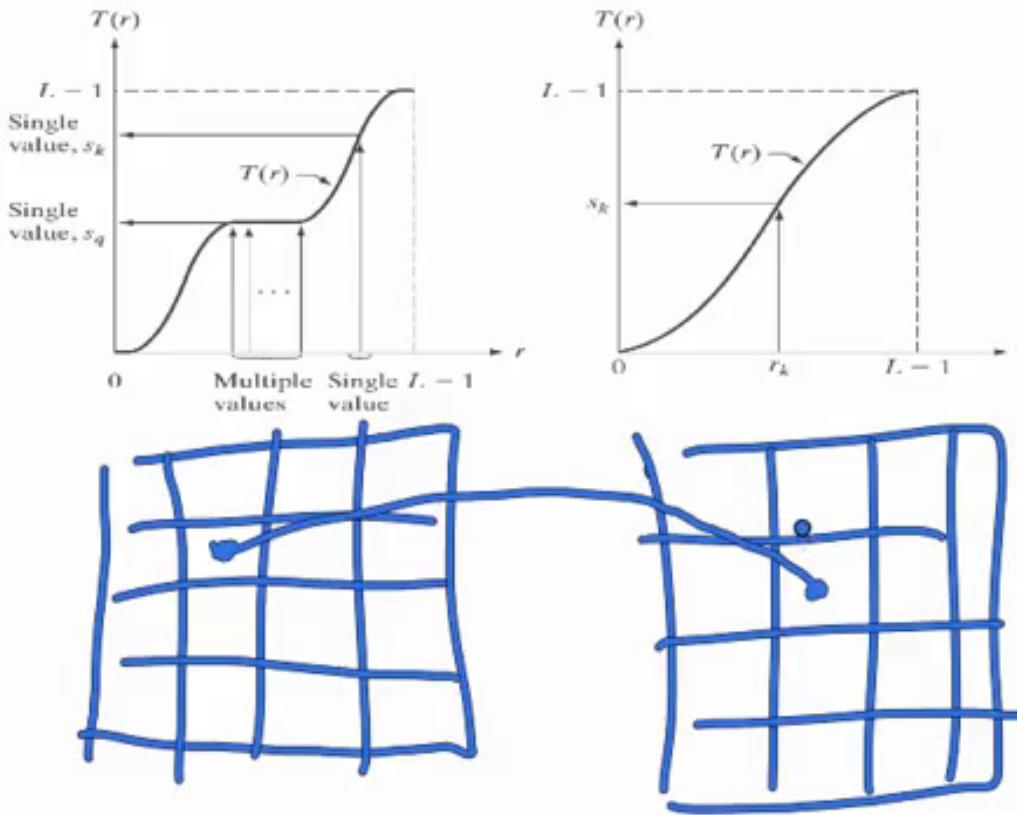
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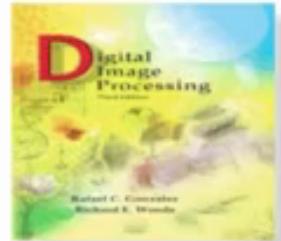
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a b

FIGURE 3.17
(a) Monotonically increasing function, showing how multiple values can map to a single value.
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.



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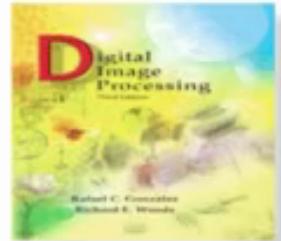
www.ImageProcessingPlace.com



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$$S = T(r) \quad P_S(s) = P_r(r) \left| \frac{ds}{dr} \right|$$
$$S = T(r) = (-i) \int_{-\infty}^r P_r(w) dw$$
$$\frac{ds}{dr} = \frac{\int T(r)}{dr} = \frac{\int (-i) \int P_r(w) dw}{dr}$$



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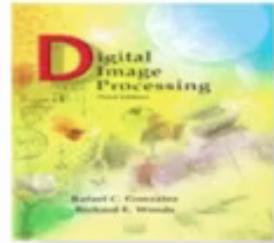
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$$\begin{aligned} P_S(\xi) &= P_r(r) \left| \frac{\partial r}{\partial \xi} \right| = \\ &= P_r(r) \left| \frac{1}{(-1) P_r(r)} \right| = \frac{1}{L-1} \\ S = T(r) &= (-1) \int_0^r P_r(w) dw \end{aligned}$$



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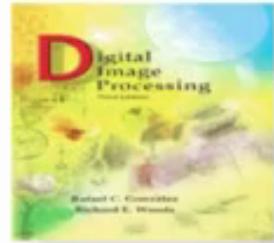


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r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1
Intensity distribution and histogram values for a 3-bit, 64×64 digital image.



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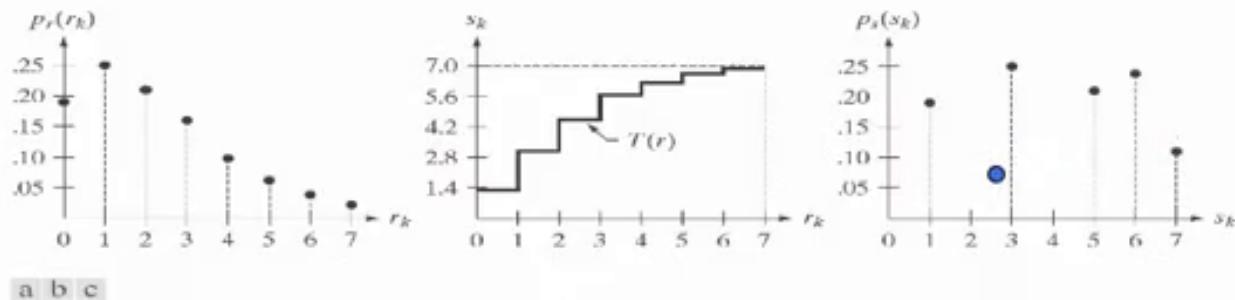
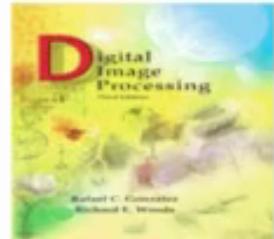


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

$$\int_D p_r(w) dw$$



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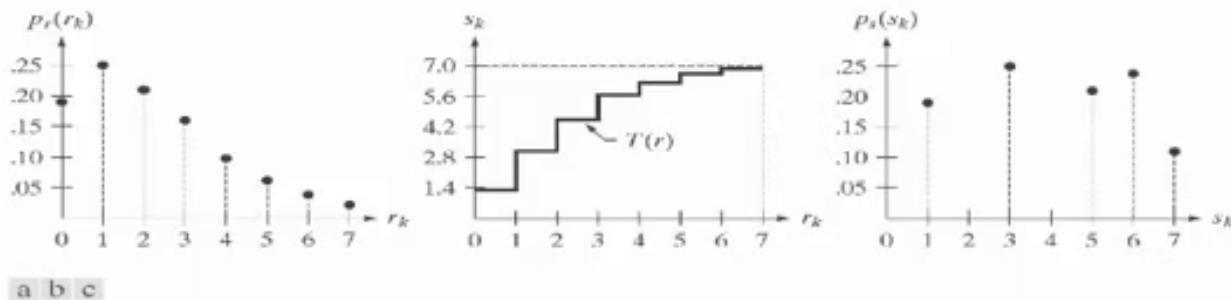
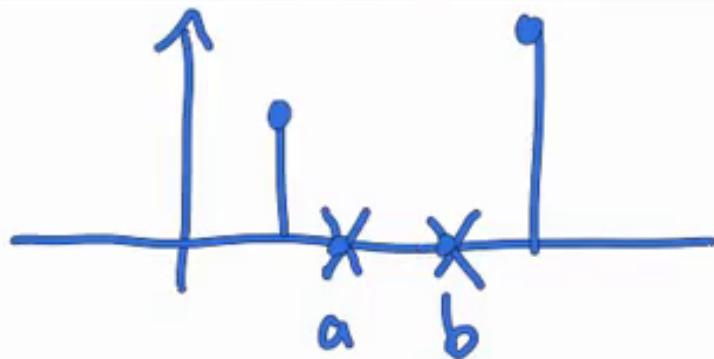
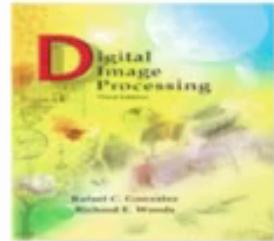


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.



$$\begin{aligned} T(a) &\stackrel{?}{=} T(b) \\ \downarrow & \\ T(a) &= T(b) \end{aligned}$$



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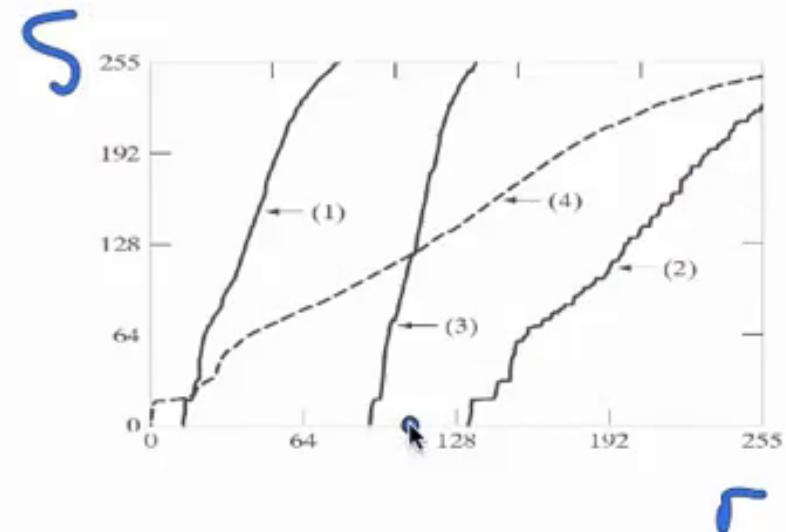
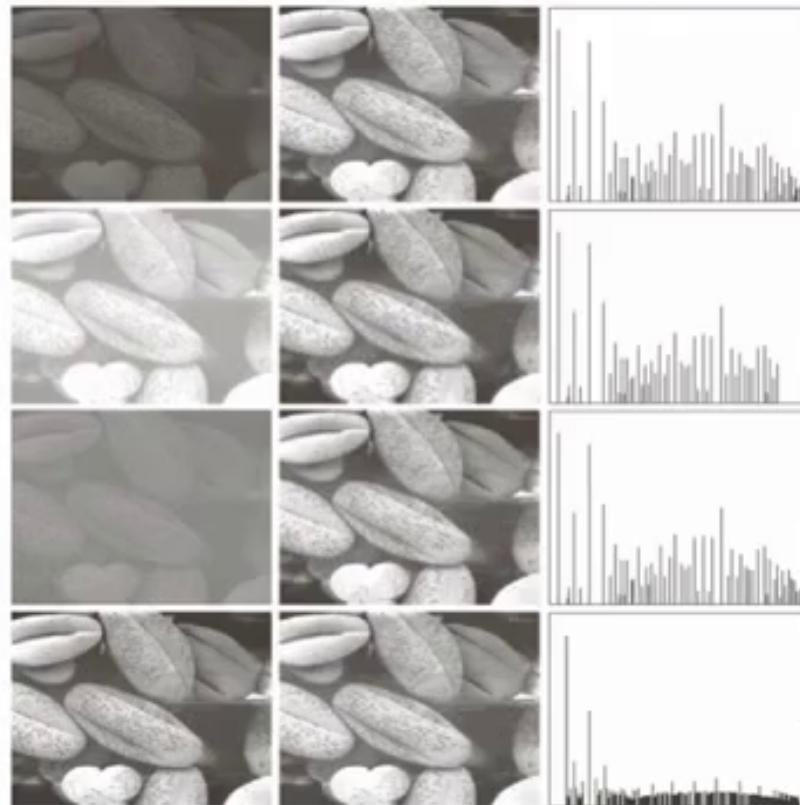
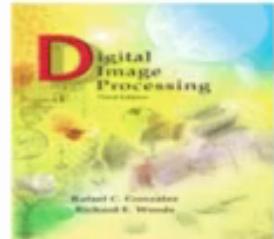


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.



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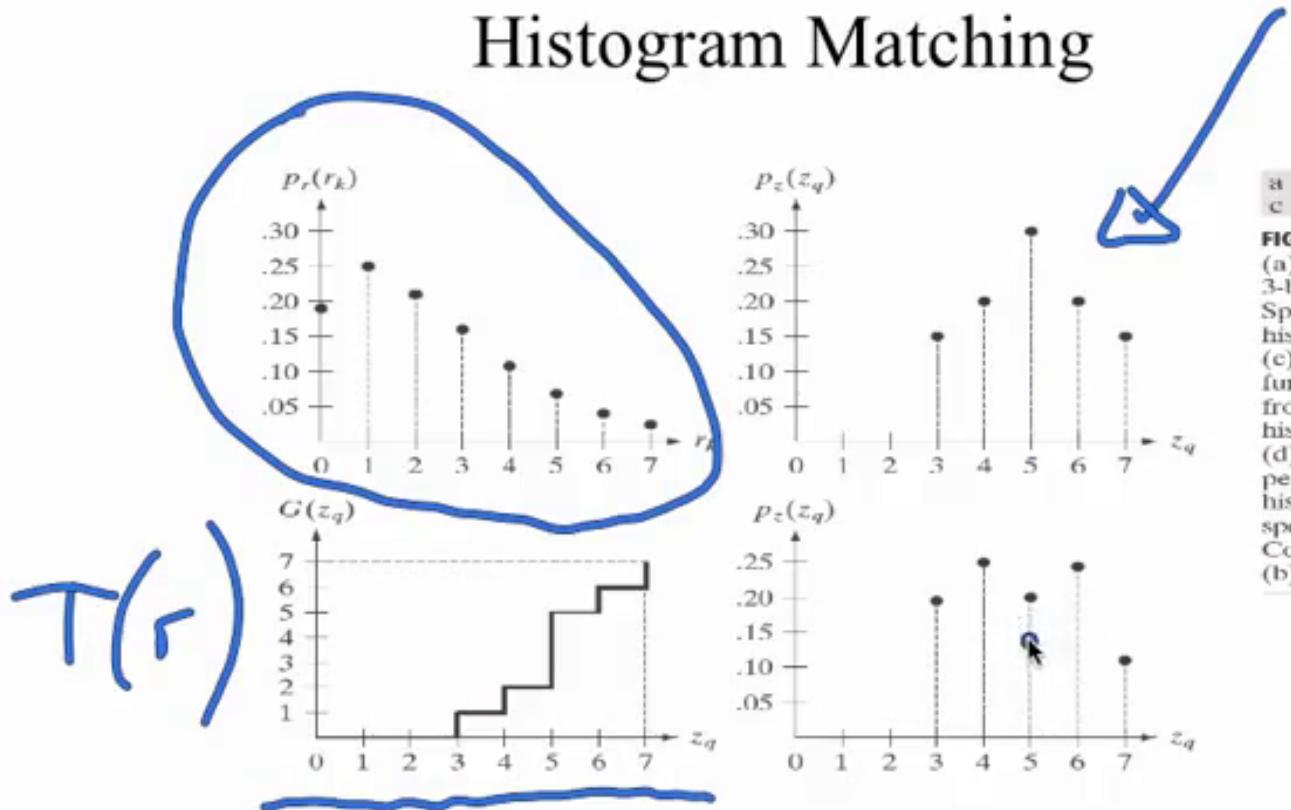
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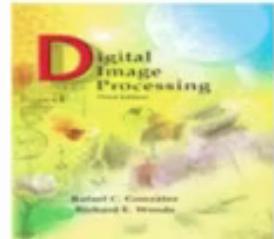
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Histogram Matching



a b
c d

FIGURE 3.22
(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).



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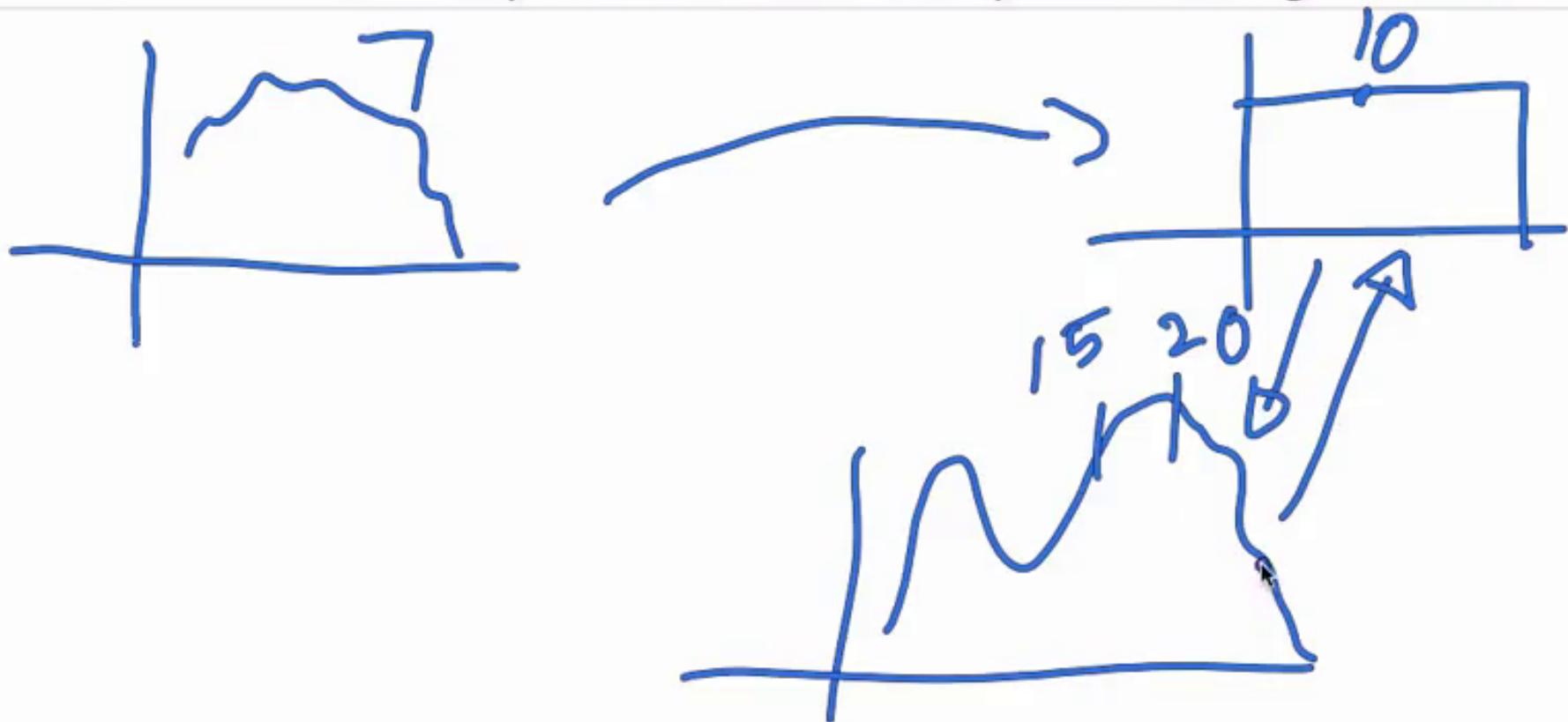
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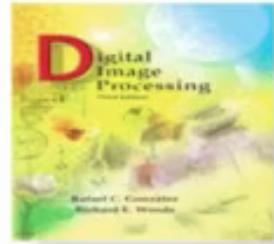
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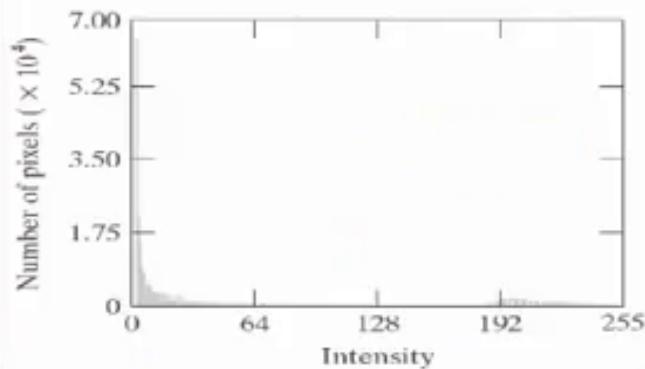
Chapter 3

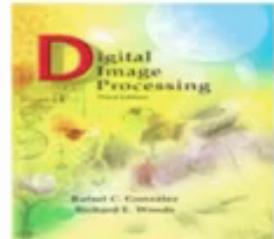
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a b

FIGURE 3.23
(a) Image of the
Mars moon
Phobos taken by
NASA's *Mars
Global Surveyor*.
(b) Histogram.
(Original image
courtesy of
NASA.)





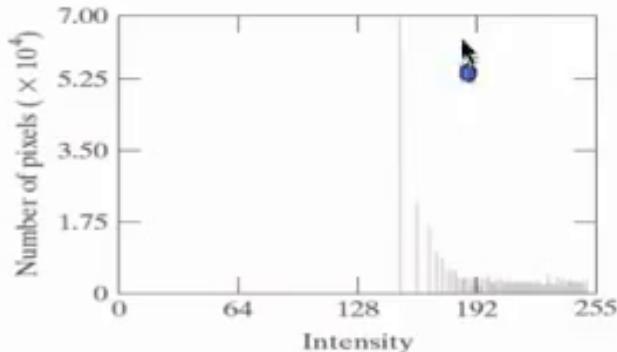
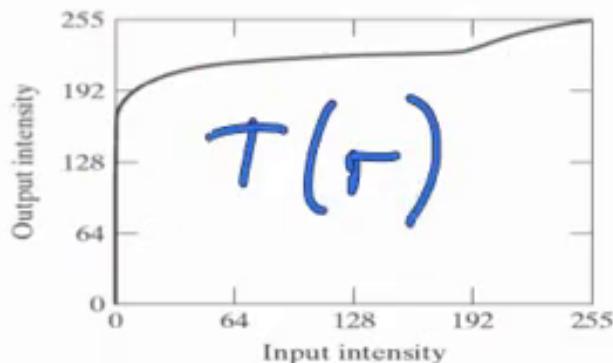
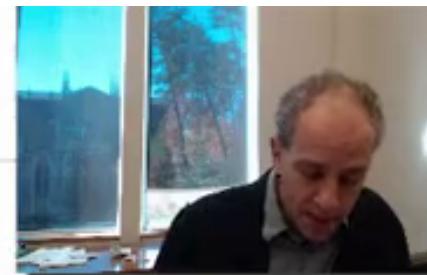
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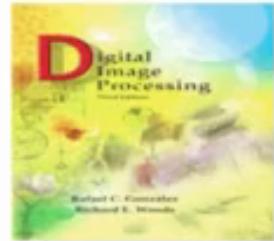
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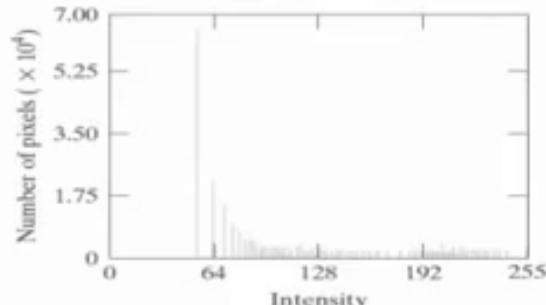
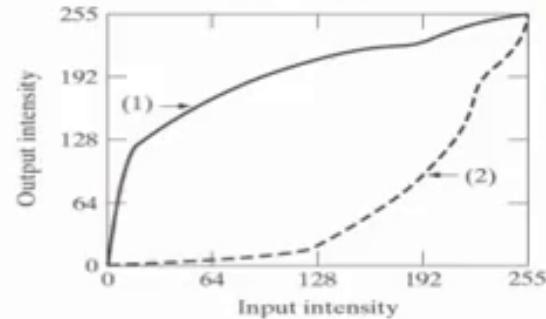
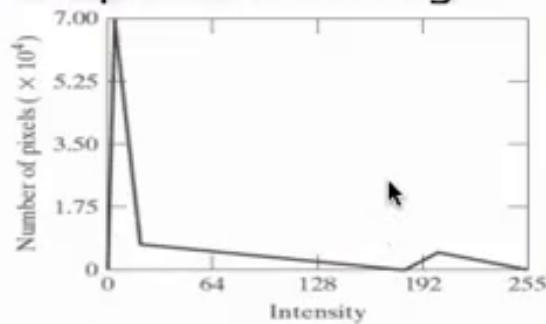
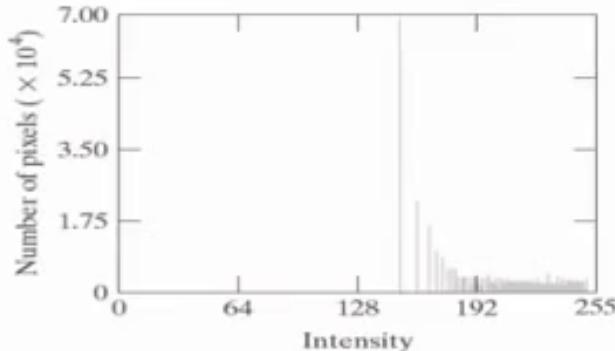
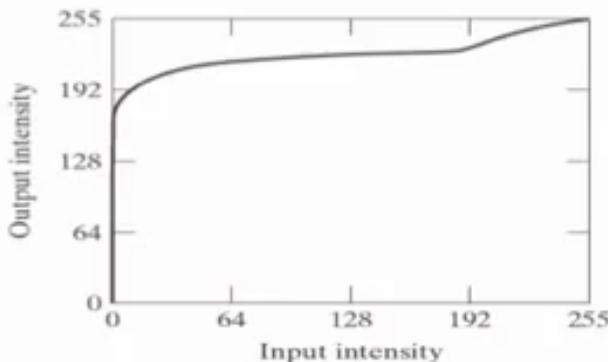
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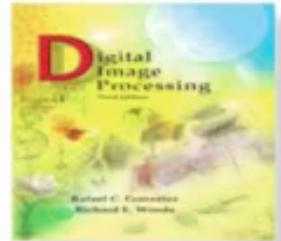
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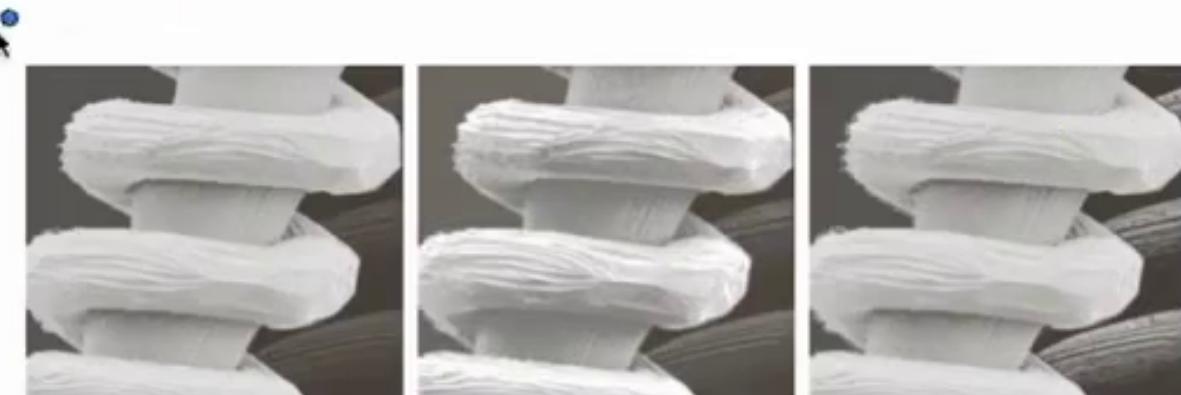
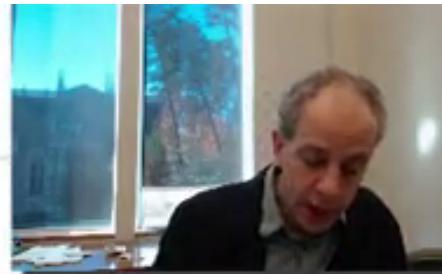
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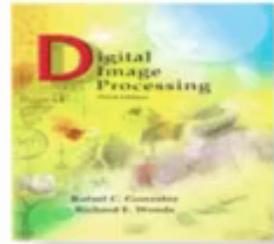
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a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)



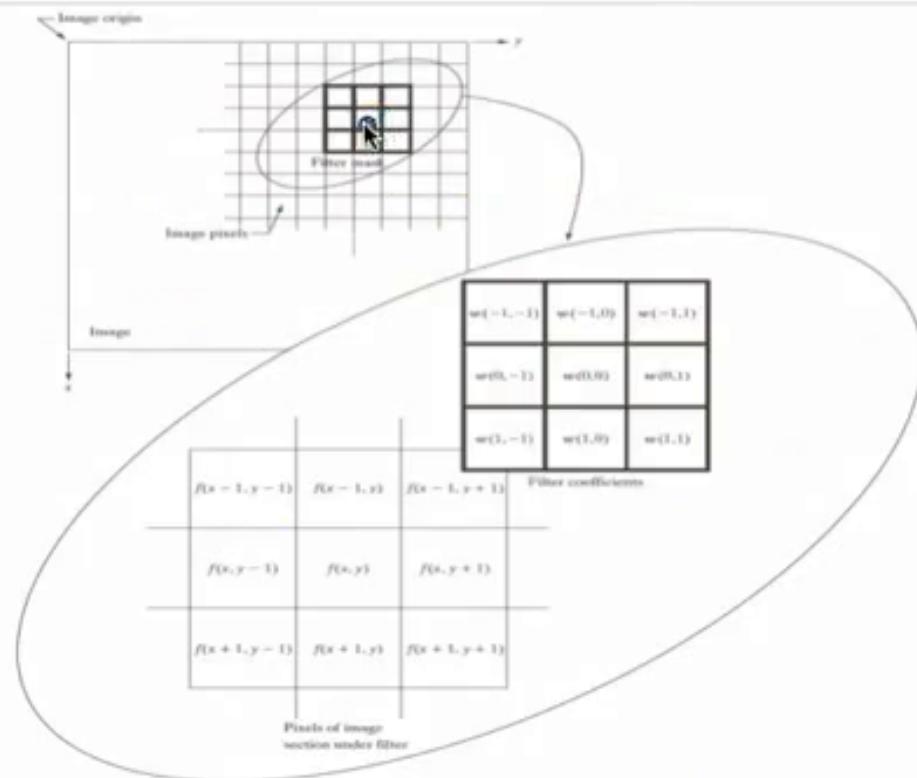
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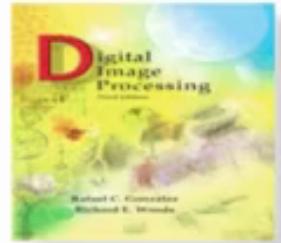
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w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

FIGURE 3.28 The mechanics of linear spatial filtering using a 3×3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.



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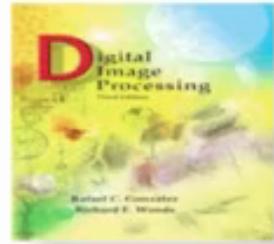
Chapter 3

Intensity Transformations & Spatial Filtering


$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$
$$\frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.



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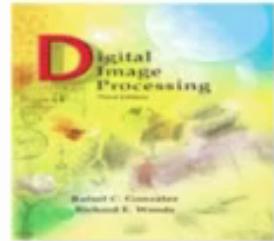
Chapter 3

Intensity Transformations & Spatial Filtering



FIGURE 3.33 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15, 35$, and 65 , respectively. The black squares at the top are of sizes $3, 5, 9, 15, 25, 35, 45$, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.





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Chapter 3 Intensity Transformations & Spatial Filtering



a b c

FIGURE 3.34 (a) Image of size 528×485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15×15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Averaging, Gaussian Filtering, Heat Flow

$$(a-1)^2 + (a-2)^2 + (a-3)^2$$

1, 2, 3 a

No

Yes - 0

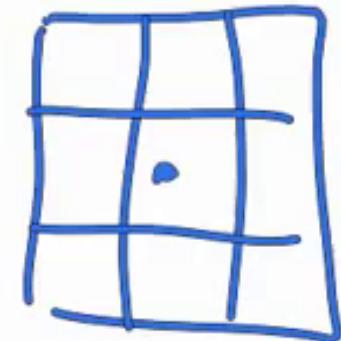
Yes - 2

Yes - 4

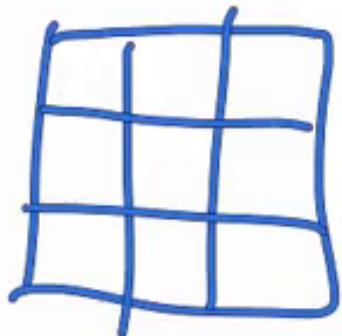


Averaging, Gaussian Filtering, Heat Flow

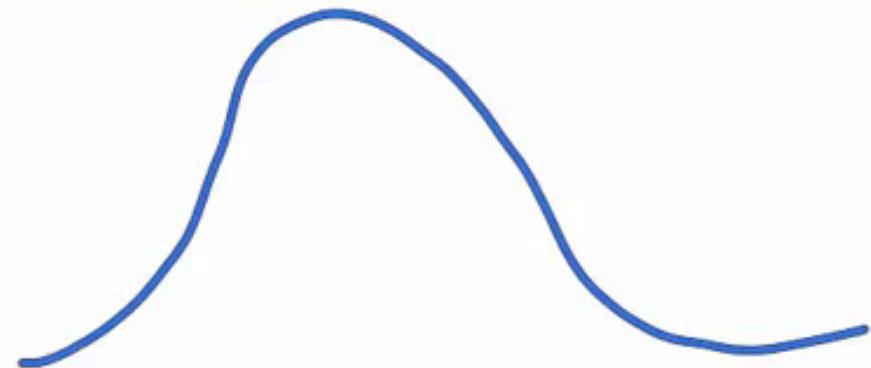
$$N = \sum_i (a - a_i)^2$$
$$a = \bar{a}_i$$



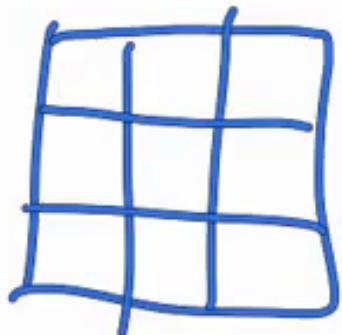
Averaging, Gaussian Filtering, Heat Flow



$$f(x, y; \sigma) = f(x, y) * G(0, \sigma)$$



Averaging, Gaussian Filtering, Heat Flow



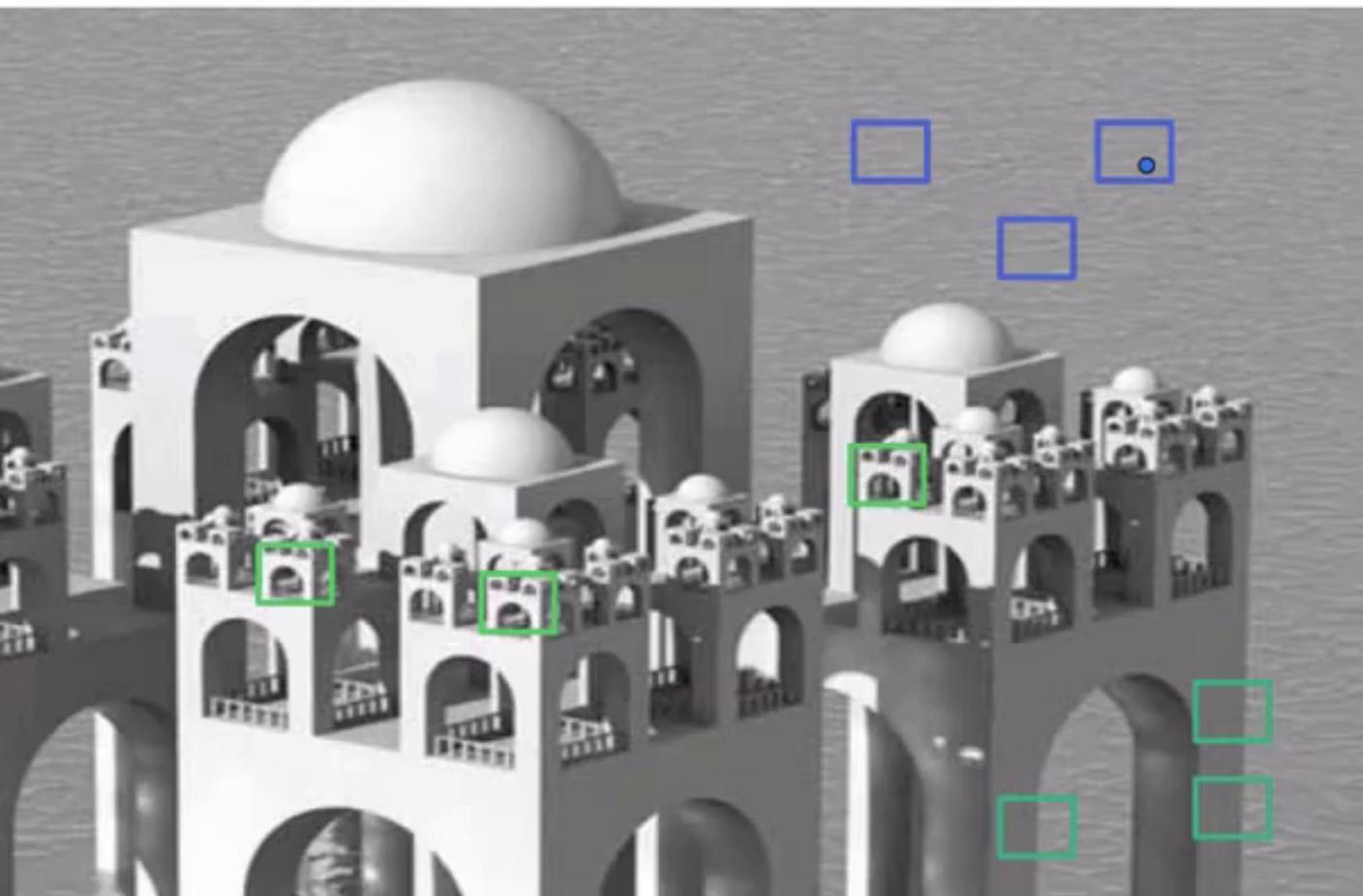
$$f(x, y; \sigma) = f(x, y) * G(0, \sigma)$$



$$\frac{df}{dt} = \Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



Non Local Means/Average



Credits: Glasner et al. – Buades et al.



The algorithm result is displayed hereafter. It ran in 5.82s.
You can run again this algorithm with new data.

Run again?: new input different parameter or subimage

Results (sigma: 25)

Noisy



Denoised

Original

Difference



The algorithm result is displayed hereafter. It ran in 5.82s.
You can run again this algorithm with new data.

Run again?: new input different parameter or subimage

Results (sigma: 25)

Noisy

Denoised 

Original

Difference



The algorithm result is displayed hereafter. It ran in 5.82s.
You can run again this algorithm with new data.

Run again?: new input different parameter or subimage

Results (sigma: 25)

Noisy

Denoised

Original 

Difference



The algorithm result is displayed hereafter. It ran in 5.82s.

You can run again this algorithm with new data.

Run again?: new input different parameter or subimage

Results (sigma: 25)

Noisy

Denoised

Original

Difference



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The algorithm result is displayed hereafter. It ran in 16.33s.

You can run again this algorithm with new data.

Run again?:

Results (sigma: 35)

Noisy



Denoised

Original

Difference



← → C demo.ipol.im/demo/bcm_non_local_means_denoising/result?key=5ABA2A4448E8980B01B51B37F6486422

article demo archive

Please cite this article if you publish results obtained with this online demo.

The algorithm result is displayed hereafter. It ran in 16.33s.
You can run again this algorithm with new data.

Run again?:

Results (sigma: 35)

Noisy

Denoised

Original

Difference



[←](#) [→](#) [C](#) demo.ipol.im/demo/bcm_non_local_means_denoising/result?key=5ABA2A4448E8980B01B51B37F6486422

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The algorithm result is displayed hereafter. It ran in 16.33s.
You can run again this algorithm with new data.

Run again?:

Results (sigma: 35)

Noisy
Denoised
Original 
Difference



← → C demo.ipol.im/demo/bcm_non_local_means_denoising/result?key=5ABA2A4448E8980B01B51B37F6486422

article demo archive

Please cite this article if you publish results obtained with this online demo.

The algorithm result is displayed hereafter. It ran in 16.33s.
You can run again this algorithm with new data.

Run again?:

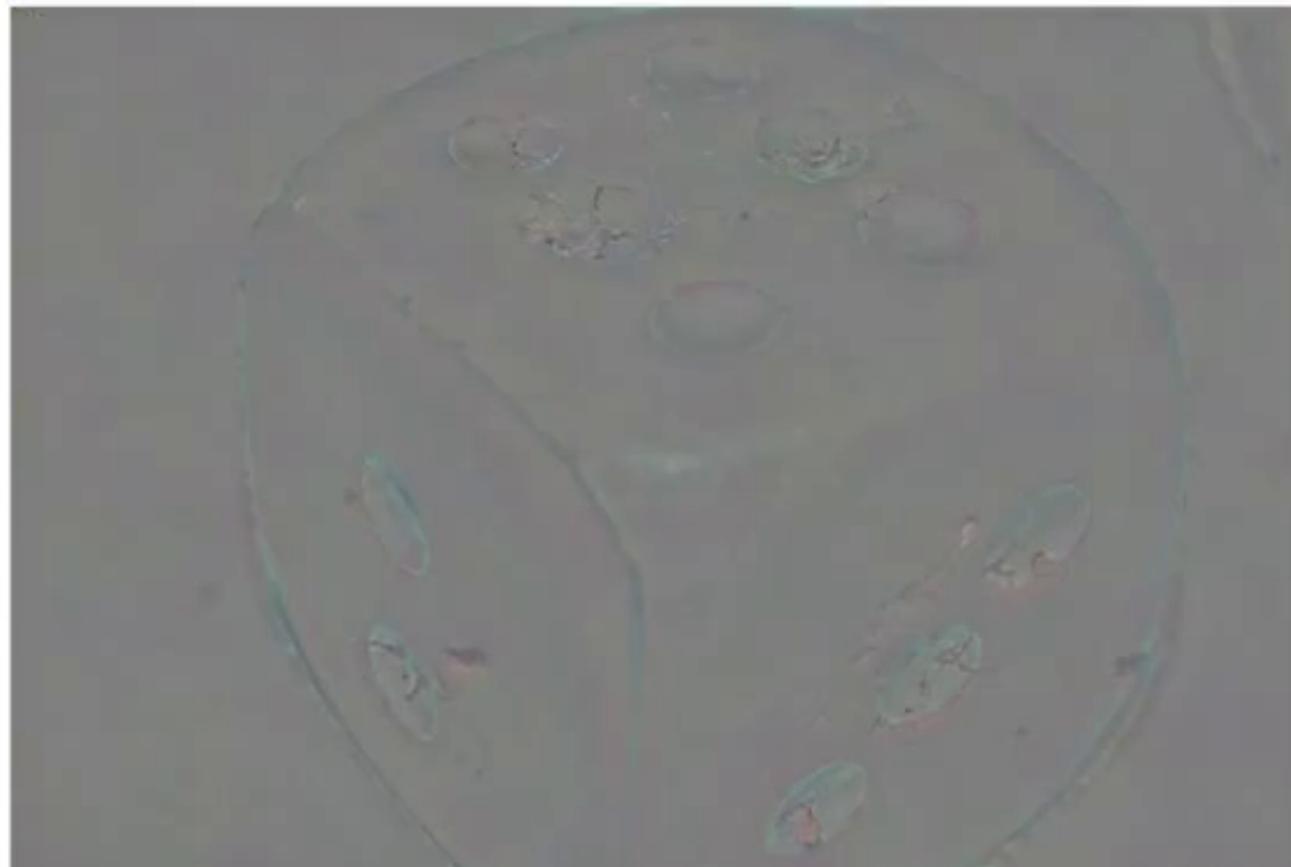
Results (sigma: 35)

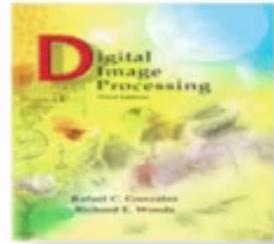
Noisy

Denoised

Original

Difference





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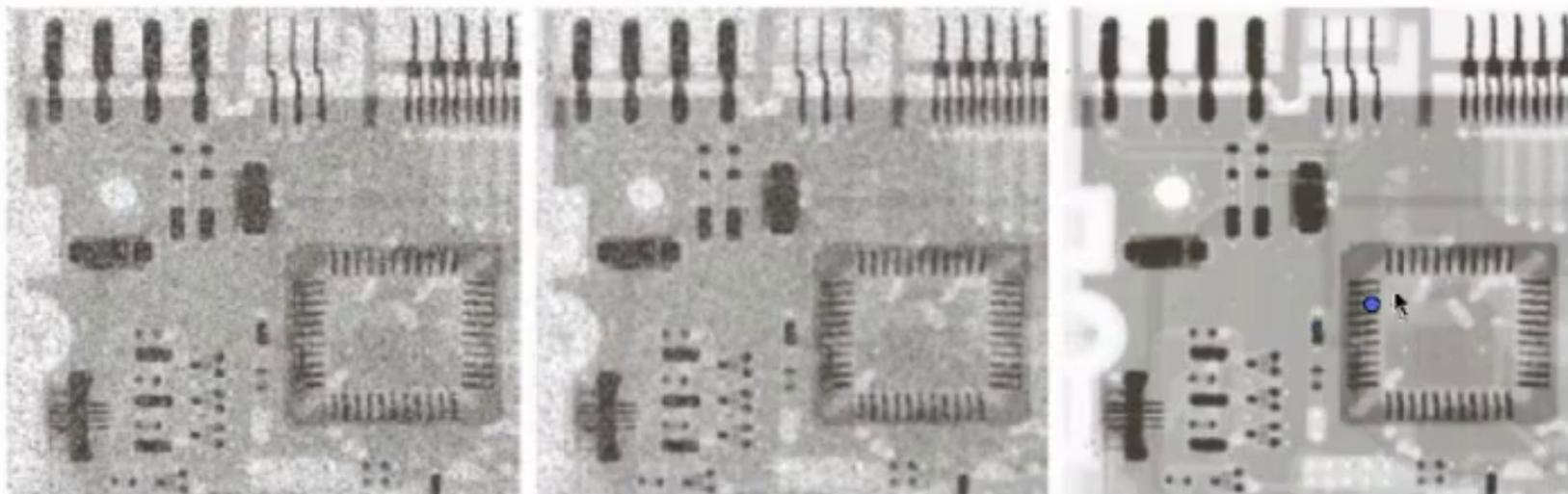
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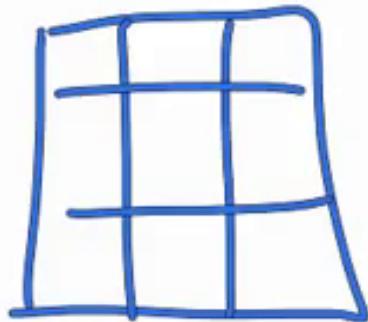
Chapter 3

Intensity Transformations & Spatial Filtering



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



~~X~~

• • • • • • •

$$\sum_i (a - a_i)^2 \quad \sum f(a - a_i)$$

Mean

$$\sum |a - a_i|$$



Image and Video Processing: From Mars to Hollywood with a Stop at the Hospital

Guillermo Sapiro



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>> I = imread('eight.tif');
>> J = imnoise(I,'salt & pepper',0.09);
>> K = medfilt2(J);
fx>> figure, imshow(I); figure, imshow(J), figure, imshow(K)
```

Select a file to view details

Command Window

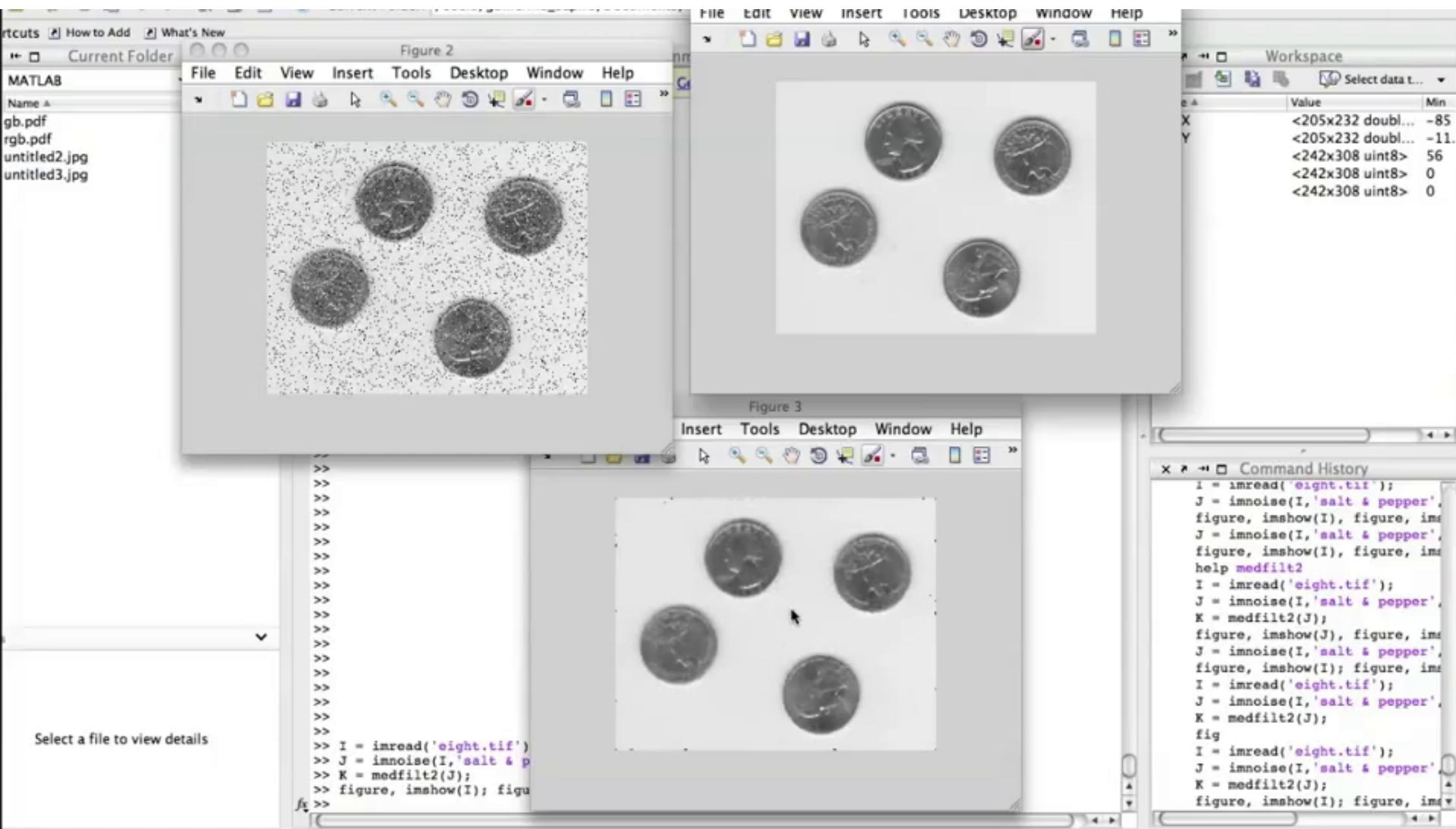
Watch this [Video](#), see [Demos](#), or read [Getting Started](#).

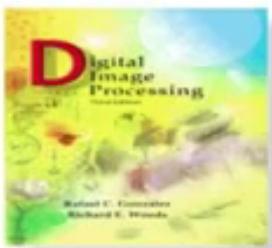
Workspace

Name	Value	Min
GX	<205x232 double	-85
GY	<205x232 double	-11.
I	<242x308 uint8>	56
J	<242x308 uint8>	0
K	<242x308 uint8>	0

Command History

```
help images
I = imread('eight.tif');
J = imnoise(I,'salt & pepper');
figure, imshow(I), figure, imshow(J)
J = imnoise(I,'salt & pepper');
figure, imshow(I), figure, imshow(J)
help medfilt2
I = imread('eight.tif');
J = imnoise(I,'salt & pepper');
K = medfilt2(J);
figure, imshow(J), figure, imshow(K)
J = imnoise(I,'salt & pepper');
figure, imshow(I); figure, imshow(J)
I = imread('eight.tif');
J = imnoise(I,'salt & pepper');
K = medfilt2(J);
fig
I = imread('eight.tif');
J = imnoise(I,'salt & pepper');
K = medfilt2(J);
```





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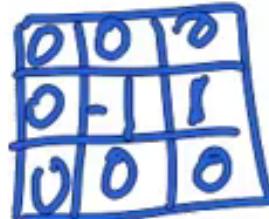
www.ImageProcessingPlace.com



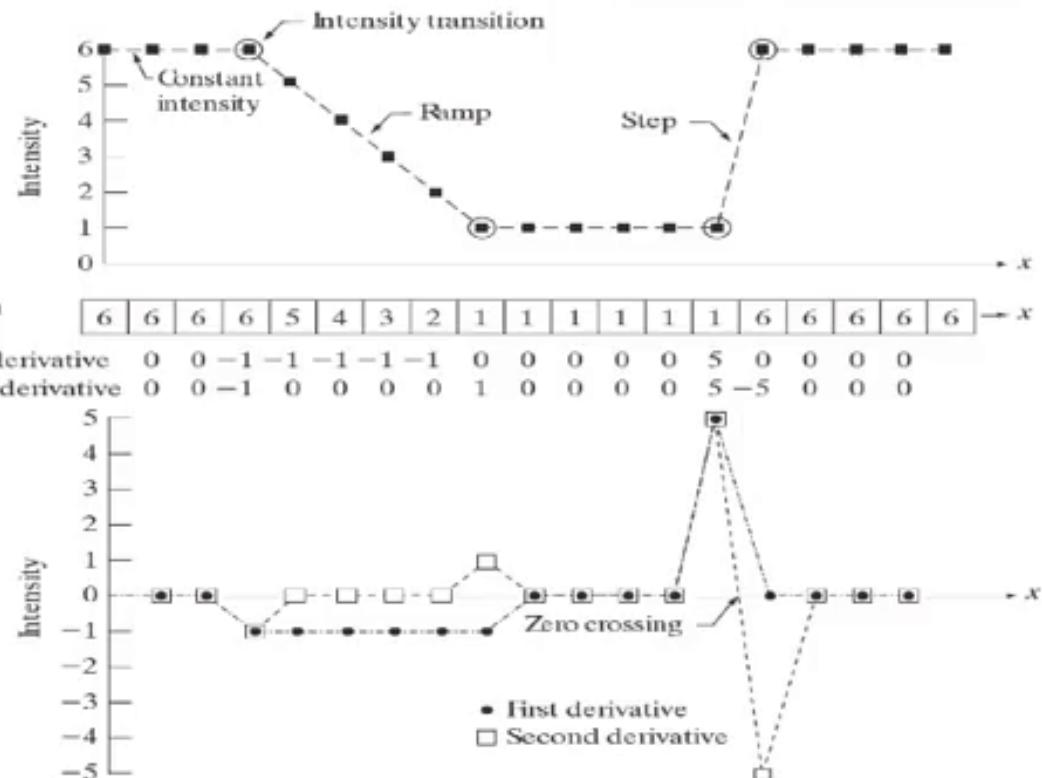
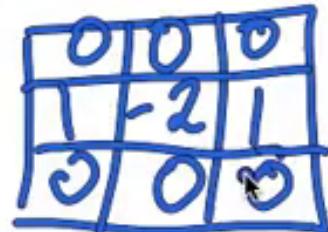
Chapter 3

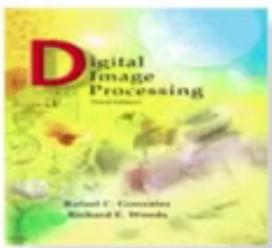
Intensity Transformations & Spatial Filtering

$$\frac{\partial f}{\partial x} \approx f(x+1) - f(x)$$



$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$





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Intensity Transformations & Spatial Filtering

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \Delta f$$
$$= f(x+1) + f(x-1) - 2f(x)$$
$$f(y+1) + f(y-1) - 2f(y)$$

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1



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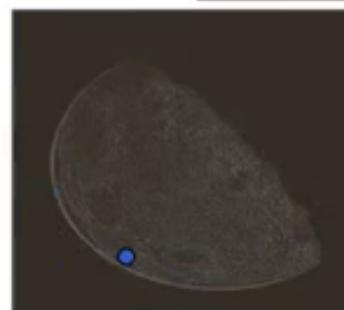
Intensity Transformations & Spatial Filtering

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

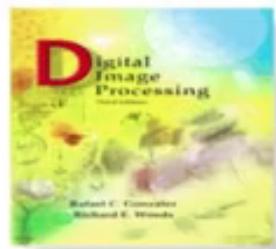
0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1



a
b c
d c

FIGURE 3.38
(a) Blurred image of the North Pole of the moon.
(b) Laplacian without scaling.
(c) Laplacian with scaling.
(d) Image sharpened using the mask in Fig. 3.37(a).
(e) Result of using the mask in Fig. 3.37(b).
(Original image courtesy of NASA.)



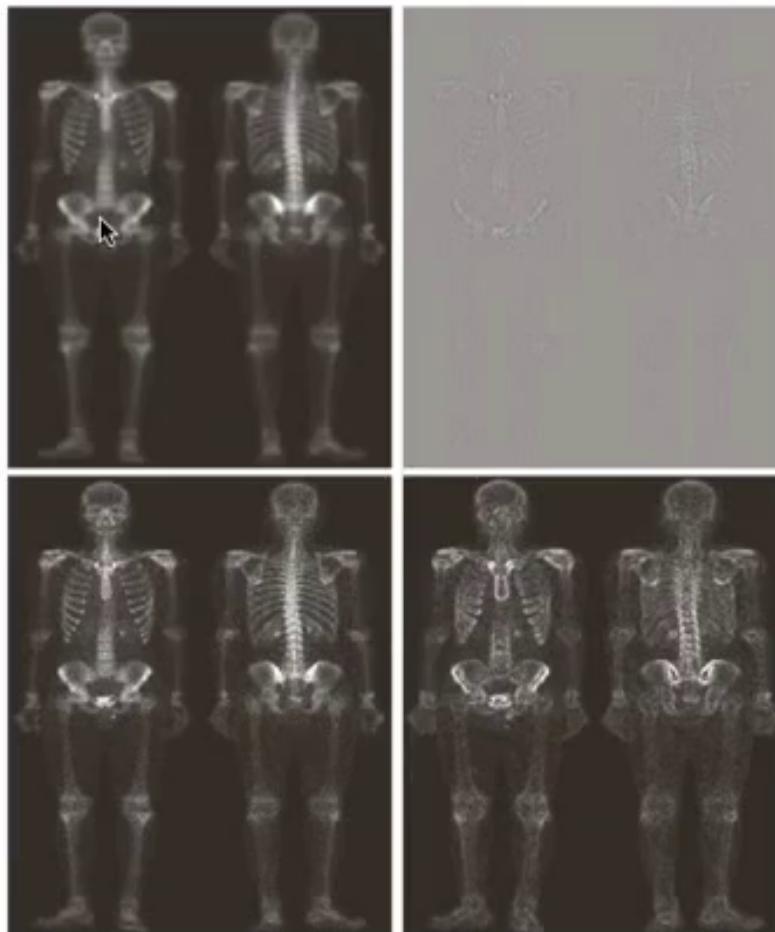
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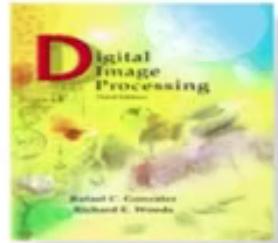
Chapter 3

Intensity Transformations & Spatial Filtering



a b
c d

FIGURE 3.43
(a) Image of
whole body bone
scan.
(b) Laplacian of
(a). (c) Sharpened
image obtained by
adding (a) and (b).
(d) Sobel gradient
of (a).



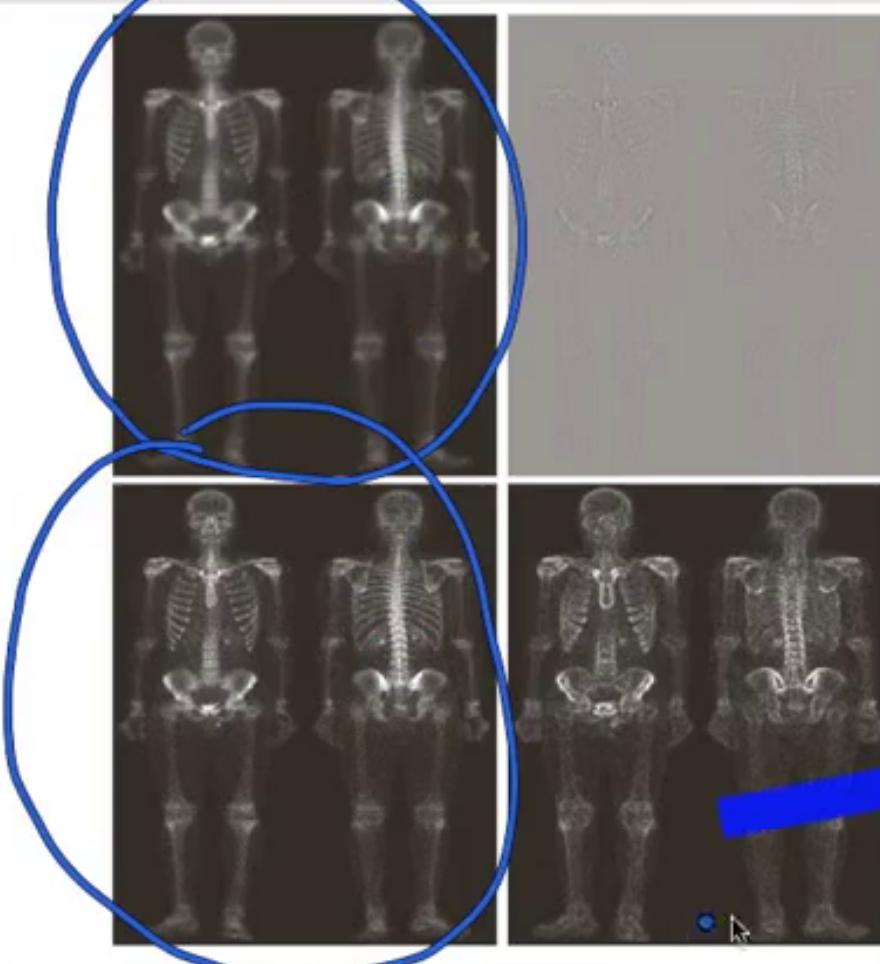
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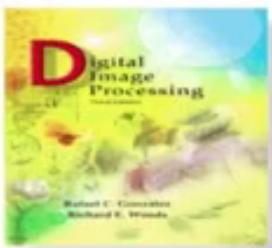
Intensity Transformations & Spatial Filtering



a b
c d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a).
(c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of (a).

Sobel
Edges



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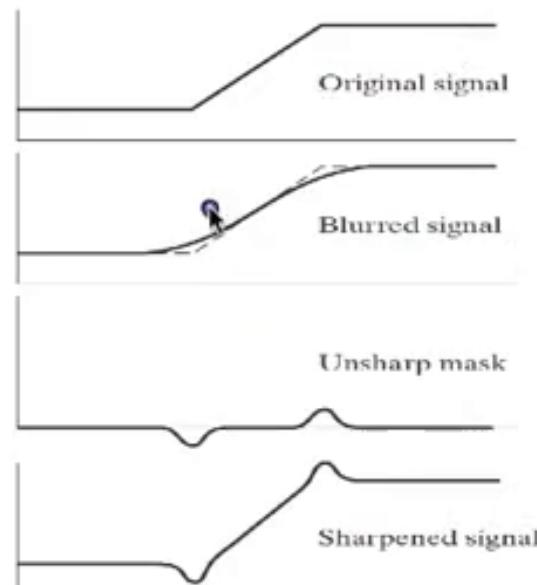
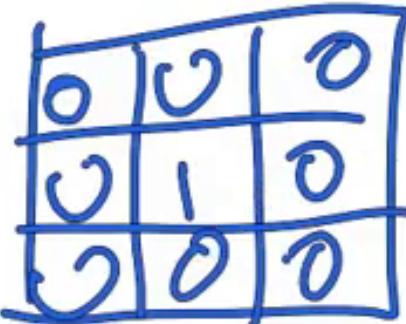
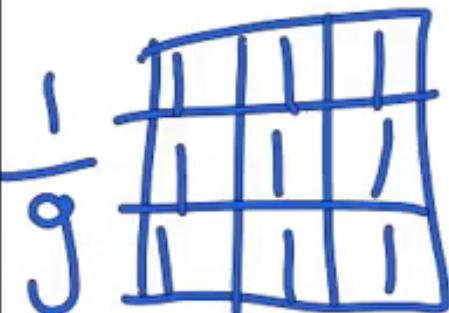
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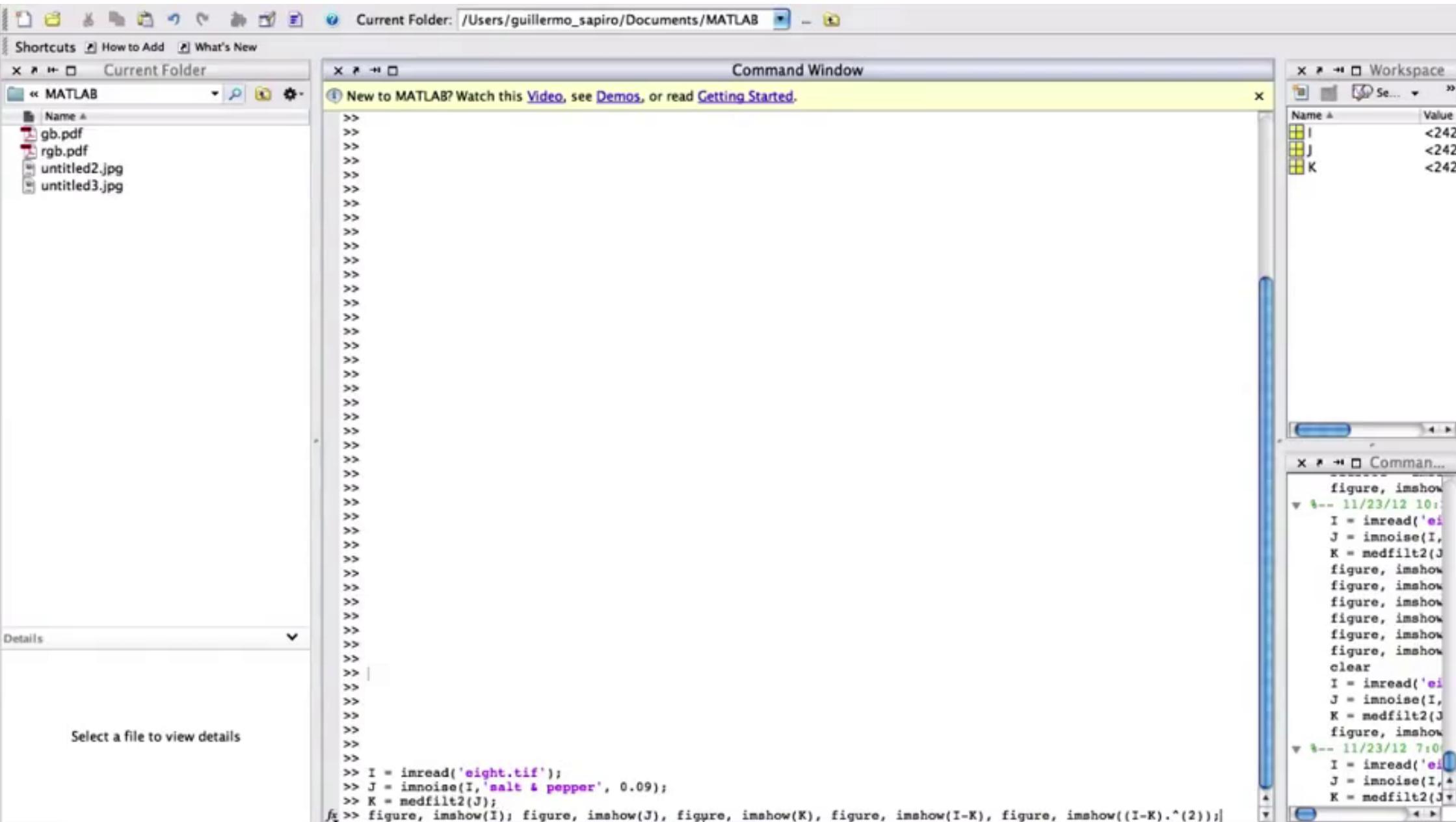
Intensity Transformations & Spatial Filtering

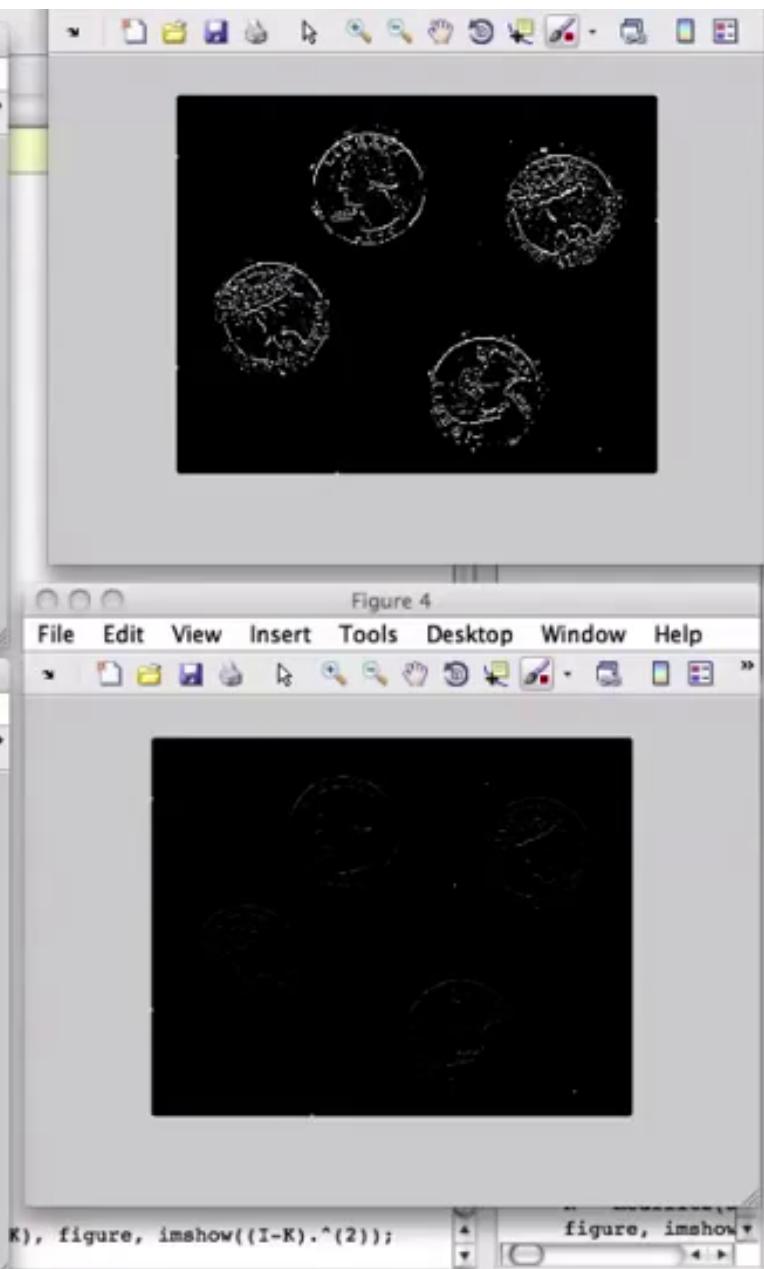
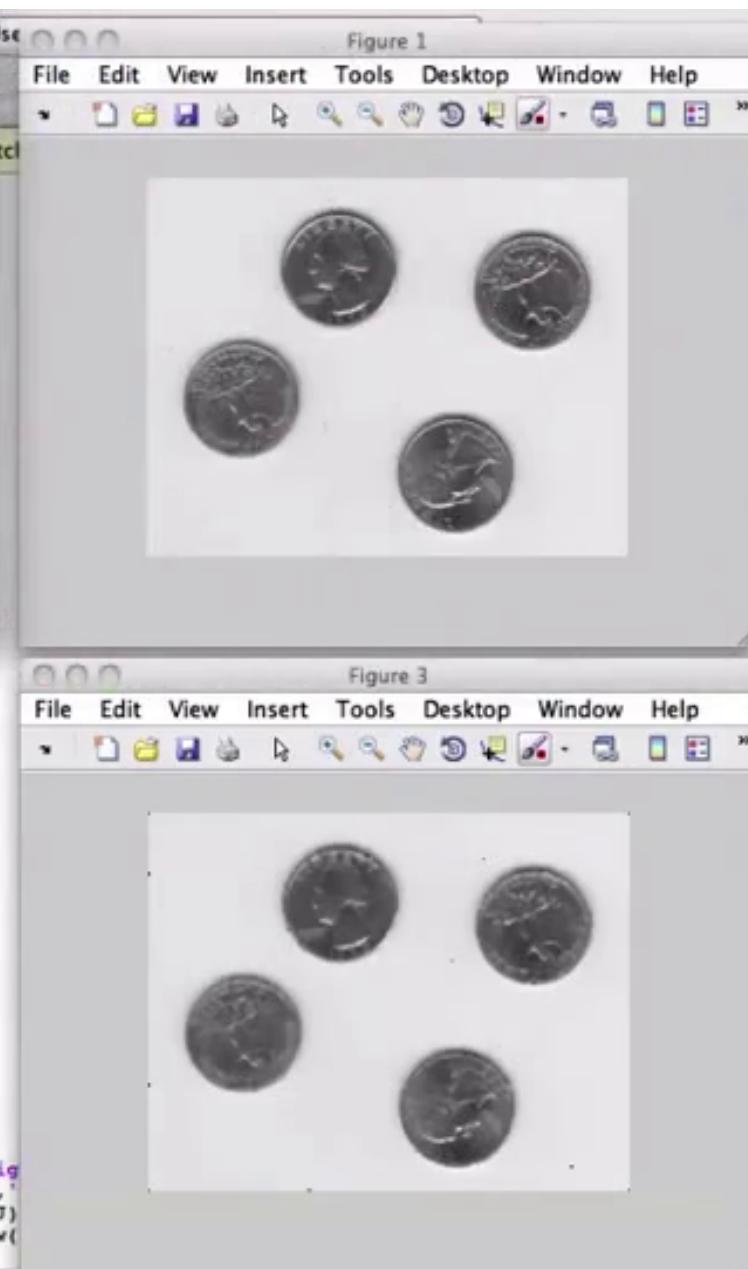
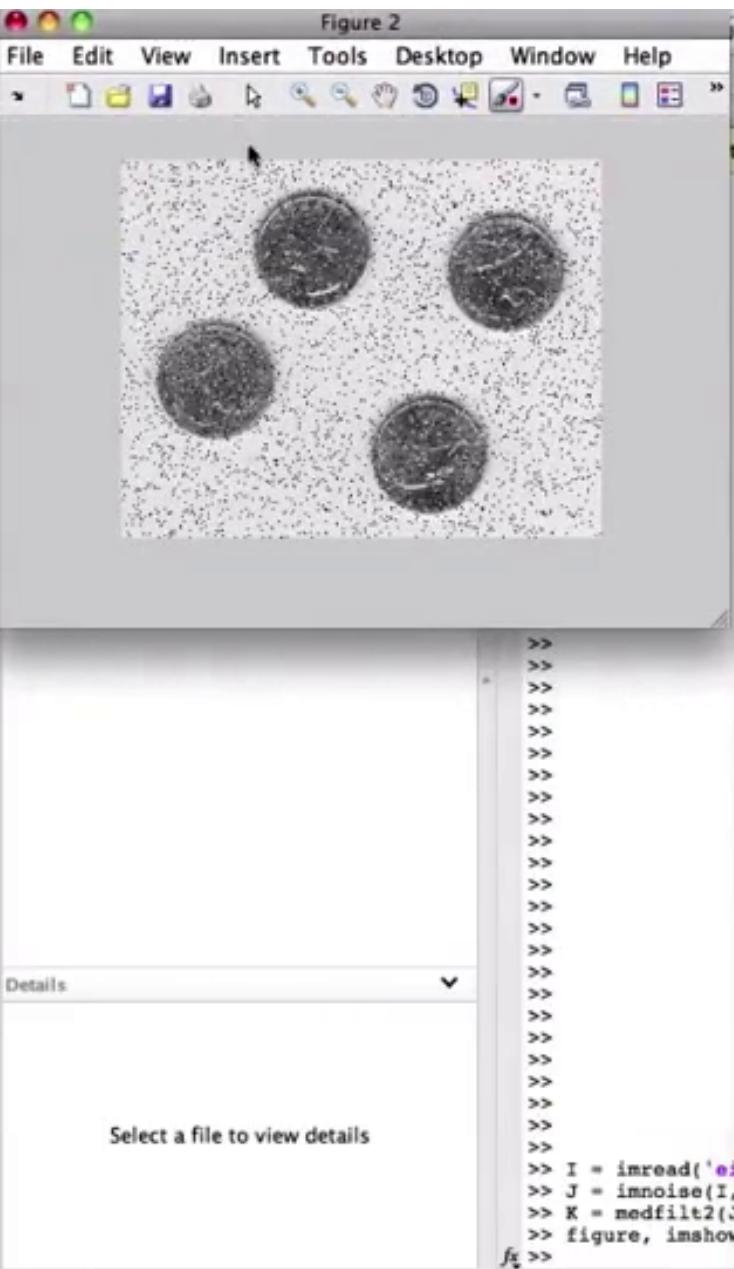
Unsharp



a
b
c
d

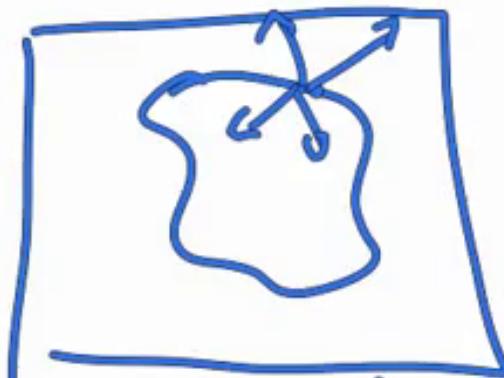
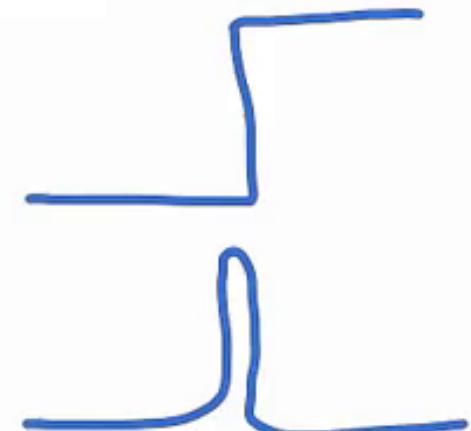
FIGURE 3.39 1-D illustration of the mechanics of unsharp masking.
(a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).





Edge Detection and Color Edge Detection

$$\nabla F(x, y) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right)$$



$$|\nabla F| = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2}$$

(R, G, B)

