



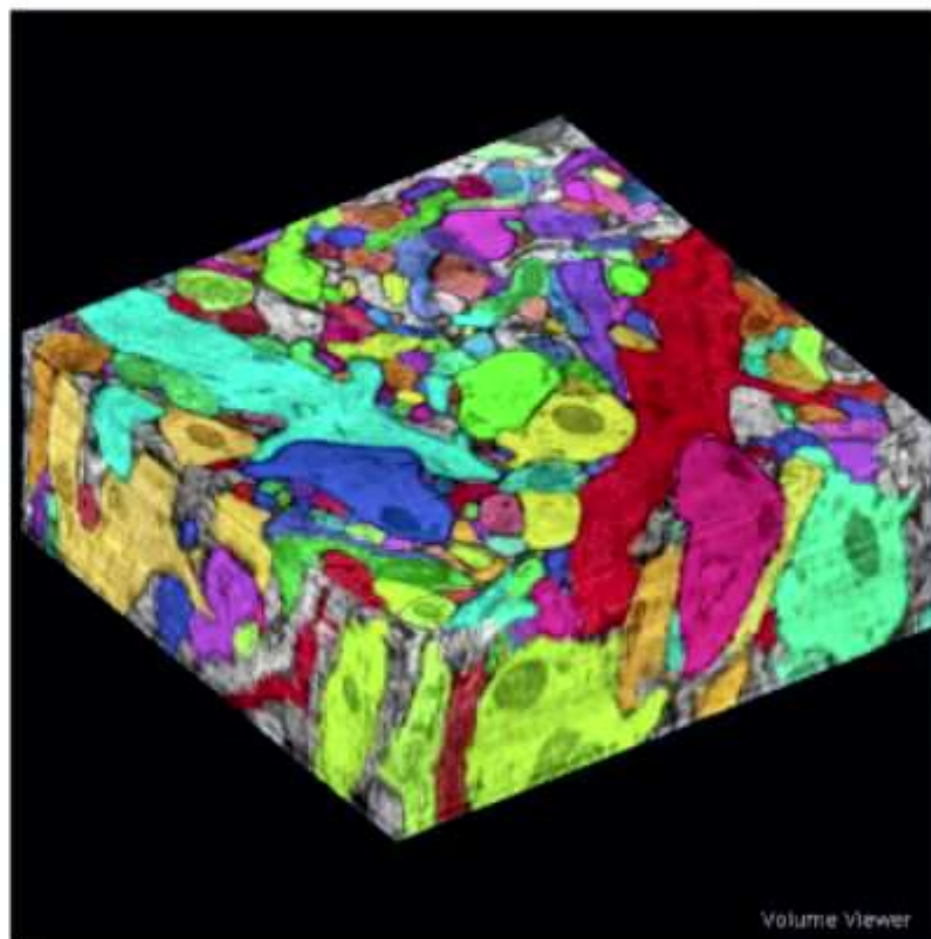
Courtesy of IEEE and IJCV



Courtesy of IEEE and IJCV



Courtesy of IEEE and IJCV



Courtesy of [openconnectomeproject.org](https://openconnectomeproject.org)





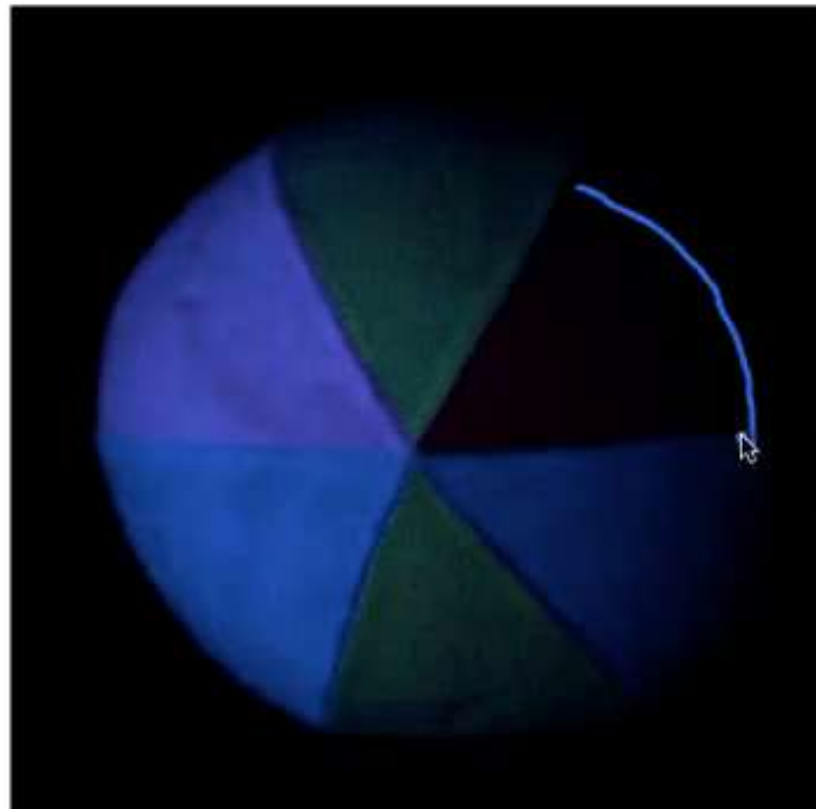
Original SIGGRAPH project videos courtesy of Artbeats ([www.artbeats.com](http://www.artbeats.com)), Mike Wilbur, Jon Goldman, and Jiawen Chen



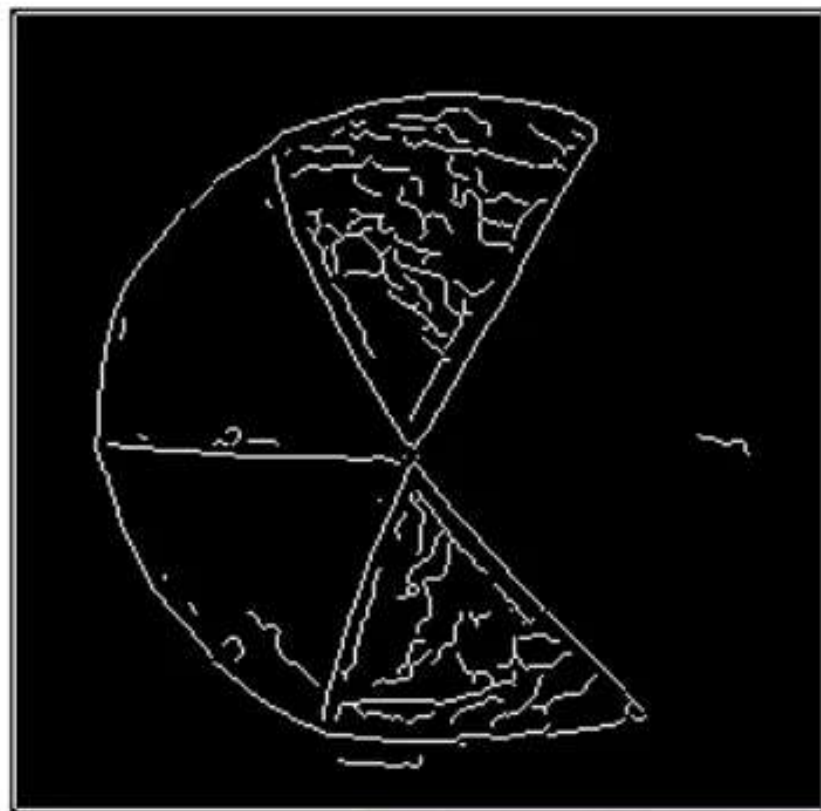
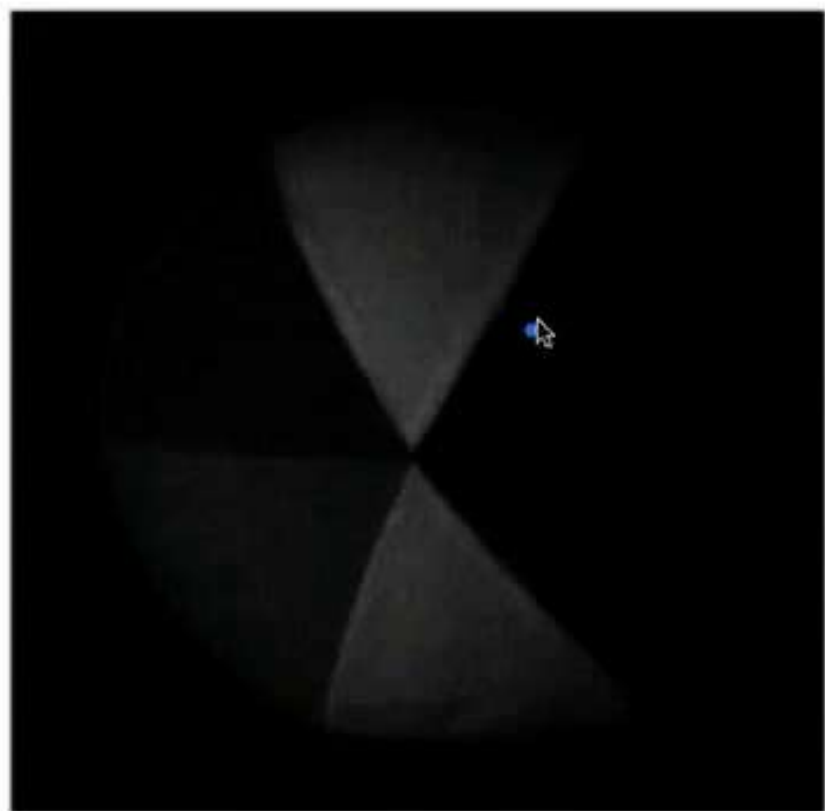
$$\begin{aligned} \nabla I &= \\ &= \left\langle \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right\rangle \\ &= \begin{bmatrix} -1 & +1 & -1 \end{bmatrix} \end{aligned}$$

Images courtesy of ipol.im





Images courtesy of ipol.im







# Digital Image Processing, 3rd ed.

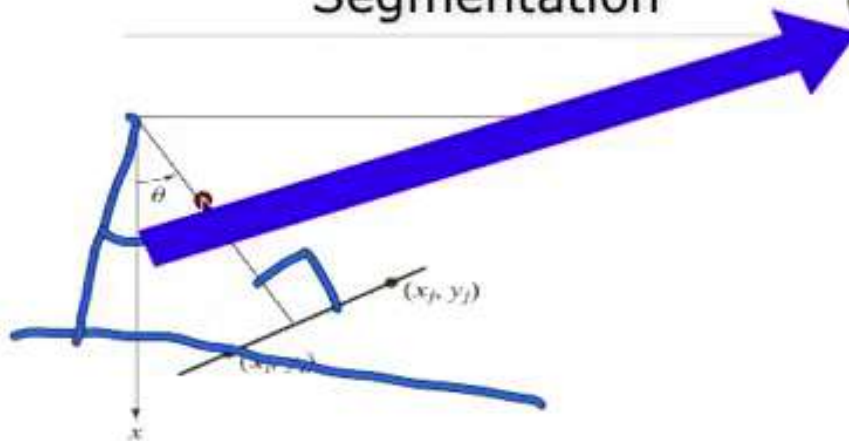
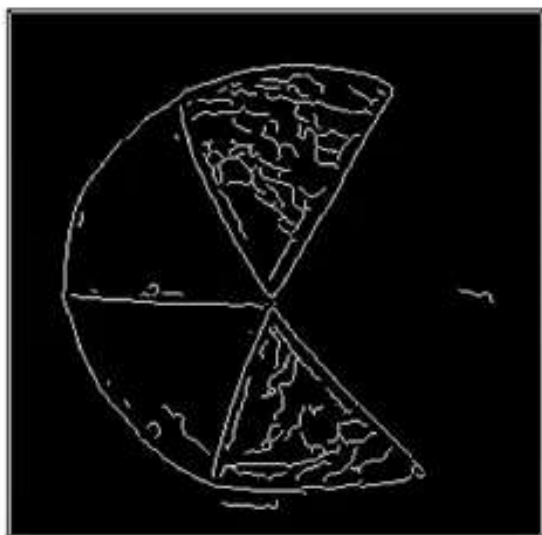
Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10

## Segmentation

Angle measured to x axis.



$$\rho = x \cos \theta + y \sin \theta$$





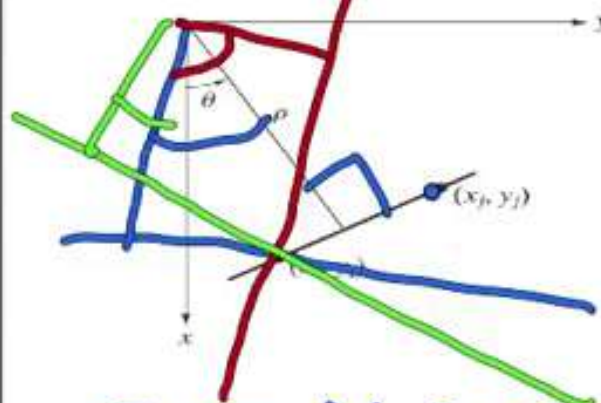
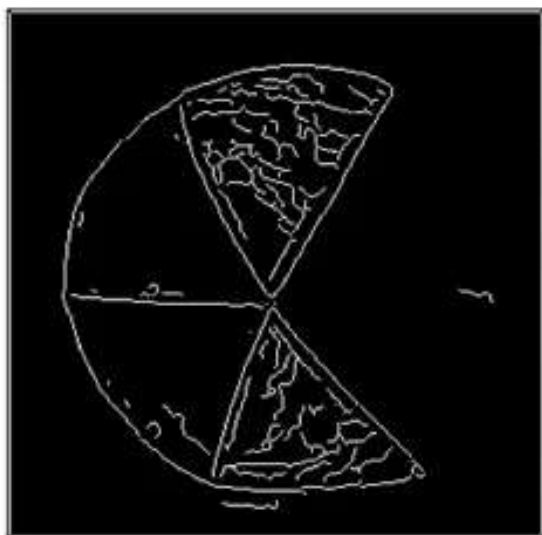
# Digital Image Processing, 3rd ed.

Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10

### Segmentation



$$\rho = x \cos \theta + y \sin \theta$$
$$(\rho, \theta)$$





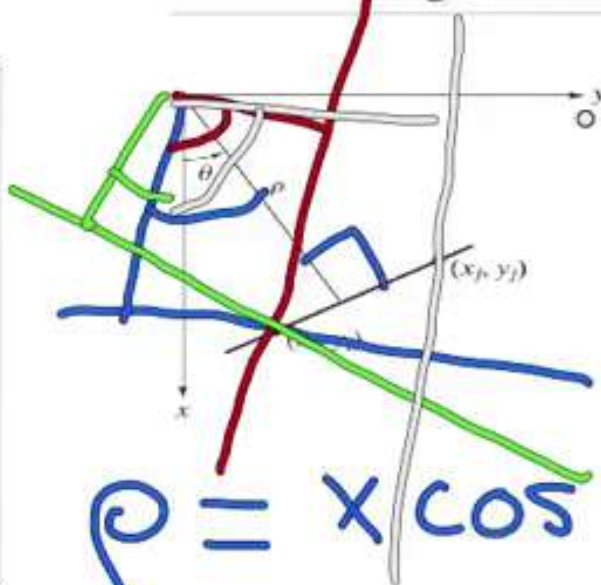
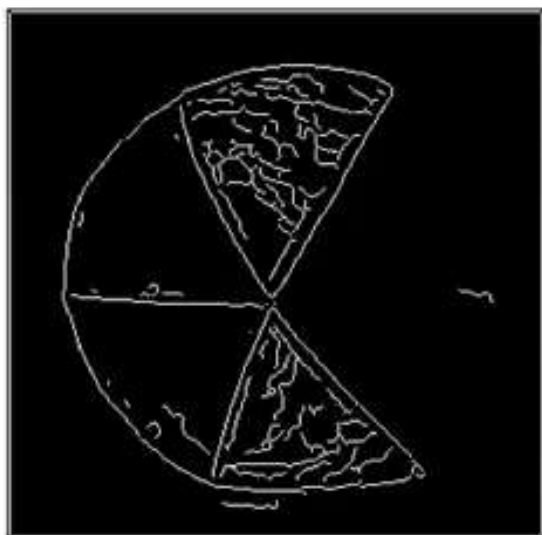
# Digital Image Processing, 3rd ed.

Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10

### Segmentation



$$\rho = x \cos \theta + y \sin \theta$$
$$(\rho, \theta)$$





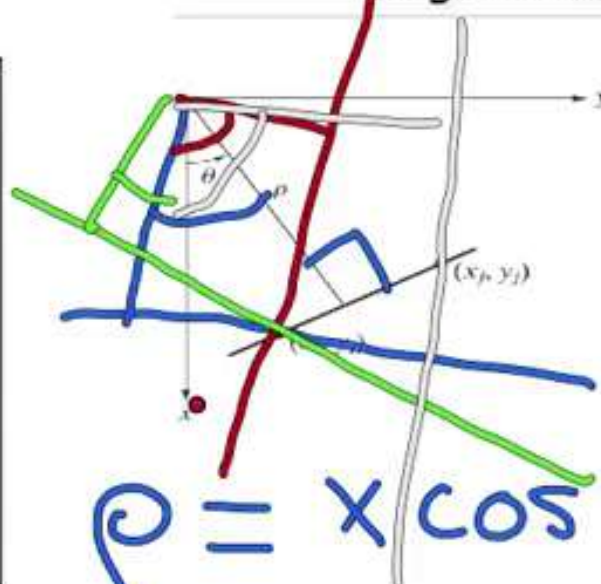
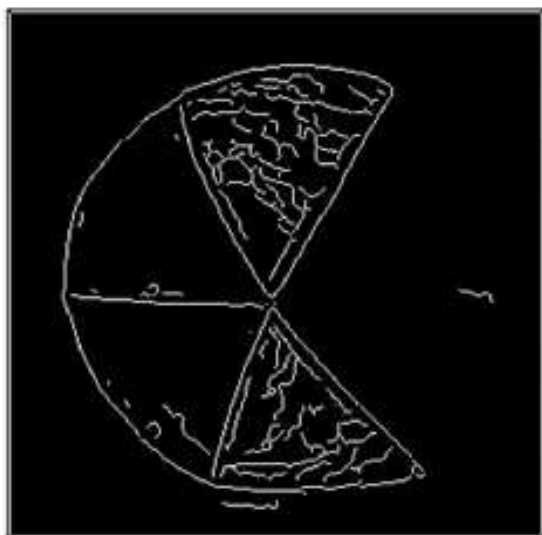
# Digital Image Processing, 3rd ed.

Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10

### Segmentation



$$\rho = x \cos \theta + y \sin \theta$$

$(\rho, \theta)$   $(\rho, \theta)$







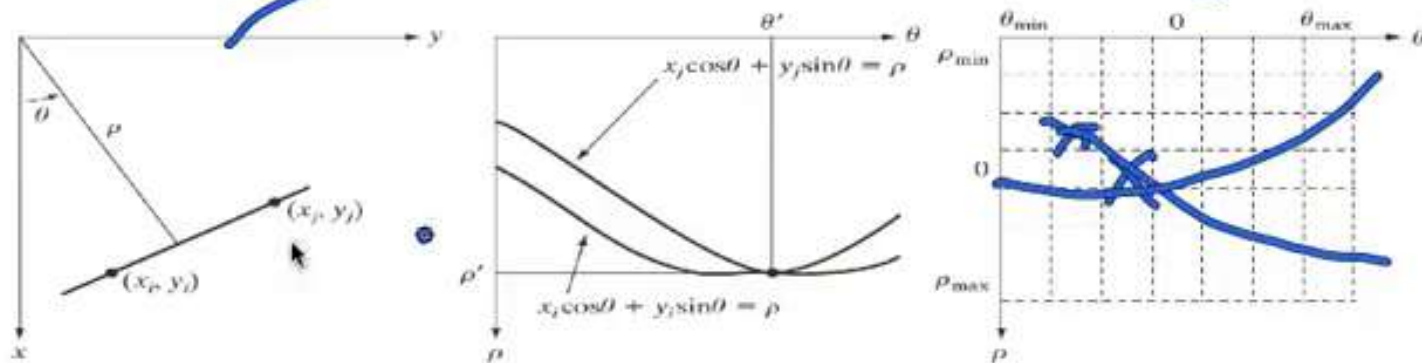
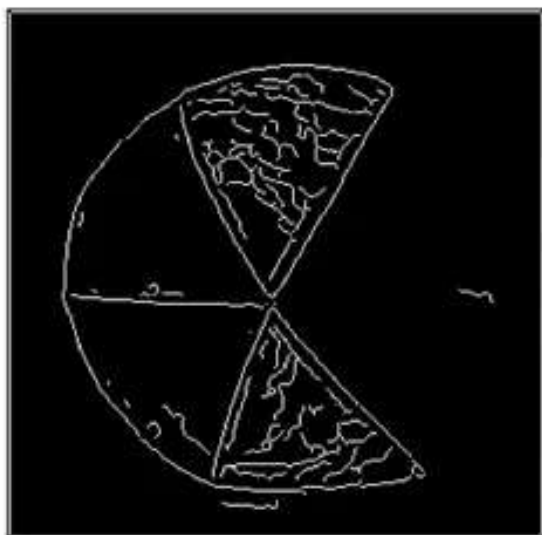
# Digital Image Processing, 3rd ed.

Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10

## Segmentation



$$\rho = x \cos \theta + y \sin \theta$$





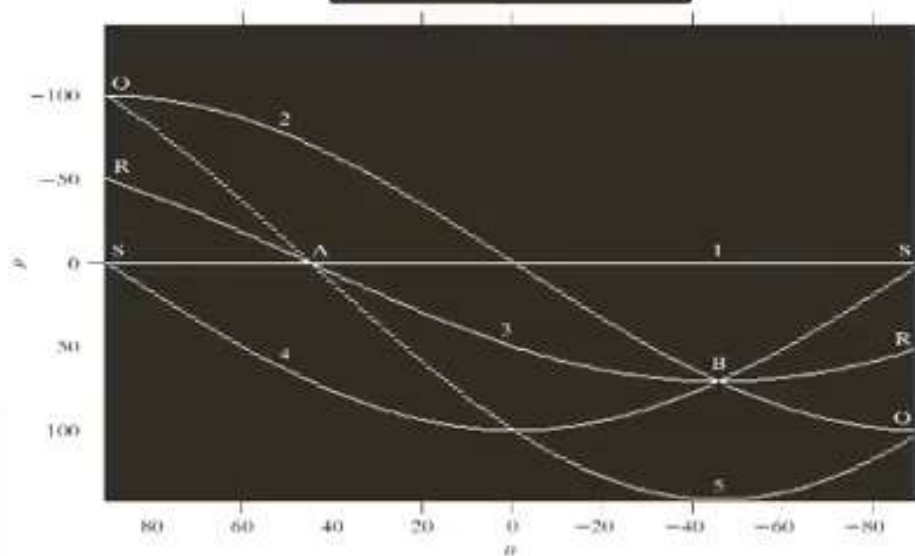
# Digital Image Processing, 3rd ed.

Gonzalez & Woods

www.ImageProcessingPlace.com

## Chapter 10

## Segmentation



a  
b

**FIGURE 10.33**

(a) Image of size  $101 \times 101$  pixels, containing five points.  
(b) Corresponding parameter space. (The points in (a) were enlarged to make them easier to see.)





# *Digital Image Processing, 3rd ed.*

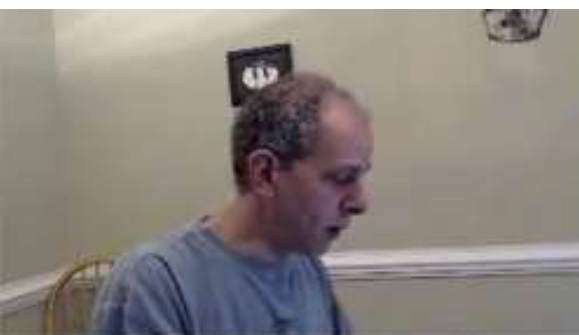
Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10

## Segmentation





Folder: /Users/guillermo\_sapiro/Documents/MATLAB

Command Window

MATLAB? Watch this [Video](#), see [Demos](#), or read [Getting Started](#).

```
len = norm(lines(k).point1 - lines(k).point2);
if ( len > max_len)
    max_len = len;
    xy_long = xy;
end
end

% highlight the longest line segment
plot(xy_long(:,1),xy_long(:,2),'LineWidth',2,'Color','cyan');
```

See also [hough](#) and [houghpeaks](#).

Reference page in Help browser  
[doc houghlines](#)

```
>>
I = imread('circuit.tif');
rotI = imrotate(I,33,'crop');
BW = edge(rotI,'canny');
[H,T,R] = hough(BW);
imshow(H,[],'XData',T,'YData',R,'InitialMagnification','fit');
xlabel('\theta'), ylabel('\rho');
axis on, axis normal, hold on;
P = houghpeaks(H,5,'threshold',ceil(0.3*max(H(:)))));
x = T(P(:,2));
y = R(P(:,1));
plot(x,y,'s','color','white');

% Find lines and plot them
lines = houghlines(BW,T,R,P,'FillGap',5,'MinLength',7);
figure, imshow(rotI), hold on
max_len = 0;
for k = 1:length(lines)
    xy = [lines(k).point1; lines(k).point2];
    plot(xy(:,1),xy(:,2),'LineWidth',2,'Color','green');

    % plot beginnings and ends of lines
    plot(xy(1,1),xy(1,2),'x','LineWidth',2,'Color','yellow');
    plot(xy(2,1),xy(2,2),'x','LineWidth',2,'Color','red');

    % determine the endpoints of the longest line segment
    len = norm(lines(k).point1 - lines(k).point2);
    if ( len > max_len)
        max_len = len;
        xy_long = xy;
    end
end
```

Select a file to view details

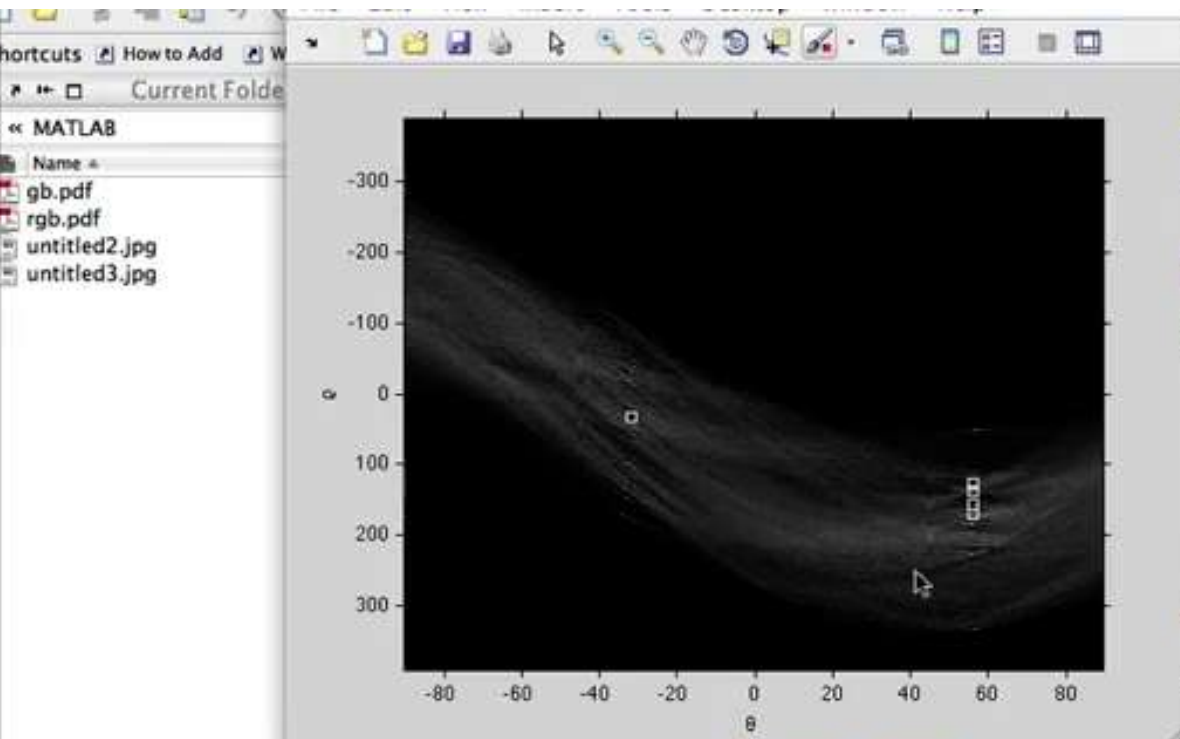
Workspace

Name
BW
H
I
P
R
T
ball
e
im
k
len
lines
max_len
radii
rotI
x
xy

Command Window

```
x = T(P(:,2));
y = R(P(:,1));
plot(x,y,'s','color','white');
% Find lines
lines = houghlines(BW,T,R,P,'FillGap',5,'MinLength',7);
figure, imshow(rotI), hold on
max_len = 0;
for k = 1:length(lines)
    xy = [lines(k).point1; lines(k).point2];
    plot(xy(:,1),xy(:,2),'LineWidth',2,'Color','green');
    % plot beginnings and ends of lines
    plot(xy(1,1),xy(1,2),'x','LineWidth',2,'Color','yellow');
    plot(xy(2,1),xy(2,2),'x','LineWidth',2,'Color','red');
    % determine the endpoints of the longest line segment
    len = norm(lines(k).point1 - lines(k).point2);
    if ( len > max_len)
        max_len = len;
        xy_long = xy;
    end
end
```





```

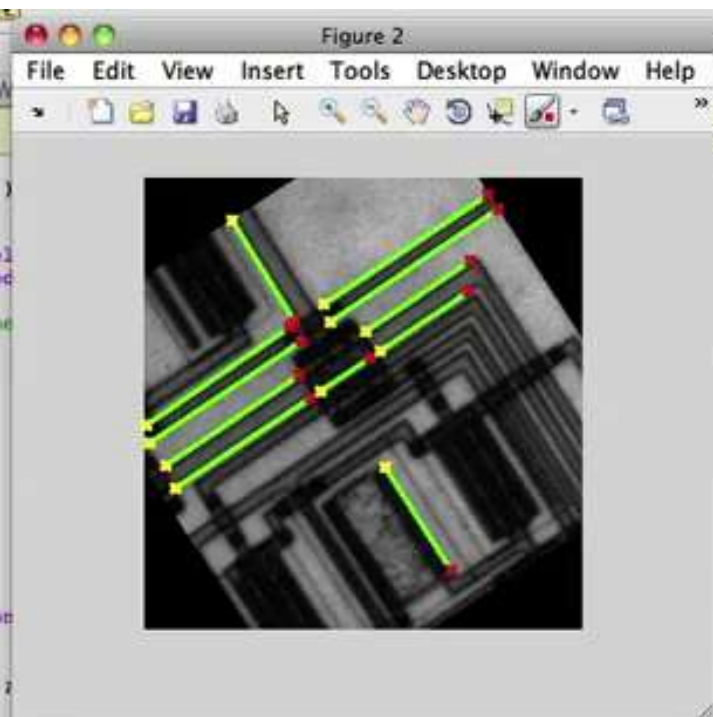
plot(x,y,'s','color','white');

% Find lines and plot them
lines = houghlines(BW,T,R,P,'FillGap',5,'MinLength',7);
figure, imshow(rotI), hold on
max_len = 0;
for k = 1:length(lines)
    xy = [lines(k).point1; lines(k).point2];
    plot(xy(:,1),xy(:,2),'LineWidth',2,'Color','green');

    % plot beginnings and ends of lines
    plot(xy(1,1),xy(1,2),'x','LineWidth',2,'Color','yellow');
    plot(xy(2,1),xy(2,2),'x','LineWidth',2,'Color','red');

    % determine the endpoints of the longest line segment
    len = norm(lines(k).point1 - lines(k).point2);
    if ( len > max_len)
        max_len = len;
        xy_long = xy;
    end
end

```



Workspace

Name	Class
BW	logical
H	double
I	double
P	double
R	double
T	double
ball	double
e	double
im	double
k	double
len	double
lines	double
max_len	double
radii	double
rotI	double
x	double
xy	double

Command Window

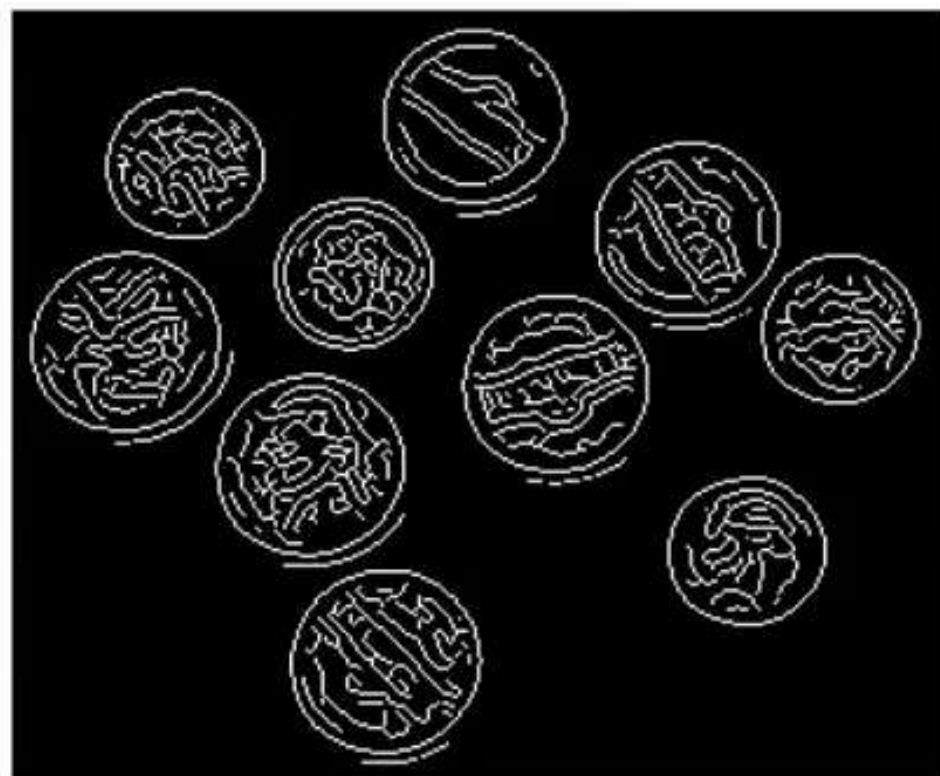
```

x = T(P(:,2));
y = R(P(:,1));
plot(x,y,'s');
% Find lines
lines = houghlines(BW,T,R,P,'FillGap',5,'MinLength',7);
figure, imshow(rotI), hold on
max_len = 0;
for k = 1:length(lines)
    xy = [lines(k).point1; lines(k).point2];
    % plot beginnings and ends of lines
    plot(xy(1,1),xy(1,2),'x','LineWidth',2,'Color','yellow');
    plot(xy(2,1),xy(2,2),'x','LineWidth',2,'Color','red');
    % determine the endpoints of the longest line segment
    len = norm(lines(k).point1 - lines(k).point2);
    if ( len > max_len)
        max_len = len;
        xy_long = xy;
    end
end

```

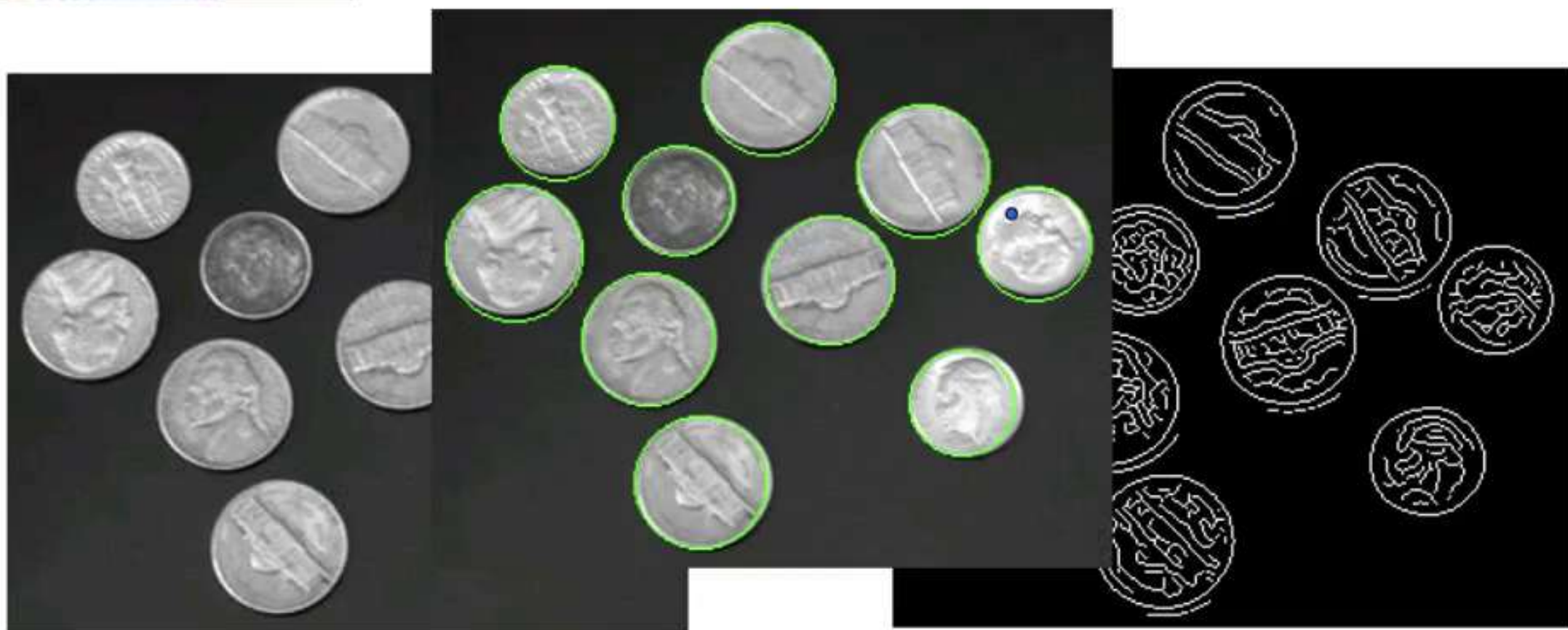


# What About Circles?





# What About Circles?



Images courtesy of D. Young and Mathworks





## LSD: a Line Segment Detector

Rafael Grompone von Gioi, J  r  mie Jakubowicz, Jean-Michel Morel, Gregory Randall

[article](#) [demo](#) [archive](#)

published - 2012-03-24

reference - Grompone von Gioi, Rafael, J  r  mie Jakubowicz, Jean-Michel Morel, and Gregory Randall. "LSD: a Line Segment Detector." *Image Processing On Line* 2012 (2012). <http://dx.doi.org/10.5201/ipol.2012.gjmr-lsd>

- full text manuscript: PDF  high-res.  <sup>[?]</sup>
- source code: ZIP 

*Communicated by* Lionel Moisan

*Demo edited by* Rafael Grompone

### Abstract

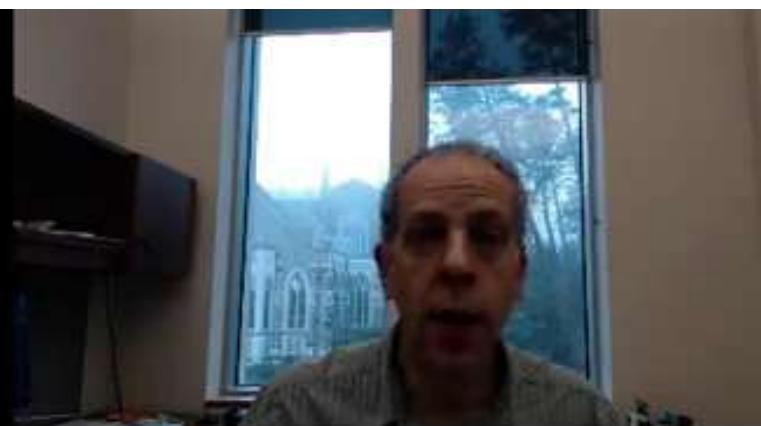
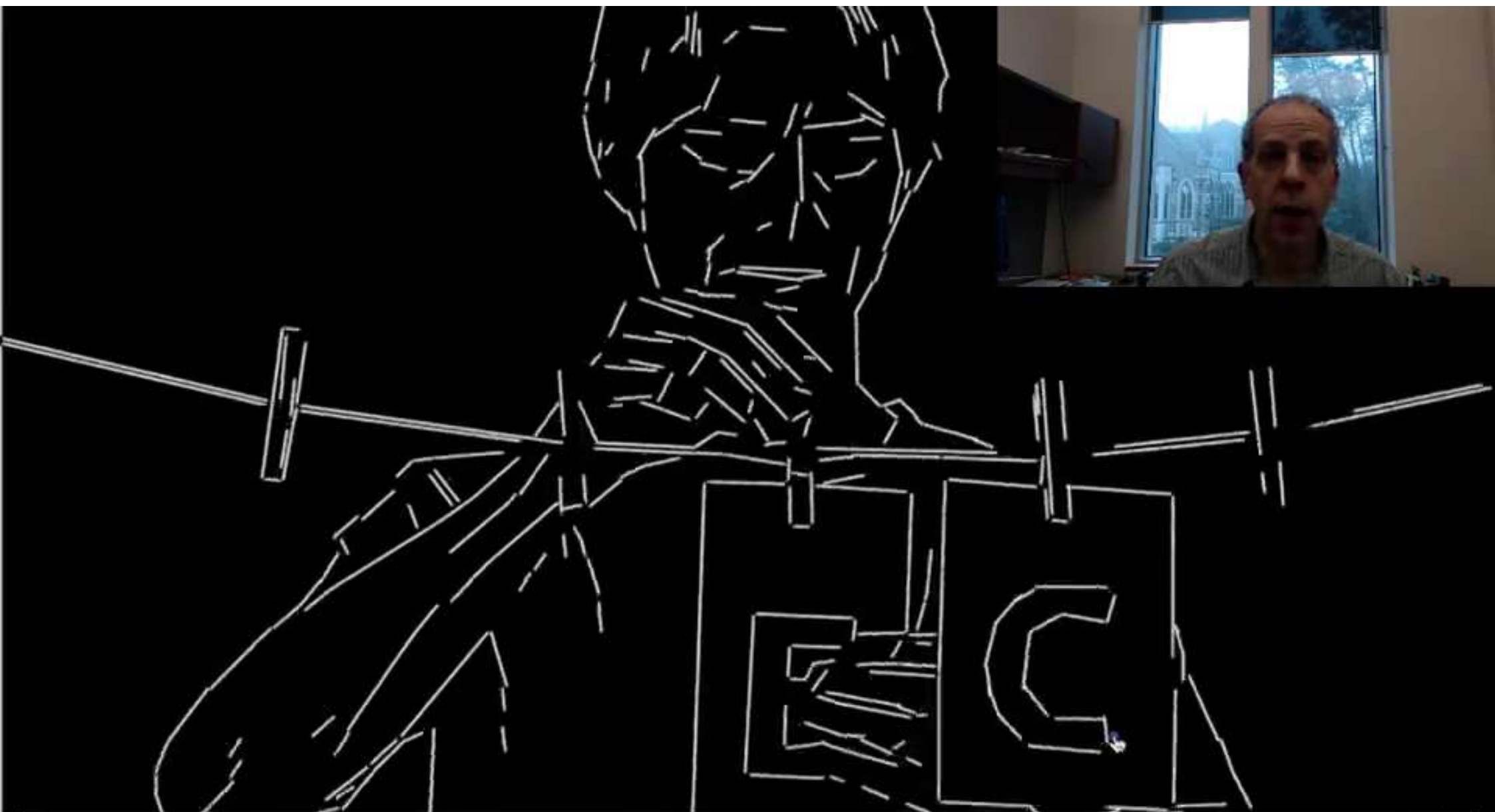
LSD is a linear-time Line Segment Detector giving subpixel accurate results. It is designed to work on any digital image without parameter tuning. It controls its own number of false detections: On average, one false alarms is allowed per image. The method is based on Burns, Hanson, and Riseman's method, and uses an a-contrario validation approach according to Desolneux, Moisan, and Morel's theory. The version described here includes some further improvement over the one described in the original article.

### Supplementary Material

- sample video: MP4  <sup>[?]</sup>







demo.ipol.im/demo/gjmr\_line\_segment\_detector/input\_select?chairs.x=50&chairs.y=55



Image Processing On Line

[HOME](#) · [ABOUT](#) · [ARTICLES](#) · [PREPRINTS](#) · [NEWS](#) · [SEARCH](#)

## LSD: a Line Segment Detector

[article](#) [demo](#) [archive](#)

Please cite the reference article if you publish results obtained with this online demo.

The image was converted to gray level values.

Run the algorithm:

Or you can run it after selecting a subimage by clicking two opposite corners of the subimage.



demo.ipol.im/demo/gjmr\_line\_segment\_detector/result?key=EB02C5128E963A9338DAD05925CBB4CE

[article](#) [demo](#) [archive](#)

Please cite the reference article if you publish results obtained with this online demo.

Run again? [new image](#) [different subimage](#)

## Result

698 Line Segments were detected. The algorithm ran in 0.22s.

You can download the result in [EPS](#) format, in [SVG](#) format, or an [ASCII](#) file (see description below).

[output](#)

[input](#)



## Result

847 Line Segments were detected. The algorithm ran in 0.28s.

You can download the result in [EPS](#) format, in [SVG](#) format, or an [ASCII](#) file (see description below).

output

input





## Result

847 Line Segments were detected. The algorithm ran in 0.28s.

You can download the result in [EPS](#) format, in [SVG](#) format, or an [ASCII](#) file (see description below).

output



input





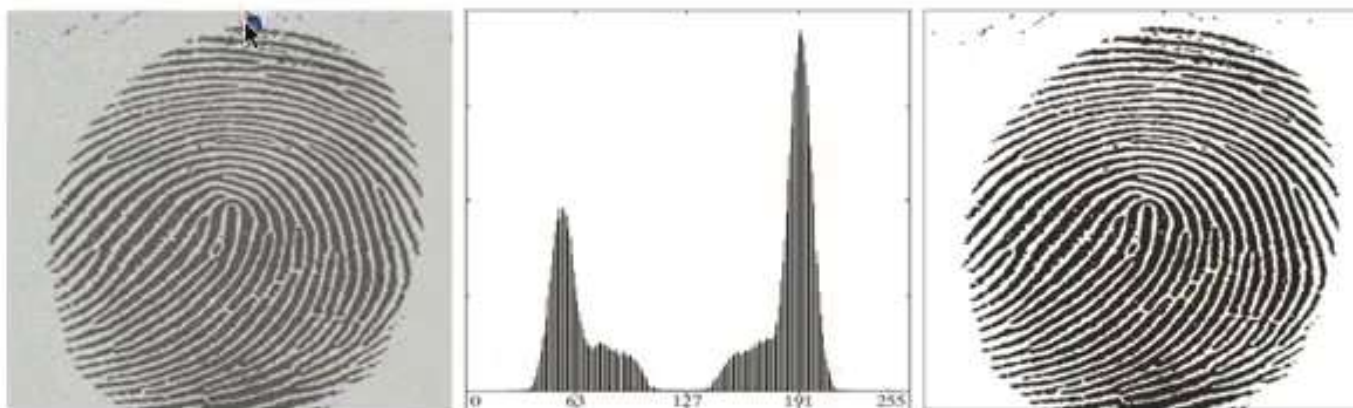
# *Digital Image Processing, 3rd ed.*

Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10

## Segmentation



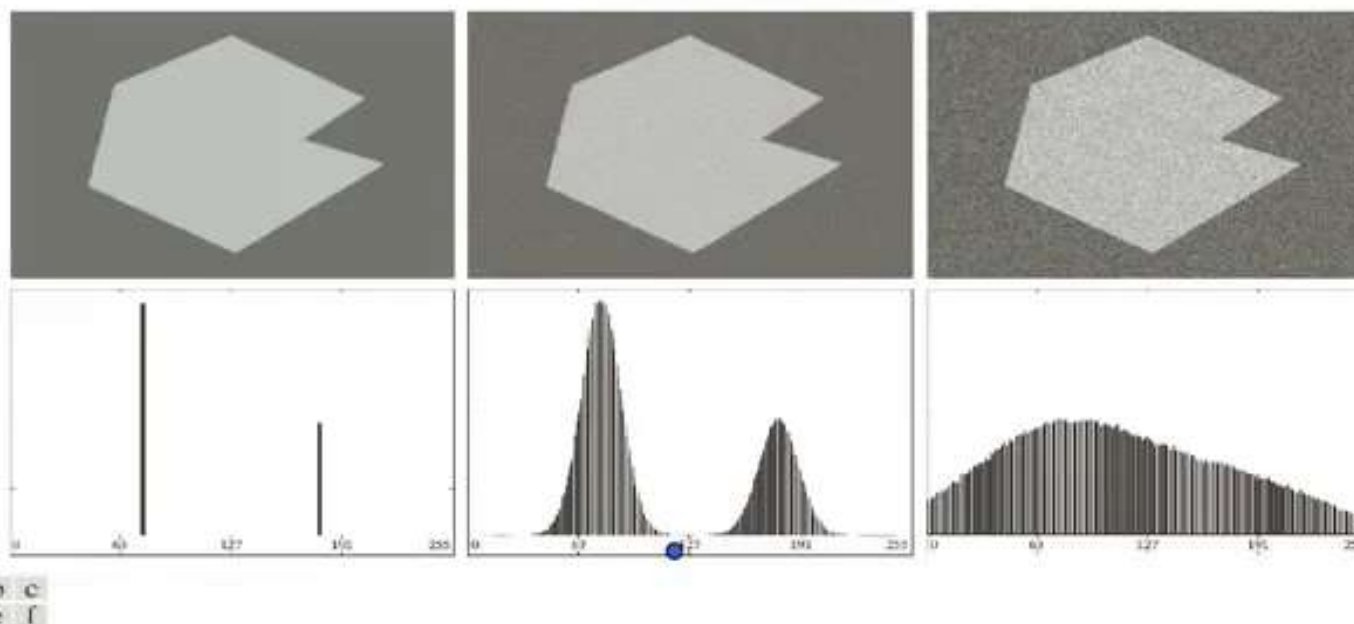


# Digital Image Processing, 3rd ed.

Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10 Segmentation



**FIGURE 10.36** (a) Noiseless 8-bit image. (b) Image with additive Gaussian noise of mean 0 and standard deviation of 10 intensity levels. (c) Image with additive Gaussian noise of mean 0 and standard deviation of 50 intensity levels. (d)–(f) Corresponding histograms.



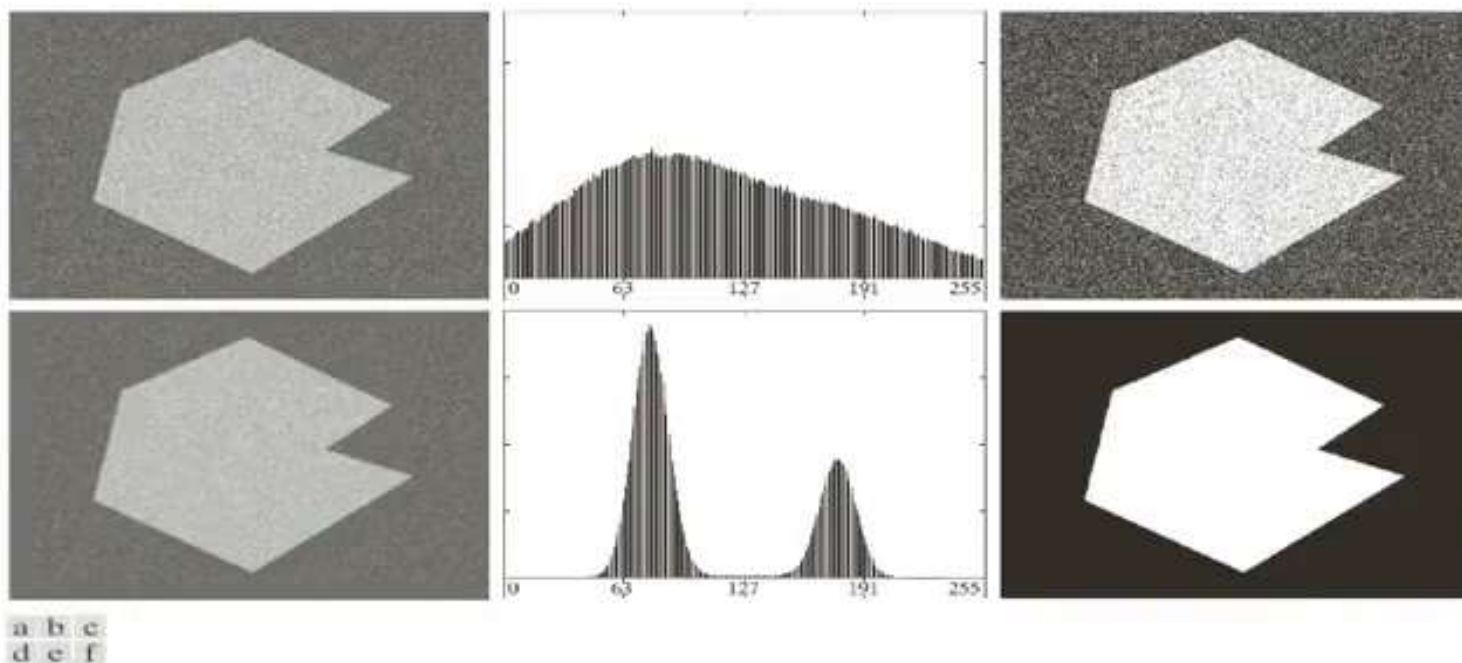
# *Digital Image Processing, 3rd ed.*

Gonzalez & Woods

[www.ImageProcessingPlace.com](http://www.ImageProcessingPlace.com)

## Chapter 10

## Segmentation



**FIGURE 10.40** (a) Noisy image from Fig. 10.36 and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a  $5 \times 5$  averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method.



Minimize the *weighted within-class variance*:

$$\sigma_w^2(t) = q_1(t)\sigma_1^2(t) + q_2(t)\sigma_2^2(t)$$

$$q_1(t) = \sum_{i=1}^t P(i)$$

$$q_2(t) = \sum_{i=t+1}^I P(i)$$

$$\mu_1(t) = \sum_{i=1}^t \frac{iP(i)}{q_1(t)}$$

$$\mu_2(t) = \sum_{i=t+1}^I \frac{iP(i)}{q_2(t)}$$

$$\sigma_1^2(t) = \sum_{i=1}^t [i - \mu_1(t)]^2 \frac{P(i)}{q_1(t)}$$

$$\sigma_2^2(t) = \sum_{i=t+1}^I [i - \mu_2(t)]^2 \frac{P(i)}{q_2(t)}$$

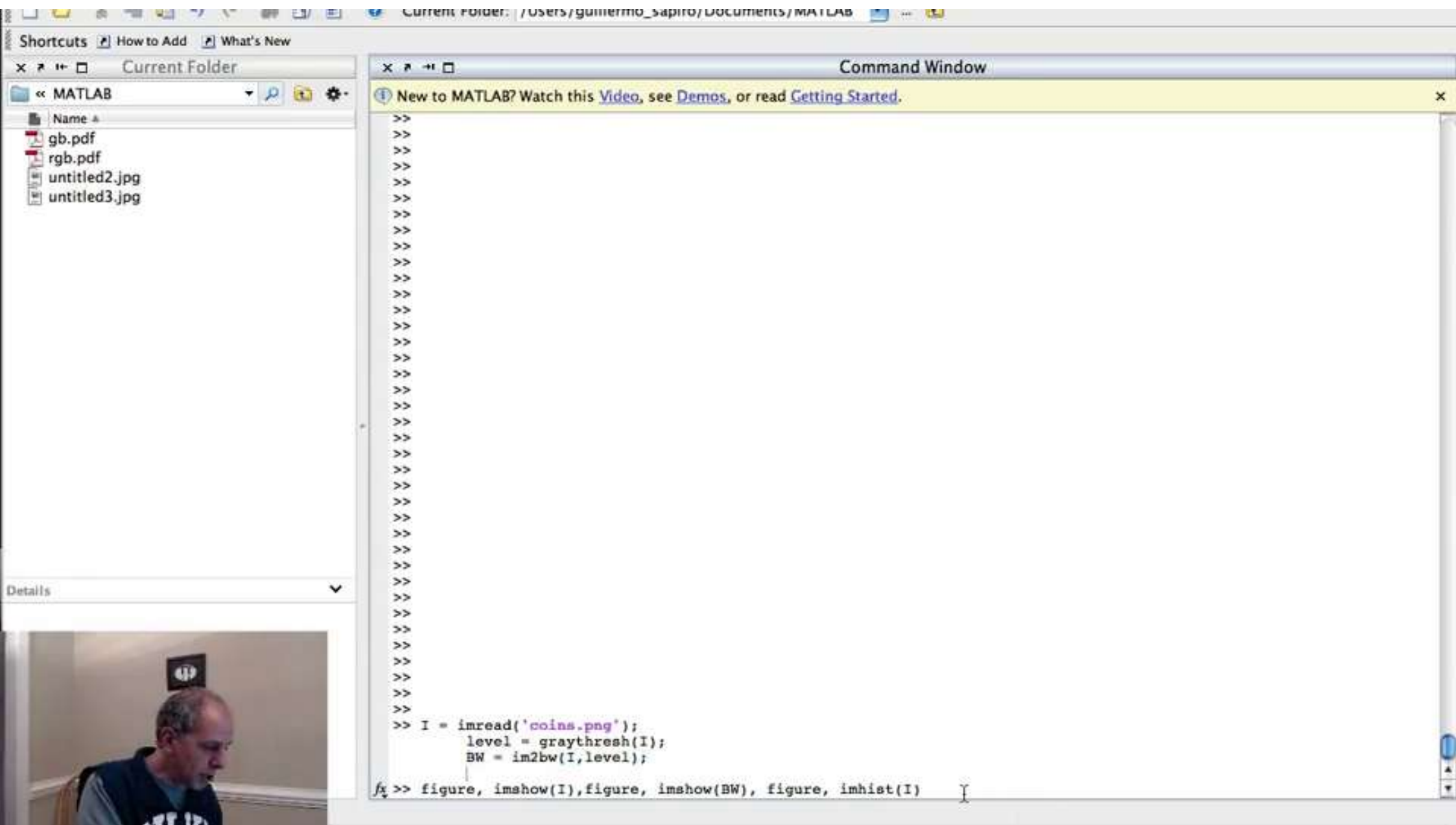


Minimize the *weighted within-class variance*:

$$\sigma_w^2(t) = q_1(t)\sigma_1^2(t) + q_2(t)\sigma_2^2(t)$$

$$\sigma^2 = \underbrace{\sigma_w^2(t)}_{\text{Within-class}} + \underbrace{q_1(t)[1 - q_1(t)][\mu_1(t) - \mu_2(t)]^2}_{\text{Between-class}}$$









# Automatic Segmentation is Tough!



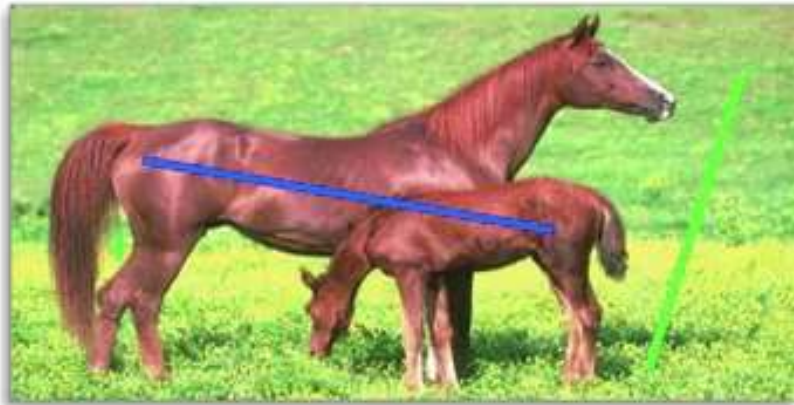
By Doolittle



# Automatic Segmentation is Tough!

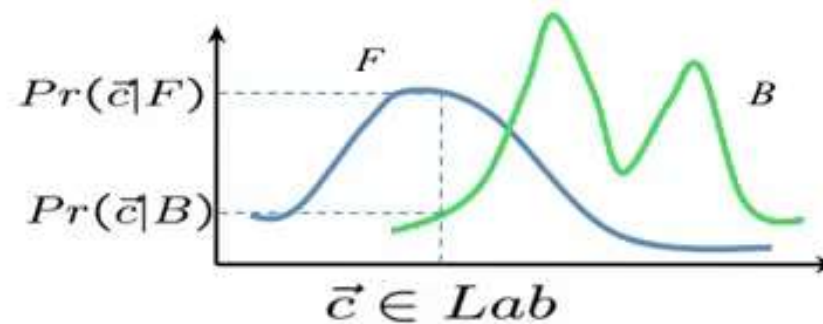


# Interactive image segmentation



# Step1 – Feature Distribution Estimation

- Estimate the color distribution on scribbles
- Each pixel is assigned a probability to belong to F or B:



$$\leftarrow P_F(x) = \frac{Pr(\vec{c}_x|F)}{Pr(\vec{c}_x|F) + Pr(\vec{c}_x|B)}$$

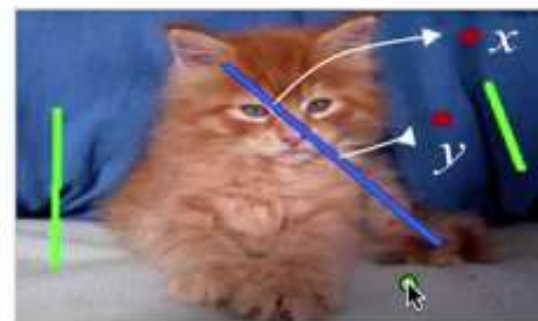
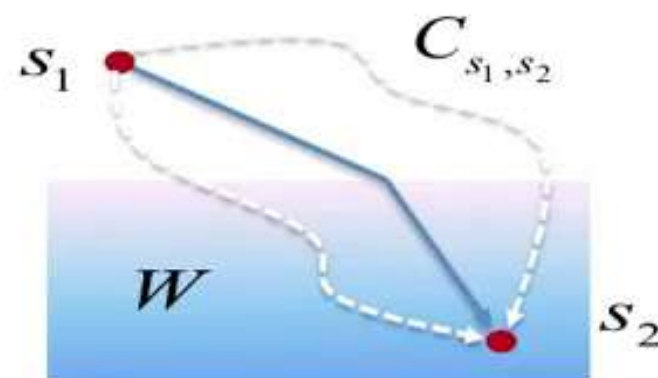


## Step2 – Weighted Distance Transform

### Weighted Geodesic Distance

$$d(s_1, s_2) := \min_{C_{s_1, s_2}} \int_{C_{s_1, s_2}} W ds$$

- Computed in linear time!



# Weighted Distance Transform (cont' d)

$$W := |\nabla P_F(x) \cdot \vec{C}'_{s_1, s_2}(x)|$$

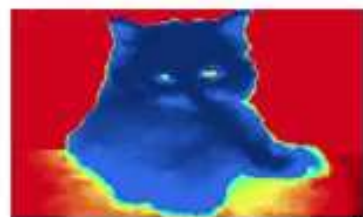
$$D_l(x) := \min_{s \in \Omega_l} d(s, x), \quad l \in \{F, B\}$$



- Pixels are classified by comparing  $D_F(x)$  and  $D_B(x)$



$P_F(x)$



$D_F(x)$



$D_B(x)$



# Weighted Distance Transform



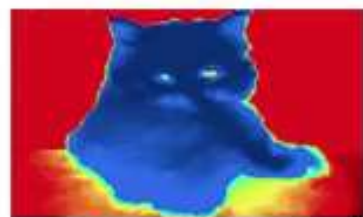
$$W := |\nabla P_F(x) \cdot \vec{C}'_{s_1, s_2}(x)|$$

$$D_l(x) := \min_{s \in \Omega_l} d(s, x), \quad l \in \{F, B\}$$

- Pixels are classified by comparing  $D_F(x)$  and  $D_B(x)$



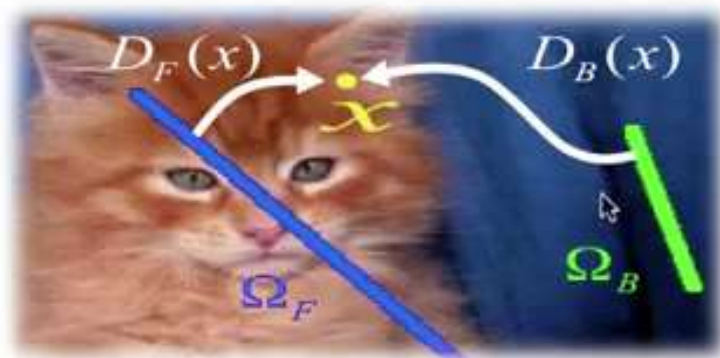
$P_F(x)$



$D_F(x)$



$D_B(x)$



# Weighted Distance Transform



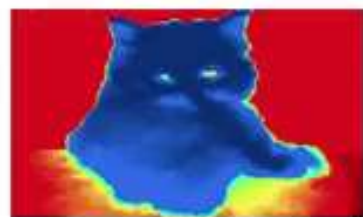
$$W := |\nabla P_F(x) \cdot \vec{C}'_{s_1, s_2}(x)|$$

$$D_l(x) := \min_{s \in \Omega_l} d(s, x), \quad l \in \{F, B\}$$

- Pixels are classified by comparing  $D_F(x)$  and  $D_B(x)$



$P_F(x)$



$D_F(x)$



$D_B(x)$



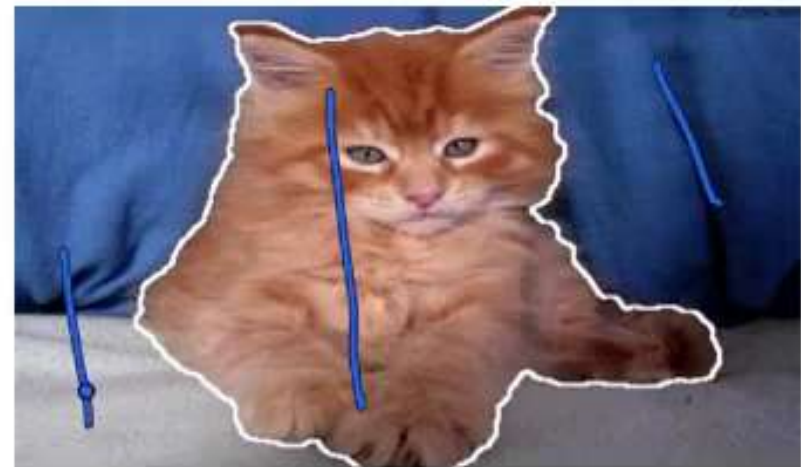
*binary segmentation*



## Step3 – Refine



- Automatically create a narrow band and new scribbles.
  - Band boundaries serve as “new scribbles”

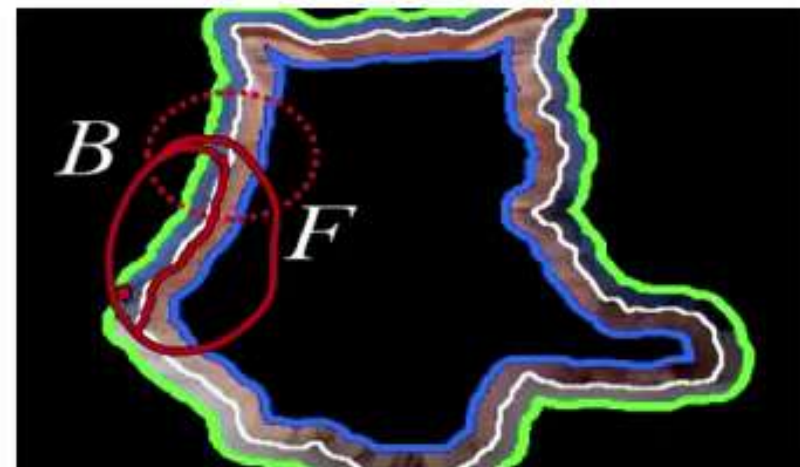


## Step3 – Refine



- Automatically create a narrow band and new scribbles.
  - Band boundaries serve as “new scribbles”

$\rho_F$



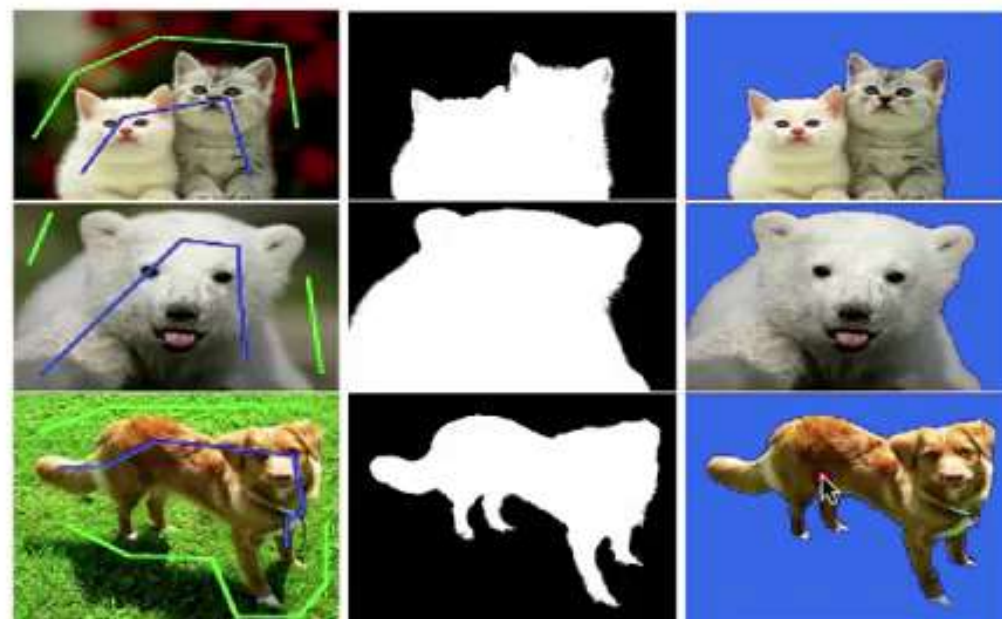
## Step3 – Refine



- Automatically create a narrow band and new scribbles.
  - Band boundaries serve as “new scribbles”

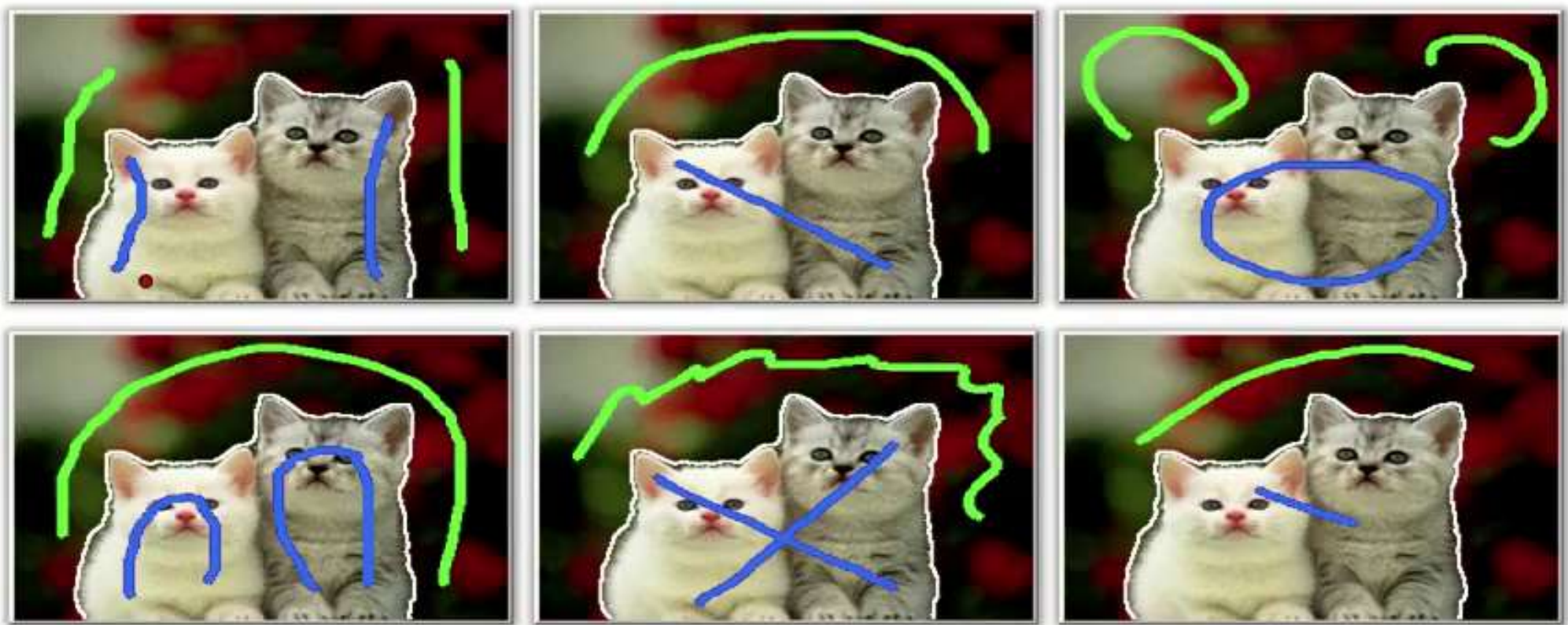
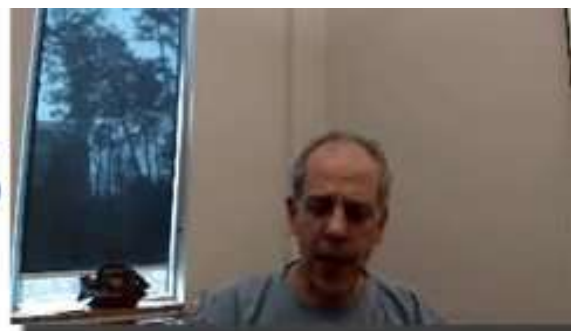


# Examples



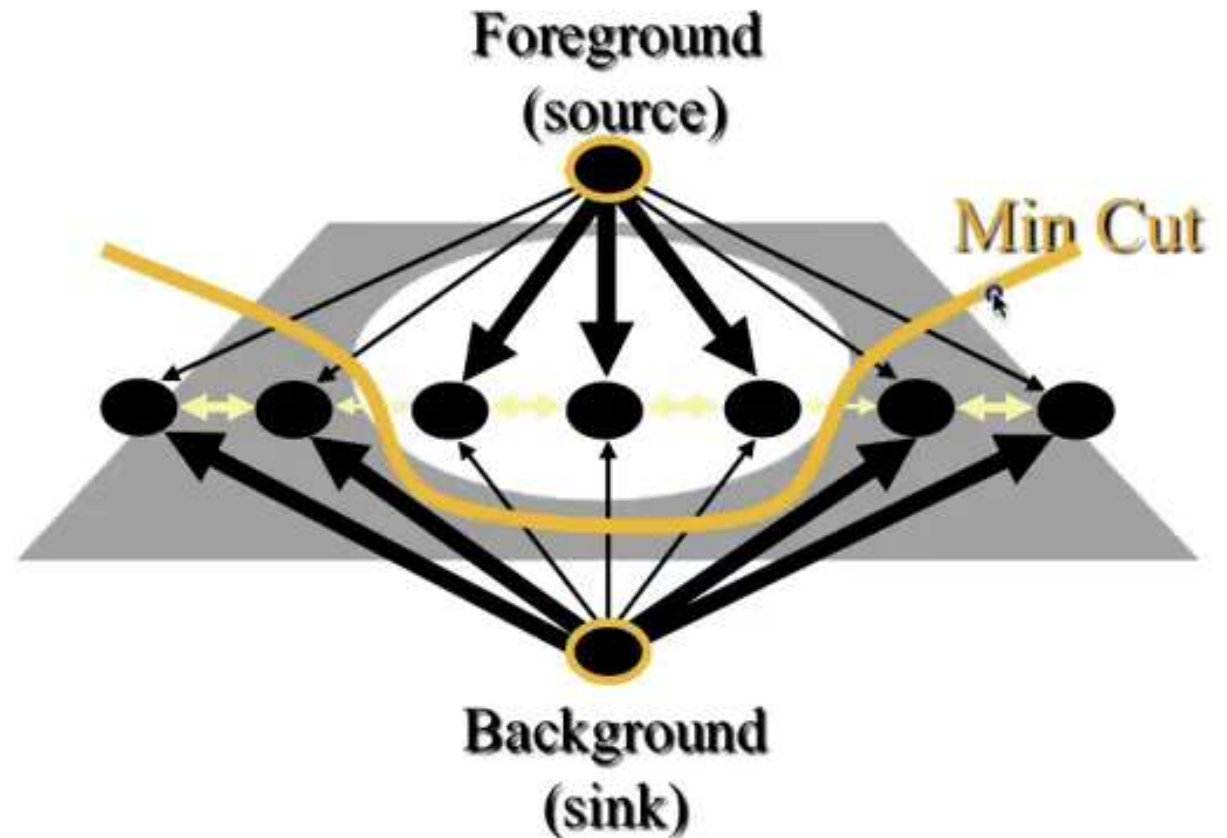
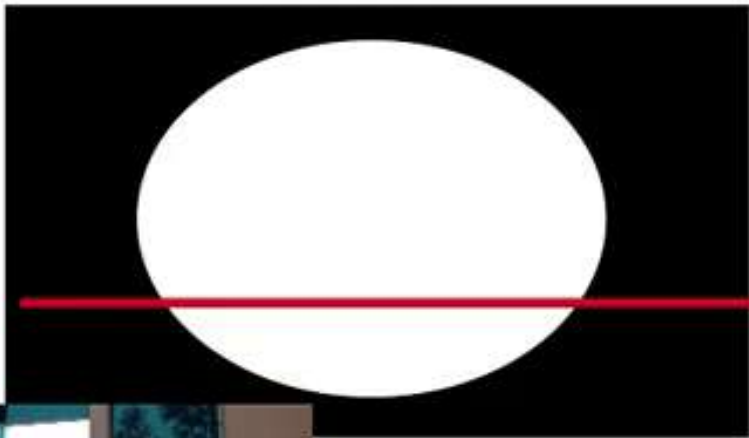


# Scribble Robustness



# Graph Cuts - Boykov and Jolly (2001)

Image



**Cut:** separating source and sink; Penalty: collection of edges

**Min Cut:** Global minimal energy in polynomial time





# Photomontage



Courtesy of Carsten Rother



# Photomontage



Courtesy of Carsten Rother

# Photomontage



Courtesy of Carsten Rother



# Photomontage



Courtesy of Carsten Rother

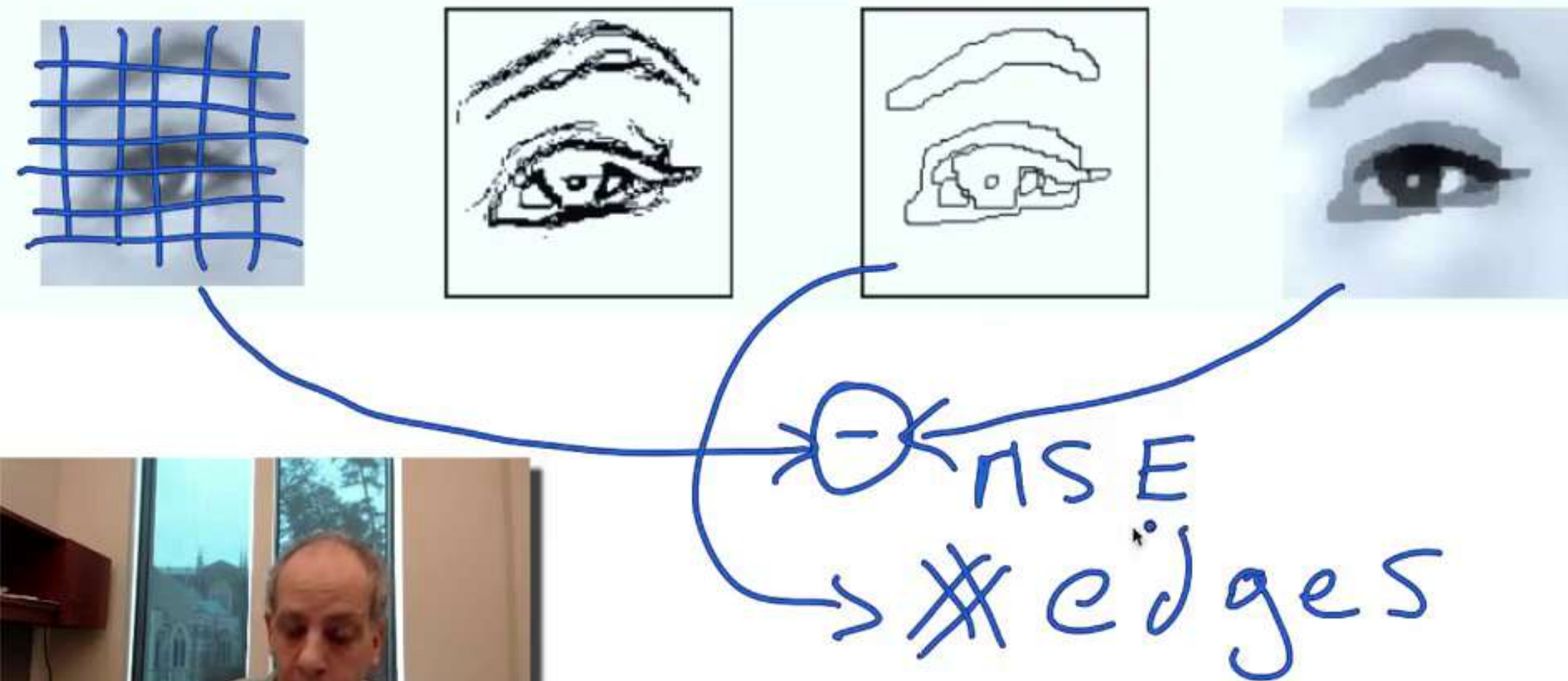


Image courtesy of Alan Yuille



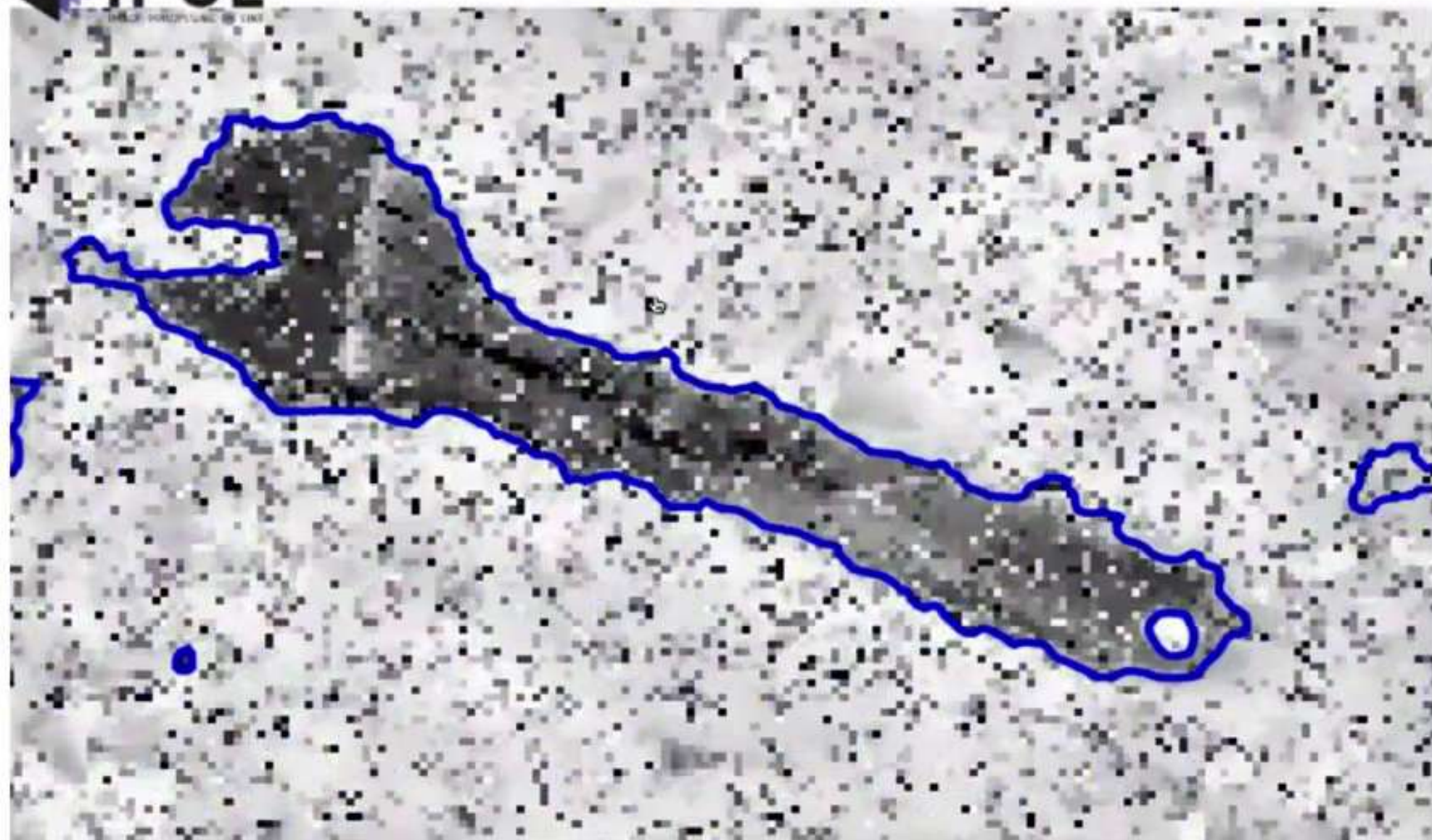


Image courtesy of ipol.im

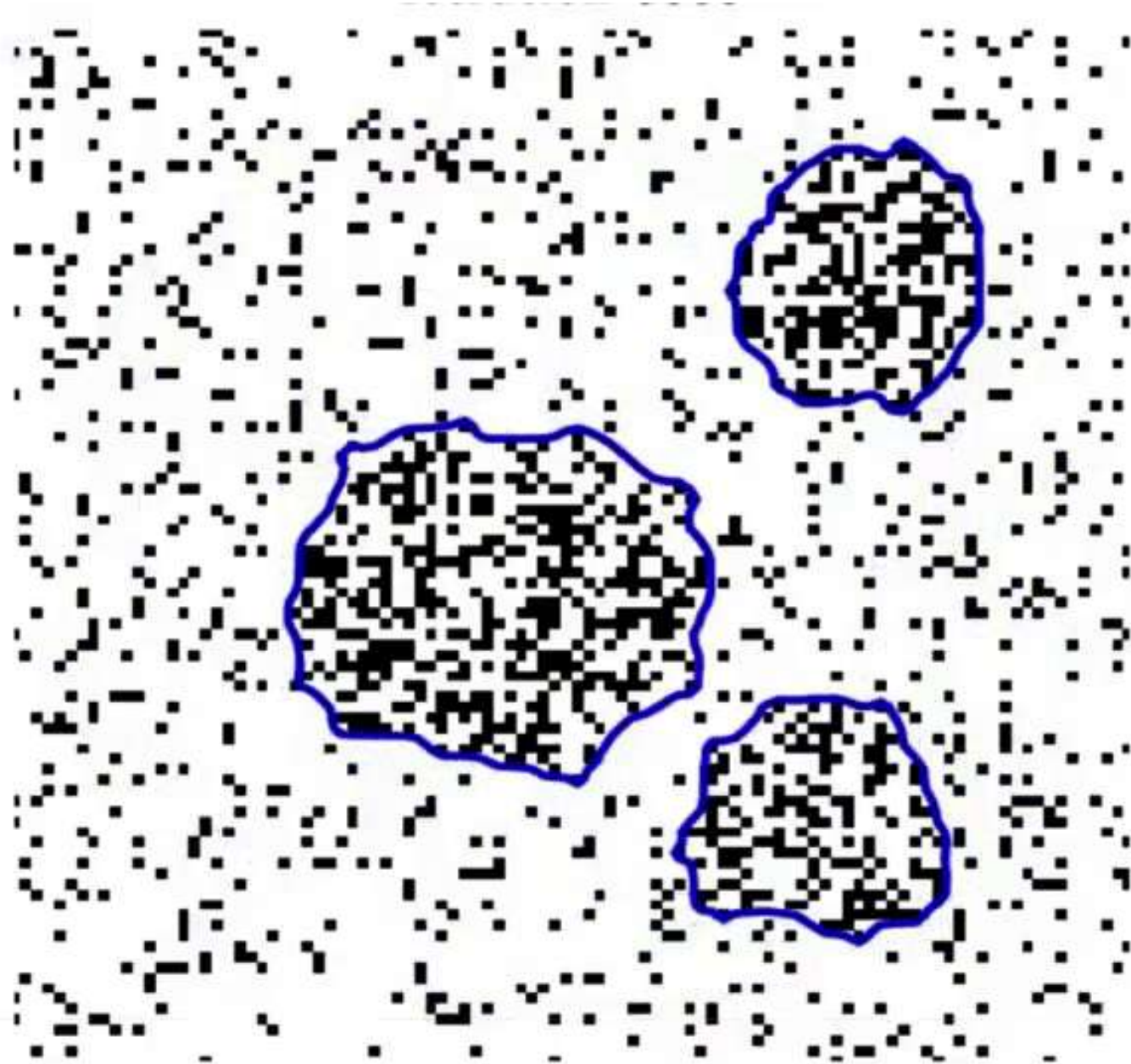


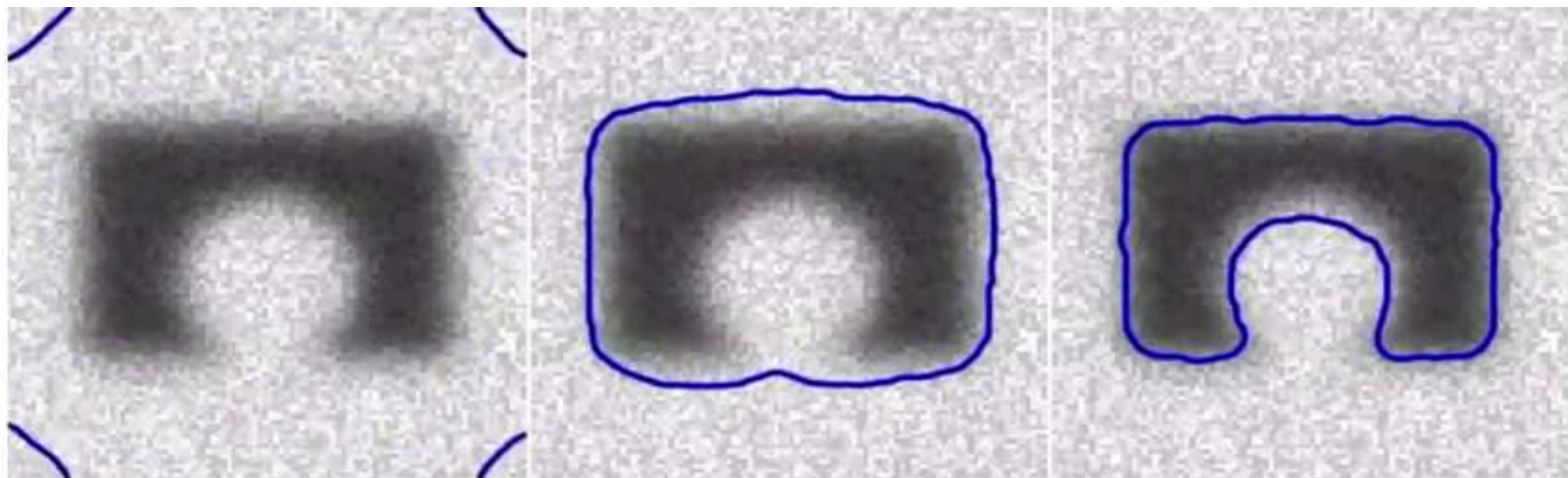
IPOL

FIGURE 112



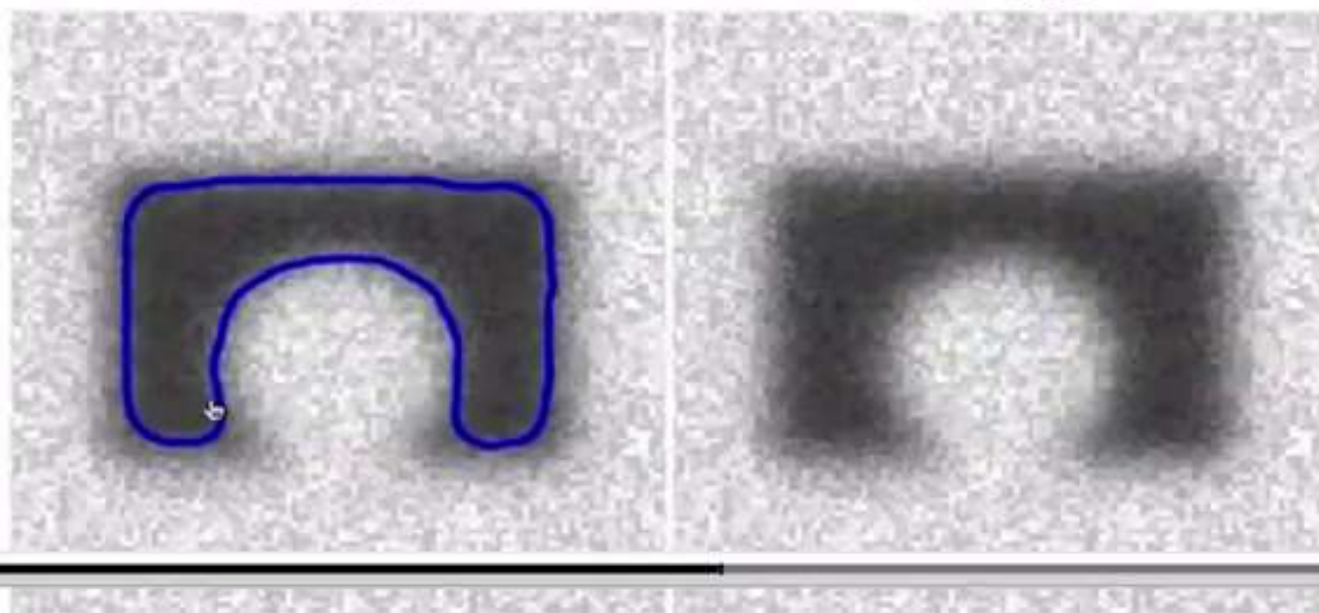






$\nu = 0.1$

$\nu = 0.3$





Evolution

Segmentation

Input



# Behind Roto Brush:

Image and Video Processing: From Mars  
to Hollywood with a Stop at the Hospital

Guillermo Sapiro

Duke  
UNIVERSITY

## Adobe's Video Segmentation



# After Effects

Images Courtesy of: ACM – SIGGRAPH and ECCV; Xue Bai  
Jue Wang; David Simons; Adobe



# Problem: Interactive Video Segmentation

- Pixel level accuracy
- Minimal user intervention
- Interactive real-time



# High-Quality Video Object Cutout



Original





# Challenges



overlapping color distributions



weak boundaries



topology changes, dynamic backgrounds



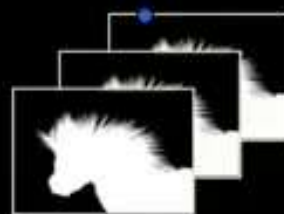
# Features

- **Accuracy**  
work with complicated scenes
- **Robustness**  
on diverse data
- **Practical workflow**  
easy to converge/interact
- **Computational efficiency**  
Faster than 2 frames per second



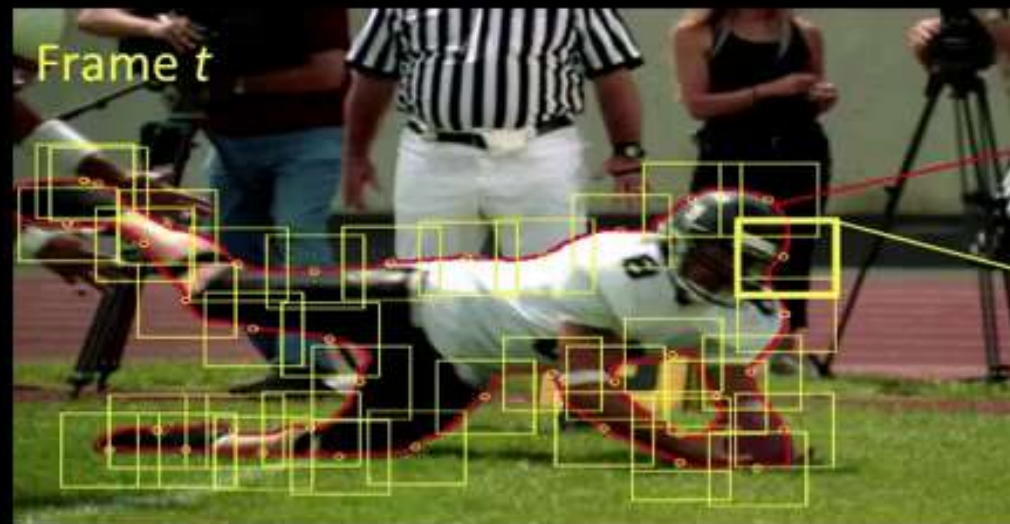
# Algorithm Overview

- Localized classifiers
- Multi-frame propagation
- Local correction
- Post-processing





# Localized Classifiers



Existing  
segmentation

Local classifier  
window



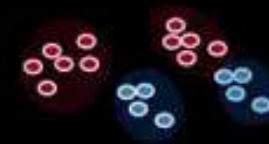
Local window  $i$



Shape Prior

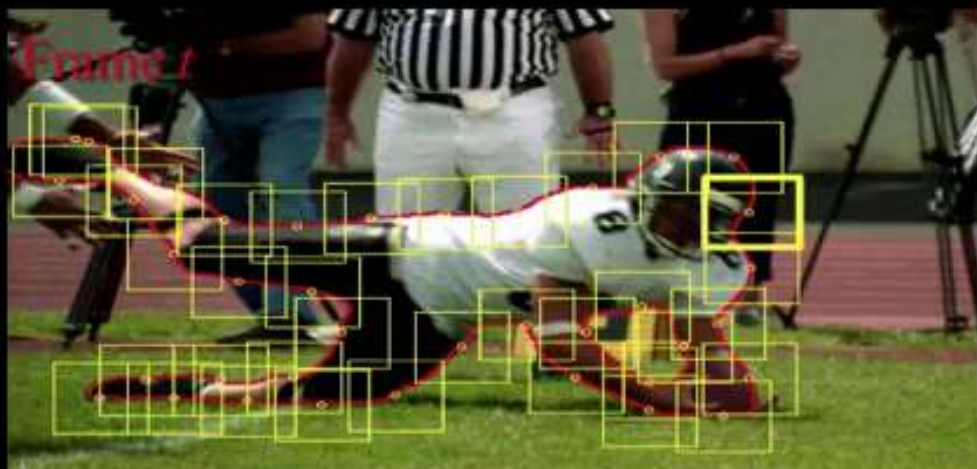


Color Models



Local classifier  $i$

# Localized Classifiers



Frame  $t+1$



Graph cut

training



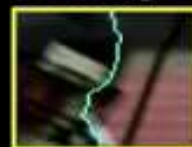
shape



GMM's



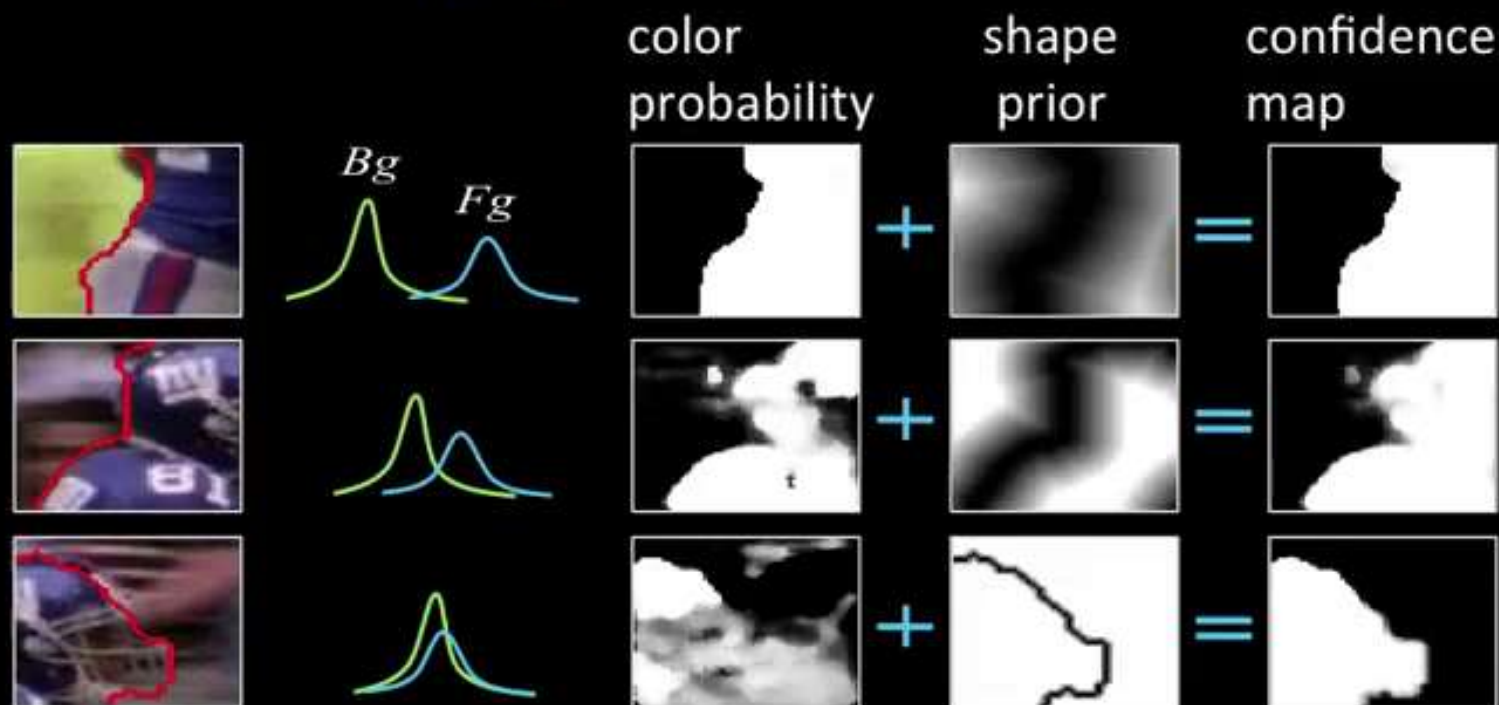
testing



Integration

# Adaptive Color-Shape Integration

- If colors are separable, trust color model
- If not, trust shape prior






# Single-Frame Propagation Example



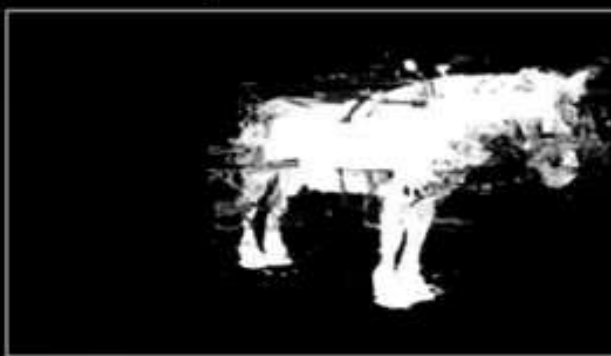
frame  $t$  (segmented)



frame  $t+1$  



global color model



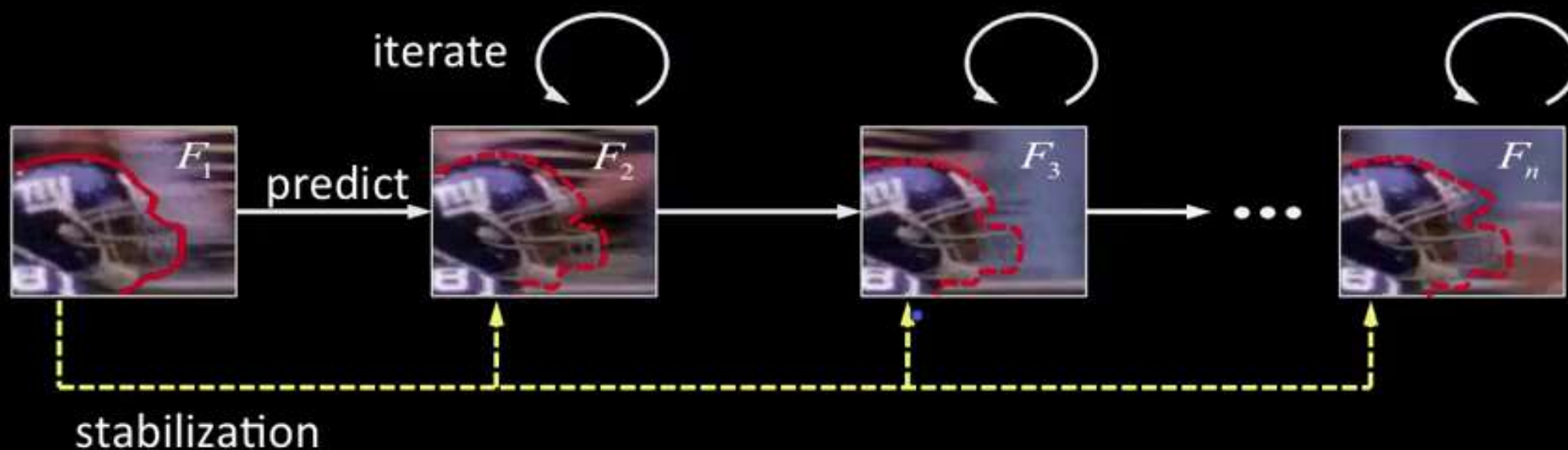
local color models



local color models +  
local shape prior

# Multi-Frame Propagation

- Practical workflow
- Stabilization

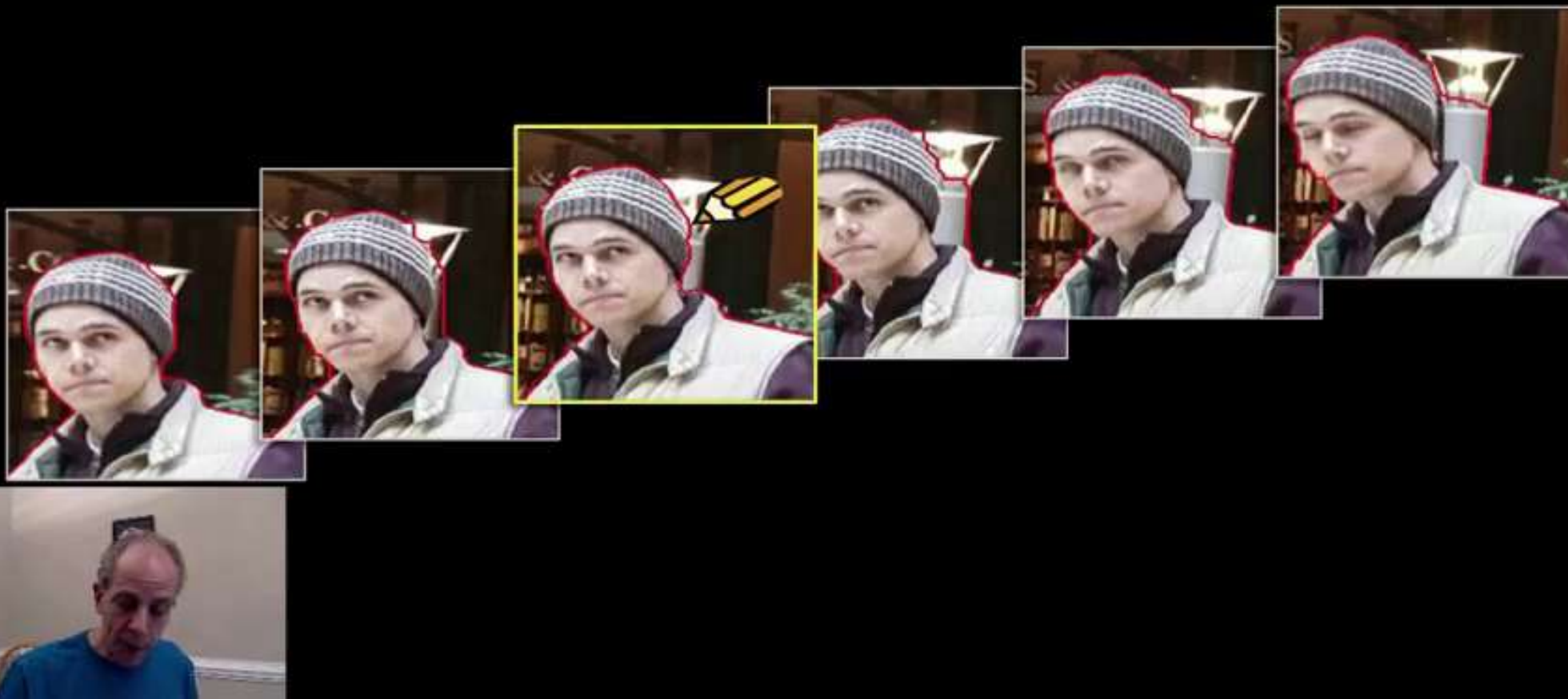


# Spatial-Temporal Local Correction





# Spatial-Temporal Local Correction



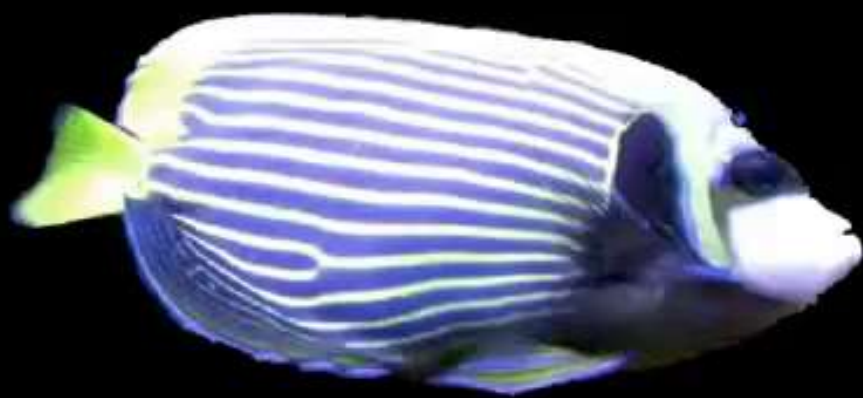
# Spatial-Temporal Local Correction



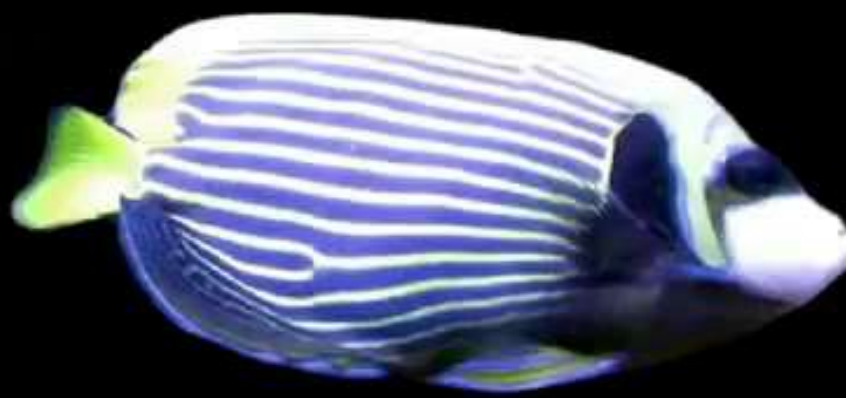
Affects only a few local windows

Propagates changes temporally

# Post-processing



before



after





# Results



720×406 185 frames

# Results



720×576 50 frames

# Results



848×480 232 frames



original



filter



relight



composite





Original

Composite



473×588





Original



Composite



504×380