Department of Mathematics
Probability Theory (MAL-205)
Assignment on
CDF, PMF, PDF

1. Determine the constant A in the following functions, so that those functions are pmfs/pdfs. Find CDF ($F_X(x)$) in each case.
(i)$f_X(x) = \begin{cases} A\left(\frac{2}{3}\right)^{x-1}, & \text{if } x = 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$

(ii) $f_X(x) = \begin{cases} A\binom{7}{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{7-x}, & \text{if } x = 0, 1, 2, ..., 7 \\ 0, & \text{otherwise} \end{cases}$

(iii)$f_X(x) = \begin{cases} \frac{A}{x}e^{\frac{-(log x)^2}{2}}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$

(iv)$f_X(x) = \begin{cases} Ax^{\frac{-1}{2}}, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

(v) $f_X(x) = Ae^{-|x|}, \qquad x \in \mathbb{R}.$

2. Check if the following functions define CDFs :
(a) $F_X(x) = 0,$ if $x < 0,$ $= x,$ if $0 \le x \le 1/2,$ and $= 1,$ if $x > 1/2.$
(b) $F_X(x) = (1/\pi)\tan^{-1} x, \quad -\infty < x < \infty.$
(c) $F_X(x) = 1 - e^{-x},$ if $x \ge 0,$ and $= 0,$ if $x < 0.$

3. For a certain rv X, CDF is defined as $F_X(x) = \begin{cases} 0, & \text{if } x \le 1 \\ K(x-1)^4 & \text{if } 1 < x \le 3 \\ 1, & \text{if } x > 3. \end{cases}$

   (a) Determine the value of K.

   (b) Find pdf $f_X(x)$ and $\Pr\left(\frac{3}{2} \le x \le \frac{9}{2}\right)$.

4. Find the CDF of the distribution whose pdf is given by $f_X(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a \le x \le b \\ 0, & \text{otherwise.} \end{cases}$

5. The pdf for a continuous 'Rayleigh' rv X is given by $f_X(x) = \begin{cases} \alpha^2 x e^{-\frac{\alpha^2 x^2}{2}}, & \text{if } x > 0 \\ 0, & \text{otherwise.} \end{cases}$

   Find CDF of X.

6. The distance covered by a person is assumed to be a continuous rv with pdf
$f_X(x) = \begin{cases} 6x(1-x), & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$ Find CDF of X. Compute $\Pr\left(X \le \frac{1}{2} \middle| \frac{1}{3} \le X \le \frac{2}{3}\right)$.
Determine k such that $\Pr(X > k) = P(X < k)$.

7. Two unbiased dice are thrown and the rv X denote the sum of faces turned. Construct the table for pmf and find CDF.

8. The length of time (in minutes) that a certain person speaks over the telephone is found to be a random phenomenon with pdf $f_X(x) = \begin{cases} Ke^{-\frac{x}{7}}, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$

   (a) Find the constant K.

   (b) Show that the telephone conversation will last more than $m + n$ minutes given that it has lasted for at least 'm' minutes is equal to the unconditional probability that it will last more than 'n' minutes.

9. Suppose a certain type of small data processing firm is so specialized that some have difficulty making a profit in their first year operation. The pdf that characterizes the proportion Y that make a profit is given by $f_X(x) = \begin{cases} Cy^4(1-y)^3, & \text{if } 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$
   After finding the value of C, compute the probability that at most 50% of the firms make a profit in the first year. Also find the probability that at least 80% of the firms make a profit in the first year.

10. A bag contains two fair coins and a third coin which is biased. The probability of tossing a head on this third coin is 3/4. A coin is pulled at random and tossed three times. Let X be the rv that counts the number of heads obtained in these three tosses. Give the pmf and CDF.