Visvesvaraya National Institute of Technology
Department of Mathematics
Numerical Methods and Probability Theory (MAL 205)
Assignment 5

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1. Suppose X has a binomial distribution with parameters 6 and 1/2. Show that $X = 3$ is the most likely outcome.

2. If a fair dice is successively flipped, find the probability that a 3 first appears on the fifth trial.

3. Suppose that two teams are playing a series of games, each of which is independently won by team A with probability p and by team B with probability $1-p$. The winner of the series is the first team to win 4 games. Find the probability that a total of 7 games are played. Also show that this probability is maximized when $p = 1/2$.

4. (Normal Approximation to the Binomial). Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$. Use the normal approximation and then compare it to the exact solution.

5. A manufacturer does not know the mean and SD of the diameters of ball bearings he is producing. However, a sieving system rejects all bearings larger than 2.4 cm and those under 1.8 cm in diameter. Out of 1000 ball bearings 8% are rejected as too small and 5.5% as too big. What is the mean and standard deviation of the ball bearings produced?

6. The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year? What number of burnt acres corresponds to the 38th percentile?

7. Find the probability of getting between 3 and 6 heads inclusive in 10 tosses of a fair coin by using (a) the binomial distribution, (b) the normal approximation to binomial approximation.

8. Let X be a random variable with with probability density function
$$f(x) = \begin{cases} \frac{4}{81}x(9-x^2) & \text{for } 0 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$
Find the first four moments (a) about origin and (b) about mean for X and hence calculate the coefficients of skewness and kurtosis.

9. If the probability of a defective ball is 0.05, find the coefficients of (a) skewness and (b) kurtosis for the distribution of defective balls in a total of 500.

10. Suppose that two teams are playing a series of games, each of which is independently won by team A with probability p and by team B with probability $1-p$. The winner of the series is the first team to win four games. Find the expected number of games that are played, and evaluate this quantity when $p = 1/2$.

11. Successive monthly sales are independent normal random variables with mean 100 and variance 100.

(a) Find the probability that at least one of the next 5 months has sales above 115.

(b) Find the probability that the total number of sales over the next 5 months exceeds 530.