Introduction

Observable change in any physical ability

↓

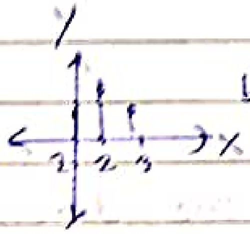
Signal → It carries information lies in the change.

eg: $i(t) = i_0 \sin \omega t$ → 2D

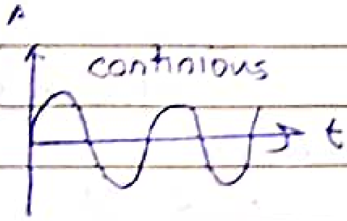
$v(t) = v_0 \sin \omega t$

$x(t) = x$ → function of time

Discrete



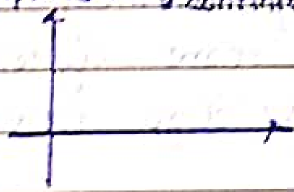
b/w 1 & 2 or 2 & 3 signal does not exists.



* Characteristic whether signal is continious / discrete is property of quantity on x-axis.

* Neighbour pt. can be determined in discrete but not in continious.

Amplitude $\begin{cases} \text{discrete} \\ \text{continious} \end{cases}$



t $\begin{cases} \text{discrete} \\ \text{continious} \end{cases}$

Amp.	Time	Signal
C	C	ANALOG
D	C	BOXCAR
C	D	SAMPLED DATA
D	D	Digital

Analog signals must be converted to digital signal for storing purposes.
(To convert continious time axis to discrete)
use switch turn on/off at T sec. → SAMPLING

Discrete pts of time are samples.
Choice of value of sample determines whether it can be converted back to analog.

Reference book

- Signal is said to be one-dimensional when its function depends only on one variable.
- It is multi-dimensional for more than one variable
  eg: image.
- A system is formally defined as the entity that manipulates one or more signals to accomplish a function, thereby yeilding new signals.

eg Message signal → Transmitter $\xrightarrow{\text{Transmitted signal}}$ Channel

Channel $\xrightarrow{\text{Recieved Signal}}$ Reciever

Reciever → Estimate of message signal

Analog vs digital signal processing

Type of signal processing

(1) Analog — continious-time approach

(2) Digital — discrete - time approach

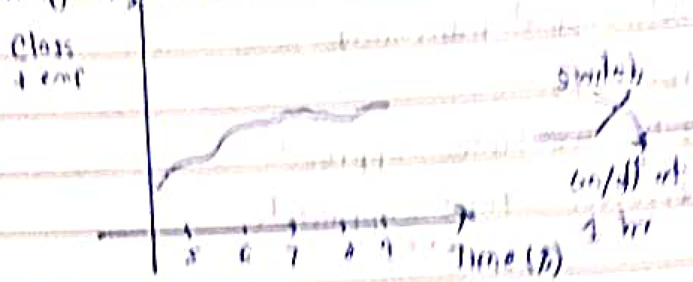
Analog → uses RLC, diodes, transistors, amplifiers etc.

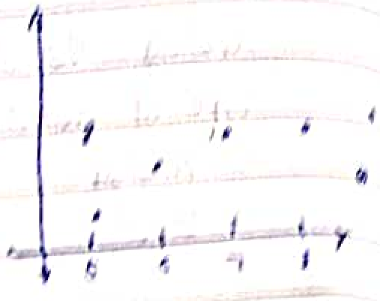
Digital → uses adders, multipliers and memory

Advantage of digital over analog

- Flexibility → same ~~signal proces~~ machine can be used to implement different version of signal processing operation of interest.
- Repeatability → prescribed signal processing operation can be repeated exactly over & over again when it is implemented by digital means. (same output for same input under identical condition)

Analog Thermometer

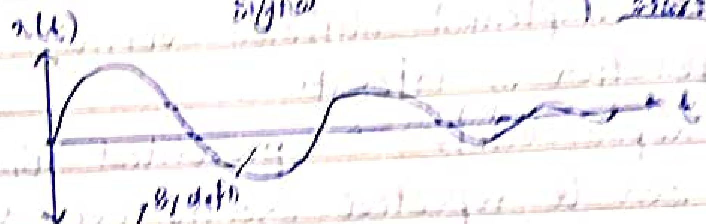




Sampling

Sampling : $x(t) = 2e^{-0.2t} \cdot \sin t$ $\quad t \geq 0$

↳ Continuous time (CT) signal



$x(nT) \triangleq x(t = nT)$

zero & +ve integer index

$x(0) = x(t=0) = 0$

$x(T) = x(t=0.5) = 0.08522$

$x(2T) = x(t=1) = 1.2468$

[Assume for this example $T = 0.5$]

$x(3T) = x(1.5) = 1.2721$

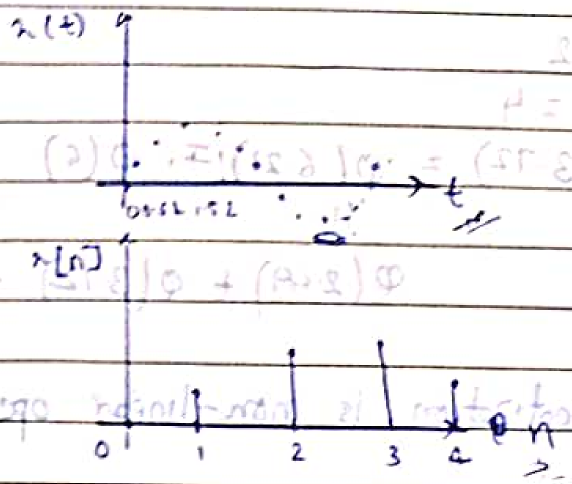
→ Given

$x[n] \triangleq x(t = nT)$

$x[1] = 0.08522$

$x[2] = 1.2468$

$x[3] = 1.2721$



On making time axis discrete and plotting sampled data, a similar pattern to analog signal is observed.

On making n discrete and plotting sampled data same effect doesn't occur

sampling period

They match when $T = 1$.

Such signal is ~~dt~~ discrete time signal (DT)

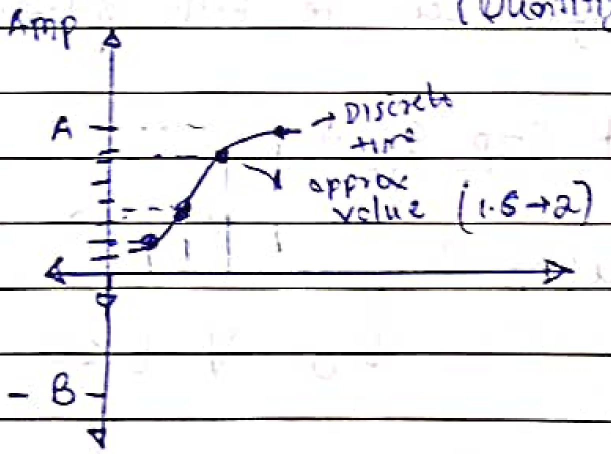
Signal can never be discrete or continious

These terms apply to variables & axes.

(converting infinite value to finite value)

<u>Discretization of Amp. axis</u>

(Quantization)



(First discrete time then discrete amplitude) eg: $\phi(2.49) = 2$

$\phi(3.72) = 4$

$\phi(2.49 + 3.72) = \phi(6.21) = \phi(6)$

$||$

$\phi(2.49) + \phi(3.72)$ → Not always $\neq$

eg $\phi(2.7) + \phi(3.7)$

$\phi(2.7+3.7)$

∴ Quantization is non-linear operation

After quantization signal becomes digital.
(discrete time & amplitude axes)

Analog signal → Sampling → Quantitization → Digital signal

ADC
Analog to digital conversion ~~converter~~

Elementary CT Signal

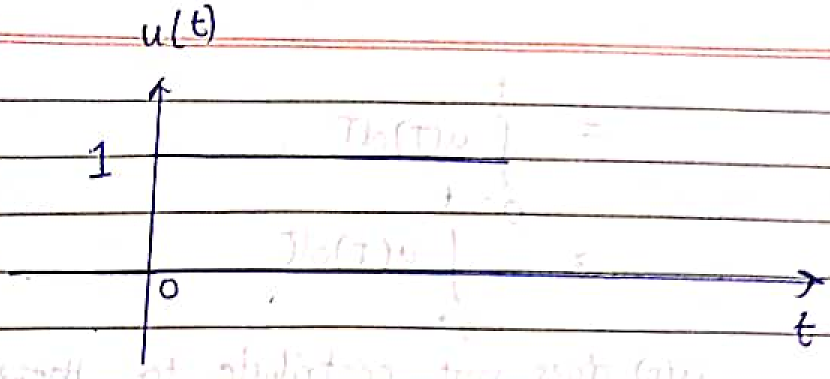
+ve sided → if $x(t) = 0 \quad \forall \; t < 0$

-ve sided → if $x(t) = 0 \quad \forall \; t > 0$

A signal can be both sided.

1) Unit Step Signal

$$u(t) \triangleq \begin{cases} 1 & \forall \; t > 0 \\ 0 & \forall \; t < 0 \end{cases}$$



Signal $u(0) = ??$

$= 1$

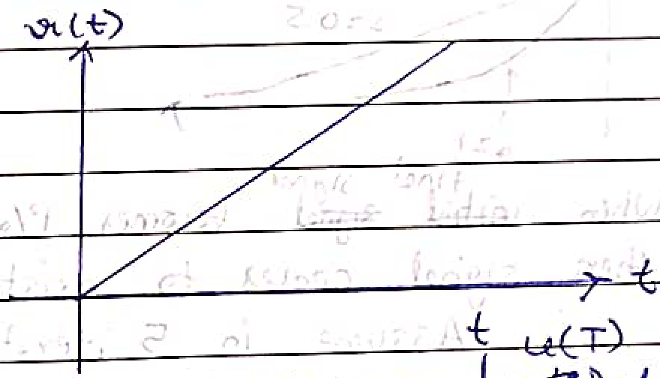
$= 1/2 \; \left\{ \frac{u(0^-) + u(0^+)}{2} \right.$

<u>Engineering perspective</u>

As $u(0)$ appears for 0 (No) time.
So values of $u(0)$ doesn't matter.

2> <u>Ramp signal</u>

$$r(t) = \begin{cases} t & \forall \; t > 0 \\ 0 & \text{otherwise} \end{cases}$$



$$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$$

$$= \int_{0}^{t} u(\tau)\, d\tau$$

$$= \int_{0^-}^{t} u(T)\,dT$$

$$= \int_{0^+}^{t} u(T)\,dT$$

u(0) does not contribute to these integrations

CT signals may have discontinuities in amplitude. like u(t) at t = 0.

Real exponential Signal

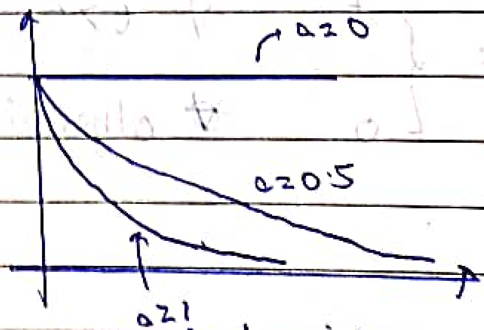
$x(t) = e^{-at} \quad \forall \; t \geq 0$

For +ve 'a'

For -ve 'a' → growing exponential

↓

hard to generate in lab



When ~~initial signal~~ final signal becomes 1% of initial signal then signal ceases to exists.

∴ Assume in 5 iteration signal ceases to exist. (0.0693 < 1% o

Magnitude of e^{-at} becomes 37% in one time constant.

Time constant = $1/a$

e^{-at} signal can be zero after 5 time constants.

$x(t)$

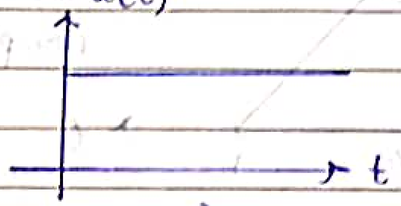
Shifting a signal in time

$x(t - t_0) \rightarrow$ delay

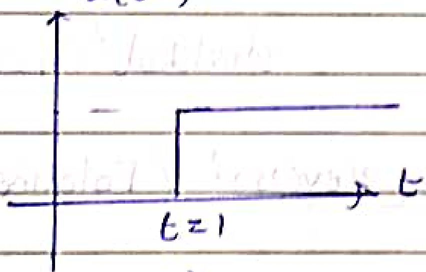
$x(t + t_0) \rightarrow$ advancement

t_0 is +ve value of time shift.

$u(t)$

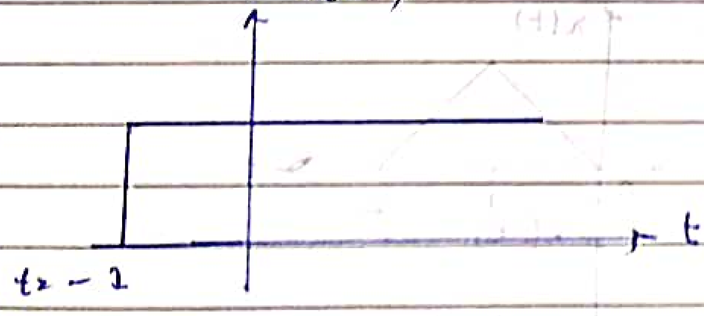


$u(t-1)$



$$x(t-1) \begin{cases} 1 & t-1 \geq 0,\ t \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$u(t+1)$



$$x(t+1) \begin{cases} 1 & t+1 \geq 0,\ t \geq -1 \\ 0 & \end{cases}$$

Scaling a signal in time

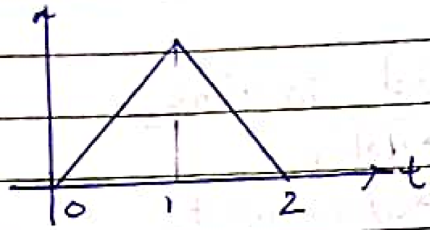
$x(at)$

$x(bt)$

$x(1) = x(t=1)$

$= x(at=1)$

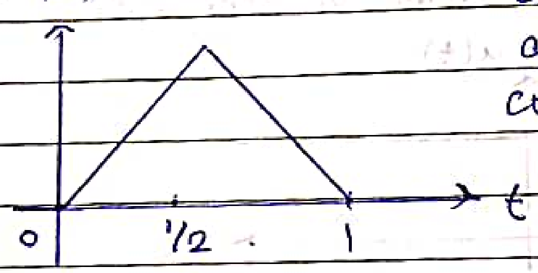
$= x(a=1/t)$



$x(2t)$

$a>1$

compressed signal

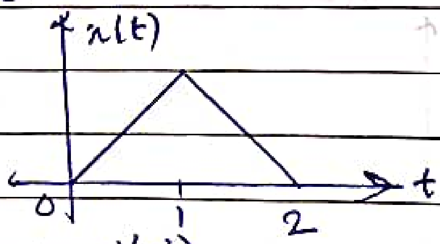


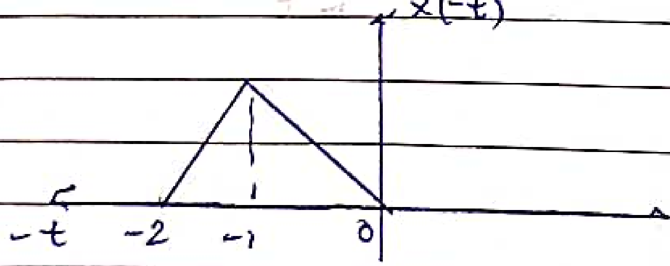
$a<1$

stretched/expanded signal

Time reversal / Folding

Scaling $a=-1$



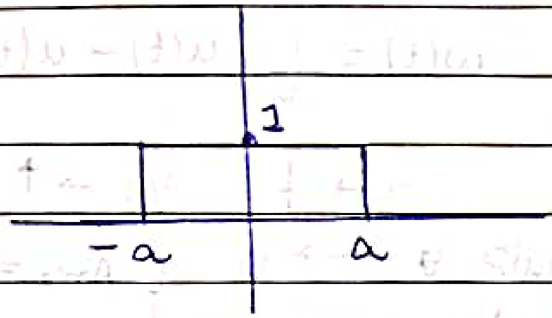


$x(t) \rightarrow x(\alpha t + \beta)$

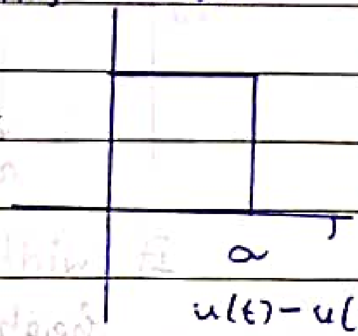
* Always perform shifting

* Then perform scaling

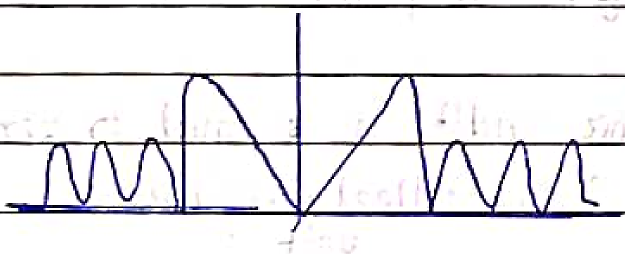
Pulses : (Windows) → Filtering signal by multiplying with window.



or

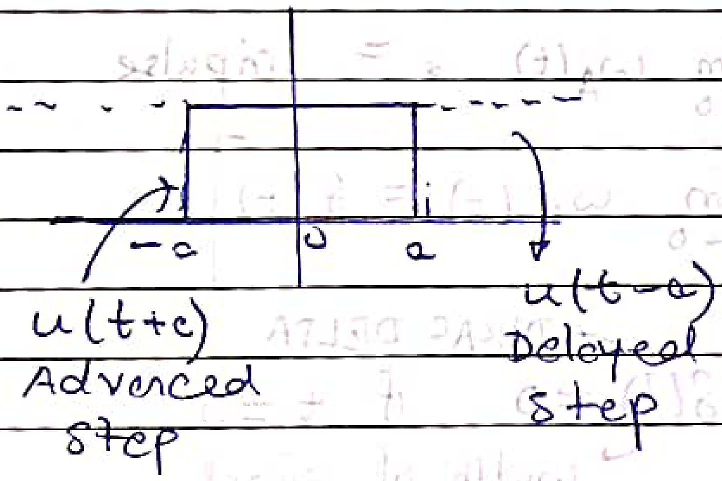


$u(t) - u(t-a) = w_a(t)$

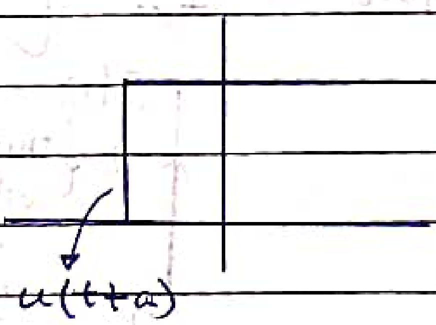


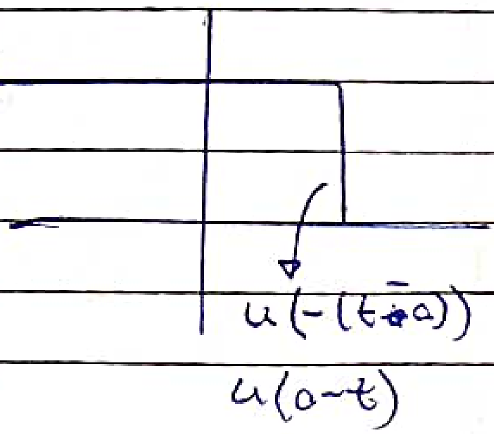
$x\{u(t)\} \rightarrow$ +ve side

$u(-t) \rightarrow$ -ve side



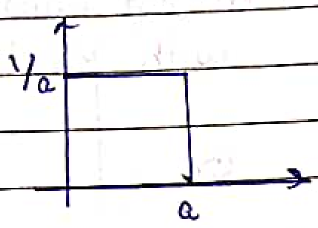
$$\boxed{w(t) = u(t+a) - u(t-a)}$$





To change amplitude to A & multiply window with A amplitude.

Consider a pulse of unit area.



$$w(t) = \frac{1}{a}\left(u(t) - u(t-a)\right)$$

$a \to \downarrow \qquad 1/a \to \uparrow$

$$\left.\begin{array}{l}\text{If width of pulse is } \longrightarrow 0 \\ \text{height of pulse } \longrightarrow \infty\end{array}\right\} \text{Area} = 1$$

The pulse whose width is equal to zero & area under pulse is equal to 1 is called unit impulse.

$$\lim_{a \to 0} w_A(t) = \text{impulse}$$

$$\boxed{\lim_{a \to 0} w_A(t) = \delta(t)}$$

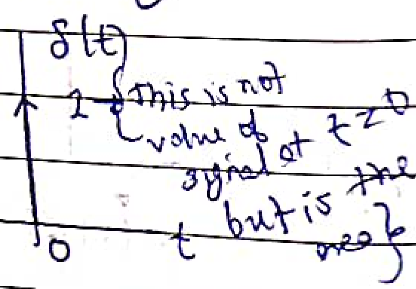
→ DIRAC DELTA

$\delta(t) = 0$ if $t \neq 0$

↳ width of signal

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

{Unit impulse ↳ Area = 1}



Impulse is used to locate a point in a signal. It is also called as sampling function. Multiply signal with impulse at a specific point

$\delta(t-0) \rightarrow$ impulse of area $= 1$ at $t = 0$

$\int A\,\delta(t-t_0)\,dt \rightarrow$ impulse of Area $= A$ at $t = t_0$

$A\delta(t-t_0)\cdot x(t)$

$= \cancel{x(t_0)}\; x(t_0)\,A\delta(t-t_0)$ ← Shifting property

impulse at t_0 of area $A\cdot x(t_0)$

Sample of signal at $t = t_0$

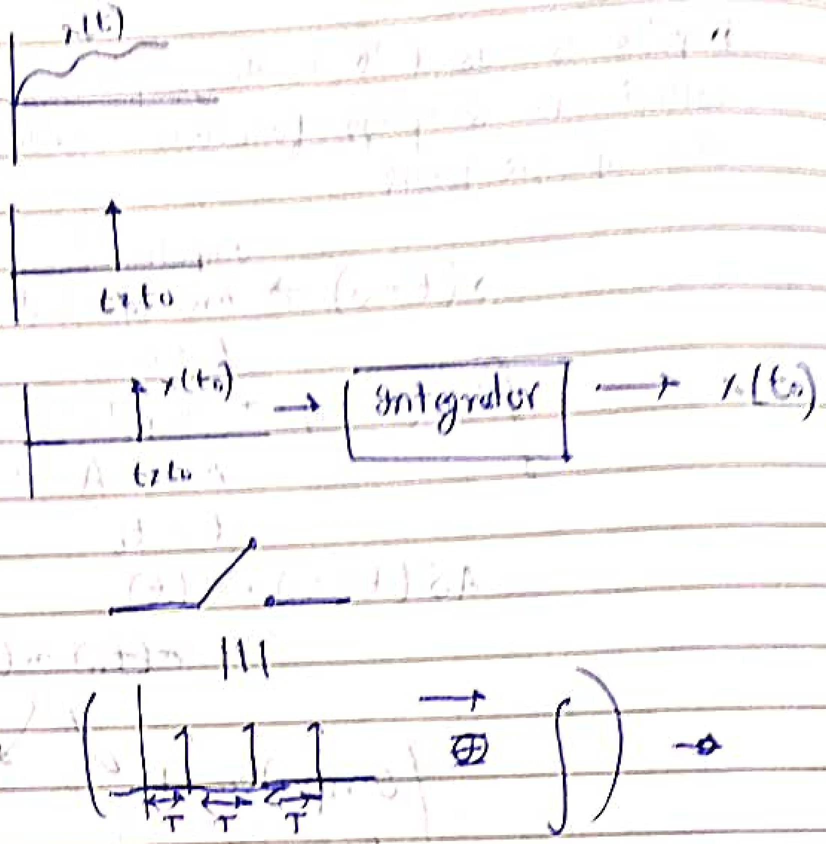
$$\int_{-\infty}^{+\infty} x(t)\cdot\delta(t-t_0)\,dt$$

$$= \int_{-\infty}^{+\infty} x(t_0)\,\delta(t_0-t_0)\,dt$$

$$= x(t_0)\underbrace{\int_{-\infty}^{+\infty}\delta(t-t_0)\,dt}_{1}$$

$= x(t_0) \rightarrow$ sampling product

eg:



eg)
$$\int_{0}^{\infty} \sin(t-2)\,\delta(t-3)\,dt$$

$$= \sin(3-2) \underbrace{\int_{0}^{\infty} \delta(t-3)\,dt}_{1}$$

$$= \sin 1$$

$$= 0.84$$

$$\int_{-\infty}^{0} \sin(t-2)\,\delta(t-3)\,dt = 0$$

(Impulse not covered in range)

eg// $\int_{-\infty}^{\infty} x(t-\tau)\, \delta(\tau-3)\,d\tau$

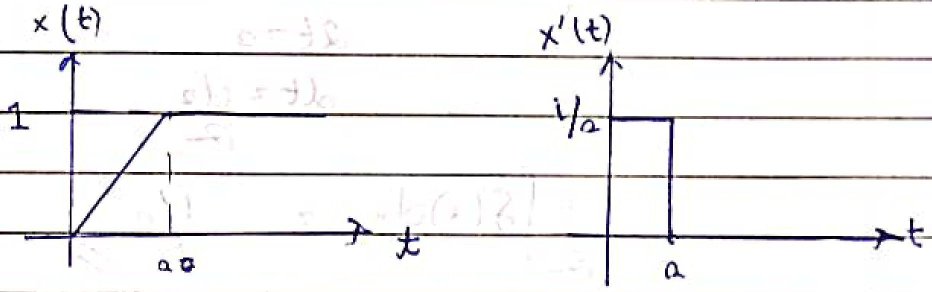
$x(t-3) \underbrace{\int_{-\infty}^{\infty} \delta(\tau-3)\,d\tau}_{1}$

$= x(t-3)$

eg// $\int_{-\infty}^{\infty} x(t-\tau)\, \delta(t-3)\,dt$

$x(3-\tau)$

$x(\cancel{t-3}) \int_{-\infty}^{\infty} \delta(t-3)\,dt = x(3-\tau)$



at $a = 0$

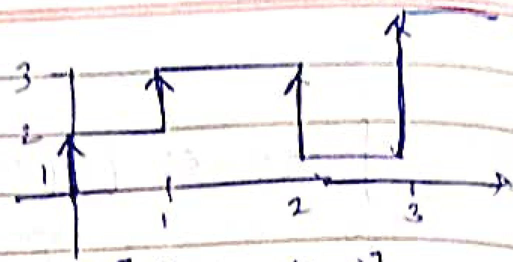
$x(t) \rightarrow$ unit step

$x'(t) \rightarrow$ impulse

$\therefore \boxed{\delta(t) = \frac{d(u(t))}{dt}}$

$\boxed{\int_{0}^{\infty} \delta(t-\tau)\,dt = u(t)}$

$\int \sum_{k=0}^{n} \delta(t-8)x(8)$



* Make impulses at point of discontinuity

$2[u(t)-u(t-1)]$
$+3[u(t)-u(t-2)]$
$+[u(t-2)-u(t-3)]$

$+ 6u(t-3)$

Area under : $\delta(2t)$

$$\int \delta(2t)\,dt$$

$2t = a$

$dt = \frac{da}{2}$

$$\therefore \frac{1}{2}\int \delta(a)\,da = 1/2$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

<u>Problem set 1</u>

1) $z = 1/2\, e^{i\pi/4}$

$$= 1/2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$\cos\frac{\pi}{4} \to Re(z)$, $\sin\frac{\pi}{4} \to Im(z)$

$$|z| = \frac{1}{2}\sqrt{\cos^2\pi/4 + \sin^2\pi/4} = 1/2$$

$$z + z^* = 2Re(z)$$

PAGE NO :
DATE : / /

Youtube

1) $$I = \int_{-\infty}^{\infty} \cos\pi t \cdot \delta(t-2) + 3\delta(t+1) + \sin\pi t \cdot \delta(2t-1)\,dt$$

$$\int_{-\infty}^{\infty} \cos\pi t \cdot \delta(t-2)\,dt + 3\int_{-\infty}^{\infty} \delta(t+1) + \int_{-\infty}^{\infty} \sin\pi t\ \delta(2t-1)$$

$$\cos 2\pi\ (1) + 3 + \int_{-\infty}^{\infty} \sin\pi t \cdot \frac{1}{2}\,\delta(t-1/2)\,dt$$

$$1 + 3 + \sin\frac{\pi}{2} \cdot \frac{1}{2}$$

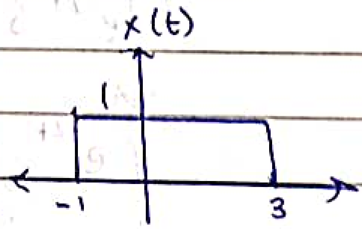
$$4 + 1/2 = 9/2$$

2) $$\int_{-\infty}^{\infty} e^{-t}\,\delta(2t-2)\,dt$$

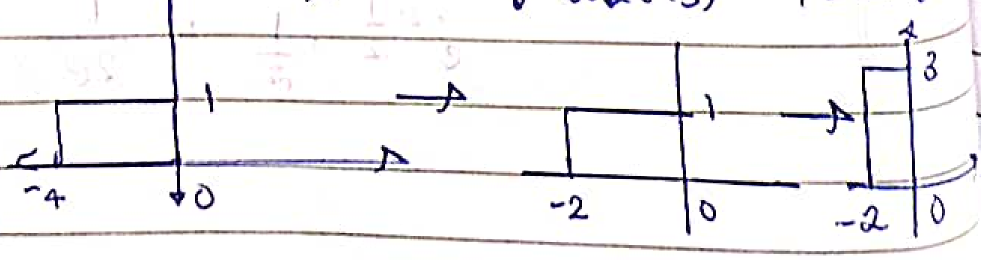
$$\int_{-\infty}^{\infty} e^{-t}\,\frac{1}{2}\,\delta(t-1)\,dt$$

$$\frac{e^{-1}}{2} = \frac{1}{2e}$$

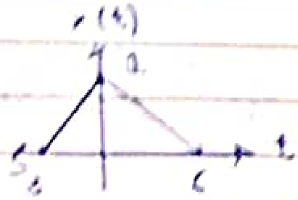
3) $y(t) = 3x(2t+3)$



$v(t+3)$ $\quad$ $v\ u(2t+3)$ $\quad$ $Y = 3x(2t+3)$

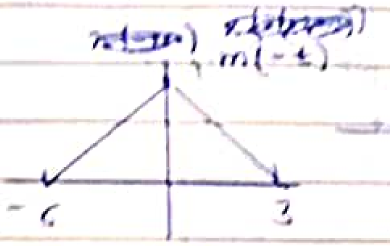


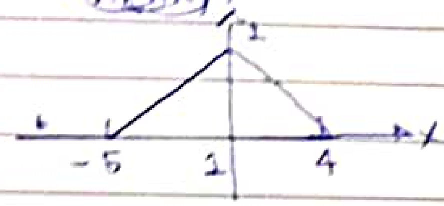
3)



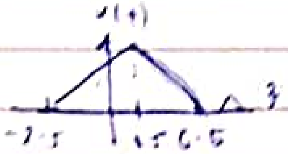
$y(t) = x(-2t+1)$

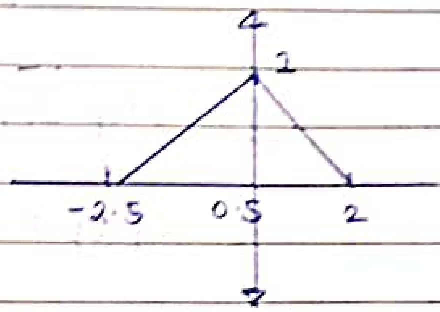
$= x(-(2t-1))$





$x(-2(t-0-1/2))$





x —— x —— x

D. Discrete – Time Signals / (Sequences)

$x[n] = x(t)\Big|_{nT}$ → sampling pt.

↓

index integers.

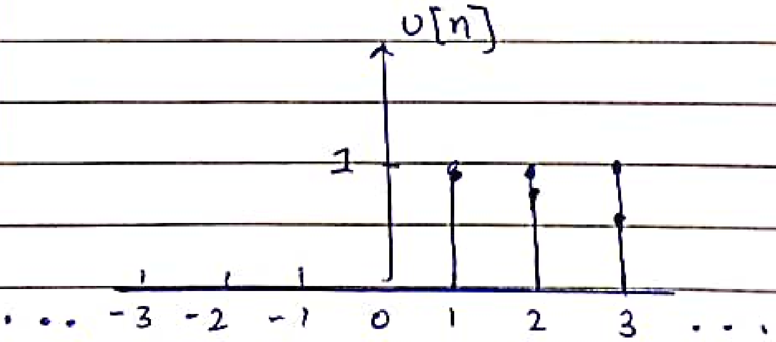
$x(T)$ → sample

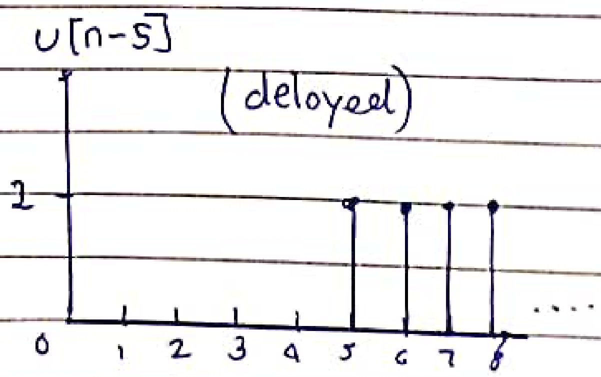
$x(2T)$

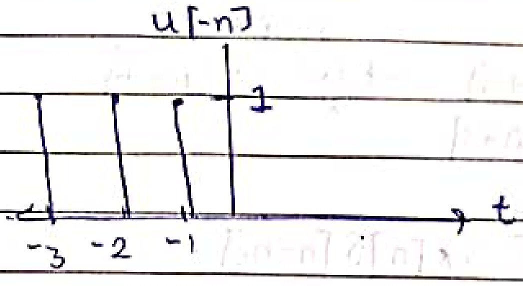
$$x[n] \triangleq x(nT)$$

DT step Sequence

$$x[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$







$u[n \pm n_0]$ exists only if $n_0 \in$ Integer

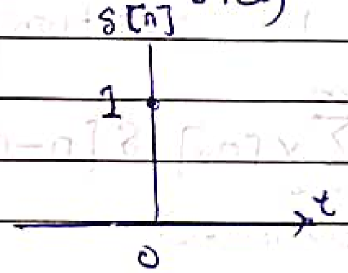
Impulse Sequences

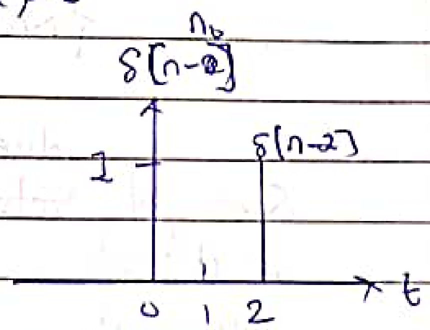
Kronecker delta

$$\delta[n] \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

$\downarrow$

value (not area)





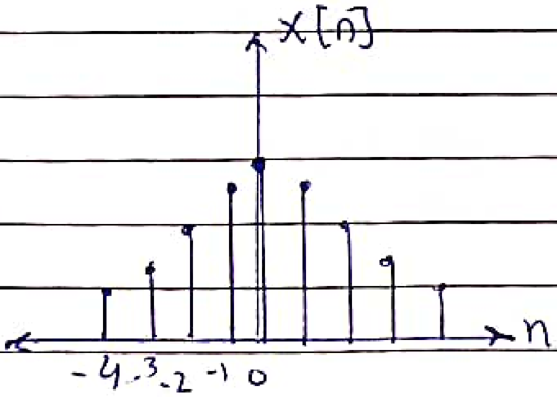
$$u[n] = \delta[n] + \delta[n-1] + \vdots = \sum_{n_0=0}^{\infty} \delta[n-n_0]$$

Step = integral of impulse

$\therefore$ Sum of pulses = unit step [D.T]

Difference of steps = pulses [D.T]

$(u[n] - u[n-1] = \delta[n])$ $\quad$ $(\delta[n])$



$x[n+4] \cdot \delta[n+4] \rightarrow \{\text{sample pt at } n=4\}$

$+ \; x[n+3] \cdot \delta[n+3]$

$$x[n] = \sum_{n=-\infty}^{\infty} x[n]\,\delta[n-n_0]$$

only integer

$x[n] = \{1, 2, 3, 4, 5, 4, 3, 2, 1\}$

$\uparrow$ indicates 0 position

{No arrow sequence starts from 1st position}

Sum of ~~valve~~ values at different positions = $\displaystyle\sum_{n=-\infty}^{\infty} x[n_0]\,\delta[n-n_0]$ (INTEGER)

↓

Discrete Signal

$x(n) = x[n_0]\,u[n] + x[n_0]\,u[-n]$ } In terms of $u[n]$

$\cancel{x[0]\,u[}$

$- \, x[n]\,\delta[n]$

↓

1 only at $n=0$

↓

$x[0]$

$$\therefore \; x[n] = x[n]u[n] + x[n]u[-n] - x[n]\delta[n]$$

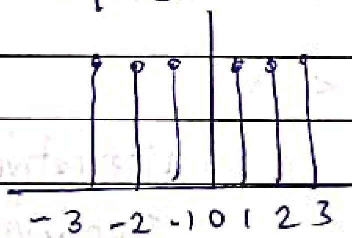
Window Sequence

$u[n] - u[n - n_0] \Rightarrow$ window of 'n_0' length

$$W_0[n] = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

$w[3]$

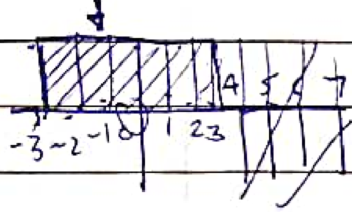
$\boxed{\text{length} = 2 \times n + 1}$ → no of samples in the sequence



☆ To make $w[3]$ use

$$w[3] = u[n+3] - u[n-4]$$

$w[n] = u$



$\boxed{u[n] - u[n-5]}$

Discrete Time Exponential Sequences

$$x[n] = C\alpha^n$$

where $C \rightarrow$ constant

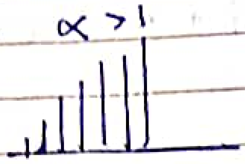
$\alpha \rightarrow e^{\beta}$

↳ can take form

if $\alpha \in$ Real then sequence is real exponential.

case 1 : $\alpha > 1$

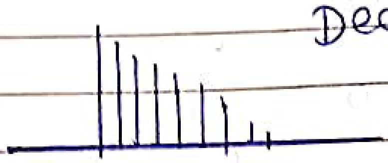
Grow



case 2 :

$0 < \alpha < 1$

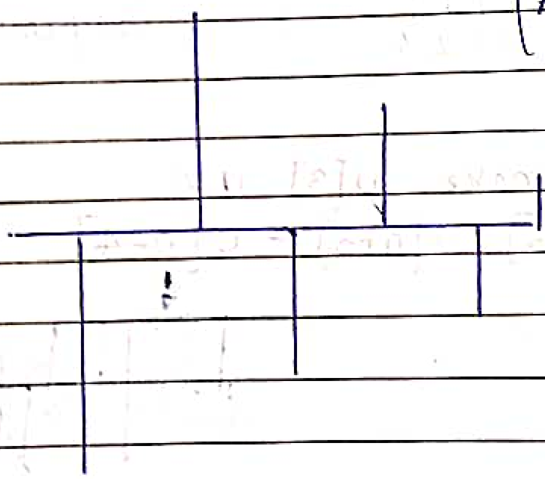
Decay



case 3

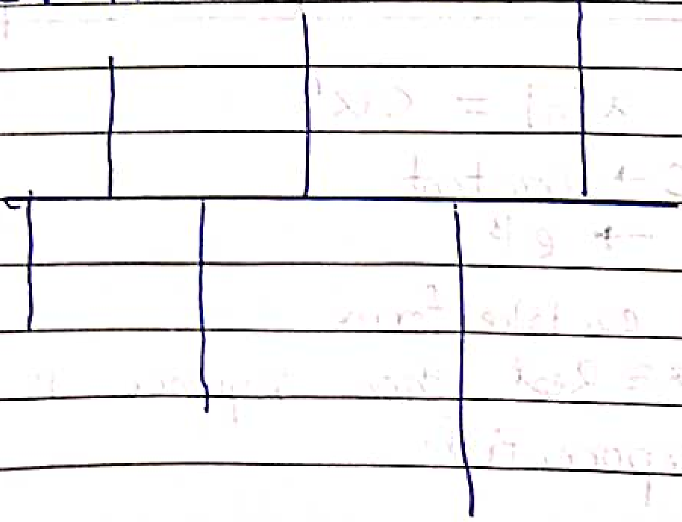
$-1 < \alpha < 0$

(Alternatively Decaying)



case IV : $\alpha < -1$

(Alternatively Growing)



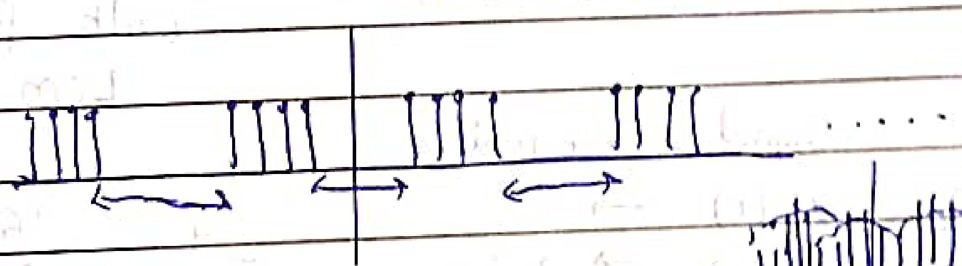
Periodicity

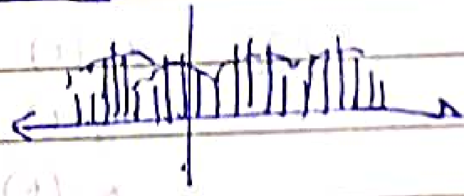
- Periodic → $x(t) = x(t+nT)$
  $T \rightarrow$ period of the signal
- Aperiodic
  {non-periodic}

Smallest value of $T \Rightarrow$ {Fundamental period}

Time limited signals cannot be periodic
Periodic signals are generated $t = -\infty$ till $t = +\infty$

$x[n] = x[n+N]$ After 'N' sample signal repeats





if → CT is periodic resulting DT should ~~also~~ also be periodic

* $x(t)$ is pdc with f.p.T.

$$\int_{a}^{a+t} x(t)\,dt = \int_{b}^{b+t} x(t)\,dt$$

$(T_n)\ (n = 1, 2 \ldots)$ with period T

Summation of m periodic signal is periodic if and only if

$$\left[\frac{T}{T_m} = n_i\right]$$ Verify using MATLAB

n_i → integer

$$\frac{T}{T_1} = n_1 \quad \& \quad \frac{T}{T_2} = n_2$$

$$\frac{T_1}{T_2} = \frac{n_2 \rightarrow \text{integer}}{n_1 \rightarrow \text{integer}}$$ then ~~sum~~ resultant signal is periodic with period $LCM(T_1, T_2)$

Signal	Period
$x_1(t)$	$\rightarrow 4$
$x_2(t)$	$\rightarrow 1.25$
$x_3(t)$	$\rightarrow \sqrt{2}$

eg

$$\frac{4}{1.25} = \frac{16}{5}$$

$\sqrt{2}$ causes aperiodicity

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

eg

$x_1(t) \rightarrow 1.08$

$x_2(t) \rightarrow 3.6$

$x_3(t) \rightarrow 2.025$

$$\frac{1.08}{3.6} = \frac{108 \times 10^{-2}}{36 \times 10^{-1}} = 3/10 \quad (30)$$

$$\frac{3.6}{2.025} = \frac{36 \times 10^{-1}}{2025 \times 10^{-3}} = \frac{12 \times 1000}{675} = \frac{40}{9}$$

$\Sigma x(t) \rightarrow$ ~~T = 2720~~

LCM

$T_1 = \frac{108}{100} = \frac{27}{25}$

$T_2 = 36/10 = 18/5$

$T_3 = \frac{2025}{1000} = \frac{81}{40}$

$$T = \frac{LCM(27, 18, 81)}{HCF(25, 5, 40)} = 162/5 = \boxed{32.4s}$$

$= 160/9 \quad (1440)$

Youtube

Imp points

CTS : $x(t) = x(t \pm nT_0)$

$\downarrow$

FTP = smollest period for which $x(t)$ is periodic

$\neq 0$

$\neq \infty$

$= +ve$

DTS : $x[n] = x[n \pm mN]$

$\downarrow$

FTP

eg : $x(t) = \sin^2(4\pi t)$

$x(t+2) = \sin^2(4\pi(t+2))$

$= \sin^2(4\pi t + 8\pi)$

$= \sin^2(4\pi t)$

$T = 2$

$x(t+0.25) = \sin^2(4\pi(t+0.25))$

$= \sin^2(4\pi t + \pi)$

$= \sin^2(4\pi t)$

$= x(t)$

$\boxed{FTP = 1/4}$

eg : $x(t) = \sin 6\pi t + \cos 5\pi t$

$x(t+2) = \sin(6\pi(t+2)) + \cos(5\pi(t+2))$

$= \sin(6\pi t + 12\pi) + \cos(5\pi t + 10\pi)$

$= \sin(6\pi t) + \cos(5\pi t)$

PAGE NO
DATE / /

$x_1(t) = \cancel{\cos}\ \sin 6\pi t$ $\qquad$ $x_2(t) = \cos 5\pi t$

$\omega = 6\pi$ $\qquad$ $\omega = 5\pi$

$\frac{2\pi}{\omega} = T_1 = \frac{1}{3}$ $\qquad$ $T = \frac{2}{5}$

$\frac{T_1}{T_2} = \frac{1/3}{2/5} = 5/6$

$LCM\ (\cancel{5,6})\ 1/3, 2/5)$

$T = \frac{LCM(1,2)}{HCF(3,5)}$

$= \frac{2}{1} = 2$

Q) $30 \sin 100t + 10 \cos 300t + 6 \sin(500t + \pi/4)$
fundamental freq = ? in rad/s

* Hint

$x_1(t) = 30 \sin 100t$ $\quad$ $\omega = 100$

$T = \frac{2\pi}{100} = \pi/50$

$x_2(t) = \cancel{8} \cdot 10 \cos 300t$ $\quad$ $\omega_2 = 300$

$T = \frac{2\pi}{300} = \frac{\pi}{150}$

$x_3(t) = 6 \sin(500t + \pi/4)$ $\quad$ $\omega = 500$

$T = \frac{2\pi}{500} = \pi/25$

$F.T.P = \frac{LCM(\pi, \pi, \pi)}{HCF(50, 150, 250)} = \boxed{\pi/50}$

$T = \cancel{\omega} = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T} = \frac{2\cancel{\pi}}{(\cancel{\pi}/50)} = 100$

PAGE NO.:
DATE: / /

Q) DT signal $x[n] = \sin^2(\pi^2 n)$; n is integer
Periodic with ____:

$\omega = \pi^2$

$T = \frac{2\pi}{\pi^2} = \frac{2}{\pi} \rightarrow$ irrational

~~$t = \pi/2$~~

so non periodic

Note: $x(t)$ CT signal is periodic Period T $\xrightarrow[\text{sampling}]{\text{uniform } (T_s)}$ $x[n]$ D.T signal is this periodic for

if $\frac{T}{T_s}$ = rational then signal is periodic

(a) $\sqrt{2} T_s$
(b) $1.2 T_s$
(c) Always
(d) Never

so option B

Periodicity in 2 domain

- Temporal — Time
- ~~Pastial~~ Spatial — space

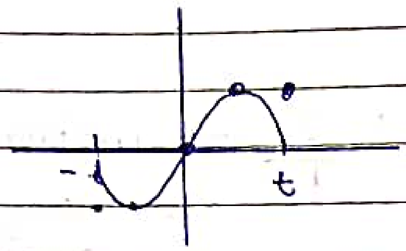
Assignment :

Yellow
Blue
Yellow
Blue

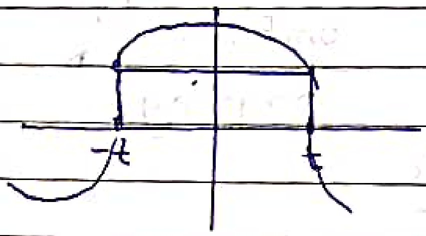
PAGE NO:
DATE:

Odd Signals & Even Signals

odd $x(-t) = -x(t)$ → Signal will pass through origin symmetrically about origin.



even $x(-t) = x(t)$



Symmetric about vertical axis

Any signal can be broken into even part and odd part.

$$x(t) = x_e(t) + x_o(t)$$

$$\left\{ \begin{aligned} x_e(t) &= \frac{x(t) + x_o(-t)}{2} \\ x_o(t) &= \frac{x(t) - x(-t)}{2} \end{aligned} \right\}$$

Power & Energy of Signal

$$E = \lim_{l \to \infty} \int_{-l}^{l} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

DT Sequences

$$\lim_{L\to\infty} \sum_{n=-L}^{L} |x[n]|^2$$

$$= \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Power of the signal

$$P_{avg} = \lim_{L\to\infty} \frac{1}{2L} \int_{-L}^{L} |x(t)|^2 dt$$

For DT signal

$$P_{avg} = \frac{b}{2} \lim_{L\to\infty} \frac{1}{2L+1} \sum_{n=-L}^{n=L} |x[n]|^2$$

If duration of signal in time is finite then it is a finite energy signal.

Energy signals

E → finite

$P_{avg} = 0$

Power Signals → infinite energy

$P_{avg} \neq 0$.

PAGE NO.:
DATE: / /

i) $x(t) \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$

& Energy signal
finite energy
$P_{avg} = 0$

ii) $x[n] = 4$
Power signal
infinite energy
$P_{avg} \neq 0$

$$E = \sum_{-\infty}^{\infty} (4)^2 = \text{Not defined}$$

$$P = \lim_{L \to \infty} \frac{1}{2L+1} \sum_{-L}^{L} (4)^2$$

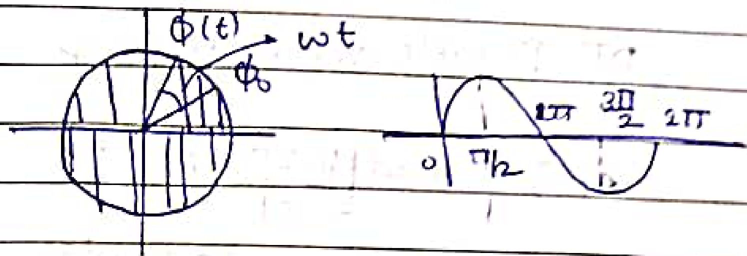
$$= \lim_{L \to \infty} \frac{16\cancel{(2L+1)}}{\cancel{2L+1}}$$

$$\boxed{P_{avg} = 16}$$

{Remember AP, GP, H
formulas}

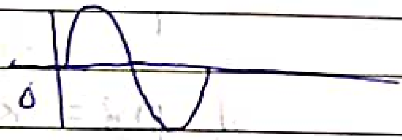
Frequencies

i) CT Frequency 'ω'.
$x(t) = A\cos(\omega t + \phi_0)$
$\omega t + \phi_0 = \text{rad}$
$\omega t = \text{rad}$
$\boxed{\omega = \text{rad/sec}}$



$\omega \rightarrow$ controls speed of rotation in a given time

if $\omega \uparrow$
same height is achieved in less time



2) <u>CT frequency in 'Hz' : 'f'</u>

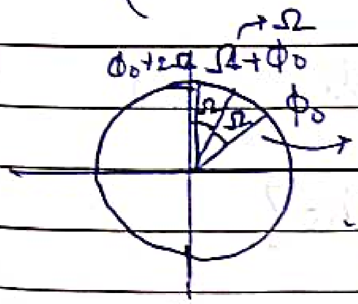
$$x(t) = A \sin(\underbrace{2\pi f t}_{\omega} + \phi)$$

$f \rightarrow$ rate of change wrt time

3) <u>Discrete time frequency (Ω)</u>

$$x[n] = A\cos[\Omega n + \phi_1]$$

($\Omega \rightarrow$ radians)



it may/may reach same position
∴ Such a signal may or may not be periodic.

PAGE NO.
DATE / /

DT Discrete value frequency

$$x[n] = A\cos\left[\frac{k \cdot 2\pi}{N} \cdot n + \phi_0\right]$$

$$(2\pi) \cdot \frac{k(2\pi)}{N} \text{ + radian}$$

$\therefore \{k \to$ frequency $\to$ unitless$\}$

if $\Omega = \frac{k \cdot 2\pi}{N}$ (distance covered in 2 jump)

$$\left[\frac{\Omega}{2\pi} = \frac{k}{N}\right]$$ condition for periodicity

(rational)

eg Consider a signal,

$x[n] = \cos \frac{n}{6}$ is it periodic

$A\cos[\Omega n + \phi_0]$

$A = 1$

$\phi_0 = 0$

$\Omega = 1/6$

$$\frac{\Omega}{2\pi} = \frac{1}{6 \cdot 2\pi} = 1/12\pi$$

irrational

$\therefore$ Aperiodic signal

$\cos \frac{2\pi n}{12}$ periodic?

$$\frac{\Omega}{2\pi} = \frac{2\pi}{2\pi \cdot 12} = \frac{1}{12} \text{ rational}$$

$\therefore$ periodic signal

Youtube

i) $x(t) = A_0$ $\quad x(-t) = x(t)$ even

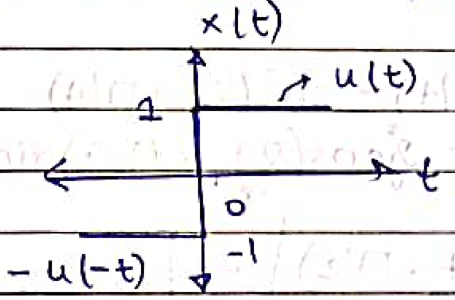
ii) $x(t) = \sin \pi/2$ even

iii) $x(z) = \sin z^2 = x(-z)$ = even

Pts//

- i) even × even = even
- ii) odd × odd = even
- Even × odd = odd
- even ± even = even
- odd ± odd = odd
- even ± odd = NENO//

Q)



even & odd parts of u(t)

u(t) are

$$x(t) = x_o(t) + x_e(t)$$

$$x(t) = u(t) - u(-t)$$

$$\left[\begin{array}{l} \text{even part } u(t) = \dfrac{u(t)+u(-t)}{2} = 1/2 \\ \\ \text{odd part } u(t) = \dfrac{u(t)-u(-t)}{2} = \dfrac{x(t)}{2} \end{array}\right]$$

Oppenheim

1.3) (a) $x_1(t) = e^{-2t}\, u(t)$

$$E = \lim_{l \to \infty} \int_{-l}^{l} |x(t)|^2 \, dt$$

$$E = \lim_{l \to \infty} \int_{-l}^{l} \frac{1}{e^{4|a|t}} |u^2(t)|\, dt \cdot \frac{1}{4} \int \frac{1}{e^{t}} \, u^2(t/4)\, dt$$

$$= 1/4$$

$P = 0$

ii) $x_2(t) = e^{j(2t + \pi/4)}$

$$= \cos(2t + \pi/4) + j \sin(2t + \pi/4)$$

$$E = \int_{-\infty}^{\infty} |x_2(t)|^2 \, dt$$

$$= \lim_{L \to \infty} \int_{-L}^{L} \cos^2(2t + \pi/4) - \sin^2(2t + \pi/4) + 2j\cos(2t + \pi/4)\sin(2t + \pi/4)\, dt$$

$$= \int_{-L}^{L} \left[\frac{1 + \cos(4t + \pi/2)}{2}\right] - \left[\frac{1 - \cos(4t + \pi/2)}{2}\right] - 2j \left[\sin(4t + \pi/2)\right] dt$$

$$= \frac{L}{2} + \frac{L}{2}\cos(4L + \pi/2)$$

$$- \left(-\frac{L}{2} - \frac{L}{2}\cos(4L + \pi/2)\right)$$

$$- \frac{L}{2} + \frac{\cos(4L + \pi/2)}{2}$$

$$+ \int_{-L}^{L} \cos(4t + \pi/2) - j\sin(4t + \pi/2)\, dt$$

$$\int_{-L}^{L} e^{-j(4t + \pi/2)}\, dt = 4 - 4e^{-j(4L + \pi/2)}\Big|_{-L}^{L}$$

$$= 0 \cdot \infty$$

PAGE NO.:
DATE: / /

$$P = \frac{1}{2L+1}\int_{-L}^{L} e^{j(2t+\pi/4)}\,dt \qquad 2t = x,\quad dt = dx/2$$

$$\left(\frac{1}{2L+1}\right)\left[\frac{1}{2}e^{j\left(2t+\frac{\pi}{4}\right)}\right]_{-L}^{L}$$

$$\lim_{L\to\infty}\left(\frac{1}{2L+1}\right)\left[\frac{1}{2}e^{j(2L+\pi/4)} - \frac{1}{2}e^{j(-2L+\pi/4)}\right]$$

$$1/2 \cdot 2 = 1$$

(4) $$\left|e^{j(2t+\pi/4)}\right| = 1$$

$$E_0 \int_{-\infty}^{\infty} |x_2(t)|^2\,dt = \text{N.D} \;\; \infty$$

$$P_\infty = \lim_{L\to\infty}\frac{1}{2L+1}(2L) = 1$$

(iii) $x_3(t) = \cos t$

$$E = \int_{-L}^{L}\{\cos^2(t)\}\,dt$$

$$= \int^{L} \frac{1-\cos 2t}{2}\,dt$$

$$= \left[\frac{L}{2} - \frac{\sin 2L}{2}\right] - \left[\frac{-L}{2} - \frac{\sin 2L}{2}\right]$$

$$P = \left(\frac{1}{2L+1}\right)L = 1/2$$

$$= \infty$$

PAGE NO.:
DATE:

d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$

$$\lim_{L\to\infty} \sum_{n=-L}^{L} \left|\left(\frac{1}{2}\right)^n u[n]\right|^2$$

$$\lim_{L\to\infty} \sum_{n=-L}^{L} \left(\frac{1}{2}\right)^{2n} \quad 1/4$$

$$\sum_{n=1}^{L} \left(\frac{1}{2}\right)^{2n} = (1/4) + (1/4)^2 + (1/4)^3 \cdots \infty$$

$$\frac{a}{1-r} = \frac{1/4}{1-1/4}$$

$$= \frac{4 \cdot 1/4}{3} = 1/3$$

$$\boxed{\sum_{n=-L}^{0} (1/4)^n = (1/4)^0 + (1/4)^{-1} + (1/4)^{-2} \cdots \infty \qquad \frac{1}{1-(1/4)^{-1}} = \frac{1}{1-4} = \frac{-3}{N.P}}$$

$$\sum_{-L}^{L} |x[n]|^2 = 1/3 + 1 = 4/3$$

$$P_\infty = 0$$

e) $x_2[n] = e^{j(\pi/2n + \pi/8)}$

$$|x_2[n]|^2 = 1$$

$$\sum_{-\infty}^{\infty} |x_2[n]|^2 = \infty$$

$$P_\infty = \lim_{L\to\infty} \frac{1}{2L+1} \cdot 2L = 1$$

2) f) $x_3[n] = \cos\left(\frac{\pi n}{4}\right)$

$E = \lim_{L\to\infty} \sum_{n=-L}^{L} |x_3[n]|^2 = \sum \cos^2\left(\frac{\pi}{4} n\right) = \infty$

$P_\infty = \frac{1}{2L+1} \sum_{-L}^{L} \frac{1+\cos(\pi/2 n)}{2}$

$= 1/2$

1.4 (B) (c) $x[n-3]$ $\quad x[n] = 0 \begin{cases} n < -2 \\ n > 4 \end{cases}$

$n < -2$

$n-3 < -2-3$ $\quad x[n] \neq 0$ if $n \in [-2, 4]$

$n-3 < -5$

$n > 4$

$n-3 > 1$

$x[n-3] = 0$ if $\{n < -5 \ \& \ n > 1\}$

(d) $x[-n+2]$

$n < -2$

$-n > 2$

$-n+2 > 4$

$-n < -4$

$-n+2 < -2$

$x[-n+2] = 0 \quad n > 4 \ \& \ n < -2$

1.5) $x(t) = 0 \quad t < 3$

$x(1-t) + x(2-t)$

$-t > -3$	$-t > -3$
$1-t > -2$	$2-t > -1$

$x(1-t) + x(2-t) = 0 \quad t \in [(-1, \infty$

1.6) a) $x_1(t) = 2e^{j(t+\pi/4)}u(t)$

$x_1(-t)$

$= \cancel{2e^{j(-t+\pi/4)}}\, \underline{u(-t)}$

$2\left[\cancel{\cos(-t+\pi/4) + \sin(-t+\pi/4)}\right] \cancel{u(t)}\ u(-t)$

$2\pi - \pi/4$

b)

$u(t+2\pi)$

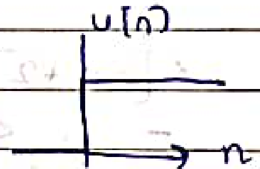
$x_1(t) = 2\left[\cos(t+\pi/4) + 2j\sin(t+\pi/4)\right]u(t)$

$x_1(t + 2\pi) = u(t+2\pi) \neq u(t)$

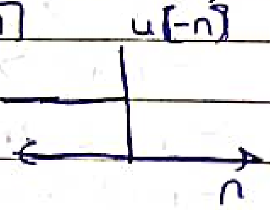
so ~~not~~ aperiodic

★ unit step is aperiodic function

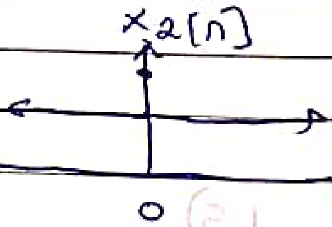
u[n]



u[-n]



$x_2[n]$ u[n] + u[-n] =
periodic $(n \neq 0)$



PAGE NO.:
DATE: / /

1.7) $x_1[n] = u[n] - u[n-4]$

$x_1[-n] = u[-n] - u[-n-4]$

even part $= \dfrac{x_1[n] + x_1[-n]}{2}$

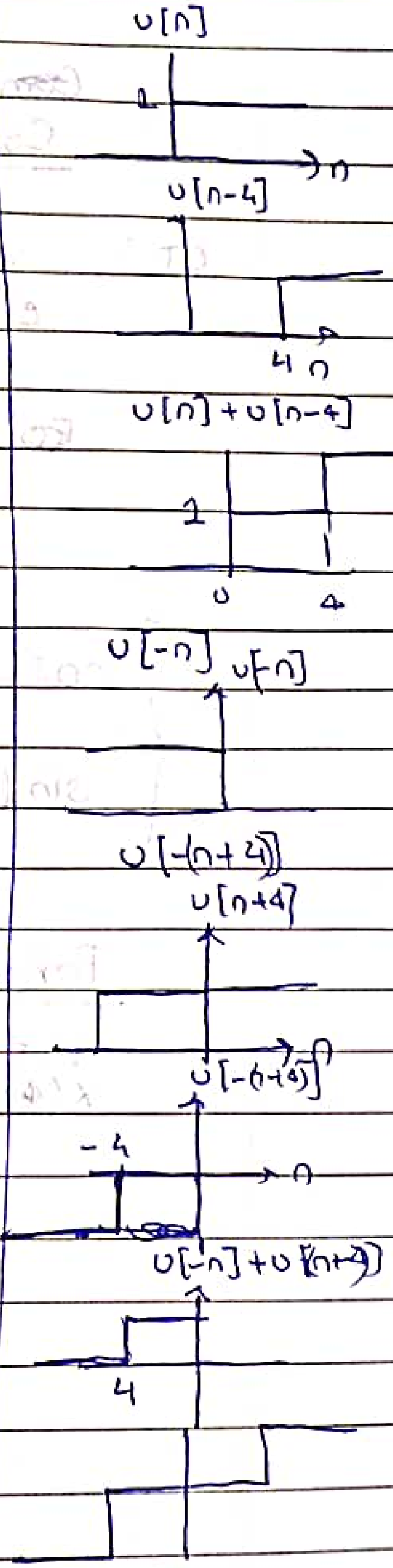
$\neq 0$

$x_2(t) = \sin(1/2t)$

$x_2(-t) = -\sin(t/2)$

$x_2(t) + x_2(-t) = 0$

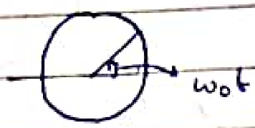
$\therefore$ even part $= 0$



~~Com~~ Complex Exponential signals

CT :

$e^{j\omega_0 t}$



~~Ex~~ Essence of -ve fixed

+ve w_0 $\longrightarrow$ phasor rotate anti-clockwi

-ve w_0 $\longrightarrow$ phasor rotates clockwise

$$\left\{ \begin{aligned} \cos(\omega_0 t) &= \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \\ \sin(\omega_0 t) &= \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \end{aligned} \right\}$$

Periodicity of complex exponentials

$x(t+T) = e^{j\omega_0(t+T)}$

$= e^{j\omega_0(t)} \cdot e^{j\omega_0 T}$

$= x(t)$ at $T = 0, 2n\pi$

$f = \frac{\omega_0}{2\pi}$ $\quad$ $\omega_0 t = 2\pi$

~~$e^{j\omega_0 T} = e^0$~~

~~$\omega_0 T = 0$~~

$\frac{2\pi}{\omega_0}$

$\cos(\omega$ $+ j\sin$ 2π

All complex exponential (C.T) are periodic period, $2\pi/\omega_0$ for all values of ω

(periodic in time)

PAGE NO:
DATE: / /

DT $x[n] = e^{j\Omega_0 n}$
$= \cos(\Omega_0 n) + j\sin(\Omega_0 n)$
~~cos(Ω₀n + Ω₀ · 2πn/Ω₀)~~

Periodicity

$e^{j\Omega_0 n} = e^{j\Omega_0(n+N)}$
$= e^{j\Omega_0 n} \cdot e^{j\Omega_0 N}$

$j\Omega_0 N = 0$

$N = \frac{1}{\Omega_0} = \frac{\Omega}{2\pi}$

$$\left[N = \left(\frac{2\pi k}{\Omega_0}\right) \right] \qquad \left[\Omega_0 = \frac{2\pi}{N} \cdot k \right]$$

$N \rightarrow$ integer, $k \rightarrow$ integer

$e^{j(\omega_0+\omega)t} = e^{j\omega_0 t}$

~~cos(ω₀+ω)(t)~~

$\cancel{e^{j\omega_0 t}} \cdot e^{j\omega t} = \cancel{e^{j\omega_0 t}}$ $\qquad \frac{2\pi}{T} = 2\pi n \pm 0$

$1/T = n$

$e^{j\omega t} = 1$ $\qquad$ {no for $e^{1/T}$}

$\cos(\omega t) + j\sin(\omega t) = 1 + 0j$

$\omega t = 2n\pi$

$\frac{2\pi}{T}$ $\{t\} = \frac{2n\pi}{\omega}$ → As ω changes new sinosoid is generated (~~contin~~ periodic in time) not periodic in oscillation

$e^{j(\Omega_0+\Omega)n}$ $\quad \Omega = \frac{2\pi}{n}$

n → irrespective

$$\cancel{e^{j\Omega_0 n}} \cdot e^{j2\pi kn} = \cancel{e^{j\Omega n}}$$

$$e^{j2\pi kn} = 1$$

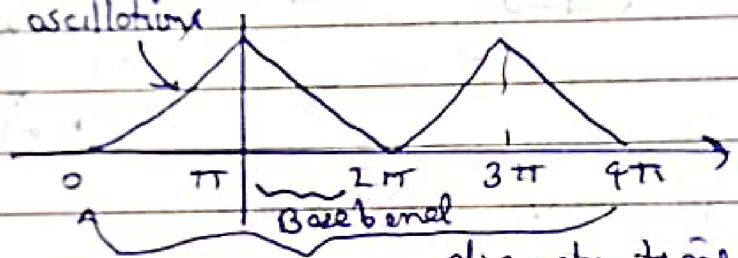
$$\cos(2\pi kn) + j\sin(2\pi kn) = 1 + 0j$$

$$2\pi kn = 2N\pi \pm 0$$

$$n = \frac{2N\pi}{2\pi k} = \frac{N}{k} \Big\}\text{ always an integer}$$

$\therefore$ Periodic

If Ω_0 is changed by 2π
Same signal will be achieved



* Signal / System analysis should be done b/w
  $0 - \pi$ or $\pi - 2\pi$. (Symmetric nature)

$e^{j\omega_0 t}$ → is always periodic $\forall\ \omega_0$
→ rate of oscillation will grow if ω_0 increases
(continious frequency)

$e^{j\Omega_0 n}$ → is not always periodic (only periodic at $\frac{\Omega_0}{2\pi} = k$)
→ Rate of oscillation ~~only~~ grows only from 0 to π
& reduces symmetrically b/w π to 2π
Repeats itself after $= \frac{2\pi k}{\Omega_0}$.

PAGE NO.:
DATE: / /

eg// $x(t) = \cos\left(\frac{8\pi t}{31}\right)$

$x[n] = \cos\left(\frac{8\pi n}{31}\right)$

find whether signals are periodic or not?

$\cos\left[\frac{8\pi}{31}(t+T)\right]$

$\cos\left[\frac{8\pi t}{31} + \frac{8\pi T}{31}\right]$

~~sin(x+T) = sin x~~

$\frac{\cancel{8}^{4}\cancel{\pi} T}{31} = \cancel{2}\cancel{\pi} k$

$\left\{T = 31k/4\right\}$

$\cos\left(\frac{8\pi(n+N)}{31}\right) = \cos\left(\frac{8\pi n}{31} + \frac{8\pi N}{31}\right)$

$\frac{\cancel{8}^{4}\cancel{\pi} N}{31} = \cancel{2}\cancel{\pi} k$

$N = \frac{31k}{4} \rightarrow 1$

$\left\{N = 31k/4\right\}$

$= \frac{31k}{4}$

$\boxed{N = 31}$

$\therefore$ Both are periodic

PAGE NO.:
DATE: / /

Oppenheim

1.7) a) $x_1(t) = je^{j10t}$

$j\cos(10t) - \sin(10t)$

$\cancel{x(t+2\pi) = j\cos}$

$10 = \frac{2\pi}{T}$

$T = \pi/5$

$x(t+\pi/5) = j\cos(10t+2\pi) - \sin(10t+2\pi)$

$= x(t)$

Periodic

b) $x_2(t) = e^{(-1+j)t}$

$= e^{-t} \cdot e^{jt}$ → periodic

non periodic

non periodic

c) $x_3[n] = e^{j7\pi n}$

$x_3 = \cos(7\pi n) + j\sin(7\pi n)$

$7\pi = \frac{2\pi k}{N}$

$N = (2/7)k$

$k = 7$

$N = 2$

$\cos(7\pi(n+2)) + j\sin(7\pi(n+2))$

$= x(t)$

Periodic

d) $x_4[n] = 3e^{j3\pi(n+1/2)/5}$

$$= \cos\left(\frac{3\pi(n+1/2)}{5}\right) + j\sin\left(\frac{3\pi(n+1/2)}{5}\right)$$

$$= \cos\left(\frac{3\pi n}{5} + \frac{3\pi}{10}\right) + j\sin\left(\frac{3\pi n}{5} + \frac{3\pi}{10}\right)$$

$$\Omega = \frac{3\pi}{5}$$

$$\Omega = \frac{2\pi k}{N}$$

$$\frac{3\pi}{5} = \frac{2\pi k}{N}$$

$$N = \frac{2}{3} \times 5k$$

$$= \frac{10k}{3} \qquad k=3$$

$$\boxed{N = 10}$$

Periodic

1.10) $x(t) = 2\cos(10t+1) - \sin(4t-1)$

$$x_1(t) = 2\cos(10t+1)$$

$$\omega = 10$$

$$\frac{2\pi}{T} = 10 \qquad T_1 = \frac{2\pi}{10} = \pi/5$$

$$x_2(t) = \sin(4t-1)$$

$$\omega = 4 = \frac{2\pi}{T_2} \quad \therefore \quad T_2 = \frac{\pi}{2}$$

$$T_1/T_2 = \frac{\pi/5}{\pi/2} = 2/5 \quad \therefore \text{Periodic}$$

$T_1 = \pi/5$
$T_2 = \pi/2$

$\text{LCM}(T_1, T_2)$

$= \dfrac{\text{LCM}(\pi, \pi)}{\text{HCF}(5, 2)}$

$= \pi/1 = \pi$

1.11) $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$

$x_1[n] = \cos\left[\dfrac{4\pi n}{7}\right] + j\sin\left[\dfrac{4\pi n}{7}\right]$

$\Omega = \dfrac{4\pi}{7} = \dfrac{2\pi k}{N}$
$\ 2$

$N_1 = \dfrac{7}{2} k_1 \qquad = 7$

$k_1 = \{2, 4, 6 \ldots\}$

$x_2[n] = \cos\left[\dfrac{2\pi n}{5}\right] + j\sin\left(\dfrac{2\pi n}{5}\right)$

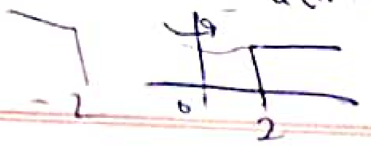
$\Omega = \dfrac{2\pi}{5} = \dfrac{2\pi k_2}{N}$

$N_2 = 5$

$\text{LCM}(N_1, N_2) = \dfrac{\text{LCM}(7, 5)}{\text{HCF}(1, 1)}$

$= 35$

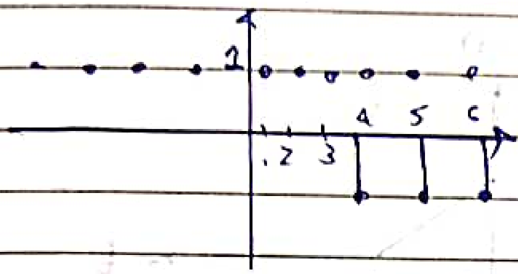
$u(n-2)$ $\quad$ $u(-(n-2))$



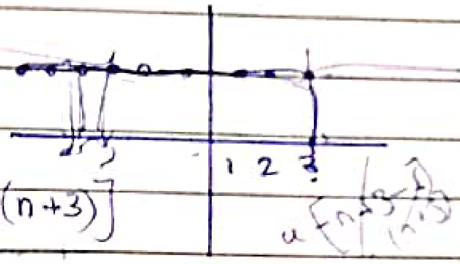
1.12) $x[n] = 1 - \sum_{k=3}^{\infty} \delta[n-1-k]$

$\cancel{x[n]} =$

$= 1 - \left(\delta[n-4] + \delta[n-5] + \delta[n-6] \dots \right)$



$=$



$u[-(n+3)]$

$x[n] = u[-n+3]$
$= u[-(n-3)]$

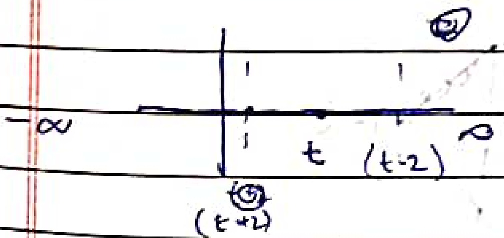
$x[n] = u[Mn - n_0]$

$$\left\{ \begin{array}{l} M = -1 \\ n_0 = 3 \end{array} \right\}$$

1.13) $x(t) = \delta(t+2) - \delta(t-2)$

$\cancel{E_\infty}$ $\quad \int_{-\infty}^{t} 1 \cancel{x(t)^3} dt$

$$y(t) = \int_{-\infty}^{t} \delta(t+2) - \delta(t-2)\, dt$$



$$= \int_{-\infty}^{t} \delta(t+2)\, dt - \cancel{\int_{-\infty}^{t} \delta(t-2)\, dt} \to 0$$

$t+2 = x$
$dt = dx$

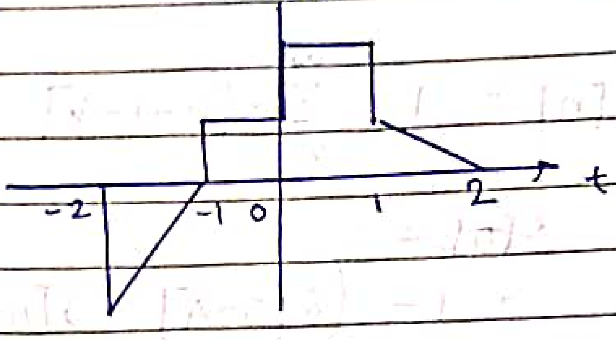
$$\int_{-\infty}^{x-2} \delta(x)\, dx - \cancel{\int_{-\infty}^{t} \delta(x_2)\, dx_2} \to 0$$

$y = 1$

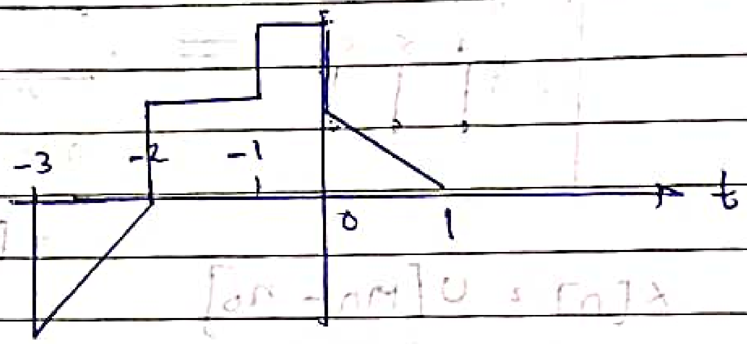
$$E_\infty = \int^{\infty} |y|^2 dt = \infty$$

1.21)

$x(t)$

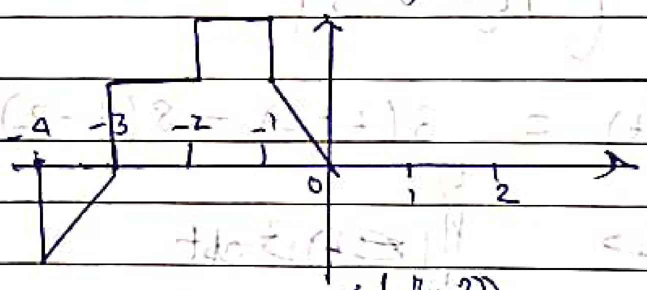


$x(t-1)$

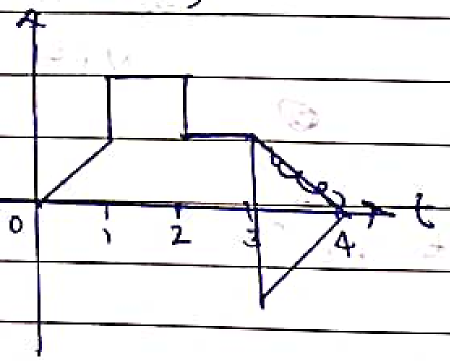


$x(2-t) = x(-(t-2))$

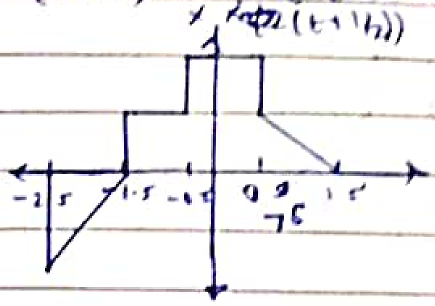
$x(t-2)$

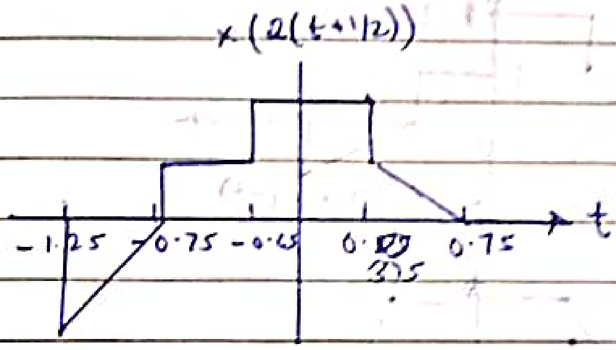


$x(-(t-2))$

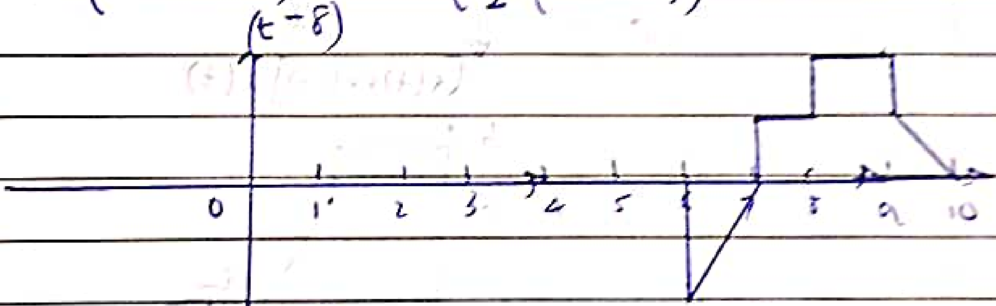


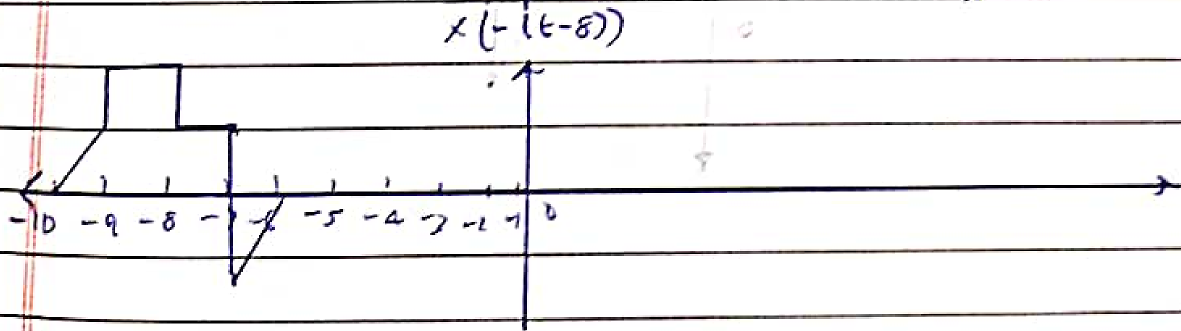
$x(2t+1) = x(2(t+1/2))$

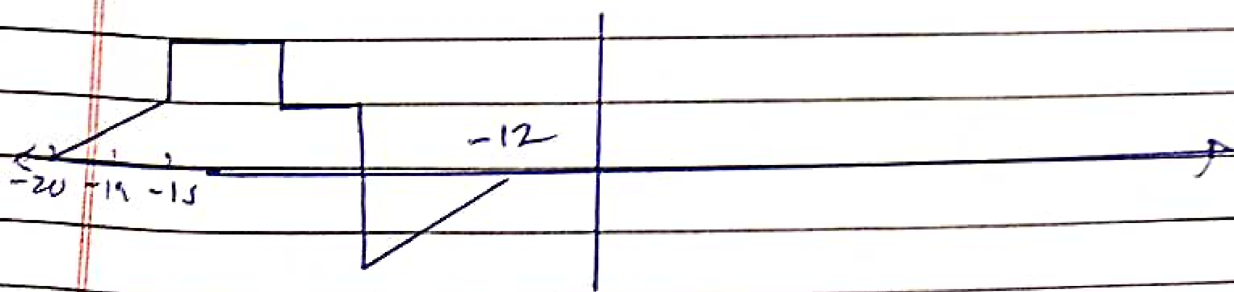




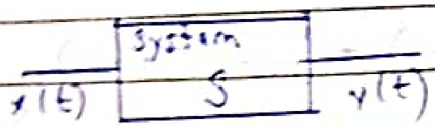
$x(4 - t/2) = x\left(\frac{-1}{2}(t-8)\right)$



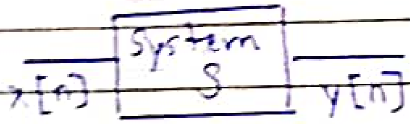




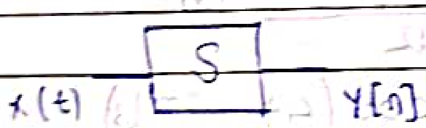
Systems :



C.T System

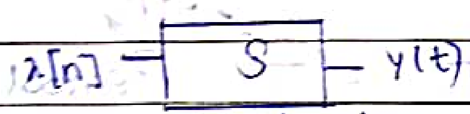


DT System



Hybrid system

eg : Sampler, ADC

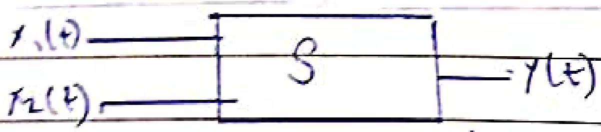


Hybrid system

eg DAC

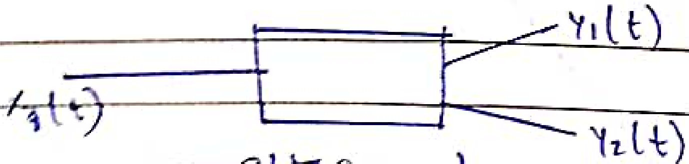
SISO

single inp
single o



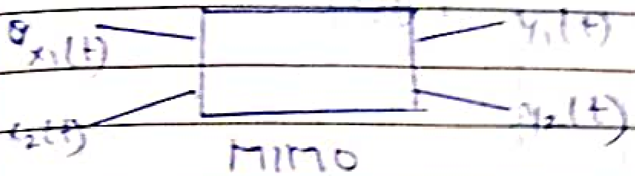
MISO system

multi input
single output

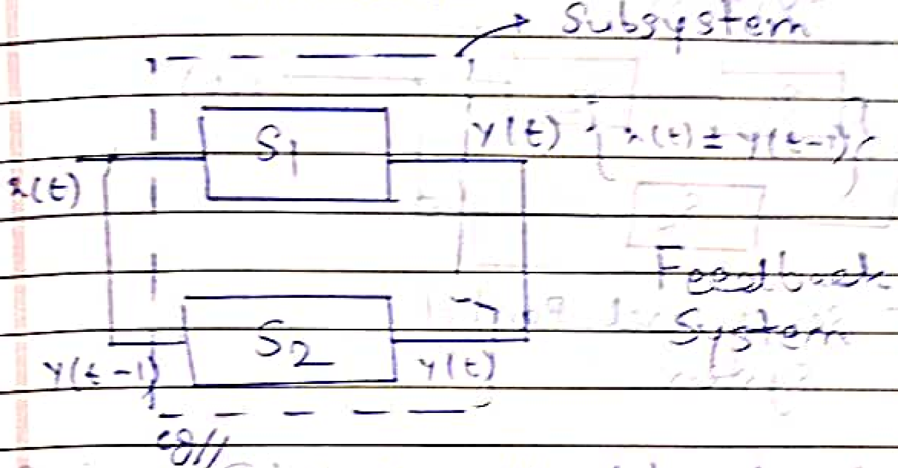


SIMO system

single input
multi output

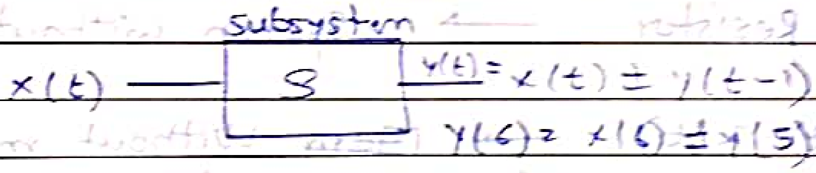


MIMO system
multi-input
multi-output

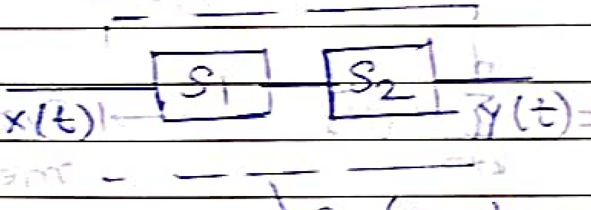


Feedback System

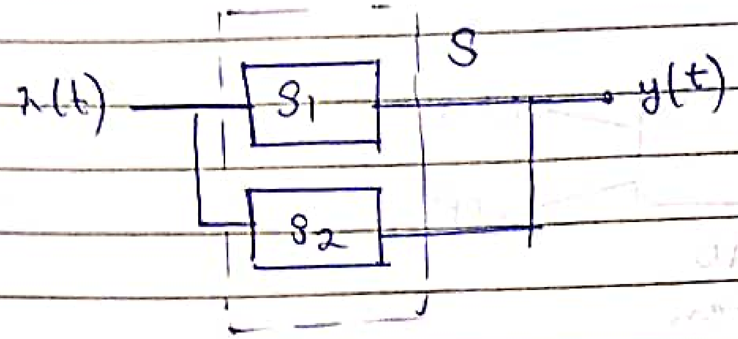
If S_2 is unit delay system



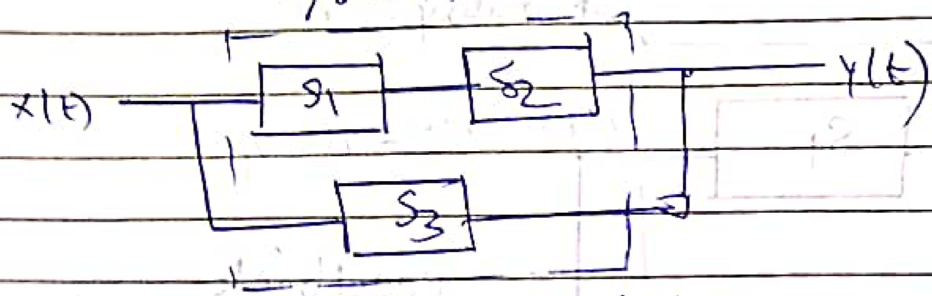
Feedback system is used for stability but some feedback systems are unstable and also useful.



S (systems in cascade)
Series connections



Porallel system



Hybrid - Series and Parallel system

☑ (Dynamic Syst

Capacitor & Inductor $\longrightarrow$ Systems with memory

Resistor $\longrightarrow$ System without memory

$y(t) = x(t)$ $\longrightarrow$ ~~Item~~ Without memory

$y(t) = x(t^2)$ $\longrightarrow$ with memory t

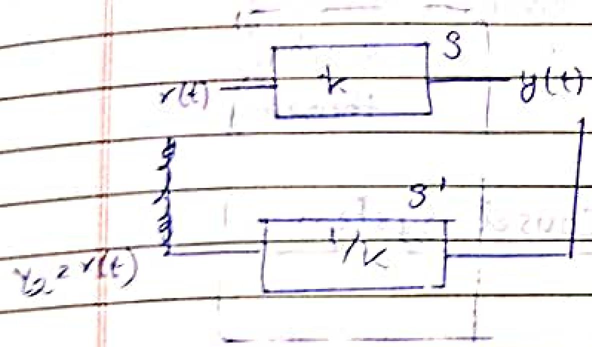
$\frac{0.2}{d^2}$ $\frac{0.04}{4}$ $\underbrace{\text{without memory}}_{\text{Preedictor System}}$ t

Eg//

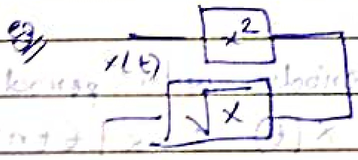
$$y[n] = \sum_{k=0}^{n} x[k] \longrightarrow \text{System with memory}$$

$$y[n] = \left[x[n] - x^2[n]\right]^{1/2} \longrightarrow \text{without memory}$$

Invertible & Non-invertible System



For a system exists another system which when cascaded with the first one resulting is unit response is called invertible system.



Non-invertible system

If S is inverse of S' then cascade of S and S' form unity system.

(i) $y[n] = \sum_{n=-\infty}^{N} x[n]$

$y[n] = \ldots x[0] + x[1] + x[2] \ldots x[n]$

$y[n-1] = \ldots x[0] + x[1] \ldots x[n-1]$

$\{ y[n] - y[n-1] = x[n]$

so invertible function

(ii) $y(t) = x(t) \rightarrow$ invertible system

Causal and Non Causal Systems

Causal → Non anticipating, non-~~precied~~ predicting

Non causal → Anticipating

eg// $y(t) = \exp(x(t) - x(t-1))$

eg $y(t) = x(t+2)$ { $y(2)$ depends on $x(4)$ }

$$y(t) = \int_{-\infty}^{t} x(t)\,dt$$

$$y[n] = \sum_{k=1}^{M} x[k]$$

{ overall non causal
$M > n$ non causal
$M \le n$ causal }

Causality is must for realisability

Stable or Unstable System

↓

BIBO — bounded input bounded output

Stability — $y(t)$ should not grow unboundedly for bounded $x(t)$.

bounded → Finite duration, finite energy

$$y[n] = \sum_{k=-\infty}^{M} x[k]$$

{ If $|x[k]| < \infty$? Then it is bounded system }

(i) $y(t) = e^{t}$ → No input

$x(t)$

$= e$ → input

so unstable

{Stability is only dependent on $x(t)$}

Useful unstable system → Oscillator

Stable system cannot generate output periodic in time. So they can't be periodic system.

Time invariant / Time variant

TI TV

Time invariance

$x(t) \longrightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$

{time shift invariance}

{Change in output = change in input for time shift}

$x(t) \to y(t)$

$x(t-t_0) \to y(t)$

time variant

{A system is time invariant if shift is invariant}

$y(t) = \sin x(t)$

$y(t-t_0) = \sin x(t-t_0)$

} TI

$y(t) = t \sin x(t)$

$y(t-t_0) = t \sin x(t-t_0)$

} T.V

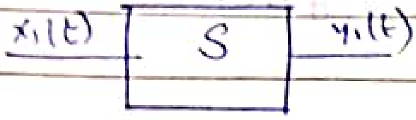
Q) $y[n] = n\,x[n] \rightarrow$ T.V

Linear and Non-linear System

Linearity $\Rightarrow$ Superposition $x_1 + x_2 \rightarrow y_1 + y_2$

+

Homogenity $\alpha x_1 \rightarrow \alpha y_1$ if $x_1 \rightarrow y_1$



$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

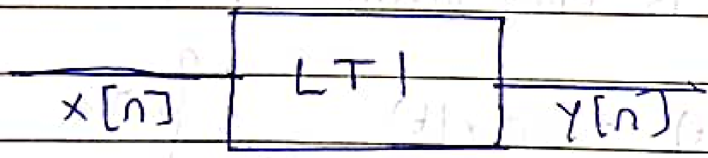
$a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)$

$y(t) = m x(t) + c$ → Non linear

$\left\{ \begin{matrix} x = 0 \\ y = c \end{matrix} \right\}$ → so non linear

{although it is eqn of line}

Q) Linear Time Invarient system (LTI)



LTI systems can be characterized by imp-

response.

↓

resp-

{ of system w-

input is -

impul-

$h[n] = y[n]$ when $x[n] = \delta[n]$

$$\delta[n] \longrightarrow \boxed{\text{LTI}} \longrightarrow h[n]$$

unique to system

$$\delta[n-k] \longrightarrow h[n-k] \rightarrow \text{TI}$$

$$x[k]\delta[n-k] \longrightarrow x[k]\cdot h[n-k] \rightarrow \text{Homogenity}$$

$$\underbrace{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]}_{x[n]} = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \rightarrow x[n]$$

Superimposition

$$x[n]\cdot\delta[n-k] = x[k]$$

$$\left\{ x[n] \longrightarrow \sum_{k=-\infty}^{\infty} x[k]h[n-k] \right\}$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Convolution of input

representation

$$y[n] = x[n] * h[n]$$

$y[n] = h[n] * x[n]$ Convolution is commutative

$(x_1+x_2) \rightarrow \boxed{h} \rightarrow h*[x_1+x_2]$

$= h*x_1 + h*x_2$

Distributive

[illegible]

$y[n] = x[n] * h[n]$

$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$x[n] = \{1, 2, -2, 0\}$

[illegible]

[illegible]

$x[0] \quad x[1] \quad x[2]$

$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

[illegible]

eg

1) $y[-5] = \sum_{k=0}^{2} x[k]h[n-k]$ $\quad (n \to -5)$

$= x[0]h[-5] + x[1]h[-6] + x[2]h[-7]$

$\underbrace{h[-5],\ h[-6],\ h[-7]}$

does not exists

so $x[-5]$ ~~does no~~ $= 0$

2) $y[-2] = \sum_{k=0}^{2} x[k]h[n-k]$ $\quad (n \to -2)$

$x[0]h[-2] + x[1]h[-3] + x[2]h[-4]$

$(-1)(1) + 0 + 0$

$= -1$

3) $y[-1] = x[0]h[-1] + x[1]h[-2] + x[3]h[-3]$

$= (-1)(2) + 0 + 0 + 0$

$= -2$

4) $y[0] = x[0]h[0] + x[1]h[-1] + x[2]h[-2]$

$= 2 + 0 + 2 = 4$

5) $y[1] = x[0]h[0] + x[1]h[-1] + x[2]h[-2]$

$= 2 + 0 + 2$

$= 4$

PAGE NO.
DATE / /

position

width of sequence = $\underbrace{RH - LH + 1}_{\text{position}}$ $W_x = 4$

$W_h = 3$

$\{W_y = W_h + W_x - 1\}$

$\{W_y = 6\}$

Y signal starts at starting position of
(s.p of x + s.p of h) = (0 + (-2))
= -2

(e.p of x + e.p of h) = (1 + 2)
= 3

eg

$x_1(t) = \{-1, 1, 0, 2, -3\}$ ↑ given
(positions −2, −1, 0, 1, 2)

$h = x_2(t) = \{-1, 0\}$
(positions 0, 1)

$y \in (-2, 3) \rightarrow$ exists

$y(-2) \rightarrow y(-3)$ exists $n-k$

$y(-1)$
$= 1$

$y(-2) = x(0)h(-2) + x(1)h(-3) + x(2)h(-4)$
$= 0 + 2(0) + 0 + 0.\ 1 + 0 = 1$
$= 0$

here $\{x = x_1, x_2 = h\}$

$y(-1) = x(0)h(-1) + x(1)h(-2) + x(2)h(-3)$
$= 0 + 2(0) + 0 + 0 = 0$

$= 0 + 2($

PAGE NO :
DATE / /

$y(-2) = 1$
$y(-1) = -1$
$y(0) = 0$
$y(1) = -2$
$y(2) = 3$
$y(3) = 0$

{Trick for convolution}

$h[n] = \{1, 2, -2, 0\}$
$x[n] = \{-1, 0, 2\}$

$x[n]$ \ $h[n]$	1	2	-2	0
-1	-1	-2	2	0
0	0	0	0	0
2	2	4	-4	0

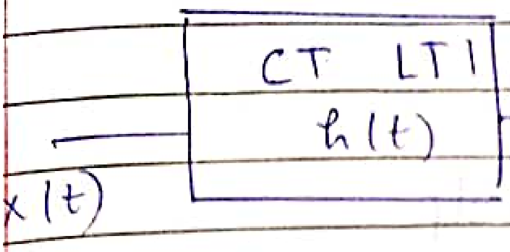
this entry is starting point

$\{-1, -2, 4, -4, -4, 0\}$

$x_2[n]$ \ $x_1[n]$	-1	1	0	2	3
1					
0	0	0	0	0	0
1	-1	1	0	2	3

x_2 \ x_1	-1	1	$\underset{\downarrow}{0}$	2	3
→ -1	1	-1	0	-2	3
0	0	0	0	0	0

$$y(n) = \{1, -1, \underset{\uparrow}{0}, -2, 3\}, 0\}$$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= x(t) * h(t)$$

$$= h(t) * x(t)$$

eg) $f_1(t) = e^{-t^2}$

$f_2(t) = 3t^2$

$\tau^2 = p$

$2\tau\, d\tau = dp$

$2\sqrt{p}\, d\tau = dp$

$d\tau = 2p^{-1/2} dp$

$$f(t) = f_1(t) * f_2(t)$$

$$= \int_{-\infty}^{\infty} e^{-\tau^2} \cdot 3(t-\tau)^2\, d\tau$$

$$3\int_{-\infty}^{\infty} e^{-p}\, 2p^{-1/2}\left(1 - \frac{\sqrt{p}}{t}\right) dp$$

$\tau/t = p$

$d\tau = t\, dp$

$$= 3\int_{-\infty}^{\infty} e^{-\tau^2}(t-\tau)^2\, d\tau$$

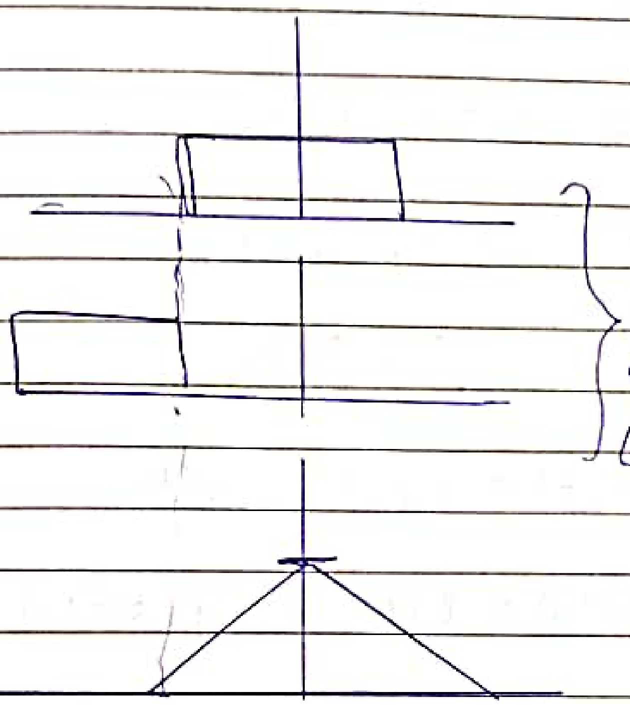
$$= 3t^2 \int_{-\infty}^{\infty} e^{-\tau^2}\left(1 - \frac{\tau}{t}\right)^2$$

$$\int_{-\infty}^{\infty} 3\tau^2\, e^{-(t-\tau)^2}\, d\tau$$

$$= 5.3t^2 + 2.659$$

$= \cancel{3}t^3\ \cancel{e^{-\tau^2}}$

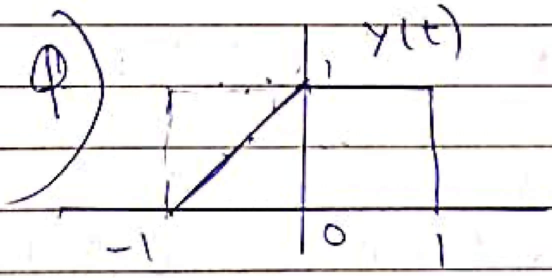
(1) $x(t) \longrightarrow \boxed{\begin{array}{c}\text{CT LTI}\\ h(t)\end{array}} \longrightarrow y(t)$

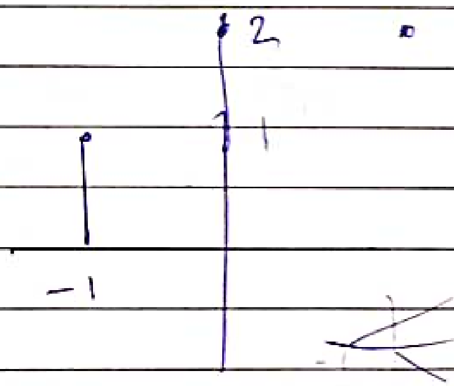


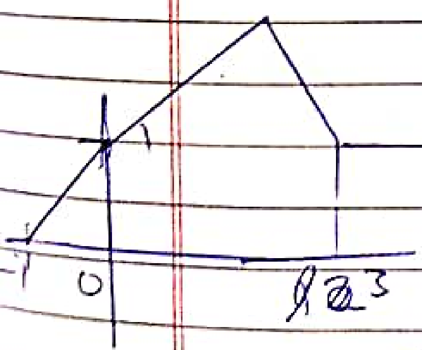
Add

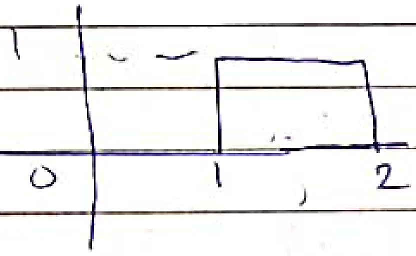
① keep one signal fixed

② move other from $-\infty$ to $+\infty$

(1) 







$x(t) \begin{cases} t \\ \end{cases}$

$y(t) = r(t+1) - r(t) + u(t) - u(t-1)$

$$y(t) \begin{cases} t & -1 \le t \le 0 \\ & t\,[t-1-t+2] \\ u(t) - u(t-1) & \text{else} \end{cases}$$

$h(t) = u(t-1) - u(t-2)$

$x(t) * \delta(t-1) = x(t-1)$

$(t-1)t$

$(t-2)t$

$\int y(t) * h(t) = \int t * u(t-1)\,dt - \int t * u(t-2)\,dt$

Properties

1) $x_1 * x_2 = x_2 * x_1$

$x_1 * [x_2 + x_3] = x_1 * x_2 + x_2 * x_3$

$x_1 * [x_2 * x_3] = x_2 * x_3 * x_2 * x_1$

$= x_3 * x_2 * x_1$

Shift CT If $x(t) * h(t) = y(t)$

$x(t-t_0) * h(t) = y(t-t_0)$

and

$x(t-t_1) * h(t-t_0) = y(t-t_1-t_2)$

DT If $x[n] * h[n] = y[n]$

then

$x[n-n_0] * h[n] = y[n-n_0]$

$x[n-n_1] * h[n-n_2] = y[n-n_1-n_2]$

Signal convolution with impulse is signal itself, $\Rightarrow$ ~~x(t)~~ $\delta(t) * x(t) = x(t)$

$$\Rightarrow \int_{-\infty}^{\infty} \delta(\tau)\, x(t-\tau)\, d\tau = x(t)$$

PAGE NO.:
DATE: / /

CT//

$x(t) \rightarrow$ LTI $\rightarrow y(t)$

$A_x \qquad A_h \qquad \{A_y = A_x \cdot A_h\}$

DT//

$x[n] \qquad h[n] \qquad y[n]$

$S_x \qquad S_h \qquad \{S_y = S_x \cdot S_h\}$

Diffrentiation

$$x(t) * h(t) = y(t)$$

$$\left\{\frac{d}{dt}x(t) * h(t)\right\} = \left\{x(t) * \frac{dh(t)}{dt}\right\}$$

$$= \frac{d}{dt}y(t)$$

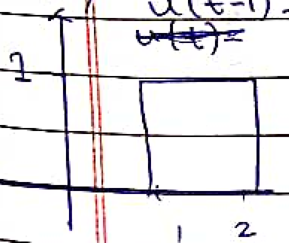
Time Scaling

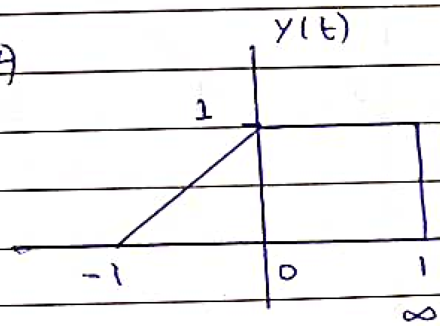
If $x(t) * h(t) = y(t)$

then

$$x(at) * h(at) = \frac{1}{|a|}y(at)$$

$x(t) = u(t-1) - u(t-2)$ ~~u(t)=~~





$$y(t) = \begin{cases} t & -1 \le t \le 0 \\ 1 & 0 \le t \le 1 \end{cases}$$

$u(t) - u(t-1)$

$$y(t) * x(t) = \int_{-\infty}^{\infty} y(t) * \frac{d}{dt}x(t)\, dt$$

$$\int_{-\infty}^{\infty} y(t) * \frac{d}{dt}[u(t-1) - u(t-2)]\, dt$$

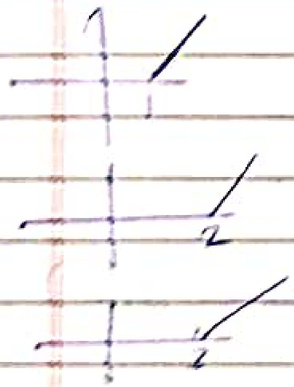
$$= \int_{-\infty}^{\infty} y(t) * [\delta(t-1) - \delta(t-2)]\, dt$$

$$\int_{-\infty}^{\infty} [y(t) * \delta(t-1) - y(t)\,\delta(t-2)]\, dt$$

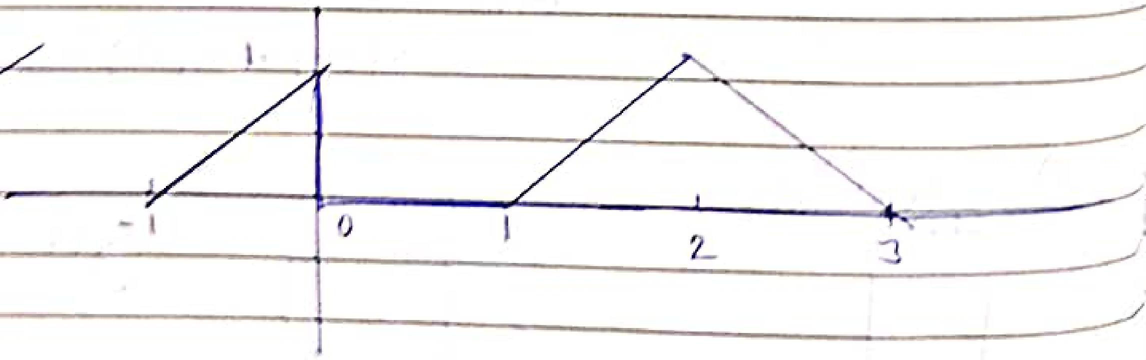
$$\int_{-\infty}^{\infty} y(t-1) - y(t-2)\, dt$$

$$\int_{-1}^{0} [y(t-1) - y(t-2)]\, dt + \int_{0}^{1} y(t-1) - y(t-2)$$

$$\int_{-1}^{0} t[(t-1) - (t-2)]\, dt + \int_{0}^{1} \begin{matrix} u(t-1) - u(t-2) \\ - u(t-2) + u(t-3) \end{matrix}$$



$$t\Big|_{-1}^{0} + [r(t-1) - 2r(t-2) + r(t-3)]_0^1$$



Youtube

Convolution

* only applicable to LTI system
* Convolution produces new signal

$$x(t) \longrightarrow \underset{}{\overset{(h(t))}{\boxed{LTI}}} \longrightarrow y(t)$$

<u>CT</u> $\quad y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$y(\omega) = x(\omega)\, h(\omega) \quad \{\text{freq. domain}\}$$

<u>D.T</u>

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} \cancel{\phi}\, [\cancel{\phi}]\, \cancel{\theta}[n-k]$$

$$= \sum_{k=-\infty}^{\infty} x[n]\, h[n-k]$$

Properties

<u>Comm</u> $\quad x_1(t) * x_2(t) = x_2(t) * x_1(t)$

<u>Dist.</u> $\quad x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$

<u>Associ</u> $\quad x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$

<u>Shift</u> $\quad x_1(t) * x_2(t) = z(t)$

$$x_1(t) * x_2(t-t_1) = z(t-t_1)$$

$$x(t-t_1) * x_2(t) = z(t-t_1)$$

$$x_1(t-t_1) * x_2(t-t_2) = z(t-t_1-t_2)$$

Results $x_1(t) * \delta(t) = x(t)$
$x(t) * \delta(t-t_1) = x(t-t_1)$

eg $x(t) * \delta(t-2)$
$= x(t-2)$

$\delta(at) = \frac{1}{|a|}\delta(t)$

$x(t) * h(t) = y(t)$

- $\frac{dy}{dt} = \frac{d(x(t))}{dt} * h(t) = x(t) * \frac{dh}{dt}$

- $u(t) * u(t) = r(t)$
$u(t-t_1) * u(t-t_2) = r(t-t_1-t_2)$
$y(-t) = x(-t) * h(-t)$

$y(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$

$x(\alpha t) * h(\alpha t) = \int_{-\infty}^{\infty} x(\alpha\tau)\,h(\alpha(t-\tau))\,d\tau$

$\alpha\tau = p$
$d\tau = \frac{dp}{\alpha}$

$\frac{1}{\alpha}\int_{-\infty}^{\infty} x(p)\,h(\alpha t - p)\,dp$

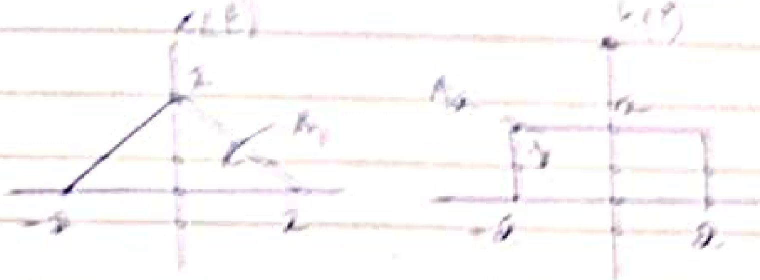
$\frac{1}{\alpha}\,y(\alpha t)$

Area

Input signal has A_x area & system $h(t)$ has area A_h

Area of convoluted signal $= A_x \cdot A_h$

Eg

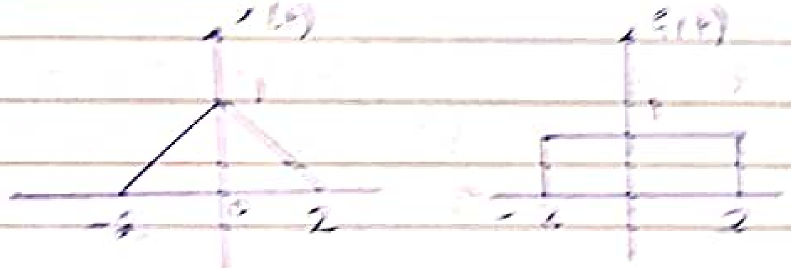


$A_x = \frac{1}{2} \times 4 \times 2 = 4$

$A_h = 2 \times 4 = 8$

$A_c = 32$

Limits of convolution



$z(t) = x(t) * g(t)$

$\left\{ \begin{array}{l} \text{lower limit} = -2-1 = -3 \\ \text{upper limit} = 3+2 = 5 \end{array} \right\}$

Slide & Shift method

$x(t) = t^2 + 2t + 1$

$g(t) = t^2 + 2t + 4$

			t^2	$2t$	1	
4	$2t$	t^2				
	4	$2t$	t^2			$= t^4$
		4	$2t$	t^2		$= 2t^3 + 2t^3 = 5t^3$
			4	$2t$	t^2	$= 4t^2 + 5t^2 + t^2 = 11t^2$
				4	$2t$	$= 8t + 2t = 11t$
					4	$= 4$

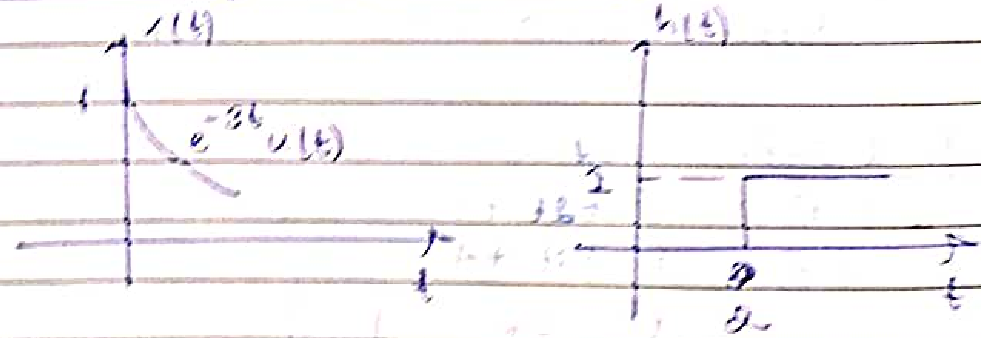
$x(t) = 3t^3 + t + 1$

$h(t) = t^2 + t + 2$

	1			$3t^3$	0	t	1	
t^2	·	t	2					
	t^2	t		2				$6t^3$
			$0t^2$	t	2			$3t^4$
				t^2	t	2		
					t^2	t	2	
						t^2	t	
							t^2	

			$3t^3$	0	t	1	
2	t	t^2					
	2	t	t^2				$3t^5$
		2	t	t^2			$3t^4$
			2	t	t^2		$6t^3 + t^3 = 7t^3$
				2	t	t^2	$t^2 + t^2 = 2t^2$
					2	t	$2t + t = 3t$
						2	2

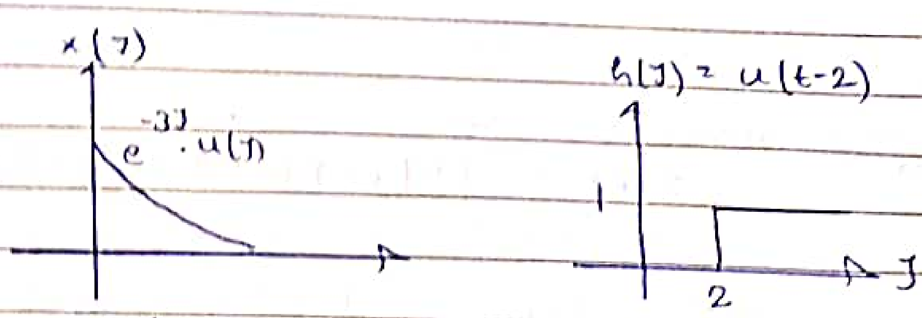
Convolution by image method

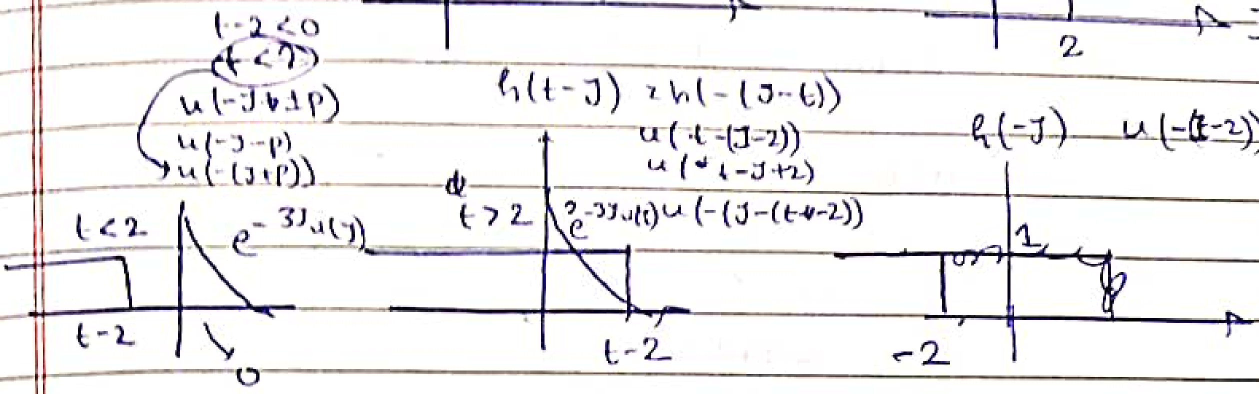


$z(t) = x(t) * h(t)$

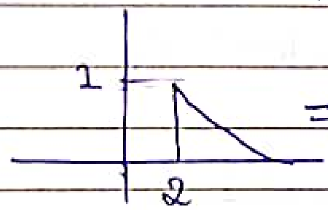
$z(t)$ limits

$2,\ 0 < t < \infty$

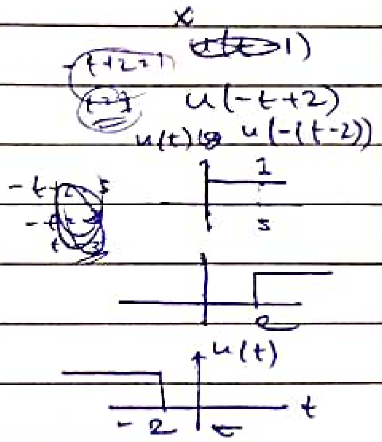




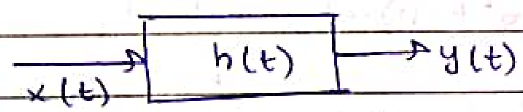
$e^{-3\tau}\, u(t-2)$



= result



eg $h(t) = \delta(t-1) + \delta(t-3)$



$x(t) = u(t)$

$h(t) = \delta(t-1) + \delta(t-3)$

$x(t) * h(t)$

$= u(t) * [\delta(t-1) + \delta(t-3)]$

$y(t) = u(t-1) + u(t-3)$

$y(2) = u(2-1) + u(2-3)$

$= 1$

Q) Given

$z(t) = \int_{-\infty}^{\infty} x(a-s+b)\, h(t+s)\, ds$

express $z(t)$ as $y(t) = x(t) * h(t)$

$y(t) = \int_{-\infty}^{\infty} x(s)\, h(t-s)\, ds = \int_{-\infty}^{\infty} x(t-s)\, h(s)\, ds$

$-s+b = p$

$-ds = dp$

$\cancel{= \int_{-\infty}^{\infty} x(p)\, h(t-p)\, dp}$

$\boxed{\cancel{x(s)\, h(t-s)\, ds}}$

$t+s = p$

$ds = dp$

$\int_{-\infty}^{\infty} x(a-(t-p)+b)\, h(p)\, dp$

$\int_{-\infty}^{\infty} x(T-p)\, h(p)\, dp = y(T)$

$= y(t+b)$

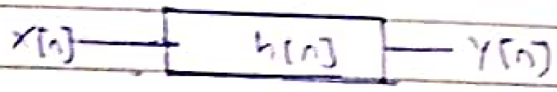
$x(t) = u(t-3) - u(t-5)$

$h(t) = e^{-3t}\, u(t)$

$\frac{d}{dt} x(t) * h(t)$

$[\delta(t-3) - \delta(t-5)] * \frac{d}{dt}\left(e^{-3t}\, u(t)\right)$

$e^{-3(t-3)}\, u(t-3) - e^{-3(t-5)}\, u(t-5)$



For memoryless system

Discrete: $y[n] = h[0]\,x[n]$

Continuous:

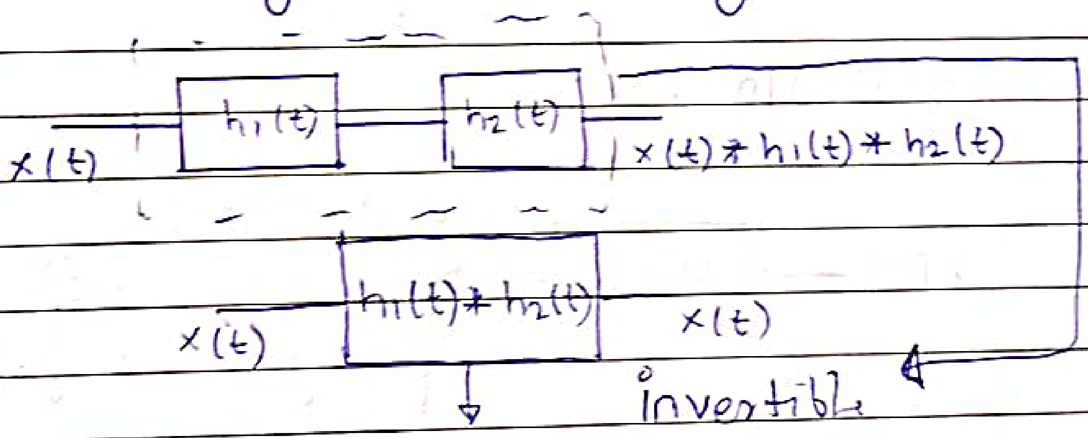
$$y(t) = \int_{-\infty}^{\infty} h(\tau).x(t-\tau)\,d\tau \qquad \text{or} \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$y(t) = h(0)\,x(t)$$

eg $h(t) = \delta(t-t_0)$

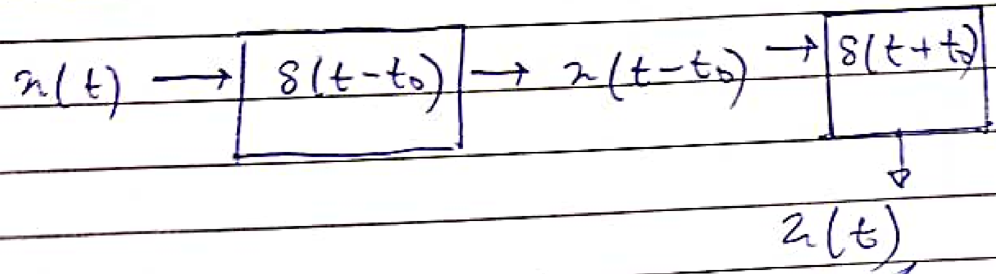
System with memory

eg



condⁿ of invertibility

eg



eg//

$x[n] \rightarrow \boxed{h[n] = u[n]} \rightarrow x[n] * u[n]$

$y[n] = \sum x[k]\, u[n-k] = x(0)u(n) + x(1)u(n-1) + \ldots$

$y[n-1] = \sum x[k]\, u[n-1-k] = x(0)u(n-1) + x(1)u(n-2) + x(2)u(n-3)$

$\cancel{y[n] - y[n-1] = x[0]u[n]}$

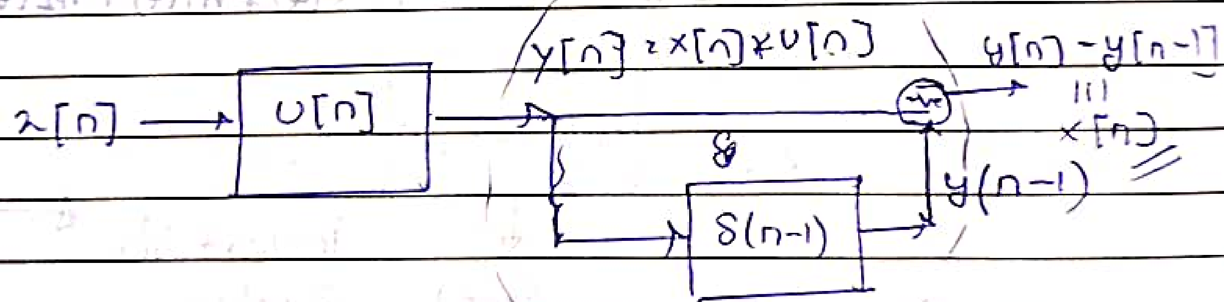
$y[n] = \sum_{k=-\infty}^{n} x[k]$ $\qquad \cancel{x[-3]\ x[-2]\ x[-1]\ x[0]}$

$y[n] - y[n-1]$

$y[n] - y[n-1] = \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k]$

$= x[n] + \cancel{\sum_{k=-\infty}^{n-1} x[k]} - \cancel{\sum_{k=-\infty}^{n-1} x[k]}$

$y[n] - y[n-1] = x[n]$



Inverse of $\boxed{u[n]}$ system

Causality

$$\vdots$$

$$y[n] = h[-2] * x[n+2]$$
$$+$$
$$h[-1] * x[n+1]$$
$$+$$
$$h[0] * x[n]$$
$$+$$
$$h[1] * x[n-1]$$
$$\vdots$$

} coeff should be zero

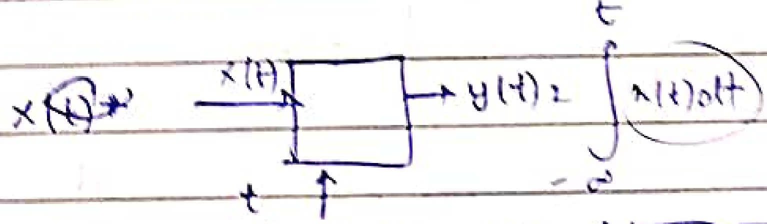
If $h[n] = 0$, $n < 0$ then system is causal

IIIly

$h(t) = 0$, $t < 0$

(i) $y(t) = \int_{-\infty}^{t} x(t)\,dt$

$$h(t) = \int_{-\infty}^{t} \delta(t)\,dt = u(t)$$



$\int_{-\infty}^{t} \delta(t)dt = u(t)$

~~Non causal~~

Causal

$x(t)$ $h(t)$

$x(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow x(t) * u(t)$

$h(t) \longrightarrow h(t) * u(t)$

$\Downarrow$

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow x(t) * h(t)$

Unit Step Response

Output when input is unit step.

$u(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t)$ $\quad \equiv \quad h(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t)$

$h(t) \uparrow$ $\qquad$ $\uparrow u(t)$

$$y(t) = h(t) * u(t)$$

A system with step signal impulse response is called as integrator.

$u(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow s(t) = y(t) = \int_{-\infty}^{t} h(t)\,dt$

$\uparrow h(t)$

$u[n] \longrightarrow \boxed{\text{LTI}} \longrightarrow s[n] = y[n] = \sum_{k=-\infty}^{N} h[k]$ (Accumulation) Accumulator

step response

$\uparrow h[n]$

impulse response

$$h[n] = s[n] - s[n-1]$$

$$h(t) = \frac{d(s(t))}{dt}$$

$v(t) - iR - L\frac{di}{dt} - \frac{1}{C}$

PAGE NO.:
DATE: / /

~~$y(t) =$~~ y_0

LTI system using Diffrential Equation

$$y(t) = y_0(t) + y_h(t)$$

$y_0(t)$ → Forced responsed

$y_h(t)$ → Natural response

$\frac{dy(t)}{dt} + y(t) = 2x(t)$

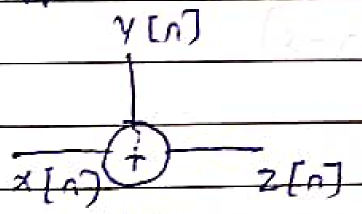
$\underline{x(t) = 0}$ (i/p = 0)

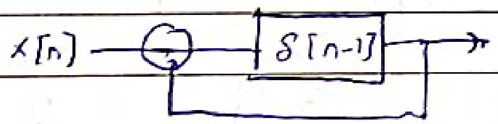
In cascade system,

overall impulse response = $h_1(t) * h_2(t) * \dots h_n(t)$

~~Find~~ Q) Find out the impulse responses of diffrent EDC devices such as RC filters, diodes. Approximation of formula $I = I_0(1 - e^{\frac{kT}{\omega}})$ (piece-wise) to in linear form then find impulse response.

(Oppenhiem solved examples)

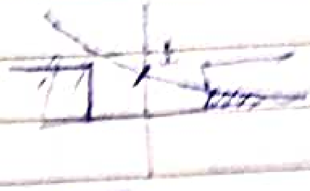




{Feedback system}

(Block diagram representation of systems)

Questions



Q) (1) $h[n] = 5^{-n} u[n-4]$

(2) $= 5^{-n} u[n+4]$

(3) $= 5^{-n} u[-n-1]$

$[-(n+1)]$

(1) ~~Causal~~ cas. Causal — Stable

(2) Non ~~causal~~ Causal — Stable

(3) ~~Causal~~ Non Causal — Unstable

★ Anticausal impulse response

Find Convolution

$h[n] = 2\delta[n+1] + 2\delta[n-1]$

$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$

$2\delta[n+1] * \delta[n] = 2\delta[n+1]$

$2\delta[n+1] * 2\delta[n-1] = 2\delta[n+1-1] = 2\delta[n]$

$2\delta[n+1] * (-\delta[n-3]) = 2\delta[n+1-3]$

$= -2\delta[n-2]$

$2\delta[n-1] \delta[n] = 2\delta[n-1]$

$2\delta[n-1] * 2\delta[n-1] = 4\delta[n-2]$

$-\delta[n-3] \cdot 2\delta[n-1] = -2\delta[n-4]$

$\boxed{2\delta[n+1] + 2\delta[n] + 2\delta[n-1]}$ $+ 2\delta[n-1] + 2\delta[n-2]$

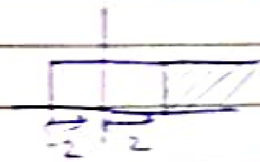
$2[\delta[n+1] + \delta[n] + \delta[n-1]] + \delta - 2\delta[n-4] - 2\delta[n-2]$
$+ 4\delta[n-2]$

$2(\delta[n+1] + 2\delta[n] + \delta[n-1] + \delta[n-2] - \delta[n-4])$

2 a) $x[n] = \frac{1}{2^{n-2}} u[n-2]$

$n > 2$

$h[n] = u[n+2]$



$\sum x(k) h(n-k)$ $\quad \frac{1}{2^{n-2}} u[n-2] * u[n+2]$

$y[n] \sum_{k=2}^{\infty} \frac{1}{2^{k-2}} u[k-2]\, u[n+2-k]$

$y[n] = \frac{u[n]}{2^0} + \frac{u[n-1]}{2^1} + \frac{u[n-2]}{2^2} \cdots \frac{1}{2^n}$

$\frac{1(2^{n-1}-1)}{2-1}$

$\frac{2^n}{2} - 1$

$2 \left(\frac{2^{n-2}}{} - \frac{1}{2} \right)$

PAGE NO.
DATE

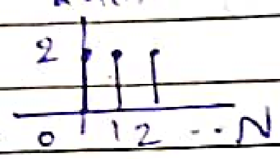
Q) $y[n] = x[n] * h[n]$

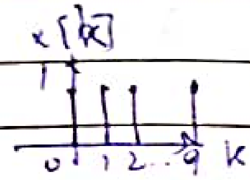
$x[n] = 1 \quad \forall \{0, 1, 2 \ldots 9\}$

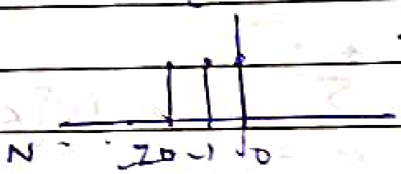
$h[n] = 2 \quad \forall \{0, 1, 2 \ldots N\}$

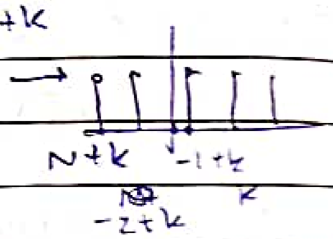
Find N

$y[n] = x[n] * h[n]$



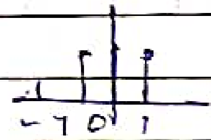


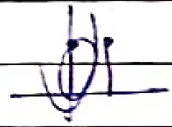




Q)

$y[0] = 2$
$y[1] = 4$
$y[2] = 6$
$y[3] = 8$



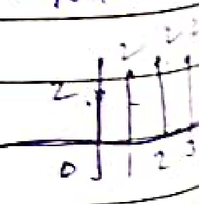


$$\sum_{n=0}^{k} 2^{n+1}$$

$$\frac{2(2^{n+1}-1)}{2-1}$$

$$= (2^{n+1}-1)2$$

$-N+4$



$y[4] = 10$
$y[14] = 0$

$\boxed{N = 4}$

$N_q = N + 10 + 1 - 1$
$= N + 10$
$N = 14 - 10$
$= 4$

PAGE NO.:
DATE: / /

P) Find signal

$220\sqrt{2}\sin(2\pi(50)t)$ is energy or power

$$48400\times 2\int_{-L}^{L}\sin^2 2\pi(100\pi t)\,dt$$

$$48400\int\left[1-\cos 200\pi t\right]dt$$

$$48400\left[t-\frac{\sin 200\pi t}{200\pi}\right]_{-L}^{L}$$

$$48400\left(L-\frac{\sin 200\pi L}{200\pi}-\left(-L+\frac{\sin 200\pi L}{200\pi}\right)\right)$$

$$=\infty$$

$$P=\frac{1}{2L}\,48400\;2L=48400$$

Q) $x(t)=\max(0,t)=t$

$$E=\infty$$

$$P=\frac{1}{2L}\int_{-L}^{L}t^2\,dt=\left.\frac{t^3}{3}\right|_{-L}^{L}$$

$$=\frac{1}{2L}\left(\frac{L^3}{3}+\frac{L^3}{3}\right)=\frac{2L^3}{3}\cdot\frac{1}{2L}$$

= Neither ~~infinite~~ energy nor power

Q) $x(t) = \sin t \, e^{-t} u(t)$

$$\int_{0}^{\infty} |\sin t \, e^{-t}|^2 \, dt$$

$$= \int_{0}^{L} \sin^2 t \, e^{-2t} \, dt$$

Q) $x(t) = e^{-j(t+\pi)}$

$$E_\infty = \int |e^{-j(t+\pi)}|^2 = \int_{-L}^{L} 1 = 2L =$$

$P_{avg} = 1$

Q) $x[n] = |e^{-4n}|^2$

$$\sum_{k=-\infty}^{\infty} e^{-8k}$$

$$\sum_{n=0}^{\infty} \frac{1}{e^{8n}} \quad n > 0$$

$$\frac{1}{1-e^{8}} \quad n > 0$$

$$e^{8k} \quad n < 0$$

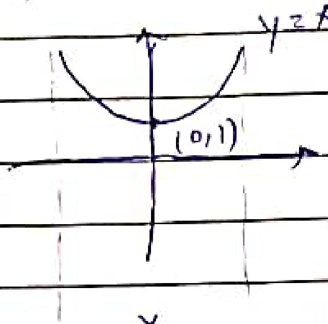
NENP

Fourier Series Representation

sampling frequency

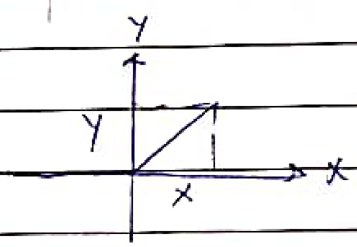
$\left\{ F_s \geq 2F \right\}$ to recover og. signal

Hence $-0.5 < f < 0.5$



Q

$y = x^2+1$



$x+jy$

Euler eqn: $re^{\pm j\theta} = r\left[\cos\theta \pm j\sin\theta\right]$

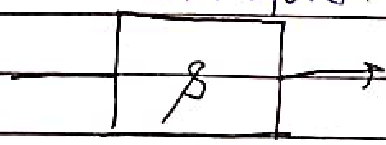
Complex exponential

$e^{j\Omega t} = e^{st}$

$s \rightarrow \underbrace{\sigma + j\Omega}_{\text{Generic exp.}}$

LTI system

e^{st}



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

PAGE NO :
DATE : / /

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= e^{st} \boxed{\int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau}$$

↓

function of s

$$\boxed{y(t) = e^{st} H(s)}$$

⇓

Eigen function where

$H(s)$ = eigen value

e^{st} = eigen function

Complex exponentials form eigen functions of LTI systems.

$$\left. e^{j\Omega_0 t} \right\} \Omega_0 = 2\pi / T_0$$

$$x_p(t) = e^{j\Omega t} + e^{2j\Omega t} + e^{3j\Omega t} + \dots + e^{kj\Omega t}$$

$$= a_0 e^{j\Omega_0 t} + a_1 e^{2j\Omega_0 t} + c_2 e^{3j\Omega_0 t} \;\; + c_k e^{kj\Omega t}$$

$$\left\{ x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t} \right\} \text{Fourier Series}$$

Multiply both sides with $e^{-jn\Omega_0 t}$

$$x_p(t)\,e^{-jn\Omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}\, e^{-jn\Omega_0 t}$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{j\Omega_0 t(k-n)}$$

Integrating both sides ~~bot~~ over a period

$$\int_0^T x_p(t)\,e^{-j\Omega_0 t}\,dt = \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\Omega_0 t}\,dt$$

$$\sum_{k=-\infty}^{\infty} a_k \left[\frac{e^{j(k-n)\Omega_0 t}}{j(k-n)\Omega_0}\right]_0^T$$

if $k=n$

$\cancel{\sum_{}^{\infty} a_k T}$

$$= \sum_{k=-\infty}^{\infty} a_k \left[\frac{e^{j(k-n)\Omega_0 T} - 1}{j(k-n)\Omega_0}\right]$$

if $k \neq n$

$$\Omega_0 = \frac{2\pi}{T}$$

$$= \sum_{k=-\infty}^{\infty} a_k \left[\frac{\left(e^{j(k-n)2\pi} - 1\right)T}{j(k-n)2\pi}\right]$$

$$\int_0^T \cos\left[(k-n)\frac{2\pi t}{T}\right] dt + j\sin\left[(k-n)\frac{2\pi t}{T}\right] dt$$

$\underbrace{k-n}_{m} \in \mathbb{Z}$

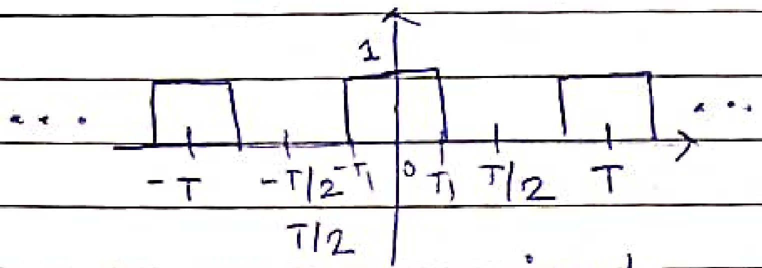
$$\frac{\sin\left(\frac{2\pi t m}{T}\right)}{2\pi m/T} = 0$$

$$\therefore \begin{cases} k=n & = t \\ k\neq n & = 0 \end{cases}$$

$$\int_0^T x_p e^{-jk\Omega_0 t}\, dt = a_k \cdot T$$

$$\boxed{a_k = \frac{1}{T}\int x_p e^{-j\Omega_0 kt}\, dt}$$

(Analysis Equation of Fourier Series)



$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x_p(t)\, e^{-jk\Omega_0 t}\, dt \qquad \Omega_0 = \frac{2\pi}{T}$$

$$= \cancel{\phi}\int_{-T/2}^{T/2} = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\Omega_0 t}\, dt$$

$$\frac{1}{T}\left[\frac{e^{-j\Omega_0 kt}}{-j\Omega_0 k}\right]_{-T_1}^{T_1}$$

$$\frac{1}{T}\left[\frac{e^{-j\Omega_0 kT_1} - e^{j\Omega_0 kT_1}}{-j\Omega_0 k}\right]$$

$$\frac{1}{T}\left[\frac{e^{j\Omega_0 kT_1} - e^{-j\Omega_0 kT_1}}{j\Omega_0 k}\right]$$

$$= \frac{1}{T}\left(\frac{e^{j\Omega_0 kT_1} - e^{-j\Omega_0 kT_1}}{j\Omega_0 k}\right)$$

$$= \frac{1}{T\cancel{j}\Omega_0 k} 2\cancel{j}\sin(\Omega_0 kT_1)$$

$$= \frac{2\sin(\Omega_0 kT_1)}{T\Omega_0 k}$$

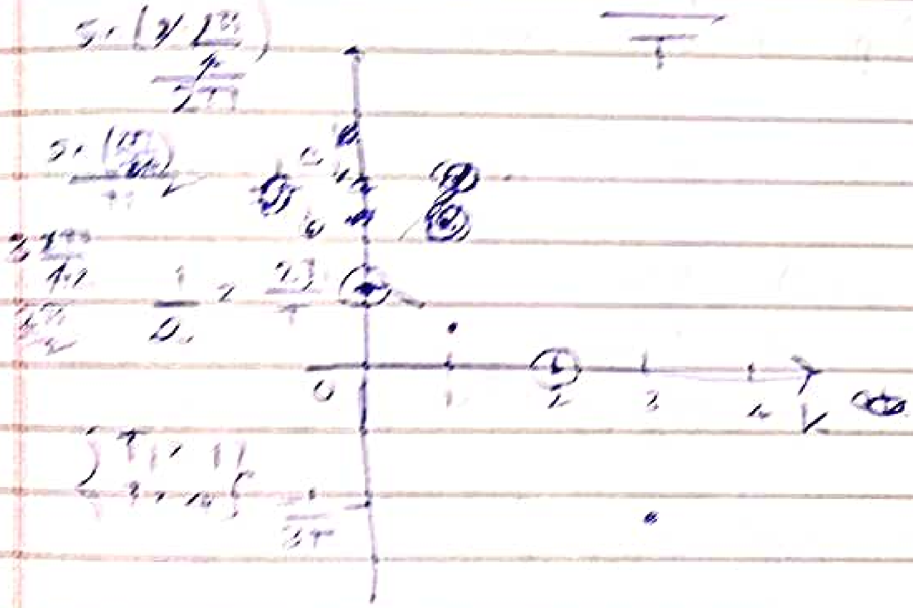
$$\frac{\cancel{2}\sin\left(\frac{2\pi kT_1}{T}\right)}{\cancel{T}\,\frac{\cancel{2}\pi k}{\cancel{T}}}$$

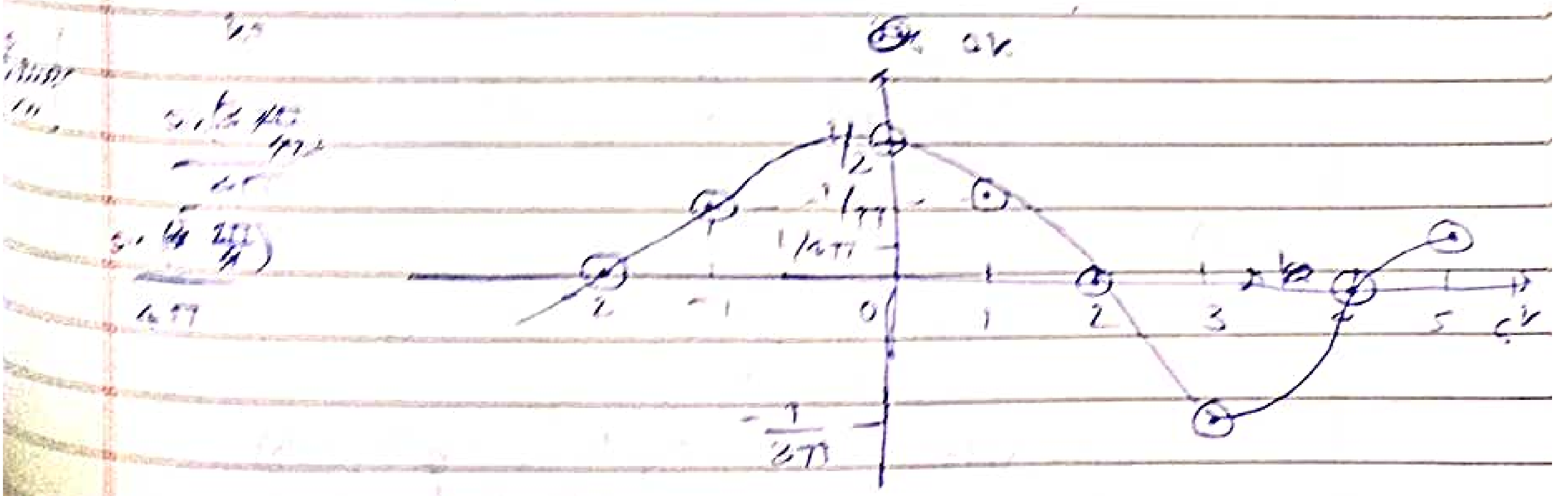
$$\left\{ e_k = \frac{\sin\left(2\pi k\frac{T_1}{T}\right)}{\pi k} \right\}$$

$$x_p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$

$$= \left[\frac{\sin(k\Omega_0 T_1)}{k\pi}\right] e^{jk\Omega_0 t} \quad \text{when } k \neq 0$$

$$a_k \begin{cases} \dfrac{\sin(kT_1\Omega_0)}{k\pi} & k \neq 0 \\[2ex] \dfrac{2T_1}{T} & k = 0 \end{cases}$$





repeated with each ...

$$x_p[n] = \sum_{k=-\infty}^{\infty} a_k e^{-jk\Omega_0 n}$$

$$a_k = \cancel{\frac{1}{T}\int} x_p$$

$$a_k = \frac{1}{N} \sum_{\langle N \rangle} x_p[n] e^{-jk\Omega_0 n}$$

$$a_k = \frac{1}{T} \int_{\langle T \rangle} x_p(t) e^{-jk\Omega_0 t} dt$$

[HW Prove all]

Properties of Fourier Series

1) Fourier series satisfies linearity / superposition

{$x_1, x_2 \to$ periodic}

$$x_1(t) \to a_k$$
$$x_2(t) \to b_k$$
$$A x_1(t) + B x_2(t) \xrightarrow{\mathcal{F}} A a_k + B b_k$$

2) Time shift property

$$x(t) \xleftrightarrow{\mathcal{F}} a_k$$
$$x(t - t_0) \xleftrightarrow{\mathcal{F}} a_k e^{-jk\Omega t_0}$$

3) Time Multiplication

$$x_1(t) \xrightarrow{\mathcal{F}} a_k$$
$$x_2(t) \xrightarrow{\mathcal{F}} b_k$$
$$x_1(t) \cdot x_2(t) \xleftrightarrow{\mathcal{F}} \sum_{l=-\infty}^{l=\infty} a_l \, b_{(k-l)}$$

$l<>$

(iii) HW explore other properties

4) Parseval's Theorem

$$\int_0^T |x(t)|^2 dt = \sum_{\langle k\rangle} |a_k|^2 \quad \{\text{No energy loss}\}$$

$$\text{error } e(t) = x_p(t) - \sum_{k=-\infty}^{\infty} a_k e^{-jk\Omega_0 t} = 0$$

$$e(t) = x_p(t) - \sum_{k=-N}^{N} a_k e^{-jk\Omega_0 t}$$

Fourier Analysis of LTI system

$$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$$

$h(t)$

$$e^{st} \longrightarrow H(s)e^{st}$$

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$

$$s = \sigma + j\Omega$$

Assume $\sigma = 0$

$$s = j\Omega$$

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-j\Omega t} dt$$

$h(t) = e^{-t} u(t)$

$$H(s) = \int_{-\infty}^{\infty} e^{-t} \cdot u(t)\, e^{-j\Omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-t(1+j\omega)}\, dt$$

$$\frac{e^{-t}}{(1+j\omega)} \Bigg|_{\infty}^{0}$$

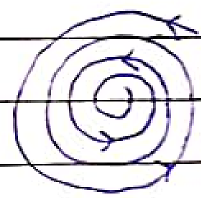
$$H(s) = \frac{1}{1+j\omega}$$

if $h(t) = e^{-\alpha t} u(t)$ $\qquad e^{-(\alpha + j\Omega t)}$

$\cancel{\frac{1}{(1+j\omega)} \cdot \frac{(1-j\omega)}{(1-j\omega)}}$

$$e^{-\infty} \cdot e^{-j\omega\infty}$$

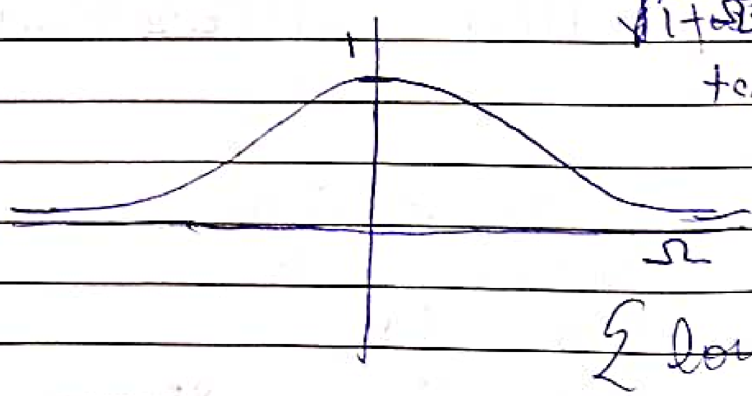
$\downarrow$ $re^{j\theta}$ $\qquad$ $\downarrow$ ∞ times rotation



$$H(s) = \frac{1}{1+j\omega} = \frac{1-j\omega}{\sqrt{1+\omega^2}}$$

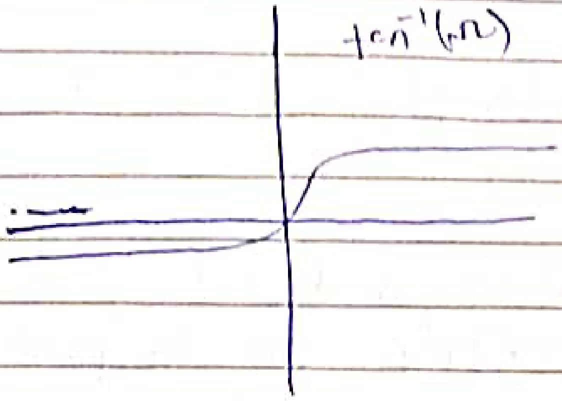
$$H(s) = \frac{1}{\sqrt{1+\Omega^2}} - \frac{\Omega}{\sqrt{1+\Omega^2}}\, j$$

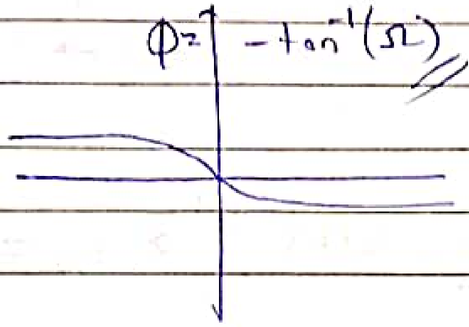
$\tan^{-1}\left(\frac{-\Omega}{1}\right)$



$-\tan^{-1}(\Omega)$

{low pass filter}

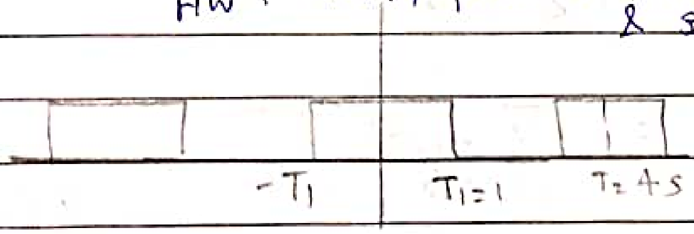




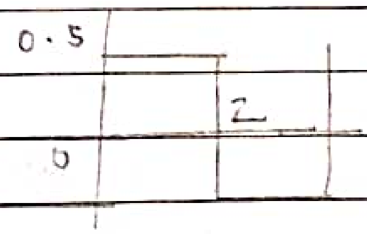
{Time Shift}

If signal is made to pass through $H(S)$
It attenuates the higher frequency & acts as a low pass filter.

HW : Verify parseval's theorem & ~~shift~~



a_k



$a_0 + a_k e^{-jk\omega_0 \Omega}$

<u>Properties</u>: ~~x(t)~~

Conjugate : $x^*(t) \xrightarrow{\mathcal{F}} a^*_{-k}$

Time reversal : $x(-t) \xrightarrow{\mathcal{F}} a^*_{-k}$

Convolution: $\int_{\langle T \rangle} x(\tau)\, y(t-\tau)\, d\tau \longleftrightarrow Ta_k b_k$

If convolution is difficult go to frequency domain & to multiply point by point fourier coefficients

2 Differentiation: $\frac{dx(t)}{dt} \rightarrow (jk\Omega_0)\, a_k$

Real signal: $x(t) \longleftrightarrow a_k = a^*_{-k}$

2

Discrete Time Fourier Series

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x_p[n]\, e^{-jk\omega_0 n}$$

$$x_p[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}$$

$x_p[n] = x[n+pN]$

$a_k = a_{k+pN}$

$$a_{k+pN} = \frac{1}{N} \sum_{n=\langle N \rangle} x_p[n]\, e^{-j\omega_0 (k+pN)}$$

$$\frac{1}{N} \sum_{k=\langle N \rangle} x_p[n]\, e^{-j\omega_0 k} \cdot e^{-j\omega_0 pN}$$

$$e^{-j\omega_0 pN} \frac{1}{N} \sum_{k=\langle N \rangle} x_p[n]\, e^{-j\omega_0 k}$$

$$\left[\cos\left(\frac{2\pi}{N} pN\right) - j\sin(\omega_0 pN)\, a_k\right]$$

$(2\pi k)$ → multiple of 2π

$\cos(2\pi k) = 1$, $\sin(2\pi k) = 0$

$$(1-0)\, a_k$$
$$= a_k$$

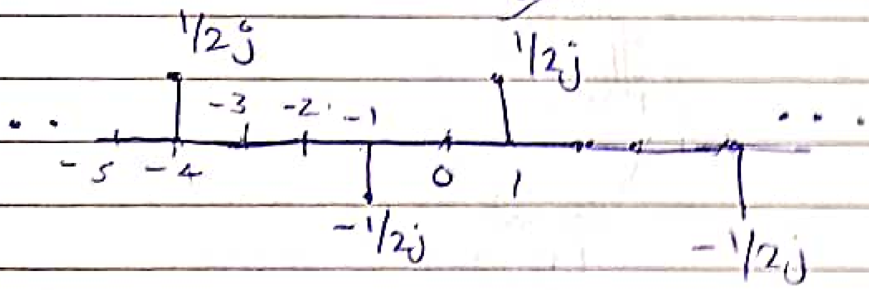
$$\therefore \{a_{k+pN} = a_k\}$$

Proved

$\therefore$ Fourier Series of a periodic signal is also periodic

eg: $\sin\left(\frac{2\pi n}{5}\right)$

$$N = \frac{2\pi}{2\pi} 5k = 5k$$

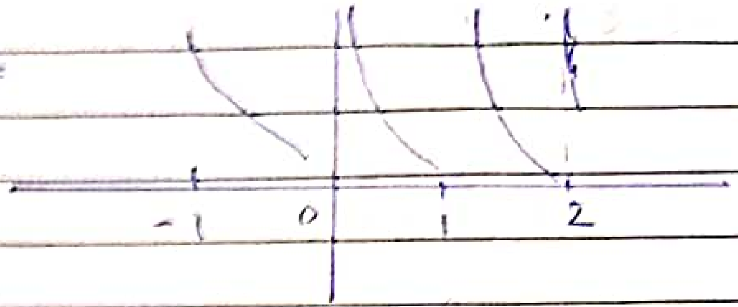


Dirichlet Conditions

* Signal must be absolutely integrable

$$\int_{-\infty}^{+\infty} |x(t)|\, dt < \infty$$

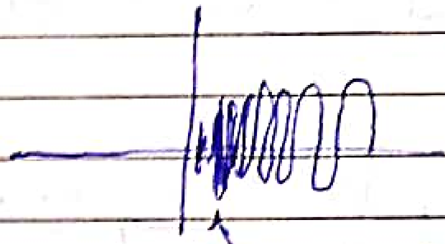
eg



$x(t) = \frac{1}{t} \quad (0 < t \leq 1)$ Violated

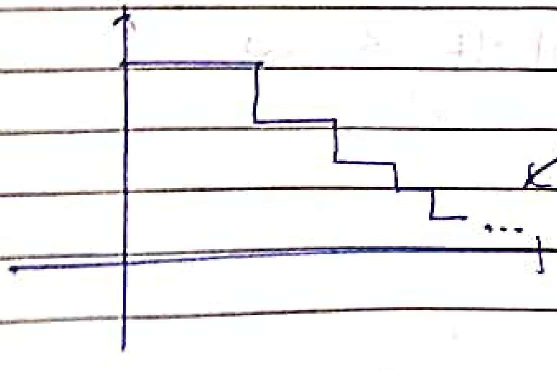
2) It should have finite number of maxima and minimas.

$x(t) = \sin\left(\frac{2\pi}{t}\right) \; ; \; 0 < t \leq 1$



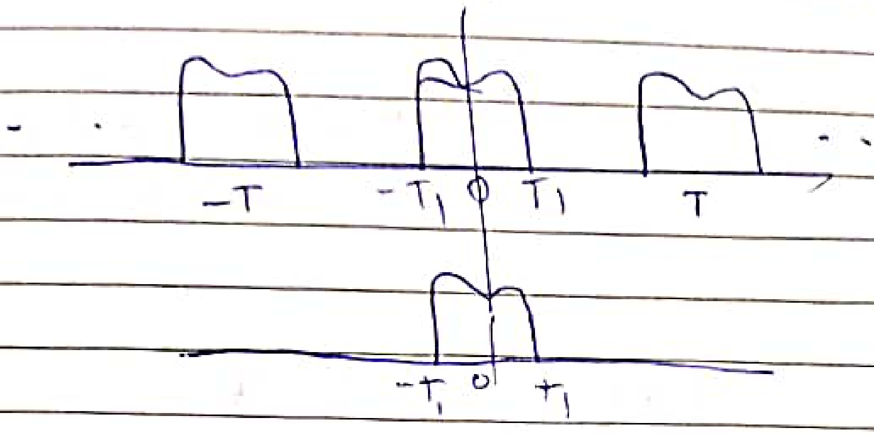
Cannot differentiable / Infinite maxima / minima

3) Signal must have finite number of finite discontinuity.



PAGE NO.:
DATE: / /

HW) Dirichlet condition for discrete time FS.



$$a_k = \frac{1}{T}\int_{\langle T \rangle} x_p(t)\, e^{-jk\Omega_0 t}\, dt$$

$$= \frac{1}{T}\int_{-\infty}^{\infty} x(t)\, e^{-jk\Omega_0 t}\, dt$$

$$c_k = \frac{X(k\Omega_0)}{T}$$

$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t}$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} X(k\Omega_0)\, e^{jk\Omega_0 t}$$

$$X(k\Omega_0) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega_0 k t}\, dt$$

$$\Omega_0 = \frac{2\pi}{T}$$

$T \to \infty \quad \Omega_0 \to 0$

$k\Omega_0 = \omega$ (continious case)

PAGE NO.
DATE / /

$$x_p(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)e^{j\Omega t}\, d\Omega$$

Synthesis eqn for a continious time aperiodic signal

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)e^{j\Omega t}\, d\Omega$$

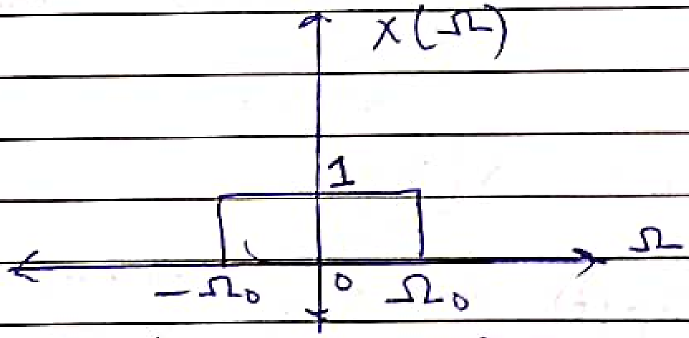
S

H(ω)

(DTFT)

Discrete time fourier transform

eg



Calculate x(t)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t}\, d\Omega$$

$$= \frac{1}{2\pi}\int_{-\Omega_0}^{\Omega_0} e^{j\Omega t}\, d\Omega$$

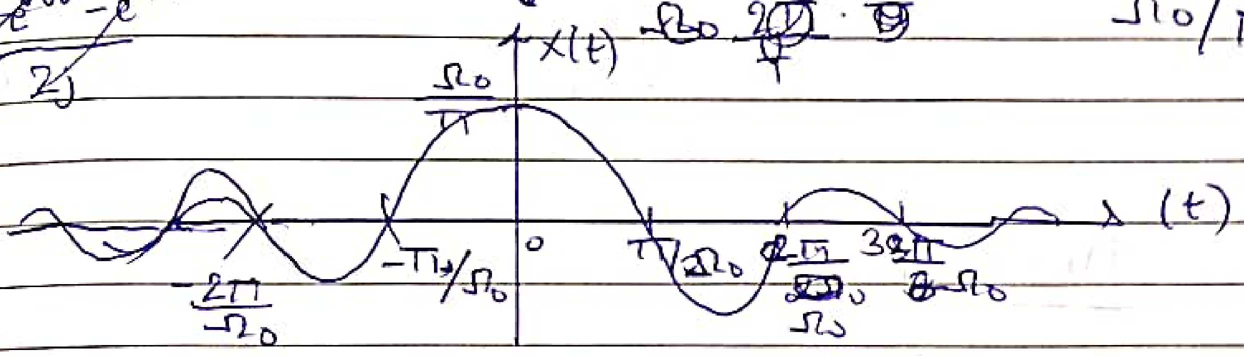
$$\frac{1}{2\pi} \left.\frac{e^{j\Omega_0 t}}{(jt)}\right|_{-\Omega_0}^{\Omega_0}$$

$$= \frac{1}{2\pi j t}\left(e^{j\Omega_0 t} - e^{-j\Omega_0 t}\right)$$

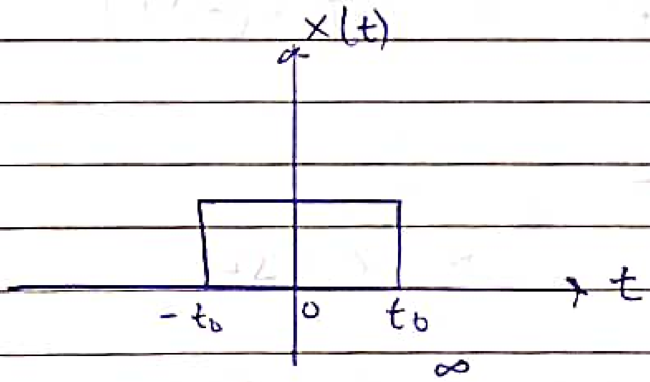
$$= \frac{1}{\pi t}\left(\sin(\Omega_0 t)\right) \quad \text{at } t \neq 0$$

at $t = 0$ $\quad \Omega_0/\pi$

$\sin(t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$



ii)

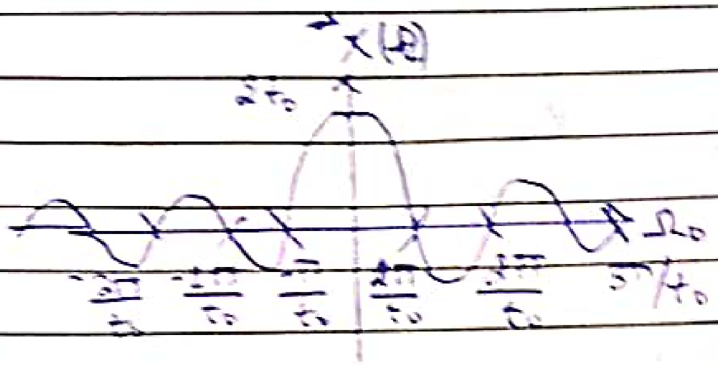


$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega_0 t}\, dt$$

$$= \int_{-t_0}^{t_0} e^{-j\Omega_0 t}\, dt$$

$$= \frac{1}{-\Omega j} \left. e^{-j\Omega_0 t} \right|_{t_0}^{-t_0} = \frac{1}{\Omega j}\left(e^{j\Omega_0 t_0} - e^{-j\Omega_0 t_0}\right)$$

$$= \frac{2}{\Omega} \sin(\Omega_0 t_0)$$



$$X(\Omega_0) = \frac{2}{\Omega_0}\sin(\Omega_0 t_0) \qquad \Omega_0 \neq 0$$

$$\qquad\quad 2t_0 \qquad\qquad\qquad \Omega_0 = 0$$

Duality property

$x(t) \longrightarrow X(\Omega) \implies$

$X(t) \longrightarrow 2\pi\, x(-\Omega)$

Linearity

$x_1(t) \xleftrightarrow{F} X_1(\Omega)$

$x_2(t) \xleftrightarrow{F} X_2(\Omega)$

$Ax_1(t) + Bx_2(t) \longrightarrow AX_1(\Omega) + BX_2(\Omega)$

Time Shift

$x(t) \xleftrightarrow{F} X(\Omega)$

$x(t-t_0) \longleftrightarrow X(\Omega)e^{-j\Omega t_0}$

$$X(\Omega) = \int_{-\infty}^{\infty} x(t-t_0) \cdot e^{-j\Omega(t-t_0)} dt$$

$t - t_0 = s$

$$= \int_{-\infty}^{\infty} x(s)\, e^{-j\Omega(s+t_0)} ds$$

$$e^{-j\Omega t_0} \int_{-\infty}^{\infty} x(s)\, e^{-j\Omega s} ds$$

$$x(\Omega)\ e^{-j\Omega t_0}\ X(t)$$

Time Convolution

$$x(t) * h(t) \longleftrightarrow X(\Omega)\, H(\Omega)$$

eg $\quad x(t) = \delta(t)$

$$X(\Omega) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\Omega t} dt$$

$\{at\ t = 0\}$

$=$ Area under impulse

$= 1$ $\quad \left\{ \int_{-\infty}^{\infty} \delta(t) dt = 1 \right\}$

$x(t)$

$X(\Omega) = \delta(\Omega)$

$X(t) = 2\pi x(-t)$

$= 2\pi\, \delta(-\Omega)$

$= 2\pi$

PAGE NO.:
DATE: / /

$$\cancel{2\pi \longleftrightarrow \delta(\Omega)}$$
$$\cancel{\longleftrightarrow \delta(\Omega-\Omega_0)}$$

δ

$$\underset{(\delta(t))}{x(t)} \longrightarrow \underset{(1)}{X(\Omega)}$$

$$\underset{x(1)}{X(t)} \longrightarrow \underset{2\pi}{2\pi x(-\Omega)}$$

$$2\pi \longrightarrow \delta(\Omega)$$
$$2\pi e^{j\Omega_0 t} \longrightarrow \delta(\Omega-\Omega_0)$$

$$x(t-t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega(t-t_0)}\, d\Omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{-j\Omega t_0} \cdot e^{j\Omega t}\, d\Omega$$

$$= 2\pi\, X(\Omega)\, e^{j\Omega t}$$

$$= 2\pi\, e^{j\Omega t}$$

PAGE NO.
DATE: / /

$$x(t) \rightarrow x(t-t_0)$$

$$X'(\Omega) = \int_{-\infty}^{\infty} x(t-t_0)\, e^{-j\Omega t}\, dt$$

$$t - t_0 = p$$
$$t = p + t_0$$

$$\int_{-\infty}^{\infty} x(p)\, e^{-j\Omega(p+t_0)}\, dp$$

$$= e^{-j\Omega t_0} \int_{-\infty}^{\infty} x(p)\, e^{-j\Omega p}\, dp$$

$$\boxed{X'(\Omega) = e^{-j\Omega t_0}\, X(\Omega)}$$ {Time Shift derivation}

$$X(\Omega - \Omega_0) = e^{j\Omega_0 t}\, x(t)$$ {Frequency shift}

$$X'(\Omega-\Omega_0) = \int x(t)\, e^{-j(\Omega-\Omega_0)t}\, dt$$
$$= e^{-j\Omega_0 t} \int x(t)\, e^{-j\Omega t}\, dt$$
$$= e^{-j\Omega_0 t}\, X(\Omega)\, X(\Omega-\Omega_0) =$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t}\, d\Omega$$

$$\boxed{e^{j\Omega_0 t}\, x(t) = X(\Omega)}$$

$$x'(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega-\Omega_0)\, e^{j\Omega t}\, d\Omega$$
$$\Omega - \Omega_0 = p$$
$$\frac{e^{j\Omega t}}{2\pi} \int X(p)\, e^{jpt}\, d\Omega$$
$$e^{j\Omega_0 t}\, x(t)$$

$$\frac{1}{2\pi} \int X(\Omega)\, e^{j\Omega t} \cdot e^{j\Omega_0 t}\, d\Omega$$
$$\frac{1}{2\pi} \int X(\Omega)\, e^{jt(\Omega+\Omega_0)}\, d\Omega$$
$$\Omega + \Omega_0 = p$$
$$\frac{1}{2\pi} \int X(p - \Omega_0)\, e^{j}$$

Frequency Derivative $-jt\,x(t) \longleftrightarrow \frac{d}{d\Omega}X(\Omega)$

Integral : $\int_{-\infty}^{t} x(t)\,dt \longleftrightarrow \frac{X(\Omega)}{j\Omega} + \pi X(0)\,\delta(\Omega)$

Parseval's Theorem : $\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\Omega)|^2\,d\Omega$

Time derivative : $\frac{d}{dt}x(t) \longleftrightarrow j\Omega X(\Omega)$

Time Multiply $x(t)\cdot h(t) = \frac{1}{2\pi}\int X(\theta)H(\Omega-\theta)\,d\theta$

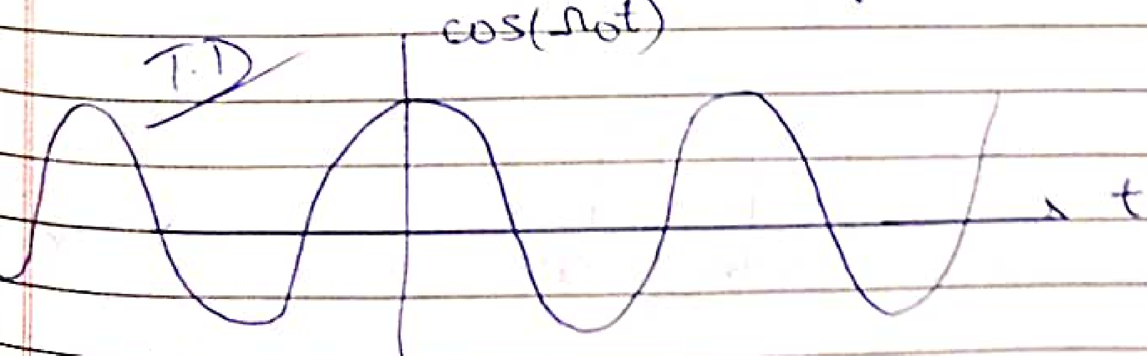
$\delta(\Omega) \xleftrightarrow{\mathcal{F}} 2\pi$

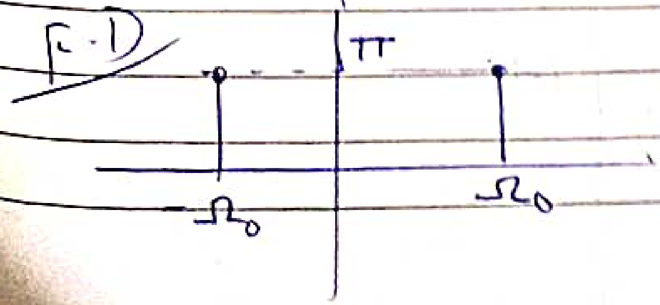
$\delta(\Omega-\Omega_0) \longleftrightarrow e^{j\Omega_0 t}\cdot 2\pi$

$e^{j\Omega_0 t} \xleftrightarrow{\mathcal{F}} 2\pi[\delta(\Omega-\Omega_0)]$

$e^{j\Omega_0 t} + e^{-j\Omega_0 t} \xrightarrow{\mathcal{F}} 2\pi[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)]$

$\cos(\Omega_0 t) \xleftrightarrow{\mathcal{F}} \pi[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)]$





$$1 \longleftrightarrow 2\pi\delta(\Omega)$$

$$e^{j\Omega kt} \longleftrightarrow 2\pi\delta(\Omega - k\Omega_0)$$

$$\sum_{k=-\infty}^{\infty} a_k e^{j\Omega_0 kt} \longleftrightarrow 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\Omega - k\Omega_0)$$

Fourier Series

$a_k \Rightarrow$ known

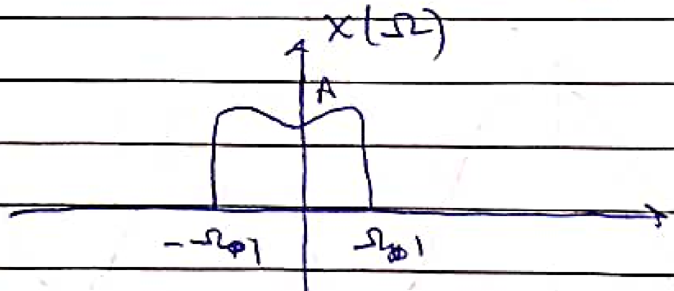
Then we can find the Fourier transform as an integral multiple of fundamental frequency.

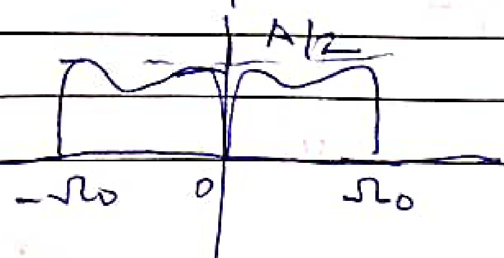
$$x(t) \longleftrightarrow X(\Omega)$$

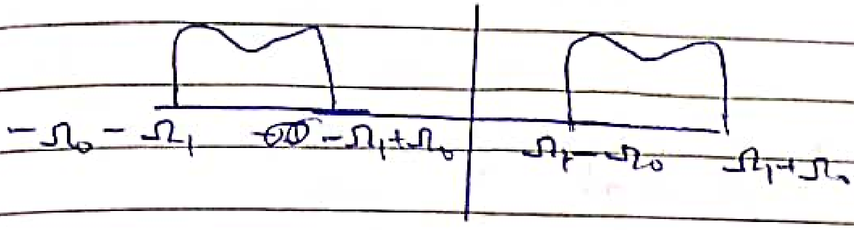
$$x(t)\cdot\cos(\Omega_0 t) \longleftrightarrow \frac{1}{2\pi}\left[X(\Omega) * \pi\left[\delta(\Omega-\Omega_0) + \delta(\Omega+\Omega_0)\right]\right]$$

$$= \frac{1}{2\cancel{\pi}}\left[\cancel{\pi}\left[X(\Omega-\Omega_0) + X(\Omega+\Omega_0)\right]\right]$$

$$= \frac{1}{2}\left(X(\Omega-\Omega_0) + X(\Omega+\Omega_0)\right)$$







Revision

CTFS

$$c_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\Omega_0 t}\, dt$$

$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\Omega_0 t}$$

DTFS

$$c_k = \frac{1}{N}\sum_{\langle N\rangle} x_p[n]\, e^{-jk\omega_0 n}$$ (Periodic in nature)

$$x_p[n] = \sum_{k=\langle N\rangle} c_k\, e^{jk\omega_0 n}$$

$0<f<1$

$\omega = 2\pi f$, $f = \frac{F_0}{F_s}$

$\{0<\omega<2\pi\}$

CTFT

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega_0 t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega_0 t}\, dt$$

(Continious time Aperiodic)

DTFT

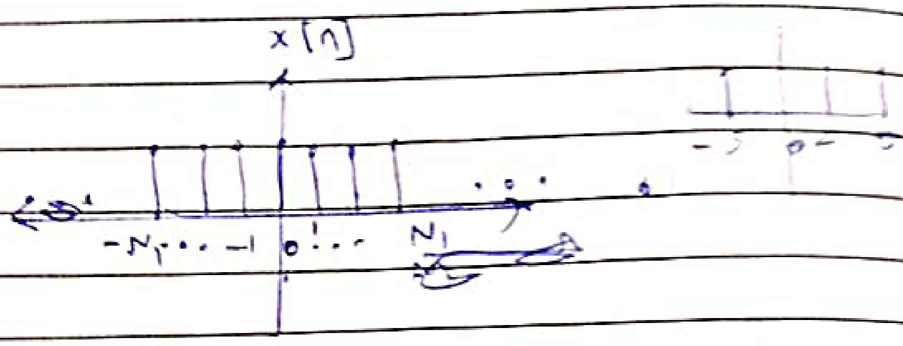
$$X(w) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega_0 n}$$

only time is discrete, ω is continious

ω is cont. ← $$x(t) = \frac{1}{2\pi}\int_{0}^{2\pi} X(w)\, e^{jwn}\, dw$$

PAGE NO.
DATE

eg



$$X(\Omega) = \sum_{-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{-N_1}^{2N_1} e^{-j\omega n}$$

$$= e^{j\omega N_1} + e^{j\omega(N-1)} + e^{j\omega(N-2)} + e^{0} + e^{-j\omega} + e^{-2j\omega}$$

$$X(w) = \frac{e^{j\omega N_1}\left(e^{-j\omega(2N+1)} - 1\right)}{e^{-j\omega} - 1}$$

$$m = n + N_1$$

$$X(w) = \sum_{m=0}^{2N_1} e^{-j\omega(m-N_1)}$$

$$= e^{j\omega N_1} \sum_{m=0}^{2N_1} e^{-j\omega m}$$

$$= e^{j\omega N_1}\left[\frac{1 - e^{-j\omega(2N_1+1)}}{1 - e^{-j\omega}}\right]$$

$$= \frac{e^{j\omega N_1} \cdot e^{-j\omega\frac{(2N_1+1)}{2}}}{e^{-j\omega/2}}\left[\frac{e^{j\omega\frac{(2N_1+1)}{2}} - e^{-j\omega\frac{(2N_1+1)}{2}}}{e^{j\omega/2} - e^{-j\omega/2}}\right]$$

$$\frac{e^{j\omega N_1} \cdot e^{-j\omega N_1}}{e^{j\omega/2} \cdot e^{\frac{-j\omega}{2}}} \cdot \frac{2j \sin(\omega N + 1/2)}{2j \sin(\omega/2)} = \frac{\sin(\omega N}{\sin(\omega}$$

PAGE NO.
DATE / /

HW { what if both ~~aper~~ time & periodic is discrete Come up with similar fourier equations (Analysis & Synthesis) for discrete time aperiodic for discrete spectrum / set of frequencies }.

To find fourier series

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$$

$$f(x) \begin{cases} 3 & 0 < x < 1 \\ -3 & -1 < x < 0 \end{cases} \qquad f(x) = f(x+2)$$

$$2L = 2$$

$$L = 1$$

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}\,dx$$

$$a_0 = \frac{1}{2L} \int_{-1}^{0} -3\,dx + \int_{0}^{1} 3\,dx$$

$$= \frac{1}{2} \left(0 - 3x^0 + 3x - 0\right) = 0$$

PAGE NO
DATE

$$a_n = \frac{1}{2L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$= \frac{1}{2}\int_{-1}^{0}(-3)\cos(n\pi x)\,dx + \int_{0}^{1} 3\cos(n\pi x)\,dx$$

$$\frac{1}{2}\left(\left[-3\sin(n\pi x)\right]_{-1}^{0} + \left[3\sin(n\pi x)\right]_{0}^{1}\right)$$

$$\frac{1}{2}\left(0 - 3\sin(n\pi) + 3\sin\pi - 0\right)$$

$$= 0$$

$$b_n = \frac{1}{2L}\int_{-L}^{L} f(x)\sin n\pi x\,dx$$

$$\int_{-1}^{0} -3\sin n\pi x\,dx + \int_{0}^{1} 3\sin n\pi x\,dx$$

$$= \left.\frac{3\cos n\pi x}{n\pi}\right|_{-1}^{0} + \left.-\frac{3\cos n\pi x}{n\pi}\right|_{0}^{1}$$

$$\frac{3}{n\pi} - \frac{3\cos n\pi}{n\pi} - \frac{3\cos n\pi}{n\pi} + \frac{3}{n\pi}$$

$$\frac{6}{n\pi} - \frac{6\cos n\pi}{n\pi} = \frac{6}{n\pi}(1-\cos n\pi)$$

$$\cos n\pi = 1 \quad n = \text{odd}$$
$$= -1 \quad n = \text{even}$$

$$b_n = \frac{6}{n\pi}\left(1-(-1)^n\right)$$

$$b_n = \frac{12}{n\pi} \quad n = \text{odd}$$

$$b_n = 0 \quad n = \text{even}$$

PAGE NO.:

DATE: / /

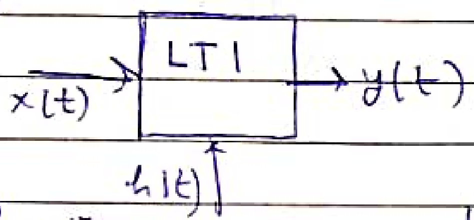
$$f(x) = 0 + 0 + \sum_{\substack{n=odd \\ n=1}}^{\infty} b_n \sin \frac{n\pi}{L} x$$

$$= \frac{12}{\pi} \sin \frac{\pi}{\cancel{2}} x + \frac{12}{3\pi} \sin \frac{3\pi}{} x$$

$$X(W) = \sum_{k=-N}^{N} 2\pi a_k \, \delta(\omega - kW_0)$$

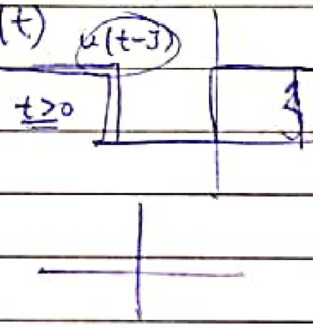
{Fourier Transform discrete ~~b~~ periodic signal}

Eg// $x(t) = e^{-t} u(t)$



$$y(t) = \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{-2(t-\tau)} u(t-\tau) d\tau$$

$h(t) = e^{-2t} u(t)$ $t>0$



$$\int_0^t e^{-\tau} e^{-2t} \cdot e^{2\tau} d\tau$$

$$e^{-2t} \int_0^t e^{\tau} d\tau$$

$$e^{-2t} \cdot e^{t} - e^{-2t} \cdot 1$$

$$(e^{-t} - e^{-2t}) u(t)$$

$$X(\Omega) = \int_{-\infty}^{\infty} e^{-t} u(t) e^{-j\Omega t} dt$$

$$= \int_0^{\infty} e^{-(1+j\Omega)t} dt$$

$$= \left. \frac{e^{-(1+j\Omega)t}}{-(1+j\Omega)} \right|_0^{\infty} = \frac{1}{1+j\Omega} = \frac{1}{1+j\Omega}$$

$$H(\Omega) = \int_0^{\infty} e^{-2t} e^{-tj\Omega} dt = \left. \frac{e^{-(2+j\Omega)t}}{-(2+j\Omega)} \right|_0^{\infty} = \frac{1}{2+j\Omega}$$

PAGE NO
DATE / /

$x(t) * h(t) \longleftrightarrow X(w) \cdot H(w)$

$$Y(\Omega) = H(\Omega) \cdot X(\Omega)$$
$$= \frac{1}{(2+j\Omega)} \cdot \frac{1}{(1+j\Omega)}$$
$$= \frac{1}{2+3j\Omega - \Omega^2}$$

$$Y(w) = \frac{1}{1+jw} - \frac{1}{2+jw}$$

IFT

$$y(t) = e^{-t}u(t) - e^{-2t}u(t)$$
$$= \left[e^{-t} - e^{-2t}\right]u(t)$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{1}{1+j\Omega} - \frac{1}{2+j\Omega}\right)e^{j\Omega t}\,d\Omega$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{j\Omega t}}{1+j\Omega}\,d\Omega - \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{j\Omega t}}{2+j\Omega}\,d\Omega$$

$$\left(\frac{e^{j\Omega t}}{\partial t} \cdot \frac{1}{1+j\Omega}\right)\Bigg|_{-\infty}^{\infty} - e^{j\Omega t}$$

Generalised Fourier Transform

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}\,dt$$

$$j\Omega = s$$

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\,e^{-st}\,dt$$

$$\therefore\ X(s) = \int_{-\infty}^{\infty} x(t)\,e^{-st}\,dt$$

{Laplace Transform}

eg :

$$x(t) = e^{-at}u(t)$$

$$X(s) = \int_{0}^{\infty} e^{-at}\cdot e^{-st}\,dt$$

$$= \int_{0}^{\infty} e^{-(s+a)t}\,dt$$

$$= \left.\frac{e^{-(s+a)t}}{-(s+a)}\right|_{\infty}^{0}$$

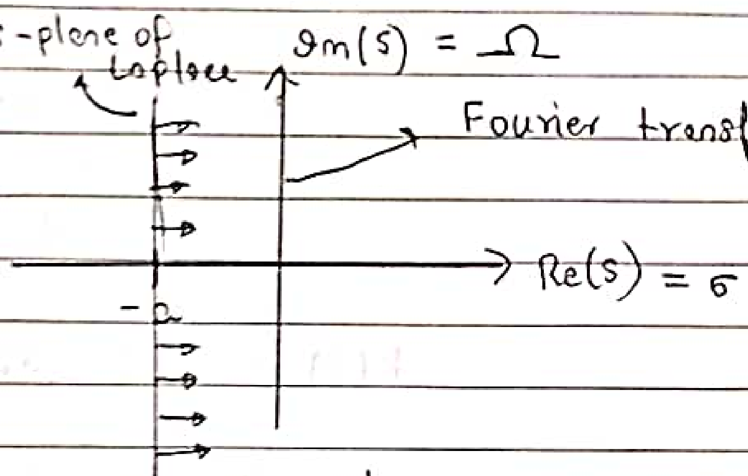
$$= \frac{1}{s+a} \quad \left\{\text{only if } s+a>0\right\}$$

$$\boxed{Re(s) > -a}$$

$$e^{-st} = e^{-\sigma t} \cdot e^{j\Omega t}$$

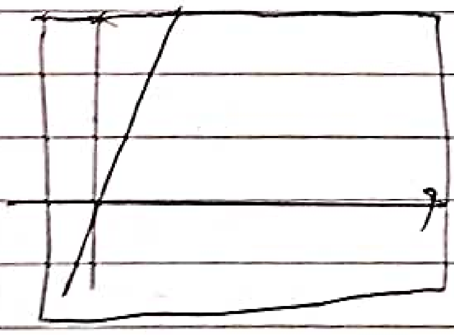
Real part

$$[\sigma > -a]$$



For any value > -a signal will converge.

when $\sigma = 0$ we obtain Fourier transform from laplace transform.



$Re\left(\frac{1}{s+a}\right)$

σt

$$X(s) = \frac{N(s)}{D(s)}$$

e.g.: $x(t) = e^{-2t}u(t)$

$h(t) = e^{-3t}u(t)$

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

PAGE NO.
DATE / /

$$x(s) = \int_0^{\infty} e^{-2t} e^{-st} dt$$

$$= \int_0^{\infty} e^{-(2+s)t} dt$$

$$\left. \frac{e^{-(2+s)t}}{2+s} \right|_{\infty}^{0}$$

$$\frac{1}{s+2} \qquad s+2>0$$

$$\sigma > -2$$

$$h(s) = \int_0^{\infty} e^{-3t} e^{-st} dt$$

$$= \frac{1}{s+3}$$

$$Y(s) = X(s) \cdot h(s) = \frac{1}{(s+2)(s+3)}$$

$$= \frac{1}{s+2} - \frac{1}{s+3}$$

$$y(t) = \left(e^{-2t} - e^{-3t}\right) u(t)$$

eg: $x(t) = e^{-2t} u(t)$

$h(t) = -e^{-2t} u(-t)$

$$-\int_{-c}^{d} x(t) dt = \int_{-d}^{c} x(t) dt$$

$$x(s) = \frac{1}{s+2}$$

$$h(s) = \int_{-\infty}^{0} -e^{-2t} \cdot e^{-st} dt$$

$$= -\int_{-\infty}^{0} e^{-(s+2)t} dt = \frac{e^{-(s+2)t}}{s+2}$$

$$\frac{1}{s+2} \qquad s+2<0 \qquad \sigma < -2$$

$$\frac{1}{s+2}$$

PAGE NO.
DATE

$-3 \quad -2$

ROC → Region of convergence

$\cdot\ y(t) \Rightarrow$ ~~BRC~~ Assuming
ROC = $\sigma > -2$

$x(t) = e^{-at} u(t)$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-at} e^{-j\Omega_0 t} dt$$

$$= \int_{0}^{\infty} e^{-t(a+j\Omega_0)} dt$$

$$= \left. \frac{e^{-(a+j\Omega_0)t}}{a+j\Omega_0} \right|_{0}^{\infty}$$

$$X(\Omega) = \frac{1}{a+j\Omega_0} = \frac{1}{a+j\omega_0}$$

$x(t) = te^{-at} u(t)$

$X(\omega) = \int te^{-(a+j\omega)t} dt \qquad e^{-(a+j\omega)t} = P$

$e^{-(a+j\omega)t}$

$x(t) = te^{-at} u(t)$

differentiation in frequency domain,

$-jt\,x(t) = \frac{d}{d\omega} X(\omega)$

$\left\{ t\,x(t) = j \frac{d}{d\omega} X(\omega) \right\}$

$$\frac{1}{2\pi}\int_{\cancel{-\infty}\,-900\pi}^{\cancel{\infty}\,900\pi} e^{j\omega t}\, d\omega$$

$$\frac{1}{2\pi} \left. \frac{e^{j\omega t}}{j\omega t} \right|_{-900\pi}^{900\pi}$$

$$\frac{1}{2\pi}\left[\frac{e^{j900\pi t} - e^{-j900\pi t}}{j\omega t}\right]$$

$$h(t) = \frac{\sin(900\pi t)}{\pi t}$$

Q) $x(t) = \frac{\sin(Bt)}{Bt}$ energy = ?

$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$

$X(t) \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\, x(-\omega)$

——— x ——— x ——— x ———

$x(n) \longleftrightarrow X(\Omega)$

$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$ { Frequency is continuous of discrete time instances }

$x(n) = \frac{1}{2\pi}\int_{\langle 2\pi \rangle} X(\Omega)\, e^{j\Omega n}\, d\Omega$ //

$x(Z)\big|_{Z = e^{j\Omega}} = X(\Omega)$ ← only

PAGE NO.:
DATE: / /

$$x(n-n_0) = e^{-j\Omega n_0} X(W)$$

$$e^{j\Omega_0 n} x(n) \leftrightarrow X(\Omega-\Omega_0)$$

$$x^*(n) \leftrightarrow X^*(-\Omega)$$

$$x(-n) \leftrightarrow X(-\Omega)$$

$$x(n/m) \leftrightarrow X(m\Omega)$$

$$x(n) \leftrightarrow X(\Omega)$$

$$x(t) \leftrightarrow X(-m)$$

$$n x(n) \leftrightarrow j \frac{d}{d\Omega} X(\Omega)$$

Q)

$x(n) = 1 \qquad -N_1 \le n \le N_1$ Comment whether $x(n)$ is low pass / high pass filter

$h(n) = x(n) = 1$ let $N_1 \Rightarrow$ odd

$$H(\Omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\Omega n}$$

$$= \sum_{n=-N_1}^{N_1} h(n) e^{-j\Omega n}$$

$$= h(0) e^0 + \sum_{n=1}^{N_1} h(n) e^{-j\Omega n} + \sum_{n=-N_1}^{-1} e^{-j\Omega n}$$

$$= 1 + \sum_{n=1}^{N_1} e^{-j\Omega n} + \sum_{m=1}^{N_1} e^{j\Omega m}$$

$$= 1 + \sum_{n,m=1}^{N_1} 2\left(\frac{e^{j\Omega m} + e^{-j\Omega n}}{2}\right)$$

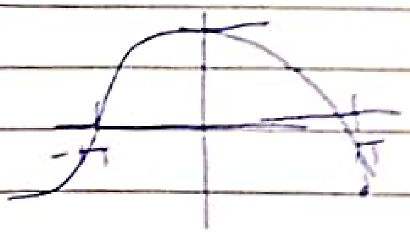
$$= 1 + 2\sum_{-N_1}^{N_1} \cos(\Omega n)$$

$\Omega = 0$

$$H(W) = 1 + 2N$$

$\Omega = \pi$ N odd

$$H(W) = 1 + 2(-1)^N = -1$$



low pass filter

Q) $\cancel{x_p(n)}$ is $x(n) \cdot \cos\left(\frac{\pi n}{5}\right)$

$$x(n)\ \cancel{X(\Omega)} = \frac{e^{\frac{j\pi n}{5}} + e^{-\frac{j\pi n}{5}}}{2}$$

$$= \frac{1}{2} e^{\frac{j\pi n}{5}} + \frac{1}{2} e^{-\frac{j\pi n}{5}}$$

$$A \longleftrightarrow 2\pi A \delta(\Omega)$$

$$\cancel{X(\Omega)}\ \cancel{\Leftrightarrow}\quad A e^{j\pi n/5} \longleftrightarrow 2\pi A\, \delta\left(\Omega - \frac{\pi n}{5}\right)$$

$$\cancel{2}\pi \left(\frac{1}{\cancel{2}}\right) \delta\left(\Omega - \frac{\pi n}{5}\right) + \cancel{2\pi} \left(\frac{1}{\cancel{2}}\right) \delta\left(\Omega + \frac{\pi n}{5}\right)$$

$$X(\Omega) = \pi \delta(\Omega - \pi n/5) + \pi \delta(\Omega + \pi n/5)$$

Q) If $x_p(n)$ is a discrete periodic signal
Find DTFT.

Period $= N_0$

$$x(n - N_0) = x(n)$$

$$\Omega = \frac{2\pi}{N_0}$$

$$A \overset{\mathcal{F}}{\longleftrightarrow} 2\pi A \delta(\Omega)$$

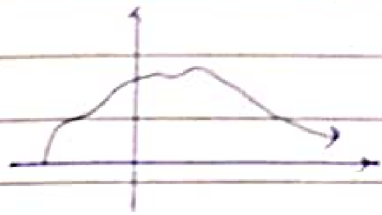
$$A e^{jk\Omega_0 n} \longleftrightarrow 2\pi A \delta(\Omega - k\Omega_0)$$

$$x_p(n) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 n}$$

$$X_p(\omega) = \sum_{\langle k \rangle} c_k\, \mathcal{F}\{e^{jk\Omega_0 n}\}$$

$$= \sum_{\langle k \rangle} c_k\, 2\pi \delta(\Omega - k\Omega_0)$$

Properties

1) ROC is governed by poles
2) Poles by definition ~~cannot~~ are never part of ROC, zeroes however can be present in the ROC.
3) Fourier transform to exists, $j\Omega$ axis must be included in the ROC
4) 

For RHS signals the ROC is always towards the right of the rightmost pole.

5) For LHS signals the ROC is always towards the left of the leftmost ~~pole~~ pole.

6) Eg:
$$x(t) = e^{-a|t|}$$

$$X(s) = \int_{0}^{\infty} e^{-at} e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-t(s+a)}\, dt$$

$$= \left. \frac{e^{-(s+a)t}}{-(s+a)} \right|_{0}^{\infty}$$

$$= \frac{1}{s+a} \qquad s+a > 0$$

$$s > -a$$

$$\int_{-\infty}^{0} e^{at} \cdot e^{-st}\, dt = \left. \frac{e^{(a-s)t}}{a-s} \right|_{-\infty}^{0} = \frac{1}{a-s} = \frac{-1}{s-a}$$

$$a - s > 0$$

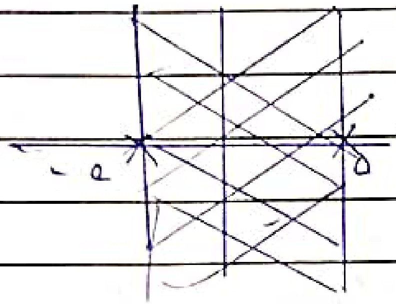
$$\boxed{s < a}$$

$$-a < s < a$$

$$\boxed{|s| < a}$$

$$X(s) = \frac{1}{s+a} - \frac{1}{s-a} \qquad |s| < a$$

only for $a>0$



if $a = 0$

$x(t) = 1$

$$X(s) = \int_{-\infty}^{\infty} e^{-st}\,dt$$

$$= \left.\frac{e^{-st}}{-s}\right|_{-\infty}^{\infty} = \text{Diverges}$$

∴ For 2 sided signals ROC is always a strip

eg $x(t) = \cos(2t)$

$$\cancel{\frac{e^{2jt} - e^{-2jt}}{2}}$$

$$\cancel{\frac{1}{2}\int (e^{2jt} - e^{-2jt})}\, e^{-st}\,dt$$

$$\int_0^{\infty} \cos 2t\, e^{-st}\,dt = \left.\left(\frac{\cos 2t\, e^{-st}}{-s}\right)\right|_0^{\infty} - 2\int_0^{\infty} \frac{\sin 2t\, e^{-st}}{s}$$

$$I = \left[0 + 1\right] - \left.\left(\frac{\sin 2t\, e^{-st}}{-s^2}\right)\right|_0^{\infty}$$

$$- 2\int_0^{\infty} \frac{\cos 2t\, e^{-st}}{s^2}$$

PAGE NO.
DATE / /

2D = [illegible] 19

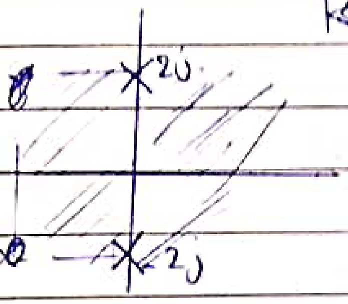
$$\boxed{I = \frac{s}{s^2+4}} \quad s>0$$

Generalized form

$$\boxed{I = \frac{s}{s^2+\Omega^2}} \quad \text{for } \cos(\Omega t)$$

$\cancel{|\sigma| < 2j}$

$|s| \le 2$



σ

$s = \pm 2j$

$$\sin(\Omega t)u(t) \overset{L}{\longleftrightarrow} \frac{\Omega}{s^2+\Omega^2} \qquad s>0$$

eg. $x(t) = u(t)$;

$X(s) = \frac{1}{s}$, $\sigma > 0$

$\delta(t) \longleftrightarrow 1$; $\sigma = 0$

Properties

1) Linearity :- $x_1(t) \overset{L}{\longleftrightarrow} X_1(s)$; R_1

$x_2(t) \overset{L}{\longleftrightarrow} X_2(s)$; R_2

$$a x_1(t) + b x_2(t) \leftrightarrow a X_1(s) + b X_2(s)$$

Time - shift $\quad x(t) \longleftrightarrow X(s)$

$$x(t-t_0) \longleftrightarrow e^{-st_0} X(s)$$

HW Find if region of convergence is affected from

Convolution : $\quad x(t) \xleftrightarrow{L} X(s) \; ; \; R_1$

$$h(t) \xleftrightarrow{L} H(s) \; ; \; R_2$$

$$x(t) * h(t) \xleftrightarrow{L} X(s) \cdot H(s) \; ; \; R_1 \cap R_2$$

Derivation $\quad x(t) \longleftrightarrow X(s)$

$$\frac{d}{dt} x(t) \longrightarrow s X(s)$$

R.O.C = ?

Integrate : $\quad \int x(t)\,dt \longleftrightarrow \frac{X(s)}{s}$

R.O.C = ?

Initial Value Theorem :

Initial : $\quad x(0^+) = \lim_{s \to \infty} \cancel{s0} \; s X(s)$

Final : $\quad \lim_{x \to \infty} x(t) = \lim_{x \to 0} s X(s)$

Z - Transform -

$$\cancel{x} \quad \cancel{X} \quad X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} \cancel{x(\omega)} \, X(\omega) e^{j\omega n} \, d\omega$$

$r e^{-j\omega n} \rightarrow Z$

$= Z^{-n}$

$$\boxed{X(Z) = \sum_{n=-\infty}^{\infty} x[n] Z^{-n}}$$

eg: a $x[n] = a^n u(n)$; $a > 0$

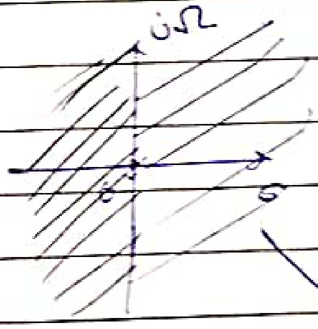
$$X(Z) = \sum_{n=0}^{\infty} a^n Z^{-n}$$

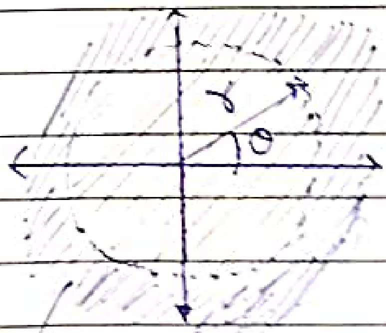
$$= \sum_{n=0}^{\infty} (aZ^{-1})^n$$

$$\frac{1}{1 - \left(\frac{a}{Z}\right)}$$ only if $|aZ^{-1}| < 1$

$|a| < |Z|$

$s = \sigma + j\omega$	$Z = re^{j\omega}$
C.T	D.T
Cartesian	Polar





when $r = 1$ it signifies discrete time fourier transform

Right hand side signals

Left hand side signals are mapped inside the circle

Z- Transform

(i) $x(n) = a^{n} u(-n)$

$$X(z) = \sum_{-\infty}^{0} a^{n} z^{-n}$$

$$= \sum_{-\infty}^{0} \left(\frac{1}{a^{-1}z}\right)^{n}$$

$1 + az + a^2z^2 + \dots$

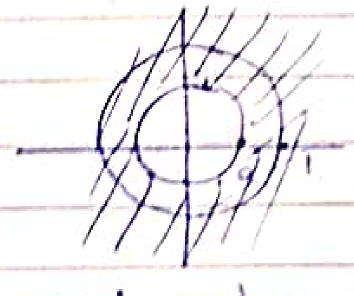
$$= 1 + \left(\frac{1}{a^{-1}z}\right)^{-1} + \left(\frac{1}{a^{-1}z}\right)^{-2} \dots \infty$$

$$X(z) = \frac{1}{1 - (a^{-1}z)} , \quad |a^{-1}z| < 1$$

$$\boxed{|z| < |a|}$$

At $a = z$, pole is observed.

$$X(z) = \frac{1}{1 - az^{-1}} = \frac{z}{z - a}$$



F.T ~~exists~~ is possible

For a causal system, it is right hand sided

$h(t) = 0 \quad t < 0$

For a stable system
$h(t)$ must be absolutely integrable

$$\int_{-\infty}^{\infty} h(t) = B \text{ (finite)}$$

For discrete one,
$h[n] = 0, \quad n < 0$
outside the circle

For a stable system, the jw axis / ~~circle~~ unit radius must be included in ROC
& Laplace

For both stability & causal,
poles must lie left of jw axis / inside unit circle.

eg

$x[n] = u[n]$

$a^n u(n)$

$X(Z) = \dfrac{1}{1 - \frac{1}{Z}}, \quad |Z| > 1$

e.g

$x[n] = -a^n u[-n-1]$

HW

$-a^n u[-(n+1)]$

$X(Z) = \dfrac{1}{1 - \frac{a^n}{Z}}$

eg

$x[n] = \delta(n)$

$X(Z) = 1 \quad ; \quad \forall Z$

entire Z plane

$$\sum \left[x[n] r^{-n}\right] e^{-j\omega n}$$

restriction on r → creates tapering

e.g.

$$x[n] = u[n-1]$$

$u[0]$

$u[n] \longleftrightarrow \dfrac{1}{1 - 1}$

$$\sum_{1}^{\infty} z^{-n} = \frac{\frac{1}{z}}{1 - \frac{1}{z}} = \frac{1}{z-1}$$

$z^{-1} + z^{-2} + z^{-3} \ldots$

$$= z^{-1}\left(\frac{1}{1 - 1/z}\right) = z^{-1}\, u[n]$$

z-transform

★ $x[n] \overset{z}{\longleftrightarrow} X(z)$; ROC $\equiv R_1$

$x[n-n_0] \overset{z}{\longleftrightarrow} z^{-n_0} X(z)$

e.g.

$$x[n] = [1, 2, 3, 4]$$

Calculate z transform

$$X(z) = \sum_{0}^{3} x(n)\, z^{-n} = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

$$\forall\ z \neq \{0\}$$

~~All~~ Entire z plane except at $z = 0$

eg $x[n] = [1, 2, 3, \underset{\uparrow}{4}]$

$X(z) = 4 + 3z^{-1} + 2z^{-2} + z^{-3}$

entire z plane ~~except~~

eg

$\cancel{x[n]} = X(z) = \dfrac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$

$\dfrac{A}{\left(1-\frac{1}{2}z^{-1}\right)} + \dfrac{B}{\left(1-2z^{-1}\right)}$

$= A - 2Az^{-1} + B - \frac{1}{2}z^{-1}B$

$= A + B - z^{-1}\left(2A + \frac{1}{2}B\right)$

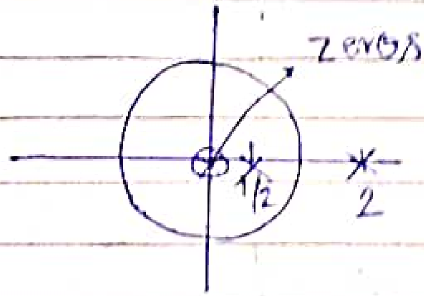
$A + B = 1$ $\quad -3A = 1$

$2A = -\frac{1}{2}B$ $\quad \boxed{A = -1/3}$

$-4A = B$ $\quad \boxed{B = 4/3}$

$\dfrac{-1}{3\left(1-\frac{1}{2}z^{-1}\right)} + \dfrac{4}{3\left(1-2z^{-1}\right)}$ $\quad$ $\left.\begin{array}{l} z = 1/2 \\ z = 2 \end{array}\right\}$ poles

$x[n] = \dfrac{-1}{3}\left(\dfrac{1}{2}\right)^{n} u(n) + \dfrac{4}{3} 2 u(n)$



$-\left(\frac{1}{2}\right)^n u(n)$

Properties:

$x[n] \xleftrightarrow{Z} X(Z)$

$x[-n] \xleftrightarrow{Z} X(Z^{-1})$ ROC is flipped

$n x[n] \xleftrightarrow{Z} -Z \frac{d}{dZ} X(Z)$

eg: ~~$n x(n) Z^{-n}$~~ $y[n] - \frac{1}{2} y[n-1] = x[n] + \frac{1}{3} x[n-1]$

$$Y(Z) - \frac{1}{2} Y(Z-1) = X(Z) + \frac{1}{3} X(Z-1)$$

$$Y(Z) - \frac{1}{2} Y(Z) \cdot Z^{-1} = X(Z) + \frac{1}{3} X(Z) \cdot Z^{-1}$$

$$H(Z) - \frac{1}{2} H(Z) Z^{-1} = 1 + \frac{1}{3} Z^{-1}$$

$$H(Z) = \frac{1 + 1/3\, Z^{-1}}{1 - \frac{1}{2} Z^{-1}}$$

For LTI system

$x[n] \longrightarrow$ LTI system $\longrightarrow y[n]$

$h[n]$

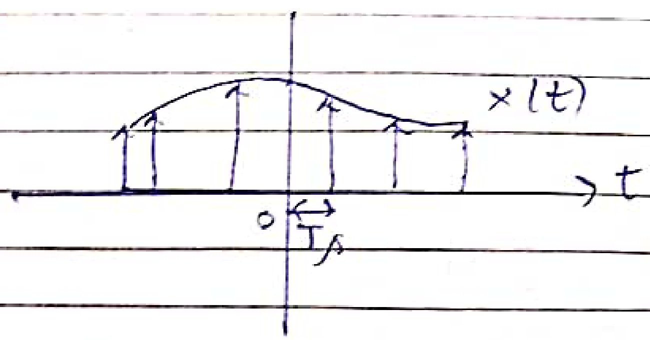
$H(z)$

$$x(n) * h(n) = X(Z) \cdot H(Z) =$$
$$y(n) \qquad\qquad Y(Z)$$

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{l=0}^{M} b_l\, x[n-l]$$

$$\boxed{H(Z) = \frac{\sum_{l=0}^{M} b_l\, Z^{-l}}{\sum_{k=0}^{N} a_k\, Z^{-k}}} \longrightarrow \text{General eqn.}$$

Sampling



$t = nT_s$

Date ______
Page ______

T_s = Sampling duration

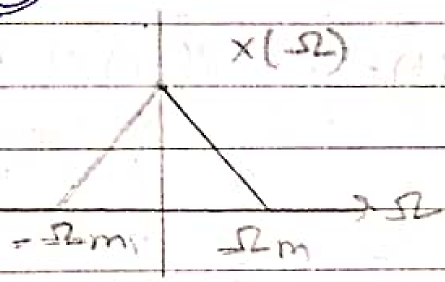
$F_s = \frac{1}{T_s}$ Sampling frequency

$\delta(t)$ being placed at integer multiple of nT_s.

$$p(t) = \sum_{t=-\infty}^{\infty} \delta(t - nT_s)$$

$\int_{-0.5}^{0.5} \delta(t) e^{-j\Omega t} dt$ 1

$\frac{1}{T_s}\int_{-0.5T_s}^{0.5T_s} \delta(t) e^{-j\Omega t} dt$



$P(\Omega) =$

OK $1/T_s$

$$x(t) \cdot p(t) = x(t)\left[\sum_{t=-\infty}^{\infty} \delta(t - nT_s)\right]$$

$$P(\Omega) = \frac{2\pi}{T_s} \sum \delta(\Omega - k\Omega_s)$$

$$= \sum_{t=-\infty}^{\infty} x[nT_s] \cdot \delta(t - nT_s)$$

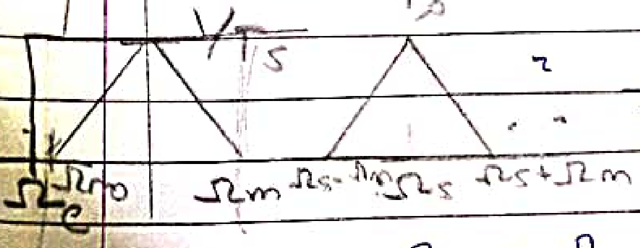
$$P(\Omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_0)$$

$X(\Omega) * P(\Omega)$

$$X(\Omega) * P(\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega') P(\Omega - \Omega') d\Omega'$$

filter having height T_s

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(\Omega - k\Omega_s)$$



$$\Omega_s - \Omega_m > \Omega_m$$

$$\therefore \boxed{\Omega_s > 2\Omega_m}$$

For square wave f [h(t)]

$$h(t) = \frac{\sin(\Omega_c t)}{\pi t} \rightarrow \left\{ \text{sinc function} \right\}$$

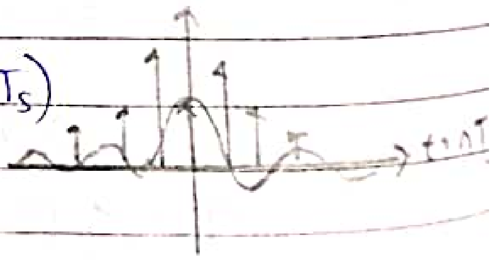
$$h(t) * P(\Omega) X(\Omega)$$

$$\Big\downarrow \text{convert to Time domain}$$

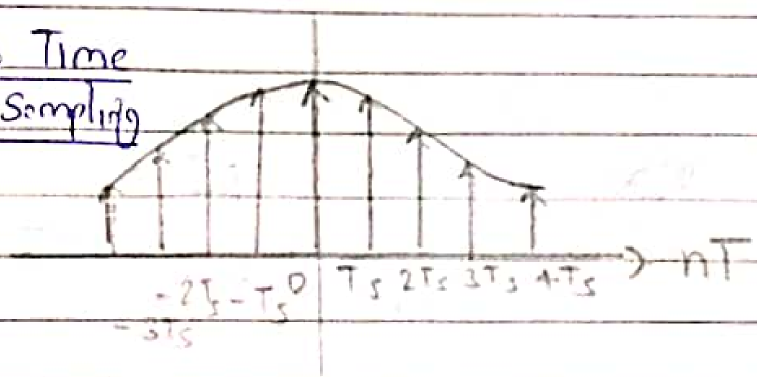
$$\frac{\sin(\Omega_c t)}{\pi t} * \left[x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right]$$

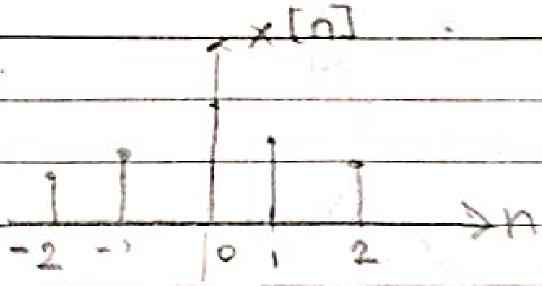
$$\frac{\sin(\Omega_c t)}{\pi t} * \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t - nT_s)$$

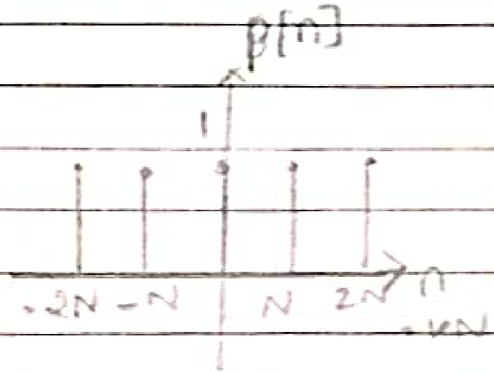
$$= \sum x(nT_s) \frac{\sin(\Omega_c nT_s)}{\pi n T_s} \delta(t - nT_s)$$



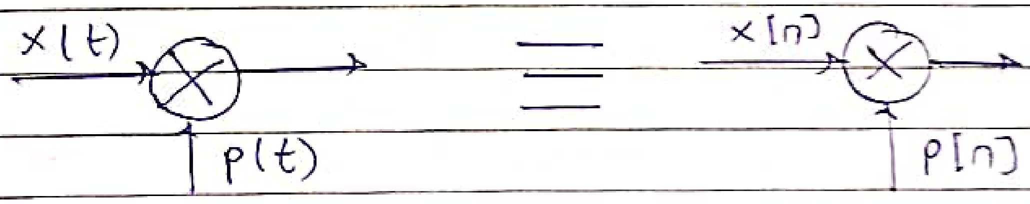
Discrete Time Signals Sampling







$$p[n] = \sum_{\substack{k \in Z \\ k=-\infty}}^{\infty} \delta(n - kN)$$



$$x[n] \cdot \sum \delta[n - kN_0] \qquad \omega =$$

$$a_k = \frac{1}{N_0} \sum_{\langle N_0 \rangle} x[n] e^{-jk\omega_0 n}$$

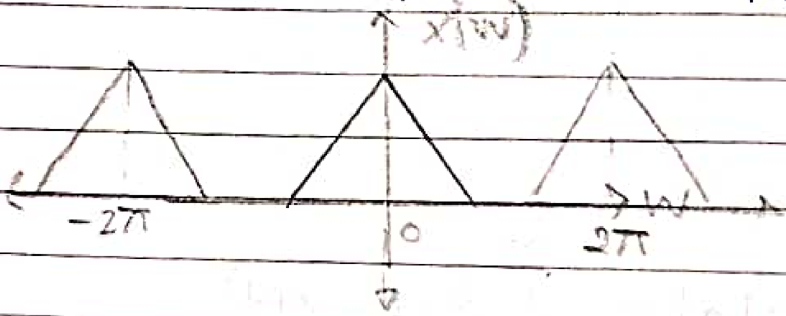
$$= \frac{1}{N_0} \sum_{n=-1}^{1} \delta[n] e^{-jk\omega n}$$

$$= \frac{1}{N_0}$$

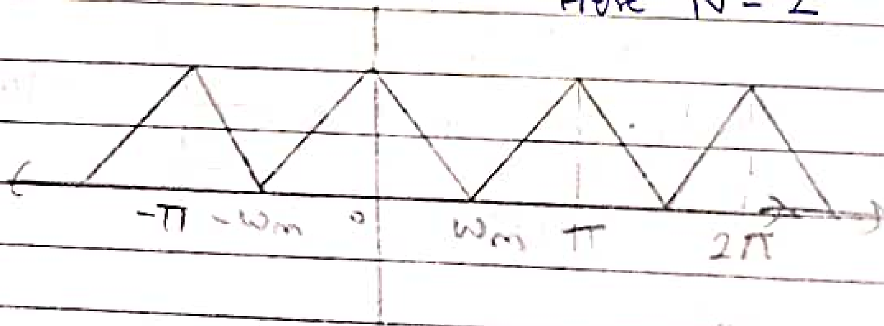
$$P(\omega) = \frac{2\pi}{N} \sum_{k=0}^{k=N-1} \delta(\omega - k\omega_0)$$

$\omega_0 = \frac{2\pi}{N}$

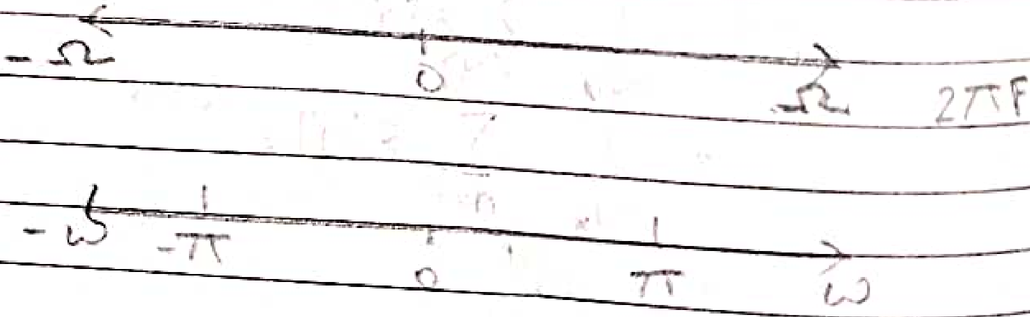
No. of replicas/samples $= N-1$



Here $N = 2$ $\therefore \omega_0 = \pi$



Note:

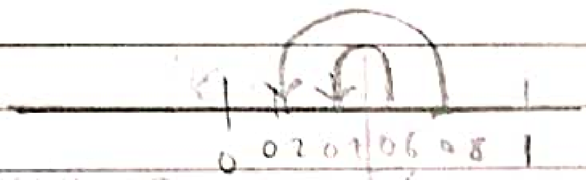


eg: $\cos(2\pi(0.2)n)$ alias

$\cos(2\pi(0.8)n)$

$= \cos(2\pi(1-0.2)n)$

$= \cos(0.2\pi\ 2\pi n)$



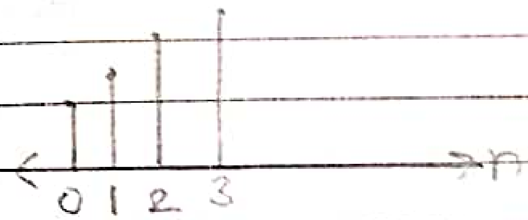
$f = F/F_s$

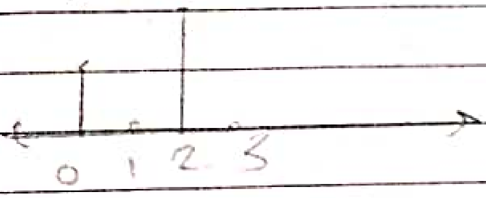
$= \dfrac{\cancel{F_s}/2}{\cancel{F_s}}$

$f = 0.5$

f:	-0.5	to	0.5	|	0	to	1
F:	$-\pi$	to	π	|	0	to	2π

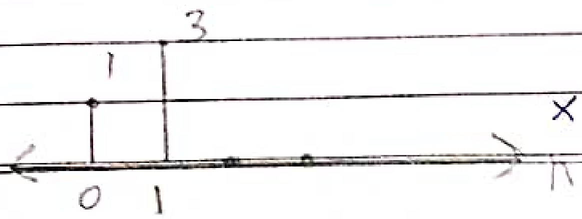
① Sampling:





② Decimation:

N = 2



$x[nN]$

~~x[n]~~ =

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$n \to nN + m$$

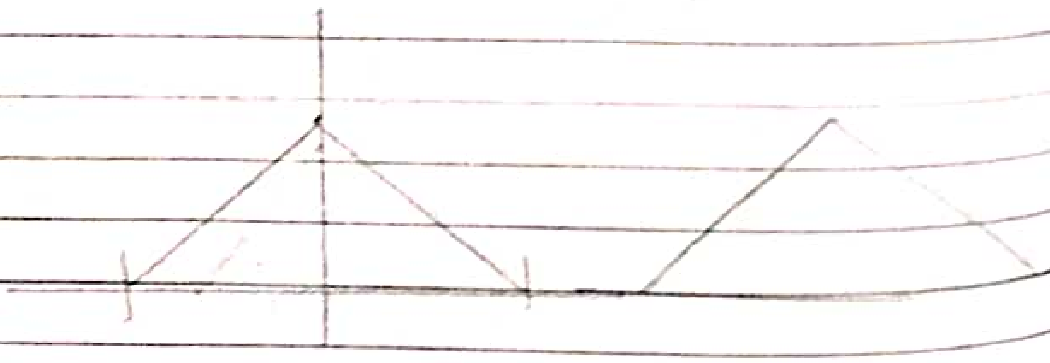
$$\sum_{n=-\infty}^{\infty} x[nN]\, e^{-j\omega nN}$$

$$\circled{X} \quad e^{-j\omega N} \sum_{n=-\infty}^{\infty} x[nN]\, e^{-j\omega n}$$

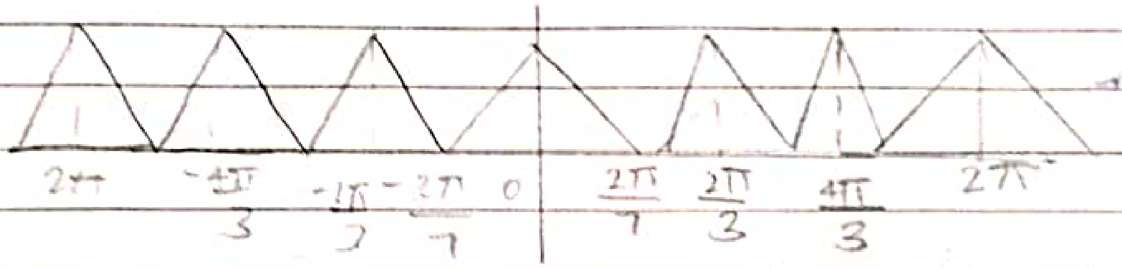
$$W = \frac{2\pi}{N}$$

$$e^{-2\pi j} \sum x[nN]\, e^{-j\frac{\omega n}{N}}$$

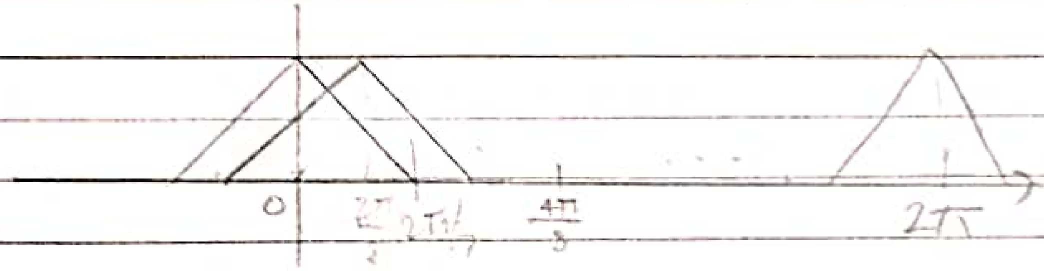
$$\sum_{m=-\infty}^{\infty} x[m]\, e^{-j\frac{\omega n}{N}}$$



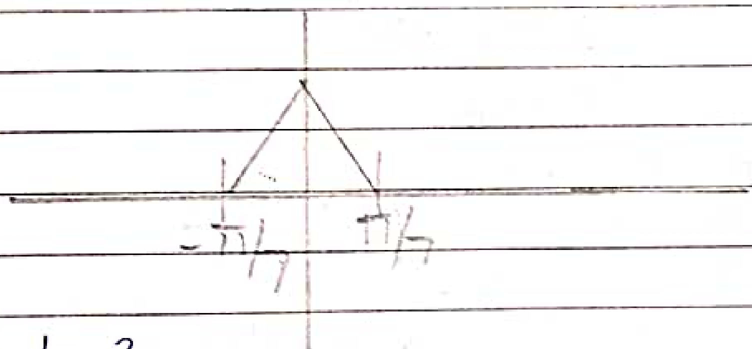
N=3



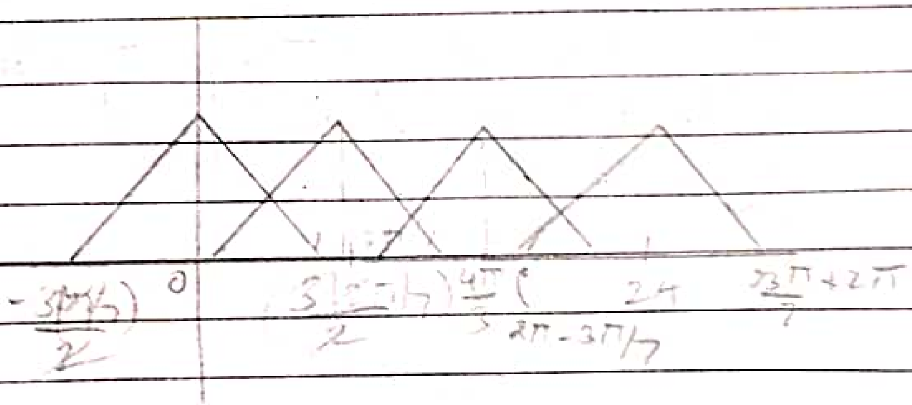
N=8



Interpolation by 2



Downsample by 3
& interpolation by 2



9960402266 → Ankit Bhurere Sir