Assignment 5: Numerical methods for boundary value problems

1. Using second order finite difference method:

   (a) Solve the BVP:

   $$(x^2 + 1)y'' + x^2y' - 4xy = 2, \quad y(0) = 0, \; y(2) = 4,$$

   with step size $h = \frac{1}{2}$.

   (b) Solve:

   $$y'' = xy + 1, \; x \in [0, 1], \quad y'(0) + y(0) = 1, \; y(1) = 1.$$

   Find $y(0), y(0.25), y(0.5)$ and $y(0.75)$.

   (c) Solve:
   $$y'' - \frac{y}{(1 + x^2)} = \frac{1}{(1 + x^2)}, \quad y'(0) = 0, \; y(1) = 0,$$

   with step-size $h = 0.25$.

   (d) Solve:

   $$y'' + (1 + x)y' - y = 0, \quad y(0) - y'(0) = 0, \; y(1) + y'(1) = 1,$$

   with step-size $h = 0.5$.

2. Use Shooting method to solve the following linear BVPs:
   (Solve the associated IVPs using Taylor's method of order 2)

   (a) Solve:

   $$y'' = \frac{2x}{1 + x^2}y' - \frac{2}{1 + x^2}y + 1, \quad y(0) = 1.25, \; y(2) = -0.95,$$

   with step-size $h = 1$.

   (b) Solve:

   $$y'' + \frac{1}{x}y' + (1 - \frac{1}{4x^2})y = 0, \quad x \in [1, 4] \text{ with } y(1) = 1, \; y(4) = 0,$$

   with step-size $h = 1$.