B. Tech. (II year), Final Examination

Subject: NM and PT (MAL-205)

Max. Marks: 50

Duration: 180 Mins.

NOTE: 1. Answer the questions, legibly, with rigorous arguments for step marking.

2. Only scientific calculators allowed, all calculation must be up to 5 (five) decimal accuracy.

3. Q1 is from CO2, Q2 and Q3 are from CO3, Q4 and Q5 are from CO4.

Q. 1 (a) Solve the IVP $y' = -2xy^2, y(0) = 0$, find $y(0.2)$ with $h = 0.2$ on the interval $[0, 0.4]$ using Runge-Kutta fourth order method and use it to find $y(0.4)$ using predictor-corrector method with $(1-2xy)$ **[5]**

$$\text{Predictor}: y_{i+1} = y_i + \frac{h}{2}\left[3y_i' - y_{i-1}'\right]$$

$$\text{Corrector}: y_{i+1} = y_i + \frac{h}{2}\left[3y_{i+1}' - y_i'\right].$$

(b) Use second order finite difference method with $h = 0.75$ to solve the BVP **[5]**

$$2x^2y'' + 3xy' - y = x, 1 < x < 4,$$
$$y(1) = 1, y(4) = \frac{41}{16}.$$

Q. 2 (a) For the Laplace PDF $f(x) = \frac{1}{2\lambda}e^{-|x-\mu|/\lambda}, x, \mu \in \mathbb{R}, \lambda > 0$, find the MGF, if it exists. **[3]**

(b) The probability density of the velocity, V, of a gas molecule, according to the Maxwell–Boltzmann law, is given by

$$f_V(v, \beta) = \begin{cases} cv^2e^{-\beta v^2}, & v > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

where c is appropriate constant and β depends on the mass of the molecule and the absolute temperature. Find c and the density function of the kinetic energy E, which is given by $E = g(V) = \frac{1}{2}mV^2$. **[4]**

(c) Let X be a RV with the PDF

$$f(x) = \begin{cases} \frac{1}{3}e^{-x/3} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}.$$

Find k, such that $P\{3 - k < X < 3 + k\} \geq 0.75$. **[3]**

Q. 3 (a) Let

$$f(x, y) = \frac{c}{(1 + x^2)\sqrt{1 - y^2}}, -\infty < x < \infty, -1 < y < 1.$$

Find the c that makes $f(x, y)$ the joint PDF of the RV (X, Y). Determine whether X and Y are independent. **[3]**

(b) A fair coin is tossed three times, let X = number of heads in three tossings, and Y = difference, in absolute value, between number of heads and number of tails. Find the joint PMF and marginal PMF's of X and Y. **[3]**

(c) Let the joint PDF of bivariate RV (X, Y) be

$$f(x,y) = \begin{cases} 3x, & 0 \le y \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find $\mathcal{P}\left\{X \le \frac{1}{2}, \frac{1}{4} < Y < \frac{3}{4}\right\}$, and the conditional PDF $f_{X|Y}(x|y)$. Also compute $f_{X|Y}\left(x|Y = \frac{1}{2}\right)$. [4]

Q. 4 (a) Let X and Y be independent random variables with common PDF

$$f(x) = \begin{cases} e^{-x} & x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Find the joint PDF of $U = \dfrac{X}{X+Y}$, $V = X + Y$ and the marginal PDF's of U and V. [5]

(b) Let X and Y be random variables and $U = aX + b$, $V = cY + d$, where a, b, c, d are constants. Show that $\rho_{U,V} = \begin{cases} \rho_{X,Y} & \text{if } ac > 0 \\ -\rho_{X,Y} & \text{otherwise} \end{cases}$. ($\rho$ correlation coefficient) [5]

Q. 5 (a) Let X be a RV with PMF $\mathcal{P}\{X = x\} = p(1-p)^x, x = 0, 1, 2, \cdots$. Find $\mathcal{P}\{X > m + n | X > m\}$, where m, n are any two non-negative integers. [3]

(b) Let $X \sim N(\mu, \sigma^2)$. Find the PDF of $Y = e^X$. [3]

(c) Let $X_i, i = 1, \ldots, n$ be independent RV's with $X_i \sim b(n_i, p)$ (binomial RV's). Find the distribution of $S_n = X_1 + \cdots + X_n$. [4]

end of the paper

$\int_{y}^{1} 3x \, dx$

B. Tech. (II year); Mid Examination

Subject: NM and PT (MAL-205)

Max. Marks: 30(6 × 5)

Duration: 90 Mins.

NOTE: 1. *Answer the questions, legibly, with rigorous arguments for step marking.*

2. *Only scientific calculators allowed, all calculation must be up to 5 (five) decimal accuracy.*

3. *Questions 1 to 3 are from CO1 and 4 to 6 are from CO2.*

Q. 1 Verify with proper reasoning which of the following one point iterative formula(s)

$$\text{i) } x_{n+1} = \frac{e^{x_n} - 1}{2}$$

$$\text{ii) } x_{n+1} = \ln(1 + 2x_n)$$

$x_0 = 0$ $\frac{1}{1+2x_n} < 2$

$n \geq 0, n \in \mathbb{Z}$, is/are suitable for finding possible solution of $f(x) = e^x - 1 - 2x = 0$ in some interval I, determine the interval I. Further, find the solution with the initial guess $x_0 \in I$.

Q. 2 Solve the equations $\begin{cases} x^2 + xy = 10, \\ y + 3xy^2 = 57. \end{cases}$ Perform two iterations of the numerical method with the initial guess $(x_0, y_0) = (1.5, 3.5)$.

Q. 3 Show that the Gauss-Jacobi method diverges whereas Gauss-Seidel method converges to approximate the solution of the system of equation $\begin{bmatrix} 2 & -1 & 1 \\ 2 & 2 & 2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -5 \end{bmatrix}$ by finding the eigenvalues of the corresponding iterative matrices.

Q. 4 Find the first three iterations of the suitable power method to the find the eigenvalue closest to $+6$ and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 2 & -4 & 5 \\ 3 & 4 & -6 \end{bmatrix}$, starting with an initial vector $(1, 1, 1)^t$.

Q. 5 Find the LU factorization, which can be obtained from the solution of $Ax = b$ using Gauss-elimination procedure, where

$$A = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 4 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

Further, solve $Ax = b_1$, $b_1 = [2, 1.5, -1, -1.5]^t$.

Q. 6 Using fourth order Runge-Kutta method, obtain the approximate value of $y(0.2)$, given that

$$y' = x + \sqrt{y}, y(0) = 1$$

with $h = 0.1$.