Department of Mathematics
Probability Theory (MAL-205)
Assignment on
Moments, MGF, Characteristic Function

1. Find expectation, variance, moment generating function, characteristic function, coefficient of skewness and kurtosis of the following pmfs/pdfs.

(i) $p_X(x) = \begin{cases} p\,(1-p)^{x-1}, & \text{if } x = 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$

(ii) $p_X(x) = \begin{cases} \binom{n}{x} p^x\,(1-p)^{n-x}, & \text{if } x = 0, 1, 2, ..., n \\ 0, & \text{otherwise} \end{cases}$

(iii) $p_X(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & \text{if } x = 0, 1, 2, ... \\ 0, & \text{otherwise} \end{cases}$

(iv)$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x > 0,\ \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$

(v) $f_X(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a < x < b \\ 0, & \text{otherwise} \end{cases}$

(vi) $f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\ \sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, & \text{if } \ x \in \mathbb{R},\ \mu \in \mathbb{R},\ \sigma > 0 \\ 0, & \text{otherwise} \end{cases}$

(vii) $f_X(x) = \begin{cases} \frac{e^{-x}\ x^{\ell-1}}{\Gamma(\ell)}, & \text{if } x > 0,\ \ell > 0 \\ 0, & \text{otherwise} \end{cases}$

2. Find the expectation and variance of the continuous rv X having pdf as $f_X(x) = \frac{1}{\pi(1+x^2)}, \quad x \in \mathbb{R}$.

3. A salesman for a medical device company has two appointments on a given day. At the first appointment, he believes that he has a 70% chance to make the deal, from which he can earn \$1000 commission if successful. On the other hand, he thinks that he only has a 40% chance to make the deal at the second appointment, from which, if successful, he can make \$1500. What is his expected commission based on his own probability belief? Assume that the appointment results are independent of each other.

4. Let X denote the number of accidents in a factory per week having pmf

$$p_X(x) = \frac{k}{(x+1)(x+2)}, \qquad x = 0, 1, 2, \ldots$$

Find the value of k, $E(X)$ and the median of X.

5. Find mean and variance of the continuous rv X whose pdf is given by
$f_X(x) = \begin{cases} \frac{1}{\beta(m,n)}\ x^{m-1}(1-x)^{n-1}, & \text{if } \ 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

6. For a rv X having pdf $f_X(x) = \frac{2a}{\pi}\left(\frac{1}{a^2+x^2}\right), \ -a \le x \le a$, show that
$$\mu_2 = \frac{a^2(4-\pi)}{\pi}, \ \mu_4 = a^4\left(1 - \frac{8}{3\pi}\right)$$
.

7. Let X be a continuous random variable with the probability density function
$$f_X(x) = \begin{cases} \frac{x+1}{4}, & \text{if } \ -1 \le x \le 1 \\ \frac{3-x}{4}, & \text{if } \ 1 \le x \le 3 \\ 0, & \quad \text{otherwise} \end{cases}$$
Find the mean and variance of X. Also evaluate $P\left(-\frac{1}{2} \le X \le \frac{5}{2}\right)$ and $P\left(|X| < \frac{3}{2}\right)$.

8. Consider the experiment of observing a sequence of trials in which probability of success is $p, \ (0 < p < 1)$ and in each trial probability of success is fixed. Let X be the rv denoting the number of trials required to get r^{th} success. Find the probability mass function, expectation and variance of the rv X.

9. For a rv X with pdf $f_X(x) = \frac{e^{-x}x^\lambda}{\lambda!}, \ x > 0, \ \lambda \ge 0$ is an integer, show that
$$P\left[0 < X < 2(\lambda+2)\right] > \frac{\lambda}{\lambda+1}$$

10. A person with n keys wants to open the door and tries the keys independently at random. Find the expectation and variance of the number of trials required to open the door.