Department of Mathematics
Probability Theory (MAL-205)
Assignment on
Special discrete and continuous distributions

1. A family has eight children. What is the probability that there being seven boys and one girl, assuming that a boy is as likely to be born as a girl? What is the probability of there being between three to five boys? How many children should be born to be 95% sure of having a girl? Ans: 0.71, 5

2. A basketball player at the free-throw line has a chance of making the basket, a statistic that does not change during the course of the game. What is the average number of free throws he makes before his first miss? What is the probability that his first miss comes on his fourth try? Ans: 5, 0.10

3. The boulevards leading into Paris appear to drivers as an endless string of traffic lights, one after the other, extending as far as the eye can see. Fortunately, these successive traffic lights are synchronized. Assume that the effect of the synchronization is that a driver has a 95% chance of not being stooped at any light, independent of the number of green lights she has already passed. Let X be the random variable that counts the number of green lights she passes before being stopped by a red light. What is the probability distribution of X? How many green lights does she pass on average before having to stop at a red light? What is the probability that she will get through 20 lights before having to stop for the first time? What is the probability that she will get through 50 lights before stoping for the fourth time? Ans: 19, 0.02, 0.0116

4. On average, one widget in 100 manufactured at a certain plant is defective. Assuming that defects occur independently, what is the distribution of defective parts in a batch of 48? Use the Poisson approximation to the binomial to compute the probability that there is more than one defective part in a batch of 48? Ans: 0.38

5. A point is chosen at random on a line segment AB of length $2a$. Find the probability that the area of the rectangle with sides AP and PB will exceed $\frac{1}{2}a^2$. Ans: $\frac{1}{\sqrt{2}}$

6. If a real number X is chosen at random from [1,50], then find the probability that $X + \frac{125}{X} > 40$. Ans: 0.32

7. Suppose that a system contains a certain type of component whose time, in years, to failure is given by T. The random variable T is modeled by the exponential distribution with mean time to failure $\lambda = 5$. If five of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years? Ans: 0.26

8. What is the probability that the value of an exponential random variable with parameter λ lies within 2σ of its expectation? Ans: 0.95

9. The lead time for orders of diodes from a certain manufacturer is known to have a gamma distribution with a mean of 40 days and a standard deviation of 20 days. Determine the probability of receiving an order within 20 days of placement date. Ans: 0.1429

10. The weight of items (in kilograms) are assumed to follow the normal distribution. From a market the following data are recorded: 10.03% of the items are under 25 kilogram weight and 89.97% of the items are under 70 kilograms weight. What are the mean and standard deviation of the distribution? Ans: 17.58, 47.5

11. Show that all odd ordered central moments are zero for a normal distribution with mean μ and variance σ^2. Also deduce $\mu_{2n} = \sigma^{2n}(2n-1)(2n-3)\ldots 5.3.1$.

12. The age of a randomly selected person in a certain population is normally distributed random variable X. Furthermore, it is known that $P(X \leq 40) = 0.5$ and $P(X \leq 30) = 0.25$. Find the mean μ and standard deviation σ of X. Also find $P(X > 65)$ and the percentage of those over 65 and than 85. Ans: 40, 14.8, 0.0457, 0.026

13. Find the median, mode and mean deviation about mean of $N(\mu, \sigma^2)$. Ans: μ, μ, $\sqrt{\frac{2}{\pi}}\ \sigma$

14. If X and Y are independent normal variates with means 6, 7 and variances 9, 16 respectively, determine λ such that $P(2X + Y \leq \lambda) = P(4X - 3Y \geq 4\lambda)$. Ans: 13.14

15. In an engineering college, it is observed that there are 500 students taking a Mathematics course. The probability of the need for a particular book from college library for any students is 0.07. How many copies of that book should be kept in the library so that the probability may be greater than 0.95 that none of the students needing a copy from the library has to go back disappointed? (Assume normal distribution.) Ans: 45

16. Suppose X_i $(i = 1, 2, ..., n)$ are independent and identically distributed normal variables with mean μ and variance σ^2. Prove that $\frac{\sqrt{n}(\overline{X} - \mu)}{\sigma} \sim N(0, 1)$.

17. In an examination, it is laid down that a student passes if he secures 30% or more marks. He is passed in the first, second or third division according as he secures 60% or more marks, between 45% and 60% marks, and marks between 30% and 45%, respectively. He gets distinction in case he secures 80% or more marks. It is noticed from the result that 8% of the students failed in the examination, whereas 8% of them obtained distinction. Calculate the percentage of students placed in the second division. (Assume normal distribution of marks.) Ans: 55, 17.79, 32%