Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
Mathematics (MAL-205)
Assignment - 2

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1. Determine p, q and r so that order of the iterative method
$$x_{n+1} = px_n + qa/x_n^2 + ra^2/x_n^5$$
for $a^{1/3}$ becomes as high as possible. For this choice of p, q and r, indicate how the error in x_{n+1} depends on the error in x_n.

2. A sequence $\{x_n\}_1^\infty$ is defined by
$$x_0 = 5$$
$$x_{n+1} = \frac{1}{16}x_n^4 - \frac{1}{2}x_n^3 + 8x_n - 12.$$
Show that it gives cubic convergence to $\alpha = 4$.

3. The system of equations $x^2y + y^3 = 10,\ xy^2 - x^2 = 3$ has a solution near $x = 0.8,\ y = 2.2$. Perform two iterations of Newton's method to obtain this root.

4. The system of equations $\log_e(x^2 + y) - 1 + y = 0,\ \sqrt{x} + xy = 0$ has one approximate solution $(x_0, y_0) = (2.4, -0.6)$. Improve this solution and estimate the accuracy of the result.

5. The system of equations $y\cos(xy) + 1 = 0,\ \sin(xy) + x - y = 0$ has one solution close to $(x, y) = (1, 2)$. Calculate this solution correct to four decimal places.

6. Calculate the solution of the system $x^2 + y^2 = 1.12,\ xy = 0.23$ correct up to three decimal place(take $(x_0 = y_0 = 1)$).

7. Calculate the solution of the system of equations $x^3 + y^3 = 53,\ 2y^3 + z^4 = 69,\ 3x^5 + 10z^2 = 770$, which is close to $(x, y, z) = (3, 3, 2)$.

8. Solve the system using Gauss Elimination method (Check the result by back substitution)

(i)
$$\begin{aligned} 8x_2 + 2x_3 &= -7\\ 3x_1 + 5x_2 + 2x_3 &= 8\\ 6x_1 + 2x_2 + 8x_3 &= 26, \end{aligned}$$

(ii)
$$\begin{aligned} 6x_2 + 13x_3 &= 61\\ 6x_1 - 8x_3 &= -38\\ 13x_1 - 8x_2 &= 79, \end{aligned}$$

(iii)
$$\begin{aligned} 10x_1 - x_2 + 2x_3 &= 4\\ x_1 + 10x_2 - x_3 &= 8\\ 2x_1 + 3x_2 + 20x_3 &= 7. \end{aligned}$$

9. Solve the system of equations by LU decomposition (Doolittle's method)

(i)
$$\begin{aligned} 5x_1 + 4x_2 + x_3 &= 3.4\\ 10x_1 + 9x_2 + 4x_3 &= 8.8\\ 10x_1 + 13x_2 + 15x_3 &= 19.2, \end{aligned}$$

(ii)
$$\begin{aligned} x_1 + x_2 + x_3 &= 1\\ 4x_1 + 3x_2 - x_3 &= 6\\ 3x_1 + 5x_2 + 3x_3 &= 4. \end{aligned}$$

10. Solve the system of equations by LU decomposition (Crout's method)

$$(i)\quad \begin{aligned} x_1 - 4x_2 + 2x_3 &= 81 \\ -4x_1 + 25x_2 + 4x_3 &= -153 \\ 2x_1 + 4x_2 + 15x_3 &= 324, \end{aligned} \qquad (ii)\quad \begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 4x_1 + 3x_2 - x_3 &= 6 \\ 3x_1 + 5x_2 + 3x_3 &= 4. \end{aligned}$$

11. Show that the LU decomposition method fails to solve the system of equations

$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ 2x_1 + 2x_2 + 5x_3 &= -3 \\ 3x_1 + 2x_2 - 3x_3 &= 6. \end{aligned}$$

12. Let $Ax = b$ (for arbitrary b). If $A = \begin{bmatrix} 1 & -k \\ -k & 1 \end{bmatrix}$, $k \in \mathcal{R}$, then determine k such that Gauss Seidel method converges.

13. Find the sufficient condition on k so that the Gauss-seidel iterative method converges for solving the system of equations $Ax = b$, where $A = \begin{bmatrix} 1 & 0 & k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ and b is arbitrary.

14. Discuss the convergence of the Gauss-Seidel iterative method for solving the system of equations $Ax = b$, and hence solve the system, where $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

15. Solve the system of equations $\begin{aligned} 2x - y &= 1 \\ -x + 2y - z &= 0 \\ -y + 2z - w &= 0 \\ . - z + 2w &= 1 \end{aligned}$ using Gauss-Seidel iterative method taking initial guess $x^{(0)} = (0, 0, 0, 0)^T$.(perform three iterations)

16. Using Jacobi method find all eigen values and the corresponding eigen vectors of the matrices $(i)\begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ $(ii)\begin{bmatrix} 2 & \sqrt{2} & 4 \\ \sqrt{2} & 6 & \sqrt{2} \\ 4 & \sqrt{2} & 2 \end{bmatrix}$ $(iii)\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ (In(i) for first rotation use a_{13} as a largest off diagonal element, for (iii) Iterate till the off-diagonal elements in magnitude are less than 0.005).