Visvesvaraya National Institute of Technology, Nagpur

Assignment-2 **Department of Mathematics**

1. If X is a continuous RV having distribution function $F(x) = \begin{cases} 1 - e^{-2x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$.
   (i) Find the PDF $f(x)$, (ii) Find $P\{X > 2\}$ and $P\{-3 < X < 4\}$.

2. The probability function for a RV X is given by $P\{X = n\} = \dfrac{1}{2^n}, n = 1, 2, 3, ...$
   (i) Find the PMF $f(x)$. (ii) Find the DF $F(x)$ of X.
   (iii) Evaluate (a) $P\{X = \text{even number}\}$ (b) $P\{X > 5\}$ (c) $P\{X = \text{ number divisible by } 3\}$.

3. Let X be an RV with PMF given by $P\{X = r\} = \begin{pmatrix} n \\ r \end{pmatrix} p^r(1-p)^{n-r}, r = 0, 1, 2, 3.., 0 < p < 1$.
   Find the PMF of the RV's (i) $Y = aX + b$ (ii) $Y = X^2$ (iii) $Y = \sqrt{X}$.

4. Let X be an RV with PDF given by $f(x) = \begin{cases} \dfrac{1}{2\pi} & \text{if } 0 < x < 2\pi \\ 0 & \text{otherwise} \end{cases}$. Let $Y = \sin X$. Find the DF and PDF of Y.

5. Suppose that a projectile is fired at an angle θ above the earth with a velocity V. Assuming that θ is an RV with PDF $f(\theta) = \begin{cases} \dfrac{12}{\pi} & \text{if } \dfrac{\pi}{6} < \theta < \dfrac{\pi}{4} \\ 0 & \text{otherwise} \end{cases}$, find the PDF of the range R of the projectile, where $R = \dfrac{V^2 \sin 2\theta}{g}$, g being the gravitation constant.

6. Let X be an positive RV of the continuous type with PDF $f(x)$. Find the PDF of the RV $U = \dfrac{X}{X+1}$.
   If, in particular, X has the PDF $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$. What is the PDF of U.

7. Let X be an RV with PDF $f(x) = \begin{cases} \dfrac{x^2}{81} & \text{if } -3 < x < 6 \\ 0 & \text{elsewhere} \end{cases}$. Find the PDF of the RV's $U = X^2$ and $Y = \dfrac{1}{3}(12 - X)$.

8. Let X be an RV with PMF/PDF given as following, then find the $E[X]$ and $Var[X]$
   (i) $f(x) = e^{-\lambda}\dfrac{\lambda^x}{x!}, x = 0, 1, 2, 3...$ (ii) $f(x) = \left(\dfrac{1}{2}\right)^x, x = 1, 2, 3, 4...$ (iii) $f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$
   (iv) $f(x) = \begin{pmatrix} n \\ k \end{pmatrix} p^k(1-p)^{n-k}, k = 0, 1, 2, 3.., 0 < p < 1$ (v) $f(x) = \dfrac{1}{\pi(1+x^2)}, -\infty < x < \infty$
   (vi) $f(x) = \begin{cases} 6x(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ (vii) $f(x) = \begin{cases} \dfrac{1}{2}x & \text{if } 0 \leq x < 1 \\ \dfrac{1}{2} & \text{if } 1 < x \leq 2 \\ \dfrac{1}{2}(3-x) & \text{if } 2 < x \leq 3 \end{cases}$

9. For any RV X with $E\left[|x|^4\right] < \infty$, define $\alpha_3 = \frac{\mu_3}{(\mu_2)^{3/2}}, \alpha_4 = \frac{\mu_4}{\mu_2^2}$ $\left(\mu_k = E\left[(x-\mu)^k\right]\right)$. Here α_3 is known as the coefficient of skewness and α_4 is known as kurtosis and is used to measure the peakedness of a distribution. Compute α_3 and α_4 for the PMF/PDF of (1).
10. Find skewness and kurtosis for the RV X with PDF $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty$.
11. Find the moment generating function (MGF) for the PDF/PMF in (1).
12. For the RV X with PDF $f(x;\lambda) = \frac{e^{-x}x^{\lambda}}{\lambda!}, x > 0$ where $\lambda \geq 0$ is an integer, show that

$$P\left\{0 < X < 2(\lambda+2)\right\} > \frac{\lambda}{\lambda+1}.$$