Introduction

Evaluation:

25 → midsem

55 → Endsem

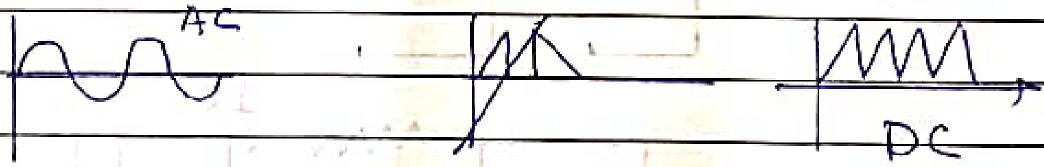
~~25~~ 20 → Teacher's assessment
- → 10 Assignment
- → 10 viva

Additivity ⎫ Linear Network
Homogeneity ⎭

If any ckt satisfies superposition principle it is linear ~~Netw~~ circuit.

Non-linear : diodes & transistor
↳ exponential, logarithm, trigo terms

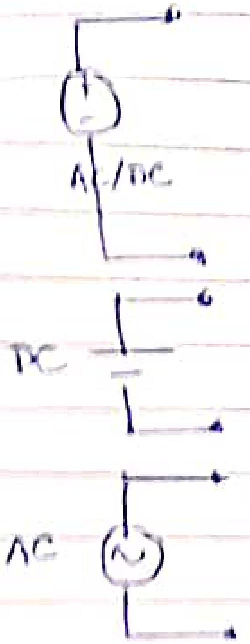
Refer: Network Analysis → Von Volkenberg
Network & System → Roy Chaudhery



DC → unidirectional

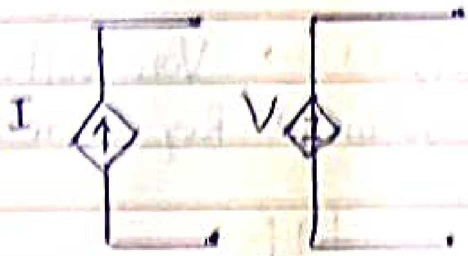
Elements –
- → Active : supply power/energy
  battery, generator, op-amp
- → Passive : do not supply energy
  eg: R, L, C

Sources
- → dependent :
- → ~~not~~ independent : An ideal independent source is an active element that provides a specified voltage/current ~~that~~ ie completely independent other ckt. variables.



Dependent Sources

An ideal dependent source is an active element in which the source quantity is controlled by another voltage or current.



V_x
Voltage controlled voltage source

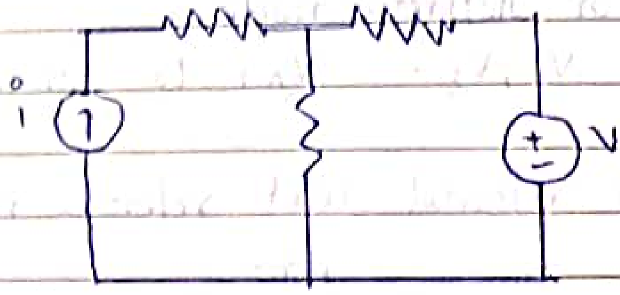
I_x
~~Current~~ Current controlled voltage source

V_x — cu voltage controlled current source

I_x — current controlled current source

PAGE NO.:
DATE: / /

Nodal Analysis, – is a technique that uses systematic application of KCL.



Branch : Represents a single element in a circuit such as voltage source in an element.

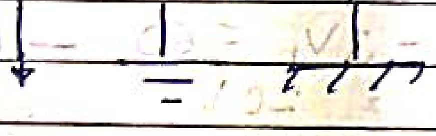
Node : A point of connection b/w 2 or more branches

loop : A closed form of a circuit.

Note : A network with 'b' branches, 'n' no. of nodes and 'l' independent loops will satisfy fundamental theorem.

$$b = l + n - 1$$

Note 2 : Reference node is also called as Datum nodes or ground. It is assumed to have 0 potential.



PAGE NO:
DATE: / /

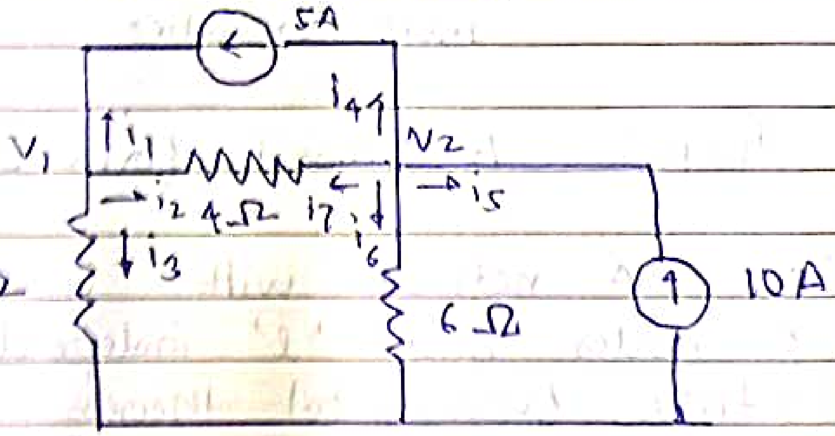
Steps

① Select a node as reference node
Assign voltages $V_1, V_2 \cdots V_{n-1}$ to remaining (n-
nodes.
All voltages are referred wrt reference node

② Apply KCL to each (n-1) non-reference nodes.
Use Ohm's law to express branch current
in terms of voltages.

③ Solve resulting simultaneous equation to obtain
the unknown load voltage.

eg



$$\frac{V_1}{2} + \frac{V_1}{4} - \frac{V_2}{4} - 5 = 0$$

$$3V_1 - V_2 = 20 \quad \text{—— ①}$$

$$\frac{V_2}{6} + \frac{V_2 - V_1}{4} + 5 - 10 = 0$$

$$5V_2 - 3V_1 = 60 \quad \text{—— ②}$$

$$V_1 = 20V$$

$$V_2 = 40/3 \ V$$

<u>Note</u> : Cremer's rule

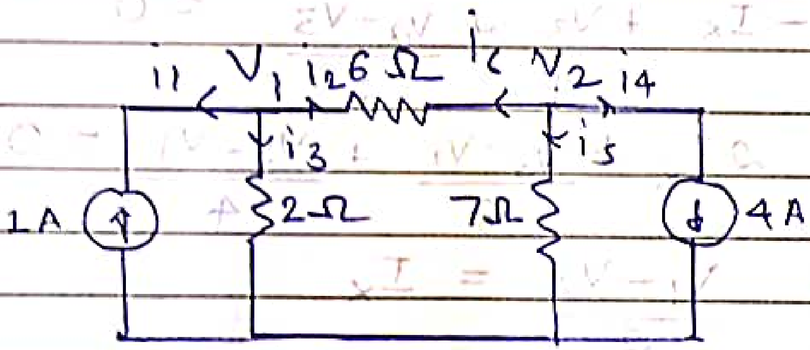
$$\begin{bmatrix} 8 & -1 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

$$\Delta = 12$$

$$V_1 = \frac{\Delta 1}{\Delta} = \begin{bmatrix} 20 & -1 \\ 60 & 5 \end{bmatrix} \Big/ 12$$

same for $V_2 = \frac{\Delta 2}{\Delta}$

<u>eg</u> :



$$\frac{V_1}{2} + \frac{V_1 - V_2}{6} - 1 = 0$$

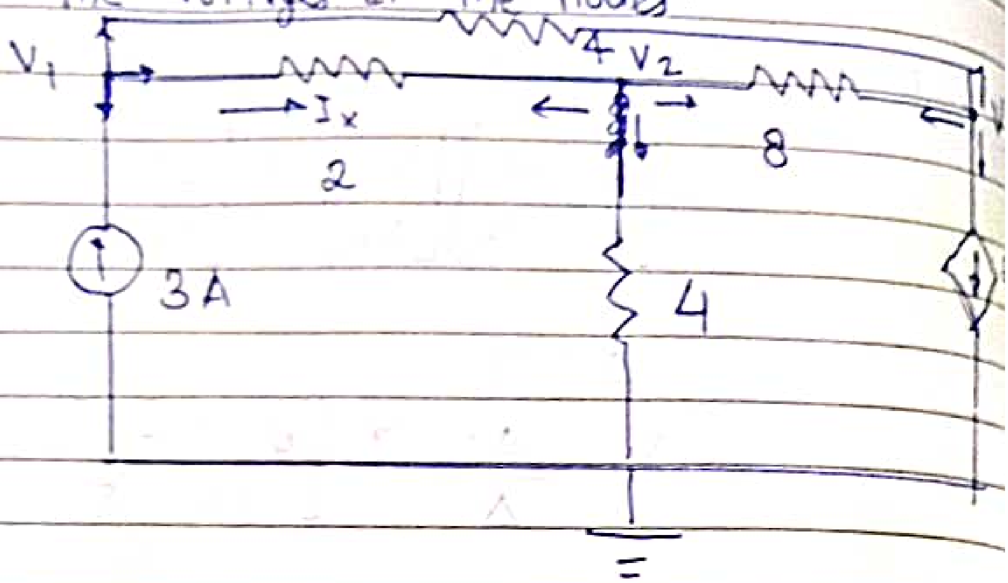
$$4V_1 - V_2 = 6 \quad \text{— (1)}$$

$$\frac{V_2 - V_1}{6} + \frac{V_2}{7} + 4 = 0$$

$$7V_1 - 13V_2 = 168 \quad \text{— (2)}$$

$$V_1 = -2V$$
$$V_2 = -14V$$

Q) Determine the voltages at the nodes.



$$-3 + I_x + \frac{V_1 - V_3}{4} = 0$$

$$-I_x + \frac{V_2}{4} + \frac{V_2 - V_3}{8} = 0$$

$$2I_x + \frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} = 0$$

$$\frac{V_1 - V_2}{2} = I_x$$

$$-12 + 4I_x + V_1 - V_3 = 0$$

$$V_1 - V_3 + 4I_x = 12 \quad — ①$$

$$-8I_x + 2V_2 + V_2 - V_3 = 0$$

$$3V_2 - V_3 - 8I_x = 0 \quad — ②$$

$$16I_x + V_3 - V_2 + 2V_3 - 2V_1 = 0$$

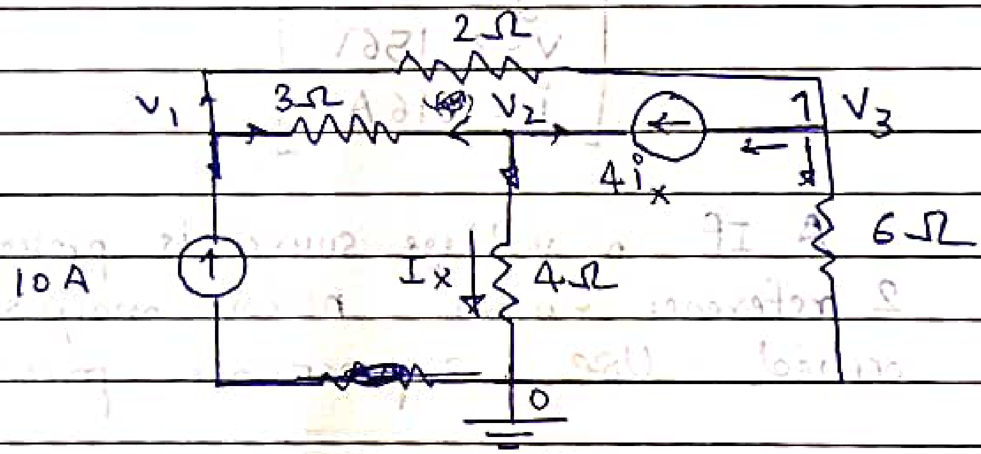
$$3V_3 - V_2 - 2V_1 \cancel{2\,8V_2 - 2V_1 - V_2} + 16I_x = 0 \quad — ③$$

$$V_1 - V_2 - 2I_x = 0 \quad — ④$$

$$\begin{aligned} V_1 &= -96/11 = -8.727 \\ V_2 &= -36/11 = -3.272 \\ V_3 &= -12/11 = -1.090 \\ I_x &= -1.09\,A \end{aligned}$$

$$\begin{aligned} V_1 &= 24/5 = 4.8\,V \\ V_2 &= 12/5 = 2.4\,V \\ V_3 &= -12/5 = -2.4\,V \\ I_x &= 6/5 = 1.2\,A \end{aligned}$$

Q) Determine voltages at nodes



$$-10 + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2} = 0$$

$$-60 + 2V_1 - 2V_2 + 3V_1 - 3V_3 = 0$$

$$5V_1 - 2V_2 - 3V_3 = 60 \quad \text{— (1)}$$

$$\frac{V_2 - V_1}{3} + I_x - 4I_x = 0$$

$$V_2 - V_1 - 9I_x = 0 \quad \text{— (2)}$$

PAGE NO.
DATE

$$\frac{V_2-0}{4} = I_x$$

$$V_2 - 4I_x = 0 \quad \text{--- (3)}$$

$$\frac{V_3-V_1}{2} + 4I_x + \frac{V_3}{6} = 0$$

$$3V_3 - 3V_1 + 24I_x + V_3 = 0$$

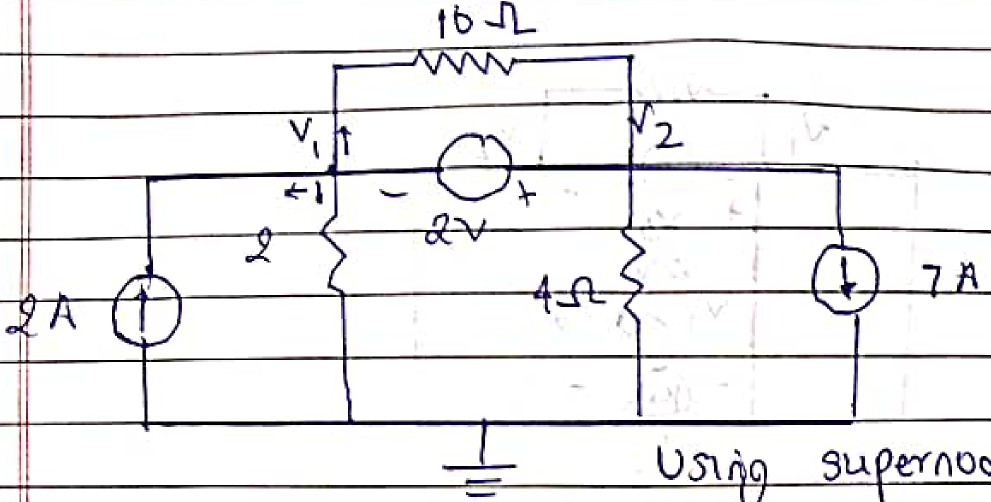
$$4V_3 - 3V_1 + 24I_x = 0 \quad \text{--- (4)}$$

$$\begin{bmatrix} V_1 = 80V \\ V_2 = -64V \\ V_3 = 156V \\ i = 16A \end{bmatrix}$$

* If a voltage source is present b/w 2 reference nodes nodal analysis is not applied. Use supernode principle.

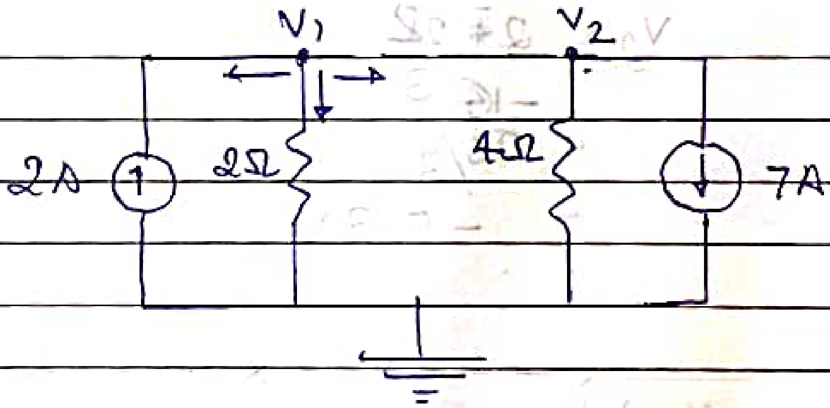
* If a voltage source is present b/w 2 reference nodes it forms a supernode. In such case apply both KCL and KVL to determine node volt

* A supernode is formed by enclosing dependent or independent voltage source connected between two non-reference nodes and any element connected in parallel with it.



Using supernode

- Remove voltage source & apply KCL
- V_1 & V_2 become supernode
- Redraw ckt by eliminating voltage source & any branch parallel with it.



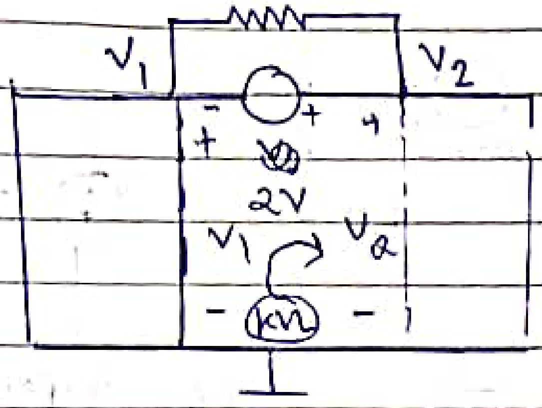
$$-2 + \frac{V_1}{2} + \frac{V_2}{4} + 7 = 0$$

$\boxed{\cancel{V_1 = 4V}}$

$$2V_1 + V_2 + 20 = 0$$

9. Restore voltage source & apply KVL & redraw the ckt.

PAGE NO.:
DATE: / /



$2V_1 + V_2 + 20 = 0$

$V_1 + 2V - V_2 = 0$ — (2)

$V_2 - V_1 = 2$ — (1)

$3V_1 = -20$

$V_1 = -22/3 = -7.3V$

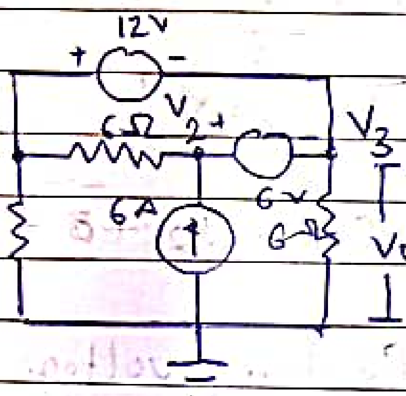
$V_2 = 2 - \frac{22}{3}$

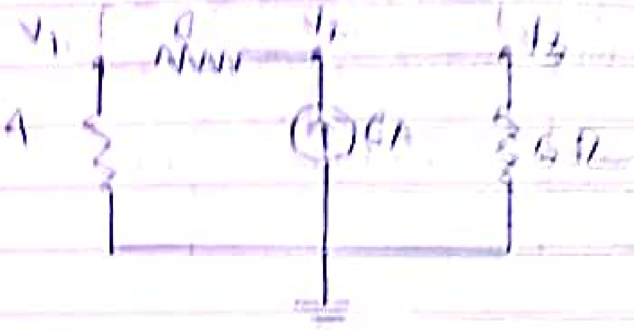
$= -16/3$

$= -5.3V$

Youtube

1)





$$\cancel{\frac{V_1-V_2}{6} + 1 \quad \frac{V_1}{4} + \frac{V_3-V_1}{}}$$

$$\frac{V_1-V_2}{6} + \frac{V_1}{4} + \frac{V_3}{6} = 0$$

$$4V_1 - 4V_2 + 6V_1 + 4V_3 = 0$$

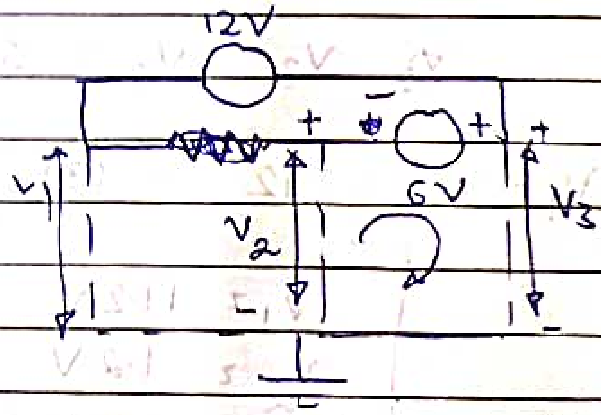
$$10V_1 - 4V_2 + 4V_3 = 0 \quad \text{— (1)}$$

$$\frac{V_2-V_1}{6} - 6 + \frac{V_3}{6} = 0$$

$$V_2 - V_1 - 36 + V_3 = 0$$

$$V_2 + V_3 - V_1 = 36 \quad \text{— (2)}$$

Don't solve like this



$$V_2 + 6 - V_3 = 0$$

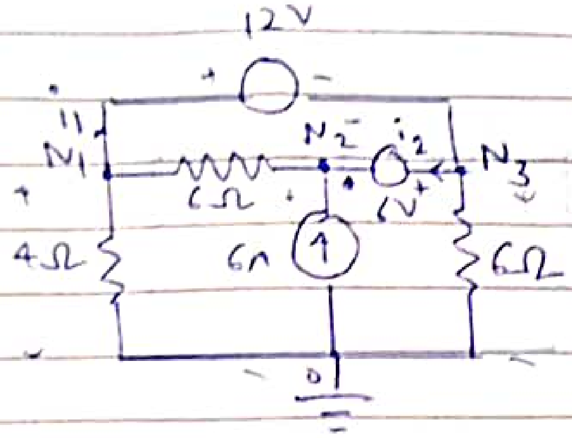
$$V_3 - V_2 = 6$$

$$V_1 = -12/5 = -2.4\,V$$

$$V_2 = 69/5 = 13.8\,V$$

$$V_3 = 99/5 = 19.8\,V$$

PAGE NO:
DATE: / /



$\underline{\underline{N_1}}$ $$\frac{V_1}{4} + \cancel{\frac{V_1 - V_2}{6}} + \cancel{i_1} = 0$$

$\underline{\underline{N_3}}$ $$\cancel{-i_1} + \cancel{i_2} + \frac{V_3}{6} = 0$$

$\underline{N_2}$ $$\cancel{\frac{V_2 - V_1}{6}} - 6 - \cancel{i_2} = 0$$

$$\frac{V_1}{4} + \frac{V_3}{6} = 6 \quad (1)$$

$$6V_1 + 4V_3 = 144 \quad (1)$$

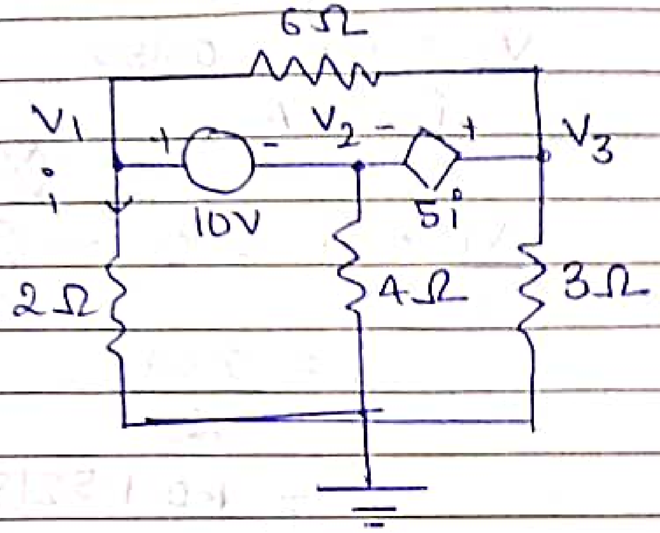
$$V_2 + 6 - V_3 = 0$$

$$V_3 - V_2 = 6 \quad (2)$$

$$V_1 - V_3 = 12 \quad (3)$$

$$V_1 = 19.2V$$
$$V_2 = 1.2V$$
$$V_3 = 7.2V$$

2) Current in 2Ω = ?



$$V_1 - V_2 = 10V \quad \text{— ①}$$

$$V_3 - V_2 - 5i = 0 \quad \text{— ②}$$

$$\frac{V_1}{2} + \cancel{i_2} + \frac{V_1 - V_3}{6} = 0 \quad \text{— ③}$$

$$\cancel{6i_2 + V_1 - V_3 = 0} \quad \text{— ③}$$

$$-\cancel{i_2} + \frac{V_2}{4} - \cancel{i_3} = 0 \quad \text{— ④}$$

$$\frac{V_3 - V_1}{6} + \cancel{i_3} + \frac{V_3}{3} = 0 \quad \text{— ⑤}$$

Add

$6V_1$ 4

$+3V_2 + 2V_3 = 0$

$$\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_3}{3} = 0 \quad \text{— ③}$$

$\cancel{12i + 3V_2 + 4V_3 = 0}$

$$\boxed{\begin{gathered} V_1 - V_3 - 5i + 10 = 0 \\ V_3 + 5i - V_1 = 10 \quad \text{— ④} \end{gathered}}$$

$$6V_1 + 3V_2 + 4V_3 = 0 \quad \text{— ③}$$

$V_1 = 6/7 = 0.85V$

$V_2 = -64/7 V = -9.14 V$

$V_3 = 6/7 = 0.85V$

$i = 2A$

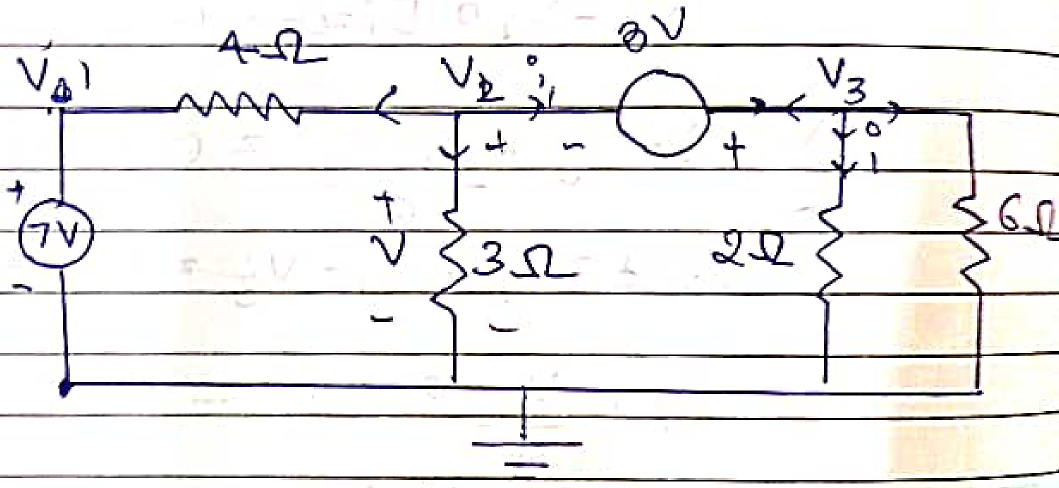
$\frac{V_1}{2} - i = 0$ — (4)

$i = \frac{3.043}{2}$

$= 1.5215 A$

x — x — x — x

Q) Find 'v' and 'i' in the ckt.



$V_1 = 7V$ — (1)

~~$V_2 = V_3$~~ $V_3 - V_2 = 3$ — (2)

$$\frac{V_2 - V_1}{4} + \frac{V_2}{3} + \cancel{i} = 0$$

$$\frac{V_3}{2} - \cancel{i} + \frac{V_3}{6} = 0$$

$$\frac{V_2 - V_1}{4} + \frac{V_2}{3} + \frac{V_3}{2} + \frac{V_3}{6} = 0$$

$$\frac{V_2}{4} - \frac{7}{4} + \frac{V_3}{2} + \frac{V_3}{6} = 0$$

$$3V_2 - 21 + 6V_3 + 2V_3 = 0 \qquad 3V_3 + 8V_3 = 21$$

$$3V_2 + 8V_3 = 21 \quad \text{--- (3)}$$

$$(-)\ 3V_2 - 3V_3 = -9$$

$$11V_3 = 12$$

$$V_3 = \frac{12}{11} V = 1.09V$$

$$V_2 = \frac{45}{11} V = 4.09V$$

$$V_1 = 7V$$

$$i = \frac{V_3}{2} = 0.545A$$

$$3\cancel{V_2} + 8V_3 = 21$$

$$3V_3 - 3\cancel{V_2} = 9$$

$$11V_3 = 30$$

$$V_3 = 2.72V$$

$$V_2 = -0.28V$$

$$i = 1.4A$$

PAGE NO.:
DATE:

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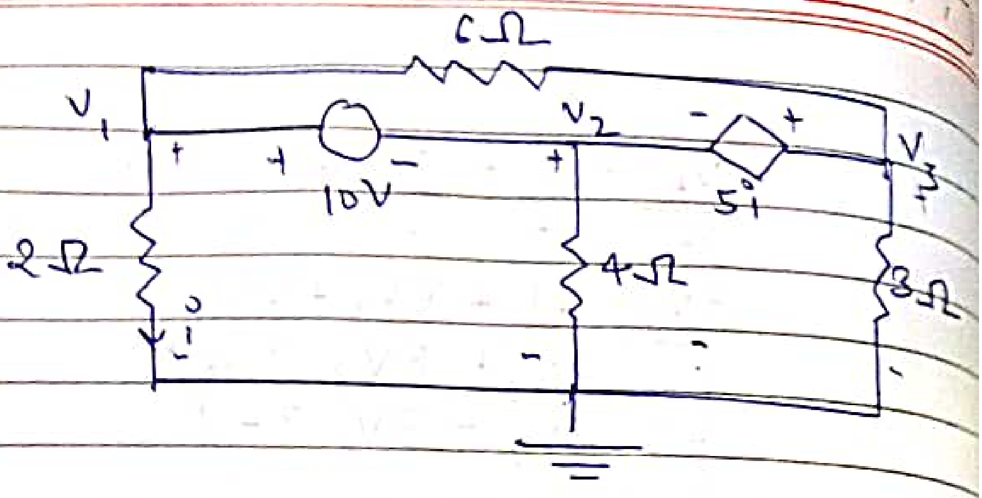


$$V_1 - 10 - V_2 = 0$$

$$V_1 - V_2 = 10 \quad \text{—— (1)}$$

$$V_2 + 5i - V_3 = 0$$

$$V_3 - V_2 = 5i \quad \text{—— (2)}$$

$$\frac{V_1}{2} = i$$

$$V_1 = 2i \quad \text{—— (3)}$$

$$V_3 - V_2 = \frac{5V_1}{2}$$

$$\frac{5V_1}{2} + V_2 - V_3 = 0 \quad \text{—— (2)}$$

$$\frac{V_1}{2} + \cancel{\frac{V_1 - V_3}{6}} + \cancel{i_1} = 0$$

$$-\cancel{i_1} + \frac{V_2}{4} + \cancel{i_2} = 0$$

$$-\cancel{i_2} + \cancel{\frac{V_3 - V_1}{6}} + \frac{V_3}{3} = 0$$

$$\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_3}{3} = 0$$

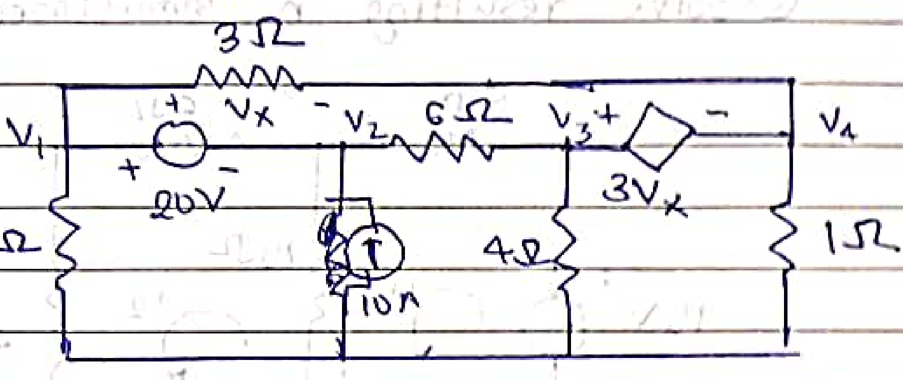
$$6V_1 + 3V_2 + 4V_3 = 0 \quad \text{—— (4)}$$

PAGE NO.:
DATE: / /

$V_1 = 3.043$ V
$V_2 = -6.956$ V
$V_3 = 0.6529$ V

$i = \frac{V_1}{2} = 1.5215$ A

HW :



Ans:
$V_1 = 26.67$V
$V_2 = 6.67$V
$V_3 = 173.33$V
$V_4 = -46.66$V

Mesh - Analysis

A mesh is a loop that does not contain any other loop within it. Mesh analysis is applicable to planar ckts. only.

A planar ckt is one that can be drawn in a plane with not branches crossing one another.

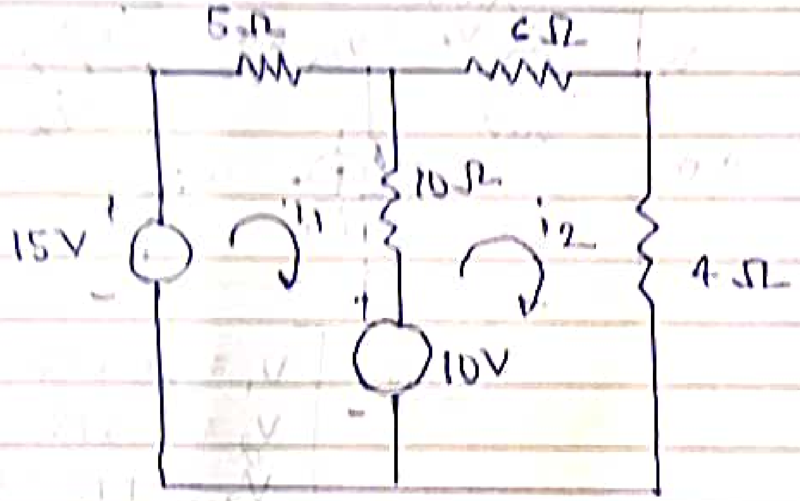
PAGE NO.
DATE / /

Steps

① Assign mesh currents i_1, i_2, i_3 to n meshes.

② Apply KVL and use ohm's law to express the voltages in terms of mesh current.

③ Solve resulting n simultaneous equations



$$15 - 5i_1 - 10(i_1 - i_2) - 10 = 0$$

$$15 - 5i_1 - 10i_1 + 10i_2 - 10 = 0$$

$$10i_2 - 15i_1 + 5 = 0$$

$$3i_1 - 2i_2 = 1 \quad —— ①$$

$$-6i_2 - 4i_2 + 10 - 10(i_2 - i_1) = 0$$

$$-10i_2 + 10 - 10i_2 + 10i_1 = 0$$

$$10i_1 - 20i_2 + 10 = 0$$

$$2i_2 - i_1 = 1 \quad —— ②$$

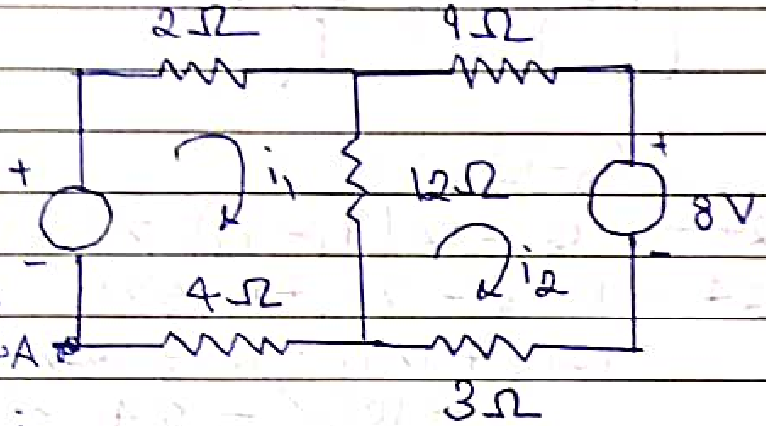
$2i_1 = 2$

$\boxed{i_1 = 1\,A}$

$\boxed{i_2 = 1\,A}$

HW :

Q)



$i_1 = 0.66A$
$i_2 = 0.33A$

HW1 : 12

$$V_1 + 20 - V_2 = 0$$

$$V_2 - V_1 = 20 \quad \text{—①}$$

$$V_3 - V_4 = 3V_x \quad \text{—②}$$

$$\cancel{\frac{V_4}{1} - V_1 = V_x} \quad \text{—③}$$

$$V_4 - V_1 = V_x \quad \text{—③}$$

$$\frac{V_1}{2} + \frac{V_1 - V_4}{3} + i_1 = 0$$

$$\frac{V_2 - V_3}{6} - 10 - i_1 = 0$$

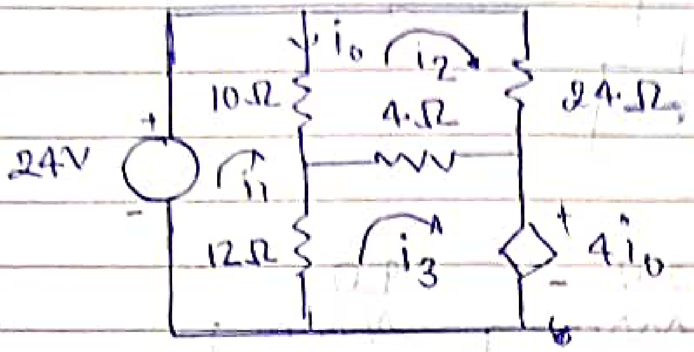
$$\frac{V_3 - V_2}{6} + \frac{V_3}{4} + i_2 = 0$$

$$\frac{V_4 - V_1}{3} + \frac{V_4}{1} - i_2 = 0$$

$$\frac{V_1}{2} + \frac{V_2}{2} - \frac{V_3}{6}$$

$$\frac{V_1}{2} - 10 + \frac{V_3}{4} + \frac{V_4}{1} = 0 \quad \text{—④}$$

Q) Find current i_0 in the ckt.



$(i_0 - i_2)$

$24 - 10 i_0 - 12(i_1 - i_3) = 0$

$24 - 10 i_0 - 12 i_1 + 12 i_3 = 0$

$10 i_0 + 12 i_1 - 12 i_3 = 24 \rightarrow ①$

$- 10 i_2 = 24$

$-24 i_2 - 4(i_2 - i_3) - 10(i_2 - i_0)$

$-24 i_2 - 4 i_2 + 4 i_3 - 10 i_2 + 10 i_0$

$10 i_0 - 38 i_2 + 4 i_3 = 0 — ②$

$12(i_3 - i_1) - 4(i_3 - i_2) - 4 i_0 = 0$

$12 i_3 - 12 i_1 - 4 i_3 + 4 i_2 - 4 i_0 = 0$

$-12 i_1 + 8 i_3 - 8 i_2 - 4 i_0 = 0$

$2 i_3 - 2 i_2 - i_0 = 0 — ③$

$i_1 - i_0 - i_2 = 0 — ④$

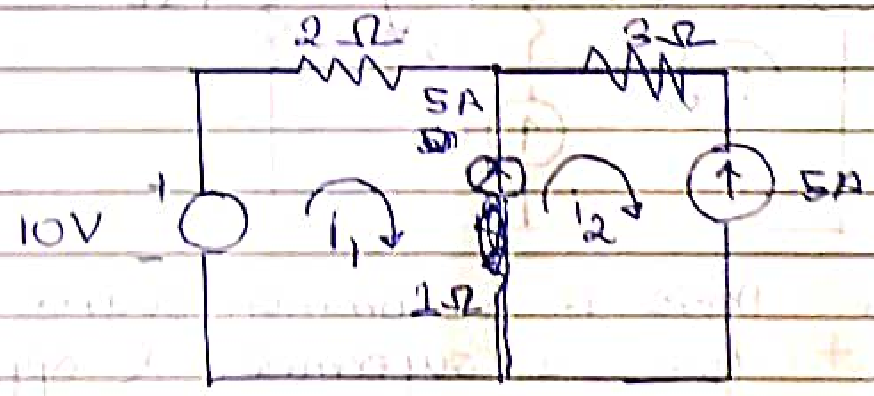
$$\left[\begin{array}{ll} i_1 = 2.603\,A & \\ i_2 = 0.679\,A & \\ i_3 = 1.641\,A & i_0 = 1.92\,A \end{array}\right]$$

$\cancel{24 - 10(i_0 - i_2)}$

$$24 - 10i_0 - 12(i_1 - i_3) = 0$$
$$24 - 10i_0 - 12i_1 + 12i_3 = 0$$
$$12i_1 - 12i_3 + 10i_0 = 24 \quad -(1)$$

$$\left[\begin{array}{l} i_1 = 2.029\text{ A} \\ i_2 = 0.529\text{ A} \\ i_3 = 1.279\text{ A} \\ i_4 \; \underline{i_0 = 1.5\text{ A}} \end{array}\right] \quad \left\{\begin{array}{l} 2.25 \\ 0.75 \\ 1.5 \\ 1.5 \end{array}\right\}$$

Q) <u>Mesh Analysis with current Sources</u>



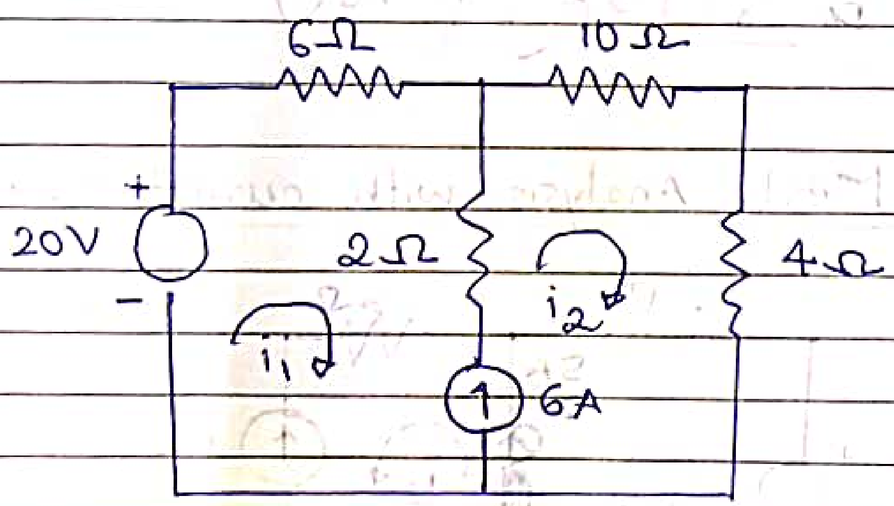
<u>Super-mesh Analysis</u>

When current source

* It exists b/w 2 meshes, then we create a supermesh by excluding the current source and any elements connected in series with it.

* Apply KVL for the supermesh without changing the branch currents.

* Restore the current source and apply KCL at the corresponding node.



Since there is a common source is present form a supermesh & apply KVL without changing current directions

$$20 - 6i_1 - 10i_2 - 4i_2 = 0$$

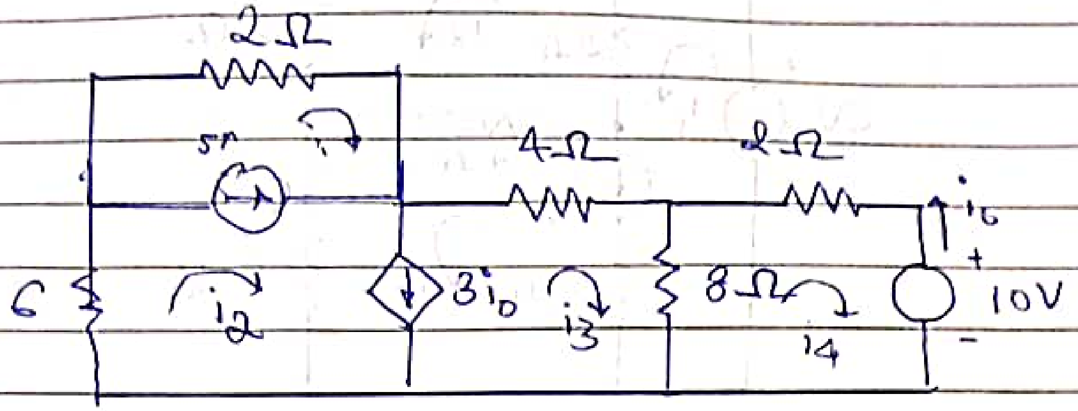
$$6i_1 + 14i_2 = 20 \quad \text{—} \ (1)$$

$$i_1 + 6 = i_2$$

$$i_2 - i_1 = 6 \quad \text{—} \ (2)$$

$$\left[i_1 = -3.2A \qquad i_2 = 2.8A \right]$$

Q) find the current



~~$2i_1$~~

$-2i_1 - 4i_3 - 2i_4 - 10 - 6i_2 = 0$

$i_1 + 3i_2 + 2i_3 + i_4 + 10 = 0$ — (1)

7.5
$i_1 = -1$
$i_2 = -2.5A$
$i_3 = 3.92A$
$i_4 = 2.14A$

~~$i_1 = -12.5A$~~
~~$i_2 = -7.5A$~~
~~$i_3 = 9.6A$~~
~~$i_4 = 5.77A$~~

$-2i_4 - 10 - 8(i_4 - i_3) = 0$

$-2i_4 - 10 - 8i_4 + 8i_3 = 0$

$8i_3 - 10i_4 = 10$

$4i_3 - 5i_4 = 5$ — (2)

$-2i_1 - 4i_3 - 8(i_3 - i_4) - 6i_2 = 0$

$-2i_1 - 4i_3 - 8i_3 + 8i_4 - 6i_2 = 0$

$-2i_1 - 6i_2 - 12i_3 + 8i_4 = 0$

$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$ — (1)

$i_2 = 5 + i_1$

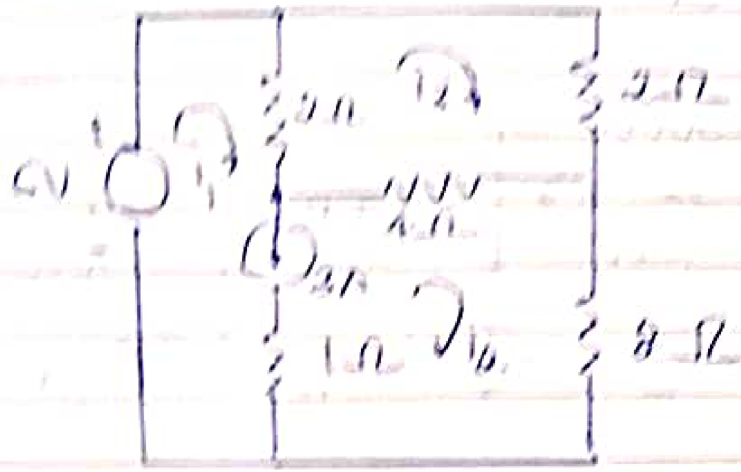
$i_2 - i_1 = 5$ — (3)

$5 + i_1 = 3i_0 + i_3$

$-i_1 + i_3 + 3i_0 = 5$ — (4)

$i_0 = -i_4$ — (5)

HW :



$$6 - 2(i_1 - i_2) - 4(i_2 - i_3) - 8i_2 = 0 \quad —①$$

$$-2(i_3 - i_1) - 2(i_3) - 4(i_3 - i_2) = 0 \quad —②$$

$$i_1 - i_3 = i_2 + 3$$

$$i_1 - i_2 - i_3 = 3 \quad —③$$

$$-2i_3 + 2i_1 - 2i_3 - 4i_3 + 4i_2 = 0$$

$$2i_1 + 4i_2 - 8i_3 = 0$$

$$i_1 + 2i_2 - 4i_3 = 0 \quad —④$$

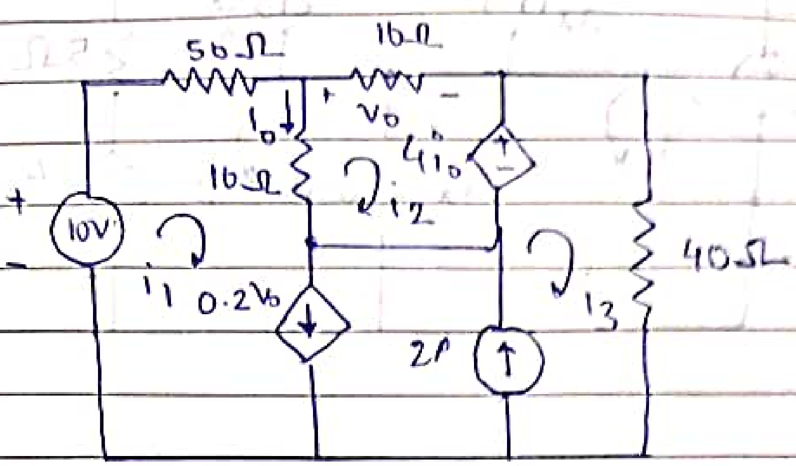
$$\cancel{-2i_3 + 2i_1 - 2i_3} \quad 6 - 2i_1 + 2i_3 - 4i_2 + 4i_3 - 8$$

$$2i_1 + 12i_2 - 6i_3 = 6$$

$$i_1 + 6i_2 - 2i_3 = 3 \quad —$$

$$\begin{bmatrix} i_1 = 4.8A \\ i_2 = 0.4A \\ i_3 = 1.4A \end{bmatrix}$$

Youtute



$$10 - 50i_1 - 10(i_1 - i_2) + 4i_0 - 40i_3 = 0$$

$$10 - 60i_1 + 10i_2 + 4i_0 - 40i_3 = 0$$

$$60i_1 - 10i_2 + 40i_3 - 4i_0 = 10 \quad \text{— (1)}$$

$$10(i_2 - i_1) - 10i_2 - 4i_0 = 0$$

$$-10i_1 - 4i_0 = 0$$

$$10i_1 + 4i_0 = 0 \quad \text{— (2)}$$

$$10i_2 = V_0 \quad \text{— (3)}$$

$$i_1 - \cancel{i_2} + \cancel{i_2} = 0.2V_0$$

$$i_1 = 0.2V_0 \quad \text{— (4)}$$

$$i_1 = 2i_2 \quad \text{— (6)}$$

$$\cancel{i_2} - i_3 + 2 = \cancel{i_2}$$

$$i_3 = 2A \quad \text{— (5)}$$

$$60i_1 - 10i_2 + 40 \times 2 - \cancel{4}\left(\frac{-10}{\cancel{4}} i_1\right) = 10$$

$$70i_1 - 10i_2 + 70 = 0 \quad \text{— (7)}$$

$i_1 = -0.928A$

$i_2 = -0.464A$

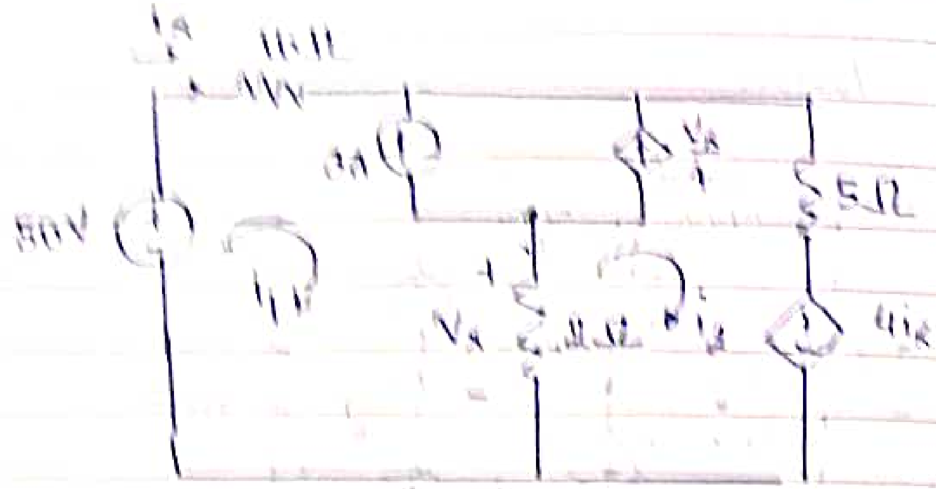
$i_3 = 2A$

$i_0 = 2.32A$

$V_0 = -4.64V$

$70i_1 - 5i_1 + 70 = 0$

$i_1 = -0.928$



$50 - 10i_1 - 5i_2 - 4i_x = 0$ — (1)

$i_1 = i_x$ — (2)

$2(i_2 - i_1) = V_x$ — (3)

$3 + \frac{V_x}{4} = i_2 - i_1$ — (4)

$3 + \frac{i_2 - i_1}{2} = i_2 - i_1$

$3 \cdot 2 = i_2 - i_1$

$i_2 - i_1 = 6$ — (5)

$\cancel{10i_1 - 5i_2 - 4i_1 = 50}$

$\cancel{6i_1 - 5i_2 = 50}$ — (6)

$\cancel{i_1 = 80A}$

$\cancel{i_2 = 86A}$

$\cancel{i_x = 80A}$

$\cancel{V_x = 12V}$

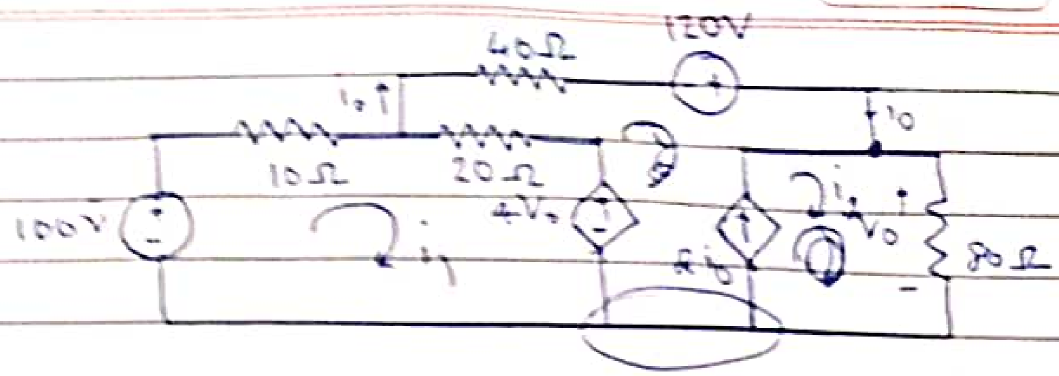
$50 = 14i_1 - 5i_2$ — (6)

$i_1 = 2.165A$

$i_2 = 4.165A$

$V_x = -4V$

PAGE NO.:
DATE / /



$100 - 10i_1 - 20(i_1 - i_0) - 4V_0 = 0$

$100 = 10i_1 + 20i_1 - 20i_0 + 4V_0$

$30i_1 - 20i_0 + 4V_0 = 100$

$15i_1 - 10i_0 + 2V_0 = 50$ — ①

$100 - 10i_1 - 40i_0 + 120 - 80i_2 = 0$

$i_1 + 8i_2 + 4i_0 = 22$ — ②

$80i_2 - V_0 = 0$ — ③

$\cancel{i_2 - 2i_0 + (i_1 - i_0)}$ $\cancel{3i_0 = i_2}$

$i_2 + \cancel{(i_1 - i_0)} - 3i_0 - \cancel{(i_1 - i_0)} = 0$

$i_2 = 3i_0$ — ④

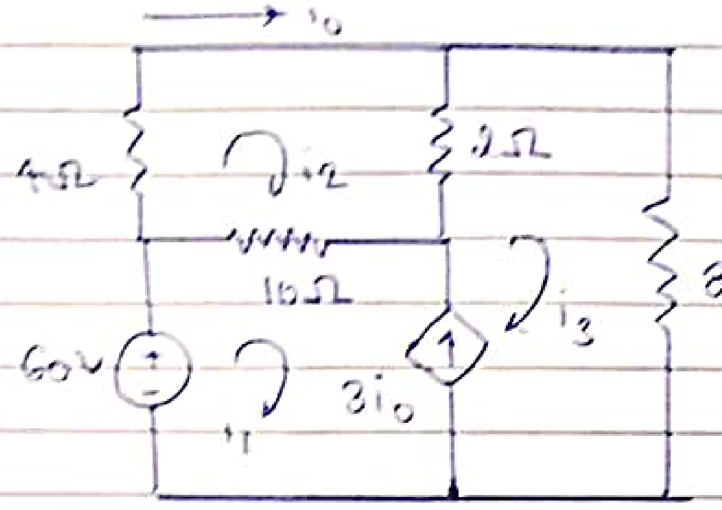
$i_1 = 178.8$ A

$i_2 = -5.6$A

$i_0 = -16.8$A

$V_0 = -1344$V

9)



$$60 - 10(i_1 - i_2) - 2(i_3 - i_2) - 8i_3 = 0$$
$$60 - 10i_1 + 10i_2 - 2i_3 + 2i_2 - 8i_3 = 0$$
$$60 - 10i_1 + 12i_2 - 10i_3 = 0$$
$$5i_1 - 6i_2 + 5i_3 = 30 \quad \text{—— (1)}$$

$$0 - 4i_2 - 2(i_2 - i_3) - 10(i_2 - i_1) = 0$$
$$-4i_2 - 2i_2 + 2i_3 - 10i_2 + 10i_1 = 0$$
$$10i_1 - \cancel{14} 16i_2 + 2i_3 = 0$$
$$5i_1 - 8i_2 + i_3 = 0 \quad \text{—— (2)}$$

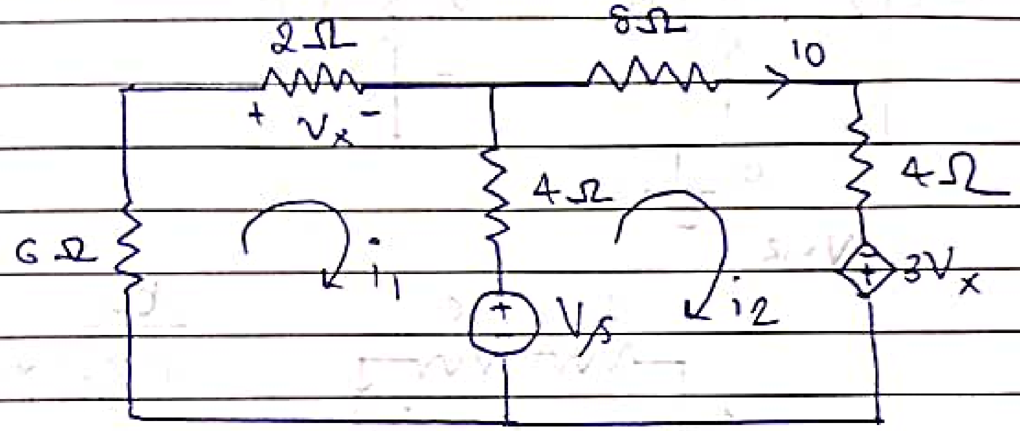
$$i_3 = 3i_0 + i_1 \quad \text{—— (3)}$$
$$i_1 - i_3 + 3i_0 = 0 \quad \text{—— (3)}$$

$$i_2 - i_0 = 0 \quad \text{—— (4)}$$

$i_1 = 1.442$ A
$i_2 = 1.731$ A
$i_3 = 6.63$ A
$i_0 = 1.73$ A

Circuit Theorems

GATE Questions



Find : i_0 when $V_s = 12V$ & $V_s = 24V$

V_s

$-6i_1 - 2i_1 - 4(i_1 - i_2) - \cancel{12}\,V_s = 0$

$-8i_1 - 4i_1 + 4i_2 - \cancel{12}\,V_s = 0$

$4i_2 - 12i_1 = \cancel{12}\,V_s$

$i_2 - 3i_1 = \frac{\cancel{3}V_s}{4}$ — (1)

$3i_1 + \frac{V_s}{4}$

$-8i_2 - 4i_2 + 3(2i_1) + V_s - 4(i_2 - i_1) = 0$

$\frac{-9V_s}{38} + \frac{V_s}{4}$

$-12i_2 + 6i_1 + V_s - 4i_2 + 4i_1 = 0$

$\frac{-36V_s + 38V_s}{4\times 38} = 2V_s/76$

$10i_1 - 16i_2 + V_s = 0$

i_0

$8i_2 - 5i_1 = \frac{V_s}{2}$ — (2)

$i_2 = 3i_1 + \frac{V_s}{4}$

$8i_2 - 24i_1 = 2V_s$

$= -\frac{3V_s}{38} + \frac{V_s}{4}$

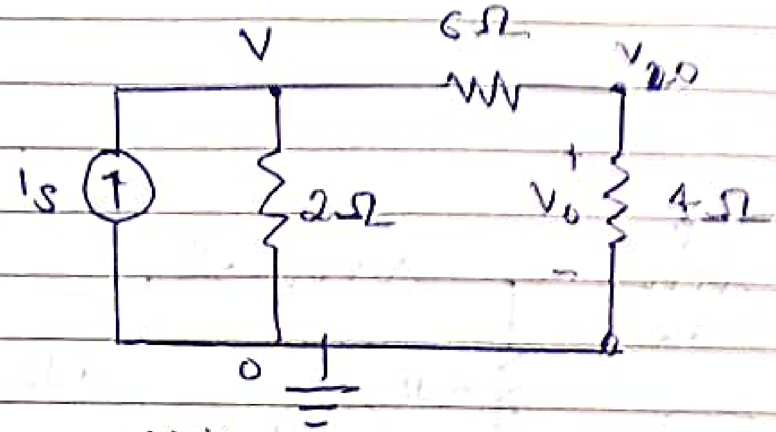
$19i_1 = -3V_s/2$

$= \frac{-12V_s + 38V_s}{4\times 38}$

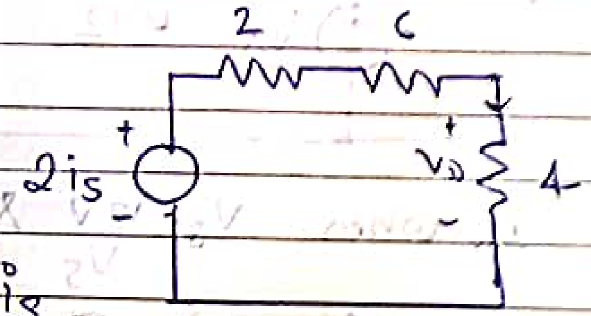
$i_1 = \frac{-3\times V_s}{38}$

$i_2 = 26V_s/4\times 38 = \boxed{13V_s/76}$

Q) Find V_0 in the ckt when $i_s = 15A$ & $30A$



$V = iR$



$\frac{12}{9}$ 2/4

$\frac{24}{15}$

$i = \frac{12}{2i_s}$

$= \frac{6}{i_s}$ ~~Req = 12 Ω~~

10V

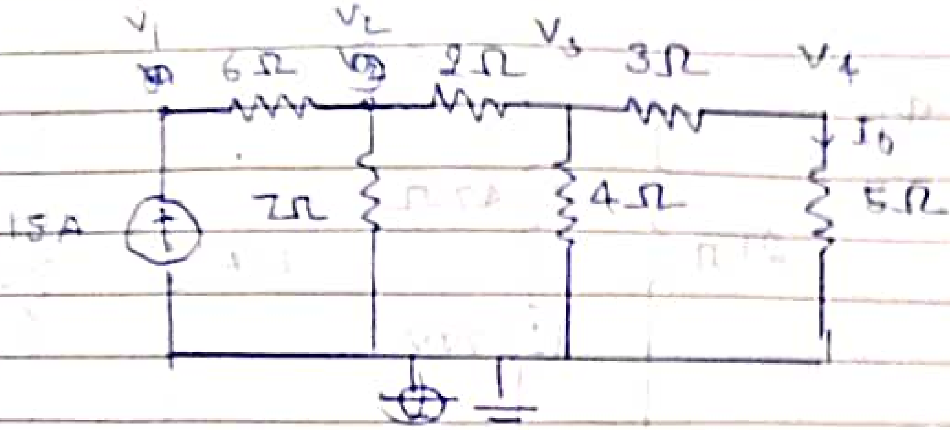
20V

$\left(\frac{6}{i_s}\right)4$ $\quad V_0 = \cancel{\left(\frac{2i_s}{3}\right)4}$

$\frac{24}{i_s}$ $\quad = \cancel{\frac{8i_s}{3}}$

$\frac{V}{2} + \frac{V - V_0}{6} - i_s = 0$

$\frac{V_0}{4} + \frac{V_0 - V}{6} = 0$



$$15 = \frac{V_1 - V_2}{6}$$

$$V_1 - V_2 = 90 \quad \text{--- (1)}$$

$$\frac{V_2 - V_1}{6} + \frac{V_2 - V_3}{2} + \frac{V_2}{7} = 0 \quad \text{--- (2)}$$

$$\frac{V_3 - V_2}{2} + \frac{V_3 - V_4}{3} + \frac{V_3}{4} = 0 \quad \text{--- (3)}$$

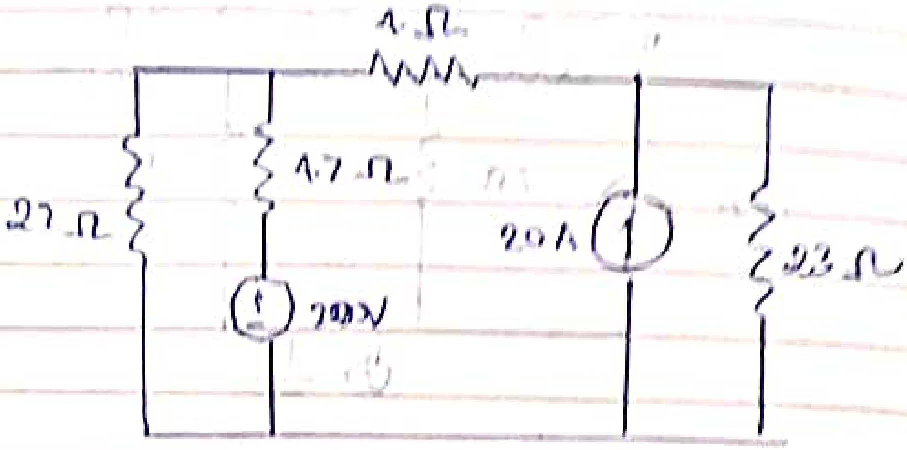
$$\frac{V_4}{5} = \frac{V_4 - V_3}{3} \quad \text{--- (4)}$$

Superposition Theorem

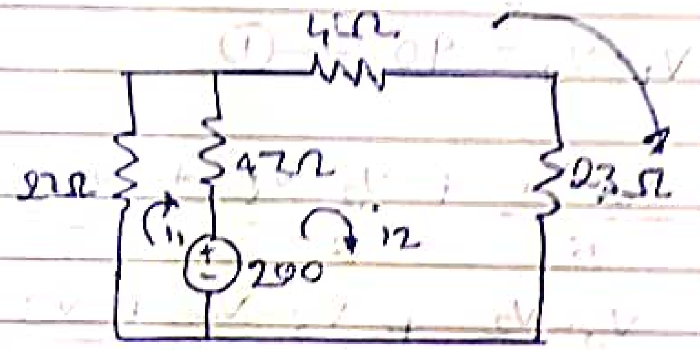
It states that the response in a linear circuit having more than 1 independent source can be obtained by adding the responses caused by the sources each acting one at a time.

When a single source is considered the other independent voltage sources are short circuited and independent current sources are open circuited.

Q)

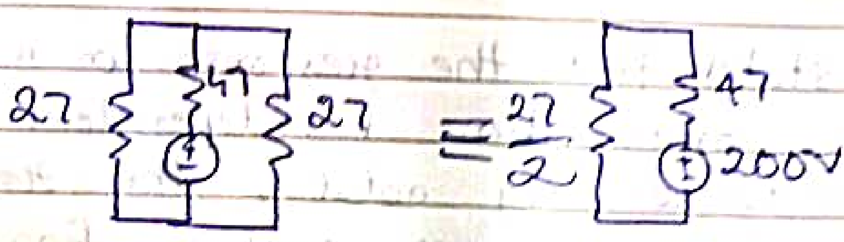


find current in 23 Ω resistor



$$-27i_1 - 47(i_1 - i_2) - 200 = 0$$
$$-27i_1 - 47i_1 + 47i_2 = 200$$
$$47i_2 - 74i_2 = 200 \quad \text{--- (1)}$$

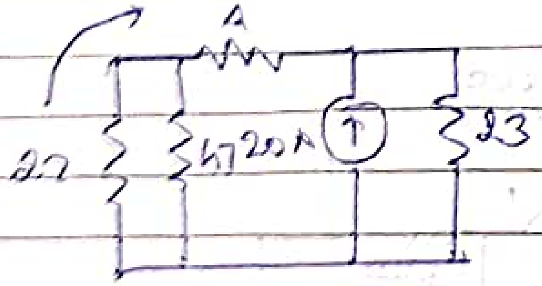
$$200 - 47(i_2 - i_1) - 4i_2 - 28i_2 = 0$$

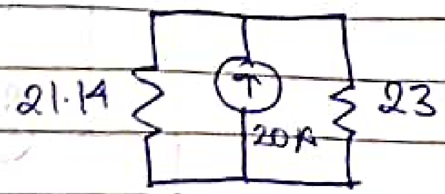


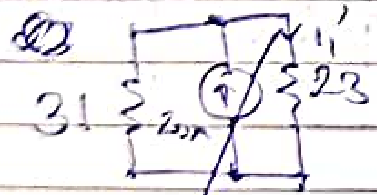
$$i = \frac{200}{60.5} = 3.3\ A$$

$i = 20.25\ A$

$$i_1 = \frac{3.3}{2} = 1.65\ A$$



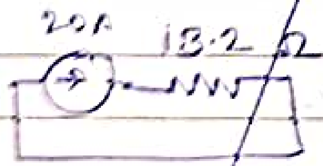




$R_{eq} = 11.015$

$V = 220.308$

$\left[i_1' = 9.58\,A \right]$



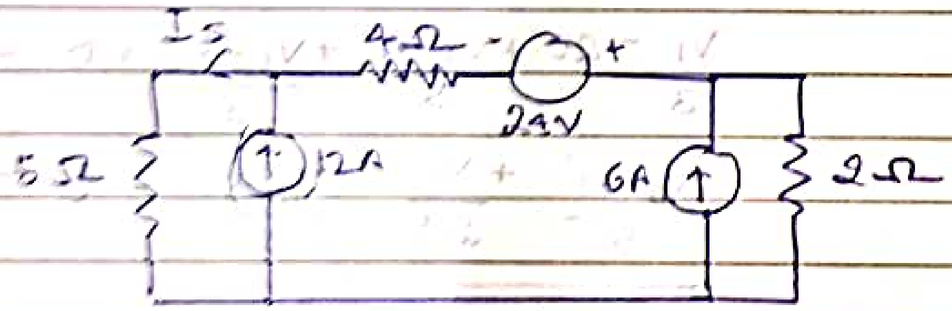
$V = 264.07\,V$

$\left[\cancel{i_1' = 11.48\,A} \right]$

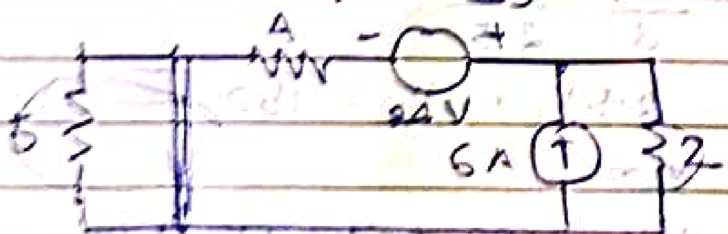
$i_T = i_1 + i_1'$

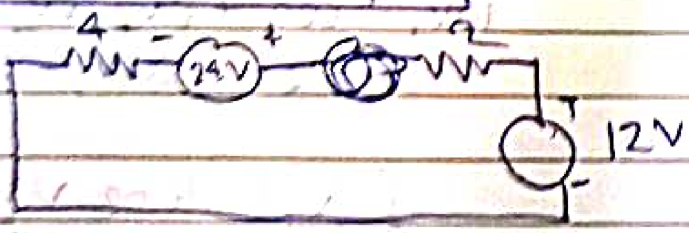
$= 1.65 + 9.58$

$\underline{11.23\,A}$



find I_5 in ckt ?



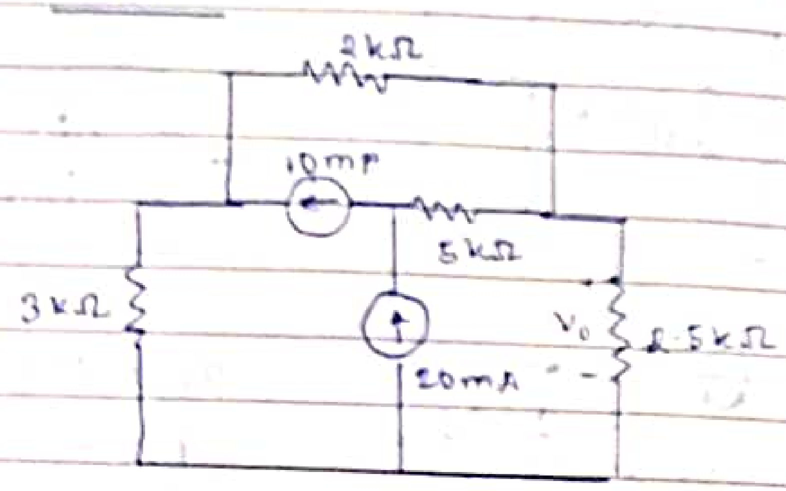


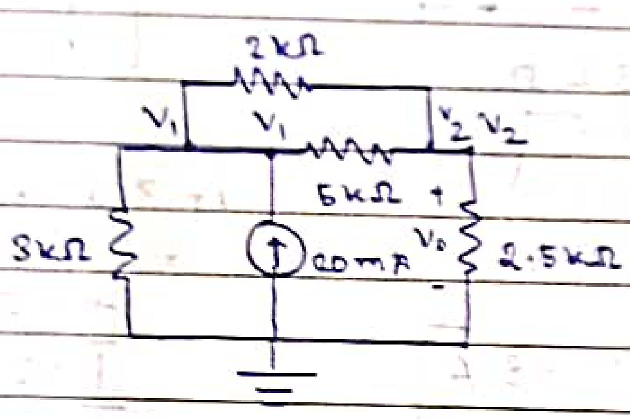
$-4i + 24 - 2i - 12 = 0$

$12 = 6i$

$\left[i = 2A \right]$

Youtube





$$\frac{V_1}{3} - 20 + \frac{V_1 - V_2}{5} + \frac{V_1 - V_2}{2} = 0 \quad \text{— (1)}$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{2.5} + \frac{V_2 - V_1}{2} = 0 \quad \text{— (2)}$$

$$\frac{V_1}{3} + \frac{V_2}{2.5} = 20$$

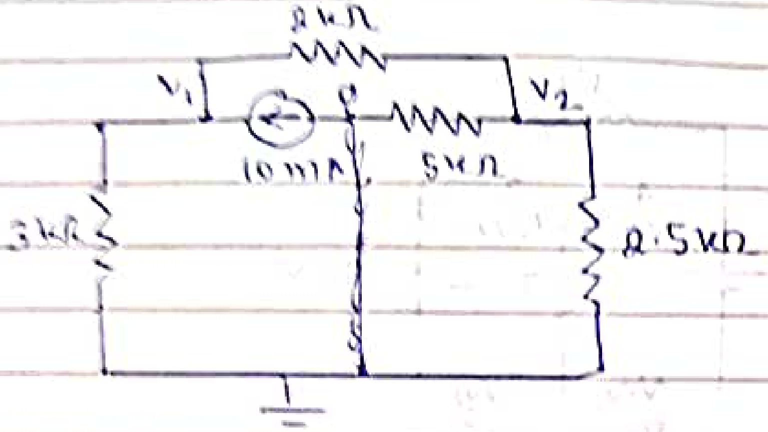
$$2.5V_1 + 3V_2 = 150 \quad \text{— (1)}$$

$$10V_1 - 600 + 6V_1 - 6V_2 + 15V_1 - 15V_2 = 0$$

$$31V_1 - 21V_2 = 600 \quad \text{— (2)}$$

$$\begin{cases} V_1 = 84.02\ V \\ V_2 = 21.64\ V \end{cases}$$

$$i_{01} = \frac{V_2}{2.5} = 13.608\ mA$$



$$\frac{V_1}{3} + \frac{V_1 - V_2}{2} - 10 = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{2.5} + 10 = 0$$

$$\frac{V_1}{3} + \frac{V_2}{2.5} = 0$$

$$2.5V_1 = -3V_2 \quad — \text{①}$$

$$5V_1 = -6V_2$$

$$2V_1 + 3V_1 - 3V_2 - 60 = 0$$

$$5V_1 - 3V_2 = 60 \quad — \text{②}$$

$$-9V_2 = 60$$

$$V_2 = \frac{-20}{3} = -6.67\,V$$

$$\boxed{V_1 = 8\,V}$$

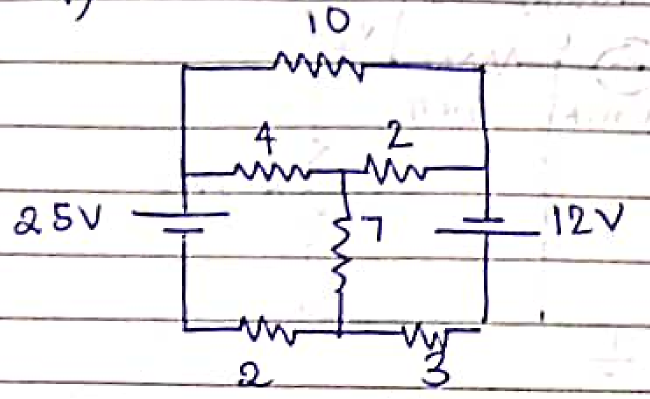
$$i_{02} = \frac{V_2}{2.5} = 2.665\,mA$$

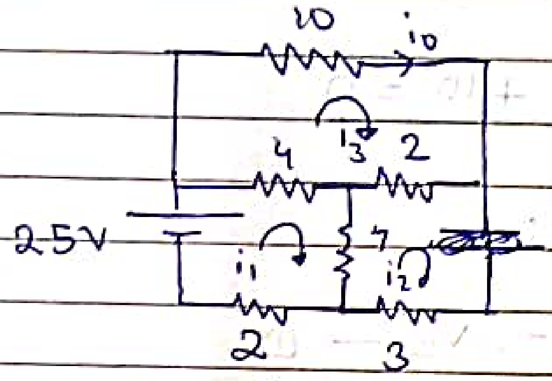
$$i = 10.94\,mA$$

$$V_0 = i_0 R$$

$$= 27.35\,V$$

Q)





$25 - 4(i_1 - i_3) - 7(i_1 - i_2) - 2i_1 = 0$

$25 - 4i_1 + 4i_3 - 7i_1 + 7i_2 - 2i_1 = 0$

$-13i_1 + 7i_2 + 4i_3 = -25$ — (1)

$-7(i_2 - i_1) - 2(i_2 - i_3) - 3i_2 = 0$

$-7i_2 + 7i_1 - 2i_2 + 2i_3 - 3i_2 = 0$

$7i_1 - 12i_2 + 2i_3 = 0$ — (2)

$-10i_3 - 2(i_3 - i_2) - 4(i_3 - i_1) = 0$

$-10i_3 - 2i_3 + 2i_2 - 4i_3 + 4i_1 = 0$

$4i_1 + 2i_2 - \cancel{18}\ 16i_3 = 0$ — (3)

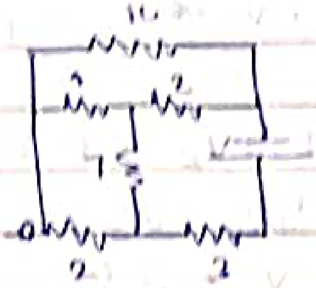
$i_1 = 3.466$ A

$i_2 = 2.21$ A

$i_3 = 1.14$ A

$i_{01} = i_3 = 1.14$ A

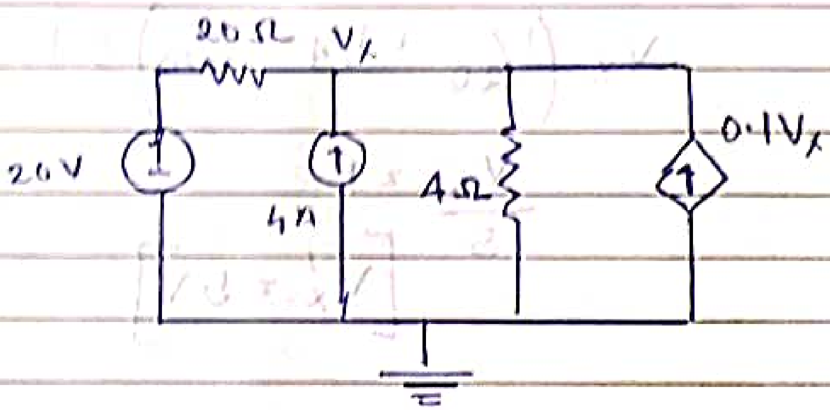
PAGE NO.:
DATE / /

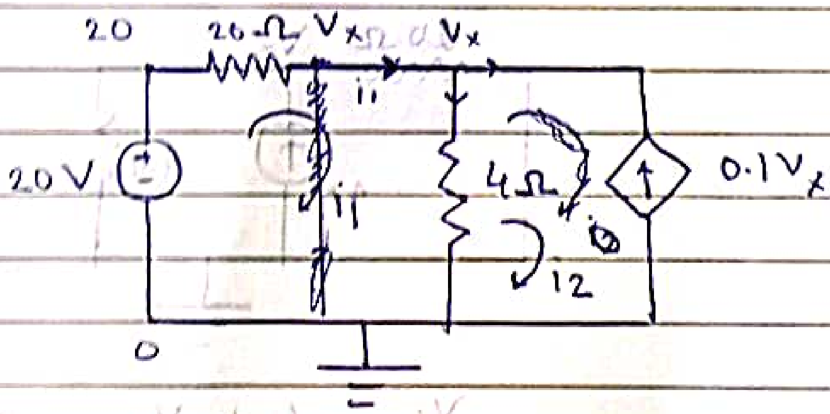


iii) $i_{02} = 0.47A \rightarrow$

$i_0 = 1.861\,A$

(Q)





$$20 - 20i_1 - 4(i_1 - i_2) = 0$$

$$20 - 20i_1 - 4i_1 + 4i_2 = 0$$

$$24i_1 - 4i_2 = 20$$

$$6i_1 - i_2 = 5 \quad \text{— (1)}$$

$$\cancel{i_1} = \cancel{i_1} - i_2 - 0.1V_x$$

$$i_2 = -0.1V_x \quad \text{— (2)}$$

V_x

$$\frac{V_x - 20}{20} + \frac{V_x - \cancel{\ \ }}{4} - 0.1V_x = 0$$

$$\cancel{V_x - 20 + 5V_x - 0.5V_x = 0}$$

$$\cancel{5.5V_x = 20}$$

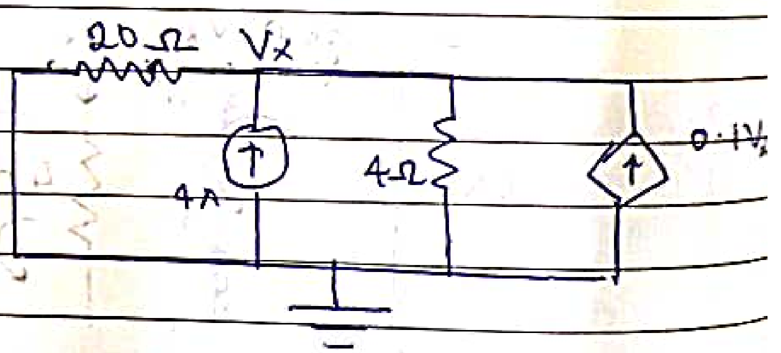
$$\cancel{V_x = 3.63}$$

$$\frac{V_x}{20} - 1 + \frac{V_x}{4} - \frac{V_x}{10} = 0$$

$$V_x\left(\frac{1}{20} + \frac{1}{4} - \frac{1}{10}\right) = 1$$

$$\frac{V_x}{5} = 1$$

$$\boxed{V_x' = 5V}$$



$$\frac{V_x}{20} - 4 + \frac{V_x}{4} - \frac{V_x}{10} = 0$$

$$\frac{5V_x}{20} - \frac{V_x}{20} = 4$$

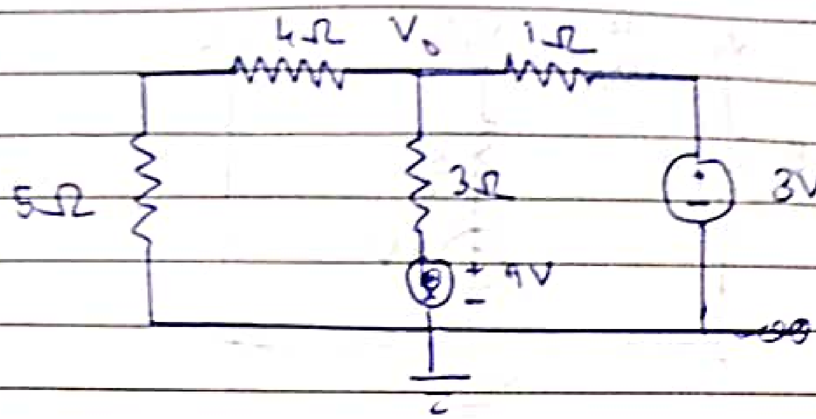
$$\frac{4V_x}{20} = 4$$

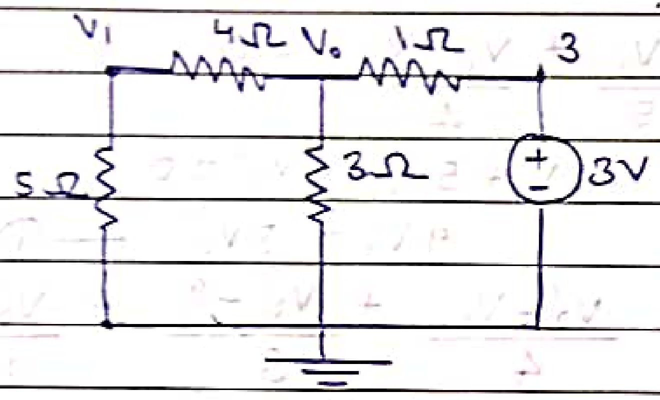
$$V_x'' = 20V$$

$$\boxed{V_x = 25V}$$

PAGE NO.:
DATE / /

Q)





$$\frac{V_0-3}{1} + \frac{V_0}{3} + \frac{V_0-V_1}{4} = 0$$

$$12V_0 - 36 + 4V_0 + 3V_0 - 3V_1 = 0$$

$$19V_0 - 3V_1 = 36 \quad \text{—} \; (1)$$

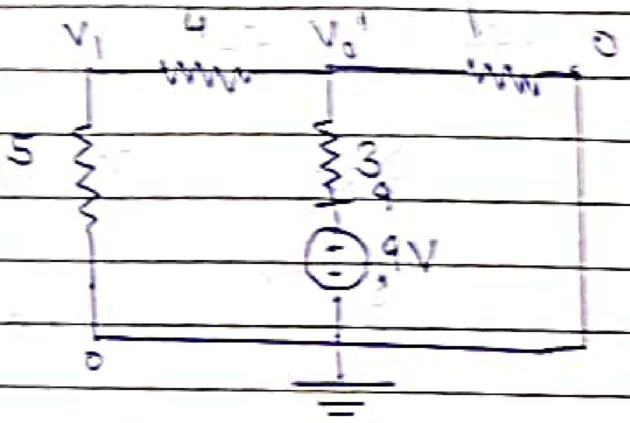
$$\frac{V_1}{5} + \frac{V_1 - V_0}{4} = 0$$

$$\frac{9V_1}{20} - \frac{V_0}{4} = 0$$

$$9V_1 - 5V_0 = 0 \quad \text{—} \; (2)$$

$$V_0' = 2.07\,V$$

$$V_1 = 1.153\,V$$



$$\frac{V_1}{5} + \frac{V_1 - V_0''}{4} = 0$$

$$4V_1 + 5V_1 - 5V_0'' = 0$$

$$9V_1 = 5V_0'' \quad — (1)$$

$$\frac{V_0'' - V_1}{4} + \frac{V_0'' - 9}{3} + \frac{V_0''}{1} = 0$$

$$\frac{V_0''}{4} + \frac{V_0''}{3} + V_0'' - \frac{V_1}{4} - 3 = 0$$

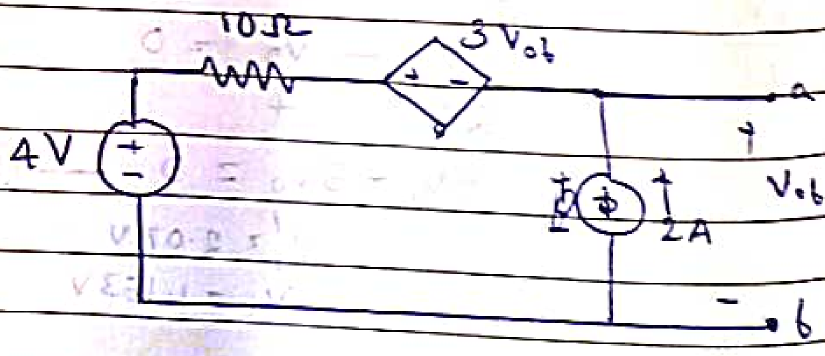
$$\frac{19V_0''}{12} - \frac{3V_1}{12} - \frac{36}{12} = 0$$

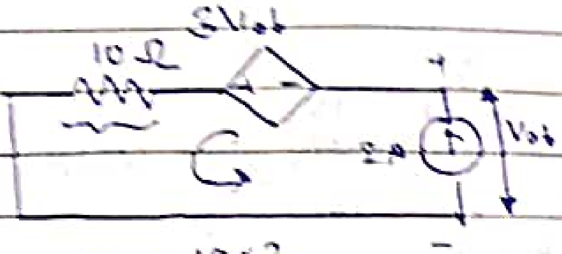
$$19V_0'' - 3V_1 = 36 \quad — (2)$$

$$V_0'' = 2.07V$$

$$V_0 = 4.14V$$

Q)





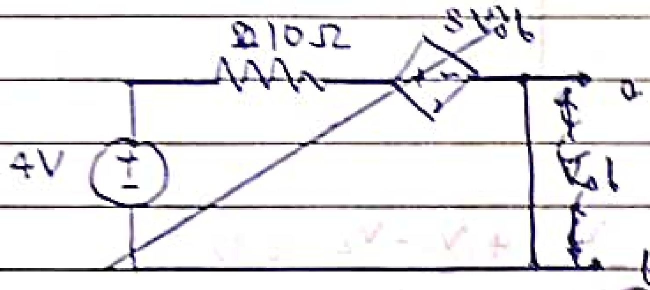
$V_R = 10 \times 2$
$= 20V$

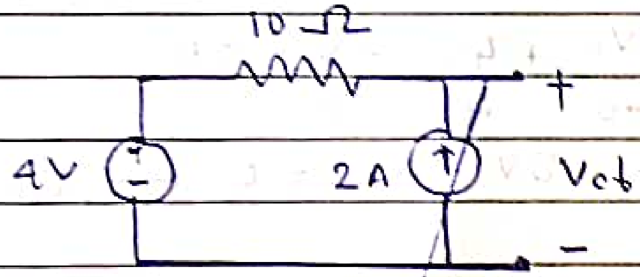
$V_{ab} = V'_{ab} + V''_{ab}$

$\therefore \; 3V'_{ab} - V_R + V'_{ab} = 0$

$4V'_{ab} = 20$

$V'_{ab} = 5V$





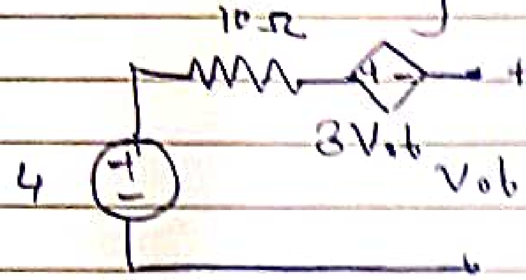
{Do not remove current/voltage dependent sources}

$V_R = 20V$

$V''_{ab} - V_R - 4 = 0$

$V''_{ab} = 24 \; V$

$V_{ab} = 29V$

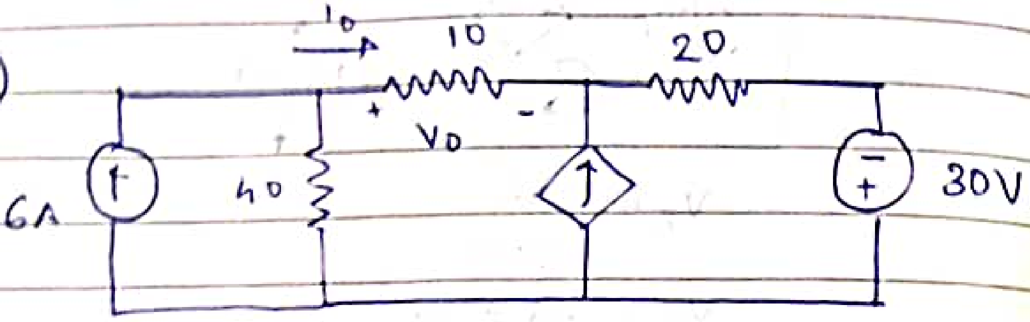


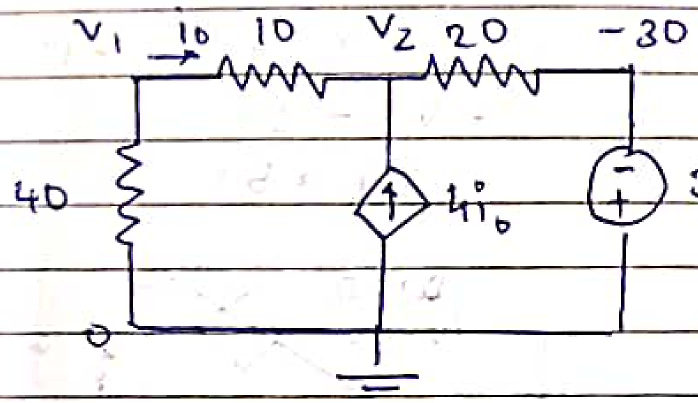
$4 - 0 - 3V'''_{ab} - V'''_{ab} = 0$

$V'''_{ab} = 1V$

$\boxed{V_{ab} = 6V}$

Q)





$$\frac{V_1}{40} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_1}{40} + \frac{4V_1}{40} - \frac{V_2}{10} = 0$$

$$5V_1 - 4V_2 = 0 \quad \text{—} \quad (1)$$

V_2: $$\frac{V_2 - V_1}{10} + \frac{V_2 + 30}{20} - 4\left(\frac{V_1 - V_2}{10}\right) = 0$$

$$\left(\frac{V_2 - V_1}{10}\right) + 4\left(\frac{V_2 - V_1}{10}\right) + \frac{V_2 + 30}{20} = 0$$

$$2\left(\frac{5V_2 - 5V_1}{20}\right) + \frac{V_2 + 30}{20} = 0$$

$$10V_2 - 10V_1 + V_2 + 30 = 0$$

$$11V_2 - 10V_1 = -30 \quad \text{—} \quad (2)$$

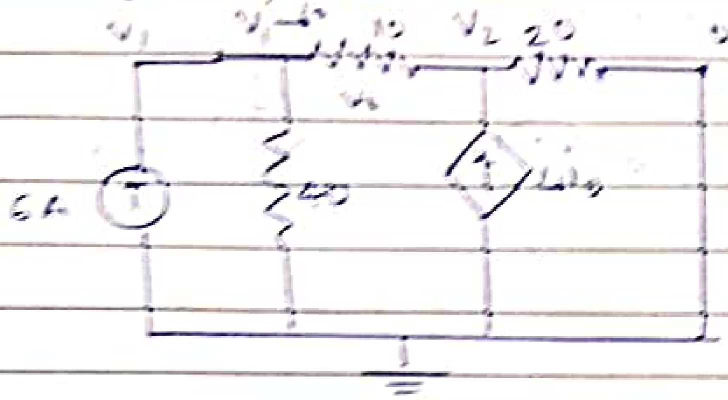
$V_1 = -8$

$V_2 = -10V$

$V_0' = V_1 - V_2$

$= 2V$

$i_0' = 2/10 = 0.2A$



$$\frac{V_1 - V_2}{10} + \frac{V_1}{40} = 6$$

$$\frac{4V_1 + V_1}{40} - \frac{4V_2}{40} = 6$$

$$5V_1 - 4V_2 = 240 \quad \text{—— (1)}$$

$$\frac{V_2 - V_1}{10} + \frac{V_2}{20} - 4\left(\frac{V_1 - V_2}{10}\right) = 0$$

$$5\left(\frac{V_2 - V_1}{10}\right) + \frac{V_2}{20} = 0$$

$$\Rightarrow \frac{10V_2 - 10V_1 + V_2}{20} = 0$$

$$11V_2 = 10V_1 \quad \text{—— (2)}$$

$V_1 = 176V$

$V_2 = 160V$

$\cancel{V_0 = V_2 - V_1 = -16V}$

$\cancel{i_0'' = -16/2 = -8A}$

$\cancel{i_0 =}$

$V_1 - V_2 = 16V$

$i_0'' = 16/10 = 1.6$

$$\left\{ \begin{aligned} V_0 &= 16 + 2 \\ &= 18V \\ i_0 &= 1.8A \end{aligned} \right\}$$

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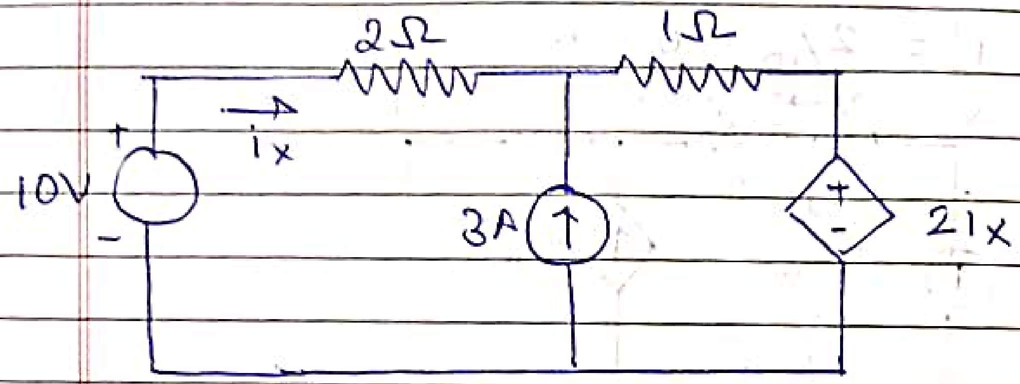
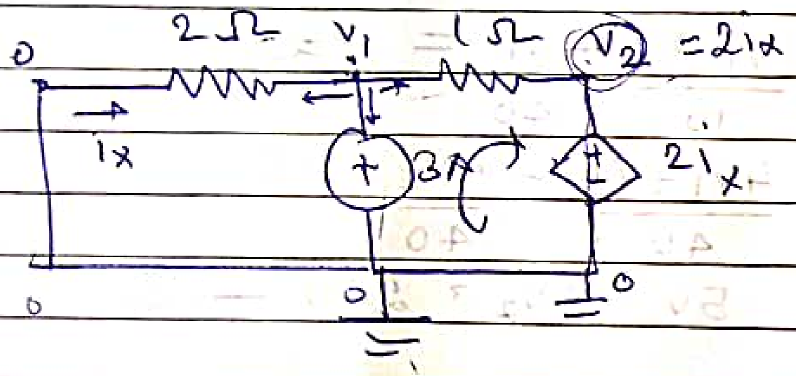
Q) Find i_x in ckt using superposition theorem





$\frac{V_1}{2} - 3 + \frac{V_1 - V_2}{1} = 0$ $\quad 3 \cancel{(-2i_x) = 6}$

$V_1 - 6 + 2V_1 - 2V_2 = 0$

$3V_1 - 2V_2 = 6$ —— ①

$\frac{V_1}{2} - 3 + V_1 - 2i_x$

$V_1 - 6 \cancel{+2V_1} \cancel{4i_x} = 0$

$3V_1 \cancel{- 4i_x = 6}$

$V_2 = 2i_x$ —— ②

$-\frac{V_1}{2} = i_x$ —— ③

$V_1 = -2i_x$

$\cancel{-6i_x + 2}$

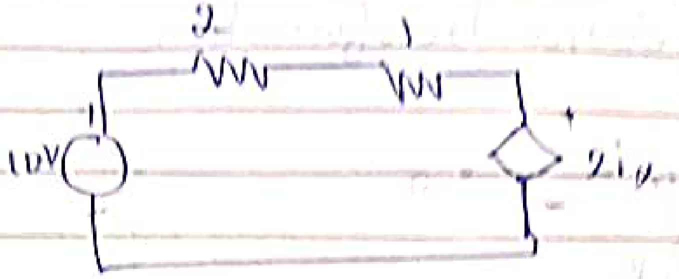
$-6i_x \cancel{- 2i_x = 6}$

$i_x = -\cancel{0.75}A$

$i_x = -0$

$V_1 = -2i_x$

$-8i_x = 6$ $\quad V_2 = 2i_x$



$10 - 2i_x - i_x - 2i_x = 0$

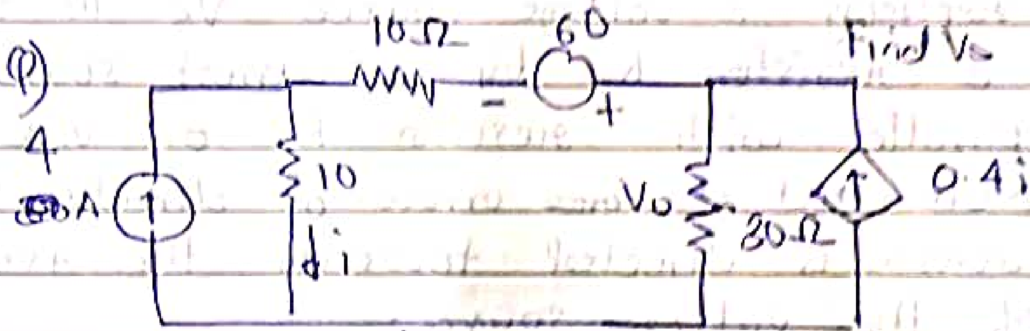
$i_{x_2} = \cancel{5A}\ \cancel{8A}\ 2A$

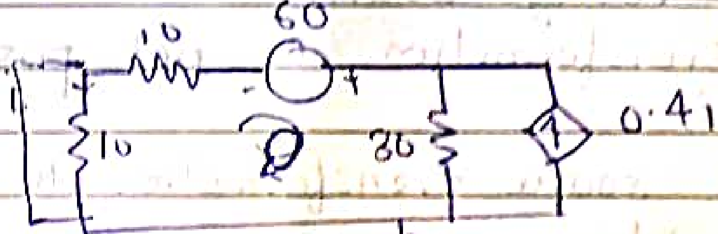
$i_x = i_{x1} + i_{x2}$

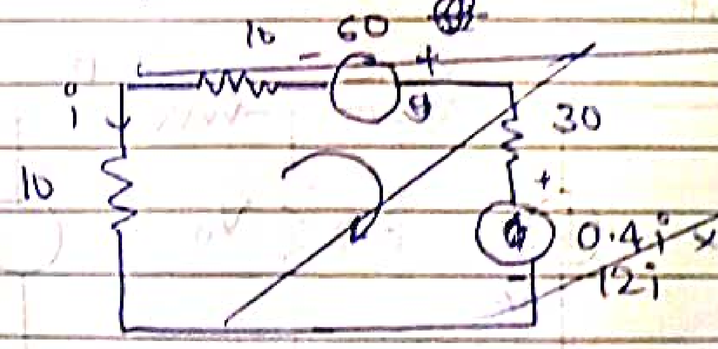
$= \cancel{8A}\ \ 1.25A$

(P) 4

Find V_0



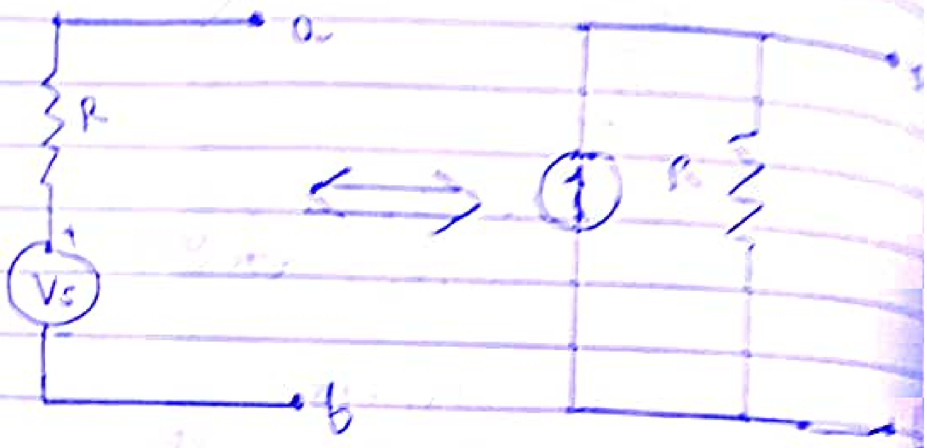




$60 - 30i_1 - 12i$
$- 10i_1 - 10i_1 = 0$

$60 - \cancel{40}\ \cancel{40}\ 50i_1$

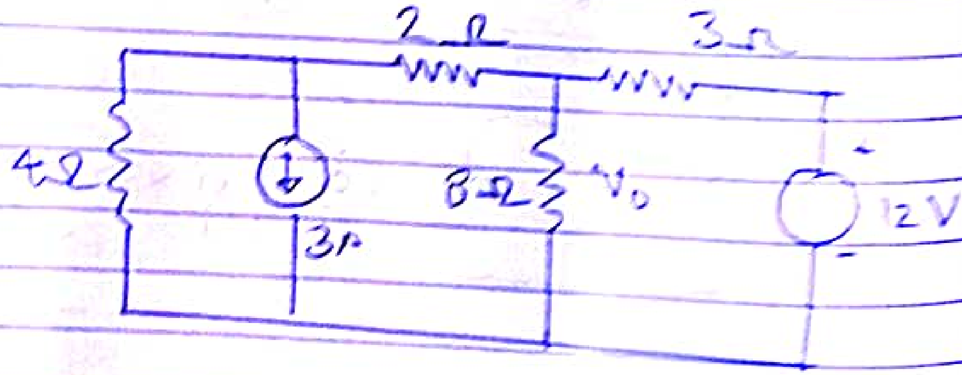
Source Transformation

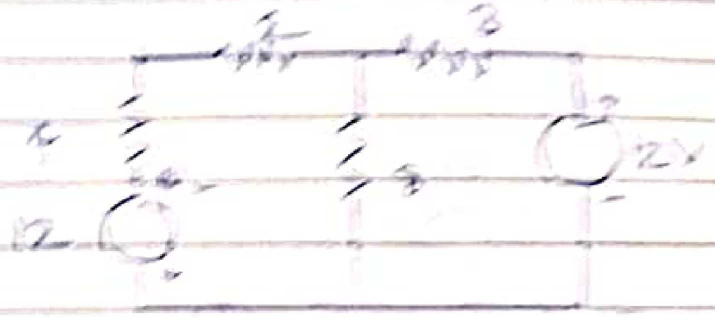


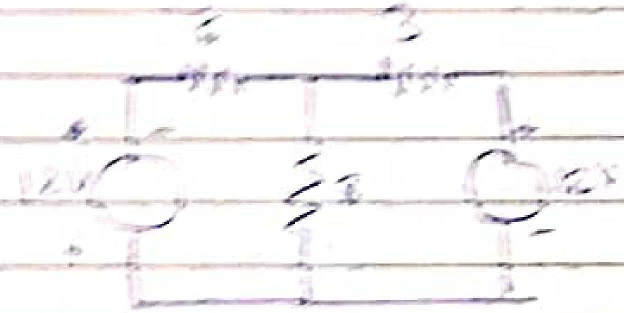
A source transformation is the process of replacing a voltage source V_s in series with a resistor R by a current source I_s in parallel with resistor R or vice-versa.
Note that → ~~the~~ arrow of direction of current source is directed towards the +ve terminal of the voltage source.
Source transformation is not possible R=0

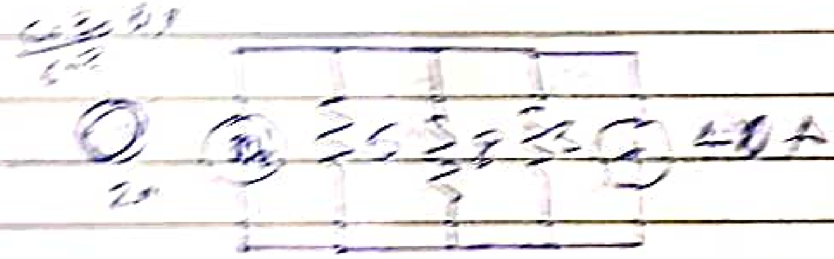
Use the source transformation to find V_o in the circuit.

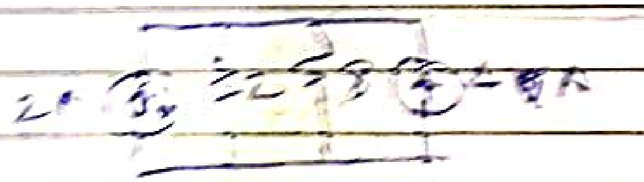
Q)











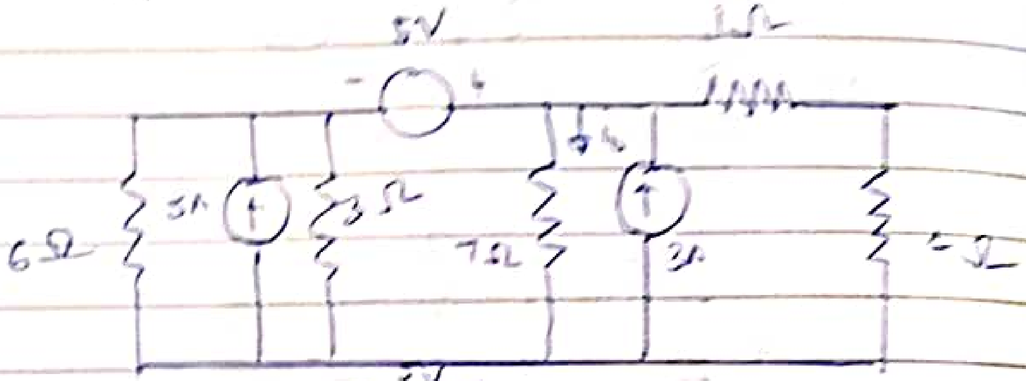
$4 = 10i$

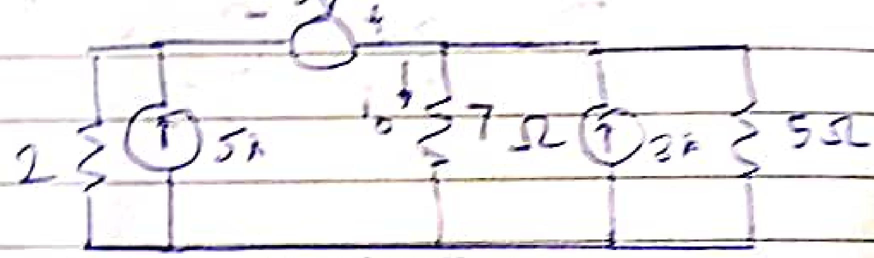
$i = 0.4A$

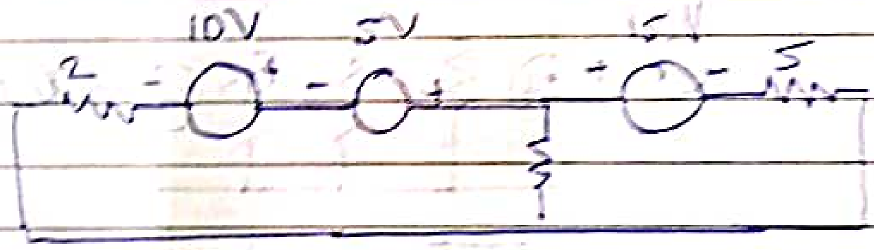
$V_0 = i \times 8$

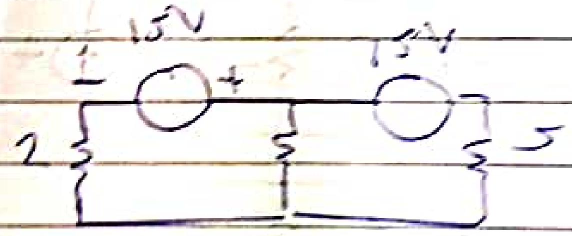
$= 3.2V$

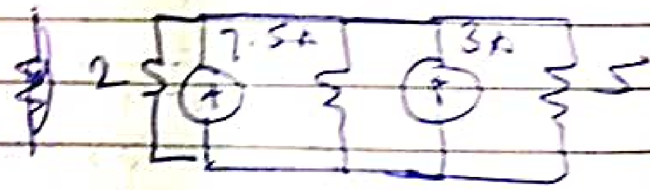
Q) Find i_0 in ckt

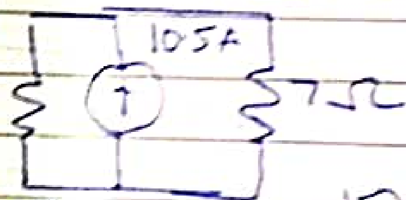






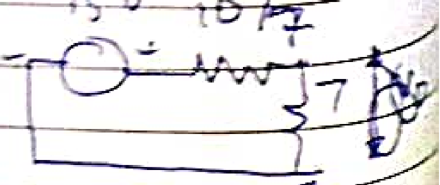






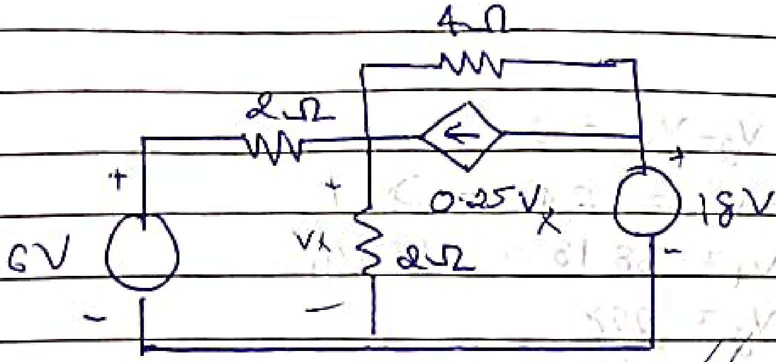
$\frac{15\times7}{1.59}$

=

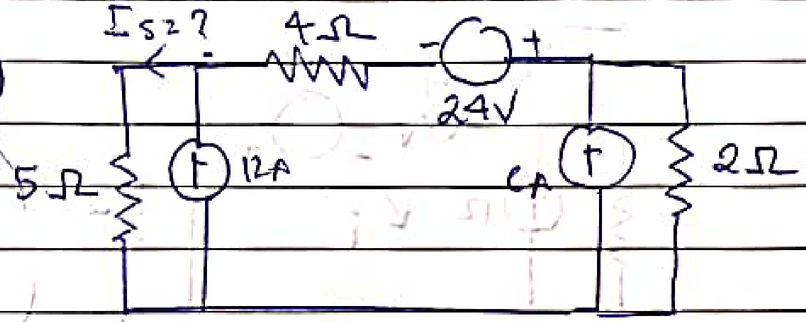


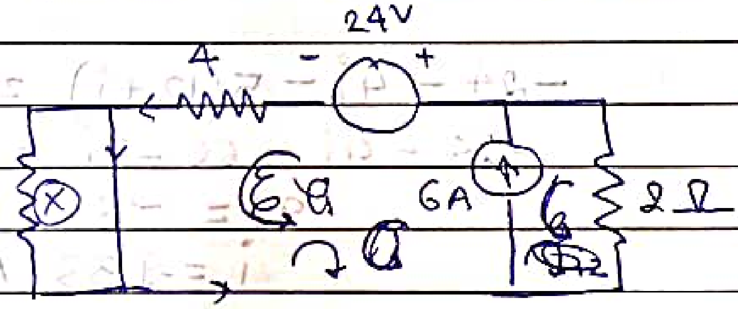
$\boxed{I = 1.78A}$

Q)



Q)





$\cancel{-24 - 4i_1 - 2i_2 = 0}$

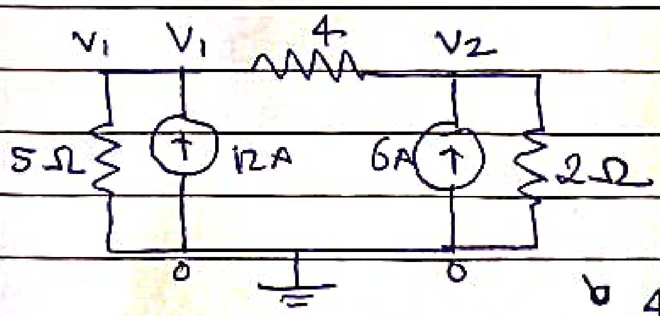
$\cancel{4i}$

$-4i_0 + 24 - 2(6+i) = 0$

$-4i_0 + 24 - 12 - 2i = 0$

$12 = 6i$

$\boxed{i = 2A} \longrightarrow$



$\frac{V_1}{5} - 12 + \frac{V_1 - V_2}{4} = 0$

$\frac{V_2}{2} - 6 + \frac{V_2 - V_1}{4} = 0$

$4V_1 - 240 + 5V_1 - 5V_2 = 0$

$9V_1 - 5V_2 = 240$ —(1)

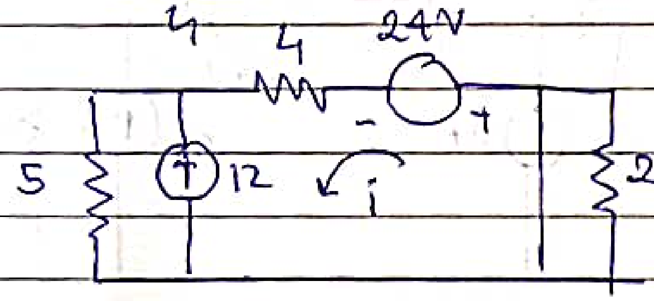
$$2V_2 - 24 + V_2 - V_1 = 0$$

$$3V_2 - V_1 = 24 \quad \text{--- (2)}$$

$$V_1 = 38.18 = 420/11$$

$$V_2 = \frac{228}{11}$$

$$\frac{V_1 - V_2}{4} = 4.363\ A (\longrightarrow)$$



$$-24 - 4i - 5(12+i) = 0$$

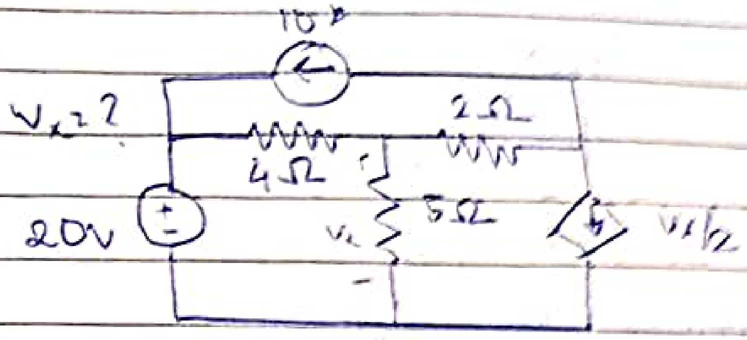
$$-24 - 4i - 60 - 5i = 0$$

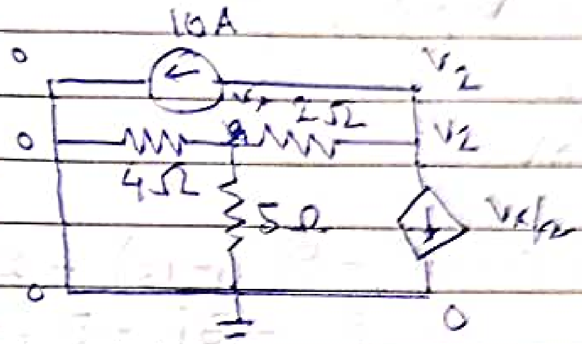
$$9i = -84$$

$$i = -9.33\ A$$

$$i = 9.33\ A \ (\longrightarrow)$$

$$\{ i_s = 3.693\ A \}$$





$$\frac{V_{x_1}}{4} + \frac{V_{x_1}}{5} + \frac{V_{x_1}-V_2}{2} = 0$$

$$5V_{x_1} + 4V_{x_1} + 10V_{x_1} - 10V_2 = 0$$

$$10V_2 = 19V_{x_1} \quad \text{— (1)}$$

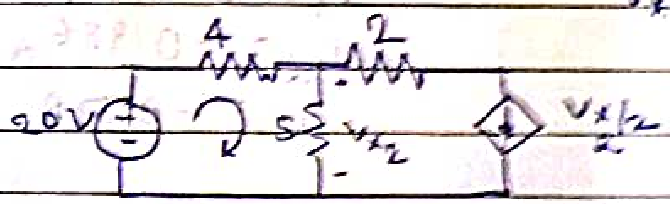
$$\frac{V_2 - V_x}{2} + \frac{V_x}{2} + 10 = 0$$

$$V_2 - \cancel{V_x} + \cancel{V_x} + 20 = 0$$

$$\boxed{V_2 = -20V}$$

$$\left\{ V_{x_1} = \frac{-200V}{19} \right\} = -10.526V$$

$$V_x = -5.26V$$



$V_x = 5\left(i - \frac{V_x}{2}\right)$

$2V_x = 10i - 5V_x$

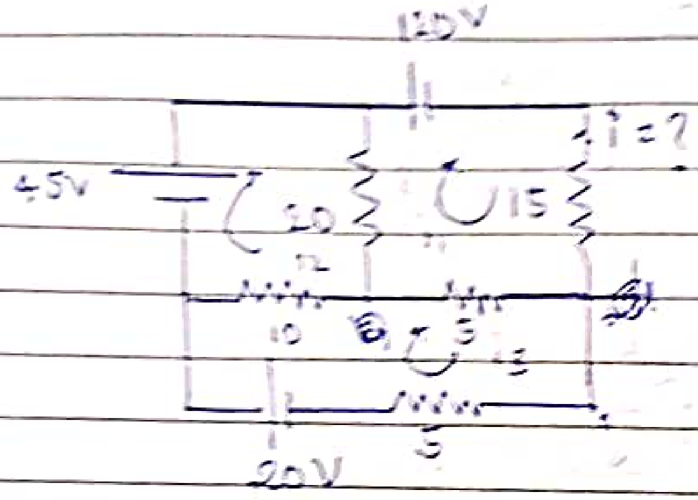
$7V_x = 10i$

$$20 - 4i - 5\left(i - \frac{V_x}{2}\right) = 0$$

$$40 - 8i - 10i + 5V_x = 0$$

$$40 - 18i + 5V_x = 0 \quad \text{— (1)}$$

$$V_{x_2} = 5.26V \qquad i = 3.68A$$



$$-120-15i_1-5(i_1-i_3)-20(i_1-i_2)=0$$
$$-120-15i_1-5i_1+5i_3-20i_1+20i_2=0$$

$$-40i_1+20i_2+5i_3=120 \quad -(1)$$

$$45-20(i_2-i_1)-10(i_2-i_3)=0$$
$$45-20i_2+20i_1-10i_2+10i_3=0$$
$$20i_1-30i_2+10i_3=-45 \quad -(2)$$

$$20-10(i_3-i_2)-5(i_3-i_1)-5i_3=0$$
$$20-10i_3+10i_2-5i_3+5i_1-5i_3=0$$

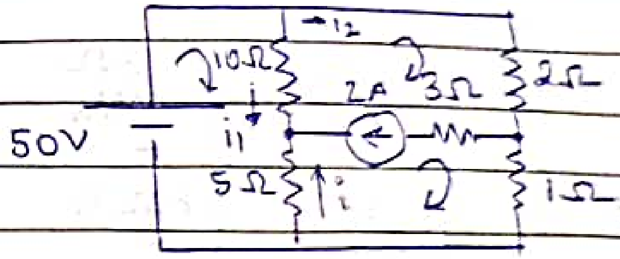
$$5i_1+10i_2-20i_3=-20 \quad -(3)$$

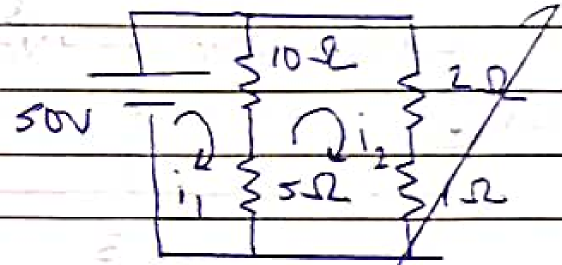
$i_1 = -3.54\,A$ 　　 $3.54\,A\uparrow$

$i_2 = -0.986\,A$

$i_3 = -0.37\,A$

Q)





$$50 - 10(i_1 - i_2) - 5(i_1 - i_2) = 0$$
$$50 - 15i_1 + 15i_2 = 0$$
$$\cancel{15}\ 3i_1 - 3i_2 = 10 \quad \text{—— (1)}$$

$$50 - 10(i_1 - i_2) - 5(i_1 - i_3) = 0$$
$$\cancel{50}\ 10 - 2i_1 + 2i_2 - i_1 + i_3 = 0$$
$$-3i_1 + 2i_2 + i_3 = -10 \quad \text{—— (1)}$$

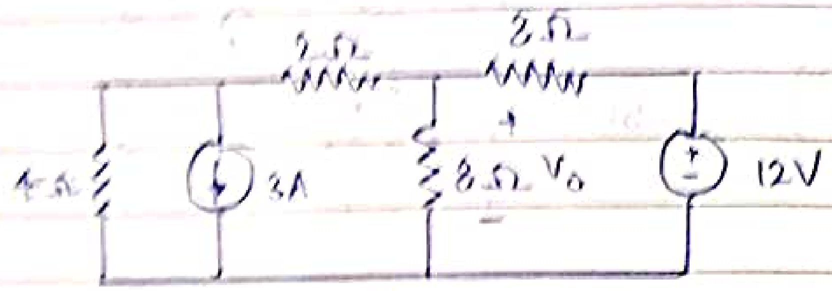
$$-10(i_2 - i_1) - 2i_2 - i_3 - 5(i_3 - i_1) = 0$$
$$-10i_2 + 10i_1 - 2i_2 - i_3 - 5i_3 + 5i_1 = 0$$

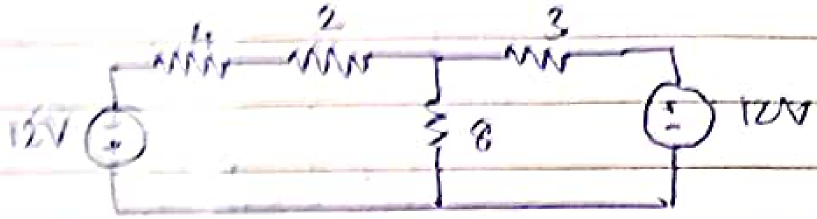
$$15i_1 - 12i_2 - 6i_3 = 0$$

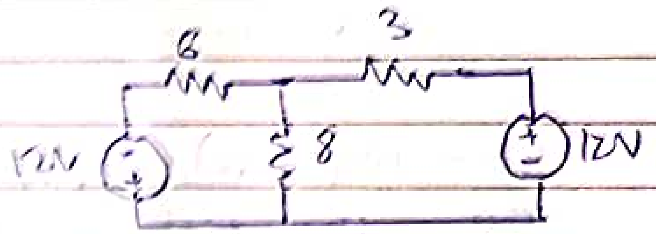
$$5i_1 - 4i_2 - 2i_3 = 0 \quad \text{—— (2)}$$

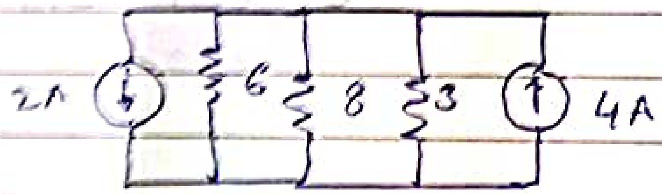
$$i_2 = 2 + i_3$$
$$i_2 - i_3 = 2 \quad \text{—— (3)}$$

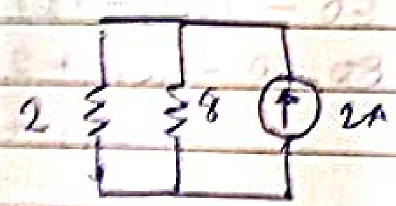
(ii)

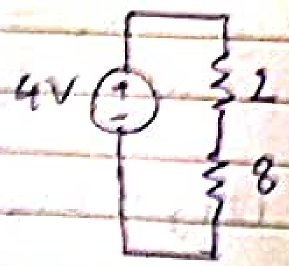










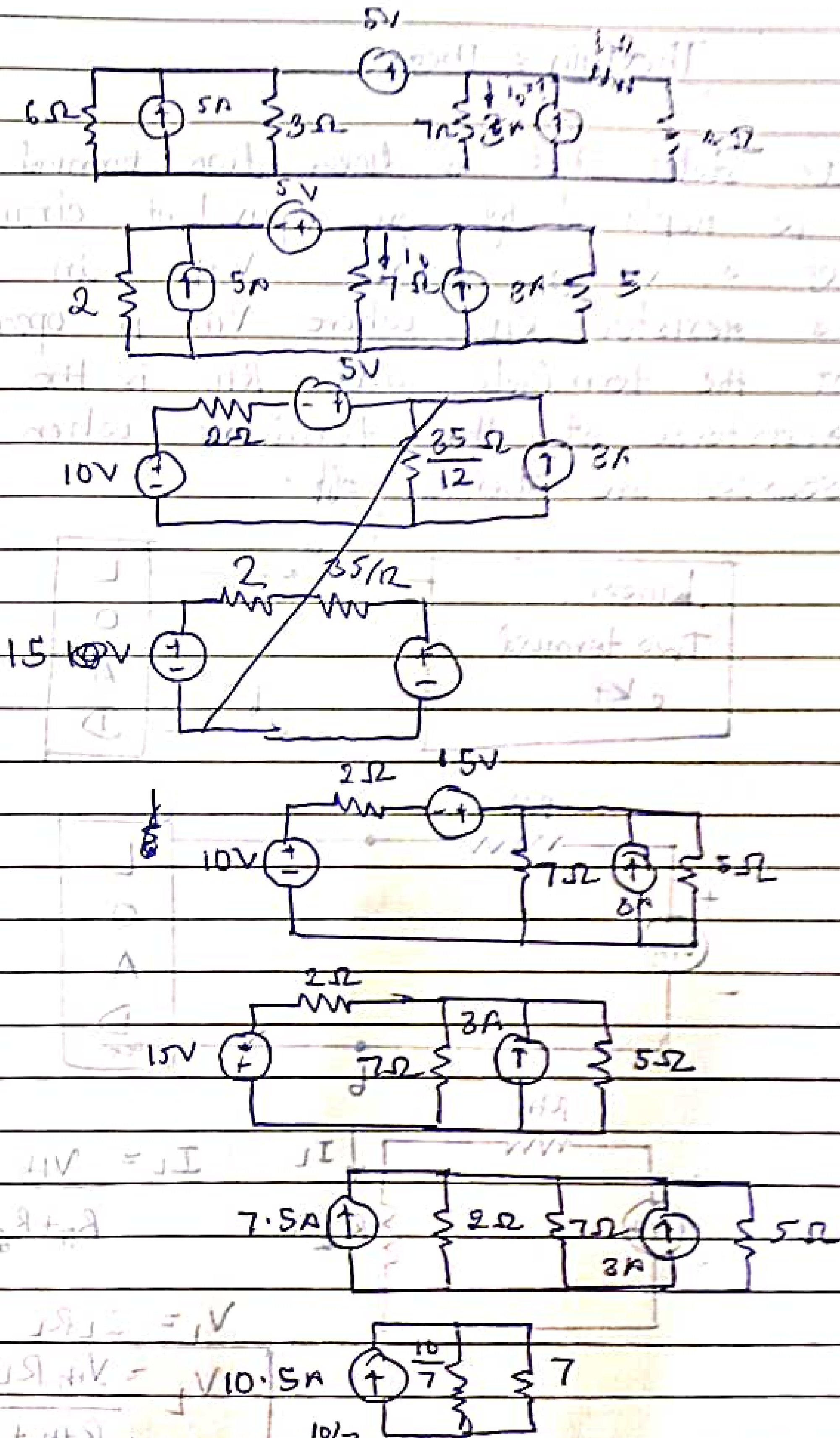


$i = 4/10$

$V_0 = 4/10 \times 8$

$= 3.2V$

Q)



$\overset{1.5}{\cancel{10.5}} \times \frac{10}{7}$

$= 15V$

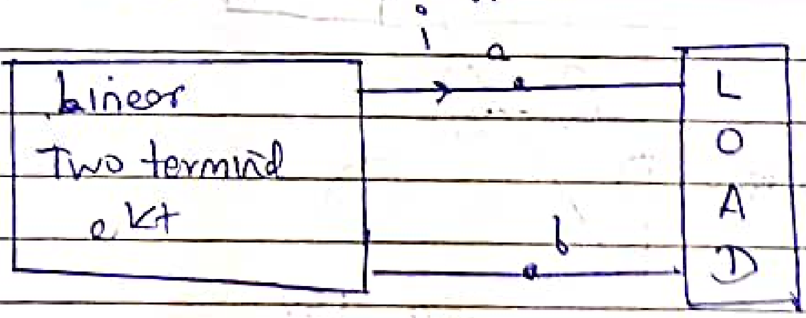
$15 = (10/7 + 7)i$;

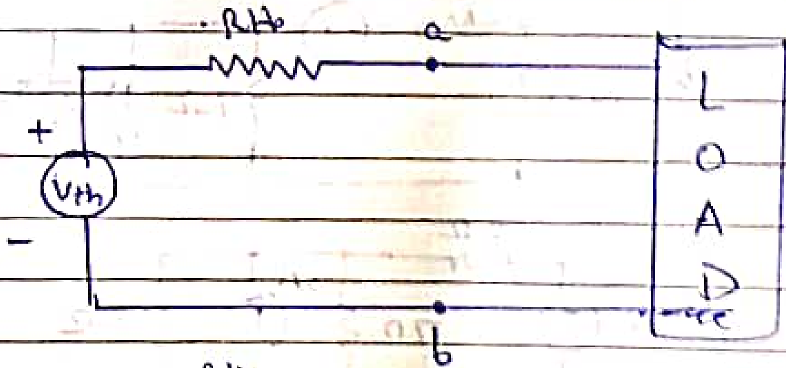
$i = \frac{15 \times 7}{59}$

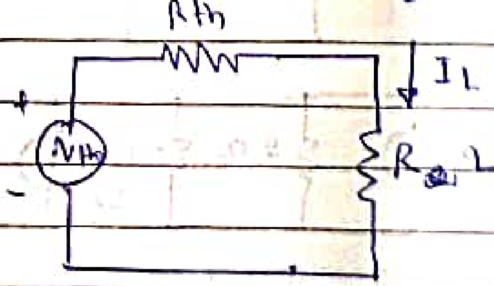
$= 1.77\,A$

Thevinin's Theorem

It states that a linear two terminal ckt. can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th} where V_{th} is open ckt voltage at the terminals and R_{th} is the equivalent resistance at the terminals when the independent sources are turned off.





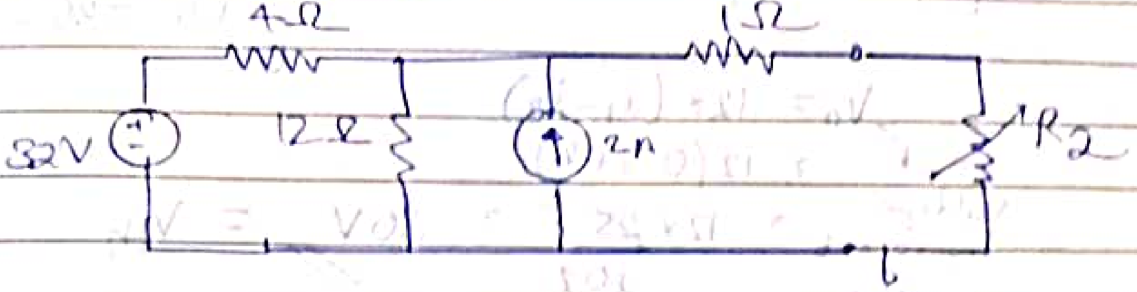


$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$V_L = I_L R_L$$

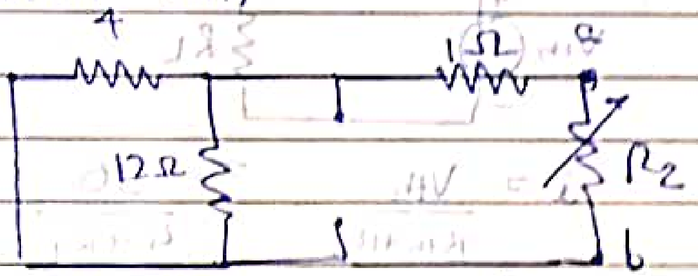
$$\boxed{V_L = \frac{V_{th} R_L}{R_{th} + R_L}}$$

Q1) Find the thevenin's equivalent ckt to the left of the terminals a, b. Find current through R_L when $R_L = 6, 16 \ \& \ 36\,\Omega$



Steps

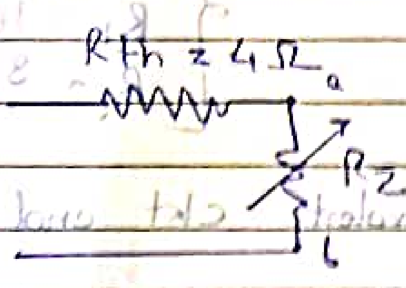
① Calculate R_{th}



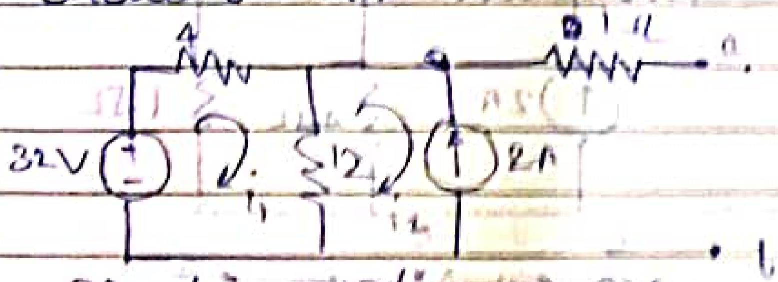
$\frac{4\times12}{4+12}$

$48/$

Turn off all sources & redraw ckt



② calculate V_{th}



$$32 - 4i_1 - 12(i_1 - i_2) = 0$$
$$32 - 4i_1 - 12i_1 + 12i_2 = 0$$
$$16i_1 - 12i_2 = 32$$
$$4i_1 - 3i_2 = 8 \quad \text{— (1)}$$

$i_2 = -2A$

$4i_1 + 6 = 8$

$\boxed{i_1 = 0.5A}$

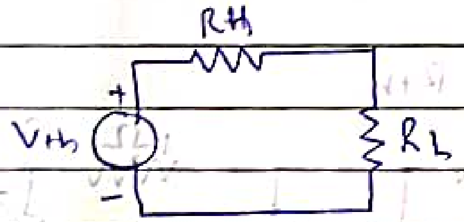
Alternatively use nodal analysis:

$$\frac{V_{th}-32}{4} + \frac{V_{th}-2}{12} = 0$$

$$\{V_{th} = 30V\}$$

$$V_0 = 12 \times (i_1 - i_2)$$
$$= 12(0.5+2)$$
$$= 12 \times \frac{\cancel{25}\,5}{\cancel{10}\,2} = 30V = V_{th}$$

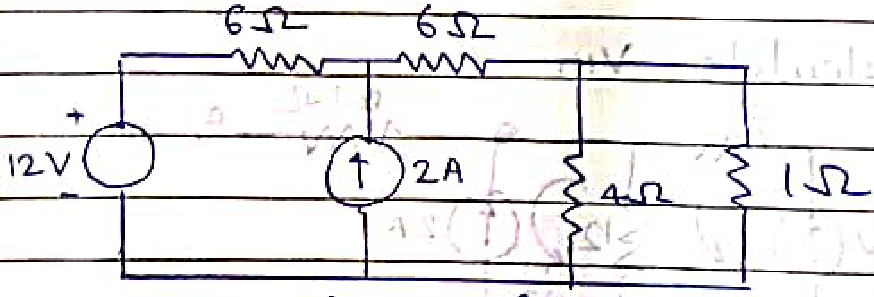
Voltage across a,b is voltage across 12 Ω.

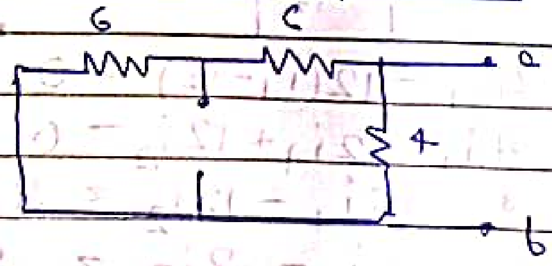


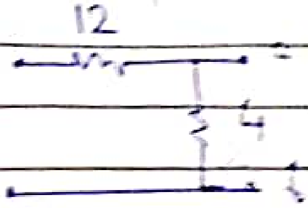
$$i = \frac{V_{th}}{R_{th}+R_L} = \frac{30}{4+R_L}$$

$R_L = 6$	$i = 3A$
$R_L = 16$	$i = 3/2\ A$
$R_L = 36$	$i = 3/4\ A$

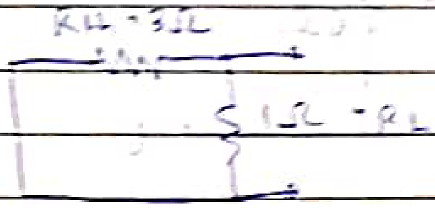
Q) Find equivalent ckt and current?

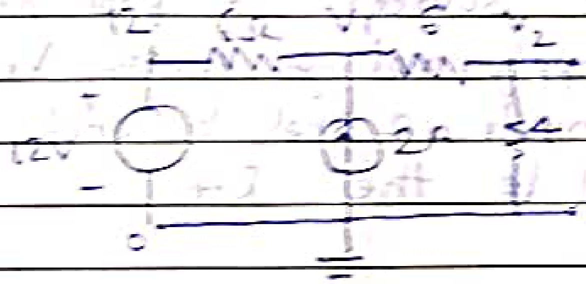






$\frac{12\times4}{12+4} = 48/16 = 3\Omega$





$$\frac{V_1-12}{6} - 2 + \frac{V_1-V_2}{6} = 0$$

$$2V_1 - 12 - V_2 - 12 = 0$$

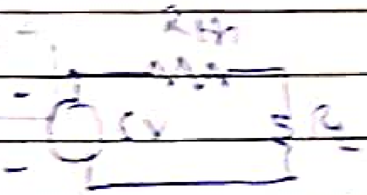
$$2V_1 - V_2 = 24 \quad \text{— (1)}$$

$$\frac{V_2-V_1}{6} + \frac{V_2}{4} = 0$$

$$4V_2 - 4V_1 + 6V_2 = 0$$

$$10V_2 = 4V_1$$

$$2.5V_2 = V_1$$



(1) — $5V_2 - V_2 = 24$

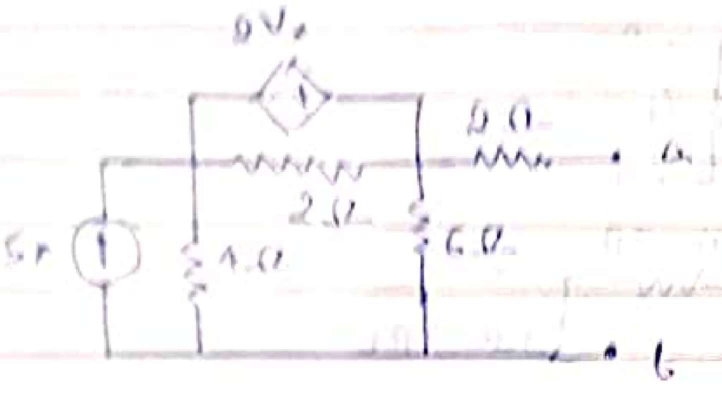
$V_2 = 6V$ $\quad V_{th} = 6V$

$i = \frac{6}{3+1} = 1.5A$

$V_1 = 15V$

$i = \frac{V_2}{4} = \frac{6}{4} = 1.5A$

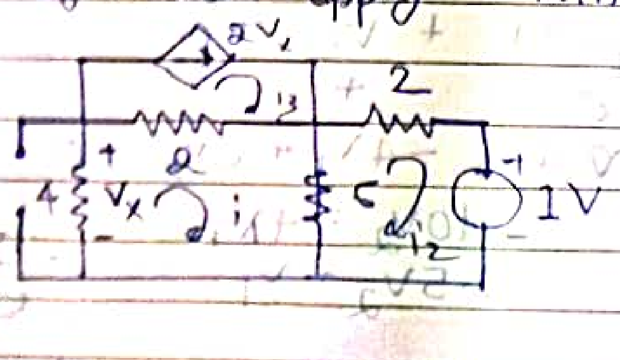
Q) Find the thevnin's theorem equivalent ckt.



In dependent sources to find Rth we need to apply voltage source $V_0 (\simeq 1V)$ across terminals a and b and find $\frac{V_0}{i_0}$ will be the Rth.

Assume current flowing b/w terminals a and b is i_0 then $R_{th} = V_0/i_0 = 1/i_0$

~~Alt~~ Alternatively we will assume 1 A current source to find the corresponding voltage V_0 and apply $R_{th} = V_0/1$.



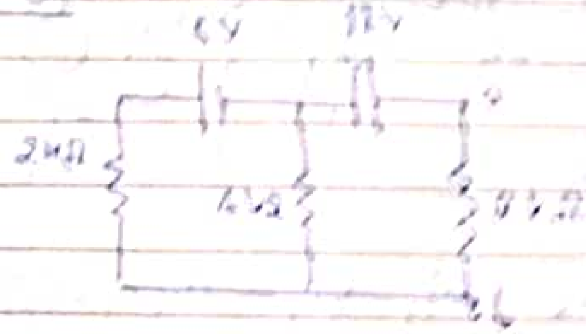
$$-4i_1 - 2(i_1 - i_3) - 6(i_1 - i_2) = 0 \quad (1)$$

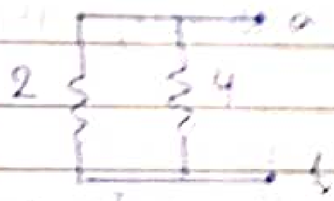
$$2V_x - 2(i_3 - i_1) = 0 \quad (2)$$

$$-2i_2 - 1 - 6(i_2 - i_1) = 0 \quad (3)$$

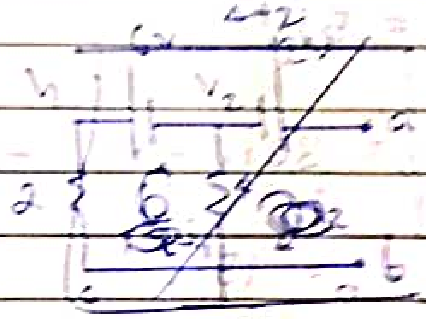
$$4i_1 = V_x \quad (4)$$

Youtube





$R_{th} = \frac{4\times 2}{4+2} = 4/3\ \Omega$



$6 - 2i_1 - 4(i_1 - i_2) = 0$

$V_1 + 12 - V_2$

$= 12 + (V_1 - V_2)$

$= 12 + 12$

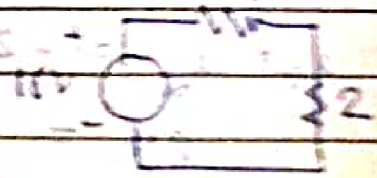
$= 12 + 4$

$i_2 = 0$

$6 - 2i_1 - 4i_1 = 0$

$\boxed{i_1 = 1}$

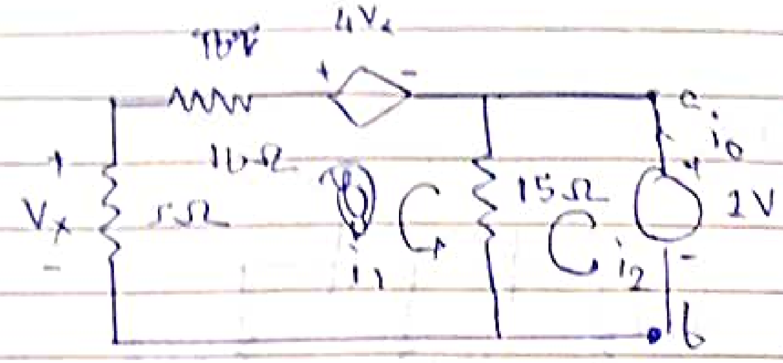
$\boxed{V_{th} = 8\tfrac{1}{3} V}$



$i = \frac{a3}{4/3 + 2}$

$\frac{8\tfrac{1}{3}}{4/3 + 2} = \frac{a3}{10/3} = \frac{a}{5} \times 3$

$= 24/5$



$$4V_x - 10i_1 - 5i_1 - 15(i_1 - i_2) = 0$$
$$4V_x - 15i_1 - 15i_1 + 15i_2 = 0$$
$$4V_x = 30i_1 - 15i_2 \quad \text{— (1)}$$

$$1 - 15(i_2 - i_1) = 0$$
$$1 = 15i_2 - 15i_1 \quad \text{— (2)}$$

$$V_x = 5i_1$$

$$20i_1 = 30i_1 - 15i_2 \quad \text{— (3)}$$

$$20i_1 + 1 = 15i_1$$
$$-\frac{1}{5} = i_1 = -0.2\,A$$

$$V_x = -1\,V$$

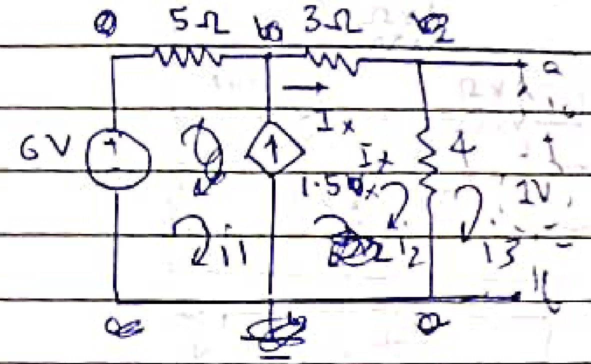
i_2

$$1 = 15i_2 - (-3)$$
$$i_2 = \frac{1-3}{15} = -\frac{2}{15}\,A$$

$$i_0 = i_2 = -2/15\,A$$
$$R_{th} = -15/2$$
$$= -7.5\,\Omega$$

Since no independent source
$V_{th} = 0$

Q)



$R_{th} = 1/i_0$

$$6 - 5i_1 - 3i_2 - 4(i_2 - i_3) = 0$$
$$6 - 5i_1 - 3i_2 - 4i_2 + 4i_3 = 0$$
$$5i_1 + 7i_2 - 4i_3 = 6 \quad \text{—— (1)}$$

$-5+14-4i_3=c$

$9-6=4i_3$

$$-1 - 4(i_3 - i_2) = 0$$
$$4i_2 - 4i_3 = 1 \quad \text{—— (2)}$$

$-5+14-4i_3=76$

$-4i_3 = 9/11-14$

$\boxed{i_3 = 3/4 \text{ A}}$

$$i_2 = i_x \quad \text{—— (3)}$$
$$i_1 + 1.5i_x = i_2$$
$$2i_1 = -i_2 \quad \text{—— (4)}$$

$i_0 = -i_3$

$= -0.75A$

$R_{th} = 4/3\,\Omega$

$$5i_1 + 7(-2i_1) - 4i_3 = 6$$
$$5i_1 - 14i_1 - 4i_3 = 6$$
$$-9i_1 - 4i_3 = 6$$
$$4i_2 - 4i_3 = 1$$

$1 = 4i_2$

$1/4 = i_3 - i_2$

$1/4 = i_3 - 2$

$2 + 1/4$

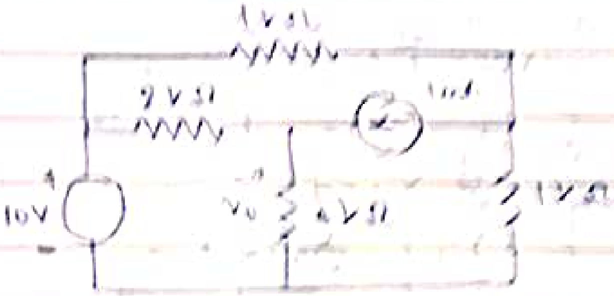
$\boxed{i_3 = 9/4 \text{ A}}$

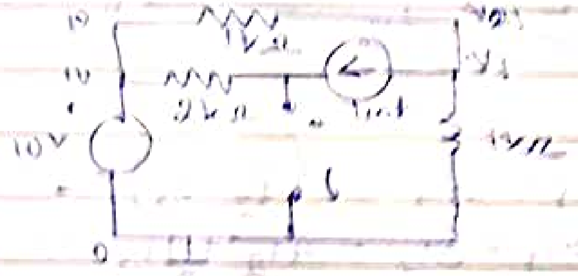
$$-13i_1 - 4i_2 = 5$$
$$-13i_1 - (4(-2i_1)) = 5$$
$$-13i_1 + 8i_1 = 5$$

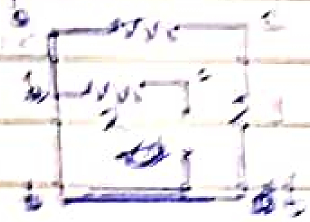
$\boxed{i_1 = -1 \text{ A}}$ $\boxed{i_2 = 2 \text{ A}}$

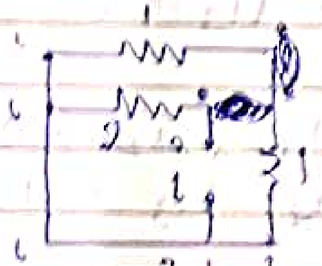
$\cancel{-4i_3 = (-5+14)}$

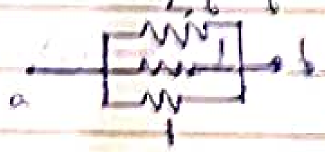
1)











$\frac{1}{R_{th}} = \frac{1}{4} + \frac{1}{2}$

$R_{th} = \frac{2}{3}\,k\Omega$

$10 - V_1 = (10^{-3})\, 2\times 10^3$

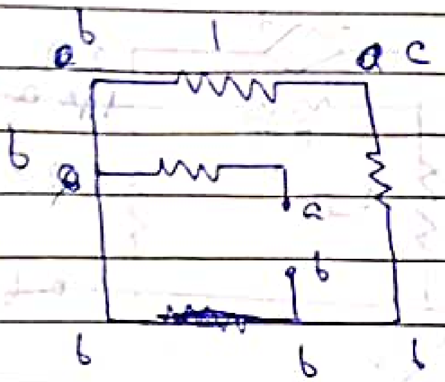
$\boxed{V_1 = 8\,V}$

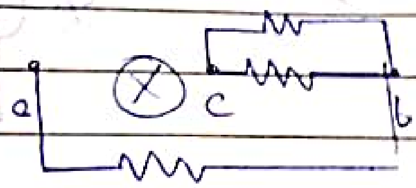
$V_{ab} - 2 - 10 = 0$

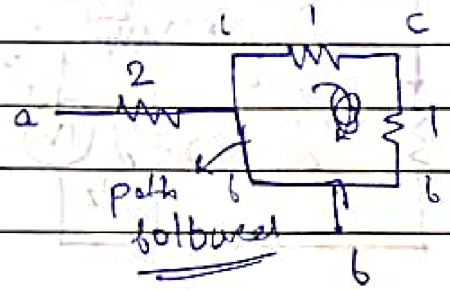
$V_{ab} = 12\,V = V_{th}$

$V_o = 4 \times$

$8\,V$



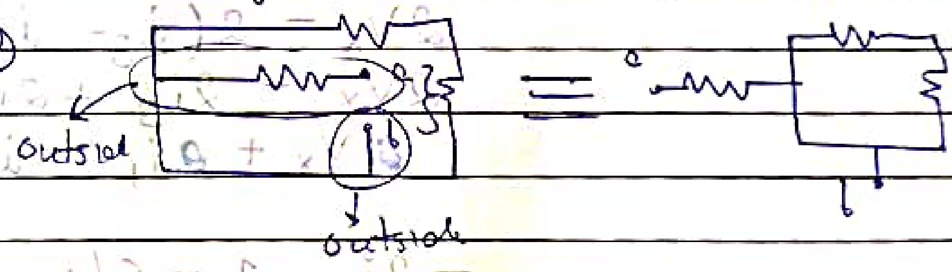




$R_{th} = 2\,\Omega$

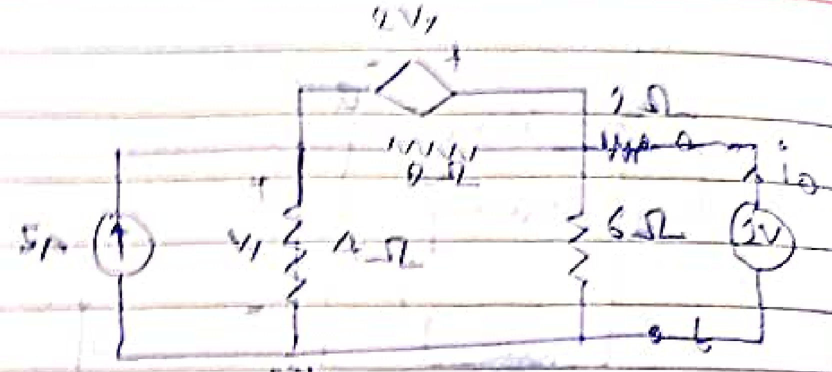
IMP points

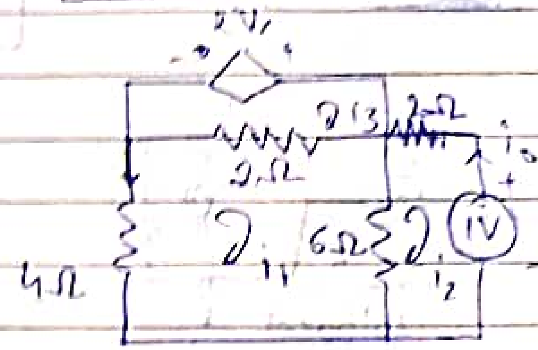
1. Current sources are ~~shor~~ open ckt
   Voltage sources are short ckt

2. 

3. Use short ckt path (if exists) in R_{th}

(P)





$R_{th} = \frac{1}{i}$

$$-4i_1 - 2(i_1 - i_3) - 6(i_1 - i_2) = 0$$
$$-4i_1 - 2i_1 + 2i_3 - 6i_1 + 6i_2 = 0$$

$$6i_2 - 12i_1 + 2i_3 = 0$$

$$3i_2 - 6i_1 + i_3 = 0 \quad \text{— (1)}$$

$$2V_x - 2(i_3 - i_1) = 0$$
$$2V_x - 2i_3 + 2i_1 = 0$$
$$V_x + i_1 - i_3 = 0 \quad \text{— (2)}$$

$$-2i_2 - 1 - 6(i_2 - i_1) = 0$$
$$-2i_2 - 1 - 6i_2 + 6i_1 = 0$$
$$6i_1 - 8i_2 = 1 \quad \text{— (3)}$$

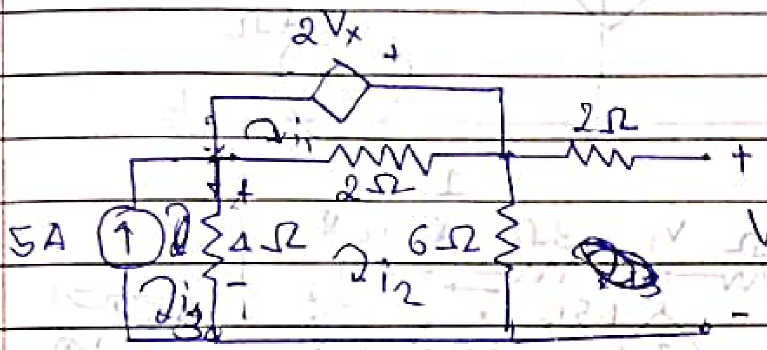
$$-4i_2 = V_x \quad \text{— (4)}$$

$i_1 = 0.02A$
$i_2 = -0.108A$
$i_3 = 0.45A$
$V_x = 0.43V$

~~$i_1 = 0.02A$~~
~~$i_2 = -0.108A$~~
~~$i_3 =$~~

$i_0 = -i_2$
$= 0.1067\ A$

$R_{th} = \frac{V_0}{i_0} = 6\,\Omega$



$\cancel{2}V_x - \cancel{2}(i_1 - i_2) = 0$

$V_x + i_2 - i_1 = 0$ — (1)

$4(i_2 - i_3) = V_x$

$-2(i_2 - i_1) - 6i_2 - 4(i_2 - i_3) = 0$

$-2i_2 + 2i_1 - 6i_2 - 4i_2 + 4i_3 = 0$

$2i_1 - 12i_2 + 4i_3 = 0$

$i_1 - 6i_2 + 2i_3 = 0$ — (2)

$5 = i_3 + \cancel{\ } + i_1$

$5 = i_3 - \cancel{i_2} + \cancel{i_2} - \cancel{i_1} + \cancel{i_1}$

KVL

$V_x + (i_2 - i_3)4 = 0$

$V_x = (i_3 - i_2)4$

$V_x = 20 - 4i_2$ — (5)

$i_3 = 5A$ — (3)

$i_1 - 6i_2 + 10 = 0$

$6i_2 - i_1 = 10$ — (4)

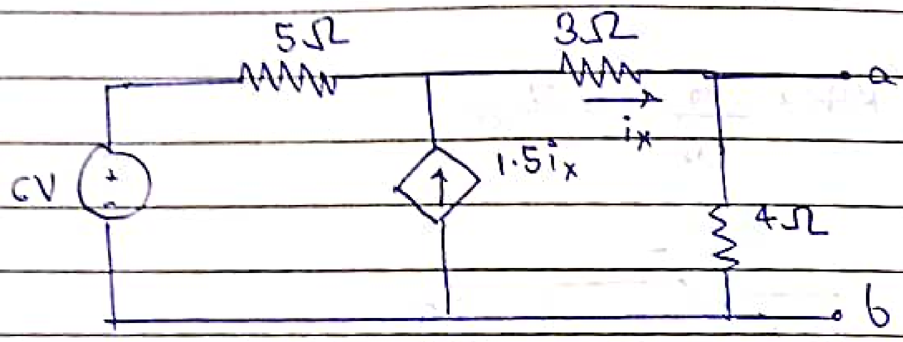
$V_{Th} = 6i_2 = \cancel{6} \times \frac{10}{\cancel{3}} = 20\ V$

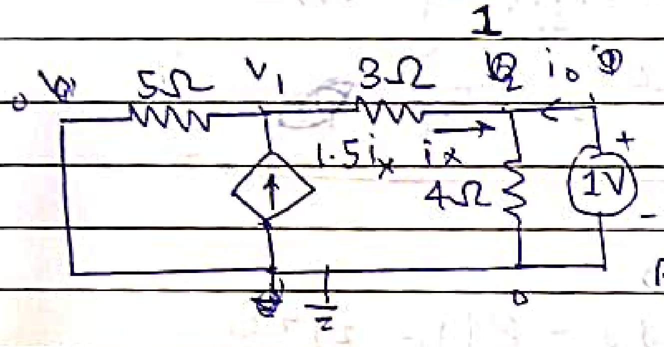
$i_1 = 10\ A$

$i_2 = 10/3\ A$

$V_x = 20/3\ V$

Q) Find thevenin's equivalent ckt





$R_{Th} = 1/i_0$

$$\frac{V_1}{5} - 1.5 i_x + \frac{V_1 - V_2}{3} = 0$$

$$\cancel{\frac{V_2 - V_1}{3} + \frac{V_2}{4}}$$

$$\frac{V_1}{5} - 1.5 i_x + \frac{V_1 - 1}{3} = 0$$

$V_2 = 1$ V

$$3V_1 - 22.5 i_x + 5V_1 - 5$$

(3) $8V_1 - 22.5 i_x$

(1) $$\frac{0 - V_1 - 1}{3} = i_x$$

$V_1 - 1 = 3 i_x$

$V_1 - 3 i_x = 1$ — (2)

$\cancel{V_1 = -5V}$

$\cancel{i_x = -2A}$

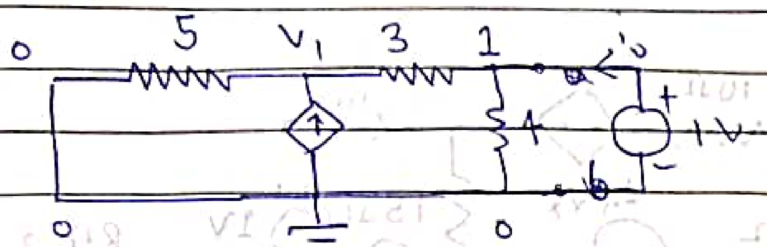
$R_{Th} = 1/$

(2) $V_1 - 3i_x = 1$

$V_1 = -0.161$

$i_x = -0.279$

PAGE NO.:
DATE: / /



$$\frac{V_1}{5} + \frac{V_1 - 1}{3} - \frac{3}{2} i_x = 0$$

$$6V_1 + 10V_1 - 10 - 45 i_x = 0$$

$$16V_1 - 45 i_x = 10 \quad \text{—— (1)}$$

$$\frac{V_1 - 1}{3} = i_x$$

$$V_1 - 1 = 3 i_x$$

$$V_1 - 3 i_x = 1 \quad \text{—— (2)}$$

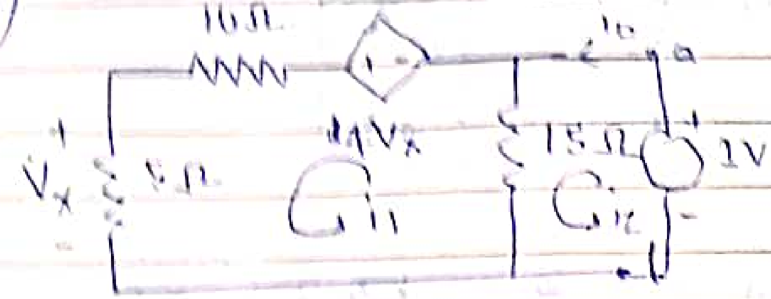
$$V_1 = -5V$$

$$i_x = -2A$$

$$i_0 = -i_x = 2A$$

$$R_{th} = 1/i_0 = 0.5\,\Omega$$

(P)



$R_{th} = 1/i_o$

$$4V_x - 10i_1 - 5i_1 - 15(i_1 - i_2) = 0$$
$$4V_x - 15i_1 - 15i_1 + 15i_2 = 0$$

$$4V_x - 30i_1 + 15i_2 = 0 \quad \text{—} \ (1)$$

$$1 - 15(i_2 - i_1) = 0$$
$$15i_1 - 15i_2 + 1 = 0 \quad \text{—} \ (2)$$

$$-5i_1 + V_x = 0$$
$$V_x = 5i_1 \quad \text{—} \ (3)$$

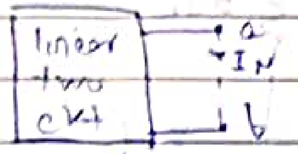
$$\left\{ \begin{array}{l} i_1 = -0.2A \\ i_2 = -0.133A \\ V_x = -1V \end{array} \right\} \qquad \{ R_{th} = -7.5\,\Omega \} \qquad \{ V_{th} = 0 \}$$

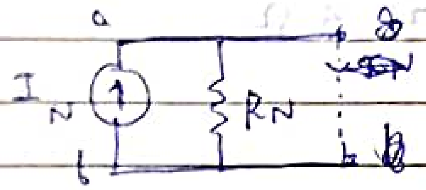
Norton's Theorem

A linear two terminal ckt can be replaced by an equivalent ckt consisting of a current source I_N in parallel with a resistor R_N. Where I_N is the short ckt current through the terminals and R_N is the equivalent resistance

PAGE NO.:
DATE: / /

at terminals when the independent sources are turned off.



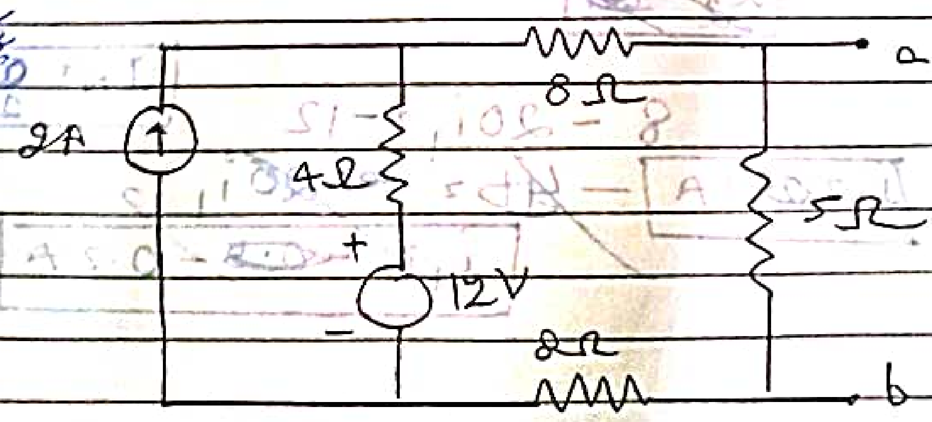


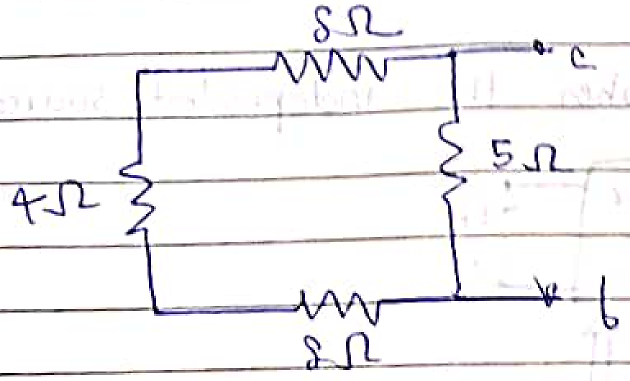
Steps

① R_N is same as R_{th}.
② I_N is calculated by short circuiting the terminals a and b.

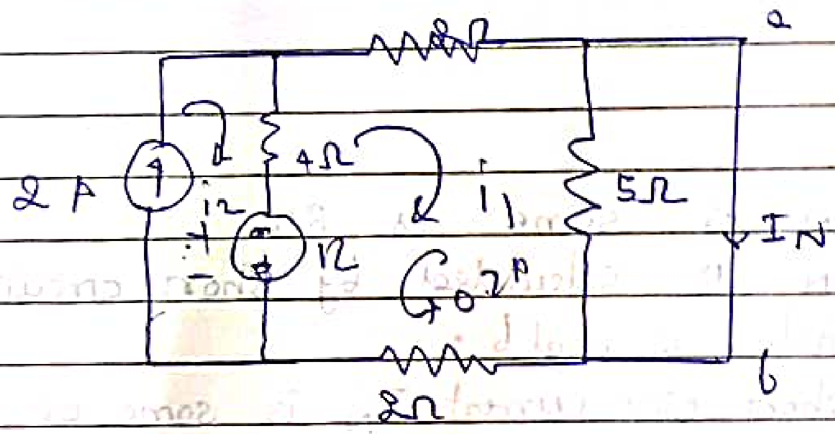
Note (i) The short ckt current I_N is same as Norton's current.
(ii) We can apply source transformation b/w Norton & Thevenin ckt.

Q) Find Norton's equivalent ckt for-





$$R_N = \frac{5\times 20}{5+20} = 4\Omega$$



$$12 - 8i_1 - 8i_1 - 4(i_1 - i_2) = 0$$
$$-16i_1 - 4i_1 + 4i_2 = -12$$
$$4i_2 - 20i_1 = -12$$

$\cancel{i_2 = 2A}$

$$\boxed{I_N = 0.2A}$$

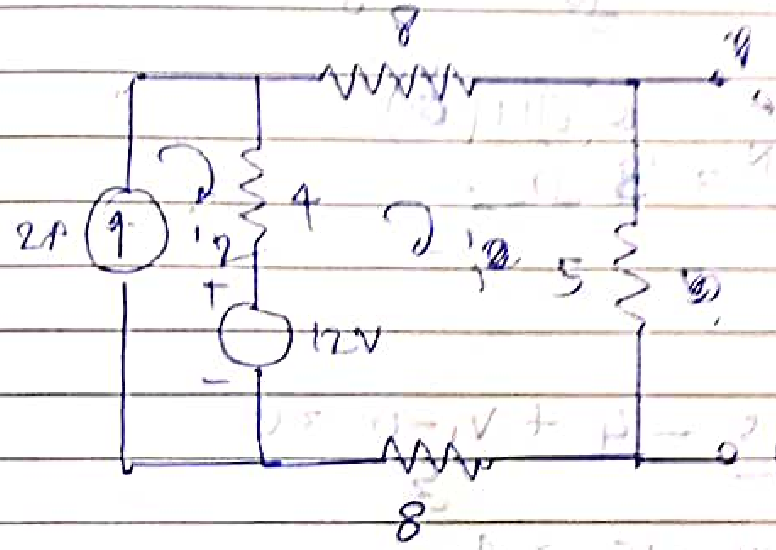
$$8 - 20i_1 = -12$$

$\boxed{i_1 = 1A}$ $\cancel{-4 = 20i_1}$

$\boxed{\cancel{i_1 = -0.2A}}$

Alternative Method

Using Thevenin's Voltage



$$12 - 4(i_1 - i_2) - 8i_1 - 5i_1 - 8i_1 = 0$$

$$12 - 4i_1 + 4i_2 - 21i_1 = 0$$

$$12 = 25i_1 - 4i_2 \quad \text{--- (1)}$$

$$\{i_2 = 2A\}$$

$$12 = 25i_1 - 8$$

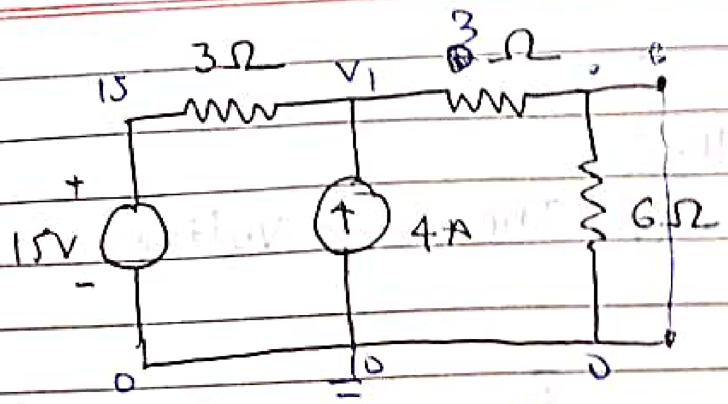
$$i_1 = \frac{20}{25} A$$

$$V_{ab_{th}} = \frac{20}{25} \times 5$$

$$= 4V$$

$$i_{ab} = \frac{V_{ab\,th}}{R_{ab\,th}} = 4/4 = 1A$$

PAGE NO
DATE



$R_{N} = (6) \| (6)$

$N = 3\ \Omega$

i_N

$$\frac{V_1-15}{3} - 4 + \frac{V_1-0}{3} = 0$$

$$\frac{2V_1-15}{3} = 4$$

$2V_1 = 12+15$

$V_1 = 27/2$

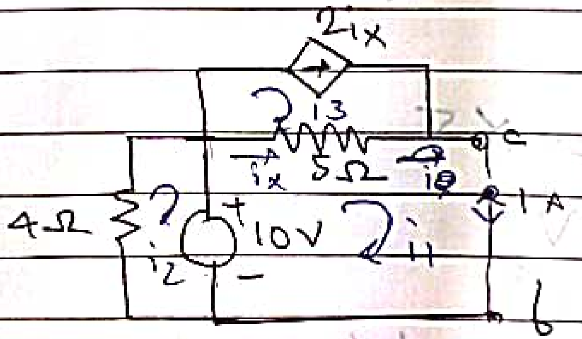
$i_N = \frac{V_1}{3} = 9/2 = 4.5A$

$\{i_N = 4.5A\}$

$(1-1/3)5$

$\frac{2}{3} \cdot 5$

$10/3$



$R_N = \frac{V_{ab}}{1}$

$\{i_x = 1/3A\}$

$10 - 5(i_1 - i_3) = 0$

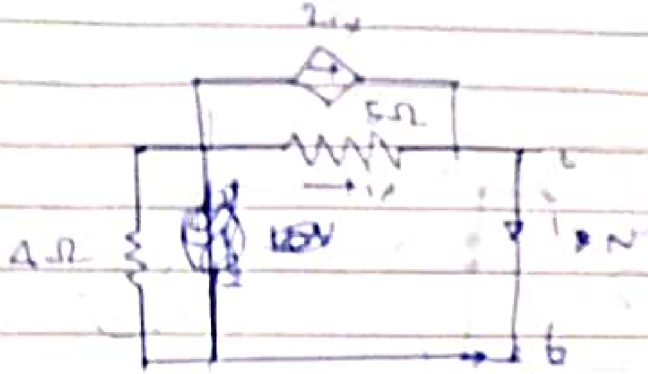
$i_1 - i_3 = i_x$

$-10 - 4i_2 = 0$

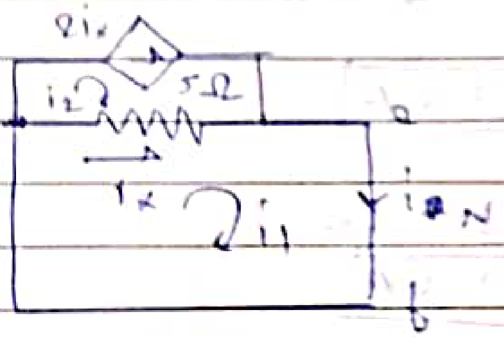
$\{i_1 = 3i_x\}$

$i_3 = 2i_x$

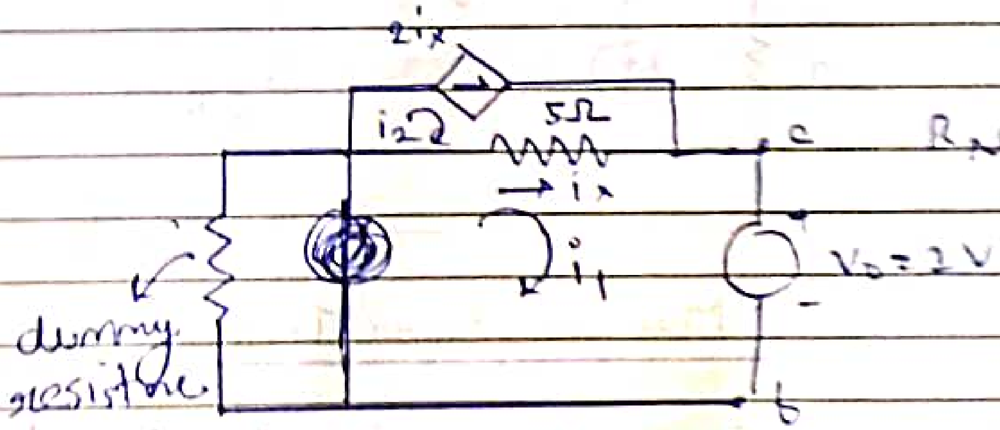
$\{i_1 = 1A\}$



$R_N = V_{ab}$



$i_N = 3i_x$



$R_N = \frac{V_0}{i_0}$

$1 - 5(i_1 - i_2) = 0$

$1 = 5(i_2 - i_1)$

$i_2 = 2i_x$

$i_1 - i_2 = i_x$

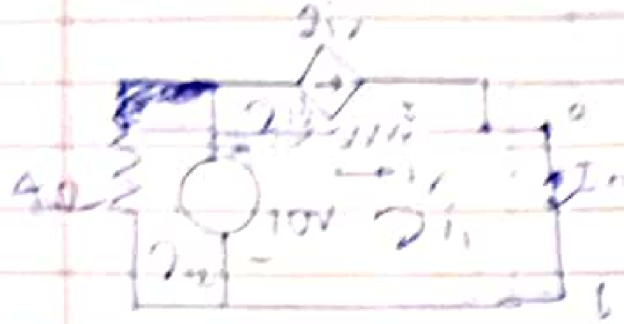
$i_1 = 3i_x$

$i_0 = 3i_x = 0.6\,A$

$1 = 5i_x$

$\{ i_x = 0.2\,A \}$

$R_N = 10/6 = 1.66\,\Omega$



i $10-5(i_1-i_2)=0$

$i_1-i_2=i_x$

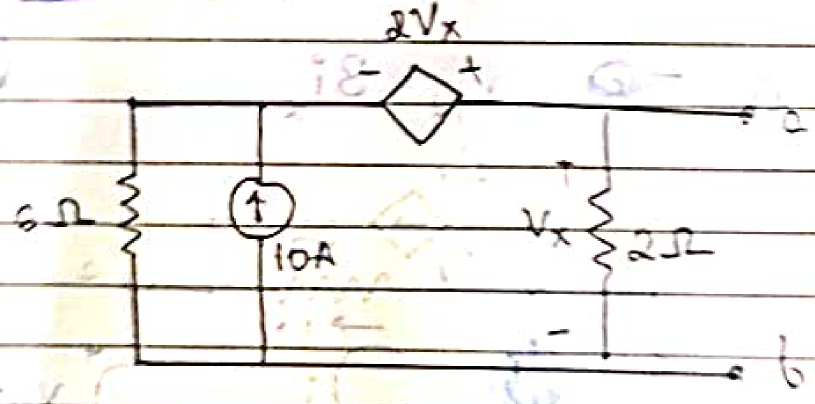
$\boxed{i_x = 2A}$

$2i_x+i_x = I_N$

$\boxed{I_N = 6A}$

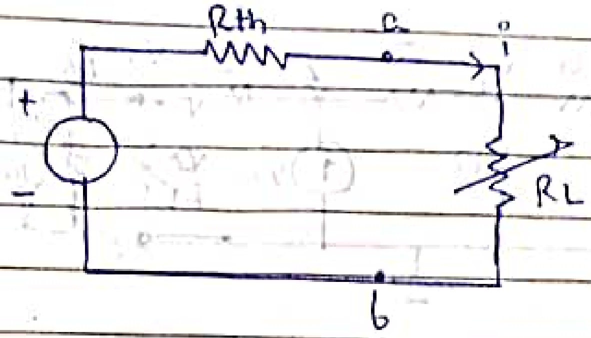
Q)

HW



Maximum Power Transfer Theorem

The maximum power is transferred to the load when load resistance is equal to source resistance as seen from the load.



$P = i^2 R$

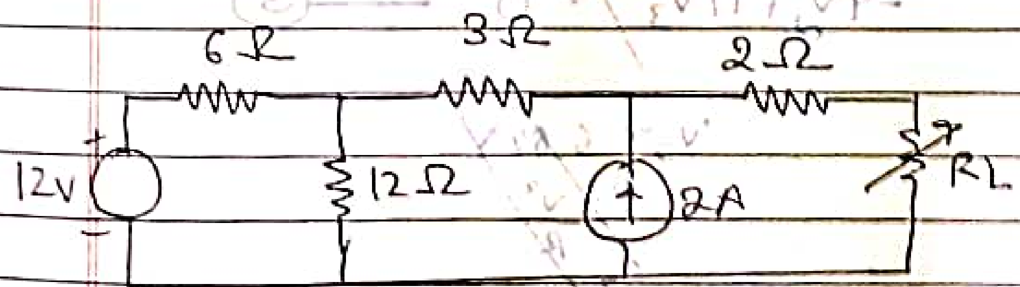
$\left(\frac{V_{th}}{R_{th}+R_L}\right)^2 \cdot R_L \quad \frac{\partial P}{\partial R} = V_{th}^2 \frac{\left[(R_{th}+R_2)^2 - 2R_2(R_{th}+R_2)\right]}{(R_{th}+R_2)^4}$

$$V_{th}^2 \left[\frac{R_{th} + R_L - 2R_L}{(R_{th}+R_2)^2}\right] = 0$$

$$\boxed{R_{th} = R_L}$$

$$\left\{ P_{max} = \frac{V_{th}^2}{4R_{th}} \right\}$$

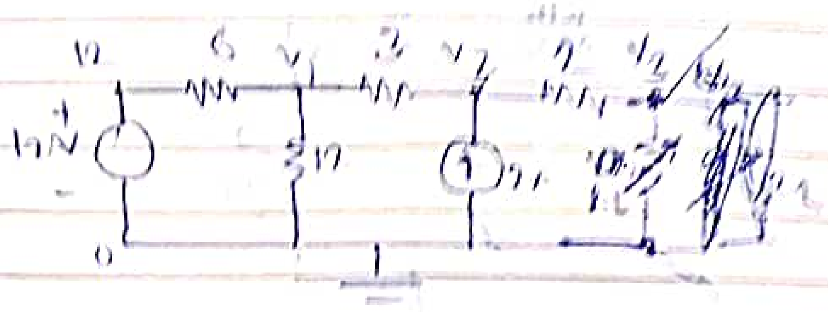
1) Find the R_L for max power transfer in a ckt. Also calculate max. power.



$R_{th} = \frac{6 \times 12}{6+12} + 5$

$= 4 + 5$

$= 9\Omega = R_L$



$$\frac{V_1-12}{6}+\frac{V_1}{12}+\frac{V_1-V_2}{3}=0$$

$$2V_1-24+V_1+4V_1-V_2=0$$

$$7V_1-V_2=24 \quad — (1)$$

$$\frac{V_2-V_1}{3}-2+\frac{V_2-V_3}{2}=0$$

$$2V_2-2V_1-12+3V_2-3V_3=0$$

$$-2V_1+5V_2-3V_3=12 \quad —— (2)$$

$$\frac{V_3-V_2}{2}+\frac{V_3}{9}=0$$

$$9V_3-9V_2+2V_3=0$$

$$-9V_2+11V_3=0 \quad —— (3)$$

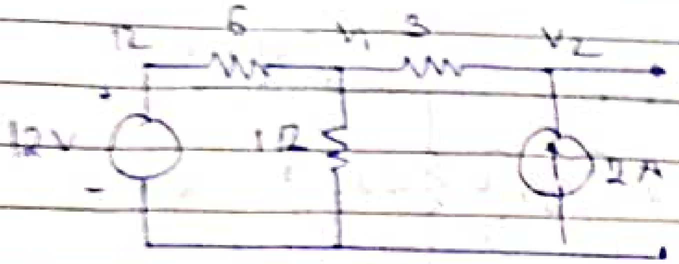
$V_3 = 6.62V$

$P = \frac{V^2}{R}$

$$\frac{V_2-V_1}{3}-2=0$$

$$V_2-V_1=6$$

PAGE NO.:
DATE / /



$$\frac{V_1-12}{6} + \frac{V_1}{12} + \frac{V_1-V_2}{3} = 0$$

$$2V_1 - 24 + V_1 + 4V_1 - 4V_2 = 0$$

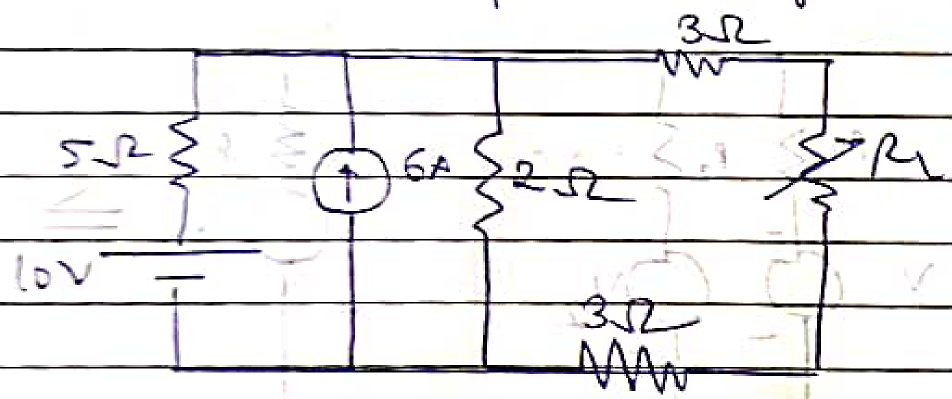
$$7V_1 - 4V_2 = 24 \quad \text{--- (1)}$$

$$\frac{V_2 - V_1}{3} - 2 = 0$$

$$V_2 - V_1 = 6 \quad \text{--- (2)}$$

$$V_{th} = V_2 = 22V$$

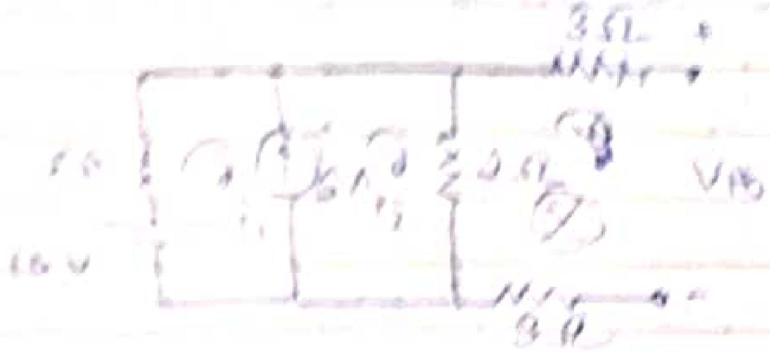
(1) Find max power transfer in R_L



$$R_{th} = \frac{5\times 2}{5+2} + 3 + 3$$

$$= 6 + 1.428$$

$$= 7.428\,\Omega$$

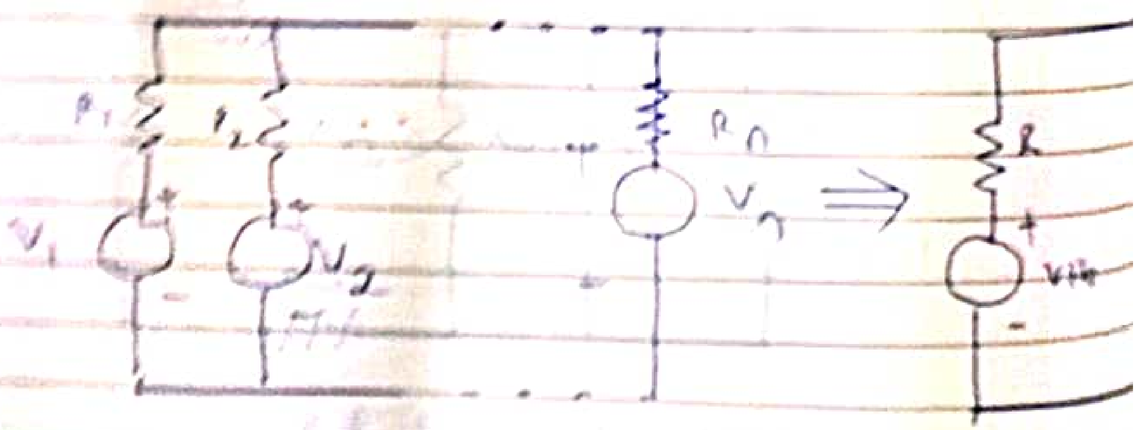


$$10 - 5i_1 - 2i_2 = 0$$

$$i_1 + 6 = i_2$$

$$i_1 = -0.73\ A$$
$$i_2 = 5.71\ A$$

$$V_{th} = \ldots$$
$$= 42.43\ V$$
$$\{P = 60.72\ W\}$$



$$G_i = 1/R_i$$

Millman's Theorem states that in any network if the voltage sources $V_1, V_2, V_3 \ldots V_n$ in series with internal resistance $R_1, R_2, R_3 \ldots R_n$ are connected in parallel, they can be replaced by a single source V in series with a resistance R where

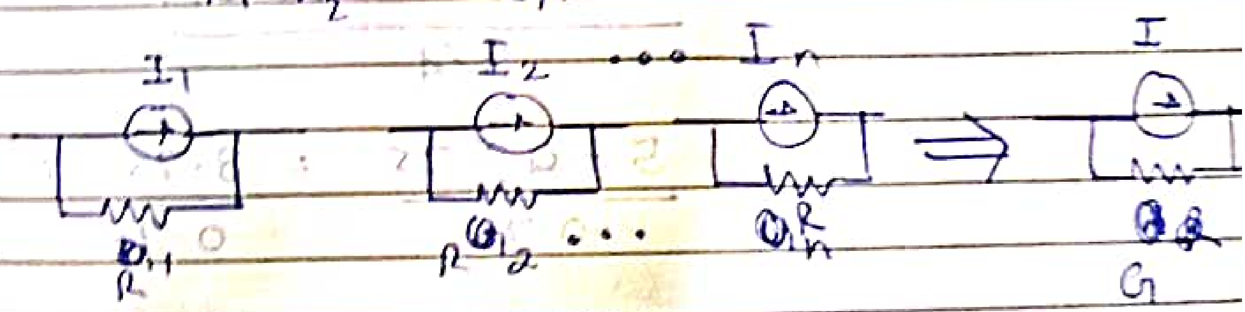
$$V = \frac{V_1 G_1 + V_2 G_2 \ldots V_n G_n}{G_1 + G_2 + G_3 \ldots G_n}$$

$$\& \quad R = \frac{1}{G_1 + G_2 \ldots G_n}$$

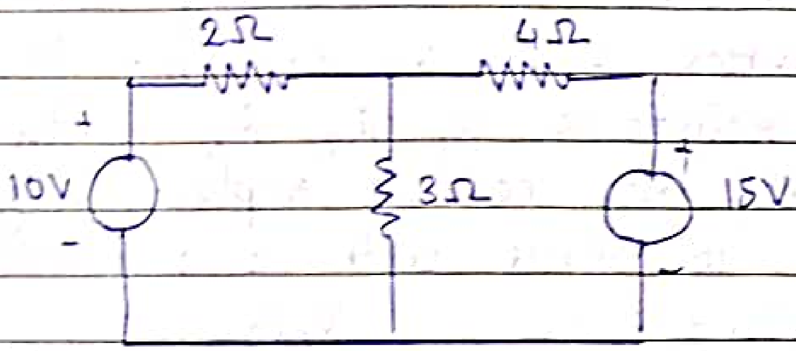
Similarly if n number of current sources are connected as shown can be replaced by a single current source $(i_1, i_2 \ldots i_n)$ I where

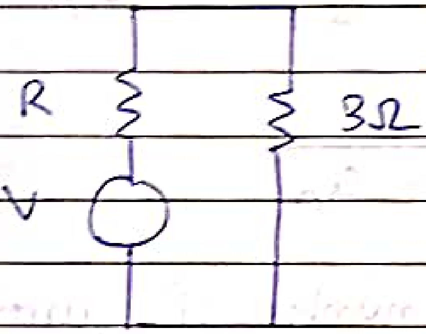
$$I = \frac{I_1 R_1 + I_2 R_2 \ldots + I_n R_n}{R_1 + R_2 \ldots + R_n}$$

$$G = \frac{1}{R_1 + R_2 \ldots + R_n}$$



1) Find current through 3Ω resistence.





$$G_1 = 1/2$$

$$G_2 = 1/4$$

$$G_1 + G_2 = \frac{2}{4} + \frac{1}{4} = 3/4$$

$$R = \frac{1}{G_1 + G_2} = 4/3 = 1.33\Omega$$

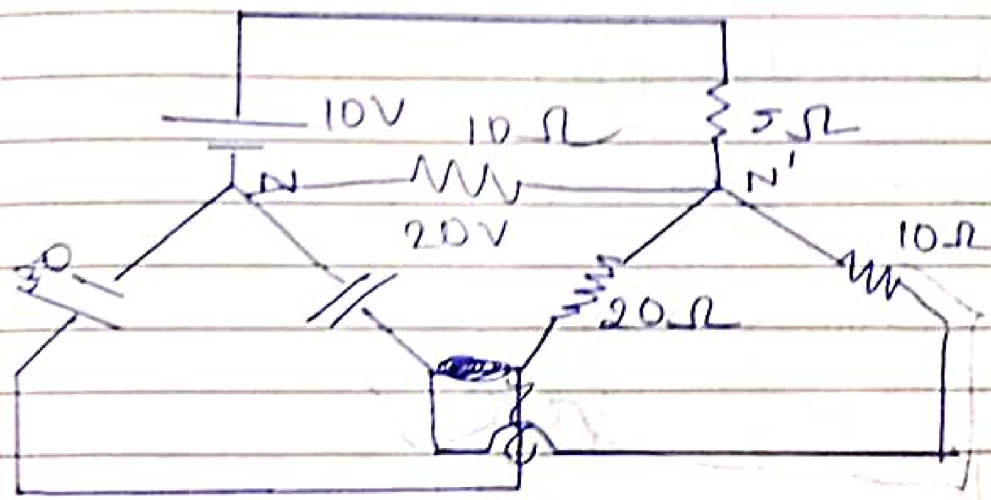
$$V = \frac{10 \times 1/2 + 15 \times 1/4}{3/4}$$

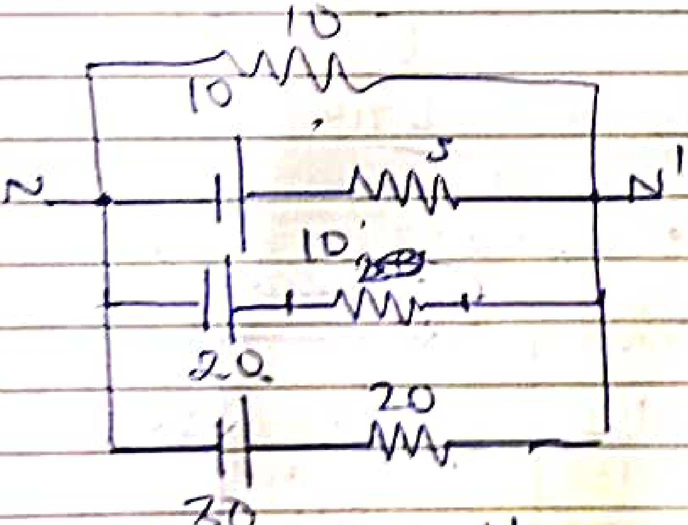
$$= \frac{5 + 3.75}{0.75} = \frac{8.75}{0.75} = 11.67V$$

$$i = \frac{V}{R+3} = 2.69\ A$$

Q) Find the current through 10Ω connected between terminals.

N−N'





$G_1 = 0\ 1/5 = 0.2$

$G_2 = \cancel{0.1}\ 0.1$

$G_3 = 1/20 = 0.05$

$G_n = \cancel{0.35}\ 0.25$

$R = \cancel{\ldots}\ \Omega$

$$\frac{10\times 0.2 + 30\times 0.05}{0.25}$$

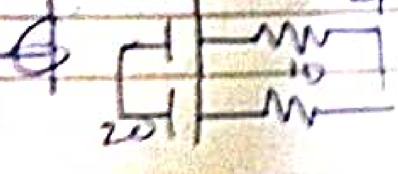
$= 4(2+1.5)$

$= 14$

$R = 4\Omega$

14V

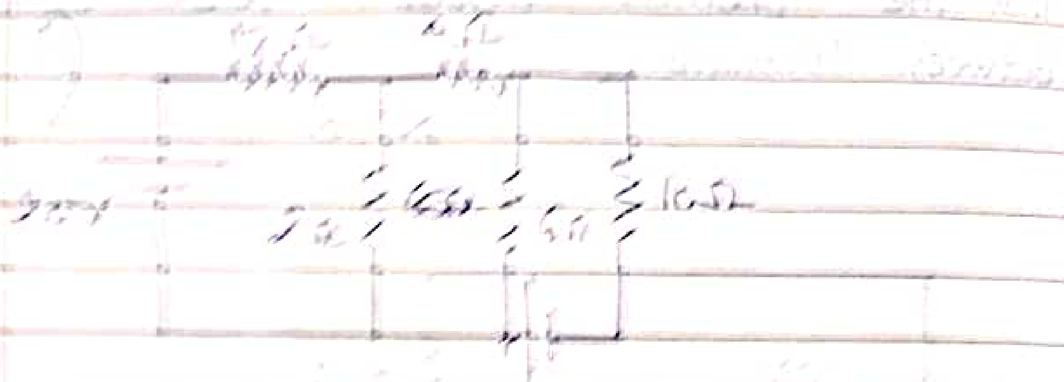
4

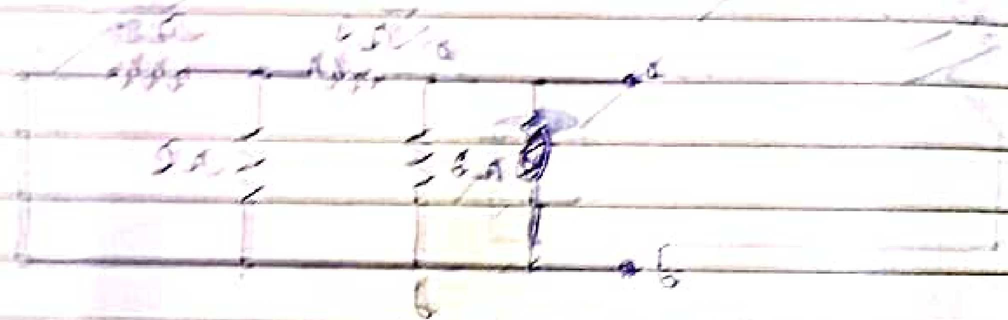


$14 - 4i - 10i - 20 = 0$

$i = \ldots$

Norton





$\frac{12 \times 4}{12+4} = 5.714\ \Omega$

$R_N = R_{th} = 2.92\ \Omega$

$40 - 2(i_1 - i_2) - 4i_1 = 0$

$40 - 6i_1 + 2i_2 = 0$

$-6i_1 + 2i_2 = -40$ — (1)

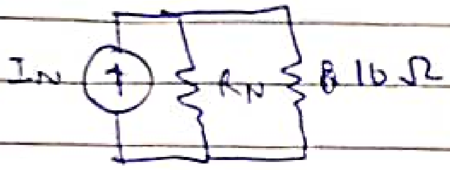
$20 - 10i_2 - 2(i_2 - i_1) = 0$

$2i_1 - 14i_2 = -20$ — (2)

$i_2 = 1.5A = i_N$

$i_1 = 3.5A$

PAGE NO.
DATE / /

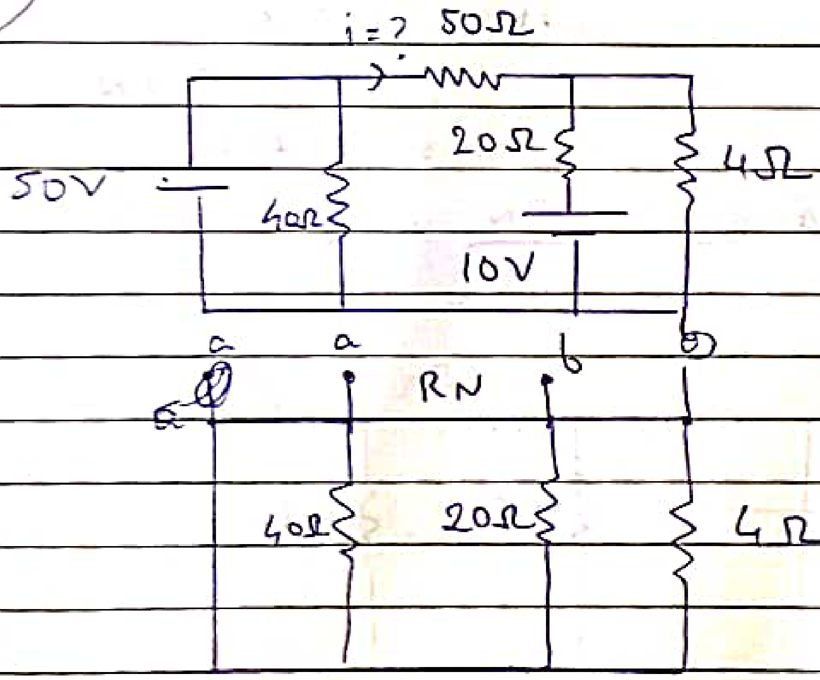


$$i_{10\Omega} = \frac{R_N}{10+R_N} \times i_N$$

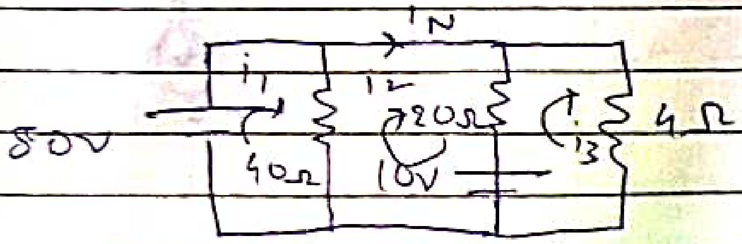
$$= 1.699\ A$$

Gotti – V 40V is b/w I_N branch & 6Ω so cannot neglected 6Ω

2)



$$R_N = \frac{20 \times 4}{20+4} = 3.33\ \Omega$$



$50 - 40(i_1 - i_2) = 0$

$= -40i_1 + 40i_2 = -50$ — (1)

$-40(i_2 - i_1) - 20(i_2 - i_3) - 10 = 0$

$= 40i_1 - 60i_2 + 20i_3 = 10$ — (2)

$10 - 20(i_3 - i_2) - 4i_3 = 0$

$= 20i_2 - 24i_3 = -10$ — (3)

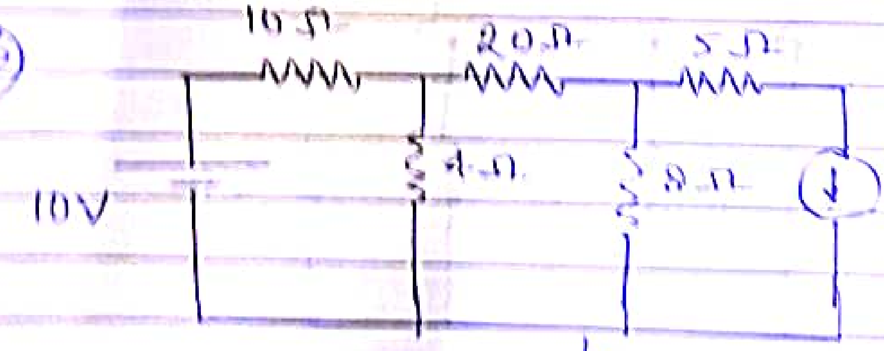
$\therefore i_1 = 15.75A$

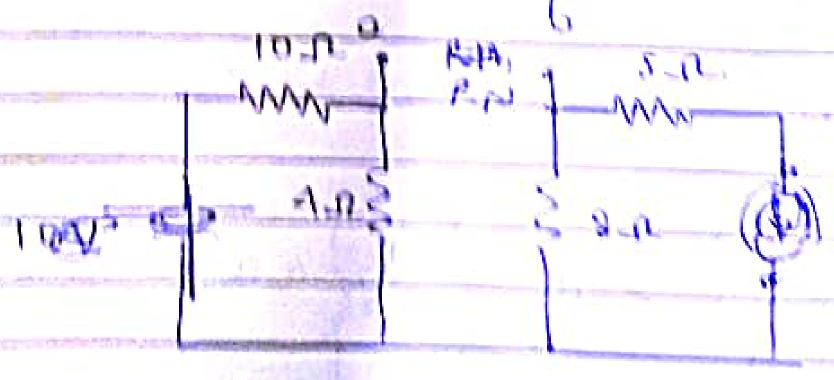
$i_2 = 14.5A = i_N$

$i_3 = 12.25A$

$i_1 \ (R_1) = \frac{R_N}{R_N + R_1} \times i_N = 0.905 \ A$

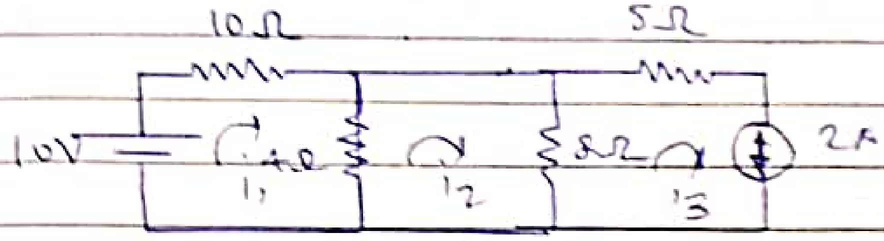
3)





$$R_p = \frac{10 \times 4}{10+4} = 2.857\,\Omega$$

$$R_N = 10.857\,\Omega$$



$$10 - 10i_1 - 4(i_1 - i_2) = 0$$

$$-14i_1 + 4i_2 = -10 \quad \text{—— (1)}$$

$$-4(i_2 - i_1) - 8(i_2 - i_3) = 0$$

$$-4i_2 + 4i_1 - 8i_2 + 8i_3 = 0$$

$$4i_1 - 12i_2 + 8i_3 = 0 \quad \text{—— (2)}$$

$$i_3 = 2A \quad \text{—— (3)}$$

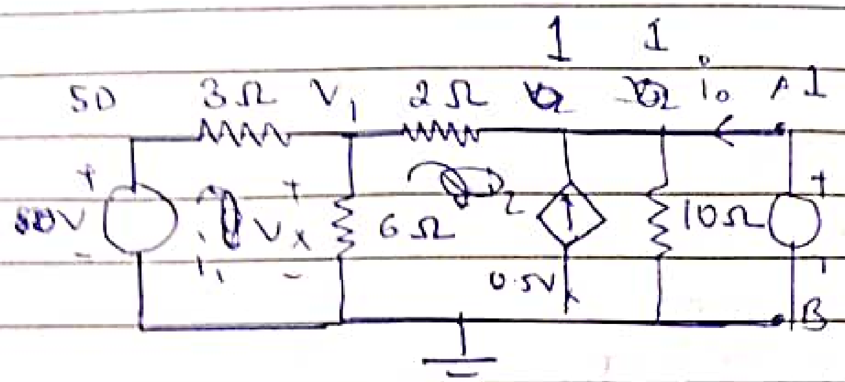
$$4i_1 - 12i_2 = -16 \quad \text{—— (4)}$$

$$i_1 = 1.21\,A \qquad i_2 = 1.736\,A$$

$$i_N = i_2 = 1.736A$$

$$i_L = \frac{R_N}{R_N + R_L} \times i_N = 0.6A$$

4)



$R_N = V_0/i$

$$\frac{V_1-50}{3} + \frac{V_1}{6} + \frac{V_1-1}{2} = 0$$

$$2V_1 - 100 + V_1 + 3V_1 - 3 = 0$$

$$6V_1 = 103$$

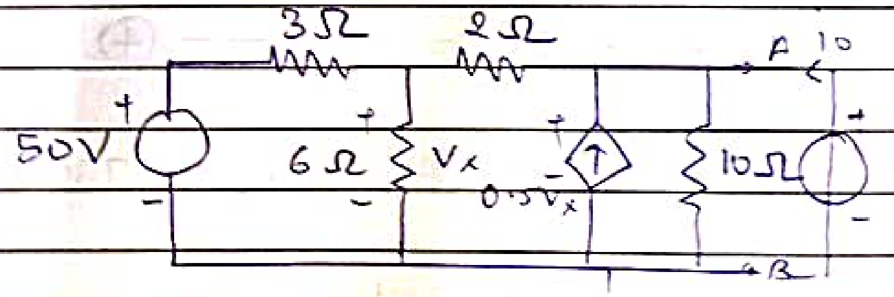
$$\{V_1 = 17.167\ V\}$$

$$i_{2\Omega} = \frac{V_1-1}{2} = 8.08\ A\ (\longrightarrow)$$

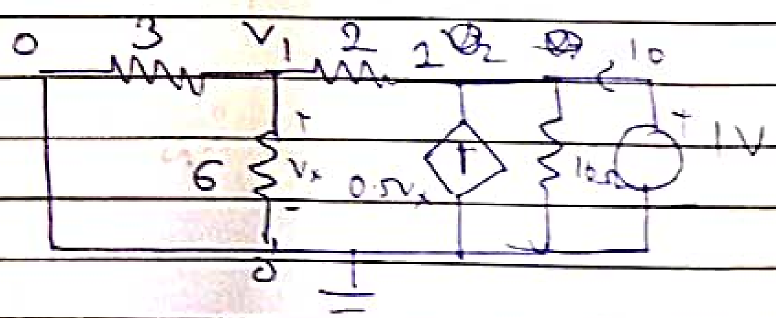
$$V_x = V_1 = 17.167\ V$$

$$i' = 8.835\ A\ (\uparrow)$$

4)



$R_N =$



PAGE NO.
DATE

$$\frac{V_1}{3} + \frac{V_1}{6} + \frac{V_1 - 1}{2} = 0$$

$$2V_1 + V_1 + 3V_1 - 3 = 0$$

$$V_1 = 3/6 = 0.5V$$

$V_x = 0.5V$

$i_{dS} = 1.5A$

$$i_1 = \frac{V_1 - 1}{2} = \frac{0.5 - 1}{2} = -0.25A$$

$$i_2 = \frac{1}{10} = 0.1A$$

2

$$-0.25 + 1.5 + i_n + 0.1 = 0$$

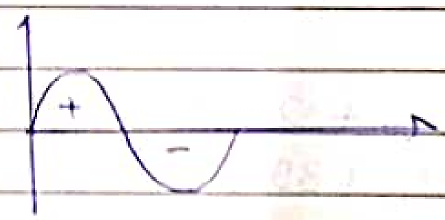
$$i_n = 1.35A$$

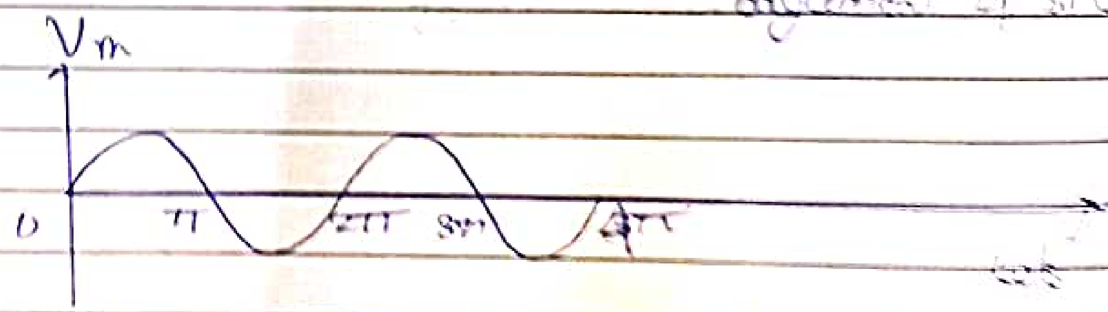
$R_N = 1/1.35$

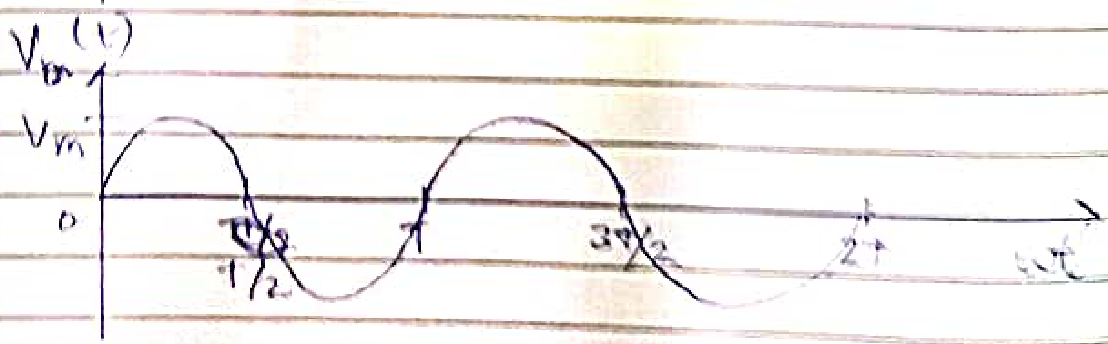
AC Circuits

The sinosoid is a signal that has the form of sine or cosine function.
Sinosodial current is usually referred to as alternating current or AC.
Sinosoidal signal is easy to generate or transmit. It is easy to handle mathematically i.e the derivative or integral of a sinosoid is also a sinosoid.

$V(t) = V_m \sin(\omega t)$ where V_m is amplitude of sinosoid,
ω is angular frequency in radians/second,
(ωt) is called argument of sinosoid.







The sinusoidal repeats after every T seconds where T is called period.

$\omega T = 2\pi$

$T = \frac{2\pi}{\omega}$ → time period

$v(t) = V_m \sin \omega t$

$v(t+T) = V_m \sin \omega (t+T)$

$= V_m \sin(\omega t + \omega T)$

$= V_m \sin(\omega t + 2\pi)$

$= V_m \sin \omega t$

$v(t+T) = v(t)$

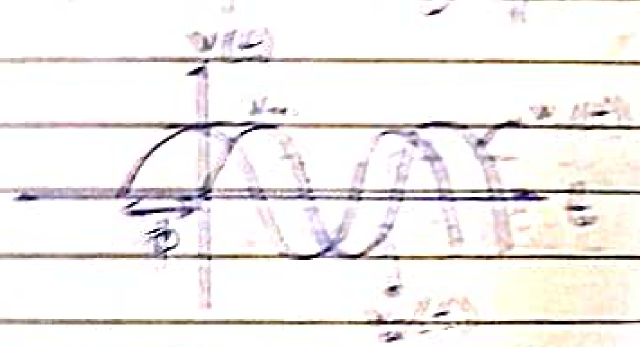
∴ $v(t)$ is periodic function with period $= T$

<u>General expression of sinusoid</u>

$v(t) = V_m \sin(\omega t - \phi)$

$(\omega t - \phi)$ → argument

ϕ → phase angle / phase of sinusoid



$v_1(t) = V_m \sin \omega t$

$v_2(t) = V_m \sin(\omega t - \phi)$

PAGE NO.
DATE

v_2 leads v_1 by ϕ deg
or v_1 lags v_2 by ϕ

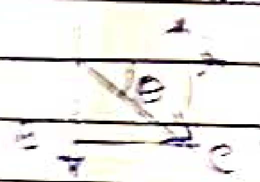
when you are comparing two sinusoids make sure that both are in sine or cosine form with positive amplitude.

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(\omega t \pm \pi) = -\sin \omega t$
$\cos(\omega t \pm \pi) = -\cos \omega t$
$\sin(\omega t \pm 90) = \pm \cos \omega t$
$\cos(\omega t \pm 90) = \mp \sin \omega t$

Adding two sinusoid

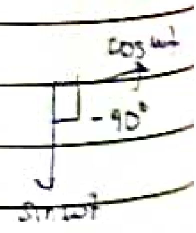
$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$



$C = \sqrt{A^2 + B^2}$
$\theta = \tan^{-1}(B/A)$

$\tan^{-1}(4/3)$

Q) $3 \cos \omega t + 4 \sin \omega t$
$5 \cos(\omega t + 53)$



Q) $v(t) = 12 \cos(50t + 10^\circ)$
A = 12 unit rad/s
$\omega = 50$; T = 0.125 s
$\phi = 10^\circ$; f = 7.957 Hz

Calculate phase angle b/w

$v_1 = -15\cos(\omega t + 50^\circ)$

$v_2 = 12\sin(\omega t - 10^\circ)$

$v_1 = -10\cos(\omega t + 50 - 40)$

$= 10\cos(\omega t - 40)$

$v_2 = 12\sin(\omega t - 10)$

$\theta_1 - \theta_2 = -30^\circ$

$\therefore v_1$ is lagging by 30°

$\cancel{v_1 = -10\cos(\omega t + 50)}$

$\cancel{= -10\cos(\omega t [illegible])}$

$i_1 = -4\sin(377t + 55)$

$i_2 = 5\cos(377t - 40)$

$- 50$

$= 5\cos(377t + 90 + 50)$

$\cancel{i_2 = 5\sin(377t + 50)}$

180

$\underline{55}$

125

$-5\sin(377t - [illegible]$

$i_2 = 5\sin(377t + 130)$

$i_1 = -4\sin(377t + 180 - 155)$

$= 4\sin(377t + [illegible] - 155)$

$i_1 = -4\sin(377t + 25)$

$i_2 = 5\cos(377t + 40)$
$+50$

$i_2 = 5\cos(377t + 90 + 50)$

$i_2 = 5\sin(377t + 50)$

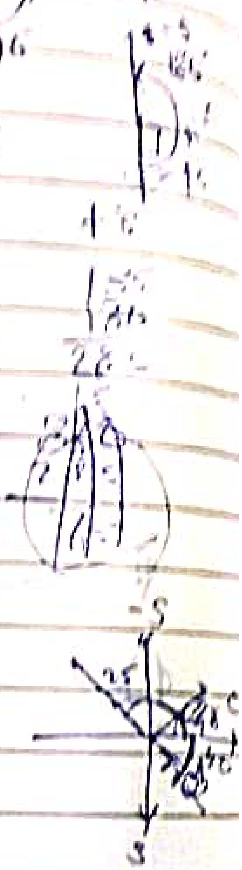
$i_1 = -4\sin(377t + 25)$

$= -4\sin(377t + 180 - 180 + 25)$

$= 4\sin(377t - 155)$

$\phi_1 - \phi_2 = 205 - 360$

$= -155°$



Euler's identity

$$e^{j\phi} = \cos\phi + j\sin\phi$$

Phasor is a complex number that represents the amplitude and phase of a sinosoid. Sinosoid's are easily expressed in terms of phasors which are more convinient to work with than sine and cosine functions. A complex number Z can be written in rectangular form when it equal to

$Z = x + jy$ (rectangular form)

$j = \sqrt{-1}$

$Z = r\angle\phi$ → Polar form

$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$

PAGE NO.:
DATE: / /

The idea of phasor representation is based on Euler identity.

$$\{e^{j\phi} = \cos\phi + j\sin\phi\}$$

$$z = re^{j\phi} \rightarrow \text{exponential form}$$

$$\cos\phi = \text{real}(e^{j\phi})$$

$$V(t) = V_m \cos(\omega t + \phi)$$

$$= Re\left[V_m e^{j(\omega t + \phi)}\right]$$

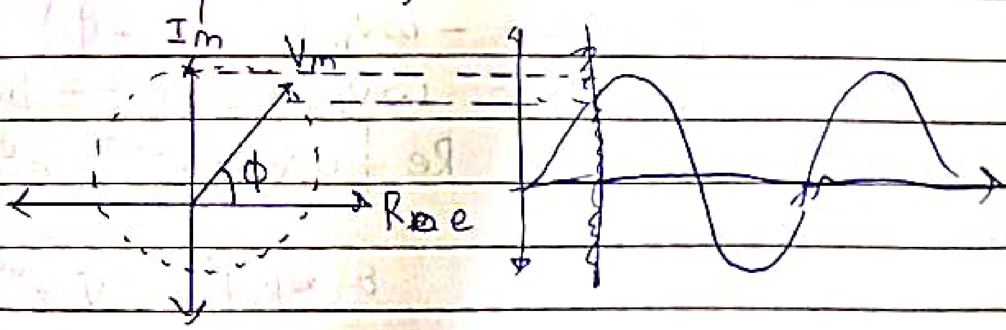
$$= Re\left[V_m e^{j\phi} \cdot e^{j\omega t}\right]$$

$$= Re\left[\mathbf{V} e^{j\omega t}\right] \text{ --- (1)}$$

where

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle\phi \text{ --- (2)}$$

$\mathbf{V}$ is the phasor representation of sinosid $v(t)$.
A phasor is a complex representation of the magnitude and phase of the sinosoid.



If we project my point of $V e^{j\omega t}$ on real axis it will appear like in figure b

To get the phasor corresponding to a sinosoid express the sinosoid in cosine form so that sinosoid can be written as a real part of an a complex number. Then take out the time factor $e^{j\omega t}$ and whatever is left is a phasor corresponding to the sinosoid.

Time Domain	Phasor Domain
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

$$V(t) = V_m \sin(\omega t + \phi)$$
$$= Re(V e^{j\omega t})$$

$$\frac{\partial V(t)}{\partial t} = -\omega V_m \sin(\omega t + \phi)$$
$$= \omega V_m \cos(\omega t + \phi + 90)$$
$$Re\left[\omega V_m e^{j\omega t} \cdot e^{j\phi} \cdot e^{j90^\circ}\right]$$

$$= Re\left[j\omega V e^{j\omega t}\right]$$

This shows that derivative of $V(t)$ is transformed to the phasor domain is $j\omega V$.

PAGE NO.:
DATE: / /

$$\frac{\partial V(t)}{\partial t} \rightarrow j\omega V$$

$$V = L\frac{di}{dt}$$

$$= L\,\omega j I$$

$$= X_L I$$

$v(t) = v_m \sin(\omega t)$ $\qquad |v_m e^{j\omega t}| \rightarrow Im$

$\frac{d(v(t))}{dt} = v_m \cos(\omega t),$ $\qquad |v_m e^{j\omega t}|$ $\quad Re$ $\quad \vec{I}+90$

$= v_0(t)(j\omega)$

$v_m(\omega t + \phi_2 + 90 - 90)$

$$v(t) = v_m \cos(\omega t + 0 + \phi_1) + v_m \sin(\omega t + \phi_2)$$
$$= v_{m_1}\angle\phi_1 + v_{m_2}\angle(\phi_2 - 90)$$

Time Domain	Phasor Time
$v(t)$ is instanteneous representation	$V(P)$ is instantaneous phasor domain representation
Time dependent	Not time dependent
Always real	Phasor is always complex

Note: Phasor analysis applies only when frequency is constant.

Evaluate

(i) $(40\angle 50 + 20\angle -30)^{1/2}$

$$\left[(25.71 + j\,30.64) + (17.32 - j\,10)\right]^{1/2}$$

$$(43.03 + j\,20.64)^{1/2}$$

$$= \left[47.724 \angle 25.62\right]^{1/2}$$

$$= 6.91 \angle 12.81^\circ$$

(ii) $\dfrac{10\angle -30 + (3 - j4)}{(2+j4)(3-j5)^*}$

$$= \frac{(8.66 - 5j) + (3 - 4j)}{(2+j4)(3+j5)}$$

$$= \frac{(11.66 - 9j)}{(-14 + 22j)}$$

$$= \frac{14.729 \angle -37.66}{26.076 \angle 122.471}$$

$$= 0.5648 \angle -160.131$$

PAGE NO:
DATE: / /

Transform the sinusoidal to phasor.

$$v = -4\sin(30t + 50^\circ)$$

$$= -4\,\overset{\sin}{\cancel{\cos}}(30t + 50 + 90 - 90)$$

$$= -4\cos(30t - 40)$$

$$= -4\cos(30t - 40 + 180 - 180)$$

$$= 4\cos(30t - 220)$$

$$= 4\cos(30t + 140)$$

360
220
140

$\cancel{4\angle 140}$

$$r\angle\theta = 4\angle 140^\circ$$

Q) (i) $= 6\cos(50t - 40^\circ)$

time domain $I = 6\angle -40^\circ$

But $I = -3 + j4$

Polar $r\angle\phi = 5\angle 126.86$

$= 5\cos(\omega t + 126.86)$
Sinusoid.

Q)
$i_1 = 4\cos(\omega t + 30)$
$i_2 = 5\sin(\omega t - 20) = 5\sin(\omega t - 20 + 90 - 90)$
$= 5\cos(\omega t - 110)$
Find the sum of sinusoids

$4\angle 30^\circ + 5\angle -110^\circ$
$= 3.46 + 2j + (-1.71) - j4.698$

$1.75 - 2.698j$
$= 3.215\angle -57.031$
$= 3.215\cos(\omega t - 57.031)$

$v_1 = -10\sin(\omega t + 30^\circ)$
$v_2 = 20\cos(\omega t - 45^\circ)$

$= -10\sin(\omega t + 30 - 90 + 90)$
$v_1 = 10\cos(\omega t + 120)$

$v_1 + v_2 = 20\angle -45^\circ + 10\angle 120^\circ$
$= 14.14 - j14.14 - 5 + j8.66$
$= 9.14 - j5.48$
$= 10.65\angle -30.94$
$= 10.65\cos(\omega t - 30.94^\circ)$

Q) Determine the current in ckt using phasor approach

$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$

$4I + \frac{\overset{4}{\cancel{8}}\, I}{2j\omega} - 3I j\omega^2 = 50\angle 75^\circ$

$4I - 10Ij\omega = 12.94 + 48.296j$
$4I - j\frac{\cancel{22}}{10} I = 12.94 + 48.296j$
$I = \frac{12.94 + 48.296}{4 - j10} = \frac{0.50\angle 75^\circ}{10.77\angle -68}$

$4.64 \angle 143.19^\circ$

$= 4.64 \cos(2t + 143.19^\circ)$

Q) Find v(t) in the ckt

$$2\frac{dv}{dt} + 5v + 10\int v\,dt = 20\cos(5t - 20^\circ)$$

$$2Vj\omega + 5V + \frac{10V}{j\omega} = 20 \angle -20^\circ$$

$$10Vj + 5V - 2Vj = 20 \angle -20$$

$$V = \frac{20 \angle -20}{5 + 8j}$$

$$= \frac{20 \angle -20}{9.43 \angle 57.99}$$

$$= 2.121 \angle -87.99$$

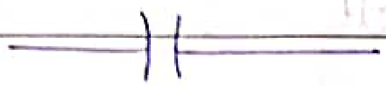
$$= 2.121 \cos(5t - 87.99)$$

Phasor Relationship for circuit elements

	Time Domain	Phasor Domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega L I$
C	$v = \frac{1}{C}\int i\,dt$	$V = \frac{I}{j\omega L}$

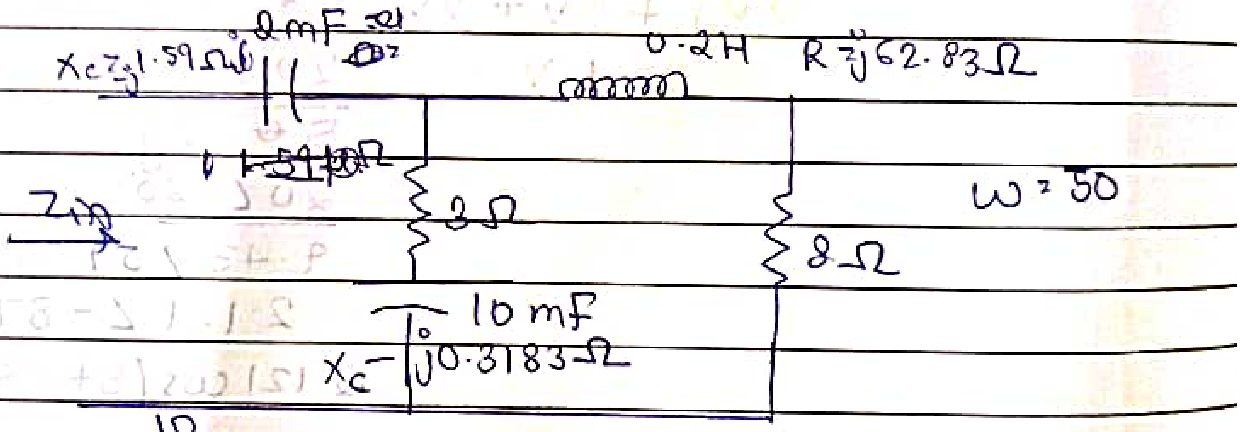
$Z = \frac{V}{I}$ = Impedence of the ckt.

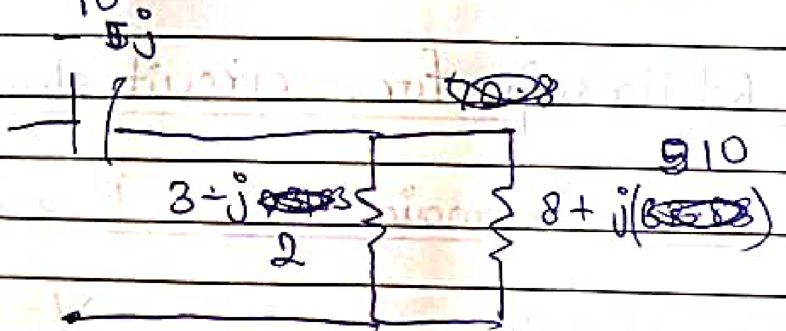
when $\omega = 0$ $Z_L = 0$ {short ckt}
$\omega = \infty$ $Z_L = \infty$ {open ckt}



when $\omega = 0$ $Z_C = \infty$ {open ckt}
$\omega = \infty$ $Z_L = 0$ {short ckt}

Q) Find the input impedence of the circuit assumming that ckt operates at $\omega = 50$ rad





$3.01 \angle -0.102$ $63.33 \angle 1.44°$

$R_{eq} = \frac{190.62 \angle 1.338}{11 - j62.517}$

$= \frac{190.62 \angle 1.338}{63.477 \angle -1.396}$

$3 \angle 2.734$ $2.754 + j1.189$

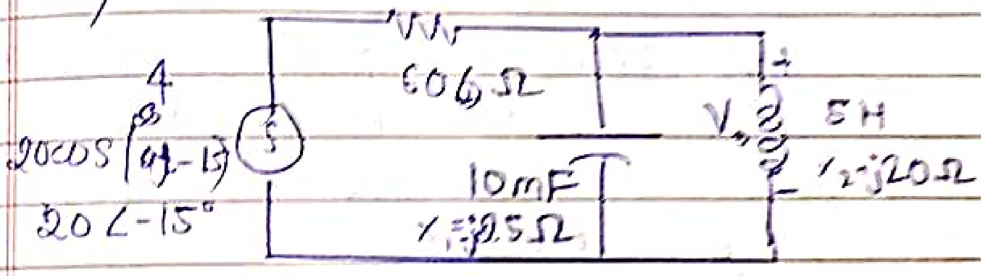
4.058

$$R_{eq} = \frac{(8+10j)(3-2j)}{(8+10j+3-2j)} = \frac{(12.8\angle 0.89)(3.605\angle -0.59)}{13.6\angle 0.62}$$

$$Z_{eq} = R_{eq} - 10j$$

$$= \cancel{3.22-j}\ (8.22 - 11.07j)\ \Omega$$

(b) Find $v_o(t)$



$$R_p = \frac{(-j2.5)(j20)}{-5j}$$

$$\frac{\cancel{25}\angle -1.57 \times 20\angle 1.57}{\cancel{5}\angle -1.57}$$

$$R_p = 100\angle 1.57\ \Omega$$

$$R_{net} = (j100 + 60)\ \Omega$$

$$i = V/R$$

$$= \frac{20\angle -15^\circ}{116.6\angle 59.03} = 0.171\angle -74.03^\circ \quad \cancel{0.171\angle -13.97}$$

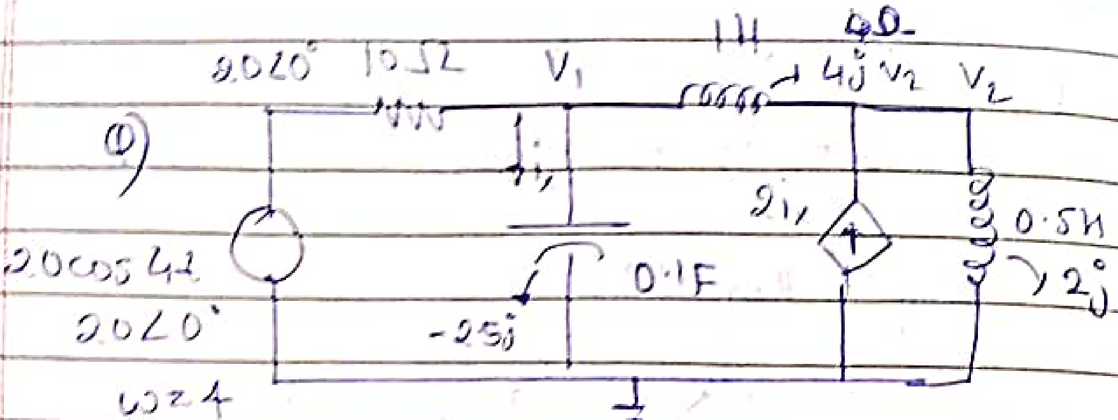
$$= \cancel{0.2\angle -12.43}$$

$$= \cancel{(0.129 - j0.152)}\ A$$

$$V_o = (0.171\angle -74.03)(100\angle 1.57)$$

$$= 17.1\angle -18.03\ V$$

1)



Steps
1) Transform the ckt to phasor or frequency domain

2) Solve the problem using any of the known technique mesh, nodal, superposition etc.

3) Transform resulting phasor to the time domain

$$\frac{20-V_1}{10} + \frac{V_1}{-2.5j} + \frac{V_2-V_1}{4j} = 0$$

$$2 - 0.1V_1 + (0.4j)V_1 + j0.25(V_2 - V_1) = 0$$

$$2 + V_1(0.1 + 0.4j + 0.25j) + 0.25V_2 j = 0$$

$$V_1(0.1 + 0.15j) - 0.25V_2 j = -2 \quad (1)$$

$$\frac{V_2-V_1}{4j} + 2i_x + \frac{V_2}{2j} = 0$$

$$j(V_1 - V_2)(0.25) - 2i_x - 0.5jV_2 = 0$$

$$0.25jV_1 - j(0.75)V_2 - 2i_x = 0 \quad (2)$$

$$\frac{V_1}{-2.5j} = i_x$$

$$0.4jV_1 - i_x = 0 \quad (3)$$

PAGE NO.:
DATE: / /

$$jV_1 - j(0.75)V_2 - 2(0.4j)V_1 = 0$$

$$\cancel{j}0.2V_1 = \cancel{j}0.75V_2$$

$$\{V_1 = 3.5V_2\}$$ $i =$

$$3.5V_2 (-0.1j + 0.65j) - j0.25V_2 = -2$$

$$\frac{V_1 - 20}{10} + \frac{V_1 - V_2}{4j} + \frac{V_1}{-2.5j} = 0$$

$$0.1V_1 - 2 - 0.25V_1 j + 0.25V_2 j + 0.4V_1 j = 0$$

$$V_1(0.1 - 0.25j + 0.4j) + 0.25V_2 j = 2$$

$$V_1(0.1 + 1.5j) + 0.25V_2 j = 2 \quad \text{— (1)}$$

$$\frac{V_2 - V_1}{4j} - 2i_x + \frac{V_2}{2j} = 0$$

$$-0.25V_2 j + 0.25V_1 j - 2i_x + 0.5V_2 j = 0$$

$$\cancel{V_1(0.25)}$$

$$0.25V_1 j - V_2(0.25j + 0.5j) - 2i_x = 0$$

$$0.25V_1 j - 0.75V_2 j - 2i_x = 0 \quad \text{— (2)}$$

$$\cancel{0 - V_2} \quad \frac{V_1}{-2.5j} = i_x$$

$$0.4V_1 j = i_x \quad \text{— (3)}$$

0.75

$$0.25V_1 j - \cancel{0.5}V_2 j - 0.8V_1 j = 0$$

$$-0.55V_1 j = 0.75V_2 j \qquad \underline{V_1 = -1.36V_2}$$

$V_1(0.01 + 1.5j) + 0.25V_2 j = 2$

$-1.36V_2(0.1 + 1.5j) + 0.25V_2 j = 2$

$V_2(-0.136 - 2.04j + 0.25j) = 2$

$V_2 = \dfrac{2}{-0.136 - 1.79j}$

$= -0.084 + 1.11j$

$V_1 = 0.114 - 1.509j$

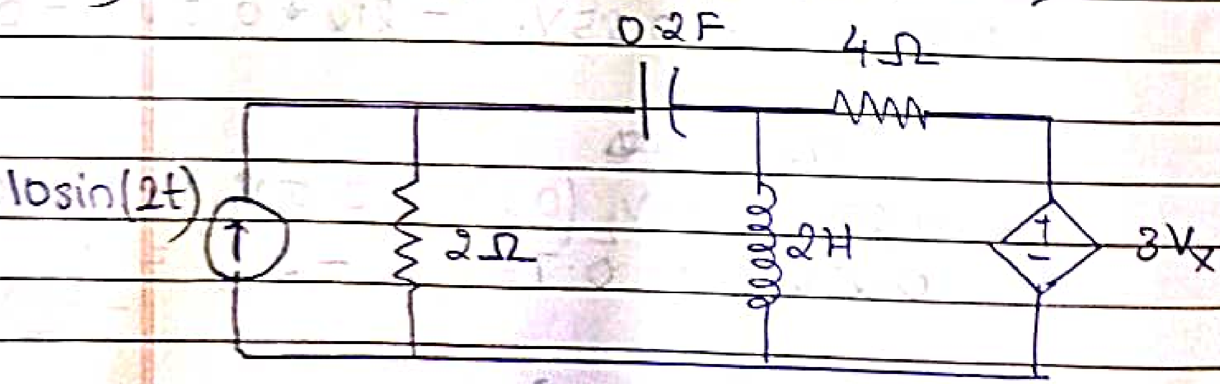
$i_x = 0.4V_1 j$

$= 0.603 + 0.045\hat{i}$

$\{ i_x = 7.59\angle 108.4^\circ \}$

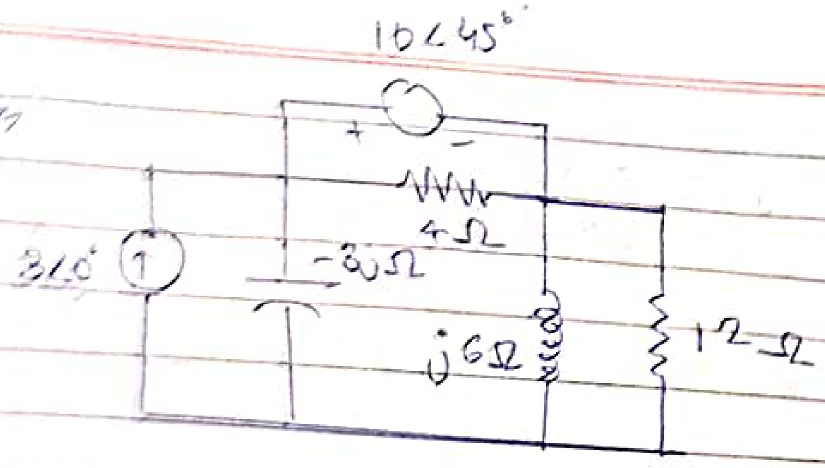
$i_x = 7.59\cos(\omega t + 108.4^\circ)$

9) Find V_1, V_2 in the ckt. HW



$V_1 = 20.96\sin(2t + 58^\circ)\ V$

$V_2 = 44.11\sin(2t + 41^\circ)\ V$



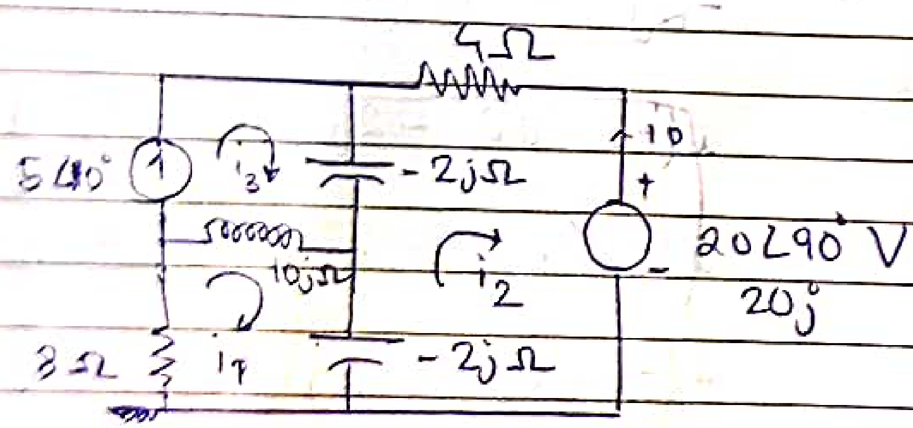
Find V_1 & V_2

$V_1 = 25.78 \angle -70.48$

$V_2 = 31.41 \angle -87.18$

{Use supernode}

Q) Determine i_o using mesh analysis



$\cancel{-3i_1 - i10j + 2j = 0}$

$-8i_1 - 10j(i_1 - i_3) + 2j(i_1 - i_2) = 0$

$-8i_1 - 10j\,i_1 + 10j\,i_3 + 2j\,i_1 - 2j\,i_2 = 0$

$i_1(-8 - 10j + 2j) - 2j\,i_2 + 10j\,i_3 = 0$

$i_1(-8 - 8j) - 2j\,i_2 + 10j\,i_3 = 0$ —(1)

PAGE NO.:
DATE: / /

$i_3 = 5\angle 90^\circ$ A
$= 5A$ ~~4.924 + 0.868j~~ — (2)

$\cancel{-4i_2 - 20j + 2j(i_2 - i_1) + 2j(i_2 - i_3) = 0}$

$-4i_2 - 20j + 2ji_2 - 2ji_1 + 2ji_2 - 2ji_3 = 20$

$i_2(-4 + 2j + 2j) - 2ji_1 - 2ji_3 = 20j$

$-2ji_1 + i_2(-4 + 4j) - 2ji_3 = 20j$ — (3)

$-2ji_1 + i_2(-4 + 4j) - 10j = 20j$

$-2ji_1 + i_2(-4 + 4j) = 30j$ — (4)

$\boxed{\begin{array}{l} \cancel{i_1(-8-8j) - 2ji_2 + (10j)(5) = 0} \\ \cancel{i_1(-8-8j) - 2ji_2 = -50j} \text{ — (5)} \end{array}}$

$i_1(-8-8j) - 2ji_2 = -50j$ — (5)

$-32j\, i_1 = i_2(-4+4j) = 400$ — (6)

$-34j i_1 = 400 + 30j$

$i_1 = -0.88 + 11.76j$

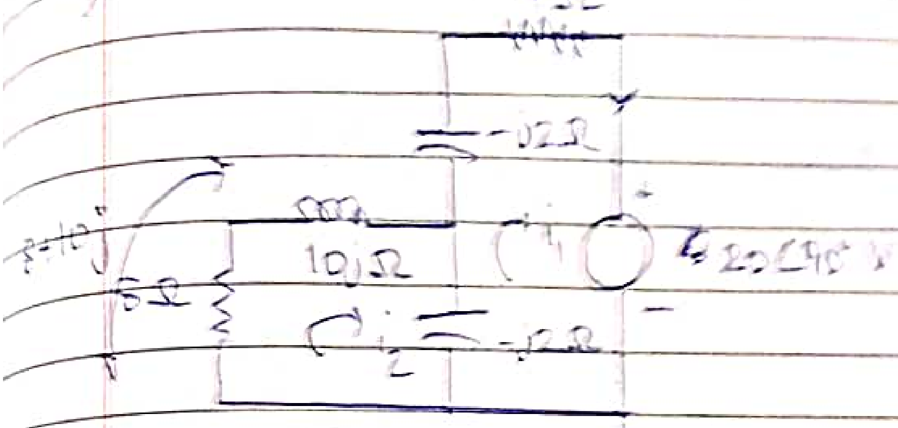
$i_2 = \dfrac{-23.52 + 25.24j}{(-4 + 4j)} = 6.47 - 0.5$

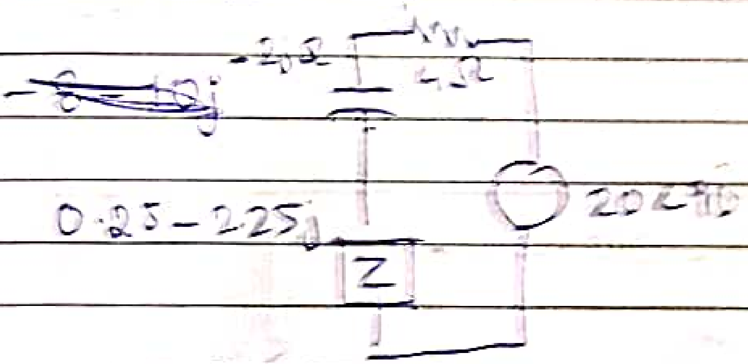
$i_2 = 6.42 \angle -5.21$

$\cancel{6.4 \cos(\omega t - 5.21^\circ)}$

$i_2 = 6.42 \cos(\omega t - 144.78)$

(1) Solve same question using superposition theorem



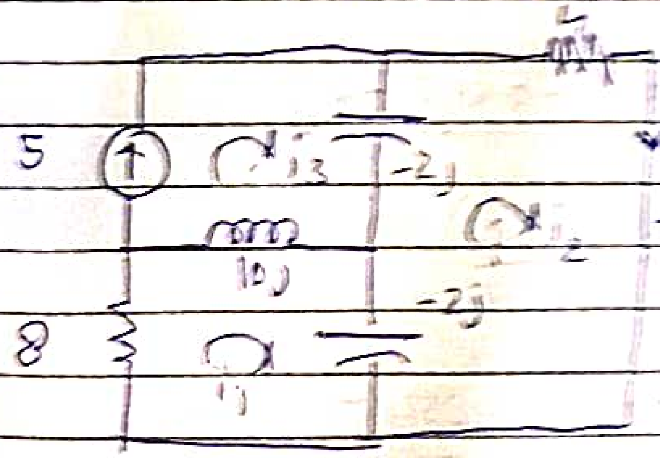


$Z_{eq} = 4.25 - 4.25j$

$V = 20j$

$i_o' = -2.353 + 2.353j$

$= 3.32 \angle 135°$



$i_3 = 5A$

$-2i_2 - 4i_2 - 5j$

$-2j(i_2 - i_1) = 0$

$-8i_1 - 10ji_1 - 2 \cdot 10j$

$-2j i_1 - 2j i_2 = 0$

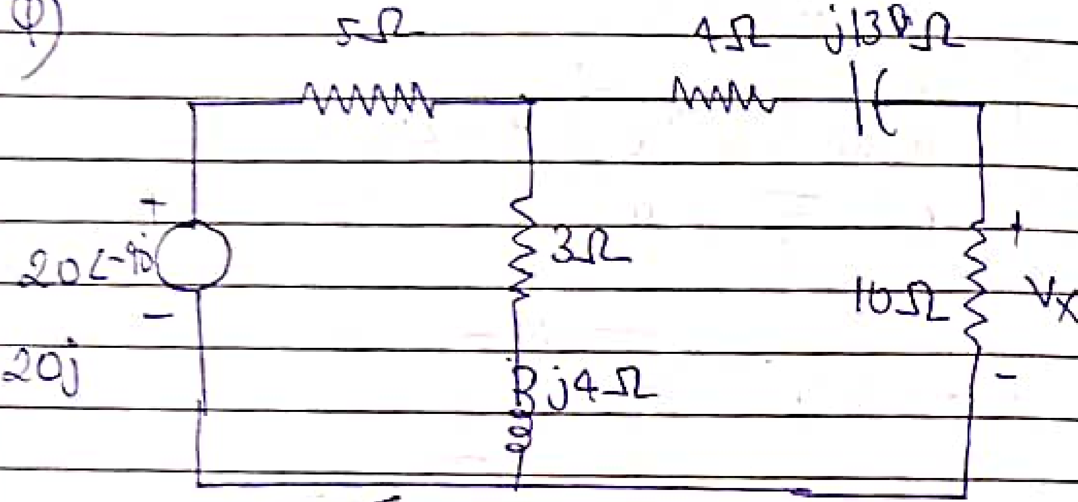
$i_1(-8 - 10j + 2j) + i_2(-2j + 5j) = 0$

$i_1(-8 - 8j) + i_2(3j) = 0$

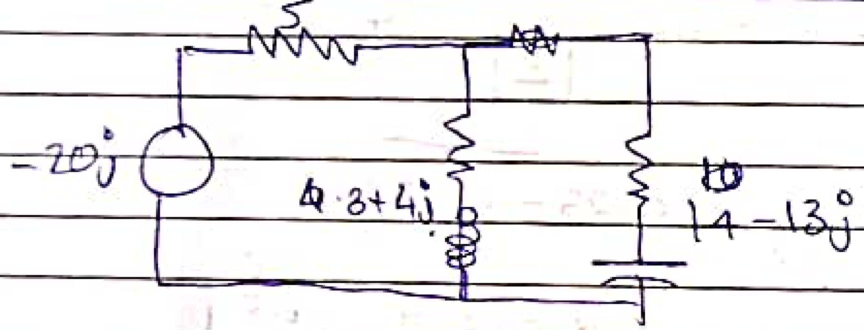
$2i_1(1 - j) = i_2$

$\cancel{-2j(i_2-i_1)}\ \cancel{-2j(1}$

Q)



−20j



$Z_{eq} = 8.905 + 3.067j$

$i = -0.6915 - 2j$

$i_2 = \left(\frac{R_1}{R_1+R_2}\right) \times i$

$i_2 = 0.4854 - 0.2586$

$= 0.5499 \angle -28.04°$

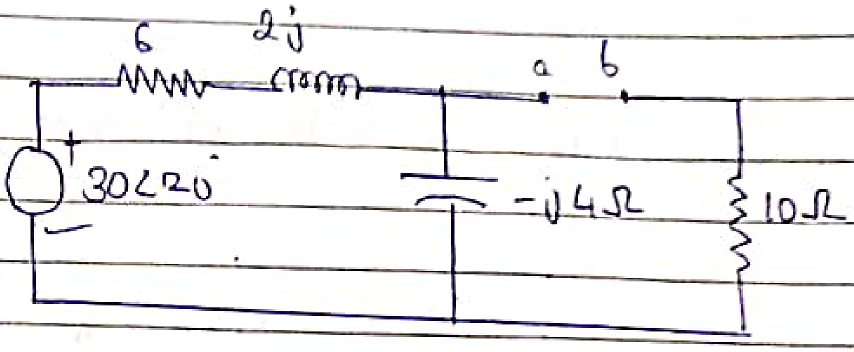
$V_x = i_2 \times R_2$

$= 5.499 \angle -28.04°$

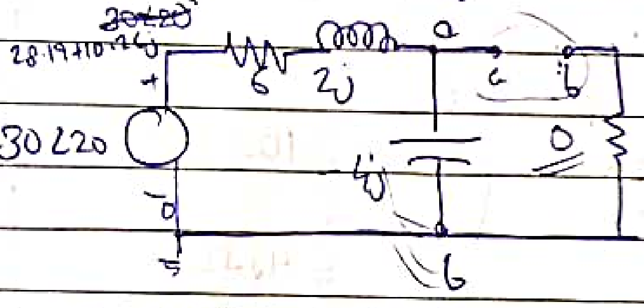
$V = 5.499 \cos(\omega t - 28.04°)$

PAGE NO.:
DATE: / /

Q) Find thevenin eq. ckt across a & b



$Z_{Th} = 12.4 - 3.2j$



$V = 28.19 + 10.26j$

$i = \frac{V}{Z} = 3.715 + 2.948j$

$11.794 + 35.352j$

$\cancel{\Delta V = 16.394 + 25.48j}$

$V_{Th} = V_{ab} = 11.796 - 24.092j$

$\cancel{\Delta V = 148.62 + 117.94i}$

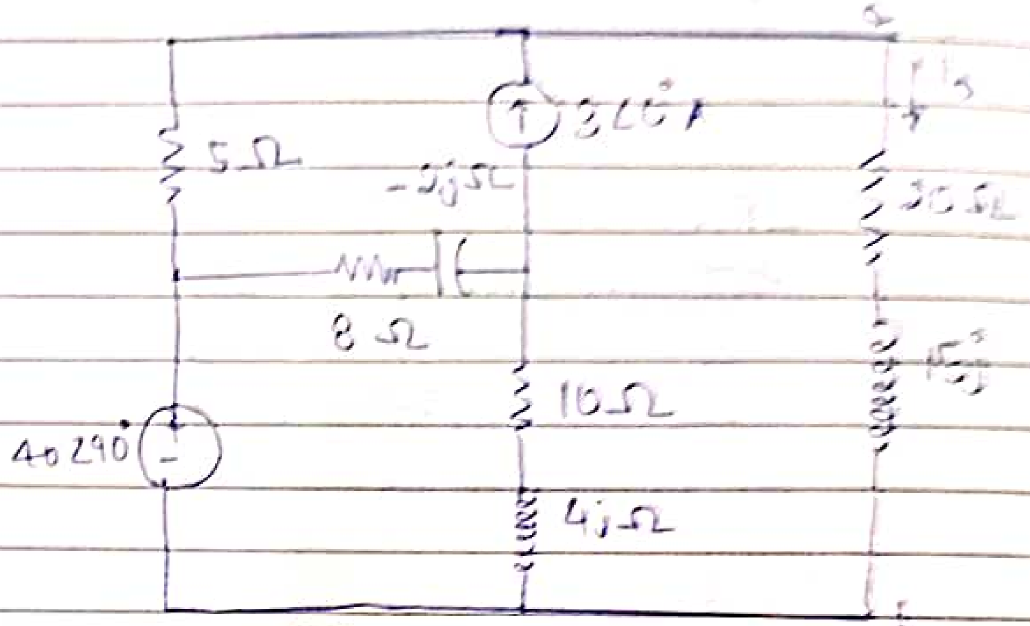
$\cancel{V_a =}$

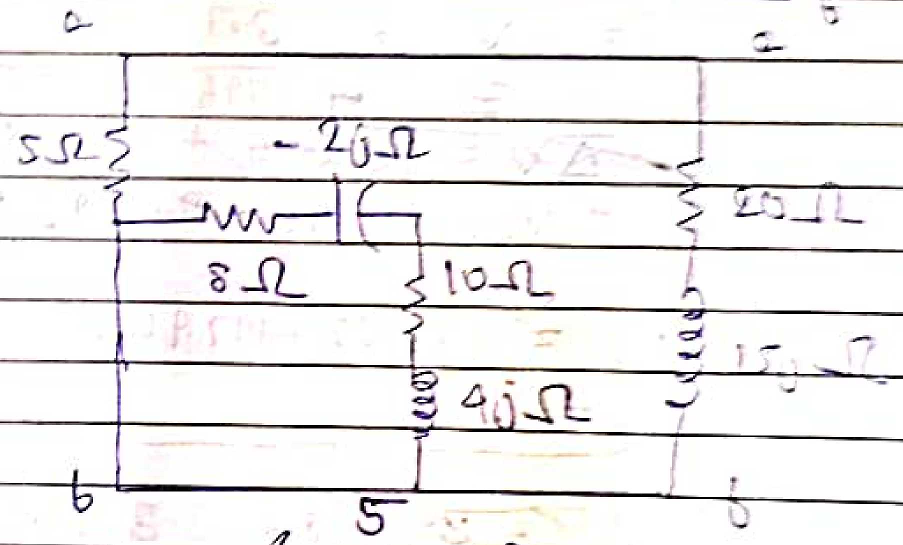
$\cancel{\Delta V = 89.478 + 13.326j}$

$\Delta V_a = 11.78 - 14.86j$

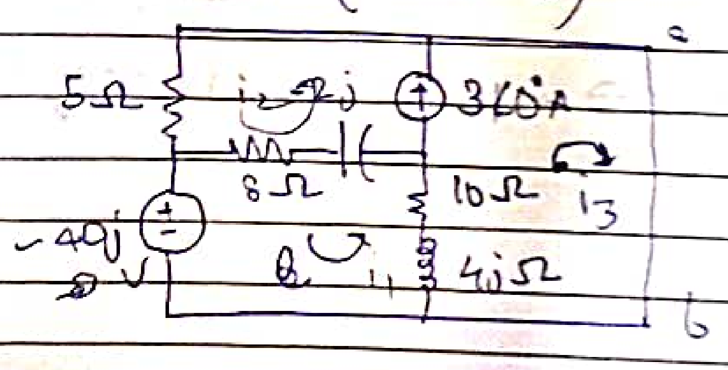
$\underline{\underline{V_{Th}}} = \left\{ r\angle\theta = 18.97\angle -51.56\,V \right\}$

Q) Calculate i_o in the ckt using Norton's Theorem





$R_N = (\ldots) \ \Omega$



PAGE NO.:

DATE: / /

$$-(-40j)-(4j+10)i_1+2j(i_1-i_2)-8(i_1-i_2)=0$$

$$i_2=3A$$

$$40j-(4j+10)i_1+2ji_1-6j-8i_1+24=0$$

$$(-4j-10+2j-8)i_1=-34j-24$$

$$i_1=\frac{34j+24}{2j+18}$$

$$=1.527+1.719j$$

$$-(-40j)-4j(i_1-i_3)-10(i_1-i_3)+2j(i_1-i_2)$$
$$-8(i_1-i_2)=0$$

$$40j-4ji_1+4ji_3-10i_1+10i_3+2ji_1-2ji_2$$
$$-8i_1+8i_2=0$$

$$i_1(-4j-10+2j-8)+i_2(-2j+8)+$$
$$i_3(4j+10)+40j=0$$

$$i_1(-18-2j)+i_2(8-2j)+i_3(4j+10)+40j=0 \quad —(1)$$

$$-5i_2-8(i_2-i_1)+2j(i_2-i_1)-10(i_2-i_1)-4j(i_2-i_1)$$

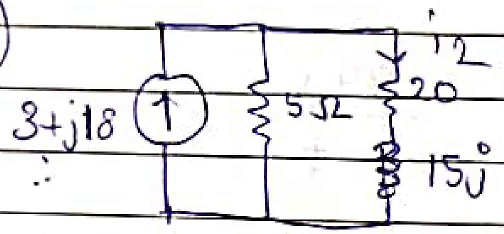
$$i_1(8-2j+10+4j)+i_2(-5-8+2j)$$
$$+i_3(-10-4j)=0 \quad —(2)$$

$i_2 + 3 = i_3$ — (3)

$\boxed{i_2 = j8}$ A

$\boxed{i_3 = 3 + j8}$ A

Q)



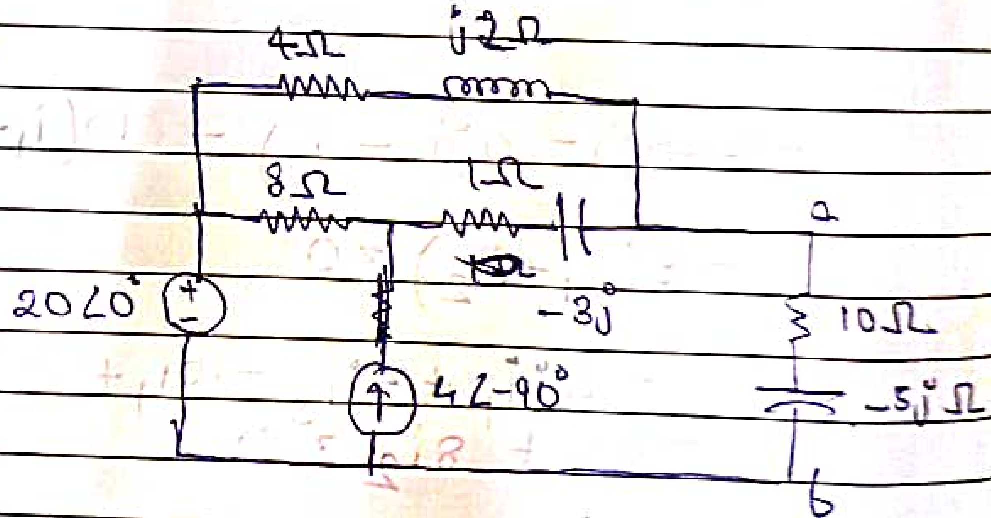
1.147 0.911

$i_2 = \cancel{} + \cancel{}j$

1.465 38.45

$i_2 = \cancel{0.445} \angle \cancel{}$ A

HW



$Z_N = 3.1 + j\,0.7\,\Omega$

$I_N = 8.39 \angle -32.68°$ A

$i_o = 1.97° \angle -2.1$ A

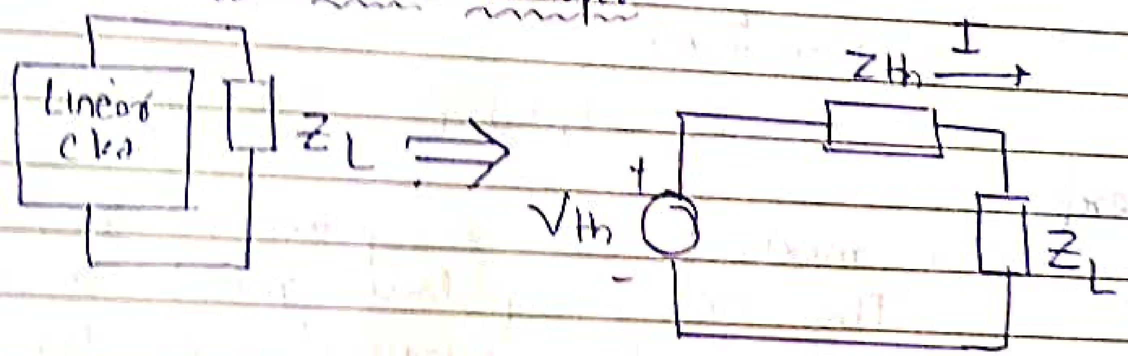
Instantaneous power $P = vi$

Average ~~in~~ or Real Power in Watts is the average of ~~instan~~ instantaneous power $P = \frac{1}{T}\int_0^t P.dt$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Max Avg. Power Transfer



For the above ckt let $Z_{th} = R_{th} + jX_{th}$
$Z_L = R_L + jX_L$

The current through the load $i = \frac{V_{th}}{Z_{th} + Z_L}$

$$\left[I = \frac{V_{th}}{(R_{th}+R_L) + j(X_{th}+X_L)} \right] = \frac{V_{th}}{\sqrt{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}} = |I|$$

The average power delivered to the load is

$$P = \frac{1}{2} |I|^2 R_L$$

$$\left\{ P = \frac{V_{th}^2 R_L}{2\left[(R_{th}+R_L)^2 + (X_L+X_{th})^2\right]} \right\}$$

PAGE NO.
DATE:

$$\frac{dP}{dX_L} = \frac{-V_{th}^2 R_L \, \cancel{2(X_{th}+X_L)}}{[(R_{th}+R_L)^2 + (X_{th}+X_L)^2]^2} \cancel{= 0} \quad 2(X_{th}+X_L) = 0$$

$$\{X_{th} = -X_L\}$$

$$\frac{\partial P}{\partial R_L} = 0 \Rightarrow R_L = \sqrt{R_{th}^2 + (X_L + X_{th})^2} \quad \text{— (A)}$$

$\therefore$ To meet requirements of avg. power

results in

$X_L = -X_{th}$ and $R_L = R_{th}$ i.e

{put this in (A)}

$$Z_L = R_L + jX_L = R_{th} - jX_{th} = \overline{Z}_{th} \text{ or } Z_{th}^{*}$$

Therefore

For maximum average power transfer;
The load impedance must be equal to the complex conjugate of the thevenin's impedance Z_{th}.

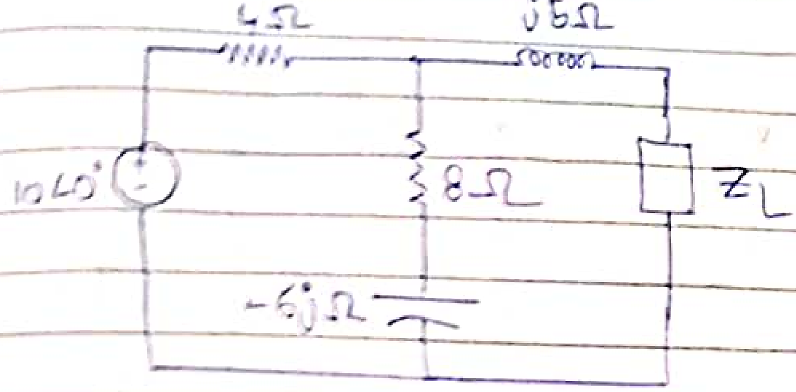
$\therefore$

$$\boxed{P_{max} = \frac{(V_{th})^2}{8R_{th}}}$$

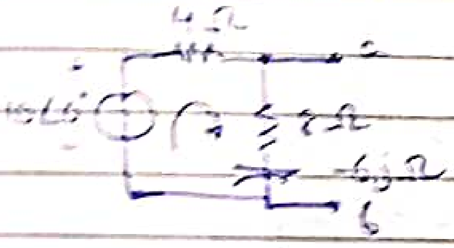
No capacitor / inductor consumes power.

Q) Determine the load impedance Z_L that maximizes the average power from the circuit. Also calculate the maximum average power.

PAGE NO.:
DATE: / /



$Z_{th} = 2.933 + 4.467j$



$V_{th} = ?$

$10 - 4i - (8-6j)i = 0$

$\frac{10}{12-6j} = i$

$i = 0.667 + 0.333j$

$V_{oc} = V_{th} = i \times (8-6j)$

$= 7.33 - 1.33j = 7.449 \angle -10.28° V$

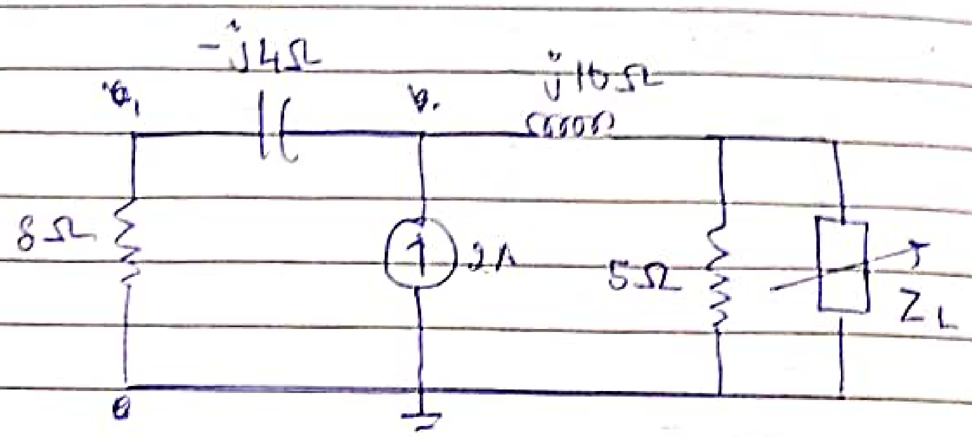
$\{Z_L = Z_{th}^* = 2.933 - 4.467j\} \rightarrow *$

$P_{max} = \frac{(V_{th})^2}{8 R_{th}}$

$= \frac{(7.449)^2}{8 \times 2.933}$

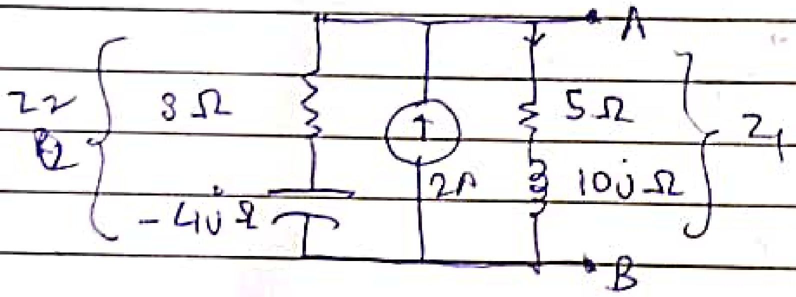
$\{P_{max} = 2.317 W\}$

HW



$\Rightarrow$

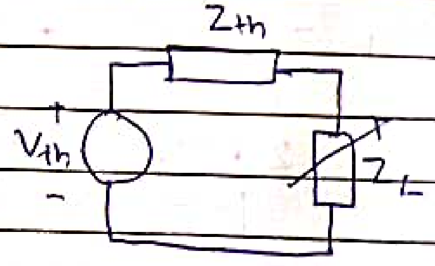
$$Z_{th} = (3.415 + 0.732j)\ \Omega$$



$$V_{th} = V_{AB} = i_{AB} \times (5+10j)$$

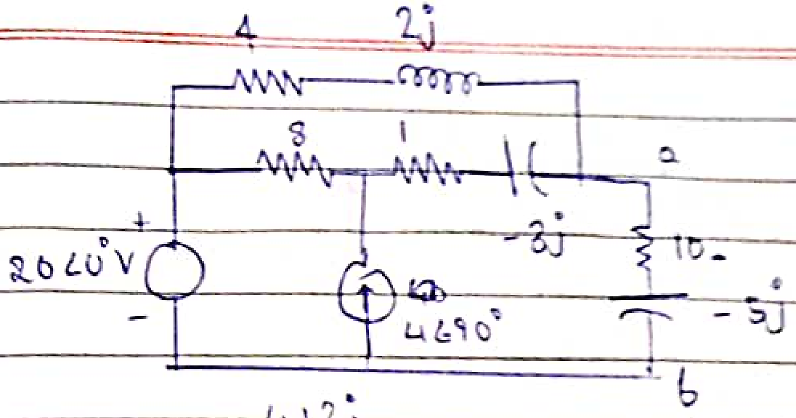
$$= \left(\frac{Z_2}{Z_1+Z_2}\right) \times i_T \times (5+10j)$$

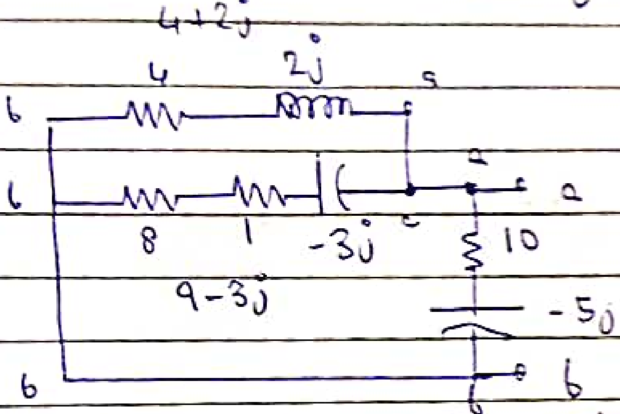
$$V_{th} = 13.658 + 2.926j = 13.967\angle 12.091^\circ$$



$$P_m = \frac{V^2}{8R_{th}} = 6.828\ \text{W}$$

Q)

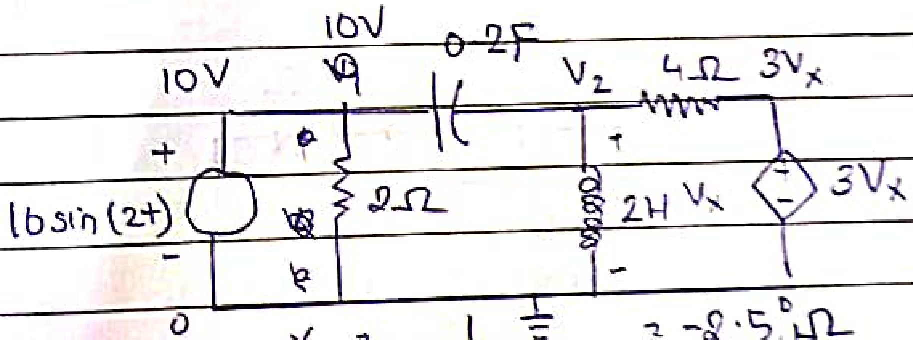




$$Z_{ab} = Z_N = (4+2j) \,||\, (9-3j) \,||\, (10-5j)$$

$$Z_{ab} = Z_N = 2.618 + 0.184j$$

Q)



$$X_C = \frac{1}{0.2\times 2} = -2.5j\,\Omega$$

$$X_L = 2\times 2 = 4j\,\Omega$$

$$\{V_1 = 10V\}$$

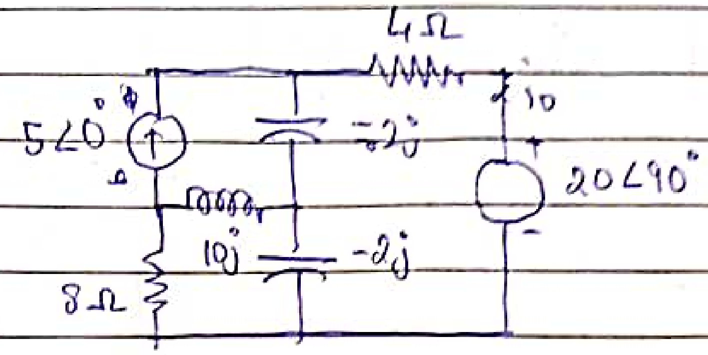
$$V_2(-0.5+0.15j) = 4j \qquad \frac{V_2-10}{-2.5j} + \frac{V_2-0}{4j} + \frac{V_2-3V_x}{4} = 0$$

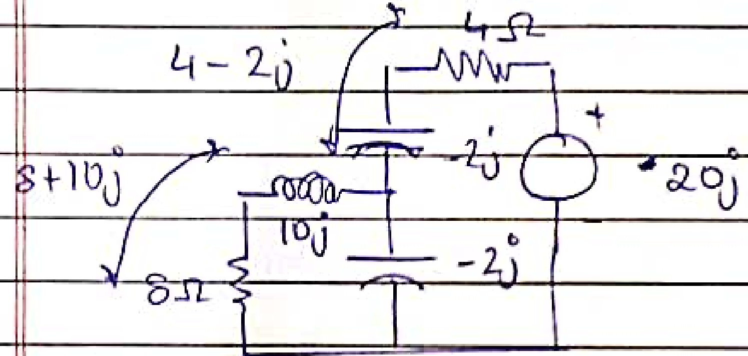
$$\frac{V_2 j}{2.5} - 4j + \frac{V_2}{4j} - \frac{V_2}{2} = 0 \qquad V_2 = V_x$$

$$V_2 = 2.201 - 7.33j \qquad \frac{V_2}{-2.5j} - 4 + \frac{V_2}{4} - \frac{V_2}{2} = 0$$

$$0.15V_2 = 4 \qquad \{V_2 = 26.67V\}$$

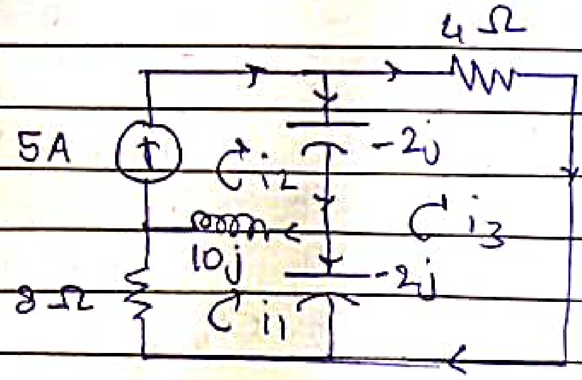
Q)





$$Z_{Req} = 4.25 - 4.25j$$

$$i_o' = \frac{20j}{Z_{eq}} = -2.35 + 2.35j$$



$$-8i_1 - 10j(i_1 - i_2) + 2j(i_1 - i_3) = 0$$

$$-8i_1 - 10ji_1 + 10ji_2 + 2ji_1 - 2ji_3 = 0$$

$$(-8 - 8j)i_1 + 10ji_2 - 2ji_3 = 0 \quad \text{--- } (1)$$

$$i_2 = 5A \quad \text{--- } (2)$$

$$(-8 - 8j)i_1 - 2ji_3 = -50j \quad \text{--- } (3)$$

$$-4i_3 + 2j(i_3 - i_1) + 2j(i_3 - 5) = 0$$

$$-4i_3 + 2ji_3 - 2ji_1 + 2ji_3 - 10j = 0$$

PAGE NO.:
DATE: / /

$i_3(-4+\overset{4}{8}j) - 2ji_1 = 10j$ — (4)

$-(8+8j) \quad -2j$

$-(4+4j) \quad -2$

$$\begin{bmatrix} -(8+8j) & -2j \\ -2j & -(4+4j) \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} -50j \\ 10j \end{bmatrix}$$

$$\Delta_0 = 8(1+j)\cdot 4(1+j) - (-2j)(-2j)$$
$$= 32(1+j)^2 - (4j^2)$$
$$= 4+64i$$

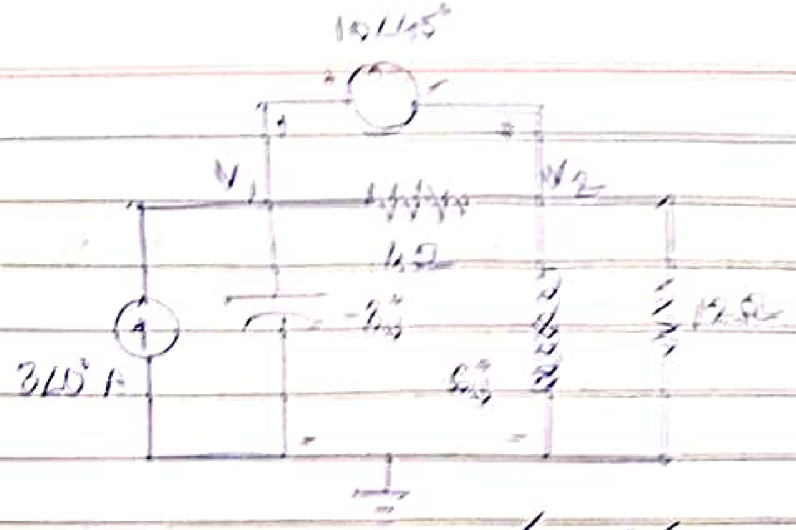
$$\Delta_2 = \left|\begin{bmatrix} -(8+8j) & -50j \\ -2j & 10j \end{bmatrix}\right| = 180-80j$$

$$\Delta_1 = \left|\begin{bmatrix} -50j & -2j \\ 10j & -(4+4j) \end{bmatrix}\right| = -220+200i$$

$$i_0'' = i_3 = \frac{\Delta_2}{\Delta_0} = -1.085 - 2.88j$$

$$i_0 = i_0' + i_0''$$
$$= -3.39 - 0.53j$$
$$= 3.43 \angle 171^\circ$$

Q)



$$-3 + \frac{V_1}{-3j} + \cancel{\frac{V_1 - V_2}{4}} + \cancel{i} = 0$$

$$\cancel{\frac{V_2 - V_1}{4}} + \frac{V_2}{6j} + \frac{V_2}{12} \cancel{- i} = 0$$

$$-3 + \frac{V_1}{-3j} + \frac{V_2}{6j} + \frac{V_2}{12} = 0$$

$$-3 + \frac{V_1 j}{3} - \frac{V_2 j}{6} + \frac{V_2}{12} = 0$$

$$-36 + 4V_1 j - 2V_2 j + V_2 = 0$$

$$V_2(1-2j) + V_1(4j) = 36 \quad \text{— (1)}$$

$$V_1 - (7.07 + 7.07j) - V_2 = 0$$

$$V_1 - V_2 = 7.07 + 7.07j \quad \text{— (2)}$$

$$(1-2j)V_1 - (1-2j)V_2 = 21.21 - 7.07j \quad \text{— (3)}$$

$$(1 - 2j + 4j)V_1 = 57.21 - 7.07j$$

$$V_1 = 8.614 - 24.298j$$

$$= 25.78 \angle -70.4°$$

Exam Pattern

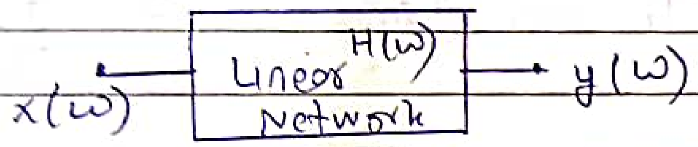
① Nodal/Mesh Analysis — [supermesh, supernode] A 6

② Superposition/Thevenin/Max. Power 6

AC extra

+ Phasor Analysis (Add, Multiply)

Frequency Response Analysis



* The frequency response of this circuit is the variation in its behaviour with the change in input signal frequency.

* The frequency response analysis is very significant in communication systems and control systems.

+ ~~Filters~~ Applications are in filters, TV, radio etc.

The transfer function H(ω) also called network function is ~~a~~ useful tool for finding frequency response of ckt. It can be expressed as

$$H(\omega) = \frac{y(\omega)}{x(\omega)}$$

Since the input and output variable can either voltage or current, therefore the possible transfer function

$$H(\omega) = \text{Voltage gain} = \frac{V_o(\omega)}{V_i(\omega)}$$

$$H(\omega) = \text{Current gain} = \frac{I_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer impedance} = \frac{V_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Admittance} = \frac{I_o(\omega)}{V_i(\omega)}$$

Transfer function $H(\omega)$ can be expressed in terms of its numerator polynomial $N(\omega)$ and denominator polynomial $D(\omega)$ and

$$H(\omega) = \frac{N(\omega)}{D(\omega)}$$

The roots of $N(\omega) = 0$ are called zeros of the system.

The roots of $D(\omega) = 0$ are called poles of the system.

Frequency at which overall frequency of system becomes zero is called zero of system.

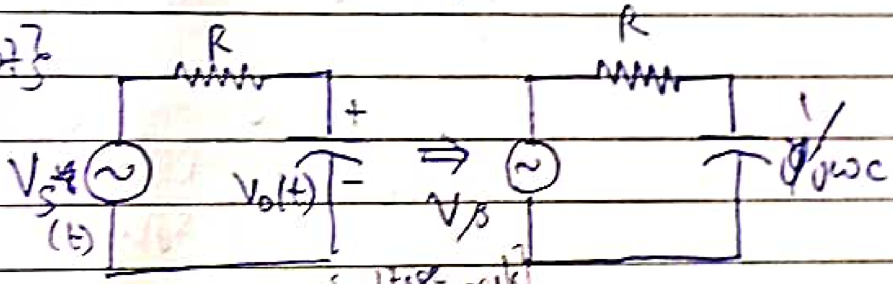
Frequency at which overall frequency of system becomes ∞ is called pole of system.

* A Zero is a value that results in zero value of the function.

* A pole is a value that results in the function as ∞.

eg: For the R-C circuit, obtain a transfer function $\frac{V_o}{V_s}$ and its frequency response.

$\{V_s = V_m \cos \omega t\}$



{voltage division rule}

$$H(\omega) = \frac{V_o}{V_s} = \frac{1/j\omega C}{R + \frac{1}{j\omega C}}$$

where $\omega_0 = \frac{1}{RC}$

$$= \frac{1}{1 + jR\omega C}$$

$$H = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}} \angle -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$\omega = 0$, $H = 1$, $\phi = 0°$

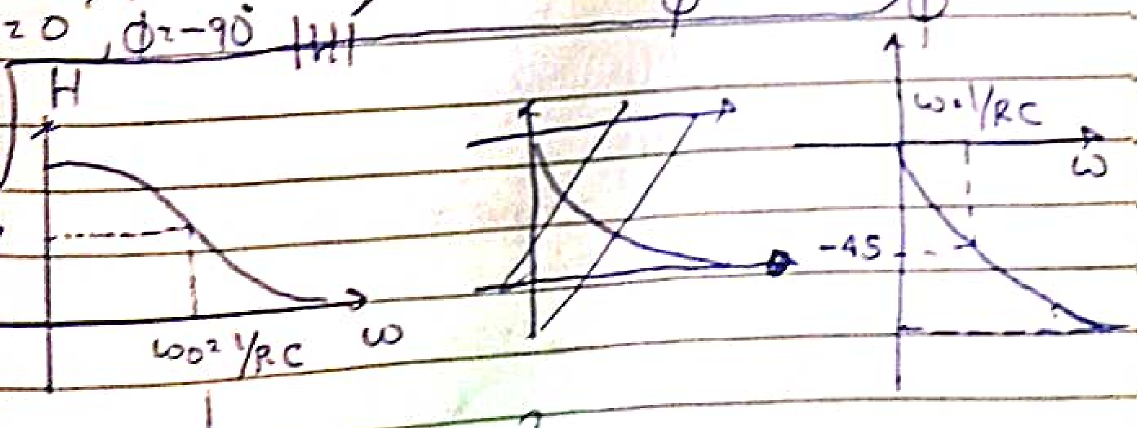
$\omega = \infty$, $H = 0$, $\phi = -90°$

$\omega = \omega_0$

$H = 1/\sqrt{2}$

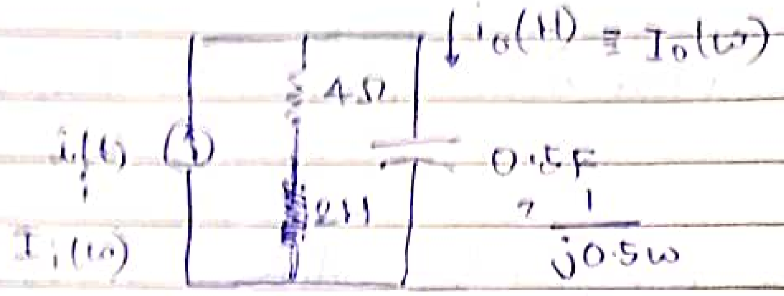
$\phi = -45°$

0.707



{cutoff frequency}

find
For the ckt, $\frac{I_o(\omega)}{I_i(\omega)}$ and it's poles and zeroes



$$\frac{I_o(\omega)}{I_i(\omega)} = \frac{\cancel{1/j0.5\omega}}{}$$

$$= \frac{\dfrac{4 + j2\omega}{4 + j2\omega + \dfrac{1}{0.5j\omega}} \times \cancel{I_i(\omega)}}{\cancel{I_i(\omega)}}$$

$$= \frac{(4 + 2j\omega)(0.5j\omega)}{2j\omega + \omega^2(j)^2 + 1}$$

Transfer function $= \dfrac{2j\omega - \omega^2}{2j\omega - \omega^2 + 1} = \dfrac{2js - as^2}{2js - s^2 + 1}$

$N(\omega) = 0$

$2j\omega - \omega^2 = 0$

$\omega = 0$ or $\omega = 2j$

$D(\omega) = 0$

$2j\omega - \omega^2 + 1 = 0$

$\omega^2 - 2j\omega - 1 = 0$

$\cancel{(\omega - 1)^2 = 0}$

$\omega^2 - j\omega - i\omega - 1 = 0$

$j \quad \omega = \pm 1$

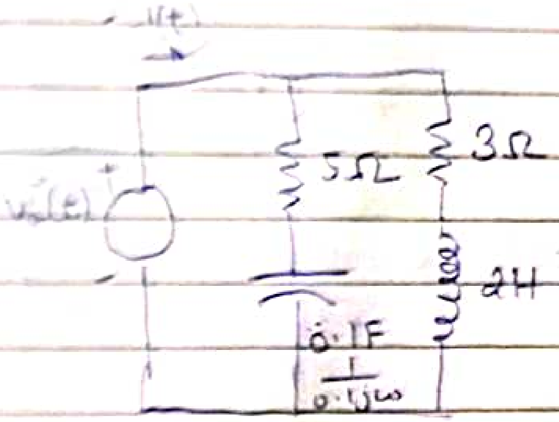
PAGE NO.
DATE / /

$j\omega = s$

$$\frac{(4+7s)(0.5s)}{2s+s^2+1} = \frac{2s+s^2}{2s+s^2+1}$$

$$2s+s^2=0$$
$$s=0 \text{ or } s=-2 \quad \text{(zeros)}$$

$$s^2+2s+1=0$$
$$s=-1,-1 \quad \text{(poles)}$$



$$TF = \frac{V_o(t)}{i_i(t)} = \frac{\left[\dfrac{(3+2j\omega)\left(5+\frac{1}{0.1j\omega}\right)}{3+2j\omega+5+\frac{1}{0.1j\omega}}\right] \times \cancel{I(t)}}{\cancel{I(t)}}$$

$j\omega = s$

$$= \frac{(3+2s)\left(5+\frac{10}{s}\right)}{3+2s+5+\frac{10}{s}}$$

$$= \frac{15+\frac{30}{s}+10s+20}{3+2s+5+10/s}$$

$$= \frac{35s+30+10s^2}{\ldots}$$

$10s^2+35s+10=0$

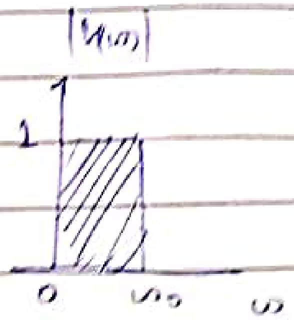
$s = \frac{-7\pm\sqrt{33}}{4}$

$2s^2+8s+10=0$

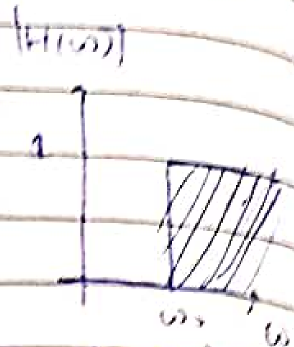
$s=-2\pm j$

s^2+4s+6
s^2+4s+5

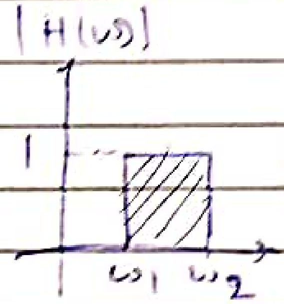
Filters
- → Passive Filters
- → Active Filters

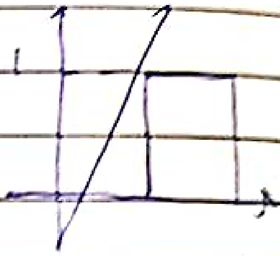


LB
(LPF)
low pass filter

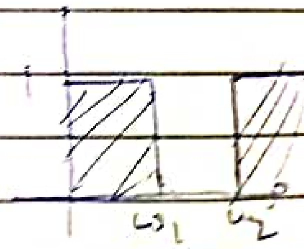


(HPF)
high pass filter





(BPF) Band pass filter



(BSF) Band stop filter

Passive Filters

A filter is a circuit that is designed to pass the signals ~~of~~ with desired frequencies and attenuate other frequency.

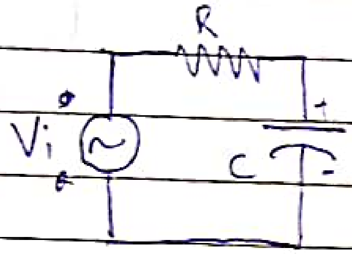
A filter is a passive filter which consist of only RLC elements.

Active filter consists of active elements such as operational amplifiers, transistors etc

PAGE NO.:
DATE: / /

Design of Low Pass filter

A low pass filter is formed when output of RC circuit is taken out of the capacitor.

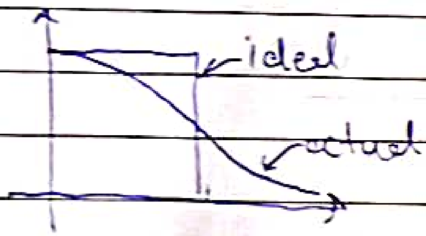


Transfer function

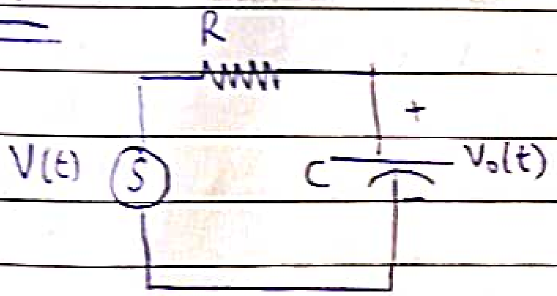
$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

$$= \frac{1/j\omega C}{R + 1/j\omega C}$$

$H(0) = 1$, $H(\infty) = 0$



LPF



$$H(\omega) = \frac{V_o(\omega)}{V_t(\omega)} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$H(\omega) = \frac{1}{1 + j\omega CR}$$

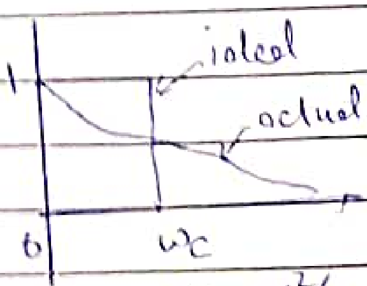
$|H(\omega)| = 1$

$|H(\infty)| = 0$

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$\omega c = 1/RC$

PAGE NO.:
DATE: / /



$P = V^2/R$

when $\omega = \omega_c$

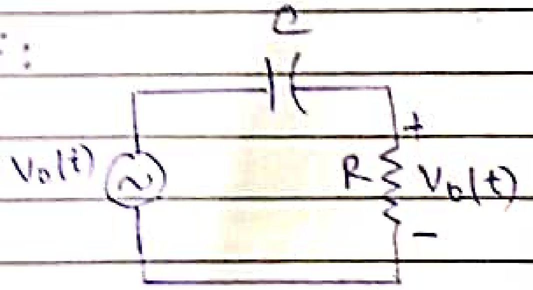
$P = \dfrac{0\cdot\left(\frac{1}{\sqrt{2}}V\right)^2}{R} = \dfrac{1}{2} V^2/R$

$= 0.5P$

$\omega_c \rightarrow$ half power frequency
OR
Cutoff frequency

This is $\boxed{\text{Low Pass Filter}}$.

HPF:



$H(\omega) = \dfrac{V_o(\omega)}{V_i(\omega)}$

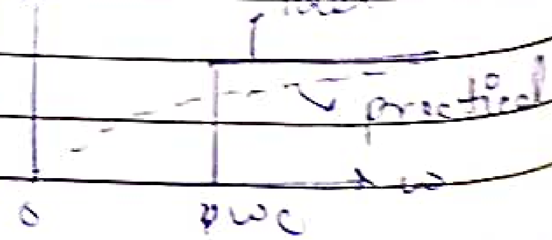
$= \dfrac{R}{R + \frac{1}{j\omega C}}$

$$A(\omega) = \frac{Rj\omega C}{Rj\omega C + 1}$$

$H(0) = 0$

$H(\infty) = 1$

$\omega_c = \dfrac{1}{RC}$



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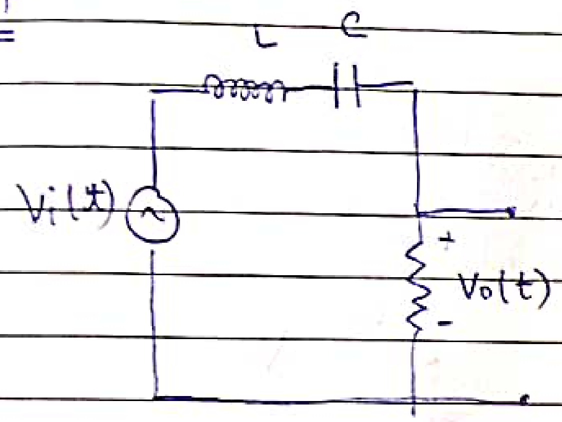
☆

A low pass filter can also be formed of an R-L ckt by taking output across resistor.

A high pass filter can also be formed by taking output across the inductor.

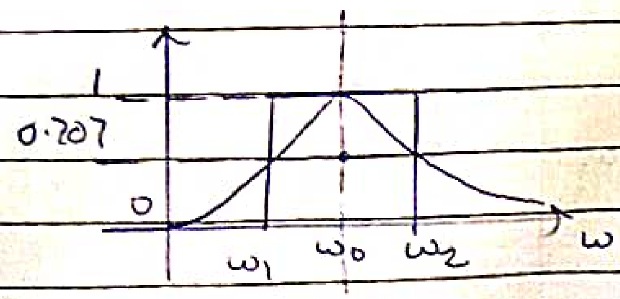
BPF



$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

$$= \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

BPF passes a band of freq. $(\omega_1 < \omega < \omega_2)$ ~~centre~~ centered on ω_0, when ω_0 is a resonant frequency



$$\frac{R^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{1}{2}$$

$$2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$R = \omega L - \frac{1}{\omega C}$$

$$R = \pm \frac{1}{\omega C} - \omega L$$

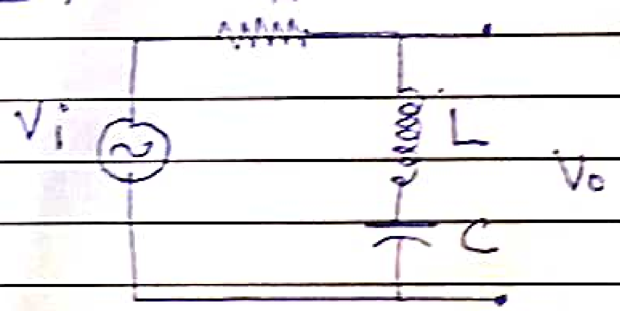
$$\left(\omega L - \frac{1}{\omega C} + R\right)\left(\omega L - \frac{1}{\omega C} - R\right) = 0$$

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$ {This is Band pass Filter}

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

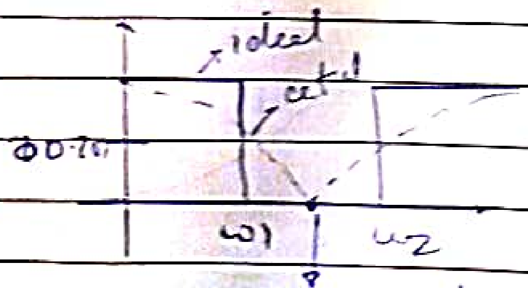
BSF / Notch filter



$$H(\omega) = V_o/V_i$$

$$= \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\Rightarrow H(0) = 1$$
$$H(\infty) = 1$$

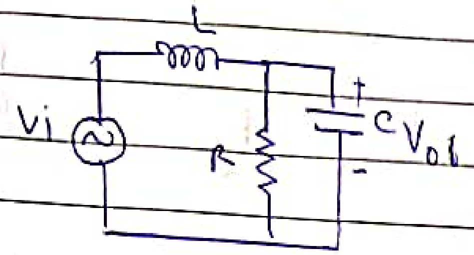


$$\omega_0 = \frac{1}{\sqrt{LC}}$$

At corner frequency $|H(\omega)| = \frac{1}{\sqrt{2}} H(\omega_c)$

PAGE NO.:
DATE: / /

Q) Determine the type of filter shown in the figure. Also calculate the cutoff frequencies



$L = 2H$

$R = 2k\Omega$
$C = 2\mu F$

$S = j\omega$

$$H(\omega) = \frac{V_o}{V_i} = \left[\frac{\left[\frac{(R)(1/j\omega C)}{R + 1/j\omega C}\right]}{j\omega L + \frac{R\,1/j\omega C}{R + 1/j\omega C}}\right] \cancel{V_i} \Big/ \cancel{V_i}$$

$\left\{ \begin{array}{l} H(\omega) = 1/\sqrt{2} \\ \text{To find } \omega_c \end{array} \right\}$ corner frequency

$$= \frac{\frac{\frac{R \times 1}{SC}}{R + 1/SC}}{SL + \frac{R\,1/SC}{R + 1/SC}} = \frac{\frac{\frac{R \times 1}{\cancel{SC}}}{\frac{RSC+1}{\cancel{SC}}}}{SL + \frac{R/\cancel{SC}}{\frac{RSC+1}{\cancel{SC}}}}$$

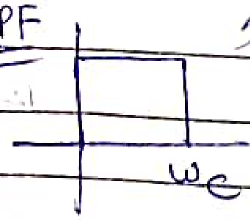
It is a 2nd order, low pass filter

$$= \frac{\left(\frac{R}{\cancel{RSC+1}}\right)}{\frac{(RSC+1)SL + R}{\cancel{(RSC+1)}}} = \frac{R}{S^2RLC + SL + R}$$

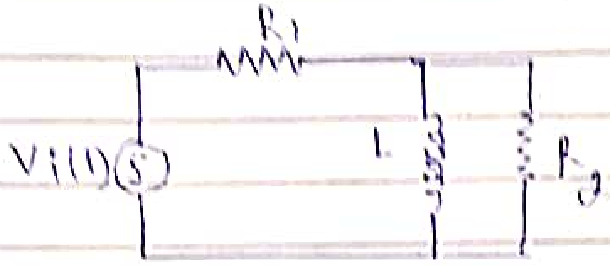
LPF

$H(0) = 1$

$H(\infty) = 0$



Q) Find type of filter and corner frequency.



$R_1 = 100\,\Omega = R_2$
$L = 2\,mH$

$$H(\omega) = \frac{V_o}{V_i} = \frac{\left[\dfrac{\dfrac{(j\omega L)R_2}{j\omega L + R_2}}{R_1 + \dfrac{(j\omega L)R_2}{j\omega L + R_2}}\right]\cancel{V_i}}{\cancel{V_i}}$$

$\omega j = s$

$$\frac{\dfrac{sR_2L}{\cancel{sL + R_2}}}{\dfrac{R_1sL + R_1R_2 + sLR_2}{\cancel{sL + R_2}}} = \frac{sR_2L}{R_1sL + R_1R_2 + sLR_2}$$

$$H(\omega) = \frac{sR_2L}{R_1sL + R_1R_2 + sLR_2} = \frac{sR_2L}{s(R_1L + R_2L) + R_1R_2}$$

$$H(0) = 0$$

$$H(\infty) = \frac{R_2L}{R_1L + R_2L} = \frac{R_2}{R_1 + R_2} = 1/2$$

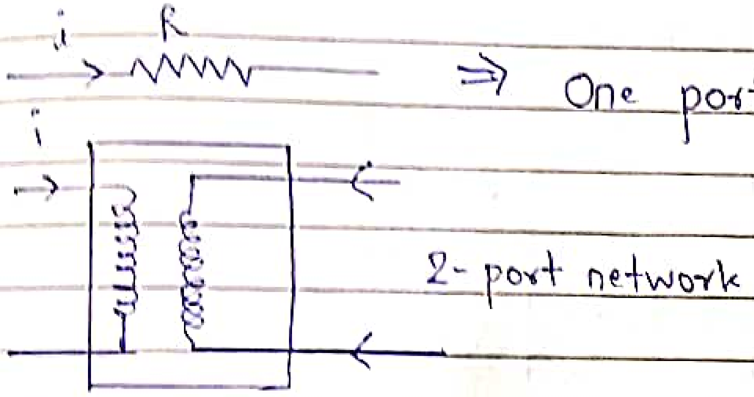
$\{\omega_c = 25\,k\,rad/s\}$

PAGE NO.:
DATE: / /

Two - Port Network

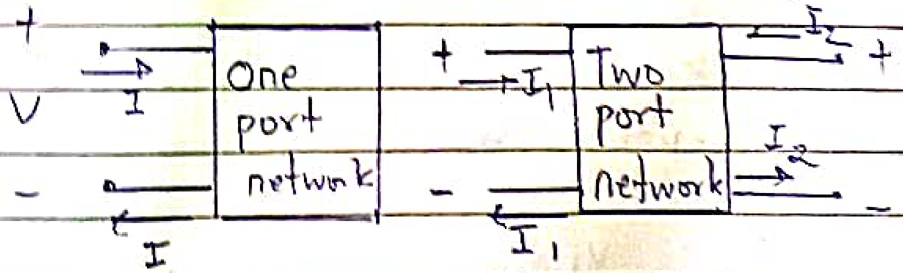
A pair of terminals through which current can enter or leave a network is called as a port.

Pair of terminals



Two terminal devices (R-L-C) results in one port network.

Two port network is a electric network having 2 seperate ports for input and output.



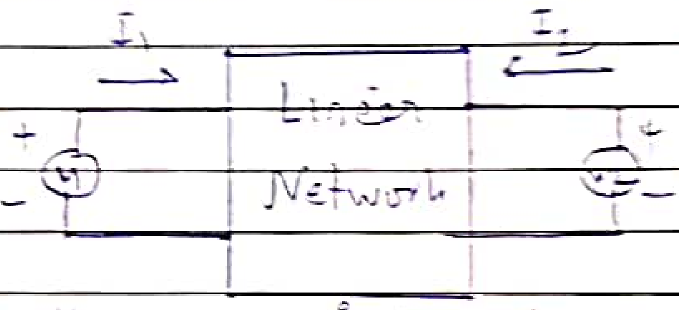
Applications are : communication systems
control systems
Power system networks
etc..

The various terms that relate voltage and current of a network are called parameters

eg// V/I = Resistance

Z parameter

Impedence Parameters



$$V_1 = Z_{11} i_1 + Z_{12} i_2$$

$$V_2 = Z_{21} i_1 + Z_{22} i_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = [Z][I]$$

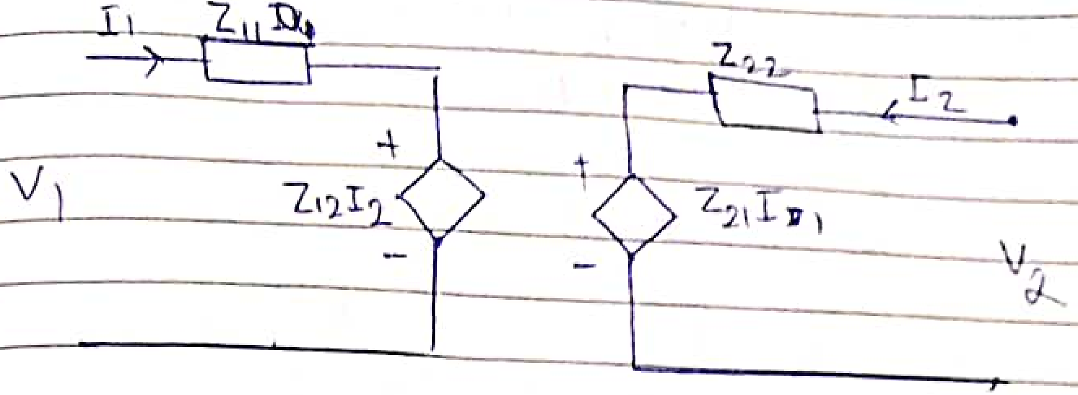
where Z terms are called impedance parameters or Z-parameters.

$Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$ open ckt i/p impedance

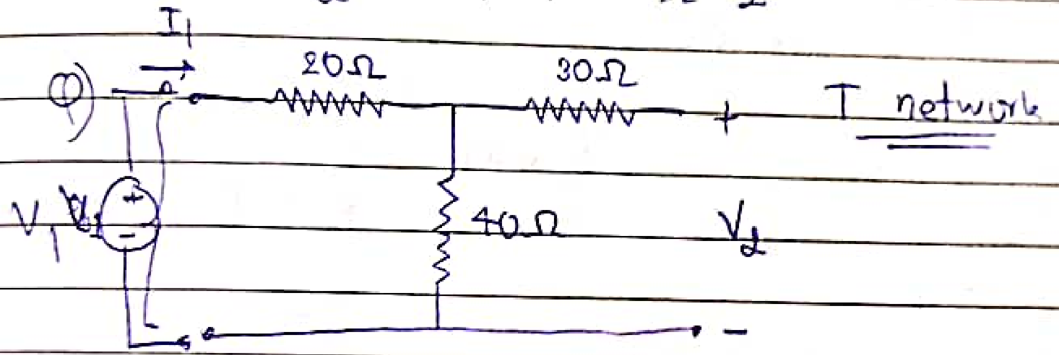
$Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$ O.C transfer impedance from port 1 to port

$Z_{21} = V_2/I_1 \Big|_{I_2=0}$ O-C transfer impedance from port 2 to port

$Z_{22} = V_2/I_2 \Big|_{I_1=0}$ O.C o/p impedance



$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$



① To determine Z_{11} & Z_{21} apply a voltage source V_1 to the input nodes and keep the output load open circuited.

$$Z_{11} = \frac{V_1}{I_1} = \frac{(20+40)I_1}{I_1}$$
$$= 60\,\Omega$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{40I_1}{I_1} = 40\,\Omega$$

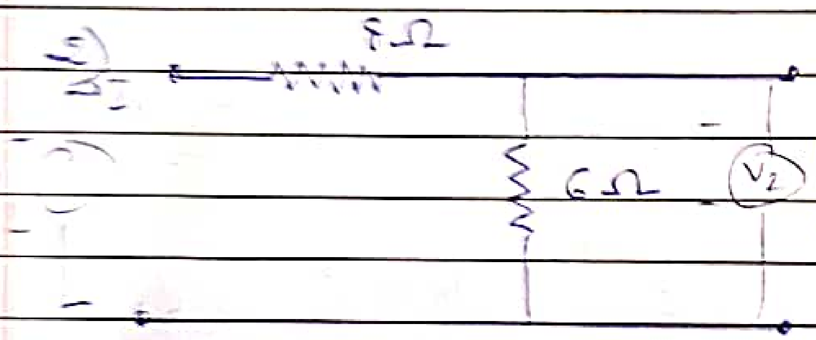
② To find Z_{12} & Z_{22} apply a voltage source V_2 to the output port and keep the input port open circuited

$Z_{12} = V_1/I_2 = 40I_2/I_2 = 40\Omega$

$$Z_{22} = \frac{(40+30)I_2}{I_2}$$

$= 70\Omega$

$$Z = \begin{bmatrix} 60\Omega & 40\Omega \\ 40\Omega & 70\Omega \end{bmatrix}$$



Find Z parameters

$$Z_{11} = \frac{V_1}{I_1} = 14\Omega$$

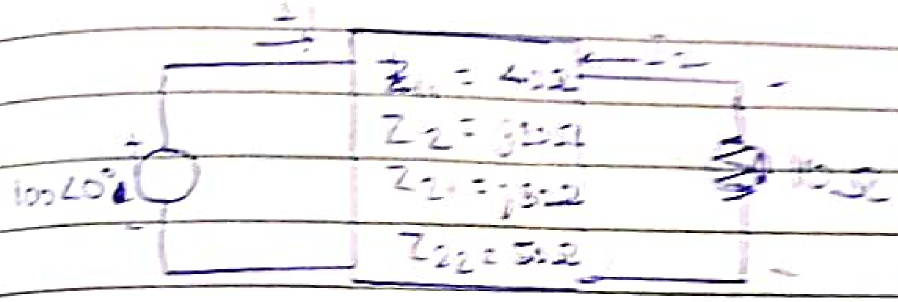
$$Z_{21} = \frac{V_2}{I_1} = \frac{6I_1}{I_1} = 6\Omega$$

$$Z_{12} = \frac{V_1}{I_2} = 6I_2/I_2 = 6\Omega$$

$$Z_{22} = \frac{V_2}{I_2} = 6I_2/I_2 = 6\Omega$$

$$\begin{bmatrix} 14 & 6 \\ 6 & 6 \end{bmatrix}$$

Q) Find i_1 & i_2 in the ckt



~~so ⇒ $Z_{11} = \frac{V_1}{I_1}$~~

~~$I_1 = \frac{40}{100} = 0.4\ A$~~

$$V_1 = Z_{11} i_1 + Z_{12} i_2$$

$$100 + 0j = 40 i_1 + j20 i_2$$

$$V_2 = j30 i_1 + 50 i_2 = -10 i_2$$

~~$500 = 200 i_1 + 100 j i_2$~~

$$300 = 120 i_1 + 60 j i_2 \quad —(1)$$

$$0 = 30j i_1 + 60 i_2$$

$$0 = -30 i_1 + 60 j i_2 \quad —(2)$$

(1) − (2)

$$300 = 150 i_1 + 0$$

$$i_1 = 2\ A \angle 0°\ A$$

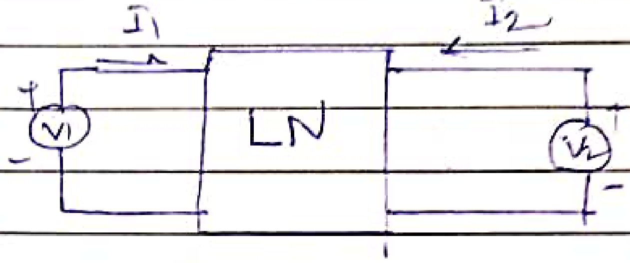
$$i_2 = \frac{30 i_1}{60 j} = \frac{1}{2} \cdot \frac{2}{j} = -j\ A$$

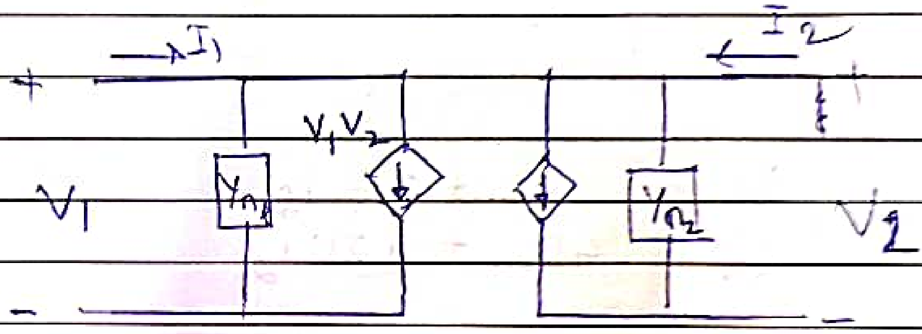
$$= 1 \angle -90°\ A$$

$(I_1, I_2) \equiv (V_1, V_2)$

$$\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{cases}$$

Admittance Parameters / Y parameters





$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

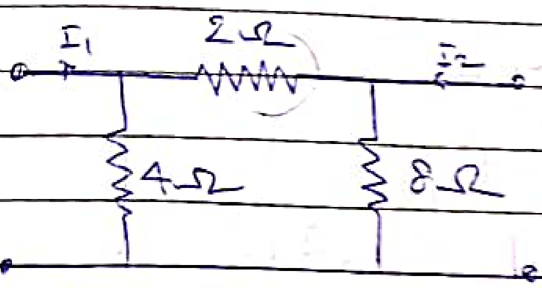
$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}$ short ckt input ~~input~~ admittance

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$ short ckt ~~short ckt~~ transfer admittance from port 1 to port 2

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$ short ckt transfer admittance from port 2 to port 1

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$ Short circuit output admittance.

Q) Obtain y-parameters of the π network



$2\left(\frac{8}{10}\right)I_2 = 5\Omega$

$\cancel{Y_{11} = \frac{I_1}{V_1} = 2 \quad \frac{1}{Z} = \frac{1}{2}}$

① To find Y_{11} & Y_{21} apply a current source I_1 to the input port and make the output port short circuited.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{11} = I_1/V_1 = \frac{I_1}{(8/6)I_1} = 6/8 = 3/4 = 0.75 \text{ S (mho)}$$

$$\boxed{Y_{12} = I_1/V_2 = I_1 \Big/ \frac{16}{10}I_2 = \frac{10}{16} = \frac{5}{8} = 0.625 \text{ S}}$$

$$Y_{21} = I_2/V_1 = I_2/V_1$$

$$I_2 = -4/6\, I_1$$
$$= -\frac{4}{6}I_1 \Big/ \frac{8}{6}I_1 = -0.5 \text{ S}$$

To get Y_{12} & Y_{22} short ckt the input port and apply a current source I_2 at the output

$Y_{12} = I_1 / V_2$

$I_1 = -\frac{8}{10} I_2$

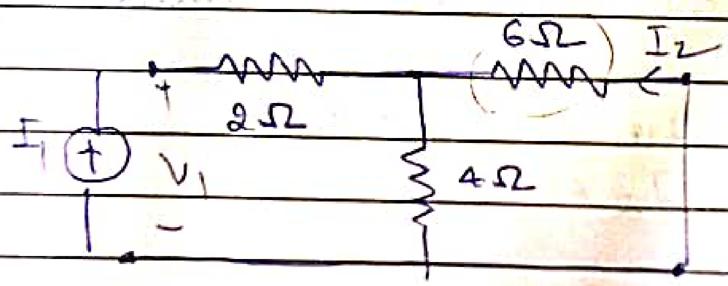
$\therefore Y_{12} = \frac{-8/10 \cancel{I_2}}{16/10 \cancel{I_2}}$

$= \frac{-1}{2} S = 0.5 S$

$Y_{22} = I_2 / V_2$

$= I_2 / \frac{16}{10} I_2 = 10/16$

$= 0.625 S$



$\frac{6 \times 4}{6+4} = \frac{24}{10} + 2$

$= 44/10$

$Y_{11} = \frac{I_1}{V_1} = \frac{I_1}{\frac{44}{10} I_1} = \frac{10}{44}$

$= 0.227 S$

$Y_{21} = \frac{I_2}{V_1}$

$\frac{4}{10}$

$I_2 = -\frac{2}{5} I_1$

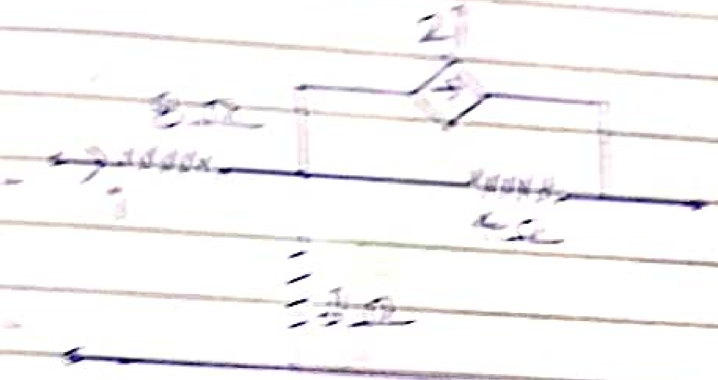
$Y_{21} = \frac{-2/\cancel{5} I_1}{22\,\cancel{44}/\cancel{10} I_1} = \frac{-1}{11} S = -0.091 S$

$y_{12} = \frac{I_1}{V_2}$ [illegible]

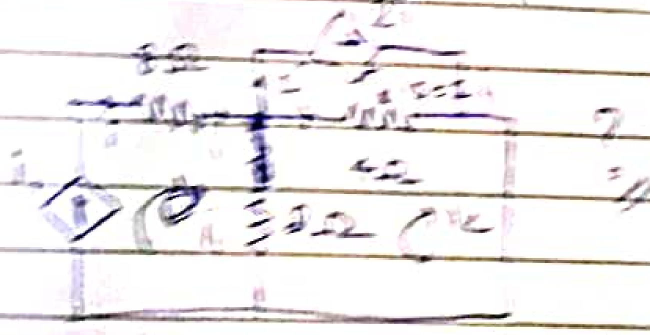
$= -\frac{1}{2}$ [illegible] $= -\frac{1}{11}$ [illegible] $= -0.09$ [illegible]

$y_{22} = \frac{I_2}{V_2} = \frac{3}{22}$ S

$= 0.136$ [illegible]



c) Obtain S-C parameters



[illegible] $- 8i_x - 2i_x -$ [illegible]

[illegible] $- 7i_x - 2i_x - 4i_x = 0$

$V_x = 9i_x$

$-2(v_x - i_x) - 4(i_x - 2i_x) = 0$

$-2i_x - 5$ [illegible] $- 4i_x + 8i_x = 0$

$\cancel{\frac{V_x}{6}}$

$V_x = 2i_x$

Hybrid Parameters

$(V_1, I_2) = f(V_2, I_1)$

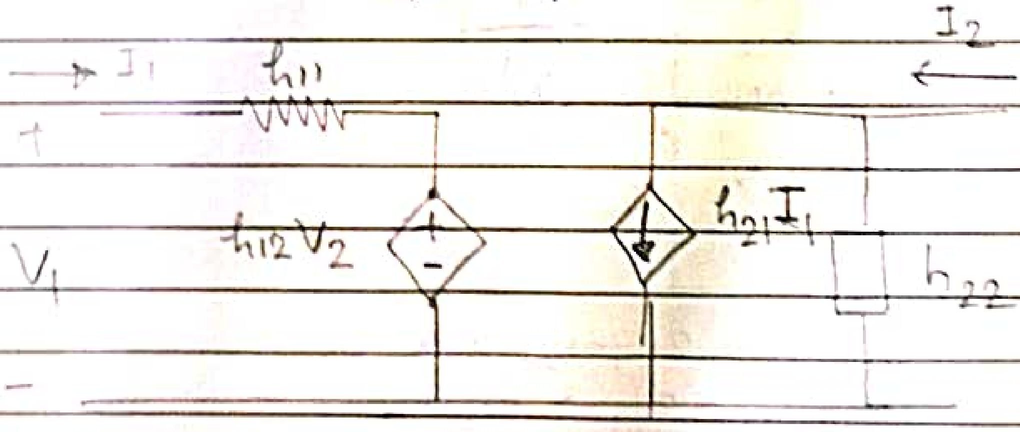
$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \Rightarrow$ S.C I/p Impedance

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \Rightarrow$ O.C reverse voltage gain

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \Rightarrow$ S.C forward current gain

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \Rightarrow$ O.C output admittance



g - parameters

$(I_1, V_2) = f(V_1, I_2)$

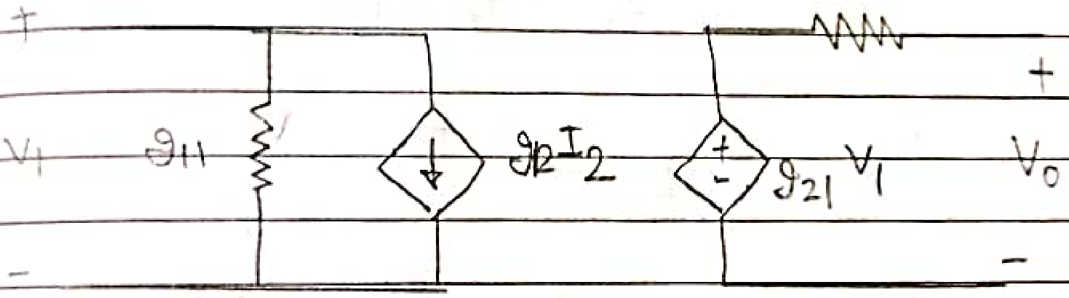
PAGE NO.:

DATE: / /

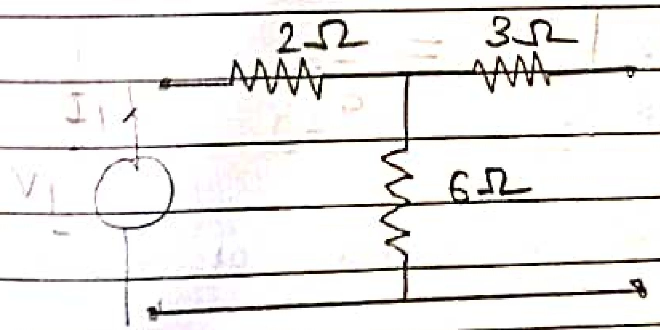
$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$g_{11} \Rightarrow$ O.C i/p Admittence
$g_{12} \Rightarrow$ S.C Reverse current gain
$g_{21} \Rightarrow$ O.C forward volt gain
$g_{22} \Rightarrow$ S.C o/p impedence



Q) Find the hybrid parameters



$\cancel{h_{11}} = \cancel{\frac{V_1}{I_1}}$

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$h_{11} = \frac{V_1}{I_1} = \frac{8 \times \cancel{I_1}}{\cancel{I_1}} = 8\ \Omega$

h

$\frac{6\times 3}{6+3} = 2$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{4I_1}{I_1} = 4\Omega$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{4I_1}{9I_2}$$

$$\left\{ \frac{6\, I_1}{3+6} = -I_2 \right\}$$

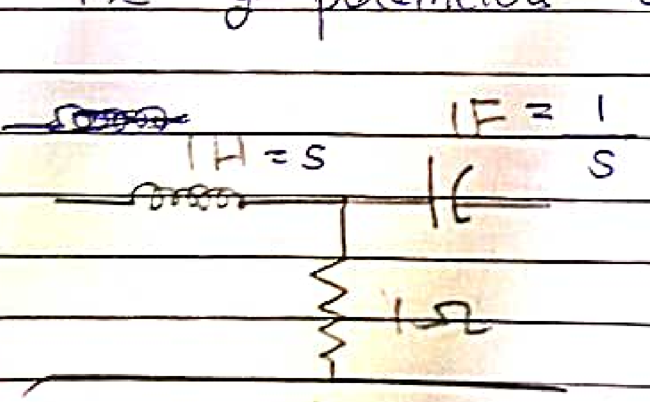
$$\left\{ I_2 = -\tfrac{2}{3} I_1 \right\}$$

$$h_{12} = \frac{4I_1}{3\cancel{9}\left(-\cancel{2}/\cancel{3}\right)I_1} = \frac{2}{3}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-\tfrac{2}{3} I_1}{I_1} = \frac{-2}{3}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{I_2}{9I_2} = \tfrac{1}{9} \text{ mho/S}$$

Q) Find the g-parameters as a function of '



$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

PAGE NO.
DATE / /

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$

$$= \frac{I_1}{(s+1)I_1} = \frac{1}{s+1} \text{ mho}|S.$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

$$I_2 = -\left(\frac{1}{1+1/s}\right)I_1$$

$$g_{12} = \frac{-1}{\frac{1}{1+1/s}} = -\left(1+\frac{1}{s}\right)$$

$$I_1 = -\frac{1}{1+s} I_2$$

$$g_{12} = \frac{I_1}{I_2} = -\frac{1}{1+s}$$

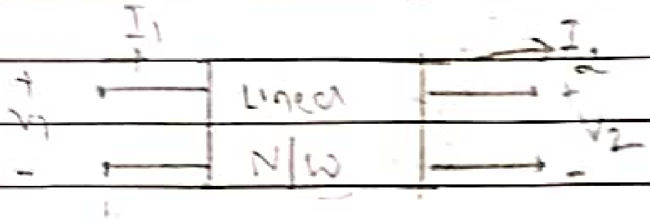
$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \left(\frac{1}{1+s}\right)$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{\left(\frac{s}{1+s} + \frac{1}{s}\right)\cancel{I_2}}{\cancel{I_2}}$$

$$= \frac{s^2+s+1}{s^2+s} \text{ in } \Omega$$

Transmission Parameters

$$(V_1, I_1) = f(V_2, -I_2)$$



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

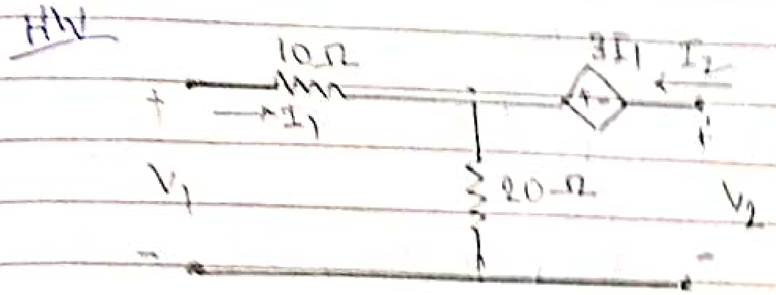
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$ O.C volt ratio

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$$ Negative S.C impedance

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$ O C Transfer admittance

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$ Negative S.C current ratio.

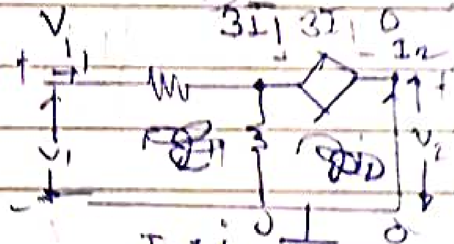
HW

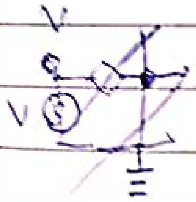


$$A = \frac{\left(\frac{20}{30}\right)V_2'}{V_2} = \frac{2}{3}$$

$$\cancel{B}\ C = \frac{I_1}{V_2} = \frac{I_1}{3/2\,V_1} = \frac{I_1}{3/2 \times 30\, I_1} = \frac{1}{45}$$

$$V_1 = 30 I_1$$





$I_1 = i_1$

$I_2 = -i_2$

$$V_2 = 20 I_1 - 3 I_1 = 17 I_1$$

$$A = \frac{V_1}{V_2} = \frac{30}{17} = 1.76$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{17 I_1} = 1/17$$

To get B & D short ckt o/p and apply voltage source at i/p

$$B = \frac{-V_1}{I_2} = \frac{-13 I_1}{-17 I_1/20}$$

$$\frac{V_1 - 3I_1}{10} = I_1 = \frac{20 \times 13}{17}\ \text{mho}$$

$$V_1 = 13 I_1$$

$$\frac{3I_1}{20} = i' \qquad I_2 + I_1 = i'$$

$$I_2 = \frac{3I_1}{20} - I_1 = \frac{-17 I_1}{20}$$

$$D = \frac{-I_1}{I_2} = \frac{-I_1}{\frac{-17 I_1}{20}} = 20/17$$

PAGE NO.:
DATE: / /

Relationship between Parameters

Given the Z-parameters, let us obtain the Y parameter.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{—— (1)}$$

We know that,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{—— (2)}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Y]^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\boxed{[Y] = [Z]^{-1}}$$

$$[Z]^{-1} = \frac{1}{Z_{11}Z_{22} - Z_{12}Z_{21}} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

PAGE NO.
DATE

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$

$$\left\{ \begin{array}{ll} Y_{11} = \dfrac{z_{22}}{\Delta_z} & Y_{12} = \dfrac{-z_{12}}{\Delta_z} \\ Y_{21} = \dfrac{-z_{21}}{\Delta_z} & Y_{22} = \dfrac{z_{11}}{\Delta_z} \end{array} \right]$$

~~Y_P~~

Given Z parameters, let us obtain h parameters:

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad — ③$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$I_2 = \frac{-z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2 \quad — ④$$

put ④ in ③

$$V_1 = z_{11}I_1 + z_{12}\left(\frac{V_2 - z_{21}I_1}{z_{22}} \right)$$

$$= \frac{(z_{11}z_{22} - z_{21}z_{12}) I_1 + z_{12}V_2}{z_{22}} \quad — ⑤$$

eqn ④ & ⑤ can be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta z}{z_{22}} & \dfrac{z_{12}}{z_{22}} \\ \dfrac{-z_{21}}{z_{22}} & \dfrac{1}{z_{22}} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad — ⑦$$

the h- parameter are

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad —(7)$$

$$\left\{ h_{11} = \frac{\Delta z}{z_{22}} \quad h_{12} = \frac{z_{12}}{z_{22}} \quad h_{21} = \frac{-z_{21}}{z_{22}} \quad h_{22} = \frac{1}{z_{22}} \right\}$$

Q) Find [Z] and [Y] of a two port N/w if

$$[T] = \begin{bmatrix} 10 & 1.5\Omega \\ 2 & 4 \end{bmatrix}$$

A
$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}\right\} T$$

Z
$$\left.\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}\right\} Z$$

$$V_2 = \frac{I_1 + DI_2}{C} = \frac{1}{C}I_1 + \frac{D}{C}I_2$$

$$V_1 = AV_2 - BI_2$$

$$= -BI_2 + \frac{A}{C}(I_1 + DI_2)$$

$$V_2 = \frac{I_1}{C} + \frac{D}{C}I_2$$

$$V_1 = I_2\left(-B + \frac{AD}{C}\right) + \frac{A}{C}I_1$$

$$z_{11} = -B + \frac{AD}{C} = -1.5 + \frac{10 \times 4}{2}$$

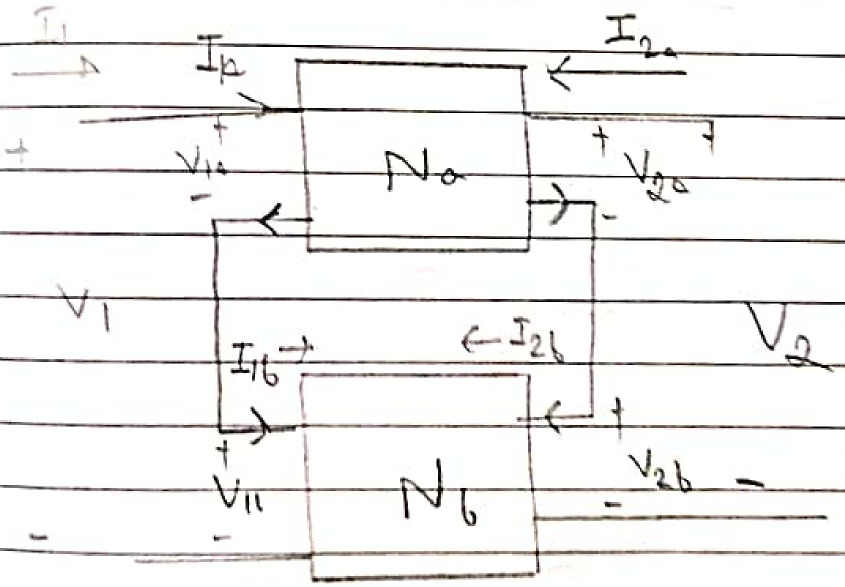
$$= 18.5\,\Omega$$

PAGE NO.:
DATE: / /

$$\left\{\begin{array}{l} Z_{11} = A/C \\ Z_{12} = \dfrac{AD-BC}{C} = \dfrac{\Delta T}{C} \\ Z_{21} = 1/C \\ Z_{22} = D/C \end{array}\right\}$$

Interconnection of Networks

Series Connection



For n/w 'a'

$$V_{1a} = Z_{11a} I_{1a} + Z_{12a} I_{2a}$$
$$V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}$$

For n/W 'b'

$$V_{1b} = Z_{11b} I_{1b} + Z_{12b} I_{2b}$$
$$V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$$

$$\left[I_1 = I_{1a} = I_{1b} \quad \& \quad I_2 = I_{2a} = I_{2b} \right]$$

$$V_1 = V_{1a} + V_{1b} = (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$$

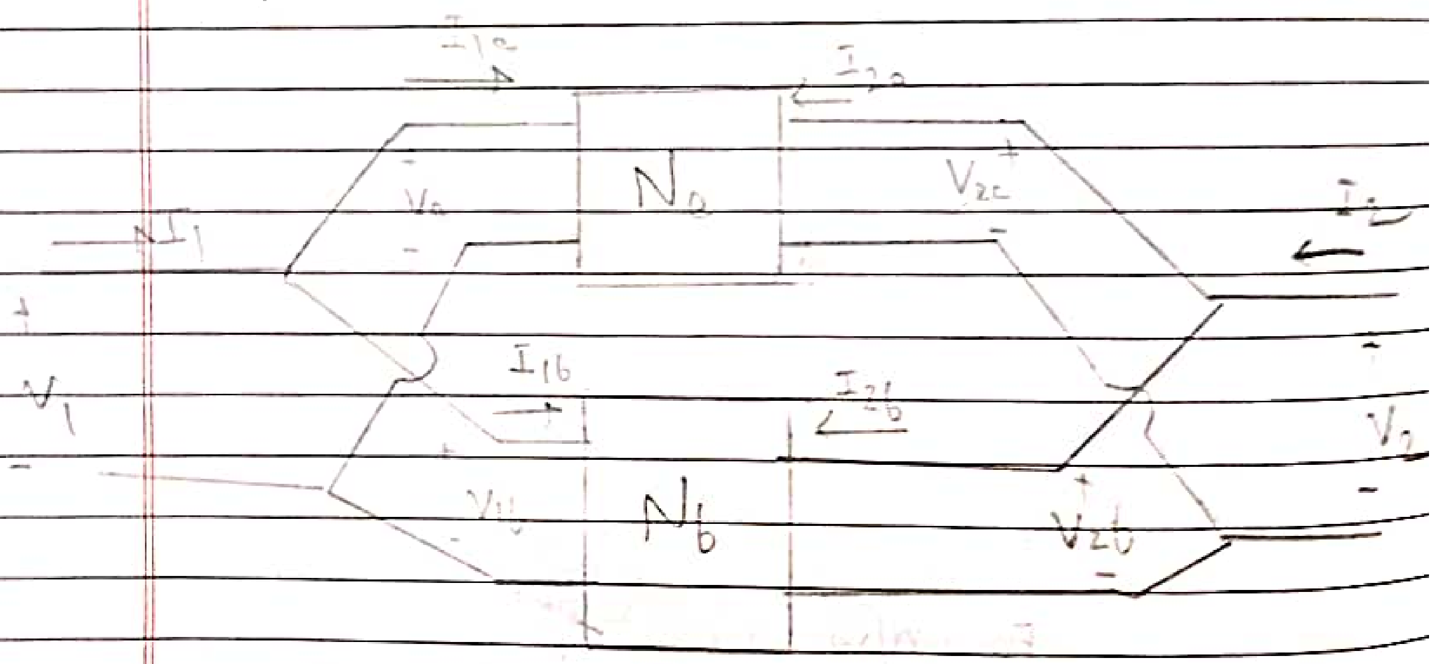
$$V_2 = V_{2a} + V_{2b} = (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$$

the Z parameters overall are:-

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11a} + Z_{11b} & Z_{12a} + Z_{12b} \\ Z_{21a} + Z_{21b} & Z_{22a} + Z_{22b} \end{bmatrix}$$

$$[Z] = [Z_a] + [Z_b]$$

<u>Parallel Connections</u>



<u>For N/W - A</u> :

$$I_{1a} = Y_{11a} V_{1a} + Y_{12a} V_{2a}$$
$$I_{2a} = Y_{21a} V_{2a} + Y_{22a} V_{2a}$$

For N/w - b

$$I_{1b} = Y_{11b} V_{1b} + Y_{12b} V_{2b}$$
$$I_{2b} = Y_{21b} V_{1b} + Y_{22b} V_{2b}$$

$V_1 = V_{1a} = V_{1b}$ $\quad$ $V_2 = V_{2a} = V_{2b}$

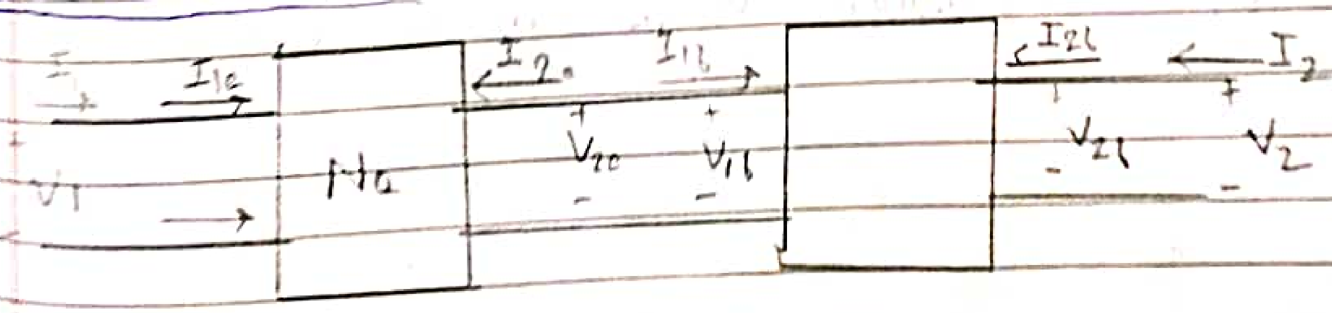
$$I_1 = I_{1a} + I_{1b}$$
$$I_2 = I_{2a} + I_{2b}$$

$$I_1 = (Y_{11a} + Y_{11b}) V_1 + (Y_{12a} + Y_{12b}) V_2$$
$$I_2 = (Y_{21a} + Y_{21b}) V_1 + (Y_{22a} + Y_{22b}) V_2$$

$$\cancel{\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} =} \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{22a} + Y_{22b} \end{bmatrix}$$

$$Y = [Y_a] + [Y_b]$$

Cascade Connections



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \quad \text{--- (1)}$$

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix} \quad \text{--- (2)}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} \qquad \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} \qquad \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix} = \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

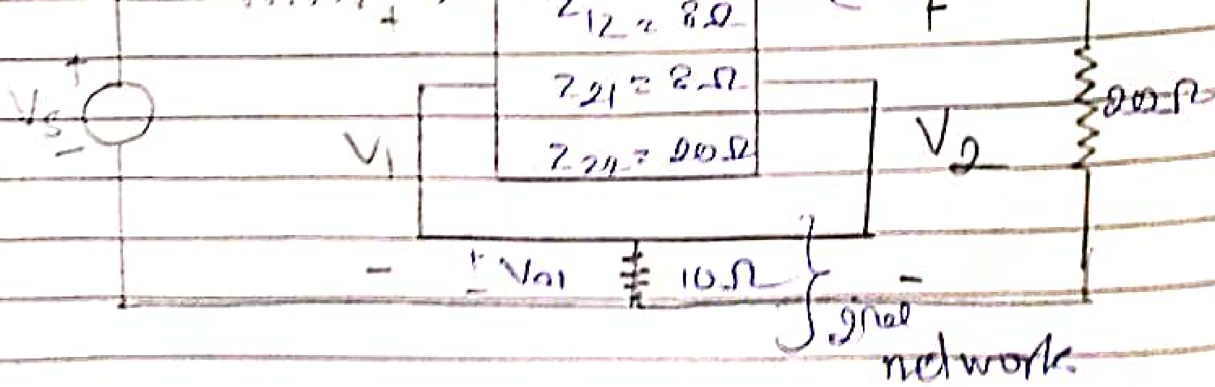
substitute (2) in (1)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} //$$

Transmission parameters of overall network is product of individual transmission parameters of networks in cascade connection.

Q) Evaluate V_2/V_s in the ckt



PAGE NO:
DATE / /

$$Z'_{11} = \frac{V_{a1}}{I_1} = \frac{10 I_1}{\cancel{I_1}} = 10$$

$$Z'_{12} = \frac{V_{a1}}{I_2} = \frac{10 I_1}{I_2} = 10$$

$\{I_1 = -I_2\}$

$$Z'_{21} = \frac{V_{a2}}{I_1} = 10$$

$$Z'_{22} = V_{b2}/I_2 = 10$$

$$[Z]' = \begin{bmatrix} Z'_{11} & Z'_{12} \\ Z'_{21} & Z'_{22} \end{bmatrix} = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$$

$$[Z]_{net} = [Z] + [Z]'$$

~~$V_S - 5I_1 = V_1$~~

$$= \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

$$V_1 = 22I_1 + 18I_2 \quad -(1)$$

$$V_2 = 18I_1 + 30I_2 \quad -(2)$$

$$V_1 = V_S - 5I_1 \quad -(3)$$

put (3) & (4) in (1)

$$V_S = 27I_1 + 18I_2$$

$$-20I_2 = V_2 = 18I_1 + 30I_2$$

$$I_2 = -V_2/20 \quad -(4)$$

$$V_S - 5I_1 = 22I_1 - \frac{18V_2}{20}$$

$$V_S = 27I_1 - 0.9V_2 \quad -(5)$$

PAGE NO.:
DATE: / /

$$V_2 = 18I_1 + 3\cancel{0}\left(\frac{-V_2}{2\cancel{0}}\right)$$

$$V_2 = 18I_1 - 1.5V_2$$

$$2.5V_2 = 18I_1$$

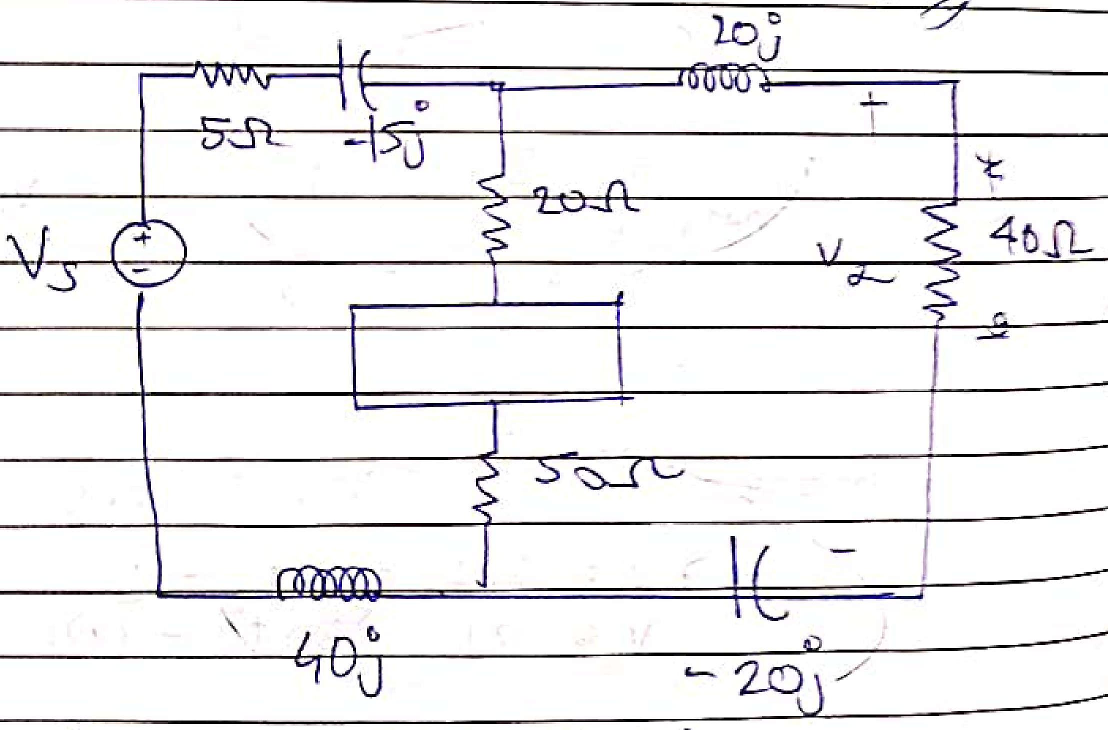
$$V_s = \frac{\cancel{27}^{3}}{\cancel{18}_{2}}(2.5V_2) - 0.9V_2$$

$$V_s = 2.85(V_2)$$

$$\frac{V_2}{V_s} = \frac{1}{2.85}$$

$$= 0.35$$

Q)



Ans $0.58 \angle -40^\circ$

PAGE NO.:
DATE: / /

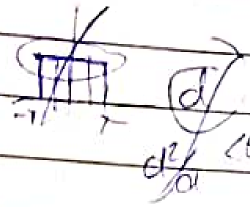
$$f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \dots + b_0 + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \dots$$

$$f(t) = a_0 + \sum_{k=-\infty}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

{Fourier Analysis}

where $\omega_0 = \dfrac{2\pi}{T}$

~~f(t)=~~



Useful trignometry identities

1) $\int_0^T (\sin n\omega_0 t)\, dt = 0$

2) $\int_0^T \cos n\omega_0 t \, dt = 0$

3) $\int_0^T \sin n\omega_0 t \cos n\omega_0 t \, dt = 0$

4) $\int_0^T \sin n\omega_0 t \sin m\omega_0 t \, dt = 0 \quad m \neq n$

5) $\int_0^T \cos n\omega_0 t \cos m\omega_0 t \, dt = 0 \quad m \neq n$

6) $\int_0^T \sin^2 n\omega_0 t \, dt = T/2 = \int_0^T \cos^2 n\omega_0 t \, dt$

7) $\int \sin at \, dt = -1/a \cos at$

8) $\int t \cdot \cos at \, dt = 1/a^2 \cos at + \frac{1}{a} t \sin at$

9) $\int t \cdot \sin at \, dt = 1/a^2 \sin at - \frac{1}{a} \cdot t \cdot \cos at$

$\cos 2n\pi = 1$ $\quad\quad \cos n\pi = (-1)^n$

$\sin 2n\pi = 0$ $\quad\quad \sin n\pi = 0$

$e^{j2\pi n} = 1$ $\quad\quad e^{j\pi} = (-1)^n$

$e^{j\pi n/2} = (-1)^{n/2}$ $\quad$ n - even

$= j(-1)^{\frac{n-1}{2}}$ $\quad$ n - odd

$\cos \frac{n\pi}{2} = (-1)^{n/2}$ $\quad$ n - even

0 $\quad$ n - odd

$\sin \frac{n\pi}{2} = (-1)^{n-1/2}$ $\quad$ n - ~~even~~ odd

0 $\quad$ n - even

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\cdot \sin n\omega_0 t\,dt$$

Amplitude form :- We know that,

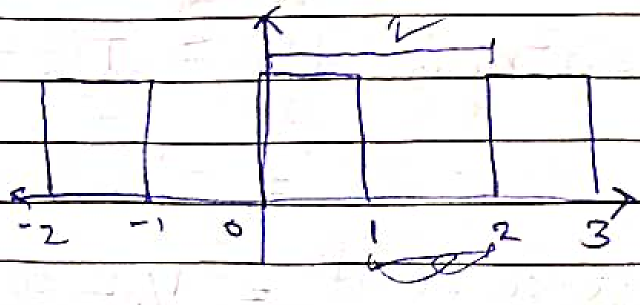
$$A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$$

$$C = \sqrt{A^2+B^2} \ \& \ \theta = \tan^{-1}(B/A)$$

① can be written as

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n)$$

Q) Determine the fourier series of waveform and obtain the amplitude & phase spectrum.



PAGE NO.:
DATE: / /

$$f(t) = \begin{cases} 1 & 0<t<1 \\ & 2<t<3 \\ & -2<t<-1 \\ 0 & \text{otherwise} \end{cases}$$

$a_0 + \sum a_n \cos$ cos

$$x(t) = c_0 + \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t) e^{-jk\Omega_0 t}\,dt$$

$$= \frac{1}{2}\int_0 e^{-jk\Omega_0 t}\,dt = \frac{e^{-jk\Omega_0 t}}{2jk\Omega_0}\Big|_1^0 \quad \frac{2\pi}{2}$$

$$= \frac{1-e^{-jk\Omega_0}}{2jk\Omega_0} \quad k \neq 0$$

where

$c_0 = 1/2$ $\qquad 1/2 \quad \frac{2t}{\ }$ $\quad k=0$

$$a_1 = \frac{1-e^{2j\Omega_0}}{2j\Omega_0} = \frac{1-e^{j\Omega_0}}{2j\frac{2\pi}{2}}$$

$$= \frac{1}{2j\pi}\left(e^{j\Omega_0 t} - e^{-j\Omega_0 t}\right)$$

$= \sin$

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\sin \pi t + \frac{2}{3\pi}\sin 3\pi t$$

$\pi, 3\pi, 5\pi, 7\pi$

$$a_k = \frac{1-(\cos(\pi k) - \overset{0}{\sin(\pi k)})}{2j\pi k} \quad k \neq 0$$

$$\frac{1-\cos \pi k}{2j\pi k} \quad k = \text{odd}$$

$$= 2/2j\pi k \quad \frac{1}{j\pi k} \qquad 0 \quad k=\text{even}$$

$$a_k = \frac{1}{2}\int_0^1 x(t)\, e^{-jk\Omega_0 t}$$

$$x(t) = a_0 + a_1 e^{j\omega_0 \pi t} + a_2 e^{2j\pi t} + a_3 e^{3j\pi t}$$

$$= \frac{1}{8} + \frac{1}{j\pi 8}(\cos \pi t + j\sin \pi t) + \frac{1}{2\pi}(\cos 2\pi t + j\sin 2\pi t)$$

$$\frac{1}{2} \times 0 \int_0^1 e^{-j\Omega_0 t k} dt = -\frac{1}{2} \left. \frac{e^{-j\Omega_0 t k}}{j\Omega_0 k} \right|_0^1$$

$$= \frac{1 - e^{-j\Omega_0 k}}{2j\Omega_0 k}$$

$$= \frac{1}{2j \frac{2\pi}{2} k}\left(1 - e^{-j\Omega_0 k}\right)$$

$$= \frac{1}{2\pi j k}\left(1 - e^{-j\pi k}\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T}\int_0^1 dt + \int_1^2 0\,dt$$

$$= 1/2$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$$

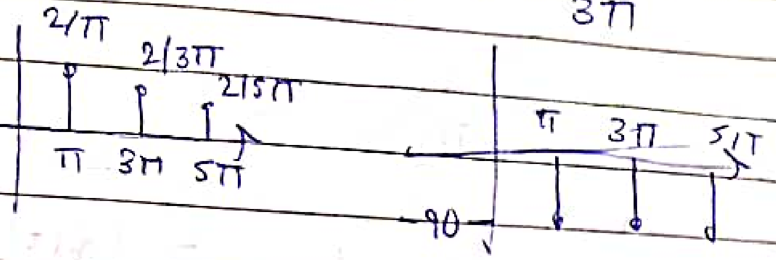
$$= \frac{2}{TT}\int_0^T \cos(n\omega_0 t)\,dt = 0$$

$$b_n = \frac{2}{T}\int_0 \sin(n\omega_0 t)\,dt = -\frac{\cos(n\pi t)}{n\pi}$$

$$\frac{1 - \cos n\pi}{n\pi} = 2/n\pi \quad 0$$

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sin(\pi t) + \frac{2}{3\pi}\sin(3\pi t) + \dots$$

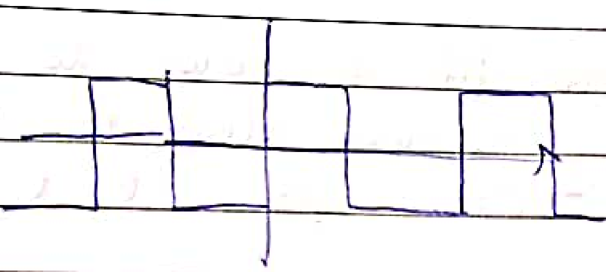
HW



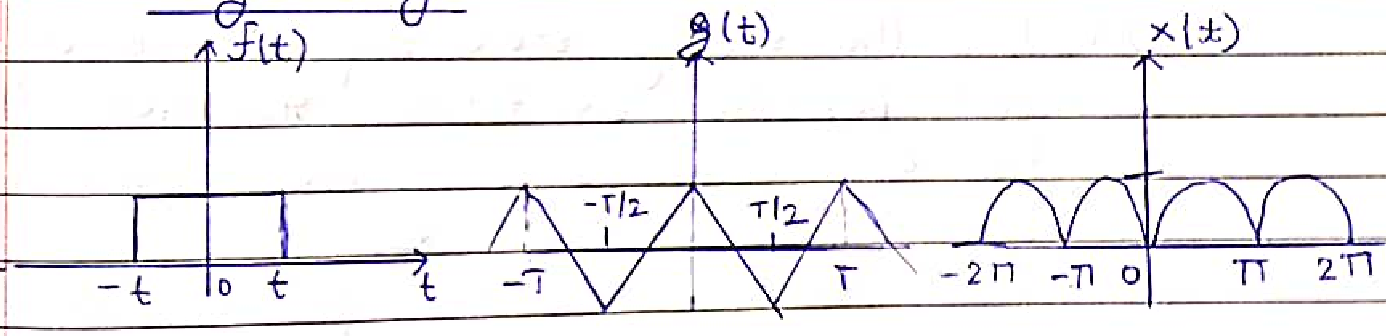
$\phi = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$

$= \phi$

-90



Symmetry



The property of an even function $f_e(t)$ is $\int_{-T/2}^{T/2} f_e(t)\,dt = 2\int_{0}^{T/2} f_e(t)\,dt$

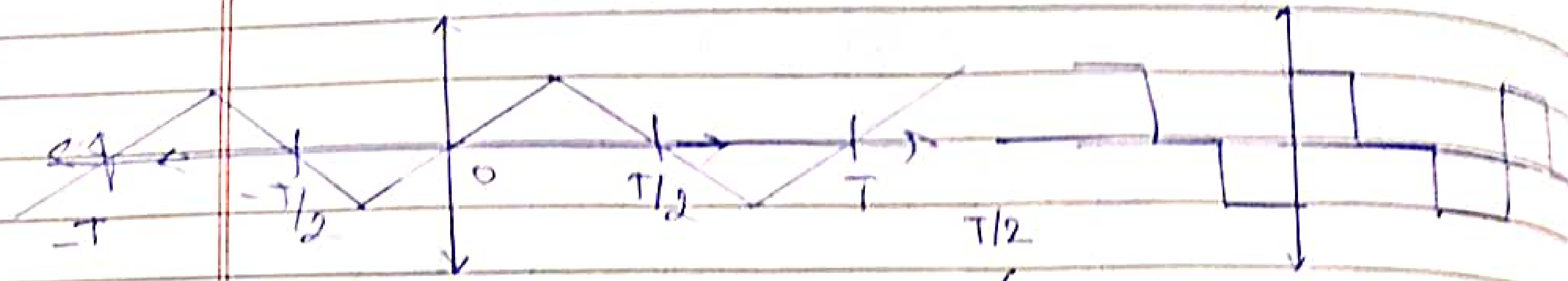
$\therefore$ Fourier coefficients of even function

$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt \qquad a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(\omega_0 n t)\,dt$

$b_n = 0$

A function $f(t)$ is even if it's plot is symmetrical about vertical axis. eg: t^2, t^4, cost

Symmetry



$a_0 = 0$ $\quad b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t \, dt$

$a_n = 0$

A function f(t) is said to be odd if it's plot is antisymmetrical about the vertical ~~axes~~ axis.

$f(t) = -f(-t)$. $f(t) = t, t^3, \sin t$.

Steps for Fourier Series Application to Circuit

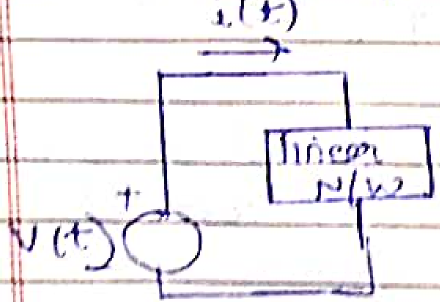
(0) To find the steady state response to a non-sinusoidal periodic excitation requires the application of fourier series, AC phasor analysis and superposition principle.

(1) Express the given excitation function in a fourier series.

(2) Find the response for every term in the Fourier series.

(3) Add the individual responses using superposition principle.

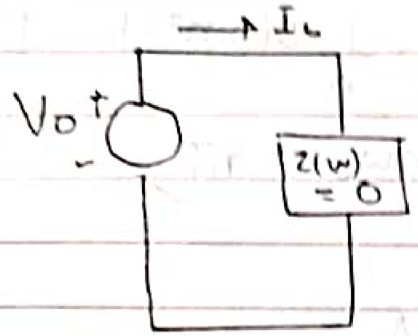
(4) $V(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \phi_n)$ —— (1)



dc components = V_0

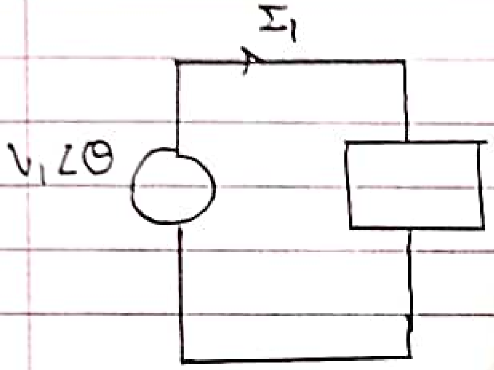
ac components = $V_n = V_n \angle\phi$ with several harmonics.

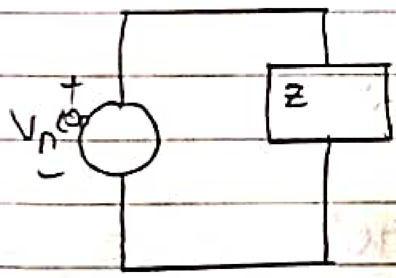
Fourier Analysis



Final Resp = $i(t) = i_0(t) + i_1(t) + \dots + i_n(t)$

$$= I_0 + \sum_{n=1}^{\infty} |I_n| \cos(n\omega_0 t + \phi_n$$





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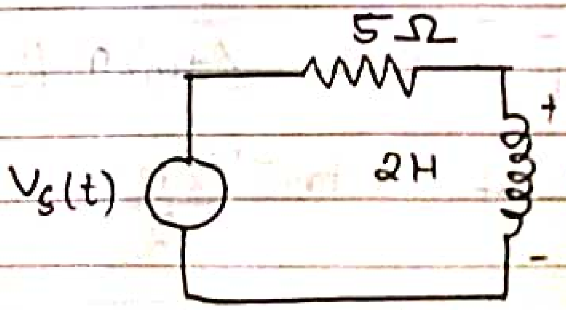
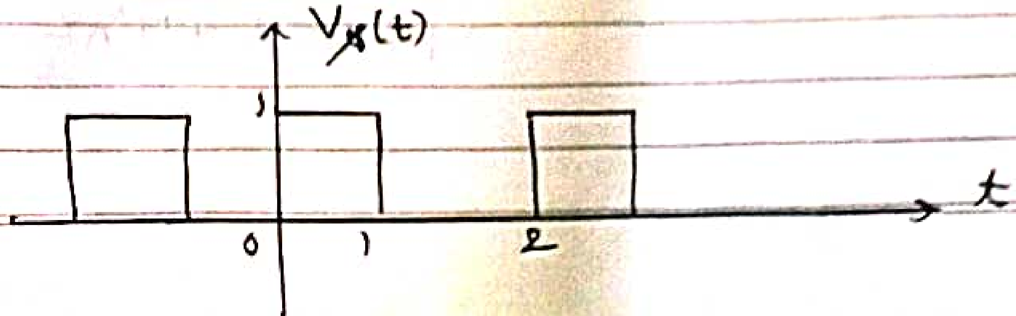


Final response $V_0(t)$



$$V_0(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{1}{n} \sin n\pi t \quad \{n = 2k-1\}$$

$k = 1, 2, 3$

$$\omega_n = n\omega_0 = n\pi$$

$V_0(t) =$

$$V_0(t) = \frac{j\omega_n L}{R + j\omega_n L} \cdot V_s$$

<u>For d.c components</u>

$V_s = 1/2$

$\omega_0 = 0$

$\therefore V_0 = 0$

For n^{th} harmonic,

$$V_s = \frac{2}{n\pi} \angle -90^\circ$$

$$V_0 = \frac{\frac{2\pi \angle 90^\circ}{} \cdot \frac{2}{n\pi} \angle -90^\circ}{\sqrt{25 + 4\pi^2 n^2} \angle \tan^{-1} 2n\pi/5}$$

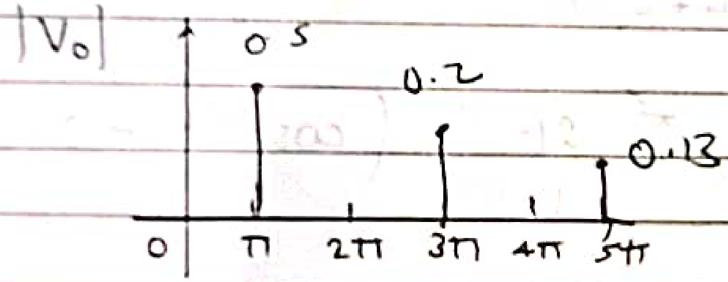
$$= \frac{4 \angle -\tan^{-1}\left(\frac{2\pi}{5}\right)}{\sqrt{25 + 4\pi^2 n^2}}$$

In time domain,

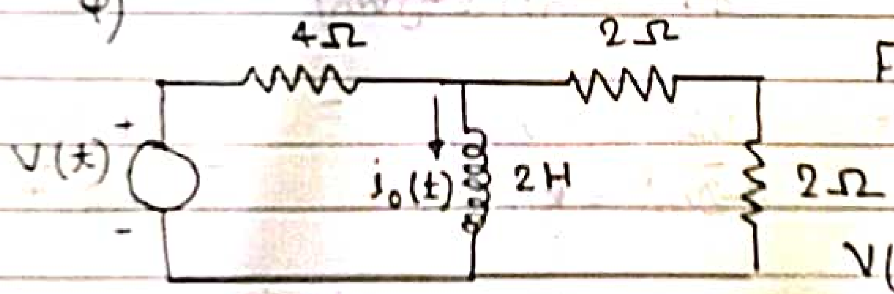
$$V_o(t) = \sum_{k=1}^{\infty} \frac{4}{\sqrt{25+4n^2\pi^2}} \cos\left(n\pi t - \frac{2n\pi}{5}\right)$$

$n = 2k-1$
$k = 1, 2, 3 \ldots$

$$V_o(t) = 0.49\cos(\pi t - 51.4^\circ) + 0.2\cos(3\pi t - 75.1^\circ)$$
$$+ 0.12\cos(5\pi t - 80.9^\circ) + \ldots$$



Q)



Find the response $i_o(t)$ in the ckt if

$$V(t) = 1 + \sum_{n=1}^{\infty} \frac{2(-1)^n}{1+n^2} \cdot (\cos(nt) - n\sin(nt))$$

$w_n = n(1) = n$

$V_o(t) = 1$

$\omega_o = 0$

$i_o(t) = 0$

$$Z_{eq} = 4 + \frac{4 \times 2jw_n}{\cancel{6}\ 4+2jw_n}$$

$$\cancel{\frac{V_1}{I_1}} \quad V_1 = \left[4 + \frac{8jw_n}{4+2jw_n}\right] I_1 = \left[\frac{16+\cancel{8}jw_n+8jw_n}{4+2jw_n}\right] I_1$$

$$= \left[\frac{8+8jw_n}{2+jw_n}\right] I_1$$

$$I_0 = \left(\frac{4}{4+2jw_n}\right) I_1$$

$$I_0 = \left(\frac{4}{4+2jw_n}\right)\left(4 + \frac{8jw_n}{4+2jw_n}\right) V_1$$

$$\boxed{\cancel{V_1 = \sum \frac{2(-1)^n}{1+n^2}\left(\cos nt - \sin nt\right)}}$$

$$I_0 = \frac{32+32jw_n}{(4+2jw_n)(2+jw_n)} V_1$$

$$= \frac{32+32w_nj}{8-2w_n^2+8jw_n} = \frac{16+16w_nj}{4-w_n^2+4w_nj} V_1$$

$$V_1 = \frac{2(-1)^n}{1+n^2} \angle \tan^{-1}(n)$$

$I_0 =$

Date ____
Page ____

Laplace Transform

$$L[f(t)] = F(s) = \int_0^\infty f(t) e^{-st}\, dt$$

where s is a complex veriable given by
$s = \sigma + j\omega$

1) $$L[u(t)] = \int_0^\infty e^{-st}\, dt$$

$$= \left.\frac{e^{-st}}{s}\right|_\infty^0 = \frac{1}{s}$$

2) $$L[e^{-at}] = \int_0^\infty e^{-at} \cdot e^{-st}\, dt$$

$$= \int_0^\infty e^{-(a+s)t}\, dt = \left.\frac{e^{-(a+s)t}}{a+s}\right|_\infty^0$$

$$= \frac{1}{a+s}$$

3) $$L[e^{at}] = \frac{1}{s-a}$$

4) $$L[\sin(\omega t)] = \frac{\omega}{s^2+\omega^2}$$

5) $$L[\cos(\omega t)] = \frac{s}{s^2+\omega^2}$$

6) $L[t] = \frac{1}{s^2}$

7) $L[f(at)] = \frac{1}{a} F\left[\frac{s}{a}\right]$ $\qquad L\left[\int f(t)\,dt\right] = \frac{1}{s} F(s)$

8) $L[e^{-at} \cdot f(t)] = F(s+a)$ $\qquad L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$

9) $L\left[\frac{d}{dt} f(t)\right] = s \cdot F(s) - f(0)$

10) $L[t \cdot f(t)] = \frac{-d}{ds} F(s)$

11) Initial value theorem $\quad \underset{t=0^+}{f(t)} = \lim_{s \to \infty} s \cdot F(s)$

Final value theorem $\quad \underset{t=\infty}{f(t)} = \lim_{s \to 0} s \cdot F(s)$

Convolution : $f_1(t) * f_2(t) = F_1(s) \cdot F_2(s)$

$L[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(s) + a_2 F_2(s)$

$L[\delta(t)] = 1$

Q) Apply laplace transform of $f(t) = \delta(t) + 2u(t) - 3e^{-2t}$, $t \geq 0$

$$L(f(t)) = F(s) = 1 + \frac{2}{s} - \frac{3}{s+2}, \quad s > 0$$

$$= \frac{s^2 + s + 4}{s(s+2)}$$

Q) Find laplace of $\cos 2t + e^{-3t}$, $t \geq 0$

$$F(s) = \frac{s}{4+s^2} + \frac{1}{s+3}, \quad s > -3$$

$$= \frac{s^2 + 3s + 4 + s^2}{(4+s^2)(3+s)}$$

$$= \frac{2s^2 + 3s + 4}{(4+s^2)(3+s)}$$

Q) $$F(s) = \frac{N(s)}{(s+p_1)(s+p_2)\cdots(s+p_n)} = \frac{k_1}{(s+p_1)} + \frac{k_2}{(s+p_2)} \cdots + \frac{k_n}{(s+p_n)}$$

where $k_1, k_2 \cdots k_n$ are called Residues of $F(s)$

$$K_1 = (s+p_1)F(s)\Big|_{s=-p_1}$$

$$k_i = (s+p_i)F(s)\Big|_{s=-p_i}$$

$$F_s = \frac{k_n}{(s+p)^n} + \frac{k_{n-1}}{(s+p)^{n-1}} \cdots + \frac{k_1}{(s+p)} + F_1$$

$$k_n = (s+p)^n F_s \Big|_{s=-p}$$

$$k_{n-1} = \frac{d}{ds}\left[(s+p)^n \cdot F(s)\right]\Big|_{s=-p}$$

$$k_{n-2} = \frac{d^2}{ds^2}\left[(s+p)^n \cdot F(s)\right]\Big|_{s=-p}$$

1) Find I.L.T of $F(s) = \frac{3}{s} - \frac{5}{s+1} + \frac{6}{s^2+4}$

$L^{-1}[F(s)] = f(t) = 3u(t) - 5e^{-t} + 3\sin(2t)$

Q) Find the inverse laplace transform of

$F(s) = 1 + \frac{4}{s+3} - \frac{5s}{s^2+16}$

$L^{-1}[F(s)] = f(t) = \delta(t) + 4e^{-3t} - \frac{5}{} \cos(4t)$

Q) Find $f(t)$ if $F(s) = \frac{s^2+12}{s(s+2)(s+3)}$

$F(s) = \frac{s}{(s+2)(s+3)} + \frac{12}{s(s+2)(s+3)}$

Date
Page

$\frac{12}{s}$ − $12(s+3)$ $\frac{12}{s(s+2)} - \frac{12}{s(s+3)}$

$\frac{12(s+3)-...}{s(s+2)}$

$$F(s) = \frac{s}{s+2} - \frac{s}{s+3} + \frac{12}{s(s+2)(s+3)}$$

$(s+3)-(s+2)$

$$= \cancel{1} - \frac{2}{s+2} - \left[\cancel{1} - \frac{3}{s+3}\right] + \frac{12}{s(s+2)(s+3)}$$

$$= \frac{3}{s+3} - \frac{2}{s+2} + 12\left[\frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}\right]$$

$12 / s(s+2)(s+3)$

$$12\left[\cancel{\frac{A}{s}} + \frac{B}{s+2} + \cancel{\frac{C}{s+3}}\right] = \left[\frac{A(s+2)(s+3) + B(s)(s+3) + C(s)(s+2)}{(s+2)(s)(s+3)}\right]$$

$$\frac{s^2+12}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = s\,F(s)\Big|_{s=0} = \frac{12}{6} = 2$$

$$B = (s+2)F(s)\Big|_{s=-2} = \frac{s^2+12}{s(s+3)} = \frac{16}{-2} = -8$$

$$C = (s+3)F(s)\Big|_{s=-3} = \frac{s^2+12}{s(s+2)} = \frac{21}{3} = 7$$

$$F(s) = 2/s - 8/s+2 + 7/s+3$$

$$= 2u(t) - 8e^{-2t} + 7e^{-3t}$$

Q) Find I.L.T of $\frac{10s^2+4}{s(s+1)(s+2)^2}$

$$\frac{10s^2+4}{s(s+1)(s+2)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{k_1}{(s+2)^2} + \frac{k_2}{(s+2)}$$

$$A = s\cdot F(s)\Big|_{s=0} = \frac{4}{4} = 1$$

$$B = (s+1)F(s)\Big|_{s=-1} = \frac{10+4}{(-1)(1)^2} = -14$$

$$k_1 = (s+2)^2 F(s)\Big|_{s=-2} = \frac{10(4)+4}{-2(-1)} = \frac{44}{2} = 22$$

$$k_2 = \frac{d}{ds}\left(\frac{10s^2+4}{\underbrace{(s+1)(s)}_{s^2+s}}\right)$$

$$k_2 = \frac{(s^2+s)(20s) - (2s+1)(10s^2+4)}{(s^2+s)^2}\Bigg|_{s=-2}$$

$$= \frac{(4-2)(-40) - (-4+1)(40+4)}{(4-2)^2}$$

$$= \frac{2(-40) - (-3)(44)}{4}$$

$$= \frac{-80+132}{4} = 13$$

Date
Page

$$F(s) = \frac{1/s}{\cancel{s}} - \frac{14}{s+1} + \frac{22}{(s+2)^2} + \frac{13}{(s+2)}$$

$$= u(t) - 14e^{-t} + 22te^{-2t} + 13e^{-2t}$$

Q) Find ILT $F(s) = \dfrac{20}{(s+3)(s^2+8s+25)}$

$$F(s) = \frac{A}{(s+3)} + \frac{Bs+C}{(s^2+8s+25)}$$

$$A = \Big[(s+3)F(s)\Big]\Big|_{s=-3}$$

$$= \frac{20}{9-24+25} = \frac{20}{10} = 2$$

$\cancel{B = (s^2+8s+25)F(s)}$

$$F(s) = A(s^2+8s+25) + Bs(s+3) + C(s+3)$$

$As^2 + Bs^2 = 0$

$A = -B$

$\boxed{B = -2}$

$s = 0$

$20 = 25A + 3C$

$20 = 50 + 3C$

$20 = 50 + 3C$

$\boxed{C = -10}$

$$F(s) = \frac{2}{s+3} - \frac{2s+10}{s^2+8s+25}$$

$$= \frac{2}{s+3} - \frac{2s+10}{s^2+8s+16+9}$$

$$= \frac{2}{s+3} - \frac{2s+10}{(s+4)^2+3^2}$$

$x(t) \to X(s)$
$e^{at}x(t) \to X(s-a)$

$$= \frac{2}{s+3} - 2\left[\frac{s+5}{(s+4)^2+3^2}\right]$$

$$= \frac{2}{s+3} - 2\left[\frac{s+4}{(s+4)^2+3^2}\right] - \frac{2\times 3\ \frac{1}{3}}{(s+4)^2+3^2}$$

$$f(t) = \left[2e^{-3t} - 2\,\cancel{\sin}\,e^{-4t}\cos\left(\cancel{s+4}\,3t\right) - \frac{2}{3}e^{-4t}\sin(3t)\right]$$

(2) $$F(s) = \frac{10}{(s+1)(s^2+4s+13)}$$

$$Fs = \frac{A}{s+1} + \frac{Bs+C}{s^2+4s+13}$$

$$A = (s+1)F_s\Big|_{s=-1} = \frac{10}{1-4+13} = \frac{10}{10} = 1$$

$$10 = A(s^2+4s+13) + Bs(s+1) + C(s+1)$$

$s = 0$

$$10 = A(13) + 0 + C$$

$$10 = 13 + C$$

$$\boxed{C = -3}$$

$$A\cancel{s^2} + B\cancel{s^2} = 0$$

$$\boxed{B = -1}$$

$$F(s) = \frac{1}{s+1} - \frac{s+3}{s^2+4s+13}$$

$$\frac{1}{s+1} - \frac{s+3}{s^2+4s+4+9}$$

$$\frac{1}{s+1} - \frac{s+3}{(s+2)^2+3^2}$$

$$\cancel{f(t) = e^{-t} - e^{-2t}\cos(3t)}$$

$$\frac{1}{s+1} - \frac{s+2}{(s+2)^2+3^2} - \frac{1}{3}\frac{3}{(s+2)^2+3^2}$$

$$f(t) = e^{-t} - e^{-2t}\cos(3t) - \frac{1}{3}e^{-2t}\sin(3t)$$

Application of Laplace Transform to circuits:

1) Transform the given ckt from time domain to S - domain.

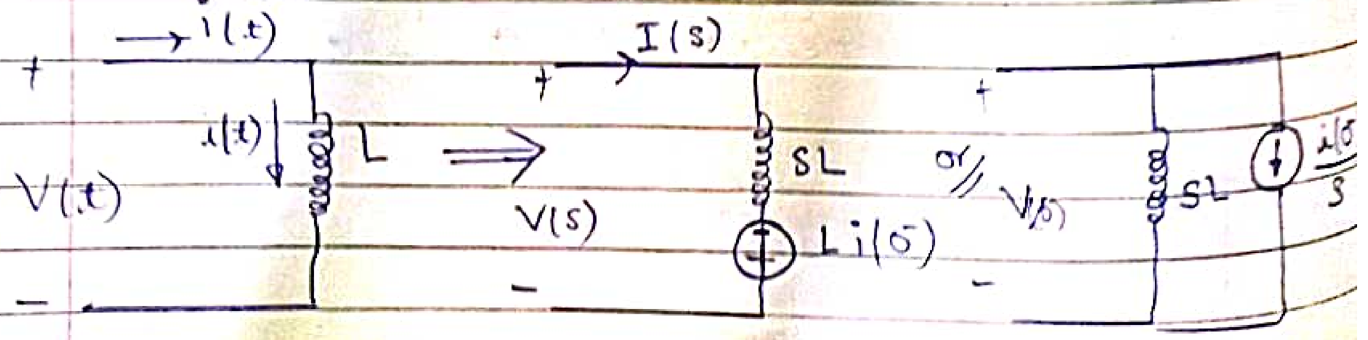
2) Solve the ckt with any of the 1 node technique

3) Take inverse laplace transform of the solution to apply the solution in time domain.

Resistor -
$$V_R(t) = i(t) \cdot R$$
$$V_R(S) = R \cdot I(S)$$

Inductance -
$$V_L(t) = L \cdot \frac{di(t)}{dt}$$
$$V_L(s) = L\left[S(I(s)) - I(0^-)\right]$$
$$= SLI(s) - L\,i(0^-)$$

$$I(S) = \frac{1}{SL} V_L(S) + \frac{i(0^-)}{S}$$

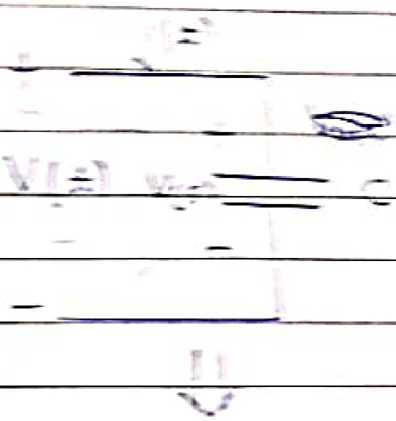


Capacitor —

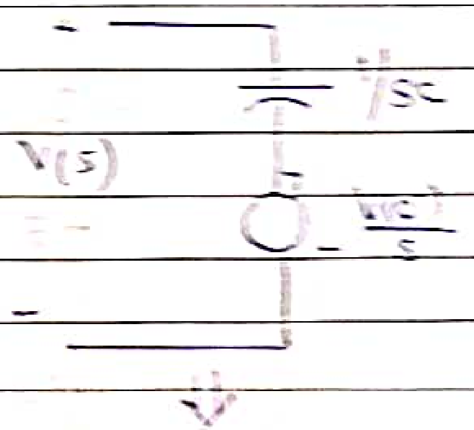
$$i(t) = C \frac{dv(t)}{dt}$$

$$I(s) = C\left[sV(s) - v(0^-)\right]$$

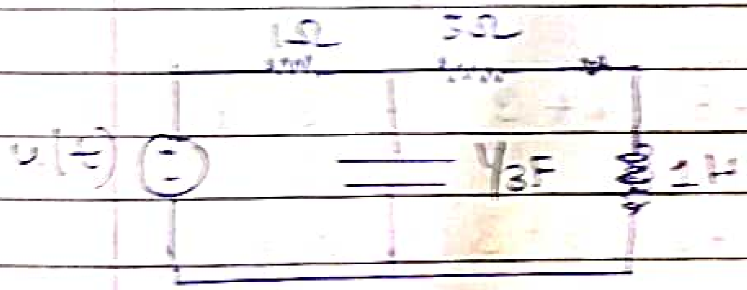
$$= sCV(s) - Cv(0^-)$$

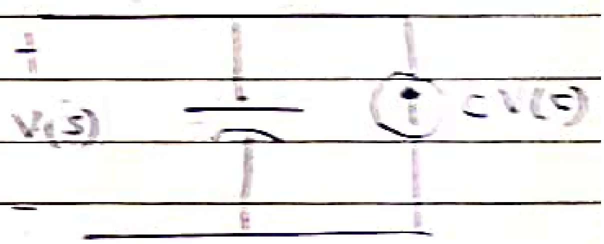


$$V_c(s) = \frac{1}{sC} I(s) + \frac{v(0^-)}{s}$$

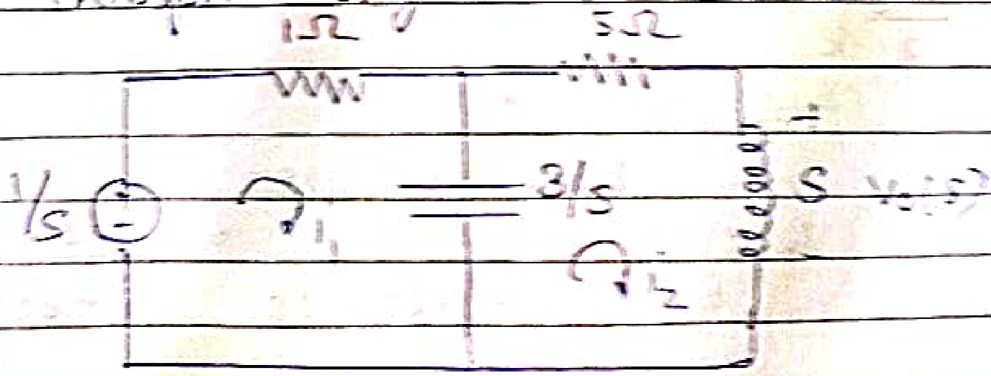


Q) Find $v_0(t)$ in the ckt, assuming zero input condition.





Transform every element to s domain



Date
Page

$$\frac{1}{s} - i_1 - \frac{3}{s}(i_1 - i_2) = 0$$

$$\frac{1}{s} - i_1 - \frac{3}{s} i_1 + \frac{3}{s} i_2 = 0$$

$$\frac{3}{s} i_2 - i_1 (1 + 3/s) = -\frac{1}{s}$$

$$i_1 \left(1 + \frac{3}{s}\right) - \frac{3}{s} i_2 = \frac{1}{s} \quad — (1)$$

$$-5i_2 - si_2 - \frac{3}{s}(i_2 - i_1) = 0$$

$$-5i_2 - si_2 - \frac{3}{s} i_2 + \frac{3}{s} i_1 = 0$$

$$i_2 \left(5 + s + \frac{3}{s}\right) = \frac{3}{s} i_1$$

$$i_2 (5s + s^2 + 3) = 3 i_1 \quad — (2)$$

$$\frac{i_2}{3} (5s + s^2 + 3)\left(1 + \frac{3}{s}\right) - \frac{3}{s} i_2 = \frac{1}{s}$$

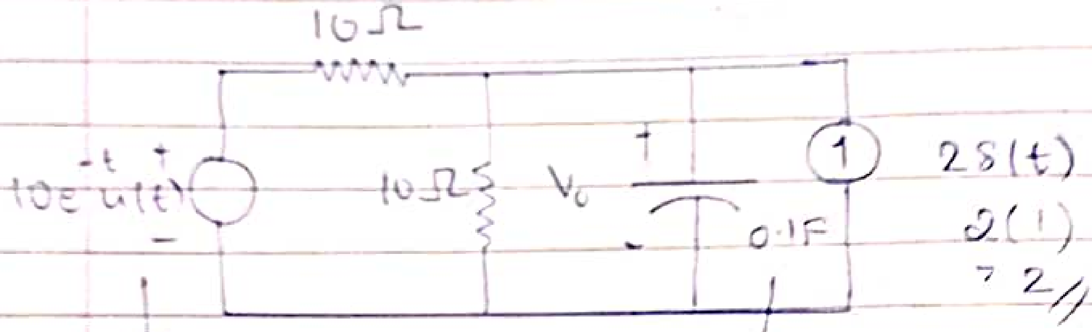
$$\frac{i_2}{3} (5s + s^2 + 3)(s + 3) - 3i_2 = 1$$

$$\frac{i_2}{3}\left(5s^2 + 15s + s^3 + 3s^2 + 3s + 9\right) - 3i_2 =$$

$$i_2(s) = \frac{3}{s^3 + 8s^2 + 18s}$$

$$V_0(s) = s \times \frac{3}{s^3 + 8s^2 + 18s} = \frac{3}{s^2 + 8s + 18}$$

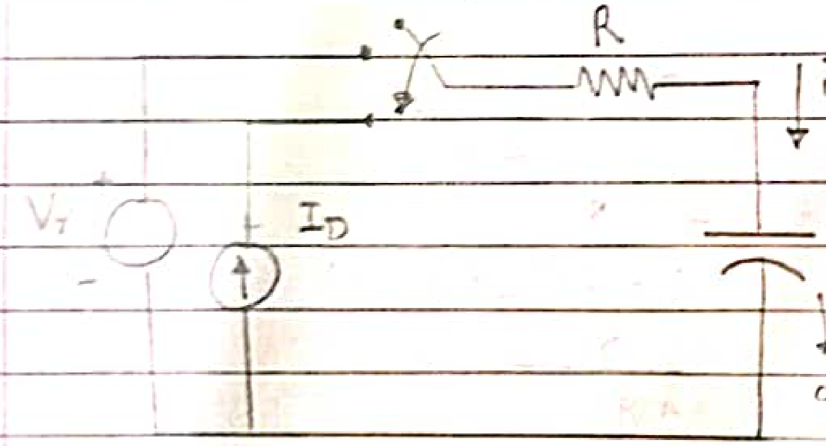
Q) Find $V_o(t)$ in the ckt assuming the initial condition $V_o(0^-) = 5V$



$2\delta(t)$
$2(1)$
$= 2$

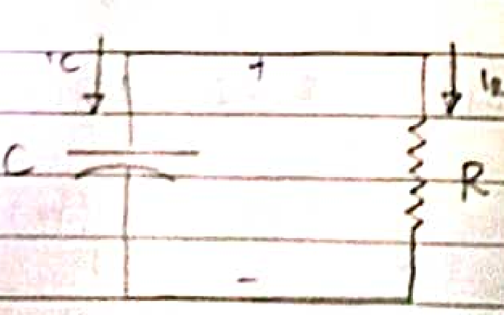
$10\left(\frac{1}{s+1}\right)$

$\frac{1}{0.1s} = 10/s$



discharge of capacitor after switch is moved

|||



Consider a series combination of resistor and initially charged capacitors. The initial voltage of capacitor at $t=0$, $V(0) = V_0$. So energy stored $W(0)/E = \frac{1}{2}CV_0^2$. On applying KCL in the loop

$$i_C + i_R = 0$$

$$C\frac{dV}{dt} + \frac{V}{R} = 0$$

$$\frac{dV}{dt} + \frac{V}{RC} = 0$$

$$\int_{V_0}^{V} \frac{dV}{V} = \int_0^t -\frac{1}{RC}\,dt$$

$$\ln\left(\frac{V}{V_0}\right) = -\frac{1}{RC}t$$

$$\boxed{V = V_0 e^{-t/RC}}$$

OR

$$\ln(V) = \frac{-t}{RC} + \ln A_0$$

where A_0 is integral constant

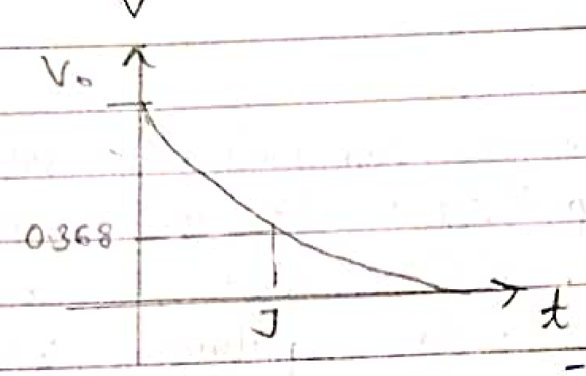
$$\ln\frac{V}{A} = -t/RC$$

$$V = Ae^{-t/RC}$$

using initial conditions

$$V(0) = A = V_0$$

$$\boxed{V(t) = V_0 e^{-t/RC}}$$



when $t = RC$, $V(t) = V_0 e^{-1} = 0.368\,V_0$

$\{\tau = RC\}$

t	$V(t)/V_0$
τ	0.367
2τ	0.13
3τ	0.04
4τ	0.018
5τ	0.0067

{Time req. to reach 36% of initial value is called time constant}

The voltage $V(t)$ is less than 1% of V_0 after 5τ. Thus it is customary to assume that capacitor is fully discharged after 5 time constant.

$$P_R(t) = \frac{V(t)}{R} = \frac{V_0}{R} e^{-t/\tau}$$

Power dissipated in Resistor

$$P(t) = V \cdot i_R = \frac{V_0^2}{R} e^{-2t/\tau}.$$

Energy absorbed upto 't' is—

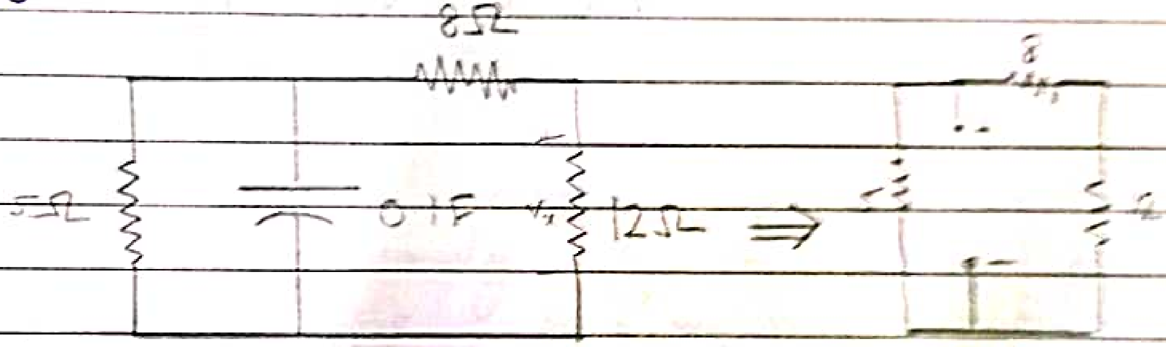
$$W_R(t) = \int_0^{\tau} P \cdot dt = \int_0^{t} \frac{V_0^2}{R} e^{-2t/\tau}\, dt$$

$$\frac{1}{2} C V_0^2 \left(1 - e^{-2t/\tau}\right)$$

As $t \to \infty$

$$w_R = \frac{1}{2} C V_0^2$$

Q) let $V_c(0) = 15V$. Find V_c, V_x and i_x $t > 0$.



$20 \times 5/25 = 4$

$$V_c = V_0 e^{-t/RC}$$

$R_{eq} = 4\Omega$

$$\left\{ V_c = 15 e^{-t/0.4} \; V \right\}$$

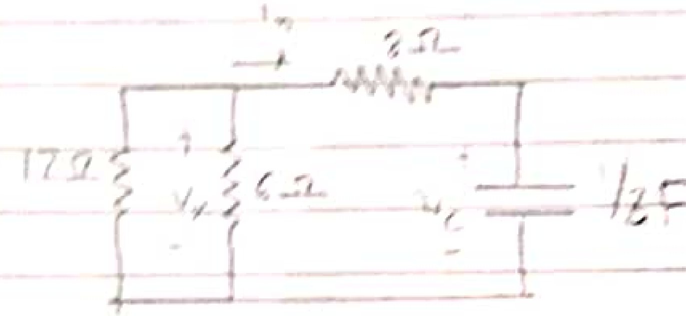
$$V_x = \frac{12}{20} \times V_0 e$$

$$= \frac{3}{5} \times 15 e^{-t/0.4}$$

$$\left\{ V_x = 9 e^{-t/0.4} \; V \right\}$$

$$\left\{ i_x = \frac{V_x}{R} = 0.75 e^{-t/0.4} \; A \right\}$$

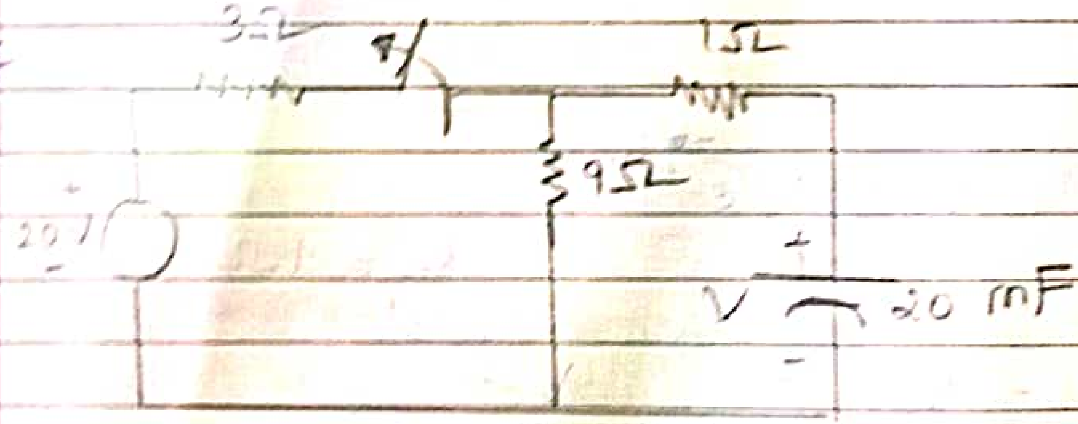
eg



$V_c(0) = 30\,V$

Find V_c, V_x, i_o for $t>0$

eg



The switch in the ckt has been closed for a long time and it has been opened at $t=0$. Find $V(t)$ at $t=0$ also calculate the initial energy stored in the capacitor.

For $t<0$ switch is closed

cap → open ckt.

$$i = \frac{\cancel{20}}{\cancel{12}}$$

$$= \frac{5}{3}$$

$$V_C = \frac{5}{\cancel{3}} \times \cancel{9}^{3}$$

$$= 15\ V$$

∴ Voltage across cap = 15 V

Since the voltage across capacitor cannot change instantaneously ∴ The voltage at $t=0$ is same as ~~voltage~~ $t=0^+$.

$$V_C(0) = 15V$$

$$V = V_0 e^{-t/RC}$$

$$= 15 e^{-t/10 \times 20 \times 10^{-3} 10^{-1}}$$

$$= 15 e^{-5t}\ V$$

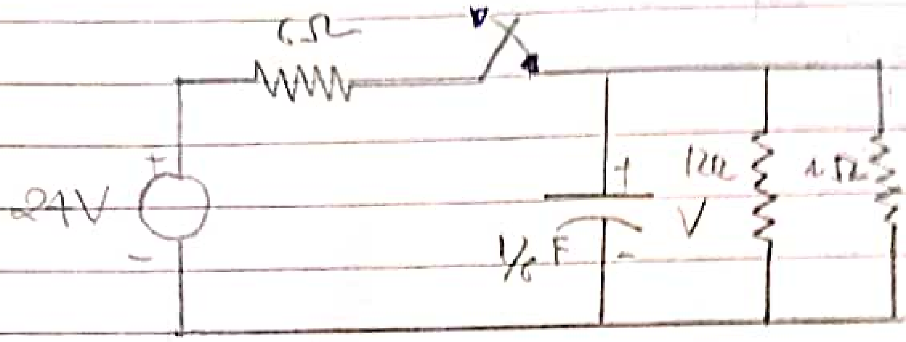
$$\text{Energy} = W_C(0)$$

$$= \frac{1}{2} \times C \times V_0^2$$

$$= \frac{1}{\cancel{2}} \times \cancel{20}^{10} \times 10^{-3} \times 10^{-2} \times 225$$

$$= 2.25\ J$$

eg: Find $V(t)$ for $t \geq 0$; $W_c(0) = ?$



$$\frac{12\times 4}{16} = 3\Omega$$

$$V_c = \frac{\cancel{3}}{\cancel{9}3} \times 24$$

$$= 8V$$

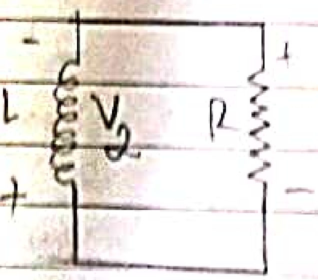
$$\tau = R_{eq}.C$$

$$= \frac{1}{6} \times 3 = 0.5$$

$$V = 8e^{-2t} \ V$$

$$E(0) = \frac{1}{\cancel{2}} \times \frac{1}{6} \times \overset{32}{\cancel{64}} = 16/3 \ J$$

eg: Source Free R-L ckt



$$i(0) = I_0$$

$$w(0) = \frac{1}{2} L I_0^2$$

Applying KVL in the loop,

$$V_L + V_R = 0$$

$$L\frac{di}{dt} + i \cdot R = 0$$

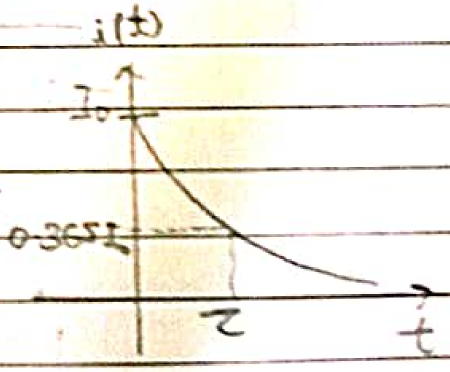
$$\frac{di}{dt} + \frac{Ri}{L} = 0$$

$$\int_{I_0}^{i} \frac{di}{i} = -\int_{0}^{t} \frac{R}{L}\,dt$$

$$\ln(i/I_0) = -\frac{Rt}{L}$$

$$\frac{i}{I_0} = e^{-tR/L}$$

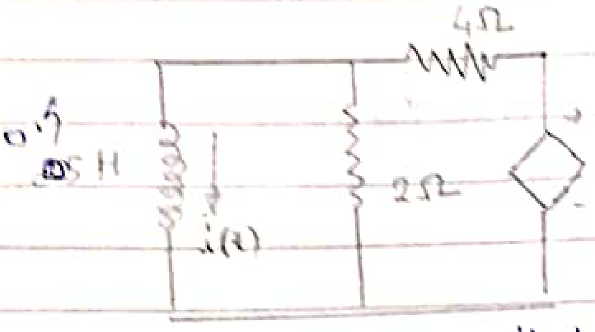
$$\therefore\ i = I_0 e^{-tR/L}$$



$$\{\tau = L/R\}$$

$$\therefore\ i = I_0 e^{-t/\tau}$$

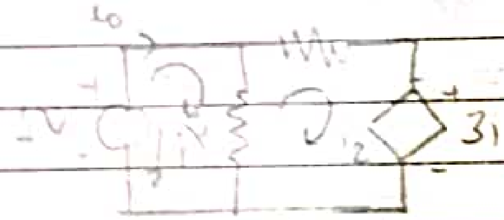
(1)



{Natural response of ckt}

Assume that $i(0) = 10\,\mu A$,
calculate
$i(t)$ and $i_x(t)$ in the ckt.

Q



$i_1 = -i$

$-3i - 2(i_2 - i_1) - 4(i_2) = 0$

$-3i - 2i_2 + 2(-i) - 4i_2 = 0$

$6i_2 = -5i$

$i_2 = -5i/6$

$1 - (i_1 - i_2)2 = 0$ $\quad i_1 = -i$

$= 6/7\ A$

$1 - 2(-i) + 2i_2 = 0$

$1 + 2i - \frac{5i}{6} = 0$ $\quad R = 1/(6/7)$

$= 7/6\ \Omega$

$1 + 7i/6 = 0$

$i = -6/7\ A$

Date
Page

$\frac{1}{2} = i_1 - i_2$

$\frac{1}{2} = i_1 - \frac{5}{6} i_1$

$i_1 = 3A$

$i_2 = \frac{-5 \times \cancel{3}}{\cancel{6}\,2}$

$i_2 = -2.5A$

$R_{eq} = 1/3\ \Omega$

$\tau = L/R_{eq} = 3/2\ s$

$i(t) = 10e^{-2/3 t}$

$v(t) = L \frac{di(t)}{dt}$

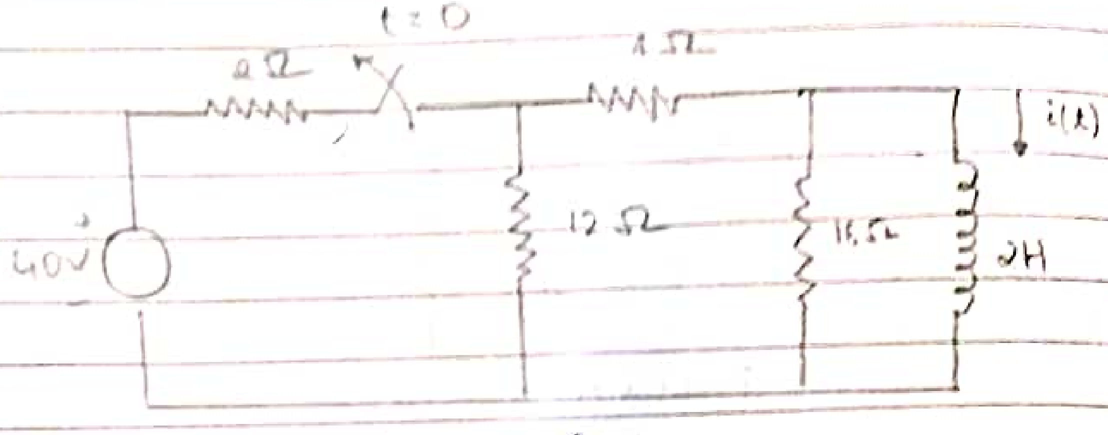
$= \cancel{-\frac{10}{3}\,2\left(\frac{-2}{\ }\right)}$

$\left(\frac{1}{2}\right) 10 \left(\frac{-2}{3}\right) e^{-2/3 t}$

$v(t) = -10/3\ e^{-2t/3}\ V$

$i_x(t) = \frac{v(t)}{2} = -1.677 e^{-2t/3}\ A$

eg



i (t) = ?

for t < 0 switch is closed

R_{eq} = 5 Ω

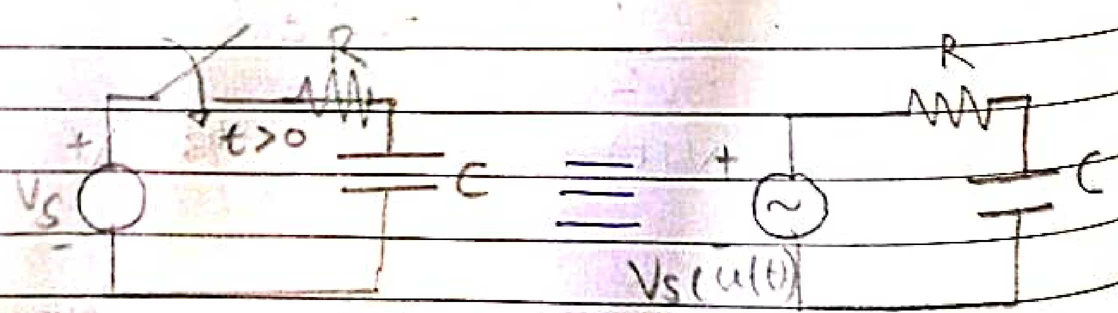
$i_0 = \frac{40}{5} = 8A$

$i(t) = \frac{12 \times 8}{16} = 6A$

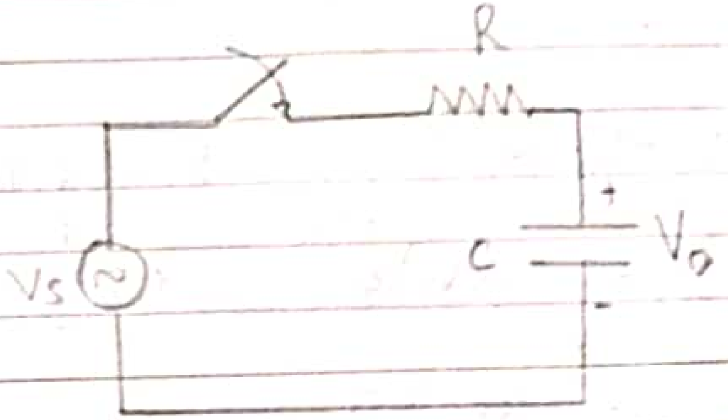
$i = 6e^{-t/\tau}$

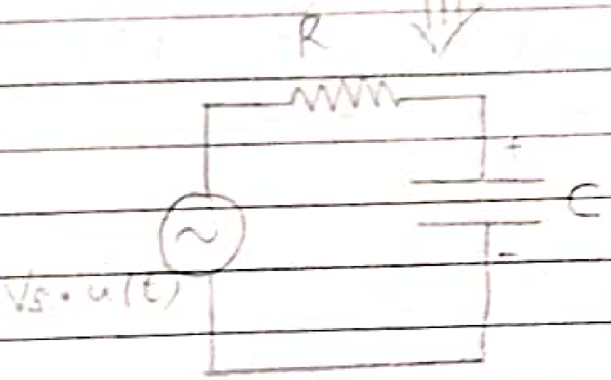
$\tau = \frac{L}{R} = \frac{2}{8} = 0.25$

$i = 6e^{-4t}$



Switching action is replaced by unit step.





Assume that initial voltage across capacitor
~~V(0)~~

$V(0^-)$ = voltage across capacitor just befor switching
$V(0^+)$ = voltage across cap. just after switching

$$\frac{dV}{dt} + \frac{V}{RC} = \frac{V_s}{R \cdot C} u(t) \quad \text{(1)}$$

For $t > 0$

$$\frac{dV}{dt} = \left\{ \frac{-V}{R \cdot C} + \frac{V_s(u(t))}{R \cdot C} \right\}$$

$$\int_{V_0}^{V(t)} \frac{dV}{V_s - V} = \frac{1}{RC} \int_0^t dt$$

$$\ln(V - V_s) = -t/RC$$

$$\ln\left(\frac{V_t - V_s}{V_0 - V_s}\right) = \frac{-t}{RC}$$

$$V_t - V_s = (V_0 - V_s)e^{-t/RC}$$

$$V(t) = V_s + (V_0 - V_s)e^{-t/RC} \quad ; \quad t \geq 0$$

let $RC = \tau$

$$\boxed{V(t) = V_s + (V_0 - V_s)e^{-t/\tau}} \quad t > 0$$

for $t < 0$

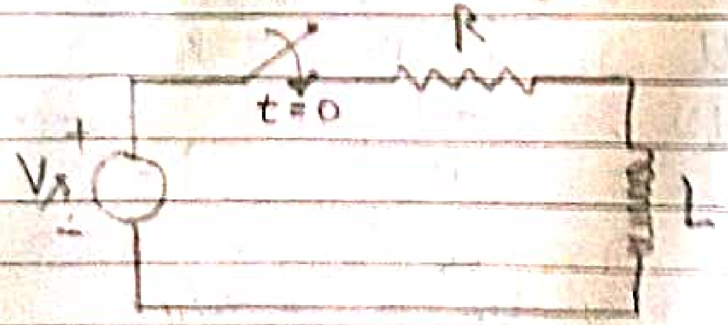
$$V(t) = V_0 \quad , t \leq 0$$

$$= \underbrace{V_s}_{\text{Forced response}} + \underbrace{(V_0 - V_s)e^{-t/\tau}}_{\text{Natural response}} \quad , t \geq 0$$

$$V(\infty) = V_s$$

$$\therefore \boxed{V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}} \quad t > 0$$

Step Response of R-L ckt



Date
Page

~~v(t)~~ $$i(t) = i_n + i_F \quad \text{—} (1)$$

Natural response,

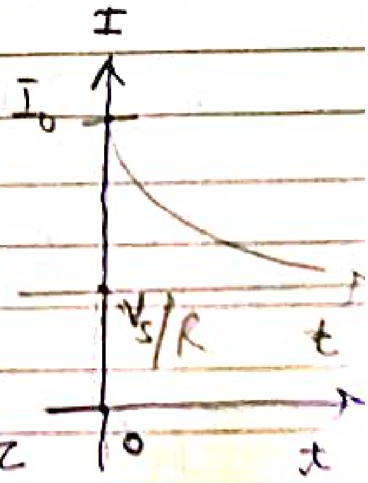
$$i_n = A \cdot e^{-t/\tau} \qquad \tau = L/R \quad \text{—} (2)$$

$$i_F = \frac{V_0}{R} \quad \text{—} (3)$$

$$i(t) = A \cdot e^{-t/\tau} + \frac{V_0}{R} \quad \text{—} (4)$$

let I_0 be the initial ~~condition~~ current through inductor.

$$i(0^-) = i(0^+) = i(0)$$



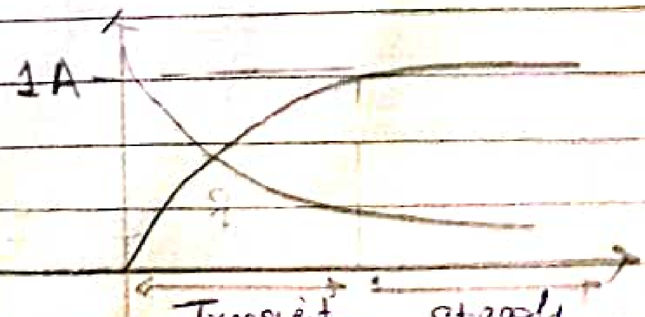
at $t = 0$

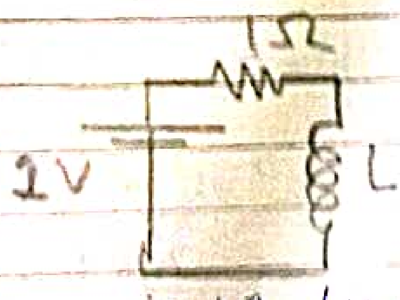
$$I_0 = A + \frac{V_{0s}}{R}$$

$$A = I_0 - \frac{V_{0s}}{R} \quad \text{—} (5)$$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_{0s}}{R}\right) e^{-t/\tau}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}.$$

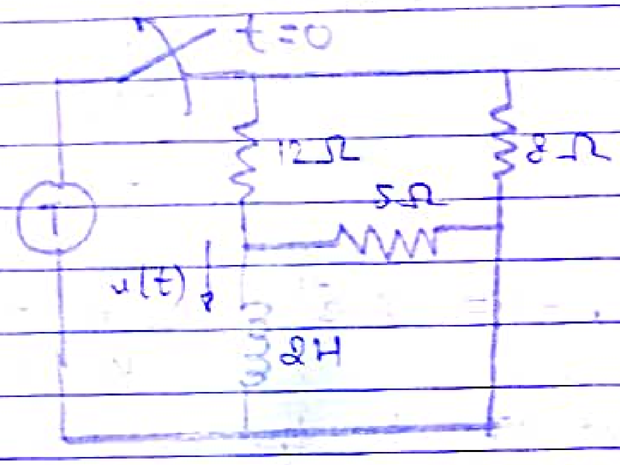




$i = 1A$ (in steady state)

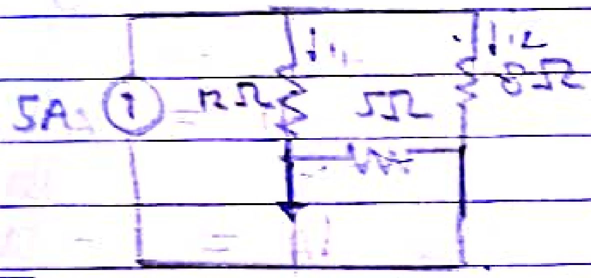
initially it flows on exponential increase only after 5 time constants, I reaches steady state at 1 A and remains constant.

Q)



Find i(t) for t > 0.

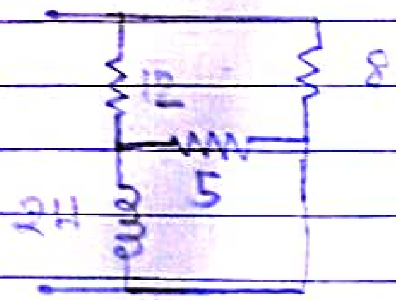
For t < 0



$i_1 = \frac{8}{\cancel{20}_{4}} \times \cancel{5} = 2A$

$\boxed{i(t) = 2A}$ for t < 0

For t > 0,



$E_L = \frac{1}{2} L i^2$

$= \frac{1}{2} \times 2 \times 4$

$R = \frac{20 \times 5}{25} = 4\Omega$

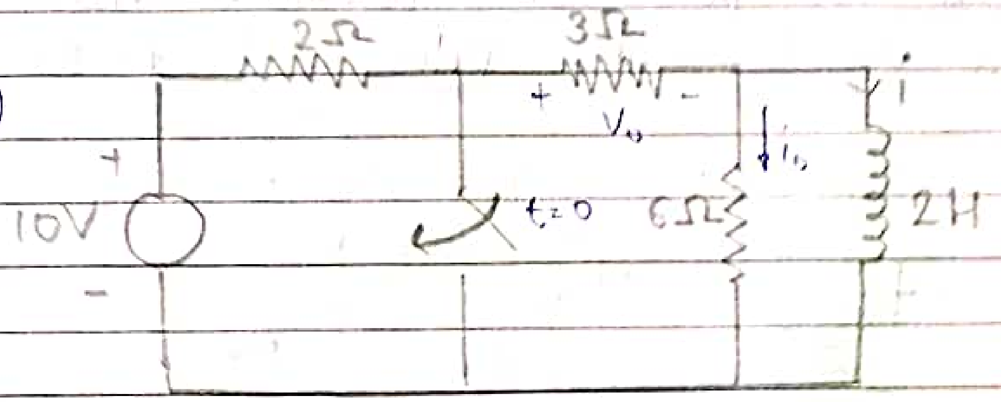
$i = i_0 e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{2}{4} = 0.5$

$i = 2e^{-t/0.5}$

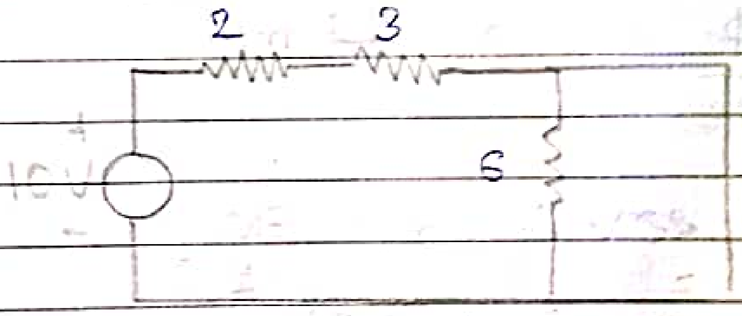
$i = 2e^{-2t}$ A

Q)

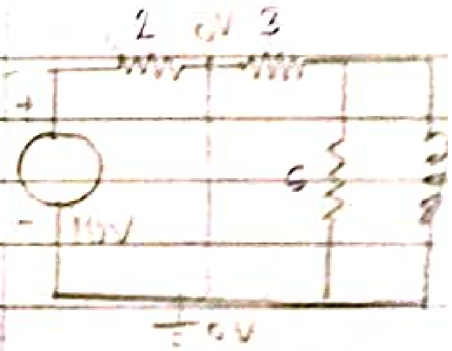


Find $i_0(t)$, $i(t)$, $V_0(t)$ for $t<0$, $t>0$

For $t<0$



For $t>0$



$i = \frac{10}{5} = 2A = i$

$V_0 = 2\times 3 = 6V$

$i_0 = 0A$

$t>0$

$i = i_0 e^{-t/\tau}$

$R = 18/9 = 2\Omega$

$\tau = L/R$

$\{\tau = 2/2 = 1s\}$

$i = 2e^{-t}$

$\frac{R}{2}\times 2e^{-t}\times$

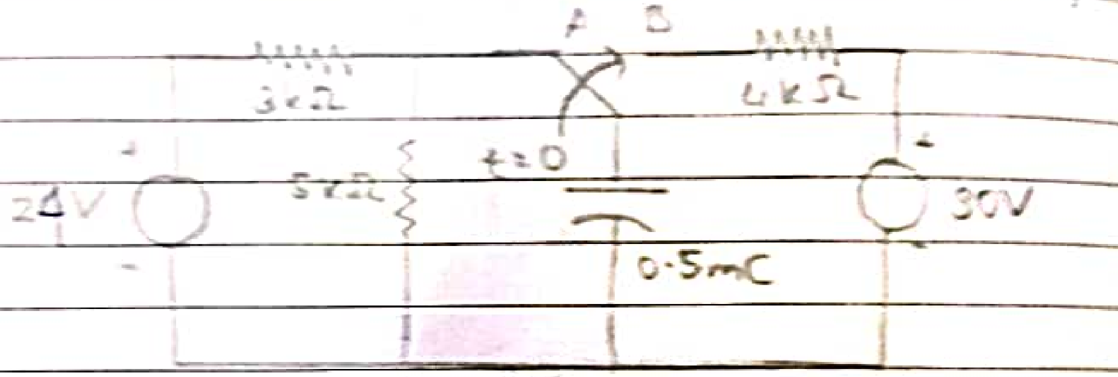
$\{V_0 = -4e^{-t}\ V\}$

$i_0(t) = V_L/6$

$\boxed{V_0 = -V_L \text{ for } t>0}$

$i_0(t) = -\frac{2}{3}e^{-t}$

Q) The given ckt, the switch has been in position A for a long time. At $t=0$, the switch moves to position B. Determine $V(t)$, $t>0$ and calculate its value at $t = 1s$ & $4s$.



At $t<0$,

$$i = \frac{\cancel{24}}{8}\ mA = 3\ mA$$

$$V_c = V \text{ across } 5k\Omega$$

$$= \frac{5}{8} \times \frac{24}{1}$$

$$V(0) = 15V$$

For $t>0$,

$$v(t) = V(\infty) + (V_0 - V_\infty)e^{-t/\tau}$$

$$\tau = RC = 4 \times 0.5 \times 10^{3}$$
$$= 2\ s$$

$$V_\infty = 30V$$

$$V(t) = 30 + -15e^{-t/2} = 30 - 15e^{-0.5t}$$

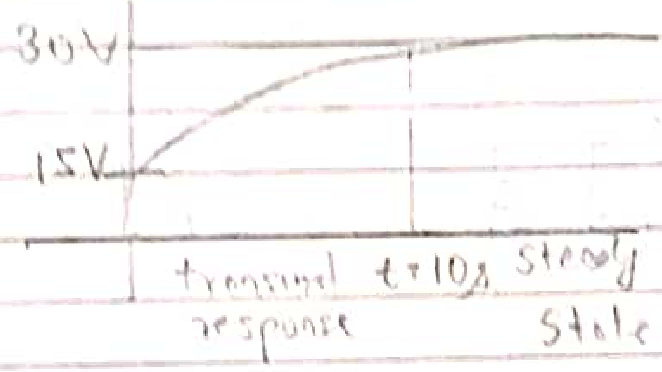
$t = 1\,s$

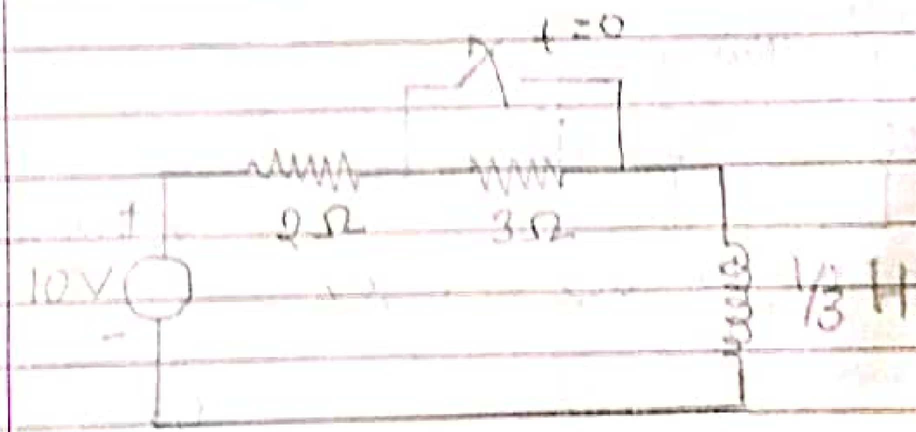
$V(t) = 30 - 15e^{-1/2}$

$= 30 - 15e^{-0.5}\,V = 20.9\,V$

$t = 4\,s$

$V(t) = 30 - 15e^{-4/2}\,V = 27.97\,V$





Find $i(t)$ for $t > 0$, assume that switch has been closed for a long time.

For $t < 0$,

$i = \frac{10}{2} = 5A$

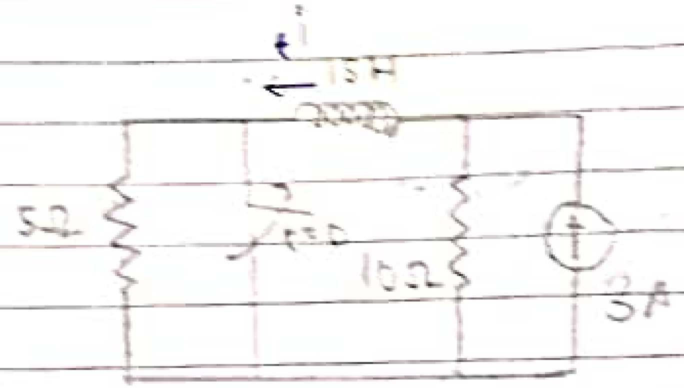
$i(0) = 5A$

for $t > 0$ $\quad i_\infty = 2A$, $\tau = L/R = 1/15\,s$

$i(t) = i_\infty + (i_0 - i_\infty)e^{-t/\tau}$

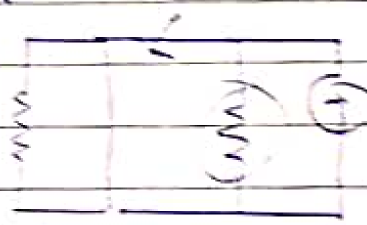
$= 2 + 3e^{-t/(1/15)} = 5 + 3e^{-15t}\,A \quad t > 0$

Q)



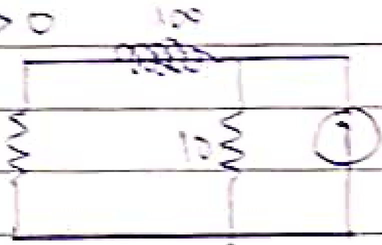
Find $i_o(t)$ for $t>0$

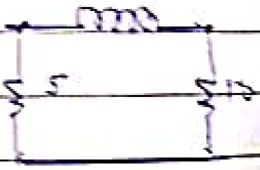
$t<0$



$i_0 = 3A$

$t>0$





$i_{(\infty)} = \frac{10}{15} \times 3 = 2A$

$i(t) = i_\infty + (i_0 - i_\infty)e^{-t/\tau}$

$= 2 + e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{1.5}{15} = 0.1 \text{ sec}$

$\boxed{i(t) = 2 + e^{-10t} A}$