Marks: 30 **Time: 1.5 hour**

Note: Attempt all questions, write clearly for step marking. Symbols are having their usual meaning.

1. (a) Let X be distributed with PDF $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$. Find $P\left\{ \left| X - \frac{1}{2} \right| < 2\sqrt{\frac{1}{12}} \right\}$ and compare the answer with the Chebychev's inequality.

    (b) A continuous random variable X has a pdf $f(x) = 3x^2, 0 \leqslant x \leqslant 1$. Find a and b s.t. (i) $P(X \leqslant a) = P(X > a)$ (ii) $P(X > b) = 0.05$.

    (CO 3) (2+3 marks)

2. If the joint cumulative distribution function of X and Y is given by:

$$F_{XY}(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)} & ; x > 0, y > 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

    (a) Find the marginal densities of X and Y and check whether they are independent or not.

    (b) Find $P(X \leq 1 \cap Y \leq 1)$ and $P(X + Y \leq 1)$.

    (CO 3) (2+3 Marks)

3. A random variable X has the following probability distribution

x:	0	1	2	3	4	5	6	7
p(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

    (i) Find k (ii) Evaluate $P(X < 6), P(X \geqslant 6)$ and $P(0 < X < 5)$ (iii) If $P(X \leq C) > \frac{1}{2}$, find the minimum of C. (iv) Determine the distribution function of X.

    (CO 3)(5 Marks)

4. Let X be a random variable with PDF,

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

    where $\lambda > 0$. Show that MGF of X exists and find its moments $E(X^k)$.   (CO 3) (5 marks)

5. Suppose a pair of die is rolled and let X be the sum of the numbers that appear. Write down the PMF of X. Then, find $P(X \geq 8), P(3 \leq X \leq 10), E(X)$ and $Var(X)$.

    (CO 3) (5 marks)

6. (a) If a string of one meter is cut into two pieces at a random point along its length, what is the probability that the longer piece is at least twice the length of the shorter ?

    (b) Let X be the normal variable with mean μ and standard deviation σ. If Z is the standard normal variable such that $Z = -0.8$ when $X = 26$ and $Z = 2$ when $X = 40$, then find μ and σ. Also, find $P(X > 45)$ and $P(|X - 30| > 5)$.

    (CO 4) (2+3 marks)

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IV Semester, B.Tech (ECE, EEE & CHEM) Slot-E Max. Marks: 50

Subject: Numerical Methods and Probability Theory (MAL 205) Duration: 3 Hours

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Instructions: *Answer all questions. All numerical calculation has to be four decimal accuracy.*

1. State Chebyshev's inequality. For the RV with PDF $f(x; \lambda) = \dfrac{e^{-x} x^{\lambda}}{\lambda!}, x > 0$ where $\lambda \geq 0$ is an integer, show that $P\{0 < X < 2(\lambda+1)\} > \dfrac{\lambda}{1+\lambda}$. [CO3, 5 Marks]

2. Derive the relation between Binomial and Poisson distribution. [CO4, 5 Marks]

3. Define rate of convergence and derive rate of convergence for Secant Method. [CO1, 5 Marks]

4. Using the Newton-Raphson method, derive a formula to find $N^{\frac{1}{q}}$, where N, q are positive integers. Hence find an approximation for $18^{\frac{1}{3}}$ correct to four decimals. [CO1, 5 Marks]

5. Using the Crout's method, solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 7 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}.$$

[CO2, 5 Marks]

6. Using the Gauss-Seidel method with initial guess $X^{(0)} = (0, 0, 0, 0)^T$, solve the system of equations $2x - y = 1, -x + 2y - z = 0, -y + 2z - w = 0, -z + 2w = 1$. Perform three iterations of the method. [CO2, 5 Marks]

7. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} -1 & 3 & 5 \\ 3 & 4 & 3 \\ 5 & 3 & -1 \end{bmatrix}$$

using the Jacobi method. [CO3, 5 Marks]

8. Using a suitable numerical method, find the smallest eigenvalue in magnitude of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \text{ initial eigenvector } X^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

[CO3, 5 Marks]

9. Using Milne's predictor-corrector method with the step size $h = 0.1$, find $y(0.4)$ for the initial value problem

$$y' = x + y, \qquad y(0) = 1.$$

Calculate the required initial values using Euler's method. Perform one iteration of the corrector. [CO3 5 Marks]

10. Solve the boundary value problem

$$xy'' + y = 0, \ y(1) = 1, \ y(2) = 2$$

by second order finite difference method with $h = 0.25$. Solve the resulting system of equation by Gauss elimination method. [CO3, 5 Marks]

Department of Mathematics
Probability Theory (MAL-205)
Assignment on
CDF, PMF, PDF

1. Determine the constant A in the following functions, so that those functions are pmfs/pdfs. Find CDF ($F_X(x)$) in each case.

   (i) $f_X(x) = \begin{cases} A\left(\frac{2}{3}\right)^{x-1}, & \text{if } x = 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$

   (ii) $f_X(x) = \begin{cases} A\binom{7}{x}\left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{7-x}, & \text{if } x = 0, 1, 2, ..., 7 \\ 0, & \text{otherwise} \end{cases}$

   (iii) $f_X(x) = \begin{cases} \frac{A}{x}e^{\frac{-(\log x)^2}{2}}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$

   (iv) $f_X(x) = \begin{cases} Ax^{\frac{-1}{2}}, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

   (v) $f_X(x) = Ae^{-|x|}, \quad x \in \mathbb{R}.$

2. Check if the following functions define CDFs :
   (a) $F_X(x) = 0$, if $x < 0$, $= x$, if $0 \leq x \leq 1/2$, and $= 1$, if $x > 1/2$.
   (b) $F_X(x) = (1/\pi)\tan^{-1}x$, $-\infty < x < \infty$.
   (c) $F_X(x) = 1 - e^{-x}$, if $x \geq 0$, and $= 0$, if $x < 0$.

3. For a certain rv X, CDF is defined as $F_X(x) = \begin{cases} 0, & \text{if } x \leq 1 \\ K(x-1)^4 & \text{if } 1 < x \leq 3 \\ 1, & \text{if } x > 3. \end{cases}$

   (a) Determine the value of K.
   (b) Find pdf $f_X(x)$ and $\Pr\left(\frac{3}{2} \leq x \leq \frac{9}{2}\right)$.

4. Find the CDF of the distribution whose pdf is given by $f_X(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}$

5. The pdf for a continuous 'Rayleigh' rv X is given by $f_X(x) = \begin{cases} \alpha^2 x e^{-\frac{\alpha^2 x^2}{2}}, & \text{if } x > 0 \\ 0, & \text{otherwise.} \end{cases}$
   Find CDF of X.

6. The distance covered by a person is assumed to be a continuous rv with pdf
   $f_X(x) = \begin{cases} 6x(1-x), & \text{if } 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$ Find CDF of X. Compute $\Pr\left(X \leq \frac{1}{2} | \frac{1}{3} \leq X \leq \frac{2}{3}\right)$.
   Determine k such that $\Pr(X > k) = P(X < k)$.

7. Two unbiased dice are thrown and the rv X denote the sum of faces turned. Construct the table for pmf and find CDF.

1. Three newspapers A, B, C are published in a city and a survey of readers indicates the following: 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all the three. For a person chosen at random, find the probability that he reads none of the paper. [Ans: 13/20]

2. For any two event A and B, prove that
   (a) $P(A^c \cap B) = P(B) - P(A \cap B)$.
   (b) $P(A \cap B^c) = P(A) - P(A \cap B)$.

3. Events A and B are such that
$$P(A \cup B) = \frac{3}{4}, \quad P(A \cap B) = \frac{1}{4}, \quad \text{and} \quad P(A^c) = \frac{2}{3}.$$
   Find $P(B)$ and $P(A \cap B^c)$. [Ans: 2/3 and 1/12]

4. If $P(A) = a$ and $P(B) = b$, then show that
$$P(A \mid B) \geq (a + b - 1)/b$$

5. Three fair dice are thrown once. Given that no two show the same face.
   (a) What is the probability that the sum of faces is 7? [Ans: 1/20]
   (b) What is the probability that one is an ace? [Ans: 1/2]

6. A single die is tossed; then n coins are tossed, where n is the number shown on the die. What is the probability of exactly two heads? [Ans: 99/384]

7. An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two are drawn from the first urn and put into the second urn and then a ball is drawn from the latter. What is the probability that it is a white ball? [Ans: 59/130]

8. The contents of Urn I, II, and III are as follows:
   1 white, 2 red, and 3 black balls,
   2 white, 2 red, and 2 black balls, and
   3 white, 1 red, and 2 black balls.
   One urn is chosen at random and two balls are drawn. They happen to be white and red. What is the probability that they come from urns I, II, or III? [Ans: 2/11, 6/11, 3/11].

9. Suppose 20% of the items produced by a factory are defective. Suppose 4 items are chosen at random. Find the probability that (a) 2 are defective, (b) none is defective.

10. The probability that John hits a target is 1/4. He fires 6 times. Find the probability that he hits the target: (a) exactly 2 times, (b) more than 4 times, (c) at least once.