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Q1. Markov Chain

(a) Transition matrix Q

State = {1,2,3,4}

~~Pb~~ Transition Probabilities:

From State 1 : $P(1 \to 1) = 0.5$

$P(1 \to 2) = 0.5$

From State 2:

$P(2 \to 1) = 0.25$

$P(2 \to 3) = 0.75$

From state 3:

$P(3 \to 3) = 0.75$

$P(3 \to 4) = 0.25$

From state 4:

$P(4 \to 3) = 0.25$

$P(4 \to 4) = 0.75$

Now

$$Q = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0 & 0.75 & 0 \\ 0 & 0 & 0.75 & 0.25 \\ 0 & 0 & 0.75 & 0.75 \end{pmatrix}$$

(b) Recurrent & transient states:

1. State 1 & 2

→ $1 \leftrightarrow 2$ (they communicate with each other)

→ From state 1, you can return to state 1

→ From State 2, you can return to state 2 via State 1.

→ Thus, states 1 & 2 form a recurrent class.

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(2.) State 3 & 4

→ $3 \leftrightarrow 4$ (they can communicate)

→ from state 3, you can return to state 3.

→ from state 4, you can return to state 4.

→ Thus, states 3 & 4 are recurrent

(c) Stationary distribution

Stationary distribution satisfies : $\pi Q = \pi$ & $\sum \pi_i = 1$

(1.) For recurrent class {1,2}

let π_1 & π_2 be probabilities for state 1 & 2

eq^ns:

$$\pi_1 = 0.5\pi_1 + 0.25\pi_2 \quad \text{—(1)}$$

$$\pi_2 = 0.5\pi_1 + 0\pi_2 \Rightarrow \boxed{\pi_2 = 0.5\pi_1} \quad \text{—(2)}$$

By substitution from (2) in (1)

$$\pi_1 = 0.5\pi_1 + 0.25 \times 0.5\pi_1 \Rightarrow \pi_1 = 0.625\pi_1$$

⇒ $\pi_1 = 0$ [this isn't valid].

we note that: $\boxed{\pi_1 + \pi_2 = 1}$ $\boxed{\pi_2 = 0.5\pi_1}$

Solving the above 2 :→ $\pi_1 = 2/3$ $\pi_2 = 1/3$

thus, one stationary distribution is :

$$\boxed{\pi^{(1)} = \left(2/3, 1/3, 0, 0\right)}$$

Similarly, for recurrent class.

$$\pi_3 = 0.75\pi_3 + 0.25\pi_4$$

$$\pi_4 = 0.25\pi_3 + 0.75\pi_4$$

as $\pi_3 + \pi_4 = 1$ so $\pi_3 = \pi_4 = 0.5$

$$\boxed{\pi(2) = (0, 0, 0.5, 0.5)}$$

Q3. Cat chain:

Cat moves 2 rooms ~~(Room 1~~ & (Room 1 & Room 2)

It moves to other room with probability 0.8, & it stays in the same room with probability 0.2.

$\pi_c(1) =$ long-run probability, cat is in room 1

$\pi_c(2) = 1 - \pi_c(1)$

Transition matrix $P_c = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$

Stationary distribution $\pi_c = [\pi_1, \pi_2]$

$\pi_c P_c = \pi_2$, $\pi_2 + \pi_1 = 1$

From eq's:

$\pi_1 = 0.2\pi_1 + 0.8\pi_2$

$\Rightarrow \boxed{\pi_1 = \pi_2 = 0.5}$

$\pi_c = [0.5, 0.5]$

Mouse distribution

$\pi_M = [\pi_1, \pi_2]$

$P_M = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$

$\pi_1 = 0.7\pi_1 + 0.6\pi_2$
$\pi_1 + \pi_2 = 1$
$\Rightarrow \pi_M = \left[\frac{2}{3}, \frac{1}{3}\right]$

(b) 4 possible joint states: (1,1), (1,2), (2,1), (2,2)

Let Z_n be the joint state of (Cat room, mouse room)

$\Rightarrow Z_n$ is a Markov chain with 4 states, as transition to next state depends on previous one.

Q5 (a) Is the stock price recurrent?

We model this as a markov chain with discrete steps

$120 + 0.01 \times k \quad , k \in \mathbb{Z}$

(i) 0.10 probability of moving up by 0.01

(ii) 0.85 probability of staying at current price

(iii) 0.05 probability of moving down by 0.01

This is a random walk with a bias upward because the upward movement probability (0.10) is greater than downward movement probability (0.05). Since the walk is on an infinite countable space with ~~ass~~ asymmetric transition probabilities.

The chain isn't symmetric ; positive drift

Ans: Stock price isn't recurrent. It is transient.

(b) A stationary distribution for a Markov chain exists only if the chain is positive recurrent.

Since, chain is "transient" the ~~state~~ stock tends to drift upward indefinitely.

∴ No, a stationary distribution doesn't exists.

(c) American Option simulation

Stock touches Rs130 (Strike price = 125, profit = 5) before 1:00 pm

(10:00am to 1:00pm)
Time = 3hrs = 10800 seconds

Step size = 5 seconds ⇒ No of steps = 10800/5 = 2160

Starting price = ₹120
Target Price = ₹130

Each step Up with probability = 0.1 (₹0.01)
Same with probability = 0.85
Down with probability = 0.05