(2)

(a) Say, W = team wins a game
L = team loses a game

the transitions,

$P(W \to W) = 0.8$
$P(W \to L) = 0.2$
$P(L \to W) = 0.3$
$P(L \to L) = 0.7$

So, the transition matrix P is,

$$P = \begin{pmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{pmatrix}$$

we find stationary distribution $\pi = (\pi_W, \pi_L)$ satisfying,

$$\pi = \pi P \quad , \quad \pi_W + \pi_L = 1$$

eqs,
$$\pi_W = 0.8\pi_W + 0.3\pi_L$$
$$\pi_L = 0.2\pi_W + 0.7\pi_L$$

now substitute $\to$ $\pi_L = 1 - \pi_W$:

$$\pi_W = 0.8\pi_W + 0.3(1 - \pi_W)$$
$\hookrightarrow \pi_W = \underline{0.6}$; $\pi_L = \underline{0.4}$ (long run proportion of wins π_W)

(b) the dinner probabilities -
after a win - 0.7
after a loss - 0.2

the expected proportion of dinners:
$= \pi_W \cdot 0.7 + \pi_L \cdot 0.2$
$= (0.6)(0.7) + (0.4)(0.2) = \underline{0.5}$.

(long-run proportion of games with dinner)

(c) The expected waiting time until a dinner is the inverse of the probability of dinner in a single game.

expected no. of games ~~&~~ to dinner $= 1/0.5 = 2$

(4)

Ans. The stationary probability distribution is vector π such that $\pi P = \pi$, where P is the transition matrix. The probability of being in any given state does not change over time.

Classifying the square -

- chessboard can be classified into 'types' based on their position which determine how many neighbour (legal moves) the king as -

- corner sq. (Type C) : 4 corners. each 3 neighbour
  degree = 3.
- edge sq. (Type E) : 24 sq. on edge but not corner. each has 5 neighbour.
  degree = 5
- inner sq. (Type I) : 36 sq. not on edges or corners. each 8 neighbour.
  degree = 8

$\Rightarrow$ Z (normalization const.) $= 4 \times 3 + 24 \times 5 + 36 \times 8 = 420$.

- Type C sq. $= 3/420 = 1/140$
- Type E sq. $= 1/84$
- Type I $= 2/105$

(Q6) Transition probability & stationary distribution.

(a) for any 2 permutations g & h, the transition probability $q(g, h)$ is.

$$q(g, h) = \begin{cases} \frac{2}{26 \cdot 25} & \text{if } h \text{ is obtained from } g \text{ by swapping 2 letters.} \\ 0 & \text{otherwise} \end{cases}$$

because their are $\binom{26}{2} = 325$ possible swaps, & the probability of picking any pair (i, j) with $i \neq j$ is $2/26 \cdot 25$

the distribution is <u>uniform</u> over all permutations, as the chain is symmetric & all states are equally likely in the long run.

(b) The proposal probability is $q(g, h) = 2/26 \cdot 25$ if h can be reached by swapping two letters in g & zero otherwise.

- the acceptance probability is

$$A(g \to h) = \begin{cases} 1 & \text{if } S(h) \geq S(g) \\ \frac{S(h)}{S(g)} & \text{if } S(h) < S(g) \end{cases}$$

transition probability is:

~~$A(g \to h) =$~~ $P(g, h) = q(g, h) \cdot A(g \to h)$

for reversibility,

$$S(g)\, q(g, h)\, A(g \to h) = S(h)\, q(h, g)\, A(h \to g)$$

& we have -

$$S(g)\, q(g, h)\, A(g \to h) = S(h)\, q(h, g)\, A(h \to g)$$

Thus, the chain is reversible with respect to the distribution $\pi(g) \propto S(g)$. So the stationary distribution is proportional to $S(g)$