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classmate

Date

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Q $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ find the eigen values and corresponding eigen vectors of the above matrix

Sol Characteristic equation is represented as.

$|A - \lambda I| = 0$

$$\begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix} = 0$$

$\Rightarrow \lambda^3 - \begin{pmatrix}\text{Sum of} \\ \text{Diagonal element}\end{pmatrix}\lambda^2 + \begin{pmatrix}\text{Sum of} \\ \text{Diagonal minor}\end{pmatrix}\lambda - |A| = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 + \begin{pmatrix}\text{Sum of Diagonal} \\ \text{minor}\end{pmatrix}\lambda - |A| = 0$

$|A| = 8[(-3)(1) - (-2)(-4)] - (-8)[4 - (-2)(3)]$
$+ (-2)[(4)(-4) - (-3)(3)]$

$\Rightarrow 8[-3-8] + 8[4+6] - 2[(-16) - (-9)]$

$\Rightarrow 8(-11) + 8(10) + 14 = 6$

$\Rightarrow \therefore |A| = 6$

Sum of Diagonals minor

$M_1 = \begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix}$ $M_2 = \begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix}$

$M_3 = \begin{vmatrix} 8 & -8 \\ 4 & -3 \end{vmatrix}$

$M_1 = -11$ $M_3 = 8$

$M_2 = 14$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\boxed{\lambda = 1, 2, 3}$$

for $\lambda = 1$ Eigen vectors

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$7x - 8y - 2z = 0$$
$$4x - 4y - 2z = 0$$

Applying Cramers rule

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$

$$\frac{x}{8} = \frac{+y}{+6} = \frac{z}{4}$$

$$v_1 = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

For $\lambda = 2$, the eigen vector

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$6x - 8y - 2z = 0$$
$$4x - 5y - 2z = 0$$

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -5 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 6 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 6 & -8 \\ 4 & -5 \end{vmatrix}}$$ (Applying Cramers rule)

$$\frac{x}{6} = \frac{+y}{+4} = \frac{z}{2}$$

$$v_2 = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

For $\lambda = 3$, the eigen vector

$$\begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$5x - 8y - 2z = 0$$
$$4x - 6y - 2z = 0$$

Applying Cramers rule

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -6 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 5 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 5 & -8 \\ 4 & -6 \end{vmatrix}}$$

$$\frac{x}{4} = \frac{+y}{+2} = \frac{z}{2}$$

$$v_3 = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$