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AP24122040004

1. Compute the PCA for the following dataset.

$X = (X_1, X_2) = \{(1,2), (3,3), (3,5), (5,4), (5,6), (6,5), (8,7), (9,8)\}$

Sol

		X	Y	
X_1	X_2	$X_1 - \bar{X}_1$	$X_2 - \bar{X}_2$	$X \cdot Y$
1	2	-4	-3	12
3	3	-2	-2	4
3	5	-2	0	0
5	4	0	-1	0
5	6	0	1	0
6	5	1	0	0
8	7	3	2	6
9	8	4	3	12

$\bar{X}_1 = \frac{40}{8} \qquad \bar{X}_2 = \frac{40}{8}$

$\bar{X}_1 = 5 \qquad \bar{X}_2 = 5$

$Cov(X \cdot Y) = \frac{\Sigma (X \cdot Y)}{n-1} = \frac{34}{7} = 4.857$

$Cov(Y \cdot X) = \frac{\Sigma (Y \cdot X)}{n-1} = \frac{34}{7} = 4.857$

$Cov(X \cdot X) = \frac{\Sigma (X \cdot X)}{n-1} = \frac{50}{7} = 7.142$

$Cov(Y \cdot Y) = \frac{\Sigma (Y \cdot Y)}{n-1} = \frac{28}{7} = 4$

$$\text{Cov matrix (S)} = \begin{bmatrix} 7.142 & 4.857 \\ 4.857 & 4 \end{bmatrix}$$

Eigen values of Cov matrix (S)

$$= \begin{bmatrix} 7.14 - \lambda & 4.85 \\ 4.85 & 4 - \lambda \end{bmatrix}$$

$$(7.14-\lambda)(4-\lambda) - (4.85)^2$$

$$\boxed{\begin{aligned}\lambda_1 &= 10.67\\ \lambda_2 &= 0.46\end{aligned}}$$

For eigen vector calculation

$$(Cov - \lambda I)\, U_k = 0$$

for $\lambda_1 = 10.67$.

$$\begin{bmatrix} -3.52 & 4.85 \\ 4.85 & -6.66 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = 0$$

$$-3.52\,U_1 + 4.85\,U_2 = 0 \quad \text{— (i)}$$
$$4.85\,U_1 - 6.66\,U_2 = 0 \quad \text{— (ii)}$$

$$\Rightarrow 3.52\,U_1 = 4.85\,U_2$$

$$\Rightarrow \frac{U_1}{4.85} = \frac{3.52\,U_2}{3.52} = t$$

$$U_1 = \begin{bmatrix} 4.85 \\ 3.52 \end{bmatrix} x$$

$$|U_1| = \sqrt{(4.85)^2 + (3.5)^2}$$

$$= \sqrt{23.52 + 12.39} = 5.99$$

$$|U_2| = \frac{4.85U_1 = 6.66U_2}{\frac{U_1}{6.66} = \frac{U_2}{4.85}}$$

$$|U_2| = \begin{bmatrix} 6.66 \\ 4.85 \end{bmatrix}$$

$$|U_2| = \sqrt{(6.66)^2 + (4.85)^2} = 8.238$$

$$C_1 = \begin{bmatrix} 5.992 \\ 8.238 \end{bmatrix}$$

with $\lambda_1 = 0.46$

$$\begin{bmatrix} 6.68 & 4.85 \\ 4.85 & 3.54 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0$$

$$= \quad 6.68\,u_1 + 4.85\,u_2 = 0 \quad \text{— (iii)}$$
$$4.85\,u_1 + 3.54\,u_2 = 0 \quad \text{— (iv)}$$

$$\Rightarrow \quad 6.68\,u_1 = -4.85\,u_2 \quad \text{(from eq iii)}$$

$$\Rightarrow \quad \frac{u_1}{-4.85} = \frac{u_2}{6.68} = t$$

$$v_1 = \begin{bmatrix} -4.85 \\ 6.68 \end{bmatrix}$$

$$\|v_1\| = \sqrt{(-4.85)^2 + (6.68)^2}$$
$$= \sqrt{23.52 + 44.62}$$
$$= \boxed{8.25}$$

$$4.85\,v_1 = -3.54\,v_2 \quad \text{(from iv)}$$

$$\frac{v_1}{-3.54} = \frac{v_2}{4.85}$$

$$v_2 = \begin{bmatrix} -3.54 \\ 4.85 \end{bmatrix}$$

$$\|v_2\| = \sqrt{(-3.54)^2 + (4.85)^2}$$
$$= \sqrt{12.53 + 23.52}$$
$$= \boxed{6.004}$$

$$C_2 = \begin{bmatrix} 8.25 \\ 6.004 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -3 \\ -2 & -2 \\ -2 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ 3 & 2 \\ 4 & 3 \end{bmatrix}_{8\times 2} \qquad \begin{bmatrix} 5.992 & 8.25 \\ 8.238 & 6.004 \end{bmatrix}_{2\times 2}$$

$$\begin{bmatrix} -48.682 & -51.012 \\ -28.46 & -28.508 \\ -11.984 & -16.5 \\ -8.238 & -6.004 \\ 8.238 & 6.004 \\ 5.992 & 8.25 \\ 34.452 & 36.758 \\ 48.682 & 51.012 \end{bmatrix}$$