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# ISMAT401

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# M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE – 560 054

## SEMESTER END EXAMINATIONS – MAY / JUNE 2013

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|---|---|---|---|
| Course & Branch | : B.E.- Information Science and Engg | Semester | : IV |
| Subject | : Engineering Mathematics-IV | Max. Marks | : 100 |
| Subject Code | : ISMAT401 | Duration | : 3 Hrs |

**Instructions to the Candidates:**

- Answer one full question from each unit.

### UNIT – I

1. a) Find the real root of the equation $3x = Cosx + 1$ by Newton-Raphson iterative method, correct to four decimal places. (06)

b) Find a parabola of the form $y=a+bx+cx^2$ which fits most closely with the observations and hence estimate $y$ at $x$ =7.5 (07)

| $y$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $x$ | 4.63 | 2.11 | 0.67 | 0.09 | 0.63 | 2.15 | 4.58 |

c) In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are eligible: Variance of $x$=9, Regression lines: $8x-10y+66=0, 40x-18y=214$. What were i) the means of $x$ and $y$ ii) The coefficient of correlation between $x$ and $y$ iii) The standard deviation $y$. (07)

2. a) Find a real root of the equation $x\log_{10}x=1.2$ by Regula- Falsi method, correct to four decimal places. (06)

b) At constant temperature, the pressure P and the volume V of a gas are connected by the relation $PV^{\gamma}=K$. Find the best fitting equation of this form to the following data and estimate V when P=4 (07)

| P(Kg.sq cm) | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
|---|---|---|---|---|---|---|
| V(c.c) | 1620 | 1000 | 750 | 620 | 520 | 460 |

c) If $z=ax+by$ and r is the correlation coefficient between $x$ and $y$, show that $\sigma_z^{\ 2}=a^2\sigma_x^{\ 2}+b^2\sigma_y^{\ 2}+2abr\sigma_x\sigma_y$ and hence show that $r=\dfrac{\sigma_x^{\ 2}+\sigma_y^{\ 2}-\sigma^2{}_{x-y}}{2\sigma_x\sigma_y}$. Also explain the significance of the formula when r = 0 and r = ±1. (07)

## UNIT – II

3. a) A and B throw alternatively with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chance of winning. (06)

b) State and prove Baye's theorem. (07)

c) The pdf of a random variable X is given by $f(X = x) = \begin{cases} x & ,0 \le x \le 1 \\ 2 - x & ,1 < x \le 2 \\ 0 & ,elsewhere \end{cases}$ (07)

Find (i) Cumulative distribution function F(X) and (ii) $P(X \ge 1.5)$

4. a) A problem in mechanics is given to three students A, B and C whose chances of solving it are 1/2, 1/3, 1/4 respectively. What is the probability that i) the problem will be solved ii) at least two will be solved iii) at most one will be solved. (06)

b) In a bolt factory there are four machine A,B,C,D manufacturing respectively 20%, 15%, 25%, 40% of the total production .Out of these 5%,4%,3%,2% are defective. If a bolt drawn at random was found defective. What is the probability that it was manufactured by A or D? (07)

c) The probability density function of a variate X is (07)

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P(X) | k | 3k | 5k | 7k | 9k | 11k | 13k |

Find (i) k (ii) $P(X < 4)$, $P(X \ge 5)$, $P(3 < X < 6)$ (iii) Mean and variance.

## UNIT – III

5. a) In a bombing action there is a 50% chance that any bomb will strike the target. Two direct hits are needed to destroy the target completely. How many bombs are required to be dropped to give a 99% chance or better chance of completely destroying the target? (06)

b) Define pdf of exponential distribution and hence find its mean and variance. (07)

c) In an examination 7% of students score less than 35 marks and 89% of students score less than 60. Find the mean and the standard deviation if the marks are normally distributed, given that A (1.22263) = 0.39 and A (1.4757) = 0.43 in the usual notation. (07)

6. a) A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probability of (i) no error during a micro second (ii) at least one error per micro second (iii) at most two errors per micro second. (06)

b) The daily sales of a certain brand of bicycles in a city in excess of 1000 pieces is distributed as the Gamma distribution with parameters $\alpha = 2$ and $\beta = 500$. The city has a daily stock of 1500 pieces of the brand. Find the probability that the stock is insufficient on a particular day. (07)

c) A fair coin is tossed three times. Let X denotes 0 or 1 according as a head or a tail occurs on the first toss. Let Y denote the number of heads which occur. Determine (i) the marginal distributions of X and Y and (ii) the joint distribution of X and Y (iii) the expected values of X, Y and XY. (07)

## UNIT – IV

7. a) A die was thrown 9000 times and a throw of 5 or 6 was observed 3240 times. On the assumption of random throwing, do the data indicate an unbiased die? (06)

b) A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure.5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6. Can it be concluded that the stimulus will increase the blood pressure? (07)

c) A set of five similar coins is tossed 320 times and the result is (07)

| No.of heads | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 6 | 27 | 72 | 112 | 71 | 32 |

Test the hypothesis that the data follow a binomial distribution.

8. a) Define (i) Type-I and Type – II errors. (ii) Testing hypothesis. (iii) Power of the test (iv) Level of significance. (06)

b) A sample of 12 measurements of the diameter of a metal ball gave the mean $\overline{X}$ = 7.38mm With S.D. S = 1.24mm. Find a) 95% and b) 99% confidence limits for the actual diameter. (07)

c) In one sample of 8 observations the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in another sample of 10 observations, it was 102.6. Test whether the difference in variance is significant at 5% level using F-test. (07)

## UNIT – V

9. a) Two boys $B_1$ and $B_2$ and two girls $G_1$ and $G_2$ are throwing a ball from one to the other. Each boy throws the ball to the other with probability ½ and to each girl with probability ¼. On the other hand, each girl throws the ball to each boy with probability ½ and never to the girl. In the long run, how often does each receive the ball? (06)

b) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probabilities that (i) A has the ball, (ii) B has the ball and (iii) C has the ball. (07)

c) Customers arrive in a telephone booth at intervals of 10 minutes on the average. The length of a phone call is 3 minutes on the average. (i) What is the probability that a person arriving at the booth will have to wait? (ii) What is the average length of the queue that forms from time to time? (iii)The owner of the booth will install a second booth when convinced that an arrival would expect to wait at least three minutes for the phone. By how much must the flow of arrivals be increased in order to justify a second booth? (07)

10. a) In a twenty four hour service station, vehicles arrive at the rate of 30 per day on the average. The average servicing time for a vehicle is 36 minutes. Find (i) the mean queue size (ii) the probability that queue size exceeds nine. (06)

b) There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process a marble is selected from each urn and the two marbles selected are interchanged. Let the state $a_i$ of the system be the number i of red marbles in urn A. (i) Find the transition matrix P (ii) What is the probability that there are 2 red marbles in urn A after 3 steps? (07)

c) A person repairing radios finds that the time spent on the radio sets has been exponential distribution with mean 20 minutes. If the radios are repaired in the order in which they come in and their arrival is approximately Poisson with an average rate of 15 for 8 hour day. What is the repairman's expected idle time each day? How many jobs are ahead if the average set just brought in? (07)

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