Linear Algebra

Q.36

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Question 31

Find the equation of the tangent to the circle at the point $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ whose centre is point of intersection of straight lines $\begin{bmatrix} 2 & 1 \end{bmatrix} x = 3$ and $\begin{bmatrix} 1 & -1 \end{bmatrix} x = 1$

Solution

- Let A=$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and B=$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- Let O be the solution of Ax=3 and Bx=1 This can be written as $\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} x = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$
- Therefore, $O = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$
- $O = \begin{bmatrix} \frac{4}{3} \\ \frac{1}{3} \end{bmatrix}$

Solution

- Given a point P on circle as P=$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and we have O=$\begin{bmatrix} \frac{4}{3} \\ \frac{1}{3} \end{bmatrix}$
- Let us define matrix T=$\begin{bmatrix} O & P \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & 1 \\ \frac{1}{3} & -1 \end{bmatrix}$
- The direction vector OP is given by D $=$ P-O $=$ [T]$\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & 1 \\ \frac{1}{3} & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{4}{3} \end{bmatrix}$ which is a radial vector.

Solution

- The direction vector for tangent line will be the normal vector to radial vector. Let normal vector = N
- By definition, $N^T D = 0$
- Therefore, $\mathrm{N} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} [\mathrm{D}] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} \\ -\frac{4}{3} \end{bmatrix} = \begin{bmatrix} -\frac{4}{3} \\ \frac{1}{3} \end{bmatrix}$
- Tangent line : x = P + (t)N
- $\mathrm{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + (t) \begin{bmatrix} -\frac{4}{3} \\ \frac{1}{3} \end{bmatrix}$

Figure