Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. An amplifier having a low-frequency gain of 1000 and poles at 10^4 Hz and 10^5 Hz is operated in a closed negative-feedback loop with a frequency-independent β .
(a) For what value of H do the closed-loop poles become coincident? At what frequency?
(b) What is the low-frequency gain corresponding to the situation in (a)? What is the value of the closed-loop gain at the frequency of the coincident poles?
(c) What is the value of Q corresponding to the situation in(a)?
(d) If H is increased by a factor of 10, what are the new pole locations? What is the corresponding pole Q?
Solution: Given, low frequency Gain and two poles.Therefore,G(s) can be written as..,

$$G(s) = \frac{G_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)} \tag{1.0.1.1}$$

where,-p1 and -p2 are poles of G(s). Parameters given are shown in Table.1.0.1:1

$$G_0 = 1000 \tag{1.0.1.2}$$

$$\text{Therefore.,}G(s) = \frac{1000}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)} \tag{1.0.1.3}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

Parameter	Value
Low Frequency Gain	1000
Pole 1 (p1)	10^4 Hz
Pole 2 (p2)	10^5 Hz

TABLE 1.0.1: 1

Where p_1 and p_2 are

$$p_1 = 2\pi 10^4 rad/sec \tag{1.0.1.4}$$

$$p_2 = 2\pi 10^5 rad/sec \tag{1.0.1.5}$$

Now, we connect the system in a negative feedback of feedback factor H. We know that the closed loop gain of a negative feedback system is.,

$$T(s) = \frac{G(s)}{1+G(s)H} \tag{1.0.1.6}$$

1.0.2. For what value of H do the closed-loop poles become coincident? At what frequency?
Solution: Given the negative feedback system, the closed loop transfer function is:

$$T(s) = \frac{\frac{G_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}}{1+\frac{HG_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}} \tag{1.0.2.1}$$

$$\tag{1.0.2.2}$$

$$T(s) = \frac{G_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)+HG_0} \tag{1.0.2.3}$$

$$T(s) = \frac{G_0}{\frac{s^2}{p_1 p_2}+\left(\frac{1}{p_1}+\frac{1}{p_2}\right)s+HG_0} \tag{1.0.2.4}$$

$$\tag{1.0.2.5}$$

For this equation to have equal and repetitive roots, D=0

$$b^2 - 4ac = 0 \tag{1.0.2.6}$$

$$\left(\frac{1}{p_1} + \frac{1}{p_2}\right)^2 = \frac{4}{p_1 p_2}\left(G_0 H + 1\right) \tag{1.0.2.7}$$

$$H = \frac{(p_1 - p_2)^2}{4G_0 p_1 p_2} \tag{1.0.2.8}$$

Value of H on plugging in values:

$$H = 0.002025 \tag{1.0.2.9}$$

Using this our final closed loop transfer function is:

$$T(s) = \frac{1000}{2.533x10^{-11}s^2 + 1.750x10^{-5}s + 3.025} \tag{1.0.2.10}$$

Verify the poles of above transfer function using python code.

```
codes/ee18btech11036/ee18btech11036_1.py
```

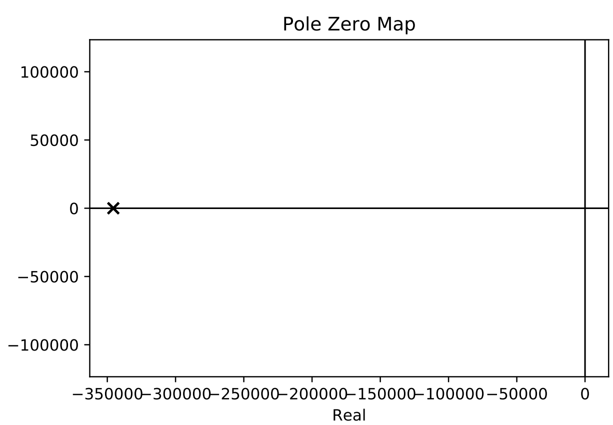

Fig. 1.0.2: 1

From the above graph is is evident that the system has two coinciding poles at frequency 345575.1 rad/sec

1.0.3. What is the low-frequency gain corresponding to the situation in (a)? What is the valueof the closed-loop gain at the frequency of the coincident poles?
Solution: Putting s=0 in equation 1.0.2.10

$$Gain = \frac{1000}{3.025} \implies 330.578/50.378(dB) \tag{1.0.3.1}$$

$$\tag{1.0.3.2}$$

Plot the transfer function above using the given code

```
codes/ee18btech11036/ee18btech11036_2.py
```

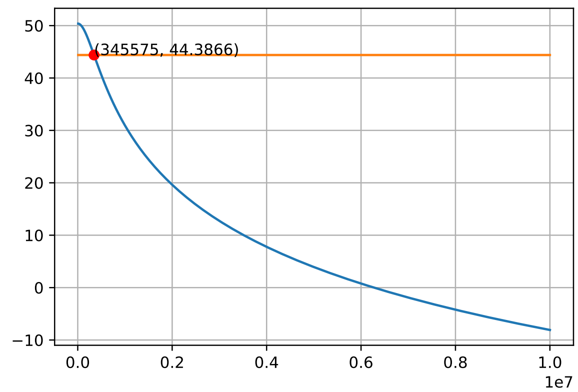

Fig. 1.0.3: 1

From the plot , the gain (dB) at pole frequency is 44.3866

1.0.4. What is the value of Q corresponding to the situation in(a)?
Solution:
In general, For a second order amplifier the C.E is.,

$$C.E = s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{1.0.4.1}$$

The quality factor Q of equation.1.0.4.1, is given by

$$Q = \frac{1}{2\zeta} \tag{1.0.4.2}$$

From equation.1.0.2.5 and 1.0.1.5

$$\omega_n = \pm\sqrt{(HG_0 + 1)\,p_1 p_2} \tag{1.0.4.3}$$

$$\zeta = \pm\frac{2p_1 p_2}{\sqrt{(HG_0 + 1)\,p_1 p_2}} \tag{1.0.4.4}$$

Therefore,For the given second order amplifier with characteristic equation.1.0.2.6,the Q factor is,

$$Q = \pm\frac{\sqrt{(1 + HG_0)\,p_1 p_2}}{p_1 + p_2} \tag{1.0.4.5}$$

From equations 1.0.2.9 , 1.0.1.5

$$Q = 0.5 \tag{1.0.4.6}$$

$$\tag{1.0.4.7}$$

Parameter	Value
H	0.002025
Gain (0 freq)	50.378
Gain (pole freq)	44.38
Q	0.5

TABLE 1.0.4: 1

1.0.5. If H is increased by a factor of 10, what are the new pole locations? What is the corresponding pole Q?
Solution: H is increased by factor of 10, hence

$$H = 0.02025 \tag{1.0.5.1}$$

Using 1.0.2.5 we get :

$$T(s) = \frac{1000}{2.533x10^{-11}s^2 + 1.750x10^{-5}s + 21.25} \tag{1.0.5.2}$$

Find the poles of the given Tf using the following code:

```
codes/ee18btech11036/ee18btech11036_3.py
```

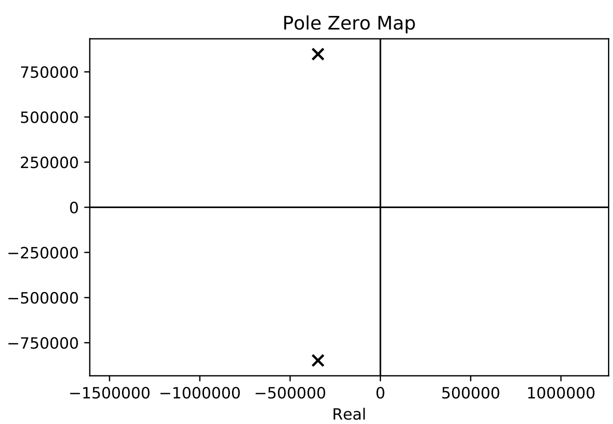

Fig. 1.0.5: 1

Poles are :
From equation 1.0.4.5

$$Q = \pm\frac{\sqrt{(1 + 10HG_0)\,p_1p_2}}{p_1 + p_2} \tag{1.0.5.3}$$

Solving this we get :

$$Q = 1.325 \tag{1.0.5.4}$$

1.0.6. Find the step response of $T(s)$
Solution: The following code generates the desired response of in Fig. 1.0.3.

```
codes/ee18btech11036/ee18btech11036_4.py
```

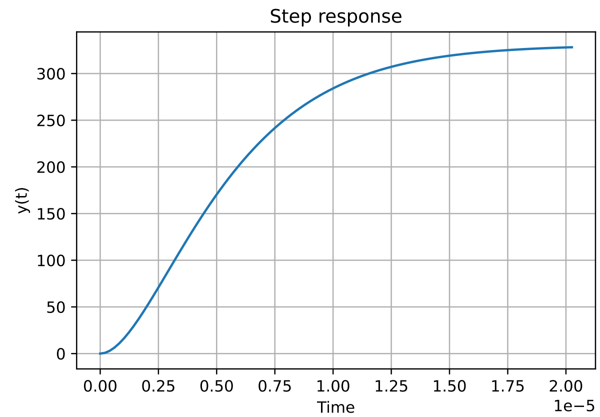

Fig. 1.0.6

1.0.7. Design a circuit that represents the transfer function 1.0.2.10
Solution: The circuit can be designed using an operational amplifiers having negative feedback.Consider the circuit shown in figure.1.0.7:1.Assume the gain of all the amplifiers are large.And assume no zero state response.Take the parameters in s-domain.
For the first amplifier.., Applying KCL at

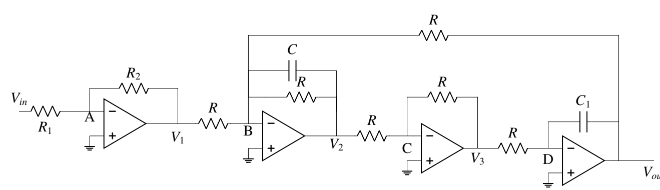

Fig. 1.0.7: 1

node A., Since, the opamp has large gain, potential at node A is assumed to be zero due to virtual short at node A.

$$\frac{0 - V_{in}(s)}{R_1} + \frac{0 - V_1(s)}{R_2} = 0 \tag{1.0.7.1}$$

$$\frac{V_{in}(s)}{R_1} = \frac{V_1(s)}{R_2} \tag{1.0.7.2}$$

$$\implies V_{in} = -\frac{V_1(s)R_1}{R_2} \tag{1.0.7.3}$$

For the second amplifier.., Applying KCL at node B.., Similarly potential at node B is zero.

$$\frac{-V_1(s)}{R} + \frac{-V_2(s)}{R} - sCV_2(s) + \frac{-V_{out}(s)}{R} = 0 \tag{1.0.7.4}$$

$$\frac{-V_1(s)}{R} + \frac{-V_2(s)}{R} - sCV_2(s) = \frac{V_{out}(s)}{R} \tag{1.0.7.5}$$

$$\frac{-V_1(s)}{R} = V_2(s)\left[sC + \frac{1}{R}\right] + \frac{Vout(s)}{R} \tag{1.0.7.6}$$

For the third amplifier.., Potential at node C is zero(Due to high gain of amplifier).Applying KCL at node C.

$$\frac{-V_2(s)}{R} + \frac{-V_3(s)}{R} = 0 \tag{1.0.7.7}$$

$$\implies V_2(s) = -V_3(s) \tag{1.0.7.8}$$

For the Fourth amplifier., Potential at node D is zero.Applying KCL at node D.

$$\frac{-V_3(s)}{R} + sC_1(-V_{out}(s)) = 0 \tag{1.0.7.9}$$

$$V_3(s) = -sC_1RV_{out}(s) \tag{1.0.7.10}$$

From equation.1.0.7.10 and equation. 1.0.7.8..,

$$V_2(s) = sC_1RV_{out}(s) \tag{1.0.7.11}$$

Substituting the equation.1.0.7.6 and equation.1.0.7.11,

$$\frac{-V_1(s)}{R} = \left(s^2C_1CR + sC_1\right)V_{out}(s) + \frac{V_{out}(s)}{R} \tag{1.0.7.12}$$

$$V_1(s) = -\left(s^2C_1CR^2 + sC_1R + 1\right)V_{out}(s) \tag{1.0.7.13}$$

from equation.1.0.7.3 and equation.1.0.7.13.

$$V_1(s) = \frac{R_1}{R_2}\left(s^2C_1CR^2 + sC_1R + 1\right)V_{out}(s) \tag{1.0.7.14}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1\left(s^2C_1CR^2 + sC_1R + 1\right)} \tag{1.0.7.15}$$

Comparing equation.1.0.2.10 and equation.1.0.7.15

$$R_2 = 33k\Omega \tag{1.0.7.16}$$

$$R_1 = 100\Omega \tag{1.0.7.17}$$

$$C_1CR^2 = 0.8373x10^{-11} \tag{1.0.7.18}$$

$$C_1R = 0.5785x10^{-5}F \tag{1.0.7.19}$$

$$\text{Let.,}C_1 = 4nF \tag{1.0.7.20}$$

$$\implies C = 1nF \tag{1.0.7.21}$$

$$\text{and.,}C_1R = 0.5785x10^{-5}F \tag{1.0.7.22}$$

$$\implies R = 1.446k\Omega \tag{1.0.7.23}$$

Parameter	Value
R_1	100 Ω
R_2	33 kΩ
R	1.446 kΩ
C	1 nF
C_1	4 nF

TABLE 1.0.7: 1

From Table.1.0.7:1. The Final circuit is shown in figure.1.0.7

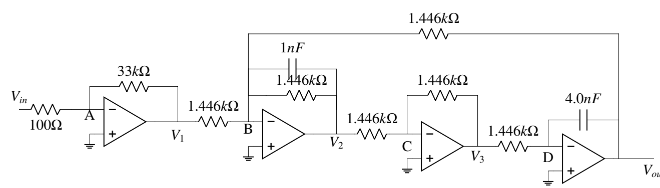

Fig. 1.0.7: 1

1.0.8. Verify the step response of the output from ngspice simulation.
Solution: The following netlist file does the transient anaylsis and store the Vout values with respect to time in a dat file.

```
codes/ee18btech11036/spice/ngspice_db.net
```

Following python code is to plot the step response.

```
codes/ee18btech11036/spice/
    ee18btech11036_spice.py
```

The step response obtained is shown in the figure.1.0.8.The graph has steady state value equal to 50.3 dB.

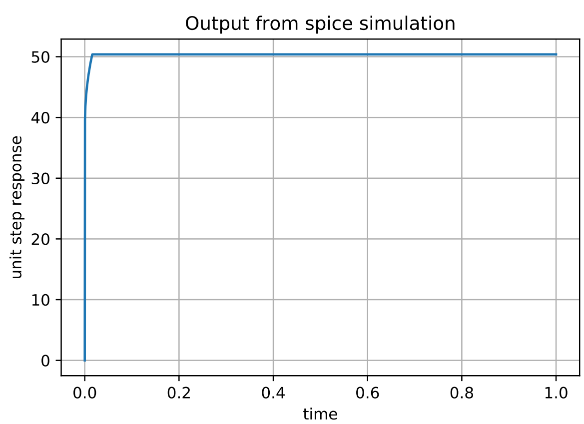
Output from spice simulation
unit step response
time
0
10
20
30
40
50
0.0
0.2
0.4
0.6
0.8
1.0

Fig. 1.0.8