Feedback and Poles

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An amplifier having a low-frequency gain of 1000 and poles at 10^4 Hz and 10^5 Hz is operated in a closed negative-feedback loop with a frequency-independent H.

a. For what value of H do the closed-loop poles become coincident? At what frequency?

b. What is the low-frequency gain corresponding to the situation in (a)? What is the value of the closed-loop gain at the frequency of the coincident poles?

c. What is the value of Q corresponding to the situation in(a)?

d. If H is increased by a factor of 10, what are the new pole locations? What is the corresponding pole Q?

1. Find $G(s)$
Solution: Given, low frequency Gain and two poles.Therefore,G(s) can be written as..,
$$G(s) = \frac{G_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)} \tag{1.1}$$
where the parameters are tabulated in Table 1

Parameter	Value
Low Frequency Gain	1000
Pole 1 (p1)	10^4 Hz
Pole 2 (p2)	10^5 Hz

TABLE 1

2. For what value of H do the closed-loop poles become coincident? At what frequency?

Solution: The closed loop transfer function is
$$T(s) = \frac{G(s)}{1+G(s)H} \tag{2.1}$$
$$T(s) = \frac{G_0}{\frac{s^2}{p_1p_2}+\left(\frac{1}{p_1}+\frac{1}{p_2}\right)s+HG_0+1} \tag{2.2}$$
upon substituting from (1.1) and simplifying. For (2.2) to have equal roots,
$$\left(\frac{1}{p_1}+\frac{1}{p_2}\right)^2 = \frac{4}{p_1p_2}\left(G_0H+1\right) \tag{2.3}$$
$$\implies H = \frac{\left(p_1-p_2\right)^2}{4G_0p_1p_2} \tag{2.4}$$
$$H = 2.025\times10^{-3} \tag{2.5}$$
yielding
$$T(s) = \frac{1000}{2.533\times10^{-11}s^2+1.750\times10^{-5}s+3.025} \tag{2.6}$$
The following code computes the poles to be at $\omega_p = 345575.1.$ rad/sec

```
codes/ee18btech11036/ee18btech11036_1.py
```

3. Find $T(0)$ and $|T(\jmath\omega_p)|$.
Solution: From 2.6,
$$T(0) = \frac{1000}{3.025} = 50.378\,dB \tag{3.1}$$
$$|T(\jmath\omega_p)| = 44.3866 \tag{3.2}$$

4. Find Q from (2.2).
Solution: If the denominator of (2.2) is expressed as
$$\frac{s^2}{p_1p_2}+\left(\frac{1}{p_1}+\frac{1}{p_2}\right)s+HG_0+1 = s^2+2\zeta\omega_ns+\omega_n^2 \tag{4.1}$$

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then

$$\omega_n = \sqrt{(HG_0 + 1)\, p_1 p_2} \tag{4.2}$$

$$\zeta = \frac{2p_1p_2}{\sqrt{(HG_0 + 1)\, p_1 p_2}} \tag{4.3}$$

and

$$Q = \frac{1}{2\zeta} \tag{4.4}$$

$$= \pm \frac{\sqrt{(1 + HG_0)\, p_1 p_2}}{p_1 + p_2} \tag{4.5}$$

$$= 0.5 \tag{4.6}$$

5. If H is increased by a factor of 10, what are the new pole locations? What is the corresponding pole Q?

Parameter	Value
H	0.002025
Gain (0 freq)	50.378
Gain (pole freq)	44.38
Q	0.5

TABLE 5

Solution: From (2.5),

$$H = 0.02025 \tag{5.1}$$

which, upon substitution in 2.2 yields

$$T(s) = \frac{1000}{2.533 \times 10^{-11} s^2 + 1.750 \times 10^{-5} s + 21.25} \tag{5.2}$$

The new pole locations are given by the following code

```
codes/ee18btech11036/ee18btech11036_3.py
```

and

$$Q = 1.325 \tag{5.3}$$

6. Find the step response of $T(s)$
Solution: The following code generates the desired response in Fig. 6.

```
codes/ee18btech11036/ee18btech11036_4.py
```

7. Design a circuit for $T(s)$ in (2.6).
Solution: A modular approach to make circuit is used here.The first op-amp corresponds to a simple voltage subtractor while the other two correspond to each of the poles p_1 and p_2. Consider the circuit in Fig. 7.2.

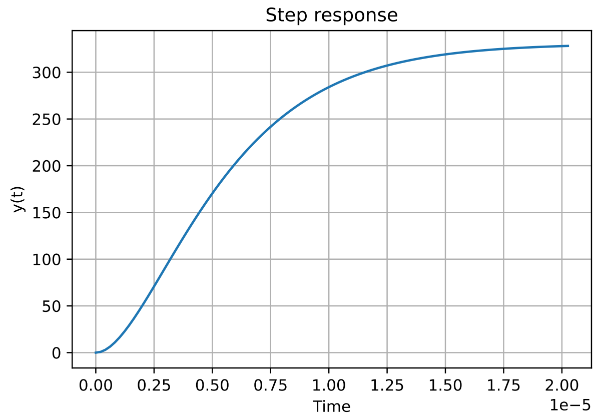

Fig. 6

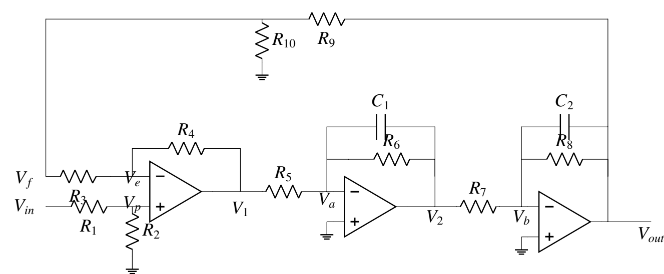

Fig. 7.2

$$T(s) = \frac{R_6R_8}{(R_5R_7\,(s^2C_1C_2R_6R_8 + s(C_1R_6 + C_2R_8) + 1)) + HR_6R_8} \tag{7.1}$$

Using parameters from 7 in (7.1), (2.6) is obtained.

Parameter	Value
R_1, R_2, R_3, R_4	$1k\Omega$
R_5, R_7	100Ω
R_6	$10k\Omega$
R_8	$1k\Omega$
R_9	4928Ω
R_{10}	10Ω
C_1, C_2	1.59 nF

TABLE 7

The value of negative feedback gain:
The circuit can be represented as block diagram as shown in Fig. 7.4

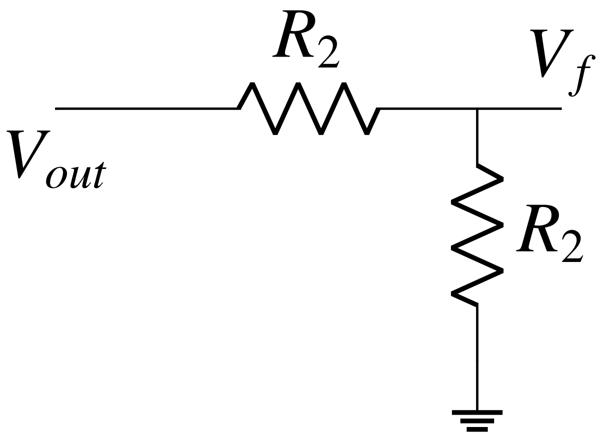

Fig. 7.3

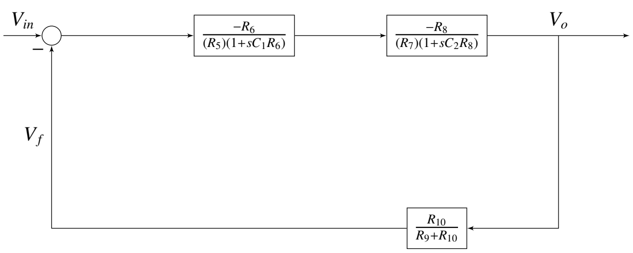

Fig. 7.4

The negative feedback gain :

$$I = \frac{V_{out}}{R_9 + R_{10}} \tag{7.2}$$

$$V_f = IR_{10} \tag{7.3}$$

$$\implies \frac{V_f}{V_{out}} = \frac{R_{10}}{R_{10} + R_9} \tag{7.4}$$

Solving this block diagram:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{7.5}$$

$$T(s) = \frac{\frac{R_6R_8}{(R_5R_7)(1+sC_1R_6)(1+sC_2R_8)}}{1 + \frac{R_6R_8}{(R_5R_7)(1+sC_1R_6)(sC_2R_8)}(H)} \tag{7.6}$$

$$\implies T(s) = \frac{R_6R_8}{R_5R_7\left(s^2C_1C_2R_6R_8 + s(C_1R_6 + C_2R_8) + 1\right) + HR_6R_8} \tag{7.7}$$

Transfer function was found to be same as given in equation 7.1

8. Verify the step response of the output from ngspice simulation.
Solution: The following netlist file does the transient anaylsis and store the Vout values with respect to time in a dat file.

```
codes/ee18btech11036/spice/ngspice_db.net
```

Following python code is to plot the step response.

```
codes/ee18btech11036/spice/
    ee18btech11036_spice.py
```

The step response obtained is shown in Fig. 8.The graph has steady state value equal to 50.3 dB/330 V.

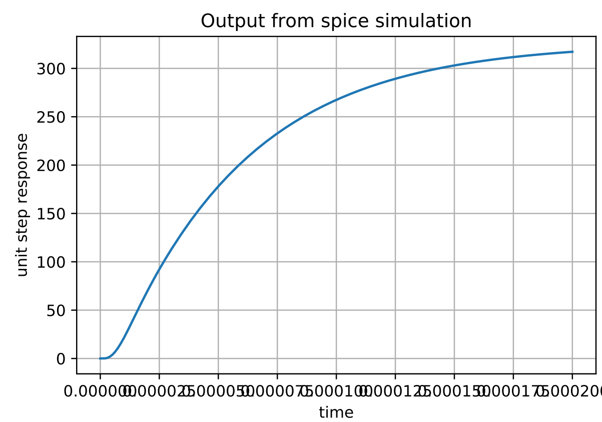

Fig. 8