Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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8.1 Stability

8.1. For a unity feedback system shown in Fig. 1

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)} \tag{8.1.1}$$

Design a lead compensator to yield a $K_v = 2$ and a phase margin of 30.

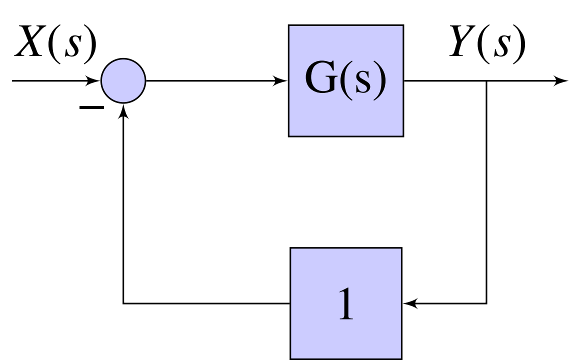

Fig. 8.1

Solution: For unity feedback we have Velocity error constant (K_v)

$$K_v = \lim_{s \to 0} sG(s) \tag{8.1.2}$$

$$\lim_{s \to 0} \left(\frac{K}{(2+s)(4+s)(6+s)} \right) = 2 \tag{8.1.3}$$

$$\implies K = 96 \tag{8.1.4}$$

Check the phase margin and gain crossover frequency by running the following code

```
codes/ee18btech11036_1.py
```

- The Phase margin: 19.76°
- Gain Crossover Frequency:1.469 rad/sec

The Bode plot of system is as shown,

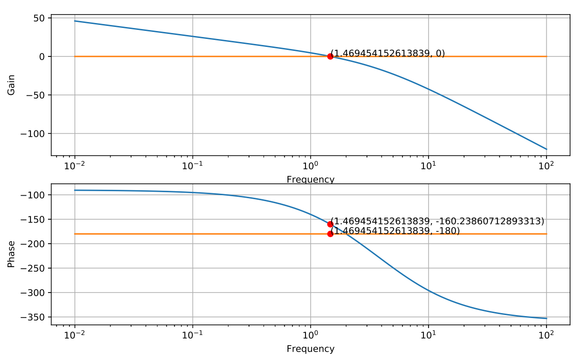

Fig. 8.1

Therefor amount of phase to be added: 30-19.76=10.24

8.2. The circuit of lead compensator is given by

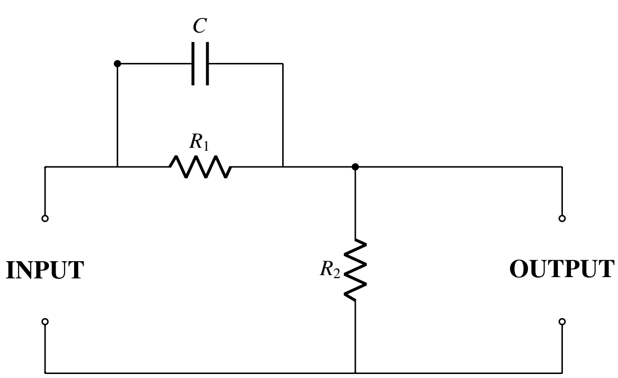

Fig. 8.2

Transfer function:

$$C(s) = \beta \left(\frac{1 + j\tau\omega}{1 + j\beta\tau\omega} \right) \tag{8.2.1}$$

$$\beta = \left(\frac{R_2}{R_1 + R_2} \right) \tag{8.2.2}$$

$$\tau = R_1 C \tag{8.2.3}$$

Find the values of β and τ

Solution: The maximum phase lead compensated by a lead compensator is given by

$$\phi = \sin^{-1} \frac{1 - \beta}{1 + \beta} \tag{8.2.4}$$

at

$$\omega = \frac{1}{\sqrt{\beta}\tau} \tag{8.2.5}$$

Now we know that from Gain crossover frequency

$$\omega = 1.469 rad/sec \tag{8.2.6}$$

and the phase margin to be added:

$$\phi = 10.24^\circ \tag{8.2.7}$$

But to compensate for the added magnitude of lead compensator, a correction factor of $10^\circ - 20^\circ$ is added.Hence

$$\phi = 30.24^\circ \implies \beta = 0.33 \tag{8.2.8}$$

From the bode plot ω is chosen at which gain of original system is

$$-20 \log \left(1/\sqrt{\beta} \right) = -4.81 \tag{8.2.9}$$

Find the plot using the following code

```
codes/ee18btech11036_4.py
```

From plot ω=2.009 rad/sec

Solving equations 8.2.4 and 8.2.5:

$$\tau = 0.828 \tag{8.2.10}$$

$$\beta = 0.33 \tag{8.2.11}$$

$$\tag{8.2.12}$$

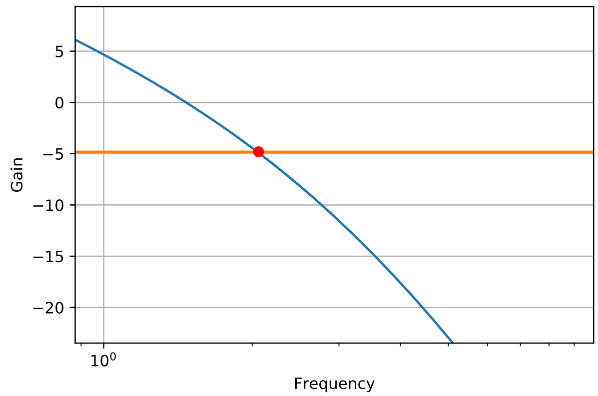

Fig. 8.2

New Transfer Function:

$$G(s) = \frac{96\,(1 + 0.828s)}{(s)\,(2 + s)\,(4 + s)\,(6 + s)\,(1 + 0.273s)} \tag{8.2.13}$$

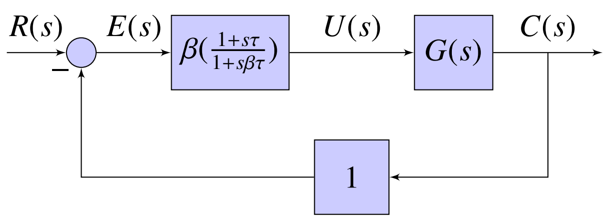

Fig. 8.2

8.3. Verify your results from the following code:

```
codes/ee18btech11036_2.py
```

- The Phase margin: 29.269°
- The Gain Crossover Frequency: 2.02 rad/sec

The Bode plot is as shown,

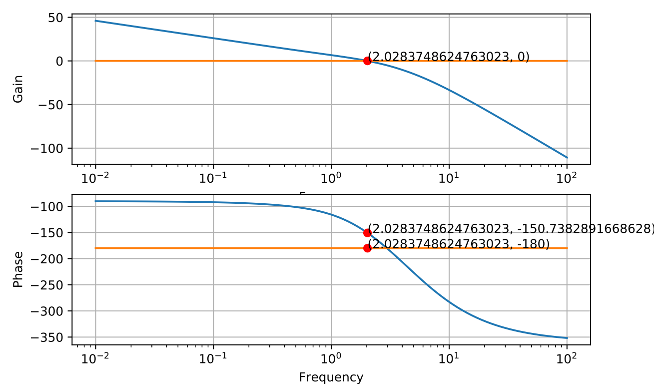

Fig. 8.3