

Gate Problems on Control Systems

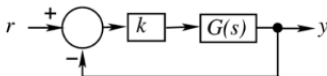
EC 2016 Q46

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Question

Q.46 In the feedback system shown below $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$.



The positive value of k for which the gain margin of the loop is exactly 0 dB and the phase margin of the loop is exactly zero degree is _____

The gain margin is defined as the change in open-loop gain required to make the closed-loop system unstable. Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed-loop.

The phase margin is defined as the change in open-loop phase shift required to make the closed-loop system unstable. The phase margin also measures the system's tolerance to time delay

Technique Description

As given in question we see that gain margin is 0 dB and phase margin is 0 degrees. This implies that system is just enough stable and will become destabilized on just small increase in gain. Hence the system is marginally stable.

The stability of the system can be checked by Routh-Hurwitz Stability Criterion

Routh-Hurwitz Stability Criterion

Necessary condition for Routh-Hurwitz Stability Criterion: The necessary condition is that the coefficients of the characteristic polynomial should be positive. This implies that all the roots of the characteristic equation should have negative real parts.

Sufficient condition for Routh-Hurwitz Stability Criterion: The sufficient condition is that all the elements of the first column of the Routh array should have the same sign. This means that all the elements of the first column of the Routh array should be either positive or negative.

The Routh array for characteristic equation

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n = 0$$

$$\begin{array}{c|cccc} s^n & a_0 & a_2 & a_4 & \cdots \\ s^{n-1} & a_1 & a_3 & a_5 & \cdots \\ s^{n-2} & b_1 & b_2 & b_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array}$$

$$\text{where } b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \quad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \quad c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \quad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

Solution

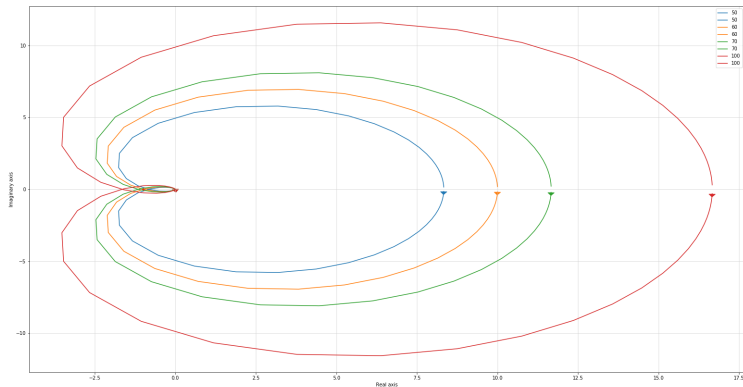
The Routh array for equation $s^3 + 6s^2 + 11s^1 + (6 + k)$

$$\begin{array}{ccc} s^3 & 1 & 11 \\ s^2 & 6 & (6+k) \\ s^1 & \frac{66-(6+K)}{6} & 0 \\ s^0 & (6+k) & 0 \end{array}$$

Now since the system is marginally stable therefore s^1 row is ≥ 0

Hence $\frac{66-(6+K)}{6} \geq 0$ Hence $k=60$

Verification using Plots



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