Gate Problems on Control Systems

EC 2016 Q46

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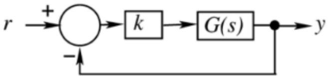
Question

Q.46 In the feedback system shown below $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$.



The positive value of k for which the gain margin of the loop is exactly 0 dB and the phase margin of the loop is exactly zero degree is ________

Theory

The gain margin is defined as the change in open-loop gain required to make the closed-loop system unstable. Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed-loop.

The phase margin is defined as the change in open-loop phase shift required to make the closed-loop system unstable. The phase margin also measures the system's tolerance to time delay

Technique Description

As given in question we see that gain margin is 0 dB and phase margin is 0 degrees. This implies that system is just enough stable and will become destabilized on just small increase in gain. Hence the system is marginbally stable.

The stability of the system can be checked by Routh-Hurwitz Stability Criterion

Routh-Hurwitz Stability Criterion

Necessary condition for Routh-Hurwitz Stability Criterion: The necessary condition is that the coefficients of the characteristic polynomial should be positive. This implies that all the roots of the characteristic equation should have negative real parts.
Sufficient condition for Routh-Hurwitz Stability Criterion:The sufficient condition is that all the elements of the first column of the Routh array should have the same sign. This means that all the elements of the first column of the Routh array should be either positive or negative.

The Routh array for characteristic equation
$a_0 s^n + a_1 s^(n-1) + a_2 s^(n-2) + a_3 s^(n-3)..... + a_n$

s^n	a_0	a_2
$s^(n-1)$	a_1	a_3
$s^(n-2)$	2	0
$s^(n-3)$	0	0

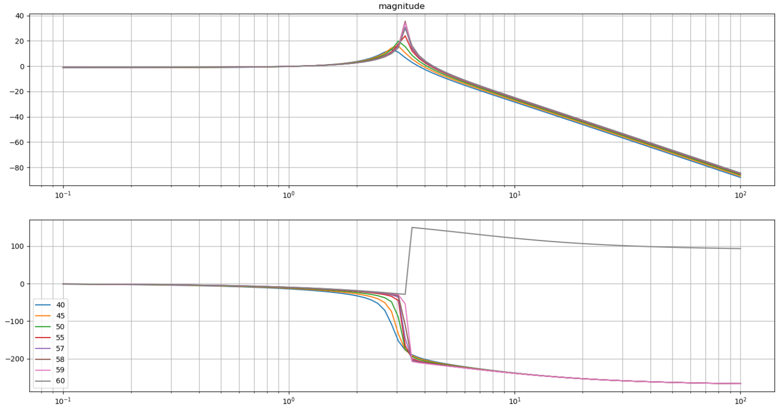
Solution

The Routh array for equation $s^3 + 6s^2 + 11s^1 + (6 + k)$

$$\begin{array}{lll} s^3 & 1 & 11 \\ s^2 & 6 & (6+k) \\ s^1 & \frac{66-(6+K)}{6} & 0 \\ s^0 & (6+k) & 0 \end{array}$$

Now since the system is marginally stable therefore s^1 row is ≥ 0
Hence $\frac{66-(6+K)}{6} \geq 0$ Hence k=60

Verification using Plots



Verification using Plots