Assignment 1

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Download all python codes from

https://github.com/piyushSTK/EE3025/tree/main/Assignment1/codes

and latex-tikz codes from

https://github.com/piyushSTK/EE3025/tree/main/Assignment1

1 Problem

(7.1) The command

```
output_signal = signal.lfilter(b, a, input_signal)
```

in Problem (2.3) is executed through the following difference equation

$$\sum_{m=0}^{M} a(m)\, y(n-m) = \sum_{k=0}^{N} b(k)\, x(n-k) \tag{1.0.1}$$

where the input signal is $x(n)$ and the output signal is $y(n)$ with initial values all 0. Replace **signal.filtfilt** with your own routine and verify.

2 Solution

This problem can be solved by using Z transfrom on the difference equation and then applying the properties of Z transform. Let take $X(z)$ and $Y(z)$ as the z-transforms of $x(n)$ and $y(n)$.
Using the time shifting property,

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{2.0.1}$$

$$\mathcal{Z}\{y(n-m)\} = z^{-m}Y(z) \tag{2.0.2}$$

We get:

$$Y(z)\sum_{m=0}^{M} a(m)\, z^{-m} = X(z)\sum_{k=0}^{N} b(k)\, z^{-k} \tag{2.0.3}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b(k)\, z^{-k}}{\sum_{m=0}^{M} a(m)\, z^{-m}} \tag{2.0.4}$$

Now to obtain DFT from Z transform, we evaluate (2.0.4) at $z = e^{j\omega}$ where $\omega = 2\pi k/N$
Hence $H(k)$:

$$H(k) = H\left(z = e^{j2\pi k/N}\right). \tag{2.0.5}$$

$X(k)$ is calculated from our input signal $x(n)$ using **np.fft.fft**.
Therefore $Y(k)$:

$$Y(k) = H(k)\, X(k) \tag{2.0.6}$$

Since this is in frequency domain, $y(n)$ is found from $Y(K)$ using **np.fft.ifft** command.
Below is the python code for this question. This code plots the output signals and returns the corresponding soundfiles.

codes/ee18btech11036.py

3 Verification

Plotting the time domain output signal $y(n)$ obtained using signal.filtfilt as well as constructed filter.

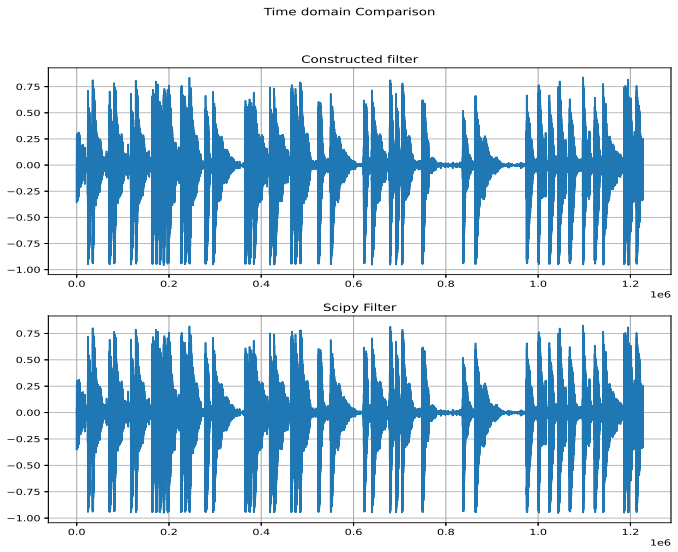

Fig. 0: Time domain response

Frequency Response of both, constructed filter, and signal.filtfilt for further verification

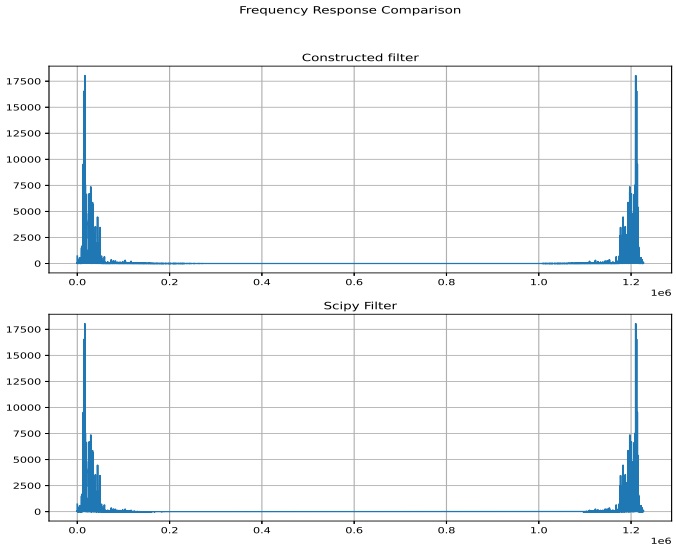

Fig. 0: Frequency domain response