

ASSIGNMENT-4

Date :

P. No. :

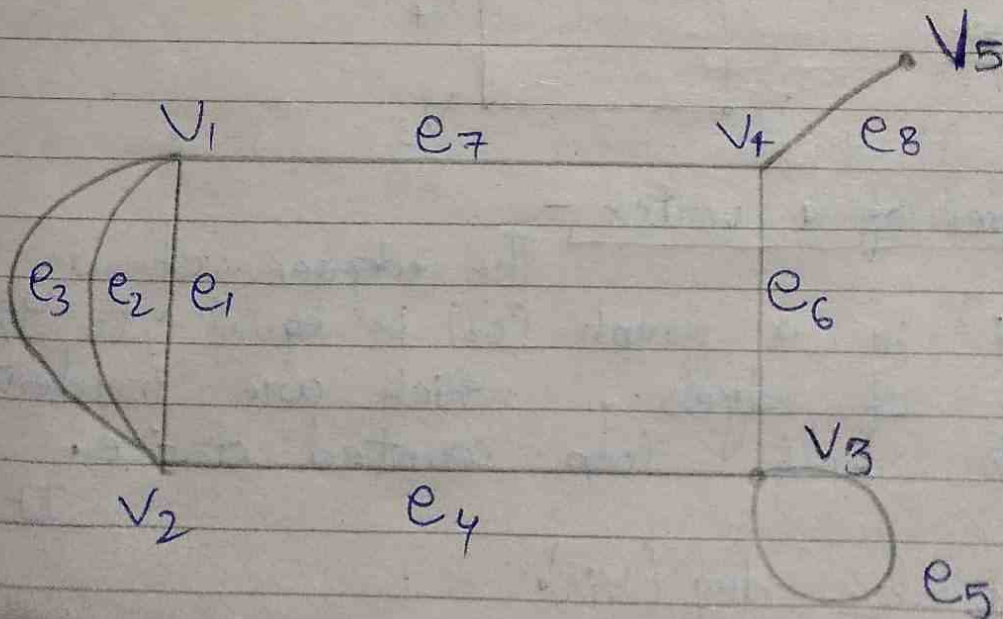
UNIT - 5

"BOOLEAN ALGEBRA & GRAPH THEORY"

IMP

★ Graph -

A graph $G = (V, E)$ consists of a set of object $V = \{V_1, V_2, V_3, \dots\}$ whose elements are called vertices and a another set $E = \{E_1, E_2, E_3, \dots\}$ whose elements are called edges and set of $\{V, E\}$ is called a graph.



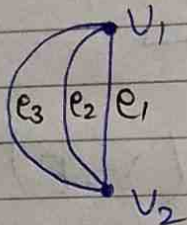
★ Self loop -

An edge is said to self loop if its both vertices are same.

★ Parallel graph -

If there are two (OR) more than two having same pair of vertices, then such edges are called parallel edges.

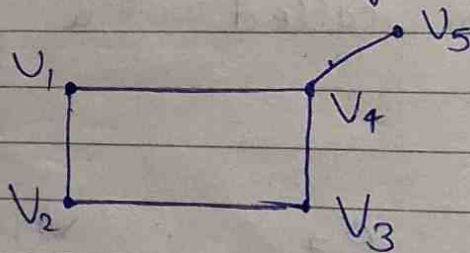
Ex. \Rightarrow



★ Simple Graph -

A graph that has neither self loop nor parallel edge are called simple edge.

Ex. \Rightarrow



★ Degree of a vertex -

The degree of a vertex $\deg(V_i)$ in a graph (G) is equal to the no. of edges, which are incident on V_i with self loop counted twice.

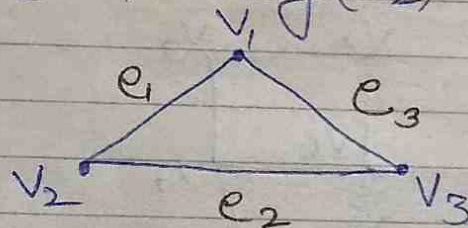
It is denoted by $\deg(V_i)$.

ex. $\Rightarrow \deg(v_1) = \deg(v_2) = \deg(v_3) = 4$ ★
 $\deg(v_4) = 3, \deg(v_5) = 1$

★ Regular Graph, -

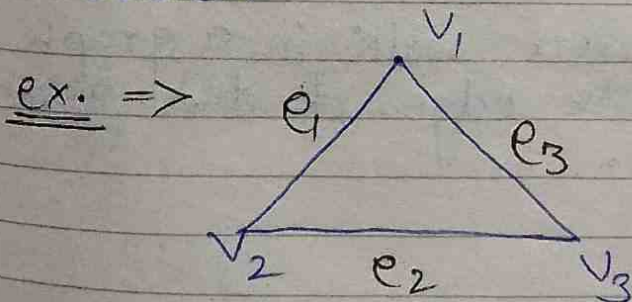
A graph (G) in which all vertices are of equal degree is called regular graph.

ex. $\Rightarrow \deg(v_1) = \deg(v_2) = \deg(v_3)$



★ Isolated Vertex, -

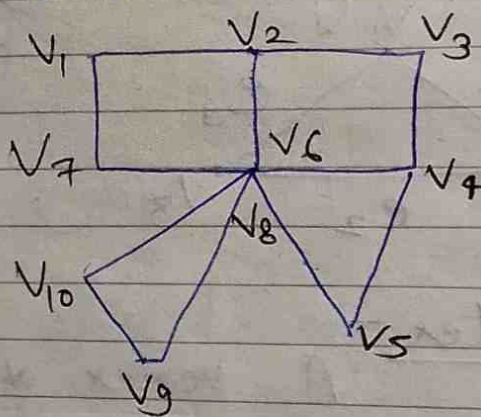
A vertex of degree zero is called isolated vertex, or an end vertex.



• $v_4 \rightarrow (v_4 \text{ is isolated vertex})$

★ Open Walk - When terminal vertices are different, then it is an open walk.
 $V_5 \quad e_1 \quad V_4 \quad e_5 \quad V_7$ is an open walk.

★ Connected Graph - A graph (G) is called connected, if there is at least one path b/w every pair of vertices. Otherwise it is disconnected.



★ Euler graph - A closed walk in a graph which includes all the edges of the graph is an Euler graph.

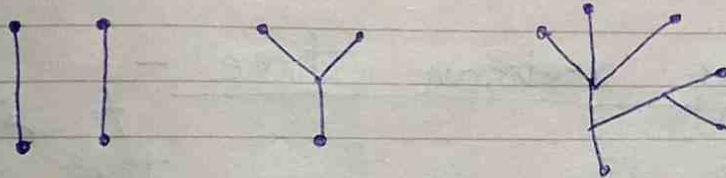
★ Euler ~~Tree~~ -

A closed walk in a graph which include all the edges of the graph in euler graph.

IMP

★ Tree -

A connected graph having no circuit is called tree.



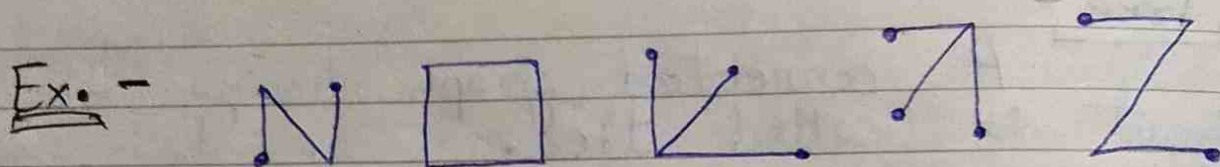
★ Decision Tree -

A decision tree are labelled rooted tree which occur in application specially in computer programming & computer algorithm.

The root represent a starting point later vertices represent later decision point, and one proceed downward to the tree.

④ Spanning Tree -

If $G = (V, E)$ is any connected graph, a spanning tree in G is a subgraph $T = (V, E')$ which is a tree.



⑤ Binary decision tree -

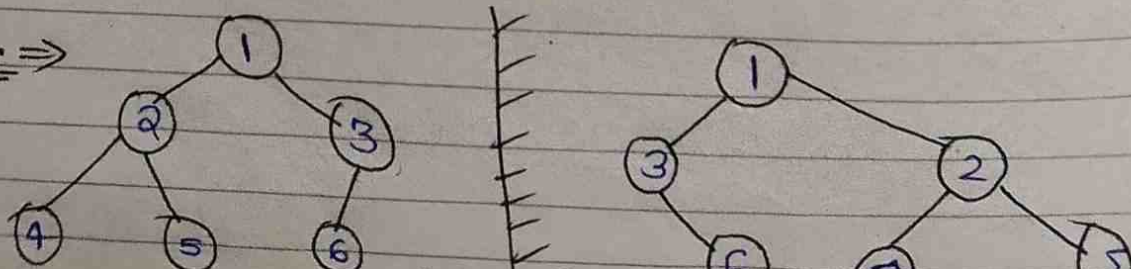
A student must pass two of three test to pass a course. The binary decision tree appears in adjoining figure with F denoting fail, P denoting pass.

M.T. ^{IMP}

⑥ Isomorphic Tree -

Two trees are called isomorphic if one of them can be obtained from other by a series of flips, i.e. by swapping left and right children of a no. of nodes. Two empty trees are isomorphic.

Ex. \Rightarrow



★ Theorem - 1 \Rightarrow

The sum of the degrees of all vertices in a graph is equal to twice the no. of edges.

★ Theorem - 2 \Rightarrow

The max. no. of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

★ Theorem - 3 \Rightarrow A graph (G) is disconnected if and only if its vertex set V can be partitioned into two non-empty, disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and other end vertex is in subset V_2 .

★ Theorem - 4 \Rightarrow

If a graph G (G may be connected or disconnected) has exactly two odd vertices, there must be path joining these two vertices.

★ Theorem - 5 \Rightarrow Let G be simple graph with n vertices. If G has k components, then the max. no. of edges that G can have are

$$\frac{(n-k)(n-k+1)}{2}.$$

• Theoram-6 -

The sum of the degrees of all vertices in a graph is equal to twice no. of edges (i.e even).

★ Walk -

A walk in graph G is defined as a finite alternating sequence of vertices & edges and to which begins to ends with vertices