NAME — PIYUSH AGRAWAL

BRANCH → IT

SUBJECT → PHYSICS

Quantum Mechanics

Date: 28/03/23
P. No:

"Unit-1"

★ Quatum Mechanics:- 14 Dec. 1900 quatum mechanics was discovered.

→ Black-Body Radiation → Nobel Prize

Classical Physics — Quantum Physics — Modern Physics

★ Quantum Mechanics:- It is the branch of physics: which explain about the motion of microscopic particles like electron, proton etc. It was introduced by Max planck in 1990.

★ Classical Mechanics:- It is the branch of physics which explains the motion of macroscopic objects. It was introduced by Issac Newton.

★ Application of Quantum ~~Mech.~~ Physics -

- Computer System Network
- Mob. phones, transistor, laser, microscope etc
- Digital Camera, LED.
- Telecommunications
- Nuclear power plants

Date: 29/03/23
P. No:

Date - 28/03/23
H.W → Definition photon.
Write a short note of Max planck.
Properties of photons.

★ **Photon** -

A particle representing a quantum of light and made up of electromagnetic waves. They have no mass and no charge. A photon carry energy proportional to the radiation frequency.

$$\boxed{E = h\nu}$$

▸ **Examples** - Radio waves, microwaves, infrared, UV-light etc.

★ **Properties of photon** -

(1) The energy of photon given as

$$\boxed{E = h\nu}$$

Here,
ν = frequency, h = planck's constant
E = Energy of photon

(2) We know that speed of light ($c = 3 \times 10^8$). Therefore the speed of photon is also equa to $c = 3 \times 10^8$ m/s.

Date:
P. No:

(3) The rest-mass of photon is zero.
(4) Photons are stable particles.
(5) They do not have any electric change.

★ Max-planck :-

Max-planck is a german physician who introduced to quantum physics. He got nobel prize in 1918. Max-planck was born on 28 april in 1858. He work on 2nd law of thermo dynamic.

After using planck theory he analysis results come different in every experiment then he give a new theory is called max-planck quantum theory, which mean he give relation b/w energy of photon & frequency of radiation.

$$\boxed{E = h\nu} \quad (1)$$

h = planck's constant = 6.62×10^{-34} Js

ν = frequency of radiation

We know that =>

$$\boxed{E = \frac{hc}{\lambda}} \quad (3) \qquad \nu = \frac{C}{\lambda} \quad (2)$$

C → Speed of light
λ → Wavelength of radiatio[n]

Date: 31/03/23
P. No:

★ Planck's Quantum hypothesis -

* In 1900, plank reported his discovery of a formula that accurately described the shape of a blackbody spectrum for all wavelength & temperature.

* When a blackbody is heated, it emits thermal radiations of different wavelength & frequency. To explain these radiations. Max planck put forward a theory known as planck quantum theory.

• According to planck's quantum theory -

(1) Substances radiate (or) absorb energy discontinuously in the form of small packets (or) bundles of energy.

(2) The smallest packet of energy is called quantum quanta. In case of light the quanta is known as photon.

(3) The energy of quanta is directly proportional to the frequency of quanta radiation

Energy of Photon $\propto$ frequency of radiation

P. No:

★ Mass & Momentum of a photon :-

- Rest mass (m_0) of photon is zero. If photon is in rest then it's mass is zero.

- When photon is in motion, then its mass is known as Kinetic mass ~~and~~ let it is denoted by (m).

Then according to einsteins mass energy eqn -

$$\boxed{E = mc^2} \quad \text{(4)}$$

From eq. (4) & (1), we get -

$$mc^2 = h\nu \Rightarrow \boxed{mc^2 = \frac{hc}{\lambda}} \quad \text{(5)}$$

Mass of photon is given by -

$$m = \frac{h\nu}{c^2} = \frac{h}{c^{\cancel{2}}} \cdot \frac{\cancel{c}}{\lambda} = \frac{h}{c\lambda} \Rightarrow \boxed{m = \frac{h}{c\lambda}} \quad \text{(6)}$$

We know that, momentum (P) = mass × Velocity

$$P = \frac{h}{\cancel{c}\lambda} \times \cancel{c}$$

$$\boxed{P = \frac{h}{\lambda}}$$

Date:
P. No:

★ Properties of photon :-

1. They have zero mass & rest energy.
2. They have no electric charge (neutral).
3. They only exist as a moving particles.
4. They are stable.
5. They can destroyed & Created by many natural process (like radiation, absorption, or immetiated or emission).
6. When in empty space. They travel at the speed of light (in vacuum).

★ Wave particle duality of radiation :-

1. Wave -
A wave nothing but spreading of disturbance in a medium. The characteristics / properties of wave are -

(i) Amplitude (ii) Time Period (iii) frequency (iv) Wave length (v) Phase (vi) Intensity.

2. Phase :- A particle is a point in space which has mass & occupies space or Region. The Charac. properties of a particles.

1. Mass 2. Velocity 3. Momentum 4. Energy.

Date:
P. No:

★ RADIATION (LIGHT)

Wave
(Vissible / Infrared / UV / X-rays)
(λ OR frequency)
↓ Shows
Interference / Diffraction
↓ When
(Two waves at the same position at the same time.)
↓ Conclusion
The radiations behaves like Wave

Particle
(Photon / Quanta)
$E = h\nu$ $\left(P = \frac{h\nu}{C}\right)$
Shows ↓
Blackbody Radiation, Photoelectric effect.
↓ When
(The Radient energy in its interaction with matter in the form of photon)
↓ Conclusion
The radiations behaves like particles

Waveparticle dulity

1924 Lewis de-Broglie

Date: 04/04/23
P. No:

★ De-Broglie Matter Waves -

In 1924, Lewis-de-Broglie suggested that matter-wave also Exhibit dual nature like Radiation (Light), they are →
(1) Wave Nature
(2) Particle Nature

- Wave nature of ~~matter~~ is Verified by Davison & German experiment, G.P. Thomson experiment etc.

- Particles nature of matters waves is verified by photo-electric effect, Compton effect etc.

★ Matterwave (or) De-Broglie - Waves -

The waves associated with a material particles are called as Matter wave.

De-Broglie Relation ⇒ $\boxed{\lambda = \frac{h}{p}}$

h → Planck's constant

p → Momentum

λ → Wavelength

Date:
P. No:

Difference b/w matter waves & Electro-magnetic waves.

Ans-)

Matter Wave (MW)	(E-M) waves
(1) MW is associated with moving particles (or) material particles.	(1) Oscillating charged particle gives rise to E-M Waves.
(2) Wavelength of MW. is given as $\boxed{\lambda = \frac{h}{mv}}$.	(2) Wavelength of an EM waves is given by $\boxed{\lambda = \frac{hC}{E}}$.
(3) Wavelength of MW depends upon mass of the particle & velocity.	(3) Wavelength of EM depends upon the energy of the photon.
(4) It can travel with a velocity > ~~velocity of light in vacuum.~~	(4) It can travel with a velocity = Speed of light in vacuum.
(5) It is not EM Wave.	(5) Electric field & magnetic field oscillate $\perp^{R}$ to each other and generate em wave.

P. No:

★ Heisenberg Uncertainity Principle & its applications

On the Basic of dual Nature of matter, in 1927, Heisenberg propounded a principle called "Heisenberg - Uncertainty Principle".

According to this principle "It is impossible to measure the exact position & momentum of a particle simultaneously with accuracy".

$$\boxed{\Delta x \cdot \Delta p \geq \frac{h}{4\pi} \text{ (or) } \frac{h}{2\pi}}$$

x = Position
P = Momentum

So, the product of uncertainties in measuring the position & momentum of the particle can never be smaller than the number of the order $\left(\frac{\hbar}{2}\right)$ (OR) $\left(\frac{h}{2\pi}\right)$. $\boxed{\hbar = \frac{h}{\pi}}$

- In terms of energy & time -

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06/04/23

$$\boxed{\Delta E \cdot \Delta t \geq \frac{h}{2\pi} \text{ (OR) } \frac{h}{4\pi}}$$

- In terms of angular momentum -

$$\boxed{\Delta J \cdot \Delta \theta \geq \frac{h}{2\pi} \text{ (OR) } \frac{h}{4\pi}}$$

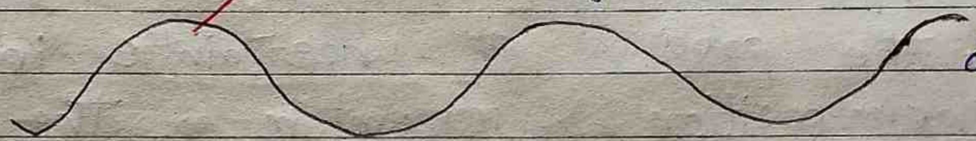
Confirm ★★★ M.I★★

★ Phase & Group Velocity :-

● **Phase Velocity / Wave velocity :-** The velocity of individual wave forming the wave packet is called the 'Phase velocity / Wave velocity'. It is denoted by the symbol 'V_p'.

(OR)

The phase velocity of a wave is the rate at which a definite phase of a wave propagate through a medium denoted by 'V_p'.



displace.

★ Derivation :-

We know that simple harmonic wave equation

$$\boxed{y = a \sin (\omega t - kx)}$$

displacement (y), Amplitude (a), Phase (ϕ) ($\omega t - kx$)

$$\phi(x,t) = \omega t - kx \qquad \begin{cases} \omega = 2\pi \nu \\ k = \dfrac{2\pi}{\lambda} \end{cases}$$

differen. w.r.t time(t)

$$\frac{d\phi}{dt} = \omega - k\frac{dx}{dt}$$

$$\left[\text{Phase} = \text{Constant} \quad \frac{d\phi}{dt} = 0 \right]$$

$$0 = \omega - k\frac{dx}{dt}$$

$$\boxed{\left(\frac{dx}{dt}\right)_{\phi} = \frac{\omega}{k}} \Rightarrow \boxed{v_p = \frac{\omega}{k}}$$

~~OR v_p~~

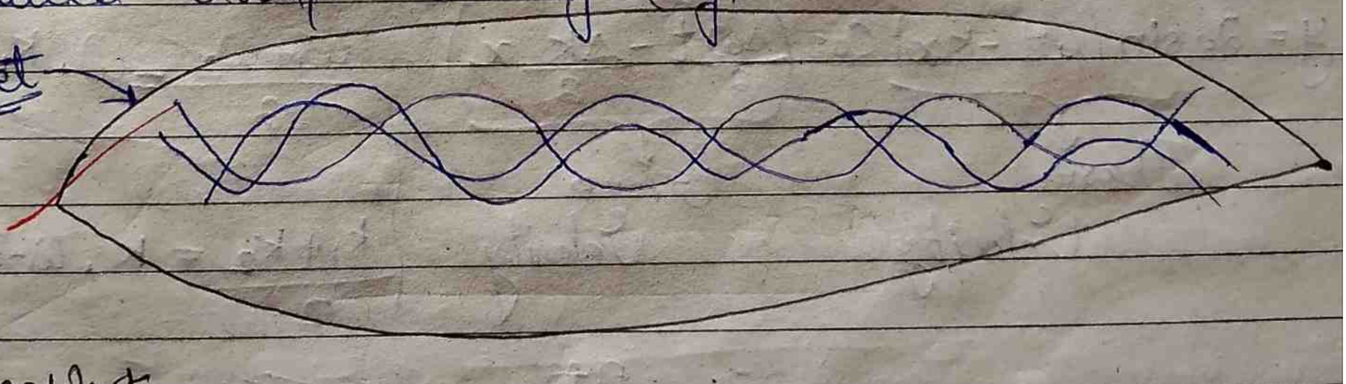
↳ "PHASE VELOCITY"

★ Group Velocity :- The velocity of wave packet formed by the individual waves is called the Group Velocity. It is denoted by the symbol (V_g).

OR

The velocity with which the overall envelope (packet) of the wave propagate through a medium is called Group Velocity (v_g).

Packet



• HomeWork

(1)

Date:
P. No:

★ Derivation - $[y = y_1 + y_2] \rightarrow (1)$

Put values in eq. (1)

$$y = a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$

$$\left.\begin{cases} y_1 = a\sin(\omega_1 t - k_1 x) \\ y_2 = a\sin(\omega_2 t - k_2 x) \end{cases}\right\} \rightarrow 2\text{ SHM}$$

Formula Used $\Rightarrow$ $\left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$$Y = 2a\left[\sin\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{k_1+k_2}{2}\right)x\right]\left[\cos\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right]$$

Put values !!

let $\frac{\omega_1+\omega_2}{2} = \omega$, $k_1 - k_2 = \Delta k$

$$y = 2a\underbrace{\sin(\omega t - kx)}_{\text{Phase Velocity}}\underbrace{\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)}_{\text{Group Velocity}}$$

$\frac{k_1+k_2}{2} = k$, $\omega_1 - \omega_2 = \Delta\omega$

$$V_g = \frac{\Delta\omega/\cancel{2}}{\Delta k/\cancel{2}} = \frac{\Delta\omega}{\Delta k}$$

Put $\boxed{\Delta = d}$

$$\boxed{V_g = \frac{d\omega}{dk}}$$

Date:
P. No:

Que. Difference b/w Group velocity and phase velocity.

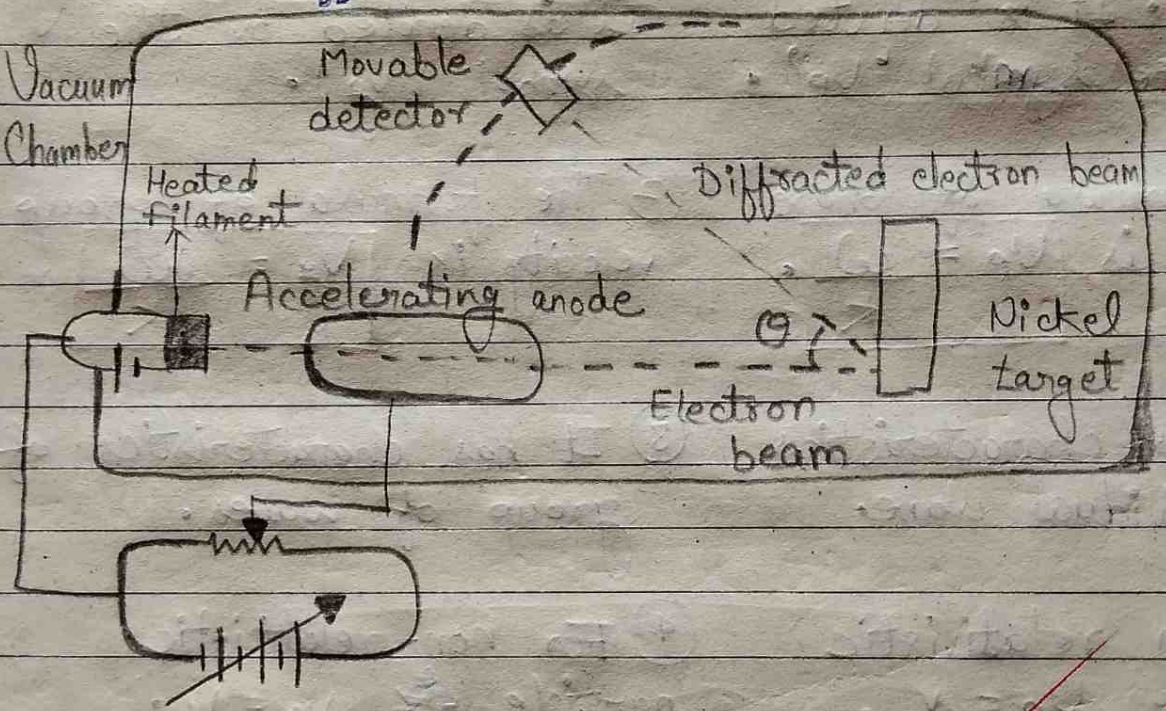
Phase Velocity	Group Velocity
(1) The velocity of individual wave forming the wave packet is called the 'Phase velocity / Wave Velocity'. It is denoted by the symbol 'V_p'.	(1) The velocity of wave packet formed by the individual waves is called the group velocity. It is denoted by the symbol (V_g).
(2) The formula of phase velocity is $\boxed{V_p = \frac{\omega}{k}}$.	(2) The formula of group velocity is $\boxed{V_g = \frac{d\omega}{dk}}$
(3) It has characteristics of individual wave.	(3) It has characteristics of group of waves.
(4) For non relativistic particle $V_p = \frac{v}{2}$.	(4) For non-relativistic particle $V_g = V$.
(5) For relativistic particle $V_p = \frac{c^2}{v}$.	(5) For relativistic particle $V_g = V$.

Date: 11/04/23
P. No:

H.W.
5 marks

Que. Davisson & Germer's electron diffraction experiment.

Ans-1 Initial atomic models proposed by scientists could only explain the particle nature of electron but failed to explain the properties related to their wave nature. C. J. Davission & L.H. Germer, in the year 1927, carried out an experiment, popularly known as Davission Germer's experiment, to explain the wave nature of electrons through electron diffraction.



- We obtained the variation of the intensity (I) of the scattered electrons by changing the angle of scattering θ.
- By changing the accelerating potential difference, the accelerated voltage was varied from 44V to 68V

Date:
P. No:

★ Relationship b/w Group & phase velocity —

We know that,

phase / wave velocity $\boxed{v_p = \frac{\omega}{k}}$

$$\boxed{\omega = k V_p} \quad (1)$$

& Group Velocity

$$\boxed{v_g = \frac{d\omega}{dk}} \quad (2)$$

Put values eq. (2) from eq. (1) —

$$v_g = \frac{d(kV_p)}{dk} = \frac{dk}{dk}V_p + k\frac{dV_p}{dk}$$

$$V_g = V_p + k\frac{dV_p}{dk} = V_p + \left(\frac{2\pi}{\lambda}\right)\frac{d(V_p)}{d\left(\frac{2\pi}{\lambda}\right)} \qquad \left[\because k = \frac{2\pi}{\lambda}\right]$$

$$\boxed{V_g = V_p - \lambda\frac{dV_p}{d\lambda}} \quad (3) \leftarrow V_g = V_p + \left(\frac{1}{\lambda}\right)\frac{d(V_p)}{d(\lambda)^{-1}}$$

The above eq. (3) related the group velocity (V_g) & the wave velocity V_p. Thus, the group velocity depends ~~V_p~~ & the variations of wave velocity with the wavelength $\left(\text{i.e. } \frac{dV_p}{d\lambda}\right)$.

Date:
P. No:

M.I*
What is ψ & its properties.

★ Wave Function:-
It is an essential element of a quantum mechanical system by using it we can get any meaningful information about the system. It is denoted by (ψ) OR $\psi(x,t)$.

★ ψ is related to the probability of finding the particle. Max Born (1926) put these ideas forward for the first time.

★ The wave func.^{n} ψ indicates the state of the particle.

★ The square of the absolute value of the wave func.^{n} ψ is considered & is known as probability density denoted by $|\psi|^2$.

★ ψ describes the state of matter-wave as func.^{n} of position & time.

★ ψ itself has no direct physical significance

$$\int_{-}^{+} |\psi|^2 \, dv = 1$$

$$\boxed{\int_{-\infty}^{+\infty} \psi \psi^* \, dv = 1}$$

Normalization condition

$\psi \rightarrow$ Normalized wave func.^{n}

Date:
P. No:

IMP

★ Properties of wave funcⁿ :-

ψ should

→ ① Satisfy the law of conservation of energy.

i.e ⇒ Total Energy = P.E. + K.E.

→ ②* be consistent with de-Broglie hypothesis
i.e. $\lambda = \frac{h}{p}$

→ ③ be single valued because probability is Unique.

→ ④* be Continuous.

→ ⑤* be finite.

→ ⑥* be linear so that de-Broglie waves have the important superposition property.

★ Eigen value and Eigen function :-

Meaning of Eigen :- Proper, characteristics.

Eigen function :- Proper funcⁿ → proper value Res

Eigen funcⁿ of a system is a funcⁿ that response to Complex scalar eigen value.

i.e $\boxed{y(t) = \lambda f}$

Eigen funcⁿ

Eigen funcⁿ

Eigen Value

Date:
P. No:

Eigen value :- Special set of scalars quantities associated with a linear system of eq.^{n} (i.e. a matrix eq.^{n}).

Ex. => $3x + 4y$ $\left[\therefore x, y \rightarrow \text{eigen func.}^n\right]$

$\underbrace{3 \quad 4}_{\text{Eigen values}}$

Concept of eigen value & eigen functions :-

If (ψ) is well defined func.^{n}, that operates on operator ($\hat{p}$) in two different ways depending upon nature of ψ as -

(i) The operator $\hat{p}$ operating an a func.^{n} ψ may change into a new func.^{n} (say ϕ).

i.e. $\boxed{\hat{p}\psi = \phi}$ — (1)

the new func.^{n} ϕ is linearly independent of ψ.

(ii) The operator $\hat{p}$ operating an a func.^{n} ψ is equal to λ-times of func.^{n} ψ.

i.e. $\boxed{\hat{p}\psi = \lambda\psi}$ — (2)

Where $\begin{cases} \lambda = \text{real or Complex number}, \ \lambda = \text{Eigen value} \\ \psi = \text{eigen func.}^n \end{cases}$

Date:
P. No:

★ ~~Eige~~ Example :- $\frac{d(e^{-4x})}{dx} = -4e^{-4x}$

Comparing above value to eq. (2) we get -

* Eigen func.^{n} = e^{-4x}
* Eigen value = -4
* operator = $\frac{d}{dx}$

Eigen value used for :-

allow us to 'reduce' a linear operation (or) used to make linear transformation understandable.

★ Eigen value & Eigen function :- It is defined as :

$$\boxed{\hat{A}\,\Psi_m = a_m\,\Psi_m} \rightarrow \text{Eigen Value equation}$$

operator Eigen Func.^{n} Constant → Eigen Value Eigen Func.^{n}

• In this eigen value equation -

• Ψ_m is eigen func.^{n} with operator $\hat{A}$ and a_m is eigen value.

Date:
P. No:

For ex.- Schrodinger eq.^{n} for a wave → (In 3D)

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{-8\pi^2m(E-V)\psi}{h^2}$$

When solved can given many values of Ψ. When the wave func.^{n} Ψ all some conditions, then that func.^{n} is called "Well behaved func.^{n}". Such wave func.^{n} are called "Eigen functions".

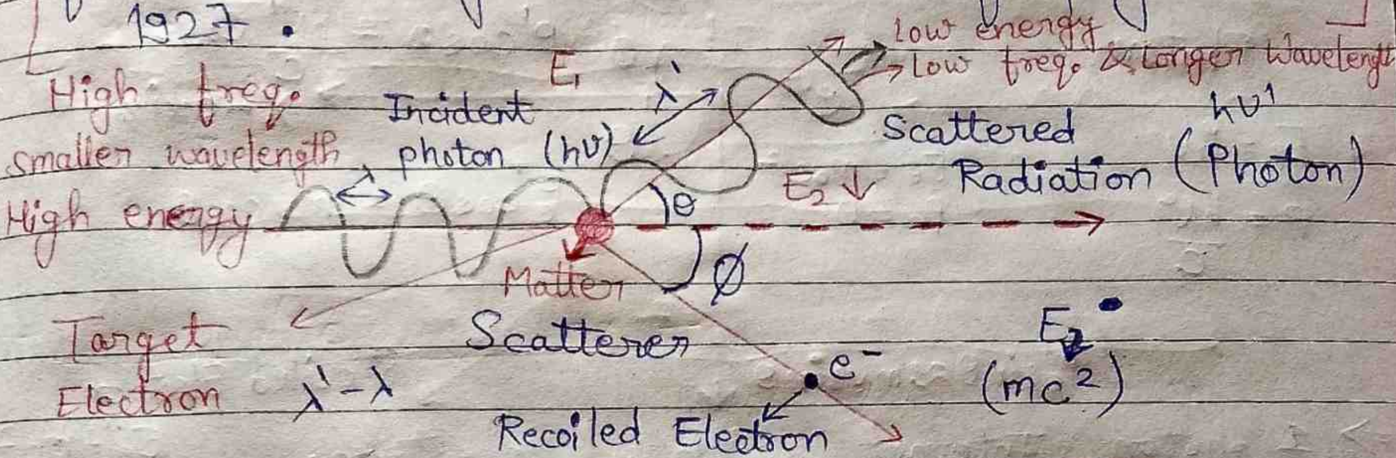
In case of energy (E) these eigen func.^{n} are called "eigen values".

"If an operator $\hat{A}$ operates on a well-behaved func.^{n} Ψ to give the same func.^{n} by multiplying with the constant, then that func.^{n} φ is called "Eigen func.^{n}" and that multiple constant is called as "Eigen value".

Date:
P. No:

★ Compton's effect :- (high freq., x-ray, γ-ray etc.) ★

A.H Compton discovered Compton effect in 1923 for which he got Nobel Prize of physics in 1927.



When high frequency radiations are scattered by electrons of the scatterer, the frequency of the scattered wave is smaller than the frequency of incident radiations, this phenomenon is called "COMPTON - EFFECT".

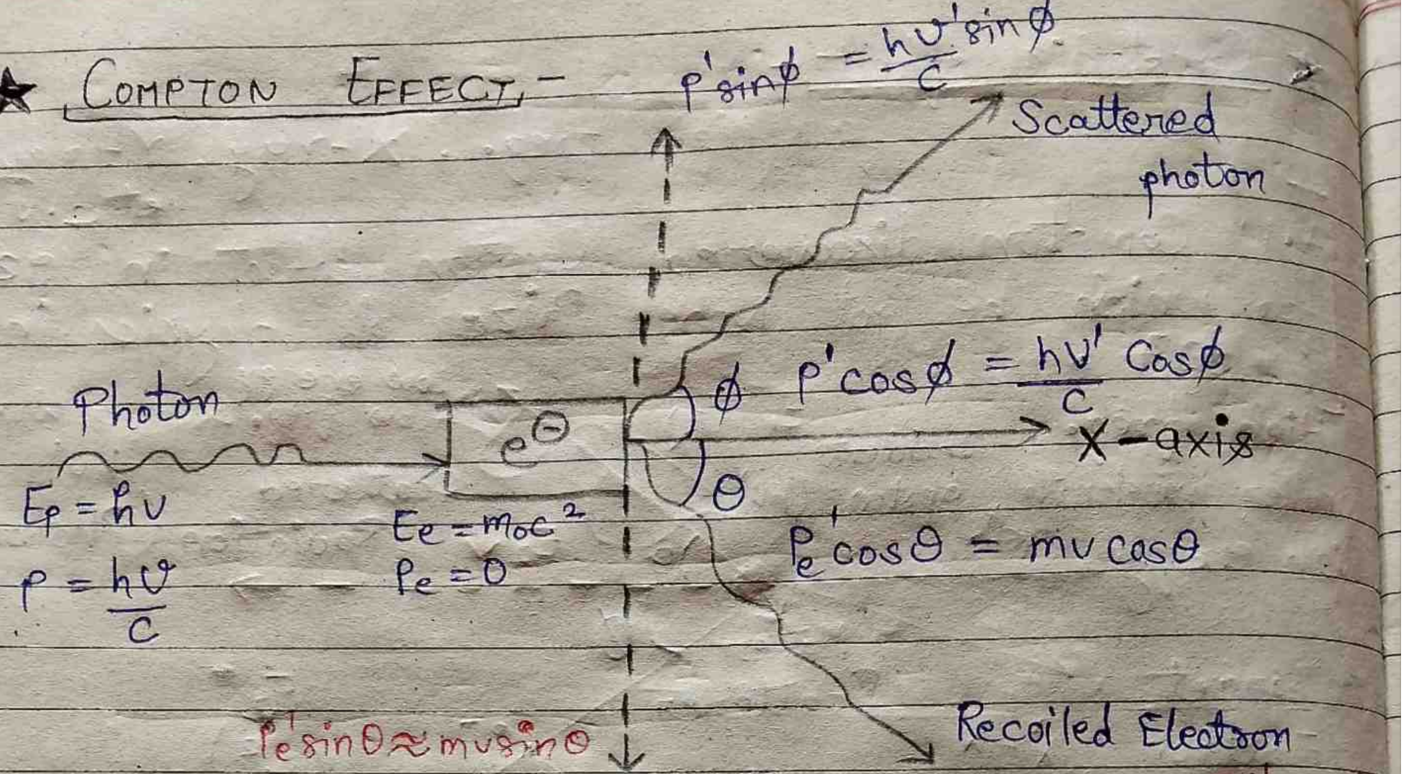
A.H. Compton in 1923 explained this effect by considering the particle name of electromagnetic wave.

The change in wavelength of high-energy radiations during scattering process is called "COMPTON SHIFT" & is given by -

$$\boxed{\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)} \quad \text{OR} \quad \boxed{\Delta\lambda = \lambda' - \lambda}$$

This is called the "compton shift eq.ⁿ"

★ Compton Effect :-



⇒ 1 photon Interact with 1 electron. $E_e' = mc^2$

⇒ ϕ = Scattering Angle $P_e' = mv$

⇒ θ = Recoiled Angle.

⇒ A photon which is incident on graphite block emit two types of photon wavelength (λ & λ').

where $\boxed{\Delta\lambda = \lambda' - \lambda}$ $\lambda' > \lambda$

$\Delta\lambda$ = Compton shift

★ Schrodinger Wave equations :-

In 1925, Schrodinger established a wave ~~func~~ func.^{n} eq.^{n} assuming the concept of wave func.^{n} on the basis of de-Broglie's wave hypothesis and planck's quantum theory. This wave eq.^{n} is called the schrodinger Wave equations. This equation is capable to explain successfully all types of problems.

★ The role of schrodinger's wave eq.^{n} in quantum mechanics is similiar to that of newton laws of motion in classical mechanics.

"SCHRODINGER WAVE EQUATIONS"

(1) Time Dependent Schrodinger Wave Equation

(2) Time Independent Schrodinger Wave eq.^{n}

★ Time Independent Schrodinger Wave eq.^{n} –

General wave expression in differential form is given by –

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 \psi}{\partial t^2}} \quad (1)$$

1D

★ In 3-D form:-

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2 \psi}{\partial t^2}}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi = \frac{1}{V^2}\frac{\partial^2 \psi}{\partial t^2}$$

$$\boxed{\nabla^2 \psi = \frac{1}{V^2}\frac{\partial^2 \psi}{\partial t^2}} \quad (2) \quad \left[\because \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} = \nabla\right]$$

Let if ψ is a complex func.^{n} of space cordinates of the particle & time t, than one of the sol.^{n} of above diff. eq.^{n}.

$$\boxed{\psi_{(x,y,z)} = \psi_0 e^{-i\omega t}} \quad (3)$$

Date:
P. No:

diff. eq. (3) w.r.t (t)

$$\frac{\partial \psi}{\partial t} = \psi_0(-i\omega)e^{-i\omega t}$$

Constant

Again diff. w.r.t (t)

$$\frac{\partial^2 \psi}{\partial t^2} = \psi_0(-i\omega)(-i\omega)e^{-i\omega t}$$

$$= \psi_0 e^{-i\omega t}(-i\omega)(-i\omega) \quad \text{[From eq. (3)]}$$

$$\frac{\partial^2 \psi}{\partial t^2} = i^2\omega^2\psi$$

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = -\omega^2\psi} \quad \left[\because i^2 = -1\right]$$

- where $\omega = 2\pi\nu = 2\pi\frac{v}{\lambda}$

"(Angular Freq.)" $\nu \downarrow$ (f)

So put ~~value~~ $\rightarrow \frac{\partial^2 \psi}{\partial t^2} = \frac{-(2\pi)^2 v^2\psi}{\lambda^2}$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{-(2\pi)^2 v^2 \psi}{\left(\frac{h}{p}\right)^2} \quad \left[\because \text{"Matter" Wave } \lambda = \frac{h}{p}\right]$$

$$\frac{\partial^2 \psi}{\partial t^2} = \left[\frac{-(2\pi)^2 \nu^2 p^2}{h^2}\right] \qquad \left[\because \hbar = \frac{h}{2\pi}\right]$$

$$\frac{\partial^2 \psi}{\partial t^2} = \left[\frac{-(2\pi)^2 \nu^2 p^2}{h^2}\right] \qquad \left[\because p^2 = m^2 v^2\right]$$

$$\frac{\partial^2 \psi}{\partial t^2} = \left[\frac{-V^2 m^2 v^2}{\hbar^2}\right] \psi$$

Put this value in eq. (2)

$$\nabla^2 \psi = \frac{1}{\cancel{v^2}} \left[\frac{-\cancel{v^2} m^2 v^2}{\hbar^2}\right] \psi$$

$$\boxed{\nabla^2 \psi = -\left(\frac{m^2 v^2}{\hbar^2}\right) \psi} \quad (4)$$

To fine values of $m^2 v^2$, we use

Total Energy $\Rightarrow E = K.E + P.E.$

Kinetic Energy   Potential Energy

$$E = \frac{1}{2} m v^2 + V$$

$$E - V = \frac{1}{2} m v^2$$

Date:
P. No:

$$2m(E-V) = m^2v^2$$

$$m^2v^2 = 2m(E-V)$$ put in eq. (4)

$$\nabla^2\psi = -\frac{2m(E-V)}{\hbar^2}\psi$$

***

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad (5)$$

Final Eq.^{n}

This equation is called a schrodinger time Independent wave equation.

★ Time dependent schrodinger's wave equation –

We Know that from schroodinger time independent wave eq.^{n} –

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad (1)$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E\psi - V\psi) = 0$$

▶ From general sol.^{n}, ⇒ $\boxed{\psi = \psi_0 e^{-i\omega t}}$ – (3)

diff. above eq.^{n} w.r.t (t) :

$$\frac{\partial\psi}{\partial t} = \psi_0(-i\omega)e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = \psi(-i\omega)$$ [where $\omega = 2\pi\vartheta$, $\omega = 2\pi \frac{E}{h}$]

Doubt: $E = h\vartheta$, $\vartheta = \frac{E}{h}$ Bcoz $\hbar = \frac{h}{2\pi}$

$$\boxed{\omega = \frac{E}{\hbar}}$$

So that $\Rightarrow$

$$\frac{\partial \psi}{\partial t} = \psi(-i)\frac{E}{\hbar}$$

Hence $E\psi \frac{(-i)}{\hbar} = \frac{\partial \psi}{\partial t} \Rightarrow E\psi = \frac{\partial \psi}{\partial t} \times \frac{\hbar}{-i} \times \left(\frac{i}{i}\right)$

$$\boxed{E\psi = \hbar i \frac{\partial \psi}{\partial t}}$$ $\rightarrow$ Put this value in eq. (2) *

$$\nabla^2 \psi + \frac{2m}{\hbar^2}\left(\hbar i \frac{\partial \psi}{\partial t} - V\psi\right) = 0$$

$$\boxed{\frac{\hbar^2}{2m}\nabla^2\psi + \hbar i \frac{\partial \psi}{\partial t} - V\psi = 0} \quad (3)$$

(OR)

This is called a schrodinger time dependent eq.^{n}

$$\hbar i \frac{\partial \psi}{\partial t} = V\psi - \frac{\hbar^2}{2m}\nabla^2\psi$$

$$\hbar i \frac{\partial \psi}{\partial t} = \left(V - \frac{\hbar^2}{2m}\nabla^2\right)\psi$$

where $H = V - \frac{\hbar^2}{2m}\nabla^2$

Hamiltonian Operator !

** $\boxed{E\psi = H\psi} \rightarrow (4)$

★ Schrodinger wave eq.^{n} applications :-

Particle in 1-D potential box/Well :-

(Free Particle)

Consider a particle moving in a one-dimensional potential box shown in fig-1. The potential region can be expressed as below :-

$$V = \begin{cases} 0 & 0 < x < L \\ \infty & 0 > x > L \end{cases}$$

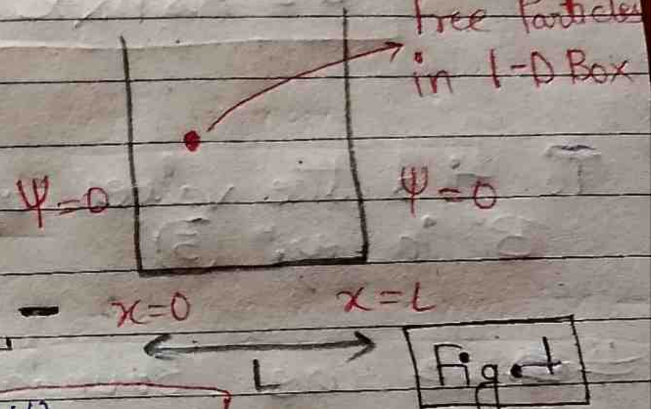


Fig-1

* from Schrodinger wave eq.^{n} -

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0} \quad (1)$$

* for One dimensional box :-

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E\psi - V\psi) = 0} \quad (2)$$

for free particle $\boxed{V = 0}$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0} \quad (3)$$

$$\boxed{\frac{d^2\psi}{\partial x^2} + k^2\psi = 0} \quad (4)$$

Proper diff. eq.ⁿ

Since eq.ⁿ (4) is a differential eq.ⁿ

Hence one of sol.ⁿ of eq.ⁿ (4) is -

$$\boxed{\psi = A\sin kx + B\cos kx} \quad (5)$$

where $\boxed{K^2 = \frac{2mE}{\hbar^2}}$

To find the values of A & B in eq.ⁿ (5)

we use Boundary conditions :-

- for first Boundary conditions $x=0$ & $x=L$

* (i) $x=0$, $\psi=0$

put these values in eq. (5)

$$0 = A\sin 0 + B\cos 0$$

$$0 = 0 + B$$

$$B = 0$$

(ii) For Second Boundary Condit[ion]

$\boxed{x=L}$

$\boxed{x=L}$, $\boxed{\psi=0}$ & $\boxed{B=0}$

So from eq. (5) -

$$0 = A\sin KL + 0$$

$$A\sin KL = 0$$

$$\sin KL = 0$$

$\sin n\pi$ $\quad n = 1, 2, 3,$ satisfied

Date:
P. No:

∴ By Comparing, $kL = n\pi$

$$\boxed{k = \frac{n\pi}{L}} \quad \text{OR} \quad \boxed{k^2 = \frac{n^2\pi^2}{L^2}}$$

Now put this value of k^2 in eq. 6

$$\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2} \qquad \boxed{\because \hbar = \frac{h}{2\pi}}$$

$$E = \frac{\hbar^2 n^2 \pi^2}{2mL^2} \Rightarrow \frac{h^2 n^2 \cancel{\pi^2}}{(2\cancel{\pi})^2\, 2mL^2}$$

$$\boxed{E = \frac{n^2 h^2}{8mL^2}} \quad \text{Where } n = 1, 2, 3, 4, \text{-----}$$

So,

$$\boxed{E_n = \frac{n^2 h^2}{8mL^2}} \text{ —— } (7)$$

Energy Eigen values

Energy of n^{th} state

for $\boxed{n=1}$

$$E = \frac{h^2}{8ML^2}$$

from eq.^{n} (7), it is clear that in a 1-D potential well of infinite depth, only the discreate energy states are possible. These Energy states

Date:
P. No:

are "not equispaced", but the energy of n^{th} state is directly proportional to (n^2).

Good

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