S.A.T.I.

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"UNIT - 1"

"PROBABILITY DISTRIBUTION"

Que.1 Find the probability that at most 5 defective fuses will be found in a box of 200 fuses it experience show that 2% of such fuses are defective.

Sol.ⁿ ⇒ The probability of finding defective fuses

$$p = 2\% = \frac{2}{100}$$

Therefore, avg. no. of defective fuses in a box of 200 fuses (Mean) = np

$$n = 200$$

$$\text{Mean} = 200 \times \frac{2}{100}$$

$$\text{Mean} = 4 \Rightarrow m = 4$$

$$P(x \le 5) = \sum_{r=0}^{5} \frac{e^{-4} 4^r}{r!}$$

$$P(x \le 5) = e^{-4}\left(1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!}\right)$$

$$P(x \le 5) = 0.7845$$ Ans.

॥स्वच्छ भारत, स्वस्थ भारत॥

Que.2 Six dice are thrown 729 times. How many times do you expect atleast 3 dice to show a five (OR) Six.

Sol.ⁿ ⇒ p = Probability of sucess (dice shows 5 (OR) 6) $= \frac{2}{6} = \frac{1}{3}$

q = Probability of failure (dice does not show 5 (OR) 6) $= \frac{4}{6} = \frac{2}{3}$ $\left[\begin{matrix} 1-p=q \\ \therefore\ 1-\frac{1}{3}=\frac{2}{3} \end{matrix}\right]$

$n = 6$

Probability of getting atleast 3 dice to show 5 (OR) 6 =

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[{}^6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{6-0} + {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{6-1} + {}^6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{6-2} \right]$$

$$= 1 - \left[1 \times \left(\frac{2}{3}\right)^6 + 6 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^5 + \frac{6 \times 5 \times 4!}{2! \times 4!} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 \right]$$

$$= 1 - \left[\left(\frac{2}{3}\right)^6 + 2 \times \left(\frac{2}{3}\right)^5 + \frac{1}{3} \times \left(\frac{2}{3}\right)^4 \right]$$

$$= 1 - \left[\left(\frac{2}{3}\right)^4 \left(\left(\frac{2}{3}\right)^2 + 2 \times \left(\frac{2}{3}\right) + \frac{1}{3} \right] \Rightarrow 1 - \left[\frac{16}{81} \left[\frac{4}{9} + \frac{4}{3} + \frac{1}{3} \right] \right]$$

$$= 1 - \left[\frac{16}{81} \left(\frac{31}{9} \right) \right] = 1 - \frac{496}{729} = \frac{729 - 496}{729} = \frac{233}{729}$$

← We are Getting
← Total

$$= \boxed{233 \text{ Times}} \text{ Ans.}$$

SESSIONAL PAPERS

Que.3 Find the mean, S.D and variance of binomial distribution and poison distribution.

Ans-3 Binomial Distribution :-

The probability that in one set of (n) tries, (r) tries are success the remaining (n-r) ~~times~~ tries is -

$$P(x=r) = {}^{n}C_r \, p^r \, q^{n-r}$$

p = prob. of success
q = prob. of failure
n = no. of trial
$p+q=1$

- Mean $(\bar{x}) = np$
- Variance $(\sigma^2) = npq$
- Standard Deviation $(\sigma) = \sqrt{npq}$

★ Poison Distribution :-

It is a particular limiting part of the binomial distribution when p & q is very small & n is very large.

$$\text{(Mean) } m = np$$

$$P(x=r) = \frac{e^{-m} \cdot m^r}{r!}$$

S 479

Ques If the probability that an individual suffers a bad reaction from a certain injection is 0.001. determine the probability that out of 2000 individuals —

(i) Exactly 3 (ii) More than 2 (iii) None
(iv) More than one individual will suffer a bad reaction.

Sol.ⁿ⇒ Given ⇒ $p = 0.001$

$q = 1 - 0.001 = 0.999$ $(1-p)$

$n = 2000$

- Mean $(m) = np = \cancel{2000} \times \frac{0.001}{\cancel{1000}} = 2$ $\boxed{m = 2}$

(i) Exactly 3 ⇒

$\boxed{r = 3}$

$$\boxed{P(x = r) = \frac{e^{-m} \cdot m^r}{r!}}$$

$$P(x=3) = \frac{e^{-2} \cdot 2^3}{3!} = \frac{4\cancel{8}}{3\cancel{6}} \cdot e^{-2} = \frac{4}{3e^2}$$ Ans.

(ii) More than 2

Prob. of more than 2 — Prob. of less than or equal to 2

$$P(x > 2) = 1 - P(x \leq 2)$$

$$P(x>2) = 1 - [P(0) + P(1) + P(2)]$$

$$P(x>2) = 1 - \left[\frac{e^{-2} \cdot m^0}{0!} + \frac{e^{-2} \cdot (2)^1}{1!} + \frac{e^{-2} \cdot (2)^2}{2!}\right]$$

$$P(x>2) = 1 - \left[e^{-2} + 2e^{-2} + \frac{4e^{-2}}{2}\right] = 1 - [5e^{-2}$$

$$P(x>2) = 1 - \frac{5}{e^2} = \boxed{0.32}$$ Ans.

(iii) None

$$P(x=0) = \frac{e^{-2}\cdot(2)^0}{0!} = \frac{e^{-2}(1)}{1} = \frac{1}{e^2}$$ Ans

(iv) More than one individual will suffer a bad rxn.

Prob. of more the one indiv. Prob. of less than (or) equal to 1 one indiv.

$$P(x>1) = 1 - (P \le 1)$$

$$P(x>1) = 1 - [P(0) + P(1)]$$

$$P(x>1) = 1 - \left[\frac{e^{-2}\cdot(2)^0}{0!} + \frac{e^{-2}\cdot(2)^1}{1!}\right]$$

$$P(x>1) = 1 - \left[e^{-2} + 2e^{-2}\right]$$

$$P(x>1) = 1 - \frac{3}{e^2} = 0.59$$

$$\boxed{P(x>1) = 0.59}$$

Prob. of more than one individual will suffer a bad rxn.

SAG

Que. 5 Fit a second degree parabola to the following data:

x	0	1	2	3	4
y	1	5	10	22	38

Sol.ⁿ ⇒

Second degree parabola eq.ⁿ ⇒ $\boxed{y = a + bx + cx^2}$ — (1)

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad — (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad — (3)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad — (4)$$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	5	1	1	1	5	5
2	10	4	8	16	20	40
3	22	9	27	81	66	198
4	38	16	64	256	152	608
10	76	30	100	354	243	851

$$76 = (5)a + (10)b + (30)c \quad — (5)$$
$$243 = (10)a + (30)b + (100)c \quad — (6)$$
$$851 = (30)a + (100)b + (354)c \quad — (7)$$

From eq. (5) & (6) [Multiply 2 in eq. (5)]

$$152 = 10a + 20b + 60c$$
$$-243 = -10a - 30b - 100c$$
$$-91 = -10b - 40c$$

$$\boxed{91 = 10b + 40c} \quad (8)$$

From eq. (6) & (7) [Multiply 3 in eq. 6]

$729 = 30a + 90b + 300c$ — (6) × 3

$851 = 30a + 100b + 354c$ — (7)

$-122 = -10b - 54c$

$122 = 10b + 54c$ — (9)

From eq. (8) & (9)

$91 = 10b + 40c$ — (8)

$122 = 10b + 54c$ — (9)

$-31 = -14c$

$c = \frac{31}{14} = 2.21$

$c = 2.21$

Put the value of a, b & c in eq. (1)

Put value of c in eq. (8)

$91 = 10b + 40(2.21)$

$91 = 10b + 88.4$

$91 - 88.4 = 10b$

$2.6 = b$

$b = 0.26$

Put value of b & c in eq. (5)

$76 = 5a + 10(0.26) + (30)(2.21)$

$76 = 5a + 2.6 + 66.3$

$76 = 5a + 68.9$

$76 - 68.9 = 5a$

$a = 1.42$

Ans.

Second order ⇒ Parabola eqn is $y = 1.42 + 0.26x + 2.21x^2$

Que.6 Find the mean and standard deviation for the table of death of women over 85 years old recorded in the 3 year period.

No. of death :	0	1	2	3	4	5	6	7
No. of days :	364	375	218	89	33	13	2	1

No. of Death (x)	No. of days (f)	$f.(x)$
0	364	0
1	375	375
2	218	436
3	89	267
4	33	132
5	13	65
6	2	12
7	1	7
$\Sigma x = 28$	$\Sigma f = 1094$	$\Sigma fx = 1294$

$$\text{Mean } (\bar{x}) = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Mean } (\bar{x}) = \frac{1294}{1094}$$

$$\boxed{\text{Mean } (\bar{x}) = 1.18}$$