"DIFFERENTIAL EQUATION"

DATE 27/03/23

UNIT - 3

★ **Differential equation** - Jismai derivative $\left(\frac{dy}{dx}\right)$ ho..

▸ Ex. -

① $dy/dx + y = 0$

② $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = e^x$

Firstly Say "order" then "degree"

★ **ORDER of a D.E.** - Highest $\frac{dy}{dx}$.

(i) $\frac{d^2y}{dx^2} + y = 0 \rightarrow$ (order-2)

(ii) $\frac{d^3y}{dx^3} + 4y = 0 \rightarrow$ (order-3)

★ **Degree of a D.E.** - Hightest $\frac{dy}{dx}$ power Ki Value.

(i) $\left(\frac{d^2y}{dx^2}\right)^1 + \left(\frac{dy}{dx}\right)^3 + y = 0 \longrightarrow$ (deg - 1)

(ii) $\left(\frac{d^3y}{dx^3}\right)^1 + \frac{dy}{dx} + y = 0 \longrightarrow$ (deg - 1)

★ Differential eqn of 1^{st} order & 1^{st} degree :-

▶ [METHOD - 1] ⇒ Separation of Variable :-

Que.1 $\frac{dy}{dx} = \frac{y}{x}$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\log y = \log(x) + \log C$$

$$\log(y) = \log(xC)$$

$$\boxed{y = xC}$$ Ans.

Que.2 $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

[∘ $\tan^{-1} x = \int \frac{dx}{1+x^2}$]

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2} + C$$

$$\tan^{-1} y = \tan^{-1} x + \tan^{-1} C$$

$$\cancel{\tan^{-1}} y = \cancel{\tan^{-1}} \left(\frac{x+C}{1-xC} \right)$$

$$\boxed{y = \frac{x+C}{1-xC}}$$ Ans.

[∘ Formula Used

∘∘ $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$]

[METHOD - 2] ⇒ Linear Differential EQUATION -

Jinki $\left(\frac{dy}{dx}\right)^{1}$ ki power/degree one ho.

y - form linear	x - form linear
$\boxed{\frac{dy}{dx} + Py = Q}$	$\boxed{\frac{dx}{dy} + Px = Q}$
[x (OR) May be Constant]	[y (OR) May be Constant]
$\boxed{I.F = e^{\int P dx}}$	$\boxed{I.F. = e^{\int P dy}}$
$\boxed{y(I.F.) = \int Q(I.F.)dx + C}$	$\boxed{x(I.F.) = \int Q(I.F.)dy + C}$

Que.1 $\frac{dy}{dx} + \frac{y}{x} = x \Rightarrow \boxed{P = \frac{1}{x}}, \boxed{Q = x}$

$$y(I.F.) = \int Q.(I.F.)\,dx + C$$

$$y.(x) = \int x\,(x)\,dx + C$$

$$yx = \int x^2 dx + C$$

$$yx = \frac{x^3}{3} + C$$

$$\boxed{y = \frac{x^2}{3} + \frac{C}{x}}$$ Ans.

$$I.F. = e^{\int P dx}$$

$$I.F. = e^{\int \frac{1}{x} dx}$$

$$I.F. = e^{\log x}$$

$$I.F. = x^{\cancel{\log_e e}^{1}}$$

$$\boxed{I.F. = x}$$

Que.2 $\frac{dy}{dx} + \frac{y}{x} = e^x$ $\boxed{\frac{dy}{dx} + Py = Q}$

$\boxed{P = \frac{1}{x}}$ $\boxed{Q = e^x}$

$$I.F. = e^{\int P dx}$$
$$I.F. = e^{\int \frac{1}{x} dx}$$
$$I.F. = e^{\log x}$$
$$I.F. = x^{\log_e e}$$
$$\boxed{I.F. = x}$$

$$y(I.F) = \int Q(I.F) dx + C$$
$$y(x) = \int e^x (x) dx + C$$
$$xy = \int x e^x dx + C$$

- Integration By Parts :-

$$\boxed{\int I.II\, dx = I \int II\, dx - \int \left[\frac{dI}{dx} \left(\int II\, dx \right) \right] dx}$$

Lograth. → Trigono.

ILATE → Select I function by this Rule!

Inverse Algebra Exponential

$$xy = \int \underset{I}{x} \underset{II}{e^x} dx + C$$
$$xy = x \int e^x dx - \int \left[\frac{dx}{dx} \int e^x dx \right] dx + C$$
$$xy = x e^x - \int e^x dx + C$$
$$xy = x e^x - e^x + C$$
$$\boxed{xy = e^x (x-1) + C} \text{ Ans.}$$

Que.3 $\frac{dy}{dx} + \frac{y}{x} = e^{x^2} \Rightarrow \boxed{P = \frac{1}{x}}$ $\boxed{Q = e^{x^2}}$

$$\boxed{y(I.F.) = \int Q(I.F.)\,dx + C}$$ $$\boxed{\frac{dy}{dx} + Py = Q}$$

$$I.F. = e^{\int P\,dx}$$

$$I.F. = e^{\int \frac{1}{x} dx}$$

$$I.F. = e^{\log x}$$

$$\boxed{I.F. = x}$$

$$y(x) = \int e^{x^2}(x)\,dx + C$$

$$xy = \int x \cdot e^{x^2}\,dx + C$$

Let, $\boxed{x^2 = t}$

Differen. both side w.r.t (x)

$$\frac{d(x^2)}{dx} = \frac{dt}{dx}$$

$$\boxed{2x = \frac{dt}{dx}}$$ $$\boxed{x\,dx = \frac{dt}{2}}$$

$$xy = \int e^t \frac{dt}{2} + C$$

$$xy = \frac{1}{2}\int e^t\,dt + C$$

$$xy = \frac{1}{2}e^t + C$$

Ans $\boxed{xy = \frac{1}{2}e^{x^2} + C}$

$$\cancel{xy = \frac{1}{2}\left(\frac{e^{t^2}}{2}\right) + C}$$

$$\boxed{\cancel{xy = \frac{e^{x^4}}{4} + C}}$$

Ans.

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Que.1 $(1+y^2)dx = (\tan^{-1}y - x)dy$

$$\frac{dy}{dx} = \frac{1+y^2}{\tan^{-1}y - x} \Rightarrow \frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2}$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{x}{1+y^2} + \frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2}$$

$$\boxed{\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}} \quad \left[\frac{dx}{dy} + Px = Q\right]$$

$$\boxed{P = \frac{1}{1+y^2}} \quad \boxed{Q = \frac{\tan^{-1}y}{1+y^2}}$$

$$I.F = e^{\int P\,dy}$$

$$I.F. = e^{\int \frac{1}{1+y^2}dy}$$

$$\boxed{I.F. = e^{\tan^{-1}y}}$$

$$\Rightarrow \boxed{x(I.F.) = \int Q(I.F.)\,dy + C}$$

$$x\left(e^{\tan^{-1}y}\right) = \int \frac{\tan^{-1}y}{1+y^2}\cdot\left(e^{\tan^{-1}y}\right)dy + C$$

Let, $\boxed{\tan^{-1}y = t}$ → diffren. w.r.t (y) ⇒ $\frac{d(\tan^{-1}y)}{dy} = \frac{dt}{dy}$

$$\frac{1}{1+y^2} = \frac{dt}{dy}$$

$$\boxed{\frac{dy}{1+y^2} = dt}$$

$$xe^t = \int t\cdot e^t\,\cancel{dy}\,dt + C$$

$$x\cdot e^t = \left[e^t(t-1)\right] + C$$

Replace $\boxed{t = \tan^{-1}y}$

Divide by this

Ans. $\boxed{x = \tan^{-1}y - 1 + \frac{C}{e^{\tan^{-1}y}}}$

For ex. $\Rightarrow$ $\boxed{\tan^{-1}x = t}$ | Make Que. very easy!!

New Que.

Que. 2 $\cos^2 x \frac{dy}{dx} + y = \tan x$ $\boxed{P = \sec^2 x}$ $\boxed{Q = \frac{\tan x}{\cos^2 x}}$

Divide whole eq.n by $\cos^2 x$

$$\boxed{\frac{dy}{dx} + y \cdot \sec^2 x = \frac{\tan x}{\cos^2 x}}$$

$$\boxed{\frac{dy}{dx} + Py = Q}$$

$\Rightarrow y(I.F.) = \int Q(I.F.)\,dx + C$

I.F. $= \int e^{\int P dx}$

I.F. $= \int e^{\sec^2 x dx}$

$$\boxed{I.F. = e^{\tan x}}$$

$y(e^{\tan x}) = \int \frac{\tan x}{\cos^2 x} \cdot (e^{\tan x})\,dx + C$

$\Rightarrow$ Let $\boxed{t = \tan x}$

diffren. both side by $dx \longrightarrow \frac{dt}{dx} = \frac{d(\tan x)}{dx}$

$\frac{dt}{dx} = \sec^2 x$

$$\boxed{dt = \sec^2 x\,dx}$$

$y \cdot e^t = \int \cancel{\tan x}\, t \cdot e^t\,dt + C$

$y \cdot e^t = e^t(t-1) + C$

Replace (t) with $\tan x$

$$\boxed{y \cdot e^{\tan x} = e^{\tan x}(\tan x - 1) + C}$$

Ans.

★ Bernqullis equation:-

$$\boxed{\frac{dy}{dx} + py = Q \cdot y^n}$$ Divide by y^n.

$$\boxed{\frac{1}{y^n}\frac{dy}{dx} + \frac{P}{y^{n-1}} = Q} \quad (1)$$

$$\frac{1}{y^{n-1}} = V \Rightarrow \boxed{y^{1-n} = V}$$

Diffren. w.r.t (x)

$$\frac{d(y^{1-n})}{dx} = \frac{dV}{dx} \Rightarrow (1-n)\,y^{1-n-1}\frac{dy}{dx} = \frac{dv}{dx}$$

Chain Rule Apply

$$\frac{1}{y^n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{dv}{dx}$$

Put $\frac{1}{y^{n-1}}$ & $\frac{1}{y^n}\frac{dy}{dx}$ put in eq. (1)

Solve it ← Linear

Que.1 $xy(1+xy^2)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{xy + x^2y^3} \Rightarrow \frac{dx}{dy} = xy + x^2y^3$

$\frac{dx}{dy} - xy = x^2y^3 \Rightarrow \boxed{P=-y}\ \boxed{Q = x^2y^3}$

Divided by x^2

$\frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3$

Put $\boxed{v = \frac{1}{x}}$ & $\boxed{\frac{1}{x^2}\frac{dx}{dy} = -\frac{dv}{dy}}$

$\frac{-dv}{dy} - v.y = y^3 \Rightarrow \boxed{\frac{dv}{dy} + v.y = -y^3}$

$P = y$ $Q = -y^3$

$I.F. = \int e^{P dy} = \int e^{\int y dy} = e^{y^2/2}$

$\boxed{I.F. = e^{y^2/2}}$

$v(I.F.) = \int Q(I.F.)\,dy + C$

$v.(e^{y^2/2}) = \int -y^3.(e^{y^2/2})\,dy + C$

Let $\boxed{\frac{y^2}{2} = t}$

diff. w.r.t dy

$\frac{1}{2}\frac{d(y^2)}{dy} = \frac{dt}{dy} \Rightarrow \boxed{y = \frac{dt}{dy}}$

$v.e^t = \frac{-2}{2}\int y^2.y\, e^t dy + C$

$v.e^t = -2\int t\,dt + C$

$v.e^t = -2\,e^t(t-1) + C$

Replace $\boxed{v = \frac{1}{x}}$ & $\boxed{t = \frac{y^2}{2}}$

$\frac{1}{x}.e^{y^2/2} = -2e^{y^2/2}\left(\frac{y^2}{2} - 1\right) + C$

Ans.

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★ Exact differential Eq.^{n} -

Standard Exact Differen. EQUATION

$$m(x,y)\,dx + N(x,y)\,dy = 0$$

$\boxed{\dfrac{\partial m}{\partial y} = \dfrac{\partial N}{\partial x}}$ [Partial differentiation]

Sol.^{n} => $\boxed{\int_{y} M\,dx + \int_{x} N\,dy = C}$

[terms of (N) not containing x]

Que.1 $(x^2 - ay)\,dx = (ax - y^2)\,dy$

$(x^2 - ay)\,dx - (ax - y^2)\,dy = 0$ — (1)

$\Rightarrow M(x,y) = x^2 - ay$

$\Rightarrow N(x,y) = -(ax - y^2) = y^2 - ax$

$\boxed{\dfrac{\partial M}{\partial y} = -a}$, $\boxed{\dfrac{\partial N}{\partial x} = -a}$

$\boxed{\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}}$ Given diff eq. (1) is =>

$$\int_{y\ \text{const.}} M\,dx + \int_{x\ \text{Const.}} N\,dy = C$$

$$\int_{y\ \text{Const.}} (x^2 - ay)\,dx + \int_{x\ \text{Const.}} (y^2 - ax)\,dy = C$$

$$\frac{x^3}{3} - \cancel{ayx} + \frac{y^3}{3} - \cancel{axy} = C$$

$$\boxed{\frac{x^3}{3} + \frac{y^3}{3} = C}$$ Ans.

$$\boxed{\frac{x^3}{3} + \frac{y^3}{3} - 2ayx = C}$$ Ans.

Homogenous form → Jiski degree same ho

Har term ki same degree honi chahiye

$M = (x^2y - 2xy^2)$ = 3 degree

$N = (x^3 - 3x^2y)$ = 3 degree

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★ RULE - I –

$\boxed{Mdx + Ndy = 0}$ be a homogeneous equation then I_f is $\dfrac{1}{Mx + Ny}$. $\boxed{Mx + Ny \neq 0}$

Ex.– $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ —— (1)

$M = (x^2y - 2xy^2)$

$N = -(x^3 - 3x^2y)$

Partial differen

$\dfrac{\partial M}{\partial y} = x^2 - 4xy$, $\dfrac{\partial N}{\partial x} = -3x^2 + 6xy$

$\boxed{\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}}$ eq. (1) is not exact !!

$$I.F. = \frac{1}{Mx + Ny} = \frac{1}{(x^2y - 2xy^2)x + [-(x^3 - 3x^2y)]y}$$

$$\boxed{I.F. = \frac{1}{x^2y^2}}$$

From eq. (1) × I.F. ⟹ $\left(\dfrac{x^2y - 2xy^2}{x^2y^2}\right)dx - \left(\dfrac{x^3 - 3x^2y}{x^2y^2}\right)dy = 0$

$$\left(\frac{1}{y} - \frac{2}{x}\right)dx - \left(\frac{x}{y^2} - \frac{3}{y}\right)dy = 0 \quad \text{—— (2)}$$

New $M = \left(\dfrac{1}{y} - \dfrac{2}{x}\right)$, New $N = \left(\dfrac{x}{y^2} - \dfrac{3}{y}\right)$

Formula $\Rightarrow \int M\,dx + \int (\text{term of } N \text{ not containing } x)\,dy$
(y constant)

$$\frac{\partial M}{\partial y} = \frac{-1}{y^2}, \quad \frac{\partial N}{\partial x} = \frac{-1}{y^2}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$\int M\,dx + \int N\,dy = C$$

$$\int \left(\frac{1}{y} - \frac{2}{x}\right) \partial x + 3\int \frac{1}{y}\,dy = C$$

$$\boxed{\frac{x}{y} - 2\log x + 3\log y = C}$$ Ans

▶ RULE - II -

$M\,dx + N\,dy = 0$ is of line form

$\boxed{F_1(x,y)\,y\,dx + F_2(x,y)\,x\,dy = 0}$ then

$$\boxed{I.F. = \frac{1}{Mx - Ny}}, \quad \boxed{Mx - Ny \neq 0}$$

▶ RULE - III -

If $\boxed{M\,dx + N\,dy = 0}$ then

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ is a function of x.

then $\boxed{I.F. = e^{\int f(x)\,dx}}$

- **Rule - IV** :-

$$\boxed{Mdx + Ndy = 0} \text{ then}$$

$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$ is a function of y

then $\boxed{I.F. = e^{\int f(y)dy}}$

- **Ex.** →

$$(xy^2 - e^{1/x^3})dx - x^2y\,dy = 0 \quad \text{--- (1)}$$

$$M = (xy^2 - e^{1/x^3}) \rightarrow \boxed{\frac{\partial M}{\partial y} = 2xy}$$

$$N = -x^2y$$

$$\hookrightarrow \boxed{\frac{\partial N}{\partial x} = -2xy}$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

eq(1) is not exact

$$I.F. = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$$

$$I.F. = \frac{1}{-x^2y}(2xy + 2xy) = \boxed{-\frac{4}{x} = f(x)}$$

$$\Rightarrow I.F. = e^{\int f(x)dx} = e^{-\int \frac{4}{x}dx} = e^{-4\log x}$$

$$\boxed{I.F. = \frac{1}{x^4}} \text{ Ans.}$$

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Que.★

H.W Que.

Ex.- $\left(\frac{y^2}{x^3} - \frac{e^{1/x^3}}{x^4}\right)dx - \frac{y}{x^2}dy = 0$

Que. $(y + xy^2)dx + (x - x^2y)\,dy = 0$

$M = (y + xy^2)$, $\frac{\partial M}{\partial y} = 1 + 2xy$

$N = (x - x^2y)$

$\frac{\partial N}{\partial x} = 1 - 2xy$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

$$\boxed{(1 + xy)y\,dx + (1 - xy)x\,dy = 0} \quad (1)$$

$$\boxed{I.F. = \frac{1}{Mx - Ny}} = \frac{1}{xy + x^2y^2 - xy + x^2y^2} = \frac{1}{2x^2y^2}$$

From eq. (1) $\times$ I.F.

$$\underbrace{\left(\frac{1}{2x^2y} + \frac{1}{2x}\right)}_{\text{New } M}\partial x + \underbrace{\left(\frac{1}{2xy^2} - \frac{1}{2y}\right)}_{\text{New } N}\partial y = 0 \quad (2)$$

$$\frac{\partial M}{\partial y} = \frac{-1}{2x^2y^2}, \quad \frac{\partial N}{\partial x} = \frac{-1}{2x^2y^2}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$\int M dx + \int N dy = C$$

$$\int \left(\frac{1}{2x^2y} + \frac{1}{2x}\right)dx + \int -\frac{1}{2y}\,dy = C$$

$$\boxed{\frac{-1}{2xy} + \frac{1}{2}\log x - \frac{1}{2}\log y = C} \quad \text{Ans.}$$

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$$eq\ (1) \times I.F$$

$$\frac{(xy^2 - e^{1/x^3})}{x^4}\, dx - \frac{x^2 y}{x^4}\, dy = 0$$

$$\boxed{\underbrace{\left(\frac{y^2}{x^3} - \frac{e^{1/x^3}}{x^4}\right)}_{M} dx - \underbrace{\frac{y}{x^2}}_{N}\, dy = 0} \longleftarrow (II)$$

$$\frac{\partial M}{\partial y} = \frac{2y}{x^3}, \quad \frac{\partial N}{\partial x} = \frac{2y}{x^3} \Rightarrow \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

eq. (II) is exact

General Sol.^{n} $\Rightarrow \int M\,dx + \int N\,dy = C$

$$\int \left(\frac{y^2}{x^3} - \frac{e^{1/x^3}}{x^4}\right) dx - \int 0 \cdot dy = C$$

$$\frac{y^2}{-2x^2} - \int \frac{e^{1/x^3}}{x^4}\, dx = C$$

Put $1/x^3 = t$

$-\frac{3}{x^4}\, dx = dt$

$\frac{dx}{x^4} = \frac{-dt}{3}$

$$\frac{y^2}{-2x^2} + \frac{1}{3}\int e^t\, dt = C$$

$$\boxed{\frac{-y^2}{2x^2} + \frac{1}{3}\left[e^{1/x^3}\right] = C} \quad \text{Ans.}$$

Que. $$\underbrace{y\log y}_{M}\,dx + \underbrace{(x - \log y)}_{N}\,dy = 0 \quad \text{--- (1)}$$

$$\frac{\partial M}{\partial y} = y \times \frac{1}{y} + \log y\,(1) = 1 + \log y$$

$$\frac{\partial N}{\partial x} = 1$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

eq. (1) is not exact

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{x - \log y}\left(1 + \log y - 1\right) = \frac{\log y}{x - \log y}$$

it is not $f(x)$.

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{y\log y}\left(\cancel{1} - \cancel{1} - \log y\right) = \frac{-\cancel{\log y}}{y\,\cancel{\log y}}$$

$$\boxed{f(y) = -\frac{1}{y}} \qquad I.F. = e^{\int f(y)\,dy}$$

$$I.F. = e^{-\int \frac{1}{y} dy} = e^{-\log y} = \frac{1}{y}.$$

eq (1) × I.F.

$$\underbrace{\log y}_{M}\,dx + \underbrace{\left(\frac{x}{y} - \frac{\log y}{y}\right)}_{N} dy = 0 \quad \text{--- (2)}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{1}{y}}, \quad \boxed{\frac{\partial N}{\partial x} = \frac{1}{y}}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ eq (II) is exact

$$\int M\,dx + \int N\,dy = 0$$

$$\int \log y\,dx - \int \frac{\log y}{y}\,dy = C$$

Put $\log y = t$

$\frac{dy}{y} = dt$

$$x\log y - \int t\,dt = C$$

Ans. $\boxed{x\log y - \frac{(\log y)^2}{2} = C}$

★ First order and higher degree D.E. :-

Standard form :-

Higher degree

$$\left(\frac{dy}{dx}\right)^n + P_1\left(\frac{dy}{dx}\right)^{n-1} + P_2\left(\frac{dy}{dx}\right)^{n-2} + \dots + P_n = 0$$

1st order

Where $P_1, P_2, P_3, \dots, P_n$ are funcⁿ of x & y.

→ factorization → find → Solⁿ

Que $\left(\frac{dy}{dx}\right)^2 + 3\left(\frac{dy}{dx}\right) + 2 = 0$

Put $\boxed{\frac{dy}{dx} = P}$

$$P^2 + 3P + 2 = 0$$
$$P^2 + 2P + P + 2 = 0$$
$$P(P+2) + 1(P+2) = 0$$
$$(P+2)(P+1) = 0$$
$$P = -2 \ \& \ -1$$

$\frac{dy}{dx} = -2$	$\frac{dy}{dx} = -1$
$dy = -2dx$	$dy = -dx$
$\int dy = -2\int dx + C_1$	$\int dy = -\int dx + C_2$
$y = -2x + C_1$	$y = -x + C_2$

$(y + 2x - C_1 = 0)$ $(y + x - C_2 = 0)$

General Solⁿ ⇒ $\boxed{(y + 2x - C_1)(y + x - C_2) = 0}$

Ans.

Que. $y\left(\frac{dy}{dx}\right)^2 + (x-y)\left(\frac{dy}{dx}\right) - x = 0$

Let,

$\boxed{\frac{dy}{dx} = p}$ $\quad y(p)^2 + (x-y)p - x = 0$

$$yp^2 + xp - yp - x = 0$$

$$yp(p-1) + x(p-1) = 0$$

$$(p-1)(yp+x) = 0$$

$p - 1 = 0 \Rightarrow \boxed{\frac{dy}{dx} = 1} = \int dy = \int dx$

$y = x + C_1$

$\boxed{y - x - C_1 = 0}$

$yp + x = 0$

$p = -\frac{x}{y}$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$y\,dy = -x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + C_2$$

$$\frac{x^2}{2} + \frac{y^2}{2} = C_2$$

$$x^2 + y^2 = 2C_2$$

$$x^2 + y^2 - 2C_2 = 0$$

Ans. $\Rightarrow (y - x - C_1)(y^2 + x^2 - C_2) = 0$

★ Method - 2 → If function can't be factorized

$f(x, y, p) = 0$ — (1)

$y = f(x, p)$

↓

diff. w.r.t x

↓

Find p ⇒ Sol.^{n} ⇒

Factor Cancel hote hai!!

Que $y - 2px = \tan^{-1}(xp^2)$

$y = 2px + \tan^{-1}(xp^2)$ $\boxed{p = \frac{dy}{dx}}$

diff. w.r.t. x

$$\frac{dy}{dx} = 2\left[p.1 + x\frac{dp}{dx}\right] + \frac{1}{1+(xp^2)^2} \times \left[2xp\frac{dp}{dx} + p^2.1\right]$$

$$P = 2P + 2x\frac{dp}{dx} + \frac{2xP}{1+x^2p^4}\frac{dP}{dx} + \frac{p^2}{1+x^2p^4}$$

$$P - 2P - \frac{p^2}{1+x^2p^4} = \left(2x + \frac{2xP}{1+x^2p^4}\right)\frac{dp}{dx}$$

$$-p - \frac{p^2}{1+x^2p^4} = \left(\frac{2x + 2x^3p^4 + 2xP}{1+x^2p^4}\right)\frac{dp}{dx}$$

$$\frac{-p - x^2p^5 - p^2}{\cancel{1+x^2p^4}} = \left(\frac{2x + 2x^3p^4 + 2xp}{\cancel{1+x^2p^4}}\right)\frac{dP}{dx}$$

$$\frac{-p(p+x^2p^4+1)}{\cancel{(1+x)}} = 2x(1+x^2p^4+p)\frac{dp}{dx}$$

$$-p = 2x\frac{dp}{dx}$$

$$2\frac{dp}{p} = -\frac{dx}{x}$$

$$2\int\frac{dp}{p} = -\int\frac{dx}{x} + C$$

$$2\log p = -\log x + \log C$$

$$\log p^2 + \log x = \log C$$

$$\log(p^2 \cdot x) = \log C$$

$$p^2 \cdot x = C$$

$$p^2 = \frac{C}{x} \Rightarrow \boxed{p = \sqrt{\frac{C}{x}}}$$

Put (p) in eq (1)

$$y = 2\sqrt{\frac{C}{x}} \times x + \tan^{-1}\left(\cancel{x} \times \frac{C}{\cancel{x}}\right)$$

$$\boxed{y = 2\sqrt{Cx} + \tan^{-1} C}$$

- Method-3 -

When we apply Method-2

& (Factor Not Cancel)

↓

We should Apply Use (Linear D.E)

OR

(Separation of Variable)

Que. $y = 2px + p^n$ —(1)

$$\frac{dy}{dx} = 2\left[p\cdot 1 + x\frac{dp}{dx}\right] + np^{n-1}\frac{dp}{dx}$$

$$p = 2p + 2x\frac{dp}{dx} + np^{n-1}\frac{dp}{dx}$$

$$-p = (2x + np^{n-1})\frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{-p}{2x + np^{n-1}}$$

$$\frac{dx}{dp} = \frac{2x + np^{n-1}}{-p}$$

$$\frac{dx}{dp} = \frac{2x}{-p} - np^{n-2}$$

$$\frac{dx}{dp} + \frac{2}{p}x = -np^{n-2}$$

$$\boxed{P_1 = \frac{2}{p}}, \boxed{Q_1 = -np^{n-2}}$$

$$I.F. = e^{\int P dp}$$

$$= e^{2\int \frac{1}{p} dp}$$

$$= e^{2\log p} = e^{\log p^2} = p^2$$

$$\boxed{I.F. = p^2}$$

$$x\cdot(p^2) = \int -np^{n-2}\cdot p^2 dp$$

$$xp^2 = -n\int p^n dp + c$$

$$xp^2 = -n\left[\frac{p^{n+1}}{n+1}\right]$$

$$xp^2 = \left(\frac{-n}{n+1}\right)p^{n+1}$$

$$\boxed{x = \left(\frac{-n}{n+1}\right)p^{n-1}}$$ —(2)

(I) & (II) are Required eq.^{n}

Higher Order & Const coeff.

★ **Linear Differential equation with const. coeff.**

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_n y = R(x)$$

Where $a_1, a_2, a_3, \dots a_n$ are Constant

Put $\frac{d}{dx} = D$ $\quad (D^n + a_1 D^{n-1} + a_2 D^{n-2} + \dots a_n) \cdot y = R(x)$

Operator Form

$D = m, \; y = 1$

"Auxillary Equation" —

$m^n + a_1 m^{n-1} + a_2 m^{n-2} + \dots + a_n = 0$

→ Factor → Roots (m) → $\boxed{y = C.F + P.I}$

C.F. = Complementary func.^{n}
★ (L.H.S se Jo sol.^{n} milega)

P.I. = Particular integral ★ (R.H.S se Jo sol. milega)

When (i) $\boxed{R(x) = 0}$ Hence eq. ① is called Homogeneous D.E.

When (ii) $\boxed{R(x) \neq 0}$ Hence eq. ① is called

Non-Homogeneous D.E.

M.I.* → Jitne order ki eq.^{n} hoti hai utne Roots hote hai !!

Ex. → $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$ $\frac{d}{dx} = D$

$D^2 + 4D + 3y = 0$

Put $D = m,\ y = 1$

$m^2 + 4m + 3 = 0$

$m^2 + 3m + m + 3 = 0$

$m(m+3) + 1(m+3) = 0$

$(m+3)(m+1) = 0$

$\boxed{m = -1, -3}$

$y_c = C_1e^{-x} + C_2e^{-3x}$

$y_p = 0$

$y = y_c + y_p$

$\boxed{y = C_1e^{-x} + C_2e^{-3x}}$

Ans.

Ex. → $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$D^2 + 2D + y = 0$

Put $D = m,\ y = 1$

Auxilliary Sol.^{n}

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$\boxed{m = -1, -1}$

$y_c = (C_1 + C_2x)e^{-x}$

$y_p = 0$

$y = y_c + y_p$

$\boxed{y = (C_1 + C_2x)e^{-x}}$

Ans.

★ Find C.F. :-

① When two roots are real & distinct ($m = m_1, m_2, \ldots$)

$$C.F. = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$(m = m_1, m_2, m_3)$$

$$C.F. = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x} + \ldots$$

② When two roots are equal and real

$$C.F. = (C_1 + C_2 x) e^{m_1 x}$$

$$(m = m_1, m_2, m_3, \ldots)$$

$$C.F. = (C_1 + C_2 x + C_3 x^2) e^{m_1 x}$$

③ When roots are imaginary ($m = a + ib$)

$$C.F. = e^{ax} (C_1 \cos bx + C_2 \sin bx)$$

Que. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

O.E. $\Rightarrow D^2 + 2D + 2y = 0$

Put $D = m$, $y = 1$

A.E. $\Rightarrow m^2 + 2m + 2 = 0$

We use Quadratic Formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$a = 1$, $b = 2$, $c = 2$

$$m = \frac{-2 \pm \sqrt{4 - 4 \times 1 \times 2}}{2 \times 1}$$

$$m = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$m = \frac{-2 \pm \sqrt{4 i^2}}{2}$$

$$m = \frac{-2 \pm 2i}{2}$$

$$\boxed{m = -1 \pm i}$$

$y = C.F. + I.F.$

$y_c = C.F. \Rightarrow$

$y_c = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

$y_p = I.F. = 0$

$$\boxed{y = e^{-x}(c_1 \cos x + c_2 \sin x)}$$

Ans.

DATE 12/04/23

★ Finding P.I :- $P.I = \frac{1}{f(D)} R(x)$

(1) When $R(x) = e^{ax}$

$$P.I. = \frac{1}{f(D)} \cdot e^{ax} = \frac{1}{f(a)} \cdot e^{ax}$$

$\boxed{D = a}$, $\boxed{f(a) \neq 0}$

Que. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x}$ $\quad \frac{d}{dx} = D$

$$(D^2 + 4D + 3)y = e^{2x}$$

$D = m$, $y = 1$

A.E. ⇒ $m^2 + 4m + 3 = 0$

$$(m+1)(m+3) = 0$$

$$\boxed{m = -1, -3}$$

$$y_c = C_1 e^{-x} + C_2 e^{-3x}$$

$$y_P = \frac{1}{f(D)} R(x) = \frac{1}{D^2 + 4D + 3} \cdot e^{2x}$$

$\hookrightarrow \boxed{D = a = 2}$

$$y_P = \frac{1}{4 + 8 + 3} \cdot e^{2x} = \frac{e^{2x}}{15}$$

$y = y_c + y_P$ ⇒ $\boxed{y = C_1 e^{-x} + C_1 e^{-3x} + \frac{e^{2x}}{15}}$

Que. $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = \cosh 2x$

$$(D^2+2D+1)y = \frac{e^{2x}+e^{-2x}}{2}$$

$\boxed{D=m,\ y=1}$

$$m^2+2m+1=0$$
$$(m+1)^2=0$$
$$m=-1,-1$$

$$y_c = (C_1+C_2x)e^{-x},\quad y_p = \frac{1}{D^2+2D+1}\left(\frac{e^{2x}+e^{-2x}}{2}\right)$$

$$y_p = \left[\frac{1}{D^2+2D+1}(e^{2x}+e^{-2x})\right]\left(\frac{1}{2}\right)$$

$$y_p = \frac{1}{2}\left[\frac{e^{2x}}{\underbrace{D^2+2D+1}_{\boxed{D=2}}} + \frac{e^{-2x}}{\underbrace{D^2+2D+1}_{\boxed{D=-2}}}\right]$$

$\left[\because\ e^{ax}\ D=a\right]$

$$y_p = \frac{1}{2}\left[\frac{e^{2x}}{9}+\frac{e^{-2x}}{1}\right]$$

$$\boxed{y = y_c + y_p}$$

$$\Rightarrow \boxed{y = (C_1+C_2x)e^{-x} + \frac{1}{2}\left[\frac{e^{2x}}{9}+\frac{e^{-2x}}{1}\right]}$$

Ans

(ii) (2) When $R(x) = \sin ax$ (OR) $\cos ax$

$$P.I = \frac{1}{f(D^2)} \cdot \cos ax \Rightarrow \boxed{\frac{1}{f(-a^2)} \cos ax}$$

$$\boxed{D^2 = -a^2}$$

Que. $(D^2+1)y = \cos 2x$

$i = \sqrt{-1}$
$i^2 = -1$

$$m^2 + 1 = 0$$
$$m^2 = -1$$
$$m = \pm i = 0 \pm i$$

a , b

$$y_c = e^{ax}(C_1 \cos x + C_2 \sin x) \quad [\because a = 0]$$
$$y_c = e^{0x}(C_1 \cos x + C_2 \sin x)$$

$$y_p = \frac{1}{D^2+1} \cos 2x \quad [\because D^2 = -a^2]$$

a

$$\boxed{D^2 = -4} = -(2)^2$$

$$y_p = \frac{1}{-4+1} \cos 2x = \frac{\cos 2x}{-3}$$

$$y = y_c + y_p$$

$$\boxed{y = (C_1 \cos x + C_2 \sin x) + \frac{\cos 2x}{-3}}$$

Ans

Que. $(D^2+D+1) = \sin 2x$

$$m^2+m+1=0$$

$$\left(m^2+m+\frac{1}{4}\right)+1-\frac{1}{4}=0$$

$$\left(m+\frac{1}{2}\right)^2+\frac{3}{4}=0$$

$2ab = 1$
$ab = 1/2$

$\frac{1}{2}$ ka square add (OR) Subtract kar do...!

$$\left(m+\frac{1}{2}\right)^2 = -\frac{3}{4} \Rightarrow m+\frac{1}{2} = \frac{\sqrt{3}}{2}i$$

$$\boxed{m = \underbrace{\frac{-1}{2}}_{a} + \underbrace{\frac{\sqrt{3}}{2}}_{b}i}, \quad y_c = e^{-x/2}\left(C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right)$$

$$y_p = \frac{1}{D^2+D+1}\sin 2x \Rightarrow \frac{1}{-4+D+1}\sin 2x \Rightarrow \frac{1}{D-3}\sin 2x$$

$D^2 = -4$ $[\because D^2 = -a^2]$

$$y_p = \frac{(D+3)\sin 2x}{(D^2-9)} \Rightarrow \frac{(D+3)\sin 2x}{(-4-9)} = \frac{-1}{13}\left[(D+3)\sin 2x\right]$$

$$y_p = \frac{-1}{13}\left[\frac{d(\sin 2x)}{dx} + 3\sin 2x\right]$$

$$y_p = -\frac{1}{13}\left[2\cos 2x + 3\sin 2x\right]$$

$$\boxed{y = y_c + y_p}$$

Ans. $$y = e^{-x/2}\left(C_1\cos\frac{\sqrt{3}}{2}x + C_2\sin\frac{\sqrt{3}}{2}x\right) + \left(\frac{-1}{13}\right)(2\cos 2x + 3\sin 2x)$$

M.I* → ① $(1-D)^{-1} = 1 + D + D^2 + D^3 + \cdots$
② $(1+D)^{-1} = 1 - D + D^2 - D^3 + \cdots$

DATE 17/04/23

(iii) When $R(x) = x^m$ $(\forall\, m \in Z)$

$$P.I. = \frac{1}{f(D)} \times x^m \Rightarrow [f(D)^{-1}]\, x^m$$

Que/ $(D^2 + 4)y = x^2$

$D = m, \; y = 1$

A.E. :=> $m^2 + 4 = 0$

$m^2 = -4$

$m = \pm 2i = 0 \pm 2i$

$$y_c = e^{ax}(C_1 \cos bx + C_2 \sin bx)$$

$$y_c = e^{ax}(C_1 \cos 2x + C_2 \sin 2x)$$

$$y_p = \frac{1}{f(D)} R(x) \Rightarrow \frac{1}{D^2 + 4} x^2$$

$$y_p = \frac{1}{4\left(\frac{D^2}{4} + 1\right)} x^2 \Rightarrow y_p = \frac{1}{4}\left(\frac{D^2}{4} + 1\right)^{-1} x^2$$

$$y_p = \frac{1}{4}\left[1 - \left(\frac{D^2}{4}\right) + \left(\frac{D^2}{4}\right)^2\right] x^2$$ [See M.I on top]

$$y_p = \frac{1}{4}\left[x^2 - \left(\frac{2}{4}\right) - 0\right]$$

$$y_p = \frac{1}{4}\left[x^2 - \frac{2}{4}\right] = \frac{1}{4}\left(x^2 - \frac{1}{2}\right)$$

$$\boxed{y = y_c + y_p}$$

$$M.I \Rightarrow \left[\frac{1}{D} = \int (\text{Integration})\right]$$

Marks Plus DATE PAGE

Que. 2 $(D^3 - D^2 - 6D)y = 1+x^2$

$D = m,\ y = 1$

$(m^3 - m^2 - 6m) = 0 \leftarrow$ Auxillary eq^n

$m(m^2 - m - 6) = 0$

$m(m^2 - 3m + 2m - 6) = 0$

$m[m(m-3) + 2(m-3)] = 0$

$\boxed{m = 0,\ m = 3,\ m = -2}$ ← ∵ The Roots are real & distinct

$$\left[y_c = C_1 e^{0x} + C_2 e^{-2x} + C_3 e^{3x}\right]$$

$$y_p = \frac{1}{(D^3 - D^2 - 6D)}(1+x^2) \Rightarrow \frac{1}{-6D\left[1 - \frac{(D^3 - D^2)}{6D}\right]}(1+x^2)$$

$$y_p = \frac{-1}{6D} \times \left[1 - \left(\frac{D^2}{6} - \frac{D}{6}\right)\right]^{-1}(1+x^2)$$ ∵ See M.I★ of previous page

$$y_p = \frac{-1}{6D}\left[1 + \left(\frac{D^2}{6} - \frac{D}{6}\right) + \left(\frac{D^2}{6} - \frac{D}{6}\right)^2\right](1+x^2)$$

$$y_p = \frac{-1}{6D}\left[1 + \frac{D^2}{6} - \frac{D}{6} + \frac{D^4}{36} + \frac{D^2}{36} - 2 \times \frac{D^2}{6} \times \frac{D}{6}\right](1+x^2)$$

$$y_p = -\frac{1}{6D}\left[1 + x^2 + \left(\frac{2}{6} - \frac{2x}{6}\right) + \left(0 + \frac{2}{36} + 0\right)\right]$$

$\left[\because \frac{1}{D} = x\right]$

$$y_p = \frac{-1}{6D}\left[\frac{x^2}{D} - \frac{x}{3D} + \frac{25}{18D}\right] \Rightarrow \frac{-1}{6}\left[\frac{x^3}{3} - \frac{x^2}{3} - \frac{25}{18}x\right]$$

$\left[\because \frac{1}{D} = \int\right]$

$\boxed{y = y_c + y_p}$ Ans.

Que.3 $(D^2+3D+2)y = e^{-x}$

$D = m,\ y = 1$

$(m^2+3m+2) = 0$

$(m+1)(m+2) = 0$

$m = -1, -2$

$$\boxed{y_c = C_1 e^{-x} + C_2 e^{-2x}}$$

Now,

$$P.I. = \frac{1}{f(D)} R(x)$$

$$y_p = \frac{1}{D^2+3D+2}(e^{-x}) \Rightarrow \frac{1}{1-3+2} e^{-x} = \frac{e^{-x}}{0}$$

$\boxed{D=a}$ here $(a=-1)$

Diffren. denominator w.r.t to D & multiple numerator by x

$$y_p = \frac{x}{2D+3} e^{-x} = \frac{x}{2(-1)+3} e^{-x} = \frac{x e^{-x}}{1}$$

$$y = y_c + y_p$$

Ans:

$$\boxed{y = C_1 e^{-x} + C_2 e^{-2x} + x e^{-x}}$$

(4) $(D^2+4D+3)y = e^{-3x}+e^{2x}$

$$m^2+4m+3=0$$
$$m^2+3m+m+3=0$$
$$m(m+3)+1(m+3)=0$$
$$m=-1,-3$$

$$y_c = C_1 e^{-x} + C_2 e^{-3x}, \quad y_p = \frac{1}{f(D)} R(x)$$

$$y_p = \frac{1}{D^2+4D+3}(e^{-3x}+e^{2x}) = \frac{e^{-3x}}{D^2+4D+3} + \frac{e^{2x}}{D^2+4D+3}$$

$\hookrightarrow D=-3$ $\quad \hookrightarrow D=2$

$$y_p = \frac{e^{-3x}}{9+(-12)+3} + \frac{e^{2x}}{4+8+3} = \frac{e^{-3x}}{0} + \frac{e^{2x}}{15}$$

$$y_p = \frac{x e^{-3x}}{2(-3)+4} + \frac{e^{2x}}{15} \Rightarrow \frac{xe^{-3x}}{-2} + \frac{e^{2x}}{15}$$

$$y = y_c + y_p$$

$$\boxed{y = C_1 e^{-x} + C_2 e^{-3x} + \frac{xe^{-3x}}{-2} + \frac{e^{2x}}{15}}$$ Ans.

(5) $(D^2+4)y = \cos 2x$ (a)

$$m^2+4=0$$
$$m = \pm 2i$$

$$y_c = C_1 \cos 2x + C_2 \sin 2x$$

$$y_p = \frac{1}{D^2+4}\cos 2x$$

$$D^2 = -a^2 = -4$$

$$y_p = \frac{1}{-4+4}\cos 2x$$

$$y_p = \frac{1}{D} \cos 2x$$

then $\frac{x \cos 2x}{2D} \longrightarrow$ Integration

$$\frac{x}{2} \times \frac{1}{D} (\cos 2x) \Rightarrow \frac{x}{2} \times \frac{\sin 2x}{2} = \frac{x \sin 2x}{4}$$

$$\boxed{y = C_1 \cos 2x + C_2 \sin 2x + \frac{x \sin 2x}{4}}$$

H.W (1) $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$

Incomplete

$$m(m^2 + 2m + 1) = 0$$
$$m(m+1)^2 = 0$$
$$m = 0, -1, -1$$

$$y_c = C_1 e^{0x} + (C_2 + C_3 x) e^{-x}$$

$$\left[y_p = \frac{1}{f(D)} x^2 \right]$$

$$y_p = \frac{1}{D^3 + 2D^2 + D} (e^{2x} + x^2 + x)$$

$$y_p = \frac{e^{2x}}{D^3 + 2D^2 + D} + \frac{x^2}{D^3 + 2D^2 + D} + \frac{x}{D^3 + 2D^2 + D}$$

$\hookrightarrow D = 2$

$$y_p = \frac{e^{2x}}{8+8+2} + \frac{x^2}{D(D^2 + 2D + 1)} + \frac{x}{D(D^2 + 2D + 1)}$$

DATE 18/04/23

(iv) (4) When $\boxed{R(x) = e^{ax} \cdot v}$ [v is a funcⁿ of x]

$$\boxed{P.I. = \frac{1}{f(D)} \cdot e^{ax} v \Rightarrow \frac{e^{ax}}{f(D+a)} \cdot v}$$

$\boxed{D = D \neq a}$

Ex. 1 $(D^2-4)y = e^x \cdot \sin x$

A.E. $\Rightarrow m^2 - 4 = 0$
$m^2 = 4$
$m = \pm 2$

$y_c = C_1 e^{-2x} + C_2 e^{2x}$

$y_p = \frac{1}{(D^2-4)}$

$$y_p = \frac{1}{D^2-4} e^{1x} \sin x \quad , \quad \boxed{D = D+a = D+1}$$

e^{ax}

$$y_p = \frac{e^x}{(D+1)^2 - 4} \sin x \Rightarrow \frac{e^x}{D^2+2D-3} \sin x$$

$$\boxed{D^2 = -1} \quad y_p = \frac{e^x}{-1+2D-3} \sin x \Rightarrow \frac{e^x}{(2D-4)} \sin x$$

$$y_p = \frac{e^x}{2(D-2)} \sin x \Rightarrow \frac{e^x}{2}\left[\frac{1}{(D-2)} \sin x\right] \Rightarrow \frac{e^x}{2}\left[\frac{(D+2)}{D^2-4} \sin x\right]$$

$$y_p = \frac{e^x}{2}\left[\frac{D+2}{-5} \sin x\right] \Rightarrow \frac{e^x}{10}\left[\cos x + 2 \sin x\right]$$

$$y = y_c + y_p$$

(2) $(D^2-6D+13)y = 8e^{3x}\cdot \cos 2x$

$m^2-6m+13=0$

$(m^2-6m+9)+13-9=0$

$(m-3)^2+4=0$

$(m-3)^2=-4$

$m-3=\pm 2i$

$\boxed{m=3\pm 2i}$

$y_c = e^{3x}(C_1\cos 2x + C_2 \sin 2x)$

$y_P = \dfrac{1}{D^2-6D+13}\cdot 8e^{3x}\cos 2x$

$$y_P = 8\left[\frac{1}{D^2-6D+13}\cdot e^{3x}\cos 2x\right]$$

$\boxed{D=D+3}$

$$y_P = 8\left[\frac{e^{3x}\cdot\cos 2x}{(D+3)^2-6(D+3)+13}\right]$$

$$y_P = 8\left[\frac{e^{3x}\cdot\cos 2x}{D^2+\cancel{6D}+9-\cancel{6D}-18+13}\right]$$

$$y_P = 8\left[\frac{e^{3x}}{D^2+4}\cos 2x\right] \quad \boxed{D^2=-4}$$

$$y_P = 8e^{3x}\left[\frac{1}{D^2+4}\cos 2x\right] \Rightarrow 8e^{3x}\left[\frac{x}{2D}\cos 2x\right]$$

$\frac{1}{D}=\int$

$$y_P = 8e^{3x}\left[\int x\cos 2x\right] \Rightarrow 8e^{3x}\left[\frac{x}{2}\times\frac{\sin 2x}{2}\right]$$

Ans $$\boxed{y_P = 8e^{3x}\left[\frac{x\sin 2x}{4}\right]}$$

(v) ★ (5) When $\boxed{R(x) = x \cdot v}$

$$P.I. = \frac{1}{f(D)} x \cdot v \Rightarrow x \cdot \frac{1}{f(D)} v - \frac{f'(D)}{[f(D)]^2} v$$

Que.1 $(D^2 - 2D + 1)y = x\sin x$

$$m^2 - 2m + 1 = 0$$
$$(m-1)^2 = 0$$
$$m = 1, 1$$

$$y_c = (C_1 + C_2 x)e^x$$

$$y_P = \frac{1}{D^2 - 2D + 1} x \cdot \sin x$$

$$y_P = x \times \frac{1}{(D^2 - 2D + 1)} \sin x - \frac{(2D-2)\sin x}{(D^2 - 2D + 1)^2}$$

$$y_P = \left[\frac{x\sin x}{-1 - 2D + 1} - \frac{(2D-2)\sin x}{4D^2}\right]$$

$$y_P = \left[\frac{x}{-2D}\sin x - \frac{(2D-2)}{4D^2}\sin x\right]$$

$$\left[\frac{1}{D} = \int\right]$$

$$y_P = \left[\frac{x\cos x}{2} + \frac{(2D-2)}{4}\sin x\right]$$

$$y_P = \left[\frac{x\cos x}{2} + \frac{2\cos x}{4} - 2\sin x\right]$$

$$\boxed{y = y_c + y_P}$$

★ First Order & higher degree :-

* Clairaut's equation :- (Method - 02)

$$\boxed{y = px + f(p)}$$

diff. w.r.t (x)

$p = C \rightarrow$ Sol.^{n}

Que. $\cos y \cos px + \sin y \sin px = p$

$$\cos(y - px) = p$$

$$y - px = \cos^{-1} p$$

$$\boxed{y = \cos^{-1} p + px} \quad (1)$$

$$\frac{dy}{dx} = p \cdot 1 + x\frac{dp}{dx} + \frac{(-1)}{\sqrt{1-p^2}}\frac{dp}{dx}$$

$$0 = \left(x - \frac{1}{\sqrt{1-p^2}}\right)\frac{dp}{dx}$$

$$\frac{dp}{dx} = 0 \Rightarrow dp = 0 \Rightarrow \int dp = \int 0$$

$$\boxed{p = C}$$ Ans.