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Chapter-1

Differential Equation

★ **Taylor's theoram** →

If $f(a+h)$ where a is independent of (h), be a func.^{n} of (h) Such that it can be expanded ascending powers of (h) and this expansion can be differentiable at no. of times then theoram state that:

$$f(a+h) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \dots \quad (1)$$

• **Cor. 1** → Putting $\boxed{a = x}$ in eq. (1)

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots$$

• **Cor. 2** → Putting $\boxed{h = x-a}$, in eq. (1), we get

$$f(x) = f(a) + f'(a)(x-a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \dots$$

▸ Put $a = 0$ and $h = x$ in eq. (1) → it becomes Maclaurin's theoram

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Que Expand $\sin x$ in powers of $\left(x-\frac{\pi}{2}\right)$?

Sol$^n \Rightarrow$ $f(x) = \sin x$

$$f(a+h) = f(a+x-a) = f\left(\frac{\pi}{2}+x-\frac{\pi}{2}\right) = f(x)$$

$$f(x) = \sin x \Rightarrow f\left(\frac{\pi}{2}\right) = 1$$

$$f'(x) = \cos x \Rightarrow f'\left(\frac{\pi}{2}\right) = 0$$

$$f''(x) = -\sin x \Rightarrow f''\left(\frac{\pi}{2}\right) = -1$$

$$f'''(x) = -\cos x \Rightarrow f'''\left(\frac{\pi}{2}\right) = 0$$

$$f^{IV}(x) = \sin x \Rightarrow f^{IV}\left(\frac{\pi}{2}\right) = 1$$

By Taylor's theoram

Cor. 2 $\quad f(x) = f(a) + f'(a)(x-a) + \frac{(x-a)^2}{2!}f''(a) + \ldots$

$$\sin x = 1 + \left(x-\frac{\pi}{2}\right)\cdot 0 + \frac{\left(x-\frac{\pi}{2}\right)^2}{2}(-1) + \cancel{0} + \frac{\left(x-\frac{\pi}{2}\right)^4}{4!}(1) + \ldots$$

$$\therefore \frac{\left(x-\frac{\pi}{2}\right)^3}{3!}\cdot 0$$

$$\boxed{\sin x = 1 - \frac{\left(x-\frac{\pi}{2}\right)^2}{2!} + \frac{\left(x-\frac{\pi}{2}\right)^4}{4!} + \ldots}$$

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Assignment!

Que. Expand $\tan x$ in powers of $\left(x+\frac{\pi}{4}\right)$ As far as x^4 terms and evalute $\tan(46.5^\circ)$ Singnificant to 4 digits.

Que. $\tan\left(x+\frac{\pi}{4}\right)$

Ans. $\Rightarrow \tan\left(x+\frac{\pi}{4}\right) = 1+2x+2x^2+\frac{8}{3}x^3+\frac{10}{3}x^4+\ldots$

Taylor's theoram

$$f(a+h) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a)$$

$$\boxed{a=\frac{\pi}{4}, \quad h=x}$$

$$f\left(x+\frac{\pi}{4}\right) = \tan\left(x+\frac{\pi}{4}\right)$$

$$f(x) = \tan x$$

$$f(a) = \tan a$$

$$f\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$f'(x) = \sec^2 x$$

$$f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right)$$

$$f'\left(\frac{\pi}{4}\right) = (2)^2 = \underline{\underline{4}}$$

put $x = 1.5$

$$\tan(46.5^\circ) = 1 + 2(1.5) + 2(1.5)^2 + \frac{8}{3}(1.5)^3 + \ldots$$

$$\tan(46.5) = 1.0537$$

★ **Maxima - Minima of func^n of two variables →**

Let $f(x,y)$

★ **Working Rule →**

(1) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(2) Solve the eq.^{n} $\frac{\partial f}{\partial x}=0$, $\frac{\partial f}{\partial y}=0$, we get x & y.

then,

The Pair of x & y thus obtain stationary value of $f(x,y)$. Let (a,b) be one of these pairs

(3) Find $r=\left(\frac{\partial^2 f}{\partial x^2}\right)_{(a,b)}$, $t=\left(\frac{\partial^2 f}{\partial y^2}\right)_{(a,b)}$, $s=\left(\frac{\partial^2 f}{\partial x\,\partial y}\right)_{(a,b)}$ at the point (a,b).

and Calculate $rt-s^2$.

(4) (i) If $rt-s^2$ is positive and r is greater than to 0, then $f(x,y)$ has ~~fun~~ minimum at (a,b).

(ii) If $rt-s^2$ is positive and r is smaller than to 0, then $f(x,y)$ has maximum at (a,b).

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(iii) If $rt - s^2$ is negative, then $f(x,y)$ has neither maximum nor minimum at (a,b).

(iv) If $rt - s^2$ is zero, then ~~$f(x,y)$~~ has case is doubtful.

Que. Discuss the maximum or minimum value of the func^n $f(x,y)$ is equals to $x^3 - 4xy + 2y^2$.

$$f(x,y) = x^3 - 4xy + 2y^2$$
$$f = x^3 - 4xy + 2y^2$$

$$\frac{\partial f}{\partial x} = 3x^2 - 4y, \quad \frac{\partial f}{\partial y} = -4x + 4y$$

$$3x^2 - 4y = 0$$
$$3x^2 - 4x = 0$$
$$x(3x-4) = 0$$
$$\boxed{x=0} \quad 3x - 4 = 0$$
$$\boxed{x = \frac{4}{3}}$$

$$-4x + 4y = 0$$
$$\boxed{4y = 4x}$$
$$\boxed{x = y}$$

$$\boxed{y = \frac{4}{3}} \quad \boxed{y = 0}$$

Points are $\rightarrow \left(\frac{4}{3}, \frac{4}{3}\right) \& (0,0)$ are critical points.

$$r = \left(\frac{\partial^2 f}{\partial x^2}\right)_{(0,0)} = (6x)_{(0,0)} = 0 \Rightarrow \boxed{r = 0}$$

$$t = \left(\frac{\partial^2 f}{\partial y^2}\right)_{(0,0)} = (4)_{(0,0)} = 4 \Rightarrow \boxed{t = 4}$$

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$$s = \left(\frac{\partial^2 f}{\partial x \partial y}\right)_{(0,0)} = \left[\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\right]_{(0,0)} = \left[\frac{\partial}{\partial x}(-4x+4y)\right]_{(0,0)}$$

$s = (-4)_{(0,0)} = -4 \Leftrightarrow \boxed{s = -4}$

Calculate $rt - s^2 = 0 - 16 = -16$ (-ve)

If $rt - s^2$ is (-ve) then $f(x,y)$ is neither max. nor minimum at (0,0)

Calculate At $= \left(\frac{4}{3}, \frac{4}{3}\right)$ $\underbrace{r = 8,\ t = 4,\ s = -4}_{\text{Same as phle Jaisa}}$

$rt - s^2 = 8 \times 4 - 16 = 32 - 16$

$rt - s^2 = 16 = +ve \Rightarrow \left\{ \begin{matrix} rt - s^2 = +ve \\ r = +ve \end{matrix} \right\}$

Hence, $f(x,y)$ is minimum at $\left(\frac{4}{3}, \frac{4}{3}\right)$.

The minimum Value of $f\left(\frac{4}{3}, \frac{4}{3}\right) = x^3 - 4xy + 2y^2$

$$= \left(\frac{4}{3}\right)^3 - 4\left(\frac{4}{3}\right)\left(\frac{4}{3}\right) + 2\left(\frac{4}{3}\right)^2$$

$$= \frac{-32}{27}.$$

$(-4x+4y)$

$(0,0)$

$ax.$

Faisa

Que. Discuss the maximum and minimum value of the func.^{n}.

① $u = x^3 + y^3 - 3axy$ ② Assignment! $u = x^3y^2(1-x-y)$

$f = x^3 + y^3 - 3axy$

$$\frac{\partial f}{\partial x} = 3x^2 - 3ay \quad , \quad \frac{\partial f}{\partial y} = 3y^2 - 3ax$$

$$0 = 3x^2 - 3ay \quad , \quad 0 = 3y^2 - 3ax$$

$$3ay = 3x^2 \quad , \quad 3ax = 3y^2$$

$$ay = x^2 \qquad ax = y^2$$

$$\boxed{a = \frac{x^2}{y}}$$

$$\frac{x^2}{y} \cdot x = y^2$$

$$x^3 = y^3$$

$$\boxed{x = y}$$

$$r = \left(\frac{\partial^2 f}{\partial x^2}\right) = (6x)_{(a,a)} = 6a$$

$$t = \left(\frac{\partial^2 f}{\partial y^2}\right) = (6y)_{(a,a)} = 6a$$

$$s = \left(\frac{\partial^2 f}{\partial x \partial y}\right) = \left[\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\right] = \left[\frac{\partial}{\partial x}(3y^2 - 3ax)\right]$$

$$\boxed{s = -3a}$$

Calculate $rt - s^2 = 6a \times 6a - (-3a)^2 = 36a^2 - 9a^2$

$rt - s^2 = +ve$ $= 27a^2$

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Hence $rt - s^2$ is +ve and r is −ve or +ve, according as (a). We have maximum and minimum according as (a) is −ve or +ve,

For $(x = y = a)$.

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★ Curvature, Radius of Curvature, Centre of curvature

① Curvature (K) :- Let P & Q are two neighbouring point. On a given continuous curve such that arc PQ = δS and arc $A'P = S$ where A' is fixed point on the curve. Again suppose that the tangent at the point P and Q makes angle ψ, $\psi + \delta\psi$ respectively with the x-axis then $\delta\psi$ is called total curvature of the arc PQ.

② The ratio $\frac{\delta \psi}{\delta S}$ is called avg. curvature

③ $\lim_{\delta x \to 0} \frac{\delta \psi}{\delta S}$ or $\frac{\delta\psi}{\delta s}$ is called curvature of the arc PQ.

It is denoted by Kappa (K).

④ Reciprocal of the curvature is called Radius of curvature. Radius of curvature is denoted by (ρ)

★ Carterion formula of Radius of Curvature

$$\boxed{(\rho) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}}$$

★ Pedal formula for Radius of Curva. :-

~~$\rho = r\frac{dp}{dr}$~~

$$\boxed{\rho = r\left(\frac{dr}{dp}\right)}$$

where $\boxed{p = r\sin\phi}$

★ Centre of Curvature (β) :-

$$\boxed{\beta = y + \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}}$$

and $$\boxed{\alpha = x - \frac{\left(\frac{dy}{dx}\right)\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}}$$

★ Circle of Curvature ( ) :-

$$\boxed{(x-\alpha)^2 + (y-\beta)^2 = (\rho)^2}$$

- evolute locus of centre of Curv. (α, β) is called evolute of curve.

Que. Find the Radius of Curvature of the given curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$, at the point $\left(\frac{1}{4},\frac{1}{4}\right)$.

$$\sqrt{x}+\sqrt{y}=\sqrt{a} \quad \text{— (1)}$$

diff. w.r.t (x)

$$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-\sqrt{y}}{\sqrt{x}}=-y^{1/2}x^{-1/2}$$

diff. w.r.t (x)

$$\frac{d^2y}{dx^2}=+\frac{1}{2}y^{1/2}x^{-3/2}-\frac{1}{2}x^{-1/2}y^{-1/2}\frac{dy}{dx}$$

$$=\frac{1}{2}\frac{\sqrt{y}}{x^{3/2}}-\frac{1}{2\sqrt{x}\cdot\cancel{\sqrt{y}}}\cdot\left(\frac{-\cancel{\sqrt{y}}}{\sqrt{x}}\right)$$

$$=\frac{1}{2}\left[\frac{\sqrt{y}+\sqrt{x}}{x\sqrt{x}}\right]=\frac{1}{2}\frac{\sqrt{a}}{x^{3/2}} \text{ from eq. (1)}$$

Radius of Curvature

$$\rho(x,y)=\frac{\left[1+\left(\frac{-\sqrt{y}}{\sqrt{x}}\right)^2\right]^{3/2}}{\frac{\sqrt{a}}{2x^{3/2}}}=\frac{2x^{3/2}\left(1+\frac{y}{x}\right)^{3/2}}{\sqrt{a}}$$

$$\boxed{\rho(x,y)=\frac{2(x+y)^{3/2}}{\sqrt{a}}} \leftarrow \frac{2x^{3/2}\left(\frac{x+y}{x}\right)^{3/2}}{\sqrt{a}}$$

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$$\rho\left(\frac{1}{4},\frac{1}{4}\right) = \frac{2\left(\frac{1}{4}+\frac{1}{4}\right)^{3/2}}{\sqrt{a}} = \frac{2\left(\frac{\cancel{2}\times 1}{2\ \cancel{4}}\right)^{3/2}}{\sqrt{a}}$$

$$\rho\left(\frac{1}{4};\frac{1}{4}\right) = \frac{2\left(\frac{1}{8}\right)^{\frac{1}{2}}}{\sqrt{a}} = \frac{\cancel{2}\ 1}{\cancel{2}\sqrt{2a}}$$

$$\rho\left(\frac{1}{4},\frac{1}{4}\right) = \frac{1}{\sqrt{2a}}$$

Assignment.

Que. Find the Radius of Curva at the point P the given is cycloid $x = a(t+\sin t)$ $y = a(1-\cos t)$.

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Que. Find the coordinates of the centre of curvature for the point (x, y) on the Parabola $y^2 = 4ax$ and also find eq.^{n} of the evolute of parabola.

Here $y^2 = 4ax$ — (1)

diff. w.r.t. (x)

$$2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y} = \frac{\sqrt{a}}{\sqrt{x}}$$

diff. w.r.t (x)

$$\frac{d^2y}{dx^2} = -\frac{\sqrt{a}}{2}x^{-3/2} = -\frac{\sqrt{a}}{2x^{3/2}} \Rightarrow \boxed{\frac{d^2y}{dx} = \frac{-\sqrt{a}}{2x^{3/2}}}$$

At point (x, y) the coordinates of Centre of curvature

$$\beta = y + \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}} = y + \frac{\left[1 + \left(\frac{\sqrt{a}}{\sqrt{x}}\right)^2\right]}{\frac{-\sqrt{a}}{2x^{3/2}}}$$

$$\beta = y + \left[\left[\frac{(x+a)}{x}\right] \frac{\times 2x^{3/2}}{-\sqrt{a}}\right] = y + \left[\frac{x+a}{-\sqrt{a}} \times 2x^{\frac{3}{2}-1}\right]$$

$$\beta = y + \frac{(x+a)}{-\sqrt{a}} \times 2x^{1/2}$$

$$\beta = \frac{-\sqrt{a}y + 2x^{3/2} + 2x^{1/2}a}{-\sqrt{a}} = \frac{-\sqrt{a}\sqrt{4ax} + 2x^{3/2} + 2x^{1/2}a}{-\sqrt{a}}$$

$$\beta = -\left(\frac{4ax^{1/2} + 2x^{3/2}}{\sqrt{a}}\right) \qquad \beta = -\left(\frac{2a\sqrt{x} + 2x^{3/2} + 2x^{1/2}a}{\sqrt{a}}\right)$$

$$\boxed{\beta = -2x^{3/2}a^{-1/2}} \quad — (3)$$

$$\boxed{\alpha = 3x + 2a} \quad — (4)$$

Thus the coordinates of centre of curvature is $\left(3x+2a, \frac{-2}{\sqrt{a}}x^{3/2}\right)$.

To evolute

$$3x = \alpha - 2a$$

$$\boxed{x = \frac{\alpha - 2a}{3}} \quad — (5)$$

$$\boxed{\beta = \frac{-2x^{3/2}}{\sqrt{a}}}$$

Both side square

$$\beta^2 = \frac{4x^3}{a} =$$

$$\boxed{x^3 = \frac{a\beta^2}{4}} \text{ from eq. (6)}$$

From eq. (5) & (6)

$$\left(\frac{\alpha-2a}{3}\right)^3 = \frac{a\beta^2}{4}$$

$$27a\beta^2 = 4(\alpha - 2a)^3$$

The eq.^{n} of locus of (α, β)

$$\boxed{27ay^2 = 4(x-2a)^3}$$

"श्री गणेशाय नमः"

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UNIT - II

"Partial Differentiation"

(1) Euler's theoram (2) Error & approximation
(3) Partial differention. → 12 marks

Ques. If $z(x+y) = x^2 + y^2$, show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

Here ⇒ $z = \dfrac{x^2+y^2}{(x+y)}$

$$\frac{\partial z}{\partial x} = \frac{\frac{\partial(x^2+y^2)}{\partial x}(x+y) - \frac{\partial(x+y)}{\partial x}(x^2+y^2)}{(x+y)^2}$$

$$\frac{\partial z}{\partial x} = \frac{2x(x+y) - (x^2+y^2)}{(x+y)^2}$$

$$\frac{\partial z}{\partial x} = \frac{2x^2 + 2xy - x^2 - y^2}{(x+y)^2}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{x^2 - y^2 + 2xy}{(x+y)^2}}$$

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$$z = \frac{x^2+y^2}{(x+y)}$$

$$\frac{\partial z}{\partial y} = \frac{\frac{d(x^2+y^2)}{d\cancel{x}y}(x+y) - \frac{d(x+y)}{dy}(x^2+y^2)}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{2y(x+y) - (x^2+y^2)}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{2xy + 2y^2 - x^2 - y^2}{(x+y)^2}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{y^2 - x^2 + 2xy}{(x+y)^2}}$$

$$z = \frac{x^2+y^2}{(x+y)}$$

$$\left[\left(\frac{\partial z}{\partial x}\right) - \frac{\partial z}{\partial y}\right]^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

$$\left[\frac{x^2-y^2+2xy}{(x+y)^2} - \frac{(y^2-x^2+2xy)}{(x+y)^2}\right]^2$$

$$= 4\left(1 - \left[\frac{x^2-y^2+2xy}{(x+y)^2}\right] - \left(\frac{y^2-x^2+2xy}{(x+y)^2}\right)\right)$$

$$\left[\frac{x^2-y^2\cancel{+2xy}-y^2+x^2\cancel{-2xy}}{(x+y)^2}\right]^2 = \underbrace{\quad}_{R.H.S}$$

$$\left[\frac{2x^2-2y^2}{(x+y)^2}\right]^2 \Rightarrow \left[\frac{2(x^2-y^2)}{(x+y)^2}\right]^2 = \frac{4\cancel{(x+y)^2}(x-y)^2}{\cancel{(x+y)^2}(x+y)^2}$$

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$$= \frac{4(x-y)^2}{(x+y)^2} \Leftarrow L.H.S$$

Now $R.H.S \rightarrow$

$$4\left(1-\left(\frac{dz}{dx}+\frac{dz}{dy}\right)\right] = 4\left[1-\frac{4xy}{(x+y)^2}\right]$$

$$= 4\left[\frac{(x+y)^2-4xy}{(x+y)^2}\right] = \frac{4(x-y)^2}{(x+y)^2}$$

R.H.S

$\boxed{L.H.S = R.H.S}$ Hence Proved.

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Que. 2 If $u = (x^2+y^2+z^2)^{-1/2}$, $x^2+y^2+z^2 \neq 0$

Prove that

(1) $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -u$$

Assignment (2) $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 z}{\partial z^2} = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\cancel{2}x}{\cancel{2}}(x^2+y^2+z^2)^{-3/2} = -\frac{x}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial u}{\partial y} = -\frac{\cancel{2}y}{\cancel{2}}(x^2+y^2+z^2)^{-3/2} = -\frac{y}{(x^2+y^2+z^2)^{3/2}}$$

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$$\frac{\partial u}{\partial z} = \frac{-z}{(x^2+y^2+z^2)^{3/2}}$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = \frac{-1}{(x^2+y^2+z^2)^{3/2}}(x^2+y^2+z^2)$$

$$= -(x^2+y^2+z^2)^{-1/2} = -u$$

Hence Proved.

Assignment Que. 2 If $u = e^{xyz}$, Prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1+3xyz+x^2y^2z^2)e^{xyz}$

Assignment Que. 3 If $u = \log(x^3+y^3+z^3-3xyz)$

Prove That (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 \cdot u = \frac{-9}{(x+y+z)^2}$.

Que. 4 Assignment If $x^x y^y z^z = C$ then. Prove that

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}.$$

$$\log(x^x y^y z^z) = \log C$$

$$x\log x + y\log y + z\log z = \log C$$

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$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$^2+z^2)$

$$\frac{\partial z}{\partial y} \Rightarrow \frac{\partial \left(x\log x + y\log y + z\log z\right)}{\partial y} = \frac{\partial (\log C}{\partial y}$$

$$\frac{\partial (x\log x)}{\partial y} + \frac{\partial (y\log y)}{\partial y} + \frac{\partial (z\log z)}{\partial y} = 0$$

$$0 + \left(\log y + y\cdot\frac{1}{y}\right) + \frac{\partial z}{\partial y}\cdot\log z + \frac{\partial(\log z)\cdot z}{\partial y} = 0$$

$$\log y + y\cdot\frac{1}{y} + \frac{\partial z}{\partial z}\cdot\frac{dz}{\partial y}\cdot\log z + \frac{\partial(\log z)}{\partial z}\cdot\frac{\partial z}{\partial y}\cdot z = 0$$

$$\log y + y\cdot\frac{1}{y} + \log z\cdot\frac{\partial z}{\partial y} + z\cdot\frac{1}{z}\cdot\frac{dz}{\partial y} = 0$$

$$\frac{\partial z}{\partial y} = -\frac{[1+\log y]}{[1+\log z]}$$

$$\frac{\partial z}{\partial x} = -\frac{[1+\log x]}{[1+\log z]}$$

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★ Euler's theoram –

If $f(x,y)$ is a homogenous func^n of x,y is ~~f(x,y)~~ of degree n then,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

Relation b/w second order derivative.

$$x^2\frac{\partial^2 f}{\partial x^2} + xy\frac{\partial^2 f}{\partial x\,\partial y} = (n-1)\frac{\partial f}{\partial x}$$

$$xy\frac{\partial^2 f}{\partial y\,\partial x} + y^2\frac{\partial^2 f}{\partial y^2} = (n-1)\frac{\partial f}{\partial y}$$

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x\,\partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f$$

Que. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$

(i) $\frac{x\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(ii) $\frac{x^2\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$

Assignment

$= 2\cos 3u \sin u$

$\frac{\partial y}{\partial x} = f\left(\frac{y}{x}\right)$

$\tan u = \frac{x^3+y^3}{x-y} = \frac{x^{\cancel{3}2}}{\cancel{x}}\left[\frac{1+(y/x)^3}{1-(y/x)}\right]$

$\boxed{n=2}$

Let $f = \tan u$

→ By Euler's theoram,

$$\frac{x\partial f}{\partial x} + \frac{y\partial f}{\partial y} = nf$$

$$x\frac{\partial(\tan u)}{\partial x} + y\frac{\partial(\tan u)}{\partial y} = 2\tan u$$

$u = f\left(\frac{y}{x}\right) \quad x\left(\frac{du}{\partial x}\right) + y\left(\frac{du}{dy}\right)$

$$x \sec^2 u \frac{\partial u}{\partial x} + y \sec^2 u \frac{\partial u}{\partial y} = 2\tan u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u \cdot \cos^2 u$$
$$= 2 \sin u \cos u$$
$$= \sin 2u \text{ Ans.}$$

sin2u

Assign. Que. 1 If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

Assign. Que 2 If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, verify the Euler's theoram

Ans-1) $\sin u = \frac{x^2+y^2}{(x+y)} = \frac{x^2}{x}\left(\frac{1+\left(\frac{y}{x}\right)^2}{1+\frac{y}{x}}\right)$ $\boxed{n=1}$

$f = \sin u$

By Euler's theoram

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

$$x \cos u \frac{du}{dx} + y \cos u \frac{du}{dy} = \sin u$$

$$x\frac{du}{dx} + y\frac{du}{dy} = \frac{\sin u}{\cos u} = \tan u$$

Hence Proved!

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Que. Evalute $\sqrt{99}$ approximately.

$\Rightarrow$ Let, $y = f(x) = \sqrt{x}$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

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Que. Find the percentage error in the area of the rectangle when an error of +1 % made in measure its length & breath.

Sol.^{n} => Let $A =$ Area
x and y are length breath

$$A = xy$$

$$\log A = \log x + \log y$$

$$\frac{\delta A}{A} = \frac{\delta x}{x} + \frac{\delta y}{y}$$

$$\frac{\delta A}{A} \times 100 = \frac{\delta x}{x} \times 100 + \frac{\delta y}{y} \times 100$$

$$A\% = 1 + 1 = 2\% \text{ Ans.}$$

Que. The $T = 2\pi\sqrt{\frac{l}{g}}$ of a simple pendullam

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Discuss the maxima & minima

Que. $f(x,y) = x^3y^2(1-x-y)$

Sol.ⁿ $\Rightarrow$

$$f = x^3y^2 - x^4y^2 - x^3y^3$$

$$\frac{\partial f}{\partial x} = 3x^2y^2 - 4x^3y^2 - 3x^2y^3 = 0$$

$$\frac{\partial f}{\partial y} = 2x^3y - 2x^4y - 3x^3y^2 = 0$$

$$x^2y^2(3-4x-3y)=0 \qquad x^3y(2-2x-3y)=0$$

$3-4x-3y=0$ $\times(-1)$

$4x+3y-3=0$ — (1)

$2-2x-3y=0$

$-2x-3y+2=0$ — (2)

from eq.ⁿ (1) & (2)

$4x+\cancel{3y}-3=0$ — (1)

$-2x-\cancel{3y}+2=0$ — (2)

$2x-1=0$

$$\boxed{x=\frac{1}{2}}$$

Put in eq. (1)

$\overset{2}{\cancel{4}}\left(\frac{1}{\cancel{2}}\right)+3y-3=0$

$3y-1=0$

$$\boxed{y=\frac{1}{3}}$$

$$f = 3x^2y^2 - 4x^3y^2 - 3x^2y^3$$

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$x = \frac{1}{2}$

$y = \frac{1}{3}$

$$\frac{df}{dx} = 3x^2y^2 - 2x^4y - 3x^3y^2$$

$$\frac{df}{dy} = 2x^3y - 2x^4y - 3x^3y^2$$

$$r = \left(\frac{\partial^2 f}{\partial x^2}\right)_{(a,b)} = \left(6xy^2 - 12x^2y^2 - 6xy^3\right)_{\left(\frac{1}{2}, \frac{1}{3}\right)}$$

$$r = \left(\cancel{6} \times \frac{1}{\cancel{2}} \times \frac{1}{\cancel{9}}\right) - \left(\cancel{12} \times \frac{1}{\cancel{4}} \times \frac{1}{\cancel{9}}\right) - \left(\cancel{6} \times \frac{1}{\cancel{2}} \times \frac{1}{\cancel{27}}\right)$$

$$r = \cancel{\frac{1}{3}} - \cancel{\frac{1}{3}} - \frac{1}{9} \Rightarrow \boxed{r = -\frac{1}{9}}$$

$$t = \left(\frac{\partial^2 f}{\partial y^2}\right)_{(a,b)} = \left(2x^3 - 2x^4 - 6x^3y\right)_{\left(\frac{1}{2}, \frac{1}{3}\right)}$$

$$t = 2\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^4 - \cancel{6}\left(\frac{1}{2}\right)^3\left(\frac{1}{\cancel{3}}\right)$$

$$t = \cancel{\frac{2}{8}} - \frac{\cancel{2}}{\cancel{16}}\frac{1}{8} - \cancel{\frac{2}{8}} \Rightarrow \boxed{t = -\frac{1}{8}}$$

$$S = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial\left(2x^3 - 2x^4 - 6x^3y\right)}{\partial x} =$$

$$S = \left(6x^2 - 8x^3 - 18x^2y\right)_{\left(\frac{1}{2}, \frac{1}{3}\right)}$$

$$S = \cancel{6} \times \frac{1}{\cancel{4}} - 8 \times \frac{1}{\cancel{27}\,8} - \cancel{18} \times \frac{1}{\cancel{4}} \times \frac{1}{\cancel{3}} = \cancel{\frac{3}{2}} - \frac{8}{27} - \cancel{\frac{3}{2}}$$

$$\boxed{S = \frac{-8}{27}}$$

"UNIT - IV"

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"MATRICES"

★ **Matrix** :- A system of mn number (real or Complex) arranged in a rectangular array of m rows & n columns is called ~~of~~ a matrix of order $m \times n$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & \cdots & & \\ \vdots & & & \\ a_{m1} & & & \end{bmatrix}$$

$$rt - s^2$$

$$\left(+\frac{1}{9}\right) \times \left(+\frac{1}{8}\right) - \left(\frac{8 \times 8}{27 \times 27}\right)$$

$$\frac{1}{72} - \frac{64}{729} = \_\_\_\_$$

$(27)^2$
$$\begin{array}{r} 49 \\ 28 \\ \hline 729 \end{array}$$

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★ **Elementary Row operations :-** The elementary rows operations are as follows—

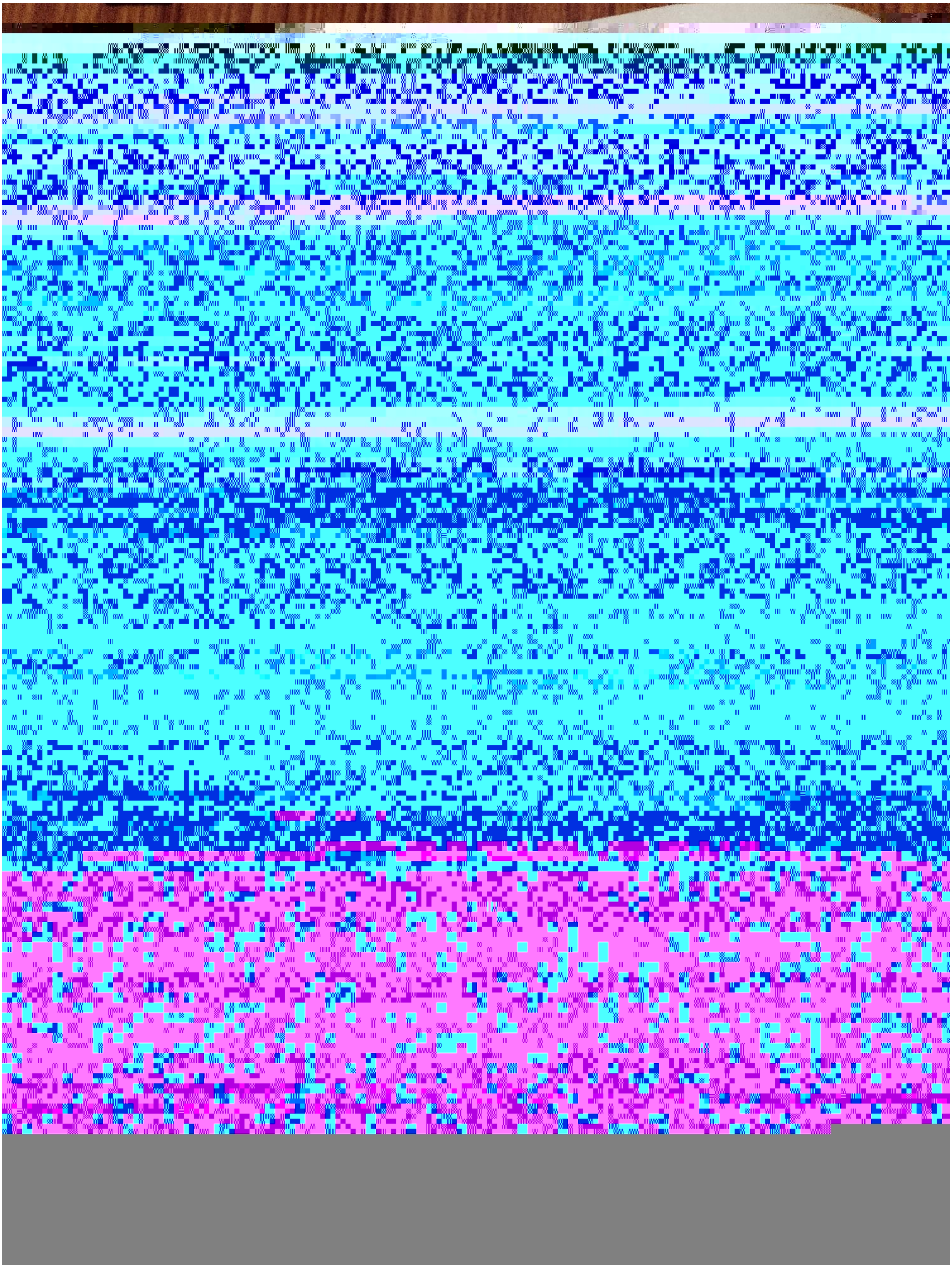
★ Minor of a matrix:-

Let ~~a,b be any~~ A be a matrix rectangular or square from this matrix A delete all columns or rows leaving a certain p columns & p rows. Now if $p > 1$, then the elements which have been left consitute a square matrix of order p. The determinant of this square matrix is called minor of (A) of order p.

★ Rank of a Matrix -

Let A be any matrix then no. r is called the rank of the matrix. If it obeys to following 2 properties -

(1) There is atleast 1 minor of A of order (r). which does not vani



[Non - zero rows = Rank hogi
utni]

→ Rank find krna ka best form!

Mark "IMP"

★ Echelon form :-

① A matrix is called Echelon form. If all the non-zero rows, if any preceed the zero rows.

② The no. of zeroes preceeding the first non-zero element in a row is less than the no. of such zeroes in the next row.

③ The first non-zero element in each row is unity.

Que 1| $A = \begin{bmatrix} 1 & 5 & 6 & 7 & 0 & 8 \\ 0 & 1 & 2 & 5 & -1 & 2 \\ 0 & 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

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• Normal form :- If A is the matrix of order $m \times n$ and rank r then a can be reduced by the no. of elementary transformation. to any of the following form.

(i) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} I_r & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ (iv) $\begin{bmatrix} I_r \end{bmatrix}$

Que $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ Find the rank & nullity of the following matrix.

$R_{31}(-1)$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

$R_{21}(-3)$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

R_{32} $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2\left(-\frac{1}{2}\right)$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Nullity of matrix $= n - r$
$= 3 - 2$
$= 1$ Ans.

n = order of matrix

r = rank

$\rho(A) = 2$

$\boxed{\text{Rank} = 2}$

Que
a

6 marks ⋆IMP

Que Find the normal form of the matrix (A) and hence find its rank.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$$R_{12} \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \to R_2 + R_1(-2)$

$R_3 \to R_3 + R_1(-3)$

$R_4 \to R_4 + R_1(-6)$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_4 \to R_4 + R_3(-1)$

$R_4 \to R_4 + R_2(-1)$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_{21}(1) = C_2 \to C_2 + C_1(1)$

$C_{31}(2) = C_3 \to C_3 + C_1(2)$

$C_{41}(4) = C_4 \to C_4 + C_1(4)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_{23}(-1) = R_2 \rightarrow R_2 + (-1)R_3$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_{32}(-4) = R_3 \rightarrow R_3 + R_2(-4)$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

• Maths Assignment Questions (Unit-4)

① Find the normal form of the matrix (A) and hence find its rank.

$$A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

Que ② Find the normal form of the matrix (A) and hence find its rank. $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$.

Que ③ ~~Find that~~ For what value of λ & μ, the eq. are –

$x+y+z=6$ (i) no sol.^{n}.

$x+2y+3z=10$ (ii) a unique sol.^{n}

$x+2y+\lambda z=4$ (iii) Infinite many sol.^{n}

Que 4 $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ If matrix A = then verify Cayley-Hamilton theoram Hence find A^{-1}.

Que 5 $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ If matrix A = then satifies cayley-Hamilton theoram Hence find A^{-1}.

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★ Solution of simultaneous eq.^{n} by elementary Transformation and consistency of the eqns:-

$$a_{11}x + a_{12}y + a_{13}z + \ldots = b_1$$

$$a_{21}x + a_{22}y + a_{23}z + \ldots = b_2$$

$AX = B$

$X = A^{-1}B$

$\hookrightarrow$ unknown

$\rho[A \vdots B] = \rho(A)$

$\downarrow$ Augmented

Consistant

$$\boxed{\rho[A \vdots B] = \rho(A)}$$

$\uparrow$ Rank

$$\boxed{\rho(A \vdots B) \neq \rho[A]}$$ Inconsistant

• Working Rule of Upper topics,-

First of all, right the augmented matrix $(A \vdots B)$. Now reduce $A \vdots B$ to the ~~echlon~~ form by the application of elementary row operation only it gives the rank of $A \vdots B$. Now obtain $\rho(A)$ after deleting last coloum from the echelon's form of $(A \vdots B)$. It gives the rank of (A).

Now Following two cases arrived.

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① Rank of A ≠ Rank of (A ⋮ B) – In this position the eq.ⁿ are called Incosistent, then they don't have no sol.ⁿ.

② Rank of A = Rank of (A ⋮ B) – In this position, the eq.ⁿ are called consistent, they have solution

★ Now following sub-cases are arrived –

① If $\gamma = n$ – (γ is rank & n is unknowns) ($n \rightarrow$ no. of unknown ~~value of x, y,~~ z)

In this case, the eq.ⁿ have unique solution.

② If $\gamma < n$, → In this case, the eq.ⁿ have Infinite solution.

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Que. Examine the following eq.^{n} for consistency.

$$5x + 3y + 14z = 4$$
$$y + 2z = 1$$
$$x - y + 2z = 0$$
$$2x + y + 6z = 2$$

Rows mai only operation lagege
columns mai nhi

convert → eschelon form's

$$AX = B$$

$$[A:B] = \left[\begin{array}{ccc:c} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \\ 2 & 1 & 6 & 2 \end{array}\right]$$

$$R_{31} \left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4 \\ 2 & 1 & 6 & 2 \end{array}\right] \Rightarrow$$

$R_4 \to R_4 + R_1(-2)$

$R_3 \to R_3 + R_1(-5)$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \\ 0 & 3 & 2 & 2 \end{array}\right]$$

$R_{32}(-8) = R_3 \to R_3 + R_2(-8)$

$R_{42}(-3) = R_4 \to R_4 + R_2(-3)$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -12 & -4 \\ 0 & 0 & -4 & -1 \end{array}\right]$$

$R_3 \to R_3 + R_4(-3)$

$$\left[\begin{array}{ccc:c} 0 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1/4 \end{array}\right] \Rightarrow$$

$R_{34} \Rightarrow R_3 \to R_3 + R_4$

$R_4(-1)$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1/4 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$\boxed{\rho(A:B) \neq \rho(A)}$ Inconsistant

$\boxed{\rho(A:B) = 4}$

$\boxed{\rho(A) = 3}$

Que. Examine the consistency for following eq^n, if consistent. Find the complete solution.

$$x+2y-z=3$$
$$3x-y+2z=1$$
$$2x-2y+3z=2$$
$$x-y+z=-1$$

$[A:B]$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

$R_{21}(-3) \Rightarrow R_2 \to R_2 + R_1(-3)$
$R_{31}(-2) \Rightarrow R_3 \to R_3 + R_1(-2)$
$R_{41}(-1) \Rightarrow R_4 \to R_4 + R_1(-1)$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right]$$

$$\begin{array}{l} R_{23}(-1) \\ R_2(-1) \end{array} \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right] \Longrightarrow \begin{array}{l} R_{32}(6) \\ R_{42}(3) \end{array} \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 2 & 8 \end{array}\right]$$

$$R_3\left(\frac{1}{5}\right) \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 2 & 8 \end{array}\right] \Longrightarrow R_{43}(-2) \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\rho(A:B) = \rho(A) = 3}$$

Consistant

$\boxed{r=n}$, $3=3$, It has unique sol.^{n}

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$$

$$\begin{cases} x+2y-z=3 \\ y=4 \\ z=4 \\ x=-1 \end{cases}$$

Que. Test the consistenly & solve.

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

$[A : B]$

$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$R_{12}(-2)$ $\left[\begin{array}{ccc|c} 1 & -49 & 3 & -14 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$

$R_2(-3)$, $R_3(-7)$ $\left[\begin{array}{ccc|c} 1 & 49 & -3 & 14 \\ 0 & 121 & 11 & -33 \\ 0 & -341 & 31 & -93 \end{array}\right]$

$R_2\left(\frac{1}{11}\right)$, $R_3\left(\frac{-1}{31}\right)$ $\left[\begin{array}{ccc|c} 1 & 49 & -3 & 14 \\ 0 & -11 & 1 & -3 \\ 0 & 11 & -1 & +3 \end{array}\right]$ $\Rightarrow$ $R_{32}(1)$ $\left[\begin{array}{ccc|c} 1 & 49 & -3 & 14 \\ 0 & -11 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$R_2\left(\frac{-1}{11}\right)$ $\left[\begin{array}{ccc|c} 1 & 49 & -3 & 14 \\ 0 & 1 & -1/11 & 3/11 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\boxed{\rho(A:B) = \rho(A) = 2}$

it is Consistent

Here, $\boxed{r < n}$. it has infinite sol.^{n}

$$\begin{bmatrix} 1 & 49 & -3 \\ 0 & 1 & -1/11 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 3/11 \\ 0 \end{bmatrix}$$

$$x + 49y - 3z = 14$$
$$y - \frac{1}{11} z = \frac{3}{11}$$
$$y = \frac{3}{11} + \frac{1}{11} z$$

Take $(z = k)$ (Constant)

$y = \frac{3}{11} + \frac{1}{11}k = \frac{3+k}{11} \Rightarrow \boxed{x = \frac{7-16k}{11}}$

★ Eigen Values and eigen vector :-

The eq.^{n} $|A - \lambda I| = 0$ is called Characteristics eq.^{n} of A, The roots of this eq.^{n} are called "roots or eigen values of matrix A.

Characteristics vector / eigen vector : If $\lambda = \lambda_1$ is char. root of 'A' then the non-zero sol.^{n} of $x = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$ of the eq.^{n} . $(A - \lambda_1 I)X = 0$ is said to be eigen vector of 'A' corresponding to charac. root $\lambda = \lambda_1$.

Que/ Find char. root and eigen vector $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left| \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \Rightarrow \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$

$10 - 5\lambda - 2\lambda + \lambda^2 - 4$

$\lambda^2 - 7\lambda + 6 = 0$

$\lambda^2 - \lambda - 6\lambda + 6 = 0$

$\lambda(\lambda - 1) - 6(\lambda - 1) = 0$

$\boxed{\lambda = 1, 6}$

Eigen value of A $\Rightarrow$ 1 and 6

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Eigen vector corresponding the eigen value $\boxed{\lambda = 1}$

$(A - \lambda_1 I) x = 0$

$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$x_1 + x_2 = 0$

$x_1 = -x_2$

$\boxed{\frac{x_1}{x_2} = -1}$

For $\lambda = 6$

$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \rightarrow 4x_2 - x_1 = 0$

$\cancel{4x_2 + x_1 = 0}$

$4x_2 = x_1$

$\boxed{\frac{x_2}{x_1}} = \frac{1}{4}$

$\boxed{X = \begin{bmatrix} 1 \\ 4 \end{bmatrix}}$

Que. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ Find characteristics root and eigen vector.

$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix}$

$\begin{bmatrix} 6-\lambda & -2 & -2 \\ 0 & 2-\lambda & 2-\lambda \\ 2 & 1 & 3-\lambda \end{bmatrix}$

$(6-\lambda)(6 - 5\lambda + \lambda^2 + 2 - \lambda)$
$+ 2(-4 + 2\lambda - 4 + 2\lambda)$

$6\lambda^2 - 36\lambda + 48 - \lambda^3 + 6\lambda^2 \cancel{- 8\lambda} \cancel{+ 8\lambda} - 16$

$-\lambda^3 + 12\lambda^2 - 36\lambda + 32$

$\boxed{\lambda^3 - 12\lambda^2 + 36\lambda - 32}$

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Maths,

Assign.
Que. 1

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Find characteristic, root and eign vector.

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$8-\lambda\left[(7-\lambda)(3-\lambda) - (-4)\times(-4)\right] + 6\left[-6(3-\lambda) - (-4)\times 2\right]$$
$$+ 2\left[24 - (14 - 2\lambda)\right]$$

$$8-\lambda\left[21 - 10\lambda + \lambda^2 - 16\right] + 6\left[-18 + 6\lambda + 8\right]$$
$$+ 2\left[24 - 14 + 2\lambda\right]$$

$$(8-\lambda)\left[\lambda^2 - 10\lambda + 5\right] + (36\lambda - 60) + 20 + 4\lambda$$

$$8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda - 60 + 20$$

$$\boxed{-\lambda^3 + 18\lambda^2 - 45\lambda = 0}$$

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$$
$$-\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$-\lambda[\lambda^2 - 15\lambda - 3\lambda + 45] = 0$$
$$-\lambda[\lambda(\lambda-15) - 3(\lambda-15)] = 0$$

$$-\lambda[(\lambda-3)(\lambda-15)] = 0$$

$$\boxed{\lambda = 0, 3, 15} \leftarrow \text{Ans.}$$

$2\times2+8\times3$

3×1

$8+9$

17

$$\boxed{\lambda = 0}$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 4 & -3 & 1 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 4x_1 - 3x_2 + x_3 \\ -6x_1 + 7x_2 - 4x_3 \\ 2x_1 - 4x_2 + 3x_3 \end{bmatrix} = 0 \quad \begin{matrix} (1) \\ (2) \\ (3) \end{matrix}$$

~~(1) + (3)~~

~~$6x_1 - 7x_2 + 4x_3$~~

~~$-6x_1 + 7x_2 - 4x_3$ — (2)~~

$$\begin{bmatrix} 4 & -3 & 1 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(2) + (3) × 3

$$-6x_1 + 7x_2 - 4x_3 = 0$$
$$6x_1 - 12x_2 + 9x_3 = 0$$

$$-5x_2 + 5x_3 = 0$$
$$5x_2 = 5x_3$$
$$\boxed{x_2 = x_3}$$

eq. (1) $[x_2 = x_3]$

$$4x_1 - 3x_2 + x_3 = 0$$
$$4x_1 - 3x_2 + x_2 = 0$$
$$4x_1 - 2x_2 = 0$$

~~eq. (3) × 2 $[x_2 = x_3]$~~

$4x_1 - 2x_2 = 0$

$4x_1 = 2x_2$

$\boxed{x_1 = \frac{x_2}{2}}$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 1 \end{bmatrix}$$

easiest

★ Cayley - Hamilton theoram:- Every square matrix satisfied its character. eq.^{n} → $|A - \lambda I| = 0$.

$$\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots + a_n I = 0$$

then the eq.^{n},

put $\lambda = A$

$$① \boxed{A^n + a_1 A^{n-1} + a_2 A^{n-2} + \dots + a_{n-1} A + a_n I = 0}$$

Multiply eq. ① by A^{-1}, we get

$$A^{n-1} + a_1 A^{n-2} + \dots + a_{n-1} I + a_n A^{-1} = 0$$

★★

$$\boxed{A^{-1} = \frac{-1}{a_n} [A^{n-1} + a_1 A^{n-2} + \dots + a_{n-1}}$$

$A^2 = A \cdot A$

$A^3 = A^2 \cdot A$

$A^4 = A^3 \cdot A$

⋮

$A^n = A^{n-1} \cdot A$

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Que Find the characteristics eq.^{n} of matrix A. and verify that it satifies by A & find A^{-1}.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\boxed{|A - \lambda I| = 0}$$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$C_2 \to C_2 + C_3 = C_{23}$$

$C_{23}(1)$
$$\begin{vmatrix} 2-\lambda & 0 & 1 \\ -1 & 1-\lambda & -1 \\ 1 & 1-\lambda & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\left[(1-\lambda)(2-\lambda) - (-1)(1-\lambda)\right] + 1\left[(-1(1-\lambda) - 1(1-\lambda)\right]$$

$$(2-\lambda)\left[2 - \lambda - 2\lambda + \lambda^2 + (1-\lambda)\right] + -1 + \lambda - (1-\lambda)\Big]$$

$$\cancel{(2-\lambda)\left[2 - 3\lambda + \lambda^2 + 1 - \lambda + -1 + \lambda - 1 + \lambda\right.}$$

$$\cancel{(2-\lambda)\left[\lambda^2 - 2\lambda + 1\right)}$$

$$\cancel{2\lambda^2 - 4\lambda + 2 - \lambda^3 + 2\lambda^2 - \lambda = 0}$$

$$-\lambda^3 + 4\lambda^2 - 5\lambda$$

$$(2-\lambda)\left[\lambda^2 - 4\lambda + 2\right] + \left[2\lambda - 2\right] = 0$$

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The charac. eq.ⁿ

$$\boxed{\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0}$$

Put $\boxed{\lambda = A}$

$$A^3 - 6A^2 + 9A - 4 = 0$$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -6 & 6 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Put the values

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Hence Proved..

$\left[A \cdot A^{-1} = I, \quad A \cdot I = A\right]$

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Now Find A^{-1}

Multiply eq. ① by A^{-1}, on both side

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

$$4A^{-1} = [A^2 - 6A + 9I]$$

$$4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -6 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$4A^{-1} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}}$$

$$A^3 - 6A^2 + 9A - 4 = 0$$

$$A^2(A \cdot A^{-1}) - 6A(A \cdot A^{-1}) - 9A \cdot A^{-1} - 4A^{-1} = 0$$

$$A(A \cdot I)$$

$$A^2 - 6A - 9A - 4A^{-1} = 0$$

$4A^{-1}$

Que. Find the characteristics of matrix A and verify that it satifies by A λ find A^{-1}.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$1-\lambda\left((2-\lambda)(3-\lambda) - 1\times 0\right) - 0(\quad) + 2(0-2(2-\lambda))$$

$$1-\lambda(6-2\lambda-3\lambda+\lambda^2) + 2(-4+2\lambda) = 0$$

$$(1-\lambda)(\lambda^2-5\lambda+6) - 8 + 4\lambda = 0$$

$$\lambda^2-5\lambda+6-\lambda^3+5\lambda^2-6\lambda-8+4\lambda = 0$$

$$-\lambda^3+6\lambda^2-11\lambda+4\lambda-2 = 0$$

$$\lambda^3-6\lambda^2+7\lambda+2 = 0$$

$$\boxed{A^3-6A^2+7A+2I = 0}$$

Multiply both side by A^{-1}

$$A^2A\cdot A^{-1} - 6A\cdot A\cdot A^{-1} + 7A\cdot A^{-1} + 2A^{-1} = 0$$

$$A^2I - 6AI + 7I + 2A^{-1} = 0$$

$$A^2 - 6A + 7I + 2A^{-1} = 0$$

~~$A^2 - 6A + 7$~~

~~$A^2 - 7A + A + 7$~~

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$$2A^{-1} = -A^2 + 6A - 7I$$

$$A^2 = A \cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$2A^{-1} = \begin{bmatrix} -5 & 0 & -8 \\ -2 & -4 & -5 \\ -8 & -0 & -13 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2A^{-1} = \begin{bmatrix} -5 & 0 & -8 \\ -2 & -4 & -5 \\ -8 & -0 & -13 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 12 \\ 0 & 12 & 6 \\ 12 & 0 & 18 \end{bmatrix} + \begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$$

$$2A^{-1} = \begin{bmatrix} -6 & 0 & 4 \\ -2 & 1 & 1 \\ 4 & 0 & -2 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ -1 & 1/2 & 1/2 \\ 2 & 0 & -1 \end{bmatrix}}$$

Ans.

$$A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

UNIT - 5

"GRAPH THEORY"

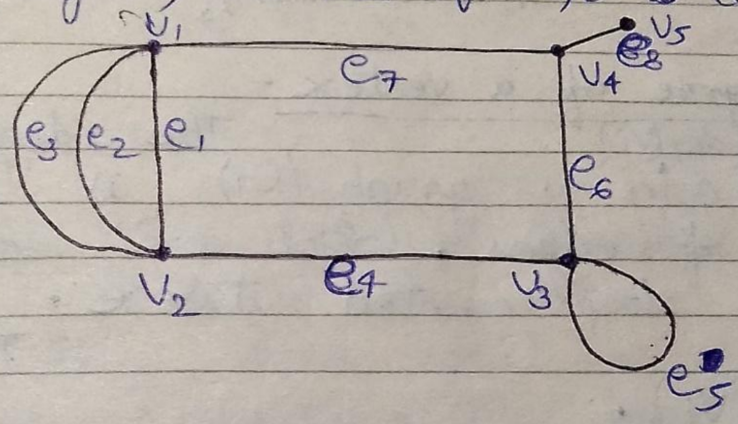
Date 11/01/23

PYQs

★ **Graph** :- A Graph $\boxed{G = (V, E)}$ consist of a set of object $V = \{v_1, v_2, v_3 - - - -\}$ whose elements are called vertices and a another ~~se~~ set $E = \{e_1, e_2, e_3, - - -\}$ whose elements are called edges, and set of V,E is called a graph.

Example :-

Sari defination mai example likhna hai

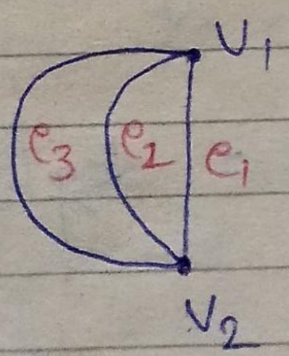


② **Self loop** :- (Self loop) An edge is said to be a self loop, if its both ~~and~~ end vertices are same.

example :-

③ **Parallel edge** :- If there are two or more than two having same pair of vertices, then such edges are called parallel edg

example :-

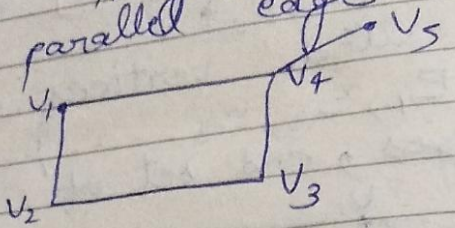


graph

PYQs

④ **Simple graph :-** A graph that has neither self loop nor parallel edge are called simple edge.

example :-



PYQs

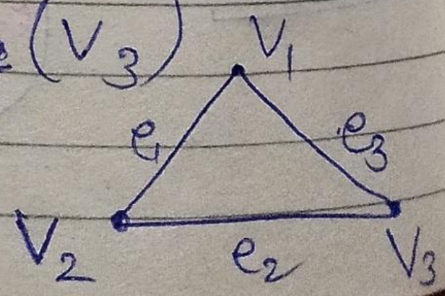
⑤ **Degree of a vertex :-** The degree of a vertex $deg(V_i)$ in a graph (G) is equal to the no. of edges, which are incident on V_i with self loop counted twice.
It is denoted by $deg(V_i)$.

ex ⇒ $deg(V_1) = deg(V_2) = deg(V_3) = 4$

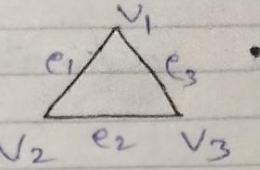
$deg(V_4) = 3, \quad deg(V_5) = 1$

⑥ **Regular graph :-** A graph (G) in which all vertices are of equal degree is called regular graph.

Ex. ⇒ $deg(V_1) = deg(V_2) = deg(V_3)$

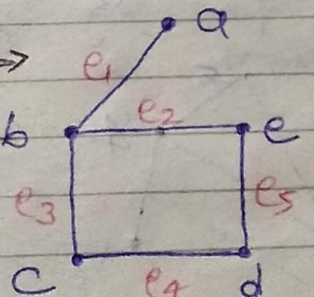


⑦ Isolated vertex :- A vertex of degree zero is called isolated vertex, or an end vertex.

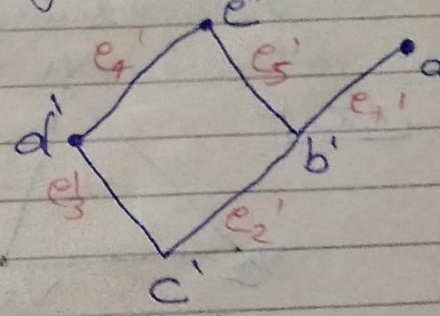
ex. ⇒ 

→ V_4 is isolated vertex.

⑧ Iso-morphic graph :- Two graph are called iso-morphic, if there graph ~~theoretic~~ properties are same that is they have same no. of edge, they have equal no. of vertices with a given degree, they have same no. of vertices.

ex. ⇒ 

g



g'

The vertices a, b, c, d, e correspond to a', b', c', d', e' respectively and the edges e_1, e_2, e_3, e_4, e_5 correspond to $e_1', e_2', e_3', e_4', e_5'$ respectively.

A graph g' is said to be sub-graph of graph g. If all the vertices and all edges of 'G'' are in G.

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Each edge of (g') has the same end vertices in (g') as in g.

GATE → MCQs

Note :- From the defination of sub-graph

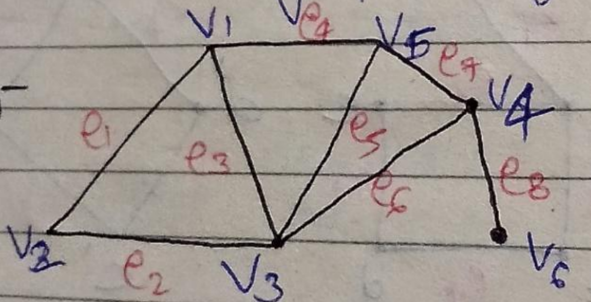
① Every graph is ~~its~~ own sub-graph.

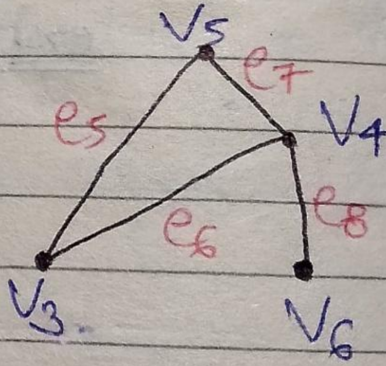
② A single vertex in (g) is a sub-graph of (g).

③ A single edge in (g) ~~is a~~ together with it end vertices is a sub-graph of (g).

④ A sub-graph of a sub-graph of (g) is also a sub-graph of (g).

ex. :-





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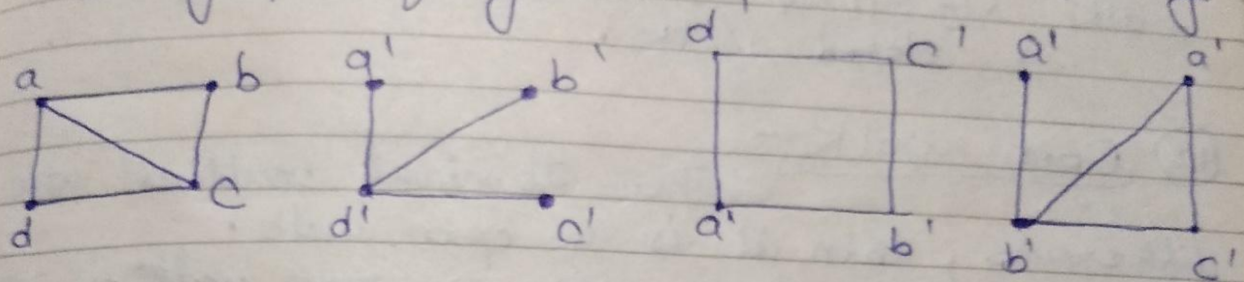
(9) Spanning sub-graph :-

Let $G' = (V', E')$ be subgraph of a graph $G = (V, E)$. If $V' = V$, then G' is said to be spaning sub-graph of g.



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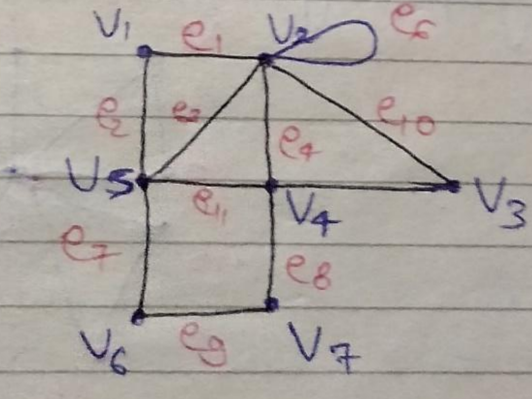
(10) Walk :-

A walk in a graph (g) is defined as a finite alternating sequence of vertices & edges which begins and ~~ends~~ ends, with vertices.



In a walk, no edge appear more than one however, a ~~vertice~~ vertex may appear more than once.

$\hookrightarrow V_1 \, e_1 \, V_2 \, e_6 \, V_2 \, e_{10} \, V_3 \, e_5 \, V_4 \, e_8 \, V_7$

walk ↗

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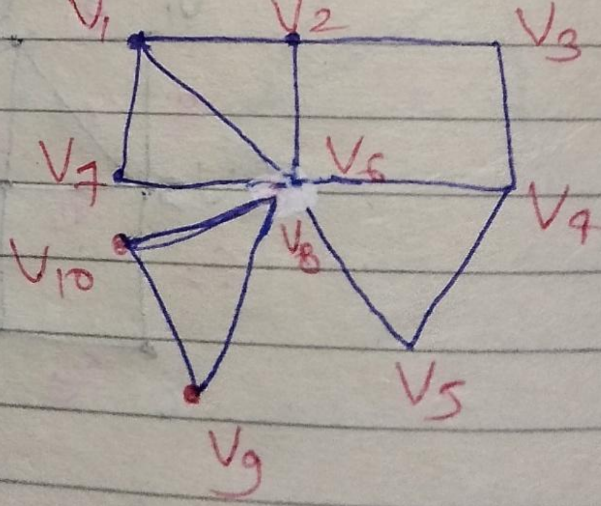
(11) Closed Walk :- If terminal vertices are same then walk is called closed walk. A walk $V_5 e_{11} V_4 e_5 V_2 e_6 V_2 e_1 V_1 e_2 V_5$ is a closed walk.

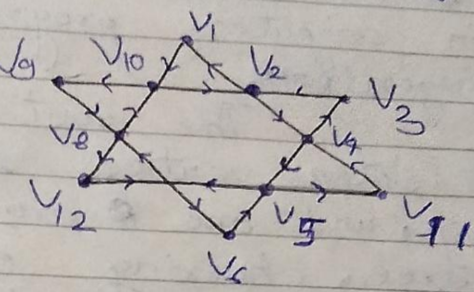
(12) Open walk :- When terminal vertices are difference, then it is a open walk. $V_5 e_1 V_4 e_5 V_7$ is a open walk.

(13) Connected Graph :- A graph (G) is called connected, if there is atleast one path b/w every pair of vertices. Otherwise ~~graph~~ (g) it is disconnected.



★ Euler graph :-

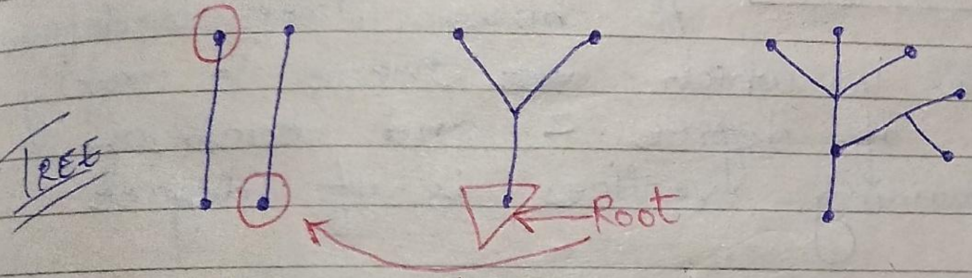
A closed walk in a graph which include all the edges of the graph is euler graph.



IMP PYQS

★ Tree :-

A connected graph having no circuit is called tree.



- **Some Types of Trees :-**

① Rooted tree :-

A rooted tree is a tree with distinguish vertices called the root.

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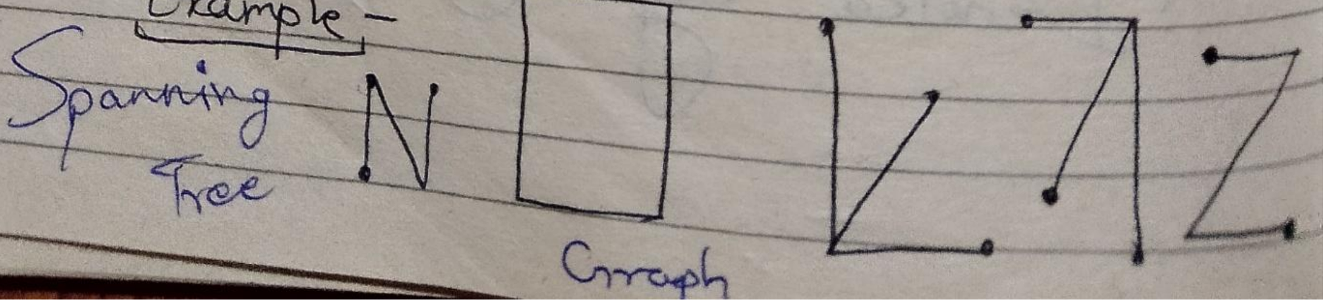
② Decision tree :- A decision tree are labelled rooted tree which occur ~~labelled~~ ~~rooted tree~~ in application specially in computer programming & computer algorithm.

The root represent a starting point later vertices represent later decision point, and one proceed downward to the tree, chossing edge at each step occur to observed data.

③ Binary Tree :- A binary tree is defined as a tree in which there is exactly one vertex of degree 2 and each of the remaining vertices is of degree 1 or 3.

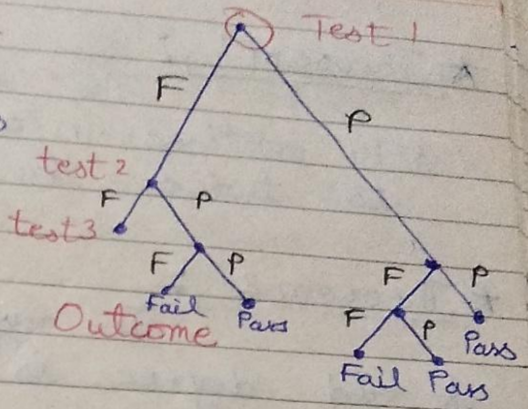
④ Spanning Tree :- If $g = (V, E)$ is any connected graph, a spanning tree in g is a sub-graph $T = (V, E')$ which is a tree.

Example -



(5) Binary decision tree :-

A student must pass two of three test to pass a course. The binary decision reappears in adjoining figure with F denoting Fail, P denoting pass.



If a student passes both of the first two test or fail both there is no decision, needed from test 3. We have ~~improvised~~ Emphasized. The path to the outcome for a student who passes the first test fail the second and passes the third: the student passes the course.

In a binary tree, each decision has only two possible outcomes that is yes OR NO, True OR False, 0 OR 1).

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6-Marks Que

★ Theorem 1 :- The sum of the degrees of all vertices in a graph is equal to equal to twice the no. of edges.

★ Theorem 2 :- In any graph, the no. of vertices of odd degree is always even.

★ Theoram 5 :

The maximum no. of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

Properties of double integration —

★ Properties of double integration

① If K is constant func^n then, double integration are $\iint_R K f(x,y)\,dx\,dy = K\iint_R f(x,y)\,dx\,dy$

② Linear property of double integral

$$\iint_R \{K_1 f(x,y) + K_2 f(x,y)\}\,dx\,dy$$

$$= K_1 \iint_R f_1(x,y)\,dx\,dy + K_2 \iint_R f_2(x,y)\,dx\,dy$$

③ If the ~~reason are~~ region (R) is par into R_1, R_2

$$\iint_R f(x,y)\,dx\,dy = \iint_{R_1} f(x,y)\,dx\,dy) + \iint_{R_2} f(x,y)\,dx\,dy$$

④ The double integral $\int_a^b \int_{f_1(x)}^{f_2(x)} dx\,dy$ defines the area enclosed by the $y = f_1(x)$, $y = f_2(x)$ & the ordinates $x = a$, $x = b$.

(DNF) & (CNF)
IMP

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★ Invalution law – $(a')' = a$

★ Demorgan's law – $(a+b)' = a'b'$
$(a.b)' = a' + b'$

★ In the Boolean Algebra, prove the following.

(i) $a' + ab = a' + b$
(ii) $a \cdot b \Rightarrow a \cdot b' = 0 \;\forall\; a, b \in B$
(iii) $a \cdot b + b \cdot c + c \cdot a = (a+b)(b+c)(c+a)$

★ Boolean func.^{n} –
An expression obtained by the application of binary operations '+' & '·' and a uniary operation (') on the finite number of elements of boolean-algebra $(B, +, ')$ is called Boolean func.^{n} OR polynomial.

★ Minimal Boolean function –
A minimal boolean func.^{n} is (n) variable $x_1, x_2, x_3, \ldots x_n$ is product of (n) letters.

★ Disjunctive normal form (DNF) –
A boolen polynomial which can be written OR sum of the minimal boolean function called disjuntive normal form (or conormial form)

$f(x,y) = x.y + x'y + xy' + x'y'$

2 mai comm

★ **Conjuctive Normal form :-** A boolean polynomial is called a CNF (or dual canonical form) if it is the product of distinct factors where each factor is the sum of variables x_1, x_2, x_3 and their products $x_1', x_2', x_3', \ldots$ and in each factor variables or their complements do not occur more than once

$$f(x_1, x_2) = (x_1 + x_2)(x_1' + x_2)(x_1 + x_2')(x_1' + x_2')$$

Ex.- Convert $x + x'y$ to DNF.

$$x(1) + x' \cdot y$$
$$x(y+y') + x'y$$
$$xy + xy' + x'y \quad \text{Ans}$$

▸ Disjunctive (DNF)

Ex.- Write the following func.^{n} into conjuctive normal form, in which maximum number of variable are used.

(i) $f(x, y, z) = x \cdot y' + xz + xy$

Que. Test for Consistency $\boxed{AX = B}$ Solve :-

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 22$$

$C = [A \vdots B]$

Rank of A = Rank of C then consistent

★ If Rank = no. of unknown (x, y, z) then unique $sol^n \Rightarrow$

★ Rank < no. of unknown $\Rightarrow$ Infinite sol^n

$$AX = B$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 22 \end{bmatrix}$$

Now, Augumented Matrix

$$C = [A \vdots B]$$

$$C = \begin{bmatrix} 2 & -3 & 7 & \vdots & 5 \\ 3 & 1 & -3 & \vdots & 13 \\ 2 & 19 & -47 & \vdots & 22 \end{bmatrix}_{3 \times 4}$$

For Finding Rank Convert in upper triangular Matrix

(—) Lower triangluar matrix

$$R_2 = R_2 - R_2 \quad \text{OR} \quad 2R_2 - 3R_1$$
$$R_3 = R_3 - R_1$$

$$C = \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 22 & -54 & 17 \end{array}\right] \qquad R_3 = R_3 - 2R_2$$

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$27 \times 2 = 54$

$$C = \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ \underbrace{0 \quad 0 \quad 0}_{A} & & & \underbrace{-5}_{B} \end{array}\right]$$

← Now it is upper-triangular matrix

★ Rank of a matrix = No. of non-zero row is called rank of a matrix.

Now, In C matrix

No. of non-zero row = 3

$\boxed{\text{Rank of Matrix} = 3}$

Rank of a matrix find krne ka Yeh rule hai Matrix Upper triangular matrix hon chahiye!!

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 0 & 11 & -27 \\ 0 & 0 & 0 \end{bmatrix}$$

Upper triang. matrix

Rank of a matrix

= No. of Non. zero Row

$\boxed{\text{Rank} = 2}$

of matrix A

[∴ Rank of C ≠ Rank of A]

[∴ The given system is not consistent and there exist No Sol.ⁿ]