Assignment 2

LA Monsoon 2020, IIIT Hyderabad

October 21, 2020

Due date: October 27, 2020

General Instructions: All symbols have the usual meanings (example: F is an arbitrary field, $\mathbb{R}$ is the set of reals, $\mathbb{N}$ the set of natural numbers, and so on.) Remember to prove all your intermediate claims, starting from basic definitions and theorems used in class to show whatever is being asked. You may use any other non-trivial theorems not used in class, as long as they are well known and a part of basic linear algebra texts. It is always best to try to prove everything from definitions. Arguments should be mathematically well formed and concise.

---

1. [1.5 points] Let S be a linearly independent subset of a vector space V. Suppose β is a vector in V which is not in the subspace spanned by S. Then, prove that the set obtained by adjoining β to S is linearly independent.

2. [1 point] Prove that if S is a subspace of a vector space V, then $span(S) = S$.

3. [3 points] For the given Matrices, calculate their nullspaces.

   (a) $\begin{bmatrix} 12 & 4 & 4 \\ 5 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

   (c) $\begin{bmatrix} 12 & 3 & 9 \\ 20 & 5 & 15 \\ 16 & 4 & 12 \end{bmatrix}$

4. [3 points] Let W be a subspace of V with a basis $\{\alpha_i : i \in [m]\}$. Let $\beta \in V \setminus W$. Show that the set $\{\alpha_i + \beta : i \in [m]\}$ spans an m dimensional subspace of V.

5. [1.5 points] Suppose F is a finite field with p^n elements (such that p is a prime.) Let V be a k-dimensional vector space over F. Then count the cardinality of the following:

   1. The number of linear transformations $T : V \to V$.

   2. The number of invertable linear transformations $T : V \to V$.

   3. The number of linear transformations $T : V \to V$ with determinant 1.