Assignment 3

LA Monsoon 2020, IIIT Hyderabad

October 29, 2020

Due date: November 3, 2020

General Instructions: All symbols have the usual meanings (example: F is an arbitrary field, $\mathbb{R}$ is the set of reals, $\mathbb{N}$ the set of natural numbers, and so on.) Remember to prove all your intermediate claims, starting from basic definitions and theorems used in class to show whatever is being asked. You may use any other non-trivial theorems not used in class, as long as they are well known and a part of basic linear algebra texts. It is always best to try to prove everything from definitions. Arguments should be mathematically well formed and concise.

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1. [2 points] (a) Using Gaussian elimination, solve for x, y and z in

$$\begin{aligned} x + 3y + 5z &= 14 \\ 2x - y - 3z &= 3 \\ 4x + 5y - z &= 7 \end{aligned}$$

   (b) Using Gauss-Jordan elimination, solve for x, y and z in

$$\begin{aligned} y + z &= 4 \\ 3x + 6y - 3z &= 3 \\ -2x - 3y + 7z &= 10 \end{aligned}$$

2. [4 points] Suppose $V = \mathbb{R}^\infty$ is a vector space over $\mathbb{R}$ of all sequences of real numbers, such that addition and multiplication are defined coordinate-wise. Do the following form a subspace of V? Prove all your claims.

   1. Absolutely summable sequences. ((x_i) such that $\sum_{i=1}^{\infty} |x_i| < \infty$, i.e. the sum is finite).
   2. Bounded sequences. ((x_i) such that $\exists M > 0$ s.t. $|x_i| \leq M \forall i$, i.e. the sum is bounded above).
   3. Arithmetic sequences. ((x_i) such that $x_i = a + di$ for some fixed a and d).
   4. Geometric sequences. ((x_i) such that $x_i = ar^i$ for some fixed a and r).

3. [2.5 points] Show that the set of all continuous real valued functions on the domain $[0,1] \subset \mathbb{R}$ denoted by $\mathbb{R}[0,1] = \{f : [0,1] \to \mathbb{R} | f \text{ is continuous } \}$ forms a vector space over $\mathbb{R}$.

4. [1.5 points] Define three vectors f, g, h as $f(x) = x, g(x) = e^x, h(x) = e^{-x}; x \in [0,1]$ from $\mathbb{R}[0,1]$. Show that these 3 functions are linearly independent.