In [51]:
```
import numpy as np
```

In [52]:
```
X = np.random.randint(-10,10,200)
```

In [53]:
```
X
```

Out[53]:
```
array([ -8,   9,   2,   9, -10,   1,  -6,  -6,  -6,  -2,  -7,   9,   7,
         5,  -8,  -1,   6,  -1,  -8,   1,   9,   3,   0,   8,  -4,   3,
         6,  -4, -10,  -9,   3, -10,   2,   4,  -9,  -5,   1,  -2,  -2,
        -8, -10,   5, -10,  -5,  -8,   5,   3,   1,   0,   9,   1,   2,
       -10,   8,   5,   9,  -2,   4,   0,  -6,   3,  -5,   8,  -4,   9,
        -4, -10,   8,  -8,  -8,  -7,   4,  -2,   1,   1,  -5,   7,   9,
         1,  -3,   3,  -4,   1,  -1,   7,  -5,   0,  -1, -10,  -1,   9,
         3,   5,   2,  -4,  -6,  -2,   7,  -6,  -5,  -3,   2,  -5,  -7,
        -6,  -8,   6,  -3,   9,   3,   3,  -3,   0,   4,   6,  -2,   1,
         9,  -2,   7,   7,  -8,  -7, -10,   8,  -6,  -7,  -9,   1,  -6,
         6,   1,   9,   0,  -9,  -4,  -1,   9,  -1,  -8,  -9,   9,   7,
         5,   9,   3, -10, -10,   1,   0,   8,   0,   6,  -8,   5,   6,
         0,   0,   8,   4,  -5,   0,   1,   1,  -8,  -6,  -3,  -4,  -5,
         6,  -6,  -4,   5,   6,   3,   1,  -5,   6,   6,  -5,  -3,   4,
        -3,   3,  -8,  -9,   7,   0,   6,  -2,  -9,   4,  -4,  -4,   3,
        -6,  -8,  -8,   9,   9])
```

In [54]:
```
E = np.random.normal(-40,40,200)
```

In [55]:
```
E
```

Out[55]:
```
array([ -44.36171464,  -46.07193786, -134.49159837,  -44.01405763,
        -48.2127552 ,  -55.41676241,  -84.11774042,  -16.98635178,
        -48.3647891 ,  -74.33107814,  -38.98082854,   50.53197061,
        -50.7469327 ,  -23.04013232,   -6.09205335,  -56.84177202,
        -65.61487332,   -5.89948178, -103.15896861,  -41.90866629,
        -78.34230112,   -4.09821686,  -43.67493779,  -27.2031704 ,
        -65.12629008,  -61.11009606,  -58.57467601,  -24.06764873,
          6.48091041, -132.39362213,  -94.25002295,   15.79027343,
        -73.22731879,   34.26717319,  -52.98776076,   10.54723418,
        -57.1451044 ,  -37.93660361,  -21.69990645,  -58.66017936,
        -31.97996572,  -55.16225891,  -31.73198885,  -26.43788905,
        -69.99481608,  -17.89574684,  -98.95115113,  -16.1002098 ,
         -8.6651039 , -114.134225  ,   52.69705939,  -13.59723975,
        -54.14150782,   12.05836482,  -52.56021681,  -49.13857491,
        -85.17525187,  -60.48227258,  -40.06623851,  -32.7113197 ,
        -87.25862997,  -19.05313043,  -31.54317714,  -48.62307868,
        -35.54157717,  -31.36125088,  -36.37885434,  -89.04760249,
        -98.08975326,   43.84772235, -110.33017695,  -29.27630224,
        -10.00044899,  -49.25768124,  -59.17603824,  -26.96511722,
        -60.43402426,  -36.29650687,  -61.04538172,  -38.86874383,
         20.57761918, -181.72537846,  -77.68453504,   40.14863833,
        -35.04780536,  -29.75592703,  -34.83611206,  -94.38595097,
        -66.1278602 ,  -23.53898795,  -81.39631488,  -52.12273165,
        -66.18086187,  -29.86908085,  -98.9535578 ,   -9.45137855,
        -67.31243878,  -10.43258191,  -41.85151077,  -79.14012627,
        -28.02851776,  -59.74844986,  -53.82538505,  -15.18383661,
         40.67600369,  -21.62637174,  -84.94845388,  -65.46402462,
         51.50731283,  -43.50767492, -108.81154911,  -11.23757047,
        -56.53857524,  -16.49130661,  -56.90214211,  -49.6591721 ,
        -84.36169383,  -68.0971888 ,  -22.79931397,    3.8421679 ,
         56.10623452,  -47.45900194,  -12.93419578,   37.83924743,
         -8.90201612,  -35.6107933 ,  -26.19847524,  -40.17408575,
        -86.58734746,  -62.87755623,  -36.81650995,  -59.30614122,
         25.12806088,   -0.81241901,  -37.96423255, -104.63748207,
        -71.73842361, -100.267218  ,  -55.63790122,  -29.40524968,
          0.90925212,   -3.58951986,  -96.28830993,  -22.07641662,
        -85.54031434,  -55.30394903,  -79.3761888 ,   10.50964898,
         10.55800591,   68.20425284,  -86.31304133, -146.52630589,
          5.02346835,  -36.88174302,   12.84804856,  -47.01779459,
        -29.38882473,  -25.07752977,  -48.91270033,  -42.02066103,
         29.47603465,  -33.05573877,   21.49068086,   10.42897151,
        -52.31731178,  -76.84923364,  -61.27575218,  -49.60639882,
         18.45804271,  -25.90653553,  -64.49122681,   -1.52380357,
        -92.73988908, -104.83824044,  -50.81705849,  -79.18047688,
        -22.15250369,  -73.42830783,  -18.76561364,  -40.70334508,
         -3.86329246,  -28.86271479,  -31.24096056,    8.06655211,
        -47.51886222,   48.39153617,   12.67887539,  -69.00084269,
        -18.14654176,  -52.73329745,   44.94983809,   20.27265034,
        -55.14532272,  -49.74536301,  -67.14943615,  -25.7398257 ,
        -62.74745206,  -22.03869274,  -77.07034501,  -49.67299184])
```

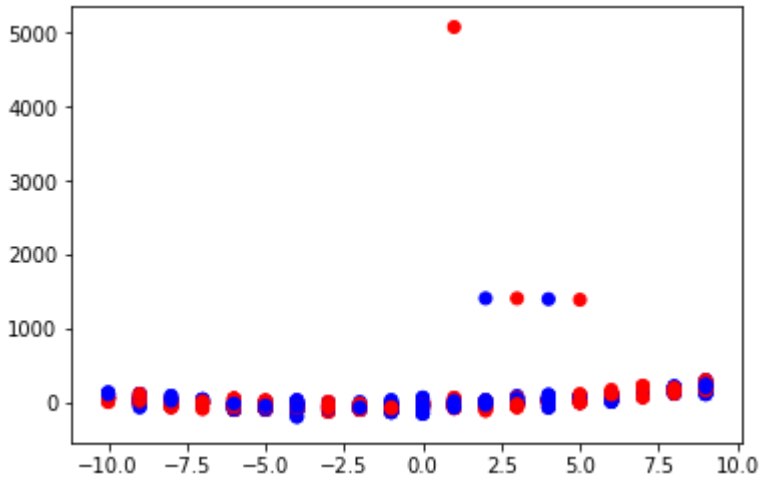
In [56]:
```
Y = 10*X + 2*(X**2) + E
```

In [57]:
```
Y
```

Out[57]:
```
array([   3.63828536,  205.92806214, -106.49159837,  207.98594237,
           51.7872448 ,  -43.41676241,  -72.11774042,   -4.98635178,
          -36.3647891 ,  -86.33107814,  -10.98082854,  302.53197061,
          117.2530673 ,   76.95986768,   41.90794665,  -64.84177202,
           66.38512668,  -13.89948178,  -55.15896861,  -29.90866629,
          173.65769888,   43.90178314,  -43.67493779,  180.7968296 ,
          -73.12629008,  -13.11009606,   73.42532399,  -32.06764873,
          106.48091041,  -60.39362213,  -46.25002295,  115.79027343,
          -45.22731879,  106.26717319,   19.01223924,   10.54723418,
          -45.1451044 ,  -49.93660361,  -33.69990645,  -10.66017936,
           68.02003428,   44.83774109,   68.26801115,  -26.43788905,
          -21.99481608,   82.10425316,  -50.95115113,   -4.1002098 ,
           -8.6651039 ,  137.865775  ,   64.69705939,   14.40276025,
           45.85849218,  220.05836482,   47.43978319,  202.86142509,
          -97.17525187,   11.51772742,  -40.06623851,  -20.7113197 ,
          -39.25862997,  -19.05313043,  176.45682286,  -56.62307868,
          216.45842283,  -39.36125088,   63.62114566,  118.95239751,
          -50.08975326,   91.84772235,  -82.33017695,   42.72369776,
          -22.00044899,  -37.25768124,  -47.17603824,  -26.96511722,
          107.56597574,  215.70349313,  -49.04538172,  -50.86874383,
           68.57761918, -189.72537846,  -65.68453504,   32.14863833,
          132.95219464,  -29.75592703,  -34.83611206, -102.38595097,
           33.8721398 ,  -31.53898795,  170.60368512,   -4.12273165,
           33.81913813,   -1.86908085, -106.9535578 ,    2.54862145,
          -79.31243878,  157.56741809,  -29.85151077,  -79.14012627,
          -40.02851776,  -31.74844986,  -53.82538505,   12.81616339,
           52.67600369,   26.37362826,   47.05154612,  -77.46402462,
          303.50731283,    4.49232508,  -60.81154911,  -23.23757047,
          -56.53857524,   55.50869339,   75.09785789,  -61.6591721 ,
          -72.36169383,  183.9028112 ,  -34.79931397,  171.8421679 ,
          224.10623452,    0.54099806,   15.06580422,  137.83924743,
          199.09798388,  -23.6107933 ,    1.80152476,   31.82591425,
          -74.58734746,  -50.87755623,   95.18349005,  -47.30614122,
          277.12806088,   -0.81241901,   34.03576745, -112.63748207,
          -79.73842361,  151.732782  ,  -63.63790122,   18.59475032,
           72.90925212,  248.41048014,   71.71169007,   77.92358338,
          166.45968566,   -7.30394903,   20.6238112 ,  110.50964898,
           22.55800591,   68.20425284,  121.68695867, -146.52630589,
          137.02346835,   11.11825698,  112.84804856,   84.98220541,
          -29.38882473,  -25.07752977,  159.08729967,   29.97933897,
           29.47603465,  -33.05573877,   33.49068086,   22.42897151,
           -4.31731178,  -64.84923364,  -73.27575218,  -57.60639882,
           18.45804271,  106.09346447,  -52.49122681,   -9.52380357,
            7.26011092,   27.16175956,   -2.81705849,  -67.18047688,
          -22.15250369,   58.57169217,  113.23438636,  -40.70334508,
          -15.86329246,   43.13728521,  -43.24096056,   56.06655211,
            0.48113778,  120.39153617,  180.67887539,  -69.00084269,
          113.85345824,  -64.73329745,  116.94983809,   92.27265034,
          -63.14532272,  -57.74536301,  -19.14943615,  -13.7398257 ,
          -14.74745206,   25.96130726,  174.92965499,  202.32700816])
```

In [58]:
```
import matplotlib.pyplot as plt
```

In [60]:
```
plt.scatter(X,Y,c=['red','blue'])
plt.show()
```



In [61]:
```
X1=X
X2=X**2
X3=X**3
X4=X**4
X5=X**5
```

In [62]:
```
X = np.stack((X1,X2,X3,X4,X5), axis = -1)
```

In [63]:
```
X
```

```
Out[63]: array([[     -8,      64,    -512,    4096,  -32768],
                [      9,      81,     729,    6561,   59049],
                [      2,       4,       8,      16,      32],
                [      9,      81,     729,    6561,   59049],
                [    -10,     100,   -1000,   10000, -100000],
                [      1,       1,       1,       1,       1],
                [     -6,      36,    -216,    1296,   -7776],
                [     -6,      36,    -216,    1296,   -7776],
                [     -6,      36,    -216,    1296,   -7776],
                [     -2,       4,      -8,      16,     -32],
                [     -7,      49,    -343,    2401,  -16807],
                [      9,      81,     729,    6561,   59049],
                [      7,      49,     343,    2401,   16807],
                [      5,      25,     125,     625,    3125],
                [     -8,      64,    -512,    4096,  -32768],
                [     -1,       1,      -1,       1,      -1],
                [      6,      36,     216,    1296,    7776],
                [     -1,       1,      -1,       1,      -1],
                [     -8,      64,    -512,    4096,  -32768],
                [      1,       1,       1,       1,       1],
                [      9,      81,     729,    6561,   59049],
                [      3,       9,      27,      81,     243],
                [      0,       0,       0,       0,       0],
                [      8,      64,     512,    4096,   32768],
                [     -4,      16,     -64,     256,   -1024],
                [      3,       9,      27,      81,     243],
                [      6,      36,     216,    1296,    7776],
                [     -4,      16,     -64,     256,   -1024],
                [    -10,     100,   -1000,   10000, -100000],
                [     -9,      81,    -729,    6561,  -59049],
                [      3,       9,      27,      81,     243],
                [    -10,     100,   -1000,   10000, -100000],
                [      2,       4,       8,      16,      32],
                [      4,      16,      64,     256,    1024],
                [     -9,      81,    -729,    6561,  -59049],
                [     -5,      25,    -125,     625,   -3125],
                [      1,       1,       1,       1,       1],
                [     -2,       4,      -8,      16,     -32],
                [     -2,       4,      -8,      16,     -32],
                [     -8,      64,    -512,    4096,  -32768],
                [    -10,     100,   -1000,   10000, -100000],
                [      5,      25,     125,     625,    3125],
                [    -10,     100,   -1000,   10000, -100000],
                [     -5,      25,    -125,     625,   -3125],
                [     -8,      64,    -512,    4096,  -32768],
                [      5,      25,     125,     625,    3125],
                [      3,       9,      27,      81,     243],
                [      1,       1,       1,       1,       1],
                [      0,       0,       0,       0,       0],
                [      9,      81,     729,    6561,   59049],
                [      1,       1,       1,       1,       1],
                [      2,       4,       8,      16,      32],
                [    -10,     100,   -1000,   10000, -100000],
                [      8,      64,     512,    4096,   32768],
                [      5,      25,     125,     625,    3125],
                [      9,      81,     729,    6561,   59049],
                [     -2,       4,      -8,      16,     -32],
```

```
[      4,      16,      64,     256,    1024],
[      0,       0,       0,       0,       0],
[     -6,      36,    -216,    1296,   -7776],
[      3,       9,      27,      81,     243],
[     -5,      25,    -125,     625,   -3125],
[      8,      64,     512,    4096,   32768],
[     -4,      16,     -64,     256,   -1024],
[      9,      81,     729,    6561,   59049],
[     -4,      16,     -64,     256,   -1024],
[    -10,     100,   -1000,   10000, -100000],
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[     -2,       4,      -8,      16,     -32],
[      1,       1,       1,       1,       1],
[      1,       1,       1,       1,       1],
[     -5,      25,    -125,     625,   -3125],
[      7,      49,     343,    2401,   16807],
[      9,      81,     729,    6561,   59049],
[      1,       1,       1,       1,       1],
[     -3,       9,     -27,      81,    -243],
[      3,       9,      27,      81,     243],
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[      1,       1,       1,       1,       1],
[     -1,       1,      -1,       1,      -1],
[      7,      49,     343,    2401,   16807],
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[      0,       0,       0,       0,       0],
[     -1,       1,      -1,       1,      -1],
[    -10,     100,   -1000,   10000, -100000],
[     -1,       1,      -1,       1,      -1],
[      9,      81,     729,    6561,   59049],
[      3,       9,      27,      81,     243],
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[      2,       4,       8,      16,      32],
[     -4,      16,     -64,     256,   -1024],
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[      2,       4,       8,      16,      32],
[     -5,      25,    -125,     625,   -3125],
[     -7,      49,    -343,    2401,  -16807],
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[     -8,      64,    -512,    4096,  -32768],
[      6,      36,     216,    1296,    7776],
[     -3,       9,     -27,      81,    -243],
[      9,      81,     729,    6561,   59049],
[      3,       9,      27,      81,     243],
[      3,       9,      27,      81,     243],
[     -3,       9,     -27,      81,    -243],
[      0,       0,       0,       0,       0],
[      4,      16,      64,     256,    1024],
```

```
[      6,      36,     216,    1296,    7776],
[     -2,       4,      -8,      16,     -32],
[      1,       1,       1,       1,       1],
[      9,      81,     729,    6561,   59049],
[     -2,       4,      -8,      16,     -32],
[      7,      49,     343,    2401,   16807],
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[     -9,      81,    -729,    6561,  -59049],
[      1,       1,       1,       1,       1],
[     -6,      36,    -216,    1296,   -7776],
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[     -9,      81,    -729,    6561,  -59049],
[     -4,      16,     -64,     256,   -1024],
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[     -1,       1,      -1,       1,      -1],
[     -8,      64,    -512,    4096,  -32768],
[     -9,      81,    -729,    6561,  -59049],
[      9,      81,     729,    6561,   59049],
[      7,      49,     343,    2401,   16807],
[      5,      25,     125,     625,    3125],
[      9,      81,     729,    6561,   59049],
[      3,       9,      27,      81,     243],
[    -10,     100,   -1000,   10000, -100000],
[    -10,     100,   -1000,   10000, -100000],
[      1,       1,       1,       1,       1],
[      0,       0,       0,       0,       0],
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[     -8,      64,    -512,    4096,  -32768],
[      5,      25,     125,     625,    3125],
[      6,      36,     216,    1296,    7776],
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[      0,       0,       0,       0,       0],
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[      0,       0,       0,       0,       0],
[      1,       1,       1,       1,       1],
[      1,       1,       1,       1,       1],
[     -8,      64,    -512,    4096,  -32768],
[     -6,      36,    -216,    1296,   -7776],
[     -3,       9,     -27,      81,    -243],
[     -4,      16,     -64,     256,   -1024],
[     -5,      25,    -125,     625,   -3125],
[      6,      36,     216,    1296,    7776],
[     -6,      36,    -216,    1296,   -7776],
```
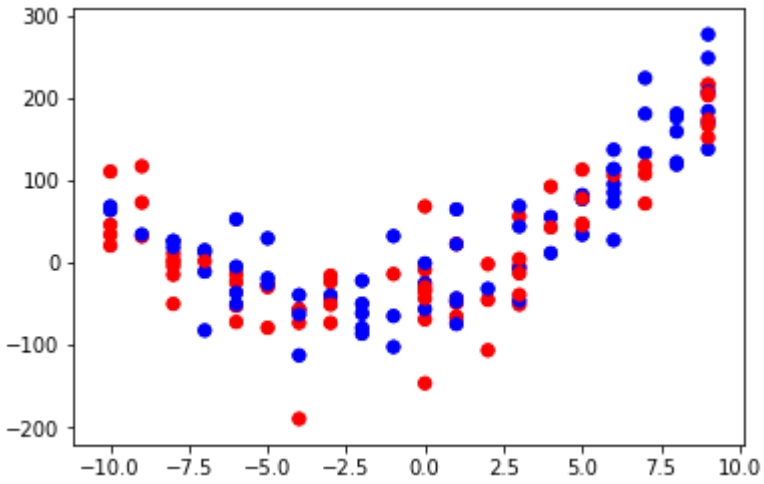

```
[     -4,      16,     -64,     256,   -1024],
[      5,      25,     125,     625,    3125],
[      6,      36,     216,    1296,    7776],
[      3,       9,      27,      81,     243],
[      1,       1,       1,       1,       1],
[     -5,      25,    -125,     625,   -3125],
[      6,      36,     216,    1296,    7776],
[      6,      36,     216,    1296,    7776],
[     -5,      25,    -125,     625,   -3125],
[     -3,       9,     -27,      81,    -243],
[      4,      16,      64,     256,    1024],
[     -3,       9,     -27,      81,    -243],
[      3,       9,      27,      81,     243],
[     -8,      64,    -512,    4096,  -32768],
[     -9,      81,    -729,    6561,  -59049],
[      7,      49,     343,    2401,   16807],
[      0,       0,       0,       0,       0],
[      6,      36,     216,    1296,    7776],
[     -2,       4,      -8,      16,     -32],
[     -9,      81,    -729,    6561,  -59049],
[      4,      16,      64,     256,    1024],
[     -4,      16,     -64,     256,   -1024],
[     -4,      16,     -64,     256,   -1024],
[      3,       9,      27,      81,     243],
[     -6,      36,    -216,    1296,   -7776],
[     -8,      64,    -512,    4096,  -32768],
[     -8,      64,    -512,    4096,  -32768],
[      9,      81,     729,    6561,   59049],
[      9,      81,     729,    6561,   59049]])
```

In [64]:
```
from sklearn.model_selection import train_test_split
```

In [65]:
```
X_train, X_test, Y_train, Y_test = train_test_split(X,Y, test_size=0.3, random
_state=1)
```

In [67]:
```
plt.scatter(X_train[:,0],Y_train,c=['RED','BLUE'])
plt.show()
```



In [68]:
```
from sklearn import linear_model
```

In [69]:
```
lR1 = linear_model.LinearRegression()
lR2 = linear_model.LinearRegression()
lR3 = linear_model.LinearRegression()
lR4 = linear_model.LinearRegression()
lR5 = linear_model.LinearRegression()
```

In [70]:
```
lR1.fit(X_train[:,0:1],Y_train)
lR2.fit( X_train[:,0:2],Y_train)
lR3.fit( X_train[:,0:3],Y_train)
lR4.fit( X_train[:,0:4],Y_train)
lR5.fit( X_train[:,0:5],Y_train)
```

Out[70]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=1, normalize=False)
```

In [71]:
```
from sklearn.metrics import mean_squared_error
```

In [72]:
```
Y1_pred = lR1.predict(X_train[:,0:1])
Y2_pred = lR2.predict(X_train[:,0:2])
Y3_pred = lR3.predict(X_train[:,0:3])
Y4_pred = lR4.predict(X_train[:,0:4])
Y5_pred = lR5.predict(X_train[:,0:5])
```

In [73]:
```
mse1 = mean_squared_error(Y_train, Y1_pred)
```

In [75]:
```
mse1
```

Out[75]:
```
5252.2402112628306
```

In [76]:
```
mse2 = mean_squared_error(Y_train, Y2_pred)
```

In [77]:
```
mse2
```

Out[77]:
```
1565.5449574007125
```

In [78]:
```
mse3 = mean_squared_error(Y_train, Y3_pred)
```

In [79]:
```
mse3
```

Out[79]:
```
1552.8356617790018
```

In [80]:
```
mse4 = mean_squared_error(Y_train, Y4_pred)
```

In [81]:
```
mse4
```

Out[81]:
```
1543.0623923963494
```

In [82]:
```
mse5 = mean_squared_error(Y_train, Y5_pred)
```

In [83]:
```
mse5
```

Out[83]:
```
1541.9611214264285
```

In [84]:
```
error_train = [mse1,mse2,mse3,mse4,mse5]
```

In [85]:
```
error_train
```

Out[85]:
```
[5252.2402112628306,
 1565.5449574007125,
 1552.8356617790018,
 1543.0623923963494,
 1541.9611214264285]
```

In [86]:
```
Y1_pred = lR1.predict(X_test[:,0:1])
Y2_pred = lR2.predict(X_test[:,0:2])
Y3_pred = lR3.predict(X_test[:,0:3])
Y4_pred = lR4.predict(X_test[:,0:4])
Y5_pred = lR5.predict(X_test[:,0:5])
```

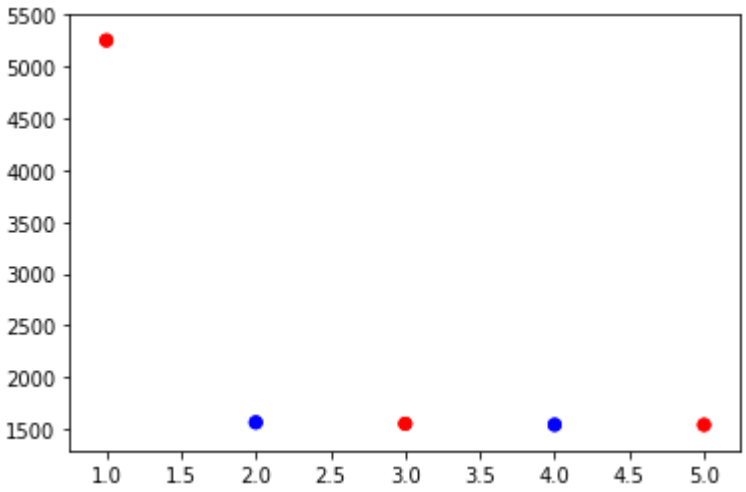
In [87]:
```
mse1 = mean_squared_error(Y_test, Y1_pred)
mse2 = mean_squared_error(Y_test, Y2_pred)
mse3 = mean_squared_error(Y_test, Y3_pred)
mse4 = mean_squared_error(Y_test, Y4_pred)
mse5 = mean_squared_error(Y_test, Y5_pred)
```

In [88]:
```
error_test = [mse1,mse2,mse3,mse4,mse5]
```

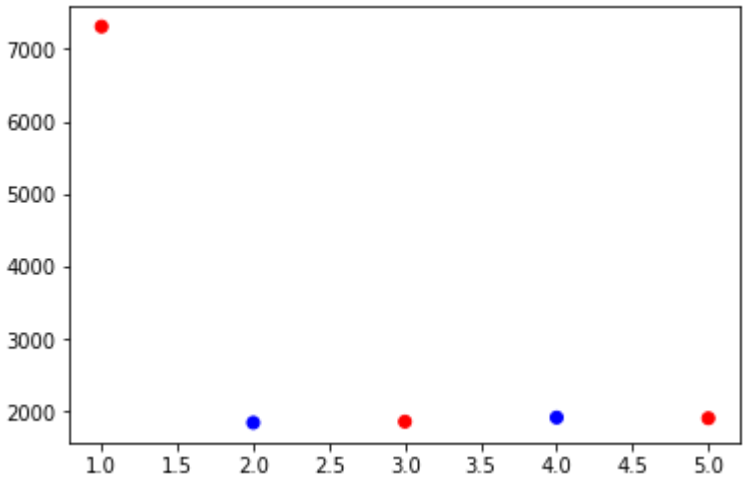
In [89]:
```
error_test
```

Out[89]:
```
[7308.7370165461152,
 1847.4058764987249,
 1861.8171469205558,
 1918.2861566918041,
 1908.7291955092471]
```

In [91]:
```
plt.scatter([1,2,3,4,5],error_train,c=['red','blue'])
plt.show()
```



In [92]:
```
plt.scatter([1,2,3,4,5],error_test,c=['red','blue'])
plt.show()
```



In [ ]:
```
Answers to Step 8:

Training error increases as the number of features k increases  from 1,2...5.
1) Whereas, test error decreases at the start and later increases as k increas
es
2) When sample size is 40: Training error and test error both decrease and aft
er few iterations become Almost constant
3) When sample size is 200: Training error and test error both decrease and af
ter few iterations become Almost constant
```