Enhanced Dynamic Spectrum Access in Multiband Cognitive Radio Networks via Optimized Transmission HyperspaceTM

Journal:	*IEEE Transactions on Wireless Communications*
Manuscript ID:	Paper-TW-Mar-15-0319
Manuscript Type:	Original Transactions Paper
Date Submitted by the Author:	11-Mar-2015
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Keyword:	

Enhanced Dynamic Spectrum Access in Multiband Cognitive Radio Networks via Optimized Transmission Hyperspace^{TM}

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Abstract

The "Transmission Hyperspace^{TM} (TH)" concept developed by Drozd et al. enhances dynamic spectrum access by exploiting multiple orthogonal communication dimensions. Practical implementation in terms of resource assignment in the TH is a very challenging task, since the solution may depend heavily on the given wireless network specifications. In this paper, we show the performance enhancement achieved by exploiting multiple dimensions of TH by considering a number of representative problems and by developing suitable optimization algorithms to enable practical implementations. In particular, we employ the three dimensions, namely frequency, power and transmit direction, and consider the problem of total sum-rate maximization of the secondary users (SUs) under the quality-of-service (QoS)

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constraints of the primary users (PUs) and the SUs. Since this problem is NP-hard, we develop heuristic algorithms which offer different trade-offs in terms of goodness of their solutions and their computational complexity. We also consider the problem subject to simultaneous multiple conflicting objectives such as maximizing the total sum-rate of SUs while minimizing the total emitted power from the SUs. Several bi-objective optimization problems for the TH are formulated, and a multiobjective evolutionary algorithm to solve these problems in a time-efficient manner is developed. Simulation results are provided to illustrate performance enhancement by using the TH paradigm as well as to investigate the computational efficiency of the proposed optimization algorithms.

I. INTRODUCTION

The emerging paradigm of Dynamic Spectrum Access (DSA) networks has been proposed as a solution to the problem of spectrum inefficiency especially with the rapid growth in demand for wireless. These DSA networks are implemented via cognitive radio networks that enable multiple radios operating with different priority levels to co-exist and co-operate in an environment without interfering with each other, so as to increase spectrum efficiency [1]. The overall objective of cognitive radio networking is to achieve maximized network efficiency without interrupting higher priority transmissions and without compromising security while jointly satisfying heterogeneous quality-of-service (QoS) requirements of multiple users.

Conventional Cognitive Radio Networks (CRNs) attempt to enhance spectral efficiency by assigning the frequency spectrum in an intelligent manner. The *"Transmission Hyperspace^{TM} (TH)"* system, developed by Drozd et al. [2], [3], represents a new technology for achieving enhanced dynamic spectrum access and spectrum maneuverability in the wireless applications by going beyond simply assigning or allocating frequency spectrum to networked communications systems. The Transmission Hyperspace^{TM} (TH) concept has been proposed to address the fundamental problems of spectrum crowding by providing control of multiple orthogonal communication dimensions such as frequency, time, space, coding, and antenna directionality using a system optimization approach [2], [3]. This is intended to maximize desired connectivity and throughput for intended users while concurrently denying access to unauthorized or malicious users. The TH concept is useful not only for single communication networks but also in environments where multiple communication networks co-exist with radars and multi-sensor systems. These radars and multi-sensor systems put additional constraints on the system design which complicates the shared dynamic spectrum access problem. The resulting TH based optimization problems usually boil down to spectrum sharing problems involving additional dimensions such as time and antenna parameters.

The TH paradigm regards the RF resource space as an electromagnetically occupied hypercube volume (Fig. 1) existing in multiple dimensions (time, space, frequency, beam direction, code/modulation, etc.) [4]. Here, the space is constantly changing with "cells" of network resources that have been assigned, used, and released. The system parameters are continuously adapted based upon feedback from sensed returns. Several key capabilities are brought together under the TH concept: (i) the ability to achieve dynamic "spectrum" access that goes beyond just allocating frequencies by employing a sense and adapt approach over multiple communication dimensions to "optimize" the RF transmission plan; (ii) the use of embedded algorithms that characterize the EM environment focusing on the physical (PHY) layer; and (iii) models to study the impact on the upper layers (data, network) due to incident electromagnetic interference and disruptive jamming at the PHY layer.

As indicated earlier, the frequency dimension is the fundamental parameter addressed by dynamic spectrum allocation techniques. It is the only dimension that is allocated under the condition that the secondary user (SU) must respect the legal rights of primary users (PUs) while also compensating for space, time and frequency variations due to multipath propagation, mobility, and location dependent shadowing. The TH concept considers additional dimensions that allows secondary users to continue transmitting even when primary users access the frequency channel by maintaining orthogonality in other communication dimensions. Furthermore, the use of directional antennas drastically increases the number of messages that can be sent at a given time. The antenna at each node can then be oriented in the direction of the node with which it is to communicate, and the signal is transmitted in a beam of limited angle originating from the sender. Consequently, such a message will not significantly interfere with the messages being received simultaneously by other nodes that do not lie close to the axis of the beam, or whose antennas are facing in substantially different directions. In our previous work [3], we have shown by simulations that for a 10×10 array of nodes, the number of time slots required to transmit a message from every node to every other node can be reduced by a factor of over 23 by the use of directional antennas with a beam-angle of 25 degrees, using the heuristic of allotting to each time slot those messages that interfere the least with other messages already allocated to that time slot. As the beam-angle reduces, this factor improves further. However, the beam-angle is a function of the hardware, possibly not tunable to fit a problem. In our earlier work [5], we derived explicit expressions for the successful communication probability (SCP) in a multi-hop CRN and we proved that the proposed TH concept significantly improves network utility in terms of SCP. In [5], we considered time, frequency, power, antenna directionality and beamwidth as the TH dimensions to improve SCP.

Although the advantages of using multiple transmit dimensions in the context of TH are obvious, the

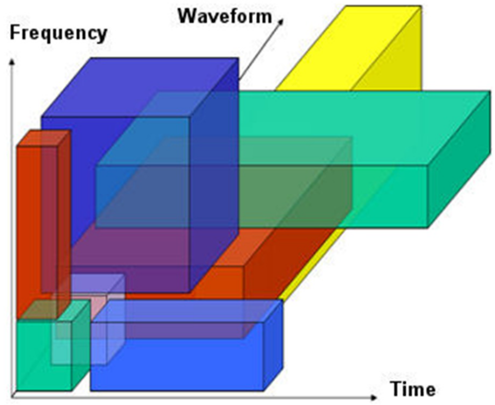


Fig. 1. Transmission Hyperspace^{TM} provides multidimensional DSA.

resulting optimization problems involving TH are more complex requiring the design of optimization approaches that yield high quality solutions in a computationally efficient manner. Furthermore, the solution (as well as the methodology to obtain that solution) heavily depends on the given problem specifications. In general, there is no unique (and global) method that can be applied to every cognitive radio networking optimization problem. In this paper, we assume that the SUs employ orthogonal frequency division multiplexing (OFDM) since it has been advocated as a promising candidate technology for cognitive radio networks [6]. We focus on three dimensions of the Transmission Hyperspace^{TM}. First, we consider only power and frequency allocations of SUs under the quality-of-service (QoS) constraints of the PUs and the SUs. Then, we add the third dimension of antenna directionality and solve the problem with power, frequency and directional antenna, i.e., spatial, dimensions. Our goal in this paper is to investigate the problem of spectrum sharing in CRNs utilizing TH. We discuss the challenges associated with the solution of these problems. Many relevant problems can be formulated but in this paper we consider five representative optimization problems and develop algorithms to solve them. We first show that all these problems are NP-hard [7] which requires the development of efficient heuristic algorithms. The first problem involves the maximization of total sum-rate of the SUs while satisfying the QoS constraints of PUs and SUs. For this problem, we propose three different suboptimal algorithms based on convex relaxation with tree pruning (CRTP), convex relaxation with gradual removal (CRGR), and genetic algorithms (GA). These algorithms offer different tradeoffs in terms of goodness of their solutions and computational complexity. In the second problem, we add antenna directionality to the first problem. The other three problems are multiobjective optimization problems, where along with total sum-rate maximization, we include total power minimization of the SUs, maximization of fairness among the SUs and maximization of the number of active SU links respectively. For these bi-objective problems, we propose a multiobjective evolutionary algorithm which generates the Pareto front between conflicting objectives in a time-efficient manner.

Our preliminary results were presented in [8] where we considered two TH dimensions, namely frequency and power, and presented one multi-objective optimization problem where the objectives were maximization of the total sum rate of the secondary users and the minimization of the total power of the SUs. In this paper, we first present and build on the work in [8] in Section III, and add one more TH dimension, transmit antenna direction, in Section IV. Furthermore, we formulate two new multiobjective problems in CRNs: i) we jointly maximize the total sum rate of the secondary users and the fairness between SUs, and ii) we jointly maximize the total sum rate of the secondary users and the total number of active SUs. Numerical results show the efficiency, in terms of solution quality and computational efficiency, of the proposed formulations for both the cases of omnidirectional and directional antennas.

Thus, the main contribution of this paper is to show the performance enhancement achieved by exploiting multiple dimensions of TH. We consider a number of representative problems and by developing suitable optimization algorithms to enable practical implementations. The rest of the paper is organized as follows. In Section II, we present the system model for the CRN and the problem statement. In Section III, we introduce the Maximization of Sum-Rate Under QoS Constraints with Omnidirectional Antennas and in Section IV, we state the same problem with directional antennas. In Section V, we introduce different multiobjective optimization problems in CRNs. Section VI presents our numerical results, and finally Section VII is devoted to our conclusions and future research directions.

II. System Model and Problem Statement

We consider a CRN, where the available frequency band is shared between SUs and PUs in the network under the constraint that SU transmitters do not create harmful interference at PU receivers. We assume that the shared spectrum is divided into K discrete frequency subbands and, without loss of generality, each subband has an identical bandwidth of B Hz. This set of assumptions is applicable to systems using orthogonal-frequency-division-multiplexing (OFDM) technology. We number each secondary and primary transmitter-receiver pair by the indices $n \in \mathcal{N} = \{SU_1, \ldots, SU_N\}$ and $m \in \mathcal{M} = \{PU_1, \ldots, PU_M\}$, respectively, and refer to them as users. Throughout the paper, the terms subband and carrier are used interchangeably. Our formulations are based on the physical model [9], which provides a realistic modeling of the physical communication environment by utilizing a path-loss model. The general path loss between any transmitter i and any receiver j is given as

$$L_{i,j}(k) = \frac{G_{t,i}G_{r,j}}{d_{i,j}^{\alpha}} \left(\frac{c}{4\pi f(k)}\right)^2, \tag{1}$$

where $G_{t,i}$ and $G_{r,j}$ are the transmit and receiver antenna gains respectively, c is the speed of light, $f(k)$ is the carrier frequency of subband k, $d_{i,j}$ is the distance between transmitter i and receiver j, and α is the attenuation constant. We assume that the path-loss in the received power is the dominant loss factor, and therefore, we neglect the effects of shadowing and multi-path fading. We assume an additive white Gaussian (AWGN) channel with zero mean and variance N_0. Under these assumptions, the achievable data rate of secondary user n can be expressed [10] as:

$$R_n = B \sum_{k=1}^{K} \log[1 + \gamma_n(k)], \tag{2}$$

where $\log$ is defined in base 2 and $\gamma_n(k)$ is the signal-to-interference-plus-noise ratio (SINR) of secondary user n on carrier k,

$$\gamma_n(k) \triangleq \frac{p_n(k)L_{n,n}(k)}{N_0 + \sum_{l\in\mathcal{N}\cup\mathcal{M}, l\neq n} p_l(k)L_{l,n}(k)}. \tag{3}$$

In (3), $p_n(k)$ and $p_l(k)$ represent transmit powers of the n-th secondary user and the l-th primary or secondary user, respectively. The SINR condition for establishing a successful communication link n on carrier k is given by $\gamma_n(k) \geq \gamma_n^*$.

We assume a narrowband primary network, where a single channel with a predetermined transmit power value is allocated to each primary user. This scenario is applicable to networks where legacy radios have the licenses to operate on narrowband channels. Generalization to a wideband primary network is straightforward and does not affect the methodology. Secondary users utilize multiband techniques where each band corresponds to a primary narrowband channel, to access the spectrum and each secondary user has a power budget denoted by P^B.

III. P1:Maximization of Sum-Rate Under QoS Constraints with Omnidirectional Antennas

In this section, we consider omnidirectional antennas based on the path loss model in (1) with $G_{t,i} = G_{r,j} = 1$, $\forall i, j$. We assume that each PU occupies a single subband and PUs operate on disjoint subbands. This results in the equality $M = K$. Given primary network activity and location of users, the optimization problem is to maximize the sum-rate (or achievable capacity) of the secondary network. The optimization variables are power levels allocated to each secondary user over each shared frequency subband.

Define $\mathbf{p}_n \triangleq [p_n(1), \ldots, p_n(K)]^T$ as the power allocation vector where each element represents the power level allocated to secondary user n over each subband. User n is said to be inactive over frequency band k if $p_n(k) = 0$. A secondary user is said to be active if it is transmitting on at least one of the K subbands. Each primary user occupies one of the K subbands and primary users operate at disjoint subbands. An SU is allowed to transmit on a subband, if and only if it does not violate any SINR of PU and SU pairs or its own power budget constraints.

The sum-rate maximization problem, P1, is formulated as follows:

$$\begin{aligned}
\text{Find} \quad & \mathbf{p}_n, \quad \forall n \in \mathcal{N} \\
\text{Maximize} \quad & \sum_{n=1}^{N} R_n(\mathbf{p}_n) && (4)\\
\text{Subject to} \quad & \gamma_m(k) \geq \gamma_m^*, \quad \forall m \in \mathcal{M}, && (5)\\
& \gamma_n(k) \geq \mathbf{I}(p_n(k))\gamma_n^*, \quad \forall n \in \mathcal{N}, \forall k \in \{1, \ldots, K\} && (6)\\
& p_n(k) \geq 0, \quad \forall n \in \mathcal{N}, \forall k \in \{1, \ldots, K\} && (7)\\
& \sum_{k=1}^{K} p_n(k) \leq P^B, \quad \forall n \in \mathcal{N}. && (8)
\end{aligned}$$

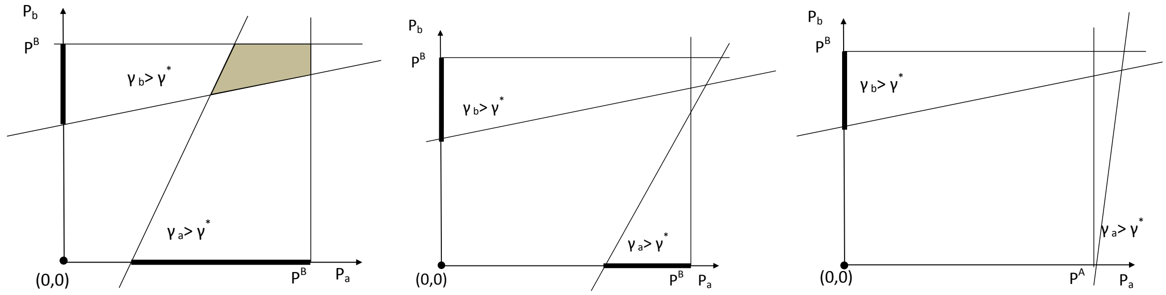
In P1, $\mathbf{I}(\cdot)$ is the indicator function for the set of positive real numbers. Inequality (5) represents a set of $K(= M)$ SINR constraints for the primary users. The inequality (6) represents a set of $N \times K$ SINR constraints for each of the N SUs over each of the K frequency subbands. The SU n will transmit at a frequency k, if and only if the SINR corresponding to that link is greater than or equal to the threshold SINR. If the link is not active over that subband, i.e., if $p_n(k) = 0$, the SINR constraint is automatically satisfied. It should be noted that P1 is in fact a *soft-spectrum allocation* and *power allocation* problem. The difference between conventional spectrum and power allocation formulations that have been considered in the literature, e.g. [11], [12], and our problem P1 is that conventional formulations treat spectrum allocation as a hard allocation problem such that no two users share the same spectrum. In conventional settings, spectrum allocation is carried out first based on channel conditions followed by power allocation [11], [12]. However, in our formulation, the constraint in (6) provides a way to allocate the spectrum in such a way that multiple SUs can share the spectrum via other orthogonal dimensions of TH as long as their QoS constraints are not violated.

Proposition 1: The sum-rate maximization problem with QoS constraints P1 is NP-hard.

Proposition 1 can be proved by noticing that P1 is a generalization of the sum-rate maximization problem without the QoS constraints which was shown to be NP-hard [13].

The NP-hardness of the above problem motivates the development of efficient suboptimal algorithms. In order to understand the problem better, let us consider a simple example of two SUs, one PU and one frequency subband. For a given set of locations, we draw the SU SINR constraints given in (6) in Fig. 2(a). For now, we ignore the SINR constraints for PUs. The area inside the square formed by the bounds on the SU power budget is the region where power budget constraints are satisfied. There are four possibilities: both SUs transmit, only the first SU transmits, only the second SU transmits, or both SUs are off. When both SUs transmit, SINR constraints for both SUs must be satisfied simultaneously which is represented by the gray region in Figure 2(a). Let SU_a and SU_b be two SU links operating at the same frequency with powers p_a and p_b, respectively. The SINRs for SU_a and SU_b, γ_a and γ_b, should satisfy a certain value, so that $\gamma_a > \gamma^*$ and $\gamma_b > \gamma^*$. The maximum power levels for SU_a and SU_b are limited to P^B. When only SU_a transmits, only the SINR constraint for SU_a needs to be satisfied. In that case, the constraint in (6) for SU_b is automatically satisfied because $p_b = 0$. Here, the dark line on the horizontal axis is feasible. Similar results hold when only SU_b transmits. When both are off, the constraints in (6) are automatically satisfied and this region is a single point $(0, 0)$. Now, with a different set of locations, it is possible that the receivers of both SUs are close to each other. Then the constraint region may look as shown in Fig. 2(b). Here, there is no region where both SU SINR constraints for SUs and power budget constraints are satisfied simultaneously. In other words, it is not feasible for both SU_a and SU_b to transmit simultaneously. Another scenario could arise when some secondary transmitters and receivers are so far apart, or the secondary receiver is so close to the PU transmitter such that there is no region where that user can be active, i.e, there is no region where the power budget as well as the SINR constraint for that SU is satisfied simultaneously. This scenario is shown in Fig. 2(c). Note that, if this is the case, i.e., an SU cannot transmit even when other SUs are OFF, then the possibility of that SU transmitting when others are ON is automatically eliminated. We now consider the PU constraints defined in (5). For a given constant PU transmit power, the constraints in (5) define halfspaces. Therefore, in the above three scenarios, the addition of linear PU constraints will only reduce the volume of the feasible regions shown in Figures 2(a)-2(c) for a two dimensional case. Resulting feasible regions will be the intersection of the previous regions and the additional halfspaces.

We propose three different algorithms to solve Problem P1 based on the following methods: 1) convex relaxation with tree pruning; 2) convex relaxation with gradual removal; 3) genetic algorithms. The comparison of these three algorithms in terms of finding good quality solutions and time complexity will be investigated later. The following subsections provide a detailed description of the algorithms.



(a) Scenario-I (b) Scenario-II (c) Scenario-III

Fig. 2. Possible Scenarios in Secondary Power Allocations

A. CRTP: Convex Relaxation with Tree Pruning

We start by noting that different combinations of ON and OFF power variables define different disjoint regions in space. There are a total of $2^{N\times K}$ ON/OFF combinations and hence that many regions. However, the number of these regions which are feasible depends on the locations of secondary and primary users in the network.

For each feasible region, there are M linear PU SINR constraints, N linear power budget constraints and some linear SU SINR constraints whose number depends upon the number of non zero elements of the power vector $\mathbf{p}_n$. Furthermore, we note that the objective function is nonconvex. In order to alleviate the nonconvexity issue, we first use the following approximation $\log(1+x) \approx \log(x)$ which is tight for values of $SINR \geq 5$. The same approximation has been used in [14], [15] for different problems. In a recent work [16], the authors have shown that $p_k = ce^{\mu s_k}$, $c, \mu > 0$ is the unique transformation that transforms the objective function defined by $g(\mathbf{p}) = \sum_{n=1}^{N} \log\left(\frac{p_n}{N_n + \sum_{l\neq n} a_{l,n} p_l}\right)$ into a convex function. Note that in $g(.)$, we set $K = 1$ for notational convenience and $a_{l,n} \triangleq L_{l,n}/L_{n,n}$ and $N_n \triangleq N_0/L_{n,n}$. The same transformation has been used by others in similar formulations (e.g., [14], [17]). Using the high SINR approximation and applying the transformation $p_n = e^{s_n}$, the resulting problem becomes convex in each feasible region. However, the problem as a whole remains nonconvex as all these feasible regions are disjoint, resulting in a nonconvex global feasible set.

In order to solve this problem globally, we need to find the regions that are feasible. Investigating all the possible regions (2^{MN}) is a computationally expensive task especially for large number of variables. There is an efficient way of reducing the computational effort by eliminating infeasible regions of the 2^{MN} possible disjoint regions. For a given region, observe that the non-trivial SU SINR constraints consist of a group of linear inequalities with the number of variables being equal to the number of inequalities. We can consider these linear inequalities as equalities and solve for $\mathbf{p}_n$. The resulting solution, say (P^*), is the minimum feasible solution, in the given region, which satisfies the SU SINR constraints. This means any power vector p' in this region which satisfies non-trivial SU SINR constraints follows $p' \succeq P^*$, where $\succeq$ is element-wise inequality [18], [19].

Once we have the solution (P^*) for a given region we check to see if it satisfies M PU SINR constraints and also the N power budget constraints. If these constraints are violated at $p = P^*$, it means that there is no feasible p that satisfies the PU SINR and the power budget constraints without violating the SU SINR constraints. Using this reasoning, we can eliminate all the infeasible regions using a tree pruning method. We start with the lowest branch of the tree which has one variable. Note that the lowest branches define regions in space where only one SU is active over exactly one subband. We then solve the SU SINR linear equations for that region and check if the solution violates the PU SINR or power budget constraints. If it violates any of these constraints, the region is eliminated and is not considered any further. We know that if a combination of some users active over some frequency band is infeasible then all combinations that are supersets of that combination are also infeasible. Then, we can prune that branch of the tree.

For each of the remaining feasible regions, we have a convex optimization problem which can be solved globally and efficiently. The optimum solutions for each of these regions are stored and then compared to find the best solution, i.e., maximum sum-rate, and the corresponding power allocation vector.

If the SINR threshold γ_n^* is sufficiently large (i.e., $\log(1+x) \approx \log(x)$), the CRTP algorithm will find solutions that are very close to the global optimum since the relaxed convex problem approximates the original problem with high accuracy in the feasible regions. Note that the tree pruning approach helps reduce the computational complexity, which can be a bottleneck for large problems with too many variables. Table I shows an example of how the number of regions and the number of feasible regions scale with the problem size for a specific realization of the problem. The advantage we get from the tree pruning method can be deduced from the table.

B. CRGR - Convex Relaxation with Gradual Removal

We note that the worst-case complexity of the CRTP algorithm is exponential. In order to find the solutions faster, we propose a heuristic algorithm, referred to as CRGR, based on convex relaxation with gradual removal. This algorithm is very similar to the one proposed in [14] without the QoS constraints. Although this method does not guarantee an optimum solution, it can give good solutions with less computational effort.

TABLE I

COMPARISON OF NUMBER OF FEASIBLE REGIONS WITH PROBLEM SIZE

N	K	No. variables	No. regions	Feasible regions
5	2	10	1024	255
4	3	12	4096	1535
7	2	14	16384	2319
8	2	16	65536	16383
9	2	18	262144	31645

Several different disjoint regions exist in the problem because of SU SINR constraints, which differ from region to region. Without SU SINR constraints, the remaining constraints define a space which is an intersection of halfspaces. This means that without the SU SINR constraints, the optimization space becomes a convex set. The optimization problem P1 can be easily solved if SU SINR constraints are excluded. In CRGR, we use the same high SINR approximation as before ($\log(1+x) \approx \log(x)$) and apply the $p = e^s$ transformation. The resulting problem is convex. After solving this constrained optimization problem without the SU SINR constraints, we switch off the link over the subband which has the highest (SU SINR) constraint violation, and also switch off the links over subbands for which the power level as a result of optimization is close to zero. This process reduces the number of variables as the power level for some links over certain subbands is fixed at zero. The optimization problem P1 is again solved without the SU SINR constraints with the reduced variable set. This process is repeated until we get a solution which satisfies all the PU, SU and power budget constraints. The steps of the algorithmic procedure are shown in Algorithm 1.

C. GA - Genetic Algorithm

Heuristic artificial intelligence techniques have been used to solve NP-hard problems which tend to exploit the structure of the problem and arrive at a good solution. In this section, we solve P1 with a genetic algorithm (GA) which belongs to the evolutionary class of artificial intelligence techniques. This algorithm is based on the domination principle. A feasible solution always dominates an infeasible solution. Among two infeasible solutions, the one with lower overall constraint violation dominates the other. Among two feasible solutions, one with higher objective value dominates the other (for a maximization problem). The aim of the algorithm is to iteratively improve a set of solutions which are not dominated by other solutions. Given enough number of iterations, GAs have been empirically shown to produce optimal or close to optimal results for highly complex optimization problems. Following is

Algorithm 1 CRGR algorithm

1: **repeat**
2: Solve P1 without SU SINR constraints
3: Maximum SU SINR constraint violation $= \max(0, \mathbf{I}(p_n(k))\gamma_n^*) - \gamma_n(k)), \quad \forall n \in \mathcal{N}, \quad \forall k \in \{1, \ldots, K\}$
4: Fix $p_n(k) = 0$ for link with maximum SU SINR constraint violation
5: **if** $p_n(k) < 10^{-3} \quad \forall n \in \mathcal{N}, \quad \forall k \in \{1, \ldots, K\}$ **then**
6: Set $p_n(k) = 0$
7: **end if**
8: **until** Maximum SU SINR constraint violation $= 0$
9: $p_n(k) \quad \forall n \in \mathcal{N}, \quad \forall k$ is the sub-optimal solution

the description of our GA, which is a modified version of [20], employed in this paper:

1) Representation: A gene/solution is a structure comprising of $N \times K$ real variables corresponding to the powers of SUs. Let $\mathbf{p}_i^t$ be a solution with elements $p_n^t(k)$ at time (or generation) t and $i \in \{1, \ldots, \mathcal{S_P}\}$ where $\mathcal{S_P}$ is the population size.

2) Initialization: The population is created by initializing each variable in each gene according to uniform random distribution between its lower and upper bounds.

Note that given the variable bounds $p_n(k) \in [0, p_{max}]$, it is very unlikely to achieve some solutions which have the form $p_n(k) = 0$. In this form, $p_n(k) \in [0, p_{max}]$, GA may fail to provide good solutions since it becomes hard to discover all the feasible regions. This fact motivates modifications that would increase the probability of creating individuals in each and every feasible region. Therefore, we change the variable bounds from $p_n(k) \in [0, p_{max}]$ to $p_n(k) \in [-p_{max}, p_{max}]$. Whenever a variable is less than zero, $p_n(k) \leq 0$, we treat it as equal to zero, $p_n(k) = 0$, while evaluating the objective function. This modification provides more homogeneous distribution for generating different types of SU admission strategies (some SU links are on and some of them off) and GA explores the feasible regions better.

3) Selection: The individuals that will go into the mating pool are chosen according to tournament selection [21]. $2 \times \mathcal{S_P}$ pairs of individuals are selected randomly without replacement and the best of each pair goes into the mating pool. In case both individuals are non-dominated, one is selected with probability 0.5. In case of a single objective, this means that the best individual gets two copies in the mating pool.

4) Crossover: From the mating pool, $2 \times \mathcal{S}_{\mathcal{P}}$ random pairs of individuals are created and the best of each pair is passed on into the mating pool. The individuals in the mating pool undergo Simulated Binary Crossover (SBX)[21] to form the offspring solutions. Let $\mathbf{p}_1^t$ and $\mathbf{p}_2^t$ be two parent solutions. The offspring solutions are created according to,

$$\mathbf{p}_1^{t+1} = 0.5[(1+\beta_q)\mathbf{p}_1^t + (1-\beta_q)\mathbf{p}_1^t],$$
$$\mathbf{p}_2^{t+1} = 0.5[(1-\beta_q)\mathbf{p}_2^t + (1+\beta_q)\mathbf{p}_2^t],$$

where β_q follows the probability distribution

$$c(\beta_q) = \begin{cases} 0.5(\eta_c+1)\beta_q^{\eta_c}, & \text{if } \beta_q \leq 1 \\ 0.5(\eta_c+1)/\beta_q^{\eta_c+2} & \text{otherwise} \end{cases} \tag{9}$$

as defined in [21].

In order to promote diversity, the negative elements of $\mathbf{p}_1^t$ and $\mathbf{p}_2^t$ are first updated by a new value which is uniformly generated within the interval $[-p_{max}/10, 0]$. The division by a factor of 10 generates values closer to zero, which then increase the chances of generating offspring with positive value.

5) Mutation: Each variable of each gene is mutated with a probability p_m. If $p_n^t(k) > 0$, polynomial mutation is performed [21]. Mutated value is equal to

$$p_n^t(k) = p_n^t(k) + (p_{max} - 0)\delta$$

where

$$\delta = \begin{cases} (2u)^{1/(\eta_m+1)} - 1, & \text{if } u < 0.5 \\ 1 - |(2(1-u))|^{1/(\eta_m+1)} & \text{otherwise} \end{cases} \tag{10}$$

If $p_n^t(k) < 0$ then any of the following mutations is performed by equal probability: (i) $p_n^t(k) \sim U[0; p_{max}]$, where $U[a,b]$ denotes uniform distribution between a and b. This means, if we mutate a variable which is OFF, we turn it ON and assign it a random power. (ii) The user is turned ON at the smallest power level that satisfies its SINR constraint. (iii) The user is turned ON at a power which is equal to the average of its power at other channels.

6) Elitism: All solutions of generation t and $t-1$ are combined and the best solutions from this set go to population of $t+1$.

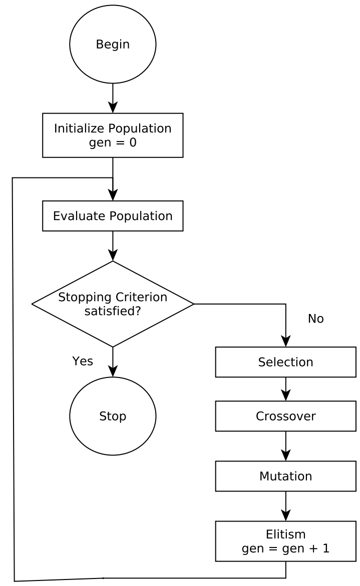


Fig. 3. Flowchart of Genetic Algorithm [21]

7) Evaluate the function value: Repeat Steps 2 through 7 until the solution does not improve significantly. A common approach is to run the program for a specified large number of iterations.

A flowchart representation of GA is illustrated in Fig. 3.

IV. P2:Maximization of Sum-Rate under QoS Constraints with Directional Antennas

In the previous section, we assumed that all users transmit with omnidirectional antennas. In this section, we no longer operate under that assumption and consider directional antennas at transmitters. In this setting, users transmit using directional antennas and receive using omnidirectional antennas. This can be realized by transmit beamforming techniques. Transmit directionality is expected to increase network throughput and decrease delay by providing enhanced spatial reuse and increased range [22]. Modeling a precise antenna pattern has two drawbacks. First, it is an extremely difficult task that is not the focus of this paper. Second, a precise antenna pattern would make the optimization problem prohibitively complex to tackle. To alleviate these drawbacks, we consider the so-called keyhole model [22], [23], [5], a two-dimensional antenna pattern approximation. Previous research works, e.g. [22], [23], [5], have used similar antenna pattern approximations in the context of similar but different network optimization and analysis problems. The keyhole model is simple enough for our problem, but still reflects the main characteristics of a realistic antenna pattern as shown in Fig. 4. Using this keyhole pattern, we are able to model both the enhanced power gain in the main-lobe region and the reduced power gain in the side-lobe region compared to an isotropic antenna pattern used in the previous section.

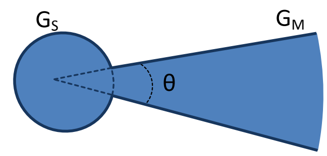


Fig. 4. Keyhole Antenna Pattern.

The derivation of different antenna gains that provide a fair comparison with omnidirectional antennas is provided in [5]. Here, we briefly explain how we can simulate different antenna gains G_M and G_S to make such a comparison. Let θ denote the main lobe beamwidth in radians. We define normalized beamwidths α and η such that $\alpha = \theta/2\pi$ and $0 \leq \eta \leq (1-\theta)/2\pi$. After some manipulations, the transmit antenna gains are calculated as

$$G_m = \frac{1}{\alpha + \eta}, \; G_s = \frac{\eta}{(1-\alpha)(\alpha + \eta)}. \tag{11}$$

The purpose of introducing η is that the main lobe and the side lobe gains can be controlled by adjusting η. For instance, $\eta = (1-\theta)/2\pi$ corresponds to an omnidirectional antenna pattern. Similarly as $\eta \to 0$, the side lobe power vanishes, resulting in an antenna pattern pointing all its transmitted power towards the main lobe direction. If we denote the isotropic transmission power defined in the previous section as P_t, the resulting equivalent directional power in the direction of the main lobe as well as in the direction of the side lobe can be calculated, respectively, as

$$P_t^m = \frac{P_t}{\alpha + \eta}, \; P_t^s = \frac{\eta P_t}{(1-\alpha)(\alpha + \eta)} \tag{12}$$

In this problem formulation, everything stays the same as in the previous formulation (in Section III) except for different transmit gains G_M and G_S in the main lobe and the side lobe, respectively. Given the locations of users, the new path loss for user n is now a function of the transmit antenna direction ϕ_n. A communication link setup with transmit directionality is illustrated in Fig. 5.

Note that the interference term in (3) which is the summation of the power terms from all other transmitters is now a function of other transmitters' transmit directions. Therefore, we augment the power vector defined in Section III with the transmit direction angle and define the new vector $\mathbf{q}_n \triangleq$

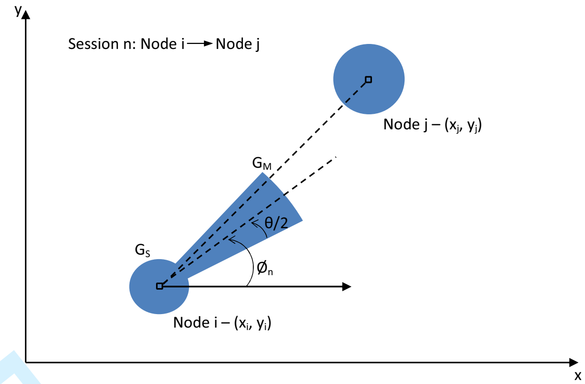


Fig. 5. Illustration of Transmit Directionality

$[p_n(1), \ldots, p_n(K), \phi_n]^T$ for user n. The sum-rate maximization problem P2 is defined as:

$$\text{Find} \quad \mathbf{q}_n, \quad \forall n \in \mathcal{N}$$

$$\text{Maximize} \quad \sum_{n=1}^{N} R_n(\mathbf{q}_n) \tag{13}$$

$$\text{Subject to} \quad \gamma_m(k) \geq \gamma_m^*, \quad \forall m \in \mathcal{M} \tag{14}$$

$$\gamma_n(k) \geq \mathbf{I}(p_n(k))\gamma_n^*, \quad \forall n \in \mathcal{N}, \quad \forall k \in \{1, \ldots, K\} \tag{15}$$

$$p_n(k) \geq 0, \quad \forall n \in \mathcal{N}, \quad \forall k \in \{1, \ldots, K\} \tag{16}$$

$$\sum_{k=1}^{K} p_n(k) \leq P^B, \quad \forall n \in \mathcal{N}. \tag{17}$$

The problem P2 is also NP-hard and it is even more complex than the previous problem because of the additional optimization dimensions ϕ_n, $n = 1, \ldots, N$. We propose two algorithms to solve P2: i) a heuristic algorithm based on modification of the CRGR, and ii) a genetic algorithm.

A. D-CRGR: Directional Convex Relaxation with Gradual Removal

In the heuristic based approach, we partition variables of the problem in power levels ($\mathbf{p}_n$) and direction variable (ϕ_n). The problem is then solved by alternating between $\mathbf{p}_n$ and ϕ_n. We observe that given the power vector $\mathbf{p}_n$, we can vary the direction ϕ_n of the n^{th} secondary transmitter, so as to maximize the received power at the corresponding receiver while minimizing the interference at other secondary receivers. We solve the following single objective problem for each secondary user:

$$\underset{\phi_n}{\text{Maximize}} \quad \left\{ L_{n,n}(k,\phi_n) - \sum_{j\in\mathcal{S}-\{n\}} L_{n,j}(k,\phi_n) \right\}, \quad \forall n = 1,\ldots,N \tag{18}$$

where k is a pre-selected fixed arbitrary frequency channel, $L_{n,n}(k,\phi_n)$ and $L_{n,j}(k,\phi_n)$ are path losses dependent on ϕ_n, and $\mathcal{S}$ is the set of all secondary users with non-zero power assignment on at least one frequency channel. The first term in (18) is directly proportional to the power received at the receiver corresponding to the n^{th} user. The second term is proportional to the interference caused by the transmitter corresponding to the n^{th} user at all the secondary receivers which are active. We need not consider inactive secondary users in (18) as interference at these secondary receivers doesn't impact the sum-rate.

It is worth noting that there is no coupling between the problems given in (18) for different n. Therefore, each optimization problem can be solved independently. Moreover, we can fix k to a constant value, i.e., $k = c$, since the optimization is independent of the frequency channel k. Due to the keyhole model, each function in (18) is a piecewise constant function and there may be multiple optimal solutions. We do an exhaustive search by varying ϕ_n from 0° to 360° in steps of 1° and choosing one of the optimal solutions in each iteration. We observe that, after finding ϕ_n, P2 reduces to an instance of P1. We use CRGR to solve this instance of P1 for power vector $\mathbf{p}_n$, $\forall n = 1,\ldots,N$. We could also have used CRTP or GA here, but using them iteratively would not be efficient. In summary, P2 is solved by alternating between (18) and CRGR until convergence is achieved. The steps of the algorithmic procedure are shown in Algorithm 2.

Algorithm 2 D-CRGR algorithm

1: Initialize $p_n(k)$ randomly between 0 and P^B $\forall n \in \mathcal{N}\, \forall k \in \{1,\ldots,K\}$
2: **repeat**
3: Solve $\phi_n = \arg\max_{\phi_n} \left\{ L_{n,n}(k,\phi_n) - \sum_{j\in\mathcal{S}-n} L_{n,j}(k,\phi_n) \right\}, \quad \forall n = 1,\ldots,N$
4: $p_n(k) = CRGR(\phi_n), \quad \forall n \in \mathcal{N}, \quad \forall k \in \{1,\ldots,K\}$
5: $q_n = [p_n(1),\ldots,p_n(K),\phi_n]^T, \quad \forall n \in \mathcal{N}$
6: **until** $\sum_{n=1}^{N} R_n(\mathbf{q}_n)$ converges
7: $q_n, \quad \forall n = 1,\ldots,N$ is the sub-optimal solution

GA for solving P2 is exactly same as GA for solving P1 except for the introduction of N new continuous variables for ϕ_n. Each of the ϕ_n is uniformly and randomly initialized in $[0, 360)$. Modified crossover and mutation as defined in Section III-C are used.

V. Multiobjective Optimization in CRNs

In a multiobjective optimization problem (MOP) with two objectives, one needs to find a set of Pareto optimal (trade-off) solutions (non-dominated by any other solution) between two objectives. After finding the set of trade-off solutions, the final decision is made by the network controller to select one of these solutions. A well-known technique for solving MOPs is to minimize a weighted sum of the objectives. However, minimizing the weighted sum of the objectives suffers from several drawbacks. As an important example, a uniform spread of weights rarely produces a uniform spread of points on the Pareto front hence the entire Pareto-optimal front cannot be exploited. Therefore, the use of alternate approaches are desirable for MOPs.

CRTP and CRGR, as explained in Section III, can handle the single objective optimization problem but there is no natural extension to MOPs. GA on the other hand can be extended to handle MOPs. NSGA-II is a state of the art multi objective evolutionary algorithm, which is a popular extension of GA to handle MOPs [20]. NSGA-II is an elitist algorithm which uses a non-domination ranking approach where solutions in the duplicated population are ranked according to fronts they belong to. For instance, a solution belongs to the first front, if no other solution in the population dominates it. Similarly, the second front is composed of only the solutions that are dominated by the first front and so on. After several iterations, the NSGA-II population consists of non-dominated solutions which belong to the Pareto-optimal front.

For this problem, motivated by the GA presented in the previous sections, we use a modified version of NSGA-II [20]. The population of size $\mathcal{S}_\mathcal{P}$ is duplicated by crossover and mutation operations as explained in Section III-C. In the initial population there are solutions from all over the variable space (randomly distributed between lower and upper bound of variables), i.e., there are also infeasible solutions. After crossover or mutation, we may still have many infeasible solutions which are not removed from the population. Here, the main idea is that the number of infeasible solutions should continue to drop as the generations increment. This is because after performing crossover and mutation, we combine the populations of generation t and $t-1$. We then choose the best individuals to go into population $t+1$. When choosing between feasible and infeasible solutions, we always prefer the feasible ones. Due to this step, the number of infeasible solutions is likely to vanish over time. Moreover, if the non-dominated set of combined population contains more members than the population size, then the least crowded solutions (in the objective function space) are chosen and passed onto the next iteration. This approach helps in maintaining diversity in the population, thereby giving more choices to the user in selecting a solution.

Measuring convergence of the solutions returned by NSGA-II to the true Pareto optimal front is not a trivial task. Several metrics have been proposed as stopping criteria for multiobjective evolutionary algorithms (MOEA) but the standard approach remains running the algorithm for a fixed number of function evaluations. In our simulations, we use the standard approach but to gauge the quality of solutions we do an offline convergence study. The two major aspects on which a MOEA can be judged are nearness to the Pareto optimal front and the spread of the solutions on the front. We use hypervolume indicator and generational distance (GD) which are two popular metrics in the evolutionary multiobjective optimization community for evaluating the performance of MOEA.

Hypervolume indicator, proposed by Zitzler et al. [24], measures the volume of the objective space dominated by the non-dominated set of points. Given a set of points, hypervolume indicator measures the volume occupied by the union of hypercubes defined by each point in the set and an input reference point. Fleischer [25] showed that for a given problem the solutions covering the entire Pareto optimal front maximize the hypervolume measure. In our simulations, we compute $HV(i)$, the value of hypervolume indicator at i^{th} generation, and empirically show the it increases as the algorithm proceeds.

A popular heuristic approach for terminating evolutionary single objective optimization algorithms is to stop when there is no further improvement in the objective function value. An equivalent metric for multiobjective problems can be constructed by using the GD metric [26]. $GD(i,j)$ defined as

$$GD(i,j) = \sqrt{\sum_{k=1}^{\mathcal{S}_{\mathcal{P}}} g_k^2} \tag{19}$$

measures the distance between non-dominated solutions in generation i and generation j repectively, where g_k is the Euclidean distance between the k^{th} solution in generation i and the nearest solution in non-dominated set of generation j. The value of this metric decreases as the iterations progress.

A. P3:Maximization of Sum-Rate and Minimization of Network Power Under QoS Constraints

In this problem, we consider joint minimization of total power consumption and maximization of sum-rate of the CRN. Minimizing total network power is an important problem especially when the network has a constraint on the interference it can cause to other neighboring networks. Our bi-objective optimization problem referred to as P3 is defined as follows.

$$\begin{aligned} &\text{Find} && \mathbf{q}_n, \quad \forall n \in \mathcal{N} \\ &\text{Minimize} && \left\{-\sum_{n=1}^{N} R_n(\mathbf{q}_n)\right\}, \left\{\sum_{n=1}^{N}\sum_{k=1}^{K} p_n(k)\right\} \qquad (20) \\ &\text{Subject to} && \text{Constraints of P1} \qquad (21) \end{aligned}$$

B. P4:Maximization of Sum-Rate and Maximization of Fairness Under QoS Constraints

Allocating power in order to maximize the total sum-rate of the network, can result in skewed power distributions. Here, we consider the trade-off between sum-rate and fairness. We model fairness by the variance of the sum-rate capacities of SUs as

$$\sum_{n\in\mathcal{S}} (R_n - \mu_R)^2$$

where the average sum-rate is defined by,

$$\mu_R \triangleq \frac{1}{|\mathcal{S}|}\sum_{n=1}^{|\mathcal{S}|} R_n$$

Here, $\mathcal{S} \subset \mathcal{N}$ denotes the set of operating SU users and $|\mathcal{S}|$ is the cardinality of set $\mathcal{S}$. Minimizing the sum-rate variance and making it tend to zero forces the SU users to operate close to the average sum-rate capacity, μ_R. Fairness may also conflict with the total sum-rate objective because of the non-uniform distribution of channel gains due to the path loss among the users. The SU links where transmitter and receiver are close to each other may obtain most of the resources in order to maximize the total sum-rate yielding much smaller capacity to other SU links. In such cases, the variance of the sum-rate of individual SU links may become large. Other metrics of fairness such as the Max-Min fairness can be used [27] as well. Here, Problem P4 is defined as:

$$\begin{aligned} &\text{Find} && \mathbf{q}_n, \quad \forall n \in \mathcal{N} \\ &\text{Minimize} && \left\{-\sum_{n=1}^{N} R_n(\mathbf{q}_n)\right\}, \left\{\sum_{n\in\mathcal{S}} (R_n - \mu_R)^2\right\} \qquad (22) \\ &\text{Subject to} && \text{Constraints of P1} \qquad (23) \end{aligned}$$

C. P5:Maximization of Sum-Rate and Maximization of Number of Active Links Under QoS Constraints

For our last problem, we consider the trade-off between sum-rate and number of active users in a network. A user is considered active if it is able to transmit on a frequency channel. Whether these two objectives are conflicting depends on the topology of the network as well as on the SINR constraints of the secondary users. If the topology is set up in such a way that some users have very high channel gains as compared to other users, then all the power would be allocated to these high-gain users and the second objective will not be satisfied. If the SINR constraints for SUs are very high, then also only very few users will be admitted. Therefore, the trade-off between the number of active users and the sum-rate of the network, is relevant when some users have higher channel gains as compared to others, or when the SU-SINR threshold is low. Problem P5 is defined as:

$$\begin{aligned} \text{Find} \quad & \mathbf{q}_n, \quad \forall n \in \mathcal{N} \\ \text{Minimize} \quad & \left\{ -\sum_{n=1}^{N} R_n(\mathbf{q}_n) \right\}, \left\{ -\sum_{n=1}^{N} i_n \right\} \qquad (24) \\ \text{Subject to} \quad & \text{Constraints of P1} \\ \text{Where} \quad & i_n = \begin{cases} 1, & \text{if} \quad \exists n : \max\{p_n(1), \ldots, p_n(K)\} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (25) \end{aligned}$$

VI. Numerical Results

In this section, we provide numerical results to evaluate the performance of our proposed algorithms in terms of solution quality and time complexity. The transmit power of each primary transmitter and the power budget P^B for each secondary transmitter are set to 6 dB and -3 dB, respectively. The SINR thresholds for PUs and SUs are $\gamma_m^* = 20$ dB and $\gamma_n^* = 10$ dB for $n = 1, \ldots, N$ and $m = 1, \ldots, M$, respectively. Fig. 6 illustrates an example deployment of PU and SU transmitter receiver pairs. We consider an area of 5×5 kilometers. PU and SU transmitters are randomly deployed in the area. PU receivers are randomly deployed within ζ_k distance of their respective transmitters, where ζ_k is chosen to provide 20 dB at the boundary of their deployment region. Note that we set $M = K$ as explained in Section III, i.e, the number of subbands is equal to the number of primary users. Each subband has a bandwidth of $B = 6$ MHz. Attenuation constant $\alpha = 4$. SU receivers are randomly deployed within Δ distance of their respective transmitters. For directional antennas, main lobe antenna gains and side lobe antenna gains are $G_M = 4$ and $G_S = 0.4$, respectively. The main lobe beam width for directional antenna

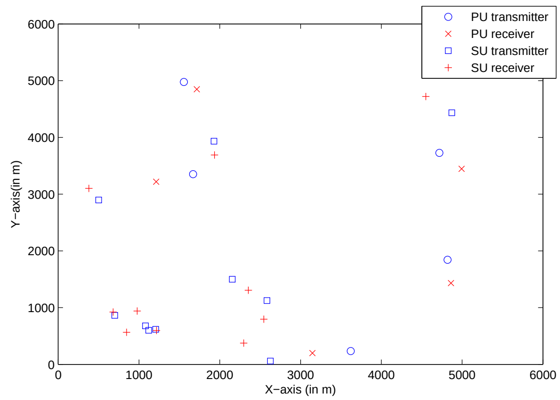


Fig. 6. An example deployment of PU and SU transmitter-receiver pairs (SU and PU locations for the Figures 9, 10 and 11)

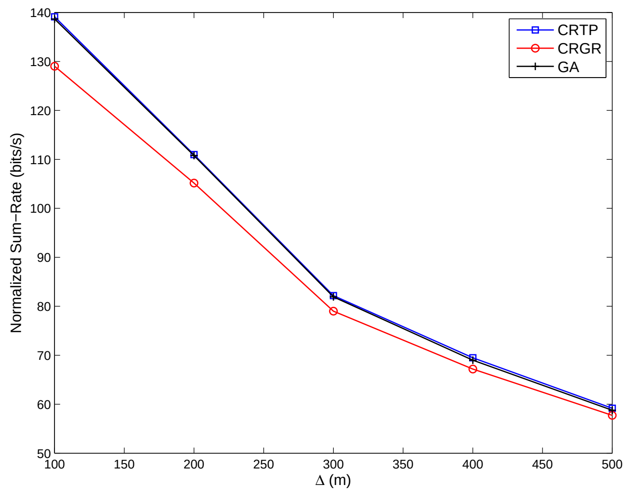


Fig. 7. Sum-rate performance of proposed algorithms as a function of Δ for P1 (N=M=K=3)

is $\theta = \frac{\pi}{3}$. Following parameters are used to run GA for all the problems – populations size $= 1000$, number of generations $= 2000$, crossover probability $= 0.9$, mutation probability$= 0.05$, η_c and η_m as defined in (9) and (10) are fixed at 15 and 70, respectively. Single objective and multi-objective genetic algorithms were implemented in C programming language and their results were linked to Matlab. All the other simulations were performed using Matlab on an Intel Core i5 dualcore CPU with 2.6 Ghz clockspeed and 5.4GB of RAM.

In Fig. 7, we present the sum-rate performance achieved by the proposed algorithms for P1. Here, we vary Δ from 100 meters to 500 meters and provide sum-rate results averaged over 100 Monte Carlo simulations for each Δ value. The normalized sum-rate results, i.e., $\frac{1}{B}\sum_n R_n$, are obtained for the case

TABLE II

COMPARISON OF RUN TIME WITH PROBLEM SIZE

N	K	T_{CRTP} (in sec)	T_{CRGR} (in sec)	T_{GA} (in sec)
5	2	22	1	45
4	3	274	3	48
7	2	936	4	57
8	2	4275	11	128
9	2	11219	47	129

when $N = M = K = 3$. As expected, the maximum sum-rate values decrease as Δ increases, because high Δ values imply low SINR at the secondary receivers. Since SU SINR thresholds are selected to be $\gamma_n^* = 10$ dB, one should note that the function approximation $\log(1 + SINR) \approx \log(SINR)$ is highly accurate in feasible regions. Therefore, CRTP should provide solutions that are very close to the global optimum. In this case, the solutions obtained by CRTP can be used as benchmarks for those obtained by CRGR and GA. As is clear from Fig. 7, GA performs very close to CRTP. Among the three algorithms, CRGR performs the worst in terms of maximizing sum-rate. The complexity of the CRTP algorithm is worst-case exponential, which can be computationally prohibitive for large problems. In contrast, the CRGR algorithm has the lowest computational complexity because of its greedy nature. Therefore, it is scalable for larger problems with a large number of users and frequency bands. GA provides a balance between optimality and computational complexity, since it can provide solutions that are very close to that of the CRTP with reduced computational complexity. Table II shows an empirical comparison between the run times of the three algorithms for randomly chosen PU and SU locations.

In Fig. 8, we compare the sum-rate solutions obtained by CRGR and GA for different numbers of shared frequency bands. It is clear from the figure that as K increases, i.e., as the shared bandwidth gets larger, the maximum sum-rate increases as expected. Furthermore, the performance of the CRGR algorithm is slightly worse than that of the GA. By keeping the same numbers of shared frequency bands, in Fig. 9, we compare sum-rate solutions obtained by D-CRGR and GA with directional antennas. Again, GA is a little better than D-CRGR in terms of solution quality. Simulation results corroborate that directional antennas improve the total sum-rate of the SU communication.

For multiobjective problems P3, P4 and P5, we first estimate the number of generations required for convergence. We evaluate the time required for convergence considering the network given in Fig. 6 using the hypervolume indicator and GD as described in Section V. Table III gives the convergence metrics for five independent runs of NSGA-II. We use the implementation developed by Fonseca et al. [28] for

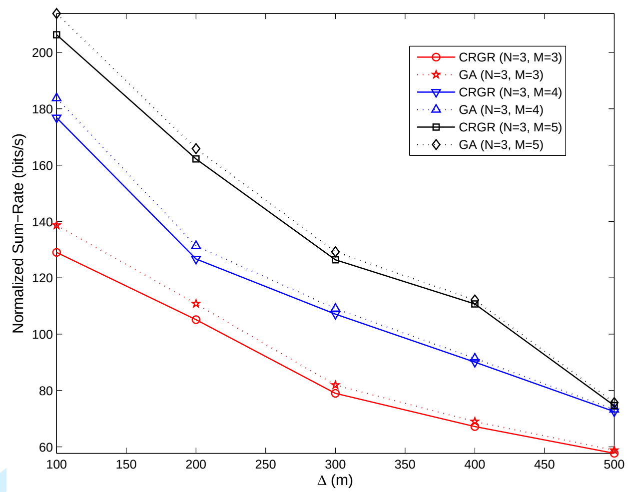


Fig. 8. Performance of CRGR versus GA for P1

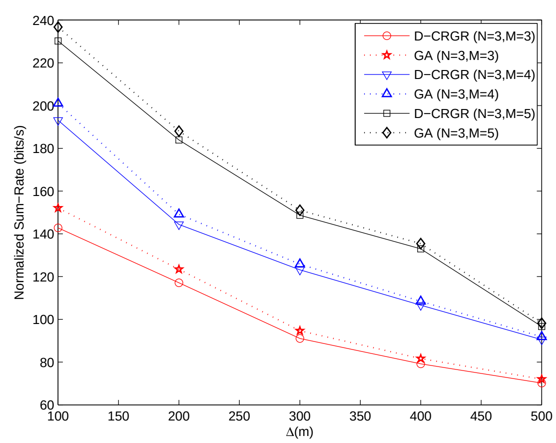


Fig. 9. P2: Performance Results for Δ vs Normalized Sum rate using directional GA and Directional CRGR algorithm for $\theta = \frac{\pi}{3}$

hypervolume computation. Maximum value of each objective function in the input non-dominated set is used as a reference point for hypervolume computation. Table III shows that convergence for all the problems is achieved in around 1000 generations, which takes less than 40 seconds.

We obtain the Pareto front containing 1000 non-dominated solutions of P3, P4 and P5 when $N = 10$ and $M = K = 5$, the same network deployment given in Fig. 6. The Pareto fronts for P3, P4 and P5 are obtained after 1,000,000 function evaluations, corresponding to a population size of 1000 with 1000 generation runs. We show these pareto fronts in Figs. 10, 11, and 12, respectively. It is interesting to see

TABLE III

PERFORMANCE METRICS OF NSGA-II FOR P3, P4 AND P5

Algorithm	Generations	HV (mean)	HV (stddev)	GD(t,t+100) (mean)	GD(t,t+100) (std dev)	Run time (in sec) (mean)
P3 Omnidirectional	100	0.399	0.032	636.159	20.12	1.54
	400	0.455	0.032	0.012	0.003	6.45
	700	0.458	0.032	0.001	0.001	12.54
	1000	**0.459**	**0.032**	**0.0**	**0.0**	**18.52**
	1500	0.460	0.032	0.0	0.0	28.39
P3 Directional	100	0.568	0.015	446.944	5.154	2.16
	400	0.701	0.011	0.052	0.009	7.79
	700	0.714	0.013	0.011	0.010	14.49
	1000	**0.718**	**0.014**	**0.001**	**0.0**	**21.83**
	1500	0.720	.013	0.0	0.0	34.35
P4 Omnidirectional	100	0.338	0.024	1.639	0.529	2.18
	400	0.408	0.032	0.049	0.036	7.38
	700	0.416	0.030	0.003	0.0	13.63
	1000	**0.418**	**0.030**	**0.0**	**0.0**	**20.89**
	1500	0.419	0.030	0.0	0.0	33.12
P4 Directional	100	0.569	0.149	3.310	1.756	2.20
	400	0.789	0.155	0.245	0.186	8.23
	700	0.804	0.151	0.007	0.005	15.27
	1000	**0.807**	**0.151**	**0.001**	**0.0**	**23.31**
	1500	0.809	0.151	0.0	0.0	37.19
P5 Omnidirectional	100	0.010	0.002	6.768	3.974	2.76
	400	0.041	0.010	0.037	0.040	10.59
	700	0.043	0.011	0.0	0.0	18.20
	1000	**0.043**	**0.011**	**0.0**	**0.0**	**25.57**
	1500	0.044	0.011	0.0	0.0	37.72
P5 Directional	100	0.001	0.001	7.993	1.704	3.07
	400	0.043	0.009	0.031	0.030	14.39
	700	0.054	0.015	0.0	0.0	25.49
	1000	**0.054**	**0.015**	**0.0**	**0.0**	**36.17**
	1500	0.054	0.015	0.0	0.0	54.05

from Fig. 10 that the normalized sum-rate can be increased from 0 to about 100 bits/s with an increase of 0.4 Watts in the total network power. On the other hand, by using directional antennas and selecting appropriate transmit direction angle, the normalized sum-rate is increased from 0 to about 100 bits/s with an incremental increase of 0.1 Watts in the total network power. The Pareto front obtained by our modified NSGA-II algorithm helps the network controller clearly see the trade offs between different solutions. The advantage of formulating multiobjective optimization problems for CRNs is clear in this example. Next in Fig. 11, we illustrate the trade-off between Sum-rate and Fairness. By minimizing the variance of the sum rate of the SU links, our fairness criteria forces the sum rate of SU links to be close to the average sum rate. For the normalized sum-rate of 120 bits/sec, use of directional antennas provides a fairness value near zero which means that all the SU links operate with very similar capacity. Simulations indicates that at the same fairness value, using directional antennas results in higher sum-rate. Finally, Fig. 12 shows the trade-off between the number of active SUs and the normalized sum rate of the secondary network. Though it may seem that increasing the number of users should increase the sum-rate of the network but depending on the SU and PU locations, it is possible that users may have to transmit at lower power levels (to satisfy the QoS constraints) leading to overall lower sum-rate. One such example is shown in Fig. 12, where the maximum sum-rate with 9 active SUs is lower than with 7 active SUs. Also, the use of directional antennas improves the normalized sum-rate of the secondary links due to the fact that directional antennas decrease the interference from other SUs. For other SU deployments, directional antennas may not only improve the sum-rate of the secondary network but may also improve the total number of active SUs as well.

VII. CONCLUSION

In this paper, we have studied the spectrum sharing problem in a dynamic spectrum access network and enhanced the performance of the network by exploiting the frequency, power and antenna direction dimensions of TH. We have considered five representative single objective and multi-objective problems and developed suitable optimization algorithms which enable practical implementations. The single objective problem of sum-rate maximization in multiband cognitive radio networks was considered under the PU and SU QoS constraints. Motivated by the fact that these problems are NP-hard, we have developed three suboptimal algorithms based on convex relaxation with tree pruning (CRTP), convex relaxation with gradual removal (CRGR), and genetic algorithms (GA). Each algorithm offers different trade offs in optimality and computational complexity. CRTP provides the best performance in terms of optimality; however, it is computationally the most expensive algorithm. CRGR offers the lowest computational

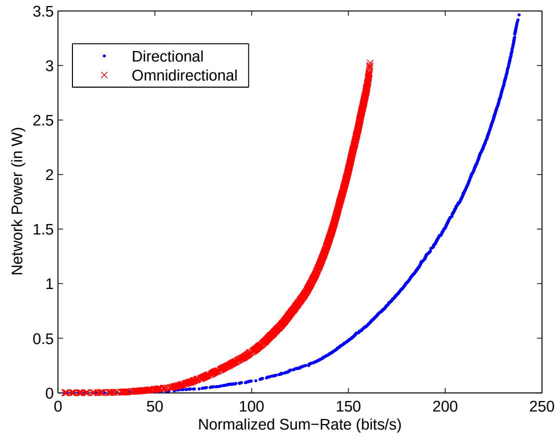


Fig. 10. Trade-off between Sum-rate and Total Network Power, P3 (N=10, K=M=5)

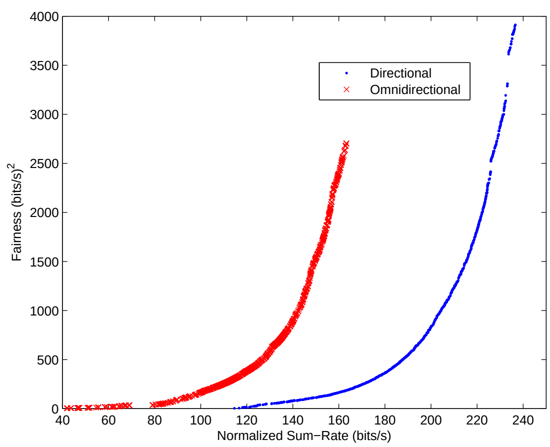


Fig. 11. Trade-off between Sum-rate and Fairness, P4 (N=10, K=M=5)

complexity; however it suffers from high degree of suboptimality. Numerical results showed that GA provides a balance between optimality and computational complexity.

The spectrum sharing problem in cognitive radio networks may involve multiple conflicting objectives such as the maximization of the capacity of the secondary network and the minimization of the interference to the primary network. We have also shown that the GA developed for solving the single-objective optimization problem can be extended for solving such multiobjective optimization problems in a time-efficient manner. We have obtained solutions which achieve different trade-offs between critical and conflicting objective values for the secondary network. Simulations results offer various alternative

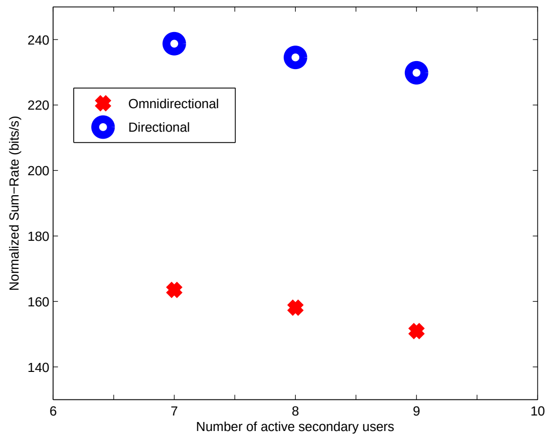


Fig. 12. Trade-off between Sum-rate and number of active users, P5 (N=10, K=M=5)

solutions to the network controller.

In the future, we will extend our formulations to other communication scenarios in cognitive radio networks such as other possible dimensions of the TH such as time, modulation, coding parameters as well as new objective functions such as bit error rate, packet error rate, throughput, latency, etc. Such scenarios may include heterogeneous QoS constraints for different users such as combinations of data rate and SINR constraints.

Acknowledgement of Support and Disclaimer

(a) ANDRO Computational Solutions, LLC acknowledges the U.S. Government's support in the publication of this paper. This material is based upon work funded by AFRL, under AFRL Contract No. FA8750-10-C-0221; (b) Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of AFRL.

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Enhanced Dynamic Spectrum Access in Multiband Cognitive Radio Networks via Optimized Transmission Hyperspace^{TM}

Piyush Bhardwaj, Ankita Panwar, Onur Ozdemir, Engin Masazade, Irina Kasperovich, Andrew L. Drozd, Chilukuri K. Mohan, and Pramod K. Varshney

***Abstract*—The "Transmission Hyperspace^{TM} (TH)" concept developed by Drozd et al. enhances dynamic spectrum access by exploiting multiple orthogonal communication dimensions. Practical implementation in terms of resource assignment in the TH is a very challenging task, since the solution may depend heavily on the given wireless network specifications. In this paper, we show the performance enhancement achieved by exploiting multiple dimensions of TH by considering a number of representative problems and by developing suitable optimization algorithms to enable practical implementations. In particular, we employ the three dimensions, namely frequency, power and transmit direction, and consider the problem of total sum-rate maximization of the secondary users (SUs) under the quality-of-service (QoS) constraints of the primary users (PUs) and the SUs. Since this problem is NP-hard, we develop heuristic algorithms which offer different trade-offs in terms of goodness of their solutions and their computational complexity. We also consider the problem subject to simultaneous multiple conflicting objectives such as maximizing the total sum-rate of SUs while minimizing the total emitted power from the SUs. Several bi-objective optimization problems for the TH are formulated, and a multiobjective evolutionary algorithm to solve these problems in a time-efficient manner is developed. Simulation results are provided to illustrate performance enhancement by using the TH paradigm as well as to investigate the computational efficiency of the proposed optimization algorithms.**

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Transmission Hyperspace^{TM} is a registered trademark of ANDRO Computational Solutions, LLC. This material is based on research sponsored in part by the Air Force Research Laboratory, under prime contract number FA8750-10-C-0221, and CASE: The Center for Advanced Systems and Engineering, a NYSTAR center for advanced technology at Syracuse University. The research was conducted at ANDRO Computational Solutions, LLC in cooperation with Syracuse University. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Research Laboratory or the U.S. Government. Approved for Public Release; Distribution Unlimited: 88ABW-2015-0874, 06Mar15.

I. Introduction

The emerging paradigm of Dynamic Spectrum Access (DSA) networks has been proposed as a solution to the problem of spectrum inefficiency especially with the rapid growth in demand for wireless. These DSA networks are implemented via cognitive radio networks that enable multiple radios operating with different priority levels to co-exist and co-operate in an environment without interfering with each other, so as to increase spectrum efficiency [1]. The overall objective of cognitive radio networking is to achieve maximized network efficiency without interrupting higher priority transmissions and without compromising security while jointly satisfying heterogeneous quality-of-service (QoS) requirements of multiple users.

Conventional Cognitive Radio Networks (CRNs) attempt to enhance spectral efficiency by assigning the frequency spectrum in an intelligent manner. The *"Transmission Hyperspace^{TM} (TH)"* system, developed by Drozd et al. [2], [3], represents a new technology for achieving enhanced dynamic spectrum access and spectrum maneuverability in the wireless applications by going beyond simply assigning or allocating frequency spectrum to networked communications systems. The Transmission Hyperspace^{TM} (TH) concept has been proposed to address the fundamental problems of spectrum crowding by providing control of multiple orthogonal communication dimensions such as frequency, time, space, coding, and antenna directionality using a system optimization approach [2], [3]. This is intended to maximize desired connectivity and throughput for intended users while concurrently denying access to unauthorized or malicious users. The TH concept is useful not only for single communication networks but also in environments where multiple communication networks co-exist with radars and multi-sensor systems. These radars and multi-sensor systems put additional constraints on the system design which complicates the shared dynamic spectrum access problem. The resulting TH based optimization problems usually boil down to spectrum sharing problems involving additional dimensions such as time and antenna parameters.

The TH paradigm regards the RF resource space as an electromagnetically occupied hypercube volume (Fig. 1) existing in multiple dimensions (time, space, frequency, beam direction, code/modulation, etc.) [4]. Here, the space is constantly changing with "cells" of network resources that have been assigned, used, and released. The system parameters are continuously adapted based upon feedback from sensed returns. Several key capabilities are brought together under the TH concept: (i) the ability to achieve dynamic "spectrum" access that goes beyond just allocating frequencies by employing a sense and adapt approach over multiple communication dimensions to "optimize" the RF transmission plan; (ii) the use of embedded algorithms that characterize the EM environment focusing on the physical (PHY) layer; and (iii) models to study the impact on the upper layers (data, network) due to incident electromagnetic interference and disruptive jamming at the PHY layer.

As indicated earlier, the frequency dimension is the fundamental parameter addressed by dynamic spectrum allocation techniques. It is the only dimension that is allocated under the condition that the secondary user (SU) must respect the legal rights of primary users (PUs) while also compensating for space, time and frequency variations due to multipath propagation, mobility, and location dependent shadowing. The TH concept considers additional dimensions that allows secondary users to continue transmitting even when primary users access the frequency channel by maintaining orthogonality in other communication dimensions. Furthermore, the use of directional antennas drastically increases the number of messages that can be sent at a given time. The antenna at each node can then be oriented in the direction of the node with which it is to communicate, and the signal is transmitted in a beam of limited angle originating from the sender. Consequently, such a message will not significantly interfere with the messages being received simultaneously by other nodes that do not lie close to the axis of the beam, or whose antennas are facing in substantially different directions. In our previous work [3], we have shown by simulations that for a 10×10 array of nodes, the number of time slots required to transmit a message from every node to every other node can be reduced by a factor of over 23 by the use of directional antennas with a beam-angle of 25 degrees, using the heuristic of allotting to each time slot those messages that interfere the least with other messages already allocated to that time slot. As the beam-angle reduces, this factor improves further. However, the beam-angle is a function of the hardware, possibly not tunable to fit a problem. In our earlier work [5], we derived explicit expressions for the successful communication probability (SCP) in a multi-hop CRN and we proved that the proposed TH concept significantly improves network utility in terms of SCP. In [5], we considered time, frequency, power, antenna directionality and beamwidth as the TH dimensions to improve SCP.

Although the advantages of using multiple transmit dimensions in the context of TH are obvious, the resulting optimization problems involving TH are more complex requiring the design of optimization approaches that yield high quality solutions in a computationally efficient manner. Furthermore, the solution (as well as the methodology to obtain that solution) heavily depends on the given problem specifications. In general, there is no unique (and global) method that can be applied to every cognitive radio networking optimization problem. In this paper, we assume that the SUs employ orthogonal frequency division multiplexing (OFDM) since it

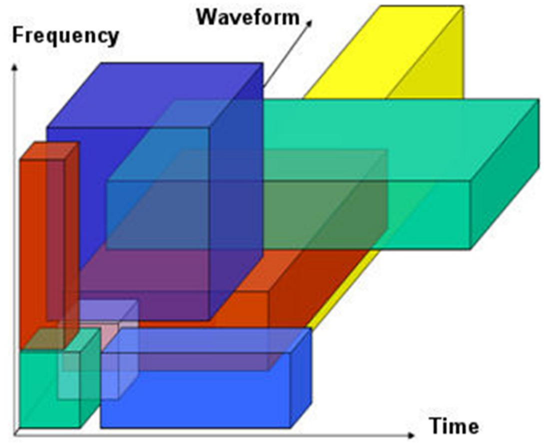


Fig. 1. Transmission Hyperspace^{TM} provides multidimensional DSA.

has been advocated as a promising candidate technology for cognitive radio networks [6]. We focus on three dimensions of the Transmission Hyperspace^{TM}. First, we consider only power and frequency allocations of SUs under the quality-of-service (QoS) constraints of the PUs and the SUs. Then, we add the third dimension of antenna directionality and solve the problem with power, frequency and directional antenna, i.e., spatial, dimensions. Our goal in this paper is to investigate the problem of spectrum sharing in CRNs utilizing TH. We discuss the challenges associated with the solution of these problems. Many relevant problems can be formulated but in this paper we consider five representative optimization problems and develop algorithms to solve them. We first show that all these problems are NP-hard [7] which requires the development of efficient heuristic algorithms. The first problem involves the maximization of total sum-rate of the SUs while satisfying the QoS constraints of PUs and SUs. For this problem, we propose three different suboptimal algorithms based on convex relaxation with tree pruning (CRTP), convex relaxation with gradual removal (CRGR), and genetic algorithms (GA). These algorithms offer different tradeoffs in terms of goodness of their solutions and computational complexity. In the second problem, we add antenna directionality to the first problem. The other three problems are multiobjective optimization problems, where along with total sum-rate maximization, we include total power minimization of the SUs, maximization of fairness among the SUs and maximization of the number of active SU links respectively. For these bi-objective problems, we propose a multiobjective evolutionary algorithm which generates the Pareto front between conflicting objectives in a time-efficient manner.

Our preliminary results were presented in [8] where we considered two TH dimensions, namely frequency and power, and presented one multi-objective optimization problem where the objectives were maximization of the total sum rate of the secondary users and the minimization of the total power of the SUs. In this paper, we first present and build on the work in [8] in Section III, and add one more TH dimension, transmit antenna direction, in Section IV. Furthermore, we formulate two new multiobjective problems in CRNs: i) we jointly maximize the total sum rate of the secondary users and the fairness between SUs, and ii) we jointly maximize the total sum rate of the secondary users and the total number of active SUs. Numerical results show the efficiency, in terms of solution quality and computational efficiency, of the proposed formulations for both the cases of omnidirectional and directional antennas.

Thus, the main contribution of this paper is to show the performance enhancement achieved by exploiting multiple dimensions of TH. We consider a number of representative problems and by developing suitable optimization algorithms to enable practical implementations. The rest of the paper is organized as follows. In Section II, we present the system model for the CRN and the problem statement. In Section III, we introduce the Maximization of Sum-Rate Under QoS Constraints with Omnidirectional Antennas and in Section IV, we state the same problem with directional antennas. In Section V, we introduce different multiobjective optimization problems in CRNs. Section VI presents our numerical results, and finally Section VII is devoted to our conclusions and future research directions.

II. System Model and Problem Statement

We consider a CRN, where the available frequency band is shared between SUs and PUs in the network under the constraint that SU transmitters do not create harmful interference at PU receivers. We assume that the shared spectrum is divided into K discrete frequency subbands and, without loss of generality, each subband has an identical bandwidth of B Hz. This set of assumptions is applicable to systems using orthogonal-frequency-division-multiplexing (OFDM) technology. We number each secondary and primary transmitter-receiver pair by the indices $n \in \mathcal{N} = \{SU_1, \ldots, SU_N\}$ and $m \in \mathcal{M} = \{PU_1, \ldots, PU_M\}$, respectively, and refer to them as users. Throughout the paper, the terms subband and carrier are used interchangeably. Our formulations are based on the physical model [9], which provides a realistic modeling of the physical communication environment by utilizing a path-loss model. The general path loss between any transmitter i and any receiver j is given as

$$L_{i,j}(k) = \frac{G_{t,i} G_{r,j}}{d_{i,j}^{\alpha}} \left(\frac{c}{4\pi f(k)} \right)^2, \tag{1}$$

where $G_{t,i}$ and $G_{r,j}$ are the transmit and receiver antenna gains respectively, c is the speed of light, $f(k)$ is the carrier frequency of subband k, $d_{i,j}$ is the distance between transmitter i and receiver j, and α is the attenuation constant. We assume that the path-loss in the received power is the dominant loss factor, and therefore, we neglect the effects of shadowing and multi-path fading. We assume an additive white Gaussian (AWGN) channel with zero mean and variance N_0. Under these assumptions, the achievable data rate of secondary user n can be expressed [10] as:

$$R_n = B \sum_{k=1}^{K} \log[1 + \gamma_n(k)], \tag{2}$$

where $\log$ is defined in base 2 and $\gamma_n(k)$ is the signal-to-interference-plus-noise ratio (SINR) of secondary user n on carrier k,

$$\gamma_n(k) \triangleq \frac{p_n(k) L_{n,n}(k)}{N_0 + \sum_{l \in \mathcal{N} \cup \mathcal{M}, l \neq n} p_l(k) L_{l,n}(k)}. \tag{3}$$

In (3), $p_n(k)$ and $p_l(k)$ represent transmit powers of the n-th secondary user and the l-th primary or secondary user, respectively. The SINR condition for establishing a successful communication link n on carrier k is given by $\gamma_n(k) \geq \gamma_n^*$.

We assume a narrowband primary network, where a single channel with a predetermined transmit power value is allocated to each primary user. This scenario is applicable to networks where legacy radios have the licenses to operate on narrowband channels. Generalization to a wideband primary network is straightforward and does not affect the methodology. Secondary users utilize multiband techniques where each band corresponds to a primary narrowband channel, to access the spectrum and each secondary user has a power budget denoted by P^B.

III. P1: Maximization of Sum-Rate Under QoS Constraints with Omnidirectional Antennas

In this section, we consider omnidirectional antennas based on the path loss model in (1) with $G_{t,i} = G_{r,j} = 1, \forall i, j$. We assume that each PU occupies a single subband and PUs operate on disjoint subbands. This results in the equality $M = K$. Given primary network activity and location of users, the optimization problem is to maximize the sum-rate (or achievable capacity) of the secondary network. The optimization variables are power levels allocated to each secondary user over each shared frequency subband.

Define $\mathbf{p}_n \triangleq [p_n(1), \ldots, p_n(K)]^T$ as the power allocation vector where each element represents the power level allocated to secondary user n over each subband. User n is said to be inactive over frequency band k if $p_n(k) = 0$. A secondary user is said to be active if it is transmitting on at least one of the K subbands. Each primary user occupies one of the K subbands and primary users operate at disjoint subbands. An SU is allowed to transmit on a subband, if and only if it does not violate any SINR of PU and SU pairs or its own power budget constraints.

The sum-rate maximization problem, P1, is formulated as follows:

$$\begin{aligned}
&\text{Find} && \mathbf{p}_n, \quad \forall n \in \mathcal{N} \\
&\text{Maximize} && \sum_{n=1}^{N} R_n(\mathbf{p}_n) && (4)\\
&\text{Subject to} && \gamma_m(k) \geq \gamma_m^*, \quad \forall m \in \mathcal{M}, && (5)\\
& && \gamma_n(k) \geq \mathbf{I}(p_n(k))\gamma_n^*, \quad \forall n \in \mathcal{N}, \forall k \in \{1, \ldots, K\} && (6)\\
& && p_n(k) \geq 0, \quad \forall n \in \mathcal{N}, \forall k \in \{1, \ldots, K\} && (7)\\
& && \sum_{k=1}^{K} p_n(k) \leq P^B, \quad \forall n \in \mathcal{N}. && (8)
\end{aligned}$$

In P1, $\mathbf{I}(\cdot)$ is the indicator function for the set of positive real numbers. Inequality (5) represents a set of $K(= M)$ SINR constraints for the primary users. The inequality (6) represents a set of $N \times K$ SINR constraints for each of the N SUs over each of the K frequency subbands. The SU n will transmit at a frequency k, if and only if the SINR corresponding to that link is greater than or equal to the threshold SINR. If the link is not active over that subband, i.e., if $p_n(k) = 0$, the SINR constraint is automatically satisfied. It should be noted that P1 is in fact a *soft-spectrum allocation* and *power allocation* problem. The difference between conventional spectrum and power allocation formulations that have been considered in the literature, e.g. [11], [12], and our problem P1 is that conventional formulations treat spectrum allocation as a hard allocation problem such that no two users share the same spectrum. In conventional settings, spectrum allocation is carried out first based on channel conditions followed by power allocation [11], [12]. However, in our formulation, the constraint in (6) provides a way to allocate the spectrum in such a way that multiple SUs can share the spectrum via other orthogonal dimensions of TH as long as their QoS constraints are not violated.

Proposition 1: The sum-rate maximization problem with QoS constraints P1 is NP-hard.

Proposition 1 can be proved by noticing that P1 is a generalization of the sum-rate maximization problem without the QoS constraints which was shown to be NP-hard [13].

The NP-hardness of the above problem motivates the development of efficient suboptimal algorithms. In order to understand the problem better, let us consider a simple example of two SUs, one PU and one frequency subband. For a given set of locations, we draw the SU SINR constraints given in (6) in Fig. 2(a). For now, we ignore the SINR constraints for PUs. The area inside the square formed by the bounds on the SU power budget is the region where power budget constraints are satisfied. There are four possibilities: both SUs transmit, only the first SU transmits, only the second SU transmits, or both SUs are off. When both SUs transmit, SINR constraints for both SUs must be satisfied simultaneously which is represented by the gray region in Figure 2(a). Let SU_a and SU_b be two SU links operating at the same frequency with powers p_a and p_b, respectively. The SINRs for SU_a and SU_b, γ_a and γ_b, should satisfy a certain value, so that $\gamma_a > \gamma^*$ and $\gamma_b > \gamma^*$. The maximum power levels for SU_a and SU_b are limited to P^B. When only SU_a transmits, only the SINR constraint for SU_a needs to be satisfied. In that case, the constraint in (6) for SU_b is automatically satisfied because $p_b = 0$. Here, the dark line on the horizontal axis is feasible. Similar results hold when only SU_b transmits. When both are off, the constraints in (6) are automatically satisfied and this region is a single point $(0, 0)$. Now, with a different set of locations, it is possible that the receivers of both SUs are close to each other. Then the constraint region may look as shown in Fig. 2(b). Here, there is no region where both SU SINR constraints for SUs and power budget constraints are satisfied simultaneously. In other words, it is not feasible for both SU_a and SU_b to transmit simultaneously. Another scenario could arise when some secondary transmitters and receivers are so far apart, or the secondary receiver is so close to the PU transmitter such that there is no region where that user can be active, i.e, there is no region where the power budget as well as the SINR constraint for that SU is satisfied simultaneously. This scenario is shown in Fig. 2(c). Note that, if this is the case, i.e., an SU cannot transmit even when other SUs are OFF, then the possibility of that SU transmitting when others are ON is automatically eliminated. We now consider the PU constraints defined in (5). For a given constant PU transmit power, the constraints in (5) define halfspaces. Therefore, in the above three scenarios, the addition of linear PU constraints will only reduce the volume of the feasible regions shown in Figures 2(a)-2(c) for a two dimensional case. Resulting feasible regions will be the intersection of the previous regions and the additional halfspaces.

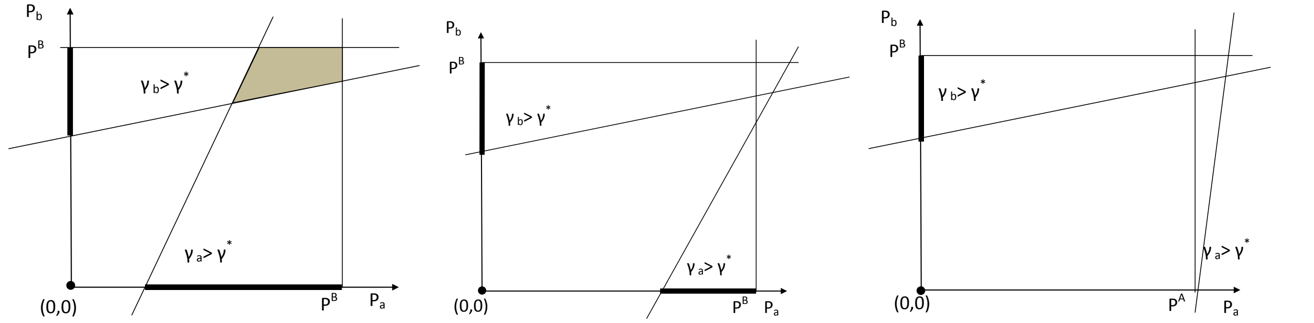
We propose three different algorithms to solve Problem P1 based on the following methods: 1) convex relaxation with tree pruning; 2) convex relaxation with gradual removal; 3) genetic algorithms. The comparison of these three algorithms in terms of finding good quality solutions and time complexity will be investigated later. The following subsections provide a detailed description of the algorithms.

A. CRTP: Convex Relaxation with Tree Pruning

We start by noting that different combinations of ON and OFF power variables define different disjoint regions in space. There are a total of $2^{N\times K}$ ON/OFF combinations and hence that many regions. However, the number of these regions which are feasible depends on the locations of secondary and primary users in the network.

For each feasible region, there are M linear PU SINR constraints, N linear power budget constraints and some linear SU SINR constraints whose number depends upon the number of non zero elements of the power vector $\mathbf{p}_n$. Furthermore, we note that the objective function is nonconvex. In order to alleviate the nonconvexity issue, we first use the following approximation $\log(1 + x) \approx \log(x)$ which is tight for values of $SINR \geq 5$. The same approximation has been used in [14], [15] for different problems. In a recent work [16], the authors have shown that $p_k = ce^{\mu s_k}$, $c, \mu > 0$ is the unique transformation that transforms the objective function defined by $g(\mathbf{p}) = \sum_{n=1}^{N} \log\left(\frac{p_n}{N_n + \sum_{l\neq n} a_{l,n} p_l}\right)$ into a convex function. Note that in $g(.)$, we set $K = 1$ for notational convenience and $a_{l,n} \triangleq L_{l,n}/L_{n,n}$ and $N_n \triangleq N_0/L_{n,n}$. The same transformation has been used by others in similar formulations (e.g., [14], [17]). Using the high SINR approximation and applying the transformation $p_n = e^{s_n}$, the resulting problem becomes convex in each feasible region. However, the problem as a whole remains nonconvex as all these feasible regions are disjoint, resulting in a nonconvex global feasible set.

In order to solve this problem globally, we need to find the regions that are feasible. Investigating all the possible regions (2^{MN}) is a computationally expensive task especially for large number of variables. There is an efficient way of reducing the computational effort by eliminating infeasible regions of the 2^{MN} possible disjoint regions. For a given region, observe that the non-trivial SU SINR constraints consist of a group of linear inequalities with the number of variables being equal to the number of inequalities. We can consider these linear inequalities as equalities and solve for $\mathbf{p}_n$. The resulting



(a) Scenario-I (b) Scenario-II (c) Scenario-III

Fig. 2. Possible Scenarios in Secondary Power Allocations

solution, say (P^*), is the minimum feasible solution, in the given region, which satisfies the SU SINR constraints. This means any power vector p' in this region which satisfies non-trivial SU SINR constraints follows $p' \succeq P^*$, where $\succeq$ is element-wise inequality [18], [19].

Once we have the solution (P^*) for a given region we check to see if it satisfies M PU SINR constraints and also the N power budget constraints. If these constraints are violated at $p = P^*$, it means that there is no feasible p that satisfies the PU SINR and the power budget constraints without violating the SU SINR constraints. Using this reasoning, we can eliminate all the infeasible regions using a tree pruning method. We start with the lowest branch of the tree which has one variable. Note that the lowest branches define regions in space where only one SU is active over exactly one subband. We then solve the SU SINR linear equations for that region and check if the solution violates the PU SINR or power budget constraints. If it violates any of these constraints, the region is eliminated and is not considered any further. We know that if a combination of some users active over some frequency band is infeasible then all combinations that are supersets of that combination are also infeasible. Then, we can prune that branch of the tree.

For each of the remaining feasible regions, we have a convex optimization problem which can be solved globally and efficiently. The optimum solutions for each of these regions are stored and then compared to find the best solution, i.e., maximum sum-rate, and the corresponding power allocation vector.

If the SINR threshold γ_n^* is sufficiently large (i.e., $\log(1 + x) \approx \log(x)$), the CRTP algorithm will find solutions that are very close to the global optimum since the relaxed convex problem approximates the original problem with high accuracy in the feasible regions. Note that the tree pruning approach helps reduce the computational complexity, which can be a bottleneck for large problems with too many variables. Table I shows an example of how the number of regions and the number of feasible regions scale with the problem size for a specific realization of the problem. The advantage we get from the tree pruning method can be deduced from the table.

B. CRGR - Convex Relaxation with Gradual Removal

We note that the worst-case complexity of the CRTP algorithm is exponential. In order to find the solutions faster, we propose a heuristic algorithm, referred to as CRGR, based on convex relaxation with gradual removal. This algorithm is very similar to the one proposed in [14] without the QoS constraints. Although this method does not guarantee an optimum solution, it can give good solutions with less computational effort.

TABLE I
COMPARISON OF NUMBER OF FEASIBLE REGIONS WITH PROBLEM SIZE

N	K	No. variables	No. regions	Feasible regions
5	2	10	1024	255
4	3	12	4096	1535
7	2	14	16384	2319
8	2	16	65536	16383
9	2	18	262144	31645

Several different disjoint regions exist in the problem because of SU SINR constraints, which differ from region to region. Without SU SINR constraints, the remaining constraints define a space which is an intersection of halfspaces. This means that without the SU SINR constraints, the optimization space becomes a convex set. The optimization problem P1 can be easily solved if SU SINR constraints are excluded. In CRGR, we use the same high SINR approximation as before ($\log(1 + x) \approx \log(x)$) and apply the $p = e^s$ transformation. The resulting problem is convex. After solving this constrained optimization problem without the SU SINR constraints, we switch off the link over the subband which has the highest (SU SINR) constraint violation, and also switch off the links over subbands for which the power level as a result of optimization is close to zero. This process reduces the number of variables as the power level for some links over certain subbands is fixed at zero. The optimization problem P1 is again solved without the SU SINR constraints with the reduced variable set. This process is repeated until we get a solution which satisfies all the PU, SU and power budget constraints. The steps of the algorithmic procedure are shown in Algorithm 1.

C. GA - Genetic Algorithm

Heuristic artificial intelligence techniques have been used to solve NP-hard problems which tend to exploit the structure of the problem and arrive at a good solution. In this section, we solve P1 with a genetic algorithm (GA) which belongs to the evolutionary class of artificial intelligence techniques. This algorithm is based on the domination principle. A feasible solution always dominates an infeasible solution. Among two infeasible solutions, the one with lower overall constraint violation dominates the other. Among two feasible solutions,

Algorithm 1 CRGR algorithm

```
repeat
  Solve P1 without SU SINR constraints
  Maximum SU SINR constraint violation = max(0, I(p_n(k))γ_n^*) − γ_n(k)),  ∀n ∈ N,  ∀k ∈ {1,...,K}
  Fix p_n(k) = 0 for link with maximum SU SINR constraint violation
  if p_n(k) < 10^{-3}  ∀n ∈ N,  ∀k ∈ {1,...,K} then
    Set p_n(k) = 0
  end if
until Maximum SU SINR constraint violation = 0
p_n(k)  ∀n ∈ N,  ∀k is the sub-optimal solution
```

one with higher objective value dominates the other (for a maximization problem). The aim of the algorithm is to iteratively improve a set of solutions which are not dominated by other solutions. Given enough number of iterations, GAs have been empirically shown to produce optimal or close to optimal results for highly complex optimization problems. Following is the description of our GA, which is a modified version of [20], employed in this paper:

1) Representation: A gene/solution is a structure comprising of $N \times K$ real variables corresponding to the powers of SUs. Let $\mathbf{p}_i^t$ be a solution with elements $p_n^t(k)$ at time (or generation) t and $i \in \{1, \ldots, \mathcal{S_P}\}$ where $\mathcal{S_P}$ is the population size.

2) Initialization: The population is created by initializing each variable in each gene according to uniform random distribution between its lower and upper bounds.

Note that given the variable bounds $p_n(k) \in [0, p_{max}]$, it is very unlikely to achieve some solutions which have the form $p_n(k) = 0$. In this form, $p_n(k) \in [0, p_{max}]$, GA may fail to provide good solutions since it becomes hard to discover all the feasible regions. This fact motivates modifications that would increase the probability of creating individuals in each and every feasible region. Therefore, we change the variable bounds from $p_n(k) \in [0, p_{max}]$ to $p_n(k) \in [-p_{max}, p_{max}]$. Whenever a variable is less than zero, $p_n(k) \leq 0$, we treat it as equal to zero, $p_n(k) = 0$, while evaluating the objective function. This modification provides more homogeneous distribution for generating different types of SU admission strategies (some SU links are on and some of them off) and GA explores the feasible regions better.

3) Selection: The individuals that will go into the mating pool are chosen according to tournament selection [21]. $2 \times \mathcal{S_P}$ pairs of individuals are selected randomly without replacement and the best of each pair goes into the mating pool. In case both individuals are non-dominated, one is selected with probability 0.5. In case of a single objective, this means that the best individual gets two copies in the mating pool.

4) Crossover: From the mating pool, $2 \times \mathcal{S_P}$ random pairs of individuals are created and the best of each pair is passed on into the mating pool. The individuals in the mating pool undergo Simulated Binary Crossover (SBX)[21] to form the offspring solutions. Let $\mathbf{p}_1^t$ and $\mathbf{p}_2^t$ be two parent solutions. The offspring solutions are created according to,

$$\mathbf{p}_1^{t+1} = 0.5[(1+\beta_q)\mathbf{p}_1^t + (1-\beta_q)\mathbf{p}_1^t],$$
$$\mathbf{p}_2^{t+1} = 0.5[(1-\beta_q)\mathbf{p}_2^t + (1+\beta_q)\mathbf{p}_2^t],$$

where β_q follows the probability distribution

$$c(\beta_q) = \begin{cases} 0.5(\eta_c+1)\beta_q^{\eta_c}, & \text{if } \beta_q \leq 1 \\ 0.5(\eta_c+1)/\beta_q^{\eta_c+2} & \text{otherwise} \end{cases} \tag{9}$$

as defined in [21].

In order to promote diversity, the negative elements of $\mathbf{p}_1^t$ and $\mathbf{p}_2^t$ are first updated by a new value which is uniformly generated within the interval $[-p_{max}/10, 0]$. The division by a factor of 10 generates values closer to zero, which then increase the chances of generating offspring with positive value.

5) Mutation: Each variable of each gene is mutated with a probability p_m. If $p_n^t(k) > 0$, polynomial mutation is performed [21]. Mutated value is equal to

$$p_n^t(k) = p_n^t(k) + (p_{max} - 0)\delta$$

where

$$\delta = \begin{cases} (2u)^{1/(\eta_m+1)} - 1, & \text{if } u < 0.5 \\ 1 - |(2(1-u))|^{1/(\eta_m+1)} & \text{otherwise} \end{cases} \tag{10}$$

If $p_n^t(k) < 0$ then any of the following mutations is performed by equal probability: (i) $p_n^t(k) \sim U[0; p_{max}]$, where $U[a,b]$ denotes uniform distribution between a and b. This means, if we mutate a variable which is OFF, we turn it ON and assign it a random power. (ii) The user is turned ON at the smallest power level that satisfies its SINR constraint. (iii) The user is turned ON at a power which is equal to the average of its power at other channels.

6) Elitism: All solutions of generation t and $t-1$ are combined and the best solutions from this set go to population of $t+1$.

7) Evaluate the function value: Repeat Steps 2 through 7 until the solution does not improve significantly. A common approach is to run the program for a specified large number of iterations.

A flowchart representation of GA is illustrated in Fig. 3.

IV. P2: Maximization of Sum-Rate under QoS Constraints with Directional Antennas

In the previous section, we assumed that all users transmit with omnidirectional antennas. In this section, we no longer operate under that assumption and consider directional antennas at transmitters. In this setting, users transmit using directional antennas and receive using omnidirectional antennas. This can be realized by transmit beamforming techniques. Transmit directionality is expected to increase network throughput and decrease delay by providing enhanced spatial reuse and increased range [22]. Modeling a precise antenna pattern has two drawbacks. First, it is an extremely difficult task that is not the focus of this paper. Second, a precise antenna pattern would make the optimization problem prohibitively complex to tackle. To alleviate these drawbacks,

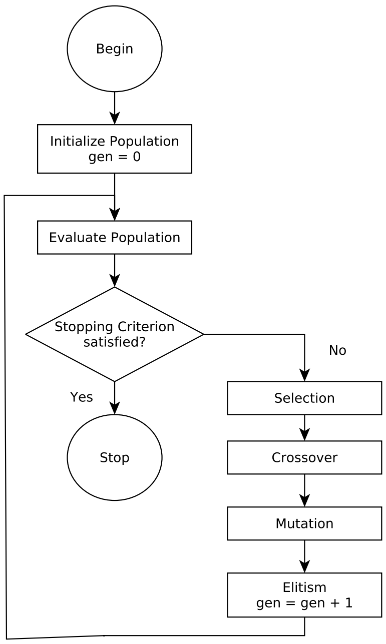


Fig. 3. Flowchart of Genetic Algorithm [21]

we consider the so-called keyhole model [22], [23], [5], a two-dimensional antenna pattern approximation. Previous research works, e.g. [22], [23], [5], have used similar antenna pattern approximations in the context of similar but different network optimization and analysis problems. The keyhole model is simple enough for our problem, but still reflects the main characteristics of a realistic antenna pattern as shown in Fig. 4. Using this keyhole pattern, we are able to model both the enhanced power gain in the main-lobe region and the reduced power gain in the side-lobe region compared to an isotropic antenna pattern used in the previous section.

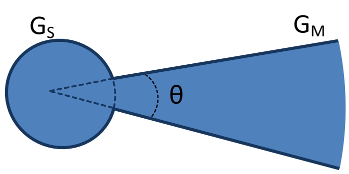


Fig. 4. Keyhole Antenna Pattern.

The derivation of different antenna gains that provide a fair comparison with omnidirectional antennas is provided in [5]. Here, we briefly explain how we can simulate different antenna gains G_M and G_S to make such a comparison. Let θ denote the main lobe beamwidth in radians. We define normalized beamwidths α and η such that $\alpha = \theta/2\pi$ and $0 \leq \eta \leq (1-\theta)/2\pi$. After some manipulations, the transmit antenna gains are calculated as

$$G_m = \frac{1}{\alpha+\eta},\ G_s = \frac{\eta}{(1-\alpha)(\alpha+\eta)}. \tag{11}$$

The purpose of introducing η is that the main lobe and the side lobe gains can be controlled by adjusting η. For instance, $\eta = (1-\theta)/2\pi$ corresponds to an omnidirectional antenna pattern. Similarly as $\eta \to 0$, the side lobe power vanishes, resulting in an antenna pattern pointing all its transmitted power towards the main lobe direction. If we denote the isotropic transmission power defined in the previous section as P_t, the resulting equivalent directional power in the direction of the main lobe as well as in the direction of the side lobe can be calculated, respectively, as

$$P_t^m = \frac{P_t}{\alpha+\eta},\ P_t^s = \frac{\eta P_t}{(1-\alpha)(\alpha+\eta)} \tag{12}$$

In this problem formulation, everything stays the same as in the previous formulation (in Section III) except for different transmit gains G_M and G_S in the main lobe and the side lobe, respectively. Given the locations of users, the new path loss for user n is now a function of the transmit antenna direction ϕ_n. A communication link setup with transmit directionality is illustrated in Fig. 5.

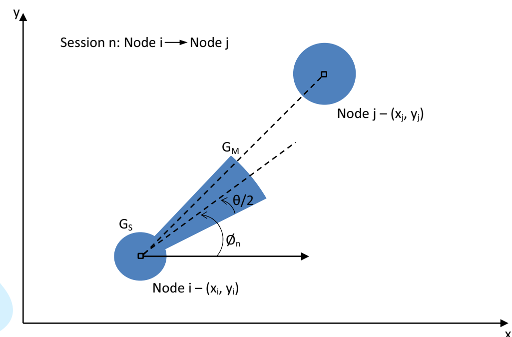


Fig. 5. Illustration of Transmit Directionality

Note that the interference term in (3) which is the summation of the power terms from all other transmitters is now a function of other transmitters' transmit directions. Therefore, we augment the power vector defined in Section III with the transmit direction angle and define the new vector $\mathbf{q}_n \triangleq [p_n(1), \ldots, p_n(K), \phi_n]^T$ for user n. The sum-rate maximization problem P2 is defined as:

$$\begin{aligned}
\text{Find} \quad & \mathbf{q}_n, \quad \forall n \in \mathcal{N} \\
\text{Maximize} \quad & \sum_{n=1}^{N} R_n(\mathbf{q}_n) && (13)\\
\text{Subject to} \quad & \gamma_m(k) \geq \gamma_m^*, \quad \forall m \in \mathcal{M} && (14)\\
& \gamma_n(k) \geq \mathbf{I}(p_n(k))\gamma_n^*, \quad \forall n \in \mathcal{N}, \quad \forall k \in \{1,\ldots,K\} && (15)\\
& p_n(k) \geq 0, \quad \forall n \in \mathcal{N}, \quad \forall k \in \{1,\ldots,K\} && (16)\\
& \sum_{k=1}^{K} p_n(k) \leq P^B, \quad \forall n \in \mathcal{N}. && (17)
\end{aligned}$$

The problem P2 is also NP-hard and it is even more complex than the previous problem because of the additional optimization dimensions ϕ_n, $n = 1, \ldots, N$. We propose two algorithms to solve P2: i) a heuristic algorithm based on modification of the CRGR, and ii) a genetic algorithm.

A. D-CRGR: Directional Convex Relaxation with Gradual Removal

In the heuristic based approach, we partition variables of the problem in power levels ($\mathbf{p}_n$) and direction variable (ϕ_n). The problem is then solved by alternating between $\mathbf{p}_n$ and ϕ_n. We observe that given the power vector $\mathbf{p}_n$, we can vary the direction ϕ_n of the n^{th} secondary transmitter, so as to maximize the received power at the corresponding receiver while minimizing the interference at other secondary receivers. We solve the following single objective problem for each secondary user:

$$\underset{\phi_n}{\text{Maximize}} \quad \left\{ L_{n,n}(k,\phi_n) - \sum_{j \in \mathcal{S}-\{n\}} L_{n,j}(k,\phi_n) \right\}, \quad \forall n = 1,\ldots,N \tag{18}$$

where k is a pre-selected fixed arbitrary frequency channel, $L_{n,n}(k,\phi_n)$ and $L_{n,j}(k,\phi_n)$ are path losses dependent on ϕ_n, and $\mathcal{S}$ is the set of all secondary users with non-zero power assignment on at least one frequency channel. The first term in (18) is directly proportional to the power received at the receiver corresponding to the n^{th} user. The second term is proportional to the interference caused by the transmitter corresponding to the n^{th} user at all the secondary receivers which are active. We need not consider inactive secondary users in (18) as interference at these secondary receivers doesn't impact the sum-rate.

It is worth noting that there is no coupling between the problems given in (18) for different n. Therefore, each optimization problem can be solved independently. Moreover, we can fix k to a constant value, i.e., $k = c$, since the optimization is independent of the frequency channel k. Due to the keyhole model, each function in (18) is a piecewise constant function and there may be multiple optimal solutions. We do an exhaustive search by varying ϕ_n from $0°$ to $360°$ in steps of $1°$ and choosing one of the optimal solutions in each iteration. We observe that, after finding ϕ_n, P2 reduces to an instance of P1. We use CRGR to solve this instance of P1 for power vector $\mathbf{p}_n$, $\forall n = 1,\ldots,N$. We could also have used CRTP or GA here, but using them iteratively would not be efficient. In summary, P2 is solved by alternating between (18) and CRGR until convergence is achieved. The steps of the algorithmic procedure are shown in Algorithm 2.

GA for solving P2 is exactly same as GA for solving P1 except for the introduction of N new continuous variables for ϕ_n. Each of the ϕ_n is uniformly and randomly initialized in $[0, 360)$. Modified crossover and mutation as defined in Section III-C are used.

Algorithm 2 D-CRGR algorithm

Initialize $p_n(k)$ randomly between 0 and P^B $\forall n \in \mathcal{N}$, $\forall k \in \{1,\ldots,K\}$
repeat
  Solve $\phi_n = \arg\max_{\phi_n} \left\{ L_{n,n}(k,\phi_n) - \sum_{j \in (\mathcal{S}-n)} L_{n,j}(k,\phi_n) \right\}$, $\forall n = 1,\ldots,N$
  $p_n(k) = CRGR(\phi_n)$, $\forall n \in \mathcal{N}$, $\forall k \in \{1,\ldots,K\}$
  $q_n = [p_n(1),\ldots,p_n(K),\phi_n]^T$, $\forall n \in \mathcal{N}$
until $\sum_{n=1}^{N} R_n(\mathbf{q}_n)$ converges
q_n, $\forall n = 1,\ldots,N$ is the sub-optimal solution

V. Multiobjective Optimization in CRNs

In a multiobjective optimization problem (MOP) with two objectives, one needs to find a set of Pareto optimal (trade-off) solutions (non-dominated by any other solution) between two objectives. After finding the set of trade-off solutions, the final decision is made by the network controller to select one of these solutions. A well-known technique for solving MOPs is to minimize a weighted sum of the objectives. However, minimizing the weighted sum of the objectives suffers from several drawbacks. As an important example, a uniform spread of weights rarely produces a uniform spread of points on the Pareto front hence the entire Pareto-optimal front cannot be exploited. Therefore, the use of alternate approaches are desirable for MOPs.

CRTP and CRGR, as explained in Section III, can handle the single objective optimization problem but there is no natural extension to MOPs. GA on the other hand can be extended to handle MOPs. NSGA-II is a state of the art multi objective evolutionary algorithm, which is a popular extension of GA to handle MOPs [20]. NSGA-II is an elitist algorithm which uses a non-domination ranking approach where solutions in the duplicated population are ranked according to fronts they belong to. For instance, a solution belongs to the first front, if no other solution in the population dominates it. Similarly, the second front is composed of only the solutions that are dominated by the first front and so on. After several iterations, the NSGA-II population consists of non-dominated solutions which belong to the Pareto-optimal front.

For this problem, motivated by the GA presented in the previous sections, we use a modified version of NSGA-II [20]. The population of size $\mathcal{S}_{\mathcal{P}}$ is duplicated by crossover and mutation operations as explained in Section III-C. In the initial population there are solutions from all over the variable space (randomly distributed between lower and upper bound of variables), i.e., there are also infeasible solutions. After crossover or mutation, we may still have many infeasible solutions which are not removed from the population. Here, the main idea is that the number of infeasible solutions should continue to drop as the generations increment. This is because after performing crossover and mutation, we combine the populations of generation t and $t-1$. We then choose the best individuals to go into population $t+1$. When choosing between feasible and infeasible solutions, we always prefer the feasible ones. Due to this step, the number of infeasible solutions is likely to vanish over time. Moreover, if the non-dominated set of combined population contains more members than the population size, then the least crowded solutions (in the objective function space) are chosen and passed onto the next iteration. This approach helps in maintaining diversity in the population, thereby giving more choices to the user in selecting a solution.

Measuring convergence of the solutions returned by NSGA-II to the true Pareto optimal front is not a trivial task. Several metrics have been proposed as stopping criteria for multi-objective evolutionary algorithms (MOEA) but the standard approach remains running the algorithm for a fixed number of function evaluations. In our simulations, we use the standard approach but to gauge the quality of solutions we do an offline convergence study. The two major aspects on which a MOEA can be judged are nearness to the Pareto optimal front and the spread of the solutions on the front. We use hypervolume indicator and generational distance (GD) which are two popular metrics in the evolutionary multiobjective optimization community for evaluating the performance of MOEA.

Hypervolume indicator, proposed by Zitzler et al. [24], measures the volume of the objective space dominated by the non-dominated set of points. Given a set of points, hypervolume indicator measures the volume occupied by the union of hypercubes defined by each point in the set and an input reference point. Fleischer [25] showed that for a given problem the solutions covering the entire Pareto optimal front maximize the hypervolume measure. In our simulations, we compute $HV(i)$, the value of hypervolume indicator at i^{th} generation, and empirically show the it increases as the algorithm proceeds.

A popular heuristic approach for terminating evolutionary single objective optimization algorithms is to stop when there is no further improvement in the objective function value. An equivalent metric for multiobjective problems can be constructed by using the GD metric [26]. $GD(i,j)$ defined as

$$GD(i,j) = \sqrt{\sum_{k=1}^{\mathcal{S}_{\mathcal{P}}} g_k^2} \tag{19}$$

measures the distance between non-dominated solutions in generation i and generation j repectively, where g_k is the Euclidean distance between the k^{th} solution in generation i and the nearest solution in non-dominated set of generation j. The value of this metric decreases as the iterations progress.

A. P3:Maximization of Sum-Rate and Minimization of Network Power Under QoS Constraints

In this problem, we consider joint minimization of total power consumption and maximization of sum-rate of the CRN. Minimizing total network power is an important problem especially when the network has a constraint on the interference it can cause to other neighboring networks. Our bi-objective optimization problem referred to as P3 is defined as follows.

$$\begin{aligned} \text{Find} \quad & \mathbf{q}_n, \quad \forall n \in \mathcal{N} \\ \text{Minimize} \quad & \left\{ -\sum_{n=1}^{N} R_n(\mathbf{q}_n) \right\}, \left\{ \sum_{n=1}^{N} \sum_{k=1}^{K} p_n(k) \right\} && (20) \\ \text{Subject to} \quad & \text{Constraints of P1} && (21) \end{aligned}$$

B. P4:Maximization of Sum-Rate and Maximization of Fairness Under QoS Constraints

Allocating power in order to maximize the total sum-rate of the network, can result in skewed power distributions. Here, we consider the trade-off between sum-rate and fairness. We model fairness by the variance of the sum-rate capacities of SUs as

$$\sum_{n\in\mathcal{S}} (R_n - \mu_R)^2$$

where the average sum-rate is defined by,

$$\mu_R \triangleq \frac{1}{|\mathcal{S}|} \sum_{n=1}^{|\mathcal{S}|} R_n$$

Here, $\mathcal{S} \subset \mathcal{N}$ denotes the set of operating SU users and $|\mathcal{S}|$ is the cardinality of set $\mathcal{S}$. Minimizing the sum-rate variance and making it tend to zero forces the SU users to operate close to the average sum-rate capacity, μ_R. Fairness may also conflict with the total sum-rate objective because of the non-uniform distribution of channel gains due to the path loss among the users. The SU links where transmitter and receiver are close to each other may obtain most of the resources in order to maximize the total sum-rate yielding much smaller capacity to other SU links. In such cases, the variance of the sum-rate of individual SU links may become large. Other metrics of fairness such as the Max-Min fairness can be used [27] as well. Here, Problem P4 is defined as:

$$\begin{aligned} \text{Find} \quad & \mathbf{q}_n, \quad \forall n \in \mathcal{N} \\ \text{Minimize} \quad & \left\{ -\sum_{n=1}^{N} R_n(\mathbf{q}_n) \right\}, \left\{ \sum_{n\in\mathcal{S}} (R_n - \mu_R)^2 \right\} && (22) \\ \text{Subject to} \quad & \text{Constraints of P1} && (23) \end{aligned}$$

C. P5:Maximization of Sum-Rate and Maximization of Number of Active Links Under QoS Constraints

For our last problem, we consider the trade-off between sum-rate and number of active users in a network. A user is considered active if it is able to transmit on a frequency channel. Whether these two objectives are conflicting depends on the topology of the network as well as on the SINR constraints of the secondary users. If the topology is set up in such a way that some users have very high channel gains as compared to other users, then all the power would be allocated to these high-gain users and the second objective will not be satisfied. If the SINR constraints for SUs are very high, then also only very few users will be admitted. Therefore, the trade-off between the number of active users and the sum-rate of the network, is relevant when some users have higher channel gains as compared to others, or when the SU-SINR threshold is low. Problem P5 is defined as:

$$
\begin{aligned}
\text{Find} \quad & \mathbf{q}_n, \quad \forall n \in \mathcal{N} \\
\text{Minimize} \quad & \left\{ -\sum_{n=1}^{N} R_n(\mathbf{q}_n) \right\}, \left\{ -\sum_{n=1}^{N} i_n \right\} \qquad (24)\\
\text{Subject to} \quad & \text{Constraints of P1} \\
\text{Where} \quad & i_n = \begin{cases} 1, & \text{if} \quad \exists n : \max\{p_n(1), \ldots, p_n(K)\} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (25)
\end{aligned}
$$

VI. Numerical Results

In this section, we provide numerical results to evaluate the performance of our proposed algorithms in terms of solution quality and time complexity. The transmit power of each primary transmitter and the power budget P^B for each secondary transmitter are set to 6 dB and -3 dB, respectively. The SINR thresholds for PUs and SUs are $\gamma_m^* = 20$ dB and $\gamma_n^* = 10$ dB for $n = 1, \ldots, N$ and $m = 1, \ldots, M$, respectively. Fig. 6 illustrates an example deployment of PU and SU transmitter receiver pairs. We consider an area of 5×5 kilometers. PU and SU transmitters are randomly deployed in the area. PU receivers are randomly deployed within ζ_k distance of their respective transmitters, where ζ_k is chosen to provide 20 dB at the boundary of their deployment region. Note that we set $M = K$ as explained in Section III, i.e, the number of subbands is equal to the number of primary users. Each subband has a bandwidth of $B = 6$ MHz. Attenuation constant $\alpha = 4$. SU receivers are randomly deployed within Δ distance of their respective transmitters. For directional antennas, main lobe antenna gains and side lobe antenna gains are $G_M = 4$ and $G_S = 0.4$, respectively. The main lobe beam width for directional antenna is $\theta = \frac{\pi}{3}$. Following parameters are used to run GA for all the problems – populations size $= 1000$, number of generations $= 2000$, crossover probability $= 0.9$, mutation probability$= 0.05$, η_c and η_m as defined in (9) and (10) are fixed at 15 and 70, respectively. Single objective and multi-objective genetic algorithms were implemented in C programming language and their results were linked to Matlab. All the other simulations were performed using Matlab on an Intel Core i5 dualcore CPU with 2.6 Ghz clockspeed and 5.4GB of RAM.

In Fig. 7, we present the sum-rate performance achieved by the proposed algorithms for P1. Here, we vary Δ from 100 meters to 500 meters and provide sum-rate results averaged over 100 Monte Carlo simulations for each Δ value. The normalized sum-rate results, i.e., $\frac{1}{B}\sum_n R_n$, are obtained for the case when $N = M = K = 3$. As expected, the maximum sum-rate values decrease as Δ increases, because high Δ values imply low SINR at the secondary receivers. Since SU SINR thresholds are selected to be $\gamma_n^* = 10$ dB, one should note that the function approximation $\log(1+SINR) \approx \log(SINR)$ is highly accurate in feasible regions. Therefore, CRTP should provide solutions that are very close to the global optimum. In this case, the solutions obtained by CRTP can be used as benchmarks for those obtained by CRGR and GA. As is clear from Fig. 7, GA performs very close to CRTP. Among the three algorithms, CRGR performs the worst in terms of maximizing sum-rate. The complexity of the CRTP algorithm is worst-case exponential, which can be computationally prohibitive for large problems. In contrast, the CRGR algorithm has the lowest computational complexity because of its greedy nature. Therefore, it is scalable for larger problems with a large number of users and frequency bands. GA provides a balance between optimality and computational complexity, since it can provide solutions that are very close to that of the CRTP with reduced computational complexity. Table II shows an empirical comparison between the run times of the three algorithms for randomly chosen PU and SU locations.

In Fig. 8, we compare the sum-rate solutions obtained by CRGR and GA for different numbers of shared frequency bands. It is clear from the figure that as K increases, i.e., as the shared bandwidth gets larger, the maximum sum-rate increases as expected. Furthermore, the performance of the

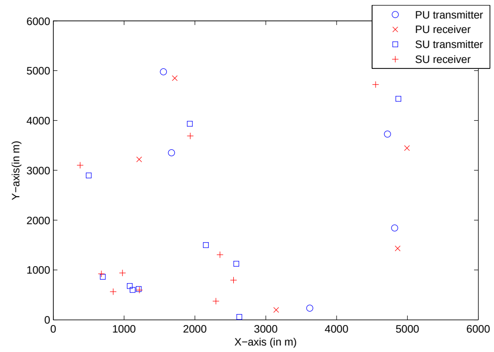


Fig. 6. An example deployment of PU and SU transmitter-receiver pairs (SU and PU locations for the Figures 9, 10 and 11)

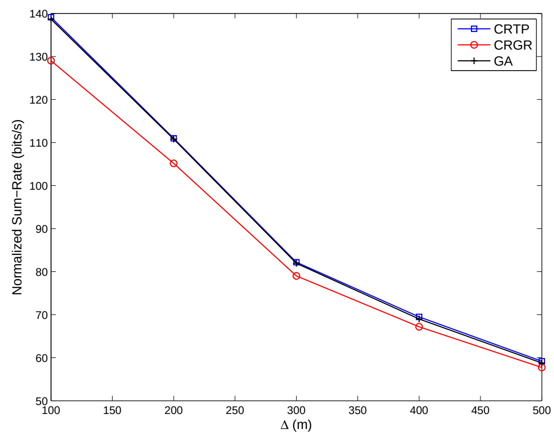


Fig. 7. Sum-rate performance of proposed algorithms as a function of Δ for P1 (N=M=K=3)

TABLE II
Comparison of Run Time with Problem Size

N	K	T_{CRTP} (in sec)	T_{CRGR} (in sec)	T_{GA} (in sec)
5	2	22	1	45
4	3	274	3	48
7	2	936	4	57
8	2	4275	11	128
9	2	11219	47	129

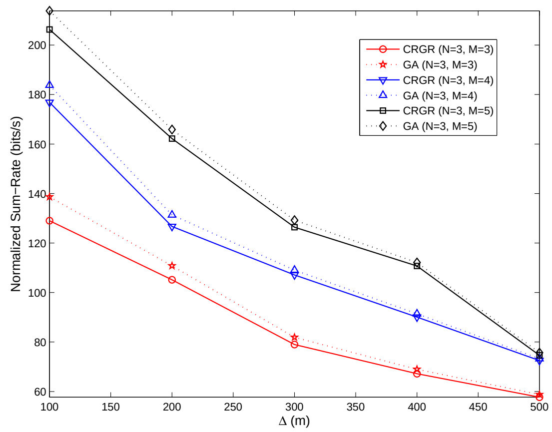


Fig. 8. Performance of CRGR versus GA for P1

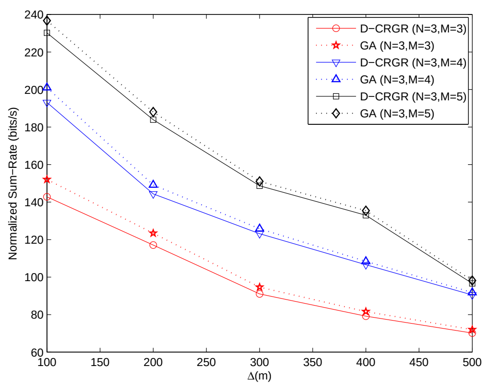


Fig. 9. P2: Performance Results for Δ vs Normalized Sum rate using directional GA and Directional CRGR algorithm for $\theta = \frac{\pi}{3}$

CRGR algorithm is slightly worse than that of the GA. By keeping the same numbers of shared frequency bands, in Fig. 9, we compare sum-rate solutions obtained by D-CRGR and GA with directional antennas. Again, GA is a little better than D-CRGR in terms of solution quality. Simulation results corroborate that directional antennas improve the total sum-rate of the SU communication.

For multiobjective problems P3, P4 and P5, we first estimate the number of generations required for convergence. We evaluate the time required for convergence considering the network given in Fig. 6 using the hypervolume indicator and GD as described in Section V. Table III gives the convergence metrics for five independent runs of NSGA-II. We use the implementation developed by Fonseca et al. [28] for hypervolume computation. Maximum value of each objective function in the input non-dominated set is used as a reference point for hypervolume computation. Table III shows that convergence for all the problems is achieved in around 1000 generations, which takes less than 40 seconds.

We obtain the Pareto front containing 1000 non-dominated solutions of P3, P4 and P5 when $N = 10$ and $M = K = 5$, the same network deployment given in Fig. 6. The Pareto fronts for P3, P4 and P5 are obtained after 1,000,000 function evaluations, corresponding to a population size of 1000 with

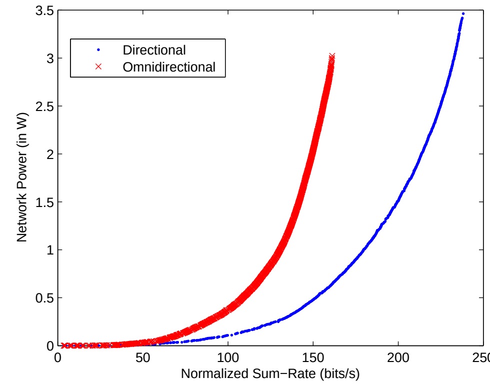


Fig. 10. Trade-off between Sum-rate and Total Network Power, P3 (N=10, K=M=5)

1000 generation runs. We show these pareto fronts in Figs. 10, 11, and 12, respectively. It is interesting to see from Fig. 10 that the normalized sum-rate can be increased from 0 to about 100 bits/s with an increase of 0.4 Watts in the total network power. On the other hand, by using directional antennas and selecting appropriate transmit direction angle, the normalized sum-rate is increased from 0 to about 100 bits/s with an incremental increase of 0.1 Watts in the total network power. The Pareto front obtained by our modified NSGA-II algorithm helps the network controller clearly see the trade offs between different solutions. The advantage of formulating multiobjective optimization problems for CRNs is clear in this example. Next in Fig. 11, we illustrate the trade-off between Sum-rate and Fairness. By minimizing the variance of the sum rate of the SU links, our fairness criteria forces the sum rate of SU links to be close to the average sum rate. For the normalized sum-rate of 120 bits/sec, use of directional antennas provides a fairness value near zero which means that all the SU links operate with very similar capacity. Simulations indicates that at the same fairness value, using directional antennas results in higher sum-rate. Finally, Fig. 12 shows the trade-off between the number of active SUs and the normalized sum rate of the secondary network. Though it may seem that increasing the number of users should increase the sum-rate of the network but depending on the SU and PU locations, it is possible that users may have to transmit at lower power levels (to satisfy the QoS constraints) leading to overall lower sum-rate. One such example is shown in Fig. 12, where the maximum sum-rate with 9 active SUs is lower than with 7 active SUs. Also, the use of directional antennas improves the normalized sum-rate of the secondary links due to the fact that directional antennas decrease the interference from other SUs. For other SU deployments, directional antennas may not only improve the sum-rate of the secondary network but may also improve the total number of active SUs as well.

VII. Conclusion

In this paper, we have studied the spectrum sharing problem in a dynamic spectrum access network and enhanced the performance of the network by exploiting the frequency,

TABLE III
PERFORMANCE METRICS OF NSGA-II FOR P3, P4 AND P5

Algorithm	Generations	HV (mean)	HV (stddev)	GD(t,t+100) (mean)	GD(t,t+100) (std dev)	Run time (in sec) (mean)
P3 Omnidirectional	100	0.399	0.032	636.159	20.12	1.54
	400	0.455	0.032	0.012	0.003	6.45
	700	0.458	0.032	0.001	0.001	12.54
	1000	**0.459**	**0.032**	**0.0**	**0.0**	**18.52**
	1500	0.460	0.032	0.0	0.0	28.39
P3 Directional	100	0.568	0.015	446.944	5.154	2.16
	400	0.701	0.011	0.052	0.009	7.79
	700	0.714	0.013	0.011	0.010	14.49
	1000	**0.718**	**0.014**	**0.001**	**0.0**	**21.83**
	1500	0.720	.013	0.0	0.0	34.35
P4 Omnidirectional	100	0.338	0.024	1.639	0.529	2.18
	400	0.408	0.032	0.049	0.036	7.38
	700	0.416	0.030	0.003	0.0	13.63
	1000	**0.418**	**0.030**	**0.0**	**0.0**	**20.89**
	1500	0.419	0.030	0.0	0.0	33.12
P4 Directional	100	0.569	0.149	3.310	1.756	2.20
	400	0.789	0.155	0.245	0.186	8.23
	700	0.804	0.151	0.007	0.005	15.27
	1000	**0.807**	**0.151**	**0.001**	**0.0**	**23.31**
	1500	0.809	0.151	0.0	0.0	37.19
P5 Omnidirectional	100	0.010	0.002	6.768	3.974	2.76
	400	0.041	0.010	0.037	0.040	10.59
	700	0.043	0.011	0.0	0.0	18.20
	1000	**0.043**	**0.011**	**0.0**	**0.0**	**25.57**
	1500	0.044	0.011	0.0	0.0	37.72
P5 Directional	100	0.001	0.001	7.993	1.704	3.07
	400	0.043	0.009	0.031	0.030	14.39
	700	0.054	0.015	0.0	0.0	25.49
	1000	**0.054**	**0.015**	**0.0**	**0.0**	**36.17**
	1500	0.054	0.015	0.0	0.0	54.05

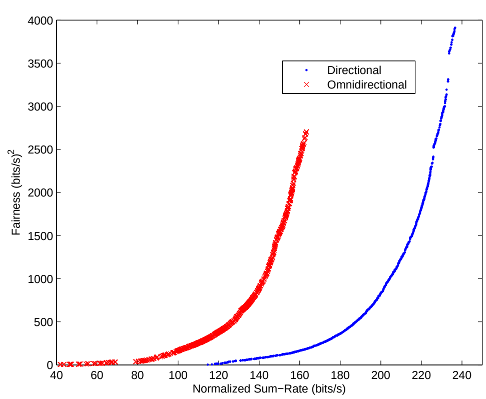


Fig. 11. Trade-off between Sum-rate and Fairness, P4 (N=10, K=M=5)

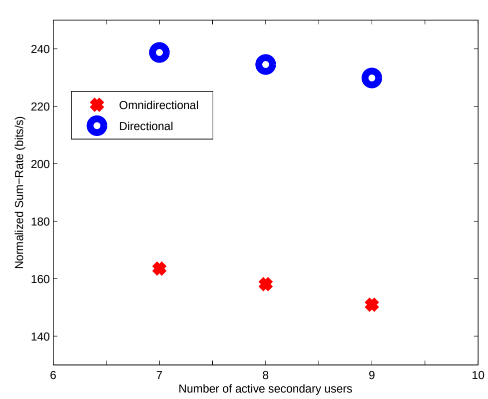


Fig. 12. Trade-off between Sum-rate and number of active users, P5 (N=10, K=M=5)

power and antenna direction dimensions of TH. We have considered five representative single objective and multi-objective problems and developed suitable optimization algorithms which enable practical implementations. The single objective problem of sum-rate maximization in multiband cognitive radio networks was considered under the PU and SU QoS constraints. Motivated by the fact that these problems are NP-hard, we have developed three suboptimal algorithms based on convex relaxation with tree pruning (CRTP), convex relaxation with gradual removal (CRGR), and genetic algorithms (GA). Each algorithm offers different trade offs in optimality and computational complexity. CRTP provides the best performance in terms of optimality; however, it is computationally the most expensive algorithm. CRGR offers the lowest computational complexity; however it suffers from high degree of suboptimality. Numerical results showed that GA provides a balance between optimality and computational complexity.

The spectrum sharing problem in cognitive radio networks may involve multiple conflicting objectives such as the maximization of the capacity of the secondary network and the minimization of the interference to the primary network. We have also shown that the GA developed for solving the single-objective optimization problem can be extended for solving such multiobjective optimization problems in a time-efficient manner. We have obtained solutions which achieve different trade-offs between critical and conflicting objective values for the secondary network. Simulations results offer various alternative solutions to the network controller.

In the future, we will extend our formulations to other communication scenarios in cognitive radio networks such as other possible dimensions of the TH such as time, modulation, coding parameters as well as new objective functions such as bit error rate, packet error rate, throughput, latency, etc. Such scenarios may include heterogeneous QoS constraints for different users such as combinations of data rate and SINR constraints.

Acknowledgement of Support and Disclaimer

(a) ANDRO Computational Solutions, LLC acknowledges the U.S. Government's support in the publication of this paper. This material is based upon work funded by AFRL, under AFRL Contract No. FA8750-10-C-0221; (b) Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of AFRL.

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