Blind Signal Separation

Introduction:

Blind signal separation (BSS), also known as blind source separation, is the separation of a set of source signals from a set of mixed signals, without the aid of information (or with very little information) about the source signals or the mixing process. This problem is in general highly underdetermined, but useful solutions can be derived under a surprising variety of conditions. Much of the early literature in this field focuses on the separation of temporal signals such as audio. However, blind signal separation is now routinely performed on multidimensional data, such as images and tensors, which may involve no time dimension whatsoever.

Technology Used: MATLAB

Related Terminologies:

Probability density function:

A function of a continuous random variable, whose integral across an interval gives the probability that the value of the variable lies within the same interval.

Cumulative distribution function (CDF)

In probability theory and statistics, the cumulative distribution function (CDF) of a real-valued random variable X, or just distribution function of X, evaluated at x, is the probability that X will take a value less than or equal to x.

Statistical Independence:

In probability theory, two events are independent, statistically independent, or stochastically independent if the occurrence of one does not affect the probability of occurrence of the other.

Degree of statistical independence:

The number of independent pieces of information that go into the estimate of a parameter are called the degrees of statistic independence.

Assumption:

All the signals are statistically independent.

Mathematical representation

$S(t) = (S_1(t), S_2(t) \ldots\ldots S_n(t))^T$ → set of individual source signals.

$S(t)$ is mixed using matrix $A = [a_{ij}] \in R^{m \times n}$

to produce set of 'mixed' signals. $x(t)$

$x(t) = (x_1(t), \ldots\ldots x_m(t))^T$ → set of mixed signals

usually $n = m$;

if $m > n$;

unmixing is done using conventional linear method.

if $n > m$;

unmixing is done using non-linear method.

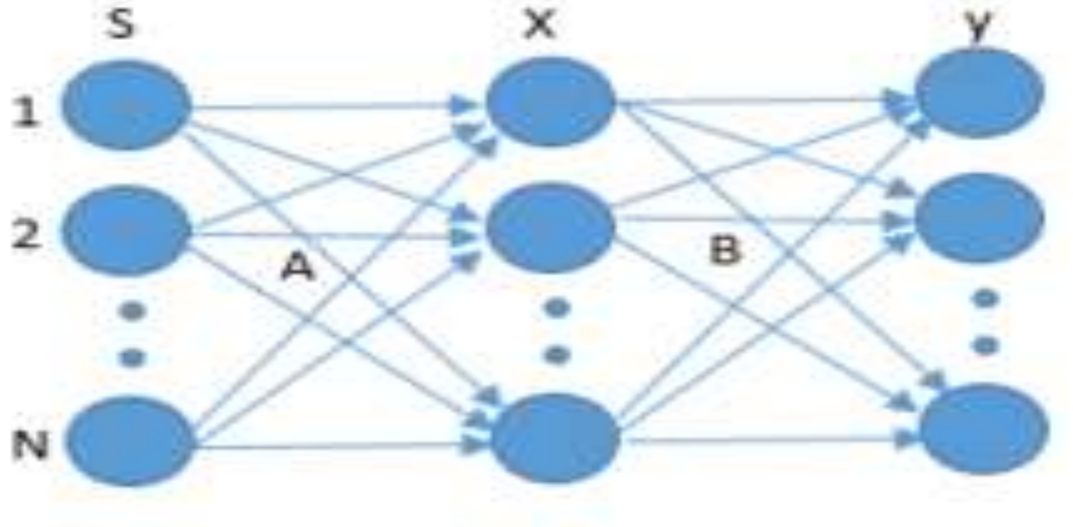
⇒ In our case $\underline{\underline{n = m}}$

$$\boxed{x(t) = A \cdot S(t)}$$

The above equation is effectively 'inverted' as follows. Blind source separation separates the set of mixed signals, x(t) through the determination of an 'un-mixing' matrix, $B=B_{[i\,j]}$ belongs to $(R^{(m*n)})$, to 'recover' an approximation of the original signals, **$y(t)=y_1(t),\ldots y_n(t)$.**

$$\mathbf{Y(t)=B.X(t)}$$

Basic flowchart of BSS



Applications:

At a cocktail party, there is a group of people talking at the same time. You have multiple microphones picking up mixed signals, but you want to isolate the speech of a single person. BSS can be used to separate the individual sources by using mixed signals.

BSS is used to separate the mixed signals with only knowing mixed signals and nothing about original signal or how they were mixed. The separated signals are only approximations of the source signals.

Brain imaging is another ideal application for BSS. In electroencephalogram (EEG) and Magnetoencephalography (MEG), the interference from muscle activity masks the desired signal from brain activity. BSS, however, can be used to separate the two so an accurate representation of brain activity may be achieved.

Approaches:

Since the chief difficulty of the problem is its under-determination, methods for blind source separation generally seek to narrow the set of possible solutions in a way that is unlikely to exclude the desired solution. In one approach, exemplified by principal and independent component analysis, one seeks source signals that are minimally correlated or maximally independent in a probabilistic or information-theoretic sense. A second approach, exemplified by nonnegative matrix factorization, is to impose structural constraints on the source signals. These structural constraints may be derived from a generative model of the signal, but are more commonly heuristics justified by good empirical performance. A common theme in the second approach is to impose some kind of low-complexity constraint on the signal, such as sparsity in some basis for the signal space. This approach can be particularly effective if one requires not the whole signal, but merely its most salient features.

Methods

There are different methods of blind signal separation:

Principal components analysis

Singular value decomposition

Independent component analysis

Dependent component analysis

Non-negative matrix factorization

Stationary subspace analysis

Common spatial pattern

The Method in which we are focussing on:

Blind Source Separation Using Temporal Predictability (principal component analysis):

A measure of temporal predictability is defined, and used to separate linear mixtures of signals. Given any set of statistically independent source signals, it is conjectured here that a linear mixture of those signals has the following property: the temporal predictability of any signal mixture is less than (or equal to) that of any of its component source signals. It is shown that this property can be used to recover source signals from a set of linear mixtures of those signals by finding an un-mixing matrix which maximises a measure of temporal predictability for each recovered signal. This matrix is obtained as the solution to a generalised eigenvalue problem; such problems have scaling characteristics of **O(N^3),** where N is the number of signal mixtures. In contrast to independent component analysis, the temporal predictability method requires minimal assumptions regarding the probability density functions of source signals. It is demonstrated that the method can separate signal mixtures in which each mixture is a linear combination of source signals with super-Gaussian, sub-Gaussian, and Gaussian probability density functions, and on mixtures of voices and music.

Separating Mixtures of Signals with Different Pdfs:

Three source signals s = {s1 | s2 | s3} t

(1)**Super-Gaussian Signal (speech signal)**

(2) **Sub-Gaussian Signal (a sine wave)**

(3)**Gaussian Signal.** Signal 3 was generated using the randn procedure in MatLab, and temporal structure was imposed on the signal by sorting its values in ascending order. These three signals were mixed using a random matrix A to yield a set of three signal mixtures: x = As.

The resultant mixtures exemplify three universal properties of linear mixtures of statistically independent source signals:

1. Temporal predictability (conjecture)— The temporal predictability of any signal mixture is less than (or equal to) that of any of its component source signals.

2. Gaussian probability density function— The central limit theorem ensures that the extent to which the probability density function (pdf) of any mixture approximates a gaussian distribution is greater than (or equal to) any of its component source signals.

3. Statistical Independence—The degree of statistical independence between any two signal mixtures is less than (or equal to) the degree of independence between any two source signals.

Problem Definition and Temporal Predictability.

Consider a set of K statistically independent source signals s = {s1 | s2 | ··· | sK} t ,where the ith row in s is a signal si measured at n time points (the superscript t denotes the transpose operator).It is assumed throughout this article that source signals are statistically independent, unless stated otherwise. A set of M ≥ K linear mixtures x = {x1 | x2 | ··· | xM} t of signals in s can be formed with an M×K mixing matrix A: x = As.

If the rows of A are linearly independent,3 then any source signal si can be recovered from x with a 1 × M matrix Wi : si = Wix.

The problem to be addressed here consists in finding an unmixing matrix W = {W1 | W2 |···| WK} t such that each row vector Wi recovers a different signal yi , where yi is a scaled version of a source signal si , for K = M signals.

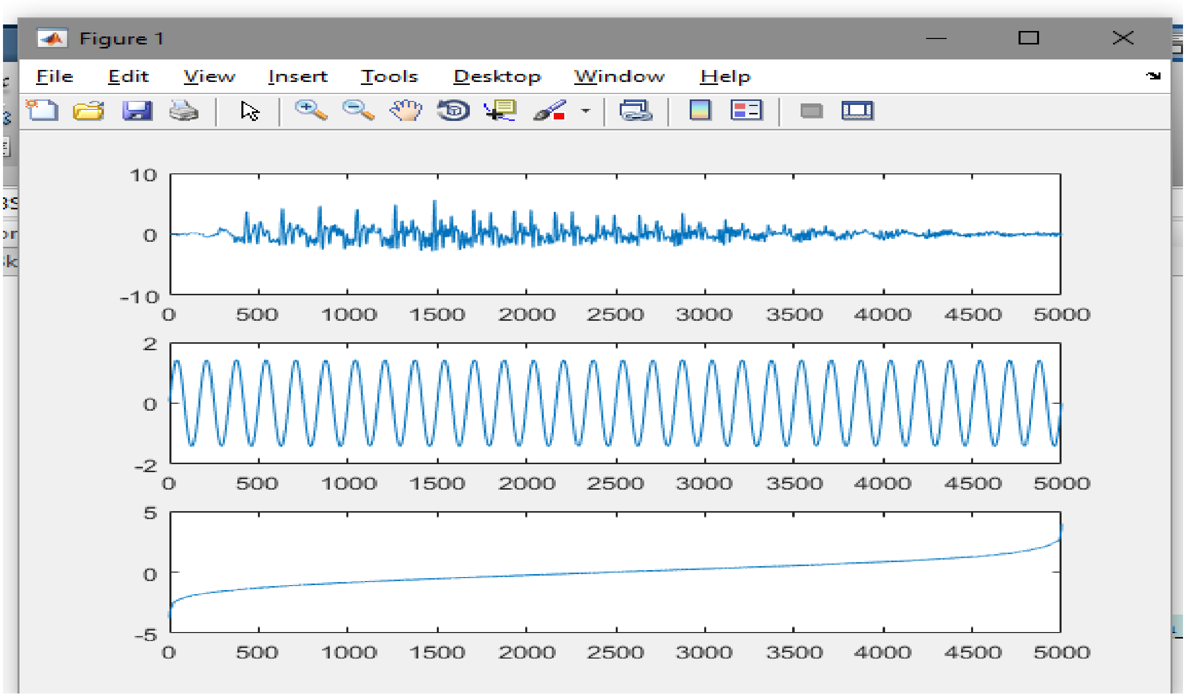
Code that takes input of the above defined three signals

```
if demo_id==2
% SUB-GAUSSIAN == SINE
periods=30;x=1:num_ip_vecs ; x=x/num_ip_vecs *2*pi*periods; s=sin(x);
sources(2,:)=s;

% SUPER-GAUSSIAN == a sound
[y]=audioread('why.wav'); y=y(1:num_ip_vecs)';
sources(1,:)=y;

% GAUSSIAN == GAUSS
s=randn(1,num_ip_vecs);s=sort(s);
sources(3,:)=s;
```

```
end;
```

Code Making Mixture Matrix:

```
sources=sources';
% Make mixing matrix A.
A = randn(num_mixtures,num_sources);
mixtures=sources*A;
```



Solution strategy:

The method for recovering source signals is based on the following conjecture: the temporal predictability of a signal mixture xi is usually less than that of any of the source signals that contribute to xi.

For example, the waveform obtained by adding two sine waves with different frequencies is more complex than either of the original sine waves. This observation is used to define a measure F(Wi, x) of temporal predictability, which is then used to estimate the relative predictability of a signal yi recovered by a given matrix Wi ,

where yi = Wix.

If source signals are more predictable than any linear mixture yi of those signals, then the value of Wi , which maximizes the predictability of an extracted signal yi , should yield a source signal (i.e., yi = csi , where c is a constant). An information-theoretic analysis of the function F proves that maximizing the temporal predictability of a signal amounts to differentially maximizing the power of Fourier components with the lowest (nonzero) frequencies .The function F is invariant with respect to the power of low-frequency components in signal mixtures and therefore tends to amplify differentially even very low power components, which have the lowest (nonzero) temporal frequency.

Ultimate Aim:

The three signals were mixed using a random matrix A to yield a set of three signal mixtures: x = As. Each signal consisted of 3000 samples; The correlations between source signals and recovered signals are given in Table 1. The three recovered signals each had a correlation of r > 0.99, with only one of the source signals, and other correlations were close to zero. Note that the mixtures used here do not include any temporal delays or echoes.

Output Co-relation Matrix:

Source Signals vs Signals Recovered

	s1	s2	s3
y1	0.000	0.001	1.000
y2	1.000	0.000	0.000
y3	0.042	0.999	0.002