

2

Vector and Calculus

JEE-Main

Addition and Subtraction of Two Vectors

1. If two vectors \vec{A} and \vec{B} having equal magnitude R are inclined at an angle θ , then [31 Jan, 2024 (Shift-II)]

$$(a) |\vec{A} - \vec{B}| = \sqrt{2} R \sin \frac{\theta}{2} \quad (b) |\vec{A} + \vec{B}| = 2 R \sin \frac{\theta}{2}$$

$$(c) |\vec{A} + \vec{B}| = 2 R \cos \frac{\theta}{2} \quad (d) |\vec{A} - \vec{B}| = 2 R \cos \frac{\theta}{2}$$

2. A vector has magnitude same as that of $\vec{A} = 3\hat{i} + 4\hat{j}$ and is parallel to $\vec{B} = 4\hat{i} + 3\hat{j}$. The x and y components of this vector in first quadrant are x and 3 respectively where $x = \underline{\hspace{2cm}}$. [30 Jan, 2024 (Shift-II)]

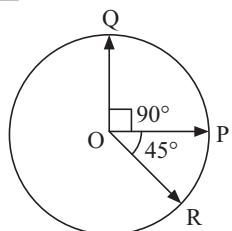
3. Two forces \vec{F}_1 and \vec{F}_2 are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two forces is equal to the force of larger magnitude. The angle between \vec{F}_1 and \vec{F}_2 is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of $|n|$ is $\underline{\hspace{2cm}}$. [04 April, 2024 (Shift-I)]

4. The angle between vector \vec{Q} and the resultant of $(2\vec{Q} + 2\vec{P})$ and $(2\vec{Q} - 2\vec{P})$ is: [05 April, 2024 (Shift-I)]

$$(a) 0^\circ \quad (b) \tan^{-1}\left(\frac{2\vec{Q} - 2\vec{P}}{2\vec{Q} + 2\vec{P}}\right)$$

$$(c) \tan^{-1}\left(\frac{P}{Q}\right) \quad (d) \tan^{-1}\left(\frac{2Q}{P}\right)$$

5. Three vectors \vec{OP} , \vec{OQ} and \vec{OR} each of magnitude A are acting as shown in figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is $\underline{\hspace{2cm}}$. [08 April, 2024 (Shift-I)]



6. If \vec{a} and \vec{b} makes an angle $\cos^{-1}\left(\frac{5}{9}\right)$ with each other, then $|\vec{a} + \vec{b}| = \sqrt{2} |\vec{a} - \vec{b}|$ for $|\vec{a}| = n |\vec{b}|$. The integer value of n is $\underline{\hspace{2cm}}$. [09 April, 2024 (Shift-I)]

7. The resultant of two vectors \vec{A} and \vec{B} is perpendicular to \vec{A} and its magnitude is half that of \vec{B} . The angle between vectors \vec{A} and \vec{B} is \dots . [09 April, 2024 (Shift-II)]

8. When vector $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is subtracted from vector \vec{B} , it gives a vector equal to $2\hat{j}$. Then the magnitude of vector \vec{B} will be: [11 April, 2023 (Shift-II)]

$$(a) \sqrt{13} \quad (b) 3 \quad (c) \sqrt{6} \quad (d) \sqrt{5}$$

9. Match List-I with List-II.

List-I		List-II	
A.	$\vec{C} - \vec{A} - \vec{B} = 0$	I.	
B.	$\vec{A} - \vec{C} - \vec{B} = 0$	II.	
C.	$\vec{B} - \vec{A} - \vec{C} = 0$	III.	
D.	$\vec{A} + \vec{B} = -\vec{C}$	IV.	

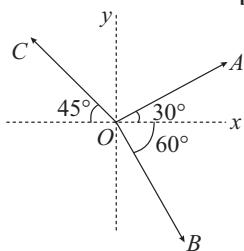
Choose the correct answer from the options given below:

[25 July, 2021 (Shift-I)]

- (a) A-IV, B-I, C-III, D-II (b) A-IV, B-III, C-I, D-II
(c) A-I, B-IV, C-II, D-III (d) A-III, B-II, C-IV, D-I

10. The magnitude of vectors \overrightarrow{OA} , \overrightarrow{OB} and \overrightarrow{OC} in the given figure are equal. The direction of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$ with x -axis will be

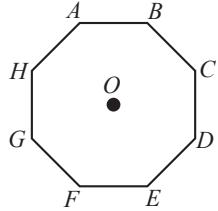
[26 Aug, 2021 (Shift-I)]



- (a) $\tan^{-1}\frac{(\sqrt{3}-1+\sqrt{2})}{(1-\sqrt{3}+\sqrt{2})}$ (b) $\tan^{-1}\frac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$
 (c) $\tan^{-1}\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$ (d) $\tan^{-1}\frac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$

11. In an octagon ABCDEFGH of equal side, what is the sum of $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$, if, $\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

[25 Feb, 2021 (Shift-I)]

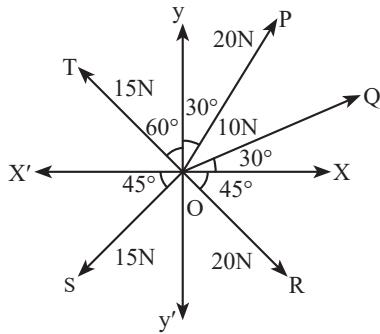


- (a) $16\hat{i} + 24\hat{j} - 32\hat{k}$ (b) $16\hat{i} - 24\hat{j} + 32\hat{k}$
 (c) $-16\hat{i} - 24\hat{j} + 32\hat{k}$ (d) $16\hat{i} + 24\hat{j} - 32\hat{k}$

12. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and \overrightarrow{OT} is approximately N.

[27 Aug, 2021 (Shift-I)]

[Take $\sqrt{3} = 1.7, \sqrt{2} = 1.4$. Given \hat{i} and \hat{j} unit vectors along x, y axis]



- (a) $9.25\hat{i} + 5\hat{j}$ (b) $2.5\hat{i} - 14.5\hat{j}$
 (c) $-1.5\hat{i} - 15.5\hat{j}$ (d) $3\hat{i} + 15\hat{j}$

13. **Statement-I:** If three forces \vec{F}_1, \vec{F}_2 and \vec{F}_3 are represented by three sides of a triangle and, $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$ then these three forces are concurrent forces and satisfy the condition for equilibrium.

[31 Aug, 2021 (Shift-II)]

Statement-II: A triangle made up of three forces \vec{F}_1, \vec{F}_2 and \vec{F}_3 as its sides taken in the same order, satisfy the condition for translatory equilibrium. In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Statement-I is false but Statement-II is true
 (b) Both Statement-I and Statement-II are true
 (c) Statement-I is true but Statement-II is false
 (d) Both Statement-I and Statement-II are false

14. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is [8 April, 2023 (Shift-I)]

- (a) $\frac{\sqrt{5}A}{4}$ (b) $\frac{5A}{2}$ (c) $\frac{\sqrt{5}A^2}{2}$ (d) $\frac{\sqrt{5}A}{2}$

15. A vector in $x-y$ plane makes an angle of 30° with y -axis. The magnitude of y -component of vector is $2\sqrt{3}$. The magnitude of x -component of the vector will be: [15 April, 2023 (Shift-I)]

- (a) $\frac{1}{\sqrt{3}}$ (b) 6 (c) $\sqrt{3}$ (d) 2

16. A mosquito is moving with a velocity $\vec{v} = 0.5t^2\hat{i} + 3\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s?

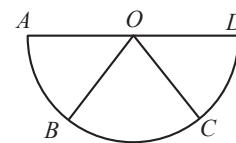
- [16 March, 2021 (Shift-II)]
 (a) $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y -axis (b) $\tan^{-1}\left(\frac{5}{2}\right)$ from y -axis
 (c) $\tan^{-1}\left(\frac{2}{3}\right)$ from x -axis (d) $\tan^{-1}\left(\frac{5}{2}\right)$ from x -axis

17. **Assertion A:** If A, B, C and D are four points on a semicircular arc with centre at ' O ' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$$

Reason R: Polygon law of vector addition yields

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$$

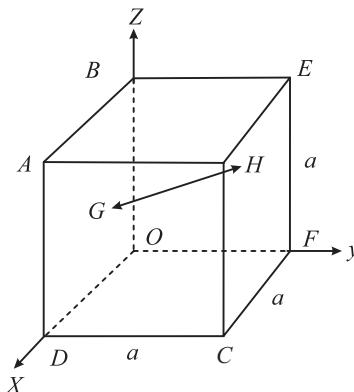


In the light of the above statements, choose the most appropriate answer from the options given below: [27 July, 2021 (Shift-I)]

- (a) A is not correct but R is correct.
 (b) A is correct but R is not correct.
 (c) Both A and R are correct and R is the correct explanation of A.
 (d) Both A and R are correct but R is not the correct explanation of A.

18. In the cube of side ' a ' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be

[10 Jan, 2019 (Shift-I)]



- (a) $\frac{1}{2}a(\hat{k} - \hat{i})$ (b) $\frac{1}{2}a(\hat{i} - \hat{k})$
 (c) $\frac{1}{2}a(\hat{j} - \hat{i})$ (d) $\frac{1}{2}a(\hat{j} - \hat{k})$

Dot Product

19. If two vectors $\vec{P} = \hat{i} + 2m\hat{j} + m\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + m\hat{k}$ are perpendicular to each other. Then, the value of m will be:

[24 Jan, 2023 (Shift-II)]

- (a) 1 (b) -1 (c) -3 (d) 2

20. Vectors $a\hat{i} + b\hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $\frac{x}{2}$. The value of x is _____.

[24 Jan, 2023 (Shift-I)]

21. Which of the following relations is true for two unit vector \hat{A} and \hat{B} making an angle θ to each other? [25 June, 2022 (Shift-I)]

- (a) $|\hat{A} + \hat{B}| = |\hat{A} - \hat{B}| \tan \frac{\theta}{2}$ (b) $|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}| \tan \frac{\theta}{2}$
 (c) $|\hat{A} + \hat{B}| = |\hat{A} - \hat{B}| \cos \frac{\theta}{2}$ (d) $|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}| \cos \frac{\theta}{2}$

22. Two vectors \vec{A} and \vec{B} have equal magnitudes. If magnitude of $\vec{A} + \vec{B}$ is equal to two times the magnitude of $\vec{A} - \vec{B}$, then the angle between \vec{A} and \vec{B} will be: [29 June, 2022 (Shift-I)]

- (a) $\sin^{-1}\left(\frac{3}{5}\right)$ (b) $\sin^{-1}\left(\frac{1}{3}\right)$
 (c) $\cos^{-1}\left(\frac{3}{5}\right)$ (d) $\cos^{-1}\left(\frac{1}{3}\right)$

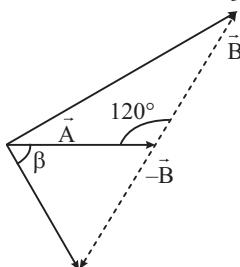
23. If $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})m$ and $\vec{B} = (\hat{i} + 2\hat{j} - 2\hat{k})m$. The magnitude of component of vector \vec{A} along vector \vec{B} will be _____ m. [26 July, 2022 (Shift-II)]

24. If the projection of $2\hat{i} + 4\hat{j} - 2\hat{k}$ on $\hat{i} + 2\hat{j} + \alpha\hat{k}$ is zero. Then, the value of α will be _____. [28 July, 2022 (Shift-I)]

25. If \vec{A} and \vec{B} are two vectors satisfying the relation $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$. Then the value of $|\vec{A} - \vec{B}|$ will be: [20 July, 2021 (Shift-I)]

- (a) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$ (b) $\sqrt{A^2 + B^2}$
 (c) $\sqrt{A^2 + B^2 + 2AB}$ (d) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

26. The angle between vector (\vec{A}) and $(\vec{A} - \vec{B})$ is: [26 Aug, 2021 (Shift-II)]



- (a) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$ (b) $\tan^{-1}\left(\frac{B \cos \theta}{A - B \sin \theta}\right)$
 (c) $\tan^{-1}\left(\frac{A}{0.7B}\right)$ (d) $\tan^{-1}\left(\frac{-B}{A - B \frac{\sqrt{3}}{2}}\right)$

27. What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$? [22 July, 2021 (Shift-II)]

- (a) $2(\hat{i} + \hat{j} + \hat{k})$ (b) $\sqrt{2}(\hat{i} + \hat{j})$
 (c) $(\hat{i} + \hat{j})$ (d) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$

28. Two vectors \vec{A} and \vec{B} have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is ' n ' times the magnitude of $(\vec{A} - \vec{B})$. The angle between \vec{A} and \vec{B} is:

- (a) $\cos^{-1}\left[\frac{n^2 - 1}{n^2 + 1}\right]$ (b) $\cos^{-1}\left[\frac{n - 1}{n + 1}\right]$
 (c) $\sin^{-1}\left[\frac{n^2 - 1}{n^2 + 1}\right]$ (d) $\sin^{-1}\left[\frac{n - 1}{n + 1}\right]$

29. Two vectors \vec{X} and \vec{Y} have equal magnitude. The magnitude of $(\vec{X} - \vec{Y})$ is n times the magnitude of $(\vec{X} + \vec{Y})$. The angle between \vec{X} and \vec{Y} is: [25 July, 2021 (Shift-II)]

- (a) $\cos^{-1}\left(\frac{n^2 + 1}{n^2 - 1}\right)$ (b) $\cos^{-1}\left(\frac{-n^2 - 1}{n^2 - 1}\right)$
 (c) $\cos^{-1}\left(\frac{n^2 - 1}{-n^2 - 1}\right)$ (d) $\cos^{-1}\left(\frac{n^2 + 1}{-n^2 - 1}\right)$

30. **Statement-I:** Two forces $(\vec{P} + \vec{Q})$ and $(\vec{P} - \vec{Q})$ where $\vec{P} \perp \vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2 + Q^2)}$, when they act at an angle θ_2 , the magnitude of their resultant becomes $\sqrt{2(P^2 + Q^2)}$. This is possible only when $\theta_1 < \theta_2$. [31 Aug, 2021 (Shift-II)]

Statement-II: In the situation given above, $\theta_1 = 60^\circ$ and $\theta_2 = 90^\circ$

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both Statement-I and Statement-II are true
 (b) Both Statement-I and Statement-II are false
 (c) Statement-I is true but Statement-II is false
 (d) Statement-I is false but Statement-II is true

31. The sum of two forces \vec{P} and \vec{Q} is \vec{R} such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and \vec{Q} will make with \vec{Q} is [7 Jan, 2020 (Shift-II)]

32. Let $|\vec{A}_1| = 3$, $|\vec{A}_2| = 5$ and $|\vec{A}_1 + \vec{A}_2| = 5$. The value of $(2\vec{A}_1 + 3\vec{A}_2) \cdot (3\vec{A}_1 - 2\vec{A}_2)$ is: [8 April, 2019 (Shift-II)]

- (a) -112.5 (b) -106.5
 (c) -118.5 (d) -99.5

33. Two forces P and Q , of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is: [10 Jan, 2019 (Shift-II)]

- (a) 120° (b) 60° (c) 90° (d) 30°

Cross Product

34. For three vectors $\vec{A} = (-x\hat{i} - 6\hat{j} - 2\hat{k})$, $\vec{B} = (-\hat{i} + 4\hat{j} + 3\hat{k})$ and $\vec{C} = (-8\hat{i} + \hat{j} + 3\hat{k})$, if $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$, then value of x is _____.

[06 April, 2024 (Shift-I)]

35. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, the unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$. The value of x is _____.

[25 Jan, 2023 (Shift-I)]

36. \vec{A} is a vector quantity such that $|\vec{A}|$ = non-zero constant. Which of the following expression is true for \vec{A} ?

[25 June, 2022 (Shift-I)]

- (a) $\vec{A} \cdot \vec{A} = 0$ (b) $\vec{A} \times \vec{A} < 0$
 (c) $\vec{A} \times \vec{A} = 0$ (d) $\vec{A} \times \vec{A} > 0$

37. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle having position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be:

[25 June, 2022 (Shift-I)]

- (a) $3\hat{i} + 4\hat{j} - 2\hat{k}$ (b) $-10\hat{i} + 10\hat{j} + 5\hat{k}$
 (c) $10\hat{i} + 5\hat{j} - 10\hat{k}$ (d) $10\hat{i} + \hat{j} - 5\hat{k}$

38. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$ the angle between \vec{P} and \vec{Q} is θ ($0^\circ < \theta < 360^\circ$). The value of ' θ ' will be _____.

[25 Feb, 2021 (Shift-II)]

Function, Differentiation as a Rate Measurement

39. A person of height 1.6 m is walking away from a lamp post of height 4m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm s^{-1} , the speed of the tip of the person's shadow on the ground with respect to the person is _____ cm s^{-1} .

[JEE Adv, 2023]

40. If $t = \sqrt{x} + 4$, then $\left(\frac{dx}{dt}\right)_{t=4}$ is: [29 July, 2022 (Shift-I)]

- (a) 4 (b) zero
 (c) 8 (d) 16

41. Three particles P , Q and R are moving along the vectors $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on point and start to move in different directions. Now particle P is moving normal to the plane which contains vector \vec{A} and \vec{B} . Similarly particle Q is moving normal to the plane which contains vector \vec{A} and \vec{C} . The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is _____.

[22 July, 2021 (Shift-II)]

ANSWER KEY

JEE-Main

- | | | | | | | | | | |
|------------------|---------|---------|---------|---------|---------|----------|-----------|----------|---------|
| 1. (c) | 2. [4] | 3. [6] | 4. (a) | 5. (c) | 6. (c) | 7. [150] | 8. (*) | 9. (b) | 10. (c) |
| 11. (a) | 12. (a) | 13. (b) | 14. (d) | 15. (d) | 16. (a) | 17. (d) | 18. (c) | 19. (d) | 20. [1] |
| 21. (b) | 22. (c) | 23. [2] | 24. [5] | 25. (a) | 26. (a) | 27. (c) | 28. (a) | 29. (c) | 30. (a) |
| 31. $[90^\circ]$ | 32. (c) | 33. (a) | 34. [4] | 35. [4] | 36. (c) | 37. (b) | 38. [180] | 39. [40] | 40. (b) |
| 41. [3] | | | | | | | | | |