Indian Institute of Technology, Kanpur.

Odd semester, 2021 - 2022.

Optimal Control

Assignment

(Deadline -) **Answer all questions** **Marks - 20**

1. All planets, minor-planets and asteroids orbit the sun in the ecliptic plane in near-circular orbits. Space missions from one to body's orbit to another do a two-impulse, optimal maneuver (Hohmann transfer) as shown in the figure. This, however, is inefficient when the two orbits are separated by a large distance (as in the case of Earth and Ceres) due to the requirement of very large impulses. To solve this, you will now design a continuous-thrust, fixed time, optimal control to reach Ceres from Earth.

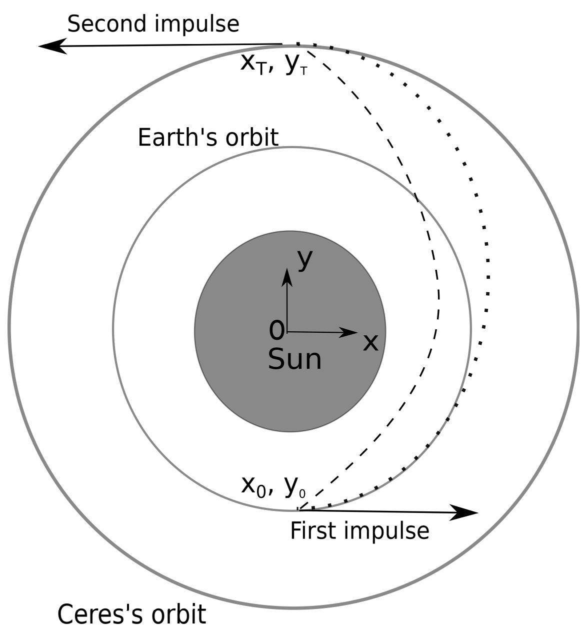

Figure 1: The dotted lines correspond to the orbit of Hohmann transfer. The first and second impulses for Hohmann transfer have $\Delta \boldsymbol{v_1} = \sqrt{\frac{GM_{sun}}{r_E}}\left(\sqrt{\frac{2r_C}{r_E+r_C}}-1\right)$ and $\Delta \boldsymbol{v_2} = \sqrt{\frac{GM_{sun}}{r_C}}\left(1-\sqrt{\frac{2r_E}{r_E+r_C}}\right)$ where r_E and r_C are the radii of Earth and Ceres respectively. Dashed lines represent your design. Note that your design has continuous thrust and not impulses.

The dynamical model of the spacecraft is

$$\dot{x} = v_x \tag{1}$$

$$\dot{y} = v_y \tag{2}$$

$$\dot{v_x} = -\frac{GM_{sun}x}{r^3} + a_x(t) \tag{3}$$

$$\dot{v_y} = -\frac{GM_{sun}y}{r^3} + a_y(t) \tag{4}$$

where G and M_{sun} are the universal gravitational constant ($6.67 \times 10^{-11}\ Nm^2kg^{-2}$) and mass of the sun ($2 \times 10^{30}\ kg$) respectively. and

$$r = \sqrt{x^2 + y^2} \tag{5}$$

The initial states are $x_0 = 0$, $y_0 = -1.5 \times 10^8\ km$, $v_{x0} = 30\ kms^{-1}$, $v_{y0} = 0$. The target states $x_T = 0$, $y_T = 4.15 \times 10^8\ km$, $v_{xT} = -18\ kms^{-1}$, $v_{yT} = 0$. The transfer time T is required be 473 Earth days.

(a) Plot the control solution ($a_x(t)$and $a_y(t)$) **(10 marks)**

(b) Calculate the net $\Delta \boldsymbol{v}$ of your solution by time integrating the accelerations. **(5 marks)**

(c) Compare the orbit and performance with those of Hohmann transfer. **(5 marks)**

(Hint: Neglect the phasing and gravity of the planets. Choose 1 $AU = 1.5 \times 10^8\ km$ as the unit for distance and 1 day as the unit for time. You need to express also G in this unit system.)