Q1. Derive equations for optimal finite- and infinite-time linear quadratic regulator problems. 20 marks

Q2. Consider the following attitude control problem: 20 marks

$$\left.\begin{aligned} \ddot{\phi} &= l \\ \ddot{\theta} &= m \\ \ddot{\psi} &= n \end{aligned}\right] \quad -(1)$$

where \phi, \theta, and \psi are the attitude angles and l, m, and n are the rolling, pitching, and yawing moments. Formulate both finite- and infinite-time LQR problem and simulate the attitude dynamics with the optimal control. Please show the all steps and discuss the results with different Q and R. [Suggestion: Q and R could be chosen diagonal matrices]

Q3. Consider an infinite-time LQR problem. Derive transfer function of the closed-loop systems and discuss about stability margins. [Hint: you can use Nyquist stability criterion] 20 marks

Q4. Consider the following optimal control problem of flip maneuver of variable pitch quadcopter: 60 marks

$$J = \frac{1}{2}\int_0^{T_f} v^2 \, dt$$

$$\dot{\theta} = q$$

$$J\dot{\omega} + \omega \times J\omega = M$$

$$M = [l, m, n]$$

$$T = \nu K (C_{T_1} + C_{T_2} + C_{T_3} + C_{T_4})$$

$$l = \nu K l (C_{T_1} - C_{T_2} - C_{T_3} + C_{T_4})$$

$$m = \nu K l (C_{T_1} + C_{T_2} - C_{T_3} - C_{T_4})$$

$$n = \nu \frac{KR}{\sqrt{2}} \left(C_{T_1}^{3/2} - C_{T_2}^{3/2} + C_{T_3}^{3/2} - C_{T_4}^{3/2}\right)$$

$$\nu \in \{+1, -1\}$$

$$J = \mathrm{diag}(0.012, 0.026, 0.038)$$

$$K = \rho \pi R^2 V_{tip}^2, \quad m = 1.4\,kg$$

$$l = 0.18m, \quad R = 0.14m, \quad \Omega = 418.9\,rad/s$$

Solve it.