Covariance Steering

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Abstract

Covariance Steering belong to the class of stochastic optimal control for discrete-time linear systems. The goal is to formulate optimal control input that will steer state from initial gaussian distribution to a final distribution.

Index Terms

Stochastic, Optimal Control, Probability Distribution, Covariance

I. Introduction

LQR is an optimal regulator for linear state feedback control but it designed for deterministic environment. In a real world scenario both system dynamics and state measurements are corrupted by noise (assumed to be gaussian). Linear Quadratic Gaussian (LQG) and Covariance steering are optimal techniques used under such stochastic environment. LQG is a combination of LQR controller and Linear Quadratic Estimator (Kalman filter). Since both Controller and estimator are optimal their combination also remains optimal. On the other hand, covariance steering is a stochastic optimal control technique that handles the problem in a different way. Here, control commands are computed for multiple states that start from an initial gaussian distribution and converges to a target gaussian distribution. Since the e Gaussian distributions can be fully described by the first two moments, the optimal covariance steering technique controls the complete distribution.

II. Problem Statement

We assume a linear discrete time stochastic system:

$$x_{k+1} = A_k x_k + B_k u_k + D_k w_k \tag{1}$$

where, $k = [0, 1, 2, \ldots, N-1]$ is the time index, $x_k \in \mathbb{R}^n$ is the state at time k, $u_k \in \mathbb{R}^m$ is the control at time k and $w_k \in \mathbb{R}^r$ is the disturbance at time k. Also, A_k, B_k and D_k vary with time

The boundary conditions are in the form of probability distribution of state at initial (x_0) and final (x_f) time:

$$x_0 \sim \mathcal{N}(\mu_0, \Sigma_0) \tag{2}$$

$$x_f \sim \mathcal{N}(\mu_f, \Sigma_f) \tag{3}$$

The noise / disturbance is a zero mean gaussian noise i.e. $w_k \sim \mathcal{N}(0, \mathbb{1})$. It is also assumed that the the state and noise disturbance are uncorrelated i.e. $\mathbb{E}[x_k, w_k] = 0$. The form of cost function taken as:

$$J = \mathbb{E}\left[\sum_{k=0}^{N-1} \left(x_k^T Q_k x_k + u_k^T R_k u_k\right)\right] \tag{4}$$

where, $Q_k (\in \mathbb{R}^{n\times n}) \succeq 0$ and $R_k (\in \mathbb{R}^{m\times m}) \succ 0$ for all $k = [0, 1, 2, \ldots, N-1]$.
In order to find an optimal control sequence that minimizes (4) subject to dynamic equation (1) and boundary conditions in (2) and (3), the above set of equations are restructured to form its augmented version in the following way:

At k=0: $x_k = x_0$

$$x_1 = A_0 x_0 + B_0 u_0 + D_0 w_0$$

At k=1: $x_k = x_1$

$$\begin{aligned} x_2 &= A_1 x_1 + B_1 u_1 + D_1 w_1 \\ x_2 &= (A_1 A_0) x_0 + \begin{bmatrix} A_1 B_0 & B_1 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} + \begin{bmatrix} A_1 D_0 & D_1 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} \end{aligned} \tag{5}$$

$$\vdots$$

At k=k: $x_k = x_k$ (in general)

$$x_k = \hat{A}_{k-1} x_0 + \hat{B}_{k-1} \hat{U}_{k-1} + \hat{D}_{k-1} \hat{W}_{k-1} \tag{6}$$

where, we have following definitions:

1)

$$\hat{A}_k = A_{k-1,0} = A_{k-1}.A_{k-2} \ldots A_2 A_1 A_0$$

2)
$$\hat{B}_k = \begin{bmatrix} B_{k-1,0} & B_{k-1,1} & \dots & B_{k-1} \end{bmatrix}$$

where each element is defined as: $B_{k1,k0} = A_{k1,k0+1}B_{k0}$ and $A_{k1,k0} = A_{k1}.A_{k1-1}\dots A_{k0}$

3)
$$\hat{D}_k = \begin{bmatrix} D_{k-1,0} & D_{k-1,1} & \dots & D_{k-1} \end{bmatrix}$$

where each element is defined as: $D_{k1,k0} = A_{k1,k0+1}D_{k0}$ and $A_{k1,k0} = A_{k1}.A_{k1-1}\dots A_{k0}$

4)
$$\hat{U}_k = \begin{bmatrix} u_0 & u_1 & \dots & u_{k-1} \end{bmatrix}^T$$

5)
$$\hat{W}_k = \begin{bmatrix} w_0 & w_1 & \dots & w_{k-1} \end{bmatrix}^T$$

Now utilizing the form of (6) an augmented version of the whole system is defined as follows:

Augmented System Dynamic

$$X = \mathcal{A}x_0 + \mathcal{B}U + \mathcal{D}W \tag{7}$$

where, we have following definitions:

1) The augmented state is defined as: $X \in \mathbb{R}^{n(N+1)\times 1}$

$$X = \begin{bmatrix} x_0 & x_1 & \dots & x_N \end{bmatrix}^T$$

2) The augmented $\mathcal{A}$ is defined as: $\mathcal{A} \in \mathbb{R}^{n(N+1)\times 1}$

$$\mathcal{A} = \begin{bmatrix} I_{n\times n} & \hat{A}_1 & \hat{A}_2 & \dots & \hat{A}_N \end{bmatrix}^T$$

3) The augmented $\mathcal{B}$ is defined as: $\mathcal{B} \in \mathbb{R}^{nN\times mN}$

$$\mathcal{B} = \begin{bmatrix} 0_{n\times m} & 0_{n\times m} & \dots & 0_{n\times m} \\ \hat{B}_0 & 0_{n\times m} & \dots & 0_{n\times m} \\ \hat{B}_{1,0} & \hat{B}_0 & \dots & 0_{n\times m} \\ \vdots & & & \\ \hat{B}_{N-1,0} & \hat{B}_{N-1,1} & \dots & \hat{B}_{N-1} \end{bmatrix}$$

4) The augmented $\mathcal{D}$ is defined as: $\mathcal{D} \in \mathbb{R}^{nN\times rN}$

$$\mathcal{D} = \begin{bmatrix} 0_{n\times r} & 0_{n\times r} & \dots & 0_{n\times r} \\ \hat{D}_0 & 0_{n\times r} & \dots & 0_{n\times r} \\ \hat{D}_{1,0} & \hat{D}_0 & \dots & 0_{n\times r} \\ \vdots & & & \\ \hat{D}_{N-1,0} & \hat{D}_{N-1,1} & \dots & \hat{D}_{N-1} \end{bmatrix}$$

5) The augmented control is defined as: $U \in \mathbb{R}^{m(N+1)\times 1}$

$$U = \begin{bmatrix} u_0 & u_1 & \dots & u_N \end{bmatrix}^T$$

6) The augmented noise is defined as: $W \in \mathbb{R}^{r(N+1)\times 1}$

$$W = \begin{bmatrix} w_0 & w_1 & \dots & w_N \end{bmatrix}^T$$

In order to formulate the terminal constraints (i.e. $x_0 \sim \mathcal{N}(\mu_0, \Sigma_0)$ and $x_N = x_f \sim \mathcal{N}(\mu_f, \Sigma_f)$)

compatible to above notations, we define $E_0(\in \mathcal{R}^{n\times nN}) = \begin{bmatrix} I_{n\times n} & 0_{n\times n} & \dots & 0_{n\times n} \end{bmatrix}$ and $E_N(\in \mathcal{R}^{n\times nN}) = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} & \dots & I_{n\times n} \end{bmatrix}$ such that E_0 and E_N are used to pick the 0^{th} and N^{th} component of augmented state vector.

Augmented Boundary Conditions

1) Boundary Condition: Initial state Mean and Covariance are defined as:

$$\mu_0 = E_0 \mathbb{E}[X] \tag{8}$$

$$\Sigma_0 = E_0 \Sigma_{xx} E_0^T \tag{9}$$

2) Boundary Condition: Final state Mean and Covariance are defined as:

$$\mu_f = E_N \mathbb{E}[X] \tag{10}$$

$$\Sigma_f = E_N \Sigma_{xx} E_N^T \tag{11}$$

where, $\Sigma_{xx} = \mathbb{E}[(X - \mathbb{E}[X])(X - \mathbb{E}[X])^T]$

Likewise the augmented Cost function in terms of augmented state and augmented control can be defined as:

Augmented Cost Function

1) Cost Function in terms of augmented state and augmented control is defined as:

$$J = \mathbb{E}[X^T Q X + U^T R U] \tag{12}$$

where $Q = blkdiag(Q_0, Q_1, \dots, Q_{N-1}, 0)$ and
$R = blkdiag(R_0, R_1, \dots, R_{N-1})$

Next, the controller is defined as a sum of close loop and open loop controller. We can consider two forms of controller:

1) Form 1: Controller utilizes full state history. Such a controller is optimal and computational intensive as it utilizes previous state history.

$$u_k = \sum_{j=0}^{k} K_{kj} \tilde{x}_k + v_k \tag{13}$$

2) Form 2: Controller utilizes only the current state. Such a controller may be sub-optimal but provides computational advantage.

$$u_k = K_k \tilde{x}_k + v_k \tag{14}$$

In our study we consider the form 2 controller for its computational benefits, Thus,

$$u_k = K_k \tilde{x}_k + v_k$$

where $\tilde{x}_k = (x_k - \bar{x}_k)$ is deviation of state at time step k, $\bar{x}_k = \mathbb{E}[x_k]$ is mean at time k and K_k is close loop gain at time step k. Thus we define Mean and Deviation controller as follows:

$$\text{Mean Controller} = \bar{u}_k = v_k \tag{15}$$

$$\begin{aligned} \text{Deviation Controller} =& \tilde{u}_k = u_k - \bar{u}_k \\ & \tilde{u}_k = K_k \tilde{x}_k \end{aligned} \tag{16}$$

This leads us to augmented form of controller:

$$U = K\tilde{X} + V \tag{17}$$

Thus, its evident that Mean controller depends on the open loop part of control while Deviation controller depends upon the close loop part. Also the above definition of Mean and Deviation controller can help to split the system dynamics in two components name Mean motion and Deviation motion as follows:

$$\text{Mean Motion} \equiv \bar{X} = \mathcal{A}\mu_0 + \mathcal{B}\bar{U} \tag{18}$$

$$\text{Deviation Motion} \equiv \tilde{X} = \mathcal{A}x_0 + \mathcal{B}\tilde{U} + \mathcal{D}W \tag{19}$$

In summary we now have,

Augmented Controller

1) As discussed before the Controller is defined as:

$$U = K\tilde{X} + V \tag{20}$$

where

$$K = diag(k_0, k_1, \ldots, k_{N-1})$$

Note that in case of Form 1 controller, K matrix would be a lower diagonal matrix rather than a diagonal matrix. $\tilde{X}$ is the augmented deviation, whereas K and V are our design variables.

We now aim to utilize decoupled Mean and Deviation dynamics using (18) and (19) and explicitly define both the dynamics in appropriate form. Plugging in the Mean controller (from (15)) and Deviation controller (from (16)) in (18) and (19) respectively we get,

$$\bar{X} = \mathcal{A}\mu_0 + \mathcal{B}V \tag{21}$$

$$\tilde{X} = (\mathbb{1} - \mathcal{B}K)^{-1}\mathcal{A}x_0 + (\mathbb{1} - \mathcal{B}K)^{-1}\mathcal{D}W \tag{22}$$

The form of (22) is not appropriate to formulate a convex optimization problem due to the presence of $(\mathbb{1} - \mathcal{B}K)^{-1}$. The following transformation is proposed:

$$\begin{aligned} L &= K(\mathbb{1} - \mathcal{B}K)^{-1} \\ \implies K &= L(\mathbb{1} + \mathcal{B}L)^{-1} \end{aligned} \tag{23}$$

With this transformation, the deviation dynamics and deviation controller would become,

$$\tilde{X} = (\mathbb{1} + \mathcal{B}L)\mathcal{A}x_0 + (\mathbb{1} + \mathcal{B}L)\mathcal{D}W \tag{24}$$

$$\tilde{U} = K\tilde{X} = L(\mathbb{1} + \mathcal{B}L)^{-1}\tilde{X} \tag{25}$$

Summary of decoupled dynamics is as follows:

Decoupled Controller and Dynamics

1) Mean Controller:

$$\bar{U} = V \tag{26}$$

2) Deviation Controller:

$$\tilde{U} = L(\mathbb{1} + \mathcal{B}L)^{-1}\tilde{X} \tag{27}$$

3) Mean Dynamics:

$$\bar{X} = \mathcal{A}\mu_0 + \mathcal{B}V \tag{28}$$

4) Deviation Dynamics:

$$\tilde{X} = (\mathbb{1} + \mathcal{B}L)\mathcal{A}x_0 + (\mathbb{1} + \mathcal{B}L)\mathcal{D}W \tag{29}$$

Note that our design variables are now L and V

Now in order to formulate the boundary conditions in terms of design variables L and V, Terminal mean and covariance are plugged in the boundary condition:

$$\begin{aligned} \mu_f &= E_N \mathbb{E}[X] \\ &= E_N(\mathcal{A}\mu_0 + \mathcal{B}V) \end{aligned} \tag{30}$$

By utilizing (29), overall augmented covariance can be written as:

$$\begin{aligned} \Sigma_{xx} &= \mathbb{E}[\tilde{X}\tilde{X}^T] \\ &= \mathbb{E}\left[(\mathbb{1} + \mathcal{B}L)(\mathcal{A}x_0 + \mathcal{D}W).(\mathbb{1} + \mathcal{B}L)^T(\mathcal{A}x_0 + \mathcal{D}W)^T\right] \\ &= (\mathbb{1} + \mathcal{B}L)\,\mathbb{E}\left[(\mathcal{A}x_0 + \mathcal{D}W).(\mathcal{A}x_0 + \mathcal{D}W)^T\right](\mathbb{1} + \mathcal{B}L)^T \\ &= (\mathbb{1} + \mathcal{B}L)(\mathcal{A}\Sigma_0\mathcal{A}^T + \mathcal{D}\mathcal{D}^T)(\mathbb{1} + \mathcal{B}L)^T \\ &= (\mathbb{1} + \mathcal{B}L)\,S\,(\mathbb{1} + \mathcal{B}L)^T \end{aligned} \tag{31}$$

where $S = (\mathcal{A}\Sigma_0\mathcal{A}^T + \mathcal{D}\mathcal{D}^T)$

The terminal covariance would therefore be:

$$
\begin{aligned}
\Sigma_f &= E_N \Sigma_{xx} E_N^T \\
&= E_N (\mathbb{1} + \mathcal{B}L)\, S\, (\mathbb{1} + \mathcal{B}L)^T E_N^T
\end{aligned}
\tag{32}
$$

The terminal covariance defined in (32) is an equality constraint which is quadratic in terms of "L". Since such a quadratic equality constraints are not preferred in optimization, the condition is relaxed to following:

$$
\Sigma_f \leq E_N (\mathbb{1} + \mathcal{B}L)\, S\, (\mathbb{1} + \mathcal{B}L)^T E_N^T \tag{33}
$$

This is a relaxed criteria which means that the terminal covariance should be anything smaller than R.H.S of (33).

Summary of updated terminal condition is as follows:

Updated Terminal Condition

1) Terminal Mean:

$$
\mu_f = E_N (\mathcal{A}\mu_0 + \mathcal{B}V) \tag{34}
$$

2) Terminal Covariance:

$$
\Sigma_f \leq E_N (\mathbb{1} + \mathcal{B}L)\, S\, (\mathbb{1} + \mathcal{B}L)^T E_N^T \tag{35}
$$

Note that our design variables are L and V. Terminal Mean is a linear function of v whereas Σ_f is a Linear Matrix Inequality (LMI) in L

Hence, we arrive at our resulting optimization problem as follows:

Resulting Optimization Problem

$$\min_{L,V} \quad J = \underbrace{(\bar{X}^T Q \bar{X} + \bar{U}^T R \bar{U})}_{\text{Mean Cost}(J_\mu)} + \underbrace{(tr(Q\Sigma_{xx}) + tr(R\Sigma_{uu}))}_{\text{Covariance Cost}(J_\Sigma)}$$

$$\begin{aligned} \text{s.t.} \quad & \mu_f = E_N(\mathcal{A}\mu_0 + \mathcal{B}V) \\ & \Sigma_f \leq E_N(\mathbb{1} + \mathcal{B}L)\, S\, (\mathbb{1} + \mathcal{B}L)^T E_N^T \end{aligned}$$

where

$$\begin{aligned} \bar{X} &= \mathcal{A}\mu_0 + \mathcal{B}V \\ \bar{U} &= V \\ \Sigma_{xx} &= (\mathbb{1} + \mathcal{B}L)\, S\, (\mathbb{1} + \mathcal{B}L)^T \\ S &= (\mathcal{A}\Sigma_0\mathcal{A}^T + \mathcal{D}\mathcal{D}^T) \\ \Sigma_{uu} &= LSL^T \\ Q &= blkdiag(Q_0, Q_1, \ldots, Q_{N-1}, 0) \\ R &= blkdiag(R_0, R_1, \ldots, R_{N-1}) \end{aligned}$$

III. Implementation

A. Modified Mathematical Formulation

From the above formulation, it is evident that Mean and Covariance steering problems can be independently handled. Okamoto and Tsiotras (2019) demonstrated that the Mean Steering problem can also be derived in a close form (apart from optimization problem formulation as above).

$$\bar{U} = \mathcal{R}^{-1}(\mathcal{B}^T Q \mathcal{A}\mu_0 + \bar{B}_N^T(\bar{B}_N \mathcal{R}^{-1} \bar{B}_N^T)^{-1}(\mu_N - \bar{A}_N\mu_0 - \bar{B}_N\mathcal{R}^{-1}\mathcal{B}^T Q \mathcal{A}\mu_0)) \tag{36}$$

where $\mathcal{R} = \mathcal{B}^T Q \mathcal{B} + R$

For Covariance steering, Rastogi (2021) showed that the optimization problem (for Covariance) can be formulated in the frobenius norm (which is more suitable to feed in the CVX software)

$$\min_{L} \quad J_{cov}(L)$$

$$\text{subject to} \quad E_0(\mathbb{1}+\mathcal{B}L)\Sigma_{xx}(\mathbb{1}+\mathcal{B}L)^T E_0^T = \Sigma_0$$
$$1 - \|\bar{\bar{\Sigma}}(I+BL)^T E_N^T \Sigma_f^{1/2}\| \geq 0$$

where

$$\begin{aligned}
J_{cov}(L) &= \|\bar{\bar{Q}}(\mathbb{1}+BL)\bar{\bar{\Sigma}}\|_F^2 + \|\bar{\bar{R}}L\bar{\bar{\Sigma}}\| \\
\Sigma_{xx} &= V_{xx}\Lambda_{xx}V_{xx}^T \quad \text{(Eigen value decomposition)} \\
Q &= V_Q\Lambda_Q V_Q^T \quad \text{(Eigen value decomposition)} \\
R &= V_R\Lambda_R V_R^T \quad \text{(Eigen value decomposition)} \\
\bar{\bar{\Sigma}} &= V_{xx}\Lambda_{xx}^{1/2} \\
\bar{\bar{Q}}^T &= V_Q\Lambda_Q^{1/2} \\
\bar{\bar{R}}^T &= V_R\Lambda_R^{1/2}
\end{aligned}$$

B. Numerical Example

The Initial mean and covariance is taken as:

$$\mu_0 = \begin{bmatrix} -10 & 1 & 0 & 0 \end{bmatrix}^T \qquad \Sigma_0 = \begin{bmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix} \tag{37}$$

The Final mean and covariance is taken as:

$$\mu_f = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \qquad \Sigma_f = 0.2\begin{bmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix} \tag{38}$$

The transition matrix for state, input and disturbance are assumed as follows:

$$A_k = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B_k = \begin{bmatrix} 0.5\Delta t^2 & 0 \\ 0 & 0.5\Delta t^2 \\ \Delta t^2 & 0 \\ 0 & \Delta t^2 \end{bmatrix} \qquad D_k = \eta I_4 \tag{39}$$

$$Q_k = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0.05 \end{bmatrix} \qquad R_k = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix} \tag{40}$$

C. Results

The following are the results when Mean was steered but the covariance was not steered. Three cases were considered where Number of time steps was varied considering N=20, N=30 and N=50. The red ellipses show the terminal co-variance ellipse constraints.

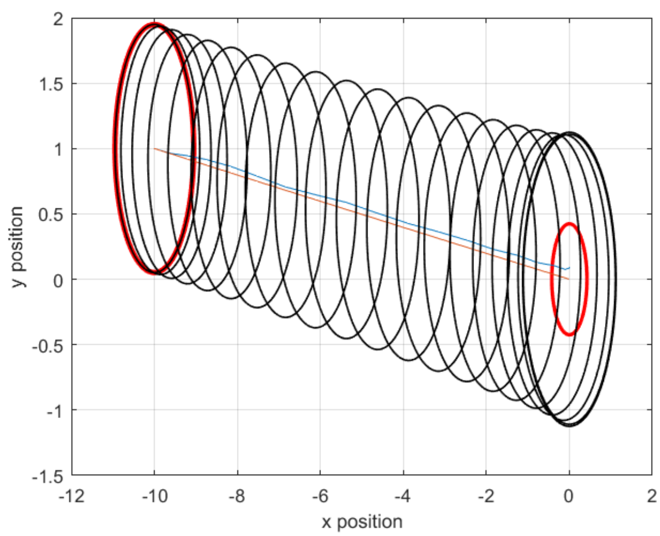

Fig. 1. This shows the result of mean steering (without co-variance steering) for N=20 time steps.

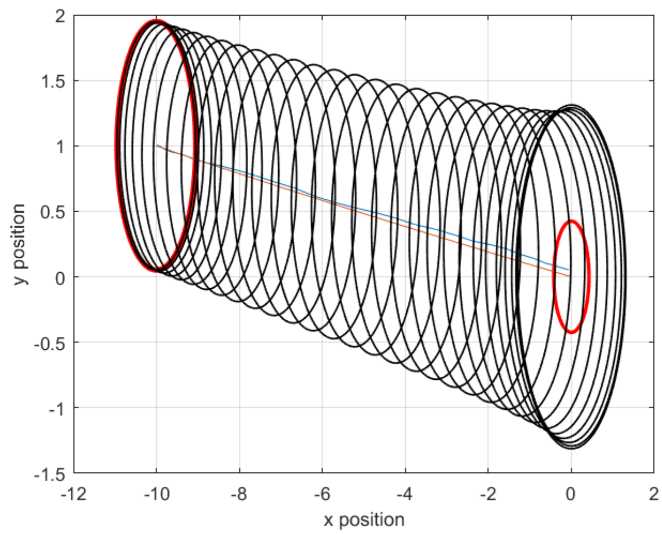

Fig. 2. This shows the result of mean steering (without co-variance steering) for N=30 time steps.

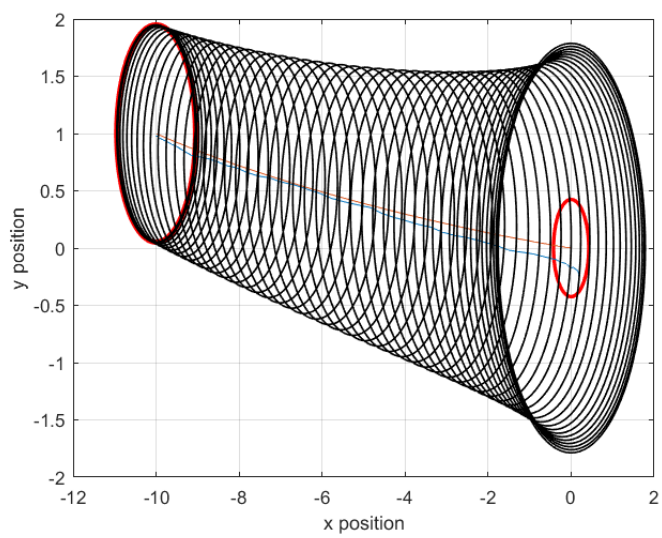

Fig. 3. This shows the result of mean steering (without co-variance steering) for N=50 time steps.

The following are the results when both Mean and Covariance are steered. Once again, three cases were considered where Number of time steps was varied considering N=20, N=30 and N=50.The red ellipses show the terminal co-variance ellipse constraints.

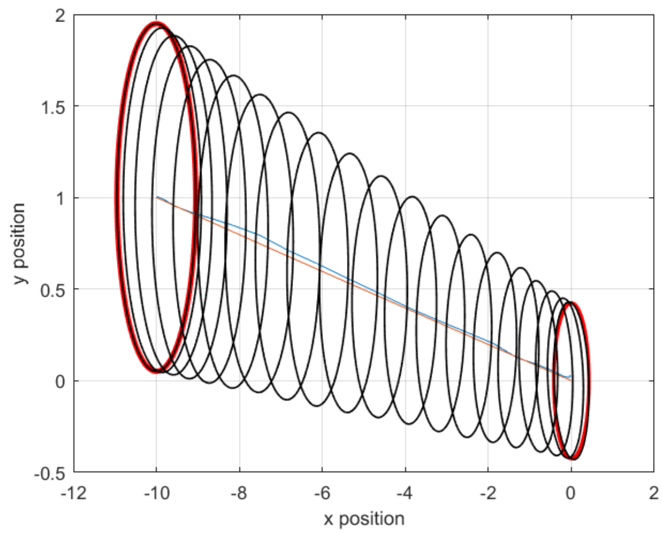

Fig. 4. This shows the result of both mean and co-variance steering for N=20 time steps.

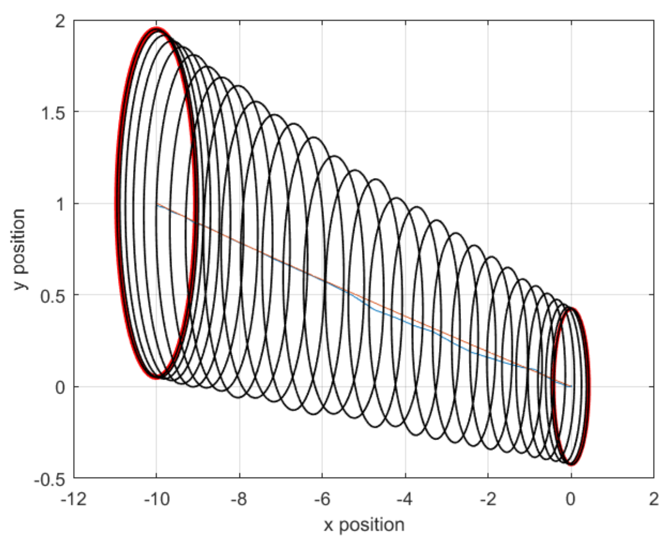

Fig. 5. This shows the result of both mean and co-variance steering for N=30 time steps.

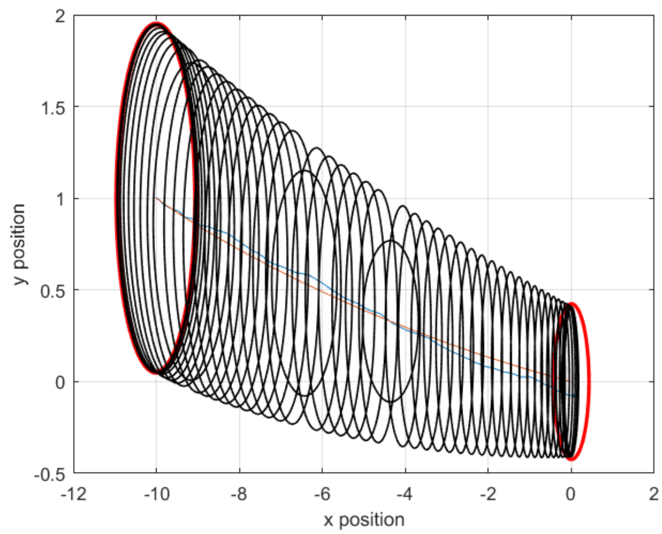

Fig. 6. This shows the result of both mean and co-variance steering for N=50 time steps.

IV. Additional Future Work Covariance Steering with Chance Constraints

In real life, systems would face additional constraints like:

1) Limitations on Controller: For instance, Control $\in [U_{max}, U_{min}]$
2) Path Constraints: We may not want to deviate away from a region along the entire path.
3) Sensor Viewing Constraints: Sensor's field of view may induce path constraints. These path constraints could be polytopes or cones. In chance constraints we deal with polytopes.

In chance constraints, we deal with polytopes. Thus an additional path constraint is introduced such that through out the trajectory the system tends to lie inside the prescribed reqion with a very high probability.

Mathematical interpretation of polytopes and cone:

1) Half space interpretation of Polytopes:

$$\mathcal{X}^P = \bigcap_{j=1}^{Nx} \{x : \alpha_j^T \leq \beta_j\}$$

2) Cones:

$$\mathcal{X}^C = \{x : \|Ax + b\|_2 \leq c^T x + d\}$$

A. Chance Constraints

It is preferred that our system trajectories lie within the costrained region. Since the systems are stochastic enforcing Hard constraint is difficult. Hence the constraints are modelled in probabilistic sense. Constraints implies that the system state lie within the polytope region ($x_k \in \mathcal{X}^P$). This can be modelled in probabilistic sense as:

$$\mathbb{P}[x_k \in \mathcal{X}^P] \geq 1 - \Delta$$

where Δ is very small (close to zero e.g. 0.01) . Such probabilistic constraints are called **Chance Constraints**. The Joint chance constraint (chance constraint satisfied for all time) can thus be represented as

$$\mathbb{P}[\forall_{k=1}^{N} x_k \in \mathcal{X}^P] \geq 1 - \Delta$$

Avoiding the derivation further and reaching to the optimization problem formulation, we get following additional constraints (along with those derived earlier in Mean and Covariance steering):

1) Additional constraint 1:

$$\alpha_j^T E_k(\mathcal{A}\mu_0 + \mathcal{B}V) + \|S^{1/2}(\mathbb{1} + \mathbb{B}L)^T E_k^T \alpha_j\|_2^{-1}(1 - \delta_{jk} \leq \beta_j)$$

2) Additional Constraint 2:

$$\sum_{k=1}^{N} \sum_{j=1}^{Nx} \delta_{jk} \leq \Delta$$

where $k \in [1, N]$ and $j \in [1, N_x]$. L and V are again optimization variables

References

K. Okamoto and P. Tsiotras. Optimal stochastic vehicle path planning using covariance steering. *IEEE Robotics and Automation Letters*, 4(3):2276–2281, 2019. doi: 10.1109/LRA.2019.2901546.

K. Okamoto, M. Goldshtein, and P. Tsiotras. Optimal covariance control for stochastic systems under chance constraints. *IEEE Control Systems Letters*, 2(2):266–271, 2018. doi: 10.1109/LCSYS.2018.2826038.

A. Rastogi. *A Comparative study of LQG and Optimal Covariance Steering Techniques*. M.Tech. thesis, IIT Kanpur, Department of Aerospace Engineering, August 2021.