CS 4820, Spring 2014	**Homework 9, Problem 3**
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3. *(10 points in total)* Show that each of following problems is semidecidable but not decidable. (A problem is called *decidable* if the set of YES instance is recursive.)

3.a. *(5 points)* Given a string of the form $x\#y\#q$, determine if the Turing machine M_x on input y enters state q at some point during its computation.

3.b. *(5 points)* Given a string x, decide if the Turing machine M_x accepts at least one string y that contains a 0 in some position.

Solution

3.a. First we will prove that the problem P is undecidable. We will reduce the Membership Problem to this problem. Since we know that MP is undecidable, hence we would know that P is undecidable

Given a input x#y, we want to find out if y belongs to L(M_x). Let the accept state of M_x be t. Now suppose we have a universal turing machine U which solves the problem P of determining given a string of the form a#b#b, if the turing machine M_a on input b enters state c.
We claim that we can solve this membership problem by running the universal turing machine with input x#y#t.

Proof. Consider a YES instance of the MP. This means that input y belongs to L(M_x). This means if we execute y on M_x, we would eventually reach the state t which is the accept state. Hence this would be a YES instance of U.

Now consider a YES instance of U. This means that the input when run on machine M_x reaches a state t. This means that M_x accepts y. Hence it's an YES instance of the MP.

This proves that the reduction is correct. □

Now we will show that the problem P is semi decidable. We can show this by showing that there exists a TM which accepts all YES instances of the problem P. For a given input x#y#q, construct the turing machine M_x such that it accepts whenever it reaches the state q. We claim this turing machine M_x accepts all YES instances of P. This is because if the input is an YES instance of P, it will reach state q at some point during its computation. And as soon as it reaches the state q M_x would accept it. This proves that there exists such a turing machine (namely M_x) and the problem P is semi decidable.

Hence we have proved that the problem P us semi decidable but not decidable.

3.b. First we will prove that this property is undecidable. We will use Rice's Theorem(proved in class) to show that this property is not trivial. We will show that there exist a r.e. L which contains the property and a r.e. L which does not contain the property.

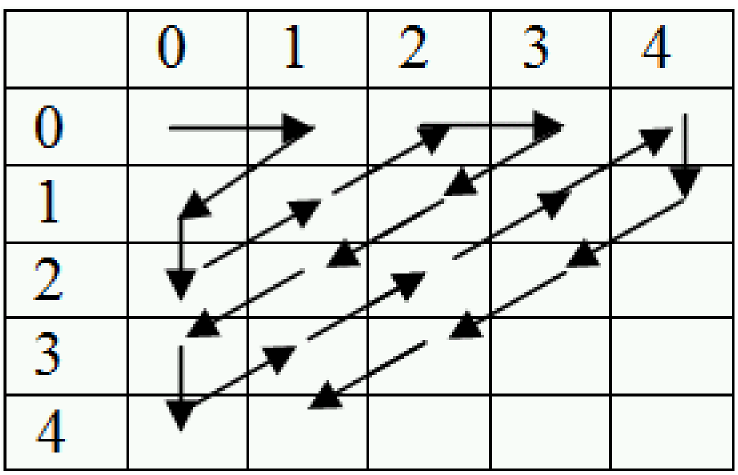
Consider a TM M whose L(M) = {1}. Clearly the set is r.e. Also the set does not contain this property since it does not accept any string y that contains 0 in some position.

Consider a TM M' whose L(M') = {0}. Clearly this set is r.e. The language of this turing machine contains this property since it does accept a string y (0) which contains 0 in some position.

Hence using Rice's Theorem we can say that this property is undecidable.

Now we will show that this property is semidecidable. We will show this by showing that there exists a turing machine M which would accept an YES instance of this problem which means that it would accept an input string which satisfies this property.

Consider a universal turning machine U which when given an input x runs the machine M_x in the following manner : In each iteration it fixes a string y and number of steps t and runs M_x on y for t steps. If it accepts U accepts otherwise U rejects. The order in which it chooses the string y and the number of steps t is defined by this table.



Assume that all different combinations of y run on the horizontal axis and all different values of t runs on the vertical axis. Since for a YES instance x, we know that M_x accepts a y(which contains a 0 at some position) in finite steps, we know that if we can enumerate all possible pairs of (y,t), we will eventually find that particular y and the machine M_x would accept. The reason for choosing the values of t and v in such a manner would ensure we cover all possible combinations of these pairs. In other words we can number these pairs in a definite manner which would allow us to cover all of them (This set of pairs is a infinitely countable set).

So we have shown that there exists a turning machine U which would accept all YES strings which satisfy this property, and hence this property is semi decidable and not decidable (proved above).