CS 4820, Spring 2014	**Homework 9, Problem 1**
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1. *(10 points)* The class **NP** is defined in terms of polynomial-time verifiers and polynomial-length certificates. In this exercise, we will see that semidecidable problems can be characterized in terms of verifiers and certificates of unbounded length. (A problem is called *semidecidable* if the set of YES instance is recursively enumerable.)

Show that a problem A is semidecidable if and only if there exists a Turing machine V that halts on every input and satisfies the following property: a string x is a YES instance of A if and only there exists some string y such that V on input $x\#y$ accepts.

Solution

Given a problem A which is semidecidable we know that there exists a turing machine M which accepts x if x $\in$ A

Construct a turing machine V which is basically the same machine M which when given a input of the form x#y simulates x on M for y steps. If M does not halt within y steps, V just halts and rejects. It accepts if M accepts x within y steps. If the input does not contain a finite y, then the V halts.

We will show below that V halts on every input and satisfies the property mentioned in the question.

Proof of Correctness

Proof. First we have to prove that the machine V that we have constructed halts on every input. We know that the machine V runs for maximum y steps. If M accepts before that, V also accepts. Otherwise V halts and rejects. Also if the value of y is not a finite valid value, then V simply halts. Hence V halts on any input. □

Proof. Now we prove that given a semi decidable problem A, there exists such a turing machine V which satisfies the property that it accepts on input x#y if x is an YES instance of A.
We know the machine M described above exists since A is semi decidable (by definition of semi decidable problem). Since V is a modified version of M, it also exists. Also we know that a YES instance of x is accepted by the machine M in a finite number of steps. Now if we run V on inputs x#y by varying y from 1,2,..∞, we know that we will find a finite value of y on which V accepts and halts. This value would be the number of steps that M takes to accept the YES instance x. Hence if A is semi decidable then turning machine V exists which halts on every input and satisfies the given property.

Now we prove the reverse direction of statement - if such a machine V exists which follows this property, then A is semi decidable. Now we know that if V accepts a input x#y, this means that the string x is an YES instance of A (using the iff condition of this property). We also know that V halted within y number of steps. This means that if there is a YES instance of the problem A, we can construct a TM M (which is basically V), which accepts x in a finite number of steps (which is y). This means that A is semi decidable.

This completes the proof. □