

Modular arithmetic

$A \% B$ = remainder when A is divided by B.

1. $A = q_1.C + r_1 \dots B = q_2.C + r_2$.
2. $(a*b) \bmod C = (q_1.C + r_1)(q_2.C + r_2) \bmod C = (r_1 * r_2) \bmod C = ((a \bmod C) * (b \bmod C)) \% C$.
3. $(a-b) \bmod C = (a \% C - b \% C) \% \bmod C$.
4. $(a+b) \bmod C = (a \% C + b \% C) \% \bmod C$.
5. $(a^b) \bmod C = (a \bmod C)^b \bmod C$. (using binomial theorem).
6. If we have to calculate modulo of a negative number we have to first add it with the no given for modulo no of times so that the no becomes positive and then take modulo.
7. $(a/b) \% C = (a * b^{-1}) \bmod C$
8. $ax \text{ congruent } 1 \bmod C$ (x is modulo inverse). $(3^{-1} \bmod 5 \Rightarrow 3x = 1 \bmod 5 \Rightarrow x = (3^{-1} \bmod 5))$
9. Fermat's little theorem $(a^{-1} \bmod P)$ (P is Prime), then P divides $(a^p - a) \dots X = a^{(P-2)} \bmod P$.
 - $p / (a^p - a)$
 - $p / (a^{(p-1)} - 1) a \rightarrow - p / (a^{(p-1)} - 1)$
 - $a^{(p-1)} - 1 \text{ congruent } 0 \bmod P$
 - $a^{(p-1)} \text{ congruent } 1 \bmod P$ (adding 1 both side)
 - $a * a^{(p-2)} \text{ congruent to } 1 \bmod P$ (ax congruent to 1 mod P)So, $X = a^{(P-2)} \bmod P$.
10. So $(a/b) \bmod P = (a * (b^{(P-2)} \bmod P)) \bmod P$.