# Time Series forecasting

| x | y |
|---|---|
| a | ← p |
| b | ← q |
| c | ← r |
| d | ← s |
| e | ← t |

$y_t$

$y_{t+1}$

| GC | $x_t$ | $y_t$ | |
|---|---|---|---|
| 0 | $a_1$ | $p_1$ | $y_{n-2}$ |
| 0 | $b_1$ | $q_1$ | $y_{n-1}$ |
| 0 | $c_1$ | $r_1$ | $y_n$ |
| 1 | $d_1$ | $s_1$ | |
| 1 | $e_1$ | $t_1$ | |

Date

$p_1 = m a_1 + c$

28/05/2024

$r_1 = m q_1 + n p_1$

* Time dependent.
* Sequence
* Trend, Seasonality
  Level.

* Level :

* Trend :

* Seasonality.

## Exponential Smoothing

1) Single Exp. Smoothing
2) Double Exp Smoothing.
   * Level & Trend.
   Holt's Model
3) Triple Exp Smoothing.
   * Level, Trend, Seasonality
   Holt's Winter Model
4) ARIMA
5) SARIMA
6) SARIMAX ←

* Simple Exponential :

$$f_{t+1} = \alpha y_t + (1-\alpha) f_t$$

$$= \alpha y_t + f_t - \alpha f_t$$

$$= f_t + \alpha (y_t - f_t)$$

$$f_t = f_{t-1} + \alpha (y_{t-1} - f_{t-1})$$

$$f_{t+1} = f_t + \alpha (y_t - (f_{t-1} + \alpha (y_{t-1} - f_{t-1})))$$

$$f_{t+1} = \alpha f_t + \underline{\alpha}(1-\alpha) f_{t-1} + \underline{\alpha (1-\alpha)^2} f_{t-2} \cdots$$

→ Only Level in data

* Trend
* Seasonality

$0 < \alpha < 1$

$f_{t+1}$

$f_t$ $\alpha$ $y_2$

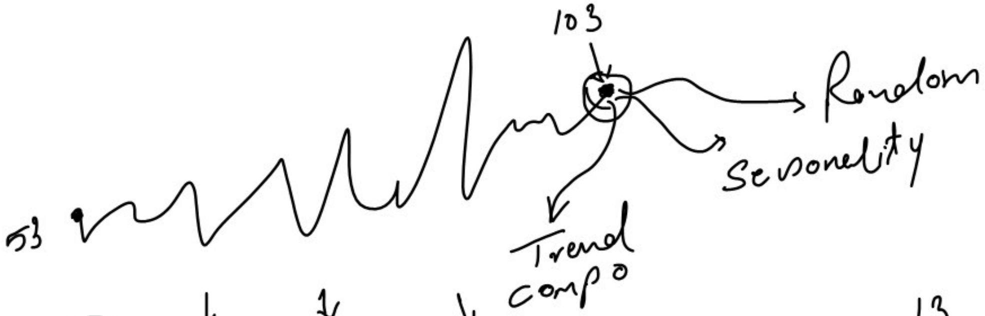
$\alpha(1-\alpha)$ $y_4$

$\alpha(1-\alpha)^2$ $y_8$

$\vdots$

New

# Decomposition.




$$103 = \underbrace{\overset{47}{\text{Trend}}}_{} + \overset{29}{\text{Seaso}} + \overset{27}{\text{Rand}}.$$

**Additive**

$$130 = \overset{13}{\text{Trend}} \times \overset{5}{\text{Seaso}} \times \overset{2}{\text{Rand}}.$$

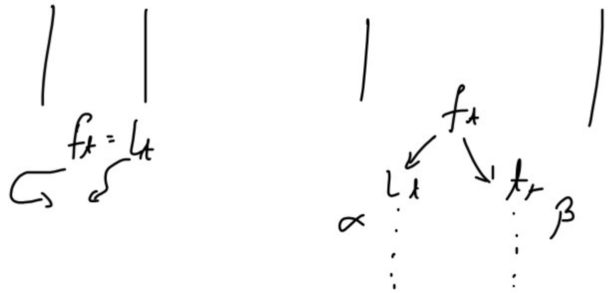
**Multiplicative**

* Double Exp Smoothing

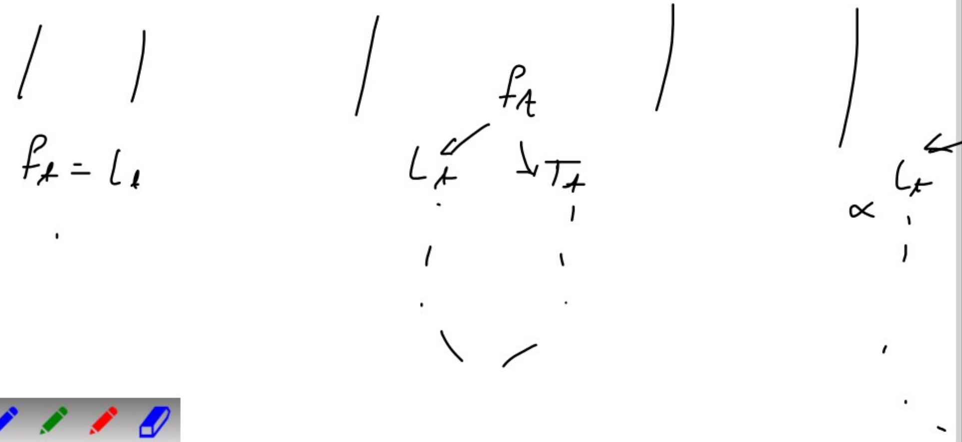
$$f_{t+1} = L_t + T_n$$

$$L_{t} = \alpha y_{t-1} + (1-\alpha) f_{t-1}$$

$$T_t = \beta (L_t - L_{t-1}) + (1-\beta) T_{t-1}$$




* Triple Ex Smoothing



# ARIMA

Auto Regression ($\alpha$)

Moving Average ($q$)

$$Y_t = \phi Y_{t-1} + \varepsilon$$

y
a
b
c
d
e
f
g
h

[Stationarity]

$$Y_2 = \phi Y_1 + \varepsilon$$

$$Y_3 = \phi Y_2 + \varepsilon$$

$$Y_4 = \phi Y_3 + \varepsilon$$

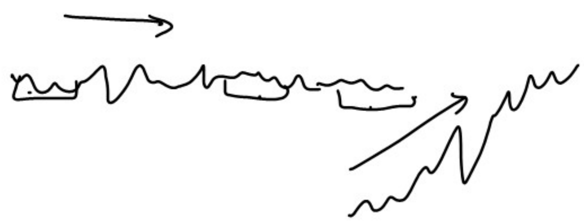
$$\vdots$$

$$Y_{80} = \phi Y_{79} + \varepsilon$$

$$Y_t = \phi Y_{t-1} + \varepsilon$$

$$Y_{16} = \phi Y_{15} + \varepsilon$$

$$Y_{27} = \phi_1 Y_{26} + \phi_2 Y_{25} + \phi_3 Y_{24} + \varepsilon$$



* Most of the Series are non-stationary:

Check for stationarity.

Aug. Dicky fuller Test

Adf test:

$H_0$: Series is non-stationary

$H_1$: Series is stationary

P-value > 0.05

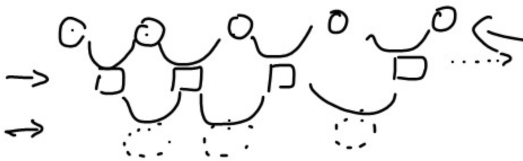
P-value < 0.05

Yes

Continue

No

Differencing.



Yes

Continue

Repeat Differencing

# ARIMA

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \dots + \phi_p y_{t-p} + \varepsilon$$

$p \rightarrow$ Defines how many previous term the current term depends upon.

$p = 1$ : $y_t = \phi y_{t-1} + \varepsilon$

$p = 2$ : $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon$

$p = 3$ : $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon$

$\vdots$

AR I MA

$$y_t = (\phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p}) + (m_1 \varepsilon_{t-1} + m_2 \varepsilon_{t-2} + m_3 \varepsilon_{t-3} + \dots m_q \varepsilon_{t-q})$$

$q \rightarrow$ Defines how many previous error terms must the current Error depends upon.

$q_a = 1$ : $y_t = AR + m_1 \varepsilon_{t-1}$

$q = 2$ : $y_t = AR + m_1 \varepsilon_{t-1} + m_2 \varepsilon_{t-2}$

$\vdots$

Differencing (d)

$\rightarrow d = 0$ : means no differencing (Already stationary)

$\rightarrow d = 1$ : means 1 term diff to make series stationary.

1) Check for stationarity : →

'd'

2) Consider PACF

'p'

ARIMA(p, d, q)

$\begin{array}{ccc} p & d & q \\ \end{array}$

↳ ARIMA(2, 1, 1)

3) Consider ACF

'q'

SARIMA

Seasonality

Auto Regression

Moving average

SARIMA (p, d, q) × (P, D, Q, S)

12

d = 0

p = 2 ; q = 1

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y1 → | 23 | 46 | 44 | 52 | 66 | 33 | 48 | 59 | 60 | 63 | 66 | 70 |
| Y2 → | 26 | 34 | 54 | 49 | 63 | 37 | 45 | 56 | 63 | 66 | 68 | 72 |
| Y3 → | 24 | 43 | 48 | 54 | 68 | 35 | 46 | 56 | 62 | 64 | 67 | 71 |

36

A24

Jl 24

June 24

Au 2023

Au 2022

P = 2

P = 2

P

D = 0

Q

p = 2 P = 2

d = 0 D = 0

q = 1 Q = 1

| Time | Demand | No. of Computers | Level avrov. | Populat. | Growth nom |
|---|---|---|---|---|---|

Exogenous Variable

SARIMAX