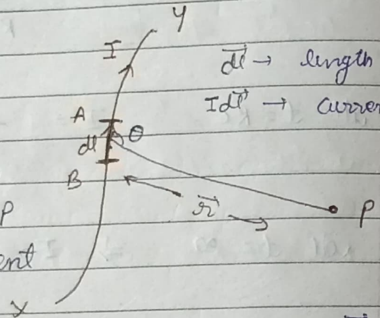
MAGNETOSTATICS

* BioT - Severts law

This law deals with the magnetic field induction at a point due to a small current element.

Let us consider a small element AB of length dl of the conducting wire XY carrying current I.

~~let us consider~~



$\vec{dl} \rightarrow$ length element

$I\vec{dl} \rightarrow$ current " "

Let $\vec{r}$ be the position vector of the point P from the current element $I\vec{dl}$ and θ be the angle between $\vec{dl}$ & $\vec{r}$

⇒ Acc. to BioT severts law, the magnitude of magnetic field induction dB at point P due to small current element depends upon following factors.

(i) $\vec{dB} \propto I$ (ii) $\vec{dB} \propto \vec{dl}$

(iii) $\vec{dB} \propto \frac{1}{r^2}$ (iv) $\vec{dB} \propto \sin\theta$.

Combining (i) (iii) (ii) (iv)

$$dB \propto \frac{I\,dl\,\sin\theta}{r^2} \Rightarrow dB = \frac{K\,I\,dl\,\sin\theta}{r^2}$$

$K \rightarrow$ Constant of proportionality

In SI unit $K = \frac{\mu_0}{4\pi} = 10^{-7}\,T\,m\,A^{-1}$

$\mu_0 \rightarrow$ magnetic permeability of free space

$$\boxed{\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2}}$$

$$\boxed{\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,(\vec{dl} \times \vec{r})}{r^3}} \rightarrow \text{Vector form.}$$

Ampere's Circuital law.

Ampere's circuital law in magnetostatics is analogous to gauss's law in electrostatics.

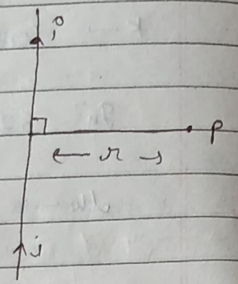
It states that " The line integral of magnetic field around a closed loop is equal to μ_0 times the current passing through the loop.

$$\boxed{\oint B \cdot dl = \mu_0\, i_{inside}}$$

Date ____
Page ____

Proof

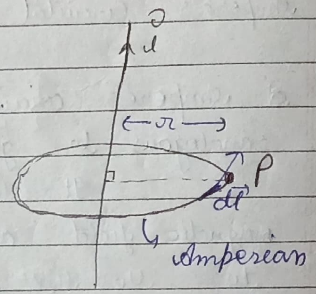
Let i be the current flowing through a infinitimly long straight conductor.



Acc. to BIOT Savart's law, the magnitude of magnetic field at any point P at a perpendicular distance r from the conductor is given by

$$B = \frac{\mu_0}{4\pi} \frac{2i}{r} \quad \text{— (1)}$$

Now consider a loop (circular) of radius r with center is wire and passes through point P.



$$\oint B \cdot dl = \oint B\,dl \cos 0 = \oint B \cdot dl$$

$$= B \oint \vec{dl}$$

$$\oint \vec{B} \cdot \vec{dl} = B\, 2\pi r \quad \text{—(2)}$$

using (1) & (2)

$$\oint \vec{B} \cdot \vec{dl} = \frac{\mu_0}{4\pi} \frac{2i}{r} (2\pi r)$$

$$\oint \vec{B} \cdot \vec{dl} = \frac{\cancel{\mu_0}}{\cancel{4\pi}} \quad \mu_0 i$$

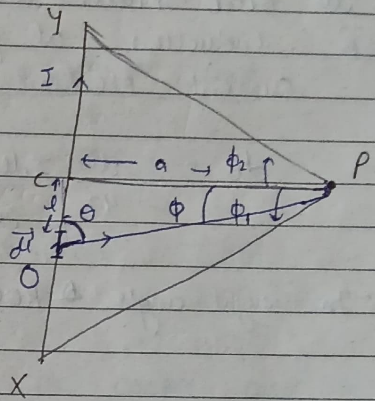
Date ____
Page ____

$$\boxed{\oint B \cdot dl = \mu_0 i}$$

Magnetic field due to a current carrying wire

Consider a straight wire conductor XY carrying current I in the direction X to Y. Let 'P' be a point at perpendicular direction from the straight wire conductor clearly $PC = a$.

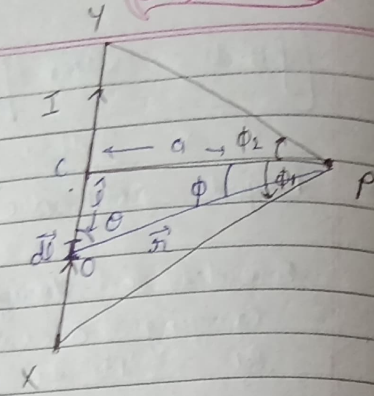
Let the conductor is made up of large no. of small current element



Consider one such small current element $I\vec{dl}$ of the straight wire.

Let $\vec{r}$ be the position vector of point P w.r.t small current element Idl and 'θ' be ~~the~~ the small angle btw $I\vec{dl}$ & $\vec{r}$

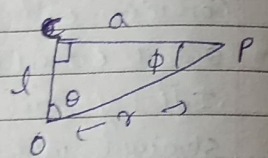
Let $CO = l$



Acc to Biot Savarts law the magnitude of $\vec{B}$. induction at point P due to small current element $Id\vec{l}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2} \quad \text{--- (i)}$$

In right angle $\triangle$ POC



$$90+\theta+\phi = 180$$
$$\theta = 90-\phi.$$

So $$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin(90-\phi)}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl\cos\phi}{r^2}$$

and $$\cos\phi = \frac{a}{r}$$

$$r = \frac{a}{\cos\phi} \quad \text{--- (ii)}$$

also, $$\tan\phi = \frac{l}{a}.$$

$$l = a\tan\phi.$$

differentiate both side.

$$dl = a\sec^2\phi \cdot d\phi$$

$$\boxed{d\vec{l} = a\sec^2\phi \cdot d\phi} \quad \text{--- (iii)}$$

Put (ii) & (iii) in eq (i)

$$d\vec{B} = \frac{\mu_0}{4\pi}\, I\, \frac{(a\sec^2\phi\, d\phi)\cos\phi}{\left(\frac{a}{\cos\phi}\right)^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a}\cos\phi\, d\phi. \quad \text{--- (iv)}$$

Magnetic field due to complete wire $\Rightarrow$ Integral eq (iv) from limit $\boxed{-\phi_1 \text{ to } +\phi_2}$

$$B = \int d\vec{B} = \int_{-\phi_1}^{\phi_2} \frac{\mu_0}{4\pi}\frac{I}{a}\cos\phi\, d\phi$$

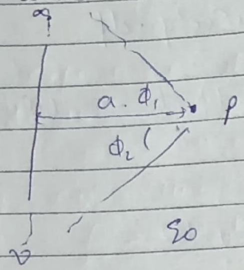
$$= \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin\phi\right]_{-\phi_1}^{\phi_2}$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin\phi_2 + \sin\phi_1\right]}$$

Scanned with OKEN Scanner

* $B = \frac{\mu_0}{4\pi} \frac{I}{a} [\sin\phi_1 + \sin\phi_2]$

* if wire ~~are~~ is infinite

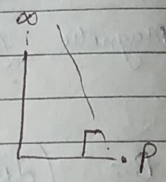


then ϕ_1 & ϕ_2 are nearly equals to $90°$.

$$\text{So } B_\infty = \frac{\mu_0}{4\pi} \frac{I}{a} [\sin 90 + \sin 90]$$

$$\boxed{B_\infty = \frac{\mu_0}{4\pi} \frac{2I}{a}}$$

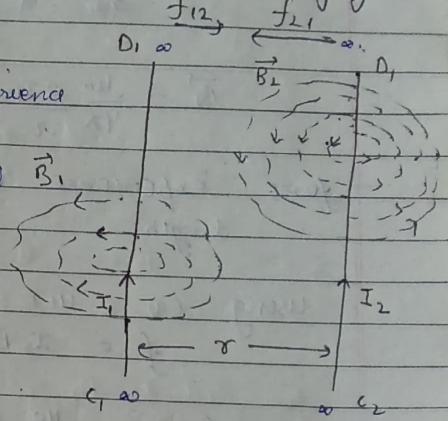
Case (II) when the straight wire is ∞. but the point is near one of the end.



$$B = \frac{\mu_0}{4\pi} \frac{I}{a} [\sin 90 + \sin 0]$$

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{I}{a}}$$

force between two parallel conductor carrying current.



- The force ~~experiment~~ experience by a conductor of length 'l' carrying current $\vec{B}_1$ 'I' place in external magnetic field $\vec{B}$ is given as:

$$\vec{F} = IBl\sin\theta \quad \text{— (i)}$$

the force experience by unit length of the conductor carrying current I place in external field is given as:

$$\frac{\vec{F}}{l} = f = IB\sin\theta \quad \text{— (ii)}$$

Consider C_1D_1 and C_2D_2 are two infinite long straight conductors carrying current I_1 & I_2 resp. They are held ∥ to each other at a dist. of 'r' apart.

Since each conductor is in the region of magnetic field due to other conductor so it experience a force.

for wire C_1D_1 the magnetic field at a dist r from is given by

$$\vec{B_1} = \frac{\mu_0}{4\pi} \frac{2I}{a} = \frac{\mu_0}{4\pi} \frac{2I_1}{r}$$

$$\vec{B_1} = \frac{\mu_0}{4\pi} \frac{2I_1}{r}$$

force experience by wire C_2D_2 per unit length

using eq (11)

$$f = I_2 B_1 \sin 90^\circ = I_2 \times \frac{\mu_0}{4\pi} \frac{2I_1}{r}$$

$$f_{21} = \frac{\mu_0}{4\pi} \frac{2I_1I_2}{r} \quad (\text{towards wire } 1)$$

the ~~g~~ direction of force is given by right hand palm rule which closer the another wire

• For wire C_2D_2

$$\vec{B_2} = \frac{\mu_0}{4\pi} \frac{2I_2}{r}.$$

$$f_{12} = \vec{B_2} I_1 \sin 90 = \frac{\mu_0}{4\pi} \frac{2I_1I_2}{r} \quad \left[\begin{array}{l}\text{towards} \\ \text{wire } 2\end{array}\right]$$

\# If ~~the~~ both wires carrying current in same direction they attracted each other

\# when the currents in C_1D_1 & C_2D_2 are in opposite directions then they will repell each other.

Definition of 1 ampere.

let $I_1 = I_2 = 1$ ampere & $r = 1\,m$.

$$\vec{F} = \cancel{\ } \quad \vec{f} = \frac{\mu_0}{4\pi} \frac{2I_1I_2}{r} = \frac{\mu_0}{4\pi} 2$$

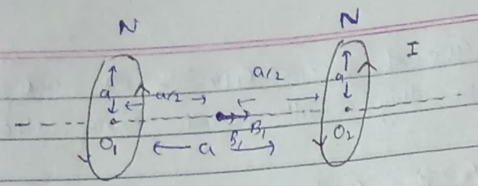
$$\vec{f} = 2 \times 10^{-7} \; N/m.$$

Thus one ampere is that much current which were flowing through each of two parallel infinitely long conductors placed in free space at a ~~st~~ separation of 1 meter from each other will attract or repel each other by a force of 2×10^{-7} N/m

\# Helmholtz Coils

A helmholtz coil is a device consisting of two identical parallel circular coils of wire, spaced apart by a distance equal to its radius that when carrying equal current in the same direction, creates a region of neary uniform magnetic field b/w them

$\frac{\mu_0 N I a^2}{2(a^2+a/2)^{3/2}}$



→ Magnetic field on the axis of a current carrying loop is given by

$$\boxed{B = \frac{\mu_0 N I a^2}{2(a^2+x^2)^{3/2}}}$$

for coil 1 $B_1 = \frac{\mu_0 N I a^2}{2\left(a^2+\frac{a^2}{4}\right)^{3/2}}$

$$= \frac{\mu_0 N I a^2}{2\left(\frac{5a^2}{4}\right)^{3/2}}$$

$$= \frac{4\mu_0 N I a^2}{5^{3/2} a^3} = \frac{4}{5\sqrt{5}}\frac{\mu_0 N I}{a}$$

for coil 2 $B_2 = \frac{4}{5\sqrt{5}}\frac{\mu_0 N I}{a}$

the field at center by the both coils are in same direction

So net magnetic field

$$B = B_1 + B_2$$

$$B = \frac{8}{5\sqrt{5}}\frac{\mu_0 N I}{a}$$

$$\boxed{B = \frac{8}{5\sqrt{5}}\frac{\mu_0 N I}{a}} = 0.716\frac{\mu_0 N I}{a}$$

B at center of a coil $B_c = \frac{\mu_0 N I}{2a}$

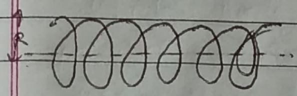
$$\frac{B}{B_c} = \frac{0.716}{1/2} = 1.432$$

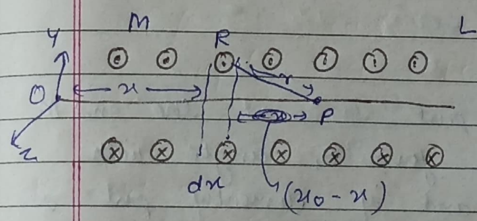
$$\boxed{B = 1.432\, B_c}$$

Magnetic Field due to a solenoid

n: Turns per unit length

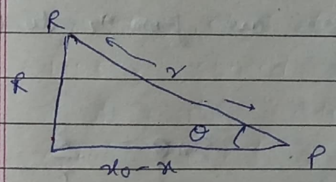
R: Radius of each turn





Let consider an elementary part of the solenoid of length dx at a distance of x from O

$OP = x_0$ $RP = r$

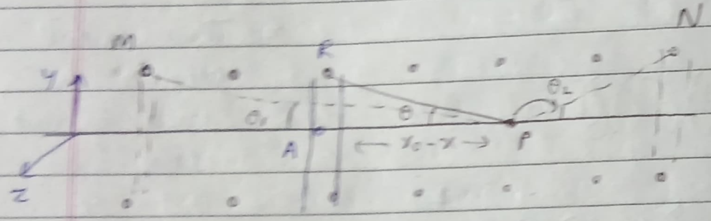


Magnetic field due to a ring on its axis is given by

$$\vec{B} = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$

$$\Rightarrow dB = \frac{\mu_0 I R^2}{2[R^2+(x_0-x)^2]^{3/2}}$$

$$d\vec{B} = \frac{\mu_0 I R^2}{2r^3} \quad \text{— (i)}$$



If the elementary part is at the end M then it will form θ_1 angle with observation point and if elementary part is at end N then it will form angle θ_2.

So limits of integration are from θ_1 to θ_2

From $\triangle APR$ =

$$\sin\theta = \frac{R}{r}$$

$$\boxed{r = R\,\mathrm{Cosec}\,\theta}$$

~~Now~~ ~~replace~~ ~~r in~~

$n \rightarrow$ no. of turn per unit length

Total no. of turns in dx length $= n\,dx$.

$$d\vec{B} = \frac{\mu_0 I R^2}{2r^3} \cdot n\,dx$$

$\cancel{r = R\,\mathrm{Cosec}\,\theta}$

$\cancel{dr = -R\,\mathrm{Cosec}\,\theta\cot\theta\, d\theta}$

$$\tan\theta = \frac{R}{x_0 - x} \qquad x_0 - x = R\cot\theta$$

differentiate $\Rightarrow$ $\boxed{+dx = R + \mathrm{Cosec}^2\theta \cdot d\theta}$

$$\text{So } d\vec{B} = \frac{\mu_0 I R^2}{2R^3\,\mathrm{Cosec}^3\theta} \cdot n \cdot R\,\mathrm{Cosec}^2\theta \cdot d\theta$$

$$d\vec{B} = \frac{\mu_0 I R^3\, n \sin\theta \cdot d\theta}{2R^3}$$

$$d\vec{B} = \frac{\mu_0 I n \sin\theta \cdot d\theta}{2}$$

Magnetic field due to whole solenoid

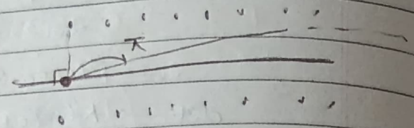
$$\int d\vec{B} = \frac{\mu_0 I n}{2}\int_{\theta_1}^{\theta_2} \sin\theta \cdot d\theta$$

$$\vec{B} = \frac{\mu_0 I n}{2}\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\boxed{\vec{B} = \frac{\mu_0 n I}{2}\left[\cos\theta_1 - \cos\theta_2\right]}$$

Case 1 :- if point lies on the one end of solenoid and solenoid is ∞ long.

$\theta_1 \to 90 \quad \theta_2 \to \pi$.



$$\vec{B} = \frac{\mu_0 n I}{2}(+1) = \frac{\mu_0 n I}{2}.$$

if point on other end.

$\theta_1 \longrightarrow 0 \qquad \theta_2 \longrightarrow 90$.

$$\vec{B} = \frac{\mu_0 n I}{2}.$$

Case 2 point at the middle (center)

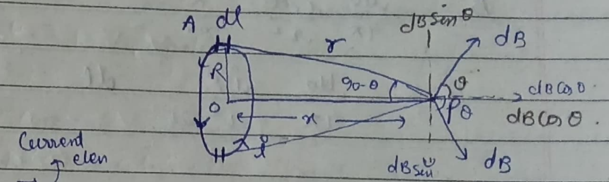
$\theta_1 \longrightarrow 0 \qquad \theta_2 \longrightarrow \pi$.

$$B = \frac{\mu_0 n I}{2}\,2 = \mu_0 n I$$

$$\boxed{B = \mu_0 n I}$$

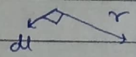
Magnetic field intensity at a point on the axis of a current carrying ring

Consider a Ring of radius R carrying a current i.



Consider a $\vec{dl}$ elementer which is r distance from point P

angle btw dl and r is 90°.



The dB sinθ component cancelled each other thus the net magnetic field is due to dBcosθ component

$$\vec{B} = dB\cos\theta. \quad \text{—— (i)}$$

using BiOT severts law

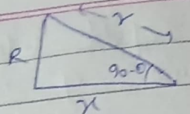
$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\,dl}{r^2}$$

Put in eq (i),

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,dl}{r^2}.\cos\theta$$

from $\triangle AOP$



$$\sin 90-\theta = \frac{R}{r}$$

$$B = \frac{\mu_0}{4\pi}\frac{i\,dl}{r^2}\,\frac{R}{r}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{iR}{r^2 \cdot r}\,dl \quad \text{—(ii)}$$

net magnetic field integrate eq (ii)

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{iR}{r^3}\int dl = \frac{\mu_0 i R\, 2\pi R}{4\pi\, r^3}$$

$$\boxed{\vec{B} = \frac{\mu_0}{2}\frac{iR^2}{(R^2+x^2)^{3/2}}}$$

$\vec{B}$ at centre

$$\vec{B}_c = \frac{\mu_0}{2}\frac{iR^2}{(R^2)^{3/2}} = \frac{\mu_0 i}{2R}$$

⇒ Magnetic Vector Potential

we know that

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{—(i)}$$

Also by definition divergence of curl of vector $\vec{A}$ is zero.

div (curl $\vec{A}$) = 0 ⇒ $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$ —(ii)

equating (i) & (ii).

$\vec{B} = \vec{\nabla}\times\vec{A}$ here $\vec{A}$ ~~represent~~ is called Magnetic vector potential

Thus magnetic vector potential $\vec{A}$ is defined in such a way that it's curl gives the magnetic flux density.

So,

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

Also we know that $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \mu_0\vec{J}$$

$$(\vec{\nabla}\cdot\vec{A})\vec{\nabla} - (\vec{\nabla}\cdot\vec{\nabla})\vec{A} = \mu_0\vec{J}$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0\vec{J} \quad \text{—— (iii)}$$

If we choose $\vec{A}$ in such a way that $\vec{A}$ is of solenoidal nature i.e $\vec{\nabla}\cdot\vec{A} = 0$.

then eq (iii)

$$-\nabla^2\vec{A} = \mu_0\vec{J}$$

$$\boxed{\nabla^2\vec{A} = -\mu_0\vec{J}} \quad \text{—— (iv)}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

* In electrostatics the poisson's equation for electrostatic potential is -

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

$$\text{and } V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}.$$

→ Similarly in magnetostatic the poisson's eq. for magnetostatic potential.

$$\nabla^2 \vec{A} = -\mu_0 \vec{J} \longrightarrow \text{Current density}$$

$$\nabla^2 [A_x \hat{i} + A_y \hat{j} + A_z \hat{k}] = -\mu_0 [J_x \hat{i} + J_y \hat{j} + J_z \hat{k}]$$

$$\nabla^2 A_x = -\mu_0 \vec{J_x} \quad \text{— (VI)}$$

$$\nabla^2 A_y = -\mu_0 \vec{J_y} \quad \text{— (VII)}$$

$$\nabla^2 A_z = -\mu_0 \vec{J_z} \quad \text{— (VIII)}$$

→ $\nabla^2 A_x = -\mu_0 \vec{J_x}$

$$\boxed{A_x = \frac{\mu_0}{4\pi} \int_V \frac{J_x \, dv}{r}}$$

from $V = \frac{q}{4\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0} \frac{\int_V \rho \, dv}{r}$

Similarly $A_y = \frac{\mu_0}{4\pi} \int_V \frac{J_y \, dv}{r}$

$$A_z = \frac{\mu_0}{4\pi} \int_V \frac{J_z \, dv}{r}$$

in general $\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J} \cdot dv}{r}}$

\# $\phi_B = \int_S \vec{B} \cdot d\vec{S}$

$\phi_B = \int_S \vec{\nabla} \times \vec{A} \cdot ds \qquad \because \vec{B} = \vec{\nabla} \times \vec{A}$

using stokes theor $\int_S (\nabla \times \vec{A}) \cdot ds = \oint \vec{A} \cdot dl$

$$\phi_B = \oint_l \vec{A} \cdot \vec{dl}$$

$$\boxed{\vec{A} = \frac{d\phi_B}{dl}} \text{ weber}$$

SI unit → weber/meter.

* Divergence of magnetic field :-

acc. to BioT severts law

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin\theta}{r^3} \vec{r}$$

$$\vec{dB} = \frac{\mu_0 I}{4\pi} dl \times \frac{\vec{r}}{r^3} = -\frac{\mu_0 I}{4\pi} \vec{dl} \times$$