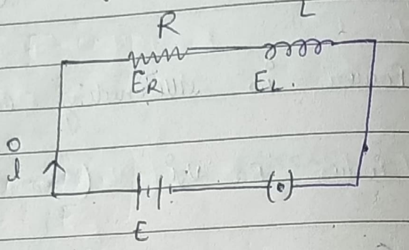
STEADY & VARYING CURRENT

growth and decay of current through R-L circuit.



Potential drop at Resistance

$$E_R = iR.$$

Potential drop at inductor by Faradays law.

$$E_L = L\frac{di}{dt}$$

$$\boxed{E_L = L\frac{di}{dt}}$$

. total potential drop in circuit

= Sum of Potential drop in R & L.

$$E = E_R + E_L.$$

$$E = iR + L\frac{di}{dt}.$$

$$E = iR + L\frac{di}{di}.$$

$$E - iR = L\frac{di}{dt}$$

$$\frac{(E - iR)}{L} = \frac{di}{dt}.$$

$$\frac{dt}{L} = \frac{di}{E - iR}$$

let $E - iR = t$.

$$-R\,di = dt$$

$$\boxed{di = \frac{-dt}{R}}$$

$\int \frac{dt}{L}$ integrate both side

$$\int \frac{dt}{L} = \int \frac{-dt}{R \cdot t} = \frac{-1}{R}\log t + C.$$

$$\frac{t}{L} = \frac{-1}{R}\log(E - iR) + C. \quad \text{— (i)}$$

at $t = 0$ current $i = 0$

$$C = \frac{1}{R}\log(E)$$

$$\boxed{C = \frac{1}{R}\log E} \quad \text{— (ii)}$$

Put (ii) in (i)

$$\boxed{\frac{1}{e} = 0.368}$$

$$\frac{t}{L} = \frac{\log(E - iR)}{-R} + \frac{1}{R}\log E.$$

$$\frac{t}{L} = \frac{1}{R}\log\left(\frac{E}{E - iR}\right) \qquad \because \log a - \log b = \log\frac{a}{b}$$

$$\frac{t}{L} = \frac{-1}{R}\log\left[\frac{E - iR}{E}\right].$$

$$\frac{-Rt}{L} = \log\left[1 - \frac{iR}{E}\right]$$

$$1 - \frac{iR}{E} = e^{-\frac{Rt}{L}}$$

$$\frac{iR}{E} = 1 - e^{-\frac{Rt}{L}}$$

$$i = \frac{E}{R}\left[1 - e^{-\frac{Rt}{L}}\right]$$

$\frac{E}{R}$ = maximum current $= i_0$ initial current

$$i = i_0\left(1 - e^{-R\frac{t}{L}}\right)$$

$\frac{L}{R} = \gamma$ time constant

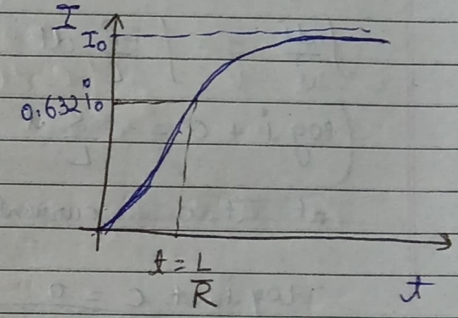
$$\boxed{\frac{L}{R} = t}$$

$$i = i_0\left[1 - e^{-t/t}\right].$$

$$i = i_0\left[1 - \frac{1}{e}\right] = i_0\left[1 - \frac{1}{2.718}\right].$$

$$i = i_0(0.632)$$

$t = t$ $\boxed{i = 0.632\, i_0}$

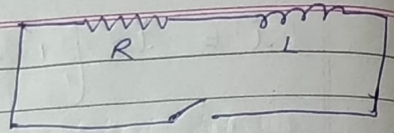


Decay of current :

when the key is pressed for $t \gg \frac{L}{R}$ current attains its maximum value I_0. On releasing key, current in the circuit is decreases from i_0 to 0. in this case external e.m.f $E = 0$.

let i be the value of current at any instant of time 't'.

$$E = V_L + V_R.$$



$E_R = -E_L$

$iR = -L\frac{di}{dt}$

$\frac{di}{i} = -\frac{R}{L}\,dt$

integrate both side

$\int \frac{di}{i} = \int \frac{-R}{L}\,dt$

$\log i + C = \frac{-R}{L}t$ —— (i)

at $t = 0$ current $i = i_0$ (max).

$\log i_0 + C = 0$

$\boxed{C = -\log i_0}$

put value of integration constant in eq(i)

$\log i - \log i_0 = -\frac{Rt}{L}$

$\log \frac{i}{i_0} = \frac{-Rt}{L}$

$\frac{i}{i_0} = e^{-\frac{R}{L}t}$

$i = i_0 e^{-\frac{R}{L}t}$

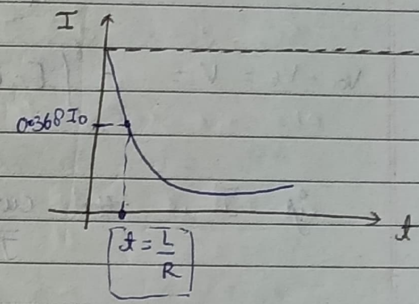
$\boxed{I = I_0 e^{-\frac{R}{L}t}}$

Here $\frac{L}{R}$ is called inductive time constant

when $\frac{L}{R} = t$

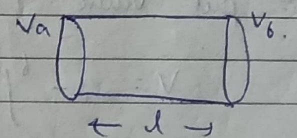
$I = I_0 e^{-t/t} = \frac{I_0}{e}$

$\boxed{I = 0.368\, I_0}$

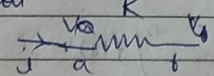


Resistance of a cylindrical conductor and specific resistance.

Consider a cylindre of length l let V_a and V_b be the potential diff across the ends of the conductor.



we know that



Resistance is given by $\boxed{R = \frac{V_b - V_a}{i}}$

$$R_{ab} = \frac{V_b - V_a}{i}$$

also we know the relation between potential diff and electric field

$$\boxed{dV = -E \cdot dr}$$

So $V_a - V_b = -\int_a^b E \cdot dl$. (for cylinder)

$$V_a - V_b = V = -\int_a^b E \cdot dl \quad \text{--- (1)}$$

if $\vec{J}$ is the current density than

$$\vec{J} = \sigma \vec{E}$$

wher $\sigma \rightarrow$ electrical conductivity

$$E = \frac{\vec{J}}{\sigma} = \frac{i}{A\sigma} \qquad \therefore \vec{J} = \frac{i}{A}$$

$$\boxed{\vec{E} = \frac{i}{A\sigma}}$$

V= put value of $\bar{E}$ in eq (1)

$$V = -\int_a^b \frac{i}{A\sigma} \cdot dl$$

$$\therefore \int_a^b f(x) = -\int_b^a f(x)$$

$$V = \int_b^a \frac{i}{\pi r^2 \sigma} \cdot dl$$

$$V = \frac{i}{\pi r^2 \sigma} \int dl = \frac{i\,l}{\pi r^2 \sigma}$$

$$\frac{V}{i} = \frac{l}{\pi r^2 \sigma}$$

we know $\frac{V}{i} = R = \frac{l}{A\sigma} \Rightarrow \frac{1}{\sigma} = \frac{RA}{l}$

$$\frac{1}{\text{Conductivity}} = \text{resistivity} = \rho = \frac{RA}{l}$$

$$\boxed{\rho = \frac{RA}{l}} \qquad \boxed{R = \frac{\rho l}{A}}$$

Weidmann's Frenz law

The ratio of Thermal conductivity (K) to the electrical conductivity is directly proportional to the temprature.

$$\frac{K}{\sigma} \propto T$$

$$\frac{K}{\sigma} \propto T \qquad \boxed{\frac{K}{\sigma} = LT}$$

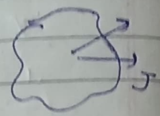
Lorentz number

Equation of continuity

we know that the electrical charge is indestructible, it is never lost or created that is the charge is conserved

we know current density (J)

$$J = \frac{i}{A}$$



So ~~$i \cdot Q$~~ $i = J \cdot A$.

also we can write $i = \int_S J \cdot ds$ —(I)

current $i = -\frac{dQ}{dt}$

$\rho = \frac{Q}{V}$

and ~~charge (Q) =~~

~~$d\sigma$~~ electrical charge density $(\rho) = \frac{Q}{V}$

$$Q = \rho V$$

$$\boxed{Q = \int_{dv} \rho \, dv} \text{ — (II)}$$

So $i = -\frac{d}{dt} \int_{dv} \rho \, dv$ — (III)

~~ex~~ Compare (III) with (I)

$$\int_S \vec{J} \cdot ds = -\frac{d}{dt} \int_{dv} \rho \, dv.$$

$$\int_S \vec{J} \cdot ds = -\int \frac{\partial}{\partial t} \rho \, dv. \text{ — (IV)}$$

using gauss divergence theorem.

$$\int_S J \cdot ds = \int_V \vec{\nabla} \cdot \vec{J} \, dv$$

So eq (IV) can be written as.

$$\int_V \vec{\nabla} \cdot \vec{J} \, dv = -\int \frac{\partial}{\partial t} \rho \, dv.$$

$$\int_V \left(\vec{\nabla} \cdot \vec{J} + \frac{\partial}{\partial t} \rho \, \cancel{dv} \right) dv = 0.$$

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial}{\partial t} \rho \, \cancel{dv} = 0$$

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial}{\partial t} \rho \, \cancel{dv}} \rightarrow \text{~~con~~ equation of continuity}$$

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$

~~# Stead~~ This is an imp equation regarding "charge flow called continuity equation and the fact the charge is conserved.

A ~~stead~~ stedy state in which all time dereviatives are 0 so we must have.

$$\frac{\partial \rho}{\partial t} = 0 \quad \text{and} \quad \vec{\nabla}\cdot\vec{J} = 0.$$

$$\int J\cdot ds = 0$$

$$\vec{\nabla}\cdot\vec{J} = \vec{\nabla}\cdot\sigma\vec{E} = \sigma\vec{\nabla}\cdot\vec{E} = 0.$$

So for a Steady state we conclude the E has 0 divergence.

$$\vec{E} = -\nabla\cdot \underset{\text{Potential}}{V}$$

$$\sigma\vec{\nabla}\cdot\nabla V = \sigma\nabla^2 V = 0$$

$$\boxed{\vec{\nabla}^2 V = 0} \rightarrow \text{laplace equation}$$

Relexation time

equation of continuity

$$\text{div } J + \frac{\partial \rho}{\partial t} = 0$$

$$\vec{\nabla}\cdot\vec{J} + \frac{\delta\rho}{\delta t} = 0.$$

where $\vec{J} \rightarrow$ current density

$\rho \rightarrow$ charge density

vector form of ohm's law

$$\vec{J} = \sigma\vec{E}$$

$$\vec{\nabla}\cdot(\sigma\vec{E}) + \frac{\partial\rho}{\delta t} = 0$$

$$\sigma(\vec{\nabla}\cdot\vec{E}) + \frac{\delta\rho}{\delta t} = 0$$

σ → Conductivity of conductor

$$\boxed{\text{div } E = \frac{\rho}{\varepsilon_0}} \rightarrow \text{differential form of ohm's law}$$

$$\sigma(\vec{\nabla}\cdot\vec{E}) + \frac{\delta\rho}{\delta t} = 0$$

$$\sigma\frac{\rho}{\varepsilon_0} + \frac{\delta\rho}{\delta t} = 0$$

$$\frac{\delta\rho}{\delta t} = -\frac{\sigma\rho}{\varepsilon_0}.$$

$$\frac{\delta\rho}{\rho} = -\frac{\sigma}{\varepsilon_0}\delta t$$

integrate both side

$$\int\frac{\delta\rho}{\rho} = \int -\frac{\sigma}{\varepsilon_0}\delta t$$

$$\log|\rho| = -\frac{\sigma}{\varepsilon_0}t + A$$

integration constant

$$\log \rho = -\frac{\sigma t}{\varepsilon_0} + A$$

at time $t = 0$ $\quad \rho = \rho_0$

$$\log \rho_0 = A$$

$$\log \rho = -\frac{\sigma t}{\varepsilon_0} + \log \rho_0$$

$$\log \frac{\rho}{\rho_0} = -\frac{\sigma t}{\varepsilon_0}$$

$$\frac{\rho}{\rho_0} = e^{-\frac{\sigma t}{\varepsilon_0}}$$

$$\boxed{\rho = \rho_0 \, e^{-\frac{t}{\Upsilon}}}$$

where Υ = relexation time $= \frac{\varepsilon_0}{\sigma}$.

Kirchoff's law

(i) Kirchoff's first law / current law.

It applies to the circuit nodes (or Junction) and states that in any network or Junction the Algebric sum of the currents at any Junction in a circuit is zero.

$$\boxed{\Sigma i = 0}$$

Kirchoff's Voltage law or loop Rule

* It applies to close circuit (meshes) and states that in any closed circuit, algebric sum of the products of the current & resestanc of the each part of the circuit is equals to the total emf acting in circuit

$$\boxed{\Sigma iR = \Sigma \mathcal{E}}$$

lorentz - drude theory

V_d, Υ, I & V_d, J, Deduction to Ohm's law.

* If $U_1, U_2, U_3 - - - U_n$ are random velocity of e^- in an conductor.

average velocity $= U = \frac{U_1 + U_2 + - - - + U_n}{n}$

no net flow of charge in any direction because with same speed e^- moves in opposite direction

On applying $\vec{E}$, then the force applied on electron

$$\boxed{f = -e\vec{E}}$$

acceleration $= a = \frac{f}{m} = \frac{f_{\cancel{m}}}{a} = \frac{-eE}{a}$

$$\boxed{a = \frac{-eE}{M}}$$

If an e^- having random thermal velocity $\vec{U_1}$ and accelerate for time τ_1

then $[\vec{V_1} = \vec{U_1} + a\vec{\tau_1}]$

Similarly, $[\vec{V_2} = \vec{U_2} + a\tau_2]$

$(\vec{V_3} = \vec{U_3} + a\tau_3)$

$\vdots$

$\vec{V_n} = U_n + a\tau_3.$

average velocity = drift velocity

$$V_d = \frac{\vec{V_1} + \vec{V_2} + \cdots + \vec{V_n}}{n} = \frac{\vec{U_1} + \vec{U_2} + \cdots U_n + a(\vec{\tau_1} + \cdots \tau_n)}{n}$$

0 (initial velocity)

$$\vec{V_d} = \frac{a(\tau_1 + \tau_2 - \cdots \tau_n)}{n}$$

$\vec{V_d} = \frac{a/\tau}{n})$ $\qquad \frac{\tau}{n} \rightarrow$ average relaxation time

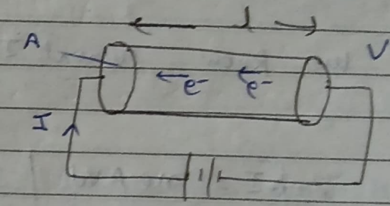
$\vec{V_d} = \vec{a}\tau$

$\vec{V_d} = \frac{-eE\tau}{M}$

$$\boxed{\vec{V_d} = \frac{-eE\tau}{Me}}$$

where.
$E \rightarrow$ Electric field
$\tau \rightarrow$ Relaxation time
$Me \rightarrow$ Mass of e^-

* Consider a cylinder (conductor) of length l which is connected to a external EMF E.



Electric field $\vec{E} = \frac{V}{l}$

Let the e^- density $= n$ (electron in per unit volume)

Charge on $e^- = e$

No. of e^- in l length conductor
$= n \times (l \times A)$ volume of con.
$= nAl$

total charge $= q = ne$

$q = nAl \times e^-$

$\boxed{q = enAl}$ ⟶ charge on condr.

$t = \frac{\text{distance}}{\text{velocity}}$

$\boxed{t = \frac{l}{V_d}}$

we know current $I = \frac{q}{t} = \frac{enAl}{\left(\frac{l}{V_d}\right)}$

Current $\boxed{I = neAV_d}$ drift velocity

deduction to ohm's law.

$V_d = \frac{eEY}{Me}$ — Mass of $\vec{e}$

$V_d = \frac{e\left(\frac{V}{l}\right)Y}{Me}$

$V_d = \frac{eVY}{M_e l}$

$I = neAV_d.$

$I = neA\left(\frac{eVY}{M_e l}\right)$

$I = neA\left(\frac{eVY}{Ml}\right) = \frac{ne^2AVY}{Ml}.$

$\boxed{\frac{V}{I} = \frac{Ml}{ne^2AY}}$ ⟶ Constant

by ohms law $\boxed{R = \frac{Ml}{ne^2AY}}$

Statement

The lorentz - Drude theory is a classical theory that explains how electrons move in a metal and how this movement contributes to electric current.

Proof of Wiedmands - Franz law.

For metals, the ratio of thermal conductivity to electrical conductivity is directly proportional to the absolute temprature. This ratio is a constant for all metal at a given temperature.

$$\frac{K}{\sigma} \propto T$$

$\frac{K}{\sigma} = LT$, where L is proportionality const. known as Lorentz number.

Derivation

This law is derived from the expression of thermal conductivity (K) and electrical conduct (σ) of a metal.

We know electrical conductivity of a metal is given by

$$\sigma = \frac{ne^2 \gamma}{m} \quad \text{--- (i)}$$

We know that thermal conductivity (K) of a metal is given by.

$$K = \frac{1}{2} n v^2 \underbrace{k}_{\text{boltzmen constant}} \gamma \quad \text{--- (ii)}$$

Dividing (2) by (1) we get.

$$\frac{K}{\sigma} = \frac{\frac{1}{2} n v^2 k \gamma}{n e^2 \gamma / m} = \left(\frac{1}{2} m v^2\right) \frac{k}{e}$$

$$\frac{K}{\sigma} = \left(\frac{1}{2} m v^2\right) \frac{k}{e^2} \quad \text{--- (3)}$$

The K.E of free e^- in a metal is given by

$$\frac{1}{2} m v^2 = \frac{3}{2} kT \quad \text{--- (4)}$$

using 4 in 3

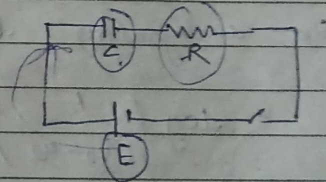
$$\frac{K}{\sigma} = \frac{3}{2} kT \frac{k}{e^2}$$

$$\frac{K}{\sigma} = \left(\frac{3k^2}{2e^2}\right) T$$

$$\boxed{\frac{K}{\sigma} \propto T} \qquad \boxed{L = \frac{3k^2}{2e^2}}$$

Charging of capacitor through resistance.

The induced emf. of capacitor $= -\frac{q}{c}$



Inducing E.M.F across capacita $= \frac{q}{c}$

P.D across $R = RI = R\frac{dq}{dt}$

Equation of R.C circuit for charging becomes.

$$\frac{q}{C} + R\frac{dq}{dt} = E \quad \text{--- (1)}$$

when ($q = q_0$ peak or max charge).

$\frac{dq}{dt} = 0$ [at max after that there is not change in charge].

eq (1) become $\boxed{E = \frac{q_0}{C}}$

$$\frac{q}{C} + R\frac{dq}{dt} = \frac{q_0}{C}$$

$$R\frac{dq}{dt} = \frac{q_0 - q}{C}$$

$$\int \frac{dq}{q_0 - q} = \frac{dt}{RC}$$

On integrating both side.

$$-\log\left(\frac{1}{q_0 - q}\right) = \frac{t}{RC} + C.$$

$$\log(q_0 - q) = \frac{-t}{RC} + C$$

at $t = 0$, $q = 0$.

$$\boxed{\log q_0 = C}$$

$$\log(q_0 - q) = \frac{-t}{RC} + \log q_0.$$

$$\log\left(\frac{q_0 - q}{q_0}\right) = \frac{-t}{RC}$$

$$\log\left(1 - \frac{q}{q_0}\right) = \frac{-t}{RC}$$

$$1 - \frac{q}{q_0} = e^{-t/RC}$$

$$\frac{q}{q_0} = 1 - e^{-t/RC}$$

$$\boxed{q = q_0\left(1 - e^{-t/RC}\right)}$$

* The charging ~~is~~ of capacitor is exponantial in nature.

* At $t = 0$ eq (1) gives $q = q_0(1 - e^0)$

$$\boxed{q = 0}$$

* At $t = RC$ → [time constant for RC circuit]

$$q = q_0(1 - e^{-1}) = q_0\left(1 - \frac{1}{e}\right)$$

$$q = q_0(1 - 0.368) = 0.632\, q_0.$$

$$\boxed{q = 0.632\, q_0}$$

the time constant of the RC circuit is ~~at~~ the time at which the

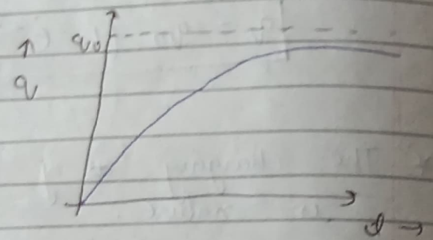
the charge on the capacitor become 0.632 time the maximum charge.

* at $t = \infty$

$$q = q_0(1 - e^{-\infty}) = q_0$$

$$\boxed{q = q_0}$$

Capacitor takes ∞ time for fully charge



for current

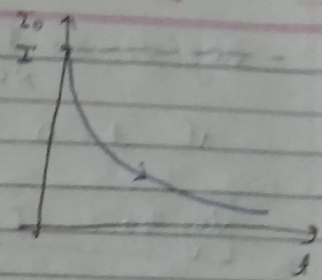
$$I = \frac{dq}{dt}$$

$$I = \frac{d}{dt}\left[q_0(1 - e^{-t/RC})\right]$$

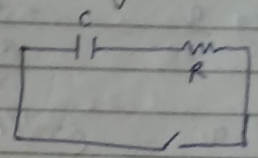
$$I = -q_0 e^{-t/RC} \cdot \frac{-1}{RC}$$

$$I = +\frac{q_0}{RC} e^{-t/RC}$$

$$\boxed{I = \frac{q_0}{RC} e^{-t/RC}}$$



Discharging of Capacitor through Resistance



The induced e.m.f across of C

$$= -\frac{q}{C}$$

inducing e.m.f across C $= \frac{q}{C}$

P.D across the R $= R\frac{dq}{dt}$ [by ohm's law]

Equation of discharging

$$\frac{q}{C} + R\frac{dq}{dt} = 0 \quad \text{[external Emf is Removed]}$$

$$R\frac{dq}{dt} = \frac{-q}{C}$$

$$\frac{dq}{q} = \frac{dt}{RC}$$

integrate :> $\int \frac{dq}{q} = \int \frac{dt}{RC}$

$$\log(q) = -\frac{t}{RC} + C$$

at $t = 0$ $\quad q = q_0$.

$$\boxed{\log q_0 = C}$$

→ $\log q = \frac{t}{RC} + \log q_0$.

$$\log \frac{q}{q_0} = \frac{-t}{RC}$$

$$\frac{q}{q_0} = e^{-t/RC}$$

$$\boxed{q = q_0 e^{-t/RC}}$$

$q = q_0 e^{-t/RC}$ —— (i)

* Exponential in nature

* at $t = 0$, $\boxed{q = q_0}$.

* at $t = RC$

$$q = \frac{q_0}{e} = 0.368\, q_0$$

$$\boxed{q = 0.368\, q_0}$$

* at $t = \infty$

$$q = \frac{q_0}{e^{\infty}} = q_0 e^{-\infty} \doteq 0.$$

Current variation in discharging.

$$I = \frac{dq}{dt}$$

$$I = \frac{d}{dt}\left[q_0 e^{-t/RC}\right] = -q_0 e^{-t/RC} \cdot \frac{1}{RC}$$

$$\boxed{I = -\frac{q_0}{RC} e^{-t/RC}}$$

* At $t = 0$ $\quad \boxed{I = -\frac{q_0}{RC}} = I_0$

* At $t = RC$ $\quad I = -0.368 \frac{q_0}{RC} = 0.368\, I_0$

* At $t = \infty$ ⇒ $I = 0$

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