Electrostatics

Study of effects produced by charges at rest.

Point charge.
↳ very small charge.

Coulomb's law :- gives us the force of attraction or repulsion b/w the two charges

$$+q_1 \xrightarrow{\quad r \quad} q_2$$

$$F = k \frac{q_1 q_2}{r^2}$$

$\boxed{k = 9 \times 10^9 \ Nm^2/C^2}$ → in vaccume.

$\boxed{k = \frac{1}{4\pi \varepsilon_m}}$ k is constant of perpostion. It's value depends on system of units.

$K = 1$ in cgs
$K = 9 \times 10^9$ in SI

ε_m → Permitivity of that medium
ε_0 → " " free space.

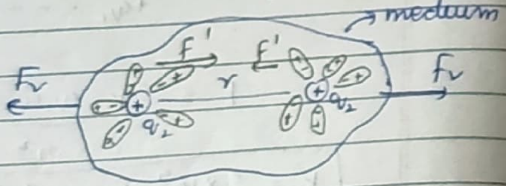
$$F = \frac{1}{4\pi \varepsilon_0 \varepsilon_r} \frac{q_1 q_2}{r^2}$$

$\varepsilon_0 = 8.854 \times 10^{-12}\ N^{-1} m^{-2} C^2$

Permitivity :- Permitivity is a material's property that measures how much it resists an electric field

· ε_0 is permitivity of free space. and smallest value of permitivity of free space

Electrostatic force b/w two charges in a medium.



$F_m = \frac{1}{4\pi \varepsilon_m} \frac{q_1 q_2}{r^2}$

$F_v = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2}$

$\frac{F_v}{F_m} = \frac{\varepsilon_m}{\varepsilon_0} = \varepsilon_r$ or $K \rightarrow$ Dielectric Constant or relative permitivity

$$\boxed{\varepsilon_r = \frac{\varepsilon_m}{\varepsilon_0}}$$

$$\boxed{F_m = \frac{F_v}{\varepsilon_r}}$$

$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

Dimention :-

$[\varepsilon_0]$

$\varepsilon_0 = \frac{q_1 q_2}{F r^2}$

$\varepsilon_0 = \frac{[q^2]}{[MLT^{-2}][L^2]}$

Charge = Current × Time
= AT.

$\varepsilon_0 \rightarrow [M^{-1} L^{-3} T^{-2} A^2]$

$\varepsilon_0 = \frac{[A^2 T^2]}{[MLT^{-2}][L^2]} = [A^2 M^{-1} L^{-3} T^4]$

$\varepsilon_0 = [M^{-1} L^{-3} T^4 A^2]$

ε_r is dimentionless

→ Limitation of Coulomb's law :-

* It is valid for charges at rest

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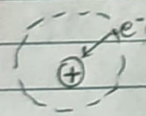
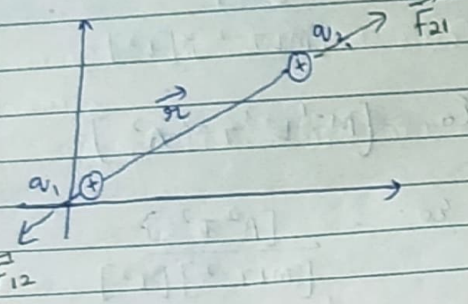


$$\boxed{\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{z e^2}{r^2}}$$

* It is valid only for point charges not valid for extended bodies

For application of coulomb's law we ~~that~~ assume that the charges are placed at sepration r for a very long time $\left(t \to \frac{r}{c}\right)$

- Direction of Coulomb's force (vector form)



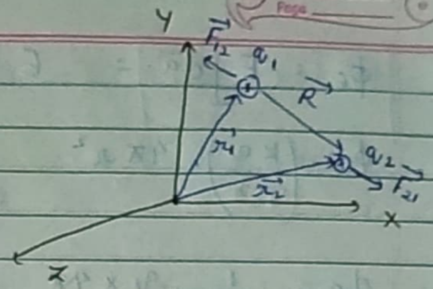
$\vec{F_{12}}$ = force on q_1 due to q_2

$= \frac{k q_1 q_2}{r^2} (-\hat{r})$ opposite of to $\vec{r}$.

$$\vec{F_{12}} = -k \frac{q_1 q_2}{r^2} \left(\frac{\vec{r}}{r}\right)$$

$$\boxed{\vec{F_{12}} = \frac{-k q_1 q_2}{r^3} \vec{r}}$$

$$\vec{F_{21}} = + \frac{k q_1 q_2}{r^3} \vec{r}$$



by Δ law

$\vec{R} = \vec{r_2} - \vec{r_1}$

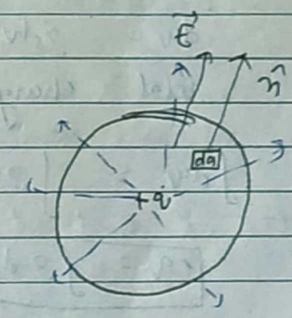
$$\vec{F_{21}} = \frac{k q_1 q_2 (\vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}|^3}$$

$$\vec{F_{12}} = \frac{k q_1 q_2 (\vec{r_1} - \vec{r_2})}{|\vec{r_2} - \vec{r_1}|^3}$$

Gauss's law :-

The electric flux passing through any closed Surface is equal to $\frac{1}{\varepsilon_0}$ time charge enclosed by that surface.

$$\boxed{\phi_E = \frac{q}{\varepsilon_0}}$$



$d\phi_E = \vec{E} \cdot d\vec{a}$

integrale

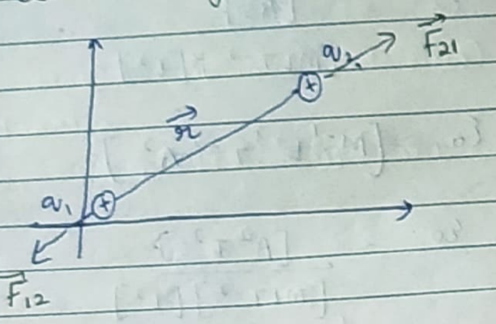
$\int d\phi_E = \int \vec{E} \cdot d\vec{a}$

$\phi_E = \int E \, da \cos\theta$

$\phi_E = \int E \, da \cos 0^\circ = \phi_E = E \int d\vec{a}$

For application of coulomb's law we ~~that~~ assume that the charges are placed at sepration r for a very long time $\left(t > \frac{r}{c}\right)$

- Direction of Coulomb's force (vector form)



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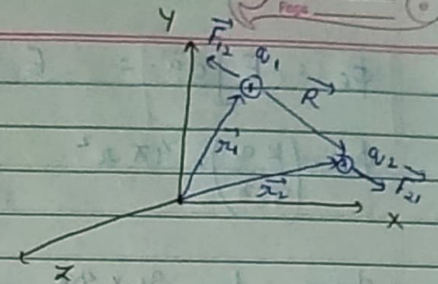
by Δ law

$\vec{R} = \vec{r}_2 - \vec{r}_1$



$$\vec{F}_{21} = \frac{k q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

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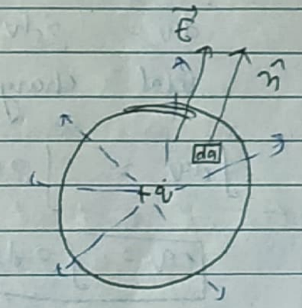


$d\phi_E = \vec{E} \cdot d\vec{a}$

integrate

$$\int d\phi_E = \int \vec{E} \cdot d\vec{a}$$

$$\phi_E = \int E \, da \cos\theta$$

$$\phi_E = \int E \, da \cos 0^\circ \; = \; \phi_E = E \int d\vec{a}$$

$$\phi_E = E\int d\vec{a} = E\,4\pi r^2$$

$$\phi_E = \left(\frac{kq}{r^2}\right) \cdot 4\pi r^2$$

$$\phi_E = \frac{1}{4\pi\varepsilon_0} \cdot q \times 4\pi$$

$$\boxed{\phi_E = \frac{q}{\varepsilon_0}}$$

Integral form of Gauss's law

⇒ Volume charge density

$$\rho = \frac{dq}{dV}$$

$$dq = \rho\, dV \quad \text{— (i)}$$

total charge integrate (i)

$$\int dq = \int \rho\, dV$$

$$\boxed{q = \int \rho\, dV} \quad \text{— (ii)}$$

$$\phi_E = \oint \vec{E}\cdot d\vec{a} \quad \text{— (iii)}$$

put eq (ii) and (iii) in gauss's law

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{\int \rho\, dV}{\varepsilon_0}$$

$$\boxed{\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int \rho\, dV}$$ — Integral form of gauss's law.

Differential form of Gauss's law

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int \rho\, dV \quad \text{— (i)}$$

Divergence theorem $\iint \vec{A}\cdot d\vec{s} = \iiint (\nabla\cdot\vec{A})\cdot dV$

$$\oint_S \vec{E}\cdot d\vec{a} = \int_V (\vec{\nabla}\cdot\vec{E})\cdot dV \quad \text{— (ii)}$$

Put value of $\oint_S \vec{E}\cdot d\vec{a}$ in eq (i)

$$\int_V (\vec{\nabla}\cdot\vec{E})\cdot dV = \frac{1}{\varepsilon_0}\int \rho\, dV$$

$$\int \left(\vec{\nabla}\cdot\vec{E} - \frac{\rho}{\varepsilon_0}\right) \cdot dV = 0$$

means one of these two is 0

$$\vec{\nabla}\cdot\vec{E} - \frac{\rho}{\varepsilon_0} = 0$$

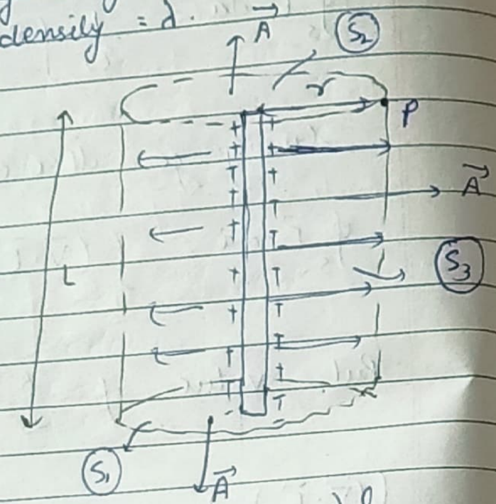
$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$ differential form of gauss law.

Electric field due to linear charge distribution / linearly charged wire.

Consider a charged wire ~~of length d~~ having the linear charge density $= \lambda$.

Electric field at P.



Consider a gaussian surface (cylindrical) of radius r and length l

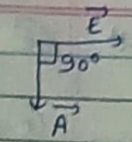
Charges enclosed by the gaussian surface $= \lambda l$

using gauss law.

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\iint \vec{E}\cdot ds = \frac{q}{\varepsilon_0}.$$

for Surface S_1 and S_2 the angle between area vector and Electric field is 90^0.

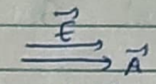


$$\oiint E\,ds\cos\theta = \frac{q}{\varepsilon_0}.$$

$\theta = 90^\circ \qquad \cos 90 = 0.$

$\boxed{E = 0}$ from the Surfaces S_1, and S_2.

for Surface S_3 angle btw area vector and $\vec{E}$ is 0°.



$$\oiint_{S_1} \vec{E}\cdot\vec{ds_1} + \oiint_{S_2} \vec{E}\cdot\vec{ds_2} + \oiint_{S_3} \vec{E}\cdot\vec{ds_3} = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$\therefore$

$$0 + 0 + \oiint \vec{E}\,\vec{ds}\cos 0 = \frac{\lambda l}{\varepsilon_0}$$

$$\int E\,ds = \frac{\lambda l}{\varepsilon_0}$$

$$E\iint_{S_3} ds = \frac{\lambda l}{\varepsilon_0}$$

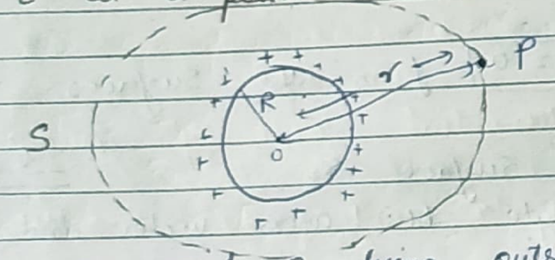
$$E\,2\pi r\cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

$\vec{E}$ due to charged Spherical Shell

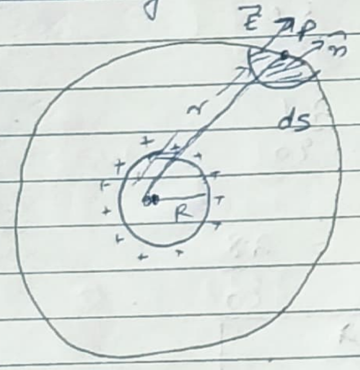
Consider a spherical shell of radius R and center O. let q be the charge on the shell

Case1 E at a point outside the shell.



Consider a point P lying outside the sphere and the distance b/w the point P and centre of spherical shell is r

draw a gaussian surface S which will be a sphere of radius r



let 'ds' be the small area element on the gaussean surface at P.

according to gauss law:

$$\oint\!\!\oint_S \vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0} \times \text{charge enclosed}.$$

$$\oint\!\!\oint_S \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

$$E\oint\!\!\oint \vec{ds}\cos 0^\circ = \frac{q}{\varepsilon_0}$$

$$= \vec{E}\cdot\oint\!\!\oint \vec{ds} = \frac{q}{\varepsilon_0}$$

$$= \vec{E}\cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}}$$

If σ be the surface charge density of the spherical shell

Charge on the spherical shell $= \sigma \times 4\pi R^2$

$$q = \sigma 4\pi R^2$$

$$\vec{E} = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0 r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

$$\boxed{\vec{E}_{out} = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

Case (ii) :- E at a point on the surface of spherical shell

Consider a point P on the surface of spherical shell at a distance r from center O. In this case $\boxed{r = R}$

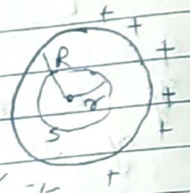
using the eq of previous derivation

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

Put r = R $\boxed{E_{surface} = \frac{\sigma}{\varepsilon_0}}$

Case (iii) inside the spherical shell.

gaussian surface of radius 'r'

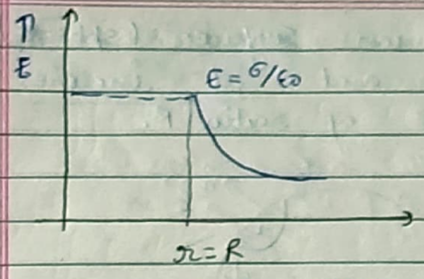


charge inside the spherical shell is 0.

$q = 0$

$$\oint_S E \cdot ds = \frac{q}{\varepsilon_0}$$

$\boxed{E = 0}$



Electric field due to charged solid sphere

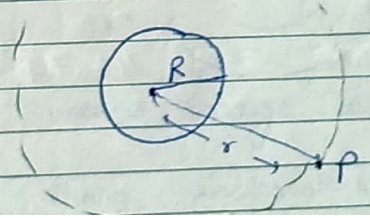
Consider a solid sphere of radius R and centre O. let q be the charge given to the solid sphere

let ρ be the volume charge density

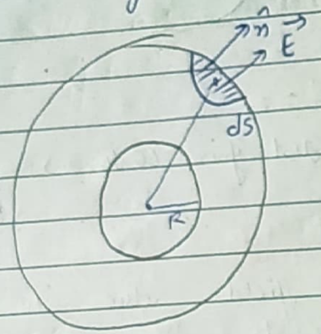
$\rho = \frac{charge}{Volume}$, $\rho = \frac{q}{\frac{4}{3}\pi R^3}$

$\boxed{q = \frac{4}{3}\pi R^3 \rho}$

Case 1 :- Point outside the sphere.



Consider a gaussian surface (sphere) of radius r and it incloses the charged sphere of radius R.



Let ds be the small area element on the gaussian surface at P.

Acc. to gauss's law.

$$\phi = \frac{q}{\varepsilon_0}$$

$$\phi_E = \oiint E \cdot ds = \oiint E\, ds \cos 0^\circ$$

$$\oiint \vec{E} ds = \frac{q}{\varepsilon_0}$$

$$\vec{E} \oiint ds = \frac{\left(\frac{4}{3}\pi R^3 \rho\right)}{\varepsilon_0}$$

$$\vec{E}\, 4\pi r^2 = \frac{4\pi R^3 \rho}{3\varepsilon_0}$$

$$\vec{E}\, r^2 = \frac{R^3 \rho}{3\varepsilon_0}$$

$$\boxed{\vec{E}_0 = \frac{\rho R^3}{3\varepsilon_0 r^2}} \qquad \boxed{\vec{E}_{outside} \propto \frac{1}{r^2}}$$

at the surface :-

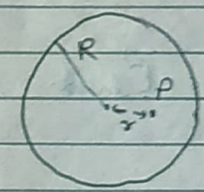
$r = R$

$$\vec{E}_0 = \frac{\rho R^3}{3\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 R^2} = \frac{\rho R}{3\varepsilon_0}$$

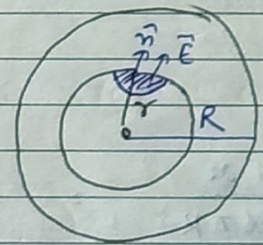
$$\boxed{\vec{E}_{surface} = \frac{\rho R}{3\varepsilon_0}}$$

point inside the sphere :-

Let a point P which is at a distance of P 'r' from the centre of sphere and $r < R$.



Take a gaussian surface (sphere) of radius 'r'.



volume charge density = ρ.

Charge enclosed by the gaussian surface $= \rho \times \frac{4}{3}\pi r^3$

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Let ds be the small area element in the gaussian surface.

flux through the area element

$$\phi_E = \oiint \vec{E} \cdot d\vec{s}$$

also by gauss law.

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\oiint \vec{E} \cdot d\vec{s} = \frac{\rho \frac{4}{3} \pi r^3}{\varepsilon_0}$$

$$\vec{E} \oiint d\vec{s} = \frac{\rho \frac{4}{3} \pi r^3}{\varepsilon_0}$$

$$\vec{E}\, 4\pi r^2 = \frac{\rho \frac{4}{3} \pi r^3}{\varepsilon_0}$$

$$\vec{E}_{inside} = \frac{\rho r}{3 \varepsilon_0}$$

$$\rho = \frac{q}{\frac{4}{3} \pi R^3}$$

$$\vec{E}_{inside} = \frac{3}{4} \frac{q\, r}{3 \varepsilon_0 \pi R^3} = \frac{q r}{4 \varepsilon_0 \pi R^3}$$

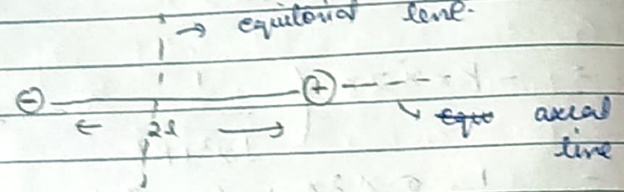
$$\boxed{\vec{E}_{inside} \propto r}$$

Electric dipole.

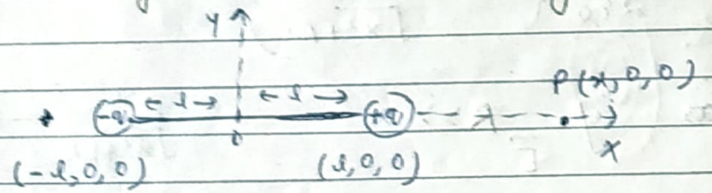
equal and opposite charge place together having a small sepration of '2l'

$+q \xleftarrow{\;\;2l\;\;} -q$, $\vec{p}$

dipole moment = $q \times 2l$ (-ve to +ve)



Electric field on the axis of dipole:



$$\vec{E} = \frac{Kq(\vec{r_1} - \vec{r_2})}{|r_1 - r_2|^3}$$

$\vec{r_1} \rightarrow$ position vector of observation pt.

$\vec{r_1} = x\hat{i}$

$\vec{r_2}$ = position v. of charge

$\vec{r_2} = \vec{r_+} = l\hat{i}$

$\vec{r_2} = \vec{r_-} = -l\hat{i}$

$$\vec{E_+} = \frac{Kq(x\hat{i} - l\hat{i})}{|x\hat{i} - l\hat{i}|^3} = \frac{Kq(x-l)\hat{i}}{(x-l)^3}$$

$$\boxed{\vec{E_+} = \frac{Kq}{(x-l)^2}\hat{i}}$$

$\vec{E}$ due to -ve charge.

$$\vec{E} = \frac{\hat{k}(-q)[x\hat{i} - (-l\hat{i})]}{(x+l)^3} = \frac{-\hat{k}q(x+l)\hat{i}}{(x+l)^3}$$

$$\vec{E_-} = \frac{+Kq(-\hat{i})}{(x+l)^2}$$

$$\vec{E_{net}} = \vec{E_+} + \vec{E_-} = Kq\left(\frac{1}{(x-l)^2} - \frac{1}{(x+l)^2}\right)\hat{i}$$

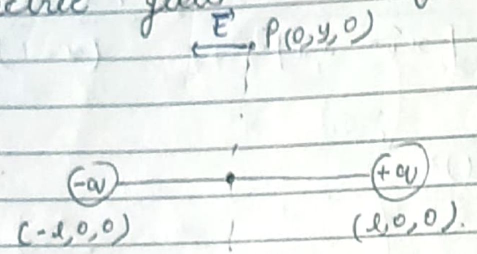
$$\vec{E_{net}} = Kq\left[\frac{(x+l)^2 - (x-l)^2}{(x^2-l^2)^2}\right]\hat{i}$$

$$\vec{E_{net}} = Kq\left[\frac{4xl}{(x^2-l^2)^2}\right]\hat{i} = \frac{2K\cancel{q}xp}{(x^2-l^2)^2}\hat{i}$$

for shot dipole $x >> l$.

$$\vec{E}_{axial} = \frac{2kpx}{(x^2-l^2)^2} = \frac{2kp}{x^3}\hat{i}.$$

o) Electric field at equitoreal.



$$\vec{E} = \frac{kq(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}.$$

$\therefore$ $r_1 \rightarrow$ P.V of observation point
$r_2 \rightarrow$ P.V of charge

$$\vec{E}_+ = \frac{kq(y\hat{j} - l\hat{i})}{|y\hat{j} - l\hat{i}|^3}$$

$$\vec{E}_+ = \frac{kq(y\hat{j} - l\hat{i})}{(y^2+l^2)^{3/2}}$$

$$\vec{E}_+ = \frac{kq(y\hat{j} - l\hat{i})}{(y^2+l^2)^{3/2}}$$

Similary $\vec{E}$ due to -ve change

$$\vec{E} = \frac{kq(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

$$\vec{E} = \frac{k(q)(y\hat{j} + l\hat{i})}{|y\hat{j} + l\hat{i}|^3}$$

$$\vec{E}_- = \frac{-kq(y\hat{j} + l\hat{i})}{(y^2+l^2)^{3/2}}$$

$$\vec{E}_{net} = \vec{E}_+ + \vec{E}_- = \frac{kq}{(y^2+l^2)^{3/2}}\left[y\hat{j} - l\hat{i} - y\hat{j} - l\hat{i}\right]$$

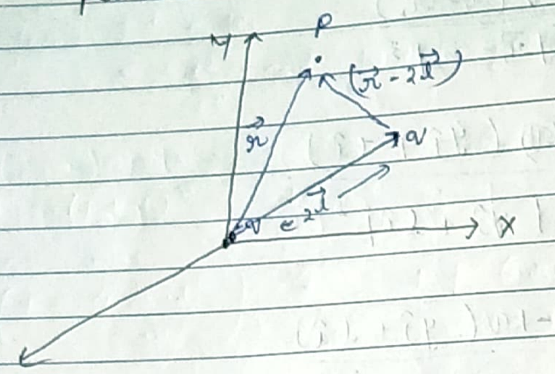
$$\vec{E}_{net} = \frac{kq}{(y^2+l^2)^{3/2}}\left[-2l\hat{i}\right) = \frac{kq2l(-\hat{i})}{(y^2+l^2)^{3/2}}$$

$$\vec{E}_{equtorial} = \frac{-kp\hat{i}}{(l^2+y^2)^{3/2}}$$

for short dipole. $l << y$.

$$\vec{E}_{eq.} = \frac{-kp}{y^3}\hat{i}$$

Electric field of dipole at any arbitary point.



E Electric field by a charge q is given by

$$\vec{E} = \frac{kq(\vec{r_1} - \vec{r_2})}{|\vec{r_1} - \vec{r_2}|^3}$$

where $\vec{r_1} \rightarrow$ P.V of observation pt.
$\vec{r_2} \rightarrow$ P.V of charge.

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0} (-q) \frac{(\vec{r} - 0)}{|\vec{r}|^3}$$

for +q

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0} q \left(\frac{\vec{r} - 2\vec{l}}{|\vec{r} - 2\vec{l}|^3} \right)$$

By superposition principle add vect.

$$\vec{E}_{net} = \vec{E}_{+q} + \vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0} q \left(\frac{\vec{r}-2\vec{l}}{|\vec{r}-2\vec{l}|^3} - \frac{\vec{r}}{|\vec{r}|^3} \right)$$

$\therefore |\vec{r}| = r.$

$$\vec{E}_{net} = \frac{q}{4\pi\varepsilon_0} \left(\frac{\vec{r}-2\vec{l}}{[(\vec{r}-2\vec{l})^2]^{3/2}} - \frac{\vec{r}}{|\vec{r}|^3} \right)$$

$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{\vec{r} - 2\vec{l}}{(r^2 + 4l^2 - 4\vec{r}\cdot\vec{l})^{3/2}} - \frac{\vec{r}}{|\vec{r}|^3} \right]$$

$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{(\vec{r} - 2\vec{l})}{r^3\left(1 + \frac{4l^2}{r^2} - \frac{4\vec{r}\cdot\vec{l}}{r^2}\right)^{3/2}} - \frac{\vec{r}}{r^3} \right]$$

$$= \frac{q}{4\pi\varepsilon_0 r^3} \left(\frac{\vec{r} - 2\vec{l}}{\left(1 + \frac{4l^2}{r^2} - \frac{4\vec{r}\cdot\vec{l}}{r^2}\right)^{3/2}} - \vec{r} \right)$$

[neglecting $\frac{4l^2}{r^2}$ as $l << r$]

$$= \frac{q}{4\pi\varepsilon_0 r^3} \left[(\vec{r} - 2\vec{l})\left(1 - \frac{4\vec{r}\cdot\vec{l}}{r^2}\right)^{-3/2} - \vec{r} \right]$$

Apply binomial theorem on $\left(1 - \frac{4\vec{r}\cdot\vec{l}}{r^2}\right)^{-3/2}$

for I^{st} order

$\therefore \frac{l^2}{r^2}$ and higher powers are very very small

$\because (1+x)^n = 1 + nx$

$$\left(1 - \frac{4\,\vec{r}\cdot\vec{l}}{r^2}\right)^{-3/2} = 1 + \frac{6\,\vec{r}\cdot\vec{l}}{r^2}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^3}\left[(\vec{r} - 2\vec{l})\left(1 + \frac{6\,\vec{r}\cdot\vec{l}}{r^2}\right) - \vec{r}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r^3}\left(\cancel{\vec{r}} + \frac{6(\vec{r}\cdot\vec{l})\vec{r}}{r^2} - 2\vec{l} - 12\left(\frac{(\vec{r}\cdot l^2)\vec{l}}{r^2}\right) - \cancel{\vec{r}}\right)$$

$l^2 << r^2$

$$= \frac{q}{4\pi\varepsilon_0 r^3}\left(\frac{6(\vec{r}\cdot\vec{l})\vec{r}}{r^2} - 2\vec{l}\right)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{6(\vec{r}\cdot\vec{l})\vec{r}q}{r^5} - \frac{2\vec{l}q}{r^3}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{3(\vec{r}\cdot\vec{l})\vec{r}}{}\right.$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{3(\vec{r}\cdot 2q\vec{l})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right)$$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{3(\vec{r}\cdot\vec{p})\vec{r}}{r^5} - \frac{2\vec{lq}}{r^3}\right)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{3(\vec{r}\cdot\vec{p})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right]$$

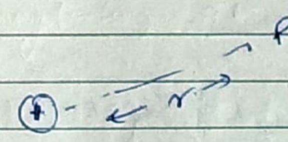
Electric scalar potential :-

The electrical potential at a point near a given charge distribution is defined as the external work per unit charge to bring it from ∞ to that point without imparting K.E.

$$V = \frac{W_{ext}}{q_0} \quad (JC^{-1} \text{ or volt})$$

Electric potential difference :- It is the work required to move a unit charge from one point to another in the region of Electric field.

$$\boxed{V_B - V_A = \frac{W_{ext}}{q_0}}$$



$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

$$dV = -\vec{E}\cdot\vec{dl}$$

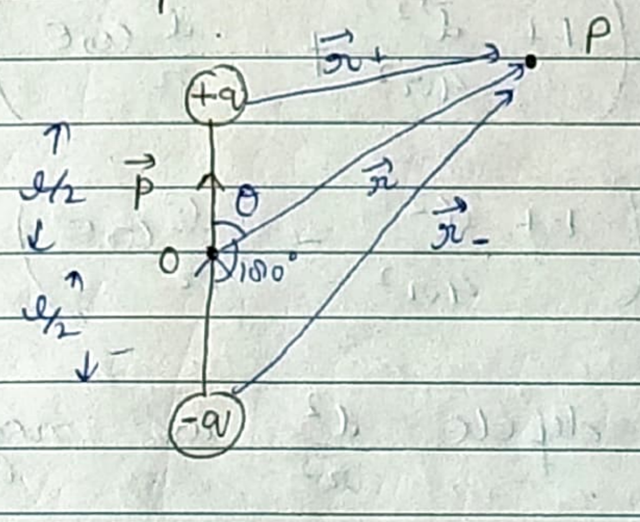
$$\int_{V(\infty)}^{V} dV = -\int_{\infty}^{r} \vec{E}\cdot\vec{dl}$$

$$V(P) - V(\infty) = -\int_{\infty}^{r} \vec{E}\cdot\vec{dl}$$

$$\boxed{V(P) = -\int_{\infty}^{r} \vec{E}\cdot dl}$$

$$\boxed{V_b - V_a = -\int_{a}^{b} \vec{E}\cdot dl}$$

Electric potential due to a dipole



$$V_P = V_+ + V_-$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} - \frac{q}{4\pi\varepsilon_0 r_-} \quad \text{—— (1)}$$

$$\vec{r} = \vec{r}_+ + \frac{\vec{l}}{2}$$

$$\boxed{\vec{r}_+ = \vec{r} - \frac{\vec{l}}{2}}$$

angle between $\vec{r}$ and $-\frac{\vec{l}}{2}$ is $180^\circ - \theta$

$$r_+^2 = r^2 + \left(\frac{\vec{l}}{2}\right)^2 - lr\cos\theta$$

from Δ law

$$\frac{\vec{l}}{2} + \vec{r}_- = \vec{r}$$

$$\vec{r}_- = \vec{r} - \frac{\vec{l}}{2}$$

$$r_-^2 = r^2 + \left(\frac{\vec{l}}{2}\right)^2 - \vec{l}\vec{r}\cos(180^\circ - \theta)$$

$$r_-^2 = r^2 + \frac{\vec{l}^2}{2} + \vec{l}\vec{r}\cos\theta$$

$$r_-^2 = r^2\left(1 + \frac{\vec{l}^2}{4r^2} + \frac{l\cos\theta}{r}\right)$$

$$r_+^2 = r^2\left(1 + \frac{l^2}{4r^2} - \frac{l\cos\theta}{r}\right)$$

for short dipole $\frac{l^2}{r^2}$ are negligible $l << r$

$$r_-^2 = r^2\left(1 + \frac{l}{r}\cos\theta\right)$$

$$r_- = r\left(1 + \frac{l}{r}\cos\theta\right)^{1/2}$$

$$r_+^2 = r^2\left(1 - \frac{l}{2}\cos\theta\right)$$

$$r_+ = r\left(1 - \frac{l}{r}\cos\theta\right)^{1/2}$$

using binomial theorem

$$\frac{1}{r_+} = \frac{1}{r}\left(1 - \frac{l}{r}\cos\theta\right)^{-1/2}$$

$$\because (1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \cdots$$

$\frac{l^2}{r^2}$ and other greater terms are neglected.

$$\frac{1}{r_+} = \frac{1}{r}\left(1 + \frac{1}{2}\frac{l}{r}\cos\theta\right) \text{ —— (ii)}$$

$$\frac{1}{r_-} = \frac{1}{r}\left(1 - \frac{l}{2r}\cos\theta\right) \text{ —— (iii)}$$

put in eq (i)

$$V_P = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + \frac{l}{2r}\cos\theta\right) - \frac{1}{r}\left(1 - \frac{l}{2r}\cos\theta\right)\right]$$

$$V_P = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}\left[1 + \frac{l}{2r}\cos\theta - 1 + \frac{l}{2r}\cos\theta\right]$$

$$V_P = \frac{q}{4\pi\varepsilon_0 r}\left(\frac{l}{r}\cos\theta\right)$$

$$V_P = \frac{\vec{p}\cos\theta}{4\pi\varepsilon_0 r^2}.$$

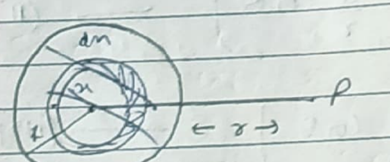
$$\boxed{V_P = \frac{\vec{p}\cos\theta}{4\pi\varepsilon_0 r^2}}$$

$$\boxed{V_P = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}}$$

$$\boxed{V_{dipole} \propto \frac{1}{r^2}} \qquad \boxed{V_{point\ charge} \propto \frac{1}{r}}$$

Electric potential due to ~~Solid~~ spherical shell

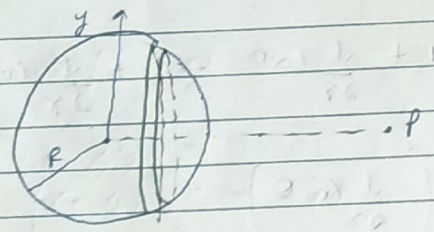
~~Let ρ be the volume charge density~~



~~Consider a shell of radius x and thickness dx~~

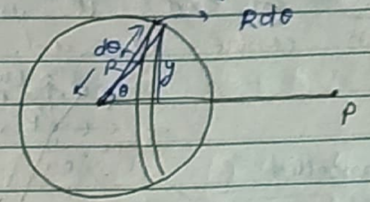
* Let σ be the surface charge density.

Consider a spherical shell of radius R.



Let the spherical shell is made up of

Let radius of the thin ring "y"



thickness of the elementary Ring $Rd\theta$

→ Potential due to Ring $= \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+x^2)^{1/2}}$

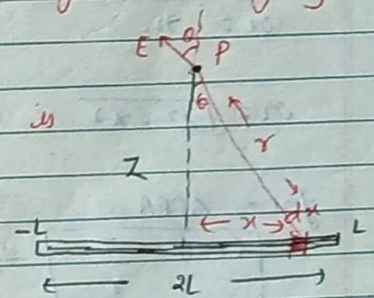
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+y^2)^{1/2}}$$

Electric field due to charged Rod

length - 2L   charge - Q

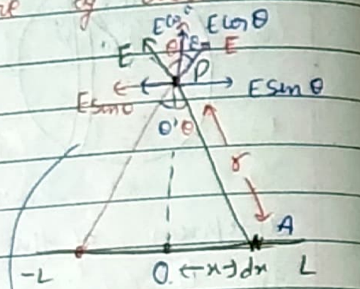
Let the charge density of the Rod is 'λ'

& $\lambda = \frac{Q}{2L}$ [charge / unit length]

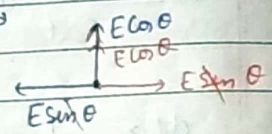


Let a point P which is ~~Set~~ at a distance of z from the centre of the rod.

take a small element in the rod of length 'dx' ~~or~~ and it is 'x' distance from centre of rod.



E Sinθ componant cancelled each other so net Electric field is due to Cosθ component



$$E = \int dE \cos\theta \quad \text{— (i)}$$

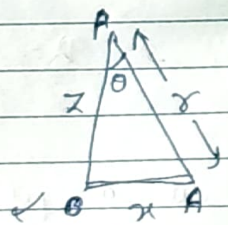
$$\text{So } dE = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

∴ ~~len~~ ∗ Charge density = λ.
∗ length of small element = dx
∗ Charge on " " = λdx

$$\boxed{Q = \lambda dx} \quad \text{— (ii)}$$

$r = r.$

$r = \sqrt{z^2 + x^2}.$



$r^2 = z^2 + x^2$

$r = \sqrt{z^2 + x^2}.$

from △OPA

$$\boxed{\cos\theta = \frac{z}{r}} \quad \text{— (iii)}$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{(\sqrt{z^2 + x^2})^2}$$

put dE in eq (i)

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{(\sqrt{z^2+x^2})^2} \cdot \left(\frac{z}{r}\right)$$

$\cos\theta$

$\sqrt{z^2+x^2}$.

$$E = \frac{1}{4\pi\varepsilon_0} \int \frac{z\lambda dx}{(z^2+x^2)^{3/2}}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{z\lambda dx}{(z^2+x^2)^{3/2}}}$$

This is even function so we use the property

$$\int_{-a}^{a} f(x) \cdot dx = 2\int_{0}^{a} f(x) \cdot dx.$$

$$E = \frac{1}{4\pi\varepsilon_0} \, 2\int_{0}^{L} \frac{z\lambda dx}{(z^2+x^2)^{3/2}}.$$

put $x = z\tan\theta$.

$dx = z\sec^2\theta \cdot d\theta$.

$$E = \frac{1}{4\pi\varepsilon_0} \, 2\int_{0}^{L} \frac{z\lambda\, dx}{(z^2 + z^2\tan^2\theta)^{3/2}}$$

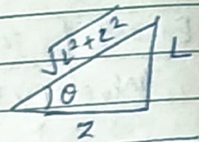
$$E = \frac{1}{4\pi\varepsilon_0}\, 2\int_0^L \frac{\lambda\, z\sec^2\theta \cdot d\theta}{(z^2)^{3/2}\,[1+\tan^2\theta)^{3/2}}$$

Since $\quad x = z\tan\theta$

when $x = 0 \quad \theta = 0$.

when $x = L \quad \tan\theta = \frac{L}{z}$

$\theta = \theta_{max}$.



$$E = \frac{1}{4\pi\varepsilon_0}\, 2\int_0^{\theta_{max}} \frac{\lambda z\sec^2\theta \cdot d\theta}{z^3\cdot \sec^3\theta}$$

$$\boxed{\sin\theta_{max} = \frac{L}{\sqrt{L^2+z^2}}}$$

$$\frac{1}{4\pi\varepsilon_0}\, 2\int_0^{\theta_{max}} \frac{\lambda\, \cancel{\sec}\cdot d\theta}{z^2 \sec\theta}.$$

$$E = \frac{1}{4\pi\varepsilon_0}\, 2\int_0^{\theta_{max}} \frac{\lambda}{z^2}\cos\theta \cdot d\theta \qquad \left(\frac{1}{\sec\theta} = \cos\theta\right)$$

$$E = \frac{1}{2\pi\varepsilon_0}\,\frac{\lambda}{z^2}\int_0^{\theta_{max}} \cos\theta\cdot d\theta$$

$$E = \frac{\lambda}{z^2\, 2\pi\varepsilon_0}\,[\sin\theta]_0^{\theta_{max}}$$

$$E = \frac{\lambda}{z^2\, 2\pi\varepsilon_0}\,[\sin\theta_{max}]$$

$$\sin\theta_{max} = \frac{L}{\sqrt{L^2+z^2}}$$

$$E = \frac{\lambda}{z^2\, 2\pi\varepsilon_0}\left(\frac{L}{\sqrt{L^2+z^2}}\right).$$

$$E = \frac{\lambda L}{2\pi\varepsilon_0\sqrt{L^2+z^2}\cdot z^2}.$$

* $\vec{E}$ due to charged disc.

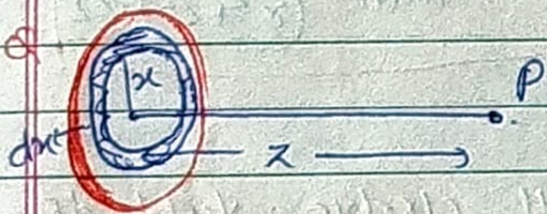
→ Consider a disk of Radius 'R' having a charge of Q distributed uniformly on its surface.

Charge = Q

Let Surface charge density of the solid disk is 'σ'

then $\sigma = \dfrac{Q}{\text{area of disk}} = \dfrac{Q}{\pi R^2}$

$$\boxed{\sigma = \frac{Q}{\pi R^2}} \quad \text{—— (i)}$$

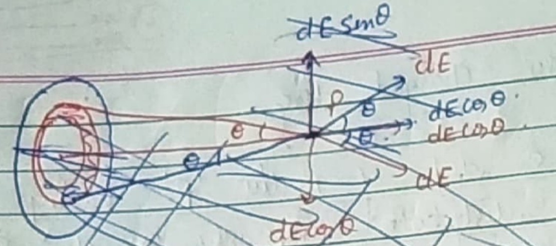


Consider a thin ring of radius x and thickness 'dx'.

So let 'dq' be the charge on thin ring.

$dq = \sigma \times \text{area of Ring} = \sigma \times 2\pi x \cdot dx$

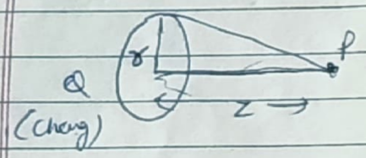
$$\boxed{dq = \sigma 2\pi x\, dx} \quad \text{—— (ii)}$$



'Sinθ' Component cancelled each other so the net Electric field is due to the 'Cosθ' Component.

$dE \cos\theta$.

We knew that electric field due to charged Ring on the axis is.



$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Qz}{(r^2+z^2)^{3/2}}}$$

from this. the Small electric field dE due to the ring

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq \cdot z}{(x^2+z^2)^{3/2}}$$

from (ii) $dq = \sigma 2\pi x \, dx$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{z \cdot \sigma 2\pi x \, dx}{(x^2+z^2)^{3/2}}$$

$$E = \int dE = \frac{1}{4\pi\varepsilon_0} \sigma 2\pi z \int_0^R \frac{x\,dx}{(x^2+z^2)^{3/2}}$$

$$E = \frac{\sigma 2\pi z}{4\pi\varepsilon_0} \int_0^R \frac{x\,dx}{(x^2+z^2)^{3/2}}$$

Put $x^2+z^2 = t$.

$2x\,dx = dt$. $\Rightarrow$ $\boxed{x\,dx = \frac{dt}{2}}$.

when $x = 0 \Rightarrow t = z^2$

" $x = R \Rightarrow t = R^2+z^2$.

$$E = \frac{\sigma z}{2\varepsilon_0} \int_{z^2}^{R^2+z^2} \frac{dt}{2 \cdot (t)^{3/2}}.$$

$$E = \frac{\sigma z}{4\varepsilon_0} \int_{z^2}^{R^2+z^2} (t)^{-3/2} \cdot dt.$$

$$E = \frac{\sigma z}{4\varepsilon_0} \left[\frac{(t)^{-1/2}}{-1/2}\right]_{z^2}^{R^2+z^2} \qquad \therefore \int t^{-3/2} = \frac{t^{-3/2+1}}{\frac{-3}{2}+1}$$

$$E = \frac{\sigma z}{2\varepsilon_0} - \left[\frac{1}{\sqrt{t}}\right]_{z^2}^{R^2+z^2}$$

$$E = \frac{\sigma z}{2\varepsilon_0} - \left[\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{\sqrt{z^2}}\right]$$

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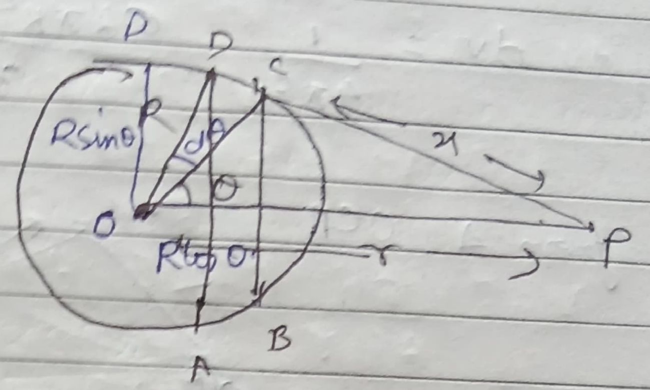
$$E = \frac{\sigma Z}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right)$$

$$\boxed{E = \frac{\sigma Z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right]}$$

** Potential due uniform charged spherical ~~sh~~ shell.



⇒ Let consider the spherical shell is made up of ~~lar~~ large no. of thin Rings consider one such ring of ~~Radius~~ thickness DC which is x distance from point P.

$\angle DOC = d\theta$

~~as~~ angle = $\frac{\text{arc}}{\text{radius}}$

arc = DC = $R d\theta$ ⟶ Thickness.

Radius of Ring = OP = $R \sin\theta$.

Let σ be the surface charge density

$$\sigma = \frac{\theta}{4\pi R^2}$$

area of the ring = 2π (radius) × Thickness

$= 2\pi R \sin\theta \cdot R d\theta$.

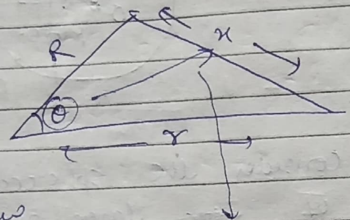
$$\boxed{\text{Charge on ring } dq = \sigma\, 2\pi R^2 \sin\theta \cdot d\theta}$$

then

Electric potential due to ring $= \frac{k\,dq}{x}$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi R^2 \sin\theta \cdot d\theta}{x} \quad \text{— (i)}$$

from Δ OCP



use cosene law

$$\cos\theta = \frac{R^2 + r^2 - x^2}{2Rr}$$

$$2Rr\cos\theta = R^2 + r^2 - x^2$$

differentiate both sides.

$$-\cancel{2}Rr\sin\theta \cdot d\theta = -\cancel{2}x\,dx \quad \because R, r \text{ are constant}$$

$$\sin\theta \cdot d\theta = \frac{x\,dx}{Rr}$$

Put $\sin\theta\, d\theta$ in eq (i)

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi R^2}{x}\frac{x\,dx}{Rr}$$

$$\boxed{dV = \frac{\sigma R}{2\varepsilon_0}\frac{R}{r}\,dx} \quad \text{— (ii)}$$

potential for comple shell integrate eq (ii) from $r-R$ to $r+R$.

$$\int dV = V = \int_{r-R}^{r+R} \frac{\sigma R}{2\varepsilon_0 r}\,dx.$$

$$\frac{\sigma R}{2\varepsilon_0 r}\left[x\right]_{r-R}^{r+R}$$

$$V = \frac{\sigma R}{2\varepsilon_0 r}\left[r + R - r + R\right]$$

$$V = \frac{\sigma 2R^2}{2\varepsilon_0 r} = \frac{\sigma R^2}{\varepsilon_0 r}$$

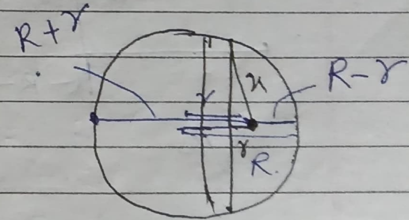
multiply 4π on numerator & denominator

$$V = \frac{\sigma R^2 \times 4\pi}{4\pi\varepsilon_0 r} = \frac{(4\pi R^2\sigma)}{4\pi\varepsilon_0 r} \rightarrow Q.$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

for the point outside the sphere ~~sh~~ charged sphere acts as point charge.

for inside



take eq (ii).

$$dV = \frac{\sigma R}{2\varepsilon_0 r}\,dx$$

we Integrate from $R-r$ to $R+r$.

$$\int dV = V = \int_{R-r}^{R+r} \frac{\sigma R}{2\varepsilon_0 r}\, dx$$

$$V = \frac{\sigma R}{2\varepsilon_0 r}\int_{R-r}^{R+r} dx = \frac{\sigma R}{2\varepsilon_0 r}\left[x\right]_{R-r}^{R+r}$$

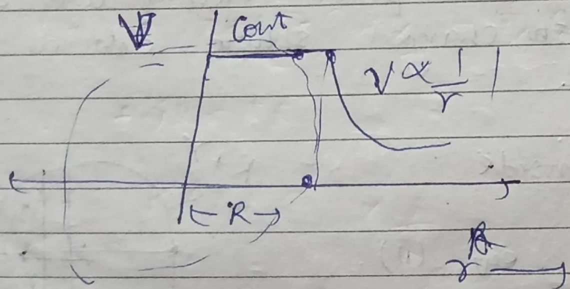
$$V = \frac{\sigma R}{2\varepsilon_0 r}\left(R+r-R+r\right)$$

$$V = \frac{\sigma R}{2\varepsilon_0}\frac{2r}{r} \qquad \frac{\sigma R}{\varepsilon_0}$$

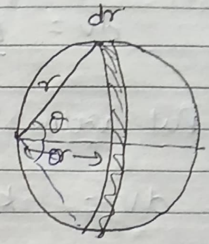
multiply $\frac{4\pi R}{4\pi R}$ — Q

$$V = \frac{(4\pi R^2\sigma)}{4\pi\varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}.$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \longrightarrow \text{Constant.}$$



Potential(V) at the edge of the ~~disc~~ charged disk (P4Q)



~~due to the~~

let ~~the~~ R be the Radius of disc and Q be the charge distributed on it

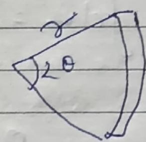
let σ be surface charg density

$$\sigma = \frac{Q}{2\pi R}.$$

Charge on the consider Ring = dQ

$$dQ = \cancel{\sigma\, 2\pi x}\, dr$$

$$\boxed{\cancel{dq = \sigma\, 2\pi r\, dr}}$$



- length of arc $= r2\theta$

area $= r2\theta \cdot dr$

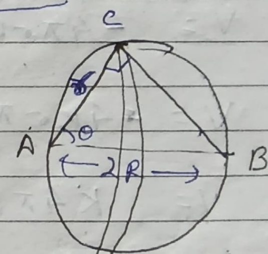
$dq = \sigma \times \text{area} = \sigma 2\theta r\, dr.$

$$\boxed{dq = \sigma 2\theta r dr}$$

from ΔABC

$$\cos\theta = \frac{AC}{AB}$$

$$\cos\theta = \frac{r}{2R}.$$



differentiate

$$-\sin\theta\, d\theta = \frac{dr}{2R}$$

$$-\sin\theta\, d\theta = \frac{dr}{2R} \qquad \boxed{dr = -2R\sin\theta\, d\theta}$$

Potential is given by

$$dV = \frac{k\,dq}{r} = \frac{K\, 2\sigma\theta r\, dr}{r}$$

Put value of dr.

$$dV = k2\sigma\theta\,(-2R\sin\theta \cdot d\theta)$$

$$dV = -4\pi\sigma\,(\theta \cdot \sin\theta) \cdot d\theta$$

for the complete disk we integrate from 90 to 0.

$$\int dV = \int_{\pi/2}^{0} -4\pi\sigma\ \theta\sin\theta \cdot d\theta$$

by parts.

$$-\theta\cos\theta - \int -\cos\theta \cdot d\theta$$

$$V = \int_{\pi/2}^{0} -4\pi\sigma k R\,(-\theta\cos\theta + \sin\theta)$$

$$-\theta\cos\theta + \sin\theta$$

$$V = -4\pi\sigma k R\Big[\sin\theta - \theta\cos\theta\Big]_{\pi/2}^{0}$$

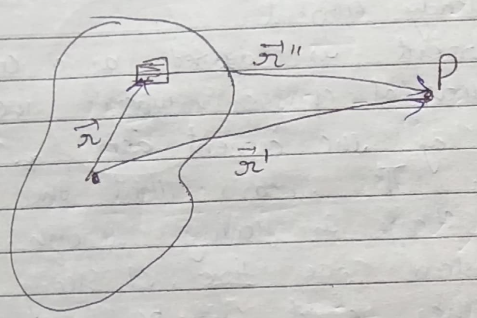
$$V = -4\pi\sigma k\left[\cdot\left(\sin\tfrac{\pi}{2} - \tfrac{\pi}{2}\cos\tfrac{\pi}{2}\right)\right].$$

$$V = 4k\sigma\pi R$$

$$V = K\,4\pi R\sigma \times \frac{R}{R}$$

$$V = \frac{k(4\pi R^2\sigma)}{R} = \frac{kQ}{R}$$

Electric potential due to arbitary charge distribution



Const a body in which charge is distributed arbitary let the charge density is ρ. Consider a element element of volume $d\tau$ and it is at a distance of $\vec{r}$ by from the origin of the body

~~we know the~~

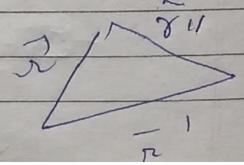
Potential due to point charge $= \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{kq}{r}$

from elementy elem $dV = \frac{k\,dq}{\vec{r}''}$

$dq = \rho\, d\tau$.

$V = k\frac{\rho\, d\tau}{\vec{r}''}$ you $\int dV = k\int\frac{\rho\, d\tau}{\vec{r}''}$

using Tri la of add.



$\vec{r} + \vec{r}'' = \vec{r}'$

$\vec{r}' = \vec{r} + \vec{r}''$ $\qquad \vec{r}'' = \vec{r}' - \vec{r}$

~~$\vec{r}'' = \vec{r}' - \vec{r}$~~

square both side (modules)

$$\vec{r}''^2 = \vec{r}'^2 + \vec{r}^2 - 2\vec{r}'\vec{r}\cos\theta.$$

$$(\vec{r}'')^2 = \vec{r}'^2\left(1 + \left(\frac{\vec{r}}{\vec{r}'}\right)^2 - 2\frac{\vec{r}}{\vec{r}'}\cos\theta\right)$$

$$(\vec{r}'')^2 = \vec{r}'^2\left[1 + \frac{\vec{r}}{\vec{r}'}\left(\frac{\vec{r}}{\vec{r}'} - 2\cos\theta\right)\right]$$

take $\frac{\vec{r}}{\vec{r}'}\left(\frac{\vec{r}}{\vec{r}'} - 2\cos\theta\right) = \varepsilon$ take $\frac{\vec{r}'}{r}\left(\frac{\vec{r}'}{r} - 2\cos\theta\right) = \varepsilon$

$$(\vec{r}'')^2 = (\vec{r}')^2(1+\varepsilon)$$

taking reciporcal

$$\left(\frac{1}{\vec{r}''}\right)^2 = \left(\frac{1}{\vec{r}'}\right)^2(1+\varepsilon)^{-1}$$

$$\frac{1}{\vec{r}''} = \frac{1}{\vec{r}'}(1+\varepsilon)^{-1/2}$$

using benomial theorm.

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2} + \frac{n(n-1)(n-2)}{6}x^3 + \ldots$$

$$(1+\varepsilon)^{-1/2} = 1 - \frac{x}{2} + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \ldots$$

$$\varepsilon = \frac{r'}{r}\left(\frac{r'}{r} - 2\cos\theta\right)$$

$$\varepsilon = \frac{r'}{r}\left(\frac{r'}{r} - 2\cos\theta\right)$$

$$(1+\varepsilon)^{-1/2} = 1 - \frac{\varepsilon}{2} + \frac{3}{8}\varepsilon^2 - \frac{5}{16}\varepsilon^3 + \cdots$$

$$= 1 - \frac{1}{2}\left(\frac{r'}{r}\left(\frac{r'}{r} - 2\cos\theta\right)\right) + \frac{3}{8}\left(\left(\frac{r'}{r}\right)^2\left(\frac{r'}{r} - 2\cos\theta\right)^2\right.$$

$$\left. - \frac{5}{16}\left(\left(\frac{r'}{r}\right)^3\left(\frac{r'}{r} - 2\cos\theta\right)^3\right]\right.$$

$$= 1 - \frac{1}{2}\left(\frac{r'}{r}\right)^2 + \cos\theta + \frac{3}{8}\left(\frac{r'}{r}\right)^2\left[\left(\frac{r'}{r}\right)^2 + 4\cos^2\theta - 4\frac{r'}{r}\cos\theta\right.$$

$$- \frac{5}{16}\left(\frac{r'}{r}\right)^3\left[\left(\frac{r'}{r}\right)^2 - 8\cos^3\theta + 3\left(\frac{r'}{r}\right)4\cos^2\theta - 3\left(\frac{r'}{r}\right)^2 2\cos\theta\right)$$

$$1 - \frac{1}{2}\left(\frac{r'}{r}\right)^2 + \frac{r'}{r}\cos\theta + \frac{3}{8}\left(\frac{r'}{r}\right)^4 + \frac{3}{2}\cos^2\theta - \frac{3}{2}\left(\frac{r'}{r}\right)^3\cos\theta$$

$$-\frac{5}{16}\left(\frac{r'}{r}\right)^5 + \frac{5}{2}\left(\frac{r'}{r}\right)^3\cos^3\theta - \frac{45}{4}$$

on negle higher ter

$$\frac{1}{r''} = \frac{1}{r}\left[\left(\frac{r'}{r}\right)^0 + \left(\frac{r'}{r}\right)^1\cos\theta + \left(\frac{r'}{r}\right)^2\left(\frac{3}{2}\cos\theta - \frac{1}{2}\right)\right.$$

$$+ \left(\frac{r'}{r}\right)^3\left(\frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta\right)$$

$$+ \cdots$$

now coeffeceants of $\frac{r'}{r}$ are legendre polegnomial which can be expressed as

$$\frac{1}{r''} = \frac{1}{r}\sum_{n=0}^{n}\left(\frac{r'}{r}\right)^n P_n\cos\theta$$

whe $P_n\cos\theta$ are legendre polynomial

$P_1\cos\theta = \cos\theta$ $\quad P_0\cos\theta = 1$

$P_2\cos\theta = \left(\frac{3}{2}\cos\theta - \frac{1}{2}\right)$

$P_3\cos\theta = \left(\frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta\right)$

Put in (1)

$$V(P) = k\int \rho\, dz\left[\frac{1}{r}\sum_{n=0}^{n}\left(\frac{r'}{r}\right)^n P_n\cos\theta\right]$$