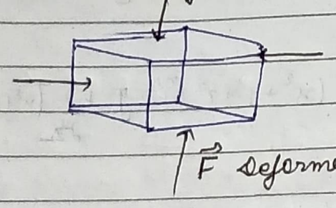
Properties of Matter (unit 5).

⇒ 

$\vec{F}$ deforming Force.

Elasticity :- The property by which the, solid regain or tends to regain the orignal configuration on removal of deforming Force is called Elasticity.

→ Restoring Force :- " The internal Force by which the body regain orignal configuration on removal of deforming force is called Restoring force.

⇒ Stress :- " The internal restoring force per unit area of body is called stress..

$$\text{Stress} = \frac{\text{Restoring Force}}{\text{Area}} = \frac{F}{A} \quad \text{unit } N/m^2$$

Types of Stress

- Normal stress
- Tangential Stress
- Shering Stress.

Normal Stress :- The internal restoring force per unit area normal to the surface of body is called Normal stress.

Tangential Stress :- The internal restoring force per unit area along the Tangent to the Surface of body is called Tangential Stress

⇒ Strain :- It is defined as the ratio of change in configuration & orignal configuration

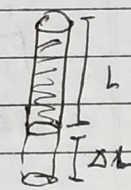
$$\text{Strain} :- \frac{\text{Change in confi.}}{\text{Orignal confi.}}$$ $\boxed{\text{No unit}}$

Types of Strain

- Longitudinal Strain
- Volumetric Strain
- Shering Strain.

⇒ Longitudinal Strain :- Ratio of change in length & orignal length

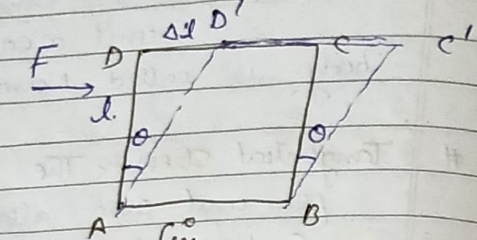
$$\text{Long. Strain} = \frac{\Delta L}{L}$$



→ Volumatric Strain :- Ratio of change in Volume & orignal volume

$$\boxed{\text{Vol. Str.} = \frac{\Delta V}{V}}$$

⇒ Shering Strain :-



in Δ DD'A & CC'B

$\tan\theta = \frac{\Delta l}{l}$

fig

Since θ is very Small

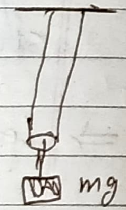
$\tan\theta \approx \theta$

$\boxed{\theta = \frac{\Delta l}{l}}$

* defined as the angle by which the side which was ⊥ to the fixed side get turned on applying Tangential stress.

⇒ Elastic limit :- —

The limit up to which the body regain orignal configuration completely on removal of deforming force is called Elastic limit.



Hooke's Law :-

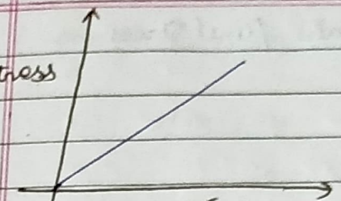
within elastic limit Stress is directly proportional to the strain

Stress ∝ Strain

Stress = E Strain

E → modulus of elasticity / Coefficient of elasticity

$E = \frac{\text{Stress}}{\text{Strain}}$ (N/m²)



⇒ Elastic Constants :-

There are four Elastic constant.

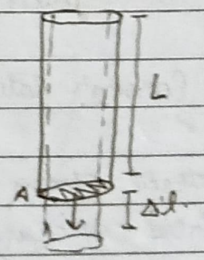
1. Young's Modulus (Y)
2. Bulk Modulus (K)
3. Modulus of rigidity or Shear modulus (η)
4. Poisson's Ratio (σ).

⇒ Young's Modulus :-

defined as the ratio of Normal stress & longitudnal Strain

$Y = \frac{\text{Normal Stress}}{\text{longitudnal Strain}}$

if $\vec{F}$ is the force applied normal to surface of area A.



• Normal stress $= \frac{\vec{F}}{A}$

• longitudal Strain $= \frac{\Delta L}{L}$

$Y = \frac{\vec{F}\ L}{A \cdot \Delta L} = \frac{\vec{F} \cdot L}{A \cdot \Delta L}$

If α is the longitudinal Strain / unit Stress

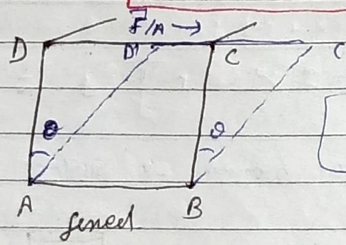
$$\boxed{Y = \frac{1}{\alpha}} \quad N/m^2$$

→ Bulk Modulus :- It is defined as the ratio of normal stress and volumetric strain.

$$\boxed{K = \frac{\text{normal Stress}}{\text{Volumetric strain}}} = \frac{F}{A} \frac{V}{\Delta V} = \frac{\vec{F} \cdot V}{A \cdot \Delta V} \quad N/m^2$$

→ Modulus of Rigidity :- Ratio of Tangential stress & shearing strain
or Shear Modulus :

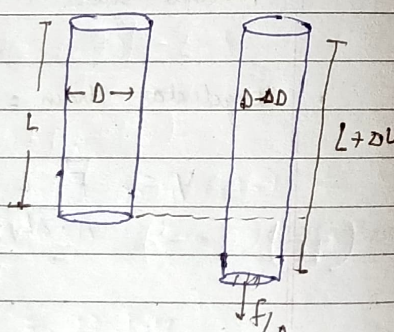
$$\boxed{\eta = \frac{\text{Tangential stress}}{\text{~~Volumetric~~ Shearing strain}}}$$



$$\boxed{\eta = \frac{F}{A \cdot \phi} \quad N/m^2}$$

→ Poisson's Ratio :-

Lateral Strain :-
"Ratio of change in configuration and original configuration in ⊥er direction of stress"



Lateral Strain :- $\frac{\Delta D}{D}$

Longitudinal Strain :- Ratio of change in configuration to original config. in the direction of stress

$$\text{Long. Strain} = \frac{\Delta L}{L}$$

→ The ratio of Lateral Strain and Longitudinal Strain is the poissons Ratio

$$\sigma = \frac{\text{Lateral Strain}}{\text{Long. Strain}} = -\left(\frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}}\right)$$

(increase in dimension in one direction thus decrease in dimension in other direction)

$$\boxed{\sigma = -\left(\frac{\Delta D}{\Delta L}\right)\left(\frac{L}{D}\right)}$$

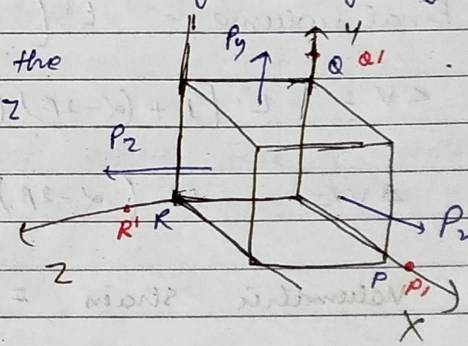
Relationship b/w Elastic Constants

⇒ Relationship b/w Y, K, and σ

$$Y = 3K(1 - 2\sigma)$$

Proof :- Consider a cubical Body each of length l

Let P_x, P_y and P_z are the stress applied in X, Y, Z direction respectively



Orignal volume of cube = L^3

If $\alpha \to$ Longitudinal Strain per unit Stress.
β – Lateral Strain per unit stress.

** Stress produced extansion in their own direction & compression in $\perp$ direction

final volume after stress = $OP' \times OQ'\ OZ'$

$OP' = \cancel{[1+\alpha]}\ L[1+\alpha P_x - \beta P_y - \beta P_z]$

$OQ' = L[1+\alpha P_y - \beta P_x - \beta P_z]$

$\cancel{OQ}\ OR' = L[1+\alpha P_z - \beta P_x - \beta P_y]$

Final volume of cuboid body

$= L[1+\alpha P_x - \beta P_y - \beta P_z] \times L[1+\alpha P_y - \beta P_x - \beta P_z]$
$L[1+\alpha P_z - \beta P_x - \beta P_y]$

$= L^3 [1 + (\alpha - 2\beta)(P_x + P_y + P_z)]$

⇒ after neglecting higher order terms $\alpha^2/\beta^2/\beta^3/\alpha^3$..

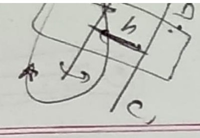
Final volume = $L^3 [1 + (\alpha - 2\beta)(P_x + P_y + P_z)]$

$\Delta V = L^3 [1 + (\alpha - 2\beta)(P_x + P_y + P_z)] - L^3$

$\Delta V = L^3 (\alpha - 2\beta)(P_x + P_y + P_z)$

Volumetric strain $= \frac{\Delta V}{V} = (\alpha - 2\beta)(3P)$

$P_x = P_y = P_z = P$.



$I_{AB} = I_{BC} + M h^2$

Bulk modulus $= K = \frac{\text{Normal Stress}}{\text{Volu. Strain}} = \frac{P}{(\alpha - 2\beta) 3P}$

$K = \frac{1}{3(\alpha - 2\beta)}$

$K = \frac{1}{3\alpha\left(1 - 2\frac{\beta}{\alpha}\right)}$

$\frac{1}{\alpha} = \frac{\text{Stress}}{\text{Logt. Strain}} = Y.$

$\frac{\beta}{\alpha} =$ Poissions Ratio $= \sigma$

$K = \frac{Y}{3(1-2\sigma)}.$

$$\boxed{Y = 3K(1-2\sigma)}$$

Relationship btw elastic constants &c.

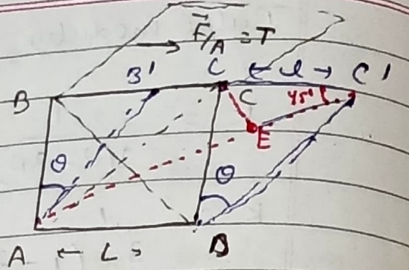
Relationship btw Y, η & σ;

$\eta = \frac{\text{Tangential Stress}}{\text{Shering Strain}}.$

Consider a cubical body each of length L.

Let $\frac{\vec{F}}{A} = \frac{\vec{F}}{L^2} = T$ is Tangential Stress

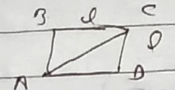
$\theta \rightarrow$ Shearing strain



Apply tangential stress due to it the body gets deformed.

Now, modulus of Rigidity $\rightarrow \boxed{\eta = \frac{T}{\theta}}$ — (i)

$\rightarrow$ initial

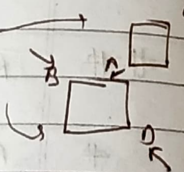


$AC^2 = L^2 + L^2 = 2L^2 \Rightarrow \boxed{AC = L\sqrt{2}}$

After deformation, the increase in AC is due to.

(I) Extensional Stress along AC.

(II) Compressional " " BD.



If α - Longitu. strain / unit stress

β - Lateral " / unit stress

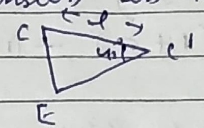
Now Extention in AC = $T \cdot \alpha \cdot (AC) + T \cdot \beta (AC)$

$= T(\alpha + \beta) \cdot AC$

$\boxed{AC = L\sqrt{2}}$

$= \boxed{T(\alpha + \beta) L\sqrt{2}}$ — (ii)

Also from fig extension is AC is EC'



$\cos 45^\circ = \frac{EC'}{l}$

$\boxed{EC' = \frac{l}{\sqrt{2}}}$ — (iii) it is also extention is AC

So we can equate eq (ii) & (iii)

$T(\alpha + \beta) L\sqrt{2} = \frac{l}{\sqrt{2}}$

$T \cdot (\alpha + \beta) L \cdot 2 = l$

$\frac{T \cdot 2}{\left(\frac{l}{L}\right)} = \frac{1}{\alpha + \beta}$

$\frac{T \cdot 2}{\left(\frac{l}{L}\right)} = \frac{1}{\alpha} \cdot \frac{1}{\left(1 + \frac{\beta}{\alpha}\right)}$

$\frac{l}{L} = \frac{\text{Change in Leng}}{\text{orignal Leng}} = \theta \rightarrow$ Shering strain

$\frac{1}{\alpha} = Y \cdot \frac{\beta}{\alpha} = \sigma$

$\left(\frac{T}{\theta}\right) \cdot 2 = \frac{Y}{(1 + \sigma)}$

$2 \cdot \eta = \frac{Y}{(1 + \sigma)} \Rightarrow \boxed{Y = 2\eta (1 + \sigma)}$

modulus of Rigidity

Poisson's

Date ____ Page ____

$\Rightarrow$ $Y = 3K(1-2\sigma)$ — (i)
$Y = 2\eta(1+\sigma)$ — (ii)

★ Relationship b/w K, η, σ :-

Since we know that
$Y = 3K(1-2\sigma)$ & $Y = 2\eta(1+\sigma)$

where Y - Young's Mod. σ - Poisson.
K - Bulk Mod. η - modulus of rigidity
shear modulus

divide eq (i) & (ii)

$$1 = \frac{3}{2}\frac{K(1-2\sigma)}{\eta(1+\sigma)}$$

$$\boxed{K = \frac{2\eta(1+\sigma)}{3(1-2\sigma)}} \quad \text{— (iii)}$$

& $$\boxed{\eta = \frac{3K(1-2\sigma)}{2(1+\sigma)}} \quad \text{(iv)}$$

→ From (iii) $3K - 6K\sigma = 2\eta + 2\sigma\eta$
$3K - 2\eta = \sigma(2\eta + 6K)$

$$\boxed{\sigma = \frac{3K - 2\eta}{2\eta + 6K}}$$

Date ____ Page ____

$\Rightarrow$ Relationship b/w Y, K & η :-

$Y = 3K(1-2\sigma)$ — (i)
$Y = 2\eta(1+\sigma)$ — (ii)

Eq (i) can be $\quad \frac{Y}{3K} = 1 - 2\sigma$

Eq (ii) $\quad \frac{Y}{2\eta} = 1 + \sigma \quad \times 2$
[multiply by 2 & add to above eq]

$$\frac{2Y}{\eta} = 2 + 2\sigma \quad \text{— (iv)}$$

$$\Rightarrow Y\left[\frac{1}{3K} + \frac{1}{\eta}\right] = 3$$

$$= Y\,\frac{\eta + 3K}{3K\eta} = 3$$

$$= Y(\eta + 3K) = 9K\eta \quad \text{— (v)}$$

$$\boxed{Y = \frac{9K\eta}{\eta + 3K}}$$

in eq (v) we can divide by $(YK\eta)$

$$\boxed{\frac{1}{K} + \frac{3}{\eta} = \frac{9}{Y}}$$ Imp

Limiting values of σ :-

→ using the relations
$Y = 3K(1-2\sigma)$
$Y = 2\eta(1+\sigma)$

⇒ when the value of σ is $-ve$.

then $1 - 2(\sigma) > 0$
$1 > 2(-\sigma)$
$2\sigma < 1$
$\boxed{\sigma < \frac{1}{2}}$

⇒ when the value of σ is $+ve$.
$1 + \sigma > 0$
$\sigma > -1$

The limiting value of σ is.
$\therefore \boxed{-1 < \sigma < \frac{1}{2}}$

If $\sigma = \frac{1}{2}$

$K = \dfrac{Y}{3(1-2\sigma)} = \dfrac{Y}{0} = \infty$

Compressibility $\frac{1}{K} = \frac{1}{\infty} = 0$

[Means Compressibility is 0 which is never valid / Possible]

If the value of (σ) is $-ve$
mean as the extension in one direction also increases the entention take place in $\perp^r$ direction (which is impossible)

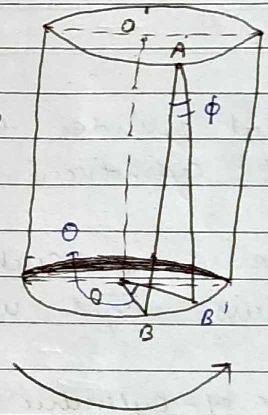
Hence σ can't be negative
so $\sigma > 0$

$\left[\ 0 < \sigma < \frac{1}{2}\ \right]$

{For most of the solids the value of σ lies from 0.2 to 0.4}

★ Difference b/w Angle of Shear and Angle of Twist :-

Consider a cylinder



Let AB is the reference line or generating line.

* it is fixed from upper end & twisted from lower end.

Twist

→ The reference line get twisted from AB to AB'

$\angle BOB' = \theta$ angle of twist

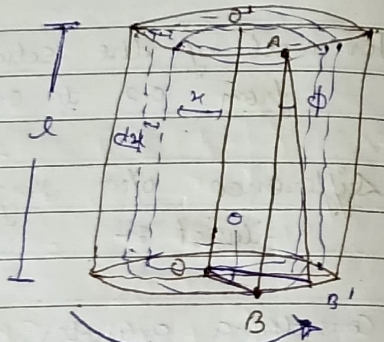
$\angle BAB' = \phi$ is the angle of shear or shearing strain

Angle of twist decreases as one move towards fixed end. & Angle of shear is indepen. of cross section.

⇒ Torque Required in twisting a cylinder

(1) Solid cylinder

Fixed



Twisting Torque.

$\angle BOB' = \theta$ = angle of Twist

ϕ → angle of shearing strain

∴ The Solid cylinder is supposed to be made up of cylindrical shells.

Consider one such cylindrical shell of radius $\boxed{x}$ & width $\boxed{dx}$.

Area of cylinder shell = $2\pi x\,dx$

Tangential stress = $\frac{F}{A} = \frac{F}{2\pi x\,dx}$

Shearing strain = ϕ

Modulus of rigidity = η = $\frac{\text{Tangential stress}}{\text{shearing strain}}$

$= \eta = \frac{F}{2\pi x\,dx\,\phi}$

$$\boxed{F = \eta \cdot 2\pi x\,dx \cdot \phi} \quad \text{— (1)}$$

Torque required in twisting cylindrical shell

$d\tau = F \times \perp$ dist.

$d\tau = \eta\, 2\pi x\,dx \cdot x\phi$ — (1)

arc = angle × radius

In Δ BOB' $\quad \theta = \cancel{BB'} x \quad BB' = \theta \times x$

In Δ BAB' $\quad BB' = l\phi$

$\theta \cdot x = l \cdot \phi$

$$\boxed{\phi = \frac{x \cdot \theta}{l}}$$

$d\tau$ $F = 2\pi\eta \frac{x\theta}{l} x^2 dx = \frac{2\pi\eta\theta}{l} x^3 dx$

$$\boxed{d\tau = \frac{2\pi\eta\theta}{l} x^3 \cdot dx}$$

In order to calculate the torque required in twi solid cylinder we intergrate eq (2) from x=

$$T = \frac{2\pi\eta\theta}{l} \int_0^r x^3 dx = \frac{2\pi\eta\theta}{l}\frac{r}{4}$$

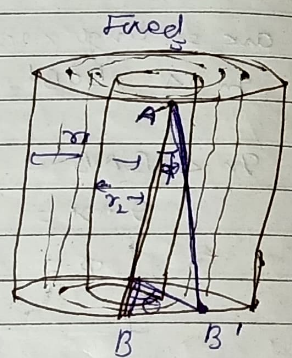
$$\boxed{\tau = \frac{\pi \eta \theta}{2l} \cdot r^4}$$

$\tau \propto r^4$
$\propto \theta$
$\propto \frac{1}{l}$

" Torque Required / unit angular twist is called Tortional Rigidity

$$\boxed{\frac{\tau}{\theta} = \frac{\pi \eta r^4}{2l}}$$

→ Hollow cylender



$$d\tau = \frac{2\pi\eta\theta}{l} x^3 dx$$

in order to calculate the expression of torque required in twisting hollow cylender, we integrate eq (2) from $x = r_2$ to $x = r_1$

$$\int d\tau = \tau = \int_{r_2}^{r_1} \frac{2\pi\eta\theta}{l}\left(x^3\right) = \frac{2\pi\eta\theta}{l}\left[\frac{x^4}{4}\right]_{r_2}^{r_1}$$

$$= \frac{2\pi\eta\theta}{l}\left[\frac{r_1^4}{4} - \frac{r_2^4}{4}\right]$$

$$\tau_{\text{hollow cylender}} = \frac{\pi\eta\theta}{2l}\left(r_1^4 - r_2^4\right)$$

Prove that hollow cylinder (shaft) is more stronger than solid cylinder or shaft of same mass & dimension

$$\tau_H = \frac{\pi\eta\theta}{2l}\left(r_1^4 - r_2^4\right) \quad \text{— (1)}$$

$$\tau_S = \frac{\pi\eta\theta r^4}{2l} \quad \text{— (2)}$$

Eq (1) ÷ Eq (2)

$$\frac{\tau_H}{\tau_S} = \frac{r_1^4 - r_2^4}{r^2}$$

So density of material is same

$$M_H = \pi(r_1^2 - r_2^2)\rho = \pi r^2 \rho$$

$$r_1^2 - r_2^2 = r^2$$

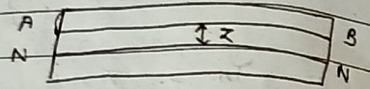
$$\frac{\tau_H}{\tau_S} = \frac{(r_1^2 + r_2^2)\cancel{(r_1^2 - r_2^2)}}{\cancel{r_1^2 - r_2^2}}$$

$$\tau_H = (r_1^2 + r_2^2)\tau_S$$

$$\boxed{\tau_H > \tau_S}$$

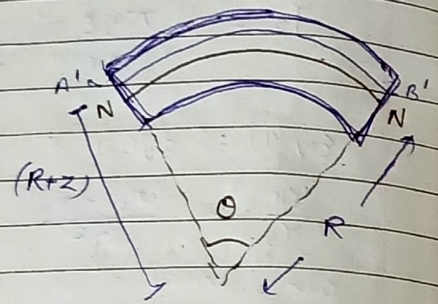
Greater torque is required to twist a hollow cylinder.

Bending Moment :-



NN → Neutral axis

AB = NN is the axis at dist. z from neutral axis.



Arc = R × Angle

$A'B' = (R+z)\cdot\theta$

$NN = AB = \theta \cdot R$

increase in length $= A'B' - AB = \theta[R+z-R] = \theta z$

$\boxed{\text{Increase in length} = \theta z}$

Original length $= \theta R$

Long. strain $= \frac{\Delta l}{l} = \frac{\theta z}{\theta R} = \frac{z}{R}$

Now young's modulus $= \frac{\text{N. Stress}}{\text{Long. strain}} = \frac{\vec{F}/a}{z/R}$

$Y = \frac{\vec{F}}{a} \cdot \frac{R}{z}$

$\vec{F} = \frac{Yaz}{R}$

for complete beam $\vec{F} = \sum \frac{Yaz}{R}$

Now. Moment of force / Bending moment

$Y = \vec{F} \times \perp \text{dis.} = \vec{F} \cdot z$

$Y = \sum \frac{Yaz}{R} \cdot z = \sum \frac{Yaz^2}{R}$

$Y = \frac{Y}{R} \sum az^2 = \frac{YI_g}{R}$

$\boxed{\sum az^2 = I_g}$ → Geometrical M.I.

(i) for rectangular beam of breadth b & thickness d.

$I_g = Ak^2$ ↳ Area of cross section

$A = b \cdot d$

$I = Mk^2 = \frac{Md^2}{12}$.

$\boxed{k^2 = \frac{d^2}{12}}$ $\quad I_g = \frac{bd \cdot d^2}{12}$

$\boxed{I_g = \frac{bd^3}{12}}$

$\boxed{Y = \frac{Ybd^2}{12R}}$

(ii) for a beam of circular crosssection of radius r.

$A = \pi r^2$

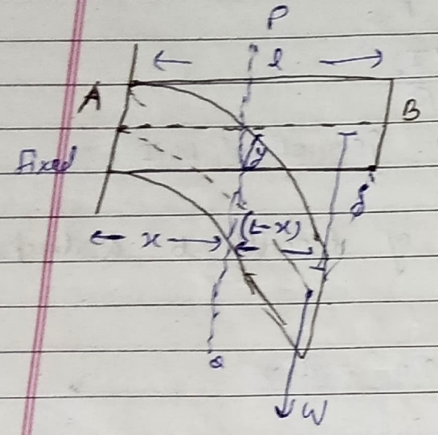
$I = Mk^2 = \frac{Mr^2}{4}$. $\quad k^2 = \frac{r^2}{4}$

$I_g = \pi r^2 \cdot \frac{r^2}{4} = \frac{\pi r^4}{4}$

$\boxed{Y = \frac{Y\pi r^4}{4R}}$

Cantilever :-

It is a beam of uniform area of cross. section which is fixed horizontally from one end & loaded at other.



Consider a cantilever of length l which is fixed at end A & loaded at end B.

① Expression of depression at loaded end when weight of cantilever is neglect

Consider a section PQ at distance x from fixed end.

Bending moment of W about PQ

$$Y = \text{Force} \times \perp \text{distance}$$

$$Y = W(l-x)$$

If Y - Young's Modulus of beam,
I_g - Geometrical M.I
R - radius of curvature

$$Y = \frac{Y I_g}{R} \quad \text{— (ii)}$$

On equating two eq

$$\frac{Y I_g}{R} = W(l-x)$$

$$\frac{1}{R} = \frac{W}{Y I_g}(l-x)$$

from differential calculus

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{Y I_g}\left[W(l-x)\right] \quad \text{— (iii)}$$

Integrate eq (iii)

$$\frac{dy}{dx} = \frac{1}{Y I_g}\int W(l-x)$$

Const- of Integration

$$\frac{dy}{dx} = \frac{1}{Y I_g} W\left(lx - \frac{x^2}{2}\right) + C_1$$

at fixed end $x = 0$ means change $\frac{dy}{dx} = 0$

$$C_1 = 0$$

$$dy = \frac{1}{Y I_g} W\left[lx - \frac{x^2}{2}\right] \cdot dx$$

$$y = \frac{1}{Y I_g} W\left[\frac{lx^2}{2} - \frac{x^3}{6}\right] + C_2$$

at fixed end $x = 0$ $y = 0$ $C_2 = 0$

Eq giv. depression at section PQ

To find equation at loaded end put $x = l$, $y = \delta$

$$y = \frac{1}{YI_g}\left[W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)\right]$$

$$y = \delta = \frac{1}{YI_g}\left[W\left[\frac{l^3}{2} - \frac{l^3}{6}\right]\right]$$

$$\delta = \frac{W}{YI_g}\frac{l^3}{3}$$

$$\delta = \frac{1}{3}\frac{Wl^3}{YI_g}$$

$$\boxed{\delta = \frac{Wl^3}{3YI_g}}$$

Case (1) for rectangular Cantilever

$$I_g = \frac{bd^3}{12}$$

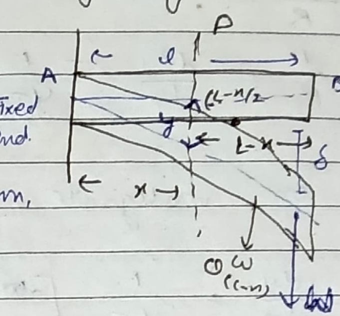
$$\delta = \frac{Wl^3}{3Ybd^3} 12 = \frac{4Wl^3}{Ybd^3}$$

Case (11) Circular cantilever

$$I_g = \frac{\pi r^4}{4} \quad \delta = \frac{Wl^3}{3Y\pi r^4} 4 = \boxed{\frac{4}{3}\frac{Wl^3}{Y\pi r^4}}$$

Weight of beam is not neglecting

W → weight loaded at end B.



ω → weight / unit length of beam.

ωl → weight of beam.

weigh of beam of length $(l-x)$ is $= \omega\cdot(l-x)$

- Moment of beam is due to weight loaded & weight of beam.

B.M = moment of W about PQ + M of $\omega(l-x)$ about PQ

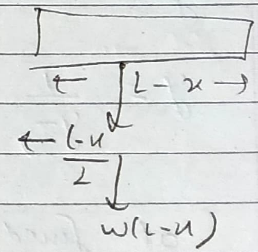
$$Y = \frac{YI_g}{R} \quad \text{(1)}$$

$Y =$ Force × ⊥ dis. =

[This weight is act at $\left(\frac{l-x}{2}\right)$ → center से act करता weigh

$$Y = W(l-x) + \omega(l-x)\frac{(l-x)}{2} \quad \text{-(ii)}$$

$$\boxed{\frac{YI_g}{R} = W(l-x) + \omega\frac{(l-x)^2}{2}}$$



$$\frac{YI_g}{R} = W(l-x) + \frac{\omega}{2}(l-x)^2$$

$$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{1}{YI_g}\left(W(l-x) + \frac{\omega(l-x)^2}{2}\right)$$

on integrate both side.

$$\frac{dy}{dx} = \frac{1}{YI_g}\left[W\left(lx - \frac{x^2}{2}\right) + \omega\left(lx - \frac{x^3}{3}\right)\right]$$

$$\int \frac{d^2y}{dx^2} = \int \frac{1}{YI_g}\left[W(l-x) + \omega\left[\frac{l^2+x^2-2lx}{2}\right]\right]$$

$$\frac{dy}{dx} = \frac{1}{YI_g}\left[W\left(lx - \frac{x^2}{2}\right) + \frac{\omega}{2}\left[xl^2 + \frac{x^3}{3} - lx^2\right]\right] + c_1$$

at fixed end $x = 0$ $\frac{dy}{dx} = 0$, $c_1 = 0$

$$\frac{dy}{dx} = \frac{1}{YI_g}\left[W\left(lx - \frac{x^2}{2}\right) + \frac{\omega}{2}\left(xl^2 + \frac{x^3}{3} - lx^2\right)\right]$$

again integrate

$$y = \frac{1}{YI_g}\left[W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + \frac{\omega}{2}\left[\frac{x^2l^2}{2} + \frac{x^4}{12} - \frac{lx^3}{3}\right]\right] + C_2$$

at fixed end $x = 0$ $y = 0 \Rightarrow C_2 = 0$

To find out depression at loaded end we put $x = l$ in eq.

$$\delta = \frac{1}{YI_g}\left[W\left[\frac{l^3}{2} - \frac{l^3}{6}\right] + \frac{\omega}{2}\left[\frac{l^4}{2} + \frac{l^4}{12} - \frac{l^4}{3}\right]\right]$$

$$\delta = \frac{1}{YI_g}\left[\frac{Wl^3}{3} + \frac{\omega}{2}\frac{l^4}{3}\right]$$

$$\boxed{\delta = \frac{1}{YI_g}\left[\frac{Wl^3}{3} + \frac{\omega}{2}\frac{l^4}{4}\right]}$$