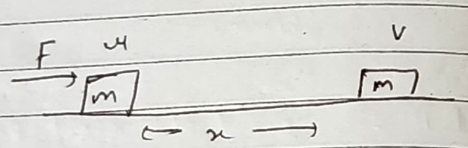
Conservation Laws :-

→ Frame of Reference & its Type

Frame of Reference is a coordinate system relative to which the position & motion of a body may be described.

Work energy theorem :-

It states that work done by a force acting on a body is equals to change in kinetic energy of body



$K_1 = \frac{1}{2} m u^2$ $\quad$ $K_2 = \frac{1}{2} m v^2$

$W = K_2 - K_1 = \frac{1}{2} m (v^2 - u^2)$

Proof

let us consider a body of mass m which is displaced from point A to point B through distance x under the action of force $\vec{F}$ [Small amount of work done in displacing the body through small distance dx is given by

$dW = F.dx$

total work done :

$W = \int_A^B F dx$ — (i)

acc. to newtons II nd Law of Motion

$F = ma = m.\frac{dv}{dt}$

Multiply by dx/dx

$F = m\left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = mv.\frac{dv}{dx}$ (velocity)

$F = mv.\frac{dv}{dx}$ ⟶ eq (ii)

$W = \int_A^B F.dx = \int_A^B mv\frac{dv}{dx}.dx$

$W = \int_A mv\,dv$

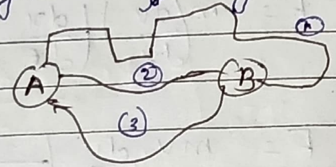
at point A velocity = u & at B v = v

$W = \int_u^v mv\,dv = \left[\frac{mv^2}{2}\right]_u^v$

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = K_B - K_A = \Delta K.}$$

Conservative & Non-Conservative forces

A force is said to be conservative if the work done by it in moving particle from one position to another position depends only on the initial and the final position & not depends on the path following by the particle



$W_1 = W_2 = W_3$

(2) also work done in a closed loop is zero.

(3) The kinetic energy of particle remains same at initial & final position

ex :- Gravitational force, ~~Ele~~ Electrostatic spring force.

Non- Conservative forces.

work done is depends on path.

ex. friction, air resistance, push-pull forces

Conservative forces as negative gradient of potential energy :-

potential energy :- $U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r}$ — (i)

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ — (ii)

$dr = dx\hat{i} + dy\hat{j} + dz\hat{k}$ — (iii)

$U = -\int_{\infty}^{r} (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$

$U(r) = -\int_{\infty}^{r} F_x dx + F_y dy + F_z dz$

$U(x,y,z) = -\int_{\infty}^{x} F_x dx - \int_{\infty}^{y} F_y dx - \int_{\infty}^{z} F_z dz$ — (iv)

partially differtial eq (iv) w.r.t x, y, z

$-\frac{\partial u}{\partial x} = +F_x \quad -\frac{\partial u}{\partial y} = +F_y \quad -\frac{\partial u}{\partial z} = F_z$

Put value of F_x, F_y, F_z in equation (ii)

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{F} = -\frac{\partial u}{\partial x}\hat{i} + -\frac{\partial u}{\partial y}\hat{j} - \frac{\partial u}{\partial z}\hat{k}$

$\vec{F} = -\left[\frac{\partial \hat{i}}{\partial x} + \frac{\partial \hat{j}}{\partial y} + \frac{\partial \hat{k}}{\partial z}\right]\cdot u.$

$\boxed{\vec{F} = -\nabla U}$ ⟶ hence proved

Poiseuille's Formula / Equation :~

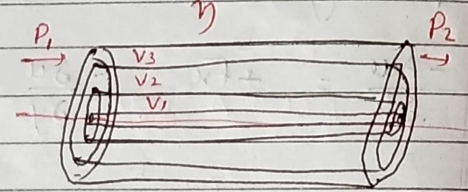
The volum (v) of the liquid flowing per second through a capillary tube varies :

(i) directly as the pressur difference (P) across the 2 ends of the tube $V \propto P$

(ii) directly as the fourth power of radius (r) of the tube $V \propto r^4$

(iii) inversely as the length (l) of the tube. $V \propto \frac{1}{l}$

(iv) inversely as the coefficient of viscosity of the liquid. $V \propto \frac{1}{\eta}$



$v_1 > v_2 > v_3$ $\boxed{P = P_1 - P_2}$

$\boxed{\text{at the boundary } v = 0}$

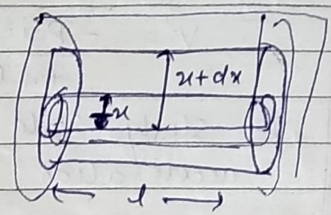
Combining the factor

$$V \propto \frac{Pr^4}{l\eta}$$

$$\boxed{V = \frac{\pi}{8} \frac{Pr^4}{\eta l}}$$

Volume of liquid flowing per second $= \boxed{V = \frac{\pi}{8} \frac{Pr^4}{\eta l}}$

⇒ derivation :~



$F \propto A$

↳ Area of the cylindrical Tube

& $F \propto \frac{dv}{dx}$ → velocity gradian

∵ $F \propto A \frac{dv}{dx}$ ⇒ $F = (-\eta) A \frac{dv}{dx}$ — (i)

[Coefficient of viscosity]

let pressure difference is P.

$$F = P \times (\text{Area}) = P \times \pi x^2 \quad \text{— (ii)}$$

equate (i) & (ii)

$$P\pi x^2 = -\eta A \frac{dv}{dx}$$

A → area of cylinder = $2\pi x l$

$$-\eta (2\pi x l) \frac{dv}{dx} = P\pi x^2$$

$$-\eta 2 l \frac{dv}{dx} = Px$$

$$dv = \frac{-Px\, dx}{\eta 2 l}$$

$$dv = \frac{-P x\, dx}{2\eta l}$$

integrate

$$V = \frac{-P x^2}{4\eta l} + C$$

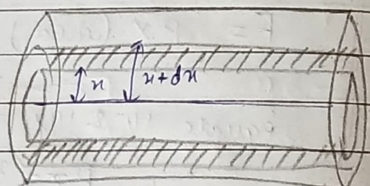
for study state $x = r$ & there is no acceleration $V = 0 \rightarrow$ [wall of the contect : $v=0$]

$$0 = \frac{-P r^2}{4\eta l} + C \Rightarrow C = \frac{P r^2}{4\eta l}$$

$$v = \frac{-P x^2}{4\eta l} + \frac{P r^2}{4\eta l} = \frac{P}{4\eta l}\left[r^2 - x^2\right]$$

$$v = \frac{P}{4\eta l}\left[r^2 - x^2\right]$$

velocity of fl



- The small volume of liquid flowing per second through the shaded region is given by

$dv =$ ~~volume~~ velocity of flow × area of shaded region

$$dv = \frac{P}{4\eta l}\left(r^2 - x^2\right) \times (2\pi x\, dx)$$

$$dv = \frac{P\pi}{2\eta l}\left[r^2 x - x^3\right] dx$$

Total volume of the liquid flowing per second through capillary tube is given by

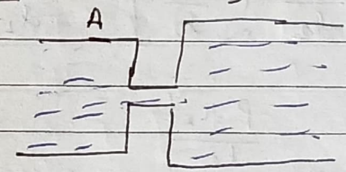
$$\int dV = \frac{P\pi}{2\eta l}\int r^2 x - \int x^3 dx = \frac{P\pi}{2\eta l}\left[\frac{r^2 x^2}{2} - \frac{x^4}{4}\right]_0^r$$

$$V = \frac{P\pi}{2\eta l}\left[\frac{r^4}{2} - \frac{r^4}{4}\right] = \frac{P\pi}{2\eta l}\left[\frac{r^4}{4}\right]$$

$$\boxed{V = \frac{\pi}{8}\frac{P r^4}{\eta l}} \rightarrow \text{volume of liquid flowin / second through capilary tuble.}$$

Rate of flow.

Capillaries is Series



If the two capallaries of length l_1, l_2 & radius r_1 & r_2 are conneil in series then obviously the rate of flow [volume of liq flowing per sec is same]

$$V_1 = V_2 = V$$

$$V_1 = \frac{\pi}{8}\frac{P_1 r_1^4}{\eta l_1} = \frac{\pi}{8}\frac{P_2 r_2^4}{\eta l_2} \quad - V_2$$

pressure differen accross the tube $P = P_1 + P_2$

$$\boxed{P_2 = P - P_1} \rightarrow eq(1)$$

$$V = \frac{\pi}{8}\frac{P_1 r_1^4}{\eta l_1} = \frac{\pi}{8}\frac{P_2 r_2^4}{\eta l_2}$$

$$\frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2} \quad \text{—— (ii)}$$

from eq (i) $P_2 = P - P_1$ put in eq (ii)

$$\frac{P_1 r_1^4}{l_1} = \frac{(P-P_1) r_2^4}{l_2} = \frac{P r_2^4}{l_2} - \frac{P_1 r_2^4}{l_2}$$

$$\frac{P_1 r_1^4}{l_2} + \frac{P_1 r_2^4}{l_2} = \frac{P r_2^4}{l_2}$$

$$P_1\left[\frac{r_1^4}{l_2} + \frac{r_2^4}{l_2}\right] = \frac{P r_2^4}{l_2}$$

$$P_1 = \frac{\frac{P r_2^4}{l_2}}{\frac{r_1^4}{l_2} + \frac{r_2^4}{l_2}}$$

putting value of P_1 is $V = \frac{\pi}{8}\frac{P_1 r_1^4}{\eta l_1}$

$$V = \frac{\pi r_1^4}{8\eta l_1}\left[\frac{\frac{P r_2^4}{l_2}}{\frac{l_2 r_1^4 + l_1 r_2^4}{l_1 l_2}}\right]$$

$$V = \frac{\pi P r_1^4 r_2^4}{8\eta\left[l_2 r_1^4 + l_1 r_2^4\right]}$$

$$V = \frac{\pi P r_1^4 r_2^4}{8\eta\, r_1^4 r_2^4\left[\frac{l_2}{r_2^4} + \frac{l_1}{r_1^4}\right]}$$

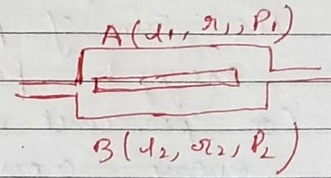
$$V = \frac{\pi P}{8\eta} \cdot \frac{1}{\left[\frac{l_2}{r_2^4} + \frac{l_1}{r_1^4}\right]} = \left(\frac{P}{R_1 + R_2}\right)$$

Capallaries in parallel combination :-

If the two capellaries are joined in parrell the pressure difference across them is the Same than the volume of the liquid flowing / second through them is given by

$V = V_1 + V_2$

$P = P_1 = P_2$



$$= \frac{\pi P_1 r_1^4}{8\eta l_1} + \frac{\pi P r_2^4}{8\eta l_2} = \frac{\pi P}{8\eta}\left[\frac{r_1^4}{l_1} + \frac{r_2^4}{l_2}\right]$$

if we put $R = \frac{8\eta}{l}\ \frac{8\eta l}{\pi r^4}$

then $P\left[\frac{\pi r_1^4}{8\eta l_1} + \frac{\pi r_2^4}{8\eta l_2}\right]$

$$P\left[\frac{1}{\frac{8\eta l_1}{\pi r_1^4}} + \frac{1}{\frac{8\eta l_2}{\pi r_2^4}}\right] = P\left[\frac{1}{R_1} + \right.$$

For series combination

$$V = \frac{P}{R_1 + R_2}$$

& for parallel combination

$$V = P\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

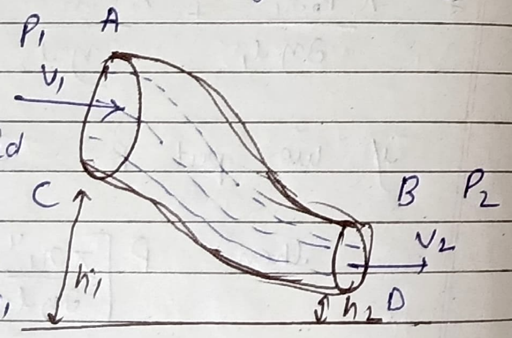
$$\boxed{R_1 = \frac{8\eta l_1}{\pi r_1^4}}$$

$$\boxed{R_2 = \frac{8\eta l_2}{\pi r_2^4}}$$

Bernoulli's Theorem :-

it is the mathematical expression of "Law of conservation of energy" in fluid dy.

It states that "for the steamline flow of an ideal liquid the total energy [the sum of pressure energy, potential energy & kinetic energy] per unit mass (or per unit volume) remains constant at every cross-section throughout the flow.



Consider a tube AB in which a ~~so~~ liquid flows streamline. at A let the pressure will be P_1 & velocity v_1 & at B, P_2 & V_2.

$V_2 > V_1$

$a_1 \to$ area at A

$a_2 \to$ " at B

So here.

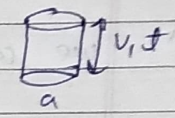
→ Increase in kinetic energy $= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

→ decrease in potential energy $= mgh_1 - mgh_2$

→ work done at ~~po~~ C = Force x displacement.

Force = Pressure x area $= P_1 \times a_1$

displacement = velocity x t = (length)



W_c ~~Force~~ $= P_1 a_1 \times v_1 \times t_1$

~~Force~~ $: W_c = P_1 V$

also ~~velocity~~ volume $= \frac{\text{mass}}{\text{density}} = \frac{m}{\rho}$

$$W_c = \frac{P_1 m}{\rho}$$

Similar workdone at D $\quad W_D = \frac{P_2 m}{\rho}$

net work done $= \frac{P_1 m}{\rho} - \frac{P_2 m}{\rho}$.

By conservation of energy

Increase in kinetic energy = decrease in potential energy + work done by liquid

$$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = mgh_1 - mgh_2 + \frac{P_1 m}{\rho} - \frac{P_2 m}{\rho}$$

$$\frac{1}{2}mv_1^2 + mgh_1 + \frac{P_1 m}{\rho} = \frac{1}{2}mv_2^2 + mgh_2 + \frac{P_2 m}{\rho}$$

both side devide by m/ρ. Total energy at B

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2$$

energy at A

~~He~~ here ~~ρ, v_1, V_2~~, g, ~~h_1, h_2~~ P_1, P_2 are constant

we can say that $\frac{1}{2}\rho v^2 + \rho g h + P = \text{Constant}$

Total energy is conserved Hence prove