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Bsc. Ist Sem

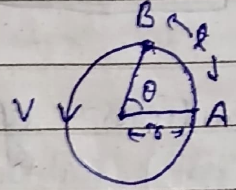
Rotational Motion ≈

angular displacement

angular velocity $(\omega) = \frac{\Delta\theta}{\Delta t}$

$$\boxed{\omega = \lim_{\Delta t \to 0} \frac{d\theta}{dt}}$$

* Relation b/w $\vec{V}$ & ω.



$\vec{V} = \frac{\widehat{AB}}{t}$

But arc $= \theta \times r$ $\quad \widehat{AB} = \theta \times r$

$\vec{V} = \frac{\theta r}{t}$

$\boxed{\vec{V} = r\omega}$ $\quad \boxed{\omega = \frac{\theta}{t}}$ $\quad \boxed{\vec{V} = \vec{\omega} \times \vec{r}}$

* Relation b/w Angular accelerat (α) with accel. (a)

$\Delta\omega$ = change in angular velocity in Δt time

$$\boxed{\alpha = \frac{\Delta\omega}{\Delta t}}$$

$\omega = \frac{d\theta}{dt}$

$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

$\alpha = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$

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Relation $a = \frac{v}{t}$ — (1)

But $v = r\omega$

$$a = \frac{r\omega}{t} = r\alpha$$

$$\boxed{a = r\alpha} \qquad \boxed{\vec{a} = \vec{\alpha} \times \vec{r}}$$

→ Equations of Rotational Motions :-

* $\omega_0 \rightarrow$ initial angular veloc
$\omega \rightarrow$ angular vel at 't'

$\alpha \rightarrow$ angular acceleration

$$\begin{bmatrix} v = u + at \\ v = ut + \frac{1}{2}at^2 \\ v^2 - u^2 = 2aS. \end{bmatrix}$$

we know $\alpha = \frac{d\omega}{dt}$.

$$d\omega = \alpha\, dt$$

Integrate

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha \cdot dt$$

$$\omega - \omega_0 = \alpha t$$

$$\boxed{\omega = \omega_0 + \alpha t}$$ I[st] Equation of Motion

$\alpha = \frac{d\omega}{dt}$

$$\alpha = \frac{d}{dt}(\omega_0 + \alpha t)$$

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$$\Rightarrow \omega = \frac{d\theta}{dt}$$

$$\Rightarrow d\theta = \omega \cdot dt$$

$$\Rightarrow d\theta = (\omega_0 + \alpha t) \cdot dt$$

$$\int_0^{\theta} d\theta = \int_0^t \omega_0 \cdot dt + \alpha t$$

$$\theta = \omega_0 t + \frac{\alpha t^2}{2}$$

$$\boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2}$$ II[nd] Law of Rotational eq.

* 3[rd] eq. of Motion :-

$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \frac{d\theta}{dt}$$

$$\frac{\alpha}{\omega} = \frac{d\omega}{d\theta}.$$

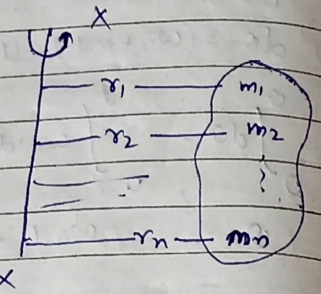
$$\int_0^{\theta} \alpha\, d\theta = \int_{\omega_0}^{\omega} \omega \cdot d\omega$$

$$\alpha\theta = \left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega} = \frac{\omega^2 - \omega_0^2}{2}$$

$$\boxed{\omega^2 - \omega_0^2 = 2\alpha\theta}$$ III[rd] Equation of Rotational eq.

* Moment of Inertia :-
Inertia of a body in rotational motion.

$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2$



* Moment of Inertia (MI) of body about an axis of rotation is equal to the sum of products of masses of each particle & squ. of their $\perp^r$ distances.

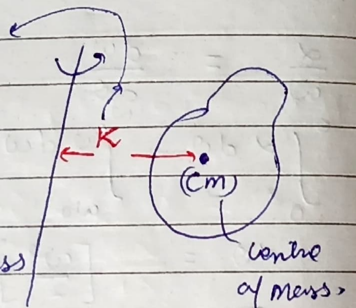
$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$

it is also a scalar quantity.

$\boxed{kg\, m^2}$

→ Radius of Gyration :-

↳ Radius of Gyrat (K) is the $\perp^r$ distance from a point in a body where its entire mass of body is assumed to be concentrated to the Rotation of axis.



$I = K^2 \times m_{(cm)}$ $\boxed{K = \sqrt{\frac{I}{m}}}$

↳ moment of Inertia of body
whole

$$\boxed{MK^2 = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2}$$

if $m_1 = m_2 = \cdots = m_n$

$MK^2 = \frac{nm}{n}\left(r_1^2 + r_2^2 + \cdots + r_n^2\right)$

* multiply & divide by n.

$\boxed{mn = \text{Mass of comple body}}$

$MK^2 = M\left[\frac{r_1^2 + r_2^2 + \cdots r_n^2}{n}\right]$

$K^2 = \frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}$

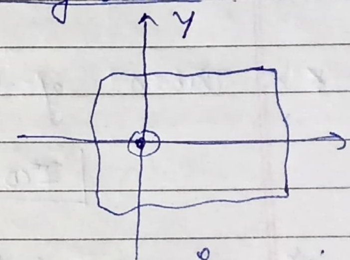
$$\boxed{K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^3 + \cdots + r_n^2}{n}}}$$

• "Radius of gyration is Root mean square of $\perp^r$ dist of different particles from axis of rotation"

→ Perpendicular theorm of moment of Inertia.

$I = I_x + I_y$

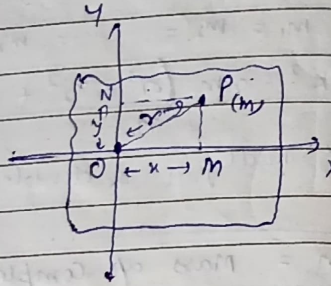
I_x & I_y are the (MI) of the body about X & Y axis.



* Moment of I of a body about an axis passing through any point and $\perp^r$ to the plane of body is equals to the sum of moment of Inertia body about two mutually $\perp$ axes passing

through a same point & lies in same plane

* Proof :-



M.I of particle p about x-axis = my^2

M.I of body about x-axis = $I_x = \sum my^2$ - (i)

* M.I of particle about y-axis = mx^2

" " body " " " = $\sum mx^2 = I_y$

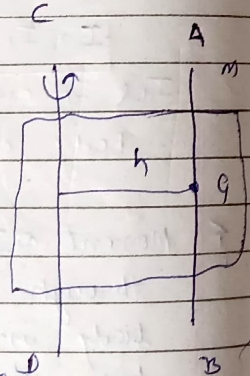
moment of inertia of body = $I_x + I_y$
$= \sum my^2 + mx^2$

$I_x + I_y = \sum m(x^2+y^2)$

→ $\boxed{x^2+y^2=r^2}$ $\boxed{I_x+I_y = \sum mr^2} = I$

→* Theorem of ∥ axes of M.I.

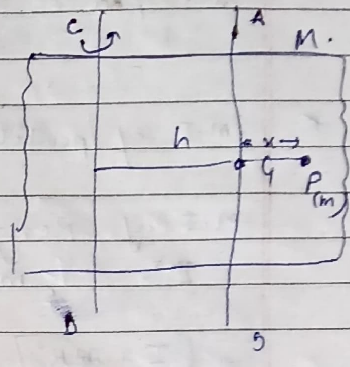
$\boxed{I_{CD} = I_{AB} + Mh^2}$



M.I of a body about an axis of rotation is equal to the sum of M.I of body about an parallel axis to given axis passes

Through center of gravity & product of mass of the body & square of the ⊥r dist. b/w two ∥ axis."

* Proof :-



I_{CD} & I_{AB}

Let P is any mass.

$I_{AB\,(P)} = mx^2$ — (i)

I_{AB} = M.I of body = $\sum mx^2$ — (ii)

M.I of P along CD axis = $m(h+x)^2$

M.I of body " " " = $\sum m(h+x)^2 = \sum mx^2 + mh^2 + 2mxh$

$I_{CD} = \sum mx^2 + \sum mh^2 + \sum 2mxh$

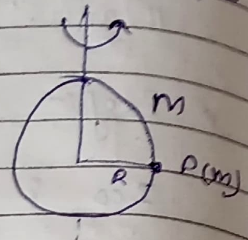
$I_{CD} = I_{AB} + Mh^2$ ∵ $\sum mh^2 = Mh^2$

(c) $\sum 2mxh = 0$

as the body balences about center of gravity

$\boxed{I_{CD} = I_{AB} + Mh^2}$

* Moment of Inertia of a Ring: about axis passing through its center & ⊥ to its plane.



* M.I of particle (P) = mR^2

M.I of Ring about axis of

$I = \sum mR^2$

$\boxed{I = MR^2}$

(b) about its diameter.

· using ⊥ theorm of axis.

$I = I_x + I_y$

$I_x = \cancel{MR^2} I_y$

$I_y = \cancel{MR^2}$.

$\boxed{I \cancel{(\text{diameter}) = 2MR^2}}$

* Since it is symmetric

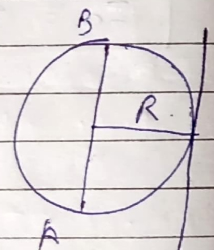
$I_x + I_y = I$

$2I_x = MR^2$

$\boxed{I_x = \frac{MR^2}{2}}$

(c) about tangent

using theorm of plane



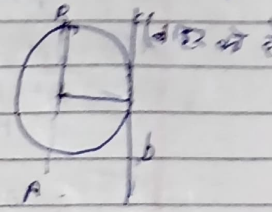
→ $I = I_{AB} + mh^2$

$I = \frac{MR^2}{2} + MR^2$

$\boxed{I_{ring} = \frac{3}{2}MR^2}$

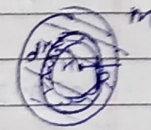
*(d) M.I of Ring about the Tangent ⊥ to its plane

$I_{\cancel{AB}} = I_{AB} + Mh^2$ [Parallel to the axis (circled)]



∴ $I = MR^2 + MR^2 = 2MR^2$

(e) M.I of Solid Disc :-



$\boxed{\sigma = \frac{M}{A} = \frac{M}{4\pi R^2}}$

length of the thin disc = $2\pi x$.

area of " " = $2\pi x \cdot dx$

Mass of thin disc = $2\pi x \cdot dx \cdot \sigma = 2\pi\sigma x \cdot dx$.

Moment of inertia of entire disc = $\int_0^R 2\pi\sigma x\,dx \cdot x^2$

$\cancel{2\pi\sigma\left[\frac{x^4}{4}\right]_0^R}$ $\cancel{\frac{2\pi\sigma R^4}{4}}$

$= \cancel{\pi\sigma R^2} = \cancel{\frac{\pi \times M}{4\pi R^2} \pi R^2}$ $\frac{\pi\sigma R^4}{2} = \frac{\pi R^4}{2} \times \frac{M}{4\pi R^2}$

$\int_0^R 2\pi\sigma x^3 dx$

$\frac{MR^2}{2}$

$2\pi\sigma\left[\frac{x^4}{4}\right]_0^R = \frac{2\pi\sigma R^4}{4}$ $\frac{\pi\sigma R^4}{2}$

$$dI = \frac{M}{\pi R^2} x\, 2\pi x$$

$$dI = \frac{2M}{R^2} \cdot x\,dx \cdot x^2 \qquad I = MR^2$$

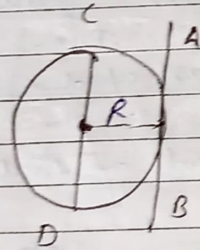
$$\int_0^R \frac{2M}{R^2} \cdot x^3 dx$$

$$= \frac{2M}{R^2} \cdot \frac{R^4}{4} = \boxed{\frac{MR^2}{2}}$$

* M.I about its diameter $= I = I_x + I_y$

$$\boxed{I_x = \frac{MR^2}{4}}$$

about a tangent



Theorem of || axis

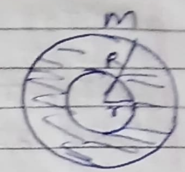
$$I = I_{CO} + Mh^2$$

$$= \frac{MR^2}{4} + MR^2$$

$$\boxed{I = \frac{5MR^2}{4}}$$

— x — x — x — x — x — x — x —

* M.I of Annular disc.



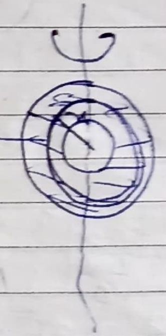
- Area $= \cancel{4}\pi R^2 - \cancel{4}\pi r^2$

area of the disc $= \cancel{4}\pi(R^2 - r^2)$

$$\sigma = \frac{M}{A} = \frac{M}{\cancel{4}\pi(R^2-r^2)}$$

$$\boxed{\sigma = \frac{M}{4\pi(R^2-r^2)}}$$

* about an axis passing through its centre & ⊥ to its plane.



$2\pi x \cdot dx$

area of ring $= 2\pi x\,dx$

Mass of ring $= \frac{2\cancel{\pi} x\,dx \cdot M}{\cancel{\pi}(R^2-r^2)}$

$$M_{ri} = \frac{2x\,dx\,M}{R^2-r^2}$$

M.I of disc $= MR^2 = \frac{M \cdot 2x^3 dx}{R^2-r^2}$

M.I of whole ring $= \int_r^R \frac{2Mx^3 dx}{R^2-r^2}$

$$= \frac{2M}{R^2-r^2}\left[\frac{x^4}{4}\right]_r^R$$

$$= \frac{2M}{(R^2-r^2)} \cdot \frac{R^4-r^4}{\cancel{4}\,2} = \frac{(R^2+r^2)M}{2}$$

M.I about its diameter -

$I = I_x + I_y$ ⟶ since disc is symmetric

$I = 2I_x = \frac{M(R^2+r^2)}{2}$

$$\boxed{I_x = \frac{M}{4}(R^2+r^2)}$$

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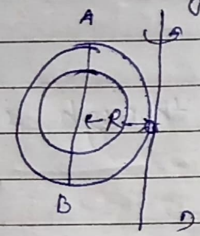
* about the tangent in its plane :-



Theorm of ⊥r plane

$= I = I_{AB} + Mh^2$.

$I = \frac{M}{4}(R^2+r^2) + MR^2$

$I = \frac{MR^2+Mr^2+4MR^2}{4}$

$I = \frac{5MR^2+Mr^2}{4}$

$$\boxed{I_{CD} = \frac{M(5R^2+r^2)}{4}}$$

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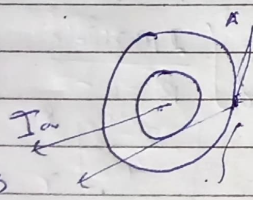
* about the tangent ⊥ to its plane.



Theor of ∥ axis

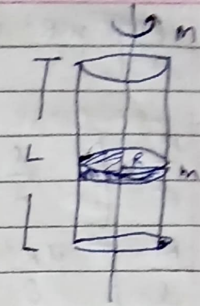
$I = I_a + Mh^2$

$= \frac{M}{2}(R^2+\cancel{r^2}) + MR^2$

$$\boxed{I = \frac{M}{2}[3R^2+r^2]}$$

* M.I of solid cylinder.

a) about its axis of symmetry

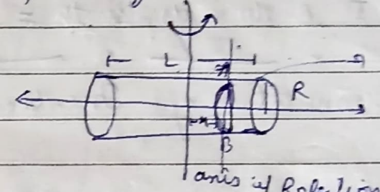


Moment of Inertia of solid disc

$= \frac{MR^2}{2}$

MI of Solid cylinder $= \sum \frac{mR^2}{2} = \frac{MR^2}{2}$

(b) about an axis passing through its centre ⊥ to its length



mass / unit length

$= \frac{M}{l}$

$\frac{M}{l} \times 2\pi r$

→ M.I of the disc along the axis =

mass of solid disk $= \frac{M}{l}dx$.

M.I $= \frac{mR^2}{4} = \left(\frac{M}{l}\right)\frac{R^2(dx)}{4}$

M.I of the Rod using ∥ axis theorm

$= \frac{M}{l}\frac{R^2 dx}{4} + \left(\frac{M}{l}dx\right).x^2$

$= \int_{-l/2}^{l/2} \frac{M}{l}\left[\frac{R^2 dx}{4} + dx.x^2\right]$

$\frac{M}{l}\left[\frac{R^2 x}{4} + \frac{x^3}{3}\right]_{-l/2}^{l/2}$

$$\frac{M}{l}\left[\frac{xR^2}{4}+\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

$$\frac{M}{l}\left[\frac{l}{2}\frac{R^2}{4}+\frac{l^3}{24}-\left[-\frac{lR^2}{8}-\frac{l^3}{24}\right]\right]$$

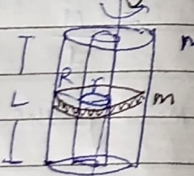
$$\frac{M}{l}\left[\frac{lR^2}{8}+\frac{l^3}{24}+\frac{lR^2}{8}+\frac{l^3}{24}\right]$$

$$\frac{M}{l}\left[\frac{lR^2}{4}+\frac{l^3}{12}\right] \qquad \boxed{I_0 = M\left[\underbrace{\frac{R^2}{4}}_{\text{disc}}+\underbrace{\frac{l^2}{12}}_{\text{Rod}}\right]}$$

M.I ⊥ to its length $= M\left[\frac{R^2}{4}+\frac{l^2}{12}\right]$

—x—x—x—

* Moment of Inertia of hollow cylinder

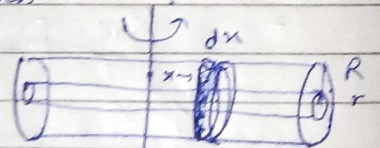


↳ Moment of Inert of annular dis $= \frac{m(R^2+r^2)}{2}$

MI of hollow cylinder $= \sum \frac{m(R^2+r^2)}{2}$

$$\boxed{I=\frac{M(R^2+r^2)}{2}}$$

(b) about an axis passing through its centre and ⊥ to its length.



~~MI of the solid disc~~ $= \frac{M}{4}(R^2+r^2)$

M/L of Hollow cylinder = M/L

Mass of Solid disc $= \frac{M}{L}dx$

MI of annular disc about its diameter $= \frac{M(R^2+r^2)}{4}$

$$= \frac{M\,dx}{4L}(R^2+r^2)$$

M.I about an $= I_{AB} + Mx^2$

$$= \frac{M}{l}\frac{dx(R^2+r^2)}{4} + \frac{M}{l}dx\cdot x^2$$

$$dI = \frac{M}{l}\left[\frac{(R^2+r^2)dx}{4}+dx\cdot x^2\right]$$

So $I=\int dI = \int_{-l/2}^{l/2}\left[\frac{R^2+r^2}{4}\right]x + \frac{x^3}{3}$

$$I = \left[\frac{(R^2+r^2)}{4}x+\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

$$I = \left[\left(\frac{R^2+r^2}{4}\right)\frac{L}{2}+\frac{L^3}{24}+\frac{L}{2}\left(\frac{R^2+r^2}{4}\right)+\frac{L^3}{24}\right]$$

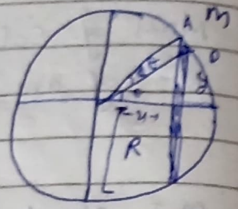
$$I = \left[\frac{(R^2+r^2)\cdot L}{4}+\frac{L^3}{12}\right]\frac{m}{l}$$

$$I = \frac{L}{2}(R^2+r^2)$$

$$\boxed{I = \frac{R^2+r^2}{4}+\frac{l^2}{12}}$$

* M·I of Spherical shell

(a) about its diameter



$\sigma = \frac{M}{A} = \frac{M}{4\pi R^2}$

Area of ring = $2\pi y \cdot AB$

Mass of Ring = $\frac{M}{4\pi R^2} \cdot 2\pi y\, AB = \frac{M y AB}{2R^2}$

MI of Ring about an axis passing through its center & ⊥ to its plane $= MR^2 = \frac{M}{2R^2} y \cdot AB \cdot y^2$

$M.I = dI = \frac{M}{2R^2} y^3 \cdot (AB)$

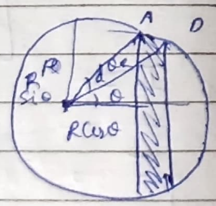
arc = $\theta \times r$ = $AB = d\theta \cdot R$

$\sin\theta = \frac{y}{R}$ $\quad y = R\sin\theta$.

$dI = \frac{M}{2R^2} \cdot [R^3 \sin^3\theta] \cdot R\, d\theta$

— x — x — x — x — x — x — x —

or



$AB = R d\theta$

Radius of disc = $R\sin\theta$.

Length of disc = $2\pi R\sin\theta$

area = $2\pi R^2 \sin\theta \cdot d\theta$ $\qquad \sigma = \frac{M}{4\pi R^2}$

Mass of Ring = $\sigma \times 2\pi R^2 \sin\theta\, d\theta$

$M.I = dI = \sigma \times 2\pi R^2 \sin\theta\, d\theta \cdot (R\sin\theta)^2$

M.I of whole spherical shell = $\int dI = \int_0^\pi \sigma\, 2\pi R^4 \sin^3\theta \cdot d\theta$

$= \sigma 2\pi R^4 \int_0^\pi \sin\theta(1-\cos^2\theta)\, d\theta$

$= \sigma 2\pi R^4 \cdot \int_{-1}^{1} t^2 - 1$

$= \sigma 2\pi R^4 \int_{-1}^{1} 1 - t^2 = \sigma 2\pi R^4 \left[t - \frac{t^3}{3}\right]_{-1}^{1}$

$= \sigma 2\pi R^4 \left[1 - \frac{1}{3} - \left[-1 + \frac{1}{3}\right]\right]$

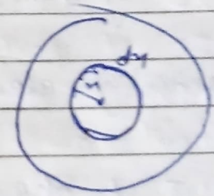
$= \sigma 2\pi R^4 \left[\frac{2}{3} + \frac{2}{3}\right] = \frac{4}{3} \cdot \frac{M}{4\pi R^2} \cdot 2\pi R^4$

$\boxed{M.I = \frac{2}{3} M R^2}$

* M.I of a sphere about a diameter :-

Mass = M $\qquad$ volume mass density = $\frac{M}{\frac{4}{3}\pi R^3}$

Radius = R



M.I of the small sphere

$dI = \frac{2}{3} m x^2$

~~M.I of the whole sphere~~

mass of spherical cell = $\rho \times \frac{4}{3}\pi x^3 \cdot dx$

$= dI = \frac{2}{3} \rho \times \frac{4}{3}\pi x^2 \cdot dx$

$$\int dI = \int_0^R \left(\rho \frac{2}{3} 4\pi x^2 dx\right) \cdot x^2$$

$$I = \frac{2}{3} 4\pi \rho \left[\frac{x^5}{5}\right]_0^R = \frac{2}{3} 4\pi \rho \frac{R^5}{5}$$

$$I = \frac{2}{3} \times 4 \times \pi \times \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{R^5}{5}$$

$$\boxed{I = \frac{2}{5} \times MR^2}$$

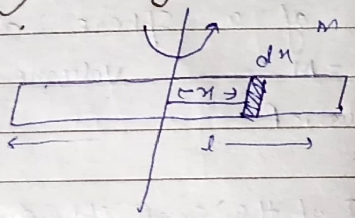
$$\boxed{\text{M.I of Solid sphere} = \frac{2}{5} MR^2}$$

$$I = \frac{2MR^2 + 5MR^2}{5} = \frac{7}{5} MR^2$$

* Moment of Inertia of thin bar or Rod.

(a) about an axis passing through its center & ⊥ to the length.

↳



M/L of thin Rod = $\frac{M}{L}$.

Mass of element = $\frac{M}{L} dx$

M.I of the ~~roo~~ elem = $MR^2 = \frac{M}{L} dx \, (x^2)$

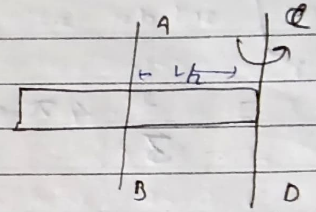
M.I of the Rod = $\int_{-L/2}^{L/2} \frac{M}{L} dx \, x^2$

$$I = \frac{M}{L} \left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{M}{L} \left(\frac{L^3}{24} + \frac{L^3}{24}\right)$$

$$= \frac{M}{12} L^2$$

$$\boxed{\text{M.I of thin Rod} = \frac{ML^2}{12}}$$

(b) about the tangent passing through one end & ⊥ to its length.



$I_{CD} = I_{AB} + ML^2$

$I_{CD} = \frac{M}{12} L^2 + \frac{ML^2}{4}$

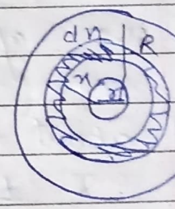
$I_{CD} = \frac{ML^2 + 3ML^2}{12}$

$$\boxed{I_{CD} = \frac{ML^2}{3}}$$

x —— x —— x —— x —— x

M.I of hollow sphere

a) about its diameter



mass / unit volume = $\frac{M}{\frac{4}{3}\pi [R^3 - r^3]}$

$\rho = \frac{3}{4} \frac{M}{\pi [R^3 - r^3]}$

Mass of the smaller disc we taper
$= 4\pi x^2 \cdot dx \cdot \times \sigma$,
volume, vol/m/m

$M = 4\pi x^2 dx \times \sigma$,

M.I of the sphere $= \frac{2}{3} MR^2$

$dI = \frac{2}{3} (4\pi x^2 dx \cdot \sigma) \cdot x^2$

$\frac{2}{3} 4\pi\sigma \int_r^R x^4 \cdot dx = \frac{2}{\cancel{3}} 4\pi\sigma \left[\frac{x^5}{5}\right]_r^R$

$= \frac{2}{3} 4\pi\sigma \left[\frac{R^5 - r^5}{5}\right]$

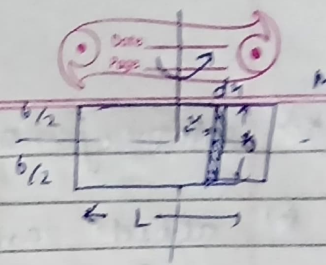
$I = \frac{2}{3} \cancel{4\pi} \frac{3}{4} \frac{M}{\cancel{\pi}(R^3 - r^3)} \left(\frac{R^5 - r^5}{5}\right)$

$I = \frac{2M}{5} \left(\frac{R^5 - r^5}{R^3 - r^3}\right)$

$I = \frac{2}{5} M \left[\frac{R^5 - r^5}{R^3 - r^3}\right]$

M.I of Ractangular lamena:-

a) about an axis passing through its center & $\perp$ to its plane.



* Mass / unit long $= \frac{M}{L}$

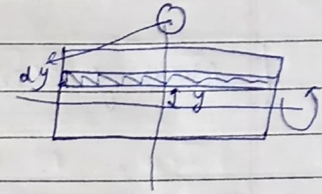
Mass of this small lamena $= \frac{M}{L} dx$

dI_y (M.I about y axis) $= \frac{M}{L} dx \cdot x^2$

$I_y = \int_{-L/2}^{L/2} \frac{M}{L} x^2 \cdot dx$.

$I_y = \frac{M}{L} \left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{ML^2}{12}$

$\boxed{I_y = \frac{ML^2}{12}}$



dI_x = M.I about x axis.
Mass / unit breath $= \frac{M}{b}$

Mass of small element $= \frac{M}{b} \cdot dy$

M.I x = $dI_x = \frac{M}{b} dy \cdot y^2$

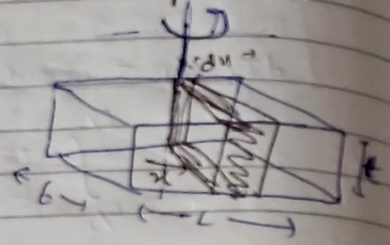
$\therefore I_x = \int_{-b/2}^{b/2} \frac{M}{b} dy^2 \cdot dy = \frac{M}{12} b^2$

→ M.I about axis $\perp$ to plane using $\perp^r$ theorem
$I = I_x + I_y$
$I = \frac{M}{12} [L^2 + b^2]$

$\boxed{I = \frac{M}{12} (L^2 + b^2)}$ → MI of Rectangular lamina

-*) MI of Solid rectangular Bar

(a) about an axis passing through its centre & $\perp$ to its plane.

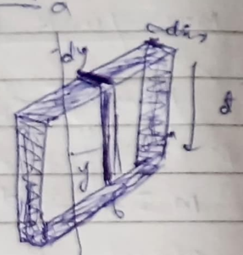


ρ = density of material

mass of bar $M = lb \cdot t \cdot \rho$

The rectangular ~~lamina~~ Bar is made of a no. of Small Bar of dimension dx, b, t & take a small rectn Bar which is x dist. from axis of rotation.

$$\boxed{\text{Mass of element} = dx \cdot b \cdot t \cdot \rho}$$



This small ~~lo~~ rectangular Bar is made up of Small rectn bar of thickness dy, t, dx

mass of another small Rs = $dx\, dy\, t \cdot \rho$.

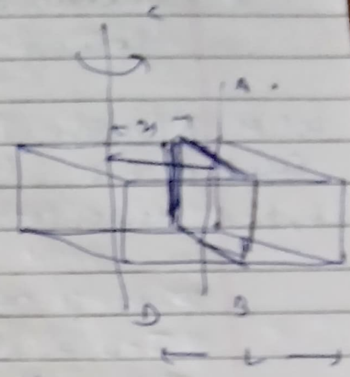
M.I of this small element = $dI_y = (dx\, dy\, t\rho) \cdot y^2$

M.I of the first element = $\int_{-b/2}^{b/2} dx\, dy .\, t\rho\, y^2$

M.I = $dx\, t \cdot \rho \left[\frac{y^3}{3}\right]_{-b/2}^{b/2}$

MI = $\frac{dx\, t\, \rho}{3} \cdot \frac{b^3}{4}$

$$\boxed{I_{AB} = \frac{dx\, t\, \rho\, b^3}{12}}$$



$\cdot\ I_{CD} = I_{AB} + \text{Mass} \times h^2$

$dI = \frac{t\rho b^3}{12} dx + b \cdot t \cdot \rho \cdot x^2 \cdot dx$

$dI = t\rho b \left[\frac{b^2}{12} \cancel{dx} + x^2\right] \cdot dx$

$I = \int dI = \int_{-L/2}^{L/2} t\rho b \left[\frac{b^2}{12} + x^2\right]$

$= t\rho b \left[\frac{b^2 x}{12} + \frac{x^3}{3}\right]_{-L/2}^{L/2}$

$I = t\rho L \left[\frac{b^2 L}{24} + \frac{L^3}{24} - \left[\frac{-b^2 L}{24} - \frac{L^3}{24}\right]\right]$

$I = t\rho b \left[\frac{b^2 L}{12} + \frac{L^3}{12}\right]$

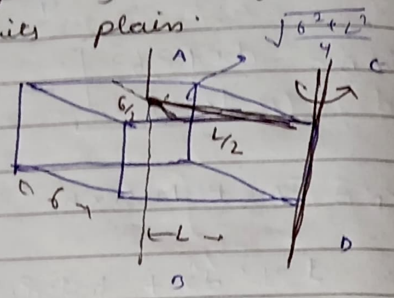
$I = \frac{t\rho L b}{12}(b^2 + L^2)$

$I = \frac{b t \rho L}{12}\left[b^2 + L^2\right]$

$$I = \frac{M}{12}\left[b^2 + L^2\right]$$

* M.I of Solid rectangular bar.

(i) about an axis passing through its one corners & ⊥ to its plain.



Let M is mass of Solid □ bar L, b, — demention

using || axis theorem

$I_{CD} = I_{AB} + Mh^2$

$I_{CD} = \frac{M}{12}[b^2+L^2] + M\left[\frac{b^2+L^2}{4}\right]$

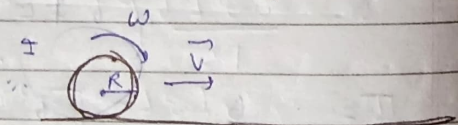
$I_{CD} = \frac{M(b^2+L^2)}{12} + \frac{M(b^2+L^2)}{4}$

$I_{CD} = M[b^2+L^2]\left(\frac{1}{3}\right)$

$\boxed{I_{CD} = \frac{M}{3}[b^2+L^2]}$

K.E of a body Rolling on a Surface;

* On Horizontal Surface.



in this motion it is in Rotational motion and as well as in Translation motion

So Total K.E of the body is due to both Rotational & Transitional

→ Total K.E = Rotational K.E + Transitional K.E.

$= \frac{1}{2} I\omega^2 + \frac{1}{2} MV^2$. — (i)

we know that $I = MK^2$ ∀ K is Radius of gyration

and $V = r\omega$

$\boxed{\omega = \frac{V}{r}}$

$K.E = \frac{1}{2} MK^2\left[\frac{V}{R}\right]^2 + \frac{1}{2} MV^2$

$K.E = \frac{1}{2} MV^2\left[1+\frac{K^2}{R^2}\right]$

* for some spacial cases :-

→ if the rotator object is Ring.

then $I = MK^2$ & $I = MR^2$

$MK^2 = MR^2$ $\boxed{\frac{K^2}{R^2} = 1}$

$K.E = \frac{1}{2} MV^2[1+1] = MV^2$.

→ if a disc.

$I = MK^2 = \frac{MR^2}{2}$

$\frac{K^2}{R^2} = \frac{1}{2}$

$\boxed{K.E = \frac{3}{4} MV^2}$

→ If spherical shell

$I = MK^2 = \frac{2}{3} MR^2$

$\frac{K^2}{R^2} = \frac{2}{3}$ $K.E = \frac{1}{2} MV^2 \left[1 + \frac{K^2}{R^2}\right]$

$\boxed{K.E = \frac{5}{6} MV^2}$

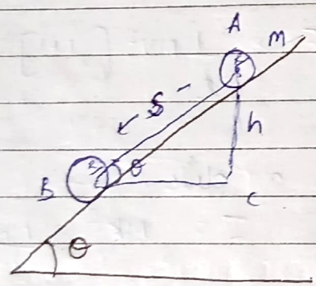
→ For Solid Sphere

$I = MK^2 = \frac{2}{5} MR^2$

$\boxed{\frac{K^2}{R^2} = \frac{2}{5}}$

$\boxed{K.E = \frac{7}{10} MV^2}$

* Acceleration produced in a body rolling down on an inclined plane without slipping.



- So here object has some initial potential energy and as the object moves down it losses its P.E and gains kinetic Energy. By conservation of energy

Loss in PE = gain in K.E

PE = K.E by Rotation M. + K.E by Translation M

$PE = \frac{1}{2} I\omega^2 + \frac{1}{2} MV^2$

from ΔABC we can find $h = S \sin\theta$.

and $PE = mgh$

$Mg(S \sin\theta) = \frac{1}{2} I\omega^2 + \frac{1}{2} MV^2$

also $I = MK^2$ & $\omega = \frac{V}{R}$

$Mg(S \sin\theta) = \frac{1}{2} MV^2 \left[1 + \frac{K^2}{R^2}\right]$

$g(S \sin\theta) = \frac{1}{2} V^2 \left[1 + \frac{K^2}{R^2}\right]$

- On differentiating both side w.r.t 't'

$\frac{d}{dt} g(S \sin\theta) = \frac{d}{dt}\left(\frac{1}{2} V^2 \left(1 + \frac{K^2}{R^2}\right)\right)$

$g \sin\theta . \frac{dS}{dt} = \frac{1}{2} 2.V . \frac{dV}{dt} \left[1 + \frac{K^2}{R^2}\right]$

* we know that $\frac{dS}{dt}$ = velocity & $\frac{dV}{dt}$ = acceleration

$g \sin\theta . \cancel{V} = \cancel{V} . a \left[1 + \frac{K^2}{R^2}\right]$

$$\boxed{a = \frac{g \sin\theta}{\left[1 + \frac{K^2}{R^2}\right]}}$$

For different cases

if rolling body is a Ring:

$I = MK^2 = MR^2$

$\frac{K^2}{R^2} = 1$

$a = \frac{g \sin\theta}{\left[1 + \frac{K^2}{R^2}\right]} = \frac{g \sin\theta}{2}$ $\boxed{a = \frac{g \sin\theta}{2}}$

if its is disc

$MK^2 = \frac{MR^2}{2} = \frac{K^2}{R^2} = \frac{1}{2}$.

$a = \frac{g \sin\theta}{1 + \frac{K^2}{R^2}} = \frac{2 g \sin\theta}{3}$

$\boxed{a = \frac{2}{3} g \sin\theta} = 0.667\, g \sin\theta$

Spherical Shell.

$I = MK^2 = \frac{2}{3} MR^2$

$\boxed{\frac{K^2}{R^2} = \frac{2}{3}}$

$a = \frac{g \sin\theta}{\left[1 + \frac{K^2}{R^2}\right]} = \frac{3}{5} g \sin\theta = 0.6\, g \sin\theta$

$\boxed{a = \frac{3}{5} g \sin\theta}$

Solid sphere.

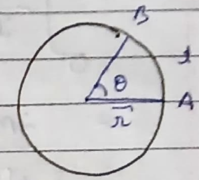
$I = MK^2 = \frac{2}{5} MR^2$ $\frac{K^2}{R^2} = 2/5$

$a = \frac{g \sin\theta}{\left[1 + \frac{K^2}{R^2}\right]} = \frac{5}{7} g \sin\theta$

$\boxed{a = \frac{5}{7} g \sin\theta}$

→ Relationship b/w linear velocity angular velocity

rate of change of angular displacement is ~~and~~ angular velocity is ω.



$\omega = \frac{d\theta}{dt}$.

$\vec{v} = \frac{\overline{AB}}{t}$

$\vec{v} = \frac{\theta \times \vec{r}}{t}$

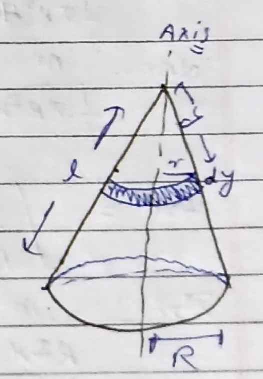
$\boxed{\vec{v} = \omega \vec{r}}$ $\boxed{\vec{v} = \vec{r} \times \vec{\omega}}$

$\boxed{\vec{v} = r\omega}$

→ M.I for Hollow Cone.

Mass - M   Mass/unit area $= \frac{M}{\pi R l}$

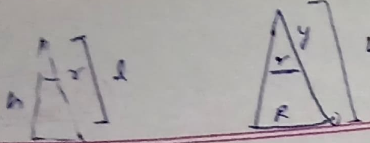
Radius - R



$I_{HC} = \int dI_{ring} = \int dm \cdot r^2$

$dm = \frac{M}{\pi R l} (2\pi r \cdot dy)$

$I_{HC} = \int \cancel{dm} \cdot \frac{M}{Rl} 2 r^3 . dy$



using concept of similar Δ = $\frac{r}{R} = \frac{y}{l}$

$r = \frac{yR}{l}$

$I_{HC} = \int \frac{2M}{Rl} r^3 . dy = \int \frac{2M}{Rl} \frac{y^3 R^3}{l^3} . dy$

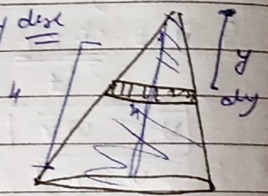
$I_{HC} = \int_0^l \frac{2MR^2}{l^4} . y^2 . dy = \frac{2MR^2}{l^4} \int_0^l y^3 . dy$

$= \frac{2MR^2}{l^4} . \left[\frac{y^4}{4}\right]_0^l = \frac{2MR^2}{4} = \frac{MR^2}{2}$

$\boxed{\text{M.I of Hollow cone} = \frac{MR^2}{2}}$

* Solid cone

$I_{SC} = \int dI_{disc} = \int \frac{dm\, r^2}{2}$ M.I. of disc



mass/unit vol = $\frac{M}{\frac{1}{3}\pi R^3 H}$ $V = \frac{1}{3}\pi r^2 h$

Area Thickness

$dm = \frac{M}{\frac{1}{3}\pi R^3 H} . (\pi r^2 . dy)$

similarity of Δ

$\left[\frac{r}{R} = \frac{y}{H}\right]$

$r = \frac{Ry}{H}$

$I_{SC} = \int \frac{3M}{R^3 H\, 2} r^4 . dy$

$I_{SC} = \frac{3M}{R^3 H} \int \frac{R^4 y^4 . dy}{H^4}$

$I_{SC} = \frac{3MR^2}{2H^5} \left[\frac{R^5}{5}\right]_0^H = \boxed{\frac{3}{10} MR^2}$

$\boxed{\text{M.I of Solid Cone} = \frac{3}{10} MR^2}$

* here $R \rightarrow$ Radius $b \rightarrow$ breath
$r \rightarrow$ inner Radius
$L \rightarrow$ length

Shape	MI about axis	M.I about diameter	M.I about Tangent
* Ring	MR^2	$MR^2/2$	$3/2\, MR^2$
* Solid disc	$MR^2/2$	$MR^2/4$	$5/4\, MR^2$
* Annular disc	$(R^2+r^2)M/2$	$M/4\,(R^2+r^2)$	$M/4\,[5R^2+r^2]$
* Solid cylinder	axis of symmetry $\frac{MR^2}{2}$	$M\left[\frac{R^2}{4}+\frac{l^2}{12}\right]$	
* hollow Cylinder	" $M(R^2+r^2)/2$	" $\frac{R^2+r^2}{4}+\frac{l^2}{12}$	
* Spherical Shell		$2/3\, MR^2$	
* Sphere		$\frac{7}{5} MR^2$	
* Rod / bar	$\frac{ML^2}{12}$		$\frac{ML^2}{3}$
* Hollow sphere		$\frac{2}{5} M \frac{[R^5-r^5]}{[R^3-r^3]}$	
* Rectangular lamina	$\frac{M}{12}[L^2+b^2]$		