# Electro magnetic Induction

The phenominon in which an emf is induced in a coil due to the change in magnetic flux linked with a coil is called Electromagnetic Induction.

and the direction of induced emf can ~~by~~ be obtained by any of the lenz law or flamings Right hand Rule.

# Faraday's ~~first~~ observations.

- whenever there is change in magnetic flux linked with a circuit or a conductor is placed in a varying magnetic field then an E.M.F is induced if the circuit is closed. a current is induced which is called induced current.

$$\boxed{\phi \propto i}$$

- The induced emf in a coil is proportional to the rate of change of magnetic flux.

$$\varepsilon \propto \frac{d\phi_B}{dt}$$

$$\boxed{\varepsilon = -\frac{d\phi_B}{dt}}$$

Here the negative sign shows that the induced e.m.f always opposes the change in the magnetic flux.

• If N is the no. of turns in the coil then total induced emf is given by

$$\boxed{\mathcal{E} = -N\frac{d\phi}{dt}}$$

# If 'R' is total resistance of the circuit then the induced current is given by

$$\boxed{i = \frac{\mathcal{E}}{R} = \frac{-N}{R}\frac{d\phi_B}{dt}}$$

# Integral and differential form of faraday's law.

Change in Magnetic flux → Emf

Electric field

Acc to faradays law

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{l}$$

~~also $\mathcal{E} = \oint \vec{E}\cdot d\vec{l}$~~

Magnetic flux $= \phi_B = \oint_S \vec{B}\cdot d\vec{s}$ — (i)

By the definition of emf the induced e.m.f is equal to the line integral of Electric field induced due to varying magnetic field

$$\mathcal{E} = \oint_C \vec{E}\cdot dl \quad \text{— (ii)}$$

$$\mathcal{E} = \frac{-d\phi_B}{dt} = \frac{-d}{dt}\oint_S \vec{B}\cdot d\vec{s}$$

$$\boxed{\oint_C \vec{E}\cdot d\vec{l} = \frac{-d}{dt}\oint_S \vec{B}\cdot d\vec{s}}$$

This is the integral form of fradays law and it shows that the line integral of $\vec{E}$ around the electric circuit is equals to the negative rate of change of magnetic flux passing through the circuit.

By stokes law

$$\oint \vec{E}\cdot dl = \oint_S (\vec{\nabla}\times\vec{E})\cdot ds \quad \text{— (iii)}$$

So $\oint_S (\vec{\nabla}\times\vec{E})\cdot ds = \frac{-d}{dt}\oint_S \vec{B}\cdot d\vec{s}$

$$\oint\left(\vec{\nabla}\times\vec{E} + \frac{dB}{dt}\right)\cdot ds = 0$$

$$\vec{\nabla} \times \vec{E} + \frac{d\vec{B}}{dt} = 0$$

$$\text{Curl } \vec{E} = -\frac{d\vec{B}}{dt}$$

Since the magnetic field $\vec{B}$ depends on time as well as on position, so instead of $\frac{d\vec{B}}{dt}$, we can write $\frac{\partial \vec{B}}{\partial t}$.

So, $\boxed{\text{Curl } \vec{E} = \vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}}$

# Self Induction

whenever there is change in a circuit the current in a circuit or coil [ change the no. of field lines passing through a circuit or coil ) than an e.m.f is induced in that circuit or coil itself. this phenomenon is called self induction

Coefficient of self Induction

- Acc. to faradays law the flux linked with the circuit or coil due to the magnetic field is proportional to the current in the coil

$\phi \propto i$

$\phi = Li$

Coefficien of self Induction / self Inductor

If $i = 1$ then $\boxed{\phi = L}$

* the Coefficiant of self inductance ( L ) is equals to the magnetic flux passe through the circuit when 1 ampere current is flows.

$i = 1 \Rightarrow \phi = L$

# Acc. to faradays Second law, the induced emf in the coil is given by

$$\varepsilon = -\frac{d\phi}{dt} \quad \text{—— (1)}$$

$$\varepsilon = -\frac{d}{dt}(LI) = -L\frac{di}{dt}$$

$$\boxed{\varepsilon = -L\frac{di}{dt}}$$

If $\frac{di}{dt} = 1$ then $\boxed{\varepsilon = -L}$

- the Coefficiant of self Inductance of a coil is numerically equal to the induced E.m.f in the coil / circuit when there is unit rate of change of current in the coil.

# Self Induction of plain circular coil.

Consider a plain circular coil of radius 'R' having N no. of turns and carrying a current 'i'.

then we know that the magnetic field at the centre of coil is given by

$$B = \frac{\mu_0 N I}{2R} \quad \text{— (i)}$$

# If we assume that the intensity of $\vec{B}$ is uniform over the entire plane of the coil, then the magnetic flux linked with coil.

$\phi_B = B \times$ effective area of the coil

$\phi_B = B \times (NA)$ $\qquad A = \pi R^2$

$\phi_B = B \times N(\pi R^2)$

$$\phi_B = \frac{\mu_0 N I}{2R} N \pi R^2$$

$$\phi_B = \frac{\mu_0 N^2 I \pi R}{2} = \frac{\mu_0 \pi N^2 I R}{2}$$

$$\phi = \frac{\mu_0 N^2 \pi I R}{2} = \frac{\mu_0 \pi N^2 I R}{2}$$

or $\boxed{\frac{\phi}{I} = \frac{\mu_0 \pi N^2 R}{2}}$ — (iii)

we know that ~~$L \propto \phi$~~ $\phi = Li$

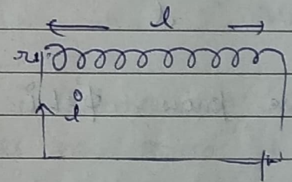
$$\boxed{L = \frac{\phi}{i}}$$

So, eq (ii) can be written as

$\boxed{L = \frac{\mu_0 \pi N^2 R}{2}} \rightarrow$ Self Inductance of plain circular coil

** Self Inductance of a current carrying solenoid

Consider a solenoid having radius r, length l and have total N no. of turns and carries a current I.




~~the~~ kno

We know that magnetic field on the axis of solenoid

$$B = \mu_0 n i = \frac{\mu_0 N i}{l}$$

$\because n =$ no. of turns / unit length

$$B = \frac{\mu_0 N i}{l} \quad \text{--- (i)}$$

Now we assume that the solenoid is very long. the intensity of magnetic field can be assumed to be uniform inside it at each point.

So, the total magnetic flux linked with the solenoid.

$$\phi = B \times (\text{effective area of Solenoid})$$

$$\phi = B \times (N \pi r^2)$$

$$\phi = \frac{\mu_0 N i}{l} (N \pi r^2) \quad \because \text{from eq (i)}$$

$$\phi = \frac{\mu_0 N^2 i \pi r^2}{l}$$

$$\frac{\phi}{i} = \frac{\mu_0 N^2 \pi r^2}{l} \quad \text{--- (ii)}$$

$\because$ we know $\phi = Li \Rightarrow L = \frac{\phi}{i}$

eq (ii) Can be written as.

$$L = \frac{\mu_0 N^2 \pi r^2}{l}$$

multiply & devide by $l$

$$L = \frac{\mu_0 N^2}{l^2} \pi r^2 l$$

$$L = \mu_0 n^2 \pi r^2 l = \mu_0 n^2 A l$$

$$\boxed{L = \mu_0 n^2 A . l}$$

where $\boxed{n = \frac{N}{l}}$ & $A$ = area of solenoid
$A = \pi r^2$

$$L = \frac{\mu_0 N^2}{l} (\pi r^2)$$

let $\mu_r$ be the relative permitivity of any medium

$\mu_m = \mu_0 \mu_r$

Permeability of medium.

So $\boxed{L = \frac{\mu_0 \mu_r N^2}{l} (\pi r^2)}$ Self Inductance of Solenoid

* Equivalent self inductance of two inductive coils in series & in parallel.

** $L = L_1 + L_2 + L_3 + \cdots$ (in series)

** $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots$ (in parallel)

# Mutual Induction :-

If the two coils are placed near each other and the current flowing in one coil is changed then an emf is induced in the other coil. "this phenomenon of Electromagnetic Induction is called the mutual Induction."

the coil in which current is changed, is called primary coil and the other coil kept nearby in which the e.m.f is induced is called secondary coil.

→ Mutual Inductance ~~or~~ Coefficient of Mutual Inductance

- Acc to faraday's law, if "$i_1$" is the current flowing in the primary coil then the magnetic flux linked with the secondary coil is proportional to the '$i_1$'

$$\therefore \quad \phi_2 \propto i_1$$

$$\boxed{\phi_2 = M i_1}$$

M → coefficient of Mutual Induction / Mutual Inductance

when $i_1 = 1 \Rightarrow \phi_2 = M$

∴ Therefore the mutual Inductance b/w the two coil is numerically equal to the magnetic flux linked with the secondary coil when a unit current flows in the primary coil.

# Acc. to faraday's Second law :-

If the current flowing in the primary coil is changed, then the induced E.m.f in the secondary coil is .

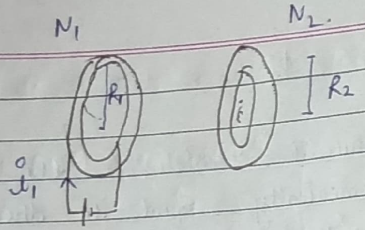
$$\varepsilon = -\frac{d\phi}{dt}$$

$$\varepsilon = -\frac{d}{dt} M i_1$$

$$\boxed{\varepsilon = -M \frac{di_1}{dt}} \quad \text{—— (i)}$$

# Mutual Inductance of two circular ~~disk~~ plain coil

let consider two coils Primary & secondary coil, $N_1$, $R_1$ are the no. of Turns, ~~length~~ Radius of coil of Primary coil and $N_2$, $R_2$ is for secondary coil.




Magnetic field due to the primarly coil is given by = $\frac{\mu_0 N i}{2R}$

for primary coil: $B = \frac{\mu_0 N_1 i_1}{2R_1}$.

→ area of the secondary coil = $N_2 \pi R_2^2$

flux linked with secondary coil

$= B \times$ area

$= \cancel{\phi}$

$$\phi = \frac{\mu_0 N_1 i_1}{2R_1} \times N_2 \pi R_2^2 \quad \rightarrow (1)$$

by the definition of Mutual Inductance

$$\phi_2 \propto i_1$$

$$\phi_2 = M i_1$$

$$\boxed{M = \frac{\phi_2}{i_1}}$$

from eq (i)

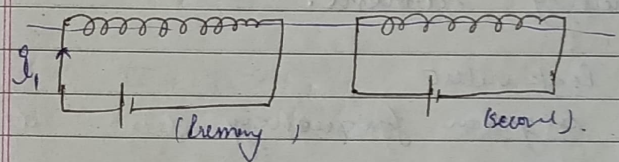
$$\frac{\phi}{i_1} = \frac{\mu_0 N_1 N_2 (\pi R_2^2)}{2R_1}$$

$$\boxed{M = \frac{\mu_0 N_1 N_2 (\pi R_2^2)}{2R_1}}$$

• Mutual Inductance of two long solenoids

Primary solenoid : $N_1, l_1, i_1, r_1$

Secondary " : $N_2, l_2, r_2$




Magnetic field due to primary solenoid.

$$B = \mu_0 n i = \mu_0 n_1 i_1 = \frac{\mu_0 N_1}{l_1} i_1$$

area of soleoneid (secondary) = $N_2 \pi r_2^2$

flux linked with sec. solenoid = $B \times$ area of sol.

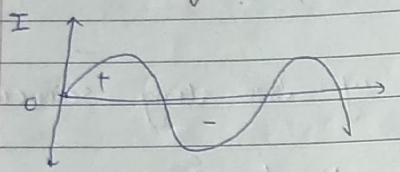
$$\cancel{\phi} = \frac{\mu_0 N_1 i_1}{l_1} \times N_2 (\pi r_2^2)$$

$$\frac{\phi}{i_1} = M = \frac{\mu_0 N_1 N_2 \pi r_2^2}{l_1} = \frac{\mu_0 N_1 N_2 A}{l_1}$$

Mutual Inductance for selenoid

$$\boxed{M = \frac{\mu_0 N_1 N_2 A}{l_1}}$$

# Alternating Current :-




$I = I_0 \sin \omega t$

The electric current whose magnitude change continuesly & periodically reverse its direction is called alternating current.

$I_0 \rightarrow$ peak value
$\omega \rightarrow$ angular frequency

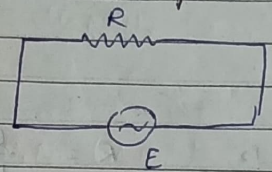
$E = E_0 \sin \omega t$
$\downarrow$ Emf ; $\downarrow$ max Emf

Complex representation of Alternating EMF & Current

$$\boxed{\begin{aligned} I &= I_0 e^{j\omega t} \\ E &= E_0 e^{j\omega t} \end{aligned}}$$

where $j = \sqrt{-1}$

A.C Circuit with pure resistance



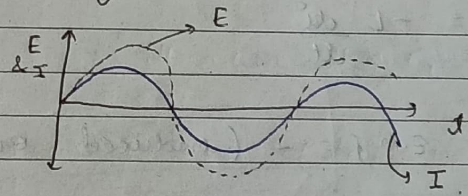

The instantaneous value of alternating e.m.f is given by $\boxed{E = E_0 e^{j\omega t}}$

Acc. to ohm's law. $\boxed{I = \frac{V}{R}}$

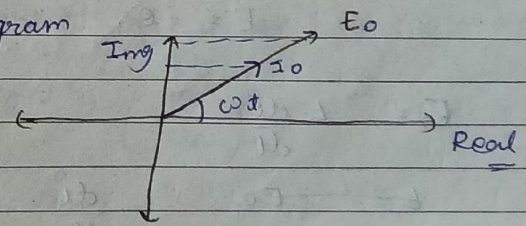
$$I = \frac{E}{R} = \frac{E_0 e^{j\omega t}}{R} = \left(\frac{E_0}{R}\right) e^{j\omega t}$$

$$\boxed{I = I_0 e^{j\omega t}}$$

graphically




Phasor diagram



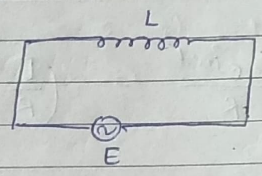
Resistance

$$R = \frac{E_0}{I_0}$$

devide by $\sqrt{2}$ $\quad R = \dfrac{\frac{E_0}{\sqrt{2}}}{\frac{I_0}{\sqrt{2}}} = \dfrac{E_v}{I_v}$

$R = \dfrac{E_v}{I_v}$ — $E_v$ → virtual Emf, $I_v$ → virtual current.

# A.C circuit with Inductor Only.




$$E = E_0 e^{j\omega t} \quad — (1)$$

~~to~~ Acc. to faradays law the induced emf is given by

$$e = -L\frac{di}{dt}$$

Inducing E.m.f = - (induced emf)

$$E = -e$$

$$E_0 = L\frac{di}{dt}$$

$\cancel{L = E_0}$ $\quad \dfrac{di}{dt} = \dfrac{E}{L}$

$$\frac{di}{dt} = \frac{E}{L} = \frac{E_0 e^{j\omega t}}{L}$$

$$di = \frac{E_0}{L} e^{j\omega t} \cdot dt$$

On integrating both sides

$$\int di = i = \frac{E_0}{L}\int e^{j\omega t} \cdot dt = \frac{E_0}{j\omega L} e^{j\omega t}$$

$$I = \frac{E_0 e^{j\omega t}}{j\omega L}$$

devide & multiply by $j$ ⇒ $I = \dfrac{E_0 j e^{j\omega t}}{j^2 \omega L} = \dfrac{-E_0 j e^{j\omega t}}{\omega L}$

$$I = \frac{-E_0 j e^{j\omega t}}{\omega L} \quad — (ii)$$

$$I = -j\left(\frac{E_0}{\omega L}\right)\left[\cos\omega t + \sin\omega t\right] \quad —$$

$$\therefore e^{-j\pi/2} = \cos\pi/2 - j\sin\pi/2$$

$$\boxed{e^{-j\pi/2} = -j}$$

Put value of $-j$ in eq (ii)

$$I = e^{-j\pi/2}\left(\frac{E_0}{\omega L}\right) e^{j\omega t}.$$

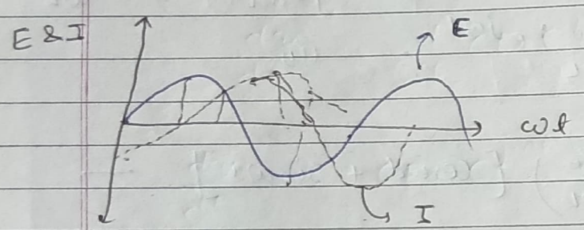
$$I = \left(\frac{E_0}{\omega L}\right) e^{j(\omega t - \pi/2)}$$

$$\boxed{I = I_0 \, e^{j(\omega t - \pi/2)}} \quad \text{--- (iii)} \qquad \mathcal{E} = E_0 e^{j\omega t}$$

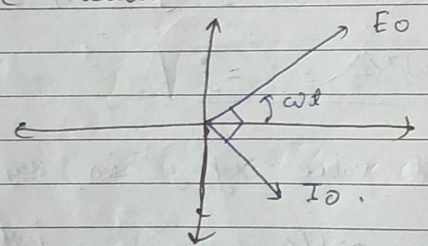
$$I_0 = \frac{E_0}{\omega L} \qquad \therefore \omega L = X_L \left[\begin{matrix}\text{inductive} \\ \text{reactance}\end{matrix}\right]$$

From eq (1) & (iii) the current lack from Emf by the phase difference of $\pi/2$.

# Graph




# Vector notation




- Reactance due to Inductor.

Inductive Reactance.

$$\omega L = \frac{E_0}{I_0} = \frac{\frac{E_0}{\sqrt{2}}}{\frac{I_0}{\sqrt{2}}} = \frac{E_v}{I_v}.$$

$$\omega L = \boxed{X_L = \frac{E_v}{I_v}}$$ virtual value of E.M.f
" " " Current.

As $X_L = \omega L$

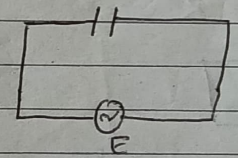
$$\boxed{X_L = 2\pi \nu L}$$

$X_L \propto \nu$ (frequency)

for DC $\nu = 0$

$$\boxed{X_L = 0}$$

# A.C circuit with Capacitor Only.




$$\boxed{E = E_0 \, e^{j\omega t}} \quad \text{———— (1)}$$

Induced emf is given by

$$e = -\frac{q}{c}$$

$\boxed{E = -e}$ ⟶ by lenz law.

$$\boxed{E = \frac{q}{C}}$$

$$\boxed{q = CE}$$

$$q = CE_0 e^{j\omega t}$$

we know that current $I = \frac{dq}{dt}$.

$$\frac{dq}{dt} = \frac{d}{dt} CE_0 e^{j\omega t}$$

$$\frac{dq}{dt} = CE_0 e^{j\omega t} \cdot j\omega$$

$$\frac{dq}{dt} = j\omega C E_0 \; e^{j\omega t}$$

$$\boxed{e^{j\pi/2} = \hat{j}}$$

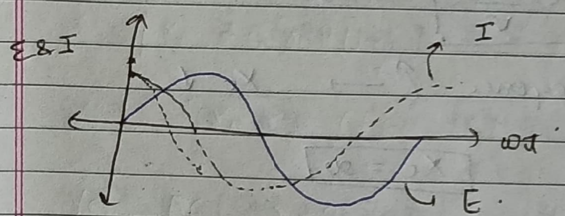
$$\frac{dq}{dt} = \mathring{i} = e^{j\pi/2} \cdot \omega C E_0 \; e^{j\omega t}$$

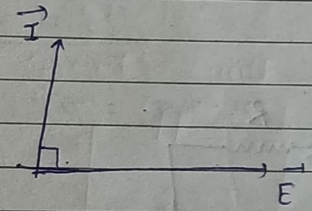
$$\mathring{i} = \frac{E_0}{\left(\frac{1}{\omega C}\right)} e^{j(\omega t + \pi/2)}$$

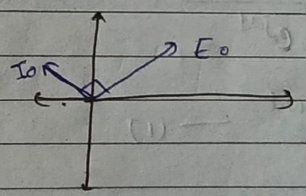
$$\boxed{I_0 = \frac{E}{1/\omega C}}$$

$\boxed{\mathring{i} = \mathring{i}_0 \; e^{j(\omega t + \pi/2)}}$ —— (ii)

from eq (i) & (ii) the alternating current leads the EMF by the phase angle of $\pi/2$.




Phaser diagram

Resistance due to Capacitor

Capacitive Reactance

$$\frac{1}{\omega C} = X_C = \frac{E_0}{I_0} = \frac{E_v}{I_v}.$$

$$\boxed{X_C = \frac{1}{\omega C} = \frac{E_v}{I_v}}$$

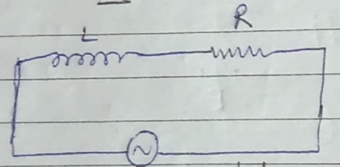
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$

$$\boxed{X_C \propto \frac{1}{\nu}}$$

as frequency $\uparrow \longrightarrow X_C \downarrow$.

for DC. $\boxed{X_C = \infty}$

# L-R circuit




$$E = E_0 e^{j\omega t}$$

$$E = E_0 e^{j\omega t} \quad \text{— (1)}$$

Potential diff. across R $= I_0 R$

P.D across Inductor $= I_0 j\omega L$

$(e^{j\pi/2})$ Current lags behind in inductor by $\pi/2$.

The peak value is given by

$$E_0 = I_0 R + I_0 j\omega L.$$

$$E_0 = I_0 [R + j\omega L]$$

$$E_0 = I_0 Z$$

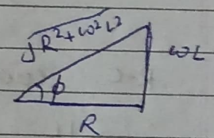
where $Z \rightarrow$ impedence (complex) of L-R circuit

$$Z = [R + j\omega L]$$

$$|Z| = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + X_L^2}$$

$$Z = |Z| (\cos\phi + j \sin\phi) = |Z| e^{j\phi}$$

$$\boxed{Z = |Z| e^{j\phi}}$$




$$\phi = \tan^{-1} \frac{\omega L}{R}$$

instantaneous value of current is given by

$$I = \frac{E}{Z} = \frac{E_0 e^{j\omega t}}{|Z| e^{j\phi}}$$

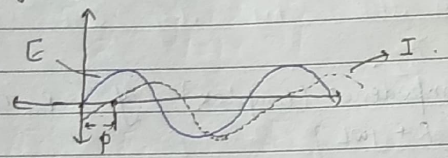
$$I = \frac{E_0}{|Z|} e^{j(\omega t - \phi)} \quad \text{— (II)}$$

$I = I_0 e^{j(\omega t - \phi)}$     $E = E_0 e^{j\omega t}$

Current in L-R circuit ~~the~~ lags behind the E.m.f by the phase difference of $\phi$:

$$I_0 = \frac{E_0}{|Z|} = \frac{E_0}{\sqrt{R^2 + X_L^2}}$$

•) graph

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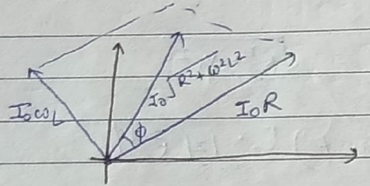
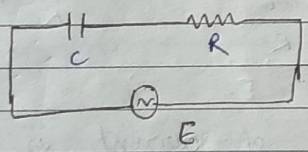



# R-C circuit




~~$E = E_0 \sin$~~     $E = E_0 e^{j\omega t}$

Peak value of alternating Emf across R

$$V_R = I_0 R.$$

the emf across C $= \frac{I_0}{j\omega C}$

$\frac{1}{j} \rightarrow$ Current leads in capacitive circuit by $\pi/2$

$$V_C = \frac{-j I_0}{\omega C}$$

So $E_0 = \frac{-I_0 j}{\omega C} + I_0 R$.

$$E_0 = I_0 \left[ R - \frac{j}{\omega C} \right]$$

$$E_0 = I_0 Z$$

$Z = R - \frac{j}{\omega C}$ = impedence of R-C circuit

$$|Z| = \sqrt{R^2 + \frac{1}{(\omega C)^2}} = \sqrt{R^2 + X_C^2}$$

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$Z = |Z| e^{j\phi} = |Z| [\cos\phi + j \sin\phi]$$

$$Z = \sqrt{R^2 + X_C^2}\, e^{j\phi}$$

~~Current~~ is given by $i = \frac{E}{Z} = \frac{E_0 e^{j\omega t}}{\ }$

To potential drop across capacitor and Resistor

$$V_T = E = V_R + V_C$$

$$E = IR - \frac{jI}{\omega C}$$

$$\frac{E}{I} = R - \frac{j}{\omega C} = Z.$$

Z can be written as

$$Z = |Z|\cos\phi - j|Z|\sin\phi = |Z| e^{-j\phi}$$

$$Z = |Z| e^{-j\phi}$$

$$I = \frac{E}{Z} = \frac{E_0 e^{j\omega t}}{|Z| e^{-j\phi}}$$

$$\boxed{I = I_0 e^{j(\omega t + \phi)}}$$

$$I_0 = \frac{E_0}{\sqrt{R^2 + X_C^2}}.$$

Current leads potential by the phase difference of $\phi$.

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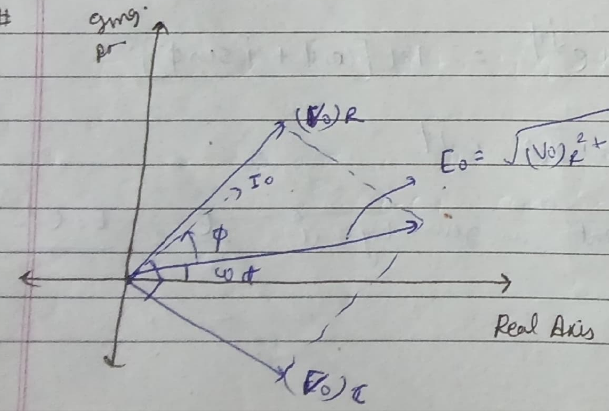
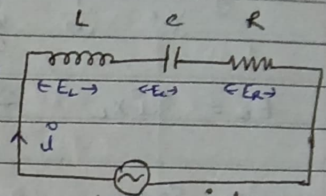



$$E_0 = \sqrt{(V_0)_R^2 + (V_0)_C^2}$$

# Series L-C-R




Let a source of alternating emf given by $E = E_0 e^{j\omega t}$ be connected to a series combination of a resistor, inductor, capacitor.

$E = E_0 e^{j\omega t}$

Let '$i$' be the current drawn in the circuit at time '$t$'

- Voltage drop across capacitor C at time '$t$' is given by

$$E_C = I \times \text{Resistance by Capacitor} = i \times X_C$$

$$E_C = iX_C \quad \text{—— (1)}$$

in capacitor the emf lags behind current by the phase diff. of $\pi/2$

so we multiply $-j$ on R.H.S in eq (1).

$$\boxed{E_C = -j\,i\,X_C} \longrightarrow \text{eq (ii)}$$

- Voltage drop across resistor in the circuit at time $t$.

$$E_R = iR =$$

$$\boxed{E_R = iR} \longrightarrow \text{eq (iii)}$$

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Voltage drop across inductor L at time 't' is given by

$$\boxed{E_L = i X_L} \quad \text{— (iv)}$$

~~that~~ Since in inductive circuit the current lacks behind by the Emf. with a phase diff. of $\pi/2$ so we multiply by 'j' on R.H.S of eq (iv)

$$\boxed{E_L = j\, i X_L} \longrightarrow (V)$$

total voltage drop in the circuit

$$E = E_L + E_C + E_R$$

∴ So add eq (ii), (iii), (V)

$$\cancel{E = E_0 e^{j\omega t}} = \quad E = i[R + jX_L - jX_C]$$

$$E = i[R + j(X_L - X_C)]$$

$$\frac{E}{i} = Z = R + j(X_L - X_C)$$

where Z is the impedence of the L-C-R circuit.

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

also we can write

$$Z = |Z|[\cos\phi + j\sin\phi]$$

$$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2} \cdot e^{j\phi}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \cdot e^{j\phi}$$

we know that $E = iZ$

from this $i = \dfrac{E}{Z} = \dfrac{E_0 e^{j\omega t}}{\sqrt{R^2 + (X_L - X_C)^2} \cdot e^{j\phi}}$

$$\frac{E}{Z} = \frac{E_0}{\sqrt{R^2 + (X_L - X_C)^2}} \cdot e^{j(\omega t - \phi)}$$

$$i = i_0 e^{j(\omega t - \phi)}$$

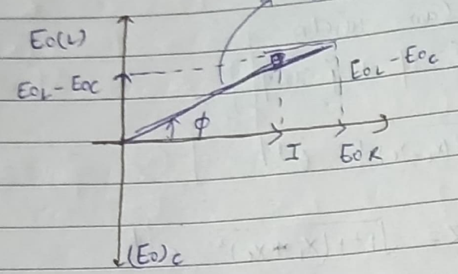
$$\boxed{I = I_0 e^{j(\omega t - \phi)}}$$

here $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right)$

if $X_L > X_C$, the current lacks behind by emf with phase diff of $\phi$.

if $X_C > X_C$ then current leads ~~to~~ emf by the phase of $\phi$.

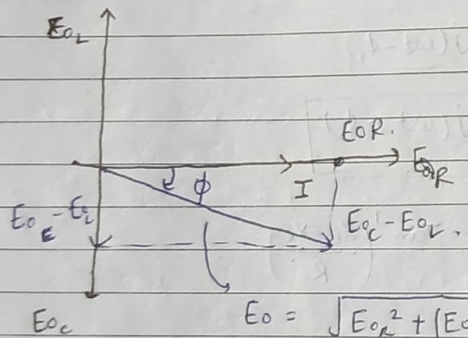
(i) If $X_L > X_C$




$$E_0 = \sqrt{E_{0R}^2 + (E_{0L} - E_{0C})^2}$$

Case II If $X_C > X_L$

$$\frac{1}{\omega C} > \omega L$$




$$E_0 = \sqrt{E_{0R}^2 + (E_{0C} - E_{0L})^2}$$

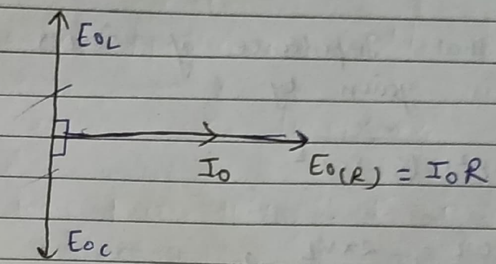
Case III when $X_L = X_L$.

$$\omega L = \frac{1}{\omega C}$$

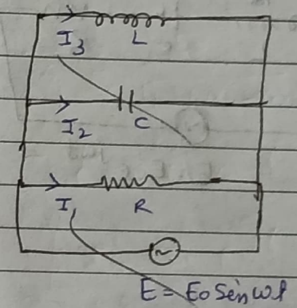
$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 0$$

Current & Emf are in same phase.

$$I_0 = \frac{E_0}{R} \quad \text{Max current}$$




# ~~Parallel~~ L-C-R circuit




$$I_1 = \frac{E_0}{R} \sin\omega t \quad \text{— (i)}$$

$$I_2 = \frac{E_0}{X_C} \sin(\omega t + \pi/2) \quad \text{— (ii)}$$

$$I_3 = \frac{E_0}{X_L} \sin(\omega t - \pi/2) \quad \text{— (iii)}$$

$\boxed{V_C = V_L = V_R}$

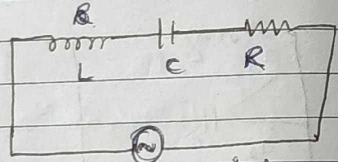
* Series Rasonant Circuit

We know that Impedance of series LCR circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where $X_L = \omega L = 2\pi \nu L$,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$




$E = E_0 e^{j\omega t}$

• at Rasonace

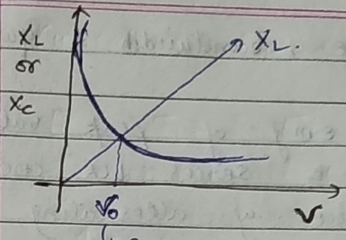
$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$\omega = 2\pi \nu = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\nu = \frac{1}{2\pi\sqrt{LC}}}$$

Rasonant frequency

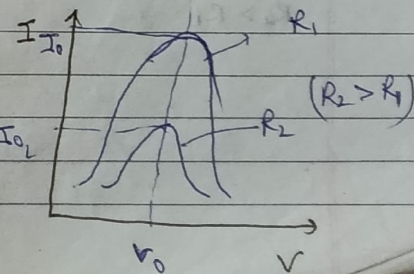



The value of Rasonant frequency of the circuit depends only on the value of Inductance (L) & Capacitance (C)

• If $\nu = \nu_0$, whe have $Z = R =$ ~~maximum~~ minimum

for the frequency $\nu_0$, the impedance of the circuit is minimum and the corresponding current is maximum. The variation of alternaty current with frequency $\nu$ is shown in fig

• In Series rasonant circuit, the current flowing in the circuit is max. hence it is called acceptor circuit. Such circuits are used in tuning of radio & t.v.



Sharpness of resonance, bandwidth & quality factor

when an alternating emf of peak value $E_0$ is applied to a series LCR circuit, then the peak value of alternating current in it is given by.

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

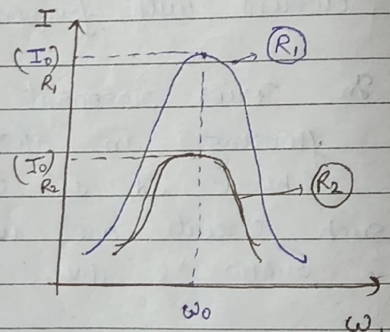
where Z is the impedence of LCR circuit

At resonance,

$$\omega L = \frac{1}{\omega C}, \quad Z = R \text{ (minimum)}$$

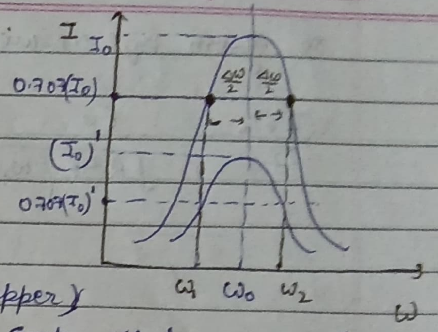
$$I_0 = \frac{E_0}{R} \text{ (max)} \quad \& \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

The variation of $I_0$ with '$\omega$' is shown in fig for 2 diff value of R




$R_2 > R_1$

$\frac{I_0}{\sqrt{2}} = 0.707 I_0 \rightarrow$ Rms current




for each curve, there are 2 frequencies $\omega_1$ and $\omega_2$ (called lower and upper cut of frequencies) such that current falls to $\frac{1}{\sqrt{2}}$ times the max current $I_0$

then the difference $\omega_1 - \omega_2$ is called bandwith $(\Delta\omega)$ of series LCR circuit

$$\boxed{\Delta\omega = \omega_2 - \omega_1}$$

$$\text{Sharpness of Resonance} = \frac{\text{Resonent frequency}}{\text{Bandwidth}}$$

$$= \frac{\omega_0}{\omega_2 - \omega_1 \; (\Delta\omega)}$$

from graph : $\omega_2 = \omega_0 + \frac{\Delta\omega}{2}$

& $\omega_1 = \omega_0 - \frac{\Delta\omega}{2}$

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{—(i)}$$

at $\omega = \omega_2$ we have $I = \frac{I_0}{\sqrt{2}}$

so from eq$^n$ (i) we get.

$$\frac{(I_0)_m}{\sqrt{2}} = \frac{E_0}{\sqrt{R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2}}$$

$$\boxed{I_{0m} = \frac{E_0}{R}}$$

$$\frac{E_0}{R\sqrt{2}} = \frac{E_0}{\sqrt{R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2}}$$

$$R\sqrt{2} = \sqrt{R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2}$$

Square both sides

$$2R^2 = R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2$$

$$R^2 = \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2$$

$$R = \omega_2 L - \frac{1}{\omega_2 C}$$

$\therefore$ we know $\omega_2 = \omega_0 + \frac{\Delta\omega}{2}$

$$R = \left(\omega_0 + \frac{\Delta\omega}{2}\right)L - \frac{1}{\left(\omega_0 + \frac{\Delta\omega}{2}\right)C}$$

$$R = \omega_0 \left[1 + \frac{\Delta\omega}{2\omega_0}\right)L - \frac{1}{\omega_0 C\left[1 + \frac{\Delta\omega}{2\omega_0}\right)}$$

$$R = \omega_0 L\left(1 + \frac{\Delta\omega}{2\omega_0}\right) - \frac{1}{\omega_0 C\left(1 + \frac{\Delta\omega}{2\omega_0}\right)}$$

at Resonance

$$\boxed{\omega_0 L = \frac{1}{\omega_0 C}}$$

$$R = \omega_0 L\left(1 + \frac{\Delta\omega}{2\omega_0} - \frac{1}{1 + \frac{\Delta\omega}{2\omega_0}}\right)$$

$$R = \omega_0 L\left[\left(1 + \frac{\Delta\omega}{2\omega_0}\right) - \left(1 + \frac{\Delta\omega}{2\omega_0}\right)^{-1}\right]$$

$\boxed{(1+x)^{-n} = 1+nx}$, Highest power are neglig

$$R = \omega_0 L\left[\left(1 + \frac{\Delta\omega}{2\omega_0}\right) - 1 + \frac{\Delta\omega}{2\omega_0}\right]$$

$$R = \omega_0 L\left(\frac{2\Delta\omega}{2\omega_0}\right)$$

$$R = L\,\Delta\omega.$$

$$\boxed{\Delta\omega = \frac{R}{L}}$$

$$\boxed{\Delta\omega = \frac{R}{L}}$$

Sharpness of resonance $= \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{\frac{R}{L}} = \frac{\omega_0 L}{R}$

$$\boxed{\text{Sharpness of resonance} = \frac{\omega_0 L}{R}}$$

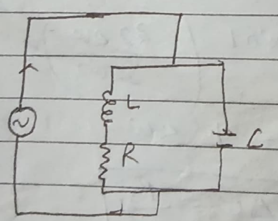
Quality factor (Q) of series LCR circuit is given by

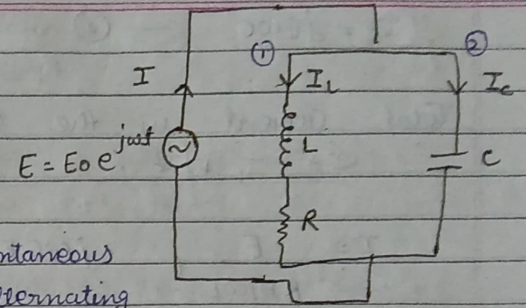
$$Q = \frac{\omega_0 L}{R} = \frac{\omega_0}{\frac{R}{L}} = \frac{\omega_0}{\Delta\omega}$$

$$\boxed{Q = \frac{\omega_0}{\Delta\omega}}$$

# Parallel Resonant Circuit

A parallel resonant circuit consists of a series combination of inductor and resistor connected in parallel to a capacitor. This parallel combination is then connected to a source of alternating emf.




Let the instantaneous value of alternating e.m.f at time 't' is given by

$$\boxed{E = E_0 e^{j\omega t}}$$

Voltage drop across branch ①

$$V_L + V_R$$
$$= I_L j\omega L + I_L R$$

$E = I_L (j\omega L + R)$ [ voltage leades current by $\pi/2$ so multiply by $j$

$$E = I_L (j\omega L + R)$$

$$E = I_L (j\omega L + R)$$

$$I_L = \frac{E}{j\omega L + R} \longrightarrow (i)$$

Voltage drop across branch ② $= V_c$.

$$E = I_c X_c \, {-j} = \frac{-I_c j}{\omega L}$$

$$I_c = \frac{E\omega L}{-j} = jE\omega L$$

$I_C = jE\omega C$ — (2)

Total current in the circuit is given by

$I = I_L + I_C$.

$$I = \frac{E}{R+j\omega L} + j\omega C E$$

$$I = E\left[\frac{1}{R+j\omega L} + j\omega C\right]$$

$$\frac{I}{E} = \frac{1}{R+j\omega L} + j\omega C = \frac{1}{Z}$$

the reciprocal of impedence (Z) of a circuit is known as admittance (Y).

$$Y = \frac{1}{Z} = \frac{1}{\frac{E}{I}} = \frac{I}{E}.$$

$$\boxed{Y = \frac{1}{R+j\omega L} + j\omega C}$$

$$\frac{1}{Z} = \frac{1}{R+j\omega L} + j\omega C$$

$$Y = \frac{1}{Z} = \frac{R-j\omega L}{R^2+\omega^2L^2} + j\omega C$$

$$Y = \frac{1}{Z} = \frac{R-j\omega L}{R^2+\omega^2L^2} + j\omega C$$

$$Y = \frac{1}{Z} = \frac{R-j\omega L + j\omega C(R^2+\omega^2L^2)}{R^2+\omega^2L^2}$$

$$Y = \frac{1}{Z} = \frac{R-j\omega L + j\omega CR^2 + j\omega^3CL^2}{R^2+\omega^2L^2}$$

$$Y = \frac{1}{Z} = \frac{R-j(\omega L - \omega CR^2 - \omega^3CL^2)}{R^2+\omega^2L^2}.$$

from above equation it is clear that for a given values of L, C & R the admittance (Y) will by maximum or the impedence maximum if.

$$\omega L - \omega CR^2 - \omega^3CL^2 = 0$$

$$\omega(L - CR^2 - \omega^2CL^2) = 0$$

$$L - CR^2 - \omega^2CL^2 = 0$$

$$L - CR^2 = \omega^2CL^2$$

$$\frac{L}{CL^2} - \frac{CR^2}{CL^2} = \omega^2$$

$$\frac{1}{CL} - \frac{R^2}{L^2} = \omega^2.$$

$$\omega = \sqrt{\frac{1}{CL} - \frac{R^2}{L^2}}.$$

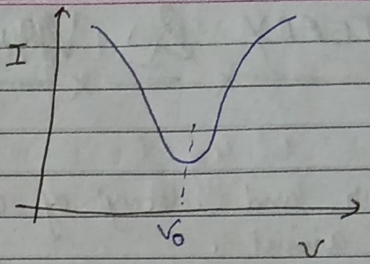
$$2\pi\nu_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

$$\boxed{\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}} \longrightarrow \text{Rejecter Circuit.}$$

$\nu_0$ is called "Rasonant Frequency" In parallel resonant circuit at frequency equals to rasonant frequency ($\nu_0$) the (Z) of the circuit is max, ~~to~~ So current in it will be minimum

So parallel resonant circuits are called "Rejector circuits"

These circuits are used as "filter circuits" in radios to block the currents of undesired frequency.




Power in Alternating current

Power $P = Vi$ ( in current electricity )

So, here in AC both V & i varies with time power with also vary with time

$P(t) = V(t)\, i(t)$

let $V = V_0 \sin\omega t$ $\quad I = i_0 \sin(\omega t + \phi)$

$$\boxed{P(t) = V_0 \sin\omega t \; i_0 \sin(\omega t + \phi)}$$