<u>EE 536a 8/28/24 Notes</u>

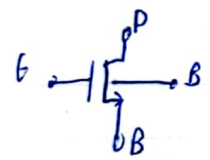
Small Signal Analysis

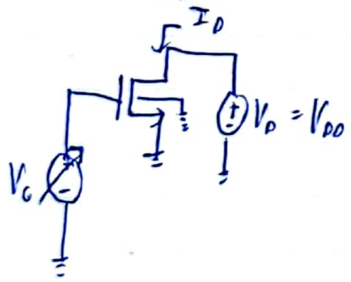
$$\begin{array}{ccccc} x & \longrightarrow & \boxed{f(\cdot)} & \longrightarrow & y = f(x) \\ x_0 + \delta x & & & & Y_0 + \delta y \end{array}$$

$$y\big|_{x_0+\delta x} = f(x_0 + \delta x) \approx \underbrace{f(x)}_{Y_0} + \underbrace{f'(x)\big|_{x=x_0} \cdot \delta x}_{\delta y}$$

1) DC
2) Calculate $f'(x)\big|_{x_0}$
3) Small signal analysis

NFET





When $V_{GS} < V_{TH}$, $I_D \approx 0 \Rightarrow$ Cutoff region

When $V_{GS} \geq V_{TH}$,

$$I_{DS} = \begin{cases} \dfrac{\mu_N C_{ox}}{2} \dfrac{W}{L} \dfrac{2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2}{1 + \frac{V_{DS}}{E_c L}} & ① \\[2ex] \dfrac{\mu_N C_{ox}}{2} \dfrac{W}{L} \dfrac{(V_{GS} - V_{TH})^2}{1 + \frac{V_{GS} - V_{TH}}{E_c L}} (1 + \lambda V_{DS}) & ② \end{cases}$$

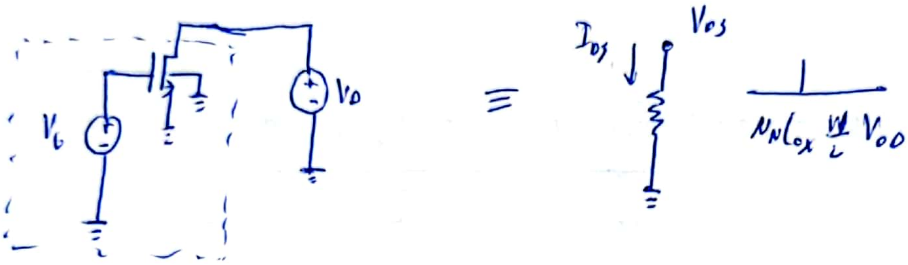
μ_N: mobility of electrons
C_{ox}: cap/unit ~~width~~ area
W: width
L: length
E_c: critical field
λ: channel length modulation coeff., $\propto \frac{1}{L}$

① $V_{DS} < V_{GS} - V_{TH}$: triode
② $V_{DS} \geq \underbrace{V_{GS} - V_{TH}}_{V_{OD}}$: saturation

~~[illegible]~~ Triode

Assume $V_{DS} \ll E_c L$ $\Rightarrow$ $I_{DS} \approx \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) V_{DS}$

$V_{DS} \ll V_{OD}$



$$\left.\frac{\partial I_{DS}}{\partial V_D}\right|_{V_{DS}=0} (\text{triode}) = \frac{\cancel{2}\,\mu_n C_{ox}}{\cancel{2}} \frac{W}{L} \left(\frac{\left(1+\frac{V_{DS}}{E_c L}\right)\left(2V_{DS} - 2(V_{GS}-V_{TH})\right) - \cancel{\left(2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right)\left(\frac{1}{E_c L}\right)}}{\left(1+\frac{V_{DS}}{E_c L}\right)^2} \right)$$

~~If~~ $\cancel{\left.\frac{\partial I_{DS}}{\partial V_D}\right|_{V_{DS}=0} = \mu_n C_{ox} \frac{W}{L} (-2(V_{GS}-V_{TH}))}$

$$\frac{\partial I_{DS}}{\partial V_D} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

Sat.

Case 1

longer CLM ($\lambda = 0$) $\in$ generally a bad assumption!

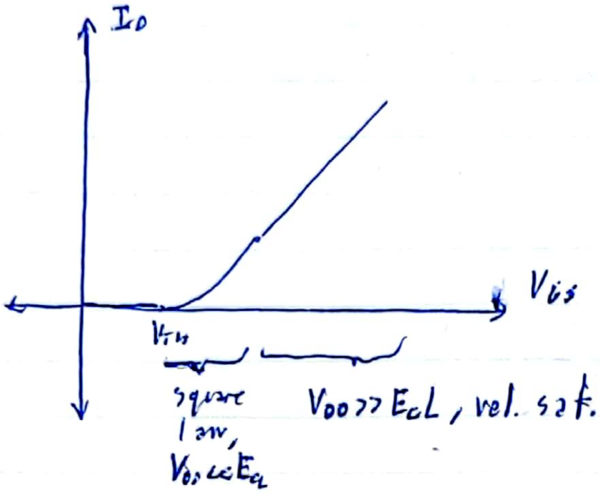
Assume $V_{GS} - V_{TH} \ll E_c L$

$I_{DS} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2$

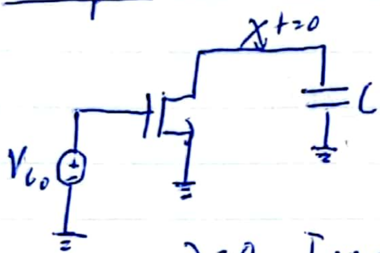
Case 2

Assume $V_{GS} - V_{TH} \gg E_c L$

$I_{DS} = \frac{\mu_n C_{ox}}{2} E_c W (V_{GS} - V_{TH})$



Example

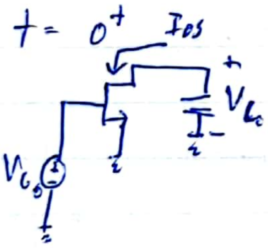


$$V_C\Big|_{t=0} = V_{C0}\ , \quad V_{C0} > V_{G0} > V_{TH}$$

Plot $V_C(t),\ t>0$

$\lambda = 0$, Ignore vel. sat.

$t = 0^+$



$V_{DS} > V_{GS} - V_{TH}$

$V_{C0} > V_{G0} \Rightarrow V_{C0} > V_{GS} - V_{TH} \Rightarrow$ sat.

$$I_{DS} = \frac{\mu_N C_{ox}}{2} \frac{W}{L} (V_{G0} - V_{TH})^2 \Rightarrow I \approx \text{fixed until triode region}$$

$$I_C(t) = C\frac{dV_C(t)}{dt}$$

$$V_C(t) = \frac{1}{C}\int_0^t I_C(t)\,dt + V_C(0)$$

$$I_{DS} = -I_C$$

$$\frac{\mu_N C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 = -C\frac{d}{dt}V_C(t)$$

$$\int dV_C(t) = \int -\frac{1}{C}\frac{\mu_N C_{ox}}{2}\frac{W}{L}(V_{G0} - V_{TH})^2\,dt$$

$$\boxed{V_C(t) = V_C(0) - \frac{1}{C}\frac{\mu_N C_{ox}}{2}\frac{W}{L}(V_{G0} - V_{TH})^2 t}$$ in sat, $t \le t_0$

When $\underbrace{V_{DS}}_{V_C(t)} = \underbrace{V_{GS} - V_{TH}}_{V_{G0}-V_{TH}}$, transistor enters triode

at $t = t_0$: $V_C(t) = V_{G0} - V_{TH}$

$$V_{C0} - \frac{k_n}{C} t = V_{G0} - V_{TH}$$

$$\boxed{t_0 = \frac{C}{k_n} \frac{(V_{C0} - V_{G0} + V_{TH})}{(V_{G0} - V_{TH})^2}}$$

$t > t_0$: $\underbrace{I_{DS}}_{-I_C(t)} = \frac{\mu_N C_{ox}}{2} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right]$

$$\frac{\mu_N C_{ox}}{2} \frac{W}{L} \left[2(V_{G0} - V_{TH}) V_C(t) - V_C^2(t) \right] = -C \frac{d V_C(t)}{dt}$$

$$-\frac{1}{C} k_n \, dt = \frac{d V_C(t)}{2(V_{G0} - V_{TH}) V_C(t) - V_C^2(t)}$$

$$-\frac{k_n}{C} t \Big|_{t_0}^{t} = \frac{\frac{1}{2(V_{G0} - V_{TH})}}{V_C(t)} + \frac{\frac{1}{2(V_{G0} - V_{TH})}}{2(V_{G0} - V_{TH}) - V_C(t)}$$

$$-\frac{k_n}{C} t \Big|_{t_0}^{t} = \frac{1}{2(V_{G0} - V_{TH})} \left[\ln(V_C(t)) + \ln\left(2(V_{G0} - V_{TH}) - V_C(t)\right) \right]_{V_C(t_0)}^{V_C(t)}$$

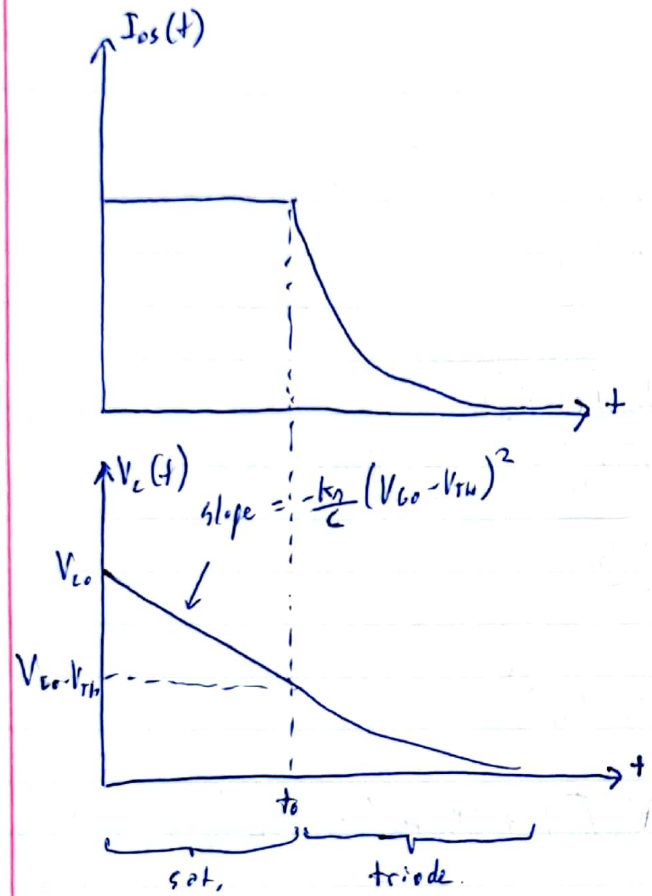
$$= \frac{1}{2(V_{G0} - V_{TH})} \ln\left(\frac{V_C(t)}{V_C(t) - 2(V_{G0} - V_{TH})} \right) \Bigg|_{V_C(t_0)}^{V_C(t)}$$

$$\frac{k_n}{C}(t - t_0) = \frac{1}{2(V_{G0} - V_{TH})} \left(\ln\left(\frac{V_C(t)}{V_C(t) - 2(V_{G0} - V_{TH})} \right) - \ln\left(\frac{V_C(t_0)}{V_C(t_0) - 2(V_{G0} - V_{TH})} \right) \right)$$

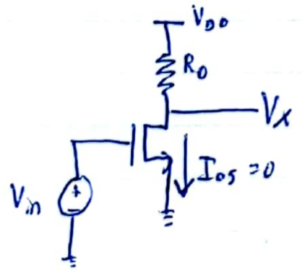
$$-\frac{k_n}{C} t = \frac{1}{2(V_{G0} - V_{TH})} \ln \frac{V_C(t)}{2(V_{G0} - V_{TH}) - V_C(t)} + \square$$

$$\frac{V_C(t)}{2(V_{G0} - V_{TH}) - V_C(t)} = e^{2(V_{G0} - V_{TH})\left(-\frac{k_n}{C} t - \square\right)}$$

$$\boxed{V_C(t) = \frac{2(V_{G0} - V_{TH}) e^{2(V_{G0} - V_{TH})\left[-\frac{k_n}{C} t - \square\right]}}{1 + e^{2(V_{G0} - V_{TH})\left[-\frac{k_n}{C} t - \square\right]}}}$$ in triode, $t > t_0$



Example



Derive & Plot $V_X(V_{in})$ When $V_{in} = [0, V_{DD}]$
ignore vel. sat. for HW #! Mostly ignore λ.

$\lambda = 0$, ignore vel. sat.

$V_{in} < V_{TH} \Rightarrow$ Cutoff $\Rightarrow I_{DS} = 0$

KVL: $V_X = V_{DD} - R_D I_{DS}$

$I_{DS} = 0 \Rightarrow V_X = V_{DD}$

$V_{in} > V_{TH} \Rightarrow$ sat.

$I_{DS} = k_n (V_{in} - V_{TH})^2$

$V_X = V_{DD} - R_D I_{DS} = V_{DD} - R_D k_n (V_{in} - V_{TH})^2$

until $\underbrace{V_{DS}}_{V_X} < V_{in} - V_{TH}$

$V_x = V_{in} - V_{TH}$

$V_{DD} - R_D k_n (V_{in} - V_{TH})^2 = V_{in} - V_{TH}$

$R_D k_n (V_{in} - V_{TH})^2 + (V_{in} - V_{TH})^2 \cancel{\ldots} - V_{DD} = 0$

$$V_{in} - V_{TH} = \frac{-1 \pm \sqrt{1 + 4 V_{DD} R_D k_n}}{2 R_D k_n}$$

$$\boxed{V_{in_1} = V_{TH} + \frac{-1 + \sqrt{1 + 4 V_{DD} R_D k_n}}{2 R_D k_n}}$$

$$V_{in} > V_{in_1} = V_{TH} + \frac{-1 + \sqrt{1 + 4 V_{DD} R_D k_n}}{2 R_D k_n}$$

triode $\quad I_{DS} = k_n [2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2]$

$V_x = V_{DS} = V_{DD} - R_D I_{DS}$

$V_x = V_{DD} - R_D k_n \cancel{V} V_x [1 + 2 R_D k_n (V_{in} - V_{TH})] - (R_D k_n) V_x^2$

$V_x = \ldots$