

CHAPTER -1

Q- 1 Properties of Regular expression

Ans -

- It is a way of representing regular languages.
- The algebraic description for regular languages is done using regular expressions.
- They can define the same language that various forms of finite automata can describe.
- Regular expressions offer something that finite automata do not, i.e. it is a declarative way to express the strings that we want to accept. They act as input for many systems. They are used for string matching in many systems (Java, python etc.)
- For example, Lexical-analyzer generators, such as Lex or Flex.

The widely used operators in regular expressions are Kleene closure(*), concatenation(.), Union(+).

- The set of regular expressions is defined by the following rules.
- Every letter of Σ can be made into a regular expression, null string, ϵ itself is a regular expression. If r_1 and r_2 are regular expressions, then (r_1) , $r_1.r_2$, r_1+r_2 , r_1^* , r_1^+ are also regular expressions.

$\Sigma = \{a, b\}$ and r is a regular expression of language made using these symbols

\emptyset	$\{\}$
ϵ	$\{\epsilon\}$
a^*	$\{\epsilon, a, aa, aaa, \dots\}$
$a + b$	$\{a, b\}$
$a.b$	$\{ab\}$
$a^* + ba$	$\{\epsilon, a, aa, aaa, \dots, ba\}$

The union of two regular languages, L_1 and L_2 , which are represented using $L_1 \cup L_2$, is also regular and which represents the set of strings that are either in L_1 or L_2 or both. $L_1 = (1+0).(1+0) = \{00, 10, 11, 01\}$ and $L_2 = \{\epsilon, 100\}$ then $L_1 \cup L_2 = \{\epsilon, 00, 10, 11, 01, 100\}$.

The concatenation of two regular languages, L_1 and L_2 , which are represented using $L_1.L_2$ is also regular and which represents the set of strings that are formed by taking any string in L_1 concatenating it with any string in L_2 . $L_1 = \{0, 1\}$ and $L_2 = \{00, 11\}$ then $L_1.L_2 = \{000, 011, 100, 111\}$.

-If L_1 is a regular language, then the Kleene closure i.e. L_1^* of L_1 is also regular and represents the set of those strings which are formed by taking a number of strings from L_1 and the same string can be repeated any number of times and concatenating those strings. $L_1 = \{0, 1\} = \{\epsilon, 0, 1, 00, 01, 10, 11, \dots\}$, then L_1^* is all strings possible with symbols 0 and 1 including a null string

Kleene closure is an unary operator and Union(+) and concatenation operator(.) are binary operators.

If r_1 and r_2 are regular expressions(RE), then

- r_1^* is a RE
- $r_1 + r_2$ is a RE
- $r_1.r_2$ is a RE
- $(r^*)^* = r$, closing an expression that is already closed does not change the language.
- $\emptyset^* = \epsilon$, a string formed by concatenating any number of copies of an empty string is empty itself.
- $r_+ = r.r^* = r^*r$, as $r^* = \epsilon + r + rr + rrr \dots$ and $r.r^* = r + rr + rrr \dots$
- $r^* = r^* + \epsilon$

If r_1, r_2, r_3 are RE, then

- : $r_1 = a, r_2 = b, r_3 = c$, then
- The resultant regular expression in LHS becomes $a+(b+c)$ and the regular set for the corresponding RE is $\{a, b, c\}$.
- for the RE in RHS becomes $(a+b)+c$ and the regular set for this RE is $\{a, b, c\}$, which is same in both cases. Therefore, the associativity property holds for union operator.
- $r_1 = a, r_2 = b, r_3 = c$
- Then the string accepted by RE $a.(b.c)$ is only abc .
- The string accepted by RE in RHS is $(a.b).c$ is only abc , which is same in both cases. Therefore, the

Associativity property does not hold for Kleene closure($*$) because it is unary operator.

In the case of union operators if $r + x = r \Rightarrow x = \emptyset$ as $r \cup \emptyset = r$, therefore \emptyset is the identity for $+$. Therefore, \emptyset is the identity element for a union operator. In the case of concatenation operator -if $r.x = r$, for $x = \epsilon$. $\epsilon = r \Rightarrow \epsilon$ is the identity element for concatenation operator($.$).

- If $r + x = r \Rightarrow r \cup x = r$, there is no annihilator for $+$
- In the case of a concatenation operator, $r.x = x$, when $x = \emptyset$, then $r.\emptyset = \emptyset$, therefore \emptyset is the annihilator for the ($.$) operator. For example $\{a, aa, ab\}.\{\} = \{\}$

If r_1, r_2 are RE, then

- $r_1 + r_2 = r_2 + r_1$. For example, for $r_1 = a$ and $r_2 = b$, then RE $a + b$ and $b + a$ are equal.
- $r_1.r_2 \neq r_2.r_1$. For example, for $r_1 = a$ and $r_2 = b$, then RE $a.b$ is not equal to $b.a$.

are regular expressions, then

- $(r_1 + r_2).r_3 = r_1.r_3 + r_2.r_3$ i.e. Right distribution
- $r_1.(r_2 + r_3) = r_1.r_2 + r_1.r_3$ i.e. left distribution
- $(r_1.r_2) + r_3 \neq (r_1 + r_3)(r_2 + r_3)$

- $r_1 + r_1 = r_1 \Rightarrow r_1 \cup r_1 = r_1$, therefore the union operator satisfies idempotent property.
- $r.r \neq r \Rightarrow$ concatenation operator does not satisfy idempotent property.

There are many identities for the regular expression. Let p, q and r are regular expressions.

- $\emptyset + r = r$
- $\emptyset.r = r.\emptyset = \emptyset$
- $\epsilon.r = r.\epsilon = r$
- $\epsilon^* = \epsilon$ and $\emptyset^* = \epsilon$
- $r + r = r$
- $r^*.r^* = r^*$
- $r.r^* = r^*.r = r_+$
- $(r^*)^* = r^*$
- $\epsilon + r.r^* = r^* = \epsilon + r.r^*$

- $(p.q)^*.p = p.(q.p)^*$
- $(p + q)^* = (p^*.q^*)^* = (p^* + q^*)^*$
- $(p + q).r = p.r + q.r$ and $r.(p + q) = r.p + r.q$

Q-2 Convert dfa to regular expression

Ans- 2015 wale m h or rgpv m h imp h

Q-3 Pumping lemma for regular language

Ans- For any regular language L , there exists an integer n , such that for all $x \in L$ with $|x| \geq n$, there exists $u, v, w \in \Sigma^*$, such that $x = uvw$, and (1) $|uv| \leq n$ (2) $|v| \geq 1$ (3) for all $i \geq 0$: $uv^i w \in L$. In simple terms, this means that if a string v is 'pumped', i.e., if v is inserted any number of times, the resultant string still remains in L . Pumping Lemma is used as a proof for irregularity of a language. Thus, if a language is regular, it always satisfies pumping lemma. If there exists at least one string made from pumping which is not in L , then L is surely not regular. The opposite of this may not always be true. That is, if Pumping Lemma holds, it does not mean that the language is regular. <https://media.geeksforgeeks.org/wp-content/cdn-uploads/gq/2016/03/p1.png>

For example, let us prove $L_0 = \{0^n 1^n \mid n \geq 0\}$ is irregular. Let us assume that L is regular, then by Pumping Lemma the above given rules follow. Now, let $x \in L$ and $|x| \geq n$. So, by Pumping Lemma, there exists u, v, w such that (1) – (3) hold. We show that for all u, v, w , (1) – (3) does not hold. If (1) and (2) hold then $x = 0^n 1^n = uvw$ with $|uv| \leq n$ and $|v| \geq 1$. So, $u = 0^a, v = 0^b, w = 0^c 1^n$ where: $a + b \leq n, b \geq 1, c \geq 0, a + b + c = n$. But, then (3) fails for $i = 0$: $uv^0 w = uw = 0^a 0^c 1^n = 0^{a+c} 1^n \notin L$, since $a + c \neq n$. <https://media.geeksforgeeks.org/wp-content/cdn-uploads/gq/2016/03/p2.png>

Q-4 explain dfa and nfa with example

Ans- DFA refers to deterministic finite automata. Deterministic refers to the uniqueness of the computation. The finite automata are called deterministic finite automata if the machine is read an input string one symbol at a time.

- In DFA, there is only one path for specific input from the current state to the next state.
- DFA does not accept the null move, i.e., the DFA cannot change state without any input character.
- DFA can contain multiple final states. It is used in Lexical Analysis in Compiler.

In the following diagram, we can see that from state q_0 for input a , there is only one path which is going to q_1 . Similarly, from q_0 , there is only one path for input b going to q_2 .

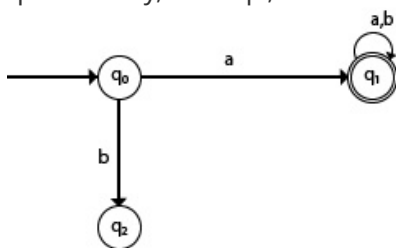


Fig:- DFA

Formal Definition of DFA

A DFA is a collection of 5-tuples same as we described in the definition of FA.

1. Q : finite set of states
2. Σ : finite set of the input symbol
3. q_0 : initial state
4. F : **final** state
5. δ : Transition function

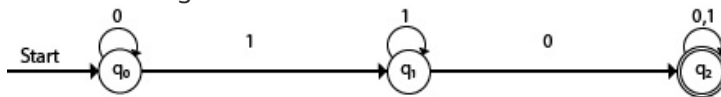
Transition function can be defined as:

$$1. \delta: Q \times \Sigma \rightarrow Q$$

Example 1:

1. $Q = \{q_0, q_1, q_2\}$
2. $\Sigma = \{0, 1\}$
3. $q_0 = \{q_0\}$
4. $F = \{q_2\}$

Transition Diagram:



Present State	Next state for Input 0	Next State of Input 1
→q0	q0	q1
q1	q2	q1
*q2	q2	q2

NFA (Non-Deterministic finite automata)

- NFA stands for non-deterministic finite automata. It is easy to construct an NFA than DFA for a given regular language.
- The finite automata are called NFA when there exist many paths for specific input from the current state to the next state.
- Every NFA is not DFA, but each NFA can be translated into DFA.
- NFA is defined in the same way as DFA but with the following two exceptions, it contains multiple next states, and it contains ϵ transition.

In the following image, we can see that from state q_0 for input a , there are two next states q_1 and q_2 , similarly, from q_0 for input b , the next states are q_0 and q_1 . Thus it is not fixed or determined that with a particular input where to go next. Hence this FA is called non-deterministic finite automata.

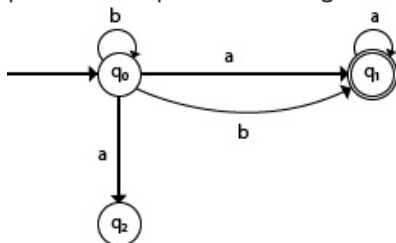


Fig:- NDFA

Formal definition of NFA:

NFA also has five states same as DFA, but with different transition function, as shown follows:

$$\delta: Q \times \Sigma \rightarrow 2Q$$

where,

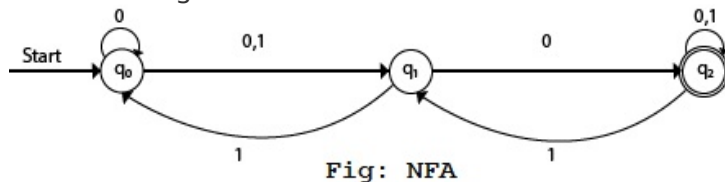
1. Q : finite set of states
2. Σ : finite set of the input symbol
3. q_0 : initial state

4. F: **final** state
5. δ : Transition function

Example 1:

1. $Q = \{q_0, q_1, q_2\}$
2. $\Sigma = \{0, 1\}$
3. $q_0 = \{q_0\}$
4. $F = \{q_2\}$

Transition diagram:



Transition Table:

Present State	Next state for Input 0	Next State of Input 1
→q0	q0, q1	q1
q1	q2	q0
*q2	q2	q1, q2

In the above diagram, we can see that when the current state is q_0 , on input 0, the next state will be q_0 or q_1 , and on 1 input the next state will be q_1 . When the current state is q_1 , on input 0 the next state will be q_2 and on 1 input, the next state will be q_0 . When the current state is q_2 , on 0 input the next state is q_2 , and on 1 input the next state will be q_1 or q_2 .

Q-5 How dfa equivalence to nfa

Ans -

Q- 6 Difference between mealy and moore machine

Ans -

Difference Between Moore and Mealy Machine

Moore Machine –

- Output depends only upon present state.
- If input changes, output does not change.
- More number of states are required.
- There is less hardware requirement for circuit implementation.
- They react faster to inputs.
- Synchronous output and state generation.
- Output is placed on states.
- Easy to design.

Mealy Machine –

- Output depends on present state as well as present input.
- If input changes, output also changes.
- Less number of states are required.
- There is more hardware requirement for circuit implementation.
- They react slower to inputs (One clock cycle later).
- Asynchronous output generation.
- Output is placed on transitions.
- It is difficult to design.

Q-7 Mylle nerode theorem

Ans- Copy m se dekh lena

Q-8 application of pumping lemma

Ans - Applications of Pumping Lemma

Pumping Lemma is to be applied to show that certain languages are not regular. It should never be used to show a language is regular.

- If L is regular, it satisfies Pumping Lemma.
- If L does not satisfy Pumping Lemma, it is non-regular.

chapter – 2

Q-1 CFL generated by grammer

Ans – dekhna h kese krna h

Q-2 Regular grammmer

Ans – Copy m h

Q-3 CNF (CHOMSKY NORMAL FORM)

ans - Chomsky's Normal Form (CNF)

CNF stands for Chomsky normal form. A CFG(context free grammar) is in CNF(Chomsky normal form) if all production rules satisfy one of the following conditions:

- Start symbol generating ϵ . For example, $A \rightarrow \epsilon$.
- A non-terminal generating two non-terminals. For example, $S \rightarrow AB$.
- A non-terminal generating a terminal. For example, $S \rightarrow a$.

For example:

1. $G_1 = \{S \rightarrow AB, S \rightarrow c, A \rightarrow a, B \rightarrow b\}$
2. $G_2 = \{S \rightarrow aA, A \rightarrow a, B \rightarrow c\}$

The production rules of Grammar G_1 satisfy the rules specified for CNF, so the grammar G_1 is in CNF. However, the production rule of Grammar G_2 does not satisfy the rules specified for CNF as $S \rightarrow aZ$ contains terminal followed by non-terminal. So the grammar G_2 is not in CNF.

Steps for converting CFG into CNF

Eliminate start symbol from the RHS. If the start symbol T is at the right-hand side of any production, create a new production as:

1. $S_1 \rightarrow T$

Where S_1 is the new start symbol.

In the grammar, remove the null, unit and useless productions. You can refer to the [Simplification of CFG](#)

Eliminate terminals from the RHS of the production if they exist with other non-terminals or terminals. For example, production $S \rightarrow aA$ can be decomposed as:

1. $S \rightarrow RA$
2. $R \rightarrow a$

Eliminate RHS with more than two non-terminals. For example, $S \rightarrow ASB$ can be decomposed as:

1. $S \rightarrow RS$
2. $R \rightarrow AS$