

13100673
EH-50

B.E. II Semester (CGPA) CSE Exam. 2014

DISCRETE STRUCTURE

Paper : CS-205

Time Allowed : Three Hours

Maximum Marks : 60

Note : Attempt all the questions.

All questions carry equal marks.

- Q.1. a) Let $N = \{0, 1, 2, 3, \dots\}$. Define functions f, g and h from set N to N by

$$f(n) = n + 1$$

$$g(n) = 2n$$

$$h(n) = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$$

Compute $go(fog)oh$. Is the function h is inversible? Is the function f is onto?

- b) Prove that every transposition (x, y) is its own inverse. That is $(x y)^{-1} = (x y)$

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P.T.O.

(2)

OR

- a) Prove that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are surjections, then $g \circ f: A \rightarrow C$ is a surjection.
- b) Two finite sets have x and y number of elements. The total number of subsets of the first set is four times the total number of subsets of the second set. Find the value of $x-y$.

- Q.2. a) Determine whether the following statements are tautologies?

i) $r: (p \wedge \sim q) \vee (q \wedge \sim p)$

ii) $(p \rightarrow q) \vee (q \rightarrow p)$

- b) Write a short note (300 words) on "Proof Systems"

OR

- a) Prove that "If $x, y \in \mathbb{Z}$ (set of integer) such that xy is odd then both x and y are odd, by proving its contra positive.
- b) Use mathematical induction to prove that $2^n < n!$ for every positive integer n with $n \geq 4$.

- Q.3. a) Prove that Intersection of two sublattices is a sublattice.
- b) If L and M are two distributive lattices then $L \times M$ is also distributive.

OR

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Contd.....

(3)

- Q3/ a) Let L be a lattice. Then prove that for $a, b, c \in L$,
 $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.
- b) Explain Hasse diagram with its properties. If N be a positive integer and D_N denote the set of all divisions of N . Consider the partial order 'divides' in D_N . Then draw the Hasse diagram for D_{30} .

- Q4/ a) Prove that a subgroup H of a group G is normal iff
 $g^{-1}hg \in H \quad \forall h \in H, g \in G$.
- b) For any a, x in a group, show that $(x^{-1}ax)^n = x^{-1}a^n x$ where n is a positive integer.

OR

- a) If G is a finite group and H is a subgroup of G then $O(H)$ divides $O(G)$. [Lagrange's Theorem]
- b) Prove that a Boolean ring is commutative.

- Q5/ a) State generalized pigeonhole principle. Find nP_r and nC_r .
How many three digit numbers can be formed using 1, 2, 3, 4, 5?
- b) Define tree. Prove that a tree of a connected graph has no circuit.

OR

- a) Write Prim's algorithm.
- b) Prove that a complete graph of five vertices is non planar.

(4)

- Q1/ Define Hamiltonian walk with an example.
- ii) Relationship between permutation and combination can be expressed as _____.
- iii) Define modulus function with an example.
- iv) If A and B are finite sets then $A \cup B$ and $A \cap B$ are also finite and $n(A \cup B) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} - \underline{\hspace{1cm}}$.
- v) State Inclusion - Exclusion principle.

