LESSON 6.1 – EXPONENTS

OVERVIEW

Here's what you'll learn in this lesson:

Properties of Exponents

a. Definition of exponent, power, and base

b. Multiplication Property

c. Division Property

d. Powers raised to powers

e. Products raised to powers

f. Quotients raised to powers

g. The zero exponent

If a friend agrees to give you one penny today, two pennies tomorrow, four pennies the next day, eight after that, and every day thereafter gives you double the amount he gave you the day before, by the end of a week he will have given you $1.27, and by the end of a month, $10,737,418.23!

While doubling your money every day may not be something you can relate to, it is an example of a situation that can be described using exponents. Another, perhaps more relevant example is the growth of your money in an account earning compound interest.

In this lesson, you will learn some general properties of exponents. These properties will help you later when you learn to simplify expressions involving exponents.

EXPLAIN

PROPERTIES OF EXPONENTS

Summary

You have seen how to use exponential notation as a shortcut for writing multiplication of repeated factors. Now you'll learn some basic properties involving exponents.

Multiplication Property

There is a rule for multiplying exponential expressions with the same base.

For example, find $3^2 \cdot 3^3$:

$$3^2 \cdot 3^3 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^{2+3} = 3^5$$

In general:

$$x^m \cdot x^n = x^{m+n}$$

To multiply two expressions with the same base, add their exponents.

Here, m and n are positive integers.

This property is called the multiplication property of exponents.

Division Property

There are also rules for dividing two expressions with the same base.

For example, find $\frac{5^7}{5^4}$:

$$\frac{5^7}{5^4} = \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot 5 \cdot 5 \cdot 5}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}}} = 5^{7-4} = 5^3$$

In general:

$$\frac{x^m}{x^n} = x^{m-n}$$

To divide two expressions with the same base, subtract their exponents.

Here, $x \neq 0$, m and n are positive integers, and $m > n$.

You can also divide expressions where the exponent in the denominator is greater than the exponent in the numerator.

For example, find $\frac{5^4}{5^7}$:

$$\frac{5^4}{5^7} = \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot 5 \cdot 5 \cdot 5} = \frac{1}{5^{7-4}} = \frac{1}{5^3}$$

In general:

$$\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$$

Here, $x \neq 0$, m and n are positive integers, and $m < n$.

These are called the division properties of exponents.

Power of a Power Property

There is a rule for raising an exponential expression to a power.

For example, find $(2^4)^3$:

$$(2^4)^3 = 2 \cdot 2 \cdot 2 \cdot 2 \quad \cdot \quad 2 \cdot 2 \cdot 2 \cdot 2 \quad \cdot \quad 2 \cdot 2 \cdot 2 \cdot 2 = 2^{4 \cdot 3} = 2^{12}$$

In general:

$$(x^m)^n = x^{m \cdot n}$$

When an expression raised to a power is itself raised to a power, multiply the exponents.

This is called the power of a power property of exponents.

Power of a Product Property

There is a rule for raising a product to a power.

For example, find $(4 \cdot 5)^2$:

$$\begin{aligned}(4 \cdot 5)^2 &= (4 \cdot 5) \cdot (4 \cdot 5) \\ &= 4 \cdot 5 \cdot 4 \cdot 5 \\ &= 4 \cdot 4 \cdot 5 \cdot 5 \\ &= 4^2 \cdot 5^2 \\ &= 16 \cdot 25 \\ &= 400\end{aligned}$$

Here you could have done the multiplication first and then applied the exponent: $(4 \cdot 5)^2 = (20)^2 = 400$. This may seem easier, but when you have an expression that includes variables as well as numbers, the other way is more useful.

In general:

$$(xy)^n = x^n \cdot y^n$$

To raise a product to a power, raise each factor to the power.

This is called the power of a product property of exponents.

Power of a Quotient Property

There is a rule for raising a quotient to a power.

For example, find $\left(\frac{2}{5}\right)^3$:

$$\left(\frac{2}{5}\right)^3 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}$$
$$= \frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5}$$
$$= \frac{2^3}{5^3}$$
$$= \frac{8}{125}$$

In general:

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

Here, $y \neq 0$.

This is called the power of a quotient property of exponents.

To raise a quotient to a power, raise the numerator and the denominator to the power.

Zero Power Property

There is a rule for raising a nonzero quantity to the zero power.

You know that:

$$\frac{x^m}{x^n} = x^{m-n} \text{ when } x \neq 0 \text{ and } m > n$$

and

$$\frac{x^m}{x^n} = \frac{1}{x^{n-m}} \text{ when } x \neq 0 \text{ and } m < n$$

But what happens when $m = n$?

For example, find $\frac{3^2}{3^2}$ in two ways:

1. Calculate the value of the numerator and denominator.
$$\frac{3^2}{3^2} = \frac{3 \cdot 3}{3 \cdot 3} = \frac{9}{9} = 1$$
2. Use the division property of exponents.
$$\frac{3^2}{3^2} = 3^{2-2} = 3^0$$

So, $3^0 = 1$.

In general:

$$x^0 = 1$$

Here, $x \neq 0$.

This is called the zero power property of exponents.

Any nonzero number raised to the zero power is equal to 1.

Sample Problems

1. Find: $b^3 \cdot b^8 \cdot b^6$

☑ a. Add the exponents. The base stays the same. $= b^{3+8+6}$

$= b^{17}$

2. Find: $\frac{x^9}{x^5}$

☐ a. Subtract the exponent in the denominator from the exponent in the numerator. The base stays the same. $=$ _____

3. Find: $(3^4)^6$

☐ a. Use the power of a power property to multiply the exponents. $= 3^{4 \cdot 6}$

$=$ _____

4. Find: $(b^2 \cdot c)^5$

☐ a. Use the power of a product property to raise each factor to the 5th power. $=$ _____

☐ b. Use the power of a power property to multiply the exponents. $=$ _____

5. Find: $\left(\frac{x}{y^3}\right)^2$

☐ a. Use the power of a quotient property to raise the numerator and denominator to the 2nd power. $=$ _____

☐ b. Use the power of a power property to multiply the exponents. $=$ _____

6. Find: $(3x^2y^5)^0$

☐ a. Use the zero power property to evaluate this expression. $=$ _____

Answers to Sample Problems

a. x^4

a. 3^{24}

a. $(b^2)^5 \cdot c^5$

b. $b^{10} \cdot c^5$ *or* $b^{10}c^5$

a. $\frac{x^2}{y^{3 \cdot 2}}$

b. $\frac{x^2}{y^6}$

a. *1*

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Properties of Exponents

Use the appropriate properties of exponents to simplify the expressions in problems 1 through 12. (Keep your answers in exponential form where possible.)

1. Find:

   a. $3^2 \cdot 3^5$ b. $5^2 \cdot 5^5$

   c. $7^2 \cdot 7^5$

2. Find:

   a. $\frac{3^9}{3^5}$ b. $\frac{3^5}{3^9}$

   c. $\frac{3^9}{3^9}$

3. Find:

   a. $(7^3)^2$ b. $(7^2)^3$

4. Find:

   a. $(5 \cdot x)^3$ b. $(3 \cdot y)^2$

   c. $(a^2 \cdot b)^4$

5. Find:

   a. $\left(\frac{x^3 \cdot x^5}{x^4}\right)^2$ b. $\left(\frac{a^{12} \cdot a^6}{a^9 \cdot a^7}\right)^4$

   c. $\left(\frac{b^6 \cdot b^5}{b^3 \cdot b^8}\right)^3$ d. $\frac{2^3 \cdot x^5}{2^5 \cdot x^2}$

6. Find:

   a. $(a^2 \cdot a^3)^2 + (a^2 \cdot a^3)^2$

   b. $\frac{y^4 \cdot 3y^2}{y^8}$

   c. $x^4 \cdot x^9 \cdot x \cdot y^5 \cdot y^{11}$

7. Find:

   a. $(b^3)^2 \cdot (b^4)^3$

   b. $\frac{y^6}{y^{17}} \cdot (y^5)^2 \cdot (y^3)^4$

   c. $\frac{a^4 \cdot b^6}{a^{11} \cdot b^3}$

8. Find:

   a. $\frac{(xy)^4}{y^9 \cdot x^7}$ b. $\frac{(3b)^6}{(3b^2)^4}$

9. As animals grow, they get taller faster than they get stronger. In general, this proportion of increase in height to increase in strength can be written as $\frac{x^2}{x^3}$. Simplify this fraction.

10. An animal is proportionally stronger the smaller it is. If a person is 200 times as tall as an ant, figure out how much stronger a person is, pound for pound, by simplifying the expression $\frac{200^2}{200^3}$.

11. Find:

    a. $\left(\frac{4xy^2z}{5x^2yz^3}\right)^0$ b. $\frac{y^7 \cdot y}{y^9 \cdot y^2}$

    c. $\left(\frac{b^3 \cdot b^5}{b^6 \cdot b^3}\right)^4$ d. $-2x^0 + 5y^0$

12. Find:

    a. $\left(\frac{(x^3 \cdot x^4)^2}{x^7}\right)^5$ b. $\frac{(4a^2)^0 - 3b^0}{2}$

    c. $\left(\frac{(3x \cdot 3x^2)^2}{3^{11} \cdot x^7}\right)^3$ d. $\left(\frac{b^8}{(b^2 \cdot b^7)^3}\right)^4$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Properties of Exponents

1. Find: $7^5 \cdot 7^3$. Leave your answer in exponential notation.
2. Find: $6^3 \cdot 6^4$. Leave your answer in exponential notation.
3. Find: $b^{12} \cdot b^3$
4. Find: $c^9 \cdot c^4$
5. Find: $a^6 \cdot a^5$
6. Find: $5^7 \div 5^3$. Leave your answer in exponential notation.
7. Find: $9^{10} \div 9^4$. Leave your answer in exponential notation.
8. Find: $\frac{m^{10}}{m^4}$
9. Find: $\frac{n^{20}}{n^{15}}$
10. Find: $\frac{b^{12}}{b^5}$
11. Find: $(5^3)^4$. Leave your answer in exponential notation.
12. Find: $(8^2)^5$. Leave your answer in exponential notation.
13. Find: $(13^5)^6$. Leave your answer in exponential notation.
14. Find: $(y^8)^3$
15. Find: $(z^{12})^4$
16. Find: $(x^9)^4$
17. Find: $(3 \cdot a)^4$
18. Find: $(4 \cdot b)^2$
19. Find: $(2 \cdot y)^3$
20. Find: $\frac{a^6b^5}{a^8b^2}$
21. Find: $\frac{m^7n^4}{m^3n^{10}}$
22. Find: $\frac{x^3y^7z^{12}}{xy^8z^5}$
23. Find: 5^0
24. Find: 348^0
25. Find: x^0
26. Find: $5^1 + (4z)^0$
27. Find: $a^0 - (xyz)^0 + 3^1$
28. Find: $2^1 - (3x)^0 + y^0$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Rewrite each expression below. Keep your answer in exponential form where possible.

   a. $11 \cdot 11 \cdot 11 \cdot 11$

   b. $3 \cdot 3 \cdot y \cdot y \cdot y \cdot y \cdot y$

   c. $5^{12} \cdot 5^{8} \cdot 5^{23}$

   d. $x^{7} \cdot y \cdot y^{19} \cdot x^{14} \cdot y^{6}$

   e. $7^{8} \cdot b^{5} \cdot b^{8} \cdot 7^{10} \cdot b$

2. Rewrite each expression below in simplest form using exponents.

   a. $\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$

   b. $\frac{b^{20}}{b^{14}}$

   c. $\frac{3^{12} \cdot x^{7}}{3^{9} \cdot x^{16}}$

   d. $\frac{y^{17}}{y^{14} \cdot y^{3} \cdot y^{4}}$

3. Circle the expressions below that simplify to $\frac{x^3}{y^5}$.

   $\frac{x^6y^2}{x^3y^7}$ $\frac{y^{11}x^5}{y^2x^4}$

   $\frac{xy^9}{x^6y^4}$ $\frac{x^7y}{x^4y^6}$

4. Circle the expressions below that simplify to $5y$.

   $(31x^8)^0 \cdot 5y$

   $-(-5y)^0$

   $\frac{5y^2}{y}$

   $\frac{(5y)^2}{5y}$

   $\frac{5 \cdot 5 \cdot 5 \cdot y \cdot y \cdot y \cdot y}{5 \cdot 5 \cdot y \cdot y}$

5. Simplify each expression below.

   a. $(b^4 \cdot b^2)^8$

   b. $(3^5 \cdot a^6)^2$

   c. $(2^9 \cdot x^4 \cdot y^6)^{11}$

6. Simplify each expression below.

   a. $\left(\frac{5y^{10}}{3x^8}\right)^4$

   b. $\left(\frac{7a^3b^4}{5a^2}\right)^6$

7. Calculate the value of each expression below.

   a. $(4x)^0 - 2y^0$

   b. $(5xy^2 \cdot 4x^3)^0$

   c. $-2x^0 - y^0$

   d. $\frac{(4x)^0}{2} + \frac{3x^0}{2} + \frac{-2x^0}{2}$

8. Rewrite each expression below using a single exponent.

   a. $\left(\frac{a^4 \cdot a^5}{a \cdot a^3}\right)^7$

   b. $\left(\frac{a \cdot a^3}{a^4 \cdot a^5}\right)^7$