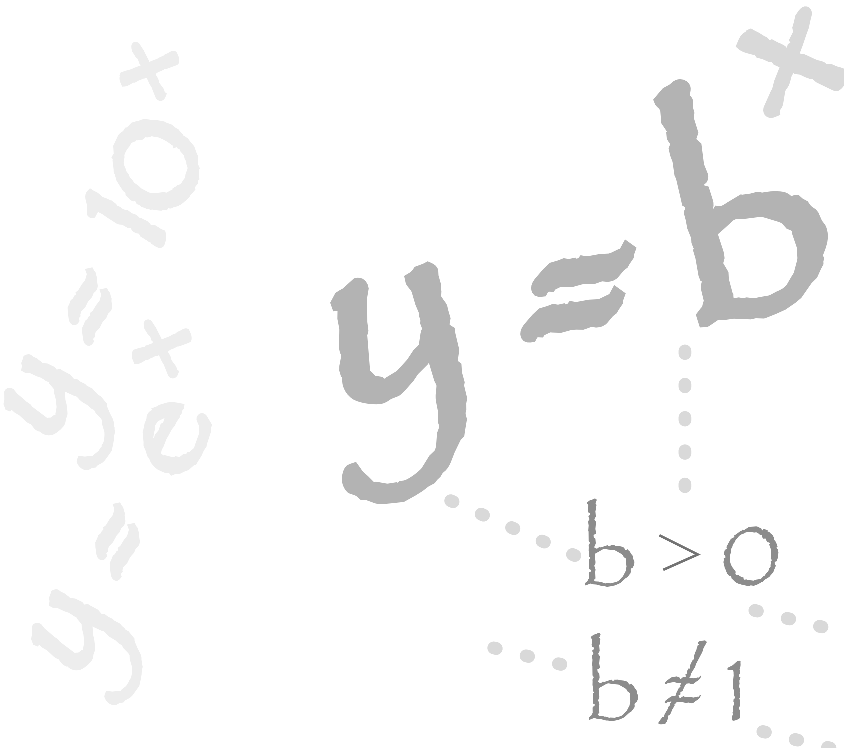
LESSON 12.2 – LOGS AND THEIR PROPERTIES



OVERVIEW

Here's what you'll learn in this lesson:

The Logarithm Function

a. Converting from exponents to logarithms and from logarithms to exponents

b. Graphing a logarithmic function

Logarithmic Properties

a. The algebra of logarithmic functions

Suppose you see a newspaper ad for a savings and loan that promises to multiply your savings by 10 in 30 years. Before you open an account, you want to know if the claim is true. Or, suppose a truck load of hazardous materials is accidentally spilled into a lake that supplies drinking water to your community. Local health officials say that the water will be safe to drink after 10 days, but you want to be sure that this is true. In both of these situations, you can use logarithms to help you find the answer you are looking for.

In this lesson, you'll learn about logarithms and how to graph logarithmic functions. In addition, you'll learn some properties of logarithms.

EXPLAIN

THE LOGARITHM FUNCTION

Summary

An Introduction to the Logarithmic Function

You have already worked with exponential functions, and you have learned many of the properties of exponential functions. You have also learned how to graph exponential functions. For example, Figure 12.2.1 shows the graph of the exponential function $y = 3^x$.

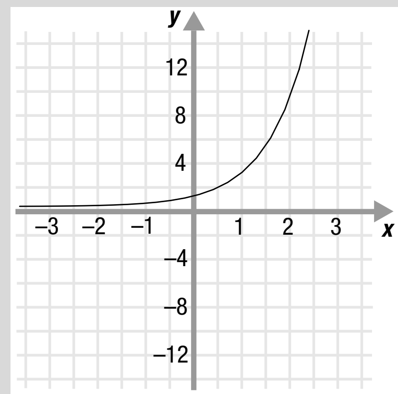


Figure 12.2.1

Here is a table of ordered pairs that satisfy $y = 3^x$:

x	y
0	1
1	3
2	9
–1	$\frac{1}{3}$
–2	$\frac{1}{9}$

When you graph $y = 3^x$, you begin by making a table of ordered pairs. To do this, you choose values for x, and then calculate their corresponding values for y.

For example, if you choose $x = 2$, then $y = 3^2 = 9$.

If you go in the reverse direction – that is, if you start with a value for y and calculate the corresponding value for x – then you obtain a new function called a logarithm. The logarithmic function is the inverse function of the exponential function. That is, the logarithmic function "undoes" what the exponential function does.

In the example above, if you choose $y = 9$, then $9 = 3^x$.

To solve for x, you can write this as a logarithm: $x = \log_3 9$.

Switching Between Exponential and Logarithmic Notation

So you now have two equivalent ways of writing the same information. Here is a statement in exponential form:

exponent

$$3^2 = 9$$

base

You can also write the same statement in logarithmic form:

logarithm

$$\log_3 9 = 2$$

base

You've already seen this idea of writing the same information in two different ways when you studied powers and roots. For example, the statements $3^2 = 9$ and $\sqrt{9} = 3$ express the same information.

In both cases the base is 3. In exponential form the number 2 is called the exponent. In logarithmic form it is called the logarithm.

Here's a way to generalize this:

$b^L = x$ is the same as $\log_b x = L$

These two statements are equivalent. They contain the same information written in different ways. In both cases, b is the base and is a positive number and $b \neq 1$. The x is also a positive number.

Here are some more examples:

Exponential Form	Logarithmic Form
$b^L = x$	$\log_b x = L$
$5^2 = 25$	$\log_5 25 = 2$
$\left(\frac{1}{3}\right)^4 = \frac{1}{81}$	$\log_{\frac{1}{3}}\left(\frac{1}{81}\right) = 4$

So any exponential statement can be written as a logarithmic statement. And any logarithmic statement can be written as an exponential statement. To do this:

1. Rewrite the statement, using the fact that $b^L = x$ is the same as $\log_b x = L$.

For example, to write $\left(\frac{1}{2}\right)^{-3} = 8$ in logarithmic form:

1. Rewrite the statement, using the fact that $b^L = x$ is the same as $\log_b x = L$. $\quad \log_{\frac{1}{2}} 8 = -3$

Similarly, to write $\log_{10} 10{,}000 = 4$ in exponential form:

1. Rewrite the statement, using the fact that $b^L = x$ is the same as $\log_b x = L$. $\quad 10^4 = 10{,}000$

Finding Logarithms

By rewriting a logarithmic statement in exponential form, you can find the value of the logarithm. Here are the steps:

1. Set the logarithm equal to L to create an equation. The variable L is the unknown value of the logarithm.
2. Rewrite the equation in exponential form.
3. Rewrite the constant term using the base.
4. Solve for L.

For example, to find $\log_6 36$:

1. Set the logarithm equal to L to create an equation. $\log_6 36 = L$
2. Rewrite in exponential form. $6^L = 36$
3. Rewrite the constant term, 36, using the base, 6. $6^L = 6^2$
4. Solve for L. $L = 2$

So, $\log_6 36 = 2$.

Here's a similar example that uses fractions. To find $\log_3\left(\frac{1}{81}\right)$:

1. Set the logarithm equal to L to create an equation. $\log_3\left(\frac{1}{81}\right) = L$
2. Rewrite in exponential form. $3^L = \frac{1}{81}$
3. Rewrite the constant term, $\frac{1}{81}$, using the base, 3. $3^L = 3^{-4}$
4. Solve for L. $L = -4$

So, $\log_3\left(\frac{1}{81}\right) = -4$.

When the constant cannot be easily rewritten as a power of the base, you will need to use a calculator. You'll learn how to do this in another lesson.

Graphing Logarithmic Functions

You can graph a logarithmic function by using your knowledge of inverses and exponential functions. Here are the steps for graphing a logarithmic function:

1. Write the logarithmic function in the form $y = \log_b x$.
2. Switch x and y to get $x = \log_b y$.
3. Rewrite in exponential form, $y = b^x$.
4. Graph this exponential function, $y = b^x$.
5. Graph the original logarithmic function $y = \log_b x$ by reflecting the exponential function $y = b^x$ about the line $y = x$.

For example, to graph the logarithmic function $y = \log_5 x$:

1. The logarithmic function is in the form $y = \log_b x$. $y = \log_5 x$
2. Switch x and y. $x = \log_5 y$
3. Rewrite in exponential form, $y = b^x$. $y = 5^x$
4. Graph this exponential function, $y = 5^x$. See Figure 12.2.2.
5. Graph $y = \log_5 x$ by reflecting $y = 5^x$ about the line $y = x$. See Figure 12.2.3.

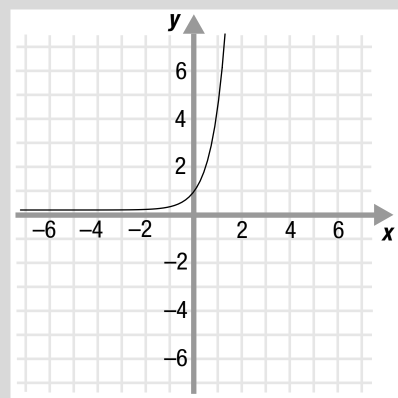


Figure 12.2.2

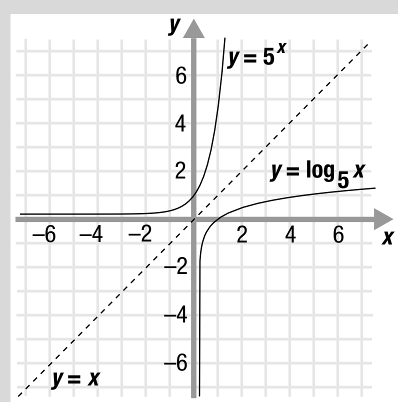


Figure 12.2.3

Sample Problems

1. Write in logarithmic form: $7^3 = 343$

   ☑ a. Rewrite the statement, using the fact that $b^L = x$ is the same as $\log_b x = L$. — $\log_7 343 = 3$

2. Write in exponential form: $\log_{11} 200 = x$

   ☐ a. Rewrite the statement, using the fact that $b^L = x$ is the same as $\log_b x = L$. — __________

3. Find: $\log_4 1024$

   ☑ a. Set the logarithm equal to L to create an equation. — $\log_4 1024 = L$

   ☐ b. Rewrite the equation in exponential form. — __________

   ☐ c. Rewrite 1024 using the base, 4. — $4^L =$ ________

   ☐ d. Solve for L. — $L =$ ______

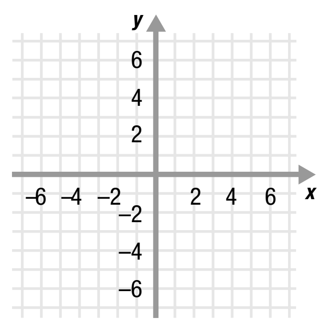
4. Graph the function $y = \log_2 x$.

   ☑ a. Write the logarithmic function in the form $y = \log_b x$. — $y = \log_2 x$

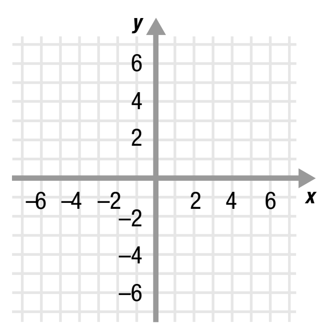
   ☑ b. Switch x and y. — $x = \log_2 y$

   ☑ c. Rewrite in exponential form, $y = b^x$. — $y = 2^x$

   ☐ d. Graph this exponential function, $y = 2^x$.



   ☐ e. Graph $y = \log_2 x$ by reflecting $y = 2^x$ about the line $y = x$.



Answers to Sample Problems

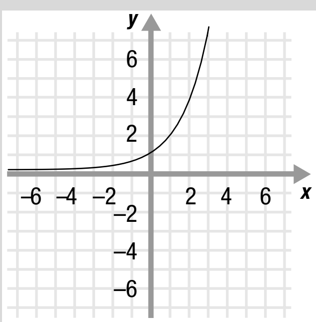
2\. *a.* $11^x = 200$

3\. *b.* $4^L = 1024$

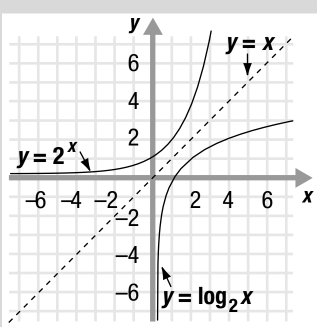
c. 4^5

d. 5

4\. *d.*



e.



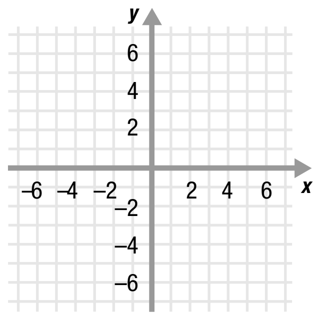
5. Graph the function $y = \log_{\frac{1}{3}} x$.

☑ a. Write the logarithmic function in the form $y = \log_b x$. $y = \log_{\frac{1}{3}} x$

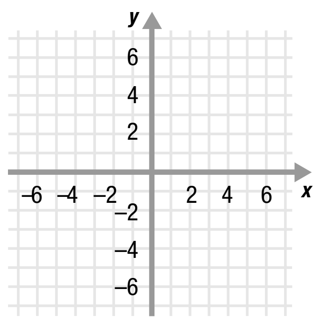
☑ b. Switch x and y. $x = \log_{\frac{1}{3}} y$

☐ c. Rewrite in exponential form, $y = b^x$. ____________

☐ d. Graph this exponential function, $y = \left(\frac{1}{3}\right)^x$.



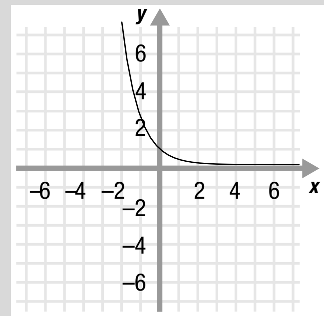
☐ e. Graph $y = \log_{\frac{1}{3}} x$ by reflecting $y = \left(\frac{1}{3}\right)^x$ about the line $y = x$.



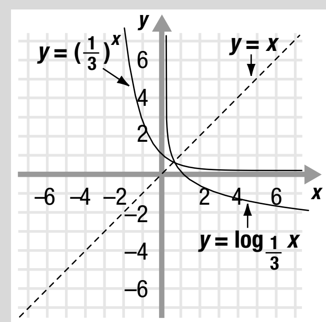
Answers to Sample Problems

c. $y = \left(\frac{1}{3}\right)^x$

d.



e.



LOGARITHMIC PROPERTIES

Summary

Logarithms can be rewritten using a variety of algebraic properties. These properties are like the properties that you already know for exponents.

Here are the fundamental properties of exponents.

Properties of Exponents	Examples
$b^1 = b$	$7^1 = 7$
$b^0 = 1$	$13^0 = 1$
$b^x b^y = b^{x+y}$	$11^5 11^8 = 11^{5+8} = 11^{13}$
$\frac{b^x}{b^y} = b^{x-y}$	$\frac{11^8}{11^5} = 11^{8-5} = 11^3$
$\left(b^x\right)^n = b^{nx}$	$\left(8^5\right)^7 = 8^{5 \cdot 7} = 8^{35}$

Here are the corresponding fundamental properties of logarithms, as well as several others.

Properties of Logarithms	Examples
$\log_b b = 1$	$\log_9 9 = 1$
$\log_b 1 = 0$	$\log_{17} 1 = 0$
Log of a Product: $\log_b uv = \log_b u + \log_b v$	$\log_{11} 17x = \log_{11} 17 + \log_{11} x$
Log of a Quotient: $\log_b \frac{u}{v} = \log_b u - \log_b v$	$\log_{11} \frac{x}{17} = \log_{11} x - \log_{11} 17$
Log of a Power: $\log_b u^n = n \cdot \log_b u$	$\log_8 13^{22} = 22 \cdot \log_8 13$
$b^{\log_b x} = x$	$5^{\log_5 23} = 23$
$\log_b \frac{1}{v} = -\log_b v$	$\log_6 \frac{1}{32} = -\log_6 32$
$\log_b b^n = n$	$\log_{14} 14^{29} = 29$

As with the properties of exponents, you can use one or several properties of logarithms to rewrite statements that contain logarithms. Here are some examples.

To find $\log_{17} 17^{2x-3}$:

1. Use the property $\log_b b^n = n$. $\quad \log_{17} 17^{2x-3} = 2x - 3$

So, $\log_{17} 17^{2x-3} = 2x - 3$.

To rewrite the expression $\log_{11}[x^3(5x + 17)]$:

1. Use the log of a product property. $\log_{11}[x^3(5x + 17)]$

$= \log_{11} x^3 + \log_{11}(5x + 17)$

2. Use the log of a power property. $= 3 \cdot \log_{11} x + \log_{11}(5x + 17)$

So, $\log_{11}[x^3(5x + 17)] = 3 \cdot \log_{11} x + \log_{11}(5x + 17)$.

To rewrite $3\log_b x - 5\log_b y + 7\log_b z$ as a single logarithm:

1. Use the log of a power property. $3\log_b x - 5\log_b y + 7\log_b z$

$= \log_b x^3 - \log_b y^5 + \log_b z^7$

2. Use the log of a quotient property. $= \log_b \frac{x^3}{y^5} + \log_b z^7$

3. Use the log of a product property. $= \log_b \frac{x^3 z^7}{y^5}$

So, $3\log_b x - 5\log_b y + 7\log_b z = \log_b \frac{x^3 z^7}{y^5}$.

You have to work from left to right when you rewrite logarithms. In this example, you subtract the two logs, then you add.

Sample Problems

1. Simplify: $\log_{3x-7}(3x - 7)$

✔ a. Use the property $\log_b b = 1$. $\log_{3x-7}(3x - 7) = 1$

2. Simplify: $y^{\log_y(13z + 6x - 119)}$

✔ a. Use the property $b^{\log_b x} = x$. $y^{\log_y(13z + 6x - 119)}$

$= 13z + 6x - 119$

3. Rewrite this expression: $\log_{37} \frac{3p}{qr}$

✔ a. Use the log of a quotient property. $\log_{37} \frac{3p}{qr}$

$= \log_{37} 3p - \log_{37} qr$

☐ b. Use the log of a product property. = ____________________

4. Write $\log_9 \frac{1}{2}$ using $\log_9 2$.

☐ a. Use the property $\log_b \frac{1}{v} = -\log_b v$. $\log_9 \frac{1}{2}$

= ____________

5. Write as a single logarithm not containing exponents: $\log_w f^7 + \log_w g^7$

☐ a. Use the log of a product property. $\log_w f^7 + \log_w g^7$

= ______________

☐ b. Use the log of a power property. = ____________

Answers to Sample Problems

3. b. $\log_{37} 3 + \log_{37} p - (\log_{37} q + \log_{37} r)$

or $\log_{37} 3 + \log_{37} p - \log_{37} q - \log_{37} r$

4. a. $-\log_9 2$

5. a. $\log_w f^7 g^7$ or $\log_w (fg)^7$

b. $7 \log_w (fg)$

EXPLORE

Sample Problems

On the computer you used the grapher to graph various logarithmic functions and compared their behavior. You also explored the algebraic properties of logarithms. Below are some additional problems to explore.

1. Graph these logarithmic functions on the same set of axes, and answer the questions below.

   $y = \log_2 x$

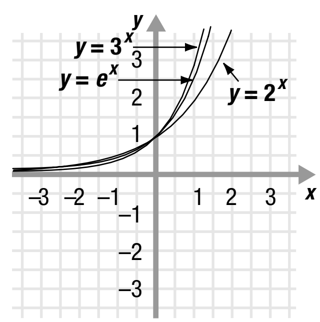
   $y = \log_3 x$

   $y = \log_e x$

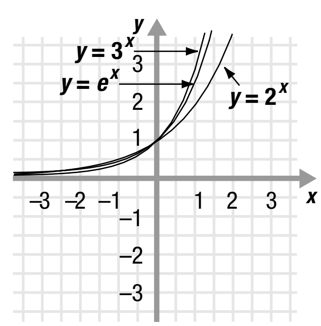
   ✔ a. To graph the functions, start by finding the inverse function of each logarithmic function.

Function	Inverse
$y = \log_2 x$	$y = 2^x$
$y = \log_3 x$	$y = 3^x$
$y = \log_e x$	$y = e^x$

   ✔ b. Then graph each exponential function.



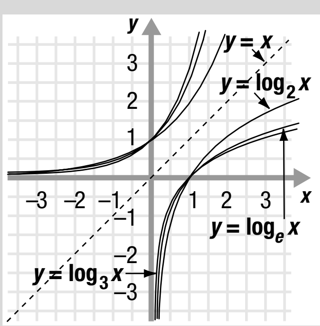
   ☐ c. Finally, reflect the graph of each exponential function about the line $y = x$ to graph each corresponding logarithmic function.



   ☐ d. What is the domain of each of the logarithmic functions? ______________

Answers to Sample Problems

c.



d. $x > 0$

Notice by looking at the graphs that they are not defined for $x \leq 0$.

☐ e. What is the range of each of the logarithmic functions? ____________

☐ f. Find the point which is common to all three logarithmic graphs. ____________

☐ g. For $x > 1$, list the three logarithmic graphs in order of their distance from the x-axis. Start with the closest one. ____________ ____________ ____________

☐ h. For $x < 1$, list the three logarithmic graphs in order of their distance from the y-axis. ____________ ____________ ____________

2. Graph these logarithmic functions on the same set of axes, and answer the questions below.

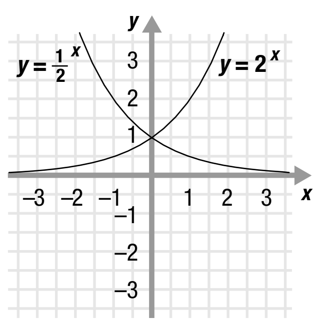
$y = \log_2 x$

$y = \log_{\frac{1}{2}} x$

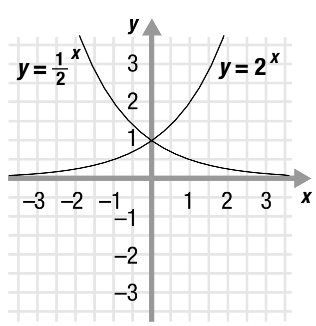
✔ a. To graph the functions, start by finding the inverse function of each logarithmic function.

Function	Inverse
$y = \log_2 x$	$y = 2^x$
$y = \log_{\frac{1}{2}} x$	$y = \left(\frac{1}{2}\right)^x$

✔ b. Then graph each exponential function.



☐ c. Finally, reflect the graph of each exponential function about the line $y = x$ to graph each corresponding logarithmic function.



☐ d. What is the domain of each of the logarithmic functions? ____________

Answers to Sample Problems

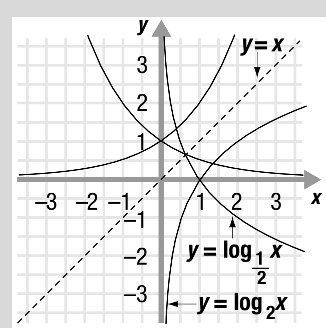
e. All real numbers.
Notice by looking at the graphs that y-values can be positive or negative.

f. (1, 0)

g. $y = \log_3 x$
$y = \log_e x$
$y = \log_2 x$

h. $y = \log_3 x$
$y = \log_e x$
$y = \log_2 x$

c.



d. $x > 0$
Notice by looking at the graphs that they are not defined for $x \le 0$.

Answers to Sample Problems

e. All real numbers. Notice by looking at the graphs that y values can be positive or negative.

f. (1, 0)

g. Increase.

Decrease.

b. 10,000

c. The contributions increase rapidly at first, but eventually taper off.

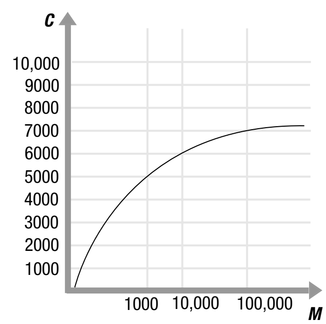
☐ e. What is the range of each of the logarithmic functions? ______________

☐ f. Find the point which is common to both logarithmic graphs. ______________

☐ g. Does the graph of $y = \log_2 x$ increase or decrease as you move from left to right along the x-axis? ______________
How about $y = \log_{\frac{1}{2}} x$? ______________

3. This is a formula for political fund raising: $C = a + b\log_{10}\frac{M}{1000}$. Here, the variable M describes the number of funding requests that have been mailed, and the variable C describes the number of contributions received in response to the requests. The numbers a and b are constants which are determined by the nature of the request and the contents of each specific mailing.

Here is a graph which illustrates the formula for a given a and b.



☑ a. Use the graph to find the number of contributions, C, corresponding to $M = 1000$ mailings.
When $M = 1000$, $C = 5000$.

☐ b. Use the graph to find the number of mailings, M, required to produce $C = 6000$ contributions.
When $M =$ ____________, $C = 6000$.

☐ c. What happens to the contributions as as the number of mailings increase? ______________________

4. Calculate the following logarithms: $\log_2 4$, $\log_2 8$, $\log_2 16$, $\log_2 32$. Use your answers to illustrate the log of a product property, the log of a quotient property and the log of a power property, respectively.

- [x] a. Calculate $\log_2 4$.
  - Set $\log_2 4$ equal to x. $\log_2 4 = x$
  - Rewrite the expression in exponential form. $2^x = 4$
  - Solve for x. $x = 2$
- [ ] b. Calculate $\log_2 8$.
  - Set $\log_2 8$ equal to x. ______________
  - Rewrite the expression in exponential form. ______________
  - Solve for x. $x =$ ___________
- [ ] c. Calculate $\log_2 16$.
  - Set $\log_2 16$ equal to x. ______________
  - Rewrite the expression in exponential form. ______________
  - Solve for x. $x =$ _________
- [ ] d. Calculate $\log_2 32$.
  - Set $\log_2 32$ equal to x. ______________
  - Rewrite the expression in exponential form. ______________
  - Solve for x. $x =$ _________
- [x] e. Illustrate the log of a product property. Substitute the values you found in (a), (b), and (d) for $\log_2 4$, $\log_2 8$, and $\log_2 32$.

$$\begin{aligned} \log_2 4 + \log_2 8 &= \log_2(4 \cdot 8) \\ &= \log_2 32 \\ 2 \quad + \quad 3 \quad &= 5 \end{aligned}$$

- [ ] f. Illustrate the log of a quotient property. Substitute the values you found in (d), (b) and (a) for $\log_2 32$, $\log_2 8$, and $\log_2 4$.

$$\begin{aligned} \log_2 32 - \log_2 8 &= \log_2 \frac{32}{8} \\ &= \log_2 4 \\ \_\_\_\_ - \_\_\_\_ &= \_\_\_\_ \end{aligned}$$

- [ ] g. Illustrate the log of a power property. Substitute the values you found in (a) and (c) for $\log_2 4$ and $\log_2 16$.

$$\begin{aligned} 2 \cdot \log_2 4 &= \log_2 4^2 \\ &= \log_2 16 \\ \_\_\_\_\_\_ &= \_\_\_\_\_ \end{aligned}$$

Answers to Sample Problems

b. $\log_2 8 = x$
$2^x = 8$
3

c. $\log_2 16 = x$
$2^x = 16$
4

d. $\log_2 32 = x$
$2^x = 32$
5

f. 5, 3, 2

g. $2 \cdot 2$, 4

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

The Logarithm Function

1. Write this exponential statement in logarithmic form: $10^4 = 10000$
2. Find: $\log_5 125$
3. In order to graph the function $y = \log_7 x$, you can use its inverse function. What is this inverse function?
4. Write this logarithmic statement in exponential form: $\log_3 500 = x$
5. Find: $\log_2 \frac{1}{64}$
6. Graph the function $y = \log_2 x$.
7. Write this exponential statement in logarithmic form: $7^{12} = y$
8. Find: $\log_{3.2} 3.2$
9. Suppose that when you graduate from college, you deposit one dollar in a savings account for your retirement, t years later. The final amount in your savings account after t years is given by A. If you receive 7% interest compounded annually, the formula which tells you how many years, t it takes to accumulate the amount A is $t = \log_{1.07} A$. Graph this function.
10. The formula for the rate of decay of a radioactive chemical is given by $T = \log_B\left(\frac{R}{S}\right)$. Here, S is the starting amount of the chemical in grams and R is the amount of the chemical in grams remaining after T years. Write this formula in exponential form.
11. Find: $\log_{\frac{1}{4}} 16$
12. Graph the function $y = \log_{\frac{1}{3}} x$.

Logarithmic Properties

13. Find: $\log_{17} 17^y$
14. Rewrite as a single logarithm: $\log_d 3x + \log_d 4y$
15. Rewrite using the log of a power property: $14\log_{13} 12$
16. Simplify: $10^{\log_{10} 12abc}$
17. Rewrite using the log of a quotient property: $\log_2 \frac{3ab}{7cd}$
18. Rewrite using the log of a product property and the log of a power property: $\log_5 x^2y^3$
19. Simplify: $\log_{3x-y} 3x - y$
20. Rewrite using the log of a product property to get an expression with four terms: $\log_B 7xyz$
21. The magnitude of an earthquake of intensity I as compared to one of minimum intensity M is measured on the Richter scale as $R = \log_{10} \frac{I}{M}$. Use a property of logarithms to rewrite this formula for R in terms of logarithms that do not contain fractions.
22. The pH of a particular fruit juice is given by the formula $x = -\log_{10}\left(1.56 \cdot 10^{-4}\right)$. Find x to four decimal places, given that $\log_{10} 1.56 = 0.1931$. (Hint: Use properties of logarithms to get your answer.)
23. Find: $\log_x\left(\frac{1}{x}\right)$
24. Write as a single logarithm: $3\log_{16} u + 5\log_{16} v - 8\log_{16} w$

Explore

25. The following functions are graphed on the grid in Figure 12.2.4:

$y = \log_3 x$

$y = \log_{15} x$

$y = \log_7 x$

Label each graph with the appropriate function.

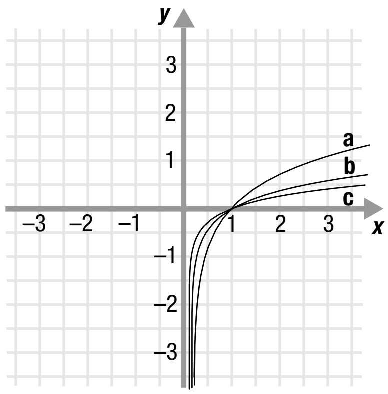


Figure 12.2.4

26. The following functions are graphed on the grid in Figure 12.2.8:

$y = \log_5 x$

$y = \log_{\frac{1}{5}} x$

Label each graph with the appropriate function. About what line can you reflect one graph in order to get the other?

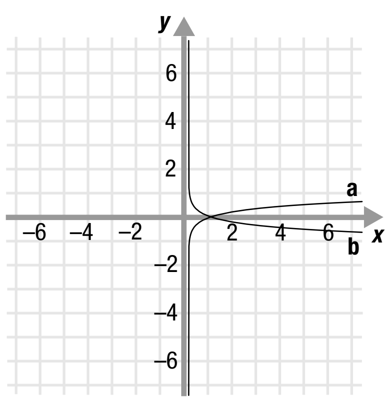


Figure 12.2.5

27. The log of a product property allows you to rewrite $\log_4 64$ as $\log_4 16 + \log_4 4$. Use numbers which are powers of 2 to:

a. rewrite $\log_4 64$ in a different way using the Log of a Product Property.

b. rewrite $\log_4 64$ using the Log of a Quotient Property.

c. rewrite $\log_4 64$ using the Log of a Power Property.

28. Describe how the graph of the logarithm $y = \log_b x$ changes as you increase the value of the base b for $b > 1$. Use the graph of $y = \log_5 x$ shown in Figure 12.2.6 to help you.

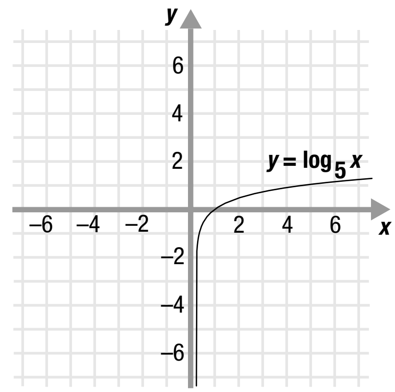


Figure 12.2.6

29. Compare a logarithmic graph $y = \log_b x$ with base $b < 1$ and another logarithmic graph with base $b > 1$. What is the same on both graphs?

30. Circle the expressions below that are equal.

$\log_{\frac{1}{2}} 4$ $\qquad -2 \qquad -\log_2 4 \qquad \log_2 \frac{1}{4}$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

The Logarithm Function

1. Write this exponential statement in logarithmic form: $2^3 = 8$
2. Write this exponential statement in logarithmic form: $4^5 = 1024$
3. Write this exponential statement in logarithmic form: $5^4 = 625$
4. Write this exponential statement in logarithmic form: $x^2 = 225$
5. Write this exponential statement in logarithmic form: $3^x = a$
6. Write this exponential statement in logarithmic form: $b^x = 36$
7. Find: $\log_2 64$
8. Find: $\log_3 27$
9. Find: $\log_3 81$
10. Find: $\log_7 49$
11. Find: $\log_5 \frac{1}{125}$
12. Find: $\log_{\frac{1}{4}} 16$
13. What is the inverse of the function $f(x) = \log_{12} x$?
14. What is the inverse of the function $f(x) = \log_7 x$?
15. What is the inverse of the function $f(x) = \log_{11} x$?
16. What is the inverse of the function $f(x) = \log_{2.5} x$?
17. What is the inverse of the function $f(x) = \log_b x$?
18. What is the inverse of the function $f(x) = \log_a x$?
19. Write this logarithmic statement in exponential form: $\log_5 125 = 3$
20. Write this logarithmic statement in exponential form: $\log_4 \frac{1}{16} = -2$
21. Write this logarithmic statement in exponential form: $\log_8 512 = x$
22. Write this logarithmic statement in exponential form: $\log_b 81 = 4$
23. Write this logarithmic statement in exponential form: $\log_6 x = 5$
24. Write this logarithmic statement in exponential form: $\log_4 1024 = x$
25. Graph the function $y = \log_3 x$.
26. Graph the function $y = \log_2 x$.
27. Graph the function $y = \log_{\frac{1}{3}} x$.
28. Graph the function $y = \log_{\frac{1}{2}} x$.

Logarithmic Properties

29. Rewrite using the log of a power property: $\log_2 33^4$
30. Rewrite using the log of a power property: $a\log_b 8$
31. Rewrite using the log of a power property: $17\log_3 25$
32. Rewrite using the log of a quotient property: $\log_7 \frac{5}{3m}$
33. Rewrite using the log of a quotient property: $\log_b \frac{7x}{8y}$
34. Rewrite using the log of a quotient property: $\log_5 \frac{6y}{11z}$
35. Rewrite using the log of a product property to get an expression with two terms: $\log_3 5x$
36. Rewrite using the log of a product property to get an expression with three terms: $\log_b 15mn$
37. Rewrite using the log of a product property to get an expression with three terms: $\log_b 21xy$
38. Rewrite as a single logarithm: $\log_c 5 + \log_c 7$
39. Rewrite as a single logarithm: $\log_b 3 + \log_b 4$

40. Rewrite as a single logarithm: $\log_b 12 + \log_b z$

41. Rewrite as a single logarithm: $\log_5 4x^7 - \log_5 2x$

42. Rewrite as a single logarithm: $\log_2 9y^5 - \log_2 3y^2$

43. Rewrite as a single logarithm: $\log_7 6xy - \log_7 12xy$

44. Rewrite as a single logarithm: $2\log_y a + 7\log_y b - \log_y c$

45. Rewrite as a single logarithm: $3\log_x a + 5\log_x b - \log_x c$

46. Rewrite as a single logarithm: $a\log_3 x - b\log_3 y - c\log_3 z$

47. Simplify: $\log_3 3$

48. Simplify: $7^{\log_7 3}$

49. Simplify: $\log_{2xy}(2xy)^{-3}$

50. Simplify: $5^{\log_5 3}$

51. Simplify: $\log_{3z}(3z)^{-2}$

52. Simplify: $(2m)^{\log_{2m} -6}$

53. Rewrite using the log of a quotient property and the log of a power property: $\log_5 \frac{z}{y^4}$

54. Rewrite using the log of a product property and the log of a power property: $\log_7 xy^5$

55. Rewrite using the properties of logarithms to get an expression with three terms: $\log_3 \frac{x}{2y}$

56. Rewrite using the properties of logarithms to get an expression with three terms: $\log_{11} \frac{x^2y^3}{z}$

EVALUATE

PRACTICE TEST

Take this practice test to make sure you are ready for the final quiz in Evaluate.

1. Write this exponential statement in logarithmic form: $18^{12x} = yz$
2. Find the value of $\log_6 \frac{1}{216}$.
3. Graph the function $y = \log_5 x$.
4. The formula for the growth rate of an experimental bacterium is given by the exponential function $P = Ie^{kt}$. Here, I is the starting population number, P is the population after t years, and k is a proportionality constant determined by the laboratory conditions. Write this formula in logarithmic form.
5. Simplify: $15^{\log_{15} xyz}$
6. Using the log of a product property twice, rewrite: $\log_{29} 19AB$
7. Write as a single logarithm: $\log_w 11 - \log_w x - \log_w y$
8. Use properties of logarithms to rewrite as an expression with three terms:

   $\log_b \frac{17x^5}{y^{12}}$
9. Graph the functions $y = \log_a x$ and $y = \log_b x$, where you know that a and b are greater than 1 and $a > b$. Use the grid in Figure 12.2.7.

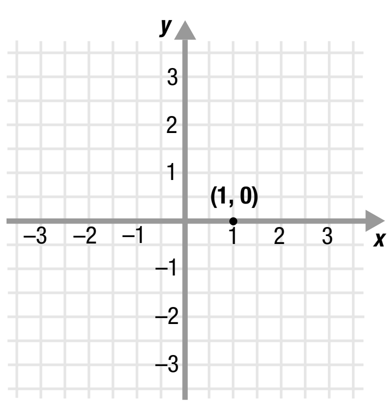


Figure 12.2.7

10. Consider the graphs shown in Figure 12.2.8.

    a. Identify which graph represents an exponential function with base less than 1.

    b. Identify which graph represents a logarithmic function with base greater than 1.

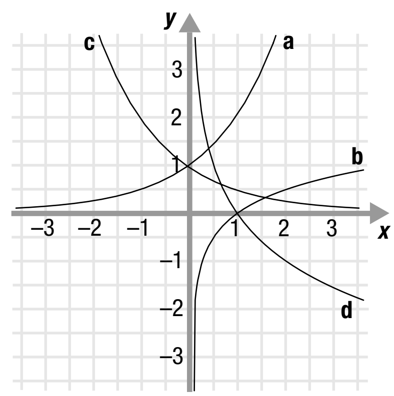


Figure 12.2.8

11. Use properties of logarithms to determine whether the following is true:

    $\log_5 25 = -\log_{\frac{1}{5}} 25$
12. Suppose that you have a formula to measure the loudness of sounds on a logarithmic scale. You do not know what base to use for your logarithms, but you have determined by experiment that $\log_b 2 = 3.4$ and that $\log_b 3 = 5.4$. Use this information and properties of logarithms to calculate $\log_b 24$.