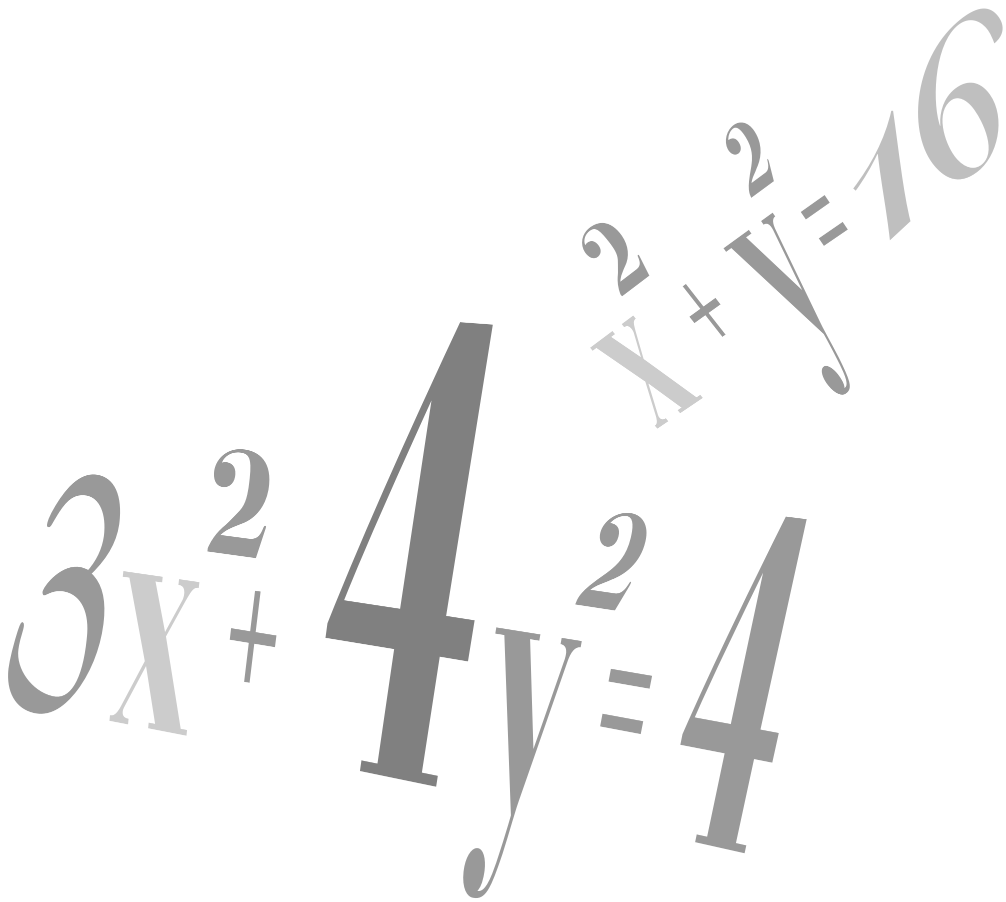
LESSON 13.2 – NONLINEAR SYSTEMS



OVERVIEW

Here's what you'll learn in this lesson:

Solving Systems

a. Recognizing the solution(s) of a nonlinear system, both graphically and algebraically.

b. Solving a nonlinear system using the substitution method.

c. Solving a nonlinear system using the elimination method.

Suppose you wanted to use your carpentry skills to make some extra money selling bookcases to your friends. One of the first things you might be interested in knowing is how many bookcases you would have to sell before you started to make a profit.

If you knew how much it would cost you to make the bookcases, and how much you planned to charge for each one, you could figure out when you might start making money by setting up and solving a nonlinear system of equations.

In this lesson, you will learn about nonlinear systems of equations. You will learn how to solve such systems both by graphing and by using algebra.

EXPLAIN

SOLVING SYSTEMS

Summary

Nonlinear Systems of Equations

A nonlinear system of equations is a system in which at least one of the equations is not the equation of a line.

You have learned how to solve systems of linear equations by graphing, by the substitution method, and by the elimination method. You can also use these methods to solve nonlinear systems of equations.

A linear system of equations can have either no solutions, one solution, or an infinite number of solutions. A nonlinear system can have no solutions, one solution, two solutions, three solutions, four solutions, five solutions, etc., up to an infinite number of solutions.

Solving Nonlinear Systems by Graphing

To solve a nonlinear system of equations by graphing:

1. Graph each equation in the system.
2. Find the points of intersection, if any. These are the real solutions of the system.

For example, to solve this nonlinear system:

$$\begin{aligned} x + y &= 5 \\ y &= x^2 - 1 \end{aligned}$$

1. Graph the line $x + y = 5$ and graph the parabola $y = x^2 - 1$. See the grid in Figure 13.2.1.
2. Find the points of intersection: (–3, 8) and (2, 3). See the grid in Figure 13.2.2.

So this nonlinear system has two real solutions, $(x, y) = (–3, 8)$ or $(x, y) = (2, 3)$.

As another example, to solve this nonlinear system:

$$\begin{aligned} y &= 5 \\ x^2 + y^2 &= 25 \end{aligned}$$

1. Graph the line $y = 5$ and graph the circle $x^2 + y^2 = 25$. See the grid in Figure 13.2.3.
2. Find the point of intersection: (0, 5). See the grid in Figure 13.2.4.

So this nonlinear system has one solution, $(x, y) = (0, 5)$.

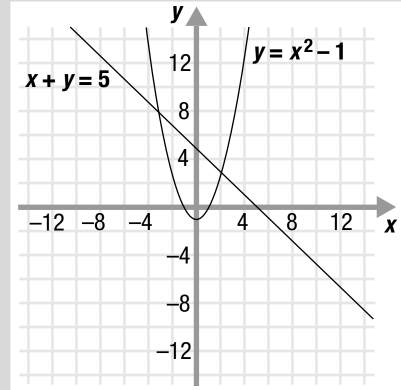


Figure 13.2.1

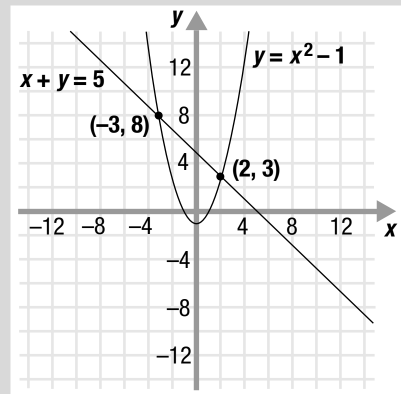


Figure 13.2.2

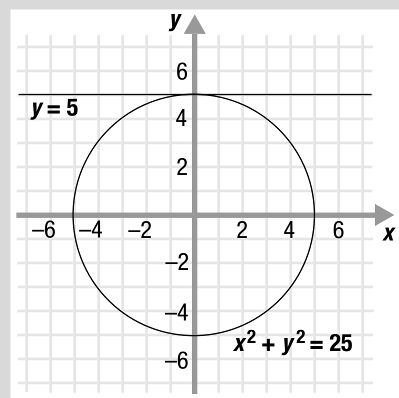


Figure 13.2.3

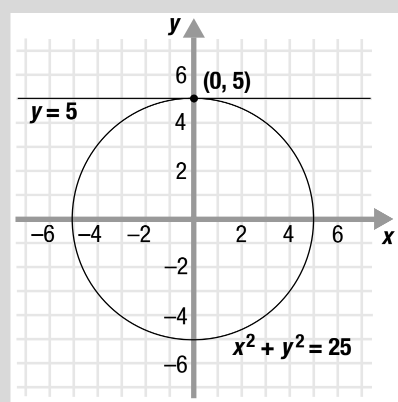


Figure 13.2.4

It is possible to get extraneous solutions when solving nonlinear systems of equations. So it is important that you check your solutions.

You have to be careful when you use graphing to solve a nonlinear system. If the solutions are fractions or irrational numbers, it may be difficult to read them accurately from the graph. If the solutions include very large numbers, you may not be able to see all of the solutions. Also, the points of intersection of the graphs represent only the **real** solutions of the system. To determine if the system has solutions that include imaginary numbers you will need to use one of the following algebraic methods.

Solving Nonlinear Systems by Substitution

To solve a nonlinear system of two equations by substitution:

1. Solve one equation for one variable in terms of the other variable.
2. Substitute into the other equation to obtain an equation in one variable.
3. Solve the resulting equation.
4. Complete the ordered pair(s) by substituting each solution into one of the original equations.
5. Check each ordered pair in both of the original equations to see if it is a solution of the system.

For example, to solve this nonlinear system by substitution:

$$\begin{aligned} y &= x^2 - 1 \\ x + y &= 1 \end{aligned}$$

1. The first equation is already solved for y in terms of x. $\quad y = x^2 - 1$
2. Substitute $y = x^2 - 1$ into the second equation. $\quad x + (x^2 - 1) = 1$
3. Solve the resulting equation.
$$x^2 + x - 2 = 0$$
$$(x - 1)(x + 2) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 1 \quad \text{or} \quad x = -2$$
4. Complete the ordered pairs.
$$\begin{aligned} x + y &= 1 & x + y &= 1 \\ 1 + y &= 1 & -2 + y &= 1 \\ y &= 0 & y &= 3 \end{aligned}$$

   So the ordered pairs are (1, 0) and (–2, 3).

5. Check each ordered pair.

   Check (1, 0):

   $y = x^2 - 1$ $\qquad$ $x + y = 1$

   Is $0 = 1^2 - 1$? $\qquad$ Is $1 + 0 = 1$?

   Is $0 = 1 - 1$? $\qquad$ Is $1 = 1$? Yes.

   Is $0 = 0$? Yes.

   Check (–2, 3):

   $y = x^2 - 1$ $\qquad$ $x + y = 1$

   Is $3 = (-2)^2 - 1$? $\qquad$ Is $-2 + 3 = 1$?

   Is $3 = 4 - 1$? $\qquad$ Is $1 = 1$? Yes.

   Is $3 = 3$? Yes.

So the solutions of this nonlinear system are the points $(x, y) = (1, 0)$ or $(x, y) = (-2, 3)$.

Sometimes nonlinear systems have equations that are expressed in terms of x or y. So you can start by solving one of the equations for x or for y. However, it is often more convenient to solve one of the equations for something other than x or y.

For example, to solve this nonlinear system by substitution:

$$x^2 + y^2 + 2y = 3$$
$$x^2 - y = 5$$

1. Solve the second equation for x^2 in terms of y. $\qquad$ $x^2 = y + 5$
2. Substitute this value for x^2 into the first equation. $\qquad$ $(y + 5) + y^2 + 2y = 3$
3. Solve this quadratic equation.

$$y^2 + 3y + 2 = 0$$
$$(y + 2)(y + 1) = 0$$
$$y + 2 = 0 \text{ or } y + 1 = 0$$
$$y = -2 \text{ or } \quad y = -1$$

4. Complete the ordered pairs.

$$x^2 - y = 5 \qquad x^2 - y = 5$$
$$x^2 - (-2) = 5 \qquad x^2 - (-1) = 5$$
$$x^2 + 2 = 5 \qquad x^2 + 1 = 5$$
$$x^2 = 3 \qquad x^2 = 4$$
$$x = \pm\sqrt{3} \qquad x = \pm 2$$

   So the ordered pairs are $(\sqrt{3}, -2)$, $(-\sqrt{3}, -2)$, $(2, -1)$, and $(-2, -1)$.

5. Check each ordered pair.

Check $(\sqrt{3}, -2)$:

$$x^2 + y^2 + 2y = 3 \qquad\qquad x^2 - y = 5$$

Is $\left(\sqrt{3}\right)^2 + (-2)^2 + 2(-2) = 3$? Is $\left(\sqrt{3}\right)^2 - (-2) = 5$?

Is $3 + 4 - 4 = 3$? Is $3 + 2 = 5$?

Is $3 = 3$? Yes. Is $5 = 5$? Yes.

Check $(-\sqrt{3}, -2)$:

$$x^2 + y^2 + 2y = 3 \qquad\qquad x^2 - y = 5$$

Is $\left(-\sqrt{3}\right)^2 + (-2)^2 + 2(-2) = 3$? Is $\left(-\sqrt{3}\right)^2 - (-2) = 5$?

Is $3 + 4 - 4 = 3$? Is $3 + 2 = 5$?

Is $3 = 3$? Yes. Is $5 = 5$? Yes.

Check $(2, -1)$:

$$x^2 + y^2 + 2y = 3 \qquad\qquad x^2 - y = 5$$

Is $2^2 + (-1)^2 + 2(-1) = 3$? Is $2^2 - (-1) = 5$?

Is $4 + 1 - 2 = 3$? Is $4 + 1 = 5$?

Is $3 = 3$? Yes. Is $5 = 5$? Yes.

Check $(-2, -1)$:

$$x^2 + y^2 + 2y = 3 \qquad\qquad x^2 - y = 5$$

Is $(-2)^2 + (-1)^2 + 2(-1) = 3$? Is $(-2)^2 - (-1) = 5$?

Is $4 + 1 - 2 = 3$? Is $4 + 1 = 5$?

Is $3 = 3$? Yes. Is $5 = 5$? Yes.

So this nonlinear system has four solutions: $(x, y) = (\sqrt{3}, -2)$

$(x, y) = (-\sqrt{3}, -2)$

$(x, y) = (2, -1)$

$(x, y) = (-2, -1)$

Solving Nonlinear Systems by Elimination

To solve a nonlinear system of two equations by elimination:

1. Eliminate a variable by multiplying the equations by appropriate numbers and adding them.
2. Solve for the other variable.
3. Complete the ordered pair(s) by substituting each solution into one of the original equations.
4. Check each ordered pair in both of the original equations to see if it is a solution of the system.

For example, to solve this nonlinear system by elimination:

$$x^2 - y^2 = 7$$
$$2x^2 + 3y^2 = 24$$

1. Eliminate the y^2-terms by multiplying the first equation by 3 and adding it to the other equation.

$$3(x^2 - y^2) = 3(7) \rightarrow \begin{aligned} 3x^2 - 3y^2 &= 21 \\ \underline{2x^2 + 3y^2} &\underline{= 24} \\ 5x^2 &= 45 \end{aligned}$$

2. Solve for x.

$$\begin{aligned} 5x^2 &= 45 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

3. Complete the ordered pairs.

$$\begin{aligned} x^2 - y^2 &= 7 \\ (3)^2 - y^2 &= 7 \\ 9 - y^2 &= 7 \\ -y^2 &= -2 \\ y &= \pm\sqrt{2} \end{aligned} \qquad \begin{aligned} x^2 - y^2 &= 7 \\ (-3)^2 - y^2 &= 7 \\ 9 - y^2 &= 7 \\ -y^2 &= -2 \\ y &= \pm\sqrt{2} \end{aligned}$$

So the ordered pairs are $(3, \sqrt{2})$, $(3, -\sqrt{2})$, $(-3, \sqrt{2})$, and $(-3, -\sqrt{2})$.

4. Check each ordered pair.

Check $(3, \sqrt{2})$:

$x^2 - y^2 = 7$	$2x^2 + 3y^2 = 24$
Is $3^2 - \left(\sqrt{2}\right)^2 = 7$?	Is $2(3)^2 + 3\left(\sqrt{2}\right)^2 = 24$?
Is $9 - 2 = 7$?	Is $18 + 6 = 24$?
Is $7 = 7$? Yes.	Is $24 = 24$? Yes.

Check $(3, -\sqrt{2})$:

$$x^2 - y^2 = 7 \qquad\qquad 2x^2 + 3y^2 = 24$$

Is $3^2 - \left(-\sqrt{2}\right)^2 = 7$? Is $2(3)^2 + 3\left(-\sqrt{2}\right)^2 = 24$?

Is $9 - 2 = 7$? Is $18 + 6 = 24$?

Is $7 = 7$? Yes. Is $24 = 24$? Yes.

Check $(-3, \sqrt{2})$:

$$x^2 - y^2 = 7 \qquad\qquad 2x^2 + 3y^2 = 24$$

Is $(-3)^2 - \left(\sqrt{2}\right)^2 = 7$? Is $2(-3)^2 + 3\left(\sqrt{2}\right)^2 = 24$?

Is $9 - 2 = 7$? Is $18 + 6 = 24$?

Is $7 = 7$? Yes. Is $24 = 24$? Yes.

Check $(-3, -\sqrt{2})$:

$$x^2 - y^2 = 7 \qquad\qquad 2x^2 + 3y^2 = 24$$

Is $(-3)^2 - \left(-\sqrt{2}\right)^2 = 7$? Is $2(-3)^2 + 3\left(-\sqrt{2}\right)^2 = 24$?

Is $9 - 2 = 7$? Is $18 + 6 = 24$?

Is $7 = 7$? Yes. Is $24 = 24$? Yes.

So this system has four solutions: $(x, y) = (3, \sqrt{2})$

$(x, y) = (3, -\sqrt{2})$

$(x, y) = (-3, \sqrt{2})$

$(x, y) = (-3, -\sqrt{2})$

Sometimes nonlinear systems have no real solutions.

As an example, to solve this system by elimination:

$$\begin{aligned} x^2 - y &= -3 \\ -x^2 - y &= -1 \end{aligned}$$

1. Eliminate the x^2-terms by adding the equations.

$$\begin{array}{r} x^2 - y = -3 \\ \underline{-x^2 - y = -1} \\ -2y = -4 \end{array}$$

2. Solve for y.

$$\begin{aligned} -2y &= -4 \\ y &= 2 \end{aligned}$$

3. Complete the ordered pairs.

$$x^2 - y = -3$$
$$x^2 - 2 = -3$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1}$$
$$x = \pm i$$

So the ordered pairs are $(i, 2)$ and $(-i, 2)$.

4. Check each ordered pair.

Check $(i, 2)$:

$x^2 - y = -3$	$-x^2 - y = -1$
Is $i^2 - 2 = -3$?	Is $-(i)^2 - 2 = -1$?
Is $-1 - 2 = -3$?	Is $-(-1) - 2 = -1$?
Is $-3 = -3$? Yes.	Is $-1 = -1$? Yes.

Check $(-i, 2)$:

$x^2 - y = -3$	$-x^2 - y = -1$
Is $(-i)^2 - 2 = -3$?	Is $-(-i)^2 - 2 = -1$?
Is $-1 - 2 = -3$?	Is $-(-1) - 2 = -1$?
Is $-3 = -3$? Yes.	Is $-1 = -1$? Yes.

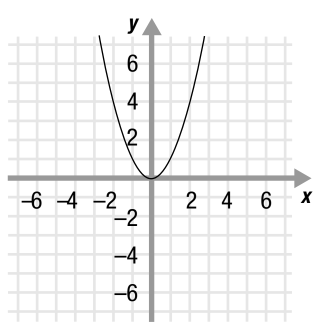
This system has no real solutions. It has two solutions that include imaginary numbers: $(x, y) = (i, 2)$ or $(x, y) = (-i, 2)$.

Sample Problems

1. Solve this nonlinear system by graphing:
$y = x^2$
$y = 2x - 1$

✔ a. Graph the parabola $y = x^2$.

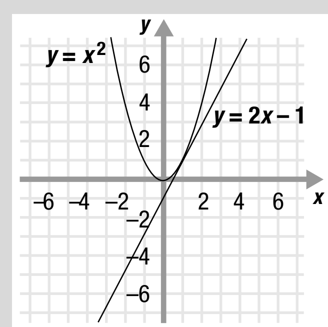
☐ b. Graph the line $y = 2x - 1$.



☐ c. Find the point of intersection. $(x, y) = (\_\_\_\_\_ , \_\_\_\_\_)$

Answers to Sample Problems

b.



c. (1, 1)

2. Solve this nonlinear system by substitution:

$$x^2 + y = 5$$
$$y = 2x^2 + 4$$

✔ a. The second equation is already solved for y in terms of x. $y = 2x^2 + 4$

✔ b. Substitute $y = 2x^2 + 4$ into the first equation.

$$x^2 + y = 5$$
$$x^2 + (2x^2 + 4) = 5$$

☐ c. Solve for x.

$$3x^2 - 1 = 0$$

$x =$ ______ or $x =$ ______

☐ d. Complete the ordered pairs.

$y =$ ______ $y =$ ______

☐ e. Check each ordered pair.

☐ f. Write the solution(s).

Answers to Sample Problems

c. $x = \pm\sqrt{\frac{1}{3}}$ or $x = \pm\frac{\sqrt{3}}{3}$

d. $\frac{14}{3}, \frac{14}{3}$

e. Here's one way to do it.

Check $\left(-\sqrt{\frac{1}{3}}, \frac{14}{3}\right)$:

$$x^2 + y = 5$$

Is $\left(-\sqrt{\frac{1}{3}}\right)^2 + \frac{14}{3} = 5$? *Yes.*

$$y = 2x^2 + 4$$

Is $\frac{14}{3} = 2\left(-\sqrt{\frac{1}{3}}\right)^2 + 4$? *Yes.*

Check $\left(\sqrt{\frac{1}{3}}, \frac{14}{3}\right)$:

$$x^2 + y = 5$$

Is $\left(\sqrt{\frac{1}{3}}\right)^2 + \frac{14}{3} = 5$? *Yes.*

$$y = 2x^2 + 4$$

Is $\frac{14}{3} = 2\left(\sqrt{\frac{1}{3}}\right)^2 + 4$? *Yes.*

f. $(x, y) = \left(-\sqrt{\frac{1}{3}}, \frac{14}{3}\right)$

$(x, y) = \left(\sqrt{\frac{1}{3}}, \frac{14}{3}\right)$

3. Solve this nonlinear system by elimination:

$$3x^2 + 4y^2 = 4$$
$$2x^2 - 3y^2 = -3$$

☑ a. Eliminate the x^2-terms by multiplying the first equation by 2 and the second equation by –3 and adding.

$$2(3x^2 + 4y^2) = 2(4) \longrightarrow 6x^2 + 8y^2 = 8$$
$$-3(2x^2 - 3y^2) = -3(-3) \longrightarrow -6x^2 + 9y^2 = 9$$
$$17y^2 = 17$$

☐ b. Finish solving for y.

$y =$ ______ or $y =$ _______

☐ c. Complete the ordered pair(s).

$x =$ _____ $x =$ _____

☐ d. Check each ordered pair.

☐ e. Write the solution(s).

Answers to Sample Problems

b. 1, –1 (in either order)

c. 0, 0

d. Check (0, 1):

$3x^2 + 4y^2 = 4$

Is $3(0)^2 + 4(1)^2 = 4$? *Yes.*

$2x^2 - 3y^2 = -3$

Is $2(0)^2 - 3(1)^2 = -3$? *Yes.*

Check (0, –1):

$3x^2 + 4y^2 = 4$

Is $3(0)^2 + 4(-1)^2 = 4$? *Yes.*

$2x^2 - 3y^2 = -3$

Is $2(0)^2 - 3(-1)^2 = 3$? *Yes.*

e. $(x, y) = (0, 1)$ *or* $(x, y) = (0, -1)$

EXPLORE

Sample Problems

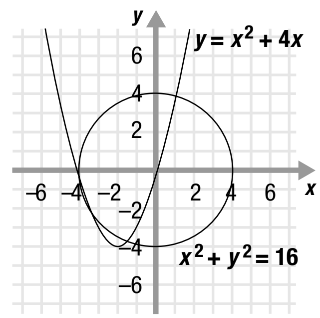
Answers to Sample Problems

On the computer, you used the Grapher to find the real solutions of nonlinear systems of equations by graphing the systems. Below are some additional exploration problems.

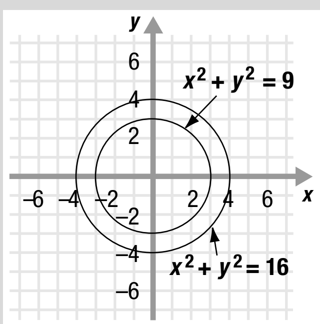
1. Graph each system below. Find the system that has only one real solution.

☑ a. circle: $x^2 + y^2 = 16$

parabola: $y = x^2 + 4x$

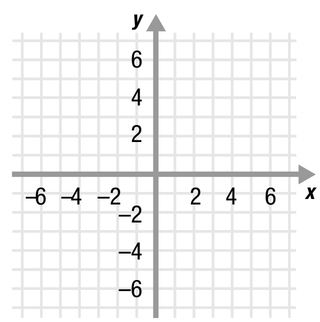


b.

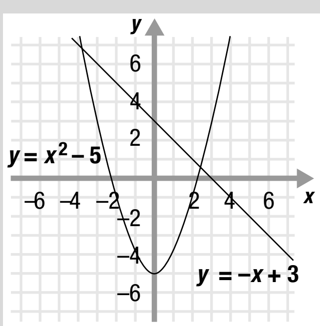


☐ b. circle: $x^2 + y^2 = 16$

circle: $x^2 + y^2 = 9$

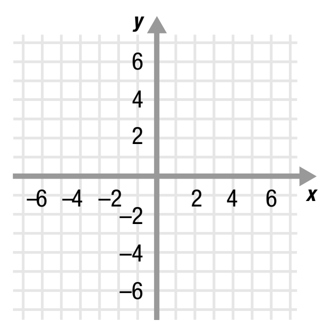


c.

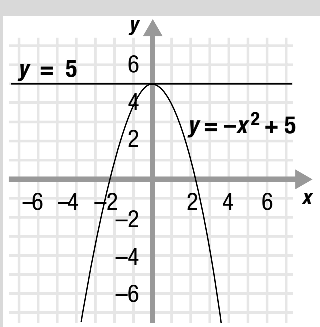


☐ c. line: $y = -x + 3$

parabola: $y = x^2 - 5$

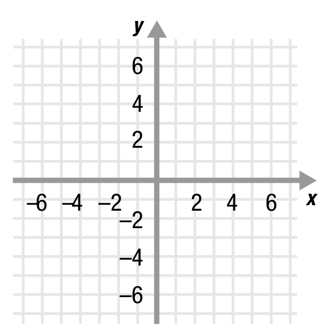


d.



☐ d. line: $y = 5$

parabola: $y = -x^2 + 5$



e. $y = 5$ and $y = -x^2 + 5$

☐ e. Write the equations of the system above whose graphs intersect in one and only one point.

2. Which of the following systems have a solution $(x, y) = (1, 1)$?

$y = x^2$	$y = x^2 - 1$	$x^2 + y^2 = 1$	$y = -x^2 + 2$
$y = x$	$y = 2x - 1$	$y = -x^2 + 1$	$y = -x + 2$

☑ a. Substitute $(x, y) = (1, 1)$ into both equations in the first system.

$y = x^2$ $\quad\quad$ $y = x$

Is $1 = 1^2$? $\quad\quad$ Is $1 = 1$? Yes.

Is $1 = 1$? Yes.

Is $(x, y) = (1, 1)$ a solution of this system? Yes.

☐ b. Substitute $(x, y) = (1, 1)$ into both equations in the second system.

$y = x^2 - 1$ $\quad\quad$ $y = 2x - 1$

Is $(x, y) = (1, 1)$ a solution of this system? _________

☐ c. Substitute $(x, y) = (1, 1)$ into both equations in the third system.

$x^2 + y^2 = 1$ $\quad\quad$ $y = -x^2 + 1$

Is $(x, y) = (1, 1)$ a solution of this system? _________

☐ d. Substitute $(x, y) = (1, 1)$ into both equations in the third system.

$y = -x^2 + 2$ $\quad\quad$ $y = -x + 2$

Is $(x, y) = (1, 1)$ a solution of this system? _________

Answers to Sample Problems

b. Is $1 = 1^2 - 1$? Is $1 = 2(1) - 1$?

Is $1 = 0$? No. Is $1 = 1$? Yes.

No.

c. Is $1^2 + 1^2 = 1$? Is $1 = -1^2 + 1$?

Is $2 = 1$? No. Is $1 = -1 + 1$?

Is $1 = 0$? No.

No.

d. Is $1 = -(1)^2 + 2$? Is $1 = -1 + 2$?

Is $1 = -1 + 2$? Is $1 = 1$? Yes.

Is $1 = 1$? Yes.

Yes.

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Solving Systems

1. Solve the following nonlinear system by graphing:

$$y = x^2 + 2x + 1$$
$$y = x + 3$$

2. Solve the following nonlinear system by substitution:

$$y = x^2 - 4$$
$$y = 6x - 13$$

3. Solve the following nonlinear system by elimination:

$$3x^2 + 4y^2 = 16$$
$$x^2 - y^2 = 3$$

4. Solve the following nonlinear system by graphing:

$$x^2 + y^2 = 9$$
$$y = -\frac{1}{3}x^2 + 3$$

5. Solve the following nonlinear system by substitution:

$$x^2 + y^2 = 13$$
$$x + y = 5$$

6. Solve the following nonlinear system by elimination:

$$4x^2 + 3y^2 = 12$$
$$x^2 + 3y^2 = 12$$

7. Solve the following nonlinear system by graphing:

$$x^2 + y^2 = 26$$
$$y = -x + 8$$

8. Solve the following nonlinear system by substitution:

$$x^2 + y^2 = 16$$
$$x^2 - 2y = 8$$

9. The supply equation for Socorro Rodriguez's hand painted jackets is $y = \frac{x}{50} + 15$, where x is the number of jackets supplied at a price of y dollars each. The demand equation is $y = \frac{12500}{x}$, where x is the number of jackets demanded at a price of y dollars each. Use substitution to find the point (x, y) where supply equals demand. How many jackets are sold and for what price?

10. Chakotay is building a house with a gabled roof,as shown in Figure 13.2.5. The plans call for a vent in the shape of an isosceles triangle in each gable. The ratio of the height to the base of these triangles must be 1 to 4. If each vent has to provide a total venting area of 3500 square inches, what should the height and base of each vent be?

    Hint: The area of a triangle is $\frac{1}{2}$base $\cdot$ height.

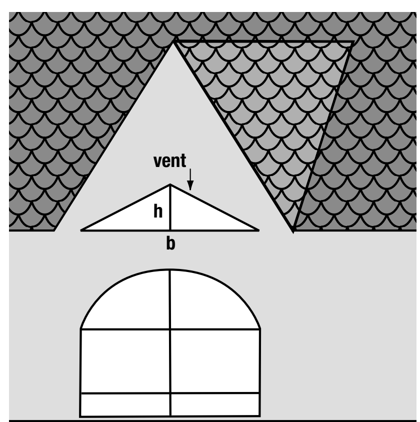


Figure 13.2.5

11. Solve the following nonlinear system by substitution:

$$x^2 + y^2 = 16$$
$$x + y = 4$$

12. Solve the following nonlinear system by elimination:

$$4x^2 - 9y^2 + 132 = 0$$
$$x^2 + 4y^2 - 67 = 0$$

Explore

13. The graph of the parabola $y = x^2 + 4x + 3$ is shown in Figure 13.2.6. What is the equation of the horizontal line that, with the parabola, forms a nonlinear system with exactly one solution?

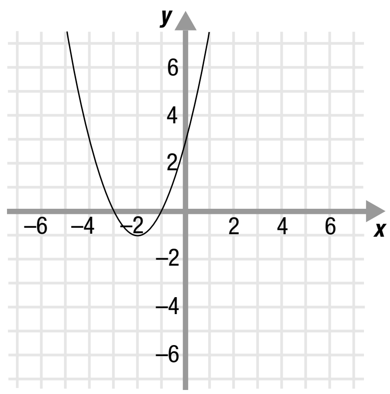


Figure 13.2.6

14. Graph this system of equations on the grid in Figure 13.2.7 to find its solutions:

    $y = x^2 - 4x + 2$

    $y = x + 2$

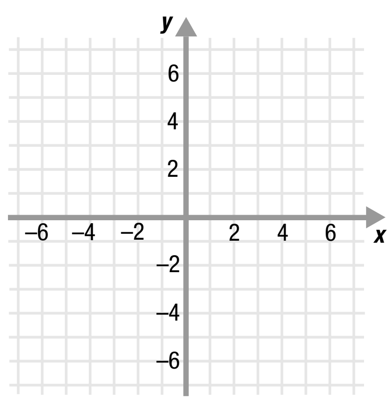


Figure 13.2.7

15. Which nonlinear system(s) below have a solution $(x, y) = (5, 0)$?

$x^2 + y^2 = 25$ $y = 0$	$y = 5$ $y = x^2 - 4x - 5$
$y = -x^2 + 4x + 5$ $y = -x + 5$	$y = x^2 + 4x - 5$ $x^2 + y^2 = 25$

16. Which of the system(s) below have only one real solution?

$(x - 3)^2 + y^2 = 18$	$y = 3$
$y = -x^2 - 3x + 4$	$y = x^2 + 3x - 4$
$y = 2x - 7$	$y = x^2 - 6$

17. Graph this system of equations to find its solutions:

    $y = -x^2 - 3x + 4$

    $y = x^2 + 3x - 4$

18. Which nonlinear system(s) below have a solution $(x, y) = (1, -8)$?

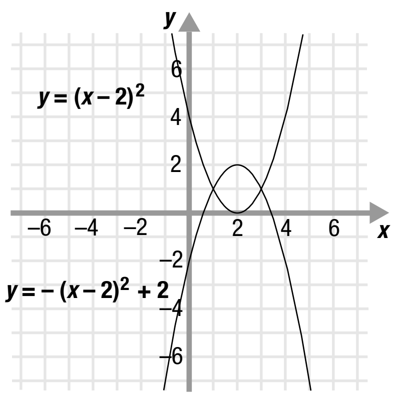
$y = x^2 - 2x - 7$ $y = -3x - 5$	$(x - 1)^2 + y^2 = 64$ $y = -8$
$y = -x^2 + 2x - 9$ $y = 2x^2 - 4x - 6$	$y = -x^2 + 8x - 15$ $y = 2x - 10$

APPLY

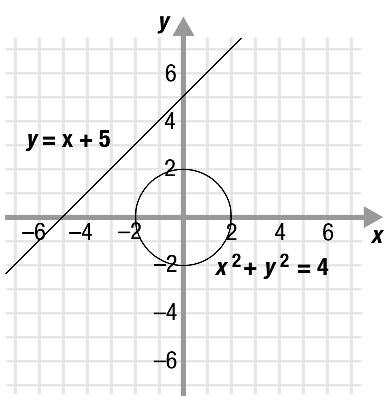
Practice Problems

Here are some additional practice problems for you to try.

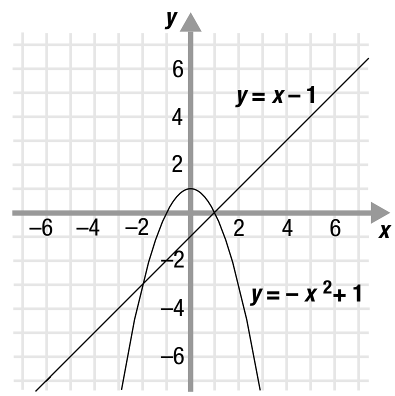
1. Use the graphs below to solve the following nonlinear system:
   $y = (x - 2)^2$
   $y = -(x - 2)^2 + 2$



2. Use the graphs below to solve the following nonlinear system:
   $x^2 + y^2 = 4$
   $y = x + 5$



3. Use the graphs below to solve the following nonlinear system:
   $y = -x^2 + 1$
   $y = x - 1$



4. Solve the following nonlinear system by substitution:
   $x^2 + y^2 = 10$
   $-3x + y = 0$

5. Solve the following nonlinear system by substitution:
   $6x^2 - 2y = 4$
   $3x^2 + y = 22$

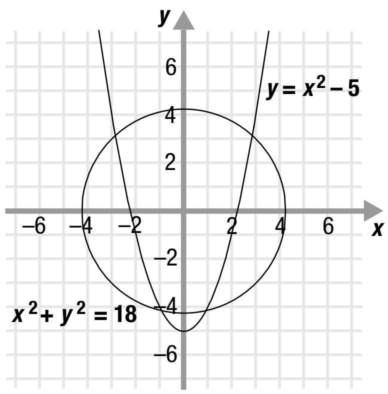
6. Solve the following nonlinear system by elimination:
   $7x^2 + 11y^2 = 28$
   $x^2 + y^2 = 4$

7. Solve the following nonlinear system by elimination:
   $10x^2 + y^2 = 14$
   $2x^2 + y^2 = 6$

8. Use the graphs below to find the number of real solutions of this nonlinear system:

$x^2 + y^2 = 18$

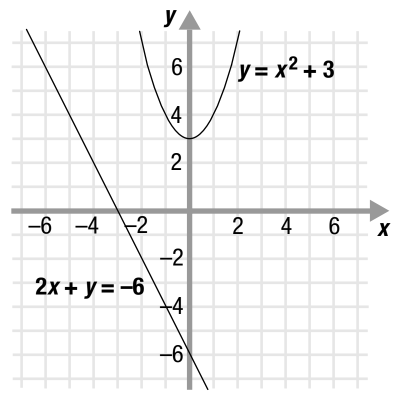
$y = x^2 - 5$



9. Use the graphs below to find the number of real solutions of this nonlinear system:

$y = x^2 + 3$

$2x + y = -6$



10. How many real solutions does this nonlinear system have?

$x^2 + y^2 = 16$

$y = x - 3$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Use the graphs in Figure 13.2.8 to find the number of solutions of this nonlinear system:

$$x^2 + y^2 = 13$$
$$y = 2x^2 - 5$$

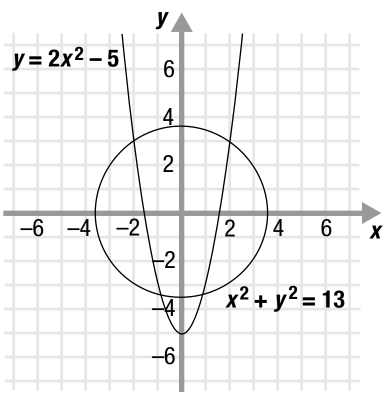


Figure 13.2.8

2. Use the substitution method to solve this nonlinear system:

$$y = x^2 - 2x + 1$$
$$y + x = 3$$

3. Use the elimination method to solve this nonlinear system:

$$x^2 - 4y = 8$$
$$x + 2y = 8$$

4. How many real solutions does this nonlinear system have?

$$x^2 + y^2 = 25$$
$$x + y = 9$$

5. The graph of the circle $x^2 + y^2 = 18$ is shown in Figure 13.2.9. Identify the equation of the line below that, with this circle, forms a nonlinear system with exactly two solutions.

$y = 2x + 11$

$y = -x$

$y = 3\sqrt{2}$

$y = 9$

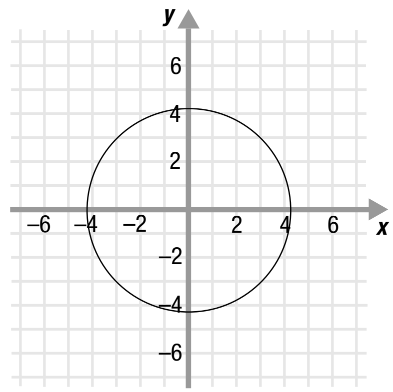


Figure 13.2.9

6. The graph of the nonlinear system below is shown in Figure 13.2.10.

$$x^2 + y^2 = 25$$
$$y = -\frac{1}{5}x^2 + 5$$

Find the solutions of this system that also satisfy the equation $y = -x + 5$.

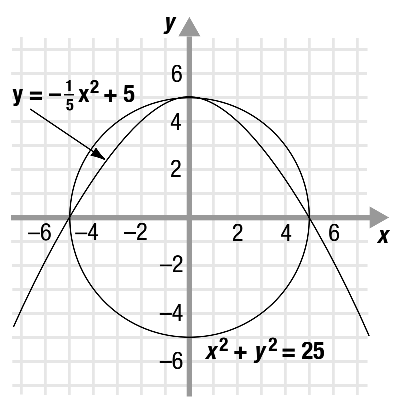


Figure 13.2.10

7. The graph of the parabola $y = x^2 - 5$ is shown in Figure 13.2.11. Circle the equation of the line below that, with the parabola, forms a nonlinear system with solutions $(x, y) = (-3, 4)$ and $(x, y) = (1, -4)$.

   $y = -2x - 2$

   $y = -2x$

   $y = \frac{4}{3}x$

   $y = 4$

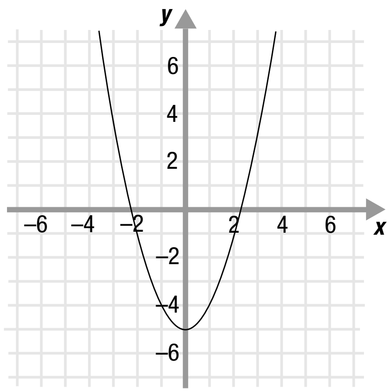


Figure 13.2.11

8. Which of the nonlinear systems below have $(x, y) = (2, -1)$ as one of their solutions?

   $y = x^2 - 5$
   $y = -2x + 3$

   $x^2 + y^2 = 5$
   $y = -x^2 + 3$

   $y = x^2 - 4$
   $y = -2x + 5$

   $x^2 + y^2 = 5$
   $y = 2$