

LESSON 8.4 – PROBLEM SOLVING





OVERVIEW

Here is what you'll learn in this lesson:

Rational Expressions

- a. Ratio and proportion*
- b. Distance problems*
- c. Work problems*
- d. Variation*

Even a day at the beach can involve algebra. For example, have you ever wondered how much faster you could get across a cove by swimming instead of jogging along the shore? Or how bad a sunburn you'd get if you used SPF 6 instead of SPF 15 sunscreen? Or how much faster you could set up the volleyball court if two people worked together?

In this lesson, you'll apply what you've learned about rational expressions to solve a variety of problems like these.



RATIONAL EXPRESSIONS

Summary

Solving Word Problems Involving Rational Expressions

In this lesson you will learn how to solve word problems when the equations contain rational expressions. Using equations with rational expressions allows you to solve a wide variety of problems.

To solve word problems using equations that contain rational expressions, you can often use this process:

1. Carefully read the problem and find the important information.
2. Determine what the problem is asking for and assign a variable to this unknown.
3. Set up an equation.
4. Solve the equation.
5. Check your answer in the original problem.

Below are some examples.

Example 1 Working alone, it would take Michael 2 hours to paint a fence. If Beth worked alone, it would take her 3 hours to paint the fence. How long will it take them to paint the fence together?

1. Find the important information.

Michael can paint the fence in 2 hours

Beth can paint the fence in 3 hours

2. Assign the variable.

Let t = the amount of time it will take them to paint the fence together

3. Set up an equation.

$$\begin{array}{ccccc} \text{fraction of the job} & & \text{fraction of the job} & & \\ \text{Michael does} & + & \text{Beth does} & = & 1 \text{ complete job} \\ \frac{t}{2} & + & \frac{t}{3} & = & 1 \end{array}$$

4. Solve the equation.

$$6\left(\frac{t}{2} + \frac{t}{3}\right) = 6(1)$$

$$3t + 2t = 6$$

$$5t = 6$$

$$t = \frac{6}{5}$$

Where did the equation come from?

Well, Beth can paint the entire fence in 3 hours, so

in 1 hour she can paint $\frac{1}{3}$ of the fence.

in 2 hours she can paint $\frac{2}{3}$ of the fence.

in t hours she can paint $\frac{t}{3}$ of the fence.

Similarly, Michael can paint the entire fence in 2 hours, so

in 1 hour he can paint $\frac{1}{2}$ of the fence.

in t hours he can paint $\frac{t}{2}$ of the fence.

If you solve the equation $d = r \cdot t$ for r you get $r = \frac{d}{t}$. You know you want to solve for r since the rates are equal.

5. Check your answer in the original problem.

$$\text{Is } \frac{\text{fraction Michael paints}}{\text{paints}} + \frac{\text{fraction Beth paints}}{\text{paints}} = 1?$$

$$\text{Is } \frac{\frac{6}{5}}{2} + \frac{\frac{6}{5}}{3} = 1?$$

$$\text{Is } \frac{6}{5} \cdot \frac{1}{2} + \frac{6}{5} \cdot \frac{1}{3} = 1?$$

$$\text{Is } \frac{3}{5} + \frac{2}{5} = 1?$$

$$\text{Is } 1 = 1? \text{ Yes.}$$

So, working together it would take them $\frac{6}{5}$ of an hour, or 1 hour and 12 minutes, to paint the fence.

Example 2 Gretchen can run as fast as Petra. Last Saturday, Gretchen ran 4 miles farther than Petra ran. If Gretchen ran for 3 hours and Petra ran for 2 hours, how far did Petra go?

1. Find the important information.

Gretchen and Petra run the same speed
Petra ran for 2 hours
Gretchen ran for 3 hours
Gretchen ran 4 miles farther than Petra

2. Assign the variable.

Let d = the distance Petra ran

3. Set up an equation.

Petra's rate = Gretchen's rate

$$\frac{\text{distance Petra ran}}{\text{time Petra ran}} = \frac{\text{distance Gretchen ran}}{\text{time Gretchen ran}}$$

$$\frac{d}{2} = \frac{d+4}{3}$$

4. Solve the equation.

$$\frac{d}{2} = \frac{d+4}{3}$$

$$3d = 2(d+4)$$

$$3d = 2d + 8$$

$$d = 8$$

5. Check your answer in the original problem.

$$\text{Is } \text{Petra's rate} = \text{Gretchen's rate} \quad ?$$

$$\text{Is } \frac{\text{distance Petra ran}}{\text{time Petra ran}} = \frac{\text{distance Gretchen ran}}{\text{time Gretchen ran}} \quad ?$$

$$\text{Is } \frac{8}{2} = \frac{12}{3} \quad ?$$

$$\text{Is } 4 = 4 \quad ? \text{ Yes.}$$

So, Petra ran 8 miles.

Sample Problems

1. It would take Ernesto 2 hours to rake the yard by himself. It would take David 5 hours to rake the yard by himself. How long will it take them working together?

- ☒ a. Find the important information. Ernesto takes 2 hours to rake
David takes 5 hours to rake
- ☒ b. Assign the variable. Let t = total time to rake the yard together
- ☐ c. Set up an equation. _____
- ☐ d. Solve the equation. $t = \underline{\hspace{2cm}}$

- ☐ e. Check your answer in the original problem. Is _____ = _____?
- Is _____ = _____?
- Is _____ = _____? _____

2. In octane, the ratio of hydrogen atoms to carbon atoms is 9 to 4. If there are 846 atoms of hydrogen in 47 molecules of octane, how many atoms of carbon are in these 47 molecules of octane?

- ☒ a. Find the important information. The ratio of hydrogen atoms to carbon atoms is 9 to 4.
47 molecules of octane contain 846 hydrogen atoms.
- ☒ b. Assign the variable. Let x = the number of carbon atoms in 47 molecules of octane

- ☒ c. Set up an equation. $\frac{\text{hydrogen atoms}}{\text{carbon atoms}} = \frac{9}{4} = \frac{846}{x}$

- ☐ d. Solve the equation. $x = \underline{\hspace{2cm}}$

- ☐ e. Check your answer in the original problem. Is _____ = _____?
- Is _____ = _____?
- Is _____ = _____? _____

Answers to Sample Problems

c. $\frac{t}{2} + \frac{t}{5} = 1$

- d. Below is one way to solve the equation.

$$10\left(\frac{t}{2} + \frac{t}{5}\right) = 10(1)$$

$$5t + 2t = 10$$

$$7t = 10$$

$$t = \frac{10}{7}$$

So it would take them $\frac{10}{7}$ hours, or about 1 hour and 26 minutes, to rake the yard working together.

e. Is $\frac{10}{2} + \frac{10}{5} = 1$?

$$\text{Is } \frac{10}{2} + \frac{10}{5} = 1?$$

$$\text{Is } \frac{5}{1} + \frac{2}{1} = 1?$$

$$\text{Is } 7 = 1? \text{ Yes.}$$

- d. Below is one way to solve the equation.

$$9 \cdot x = 4 \cdot 846$$

$$9x = 3384$$

$$x = 376$$

So, there are 376 carbon atoms in 47 molecules of octane.

e. Is $\frac{9}{4} = \frac{846}{376}$?

$$\text{Is } \frac{9}{4} = \frac{2 \cdot 3 \cdot 3 \cdot 47}{2 \cdot 2 \cdot 2 \cdot 47} ?$$

$$\text{Is } \frac{9}{4} = \frac{9}{4} ? \text{ Yes.}$$

Answers to Sample Problems

c. $50 = k(4)(25)$

d. $50 = k(100)$

$$\frac{50}{100} = k$$

$$k = \frac{1}{2}$$

$$40 = \frac{1}{2}(x)8$$

$$40 = 4x$$

$$10 = x$$

So, the base is 10 inches.

e. Is $\frac{1}{2} \cdot 10 \cdot 8 = 40$?

Is $\frac{1}{2} \cdot 80 = 40$?

Is $40 = 40$? Yes.

3. The area of a triangle varies jointly with its base and its height. If a triangle of area 50 inches² has a base of 25 inches and a height of 4 inches, what is the base of a triangle whose height is 8 inches and whose area is 40 inches²?

- ☒ a. Find the important information. area varies jointly with base and height
when the base is 25 inches and the height is 4 inches, the area is 50 inches²

- ☒ b. Assign the variable. Let x = base of the triangle
Let y = area of the triangle
Let z = height of the triangle
Let k = the constant of variation

- ☐ c. Set up an equation. $y = kxz$

- ☐ d. First find k , then $k =$ _____
substitute this value
back into the equation
 $y = kxz$ to find the
base of the triangle. base = _____

- ☐ e. Check your answer in the original problem. Is _____ = _____?

Is _____ = _____?

Is _____ = _____? _____



HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.



Explain

Rational Expressions

1. To fill up their swimming pool, the Johnsons decided to use both their high volume hose and their neighbor's regular garden hose. If they had used only their hose, it would have taken them 12 hours to fill the pool, but using both hoses it took them only 7 hours. How long would it have taken them to fill the pool using only their neighbor's hose?
2. On her camping trip, Li spent as much time hiking as she did rafting. She traveled 2.5 miles per hour when she was rafting and 3 miles per hour when she was hiking. If she went 3 miles more hiking than she did rafting, how far did she hike?
3. The ratio of jellybeans to gummy bears in a bag of candy is 7 to 2. If there are 459 pieces of candy in the bag, how many jellybeans are there?
4. The mass of an object varies jointly with its density and its volume. If 156 grams of iron has a volume of 20 cm^3 and a density of $7.8 \frac{\text{grams}}{\text{cm}^3}$, what is the volume of 312 grams of iron with the same density?
5. To empty their swimming pool, the Johnsons decided to use both the regular drain and a pump. If it takes 15 hours for the pool to empty using the drain alone and 7 hours for the pool to empty using the pump alone, how long will it take for the pool to empty using both the drain and the pump?
6. One cyclist can ride 2 miles per hour faster than another cyclist. If it takes the first cyclist 2 hours and 20 minutes to ride as far as the second cyclist rides in 2 hours, how fast can each go?
7. The ratio of the length of a rectangle to its width is 3 to 1. If the perimeter of the rectangle is 32 inches, what are its dimensions?
8. The amount of energy that can be derived from particles varies directly with their mass. If $8.184 \cdot 10^{-14} \text{ Nm}$ of energy is obtained from a particle whose mass is $9.1066 \cdot 10^{-31} \text{ kg}$, how much energy can be derived from particles whose mass is 0.001 kg ?
9. Melanie and Alex have to prune all of the trees in their yard. Working alone, it would take Melanie 7 hours to do all of the pruning. It would take Alex 11 hours to do all of the pruning by himself. How long will it take them working together?
10. A bicyclist and a horseback rider are going the same speed. The rider stops after 11.1 miles. The bicyclist goes for another hour and travels a total of 18.5 miles. How fast is each one going?
11. Fish and game wardens can estimate the population of fish in a lake if they take a sample of fish, tag them, return them to the lake, take another sample of fish, and look at the ratio of tagged fish to untagged fish. If a warden tags 117 fish in the first sample, and then finds 13 out of 642 fish have tags in the second sample, how many fish were in the lake?
12. The height of a pyramid of constant volume is inversely proportional to the area of its base. If a pyramid of volume 300 meters^3 has a base area of 90 meters^2 and a height of 10 meters, what is the height of a pyramid whose base area is 100 meters^2 ?



Practice Problems

Here are some additional practice problems for you to try.

Rational Expressions

1. Working alone it would take Josie 4 hours to paint a room. It would take Curtis 5 hours to paint the same room by himself. How long would it take them to paint the room if they work together?
2. Before the library is remodeled, all of the books must be packed in boxes. Working alone, it would take Gail 15 workdays to do the packing. It would take Rob 18 workdays. How long will it take them working together?
3. Two computers are available to process a batch of data. The faster computer can process the batch in 36 minutes. If both computers run at the same time, they can process the batch in 20 minutes. How long would it take the slower computer to process the batch alone?
4. Two copy machines are available to print final exams. The faster copy machine can do the whole job in 75 minutes. If both machines print at the same time, they can do the whole job in 50 minutes. How long would it take the slower machine to do the whole job alone?
5. Two tomato harvesters are available to harvest a field of tomatoes. The slower harvester can harvest the whole field in 7 hours. If both machines harvest at the same time, they can harvest the whole field in 3 hours. How long would it take the faster machine to harvest the whole field by itself?
6. There are two overflow pipes at a dam. The larger overflow pipe can lower the level of the water in the reservoir by 1 foot in 45 minutes. The smaller one lowers the level of water by 1 foot in 2 hours 15 minutes. If both overflow pipes are open at the same time, how long will it take them to lower the level of water by 1 foot?
7. Two fire hoses are being used to flood the skating rink at the park. The larger hose alone can flood the park in 50 minutes. The smaller hose alone can flood the park in 1 hour and 15 minutes. If both hoses run at the same time, how long will it take them to flood the park?
8. Used by itself, the cold water faucet can fill a bathtub in 12 minutes. It takes 15 minutes for the hot water faucet to fill the bathtub. If both faucets are on, how long will it take to fill the bathtub?
9. A box of chocolates contains caramel chocolates and nougat chocolates. The ratio of the number of caramels in the box to the number of nougats in the box is 4 to 3. There are a total of 42 chocolates in the box. How many caramel chocolates are in the box? How many nougat chocolates are in the box?
10. A fast food stand sells muffins and cookies. Last Monday, the ratio of the number of muffins sold to the number of cookies sold was 16 to 13. A total of 145 muffins and cookies were sold. How many muffins were sold? How many cookies were sold?
11. At a certain animal shelter, the ratio of puppies to adult dogs is 7 to 4. This week, there are a total of 55 dogs in the shelter. How many puppies are in the shelter this week? How many adult dogs are in the shelter this week?
12. In a certain cookie recipe, the ratio of cups of flour to cups of sugar is 3 to 1. If the recipe uses $2\frac{1}{4}$ cups of flour, how much sugar does it use?
13. In one multivitamin pill, the ratio of the number of units of Vitamin C to the number of units of Vitamin E is 40 to 13. If the pill contains 200 units of Vitamin C, how many units of vitamin E does it contain?

14. The ratio of the amount of caffeine, in milligrams, in a 12-ounce serving of coffee to the amount of caffeine, in milligrams, in a 12-ounce serving of cola is 25 to 9. If a 12-ounce serving of cola contains 72 milligrams of caffeine, how much caffeine does a 12-ounce serving of coffee contain?
15. Jayme can ride his bike as fast as Terry. Each day, Jayme rides his bike for one hour and 20 minutes. Each day, Terry rides his bike for two hours and rides 15 miles further than Jayme. How far does each ride?
16. Saskia runs as fast as Tanya. Each day, Tanya runs for 40 minutes. Each day, Saskia runs for one hour and runs 2 miles farther than Tanya. How far does each run?
17. Leroy rows a boat as fast as Sasha rows a boat. If Leroy rows for 30 minutes, he travels 1 mile farther than Sasha when she rows for 20 minutes. How far does each row?
18. Pietro and Maria spend the same amount of time driving to school. Pietro averages 50 miles per hour and Maria averages 30 miles per hour. Pietro drives 10 miles farther than Maria. How far does each drive to school?
19. Ranji and Paula spend the same amount of time driving to work. Ranji averages 60 miles per hour and Paula averages 40 miles per hour. Ranji drives 15 miles farther than Paula. How far does each drive to work?
20. A car averages 55 miles per hour and an airplane averages 75 miles per hour. If the airplane and the car travel for the same amount of time, the airplane travels 100 miles farther than the car. How far does each travel?
21. The accuracy of a car's speedometer varies directly with the actual speed of the car. A car's speedometer reads 24 miles per hour when the car is actually traveling at 32 miles per hour. When the speedometer reads 51 miles per hour, how fast is the car actually going?
22. The force needed to stretch a spring a certain distance varies directly with the distance. An 8 pound force stretches a spring 3.5 inches. How much force is needed to stretch the spring 12 inches?
23. A person's weight on the moon varies directly as the person's weight on Earth. A person weighing 144 pounds on Earth weighs only 24 pounds on the moon. How much does a person weigh on Earth who weighs 30 pounds on the moon?
24. The current, i , in an electrical circuit with constant voltage varies inversely as the resistance, r , of the circuit. The current in a circuit with constant voltage is 5 amperes when the resistance is 8 ohms. What is the current in the circuit if the resistance is increased to 10 ohms?
25. For storage boxes with the same volume, the area of the bottom of the box varies inversely with the height of the box. The area of the bottom of the box is 108 square inches when the height is 20 inches. What is the area when the height is 16 inches?
26. The time it takes a car to travel a fixed distance varies inversely with the rate at which it travels. It takes the car 4 hours to travel a fixed distance when it travels at a rate of 50 miles per hour. How fast does the car have to travel to cover the same distance in $2\frac{1}{2}$ hours?
27. The volume of a gas is directly proportional to the temperature of the gas and inversely proportional to the pressure exerted on the gas. Write a formula expressing this property. Use V for volume, T for temperature, and P for pressure.
28. The resistance of an electric wire is directly proportional to the length of the wire and inversely proportional to the square of its diameter. Write a formula expressing this property using R for resistance, L for length, and D for diameter.

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Caleb and Daria are going to wash windows. Working alone, it would take Daria 4 hours to wash the windows. It would take Caleb 3 hours to wash the windows by himself. How long will it take them to wash the windows working together?
2. Trisha ran to the park and then walked home. It took her $\frac{1}{2}$ hour to get to the park and 1 hour and 20 minutes to get home. If she runs 5 miles an hour faster than she walks, how far does she live from the park?
3. The ratio of raisins to peanuts in a bag of party mix is 5 to 6. If the bag contains 462 items, how many peanuts are there?
4. The area of a kite varies jointly with the lengths of its two diagonals. If a kite with area 30 inches² has one diagonal of length 10 inches and the other diagonal of length 6 inches, what is the area of a kite with diagonals of length 8 inches and 13 inches?
5. Marta is helping Ned wash dishes after a big party. If Ned could do all of the dishes by himself in 60 minutes and Marta could do all of the dishes by herself in 90 minutes, how long will it take them to do the dishes working together?
6. A harpy eagle can fly 35 kilometers per hour faster than a ruby topaz hummingbird. In the same amount of time, an eagle can fly 8.5 kilometers and a hummingbird can fly 5 kilometers. How fast can each bird fly?
7. The ratio of roses to carnations that a florist ordered was 3 to 4. If the florist received a total of 441 flowers, how many of those were roses?
8. The speed of a wave varies jointly with the wavelength and the frequency of the wave. If the speed of a wave is 20 feet per second, its wavelength is 50 feet and its frequency is 0.4 waves per second. What is the speed of a wave whose wavelength is 1 foot and whose frequency is 8 waves per second?



TOPIC 8 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic. Or you may wish to do these problems to review for a test.

1. Solve $-13 \leq 5x - 3 < 4$ for x .

2. Find: $a^2b^3c \cdot ab^2c^3$

3. Factor: $a^2 - b^2$

4. Circle the true statements.

$$\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$$

$$|19 + 4| = |19| + |4|$$

If $R = \{1, 2, 3\}$ and $S = \{1, 2, 3, 4, 5\}$, then $R \subset S$.

$$7 + 3 \cdot 6 = 60$$

$$\frac{56}{63} = \frac{8}{9}$$

The GCF and LCM of two numbers is usually the same.

5. Use the Pythagorean Theorem to find the distance between the points $(2, -5)$ and $(-3, 7)$. See Figure 8.1.

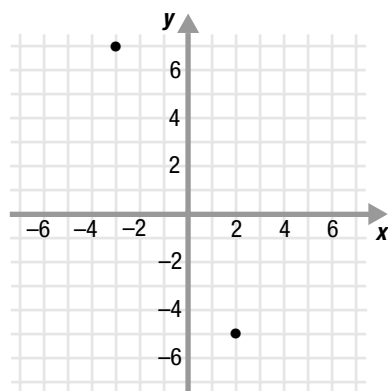


Figure 8.1

6. Factor: $18ab^4c^3 + 9a^4b^3c^2 + 12ab^2c^5$

7. Find the equation of the line parallel to the line $y = 3x + 2$ that passes through the point $(-2, 5)$.

8. Find: $(4a^2b + 3a - 9b) - (7a + 2b - 8a^2b)$

9. Solve $\frac{5}{x+3} - \frac{3}{x+3} = 1$ for x .

10. Write in scientific notation:

a) 42,789,400

b) 0.0025815

11. Find the slope and y -intercept of this line: $9x + 5y = 11$

12. Find:

a. $3^4 \cdot 3$

b. $\frac{a}{a^9}$

c. $(x^7y^0)^5$

13. It would take Kendra 4 hours to type a report. It would take Gerri $2\frac{1}{2}$ hours to type the same report. How long would it take them to type the report working together?

14. Write the equation of the circle with radius 3 whose center is at $(1, 5)$.

15. Find: $\frac{3}{x-2} + \frac{1}{x-3}$

16. Factor: $6ab - 10a + 9b - 15$

17. Factor: $x^4 - y^4$

18. Evaluate the expression $5x^2 - 6xy^4 - 4 + 7y$ when $x = 3$ and $y = 0$.

19. Solve $2(3 + y) = 5(\frac{2}{5}y + 1)$ for y .

20. Solve for x : $\frac{2}{x} - \frac{1}{x-1} = \frac{5}{2x}$

21. Use the distance formula to find the square of the distance between the points $(1, 1)$ and $(7, -4)$.

22. For what values of x is the expression $\frac{7}{x^2-9}$ undefined?

23. Find: $(9a^8b^3c - 12a^4b^3c^6) \div 3a^7b^3c^4$

Use Figure 8.2 to answer questions 24 through 26.

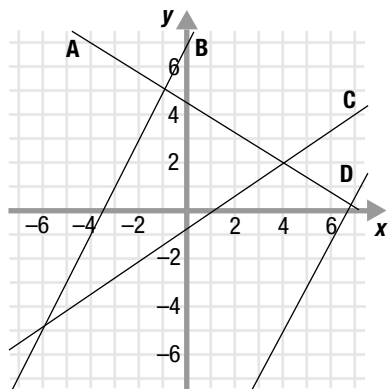


Figure 8.2

24. Which two lines form a system that has a solution of $(4, 2)$?

25. Which two lines form a system that has a solution of $(-1, 5)$?

26. Which two lines form a system that has no solution?

27. Graph the inequality $\frac{6}{5}x + 2y \geq 1$.

28. The ratio of dark brown candies to light brown candies in a bag is 3 to 1. If there are 53 light brown candies in the bag, how many are dark brown?

29. Find the slope of the line through the points $(6, -2)$ and $(4, -11)$.

30. Find: $-2s^3t(7r^2st^5 - r^3t)$

31. Factor: $6a^3b^2 - 24a^2b^3 + 24ab^2$

32. Graph the system of inequalities below to find its solution.

$$y \leq \frac{2}{3}x + 3$$

$$y > -3x - 4$$

33. Find: $\frac{x^2 - x - 6}{x^2 - 25} \cdot \frac{x^2 - 5x}{x^2 - x - 6}$

34. Factor: $8x^2y - 6xy^2 + 12x - 9y$

35. Solve $2 - \frac{x}{x+3} = \frac{-3}{x+3}$ for x .

36. Find the slope of the line that is perpendicular to the line that passes through the points $(8, -9)$ and $(-3, 11)$.

37. Find the equation of the line through the point $(-2, 6)$ that has slope $\frac{4}{3}$:

a. in point-slope form.

b. in slope-intercept form.

c. in standard form.

38. Emily withdrew \$985 in 5-dollar and 20-dollar bills from her savings account. If she had 65 bills altogether, how many of each did she have?

39. Find:

a. $2x^0 - 3x^0 + 4x^0$

b. $(x^0 \cdot x^0 \cdot x^0)^2$

c. $\frac{a^2 \cdot c}{a^5 \cdot b^6 \cdot c^4}$

40. Factor: $2x^2 + xy - 3y^2$

41. Find the radius and the center of the circle whose equation is:

$$(x - 2)^2 + (y + 6)^2 = 25$$

42. Find: $\frac{\frac{23}{9}}{\frac{4^3}{5} + \frac{5}{6^2}}$

43. Factor: $49x^2 - 14x + 1$

44. Find: $(a^2b^2 - 4a^2b - 4ab^2 + 16ab + 2b - 8) \div (b - 4)$

45. Solve $3(x + 2) = 3x + 6$ for x .

46. Graph the line $x = -2.5$.

47. Find the equation of the line perpendicular to the line $y = 2x - 5$ that passes through the point $(6, -1)$.

48. Find the slope of the line through the points $(6, -14)$ and $(22, 17)$.

49. Factor: $x^2 - 4x + 3$

50. Write in expanded form:

a. $7.1047 \cdot 10^{12}$

b. $4.294036 \cdot 10^{-8}$