

LESSON 6.2 – POLYNOMIAL OPERATIONS I

$$(x-2)^3$$



OVERVIEW

Here's what you'll learn in this lesson:

Adding and Subtracting

- a. *Definition of polynomial, term, and coefficient*
- b. *Evaluating a polynomial*
- c. *The degree of a term and a polynomial*
- d. *Writing the terms of a polynomial in descending order*
- e. *Definition of a monomial, binomial, and trinomial*
- f. *Recognizing like or similar terms*
- g. *Combining like or similar terms*
- h. *Polynomial addition*
- i. *Polynomial subtraction*

Multiplying and Dividing

- a. *Multiplying a monomial by a monomial*
- b. *Multiplying a monomial by a polynomial*
- c. *Dividing a monomial by a monomial*
- d. *Dividing a polynomial by a monomial*

Every day, people use algebra to find unknown quantities. For example, you may be interested in figuring out how long it would take you to drive across the country. Or, you may want to know why your checkbook doesn't balance.

To find these unknown quantities, you need to be able to add, subtract, multiply, and divide polynomials. That's what you will learn in this lesson.



ADDING AND SUBTRACTING

Summary

Identifying Polynomials

A polynomial is a special kind of algebraic expression which may have one or more variables and one or more terms.

For a polynomial in one variable, x , each term has the form ax^r , where the coefficient, a , is any real number, and the exponent, r , is a nonnegative integer.

For example:

$$\begin{array}{cccc} x^3 & + & 7x^2 & - & 4x & + & 2 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ a=1 & & a=7 & & a=-4 & & a=2 \\ r=3 & & r=2 & & r=1 & & r=0 \end{array}$$

For a polynomial in two variables, x and y , each term has the form ax^ry^s , where a is any real number, and r and s are nonnegative integers.

For example:

$$\begin{array}{ccc} 8x^3y^4 & + & 3xy^2 & - & 4 \\ \downarrow & & \downarrow & & \downarrow \\ a=8 & & a=3 & & a=-4 \\ r=3 & & r=1 & & r=0 \\ s=4 & & s=2 & & s=0 \end{array}$$

Polynomials with one, two, or three terms have special names:

A monomial has one term: $\frac{1}{4}t^5u^3v^2$

A binomial has two terms: $4mpd^2 + 2m^3d$

A trinomial has three terms: $-87k - 13k^2 + \sqrt{13}k^3$

An algebraic expression is not a polynomial if any of its terms cannot be written in the form ax^r .

For example, these algebraic expressions are **not** polynomials:

$$\frac{2}{3w} + 7w^2 \quad \sqrt{5t^2r} + 2t^3 \quad 6x^2 - 2\sqrt{y}$$

Remember, when $x \neq 0$:

$$x^0 = 1$$

$$x^1 = x$$

The Degree of a Polynomial

The degree of a term of a polynomial is the sum of the exponents of the variables in that term. The degree of a polynomial is the degree of the term with the highest degree.

For example, to find the degree of the polynomial $8x^3y^4 + 3xy^2 - 3$; find the degree of each term:

$$8x^3y^4 + 3xy^2 - 3 = \underset{\substack{\vee \\ 7}}{8x^3y^4} + \underset{\substack{\vee \\ 3}}{3x^1y^2} - \underset{\substack{\vee \\ 0}}{3x^0y^0}$$

The degree of the polynomial $8x^3y^4 + 3xy^2 - 3$ is the degree of the term with the highest degree, 7.

Evaluating Polynomials

Sometimes the variables in a polynomial are assigned specific numerical values. In these cases you can evaluate the polynomial by replacing the variables with the numbers.

To evaluate a polynomial:

1. Replace each variable with its assigned value.
2. Calculate the value of the polynomial.

For example, to evaluate the polynomial $6b^2 - 4bc + c^2$ when $b = 2$ and $c = 3$:

$$\begin{aligned} & 6b^2 - 4bc + c^2 \\ \text{1. Replace } b \text{ with 2 and } c \text{ with 3.} & = 6(2)^2 - 4(2)(3) + (3)^2 \\ \text{2. Calculate.} & = 24 - 24 + 9 \\ & = 9 \end{aligned}$$

So, when $b = 2$ and $c = 3$, $6b^2 - 4bc + c^2 = 9$.

Adding Polynomials

To add polynomials, combine like terms - terms that have the same variables with the same exponents.

Here is an example of two like terms:

$$3x^2y \text{ and } -2x^2y$$

Here is an example of two terms that are not like terms:

$$3x^2y \text{ and } 3xy^2$$

To add polynomials:

1. Remove the parentheses.
2. Write like terms next to each other.
3. Combine like terms.

For example, to find:

$$(3z^3 + 2zy^2 - 6y^3) + (15z^2 - 5zy^2 + 4z^2)$$

1. Remove the parentheses. $= 3z^3 + 2zy^2 - 6y^3 + 15z^2 - 5zy^2 + 4z^2$
2. Write like terms next to each other. $= 3z^3 + 2zy^2 - 5zy^2 - 6y^3 + 15z^2 + 4z^2$
3. Combine like terms. $= 3z^3 - 3zy^2 - 6y^3 + 19z^2$

Subtracting Polynomials

To subtract one polynomial from another, add the opposite of the polynomial being subtracted.

To subtract polynomials:

1. Multiply the polynomial being subtracted by -1 .
2. Distribute the -1 .
3. Simplify.
4. Write like terms next to each other.
5. Combine like terms.

For example, to find: $(6y^3 - 3z^3 + 2zy^2) - (15z^2 - 5zy^2 + 4y^3)$

1. Multiply the second polynomial by -1 . $= (6y^3 - 3z^3 + 2zy^2) + (-1)(15z^2 - 5zy^2 + 4y^3)$
2. Distribute the -1 . $= (6y^3 - 3z^3 + 2zy^2) + (-1)15z^2 - (-1)5zy^2 + (-1)4y^3$
3. Simplify. $= 6y^3 - 3z^3 + 2zy^2 - 15z^2 + 5zy^2 - 4y^3$
4. Write like terms next to each other. $= 6y^3 - 4y^3 - 3z^3 + 2zy^2 + 5zy^2 - 15z^2$
5. Combine like terms. $= 2y^3 - 3z^3 + 7zy^2 - 15z^2$

To find the opposite of a polynomial, multiply each term by -1 .

When you add the opposite, the result is the same as changing the sign of each term in the polynomial being subtracted.

b. $-2, -2$

c. $250, 60, -6$

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b. $3x^2, 5xy^2, 3$

c. $4x^2, 7xy^2, 5$

b. $3x^2, -5xy^2, 3$

c. $x^2 - 2xy^2 + 2 - 3x^2 + 5xy^2 - 3$

d. $x^2 - 3x^2 - 2xy^2 + 5xy^2 + 2 - 3$
(in any order that like terms are next to each other)

e. $-2x^2 + 3xy^2 - 1$ (in any order)

Sample Problems

1. Evaluate the polynomial $2r^3 + 3s^2r - 3s + 5$ when $r = 5$ and $s = -2$.

Evaluate:

$$2r^3 + 3s^2r - 3s + 5$$

☒ a. Replace r with 5.

$$= 2(5)^3 + 3s^2(5) - 3s + 5$$

☐ b. Replace s with -2 .

$$= 2(5)^3 + 3(\underline{\hspace{1cm}})^2(5) - 3(\underline{\hspace{1cm}}) + 5$$

☐ c. Calculate.

$$= \underline{\hspace{1cm}} + \underline{\hspace{1cm}} - \underline{\hspace{1cm}} + 5$$

$$= \underline{\hspace{1cm}}$$

2. Find: $(x^2 - 2xy^2 + 2) + (3x^2 - 5xy^2 + 3)$

☒ a. Remove the parentheses.

$$= x^2 - 2xy^2 + 2 + 3x^2 - 5xy^2 + 3$$

☐ b. Write like terms next to each other.

$$= x^2 + \underline{\hspace{1cm}} - 2xy^2 - \underline{\hspace{1cm}} + 2 + \underline{\hspace{1cm}}$$

☐ c. Combine like terms.

$$= \underline{\hspace{1cm}} - \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$

3. Find: $(x^2 - 2xy^2 + 2) - (3x^2 - 5xy^2 + 3)$

☒ a. Multiply each term in the second polynomial by -1 .

$$= (x^2 - 2xy^2 + 2) + (-1)(3x^2 - 5xy^2 + 3)$$

☐ b. Distribute the -1 .

$$= (x^2 - 2xy^2 + 2) + (-1)(\underline{\hspace{1cm}}) + (-1)(\underline{\hspace{1cm}}) + (-1)(\underline{\hspace{1cm}})$$

☐ c. Simplify. $= \underline{\hspace{2cm}}$

☐ d. Write like terms next to each other.

$$= \underline{\hspace{2cm}}$$

☐ e. Combine like terms.

$$= \underline{\hspace{2cm}}$$

MULTIPLYING AND DIVIDING

Summary

Multiplying Monomials

Monomials are easy to multiply because they each have only one term.

To multiply monomials:

1. Rearrange the factors so that the constants are next to each other and the factors with the same base are next to each other.
2. Multiply.

For example, to find: $-3p^3q^6r^2s \cdot 2p^4q^4s^5$

$$\begin{aligned} 1. \text{ Rearrange the factors.} &= -3 \cdot 2 \cdot p^3 \cdot p^4 \cdot q^6 \cdot q^4 \cdot r^2 \cdot s \cdot s^5 \\ 2. \text{ Multiply.} &= -6 \cdot p^{3+4} \cdot q^{6+4} \cdot r^2 \cdot s^{1+5} \\ &= -6 \cdot p^7 \cdot q^{10} \cdot r^2 \cdot s^6 \\ &= -6p^7q^{10}r^2s^6 \end{aligned}$$

Multiplying a Monomial by a Polynomial with More Than One Term

When multiplying a monomial by a polynomial with more than one term, you need to multiply every term in the polynomial by the monomial.

To multiply a monomial by a polynomial with more than one term:

1. Distribute the monomial to each term in the other polynomial.
2. Simplify.

For example, to find: $5x^4(3x^2y^2 + 2xy^2 - x^2y)$

$$\begin{aligned} 1. \text{ Distribute the monomial.} &= 5x^4(3x^2y^2) + 5x^4(2xy^2) - 5x^4(x^2y) \\ &= 5 \cdot 3 \cdot x^4 \cdot x^2 \cdot y^2 + 5 \cdot 2 \cdot x^4 \cdot x \cdot y^2 - 5 \cdot x^4 \cdot x^2 \cdot y \\ 2. \text{ Simplify.} &= 15 \cdot x^{4+2} \cdot y^2 + 10 \cdot x^{4+1} \cdot y^2 - 5 \cdot x^{4+2} \cdot y \\ &= 15x^6y^2 + 10x^5y^2 - 5x^6y \end{aligned}$$

In general, to multiply a monomial by a polynomial with more than one term:

$$a(b + c + d) = a \cdot b + a \cdot c + a \cdot d$$

Remember:

When $r > s$, $x^r \div x^s = \frac{x^r}{x^s} = x^{r-s}$.

When $r < s$, $x^r \div x^s = \frac{x^r}{x^s} = \frac{1}{x^{s-r}}$.

The y is on the bottom since the exponent of y in the denominator is bigger than the exponent of y in the numerator.

Dividing Monomials

To divide monomials:

1. Write the division as a fraction.
2. Cancel common numerical factors.
3. Divide factors with the same base by subtracting exponents.

For example, to find $4x^3y^4z^5 \div 10x^2y^6z^2$:

$$\begin{aligned} 1. \text{ Write the division as a fraction.} &= \frac{4x^3y^4z^5}{10x^2y^6z^2} \\ 2. \text{ Cancel common numerical factors.} &= \frac{\overset{1}{\cancel{2}} \cdot 2x^3y^4z^5}{\underset{1}{\cancel{2}} \cdot 5x^2y^6z^2} \\ 3. \text{ Divide factors with the same base.} &= \frac{2x^{3-2}z^{5-2}}{5y^{6-4}} \\ &= \frac{2x^1z^3}{5y^2} \end{aligned}$$

Dividing a Polynomial with More Than One Term by a Monomial

When you divide a polynomial with more than one term by a monomial, you must divide each term of the polynomial by the monomial.

Use this rule: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.

To divide a polynomial with more than one term by a monomial:

1. Write the division as a fraction.
2. Rewrite the fraction using the rule $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.
3. Perform the division on each of the resulting terms.

For example, to find $(15x^6y^2 + 10x^5y^3) \div 5x^4y$:

$$\begin{aligned} 1. \text{ Write the division as a fraction.} &= \frac{15x^6y^2 + 10x^5y^3}{5x^4y} \\ 2. \text{ Rewrite the fraction using} & \\ \text{the rule } \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}. &= \frac{15x^6y^2}{5x^4y} + \frac{10x^5y^3}{5x^4y} \\ 3. \text{ Divide each of the resulting terms.} &= \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} x^{6-4} y^{2-1}}{\underset{1}{\cancel{5}}} + \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{5}} x^{5-4} y^{3-1}}{\underset{1}{\cancel{5}}} \\ &= 3x^2y + 2xy^2 \end{aligned}$$

Sample Problems

1. Find: $3wx^3y^6 \cdot 7wx^2y^5z^2$

- ☒ a. Rearrange the factors so the constants are next to each other and factors with the same base are next to each other.

$$= 3 \cdot 7 \cdot w \cdot w \cdot x^3 \cdot x^2 \cdot y^6 \cdot y^5 \cdot z^2$$

- ☐ b. Multiply factors with the same base by adding the exponents.

$$= 21 \cdot w^{\rule{1cm}{0.4pt}} \cdot x^{\rule{1cm}{0.4pt}} \cdot y^{\rule{1cm}{0.4pt}} \cdot z^{\rule{1cm}{0.4pt}}$$

2. Find: $pr^2s(p^2r + pr^3s^6 - 2)$

- ☐ a. Distribute the monomial.

$$= \rule{1cm}{0.4pt} \cdot p^2r + \rule{1cm}{0.4pt} \cdot pr^3s^6 - \rule{1cm}{0.4pt} \cdot 2$$

- ☐ b. Multiply each of the resulting terms.

$$= p^{1+2}r^{2+1}s + p^{1+1}r^{2+3}s^{1+6} - 2pr^2s$$

$$= \rule{1cm}{0.4pt} + \rule{1cm}{0.4pt} - 2pr^2s$$

3. Find: $18m^6n^5p^3r \div 10m^3n^7pr$

- ☒ a. Write the division as a fraction.

$$= \frac{18m^6n^5p^3r}{10m^3n^7pr}$$

- ☒ b. Cancel common numerical factors.

$$= \frac{\overset{1}{\cancel{2}} \cdot 9m^6n^5p^3r}{\underset{1}{\cancel{2}} \cdot 5m^3n^7pr}$$

- ☐ c. Divide factors with the same base by subtracting exponents.

$$= \rule{1cm}{0.4pt}$$

4. Find: $(6w^5x^3 + 4w^3x^2y) \div 2w^2xy$

- ☒ a. Write the division as a fraction.

$$= \frac{6w^5x^3 + 4w^3x^2y}{2w^2xy}$$

- ☐ b. Rewrite the fraction using the rule

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$= \frac{6w^5x^3}{2w^2xy} + \rule{1cm}{0.4pt}$$

- ☐ c. Divide each term.

$$= \frac{3w^3x^2}{y} + \rule{1cm}{0.4pt}$$

Answers to Sample Problems

b. 2, 5, 11, 2

a. pr^2s , pr^2s , pr^2s

b. p^3r^3s , $p^2r^5s^7$

c. $\frac{9m^3p^2}{5n^2}$

b. $\frac{4w^3x^2y}{2w^2xy}$

c. $2wx$



HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.



Explain

Adding and Subtracting

1. Circle the algebraic expression that is a polynomial.

$$3\frac{1}{4}y^3 + \sqrt{3y^2 - 5}$$

$$3\frac{1}{4}y^3 + 3y^2 - \sqrt{5}$$

$$\frac{1}{4y^3} + 3y^2 - 5$$

2. Write m beside the monomial, b beside the binomial, and t beside the trinomial.

$$\underline{\hspace{1cm}} 34x + x^2 + z$$

$$\underline{\hspace{1cm}} wxy^3z^2$$

$$\underline{\hspace{1cm}} pn^2 - 13n^3$$

3. Given the polynomial $3y - 2y^3 - 4y^5 + 2$:

a. write the terms in descending order.

b. find the degree of each term.

c. find the degree of the polynomial.

4. Find: $(-3w - 12w^3 + 2) + (15w - 2w^3 + 4w^5 - 3)$

5. Find: $(2v^3 + 6v^2 + 2) - (5v + v^3 + 4v^7 - 3)$

6. Evaluate $\frac{1}{4}xy + 3y^2 - 5x^3$ when $x = 2$ and $y = 4$.

7. Find:

$$(-s^2t + s^3t^3 + 4st^2 - 27) + (3st^2 + 2st - 8s^3t^3 - 13t + 36)$$

8. Find: $(12x^3y + 9x^2y^2 + 6xy - y + 7) - (7xy - x + y - 11x^3y + 3x^2y^2 - 4)$

9. Angelina works at a pet store. Today, she is cleaning three fish tanks. These polynomials describe the volumes of the tanks:

Tank 1: xy^2

Tank 2: $x^2y - 2y^3 + 4xy^2 + 3$

Tank 3: $x^2y + 5xy^2 + 6y^3$

Write a polynomial that describes the total volume of the three tanks.

Hint: Add the polynomials.

volume = $\underline{\hspace{2cm}}$

10. Angelina has three fish tanks to clean. These polynomials describe their volumes.

Tank 1: xy^2

Tank 2: $x^2y - 2y^3 + 4xy^2 + 3$

Tank 3: $x^2y + 5xy^2 + 6y^3$

What is the total volume of the fish tanks if $x = 3$ feet and $y = 1.5$ feet?

volume = $\underline{\hspace{2cm}}$ cubic feet

11. Find:

$$(w^2yz + 3w^3 - 2wyz^2 + 4wyz) - (4wy^2z - 3w^2yz + 2wyz^2) + (2wyz + 3)$$

12. Find:

$$(tu^2v - 4t^2u^2v + 9t^3uv + 3tv) + (3t^2u^2 + 2tv - t^3) - (4t^2u^2v + 3tv + 2tu^2v) - (6t^3uv + 2tv)$$

Multiplying and Dividing

13. Find: $xyz \cdot x^2y^2z^2$

14. Find: $3p^2r \cdot 2p^3qr$

15. Find: $-6t^3u^2v^{11} \cdot \frac{1}{2}tu^2v^4$

16. Find: $3y(2x^3 + 3x^2y)$

17. Find: $5p^2r^3(2pr + p^2r^2)$

18. Find: $t^3uv^4(2tu - 3uv + 4tv + 5)$

19. Write $12w^7x^3y^2z^6 \div 4w^2x^2y^3z^6$ as a fraction and simplify.

20. Write $(36x^3y^3 + 15x^2y^5) \div 9x^2y$ as a fraction and simplify.

21. Find: $15a^7b^4d^2 \div 10a^4b^9c^3d$

22. Tony is an algebra student. This is how he answered a question on a test:

$$(2t^8u^3 - 4t^4u^9 + 6t^{12}u^6) \div 2t^4u^3 = t^2u - 2tu^3 + 3t^3u^2$$

Is his answer right or wrong? Why? Circle the most appropriate response.

The answer is right.

The answer is wrong. Tony divided the exponents rather than adding them. The correct answer is $t^{12}u^6 - t^8u^{12} + t^{16}u^9$.

The answer is wrong. The terms need to be ordered by degree. The correct answer is $3t^3u^2 + t^2u - 2tu^3$.

The answer is wrong. Tony divided the exponents rather than subtracting them. The correct answer is $t^4 - 2u^6 + 3t^8u^3$.

The answer is wrong. Tony shouldn't have canceled the numerical coefficients. The correct answer is $2t^2u - 4tu^3 + 6t^3u^2$.

23. Find: $(16x^2y^4 + 20x^3y^5) \div 12xy^2$

24. Find: $(20t^5u^{11} + 5t^3u^5 + 30tu^6v^5) \div 10t^4u^5$



Practice Problems

Here are some additional practice problems for you to try.

Adding and Subtracting

1. Circle the algebraic expressions below that are polynomials.

$$2xy + 5xz$$

$$\frac{2}{3x} + 6x$$

$$9y^2 + 13yz - 8z^2$$

$$\sqrt{24x^5}$$

$$\frac{15a^3}{5a^8}$$

2. Circle the algebraic expressions below that are polynomials.

$$8xy + \frac{3}{y}$$

$$\sqrt{17x^3}$$

$$3w - 7wz - 1$$

$$7x^2 - 13x + 8y^2$$

$$\frac{12x^2}{3x^3}$$

3. Identify each polynomial below as a monomial, a binomial, or a trinomial.

a. $17x + 24z$

b. $13ab^2 - 5$

c. $m - n + 10$

d. $42a^2b^4c$

e. $73 + 65x - 21y$

4. Identify each polynomial below as a monomial, a binomial, or a trinomial.

a. $25 - 6xyz - 4x$

b. $2xyz^3$

c. $x + y - 1$

d. $36 - 3xyz$

e. $32x^2y$

5. Find the degree of the polynomial $8a^3b^5 - 11a^2b^3 + 7b^6$.

6. Find the degree of the polynomial $12m^4n^7 - 16m^{12}$.

7. Find the degree of the polynomial $7x^3y^2z + 3x^2y^3z^4 + 6z^7$.

8. Evaluate $2x^2 - 8x + 11$ when $x = -1$.

9. Evaluate $x^3 + 3x^2 - x + 1$ when $x = -2$.

10. Evaluate $2x^2 - 5x + 8$ when $x = 3$.

11. Evaluate $x^2y + xy^2$ when $x = 2$ and $y = -3$.

12. Evaluate $5mn + 4mn^2 + 8m - n$ when $m = 4$ and $n = -2$.

13. Evaluate $3uv - 6u^2v + 2u - v + 4$ when $u = 2$ and $v = -4$.

14. Find: $(3x^2 + 7x) + (x^2 - 5)$

15. Find: $(5x^2 + 4x - 8) + (x^2 + 7x)$

16. Find: $(6a^2 + 8a - 10) + (-3a^2 - 2a + 7)$

17. Find:
 $(12m^2n^3 + 7m^2n^2 - 14mn) + (3m^2n^3 - 5m^2n^2 + 7mn)$
18. Find: $(10x^4y^3 - 9x^2y^3 + 6xy^2 - x) +$
 $(-8x^4y^3 + 14x^2y^3 + 3xy^2 + x)$
19. Find: $(13a^3b^2 + 6a^2b - 5ab^3 + b) +$
 $(2a^3b^2 - 2a^2b + 4ab^3 - b)$
20. Find: $(11u^5v^4w^3 + 6u^3v^2w) + (6u^5v^4w^3 - 11u^3v^2w)$
21. Find: $(7xy^2z^3 - 19x^2yz^2 + 26x^3y^3z) +$
 $(13xy^2z^3 - 11x^2yz^2 - 16x^3y^3z)$
22. Find: $(9a^4b^2c - 3a^2b^3c + 5abc) +$
 $(2abc - 6a^4b^2c - 2) + (3a^2b^3c + 5)$
23. Find: $(5x^3 + 7x) - (x^3 + 8)$
24. Find: $(9a^2 + 7ab + 14b) - (3a^2 - 7b)$
25. Find: $(2y^2 + 6xy + 3y) - (y^2 - y)$
26. Find: $(8x^3 + 9x^2 + 17) - (5x^3 - 3x^2 + 15)$
27. Find: $(9a^5b^3 + 8a^4b - 6b) - (-2a^5b^3 + 12a^4b + 3b)$
28. Find: $(7x^4y^2 - 3x^2y + 5x) - (9x^4y^2 + 3x^2y - 2x)$
36. Find: $4y^3(3y^2 + 5y - 10)$
37. Find: $-2a^3b^2(3a^4b^5 - 5ab^3 + 6a)$
38. Find: $2xy^3(2x^6 - 5x^4 + y^2)$
39. Find: $5a^2b^2(4a^2 + 2a^2b - 7ab^2 - 3b)$
40. Find: $-4mn^3(-3m^2n + 12mn^2 - 6m + 7n^2)$
41. Find: $4x^3y^3(3x^3 - 7xy^2 + 2xy - y)$
42. Find: $\frac{9x^3y}{3x^2}$
43. Find: $\frac{20a^5b^6}{4a^3b}$
44. Find: $\frac{12x^4y^6}{3x^2y}$
45. Find: $\frac{32a^7b^9c}{12a^5b^6c^2}$
46. Find: $\frac{15m^6n^{10}}{10n^4p^3}$
47. Find: $\frac{24x^6y^2z^7}{16wx^3z^2}$
48. Find: $\frac{27a^4b^3c^{12}d}{15ac^7d^3}$
49. Find: $\frac{42mn^6p^3q^4}{28m^2nq^5}$
50. Find: $\frac{36w^2x^3y^7z}{21w^5y^2z^2}$
51. Find: $\frac{32a^3 + 24a^5}{8a^2}$
52. Find: $\frac{21m^2 + 18mn^3}{3mn}$
53. Find: $\frac{14x + 8x^4y^2}{2xy}$
54. Find: $\frac{24a^2b^2c^3 - 4ab^4c^5}{16abc^3}$
55. Find: $\frac{32x^2y^3z^4 - 8x^5yz^7}{16x^3y^3z^4}$
56. Find: $\frac{32r^4st^2 - 3r^2st^5}{12r^3s^2t}$

Multiplying and Dividing

29. Find: $3y^4 \cdot 5y$
30. Find: $5x^3 \cdot 2x$
31. Find: $-5a^5 \cdot 9a^4$
32. Find: $-3x^3 \cdot 12x^4$
33. Find: $4x^3y^5 \cdot 7xy^3$
34. Find: $-7a^5b^6c^3 \cdot 8ab^3c$
35. Find: $-3w^2x^3y^2z \cdot 2x^2yz^2$

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Circle the expressions that are polynomials.

$$-\sqrt{325}$$

$$\frac{2}{5}p^3r - 3p^2q + \sqrt{2}r$$

$$t^2 - s + 5$$

$$\frac{5}{7}c^{15} + \frac{3}{14}c^{11} - 3\pi$$

$$m^5n^4o^3p^2r$$

$$x^2 + 3xy - \frac{2}{3x} + y^2$$

2. Write m beside the monomial(s), b beside the binomial(s), and t beside the trinomial(s).

a. ____ w^5x^4

b. ____ $2x^2 - 36$

c. ____ $\frac{1}{3}x^{17} + \frac{2}{3}x^{12} - \frac{1}{3}$

d. ____ 27

e. ____ $27x^3 - 2x^2y^3$

f. ____ $x^2 + 3xy - \frac{2}{3}y^2$

3. Given the polynomial $3w^3 - 13w^2 + 7w^5 + 8w^8 - 2$, write the terms in descending order by degree.

4. Find:

a. $(5x^3y - 8x^2y^2 + 3xy - y^3 + 13) + (-2xy + 6 + y^2 - 4y^3 - 2x^3y)$

b. $(5x^3y - 8x^2y^2 + 3xy - y^3 + 13) - (-2xy + 6 + y^2 - 4y^3 - 2x^3y)$

5. Find: $x^3y^2w \cdot x^5yw^4$

6. Find: $n^2p^3(3n + 2n^3p^2 - 35p^4)$

7. Find: $21x^5y^2z^7 \div 14xyz$

8. Find: $(15t^3u^2v - 5t^5uv^2) \div 10tuv^2$