

LESSON 8.3 — EQUATIONS WITH FRACTIONS





OVERVIEW

Here is what you'll learn in this lesson:

Solving Equations

- a. Solving equations with rational expressions*
- b. Solving for an unknown in a formula involving a rational expression*

Suppose you want to figure out a baseball pitcher's earned run average, or estimate the population of fish in a lake, or figure out how tall a building is based on a scale model. For each of these examples, you can figure out the answer by setting up an equation that involves fractions or ratios.

However, even with all the techniques you have for solving equations, solving an equation with fractions can be tricky. There may not be any solution at all, or the solution you find might not check when you plug it back into the original equation.

In this lesson you will learn how to solve equations that have fractions in them, and you will learn how to identify extraneous, or false solutions.



SOLVING EQUATIONS

Summary

Solving Equations with Rational Expressions

When you solve an equation that contains a rational expression, it helps to clear the fraction in the equation. To do this, multiply both sides of the equation by the least common denominator (LCD) of the fractions.

To solve an equation that contains rational expressions:

1. Clear the fractions by multiplying both sides of the equation by the LCD of the fractions.
2. Distribute the LCD and simplify.
3. Finish solving for the variable.

For example, to solve $2x - \frac{1}{3} = \frac{2x}{3} + \frac{2x}{5} - \frac{3}{15}$ for x :

1. Multiply by the LCD

of the fractions, 15. $15 \cdot (2x - \frac{1}{3}) = 15 \cdot (\frac{2x}{3} + \frac{2x}{5} - \frac{3}{15})$

2. Distribute the LCD

and simplify. $15 \cdot 2x - 15 \cdot \frac{1}{3} = 15 \cdot \frac{2x}{3} + 15 \cdot \frac{2x}{5} - 15 \cdot \frac{3}{15}$

$$30x - 5 = 10x + 6x - 3$$

$$30x - 5 = 16x - 3$$

3. Finish solving for x .

$$14x = 2$$

$$x = \frac{2}{14}$$

$$x = \frac{1}{7}$$

Checking for Extraneous Solutions

If an equation contains a fraction with a variable in the denominator, the solution of the equation might be extraneous (false).

To check for an extraneous solution:

1. Solve the equation for the variable.
2. Substitute the solution into the original equation and simplify.
3. Look at the denominators of the fractions. If any denominator is zero, the solution is extraneous.

Here's how to find the LCD of $\frac{1}{3}$, $\frac{2x}{5}$, and $\frac{3}{15}$:

$$\begin{array}{rcl} 3 & = & 1 \cdot 3 \\ 5 & = & 1 \cdot 5 \\ 15 & = & 1 \cdot 3 \cdot 5 \\ & & \downarrow \downarrow \downarrow \\ & & 1 \cdot 3 \cdot 5 = 15 \end{array}$$

The LCD of $\frac{4}{x+3}$, $\frac{1}{x}$, and $\frac{12}{x(x+3)}$ is $x(x+3)$.

For example, to determine if $\frac{4}{x+3} = \frac{1}{x} - \frac{12}{x(x+3)}$ has an extraneous solution:

$$\begin{aligned} 1. \text{ Solve the equation for } x. \quad & \frac{4}{x+3} = \frac{1}{x} - \frac{12}{x(x+3)} \\ & x(x+3) \cdot \frac{4}{x+3} = x(x+3) \cdot \left(\frac{1}{x} - \frac{12}{x(x+3)} \right) \\ & x(x+3) \cdot \frac{4}{x+3} = x(x+3) \cdot \frac{1}{x} - x(x+3) \cdot \frac{12}{x(x+3)} \\ & 4x = x+3 - 12 \\ & 3x = -9 \\ & x = -3 \end{aligned}$$

$$2. \text{ Substitute } -3 \text{ for } x \text{ in the original equation.} \quad \frac{4}{-3+3} = \frac{1}{-3} - \frac{12}{-3(-3+3)}$$

$$3. \text{ Check the denominators.} \quad \frac{4}{0} = \frac{1}{-3} - \frac{12}{0}$$

When you substitute $x = -3$,
two of the fractions have a
denominator of zero.

So, the solution is extraneous. This equation has no solution.

Using Cross Multiplication to Solve Proportions

An equation that sets one fraction equal to another fraction is called a proportion. An easy way to solve a proportion is to “cross multiply.”

To solve a proportion using cross multiplication:

1. Multiply the numerator of the first fraction by the denominator of the second fraction.
2. Multiply the denominator of the first fraction by the numerator of the second fraction.
3. Set the two products equal to each other.
4. Finish solving for the variable.

For example, to solve the proportion $\frac{x-1}{2} = \frac{26}{4}$:

$$1. \text{ Multiply the numerator of } \frac{x-1}{2} \text{ by the denominator of } \frac{26}{4}. \quad \frac{x-1}{2} \rightarrow \frac{26}{4} \\ (x-1) \cdot 4$$

$$2. \text{ Multiply the denominator of } \frac{x-1}{2} \text{ by the numerator of } \frac{26}{4}. \quad \frac{x-1}{2} \rightarrow \frac{26}{4}$$

$$3. \text{ Set the products equal to each other.} \quad 2 \cdot 26 = (x-1) \cdot 4 \quad 2 \cdot 26$$

$$\begin{aligned} 4. \text{ Finish solving for } x. \quad & 52 = 4x - 4 \\ & 56 = 4x \\ & 14 = x \end{aligned}$$

Proportions have exactly one term on each side of the equation. Here are some examples of proportions:

$$\frac{5}{x} = \frac{10}{20} \quad \frac{3x-2}{3} = \frac{1}{2} \quad \frac{-5y}{30} = 5$$

Sample Problems

1. Solve the equation $\frac{x-2}{4} + 3 = \frac{5}{6} - \frac{x}{6}$ for x . Determine if the solution is extraneous.

- ☐ a. Multiply both sides by the LCD of the fractions. $\frac{x-2}{4} + 3 = \frac{5}{6} - \frac{x}{6}$
- ☐ b. Distribute the LCD and simplify. $\frac{x-2}{4} + 3 = \frac{5}{6} - \frac{x}{6}$
- ☐ c. Finish solving for x . $\frac{x-2}{4} + 3 = \frac{5}{6} - \frac{x}{6}$
- ☐ d. Substitute the solution for x in the original equation. $\frac{x-2}{4} + 3 = \frac{5}{6} - \frac{x}{6}$
- ☐ e. Is the solution extraneous? $\frac{x-2}{4} + 3 = \frac{5}{6} - \frac{x}{6}$

2. Solve the equation $\frac{2}{x} - \frac{1}{x+2} = \frac{2}{x(x+2)}$ for x . Determine if the solution is extraneous.

- ☒ a. Multiply both sides by the LCD of the fractions. $\frac{2}{x} - \frac{1}{x+2} = \frac{2}{x(x+2)}$
- ☐ b. Distribute the LCD and simplify. $\frac{2}{x} - \frac{1}{x+2} = \frac{2}{x(x+2)}$
- ☐ c. Finish solving for x . $\frac{2}{x} - \frac{1}{x+2} = \frac{2}{x(x+2)}$
- ☐ d. Substitute the solution for x in the original equation. $\frac{2}{x} - \frac{1}{x+2} = \frac{2}{x(x+2)}$
- ☐ e. Is the solution extraneous? $\frac{2}{x} - \frac{1}{x+2} = \frac{2}{x(x+2)}$

3. Use cross multiplication to solve the proportion $\frac{7}{5x} = \frac{15}{10}$ for x .

- ☒ a. Multiply the numerator of $\frac{7}{5x}$ by the denominator of $\frac{15}{10}$. $\frac{7}{5x} = \frac{15}{10}$
- ☐ b. Multiply the denominator of $\frac{7}{5x}$ by the numerator of $\frac{15}{10}$. $\frac{7}{5x} = \frac{15}{10}$
- ☐ c. Set the products equal to each other. $\frac{7}{5x} = \frac{15}{10}$
- ☐ d. Finish solving for x . $\frac{7}{5x} = \frac{15}{10}$

Answers to Sample Problems

a. 12, 12

b. 12, 12, 12, 12

c. $3(x-2), 36$
 $3x - 6 + 36 = 10 - 2x$
 $5x = -20$
 $x = -4$

d. $\frac{-4-2}{4} + 3 = \frac{5}{6} - \frac{-4}{6}$

e. No

b. $2(x+2), x, 2$

c. $2x + 4 - x, 2$
 $x = -2$

d. $\frac{2}{-2} - \frac{1}{-2+2} = \frac{2}{-2(-2+2)}$

e. Yes

b. $5x, 15$ (in either order)

c. $75x$
d. $\frac{70}{75}$ or $\frac{14}{15}$



HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.



Explain

Solving Equations with Rational Expressions

In problems 1 through 12 solve for the variable. Be sure to check whether your answer is an extraneous solution.

1. Solve for x : $\frac{5}{x} - \frac{2}{x} = 1$

2. Solve for y : $\frac{4}{7}y = -\frac{2}{7}$

3. Solve for x : $\frac{3}{x} + \frac{2}{x-2} = 1$

4. Solve for y : $\frac{y}{y-5} - \frac{2}{5} = \frac{5}{y-5}$

5. Solve for x : $\frac{3x+1}{11x-9} = \frac{1}{3}$

6. Solve for x : $\frac{2}{1-x} - \frac{1}{x} = \frac{7}{6}$

7. Solve for y : $\frac{4y-9}{8} = \frac{6-8y}{16}$

8. Solve for x : $\frac{3}{2x-3} - x = \frac{2}{4x-6}$

9. A person who weighs 100 pounds on Earth would weigh 38 pounds on Mars. Use the proportion below to figure out how much someone who weighs 160 pounds on Earth would weigh on Mars.

$$\frac{\text{weight on Mars}}{\text{weight on Earth}} = \frac{38}{100} = \frac{x}{160}$$

10. A person who weighs 100 pounds on Earth would weigh 234 pounds on Jupiter. Use the proportion below to figure out how much someone who weighs 160 pounds on Earth would weigh on Jupiter.

$$\frac{\text{weight on Jupiter}}{\text{weight on Earth}} = \frac{234}{100} = \frac{x}{160}$$

11. Solve for x : $\frac{6}{x-2} - \frac{3}{x} = \frac{-5}{x-4}$

12. Solve for x : $\frac{6}{x-2} - \frac{1}{3} = \frac{3x}{x-2}$



Practice Problems

Here are some additional practice problems for you to try. Be sure to check whether your answer is an extraneous solution.

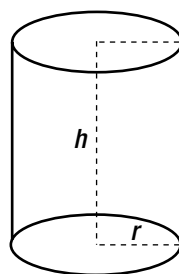
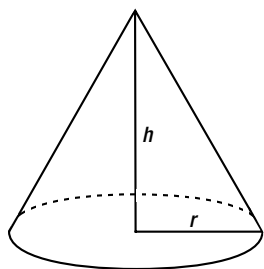
Solving Equations with Rational Expressions

1. Solve for x : $\frac{4}{x} + \frac{3}{x} = 1$
2. Solve for x : $\frac{8}{x} - \frac{4}{x} = 1$
3. Solve for x : $\frac{1}{x+1} + \frac{5}{x+1} = 2$
4. Solve for x : $\frac{10}{x-3} - \frac{4}{x-3} = -3$
5. Solve for x : $\frac{3}{x+2} + \frac{4}{x+2} = -1$
6. Solve for x : $\frac{x}{1-x} + \frac{3}{1-x} = -5$
7. Solve for x : $\frac{7}{x+3} - \frac{x}{x+3} = -3$
8. Solve for x : $\frac{4}{x-2} - \frac{x}{x-2} = -3$
9. Solve for x : $\frac{x-1}{4} + \frac{x}{3} = -2$
10. Solve for x : $\frac{x+2}{6} - \frac{x}{2} = 5$
11. Solve for x : $\frac{x+2}{3} + \frac{3x}{4} = 5$
12. Solve for x : $\frac{x-1}{5} + \frac{x}{3} = \frac{3x+2}{15}$
13. Solve for x : $\frac{x+3}{6} - \frac{x}{5} = \frac{9x-5}{30}$
14. Solve for x : $\frac{x+3}{5} + \frac{x}{4} = \frac{8x+4}{20}$
15. Solve for x : $\frac{4}{x} + \frac{2}{x+1} = 5$
16. Solve for x : $\frac{3}{x-2} - \frac{1}{x} = -4$
17. Solve for x : $\frac{2}{x} - \frac{1}{x-3} = 2$
18. Solve for x : $\frac{x}{5} + \frac{15}{3x} = \frac{x+3}{4}$
19. Solve for x : $\frac{24}{8x} - \frac{x}{3} = \frac{3-x}{6}$
20. Solve for x : $\frac{x}{6} + \frac{12}{2x} = \frac{x-2}{4}$
21. Solve for x : $\frac{x+1}{5} - \frac{3}{x} = \frac{x-2}{x}$
22. Solve for x : $\frac{3}{x} - \frac{x-2}{3} = \frac{x+1}{2x}$
23. Solve for x : $\frac{x-2}{7} - \frac{5}{x} = \frac{x+5}{x}$
24. Solve for x : $\frac{2}{3(x+4)} - \frac{4}{3} = \frac{2}{x}$
25. Solve for x : $\frac{6}{4(x-2)} + \frac{1}{4} = \frac{4}{x}$
26. Solve for x : $\frac{8}{5(x-3)} + \frac{1}{5} = \frac{5}{x}$
27. Solve for x : $5 - \frac{3}{x+4} = \frac{5x+20}{x+4}$
28. Solve for x : $\frac{2}{x-2} + 3 = \frac{3x-6}{x-2}$

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Solve $\frac{2}{3x} - \frac{1}{x} = \frac{1}{15}$ for x . Is the solution extraneous?
2. Solve $\frac{4y}{y+3} - \frac{1}{2} = \frac{9}{y+3}$ for y . Is the solution extraneous?
3. The volume V of a right circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius, and h is the height. Solve this formula for h .
6. Solve $\frac{x}{x+5} - \frac{1}{5} = \frac{3}{x+5}$ for x . Is the solution extraneous?
7. The surface area, S , of a right circular cylinder is $S = 2\pi rh + 2\pi r^2$, where r is the radius, and h is the height. Solve this formula for h .



4. Solve this proportion for x : $\frac{5x-8}{2x+1} = \frac{4}{3}$
5. Solve $\frac{5}{4y} - \frac{2}{2y} = \frac{1}{16}$ for y . Is the solution extraneous?
8. Solve this proportion for y : $\frac{6y-4}{6y+6} = \frac{4}{9}$