

Section R.2 – Linear and Rational Equations

Definition of a Linear Equation

A linear equation in one variable x is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real number, and $a \neq 0$

Addition and Multiplication Properties of Equalities

$$\text{If } a = b, \text{ then } a + c = b + c$$

$$\text{If } a = b, \text{ then } ac = bc$$

Example

$$\text{Solve: } 3(2x - 4) = 7 - (x + 5)$$

Solution

$$6x - 12 = 7 - x - 5$$

$$6x - 12 + x = 2 - x + x$$

$$7x - 12 = 2$$

$$7x - 12 + 12 = 2 + 12$$

$$7x = 14$$

$$\frac{7}{7}x = \frac{14}{7}$$

$$x = 2$$

Example

$$\text{Solve: } \frac{2x+4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$$

Solution

$$(12)\frac{2x+4}{3} + (12)\frac{1}{2}x = (12)\frac{1}{4}x - (12)\frac{7}{3}$$

$$4(2x+4) + 6x = 3x - 28$$

$$8x + 16 + 6x = 3x - 28$$

$$14x + 16 = 3x - 28$$

$$14x - 3x = -28 - 16$$

$$11x = -44$$

$$x = -4$$

Example

Solve: $\frac{5}{2x} = \frac{17}{18} - \frac{1}{3x}$

Solution

$$(18x) \frac{5}{2x} = (18x) \frac{17}{18} - (18x) \frac{1}{3x}$$

Restriction: $x \neq 0$

$$45 = 17x - 6$$

$$45 + 6 = 17x$$

$$17x = 51$$

$$\underline{x = 3}$$

Example

Solve: $\frac{x}{x-2} = \frac{2}{x-2} - \frac{2}{3}$

Solution

$$3(x-2) \frac{x}{x-2} = 3(x-2) \frac{2}{x-2} - 3(x-2) \frac{2}{3}$$

Restriction: $x \neq 2$

$$3x = 6 - 2(x-2)$$

$$3x = 6 - 2x + 4$$

$$3x + 2x = 10$$

$$\Rightarrow 5x = 10$$

$$\Rightarrow x = 2$$

No Solution **or** $\{\emptyset\}$

Identities, Conditional Equations, and Contradictions

Example

Solve: $-2(x + 4) + 3x = x - 8$

Solution

$$-2(x + 4) + 3x = x - 8$$

$$-2x - 8 + 3x = x - 8$$

$$x - 8 = x - 8$$

$$0 = 0 \quad \text{True statement}$$

Solution: All real numbers

Example

Solve: $5x - 4 = 11$

Solution

$$5x - 4 = 11$$

$$5x = 15$$

$$x = 3$$

This is a conditional equation, and its solution set is $\{3\}$

Example

Solve: $3(3x - 1) = 9x + 7$

Solution

$$3(3x - 1) = 9x + 7$$

$$9x - 3 = 9x + 7$$

$$-3 = 7 \quad \text{False statement}$$

This is a contradiction equation, and its solution set is empty set $\{\emptyset\}$ or null

Solving for a Specified Variable

Example

Solve

a) $I = Prt$ for t

$$\frac{I}{Pr} = \frac{Pr}{Pr} t$$

$$\frac{I}{Pr} = t$$

b) $A - P = Prt$ for P

$$A = Prt + P$$

$$A = P(rt + 1)$$

$$\frac{A}{rt + 1} = P \quad \text{or} \quad P = \frac{A}{rt + 1}$$

c) $3(2x - 5a) + 4b = 4x - 2$ for x

$$6x - 15a + 4b = 4x - 2$$

$$6x - 4x = 15a - 4b - 2$$

$$2x = 15a - 4b - 2$$

$$x = \frac{15a - 4b - 2}{2}$$

Example

Solve the formula $2l + 2w = P$ for w

Solution

$$2w = P - 2l$$

$$w = \frac{P - 2l}{2}$$

Example

Solve the formula $P = C + MC$ for C

Solution

$$P = C(1 + M)$$

$$\frac{P}{1 + M} = C$$

$$C = \frac{P}{1 + M}$$

Exercises Section R.2 – Linear and Rational Equations

(1 – 78) Solve

1. $5x - 8 = 72$
2. $14 - 5x = -41$
3. $2x + 6 = 3x - 2$
4. $11x - (6x - 5) = 40$
5. $9x + 11 = 7x + 1$
6. $2x - 7 = 6 + x$
7. $5x - 2 = 9x + 2$
8. $3(x - 2) + 7 = 2(x + 5)$
9. $3x + 5 - 5(x + 1) = 6x + 7$
10. $4(-2x + 1) = 6 - (2x - 4)$
11. $4(x + 7) = 2(x + 12) + 2(x + 1)$
12. $6(3x - 1) = 8 - 10(10x - 14)$
13. $5x - (2x - 8) = 35$
14. $\frac{1}{14}(3x - 2) = \frac{x + 10}{10}$
15. $\frac{5}{6}x - 2x + \frac{4}{3} = \frac{5}{3}$
16. $\frac{7}{4} + \frac{1}{5}x - \frac{3}{2} = \frac{4}{5}x$
17. $5(x + 3) + 4x - 3 = -(2x - 4) + 2$
18. $2[x - (4 + 2x) + 3] = 2x + 3$
19. $2x - \{x - [3x - (8x + 6)]\} = 2x - 2$
20. $4(2x + 7) = 2x + 22 + 3(2x + 3)$
21. $4[2x - (3 - x) + 5] = -7x - 2$
22. $3[2x - (4 - x) + 5] = 7x - 2$
23. $-4(2x - 6) + 8x = 5x + 24 + x$
24. $-8(3x + 4) + 6x = 4(x - 8) + 4x$
25. $4(x + 7) = 2(x + 12) + 2(x + 1)$
26. $-6(2x + 1) - 3(x - 4) = -15x + 1$
27. $2(x - 1) + 3 = x - 3(x + 1)$
28. $3(x - 4) - 4(x - 3) = x + 3 - (x - 2)$
29. $2 - (7x + 5) = 13 - 3x$
30. $16 = 3(x - 1) - (x - 7)$
31. $5x - 2(x + 1) = x + (3x - 5)$
32. $7(x + 1) = 4[x - (3 - x)]$
33. $2[3x - 2(2x - 3)] = 5(x - 6)$
34. $.2x - .5 = .1x + 7$
35. $.01x + 3.1 = 2.03x - 2.96$
36. $.08x - .06(x + 12) = 7.72$
37. $.04(x - 12) + .06x = 1.52$
38. $.3(x + 2) - .5(x + 2) = -.2x - .4$
39. $.6(x - 5) + .8(x - 6) = .2x - 1.8$
40. $.5x + \frac{4}{3}x = x + 10$
41. $.25x + \frac{2}{3}x = x + 2$
42. $\frac{1}{4}(x - 2) = \frac{1}{6}(x - 5)$
43. $\frac{1}{4}(3x - 2) = \frac{1}{5}(x + 5)$
44. $\frac{1}{9}(x + 2) = \frac{1}{15}(2x + 5)$
45. $\frac{1}{2}(4x + 8) - 16 = -\frac{2}{3}(9x - 12)$
46. $\frac{3}{4}(24 - 8x) - 16 = -\frac{2}{3}(6x - 9)$
47. $\frac{x - 3}{4} = \frac{5}{14} - \frac{x + 5}{7}$
48. $\frac{x + 1}{4} = \frac{1}{6} + \frac{2 - x}{3}$
49. $\frac{x - 8}{3} + \frac{x - 3}{2} = 0$
50. $\frac{5}{2x} - \frac{8}{9} = \frac{1}{18} - \frac{1}{3x}$
51. $\frac{1}{x + 4} + \frac{1}{x - 4} = \frac{22}{x^2 - 16}$
52. $\frac{3x - 1}{3} - \frac{2x}{x - 1} = x$
53. $\frac{x}{x - 2} = \frac{2}{x - 2} + 2$
54. $\frac{x}{x - 7} = \frac{7}{x - 7} + 8$
55. $\frac{3x}{5} - x = \frac{x}{10} - \frac{5}{2}$
56. $2x - \frac{2x}{7} = \frac{x}{2} + \frac{17}{2}$
57. $\frac{x + 3}{6} = \frac{2}{3} + \frac{x - 5}{4}$
58. $\frac{x + 1}{4} = \frac{1}{6} + \frac{2 - x}{3}$

$$59. \frac{x}{4} = 2 + \frac{x-3}{3}$$

$$60. 5 + \frac{x-2}{3} = \frac{x+3}{8}$$

$$61. \frac{x+1}{3} = 5 - \frac{x+2}{7}$$

$$62. \frac{3x}{5} - \frac{x-3}{2} = \frac{x+2}{3}$$

$$63. \frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x^2-2x}$$

$$64. \frac{-4x}{x-1} + \frac{4}{x+1} = \frac{-8}{x^2-1}$$

$$65. \frac{4x+3}{x+1} + \frac{2}{x} = \frac{1}{x^2+x}$$

$$66. \frac{6}{x+3} - \frac{5}{x-2} = \frac{-20}{x^2+x-6}$$

$$67. \frac{6}{x+1} - \frac{5}{x+2} = \frac{10}{x^2+3x+2}$$

$$68. 3(x-4) - 5(x+2) = 3[2 - (x+24)] - 2(x-2)$$

$$69. (2x+3)(6x-1) - 9 = 15x^2 - (3x-2)(x-2)$$

$$70. (3x-1)^2 - 2x(x-1) = 7x^2 - 5x + 2$$

$$71. (2x+3)(x-1) + (x+1)(x-4) = 3x^2$$

$$72. 4x+13 - \{2x - [4(x-3) - 5]\} = 2(x-6)$$

$$73. -2\{7 - [4 - 2(1-x) + 3]\} = 10 - [4x - 2(x-3)]$$

$$74. 2(y+2) + (y+3)^2 = y(y+5) + 2\left(\frac{17}{2} + y\right)$$

$$75. (y+1)(y-1) = (y+2)(y-3) + 4$$

$$76. 45 - [4 - 2y - 4(y+7)] = -4(1+3y) - [4 - 3(y+2) - 2(2y-5)]$$

$$77. 35 - [2 - 3y - 4(y+7)] = -3(1+3y) + 4 - 3(y+2) - 2(2y-5)$$

$$78. 25 - [2 + 5y - 3(y+2)] = -3(2y-5) - [5(y-1) - 3y + 3]$$

(79 – 100) Solve for the specific variable

$$79. V = lwh, \text{ for } h$$

$$80. A = \frac{1}{2}h(B+b), \text{ for } B$$

$$81. A = \frac{1}{2}h(a+b), \text{ for } a$$

$$82. S = 2\pi rh + 2\pi r^2 \text{ for } h$$

$$83. A = \frac{1}{2}h(b_1 + b_2), \text{ for } h$$

$$84. A = \frac{1}{2}h(b_1 + b_2), \text{ for } b_2$$

$$86. S = P + Prt \text{ for } t$$

$$87. S = 2lw + 2wh + 2hl \text{ for } h$$

$$88. S = 2lw + 2wh + 2hl \text{ for } w$$

$$89. \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \text{ for } R_1$$

$$90. \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \text{ for } R$$

$$91. V = \frac{d_1 - d_2}{t} \text{ for } d_1$$

$$92. V = \frac{d_1 - d_2}{t} \text{ for } d_2$$

$$85. \quad A = \frac{1}{2}h(b_1 + b_2), \text{ for } b_1$$

$$93. \quad z = \frac{x - \mu}{s} \text{ for } x$$

$$94. \quad z = \frac{x - \mu}{s} \text{ for } \mu$$

$$95. \quad s = \frac{1}{2}at^2 + vt \text{ for } v$$

$$96. \quad s = \frac{1}{2}at^2 + vt \text{ for } a$$

$$97. \quad L = a + (n - 1)d \text{ for } n$$

$$98. \quad L = a + (n - 1)d \text{ for } d$$

$$99. \quad A = \frac{x_1 + x_2 + x_3}{n} \text{ for } x_2$$

$$100. \quad A = \frac{x_1 + x_2 + x_3}{n} \text{ for } n$$