

Section 4.3 – Integration by Parts

Integration by Parts

Example: $\int x^2 e^x dx \quad \int x \ln x dx$

Let u and v be differentiable functions of x .

$$\int u dv = uv - \int v du$$

Guidelines for integration by Parts

1. Let dv be the most complicated portion of the integrand that fits a basic integration formula. Let u be the remaining factor.
2. Let u be the portion of the integrand whose derivative is a function simpler than u . Let dv be the remaining factor.

Example

Find: $\int x e^{2x} dx$

Solution

Let: $u = x \Rightarrow du = dx$

$$dv = e^{2x} dx \Rightarrow v = \int dv = \int e^{2x} dx = \frac{1}{2} e^{2x}$$

$$\int u dv = uv - \int v du$$

$$\begin{aligned} \int x e^{2x} dx &= \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} dx \\ &= \frac{1}{2} x e^{2x} - \frac{1}{2} \frac{1}{2} e^{2x} + C \\ &= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C \end{aligned}$$

Example

Find : $\int x \ln x dx$

Solution

Let: $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$$dv = x dx \Rightarrow v = \int dv = \int x dx = \frac{1}{2} x^2$$

$$\int x \ln x dx = \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x^2 \frac{dx}{x}$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

Example

Differentiate $y = x \ln x - x + C$ to show that it is the Antiderivative of $\ln x$.

Solution

$$\frac{dy}{dx} = \ln x + x \frac{1}{x} - 1$$

$$= \ln x + 1 - 1$$

$$= \ln x$$

Integration by Parts Repeatedly

Example

Find : $\int x^3 e^x dx$

Solution

$$\text{Let: } u = x^3 \Rightarrow du = 3x^2 dx$$

$$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$\int x^3 e^x dx = x^3 e^x - \int e^x 3x^2 dx$$

$$= x^3 e^x - 3 \int e^x x^2 dx$$

$$\text{Let: } u = x^2 \Rightarrow du = 2x dx$$

$$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$\int e^x x^2 dx = x^2 e^x - 2 \int x e^x dx$$

$$\int x^3 e^x dx = x^3 e^x - 3 \left[x^2 e^x - 2 \int x e^x dx \right]$$

$$= x^3 e^x - 3x^2 e^x + 6 \int x e^x dx$$

$$\text{Let: } u = x \Rightarrow du = dx$$

$$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x$$

$$\int x^3 e^x dx = x^3 e^x - 3x^2 e^x + 6 \left[x e^x - e^x \right] + C$$

$$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$$

$$= e^x (x^3 - 3x^2 + 6x - 6) + C$$

Exercises ***Section 4.3 – Integration by Parts***

Find each integral

1. $\int \ln x^2 dx$

2. $\int \frac{2x}{e^x} dx$

3. $\int \ln(3x) dx$

4. $\int \frac{1}{x \ln x} dx$

5. $\int \frac{x}{\sqrt{x-1}} dx$

6. $\int \frac{x^3 e^{x^2}}{(x^2+1)^2} dx$

7. $\int x^2 e^{-3x} dx$