$$\int_0^{\pi} (\cos t\,\hat{i} + \hat{j} - 2t\hat{k})\,dt = \sin t\,\hat{i} + t\hat{j} - t^2\hat{k}\Big|_0^{\pi}$$

$$= 0 - 0\,\hat{i}$$

$$= \boxed{\pi\hat{j} - \pi^2\hat{k}}$$

Ex $\vec{a}(t) = -3\cos t\,\hat{i} - 3\sin t\,\hat{j} + 2k$

Given $\begin{cases} t=0 \rightarrow (3,0,0) = \vec{r}(0) \\ \vec{v}(0) = 3\hat{j} \end{cases}$

Position as fcln of t $\Rightarrow \vec{r}(t)$?

Soln

$$\vec{v}(t) = \int \vec{a}(t)\,dt$$

$$= \int (-3\cos t\,\hat{i} - 3\sin t\,\hat{j} + 2\hat{k})\,dt$$

$$= -3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 2t\hat{k} + \vec{C}_1$$

$$v(0) = 3\hat{j} = 3\hat{j} + \vec{C}_1$$

$$\vec{C}_1 = \vec{0}$$

$$\boxed{\vec{v}(t) = -3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 2t\hat{k}}$$

$$\vec{r}(t) = \int \vec{v}(t)$$

$$= \int (-3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 2t\hat{k})\,dt$$

$$= 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + t^2\hat{k} + \vec{C}_2$$

$$\underbrace{\vec{r}(0) = 3\hat{i}}_{\text{Given}} = 3\hat{i} + \vec{C}_2$$

$$\vec{C}_2 = \vec{0}$$

$$\boxed{r(t) = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + t^2\hat{k}}$$

$$\vec{r}(t) = \underbrace{(v_0 \cos\alpha)t}_{x(t)}\,\hat{i} + \underbrace{\left((v_0 \sin\alpha)t - \tfrac{1}{2}gt^2\right)}_{y(t)}\hat{j}$$

$$y' = v_0 \sin\alpha - gt = 0 \Rightarrow t = \frac{v_0 \sin\alpha}{g}$$

Ex Given $\vec{r}(0) = 3\hat{j}$ $\quad |v_0| = 152 \quad \alpha = 20^\circ$

$\vec{w}(t) = -8.8\hat{i}$

a) $\vec{r}(t) = (152\cos 20^\circ t - 8.8t)\hat{i} + (-16t^2 + (152\sin 20^\circ)t + 3)\hat{j}$

$\approx 134.033\,t\,\hat{i} + (-16t^2 + 51.987t + 3)\hat{j}$

b) $t_{max} = \dfrac{152\sin 20^\circ}{32.2} \approx 1.62\text{ sec}$

$y_{max} = \dfrac{(152\sin 20^\circ)^2}{64.4} + 3 \approx 45.2\text{ ft}$

c) Not caught

$y(t) = -16t^2 + 152\sin 20^\circ t + 3 = 0$

$-16t^2 + 51.987t + 3 = 0$

$t = 3.3\text{ sec} \qquad \cancel{t = -.06}$

$r(3.3) = 134.033\,(3.3)$

$\approx 442\text{ ft}$

1.7

$$L = \int_a^b |\vec{v}|\, dt$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$= \int_a^b \sqrt{x'^2 + y'^2 + z'^2}\, dt$$

Ex $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$ Helix

$t = 0 \to 2\pi$

$$L = \int_0^{2\pi} \overbrace{\sqrt{(-\sin t)^2 + (\cos t)^2 + 1}}^{\sqrt{2}}\, dt \qquad \int_0^{2\pi} dt$$

$$\cancel{= \sqrt{2}\; t \Big|_0^{2\pi}}$$

$$\boxed{= 2\pi\sqrt{2} \text{ unit}}$$

Unit Tangent vector $\vec{T} = \dfrac{\vec{v}}{|\vec{v}|}$

Ex $\vec{r}(t) = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + t^2\hat{k}$

Soln $\vec{T}(t)$?

$$\vec{v}(t) = r'(t)$$
$$= -3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 2t\,\hat{k}$$
$$|\vec{v}(t)| = \sqrt{9\sin^2 t + 9\cos^2 t + 4t^2}$$
$$= \sqrt{9+4t^2}$$
$$\vec{T}(t) = \frac{-3\sin t}{\sqrt{9+4t^2}}\,\hat{i} + \frac{3\cos t}{\sqrt{9+4t^2}}\,\hat{j} + \frac{2t}{\sqrt{9+4t^2}}\,\hat{k}$$
$$= \frac{1}{\sqrt{9+4t^2}}\left(-3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 2t\,\hat{k}\right)$$

Curvature $\kappa = \dfrac{1}{v}\left|\dfrac{d\vec{T}}{dt}\right|$

Ex $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j}$

$\vec{v}(t) = -a\sin t\,\hat{i} + a\cos t\,\hat{j}$

$|\vec{v}| = \sqrt{a^2\sin^2 t + a^2\cos^2 t}$

$= a$

$\vec{T}(t) = \dfrac{\vec{v}(t)}{|\vec{v}|} = -\sin t\,\hat{i} + \cos t\,\hat{j}$

$\dfrac{d\vec{T}}{dt} = -\cos t\,\hat{i} - \sin t\,\hat{j}$

$\left|\dfrac{d\vec{T}}{dt}\right| = \sqrt{\cos^2 t + \sin^2 t}$

$= 1$

$\kappa = \dfrac{1}{a} = \dfrac{1}{\text{radius}}$

Principal Unit normal

$$\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|}$$

Ex $\vec{r}(t) = (\cos 2t)\,\hat{i} + \sin 2t\,\hat{j}$ $\quad \vec{T}?\ \vec{N}?$

$\vec{v}(t) = -2\sin 2t\,\hat{i} + 2\cos 2t\,\hat{j}$

$|\vec{v}(t)| = \sqrt{4\sin^2 2t + 4\cos^2 2t}$

$= 2$

$\boxed{\vec{T} = -\sin 2t\,\hat{i} + \cos 2t\,\hat{j}}$

$\dfrac{d\vec{T}}{dt} = -2\cos 2t\,\hat{i} - 2\sin 2t\,\hat{j}$

$\left|\dfrac{d\vec{T}}{dt}\right| = \sqrt{4\cos^2 2t + 4\sin^2 2t}$

$= 2$

$\boxed{\vec{N} = -\cos 2t\,\hat{i} - \sin 2t\,\hat{j}}$