$x^2 + y^2 = a^2$ circle $(0,0)$ center, radius a.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipse

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Hyperbolas.

$d^2 = a^2 + b^2$

$a^2x^2 + b^2y^2 = d^2$

$\frac{x^2}{b^2} \pm \frac{y^2}{a^2} = 1$

Ex $\frac{9x^2}{36} - \frac{4y^2}{36} = \frac{36}{36}$

$\frac{x^2}{4} - \frac{y^2}{9} = 1$

$a^2 = 4 \Rightarrow a = \pm 2$

$b^2 = 9 \Rightarrow b = \pm 3$

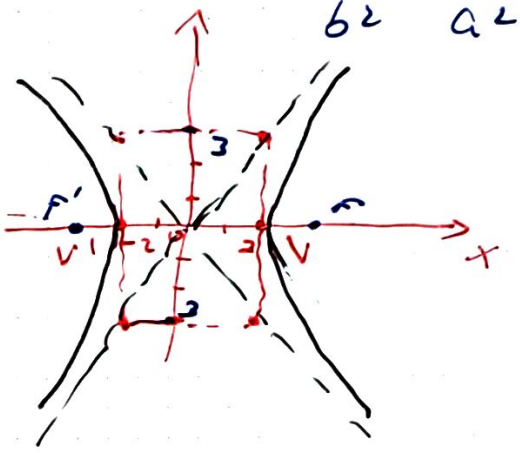
$c^2 = 13$

vertices: $V(\pm 2, 0)$

endpoints: $W(0, \pm 3)$

foci: $F(\pm\sqrt{13}, 0)$

$c^2 = a^2 + b^2$ (9+4)
$= 13$



$y = \pm\frac{3}{2}x$

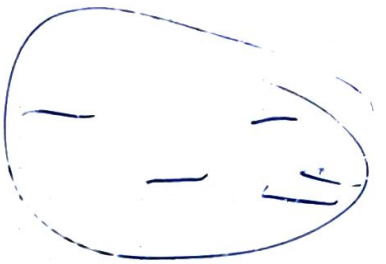
(-) $\frac{x^2}{4} - \frac{y^2}{9} = 0$

$\frac{y^2}{9} = \frac{x^2}{4}$

$y^2 = \frac{9}{4}x^2$

$y = \pm\frac{3}{2}x$ Asymptote

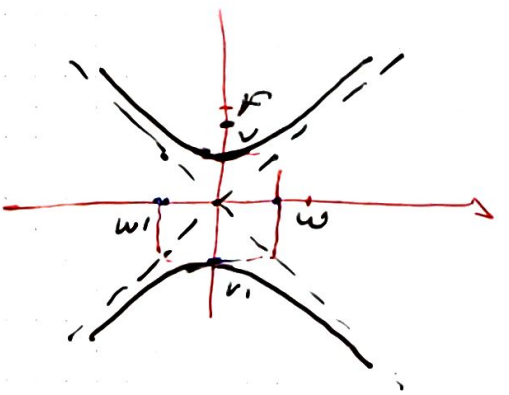
Ex $4y^2 - 2x^2 = 1$



$$\frac{y^2}{\frac{1}{4}} - \frac{x^2}{\frac{1}{2}} = 1$$

$a^2 = \frac{1}{4} \Rightarrow a = \pm \frac{1}{2}$

$b^2 = \frac{1}{2} \quad b = \pm \frac{1}{\sqrt{2}}$



Vertices: $V\left(0, \pm \frac{1}{2}\right)$

endpoints: $W\left(\pm \frac{1}{\sqrt{2}}, 0\right)$

$c^2 = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

foci: $F\left(0, \pm \frac{\sqrt{3}}{2}\right)$

$4y^2 = 2x^2 \qquad y^2 = \frac{2}{4}x^2$

$y^2 = \frac{1}{2}x^2$

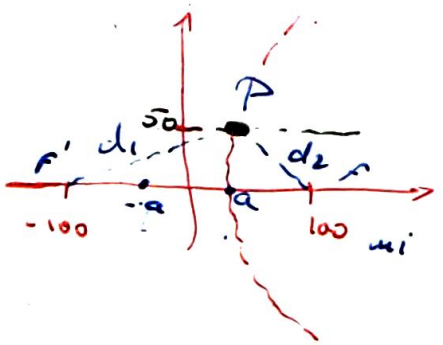
$y = \pm \frac{1}{\sqrt{2}}x$

Ex Given

$v = 980 \frac{ft}{\mu sec}$

$t = 400\ \mu sec$

? (x, y) ?



$v = \frac{mi}{hr} \frac{d}{t}$

$d_2 - d_1 = vt$

$= 980 \times 400 \quad ft$

$= 392\ 10^3 \frac{1}{5280}$

$= \frac{196}{264} 10^2$

$d_2 - d_1 = 2a = \frac{49}{66} 10^2$

$a = \frac{4900}{2(66)} = \frac{2450}{66} \approx 37.12$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{1378} - \frac{y^2}{b^2} = 1$

• $P(x, 50)$

$c = 100$

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$= (100)^2 - (37.12)^2$

$= 8,622$

$\frac{x^2}{1,378} - \frac{y^2}{8,622} = 1$

$\frac{x^2}{1,378} = 1 + \frac{50^2}{8,622}$

$\frac{a}{b} + 1 = \frac{}{b}$

$x^2 = 1378\left(1 + \frac{2020}{8622}\right)$

$\frac{11122}{8622}$

$x \approx 42.16$

$P(42, 50)$

27/ $625y^2 - 400x^2 = 250{,}000$

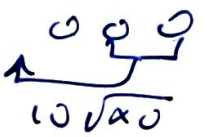
$$\frac{y^2}{400} - \frac{x^2}{625} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$a^2 = 400$

$a = \sqrt{400}$

$= 20$



distance between 2 building is $2a = 40$ yards. yd

5.5 Infinite Sequences

$a_1, a_2, a_3, \ldots, a_n, \ldots$

$a_n = \frac{n}{n+1}$

Ex 1st 4 terms & 10th term a_{10} $\left\{\frac{n}{n+1}\right\}$ $f(x) = \frac{x}{x+1}$

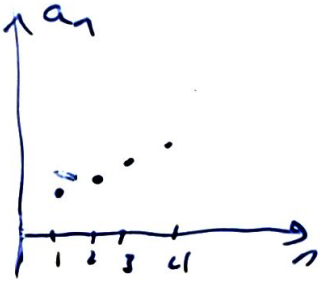
~~a_1~~: $n=1 \to \frac{1}{1+1} = \frac{1}{2}$

$n=2 \to \frac{2}{2+1} = \frac{2}{3}$

$n=3 \to a_3 = \frac{3}{3+1} = \frac{3}{4}$

$n=4 \to a_4 = \frac{4}{4+1} = \frac{4}{5}$

$n=10 \to a_{10} = \frac{10}{10+1} = \frac{10}{11}$



Ex 1st 4, a_{10}? $\{2 + (.1)^n\}$

$n=1 \to a_1 = 2 + .1 = 2.1$

$n=2 \to a_2 = 2 + (.1)^2 = 2 + .01 = 2.01$

$n=3 \Rightarrow a_3 = 2 + (.1)^3 = 2 + .001 = 2.001$

$n=4 \Rightarrow a_4 = 2 + (.1)^4 = 2 + .0001 = 2.0001$

$n=10 \Rightarrow a_{10} = 2.0000000001$

Ex 1st 4, a_{10} $\left\{(-1)^{n+1}\frac{n^2}{3n-1}\right\}$

$n=1 \Rightarrow a_1 = (-1)^2 \frac{1}{3-1} = \frac{1}{2}$

$n=2 \Rightarrow a_2 = (-1)^3 \frac{4}{6-1} = -\frac{4}{5}$

$n=3 \Rightarrow a_3 = (-1)^4 \frac{9}{9-1} = \frac{9}{8}$

$n=4 \Rightarrow a_4 = (-1)^5 \frac{16}{12-1} = -\frac{16}{11}$

$n=10 \Rightarrow a_{10} = (-1)^{11} \frac{100}{30-1} = -\frac{100}{29}$

Ex 1st 4, a_{10} } 4 }

$n=1$ $a_1 = 4$

$n=2 \rightarrow 4$

$n=3 \rightarrow 4$

$n=4 \rightarrow 4$

$n=10 \rightarrow 4$

Ex 1st 4: $a_1 = 3$ $a_{n+1} = (n+1)\,a_n$

$n=0 \rightarrow a_1 = 3$

$n=1 \rightarrow a_2 = 2\,a_1 = 2(3) = 6$

$n=2 \rightarrow a_3 = 3\,a_2 = 3(6) = 18$

$n=3 \rightarrow a_4 = 4\,a_3 = 4(18) = 72$

#27 $a_1 = 2$ $a_2 = 2$ $a_n = a_{n-1} \cdot a_{n-2}$

1st 5.

$a_1 = 2$

$a_2 = 2$

$a_3 = a_2 \cdot a_1 = 2(2) = 4$

$a_4 = a_3 \cdot a_2 = 4(2) = 8$

$a_5 = a_4 \cdot a_3 = 8(4) = 32$

#11 $\{c_n\} = \left\{ \frac{(-1)^n}{(n+1)(n+2)} \right\}$ 1st 4? c_8?

$c_1 = \frac{(-1)^1}{2(3)} = -\frac{1}{6}$

$c_2 = \frac{(-1)^2}{3(4)} = \frac{1}{12}$

$c_3 = \frac{(-1)^3}{4(5)} = -\frac{1}{20}$

$c_4 = \frac{(-1)^4}{(5)(6)} = \frac{1}{30}$

$c_{10} = \frac{(-1)^{10}}{(11)(12)} = \frac{1}{132}$

$$\begin{array}{r} 12 \\ \times\ 11 \\ \hline 132 \end{array} \qquad \begin{array}{r} 56 \\ \times\ 11 \\ \hline 616 \end{array}$$

Summation Notation

$$\sum_{n=1}^{5} a_n$$

sigma ← $\sum$; 5 ← last value ; a_n ← formula ; $n=1$ ← initial

Ex $\displaystyle\sum_{k=1}^{4} k^2(k-3) = \underbrace{1^2(1-3)}_{k=1} + \overbrace{2^2(2-3)}^{k=2} + \underbrace{3^2(3-3)}_{k=3} + \overbrace{4^2(4-3)}^{k=4}$

$= -2 - 4 + 0 + 16$

$= 10$

$$\sum_{k=1}^{n} c = nc \qquad \sum_{k=m}^{n} c = (n-m+1)c$$

$$\sum_{k=10}^{20} 5 = \overbrace{5(20-10+1)}^{=11} = 55$$

$$2^1 + 2^2 + 2^3 + \cdots + 2^{16} = \sum_{k=1}^{16} 2^k$$

$$1 \quad 3 \quad 5 \qquad \sum_{1}^{3}$$

#38 $\sum_{k=1}^{5}(2k-7) = (2-7)+(4-7)+(6-7)+(8-7)+(10-7)$

$= -5-3-1+1+3$

$= -5$

$$\sum_{k=1}^{5}(2k-7) = 2\sum_{k=1}^{5}k - \sum_{k=1}^{5}7$$

$= 2(1+2+3+4+5) - 7(5)$

$= 30 - 35$

$= -7$

#42 $\sum_{k=1}^{50} 8 = 8(50)$

$= 400$

572
253

#41 $\sum_{k=253}^{571}\left(\frac{1}{3}\right) = \frac{1}{3}(571-253+1)$

$= \frac{1}{3}(319)$

$= \frac{319}{3}$

#21 $a_1 = \sqrt{2}$ $\quad a_n = \sqrt{2+a_{n-1}}$

$a_1 = \sqrt{2}$

$a_2 = \sqrt{2+a_1} = \sqrt{2+\sqrt{2}}$

$a_3 = \sqrt{2+a_2} = \sqrt{2+\sqrt{2+\sqrt{2}}}$

$a_4 = \sqrt{2+a_3} = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$

#37 $\frac{1}{e} + \frac{2}{e^2} + \frac{3}{e^3} + \cdots + \frac{n}{e^n} = \sum_{k=1}^{n}\frac{k}{e^k}$

H 5.2 due Tomorrow
5.4

$\left\{ \frac{2^n}{n^2+2} \right\}$ 1st 4.

$n=1 \rightarrow \frac{2}{1+2} = \frac{2}{3}$

$n=2 \rightarrow \frac{2^2}{4+2} = \frac{4}{6} = \frac{2}{3}$

$n=3 \rightarrow \frac{2^3}{9+2} = \frac{8}{11}$

$n=4 \rightarrow \frac{2^4}{16+2} = \frac{16}{18} = \frac{8}{9}$

Sequences { Arithmetic, Geometric — d difference, r ratio

Arithmetic

$a_{k+1} = a_k + d \rightarrow$ common difference

$d = a_{k+1} - a_k$

Ex 1, 4, 7, 10, ..., $3n-2$, ...

$d = 4-1 = 3$ a_n

$a_{k+1} - a_k = 3(k+1) - 2 - [3k-2]$

$= 3k + 3 - 2 - 3k + 2$

$= 3$ ✓

1, 4, 7, 10, ... ? $a_n =$