Solution *Section* 6.4 – Solving Right Triangle Trigonometry

Exercise

In the right triangle ABC, $a = 29.43$ and $c = 53.58$. Find the remaining side and angles.

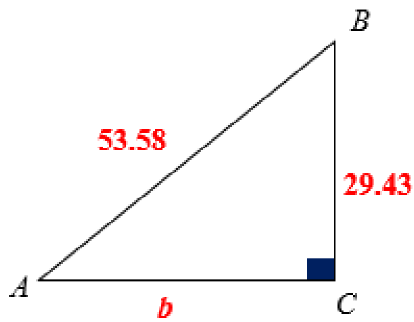
Solution

$$c^2 = a^2 + b^2$$

$$b^2 = c^2 - a^2$$

$$b = \sqrt{53.58^2 - 29.43^2}$$

$$\approx 44.77$$



$$\sin A = \frac{29.43}{53.58}$$

$$A = \sin^{-1}\left(\frac{29.43}{53.58}\right)$$

$$\approx 33.32^\circ$$

$$B = 90^\circ - A$$

$$= 90^\circ - 33.32^\circ$$

$$\approx 56.68^\circ$$

Exercise

In the right triangle ABC, $a = 2.73$ and $b = 3.41$. Find the remaining side and angles.

Solution

$$c^2 = a^2 + b^2$$

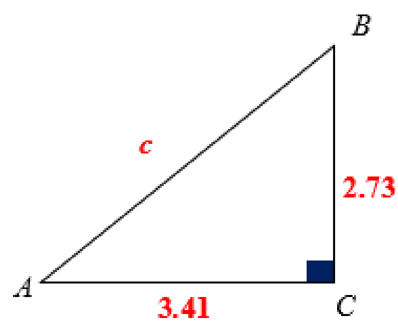
$$c = \sqrt{2.73^2 + 3.41^2} = 4.37$$

$$\tan A = \frac{a}{b} \qquad \textbf{\textit{and}} \qquad \sin A = \frac{a}{c}$$

$$= \frac{2.73}{3.41} \qquad\qquad = \frac{2.73}{4.37}$$

$$A = \tan^{-1}\left(\frac{2.73}{3.41}\right) \qquad A = \sin^{-1}\left(\frac{2.73}{4.37}\right)$$

$$= 38.7^\circ \qquad\qquad \approx 38.7^\circ$$



$$B = 90^\circ - A$$

$$= 90^\circ - 38.7^\circ$$

$$\approx 51.3^\circ$$

Exercise

The two equal sides of an isosceles triangle are each 24 *cm*. If each of the two equal angles measures 52°, find the length of the base and the altitude.

Solution

$$\sin 52° = \frac{x}{24}$$

$$x = 24\sin 52°$$

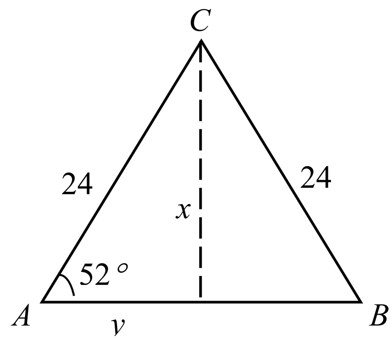
$$\underline{x \approx 19\ cm}$$

$$\cos 52° = \frac{y}{24}$$

$$y = 24\cos 52°$$

$$\underline{y \approx 15\ cm}$$

$$\Rightarrow\ \underline{AB = 2y \approx 30\ cm}$$



Exercise

The distance from A to D is 32 *feet*. Use the figure to solve x, the distance between D and C.

Solution

Triangle ***DCB***

$$\tan 54° = \frac{h}{x}$$

$$\to\ h = x\tan 54°$$

Triangle ***ACB***

$$\tan 38° = \frac{h}{x+32}$$

$$\to\ h = (x+32)\tan 38°$$

$$h = x\tan 54° = (x+32)\tan 38°$$

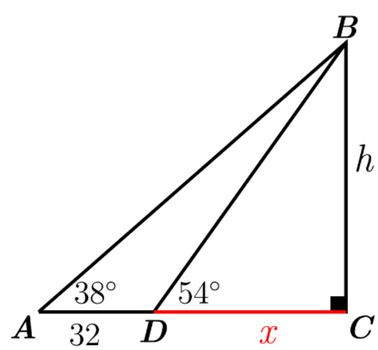
$$x\tan 54° = x\tan 38° + 32\tan 38°$$

$$x\tan 54° - x\tan 38° = 32\tan 38°$$

$$x(\tan 54° - \tan 38°) = 32\tan 38°$$

$$x = \frac{32\tan 38°}{\tan 54° - \tan 38°}$$

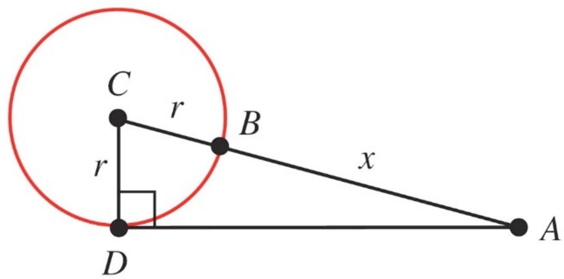
$$\underline{= 42\ ft}$$



Exercise

If C = 26° and r = 19, find x.

Solution

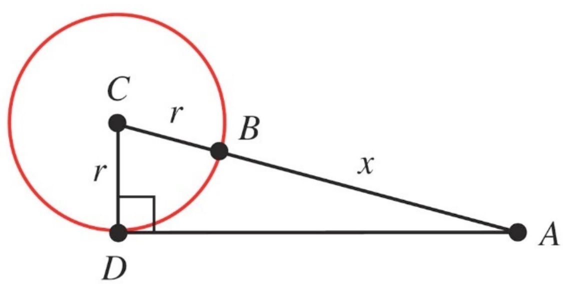


$$\cos 26° = \frac{r}{r+x}$$
$$= \frac{19}{19+x}$$
$$(19+x)\cos 26° = 19$$
$$19\cos 26° + x\cos 26° = 19$$
$$x\cos 26° = 19 - 19\cos 26°$$
$$x = \frac{19-19\cos 26°}{\cos 26°}$$
$$\approx 2.14$$

Exercise

If C = 30° and r = 15, find x.

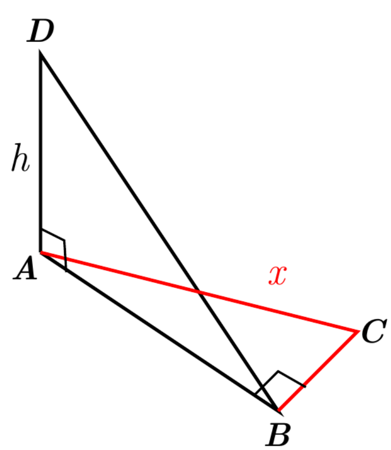
Solution



$$\cos 30° = \frac{r}{r+x}$$
$$= \frac{15}{15+x}$$
$$(15+x)\frac{\sqrt{3}}{2} = 15$$
$$15 + x = \frac{30}{\sqrt{3}}$$
$$x = 10\sqrt{3} - 15$$

Exercise

If ∠ABD = 53°, C = 48°, and BC = 42, find x and then find h.



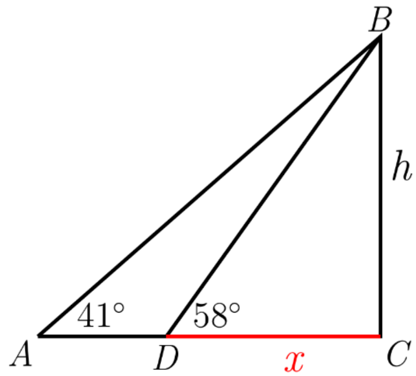
Solution

$$\tan 48° = \frac{x}{42}$$
$$x = 42\tan 48°$$
$$= 46.65 \approx 47$$
$$\tan 53° = \frac{h}{x}$$
$$h = 47\tan 53°$$
$$\approx 62$$

Exercise

If $A = 41°$, $\angle BDC = 58°$, and $AB = 28$, find $\boldsymbol{h}$, then $\boldsymbol{x}$.

Solution



$$\sin 41° = \frac{h}{AB}$$

$$h = 28 \sin 41°$$

$$\approx 18$$

$$\tan 58° = \frac{h}{x}$$

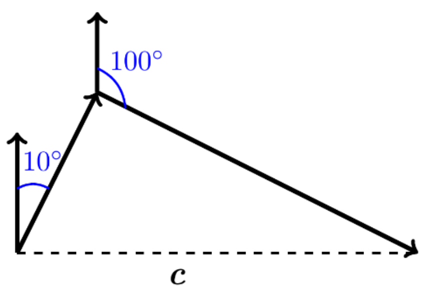
$$x = \frac{18}{\tan 58°}$$

$$\approx 11$$

Exercise

A plane flies 1.7 *hours* at 120 *mph* on a bearing of 10°. It then turns and flies 9.6 *hours* at the same speed on a bearing of 100°. How far is the plane from its starting point?

Solution



$$b = 120 \frac{mi}{hr} 1.7 hrs$$

$$= 204 \ mi$$

$$a = 120 \frac{mi}{hr} 9.6 hrs$$

$$= 1152 \ mi$$

The triangle is right triangle.

$$c = \sqrt{a^2 + b^2}$$

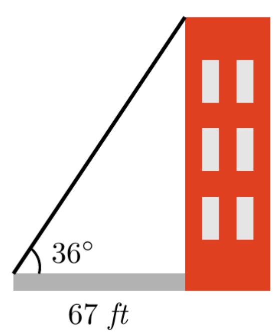
$$= \sqrt{1152^2 + 204^2}$$

$$\approx 1170 \ mi$$

Exercise

The shadow of a vertical tower is 67.0 *feet* long when the angle of elevation of the sun is 36.0°. Find the height of the tower.

Solution



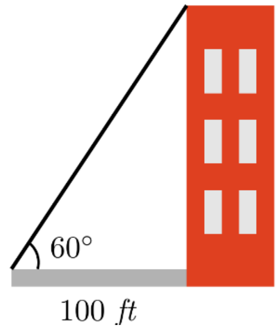
$$\tan 36° = \frac{h}{67}$$

$$h = 67 \tan 36°$$

$$\approx 48.7\ ft$$

Exercise

The shadow of a vertical tower is 100 *feet* long when the angle of elevation of the sun is 60°. Find the height of the tower.



Solution

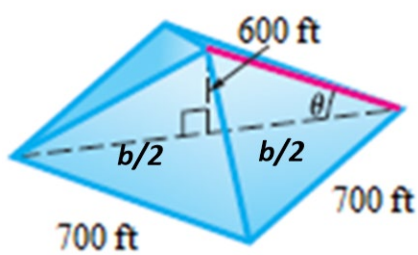
$$\tan 60° = \frac{h}{100}$$

$$h = 100 \tan 60°$$

$$= 100\sqrt{3}\ ft$$

Exercise

The base of a pyramid is square with sides 700 *feet.* long, and the height of the pyramid is 600 *feet*. Find the angle of elevation of the edge indicated in the figure to two significant digits. (Hint: The base of the triangle in the figure is half the diagonal of the square base of the pyramid.)



Solution

$$b^2 = 700^2 + 700^2$$

$$b = \sqrt{2\left(700^2\right)}$$

$$= 700\sqrt{2}$$

$$\tan\theta = \frac{600}{b/2}$$

$$= \frac{600}{\frac{700\sqrt{2}}{2}}$$

$$= 600\frac{2}{700\sqrt{2}}$$

$$= \frac{6\sqrt{2}}{7}$$

$$\theta = \tan^{-1}\left(\frac{6\sqrt{2}}{7}\right)$$

$$\approx 50.48°$$

Exercise

If a 73-*foot* flagpole casts a shadow 51 *feet* long, what is the angle of elevation of the sun (to the nearest tenth of a degree)?

Solution

$$\tan\theta = \frac{73}{51}$$

$$\theta = \tan^{-1}\left(\frac{73}{51}\right)$$

$$\approx 55.1^\circ$$

Exercise

If a 75-*foot* flagpole casts a shadow 43 *feet* long, to the nearest 10 *minutes* what is the angle of elevation of the sum from the tip of the shadow?

Solution

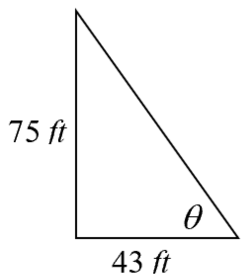
$$\tan\theta = \frac{75}{43}$$

$$\theta = \tan^{-1}\left(\frac{75}{43}\right)$$

$$= 60.17^\circ$$

$$= 60^\circ\ \ 0.17^\circ\left(\frac{60'}{1^\circ}\right)$$

$$\theta = 60^\circ\ \ 10'$$



Exercise

Suppose each edge of the cube is 3.00 *inches* long. Find the measure of the angle formed by diagonals DE and DG. *Round your answer to the nearest tenth of a degree.*

Solution

$$|DG| = \sqrt{3^2+3^2}$$

$$= 3\sqrt{2}$$

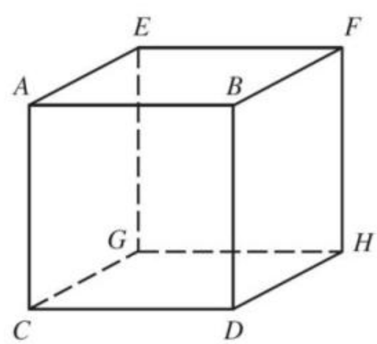
$$\tan(EDG) = \frac{EG}{GD}$$

$$= \frac{3}{3\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$EDG = \tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

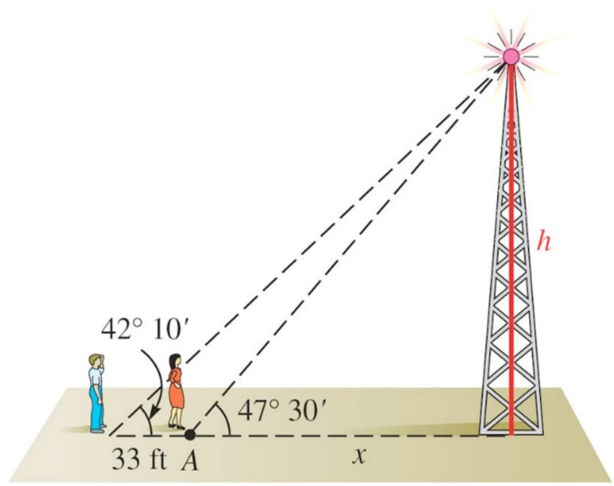
$$\angle EDG = 45^\circ$$



Exercise

A person standing at point ***A*** notices that the angle of elevation to the top of the antenna is $47^\circ\ 30'$. A second person standing 33.0 *feet* farther from the antenna than the person at ***A*** finds the angle of elevation to the top of the antenna to be $42^\circ\ 10'$. How far is the person at ***A*** from the base of the antenna?

Solution



$$47^\circ\ 30' = 47 + 30\frac{1}{60}$$

$$= 47.5^\circ$$

$$\tan 47.5^\circ = \frac{h}{x}$$

$$\Rightarrow h = x \tan 47.5^\circ \quad (1)$$

$$42^\circ\ 10' = 42 + 10\frac{1}{60} = 42.167^\circ$$

$$\tan 42.167^\circ = \frac{h}{33+x}$$

$$\Rightarrow h = (33+x)\tan 42.167^\circ \quad (2)$$

$$h = (33+x)\tan 42.167^\circ = x\tan 47.5^\circ$$

$$33\tan 42.167^\circ + x\tan 42.167^\circ = x\tan 47.5^\circ$$

$$33\tan 42.167^\circ = x\tan 47.5^\circ - x\tan 42.167^\circ$$

$$x = \frac{33\tan 42.167^\circ}{\tan 47.5^\circ - \tan 42.167^\circ}$$

$$= 162\ ft$$

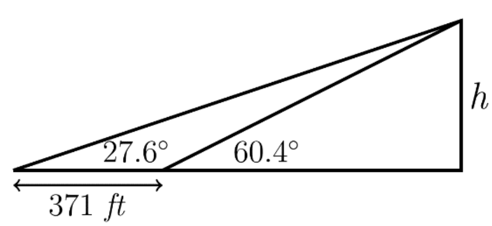
Exercise

Find ***h*** as indicated in the figure.

Solution

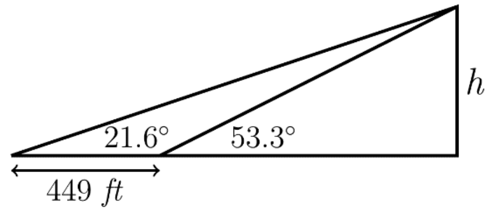
$$h = \frac{371\tan 27.6^\circ \tan 60.4^\circ}{\tan 60.4^\circ - \tan 27.6^\circ} \qquad h = \frac{x\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

$$\approx 276\ ft$$



Exercise

Find ***h*** as indicated in the figure.



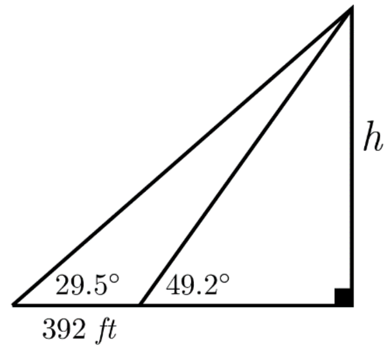
Solution

$$h = \frac{449\tan 21.6^\circ \tan 53.5^\circ}{\tan 53.5^\circ - \tan 21.6^\circ} \qquad h = \frac{x\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

$$\approx 252 \ \ ft$$

Exercise

Find h as indicated in the figure.



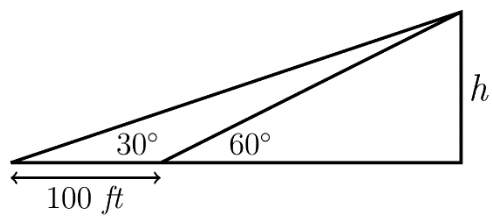
Solution

$$h = \frac{392\tan 29.5^\circ \ \tan 49.2^\circ}{\tan 49.2^\circ - \tan 29.5^\circ} \qquad h = \frac{x\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

$$\approx 433.5 \ \ ft$$

Exercise

Find h as indicated in the figure.



Solution

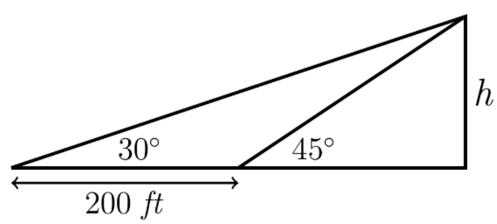
$$h = \frac{100\tan 60^\circ \ \tan 30^\circ}{\tan 60^\circ - \tan 30^\circ} \qquad h = \frac{x\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

$$= \frac{100\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$= 50\sqrt{3} \ \ ft$$

Exercise

Find h as indicated in the figure.



Solution

$$h = \frac{200\tan 45^\circ \ \tan 30^\circ}{\tan 45^\circ - \tan 30^\circ} \qquad h = \frac{x\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

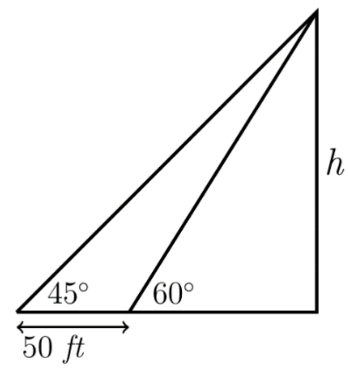
$$= \frac{100\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}}\right)}{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}}$$

$$= \frac{100}{\sqrt{3} - \sqrt{2}}$$

$$= 100\left(\sqrt{3} - \sqrt{2}\right)\ ft$$

Exercise

Find h as indicated in the figure.



Solution

$$h = \frac{50 \tan 60^\circ \ \tan 45^\circ}{\tan 60^\circ - \tan 45^\circ} \qquad h = \frac{x \tan\alpha \tan\beta}{\tan\beta - \tan\alpha}$$

$$= \frac{50\sqrt{3}\left(\frac{\sqrt{2}}{2}\right)}{\sqrt{3} - \frac{\sqrt{2}}{2}}$$

$$= \frac{50\sqrt{6}}{2\sqrt{3} - \sqrt{2}}$$

$$= \frac{50\sqrt{6}}{12-2}\left(2\sqrt{3} + \sqrt{2}\right)$$

$$= 25\left(2\sqrt{18} + \sqrt{12}\right)$$

$$= 25\left(6\sqrt{2} + 2\sqrt{3}\right)$$

$$= 50\left(3\sqrt{2} + \sqrt{3}\right)\ ft$$

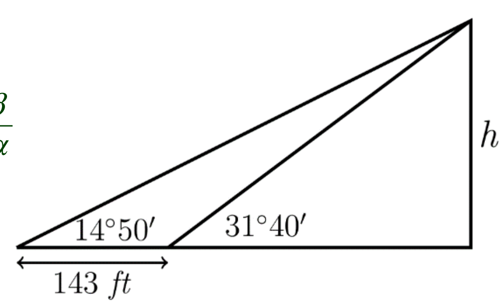
Exercise

The angle of elevation from a point on the ground to the top of a pyramid is 31° 40′. The angle of elevation from a point 143 *ft* farther back to the top of the pyramid is 14° 50′. Find the height of the pyramid.

Solution

$$h = \frac{143 \tan 14.833^\circ \tan 31.667^\circ}{\tan 31.667^\circ - \tan 14.833^\circ} \qquad h = \frac{x \tan\alpha \tan\beta}{\tan\beta - \tan\alpha}$$

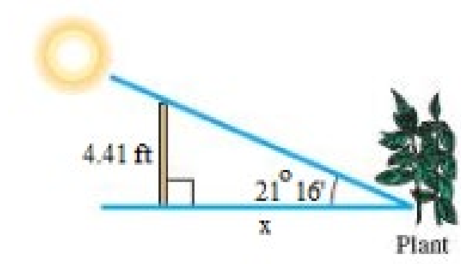
$$\approx 66\ ft$$



Exercise

In one area, the lowest angle of elevation of the sun in winter is 21° 16′. Find the minimum distance, ***x***, that a plant needing full sun can be placed from a fence 4.41 *feet.* high.

Solution



$$\tan(21°16') = \frac{4.41}{x}$$

$$x = \frac{4.41}{\tan\left(21° + \frac{16°}{60}\right)}$$

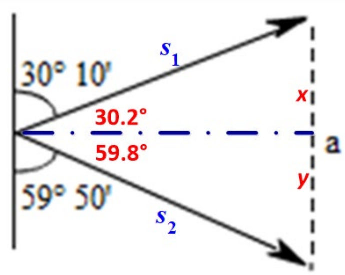
$$\approx 11.33\, ft$$

Exercise

A ship leaves its port and sails on a bearing of N 30° 10′ E, at speed 29.4 *mph*. Another ship leaves the same port at the same time and sails on a bearing of S 59° 50′ E, at speed 17.1 *mph*. Find the distance between the two ships after 2 *hrs*.

Solution

$$\begin{cases} 30°10' = 30° + \frac{10°}{60} \approx 30.16667° \\ 59°50' = 59° + \frac{50°}{60} \approx 59.8333° \end{cases}$$



After 2 *hours*:

$$\begin{cases} s_1 = 29.4 \frac{mi}{hr}.(2)\, hr = 58.8 \\ s_2 = 17.1 \frac{mi}{hr}.(2)\, hr = 34.2 \end{cases}$$

$$\begin{cases} \tan 30.2° = \frac{x}{s_1} \Rightarrow x = 58.8 \tan 30.2° \\ \tan 59.8° = \frac{y}{s_2} \Rightarrow y = 34.2 \tan 59.8° \end{cases}$$

$$a = x + y$$

$$= 58.8 \tan 30.2° + 34.2 \tan 59.8°$$

$$\approx 93 \text{ miles}$$

Exercise

Radar stations A and B are on the east-west line, 3.7 *km* apart. Station A detects a place at C, on a bearing of 61°. Station B simultaneously detects the same plane, on a bearing of 331°. Find the distance from A to C.

Solution

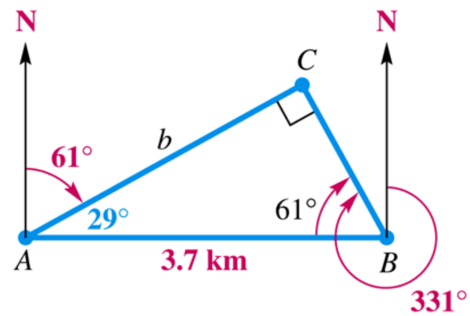
$A = 90° - 61°$

$\underline{= 29°}$

$\cos 29° = \dfrac{b}{3.7}$

$b = 3.7 \cos 29°$

$\underline{\approx 3.2\ km}$



Exercise

Suppose the figure below is exaggerated diagram of a plane flying above the earth. If the plane is 4.55 *miles* above the earth and the radius of the earth is 3,960 *miles*, how far is it from the plane to the horizon? What is the measure of angle A?

Solution

$x^2 + 3960^2 = 3964.55^2$

$x^2 = 3964.55^2 - 3960^2$

$x = \sqrt{3964.55^2 - 3960^2}$

$\underline{\approx 190}$

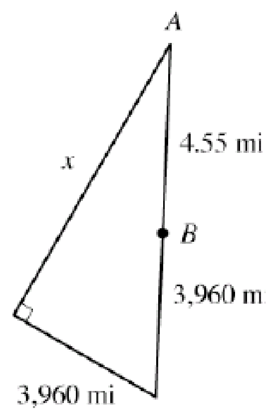
The plane is 190 *miles* from the horizon.

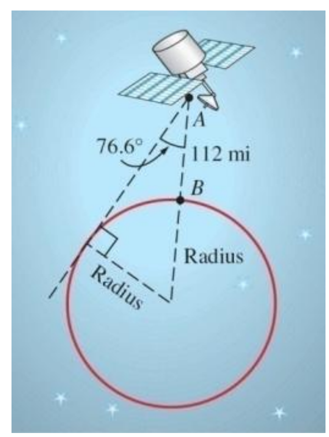
$\sin A = \dfrac{3960}{3964.55}$

≈ 0.9989

$A = \sin^{-1}(0.9989)$

$\underline{\approx 87.3°}$





Exercise

The Ferry wheel has a 250 *feet* diameter and 14 *feet* above the ground. If θ is the central angle formed as a rider moves from position P_0 to position P_1, find the rider's height above the ground h when θ is 45°.

Solution

Distance between O *and* $P_0 = radius = \dfrac{250}{2} = 125\,ft$

$$\cos\theta = \frac{OP}{OP_1}$$

$$\cos 45° = \frac{OP}{125}$$

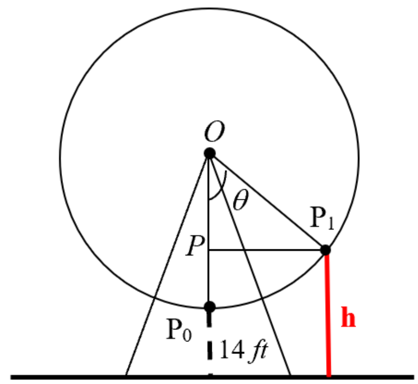
$$OP = 125\cos 45°$$

$$h = PP_0 + 14$$

$$= OP_0 - OP + 14$$

$$= 125 - 125\cos 45° + 14$$

$$\boxed{\approx 51\ ft}$$



Exercise

The length of the shadow of a building 34.09 *m* tall is 37.62 *m*. Find the angle of the elevation of the sun.

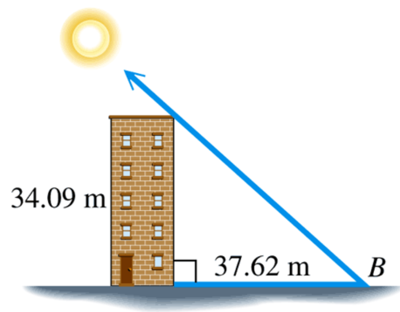
Solution

$$\tan B = \frac{34.09}{37.62}$$

$$B = \tan^{-1}\left(\frac{34.09}{37.62}\right)$$

$$\boxed{\approx 42.18°}$$

$\therefore$ The angle of elevation is $\approx 42.18°$



Exercise

The length of the shadow of a building 34.09 *m* tall is 37.62 *m*.
Find the angle of the elevation of the sun.

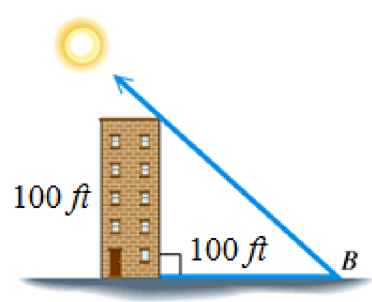
Solution

$$\tan B = \frac{100}{100}$$

$$B = \tan^{-1}(1)$$

$$\boxed{= 45°}$$

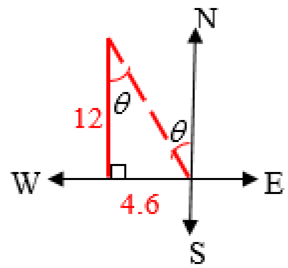
$\therefore$ The angle of elevation is $45°$



Exercise

San Luis Obispo, California is 12 *miles* due north of Grover Beach. If Arroyo Grande is 4.6 *miles* due east of Grover Beach, what is the bearing of San Luis Obispo from Arroyo Grande?

Solution



$$\tan\theta = \frac{4.6}{12} = 0.3833$$

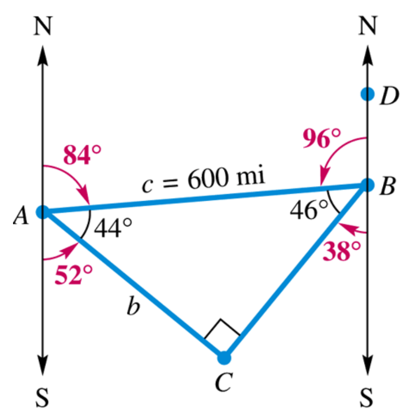
$$\theta = \tan^{-1} 0.3833 = 21°$$

The bearing of San Luis Obispo from Arroyo Grande is N 21° W

Exercise

The bearing from A to C is S 52° E. The bearing from A to B is N 84° E. The bearing from B to C is S 38° W. A plane flying at 250 *mph* takes 2.4 hours to go from A to B. Find the distance from A to C.

Solution



$$\angle ABD = 180° - 84° = 96°$$

$$\angle ABC = 180° - (96° + 38°) = 46°$$

$$\angle C = 180° - (46° + 44°) = 90°$$

$$c = rate \times time = 250(2.4) = 600\ mi.$$

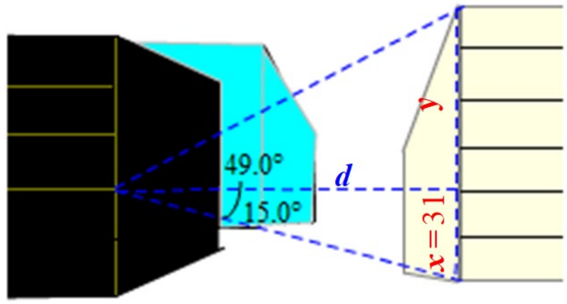
$$\sin 46° = \frac{b}{c} = \frac{b}{600}$$

$$b = 600 \sin 46° \approx 430\ \ mi$$

Exercise

From a window 31.0 *feet.* above the street, the angle of elevation to the top of the building across the street is 49.0° and the angle of depression to the base of this building is 15.0°. Find the height of the building across the street.

Solution



$$\tan 15° = \frac{31}{d}$$

$$d = \frac{31}{\tan 15°}$$

$$\tan 49° = \frac{y}{d}$$

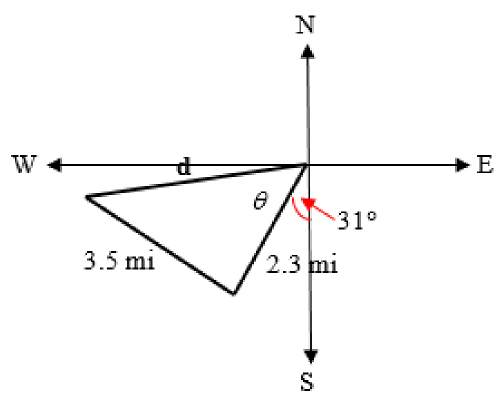
$$y = \frac{31}{\tan 15°}\tan 49°$$

$$\begin{aligned} h &= x + y \\ &= 31 + \frac{31}{\tan 15°}\tan 49° \\ &= 164\ ft \end{aligned}$$

Exercise

A man wondering in the desert walks 2.3 *miles* in the direction $S\,31°\ W$. He then turns 90° and walks 3.5 *miles* in the direction $N\,59°\ W$. At that time, how far is he from his starting point, and what is his bearing from his starting point?

Solution



$$d = \sqrt{2.3^2 + 3.5^2} = 4.2$$

$$\cos\theta = \frac{2.3}{4.2} = .55$$

$$\theta = \cos^{-1} 0.55 \approx 57°$$

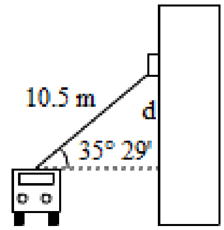
S (57°+31°) W

$\rightarrow$ Bearing $S\ 88°\ W$

Exercise

A 10.5-*m* fire truck ladder is leaning against a wall. Find the distance ***d*** the ladder goes up the wall (above the fire truck) if the ladder makes an angle of 35° 29′ with the horizontal.

Solution



$$\sin\left(35°29'\right) = \frac{d}{10.5}$$

$$d = 10.5\sin\left(35° + \frac{29°}{60}\right)$$

$$= 6.1\ m$$

Exercise

A basic curve connecting two straight sections of road is often circular. In the figure, the points ***P*** and ***S*** mark the beginning and end of the curve. Let ***Q*** be the point of intersection where the two straight sections of highway leading into the curve would meet if extended. The radius of the curve is ***R***, and the central angle denotes how many degrees the curve turns.

a) If ***R*** = 965 ft. and ***θ***= 37°, find the distance d between P and ***Q***.

b) Find an expression in terms of ***R*** and ***θ*** for the distance between points ***M*** and ***N***.

Solution

a) $\sin\frac{\theta}{2} = \frac{|PN|}{R}$

$$|PN| = 965\sin\left(\frac{37°}{2}\right)$$

$$\approx 306.2$$

$$\angle CPN = 90° - \frac{\theta}{2}$$

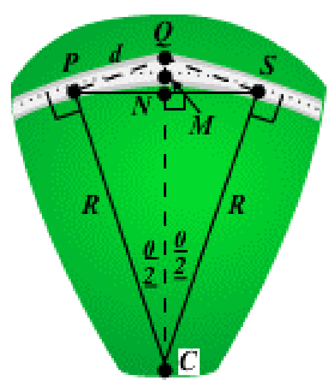
$$= 71.5°$$

$$\angle NPQ = 90° - \angle CPN$$

$$= 90° - 71.5°$$

$$= 18.5°$$

$$= \frac{\theta}{2}$$



$$\cos(NPQ) = \frac{|PN|}{d}$$

$$d = \frac{|PN|}{\cos 18.5°}$$

$$= \frac{306.2}{\cos 18.5°}$$

$$\approx 322.9$$

b) $\cos\frac{\theta}{2} = \frac{|CN|}{R}$

$|CN| = R\cos\frac{\theta}{2}$

$R = |CQ| = |CM| + 2|NM|$

$2|NM| = R - |CM|$

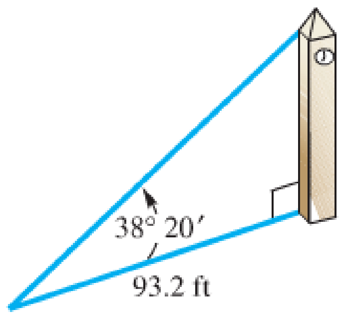
$2|NM| = R - R\cos\frac{\theta}{2}$

$$\boxed{|NM| = \frac{1}{2}R\left(1-\cos\frac{\theta}{2}\right)}$$

Exercise

The angle of elevation from a point 93.2 *feet* from the base of a tower to the top of the tower is 38° 20′. Find the height of the tower.

Solution



$\tan(38°\ 20') = \frac{h}{93.2}$

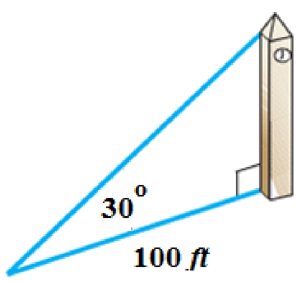
$h = 93.2\tan(38°\ 20')$

$\boxed{\approx 73.7\ ft}$

Exercise

The angle of elevation from a point 100 *feet* from the base of a tower to the top of the tower is 30°. Find the height of the tower.

Solution



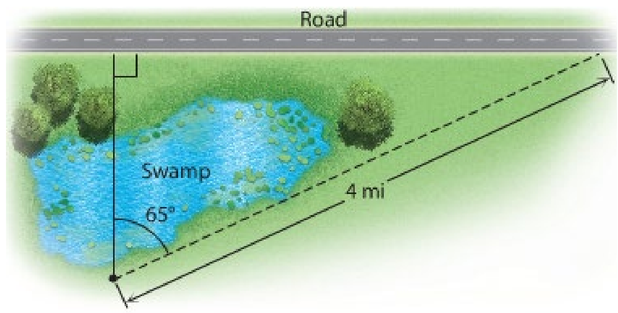
$\tan(30°) = \frac{h}{100}$

$h = 100\tan(30°)$

$\boxed{= \frac{100}{\sqrt{3}}\ ft}$

Exercise

Jane was hiking directly toward a long straight road when she encountered a swamp. She turned 65° to the right and hiked 4 *mi* in that direction to reach the road. How far was she forms the road when she encountered the swamp?



Solution

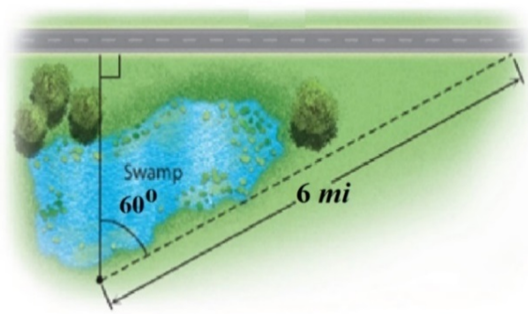
$$\cos 65° = \frac{d}{4}$$

$$d = 4\cos 65°$$

$$\approx 1.7 \ \ miles$$

Exercise

You were hiking directly toward a long straight road when you encountered a swamp. you turned 60° to the right and hiked 6 *mi* in that direction to reach the road. How far were you from the road when you encountered the swamp?



Solution

$$\cos 60° = \frac{d}{2}$$

$$d = 6\left(\frac{1}{2}\right)$$

$$= 3 \ \ miles$$

Exercise

From a highway overpass, 14.3 *m* above the road, the angle of depression of an oncoming car is measured at 18.3°. How far is the car from a point on the highway directly below the observer?

Solution

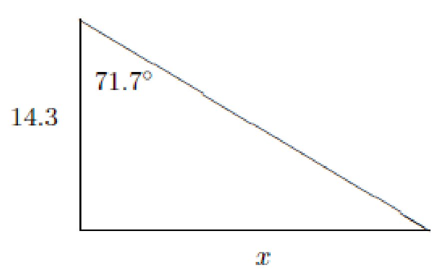
$$\alpha = 90° - 18.3°$$

$$= 71.7°$$

$$\tan(71.7°) = \frac{x}{14.3}$$

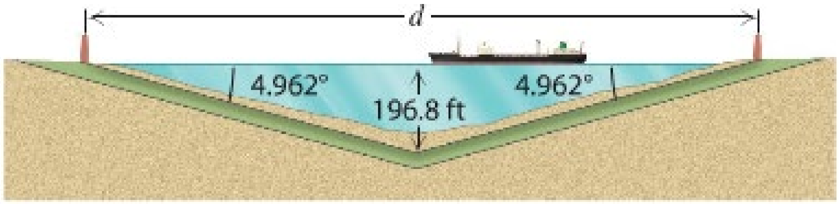
$$x = 14.3\tan(71.7°)$$

$$\approx 43.2 \ \ m$$



Exercise

A tunnel under a river is 196.8 *ft*. below the surface at its lowest point. If the angle of depression of the tunnel is 4.962° , then how far apart on the surface are the entrances to the tunnel? How long is the tunnel?



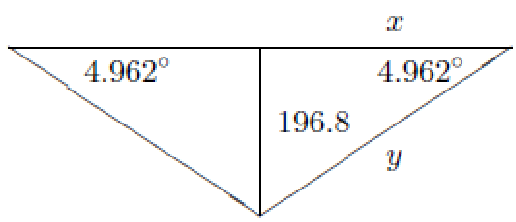
Solution

$$\tan 4.962° = \frac{196.8}{x}$$

$$x = \frac{196.8}{\tan 4.962°}$$

$$\approx 2266.75$$

$$\underline{|d} = 2x \ \underline{= 4533\, ft|}$$



$$\sin 4.962° = \frac{196.8}{y}$$

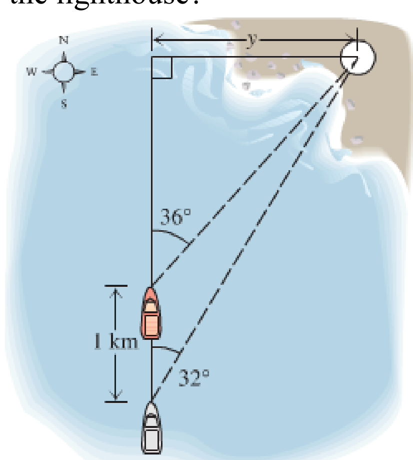
$$y = \frac{196.8}{\sin 4.962°}$$

$$\approx 2275.3$$

$\therefore$ The tunnel length: $2y = 4551 \ \ feet$

Exercise

A boat sailing north sights a lighthouse to the east at an angle of 32° from the north. After the boat travels one more *kilometer*, the angle of the lighthouse from the north is 36°. If the boat continues to sail north, then how close will the boat come to the lighthouse?

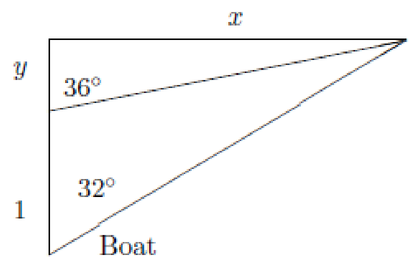


Solution

$$\tan 36° = \frac{x}{y} \Rightarrow x = y\tan 36°$$

$$\tan 32° = \frac{x}{y+1} \Rightarrow x = (y+1)\tan 32°$$

$$x = y\tan 36° = (y+1)\tan 32°$$



$$y\tan 36° = y\tan 32° + \tan 32°$$

$$y\tan 36° - y\tan 32° = \tan 32°$$

$$y(\tan 36° - \tan 32°) = \tan 32°$$

$$y = \frac{\tan 32°}{\tan 36° - \tan 32°}$$

$$\Rightarrow x = y\tan 36°$$

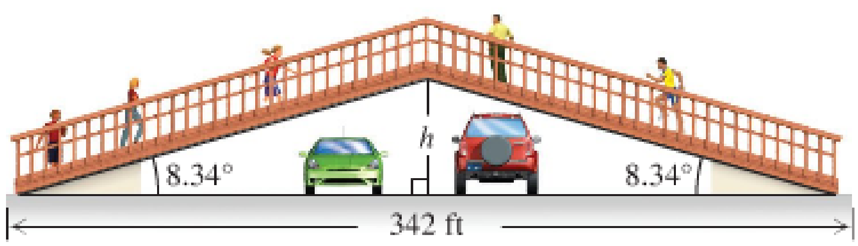
$$= \frac{\tan 32°}{\tan 36° - \tan 32°}\tan 36°$$

$$\approx 4.5\ km$$

∴ The closest will the boat come to the lighthouse is 4.5 *km*.

Exercise

The angle of elevation of a pedestrian crosswalk over a busy highway is 8.34°, as shown in the drawing. If the distance between the ends of the crosswalk measured on the ground is 342 *feet*., then what is the height h of the crosswalk at the center?



Solution

$$\frac{342}{2} = 171$$

$$\tan(8.34°) = \frac{h}{171}$$

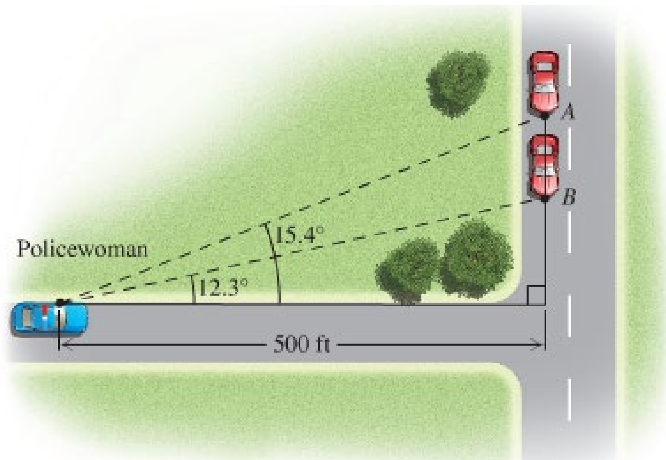
$$h = 171\tan 8.34°$$

$$\approx 25.1\ ft$$

Exercise

A policewoman has positioned herself 500 *feet*. from the intersection of two roads. She has carefully measured the angles of the lines of sight to points *A* and *B*. If a car passes from *A* to *B* is 1.75 *sec* and the speed limit is 55 *mph*, is the car speeding? (Hint: Find the distance from *B* to *A* and use *R* = *D*/*T*)

Solution



$$\tan 12.3^\circ = \frac{b}{500}$$

$$b = 500\tan 12.3^\circ$$

$$\tan 15.4^\circ = \frac{b+a}{500}$$

$$b + a = 500\tan 15.4^\circ$$

$$\begin{aligned} a &= 500\tan 15.4^\circ - b \\ &= 500\tan 15.4^\circ - 500\tan 12.3^\circ \\ &= 28.7\ ft\ \frac{1\ mi}{5280\ ft} \\ &\approx 0.0054356\ mi \end{aligned}$$

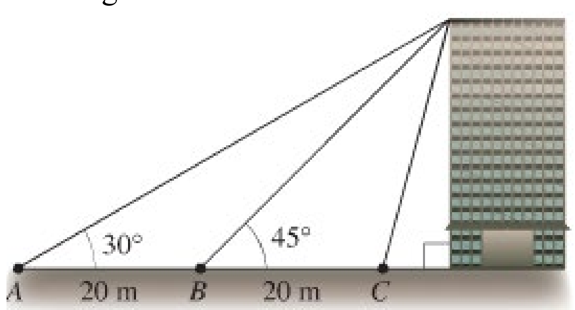
The speed is:

$$0.0054356\ mi\ \frac{1}{1.75\ \sec}\ \frac{3600\ \sec}{1\ hr} = 11.2\ \ mph$$

∴ The car is ***not*** speeding.

Exercise

From point *A* the angle of elevation to the top of the building is 30°. From point *B*, 20 meters closer to the building, the angle of elevation is 45°. Find the angle of elevation of the building from point *C*, which is another 20 *meters* closer to the building.



Solution

Let x be the distance between C and the building.

$$\tan 30^\circ = \frac{h}{40+x}$$

$$\begin{aligned} h &= (40+x)\tan 30^\circ \\ &= (40+x)\frac{1}{\sqrt{3}} \end{aligned}$$

$$\tan 45° = \frac{h}{20+x}$$

$$h = (20+x)\tan 45°$$

$$= 20 + x$$

$$\Rightarrow h = \frac{1}{\sqrt{3}}(40+x) = 20 + x$$

$$40 + x = 20\sqrt{3} + x\sqrt{3}$$

$$x - x\sqrt{3} = 20\sqrt{3} - 40$$

$$x\left(1-\sqrt{3}\right) = 20\sqrt{3} - 40$$

$$x = \frac{20\sqrt{3}-40}{1-\sqrt{3}} \approx 7.32$$

$$\Rightarrow h = (40+7.32)\frac{1}{\sqrt{3}} \approx 27.32$$

$$\tan C = \frac{h}{x} = \frac{27.32}{7.32}$$

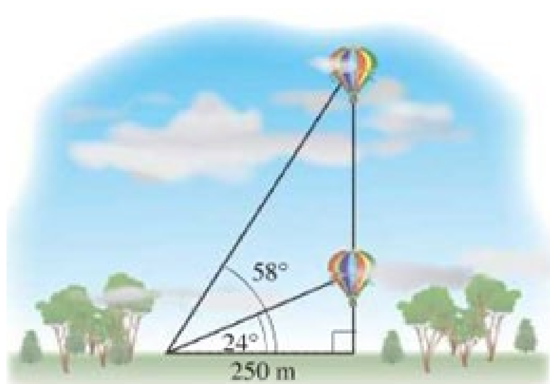
$$C = \tan^{-1}\left(\frac{27.32}{7.32}\right)$$

$$\approx 75°$$

Exercise

A hot air balloon is rising upward from the earth at a constant rate. An observer 250 *m* away spots the balloon at an angle of elevation of 24°. Two minutes later the angle of elevation of the balloon is 58°. At what rate is the balloon ascending?

Solution



$$\tan 24° = \frac{h_1}{250}$$

$$h_1 = 250\tan 24°$$

$$\tan 58° = \frac{h_2}{250}$$

$$h_2 = 250\tan 58°$$

It took 2 minutes to get from h_1 *to* h_2

$$rate = \frac{h_2 - h_1}{2}$$

$$= \frac{250\tan 58° - 250\tan 24°}{2}$$

$$\approx 144.4 \quad m/\min$$

Exercise

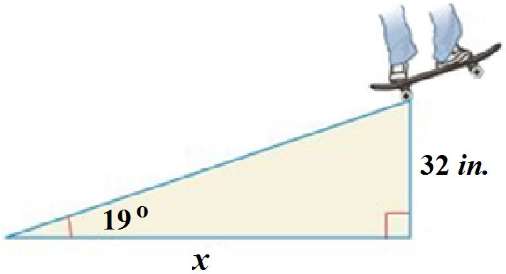
A skateboarder wishes to build a jump ramp that is inclined at a 19° angle and that has a maximum height of 32.0 *inches*. Find the horizontal width x of the ramp.

Solution

$$\tan 19^\circ = \frac{32}{x}$$

$$x = \frac{32}{\tan 19^\circ}$$

$$\boxed{\approx 92.9 \ \ in}$$



Exercise

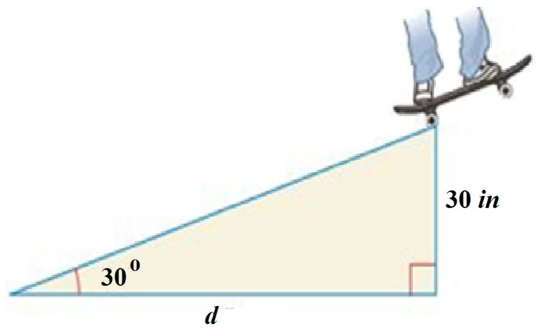
A skateboarder wishes to build a jump ramp that is inclined at a 30° angle and that has a maximum height of 30 *inches*. Find the horizontal width d of the ramp.

Solution

$$\tan 30^\circ = \frac{30}{d}$$

$$d = \frac{30}{\frac{1}{\sqrt{3}}}$$

$$\boxed{= 30\sqrt{3} \ \ in.}$$



Exercise

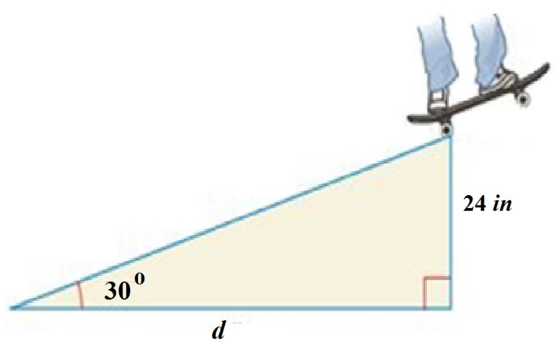
A skateboarder wishes to build a jump ramp that is inclined at a 30° angle and that has a maximum height of 24 *inches*. Find the horizontal width d of the ramp.

Solution

$$\tan 30^\circ = \frac{24}{d}$$

$$d = \frac{24}{\frac{1}{\sqrt{3}}}$$

$$\boxed{= 24\sqrt{3} \ \ in.}$$



Exercise

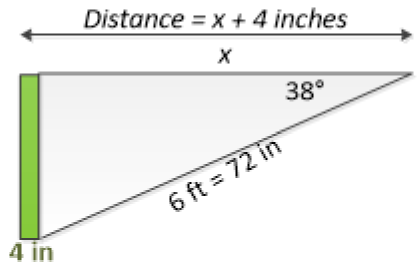
For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 *feet* from the piece of art and that the angle of depression of the light be 38°. How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 *inches* from the wall.

Solution

$$\cos 38° = \frac{x}{6}$$

$$x = 6\cos 38°$$

$$\approx 4.7 \ \ feet$$



$$distance = 4.7\,ft\,\frac{12\ in}{1\,ft} + 4in$$

$$= 60.7\ \ in$$

$$distance = \frac{60.7}{12}$$

$$\approx 5.1\ \ ft$$

Exercise

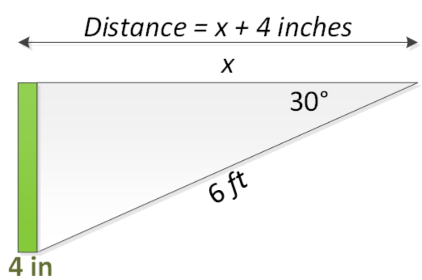
For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 *feet* from the piece of art and that the angle of depression of the light be 38°. How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 *inches* from the wall.

Solution

$$\cos 30° = \frac{x}{6}$$

$$x = 6\left(\frac{\sqrt{3}}{2}\right)$$

$$= 3\sqrt{3}\ \ ft$$



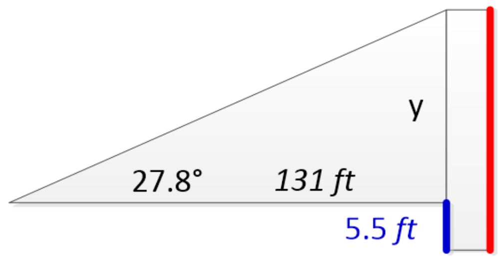
$$distance = 3\sqrt{3}\ ft\,\frac{12\ in}{1\,ft} + 4\ in$$

$$= 36\sqrt{3} + 4\quad in.$$

Exercise

A surveyor determines that the angle of elevation from a transit to the top of a building is 27.8°. The transit is positioned 5.5 *feet* above ground level and 131 *feet* from the building. Find the height of the building to the nearest tenth of a foot.

Solution



$$\tan 27.8^\circ = \frac{y}{131}$$

$$y = 131\tan 27.8^\circ$$

$$h = y + 5.5$$

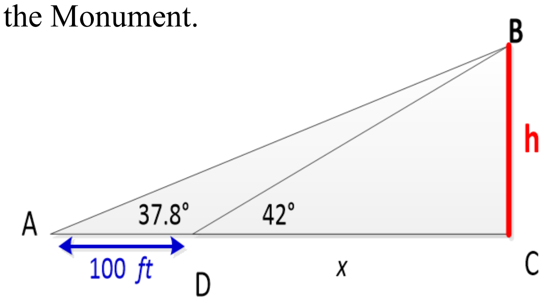
$$= 131\tan 27.8^\circ + 5.5$$

$$\approx 74.6\ ft$$

Exercise

From a point A on a line from the base of the Washington Monument, the angle of elevation to the top of the monument is 42.0°. From a point 100 *feet* away from A and on the same line, the angle to the top is 37.8°. Find the height, to the nearest foot, of the Monument.

Solution



$$h = \frac{100\tan 37.8^\circ \tan 42^\circ}{\tan 42^\circ - \tan 37.8^\circ}$$

$$\approx 560\ ft$$

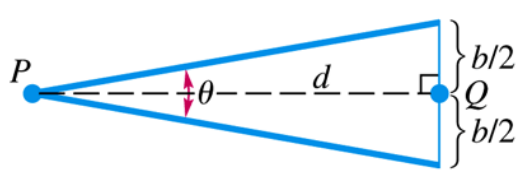
Exercise

A method that surveyors use to determine a small distance d between two points P and Q is called the ***subtense bar method***. The subtense bar with length b is centered at Q and situated perpendicular to the line of sight between P and Q. Angle θ is measured, then the distance d can be determined.

a) Find d with $\theta = 1^\circ\ 23'\ 12''$ and $b = 2.000\ cm$

b) Angle θ usually cannot be measured more accurately than to the nearest 1″. How much change would there be in the value of d if θ were measured 1″ larger?

Solution



a) $\cot\frac{\theta}{2} = \frac{d}{b/2}$

$$d = \frac{b}{2}\cot\frac{\theta}{2}$$

$$\theta = 1^\circ\ 23'\ 12''$$

$$= 1° + \frac{23°}{60} + \frac{12°}{3600}$$

$$\approx 1.38667°$$

$$d = \frac{2}{2}\cot\frac{1.38667°}{2}$$

$$\approx 82.6341\ cm$$

b) $\theta = 1°\ 23'\ 12'' + 1''$

$$= 1°\ 23'\ 13''$$

$$\approx 1.386944°$$

$$d = \frac{2}{2}\cot\frac{1.386944°}{2}$$

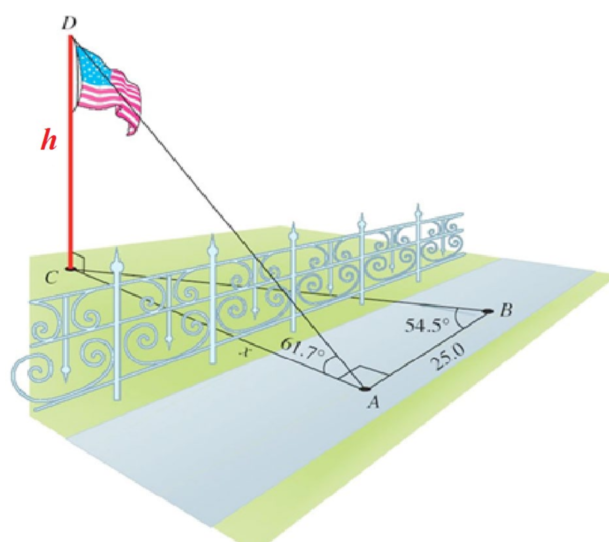
$$\approx 82.617558\ cm$$

$\therefore$ The change is: $82.6341 - 82.6175 \approx 0.0166\ cm$

Exercise

A diagram that shows how Diane estimates the height of a flagpole. She can't measure the distance between herself and the flagpole directly because there is a fence in the way. So she stands at point A facing the pole and finds the angle of elevation from point A to the top of the pole to be 61.7°. Then she turns 90° and walks 25.0 ft to point B, where she measures the angle between her path and a line from B to the base of the pole. She finds that angle is 54.5°. Use this information to find the height of the pole.

Solution



$$\tan 54.5° = \frac{x}{25.0}$$

$$x = 25.0 \tan 54.5°$$

$$\approx 35.0487\ ft$$

$$\tan 61.7° = \frac{h}{35.0487}$$

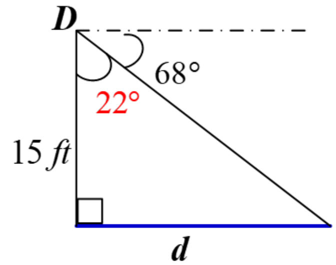
$$h = 35.0487 \tan 61.7°$$

$$\approx 65.1\ ft$$

Exercise

From a point 15 *feet* above level ground, a surveyor measures the angle of depression of an object on the ground at 68°. Approximate the distance from the object to the point on the ground directly beneath the surveyor.

Solution



$$D = 90° - 68° \underline{= 22°}$$

$$\tan 22° = \frac{d}{15}$$

$$d = 15\tan 22°$$

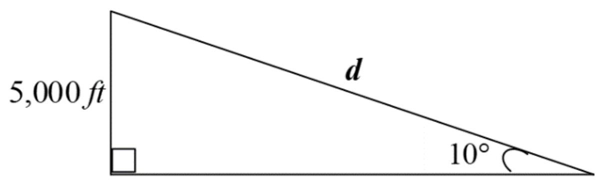
$$= 6.1 \ ft$$

Distance from the object to the point is about 6.1 *feet*.

Exercise

A pilot, flying at an altitude of 5,000 *feet* wishes to approach the numbers on a runway at an angle of 10°. Approximate, to the nearest 100 *feet*, the distance from the airplane to the numbers at the beginning of the descent.

Solution



$$\sin 10° = \frac{5,000}{d}$$

$$d = \frac{5,000}{\sin 10°}$$

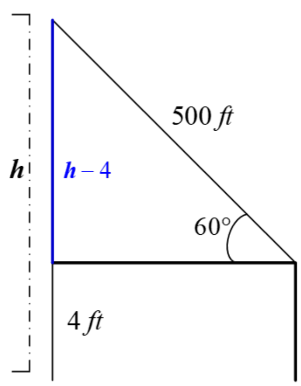
$$\approx 28,793.85$$

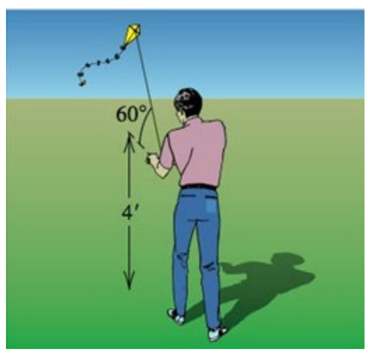
$$\approx 28,800 \ ft$$

Exercise

A person flying a kite holds the string 4 *feet* above ground level. The string of the kite is taut and make an angle of 60° with the horizontal. Approximate the height of the kite above level ground if 500 *feet* of sting is paved out.

Solution





$$\sin 60° = \frac{h-4}{500}$$

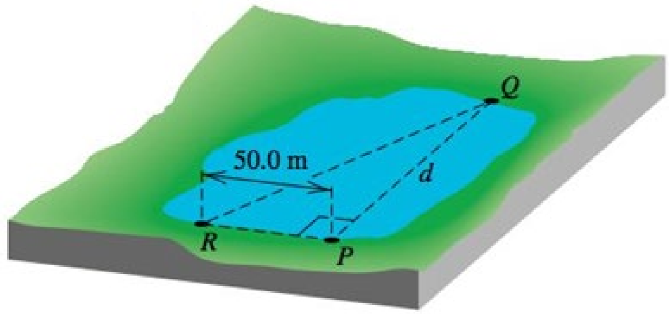
$$h - 4 = 500\frac{\sqrt{3}}{2}$$

$$h = 250\sqrt{3} + 4$$

$$\approx 437 \ ft$$

Exercise

To find the distance d between two points P and Q on opposite shores of a lake, a surveyor locates a point R that is 50.0 meters from P such that RP is perpendicular to PQ. Nest, using a transit, the surveyor measures angle PRQ as $72°\ 40'$. Find d.



Solution

Given: $\angle PRQ = 72°\ 40'$

$$\tan\left(72°\ 40'\right) = \frac{d}{50}$$

$$d = 50\tan\left(72°\ 40'\right)$$

$$\approx 160\ m$$

Exercise

A drawbridge is 150 *feet* long when stretched across a river. The two sections of the bridge can be rotated upward through an angle of 35°.

a) If the water level is 15 *feet* below the closed bridge, find the distance d between the end of a section and the water level when the bridge is fully open.

b) Approximately how far apart are the ends of the two sections when the bridge is fully opened?

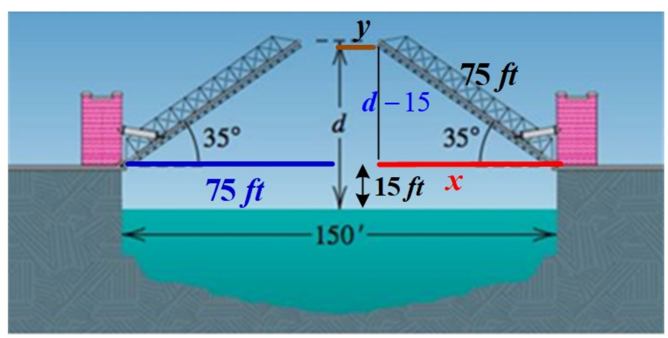
Solution

a) $\sin 35° = \dfrac{d-15}{75}$

$$d - 15 = 75\sin 35°$$

$$d = 75\sin 35° + 15$$

$$\approx 58\ ft$$



b) $\cos 35° = \dfrac{x}{75}$

$$x = 75\cos 35°$$

$$y = 75 - 75\cos 35°$$

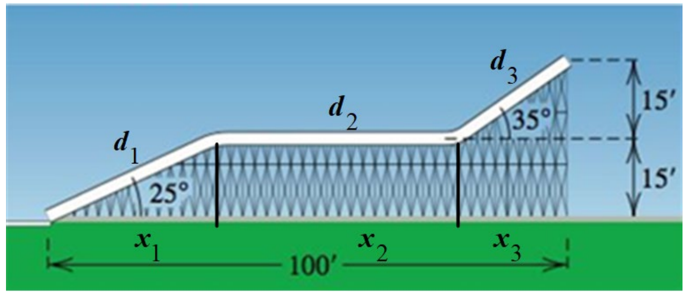
$$\approx 13.56$$

The two sections are apart: $2 \times 13.56 \approx 27\ ft$

Exercise

Find the total length of a design for a water slide to the nearest *foot*.

Solution



$$\sin 25° = \frac{15}{d_1}$$

$$d_1 = \frac{15}{\sin 25°}$$

$$\approx 35.49\ ft$$

$$\sin 35° = \frac{15}{d_3}$$

$$d_3 = \frac{15}{\sin 35°}$$

$$\approx 26.15\ ft$$

$$\tan 25° = \frac{15}{x_1} \Rightarrow x_1 = \frac{15}{\tan 25°} \approx 32.17\ ft$$

$$\tan 35° = \frac{15}{x_2} \Rightarrow x_2 = \frac{15}{\tan 35°} \approx 21.42\ ft$$

$$d_2 = 100 - x_1 - x_2$$

$$= 100 - 32.17 - 21.42$$

$$\approx 46.41\ ft$$

$$\text{Total length } = 35.49 + 26.15 + 46.41 \approx 108.05\ ft$$

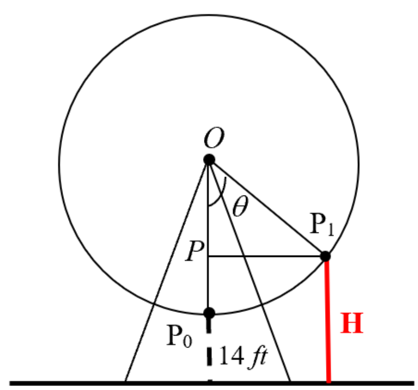
Exercise

The diameter of the Ferris wheel is 250 *feet*, the distance from the ground to the bottom of the wheel is 14 *feet*, and one complete revolution takes 20 *minutes*, find

a) The linear velocity, in miles per hour, of a person riding on the wheel.

b) The height of the rider in terms of the time *t*, where *t* is measured in minutes.

Solution



Given: $\theta = 1\ rev = 2\pi\ rad;\ t = 20\ min.;$

$$r = \frac{D}{2} = \frac{250}{2} = 125\ ft$$

a) $\omega = \dfrac{\theta}{t}$ *or* $\boxed{v = \dfrac{r\theta}{t}}$

$$= \frac{2\pi}{20}$$

$$= \frac{\pi}{10}\ rad / min$$

$$\begin{aligned} v &= r\omega \\ &= (125\, ft)\left(\frac{\pi}{10}\, rad\, /\, \min\right) \\ &\approx 39.27\, ft\, /\, \min \end{aligned}$$

$$\begin{aligned} v &\approx 39.27\ \frac{ft}{\min}\ \frac{60\min}{1hr}\ \frac{1mile}{5{,}280\, ft} \\ &\approx 0.45\ mi\, /\, hr \end{aligned}$$

b) $\cos\theta = \dfrac{OP}{OP_1} = OP_0 - OP + 14$

$$= \frac{OP}{125}$$

$$OP = 125\cos\theta$$

$$\begin{aligned} H &= PP_0 + 14 \\ &= 125 - 125\cos\theta + 14 \\ &= 139 - 125\cos\theta \end{aligned}$$

$$\omega = \frac{\theta}{t}$$

$$\theta = \omega\, t$$

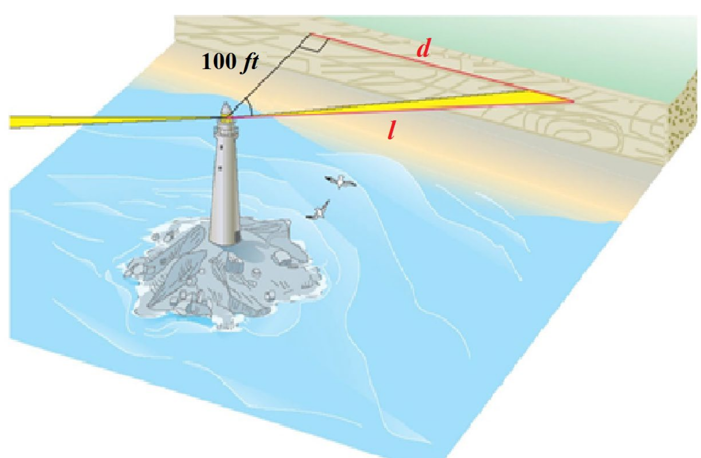
$$\theta = \frac{\pi}{10}\ t$$

$$H(t) = 139 - 125\cos\left(\frac{\pi}{10}t\right)$$

Exercise

Find an equation that expresses l in terms of time t. Find l when t is 0.5 *sec*, 1.0 *sec*, and 1.5 *sec*. (assume the light goes through one rotation every 4 *seconds*.)

Solution



$$\begin{aligned} \omega &= \frac{\theta}{t} = \frac{2\pi}{4}\frac{rad}{\sec} \\ &= \frac{\pi}{2}\ rad\, /\, \sec \end{aligned}$$

$$\frac{\theta}{t} = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{2}\ t$$

$$\cos\left(\frac{\pi}{2}\ t\right) = \frac{100}{l}$$

$$l\cos\left(\frac{\pi}{2}\ t\right) = 100$$

$$l = \frac{100}{\cos\left(\frac{\pi}{2}\ t\right)}$$

$$= 100\sec\left(\frac{\pi}{2}\ t\right)$$

For $t = 0.5\ sec$

$$\rightarrow \quad \underline{|l} = \frac{100}{\cos\left(\frac{\pi}{2}\frac{1}{2}\right)} = \frac{100}{\cos\left(\frac{\pi}{4}\right)}$$

$$= \frac{100}{\frac{1}{\sqrt{2}}}$$

$$\underline{= 100\sqrt{2}\ \ ft|} \qquad \underline{\approx 141\ \ ft|}$$

For $t = 1.0\ sec$

$$l = \frac{100}{\cos\left(\frac{\pi}{2}\right)}$$

$$= \frac{100}{0}$$

$$\underline{= Undefined\ |}$$

For $t = 1.5$ sec

$$l = \frac{100}{\cos\left(\frac{\pi}{2}\frac{3}{2}\right)}$$

$$= \frac{100}{\cos\left(\frac{3\pi}{4}\right)}$$

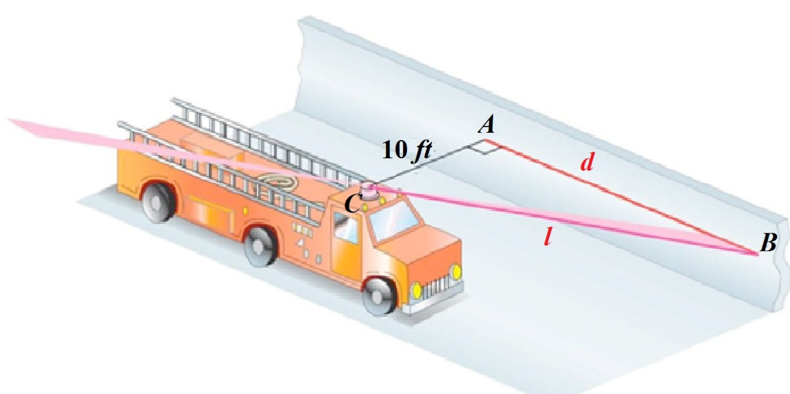
$$= -\frac{100}{\frac{1}{\sqrt{2}}}$$

$$\underline{= -100\sqrt{2}\ \ ft|} \qquad \underline{\approx -141\ \ ft|}$$

Exercise

A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top of the truck is 10 *feet* from the wall and rotates through a complete revolution every 2 *seconds*. Find the equations that give the lengths d and ℓ in terms of time.

Solution



$$\omega = \frac{\theta}{t} = \frac{2\pi}{2} = \pi\ rad\ /\ sec$$

$$\tan\theta = \frac{d}{10}$$

$$d = 10\tan\theta$$

$$= 10\tan\pi t$$

$$\sec\theta = \frac{l}{10}$$

$$l = 10\sec\theta$$

$$\boxed{= 10\sec \pi t \ \ ft}$$

Exercise

A Ferris wheel has radius 50.0 *feet*. A person takes a seat and then the wheel turns $\frac{2\pi}{3}$ *rad*.

a) How far is the person above the ground?

b) If it takes 30 *sec* for the wheel to turn $\frac{2\pi}{3}$ *rad*, what is the angular speed of the wheel?

Solution

a) $\alpha = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$

$$\cos\alpha = \frac{h_1}{r}$$

$$h_1 = r\cos\alpha$$

$$= 50\cos\frac{\pi}{6}$$

$$\boxed{= 25\sqrt{3} \ \ ft} \quad = 43.3\, ft$$

Person is $50 + 25\sqrt{3} = 93.3\, ft$ above the ground

b) $\omega = \frac{\theta}{t} = \frac{\frac{2\pi}{3}\ rad}{30\ sec}$

$$\boxed{= \frac{\pi}{45} \ \ rad\ /\ sec}$$

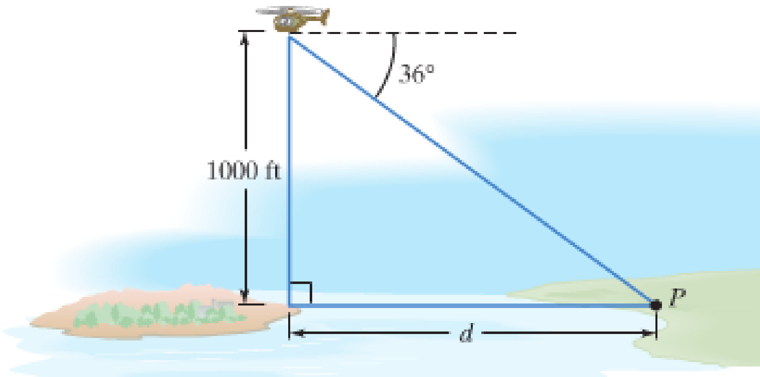
$$t = \frac{400\pi\ ft}{30\ \frac{mi}{hr}}$$

$$= \frac{40\pi}{3}\ ft\ \frac{hr}{mi}\ \frac{1mi}{5280\, ft}\ \frac{3600\sec}{1hr}$$

$$\boxed{\approx 29 \ \ \sec}$$

Exercise

A helicopter hovers 1,000 *feet* above a small island. The angle of depression from the helicopter to point P on the coast is 36°. How far off the coast is the island?



Solution

$$\tan 36° = \frac{1,000}{d}$$

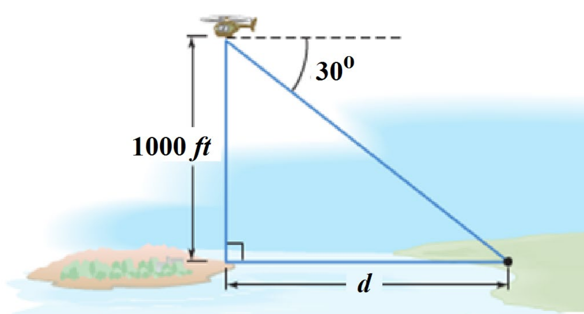
$$d = \frac{1,000}{\tan 36°}$$

$$\approx 1,376 \ \ feet$$

$\therefore$The island is approximately 1,376 *feet* off the coast.

Exercise

A helicopter hovers 1,000 *feet* above a small island. The angle of depression from the helicopter to point P on the coast is 30°. How far off the coast is the island?



Solution

$$\tan 30° = \frac{1,000}{d}$$

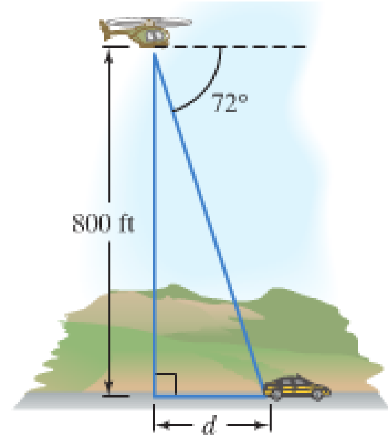
$$d = \frac{1,000}{\frac{1}{\sqrt{3}}}$$

$$= 1,000\sqrt{3} \ \ feet$$

$\therefore$The island is approximately 1,376 *feet* off the coast.

Exercise

A police helicopter is flying at 800 *feet*. A stolen car is sighted at an angle of depression of 72°. Find the distance of the stolen car from a point directly below the helicopter.



Solution

$$\tan 72° = \frac{800}{d}$$

$$d = \frac{800}{\tan 72°}$$

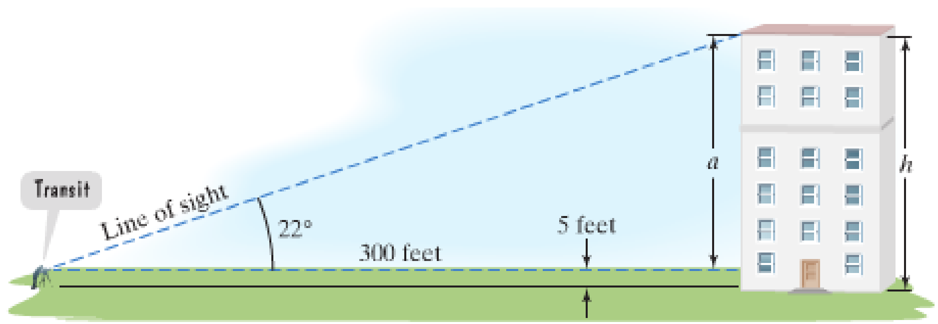
$$\approx 260 \ ft$$

∴ The stolen car is approximately 260 *feet* from a point directly below the helicopter.

Exercise

Sighting the top of a building a surveyor measured the angle of elevation to be 22°. The transit is 5 *feet* above the ground and 300 *feet* from the building. Find the building's height.

Solution



$$\tan 22° = \frac{a}{300}$$

$$a = 300 \tan 22°$$

$$\approx 121$$

$$h = 5 + 121$$

$$\approx 126 \ ft$$

Exercise

Sighting the top of a building a surveyor measured the angle of elevation to be 30°. The transit is 5 *feet* above the ground and 250 *feet* from the building. Find the building's height.

Solution

$$\tan 30° = \frac{a}{250}$$

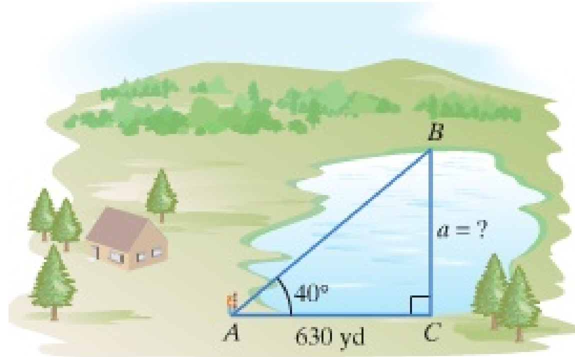
$$a = \frac{250}{\sqrt{3}}$$

$$h = 5 + \frac{250\sqrt{3}}{3}$$

$$= \frac{15 + 250\sqrt{3}}{3} \ ft$$

Exercise

Determine how far it is across the lake.



Solution

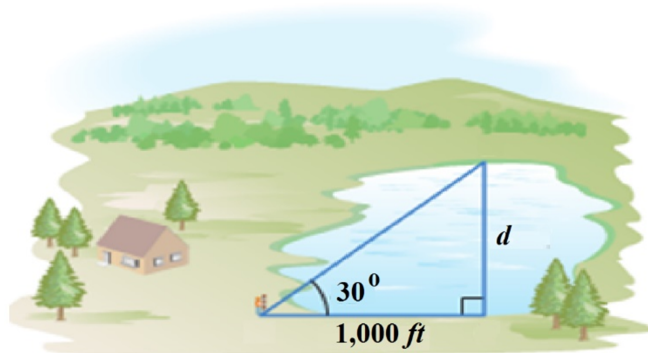
$$\tan 40° = \frac{a}{630}$$

$$a = 630\tan 40°$$

$$\boxed{\approx 529\ yd}$$

Exercise

Determine how far it is across the lake.



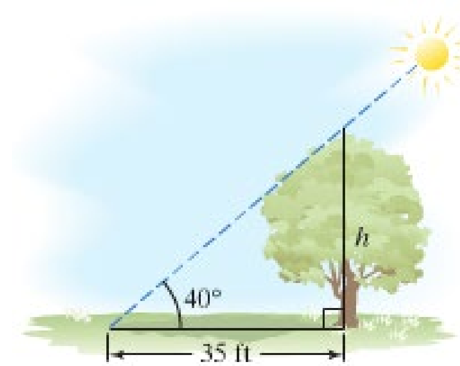
Solution

$$\tan 30° = \frac{d}{1,000}$$

$$\boxed{d = \frac{1,000}{\sqrt{3}}\ yd.}$$

Exercise

At a certain time of day, the angle of elevation of the sun is 40°. Find the height of a tree whose shadow is 35 *feet* long.



Solution

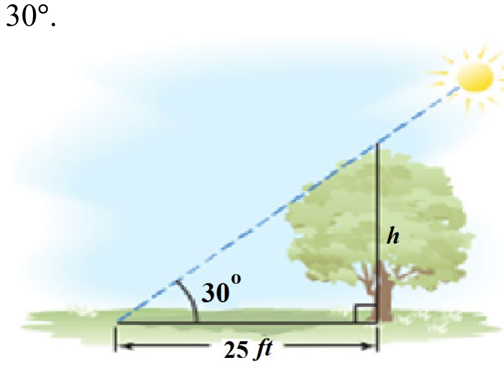
$$\tan 40° = \frac{h}{35}$$

$$h = 35\tan 40°$$

$$\boxed{\approx 29.4\ ft}$$

Exercise

At a certain time of day, the angle of elevation of the sun is 30°. Find the height of a tree whose shadow is 25 *feet* long.



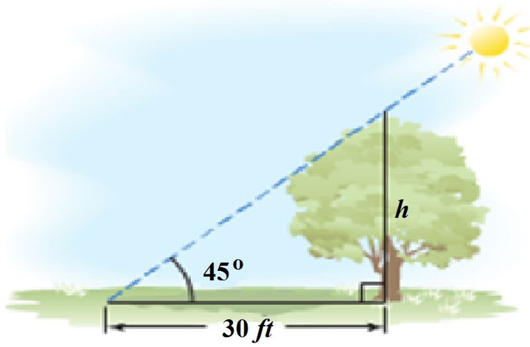
Solution

$$\tan 30° = \frac{h}{25}$$

$$\boxed{h = \frac{25}{\sqrt{3}}\ ft}$$

Exercise

At a certain time of day, the angle of elevation of the sun is 45°.
Find the height of a tree whose shadow is 30 *feet* long.



Solution

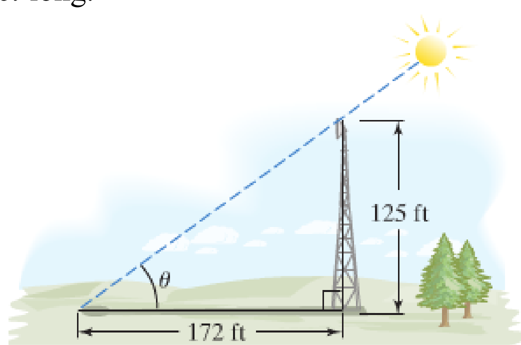
$$\tan 45^\circ = \frac{h}{25}$$

$$\boxed{h = \frac{25\sqrt{2}}{2}\ ft}$$

Exercise

A tower that is 125 *feet* casts a shadow 172 *feet* long.
Find the angle of elevation of the sun.

Solution



$$\tan\theta = \frac{125}{172}$$

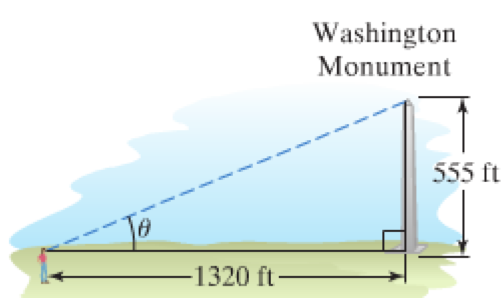
$$\theta = \tan^{-1}\left(\frac{125}{172}\right)$$

$$\boxed{\approx 36^\circ}$$

Exercise

The Washington Monument is 555 *feet* high. If you are standing one quarter of a mile, or 1,320 *feet*, from the base of the monument and looking to the top, find the angle of elevation.

Solution



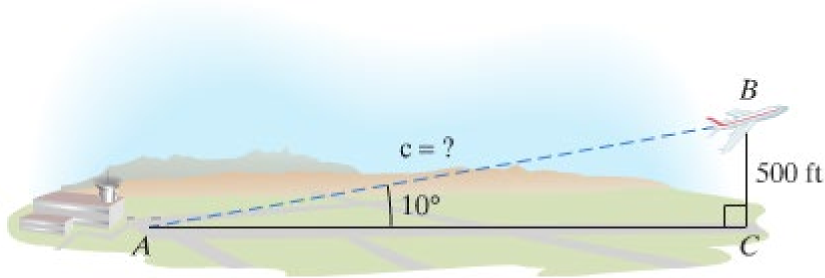
$$\tan\theta = \frac{555}{1320}$$

$$\theta = \tan^{-1}\left(\frac{555}{1320}\right)$$

$$\boxed{\approx 22.8^\circ}$$

Exercise

A plane rises from take-off and flies at an angle of $10°$ with the horizontal runway. When it has gained 500 *feet*, find the distance the plane has flown.



Solution

$$\sin 10° = \frac{500}{c}$$

$$c = \frac{500}{\sin 10°}$$

$$\approx 2,879.4\ ft$$

Exercise

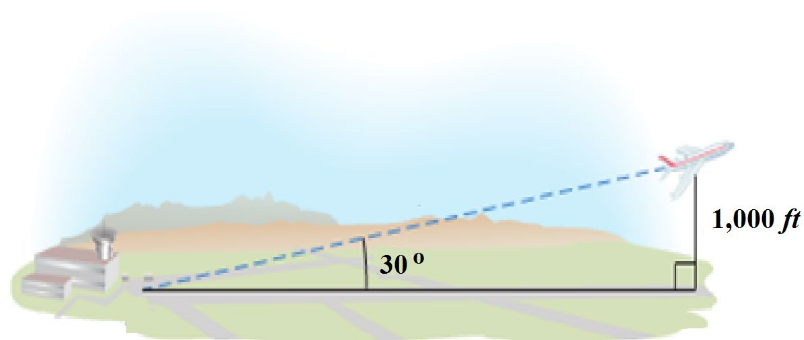
A plane rises from take-off and flies at an angle of $30°$ with the horizontal runway. When it has gained 1,000 *feet*, find the distance the plane has flown.

Solution

$$\sin 30° = \frac{1,000}{d}$$

$$d = \frac{1,000}{\frac{1}{2}}$$

$$= 2,000\ \ ft$$



Exercise

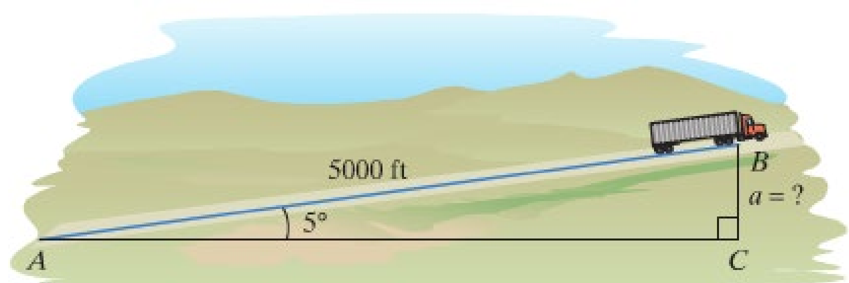
A road is inclined at an angle of 5°. After driving 5,000 *feet* along this road, find the driver's increase in altitude.

Solution

$$\sin 5° = \frac{a}{5,000}$$

$$a = 5,000 \sin 5°$$

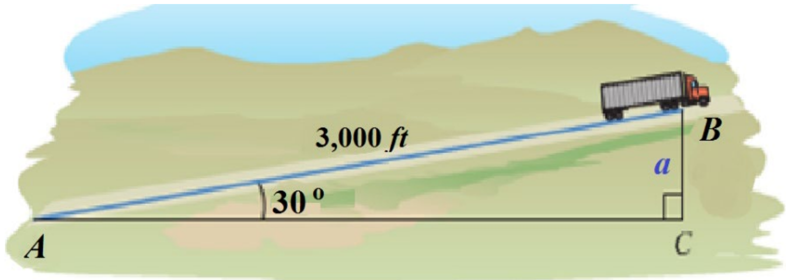
$$\approx 435.8\ \ ft$$



Exercise

A road is inclined at an angle of 30°. After driving 3,000 *feet* along this road, find the driver's increase in altitude.

Solution



$$\sin 30° = \frac{a}{3,000}$$

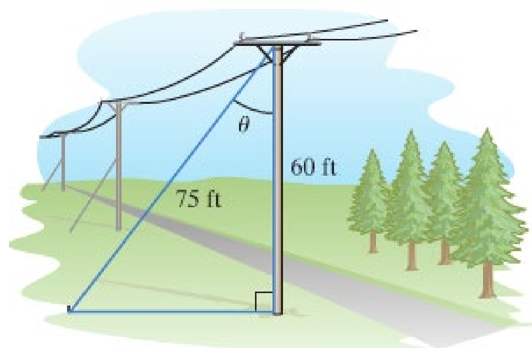
$$a = 3,000\left(\frac{1}{2}\right)$$

$$= 1,500 \ \ ft$$

Exercise

A telephone pole is 60 *feet* tall. A guy wire 75 *feet* long is attached from the ground to the top of the pole. Find the angle between the wire and the pole.

Solution



$$\cos\theta = \frac{60}{75}$$

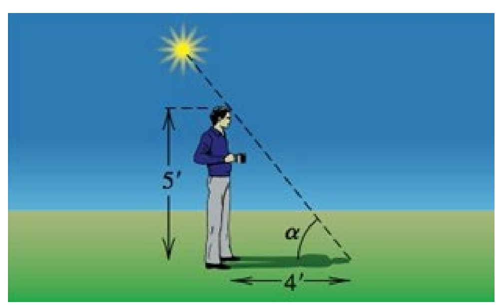
$$\theta = \cos^{-1}\left(\frac{60}{75}\right)$$

$$\approx 37°$$

Exercise

Approximate the angle of elevation α of the sun if a person 5.0 *feet* tall casts a shadow 4.0 *feet* long on level ground.

Solution



$$\tan\alpha = \frac{5}{4}$$

$$\alpha = \tan^{-1}\frac{5}{4}$$

$$\approx 51.34°$$

Exercise

A spotlight with intensity 5000 candles is located 15 *feet* above a stage. If the spotlight is rotated through an angle θ, the illuminance E (in foot-candles) in the lighted area of the stage is given by

$$E = \frac{5,000\cos\theta}{s^2}$$

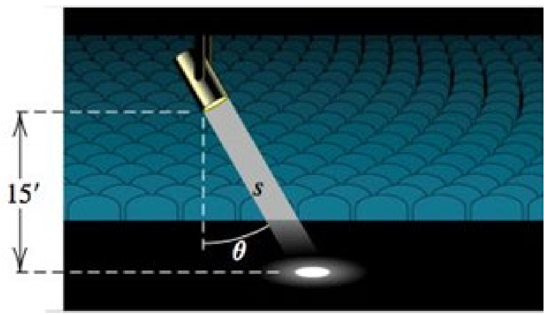
Where s is the distance (in *feet*) that the light must travel.

a) Find the illuminance if the spotlight is rotated through an angle of 30°.

b) The maximum illuminance occurs when $\theta = 0°$. For what value of θ is the illuminance one-half the maximum value.

Solution

a) $\cos\theta = \frac{15}{s} \quad \Rightarrow \quad s = \frac{15}{\cos\theta}$



$$E = \frac{5,000\cos\theta}{s^2} = 5,000\cos\theta\frac{\cos^2\theta}{15^2}$$

$$= \frac{200}{9}\cos^3\theta$$

$$= \frac{200}{9}\cos^3(30°)$$

$$= \frac{200}{9}\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{25\sqrt{3}}{3}\ ft-candles \quad \approx 14.43\ ft-candles$$

b) $E = \frac{1}{2}E_{\max}$

$$\frac{200}{9}\cos^3\theta = \frac{1}{2}\frac{200}{9}\cos^3 0°$$

$$\cos^3\theta = \frac{1}{2}$$

$$\cos\theta = \sqrt[3]{\frac{1}{2}}$$

$$\theta = \cos^{-1}\sqrt[3]{\frac{1}{2}}$$

$$\approx 37.47°$$

Exercise

A conveyor belt 9 *meters* long can be hydraulically rotated up to an angle of 40° to unload cargo from airplanes.

a) Find, to the nearest degree, the angle through which the conveyor belt should be rotated up to reach a door that is 4 *meters* above the platform supporting the belt.

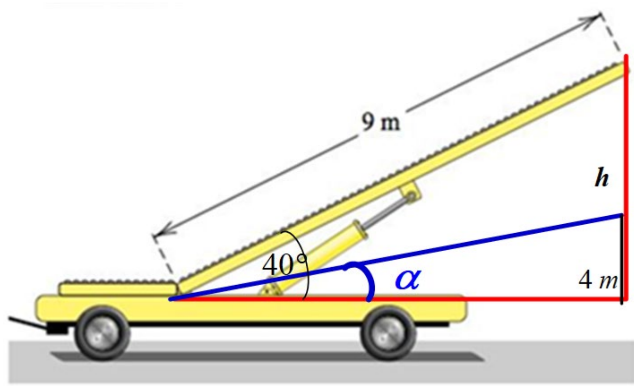
b) Approximate the maximum height above the platform that the belt can reach.

Solution

$$\sin\alpha = \frac{4}{9}$$

$$\alpha = \sin^{-1}\frac{4}{9}$$

$$\approx 26.4^\circ$$



$$\sin 40^\circ = \frac{h}{9}$$

$$h = 9\sin 40^\circ$$

$$\approx 5.785\ m$$

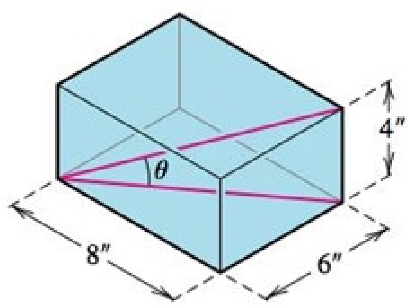
Exercise

A rectangular box has dimensions $8'' \times 6'' \times 4''$. Approximate, to the nearest tenth of a degree, the angle θ formed by a diagonal of the base and the diagonal of the box.

Solution

$$d = \sqrt{8^2 + 6^2}$$

$$= 10$$



$$\theta = \tan^{-1}\frac{4}{10}$$

$$\approx 21.8^\circ$$

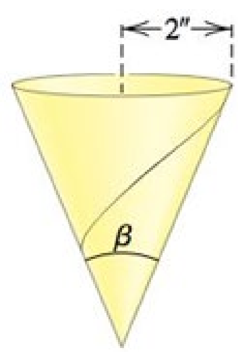
Exercise

A conical paper cup has a radius of 2 *inches*, approximate, to the nearest degree, the angle β so that the cone will have a volume of $20\ in^3$.

Solution

$$V = \frac{1}{3}\pi r^2 h$$

$$= 20\ in^3$$



$$h = \frac{60}{\pi\left(2^2\right)}$$

$$= \frac{15}{\pi} \approx 4.77\ in$$

$$\tan\frac{\beta}{2} = \frac{2}{4.77}$$

$$\frac{\beta}{2} = \tan^{-1}\frac{2}{4.77}$$

$$\approx 22.75^\circ$$

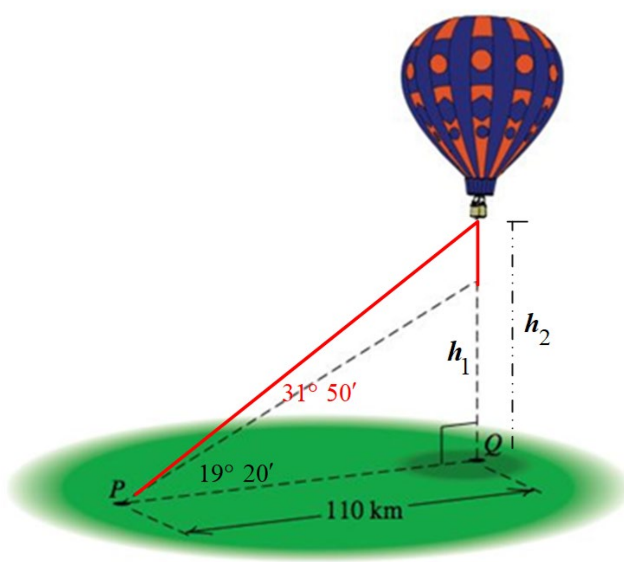
$$\beta = 2\left(22.75^\circ\right)$$

$$\approx 45.5^\circ$$

Exercise

As a hot-air balloon rises vertically, its angle of elevation from a point *P* on level ground 100 *km* from the point *Q* directly underneath the balloon changes from $19^\circ\ 20'$ to $31^\circ\ 50'$.
Approximately how far does the balloon rise during this period?

Solution



$$\tan\left(19^\circ\ 20'\right) = \frac{h_1}{100}$$

$$h_1 = 100\tan\left(19^\circ\ 20'\right)$$

$$\approx 38.59\ \ km$$

$$\tan\left(31^\circ\ 50'\right) = \frac{h_2}{100}$$

$$h_2 = 100\tan\left(31^\circ\ 50'\right)$$

$$\approx 68.29\ \ km$$

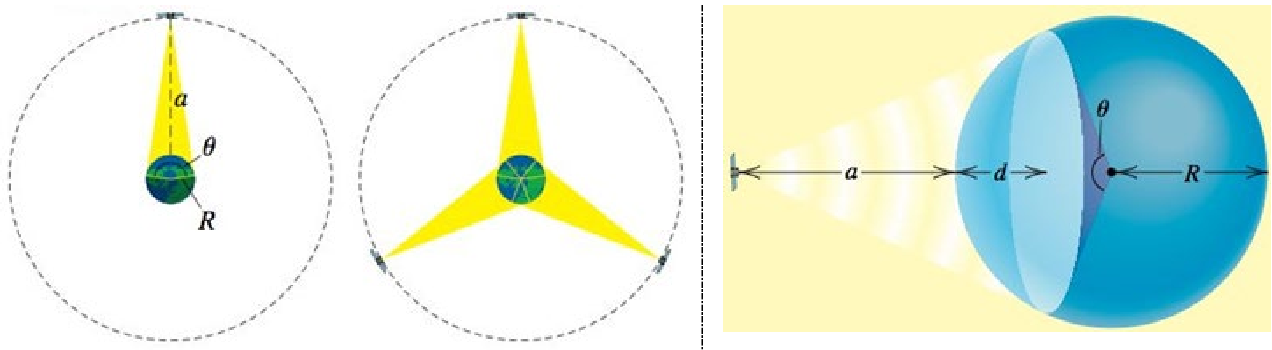
The change in elevation is:

$$h_2 - h_1 \approx 68.29 - 38.59$$

$$= 29.7\ \ km$$

Exercise

Shown in the left part of the figure is a communications satellite with an equatorial orbit–that is, a nearly circular orbit in the plane determined by Earth's equator. If the satellite circles Earth at an altitude of $a = 22{,}300\ mi$, its speed is the same as the rotational speed of Earth; to an observer on the equator, the satellite appears to be stationary–that is, its orbit is synchronous.



a ***b***

a) Using $R = 4{,}000\ mi$ for the radius of Earth, determine the percentage of the equator that is within signal range of such a satellite.

b) As shown in the right part of the figure (a), three satellites are equally spaced in equatorial synchronous orbits. Use the value of θ obtained in part (a) to explain why all points on the equator are within signal range of at least one of the three satellites.

c) The figure shows the area served by a communication satellite circling a planet of radius R at an altitude a. The portion of the planet's surface within range of the satellite is a spherical cap of depth d and surface area $A = 2\pi Rd$. Express d in terms of R and θ.

d) Estimate the percentage of the planet's surface that is within signal range of a single satellite in equatorial synchronous orbit.

Solution

a)

$$\cos\frac{\theta}{2} = \frac{R}{R+a}$$

$$= \frac{4{,}000}{26{,}300}$$

$$\frac{\theta}{2} = \cos^{-1}\left(\frac{4{,}000}{26{,}300}\right)$$

$$\approx 81.25^\circ$$

$$\theta \approx 162.5^\circ$$

The percentage of the equator that is within signal rage is:

$$\frac{162.5^\circ}{360^\circ}\times 100 \ \approx 45\%$$

b) Each satellite has a signal range of more than 120°, and this all 3 will cover all points on the equator.

c) $\cos\frac{\theta}{2} = \frac{R-d}{R}$

$R\cos\frac{\theta}{2} = R-d$

$\underline{d = R\left(1-\cos\frac{\theta}{2}\right)}$

d) $d = R\left(1-\cos 81.25^\circ\right)$

$\underline{\approx 0.8479R}$

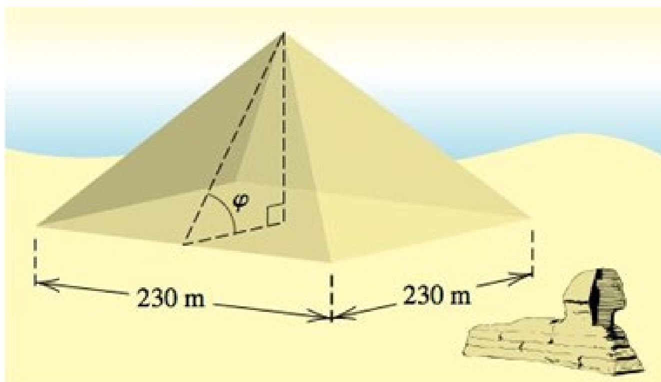
$\frac{d}{2R} \approx \frac{.8479R}{2R}$

$\underline{= 0.4239}$

The percentage of the planet's surface that is within signal range of a single satellite is: 42.39%

Exercise

The great Pyramid of Egypt is 147 *meters* high, with a square base of side 230 *meters*. Approximate, to the nearest degree, the angle φ formed when an observer stands at the midpoint of one the sides and views the apex of the pyramid.



Solution

$\tan\varphi = \frac{147}{\frac{1}{2}230}$

$\varphi = \tan^{-1}\frac{147}{115}$

$\underline{\approx 52^\circ}$

Exercise

A tunnel for a new highway is to be cut through a mountain that is 260 *feet* high. At a distance of 200 *feet* from the base of the mountain, the angle of elevation is 36°. From a distance of 150 *feet* on the other side, the angle of elevation is 47°. Approximate the length of the tunnel to the nearest foot.

Solution

Left triangle:

$\tan 36^\circ = \frac{260}{200+d_1}$

$d_1 = \frac{260}{\tan 36^\circ} - 200$

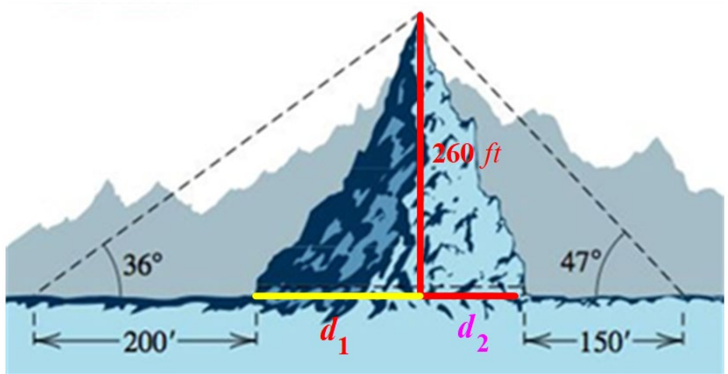
$\underline{\approx 157.86\ \ ft}$

Right triangle:

$$\tan 47° = \frac{260}{150 + d_2}$$

$$d_2 = \frac{260}{\tan 47°} - 150$$

$$\approx 92.45 \ ft$$



Length of the tunnel:

$$d = d_1 + d_2$$

$$\approx 157.86 + 92.45$$

$$= 250 \ ft$$

Exercise

When a certain skyscraper is viewed from the top of a building 50 *feet* tall, the angle of elevation is 59°. When viewed from the street next to the shorter building, the angle of elevation is 62°.

a) Approximately how far apart are the two structures?

b) Approximate the height of the skyscraper to the nearest tenth of a *foot*.

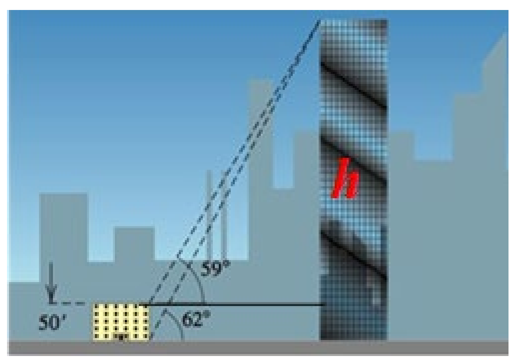
Solution

a) $\tan 59° = \dfrac{h - 50}{x}$

$\rightarrow \quad h = x \tan 59° + 50$

$\tan 62° = \dfrac{h}{x}$

$\rightarrow \quad h = x \tan 62°$



$x \tan 59° + 50 = x \tan 62°$

$x(\tan 62° - \tan 59°) = 50$

$x = \dfrac{50}{\tan 62° - \tan 59°}$

$\approx 231.0 \ ft$

b) $h = x \tan 62°$

$= 231 \tan 62°$

$\approx 434.5 \ ft$

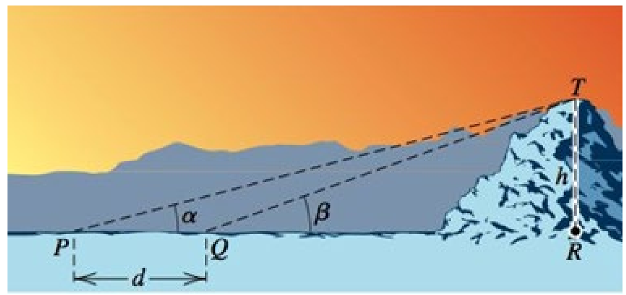
Exercise

When a mountaintop is viewed from the point P, the angle of elevation is α. From a point Q, which is d miles closer to the mountain, the angle of elevation increases to β.

a) Show that the height h of the mountain is given by: $h = \dfrac{d}{\cot\alpha - \cot\beta}$.

b) If $d = 2mi, \quad \alpha = 15°, \quad and \quad \beta = 20°$, approximate the height of the mountain.

Solution



a)
$$h = \frac{d\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

$$= d\,\frac{\frac{1}{\cot\alpha}\frac{1}{\cot\beta}}{\frac{1}{\cot\beta} - \frac{1}{\cot\alpha}}$$

$$= d\,\frac{\frac{1}{\cot\alpha\cot\beta}}{\frac{\cot\alpha - \cot\beta}{\cot\alpha\cot\beta}}$$

$$= \boxed{\frac{d}{\cot\alpha - \cot\beta}} \quad \checkmark$$

b) ***Given***: $d = 2mi, \quad \alpha = 15°, \quad and \quad \beta = 20°$

$$h = \frac{2\tan 15°\tan 20°}{\tan 20° - \tan 15°}$$

$$\boxed{\approx 2.03\ mi}$$

Exercise

An observer of height h stands on an incline at a distance d from the base of a building of height T. The angle of elevation from the observer to the top of the building is θ, and the incline makes an angle of α with the horizontal.

a) Express T in terms of h, d, α, and θ.

b) If $d = 50\ ft, \quad h = 6\ ft, \quad \alpha = 15°, \quad and \quad \theta = 31.4°$, estimate the height of the building.

Solution

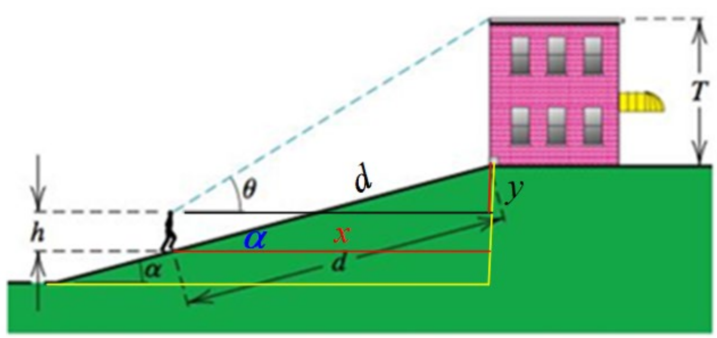
a) From ΔABD:

$$\cos\alpha = \frac{x}{d}$$

$$\rightarrow \quad x = d\cos\alpha$$

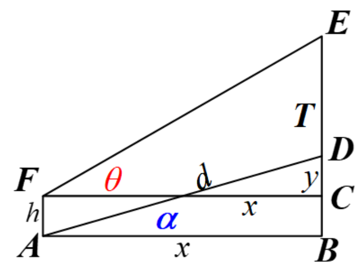
$$\sin\alpha = \frac{y+h}{d}$$

$$\rightarrow \quad y = d\sin\alpha - h$$



From ΔFCE :

$\tan\theta = \dfrac{T+y}{x} \quad \to \quad x\tan\theta = T+y$



$x\tan\theta = T+y$

$d\cos\alpha\tan\theta = T+d\sin\alpha - h$

$\boxed{T = d\left(\cos\alpha\tan\theta - \sin\alpha\right)+h}$

b) ***Given***: $d=50\,ft, \quad h=6\,ft, \quad \alpha=15°, \quad and \quad \theta=31.4°$

$T = 50\left(\cos 15°\tan 31.4° - \sin 15°\right)+6$

$\boxed{\approx 22.54 \quad ft}$