

Solution ***Section 4.1 – Antiderivatives***

Exercise

Find indefinite integral $\int v^2 dv$

Solution

$$\int v^2 dv = \underline{\frac{v^3}{3} + C}$$

Exercise

Find indefinite integral $\int x^{1/2} dx$

Solution

$$\int x^{1/2} dx = \underline{\frac{2}{3} x^{3/2} + C}$$

Exercise

Find indefinite integral $\int 4y^{-3} dy$

Solution

$$\begin{aligned} \int 4y^{-3} dy &= 4 \frac{y^{-2}}{-2} + C \\ &= \underline{-\frac{2}{y^2} + C} \end{aligned}$$

Exercise

Find indefinite integral $\int (x^3 - 4x + 2) dx$

Solution

$$\begin{aligned} \int (x^3 - 4x + 2) dx &= \frac{x^4}{4} - 4 \frac{x^2}{2} + 2x + C \\ &= \underline{\frac{1}{4} x^4 - 2x^2 + 2x + C} \end{aligned}$$

Exercise

Find indefinite integral $\int (3z^2 - 4z + 5) dz$

Solution

$$\begin{aligned}\int (3z^2 - 4z + 5) dz &= 3 \frac{z^3}{3} - 4 \frac{z^2}{2} + 5z + C \\ &= \underline{z^3 - 2z^2 + 5z + C}\end{aligned}$$

Exercise

Find indefinite integral $\int (x^2 - 1)^2 dx$

Solution

$$\begin{aligned}\int (x^2 - 1)^2 dx &= \int (x^4 - 2x^2 + 1) dx \\ &= \underline{\frac{1}{5}x^5 - \frac{2}{3}x^3 + x + C}\end{aligned}$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Exercise

Find indefinite integral $\int \frac{x^2 + 1}{\sqrt{x}} dx$

Solution

$$\begin{aligned}\int \frac{x^2 + 1}{\sqrt{x}} dx &= \int \left(\frac{x^2}{x^{1/2}} + \frac{1}{x^{1/2}} \right) dx \\ &= \int (x^{3/2} + x^{-1/2}) dx \\ &= \frac{x^{5/2}}{5/2} - \frac{x^{1/2}}{1/2} + C \\ &= \underline{\frac{2}{5}x^{5/2} - 2x^{1/2} + C}\end{aligned}$$

Exercise

Find indefinite integral $\int (\sqrt[4]{x^3} + 1) dx$

Solution

$$\int \left(\sqrt[4]{x^3} + 1 \right) dx = \int \left(x^{3/4} + 1 \right) dx$$

$$\underline{= \frac{4}{7} x^{7/4} + x + C}$$

Exercise

Find indefinite integral $\int \sqrt{x} (x+1) dx$

Solution

$$\int \sqrt{x} (x+1) dx = \int x^{1/2} (x+1) dx$$

$$= \int \left(x^{3/2} + x^{1/2} \right) dx$$

$$\underline{= \frac{2}{5} x^{5/2} + \frac{2}{3} x^{3/2} + C}$$

Exercise

Find indefinite integral $\int (1+3t) t^2 dt$

Solution

$$\int (1+3t) t^2 dt = \int \left(t^2 + 3t^3 \right) dt$$

$$\underline{= \frac{1}{3} t^3 + \frac{3}{4} t^4 + C}$$

Exercise

Find indefinite integral $\int \frac{x^2-5}{x^2} dx$

Solution

$$\int \frac{x^2-5}{x^2} dx = \int \left(1 - \frac{5}{x^2} \right) dx$$

$$= \int \left(1 - 5x^{-2} \right) dx$$

$$\underline{= x + \frac{5}{x} + C}$$

Exercise

Find indefinite integral $\int (-40x + 250)dx$

Solution

$$\int (-40x + 250)dx = \underline{-20x^2 + 250x + C}$$

Exercise

Find indefinite integral $\int \frac{x+2}{\sqrt{x}} dx$

Solution

$$\begin{aligned}\int \frac{x+2}{\sqrt{x}} dx &= \int \left[\frac{x}{x^{1/2}} + \frac{2}{x^{1/2}} \right] dx \\ &= \int \frac{x}{x^{1/2}} dx + \int \frac{2}{x^{1/2}} dx \\ &= \int x^{1/2} dx + 2 \int x^{-1/2} dx \\ &= \frac{x^{3/2}}{3/2} + 2 \frac{x^{1/2}}{1/2} + C \\ &= \underline{\frac{2}{3}x^{3/2} + 4x^{1/2} + C}\end{aligned}$$

Exercise

Find indefinite integral $\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x \right) dx$

Solution

$$\begin{aligned}\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x \right) dx &= \int \frac{1}{5} dx - \int 2x^{-3} dx + \int 2x dx \\ &= \frac{x}{5} - 2 \frac{x^{-2}}{-2} + x^2 + C \\ &= \underline{\frac{x}{5} + \frac{1}{x^2} + x^2 + C}\end{aligned}$$

Exercise

Find indefinite integral $\int (\sqrt{x} + \sqrt[3]{x}) dx$

Solution

$$\begin{aligned} \int (\sqrt{x} + \sqrt[3]{x}) dx &= \int (x^{1/2} + x^{1/3}) dx \\ &= \frac{x^{3/2}}{3/2} + \frac{x^{4/3}}{4/3} + C \\ &= \frac{2}{3} x^{3/2} + \frac{3}{4} x^{4/3} + C \end{aligned}$$

Exercise

Find indefinite integral $\int 2x(1 - x^{-3}) dx$

Solution

$$\begin{aligned} \int 2x(1 - x^{-3}) dx &= \int (2x - 2x^{-2}) dx \\ &= x^2 - 2 \frac{x^{-1}}{-1} + C \\ &= x^2 + \frac{2}{x} + C \end{aligned}$$

Exercise

Find indefinite integral $\int \left(\frac{4 + \sqrt{t}}{t^3} \right) dt$

Solution

$$\begin{aligned} \int \left(\frac{4 + \sqrt{t}}{t^3} \right) dt &= \int \left(\frac{4}{t^3} + \frac{t^{1/2}}{t^3} \right) dt \\ &= \int (4t^{-3} + t^{-5/2}) dt \\ &= 4 \frac{t^{-2}}{-2} + \frac{t^{-3/2}}{-3/2} + C \\ &= -\frac{2}{t^2} - \frac{2}{3t^{3/2}} + C \end{aligned}$$

Exercise

Find indefinite integral $\int (-2 \cos t) dt$

Solution

$$\int (-2 \cos t) dt = \underline{-2 \sin t + C}$$

Exercise

Find indefinite integral $\int 7 \sin \frac{\theta}{3} d\theta$

Solution

$$\begin{aligned} \int 7 \sin \frac{\theta}{3} d\theta &= 7 \frac{-\cos\left(\frac{\theta}{3}\right)}{\frac{1}{3}} + C \\ &= \underline{-21 \cos\left(\frac{\theta}{3}\right) + C} \end{aligned}$$

Exercise

Find indefinite integral $\int \frac{2}{5} \sec \theta \tan \theta d\theta$

Solution

$$\int \frac{2}{5} \sec \theta \tan \theta d\theta = \underline{\frac{2}{5} \sec \theta + C}$$

Exercise

Find indefinite integral $\int (4 \sec x \tan x - 2 \sec^2 x) dx$

Solution

$$\begin{aligned} \int (4 \sec x \tan x - 2 \sec^2 x) dx &= 4 \int (\sec x \tan x) dx - 2 \int (\sec^2 x) dx \\ &= \underline{4 \sec x - 2 \tan x + C} \end{aligned}$$

Exercise

Find indefinite integral $\int (2 \cos 2x - 3 \sin 3x) dx$

Solution

$$\int (2 \cos 2x - 3 \sin 3x) dx = \underline{\sin 2x + \cos 3x + C}$$

Exercise

Find indefinite integral $\int (1 + \tan^2 \theta) d\theta$

Solution

$$\begin{aligned} \int (1 + \tan^2 \theta) d\theta &= \int (\sec^2 \theta) d\theta \\ &= \underline{\tan \theta + C} \end{aligned}$$

Exercise

Find indefinite integral $\int \frac{\csc \theta}{\csc \theta - \sin \theta} d\theta$

Solution

$$\begin{aligned} \int \frac{\csc \theta}{\csc \theta - \sin \theta} d\theta &= \int \frac{1}{1 - \frac{\sin \theta}{\csc \theta}} d\theta && \text{divide by } \csc \theta \text{ \& } \csc \theta = \frac{1}{\sin \theta} \\ &= \int \frac{1}{1 - \sin^2 \theta} d\theta && \sin^2 \theta + \cos^2 \theta = 1 \Rightarrow 1 - \sin^2 \theta = \cos^2 \theta \\ &= \int \frac{1}{\cos^2 \theta} d\theta \\ &= \int \sec^2 \theta d\theta \\ &= \underline{\tan \theta + C} \end{aligned}$$

Exercise

Evaluate the integral $\int (2e^x - 3e^{-2x}) dx$

Solution

$$\int (2e^x - 3e^{-2x}) dx = \underline{2e^x + \frac{3}{2}e^{-2x} + C}$$

Exercise

Evaluate $\int \frac{dx}{\sqrt{9-x^2}}$

Solution

$$\int \frac{dx}{\sqrt{9-x^2}} = \underline{\sin^{-1}\left(\frac{x}{3}\right) + C}$$

Exercise

Evaluate $\int \frac{dx}{9+3x^2}$

Solution

$$\begin{aligned} \int \frac{dx}{9+3x^2} &= \frac{1}{3} \int \frac{dx}{3+x^2} & a^2 = 3 \rightarrow a = \sqrt{3} \\ &= \frac{1}{3} \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C \\ &= \underline{\frac{\sqrt{3}}{9} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C} \end{aligned}$$

Exercise

Find the integral $\int \frac{4x^2 - 3x + 2}{x^2} dx$

Solution

$$\begin{aligned} \int \frac{4x^2 - 3x + 2}{x^2} dx &= \int \left(\frac{4x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2} \right) dx \\ &= \int \left(4 - \frac{3}{x} + 2x^{-2} \right) dx \\ &= 4x - 3 \ln|x| - 2x^{-1} + C \\ &= \underline{4x - 3 \ln|x| - \frac{2}{x} + C} \end{aligned}$$

Exercise

Find the integral $\int (x^8 - 3x^3 + 1) dx$

Solution

$$\int (x^8 - 3x^3 + 1) dx = \underline{\frac{1}{9}x^9 - \frac{3}{4}x^4 + x + C}$$

Exercise

Find the integral $\int (2x + 1)^2 dx$

Solution

$$\begin{aligned} \int (2x + 1)^2 dx &= \int (4x^2 + 4x + 1) dx \\ &= \underline{\frac{4}{3}x^3 + 2x^2 + x + C} \end{aligned}$$

Exercise

Find the integral $\int \frac{x+1}{x} dx$

Solution

$$\begin{aligned} \int \frac{x+1}{x} dx &= \int \left(1 + \frac{1}{x}\right) dx \\ &= \underline{x + \ln|x| + C} \end{aligned}$$

Exercise

Find the integral $\int \left(\frac{1}{x^2} - \frac{2}{x^{5/2}}\right) dx$

Solution

$$\begin{aligned} \int \left(\frac{1}{x^2} - \frac{2}{x^{5/2}}\right) dx &= \int \left(\frac{1}{x^2} - 2x^{-5/2}\right) dx \\ &= -\frac{1}{x} + \frac{4}{3}x^{-3/2} + C \\ &= \underline{-\frac{1}{x} + \frac{4}{3x^{3/2}} + C} \end{aligned}$$

Exercise

Find the integral $\int \frac{x^4 - 2\sqrt{x} + 2}{x^2} dx$

Solution

$$\begin{aligned} \int \frac{x^4 - 2\sqrt{x} + 2}{x^2} dx &= \int (x^2 - 2x^{-3/2} + 2x^{-2}) dx \\ &= \underline{\frac{1}{3}x^3 + 4x^{-1/2} - \frac{2}{x} + C} \end{aligned}$$

Exercise

Find the integral $\int (1 + \cos 3\theta) d\theta$

Solution

$$\int (1 + \cos 3\theta) d\theta = \underline{\theta + \frac{1}{3}\sin 3\theta + C}$$

Exercise

Find the integral $\int 2\sec^2 \theta d\theta$

Solution

$$\int 2\sec^2 \theta d\theta = \underline{2\tan \theta + C}$$

Exercise

Find the integral $\int \sec 2x \tan 2x dx$

Solution

$$\int \sec 2x \tan 2x dx = \underline{\frac{1}{2}\sec 2x + C}$$

Exercise

Find the integral $\int 2e^{2x} dx$

Solution

$$\int 2e^{2x} dx = \underline{e^{2x} + C}$$

Exercise

Find the integral $\int \frac{12}{x} dx$

Solution

$$\int \frac{12}{x} dx = \underline{12 \ln|x| + C}$$

Exercise

Find the integral $\int \frac{dx}{\sqrt{1-x^2}}$

Solution

$$\int \frac{dx}{\sqrt{1-x^2}} = \underline{\sin^{-1} x + C}$$

Exercise

Find the integral $\int \frac{dx}{x^2 + 1}$

Solution

$$\int \frac{dx}{x^2 + 1} = \underline{\tan^{-1} x + C}$$

Exercise

Find the integral $\int \frac{1 + \tan \theta}{\sec \theta} d\theta$

Solution

$$\begin{aligned}\int \frac{1 + \tan \theta}{\sec \theta} d\theta &= \int \left(\frac{1}{\sec \theta} + \frac{\tan \theta}{\sec \theta} \right) d\theta \\ &= \int (\cos \theta + \sin \theta) d\theta \\ &= \sin \theta - \cos \theta + C\end{aligned}$$

Exercise

Find the integral $\int \left(\sqrt[4]{x^3} + \sqrt{x^5} \right) dx$

Solution

$$\begin{aligned}\int \left(\sqrt[4]{x^3} + \sqrt{x^5} \right) dx &= \int \left(x^{3/4} + x^{5/2} \right) dx \\ &= \frac{4}{7} x^{7/4} + \frac{2}{7} x^{7/2} + C \\ &= \frac{4}{7} x \sqrt[4]{x^3} + \frac{2}{7} x^3 \sqrt{x} + C\end{aligned}$$

Exercise

Find the integral $\int \left(x^{-3} + 7e^{5x} + \frac{4}{x} \right) dx$

Solution

$$\begin{aligned}\int \left(x^{-3} + 7e^{5x} + \frac{4}{x} \right) dx &= \frac{x^{-2}}{-2} + \frac{7}{5} e^{5x} + 4 \ln|x| + C \\ &= -\frac{1}{2x^2} + \frac{7}{5} e^{5x} + 4 \ln|x| + C\end{aligned}$$

Exercise

Find the integral $\int \left(\frac{2}{x} + \frac{x}{2} \right) dx$

Solution

$$\int \left(\frac{2}{x} + \frac{x}{2} \right) dx = 2 \ln|x| + \frac{1}{4} x^2 + C$$

Exercise

Find the integral $\int \frac{1}{ax} dx$

Solution

$$\int \frac{1}{ax} dx = \underline{\frac{1}{a} \ln|x| + C}$$

Exercise

Find the integral $\int x\sqrt{x} dx$

Solution

$$\begin{aligned} \int x\sqrt{x} dx &= \int x^{3/2} dx \\ &= \underline{\frac{2}{5}x^{5/2} + C} \end{aligned}$$

Exercise

Find the integral $\int \left(\frac{2}{\sqrt{x}} + 2\sqrt{x} \right) dx$

Solution

$$\begin{aligned} \int \left(\frac{2}{\sqrt{x}} + 2\sqrt{x} \right) dx &= \int \left(2x^{-1/2} + 2x^{1/2} \right) dx \\ &= 4x^{1/2} + \frac{4}{3}x^{3/2} + C \\ &= \underline{4\sqrt{x} + \frac{4}{3}x\sqrt{x} + C} \end{aligned}$$

Exercise

Find the integral $\int \left(x - 2x^2 + \frac{1}{2x} \right) dx$

Solution

$$\int \left(x - 2x^2 + \frac{1}{2x} \right) dx = \underline{\frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{2}\ln|x| + C}$$

Exercise

Find the integral $\int \left(\frac{7}{2x^3} - \sqrt[3]{x} \right) dx$

Solution

$$\begin{aligned} \int \left(\frac{7}{2x^3} - \sqrt[3]{x} \right) dx &= \int \left(\frac{7}{2} x^{-3} - x^{1/3} \right) dx \\ &= -\frac{7}{4} x^{-2} - \frac{3}{4} x^{4/3} + C \\ &= \underline{-\frac{7}{4x^2} - \frac{3}{4} x \sqrt[3]{x} + C} \end{aligned}$$

Exercise

Find the integral $\int 3e^{-2x} dx$

Solution

$$\int 3e^{-2x} dx = \underline{-\frac{3}{2} e^{-2x} + C}$$

Exercise

Find the integral $\int e^{-x} dx$

Solution

$$\int e^{-x} dx = \underline{-e^{-x} + C}$$

Exercise

Find the integral $\int e dx$

Solution

$$\int e dx = \underline{ex + C}$$

Exercise

Find the integral $\int \frac{7}{2e^{2x}} dx$

Solution

$$\begin{aligned}\int \frac{7}{2e^{2x}} dx &= \frac{7}{2} \int e^{-2x} dx \\ &= -\frac{7}{4} e^{-2x} + C \\ &= \underline{-\frac{7}{4e^{2x}} + C}\end{aligned}$$

Exercise

Find the integral $\int -3(e^{2x} + 1) dx$

Solution

$$\int -3(e^{2x} + 1) dx = \underline{-3\left(\frac{1}{2}e^{2x} + x\right) + C}$$

Exercise

Find the integral $\int \left(-3e^{-x} + 2x - \frac{1}{2}e^{5x}\right) dx$

Solution

$$\int \left(-3e^{-x} + 2x - \frac{1}{2}e^{5x}\right) dx = \underline{3e^{-x} + x^2 - \frac{1}{10}e^{5x} + C}$$

Exercise

Find the integral $\int \left(\sqrt[4]{x^3} + 1\right) dx$

Solution

$$\begin{aligned}\int \left(\sqrt[4]{x^3} + 1\right) dx &= \int \left(x^{3/4} + 1\right) dx \\ &= \underline{\frac{4}{7}x^{7/4} + x + C}\end{aligned}$$

Exercise

Find the integral $\int (5x^4 + 3x^2 + 2x + 5) dx$

Solution

$$\int (5x^4 + 3x^2 + 2x + 5) dx = \underline{x^5 + x^3 + x^2 + 5x + C}$$

Exercise

Find the integral $\int (5x^{4/3} + 3x^{2/3} + 2x^{1/3}) dx$

Solution

$$\int (5x^{4/3} + 3x^{2/3} + 2x^{1/3}) dx = \underline{\frac{15}{7}x^{7/3} + \frac{9}{5}x^{5/3} + \frac{3}{2}x^{4/3} + C}$$

Exercise

Find the integral $\int (5x^{-4/3} + 3x^{-2/3} + 2x^{-1/3}) dx$

Solution

$$\int (5x^{-4/3} + 3x^{-2/3} + 2x^{-1/3}) dx = \underline{-15x^{-1/3} + 9x^{1/3} + 3x^{2/3} + C}$$

Exercise

Find the integral $\int \frac{x^4 - 3x^2 + 5}{x^4} dx$

Solution

$$\begin{aligned} \int \frac{x^4 - 3x^2 + 5}{x^4} dx &= \int \left(1 - \frac{3}{x^2} + 5x^{-4} \right) dx \\ &= \underline{x + \frac{3}{x} - \frac{5}{3x^3} + C} \end{aligned}$$

Exercise

Find the integral $\int \left(\frac{3}{x^7} - \frac{5}{x^6} \right) dx$

Solution

$$\begin{aligned} \int \left(\frac{3}{x^7} - \frac{5}{x^6} \right) dx &= \int (3x^{-7} - 5x^{-6}) dx \\ &= -\frac{1}{2}x^{-6} + x^{-5} + C \\ &= \underline{-\frac{1}{2x^6} + \frac{1}{x^5} + C} \end{aligned}$$

Exercise

Find the integral $\int \frac{x+8}{\sqrt{x}} dx$

Solution

$$\begin{aligned} \int \frac{x+8}{\sqrt{x}} dx &= \int (x^{1/2} + 8x^{-1/2}) dx \\ &= \underline{\frac{2}{3}x^{3/2} + 16x^{1/2} + C} \end{aligned}$$

Exercise

Find the integral $\int \frac{x^2+8}{\sqrt[3]{x}} dx$

Solution

$$\begin{aligned} \int \frac{x^2+8}{\sqrt[3]{x}} dx &= \int (x^{5/3} + 8x^{-1/3}) dx \\ &= \underline{\frac{3}{8}x^{8/3} + 12x^{2/3} + C} \end{aligned}$$

Exercise

Find the integral $\int \cos\left(\frac{5\pi}{3}x\right) dx$

Solution

$$\int \cos\left(\frac{5\pi}{3}x\right) dx = \underline{\frac{3}{5\pi} \sin \frac{5\pi}{3}x + C}$$

Exercise

Find the integral $\int \sin\left(\frac{2x}{3}\right) dx$

Solution

$$\int \sin\left(\frac{2x}{3}\right) dx = \underline{-\frac{3}{2} \cos \frac{2x}{3} + C}$$

Exercise

Find the integral $\int (5 \cos x + 4 \sin x + 3 \sec^2 x) dx$

Solution

$$\int (5 \cos x + 4 \sin x + 3 \sec^2 x) dx = \underline{5 \sin x - 4 \cos x + 3 \tan x + C}$$

Exercise

Find the integral $\int \sec \theta (\sec \theta + \tan \theta) d\theta$

Solution

$$\begin{aligned} \int \sec \theta (\sec \theta + \tan \theta) d\theta &= \int (\sec^2 \theta + \sec \theta \tan \theta) d\theta \\ &= \underline{\tan \theta + \sec \theta + C} \end{aligned}$$

Exercise

Find the integral $\int (\tan^2 \theta + 1) d\theta$

Solution

$$\begin{aligned} \int (\tan^2 \theta + 1) d\theta &= \int \sec^2 \theta d\theta \\ &= \underline{\tan \theta + C} \end{aligned}$$

Exercise

Find the integral $\int (\cos^4 \theta - \sin^4 \theta) d\theta$

Solution

$$\begin{aligned} \int (\cos^4 \theta - \sin^4 \theta) d\theta &= \int (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) d\theta \\ &= \int \cos 2\theta d\theta && \cos^2 \theta - \sin^2 \theta = \cos 2\theta \quad \cos^2 \theta + \sin^2 \theta = 1 \\ &= \underline{\underline{\frac{1}{2} \sin 2\theta + C}} \end{aligned}$$

Exercise

Find the integral $\int (\cos^2 \theta - \sin^2 \theta) d\theta$

Solution

$$\begin{aligned} \int (\cos^2 \theta - \sin^2 \theta) d\theta &= \int \cos 2\theta d\theta && \cos^2 \theta - \sin^2 \theta = \cos 2\theta \\ &= \underline{\underline{\frac{1}{2} \sin 2\theta + C}} \end{aligned}$$

Exercise

Find the integral $\int (\cos^2 \theta + \sin^2 \theta) d\theta$

Solution

$$\begin{aligned} \int (\cos^2 \theta + \sin^2 \theta) d\theta &= \int (1) d\theta \\ &= \underline{\underline{\theta + C}} \end{aligned}$$

Exercise

Find the integral $\int (\cos 2x \cos 4x - \sin 2x \sin 4x) dx$

Solution

$$\int (\cos 2x \cos 4x - \sin 2x \sin 4x) dx = \int \cos 6x dx$$

$$= \frac{1}{6} \sin 6x + C$$

Exercise

Find the integral $\int (\sin 2x \cos 4x - \cos 2x \sin 4x) dx$

Solution

$$\begin{aligned} \int (\sin 2x \cos 4x - \cos 2x \sin 4x) dx &= \int \sin(-2x) dx \\ &= -\int \sin 2x dx \\ &= \frac{1}{2} \cos 2x + C \end{aligned}$$

Exercise

Find the integral $\int (\sin 3x \cos 2x + \cos 3x \sin 2x) dx$

Solution

$$\begin{aligned} \int (\sin 3x \cos 2x + \cos 3x \sin 2x) dx &= \int \sin 5x dx \\ &= -\frac{1}{5} \cos 5x + C \end{aligned}$$

Exercise

Find the integral $\int \cos 2x \sin 2x dx$

Solution

$$\begin{aligned} \int \cos 2x \sin 2x dx &= \frac{1}{2} \int \sin 4x dx \\ &= -\frac{1}{8} \cos 4x + C \end{aligned} \quad \sin \alpha = 2 \sin \alpha \cos \alpha$$

Exercise

Find the integral $\int (2 \cos^2 x - 1) dx$

Solution

$$\int (2 \cos^2 x - 1) dx = \int \cos 2x dx$$

$$= \frac{1}{2} \sin 2x + C$$

$$\cos 2x = 2 \cos^2 x - 1$$

Exercise

Find the integral $\int (1 - 2 \sin^2 x) dx$

Solution

$$\int (1 - 2 \sin^2 x) dx = \int \cos 2x dx$$

$$= \frac{1}{2} \sin 2x + C$$

$$\cos 2x = 1 - 2 \sin^2 x$$

Exercise

Find the integral $\int e^{-5x} dx$

Solution

$$\int e^{-5x} dx = -\frac{1}{5} e^{-5x} + C$$

Exercise

Find the integral $\int 4e^{4x} dx$

Solution

$$\int 4e^{4x} dx = e^{4x} + C$$

Exercise

Find the integral $\int (2 \sin \theta - 5e^\theta) d\theta$

Solution

$$\int (2 \sin \theta - 5e^\theta) d\theta = -2 \cos \theta - 5e^\theta + C$$

Exercise

Find the integral $\int \left(\frac{3}{x} + \sec^2 x \right) dx$

Solution

$$\int \left(\frac{3}{x} + \sec^2 x \right) dx = \underline{3 \ln|x| + \tan x + C}$$

Exercise

Find the integral $\int (\sin x + 2^x) dx$

Solution

$$\int (\sin x + 2^x) dx = \underline{-\cos x + \frac{2^x}{\ln 2} + C} \quad (a^x)' = a^x \ln a$$

Exercise

Find the integral $\int (2x - 3^x) dx$

Solution

$$\int (2x - 3^x) dx = \underline{x^2 - \frac{3^x}{\ln 3} + C} \quad (a^x)' = a^x \ln a$$

Exercise

Find the integral $\int \left(4x - \frac{3}{x} - \csc^2 x \right) dx$

Solution

$$\int \left(4x - \frac{3}{x} - \csc^2 x \right) dx = \underline{2x^2 - 3 \ln|x| + \cot x + C}$$

Exercise

Find the integral $\int \left(e^{4x} - \frac{3}{x} + 2 \csc x \cot x \right) dx$

Solution

$$\int \left(e^{4x} - \frac{3}{x} + 2 \csc x \cot x \right) dx = \underline{\frac{1}{4} e^{4x} - 3 \ln|x| - 2 \csc x + C}$$

Exercise

Find the integral $\int (a+b)e^{(a+b)x} dx$

Solution

$$\int (a+b)e^{(a+b)x} dx = \underline{e^{(a+b)x} + C}$$

Exercise

Find the integral $\int (a^2 - b^2)e^{(a-b)x} dx$

Solution

$$\begin{aligned} \int (a^2 - b^2)e^{(a-b)x} dx &= \frac{a^2 - b^2}{a - b} e^{(a-b)x} + C \\ &= \underline{(a+b)e^{(a-b)x} + C} \end{aligned}$$

Exercise

Find the function with the following property: $\frac{dy}{dx} = 2x - 7, \quad y(2) = 0$

Solution

$$\begin{aligned} \frac{dy}{dx} &= 2x - 7 \\ dy &= (2x - 7) dx \end{aligned}$$

$$\int dy = \int (2x - 7) dx$$

$$y = x^2 - 7x + C$$

At point (2, 0):

$$0 = 2^2 - 7(2) + C$$

$$0 = 4 - 14 + C$$

$$\rightarrow 0 = -10 + C$$

$$\underline{C = 10}$$

$$\underline{y(x) = x^2 - 7x + 10}$$

Exercise

Find the function with the following property:

$$\frac{dy}{dx} = \frac{1}{x^2} + x, \quad y(2) = 1; \quad x > 0$$

Solution

$$\frac{dy}{dx} = \frac{1}{x^2} + x$$

$$dy = (x^{-2} + x) dx$$

$$\int dy = \int (x^{-2} + x) dx$$

$$y = -x^{-1} + \frac{1}{2}x^2 + C$$

$$1 = -(2)^{-1} + \frac{1}{2}(2)^2 + C$$

$$1 + \frac{1}{2} - 2 = C$$

$$\underline{C = -\frac{1}{2} \quad |}$$

$$\underline{y(x) = -\frac{1}{x} + \frac{1}{2}x^2 - \frac{1}{2} \quad |}$$

Exercise

Find the function with the following property:

$$\frac{ds}{dt} = 1 + \cos t, \quad s(0) = 4$$

Solution

$$\frac{ds}{dt} = 1 + \cos t$$

$$ds = (1 + \cos t) dt$$

$$\int ds = \int (1 + \cos t) dt$$

$$s = t + \sin t + C$$

$$4 = 0 + \sin(0) + C$$

$$\underline{C = 4 \quad |}$$

$$\underline{s(t) = t + \sin t + 4 \quad |}$$

Exercise

Find the function with the following property: $\frac{ds}{dt} = \cos t + \sin t$, $s(\pi) = 1$

Solution

$$\frac{ds}{dt} = \cos t + \sin t$$

$$ds = (\cos t + \sin t) dt$$

$$\int ds = \int (\cos t + \sin t) dt$$

$$s = \sin t - \cos t + C$$

$$1 = \sin \pi - \cos \pi + C$$

$$1 = 0 - (-1) + C$$

$$1 = 1 + C$$

$$\underline{C = 0}$$

$$\underline{s(t) = \sin t - \cos t}$$

Exercise

Find the function with the following property: $f'(x) = 3x^2 - 1$ & $f(0) = 10$

Solution

$$f(x) = \int (3x^2 - 1) dx$$

$$= x^3 - x + C$$

$$f(0) = \underline{C = 10}$$

$$\underline{f(x) = x^3 - x + 10}$$

Exercise

Find the function with the following property: $f'(t) = \sin t + 2t$ & $f(0) = 5$

Solution

$$f(t) = \int (\sin t + 2t) dt$$

$$= -\cos t + t^2 + C$$

$$f(0) = -1 + C = 5$$

$$\underline{C = 6}$$

$$\underline{f(t) = -\cos t + t^2 + 6}$$

Exercise

Find the function with the following property: $f'(x) = x^2 + x^{-2}$ & $f(1) = 1$

Solution

$$\begin{aligned} f(x) &= \int (x^2 + x^{-2}) dx \\ &= \frac{1}{3}x^3 - \frac{1}{x} + C \end{aligned}$$

$$f(1) = \frac{1}{3} - 1 + C = 1$$

$$\underline{C = \frac{5}{3}}$$

$$\underline{f(x) = \frac{1}{3}x^3 - \frac{1}{x} + \frac{5}{3}}$$

Exercise

Find the function with the following property: $f'(x) = \sin^2 x$ & $f(1) = 1$

Solution

$$\begin{aligned} f(x) &= \int \sin^2 x \, dx & \sin^2 \theta &= \frac{1}{2}(1 - \cos 2\theta) \\ &= \frac{1}{2} \int (1 - \cos 2x) \, dx \\ &= \frac{1}{2} \left(x - \frac{1}{2} \sin 2x \right) + C \end{aligned}$$

$$f(1) = \frac{1}{2} - \frac{1}{4} \sin 2 + C = 1$$

$$\underline{C = \frac{1}{2} + \frac{1}{4} \sin 2}$$

$$\underline{f(x) = \frac{1}{2} \left(x - \frac{1}{2} \sin 2x \right) + \frac{1}{2} + \frac{1}{4} \sin 2}$$

Exercise

Derive the position function if a ball is thrown upward with initial velocity of 32 *feet* per second from an initial height of 48 *feet*. When does the ball hit the ground? With what velocity does the ball hit the ground?

Solution

$$s(t) = -16t^2 + 32t + 48$$

$$s(0) = 48$$

$$s'(0) = 32$$

$$s''(t) = -32$$

$$s'(t) = \int -32 dt$$

$$= -32t + C_1$$

$$s'(0) = -32(0) + C_1 = 32$$

$$\Rightarrow \underline{C_1 = 32}$$

$$s'(t) = -32t + 32$$

$$s(t) = \int (-32t + 32) dt$$

$$= -32 \frac{t^2}{2} + 32t + C_2$$

$$s(0) = -32 \frac{0^2}{2} + 32(0) + C_2 = 48$$

$$\Rightarrow \underline{C_2 = 48}$$

$$s(t) = -16t^2 + 32t + 48 = 0$$

$$-t^2 + 2t + 3 = 0$$

$$\underline{t_{1,2} = -1, 3}$$

The ball hits the ground in 3 *seconds*

The velocity: $v(t) = s'(t) = -32t + 32$

$$v(t = 3) = -32(3) + 32$$

$$= \underline{-64 \text{ ft/sec}^2}$$

Exercise

Suppose a publishing company has found that the marginal cost at a level of production of x thousand books is given by

$$\frac{dC}{dx} = \frac{50}{\sqrt{x}}$$

And that the fixed cost (the cost before the first book can be produced) is a \$25,000. Find the cost function $C(x)$.

Solution

$$\frac{dC}{dx} = \frac{50}{\sqrt{x}} = 50x^{-1/2}$$

$$dC = 50x^{-1/2} dx$$

$$\int dC = \int 50x^{-1/2} dx$$

$$\begin{aligned} C(x) &= 50 \frac{x^{1/2}}{1/2} + C \\ &= 50(2)x^{1/2} + C \\ &= 100\sqrt{x} + C \end{aligned}$$

$$25000 = 100\sqrt{0} + C$$

Before the first ($x = 0$) costs 25,000

$$\underline{25,000 = C}$$

$$\underline{C(x) = 100\sqrt{x} + 25,000}$$

Exercise

Find the general solution of $F'(x) = 4x + 2$, and find the particular solution that satisfies the initial condition $F(1) = 8$.

Solution

$$F(x) = \int (4x + 2) dx$$

$$= 4 \frac{x^2}{2} + 2x + C$$

$$= 2x^2 + 2x + C$$

$$F(x) = 2(1)^2 + 2(1) + C = 8$$

$$2 + 2 + C = 8$$

$$4 + C = 8$$

$$\underline{C = 4}$$

$$\underline{F(x) = 2x^2 + 2x + 4}$$

Exercise

The marginal cost function for producing x units of a product is modeled by

$$\frac{dC}{dx} = 28 - 0.02x$$

It costs \$40 to produce one unit. Find the cost of producing 200 units.

Solution

$$\begin{aligned} C &= \int (28 - 0.02x) dx \\ &= 28x - 0.02 \frac{x^2}{2} + K \end{aligned}$$

Cost \$40 for one unit

$$C(x=1) = 40$$

$$C(x=1) = 28(1) - 0.01(1)^2 + K = 40$$

$$K = 12.01$$

$$C(x) = -0.01x^2 + 28x + 12.01$$

$$\begin{aligned} C(200) &= -0.01(200)^2 + 28(200) + 12.01 \\ &= \underline{\$5212.01} \end{aligned}$$