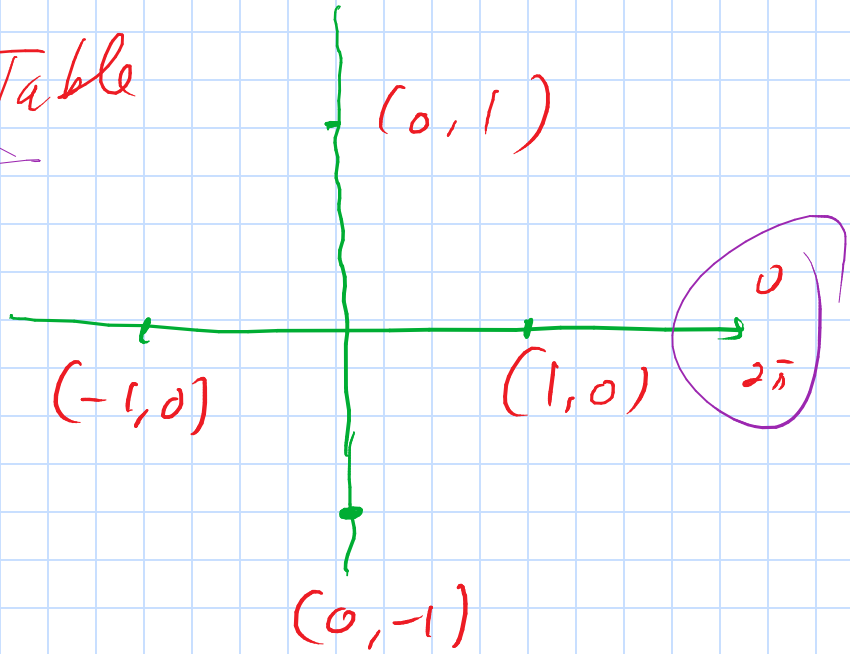
x	sine	cosine
$0+\varphi$	$0+D$	$A+D$
$\frac{1}{4}P+\varphi$	$A+D$	$0+D$
$\frac{1}{2}P$	0	$-A$
$\frac{3}{4}P$	$-A$	0
P	0	A

Table



$$y = A\sin(Bx+C)+D$$

$$y = A\cos(Bx+C)+D$$

- $P = \frac{2\pi}{|B|}$: Period
- Amplitude : $|A|$
- Phase shift : $\varphi = -\frac{C}{B}$ or $= -C\cdot\frac{1}{B}$
  or $(Bx+C=0)$ solve x
  Argument.
- Vertical Translation : VT : $y = D$

Ex $y = 3\sin 2x$ $A=3$, $B=2$, $C=0$, $D=0$

Period: $P = \frac{2\pi}{2} = \pi$

Amplitude: $|A| = 3$

phase shift: $\varphi = -\frac{C}{B} = 0$

V.T.: V.T.: $y = 0$

Ex $y = 2\sin\frac{1}{2}x$

$P = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$ $\varphi = -\frac{C}{B} = 0$

$|A| = 2$ V.T.: $y = 0$

Ex $y = -4\sin(-\pi x)$

$P = \frac{2\pi}{|-\pi|} = 2$ $\varphi = -\frac{C}{B} = 0$

$|A| = |-4| = 4$ V.T.: $y = 0$

Even & odd fctns

$\begin{cases} f(-x) = f(x) & \text{even fctn} \\ f(-x) = -f(x) & \text{odd fctn} \end{cases}$

neither

Even	Odd fctns
cosine, secant	sine, cosecant, tangent, cotangent

$y = \sin(-\theta)$

$= -\sin\theta$

$y = \cos(-\theta)$

$= \cos\theta$

Find |A|, period, phase shift, & vertical Translation, Table & graph (1 cycle)

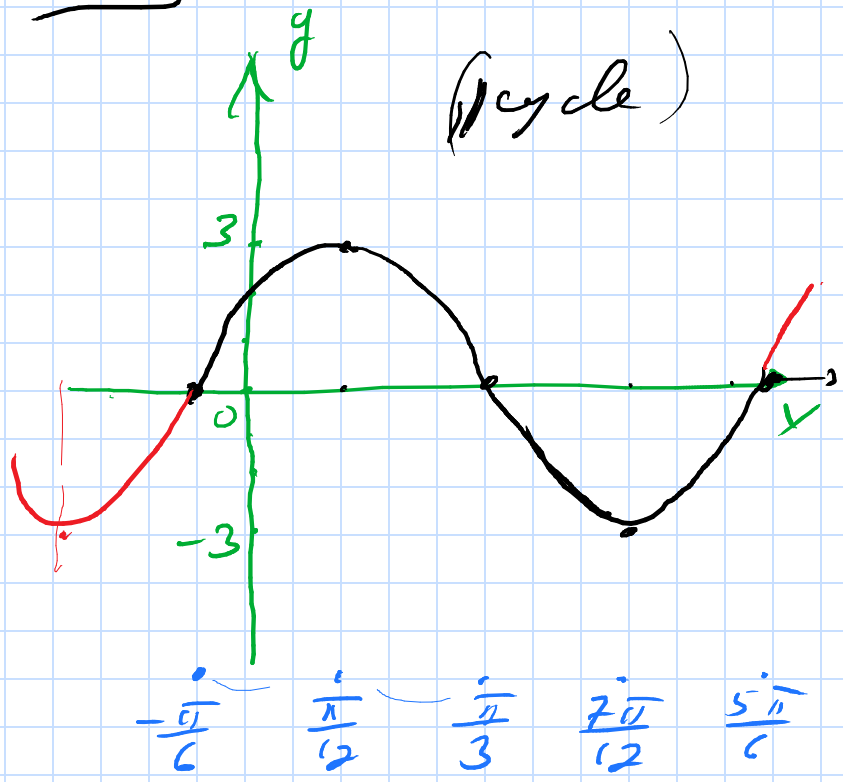
$$y = 3\sin\left(2x + \frac{\pi}{3}\right)$$

$|A| = 3$ $\quad P = \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$ ←

$\varphi = -\frac{C}{B} = -\frac{\pi}{3}\cdot\frac{1}{2} = -\frac{\pi}{6}$

V.T: $y = 0$ ✓

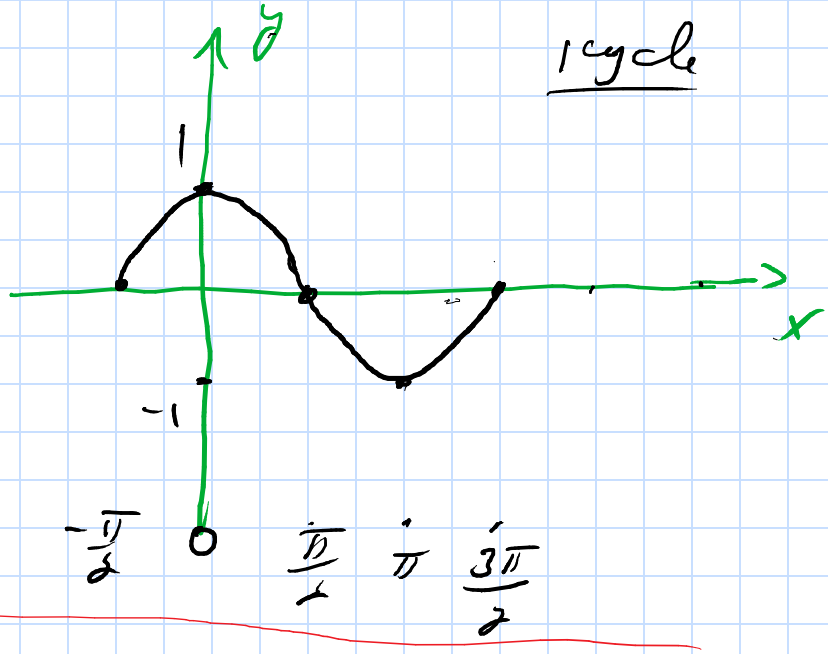
	$2x + \frac{\pi}{3}$	x	y
0	$-\frac{\pi}{6}$	$-\frac{\pi}{6}$	0
$\frac{\pi}{4}$	$-\frac{\pi}{6}$	$\frac{\pi}{12}$	3
$\frac{\pi}{2}$		$\frac{\pi}{3}$	0
$\frac{3\pi}{4}$		$\frac{7\pi}{12}$	−3
π		$\frac{5\pi}{6}$	0



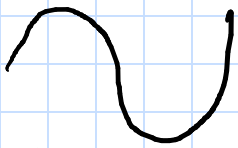
Ex $y = \sin\left(x + \frac{\pi}{2}\right)$

$|A| = 1$ $\quad P = \frac{2\pi}{b} = 2\pi$ $\quad \varphi = -\frac{c}{b} = -\frac{\pi}{2}$ $\quad V.T: y = 0$

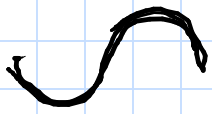
		x	y
0	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$	0
$\frac{\pi}{2}$	$-\frac{\pi}{2}$	0	1
π		$\frac{\pi}{2}$	0
$\frac{3\pi}{2}$		π	-1
2π		$\frac{3\pi}{2}$	0



Sine



+ sine



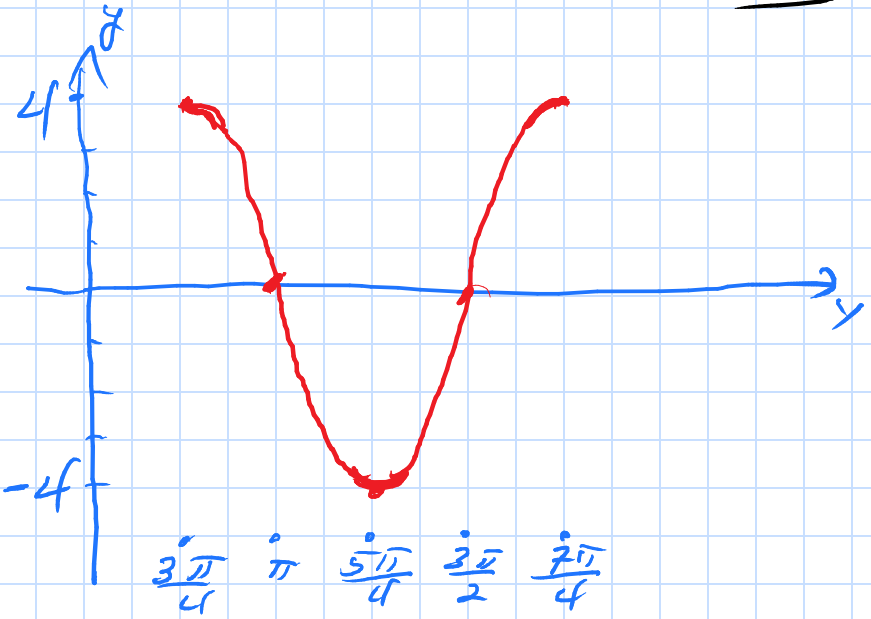
− sine

$y = 4\cos\left(2x - \frac{3\pi}{2}\right)$

$|A| = 4$ $\quad P = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ $\quad \varphi = -\frac{c}{b} = +\frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

$V.T: y = 0$

		x	y
0	$+\frac{3\pi}{4}$	$\frac{3\pi}{4}$	4
$\frac{\pi}{4}$	$+\frac{3\pi}{4}$	π	0
$\frac{\pi}{2}$		$\frac{5\pi}{4}$	-4
$\frac{3\pi}{4}$		$\frac{3\pi}{2}$	0
π		$\frac{7\pi}{4}$	4

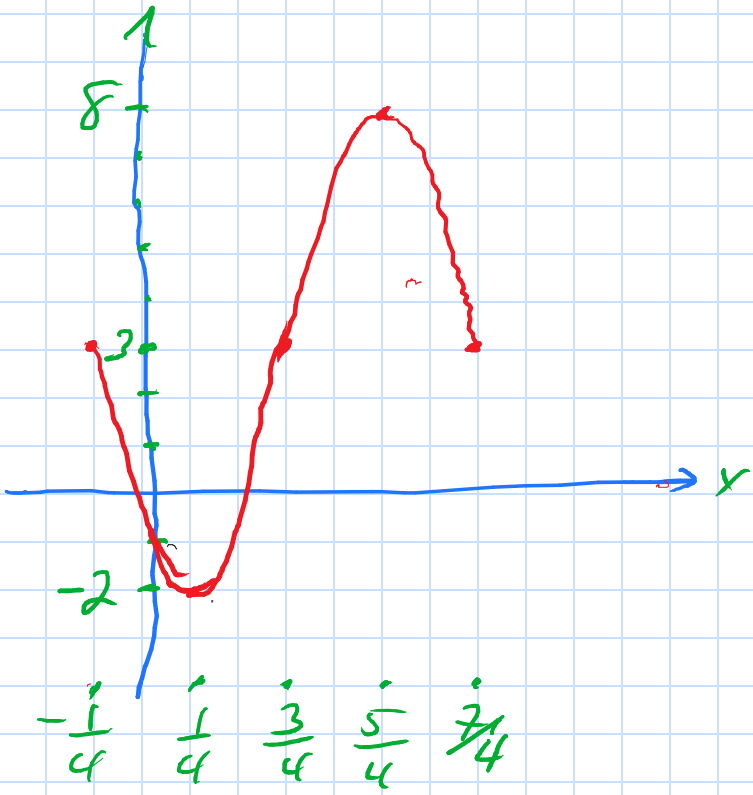


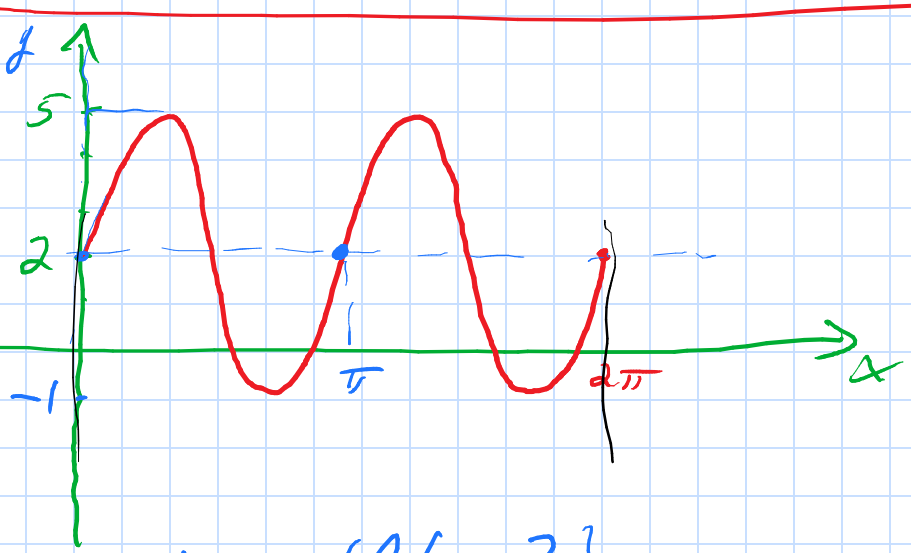
Ex $y = 3 - 5\sin\left(\pi x + \frac{\pi}{4}\right)$

$|A| = 5$ $\quad P = \frac{2\pi}{\pi} = 2$ $\quad \varphi = -\frac{\pi}{4} \cdot \frac{1}{\pi} = -\frac{1}{4}$

V.T: $y = 3$

	x	y	y
0 $-\frac{1}{4}$	$-\frac{1}{4}$	$0+3$	3
$\frac{1}{2}$ $-\frac{1}{4}$	$\frac{1}{4}$	$-5+3$	-2
1	$\frac{3}{4}$	$0+3$	3
$\frac{3}{2}$	$\frac{5}{4}$	$5+3$	8
2	$\frac{7}{4}$	$0+3$	3





$\varphi = 0 = C$ $\quad |A| = 3$

$P = \pi = \frac{2\pi}{B} \Rightarrow B = 2$

V.T. $y = 2 = D$

$y = 3\sin 2x + 2$ $\quad 0 \le x \le 2\pi$