# *Section* 1.2 – Graphing Linear Inequalities

A linear inequalities in two variables has the form

$$ax + by \le c$$
$$ax + by < c$$
$$ax + by \ge c$$
$$ax + by > c$$

1. Determine Line type
   *a.* A ***solid*** line if the original inequality $\le or \ge$
   *b.* A ***dashed*** line if the original inequality $< or >$
2. Choose a test point from on the half-planes (*Choose (0, 0) if the graph is not pass thru*)
3. If the result is True statement, shade the half-plane, otherwise, the other half

***Example***

Graph the linear inequality: $2x - 3y \le 12$

*Solution*

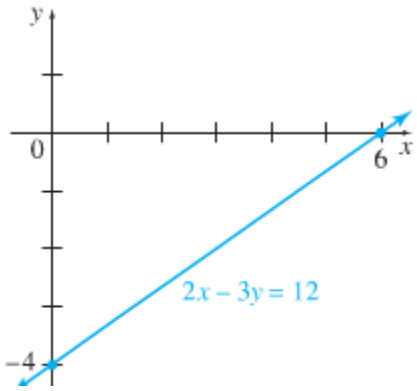
$2x - 3y = 12$

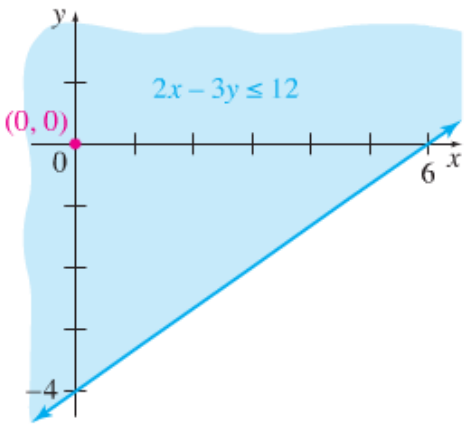
| ***x*** | ***y*** |
|---|---|
| 0 | -4 |
| 6 | 0 |

*Test*: using the point (0, 0)

$2(0) - 3(0) \le 12$

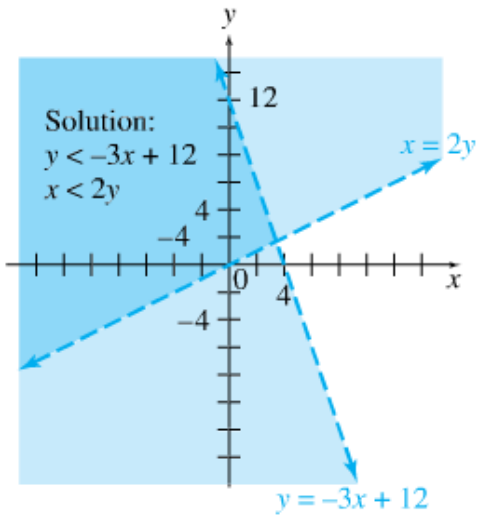
$0 \le 12 \rightarrow$ *True*



***Example***

Graph the system: $\begin{cases} y < -3x + 12 \\ x < 2y \end{cases}$




*Solution*

$y = -3x + 12$

| x | y |
|---|---|
| 0 | 12 |
| 4 | 0 |

$x = 2y$

| x | y |
|---|---|
| 0 | 0 |
| 2 | 1 |

Test: (0, 0)

$0 < -3(0) + 12 \rightarrow T$

(1,0)

$1 < 2(0) \rightarrow F$

## *Definition*

A region consisting of the overlapping parts of two or more graphs of inequalities (Heavily shaded region or the intersection) is called the ***region of feasible solution*** or the ***feasible region***.

A solution region of a system of linear inequalities

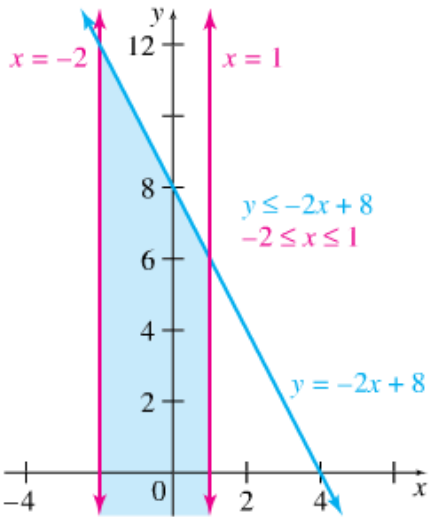
***Bounded***: if it can be enclosed within circle

***Unbounded***: $\infty$

***Corner point*** is a point in the feasible region where the boundary lines of two constraints cross.

***Example***

Graph the feasible region for the system $\begin{cases} y \le -2x + 8 \\ -2 \le x \le 1 \end{cases}$




*Solution*

$y = -2x + 8$

| x | y |
|---|---|
| 0 | 8 |
| 4 | 0 |

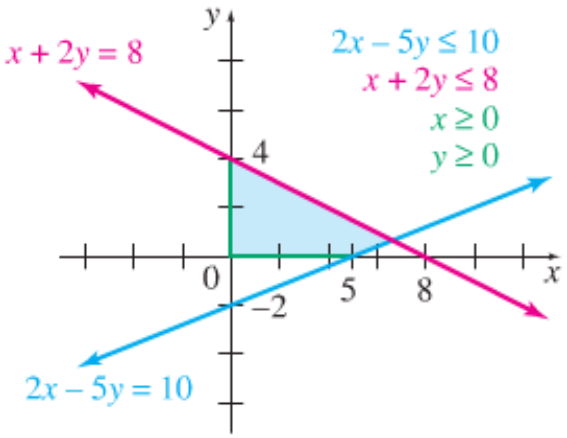
$0 \le -2(0) + 8$

$0 \le 8 \rightarrow$ *True*

***Example***

Graph the feasible region for the system $\begin{cases} 2x - 5y \le 10 \\ x + 2y \le 8 \\ x, y \ge 0 \end{cases}$

*Solution*



***Example***

Happy Ice Cream Cone Company makes cake cones and sugar cones, both of which must be processed in the mixing department and the baking department. Manufacturing one batch of cake cones requires 1 hour in the mixing department and 2 hours in the baking department, and producing one batch of sugar cones requires 2 hours in the mixing department and 1 hour in the baking department. Each department is operated for at most 12 hours per day.

*a)* Write a system of inequalities that expresses these restrictions.
*b)* Graph the feasible region
*c)* Using the graph from part (b), can 3 batches of cake cones and 2 batches of sugar cones be manufactured in one day
*d)* can 4 batches of cake cones and 6 batches of sugar cones be manufactured in one day

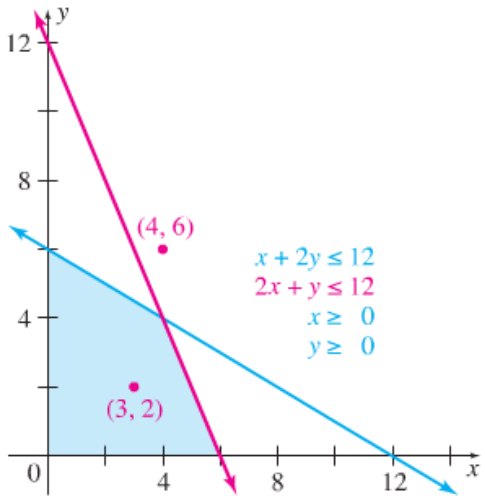
*Solution*

***a***) Let *x*: *number of cake cones*
*y*: *number of sugar cones*

| | ***Cake*** | ***Sugar*** | | ***Total*** |
|---|---|---|---|---|
| | ***x*** | ***y*** | | |
| *Hours in Mixing Dept.* | 1 | 2 | ≤ | 12 |
| *Hours in Baking Dept.* | 2 | 1 | ≤ | 12 |

$$\begin{cases} x+2y \le 12 \\ 2x+y \le 12 \end{cases}$$

***b***)




***c***) 3 batches of cake cones and 2 batches of sugar cones correspond to the point (3, 2). It is possible to manufacture these since the point (3, 2) is in the feasible region

***d***) 4 batches of cake cones and 6 batches of sugar cones correspond to the point (4, 6). It is *not* possible to manufacture these since the point (4, 6) is *not* in the feasible region

# *Exercises* *Section* 1.2 – Graphing Linear Inequalities

Graph the feasible region for the system

**1.** $\begin{cases} 3x - 2y \geq 6 \\ x + y \leq -5 \\ y \leq 4 \end{cases}$

**2.** $\begin{cases} x + y \geq 6 \\ 2x - y \geq 0 \end{cases}$

**3.** $\begin{cases} 3x + y \leq 21 \\ x - 2y \leq 0 \end{cases}$

**4.** $\begin{cases} 5x + y \geq 20 \\ x + y \geq 12 \\ x + 3y \geq 18 \\ x \geq 0 \\ y \geq 0 \end{cases}$

**5.** A manufacturing plant makes two types of inflatable boats, a two-person boat and a four-person boat. Each two-person boat requires 0.9 labor-hour in the cutting department and 0.8 labor-hour in the assembly department. Each four-person boat requires 1.8 labor-hours in the cutting department and 1.2 labor-hours in the assembly department. The maximum labor-hours available each month in the cutting and assembly departments are 864 and 672, respectively.

*a)* Summarize the information in a table

*b)* If *x* two-person boat and y four-person boats are manufactured each month, write a system of linear inequalities that reflect the conditions indicated. Find the set of feasible solutions graphically