$x^2 + y^2 = a^2$ circle $(0,0)$ center radius a.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipse

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Hyperbolas.

$d^2 = a^2 + b^2$

$\cancel{a^2x^2 + b^2y^2} = d^2$

$\frac{x^2}{b^2} \pm \frac{y^2}{a^2} = 1$

Ex $\frac{9x^2}{36} - \frac{4y^2}{36} = \frac{36}{36}$

$\frac{x^2}{4} - \frac{y^2}{9} = 1$

$a^2 = 4 \Rightarrow a = \pm 2$

$b^2 = 9 \rightarrow b = \pm 3$

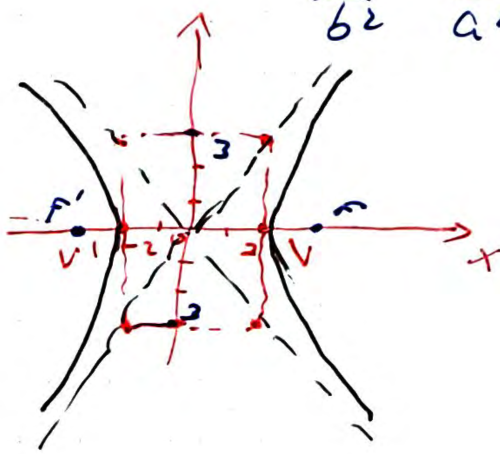
$c^2 = 13$

Vertices: $V(\pm 2, 0)$

endpoints: $W(0, \pm 3)$

foci: $F(\pm\sqrt{13}, 0)$

$c^2 = a^2 + b^2$ $(9+4)$
$= 13$



$y = \pm\frac{3}{2}x$

(2) $\frac{x^2}{4} - \frac{y^2}{9} = 0$

$\frac{y^2}{9} = \frac{x^2}{4}$

$y^2 = \frac{9}{4}x^2$

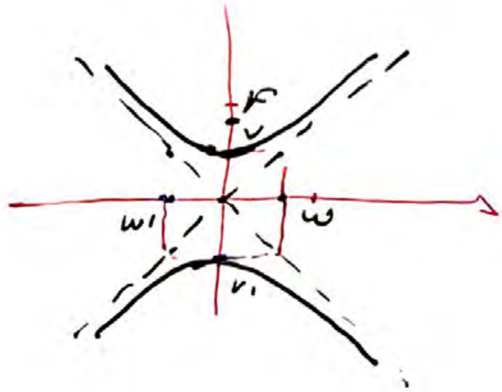
$y = \pm\frac{3}{2}x$ Asymptote

Ex $\quad 4y^2 - 2x^2 = 1$

$$\frac{y^2}{\frac{1}{4}} - \frac{x^2}{\frac{1}{2}} = 1$$

$a^2 = \frac{1}{4} \Rightarrow a = \pm \frac{1}{2}$

$b^2 = \frac{1}{2} \qquad b = \pm \frac{1}{\sqrt{2}}$



Vertices: $V\left(0, \pm \frac{1}{2}\right)$

endpoints: $w\left(\pm \frac{1}{\sqrt{2}}, 0\right)$

$c^2 = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

foci: $F\left(0, \pm \frac{\sqrt{3}}{2}\right)$

$4y^2 = 2x^2 \qquad y^2 = \frac{2}{4}x^2$

$y^2 = \frac{1}{2}x^2$

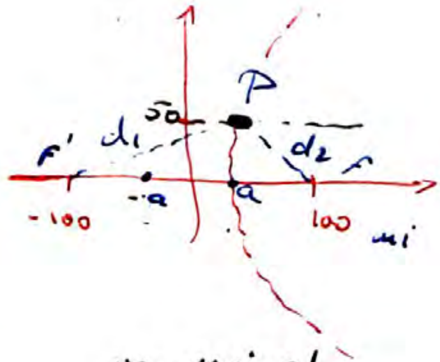
$y = \pm \frac{1}{\sqrt{2}} x$

Ex Given

$v = 980 \frac{ft}{\mu sec}$

$t = 400 \mu sec$

$P(x, y)$?



$d_2 - d_1 = vt$

$\doteq 980 \times 400 \quad ft$

$= 392 \; 10^3 \frac{1}{5280}$

$= \frac{196}{264} 10^2$

$d_2 - d_1 = 2a = \frac{49}{66} 10^2$

$v = \frac{mi}{hr} \frac{d}{t}$

$a = \frac{4900}{2(66)} = \frac{2450}{66} \approx 37.12$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{1378} - \frac{y^2}{b^2} = 1$

$\cdot P(x, 50)$

$c^2 = a^2 + b^2$

$c = 100$

$b^2 = c^2 - a^2$

$= (100)^2 - (37.12)^2$

$= 8,622$

$\frac{x^2}{1,378} - \frac{y^2}{8,622} = 1$

$\frac{x^2}{1,378} = 1 + \frac{50^2}{8,622}$

$\frac{a}{b} + 1 = \frac{}{b}$

$x^2 = 1378\left(1 + \frac{2500}{8622}\right)$

$\frac{11122}{8622}$

$x \approx 42.16$

$P(42, 50)$

27)

$$625y^2 - 400x^2 = 250{,}000$$

$$\frac{y^2}{400} - \frac{x^2}{625} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$a^2 = 400$$

$$a = \sqrt{400}$$

$$= 20$$

$10\sqrt{20}$

distance between 2 building

is $2a = \underline{40}$ yards. yd

5.5 Infinite Sequences

$a_1, a_2, a_3, \ldots, a_n, \ldots$

$a_n = \frac{n}{n+1}$

Ex 1^{st} 4 terms & $\underbrace{10^{th}\text{ term}}_{a_{10}}$ $\left\{\frac{n}{n+1}\right\}$ $f(x) = \frac{x}{x+1}$

$\cancel{a_1}$: $n = 1 \to \frac{1}{1+1} = \frac{1}{2}$

$n = 2 \to \frac{2}{2+1} = \frac{2}{3}$

$n = 3 \to a_3 = \frac{3}{3+1} = \frac{3}{4}$

$n = 4 \to a_4 = \frac{4}{4+1} = \frac{4}{5}$

$n = 10 \to a_{10} = \frac{10}{10+1} = \frac{10}{11}$

Ex 1^{st} 4, a_{10}? $\{2 + (.1)^n\}$

$n = 1 \to a_1 = 2 + .1 = 2.1$

$n = 2 \to a_2 = 2 + (.1)^2 = 2 + .01 = 2.01$

$n = 3 \Rightarrow a_3 = 2 + (.1)^3 = 2 + .001 = 2.001$

$n = 4 \to a_4 = 2 + (.1)^4 = 2 + .0001 = 2.0001$

$n = 10 \Rightarrow a_{10} = 2.0000000001$

Ex 1^{st} 4, a_{10} $\left\{(-1)^{n+1}\frac{n^2}{3n-1}\right\}$

$n = 1 \Rightarrow a_1 = (-1)^2 \frac{1}{3-1} = \frac{1}{2}$

$n = 2 \Rightarrow a_2 = (-1)^3 \frac{4}{6-1} = -\frac{4}{5}$

$n = 3 \Rightarrow a_3 = (-1)^4 \frac{9}{9-1} = \frac{9}{8}$

$n = 4 \Rightarrow a_4 = (-1)^5 \frac{16}{12-1} = -\frac{16}{11}$

$n = 10 \Rightarrow a_{10} = (-1)^{11} \frac{100}{30-1} = -\frac{100}{29}$

Ex 1^{st} 4, a_{10} $\{4\}$

$n=1$ $a_1=4$

$n=2 \to 4$

$n=3 \to 4$

$n=4 \to 4$

$n=10 \to \underline{4}$

Ex 1^{st} 4: $a_1 = 3$ $a_{n+1} = (n+1)\, a_n$

$n=0 \to a_1 = 3$

$n=1 \to a_2 = 2\, a_1 = 2(3) = \underline{6}$

$n=2 \to a_3 = 3\, a_2 = 3(6) = \underline{18}$

$n=3 \to a_4 = 4\, a_3 = 4(18) = \underline{72}$

#27 $a_1 = 2$. $a_2 = 2$ $a_n = a_{n-1} \cdot a_{n-2}$ ~~$a \cdot n-1$~~

1^{st} 5.

$a_1 = 2$

$a_2 = 2$

$a_3 = a_2 \cdot a_1 = 2(2) = \underline{4}$

$a_4 = a_3 \cdot a_2 = 4(2) = \underline{8}$

$a_5 = a_4 \cdot a_3 = 8(4) = \underline{32}$

#11 $\{c_n\} = \left\{ \frac{(-1)^n}{(n+1)(n+2)} \right\}$ 1^{st} 4? c_8 ?

$c_1 = \frac{(-1)^1}{2(3)} = -\underline{\frac{1}{6}}$

$c_2 = \frac{(-1)^2}{3(4)} = \underline{\frac{1}{12}}$

$c_3 = \frac{(-1)^3}{4(5)} = -\underline{\frac{1}{20}}$

$c_4 = \frac{(-1)^4}{(5)(6)} = \underline{\frac{1}{30}}$

$c_{10} = \frac{(-1)^{10}}{(11)(12)} = \underline{\frac{1}{132}}$

$$\begin{array}{r} 12 \\ \times 11 \\ \hline 132 \end{array} \qquad \begin{array}{r} 56 \\ \times 11 \\ \hline 616 \end{array}$$

Summation Notation

$$\text{sigma} \leftarrow \sum_{\underbrace{n=1}_{\text{initial}}}^{5 \leftarrow \text{last value}} a_n \} \text{ formula}$$

Ex $\displaystyle\sum_{k=1}^{4} k^2(k-3) = \underbrace{1(1-3)}_{k=1} + \overbrace{2^2(2-3)}^{k=2} + \underbrace{3^2(3-3)}_{k=3} + \overbrace{4^2(4-3)}^{k=4}$

$= -2 - 4 + 0 + 16$

$= 10$

$$\sum_{k=1}^{n} c = nc \qquad \sum_{k=m}^{n} c = (n-m+1)c$$

$$\sum_{k=10}^{20} 5 = 5(\overbrace{20-10+1}^{=11})$$

$= 55$

$$2^1 + 2^2 + 2^3 + \cdots + 2^{16} = \sum_{k=1}^{16} 2^k$$

1 3 5 $\displaystyle\sum_{1}^{3}$

\# 38 $\displaystyle\sum_{k=1}^{5}(2k-7) = (2-7)+(4-7)+(6-7)+(8-7)+(10-7)$

$= -5-3-1+1+3$

$= \underline{-5}$

$\displaystyle\sum_{k=1}^{5}(2k-7) = 2\sum_{k=1}^{5}k - \sum_{k=1}^{5}7$

$= 2(1+2+3+4+5) - 7(5)$

$= 30-35$

$= \underline{-7}$

---

42 $\displaystyle\sum_{k=1}^{50} 8 = 8(50)$

$= \underline{400}$

$$\begin{array}{r} 572 \\ 253 \\ \hline \end{array}$$

41 $\displaystyle\sum_{k=253}^{571}\left(\frac{1}{3}\right) = \frac{1}{3}(571-253+1)$

$= \frac{1}{3}(319)$

$= \underline{\frac{319}{3}}$

\#21 $a_1 = \sqrt{2}$ $\quad a_n = \sqrt{2+a_{n-1}}$

$a_1 = \sqrt{2}$

$a_2 = \sqrt{2+a_1} = \sqrt{2+\sqrt{2}}$

$a_3 = \sqrt{2+a_2} = \sqrt{2+\sqrt{2+\sqrt{2}}}$ ✓

$a_4 = \sqrt{2+a_3} = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$

\# 37 $\displaystyle\frac{1}{e}+\frac{2}{e^2}+\frac{3}{e^3}+\cdots+\frac{n}{e^n} = \sum_{k=1}^{n}\frac{k}{e^k}$

H 5.2 due Tomorrow
5.4

$\left\{ \frac{2^n}{n^2+2} \right\}$ 1^{st} 4.

$n=1 \rightarrow \frac{2}{1+2} = \frac{2}{3}$

$n=2 \rightarrow \frac{2^2}{4+2} = \frac{4}{6} = \frac{2}{3}$

$n=3 \rightarrow \frac{2^3}{9+2} = \frac{8}{11}$

$n=4 \rightarrow \frac{2^4}{16+2} = \frac{16}{18} = \frac{8}{9}$

Sequences $\begin{cases} \text{Arithmetic} & \underline{\underline{d}} \text{ difference} \\ \text{Geometric} & r \text{ ratio} \end{cases}$

Arithmetic

$a_{k+1} = a_k + d \rightarrow$ common difference

$d = a_{k+1} - a_k$

Ex $1, 4, 7, 10, \dots, \underbrace{3n-2}_{a_n}, \dots$

$d = 4 - 1 = 3$

$a_{k+1} - a_k = 3(k+1) - 2 - [3k-2]$

$= 3k + 3 - 2 - 3k + 2$

$= 3$ ✓

$1, 4, 7, 10, \dots$? $a_n =$