Solution   *Section* 3.7 – Probability Applications of Counting

Exercise

A basket contains 7 red apples and 4 yellow apples. A sample of 3 apples is drawn. Find the probabilities that the sample contains the following.

a) All red apples
b) All yellow apples
c) 2 yellow and 1 red apple
d) More red than yellow apples

Solution

a) $P(\text{All red apples}) = \dfrac{C_{7,3}}{C_{11,3}} = \dfrac{7}{33}$

b) $P(\text{All yellow apples}) = \dfrac{C_{4,3}}{C_{11,3}} = \dfrac{4}{165}$

c) $P(1\text{ red } \& \text{ } 2\text{ yellow apples}) = \dfrac{C_{7,1}C_{4,2}}{C_{11,3}} = \dfrac{42}{165} = \dfrac{14}{55}$

d) $P(\text{red} > \text{yellow}) = \dfrac{C_{7,2}C_{4,1} + C_{7,3}C_{4,0}}{C_{11,3}} = \dfrac{119}{165}$

Exercise

Two cards are drawn at random from a ordinary deck of 52. How many 2-card hands are possible?

Solution

$C_{52,2} = 1326$ *possibilities*

Exercise

Find the probability that the 2-card hand contains the following.

a) 2 aces
b) At least 1 ace
c) All spades
d) 2 cards of the same suit
e) Only face cards
f) No face cards
g) No card higher than 8 (count ace as 1)

Solution

a) $P(2\ aces) = \dfrac{C_{4,2}}{C_{52,2}} = \dfrac{6}{1326} \approx 0.0045$

b) $P(>1\ ace) = \dfrac{C_{4,1}C_{48,1} + C_{4,2}C_{48,0}}{C_{52,2}} \approx 0.149$

c) $P(2\ spades) = \dfrac{C_{13,2}}{C_{52,2}} \approx 0.059$

d) $P(2\ same\ suit) = \dfrac{4C_{13,2}}{C_{52,2}} \approx 0.235$

e) $P(\ face\ cards) = \dfrac{C_{12,2}}{C_{52,2}} \approx 0.0498$

f) $P(No\ face\ cards) = \dfrac{C_{40,2}}{C_{52,2}} \approx 0.588$

g) $P(<8) = \dfrac{C_{32,2}}{C_{52,2}} \approx 0.374$

Exercise

A reader wrote to the "Ask Marilyn" column in a magazine. "You have six envelopes to pick from. Two-thirds (= 4) are empty. One-third (= 2) contain a $100 bill. You're told to choose 2 envelopes at random. Which is more likely: **(1)** that you'll get at least one $100 bill, or **(2)** that you'll get no $100 bill at all?" Find the two probabilities.

Solution

$$P(at\ least\ one\ \$100\text{–}bill) = P(1\ \$100\text{–}bill) + P(2\ \$100\text{–}bill)$$

$$= \frac{C_{2,1}C_{4,1} + C_{2,2}C_{4,0}}{C_{6,2}}$$

$$= 0.6$$

$$P(no\ \$100\text{–}bill) = \frac{C_{2,0}C_{4,0}}{C_{6,2}} = 0.4$$

Exercise

After studying all night for a final exam, a bleary-eyed student randomly grabs 2 socks from a drawer containing 9 black, 6 brown, and 2 blue socks, all mixed together. What is the probability that she grabs a matched pair?

Solution

$$P(\text{matched pair}) = P(2\text{ black } or \text{ 2 brown } or \text{ 2 blue})$$
$$= P(2\text{ black}) + P(2\text{ brown}) + P(2\text{ blue})$$
$$= \frac{C_{9,2} + C_{6,2} + C_{2,2}}{C_{17,2}}$$
$$\underline{= 0.38}$$

Exercise

At a conference of writers, special-edition books were selected to be given away in contests. There were 9 books written by Hughes, 5 books by Baldwin, and 7 books by Morrison. The judge of one contest selected 6 books at random for prizes. Find the probabilities that he selection consisted of the following.

a) 3 Hughes and 3 Morrison books
b) Exactly 4 Baldwin books
c) 2 Hughes, 3 Baldwin, and 1 Morrison book
d) At least 4 Hughes books
e) Exactly 4 books written by males (Morrison is female)
f) No more than 2 books written by Baldwin

Solution

a) $P(3H \,\&\, 3M) = \dfrac{C_{9,3}C_{7,3}}{C_{21,6}} \underline{\approx 0.0542}$

b) $P(4B) = \dfrac{C_{5,4}C_{16,2}}{C_{21,6}} \underline{\approx 0.0111}$

c) $P(2H, 3B, \&1M) = \dfrac{C_{9,2}C_{5,3}C_{7,1}}{C_{21,6}} \underline{\approx 0.0464}$

d) $P(> 4H) = \dfrac{C_{9,4}C_{12,2} + C_{9,5}C_{12,1} + C_{9,6}C_{12,0}}{C_{21,6}} \underline{\approx 0.1827}$

e) $P(4 \text{ by males}) = \dfrac{C_{14,4}C_{7,2}}{C_{21,6}} \underline{\approx 0.3874}$

f) $P(< 2 \text{ by } B) = \dfrac{C_{5,2}C_{16,4} + C_{5,1}C_{16,5} + C_{5,0}C_{16,6}}{C_{21,6}} \underline{\approx 0.8854}$

Exercise

A school in Bangkok requires that students take an entrance examination. After the examination, there is a drawing in which 5 students are randomly selected from each group of 40 for automatic acceptance into the school, regardless of their performance on the examination. The drawing consists of pacing 35 red and 5 green pieces of paper into a box. Each student picks a piece of paper from the box and then does not return the piece of paper to the box. The 5 lucky students who pick the green pieces are automatically accepted into the school.

a) What is the probability that the first person wins automatic acceptance?
b) What is the probability that the last person wins automatic acceptance?
c) If the students are chosen by the order of their seating does this give the student who goes first a better chance of winning than the second, third… person?

(*Hint*: Imagine that the 40 pieces of paper have been mixed up and laid in a row so that the first student picks the first piece of paper, the second student picks the second piece of paper, and so on.)

Solution

a) $P(\text{first person}) = \frac{5}{40} \; \underline{= \frac{1}{8}}$

b) $P(\text{last person}) = \frac{5(39!)}{40!}$

$$= \frac{5}{40}$$

$$\underline{= \frac{1}{8}}$$

c) No one can have the same chance.

Exercise

A controversy arose in 1992 over the Teen Talk Barbie doll, each of which was programmed with four saying randomly picked from a set of 270 sayings. The controversy was over the saying, "Math class is tough," which some felt gave a negative message toward girls doing well in math. In an interview with Science, a spokeswoman for Mattel, the makers of Barbie, said that "There is a less than 1% chance you're going to get a doll that says math class is tough". Is this figure correct? If not, give the correct figure.

Solution

$$P(\text{Math class is tough}) = \frac{\binom{1}{1}\binom{269}{3}}{\binom{270}{4}} \; \underline{\approx .0148}$$

No, it is not correct.

The correct figure is 1.48%

Exercise

Bingo has become popular in the U.S., and it is an efficient way for many organizations to raise money. The bingo card has 5 rows and 5 columns of numbers from 1 to 75, with the center given as a free cell. Balls showing one of the 75 numbers are picked at random from a container. If the drawn number appears on a player's card, then the player covers the number. In general, the winner is the person who first has a card with an entire row, column, or diagonal covered.

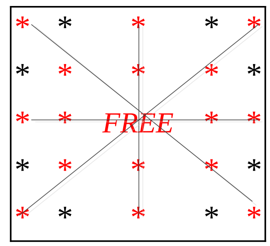
a) Find the probability that a person will win bingo after just four numbers are called.

b) An L occurs when the first column and the bottom row are both covered. Find the probability that an L will occur in the fewest number of calls.

c) An X-out occurs when both diagonals are covered. Find the probability than an X-out occurs in the fewest number of calls.

d) If bingo cards are constructed so that column one has 5 of the numbers from 1 to 15, column two has 5 of the numbers from 16 to 30, column three has 4 of the numbers from 31 to 45, column four has 5 of the numbers from 46 to 60, column five has 5 of the numbers from 61 to 75, how many different bingo cards could be constructed? (*Hint*: Order matters!)

Solution



a) There are only 4 ways to win in just 4 calls:

There are $C_{75,4}$ combinations of 4 numbers that can occur.

$$P(win\ bingo) = \frac{4}{C_{75,4}} \approx 3.291 \times 10^{-6}$$

b) There is only 1 way to get an L. It can occur in as few as 9 calls.

There are $C_{75,9}$ combinations of 9 numbers.

$$P(L\ occurs) = \frac{1}{C_{75,9}} \approx 7.962 \times 10^{-12}$$

c) There is only 1 way to get an X-out. It can occur in as few as 8 calls.

There are $C_{75,8}$ combinations of 8 numbers.

$$P(X-out\ occurs) = \frac{1}{C_{75,8}} \approx 5.927 \times 10^{-11}$$

d) Four columns contain a permutation of 15 numbers taken 5 at a time. One column contains a permutation of 15 numbers taken 4 at a time.

$$Number\ of\ \ differnt\ cards = P^{4}_{15,5} \bullet P_{15,4} \approx 5.524 \times 10^{26}$$