281-[redacted]

Ex $\quad y'' + 2y' + 2y = \cos 2t \qquad \begin{cases} y(0) = 0 \\ y'(0) = 1 \end{cases}$

$$\mathcal{L}\{y'' + 2y' + 2y\} = \mathcal{L}\{\cos 2t\}$$

$$\overbrace{s^2 Y(s) - s\,\overset{0}{\cancel{y(0)}} - \overset{=1}{y'(0)}}^{y''} + 2\overbrace{[sY(s) - \cancel{y(0)}]}^{y'}$$

$$+2Y(s) = \frac{s}{s^2+4}$$

$$(s^2+2s+2)\,Y(s) = \frac{s}{s^2+4} + 1$$

$$Y(s) = \frac{s^2+s+4}{(s^2+4)(s^2+2s+2)}$$

$$= \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+2s+2}$$

$$s^2+s+4 = (As+B)(s^2+2s+2) + (Cs+D)(s^2+4)$$

$s^3 \quad A + C = 0 \rightarrow C = -A$

$s^2 \quad 2A + B + D = 1$

$s^1 \quad 2A + 2B + 4C = 1$

$s^0 \quad 2B + 4D = 4 \rightarrow B = 2 - 2D$

$$\begin{cases} 2A + 2 - 2D + D = 1 \\ 2A + 4 - 4D - 4A = 1 \end{cases} \quad \begin{cases} 2A - D = -1 \\ -2A - 4D = -3 \end{cases}$$

$$-5D = -4$$

Ex cont

$$D = \frac{4}{5}, \quad B = 2 - \frac{8}{5} = \frac{2}{5}$$

$$2A = \frac{4}{5} - 1 = -\frac{1}{5} \Rightarrow A = -\frac{1}{10}$$

$$C = \frac{1}{10}$$

$$Y(s) = \frac{-\frac{1}{10}s + \frac{2}{5}}{s^2+4} + \frac{\frac{1}{10}s + \frac{4}{5}}{s^2+2s+2}$$

$$= \frac{-1}{10}\frac{s-4}{s^2+4} + \frac{1}{10}\frac{s+8}{s^2+2s+2}$$

$$= -\frac{1}{10}\frac{s}{s^2+4} + \frac{1}{5}\frac{2}{s^2+4}$$

$$+\frac{1}{10}\left[\frac{s+1}{(s+1)^2+1} + \frac{7}{(s+1)^2+1}\right]$$

$$\mathcal{L}^{-1}\{Y(s)\} = -\frac{1}{10}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$+\frac{1}{10}\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} + \frac{7}{10}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$

$$y(t) = -\frac{1}{10}\cos 2t + \frac{1}{5}\sin 2t$$

$$+\frac{1}{10}e^{-t}\cos t + \frac{7}{10}e^{-t}\sin t$$

Ex

$$y^{(4)} - y = 0 \qquad y(0) = 0 \quad y'(0) = 1$$
$$y''(0) = 0 \quad y'''(0) = 0$$

$$\mathcal{L}\{y^{(4)} - y\} = 0$$

$$s^4 Y(s) - s^3 y(0) - \underline{s^2 y'(0)} - s y''(0) - y'''(0)$$
$$- Y(s) = 0$$

$$(s^4 - 1)\, Y(s) = s^2$$

$$Y(s) = \frac{s^2}{(s-1)(s+1)(s^2+1)}$$

$$= \frac{A}{s-1} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}$$

$$s^2 = A(s+1)(s^2+1) + B(s-1)(s^2+1)$$
$$+ (Cs+D)(s^2-1)$$

s^3 $\quad A + B + C = 0 \leftarrow$ 1

s^2 $\quad A - B + D = 1$ 2 $\qquad A + B = 0$

s^1 $\quad A + B - C = 0 \leftarrow$ $\qquad 2A + 2B = 0$

s^0 $\quad A - B - D = 0$ $\qquad \boxed{B = -\frac{1}{4}}$

$$4A = 1 \Rightarrow \boxed{A = \frac{1}{4}}$$

1 $\rightarrow C = 0$

2 $\quad \boxed{D = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}}$

$$\mathcal{L}^{-1}\{Y(s)\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$
$$+ \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$y(t) = \frac{1}{4}e^{t} - \frac{1}{4}e^{-t} + \frac{1}{2}\sin t$$

#1 $\quad y' + y = te^{t} \qquad y(0) = -2$

$$\mathcal{L}\{y' + y\} = \mathcal{L}\{te^{t}\}$$

$$sY(s) - y(0) + Y(s) = \frac{1}{(s-1)^2}$$

$$(s+1)\,Y(s) = \frac{1}{(s-1)^2} - 2$$

$$Y(s) = \underbrace{\frac{1}{(s+1)(s-1)^2}} - \frac{2}{s+1}$$

$$\frac{1}{(s+1)(s-1)^2} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$1 = A(s^2 - 2s + 1) + B(s^2 - 1) + C(s+1)$$

$$s^2 \quad A + B = 0 \rightarrow \boxed{B = -A = -\tfrac{1}{4}}$$

$$s^1 \quad -2A + C = 0 \rightarrow \boxed{C = 2A = \tfrac{1}{2}}$$

$$s^0 \quad A - B + C = 1$$

$$\hookrightarrow A + A + 2A = 1 \Rightarrow \boxed{A = \tfrac{1}{4}}$$

$$\mathcal{L}^{-1} Y(s) = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{2}{s+1}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$+ \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\}$$

$$y(t) = +\frac{1}{4}e^{-t} - 2e^{-t} - \frac{1}{4}e^{t} + \frac{1}{2}te^{t}$$

$$\boxed{= -\frac{7}{4}e^{-t} - \frac{1}{4}e^{t} + \frac{1}{2}te^{t}}$$

$$t \rightarrow \frac{1}{s^2} \qquad t^n \rightarrow \frac{n!}{s^{n+1}}$$

$$e^{at} \rightarrow \frac{1}{s-a}$$

$$te^{at} \rightarrow \frac{1}{(s-a)^2}$$

$$\sin bt \rightarrow \frac{b}{s^2+b^2}$$

$$\cos bt \rightarrow \frac{s}{s^2+b^2}$$

#17 $y'' + y = -2\cos 2t \quad \Big| \; \begin{array}{l} y(0) = 1 \\ y'(0) = -1 \end{array}$

$$\mathcal{L}\{y'' + y\} = -2\,\mathcal{L}\{\cos 2t\}$$

$$s^2 Y(s) - s\overset{1}{y(0)} - \overset{-1}{y'(0)} + Y(s) = -2\,\frac{s}{s^2+4}$$

$$(s^2+1)\,Y(s) = \frac{-2s}{s^2+4} + s - 1$$

$$Y(s) = \underbrace{\frac{-2s}{(s^2+4)(s^2+1)}} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

$$\frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1} = \frac{-2s}{(\;)(\;)}$$

$$\begin{array}{lll} s^3 & A + C = 0 & \\ s^2 & B + D = 0 & \to -3C = +2 \\ s^1 & A + 4C = -2 & \boxed{C = -\frac{2}{3}} \\ s^0 & B + 4D = 0 & \boxed{A = +\frac{2}{3}} \end{array}$$

$$3D = 0 \Rightarrow D = 0 = B$$

$$\mathcal{L}^{-1}\{Y(s)\} = +\frac{2}{3}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - \frac{2}{3}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$$

$$+ \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$\boxed{y(t) = +\frac{2}{3}\cos 2t + \frac{1}{3}\cos t - \sin t}$$