

Ex

$$y \frac{dy}{dx} = x(y^2 + 1)$$

$$\frac{y dy}{y^2 + 1} = x dx$$

$$\frac{1}{2} \int \frac{d(y^2 + 1)}{y^2 + 1} = \int x dx$$

$$\frac{1}{2} \ln(y^2 + 1) = \frac{1}{2} x^2$$

$$\ln(y^2 + 1) = x^2 + C$$

$$y^2 + 1 = A e^{x^2}$$

Ex

$$\frac{dy}{dt} = y \cos t + y \quad y(0) = 2$$

$$\int \frac{dy}{y} = \int (\cos t + 1) dt$$

$$\ln |y| = \sin t + t + C$$

$$\ln 2 = C$$

$$\ln |y| = \sin t + t + \ln 2$$

$$y' + 4 \cot 2x y = 6 \cos 2x \quad y\left(\frac{\pi}{4}\right) = 2$$

$$e^{4 \int \cot 2x dx} = e^{2 \ln \sin 2x} = e^{\ln \sin^2 2x} \\ = \sin^2 2x$$

$$\int 6 \cos 2x \sin^2 2x dx = 3 \int \sin^2 2x d(\sin 2x) \\ = \sin^3 2x$$

$$y(x) = \frac{1}{\sin^2 2x} (\sin^3 2x + C) \\ = \sin 2x + \frac{C}{\sin^2 2x}$$

$$y\left(\frac{\pi}{4}\right) = 2 = 1 + C \Rightarrow C = 1$$

$$y(x) = \sin 2x + \frac{1}{\sin^2 2x}$$

$$y' - \frac{1}{2}y = 2\sin 3t$$

$$e^{\int -\frac{1}{2} dt} = e^{-t/2}$$

$$\int 2(\sin 3t) e^{-t/2} dt$$

	$\sin 3t$
$+ 2e^{-t/2}$	$-\frac{1}{3}\cos 3t$
$- e^{-t/2}$	$-\frac{1}{9}\sin 3t$

$$\int 2e^{-t/2} \sin 3t dt = e^{-t/2} \left(-\frac{2}{3}\cos 3t - \frac{1}{9}\sin 3t \right) + \frac{1}{2}e^{-t/2}$$

$$- \frac{1}{18} \int e^{-t/2} \sin 3t dt$$

$$\frac{37}{18}$$

$$\left(2 + \frac{1}{18}\right) \int e^{-t/2} \sin 3t dt = \frac{1}{9} (-6\cos 3t - \sin 3t) e^{-t/2}$$

$$2 \int e^{-3t/2} \sin 3t dt = \frac{2^4}{37} (-6\cos 3t - \sin 3t) e^{-t/2}$$

$$y(t) = e^{t/2} \left(-\frac{4}{37} (6\cos 3t + \sin 3t) e^{-t/2} + C \right)$$

$$= -\frac{24}{37} \cos 3t - \frac{4}{37} \sin 3t + C e^{t/2}$$

$$2xy - 9x^2 + (2y + x^2 + 1)y' = 0$$

$$M = 2xy - 9x^2$$

$$M_y = 2x$$

$$M_y = N_x = 2x \checkmark$$

$$N = 2y + x^2 + 1$$

$$N_x = 2x$$

$$\psi = \int (2xy - 9x^2) dx$$

$$= x^2y - 3x^3 + h(y)$$

$$\psi'_y = x^2 + h'(y) = 2y + x^2 + 1$$

$$h'(y) = 2y + 1$$

$$h(y) = \int (2y + 1) dy$$

$$= y^2 + y$$

$$x^2y - 3x^3 + y^2 + y = C$$

$$(x+y)^2 + (2xy + x^2 - 1) dy = 0 \quad y(1) = 1$$

$$M = x^2 + 2xy + y^2$$

$$M_y = 2x + 2y$$

$$M_y = N_x \checkmark$$

$$N = 2xy + x^2 - 1$$

$$N_x = 2y + 2x$$

$$\psi = \int (x^2 + 2xy + y^2) dx$$

$$= \frac{1}{3} x^3 + x^2 y + y^2 x + h(y)$$

$$\psi_y = x^2 + 2yx + h'(y) = 2xy + x^2 - 1$$

$$h'(y) = -1 \Rightarrow h(y) = -y$$

$$\frac{1}{3} x^3 + x^2 y + xy^2 - y = C$$

$$\frac{1}{3} + 1 + 1 - 1 = C \Rightarrow C = \frac{4}{3}$$

$$\frac{1}{3} x^3 + x^2 y + xy^2 - y = \frac{4}{3}$$
