Exercise

Solve $4^{2x-1} = 64$

Solution

$4^{2x-1} = 4^3$

$2x-1 = 3$

$2x = 4$

$x = 2$

Exercise

Solve $3^{1-x} = \frac{1}{27}$

Solution

$3^{1-x} = \frac{1}{3^3}$

$3^{1-x} = 3^{-3}$

$1 - x = -3$

$-x = -4$

$x = 4$

Exercise

Solve $9^x = \frac{1}{\sqrt[3]{3}}$

Solution

$\left(3^3\right)^x = \frac{1}{3^{1/3}}$

$3^{3x} = 3^{-1/3}$

$3x = -\frac{1}{3}$

$x = -\frac{1}{9}$

Exercise

Solve $5^{3x-6}=125$

Solution

$$5^{3x-6}=5^3$$
$$\Rightarrow 3x-6=3$$
$$\Rightarrow 3x=9$$
$$\Rightarrow x=3$$

Exercise

Solve $8^{x+2}=4^{x-3}$

Solution

$$\left(2^3\right)^{x+2}=\left(2^2\right)^{x-3}$$
$$2^{3(x+2)}=2^{2(x-3)}$$
$$3(x+2)=2(x-3)$$
$$3x+6=2x-6$$
$$3x-2x=-6-6$$
$$\underline{x=-12}$$

Exercise

Solve: $7e^{2x}-5=58$

Solution

$$7e^{2x}=63$$
$$e^{2x}=\frac{63}{7}=9$$
$$\ln e^{2x}=\ln 9$$
$$2x\ln e=\ln 9$$
$$2x=\ln 9$$
$$x=\frac{\ln 9}{2}\ \underline{\approx 1.1}$$

Exercise

Solve: $4\ln(3x) = 8$

Solution

$$\ln(3x) = 2$$

$$3x = e^2$$

$$\boxed{x = \frac{e^2}{3}}$$

Exercise

Solve: $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

Solution

$$\ln(x-3) = \ln\left(\frac{7x-23}{x+1}\right) \qquad \textit{Quotient Rule}$$

$$x - 3 = \frac{7x-23}{x+1} \qquad \textit{Cross multiply}$$

$$(x-3)(x+1) = 7x - 23$$

$$x^2 - 2x - 3 = 7x - 23$$

$$x^2 - 9x + 20 = 0 \qquad \textit{Solve for x}$$

$$\Rightarrow x = 4,\ 5$$

Check: $x = 4 \quad \Rightarrow \ln(4-3) = \ln(7(4)-23) - \ln(4+1)$

$x = 5 \quad \Rightarrow \ln(5-3) = \ln(7(5)-23) - \ln(5+1)$

Exercise

Use the properties of logarithms to rewrite $\log_b\left(\frac{x^3y}{z^2}\right)$

Solution

$$\log_b\left(\frac{x^3y}{z^2}\right) = \log_b\left(x^3y\right) - \log_b z^2$$

$$= \log_b x^3 + \log_b y - \log_b z^2$$

$$= 3\log_b x + \log_b y - 2\log_b z$$

Exercise

Use the properties of logarithms to rewrite $\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$

Solution

$$\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right) = \log_b\left(\sqrt[3]{x}y^4\right) - \log_b\left(z^5\right)$$

$$= \log_b\left(x^{1/3}\right) + \log_b\left(y^4\right) - \log_b\left(z^5\right)$$

$$= \frac{1}{3}\log_b(x) + 4\log_b(y) - 5\log_b(z)$$

Exercise

Use the properties of logarithms to rewrite $\log_b\sqrt[n]{\frac{x^3y^5}{z^m}}$

Solution

$$\log_b\sqrt[n]{\frac{x^3y^5}{z^m}} = \log_b\left(\frac{x^3y^5}{z^m}\right)^{1/n}$$

$$= \frac{1}{n}\log_b\left(\frac{x^3y^5}{z^m}\right) \qquad \textit{Power Rule}$$

$$= \frac{1}{n}\left(\log_b x^3y^5 - \log_b z^m\right) \qquad \textit{Quotient Rule}$$

$$= \frac{1}{n}\left(\log_b x^3 + \log_b y^5 - \log_b z^m\right) \qquad \textit{Product Rule}$$

$$= \frac{1}{n}\left(3\log_b x + 5\log_b y - m\log_b z\right) \qquad \textit{Power Rule}$$

$$= \frac{3}{n}\log_b x + \frac{5}{n}\log_b y - \frac{m}{n}\log_b z$$

Exercise

Use the properties of logarithms to rewrite $\log_p\sqrt[3]{\frac{m^5n^4}{t^2}}$

Solution

$$\log_p\sqrt[3]{\frac{m^5n^4}{t^2}} = \log_p\left(\frac{m^5n^4}{t^2}\right)^{1/3}$$

$$= \frac{1}{3}\log_p\left(\frac{m^5n^4}{t^2}\right)$$

$$= \frac{1}{3}\left(\log_p m^5n^4 - \log_p t^2\right)$$

$$= \frac{1}{3}\left(\log_p m^5 + \log_p n^4 - \log_p t^2\right)$$

$$= \frac{1}{3}\left(5\log_p m + 4\log_p n - 2\log_p t\right)$$

$$= \frac{5}{3}\log_p m + \frac{4}{3}\log_p n - \frac{2}{3}\log_p t$$

Exercise

Use the properties of logarithms to rewrite $\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}}$

Solution

$$\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}} = \log_a\left(\frac{m^8n^{12}}{a^3b^5}\right)^{1/4}$$

$$= \frac{1}{4}\log_a\left(\frac{m^8n^{12}}{a^3b^5}\right)$$

$$= \frac{1}{4}\left[\log_a m^8n^{12} - \log_a a^3b^5\right]$$

$$= \frac{1}{4}[\log_a m^8 + \log_a n^{12} - (\log_a a^3 + \log_a b^5)]$$

$$= \frac{1}{4}[\log_a m^8 + \log_a n^{12} - \log_a a^3 - \log_a b^5]$$

$$= \frac{1}{4}[8\log_a m + 12\log_a n - 3\log_a a - 5\log_a b]$$

$$= 2\log_a m + 3\log_a n - \frac{3}{4} - \frac{5}{4}\log_a b$$

Exercise

Solve $\log_x \frac{8}{27} = 3$

Solution

$\log_x \frac{8}{27} = 3$ *Write in exponential form*

$\frac{8}{27} = x^3$

$$x = \sqrt[3]{\frac{8}{27}}$$

$$= \frac{\sqrt[3]{8}}{\sqrt[3]{27}}$$

$$= \frac{2}{3}$$

Exercise

Solve $\log_3 \frac{1}{9} = x$

Solution

$$\frac{1}{3^2} = 3^x$$

$$3^{-2} = 3^x$$

$$x = -2$$

Exercise

Solve $3^x = 5$

Solution

$$\ln 3^x = \ln 5$$

$$x \ln 3 = \ln 5$$

$$x = \frac{\ln 5}{\ln 3}$$

$$\approx 1.465$$