Exercise

Find: $(-3,\ 0)\cap[-1,\ 2]$

Solution

$(-3,\ 0)\cap[-1,\ 2] = [-1,\ 0)$

Exercise

Find: $(-3,\ 0)\cup[-1,\ 2]$

Solution

$(-3,\ 0)\cup[-1,\ 2] = (-3,\ 2]$

Exercise

Find: $(-4,\ 0)\cap[-2,\ 1]$

Solution

$(-4,\ 0)\cap[-2,\ 1] = [-2,\ 0)$

Exercise

Find: $(-4,\ 0)\cup[-2,\ 1]$

Solution

$(-4,\ 0)\cup[-2,\ 1] = (-4,\ 1]$

Exercise

Find: $(-\infty,\ 5)\cap[1,\ 8)$

Solution

$(-\infty,\ 5)\cap[1,\ 8) = [1,\ 5)$

Exercise

Find: $(-\infty,\ 5)\cup[1,\ 8)$

Solution

$(-\infty,\ 5)\cup[1,\ 8) = \underline{(-\infty,\ 8)}$

Exercise

Solve $-3x+5>-7$

Solution

$-3x>-7-5$

$-3x>-12$

$\frac{-3}{-3}x<\frac{-12}{-3}$

∴ ***Solution***: $\underline{x<4}$ $\quad (-\infty,\ 4)$

Exercise

Solve $2-3x\le 5$

Solution

$-3x\le 3$ *Divide by – 3 both sides*

$\frac{-3}{-3}x\ge\frac{3}{-3}$

∴ ***Solution***: $\underline{x\ge -1}$ *or* $[-1,\infty)$

Exercise

Solve $4-3x\le 7+2x$

Solution

$4-3x-4\le 7+2x-4$

$-3x\le 3+2x$

$-3x-2x\le 3+2x-2x$

$-5x\le 3$

∴ ***Solution***: $\underline{x\ge -\frac{3}{5}}$ *or* $\left[-\frac{3}{5},\ \infty\right)$

Exercise

Solve the inequality equation $5x+11<26$

Solution

$5x<15$

$\therefore$ ***Solution***: $\underline{x<3}$ *or* $(-\infty,\ 3)$

Exercise

Solve the inequality equation $3x-8\geq 13$

Solution

$3x\geq 21$

$\therefore$ ***Solution***: $\underline{x\geq 7}$ *or* $[7,\ \infty)$

Exercise

Solve the inequality equation $-9x\geq 36$

Solution

$-\frac{9x}{9}\geq\frac{36}{9}$

$-x\geq 4$

$\therefore$ ***Solution***: $\underline{x\leq -4}$ *or* $(-\infty,\ -4]$

Exercise

Solve the inequality equation $-4x\leq 64$

Solution

$-\frac{4x}{4}\leq\frac{64}{4}$

$-x\leq 16$

$\therefore$ ***Solution***: $\underline{x\geq -16}$ *or* $[-16,\ \infty)$

Exercise

Solve the inequality equation $8x-11\leq 3x-13$

Solution

$5x\leq -2$

∴ ***Solution***: $\underline{x \le -\frac{2}{5}}$ *or* $\left(-\infty,\ -\frac{2}{5}\right]$

Exercise

Solve the inequality equation $18x + 45 \le 12x - 8$

Solution

$6x \le -53$

∴ ***Solution***: $\underline{x \le -\frac{53}{6}}$ *or* $\left(-\infty,\ -\frac{53}{6}\right]$

Exercise

Solve the inequality equation $4(x+1) + 2 \ge 3x + 6$

Solution

$4x + 4 + 2 \ge 3x + 6$

∴ ***Solution***: $\underline{x \ge 0}$ *or* $[0,\ \infty)$

Exercise

Solve the inequality equation $8x + 3 > 3(2x+1) + x + 5$

Solution

$8x + 3 > 6x + 3 + x + 5$

$8x + 3 > 7x + 8$

∴ ***Solution***: $\underline{x > 5}$ *or* $(5,\ \infty)$

Exercise

Solve the inequality equation $2x - 11 < -3(x+2)$

Solution

$2x - 11 < -3x - 6$

$5x < 5$

∴ ***Solution***: $\underline{x < 1}$ *or* $(-\infty,\ 1)$

Exercise

Solve the inequality equation $-4(x+2) > 3x + 20$

Solution

$-4x - 8 > 3x + 20$

$-7x > 28$

$-x > 4$

$\therefore$ ***Solution***: $\underline{x < -4}$ *or* $(-\infty, -4)$

Exercise

Solve the inequality equation $1 - (x+3) \geq 4 - 2x$

Solution

$1 - x - 3 \geq 4 - 2x$

$-x - 2 \geq 4 - 2x$

$x \geq 6$

$\therefore$ ***Solution***: $\underline{x \geq 6}$ *or* $[6, \infty)$

Exercise

Solve the inequality equation $5(3 - x) \leq 3x - 1$

Solution

$15 - 5x \leq 3x - 1$

$-8x \leq -16$

$-x \leq -2$

$\therefore$ ***Solution***: $\underline{x \geq 2}$ *or* $[2, \infty)$

Exercise

Solve the inequality equation $\frac{x}{4} - \frac{1}{2} \leq \frac{x}{2} + 1$

Solution

$4 \times \quad \frac{x}{4} - \frac{1}{2} \leq \frac{x}{2} + 1$

$x - 2 \leq 2x + 4$

$-x \leq 6$

$\therefore$ ***Solution***: $\underline{x \geq -6}$ *or* $[-6, \infty)$

Exercise

Solve the inequality equation $\frac{3x}{10}+1\geq\frac{1}{5}-\frac{x}{10}$

Solution

$10\times\ \frac{3x}{10}+1\geq\frac{1}{5}-\frac{x}{10}$

$3x+10\geq 2-x$

$4x\geq -8$

$\therefore$ ***Solution***: $\underline{x\geq -2}$ *or* $[-2,\ \infty)$

Exercise

Solve $6x-(2x+3)\geq 4x-5$

Solution

$6x-2x-3\geq 4x-5$

$4x-3\geq 4x-5$

$4x-4x\geq 3-5$

$0\geq -2\quad (true)$

$\therefore$ ***Solution***: $\mathbb{R}\ (-\infty,\ \infty)$

Exercise

Solve $\frac{2x-5}{-8}\leq 1-x$

Solution

$(-8)\frac{2x-5}{-8}\geq(-8)(1-x)$

$2x-5\geq -8+8x$

$2x-8x\geq -8+5$

$-6x\geq -3$

$\frac{-6}{-6}x\leq\frac{-3}{-6}$

$\therefore$ ***Solution***: $\underline{x\leq\frac{1}{2}}\qquad \left(-\infty,\ \frac{1}{2}\right]$

Exercise

Solve the inequality equation $1-\frac{x}{2}>4$

Solution

$2\times \quad 1-\frac{x}{2}>4$

$2-x>8$

$-x>6$

$\therefore$ ***Solution***: $\underline{x<-6}$ *or* $(-\infty,\ -6)$

Exercise

Solve the inequality equation $7-\frac{4}{5}x<\frac{3}{5}$

Solution

$5\times \quad 7-\frac{4}{5}x<\frac{3}{5}$

$35-4x<3$

$-4x>-32$

$\therefore$ ***Solution***: $\underline{x<8}$ *or* $(-\infty,\ 8)$

Exercise

Solve the inequality equation $\frac{x-4}{6}\geq\frac{x-2}{9}+\frac{5}{18}$

Solution

$18\times \quad \frac{x-4}{6}\geq\frac{x-2}{9}+\frac{5}{18}$

$3x-12\geq 2x-4+5$

$3x-12\geq 2x+1$

$\therefore$ ***Solution***: $\underline{x\geq -13}$ *or* $[-13,\ \infty)$

Exercise

Solve the inequality equation $\frac{4x-3}{6}+2\geq\frac{2x-1}{12}$

Solution

$12\times \quad \frac{4x-3}{6}+2\geq\frac{2x-1}{12}$

$8x-6+24\geq 2x-1$

$8x+18\geq 2x-1$

$6x\geq -19$

$\therefore$ ***Solution***: $\underline{x\geq -\frac{19}{6}}$ *or* $\left[-\frac{19}{6},\ \infty\right)$

Exercise

Solve the inequality equation $4(3x-2)-3x<3(1+3x)-7$

Solution

$12x-8-3x<3+9x-7$
$9x-8<9x-4$
$-8<-4$ *True*

$\therefore$ ***Solution***: $\mathbb{R}$

Exercise

Solve the inequality equation $3(x-8)-2(10-x)<5(x-1)$

Solution

$3x-24-20+2x<5x-5$
$5x-44<5x-5$
$-44<-5$ *True*

$\therefore$ ***Solution***: $\mathbb{R}$

Exercise

Solve the inequality equation $8(x+1)\le 7(x+5)+x$

Solution

$8x+8\le 7x+35+x$
$8x+8\le 8x+35$
$8\le 35$

$\therefore$ ***Solution***: $\mathbb{R}$

Exercise

Solve the inequality equation $4(x-1)\ge 3(x-2)+x$

Solution

$4x-4\ge 3x-6+x$
$4x-4\ge 4x-6$
$-4\ge -6$ *True*

$\therefore$ ***Solution***: $\mathbb{R}$

Exercise

Solve the inequality equation $7(x+4)-13>12+13(3+x)$

Solution

$7x+28-13>12+39+13x$

$7x+15>51+13x$

$-6x>36$

∴ ***Solution***: $\underline{x<-9}$ *or* $(-\infty,\ -9)$

Exercise

Solve the inequality equation $-2\left[7x-(2x-3)\right]<-2(x+1)$

Solution

$-2(7x-2x+3)<-2x-2$

$-2(5x+3)<-2x-2$

$-10x-6<-2x-2$

$-8x<4$

∴ ***Solution***: $\underline{x>\frac{1}{2}}$ *or* $\left(\frac{1}{2},\ \infty\right)$

Exercise

Solve the inequality equation $6-\frac{2}{3}(3x-12)\le\frac{2}{5}(10x+50)$

Solution

$15\times\quad 6-\frac{2}{3}(3x-12)\le\frac{2}{5}(10x+50)$

$90-10(3x-12)\le 6(10x+50)$

$90-30x+120\le 60x+300$

$210-30x\le 60x+300$

$-90x\le 90$

∴ ***Solution***: $\underline{x\ge -1}$ *or* $[-1,\ \infty)$

Exercise

Solve the inequality equation $\frac{2}{7}(7-21x)-4<10-\frac{3}{11}(11x-11)$

Solution

$77\times\ \frac{2}{7}(7-21x)-4<10-\frac{3}{11}(11x-11)$

$22(7-21x)-308<770-21(11x-11)$

$154-462x-308<770-231x+231$

$-462x-154<-231x+1,001$

$-231x<1,155$

$-x<5$

∴ ***Solution***: $\underline{x>5}$ *or* $(5,\ \infty)$

Exercise

Solve the inequality equation $3\left[3(x+5)+8x+7\right]+5\left[3(x-6)-2(3x-5)\right]<2(4x+3)$

Solution

$3(3x+15+8x+7)+5(3x-18-6x+10)<8x+6$

$3(11x+22)+5(-3x-8)<8x+6$

$33x+66-15x-40<8x+6$

$18x+26<8x+6$

$10x<-20$

∴ ***Solution***: $\underline{x<-2}$ *or* $(-\infty,\ -2)$

Exercise

Solve the inequality equation $5\left[3(2-3x)-2(5-x)\right]-6\left[5(x-2)-2(4x-3)\right]<3x+19$

Solution

$5(6-9x-10+2x)-6(5x-10-8x+6)<3x+19$

$5(-7x-4)-6(-3x-4)<3x+19$

$-35x-20+18x+24<3x+19$

$-17x+4<3x+19$

$-20x<15$

∴ ***Solution***: $\underline{x>-\frac{3}{4}}$ *or* $\left(-\frac{3}{4},\ \infty\right)$

Exercise

Solve the inequality equation $0\le 3x-1\le 10$

Solution

$0+1 \le 3x-1+1 \le 10+1$

$1 \le 3x \le 11$

$\therefore$ ***Solution***: $\boxed{\frac{1}{3} \le x \le \frac{11}{3}}$

Exercise

Solve the inequality equation $0 \le 1-3x \le 10$

Solution

$-1 \le -3x \le 9$

$-\frac{1}{3} \le -x \le 3$

$\therefore$ ***Solution***: $\boxed{-3 \le x \le \frac{1}{3}}$

Exercise

Solve the inequality equation $0 \le 2x+6 \le 54$

Solution

$-6 \le 2x \le 48$

$\therefore$ ***Solution***: $\boxed{-3 \le x \le 24}$

Exercise

Solve the inequality equation $-3 \le \frac{2}{3}x-5 \le -1$

Solution

$-3 \le \frac{2}{3}x-5 \le -1$

$-3+5 \le \frac{2}{3}x-5+5 \le -1+5$

$2 \le \frac{2}{3}x \le 4$

$2\frac{3}{2} \le \frac{3}{2}\frac{2}{3}x \le \frac{3}{2}4$

$\therefore$ ***Solution***: $\boxed{3 \le x \le 6}$

Exercise

Solve $-6 \le 6x + 3 \le 21$

Solution

$-6 - 3 \le 6x + 3 - 3 \le 21 - 3$

$-9 \le 6x \le 18$

$-\frac{9}{6} \le \frac{6}{6}x \le \frac{18}{6}$

$\therefore$ ***Solution***: $\underline{-\frac{3}{2} \le x \le 3}$ $\qquad \left[-\frac{3}{2},\ 3\right]$

Exercise

Solve the inequality equation: $1 \le 2x + 3 < 11$

Solution

$1 - 3 \le 2x + 3 - 3 < 11 - 3$

$-2 \le 2x < 8$

$-\frac{2}{2} \le \frac{2}{2}x < \frac{8}{2}$

$\therefore$ ***Solution***: $\underline{-1 \le x < 4}$

Exercise

Solve the inequality equation $\qquad |x| < 2$

Solution

$\therefore$ ***Solution***: $\underline{-2 < x < 2}$

Exercise

Solve the inequality equation $\qquad |x| \ge 2$

Solution

$\therefore$ ***Solution***: $\underline{x \le -2 \quad x \ge 2}$

Exercise

Solve the inequality equation $\qquad |x - 2| < 1$

Solution

$-1 < x - 2 < 1$

$\therefore$ ***Solution***: $1 < x < 3$

Exercise

Solve the inequality equation $|x - 1| < 4$

Solution

$-4 < x - 1 < 4$

$\therefore$ ***Solution***: $-3 < x < 5$

Exercise

Solve the inequality equation $|x + 2| \geq 1$

Solution

$x + 2 \leq -1$ $\quad$ $x + 2 \geq 1$

$x \leq -3$ $\quad$ $x \geq -1$

$\therefore$ ***Solution***: $x \leq -3 \quad x \geq -1$

Exercise

Solve the inequality equation $|x + 1| \geq 4$

Solution

$x + 1 \leq -4$ $\quad$ $x + 1 \geq 4$

$x \leq -5$ $\quad$ $x \geq 3$

$\therefore$ ***Solution***: $x \leq -5 \quad x \geq 3$

Exercise

Solve the inequality equation $|3x + 5| < 17$

Solution

$-17 < 3x + 5 < 17$

$-22 < 3x < 12$

$\therefore$ ***Solution***: $-11 < x < 4$

Exercise

Solve the inequality equation $|5x-2|<13$

Solution

$-13<5x-2<13$

$-11<5x<15$

$\therefore$ ***Solution***: $\boxed{-\frac{11}{5}<x<3}$

Exercise

Solve the inequality equation $|5x-2|\geq 13$

Solution

$5x-2\leq -13$	$5x-2\geq 13$
$5x\leq -11$	$5x\geq 15$
$x\leq -\frac{11}{5}$	$x\geq 3$

$\therefore$ ***Solution***: $\boxed{x\leq -\frac{11}{5} \quad x\geq 3}$

Exercise

Solve the inequality equation $|2(x-1)+4|\leq 8$

Solution

$-8\leq 2x-2+4\leq 8$

$-8\leq 2x+2\leq 8$

$-10\leq 2x\leq 6$

$\therefore$ ***Solution***: $\boxed{-5\leq x\leq 3}$

Exercise

Solve the inequality equation $|3(x-1)+2|\leq 20$

Solution

$-20\leq 3x-3+2\leq 20$

$-20\leq 3x-1\leq 20$

$-19\leq 3x\leq 21$

$\therefore$ ***Solution***: $\boxed{-\frac{19}{3}\leq x\leq 7}$

Exercise

Solve the inequality equation $\left|\frac{2x+6}{3}\right| > 2$

Solution

$|2x+6| > 6$

$2x+6 < -6$	$2x+6 > 6$
$2x < -12$	$2x > 0$
$x < -6$	$x > 0$

$\therefore$ ***Solution***: $x < -6 \quad x > 0$

Exercise

Solve the inequality equation $\left|\frac{3x-3}{4}\right| < 6$

Solution

$\frac{3}{4}|x-1| < 6$

$|x-1| < 8$

$-8 < x-1 < 8$

$\therefore$ ***Solution***: $-7 < x < 9$

Exercise

Solve the inequality equation $\left|\frac{2x+2}{4}\right| \geq 2$

Solution

$\frac{1}{2}|x+1| \geq 2$

$|x+1| \geq 4$

$x+1 \leq -4$	$x \geq 3$

$\therefore$ ***Solution***: $x \leq -4 \quad x \geq 3$

Exercise

Solve the inequality equation $\left|\frac{3x-3}{9}\right| \leq 1$

Solution

$\frac{1}{3}|x-1| \le 1$

$|x-1| \le 3$

$-3 \le x-1 \le 3$

$\therefore$ ***Solution***: $\underline{-2 \le x \le 4}$

Exercise

Solve the inequality equation $\left|3-\frac{2x}{3}\right| > 5$

Solution

$\frac{1}{3}|9-2x| > 5$

$|9-2x| > 15$

$9-2x < -15$	$9-2x > 15$
$-2x < -24$	$-2x > 6$
$x > 12$	$x < -3$

$\therefore$ ***Solution***: $\underline{x < -3 \quad x > 12}$

Exercise

Solve the inequality equation $\left|3-\frac{3x}{4}\right| < 9$

Solution

$\frac{3}{4}|4-x| < 9$

$|4-x| < 12$

$-12 < 4-x < 12$

$-16 < -x < 8$

$16 > x > -8$

$\therefore$ ***Solution***: $\underline{-8 < x < 16}$

Exercise

Solve the inequality equation $|x-2| < -1$

Solution

Impossible, since Absolute value can't be negative.

$\therefore$ No ***Solution***

Exercise

Solve the inequality equation $|x+2| < -3$

Solution

Impossible, since Absolute value can't be negative.

∴ No ***Solution***

Exercise

Solve the inequality equation $|x+6| > -10$

Solution

∴ ***Solution:*** $\mathbb{R}$

Exercise

Solve the inequality equation $|x+2| > -8$

Solution

∴ ***Solution:*** $\mathbb{R}$

Exercise

Solve the inequality equation $|x+2|+9 \le 16$

Solution

$|x+2| \le 5$

$-5 \le x+2 \le 5$

∴ ***Solution***: $-7 \le x \le 3$

Exercise

Solve the inequality equation $|x-2|+4 \ge 5$

Solution

$|x-2| \ge 1$

$x-2 \le -1$ $\quad$ $x-2 \ge 1$

$x \le 1$ $\quad$ $x \ge 3$

∴ ***Solution***: $x \le 1 \quad x \ge 3$

Exercise

Solve the inequality equation $2|2x-3|+10>12$

Solution

$2|2x-3|>2$

$|2x-3|>1$

$2x-3<-1$	$2x-3>1$
$2x<2$	$2x>4$
$x<1$	$x>2$

∴ ***Solution***: $\underline{x<1 \quad x>3}$

Exercise

Solve the inequality equation $3|2x-1|+2<8$

Solution

$3|2x-1|<6$

$|2x-1|<2$

$-2<2x-1<2$

$-1<2x<3$

∴ ***Solution***: $\underline{-\frac{1}{2}<x<\frac{3}{2}}$

Exercise

Solve the inequality equation $-4|1-x|<-16$

Solution

$|1-x|>4$

$1-x<-4$	$1-x>4$
$-x<-5$	$-x>3$
$x>5$	$x<-3$

∴ ***Solution***: $\underline{x<-3 \quad x>5}$

Exercise

Solve the inequality equation $-2|5-x|<-6$

Solution

$|5-x|>3$

$5-x<-3$	$5-x>3$
$-x<-8$	$-x>-2$
$x>8$	$x<2$

∴ ***Solution***: $x<2 \quad x>8$

Exercise

Solve the inequality equation $3\le|2x-1|$

Solution

$|2x-1|\ge 3$

$2x-1\le -3$	$2x-1\ge 3$
$2x\le -2$	$2x\ge 4$
$x\le -1$	$x\ge 2$

∴ ***Solution***: $x\le -1 \quad x\ge 2$

Exercise

Solve the inequality equation $9\le|4x+7|$

Solution

$|4x+7|\ge 9$

$4x+7\le -9$	$4x+7\ge 9$
$4x\le -16$	$4x\ge 2$
$x\le -4$	$x\ge \frac{1}{2}$

∴ ***Solution***: $x\le -4 \quad x\ge \frac{1}{2}$

Exercise

Solve the inequality equation: $12<\left|-2x+\frac{6}{7}\right|+\frac{3}{7}$

Solution

$(7)12<\left|-(7)2x+(7)\frac{6}{7}\right|+(7)\frac{3}{7}$ *Multiply by 7 both sides*

$84<|-14x+6|+3$

$81<|-14x+6|$

$|-14x+6|>81$

$-14x+6<-81$ $\quad$ $-14x+6>81$

$-14x<-81-6$ $\quad$ $-14x>81-6$

$-14x<-87$ $\quad$ $-14x>75$

$x>\frac{87}{14}$ $\quad$ $x<-\frac{75}{14}$

∴ ***Solution***: $\underline{x<-\frac{75}{14} \quad x>\frac{87}{14}}$ $\quad$ $\left(-\infty,-\frac{75}{14}\right)\cup\left(\frac{87}{14},\infty\right)$

Exercise

Solve the inequality equation: $4+\left|3-\frac{x}{3}\right|\geq 9$

Solution

$\left|3-\frac{x}{3}\right|\geq 9-4$

$\left|3-\frac{x}{3}\right|\geq 5$

$\left|(3)3-(3)\frac{x}{3}\right|\geq (3)5$

$|9-x|\geq 15$

$9-x\leq -15$ $\quad$ $9-x\geq 15$

$-x\leq -24$ $\quad$ $-x\geq 6$

$x\geq 24$ $\quad$ $x\leq -6$

∴ ***Solution***: $\underline{x\leq -6 \quad x\geq 24}$ $\quad$ $(-\infty,\ -6]\cup[24,\ \infty)$

Exercise

Solve the inequality equation: $|x-2|<5$

Solution

$-5<x-2<5$

∴ ***Solution***: $\underline{-3<x<7}$

Exercise

Solve the inequality equation: $|2x+1|<7$

Solution

$-7<2x+1<7$

$-7-1<2x+1-1<7-1$

$-8<2x<6$

$-\frac{8}{2}<\frac{2}{2}x<\frac{6}{2}$

$\therefore$ ***Solution***: $\boxed{-4<x<3}$

Exercise

Solve the inequality equation: $|5x+2|-2<3$

Solution

$|5x+2|<5$

$-5<5x+2<5$

$-7<5x<3$

$\therefore$ ***Solution***: $\boxed{-\frac{7}{5}<x<\frac{3}{5}}$ $\left(-\frac{7}{5},\ \frac{3}{5}\right)$

Exercise

Solve the inequality equation: $|2-7x|-1>4$

Solution

$|2-7x|>5$

$2-7x<-5$	$2-7x>5$
$-7x<-7$	$-7x>3$
$x>1$	$x<-\frac{3}{7}$

$\therefore$ ***Solution***: $\boxed{x<-\frac{3}{7} \quad x>\frac{1}{14}}$ $\left(-\infty,\ -\frac{3}{7}\right)\cup(1,\ \infty)$

Exercise

Solve the inequality equation: $|3x-4|<2$

Solution

$-2<3x-4<2$

$-2+4<3x-4+4<2+4$

$2<3x<6$

$\therefore$ ***Solution***: $\boxed{\frac{2}{3}<x<2}$ $\boxed{\left(\frac{2}{3},\ 2\right)}$

Exercise

Solve the inequality equation: $|2x+5| \geq 3$

Solution

$2x+5 \leq -3$ $\qquad$ $2x+5 \geq 3$

$2x \leq -8$ $\qquad$ $2x \geq -2$

$x \leq -4$ $\qquad$ $x \geq -1$

$\therefore$ ***Solution***: $\underline{x \leq -4 \quad x \geq -1}$ $\quad (-\infty, \ -4] \cup [-1, \ \infty)$

Exercise

Solve $|12-9x| \geq -12$

Solution

$\therefore$ ***Solution*** set: $(-\infty, \ \infty)$ because the absolute value always greater than any negative number.

Exercise

Solve $|6-3x| < -11$

Solution

$\therefore$ ***No solution***, because the absolute value cannot be less than any negative number

Exercise

Solve the inequality equation $|7+2x| < 0$

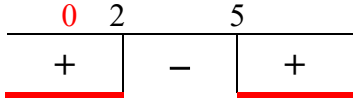
Solution

$\therefore$ ***No solution***, because the absolute value cannot be any negative number

Exercise

Solve: $x^2 - 7x + 10 > 0$

Solution



$x^2 - 7x + 10 > 0$

$(x-5)(x-2) > 0$

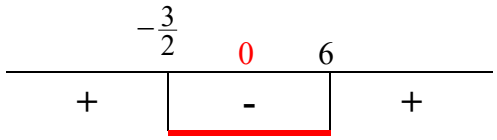
$\underline{x = 2, \ 5|}$

$\therefore$ ***Solution***: $\underline{x < 2 \quad x > 5}$ $\quad (-\infty, 2) \cup (5, \infty)$

Exercise

Solve: $2x^2 - 9x \le 18$

Solution



$2x^2 - 9x - 18 \le 0$

$(2x+3)(x-6) \le 0$

$\therefore$ ***Solution***: $-\frac{3}{2} \le x \le 6$ $\quad \left[-\frac{3}{2},\ 6\right]$

Exercise

Solve the inequality: $x^2 - 5x + 4 > 0$

Solution

$x^2 - 5x + 4 > 0$

$x = 1,\ 4$

$\therefore$ ***Solution***: $x < 1 \quad x > 4$ $\quad (-\infty,1) \cup (4,\infty)$

Exercise

Solve $\quad x^2 + x - 2 > 0$

Solution

$x^2 + x - 2 = 0 \quad \rightarrow x = -2,1$

$\therefore$ ***Solution***: $x < -2 \quad x > 1$ $\quad (-\infty,-2) \cup (1,\infty)$

Exercise

Solve $\quad x^2 - 4x + 12 < 0$

Solution

$x^2 - 4x + 12 = 0$

$$x = \frac{4 \pm \sqrt{16 - 48}}{2}$$

$$= \frac{4 \pm \sqrt{-32}}{2}$$ *Complex number*

$\therefore$ No ***Solution***

Exercise

Solve the inequality equation $x^2 + 7x > 0$

Solution

$x^2 + 7x = 0$

$x(x+7) = 0$

$x = 0, \ -7$

∴ ***Solution***: $x < -7 \quad x > 0$

Exercise

Solve the inequality equation $x^2 - 49 < 0$

Solution

$x^2 - 49 = 0$

$x = \pm 7$

∴ ***Solution***: $-7 < x < 7$

Exercise

Solve the inequality equation $x^2 - 5x \geq 0$

Solution

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0, \ 5$

∴ ***Solution***: $x \leq 0 \quad x \geq 5$

Exercise

Solve the inequality equation $x^2 - 16 \leq 0$

Solution

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

∴ ***Solution***: $-4 \leq x \leq 4$

Exercise

Solve the inequality equation $x^2 + 7x + 10 < 0$

Solution

$$x^2 + 7x + 10 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{2}$$

$$= \frac{-7 \pm 3}{2}$$

$$= \begin{cases} \frac{-7-3}{2} = -5 \\ \frac{-7+3}{2} = -2 \end{cases}$$

∴ ***Solution***: $-5 < x < 2$

Exercise

Solve the inequality equation $x^2 - 3x \geq 28$

Solution

$$x^2 - 3x - 28 = 0$$

$$x = \frac{3 \pm \sqrt{121}}{2}$$

$$= \begin{cases} \frac{3-11}{2} = -4 \\ \frac{3+11}{2} = 7 \end{cases}$$

∴ ***Solution***: $x \leq -4 \quad x \geq 7$

Exercise

Solve the inequality equation $x^2 + 5x + 6 < 0$

Solution

$$x^2 + 5x + 6 = 0$$

$$x = \frac{-5 \pm 1}{2}$$

$$= \begin{cases} \frac{-5-1}{2} = -3 \\ \frac{-5+1}{2} = -2 \end{cases}$$

∴ ***Solution***: $-3 < x < -2$

Exercise

Solve the inequality equation $x^2 < -x + 30$

Solution

$$x^2 + x - 30 = 0$$

$$x = \frac{-1 \pm 11}{2}$$

$$= \begin{cases} \frac{-1-11}{2} = -6 \\ \frac{-1+11}{2} = -5 \end{cases}$$

∴ ***Solution***: $\underline{-6 < x < -5}$

Exercise

Solve: $x^3 - 3x^2 - 9x + 27 < 0$

Solution

$$x^3 - 3x^2 - 9x + 27 = 0$$

$$x^2(x-3) - 9(x-3) = 0$$

$$(x-3)(x^2-9) = 0$$

$$\rightarrow \begin{cases} x - 3 = 0 \rightarrow \underline{x = 3} \\ x^2 - 9 = 0 \rightarrow x^2 = 9 \rightarrow \underline{x = \pm 3} \end{cases}$$

$-\infty$		-3		3		∞
	$-$		$+$		$+$	

∴ ***Solution***: $\underline{x < -3}$ $\underline{(-\infty, -3)}$

Exercise

Solve $x^3 - x > 0$

Solution

$$x(x^2 - 1) = 0$$

$$\rightarrow \begin{cases} x = 0 \\ x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1 \end{cases}$$

	-1		0		1	2
$-$		$+$		$-$		$+$

∴ ***Solution***: $\underline{-1 < x < 0 \quad x > 1}$ $\underline{(-1,0) \cup (1,\infty)}$

Exercise

$x^3+3x^2 \le x+3$

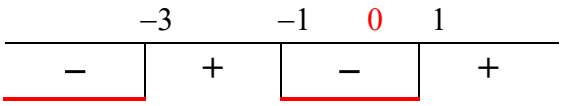
Solution

$x^3+3x^2-x-3=0$

$x^2(x+3)-(x+3)=0$

$(x+3)(x^2-1)=0$

$$\begin{cases} x+3=0 \to x=-3 \\ x^2-1=0 \to x^2=1 \to x=\pm 1 \end{cases}$$



∴ ***Solution***: $-1<x<0 \quad x>1$ $\qquad (-\infty,\ -3]\cup[-1,\ 1]$

Exercise

Solve $x^3+x^2 \ge 48x$

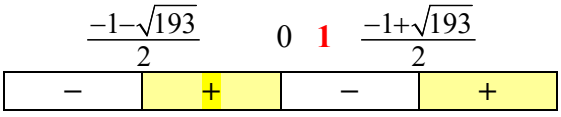
Solution

$x^3+x^2-48x=0$

$x\left(x^2+x-48\right)=0$

$x=0 \qquad x^2+x-48=0$

$x=\frac{-1\pm\sqrt{1+192}}{2}$



∴ ***Solution***: $\frac{-1-\sqrt{193}}{2}<x<0 \quad x>\frac{-1+\sqrt{193}}{2}$ $\qquad \left[\frac{-1-\sqrt{193}}{2},\ 0\right]\cup\left[\frac{-1+\sqrt{193}}{2},\ \infty\right)$

Exercise

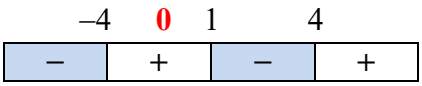
Solve the inequality equation $x^3-x^2-16x+16<0$

Solution

$x^2(x-1)-16(x-1)=0$

$(x-1)\left(x^2-16\right)=0$

$x=1,\ \pm 4$



∴ ***Solution***: $x<-4 \quad 1<x<4$

Exercise

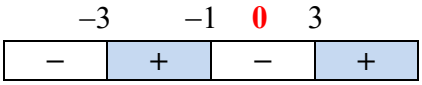
Solve the inequality equation $x^3 + x^2 - 9x - 9 > 0$

Solution

$$x^2(x+1) - 9(x+1) = 0$$

$$(x+1)\left(x^2 - 9\right) = 0$$

$$\underline{x = -1,\ \pm 3}$$



∴ ***Solution***: $\underline{-3 < x < -1 \quad x > 3}$

Exercise

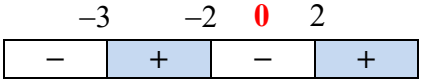
Solve the inequality equation $x^3 + 3x^2 - 4x - 12 \geq 0$

Solution

$$x^2(x+3) - 4(x+3) = 0$$

$$(x+3)\left(x^2 - 4\right) = 0$$

$$\underline{x = -3,\ \pm 2}$$



∴ ***Solution***: $\underline{-3 < x < -2 \quad x > 2}$

Exercise

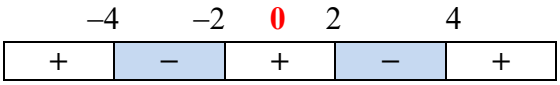
Solve the inequality equation $x^4 - 20x^2 + 64 \leq 0$

Solution

$$x^4 - 20x^2 + 64 = 0$$

$$x^2 = \frac{20 \pm \sqrt{400 - 256}}{2}$$

$$= \begin{cases} \frac{20-12}{2} = 4 \\ \frac{20+12}{2} = 16 \end{cases}$$



$$\begin{cases} x^2 = 4 \rightarrow \underline{x = \pm 2} \\ x^2 = 16 \rightarrow \underline{x = \pm 4} \end{cases}$$

∴ ***Solution***: $\underline{-4 \leq x \leq -2 \quad 2 \leq x \leq 4}$

Exercise

Solve the inequality equation $x^4 - 10x^2 + 9 \geq 0$

Solution

$x^4 - 10x^2 + 9 = 0$

$\begin{cases} x^2 = 1 \rightarrow \underline{x = \pm 1} \\ x^2 = 9 \rightarrow \underline{x = \pm 3} \end{cases}$

∴ ***Solution***: $\underline{x \leq -3 \quad -1 \leq x \leq 1 \quad x \geq 3}$

Exercise

Solve the inequality equation $\frac{x+4}{x-1} < 0$

Solution

Restriction: $x \neq 1$

$\frac{x+4}{x-1} = 0$

$\underline{x = -4}$

∴ ***Solution***: $\underline{-4 < x < 1}$

Exercise

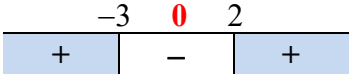
Solve the inequality equation $\frac{x-2}{x+3} > 0$

Solution

Restriction: $x \neq -3$

$\frac{x-2}{x+3} = 0$

$\underline{x = 2}$



∴ ***Solution***: $\underline{x < -3 \quad x > 2}$

Exercise

Solve the inequality equation $\frac{x-5}{x+8} \geq 3$

Solution

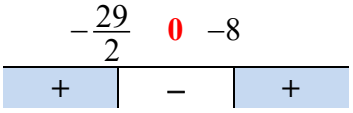
Restriction: $x \neq -8$

$$\frac{x-5}{x+8}-3=0$$

$$x-5-3x-24=0$$

$$-2x=29$$

$$\underline{\left|x=-\frac{29}{2}\right.}$$



∴ ***Solution***: $\underline{\left|x\le-\frac{29}{2}\quad x>-8\right.}$

Exercise

Solve the inequality equation $\frac{x-4}{x+6}\le 1$

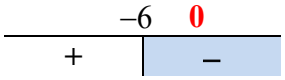
Solution

Restriction: $x\neq -6$

$$\frac{x-4}{x+6}-1=0$$

$$x-4-x-6=0$$

$$-10=0 \ \text{╳}$$



∴ ***Solution***: $\underline{\left|x>-6\right.}$

Exercise

Solve the inequality equation $\frac{x}{2x+7}\ge 4$

Solution

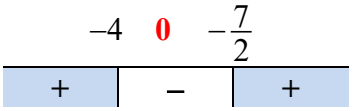
Restriction: $x\neq -\frac{7}{2}$

$$\frac{x}{2x+7}-4=0$$

$$x-8x-28=0$$

$$7x=-28$$

$$\underline{\left|x=-4\right.}$$



∴ ***Solution***: $\underline{\left|x\le-4\quad x>-\frac{7}{2}\right.}$

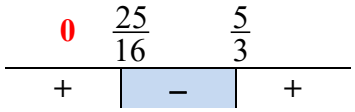
Exercise

Solve the inequality equation $\frac{x}{3x-5}\le -5$

Solution

Restriction: $x \neq \frac{5}{3}$

$$\frac{x}{3x-5}+5=0$$
$$x+15x-25=0$$
$$16x=25$$
$$\boxed{x=\frac{25}{16}}$$



∴ ***Solution***: $\boxed{\frac{25}{16} \leq x < \frac{5}{3}}$

Exercise

Solve the inequality equation $\frac{x+2}{x-5} \leq 2$

Solution

Restriction: $x \neq 5$

$$\frac{x+2}{x-5}-2=0$$
$$x+2-2x+10=0$$
$$\boxed{x=12}$$

∴ ***Solution***: $\boxed{5 \leq x < 12}$

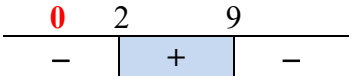
Exercise

Solve the inequality equation $\frac{3x+1}{x-2} \geq 4$

Solution

Restriction: $x \neq 2$

$$\frac{3x+1}{x-2}-4=0$$
$$3x+1-4x+8=0$$
$$\boxed{x=9}$$



∴ ***Solution***: $\boxed{2 \leq x < 9}$

Exercise

Solve the inequality equation $\frac{x}{x-3} > 0$

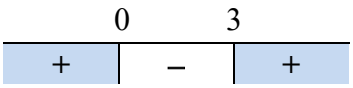
Solution

Restriction: $x \neq 3$

$\frac{x}{x-3} = 0$

$x = 0$

$\therefore$ ***Solution***: $x < 0 \quad x > 3$ $\quad (-\infty, 0) \cup (3, \infty)$



Exercise

Solve the inequality equation $\frac{x-3}{x+2} \geq 0$

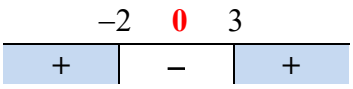
Solution

Restriction: $x \neq -2$

$\frac{x-3}{x+2} = 0$

$x = 3$

$\therefore$ ***Solution***: $x < -2 \quad x \geq 3$



Exercise

Solve the inequality equation $\frac{x-2}{x+2} \leq 2$

Solution

Restriction: $x \neq -2$

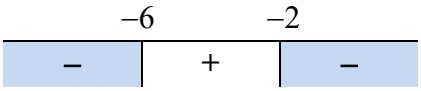
$\frac{x-2}{x+2} - 2 = 0$

$x - 2 - 2x - 4 = 0$

$-x - 6 = 0$

$x = -6$

$\therefore$ ***Solution***: $x \leq -6 \quad x > -2$ $\quad (-\infty, -6] \cup (-2, \infty)$



Exercise

Solve the inequality equation $\frac{x+2}{x-2} \geq 2$

Solution

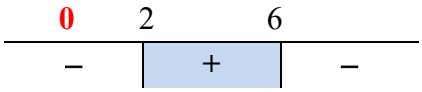
Restriction: $x \neq 2$

$\frac{x+2}{x-2} - 2 = 0$

$x + 2 - 2x + 4 = 0$

$x = 6$

$\therefore$ ***Solution***: $2 < x \le 6$



Exercise

Solve the inequality equation $\dfrac{x+2}{3+2x} \le 5$

Solution

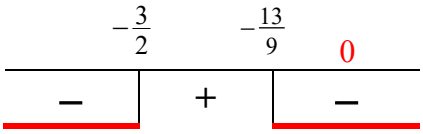
Restriction: $x \ne -\dfrac{3}{2}$

$\dfrac{x+2}{3+2x} - 5 = 0$

$x + 2 - 15 - 10x = 0$

$-9x - 13 = 0$

$x = -\dfrac{13}{9}$



$\therefore$ ***Solution***: $x < -\dfrac{3}{2} \quad x > -\dfrac{13}{9}$ $\qquad \left(-\infty, -\dfrac{3}{2}\right) \cup \left[-\dfrac{13}{9}, \infty\right)$

Exercise

Solve the inequality $\dfrac{x+6}{x-14} \ge 1$

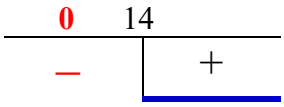
Solution

Restriction: $x - 14 \ne 0 \Rightarrow \boxed{x \ne 14}$

$\dfrac{x+6}{x-14} - 1 = 0$

$x + 6 - x + 14 = 0$

$20 = 0$ $\left(\textit{Implossible}\right)$ No Solution



$\therefore$ ***Solution***: $x > 14$ $\qquad (14, \infty)$

Exercise

Solve: $\dfrac{x-3}{x+4} \ge \dfrac{x+2}{x-5}$

Solution

Conditions: $x + 4 \ne 0 \rightarrow x \ne -4$ $\qquad$ *and* $\qquad x - 5 \ne 0 \rightarrow x \ne 5$

$\dfrac{x-3}{x+4} - \dfrac{x+2}{x-5} = 0$

$\dfrac{0-3}{0+4} - \dfrac{0+2}{0-5} = \dfrac{-3}{4} - \dfrac{2}{-5} = \dfrac{-3}{4} + \dfrac{2}{5} = -$

$$(x+4)(x-5)\left[\frac{x-3}{x+4}-\frac{x+2}{x-5}\right]=0$$

$$(x-5)(x-3)-(x+4)(x+2)=0$$

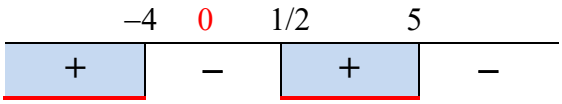
$$x^2-3x-5x+15-(x^2+2x+4x+8)=0$$

$$x^2-3x-5x+15-x^2-2x-4x-8=0$$

$$-14x+7=0$$

$$-14x=-7$$

$$x=\frac{-7}{-14}=\frac{1}{2}$$



∴ ***Solution***: $x<-4 \quad \frac{1}{2}\le x<5$ $\qquad (-\infty,-4)\cup\left[\frac{1}{2},5\right)$

Exercise

Solve: $\frac{x-4}{x+3}-\frac{x+2}{x-1}\le 0$

Solution

Conditions: $x\neq -3$ and $x\neq 1$

$$\frac{x-4}{x+3}-\frac{x+2}{x-1}=0 \qquad \frac{0-4}{0+3}-\frac{0+2}{0-1}=-\frac{4}{3}+2>0$$

$$(x+3)(x-1)\left[\frac{x-4}{x+3}-\frac{x+2}{x-1}\right]=0$$

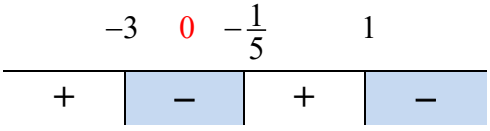
$$(x-1)(x-4)-(x+3)(x+2)=0$$

$$x^2-5x+4-(x^2+5x+6)=0$$

$$x^2-5x+4-x^2-5x-6=0$$

$$-10x-2=0$$

$$x=-\frac{1}{5}$$



∴ ***Solution***: $-3<x\le -\frac{1}{5} \quad x>1$ $\qquad \left(-3,-\frac{1}{5}\right]\cup(1,\infty)$

Exercise

Solve: $\frac{2x-1}{x+3}\ge\frac{x+1}{3x+1}$

Solution

Conditions: $x\neq -3$ and $x\neq -\frac{1}{3}$

$$\frac{2x-1}{x+3}-\frac{x+1}{3x+1}\geq 0$$

$$(x+3)(3x+1)\frac{2x-1}{x+3}-(x+3)(3x+1)\frac{x+1}{3x+1}=0$$

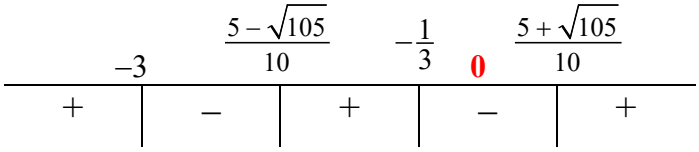
$$(3x+1)(2x-1)-(x+3)(x+1)=0$$

$$6x^2-3x+2x-1-\left(x^2+x+3x+3\right)=0$$

$$6x^2-x-1-x^2-4x-3=0$$

$$5x^2-5x-4=0$$

$$x=\frac{5\pm\sqrt{105}}{10}\rightarrow -.5 \quad 1.5$$



∴ ***Solution***: $x<-3 \quad \frac{5-\sqrt{105}}{10}\leq x<-\frac{1}{3} \quad x\geq\frac{5+\sqrt{105}}{10}$

$$(-\infty,\ -3)\cup\left[\frac{5-\sqrt{105}}{10},\ -\frac{1}{3}\right)\cup\left[\frac{5+\sqrt{105}}{10},\ \infty\right)$$

Exercise

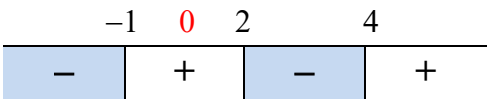
Solve the inequality equation $\frac{(x+1)(x-4)}{x-2}<0$

Solution

Restriction: $x\neq 2$

$$\frac{(x+1)(x-4)}{x-2}=0$$

$x=-1,\ 4$



∴ ***Solution***: $x<-1 \quad 2<x<4$

Exercise

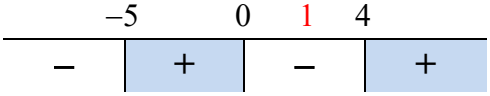
Solve the inequality equation $\frac{x(x-4)}{x+5}>0$

Solution

Restriction: $x\neq -5$

$$\frac{x(x-4)}{x+5}=0$$

$x=0,\ 4$



∴ ***Solution***: $-5<x<0 \quad x>4$

Exercise

Solve the inequality equation $\dfrac{6x^2-11x-10}{x}>0$

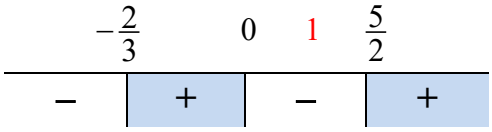
Solution

Restriction: $x\neq 0$

$6x^2-11x-10=0$

$$x=\frac{11\pm\sqrt{121+240}}{12}$$

$$=\begin{cases}\dfrac{11-19}{12}=-\dfrac{2}{3}\\ \dfrac{11+19}{12}=\dfrac{5}{2}\end{cases}$$



∴ ***Solution***: $-\dfrac{2}{3}<x<0 \quad x>\dfrac{5}{2}$

Exercise

Solve the inequality equation $\dfrac{3x^2-2x-8}{x-1}\geq 0$

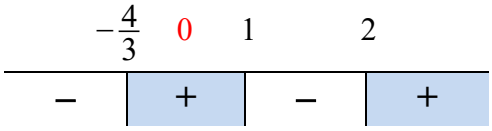
Solution

Restriction: $x\neq 1$

$3x^2-2x-8=0$

$$x=\frac{2\pm\sqrt{4+96}}{6}$$

$$=\begin{cases}\dfrac{2-10}{6}=-\dfrac{4}{3}\\ \dfrac{2+10}{6}=2\end{cases}$$



∴ ***Solution***: $-\dfrac{4}{3}\leq x<1 \quad x\geq 2$

Exercise

Solve the inequality equation $\dfrac{x^2-6x+9}{x-5}\leq 0$

Solution

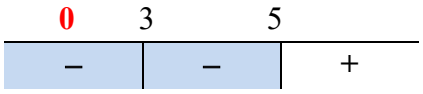
Restriction: $x\neq 5$

$x^2-6x+9=0$

$(x-3)^2=0$

$x = 3$

$\therefore$ ***Solution***: $x < 5$



Exercise

Solve the inequality equation $\dfrac{x^2 + 10x + 25}{x + 1} \le 0$

Solution

Restriction: $x \neq -1$

$x^2 + 10x + 25 = 0$

$(x + 5)^2 = 0$

$x = -5$

$\therefore$ ***Solution***: $x < -1$