$$\int_0^{\pi/2} \cos^{12} x\,dx = \frac{\pi}{2}\,\frac{11}{12}\,\frac{9}{10}\,\frac{7}{8}\,\frac{5}{6}\,\frac{3}{4}\,\frac{1}{2}$$

$$= \frac{231\,\pi}{2^{11}}$$

$$\int_0^{\pi/2} \cos^{15} x\,dx = \frac{14}{15}\,\frac{12}{13}\,\frac{10}{11}\,\frac{8}{9}\,\frac{6}{7}\,\frac{4}{5}\,\frac{2}{3}$$

$$= \frac{2^{11}}{6,435}$$

2.4. Partial Fractions

Ex $\displaystyle\int \frac{5x-3}{x^2-2x-3}\,dx$

$$\frac{5x-3}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3} \qquad A(x-3)$$

$$5x-3 = Ax-3A+Bx+B$$

$$x^1 \quad A+B = 5 \quad \rightarrow B=3 \quad 5-2$$

$$x^0 \quad -3A+B = -3$$

$$4A = 8 \Rightarrow A = 2$$

$$\int \frac{5x-3}{x^2-2x-3}\,dx = \int\left(\frac{2}{x+1} + \frac{3}{x-3}\right)dx \qquad d\,\ln(x+a) = \frac{dx}{x+a}$$

$$= 2\ln|x+1| + 3\ln|x-3| + C$$

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx$$

$$\frac{x^2+4x+1}{(x-1)(x+1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3}$$

$$x^2+4x+1 = A(x+1)(x+3) + B(x-1)(x+3) + C(x^2-1)$$

$$\begin{array}{ll} x^2 & A + B + C = 1 \\ x^1 & 4A + 2B = 4 \\ x^0 & 3A - 3B - C = 1 \end{array}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 0 \\ 3 & -3 & -1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 4 & 2 \\ 3 & -3 \end{matrix} = -16$$

$$\Delta_A = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 0 \\ 1 & -3 & -1 \end{vmatrix} = -12 \qquad A = \frac{-12}{-16} = \frac{3}{4}$$

$$\Delta_B = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 0 \\ 3 & 1 & -1 \end{vmatrix} = -8 \qquad B = \frac{1}{2}$$

$$C = -\frac{1}{4}$$

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx = \frac{3}{4}\int\frac{dx}{x-1} + \frac{1}{2}\int\frac{dx}{x+1} - \frac{1}{4}\int\frac{dx}{x+3}$$

$$= \frac{3}{4}\ln|x-1| + \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln|x+3| + C$$

$\overline{(x+a)^n}$ $\quad (x+a) \quad (x+a)^2 \cdots (x+a)^n$

Ex $\displaystyle\int \frac{6x+7}{(x+2)^2}\,dx$

$$\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

$$6x+7 = A(x+2) + B$$

$x^1 \quad \underline{A = 6}$

$x^0 \quad 2A + B = 7 \rightarrow B = \underline{-5}$

$$\int \frac{6x+7}{(x+2)^2}\,dx = 6\int \frac{dx}{x+2} - 5\int \frac{d(x+2)}{(x+2)^2} \qquad \underline{d(x+2) = dx}$$

$$\underline{= 6\ln|x+2| + \frac{5}{x+2} + C}$$

$$\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx$$

$$\begin{array}{r|l} & 2x \\ \hline x^2-2x-3 & 2x^3-4x^2-x-3 \\ & \underline{-2x^3+4x^2+6x} \\ & 5x-3 \end{array}$$

$$\frac{2x^3-4x^2-x-3}{x^2-2x-3} = 2x + \underline{\frac{5x-3}{x^2-2x-3}}$$

$$\frac{5x-3}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$5x-3 = A(x-3) + B(x+1)$$

$$x^1 \quad A + B = 5 \rightarrow \boxed{B = 3}$$

$$x^0 \quad -3A + B = -3$$

$$4A = 8 \Rightarrow \underline{A = 2}$$

$$\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx = \int 2x\,dx + 2\int \frac{dx}{x+1} + 3\int \frac{dx}{x-3}$$

$$= \underline{x^2 + 2\ln|x+1| + 3\ln|x-3| + C}$$

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$$

$$\frac{-2x+4}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$-2x+4 = (Ax+B)(x^2-2x+1) + C(x-1)(x^2+1) + D(x^2+1)$$

x^3: $A + C = 0 \rightarrow C = -A$

x^2: $-2A + B - C + D = 0$ ①

x^1: $A - 2B + C = -2 \rightarrow -2B = -2 \Rightarrow \underline{B = +1}$

x^0: $B - C + D = 4$ ②

① $\rightarrow$ $2C + 1 - C + D = 0$

② $\rightarrow$ $1 - C + D = 4$

$$\begin{cases} C + D = -1 \rightarrow \underline{C = -2} \quad \underline{A = 2} \\ -C + D = 3 \end{cases}$$

$2D = 2 \Rightarrow \underline{D = 1}$

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx = \int \frac{2x+1}{x^2+1}\,dx - 2\int \frac{dx}{x-1} + \int \frac{d(x-1)}{(x-1)^2}$$

$$= \int \frac{2x\,dx}{x^2+1} + \int \frac{dx}{x^2+1} - 2\ln|x-1| - \frac{1}{x-1}$$

$$= \int \frac{d(x^2+1)}{x^2+1} + \tan^{-1}x - 2\ln|x-1| - \frac{1}{x-1}$$

$$= \underline{\ln(x^2+1) + \tan^{-1}x - 2\ln|x-1| - \frac{1}{x-1} + K}$$

$x(x^2+1)^2$

$$\frac{1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$1 = A(x^4+2x^2+1) + (Bx+C)(x^3+x) + Dx^2+Ex$$

x^4 $\quad A + B = 0 \to \underline{B = -1}$

x^3 $\quad C = 0$

x^2 $\quad 2A + B + D = 0 \to D = -2+1 = -1$

x^1 $\quad C + E = 0 \to E = 0$

x^0 $\quad A = 1$

$$\int \frac{dx}{x(x^2+1)^2} = \int \frac{dx}{x} - \int \frac{x\,dx}{x^2+1} - \int \frac{x\,dx}{(x^2+1)^2}$$

$$= \ln|x| - \frac{1}{2}\int \frac{d(x^2+1)}{x^2+1} - \frac{1}{2}\int \frac{d(x^2+1)}{(x^2+1)^2}$$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + \frac{1}{2}\cdot\frac{1}{x^2+1} + K$$

$\frac{1}{\infty} = 0 \qquad e^{-\infty} = 0 = \frac{1}{e^{\infty}} = \frac{1}{\infty}$

$y = \frac{\ln x}{x^2} \qquad x = 1 \to \infty$

$I = \int_1^{\infty} \frac{\ln x}{x^2}\, dx \qquad u = \ln x \qquad v = \int \frac{1}{x^2} dx$

$du = \frac{1}{x} dx \qquad = -\frac{1}{x}$

$= -\frac{\ln x}{x}\Big|_1^{\infty} + \int_1^{\infty} \frac{dx}{x^2}$

$= -(0-0) - \frac{1}{x}\Big|_1^{\infty}$

$= +1$ unit^2.

$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \tan^{-1} x \Big|_{-\infty}^{\infty}$

$\frac{1}{0} = \infty$

$\frac{\pi}{2}$

$= \tan^{-1}\infty - \tan^{-1}(-\infty)$

$= \frac{\pi}{2} + \frac{\pi}{2}$

$= \pi$

$$\int_1^\infty \frac{dx}{x^p}$$

If $p \neq 1$

$$\int_1^\infty \frac{dx}{x^p} = \int_1^\infty x^{-p}\,dx$$

$$= \frac{x^{-p+1}}{1-p}\Big|_1^\infty$$

$$= \frac{1}{1-p}\left((\ )^{1-p} - 1\right)$$

$$= \begin{cases} \frac{1}{p-1} & p > 1 \\ \infty & p \le 1 \end{cases}$$

$\frac{1}{\infty} = 0$

$\infty = \infty$

$(x)^-$

$\frac{1}{x}$

$1-p<0$
$p>1$

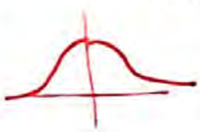
If $p = 1$

$$\int_1^\infty \frac{dx}{x} = \ln x \Big|_1^\infty = \infty$$

x^p converges when $p > 1$

$p \le 1 \to$ diverges

$\int e^{x^2}dx$?? e^{x^2} $u = e^{x^2}$, $du = 2xe^{x^2}$ $\int dx = x$



$$\int_0^1 \frac{dx}{1-x} = -\int_0^1 \frac{d(1-x)}{1-x}$$

$$= -\ln|1-x|\Big|_0^1$$

$$= -(\ln 0^+ - \ln 1)$$

$$= -(-\infty - 0)$$

$$= \infty$$

$$\int_0^3 \frac{dx}{(1-x)^{2/3}} = -\int_0^3 (1-x)^{-2/3}\, d(1-x) \qquad d(1-x) = -dx$$

$$= -3(1-x)^{1/3}\Big|_0^3$$

$$= -3\left[(-2)^{1/3} - 1\right]$$

$$= -3\left(-\sqrt[3]{2} - 1\right)$$

$$= 3 + 3\sqrt[3]{2}$$

Direct Comparison Test $0 \le f(x) \le g(x)$

$\int_a^\infty f(x)\,dx$ converges if $\int g(x)\,dx$ converges

if $f(x)$ diverges $\rightarrow$ $g(x)$ diverges

$f(x) = x^{-1} \qquad x \geq 1$

$$V = \pi \int_1^{\infty} (x^{-1})^2 \, dx \qquad \int_1^{\infty} \frac{dx}{x^2}$$

$$= -\pi \left. \frac{1}{x} \right|_1^{\infty}$$

$$= -\pi (0 - 1)$$

$$\underline{= \pi \text{ unit}^3}$$

$$S = 2\pi \int_1^{\infty} \frac{1}{x} \cdot \sqrt{1 + \frac{1}{x^4}} \, dx$$

$$= 2\pi \int_1^{\infty} \frac{1}{x} \, \frac{\sqrt{x^4 + 1}}{x^2} \, dx$$

$$= 2\pi \int_1^{\infty} \frac{\sqrt{x^4 + 1}}{x^3} \, dx \qquad \sqrt{x^4 + 1} > \sqrt{x^4} = x^2$$

$$> 2\pi \int_1^{\infty} \frac{x^2}{x^3} \, dx$$

$$= 2\pi \left. \ln x \right|_1^{\infty}$$

$$\underline{= \infty}$$