# *Section* 2.2 – Trigonometric Integrals

## Products of Powers of *Sines* and *Cosines*

We begin with integrals of the form

$$\int \sin^m x \ \cos^n x \, dx$$

### *Example*

Evaluate $\displaystyle\int \sin^3 x \cos^2 x \, dx$

***Solution***

$$\int \sin^3 x \cos^2 x \, dx = \int \sin x \sin^2 x \cos^2 x \, dx$$

$$= \int \left(1-\cos^2 x\right)\cos^2 x \ \left(-d\left(\cos x\right)\right) \qquad d\left(\cos x\right) = -\sin x dx \quad \Rightarrow \sin x dx = -d\left(\cos x\right)$$

$$= -\int \left(\cos^2 x - \cos^4 x\right) \ d\left(\cos x\right) \qquad \textit{or} \quad \textit{Assume} \quad u = \cos x$$

$$= -\left(\tfrac{1}{3}\cos^3 x - \tfrac{1}{5}\cos^5 x\right) + C$$

$$= \tfrac{1}{5}\cos^5 x - \tfrac{1}{3}\cos^3 x + C$$

### *Example*

Evaluate $\displaystyle\int \cos^5 x \, dx$

***Solution***

$$\int \cos^5 x \, dx = \int \cos^4 x \cos x \, dx \qquad \cos x dx = d\left(\sin x\right) \qquad \cos^2 x = 1 - \sin^2 x$$

$$= \int \left(1-\sin^2 x\right)^2 \, d\left(\sin x\right)$$

$$= \int \left(1 - 2\sin^2 x + \sin^4 x\right) \, d\sin x$$

$$= \sin x - \tfrac{2}{3}\sin^3 x + \tfrac{1}{5}\sin^5 x + C$$

***Example***

Evaluate $\displaystyle\int \sin^2 x \cos^4 x \, dx$

***Solution***

$$\int \sin^2 x \cos^4 x \, dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx \qquad \sin^2 x = \frac{1-\cos 2x}{2} \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \frac{1}{8}\int (1-\cos 2x)\left(1+2\cos 2x+\cos^2 2x\right)dx$$

$$= \frac{1}{8}\int \left(1+2\cos 2x+\cos^2 2x-\cos 2x-2\cos^2 2x-\cos^3 2x\right)dx$$

$$= \frac{1}{8}\int \left(1+\cos 2x-\cos^2 2x-\cos^3 2x\right)dx$$

$$= \frac{1}{8}\left[x+\frac{1}{2}\sin 2x-\int\left(\cos^3 2x+\cos^2 2x\right)dx\right]$$

$$\int \cos^3 2x dx = \int \left(1-\sin^2 2x\right)\cos 2x dx$$

$$= \frac{1}{2}\int \left(1-\sin^2 2x\right) d\left(\sin 2x\right)$$

$$= \frac{1}{2}\left(\sin 2x-\frac{1}{3}\sin^3 2x\right)$$

$$\int \cos^2 2x dx = \frac{1}{2}\int (1+\cos 4x)\,dx$$

$$= \frac{1}{2}\left(x+\frac{1}{4}\sin 4x\right)$$

$$\int \sin^2 x \cos^4 x \, dx = \frac{1}{8}\left[x+\frac{1}{2}\sin 2x-\frac{1}{2}\left(\sin 2x-\frac{1}{3}\sin^3 2x\right)-\frac{1}{2}\left(x+\frac{1}{4}\sin 4x\right)\right]+C$$

$$= \frac{1}{8}\left[x+\frac{1}{2}\sin 2x-\frac{1}{2}\sin 2x+\frac{1}{6}\sin^3 2x-\frac{1}{2}x-\frac{1}{8}\sin 4x\right]+C$$

$$= \frac{1}{8}\left(\frac{1}{2}x+\frac{1}{6}\sin^3 2x-\frac{1}{8}\sin 4x\right)+C$$

$$\boxed{= \frac{1}{16}\left(x+\frac{1}{3}\sin^3 2x-\frac{1}{4}\sin 4x\right)+C}$$

## *Example*

Evaluate $\displaystyle\int_0^{\pi/4} \sqrt{1+\cos 4x}\ dx$

**Solution**

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \quad\Rightarrow\quad 1+\cos 2\theta = 2\cos^2\theta$$

$$\theta = 2x \quad\Rightarrow\quad 1+\cos 4x = 2\cos^2 2x$$

$$\int_0^{\pi/4} \sqrt{1+\cos 4x}\ dx = \int_0^{\pi/4} \sqrt{2\cos^2 2x}\ dx \qquad \sqrt{2\cos^2 2x} = \sqrt{2}\sqrt{\cos^2 2x} = \sqrt{2}\,|\cos 2x|$$

$$= \sqrt{2}\int_0^{\pi/4} |\cos 2x|\ dx \qquad \cos 2x \ge 0 \quad \textit{on} \quad \left[0, \tfrac{\pi}{4}\right]$$

$$= \sqrt{2}\int_0^{\pi/4} \cos 2x\ dx$$

$$= \sqrt{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$$

$$= \frac{\sqrt{2}}{2}\left[\sin\frac{\pi}{2} - 0\right]$$

$$= \frac{\sqrt{2}}{2}$$

## *Example*

Evaluate $\displaystyle\int \sin^3 x \cos^{-2} x\ dx$

**Solution**

$$\int \sin^3 x\cos^{-2} x\ dx = \int \sin^2 x\cos^{-2} x\ \sin x\ dx$$

$$= -\int \left(1-\cos^2 x\right)\cos^{-2} x\ \ d(\cos x)$$

$$= -\int \left(\cos^{-2} x - 1\right)\ d(\cos x)$$

$$= -\left(-\cos^{-1} x - \cos x\right) + C$$

$$= \cos x + \sec x + C$$

## Products of Powers of *tan x* and *sec x*

### *Example*

Evaluate $\int \tan^4 x\, dx$

***Solution***

$$\int \tan^4 x dx = \int \tan^2 x \cdot \tan^2 x\, dx \qquad \tan^2 x = \sec^2 x - 1$$

$$= \int \tan^2 x \cdot \left(\sec^2 x - 1\right) dx$$

$$= \int \tan^2 x \sec^2 x dx - \int \tan^2 x\, dx$$

$$= \int \tan^2 x \sec^2 x dx - \int \left(\sec^2 x - 1\right) dx$$

$$= \int \tan^2 x \sec^2 x dx - \int \sec^2 x\, dx + \int dx$$

$$= \int \tan^2 x\, d(\tan x) - \int \sec^2 x\, dx + \int dx \qquad d(\tan x) = \sec^2 x\, dx$$

$$= \frac{1}{3}\tan^3 x - \tan x + x + C$$

### *Example*

Evaluate $\int \sec^3 x\, dx$

***Solution***

Let: $u = \sec x \qquad dv = \sec^2 x dx$

$du = \sec x \tan x dx \qquad v = \tan x$

$$\int \sec^3 x dx = \sec x \tan x - \int \tan x \left(\sec x \tan x dx\right)$$

$$= \sec x \tan x - \int \tan^2 x \sec x dx$$

$$= \sec x \tan x - \int \left(\sec^2 x - 1\right) \sec x\, dx$$

$$= \sec x \tan x - \int \left(\sec^3 x - \sec x\right) dx$$

$$= \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$$

$$\int \sec^3 x\, dx + \int \sec^3 x\, dx = \sec x \tan x - \int \sec^3 x\, dx + \int \sec^3 x\, dx + \int \sec x\, dx$$

$$2\int \sec^3 x\, dx = \sec x \tan x + \int \sec x\, dx$$

$$= \sec x \tan x + \ln\left|\sec x + \tan x\right| + C_1$$

$$\int \sec^3 x\, dx = \boxed{\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln\left|\sec x + \tan x\right| + C}$$

## Products of Sines and Cosines

Recall the identities

$$\sin mx \sin nx = \frac{1}{2}\left[\cos(m-n)x - \cos(m+n)x\right]$$

$$\sin mx \cos nx = \frac{1}{2}\left[\sin(m-n)x + \sin(m+n)x\right]$$

$$\cos mx \cos nx = \frac{1}{2}\left[\cos(m-n)x + \cos(m+n)x\right]$$

### *Example*

Evaluate $\int \sin 3x \cos 5x dx$

***Solution***

$$\int \sin 3x \cos 5x dx = \frac{1}{2}\int \left[\sin(-2x) + \sin 8x\right] dx$$

$$= \frac{1}{2}\int \left[-\sin(2x) + \sin 8x\right] dx$$

$$= \frac{1}{2}\left(\frac{1}{2}\cos 2x - \frac{1}{8}\cos 8x\right) + C$$

$$\boxed{= \frac{1}{4}\cos 2x - \frac{1}{16}\cos 8x + C}$$

## Guidelines for Cosine & Sine

***Case* 1** If ***m*** is ***odd***, we write $m$ as $2k+1$ and use the identity $\sin^2 x = 1-\cos^2 x$ to obtain

$$\sin^m x = \sin^{2k+1} x = \left(\sin^2 x\right)^k \sin x = \left(1-\cos^2 x\right)^k \sin x$$

Then we combine the single $\sin x$ with $dx$ in the integral and set $\sin x dx = -d\left(\cos x\right)$

***Case* 2** If ***m*** is ***even* and *n*** is ***odd***, in $\int \sin^m x \cos^n x dx$ we write $n$ as $2k+1$ and use the identity $\cos^2 x = 1-\sin^2 x$ to obtain

$$\cos^n x = \cos^{2k+1} x = \left(\cos^2 x\right)^k \cos x = \left(1-\sin^2 x\right)^k \cos x$$

Then we combine the single $\cos x$ with $dx$ in the integral and set $\cos x dx = d\left(\sin x\right)$

***Case* 3** If both ***m* and *n*** are ***even***, in $\int \sin^m x \cos^n x dx$, we substitute

To reduce the integrand to one in lower powers of $\cos 2x$ $\quad \int \cos ax dx = \frac{1}{a}\sin ax + C$

## Guidelines for Tangent & Secant

***Case* 1** When the power of the tangent is ***odd*** and positive.

$$\begin{aligned}\int \sec^m x\ \tan^{2k+1} x\ dx &= \int \sec^{m-1} x \left(\tan^2 x\right)^k \sec x \tan x\ dx \\ &= \int \sec^{m-1} x \left(\sec^2 x - 1\right)^k d\left(\sec x\right)\end{aligned}$$

***Case* 2** When the power of the secant is ***even*** and positive.

$$\int \sec^{2k} x\ \tan^n x\ dx = \int \left(\sec^2 x\right)^{k-1} \tan^n x\ \sec^2 x\ dx = \int \left(1+\tan^2 x\right)^{k-1} \tan^n x\ d\left(\tan x\right)$$

***Case* 3** When there are no secant factors

$$\int \tan^n x\ dx = \int \tan^{n-2} x\ \left(\tan^2 x\right) dx = \int \tan^{n-2} x\ \left(\sec^2 x - 1\right) dx$$

***Case* 4** When there are only secant, use integration by parts.

***Case* 5** Otherwise, convert to cosines and sines.

## *Wallis's Formulas*

**1.** If $n$ is odd $(n \geq 3)$, then $\displaystyle\int_0^{\pi/2} \cos^n x\,dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right) \cdots \left(\frac{n-1}{n}\right)$

**2.** If $n$ is even $(n \geq 2)$, then $\displaystyle\int_0^{\pi/2} \cos^n x\,dx = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right) \cdots \left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right)$

### *Formulas*

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

$$\int \sin^n x\,dx = \frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$$

$$\int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

# *Exercises* Section 2.2 – Trigonometric Integrals

**(1 – 149)** Evaluate the integrals

**1.** $\int \sin^5 \frac{x}{2}\, dx$

**2.** $\int \sin^4 6\theta\, d\theta$

**3.** $\int x^2 \sin^2 x\, dx$

**4.** $\int \sin^3 3x\, dx$

**5.** $\int \sin^5 x\, dx$

**6.** $\int 8\cos^4 2\pi x\, dx$

**7.** $\int x\cos^3 x\, dx$

**8.** $\int \cos^4 x\, dx$

**9.** $\int \cos^4 5x\, dx$

**10.** $\int \cos^2 3x\, dx$

**11.** $\int \cos^3 \frac{x}{3}\, dx$

**12.** $\int \cos^2 4x\, dx$

**13.** $\int \sqrt{1+\cos\frac{x}{2}}\, dx$

**14.** $\int \sec^4 2x\, dx$

**15.** $\int 6\sec^4 x\, dx$

**16.** $\int \sec^3 \pi x\, dx$

**17.** $\int \sec 4x\, dx$

**18.** $\int \csc^6 x\, dx$

**19.** $\int \tan^5 \frac{x}{2}\, dx$

**20.** $\int \tan^5 x\, dx$

**21.** $\int \tan^5 3x\, dx$

**22.** $\int \tan^6 3x\, dx$

**23.** $\int 20\tan^6 x\, dx$

**24.** $\int \tan^4 x\, dx$

**25.** $\int \tan^3 \theta\, d\theta$

**26.** $\int \tan^3 4x\, dx$

**27.** $\int \cot^3 2x\, dx$

**28.** $\int \cot^4 x\, dx$

**29.** $\int \cot^4 3x\, dx$

**30.** $\int \cot^5 3x\, dx$

**31.** $\int \sin^2 x \cos^2 x\, dx$

**32.** $\int \sin^2 x \cos^3 x\, dx$

**33.** $\int \sin^2 x \cos^4 x\, dx$

**34.** $\int \sin^2 x \cos^5 x\, dx$

**35.** $\int \sin^3 x \cos^5 x\, dx$

**36.** $\int \sin^3 x \cos^4 x\, dx$

**37.** $\int \sin^3 2x \cos^4 x\, dx$

**38.** $\int \sin^3 2x \cos^3 2x\, dx$

**39.** $\int \sin^4 x\cos^2 x\, dx$

**40.** $\int \sin^4 x\cos^3 x\, dx$

**41.** $\int \sin^4 x \cos^4 x\, dx$

**42.** $\int \sin^4 x \cos^5 x\, dx$

**43.** $\int \sin^5 x \cos^5 x\, dx$

**44.** $\int \sin^5 x \cos^{-2} x\, dx$

**45.** $\int \sin 3x\cos^6 3x\, dx$

**46.** $\int \sin^4 2x \cos 2x \, dx$

**47.** $\int \cos^3 2x \sin^5 2x \, dx$

**48.** $\int 16 \sin^2 x \cos^2 x \, dx$

**49.** $\int \sin 2x \cos 3x \, dx$

**50.** $\int \sin^2 \theta \cos 3\theta \, d\theta$

**51.** $\int \cos^3 \theta \sin 2\theta \, d\theta$

**52.** $\int \sin^{-3/2} x \ \cos^3 x \, dx$

**53.** $\int \sin^3 x \ \cos^{3/2} x \, dx$

**54.** $\int \sin \theta \sin 2\theta \sin 3\theta \, d\theta$

**55.** $\int \sin 3x \ \cos 6x \, dx$

**56.** $\int \sin 3x \cos 7x \, dx$

**57.** $\int \sin 5x \cos 4x \, dx$

**58.** $\int \cos 2\theta \ \cos 6\theta \, d\theta$

**59.** $\int \cos 5\theta \ \cos 3\theta \, d\theta$

**60.** $\int \sin 2\theta \ \cos 4\theta \, d\theta$

**61.** $\int \sin(-7\theta) \ \cos 6\theta \, d\theta$

**62.** $\int \sin \theta \ \sin 3\theta \, d\theta$

**63.** $\int \sin 5\theta \ \sin 4\theta \, d\theta$

**64.** $\int \sin x \cos^5 x \, dx$

**65.** $\int \sin^7 2x \ \cos 2x \, dx$

**66.** $\int \sin^3 2x \sqrt{\cos 2x} \, dx$

**67.** $\int \sin^3 x \cos^2 x \, dx$

**68.** $\int \frac{\cos^3 \theta}{\sqrt{\sin \theta}} \, d\theta$

**69.** $\int \frac{\cos^5 \theta}{\sqrt{\sin \theta}} \, d\theta$

**70.** $\int \frac{\cos^2 x}{\sin^5 x} \, dx$

**71.** $\int \frac{\sin^3 x}{\cos^4 x} \, dx$

**72.** $\int \frac{\sin^4 x}{\cos^6 x} \, dx$

**73.** $\int \frac{2\cos x + 3\sin x}{\sin^3 x} \, dx$

**74.** $\int \frac{\sin 2x}{1+\sin^2 x} \, dx$

**75.** $\int \frac{2 + \sin x + 2\cos x}{1+\cos x} \, dx$

**76.** $\int \frac{dx}{1-\cos x}$

**77.** $\int \frac{dx}{1-\sin x}$

**78.** $\int \frac{\sin \theta \cos \theta}{2-\cos \theta} \, d\theta$

**79.** $\int \tan^3 x \ \sec^3 x \, dx$

**80.** $\int \sec x \tan^2 x \, dx$

**81.** $\int \sec^2 x \tan^2 x \, dx$

**82.** $\int \sec^4 x \tan^2 x \, dx$

**83.** $\int \sec^6 4x \ \tan 4x \, dx$

**84.** $\int \sec^2 \frac{x}{2} \ \tan \frac{x}{2} \, dx$

**85.** $\int \tan^3 2x \ \sec^3 2x \, dx$

**86.** $\int \tan^5 2x \ \sec^4 2x \, dx$

**87.** $\int \tan^3 x \ \sec^5 x \, dx$

**88.** $\int \tan^3 x \ \sec^4 x \, dx$

**89.** $\int \tan^5 \theta \ \sec^4 \theta \, d\theta$

**90.** $\int \tan^5 \theta \ \sec^7 \theta \, d\theta$

**91.** $\int \tan^7 \theta \ \sec^5 \theta \, d\theta$

**92.** $\int \sec^4 3x \tan^3 3x \, dx$

**93.** $\int \tan^3 \frac{\pi x}{2} \ \sec^2 \frac{\pi x}{2} \, dx$

**94.** $\int \sec^{-2} x \tan^3 x \, dx$

**95.** $\int \sqrt{\tan x} \ \sec^4 x \, dx$

96. $\int \tan^5\theta\ \csc^2\theta\, d\theta$

97. $\int \csc^2 x\ \cot x\, dx$

98. $\int \csc^{10} x\ \cot x\, dx$

99. $\int (\cot 2x - \csc 2x)^2\, dx$

100. $\int \operatorname{sech}^4 x\ dx$

101. $\int \sinh^3 x\ \cosh^2 x\, dx$

102. $\int \operatorname{sech}^2 x\ \sinh x\, dx$

103. $\int \frac{\tan^3 x}{\sqrt{\sec x}}\, dx$

104. $\int \frac{\tan^2 x}{\sec x}\, dx$

105. $\int \frac{\sec x}{\tan^2 x}\, dx$

106. $\int \frac{\sec^2 x}{\tan^5 x}\, dx$

107. $\int \frac{\csc^4 x}{\cot^2 x}\ dx$

108. $\int \frac{\sec^4(\ln x)}{x}\, dx$

109. $\int e^x \sec\left(e^x + 1\right) dx$

110. $\int e^x \sec^3 e^x\ dx$

111. $\int e^x \sqrt{\tan^2 e^x + 1}\ dx$

112. $\int_0^{\sqrt{\frac{\pi}{2}}} x \sin^3\left(x^2\right) dx$

113. $\int_{\pi/6}^{\pi/3} \frac{\cos^3 x}{\sqrt{\sin x}}\, dx$

114. $\int_{\pi/6}^{\pi/2} \frac{dx}{\sin x}$

115. $\int_{\pi/6}^{\pi/3} \cot^3\theta\, d\theta$

116. $\int_0^{\pi/3} \tan^2 x\, dx$

117. $\int_0^{\pi/4} 6\tan^3 x\, dx$

118. $\int_0^{\pi/4} \tan^4 x\, dx$

119. $\int_0^{\pi} 8\sin^4 y \cos^2 y\, dy$

120. $\int_0^{\pi/6} 3\cos^5 3x\, dx$

121. $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\, d\theta$

122. $\int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}}\ dx$

123. $\int_0^{\pi} \sqrt{1-\cos^2\theta}\ d\theta$

124. $\int_0^{\pi/6} \sqrt{1+\sin x}\ dx$

125. $\int_{-\pi/3}^{\pi/3} \sqrt{\sec^2\theta - 1}\ d\theta$

126. $\int_0^{\pi} (1-\cos 2x)^{3/2}\, dx$

127. $\int_0^{\pi} \left(1-\cos^2 x\right)^{3/2}\, dx$

128. $\int_{-\pi}^{\pi} \left(1-\cos^2 x\right)^{3/2}\, dx$

129. $\int_{\pi/4}^{\pi/2} \csc^4\theta\, d\theta$

130. $\int_{-\pi}^{\pi} \sin 3x \sin 3x\, dx$

131. $\int_{-\pi/2}^{\pi/2} \cos x \cos 7x\, dx$

132. $\int_0^{\pi/4} \cos^5 2x\ \sin^2 2x\, dx$

133. $\int_0^{\pi/6} \sin^5 x\, dx$

134. $\int_{-\pi}^{\pi} \sin^2 x\, dx$

135. $\int_{-\pi/2}^{\pi/2} \left(\sin^2 x + 1\right) dx$

136. $\int_0^{\pi/3} \sec^{3/2} x\ \tan x\, dx$

137. $\int_0^{\pi/2} \frac{\cos x}{1+\sin x}\, dx$

138. $\int_{\pi/6}^{\pi/3} \sin 6x\ \cos 4x\, dx$

139. $\int_{-\pi/2}^{\pi/2} 3\cos^3 x\, dx$

**140.** $\int_0^{\pi} \sec^2 x\,dx$

**141.** $\int_0^{\ln(\sqrt{3}+2)} \frac{\cosh x}{\sqrt{4-\sinh^2 x}}\,dx$

**142.** $\int_0^{\pi/2} \cos^4 x\,dx$

**143.** $\int_0^{\pi/2} \cos^{10}\theta\,d\theta$

**144.** $\int_0^{\pi/2} \cos^7 x\,dx$

**145.** $\int_0^{\pi/2} \cos^9 x\,dx$

**146.** $\int_0^{\pi/2} \sin^5 x\,dx$

**147.** $\int_0^{\pi/2} \sin^6 x\,dx$

**148.** $\int_0^{\pi/2} \sin^8 x\,dx$

**149.** $\int_0^{\pi/2} \tan^2 \frac{x}{2}\,dx$

**150.** Find the area of the region bounded by the graphs of $y = \tan x$ and $y = \sec x$ on the interval $\left[0, \frac{\pi}{4}\right]$

Find the area of the region bounded by the graphs of the equations

**151.** $y = \sin x, \quad y = \sin^3 x, \quad x = 0, \quad x = \frac{\pi}{2}$

**152.** $y = \sin^2 \pi x, \quad y = 0, \quad x = 0, \quad x = 1$

**153.** $y = \cos^2 x, \quad y = \sin^2 x, \quad x = -\frac{\pi}{4}, \quad x = \frac{\pi}{4}$

**154.** $y = \cos^2 x, \quad y = \sin x \cos x, \quad x = -\frac{\pi}{2}, \quad x = \frac{\pi}{4}$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the *x-axis*

**155.** $y = \tan x, \quad y = 0, \quad x = -\frac{\pi}{4}, \quad x = \frac{\pi}{4}$

**156.** $y = \cos\frac{x}{2}, \quad y = \sin\frac{x}{2}, \quad x = 0, \quad x = \frac{\pi}{2}$

Find the ***volume*** of the solid generated by revolving the region bounded by the graphs of the equations about the *x-axis*, then find the ***centroid*** of the region

**157.** $y = \sin x, \quad y = 0, \quad x = 0, \quad x = \pi$

**158.** $y = \cos x, \quad y = \sin 0, \quad x = 0, \quad x = \frac{\pi}{2}$