Derivatives

Constant Rule	$\frac{d}{dx}[c]=0$, c is a constant	
Constant Multiple Rule	$\frac{d}{dx}[cu]=\mathrm{c}\frac{d}{dx}[u]$, c is a constant	
Sum and Difference Rules	$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$(u\pm v)'=u'\pm v'$
Product Rule	$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$(uv)'=u'v+v'u$
Quotient Rule	$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{\mathrm{v}\frac{du}{dx}+\mathrm{u}\frac{dv}{dx}}{v^2}$	$\left(\frac{u}{v}\right)'=\frac{u'v-v'u}{v^2}$
Power Rules	$\frac{d}{dx}[x^n]=n\,x^{n-1}$ $\frac{d}{dx}[u^n]=n\,u^{n-1}\frac{du}{dx}$	$(U^n)'=nU^{n-1}U'$
Chain Rule	$\frac{dy}{dx}=\frac{dy}{du}\bullet\frac{du}{dx}$	

$\left(\frac{1}{x}\right)'=-\frac{1}{x^2}$	$\left(\frac{1}{\sqrt{x}}\right)'=-\frac{1}{2x\sqrt{x}}$	$\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}$
$\left(\frac{1}{U}\right)'=-\frac{U'}{U^2}$	$\left(\frac{1}{\sqrt{U}}\right)'=-\frac{U'}{2U^{3/2}}$	$\left(\sqrt{U}\right)'=\frac{U'}{2\sqrt{U}}$

$$\left(\frac{1}{U^n}\right)'=-\frac{n\cdot U'}{U^{n+1}}$$

$$\left(U^m V^n W^p\right)'=U^{m-1}V^{n-1}W^{p-1}\left(mU'VW+nUV'W+pUVW'\right)$$

$$\left(\frac{ax^n+b}{cx^n+d}\right)'=\frac{n(ad-bc)x^{n-1}}{\left(cx^n+d\right)^2}$$

$$\frac{d}{dx}\left(\frac{ax^n+b}{cx^n+d}\right)^m=mn(ad-bc)x^{n-1}\frac{\left(ax^n+b\right)^{m-1}}{\left(cx^n+d\right)^{m+1}}$$

Trigonometric

$\frac{d}{dx}(\sin u) = u'\cos u$	$\frac{d}{dx}(\cos u) = -u'\sin u$	$\frac{d}{dx}(\tan u) = u'\sec^2 u$
$\frac{d}{dx}(\csc u) = -u'\csc u \ \cot u$	$\frac{d}{dx}(\sec u) = u'\sec u \ \tan u$	$\frac{d}{dx}(\cot u) = -u'\csc^2 u$

Inverse Trigonometric

$\frac{d}{dx}(\arcsin u) = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}(\arccos u) = \frac{-u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}(\arctan u) = \frac{u'}{1+u^2}$
$\frac{d}{dx}(\operatorname{arccsc} u) = \frac{-u'}{\lvert u\rvert\sqrt{u^2-1}}$	$\frac{d}{dx}(\operatorname{arcsec} u) = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	$\frac{d}{dx}(\operatorname{arccot} u) = \frac{-u'}{1+u^2}$

Hyperbolic

$\sinh(x) = \frac{e^x - e^{-x}}{2}$	$\cosh(x) = \frac{e^x + e^{-x}}{2}$	$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
$\frac{d}{dx}(\sinh u) = u'\cosh u$	$\frac{d}{dx}(\cosh u) = u'\sinh u$	$(\tanh u)' = u'\left(1-\tanh^2 u\right) = u'\left(\mathrm{sech}^2 u\right)$
$\frac{d}{dx}(\mathrm{csc\,h}u) = -u'\coth u \ \mathrm{csch}u$	$\frac{d}{dx}(\mathrm{sech}x) = -u'\tanh u \ \mathrm{sech}u$	$(\coth u)' = u'\left(1-\coth^2 u\right) = -u'\left(\mathrm{csch}^2 u\right)$

Inverse Hyperbolic

$\frac{d}{dx}\left(\sinh^{-1} u\right) = \frac{u'}{\sqrt{u^2+1}}$	$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{u'}{\sqrt{u^2-1}}$	$\frac{d}{dx}\left(\tanh^{-1} u\right) = \frac{u'}{1-u^2}$
$\frac{d}{dx}\left(\mathrm{csch}^{-1} u\right) = -\frac{u'}{\lvert u\rvert\sqrt{1+u^2}}$	$\frac{d}{dx}\left(\mathrm{sech}^{-1} u\right) = -\frac{u'}{u\sqrt{1-u^2}}$	$\frac{d}{dx}\left(\coth^{-1} u\right) = \frac{u'}{1-u^2}$

Exponential Rule

$\frac{d}{dx}\left(e^x\right) = e^x$	$\frac{d}{dx}[e^u] = u'e^u$	
$\frac{d}{dx}[a^x] = a^x \ln(a)$	$\frac{d}{dx}[a^u] = a^u \ln(a)\frac{du}{dx}$	

Derivative of Natural Log **(ln)**

$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}$	
$\frac{d}{dx}\left[\log_a x\right] = \left(\frac{1}{\ln a}\right)\frac{1}{x}$	$\frac{d}{dx}\left[\log_a u\right] = \left(\frac{1}{\ln a}\right)\left(\frac{1}{u}\right)\frac{du}{dx}$	

Increasing and Decreasing Functions

Suppose that f is continuous on $[a, b]$ and differentiable on (a, b).

- If $f'(x) > 0$ for all x in (a, b), then f is increasing on (a, b)
- If $f'(x) < 0$ for all x in (a, b), then f is decreasing on (a, b)
- If $f'(x) = 0$ for all x in (a, b), then f is constant on (a, b)

Local Extrema

- If f' changes from negative to positive at c, then f has a local minimum (***LMIN***).
- If f' changes from positive to negative at c, then f has a local maximum (***LMAX***).
- If f' doesn't change sign at c, then f has no local extremum at c.

Concavity

Let f be function whose second derivative exists on an open interval I.

- If $f''(x) > 0$ for all x in I, then f is ***concave up*** on I.
- If $f''(x) < 0$ for all x in I, then f is ***concave down*** on I.

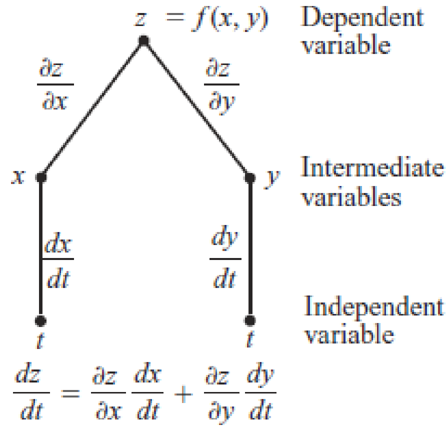
$L'Hôpital's$ Rule

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \quad \Rightarrow \quad \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Partial Derivatives

To compute $\dfrac{\partial f}{\partial x}$, differentiate $f(x, y)$ treating y as a constant. $\dfrac{\partial f}{\partial x} = f_x$, $\dfrac{\partial f}{\partial y} = f_y$

Chain Rule $\qquad \dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\dfrac{dy}{dt}$



Gradient Vector $\qquad \nabla f = \dfrac{\partial f}{\partial x}\boldsymbol{i} + \dfrac{\partial f}{\partial y}\boldsymbol{j}$

Directional Derivative: $\qquad D_u f(a, b) = \nabla f(a, b) \cdot u$

Tangent Plane for $F(x, y, z) = 0$ at $P_0(a, b, c)$:

$$F_x(P_0)(x-a) + F_y(P_0)(y-b) + F_z(P_0)(z-c) = 0$$

Tangent Plane to a Surface $z = f(x, y)$: $\qquad f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a-b)$

Second Derivative Test for Local Extrema

- If $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b), then f has a ***local maximum*** at (a, b).
- If $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b), then f has a ***local minimum*** at (a, b).
- If $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a, b), then f has a ***saddle point*** at (a, b).
- If $f_{xx}f_{yy} - f_{xy}^2 = 0$ at (a, b), then ***the Test is inconclusive*** at (a, b).

Lagrange Multipliers

One constraint: Suppose that $f(x, y, z)$ and $g(x, y, z)$ are differentiable. To find the local maximum and minimum values of f subject to the constraint $g(x, y, z) = 0$, find the values of x, y, z, and λ that simultaneously satisfy the equations

$$\nabla f = \lambda \nabla g \quad \textit{and} \quad g(x, y, z) = 0$$

Two constraints: For constraints $g(x, y, z) = 0$ and $h(x, y, z) = 0$, g and h differentiable, find the values of x, y, z, and λ that simultaneously satisfy the equations

$$\nabla f = \lambda \nabla g + \mu \nabla h, \quad g(x, y, z) = 0, \quad h(x, y, z) = 0$$