1) $\dfrac{x}{x^2+2x-3} = \dfrac{A}{x+3} + \dfrac{B}{x-1}$

$x = A(x-1) + B(x+3)$

$x^1 \quad A + B = 1$

$x^0 \quad -A + 3B = 0 \rightarrow A = 3\left(\frac{1}{4}\right)$

$4B = 1$

$B = \frac{1}{4}$

$$\frac{x}{x^2+2x-3} = \frac{3}{4}\frac{1}{x+3} + \frac{1/4}{x-1}$$

2) $\dfrac{1}{(2x+3)(4x-1)} = \dfrac{A}{2x+3} + \dfrac{B}{4x-1}$

$1 = A(4x-1) + B(2x+3)$

$x^1 \quad 4A + 2B = 0 \rightarrow (1)$

$x^0 \quad -4A + 3B = 1$ (2)

$14B = 4 \Rightarrow B = \frac{2}{7}$

(1) $4A = -2\left(\frac{2}{7}\right)$

$A = \frac{-4}{4(7)} = -1/7$

$$\frac{1}{(2x+3)(4x-1)} = \frac{-1/7}{2x+3} + \frac{2/7}{4x-1}$$

1st 4 terms? a_8?

$$a_n = (-1)^{n+1} \frac{n}{n+1}$$

$$a_1 = (-1)^2 \frac{1}{2} = \frac{1}{2}$$

$$a_2 = (-1)^3 \frac{2}{3} = -\frac{2}{3}$$

$$a_3 = (-1)^4 \frac{3}{4} = \frac{3}{4}$$

$$a_4 = (-1)^5 \frac{4}{5} = \frac{-4}{5}$$

$$a_8 = (-1)^9 \frac{8}{9} = -\frac{8}{9}$$

$$2 + 5 + 8 + \cdots + 17 = \sum_{n=1}^{6} (3n-1)$$

$d = 5 - 2 = 3$

$a_n = 2 + (n-1)(3)$

$= 2 + 3n - 3$

$= 3n - 1$

$$\frac{17-2}{3} = 5 + 1$$

Arithm. a_{12}: $a_8 = 4$ $a_{18} = -96$

$$d = \frac{-96-4}{18-8} = -\frac{100}{10} = -10$$

$a_n = a_1 + (n-1)d$

$a_8 = a_1 + 7(-10)$

$4 = a_1 - 70$

$a_1 = 74$

$a_{12} = 74 + 11(-10)$

$= 74 - 110$

$= -36$

Geom a_9: $a_2 = cf$ $a_5 = 32$

$r = \left(\frac{32}{cf}\right)^{1/5-2}$

$= (8)^{1/3}$

$= 2$

$(2^3)^{1/3}$

$a_n = a_1 r^{n-1}$

$a_2 = a_1 2^1 = 4$

$a_1 = 2$

$a_9 = 2(2)^8$

$= 2^9$

$$\sum_{n=1}^{25} 5 = 5(25)$$

$$= \underline{125}$$

$$\sum_{k=15}^{45} 8 = 8(45-15+1)$$

$$= 8(31)$$

$$= \underline{248}$$

you can avoid this step

$$\sum_{k=1}^{5} (2k+1) = (2+1)+(4+1)+(6+1)+(8+1)+11$$

$$= 3+5+7+9+11$$

$$= \underline{35}$$

$$\sum_{n=1}^{\infty} 5\left(\frac{3}{2}\right)^n = \infty \qquad \left|\frac{3}{2}\right| \geq 1$$

$$\sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right) = \frac{3}{1-\frac{2}{3}} \qquad \left|\frac{2}{3}\right| < 1$$

$$= \underline{9} \qquad \frac{3}{\frac{1}{3}} \qquad \frac{a}{1-r}$$

$4 + 8 + 12 + \cdots + 4n = 2n(n+1)$

$n = 1 \Rightarrow 4 = 2(1+1)$

$4 = 4$ ✓ P_1 is true.

Assume P_k is true: $4 + \cdots + 4k = 2k(k+1)$

is P_{k+1}: $4 + \cdots + 4k + 4(k+1) = 2(k+1)(k+2)$?

$4 + \cdots + 4k + 4(k+1) = 2k(k+1) + 4(k+1)$

$= 2(k+1)(k+2)$ ✓

P_{k+1} is also true.

∴ By the mathematical model, the given proof is completed.

$h = 15^-$ $\quad w = 10$ $\quad x = 5^-$

$\frac{x^2}{25^2} + \frac{y^2}{20^2} = 1$

$\frac{y^2}{20^2} = 1 - \left(\frac{5}{25}\right)^2$

$\left(\frac{15}{20}\right)^2 ? \quad 1 - \frac{1}{25^-}$

$\frac{225^-}{400} \cdot ? \frac{24}{25^-}$

$225^- ? \frac{24 \times 400^{16}}{25}$

$225^- < 384$

will clear

$625y^2 - 400x^2 = 25 \times 10^4$.

$\rightarrow \frac{y^2}{400} - \frac{x^2}{625} = 1$

$a^2 = 400$

$a = 20$

the closest point : $2a = \underline{40}$ yards.

$1 + 5 + 9 + \cdots + (4n-3) = n(2n-1)$

For $n = 1 \Rightarrow$ $1 = 1(2-1)$

$1 = 1$ ✓ P, is true

Assume P_k is true: $1 + \cdots + (4k-3) = k(2k-1)$

is P_{k+1}: $1 + \cdots + (4k-3) + (4(k+1)-3) =$

$(k+1)(2(k+1)-1)$

$1 + \cdots + (4k-3) + (4k+1) \stackrel{?}{=} (k+1)(2k+1)$

$1 + \cdots + (4k-3) + (4k+1) = k(2k-1) + (4k+1)$

$= 2k^2 - k + 4k + 1$

$= 2k^2 + 3k + 1$

$= (k+1)(2k+1)$ ✓

P_{k+1} is also true

∴ By the mathematical induction, the given proof is complete

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

For $n = 1 \Rightarrow 1^2 = \frac{1(2)(3)}{6}$

$1 = 1$ ✓ P_1 is true.

Assume: P_k is true: $1^2 + \cdots + k^2 = \frac{1}{6} k(k+1)(2k+1)$

is P_{k+1}: $1^2 + \cdots + k^2 + (k+1)^2 = \frac{1}{6}(k+1)(k+2)(2k+3)$

$$1^2 + \cdots + k^2 + (k+1)^2 = \frac{1}{6} k(k+1)(2k+1) + (k+1)^2 \frac{6}{6}$$

$$= \frac{1}{6}(k+1)\left[k(2k+1) + 6(k+1)\right]$$

$$= \frac{1}{6}(k+1)(2k^2 + 7k + 6)$$

$$= \frac{1}{6}(k+1)(k+2)(2k+3)$$ ✓

P_{k+1} is also true

∴ By the mathematical induction, the given proof is completed.

16 to 18 questions on Exam 1

Proof always on separate (only) paper

$$\frac{3x-1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$3x-1 = A(x+2) + B(x-1)$$

x^1 $\quad A + B = 3 \rightarrow B = 3 - \frac{2}{3} = \frac{7}{3}$

x^0 $\quad 2A - B = -1$

$$3A = 2$$

$$A = \frac{2}{3}$$

$$\frac{3x-1}{(x-1)(x+2)} = \frac{2/3}{x-1} + \frac{7/3}{x+2}$$

$$\frac{2x+1}{2x^2+x-3} = \frac{A}{2x+3} + \frac{B}{x-1}$$

$$2x+1 = A(x-1) + B(2x+3)$$

x^1 $\quad A + 2B = 2 \rightarrow A = 2 - 2\left(\frac{3}{5}\right) = 2 - \frac{6}{5} = \frac{4}{5}$

x^0 $\quad -A + 3B = 1$

$$5B = 3$$

$$B = \frac{3}{5}$$

$$\frac{2x+1}{2x^2+x-3} = \frac{4/5}{2x+3} + \frac{3/5}{x-1}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ simplify

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \qquad \left(\frac{x}{a}\right)^2$$

$$y = \frac{a_1^2 - x_1^2}{a_1^2}$$

$$y^2 = \frac{b^2}{a_1^2}(a_1^2 - x_1^2)$$

$$y^2 ? \left(\frac{b}{a_1}\right)^2 (a_1^2 - x_1^2)$$

$$\downarrow$$

simplify

$w = 20 \rightarrow x = 5$

$h = y = 9$

$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

$$\frac{y^2}{10^2} = 1 - \left(\frac{5}{20}\right)^2$$

$$= 1 - \frac{1}{16}$$

$$= \frac{15}{16}$$

$$q^2 ? \left(\frac{10}{4}\right)^2 (15)$$

$$81 ? \frac{25}{4} (15)$$

$$81 \times 4 ? (15)(25)$$

$$324 < 375$$ clear