Solution   *Section* **3.2 - Exponential Functions**

Exercise

Evaluate to four decimal places using a calculator $2^{3.4}$

Solution

$$2^{3.4} = 10.5561$$

Exercise

Evaluate to four decimal places using a calculator $5^{\sqrt{3}}$

Solution

$$5^{\sqrt{3}} = 16.2425$$

Exercise

Evaluate to four decimal places using a calculator $6^{-1.2}$

Solution

$$6^{-1.2} = 0.1165$$

Exercise

Evaluate to four decimal places using a calculator: $e^{-0.75}$

Solution

$$e^{-0.75} = .4724$$

Exercise

Evaluate to four decimal places using a calculator: $e^{2.3}$

Solution

$$e^{2.3} = 9.9742$$

Exercise

Evaluate to four decimal places using a calculator: $e^{-0.95}$

Solution

$e^{-0.95} = 0.3867$

Exercise

Evaluate to four decimal places using a calculator: $\pi^{\sqrt{\pi}}$

Solution

$\pi^{\sqrt{\pi}} = 7.6063$

Exercise

Evaluate to four decimal places using a calculator: $e^{\sqrt{2}}$

Solution

$e^{\sqrt{2}} = 4.1133$
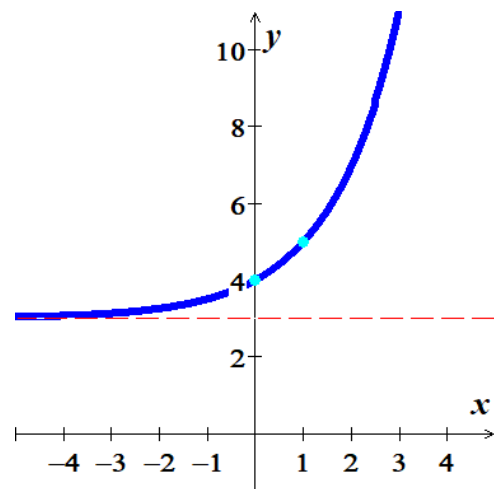
Exercise

Sketch the graph: $f(x) = 2^x + 3$

Solution

Asymptote: $y = 3$

Domain: $(-\infty, \infty)$

Range: $(3, \infty)$

x	$f(x)$
-1	3.5
0	4
1	5
2	7

Exercise

Sketch the graph: $f(x) = 2^{3-x}$

Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
1	4
2	2
0	8
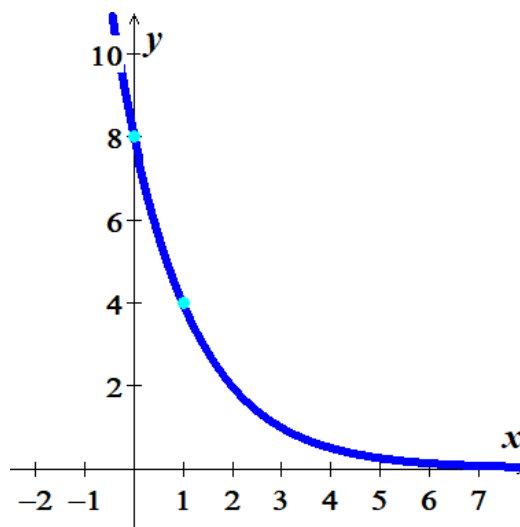
Exercise

Sketch the graph: $f(x) = \left(\dfrac{2}{5}\right)^{-x}$

Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
-1	0.4
0	1
1	2.5
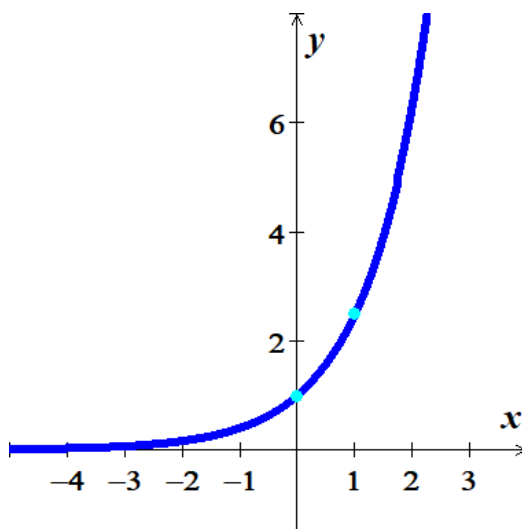
Exercise

Sketch the graph: $f(x) = -\left(\dfrac{1}{2}\right)^{x} + 4$

Solution

Asymptote: $y = 4$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4)$

x	$f(x)$
-2	0
-1	2
0	3

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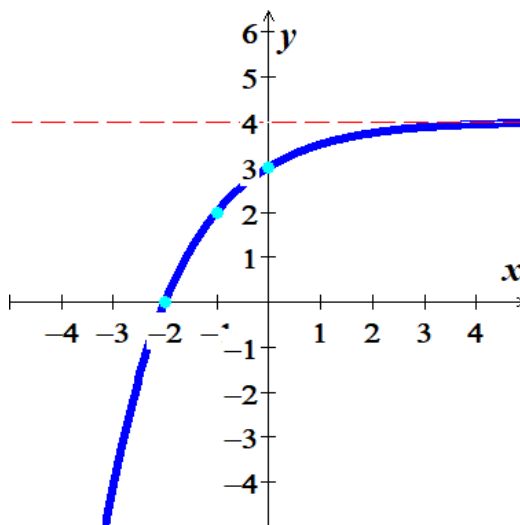
Exercise

Sketch the graph of $f(x) = 4^x$

Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
0	1
1	4
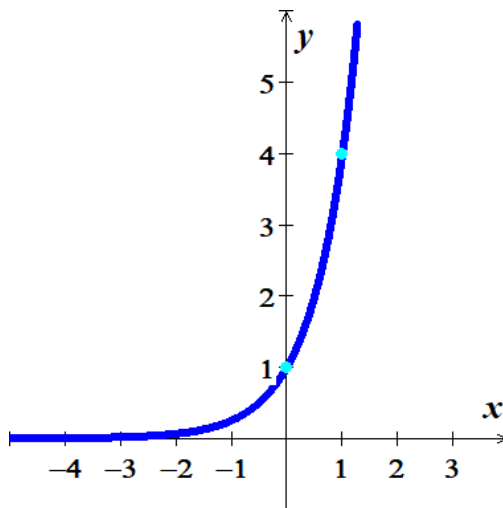
Exercise

Sketch the graph of $f(x) = 2 - 4^x$

Solution

Asymptote: $y = 2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 2)$

x	$f(x)$
0	1
1	-2
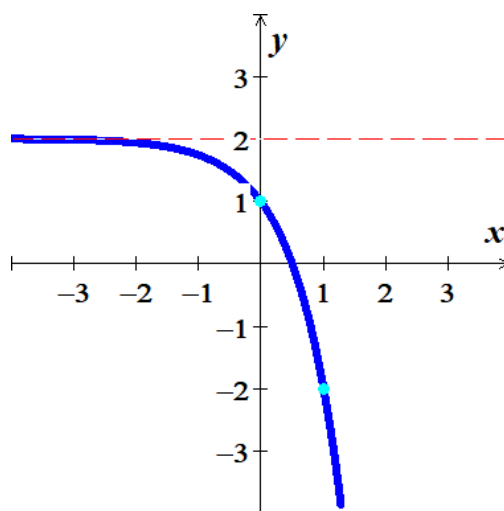
Exercise

Sketch the graph of $f(x) = -3 + 4^{x-1}$

Solution

Asymptote: $y = -3$

Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

x	$f(x)$
1	-2
2	1

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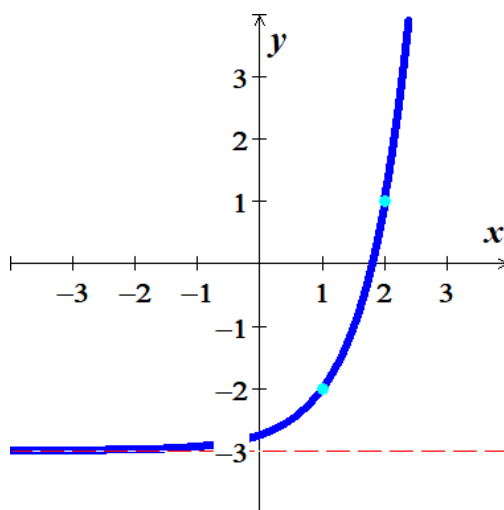
Exercise

Sketch the graph of $f(x) = 1 + \left(\dfrac{1}{4}\right)^{x+1}$

Solution

Asymptote: $y = 1$

Domain: $(-\infty, \infty)$

Range: $(1, \infty)$

x	$f(x)$
-1	2
0	$\dfrac{5}{4}$
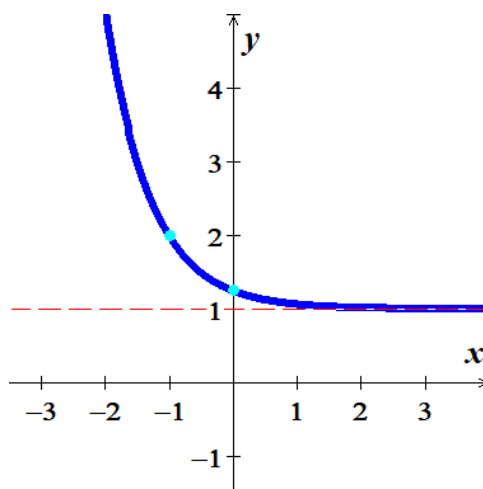
Exercise

Sketch the graph of $f(x) = e^{x-2}$

Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
2	1
3	2.7
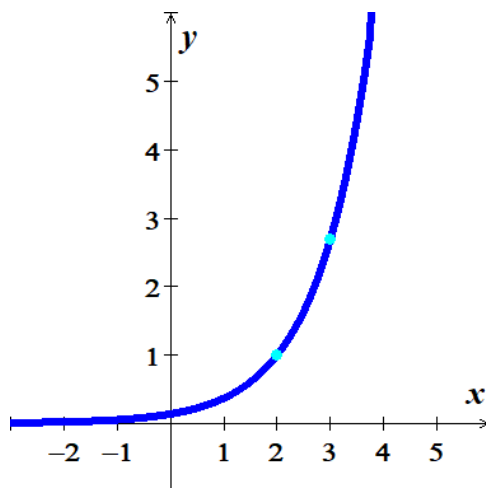
Exercise

Sketch the graph of $f(x) = 3 - e^{x-2}$

Solution

Asymptote: $y = 3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

x	$f(x)$
2	2
3	$.3$

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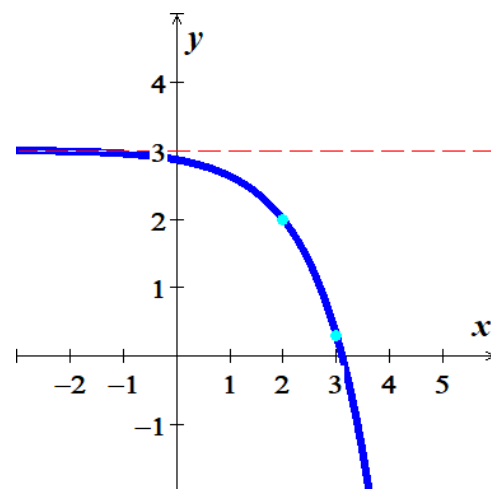
Exercise

Sketch the graph of $f(x) = e^{x+4}$

Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
-4	1
-3	2.7
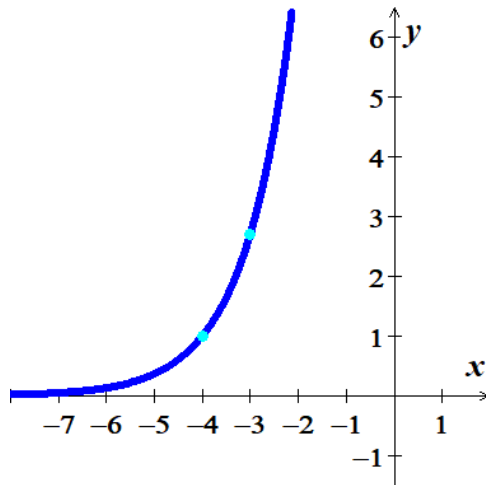
Exercise

Sketch the graph of $f(x) = 2 + e^{x-1}$

Solution

Asymptote: $y = 2$

Domain: $(-\infty, \infty)$

Range: $(2, \infty)$

x	$f(x)$
1	3
2	4.7
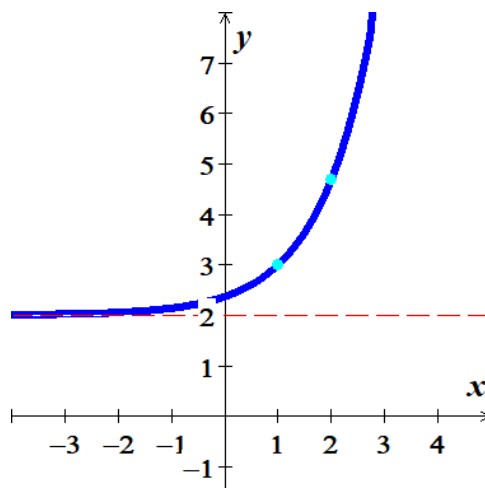
Exercise

The exponential function $f(x) = 1066e^{0.042x}$ models the gray wolf population of the Western Great Lakes, $f(x)$, in *billions*, *x years* after 1978. Project the gray population in the recovery area in 2012.

Solution

$x = 2012 - 1978 = 34$

$f(x = 34) = 1066e^{0.042(34)}$

$\qquad = 4445.6$

$\qquad \approx 4446 \ \ billions$

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Exercise

The function $f(x) = 6.4e^{0.0123x}$ describes world population, $f(x)$, in billions, x *years* after 2004 subject to a growth rate of 1.23% annually. Use the function to predict world population in 2050.

Solution

$x = 2050 - 2004 = 46$

$f(x = 46) = 6.4e^{0.0123(46)}$

$\approx 11.27 \ \ billion$

Exercise

A cup of coffee is heated to $160°F$ and placed in a room that maintains a temperature of $70°F$. The temperature T of the coffee, in *degree Fahrenheit*, after t minutes is given by

$$T(t) = 70 + 90e^{-0.0485t}$$

a) Find the temperature of the coffee 20 *minutes* after it is placed in the room
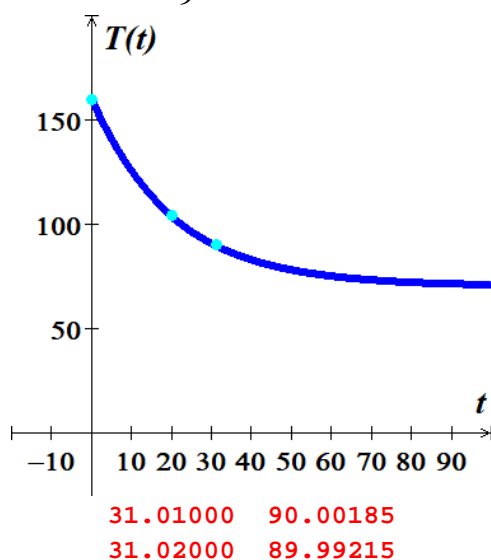*b)* Determine when the temperature of the coffee will reach $90°F$

Solution

a) $T(20) = 70 + 90e^{-0.0485(20)}$

$\approx 104°F$

b) $T(t) = 70 + 90e^{-0.0485t} = 90$

$90e^{-0.0485t} = 20$

$e^{-0.0485t} = \dfrac{2}{9}$

| 31.01000 | 90.00185 |
| 31.02000 | 89.99215 |

∴ The temperature of the coffee will reach $90°F$ in about **31.01** *minutes*.

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Exercise

A cup of coffee is heated to $180°F$ and placed in a room that maintains a temperature of $65°F$. The temperature T of the coffee, in *degree Fahrenheit*, after t minutes is given by

$$T(t) = 65 + 115e^{-0.042t}$$

a) Find the temperature of the coffee 10 *minutes* after it is placed in the room
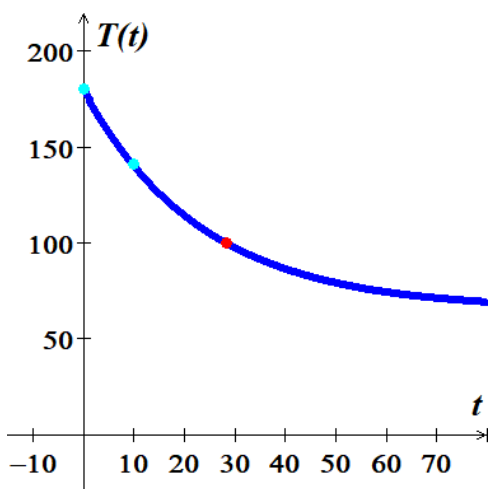b) Determine when the temperature of the coffee will reach $100°F$

Solution

a) $T(10) = 65 + 115e^{-0.042(10)}$

$\approx 141°F$

b) $T(t) = 65 + 115e^{-0.042t} = 100$

$115e^{-0.042t} = 35$

$e^{-0.042t} = \dfrac{7}{23}$


```
28.32000  100.00504
28.34000   99.97565
```

$\therefore$ The temperature of the coffee will reach $100°F$ in about **31.01** *minutes*.

Exercise

The percent $I(x)$ of the original intensity of light striking the surface of a lake that is available $x\ feet$ below the surface of the lake is given by the equation

$$I(x) = 100e^{-.95x}$$

a) What percentage of the light is available 2 *feet* below the surface of the lake?
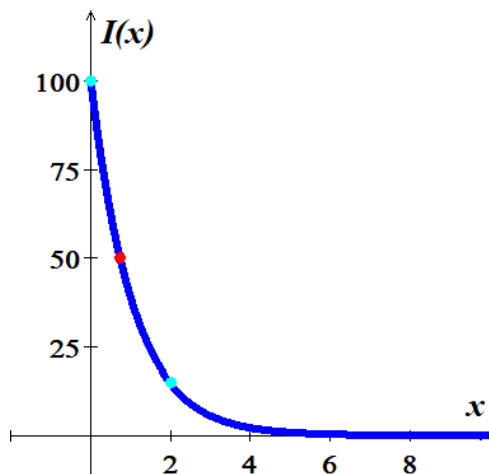b) At what depth is the intensity of the light one-half the intensity at the surface?

Solution

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a) $I(2) = 100e^{-.95(2)}$

≈ 14.96 |

∴ The percentage of the light is available 2 *feet* below the surface of the lake is **15%**

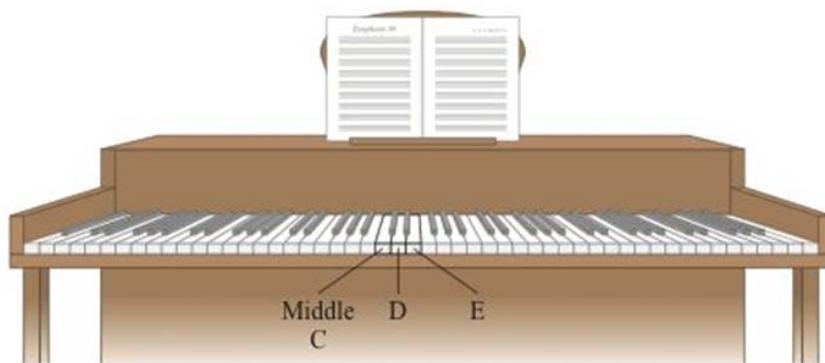b) $I(x) = 100e^{-.95x} = \dfrac{1}{2}(100)$

$e^{-.95x} = \dfrac{1}{2}$


```
0.72800    50.07742
0.73200    49.88749
```

∴ The depth is **0.73** *feet* when the intensity of the light one-half the intensity at the surface

Exercise

Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

$$f(n) = (2.75)\ 2^{\frac{n-1}{12}}$$

a) Determine the frequency of middle C, key number 40 on an 88–*key* piano.
b) Is the difference in frequency between middle C (key number 40) and D (key number 42) the same as the difference in frequency between D (key number 42) and E (key number 44)?

Solution

a) $f(40) = (2.75) \ 2^{\frac{40-1}{12}}$

≈ 26.16

the frequency of middle C is ≈ 26 *vibrations* per *second*.

b) $f(42) = (2.75) \ 2^{(41/12)}$

≈ 29.37

The difference between the frequency of middle C and D is:
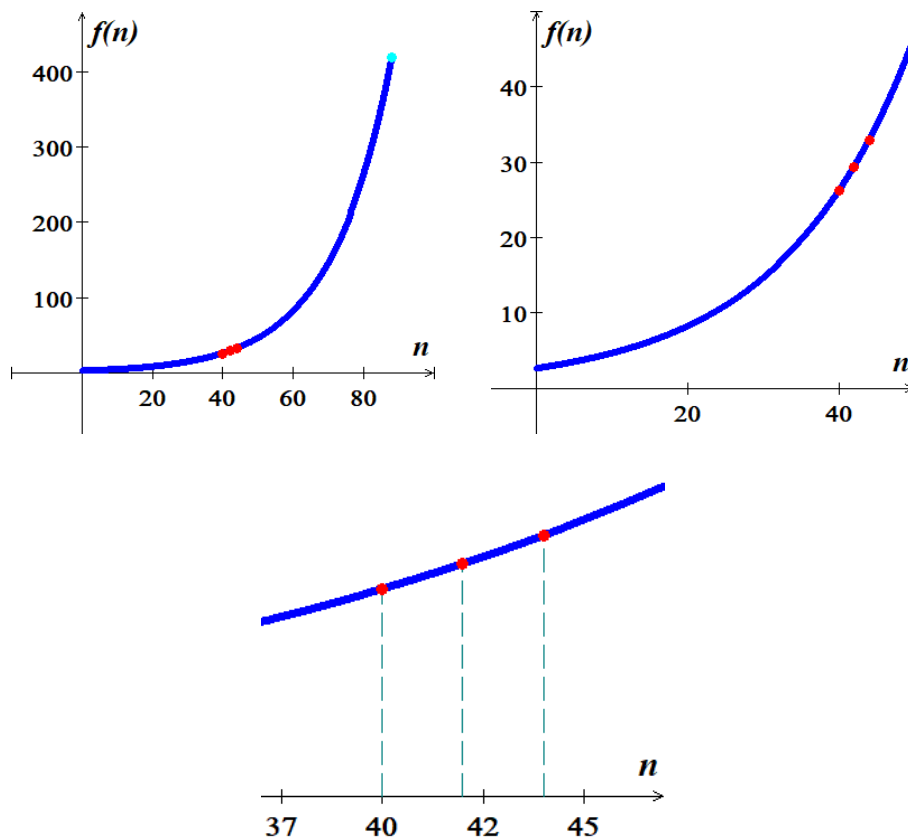$29.37 - 26.16 \approx 3.21$

$f(44) = (2.75) \ 2^{(43/12)}$

≈ 32.96

The difference between the frequency of middle D and E is:
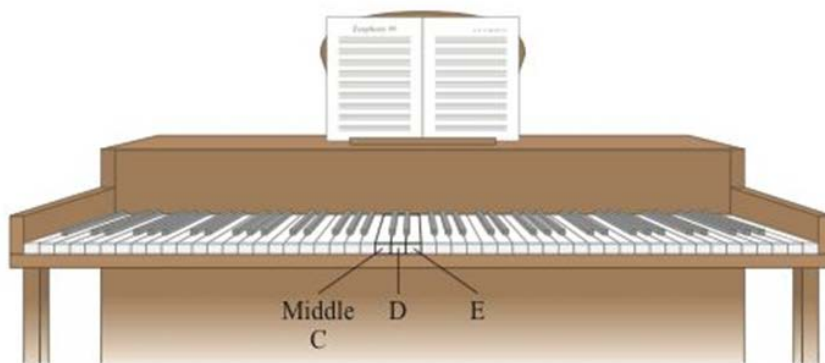$32.96 - 29.37 \approx 3.59$

$\therefore$ the differences are **not** the same since the function is *not* linear function

Exercise

Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

$$f(n) = (27.5)\ 2^{\frac{n-1}{12}}$$

Middle D   E
C

c) Determine the frequency of middle *C*, key number 40 on an 88–*key* piano.

d) Is the difference in frequency between middle *C* (key number 40) and *D* (key number 42) the same as the difference in frequency between *D* (key number 42) and *E* (key number 44)?

Solution

c) $f(40) = (27.5)\ 2^{\frac{40-1}{12}}$

≈ 261.63

The frequency of middle *C* is ≈ 262 *vibrations* per *second*.

d) $f(42) = (27.5)\ 2^{(41/12)}$

≈ 293.66

The difference between the frequency of middle *C* and *D* is: $293.66 - 261.66 \approx 32$

$f(44) = (27.5)\ 2^{(43/12)}$

≈ 329.63

The difference between the frequency of middle *D* and *E* is: $329.63 - 293.66 \approx 36$

∴ The differences are ***not*** the same since the function is *not* linear function.

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