Solution   *Section* 7.3 – Graphing Secant & Cosecant

Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \sec\left(x - \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = \frac{\pi}{2}$

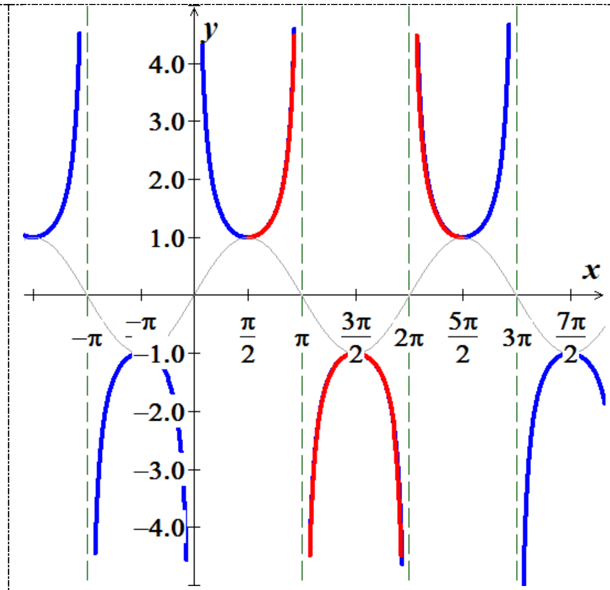
VT: $y = 0$

Asymptotes:

$$x - \frac{\pi}{2} = n\pi + \frac{\pi}{2}$$

$$x = n\pi$$

	x	$\cos\left(x - \frac{\pi}{2}\right)$
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	1
$\frac{\pi}{2} + \frac{\pi}{2}$	π	0
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	-1
$\frac{3\pi}{2} + \frac{\pi}{2}$	2π	0
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	1



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\sec\left(2x - \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

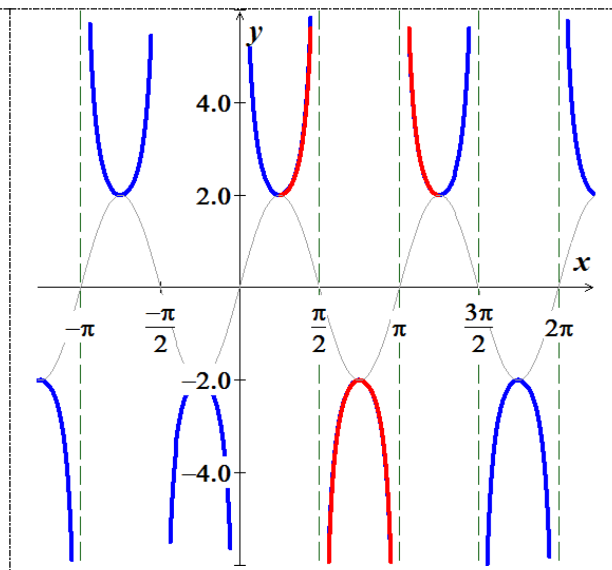
VT: $y = 0$

Asymptotes:

$$2x - \frac{\pi}{2} = n\pi + \frac{\pi}{2}$$

$$x = \left(\frac{n+1}{2}\right)\pi$$

	x	$2\cos\left(2x - \frac{\pi}{2}\right)$
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	2
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	-2
$\frac{3\pi}{4} + \frac{\pi}{4}$	2π	0
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	2



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -3\sec\left(\frac{1}{3}x + \frac{\pi}{3}\right)$

Solution

Amplitude: n/a

Period: $P = 6\pi$

Phase Shift: $\varphi = -\pi$

VT: $y = 0$

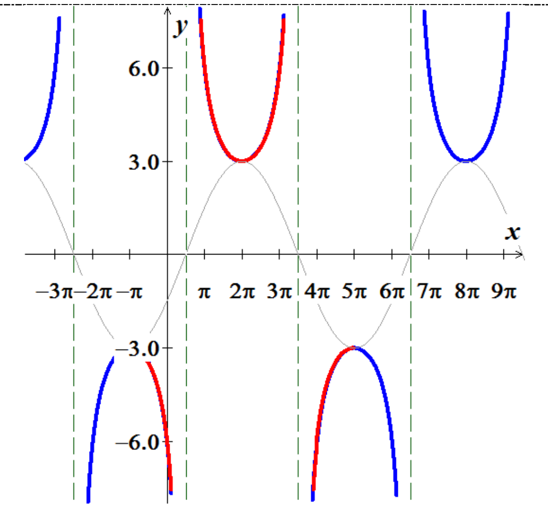
Asymptotes:

$$\frac{1}{3}x + \frac{\pi}{3} = n\pi + \frac{\pi}{2}$$

$$x = 3n\pi + \frac{\pi}{2}$$

$$x = \ldots,\ -\frac{5\pi}{2},\ \frac{\pi}{2},\ \frac{7\pi}{2},\ \ldots$$

	x	$-3\cos\left(\frac{1}{3}x+\frac{\pi}{3}\right)$
$0-\pi$	$-\pi$	-3
$\frac{3\pi}{2}-\pi$	$\frac{\pi}{2}$	0
$3\pi-\pi$	2π	3
$\frac{9\pi}{2}-\pi$	$\frac{7\pi}{2}$	0
$6\pi-\pi$	5π	-3



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \csc\left(x - \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = \frac{\pi}{2}$

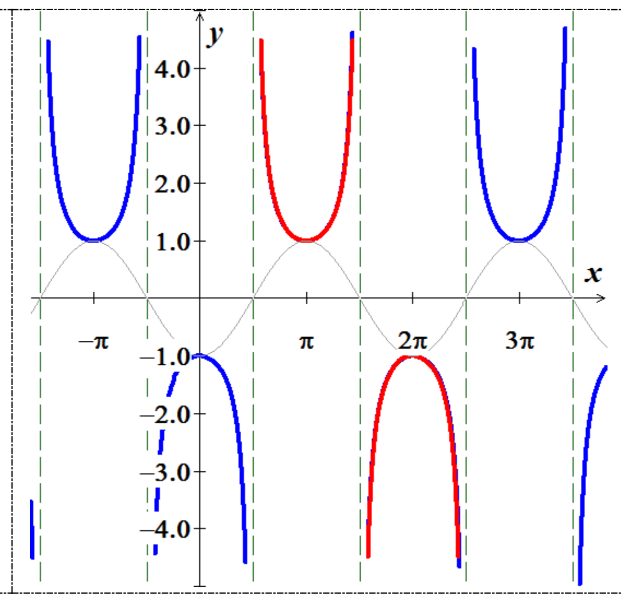
VT: $y = 0$

Asymptotes:

$$x - \frac{\pi}{2} = n\pi$$

$$x = n\pi + \frac{\pi}{2}$$

	x	$\sin\left(x-\frac{\pi}{3}\right)$
$0+\frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}+\frac{\pi}{2}$	π	1
$\pi+\frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$\frac{3\pi}{2}+\frac{\pi}{2}$	2π	-1
$2\pi+\frac{\pi}{2}$	$\frac{5\pi}{2}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\csc\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \pi$

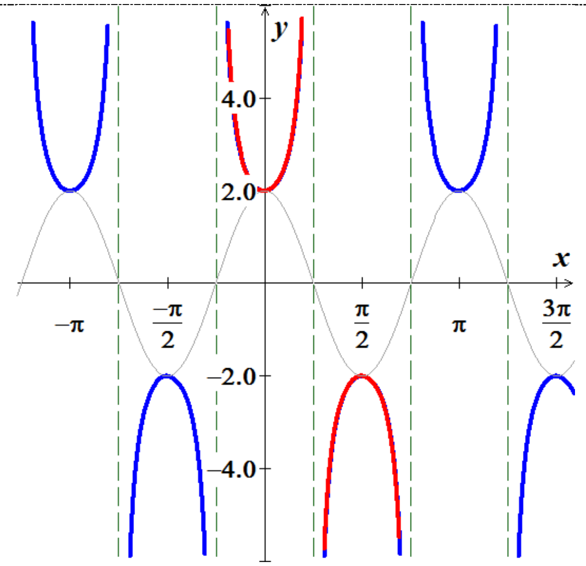
Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

Asymptotes:

$x = -\frac{\pi}{4} + n\frac{\pi}{2}$

	x	$2\sin\left(2x+\frac{\pi}{2}\right)$
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	0
$\frac{\pi}{4} - \frac{\pi}{4}$	0	2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{3\pi}{4} - \frac{\pi}{4}$	$\frac{\pi}{2}$	-2
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 4\csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = 4\pi$

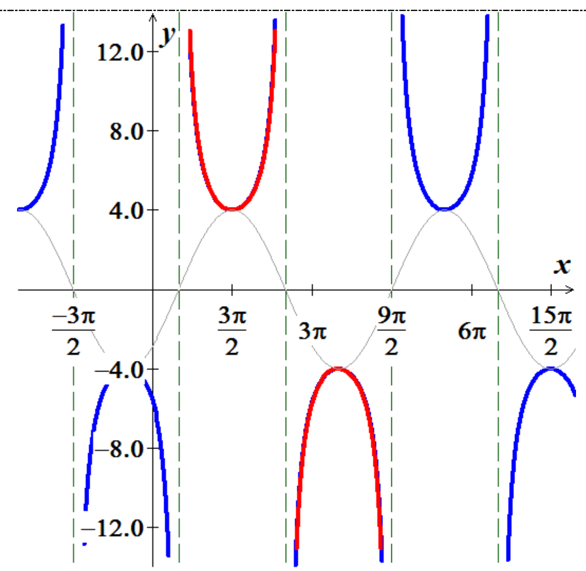
Phase Shift: $\varphi = \frac{\pi}{2}$

VT: $y = 0$

Asymptotes:

$x = \frac{\pi}{2} + 2\pi n$

	x	$4\sin\left(\frac{1}{2}x-\frac{\pi}{4}\right)$
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	4
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	0
$3\pi + \frac{\pi}{2}$	$\frac{7\pi}{2}$	-4
$4\pi + \frac{\pi}{2}$	$\frac{9\pi}{2}$	0



Exercise

Graph over a one-period interval $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

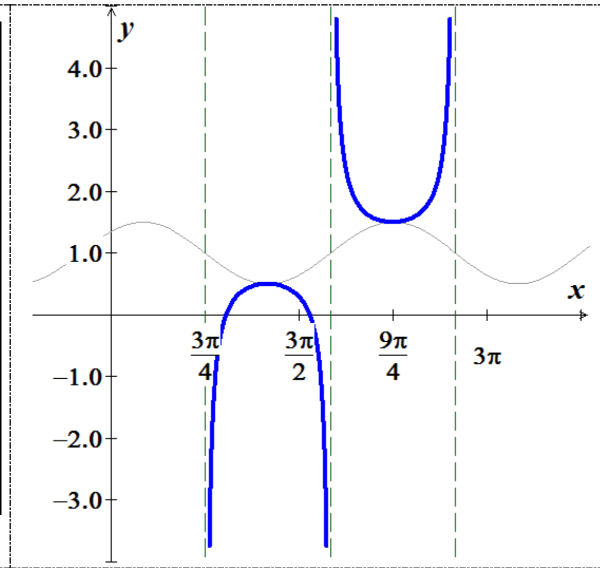
Phase Shift: $\varphi = \frac{3\pi}{4}$

VT: $y = 1$

Asymptotes:

$x = \frac{3\pi}{4} + 2\pi n$

	x	$4\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$
$0 + \frac{3\pi}{4}$	$\frac{3\pi}{4}$	1
$\frac{\pi}{2} + \frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{1}{2}$
$\pi + \frac{3\pi}{4}$	$\frac{7\pi}{4}$	1
$\frac{3\pi}{2} + \frac{3\pi}{4}$	$\frac{9\pi}{4}$	$-\frac{1}{2}$
$2\pi + \frac{3\pi}{4}$	$\frac{11\pi}{4}$	1



Exercise

Graph over a one-period interval $y = 2 + \frac{1}{4}\sec\left(\frac{1}{2}x - \pi\right)$

Solution

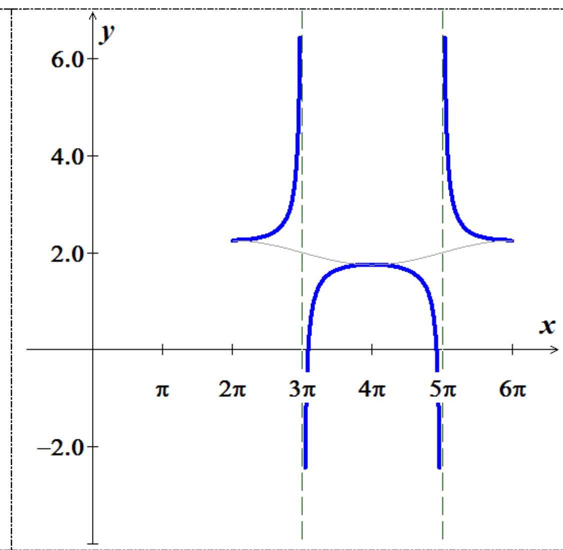
Amplitude: n/a

Period: $P = 4\pi$

Phase Shift: $\varphi = 2\pi$

VT: $y = 2$

	x	$2 + \frac{1}{4}\cos\left(\frac{1}{2}x - \pi\right)$
$0 + 2\pi$	2π	$\frac{9}{4}$
$\pi + 2\pi$	3π	2
$2\pi + 2\pi$	4π	$\frac{7}{4}$
$3\pi + 2\pi$	5π	2
$4\pi + 2\pi$	6π	$\frac{9}{4}$



Exercise

Graph $y = \frac{1}{3}\sec 2x \qquad for \quad -\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$

Solution

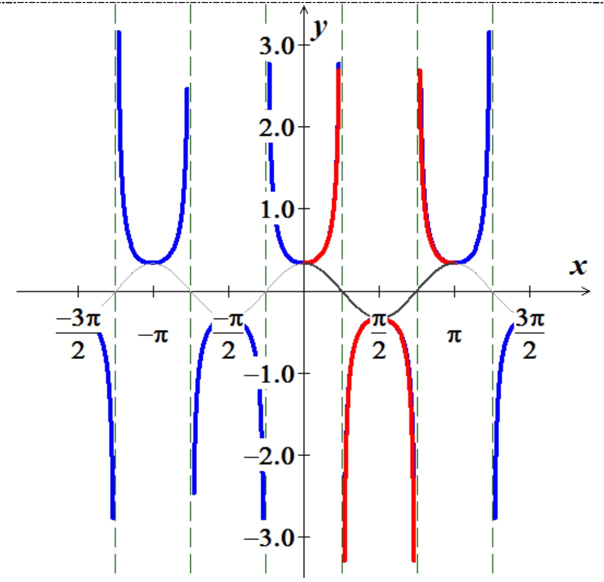
Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = 0$

VT: $y = 0$

x	$\frac{1}{3}\cos 2x$
0	$\frac{1}{3}$
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	$-\frac{1}{3}$
$\frac{3\pi}{4}$	0
π	$\frac{1}{3}$



Exercise

Graph one complete cycle $y = -1 - 3\csc\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$

Solution

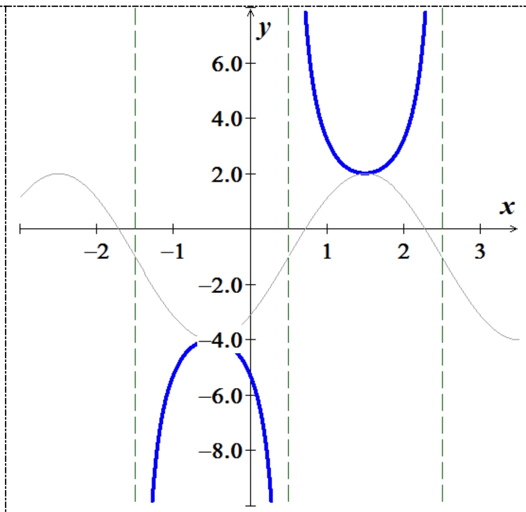
Amplitude: n/a

Period: $P = 4$

Phase Shift: $\varphi = -\frac{3}{2}$

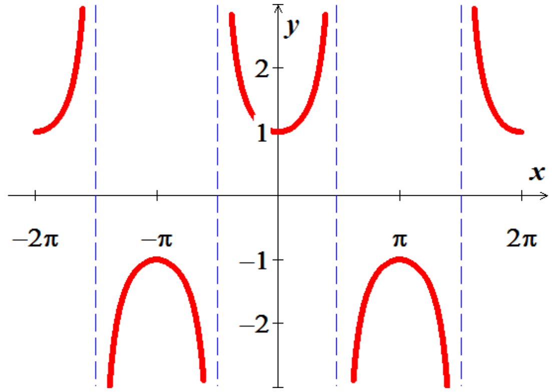
VT: $y = -1$

	x	$-1 - 3\sin\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$
$0 - \frac{3}{2}$	$-\frac{3}{2}$	-1
$1 - \frac{3}{2}$	$-\frac{1}{2}$	-4
$2 - \frac{3}{2}$	$\frac{1}{2}$	-1
$3 - \frac{3}{2}$	$\frac{3}{2}$	2
$4 - \frac{3}{2}$	$\frac{5}{2}$	-1



Exercise

Find an equation to match the graph



<u>***Solution***</u>

$P = 2\pi$

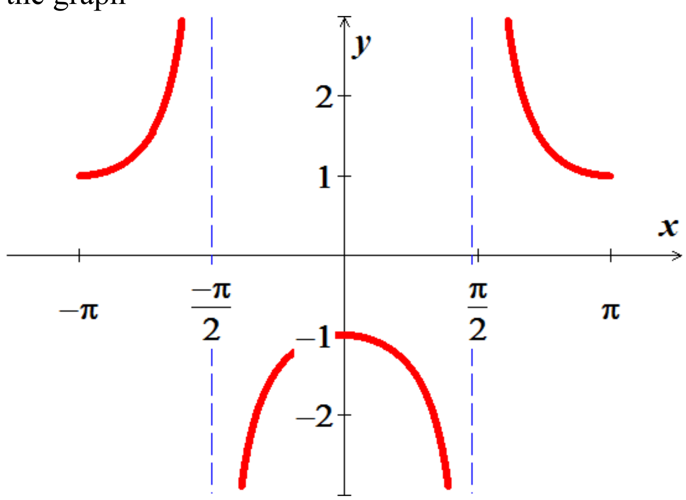
$\phi = 0$

$A = \frac{1+1}{2} = 1$

$y = \sec x \quad -2\pi \le x \le 2\pi$

Exercise

Find an equation to match the graph



<u>***Solution***</u>

$B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$

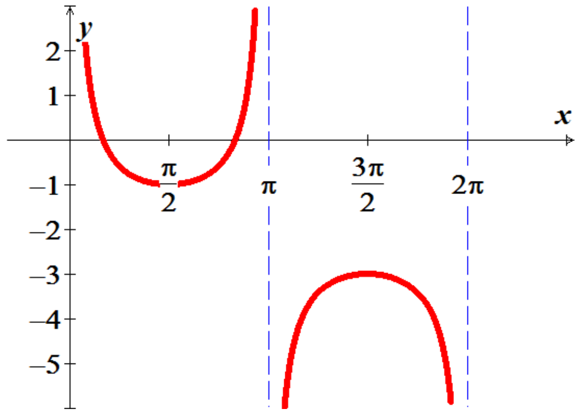
$\phi = 0 \rightarrow C = 0$

$A = \frac{1+1}{2} = 1$

$y = -\sec(x) \quad -\pi \le x \le \pi$

Exercise

Find an equation to match the graph



Solution

$$B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$$

$$\phi = 0 \to C = 0$$

$$A = \frac{-3-1}{2} = -2$$

$$y = -2 + \csc(x) \quad -2\pi \le x \le 2\pi$$

Exercise

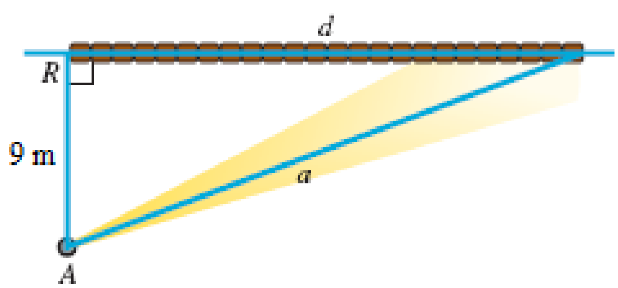
A rotating beacon is located at point A next to a long wall. The beacon is 9 m from the wall. The distance $\boldsymbol{a}$ is given by $a = 9|\sec 2\pi t|$, where t is time measured in seconds since the beacon started rotating. (When $t = 0$, the beacon is aimed at point R.) Find $\boldsymbol{a}$ for $t = 0.45$

Solution

$$a = 9\left|\sec\left(2\pi(0.45)\right)\right|$$

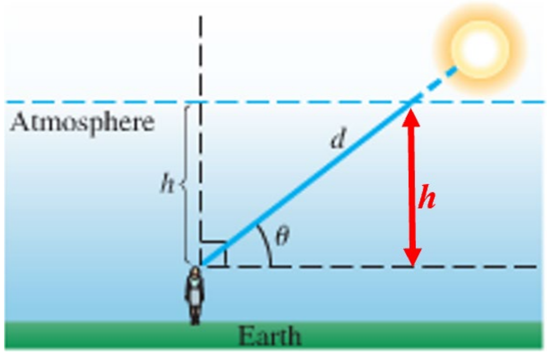
$$= \frac{9}{\left|\cos\left(2\pi(0.45)\right)\right|}$$

$$\approx 9.5\ m$$



Exercise

The shortest path for the sun's rays through Earth's atmosphere occurs when the sun is directly overhead. Disregarding the curvature of Earth, as the sun moves lower on the horizon, the distance that sunlight passes through the atmosphere increases by a factor of $\csc\theta$, where θ is the angle of elevation of the sun. This increased distance reduces both the intensity of the sun and the amount of ultraviolet light that reached Earth's surface.



a) Verify that $d = h\csc\theta$

b) Determine θ when $d = 2h$

c) The atmosphere filters out the ultraviolet light that causes skin to burn, Compare the difference between sunbathing when $\theta = \frac{\pi}{2}$ and when $\theta = \frac{\pi}{3}$. Which measure gives less ultraviolet light?

Solution

a) $\sin\theta = \frac{h}{d}$

$= \frac{1}{\csc\theta}$

$\underline{d = h\csc\theta}$ (*cross-multiplication*)

b) $\sin\theta = \frac{h}{d}$

$= \frac{h}{2h}$

$= \frac{1}{2}$

$\theta = \sin^{-1}\frac{1}{2}$

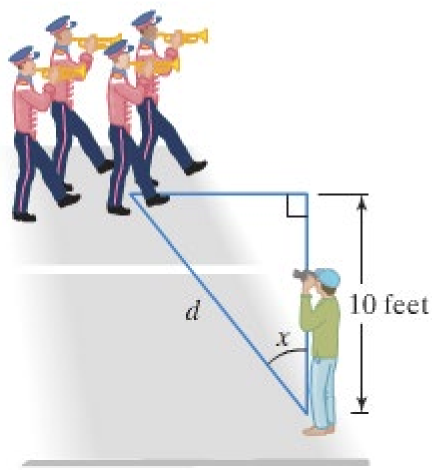
$= \frac{\pi}{6}$

c) $$\begin{cases} \csc\frac{\pi}{2} = 1 \\ \csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3} \approx 1.15 \end{cases}$$

When the distance to the sun is lager $\left(\theta = \frac{\pi}{3}\right)$, there is less ultraviolet light reaching the earth's surface. In this case, sunlight passes through 15% more atmosphere.

Exercise

Your friend is marching with a band and has asked you to film him. You have set yourself up 10 *feet* from the street where your friend will be passing from left to right. If d represents your distance, in feet, from your friend and x is the radian measure of the angle.



a) Express d in terms of a trigonometric function of x.

b) Graph the function for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Solution

a) $\cos x = \dfrac{10}{d}$

$\boxed{d = 10\sec x}$

b)

$y = 10\cos x$

Amplitude: $|A| = 10$

Period: $P = 2\pi$

Phase Shift: $\phi = 0$

VT: $y = 0$

x	y
0	10
$\frac{\pi}{2}$	0
π	−10
$\frac{3\pi}{2}$	0
2π	10