08-10-2020

Inverse Trig.

$$y = \sin x \implies \overbrace{x}^{\text{angle}} = \sin^{-1} y$$

or

$$x = \arcsin y$$

$-1 \le y \le 1$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

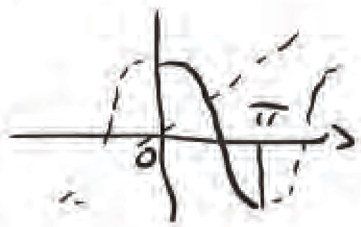
$$-1 \le y \le 1$$
$$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$\sin\left(\sin^{-1}\frac{1}{2}\right) = \frac{1}{2}$$

$$\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \checkmark$$

$$y = \cos^{-1} x \quad \text{iff} \quad x = \cos y$$

$$-1 \le x \le 1 \qquad 0 \le y \le \pi$$



Acosx

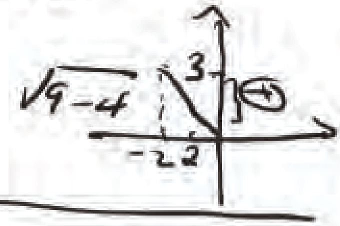
$$\cos^{-1} x = \arccos x$$

$$\cos(\cos^{-1} x) = x$$

$$\sin\Big(\underbrace{\arccos\left(-\tfrac{2}{3}\right)}_{\alpha}\Big) \qquad \alpha = \arccos\left(-\tfrac{2}{3}\right)$$

$$\cos\alpha = -\frac{2}{3} \qquad \frac{\pi}{2} < \theta < \pi$$

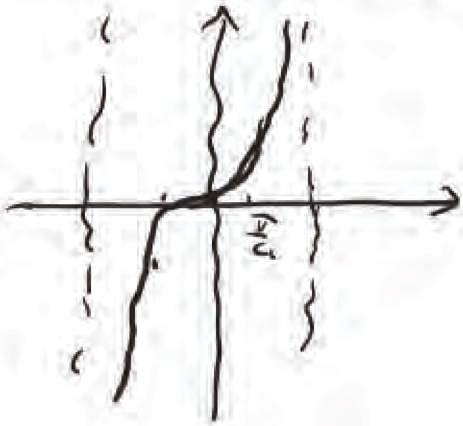
$$\sin\alpha = \frac{\sqrt{5}}{3}$$



Inverse Tangent

$$x = \tan y \Rightarrow y = \tan^{-1} x$$

$$y = \arctan x$$



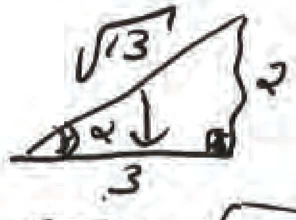
$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$\tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$$

$$\sec\Big(\underbrace{\arctan\tfrac{2}{3}}_{\alpha}\Big) \qquad \alpha = \arctan\frac{2}{3}$$

$$\tan\alpha = \frac{2}{3}$$

$$\sec\alpha = \frac{\sqrt{13}}{3}$$



$$r = \sqrt{x^2+y^2} \qquad \sqrt{9+4}$$

Ex $\sin\Big(\underbrace{\arctan\tfrac{1}{2}}_{\alpha} - \underbrace{\arccos\tfrac{4}{5}}_{\beta}\Big)$

$$\tan\alpha = \frac{1}{2}$$

$$\cos\beta = \frac{4}{5}$$

$$\sin\beta = \frac{3}{5}$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$= \frac{1}{\sqrt{5}}\,\frac{4}{5} - \frac{2}{\sqrt{5}}\cdot\frac{3}{5}$$

$$= \frac{4-6}{5\sqrt{5}}$$

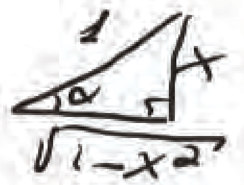
$$= -\frac{2}{5\sqrt{5}} \quad ⓐ \quad -\frac{2\sqrt{5}}{25}$$

Ex $\cos(\underbrace{\sin^{-1}x}_{\alpha}) = ?$

$\alpha = \sin^{-1}x$

$\sin\alpha = x \quad \left(\frac{x}{1}\right)$

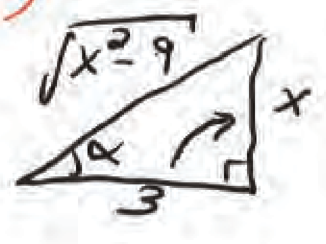
$\cos\alpha = \sqrt{1-x^2}$



#34 $\cot\left(\underbrace{\sin^{-1}\frac{\sqrt{x^2-9}}{x}}_{\alpha}\right)$

$\sin\alpha = \frac{\sqrt{x^2-9}}{x}$

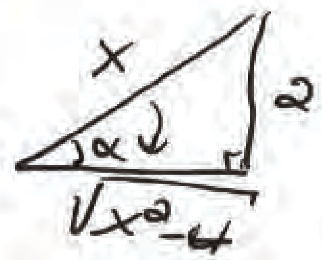
$\cot\alpha = \frac{3}{x}$



#39 $\sec\left(\underbrace{\tan^{-1}\frac{2}{\sqrt{x^2-4}}}_{\alpha}\right)$

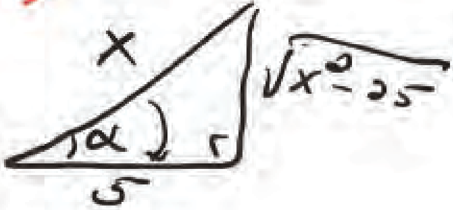
$\tan\alpha = \frac{2}{\sqrt{x^2-4}}$

$\sec\alpha = \frac{x}{\sqrt{x^2-4}}$



#22 $\sec\left(\sin^{-1}\frac{\sqrt{x^2-25}}{x}\right)$

$\sin\alpha = \frac{\sqrt{x^2-25}}{x}$

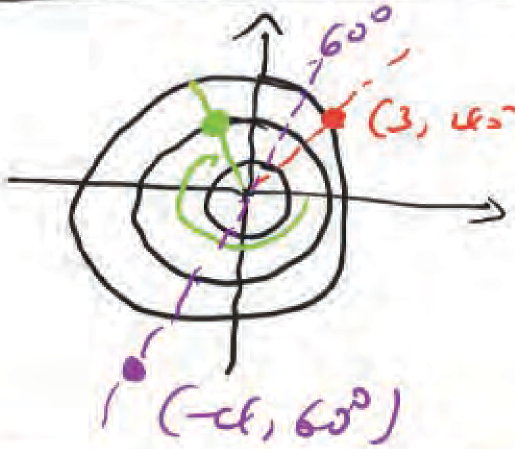


$\boxed{\sec\alpha = \frac{x}{5}}$

8.6 Polar Coordinates

$(r, \theta) \xrightarrow{\text{Relation}} (x, y)$ rectangular

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2} \\ \hat{\theta} = \tan^{-1}\frac{y}{x} \end{cases}$$



$(3, 45°)$

$(2, -240°)$

$(-4, 60°)$

Ex $(r, \theta) = (4, \frac{7\pi}{6})$ (x, y)?

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$= 4\cos\frac{7\pi}{6}$ $\qquad$ $= 4\sin\frac{7\pi}{6}$

$= 4\left(-\frac{\sqrt{3}}{2}\right)$ $\qquad$ $= 4\left(-\frac{1}{2}\right)$

$\boxed{= -2\sqrt{3}}$ $\qquad$ $\boxed{= -2}$

$$\boxed{(x, y) = (-2\sqrt{3}, -2)}$$

$(x, y) = (-1, \sqrt{3})$ $\qquad$ (r, θ)?

$r = \sqrt{x^2 + y^2}$ $\qquad$ $\hat{\theta} = \tan \frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$= \sqrt{1 + 3}$

$= 2$ $\qquad$ $\theta = \frac{2\pi}{3}$

$(r, \theta) = \left(2, \frac{2\pi}{3}\right)$ $\qquad$ 120°

$(-2, -60^\circ)$

-2

$x =$

$=$

(x, y)

---

line $ax + by = c$

$a\, r\cos\theta + b\, r\sin\theta = c$

$r(a\cos\theta + b\sin\theta) = c$

$r(a\cos\theta + b\sin\theta) = c$

$$\boxed{r = \frac{c}{a\cos\theta + b\sin\theta}}$$

$x^2 - y^2 = 16$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 16$

$r^2\cos^2\theta - r^2\sin^2\theta = 16$

$r^2(\underline{\cos^2 - \sin^2\theta}) = 16$

$$\boxed{r^2 = \frac{16}{\cos 2\theta}}$$

$r = a\sin\theta \quad (a \neq 0)$

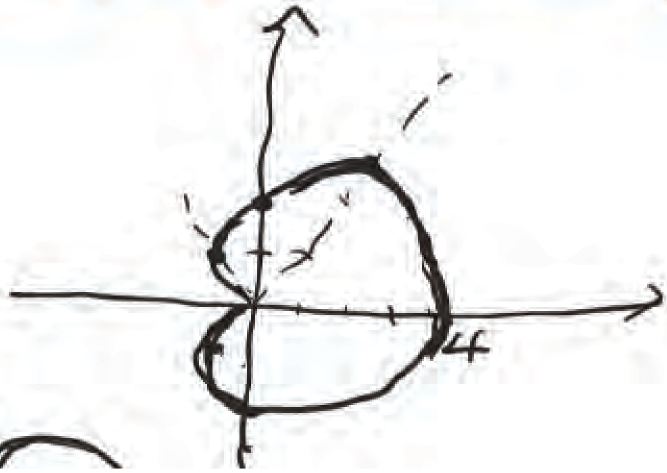
$\underline{r^2} = a r \sin\theta$

$$\boxed{x^2 + y^2 = a y}$$

Graph $\quad 0, 1, \frac{1}{2}$ < $\cos 60° = \frac{1}{2}$, $\sin 30° = \frac{1}{2}$

$r = 2 + 2\cos\theta$ (cardioid)

θ	r
0	4
60°	3
90°	2
120°	1
π	0

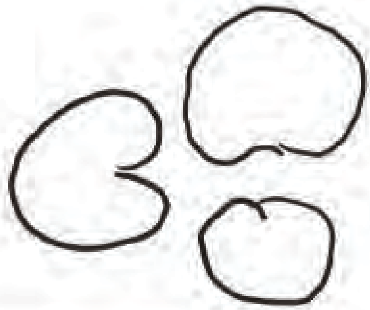


$a(1+\cos\theta)$

$a(1-\cos\theta) \longrightarrow$

$a(1+\sin\theta)$

$a(1-\sin\theta)$



#1 $(4, 30^\circ)$

$x = r\cos\theta$
$= 4\cos 30^\circ$
$= 4\left(\frac{\sqrt{3}}{2}\right)$
$= 2\sqrt{3}$

$y = r\sin\theta$
$= 4\sin 30^\circ$
$= 4\left(\frac{1}{2}\right)$
$= 2$

$(x, y) = (2\sqrt{3}, 2)$

#2 $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$

$x = r\cos\theta$
$= -\sqrt{2}\cos\frac{3\pi}{4}$
$= -\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right)$
$= 1$

$y = r\sin\theta$
$= -\sqrt{2}\sin\frac{3\pi}{4}$
$= -\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)$
$= -1$

$(x, y) = (1, -1)$

$(x, y) = (-1, \sqrt{3})$ $\quad \theta \in Q\text{II}$

$r = \sqrt{x^2 + y^2}$
$= \sqrt{1+3}$
$= 2$

$\theta' = \tan^{-1}\frac{\sqrt{3}}{1}$
$= \frac{\pi}{3}$

$\theta = 2\pi$

$$(r,\theta) = \left(2, \frac{2\pi}{3}\right)$$

#13 $(-3, -3) = (x, y)$

$$r = \sqrt{x^2+y^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$\hat{\theta} = \tan^{-1}\frac{3}{3} = \tan^{-1} 1 = \frac{\pi}{4}$$

$$\theta = \frac{5\pi}{4}$$

$$(r,\theta) = \left(3\sqrt{2}, \frac{5\pi}{4}\right)$$

#23 $r = 6\cos\theta$

$$r^2 = 6r\cos\theta$$

$$\boxed{x^2+y^2 = 6x}$$

#29 $r^2(4\sin^2\theta - 9\cos^2\theta) = 36$

$$4r^2\sin^2\theta - 9r^2\cos^2\theta = 36$$

$$4y^2 - 9x^2 = 36$$

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

#31 $r(\sin\theta + r\cos^2\theta) = 1$

$$r\sin\theta + r^2\cos^2\theta = 1$$

$$y + x^2 = 1$$

#30 $y^2 = 6x$

$(r\sin\theta)^2 = 6r\cos\theta$

$r^2\sin^2\theta = 6r\cos\theta \quad (r \neq 0)$

$r\sin\theta = 6\cos\theta$

$r = \frac{6\cos\theta}{\sin\theta}$

$= \boxed{6\cot\theta}$

#37 $(x+2)^2 + (y-3)^2 = 13$

$x^2 + 4x + \underline{4} + y^2 - 6y + \underline{9} = 13$

$\underbrace{x^2 + y^2} + 4x - 6y = 0$

$r^2 + 4r\cos\theta - 6r\sin\theta = 0$

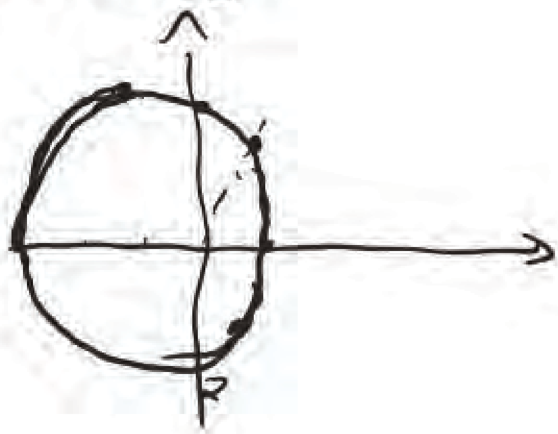
$(r \neq 0)$

$r + 4\cos\theta - 6\sin\theta = 0$

$\boxed{r = 6\sin\theta - 4\cos\theta} = \underline{f(\theta)}$

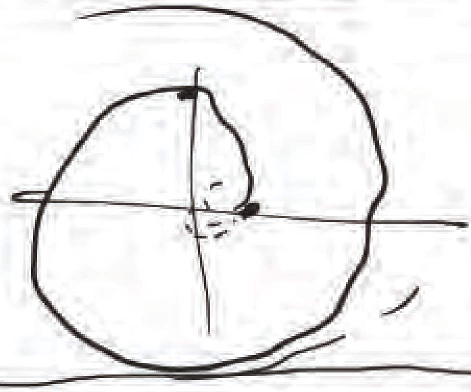
#48 $r = 2 - \cos\theta$

θ	r
0	1
$\pi/3$	$3/2$
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	$5/2$
π	3
$\frac{4\pi}{3}$	$5/2$



$\frac{11\pi}{6}$	2
$\frac{5\pi}{3}$	$\frac{3}{2}$

$r = e^{2\theta}$



8.7 Trig Form

$\mathbb{C}$

$\sqrt{-1} = i$

$-1 = i^2$

Realpart

$-i = i^3 \quad i\, i^2$

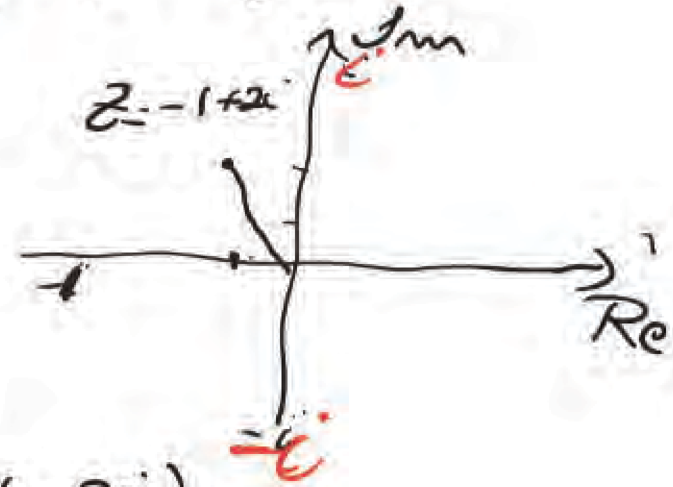
$1 = i^4$

$z = \overbrace{a}^{\text{Realpart}} + \underbrace{ib}_{Im}$ imaginary part

Re

$a, b \in \mathbb{R}$

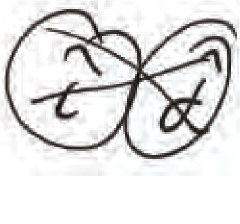
$z = -1+2i$



$(6-2i) + (-4-3i)$

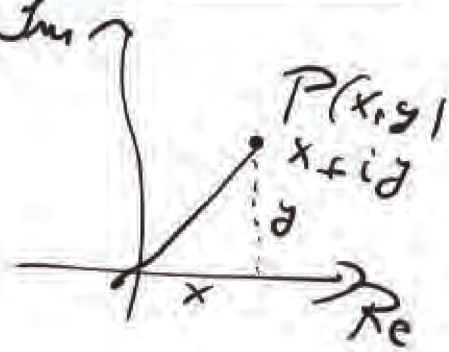
$= 6-4 + i(-2-3)$

$= 2 - 5i$



$r = \sqrt{x^2 + y^2}$

modulus.



θ: argument

$\theta = \tan^{-1}\frac{y}{x}$

$z = x + yi$

$= r\cos\theta + i(r\sin\theta)$

$= r(\cos\theta + i\sin\theta)$

$= r \text{ cis } \theta$ } Trig Form

$z = -1 + i$ Trig Form

$r = \sqrt{(-1)^2 + 1^2}$

$= \sqrt{2}$

$\bar{\theta} = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}$

$\boxed{z = \sqrt{2} \text{ cis } \frac{3\pi}{4}}$ $\left(\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right)$

$z = 6 \text{ cis } 60°$

$= 6\cos 60° + 6i\sin 60°$

$= 6\cos 60 + 6i\sin 60$

$= 6\left(\frac{1}{2}\right) + i6\left(\frac{\sqrt{3}}{2}\right)$

$= 3 + 3i\sqrt{3}$

$|r|$? $z = 5i$ $z = 7$

$r = 5$ $r = 7$

$3 + 4i \rightarrow r = 5$

$\sqrt{9+16} = \sqrt{25}$

$\cos z \leftarrow z \in \mathbb{C}$

$(r_1 \operatorname{cis} \theta_1)(r_2 \operatorname{cis} \theta_2) = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$

Ex $3(\operatorname{cis} 45^\circ)(2 \operatorname{cis} 135^\circ) = 6 \operatorname{cis}(45^\circ + 135^\circ)$

$= 6 \operatorname{cis} 180^\circ$

$= 6\cos 180^\circ + i\, 6\sin 180^\circ$

$= -6$

$a^2 + b^2 = (a+ib)(a-ib)$

$= a^2 - (ib)^2$

$a^2 - i^2 b^2$

$= a^2 - (-1)b^2$

$= a^2 + b^2$

$a^2 + b^2$ ...

$$=(a+b)^2$$

$$a+b=(\sqrt{a}-i\sqrt{b})(\sqrt{a}+i\sqrt{b})$$

$$a-b=(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$$

$$\frac{r_1 \operatorname{cis}\theta_1}{r_2 \operatorname{cis}\theta_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$$

$$\frac{10 \operatorname{cis}(-60^\circ)}{5 \operatorname{cis}(150^\circ)}=\frac{10}{5}\operatorname{cis}(-60^\circ-150^\circ)$$

$$=2\operatorname{cis}(-210^\circ)$$

$$=2\cos(-210^\circ)+i\,2\sin(-210^\circ)$$

$$=2\left(-\frac{\sqrt{3}}{2}\right)+i\,2\left(+\frac{1}{2}\right)$$

$$=-\sqrt{3}+i$$

$$\underset{(-)}{210^\circ}\ \text{III}$$

De Moivre's Theorem

$$(r\operatorname{cis}\theta)^n=r^n\operatorname{cis}n\theta$$

$$(1+i\sqrt{3})^8 \qquad \theta\in Q\text{I}$$

$$r=\sqrt{1+3}=2$$

$$\theta=\tan^{-1}\frac{\sqrt{3}}{1}=60^\circ$$

$$(1+i\sqrt{3})^8 = (2\,\text{cis}\,60^\circ)^8$$

$$= 2^8\,\text{cis}\,(480^\circ)$$

$$= 256\,\text{cis}\,(120^\circ)$$

$$= 256\,(\cos 120^\circ + i\sin 120^\circ)$$

$$= 256\left(-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right)$$

$$= -128 + i\,128\sqrt{3}$$

nth root $\quad (r\,\text{cis}\,\theta)^{1/n} = \sqrt[n]{r}\,\text{cis}\,\frac{\theta}{n}\ \alpha$

$$\alpha = \frac{\theta}{n} + \frac{360^\circ}{n}k$$

or

$$\frac{\theta + 2\pi k}{n}$$