Math 2312: **Pre-Calculus** ***Solution*** ***Sec* 5.2**

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1. $\dfrac{2s-1}{(s+1)(s-2)} = \dfrac{A}{s+1} + \dfrac{B}{s-2}$

$2s-1 = (A+B)s - 2A + B$

$$\begin{cases} A+B=2 \\ -2A+B=-1 \end{cases} \Rightarrow A=B=1$$

$$\boxed{\dfrac{2s-1}{(s+1)(s-2)} = \dfrac{1}{s+1} + \dfrac{1}{s-2}}$$

2. $\dfrac{2s-2}{(s-4)(s+2)} = \dfrac{A}{s-4} + \dfrac{B}{s+2}$

$2s-2 = (A+B)s + 2A - 4B$

$$\begin{cases} A+B=2 \\ 2A-4B=-2 \end{cases} \Rightarrow A=B=1$$

$$\boxed{\dfrac{2s-2}{(s-4)(s+2)} = \dfrac{1}{s-4} + \dfrac{1}{s+2}}$$

3. $\dfrac{7s^2+3s+16}{(s+1)(s^2+4)} = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+4}$

$7s^2+3s+16 = (A+B)s^2 + (B+C)s + 4A + C$

$$\begin{cases} A+B=7 \\ B+C=3 \\ 4A+C=16 \end{cases} \begin{matrix} \} \rightarrow A-C=4 \\ \\ 4A+C=16 \end{matrix} \Rightarrow 5A=20 \rightarrow A=4 \quad B=3 \quad C=0$$

$$\boxed{\dfrac{7s^2+3s+16}{(s+1)(s^2+4)} = \dfrac{4}{s+1} + \dfrac{3s}{s^2+4}}$$

4. $\dfrac{1}{s^2-4}=\dfrac{A}{s-2}+\dfrac{B}{s+2}$

$1=As+2A+Bs-2B$

$$\begin{cases} s^1 & A+B=0 \\ s^0 & 2A-2B=1 \end{cases}$$

$A=\dfrac{1}{4} \quad B=-\dfrac{1}{4}$

$$\underline{\left|\dfrac{1}{s^2-4}=\dfrac{\frac{1}{4}}{s-2}-\dfrac{\frac{1}{4}}{s+2}\right|}$$

5. $\dfrac{7s^2+20s+53}{(s-1)\left(s^2+2s+5\right)}=\dfrac{A}{s-1}+\dfrac{Bs+C}{s^2+2s+5}$

$7s^2+20s+53=As^2+2As+5A+Bs^2-Bs+Cs-C$

$$\begin{cases} s^2 & A+B=7 \\ s^1 & 2A-B+C=20 \\ s^0 & 5A-C=53 \end{cases}$$

$$\Delta=\begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 5 & 0 & -1 \end{vmatrix}=8 \qquad \Delta_A=\begin{vmatrix} 7 & 1 & 0 \\ 20 & -1 & 1 \\ 53 & 0 & -1 \end{vmatrix}=80$$

$\underline{A=10}$

$$\begin{cases} \underline{B=}7-A=\underline{-3} \\ \underline{C=}5A-53=\underline{-3} \end{cases}$$

$$\underline{\left|\dfrac{7s^2+20s+53}{(s-1)\left(s^2+2s+5\right)}=\dfrac{10}{s-1}+\dfrac{-3s-3}{s^2+2s+5}\right|}$$

6. $\dfrac{s^2+1}{s^3-2s^2-8s}=\dfrac{A}{s}+\dfrac{B}{s-4}+\dfrac{C}{s+2}$

$s^2+1=As^2-2As-8A+Bs^2+2Bs+Cs^2-4Cs$

$$\begin{cases} s^2 & A+B+C=1 \\ s^1 & -2A+2B-4C=0 \\ s^0 & -8A=1 \end{cases} \Rightarrow$$

$\underline{\left|A=-\dfrac{1}{8}\right|}$

$$\begin{cases} B+C=\frac{9}{8} \\ 2B-4C=-\frac{1}{4} \end{cases}$$

$$B=\frac{\begin{vmatrix} \frac{9}{8} & 1 \\ -\frac{1}{4} & -4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix}}=\frac{-\frac{17}{4}}{-6}=\underline{\frac{17}{24}} \qquad C=\frac{\begin{vmatrix} 1 & \frac{9}{8} \\ 2 & -\frac{1}{4} \end{vmatrix}}{-6}=\underline{\frac{5}{12}}$$

$$\underline{\frac{s^2+1}{s^3-2s^2-8s}=-\frac{1}{8}\frac{1}{s}+\frac{17}{24}\frac{1}{s-4}+\frac{5}{12}\frac{1}{s+2}}$$

7. $\frac{1}{x^2+2x}=\frac{A}{x}+\frac{B}{x+2}$

$1=Ax+2A+Bx$

$x \qquad 2A=1 \qquad \rightarrow A=\frac{1}{2}$

$x^0 \quad A+B=0 \quad \rightarrow B=-\frac{1}{2}$

$$\underline{\frac{1}{x^2+2x}=\frac{1}{2}\frac{1}{x}-\frac{1}{2}\frac{1}{x+2}}$$

8. $\frac{2x+1}{x^2-7x+12}=\frac{A}{x-4}+\frac{B}{x-3}$

$2x+1=Ax-3A+Bx-4B$

$x \qquad A+B=2$

$x^0 \quad -3A-4B=1$

$$A=\frac{\begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & -4 \end{vmatrix}}=\frac{-9}{-1}=\underline{9} \qquad B=\frac{\begin{vmatrix} 1 & 2 \\ -3 & 1 \end{vmatrix}}{-1}=\frac{7}{-1}=\underline{-7}$$

$$\underline{\frac{2x+1}{x^2-7x+12}=\frac{9}{x-4}-\frac{7}{x-3}}$$