Exercise

Find the limit: $\lim_{x\to 1}(2x^2 - x + 4)$

Solution

$$\lim_{x\to 1}(2x^2 - x + 4) = 2(1)^2 - (1) + 4 = 5$$

Exercise

Find the limit: $\lim_{x\to 2}\frac{x^2-4}{x-2}$

Solution

$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \frac{2^2-4}{2-2} = \frac{0}{0}$$

$$\lim_{x\to 1.9}\frac{x^2-4}{x-2} = \frac{1.9^2-4}{1.9-2} = 3.9$$

$$\lim_{x\to 2.1}\frac{x^2-4}{x-2} = \frac{2.1^2-4}{2.1-2} = 4.1$$

$$\lim_{x\to 2}\frac{x^2-4}{x-2} = 4$$

Exercise

Find the limit: $\lim_{x\to 2} f(x)$ $\qquad f(x) = \begin{cases} x^2 & , x \neq 2 \\ 0 & , x = 2 \end{cases}$

Solution

$$\lim_{x\to 1.99} x^2 = 3.96$$

$$\lim_{x\to 2.01} x^2 = 4.04$$

$$\lim_{x\to 2} f(x) = 4$$

Exercise

Find the limit: $\lim_{x\to 1}(2x^2 - x + 4)$

Solution

$$\lim_{x\to 1}(2x^2 - x + 4) = 2(1^2) - 1 + 4$$
$$= 5$$

Exercise

Find the limit: $\lim_{x\to 2}\frac{x^3-8}{x-2}$

Solution

$$\lim_{x\to 2}\frac{x^3-8}{x-2} = \frac{0}{0}$$

$$\lim_{x\to 2}\frac{x^3-8}{x-2} = \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x-2}$$
$$= \lim_{x\to 2} x^2 + 2x + 4$$
$$= 2^2 + 2(2) + 4$$
$$= 12$$

Exercise

Find the limit: $\lim_{x\to 3}\frac{x^2+x-12}{x-3}$

Solution

$$\lim_{x\to 3}\frac{x^2+x-12}{x-3} = \frac{0}{0}$$

$$\lim_{x\to 3}\frac{(x-3)(x+4)}{x-3} = \lim_{x\to 3}(x+4)$$
$$= 3 + 4$$
$$= 7$$

Exercise

Find the limit: $\lim_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$

Solution

$$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{4}-2}{0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$$

$$= \lim_{x\to 0} \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)}$$

$$= \lim_{x\to 0} \frac{x}{x\left(\sqrt{x+4}+2\right)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2}$$

$$= \frac{1}{\sqrt{4}+2}$$

$$= \frac{1}{4}$$

Exercise

Find the limit: $\lim_{x\to 0} f(x) = \begin{cases} x^2+1 & x<0 \\ 2x+1 & x>0 \end{cases}$

Solution

$$\lim_{x\to 0^-} x^2+1 = 1$$

$$\lim_{x\to 0+} 2x+1 = 1$$

$$\lim_{x\to 0} f(x) = 1$$

Exercise

Find the limit: $\lim_{x\to -2} \dfrac{5}{x+2}$

Solution

$$\lim_{x\to -2} \frac{5}{x+2} = \frac{5}{0} = \infty \text{ (}\textit{Doesn't exist}\text{)}$$

Exercise

Find the limit: $\lim_{x\to 2^-} \dfrac{|x-2|}{x-2}$

Solution

$$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \frac{(x-2)}{-(x-2)} = -1$$

Exercise

Find the limit: $\lim_{x\to 2+} \dfrac{|x-2|}{x-2}$

Solution

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \frac{(x-2)}{(x-2)} = 1$$

Exercise

$$y = \frac{3x}{1-x}$$

Solution

$VA: x = 1$

$HA: y = -3$

Exercise

$$y = \frac{x^2}{x^2+9}$$

Solution

$HA: y = 1$

Exercise

$$y = \frac{x-2}{x^2-4x+3}$$

Solution

$VA: x = 1, 3$

$HA: y = 0$

Exercise

$$y = \frac{3}{x-5}$$

Solution

VA: $x = 5$

HA: $y = 0$

Exercise

$$y = \frac{x^3 - 1}{x^2 + 1}$$

Solution

VA: *n/a*

HA: *n/a*

Exercise

$$y = \frac{3x^2 - 27}{(x+3)(2x+1)}$$

Solution

VA: $x = -3, \ -\frac{1}{2}$

HA: $y = \frac{3}{2}$

Exercise

$$y = \frac{x^3 + 3x^2 - 2}{x^2 - 4}$$

Solution

VA: $x = \pm 2$

HA: *n/a*

Exercise

$$y = \frac{x-3}{x^2-9}$$

Solution

VA: $x = -3$,

hole $x = 3$

Exercise

$$y = \frac{6}{\sqrt{x^2-4x}}$$

Solution

VA: $x = 0, 4$

Exercise

$$y = \frac{5x-1}{1-3x}$$

Solution

VA: $x = \frac{1}{3}$

HA: $y = \frac{5}{3}$