

$$\#1 \quad A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I \quad \checkmark$$

$$BA = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I$$

$\therefore B$ is the inverse of A

$$\#7 \quad \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^{-1} = \frac{1}{6} \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$$

$$\#9 \quad \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}^{-1} = \frac{1}{7-6} \begin{pmatrix} 7 & -2 \\ -3 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} 7 & -2 \\ -3 & -1 \end{pmatrix}$$

$$\#12 \quad \begin{pmatrix} -1 & 1 \\ 3 & -3 \end{pmatrix}^{-1} = \frac{1}{3-3} \nexists \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

#13

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 4 & 0 & 1 & 0 \\ 3 & 6 & 5 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -3 & 1 & 0 \\ 0 & 3 & 2 & -3 & 0 & 1 \end{array} \right] \begin{array}{l} \\ 2R_1 - R_2 \\ 2R_3 - 3R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 5 & -1 & 0 \\ 0 & 2 & 1 & -3 & 1 & 0 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array} \right] \begin{array}{l} R_1 - R_3 \\ R_2 - R_3 \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 2 & 2 & -2 \\ 0 & 2 & 0 & -6 & 4 & -2 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array} \right] \begin{array}{l} \frac{1}{2} R_1 \\ \frac{1}{2} R_2 \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array} \right]$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{pmatrix}$$

~~33~~ $\begin{pmatrix} -4 & -6 \\ 2 & 3 \end{pmatrix} = \frac{1}{-12+12} \cancel{\neq} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$

#59 $A = \begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$ $\sin^2 \theta + \cos^2 \theta = 1.$

$$\sin^2 \theta - (-\cos^2 \theta) = 1 \neq 0$$

$$A^{-1} = \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix}$$

2.3

624

 A is invertible $\Rightarrow A A^{-1} = A^{-1} A = I$

$$\Rightarrow (A^T)^{-1} = (A^{-1})^T$$

$$\begin{aligned} A^T (A^{-1})^T &= (A^{-1} A)^T \\ &= I^T \\ &= I \end{aligned}$$

$$\begin{aligned} (A^{-1})^T A^T &= (A A^{-1})^T \\ &= I^T \\ &= I \end{aligned}$$

$$\Rightarrow (A^T)^{-1} = (A^{-1})^T \rightarrow A^T \text{ is the inverse of } (A^{-1})^T$$

65

 C is invertible $\Rightarrow C C^{-1} = C^{-1} C = I$

$$\text{if } CA = CB \Rightarrow A = B$$

$$CA = CB$$

$$C^{-1}(CA) = C^{-1}(CB)$$

$$(C^{-1}C)A = (C^{-1}C)B$$

$$IA = IB$$

$$A = B \checkmark$$

66

$$\text{if } A^2 = A \Rightarrow I - 2A = (I - 2A)^{-1}$$

$$\begin{aligned} (I - 2A)(I - 2A) &= I - 2IA - 2IA + 4A^2 \quad (A^2 = A) \\ &= I - 4IA + 4A \quad (A = IA) \\ &= I - 4IA + 4IA \\ &= I \end{aligned}$$

$$\Rightarrow I - 2A = (I - 2A)^{-1}$$

#67

A is symmetric $\Rightarrow A^{-1}$ is symmetric

A is nonsingular $\Rightarrow A^{-1}$ is nonsingular too.
 $AA^{-1} = A^{-1}A = I$

A is symmetric $\Rightarrow A^T = A$

is $(A^{-1})^T = A^{-1}$?

$$\begin{aligned} A^{-1} &= (A)^{-1} \\ &= (A^T)^{-1} \\ &= (A^{-1})^T \checkmark \end{aligned}$$

$$(A^T)^{-1} = (A^{-1})^T$$

$\Rightarrow A^{-1}$ is symmetric.

#68

A, B, C square matrices

prove $ABC = I \Rightarrow B^{-1} = CA$?

$ABC = I \Rightarrow A$ is invertible $\Rightarrow AA^{-1} = I$

$$I = AA^{-1} = ABC$$

$$\Rightarrow A^{-1} = BC$$

$$A^{-1}A = BCA$$

$I = BCA \Rightarrow B$ is invertible $BB^{-1} = I$

$$I = BB^{-1} = B(CA)$$

$$\Rightarrow B^{-1} = CA \checkmark$$

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$$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$$

a) show $A^2 - 2A + 5I = 0$.

$$\begin{aligned} A^2 &= \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -3 & 4 \\ -4 & -3 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A^2 - 2A + 5I &= \begin{pmatrix} -3 & 4 \\ -4 & -3 \end{pmatrix} - 2 \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} + 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ &= 0 \checkmark \end{aligned}$$

b) $A^{-1} = \frac{1}{5} (2I - A)$

$$A^{-1} = \frac{1}{5} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$$

$$\begin{aligned} 2I - A &= 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \end{aligned}$$

$$A^{-1} = \frac{1}{5} (2I - A) \checkmark$$

c) $A \left(\frac{1}{5} (2I - A) \right) = \frac{1}{5} (2IA - A^2)$

$$A^2 - 2A + 5I = 0$$

$$5I = 2IA - A^2$$

$$\begin{aligned} A \left(\frac{1}{5} (2I - A) \right) &= \frac{1}{5} (5I) \\ &= I \Rightarrow A^{-1} = \frac{1}{5} (2I - A) \end{aligned}$$

#1 $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ Elementary $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} 2R_2$

#5 $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ ^{not} Elementary $\frac{1}{2}R_1$
 $R_2 \leftrightarrow R_3$

#7 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Elementary $R_3 - 5R_2$

#19 $E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} R_1 \leftrightarrow R_2$
 $E^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

#20 $E = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{5}R_1 \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
 $E^{-1} = \begin{pmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{pmatrix}$

#21

$$E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 \leftrightarrow R_3 \\ R_3 \leftrightarrow R_2 \end{array}$$

$$E^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

#25

$$A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

$$\begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} \quad R_1 \rightarrow R_2$$

$$E_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix} \quad R_2 - 3R_1$$

$$E_2 = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} \quad -\frac{1}{2}R_2$$

$$E_3 = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

$$A^{-1} = E_3 E_2 E_1$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix} \quad \checkmark$$

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$$\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$$

 $R_2 - R_1$

$$\begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$$

 $R_1 + R_2$

$$E_1 = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$$

 $-\frac{1}{2}R_2$

$$E_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$E_3 = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

(or)

$$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$$

 $R_1 \leftrightarrow R_2$

$$E_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

 $R_2 - R_1$

$$E_2 = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

 $\frac{1}{2}R_2$

$$E_3 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(solns are not unique)

$$\#43 \quad \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

 $\#45$

$$\begin{bmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{bmatrix} \quad \begin{array}{l} R_2 - 2R_1 : l_{21} = 2 \\ R_3 + R_1 : l_{31} = -1 \end{array}$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \quad R_3 - R_2 \quad l_{32} = 1$$

$$\begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix} = U \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$A = L U$$

#3

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 21 \end{bmatrix} \begin{bmatrix} 64 & 112 \end{bmatrix} \begin{bmatrix} 25 & 50 & 29 & 53 \\ 23 & 46 & 40 & 75 & 55 & 92 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5-6} \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$$

$$\begin{bmatrix} 11 & 21 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 64 & 112 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 16 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 50 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 25 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 29 & 53 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 14 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 23 & 46 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 23 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 40 & 75 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 25 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 55 & 92 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 18 \end{bmatrix}$$

8 1 16 16 25 0 14 5 23 0 25 5 1 18
H A P P Y - N E W - Y E A R.

#5

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 13 & 19 & 10 \end{bmatrix} \begin{bmatrix} -1 & -33 & -77 / 3 & -2 & -14 / 4 & 1 & -9 / -5 & -25 & -47 / 4 & 1 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 13 & 19 & 10 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -33 & -77 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & -14 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & -9 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -25 & -47 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & -9 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 4 \end{bmatrix}$$

9 3 5 2 5 18 7 0 4 5 14 0 18 5 14
I C E B E R G _ D E A D _ A H E A D