Section 2.3 – Present Value of an Annuity; Amortization

Principal Initial Value

PV is the present value or present sum of the payments.

PMT is the periodic payments.

Given

r = 6% semiannually, in order to withdraw \$1,000.00 every 6 months for next 3 years.

$i = \frac{r}{m} = \frac{.06}{2} = 0.03$

A = 1000 = PMT (periodic payment)

$$A = P(1+i)^n \Rightarrow P = \frac{A}{(1+i)^n} = A(1+i)^{-n} = 1000(1+.03)^{-n}$$

	Years			**1**		**2**		**3**
	Period	0	1	2	3	4	5	6
$=1000(1.03)^{-1}$								
$=1000(1.03)^{-2}$								
$=1000(1.03)^{-3}$								
$=1000(1.03)^{-4}$								
$=1000(1.03)^{-5}$								
$=1000(1.03)^{-6}$								

$$P = 1000(1.03)^{-1} + 1000(1.03)^{-2} + \cdots + 1000(1.03)^{-6}$$

$$P = R\frac{1-(1+i)^{-n}}{i}$$

Present Value (***PV***) of an ordinary annuity:

$$PV = PMT\frac{1-(1+i)^{-n}}{i}$$

i: Rate per period

n: Number of periods

Notes: Payments are made at the end of each period.

Example

A car costs \$12,000. After a down payment of \$2,000, the balance will be paid off in 36 equal monthly payments with interest of 6% per year on the unpaid balance, Find the amount of each payment.

<u>*Solution*</u>

Given: $P = 12,000 - 2,000 = 10,000$

$n = 36$

$i = \frac{.06}{12} = .005$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i} \approx \$13,577.71$$

$$10,000 = PMT\,\frac{1-(1.005)^{-66}}{.005}$$

$$PMT = \frac{10,000(.005)}{1-(1.005)^{-36}} \approx \$304.22$$

$10000(.005)/(1-(1.005)\wedge((-)36))$

Example

An annuity that earned 6.5%. A person plans to make equal annual deposits into this account for 25 years in order to them make 20 equal annual withdrawals of \$25,000 reducing the balance in the amount to zero. How much must be deposited annually to accumulate sufficient funds to provide for these payments? How much total interest is earned during this entire 45-years process?

<u>*Solution*</u>

$r = 0.065$ annually

$$1 \xrightarrow[Increasing]{PMT} \underset{FV\,=\,PV}{25\,yrs} \xrightarrow[decreasing]{25\ k} 20\,yrs\,(=45)$$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i} = 25000\,\frac{1-(1.065)^{-20}}{.065}$$

$25000(1-1.065\wedge((-)20))/.065$

$$\approx \$275,462.68$$

$\boxed{FV = PV}$ $\qquad FV = PMT\,\frac{(1+i)^n - 1}{i} \Rightarrow PMT = FV\,\frac{i}{(1+i)^n - 1}$

$$\Rightarrow PMT = FV\,\frac{i}{(1+i)^n - 1} = 275462.68\,\frac{.065}{(1.065)^{25} - 1}$$

$$= \$4,677.76$$

Total interest $= \underbrace{20(25000)}_{Withdraw} - \underbrace{25(4677.76)}_{Deposit} = \$383056.$

Amortization

Amortization debt means the debt retired in given length (= payment),

Borrow money from a bank to buy and agree to payment period (36 months)

Example

Borrow \$5000 payment in 36 months, compounded monthly @ r = 12%. How much payment?

Solution

$i=\frac{.12}{12}=.01 \qquad n=36$

$$PV=PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$\Rightarrow PMT=5000\,\frac{.01}{1-(1.01)^{-36}}$$

5000*.01 / (1-1.01^ (-)36))

$$\boxed{=\$166.07 \ \ per\ \ month}$$

Example

If you sell your car to someone for \$2,400 and agree to finance it at 1% per month on the unpaid balance, how much should you receive each month to amortize the loan in 24 months? How much interest will you receive?

Solution

$$PV=PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$PMT=2400\,\frac{.01}{1-(1+.01)^{-24}}$$

2400*.01/(1- (1 + .01) ^ (-)24))

$$\boxed{=\$112.98 \ \ per\ \ month}$$

Total interest = amount of all payment – initial loan

= 24(112.98) – 2400

= \$311.52

Amortization Schedules

Pay off earlier last payment (lump sum) = Amortization schedules

Example

If you borrow \$500 that you agree to repay in six equal monthly payments at 1% interest per month on the unpaid balance, how much of each monthly payment is used for interest and how much is used to reduce the unpaid balance

<u>Solution</u>

$$PMT = 500\frac{.01}{1-(1.01)^{-6}} = \$86.27\ per\ month$$

$500(.01/(1-1.01)\wedge(-)6$

@ The end of the 1st month interest due = 500(.01) = \$5.00

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0				\$500.00
1	\$86.27	5.00	86.27 – 5 = 81.27	500-81.27 = \$418.73
2	\$86.27	418.73(.01) = 4.19	86.27 – 4.19 = 82.08	418.73-82.08 = \$336.65
3	\$86.27	336.65(.01) = 3.37	86.27 – 3.37 = 82.90	336.65-82.90 = \$253.75
4	\$86.27	2.54	86.27 – 2.54 = 83.73	\$170.02
5	\$86.27	1.7	86.27 – 1.7 = 84.57	\$85.45
6	\$86.27	.85	86.27 – .85 = 85.54	\$0.0

Example

Construct an amortization schedule for a \$1,000 debt that is to be amortized in six equal monthly payment at 1.25% interest rate per month on the unpaid balance.

<u>Solution</u>

$$PMT = 1000\frac{.0125}{1-(1.0125)^{-6}} = \$174.03\ per\ month$$

$1000(.0125/(1-1.0125)\wedge(-)6$

1st month interest due = 1000(.0125) = \$12.50

#	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0				\$1000.00
1	\$174.03	\$12.50	\$ 161.53	\$838.47
2	\$174.03	10.48	163.55	\$ 674.92
3	\$174.03	8.44	165.59	\$ 509.33
4	\$174.03	6.37	167.66	\$ 341.67
5	\$174.03	4.27	169.76	\$ 171.91
6	\$174.03	2.15	171.91	\$0.0
	\$1044.21	**\$44.21**	**Total = \$1000**	

Equity

Equity = Current net market value – Unpaid balance

Example

A family purchase a home 10 years ago for \$80,000.00. The home was financed by paying 20% down for 30-year mortgage at 9%, on the unpaid balance. The net market of the house is now \$120,000.00 and the family wishes to sell the house. How much equity after making 120 monthly payments?

Solution

Equity = Current Net – Unpaid Balance

$$0 \longrightarrow 10 \xrightarrow{\quad Unpaid\ balance\ (20\ yrs) \quad} 30$$

Down Payment = 20 % $\Rightarrow$ Left 80% = .8(80000) = 64,000.00

$n = 12(30) = 360$

$i = \frac{.09}{12} = .0075$

Monthly Payment?

$$PMT = PV\frac{i}{1-(1+i)^{-n}}$$

$$= 64,000\frac{.0075}{1-(1.0075)^{-360}} \qquad 64,000(.0075/(1-1.0075)\wedge(12*30)$$

$$\approx \underline{\$514.96\ \ per\ month|}$$

Unpaid balance – 10 years (now) $\Rightarrow$ 30 -10 = 20 years

$$PV = PMT\frac{1-(1+i)^{-n}}{i}$$

$$= 514.96\frac{1-(1.0075)^{-240}}{.0075} \qquad 514.96((1-1.0075)\wedge 240)/.0075$$

$$\approx \underline{\$57,235.00|}$$

Equity = current – unpaid balance

= 120,000 – 57,235

= \$ 62,765.

Exercises *Section* 2.3 – Present Value of an Annuity Amortization

1. How much should you deposit in an account paying 8% compounded quarterly in order to receive quarterly payments of $1,000 for the next 4 years?

2. You have negotiated a price of $25,200 for a new truck. Now you must choose between 0% financing for 48 months or a $3,000 rebate. If you choose the rebate, you can obtain a loan for the balance at 4.5% compounded monthly for 48 months . Which option should you choose?

3. Suppose you have selected a new car to purchase for $19,500. If the car can be financed over a period of 4 years at an annual rate of 6.9% compounded monthly, how much will your monthly payments be? Construct an amortization table for the first 3 months.

4. Suppose your parents decide to give you $10,000 to be put in a college trust fund that will be paid in equally quarterly installments over a 5 year period. If you deposit the money into an account paying 1.5% per quarter, how much are the quarterly payments (Assume the account will have a zero balance at the end of period.)

5. You finally found your dream home. It sells for $120,000 and can be purchased by paying 10% down and financing the balance at an annual rate of 9.6% compounded monthly.
   a) How much are your payments if you pay monthly for 30 years?
   b) Determine how much would be paid in interest .
   c) Determine the payoff after 100 payments have been made.
   d) Change the rate to 8.4% and the time to 15 years and calculate the payment.
   e) Determine how much would be paid in interest and compare with the previous interest.

6. Sharon has found the perfect car for her family (anew mini-van) at a price of $24,500. She will receive a $3500 credit toward the purchase by trading in her old Gremlin, and will finance the balance at an annual rate of 4.8% compounded monthly.
   a) How much are her payments if she pays monthly for 5 years?
   b) How long would it take for her to pay off the car paying an extra $100 per mo., beginning with the first month?

7. Marie has determined that she will need $5000 per month in retirement over a 30-year period. She has forecasted that her money will earn 7.2% compounded monthly. Marie will spend 25-years working toward this goal investing monthly at an annual rate of 7.2%. How much should Marie's monthly payments be during her working years in order to satisfy her retirement needs?

8. American General offers a 10-year ordinary annuity with a guaranteed rate of 6.65% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $5,000 annually over the 10-year period?

9. American General offers a 7-year ordinary annuity with a guaranteed rate of 6.35% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $10,000 annually over the 7-year period?

10. You want to purchase an automobile for $27,300. The dealer offers you 0% financing for 60 months or a $5,000 rebate. You can obtain 6.3% financial for 60 months at the local bank. Which option should you choose? Explain.

11. You want to purchase an automobile for $28,500. The dealer offers you 0% financing for 60 months or a $6,000 rebate. You can obtain 6.2% financial for 60 months at the local bank. Which option should you choose? Explain.

12. Construct the amortization schedule for a $5,000 debt that is to be amortized in eight equal quarterly payments at 2.8% interest per quarter on the unpaid balance.

13. Construct the amortization schedule for a $10,000 debt that is to be amortized in six equal quarterly payments at 2.6% interest per quarter on the unpaid balance.

14. A loan of $37,948 with interest at 6.5% compounded annually, to be paid with equal annual payments over 10 years

15. A loan of $4,836 with interest at 7.25% compounded semi-annually, to be repaid in 5 years in equal semi-annual payments.