Solution   *Section* 3.3 – Absolute Extrema

Exercise

Find the absolute extrema of the function on the closed interval $f(x)=2(3-x), \ [-1,\ 2]$

Solution

$$f'=-2$$

$$f(-1)=2(3-(-1))=8$$

$$f(2)=2(3-2)=2$$

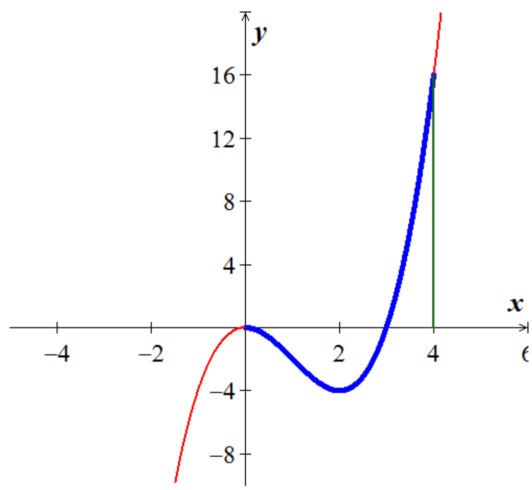
RMAX: $(-1,8)$

RMIN: $(2,2)$

Exercise

Find the absolute extrema of the function on the closed interval $f(x)=x^3-3x^2, \ [0,\ 4]$

Solution



$$f'(x)=3x^2-6x=0$$

$$3x(x-2)=0 \ \rightarrow \begin{cases} \boxed{x=0} \\ x-2=0 \Rightarrow \boxed{x=2} \end{cases}$$

$$f(0)=0^3-3(0)^2=0$$

$$f(2)=2^3-3(2)^2=-4$$

$$f(4)=4^3-3(4)^2=16$$

RMAX: $(4,\ 16)$

RMIN: $(2,\ -4)$

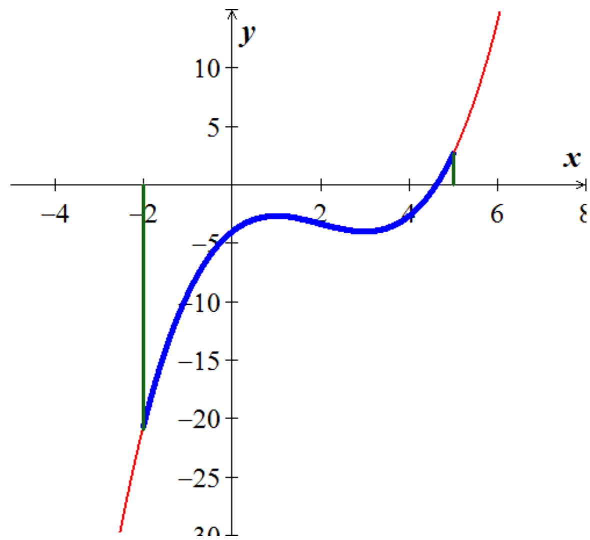
Exercise

Find the absolute extrema of the function on the closed interval $f(x)=\frac{1}{3}x^3-2x^2+3x-4, \quad [-2,\ 5]$

Solution

$$f'(x)=x^2-4x+3=0$$

$$x^2-4x+3=0 \quad \rightarrow \begin{cases} \boxed{x=1} \\ \boxed{x=3} \end{cases}$$



$$f(-2)=\frac{1}{3}(-2)^3-2(-2)^2+3(-2)-4=-20.66$$

$$f(1)=\frac{1}{3}(1)^3-2(1)^2+3(1)-4=-2.66$$

$$f(3)=\frac{1}{3}(3)^3-2(3)^2+3(3)-4=-4$$

$$f(5)=\frac{1}{3}(5)^3-2(5)^2+3(5)-4=2.66$$

RMAX: $(5,\ 2.66)$

RMIN: $(-2,\ -20.66)$

Exercise

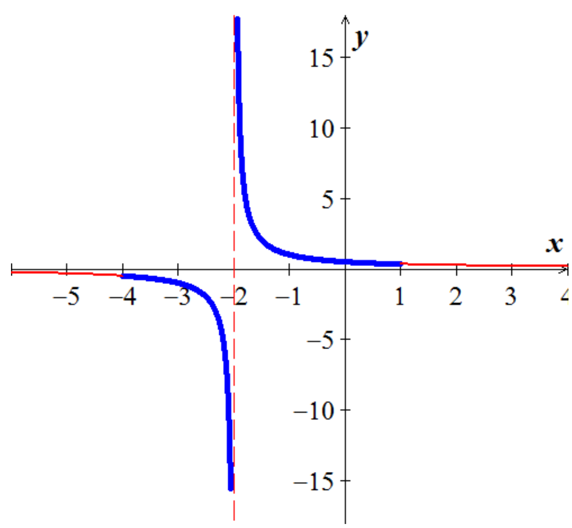
Find the absolute extrema of the function on the closed interval $f(x)=\dfrac{1}{x+2}, \quad [-4,\ 1]$

Solution

$$x+2\neq 0 \rightarrow x\neq -2 \quad (\textit{Asymptote})$$

$$f'(x)=-\frac{1}{(x+2)^2}\neq 0$$

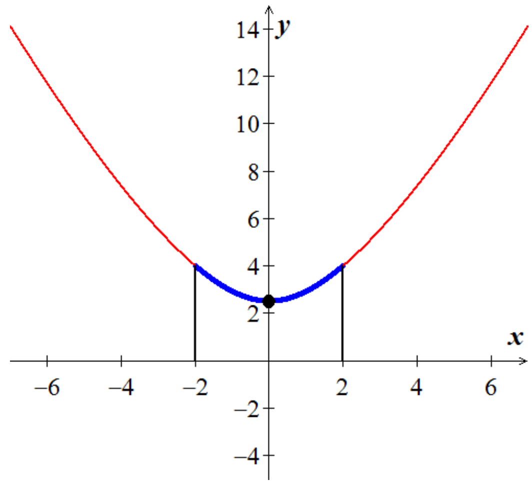
There is no Relative Extrema.



Exercise

Find the absolute extrema of the function on the closed interval $f(x)=\left(x^2+4\right)^{2/3}, \quad [-2,\ 2]$

Solution



$$f'(x)=\frac{2}{3}(2x)\left(x^2+4\right)^{2/3-1}$$

$$=\frac{4x}{3}\left(x^2+4\right)^{-1/3}$$

$$f'=\frac{4x}{3}\left(x^2+4\right)^{-1/3}=0; \quad x^2+4\neq 0$$

$$\boxed{x=0}$$

$$f\left(x=-2\right)=\left((-2)^2+4\right)^{2/3}=4$$

$$f\left(x=0\right)=\left((0)^2+4\right)^{2/3}=2.52$$

$$f\left(x=2\right)=\left((2)^2+4\right)^{2/3}=4$$

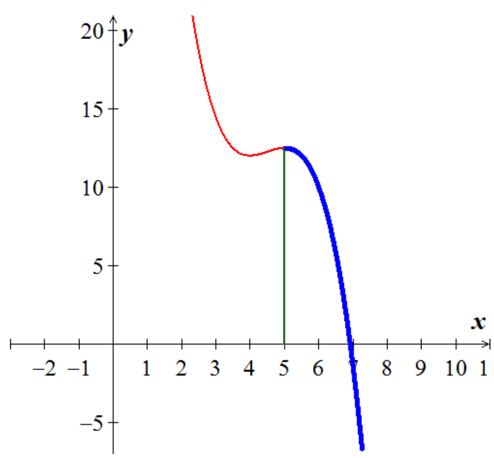
RMAX: $(-2,\ 4) \quad (2,\ 4)$

RMIN: $(0,\ 2.52)$

Exercise

$P(x)=-x^3+\frac{27}{2}x^2-60x+100, \quad x\geq 5$ is an approximation to the total profit (in thousands of dollars) from the sale of x hundred thousand tires. Find the number of hundred thousands of tires that must be sold to maximize profit.

Solution



$$P'(x)=-3x^2+27x-60=0$$

$$x=5, \quad 4\left(\textit{not in the interval}\right) \quad (x\geq 5)$$

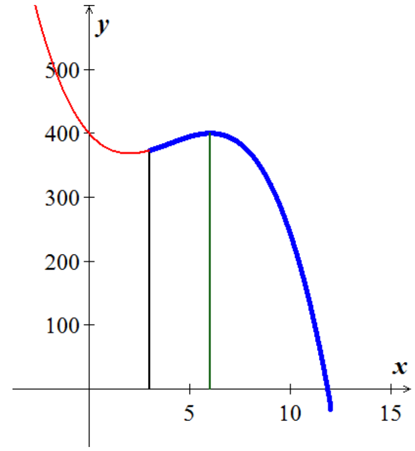
$$P(x=5)=-(5)^3+\frac{27}{2}(5)^2-60(5)+100$$

$$\underline{=12.5}$$

Exercise

$P(x) = -x^3 + 12x^2 - 36x + 400, \quad x \geq 3$ is an approximation to the total profit (in thousands of dollars) from the sale of x hundred thousand tires. Find the number of hundred thousands of tires that must be sold to maximize profit.

Solution



$$P'(x) = -3x^2 + 24x - 36 = 0$$

$$x = 6, \quad 2\left(\text{not in the interval}\right) \quad (x \geq 3)$$

$$P(x=6) = -(6)^3 + 12(6)^2 - 36(6) + 400$$

$$\underline{= 400}$$

Exercise

Researchers have discovered that by controlling both the temperature and the relative humidity in a building, the growth of a certain fungus can be limited. The relationship between temperature and relative humidity, which limits growth, can be described by

$$R(T) = -0.00008T^3 + 0.386T^2 - 1.6573T + 97.086, \quad 0 \leq T \leq 46$$

where $R(T)$ is the relative humidity (in %) and T is the temperature (in °C). Find the temperature at which the relative humidity is minimized.

Solution

$$R'(T) = -0.00024T^2 + 0.772T^2 - 1.6573 = 0$$

$$T = 2.15, \quad 3214.52\left(\text{not in the interval}\right) \quad (0 \leq T \leq 46)$$

$$R(T=0) = -0.00008(0)^3 + 0.386(0)^2 - 1.6573(0) + 97.086 \ \underline{= 97.1°C}$$

$$R(T=2.15) = -0.00008(2.15)^3 + 0.386(2.15)^2 - 1.6573(2.15) + 97.086 \ \underline{= 95.3°C}$$

$$R(T=46) = -0.00008(46)^3 + 0.386(46)^2 - 1.6573(46) + 97.086 \ \underline{= 830.5°C}$$