

Section 4.2 – Frequency Distribution; Measure of Central Tendency

Frequency Distribution

Frequency Distributions are presented as lists ordered by quantity showing the number of times each value appears.

Example

Given the data:

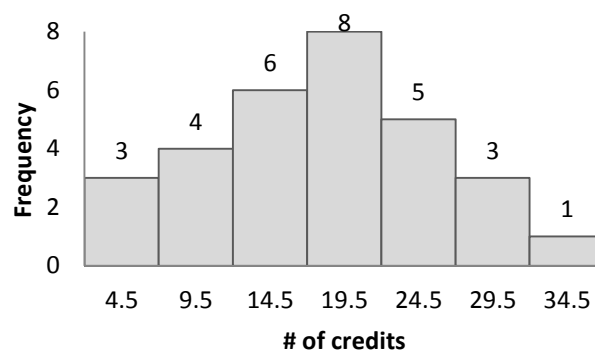
3	25	33	16	0	9	14	8	34	21
15	12	9	3	8	15	20	12	28	19
17	16	23	19	12	14	29	13	24	18

Group the data into intervals and find the frequency of each interval.

Solution

<i>Interval</i>	<i>Frequency</i>
0 – 4	3
5 – 9	4
10 – 14	6
15 – 19	8
20 – 24	5
25 – 29	3
30 – 34	1
<i>Total</i>	30

Histogram



Mean – Ungrouped Data

The mean is the quantitative data is equal to the sum of all the measurements in the data set divided by the total number of measurements in the set.

\bar{X} : Sample mean

μ : Population mean

(μ : mu)

Definition

If x_1, x_2, \dots, x_n is a set of n measurements, then the mean of the set of measurements is given by

$$[mean] = \frac{\sum_{i=1}^n x_i}{n} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

$$\bar{x} = \frac{\sum x}{n}$$

Example

The number of bankruptcy petitions (in thousands) filed in the U.S. in the years 2000-2005 are given in the table.

<i>Year</i>	<i>Petitions</i>
2000	1253
2001	1492
2002	1578
2003	1660
2004	1597
2005	2078

Find the mean number of bankruptcy petitions filed annually during this period.

Solution

$$\bar{x} = \frac{1253 + 1492 + 1578 + 1660 + 1597 + 2078}{6}$$

$$\approx 1610$$

The mean number is 1,610,000

Example

Find the mean for the sample measurements 3.2, 4.5, 2.8, 5.0, and 3.6.

Solution

$$\bar{X} = \frac{3.2 + 4.5 + 2.8 + 5 + 3.6}{5} \approx 3.8$$

Mean – Grouped Data

A data set of n measurements is grouped into k classes in a frequency table. If x_i is the midpoint of the i^{th} class interval and f_i is the i^{th} class interval.

The mean for grouped data is given by:

$$[mean] = \frac{\sum_{i=1}^n x_i f_i}{n} = \frac{x_1 f_1 + x_2 f_2 + \dots + x_n f_n}{n}$$

$$n = \sum_{i=1}^n f_i = \text{total number of measurements}$$

Example

Find the mean for the data shown in the following frequency distribution

Value (x)	Frequency (f)	$x \cdot f$
30	6	30.6 = 180
32	9	288
33	7	231
37	12	444
42	6	252
Total	40	1395

Solution

$$\bar{x} = \frac{1395}{40} = 34.875$$

Example

Find the mean from the grouped frequency distribution

Interval	Midpoint, x	Frequency, f	$x \cdot f$
0 – 4	2	3	6
5 – 9	7	4	28
10 – 14	12	6	72
15 – 19	17	8	136
20 – 24	22	5	110
25 – 29	27	3	81
30 – 34	32	1	32
Total		30	465

Solution

$$\bar{x} = \frac{465}{30} = 15.5$$

Median

Arrange the data in increasing order.

1. If the number of measurements in a set is **odd** \Rightarrow the **median** is the middle measurement.
2. If the number of measurements in a set is **even** \Rightarrow the **median** is the mean of the two middle measurements

Odd Number of Entries	Even Number of Entries
8	2
7	3
Median = 4	4
3	7
1	9
	11

$$\left. \begin{array}{l} 4 \\ 7 \end{array} \right\} \text{Median} = \frac{4+7}{2} = 5.5$$

Example

Find the median of each list of numbers

- a) 11, 12, 17, 20, 23, 28, 29
- b) 15, 13, 7, 11, 19, 30, 39, 5, 10
- c) 47, 59, 32, 81, 74, 153

Solution

a) **Median** = 20

b) {5, 7, 10, 11, 13, 15, 19, 30, 39}

Median = 13

c) {32, 47, 59, 74, 81, 153}

Median = $\frac{59+74}{2} = 66.5$

Example

Find the median and mean salary for:

\$17,000, \$18,000, \$18,000, \$20,000, \$24,000, \$28,000, \$100,000, \$120,000

Solution

Median = $\frac{20000+24000}{2} = \$22,000$

Mean = $\frac{17000+18000+18000+20000+24000+28000+100000+120000}{8} = \$43,125$

Mode

The mode is the most frequency occurring measurement in a data set. There may be a unique mode, several modes, or, if no measurement occurs more than once, essentially no mode.

Example

Find the mode for each list of numbers

- a) 57, 38, 55, 55, 80, 87, 98, 55, 57
- b) 182, 185, 183, 184, 187, 187, 185
- c) 10708, 11519, 10972, 17546, 13905, 12182

Solution

- a) **Mode:** 55
- b) **Mode:** 185; 187
- c) **Mode:** No mode

Exercises **Section 4.2 – Frequency Distribution; Measure of Central Tendency**

1. Find the mean: 9.4, 11.3, 10.5, 7.4, 9.1, 8.4, 9.7, 5.2, 1.1, 4.7
2. Find the median: 28.4, 9.1, 3.4, 27.6, 59.8, 32.1, 47.6, 29.8
3. Find the mode: 16, 15, 13, 15, 14, 13, 11, 15, 14
4. Find the mean and median: 8, 10, 16, 21, 25
5. Find the mean and median: 67, 89, 78, 86, 100, 96
6. Find the mean and median: 30,200; 23,700; 33,320; 29,410; 24,600; 27,750; 27,300; 32,680
7. Find the mean and median: 15.3, 27.2, 14.8, 16.5, 31.8, 40.1, 18.9, 28.4, 26.3, 35.3
8. The number of nations participating in the winter Olympic Games, is given below.

<i>Year</i>	<i>Participating</i>
1968	37
1972	35
1976	37
1980	37
1984	49
1988	57
1992	64
1994	67
1998	72
2002	77
2006	85

Find: Mean, Media, and Mode

9. Compute the mean for the grouped sample data listed in below table.

Class Interval	Frequency
0.5 – 5.5	6
5.5 – 10.5	20
10.5 – 15.5	18
15.5 – 20.5	4

10. Compute the mode(s), median, and mean for each data set:
 - a) 2, 1, 2, 1, 1, 5, 1, 9, 4
 - b) 2, 5, 1, 4, 9, 8, 7
 - c) 8, 2, 6, 8, 3, 3, 1, 5, 1, 8, 3

11. U.S. wheat prices and production figures for a recent decade are given in the following table.

<i>Year</i>	<i>Price</i> <i>(\$ per bushel)</i>	<i>Production</i> <i>(millions of bushels)</i>
1996	4.30	2277
1997	3.38	2481
1998	2.65	2547
1999	2.48	2296
2000	2.62	2228
2001	2.78	1947
2002	3.56	1606
2003	3.40	2345
2004	3.40	2158
2005	3.45	2105

Find the mean and median for the following

- a) Price per bushel of wheat
- b) Wheat production