# (5.1) Cramer's Rule.

Ex $\begin{cases} 5x + 7y = -1 \\ 6x + 8y = 1 \end{cases}$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$D = \begin{vmatrix} 5 & 7 \\ 6 & 8 \end{vmatrix} = 40 - 42 = -2$

$D_x = \begin{vmatrix} -1 & 7 \\ 1 & 8 \end{vmatrix} = -15 \quad (-8 - 7)$

$D_y = \begin{vmatrix} 5 & -1 \\ 6 & 1 \end{vmatrix} = 11 \quad 5 - (-6)$

$x = \frac{D_x}{D} = \frac{-15}{-2} = \frac{15}{2}$

$y = -\frac{11}{2}$

Solution: $\left(\frac{15}{2}, -\frac{11}{2}\right)$

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diagonal method (3 x 3) $AX = B$

$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ $\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$

$n! = 3! = 1 \cdot 2 \cdot 3 = 6$

copy 1st & 2nd cl only.

$= a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}$

if $D = 0 \Rightarrow$ 2x2 $\rightarrow D_y$ if $D_y = 0$ (x, y) ; $D_y \neq 0 \Rightarrow$ no sol.

3×3 $D_z \neq 0$ ⇒ $D_z = 0$ (x(z), y(z), z) infinitely many solutions
$D_z \neq 0$ unique solution

Ex

$$\begin{cases} x - 3y + 7z = 13 \\ x + y + z = 1 \\ x - 2y + 3z = 4 \end{cases}$$

$$D = \begin{vmatrix} 1 & -3 & 7 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$= 3 - 3 - 14 - 7 + 2 + 9$

$= -10$

$$D_x = \begin{vmatrix} 13 & -3 & 7 \\ 1 & 1 & 1 \\ 4 & -2 & 3 \end{vmatrix}$$

$= 39 - 12 - 14 - 28 + 26 + 9$

$= 20$

$$D_y = \begin{vmatrix} 1 & 13 & 7 \\ 1 & 1 & 1 \\ 1 & 4 & 3 \end{vmatrix}$$

$= 3 + 13 + 28 - 7 - 4 - 39$

$= -6$

$$D_z = \begin{vmatrix} 1 & -3 & 13 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix}$$

$= 4 - 3 - 26 - 13 + 2 + 12$

$= -24$

$x = \frac{20}{-10} = -2$

$y = \frac{-6}{-10} = +\frac{3}{5}$

$z = \frac{-24}{-10} = \frac{12}{5}$

$\therefore \left(-2, \frac{3}{5}, \frac{12}{5}\right)$

## 5.2 Partial Fraction Decomposition.

$$\text{——} = \frac{A_1}{x - a_1} + \frac{A_2}{x - a_2} + \cdots$$

Ex $\quad \frac{x}{x^2 - 5x + 6} = \frac{A}{x-2} + \frac{B}{x-3}$

soln

$$\frac{x}{x^2 - 5x + 6} = \frac{A(x-3) + B(x-2)}{(x-2)(x-3)}$$

$\therefore \quad x = A(x-3) + B(x-2)$

$x^1 \quad A + B = 1$

$x^0 \quad -3A - 2B = 0$

$D = \begin{pmatrix} -3 & -2 \end{pmatrix}$ $\longrightarrow$ $A = -2$

$D_A = \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} = -2$

$D_B = \begin{vmatrix} 1 & 1 \\ -3 & 0 \end{vmatrix} = 3$ $\quad B = 3$

$$\frac{x}{x^2-5x+6} = \frac{-2}{x-2} + \frac{3}{x-3}$$

$\begin{cases} {}^3A + {}^3B = {}^3 1 \\ -3A - 2B = 0 \end{cases}$

$B = 3$ $\quad A = 1 - 3 = -2$

$$\frac{}{(x-a)^n} = \frac{}{x-a} + \frac{}{(x-a)^2} + \cdots + \frac{}{(x-a)^n}$$

Ex $\quad \dfrac{x+2}{x^3-2x^2+x}$

$x^3 - 2x^2 + x = x(x^2 - 2x + 1)$

$= x(x-1)(x-1)$

$= x(x-1)^2$ ☺

$$\frac{x+2}{x^3-2x^2+x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$x+2 = A(x^2-2x+1) + Bx(x-1) + C$$

$x^2$ $\quad A + B = 0 \rightarrow B = -2$

$x^1$ $\quad -2A - B + C = 1 \rightarrow C = 1 + 4 - 2 = 3$

$x^0$ $\quad A = 2$

$C = 1 + 2A + B$

$$\frac{x+2}{x^3-2x^2+x} = \frac{2}{x} - \frac{2}{x-1} + \frac{3}{(x-1)^2}$$

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ตัวอย่าง $\quad \dfrac{x^3-8}{x^2(x-1)^3} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2} + \dfrac{E}{(x-1)^3}$

$$x^3 - 8 = Ax(x-1)^3 + B(x-1)^3 + Cx^2(x-1)^2 + Dx^2(x-1) + Ex^2$$

$= Ax$ ~~$-8$~~

$(a-b)^3 = a^3b^0 - 3a^2b^1 + 3a^1b^2 - 1a^0b^3$ (1 3 3 1)

$= a^3 - 3a^2b + 3ab^2 - b^3$

$$x^3 - 8 = Ax(x^3-3x^2+3x-1) + B(x^3-3x^2+3x-1) + Cx^2(x^2-2x+1) + Dx^3 - Dx^2 + Ex^2$$

$x^4$ $\quad A + C = 0 \rightarrow C = -2$

$x^3$ $\quad -3A + B - 2C + D = 1$ ①

$x^2$ $3A-3B+C-D+E=0$

$x^1$ $-A+3B=0 \Rightarrow A=3(8)=24$

$x^0$ $-B=-8 \Rightarrow B=8$

(1) $\rightarrow D=1+3(24)-8+2(-24)$

$=17$

(2) $E=-3(24)+3(8)+24+17$

$=-7$

$$\frac{x^3-8}{x^2(x-1)^3}=\frac{24}{x}+\frac{8}{x^2}-\frac{24}{x-1}+\frac{17}{(x-1)^2}-\frac{7}{(x-1)^3}$$

$x^{(2)}(x-1)^{(3)}$ $\quad 5-1=4$

$x^4$

$x^2(x^2+1)^3$

$2+6=8-1=7$

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Ex $\quad \frac{3x-5}{x^3-1}$ $\qquad a^3-b^3=(a-b)(a^2+ab+b^2)$

$$\frac{3x-5}{x^3-1}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$$

$$3x-5=A(x^2+x+1)+(Bx+C)(x-1)$$

$x^2$ $\quad A+B=0 \rightarrow B=-A$

$A-B+C=3$ (1)

$x^1$ $A - 5 = -5$

$x^0$ $A - C = -5 \rightarrow C = A + 5$

(1) $A + A + A + 5 = 3$

$3A = -2$

$A = -2/3$

$B = 2/3$

$C = -\frac{2}{3} + 5 = \frac{13}{3}$

3-5

$$\frac{3x-5}{x^3-1} = \frac{-2/3}{x-1} + \frac{\frac{2}{3}x + \frac{13}{3}}{x^2+x+1}$$