

2.3 #1 17

a)  $357 = 21 \times 17$  yes

c)  $35 = 2 \times 17 + \underline{1}$  No

#3 if  $a|b$  and  $b|a \Rightarrow a=b$  or  $a=-b$

if  $a|b \Rightarrow b = at$

if  $b|a \Rightarrow a = bs$

$a = ats \Rightarrow ts = 1, a \neq 0$

$s, t \in \mathbb{Z} \Rightarrow s = t = \pm 1$

$b = a$  or  $b = -a$  ✓

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$$a|b \Leftrightarrow b = at \quad (t \in \mathbb{Z})$$

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#6 a) 12-hrs 80 hours after 11:00?

$80 \bmod 12 = 8$   
hrs.

7:00

$80 \bmod 11$

$80 = 8(11) - 8$

$(80+11) \bmod 12$

$91 = 12(7) + 7$

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$-17 \bmod 2$

$-17 = 2(-9) + 1$

$-17 \bmod 2 = 1$

$= 2(-8) - 1$

(or)  $= -1$

14-a  $a \equiv -15 \pmod{23}$   $-26 < a < 0$

$a = -15 - 0(23)$   $a = -15$

$$\begin{aligned} -15 &= x(23) + a \\ -15 - (1)(23) &= a \end{aligned}$$

16-a  $(-133 \pmod{23} + 261 \pmod{23}) \pmod{23}$

$((-133 + 261) \pmod{23}) \pmod{23}$   
 $(128 \pmod{23}) \pmod{23}$

$128 = (23) \times 5 \neq 13$

$\overset{15}{13} \pmod{23} = 13$

2.4  $(10101111)_2$  binary base 2

$$\begin{aligned}
 &= 1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 \\
 &\quad + 1 \cdot 2^1 + 1 \cdot 2^0 \\
 &= 256 + 64 + 16 + 8 + 4 + 2 + 1 \\
 &= 351
 \end{aligned}$$

Base 8  $\rightarrow$  octal  
Base 16  $\rightarrow$  Hexadecimal

Ex:  $(7016)_8 = 6 \times 8^0 + 1 \times 8^1 + 0 \times 8^2 + 7 \times 8^3$   
 $\quad \quad \quad = 6 + 8 + 3584$   
 $\quad \quad \quad = 3590$

Ex  $(2A E \emptyset B)_{16} = 11 \times 16^0 + 0 \cdot 16^1 + 14 \times 16^2 + 10 \times 16^3$   
 $\quad \quad \quad + 2 \times 16^4$   
 $\quad \quad \quad = 175,627$

Ex  $(12345)_{10} = \text{octal}$

$$12345 = 8(1543) + 1$$

$$1543 = 8(192) + 7$$

$$192 = 8(24) + 0$$

$$24 = 8(3) + 0$$

$$(12345)_{10} = (30071)_8$$

Ex:  $(241)_{10} = (\quad)_2$

$$241 = 2(120) + 1$$

$$120 = 2(60) + 0$$

$$60 = 2(30) + 0$$

$$30 = 2(15) + 0$$

$$15 = 2(7) + 1$$

$$7 = 2(3) + 1$$

$$3 = 2(1) + 1$$

$$1$$

$$(241)_{10} = (1110001)_2$$

$$(11, 111, 010, 111, 00)_2$$

$$(011\ 111\ 010\ 111\ 100)_2$$

$$= (37\ 2\ 7\ 4)_F$$

$$(0011\ 1110\ 1011\ 1100)_2 = (3EBC)_{16}$$

					(1)
1	0	0	0	1	8.
2	0	0	1	0	9
3					A
4					B
					C
					D
					E
					F

0	0	1	8
0	0	1	9
0	0	1	10
0	1	0	11
0	1	0	12
0	1	1	13
1	1	1	14

$$(765)_8 = (111110101)_2$$

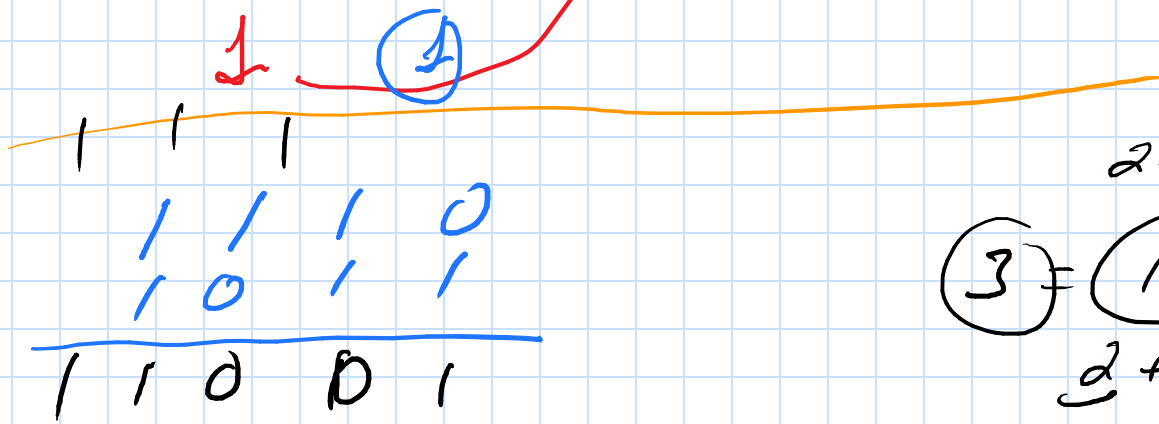
$$(A8D)_{16} = (101010001101)_2$$

0	0001	1
1	0010	2
2	0011	3
3	0100	4
4	0101	5
5	0110	6
6	0111	7
7	1000	8
8	1001	9
9	1010	A
A	1011	B
B	1100	C
C	1101	D
D	1110	E
E	1111	F

0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	0	0	0
5	1	0	1
6	1	1	0
7	1	1	1

8  
9  
A  
B  
C  
D  
E  
F

0 → 0  
1 → 1  
11 → 3



$$(3) = (11)_{2+1}$$

2.5 Prime numbers (nb)  
divide 1 or itself  $\leftarrow$   
Factor 1 or itself

1, 2, 3, 5, 7, 11, 13

$$100 = 2^2 \cdot 5^2$$

641 is prime

$$4 \times 25$$

$$999 = 3^3 \times 37$$

$$1024 = 2^{10}$$

$$9 \times 111 \\ 3^2 \times 3 \times 37$$

Trial Division

If  $n$  is a composite  $\mathbb{Z}$ ,  $n$  has a prime  
divisor  $\leq \sqrt{n}$

$n$  : composite  $\Rightarrow n \Rightarrow$  has factor  $a$   
 $n = ab \quad b > 1$

$$a > \sqrt{n}, \quad b > \sqrt{n}$$

$$ab \geq \sqrt{n} \sqrt{n} \\ = n \quad \neq$$

$$a \leq \sqrt{n} \quad b \leq \sqrt{n}$$

$$n = ab \quad \boxed{b \leq \sqrt{n}}$$

101 is prime

$$\sqrt{101} > \underline{10} \\ 2, 3, 5, 7$$

$$7007 = 7^2 \times 11 \times 13$$

$$\begin{array}{r} 1001 \\ 30 \\ 21 \end{array} \quad \begin{array}{r} 143 \\ 11 \\ 13 \times 11 \end{array}$$

$$\sqrt{7007}$$

Mersenne Primes:  $2^p - 1$

$$p=1 \quad 2^1 - 1 = 1$$

$$p=2 \quad 2^2 - 1 = 3$$

$$3 \quad 2^3 - 1 = 7$$

$$5 \quad 2^5 - 1 = 31$$

$$7 \quad 2^7 - 1 =$$

$$2^{11} - 1 = 2047 = 23 \times 89$$

$$\int \frac{x}{\ln x}$$

GCD

$$\gcd(24, 26) = 2$$

$$24 = 2^3 \times 3$$

$$26 = 2 \times 13$$

$$\gcd(24, 36) = 12$$

$$\begin{array}{r} 24 = 2^3 \times 3 \\ 36 = 2^2 \times 3^2 \\ \hline 2^2 \times 3 \end{array}$$

$$10, 17, 21$$

$$\gcd(10, 17) = 1$$

$$\gcd(10, 21) = 1$$

$$\gcd(17, 21) = 1$$

$$\left[ \begin{array}{l} 10 = 2 \times 5 \\ 17 = 17 \\ 21 = 3 \times 7 \end{array} \right]$$

LCM: Least Common Multiply

$$\begin{aligned} \text{LCM}(24, 36) &= 2^3 \times 3^2 \\ &= 8 \times 9 \\ &= 72 \end{aligned}$$

$$\begin{aligned} 24 &= 2^3 \times 3 \\ 36 &= 2^2 \times 3^2 \end{aligned}$$