Series or Test	***Form of series***	***Convergence/Divergence***	***Example***
Geometric series	$\sum_{n=0}^{\infty} ar^n$	***Convergence*** $\lvert r\rvert<1$	$\sum_{n=0}^{\infty} \frac{3}{2^n} = 3\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ $r=\frac{1}{2}<1$ The series ***converges*** and its $S = 3\frac{1}{1-\frac{1}{2}} = 6$
		Divergence $\lvert r\rvert\geq 1$	$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$; $r=\frac{3}{2}>1$ The series ***diverges***
Divergence Test	$\sum_{n=1}^{\infty} a_n$	***Divergence*** $\lim_{n\to\infty} a_n \neq 0$	$\sum_{n=0}^{\infty} (2)^n$; $\lim_{n\to\infty} 2^n = \infty$ The series ***diverges***
Integral Test	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n)$	***Convergence*** $\int_1^{\infty} f(x)\,dx$	$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$; $\int_1^{\infty} \frac{dx}{x^2+1} = \arctan x \Big\rvert_1^{\infty}$ $= \frac{\pi}{2} - \frac{\pi}{4}$ $= \frac{\pi}{4}$ The series ***converges***
		Divergence $\int_1^{\infty} f(x)\,dx = \infty$	$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$; $\int_1^{\infty} \frac{x}{x^2+1}dx = \frac{1}{2}\int_1^{\infty} \frac{d\left(x^2+1\right)}{x^2+1}$ $= \frac{1}{2}\ln\left(x^2+1\right)\Big\rvert_1^{\infty}$ $= \infty$ Series ***diverges***
$p-$series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	***Convergence*** $p>1$	$\sum_{n=1}^{\infty} \frac{1}{n^3}$; Series ***converges*** p-series $(p=3>1)$

		Divergence $p \le 1$	$\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$; Series ***diverges*** p-series $\left(p = \frac{1}{3} < 1\right)$
Ratio Test	$\sum_{n=1}^{\infty} a_n$	***Convergence*** $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$	$\sum_{n=0}^{\infty} \frac{n^2 2^{n+1}}{3^n}$; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^2 2^{n+2}}{3^{n+1}} \cdot \frac{3^n}{n^2 2^{n+1}}$ $= \lim_{n\to\infty} \frac{2}{3}\left(\frac{n+1}{n}\right)^2$ $= \frac{2}{3} < 1$ Series ***converges***
		Divergence $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$	$\sum_{n=1}^{\infty} \frac{n^n}{n!}$; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n}$ $= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n$ $= e > 1$ Series ***diverges***
Root Test	$\sum_{n=1}^{\infty} a_n$	***Convergence*** $\lim_{n\to\infty} \sqrt[n]{a_n} < 1$	$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$; $\lim_{n\to\infty} \sqrt[n]{\frac{e^{2n}}{n^n}} = \lim_{n\to\infty} \frac{e^2}{n} = 0 < 1$ Series ***converges***

		Divergence $\lim_{n\to\infty} \sqrt[n]{a_n} > 1$	$\sum_{n=1}^{\infty} \left(2\sqrt[n]{n}+1\right)^n$; $\lim_{n\to\infty} \sqrt[n]{\left(2\sqrt[n]{n}+1\right)^n} = \lim_{n\to\infty}\left(2\sqrt[n]{n}+1\right)$ $= \infty$ Series ***diverges***
Comparison Test	$\sum_{n=1}^{\infty} a_n$	***Convergence*** $0 < a_n \le b_n$ $\sum_{n=1}^{\infty} b_n$ ***converges***	$\sum_{n=1}^{\infty} \frac{1}{2+3^n}$; $a_n = \frac{1}{2+3^n} < \frac{1}{3^n} = b_n$ $\sum b_n = \left(\frac{1}{3}\right)^n$ ***converges*** Geometric $r = \frac{1}{3} < 1$ Series ***converges*** by ***Direct Comparison Test***
		Divergence $0 < b_n \le a_n$ $\sum_{n=1}^{\infty} b_n$ ***diverges***	$\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$; $b_n = \frac{1}{n^{1/2}} \le \frac{1}{2+\sqrt{n}} = a_n$ $\sum b_n = \frac{1}{n^{1/2}}$ ***diverges*** p-series $\left(p = \frac{1}{2} < 1\right)$ Series ***diverge*** by ***Direct Comparison Test***
Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$	***Convergence*** $0 \le \lim_{n\to\infty} \frac{a_n}{b_n} < \infty$ $\sum_{n=1}^{\infty} b_n$ ***converges***	$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$; $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ ***converges*** p-series $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{n}}{n^2+1}\frac{n^{3/2}}{1} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$ Series ***converges*** by ***Limit Comparison Test***

		Divergence $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ $\sum_{n=1}^{\infty} b_n$ ***diverges***	$\sum_{n=1}^{\infty} \frac{n2^n}{4n^3+1}$; $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ ***diverges*** $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n2^n}{4n^3+1}\frac{n^2}{2^n} = \lim_{n\to\infty} \frac{1}{4+\frac{1}{n^3}} = \frac{1}{4} > 0$ Series ***diverges*** by ***Limit Comparison Test***
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^n a_n$	***Convergence*** **1.** $a_n > 0$ **2.** $a_n \geq a_{n+1}$ **3.** $\lim_{n\to\infty} a_n = 0$	$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$; 1. $\frac{1}{n} > 0$ 2. $n < n+1 \;\to\; \frac{1}{n} > \frac{1}{n+1}$ 3. $\frac{1}{n} \to 0$ Series ***converges***.
		Divergence $\lim_{n\to\infty} a_n \neq 0$	$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+1}{n}$ $\frac{n+1}{n} \to 1 \neq 0$ Series ***diverges***.
Absolute Convergence	$\sum_{n=1}^{\infty} a_n$	***Convergence*** $\sum_{n=1}^{\infty} \lvert a_n \rvert$	$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ $\lim_{n\to\infty} \left\lvert \frac{(-1)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-1)^n} \right\rvert = \lim_{n\to\infty} \frac{1}{n+1} = 0$ Series ***converges*** absolutely by the ratio test