Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point (–2, 3) is on the terminal side of θ.

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

$$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} \qquad \cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{13}} \qquad \tan\theta = \frac{y}{x} = -\frac{3}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = -\frac{\sqrt{13}}{2} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{r}{y} = \frac{\sqrt{13}}{3} \qquad \cot\theta = \frac{x}{y} = -\frac{2}{3}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point (–3, –4) is on the terminal side of θ.

Solution

$3,\ 4 \ \rightarrow\ 5$

$$\sin\theta = -\frac{4}{5} \qquad \cos\theta = -\frac{3}{5} \qquad \tan\theta = \frac{-4}{-3} = \frac{4}{3}$$

$$\csc\theta = -\frac{5}{4} \qquad \sec\theta = -\frac{5}{3} \qquad \cot\theta = \frac{3}{4}$$

Exercise

Find the six trigonometry functions of θ in standard position with terminal side through the point (–3, 0).

Solution

$$r = \sqrt{(-3)^2 + 0^2} = 3 \qquad r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{0}{3} = 0 \qquad \cos\theta = \frac{-3}{3} = -1 \qquad \tan\theta = \frac{0}{-3} = 0$$

$$\csc\theta = \frac{1}{0} \rightarrow \infty \qquad \sec\theta = \frac{1}{-1} = -1 \qquad \cot\theta = \frac{1}{0} = \infty$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(12, -5)$ is on the terminal side of θ.

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{12^2 + (-5)^2} = \underline{13|}$$

$$\sin\theta = -\frac{5}{13} \qquad \cos\theta = \frac{12}{13} \qquad \tan\theta = -\frac{5}{12}$$

$$\csc\theta = -\frac{13}{5} \qquad \sec\theta = \frac{13}{12} \qquad \cot\theta = -\frac{12}{5}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(5, -12)$ is on the terminal side of θ.

Solution

$$5 \quad 12 \quad \rightarrow \quad 13$$

$$\sin\theta = -\frac{12}{13} \qquad \cos\theta = \frac{5}{13} \qquad \tan\theta = -\frac{12}{5}$$

$$\csc\theta = -\frac{13}{12} \qquad \sec\theta = \frac{13}{5} \qquad \cot\theta = -\frac{5}{12}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(9, -12)$ is on the terminal side of θ.

Solution

$$(9, -12) = 3(3, -4) \Rightarrow \quad 3 \quad 4 \quad \rightarrow \quad 5$$

$$\sin\theta = -\frac{4}{5} \qquad \cos\theta = \frac{3}{5} \qquad \tan\theta = -\frac{4}{3}$$

$$\csc\theta = -\frac{5}{4} \qquad \sec\theta = \frac{5}{3} \qquad \cot\theta = -\frac{3}{4}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(16,\ -12)$ is on the terminal side of θ.

Solution

$(16,\ -12) = 4(4,\ -3) \ \Rightarrow\ 4\ \ 3\ \ \rightarrow\ \ 5$

$\sin\theta = -\frac{3}{5}$ $\qquad$ $\cos\theta = \frac{4}{5}$ $\qquad$ $\tan\theta = -\frac{3}{4}$

$\csc\theta = -\frac{5}{3}$ $\qquad$ $\sec\theta = \frac{5}{4}$ $\qquad$ $\cot\theta = -\frac{4}{3}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(15,\ -8)$ is on the terminal side of θ.

Solution

$15\ \ 8\ \ \rightarrow\ \ 17$

$\sin\theta = -\frac{8}{17}$ $\qquad$ $\cos\theta = \frac{15}{17}$ $\qquad$ $\tan\theta = -\frac{8}{15}$

$\csc\theta = -\frac{17}{8}$ $\qquad$ $\sec\theta = \frac{17}{15}$ $\qquad$ $\cot\theta = -\frac{15}{8}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-6,\ 8)$ is on the terminal side of θ.

Solution

$(-6,\ 8) = 2(-3,\ 4) \ \Rightarrow\ 3\ \ 4\ \ \rightarrow\ \ 5$

$\sin\theta = \frac{4}{5}$ $\qquad$ $\cos\theta = -\frac{3}{5}$ $\qquad$ $\tan\theta = -\frac{4}{3}$

$\csc\theta = \frac{5}{4}$ $\qquad$ $\sec\theta = -\frac{5}{3}$ $\qquad$ $\cot\theta = -\frac{3}{4}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-15,\ 8)$ is on the terminal side of θ.

Solution

$15 \quad 8 \quad \rightarrow \quad 17$

$\sin\theta = \frac{8}{17}$ $\qquad$ $\cos\theta = -\frac{15}{17}$ $\qquad$ $\tan\theta = -\frac{8}{15}$

$\csc\theta = \frac{17}{8}$ $\qquad$ $\sec\theta = -\frac{17}{15}$ $\qquad$ $\cot\theta = -\frac{15}{8}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-7,\ 24)$ is on the terminal side of θ.

Solution

$7 \quad 24 \quad \rightarrow \quad 25$

$\sin\theta = \frac{24}{25}$ $\qquad$ $\cos\theta = -\frac{7}{25}$ $\qquad$ $\tan\theta = -\frac{24}{7}$

$\csc\theta = \frac{25}{24}$ $\qquad$ $\sec\theta = -\frac{25}{7}$ $\qquad$ $\cot\theta = -\frac{7}{24}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(10,\ -24)$ is on the terminal side of θ.

Solution

$(10,\ -24) = 2(5,\ -12) \ \Rightarrow \quad 5 \quad 12 \quad \rightarrow \quad 13$

$\sin\theta = -\frac{12}{13}$ $\qquad$ $\cos\theta = \frac{5}{13}$ $\qquad$ $\tan\theta = -\frac{12}{5}$

$\csc\theta = -\frac{13}{12}$ $\qquad$ $\sec\theta = \frac{13}{5}$ $\qquad$ $\cot\theta = -\frac{5}{12}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(7,\ 24)$ is on the terminal side of θ.

Solution

$7 \quad 24 \quad \rightarrow \quad 25$

$\sin\theta = \frac{24}{25}$ $\quad$ $\cos\theta = \frac{7}{25}$ $\quad$ $\tan\theta = \frac{24}{7}$

$\csc\theta = \frac{25}{24}$ $\quad$ $\sec\theta = \frac{25}{7}$ $\quad$ $\cot\theta = \frac{7}{24}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-7,\ -24)$ is on the terminal side of θ.

Solution

$7 \quad 24 \quad \rightarrow \quad 25$

$\sin\theta = -\frac{24}{25}$ $\quad$ $\cos\theta = -\frac{7}{25}$ $\quad$ $\tan\theta = \frac{24}{7}$

$\csc\theta = -\frac{25}{24}$ $\quad$ $\sec\theta = -\frac{25}{7}$ $\quad$ $\cot\theta = \frac{7}{24}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-24,\ -7)$ is on the terminal side of θ.

Solution

$24 \quad 7 \quad \rightarrow \quad 25$

$\sin\theta = -\frac{7}{25}$ $\quad$ $\cos\theta = -\frac{24}{25}$ $\quad$ $\tan\theta = \frac{7}{24}$

$\csc\theta = -\frac{25}{7}$ $\quad$ $\sec\theta = -\frac{25}{24}$ $\quad$ $\cot\theta = \frac{24}{7}$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(24,\ -10)$ is on the terminal side of θ.

Solution

$$(24,\ -10) = 2(12,\ -5) \ \Rightarrow\ 12\quad 5\quad \rightarrow\quad 13$$

$$\sin\theta = -\frac{5}{13} \qquad \cos\theta = \frac{12}{13} \qquad \tan\theta = -\frac{5}{12}$$

$$\csc\theta = -\frac{13}{5} \qquad \sec\theta = \frac{13}{12} \qquad \cot\theta = -\frac{12}{5}$$

Exercise

Find the values of the six trigonometric functions for an angle of $90°$.

Solution

$$\sin 90° = 1 \qquad \tan 90° = \infty \qquad \csc 90° = 1$$

$$\cos 90° = 0 \qquad \cot 90° = 0 \qquad \sec 90° = \infty$$

Exercise

Indicate the two quadrants θ could terminate in if $\cos\theta = \frac{1}{2}$

Solution

$$\cos\theta = \frac{1}{2} \qquad \rightarrow \boldsymbol{Q}\text{I} \ \&\ \boldsymbol{Q}\text{IV}$$

Exercise

Indicate the two quadrants θ could terminate in if $\csc\theta = -2.45$

Solution

$$\csc\theta = -2.45$$

$$= \frac{1}{\sin\theta} \qquad \rightarrow\ \boldsymbol{Q}\text{III} \ \&\ \boldsymbol{Q}\text{IV}$$

Exercise

Find the remaining trigonometric function of θ if $\sin\theta = \frac{12}{13}$ and θ terminates in ***Q***I

Solution

5 12 $\rightarrow$ 13

$\sin\theta = \frac{12}{13}$ $\quad$ $\cos\theta = \frac{5}{13}$ $\quad$ $\tan\theta = \frac{12}{5}$

$\csc\theta = \frac{13}{12}$ $\quad$ $\sec\theta = \frac{13}{5}$ $\quad$ $\cot\theta = \frac{5}{12}$

Exercise

Find the remaining trigonometric function of θ if $\cot\theta = -2$ and θ terminates in ***Q***II.

Solution

$\cot\theta = -2 = \frac{x}{y} \quad (\theta \in QII)$

$\underline{x = -2,\ y = 1}$

$r = \sqrt{(-2)^2 + (1)^2}$ $\qquad$ $r = \sqrt{x^2 + y^2}$

$= \sqrt{5}$

$\sin\theta = \frac{1}{\sqrt{5}}$ $\quad$ $\cos\theta = -\frac{2}{\sqrt{5}}$ $\quad$ $\tan\theta = -\frac{1}{2}$

$\csc\theta = \sqrt{5}$ $\quad$ $\sec\theta = -\frac{\sqrt{5}}{2}$

Exercise

Find the remaining trigonometric function of θ if $\tan\theta = \frac{3}{4}$ and θ terminates in ***Q***III.

Solution

$\tan\theta = \frac{3}{4} = \frac{y}{x} \quad (\theta \in QIII)$

$\underline{x = -4,\ y = -3}$

4 3 $\rightarrow$ 5

$\sin\theta = -\frac{3}{5}$ $\quad$ $\cos\theta = -\frac{4}{5}$

$\csc\theta = -\frac{5}{3}$ $\quad$ $\sec\theta = -\frac{5}{4}$ $\quad$ $\cot\theta = \frac{4}{3}$

Exercise

Find the remaining trigonometric function of θ if $\cos\theta = \frac{24}{25}$ and θ terminates in **Q**IV.

Solution

$24 \quad 7 \quad \rightarrow \quad 25$

$\cos\theta = \frac{24}{25} \quad \theta \in QIV \quad \Rightarrow y = -7$

$\sin\theta = -\frac{7}{25}$ $\qquad$ $\cos\theta = \frac{24}{25}$ $\qquad$ $\tan\theta = -\frac{7}{24}$

$\csc\theta = -\frac{25}{7}$ $\qquad$ $\sec\theta = \frac{25}{24}$ $\qquad$ $\cot\theta = -\frac{24}{7}$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = \frac{\sqrt{3}}{2}$ and θ is terminates in ***Q***IV.

Solution

$\cos\theta = \frac{\sqrt{3}}{2} = \frac{x}{r} \qquad \Rightarrow x = \sqrt{3}, \;\; r = 2$

Since θ is ***Q***IV

$y = -\sqrt{2^2 - \sqrt{3}^2}$

$= -\sqrt{4-3}$

$\underline{= -1}$

$\sqrt{3} \quad 1 \quad \rightarrow \quad 2$

$\sin\theta = -\frac{1}{2}$ $\qquad$ $\cos\theta = \frac{\sqrt{3}}{2}$ $\qquad$ $\tan\theta = -\frac{1}{\sqrt{3}}$

$\csc\theta = -2$ $\qquad$ $\sec\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\qquad$ $\cot\theta = -\sqrt{3}$

Exercise

Find the remaining trigonometric functions of θ if $\tan\theta = -\frac{1}{2}$ and $\cos\theta > 0$.

Solution

$\tan\theta = \frac{\sin\theta}{\cos\theta} < 0 \quad \& \quad \cos\theta > 0$

$\sin\theta < 0 \quad \Rightarrow \theta$ in QIV

$\Rightarrow y = 1, \quad x = 2$

$r = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$

$2 \quad -1 \quad \rightarrow \quad \sqrt{5}$

$\sin\theta = -\frac{1}{\sqrt{5}}$ $\qquad$ $\cos\theta = \frac{2}{\sqrt{5}}$ $\qquad$ $\tan\theta = -\frac{1}{2}$

$\csc\theta = -\frac{\sqrt{5}}{5}$ $\qquad$ $\sec\theta = \frac{\sqrt{5}}{2}$ $\qquad$ $\cot\theta = -2$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = \frac{3}{5}$ & $\theta \in QI$

Solution

$3 \quad 4 \quad \rightarrow \quad 5$

$\sin\theta = \frac{4}{5}$ $\qquad$ $\cos\theta = \frac{3}{5}$ $\qquad$ $\tan\theta = \frac{4}{3}$

$\csc\theta = \frac{5}{4}$ $\qquad$ $\sec\theta = \frac{5}{3}$ $\qquad$ $\cot\theta = \frac{3}{4}$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = -\frac{4}{5}$ & $\theta \in QII$

Solution

$\theta \in QII$ & $\sin\theta > 0$

$-4 \quad 3 \quad \rightarrow \quad 5$

$\sin\theta = \frac{3}{5}$ $\qquad$ $\cos\theta = -\frac{4}{5}$ $\qquad$ $\tan\theta = -\frac{3}{4}$

$\csc\theta = \frac{5}{3}$ $\qquad$ $\sec\theta = -\frac{5}{4}$ $\qquad$ $\cot\theta = -\frac{4}{3}$

Exercise

Find the remaining trigonometric functions of θ if $\sin\theta = -\frac{3}{5}$ & $\theta \in QIII$

Solution

$\theta \in QIII$ & $\cos\theta < 0$

$-4 \quad -3 \quad \rightarrow \quad 5$

$\sin\theta = -\frac{3}{5}$ $\cos\theta = -\frac{4}{5}$ $\tan\theta = \frac{3}{4}$

$\csc\theta = -\frac{5}{3}$ $\sec\theta = -\frac{5}{4}$ $\cot\theta = \frac{4}{3}$

Exercise

Find the remaining trigonometric functions of θ if $\sin\theta = -\frac{3}{5}$ & $\theta \in QIV$

Solution

$\theta \in QIV$ & $\cos\theta > 0$

$4 \quad -3 \quad \rightarrow \quad 5$

$\sin\theta = -\frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = -\frac{3}{4}$

$\csc\theta = -\frac{5}{3}$ $\sec\theta = \frac{5}{4}$ $\cot\theta = -\frac{4}{3}$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = -\frac{12}{13}$ & $\theta \in QIII$

Solution

$\theta \in QIII$ & $\sin\theta < 0$

$-12 \quad -5 \quad \rightarrow \quad 13$

$\sin\theta = -\frac{5}{13}$ $\cos\theta = -\frac{12}{13}$ $\tan\theta = \frac{5}{12}$

$\csc\theta = -\frac{13}{5}$ $\sec\theta = -\frac{13}{12}$ $\cot\theta = \frac{12}{5}$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = -\frac{5}{13}$ & $\theta \in QII$

Solution

$\theta \in QII$ & $\sin\theta > 0$

$-5 \quad 12 \quad \rightarrow \quad 13$

$\sin\theta = \frac{12}{13}$ $\quad$ $\cos\theta = -\frac{5}{13}$ $\quad$ $\tan\theta = -\frac{12}{5}$

$\csc\theta = \frac{13}{12}$ $\quad$ $\sec\theta = -\frac{13}{5}$ $\quad$ $\cot\theta = -\frac{5}{12}$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = \frac{12}{13}$ & $\theta \in QIV$

Solution

$\theta \in QIV$ & $\sin\theta < 0$

$12 \quad -5 \quad \rightarrow \quad 13$

$\sin\theta = -\frac{5}{13}$ $\quad$ $\cos\theta = \frac{12}{13}$ $\quad$ $\tan\theta = -\frac{5}{12}$

$\csc\theta = -\frac{13}{5}$ $\quad$ $\sec\theta = \frac{13}{12}$ $\quad$ $\cot\theta = -\frac{12}{5}$

Exercise

Find the remaining trigonometric functions of θ if $\sin\theta = -\frac{8}{17}$ & $\theta \in QIII$

Solution

$\theta \in QIII$ & $\cos\theta < 0$

$-15 \quad -8 \quad \rightarrow \quad 17$

$\sin\theta = -\frac{8}{17}$ $\quad$ $\cos\theta = -\frac{15}{17}$ $\quad$ $\tan\theta = \frac{8}{15}$

$\csc\theta = -\frac{17}{8}$ $\quad$ $\sec\theta = -\frac{17}{15}$ $\quad$ $\cot\theta = \frac{15}{8}$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = -\frac{15}{17}$ & $\theta \in QII$

Solution

$\theta \in QII$ & $\sin\theta > 0$

$-15 \quad 8 \quad \rightarrow \quad 17$

$\sin\theta = \frac{8}{17}$ $\quad$ $\cos\theta = -\frac{15}{17}$ $\quad$ $\tan\theta = -\frac{8}{15}$

$\csc\theta = \frac{17}{8}$ $\quad$ $\sec\theta = -\frac{17}{15}$ $\quad$ $\cot\theta = -\frac{15}{8}$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = -\frac{8}{17}$ & $\theta \in QII$

Solution

$\theta \in QII$ & $\sin\theta > 0$

$-8 \quad 15 \quad \rightarrow \quad 17$

$\sin\theta = \frac{15}{17}$ $\quad$ $\cos\theta = -\frac{8}{17}$ $\quad$ $\tan\theta = -\frac{15}{8}$

$\csc\theta = \frac{17}{15}$ $\quad$ $\sec\theta = -\frac{17}{8}$ $\quad$ $\cot\theta = -\frac{8}{15}$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = -\frac{7}{25}$ & $\theta \in QII$

Solution

$\theta \in QII$ & $\sin\theta > 0$

$-7 \quad 24 \quad \rightarrow \quad 25$

$\sin\theta = \frac{24}{25}$ $\quad$ $\cos\theta = -\frac{7}{25}$ $\quad$ $\tan\theta = -\frac{24}{7}$

$\csc\theta = \frac{25}{24}$ $\quad$ $\sec\theta = -\frac{25}{7}$ $\quad$ $\cot\theta = -\frac{7}{24}$

Exercise

Find the remaining trigonometric functions of θ if $\sin\theta = -\frac{7}{25}$ & $\theta \in QIII$

Solution

$\theta \in QIII$ & $\cos\theta < 0$

$-24 \quad -7 \quad \rightarrow \quad 25$

$\sin\theta = -\frac{7}{25}$ $\quad$ $\cos\theta = -\frac{24}{25}$ $\quad$ $\tan\theta = \frac{7}{24}$

$\csc\theta = -\frac{25}{7}$ $\quad$ $\sec\theta = -\frac{25}{24}$ $\quad$ $\cot\theta = \frac{24}{7}$

Exercise

Find the remaining trigonometric functions of θ if $\sin\theta = -\frac{24}{25}$ & $\theta \in QIV$

Solution

$\theta \in QIV$ & $\cos\theta > 0$

$7 \quad -24 \quad \rightarrow \quad 25$

$\sin\theta = -\frac{24}{25}$ $\quad$ $\cos\theta = \frac{7}{25}$ $\quad$ $\tan\theta = -\frac{24}{7}$

$\csc\theta = -\frac{25}{24}$ $\quad$ $\sec\theta = \frac{25}{7}$ $\quad$ $\cot\theta = -\frac{7}{24}$

Exercise

If $\sin\theta = -\frac{5}{13}$, and θ is ***Q***III, find $\cos\theta$ and $\tan\theta$.

Solution

$\sin\theta = -\frac{5}{13} = \frac{y}{r} \quad \rightarrow y = -5, \; r = 13$

$\Rightarrow x = \pm\sqrt{13^2 - 5^2} = \pm 12$ Since θ is Q III $\Rightarrow x = -12$ $\qquad x = \pm\sqrt{r^2 - y^2}$

$\cos\theta = -\frac{12}{13}$

$\tan\theta = \frac{5}{12}$

Exercise

If $\cos\theta = \frac{3}{5}$, and θ is ***Q***IV, find $\sin\theta$ and $\tan\theta$.

Solution

$$\cos\theta = \frac{3}{5} = \frac{x}{r} \quad (\theta \in QIV) \quad \Rightarrow \boxed{x = 3} \qquad y = \underline{-4}$$

$$\sin\theta = -\frac{4}{5}, \quad \tan\theta = -\frac{4}{3}$$

Exercise

Use the reciprocal identities if $\cos\theta = \frac{\sqrt{3}}{2}$ find $\sec\theta$

Solution

$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{2}{\sqrt{3}}$$

$$= \underline{\frac{2\sqrt{3}}{3}}$$

Exercise

Find $\cos\theta$, given that $\sec\theta = \frac{5}{3}$

Solution

$$\cos\theta = \frac{1}{\sec\theta}$$

$$= \frac{1}{\frac{5}{3}}$$

$$= \underline{\frac{3}{5}}$$

Exercise

Find $\sin\theta$, given that $\csc\theta = -\frac{\sqrt{12}}{2}$

Solution

$$\sin\theta = \frac{1}{\csc\theta}$$

$$= -\frac{2}{\sqrt{12}} \frac{\sqrt{12}}{\sqrt{12}}$$

$$= -\frac{2\sqrt{12}}{12}$$

$$\boxed{= -\frac{\sqrt{12}}{6}}$$

Exercise

Use a ratio identity to find $\tan\theta$ if $\sin\theta = \frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$

Solution

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{-\frac{4}{5}}$$

$$\boxed{= -\frac{3}{4}}$$

Exercise

If $\cos\theta = -\frac{1}{2}$ and θ terminates in ***Q***II, find $\sin\theta$

Solution

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$\boxed{= \frac{\sqrt{3}}{2}}$$

Exercise

If $\sin\theta = \frac{3}{5}$ and θ terminated in ***Q***II, find $\cos\theta$ and $\tan\theta$.

Solution

$$\boxed{\cos\theta = -\frac{4}{5}} \qquad (3,\ 4 \to 5)$$

$$\boxed{\tan\theta = -\frac{3}{4}}$$

Exercise

Find $\tan\theta$ if $\sin\theta = \frac{1}{3}$ and θ terminates in QI

Solution

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\frac{1}{9}}$$

$$= \sqrt{\frac{8}{9}}$$

$$\underline{= \frac{2\sqrt{2}}{3}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}$$

$$= \frac{1}{2\sqrt{2}}$$

$$\underline{= \frac{\sqrt{2}}{4}}$$

Exercise

Find the remaining trigonometric ratios of θ, if $\sec\theta = -3$ and $\theta \in QIII$

Solution

$$\sec\theta = \frac{1}{\cos\theta} = -3$$

$$\underline{\cos\theta = -\frac{1}{3}}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$

$$= -\sqrt{1-\frac{1}{9}}$$

$$= -\sqrt{\frac{8}{9}}$$

$$\underline{= -\frac{2\sqrt{2}}{3}}$$

$$\underline{\tan\theta = 2\sqrt{2}}$$

$$\cot\theta = \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}$$

$$\csc\theta = -\frac{3}{2\sqrt{2}} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$= -\frac{3\sqrt{2}}{4}$$

Exercise

Using the calculator and rounding your answer to the nearest hundredth, find the remaining trigonometric ratios of θ if $\csc\theta = -2.45$ and $\theta \in QIII$

Solution

$$\sin\theta = \frac{1}{-2.45} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$= -\frac{100}{245}$$

$$= -\frac{20}{49}$$

$$= -0.41$$

$$\cos\theta = -\sqrt{1-\sin^2\theta}$$

$$= -\sqrt{1-.41^2}$$

$$= -0.91$$

$$\tan\theta = \frac{-0.41}{-0.91}$$

$$= \frac{41}{91}$$

$$= 0.45$$

$$\cot\theta = \frac{1}{0.45}$$

$$= \frac{100}{45}$$

$$= \frac{20}{9}$$

$$= 2.22$$

$$\sec\theta = \frac{1}{-0.91}$$

$$= -\frac{100}{91}$$

$$= -1.1$$

Exercise

Write $\dfrac{\sec\theta}{\csc\theta}$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

Solution

$$\frac{\sec\theta}{\csc\theta} = \frac{\dfrac{1}{\cos\theta}}{\dfrac{1}{\sin\theta}}$$

$$= \frac{1}{\cos\theta}\frac{\sin\theta}{1}$$

$$= \boxed{\frac{\sin\theta}{\cos\theta}}$$

Exercise

Write $\cot\theta - \csc\theta$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

Solution

$$\cot\theta - \csc\theta = \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$$

$$= \boxed{\frac{\cos\theta - 1}{\sin\theta}}$$

Exercise

Write $\dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\sin\theta}$ in terms of $\sin\theta$ and/or $\cos\theta$, and then simplify if possible.

Solution

$$\frac{\sin\theta}{\cos\theta} + \frac{1}{\sin\theta} = \boxed{\frac{\sin^2\theta + \cos\theta}{\cos\theta\sin\theta}}$$

Exercise

Write $\sin\theta\cot\theta + \cos\theta$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

Solution

$$\sin\theta\cot\theta + \cos\theta = \sin\theta\frac{\cos\theta}{\sin\theta} + \cos\theta$$

$$= \cos\theta + \cos\theta$$

$$= \boxed{2\cos\theta}$$

Exercise

Multiply $(1-\cos\theta)(1+\cos\theta)$

Solution

$$(1-\cos\theta)(1+\cos\theta) = 1-\cos^2\theta$$
$$= \sin^2\theta$$

Exercise

Multiply $(\sin\theta+2)(\sin\theta-5)$

Solution

$$(\sin\theta+2)(\sin\theta-5) = \sin^2\theta - 3\sin\theta - 10$$

Exercise

Simplify the expression $\sqrt{25-x^2}$ as much as possible after substituting $5\sin\theta$ for x.

Solution

$$\sqrt{25-x^2} = \sqrt{25-(5\sin\theta)^2}$$
$$= \sqrt{25-25\sin^2\theta}$$
$$= \sqrt{25(1-\sin^2\theta)}$$
$$= \sqrt{25}\sqrt{\cos^2\theta}$$
$$= 5\cos\theta$$

Exercise

Simplify the expression $\sqrt{4x^2+16}$ as much as possible after substituting $2\tan\theta$ for x

Solution

$$\sqrt{4x^2+16} = \sqrt{4(2\tan\theta)^2+16}$$
$$= \sqrt{16\tan^2\theta+16}$$
$$= \sqrt{16(\tan^2\theta+1)}$$
$$= 4\sqrt{\tan^2\theta+1}$$

$$= 4\sqrt{\sec^2\theta}$$

$$= 4\sec\theta$$

Exercise

Simplify by using the table. $5\sin^2 30°$

Solution

$$5\sin^2 30° = 5\left(\frac{1}{2}\right)^2$$

$$= \frac{5}{4}$$

Exercise

Simplify by using the table. $\sin^2 60° + \cos^2 60°$

Solution

$$\sin^2 60° + \cos^2 60° = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

Exercise

Simplify by using the table. $(\tan 45° + \tan 60°)^2$

Solution

$$(\tan 45° + \tan 60°)^2 = \left(1 + \sqrt{3}\right)^2$$

$$= 1 + 3 + 2\sqrt{3}$$

$$= 4 + 2\sqrt{3}$$

Exercise

Find the exact value of $\csc 300°$

Solution

$\hat{\theta} = 360° - 300° = 60° \quad \rightarrow 300° \in QIV$

$$\csc 300° = -\frac{1}{\sin 60°}$$

$$= -\frac{1}{\frac{\sqrt{3}}{2}}$$

$$= -\frac{2}{\sqrt{3}}$$

Exercise

Find θ if $\sin\theta = -\frac{1}{2}$ and θ terminates in ***Q***III with $0° \le \theta \le 360°$.

Solution

$\hat{\theta} = \sin^{-1}\frac{1}{2} = 30°$

$\theta \in$ ***Q***III

$\Rightarrow \theta = 180° + 30°$

$= 210°$

Exercise

Find θ to the nearest degree if $\sec\theta = 3.8637$ and θ terminates in QIV with $0° \le \theta \le 360°$.

Solution

$\sec\theta = 3.8637 = \frac{1}{\cos\theta}$

$\cos\theta = \frac{1}{3.8637}$

$\hat{\theta} = \cos^{-1}\frac{1}{3.8637}$ *Calculator* : $\cos^{-1}(1 / 3.8637)$

$= 75°$

$\theta \in$ QIV

$\Rightarrow \theta = 360° - 75°$

$= 285°$

Exercise

Find the exact value of $\cos 225°$

Solution

$\hat{\theta} = 225° - 180° = 45°$

$\rightarrow 225° \in QIII$

$\cos 225° = -\cos 45°$

$= -\frac{\sqrt{2}}{2}$

Exercise

Find the exact value of $\tan 315°$

Solution

$\hat{\theta} = 360° - 315° = 45° \qquad \rightarrow 315° \in QIV$

$\tan 315° = -\tan 45°$

$= -1$

Exercise

Find the exact value of $\cos 420°$

Solution

$\hat{\theta} = 420° - 360° = 60° \qquad \rightarrow 420° \in QI$

$\cos 420° = \cos 60°$

$= \frac{1}{2}$

Exercise

Find the exact value of $\cot 480°$

Solution

$\hat{\theta} = 480° - 360° = 120°$

$\hat{\theta} = 180° - 120° = 60° \qquad \rightarrow 480° \in QII$

$\cot 480° = -\frac{\cos 60°}{\sin 60°}$

$$= -\frac{1/2}{\sqrt{3}/2}$$

$$= -\frac{1}{\sqrt{3}} \Big|$$

Exercise

Use the calculator to find the value of $\csc 166.7°$

Solution

$$\csc 166.7° = \frac{1}{\sin 166.7°}$$

$$\approx 4.3469 \Big|$$

Exercise

Use the calculator to find the value of $\sec 590.9°$

Solution

$$\sec 590.9° = \frac{1}{\cos 590.9°}$$

$$\approx -1.5856 \Big|$$

Exercise

Use the calculator to find the value of $\tan 195°\ 10'$

Solution

$$\tan(195°\ 10') = \tan\left(195° + \frac{10}{60}\right)$$

$$= \tan 195.1667°$$

$$\approx 0.271 \Big|$$

Exercise

Use the calculator to find θ to the nearest degree if $\sin\theta = -0.3090$ with $\theta \in$ QIV with $0° \le \theta \le 360°$

Solution

$\hat{\theta} = \sin^{-1}(0.3090)$ Since $\theta \in$ QIV

$\approx 18.0° \Big|$

$\theta = 180° + 40.0°$

$= 220.0° \Big|$

Exercise

Use the calculator to find θ to the nearest degree if $\cos\theta = -0.7660$ with $\theta \in$ ***Q***III with $0° \le \theta \le 360°$

Solution

$\hat{\theta} = \cos^{-1}(0.7660)$ Since $\theta \in$ ***Q***III

$\approx 40.0°$

$\theta = 180° + 40.0°$

$= 220.0°$

Exercise

Use the calculator to find θ to the nearest degree if $\sec\theta = -3.4159$ with $\theta \in$ ***Q***II with $0° \le \theta \le 360°$

Solution

$\sec\theta = -3.4159$

$\cos\theta = -\dfrac{1}{3.4159}$

$\hat{\theta} = \cos^{-1}\left(\dfrac{1}{3.4159}\right)$ Since $\theta \in$ ***Q***II

$\approx 73.0°$

$\theta \approx 180° - 73.0°$

$= 107.0°$

Exercise

Find θ to the nearest tenth of a degree if $\tan\theta = -0.8541$ and θ terminates in ***Q***IV with $0° \le \theta \le 360°$.

Solution

$\hat{\theta} = \tan^{-1} 0.8541$ $\theta \in$ ***Q***IV

$\approx 40.5°$

$\Rightarrow \theta = 360° - 40.5°$

$\approx 319.5°$

Exercise

Use the calculator to find θ to the nearest degree if $\sin\theta = 0.49368329$ with $\theta \in \boldsymbol{Q}$II with $0° \le \theta < 360°$

Solution

$\hat{\theta} = \sin^{-1} 0.49368329 \qquad \theta \in \boldsymbol{Q}$II

$= 29.6°$

$\Rightarrow \theta = 180° - 29.6°$

$= 150.4°$