Hwk 5.2

#1. $\dfrac{2s-1}{(s+1)(s-2)} = \dfrac{A}{s+1} + \dfrac{B}{s-2}$

$2s-1 = A(s-2) + B(s+1)$

$s^1 \quad A + B = 2 \Rightarrow \underline{B = 1}$

$s^0 \quad -2A + B = -1$

$3A = 3 \rightarrow \underline{A = 1}$

$\dfrac{2s-1}{(s+1)(s-2)} = \dfrac{1}{s+1} + \dfrac{1}{s-2}$

#2 $\dfrac{2s-2}{(s-4)(s+2)} = \dfrac{A}{s-4} + \dfrac{B}{s+2}$

$s^{2} \; (-1) \quad 2s-2 = A(s+2) + B(s-4)$

$s^1 \quad A + B = 2$ $\qquad \begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} = -6$

$s^0 \quad 2A - 4B = -2$

$A = \dfrac{\begin{vmatrix} 2 & 1 \\ -2 & -4 \end{vmatrix}}{-6} = \dfrac{-6}{-6} = \underline{1}$

$\underline{B = 1}$

$\dfrac{2s-2}{(s-4)(s+2)} = \dfrac{1}{s-4} + \dfrac{1}{s+2}$

#3 $\dfrac{s^2+1}{s^3-2s^2-8s}$ $\qquad s(s^2-2s-8)$

$\dfrac{s^2+1}{s^3-2s^2-8s} = \dfrac{A}{s} + \dfrac{B}{s-4} + \dfrac{C}{s+2}$

$s^2+1 = A(s^2-2s-8) + Bs(s+2) + Cs(s-4)$

$s^2 \quad A + B + C = 1$ (1)

$s^1 \quad -2A + 2B - 4C = 0$ (2)

$s^0 \quad -8A = 1 \Rightarrow \underline{A = -\frac{1}{8}}$

$\begin{cases} B + C = 1 + \frac{1}{8} = \frac{9}{8} \\ 2B - 4C = -\frac{1}{4} \end{cases}$

$$\begin{cases} B + C = \frac{9}{8} \\ B + 2C = +\frac{1}{8} \end{cases}$$

$$3C = \frac{10}{8} = \frac{5}{24} \Rightarrow \boxed{C = \frac{5}{12}}$$

$$B = \frac{9}{8} - \frac{5}{12} = \frac{27-10}{24} = \frac{17}{24}$$

$$\frac{s^2-1}{s^3-2s-8s} = \frac{1}{8s} + \frac{17/24}{s-4} + \frac{5/12}{s+2}$$

$$\begin{cases} B + C = \frac{9}{8} \\ 2B - 4C = \frac{-1}{4} \end{cases} \qquad \begin{cases} 8B + 8C = 9 \\ 8B - 16C = -1 \end{cases}$$

#4 $$\frac{1}{x^2+2x} = \frac{A}{x} + \frac{B}{x+2}$$

$$1 = A(x+2) + Bx$$

$$\begin{cases} x^1 & A + B = 0 \Rightarrow B = -\frac{1}{2} \\ x^0 & 2A = 1 \rightarrow A = \frac{1}{2} \end{cases}$$

$$\frac{1}{x^2+2x} = \frac{1}{2}\frac{1}{x} - \frac{1}{2}\frac{1}{x+2}$$

#15 $$\frac{2x+1}{x^2-7x+12} = \frac{A}{x-4} + \frac{B}{x-3}$$

$\ominus 1$ $$2x+1 = A(x-3) + B(x-4)$$

$$x^1 \quad {}^{+3}A \; {}^{+3}{+B} = \overset{+c}{2}$$

$$x^0 \quad -3A - 4B = 1$$

$$\begin{vmatrix} 1 & 1 \\ -3 & -4 \end{vmatrix} = -1$$

$$-B = 7 \Rightarrow B = -7$$

$$B = \frac{\begin{vmatrix} 1 & 2 \\ -3 & 1 \end{vmatrix}}{-1} = -7$$

$$A = 2 + \cancel{7} = 9$$

$$\frac{2x+1}{x^2-7x+12} = \frac{9}{x-4} - \frac{7}{x-3}$$

5.6 Arithmetic Seqn

$$a_n = a_1 + (n-1)d$$

Ex $20, 16.5, 13$ a_{15}?

$d = 16.5 - 20$
$= -3.5$

$a_{15} = 20 + 14(-3.5)$ $\frac{7}{2}$
$= 20 - 49$
$= -29$

$3.5 = \frac{35}{10} = \frac{7}{2}$

$3.55 = \frac{355}{100}$

Ex $a_4 = 5$ $a_9 = 20$ a_6? $f(4) = 5$ $f(9) = 20$

$a_n = a_1 + (n-1)d$
$a_4 = a_1 + 3d = 5$
$a_9 = a_1 + 8d = 20$
$5d = 15$
$d = 3$

$d = \frac{20-5}{9-4} = \frac{15}{5} = 3$

$a_1 + 3(3) = 5$
$a_1 = -4$

$a_6 = -4 + 5(3)$
$= 11$

#22 a_{20}. $a_9 = -5$ $a_{15} = 31$ AAH

$$d = \frac{31+5}{15-9} = 6$$

$$a_1 + 8(6) = -5$$

$$a_1 = -53$$

$$a_{20} = -53 + 19(6)$$
$$= -53 + 114$$
$$= 61$$

Formula: $S_n = \frac{n}{2}(2a_1 + (n-1)d)$
$$= \frac{n}{2}(a_1 + a_n)$$

Ex even from $2 \to 100$

$n = 50$

$$S_{50} = \frac{50}{2}(2+100)$$ 102
$$= 50(51)$$
$$= 2550$$

$$\frac{1}{4} + \frac{2}{9} + \frac{3}{14} + \frac{4}{19} + \frac{5}{24} + \frac{6}{29} = \sum_{n=1}^{6} \frac{n}{5n-1}$$

4, 9, 14, 19, ...

$d = 5$

$$a_n = 4 + (n-1)(5)$$
$$= 5n - 1$$

Geometric Series

$$a_{k+1} = a_k r$$

$$a_n = a_1 r^{n-1}$$

Ex $a_1 = 3 \quad r = -\frac{1}{2}$

$$a_n = 3\left(-\frac{1}{2}\right)^{n-1}$$

$$a_1 = 3$$

$$a_2 = 3\left(-\frac{1}{2}\right)^1 = -\frac{3}{2}$$

$$a_3 = 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$$

Ex 3rd term $a_3 = 5 \quad a_6 = -40 \quad a_8?$

$$r = \left(\frac{y_2}{y_1}\right)^{\frac{1}{n_2 - n_1}}$$

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 r^2 = 5$$

$$a_6 = a_1 r^5 = -40$$

$$\boxed{r = \left(-\frac{40}{5}\right)^{1/3} = (-8)^{1/3} = -2}$$

$$r = (-8)^{1/3}$$

$$\frac{a_6}{a_3} : \frac{a_1 r^5}{a_1 r^2} = \frac{-40}{5}$$

$$r^3 = -8 = (-2)^3$$

$$r = -2$$

$$5 = a_1(-2)^2$$

$$5 = 4a_1$$

$$\boxed{a_1 = \frac{5}{4}}$$

$$a_8 = \frac{5}{4}(-2)^7$$

$$= -5 \cdot 2^5$$

$$\boxed{= -160}$$

$$\frac{(-2)^7}{\cancel{4}}$$

$$2(2)$$

#64 a_{10} ; $a_4 = 4$ $a_7 = 12$

$\underline{r = (3)^{1/3}} = \sqrt[3]{3}$

$a_n = a_1 r^{n-1}$

$a_4 = 4 = a_1 (3^{1/3})^3$

$= 3a_1$

$\underline{a_1 = \frac{4}{3}}$

$a_{10} = \frac{4}{3}(3^{1/3})^9$ — $\frac{4}{3} 3^{9(\frac{1}{3})} = \frac{4}{3} 3^3$

$= \frac{4}{3} 3^3$ $4(9)$

$= 36$

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66 a_7 : $a_2 = 3$ $a_3 = -\sqrt{3}$

$\underline{r = -\frac{\sqrt{3}}{3}}$

$a_2 = a_1\left(-\frac{\sqrt{3}}{3}\right) = 3$

$a_1 = -\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\underline{= -3\sqrt{3}}$

$a_7 = (-3\sqrt{3})\left(-\frac{\sqrt{3}}{3}\right)^6$

$\frac{\cancel{3^2}}{3^6 3}$

$\underline{= -\frac{\sqrt{3}}{9}}$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

$$\begin{cases} S = \dfrac{a_1}{1-r} & \text{if } |r| < 1 \\ S = \infty & \text{if } |r| \geq 1 \end{cases}$$

Ex $\displaystyle\sum_{n=1}^{\infty} 3\left(-\frac{2}{3}\right)^{n-1}$

$|r| = +\frac{2}{3} < 1$

$$S = \frac{3}{1-\left(\frac{-2}{3}\right)}$$

$$= \frac{3}{1+\frac{2}{3}}$$

$$= \frac{3}{\frac{5}{3}} \qquad 3 \cdot \frac{3}{5}$$

$$= \frac{9}{5}$$

Ex $\displaystyle\sum_{n=1}^{\infty} 3\left(-\frac{3}{2}\right)^{n-1} = \infty \qquad |r| = \frac{3}{2} \geq 1$

$5.4\overline{27} = 5.427272727\ldots$

$= 5.4 + .027272\ldots$

$= \frac{54}{10} + .027 + .00027 + \ldots$

$r = \frac{.00027}{.027} = .01$

$$5.4\overline{27} = \frac{54}{10} + \frac{.027}{1-.01}$$

$$= \frac{54}{10} + \frac{.027}{.990}$$

$$= \frac{54}{10} + \frac{27}{990}$$

$$= \frac{54}{10} + \frac{3}{110}$$

$$= \frac{594+3}{110}$$

$$= \frac{597}{110}$$

#98 $\sum_{k=1}^{20}(3k-5) = \sum_{k=1}^{20} 3k - \sum_{k=1}^{20} 5$

$= 3\left(\frac{20}{2}(1+20)\right) - 5(20)$

$= 30(21) - 100$

$= 630 - 100$

$= \underline{530}$

#110 $\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1}$ $\quad |r| = \frac{1}{4} < 1$

$S = \frac{5}{1-\frac{1}{4}}$

$= 5 \cdot \frac{4}{3}$

$= \underline{\frac{20}{3}}$

#111 $\sum_{n=1}^{\infty} 8\left(\frac{1}{3}\right)^{n-1}$ $\quad |r| = \frac{1}{3} < 1$

$S = \frac{8}{1-\frac{1}{3}}$

$= 8\left(\frac{3}{2}\right)$

$= \underline{12}$

#112 $\sum_{k=1}^{\infty} \frac{1}{2}(3)^{k-1} = \infty$ $\quad |r| = 3 \geq 1$

#113 $\sum_{k=1}^{\infty} 6\left(\frac{-2}{3}\right)^{k-1}$ $\quad |r| = \frac{2}{3} < 1$

$S = \frac{6}{1+\frac{2}{3}}$

$= 6 \cdot \frac{3}{5}$

$= \underline{\frac{18}{5}}$

arith.. $d = \frac{y_2 - y_1}{x_2 - x_1}$ $a_n = a_1 + (n-1)d$

Geom. $r = \left(\frac{y_2}{y_1}\right)^{1/x_2 - x_1}$ $a_n = a_1 r^{n-1}$

$$\begin{cases} S = \frac{a}{1-r} & |r| < 1 \\ S = \infty & |r| \geq 1 \end{cases}$$

Sum of of $n (>0)$ numbers.

P_n: $\frac{n(n+1)}{2}$

Use Mathematical Induction

① P_1 is true

② Assume P_k is true, Prove P_{k+1} also true.

P_n: $\frac{n(n+1)}{2} = 1 + 2 + \cdots + n$

① $1 \stackrel{?}{=} \frac{1(2)}{2}$

$1 = 1$ ✓ Hence, P_1 is true

② Assume P_k: $1 + 2 + \cdots + k = \frac{k(k+1)}{2}$ is true

P_{k+1}: $1 + 2 + \cdots + k + (k+1) = \frac{(k+1)(k+2)}{2}$? true

$$\underline{1 + 2 + \cdots + k} + (k+1) = \frac{1}{2}k(k+1) + (k+1)$$
$$= (k+1)\left(\frac{k}{2} + 1\right)$$
$$= (k+1)\left(\frac{k+2}{2}\right) ✓$$

P_{k+1} is also true.

∴ By the Mathematical Induction, the proof is completed