Section 2.5 – Other Trigonometric Functions

Vertical Asymptote

A ***vertical asymptote*** is a vertical line that the graph approaches but does not intersect, while function values increase or decrease without bound as *x*-values get closer and closer to the line.

Graphing the *Tangent* and *Cotangent* Functions

The graphs of $y = k + A\tan(Bx + C)$ and $y = k + A\cot(Bx + C)$, where $B > 0$, will have the following characteristics:

No *Amplitude*

$Period = \frac{\pi}{|B|}$

Phase Shift $= -\frac{C}{B}$

Vertical translation = k

One cycle: $0 \le argument \le \pi$ $\boxed{or}$ $-\frac{\pi}{2} < argument \le \frac{\pi}{2}$

Tangent Functions

Domain: $\left\{ x \mid x \neq (2n+1)\frac{\pi}{2},\ where\ n \in \mathbb{Z} \right\}$

Range: $(-\infty,\ \infty)$

- The graph is discontinuous at values of x of the form $x=(2n+1)\frac{\pi}{2}$ and has ***vertical asymptotes*** at these values.
- Its x-intercepts are of the form $x = n\pi$.
- Its period is π.
- Its graph has no amplitude, since there are no minimum or maximum values.
- The graph is symmetric with respect to the origin, so the function is an odd function. For all x in the domain, tan(–x) = – tan(x).

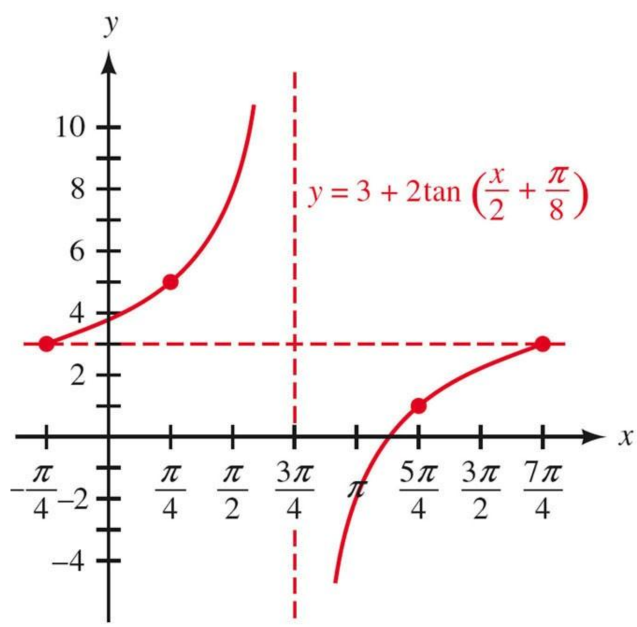
Example

Graph one complete cycle $y = 3 + 2\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)$

Period $\quad P = \frac{\pi}{1/2} = 2\pi$

Phase shift: $\phi = -\frac{\frac{\pi}{8}}{1/2} = -\frac{\pi}{4}$

x	x	$y = 3+2\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)$
$-\frac{\pi}{4}+0$	$-\frac{\pi}{4}$	3
$-\frac{\pi}{4}+\frac{1}{2}\pi$	$\frac{\pi}{4}$	5
$-\frac{\pi}{4}+\pi$	$\frac{3\pi}{4}$	∞
$-\frac{\pi}{4}+\frac{3}{2}\pi$	$\frac{5\pi}{4}$	1
$-\frac{\pi}{4}+2\pi$	$\frac{7\pi}{4}$	3



Cotangent Functions

Domain: $\{x \mid x \neq n\pi,\ \text{where } n \in \mathbb{Z}\}$

Range: $(-\infty,\ \infty)$

- The graph is discontinuous at values of x of the form $x = n\pi$ and has ***vertical asymptotes*** at these values.
- Its x-intercepts are of the form $x = (2n+1)\frac{\pi}{2}$.
- Its period is π.
- Its graph has no amplitude, since there are no minimum or maximum values.
- The graph is symmetric with respect to the origin, so the function is an odd function. For all x in the domain, $\cot(-x) = -\cot(x)$.

Example

Graph two complete cycles $y = -2 - \cot\left(x - \frac{\pi}{4}\right)$

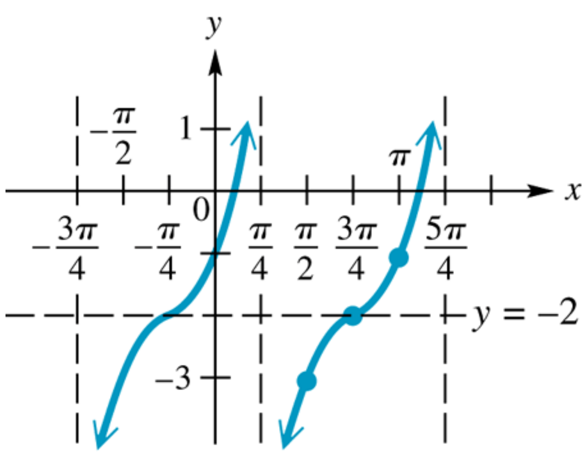
Period $P = \frac{\pi}{1} = \pi$

Phase shift: $\phi = -\frac{-\frac{\pi}{4}}{1} = \frac{\pi}{4}$

VA: $x = \frac{\pi}{4},\ \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

Vertical Translation: y = -2

x	$y = -2 - \cot\left(x - \frac{\pi}{4}\right)$
$\frac{\pi}{4}$	- ∞
$\frac{\pi}{2}$	- 3
$\frac{3\pi}{4}$	- 2
π	3
$\frac{5\pi}{4}$	∞



Graphing the *Secant* Function

Domain: $\left\{x \mid x \neq (2n+1)\frac{\pi}{2},\ where\ n \in \mathbb{Z}\right\}$

Range: $(-\infty,\ -1] \cup [1,\ \infty)$

- The graph is discontinuous at values of x of the form $x = (2n+1)\frac{\pi}{2}$ and has ***vertical asymptotes*** at these values.
- There are no x-intercepts.
- Its period is 2π.
- Its graph has no amplitude, since there are no minimum or maximum values.
- The graph is symmetric with respect to the y-axis, so the function is an even function. For all x in the domain, $sec(-x) = \sec(x)$.

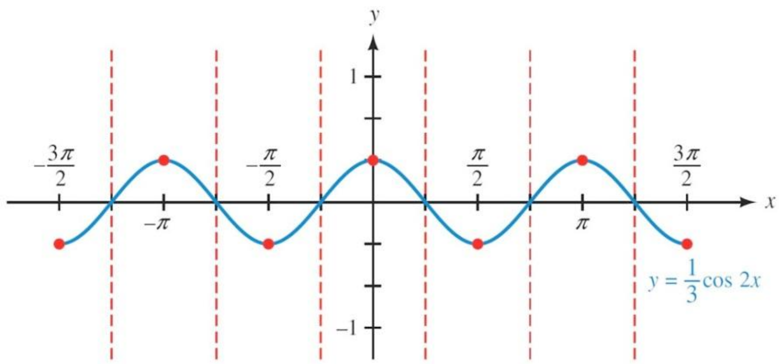
Example

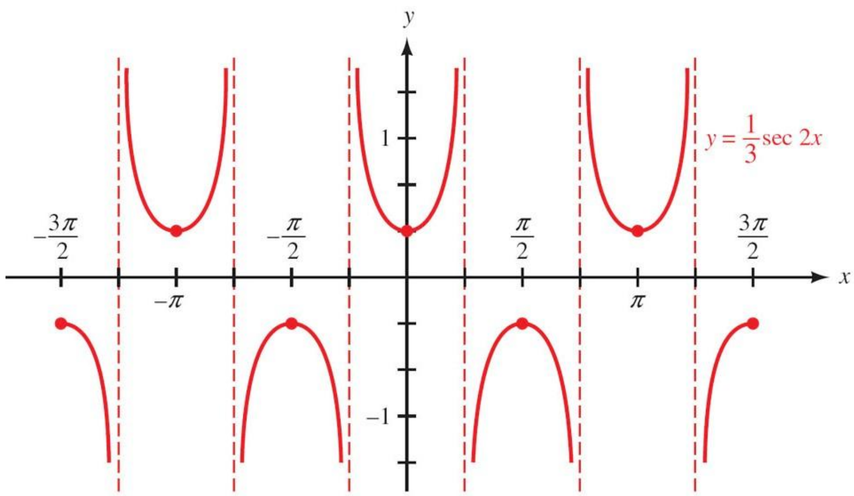
Graph $y = \frac{1}{3}\sec 2x$ *for* $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$

Period $= \frac{2\pi}{2} = \pi$

First, graph $y = \frac{1}{3}\cos 2x$

x	$y = \frac{1}{3}\cos 2x$
0	$\frac{1}{3}$
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	$-\frac{1}{3}$
$\frac{3\pi}{4}$	0
π	$\frac{1}{3}$





Graphing the *Cosecant* Function

Domain: $\{x \mid x \neq n\pi,\ where\ n \in \mathbb{Z}\}$

Range: $(-\infty,\ -1] \cup [1,\ \infty)$

- The graph is discontinuous at values of x of the form $x = n\pi$ and has ***vertical asymptotes*** at these values.
- There are no x-intercepts.
- Its period is 2π.
- Its graph has no amplitude, since there are no minimum or maximum values.
- The graph is symmetric with respect to the *origin*, so the function is an odd function. For all x in the domain $\csc(-x) = -\csc(x)$.

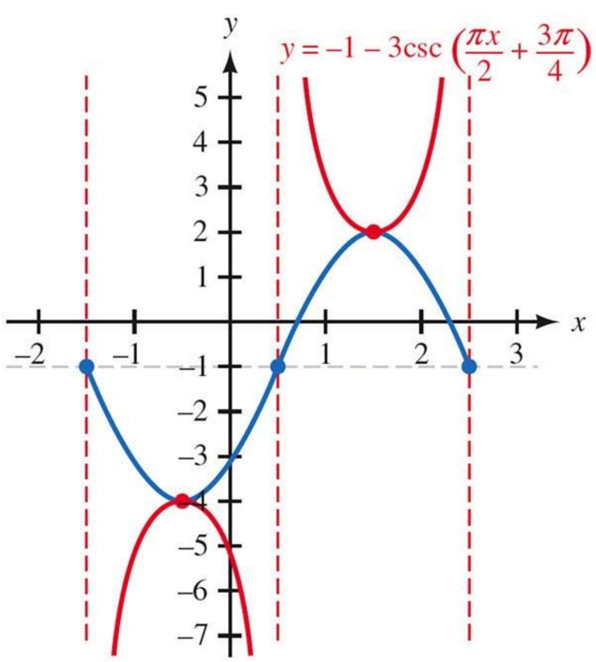
Example

Graph one complete cycle $y = -1 - 3\csc\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$

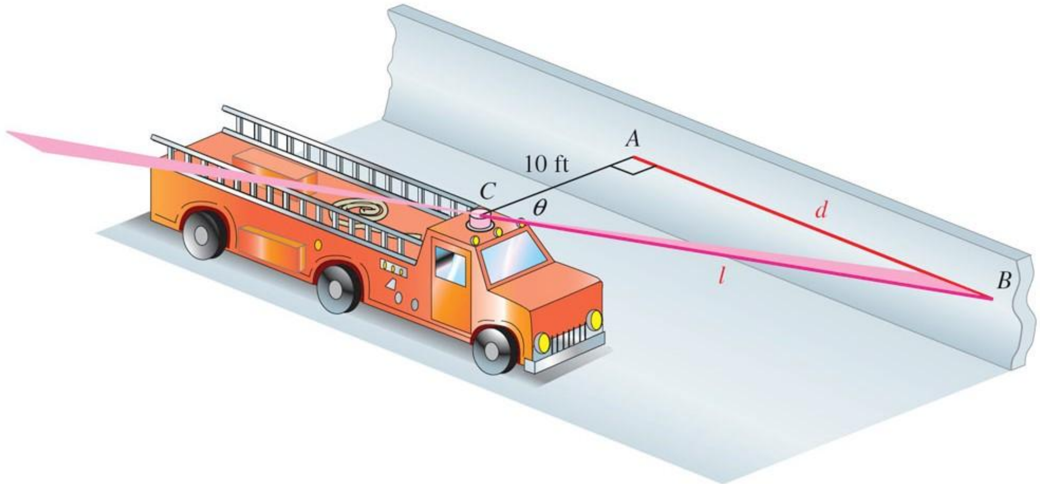
Period $= \frac{2\pi}{\pi/2} = 4$

Phase shift: $\phi = -\frac{\frac{3\pi}{4}}{\frac{\pi}{2}} = -\frac{3}{2}$

	x	$y = -1 - 3\sin\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$
$-\frac{3}{2} + 0$	$-\frac{3}{2}$	-1
$-\frac{3}{2} + 1$	$-\frac{1}{2}$	-4
$-\frac{3}{2} + 2$	$\frac{1}{2}$	-1
$-\frac{3}{2} + 3$	$\frac{3}{2}$	2
$-\frac{3}{2} + 4$	$\frac{5}{2}$	-1



Example



A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top is 10 feet from the wall and rotates through one complete revolution every 2 seconds. Graph the function that gives the length d in terms of time t from $t = 0$ to $t = 2$.

$$\omega = \frac{\theta}{t} = \frac{2\pi}{2} = \pi \ rad/sec$$

$$\tan\theta = \frac{d}{10} \quad \rightarrow \ d = 10\tan\theta$$

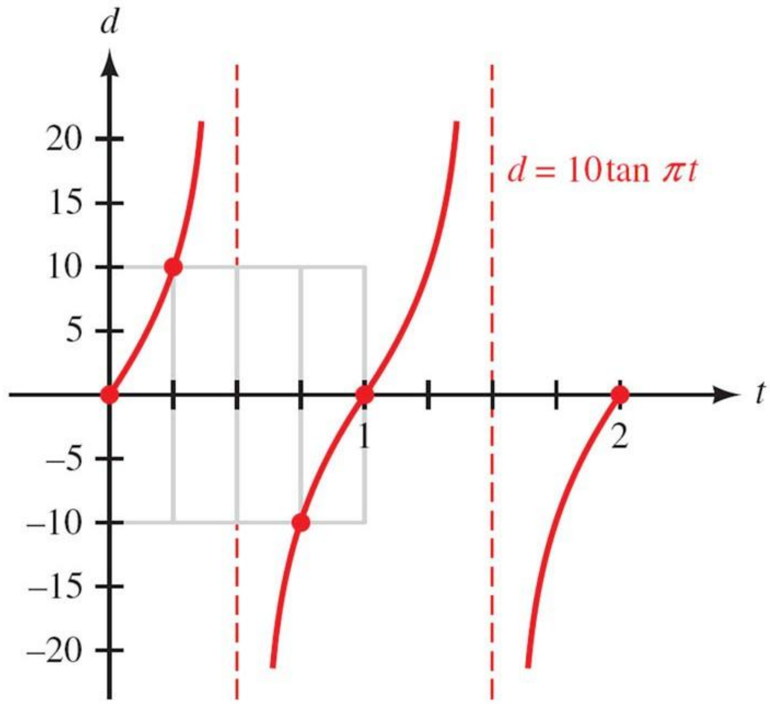
$$d(t) = 10\tan \pi t$$

t	$d = 10\tan \pi t$
0	0
$\frac{1}{4}$	10
$\frac{1}{2}$	∞
$\frac{3}{4}$	-10
1	0

Period $= \frac{\pi}{\pi} = 1$

One cycle: $0 \le \pi t \le \pi$

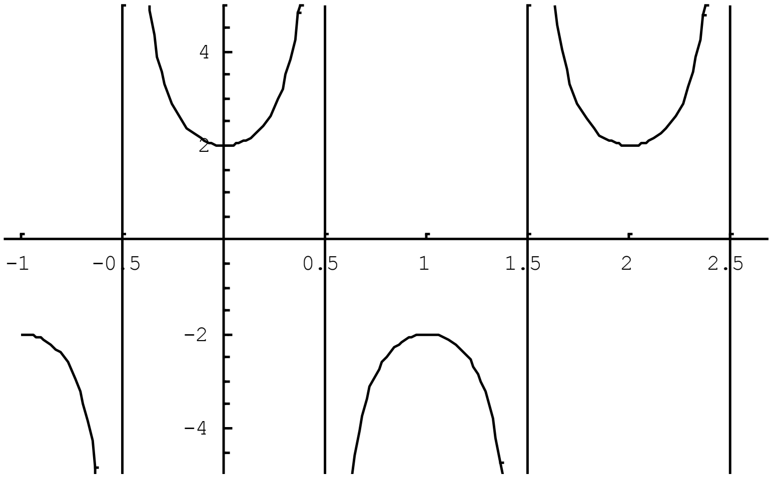
$0 \le t \le 1$



Finding the *Secant* and *Cosecant* Functions from the Graph

Example

Find an equation $y = k + A\sec(Bx + C)$ or $y = k + A\csc(Bx + C)$ to match the graph



<u>Solution</u>

For cosine:

$\boldsymbol{A} = 2$ $\quad P = 2 = \frac{2\pi}{B} \Rightarrow \boxed{B = \frac{2\pi}{2} = \pi}$ $\quad$ *Phase shift* $= -\frac{C}{B} = 0 \Rightarrow \boxed{C = 0}$

$y = 2\ \sec(\pi x)$ from –1 to 2.5.

Exercises **Section 2.5 – Other Trigonometric Functions**

1. Graph one complete cycle $y = 4\csc x$

2. Graph $y = 3\tan x \quad for \ -\pi \le x \le \pi$

3. Graph one complete cycle $y = \frac{1}{2}\cot(-2x)$

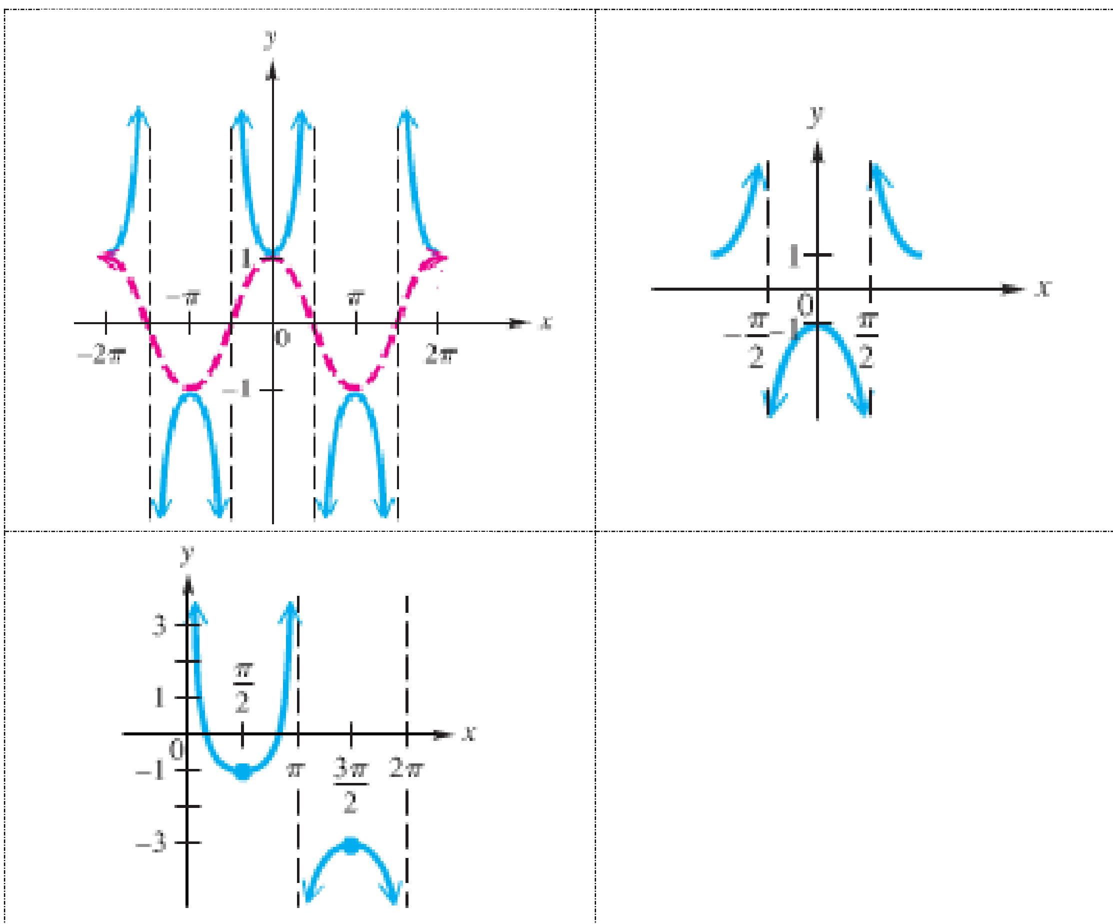
4. Graph over a 2-period interval $y = 1 - 2\cot 2\left(x + \frac{\pi}{2}\right)$

5. Graph over a 2-period interval $y = \frac{2}{3}\tan\left(\frac{3}{4}x - \pi\right) - 2$

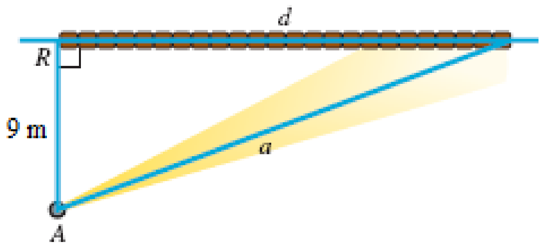
6. Graph over a one-period interval $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$

7. Graph over a one-period interval $y = 2 + \frac{1}{4}\sec\left(\frac{1}{2}x - \pi\right)$

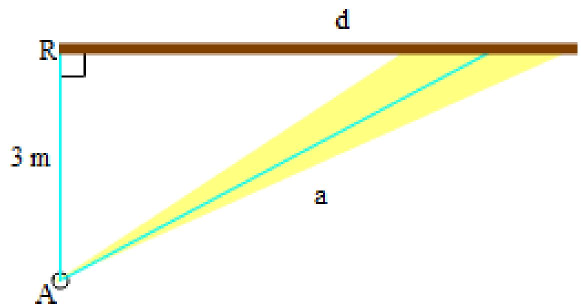
8. Find an equation to match the graph



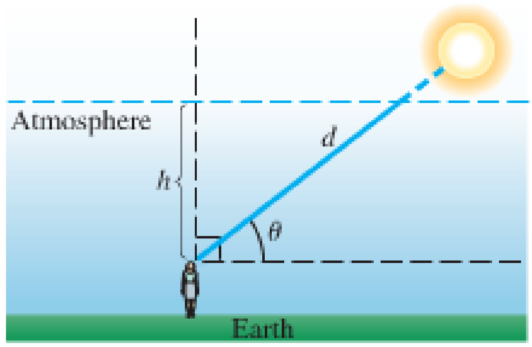
9. A rotating beacon is located at point A next to a long wall. The beacon is 9 m from the wall. The distance $\boldsymbol{a}$ is given by $a = 9|\sec 2\pi t|$, where t is time measured in seconds since the beacon started rotating. (When t = 0, the beacon is aimed at point R.) Find $\boldsymbol{a}$ for t = 0.45



10. A rotating beacon is located 3 m south of point R on an east-west wall. d, the length of the light display along the wall from R, is given by $d = 3\tan 2\pi t$, where t is time measured in seconds since the beacon started rotating. (When t = 0, the beacon is aimed at point R. When the beacon is aimed to the right of R, the value of d is positive; d is negative if the beacon is aimed to the left of R.) Find a for t = 0.8



11. The shortest path for the sun's rays through Earth's atmosphere occurs when the sun is directly overhead. Disregarding the curvature of Earth, as the sun moves lower on the horizon, the distance that sunlight passes through the atmosphere increases by a factor of $\csc\theta$, where θ is the angle of elevation of the sun. This increased distance reduces both the intensity of the sun and the amount of ultraviolet light that reached Earth's surface.

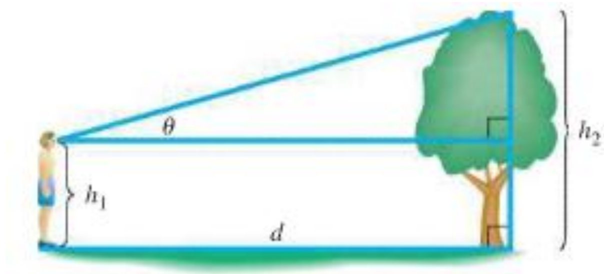


a) Verify that $d = h\csc\theta$

b) Determine θ when $d = 2h$

c) The atmosphere filters out the ultraviolet light that causes skin to burn, Compare the difference between sunbathing when $\theta = \frac{\pi}{2}$ and when $\theta = \frac{\pi}{3}$. Which measure gives less ultraviolet light?

12. Let a person whose eyes are h_1 feet from the ground stand ***d*** feet from an object h_1 feet tall, where $h_2 > h_1$ feet. Let θ be the angle of elevation to the top of the object.



a) Show that $d = \left(h_2 - h_1\right)\cot\theta$

b) Let $h_2 = 55$ and $h_1 = 5$. Graph ***d*** for the interval $0 < \theta \le \frac{\pi}{2}$