$$\sum_{k=0}^{19} \frac{k-3}{4} = \frac{1}{4}\left(\sum_{k=0}^{19} k - \sum_{k=0}^{19} 3\right)$$

$$= \frac{1}{4}\left[\frac{19(20)}{2} - 3(19-0+1)\right]$$

$$= \frac{1}{4}(190-60)$$

$$= \frac{65}{2}$$

$$\sum_{k=2}^{50} (2{,}000 - 3k) = \sum_{k=2}^{50} 2\,10^3 - 3\sum_{k=2}^{50} k$$

$n = 50 - 2 + 1$

$$= (50-2+1)\,2\times10^3 - 3\left(\frac{50(51)}{2} - 1\right)$$

$$= 98{,}000 - 3{,}822$$

$$= 94{,}178$$

$$= \frac{49}{2}(1994 + 1850)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$.\overline{78} = .78\,78\,78\,78$$

$$= .78 + .0078 + .000078 + \cdots$$

$$a = .78$$

$$r = \frac{.0078}{.78} = .01 \qquad = \frac{78 \times 10^{-4}}{78 \times 10^{-2}} = 10^{-2}$$

$$= \frac{1}{100}$$

$$.\overline{78} = \frac{.78}{1 - \frac{1}{100}}$$

$$= .78 \frac{1}{\frac{99}{100}} \qquad 100(.78)$$

$$= \frac{78}{99}$$

$$= \frac{26}{33}$$

$4+8+12+\cdots+4n = 2n(n+1)$

· For $n=1 \Rightarrow 4 \stackrel{?}{=} 2(1)(2)$

$4 = 4$ ✓ P_1 is true

· Assume $P_k : 4+8+\cdots+4k = 2k(k+1)$ is true.

Is $P_{k+1} : 4+\cdots+4k+4(k+1) = 2(k+1)(k+2)$?

$$4+\cdots+4k+4(k+1) = 2k(k+1)+4(k+1)$$
$$= 2(k+1)(k+2) \checkmark$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the proof is completed

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$1+5+9+\cdots+(4n-3) = n(2n-1)$

For $n=1 \Rightarrow 1 \stackrel{?}{=} 1(2-1)$

$1 = 1$ ✓ P_1 is true

Let $P_k : 1+5+\cdots+(4k-3) = k(2k-1)$ is true

Is $P_{k+1} : 1+\cdots+(4k-3)+(4(k+1)-3) \stackrel{?}{=} (k+1)(2(k+1)-1)$

$1+\cdots+(4k-3)+(4k+1) \stackrel{?}{=} (k+1)(2k+1)$

$$1+\cdots+(4k-3)+(4k+1) = k(2k-1)+(4k+1)$$
$$= 2k^2-k+4k+1$$
$$= 2k^2+3k+1$$
$$= (k+1)(2k+1) \checkmark$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the proof is completed

$$\frac{x}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3} \qquad (x+1)$$

$$x = A(x-3) + B(x+1)$$

$$\begin{array}{l|l} x^1 & A + B = 1 \\ x^0 & -3A + B = 0 \end{array}$$

$$\boxed{4A = 1} \Rightarrow \boxed{A = \frac{1}{4}}$$

$$B = 3A = \frac{3}{4}$$

$$\frac{x}{x^2-2x-3} = \frac{1/4}{x+1} + \frac{3/4}{x-3}$$

$$\frac{3x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$3x = A(x-1) + B(x+2)$$

$$x^1 \quad A + B = 3 \rightarrow \boxed{A = 2}$$

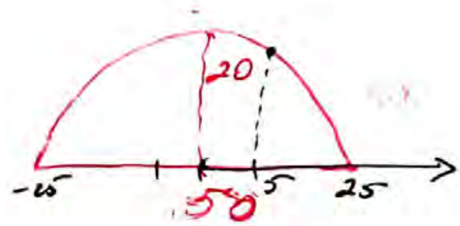
$$x^0 \quad -A + 2B = 0$$

$$\boxed{B = 1}$$

$$\boxed{\frac{3x}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}}$$

$$\frac{x^n + \ldots}{x^n + \ldots}$$ ← degree same or larger.

38 Given Truck $h = 14$, $w = 10$



$a = 25$

$b = 20$

$$\frac{x^2}{25^2} + \frac{y^2}{20^2} = 1$$

$$\frac{y^2}{20^2} = 1 - \frac{x^2}{25^2} = \left.\frac{25^2 - x^2}{25^2}\right|_{x=5}$$

$$y^2 = \frac{20^2}{25^2}\left(25^2 - 25\right)$$

$$y = \frac{\overset{4}{20}}{\underset{5}{25}} \sqrt{25(25-1)} \qquad \frac{4}{5}\,5$$

$$\boxed{= 4\sqrt{24}}$$

$$= 8\sqrt{6}$$

$$(14)^2 \cdot (8\sqrt{6})^2$$

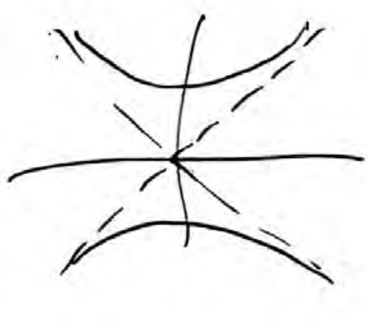
$$196 < 384$$

Truck will clear

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \longrightarrow \frac{y^2}{a^2} = \frac{x^2}{b^2}$$

$$y = \pm \frac{a}{b} x$$



$$\frac{x^2}{a^2} = \frac{y^2}{b^2}$$

$$\frac{b^2 x^2}{a^2} = y$$

$$y = \pm \frac{b}{a} x$$

5.5 1/

$\{c_n\} = \left\{ \frac{(-1)^n}{(n+1)(n+2)} \right\}$

$n=1 \rightarrow c_1 = \frac{-1}{2(3)} = -\frac{1}{6}$

$n=2 \rightarrow c_2 = \frac{1}{3(4)} = \frac{1}{12}$

$n=3 \rightarrow c_3 = \frac{(-1)^3}{(3+1)(3+2)} = \frac{-1}{20}$

$n=4 \rightarrow c_4 = \frac{(-1)^4}{5(6)} = \frac{1}{30}$

$n=8 \rightarrow c_8 = \frac{(-1)^8}{9(10)} = \frac{1}{90}$

5.6 #22

Arithmetic

a_{20}: $a_9 = -5$, $a_{15} = 31$

$a_n = a_1 + (n-1)d$

$d = \frac{y_2 - y_1}{x_2 - x_1}$

$d = \frac{31+5}{15-9} = 6$

$a_9 = a_1 + 8(6) = -5$

$a_1 = -5 - 48$

$= -53$

$a_{20} = -53 + 19(6)$

$= 61$

Geometric seq.

66/ a_7 : $a_2 = 3$ $a_3 = -\sqrt{3}$ $a_n = a_1 r^{n-1}$

$$r = -\frac{\sqrt{3}}{3}$$

$$a_2 = a_1\left(-\frac{\sqrt{3}}{3}\right) = 3$$

$$a_1 = -\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= -3\sqrt{3}$$

$$\frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{x}$$

$$\frac{9\sqrt{3}}{3}$$

$$\frac{9}{\sqrt{3}} = 9\frac{\sqrt{3}}{3}$$

$$a_7 = -3\sqrt{3}\left(-\frac{\sqrt{3}}{3}\right)^6$$

$$= -3\sqrt{3}\ \frac{3^3}{3^6} \rightarrow \frac{3^4}{36} \rightarrow 3^2$$

$$\sqrt{3} = \left(3^{\frac{1}{2}}\right)^6$$

$$= -\frac{\sqrt{3}}{9}$$

$$r = \left(\frac{y_2}{y_1}\right)^{\left(\frac{1}{x_2 - x_1}\right)}$$

107/ $$\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1} = \frac{2}{1-\frac{2}{3}}$$ $r = \left|\frac{2}{3}\right| < 1$

$$= 6$$

109/ $$\sum_{n=1}^{\infty} 3\left(\frac{3}{2}\right)^{n} = \infty$$ $r = \frac{3}{2} \geq 1$