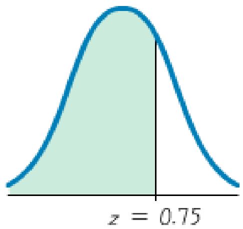
Solution Section 2.8 – Properties of the Normal Distribution

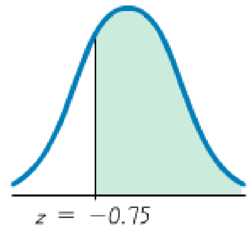
Exercise

Find the area shaded region. The graph depicts the standard distribution with mean 0 and standard deviation 1.

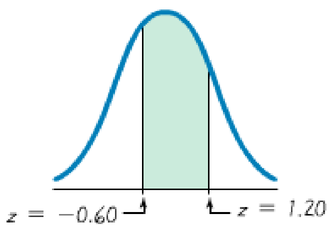
a)



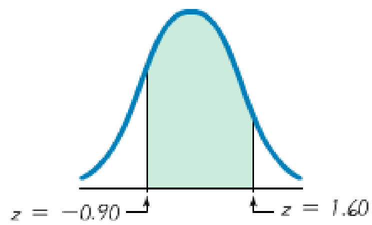
b)



c)



d)



Solution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852

a) $P(z < 0.75) \underline{= 0.7734|}$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148

b) $P(z > -0.75) = 1 - P(z < -0.75)$

$= 1 - 0.2266$

$\underline{= 0.7734|}$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451

c) $P(-0.60 < z < 1.20) = P(z < 1.20) - P(z < -0.60)$

$= 0.8849 - 0.2743$

$\underline{= 0.6106|}$

d) $P(-0.90 < z < 1.60) = P(z < 1.60) - P(z < -0.90)$

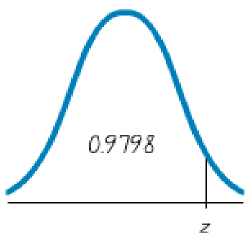
$= 0.9452 - 0.1841$

$\underline{= 0.7611|}$

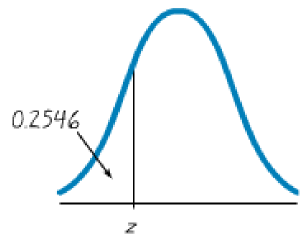
Exercise

Find the indicated z-score. The graph depicts the standard distribution with mean 0 and standard deviation 1.0.

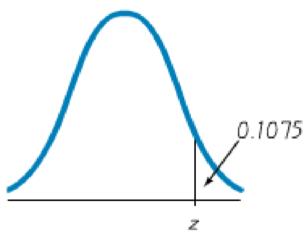
a)



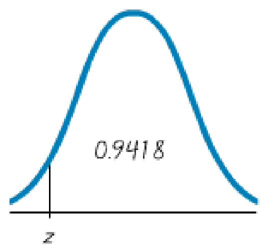
b)



c)



d)



Solution

Using Normal Distribution Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

a) For $A = 0.9798 \quad \Rightarrow \quad \underline{z = 2.05}$

b) For $A = 0.2546 \quad \Rightarrow \quad \underline{z = -0.66}$

c) Area to the right of z, then: $A = 1 - 0.1075 = 0.8925$

For $A = 0.8925 \quad \Rightarrow \quad \underline{z = 1.24}$

d) Area to the right of z, then: $A = 1 - 0.9418 = 0.0582$

For $A = 0.0582 \quad \Rightarrow \quad \underline{z = -1.57}$

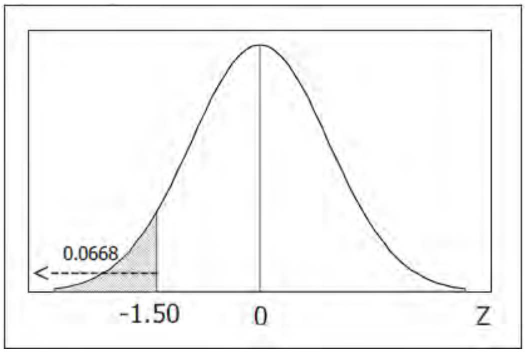
Exercise

Assume that thermometer readings are normally distributed with a mean of 0ºC and the standard deviation of the readings is 1.00ºC. A thermometer is randomly selected and tested. In each case, draw a sketch, and find the probability of each reading.

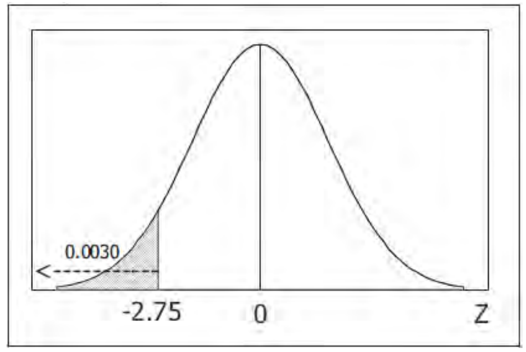
a) Less than –1.50
b) Less than –2.75
c) Less than 1.23
d) Greater than 2.22
e) Greater than 2.33
f) Greater than –1.75
g) Between 0.50 and 1.00
h) Between –3.00 and –1.00
i) Between –1.20 and 1.95
j) Between –2.50 and 5.00
k) Greater than 0
l) Less than 0

Solution

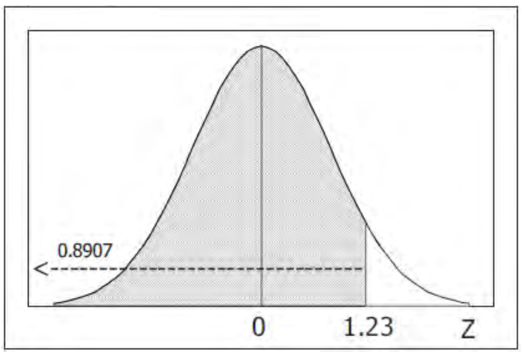
a) $P(z<-1.50)\ \underline{=0.0668}$



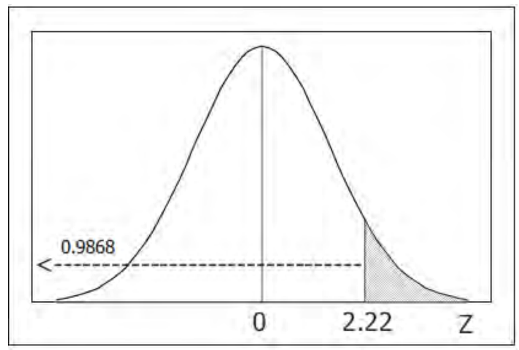
b) $P(z<-2.75)\ \underline{=0.0030}$



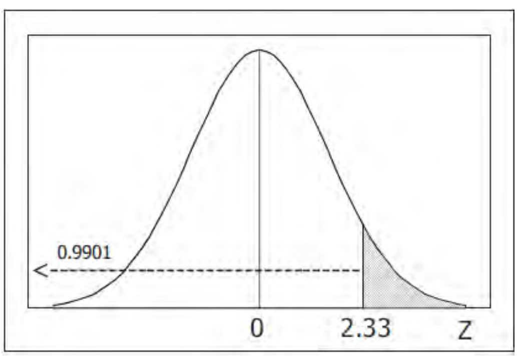
c) $P(z<1.23)\ \underline{=0.8907}$



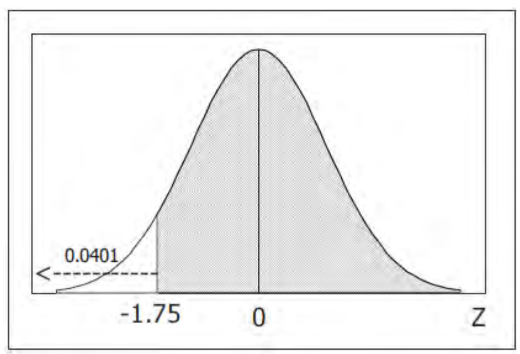
d) $P(z>2.22)=1-0.9868\ \underline{=0.0132}$



e) $P(z>2.33)=1-0.9901\ \underline{=0.0099}$



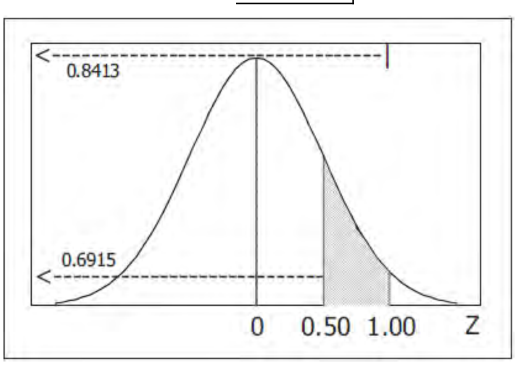
f) $P(z>-1.75)=1-0.0401\ \underline{=0.9599}$



g) $P(0.50<z<1.00)=P(z<1)-P(z<0.50)$

$=0.8413-0.6915$

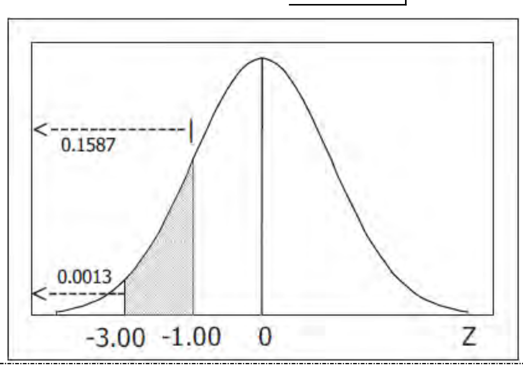
$\underline{=0.1498}$



h) $P(-3.00<z<-1.00)=P(z<-1)-P(z<-3)$

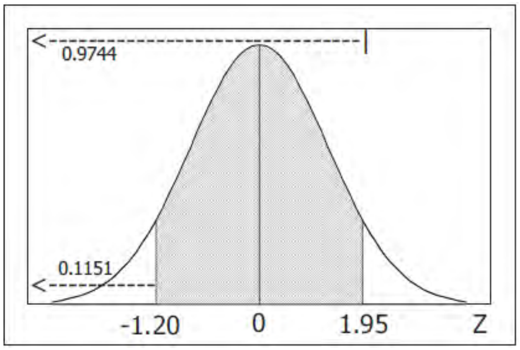
$=0.1587-0.0013$

$\underline{=0.1574}$



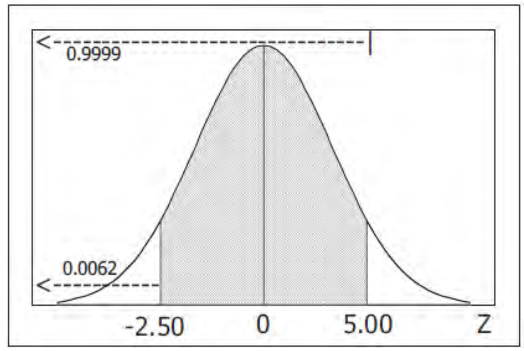
i) $P(-1.2 < z < 1.95) = P(z < 1.95) - P(z < -1.2)$

$$= 0.9744 - 0.1151$$
$$\underline{= 0.8593}$$

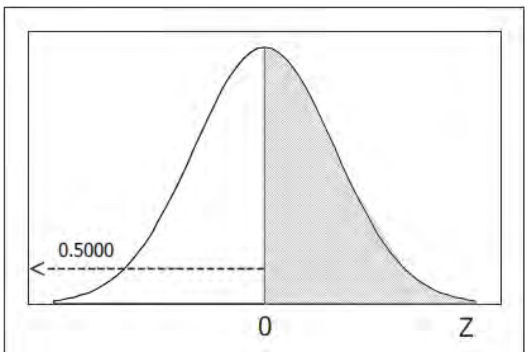


j) $P(-2.5 < z < 5) = P(z < 5) - P(z < -2.5)$

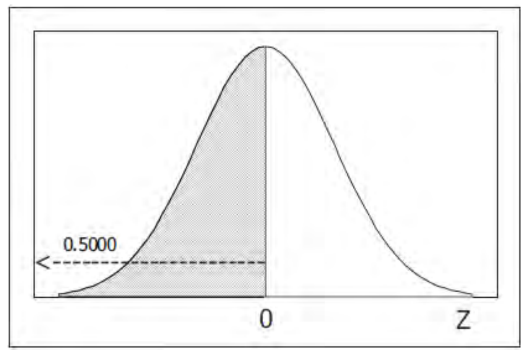
$$= 0.9999 - 0.0062$$
$$\underline{= 0.9937}$$



k) $P(z > 0) = 1 - 0.5000 \underline{= 0.5000}$



l) $P(z < 0) \underline{= 0.5000}$



Exercise

Assume that thermometer readings are normally distributed with a mean of 0°C and the standard deviation of the readings is 1.00°*C*. A thermometer is randomly selected and tested. In each case, draw a sketch, and find the temperature reading corresponding to the given information.

a) Find P_{95}, the 95th percentile. This is the temperature separating the bottom 95% from the top 5%.

b) Find P_1, the 1st percentile. This is the temperature separating the bottom 1% from the top 99%.

c) If 2.5% of the thermometers are rejected because they have readings that are too high and another 2.5% are rejected because they have readings that are too low, find the 2 readings that are cutoff values separating the rejected thermometers from the others.

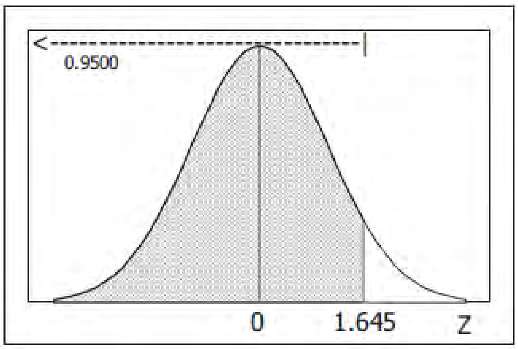
d) If 0.5% of the thermometers are rejected because they have readings that are too high and another 0.5% are rejected because they have readings that are too low, find the 2 readings that are cutoff values separating the rejected thermometers from the others.

Solution

a) For P_{95}, the cumulative area is 0.95000.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

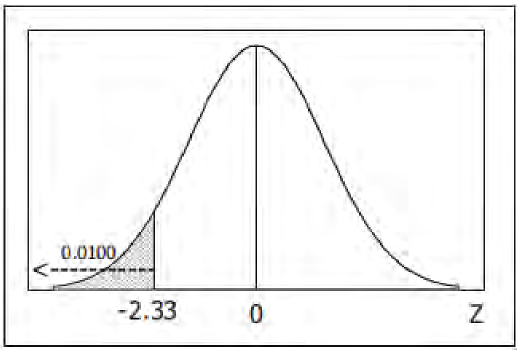
$A = 0.9500 \Rightarrow \boxed{z = 1.645}$



b) For P_1, the cumulative area is 0.0100.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084

$A = 0.0100 \Rightarrow \boxed{z = -2.33}$

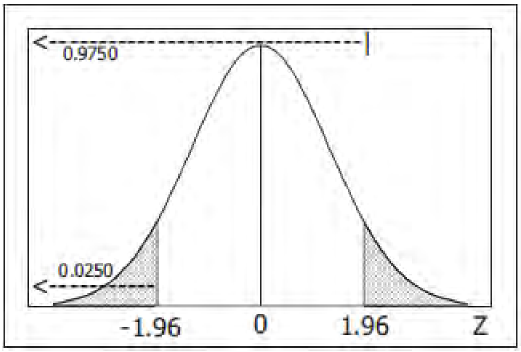


c) For the lowest 2.5%, the cumulative area is 0.0250.

$A = 0.0250 \Rightarrow \boxed{z = -1.96}$

For the highest 2.5%, the cumulative area is 1 – 0.0250 = 0.9750

$A = 0.9750 \Rightarrow \boxed{z = 1.96}$

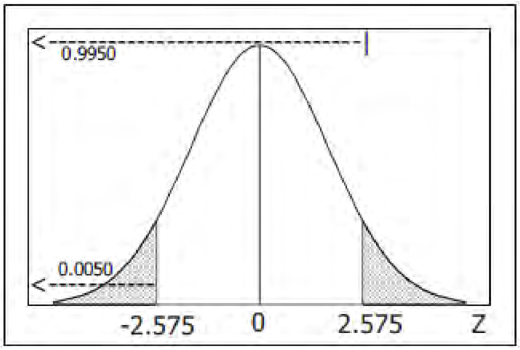


d) For the lowest 0.5%, the cumulative area is 0.0050.

$A = 0.0050 \Rightarrow \boxed{z = -2.575}$

For the highest 0.5%, the cumulative area is 1 – 0.0050 = 0.9950

$A = 0.9950 \Rightarrow \boxed{z = 2.575}$



Exercise

For a standard normal distribution, find the percentage of data that are

a) Within 2 standard deviations of the mean.

b) More than 1 standard deviation away from the mean.

c) More than 1.96 standard deviations away from the mean.

d) Between μ – 3σ and μ + 3σ.

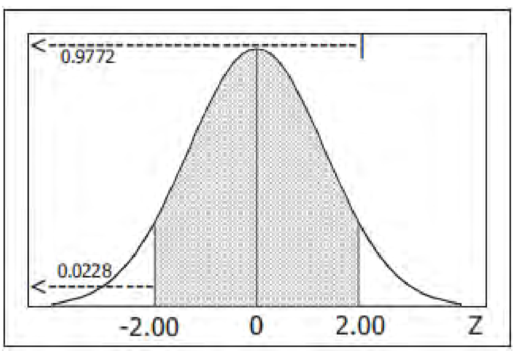
e) More than 3 standard deviations away from the mean.

Solution

a) $P(-2 < z < 2) = P(z < 2) - P(z < -2)$

$$= 0.9772 - 0.0228$$

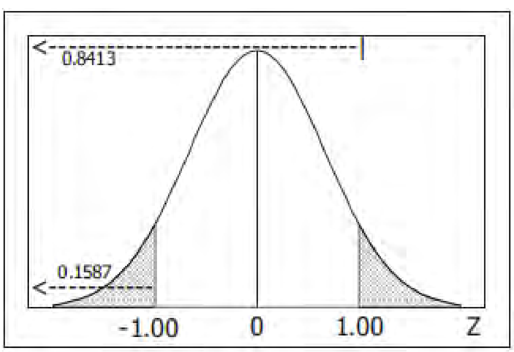
$$\underline{= 0.9544|} \quad \textit{or} \quad 95.44\%$$



b) $P(z < -1 \text{ or } z > 1) = P(z < -1) + P(z > 1)$

$$= 0.1587 - (1 - .8413)$$

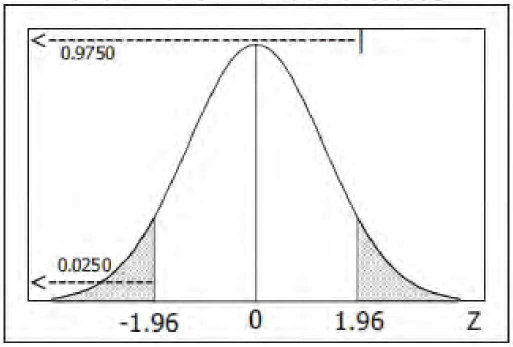
$$\underline{= 0.3174|} \quad \textit{or} \quad 31.74\%$$



c) $P(z < -1.96 \text{ or } z > 1.96) = P(z < -1.96) + P(z > 1.96)$

$$= 0.0250 - (1 - .9750)$$

$\underline{= 0.0500}$ *or* 5.00%



d) $P(-3 < z < 3) = P(z = 3) - P(z = -3)$

$$= 0.9987 - 0.0013$$

$\underline{= 0.9974}$ *or* 99.74%