2.2 #23

$$\lim_{(x,y,z)\to(\frac{\pi}{2},0,\frac{\pi}{2})} 4\cos y \sin\sqrt{xz} = 4\cos 0 \sin\sqrt{\frac{\pi^2}{4}}$$

$$= 4\sin\frac{\pi}{2}$$

$$= \boxed{4}$$

2.3 #10

$$f(x,y) = e^{xy}\ln y$$

$$f_x = y e^{xy}\ln y \qquad f_y = x e^{xy}\ln y + e^{xy}\frac{1}{y}$$

$$= \left(x\ln y + \frac{1}{y}\right)e^{xy}$$

#27

$$f(x,y,z) = \frac{xyz}{x+y}$$

$$f_x = \frac{y^2 z}{(x+y)^2} \qquad f_y = \frac{x^2 z}{(x+y)^2} \qquad f_z = \frac{xy}{x+y}$$

#30 $r(x,y) = \ln(x+y)$

$$r_x = \frac{1}{x+y} \qquad r_y = \frac{1}{x+y}$$

$$r_{xx} = -\frac{1}{(x+y)^2} \qquad r_{yy} = -\frac{1}{(x+y)^2}$$

$$r_{xy} = r_{yx} = -\frac{1}{(x+y)^2}$$

Ex/44 $u(x,y) = y(3x^2 - y^2)$

$= 3x^2y - y^3$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$u_x = 6xy$ $\quad$ $u_y = 3x^2 - 3y^2$

$u_{xx} = 6y$ $\quad$ $u_{yy} = -6y$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6y - 6y$$

$= 0$

$\therefore$ The fctn satisfies Laplace eqn.

2.4

#6 $w = xe^y + y\sin z - \cos z$

$\{ x = 2\sqrt{t},\ y = t - 1 + \ln t \quad z = \pi t$

$w = 2\sqrt{t}\, e^{t-1+\ln t} + (t - 1 + \ln t)\sin \pi t - \cos \pi t$

$\frac{dw}{dt} = w' = \left(\frac{1}{\sqrt{t}} + 2\sqrt{t}\left(1 + \frac{1}{t}\right)\right) e^{t-1+\ln t}$

$+ \left(1 + \frac{1}{t}\right)^{\sin \pi t} + \pi(t - 1 + \ln t)\cos \pi t$

$+ \pi \sin \pi t.$

$= \left(2\sqrt{t} + \frac{3}{\sqrt{t}}\right) e^{t-1+\ln t} + \left(1 + \frac{1}{t} + \pi\right)\sin \pi t$

$+ \pi(t - 1 + \ln t)\cos \pi t$

$w'\big|_{t=1} = 5$

#13 $\dfrac{\partial w}{\partial r}$ $r=1,\ s=-1$

$w=(x+y+z)^2$ $\begin{cases} x=r-s \\ y=\cos(r+s)\end{cases}$ $z=\sin(r+s)$

$=\left(r-s+\cos(r+s)+\sin(r+s)\right)^2$

$$\frac{\partial w}{\partial r}=2\left(1-\sin(r+s)+\cos(r+s)\right)\left(r-s+\cos(r+s)+\sin(r+s)\right)\Big|_{\substack{r=1\\ s=-1}}$$

$=2(1-\sin 0+\cos 0)(1+1+\cos 0+\sin 0)$

$\quad\quad 2 \quad\quad\quad 1+1+1 = 3$

$=\boxed{12}$

#31 $y\ln(x^2+y^2+4)-3=0$

$$\frac{dy}{dx}=-\frac{F_x}{F_y}$$

$$=-\frac{2xy}{x^2+y^2+4}\cdot\frac{1}{\ln(x^2+y^2+4)+\dfrac{2y^2}{x^2+y^2+4}}$$

$$=\frac{-2xy}{(x^2+y^2+4)\ln(x^2+y^2+4)+2y^2}$$

2.5

#6 $f(x,y,z) = e^{x+y}\cos z + (y+1)\sin^{-1}x$

$(0, 0, \frac{\pi}{6})$

$$\nabla f = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$$

$$= \left(e^{x+y}\cos z + \frac{y+1}{1-x^2}\right)\hat{i}$$

$$+ \left(e^{x+y}\cos z + \sin^{-1}x\right)\hat{j}$$

$$- e^{x+y}\sin z \,\hat{k} \Bigg|_{(0,0,\frac{\pi}{6})}$$

$$= \left(\cos\frac{\pi}{6} + 1\right)\hat{i} + \left(\cos\frac{\pi}{6} + \sin^{-1}0\right)\hat{j} - \sin\frac{\pi}{6}\hat{k}$$

$$= \left(\frac{\sqrt{3}}{2} + 1\right)\hat{i} + \left(\frac{\sqrt{3}}{2}\right)\hat{j} - \frac{1}{2}\hat{k}$$

2.7 #9

$$f(x,y) = \sqrt{56x^2 - 8y^2 - 16x - 31} + 1 - 8x$$

$$f_x = \frac{56x - 8}{\sqrt{56x^2 - 8y^2 - 16x - 31}} - 8 = 0$$

$$f_y = \frac{-8y}{\sqrt{56x^2 - 8y^2 - 16x - 31}} = 0 \Rightarrow y = 0$$

$$(7x - 1)^2 = \left(\sqrt{56x^2 - 8y^2 - 16x - 31}\right)^2$$

$$49x^2 - 14x + 1 = 56x^2 - 8y^2 - 16x - 31$$

$$7x^2 - 2x - 32 - 8y^2 = 0 \quad \Big|_{y=0}$$

$$7x^2 - 2x - 32 = 0 \qquad (x+2)(7x-16)$$

$$x = -2, \ \frac{16}{7}$$

CP: $(-2, 0)$ & $\left(\frac{16}{7}, 0\right)$

#3 $f(x,y) = x^3 + y^3 - 3xy + 15$

$f_x = 3x^2 - 3y = 0 \rightarrow y = x^2$ ①

$f_y = 3y^2 - 3x = 0 \rightarrow x = y^2$ ②

$y = (y^2)^2 = y^4$

$y^4 - y = 0$

$y(y^3 - 1) = 0 \rightarrow y = 0, 1$

② $\begin{cases} y = 0 \rightarrow x = 0 \\ y = 1 \rightarrow x = 1 \end{cases}$

C.P $(0,0)$ $(1,1)$

$f_{xx} = 6x \quad f_{yy} = 6y \quad f_{xy} = -3$

@ $(0,0)$ $f_{xx} = 0 \quad f_{yy} = 0 \quad f_{xy} = -3$

$f_{xx} f_{yy} - f_{xy}^2 = -9 < 0$

$f(x,y)$ has a saddle pt @ $(0,0)$

@ $(1,1)$ $f_{xx} = 6 > 0 \quad f_{yy} = 6 \quad f_{xy} = -3$

$f_{xx} f_{yy} - f_{xy}^2 = 36 - 9 > 0$

$f(x,y)$ has LMIN @ $(1,1)$ w/ value of

$f(1,1) = 1 + 1 - 3 + 15 = 14$

2.8 #37 $f(x,y,z) = x$ $g(x,y,z) = x^2+y^2+z^2 - z - 1 = 0$

$\nabla f = \lambda \nabla g$

$\hat{i} = 2x\lambda \hat{i} + 2y\lambda \hat{j} + \lambda(2z-1)\hat{k} = 0$

$\begin{cases} 2x\lambda = 1 & (1) \\ 2y\lambda = 0 \to y = 0 \text{ or } \lambda = 0 \ \# \\ \lambda(2z-1) = 0 \to \lambda = 0 \text{ or } z = \frac{1}{2} \end{cases}$

if $\lambda = 0 \Rightarrow (1)$ $0 = 1$ #

if $z = \frac{1}{2}$

$y = 0$

$(1) \to x = \frac{1}{2\lambda}$

(g) $\frac{1}{4\lambda^2} + 0 + \frac{1}{4} - \frac{1}{2} = 1$

$\frac{1}{4\lambda^2} = \frac{5}{4} \Rightarrow \lambda^2 = \frac{1}{5}$

$\lambda = \pm \frac{1}{\sqrt{5}}$

$\left(\pm \frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$

$f\left(-\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right) = -\frac{\sqrt{5}}{2}$ Min

$f\left(\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right) = \frac{\sqrt{5}}{2}$ Max