### *Exercise*

Determine all asymptotes of the function: $y = \dfrac{3x}{1-x}$

***Solution***

***VA***: $x = 1$ ***HA***: $y = -3$

***Hole***: $n/a$ ***Oblique asymptote***: $n/a$

### *Exercise*

Determine all asymptotes of the function: $y = \dfrac{x^2}{x^2+9}$

***Solution***

***VA***: $n/a$ $x^2 + 9 \neq 0$ ***HA***: $y = 1$

***Hole***: $n/a$ ***Oblique asymptote***: $n/a$

### *Exercise*

Determine all asymptotes of the function: $y = \dfrac{x-2}{x^2-4x+3}$

***Solution***

$x^2 - 4x + 3 = 0 \Rightarrow x = 1,\ 3$

$y = \dfrac{x}{x^2} \to 0$

***VA***: $x = 1,\ x = 3$ ***HA***: $y = 0$

***Hole***: $n/a$ ***Oblique asymptote***: $n/a$

### *Exercise*

Determine all asymptotes of the function: $y = \dfrac{3}{x-5}$

***Solution***

***VA***: $x = 5$ ***HA***: $y = 0$

***Hole***: $n/a$ ***Oblique asymptote***: $n/a$

## *Exercise*

Determine all asymptotes of the function: $y = \dfrac{x^3 - 1}{x^2 + 1}$

***Solution***

***VA***: *none*

***HA***: *none*

***Hole***: *n / a*

***Oblique asymptote***: $y = x$

$$\begin{array}{r|l} & x \\ x^2+1 & x^3-1 \\ & \underline{-x^3 - x} \\ & -x-1 \end{array}$$

$$y = x - \frac{x+1}{x^2+1}$$

## *Exercise*

Determine all asymptotes of the function: $y = \dfrac{x^3 + 3x^2 - 2}{x^2 - 4}$

***Solution***

***VA***: $x = \pm 2$

***HA***: *n / a*

***Hole***: *n / a*

***Oblique asymptote***: $y = x + 3$

$$\begin{array}{r|l} & x+3 \\ x^2-4 & x^3+3x^2-2 \\ & \underline{-x^3 + 4x} \\ & 3x^2+4x-2 \\ & \underline{-3x^2 \quad +12} \\ & 4x+10 \end{array}$$

$$y = x + 3 + \frac{4x+10}{x^2-4}$$

## *Exercise*

Determine all asymptotes of the function: $y = \dfrac{3x^2 - 27}{(x+3)(2x+1)}$

***Solution***

$$y = \frac{3x^2-27}{(x+3)(2x+1)} = \frac{3\left(x^2-9\right)}{(x+3)(2x+1)} = \frac{3(x+3)(x-3)}{(x+3)(2x+1)} = \frac{3(x-3)}{(2x+1)}$$

***VA***: $x = -3, \ -\frac{1}{2}$ ***HA***: $y = \frac{3}{2}$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

### *Exercise*

Determine all asymptotes of the function: $y = \dfrac{x-3}{x^2-9}$

***Solution***

$$x^2 - 9 = 0 \rightarrow \boxed{x = \pm 3}$$

$$y = \frac{x-3}{(x-3)(x+3)}$$

$$= \frac{1}{x+3}$$

***VA***: $x = 3$ ***HA***: $y = 0$

***Hole***: $x = 3 \rightarrow y = \frac{1}{6}$ ***Oblique asymptote***: *n / a*

### *Exercise*

Determine all asymptotes of the function: $y = \dfrac{6}{\sqrt{x^2 - 4x}}$

***Solution***

$$x^2 - 4x = 0$$

$$\Rightarrow x(x-4) = 0 \rightarrow \boxed{x = 0, 4}$$

***VA***: $x = 0,\ x = 4$ ***HA***: $y = 0$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

### *Exercise*

Determine all asymptotes of the function: $y = \dfrac{5x-1}{1-3x}$

***Solution***

***VA***: $x = \frac{1}{3}$ ***HA***: $y = -\frac{5}{3}$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

### *Exercise*

Determine all asymptotes of the function: $f(x) = \dfrac{2x-11}{x^2+2x-8}$

***Solution***

***VA***: $x = 2,\ x = -4$ ***HA***: $y = 0$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

### *Exercise*

Determine all asymptotes of the function: $f(x)=\dfrac{x^2-4x}{x^3-x}$

***Solution***

$$f(x)=\frac{x(x-4)}{x(x^2-1)}$$
$$=\frac{x-4}{x^2-1}$$

***VA***: $x=-1,\ x=1$ ***HA***: $y=0$

***Hole***: $x=0\rightarrow y=4$ ***Oblique asymptote***: $n/a$

### *Exercise*

Determine all asymptotes of the function: $f(x)=\dfrac{x-2}{x^3-5x}$

***Solution***

***VA***: $x=0,\ x=\pm\sqrt{5}$ ***HA***: $y=0$

***Hole***: $n/a$ ***Oblique asymptote***: $n/a$

### *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{4x}{x^2+10x}$

***Solution***

$x^2+10x=0 \quad\rightarrow\quad x=0,\ -10$

***Domain***: $(-\infty,\ -10)\cup(-10,\ 0)\cup(0,\ \infty)$

$$f(x)=\frac{4x}{x(x+10)}$$
$$=\frac{4}{x+10}$$

***VA***: $x=-10$ ***HA***: $y=0$

***Hole***: $x=0 \quad\rightarrow\quad y=\frac{4}{10}\Rightarrow$ *hole* $\left(0,\ \frac{2}{5}\right)$ ***Oblique asymptote***: $n/a$

### *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{3-x}{(x-4)(x+6)}$

***Solution***

***VA***: $x=-6$ *and* $x=4$ ***HA***: $y=0$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x^3}{2x^3-x^2-3x}$

***Solution***

$$2x^3-x^2-3x=x\left(2x^2-x-3\right)=0 \quad \rightarrow \quad x=0,\ -1,\ \frac{3}{2}$$

$$f(x)=\frac{x^3}{2x^3-x^2-3x}$$

$$=\frac{x^3}{x\left(2x^2-x-3\right)}$$

$$=\frac{x^2}{2x^2-x-3}$$

***VA***: $x=-1$ *and* $x=\dfrac{3}{2}$ ***HA***: $y=\dfrac{1}{2}$

***Hole***: $x=0 \rightarrow y=0 \Rightarrow$ *hole* $(0,\ 0)$ ***Oblique asymptote***: *n / a*

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{3x^2+5}{4x^2-3}$

***Solution***

$$4x^2-3=0 \quad \rightarrow \quad x=\pm\frac{\sqrt{3}}{2}$$

***Domain***: $\left(-\infty,\ -\dfrac{\sqrt{3}}{2}\right)\cup\left(-\dfrac{\sqrt{3}}{2},\ \dfrac{\sqrt{3}}{2}\right)\cup\left(\dfrac{\sqrt{3}}{2},\ \infty\right)$

***VA***: $x=-\dfrac{\sqrt{3}}{2}$ *and* $x=\dfrac{\sqrt{3}}{2}$ ***HA***: $y=\dfrac{3}{4}$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x+6}{x^3+2x^2}$

***Solution***

$$x^3+2x^2=x^2(x+2)=0 \quad \rightarrow \quad x=0,-2$$

***Domain***: $(-\infty,\ -2)\cup(-2,\ 0)\cup(0,\ \infty)$

***VA***: $x=0$ *and* $x=2$ ***HA***: $y=0$

***Hole***: $n/a$ ***Oblique asymptote***: $n/a$

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x^2+4x-1}{x+3}$

***Solution***

***VA***: $x=-3$

***HA***: $n/a$

***Hole***: $n/a$

***Oblique asymptote***: $y=x+1$

$$\begin{array}{r|l} & x+1 \\ \hline x+3 & x^2+4x-1 \\ & \underline{-x^2-3x} \\ & \quad x-1 \\ & \quad \underline{-x-3} \\ & \qquad -4 \end{array}$$

$$f(x)=\frac{x^2+4x-1}{x+3}=x+1-\frac{4}{x+3}$$

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x^2-6x}{x-5}$

***Solution***

$x-5=0 \quad\rightarrow\quad x=5$

***Domain***: $(-\infty,\ 5)\cup(5,\ \infty)$

$$f(x)=\frac{x^2-6x}{x-5}$$
$$=x-1-\frac{5}{x-5}$$

$$\begin{array}{r|l} & x-1 \\ \hline x-5 & x^2-6x \\ & \underline{-x^2+5x} \\ & \quad -x \\ & \quad \underline{x-5} \\ & \qquad -5 \end{array}$$

***VA***: $x=5$

***HA***: N/A

***Hole***: N/A

***Oblique asymptote***: $y=x-1$

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x^3-x^2+x-4}{x^2+2x-1}$

***Solution***

$x^2+2x-1=0 \quad \rightarrow \quad x=-1\pm\sqrt{2}$

***Domain***: $(-\infty,\ -1-\sqrt{2})\cup(-1-\sqrt{2},\ -1+\sqrt{2})\cup(-1+\sqrt{2},\ \infty)$

$$f(x)=\frac{x^3-x^2+x-4}{x^2+2x-1}$$
$$=x-3+\frac{8x-7}{x^2+2x-1}$$

***VA***: $x=-1\pm\sqrt{2}$

***HA***: $n/a$

***Hole***: $n/a$

***Oblique asymptote***: $y=x-3$

$$\begin{array}{r|l} & x-3 \\ \hline x^2+2x-1 & x^3-x^2+x-4 \\ & \underline{-x^3-2x^2+x} \\ & \quad -3x^2+2x-4 \\ & \quad \underline{3x^2+6x-3} \\ & \quad\quad 8x-7 \end{array}$$

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{4x}{x^2+10x}$

***Solution***

$x^2+10x=0 \quad \rightarrow \quad x=0,\ -10$ ***Domain***: $(-\infty,\ -10)\cup(-10,\ 0)\cup(0,\ \infty)$

$$f(x)=\frac{4x}{x(x+10)}=\frac{4}{x+10}$$

***VA***: $x=-10$ ***HA***: $y=0$

***Hole***: $x=0 \quad \rightarrow \quad y=\frac{4}{10}\Rightarrow$ *hole* $\left(0,\ \frac{2}{5}\right)$

***Oblique asymptote***: $n/a$

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{3-x}{(x-4)(x+6)}$

***Solution***

***Domain***: $(-\infty,\ -6)\cup(-6,\ 4)\cup(4,\ \infty)$

***VA***: $x=-6$ *and* $x=4$ ***HA***: $y=0$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x^3}{2x^3-x^2-3x}$

***Solution***

$2x^3-x^2-3x=x\left(2x^2-x-3\right)=0 \quad \rightarrow \quad x=0,\ -1,\ \dfrac{3}{2}$

***Domain***: $(-\infty,\ -1)\cup(-1,\ 0)\cup\left(0,\ \frac{3}{2}\right)\cup\left(\frac{3}{2},\ \infty\right)$

$f(x)=\dfrac{x^3}{2x^3-x^2-3x}=\dfrac{x^3}{x\left(2x^2-x-3\right)}=\dfrac{x^2}{2x^2-x-3}$

***VA***: $x=-1$ *and* $x=\frac{3}{2}$ ***HA***: $y=\frac{1}{2}$

***Hole***: $x=0 \quad \rightarrow \quad y=0 \Rightarrow$ *hole* $(0,\ 0)$

***Oblique asymptote***: *n / a*

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{3x^2+5}{4x^2-3}$

***Solution***

$4x^2-3=0 \quad \rightarrow \quad x=\pm\dfrac{\sqrt{3}}{2}$ ***Domain***: $\left(-\infty,\ -\frac{\sqrt{3}}{2}\right)\cup\left(-\frac{\sqrt{3}}{2},\ \frac{\sqrt{3}}{2}\right)\cup\left(\frac{\sqrt{3}}{2},\ \infty\right)$

***VA***: $x=-\frac{\sqrt{3}}{2}$ *and* $x=\frac{\sqrt{3}}{2}$ ***HA***: $y=\frac{3}{4}$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x+6}{x^3+2x^2}$

***Solution***

$x^3+2x^2=x^2(x+2)=0 \quad \rightarrow \quad x=0,-2$ ***Domain***: $(-\infty,\ -2)\cup(-2,\ 0)\cup(0,\ \infty)$

***VA***: $x=0$ *and* $x=2$ ***HA***: $y=0$

***Hole***: *n / a* ***Oblique asymptote***: *n / a*

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x^2+4x-1}{x+3}$

***Solution***

$x+3=0 \quad \rightarrow \quad x=-3$ ***Domain***: $(-\infty,\ -3)\cup(-3,\ \infty)$

$$x+3\overline{)\,x^2+4x-1}\qquad x+1$$
$$-x^2-3x$$
$$x-1$$
$$-x-3$$
$$-4$$

$$f(x)=\frac{x^2+4x-1}{x+3}=x+1-\frac{4}{x+3}$$

***VA***: $x=-3$ ***HA***: $n/a$

***Hole***: $n/a$ ***Oblique asymptote***: $y=x+1$

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x^2-6x}{x-5}$

***Solution***

$x-5=0 \quad \rightarrow \quad x=5$ ***Domain***: $(-\infty,\ 5)\cup(5,\ \infty)$

$$x-5\overline{)\,x^2-6x}\qquad x-1$$
$$-x^2+5x$$
$$-x$$
$$x-5$$
$$-5$$

$$f(x)=\frac{x^2-6x}{x-5}=x-1-\frac{5}{x-5}$$

***VA***: $x=5$

***HA***: N/A

***Hole***: N/A

***Oblique asymptote***: $y=x-1$

## *Exercise*

Determine all asymptotes of the function $f(x)=\dfrac{x^3-x^2+x-4}{x^2+2x-1}$

***Solution***

$x^2+2x-1=0 \quad \rightarrow \quad x=-1\pm\sqrt{2}$

***Domain***: $\left(-\infty,\ -1-\sqrt{2}\right)\cup\left(-1-\sqrt{2},\ -1+\sqrt{2}\right)\cup\left(-1+\sqrt{2},\ \infty\right)$

$$x^2+2x-1\overline{)\,x^3-x^2+x-4}\qquad x-3$$

$$-x^3-2x^2+x$$

$$-3x^2+2x-4$$

$$3x^2+6x-3$$

$$8x-7$$

$$f(x)=\frac{x^3-x^2+x-4}{x^2+2x-1}$$

$$=x-3+\frac{8x-7}{x^2+2x-1}$$

***VA***: $x=-1\pm\sqrt{2}$

***HA***: N/A

***Hole***: N/A

***Oblique asymptote***: $y=x-3$

## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

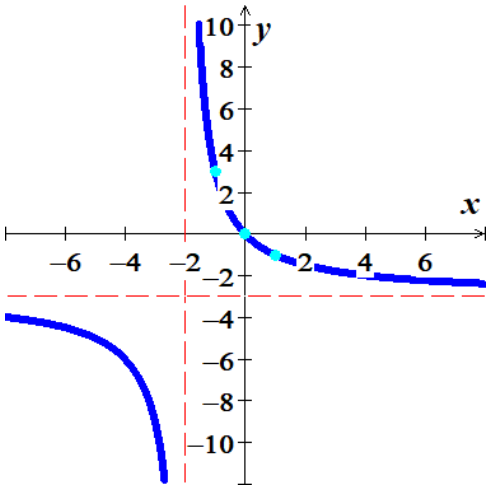
$$f(x)=\frac{-3x}{x+2}$$

***Solution***

***VA***: $x=-2$ ***HA***: $y=-3$

***Hole***: $n/a$ ***OA***: $n/a$

| *x* | *y* |
|---|---|
| 0 | 0 |
| 1 | −1 |
| −1 | 3 |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

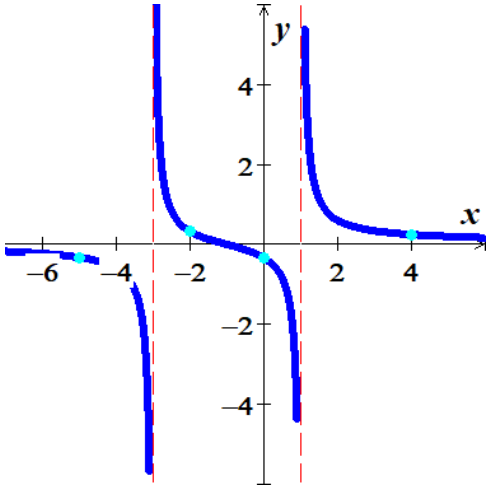
$$f(x)=\frac{x+1}{x^2+2x-3}$$

***Solution***

***VA***: $x=1,\ x=-3$ ***HA***: $y=0$

***Hole***: $n/a$ ***Oblique asymptote***: $n/a$

| $x$ | $y$ |
|---|---|
| −5 | −0.33 |
| −2 | 0.33 |
| 0 | −1/3 |
| 4 | 0.24 |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

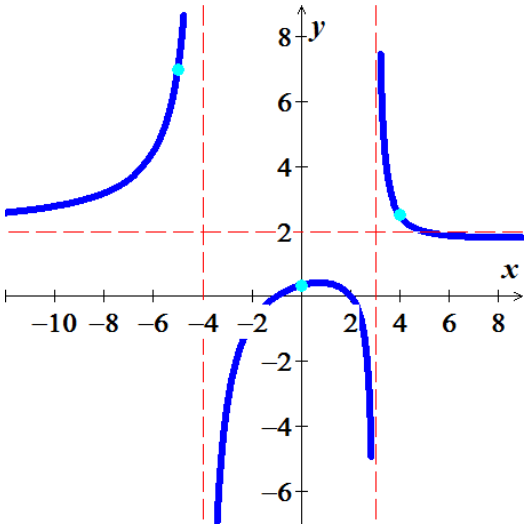
$$f(x)=\frac{2x^2-2x-4}{x^2+x-12}$$

***Solution***

***VA***: $x=-4,\ 3$ ***HA***: $y=2$

***Hole***: $n/a$ ***OA***: $n/a$

| $x$ | $y$ |
|---|---|
| −5 | 7 |
| −2 | −0.8 |
| 0 | 1/3 |
| 4 | 2.5 |
| 5 | 2 |



## *Exercise*

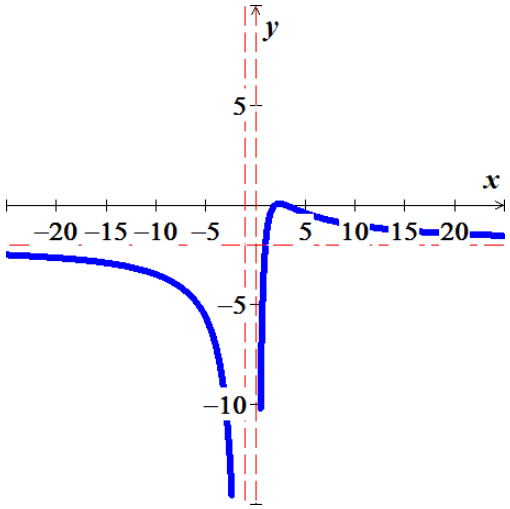
Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

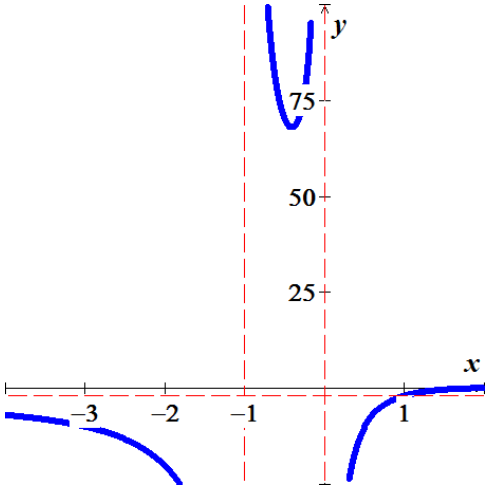
$$f(x) = \frac{-2x^2 + 10x - 12}{x^2 + x}$$

***Solution***

***VA***: $x = -1,\ 0$ ***HA***: $y = -2$

***Hole***: *n / a* ***OA***: *n / a*




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x) = \frac{x^2 - x - 6}{x + 1}$$

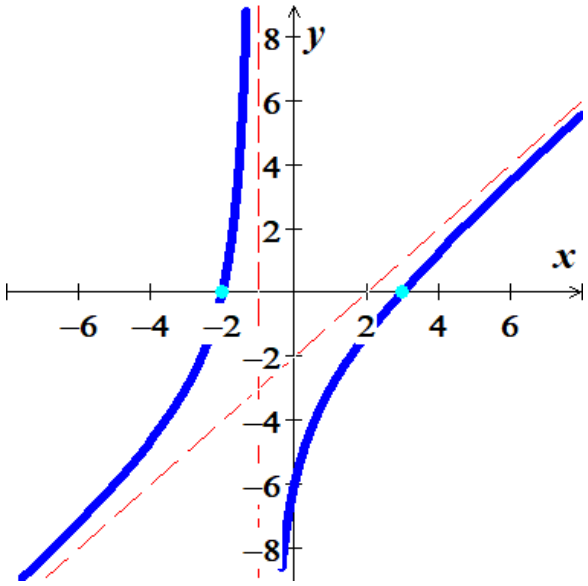
***Solution***

$$x+1 \overline{)\, x^2 - x - 6} \quad x - 2$$

$$x^2 + x$$
$$-2x - 6$$
$$-2x - 2$$
$$\boxed{-4}$$

***VA***: $x = -1$ ***HA***: *n / a*

***Hole***: *n / a* ***OA***: $y = x - 2$




| *x* | *y* |
|---|---|
| 2 | 0 |
| −2 | 0 |
| 0 | −6 |

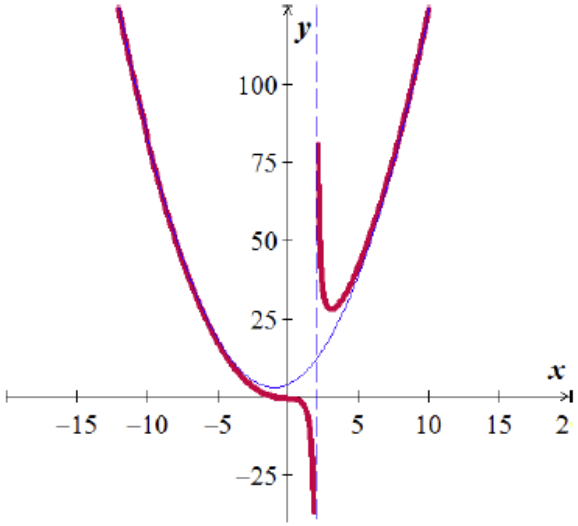
## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^3+1}{x-2}$$

***Solution***

$$\begin{array}{r} x^2+2x+4 \\ x-2\overline{)\,x^3-1} \\ \underline{x^3-2x^2} \\ 2x^2 \\ \underline{2x^2-4x} \\ 4x-1 \\ \underline{4x-8} \\ 7 \end{array}$$




***VA***: $x=2$ ***HA***: $n/a$

***Hole***: $n/a$ ***OA***: $y=x^2+2x+4$

## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{2x^2+x-6}{x^2+3x+2}$$

***Solution***

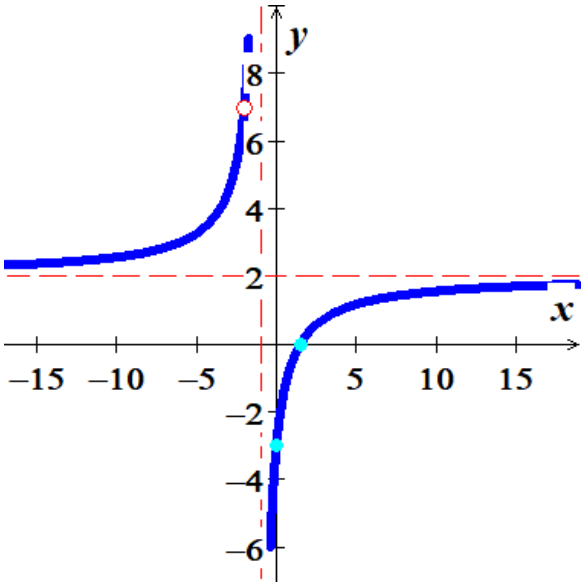
$$f(x)=\frac{(2x-3)(x+2)}{(x+1)(x+2)}$$

$$=\frac{2x-3}{x+1}$$

***VA***: $x=-1$ ***HA***: $y=2$

***Hole***: $(-2,\ 7)$ ***OA***: $n/a$

| $x$ | $y$ |
|---|---|
| 0 | −3 |
| $-\frac{3}{2}$ | 0 |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x-1}{1-x^2}$$

***Solution***

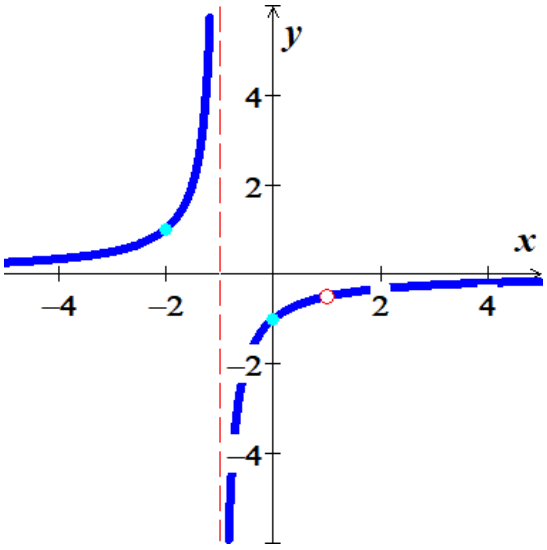
$$f(x)=\frac{x-1}{(x+1)(1-x)}$$

$$=-\frac{1}{x+1}$$

***VA***: $x=-1$ ***HA***: $y=0$

***Hole***: $\left(1,\ -\frac{1}{2}\right)$ ***OA***: *n / a*

| *x* | *y* |
|---|---|
| 0 | −1 |
| −2 | 1 |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^2+x-2}{x+2}$$

***Solution***

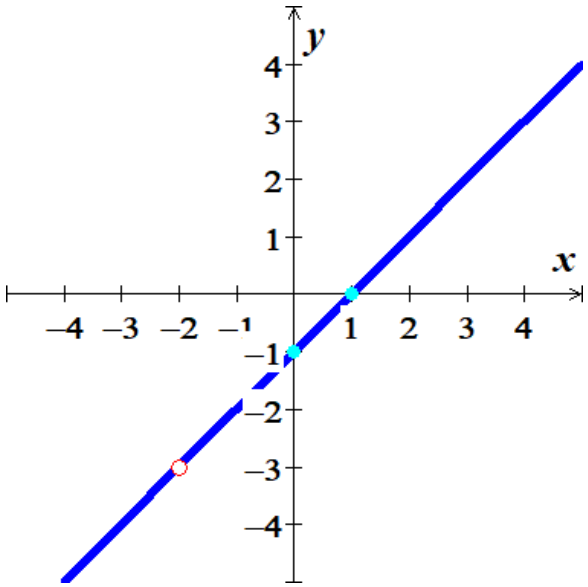
$$f(x)=\frac{(x+2)(x-1)}{x+2}$$

$$=x-1$$

***VA***: *n / a* ***HA***: *n / a*

***Hole***: $(-2,\ -3)$ ***OA***: *n / a*

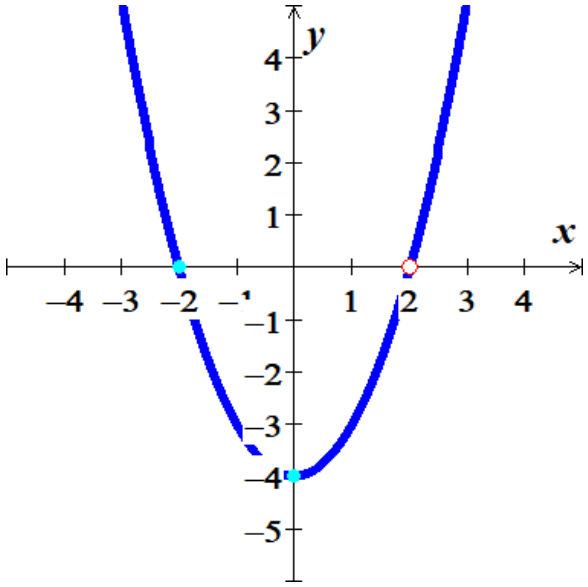
| *x* | *y* |
|---|---|
| 0 | −1 |
| 1 | 0 |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^3-2x^2-4x+8}{x-2}$$




***Solution***

$$f(x)=\frac{\left(x^2-4\right)(x-2)}{x-2}$$

$$=x^2-4$$

***VA***: $n/a$ ***HA***: $n/a$

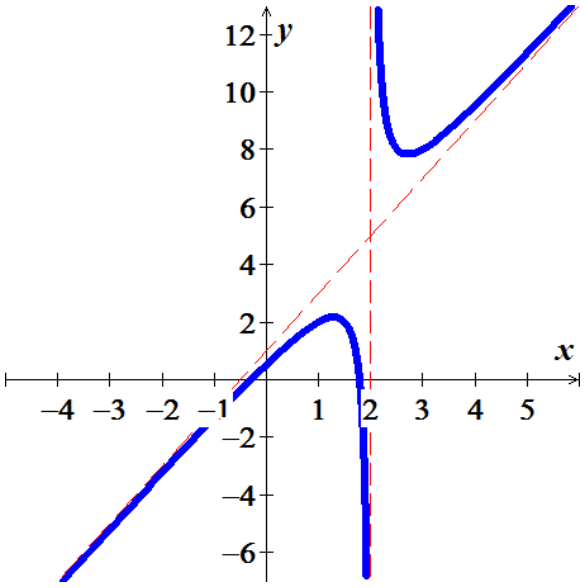
***Hole***: $(2,\ 0)$ ***OA***: $n/a$

| $x$ | $y$ |
|---|---|
| 0 | −4 |
| −2 | 0 |

## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{2x^2-3x-1}{x-2}$$




***Solution***

$$x-2\overline{)\,2x^2-3x-1}\ \ \overline{2x+1}$$

$$-2x^2+4x$$

$$x-1$$

$$-x+2$$

$$1$$

$$f(x)=\frac{2x^2-3x-1}{x-2}$$

$$=(2x+1)+\frac{1}{x-2}$$

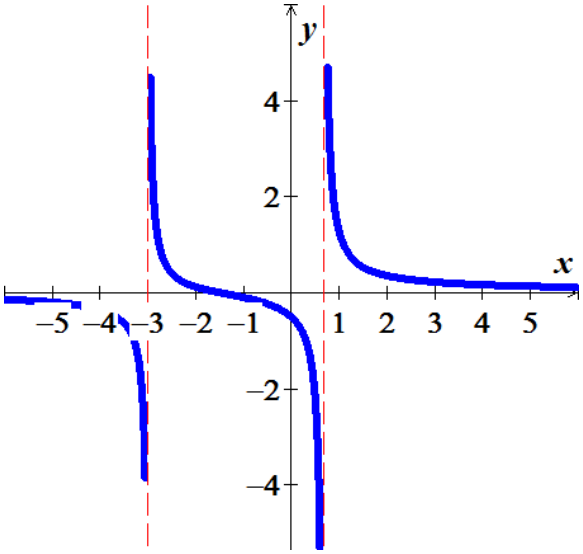
***VA***: $x=2$ ***HA***: $y=1$

***Hole***: $n/a$ ***OA***: $y=2x+1$

## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{2x+3}{3x^2+7x-6}$$




***Solution***

$$3x^2+7x-6=0 \quad \Rightarrow \quad x=-3,\ \frac{2}{3}$$

***VA***: $x=-3$ *and* $x=\frac{2}{3}$

***HA***: $y=0$

***Hole***: $n/a$

***OA:*** $n/a$

## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^2-1}{x^2+x-6}$$

***Solution***

$$x^2+x-6=0 \quad \Rightarrow \quad \underline{x=-3,\ 2}$$

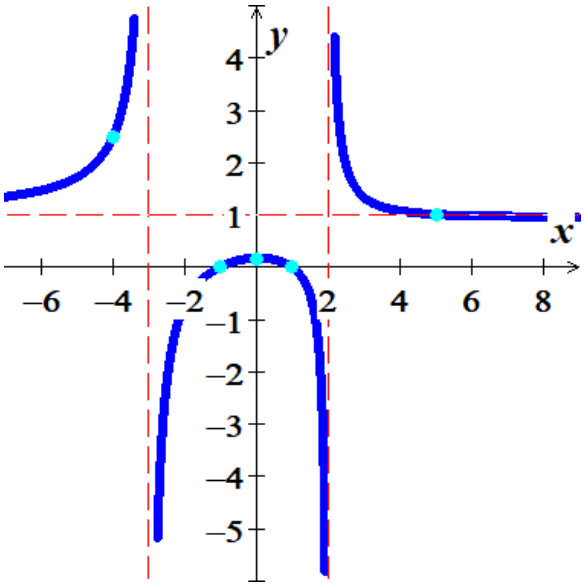
***VA***: $x=-3$ *and* $x=2$ ***HA***: $y=1$

***Hole***: $n/a$ ***OA:*** $n/a$

$$1=\frac{x^2-1}{x^2+x-6}$$

$$x^2+x-6=x^2-1$$

$$\underline{x=5}$$




| *x* | *y* |
|---|---|
| 0 | $\frac{1}{6}$ |
| 5 | 1 |
| $\pm 1$ | 0 |
| $-4$ | $\frac{5}{2}$ |

## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{-2x^2-x+15}{x^2-x-12}$$

***Solution***

$$x^2-x-12=0 \quad \Rightarrow \quad \underline{x=-3,\ 4}$$

***Domain***: $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

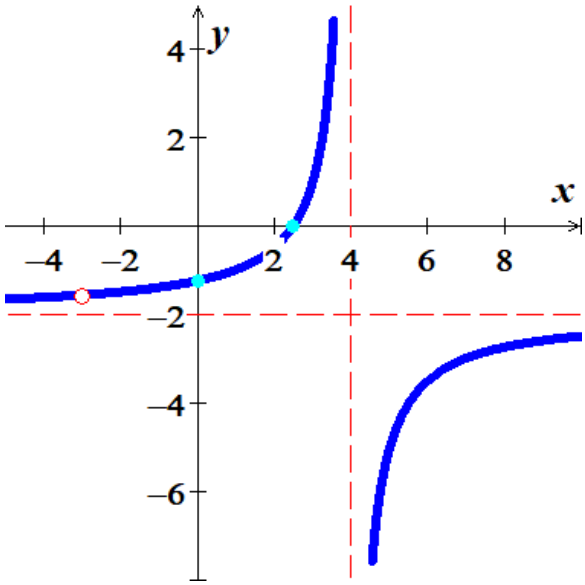
$$f(x)=\frac{(-2x+5)(x+3)}{(x-4)(x+3)}$$

$$=\frac{-2x+5}{x-4}$$

***VA***: $x=4$ ***HA***: $y=-2$

***Hole***: $\left(-3,\ -\frac{11}{7}\right)$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | $-\frac{5}{4}$ |
| $\frac{5}{2}$ | 0 |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

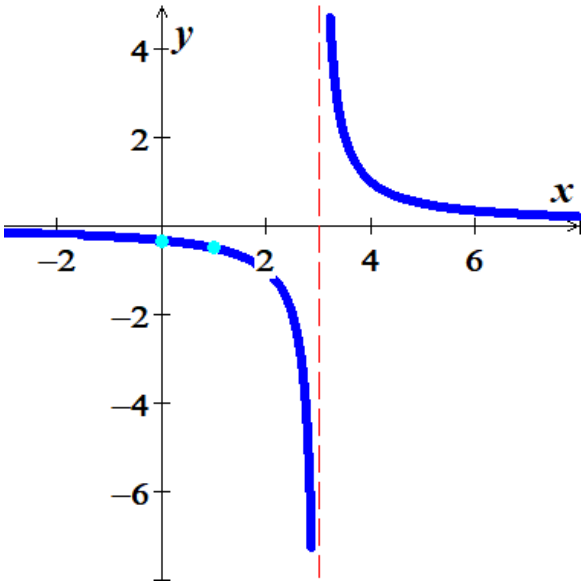
$$f(x)=\frac{1}{x-3}$$

***Solution***

***VA***: $x=3$ ***HA***: $y=0$

***Hole***: $n/a$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | $-\frac{1}{3}$ |
| 1 | $-\frac{1}{2}$ |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

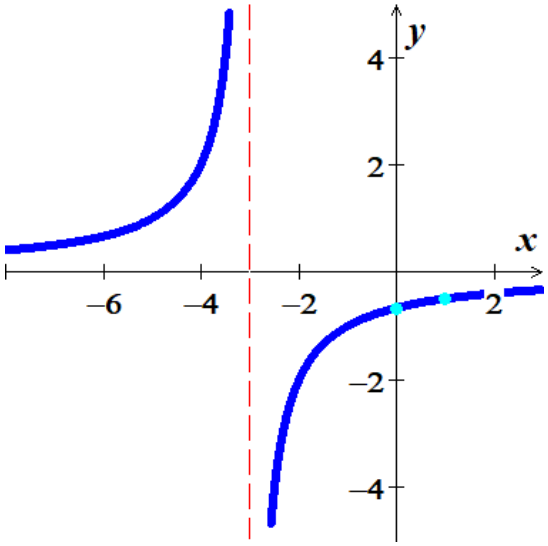
$$f(x)=\frac{-2}{x+3}$$

***Solution***

***VA***: $x=-3$ ***HA***: $y=0$

***Hole***: *n / a* ***OA:*** *n / a*

| *x* | *y* |
|---|---|
| 0 | $-\frac{2}{3}$ |
| 1 | $-\frac{1}{2}$ |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

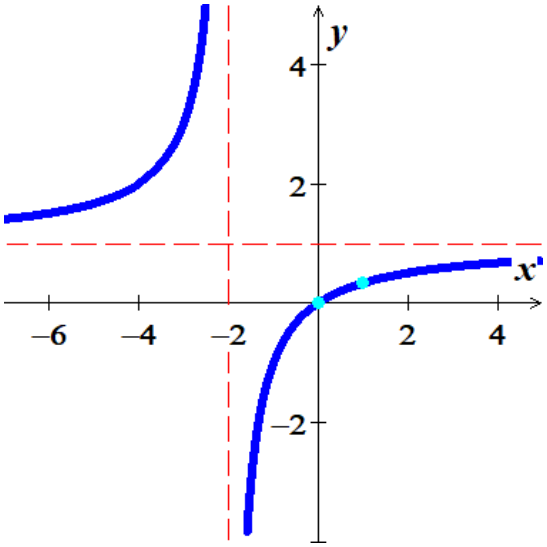
$$f(x)=\frac{x}{x+2}$$

***Solution***

***VA***: $x=-2$ ***HA***: $y=1$

***Hole***: *n / a* ***OA:*** *n / a*

| *x* | *y* |
|---|---|
| 0 | 0 |
| 1 | $\frac{1}{3}$ |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

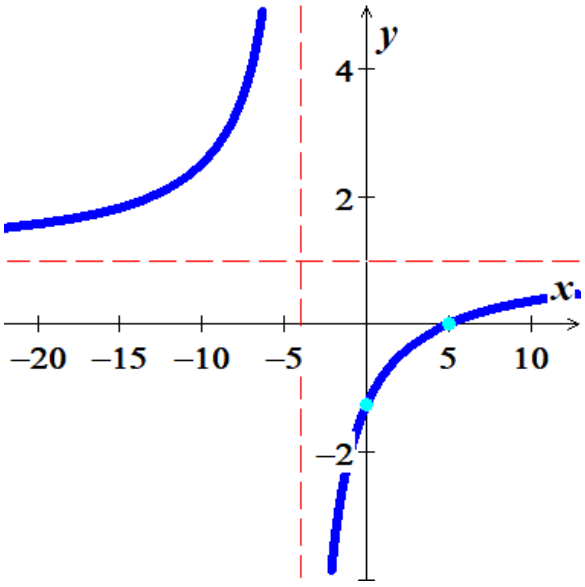
$$f(x)=\frac{x-5}{x+4}$$

***Solution***

***VA***: $x=-4$ ***HA***: $y=1$

***Hole***: *n / a* ***OA:*** *n / a*

| $x$ | $y$ |
|---|---|
| 0 | $-\frac{5}{4}$ |
| 5 | 0 |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{2x^2-2}{x^2-9}$$

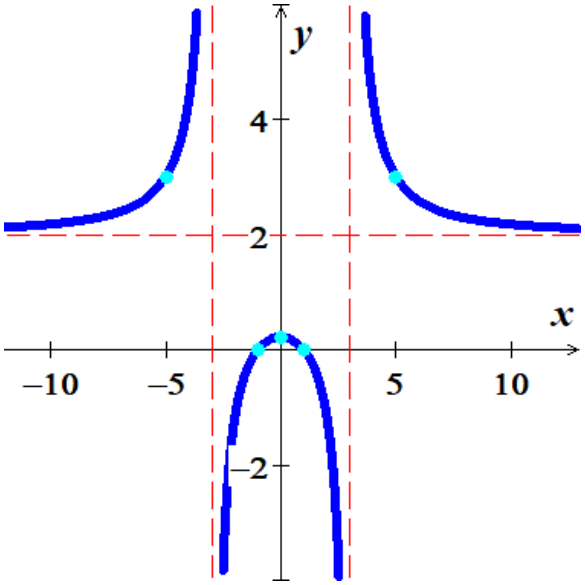
***Solution***

$x^2=9 \rightarrow \underline{x=\pm 3}$

***VA***: $x=\pm 3$ ***HA***: $y=2$

***Hole***: *n / a* ***OA:*** *n / a*

| $x$ | $y$ |
|---|---|
| 0 | $\frac{2}{9}$ |
| ±1 | 0 |
| ±5 | 3 |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

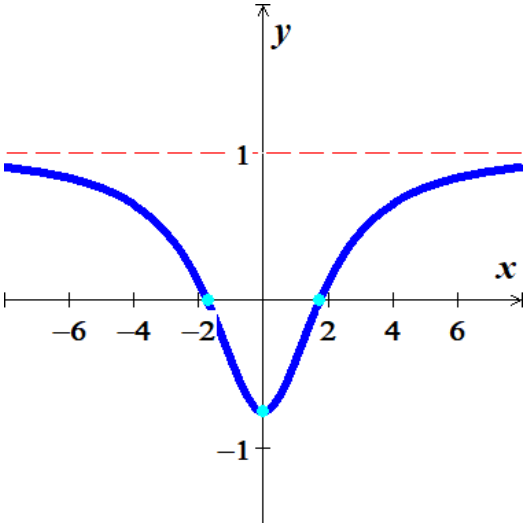
$$f(x)=\frac{x^2-3}{x^2+4}$$

***Solution***

***VA***: $n/a$ ***HA***: $y=1$

***Hole***: $n/a$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| $0$ | $-\frac{3}{4}$ |
| $\pm\sqrt{3}$ | $0$ |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2+4}{x^2-3}$$

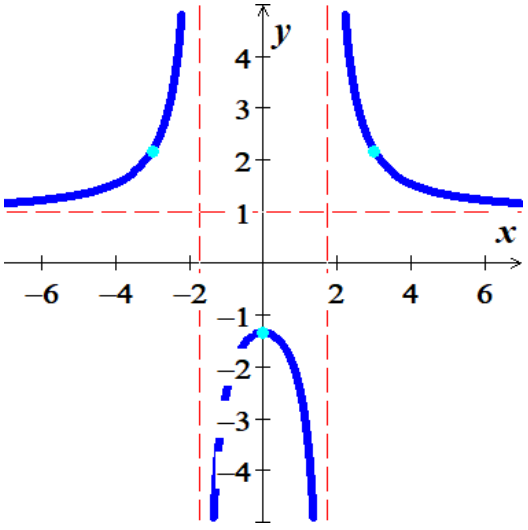
***Solution***

$x^2-3=0 \rightarrow x=\pm\sqrt{3}$

***VA***: $x=\pm\sqrt{3}$ ***HA***: $y=1$

***Hole***: $n/a$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| $0$ | $-\frac{4}{3}$ |
| $\pm 3$ | $\frac{13}{6}$ |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2}{x^2-6x+9}$$

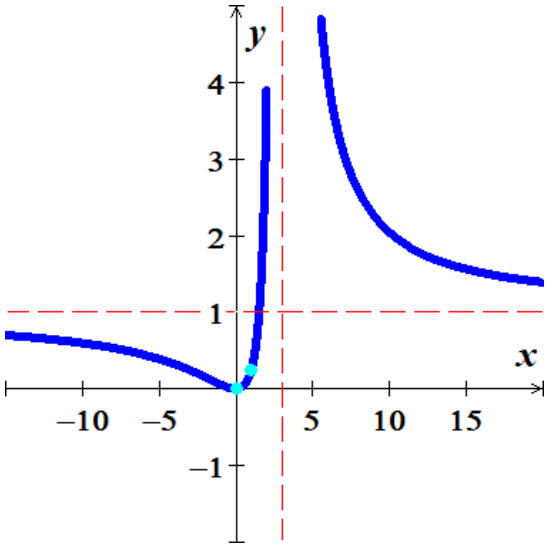
***Solution***

$x^2-6x+9=0 \quad \rightarrow \quad x=3$

***VA***: $x=3$ ***HA***: $y=1$

***Hole***: $n/a$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | $\frac{1}{4}$ |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2+x+4}{x^2+2x-1}$$

***Solution***

$x^2+2x-1=0$

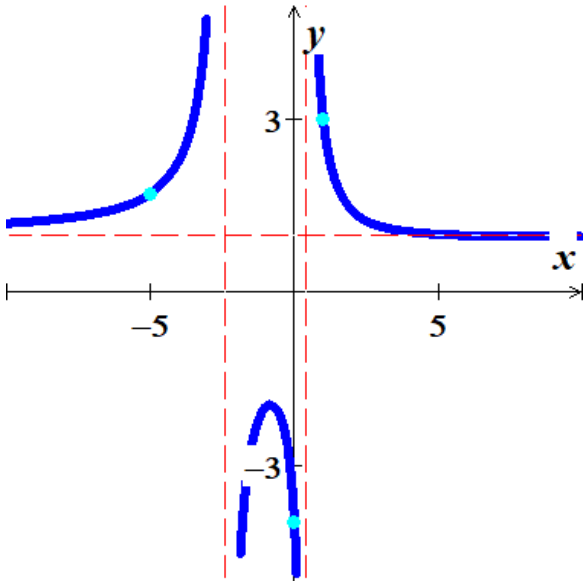
$x=\frac{-2\pm\sqrt{8}}{2}$

$=-1\pm\sqrt{2}$

***VA***: $x=-1\pm\sqrt{2}$ ***HA***: $y=1$

***Hole***: $n/a$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | −4 |
| 1 | 3 |
| −5 | $\frac{12}{7}$ |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

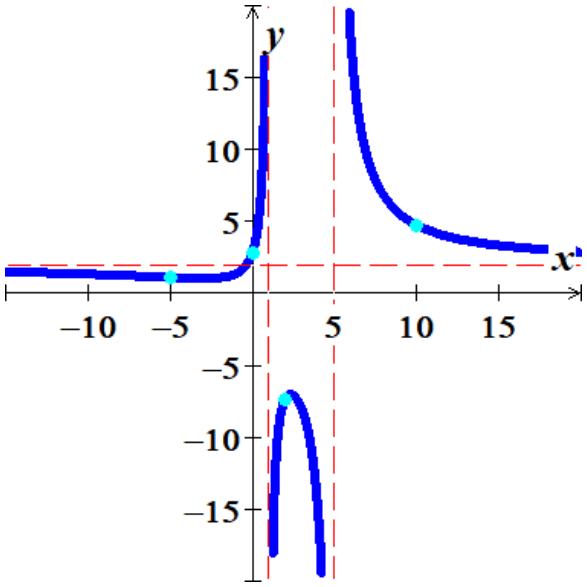
$$f(x)=\frac{2x^2+14}{x^2-6x+5}$$

***Solution***

***VA***: $x=1,\ 5$ ***HA***: $y=2$

***Hole***: $n/a$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | $\frac{14}{5}$ |
| 2 | $-\frac{22}{3}$ |
| −5 | $\frac{16}{15}$ |
| 10 | $\frac{214}{45}$ |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2-4x-5}{2x+5}$$

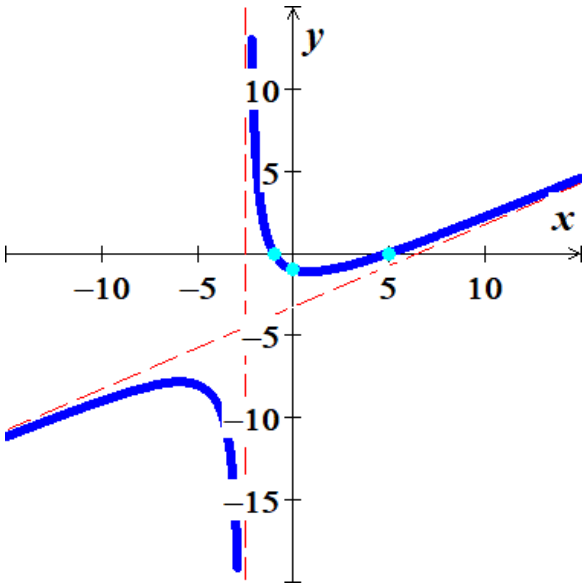
***Solution***

$$\begin{array}{r} \frac{1}{2}x-\frac{13}{4} \\ 2x+5\overline{)\,x^2-4x-5} \\ \underline{x^2+\frac{5}{2}x} \\ -\frac{13}{2}x-5 \end{array}$$

***VA***: $x=-\frac{5}{2}$ ***HA***: $n/a$

***Hole***: $n/a$ ***OA:*** $y=\frac{1}{2}x-\frac{13}{2}$

| $x$ | $y$ |
|---|---|
| 0 | −1 |
| −1, 5 | 0 |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x-3}{x^2-3x+2}$$

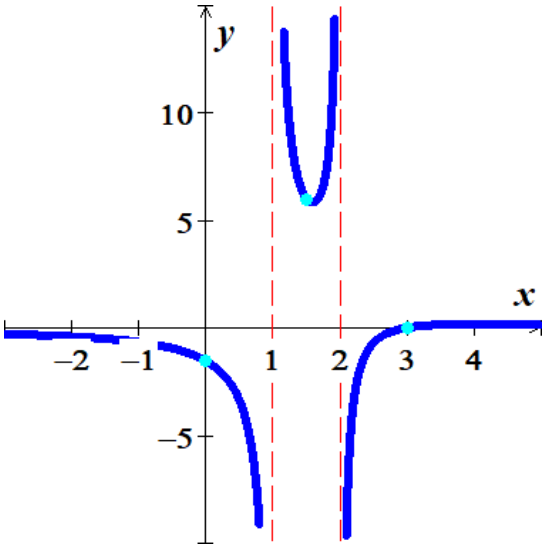
***Solution***

$x^2-3x+2 \rightarrow \underline{x=1,\ 2}$

***VA***: $x=1,\ 2$ ***HA***: $y=0$

***Hole***: $n/a$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | $-\frac{3}{2}$ |
| 3 | 0 |
| $\frac{3}{2}$ | 6 |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2+2}{x^2+3x+2}$$

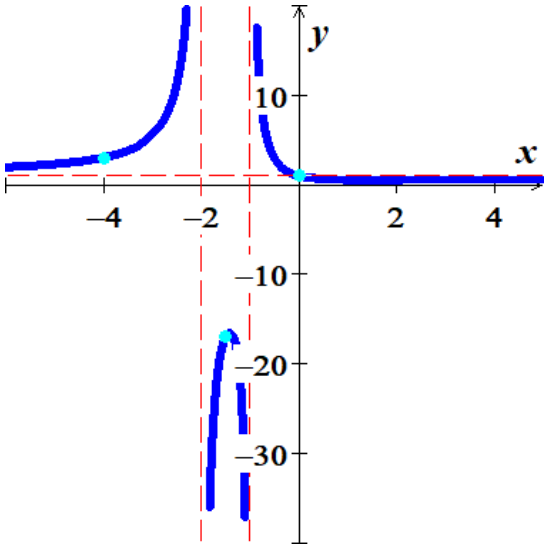
***Solution***

$x^2+3x+2 \rightarrow \underline{x=-1,\ -2}$

***VA***: $x=-1,\ -2$ ***HA***: $y=1$

***Hole***: $n/a$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | 1 |
| $-\frac{3}{2}$ | −17 |
| −4 | 3 |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x-2}{x^2-3x+2}$$

***Solution***

$$x^2-3x+2 \;\rightarrow\; \underline{x=1,\ 2}$$

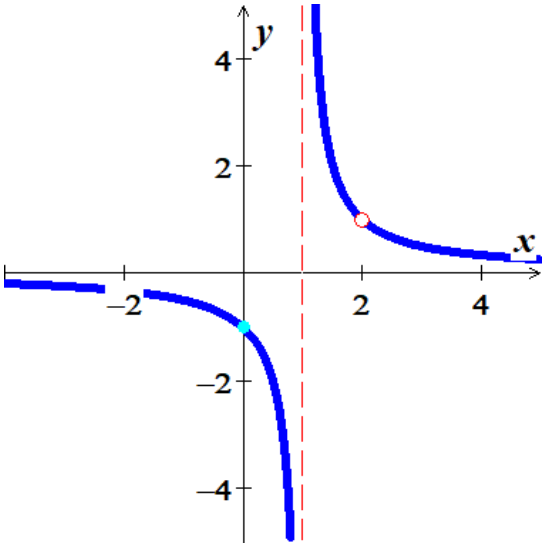
$$f(x)=\frac{x-2}{(x-2)(x-1)}$$

$$=\frac{1}{x-1}$$

***VA***: $x=1$ ***HA***: $y=0$

***Hole***: $(2,\ 1)$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | $-1$ |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2+x}{x+1}$$

***Solution***

$$f(x)=\frac{x(x+1)}{x+1}$$

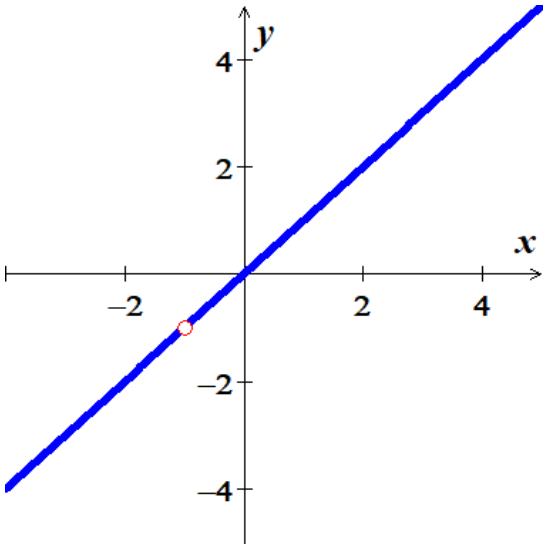
$$=x$$

***VA***: $n/a$ ***HA***: $n/a$

***Hole***: $(-1,\ -1)$ ***OA:*** $n/a$

***Hole***: $\left(-3,\ -\frac{11}{7}\right)$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | 0 |



## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2-2x}{x-2}$$

***Solution***

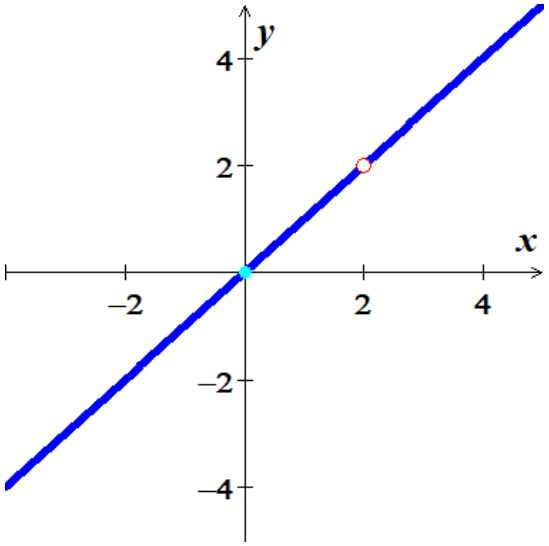
$$f(x)=\frac{x(x-2)}{x-2}$$
$$=x$$

***VA***: $n/a$ ***HA***: $n/a$

***Hole***: $(2,\ 2)$ ***OA:*** $n/a$

***Hole***: $\left(-3,\ -\frac{11}{7}\right)$ ***OA:*** $n/a$

| $x$ | $y$ |
|---|---|
| 0 | 0 |




## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2-3x}{x+3}$$

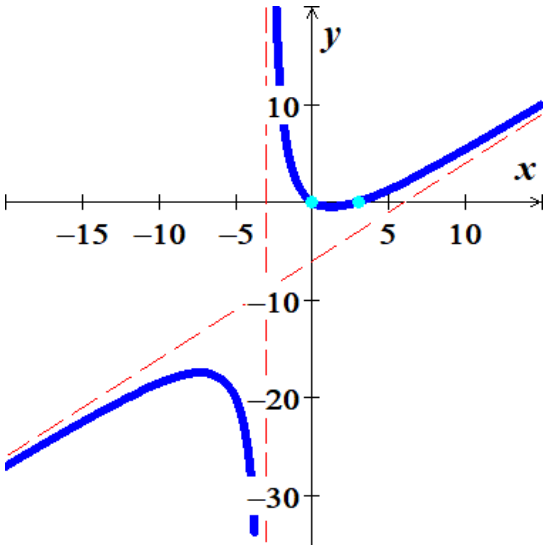
***Solution***

$$\begin{array}{r} x-6 \\ x+3\ \overline{)\,x^2-3x} \\ \underline{x^2+3x} \\ -6x-5 \end{array}$$

***VA***: $x=-3$ ***HA***: $n/a$

***Hole***: $n/a$ ***OA:*** $y=x-6$

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 3 | 0 |



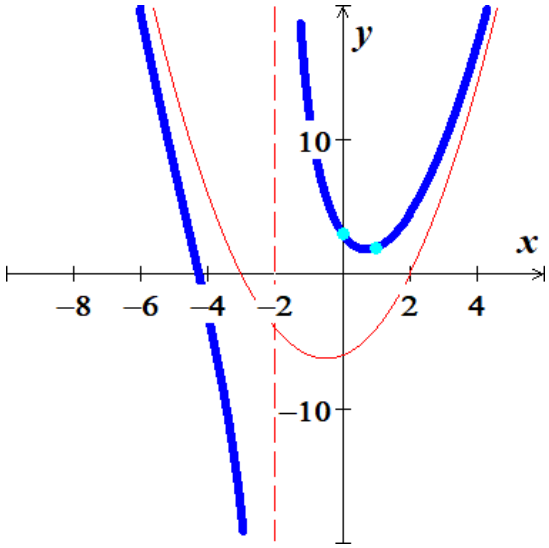
## *Exercise*

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^3+3x^2-4x+6}{x+2}$$

***Solution***

$$\begin{array}{r|l} x+2 & x^2+x-6 \\ \hline & x^3+3x^2-4x+6 \\ & \underline{x^3+2x^2} \\ & x^2-4x \\ & \underline{x^2+2x} \\ & -6x+6 \end{array}$$




***VA***: $x=-2$ ***HA***: $n/a$

***Hole***: $n/a$ ***OA:*** $y=x^2+x-6$

| *x* | *y* |
|---|---|
| 0 | 3 |
| 1 | 2 |

## *Exercise*

Find an equation of a rational function $f$ that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=4 \\ \textit{horizontal asymptote}: \ y=-1 \\ x-\textit{intercept}: \ 3 \end{cases}$$

***Solution***

***Vertical Asymptote***: $f(x)=\dfrac{\quad}{x-4}$

***Horizontal Asymptote***: $f(x)=\dfrac{-x+a}{x-4}$

***x-intercept***: $f(x=3)=\dfrac{-3+a}{3-4}=0 \ \Rightarrow \ \underline{a=3}$

$$\underline{f(x)=\frac{-x+3}{x-4}}$$

## *Exercise*

Find an equation of a rational function $f$ that satisfies the given conditions

$$\begin{cases} vertical\ asymptote: \ x=-4, x=5 \\ horizontal\ asymptote: \ y=\frac{3}{2} \\ x-intercept: \ -2 \end{cases}$$

***Solution***

***Vertical Asymptote***: $f(x)=\frac{}{(x+4)(x-5)}$

***Horizontal Asymptote***: $f(x)=\frac{3}{2}\frac{(x+a)(x+b)}{(x+4)(x-5)}$

***x-intercept***: $f(x=-2)=\frac{3}{2}\frac{(-2+a)(-2+b)}{}$

$$0=(-2+a)(-2+b)$$

$$\boxed{a=b=2}$$

$$f(x)=\frac{3}{2}\frac{(x-2)^2}{x^2-x-20}$$

$$=\underline{\left|\frac{3x^2-12x+12}{2x^2-2x-40}\right|}$$

## *Exercise*

Find an equation of a rational function $f$ that satisfies the given conditions

$$\begin{cases} vertical\ asymptote: \ x=5 \\ horizontal\ asymptote: \ y=-1 \\ x-intercept: \ 2 \end{cases}$$

***Solution***

***Vertical Asymptote***: $f(x)=\frac{}{x-5}$

***x-intercept*** : $f(x)=\frac{x-2}{x-5}$

***Horizontal Asymptote***: $f(x)=-\frac{x-2}{x-5}$

$$\underline{\left|f(x)=-\frac{x-2}{x-5}\right|}$$

## *Exercise*

Find an equation of a rational function $f$ that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=-2, \ x=0 \\ \textit{horizontal asymptote}: \ y=0 \\ x-\textit{intercept}: \ 2, \quad f(3)=1 \end{cases}$$

***Solution***

***Vertical Asymptote***: $f(x)=\dfrac{}{x(x+2)}$

***x-intercept*** : $f(x)=\dfrac{x-2}{x(x+2)}$

***Horizontal Asymptote***: $f(x)=\dfrac{a(x-2)}{x(x+2)}$

$$f(3)=1 \quad \rightarrow \frac{a(1)}{(3)(5)}=1 \quad \Rightarrow \ \underline{a=15}$$

$$\boxed{f(x)=\frac{15x-30}{x^2+2x}}$$

## *Exercise*

Find an equation of a rational function $f$ that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=-3, \ x=1 \\ \textit{horizontal asymptote}: \ y=0 \\ x-\textit{intercept}: \ -1, \quad f(0)=-2 \\ \textit{hole}: \ x=2 \end{cases}$$

***Solution***

***Vertical Asymptote***: $f(x)=\dfrac{}{(x+3)(x-1)}$

***x-intercept*** : $f(x)=\dfrac{(x+1)}{(x+3)(x-1)}$

***Horizontal Asymptote***: $f(x)=\dfrac{a(x+1)}{(x+3)(x-1)}$

$$f(0)=-2 \quad \rightarrow \frac{a}{-3}=-2 \quad \Rightarrow \ \underline{a=6}$$

***Hole at*** $x=2$**:** $f(x)=\dfrac{6(x+1)(x-2)}{\left(x^2+2x-3\right)(x-2)}$

$$\boxed{f(x)=\frac{6x^2-6x-12}{x^3-7x+6}}$$

## *Exercise*

Find an equation of a rational function $f$ that satisfies the given conditions

$$\begin{cases} vertical\ asymptote: \ x=-1, \ \ x=3 \\ horizontal\ asymptote: \ y=2 \\ x-intercept: \ \ -2, \ \ 1 \\ \ \ hole: \ \ x=0 \end{cases}$$

***Solution***

***Vertical Asymptote***: $f(x)=\dfrac{}{(x+1)(x-3)}$

***Horizontal Asymptote***: $f(x)=\dfrac{2}{(x+1)(x-3)}$

***x-intercept*** : $f(x)=\dfrac{2(x+2)(x-1)}{(x+1)(x-3)}$

***Hole at*** $x=0$**:** $f(x)=\dfrac{2x(x+2)(x-1)}{x(x+1)(x-3)}$

$$\boxed{f(x)=\frac{2x^3+2x^2-4x}{x^3-2x^2-3x}}$$