Solution

Section 2.7 - Inverse Trigonometry Functions

Exercise

Evaluate without using a calculator: $\cos\left(\cos^{-1}\frac{3}{5}\right)$

Solution

$$\cos\left(\cos^{-1}\frac{3}{5}\right) = \frac{3}{5}$$

Exercise

Evaluate without using a calculator: $\cos^{-1} \left(\cos \frac{7\pi}{6}\right)$

Solution

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

Exercise

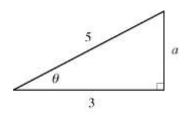
Evaluate without using a calculator: $\tan\left(\cos^{-1}\frac{3}{5}\right)$

$$\tan\left(\cos^{-1}\frac{3}{5}\right)$$

$$5^{2} = 3^{2} + a^{2} \rightarrow a = 4$$

$$\tan\left(\cos^{-1}\frac{3}{5}\right) = \tan\theta$$

$$= \frac{4}{3}$$



Exercise

Evaluate without using a calculator: $\sin \left(\cos^{-1} \frac{1}{\sqrt{5}}\right)$

Solution

$$\sin\left(\cos^{-1}\frac{1}{\sqrt{5}}\right)$$

$$(\sqrt{5})^2 = 1^2 + a^2$$

$$\Rightarrow a^2 = 5 - 1$$

$$\Rightarrow a = 2$$

$$\sin\left(\cos^{-1}\frac{1}{\sqrt{5}}\right) = \sin\theta$$

$$= \frac{2}{\sqrt{5}}$$

Exercise

Evaluate without using a calculator: $\cos\left(\sin^{-1}\frac{1}{2}\right)$

Solution

$$\cos\left(\sin^{-1}\frac{1}{2}\right)$$

$$\sin\frac{\pi}{6} = \frac{1}{2} \Rightarrow \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\cos\left(\sin^{-1}\frac{1}{2}\right) = \cos\frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2}$$

Exercise

Evaluate without using a calculator: $\sin\left(\sin^{-1}\frac{3}{5}\right)$

$$\sin\left(\sin^{-1}\frac{3}{5}\right) = \frac{3}{5}$$

Exercise

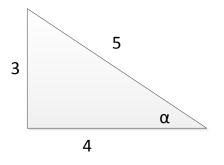
Evaluate without using a calculator: $\cos\left(\tan^{-1}\frac{3}{4}\right)$

Solution

$$\alpha = \tan^{-1} \frac{3}{4} \Rightarrow \tan \alpha = \frac{3}{4}$$

$$r = \sqrt{3^2 + 4^2} = 5$$

$$\Rightarrow \boxed{\cos \alpha = \frac{4}{5}}$$



Exercise

Evaluate without using a calculator: $\tan\left(\sin^{-1}\frac{3}{5}\right)$

Solution

$$\sin \alpha = \frac{3}{5}$$

$$\tan\left(\sin^{-1}\frac{3}{5}\right) = \frac{3}{4}$$

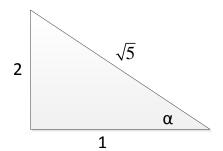
Exercise

Evaluate without using a calculator: $\sec\left(\cos^{-1}\frac{1}{\sqrt{5}}\right)$

Solution

$$\alpha = \cos^{-1} \frac{1}{\sqrt{5}} \rightarrow \cos \alpha = \frac{1}{\sqrt{5}}$$

$$|\sec \alpha| = \frac{1}{\cos \alpha} = \frac{1}{\frac{1}{\sqrt{5}}} = \frac{\sqrt{5}}{|\cos \alpha|}$$



Exercise

Evaluate without using a calculator: $\cot\left(\tan^{-1}\frac{1}{2}\right)$

$$\alpha = \tan^{-1} \frac{1}{2} \Rightarrow \tan \alpha = \frac{1}{2}$$

$$|\cot \alpha| = \frac{1}{\tan \alpha} = 2|$$

Exercise

Write an equivalent expression that involves x only for $\cos(\cos^{-1}x)$

Solution

$$\alpha = \cos^{-1} x \Rightarrow \cos \alpha = x$$

$$\cos(\cos^{-1}x) = \cos\alpha = \underline{x}$$

Exercise

Write an equivalent expression that involves x only for $\tan(\cos^{-1}x)$

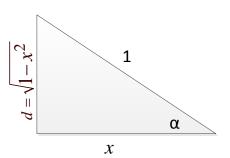
Solution

$$\alpha = \cos^{-1} x \Rightarrow \cos \alpha = x = \frac{x}{1}$$

$$x^2 + d^2 = 1 \Rightarrow d^2 = 1 - x^2$$

$$d = \sqrt{1 - x^2}$$

$$\tan(\cos^{-1}x) = \tan\alpha = \frac{\sqrt{1-x^2}}{x}$$



Exercise

Write an equivalent expression that involves x only for $\csc\left(\sin^{-1}\frac{1}{x}\right)$

$$\alpha = \sin^{-1} \frac{1}{x} \Rightarrow \sin \alpha = \frac{1}{x}$$

$$\left|\csc\left(\sin^{-1}x\right) = \csc\alpha = \frac{1}{\sin\alpha} = \underline{x}\right|$$