Solution   *Section R.1 – Derivative*

Exercise

Find the derivative of $f(t) = -3t^2 + 2t - 4$

Solution

    $\underline{f'(t) = -6t + 2}$

Exercise

Find the derivative of $g(x) = 4\sqrt[3]{x} + 2$

Solution

    $g(x) = 4x^{1/3} + 2$

    $g'(x) = \dfrac{4}{3} x^{-2/3}$

        $= \dfrac{4}{3x^{2/3}}$

        $\underline{= \dfrac{4}{3\sqrt[3]{x^2}}}$

Exercise

Find the derivative of $f(x) = x\left(x^2 + 1\right)$

Solution

    $f(x) = x^3 + x$

    $\underline{f'(x) = 3x^2 + 1}$

Exercise

Find the derivative of $f(x) = \dfrac{2x^2 - 3x + 1}{x}$

Solution

    $f(x) = \dfrac{2x^2}{x} - \dfrac{3x}{x} + \dfrac{1}{x}$

        $= 2x - 3 + \dfrac{1}{x}$

    $\underline{f'(x) = 2 - \dfrac{1}{x^2}}$ $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$

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Exercise

Find the derivative of $\quad f(x) = \dfrac{4x^3 - 3x^2 + 2x + 5}{x^2}$

$$f(x) = 4x - 3 + \frac{2}{x} + 5x^{-2} \qquad\qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$f'(x) = 4 - \frac{2}{x^2} - 10x^{-3}$$

$$= 4 - \frac{2}{x^2} - \frac{10}{x^3}$$

Exercise

Find the derivative of $\quad f(x) = \dfrac{-6x^3 + 3x^2 - 2x + 1}{x}$

$$f(x) = -6x^2 + 3x - 2 + \frac{1}{x} \qquad\qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$f'(x) = -12x + 3 - \frac{1}{x^2}$$

Exercise

Find the derivative of $f(x) = x\left(1 - \dfrac{2}{x+1}\right)$

$$f(x) = x - \frac{2x}{x+1}$$

$$\left(\frac{2x}{x+1}\right)' \Rightarrow \quad \begin{array}{ll} f = 2x & f' = 2 \\ g = x+1 & g' = 1 \end{array}$$

$$f'(x) = 1 - \frac{2(x+1) - 2x}{(x+1)^2}$$

$$= 1 - \frac{2x + 2 - 2x}{(x+1)^2}$$

$$= 1 - \frac{2}{(x+1)^2}$$

Exercise

Find the derivative of $g(s) = \dfrac{s^2 - 2s + 5}{\sqrt{s}}$

Solution

$$g(s) = \frac{s^2}{s^{1/2}} - 2\frac{s}{s^{1/2}} + \frac{5}{s^{1/2}}$$

$$= s^{3/2} - 2s^{1/2} + 5s^{-1/2}$$

$$g'(s) = \frac{3}{2}s^{1/2} - 2\frac{1}{2}s^{-1/2} + 5\left(-\frac{1}{2}\right)s^{-3/2}$$

$$= \frac{3}{2}s^{1/2} - s^{-1/2} - \frac{5}{2}s^{-3/2}$$

$$= \frac{3}{2}\sqrt{s} - \frac{1}{\sqrt{s}} - \frac{5}{2s^{3/2}}$$

$$= \frac{3}{2}\sqrt{s} - \frac{1}{\sqrt{s}} - \frac{5}{2s\sqrt{s}}$$

Exercise

Find the derivative of $f(x) = \dfrac{x+1}{\sqrt{x}}$

Solution

$$f(x) = \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}}$$

$$= x^{1/2} + x^{-1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$$

$$= \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$$

Exercise

Find the derivative to the following functions $y = 3x\left(2x^2 + 5x\right)$

Solution

$$y = 6x^3 + 15x^2$$

$$y' = 18x^2 + 30x$$

Exercise

Find the derivative to the following functions $y = 3\left(2x^2 + 5x\right)$

Solution

$$y = 6x^2 + 15x$$
$$\underline{y' = 12x + 15}$$

Exercise

Find the derivative to the following functions $y = \dfrac{x^2 + 4x}{5}$

Solution

$$y = \frac{1}{5}\left(x^2 + 4x\right)$$
$$\underline{y' = \frac{1}{5}(2x + 4)}$$

Exercise

Find the derivative to the following functions $y = \dfrac{3x^4}{5}$

Solution

$$y = \frac{3}{5}x^4$$
$$\underline{y' = \frac{12}{5}x^3}$$

Exercise

Find the derivative to the following functions $y = \dfrac{x^2 - 4}{2x + 5}$

Solution

$$\underline{y' = \frac{2x^2 + 10x + 8}{(2x + 5)^2}}$$

$$\begin{array}{ccc} 1 & 0 & -4 \\ 0 & 2 & 5 \end{array}$$

$$\left(\frac{ax^2 + bx + c}{dx^2 + ex + f}\right)' = \frac{(ae - bd)x^2 + 2(af - cd)x + bf - ce}{\left(dx^2 + ex + f\right)^2}$$

Exercise

Find the derivative to the following functions $y = \dfrac{(1+x)(2x-1)}{x-1}$

Solution

$$y = \frac{(1+x)(2x-1)}{x-1}$$

$$= \frac{2x^2 + x - 1}{x-1}$$

$$\boxed{y' = \frac{2x^2 - 4x}{(x-1)^2}}$$

$$\begin{array}{ccc} 2 & 1 & -1 \\ 0 & 1 & -1 \end{array} \qquad \left(\frac{ax^2 + bx + c}{dx^2 + ex + f}\right)' = \frac{(ae - bd)x^2 + 2(af - cd)x + bf - ce}{\left(dx^2 + ex + f\right)^2}$$

Exercise

Find the derivative to the following functions $y = \dfrac{4}{2x+1}$

Solution

$$\boxed{y' = -\frac{8}{(2x+1)^2}} \qquad \left(\frac{ax+b}{cx+d}\right)' = \frac{ad - bc}{(cx+d)^2}$$

Exercise

Find the derivative to the following functions $y = \dfrac{2}{(x-1)^3}$

Solution

$$\boxed{y' = -\frac{6}{(x-1)^4}} \qquad \left(\frac{1}{U^n}\right)' = -\frac{nU'}{U^{n+1}}$$

Exercise

Find the derivative to the following functions $y = \sqrt[3]{(x+4)^2}$

Solution

$$y = (x+4)^{2/3}$$

$$\boxed{y' = \frac{2}{3\sqrt[3]{x+4}}} \qquad \left(U^n\right)' = nU' \, U^{n-1}$$

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Exercise

Find the derivative of $f(x) = \sqrt{2t^2 + 5t + 2}$

Solution

$$f(t) = \left(2t^2 + 5t + 2\right)^{1/2}$$

$$U = 2t^2 + 5t + 2 \quad \rightarrow \quad U' = 4t + 5$$

$$f'(t) = \frac{1}{2}(4t + 5)\left(2t^2 + 5t + 2\right)^{-1/2}$$

$$\left(U^n\right)' = nU'\, U^{n-1}$$

$$= \frac{1}{2}\frac{4t + 5}{\sqrt{2t^2 + 5t + 2}}$$

Exercise

Find the derivative of $f(x) = \dfrac{1}{\left(x^2 - 3x\right)^2}$

Solution

$$f'(x) = -\frac{2(2x - 3)}{\left(x^2 - 3x\right)^3}$$

$$\left(U^n\right)' = nU'\, U^{n-1}$$

Exercise

Find the derivative of $y = t^2\sqrt{t - 2}$

Solution

$$y' = \frac{t\left(2t - 4 + \frac{1}{2}t\right)}{(t - 2)^{1/2}}$$

$$\left(U^n V^m\right)' = U^{n-1}V^{m-1}\left(nU'V + mUV'\right)$$

$$= \frac{5t^2 - 4t}{2\sqrt{t - 2}}$$

Exercise

Find the derivative of $y = \left(\dfrac{6 - 5x}{x^2 - 1}\right)^2$

Solution

$$y' = \frac{2\left(5x^2 - 12x + 5\right)(6 - 5x)}{\left(x^2 - 1\right)^3}$$

$$\frac{d}{dx}\left(\frac{ax^n + b}{cx^n + d}\right)^m = mn(ad - bc)x^{n-1}\frac{\left(ax^n + b\right)^{m-1}}{\left(cx^n + d\right)^{m+1}}$$

Exercise

Find the derivative to the following functions $y = x^2\sqrt{x^2+1}$

Solution

$$y' = \frac{x\left(2\left(x^2+1\right)+\frac{1}{2}(2x)x\right)}{\sqrt{x^2+1}}$$

$$= \frac{x\left(3x^2+2\right)}{\sqrt{x^2+1}}$$

$$\left(U^n V^m\right)' = U^{n-1} V^{m-1}\left(nU'V + mUV'\right)$$

Exercise

Find the derivative to the following functions $y = \left(\dfrac{x+1}{x-5}\right)^2$

Solution

$$y' = -\frac{12(x+1)}{(x-5)^3}$$

$$\frac{d}{dx}\left(\frac{ax^n+b}{cx^n+d}\right)^m = mn(ad-bc)x^{n-1}\frac{\left(ax^n+b\right)^{m-1}}{\left(cx^n+d\right)^{m+1}}$$

Exercise

Find the derivative to the following functions $y = x^2\sin x$

Solution

$$y' = 2x\sin x + x^2\cos x$$

$$u = x^2 \quad v = \sin x$$
$$u' = 2x \quad v' = \cos x$$

Exercise

Find the derivative to the following functions $y = \dfrac{\sin x}{x}$

Solution

$$y' = \frac{x\cos x - \sin x}{x^2}$$

$$u = \sin x \quad v = x$$
$$u' = \cos x \quad v' = 1$$

Exercise

Find the derivative to the following functions $y = \dfrac{\cot x}{1+\cot x}$

Solution

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$$u = \cot x \qquad v = 1 + \cot x$$

$$u' = -\csc^2 x \quad v' = -\csc^2 x$$

$$y' = \frac{-\csc^2 x (1 + \cot x) + \csc^2 x \cot x}{(1 + \cot x)^2}$$

$$= \frac{-\csc^2 x - \csc^2 x \cot x + \csc^2 x \cot x}{(1 + \cot x)^2}$$

$$= \frac{-\csc^2 x}{(1 + \cot x)^2}$$

Exercise

Find the derivative to the following functions $y = x^2 \sin x + 2x \cos x - 2 \sin x$

Solution

$$y' = 2x \sin x + x^2 \cos x + 2 \cos x - 2x \sin x - 2 \cos x$$

$$= x^2 \cos x$$

Exercise

Find the derivative to the following functions $y = x^3 \sin x \cos x$

Solution

$$y' = \left(x^3\right)' \sin x \cos x + x^3 (\sin x)' \cos x + x^3 \sin x (\cos x)'$$

$$= 3x^2 \sin x \cos x + x^3 \cos^2 x - x^3 \sin^2 x$$

Exercise

Find the derivative to the following functions $y = \dfrac{4}{\cos x} + \dfrac{1}{\tan x}$

Solution

$$y' = \frac{-4 \sin x}{\cos^2 x} - \frac{\sec^2 x}{\tan^2 x} \qquad\qquad \left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$= -4 \frac{\sin x}{\cos x} \frac{1}{\cos x} - \frac{1}{\cos^2 x} \frac{\cos^2 x}{\sin^2 x}$$

$$= -4 \tan x \sec x - \csc^2 x$$

Exercise

Find the derivative of $f(x) = \dfrac{\left(x^2 - 6x\right)^5}{\left(3x^2 + 5x - 2\right)^4}$

Solution

$f(x) = \left(x^2 - 6x\right)^5 \left(3x^2 + 5x - 2\right)^{-4}$ $\qquad \left(U^m V^n\right)' = U^{m-1} V^{n-1}\left(mU'V + nUV'\right)$

$f'(x) = \left(x^2 - 6x\right)^4 \left(3x^2 + 5x - 2\right)^{-5}\left[5(2x - 6)\left(3x^2 + 5x - 2\right) - 4\left(x^2 - 6x\right)(6x + 5)\right]$

$\qquad = \left(x^2 - 6x\right)^4 \left(3x^2 + 5x - 2\right)^{-5}\left[(10x - 30)\left(3x^2 + 5x - 2\right) - 4\left(6x^3 - 31x^2 - 30x\right)\right]$

$\qquad = \left(x^2 - 6x\right)^4 \left(3x^2 + 5x - 2\right)^{-5}$

$$
\begin{array}{cl}
x^3 & 30 - 24 \\
x^2 & 50 - 90 + 124 \\
x & -20 - 150 + 120 \\
x^0 & 60
\end{array}
$$

$\qquad = \dfrac{\left(x^2 - 6x\right)^4 \left(6x^3 + 84x^2 - 50x + 60\right)}{\left(3x^2 + 5x - 2\right)^5}$

Exercise

Find the derivative of $\qquad y = \ln\sqrt{x + 5}$

Solution

$y = \ln(x + 5)^{1/2}$

$\quad = \tfrac{1}{2}\ln(x + 5)$

$y' = \dfrac{1}{2(x + 5)}$

Exercise

Find the Derivatives of $\qquad y = (3x + 7)\ln(2x - 1)$

Solution

$y' = 3x\ln(2x - 1) + \dfrac{2(3x + 7)}{2x - 1}$

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Exercise

Find the Derivatives of $\qquad f(x) = \ln \sqrt[3]{x+1}$

Solution

$$f(x) = \frac{1}{3}\ln(x+1)$$

$$f'(x) = \frac{1}{3(x+1)}$$

Exercise

Find the Derivatives of $\qquad f(x) = \ln\left(x^2 \sqrt{x^2+1} \right)$

Solution

$$f(x) = 2\ln x + \frac{1}{2}\ln\left(x^2+1\right)$$

$$f'(x) = 2\frac{1}{x} + \frac{1}{2}\frac{2x}{x^2+1}$$

$$= \frac{2}{x} + \frac{x}{x^2+1}$$

Exercise

Find the Derivatives of $\qquad y = \ln\dfrac{x^2}{x^2+1}$

Solution

$$y = \ln x^2 - \ln\left(x^2+1\right)$$

$$y' = \frac{2x}{x^2} - \frac{2x}{x^2+1}$$

$$= \frac{2}{x} - \frac{2x}{x^2+1}$$

Exercise

Find the derivative of $\qquad f(x) = e^{-2x^3}$

Solution

$$f'(x) = -\frac{6x^2}{e^{2x^3}}$$

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Exercise

Find the derivative of $\qquad f(x) = 4e^{x^2}$

Solution

$\underline{f'(x) = 8xe^{x^2}}$

Exercise

Find the derivative of $\qquad f(x) = x^2 e^x$

Solution

$f'(x) = (2x)e^x + x^2 e^x$

$\underline{= xe^x(2+x)}$

Exercise

Find the derivative $\qquad f(x) = 2x^3 e^x$

Solution

$f'(x) = 6x^2 e^x + 2x^3 e^x$

$\underline{= 2x^2 e^x(3+x)}$

Exercise

Find the derivative $\qquad f(x) = \dfrac{3e^x}{1+e^x}$

Solution

$\underline{f'(x) = \dfrac{3e^x}{\left(1+e^x\right)^2}}$

$\qquad\qquad \left(\dfrac{\alpha+b}{\beta+d}\right)' = \dfrac{\alpha'd - \beta'b}{(\beta+d)^2}$

Exercise

Find the derivative $\qquad f(x) = 5e^x + 3x + 1$

Solution

$\underline{f'(x) = 5e^x + 3}$

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Exercise

Find the derivative of $f(x) = \dfrac{e^x + e^{-x}}{2}$

Solution

$$f'(x) = \frac{1}{2}e^x - e^{-x}$$

Exercise

Find the derivative of $f(x) = \dfrac{e^x}{x^2}$

Solution

$$f'(x) = \frac{x^2 e^x - 2xe^x}{x^4}$$

$$= \frac{xe^x(x-2)}{x^4}$$

$$= \frac{e^x(x-2)}{x^3}$$

Exercise

Find the derivative of $f(x) = x^2 e^x - e^x$

Solution

$$f'(x) = e^x(2x) + x^2 e^x - e^x$$

$$= e^x\left(x^2 + 2x - 1\right)$$

Exercise

Find the derivative of $f(x) = (1 + 2x)e^{4x}$

Solution

$$f'(x) = (2)e^{4x} + (1 + 2x)\left(4e^{4x}\right)$$

$$= 2e^{4x}(1 + 2 + 4x)$$

$$= 2e^{4x}(3 + 4x)$$

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Exercise

Find the derivative of $y = x^2 e^{5x}$

Solution

$$y' = x^2 \left(5e^{5x}\right) + 2x\left(e^{5x}\right)$$

$$\underline{= xe^{5x}(5x+2)}$$

Exercise

Find the derivative of $y = x^2 e^{-2x}$

Solution

$$y' = 2xe^{-2x} - 2x^3 e^{-2x}$$

$$\underline{= 2xe^{-2x}\left(1-x^2\right)}$$

Exercise

Find the derivative $\quad f(x) = \dfrac{e^x}{x^2+1}$

Solution

$$u = e^x \quad v = x^2 + 1$$

$$u' = e^x \quad v' = 2x$$

$$f'(x) = \frac{e^x\left(x^2+1\right) - 2xe^x}{\left(x^2+1\right)^2}$$

$$\underline{= \frac{\left(x^2+1-2x\right)e^x}{\left(x^2+1\right)^2}}$$

Exercise

Find the derivative $\quad f(x) = \dfrac{1-e^x}{1+e^x}$

Solution

$$u = 1-e^x \quad v = 1+e^x$$

$$u' = -e^x \quad v' = e^x$$

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$$f'(x) = \frac{-e^x\left(1+e^x\right) - e^x\left(1-e^x\right)}{\left(1+e^x\right)^2}$$

$$= \frac{-e^x - e^{2x} - e^x + e^{2x}}{\left(1+e^x\right)^2}$$

$$= -\frac{2e^x}{\left(1+e^x\right)^2}$$

Exercise

Find the Derivatives of $y = \dfrac{\ln x}{e^{2x}}$

Solution

$$y' = \frac{e^{2x}\left(\frac{1}{x}\right) - \left(2e^{2x}\right)\ln x}{e^{4x}}$$

$$= \frac{e^{2x} - 2x\, e^{2x}\ln x}{e^{4x}}$$

$$= \frac{1 - 2x\ln x}{e^{2x}}$$

Exercise

Find the Derivatives of $f(x) = e^{2x}\ln\left(xe^x + 1\right)$

Solution

$$f = e^{2x} \qquad U = 2x \to U' = 2 \qquad f' = 2e^{2x}$$

$$g = \ln\left(xe^x + 1\right) \quad U = xe^x + 1 \to U' = e^x + xe^x \quad g' = \frac{e^x + xe^x}{xe^x + 1}$$

$$f'(x) = 2e^{2x}\ln\left(xe^x + 1\right) + e^{2x}\frac{e^x + xe^x}{xe^x + 1}$$

$$= e^{2x}\left(2\ln\left(xe^x + 1\right) + \frac{e^x(1+x)}{xe^x + 1}\right)$$

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Exercise

Find the Derivatives of $f(x) = \dfrac{xe^x}{\ln\left(x^2+1\right)}$

Solution

$$u = xe^x \qquad\qquad u' = e^x + xe^x$$

$$v = \ln\left(x^2+1\right) \qquad v' = \frac{2x}{x^2+1}$$

$$f'(x) = \frac{e^x(1+x)\ln\left(x^2+1\right) - \frac{2x}{x^2+1}xe^x}{\left(\ln\left(x^2+1\right)\right)^2}$$

$$= \frac{e^x\left[(1+x)\ln\left(x^2+1\right) - \frac{2x^2}{x^2+1}\right]}{\left[\ln\left(x^2+1\right)\right]^2}$$

$$= \frac{e^x\, \dfrac{\left(x^2+1\right)(1+x)\ln\left(x^2+1\right) - 2x^2}{x^2+1}}{\ln^2\left(x^2+1\right)}$$

$$= \frac{e^x\left[\left(x^2+1\right)(1+x)\ln\left(x^2+1\right) - 2x^2\right]}{\left(x^2+1\right)\left(\ln\left(x^2+1\right)\right)^2}$$

Exercise

Find the derivative $y = \cos^{-1}\left(\dfrac{1}{x}\right)$

Solution

$$y = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \sec^{-1}(x)$$

$$y' = \frac{1}{|x|\cdot\sqrt{x^2-1}}$$

Exercise

Find the derivative $y = \sin^{-1}\left(\sqrt{2}\,t\right)$

$$y' = \frac{\sqrt{2}}{\sqrt{1 - \left(\sqrt{2}t\right)^2}}$$

$$= \frac{\sqrt{2}}{\sqrt{1 - 2t^2}}$$

Exercise

Find the derivative $y = \sec^{-1}(5s)$

Solution

$$y' = \frac{5s}{|5s|\sqrt{(5s)^2 - 1}}$$

$$= \frac{s}{|s|\sqrt{25s^2 - 1}}$$

Exercise

Find the derivative $\quad y = \cot^{-1}\sqrt{t-1}$

Solution

$$y' = -\frac{\frac{1}{2}(t-1)^{-1/2}}{1 + \left[(t-1)^{1/2}\right]^2}$$

$$= -\frac{1}{2(t-1)^{1/2}(1 + t - 1)}$$

$$= -\frac{1}{2t\sqrt{t-1}}$$

Exercise

Find the derivative $\quad y = \ln\left(\tan^{-1} x\right)$

Solution

$$y' = \frac{\frac{1}{1+x^2}}{\tan^{-1}x}$$

$$= \frac{1}{\left(1+x^2\right)\tan^{-1}x}$$

Exercise

Find the derivative $y = \tan^{-1}\left(\ln x\right)$

Solution

$$y' = \frac{\frac{1}{x}}{1+\left(\ln x\right)^2} \qquad\qquad \left(\tan^{-1}u\right)' = \frac{u'}{1+u^2}$$

$$= \frac{1}{x\left(1+\left(\ln x\right)^2\right)}$$

Solution *Section R.2 – Integration*

Exercise

Find each indefinite integral. $\displaystyle\int \frac{x+2}{\sqrt{x}}\, dx$

Solution

$$\int \frac{x+2}{\sqrt{x}}\, dx = \int \left(\frac{x}{x^{1/2}} + \frac{2}{x^{1/2}} \right) dx$$

$$= \int \left(x^{1/2} + 2x^{-1/2} \right) dx$$

$$= \frac{x^{3/2}}{3/2} + 2\frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{3}x^{3/2} + 4x^{1/2} + C$$

Exercise

Find each indefinite integral $\displaystyle\int 4y^{-3}\, dy$

Solution

$$\int 4y^{-3} dy = -\frac{2}{y^2} + C$$

Exercise

Find each indefinite integral $\displaystyle\int \left(x^3 - 4x + 2 \right) dx$

Solution

$$\int \left(x^3 - 4x + 2 \right) dx = \frac{1}{4}x^4 - 2x^2 + 2x + C$$

Exercise

Find each indefinite integral $\displaystyle\int \left(\sqrt[4]{x^3} + 1 \right) dx$

Solution

$$\int \left(x^{3/4} + 1 \right) dx = \frac{4}{7} x^{7/4} + x + C$$

Exercise

Find each indefinite integral $\int \sqrt{x} \, (x+1) \, dx$

Solution

$$\int x^{1/2} \, (x+1) \, dx = \int \left(x^{3/2} + x^{1/2} \right) dx$$

$$= \frac{2}{5} x^{5/2} + \frac{2}{3} x^{3/2} + C$$

Exercise

Find each indefinite integral $\int (1+3t) t^2 \, dt$

Solution

$$\int \left(t^2 + 3t^3 \right) dt = \frac{1}{3} t^3 + \frac{3}{4} t^4 + C$$

Exercise

Find each indefinite integral $\int \frac{x^2 - 5}{x^2} \, dx$

Solution

$$\int \frac{x^2 - 5}{x^2} \, dx = \int \left(1 - \frac{5}{x^2} \right) dx$$

$$= \int \left(1 - 5x^{-2} \right) dx$$

$$= x + 5x^{-1} + C$$

$$= x + \frac{5}{x} + C$$

Exercise

Find each indefinite integral $\int (-40x + 250)\, dx$

$$\int (-40x + 250)\, dx = -20x^2 + 250x + C$$

Exercise

Find each indefinite integral $\int \left(7 - 3x - 3x^2\right)(2x+1)\, dx$

$$\int \left(7 - 3x - 3x^2\right)(2x+1)\, dx = \int \left(14x + 7 - 6x^2 - 3x - 6x^3 - 3x^2\right)dx$$

$$= \int \left(-6x^3 - 9x^2 + 11x + 7\right)dx$$

$$= -\frac{3}{2}x^4 - 3x^3 + \frac{11}{2}x^2 + 7x + C$$

Exercise

Find the integral $\int (1 + \cos 3\theta)\, d\theta$

$$\int (1 + \cos 3\theta)\, d\theta = \theta + \frac{1}{3}\sin 3\theta + C$$

Exercise

Find the integral $\int 2\sec^2 \theta\, d\theta$

$$\int 2\sec^2 \theta\, d\theta = 2\tan \theta + C$$

Exercise

Find the integral $\displaystyle\int \sec 2x \tan 2x \; dx$

$$\int \sec 2x \tan 2x \; dx = \frac{1}{2}\sec 2x + C$$

Exercise

Find the integral $\displaystyle\int 2e^{2x} dx$

Solution

$$\int 2e^{2x} dx = e^{2x} + C$$

Exercise

Find the integral $\displaystyle\int \frac{12}{x} \; dx$

Solution

$$\int \frac{12}{x} \; dx = 12\ln|x| + C$$

Exercise

Find the integral $\displaystyle\int \frac{dx}{\sqrt{1-x^2}}$

Solution

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

Exercise

Find the integral $\displaystyle\int \frac{dx}{x^2+1}$

Solution

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$$\int \frac{dx}{x^2+1} = \tan^{-1} x + C$$

Exercise

Find the integral $\displaystyle\int \frac{1+\tan\theta}{\sec\theta}\, d\theta$

Solution

$$\int \frac{1+\tan\theta}{\sec\theta}\, d\theta = \int \left(\frac{1}{\sec\theta} + \frac{\tan\theta}{\sec\theta} \right) d\theta$$

$$= \int (\cos\theta + \sin\theta)\, d\theta$$

$$= \sin\theta - \cos\theta + C$$

Exercise

Find the general solution of the differential equation $y' = 2t + 3$

Solution

$$dy = (2t+3)\, dt$$

$$\int dy = \int (2t+3)\, dt$$

$$y = t^2 + 3t + C$$

Exercise

Find the general solution of the differential equation $y' = 3t^2 + 2t + 3$

Solution

$$\int dy = \int \left(3t^2 + 2t + 3 \right) dt$$

$$y = t^3 + t^2 + 3t + C$$

Exercise

Find the general solution of the differential equation $y' = \sin 2t + 2\cos 3t$

Solution

$$\int dy = \int \left(\sin 2t + 2\cos 3t \right) dt$$

$$y(t) = -\frac{1}{2}\cos 2t + \frac{2}{3}\sin 3t + C$$

Exercise

Find the general solution of the differential equation: $y' = x^3\left(3x^4 + 1\right)^2$

Solution

$$d\left(3x^4 + 1\right) = 12x^3 dx$$

$$\int x^3\left(3x^4 + 1\right)^2 dx = \frac{1}{12}\int \left(3x^4 + 1\right)^2 d\left(3x^4 + 1\right)$$

$$= \frac{1}{36}\left(3x^4 + 1\right)^3 + C$$

$$y(x) = \frac{1}{36}\left(3x^4 + 1\right)^3 + C$$

Exercise

Find the general solution of the differential equation: $y' = 5x\sqrt{x^2 - 1}$

Solution

$$d\left(x^2 - 1\right) = 2x\,dx$$

$$\int 5x\left(x^2 - 1\right)^{1/2} dx = \frac{5}{2}\int \left(x^2 - 1\right)^{1/2} d\left(x^2 - 1\right)$$

$$= \frac{5}{3}\left(x^2 - 1\right)^{3/2} + C$$

Exercise

Find the general solution of the differential equation: $y' = x\sqrt{x^2 + 4}$

Solution

$$\int x\sqrt{x^2 + 4}\,dx = \frac{1}{2}\int \left(x^2 + 4\right)^{1/2} d\left(x^2 + 4\right)$$

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$$= \frac{1}{3}\left(x^2 + 4\right)^{3/2} + C$$

Exercise

Evaluate the integrals $\displaystyle\int_{-2}^{2}\left(x^3 - 2x + 3\right)dx$

Solution

$$\int_{-2}^{2}\left(x^3 - 2x + 3\right)dx = \left(\frac{x^4}{4} - x^2 + 3x\right)\Bigg|_{-2}^{2}$$

$$= \left(\frac{(2)^4}{4} - (2)^2 + 3(2)\right) - \left(\frac{(-2)^4}{4} - (-2)^2 + 3(-2)\right)$$

$$= 12$$

Exercise

Evaluate the integrals $\displaystyle\int_{0}^{1}\left(x^2 + \sqrt{x}\right)dx$

Solution

$$\int_{0}^{1}\left(x^2 + \sqrt{x}\right)dx = \left(\frac{x^3}{3} + \frac{2}{3}x^{3/2}\right)\Bigg|_{0}^{1}$$

$$= \left(\frac{(1)^3}{3} + \frac{2}{3}(1)^{3/2}\right) - 0$$

$$= 1$$

Exercise

Evaluate the integrals $\displaystyle\int_{0}^{\pi/3} 4\sec u \tan u \ du$

Solution

$$\int_{0}^{\pi/3} 4\sec u \tan u \ du = 4\sec u \Bigg|_{0}^{\pi/3}$$

$$= 4\left(\sec\frac{\pi}{3} - \sec 0\right)$$

$$= 4(2-1)$$

$$\underline{= 4} \;|$$

Exercise

Evaluate the integrals
$$\int_{\pi/4}^{3\pi/4} \csc\theta \cot\theta \, d\theta$$

Solution

$$\int_{\pi/4}^{3\pi/4} \csc\theta \cot\theta \, d\theta = -\csc\theta \;\Big|_{\pi/4}^{3\pi/4}$$

$$= -\left(\csc\frac{3\pi}{4} - \csc\frac{\pi}{4}\right)$$

$$= -\left(\sqrt{2} - \sqrt{2}\right)$$

$$\underline{= 0} \;|$$

Exercise

Evaluate the integrals
$$\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt$$

Solution

$$\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt = \int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \pi t^{-2}\right) dt$$

$$= \left(4\tan t - \pi t^{-1}\right) \Big|_{-\pi/3}^{-\pi/4}$$

$$= \left(4\tan\left(-\frac{\pi}{4}\right) - \pi\left(-\frac{4}{\pi}\right)\right) - \left(4\tan\left(-\frac{\pi}{3}\right) - \pi\left(-\frac{3}{\pi}\right)\right)$$

$$= \left(4(-1) + 4\right) - \left(4\left(-\sqrt{3}\right) + 3\right)$$

$$= -\left(-4\sqrt{3} + 3\right)$$

$$\underline{= 4\sqrt{3} - 3} \;|$$

Exercise

Evaluate the integrals
$$\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} \, dy$$

Solution

$$\int_{-3}^{-1} \frac{y^5 - 2y}{y^3}\, dy = \int_{-3}^{-1} \left(y^2 - 2y^{-2}\right) dy$$

$$= \frac{1}{3}y^3 + 2y^{-1} \Big|_{-3}^{-1}$$

$$= \left(\frac{1}{3}(-1)^3 + \frac{2}{-1}\right) - \left(\frac{1}{3}(-3)^3 + \frac{2}{-3}\right)$$

$$= \frac{22}{3}$$

Exercise

Evaluate the integrals $\qquad \int_{1}^{8} \frac{\left(x^{1/3} + 1\right)\left(2 - x^{2/3}\right)}{x^{1/3}}\, dx$

Solution

$$\int_{1}^{8} \frac{\left(x^{1/3} + 1\right)\left(2 - x^{2/3}\right)}{x^{1/3}}\, dx = \int_{1}^{8} \frac{2x^{1/3} - x + 2 - x^{2/3}}{x^{1/3}}\, dx$$

$$= \int_{1}^{8} \left(2 - x^{2/3} + 2x^{-1/3} - x^{1/3}\right) dx$$

$$= 2x - \frac{3}{5}x^{5/3} + 3x^{2/3} - \frac{3}{4}x^{4/3} \Big|_{1}^{8}$$

$$= \left(2(8) - \frac{3}{5}(8)^{5/3} + 3(8)^{2/3} - \frac{3}{4}(8)^{4/3}\right) - \left(2(1) - \frac{3}{5}(1)^{5/3} + 3(1)^{2/3} - \frac{3}{4}(1)^{4/3}\right)$$

$$= \left(-\frac{16}{5}\right) - \left(\frac{73}{20}\right)$$

$$= -\frac{137}{20}$$

Exercise

Evaluate: $\qquad \int_{0}^{1} (2t + 3)^3\, dt$

Solution

$$d(2t + 3) = 2dt$$

$$\int_{0}^{1} (2t + 3)^3\, dt = \frac{1}{2} \int_{0}^{1} (2t + 3)^3\, d(2t + 3)$$

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$$= \frac{1}{8} (2t+3)^4 \Big|_0^1$$

$$= \frac{1}{8}\left(5^4 - 3^4\right)$$

$$\underline{= 68}\;|$$

Exercise

Evaluate the integral $\quad \displaystyle\int_{-1}^{1} r\sqrt{1-r^2}\; dr$

Solution

$$\int_{-1}^{1} r\sqrt{1-r^2}\; dr = -\frac{1}{2}\int_{-1}^{1}\left(1-r^2\right)^{1/2} d\left(1-r^2\right)$$

$$= -\frac{1}{3}\left(1-r^2\right)^{3/2}\Big|_{-1}^{1}$$

$$= -\frac{1}{3}(0-0)$$

$$\underline{= 0}\;|$$