

Solution

Section 1.2 – Functions

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(1, 2), (3, 4), (5, 6), (5, 8)\}$$

Solution

Not a function

Domain: $\{1, 3, 5\}$

Range: $\{2, 4, 6, 8\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(1, 2), (3, 4), (6, 5), (8, 5)\}$$

Solution

It is a Function

Domain: $\{1, 3, 6, 8\}$

Range: $\{2, 4, 5\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(9, -5), (9, 5), (2, 4)\}$$

Solution

It is ***not*** a function

Domain = $\{2, 9\}$

Range = $\{-5, 5, 4\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(-2, 5), (5, 7), (0, 1), (4, -2)\}$$

Solution

It is a function

Domain = $\{-2, 0, 4, 5\}$

Range = $\{-2, 1, 5, 7\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(-5, 3), (0, 3), (6, 3)\}$$

Solution

It is a function

$$\text{Domain} = \{-5, 0, 6\}$$

$$\text{Range} = \{3\}$$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(1, 2), (3, 4), (6, 5), (8, 5), (1, 5)\}$$

Solution

It is **not** a function

$$\text{Domain} = \{1, 3, 6, 8\}$$

$$\text{Range} = \{2, 4, 5\}$$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(-1, 3), (3, 4), (6, 5), (8, 5), (1, 5)\}$$

Solution

It is a function

$$\text{Domain} = \{-1, 1, 3, 6, 8\}$$

$$\text{Range} = \{3, 4, 5\}$$

Exercise

Find the domain and the range of the relation:

$$\{(5, 12.8), (10, 16.2), (15, 18.9), (20, 20.7), (25, 21.81)\}$$

Solution

$$\text{Domain: } \{5, 10, 15, 20, 25\}$$

$$\text{Range: } \{12.8, 16.2, 18.9, 20.7, 21.81\}$$

Exercise

Let $f(x) = -3x + 4$, find $f(0)$

Solution

$$\begin{aligned} f(0) &= -3(0) + 4 \\ &= 4 \end{aligned}$$

Exercise

Let $g(x) = -x^2 + 4x - 1$, find $g(-x)$

Solution

$$\begin{aligned} g(-x) &= -(-x)^2 + 4(-x) - 1 \\ &= -x^2 - 4x - 1 \end{aligned}$$

Exercise

Let $f(x) = -3x + 4$, find $f(a + 4)$

Solution

$$\begin{aligned} f(a + 4) &= -3(a + 4) + 4 \\ &= -3a - 12 + 4 \\ &= -3a - 8 \end{aligned}$$

Exercise

Given: $f(x) = 2|x| + 3x$, find $f(2 - h)$.

Solution

$$\begin{aligned} f(2 - h) &= 2|2 - h| + 3(2 - h) \\ &= 2|2 - h| + 6 - 3h \end{aligned}$$

Exercise

Given: $g(x) = \frac{x-4}{x+3}$, find $g(x + h)$

Solution

$$g(x + h) = \frac{x + h - 4}{x + h + 3}$$

Exercise

Given: $g(x) = \frac{x}{\sqrt{1-x^2}}$, find $g(0)$ and $g(-1)$

Solution

$$g(0) = \frac{0}{\sqrt{1-0^2}} \\ = 0$$

$$g(-1) = \frac{-1}{\sqrt{1-(-1)^2}} \\ = \frac{-1}{0} \text{ undefined}$$

Exercise

Given that $g(x) = 2x^2 + 2x + 3$. Find $g(p+3)$

Solution

$$\begin{aligned} g(p+3) &= 2(p+3)^2 + 2(p+3) + 3 \\ &= 2(p^2 + 2(p)(3) + 3^2) + 2p + 6 + 3 \\ &= 2(p^2 + 6p + 9) + 2p + 9 \\ &= 2p^2 + 12p + 18 + 2p + 9 \\ &= 2p^2 + 14p + 27 \end{aligned}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

Exercise

If $f(x) = x^2 - 2x + 7$, evaluate each of the following: $f(-5)$, $f(x+4)$, $f(-x)$

Solution

$$\begin{aligned} f(-5) &= (-5)^2 - 2(-5) + 7 \\ &= 25 + 10 + 7 \\ &= 42 \end{aligned}$$

$$\begin{aligned} f(x+4) &= (x+4)^2 - 2(x+4) + 7 \\ &= x^2 + 2(4)x + 4^2 - 2x - 8 + 7 \\ &= x^2 + 8x + 16 - 2x - 1 \\ &= x^2 + 6x + 15 \\ &= x^2 + 2x + 7 \end{aligned}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

Exercise

Find $g(0)$, $g(-4)$, $g(7)$, and $g\left(\frac{3}{2}\right)$ for $g(x) = \frac{x}{\sqrt{16-x^2}}$

Solution

$$\begin{aligned}g(0) &= \frac{0}{\sqrt{16-0^2}} \\&= \frac{0}{\sqrt{16}} \\&= 0\end{aligned}$$

$$\begin{aligned}g(-4) &= \frac{-4}{\sqrt{16-(-4)^2}} \\&= \frac{-4}{\sqrt{16-16}} \\&= \frac{-4}{0} \quad \text{undefined}\end{aligned}$$

$$\begin{aligned}g(7) &= \frac{7}{\sqrt{16-7^2}} \\&= \frac{7}{\sqrt{16-49}} \\&= \frac{7}{\sqrt{-33}} \quad \text{doesn't exist in real number}\end{aligned}$$

$$\begin{aligned}g\left(\frac{3}{2}\right) &= \frac{\frac{3}{2}}{\sqrt{16-\left(\frac{3}{2}\right)^2}} \\&= \frac{\frac{3}{2}}{\sqrt{16-\frac{9}{4}}} \\&= \frac{\frac{3}{2}}{\sqrt{\frac{4(16)-9}{4}}} \\&= \frac{\frac{3}{2}}{\frac{\sqrt{55}}{2}} \\&= \frac{3}{\sqrt{55}} \\&= \frac{3\sqrt{55}}{55}\end{aligned}$$

Exercise

$$f(x) = 3x - 4$$

$$a) f(0) \qquad b) f\left(\frac{5}{3}\right) \qquad c) f(-2a) \qquad d) f(x+h)$$

Solution

$$a) f(0) = \underline{-4}$$

$$b) f\left(\frac{5}{3}\right) = 3\frac{5}{3} - 4 \\ = 5 - 4 \\ = \underline{1}$$

$$c) f(-2a) = 3(-2a) - 4 \\ = \underline{-6a - 4}$$

$$d) f(x+h) = 3(x+h) - 4 \\ = \underline{3x + 3h - 4}$$

Exercise

$$f(x) = 3x^2 + 3x - 1$$

$$a) f(0) \qquad b) f(x+h) \qquad c) f(2) \qquad d) f(h)$$

Solution

$$a) f(0) = \underline{-1}$$

$$b) f(x+h) = 3(x+h)^2 + 3(x+h) - 1 \\ = 3(x^2 + 2hx + h^2) + 3x + 3h - 1 \\ = \underline{3x^2 + 6hx + 3h^2 + 3x + 3h - 1}$$

$$c) f(2) = 12 + 6 - 1 \\ = \underline{17}$$

$$d) f(h) = \underline{3h^2 + 3h - 1}$$

Exercise

$$f(x) = 2x^2 - 4$$

$$a) f(0) \qquad b) f(x+h) \qquad c) f(2) \qquad d) f(2) - f(-3)$$

Solution

$$a) f(0) = \underline{-4}$$

$$\begin{aligned} b) f(x+h) &= 2(x+h)^2 - 4 \\ &= 2(x^2 + 2hx + h^2) - 4 \\ &= \underline{2x^2 + 4hx + 2h^2 - 4} \end{aligned}$$

$$\begin{aligned} c) f(2) &= 8 - 4 \\ &= \underline{4} \end{aligned}$$

$$\begin{aligned} d) f(2) - f(-3) &= 8 - 4 - (18 - 4) \\ &= 4 - 14 \\ &= \underline{-10} \end{aligned}$$

Exercise

$$f(x) = 3x^2 + 4x - 2$$

$$a) f(0) \qquad b) f(x+h) \qquad c) f(3) \qquad d) f(-5)$$

Solution

$$a) f(0) = \underline{-2}$$

$$\begin{aligned} b) f(x+h) &= 3(x+h)^2 + 4(x+h) - 2 \\ &= 3(x^2 + 2hx + h^2) + 4x + 4h - 2 \\ &= \underline{3x^2 + 6hx + 3h^2 + 4x + 4h - 2} \end{aligned}$$

$$\begin{aligned} c) f(3) &= 27 + 12 - 2 \\ &= \underline{37} \end{aligned}$$

$$\begin{aligned} d) f(-5) &= 75 - 20 - 2 \\ &= \underline{53} \end{aligned}$$

Exercise

$$f(x) = -x^3 - x^2 - x + 10$$

$$a) f(0)$$

$$b) f(-1)$$

$$c) f(2)$$

$$d) f(1) - f(-2)$$

Solution

$$a) f(0) = \underline{10}$$

$$b) f(-1) = 1 - 1 + 1 + 10 \\ = \underline{11}$$

$$c) f(2) = -8 - 4 - 2 + 10 \\ = \underline{-4}$$

$$d) f(1) - f(-2) = -1 - 1 - 1 + 10 - (-8 - 4 + 2 + 10) \\ = 7 - 16 \\ = \underline{-9}$$

Exercise

For $\frac{1}{10}x^{10} - \frac{1}{2}x^6 + \frac{2}{3}x^3 - 10x$, determine

$$a) f(2) - f(-2)$$

$$b) f(1) - f(-1)$$

$$c) f(2) - f(0)$$

Solution

$$a) f(2) - f(-2) = \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2}{3}2^3 - 20 - \left(\frac{2^{10}}{10} - \frac{2^6}{2} - \frac{2}{3}2^3 + 20 \right) \\ = \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2^4}{3} - 20 - \frac{2^{10}}{10} + \frac{2^6}{2} + \frac{2^4}{3} - 20 \\ = \frac{2^5}{3} - 40 \\ = \frac{32}{3} - 40 \\ = \underline{-\frac{88}{3}}$$

$$b) f(1) - f(-1) = \frac{1}{10} - \frac{1}{2} + \frac{2}{3} - 10 - \left(\frac{1}{10} - \frac{1}{2} - \frac{2}{3} + 10 \right) \\ = \frac{1}{10} - \frac{1}{2} + \frac{2}{3} - 10 - \frac{1}{10} + \frac{1}{2} + \frac{2}{3} - 10 \\ = \frac{4}{3} - 20 \\ = \underline{-\frac{56}{3}}$$

$$\begin{aligned}
 c) \quad f(2) - f(0) &= \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2}{3}2^3 - 20 - (0) \\
 &= \frac{2^9}{5} - 2^5 + \frac{2^4}{3} - 5(2^2) \\
 &= 2^2 \left(\frac{128}{5} - 8 + \frac{4}{3} - 5 \right) \\
 &= 4 \left(\frac{384 + 20 - 195}{15} \right) \\
 &= 4 \left(\frac{209}{15} \right) \\
 &= \frac{836}{15}
 \end{aligned}$$

Exercise

For $f(x) = 3x^4 + x^2 - 4$, determine

$$a) \quad f(2) - f(-2)$$

$$b) \quad f(1) - f(-1)$$

$$c) \quad f(2) - f(0)$$

Solution

$$\begin{aligned}
 a) \quad f(2) - f(-2) &= 3(16) + 4 - 4 - (3(16) + 4 - 4) \\
 &= 48 + 4 - 4 - 48 - 4 + 4 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f(1) - f(-1) &= 3 + 1 - 4 - (3 + 1 - 4) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 c) \quad f(2) - f(0) &= 3(16) + 4 - 4 - (0) \\
 &= 48
 \end{aligned}$$

Exercise

For $f(x) = -\frac{2}{3}x^3 + 4x$, determine

$$a) \quad f(2) - f(-2)$$

$$b) \quad f(1) - f(-1)$$

$$c) \quad f(2) - f(0)$$

Solution

$$\begin{aligned}
 a) \quad f(2) - f(-2) &= -\frac{2}{3}(2^3) + 8 - \left(-\frac{2}{3}(-2)^3 - 8 \right) \\
 &= -\frac{16}{3} + 8 - \frac{16}{3} + 8 \\
 &= 2 \left(-\frac{16}{3} + 8 \right)
 \end{aligned}$$

$$\begin{aligned}
 &= 16\left(-\frac{1}{3}+1\right) \\
 &= 16\left(\frac{2}{3}\right) \\
 &= \frac{32}{3} \quad |
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f(1) - f(-1) &= -\frac{2}{3} + 4 - \left(\frac{2}{3} - 4\right) \\
 &= 2\left(-\frac{2}{3} + 4\right) \\
 &= \frac{20}{3} \quad |
 \end{aligned}$$

$$\begin{aligned}
 c) \quad f(2) - f(0) &= -\frac{16}{3} + 8 - (0) \\
 &= \frac{8}{3} \quad |
 \end{aligned}$$

Exercise

For $f(x) = \frac{2x-3}{x-4}$, determine

$$a) \quad f(0)$$

$$b) \quad f(3)$$

$$c) \quad f(x+h)$$

$$d) \quad f(-4)$$

Solution

$$a) \quad f(0) = \frac{3}{4} \quad |$$

$$\begin{aligned}
 b) \quad f(3) &= \frac{6-3}{3-4} \\
 &= -3 \quad |
 \end{aligned}$$

$$\begin{aligned}
 c) \quad f(x+h) &= \frac{2(x+h)-3}{x+h-4} \\
 &= \frac{2x+2h-3}{x+h-4} \quad |
 \end{aligned}$$

$$\begin{aligned}
 d) \quad f(-4) &= \frac{-8-3}{-4-4} \\
 &= \frac{11}{8} \quad |
 \end{aligned}$$

Exercise

For $f(x) = \frac{3x-1}{x-5}$, determine

a) $f(0)$

b) $f(3)$

c) $f(x+h)$

d) $f(-5)$

Solution

$$a) \quad f(0) = \frac{1}{5}$$

$$b) \quad f(3) = \frac{9-1}{3-5} \\ = -4$$

$$c) \quad f(x+h) = \frac{3(x+h)-1}{x+h-5} \\ = \frac{3x+3h-1}{x+h-5}$$

$$d) \quad f(-5) = \frac{-12-1}{-4-5} \\ = \frac{13}{9}$$