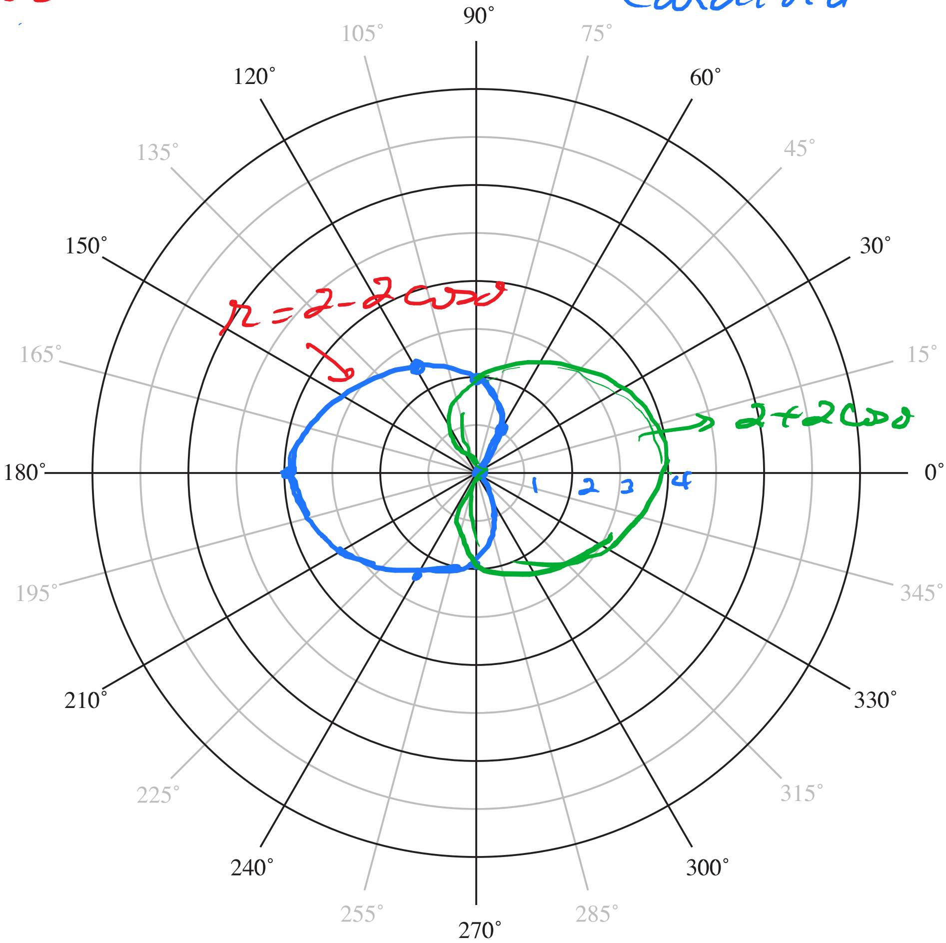
$r = 2 - 2\cos\theta$

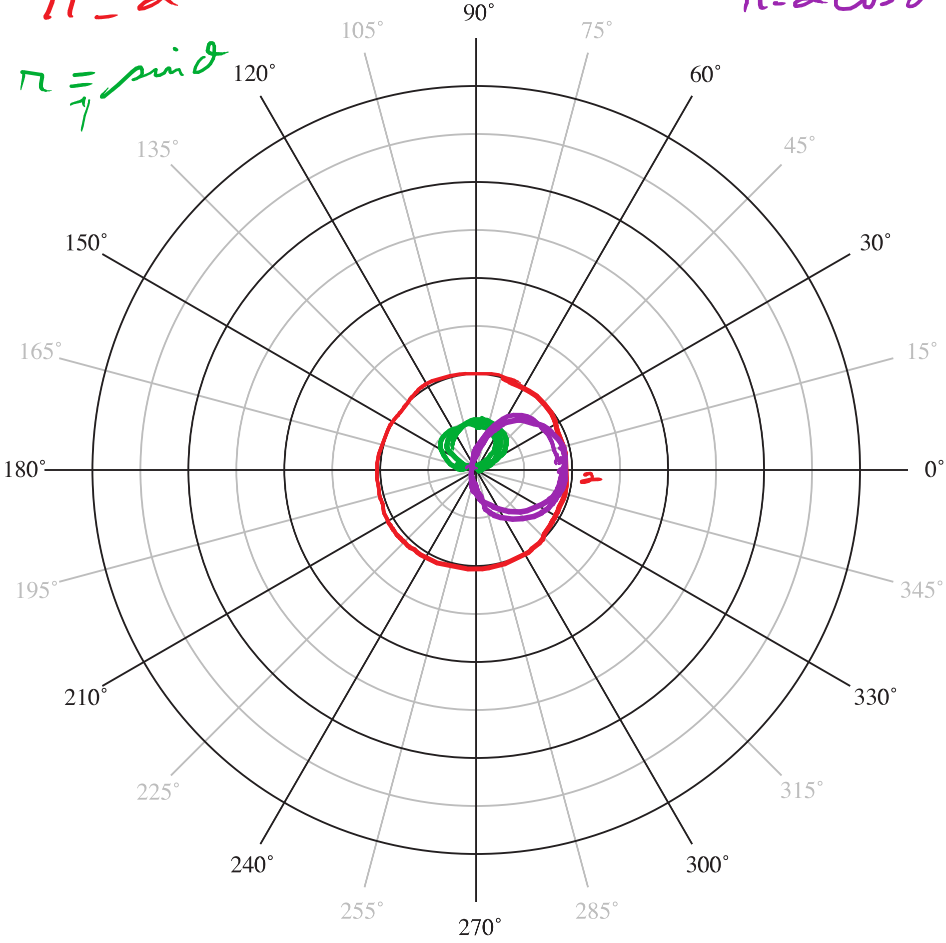
Cardioid



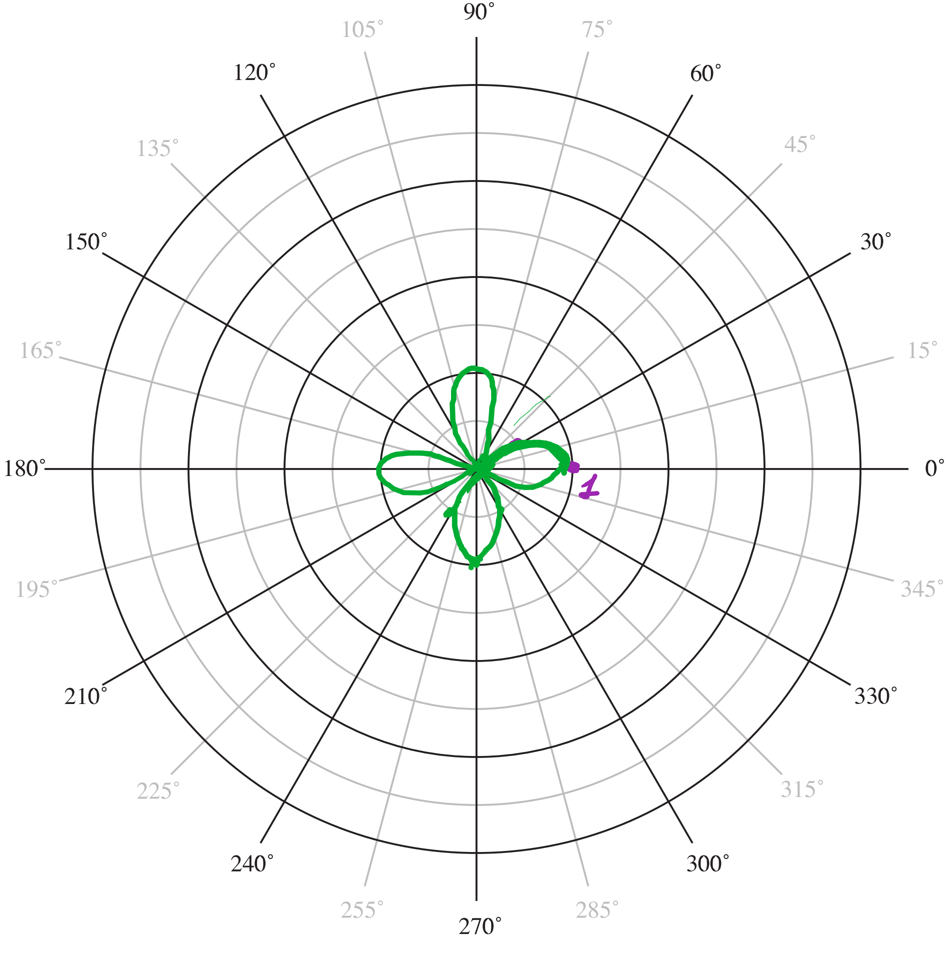
$r = 2$

$r = \sin\theta$

$r = 2\cos\theta$



$r = \cos 2\theta$



90°
105°
75°
120°
60°
135°
45°
150°
30°
165°
15°
180°
0°
1
195°
345°
210°
330°
225°
315°
240°
300°
255°
285°
270°

8.7 Trig. Form (Complex)

$\mathbb{C}$

$$\sqrt{-1} = i$$

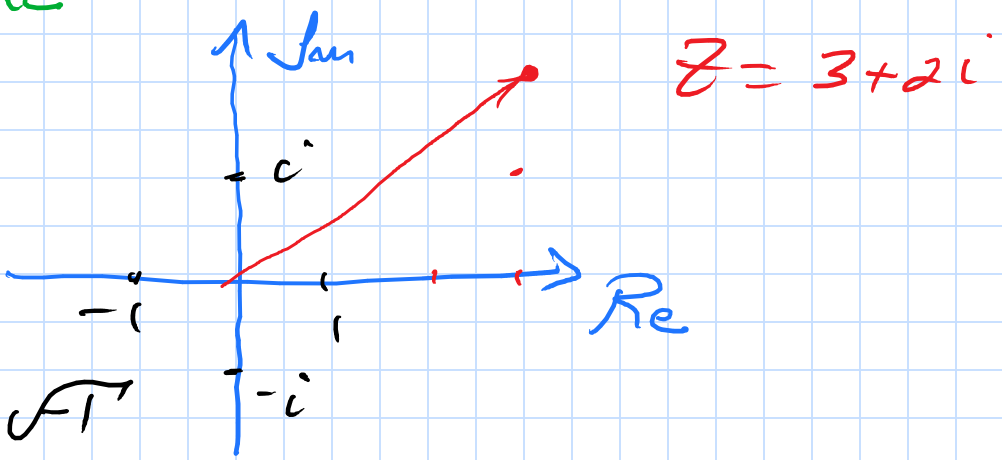
$i \quad \mathbb{C}$

j

$i \to$ current

$$z = \underbrace{x}_{\text{Real part}} + \underbrace{yi}_{\text{Imaginary}}$$

Re Im.



$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$$\boxed{x, y \in \mathbb{R}.}$$

Defn $z = x + yi$

modulus : $r = |z| = \sqrt{x^2 + y^2}$

Argument : $\hat{\theta} = \tan^{-1}\left|\frac{y}{x}\right|$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\begin{aligned} z &= x + iy \\ &= r\cos\theta + i\,r\sin\theta \\ &= r(\cos\theta + i\sin\theta) \\ &= r\,\mathrm{cis}\,\theta \end{aligned}$$

Trig. Form of z

conjugate of z : $\bar{z} = a - ib$

$$\begin{aligned} z \cdot \bar{z} &= a^2 + b^2 \\ &= (a - ib)(a + ib) \end{aligned}$$

$2\sqrt{3}$

$1 + 2i\sqrt{3}$

Ex $z = -1 + i$ $\qquad \begin{cases} x = -1 \\ y = 1 \end{cases}$ $\qquad$ Q II

$$r = \sqrt{1+1} = \sqrt{2} \qquad \theta = \tan^{-1}\left|\frac{1}{1}\right| = \frac{\pi}{4}$$

$$z = \sqrt{2}\ \text{cis}\ \frac{3\pi}{4}$$

Ex $z = 2\ \text{cis}\ 60^\circ$ $\qquad$ rect. form

$$z = 2(\cos 60^\circ + i \sin 60^\circ)$$
$$= 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$
$$= 1 + i\sqrt{3}$$

$\sqrt{1}$

Ex $z = 2(\cos 300^\circ + i \sin 300^\circ)$

$$= 2\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$
$$= 1 - i\sqrt{3}$$

Ex $z = 7 + i(0)$

$$r = 7$$

$\mathbb{R}^\circ$

Product Theorem

$$(r_1 \operatorname{cis}\theta_1)(r_2 \operatorname{cis}\theta_2) = r_1 r_2 \operatorname{cis}(\theta_1+\theta_2)$$

$$a^2+b^2 = (a-ib)(a+ib)$$

$$a+b = (\sqrt{a}-i\sqrt{b})(\sqrt{a}+i\sqrt{b})$$

Ex

$$3(\cos 45^\circ + i\sin 45^\circ)$$

$$2(\cos 135^\circ + i\sin 135^\circ)$$

$$(3\operatorname{cis}45^\circ)(2\operatorname{cis}135^\circ) = 6\operatorname{cis}(180^\circ)$$

$$= 6(\cos 180^\circ + i\sin 180^\circ)$$

$$\underline{= -6}$$

Quotient Theorem

$$\frac{r_1 \operatorname{cis}\theta_1}{r_2 \operatorname{cis}\theta_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$$

Ex

$$\frac{10\operatorname{cis}(-60^\circ)}{5\operatorname{cis}(150^\circ)} = 2\operatorname{cis}(-210^\circ)$$

$$\cancel{= 2(\cos(-210^\circ) + i\sin(-210^\circ))}$$

$$= 2(\cos 210^\circ - i\sin 210^\circ)$$

$$= 2\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$= -\sqrt{3} + i$$

De Moivre's Theorem

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} n\theta$$

Ex $\quad (1+i\sqrt{3})^8$

$r = \sqrt{1+3}$
$\quad = 2$

$\theta = \tan^{-1}\frac{\sqrt{3}}{1} = 60^\circ$

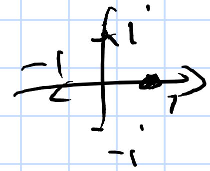
$$\begin{aligned}(1+i\sqrt{3})^8 &= (2 \operatorname{cis} 60^\circ)^8 \\ &= 2^8 \operatorname{cis} 480^\circ \\ &= 256(\cos 120^\circ + i\sin 120^\circ) \\ &= 256\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \\ &= -128 + 128i\sqrt{3}\end{aligned}$$

n^{th} Root Theorem

$$\sqrt[n]{r \operatorname{cis} \theta} = \sqrt[n]{r} \operatorname{cis} \alpha$$

$$\alpha = \frac{\theta}{n} + \frac{2\pi k}{n} \qquad \left(\frac{\theta + 360^\circ k}{n} \right.$$

Ex $\sqrt{4i}$



$z = 4i \Rightarrow r = 4$

$\theta = \frac{\pi}{2}$ $\qquad \theta = \tan^{-1}\frac{4}{0}$

$n = 2$

$$\sqrt{4i} = \sqrt{4 \operatorname{cis} \frac{\pi}{2}}$$

$$= 2\sqrt{\operatorname{cis} \frac{\pi}{2}}$$

$$\alpha = \frac{\frac{\pi}{2} + 2\pi k}{2} = \frac{\pi}{4} + \pi k$$

$k = 0 \Rightarrow \alpha = \frac{\pi}{4}$

$k = 1 \Rightarrow \alpha = \frac{5\pi}{4}$