4.1

15. $\int (\sqrt{x} + \sqrt[3]{x})\,dx = \int (x^{1/2} + x^{1/3})\,dx$

$$= \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$$

17. $\int \left(\frac{4+\sqrt{t}}{t^3}\right) dt = \int (4t^{-3} + t^{-5/2})\,dt$

$$= -2t^{-2} - \frac{2}{3}t^{-3/2} + C$$

$$= -\frac{2}{t^2} - \frac{2}{3}\frac{1}{t\sqrt{t}} + C$$

21. $\int (4\sec x \tan x - 2\sec^2 x)\,dx = 4\sec x - 2\tan x + C$

23. $\int (1+\tan^2\theta)\,d\theta = \int \sec^2\theta\,d\theta$

$$= \tan\theta + C$$

25. $\int (2e^x - 3e^{-2x})\,dx = 2e^x + \frac{3}{2}e^{-2x} + C$

38. $\int \frac{12}{x}\,dx = 12\ln|x| + C$

41. $\int \frac{1+\tan\theta}{\sec\theta}\,d\theta = \int \cos\theta\left(1+\frac{\sin\theta}{\cos\theta}\right)d\theta$

$$= \int (\cos\theta + \sin\theta)\,d\theta$$

$$= \sin\theta - \cos\theta + C$$

Ex

~~66~~ $\int (5x^{-4/3} + 3x^{-2/3} + 2x^{-1/3})\,dx$

$$= -15x^{-1/3} + 9x^{1/3} + 3x^{2/3} + C$$

62/ $\int \cos 2x \sin 2x\,dx = \frac{1}{2}\int \sin 4x\,dx$

$$= -\frac{1}{8}\cos 4x + C$$

or

$$d(\cos 2x) = -2\sin 2x\,dx$$

$$\int \cos 2x \sin 2x\,dx = -\frac{1}{2}\int \cos 2x\,d(\cos 2x)$$

$$= -\frac{1}{4}\cos^2 2x + C$$

63/ $\int (2\cos^2 x - 1)\,dx = \int \cos 2x\,dx$

$$= \frac{1}{2}\sin 2x + C$$

72/ $\int \left(e^{4x} - \frac{3}{x} + 2\csc x \cot x\right)dx = \frac{1}{4}e^{4x} - 3\ln|x| - 2\csc x + C$

74/ $\int (a^2 - b^2)e^{(a-b)x}\,dx = \frac{a^2-b^2}{a-b}e^{(a-b)x} + C$

$$= \frac{(a-b)(a+b)}{a-b}e^{(a-b)x} + C$$

$(a \neq b)$ $$= (a+b)e^{(a-b)x} + C$$

5.4

11) $\int_0^2 x(x-3)\,dx = \int_0^2 (x^2 - 3x)\,dx$

$= \frac{1}{3}x^3 - \frac{3}{2}x^2 \Big|_0^2$

$= \frac{8}{3} - 6$

$= -\frac{10}{3}$

12) $\int_0^{\pi/3} (\cos x + \sec x)^2\,dx$

$= \int_0^{\pi/3} (\cos^2 x + 2\cos x \sec x + \sec^2 x)\,dx$

$= \int_0^{\pi/3} \left(\frac{1}{2} + \frac{1}{2}\cos 2x + 2 + \sec^2 x\right)dx$

$= \int_0^{\pi/3} \left(\frac{5}{2} + \frac{1}{2}\cos 2x + \sec^2 x\right)dx$

$= \frac{5}{2}x + \frac{1}{4}\sin 2x + \tan x \Big|_0^{\pi/3}$

$= \frac{5}{2}\cdot\frac{\pi}{3} + \frac{\sqrt{3}}{8} + \sqrt{3}$

$= \frac{5\pi}{6} + \frac{9\sqrt{3}}{8}$

26/ $\displaystyle\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt$

$= 4\tan t - \frac{\pi}{t} \Big|_{-\pi/3}^{-\pi/4}$

$= -4 + 4 - (-4\sqrt{3} + 3)$

$= \boxed{4\sqrt{3} - 3}$

27/ $\displaystyle\int_1^7 \frac{dx}{x} = \ln|x| \Big|_1^7$

$= \ln 7 - \ln 1$

$= \ln 7$

28/ $\displaystyle\int_1^4 \left(\frac{x-1}{x}\right) dx = \int_1^4 \left(1 - \frac{1}{x}\right) dx$

$= x - \ln x \Big|_1^4$

$= 4 - \ln 4 - (1 - \ln 1)$

$= \boxed{3 - \ln 4}$

29/ $\displaystyle\int_{-2}^{-1} \left(3e^{3x} + \frac{2}{x}\right) dx = e^{3x} + 2\ln|x| \Big|_{-2}^{-1}$

$= e^{-3} + 2\underline{\ln 1} - (e^{-6} + 2\ln 2)$

$= e^{-3} - e^{-6} - 2\ln 2$

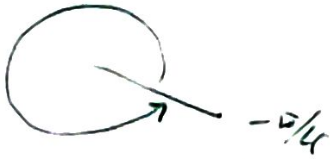
$= \frac{1}{e^3} - \frac{1}{e^6} - \ln 4$

26/ $\int_{-\pi/4}^{7\pi/4} (\sin x + \cos x)\,dx = -\cos x + \sin x \Big|_{-\pi/4}^{7\pi/4}$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$

$= 0$

$-\frac{\pi}{4} \to \frac{7\pi}{4}$



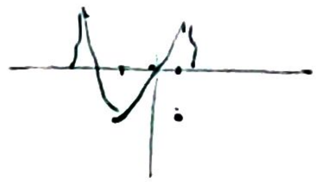
27/ $\int_{0}^{\ln 8} e^x\,dx = e^x \Big|_0^{\ln 8}$

$= e^{\ln 8} - e^0$

$= 8 - 1$

$= \boxed{7}$

31/ $y = -x^2 - 2x \qquad -3 \le x \le 2$



$-x(x+2) = 0 \to \boxed{x = 0, -2}$

Area $= -\int_{-3}^{-2} (-x^2 - 2x)\,dx + \int_{-2}^{0} (-x^2 - 2x)\,dx - \int_{0}^{2} (-x^2 - 2x)\,dx$

$= -\left(-\frac{1}{3}x^3 - x^2\right)\Big|_{-3}^{-2} + \left(-\frac{1}{3}x^3 - x^2\right)\Big|_{-2}^{0} - \left(-\frac{1}{3}x^3 - x^2\right)\Big|_{0}^{2}$

$= -\left[\frac{8}{3} - 4 - (9 - 9)\right] - \left(\frac{8}{3} - 4\right) - \left(-\frac{8}{3} - 4\right)$

$= \frac{8}{3} + 12$

$= \boxed{\frac{44}{3}}$

44) $f(x) = x^2 + 4x + 3 \qquad -3 \le x \le 0$

$x^2 + 4x + 3 = 0 \Rightarrow \underline{x = -1, -3}$

$$\text{Area} = -\int_{-3}^{-1} (x^2+4x+3)\,dx + \int_{-1}^{0} (x^2+4x+3)\,dx$$

$$= -\left(\frac{1}{3}x^3 + 2x^2 + 3x \right|_{-3}^{-1} + \left(\frac{1}{3}x^3 + 2x^2 + 3x\right|_{-1}^{0}$$

$$= -\left[-\frac{1}{3} + 2 - 3 - (-9 + 18 - 9)\right] + 0 - \left(-\frac{1}{3} + 2 - 3\right)$$

$$= \frac{4}{3} + \frac{4}{3}$$

$$= \frac{8}{3} \ \text{unit}^2$$

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45) $f(x) = x^2 - 3x + 2 \qquad 0 \le x \le 2$

$x^2 - 3x + 2 = 0 \Rightarrow \underline{x = 1, 2}$

$$\text{Area} = \int_0^1 (x^2 - 3x + 2)\,dx - \int_1^2 (x^2 - 3x + 2)\,dx$$

$$= \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x \Big|_0^1 - \left(\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x\right|_1^2$$

$$= \underbrace{\frac{1}{3} - \frac{3}{2} + 2} - \left[\frac{8}{3} - 6 + 4 - \underbrace{\left(\frac{1}{3} - \frac{3}{2} + 2\right)}\right]$$

$$= 2\left(\frac{5}{6}\right) - \frac{2}{3}$$

$$= \frac{5}{3} - \frac{2}{3}$$

$$= 1 \ \text{unit}^2$$

11.7/ $f(x) = 2x^2 - 4x + 2 \qquad 0 \le x \le 2$

$$\underbrace{2x^2 - 4x + 2}_{2(x-1)^2} = 0 \Rightarrow \boxed{x = 1, 1}$$

$$\text{Area} = \int_0^1 (2x^2 - 4x + 2) + \int_1^2 (2x^2 - 4x + 2)\, dx$$

$$= \frac{2}{3}x^3 - 2x^2 + 2x \Big|_0^1 + \left(\frac{2}{3}x^3 - 2x^2 + 2x \Big|_1^2\right.$$

$$= \frac{2}{3} - 2 + 2 + \left[\frac{16}{3} \underbrace{- 8 + 4}_{-4} - \left(\frac{2}{3} - 2 + 2\right)\right]$$

$$= \frac{16}{3} - 4$$

$$= \boxed{\frac{4}{3}}$$

4.6

7/ $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$ $\qquad d(1-\theta^2) = -2\theta\, d\theta$

$$= -\frac{1}{2}\int (1-\theta^2)^{1/4}\, d(1-\theta^2)$$

$$\boxed{= -\frac{2}{5}(1-\theta^2)^{5/4} + C}$$

30/ $\int x\sqrt{x^2+4}\, dx = \frac{1}{2}\int (x^2+4)^{1/2}\, d(x^2+4)$ $\qquad d(x^2+4) = 2x\,dx$

$$\boxed{= \frac{1}{3}(x^2+4)^{3/2} + C}$$

43/ $\int \sec x\, dx = \int \sec x \,\frac{\sec x + \tan x}{\sec x + \tan x}\, dx$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$$

$d(\sec x + \tan x) = (\sec x \tan x + \sec^2 x)\, dx$

$$\int \sec x\, dx = \int \frac{d(\sec x + \tan x)}{\sec x + \tan x}$$

$$\boxed{= \ln|\sec x + \tan x| + C}$$

65/ $\int \frac{1}{x^3} e^{1/4x^2} dx$ $\qquad d\left(\frac{1}{4} x^{-2}\right) = -\frac{1}{2} \frac{1}{x^3} dx$

$$= -2 \int e^{1/4x^2} d\left(\frac{1}{4x^2}\right)$$

$$= -2\, e^{1/4x^2} + C$$

67/ $\int \frac{-e^{3x}}{2-e^{3x}} dx$ $\qquad d(2-e^{3x}) = -3e^{3x} dx$

$$= \frac{1}{3} \int \frac{d(2-e^{3x})}{2-e^{3x}}$$

$$= \frac{1}{3} \ln |2-e^{3x}| + C$$

69/ $\int \frac{2(e^x - e^{-x})}{(e^x + e^{-x})^2} dx$ $\qquad d(e^x + e^{-x}) = (e^x - e^{-x}) dx$

$$= 2 \int \frac{d(e^x + e^{-x})}{(e^x + e^{-x})^2}$$

$$= -\frac{2}{e^x + e^{-x}} + C$$

77 $\int (6x+e^x)\sqrt{3x^2+e^x}\,dx$

$$d(3x^2+e^x) = (6x+e^x)\,dx$$

$$\int (3x^2+e^x)^{1/2}\,d(3x^2+e^x) = \boxed{\frac{2}{3}(3x^2+e^x)^{3/2} + C}$$

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180 $\int_2^3 \frac{x^2+2x-2}{x^3+3x^2-6x}\,dx$

$$d(x^3+3x^2-6x) = (3x^2+6x-6)\,dx$$
$$= 3(x^2+2x-2)\,dx$$

$$\int_2^3 \frac{x^2+2x-2}{x^3+3x^2-6x}\,dx = \frac{1}{3}\int_2^3 \frac{d(x^3+3x^2-6x)}{x^3+3x^2-6x}$$

$$= \frac{1}{3}\ln|x^3+3x^2-6x|\Big|_2^3$$

$$= \frac{1}{3}(\ln 36 - \ln 8)$$

$$= \boxed{\frac{1}{3}(2\ln 6 - 3\ln 2)}$$

$$\boxed{\frac{2}{3}\ln 6 - \ln 2}$$

81) $\int_0^{\ln 2} \frac{e^x}{1+e^{2x}}\,dx = \int_0^{\ln 2} \frac{e^x}{1+(e^x)^2}\,dx$

$$= \int_0^{\ln 2} \frac{d(e^x)}{1+(e^x)^2}$$

$$= \arctan(e^x) \Big|_0^{\ln 2}$$

$$= \arctan e^{\ln 2} - \arctan(1)$$

$$= \boxed{\tan^{-1}(2) - \frac{\pi}{4}}$$

$$\int_0^{\ln 2} \frac{e^x}{1+e^{2x}} \frac{e^{-x}}{e^{-x}}\,dx = \int_0^{\ln 2} \frac{1}{e^{-x}+e^x}\,dx$$

$$= \frac{1}{2}\int_0^{\ln 2} \operatorname{sech} x\,dx$$

$$(1+e^{2x})(1-e^{2x}) = 1 - e^{4x}$$

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212 $$\int_0^{\pi/2} \frac{\cos\theta \sin\theta}{\sqrt{\cos^2\theta + 16}}\, d\theta$$

$$d(\cos^2\theta + 16) = -2\cos\theta \sin\theta \, d\theta$$

$$\int_0^{\pi/2} \frac{\cos\theta \sin\theta}{\sqrt{\cos^2\theta + 16}}\, d\theta = -\frac{1}{2}\int_0^{\pi/2} (\cos^2\theta + 16)^{-1/2} d(16 + \cos^2\theta)$$

$$= - (\cos^2\theta + 16)^{1/2} \Big|_0^{\pi/2}$$

$$= -\left(4 - \sqrt{17}\right)$$

$$= \sqrt{17} - 4$$

222 $$\int_{-1}^{2} x^2 e^{x^3+1}\, dx \qquad d(x^3+1) = 3x^2 dx$$

$$= \frac{1}{3}\int_{-1}^{2} e^{x^3+1}\, d(x^3+1)$$

$$= \frac{1}{3} e^{x^3+1}\Big|_{-1}^{2}$$

$$= \frac{1}{3}\left(e^9 - 1\right)$$

224 $\displaystyle\int_0^4 \frac{x}{x^2+1}\,dx$ $\qquad d(x^2+1) = 2x\,dx$

$$= \frac{1}{2}\int_0^4 \frac{d(x^2+1)}{x^2+1}$$

$$= \frac{1}{2} \ln(x^2+1)\Big|_0^4$$

$$= \frac{1}{2}(\ln 17 - \ln 1)$$

$$= \frac{1}{2}\ln 17$$

215 $\displaystyle\int_0^{\pi/4} \frac{\sin\theta}{\cos^3\theta}\,d\theta$ $\qquad d(\cos\theta) = -\sin\theta\,d\theta$

$$= -\int_0^{\pi/4} \cos^{-3}\theta\, d(\cos\theta)$$

$$= \frac{1}{2}\,\frac{1}{\cos^2\theta}\Big|_0^{\pi/4}$$

$$= \frac{1}{2}(2-1)$$

$$= \frac{1}{2}$$