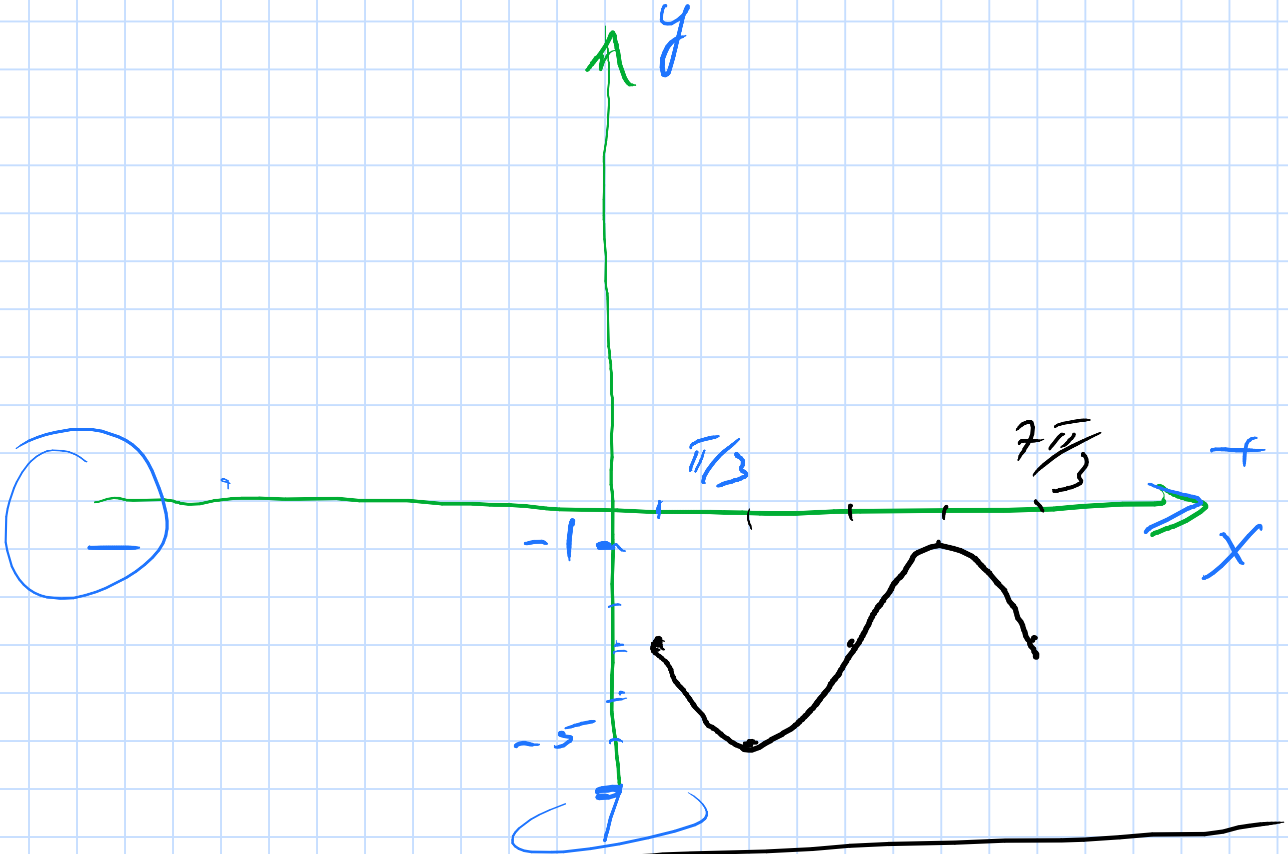
$y = -2\sin(3x - \pi) - 3$



Unit circle

8.1 Proving Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ $\qquad$ $\cos\theta\,\sec\theta = 1$

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad$ $\sin\theta\,\csc\theta = 1$

$\tan\theta = \dfrac{1}{\cot\theta}$ $\qquad$ $\tan\theta \cdot \cot\theta = 1$

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Ex

$$\sec\theta\tan\theta = \frac{1}{\cos\theta}\,\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos^2\theta}$$

Ex

$$\frac{1}{\sin\theta} + \frac{1}{\cos\theta} = \frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}$$

Ex $\tan\alpha + \cot\alpha = \dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\cos\alpha}{\sin\alpha}$

$$= \frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha \sin\alpha}$$

$$= \boxed{\frac{1}{\cos\alpha \sin\alpha}}$$

Prove $\tan x + \cos x = \sin x(\sec x + \cot x)$

$$\sin x(\sec x + \cot x) = \sin x \cdot \frac{1}{\cos x} + \sin x \frac{\cos x}{\sin x}$$

$$= \tan x + \cos x \checkmark$$

$$\tan x + \cos x = \frac{\sin x}{\cos x} + \cos x$$

$$= \frac{\sin x}{\cos x} + \cos x \frac{\sin x}{\sin x}$$

$$= \sin x\left(\frac{1}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \sin x(\sec x + \cot x) \checkmark$$

Prove $\cot\alpha + 1 = \csc\alpha(\cos\alpha + \sin\alpha)$

$$\csc\alpha(\cos\alpha + \sin\alpha) = \frac{1}{\sin\alpha}(\cos\alpha + \sin\alpha)$$

$$= \frac{\cos\alpha}{\sin\alpha} + 1$$

$$= \cot\alpha + 1 \checkmark$$

1- 1st work complicated (more trig)

2- (-) substitution

3- algebra

4- kind sine & cosine

5 Keep an eye that you're not working on

$$1 = 2$$

eye

$$2 =$$

$$1 = \text{eye}'$$

1
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^2 - b^2 = (a-b)(a+b)$$
$$a^4 - b^4 = (a^2+b^2)\underbrace{(a-b)(a+b)}_{a^2-b^2}$$
$$a^2 + b^2 \;\#$$

Prove: $\dfrac{\cos^4 t - \sin^4 t}{\cos^2 t} = 1 - \tan^2 t$

$$\frac{\cos^4 t - \sin^4 t}{\cos^2 t} = \frac{\overbrace{(\cos^2 t + \sin^2 t)}^{=1}(\cos^2 t - \sin^2 t)}{\cos^2 t}$$

$$= \frac{\cos^2 t - \sin^2 t}{\cos^2 t}$$

$$= \frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t}$$

$$= 1 - \tan^2 t \quad \checkmark$$

Prove: $1 + \cos\theta = \dfrac{\sin^2\theta}{1-\cos\theta}$

$$\frac{\sin^2\theta}{1-\cos\theta} = \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$

$$= 1+\cos\theta \quad \checkmark$$

$$1+\cos\theta = (1+\cos\theta)\frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta} \quad \checkmark$$

$$(1+\cos\theta)(1-\cos\theta) \stackrel{?}{=} \sin^2\theta$$

$$1-\cos^2\theta \stackrel{?}{=} \sin^2\theta$$

$$\sin^2\theta = \sin^2\theta \quad \checkmark$$

Prove $\tan^2\alpha\,(1+\cot^2\alpha) = \dfrac{1}{1-\sin^2\alpha}$

$$\tan^2\alpha\,(1+\cot^2\alpha) = \tan^2\alpha + \tan^2\alpha\cot^2\alpha$$
$$= \tan^2\alpha + 1$$
$$= \sec^2\alpha$$
$$= \frac{1}{\cos^2\alpha} \qquad \cos^2\alpha + \sin^2\alpha = 1$$
$$= \frac{1}{1-\sin^2\alpha} \checkmark$$

Prove $\dfrac{\sin\alpha}{1+\cos\alpha} + \dfrac{1+\cos\alpha}{\sin\alpha} = 2\csc\alpha$

$$\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + (1+\cos\alpha)^2}{\sin\alpha\,(1+\cos\alpha)}$$
$$= \frac{\sin^2\alpha + 1 + 2\cos\alpha + \cos^2\alpha}{\sin\alpha\,(1+\cos\alpha)}$$
$$= \frac{2+2\cos\alpha}{\sin\alpha\,(1+\cos\alpha)}$$
$$= \frac{2\,(1+\cos\alpha)}{\sin\alpha\,(1+\cos\alpha)}$$
$$= 2\csc\alpha \checkmark$$

Prove $\dfrac{1+\sin t}{\cos t} = \dfrac{\cos t}{1-\sin t}$

$$\frac{1+\sin t}{\cos t} = \frac{1+\sin t}{\cos t}\cdot\frac{1-\sin t}{1-\sin t}$$

$$= \frac{1-\sin^2 t}{\cos t(1-\sin t)}$$

$\cos^2 + \sin^2 = 1$

$$= \frac{\cos^2 t}{\cos t(1-\sin t)}$$

$$= \frac{\cos t}{1-\sin t} \quad ✓$$

Counterexample

$$\cot^2\theta + \cos^2\theta = \cot^2\theta \cos^2\theta$$

$$\cot^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\cot^2\frac{\pi}{4}\cos^2\frac{\pi}{4} = 1\,\frac{1}{2} = \frac{1}{2}$$

$$\frac{3}{2} \neq \frac{1}{2} \quad ✓$$

21/ $$\frac{\cot^2\theta + 3\cot\theta - 4}{\cot\theta + 4} = \cot\theta - 1$$

$$\frac{\cot^2\theta + 3\cot\theta - 4}{\cot\theta + 4} = \frac{(\cot\theta + 4)(\cot\theta - 1)}{\cot\theta + 4}$$

$$= \cot\theta - 1 \checkmark$$

25/ $$(1+\tan x)^2 + (\tan x - 1)^2 = 2\sec^2 x$$

$$(1+\tan x)^2 + (\tan x - 1)^2 = 1 + 2\tan x + \tan^2 x + 1 - 2\tan x + \tan^2 x$$

$$= 2 + 2\tan^2 x$$

$$= 2(1+\tan^2 x)$$

$$= 2\sec^2 x \checkmark$$

78/ $$\frac{\csc x - 1}{\csc x + 1} = \frac{\cot^2 x}{\csc^2 x + 2\csc x + 1}$$

$$\frac{\cot^2 x}{\csc^2 x + 2\csc x + 1} = \frac{\csc^2 x - 1}{(\csc x + 1)^2}$$

$$= \frac{(\csc x - 1)(\csc x + 1)}{(\csc x + 1)^2}$$

$$= \frac{\csc x - 1}{\csc x + 1} \checkmark$$

$3, 4 \to 5$

$5, 12 \to 13$

$8, 15 \to 17$

$7, 24 \to 25$

$20, 21 \to 29$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$-\qquad\qquad\qquad\qquad +$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$-\qquad\qquad\qquad\qquad -$$

Ex $\cos 75^\circ = \cos(30^\circ + 45^\circ)$

$$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{1}{2} \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos(x + 2\pi) = \cos x$$

$$\cos(x + 2\pi) = \cos x \cos 2\pi - \sin x \sin 2\pi$$

$$= \cos x\ (1) - \sin x\ (0)$$

$$= \cos x \ \checkmark$$

Simplify:

$$\cos 3x \cos 2x - \sin 3x \sin 2x = \cos(3x + 2x)$$
$$= \cos(5x)$$

$\cos(90^\circ - A) = \sin A$

$$\cos(90^\circ - A) = \cos 90^\circ \cos A + \sin 90^\circ \sin A$$
$$= 0 + \sin A$$
$$= \sin A \checkmark$$

Exact? $\sqrt{\ }$ leave

$$\cos(15^\circ) = \cos(60^\circ - 45^\circ)$$
$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$$
$$= \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2} + \sqrt{6}}{4} \checkmark$$

$\frac{\sin}{\cos} = \frac{3}{5}$ Q? $\frac{4}{5}$ $\qquad \frac{\sin}{\cos} =$ Q?

$\sin(A+B)$ $\quad \cos(A+B)$ $\quad \tan(A+B)$

$\sin(A-B)$ $\quad \cos(A-B)$ $\quad \tan(A-B)$