

ANSWERS

Lecture R

Section R.1

1. $f'(t) = -6t + 2$
2. $g'(x) = \frac{4}{3\sqrt[3]{x^2}}$
3. $f'(x) = 3x^2 + 1$
4. $f'(x) = 2 - \frac{1}{x^2}$
5. $f'(x) = 4 - \frac{2}{x^2} - \frac{10}{x^3}$
6. $f'(x) = -12x + 3 - \frac{1}{x^2}$
7. $f'(x) = 1 - \frac{2}{(x+1)^2}$
8. $\frac{3}{2}\sqrt{s} - \frac{1}{\sqrt{s}} - \frac{5}{2s\sqrt{s}}$
9. $f'(x) = \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$
10. $y' = 18x^2 + 30x$
11. $y' = 12x + 15$
12. $y' = \frac{1}{5}(2x + 4)$
13. $y' = \frac{12}{5}x^3$
14. $y' = \frac{2x^2 + 10x + 8}{(2x + 5)^2}$
15. $y' = \frac{2x^2 - 4x}{(x - 1)^2}$
16. $y' = -\frac{8}{(2x+1)^2}$
17. $y' = -\frac{6}{(x-1)^4}$
18. $y' = \frac{2}{3\sqrt[3]{x+4}}$
19. $f'(t) = \frac{1}{2} \frac{4t+5}{\sqrt{2t^2+5t+2}}$
20. $f'(x) = -\frac{2(2x-3)}{(x^2-3x)^3}$
21. $y' = \frac{5t^2-4t}{2\sqrt{t-2}}$
22. $\frac{2(5x^2-12x+5)(6-5x)}{(x^2-1)^3}$
23. $y' = \frac{x(3x^2+2)}{\sqrt{x^2+1}}$
24. $y' = -\frac{12(x+1)}{(x-5)^3}$
25. $y' = 2x \sin x + x^2 \cos x$
26. $y' = \frac{x \cos x - \sin x}{x^2}$
27. $y' = \frac{-\csc^2 x}{(1 + \cot x)^2}$
28. $y' = x^2 \cos x$
29. $3x^2 \sin x \cos x + x^3 \cos^2 x - x^3 \sin^2 x$
30. $-4 \tan x \sec x - \csc^2 x$
31. $xe^x(2+x)$
32. $\frac{1}{2}(e^x - e^{-x})$
33. $\frac{e^x(x-2)}{x^3}$
34. $e^x(x^2 + 2x - 1)$
35. $2e^{4x}(3 + 4x)$
36. $xe^{5x}(5x + 2)$
37. $\frac{e^{x^2+1}(20x^2 + 8x + 5)}{2\sqrt{5x+2}}$
38. $\frac{1}{3(x+1)}$
39. $\frac{2}{x} + \frac{x}{x^2+1}$
40. $\frac{2}{x} - \frac{2x}{x^2+1}$
41. $\frac{2e^x}{(1+e^x)(1-e^x)}$
42. $3^{x+1}(1+x \ln 3)$
43. $\frac{3t^{3/2} - 2(t^{3/2}+1)(\ln 8) \log_8(t^{3/2}+1)}{t^2(t^{3/2}+1) \ln 8}$
44. $\frac{(x^3-3x)^2(x^2+4x)^3}{(x^2+4x+1)^3} (11x^6 + 68x^5 + 160x^4 - 60x^3 - 219x^2 - 72x)$

Section R.2

1. $\frac{2}{3}x^{3/2} + 4x^{1/2} + C$
2. $-\frac{2}{y^2} + C$
3. $\frac{1}{4}x^4 - 2x^2 + 2x + C$
4. $\frac{4}{7}x^{7/4} + x + C$
5. $\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$
6. $\frac{1}{3}t^3 + \frac{3}{4}t^4 + C$
7. $x + \frac{5}{x} + C$
8. $-20x^2 + 250x + C$
9. $-\frac{3}{2}x^4 - 3x^3 + \frac{11}{2}x^2 + 7x + C$
10. $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$
11. $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$
12. $-x^2 \cos x - 2x \sin x + 2 \cos x + C$
13. $\left(\frac{1}{2}x^2 - \frac{3}{2}x + \frac{5}{5}\right)e^{2x} + C$
14. $\frac{e^{2x}}{13}(2 \cos 3x + 3 \sin 3x) + C$
15. $y(t) = t^2 + 3t + C$
16. $y(t) = t^3 + t^2 + 3t + C$
17. $y(t) = -\frac{1}{2} \cos 2t + \frac{2}{3} \sin 3t + C$
18. $y(x) = \frac{1}{36}(3x^4 + 1)^3 + C$
19. $\frac{5}{3}(x^2 - 1)^{3/2} + C$
20. $y(x) = \frac{1}{3}(x^2 + 4)^{3/2} + C$
21. $y(x) = e^{x^2+x} + C$
22. $y(x) = \frac{1}{6} \ln |6x - 5| + C$
23. $y(x) = \frac{1}{3} \ln |x^3 + 3x^2 + 9x + 1| + C$
24. $y(x) = -\frac{1}{\ln x} + C$
25. 12
26. 1
27. 4
28. 0
29. $4\sqrt{3} - 3$
30. $\frac{22}{3}$
31. $-\frac{137}{20}$
32. 68
33. 0
34. $F(x) = 2x^2 + 2x + 4$
35. $y(t) = \frac{1}{3}t \sin 3t - \frac{1}{9} \cos 3t + C$
36. 740.69 m
37. -64 ft/sec^2

Lecture 1

Section 1.1

5. $y(t) = \frac{1}{3}t^2 + \frac{5}{3t}$
7. $y(t) = \frac{4}{17} \cos t + \frac{1}{17} \sin t - \frac{21}{17}e^{-4t}$
8. $y(t) = te^{-t}$
9. $y(t) = \frac{6}{-3 + e^{-2t}}$
10. a) $m = -2$ b) $m = \frac{2}{5}$
c) $m = 2, 3$ d) $m = \frac{1}{2}, -4$
11. a) $c_1 = -1, c_2 = 8$ b) $c_1 = -1, c_2 = 0$
c) $c_1 = -1, \frac{\sqrt{3}}{4}c_2 = \frac{1}{4}$ d) $c_1 = -1, c_2 = 3$
12. $r = 2, 3$
13. $y(x) = x^3 - x^2 + 4x + C$
14. $y(x) = \frac{1}{3}x^3 - \frac{1}{4} \sin 2x + C_1x + C_2$
15. a) Yes b) No

Section 1.2

1. $y(x) = Ae^{x^2/2}$
2. $y(x) = Ax^2$
3. $y(x) = \ln(e^x + C)$
4. $y(x) = \tan(e^x + C)$
5. $y(x) = e^{x^2/2+x+C}$
6. $y(x) = De^{e^x+x} + 2$
7. $y(x) = -2 \pm \sqrt{x^2 + E}$
8. $y(x) = De^x |x-1|$
9. $y(t) = t \tan(\ln|t| + C)$
10. $y^4 + 16y + x^4 - 8x^2 = C$
11. $y(x) = Ae^{\ln|x^2-1|} - 1$
12. $y(x) = -\frac{1}{5}\cos 5x + C$
13. $y(x) = \frac{1}{3}x^3 + x^2 + x + C$
14. $y(x) = \frac{1}{3}e^{-3x} + C$
15. $y(x) = 1 - \frac{1}{x+C}$
16. $y(x) = Cx^4$
17. $x = \frac{1}{3}y^3 - y + C$
18. $y(x) = -\frac{1}{2}\ln(C_1 - 2x)$
19. $y(x) = \frac{1}{x^2 + C}$
20. $y(x) = -\frac{1}{2}\ln\left(C_1 - \frac{2}{3}e^{3x}\right)$
21. $(y-1)e^y = -e^{-x} - \frac{1}{3}e^{-3x} + C$
22. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 = \frac{1}{2}y^2 + 2y + \ln|y| + C$
23. $\frac{2}{2y+3} = \frac{1}{4x+5} + C$
24. $\cos y = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$
25. $y^2 = -\frac{1}{6}\sec^2 3x + C$
26. $-\frac{1}{2}\frac{1}{(e^x+1)^2} + C = \frac{1}{e^y+1}$
27. $(1+y^2)^{1/2} = (1+x^2)^{1/2} + C$
28. $y(x) = Ae^{-\cos x}$
29. $y(x) = C(1+x)^4$
30. $\arcsin y = \sqrt{x} + C$
31. $y^{3/2} = 3x^{3/2} + C_1$
32. $y^{2/3} = 2x^{4/3} + C$
33. $\sin y = x^2 + C$
34. $y(x) = C\frac{1+x}{1-x}$
35. $y(x) = \frac{A+Bx}{C_1+Cx}$
36. $y^2 = \frac{1}{-x^2+C}$
37. $y^2 = Ae^{x^2} - 1$
38. $y^4 = Ae^{4\sin x} - 1$
39. $y + \frac{2}{3}y^{3/2} = x + \frac{2}{3}x^{3/2} + C$
40. $\frac{1-6y^2}{3y^3} = \ln|x| + \frac{1}{x} + C$
41. $\sec y = C\sqrt{x^2+1}$
42. $\arctan y = -\frac{1}{x} - x + C$
43. $y(x) = Ax^{-4}$
44. $y(x) = \frac{C}{x^2+1}$
45. $y(x) = e^{x^3+3x+C}$
46. $\ln|y| = x^3 + 3x + C$
47. $y(x) = e^{e^{-x}+C}$
48. $x(t) = Ae^{t^3}$
49. $y^4 = \ln x^4 + C$
50. $y^3 = 2(x+1)^{3/2} - 6(x+1)^{1/2} + C$
51. $\ln|x| - \arctan x = t + K$

52. $(2y-1)e^{2y} = -4(x+1)e^{-x} + C$

53. $\frac{1}{2}\left(y + \frac{1}{2}\sin 2y\right) = \arctan x + C$

54. $(1-4v^2)^{-3/8} = Cx$

55. $\frac{1}{\sqrt{1+y^2}} + y = C_1 e^{x^3}$

56. $y(x) = \frac{-1}{e^{\cos x} + C}$

57. $e^{-x^2} = \ln(1+y^2) + C$

58. $y(x) = -2x$

59. $y = \sqrt{2e^{-2t^2} - 1} \quad (-\ln\sqrt{2}, \ln\sqrt{2})$

60. $y(x) = \sqrt{1-2\cos x}$

61. $y = \frac{-4}{\ln|t| + t - 5}$

62. $y = -\sqrt{2t - 2t^2} + 4$

63. $y = \frac{2-2e^{4t}}{1+e^{4t}}$

64. $y^2 - 2y = x^3 + 2x^2 + 2x + 3$

65. $y + y^2 = \frac{1}{2}x^2 - \frac{1}{2}$

66. $e^y + ye^{-y} + e^{-y} = 4 - 2\cos x$

67. $y(x) = 5 + \int_3^x e^{-t^2} dt$

68. $y = \frac{1}{2} - 2e^{-2x}$

69. $y(x) = \frac{x}{2} + \frac{\sqrt{3}}{2}\sqrt{1-x^2}$

70. $y(x) = \frac{1}{2} \frac{1-x^2}{1+x^2}$

71. $y^2 = e^{2t} + 2t - 1$

72. $y = \sqrt{t^2 + 4t + 4}$

73. $y(t) = e^{t^3/3}$

74. $y(t) = \frac{2}{e^{2t} + 1}$

75. $y(t) = 2e^{t^2+t}$

76. $y(t) = \frac{1}{2t^2 + 1}$

77. $y(x) = 2e^{e^x}$

78. $y(x) = \tan\left(x^3 + \frac{\pi}{4}\right)$

79. $y^2 = 1 + \sqrt{x^2 - 16}$

80. $y(x) = -3e^{x^4-x}$

81. $y(x) = e^{2x-2} + \frac{1}{2}$

82. $y(x) = \frac{\pi}{2}\sin x$

83. $y(x) = xe^{x^2-1}$

84. $y(x) = \frac{-1}{x^2 + x^3 + 1}$

85. $y(x) = \ln(3e^{2x} - 2)$

86. $y(x) = \tan^{-1}(\sqrt{x} - 1)$

87. $y(x) = -3e^{-3x}$

88. $y(x) = 2e^{(x+1)/2}$

89. $y(x) = \frac{3}{e}e^{x^2}$

90. $y(x) = -\sqrt{4 - 2\cos x}$

91. $y(t) = (3t+5)^{1/3}$

92. $y(x) = -1 + \sqrt{3-2x}$

93. $y(t) = -\ln\left(\frac{1}{2}t^2 + \cos t\right)$

94. $y(t) = \frac{-1}{t+1}$

95. $y(x) = \tan\left(x - \frac{\pi}{2}\right)$

96. $y(t) = \frac{3e^t - 1}{1 + 3e^t}$

97. $y(t) = (t - t^2 + 1)^{1/3}$

98. $y(x) = \arctan(e^{-x})$

99. $y^2 + \cos y = -\frac{1}{2}x^2 - \cos x + 2$

100. $ye^y = 2x - x^2 + 2e^2 - 1$

101. $y \ln y - y = x - \frac{1}{2}x^2 + e + \frac{3}{2}$

102. $y(x) = 1 - 2e^{-x^4/4}$

103. $y(x) = \tan\left(\ln|\sec x| + \frac{\pi}{3}\right)$

104. $y(x) = (\sin x + 1)^2 - 1$

105. $\frac{1}{2}y^2 + y = \ln|x| + \frac{2}{x} + 3\ln|x+1| - \frac{1}{2} - \ln 8$

106. $\frac{1}{2}y^2 + \ln|y| = -\theta \cos \theta + \sin \theta + \frac{1}{2} - \pi$

107. $y^2 = -\frac{1}{3}x^3 + 4$

108. $y(t) = \tan^{-1}(t^2 + 1)$

109. $y(x) = \frac{1}{2}\ln(4x^4 + 1)$

110. $y(x) = 4e^{x^3/3} - 1$

111. $2\sqrt{y} = -\ln|x+1| + 2$

112. $y(x) = \frac{1}{28 - 3x^2}$

113. $y^2 - 4y = x^3 + 2x^2 - 4x - 2$

114. $y(x) = \ln|x^2 - 4x - 4|$

115. $r(\theta) = \frac{2}{1 - 2\ln|\theta|}$

116. $\frac{1}{2}e^{-y}(\sin y - \cos y) = -e^{-t}(t^2 + 2t + 3) + \frac{5}{2}$

117. $\approx 70.77^\circ F$

118. $\approx 82 \text{ min} \approx 31.8 \text{ min} \approx 63.6 \text{ min}$

119. $20^\circ C$

120. $\approx -2.1 \text{ hrs}$

121. $\approx 56.18^\circ F$

122. $\approx 36.67^\circ F \approx 3.06 \text{ min}$

123. $\approx 64.461^\circ F$

124. $T(t) = 10 + 60e^{-0.462t}$

125. $T(t) = 100e^{-0.035t} \approx 70.5^\circ$

126. $\approx 82.1 \text{ sec} \approx 145.8 \text{ sec}$

127. $\approx 7.02 \text{ min}$

128. $390^\circ F$

129. $T(t) = 100 - 4(t+5)e^{-0.1t}$

130. $y(x) = \frac{4h}{L^2}x^2 + a$

131. $R(S) = AS^k$

132. $m(t) = 10e^{-\frac{3}{2000}t} + 50$

133. $t \rightarrow \infty$

134. $\approx -4.57 \text{ hrs} - 7:26 \text{ P.M}$

135. $1.13 \text{ hrs} - 8:52 \text{ P.M}$

Section 1.3

1. $v = a \tanh\left(\frac{g}{a}t\right) \quad y = \frac{a^2}{g} \ln\left(\cosh \frac{g}{a}t\right)$

2. $v = \frac{\sqrt{g}}{r} \tanh\left(r\sqrt{g}t + \tanh^{-1}\left(\frac{rv_0}{\sqrt{g}}\right)\right)$
 $\frac{1}{r^2} \ln\left(\cosh\left(r\sqrt{g}t + \tanh^{-1}\left(\frac{rv_0}{\sqrt{g}}\right)\right)\right)$
 $+ s_0 - \frac{1}{r^2} \ln\left(\cosh\left(\tanh^{-1}\left(\frac{rv_0}{\sqrt{g}}\right)\right)\right)$

3. 42.4 ft

4. $3,144 \text{ ft}$

5. 0 ft/sec

6. $\approx \pm 698.56 \text{ ft/sec}$

7. $v = 4(1 - e^{-8t}) \quad y = 4\left(t + \frac{1}{8}e^{-8t}\right) - \frac{1}{2}$

8. $7.2438 \text{ sec} = 257.1 \text{ m}$

9. 1193.7 s

10. $= -0.196 \text{ m/s}$

11. $\frac{1}{2g}v_0^2$

12. $-12.4938 \quad -14.5025 \quad 3.285$

13. $40,500$

14. $\frac{dv}{dt} = 0 \quad e = \frac{4\pi a^3}{3E}(\rho - \rho')$

15. $\approx 2150 \text{ sec}$

16. 972 sec

17. 1.31 hr

18. $V(t) = V_0\left(1 - \frac{t}{T}\right)^2 \quad V_0 = \pi r^2 h$

19. 0.028 in.

20. a) $y' = \sqrt{\frac{k^2 - y}{y}}$ b) $2k^2 \sin^2 t \, dt = dx$

c) $x(\theta) = k^2 \cdot \frac{\theta - \sin \theta}{2}$ d) $k \approx 2.194$

21. a) $\frac{a^2 kt}{kat + 1}$ b) $\frac{abe^{(a-b)kt} - ab}{ae^{(a-b)kt} - b}$

22. a) $(kt + \sqrt{H})^2$ b) $(0.1t + 0.707)^2$ c) $-\frac{\sqrt{H}}{k}$

23. a) $g - av^2$ b) $\sqrt{\frac{g}{a}}$ c) $\frac{e^{2\sqrt{ag}t} - 1}{1 + e^{2\sqrt{ag}t}} \sqrt{\frac{g}{a}}$

24. a) $-32t + 300$ b) 1406.25 ft

25. a) $1000 \frac{e^{0.126t} - 1}{25e^{0.126t} - 4}$ b) $\approx 34.78 \text{ g}$
c) $40, A: 42 \text{ g } B: 0 \text{ g}$

26. a) 29.3 g b) $\approx 34.60 \text{ g}$
c) $A: 0 \text{ g } B: 30 \text{ g } t = 70 \text{ min}$

27. a) $\approx 16.011 \text{ ft/s}^2 \approx 2510.31 \text{ ft/s}^2$
b) 16 ft/s^2 c) 13.34 min

28. a) $\left(\sqrt{H} - 4 \frac{A_h}{A_w} t\right)^2$ b) $0 \leq t \leq \frac{A_w \sqrt{H}}{4A_h}$
c) $576\sqrt{10} \text{ sec}$

29. a) 14.61 min b) 243 sec

30. $1024\sqrt{5} \text{ sec}$

31. a) $\sqrt{\frac{mg}{k}} \tanh\left(\sqrt{\frac{kg}{m}} t + \tanh^{-1}\left(\sqrt{\frac{k}{mg}} v_0\right)\right)$

b) $\sqrt{\frac{mg}{k}}$

c) $\frac{m}{k} \ln\left[\cosh\left(t + \tanh^{-1}\left(\sqrt{\frac{k}{mg}} v_0\right)\right)\right] + C_2$

32. a) $\sqrt{\frac{mg}{\kappa}} \sqrt{1 - e^{\frac{2\kappa}{m}(y - y_0)}}$ b) $\approx 400.11 \text{ ft}$

33. a) $\frac{m}{\kappa} v - \frac{1}{g} \left(\frac{mg}{\kappa}\right)^2 \ln\left|\frac{\kappa}{mg} v + 1\right|$ b) $\approx 615.93 \text{ ft}$

34. $t = 51.97 \text{ sec}$

35. $t = 181.49 \text{ sec}$

36. $t = 237 \text{ sec}$

37. $t = 1020 \text{ sec}$

38. $t = 18.7 \text{ sec}$

39. $t = 13.75 \text{ sec}$

40. a) $x(t) = 4.9t + 12.45e^{-2t} - 12.45$
b) $t \approx 22.9 \text{ sec}$

41. a) $x(t) = 4.9t - 22.55e^{-2t} + 22.55$
b) $t \approx 15.81 \text{ sec}$

42. a) $\approx 16.476 \text{ sec}$ b) $\approx 1534.81 \text{ m}$

43. a) $mv \frac{dv}{dx} + \kappa_0 x v^2 = 0$ b) $\kappa_0 = \frac{2m}{d^2} \ln(100)$

44. a) $t = 2.69 \text{ sec}$ b) $\approx 104.4 \text{ m}$

45. a) $x(t) = \frac{5}{2}e^{-12t} + 6t - \frac{5}{2}$ b) 6 m/s

c) $x(t) = \frac{25}{6}e^{-6t/5} - \frac{25}{6}$ d) 10 m/s

46. a) $\frac{dv}{dt} = -\frac{gR^2}{r^2}$ b) $v \frac{dv}{dr} = -\frac{gR^2}{r^2}$

c) $v^2 = \frac{2gR^2}{r} + v_0^2 - 2gR$ d) $v_e = \sqrt{2gR}$

e) $\approx 11.18 \text{ km/sec}$ f) $\approx 2.38 \text{ km/sec}$

47. a) -320 ft/sec b) 13.219 ft/sec

c) $\approx 179.347 \text{ ft}$ d) $\approx 179.347 \text{ ft}$

48. $-\frac{\pi}{750}\sqrt{h}; h(t) = \frac{1}{4}\left(C - \frac{\pi}{750}t\right)^2$

49. a) $-\frac{5}{6\pi h^{3/2}}$ b) $h(t) = \left(C - \frac{25}{12\pi}t\right)^{2/5}$

50. a) $r(t) = \frac{k}{\rho}t + r_0$ b) $\frac{dv}{dt} + \frac{3k}{\rho r_0 + kt}v = g$

51. $y(t) = 10 - \frac{1}{1,080\sqrt{5}}t^{3/2}$

Section 1.4

1. $y(t) = 3te^t + Ce^t$

2. $y(t) = \frac{1}{2}\sin t - \frac{1}{2}\cos t + Ce^{-t}$

3. $y(t) = e^{-t} \ln(1 + e^t) + Ce^{-t}$

4. $y(t) = e^{2t} + 1 + Ce^t$

5. $y(t) = te^{-t} + 1 + Ce^{-t}$

6. $y(x) = e^x + \frac{1}{2}e^{-x} \sin 2x + Ce^{-x}$

7. $y(x) = \frac{1}{2}\sin x + \frac{C}{\sin x}$

8. $y(t) = 1 + Ce^{\cos t}$

9. $y(x) = \frac{1}{\sec x + \tan x}(x - \cos x + C)$

10. $y(x) = \cos x \sin x + C \cos x$

11. $y(t) = (t^2 + C)\csc t$

12. $y(t) = C e^{\cos t - t}$

13. $y(x) = -3 + Ce^{-\frac{1}{2}\sin x}$

14. $y(x) = \frac{1}{4}e^{3x} + Ce^{-x}$

15. $y(t) = 1 + Ce^{\frac{1}{2}t^2}$
16. $y(x) = -\frac{1}{4}(2x^2 + 2x + 11) + Ce^{2x}$
17. $y(x) = \frac{3}{2} + \frac{C}{x^2}$
18. $y(t) = \frac{1}{2}t - \frac{7}{4} + Ce^{2t}$
19. $y(x) = \frac{1}{2} + Ce^{-2x}$
20. $y(t) = e^{-t} + Ce^{-2t}$
21. $y(t) = (t + C)e^{-2t}$
22. $y(t) = e^{3t} + Ce^{2t}$
23. $y(x) = e^{-x} + \frac{1}{2}x + \frac{1}{4} + Ce^{-2x}$
24. $y(x) = \frac{1}{2} + Ce^{-x^2}$
25. $y(t) = Ce^{t^2} - \frac{1}{2}$
26. $y(t) = \frac{5}{2} + Ce^{-t^2}$
27. $y(x) = e^{x^2}(x + C)$
28. $y(x) = \frac{1}{2}(x^2 - 1) + Ce^{-x^2}$
29. $y(t) = e^{2t}\left(\frac{1}{3}t^3 + C\right)$
30. $y(t) = t(t+1)^2 + C(t+1)^2$
31. $y(t) = \frac{1}{t^2}(\sin t + C)$
32. $y(t) = C e^{\sin 2t}$
33. $y(t) = \frac{2}{37}(6 \sin 3t + \cos 3t) + Ce^{-2t}$
34. $y(t) = -\frac{5}{3} + Ce^{3t}$
35. $y(x) = \frac{1}{e^{3x}}(x^2 + C)$
36. $y(t) = \frac{1}{3} + Ce^{-t^3}$
37. $y(x) = \frac{1}{3} + Ce^{-x^3}$
38. $y(t) = \frac{C}{t^3} - \frac{\cos t}{t^3}$
39. $y(x) = \frac{1}{4}x + \frac{1}{3}x + \frac{C}{x^3}$
40. $y(x) = \frac{1}{5}e^x + Ce^{-3x/2}$
41. $y(t) = \frac{1}{5}\left(t + \frac{6}{5}\right) + Ce^{-5t}$
42. $y(x) = -x \cos x + 2 \sin x + \frac{2}{x} \cos x + \frac{C}{x}$
43. $y(x) = \frac{1}{x}(e^x + C)$
44. $y(x) = \frac{1}{2} - \frac{1}{x} + \frac{C}{x^2}$
45. $x(y) = y^3 + \frac{C}{y^2}$
46. $y(t) = \frac{1}{t}(\sin t + C)$
47. $y(x) = \frac{1}{4}x^2 + \frac{C}{x^2}$
48. $y(x) = x^2(\sin x + C)$
49. $y(x) = \frac{C}{x^2} - \frac{1}{x^3}$
50. $y(t) = \frac{1}{4}t^2 + \frac{C}{t^2}$
51. $y(x) = \frac{1}{7}x^3 - \frac{1}{5}x + Cx^{-4}$
52. $y(x) = \frac{1}{xe^x}\left(\frac{1}{2}\cos 2x + C\right)$
53. $y(x) = \frac{1}{x} + Ce^{3x}$
54. $y(x) = 2x^3 + Cx^3e^{-2x}$
55. $y(x) = 3 + Cx^{3/2}$
56. $y(t) = 2e^{-t} + Ce^{-3t/2}$
57. $y(t) = 1 + Ce^{t^2/2}$
58. $y(x) = 4x^{1/2} + Cx^{-1/3}$
59. $y(x) = 3x + Cx^{-1/3}$
60. $y(x) = \frac{1}{x}(\ln x + C)$
61. $y(x) = \frac{1}{x^2}\left(\frac{1}{2}e^x + Ce^{-x}\right)$
62. $y(x) = \pm \sin(x + c)$
63. $y(x) = \frac{2x^{3/2}}{3(1+x)} + \frac{C}{1+x}$
64. $y(x) = \frac{\sin x + C}{x+1}$

66. $y(x) = \frac{e^{-x}}{x+1} (x^2 + C)$
67. $y(x) = \frac{x^2 + 3x + 3}{x+1} + \frac{Ce^x}{x+1}$
68. $y(x) = \frac{1}{3} (1+x^3) \ln(1+x^3) + C(1+x^3)$
69. $y(t) = t + 1 + \frac{\ln(t+1)}{(t+1)^2} + \frac{C}{(t+1)^2}$
70. $y(x) = \frac{5}{3} (x+2)^{-1} + C(x+2)^{-4}$
71. $y(x) = \frac{x+1}{x-1} (x+C)$
72. $y(x) = \frac{1}{x^2+4} \left(\frac{1}{5} x^5 + \frac{4}{3} x^3 + C \right)$
73. $y(t) = \frac{C}{1+e^t}$
74. $y(t) = \frac{C}{\sqrt{t^2+9}}$
75. $y(x) = x^2 e^{-2x} + C e^{-2x}$
76. $r(\theta) = \frac{1}{3} \sin^2 \theta + \frac{C}{\sin \theta}$
77. $y(t) = \sin t + C \cos t$
78. $y(x) = \sin x + C \cos x$
79. $y(x) = \sec x + \frac{C}{\sin x}$
80. $r(\theta) = \frac{1}{\sec \theta + \tan \theta} (\theta - \cos \theta + C)$
81. $r(\theta) = \sin \theta + C \cos \theta$
82. $y(t) = 2 + C e^{t-t^2}$
83. $x(y) = 2y^6 + Cy^4$
84. $x(y) = \frac{1}{y^2} \left((y^2 - 2y + 2) e^y + C \right)$
85. $y(x) = -x - 2 + C e^x$
86. $y(x) = e^{-4x} \left(\frac{1}{3} x^3 + C \right)$
87. $y(x) = 1 + \frac{C}{\sqrt{x^2+1}}$
88. $y(t) = 50 + C e^{-0.196t}$
89. $\frac{10(-\omega \cos \omega t + 500 \sin \omega t)}{\omega^2 + 25 \times 10^4} + C e^{-500t}$
90. $\frac{1}{122} \left(-5 \cos 60t + 6 \sin 60t + 6 e^{-50t} \right)$
91. $y(t) = -\frac{4}{3} + \frac{10}{3} e^{3t}$
92. $y(x) = 2x e^{2x} - 2e^{2x} + 5e^x$
93. $y(x) = 2 - 3(x^2 + 1)^{-\frac{3}{2}}$
94. $y(t) = \frac{1}{5} t^3 - \frac{12}{5t^2}$
95. $y(\theta) = -\frac{\cos \theta}{\theta} + \frac{\pi}{2\theta}$
96. $y(x) = 1 - \frac{7}{e^{x^2/2}}$
97. $y(t) = t^2 + \frac{1}{t^2}$
98. $y(t) = \frac{1}{(1+t^2)^2} \left(\tan^{-1} |t| - \frac{\pi}{4} \right)$
99. $y(t) = \frac{1}{2} (e^t + e^{-t})$
100. $y(t) = 2t - 4 + 5e^{-t/2}$
101. $y(x) = -\frac{1}{5} x - \frac{1}{25} + \frac{76}{25} e^{5x}$
102. $y(x) = \frac{2}{3} x - \frac{2}{9} + \frac{5}{9} e^{-3x}$
103. $y(x) = \frac{1}{x} (e^x + 2 - e)$
104. $x(y) = 2y^2 - \frac{49}{5} y$
105. $y(x) = 2x + 1 + \frac{5}{x}$
106. $y(x) = \frac{1}{18} (3x^2 - 1) e^{x^2} - \frac{17}{18} e^{2x^2}$
107. $y(x) = \frac{1}{x+1} (x \ln x - x + 21)$
108. $y(x) = -2 + \frac{3}{e^{\cos x}}$
109. $I(t) = \frac{E}{R} + \left(i_0 - \frac{E}{R} \right) e^{-Rt}$
110. $T(t) = T_m + (T_0 - T_m) e^{kt}$
111. $y(x) = 2 - 2e^{-x}$
112. $y(x) = 3x e^{2x}$
113. $y(x) = x + \frac{4}{x^2}$
114. $y(x) = \frac{7}{8} x^3 - \frac{64}{x^5}$

115. $y(x) = x \ln x + 7x$
 116. $y(x) = 0$
 117. $y(x) = \frac{1}{4}x^5 - 56x^{-3}$
 118. $y(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$
 119. $y(x) = x^4 + 9x^3$
 120. $y(x) = \frac{1}{2} - \frac{5}{2}e^{-x^2}$
 121. $y(x) = 1 + e^{-\sin x}$
 122. $y(x) = \frac{1}{1+x}(\sin x + 1)$
 123. $y(x) = -1 + e^{x + \frac{1}{2}x^2}$
 124. $y(x) = x^3 \sin x$
 125. $y(x) = e^{x^2} \left(x^3 + 5 \right)$
 126. $y(x) = \frac{1}{3} + \frac{16}{3} \left(x^2 + 4 \right)^{-3/2}$
 127. $y(x) = e^{-3x^2/2} \left(-2 + 3 \left(x^2 + 1 \right)^{3/2} \right)$
 128. $y(x) = e^{3x}$
 129. $y(x) = -2 + 3e^{3x}$
 130. $y(x) = \frac{2}{5}e^x - \frac{2}{5}e^{-3x/2}$
 131. $y(x) = 1 + \frac{1}{2}e^{-x} \sin 2x - e^{-x+\pi/2}$
 132. $y(x) = -6 + 2e^{-\frac{1}{2}\sin x}$
 133. $y = \frac{1}{x} \left(- \left(x^2 + 2x + 2 \right) e^{-x} + 5e^{-1} + e - 1 \right)$
 134. $y(x) = \frac{1}{3}e^{-x} + \frac{1}{3} \left(4e^4 - e^3 \right) e^{-4x}$
 135. $\frac{1}{6}x^3 \ln x - \frac{1}{36}x^3 + \frac{1}{2x} - \frac{17}{36x^3}$
 136. $y(x) = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{9}{10}x^{-3}$
 137. $y(x) = Ce^{-\cos x} - 2$
 138. $y(x) = \sin x \cos x - \cos x$
 139. $y(x) = \cos x \left(x^2 - \frac{\sqrt{2}\pi^2}{2} \right)$
 140. $y(x) = -\frac{1}{2} \cos 2x \cos x - \sin x + 7 \cos x$
 141. $y(t) = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{1}{2t^2}$
 $-\frac{1}{2}t^4 \cos 2t + \frac{1}{2}t^3 \sin 2t + \frac{1}{4}t^2 \cos 2t$
 142. $-t^3 + 2t^4 + \left(\pi - \frac{1}{4} \right) t^2$
 143. $-\frac{24}{37} \cos 3t - \frac{4}{37} \sin 3t + \left(y_0 + \frac{27}{37} \right) e^{t/2}$
 144. $y(t) = 1 + \frac{1}{2}e^{-4t} - \frac{1}{2}e^{-2t}$
 145. $y(t) = e^{t/2} \sin 5t$
 146. $y(t) = \frac{3}{2} - \frac{5}{2}e^{-2t}$
 147. $y(t) = 1 + e^{-\sin t}$
 148. $y(t) = 1$
 149. $y(t) = \frac{1}{e^t} \left(-\frac{1}{t} + 1 \right)$
 150. $y(x) = e^{-x} + \frac{1}{2}x + \frac{1}{4} + \frac{1}{4}e^{-2x-2}$
 151. $y(t) = \frac{1}{t^2}(\sin t - t \cos t)$
 152. $y(t) = \frac{\sin t}{t^2}$
 153. $y(t) = \sqrt{2} \csc t$
 154. $y(t) = \frac{1}{3} + \frac{5}{3}e^{-t^3}$
 155. $y(t) = \frac{1}{t}(\sin t - t \cos t - 2\pi)$
 156. $y(t) = \frac{1}{2}(\sin t - \cos t) + \frac{1}{2}e^\pi e^{-t}$
 157. $y(t) = \frac{1}{5}(2 \sin 2t + \cos 2t) + \frac{25}{5}e^{-t}$
 158. $y(t) = e^{2t}(7t + 3)$
 159. $y(t) = e^{2t}(7t + 3)$
 160. $\left(\frac{1}{2}t^2 + \frac{1}{2}t + \frac{1}{4} \right) e^{2t} + \frac{1}{6}e^{4t} - \frac{5}{12}e^{-2t}$
 161. $y(t) = -\frac{2}{3}t - \frac{2}{9} - e^{4t} + \frac{11}{9}e^{3t}$
 162. $t^3 - 3t^2 + 6t - 6 + \frac{1}{10} \sin 3t - \frac{3}{10} \cos 3t + \frac{63}{10}e^{-t}$
 163. $y(t) = \sin 2t - \frac{1}{2} \cos 2t + e^{-t} - \frac{1}{2}e^{-2t}$
 164. $y(t) = \frac{1}{4}e^{3t} + \left(y_0 - \frac{1}{4} \right) e^{-t}$
 165. $y(t) = -\frac{1}{2t} + \left(y_0 + \frac{1}{2} \right) t$

$$166. y(t) = \frac{1}{2a}e^{at} + \left(y_0 - \frac{1}{2a}\right)e^{-at}$$

$$167. y(t) = \frac{1}{3} + \left(y_0 - \frac{1}{3}\right)e^{-4t}$$

$$168. y(x) = x^2 \quad y(x) = \frac{8}{x}$$

$$169. y(t) = \frac{1}{13}(2\sin 2t + 3\cos 2t) - \frac{16}{13}e^{-3t}$$

$$170. 1 + 2e^{\cos x - 1}, \quad -1 + (2e + 2e^{-1})e^{\cos x - 1}$$

$$171. y(t) = 2t + 1 \quad y(t) = t + \frac{2}{t}$$

$$172. y(t) = 3e^{t-t^2} \quad y(t) = 3 \quad y(t) = t$$

$$173. y(x) = -\frac{1}{x}\cos x + \frac{1}{x^2}\sin x - \frac{1}{x^2}$$

$$174. y(x) = \frac{1}{2}(2x + 3)^{1/2} \ln(2x + 3)$$

$$175. \frac{10e^{3x} - 4}{3}$$

$$176. y(t) = 2t - 4 + 5e^{-t/2}$$

$$177. y(t) = \frac{1}{2}(e^t + e^{-t})$$

$$178. \frac{x_0\lambda_1}{\lambda_2 - \lambda_1}e^{-\lambda_1 t} + \frac{\lambda_2 y_0 - \lambda_1 y_0 - x_0\lambda_1}{\lambda_2 - \lambda_1}e^{-\lambda_2 t}$$

$$179. M + Ce^{-kt}$$

$$180. a) v(t) = \frac{mg}{k} + \left(v_0 - \frac{mg}{k}\right)e^{-kt/m} \quad b) \frac{mg}{k}$$

$$c) s(t) = \frac{mg}{k}t - \frac{m}{k}\left(v_0 - \frac{mg}{k}\right)e^{-kt/m} + \frac{m}{k}\left(v_0 - \frac{mg}{k}\right)$$

$$181. a) v(t) = \frac{\rho g}{4k}\left(\frac{k}{\rho}t + r_0\right) - \frac{\rho g r_0^4}{4k}\left(\frac{\rho}{kt + \rho r_0}\right)^3$$

$$b) r(t) = \frac{k}{\rho}t + r_0 \quad c) t \approx 33.3 \text{ sec}$$

$$182. a) P(t) = \frac{h}{k} + \left(P_0 - \frac{h}{k}\right)e^{kt}$$

b) increases constant decreases

$$c) T = \frac{1}{k} \ln\left(\frac{h}{h - kP_0}\right)$$

$$183. T(t) = \frac{1,300}{21} + \frac{5,000}{21}e^{-\frac{21}{20}kt}$$

Section 1.5

$$1. 43.1709 \text{ lb}$$

$$2. a) \approx 9.038 \text{ lb} \quad b) \approx 46 \text{ min} \quad c) 20$$

$$3. a) 67.032 \text{ lb} \quad b) 40.236 \text{ min} \quad c) 0$$

$$4. 21 \text{ lb}$$

$$5. 13.4 \text{ gal/min}$$

$$6. 50e^{-\frac{t}{100}}$$

$$7. 300 + t - \frac{13.5 \times 10^6}{(300 + t)^2}$$

$$8. 600 - t - 1.9647 \times 10^{-17} (600 - t)^7$$

$$9. 14.8 \text{ lb}$$

$$10. a) 50 \text{ min} \quad b) \approx 83.22 \text{ lb}$$

$$11. 279.797 \text{ lb}$$

$$12. a) \approx 7780 \text{ m} \quad b) \approx 61.65 \text{ min}$$

$$13. a) \approx 168.5 \quad b) \approx 41.13 \text{ sec}$$

$$14. 37.55 \text{ min}$$

$$15. a) 50e^{-t/20} \quad b) 150e^{-t/40} - 100e^{-t/20} \quad c) 11.51 \text{ min}$$

$$16. a) 100e^{-t/10} \quad b) 10te^{-t/10} \quad c) 36.79 \text{ gal}$$

$$17. a) e^{-t/2} \quad b) x_{n+1} = \frac{t^{n+1}e^{-t/2}}{(n+1)!2^{n+1}}; \quad \frac{n^n e^{-n}}{n!}$$

$$18. \approx 1.901 \text{ years}$$

$$19. \approx 202 \text{ lb}$$

$$20. \begin{cases} 4,000 - 3,998e^{-\frac{3}{800}t} & 0 \leq t \leq 35.5 \\ \frac{1}{320}(435.475 - t)^2 & 35.5 \leq t \leq 435.5 \end{cases}$$

$$21. 13.45 \text{ gal}$$

$$22. a) 50 \text{ min} \quad b) 82.32 \text{ lb}$$

$$23. a) 50 \text{ min} \quad b) 164.64 \text{ lb}$$

$$24. a) 25e^{-t/20} \quad b) 10.2 \text{ min} \quad c) 0$$

$$25. a) 100 \text{ min} \quad b) 50e^{-3t/100}$$

$$26. 300 - 280e^{-t/75}$$

$$27. 100c_0 + (y_0 - 100c_0)e^{-t/10}$$

$$28. a) 10c_0 + 10(1 - c_0)e^{-t/10} \quad b) 10c_0$$

$$29. a) 200 - 170e^{-t/50} \quad b) 30e^{-t/50}$$

30. a) $1,000 - 1,000e^{-t/100}$ b) 0.0975 lb/gal
 c) 2 d) 69.3 min e) 100 min
31. 64.38 lb
32. a) $500 t^2 e^{-t/50}$ b) 135.3 mg/gal
 c) 108.43 mg/gal
33. a) 400 min b) 7.5 lbs
 c) 10 lbs d) 200 min
34. a) 600 min b) $\frac{79}{160}$ c) $.4988 \text{ lb/gal}$
35. a) $\frac{3}{100}(\alpha - 1)Q$ b) ≈ 0.1472
36. a) $y' + \frac{1}{2}(10 + r)y = 4r$ b) $\frac{10}{3} \text{ gal/min}$
 c) 0.69077 min
37. a) 0.2808 kg/L b) 0.0598 kg/L
38. a) $\frac{25}{2}(1 - e^{-t/50})$ b) 4.92 lb c) 12.5 lb
39. a) $100(1 - e^{-3t/50})$ b) 78 lb c) 100 lb

Section 1.6

1. $x^2 + xy - 3y^2 = C$
2. $x^2 + 3x + y^2 - 2y = C$
3. $x + y \cos x = C$
4. $ax^2 + 2bxy + cy^2 = E$
5. $x^3 + xy - y^2 = C$
6. $x^2y - y = C$
7. $y^2 = x^2 - C_1x$
8. $y^2 = x^2(\ln x + C)$
9. $y = x(\ln x + C)^2$
10. $y^3 = 3x^3(\ln x + C)$
11. $y = -x \ln(\ln Cx)$
12. $y(x) = \frac{x}{\ln \frac{1}{Cx}}$
13. $x^2 + 2y^2 = Cx^6$
14. $y = C(y^2 + x^2)$
15. $4x^2 + y^2 = x^2(\ln x + C)^2$
16. $y + \sqrt{x^2 + y^2} = Cx^2$
17. $y^3 = 3 + Ce^{-3x^2}$
18. $y^3 = \frac{7x}{15 + 7Cx^7}$
19. $y^2 = \frac{e^{2x}}{\ln x + C}$
20. $y^3 = \frac{1}{x^3} \left(\frac{3}{2} \sqrt{1 + x^4} + C \right)$
21. $y^3 = e^{-x}(x + C)$
22. $y = \sqrt[3]{x^4 + Cx}$
23. $y = \ln(x^2 e^{2x} + Cx^2)$
24. $\sin^2 y = 4x^2 + Cx$
25. $x^2 - 2xe^y - e^{2y} = C_1$
26. $(x + y)^2 = Cxe^{\frac{y}{x}}$
27. $y = \frac{1}{-x^2 + Cx}$
28. $x^3 - x^2y + 2x + 2y^3 + 3y = C$
29. $e^x \sin y + 2y \cos x = C$
30. $y \ln x + 3x^2 - 2y = C$
31. $xe^{2y} - \sin xy + y^2 = C$
32. $x^2 + y^2 = C$
33. $x^2 - x + \frac{3}{2}y^2 + 7 = C$
34. $\frac{5}{2}x^2 + 4xy - 2y^4 = C$
35. $x \sin y + y \cos x - \frac{1}{2}y^2 = C$
36. $x^2y^2 - 3x + 4y = C$
37. $x \ln x + y \ln x - y = C$
38. $\frac{1}{2}x^2 - xy^3 - y^2 \cos x = C$
39. $\frac{1}{4}x^4 + xy^3 = C$
40. $x^3y + xe^y - y^2 = C$
41. $xy - 2xe^x + 2e^x - 2x^3 = C$
42. $x^2y^2 - 3x + 4y = C$
 $xy + x + y - 3 \ln|xy| = C$

43. $x^3y^3 - \arctan(3x) = C$
44. $-2xy + \frac{5}{2}y^2 = C$
45. $\frac{1}{2}x^2 - xy = C$
46. $\frac{1}{2}x^2 + xy = C$
47. $x^2y^2 + x = C$
48. $x + xe^x y + 2y + C = 0$
49. $x^2y^3 + x - \ln|y| = C$
50. $x^2 + xy - y^2 = C$
51. $ye^x - (x-1)e^x + y = C$
52. $e^{xy} - \frac{x}{y} = C$
53. $\ln|\sec x| + \cos x \sin y = C$
54. $\frac{1}{2}x^4 - \frac{1}{2}x^2y^2 - 2xy + 3x = C$
55. $\frac{1}{2}x^2 + x \sin y - y^2 = C$
56. $\frac{1}{2}x^2 + \sin^{-1} \frac{x}{y} + y = C$
57. $x^2 + xy^2 - \sin(x+y) - e^y = C$
58. $\sin(xy) - \frac{3}{2}y^{2/3} + 2 \arcsin x = C$
59. $x^2 + \sin(xy) - y^2 = C$
60. $e^x \sin y - x^3 + y^{1/3} = C$
61. $y \sin^2 x - xy + 2e^{xy^2} = C$
62. $x^2 - y^{-2} + \ln|y| = C$
63. $\ln|x| - \frac{x}{y} = C$
64. $\frac{1}{2}x^2y^2 - \ln x + \ln y = C$
65. $e^x \sin y + 2y \cos x = C$
66. $x^2e^x \sin y = C$
67. $x - \frac{1}{2} \ln(x^2 + y^2) = C$
68. $x^2y^3 - 2x^3y^2 = C$
69. $x^2 - y^2 = Cx$
70. $\arctan \frac{y}{x} - \frac{1}{2} \ln \left(1 + \frac{y^2}{x^2} \right) - \ln x = C$
71. $\frac{y^2}{x^2} - 4 \ln y - 2 \ln x = C$
72. $e^{3x} \left(x^2y + \frac{1}{3}y^3 \right) = C$
73. $xy + y \cos y - \sin y = C$
74. $e^x \sin y + y^2 = C$
75. $x^3y + 3x^2 + y^3 = C$
76. $x + x^3 \sin y = C$
77. $2x - \frac{y}{x} + \frac{1}{2}y^2 = C$
78. $3x - \frac{y}{x} + \frac{1}{2}y^2 = C$
79. $x^2y^2 + x^2y + x^4 = C$
80. $\frac{1}{3}x^3 - \ln|x| - \frac{y}{x} = C$
81. $x^2y^{-3} - \frac{1}{y} = C$
82. $x^2y^2 + \cos^2 x - y^2 = -3$
83. $\frac{1}{3}x^3 + x^2y + xy^2 - y = \frac{4}{3}$
84. $e^x + xy + 2y + e^y(y-1) = 3$
85. $x^2 - xy + y^2 = 7$
86. $y(x) = \frac{x - \sqrt{x^2 + 24x^3 - 8x - 16}}{4}$
87. $xy + \frac{1}{4}x^4 + \frac{1}{4}y^4 = 4$
88. $y = \tan \left(x^3 + x + \frac{\pi}{4} \right)$
89. $-2t + y \cos t + \frac{1}{4}y^4 = -\frac{3}{4}$
90. $-t^3y - t + \frac{1}{4}y^4 = -\frac{23}{4}$
91. $y = 2x + \frac{3(1 - e^{6x})}{1 + e^{6x}}$
92. $2y = x - \sqrt{4 - 3x^2}$
93. $\frac{1}{2}t^2y^2 + \sin t + \frac{1}{2}e^{2y} = \frac{3}{2}$
94. $\sin ty + t + e^{y^2} = \pi + 1$
95. $ty^2 - t + \ln|y| = 0$
96. $ye^x + x - y = e$
97. $x^2y^2 + 4x - 6y = 12$

$$98. y = \frac{1}{x^4 \left(\ln 2 - \frac{1}{16} - \ln x \right)}$$

$$99. y = \left(\frac{139e^{15x} - 3e^{-2x}}{17} \right)^{1/3}$$

$$100. y(5e^{-x} - 4x + 4)^{1/3} = -2$$

$$101. y = \left(\frac{1}{3}x + \frac{1}{3}x^{-1/2} \right)^2$$

$$102. y = -\frac{\sqrt{228 - 2x^4}}{x}$$

$$103. y = xe^{\frac{1+\ln 4}{x}-1}$$

$$104. y = \frac{12x + 9 + (1 - 4x)e^{4x}}{3 - e^{4x}}$$

$$105. e^{t+y} + t^3 + y^2 = 1$$

$$106. 4xy + x^2 - 5x + 3y^2 - y = 8$$

$$107. e^{xy} - \frac{x}{y} = e - 1$$

$$108. y^2 \ln|t| + t \cos y - 2 = 0$$

$$109. x \tan y - 2x + \ln y = 0$$

$$110. x^2y - 3x^3 + y^2 + y = 6$$

$$111. y = \frac{t^2 - 25}{\ln(t^2 + 1) - 2}$$

$$112. y^2 e^{3xy} - x = 1$$

$$113. y = -\frac{25}{1 + x^2}$$

$$114. x^2 \cos y + x^3 y - \frac{1}{2}y^2 = -2$$

$$115. x^2 y^3 - 2x^3 y^2 = C$$

$$116. 3x^4 y^2 + x^5 y^3 = C$$

$$117. x^3 y^2 + x^4 y^3 = C$$

$$118. \frac{1}{y^2} = \frac{1}{2}x - \frac{1}{20} + Ce^{-10x}$$

$$119. y = \frac{2}{x^3 + Cx}$$

$$120. y^{-2} = -\frac{1}{2}e^{2x} + Ce^{-2x}$$

$$121. \left(\frac{1}{5}(x-2)^{3/2} + \frac{C}{\sqrt{x-2}} \right)^2$$

$$122. y^3 = \frac{3}{4}e^x + Ce^{-3x}$$

$$123. \frac{1}{y^2} = -x - \frac{1}{2} + Ce^{2x}$$

$$124. -\frac{x}{\ln|x| + C}$$

$$125. 3xy^2 + x^3 = C$$

$$126. \frac{x}{\ln|x| + C}$$

$$127. \theta \arcsin(\ln|\theta| + C)$$

$$128. xe^{Cx}$$

$$129. (y+2)^2 + 2(x+1)(y+2) - 3(x+1)^2 = C$$

$$130. 2 \arctan \frac{y-3}{x+2} - \ln((x+2)^2 + (y-3)^2) = C$$

$$131. \left(x + \frac{8}{5}\right)^2 - \left(x + \frac{8}{5}\right)\left(x + \frac{6}{5}\right) - \left(x + \frac{6}{5}\right)^2 = C$$

$$132. (2x-3+2y)^3 = C_1(y+2x-2)^2$$

$$133. (3+xy)e^{x+y} = C$$

$$134. a) \frac{8}{x}\sqrt{\frac{x^3}{3}-9} \quad b) \sqrt{\frac{485}{3}} \text{ ft/s}$$

Section 1.7

$$1. y' = ky'(t)$$

$$2. y' = \frac{k}{\sqrt{y(t)}}$$

$$3. \approx 24300$$

$$4. \frac{1000}{3}$$

$$5. \approx 0.1099$$

$$6. \approx 6.3093 \text{ hrs}$$

$$7. a) 0.1P\left(1 - \frac{P}{10}\right) - 0.1 \quad b) 200$$

$$c) 484 \text{ rabbits} \quad d) \approx 153.7 M$$

$$8. 20$$

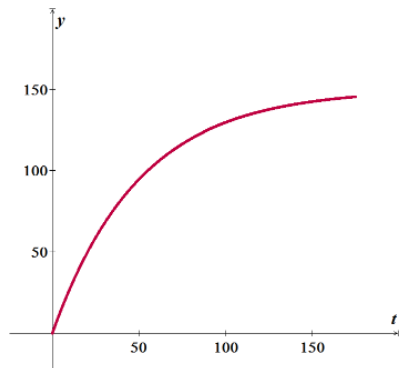
$$9. a) P(t) = \left(\frac{1}{2}kt + \sqrt{P_0}\right)^2 \quad b) 256$$

$$10. a) \frac{240}{20-t}, 2003 \quad b) \infty$$

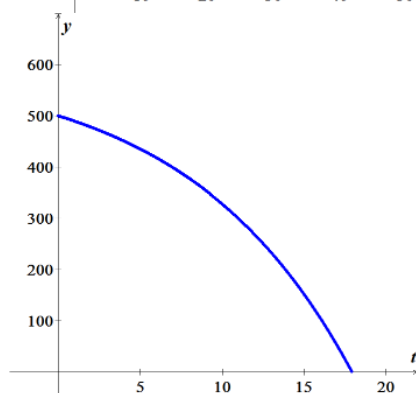
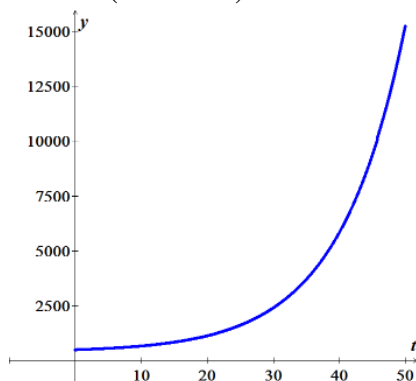
$$11. a) \frac{P_0}{1 - P_0 kt} \quad b) \infty \quad c) \frac{P_0}{1 + P_0 kt} \quad d) 0$$

12. a) $\frac{B_0 P_0}{D_0} = M$ b) ≈ 27.69 mths
c) ≈ 44.22 mths

13. a) $150(1 - e^{-0.02t})$ b) 150 mg
c) ≈ 115 hrs

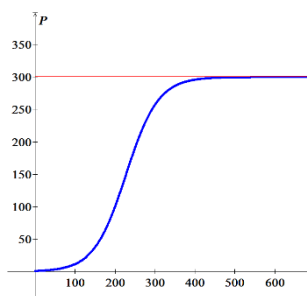


14. $10b + (500 - 10b)e^{0.1t}$



15. a) $\frac{200}{3e^{-0.08t} + 1}$ b) 200

16. a) $\frac{AP_0 e^{kt}}{A - P_0 + P_0 e^{kt}}$ b) A



17.

18. a) Ce^{-kt} b) $\frac{y_0}{1 + ky_0 t}$ c) $e^{-0.1t}$

19. a) $-M \ln \frac{M}{4}$ b) $4e^{-(\ln 4)e^{-t}}$ c) K

20. a) $\approx 3.23 \times 10^7$ kg b) ≈ 1.55 yrs

21. a) 17.5 b) ≈ 4.86 yrs

22. a) \$6,321,205.59 b) \$10,000,000

23. ≈ 9.5 yrs

24. ≈ 5 yrs

25. ≈ 7955 yrs

26. a) $50e^{-0.053t}$ b) ≈ 40.5 mg c) ≈ 13 hrs

27. 12.5%

28. ≈ 4985 yrs

Section 1.8

1. node a: $i_s - i_1 = 0$

node b: $i_1 + i_c = 0$

node c: $-i_c - i_1 = 0$

node d: $i_1 - i_s = 0$

2. node a: $i_1 + i_4 - i_2 - i_5 = 0$

node b: $i_2 + i_3 - i_1 - i_b - i_a = 0$

node c: $i_b - i_3 - i_4 - i_c = 0$

node d: $i_5 + i_a + i_c = 0$

3. path a: $-v_1 + v_2 + v_4 - v_b - v_3 = 0$

path b: $-v_a + v_3 + v_5 = 0$

path c: $v_b - v_4 - v_c - v_6 - v_5 = 0$

path d: $-v_a - v_1 + v_2 - v_c + v_7 - v_d = 0$

4. $\frac{10}{17}(\sin 2t - 4\cos 2t) + \frac{40}{17}e^{-t/2}$
5. $\frac{25}{2}(e^{-t/10} - e^{-t/2})$
6. $10 - \frac{25}{2}e^{-t/10} + \frac{5}{2}e^{-t/2}$
7. $\frac{10}{37}(6\sin 3t + \cos 3t - e^{-t/2})$
8. $I(t) = 300 - 20t - 300e^{-t/10}$
9. $\frac{40}{901}(30\sin 3t + \cos 3t - e^{-t/10})$
10. $\frac{40}{1 + 400\pi^2}(\sin 2\pi t - 20\pi \cos 2\pi t + 20\pi e^{-t/10})$
11. $\frac{1}{1010}(-10\cos 100t + \sin 100t) + \frac{1}{101}e^{-10t}$
 $\frac{10}{101}\sin 100t + \frac{1}{101}\sin 100t - \frac{1}{101}e^{-10t}$
12. $\frac{1}{2}(-\cos 100t + \sin 100t) + \frac{1}{2}e^{-100t}$
 $\frac{1}{2}(\cos 100t + \sin 100t + e^{-10t})$
13. $\frac{5}{61}(6\sin 120t + 5\cos 120t) + \frac{36}{61}e^{-100t}$
 $14.754\cos 120t - 12.295\sin 120t - 14.754e^{-100t}$
14. $\frac{1}{10^8 + 1}(\sin 100t + 10^4 \cos 100t - 10^4 e^{-10^6 t})$
 $\frac{1}{10^8 + 1}(\sin 100t + 10^4 \cos 100t - 10^4 e^{-10^6 t})$
15. $EC(1 - e^{-t/RC})$
16. $\frac{1}{R}(E + (RI_0 - E)e^{-Rt/L})$
17. $\frac{E_0}{R} - \left(\frac{E_0}{R} - I_0\right)e^{-(R/L)t}$
 $\frac{R^2 E_0}{L + E_0 L^2 \omega^2} \left(\sin \omega t - \frac{L\omega}{R} \cos \omega t\right)$
18. $\left(I_0 + \frac{R\omega E_0}{1 + E_0 L\omega^2}\right)e^{-(R/L)t}$
19. $\frac{E}{R} - \frac{E}{R}e^{-(R/L)t}$
20. $\frac{1}{10}(1 - e^{-50t})$
- 21.
- 22.
23. a) $CV(1 - e^{-t/CR})$ b) CV c) $CVe^{-(t-t_1)/CR}$
24. a) $12e^{-4t}$ b) $\frac{30}{61}(5\sin 60t + 6\cos 60t - 5e^{-50t})$
25. $E_0 e^{\frac{4-t}{RC}}$
26. a) $\frac{3}{5} - \frac{3}{5}e^{-500t}$ b) $\frac{3}{5}$
c) $\frac{10E_0(-\omega \cos \omega t + 500 \sin \omega t - \omega e^{-500t})}{\omega^2 + 25 \times 10^4}$
27. a) $\frac{1}{100} - \frac{1}{100}e^{-50t}$ b) $\frac{1}{2}e^{-50t}$
28. a) $\frac{1}{1000} - \frac{1}{500}e^{-200t}$ b) $\frac{1}{1000}$
29. $i(t) = \begin{cases} 60 - 60e^{-t/10} & 0 \leq t \leq 20 \\ 60(e^2 - 1)e^{-t/10} & t > 20 \end{cases}$
30. $E_0 C + (q_0 - E_0 C) \left(\frac{k_1}{k_1 + k_2 t}\right)^{1/Ck_2}$
31. $E = \begin{cases} 0 & 0 \leq t < 4, 6 \leq t < 10, 12 \leq t < 16 \\ 12e^{\frac{4-t}{RC}} & 4 \leq t < 6 \\ 12e^{\frac{10-t}{RC}} & 10 \leq t < 12 \\ 12e^{\frac{16-t}{RC}} & 16 \leq t < 18 \\ 12e^{\frac{22-t}{RC}} & 22 \leq t < 24 \end{cases}$

Section 1.9

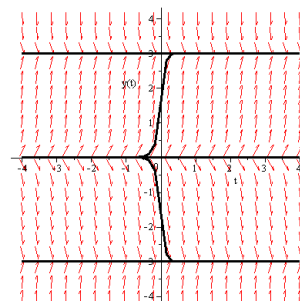
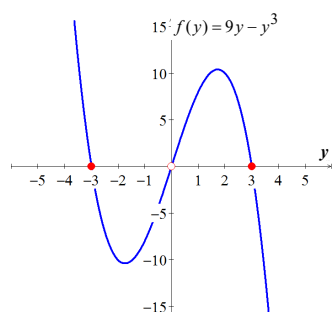
- satisfied
- not satisfied
- satisfied
- satisfied
- satisfied
- not satisfied
- satisfied
- satisfied
- satisfied
- satisfied
- satisfied
- $y(t) = \cos t \quad (2n-1)\pi \leq t \leq 2\pi$
- $f' = 2y^{-1/3}$ is not continuous at $y = 0$
- a) $-1 \pm \sqrt{t^2 - 3}$ b) $(\sqrt{3}, \infty)$

Section 1.10

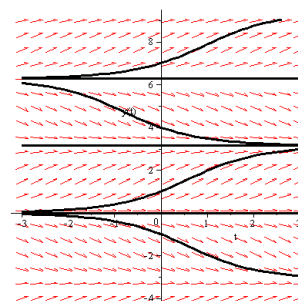
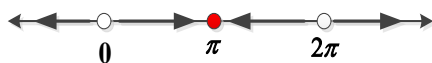
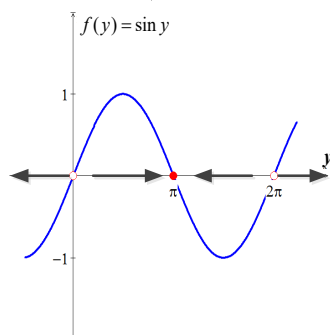
1. equilibrium at 3 – stable
2. equilibrium: $-1, 1$ both unstable
3. $-2, 1$ are asymptotically stable
 $-\frac{1}{2}, 2$ are unstable

4. smaller unstable – other is asymptotically stable
5. Asymptotically stable
6. $y = -1$ is asymptotically stable
 $y = 4$ is unstable

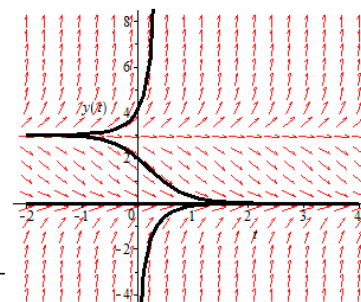
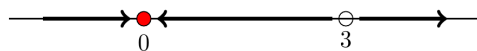
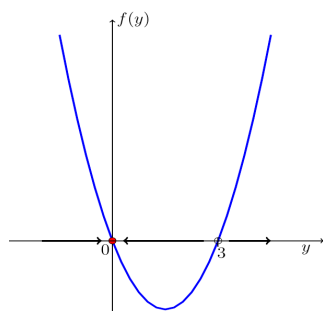
7.



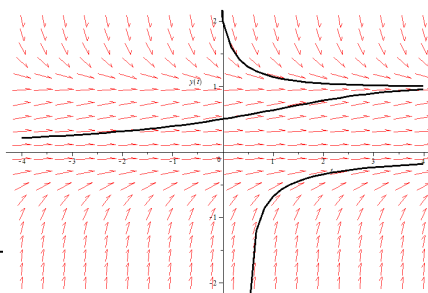
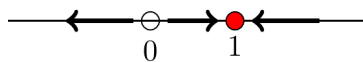
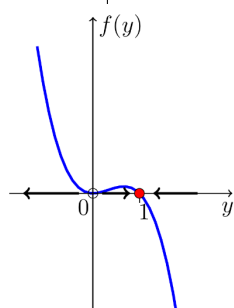
8.



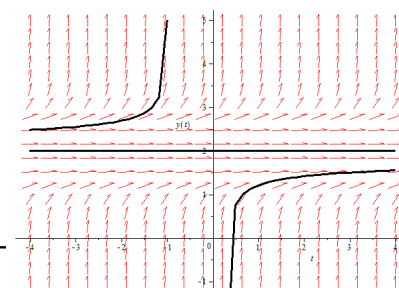
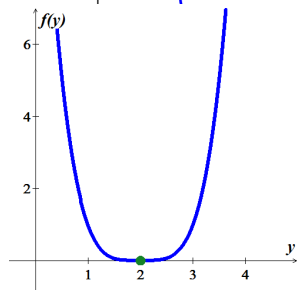
9.



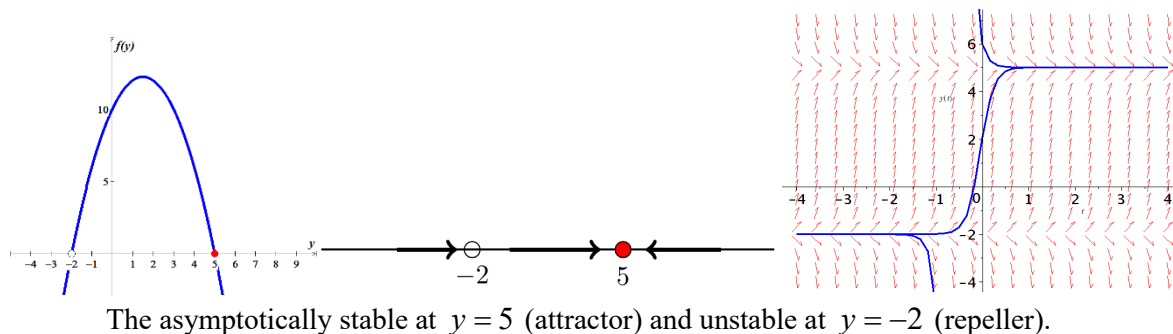
10.



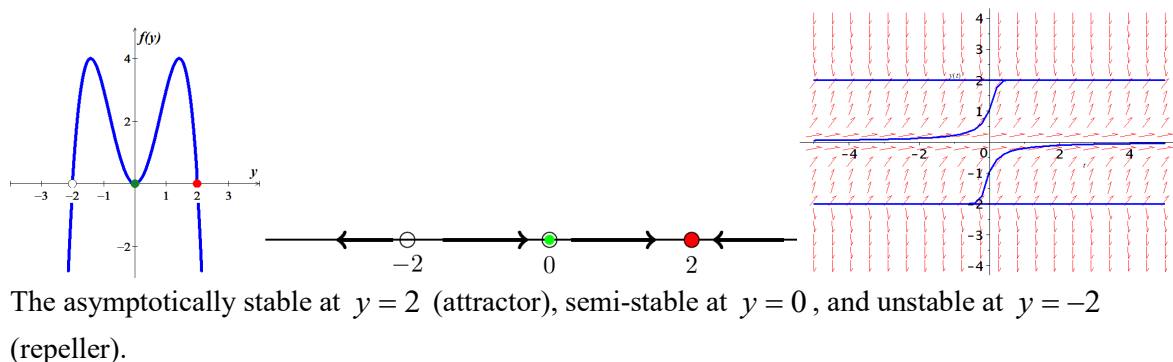
11.



12.



13.



Lecture 2

Section 2.1

1. Linear and inhomogeneous
2. Linear and inhomogeneous
3. Nonlinear
4. Linear and inhomogeneous
7. Linearly independent
8. $\frac{1}{4}e^t + \frac{3}{4}e^{-3t}$
9. Linearly independent
10. Linearly independent
11. Linearly dependent
12. Linearly independent
13. Linearly independent
14. Linearly dependent
15. Linearly dependent.
16. Linearly independent
17. Linearly independent
18. Linearly independent
19. Linearly dependent
20. Linearly dependent
21. $\begin{cases} y' = v \\ v' = -2v + 3y \end{cases}$
22. $\begin{cases} y' = v \\ v' = -3v - 4y + 2 \cos 2t \end{cases}$

23. $\begin{cases} y' = v \\ v' = -2v - 2y + 2 \sin 2\pi t \end{cases}$
24. $\begin{cases} y' = v \\ v' = -\mu(t^2 - 1)v - y \end{cases}$
25. $\begin{cases} y' = v \\ v' = -v - \frac{1}{4}y \end{cases}$
26. e^{-2t}
27. $\frac{e}{4}e^t + \frac{1}{2e^4}e^{-t+3}$
28. $C_2 \sin t$
29. $-\cos t + \sin t$
30. $2e^{2t} - 4te^{2t}$
31. $-4 + \frac{4}{e}e^{t/2}$
32. e^{2t+2}
33. $18 \ln t - 9 \ln 3t$
- 34.

35. $-\frac{1}{2}t^3 + \frac{1}{2t}$
36. $\sin \pi t$
37. $x - 3x \ln x + 2x^2$
38. $\frac{4}{3}e^x - \frac{1}{3}e^{-2x}$
39. $\frac{3}{2}e^x - 3e^{2x} + \frac{3}{2}e^{3x}$
40. $2e^x - 2xe^x + x^2e^x$
41. $-12e^x + 13e^{2x} - 10xe^{2x}$
42. $3 - \frac{1}{3}\sin 3x$
43. $2e^x - e^x \cos x - e^x \sin x$
44. $-\frac{19}{2}x + 23x^2 - \frac{15}{2}x^3$
45. $2x - x^{-2} + x^{-2} \ln x$
46. a) $2 \cos t$ b) no soln c) ∞ solns

Section 2.2

1. $C_1 + C_2 e^{-x}$
2. $C_1 e^{-2x} + C_2 e^{2x}$
3. $C_1 + C_2 e^{8x}$
4. $C_1 e^{-6x} + C_2 e^{6x}$
5. $C_1 \cos 3x + C_2 \sin 3x$
6. $C_1 \cos 4x + C_2 \sin 4x$
7. $C_1 \cos 5x + C_2 \sin 5x$
8. $C_1 e^{-8x} + C_2 e^{8x}$
9. $e^{-x/2} \left(C_1 \cos \frac{\sqrt{3}}{2} x + C_2 \sin \frac{\sqrt{3}}{2} x \right)$
10. $C_1 e^{\frac{-1-\sqrt{5}}{2} x} + C_2 e^{\frac{-1+\sqrt{5}}{2} x}$
11. $C_1 e^{-x} + C_2 e^{2x}$
12. $C_1 e^{-2x} + C_2 e^{3x}$
13. $C_1 e^{-3x} + C_2 e^{2x}$
14. $C_1 e^{\frac{1-3\sqrt{5}}{2} x} + C_2 e^{\frac{1+3\sqrt{5}}{2} x}$
15. $C_1 e^{-3t} + C_2 e^{4t}$
16. $(C_1 + C_2 t) e^{-t}$
17. $e^{-x} (C_1 \cos \sqrt{2} x + C_2 \sin \sqrt{2} x)$
18. $C_1 e^{-3x} + C_2 e^x$
19. $C_1 e^{-x} + C_2 e^{3x}$
20. $e^x (C_1 \cos \sqrt{2} x + C_2 \sin \sqrt{2} x)$
21. $e^{-x} (C_1 \cos \sqrt{3} x + C_2 \sin \sqrt{3} x)$
22. $C_1 e^{-5x} + C_2 e^{3x}$
23. $e^{-t} (C_1 \cos 4t + C_2 \sin 4t)$
24. $e^{-x} (C_1 \cos 2x + C_2 \sin 2x)$
25. $C_1 e^x + C_2 e^{2x}$
26. $C_1 e^x + C_2 e^{-4x}$
27. $C_1 e^{(-2-\sqrt{5})x} + C_2 e^{(-2+\sqrt{5})x}$
28. $(C_1 + C_2 t) e^{2t}$
29. $(C_1 + C_2 t) e^{-2t}$
30. $e^{2x} (C_1 \cos x + C_2 \sin x)$
31. $e^{-2x} (C_1 \cos x + C_2 \sin x)$
32. $C_1 e^{-5x} + C_2 e^x$
33. $e^{-2x} (C_1 \cos \sqrt{3} x + C_2 \sin \sqrt{3} x)$
34. $e^{-2x} (C_1 \cos \sqrt{5} x + C_2 \sin \sqrt{5} x)$
35. $C_1 e^{-5x} + C_2$
36. $C_1 e^{-3x} + C_2 e^{-2x}$
37. $(C_1 + C_2 x) e^{-3x}$
38. $(C_1 + C_2 t) e^{3t}$
39. $e^{3x} (C_1 \cos 4x + C_2 \sin 4x)$
40. $(C_1 + C_2 x) e^{-4x}$
41. $C_1 e^{(-4-4\sqrt{2})x} + C_2 e^{(-4+4\sqrt{2})x}$
42. $C_1 e^{4x} + C_2 e^{5x}$
43. $(C_1 + C_2 x) e^{5x}$
44. $(C_1 + C_2 x) e^{-7x}$
45. $C_1 e^{-x} + C_2 e^{3x/2}$
46. $C_1 e^{-x} + C_2 e^{x/2}$
47. $e^{x/2} (C_1 \cos \frac{1}{2} x + C_2 \sin \frac{1}{2} x)$
48. $e^{-x/2} \left(C_1 \cos \frac{\sqrt{5}}{2} x + C_2 \sin \frac{\sqrt{5}}{2} x \right)$
49. $C_1 e^{-x/2} + C_2 e^{2x}$
50. $e^{3x/4} \left(C_1 \cos \frac{\sqrt{23}}{4} x + C_2 \sin \frac{\sqrt{23}}{4} x \right)$
51. $e^x (C_1 \cos \sqrt{3} x + C_2 \sin \sqrt{3} x)$
52. $C_1 + C_2 e^{-5x/2}$

53. $C_1 e^{-3x} + C_2 e^{x/2}$
54. $C_1 e^{-4x} + C_2 e^{x/2}$
55. $C_1 \cos \frac{\sqrt{3}}{3} x + C_2 \sin \frac{\sqrt{3}}{3} x$
56. $C_1 + C_2 e^{-x}$
57. $e^{-x/3} \left(C_1 \cos \frac{\sqrt{2}}{3} x + C_2 \sin \frac{\sqrt{2}}{3} x \right)$
58. $C_1 e^{\frac{-11-\sqrt{205}}{6} x} + C_2 e^{\frac{-11+\sqrt{205}}{6} x}$
59. $C_1 e^{\frac{10-\sqrt{61}}{3} x} + C_2 e^{\frac{10+\sqrt{61}}{3} x}$
60. $C_1 + C_2 e^{-x/4}$
61. $(C_1 + C_2 x) e^{-x/2}$
62. $(C_1 + C_2 x) e^{x/2}$
63. $e^{-x/2} \left(C_1 \cos \frac{x}{2} + C_2 \sin \frac{x}{2} \right)$
64. $(C_1 + C_2 x) e^{x/2}$
65. $(C_1 \cos \sqrt{3} x + C_2 \sin \sqrt{3} x) e^{x/2}$
66. $e^x \left(C_1 \cos \frac{\sqrt{3}}{2} x + C_2 \sin \frac{\sqrt{3}}{2} x \right)$
67. $(C_1 + C_2 x) e^{3x/2}$
68. $(C_1 + C_2 x) e^{-5x/2}$
69. $C_1 e^{-4x} + C_2 e^{3x}$
70. $C_1 e^{-3t} + C_2 e^{4t/3}$
71. $C_1 e^{-4t/3} + C_2 e^{5t/2}$
72. $C_1 e^{-5x/2} + C_2 e^{x/3}$
73. $C_1 e^{-x} + C_2 e^{-7x/6}$
74. $C_1 e^{-x/4} + C_2 e^{3x/2}$
75. $C_1 e^{-x/3} + C_2 e^{x/3}$
76. $(C_1 + C_2 x) e^{-x/3} C_1 e^x + C_2 e^{7x/6}$
77. $(C_1 + C_2 x) e^{2x/3}$
78. $(C_1 + C_2 x) e^{-4x/3}$
79. $C_1 e^{-x/4} + C_2 e^{2x/3}$
80. $e^{x/4} \left(C_1 \cos \frac{\sqrt{6}}{4} x + C_2 \sin \frac{\sqrt{6}}{4} x \right)$
81. $C_1 e^{-x/4} + C_2 e^x$
82. $(C_1 + C_2 x) e^{3x/4}$
83. $(C_1 + C_2 x) e^{-x/5}$
84. $(C_1 + C_2 x) e^{x/5}$
85. $C_1 e^{-4x/5} + C_2 e^{3x/5}$
86. $(C_1 + C_2 x + C_3 x^2) e^{-3x}$
87. $C_1 e^{-3x} + C_2 e^{-x} + C_3 e^x$
88. $C_1 e^x + (C_2 + C_3 x) e^{-2x}$
89. $C_1 e^{x/3} + e^{3x} (C_2 \cos x + C_3 \sin x)$
90. $(C_1 + C_2 x + C_3 x^2) e^{2x}$
91. $C_1 e^{-3x} + C_2 e^{-x} + C_3 x e^{-x}$
92. $C_1 e^{2x} + e^{-x} (C_2 \cos 2x + C_3 \sin 2x)$
93. $C_1 e^x + C_2 e^{(-1-\sqrt{5})x} + C_3 e^{(-1+\sqrt{5})x}$
94. $C_1 e^{-x} + C_2 e^x + C_3 e^{6x}$
95. $(C_1 + C_2 x) e^{-2x} + C_3 e^{2x}$
96. $C_1 e^{-x} + C_2 e^{3x} + C_3 e^{5x}$
97. $C_1 e^{-3x} + C_2 e^{-2x} + C_3 e^{2x}$
98. $C_1 + C_2 e^{-x} + C_3 e^{5x}$
99. $C_1 e^{2x} + e^{-x/2} \left(C_2 \cos \frac{\sqrt{3}}{2} x + C_3 \sin \frac{\sqrt{3}}{2} x \right)$
100. $C_1 e^{-x} + (C_2 + C_3 x) e^{3x}$
101. $C_1 e^{-3x} + C_2 e^{-2x} + C_3 e^{2x}$
102. $C_1 e^x + e^{-x} (C_2 \cos x + C_3 \sin x)$
103. $C_1 e^{2x} + e^{-x/2} \left(C_2 \cos \frac{\sqrt{7}}{2} x + C_3 \sin \frac{\sqrt{7}}{2} x \right)$

104. $(C_1 + C_2x + C_3x^2)e^{-x}$
105. $(C_1 + C_2x + C_3x^2)e^{2x}$
106. $C_1 + C_2x + e^{-x/2}\left(C_3 \cos \frac{\sqrt{3}}{2}x + C_4 \sin \frac{\sqrt{3}}{2}x\right)$
107. $(C_1 + C_2x)e^{-x} + (C_3 + C_4x)e^x$
108. $C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x + x\left(C_3 \cos \frac{\sqrt{3}}{2}x + C_4 \sin \frac{\sqrt{3}}{2}x\right)$
109. $C_1 \cos 2x + C_2 \sin 2x + C_3 e^{-3x} + C_4 e^{3x}$
110. $C_1 \cos x + C_2 \sin x + C_3 x \cos x + C_4 x \sin x$
111. $C_1 + C_2x + e^{-x/2}\left(C_3 \cos\left(\frac{\sqrt{3}}{2}x\right) + C_4 \sin\left(\frac{\sqrt{3}}{2}x\right)\right)$
112. $e^x(C_1 \cos x + C_2 \sin x) + e^{-x}(C_3 \cos x + C_4 \sin x)$
113. $C_1 e^x + C_2 e^{-x} + e^{-x}(C_3 \cos 3x + C_4 \sin 3x)$
114. $C_1 e^{(2+i\sqrt{2})t} + C_2 e^{(2-i\sqrt{2})t} + C_3 \cos t + C_4 \sin t$
115. $(C_1 + C_2t + C_3t^2 + C_4t^3)e^{-2t}$
116. $(C_1 + C_2t)e^{-2t} + e^{2t}(C_3 \cos 2t + C_4 \sin 2t)$
117. $(C_1 + C_2t + C_3t^2 + C_4t^3)e^{-t}$
118. $C_1 e^{-t} + C_2 e^{2t} + C_3 \cos \sqrt{3}t + C_4 \sin \sqrt{3}t$
119. $(C_1 + C_2t + C_3t^2)e^{-t} + C_4 e^{2t}$
120. $(C_1 + C_2t + C_3t^2)e^t + (C_4 + C_5t)e^{-t}$
121. $(C_1 + C_2t + C_3t^2)e^t + (C_4 + C_5t)e^{-t}$
122. $C_1 e^{-x} + (C_2 + C_3x)e^{-x} + (C_4 + C_5x)e^x$
123. $C_1 + C_2x + C_3 e^{-x/2} + e^{2x}(C_4 \cos 2x + C_5 \sin 2x)$
124. $C_1 + C_2x + C_3x^2 + e^x(C_4 \cos 4x + C_5 \sin 4x)$
125. $C_1 e^t + C_2 e^{-t} + (C_3 + C_4t)e^{-2t} + e^{2t}(C_5 \cos 2t + C_6 \sin 2t)$
126. $e^{-3x}(C_1 \cos 2x + C_2 \sin 2x) + x e^{-3x}(C_3 \cos 2x + C_4 \sin 2x)$
127. $C_1 + C_2x + C_3x^2 + C_4 e^x + (C_5 + C_6x + C_7x^2)e^{2x} + C_8 \cos 3x + C_9 \sin 3x$
128. $-\sqrt{3} \cos x + \sin x$
129. $C_2 \sin x$
130. $3e^{-4x}$
131. $\frac{1}{3}e^{2t} - \frac{4}{3}e^{-t}$
132. 0
133. $(1 - 2x)e^{-x}$
134. $5(1 + x)e^x$
135. $-\frac{\sqrt{3}}{2}e^{(1-\sqrt{3})x} + \frac{\sqrt{3}}{2}e^{(1+\sqrt{3})x}$
136. No solution
137. $\frac{9}{4}e^{-t} - \frac{1}{4}e^{3t}$
138. $3e^{-4x}$
139. $e^t\left(-2 \cos 4t + \frac{5}{4} \sin 4t\right)$
140. $\cos \sqrt{2}t$
141. $y(x) = \frac{10}{7}e^{-5x} + \frac{18}{7}e^{2x}$
142. $y(x) = C_2 \sin 2x$
143. $y = -2 \cos 2x - \frac{1}{2} \sin 2x$
144. $y = -\frac{1}{2}e^{-(2+\sqrt{2})x} - \frac{1}{2}e^{(-2+\sqrt{2})x}$
145. $y(x) = \frac{4}{3}e^x - \frac{1}{3}e^{3x}$
146. $2e^{-x-2} - xe^{-x-2}$
147. $y = (1 + 5x)e^{-2x}$

148. $y = e^{2x}(\cos x + 3\sin x)$
 149. $y = e^{-2x}(\cos x + 2\sin x)$
 150. $e^{\pi-2x}\left(\cos x + \frac{1}{2}\sin x\right)$
 151. $y = -\frac{1}{3}e^{1-x} + \frac{1}{3}e^{-5x-5}$
 152. $y = e^{-1-x} + 2e^{5x+5}$
 153. $y = -\frac{8}{\sqrt{5}}e^{2t}\sin\sqrt{5}t$
 154. $y(x) = e^{2x}\left(-\cos 3x + \frac{4}{3}\sin 3x\right)$
 155. $y(x) = e^{3x-1}$
 156. $y = \frac{5e^5+3}{5e^4-1}e^{-x} - \frac{3e^4+e^5}{5e^4-1}e^{-5x}$
 157. $y(x) = e^x + 2e^{5x}$
 158. $y(x) = \left(2 + \frac{7}{3}x\right)e^{3x}$
 159. $y(x) = 5xe^{3x}$
 160. $y(x) = (2 + 4x)e^{-3x}$
 161. $y(x) = \frac{1}{5}e^{9-9x} + \frac{9}{5}e^{x-1}$
 162. $y(x) = e^{4x}(4\cos x - 17\sin x)$
 163. $y(x) = \frac{7}{6}e^{-3x} + \frac{5}{6}e^{3x}$
 164. $y(x) = \left(1 - \frac{5}{6}x\right)e^{5x}$
 165. $y(t) = (2 + 9t)e^{-5t}$
 166. $y(x) = \frac{7}{5}e^{-8x} - \frac{7}{5}e^{-3x}$
 167. $y(x) = \frac{\sqrt{3}}{3}\sin 2\sqrt{3}x$
 168. $y(x) = 2\cos 4x - \frac{1}{2}\sin 4x$
 169. $y(x) = -10\cos 4x + \frac{3}{4}\sin 4x$
 170. $y(x) = 2\cos 4x - \frac{1}{2}\sin 4x$
 171. $y(t) = 5\cos 5t - \frac{1}{5}\sin 5t$
 172. $-e^{(x+\pi)/2}\left(3\cos\frac{x}{2} + \sin\frac{x}{2}\right)$
 173. $y(x) = \frac{11}{13}e^{-7x/3} + \frac{15}{13}e^{2x}$
 174. $3e^{-2x} - 9e^{4x/3}$
 175. $y(x) = 2(1+x)e^{x/2}$
 176. $2(x-3)e^{(x-1)/2}$
 177. $y(x) = -\frac{7}{4}e^{-x/2} + \frac{11}{4}e^{3x/2}$
 178. $-\frac{1}{2}e^{(\pi-x)/2}(2\cos x + \sin x)$
 179. $y(x) = e^{-x/2}\left(-\cos 2x + \frac{3}{4}\sin 2x\right)$
 180. $y(t) = -\frac{28}{5} + \frac{28}{5}e^{5t/4}$
 181. $y(x) = (2 + 4x)e^{-3x/2}$
 182. $-6e^{3(\pi-t)}\cos\frac{t}{2} + e^{3(\pi-t)}\sin\frac{t}{2}$
 183. $y(x) = 2\sqrt{3}\cos\frac{x}{3} + 2\sin\frac{x}{3}$
 184. $y(x) = -2\cos\frac{\pi}{3}x + 3\sin\frac{\pi}{3}x$
 185. $y(x) = \frac{1}{e}(1-x)e^{x/3}$
 186. $y(x) = -e^{\pi}\sin\frac{x}{3}e^{-x/3}$
 187. $y(x) = \left(-1 + \frac{5}{3}x\right)e^{2x/3}$
 188. $y(x) = -3e^{2x/3} + 4e^{x/4}$
 189. $y(x) = (-4 + 4x)e^{x/4}$
 190. $y(x) = (-1 + x)e^{-2x/5}$
 191. $y(x) = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}xe^{-6x}$
 192. $y(x) = \frac{1}{10}e^{-3x} + \frac{1}{6}e^{-x} - \frac{4}{15}e^{2x}$
 193. $y(x) = C_1 + C_2x + C_3x^2 + C_4x^3$
 $+ C_5e^{3x} + C_6e^{-5x} + C_7xe^{-5x}$
 $+ e^{2x}\left(C_8\cos 3x + C_9\sin 3x\right)$
 $+ xe^{-5x}\left(C_{10}\cos 3x + C_{11}\sin 3x\right)$
 194. $y^{(4)} - 16y = 0$
 195. Infinite many solutions
 196. 0

Section 2.3

1. $y = \sqrt{2} \cos\left(2t - \frac{\pi}{4}\right)$
2. $y = 2 \cos 4\left(t - \frac{\pi}{12}\right)$
3. 2 N/m $y(t) = C_1 e^{-t} + C_2 e^{-2t}$
4. $v_0 = 0$
5. $x(t) = \cos 10t - \frac{1}{2} \sin 10t$ $\delta \approx 0.58195 \text{ sec}$
 $x(t) = \frac{\sqrt{5}}{2} \cos(10t - 5.8195)$
6. $x(t) = e^{-2t} \left(-\frac{1}{2} \cos 2\sqrt{3}t - \frac{1}{\sqrt{3}} \sin 2\sqrt{3}t \right)$
 $\approx -0.55 \text{ m}$
7. $x(t) = -\frac{1}{4} \cos 5t - \frac{1}{5} \sin 5t$
 $A = \frac{\sqrt{41}}{20}$, $\omega = 5$, $T = \frac{2\pi}{5}$, $t \approx 0.45 \text{ sec}$
8. $x(t) = -\frac{1}{2} \cos 5t - \frac{1}{2} \sin 5t$
 $A = \frac{\sqrt{2}}{2}$, $\omega = 4$, $T = \frac{\pi}{2}$, $\frac{1}{T} = \frac{2}{\pi}$, $t \approx 0.59$
9. $x(t) = \frac{1}{12} e^{-2t} + \frac{1}{6} e^{-5t}$,
 Mass will not return to the equilibrium position
10. $\approx 0.7567 \text{ m}$
11. $t \approx 0.08 \text{ sec}$
12. $x(t) = -3te^{-4t}$
13. $y(t) = \frac{1}{4} \cos 8t$
14. $y(t) = e^{-2t} \left(\frac{1}{4} \cos(2\sqrt{15}t) + \frac{1}{4\sqrt{15}} \sin(2\sqrt{15}t) \right)$
15. $x(t) = -e^{-t} \left(2 \cos 3t + \frac{2}{3} \sin 3t \right)$
16. $y(t) = \frac{\sqrt{10}}{6} \cos(6t - 2.81984)$
17. $y(t) = \left(-\frac{1}{2} - 2t \right) e^{-6t}$
18. $y(t) = 0.502096 e^{-5t/2} \cos\left(\frac{\sqrt{119}}{2}t - 3.2321\right)$
19. $x(t) = C_1 \cos 5t + C_2 \sin 5t$ $T = \frac{\pi\sqrt{2}}{8}$
20. $k = 320 \text{ N/m}$, $x = C_1 \cos 4t + C_2 \sin 4t$
 $x = C_1 \cos 2t + C_2 \sin 2t$
21. $y(t) = -\frac{1}{4} \cos 4\sqrt{6}t$, $y(t) = \frac{\sqrt{6}}{12} \sin 4\sqrt{6}t$
22. $k = 2$ $y = \frac{1-\sqrt{2}}{2} e^{(-2-\sqrt{2})t} + \frac{1+\sqrt{2}}{2} e^{(-2+\sqrt{2})t}$
 $k = 4$ $y(t) = (1+2t)e^{-2t}$
 $k = 6$ $y(t) = \sqrt{3} e^{-2t} \sin(\sqrt{2}t - 0.615)$
 $k = 8$ $y(t) = \sqrt{2} e^{-2t} \sin\left(2t - \frac{\pi}{4}\right)$
23. $-\frac{1}{3} \cos 5t + \frac{1}{4} \sin 5t$
 $A = \frac{5}{12}$, $\phi \approx 0.9273$, $T = \frac{2\pi}{5}$, $f = \frac{5}{2\pi}$
 $\frac{5}{12} \sin(5t - 0.9273)$
 $t = \frac{1}{5} \left(\frac{\pi}{6} + 0.9273 + 2n\pi \right)$, $\frac{1}{5} \left(\frac{5\pi}{6} + 0.9273 + 2n\pi \right)$
24. $y(t) = -\frac{2}{3} \cos 10t + \frac{1}{2} \sin 10t$
 $A = \frac{5}{6}$, $\phi \approx 0.9273$, $T = \frac{\pi}{5}$, $f = \frac{5}{\pi}$
 $y(t) = \frac{5}{6} \sin(10t - 0.9273)$
 $n = 15$ $t \approx 0.721 \text{ sec}$ $t = \frac{(2n+1)\pi}{20} + 0.09273$
 $\approx -0.597 \text{ ft}$, $\approx -5.814 \text{ ft/s}$,
 $\approx -59.685 \text{ ft/s}^2$, $\approx \pm 8.33 \text{ ft/s}$
 $\frac{1}{10} \left(\frac{\pi}{6} + 0.9273 + 2n\pi \right)$, $\frac{1}{10} \left(\frac{5\pi}{6} + 0.9273 + 2n\pi \right)$
 $\frac{1}{10} \left(\frac{5\pi}{6} + 0.9273 + 2n\pi \right)$
25. $x = -2e^{-4t} + 4e^{-2t}$ $u = 2 \cos(2\sqrt{2}t)$
26. $x = (5+10t)e^{-2t}$ $u = \frac{5\sqrt{5}}{2} \cos(4t + 0.4636)$
27. $x = 10e^{-5t} \cos(10t - 0.9273)$
 $u = 2\sqrt{14} \cos(5\sqrt{5}t - .6405)$
28. $x = 2e^{3t} \cos\left(4t - \frac{3\pi}{2}\right)$ $u = \frac{8}{5} \cos\left(5t - \frac{3\pi}{2}\right)$
29. $x = 12e^{-4t} \cos(2t - 1.176)$
 $u = \sqrt{\frac{129}{5}} \cos(2\sqrt{5}t - 0.177)$
30. $x = 4e^{-3t} - 2e^{-7t}$
 $u = 2\sqrt{\frac{22}{21}} \cos(\sqrt{21}t - 0.2149)$

$$31. x = \frac{\sqrt{313}}{3} e^{-\frac{5}{2}t} \cos(6t - 0.8254)$$

$$u = \frac{4\sqrt{233}}{13} \cos\left(\frac{13}{2}t - 0.5517\right)$$

$$32. a) -50e^{-t/2} + 50e^{-2t/5} \quad b) 4.096$$

$$33. a) 25e^{-t/5} \cos(3t - 0.6435) \quad b) \pm 25e^{-t/5}$$

$$34. t = \frac{7\pi}{18} + \frac{2n\pi}{3}, \quad \frac{\pi}{2} + \frac{2n\pi}{3}$$

$$35. a) 120 \text{ lb} / \text{ft} \quad b) x = \frac{\sqrt{3}}{12} \sin 8\sqrt{3}t$$

$$36. a) x = e^{-4t} \left(\cos 4\sqrt{3}t + \frac{\sqrt{3}}{3} \sin 4\sqrt{3}t \right)$$

$$b) x = \frac{2}{\sqrt{3}} e^{-4t} \cos\left(4\sqrt{3}t - \frac{\pi}{6}\right)$$

$$37. a) \frac{1}{2} \cos 8\sqrt{2}t + \frac{1}{8} \sin 8\sqrt{2}t$$

$$b) A = \frac{\sqrt{17}}{8}, \phi \approx 1.326, P = \frac{\pi\sqrt{2}}{8}, f = \frac{5}{\pi\sqrt{2}}$$

$$c) .16 \text{ sec}$$

$$38. a) -\frac{1}{4} \cos 5t + \frac{1}{5} \sin 5t$$

$$b) A = \frac{\sqrt{41}}{20}, \phi \approx 2.246, P = \frac{2\pi}{5}, f = \frac{5}{2\pi}$$

$$c) .179 \text{ sec}$$

$$39. a) \frac{1}{10}(\cos 10t + \sin 10t) \quad b) \frac{\pi}{40} \quad c) \frac{\sqrt{2}}{10}$$

$$40. a) 3,266.67 \text{ N/m} \quad b) 0.07 \cos 18.07t$$

$$41. a) 0.1 \cos 10t + 0.1 \sin 10t \quad b) \frac{\sqrt{2}}{10}$$

$$42. a) 1,400 \text{ N/m} \quad b) \frac{7}{100} \cos 2\sqrt{35}t$$

$$43. a) 140 \text{ N/m} \quad b) \frac{1}{5} e^{-2t} - \frac{1}{5} e^{-7t}$$

$$44. a) 6.25 \text{ lb/ft} \quad b) \frac{4}{3} e^{-4t} \sin 3t$$

$$45. a) k = 2,000 \text{ N/m} \quad b) y(t) = \frac{1}{5} \sin 10t$$

$$46. a) k = 64 \text{ N/m} \quad b) y(t) = -\frac{1}{2} \cos 4t \quad c) 3 \text{ in}$$

$$47. a) y(t) = -\frac{1}{3} \sin \frac{32}{3}t \quad b) -\frac{32}{9} \text{ ft/sec} \quad c) \frac{3\pi}{16}$$

$$48. a) x(t) = \frac{3\sqrt{5}}{10} \sin \sqrt{5}t \quad b) \omega = \sqrt{5}, f = \frac{\sqrt{5}}{2\pi}$$

$$49. x(t) = (2 - 2t)e^{-2t}$$

$$50. x(t) = \frac{1}{5} \cos 8t$$

$$51. x(t) = -0.05e^{-16t} + 0.05e^{-4t}$$

$$52. x(t) = \frac{9}{25} \sin \frac{10}{3}t$$

$$53. a) x(t) = -\frac{1}{5}e^{-6t} + \frac{6}{5}e^{-t} \quad b) \frac{49}{12} \text{ kg}$$

$$54. c = 10 \quad x = -\frac{1}{10}e^{-5t} \left(\cos 5\sqrt{3}t + \frac{\sqrt{3}}{3} \sin 5\sqrt{3}t \right)$$

$$c = 15 \quad x = e^{-15t/2} \left(-\frac{1}{10} \cos \frac{5\sqrt{7}}{2}t - \frac{3}{10\sqrt{7}} \sin \frac{5\sqrt{7}}{2}t \right)$$

$$c = 20 \quad x = (-0.1 - t)e^{-10t}$$

$$c = 25 \quad x = \frac{1}{10}e^{-10t} - \frac{1}{5}e^{-5t}$$

$$55. c = 30 \quad \frac{5-3\sqrt{5}}{100}e^{(-15-5\sqrt{5})t} + \frac{-5-3\sqrt{5}}{100}e^{(-15+5\sqrt{5})t}$$

$$56. a) y_0 = \frac{\sqrt{3}}{8} \text{ m} \quad b) = .25 \text{ m/s}$$

$$c) k = 16 \text{ N/m} \quad d) P = \pi$$

$$57. a) e^{-7t/20} \left(\frac{1}{2} \cos \frac{3\sqrt{439}}{20}t + \frac{47}{6\sqrt{439}} \sin \frac{3\sqrt{439}}{20}t \right)$$

$$b) 0$$

$$58. a) c = 2\sqrt{k} \quad b) c = \frac{16}{e}, k = \frac{64}{e^2} \quad c) c = \frac{8\sqrt{11}}{3e}$$

$$59. q(t) = \frac{q_0\sqrt{10}}{3} e^{-20t} \sin(60t + 1.249)$$

$$60. q = \frac{5\sqrt{5}}{2} e^{-20t} \sin(40t + 1.1071), t \approx 0.0509 \text{ sec}$$

$$61. t \approx -0.0275 < 0 \text{ never equal to zero}$$

$$q(t) = e^{-3t} (4 \cos 3t + 4 \sin 3t)$$

Section 2.4

$$1. y(x) = x - 3x \ln x + 2x^2$$

$$2. y(t) = 2e^{-3t}$$

$$3. y(t) = -e^{-t}$$

$$4. y(t) = 3e^{-t}$$

$$5. y(t) = -\frac{9}{4}e^{2t}$$

$$6. y(t) = -\frac{1}{5} \cos 3t$$

$$7. y_p(t) = -\frac{21}{100} \cos 2t + \frac{3}{100} \sin 2t$$

$$8. y_p(t) = \frac{3}{4}t - \frac{7}{16}$$

$$9. y_p(t) = \frac{1}{4}t - \frac{9}{16}$$

10. $y_p(t) = \frac{1}{4}t^3 - \frac{9}{16}t^2 + \frac{15}{32}t - \frac{9}{128}$
11. $y(t) = 1 - \frac{1}{10}\cos 2t + \frac{1}{5}\sin 2t$
12. $y(t) = -t + \frac{1}{2}te^{-t}$
13. $y_p = e^{-2t}\left(\frac{4}{5}\cos t - \frac{3}{5}\sin t\right)$
14. $y_p = \frac{5}{12}t^3 + \frac{1}{2}t^2 + \frac{3}{8}t + \frac{1}{15}e^{5t}$
 $+ \left(4t^2 + 8t + 16\right)e^t$
15. $y(t) = -\frac{5}{13}\cos 2t + \frac{1}{13}\sin 2t$
16. $y(t) = -\frac{1}{5}\cos 3t$
17. $y(x) = C_1 \cos x + C_2 \sin x + x \sin x$
18. $y(x) = C_1 \cos x + C_2 \sin x - \frac{1}{8}\cos 3x$
19. $y(x) = C_1 \cos x + C_2 \sin x - \frac{1}{2}x^2 \cos x$
 $+ \frac{1}{2}x \sin x$
20. $y(x) = C_1 e^{-x} + C_2 e^x - 5$
 $+ \left(\frac{1}{6}x^3 - \frac{1}{4}Bx^2 + \frac{1}{4}x\right)e^x$
21. $y(x) = C_1 + C_2 e^x - 3x$
22. $y(x) = C_1 + C_2 e^x - \cos x - \sin x$
23. $y(x) = C_1 + C_2 e^x - \frac{1}{2}\cos x - \frac{1}{2}\sin x$
24. $y(x) = C_1 + C_2 e^x + 4x^2 - 5x$
25. $y(x) = C_1 \cos x + C_2 \sin x + 2x + \frac{3}{2}e^x$
26. $y(x) = C_1 e^{-x} + \left(C_2 + \frac{1}{2}x\right)e^x - x^2 - 2$
27. $y(x) = C_1 + C_2 e^{-x} + 2x^5 - 10x^4$
 $+ 10x^3 - 120x^2 + 242x$
28. $y(x) = C_1 + C_2 e^x + 5xe^x - \frac{1}{10}\cos 2x + \frac{1}{5}\sin 2x$
29. $y(x) = C_1 \cos x + C_2 \sin x + \frac{1}{4}x \cos x$
 $+ \frac{1}{4}x^2 \sin x - \frac{1}{2}x \sin x$
30. $y(x) = C_1 + C_2 e^{-x}e^x \left(-\frac{2}{5}\cos x + \frac{1}{5}\sin x\right)$
31. $y(x) = C_1 e^{-2x} + C_2 e^{2x} - \frac{1}{4}x^2 - \frac{1}{8}$
32. $y(x) = C_1 e^{-x} + C_2 e^{2x} - 6\cos x - 2\sin x$
33. $y(x) = (C_1 + C_2 x)e^{x/2} + 12 + \frac{1}{2}x^2 e^{x/2}$
34. $y(x) = C_1 \cos x + C_2 \sin x$
 $- \left(\frac{4}{225}\cos 3x + \frac{28}{225}\sin 3x\right)e^x$
35. $y(x) = C_1 e^{-x} + C_2 e^{2x} + \frac{1}{4}e^{3x}$
36. $y(x) = C_1 e^{-2x} + C_2 e^{3x} - 4xe^{-2x}$
37. $y(x) = C_1 e^{-3x} + C_2 e^{2x} - \frac{1}{3}x - \frac{1}{18}$
38. $y(x) = C_1 e^{-2x} + C_2 e^{3x} - \frac{1}{4}e^{-x}$
 $+ \frac{49}{50}\cos x + \frac{7}{50}\sin x$
39. $y(x) = e^{-x/2}\left(C_1 \cos \frac{\sqrt{31}}{2}x + C_2 \sin \frac{\sqrt{31}}{2}x\right)$
 $+ (-3x^2 + 2x - 1)\cos 3x$
 $+ (-x^2 + 7x - 13)\sin 3x$
40. $y(x) = C_1 e^{-3x} + \left(C_2 + \frac{1}{7}x\right)e^{4x}$
41. $y(x) = C_1 + C_2 e^{-2x} + \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}$
42. $y(x) = C_1 + C_2 e^{2x} - 3x^2 + 2x$
43. $y(x) = (C_1 + C_2 x)e^{-x} - \frac{1}{2}\cos x$
 $- \frac{9}{25}\cos 2x + \frac{12}{25}\sin 2x$
44. $y(x) = (C_1 + C_2 x)e^x + 3x^2 e^x$
45. $y(x) = (C_1 + C_2 x)e^{-x} + x^2 - 4x - 2$
46. $y(x) = (C_1 + C_2 x)e^{-x} + \frac{1}{12}x^4 e^{-x}$
47. $y(x) = (C_1 + C_2 x)e^x + x^3 + 6x^2 + 22x + 32$
48. $y(x) = (C_1 + C_2 x)e^{-x} - \frac{24}{25}\cos 2x - \frac{18}{25}\sin 2x$
49. $y(x) = (C_1 + C_2 x)e^x + (x^2 - 4x + 5)e^{2x}$
 $+ \left(\frac{1}{2}x^3 + 2x^2\right)e^x$
50. $y(x) = (C_1 \cos x + C_2 \sin x)e^{-x} + \frac{1}{10}e^{6x}$

51. $y(x) = (C_1 \cos x + C_2 \sin x)e^{-x} + \frac{1}{2}x^3 - \frac{3}{2}x^2 + \frac{3}{2}x$
52. $y(x) = (C_1 \cos x + C_2 \sin x + 1)e^{-x} - \frac{2}{5}\cos x + \frac{1}{5}\sin x$
53. $y(x) = e^x(C_1 \cos x + C_2 \sin x) - \frac{1}{2}xe^x \cos x$
54. $y(x) = e^x(C_1 \cos x + C_2 \sin x) + e^{2x}\left(\frac{7}{5}\cos x + \frac{1}{5}\sin x\right)$
55. $y(x) = C_1 e^{-x} + C_2 e^{3x} + \frac{1}{3}x^2 - \frac{4}{9}x + \frac{5}{9}$
56. $y(x) = C_1 e^{-x} + C_2 e^{3x} - e^x + 3$
57. $y(x) = C_1 e^{-x} + C_2 e^{3x} - \left(\frac{2}{17}\cos x + \frac{8}{17}\sin x\right)e^{-x} - \frac{1}{3}x^2 + \frac{4}{9}x - \frac{14}{27} + \left(\frac{1}{8}x^2 - \frac{1}{16}x\right)e^{3x}$
58. $y(x) = e^x(C_1 \cos 2x + C_2 \sin 2x) + 5x^2 + 4x - 10$
59. $y(x) = e^x\left(C_1 \cos 2x + \left(C_2 + \frac{1}{4}x\right)\sin 2x\right)$
60. $y(x) = e^x\left(C_1 \cos 2x + C_2 \sin 2x + \frac{1}{3}\sin x\right)$
61. $y(x) = C_1 e^{-6x} + C_2 e^{4x} - \frac{2}{3} + \left(-\frac{1}{20}x^2 - \frac{19}{100}x\right)e^{4x}$
62. $y(x) = C_1 \cos \sqrt{3}x + C_2 \sin \sqrt{3}x + \left(-4x^2 + 4x - \frac{4}{3}\right)e^{3x}$
63. $y(x) = C_1 + C_2 e^{3x} + \frac{1}{3}xe^{3x} + 2x^2 + \frac{4}{3}x$
64. $y(x) = C_1 + C_2 e^{-3x} + \frac{2}{3}x^2 - \frac{19}{9}x$
65. $y(x) = C_1 + C_2 e^{3x} + \frac{8}{3}xe^{3x} + \frac{6}{5}\cos x - \frac{2}{5}\sin x$
- 66.
67. $y(x) = C_1 e^{-x} + C_2 e^{-2x} + 3$
68. $y(x) = C_1 e^{-x} + C_2 e^{-2x} + 2x^2 - 6x + 7$
69. $y(x) = C_1 e^x + C_2 e^{2x} - 5xe^x$
70. $y(x) = C_2 e^{2x} + x^2 + 3x + \frac{7}{2} - \left(x^2 + 3x + C_1\right)e^x + 2e^{3x}$
71. $y(x) = C_1 e^x + C_2 e^{2x} + 3\cos 2x + 2\sin 2x$
72. $y(x) = (C_1 + x)e^{-x} + (C_2 - x)e^{-2x} - \frac{1}{2}x - \frac{3}{4}$
73. $y(x) = C_1 e^x + C_2 e^{2x} + \frac{3}{10}$
74. $y(x) = C_1 e^x + C_2 e^{2x} - \frac{1}{5}x + \frac{4}{25}$
75. $y(x) = C_1 e^x + C_2 e^{2x} + \frac{3}{4}e^{4x}$
76. $y(x) = C_1 e^x + C_2 e^{2x} - \frac{1}{10}x + \frac{3}{10} - \left(\frac{1}{6}x + \frac{5}{36}\right)e^x$
77. $y(x) = A \cos 2t + B \sin 2t + \frac{3}{4}x^3 - \frac{9}{8}x$
78. $y(x) = C_1 \cos 2x + C_2 \sin 2x + \sin x$
79. $y(x) = C_1 \cos 2x + C_2 \sin 2x - \frac{3}{4}x \cos 2x$
-
80. $y(x) = C_1 \cos 2x + C_2 \sin 2x + \frac{4}{3}\cos x + \sin x - 2$
81. $y(x) = C_1 e^{-2x} + C_2 e^{2x} - \frac{1}{8}x \cos 2x + \left(-\frac{1}{8}x^2 + \frac{13}{32}\right)\sin 2x$
82. $y(x) = (C_1 + C_2 x)e^{-2x} + \frac{1}{2}x + 1$
83. $y(x) = e^{-2x}\left(C_1 \cos x + C_2 \sin x\right) + x - \frac{4}{5} + \frac{1}{2}e^{-x}$
84. $y(x) = e^{-2x}\left(C_1 \cos x + C_2 \sin x\right) + 2e^{-2x} + \frac{1}{8}\cos x + \frac{1}{8}\sin x$

85. $y(x) = C_1 + C_2 e^{-5x} + x^3 - \frac{3}{5}x^2 + \frac{6}{25}x$
86. $y(x) = C_1 + C_2 e^{5x} - \frac{1}{10}x^4 + \frac{14}{75}x^3 + \frac{53}{250}x^2 - \frac{697}{625}x$
87. $y(x) = C_1 e^{-2x} + C_2 e^{-4x} + \frac{3}{2}x e^{-2x} + \frac{1}{4}x - \frac{3}{16}$
88. $y(x) = \left(C_1 + C_2 x + \frac{1}{2}\right)e^{3x}$
89. $y(x) = \left(C_1 + C_2 x\right)e^{-3x} + \left(-\frac{1}{49}x + \frac{2}{175}\right)e^{4x}$
90. $y(x) = e^{-3x} \left(C_1 \cos 2x + C_2 \sin 2x + \frac{1}{4}x \sin 2x\right)$
93. $y(x) = C_1 \cos(2\sqrt{2}x) + C_2 \sin(2\sqrt{2}x) + \frac{5}{8}x + \frac{2}{9}e^{-x}$
94. $y(x) = \left(C_1 \cos 2x + C_2 \sin 2x\right)e^{4x} + \left(-2x - \frac{12}{13}\right)e^x + 5x^2 + 4x + \frac{11}{10}$
95. $y(x) = C_1 e^{-3x} + C_2 e^{3x} - 6$
96. $y(x) = C_1 \cos 3x + C_2 \sin 3x + \frac{1}{36}x^2 \cos 3x + \left(\frac{1}{18}x^3 - \frac{1}{108}x\right)\sin 3x + \frac{1}{2}\sin x$
97. $y(x) = \left(C_1 + C_2 x + 7x^2\right)e^{-5x}$
98. $y(x) = \left(C_1 + C_2 x\right)e^{5x} + \frac{6}{5}x + \frac{3}{5}$
99. $y(x) = C_1 e^{-4x} + C_2 e^{4x} - \frac{1}{4}x e^{4x}$
100. $y(x) = C_1 \cos 5x + C_2 \sin 5x + \frac{1}{4}\sin x$
101. $y(x) = C_1 \cos 5x + C_2 \sin 5x - 2x \cos 5x$
102. $y(x) = \left(C_1 + C_2 x\right)e^{-2x} + x^2 - 4x + \frac{7}{2}$
103. $y(x) = C_1 e^{x/2} + C_2 e^{2x} + 2x e^{x/2}$
104. $y(x) = C_1 e^x + C_2 e^{5x/2} - \frac{29}{5}$
105. $y(x) = C_1 e^{-3x/2} + C_2 e^{3x/2} + \frac{15}{9}$
106. $y(x) = C_1 e^{-x/2} + C_2 e^{3x/2} - \frac{19}{425}\cos 2x - \frac{8}{425}\sin 2x$
107. $y(x) = \left(C_1 + C_2 x + \frac{1}{6}x^3\right)e^{x/3}$
108. $y(x) = C_1 + C_2 x + C_3 e^{-x} + \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2$
91. $y(x) = C_1 + C_2 e^{7x} + \frac{3}{7}x$
92. $y(x) = C_1 + C_2 e^{-7x} + 2x^3 - \frac{6}{7}x^2 + \frac{12}{25}x$
109. $y(x) = C_1 e^{2x} + C_2 e^x + C_3 e^{2x} + \frac{5}{4} - \frac{1}{3}x e^x + \frac{1}{4}x e^{2x}$
110. $y(x) = C_1 + C_2 x + C_3 e^{-x} + 3e^x + \frac{1}{3}x^4 - \frac{4}{3}x^3 + 4x^2$
111. $y(x) = C_1 + \left(C_2 + C_3 x\right)e^{-x} + 10x$
112. $y(x) = C_1 e^{-2x} + \left(C_2 + C_3 x\right)e^{2x} + \left(\frac{1}{4}x^3 - \frac{3}{16}x^2\right)e^{2x}$
113. $y(x) = \left(C_1 + C_2 x + C_3 x^2\right)e^x + \frac{1}{2}x^3 e^x$
114. $y(x) = \left(C_1 + C_2 x + C_3 x^2\right)e^x - x - 3 - \frac{2}{3}x^3 e^x$
115. $y(x) = C_1 e^{-x} + C_2 e^{2x} + C_3 e^{3x} + \frac{3}{65}\cos 2x + \frac{11}{65}\sin 2x$
116. $y(x) = \left(C_1 + C_2 x + C_3 x^2\right)e^x + \frac{1}{6}x^3 e^x + x - 13$

$$117. \quad y(x) = C_1 + C_2 x + C_3 e^{6x} - \frac{1}{4} x^2 - \frac{6}{37} \cos x + \frac{1}{37} \sin x$$

$$118. \quad y(x) = C_1 e^x + C_2 e^{2x} + C_3 e^{3x} - \frac{1}{12} x e^{-x} - \frac{13}{144} e^{-x}$$

$$119. \quad y(x) = C_1 + C_2 x + e^{-8x} - \frac{1}{16} x^4 + \frac{7}{32} x^3 + \frac{11}{256} x^2$$

$$120. \quad y(x) = C_1 + C_2 x + C_3 \cos x + C_4 \sin x + \frac{1}{4} x^4 - 3x^2 + 2x \cos x + x \sin x$$

$$121. \quad y(x) = C_1 \cos x + C_2 \sin x + C_3 x \cos x + C_4 x \sin x + x^2 - 4x$$

$$122. \quad y(x) = C_1 + C_2 x + C_3 e^{-x} + C_4 e^x - \frac{2}{3} x^3 - \left(\frac{1}{2} x^2 + \frac{5}{2} x \right) e^{-x}$$

$$123. \quad y(x) = C_1 e^x + C_2 e^{-2x} - x - \frac{1}{2} + 6 \cos 2x - 2 \sin 2x$$

$$124. \quad y(x) = C_1 e^x + C_2 e^{2x} + \frac{3}{5} \cos x + \frac{1}{5} \sin x$$

$$125. \quad y(x) = \left(C_1 + C_2 x + C_3 x^2 \right) e^{2x} + \left(\frac{1}{780} x^5 - \frac{1}{507} x^4 + \frac{1}{2197} x^3 \right) e^{2x} - \left(\frac{23}{13,182} x^2 + \frac{251}{114,244} x + C_4 \right) \cos 3x + \left(-\frac{3}{8788} x^2 + \frac{1379}{514,098} x + C_5 \right) \sin 3x$$

$$126. \quad y(x) = \cos x - \sin x + \frac{1}{2} x \sin x$$

$$127. \quad y(x) = \frac{1}{2} + e^{x-1} + \frac{1}{2} x^2 - x$$

$$128. \quad y(x) = 2 - e^x - \frac{1}{2} x^2 + x$$

$$129. \quad y(x) = -\pi \cos t - \frac{11}{3} \sin t + 2t \cos t - \frac{8}{3} \cos 2t$$

$$130. \quad y(x) = 2e^{-x} + 2e^{2x} - 2x^2 + 2x - 3$$

$$131. \quad y(x) = \left(e - \frac{1}{12} e^4 \right) e^{-x} + \frac{1}{4} e^{3x} + \left(e^{-2} - \frac{1}{3} e \right) e^{2x}$$

$$132. \quad y(x) = \frac{1}{12} e^{-x} + \frac{2}{3} e^{2x} + \frac{1}{4} e^{3x}$$

$$133. \quad y(x) = \frac{13}{12} e^{-x} + \frac{2}{3} e^{2x} + \frac{1}{4} e^{3x}$$

$$134. \quad y(t) = (3 + 2t)e^{-t} + \sin t$$

$$135. \quad y(t) = (-23 + 5t)e^t + t^3 + 6t^2 + 18t + 24$$

$$136. \quad y(x) = (x - 3)e^x + x^2 + 3x + 1$$

$$137. \quad y(x) = e^x \left(2 \cos x - \frac{5}{2} \sin x \right) + \frac{1}{2} x + 1$$

$$138. \quad y(x) = e^{-x} \left(\frac{176}{85} \cos x + \frac{106}{85} \sin x \right) - \frac{6}{85} \cos 3x - \frac{7}{85} \sin 3x$$

$$139. \quad y(t) = e^{-t} \left(-\frac{9}{5} \cos t - \frac{13}{5} \sin t \right) - \frac{1}{5} \cos 2t + \frac{2}{5} \sin 2t$$

$$140. \quad y(x) = 2e^{-x} + \frac{3}{2} e^{3x} - \frac{1}{2} e^x - \cos x + 2 \sin x$$

$$141. \quad y(x) = e^{-x} (2 \cos 3x + 3 \sin 3x) + x^3 + 2x$$

$$142. \quad y(t) = e^t \left(2 \cos 3t - \frac{7}{3} \sin 3t \right) - \sin 3t$$

$$143. \quad y(x) = \frac{5}{2} e^{-x} - \frac{8}{3} e^{-2x} + \frac{1}{6} e^x$$

$$144. \quad y(x) = -\frac{1}{2} e^x + \frac{6}{13} e^{2x} + \frac{1}{2} e^{-x} + \frac{7}{13} \cos 3x + \frac{9}{13} \sin 3x$$

$$145. \quad y(x) = \sqrt{2} \sin 2x - \frac{1}{2}$$

$$146. \quad y(x) = \cos 2x + \frac{3}{4} \sin 2x + \frac{1}{2} x$$

$$147. \quad y(x) = (1 - 3x)e^{2x} + e^x$$

$$148. \quad y(x) = e^{2x} \left(2 \cos 2x - \frac{3}{64} \sin 2x \right) + \frac{1}{8} x^3 + \frac{3}{16} x^2 + \frac{3}{32} x$$

$$149. \quad y(t) = -\frac{5\sqrt{2}}{48} \cos 2t - \frac{\sqrt{2}}{48} \sin 2t + \frac{1}{8} + \frac{1}{24} \cos 4t$$

$$150. \quad y(x) = \left(2 + 9x + \frac{3}{2} x^2 + \frac{1}{6} x^3 \right) e^{-2x}$$

$$151. \quad y(t) = (C_1 + C_2 t) e^{-2t} - \frac{1}{4} t + \frac{1}{4}$$

$$152. \quad y(t) = -\frac{1}{14} e^{5t} - \frac{1}{2} e^{-t} + \frac{4}{7} e^{-2t}$$

$$153. y(x) = (-10 \cos x + 9 \sin x)e^{-2x} + 7e^{-4x}$$

$$154. y(t) = \left(\frac{69}{65} \cos 2t + \frac{130}{131} \sin 2t \right) e^{-2t} - \frac{4}{65} \cos t + \frac{7}{65} \sin t$$

$$155. y(t) = 2e^{-2t} + e^{6t} - \frac{3}{7}e^{5t}$$

$$156. y(t) = -\frac{1}{32}e^{-2t} + \frac{1}{160}e^{6t} + \frac{1}{40} \cos 2t - \frac{1}{20} \sin 2t$$

$$157. y(t) = -\frac{41}{125} + \frac{41}{125}e^{5t} - \frac{1}{10}t^2 + \frac{9}{25}t$$

$$158. y(x) = -\frac{3}{7}e^x + \frac{5}{28}e^{-6x} + \frac{5}{4}e^{2x}$$

$$159. y(x) = e^{-3x}(\cos x - \sin x) + 2x + 1$$

$$160. y(x) = 2e^{-4x} - 3e^{-3x} - 2 \cos 2x + \sin 2x$$

$$161. y(x) = e^{-x} - e^{-7x} - 2e^{-2x}$$

$$162. y(x) = \cos 3x - \frac{2}{15} \sin 3x + \frac{1}{5} \sin 2x$$

$$163. y(x) = \frac{5}{8}e^{-8x} + \frac{5}{8}e^{8x} - \frac{1}{4}$$

$$164. y(x) = -\frac{1}{5}e^{-2x} + \frac{186}{5}e^{x/2} - 7x^2 - 19x - 37$$

$$165. y(x) = 200e^{-x/5} - 200 + 30x - 3x^2$$

$$166. x(t) = -2 \cos 3t + 2 \cos 2t$$

$$167. x(t) = \frac{3}{2} \sin 2t - \sin 3t$$

$$168. x(t) = 372 \cos 10t - 2 \sin 10t + 3 \cos 5t + 4 \sin 5t$$

$$169. x(t) = -10 \cos 5t + 18 \sin 5t + 10 \cos 4t$$

$$170. y(x) = -\frac{9}{2} + 4e^{-x} + 2xe^{-x} + \frac{1}{2}e^{2x}$$

$$171. y(x) = -3 + 3x + 4e^{-x} + \frac{1}{6}x^3 - \frac{1}{2}x^2 + xe^{-x}$$

$$172. y(x) = 4 + x + \left(-4 + 3x - \frac{1}{2}x^2 + \frac{1}{6}x^3 \right) e^x$$

$$173. y(x) = \frac{5}{4}e^{-x} + \frac{5}{4}e^x + \frac{5}{2} \cos x - 5$$

$$174. y(x) = \frac{117}{96} + \frac{5}{4}x - \frac{3}{64}e^{-2x} - \frac{11}{64}e^{2x} - \frac{1}{48}x^4 - \frac{1}{16}x^2$$

$$175. y(x) = 2 + x - 2e^x - \frac{1}{24}x^4 - \frac{1}{6}x^3 + xe^x$$

$$176. y(x) = C_1 \cos kx + C_2 \sin kx + \frac{1}{k-b^2} \sin bx$$

Section 2.5

$$1. y(x) = C_3 e^{2x} + C_2 e^{-3x} + \frac{3}{5} x e^{2x}$$

$$2. y_p = -t - 3$$

$$3. y_p = \frac{1}{2} t^2 e^t$$

$$4. y_p = \frac{1}{2} t^2 e^{2t}$$

$$5. y_p = -2 + \sin t \ln |\sec t + \tan t|$$

$$6. y_p = \frac{2}{25} \ln |\tan 5x + \sec 5x|$$

$$7. y_p = \frac{5}{2} x^2 e^{3x}$$

$$8. y_p = \frac{1}{2} x \sin 2x$$

$$9. y_p = -4xe^{2x} - 4e^{2x} + \frac{1}{2}$$

$$10. y(t) = C_1 t + \frac{C_2}{t^3} - \frac{1}{4t}$$

$$11. y(x) = C_1 e^{-x} + C_2 e^x - \frac{1}{2} e^{-x} \int \frac{e^x}{x} dx + \frac{1}{2} e^x \int \frac{e^{-x}}{x} dx$$

$$12. y(x) = C_1 e^{-x} + C_2 e^x - \frac{1}{4} e^{-2x} + \frac{1}{6} e^{2x} + \frac{1}{12} e^{-x}$$

$$13. y(x) = C_1 e^{-x} + C_2 e^x - x$$

$$14. y(x) = C_1 e^{-x} + C_2 e^x - \frac{1}{4} x e^{-x} + \frac{1}{4} x e^x$$

$$15. y(x) = C_1 \cos x + C_2 \sin x - \frac{1}{2} x \cos x + \frac{1}{2} \sin x$$

$$16. y(x) = C_1 e^{-x} + C_2 e^x - \frac{1}{4} e^x + \frac{1}{2} x e^x$$

$$17. y(x) = C_1 \cos x + C_2 \sin x + x \sin x - \cos x \ln |\sec x|$$

$$18. y(x) = C_1 \cos x + C_2 \sin x - (\cos x) \ln |\sec x + \tan x|$$

$$19. y(x) = C_1 \cos x + C_2 \sin x - \frac{1}{2} x \cos x$$

$$20. y(x) = C_1 \cos x + C_2 \sin x - x \cos x + \sin x \ln |\sin x|$$

$$21. y(x) = C_1 \cos x + C_2 \sin x + \frac{1}{3} \cos 2x + \sin^2 x$$

$$22. y(x) = C_1 \cos x + C_2 \sin x - 1$$

$$- \cos x \ln |\csc x - \cot x|$$

$$23. y(x) = C_1 \cos x + C_2 \sin x + \cos x \sec x$$

$$+ \sin x \ln |\sec x + \tan x|$$

$$24. y(x) = C_1 \cos x + C_3 \sin x + x \cos x$$

$$+ \sin x \ln |\sec x|$$

$$25. y(x) = C_3 + C_2 e^{-x} + \frac{1}{2} x^2 + x$$

$$26. y(x) = C_1 + C_2 e^x + \cos x e^x + \frac{1}{2} \sin x e^x$$

$$27. y(x) = C_1 e^{-2x} + C_3 e^x - \frac{1}{9} x e^x + \frac{1}{3} x^2 e^x$$

$$28. y(x) = C_1 e^{-2x} + C_2 e^x + \frac{1}{10} e^{3x}$$

$$29. y(x) = C_1 e^{-x} + C_2 x e^{-x}$$

$$+ \frac{1}{2} x^2 e^{-x} \ln x - \frac{3}{4} x^2 e^{-x}$$

$$30. y(x) = C_1 e^x + C_2 x e^x - \frac{1}{2} e^x \ln(1 + x^2)$$

$$+ x e^x \tan^{-1} x$$

$$31. y(x) = \left(C_1 + C_2 x + \frac{1}{2} x^2 \right) e^{-x}$$

$$32. y(x) = \left(A_1 - \frac{x}{2} \right) e^{-2x} + C_2 e^{4x}$$

$$33. y(x) = C_1 e^{-2x} + C_2 e^{-x} - e^{-2x} \sin e^x$$

$$34. y(x) = C_1 e^{-2x} + C_2 e^{-x} + \frac{2}{3} e^x$$

$$35. y(x) = C_1 e^{-2x} + C_2 e^{-x} + e^{-x}$$

$$36. y(x) = C_1 e^{-2x} + C_2 e^{4x} + \left(\frac{x}{4} - \frac{1}{16} e^{4x} \right) \cosh 2x$$

$$37. y(x) = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{2} x \sin 2x$$

$$- \frac{1}{4} \ln |\sec x| \cos 2x$$

$$38. y(x) = A_1 \cos 2x + C_2 \sin 2x$$

$$+ \frac{1}{40} \cos 5x - \frac{1}{5} \cos 3x$$

$$39. y(x) = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{4} \sin^4 x \cos x$$

$$+ \left(\frac{1}{8} \sin 2x - \frac{x}{8} - \frac{1}{32} \sin 4x \right) \sin x$$

$$40. y(x) = C_1 e^{-2x} + C_2 e^{2x} + \frac{1}{4} e^{2x} \ln |x|$$

$$- \frac{1}{4} e^{-2x} \int \frac{e^{4x}}{x} dx$$

$$41. y(x) = C_1 e^{-2x} + C_2 x e^{2x} + \frac{1}{36} (1 - 3x) e^x$$

$$- \frac{1}{4} (x + 1) e^{-x}$$

$$42. y(t) = C_1 \cos 2t + C_2 \sin 2t + \frac{1}{6} \cos^2 2t$$

$$+ \frac{1}{12} \sin^2 2t$$

$$43. y(x) = C_1 e^{2x} + C_2 x e^{2x} + x^2 e^{2x}$$

$$44. y(x) = C_1 e^{2x} + C_2 x e^{2x}$$

$$+ \left(\frac{1}{6} x^3 + \frac{1}{2} x^2 \right) e^{2x}$$

$$45. y(x) = e^{-2x} \left(C_1 \cos x + C_2 \sin x \right) + 2$$

$$46. y(x) = C_2 e^{3x} - \left(\frac{3}{4} x^2 + \frac{1}{4} x + C_3 \right) e^{-3x}$$

$$47. y(x) = C_1 \cos 3x + C_2 \sin 3x - \frac{1}{3} x \cos 3x$$

$$+ \frac{1}{9} \sin 3x \ln |\sin 3x|$$

$$48. y(t) = C_1 \cos 3t + C_2 \sin 3t$$

$$- \frac{1}{3} (\cos 3t) \ln |\sec 3t + \tan 3t|$$

$$49. y(x) = C_1 \cos 3x + C_2 \sin 3x + \frac{1}{18} \sin^3 3x$$

$$+ \left(\frac{1}{6} x - \frac{1}{36} \sin 6x \right) \cos 3x$$

$$50. y(x) = C_1 \cos 3x + C_2 \sin 3x + \frac{x}{3} \sin 3x$$

$$+ \frac{1}{9} \cos 3x \ln |\cos 3x|$$

$$51. y(x) = C_1 \cos 3x + C_2 \sin 3x + \frac{2}{3} x \sin 3x$$

$$+ \frac{2}{9} \cos 3x \ln |\cos 3x|$$

$$52. y(x) = C_1 \cos 3x + C_2 \sin 3x - \frac{1}{12} x \cos 3x$$

$$+ \frac{1}{36} \sin 3x \ln |\sin 3x|$$

$$53. y(x) = C_1 e^{-3x} + C_2 e^{-2x} + \frac{1}{6} x^2 + \frac{1}{18} x - \frac{11}{108}$$

$$54. y(x) = C_1 e^x + C_2 e^{2x} + e^x \ln(1 + e^{-x})$$

$$- e^x - e^{2x} \ln(1 + e^{-x})$$

$$55. y(x) = C_1 + C_2 \cos x + C_3 \sin x - x \cos x$$

$$+ \ln |\sec x + \tan x| + (\sin x) \ln |\cos x|$$

$$56. y(x) = C_1 + C_2 e^x + C_3 e^{2x} + \frac{1}{2} e^x - \left(\frac{1}{2} + e^x\right) \ln(e^x + 1) - \frac{1}{2} e^{2x} \ln(1 + e^{-x})$$

$$57. y(x) = C_4 e^x + C_2 e^{2x} + C_3 e^{3x} + \frac{1}{2} x e^x$$

$$58. y(x) = C_1 x + C_2 x^2 + C_3 x^4 - \frac{1}{2} \ln x - \frac{7}{8}$$

$$59. y(t) = \cos t + 2 \sin t + \ln|\cos t| \cos t + t \sin t$$

$$60. y(t) = \frac{3}{2} \cos t + \frac{1}{2} \sin t - \frac{1}{2} \frac{\cos 2t}{\cos t}$$

$$61. y(t) = -\frac{5}{4} e^{-t} + \frac{13}{4} e^t - t - \frac{1}{2} \sin t$$

$$62. y(x) = e^x - x e^x + x e^x \ln|x|$$

$$63. y(x) = \frac{4}{9} e^{-4x} + \frac{25}{36} e^{2x} - \frac{1}{4} e^{-2x} + \frac{1}{9} e^{-x}$$

$$64. y(x) = -\frac{1}{2} e^x + \frac{6}{13} e^{2x} + \frac{1}{2} e^{-x} + \frac{9}{13} \sin 3x + \frac{7}{13} \cos 3x$$

$$65. y(t) = -\frac{5\sqrt{2}}{48} \cos 2t - \frac{\sqrt{2}}{48} \sin 2t + \frac{1}{8} + \frac{1}{24} \cos 4t$$

$$66. y(t) = -\frac{1}{6} \cos 2t + \frac{1}{8} + \frac{1}{24} \cos 4t$$

$$67. y(x) = (1 - 2x - x^3 + x^4) e^{2x}$$

$$68. y(x) = \frac{1}{3} e^{-x} + \frac{8}{3} e^{x/2} - x - 2$$

$$69. y(x) = \frac{1}{4} e^{-x/2} + \left(\frac{1}{8} x^2 - \frac{1}{4} x + \frac{3}{4}\right) e^{x/2}$$

$$70. y(t) = t + 3t \ln t + \frac{1}{2} t (\ln t)^2$$

Section 2.6

$$1. a) x(t) = \frac{4}{4-\omega^2} (\cos \omega t - \cos 2t)$$

$$b) x(t) = \frac{2}{0.19} \sin 0.1t \sin 1.9t$$

$$2. x(t) = \frac{3}{442} \cos 3t + \frac{63}{442} \sin 3t + \frac{83}{102} e^{-5t} - \frac{71}{39} e^{-2t}$$

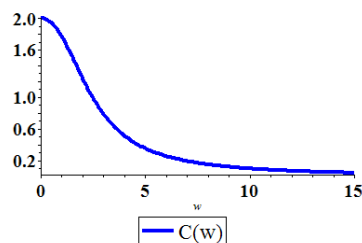
$$3. x(t) = \frac{3}{8} (\sin t - \cos t) + \frac{1}{8} e^{-2t} (3 \cos t - 21 \sin t)$$

$$4. y_p = -\frac{8}{13} \cos 3x + \frac{1}{13} \sin 3x + \frac{1}{5} e^{2x}$$

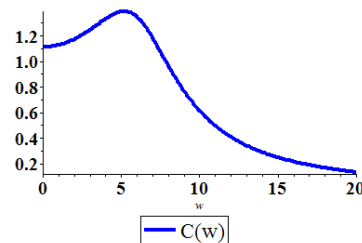
$$5. x(t) = x_0 + \frac{F_0}{k - m\omega^2} (\cos \omega t - \cos \omega_0 t)$$

$$6. x(t) = \frac{v_0}{\omega} \sin \omega t$$

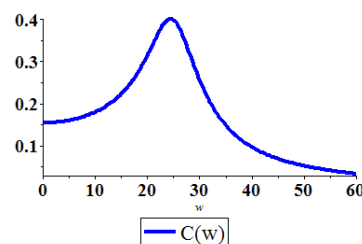
$$7. x(t) = \frac{F_0}{\omega_0 (\omega_0^2 - \omega^2)} (\omega_0 \sin \omega t - \omega \sin \omega_0 t)$$



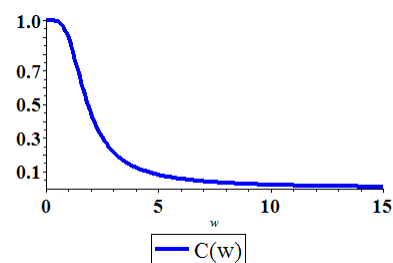
no frequency



$\omega = 3\sqrt{3}$



$\omega = 10\sqrt{6}$

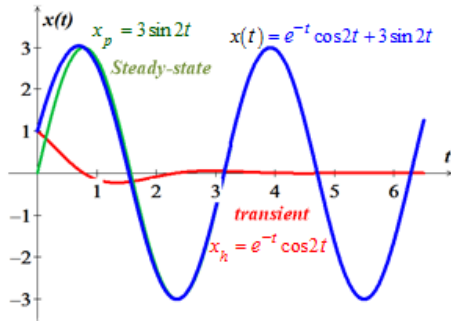


no frequency

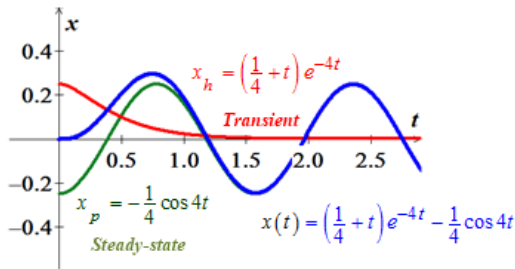
$$8. \frac{\sqrt{384}}{2\pi} \text{ Hz}$$

$$9. x(t) = e^{-t/2} \left(-\frac{4}{3} \cos \frac{\sqrt{47}}{2} t - \frac{64}{3\sqrt{47}} \sin \frac{\sqrt{47}}{2} t \right) + \frac{10}{3} (\cos 3t + \sin 3t)$$

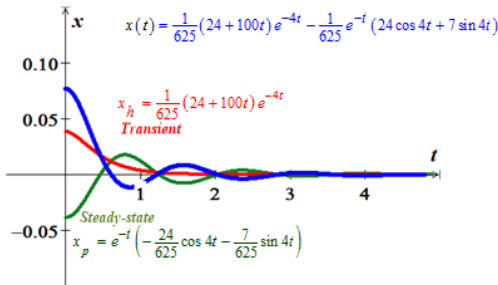
10. $x(t) = e^{-t} \cos 2t + 3 \sin 2t$



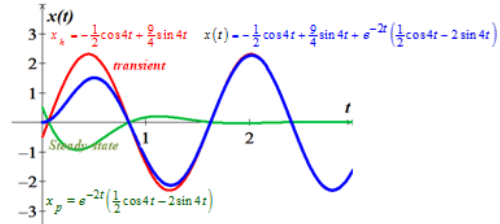
11. $x(t) = \left(\frac{1}{4} + t\right)e^{-4t} - \frac{1}{4} \cos 4t$



12. $x(t) = \frac{1}{625}(24 + 100t)e^{-4t} - \frac{1}{625}e^{-t}(24 \cos 4t + 7 \sin 4t)$



13. $x(t) = -\frac{1}{2} \cos 4t + \frac{9}{4} \sin 4t + e^{-2t} \left(\frac{1}{2} \cos 4t - 2 \sin 4t \right)$



14. $y(t) = \frac{1}{5} \cos 5t - \frac{1}{50} \sin 5t + \frac{1}{3} t \sin 5t$

15. $y(t) = e^{-3t/2} \left(-\frac{2}{25} \cos \frac{\sqrt{1271}}{2} t + \frac{4}{25\sqrt{1271}} \sin \frac{\sqrt{1271}}{2} t \right) + \frac{2}{25} \cos 2t + \frac{3}{50} \sin 2t$

16. $y(t) = e^{-5t/4} \left(\frac{1,733}{36,964} \cos \frac{\sqrt{759}}{4} t + \frac{8,641}{36,964\sqrt{759}} \sin \frac{\sqrt{759}}{4} t \right) + \frac{144}{46,205} \cos t + \frac{3}{18,482} \sin t$

17. $y_p(t) = \frac{\sqrt{2}}{50} \cos 2t + \frac{7\sqrt{2}}{50} \sin 2t$

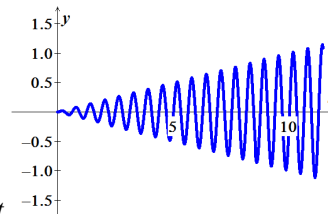
18. a) $y_p(t) = -\frac{33}{185} \cos 4t + \frac{24}{185} \sin 4t$

b) $A = \frac{9\sqrt{185}}{185} \text{ ft}, f = \frac{2}{\pi}$

19. a) $y_p(t) = -\frac{3}{1972} \cos 4t - \frac{11}{986} \sin 4t$

b) $A = 0.01 \text{ m}, \phi \approx 6.148 \text{ rad}, P = \frac{\pi}{2}$

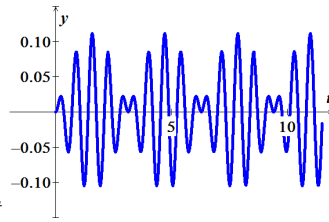
$y_p(t) = 0.01 \sin(4t + 6.148)$



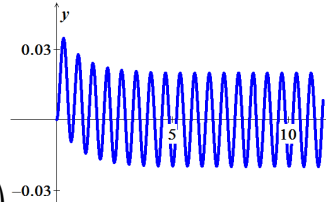
20. a) $k = 1,000 \text{ N/m}$ b) $y(t) = \frac{1}{10} t \sin 10t$

d) no maximum excursion

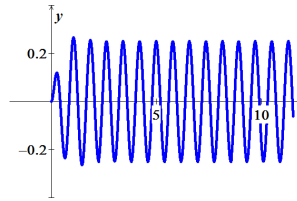
21. a) $k = 1,000 \text{ N/m}$ b) $y(t) = -\frac{1}{18} \cos 10t + \frac{1}{18} \cos 8t$ d) $\frac{1}{9}$



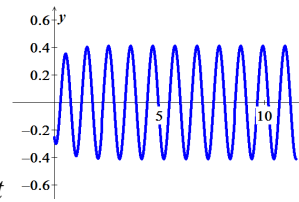
22. a) $k = 1,000 \text{ N/m}$ b) $\frac{1}{505} (\sin 10t - 10 \cos 10t + 10e^{-t})$ d) $|y_{\max}| \approx 0.03456 \text{ m}$



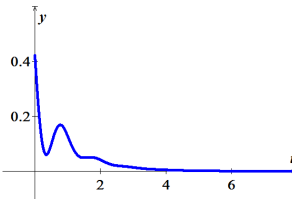
23. $y(t) = e^{-2t} \left(\frac{3}{20} \cos 6t - \frac{13}{60} \sin 6t \right) - \frac{3}{20} \cos 8t + \frac{1}{5} \sin 8t$ doesn't exist



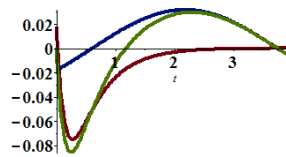
24. a) $y(t) = e^{-2t} \left(\frac{15}{37} \cos 6t + \frac{5}{74} \sin 6t \right) - \frac{15}{37} \cos 6t + \frac{5}{74} \sin 6t$ doesn't exist



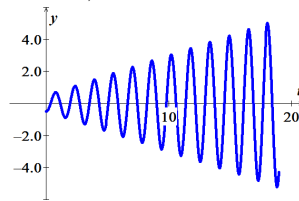
25. a) $y(t) = e^{-2t} \left(-\frac{10}{37} \cos 6t - \frac{5}{111} \sin 6t \right) + \frac{10}{37} e^{-t}$ c) 0



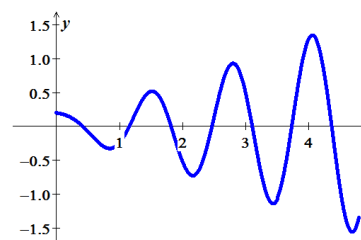
26. a) $y(t) = \frac{1}{500} (99e^{-7t} - 90e^{-2t} + 13 \sin t - 9 \cos t)$ doesn't exist



27. a) $y(t) = -\frac{1}{2} \cos 4t + \frac{1}{16} \sin 4t - \frac{1}{4} t \cos 4t$ c) ∞



28. $k = 75 \text{ N/m}$ $\omega = 5$ $y(t) = \frac{1}{5} \cos 5t - \frac{1}{50} \sin 5t + \frac{1}{3} t \sin 5t$



$$29. x(t) = e^{-t}(\cos 5t - 3\sin 5t) + 5\cos 4t + 4\sin 4t$$

$$30. mx'' + \mu x' + kx = kh(t) \quad x(t) = e^{-2t} \left(-\frac{56}{13}\cos 2t - \frac{72}{13}\sin 2t \right) + \frac{56}{13}\cos t + \frac{32}{13}\sin t$$

$$31. \omega_0 = \sqrt{\frac{k}{m} + \frac{g}{L}}$$

$$32. \omega = a \sqrt{\frac{k}{ma^2 + I}}$$

$$33. 100(1 - \cos(3.13t)) \approx 3.8 \text{ in}$$

$$34. a) -\frac{GMmr}{R^3} \quad b) r'' + \frac{g}{R}r = 0 \quad c) \approx 84.38 \text{ min} \approx 1.7691 \times 10^4 \text{ mi/hr}$$

$$35. V = LC \frac{d^2v}{dt^2} + \left(R_1 C + \frac{L}{R_2} \right) \frac{dv}{dt} + \left(\frac{R_1 + R_2}{R_2} \right) v(t)$$

$$36. V = R_1 CL \frac{d^2}{dt^2} i(t) + (R_1 R_2 C + L) \frac{d}{dt} i(t) + (R_1 + R_2) i(t)$$

$$37. q_p(t) = \frac{CE_2(\omega RC \cos \omega t + (1 - \omega LC) \sin \omega t)}{1 - 2\omega^2 LC + \omega^4 L^2 C^2 + \omega^2 R^2 C^2} \quad i_p(t) = \frac{CE_2(-\omega^2 RC \sin \omega t + (\omega - \omega^2 LC) \cos \omega t)}{1 - 2\omega^2 LC + \omega^4 L^2 C^2 + \omega^2 R^2 C^2}$$

$$38. q(t) = 10 - 10e^{-3t}(\cos 3t + \sin 3t) \quad q\left(\frac{\pi}{3}\right) = 10 + 10e^{-\pi} C$$

$$39. q(t) = (-0.012 + 1.4t)e^{-50t} + 0.012 \quad q\left(\frac{1}{35}\right) \approx 0.01871 C$$

$$40. q(t) = -\frac{1}{2}e^{-10t}(\cos 10t + \sin 10t) + \frac{3}{2} \quad \frac{3}{2}$$

$$41. q(t) = -\frac{1}{100\sqrt{7}}e^{-25t}(\sqrt{7}\cos 25\sqrt{7}t + \sin 25\sqrt{7}t) + \frac{1}{100}$$

$$i(t) = -\frac{1}{100\sqrt{7}}e^{-25t}(4,350\sqrt{7}\cos 25\sqrt{7}t - 4,400\sin 25\sqrt{7}t)$$

$$42. q(t) = \frac{150}{13}\cos t + \frac{100}{13}\sin t \quad i(t) = -\frac{150}{13}\sin t + \frac{100}{13}\cos t$$

$$43. q(t) = -\frac{1}{26}\cos 60t - \frac{3}{52}\sin 60t \quad i(t) = \frac{1}{26}\sin 60t - \frac{3}{52}\cos 60t$$

$$44. q = -\frac{1}{26}\cos 60t - \frac{3}{52}\sin 60t + \frac{1}{17}\cos 40t + \frac{4}{17}\sin 40t \quad i = \frac{1}{26}\sin 60t - \frac{3}{52}\cos 60t - \frac{1}{17}\sin 40t + \frac{4}{17}\cos 40t$$

$$45. q(t) = \sqrt{LC} \left(i_0 - \frac{E_0 C \omega}{1 - LC \omega^2} \right) \sin \frac{1}{\sqrt{LC}} t + \frac{E_0 C}{1 - LC \omega^2} \sin \omega t$$

$$i(t) = \left(i_0 - \frac{E_0 C \omega}{1 - LC \omega^2} \right) \cos \frac{1}{\sqrt{LC}} t + \frac{E_0 C \omega}{1 - LC \omega^2} \cos \omega t$$

$$46. q(t) = \left(q_0 - \frac{E_0 C}{1 - LC \omega^2} \right) \cos \frac{1}{\sqrt{LC}} t + i_0 \sqrt{LC} \sin \frac{1}{\sqrt{LC}} t + \frac{E_0 C}{1 - LC \omega^2} \cos \omega t$$

$$i(t) = -\frac{1}{\sqrt{LC}} \left(q_0 - \frac{E_0 C}{1 - LC \omega^2} \right) \sin \frac{1}{\sqrt{LC}} t + i_0 \cos \frac{1}{\sqrt{LC}} t - \frac{E_0 C \omega}{1 - LC \omega^2} \sin \omega t$$

$$47. q(t) = \frac{100}{100 - \omega^2}(10\sin \omega t - \sin 10t) \quad i(t) = \frac{1000}{100 - \omega^2}(\omega \cos \omega t - \cos 10t)$$

$$48. q(t) = \frac{72}{35} \sin \frac{1}{2}t - \frac{12}{35} \sin 3t \quad i(t) = \frac{36}{35} \cos \frac{1}{2}t - \frac{4}{35} \cos 3t$$

$$49. q(t) = -\frac{64}{5} \cos \frac{t}{2} + \frac{48}{5} \sin \frac{t}{2} + \left(8t + \frac{64}{5}\right)e^{-t} \quad i(t) = \frac{32}{5} \sin \frac{t}{2} + \frac{24}{5} \cos \frac{t}{2} - \left(8t + \frac{24}{5}\right)e^{-t}$$

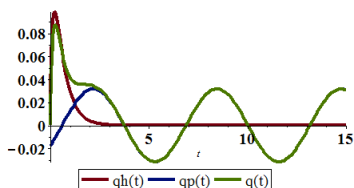
$$50. V(t) = -2e^{-t} \cos t + 2e^{-t}$$

$$52. V(t) = e^{-t}(-4 \cos t - 2 \sin t) + 4 \cos t - 2 \sin t$$

$$51. V(t) = e^{-t}(-2 \cos t - 6 \sin t) + 2 \cos t + 4 \sin t$$

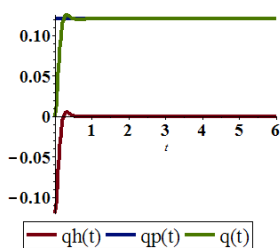
$$53. V(t) = \frac{4}{3}e^{-t} \cos \sqrt{3}t - \frac{4}{3}e^{-t}$$

$$54. q(t) = -\frac{101}{500}e^{-7t} + \frac{11}{50}e^{-2t} - \frac{9}{500} \cos t + \frac{13}{500} \sin t \quad i(t) = \frac{707}{500}e^{-7t} - \frac{22}{50}e^{-2t} + \frac{9}{500} \sin t + \frac{13}{500} \cos t$$



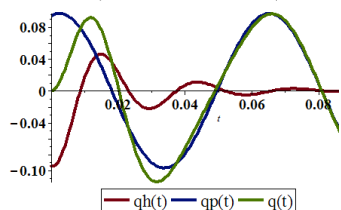
$$55. q(t) = -\frac{3}{25}e^{-10t}(\cos 10t + \sin 10t) + \frac{3}{25}$$

$$i(t) = \frac{12}{5}e^{-10t} \sin 10t$$



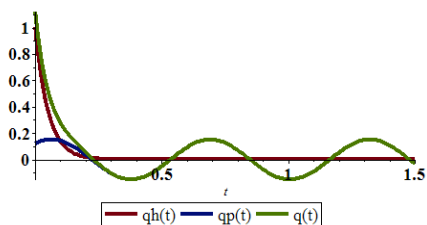
$$56. q(t) = e^{-50t} \left(-\frac{8}{85} \cos 50\sqrt{19}t - \frac{12}{85\sqrt{19}} \sin 50\sqrt{19}t \right) + \frac{8}{85} \cos 100t + \frac{2}{85} \sin 100t$$

$$I(t) = -\frac{200}{85}e^{-50t} \left(\cos 50\sqrt{19}t - \frac{41}{\sqrt{19}} \sin 50\sqrt{19}t \right) + \frac{160}{17} \sin 100t + \frac{40}{17} \cos 100t$$



$$57. q(t) = e^{-20t} \left(-\frac{84}{697} \cos 15t - \frac{464}{2091} \sin 15t \right) + \frac{84}{697} \cos 10t + \frac{64}{697} \sin 10t$$

$$i(t) = -\frac{1}{2091}e^{-20t}(1,920 \cos 15t - 13,060 \sin 15t) - \frac{840}{697} \sin 10t + \frac{640}{697} \cos 10t$$



$$58. q(t) = e^{-10t} \left(-\frac{3}{125} \cos 20t - \frac{3}{250} \sin 20t \right) + \frac{3}{125} \quad i(t) = \frac{3}{5}e^{-10t} \sin 20t$$

$$59. q(t) = e^{-10t} \left(\frac{3}{250} \cos 20t - \frac{3}{500} \sin 20t \right) - \frac{3}{250} \cos 10t + \frac{3}{125} \sin 10t$$

$$i(t) = e^{-10t} \left(-\frac{6}{25} \cos 20t - \frac{9}{50} \sin 20t \right) + \frac{3}{25} \sin 10t + \frac{6}{25} \cos 10t$$

$$60. i(t) = 0.15e^{-50t} \quad V_C = 25 - 15e^{-50t}$$

$$61. q(t) = \frac{CE_2 (\omega RC \cos \omega t + (1 - \omega LC) \sin \omega t)}{1 - 2\omega^2 LC + \omega^4 L^2 C^2 + \omega^2 R^2 C^2} \quad i(t) = \frac{CE_2 (-\omega^2 RC \sin \omega t + (\omega - \omega^2 LC) \cos \omega t)}{1 - 2\omega^2 LC + \omega^4 L^2 C^2 + \omega^2 R^2 C^2}$$

$$62. i(t) = \frac{\omega E_0}{20(\omega^2 + 25)} \left(\frac{\omega}{5} \sin \omega t + \cos \omega t - e^{-5t} \right)$$

Section 2.7

$$y(t) = e^{t^2/2}$$

Lecture 3

Section 3.1

1. $F(s) = \frac{3}{s}$
2. $F(s) = \frac{1}{s^2}$
3. $F(s) = \frac{2}{s^3}$
4. $F(s) = \frac{1}{s-6}$
5. $F(s) = \frac{1}{s+2}$
6. $F(s) = \frac{1}{(s+3)^2}$
7. $F(s) = \frac{1}{(s-3)^2}$
8. $F(s) = \frac{s-2}{9+(s-2)^2}$
9. $F(s) = \frac{3}{s^2+9}$
10. $F(s) = \frac{2}{s^2+4}$
11. $F(s) = \frac{s}{s^2+4}$
12. $F(s) = \frac{s}{s^2+b^2}$
13. $F(s) = \frac{e^7}{s-1}$
14. $F(s) = \frac{1}{e^5} \cdot \frac{1}{s+2}$
15. $F(s) = \frac{1}{(s-4)^2}$
16. $F(s) = \frac{2}{(s+2)^3}$
17. $F(s) = \frac{1}{(s+1)^2+1}$
18. $F(s) = \frac{s-2}{(s-2)^2+9}$
19. $F(s) = \frac{2}{(s+1)^2+4}$
20. $F(s) = \frac{2s}{(s^2+1)^2}$

$$21. F(s) = \frac{s^2-1}{(s^2+1)^2}$$

$$22. F(s) = \frac{48}{s^5}$$

$$23. F(s) = \frac{s}{s^2+\omega^2}$$

Section 3.2

1. $F(s) = \frac{2+4s+5s^2}{s^3} \quad s > 0$
2. $F(s) = \frac{-2s^3+12s^2-18s+12}{(s^2+1)(s^2+9)} \quad s > 0$
3. $F(s) = \frac{3s^3+6s^2+27s+150}{(s^2+9)(s^2+25)} \quad s > 0$
4. $F(s) = \frac{48}{s^5} \quad s > 0$
5. $F(s) = \frac{5!}{s^5}$
6. $F(s) = \frac{4-10s}{s^2}$
7. $F(s) = \frac{7+3s}{s^2}$
8. $F(s) = \frac{s^4-4s^2+72}{s^5}$
9. $F(s) = \frac{s^3+3s^2+6s+6}{s^4}$
10. $F(s) = \frac{48-24s+6s^2-s^3}{s^4}$
11. $F(s) = \frac{s^4-4s^3+12s^2-24s+24}{s^5}$
12. $F(s) = \frac{2s^2+6s-3}{s^3}$
13. $F(s) = \frac{9s^2+16s-8}{s^3}$
14. $F(s) = \frac{-s^3+6s-12}{s^3(s-2)}$
15. $F(s) = \frac{-s^3+9s^2+2s+18}{s^3(s+9)}$

$$16. F(s) = \frac{-2s^3 - 22s^2 + s - 3}{s^3(s+3)}$$

$$17. F(s) = \frac{5s^2 - s^3 + 12s - 24}{s^3(s-2)}$$

$$18. F(s) = \frac{2}{(s-2)^3}$$

$$19. F(s) = \frac{3s+8}{(s+2)^2}$$

$$20. F(s) = \frac{4s^2 + 11s + 9}{(s+1)^3}$$

$$21. F(s) = \frac{2s-4}{s^2-4s}$$

$$22. F(s) = \frac{s-2}{s^2-4s+8}$$

$$23. F(s) = \frac{6}{s^4} - \frac{1}{(s-1)^2} + \frac{s-4}{(s-4)^2+1}$$

$$24. F(s) = \frac{6}{s^3} - \frac{3}{s^2} - \frac{6}{(s+1)^2+9}$$

$$25. F(s) = \frac{4}{2s(s^2+4)}$$

$$26. F(s) = \frac{2}{(s-7)((s-7)^2+4)}$$

$$27. F(s) = \frac{1}{2} \frac{1}{s^2} - \frac{1}{2} \frac{s^2-4}{(s^2+4)^2}$$

$$28. F(s) = \frac{3s}{4(s^2+1)} + \frac{s}{4(s^2+9)}$$

$$29. F(s) = \frac{4(s+1)}{((s+1)^2+4)^2}$$

$$30. F(s) = \frac{s-2}{(s-2)^2+25}$$

$$31. F(s) = \frac{2}{s^3} + \frac{2}{(s-1)^2+4}$$

$$32. F(s) = \frac{s+1}{(s+1)^2+9} + \frac{1}{s-6} + \frac{1}{s}$$

$$33. F(s) = \frac{2}{(s+2)^2+4} + \frac{2}{(s-3)^3}$$

$$34. F(s) = \frac{4}{(s+2)^3} - \frac{1}{s} - \frac{4}{s^2+4}$$

$$35. F(s) = \frac{6s}{(s^2+9)^2}$$

$$36. F(s) = \frac{2s^3-24s}{(s^2+4)^3}$$

$$37. F(s) = \frac{4s^2+8s+2}{s(s+1)(s+2)}$$

$$38. F(s) = \frac{4s^2-16s+8}{s(s-2)(s-4)}$$

$$39. F(s) = \frac{s^2+8}{s(s^2-4)}$$

$$40. F(s) = \frac{-15s^3+2s^2+18}{s^5+9s^3}$$

$$41. F(s) = \frac{s^3+2s^2+4s+50}{(s^2+4)(s^2+25)}$$

$$42. F(s) = \frac{6}{(s-3)^2+36} - \frac{6}{s^4} + \frac{1}{s-1}$$

$$43. F(s) = \frac{24}{s^5} + \frac{2}{s^3} - \frac{1}{s^2} + \frac{\sqrt{2}}{s^2+2}$$

$$44. F(s) = \frac{24}{(s-5)^5} - \frac{s-1}{(s-1)^2+7}$$

$$45. F(s) = \frac{s+2}{(s+2)^2+3} - \frac{2}{(s+2)^3}$$

$$46. F(s) = \frac{6}{s+5} + \frac{1}{s-3} + \frac{5}{s^4} - \frac{9}{s}$$

$$47. F(s) = \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$

$$48. F(s) = \frac{6}{s^2-4} + \frac{6}{s^2+4}$$

$$49. F(s) = \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$$

$$50. F(s) = \frac{s^2+9}{(s^2-9)^2}$$

$$51. F(s) = \frac{12s^2-16}{(s^2+4)^3}$$

52. $F(s) = \frac{k}{s^2 - k^2}$

53. $F(s) = \frac{s}{s^2 - k^2}$

54. $F(s) = \frac{1}{2} \left(\frac{1}{s - (k+1)} - \frac{1}{s + (k-1)} \right)$

55. $F(s) = \frac{1}{2} \left(\frac{1}{s - (k-1)} + \frac{1}{s + (k-1)} \right)$

56. $Y(s) = \frac{2s}{(s+2)(s^2+1)^2} + \frac{1}{s+2}$

57. $Y(s) = \frac{2}{(s-1)(s-2)^3}$

58. $Y(s) = \frac{s^3 + 2s^2 + 6s + 1}{(s^2 + 2s + 5)(s^2 + s + 2)}$

59. $Y(s) = \frac{s+3}{(s-5)(s+2)}$

60. $Y(s) = \frac{-2s^2 + s - 8}{(s-4)(s^2+4)}$

61. $Y(s) = \frac{s^3 + 2s^2 + 5s + 8}{(s^2+4)(s^2+2s+2)}$

62. $Y(s) = \frac{-s^4 + 4s^3 + 4s^2 + s + 1}{s^2(s+1)(s^2+3s+5)}$

Section 3.3

1. $y(t) = \frac{1}{3}e^{-(2/3)t}$

2. $y(t) = -\frac{2}{5}e^{(3/5)t}$

3. $y(t) = \frac{1}{2}\sin 2t$

4. $y(t) = 3t$

5. $y(t) = 3\cos 5t + \frac{2}{5}\sin 5t$

6. $y(t) = \frac{2}{3}\sin 3t - 5\cos 3t$

7. $y(t) = \frac{5}{2}t^2e^{-2t}$

8. $y(t) = \frac{1}{120}t^5e^t$

9. $y(t) = 4e^t \cos 2t$

10. $y(t) = e^t \left(2\cos \sqrt{5}t - \frac{\sqrt{5}}{5}\sin \sqrt{5}t \right)$

11. $y(t) = e^{-t} + e^{2t}$

12. $y(t) = e^{4t} + e^{-2t}$

13. $y(t) = 4e^{-t} + 3\cos 2t$

14. $y(t) = \frac{4}{169}e^{-2t} + \frac{1}{13}te^{-2t} - \frac{4}{169}\cos 3t - \frac{5}{507}\sin 3t$

15. $y(t) = \frac{5}{169}e^{-2t} - \frac{2}{13}te^{-2t} - \frac{5}{169}\cos 3t + \frac{12}{169}\sin 3t$

16. $y(t) = -\frac{2}{15}e^{-t} + \frac{1}{5}te^{-t} - \frac{1}{60}e^{2t} + \frac{3}{20}e^{-2t}$

17. $y(t) = 10e^t - 3e^{-t} \cos 2t$

18. $f(t) = \frac{1}{2}t^2$

19. $f(t) = \frac{1}{6}t^3$

20. $f(t) = t - 2t^4$

21. $f(t) = t - 1 + e^{2t}$

22. $f(t) = 4 + \frac{1}{6}t^4 + e^{8t}$

23. $f(t) = \frac{1}{4}e^{-t/2}$

24. $f(t) = \frac{1}{5}e^{-2t/5}$

25. $f(t) = \cos \sqrt{2}t + \frac{1}{2}\sin \sqrt{2}t$

26. $f(t) = 2\cos 3t - 2\sin 3t$

27. $f(t) = 10\cos 4t$

28. $f(t) = 4t - \frac{2}{3}t^3 + \frac{1}{5!}t^5$

29. $f(t) = 1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3$

30. $f(t) = 1 + 4t + 2t^2$

31. $f(t) = \frac{\sqrt{3}}{12} \left(e^{\sqrt{3}t} - e^{-\sqrt{3}t} - \sin \sqrt{3}t \right)$

32. $f(t) = \frac{t}{5} + \frac{1}{5}\cos \sqrt{5}t$

33. $f(t) = \frac{5}{6}\sin 6t$

34. $f(t) = 10\cos 4t$

35. $f(t) = \cos \frac{1}{2}t$

36. $f(t) = \frac{1}{4}\cos \frac{1}{2}t$

37. $f(t) = \frac{1}{3}t - \frac{1}{3}e^{-3t}$

38. $f(t) = -\frac{1}{4}t + \frac{5}{4}e^{4t}$

39. $f(t) = \frac{1}{5}(t - \cos\sqrt{5}t)$

40. $f(t) = \sin 3t$

41. $f(t) = \sin 2t$

42. $f(t) = \frac{3}{16}t^2 e^{-5t/8}$

43. $f(t) = t^3 e^t$

44. $f(t) = \frac{5}{6}t^3 e^{-2t}$

45. $f(t) = e^t \cos 2t$

46. $f(t) = 3e^t \cos 3t - \frac{1}{3}e^{-t} \sin 3t$

47. $f(t) = -\frac{1}{4}e^t + \frac{3}{4}e^{-3t}$

48. $f(t) = -\frac{1}{4\sqrt{21}}e^{(-1-2\sqrt{21})t} + \frac{1}{4\sqrt{21}}e^{(-1+2\sqrt{21})t}$

49. $f(t) = e^{-t} \cos 3t$

50. $f(t) = \frac{1}{2}e^{-2t} \sin 2t$

51. $f(t) = 2e^{-2t} \cos 3t + 4e^{-2t} \sin 3t$

52. $f(t) = \frac{1}{2}e^{-t/4} \cos\left(\frac{\sqrt{47}}{4}t\right) - \frac{5}{2\sqrt{47}}e^{-t/4} \sin\left(\frac{\sqrt{47}}{4}t\right)$

53. $f(t) = -\frac{1}{8}t + \frac{17}{24}e^{4t} + \frac{5}{12}e^{-2t}$

54. $f(t) = 2\cos t + \sin t - 2\cos 2t - \frac{1}{2}\sin 2t$

55. $f(t) = \frac{1-\sqrt{3}}{2}e^{\sqrt{3}t} + \frac{1+\sqrt{3}}{2}e^{\sqrt{3}t}$

56. $f(t) = \frac{1}{3}\sin t - \frac{1}{6}\sin 2t$

57. $f(t) = -t + 3e^{-t} - 2\cos t + 3\sin t$

58. $f(t) = -\frac{1}{4}e^{-2t} + \frac{1}{4}\cos 2t + \frac{1}{4}\sin 2t$

59. $f(t) = \frac{1}{2}t - e^t - \frac{1}{3}e^{-t} + \frac{5}{6}e^{2t}$

60. $f(t) = \frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$

61. $f(t) = 2e^{-t} - 3e^{-2t} + e^{3t}$

62. $f(t) = 2e^t + 3te^t - e^{-3t}$

63. $f(t) = 3e^t \cos 2t + 4e^t \sin 2t - e^{-t}$

64. $f(t) = -4e^t - e^{-2t} + 6e^{-5t}$

65. $f(t) = -\frac{8}{3}e^t + \frac{5}{3}e^{-2t}$

66. $f(t) = 2e^t - 11e^{2t} + e^{-t}$

67. $f(t) = 2e^t + 2e^t \sin 2t$

68. $f(t) = -2e^{-2t} + 2\cos 3t - 3\sin 3t$

69. $f(t) = \frac{3}{2}t^2 + 2t + 1 - e^{-t}$

70. $f(t) = -\frac{3}{2}t^2 + 1 + 6e^{2t}$

71. $f(t) = 5e^t + 2e^{2t} \cos 3t - 5e^{2t} \sin 3t$

72. $f(t) = 6\cos\sqrt{7}t - \frac{5}{\sqrt{7}}\sin\sqrt{7}t$

73. $f(t) = \frac{13}{\sqrt{5}}e^{-4t} \sin\sqrt{5}t - 3e^{-4t} \cos\sqrt{5}t$

74. $f(t) = \frac{3}{2}e^{3t/2} \cosh\left(\frac{\sqrt{13}}{2}t\right) + \frac{5}{2\sqrt{13}}e^{3t/2} \sinh\left(\frac{\sqrt{13}}{2}t\right)$

75. $f(t) = -\frac{5}{7}e^{-2t} + \frac{12}{7}e^{5t}$

76. $f(t) = -3e^{-3t} + 2e^{4t} + e^{t/5}$

77. $f(t) = -\frac{28}{47}e^{6t} + \frac{28}{47}\cos\sqrt{11}t - \frac{67}{47\sqrt{11}}\sin\sqrt{11}t$

78. $f(t) = \frac{11}{5} - 4t + \frac{5}{2}t^2 - \frac{11}{5}e^{-2t} \cos t - \frac{2}{5}\sin t$

79. $f(t) = 5u_6(t)g(t-6) - 3u_{11}(t)g(t-11)$

Section 3.4

1. $y(t) = -\frac{7}{4}e^{-t} - \frac{1}{4}e^t + \frac{1}{2}te^t$

2. $y(t) = \frac{1}{13}e^t - \frac{1}{13}\cos 4t + \frac{5}{13}\sin 5t$

3. $y(t) = -1 + e^t + \frac{1}{2}t^2 e^t$

4. $y(t) = \frac{1}{5}e^{2t} - \frac{6}{5}e^{-3t}$

5. $y(t) = -\frac{4}{17}e^{-4t} + \frac{4}{17}e^{-9t} \cos t + \frac{1}{17}\sin t$

6. $y(t) = 2e^{-4t} + te^{-4t}$
7. $y(t) = \frac{109}{108}e^{4t} - \frac{1}{108}e^{-2t} - \frac{1}{18}te^{-2t} - \frac{1}{6}t^2e^{-2t}$
8. $y(t) = \frac{1}{8}e^{-t} - \frac{1}{8}e^{-9t}$
9. $y(t) = \frac{268}{265}e^{-16t} - \frac{3}{265}\cos 3t + \frac{16}{265}\sin 3t$
10. $y(t) = \frac{1}{3}(e^{2t} - e^{-t})$
11. $y(t) = -2t + \frac{1}{2}e^t - \frac{1}{2}e^{-t}$
12. $y(t) = 2 - t + 2e^{t-2} + 2e^{-t+2}$
13. $y(t) = t + \pi \cos t + \sin t$
14. $y(t) = -e^{-t+\pi} + 3e^{t-\pi} \cos 2t + 4e^{t-\pi} \sin 2t$
15. $y(t) = t^2 + \cos t - \sin t$
16. $y(t) = 10 \cos t + 2 \sin t - \sqrt{2} \sin \sqrt{2}t$
17. $y(t) = \frac{2}{3} \cos 2t + \frac{1}{3} \cos t - \sin t$
18. $y(t) = -\frac{1}{2}t + \frac{1}{3}e^{-t} - \frac{1}{6}e^t \cos t + \frac{1}{6}e^t \sin t$
19. $y(t) = -18 - 12t - 3t^2 - t^3$
 $+ \left(-\frac{19}{2} + \frac{43\sqrt{5}}{10}\right)e^{\left(\frac{-1+\sqrt{5}}{2}\right)t}$
 $+ \left(\frac{19}{2} - \frac{43\sqrt{5}}{10}\right)e^{\left(\frac{1+\sqrt{5}}{2}\right)t}$
20. $y(t) = -3 + 2t - 2t^2 + 2e^{-t} + 2e^{2t}$
21. $y(t) = -\frac{5}{9}e^{-t} - \frac{4}{9}e^{2t} + \frac{1}{3}te^{2t}$
22. $y(t) = e^{2t} - 3e^{-t}$
23. $y(t) = \frac{3}{5}e^{2t-\pi} - e^{-t+\frac{\pi}{2}} + \frac{7}{5}\sin t + \frac{11}{5}\cos t$
24. $x(t) = \frac{3}{5}e^{3t} + \frac{7}{5}e^{-2t}$
25. $y(t) = e^{-t} + 2te^{-t}$
26. $y(t) = -2 + t - e^{-t} + te^{-t}$
27. $y(t) = -e^{2t} + \frac{1}{2}e^t + \left(\frac{3}{4} + \frac{3\sqrt{2}}{4}\right)e^{(1+\sqrt{2})t}$
 $+ \left(\frac{3}{4} - \frac{3\sqrt{2}}{4}\right)e^{(1-\sqrt{2})t}$
28. $y(t) = 6t + 10 + 2te^{t+1} + e^{t+1}$
29. $y(t) = -\frac{1}{2}\cos t - \frac{1}{2}\sin t + \frac{3}{2}e^t + 2te^t$
30. $y(t) = 2e^t \cos 2t + e^t \sin 2t$
31. $y(t) = \frac{7}{25} + \frac{1}{5}t - \frac{7}{25}e^t \cos 2t + \frac{1}{25}e^t \sin 2t$
32. $y(t) = -\frac{7}{9} + \frac{1}{3}t - \frac{1}{2}t^2 - \frac{2}{9}e^{-3t}$
33. $y(t) = -\frac{2}{3}t + \frac{1}{3}t^3 + \frac{2\sqrt{3}}{9}\sin(\sqrt{3}t)$
34. $y(t) = \frac{1}{6}e^{-t} - \frac{1}{2}e^{t-2} + \frac{1}{3}e^{2t-3}$
35. $y(t) = \frac{1}{2}e^t - \frac{3}{5}e^{2t} + \frac{1}{10}\cos t - \frac{3}{10}\sin t$
36. $y(t) = -\frac{1}{3}e^{-t} - \frac{5}{12}e^{2t} - \frac{1}{4}e^{-2t}$
37. $y(t) = \frac{5}{4}t - \frac{1}{4}e^{4t}$
38. $y(t) = \frac{1}{20}t^5e^{2t}$
39. $y(t) = \frac{3}{4} + \frac{9}{8}t + \frac{3}{2}t^2 + \frac{3}{2}t^3 + \frac{1}{4}e^{2t} - \frac{13}{8}te^{2t}$
40. $x(t) = \frac{3}{8}t^2 - \frac{3}{4}t + \frac{9}{16} - \frac{9}{16}e^{-2t} - \frac{3}{8}te^{-2t}$
41. $y(t) = 2 - t + t^2 - 2\cos 2t + 2\sin 2t$
42. $y(t) = -t - e^{-2t} + 2te^{-2t} + e^{2t}$
43. $y(t) = \frac{43}{16} - \frac{1}{4}t + \frac{1}{2}t^2 - \frac{273}{272}e^{-4(t-3)}$
 $+ \frac{1}{17}(4\sin(t-3) - \cos(t-3))$
44. $y(t) = -\frac{4}{65}\cos t + \frac{7}{65}\sin t$
 $+ \frac{69}{65}e^{-2t}\cos 2t + \frac{131}{130}e^{-2t}\sin 2t$
45. $y(t) = 1 + \frac{1}{5}e^t + \left(-\frac{1}{10} + \frac{3\sqrt{29}}{290}\right)e^{\frac{-5+\sqrt{29}}{2}t}$
 $- \left(\frac{1}{10} + \frac{3\sqrt{29}}{290}\right)e^{\frac{-5-\sqrt{29}}{2}t}$
46. $y(t) = -e^{-6(t-1)} + 3(t-1)e^{(t-1)}$
47. $y(t) = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}$
48. $y(t) = -\frac{2}{9} + \frac{1}{6}t^2 + \frac{1}{9}\cos \sqrt{6}t - \frac{1}{\sqrt{6}}\sin \sqrt{6}t$
49. $y(t) = \frac{2}{27} + \frac{1}{9}t - \frac{2}{27}e^{3t} + \frac{10}{9}te^{3t}$

50. $y(t) = \frac{1}{10} \cos 3t + \frac{1}{30} \sin 3t$
 $-\frac{11}{10} e^{3t} \cos \sqrt{6}t - \frac{8}{10\sqrt{6}} e^{3t} \sin \sqrt{6}t$
51. $y(t) = -\frac{3}{2} e^{3t} \sin 2t$
52. $y(t) = -e^{-3t} + 3te^{-3t}$
53. $y(t) = -e^{-t} - e^{-5t} + e^t$
54. $y(t) = 8e^{2t} - 4e^{5t} + \cos t$
55. $y(t) = -2e^{-4(t-\pi)} \sin 3t$
56. $y(t) = \frac{2}{5} \sin 2t - \frac{3}{5} \sin 3t$
57. $y(t) = \frac{2}{3} \sin 2t - \frac{5}{3} \sin t$
58. $y(t) = \frac{1}{16} (\sin 4t - 4t \cos 4t) - \frac{1}{2} \cos 4t$
59. $y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81} e^{9t} - 2e^t$
60. $y(t) = -\frac{96}{125} e^{t/2} + \frac{96}{125} e^{-2t}$
 $-\frac{2}{25} te^{-2t} - \frac{1}{5} t^2 e^{-2t}$
61. $y(t) = 2e^{-5t} \cos \frac{\sqrt{2}t}{2} + 10\sqrt{2}e^{-5t} \sin \frac{\sqrt{2}t}{2}$
62. $y(t) = -1 + \frac{1}{2} e^t + \frac{1}{2} \cos t - \frac{1}{2} \sin t$
63. $y(t) = \frac{13}{60} e^t - \frac{13}{20} e^{-t} + \frac{16}{39} e^{-2t}$
 $+\frac{3}{130} \cos 3t - \frac{1}{65} \sin 3t$
64. $y(t) = 2e^t - \cos t - \sin t$
65. $y(t) = 2 + e^t - 3e^{-2t} + e^{-3t}$
66. $y(t) = -4e^{-t} + t^2 e^{-t}$
67. $y(t) = e^t \left(1 + t + \frac{1}{60} t^5 \right)$
68. $y(t) = -2e^{-t} + e^t + e^{-t} \cos 2t$
69. $y(t) = -e^{-2t} - \cos t + 2 \sin t + 2e^{-t} - 2te^{-t}$
70. $y(t) = -\frac{16}{17} e^t + \frac{28}{17} te^t + \frac{16}{17} \cos t$
 $+\frac{16}{17} \sin t + \frac{27}{17} t \sin t + \frac{4}{17} \sin t - t \cos t$
71. $y(t) = \frac{1}{4} e^t + \frac{1}{4} e^{-t} + \frac{1}{2} \cos t$
72. $y(t) = \cos \sqrt{2}t$
73. $y(t) = \left(t - 3t^2 + \frac{13}{6} t^3 \right) e^t$
74. $y(t) = 2e^t - e^{3t}$
75. $x(t) = \frac{3}{10} \sin 2t - \frac{1}{5} \sin 3t$
76. $x(t) = 5 \sin t - 4 \sin 2t + \sin 3t$
 $y(t) = 10 \sin t + 4 \sin 2t - 6 \sin 3t$
77. $x(t) = (3 \cos 5t + 2 \sin 5t) e^{-3t}$
78. $x(t) = -\frac{3}{2} e^{7t/2} \cos \frac{\sqrt{15}}{2} t - \frac{7\sqrt{15}}{10} e^{7t/2} \sin \frac{\sqrt{15}}{2} t$
79. $y(t) = \frac{5}{29} (5 \sin 2t - 2 \cos 2t)$
 $+\frac{2}{29} e^{-3t} (5 \cos 5t - 2 \sin 5t)$
80. $y(t) = -\frac{3}{1972} \cos 4t - \frac{11}{986} \sin 4t$
 $+\frac{3}{1972} e^{-3t/16} \cos \frac{\sqrt{1271}}{16} t$
 $+\frac{1417}{1972\sqrt{1271}} e^{-3t/16} \sin \frac{\sqrt{1271}}{16} t$
81. $y(t) = \frac{10}{37} e^{-t} - \left(\frac{10}{37} \cos 6t + \frac{5}{111} \sin 6t \right) e^{-2t}$
82. $y(t) = \frac{1}{500} (99e^{-7t} - 90e^{-2t} + 13 \sin t - 9 \cos t)$
83. $y(t) = \frac{1}{16} \sin 4t - \frac{1}{4} t \cos 4t - \frac{1}{2} \cos 4t$
84. $x(t) = 5 \cos 4t + 4 \sin 4t + e^{-t} (\cos 5t - 3 \sin 5t)$
85. $x(t) = \frac{1}{20} e^{-4t} - \frac{1}{20} e^{-16t}$
86. $x(t) = \frac{9}{25} \sin \frac{10}{3} t$
87. $x(t) = \frac{6}{5} e^{-t} - \frac{1}{5} e^{-6t}$
88. $q(t) = q_0 e^{-20t} \left(\cos 60t + \frac{1}{3} \sin 60t \right)$
89. $q(t) = \left(5 \cos 40t + \frac{5}{2} \sin 40t \right) e^{-20t}$
90. $q(t) = e^{-3t} (4 \cos 3t + 4 \sin 3t)$
91. $i(t) = 15e^{-10t} \sin 10t$
92. $Q(t) = \frac{1}{17} \left(40e^{-t/2} - 40 \cos 2t + 10 \sin 2t \right)$
93. $Q(t) = \frac{25}{2} \left(e^{-t/10} - e^{-t/2} \right)$
94. $Q(t) = 10 + \frac{5}{2} e^{-t/2} - \frac{25}{2} e^{-t/10}$
95. $Q(t) = \frac{1}{37} \left(-10e^{-t/2} + 10 \cos 3t + 60 \sin 3t \right)$

$$96. \quad I(t) = 300 - 20t - 300e^{-t/10}$$

$$97. \quad I(t) = \frac{1}{901}(-40e^{-t/10} + 40\cos 3t + 1200\sin 3t)$$

$$98. \quad I(t) = \frac{8}{1+400\pi^2}(100\pi e^{-t/10} - 100\pi\cos 2\pi t + 5\sin 2\pi t)$$

$$99. \quad Q(t) = EC(1 - e^{-t/RC})$$

$$100. \quad I(t) = \frac{1}{R}(E + (RI_0 - E)e^{-Rt/L})$$

$$101. \quad b) \quad k \neq \frac{1}{RC} \quad q(t) = \frac{E_0 C}{1 - kRC}(e^{-kt} - e^{-t/RC})$$

$$k = \frac{1}{RC} \quad q(t) = \frac{E_0}{R}te^{-t/RC}$$

$$102. \quad i_1(t) = \frac{6}{5} - \frac{6}{5}e^{-100t} - 60te^{-100t} \quad i_2(t) = \frac{6}{5} - \frac{6}{5}e^{-100t} - 120te^{-100t}$$

$$103. \quad x_1(t) = -\frac{\sqrt{2}}{10}\sin\sqrt{2}t + \frac{\sqrt{3}}{5}\sin 2\sqrt{3}t \quad x_2(t) = -\frac{\sqrt{2}}{5}\sin\sqrt{2}t - \frac{\sqrt{3}}{10}\sin 2\sqrt{3}t$$

$$104. \quad x_1(t) = -\sin\sqrt{3}t \quad x_2(t) = -\sin\sqrt{3}t, \quad i_1(t) = 20 - 20e^{-900t} \quad i_2(t) = \frac{100}{9} - \frac{100}{9}e^{-900t}$$

$$i_3(t) = \frac{80}{9} - \frac{80}{9}e^{-900t}$$

$$105. \quad q(t) = 50e^{-t}\sin(t-1)u(t-1) \quad i_3(t) = 50e^{-t}(\cos(t-1) - \sin(t-1))u(t-1)$$

$$106. \quad a) \quad y(x) = \frac{w_0}{4EI}(1 - \cos x \cosh x) + \frac{w_0}{8EI}(1 + \cosh \pi) \operatorname{csch} \pi (\sin x \cosh x + \cos x \sinh x)$$

$$- \frac{w_0}{4EI}(1 + \cosh \pi) \operatorname{csch} \pi (\sin x \cosh x - \cos x \sinh x)$$

$$b) \quad y(x) = \frac{w_0}{4EI} \left(\sin\left(x - \frac{\pi}{2}\right) \cosh\left(x - \frac{\pi}{2}\right) - \cos\left(x - \frac{\pi}{2}\right) \sinh\left(x - \frac{\pi}{2}\right) \right) u\left(x - \frac{\pi}{2}\right)$$

$$+ \frac{w_0}{2EI} \frac{\sinh \frac{\pi}{2}}{\sinh \pi} \sin x \sinh x - \frac{w_0}{4EI} \frac{\cosh \frac{\pi}{2}}{\sinh \pi} (\sin x \cosh x - \cos x \sinh x)$$

$$107. \quad a) \quad \theta_1(t) = \frac{1}{2}(\theta_0 + \psi_0) \cos \omega t + \frac{1}{2}(\theta_0 - \psi_0) \cos \sqrt{\omega^2 + 2K} t$$

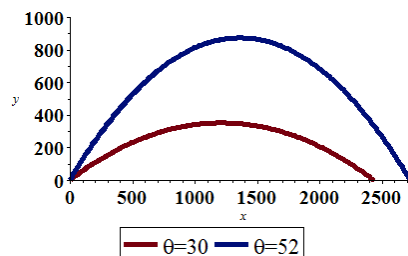
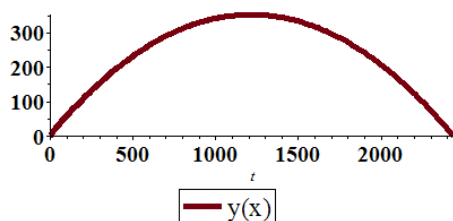
$$\theta_2(t) = \frac{1}{2}(\theta_0 + \psi_0) \cos \omega t - \frac{1}{2}(\theta_0 - \psi_0) \cos \sqrt{\omega^2 + 2K} t$$

$$b) \quad \theta_1(t) = \theta_0 \cos \omega t \quad \theta_2(t) = \theta_0 \cos \omega t$$

$$c) \quad \theta_1(t) = \theta_0 \cos \sqrt{\omega^2 + 2K} t \quad \theta_2(t) = -\theta_0 \cos \sqrt{\omega^2 + 2K} t$$

$$108. \quad a) \quad x(t) = (v_0 \cos \theta)t \quad y(t) = (v_0 \sin \theta)t - \frac{1}{2}gt^2 \quad b) \quad x = \frac{1}{g}v_0^2 \sin 2\theta \quad c) \quad \frac{1}{g}v_0^2 \sin(\pi - 2\theta)$$

$$d) \approx 2,436 \text{ ft} \quad e) t \approx 9.38 \text{ sec} \quad f) 0.57735x - 0.000237x^2 \quad g) \approx 2729 \text{ ft} \quad g) 1.2799x - 0.000469x^2$$



Section 3.5

1. Nonlinear
2. Nonlinear
3. Linear and homogeneous

4. $= -2x_1$

5. $x'_2 = 3x_2$

6. $\begin{pmatrix} -3e^{2t} - 2e^{-2t} \\ 6e^{2t} + 2e^{-2t} \end{pmatrix}$

7. $\begin{pmatrix} -te^{2t} \\ -te^{2t} + e^{2t} \end{pmatrix}$

8. $x(t) = \begin{pmatrix} \cos t + \sin t \\ 2 \sin t \end{pmatrix}$

9. $y'_4 = y_4 - 7y_1 + \cos t$

10. $-3x_3 + (\sin t)x_2 - 8x_1 + t^2$

11. $x'_6 = x_2^3 - \sin x_1 + e^{2t}$

12. $\begin{cases} x'_1 = x_2 \\ x'_2 = -\frac{5}{3}x_1 + \frac{3}{2}x_3 \\ x'_3 = x_4 \\ x'_4 = \frac{3}{2}x_1 - \frac{1}{2}x_3 \end{cases}$

13. $\begin{cases} x'_1 = x_2 \\ x'_2 = x_3 \\ x'_3 = x_4 + t \\ x'_4 = x_5 \\ x'_5 = -\frac{2}{5}x_3 + \frac{2}{5}x_4 + \frac{1}{5} \end{cases}$

14. $\begin{cases} x'_1 = x_2 \\ x'_2 = -7x_1 - 3x_2 + t^2 \end{cases}$

15. $\begin{cases} x'_1 = x_2, & x'_2 = x_3, & x'_3 = x_4 \\ x'_4 = -x_1 + 3x_2 - 6x_3 + \cos 3t \end{cases}$

16. $\begin{cases} x'_1 = x_2 \\ t^2 x'_2 = (1 - t^2)x_1 - tx_2 \end{cases}$

17. $\begin{cases} x'_1 = x_2, & x'_2 = x_3, & x'_3 = x_4 \\ t^3 x'_3 = -5x_1 - 3tx_2 + 2t^2 x_3 + \ln t \end{cases}$

18. $\begin{cases} y'_1 = y_2 \\ y'_2 = -4x_1 + 5y_1 \end{cases}$

19. $\begin{cases} y'_1 = y_2 \\ y'_2 = 3x_1 - y_1 - 2y_2 + \cos t \end{cases}$

20. $\begin{cases} x'_1 = x_2, & y'_1 = y_2, & z'_1 = z_2 \\ x'_2 = 3x_1 - y_1 + 2z_1 \\ y'_2 = x_1 + y_1 - 4z_1 \\ z'_2 = 5x_1 - y_1 - z_1 \end{cases}$

21. $\begin{cases} x'_1 = x_2, & y'_1 = y_2 \\ x'_2 = (1 - y_1)x_1 \\ y'_2 = (1 - x_1)y_1 \end{cases}$

22. X_1, X_2 , and X_3 form a fundamental set

23. X_1 and X_2 form a fundamental set

24. b) LI c) $\begin{pmatrix} 2C_1 e^t + C_2 e^{2t} \\ -3C_1 e^t - C_2 e^{2t} \end{pmatrix}$

25. b) LI c) $\begin{pmatrix} C_1 e^{3t} + 2C_2 e^{-2t} \\ 3C_1 e^{3t} + C_2 e^{-2t} \end{pmatrix}$ d) $\begin{cases} x_1 = 2e^{3t} - 2e^{-2t} \\ x_2 = 6e^{3t} - e^{-2t} \end{cases}$

26. b) LI c) $\begin{pmatrix} C_1 e^{2t} + C_2 e^{-2t} \\ C_1 e^{2t} + 5C_2 e^{-2t} \end{pmatrix}$ d) $\begin{cases} x_1 = 7e^{2t} - 2e^{-2t} \\ x_2 = 7e^{2t} - 10e^{-2t} \end{cases}$

27. b) LI c) $\begin{pmatrix} 3C_1 e^{2t} + C_2 e^{-5t} \\ 2C_1 e^{2t} + 3C_2 e^{-5t} \end{pmatrix}$

d) $\begin{cases} x_1 = \frac{72}{7}e^{2t} - \frac{16}{7}e^{-2t} \\ x_2 = \frac{48}{7}e^{2t} - \frac{48}{7}e^{-2t} \end{cases}$

$$28. \quad b) \text{ LI} \quad c) \begin{pmatrix} 2C_1 e^t - 2C_2 e^{3t} + 2C_3 e^{5t} \\ 2C_1 e^t - 2C_3 e^{5t} \\ C_1 e^t + C_2 e^{3t} + C_3 e^{5t} \end{pmatrix}$$

$$d) \begin{cases} x_1(t) = 2e^t - 4e^{3t} + 2e^{5t} \\ x_2(t) = 2e^t - 2e^{5t} \\ x_3(t) = e^t + 2e^{3t} + e^{5t} \end{cases}$$

$$29. \quad b) \text{ LI} \quad c) \begin{pmatrix} C_1 e^{2t} + C_2 e^{-t} \\ C_1 e^{2t} + C_3 e^{-t} \\ C_1 e^{2t} - C_2 e^{-t} - C_3 e^{-t} \end{pmatrix}$$

$$d) \begin{cases} x_1(t) = 7e^{2t} + 3e^{-t} \\ x_2(t) = 7e^{2t} + 5e^{-t} \\ x_3(t) = 7e^{2t} - 8e^{-t} \end{cases}$$

$$30. \quad b) \text{ LI} \quad c) \begin{pmatrix} C_1 e^{-t} + C_4 e^t \\ C_3 e^t \\ C_2 e^{-t} + 3C_4 e^t \\ C_1 e^{-t} - 2C_3 e^t \end{pmatrix}$$

$$d) \begin{cases} x_1(t) = 13e^{-t} - 12e^t \\ x_2(t) = 3e^t \\ x_3(t) = 40e^{-t} - 36e^t \\ x_4(t) = 13e^{-t} - 6e^t \end{cases}$$

$$31. \quad \begin{cases} I' = \frac{V}{L} \\ V' = -\frac{V}{CR} - \frac{I}{C} \end{cases}$$

$$32. \quad CV' = -I - \frac{V}{R_2}$$

$$33. \quad \begin{aligned} L_1 I_1' &= -R_1 I_1 - R_1 I_2 + E \\ L_2 I_2' &= -R_1 I_1 - (R_1 + R_2) I_2 + E \end{aligned}$$

$$34. \quad \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} -\frac{1}{500} & \frac{1}{500} \\ \frac{1}{500} & -\frac{1}{500} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$35. \quad \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} -\frac{1}{25} & 0 \\ \frac{1}{25} & -\frac{1}{25} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$36. \quad \begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & 0 & -\frac{k_2}{m_2} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{A}{m_2} \cos \omega t \end{pmatrix}$$

$$37. \quad \begin{cases} m_1 x_1'' = -(k_1 + k_2)x_1 + k_2 x_2 \\ m_2 x_2'' = k_2 x_1 - (k_2 + k_3)x_2 \end{cases}$$

$$38. \quad \begin{cases} ky_1'' = -2y_1 + y_2 \\ ky_2'' = y_1 - 2y_2 \end{cases}$$

$$39. \quad \begin{cases} 10x_1' = -x_1 + x_3 \\ 10x_2' = x_1 - x_2 \\ 10x_3' = x_2 - x_3 \end{cases}$$

$$40. \quad mx'' = +qBy', \quad my'' = -qBx'$$

Section 3.6

$$1. \quad \lambda_1 = -2 \rightarrow V_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \lambda_2 = 5 \rightarrow V_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$2. \quad \lambda_1 = \frac{-3 - \sqrt{17}}{2} \Rightarrow V_1 = \begin{pmatrix} \frac{5 + \sqrt{17}}{4} \\ 1 \end{pmatrix}$$

$$\lambda_2 = \frac{-3 + \sqrt{17}}{2} \Rightarrow V_2 = \begin{pmatrix} \frac{5 - \sqrt{17}}{4} \\ 1 \end{pmatrix}$$

3. $\lambda_1 = -1 \Rightarrow V_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad \lambda_2 = 2 \Rightarrow V_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
4. $\lambda_1 = -5 \Rightarrow V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \lambda_2 = -2 \Rightarrow V_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
5. $\lambda_1 = -4 \Rightarrow V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \lambda_2 = 1 \Rightarrow V_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$
6. $\lambda_1 = -1 \Rightarrow V_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \lambda_2 = 3 \Rightarrow V_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
7. $\lambda_{1,2} = 4 \Rightarrow V_1 = \begin{pmatrix} \frac{3}{2} \\ 2 \\ 1 \end{pmatrix}$
8. $\lambda = 1 \Rightarrow V_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \lambda = 2 \Rightarrow V_2 = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$
 $\lambda = 3 \Rightarrow V_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$
9. $\lambda_1 = -1 \Rightarrow V_1 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad \lambda_{2,3} = 1 \Rightarrow V_2 = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$
10. $\lambda_1 = -1 \Rightarrow V_1 = \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} \quad \lambda_2 = 1 \Rightarrow V_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$
 $\lambda_3 = 2 \Rightarrow V_3 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$
11. $\lambda_1 = 1 \Rightarrow V_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad \lambda_2 = 2 \Rightarrow V_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$
 $\lambda_3 = 3 \Rightarrow V_3 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$
12. $\lambda_1 = 0 \Rightarrow V_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \lambda_2 = -2 \Rightarrow V_2 = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$
 $\lambda_3 = 2 \Rightarrow V_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
13. $\lambda = -2 \Rightarrow V_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \lambda = -1 \Rightarrow V_2 = \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$
 $\lambda = 1 \Rightarrow V_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \\ 0 \end{pmatrix} \quad \lambda = 1 \Rightarrow V_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
14. $x_2(t) = e^{2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$
15. $\lambda_1 = -2 \Rightarrow V_1 = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \quad \lambda_2 = -1 \Rightarrow V_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
 $\lambda_3 = 1 \Rightarrow V_3 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$
16. $x(t) = C_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{5t}$
17. $x(t) = C_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^t + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$
18. $x(t) = C_1 \begin{pmatrix} 2 \\ 5 \end{pmatrix} e^t + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-3t}$
19. $x(t) = C_1 \begin{pmatrix} 4 \\ 3 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-7t/2}$
20. $x(t) = C_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 \left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} t \right)$
21. $x(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \left(\begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} t \right)$
22. $x(t) = C_1 \begin{pmatrix} \cos t \\ 2 \cos t + \sin t \end{pmatrix} e^{4t}$
 $+ C_2 \begin{pmatrix} \sin t \\ 2 \sin t - \cos t \end{pmatrix} e^{4t}$
23. $x(t) = C_1 \begin{pmatrix} \cos t \\ -\cos t - \sin t \end{pmatrix}$
 $+ C_2 \begin{pmatrix} \sin t \\ \cos t - \sin t \end{pmatrix}$

24. $x(t) = C_1 \begin{pmatrix} \cos t \\ -\cos t - \sin t \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} \sin t \\ 2\sin t + \cos t \end{pmatrix} e^{4t}$
25. $x(t) = C_1 \begin{pmatrix} 5\cos 3t \\ \cos 3t - 3\sin 3t \end{pmatrix} e^{5t} + C_2 \begin{pmatrix} 5\sin 3t \\ \sin 3t + 3\cos 3t \end{pmatrix} e^{5t}$
26. $x(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$
27. $x(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{3t}$
28. $x(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{3t}$
29. $x(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + C_2 \begin{pmatrix} 4 \\ 3 \end{pmatrix} e^{2t}$
30. $x(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + C_2 \begin{pmatrix} 4 \\ 3 \end{pmatrix} e^{3t}$
31. $x(t) = C_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^t + C_2 \begin{pmatrix} 3 \\ 4 \end{pmatrix} e^{2t}$
32. $x(t) = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + C_2 \begin{pmatrix} 5 \\ 2 \end{pmatrix} e^{2t}$
33. $x(t) = C_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t + C_2 \begin{pmatrix} 5 \\ 3 \end{pmatrix} e^{2t}$
34. $x(t) = C_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$
35. $x(t) = C_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{6t}$
36. $x(t) = C_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{5t} + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{10t}$
37. $x(t) = C_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{5t} + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{15t}$
38. $x(t) = C_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t}$
39. $x(t) = C_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t$
40. $x(t) = C_1 \begin{pmatrix} 1 \\ 3 + \sqrt{6} \end{pmatrix} e^{(2 - \sqrt{6})t} + C_2 \begin{pmatrix} 1 \\ 3 - \sqrt{6} \end{pmatrix} e^{(2 + \sqrt{6})t}$
41. $x(t) = C_1 \begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{-3t} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$
42. $x(t) = C_1 \begin{pmatrix} -2\sin 2t \\ -\cos 2t \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 2\cos 2t \\ -\sin 2t \end{pmatrix} e^{-t}$
43. $x(t) = C_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} -3 \\ 1 \end{pmatrix} e^t + \begin{pmatrix} -1 \\ 3 \end{pmatrix}$
44. $x(t) = C_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{8t} + C_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} t \right] e^{8t} + \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$
45. $y_1(t) = C_1 e^{2t} + C_2 e^{7t} - 2t - \frac{4}{7} \quad y_2(t) = -4C_1 e^{2t} + C_2 e^{7t} + 6t + \frac{10}{7}$
46. $y_1(t) = -C_1 e^{2t} - 3C_2 e^{4t} + \frac{14}{30} e^{-t} - \frac{15}{4} t + \frac{35}{32} \quad y_2(t) = C_1 e^{2t} + C_2 e^{4t} - \frac{8}{30} e^{-t} + \frac{25}{8} t - \frac{89}{32}$
47. $y_1(t) = C_1 e^{-2t} + C_2 e^{-5t} + \frac{1}{4} e^{-t} + \frac{6}{5} t - \frac{27}{50} \quad y_2(t) = C_1 e^{-2t} - 2C_2 e^{-5t} + \frac{1}{2} e^{-t} + \frac{3}{5} t - \frac{21}{50}$
48. $\begin{cases} x(t) = \left(C_1 \cos 2t - C_2 \sin 2t - \frac{1}{4} \cos 2t \cos 4t - \frac{1}{4} \sin 2t \sin 4t \right) e^{2t} \\ y(t) = \left(2C_1 \sin 2t + 2C_2 \cos 2t - \frac{1}{2} \sin 2t \cos 4t + \frac{1}{2} \cos 2t \sin 4t \right) e^{2t} \end{cases}$
49. $x(t) = 2C_1 e^t + C_2 e^{2t} + 4te^t + 3 \quad y(t) = C_1 e^t + C_2 e^{2t} + 2te^t + 3$
50. $x(t) = 2C_1 e^t + C_2 e^{2t} + \frac{1}{10} e^{-3t} - 3 \quad y(t) = C_1 e^t + C_2 e^{2t} - \frac{3}{20} e^{-3t} - 1$
51. $x(t) = -2C_1 e^{-3t} + 4C_2 e^{3t} - 12t - \frac{4}{3} \quad y(t) = C_1 e^{-3t} + C_2 e^{3t} - \frac{4}{3}$

$$52. \quad x(t) = C_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^t + C_3 \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} e^{2t}$$

$$53. \quad x(t) = C_1 \begin{pmatrix} 4 \\ 0 \\ -5 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} -7 \\ 3 \\ 5 \end{pmatrix} e^{5t} + C_3 \begin{pmatrix} -7 \\ 5 \\ 5 \end{pmatrix} e^{7t}$$

$$54. \quad x(t) = C_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^t + C_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{2t} + C_3 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{2t}$$

$$55. \quad x(t) = C_1 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} e^{8t} + C_2 \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} e^{-t} + C_3 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} e^{-t}$$

$$56. \quad x(t) = C_1 \begin{pmatrix} -\frac{3}{2} \\ 2 \\ 1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} -\frac{4}{7} \\ \frac{5}{7} \\ 1 \end{pmatrix} e^{-2t}$$

$$+ C_3 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} e^{2t}$$

$$57. \quad x(t) = C_1 \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} e^{-2t} + C_2 \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} e^t + C_3 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} e^{3t}$$

$$58. \quad \begin{cases} x_1(t) = -C_1 - e^t + \left(C_2 + \frac{1}{2}t + \frac{1}{4}\right)e^{2t} + C_3 e^{3t} \\ x_2(t) = C_1 + \left(C_2 + \frac{1}{2}t - \frac{1}{4}\right)e^{2t} + 2C_3 e^{3t} \\ x_3(t) = \left(C_3 + \frac{1}{2}t^2\right)e^{3t} \end{cases}$$

$$59. \quad y_1(t) = 6e^{2t} - 5e^{5t} \quad y_2(t) = 3e^{2t} - 5e^{5t}$$

$$60. \quad y(t) = e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + e^{3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$73. \quad y_1(t) = -\frac{2}{3}e^{-t} + \frac{11}{3}e^{2t} \quad y_2(t) = -\frac{4}{3}e^{-t} + \frac{11}{6}e^{2t}$$

$$74. \quad y_1(t) = -7e^{-t} + 6e^{-2t} \quad y_2(t) = -7e^{-t} + 9e^{-2t}$$

$$75. \quad y_1(t) = (3+4t)e^{-3t} \quad y_2(t) = (2+4t)e^{-3t}$$

$$76. \quad y_1(t) = 2 + 42t \quad y_2(t) = 4 - 14t$$

$$77. \quad y_1(t) = \left(3 + \frac{3}{2}t\right)e^{t/2} \quad y_2(t) = -\left(2 + \frac{3}{2}t\right)e^{t/2}$$

$$61. \quad y(t) = \begin{pmatrix} -4 \sin 4t \\ 2 \cos 4t + 2 \sin 4t \end{pmatrix}$$

$$62. \quad y(t) = -e^t \begin{pmatrix} \sin 2t \\ -\cos 2t - \sin 2t \end{pmatrix}$$

$$63. \quad y(t) = e^{2t} \begin{pmatrix} 2+3t \\ -1+3t \end{pmatrix}$$

$$64. \quad y(t) = e^{-2t} \begin{pmatrix} -3t \\ -3-3t \end{pmatrix}$$

$$65. \quad y(t) = e^{4t} \begin{pmatrix} 3-2t \\ 1-t \end{pmatrix}$$

$$66. \quad y(t) = \begin{pmatrix} 8e^{-3t} - 5e^{2t} \\ -4e^{-3t} + 5e^{2t} \end{pmatrix}$$

$$67. \quad y(t) = \begin{pmatrix} -\frac{4}{5} \\ -\frac{2}{5} \end{pmatrix} e^{-2t} + \begin{pmatrix} \frac{4}{5} \\ \frac{12}{5} \end{pmatrix} e^{3t}$$

$$68. \quad y(t) = \begin{pmatrix} \frac{5}{2} \\ \frac{1}{2} \end{pmatrix} e^{-2t} - \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \end{pmatrix} e^{2t}$$

$$69. \quad y(t) = \begin{pmatrix} 3 \cos 3t + 9 \sin 3t \\ 2 \cos 3t - 4 \sin 3t \end{pmatrix} e^{-2t}$$

$$70. \quad \begin{cases} y_1(t) = -e^{2t} + 2e^{3t} \\ y_2(t) = 2e^{2t} - 2e^{3t} \end{cases}$$

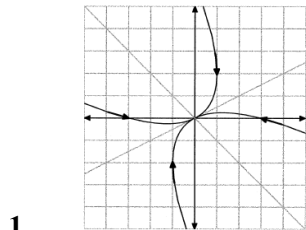
$$71. \quad \begin{cases} y_1(t) = \frac{3}{2}e^t - \frac{1}{2}e^{3t} \\ y_2(t) = -\frac{3}{2}e^t - \frac{1}{2}e^{3t} + e^{2t} \end{cases}$$

$$72. \quad y_1(t) = e^t \quad y_2(t) = -e^t$$

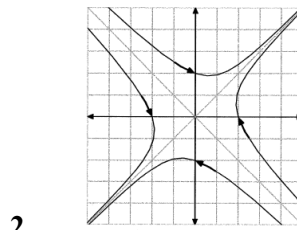
78. $y_1(t) = 6e^{-t} + 2e^t$ $y_2(t) = 2e^{-t} + e^t$
79. $y_1(t) = 2e^{-t} + e^{2t}$ $y_2(t) = e^{-t} + e^{2t}$
80. $y_1(t) = \frac{8}{5}e^{-t} - \frac{8}{5}e^{4t}$ $y_2(t) = -\frac{8}{5}e^{-t} - \frac{12}{5}e^{4t}$
81. $y_1(t) = \frac{3}{5}e^{-t} + \frac{2}{5}e^{-6t}$ $y_2(t) = \frac{12}{5}e^{-t} - \frac{2}{5}e^{-6t}$
82. $y_1(t) = 2\cos 3\sqrt{3}t + \frac{14}{\sqrt{3}}\sin 3\sqrt{3}t$ $y_2(t) = \frac{16\sqrt{3}}{3}\sin 3\sqrt{3}t - 40\cos 3\sqrt{3}t$
83. $y_1(t) = \left(3\cos 8t + \frac{133}{8}\sin 8t\right)e^{2t}$ $y_2(t) = \left(\frac{325}{104}\sin 8t - 10\cos 8t\right)e^{2t}$
84. $y_1(t) = (2-t)e^{5t}$ $y_2(t) = (-5+2t)e^{5t}$
85. $y_1(t) = \frac{1}{2}te^{-\frac{3t}{2}+3}$ $y_2(t) = \left(\frac{1}{3} - \frac{1}{6}t\right)e^{-\frac{3t}{2}+3}$
86. $y_1(t) = -\frac{7}{27}e^{-3t} - \frac{133}{108}e^{6t} - \frac{1}{6}t + \frac{1}{36}$ $y_2(t) = -\frac{14}{27}e^{-3t} + \frac{133}{108}e^{6t} - \frac{1}{6}t + \frac{81}{108}$
87. $y_1(t) = -\frac{1}{3}e^{-t} + \frac{3}{8}e^{-6t} + \frac{23}{24}e^{2t}$ $y_2(t) = -\frac{4}{3}e^{-t} - \frac{3}{8}e^{-6t} + \frac{17}{24}e^{2t}$
88. $y_1(t) = -3e^{-t} + \frac{3}{4}e^{4t} + 3t - \frac{11}{4}$ $y_2(t) = 3e^{-t} + \frac{9}{8}e^{4t} - \frac{5}{2}t + \frac{23}{8}$
89. $x_1(t) = 2e^{4t} - 2te^{4t} + 2te^{2t} - e^{2t}$ $x_2(t) = 2te^{4t} + 2te^{2t} + e^{2t}$
90. $x_1(t) = -1 + 3t + \ln t$; $x_2(t) = -4 + 3t + \ln t$
91. $x_1(t) = -5e^t + 6e^{2t} + e^{-t}$; $x_2(t) = -5e^t + 3e^{2t} + e^{-t}$
92. $y_1(t) = -e^t$; $y_2(t) = (2+4t)e^t$; $y_3(t) = 3e^{2t} - (33+24t)e^t$
93. $y_1(t) = \frac{4}{3}e^{-t/2} + \frac{2}{3}e^{-7t/2}$; $y_2(t) = \frac{4}{3}e^{-t/2} + \frac{5}{3}e^{-7t/2}$; $y_3(t) = \frac{4}{3}e^{-t/2} - \frac{7}{3}e^{-7t/2}$
94.
$$\begin{cases} x_1(t) = 4e^t - \frac{9}{4}e^{2t} - \frac{3}{4} - \frac{1}{2}t + 2te^t \\ x_2(t) = 4e^t + 2e^{2t} - 1 - t + 2te^t \\ x_3(t) = 2e^t - \frac{1}{4}e^{2t} - \frac{3}{4} - \frac{1}{2}t + 2te^t \end{cases}$$
95. $x_1(t) = -\frac{3}{4}e^t + \left(\frac{1}{2}t + \frac{19}{4}\right)e^{2t} - e^{3t}$; $x_2(t) = \frac{3}{4} + \left(\frac{1}{2}t + \frac{17}{4}\right)e^{2t} - 2e^{3t}$; $x_3(t) = \left(\frac{1}{2}t^2 - 1\right)e^{3t}$
96. $x_1(t) = 2\cos t - 2\sin t + 3$; $x_2(t) = -2\sin t - 2\cos t$
97. $x(t) = \frac{7}{2} + \frac{7}{2}t - \frac{5}{2}t^2 + \frac{1}{2}t^3$; $y(t) = \frac{17}{2} + \frac{1}{2}t - \frac{5}{2}t^2 + \frac{1}{2}t^3$
98. $x(t) = -\frac{1}{2}e^{-\sqrt{2}t} - \frac{1}{2}e^{\sqrt{2}t}$; $y(t) = \frac{1}{2}e^{-\sqrt{2}t} + \frac{1}{2}e^{\sqrt{2}t}$
99. $x(t) = e^{-t} + e^t + \cos t + \sin t$; $y(t) = e^{-t} + e^t - \cos t - \sin t$

100.
$$\begin{cases} x(t) = \frac{2}{5}\cos t + \frac{4}{5}\sin t - \frac{2}{5}\cos\sqrt{6}t + \frac{\sqrt{6}}{5}\sin\sqrt{6}t - \sin 2t \\ y(t) = \frac{4}{5}\cos t + \frac{8}{5}\sin t + \frac{1}{5}\cos\sqrt{6}t - \frac{\sqrt{6}}{10}\sin\sqrt{6}t - \frac{1}{2}\sin 2t \end{cases}$$
101. $x(t) = 2\cos t + 6\sin t - 6t - 2 \quad y(t) = 2\sin t - 2\cos t + 3$
102. $x(t) = -8\cos t - 6\sin t + 6t + 8 \quad y(t) = 4\cos t - 2\sin t - 3$
103. $(\cos 2t, \cos 2t - \sin 2t)^T \quad (\sin 2t, \sin 2t + \cos 2t)^T$
104. a) $\begin{pmatrix} x_A \\ x_B \end{pmatrix}' = \begin{pmatrix} -\frac{1}{40} & \frac{1}{90} \\ \frac{1}{40} & -\frac{1}{40} \end{pmatrix} \begin{pmatrix} x_A \\ x_B \end{pmatrix}$ b) $x(t) = 15 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{-t/120} - 15 \begin{pmatrix} -2 \\ 3 \end{pmatrix} e^{-t/24}$
- c)
$$\begin{cases} 30e^{-t/120} + 30e^{-t/24} \xrightarrow{t \rightarrow \infty} 0 \\ 45e^{-t/120} - 45e^{-t/24} \xrightarrow{t \rightarrow \infty} 0 \end{cases}$$
105.
$$\begin{pmatrix} V(t) \\ I(t) \end{pmatrix} = \begin{pmatrix} 10e^{-t}(\cos t - \sin t) \\ 20e^{-t}\sin t \end{pmatrix}$$
106. $V(t) = (4\sqrt{3} + 6)e^{(-3+\sqrt{3})t} - (4\sqrt{3} - 6)e^{(-3-\sqrt{3})t} \quad I(t) = 2\sqrt{3}e^{(-3+\sqrt{3})t} - 2\sqrt{3}e^{(-3-\sqrt{3})t}$

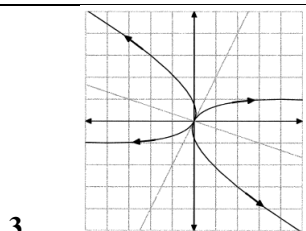
Section 3.7



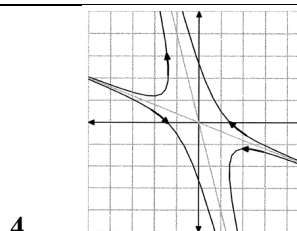
equilibrium point at the origin is a sink



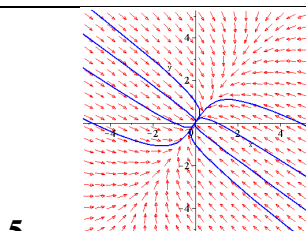
equilibrium point on the origin is a saddle



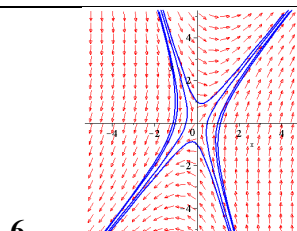
equilibrium point at the origin is a source



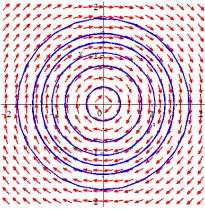
equilibrium point on the origin is a saddle



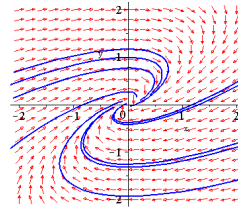
Asymptotically stable sink at the center



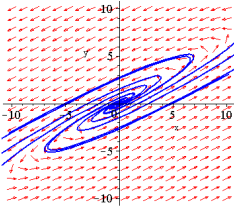
Saddle point at (0, 0); semi-stable



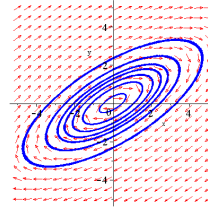
7. Equilibrium point at the origin is the center
motion is *cw*



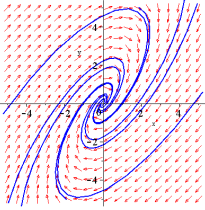
8. equilibrium point at the origin is a spiral sink
motion is *cw*



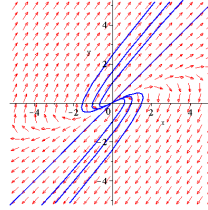
9. equilibrium point at the origin is a spiral source. motion is *ccw*



10. equilibrium point at the origin is a center.
motion is *cw*

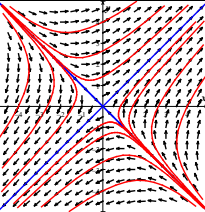


11. at the origin is a spiral sink. motion is *cw*

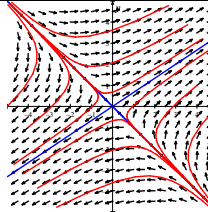


12. $y(t) = 3e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} - 5e^{5t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

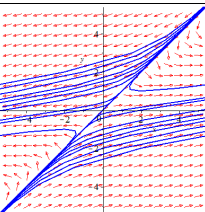
Unstable at the center (source)



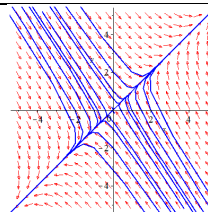
13. $x(t) = C_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t}$



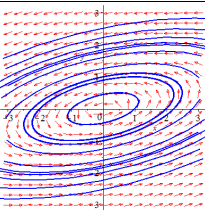
14. $x(t) = C_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{4t}$



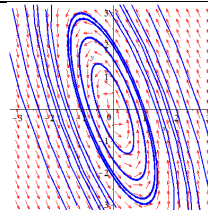
15. $x(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 7 \\ 1 \end{pmatrix} e^{5t}$



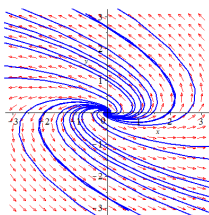
16.
$$\begin{cases} x_1(t) = 2C_1 e^{-9t} + C_2 e^t \\ x_2(t) = -3C_1 e^{-9t} + C_2 e^t \end{cases}$$



17.
$$\begin{cases} x_1 = 5C_1 \cos 2t + 5C_2 \sin 2t \\ x_2 = (C_1 - 2C_2) \cos 2t + (2C_1 + C_2) \sin 2t \end{cases}$$

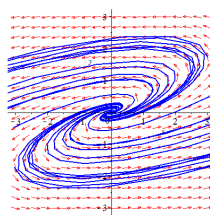


18.
$$\begin{cases} x_1 = -2C_1 \cos 3t - 2C_2 \sin 3t \\ x_2 = 3(C_1 + C_2) \cos 3t + 3(C_2 - C_1) \sin 3t \end{cases}$$



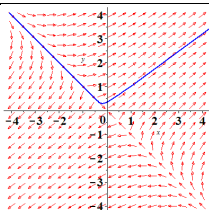
19.

$$\begin{cases} x_1 = (-5C_1 \cos 2t - 2C_2 \sin 2t) e^{2t} \\ x_2 = [(C_1 + 2C_2) \cos 2t + (C_2 - 2C_1) \sin 2t] e^{2t} \end{cases}$$



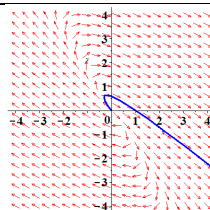
20.

$$\begin{cases} x_1 = (3C_1 \cos 3t + 3C_2 \sin 3t) e^{2t} \\ x_2 = [(C_1 - C_2) \cos 3t + (C_1 + C_2) \sin 3t] e^{2t} \end{cases}$$



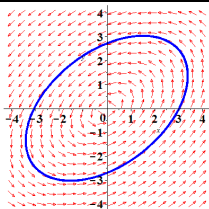
21.

$$\begin{cases} x_1(t) = -\frac{1}{7}e^{-t} + \frac{8}{7}e^{6t} \\ x_2(t) = \frac{1}{7}e^{-t} + \frac{6}{7}e^{6t} \end{cases}$$



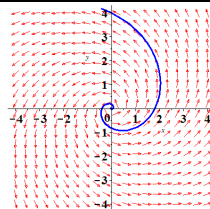
22.

$$\begin{cases} x_1(t) = -5e^{3t} + 6e^{4t} \\ x_2(t) = 6e^{3t} - 6e^{4t} \end{cases}$$



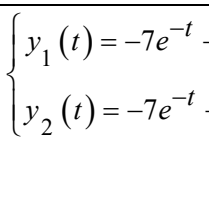
23.

$$\begin{cases} x_1(t) = 2 \cos 4t - \frac{11}{4} \sin 4t \\ x_2(t) = 3 \cos 4t + \frac{1}{2} \sin 4t \end{cases}$$



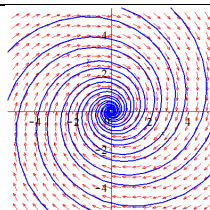
24.

$$\begin{cases} x_1(t) = -4e^t \sin 2t \\ x_2(t) = 4e^t \cos 2t \end{cases}$$



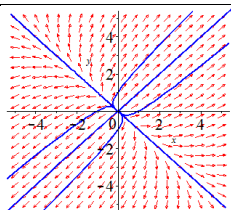
25.

$$\begin{cases} y_1(t) = -7e^{-t} + 6e^{-2t} \\ y_2(t) = -7e^{-t} + 9e^{-2t} \end{cases}$$



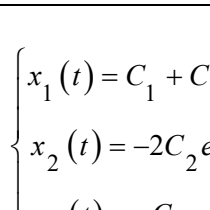
26.

$$\begin{cases} y_1(t) = (2 \sin 2t - 2 \cos 2t) e^{-t/2} \\ y_2(t) = (2 \cos 2t + 2 \sin 2t) e^{-t/2} \end{cases}$$



27.

$$\begin{cases} y_1(t) = -\frac{3}{2}e^{t/2} - \frac{1}{2}e^{2t} \\ y_2(t) = \frac{3}{2}e^{t/2} - \frac{1}{2}e^{2t} \end{cases}$$



28.

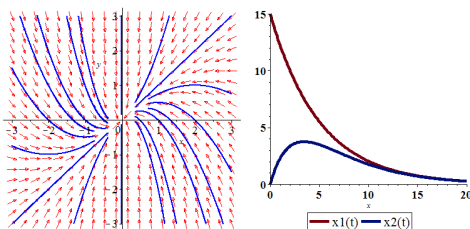
$$\begin{cases} x_1(t) = C_1 + C_2 e^{6t} + C_3 e^{9t} \\ x_2(t) = -2C_2 e^{6t} + C_3 e^{9t} \\ x_3(t) = -C_1 - C_2 e^{6t} + C_3 e^{9t} \end{cases}$$

$$29. \begin{cases} x_1(t) = -4C_1 + C_3 e^{9t} \\ x_2(t) = C_1 + C_2 e^{6t} + 2C_3 e^{9t} \\ x_3(t) = C_1 - C_2 e^{6t} + 2C_3 e^{9t} \end{cases}$$

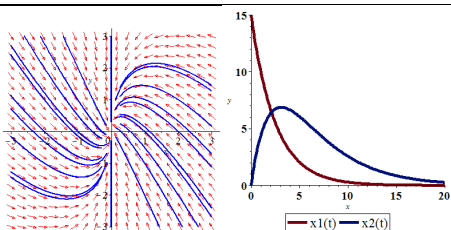
$$30. \begin{cases} x_1(t) = C_1 e^{3t} + C_2 e^{3t} + C_3 e^{6t} \\ x_2(t) = -C_1 e^{3t} + C_3 e^{6t} \\ x_3(t) = -C_2 e^{3t} + C_3 e^{6t} \end{cases}$$

$$31. \begin{cases} x_1(t) = -C_1 e^{2t} + C_2 e^{6t} + C_3 e^{9t} \\ x_2(t) = -2C_2 e^{6t} + C_3 e^{9t} \\ x_3(t) = C_1 e^{2t} + C_2 e^{6t} + C_3 e^{9t} \end{cases}$$

$$34. \begin{cases} x_1(t) = 15e^{-.2t} \\ x_2(t) = 15e^{-.2t} - 15e^{-.4t} \end{cases}$$

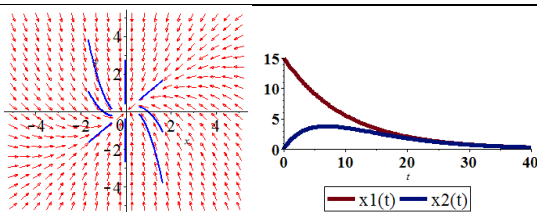


$$35. \begin{cases} x_1(t) = 15e^{-.4t} \\ x_2(t) = -40e^{-.4t} + 40e^{-.25t} \end{cases}$$

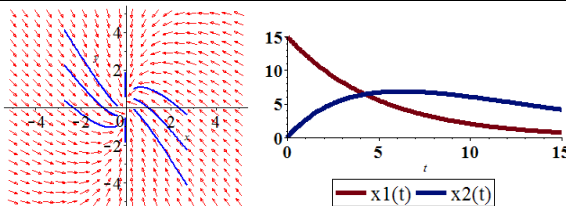


6.85 lb

$$36. \begin{cases} x_1(t) = 15e^{-t/10} \\ x_2(t) = 15e^{-t/10} - 15e^{-t/5} \end{cases}$$

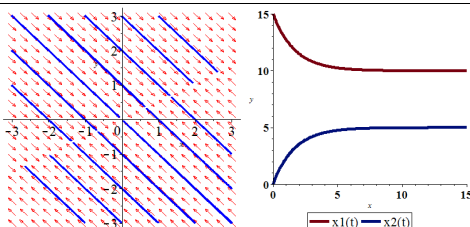
3.75 lb $t = 10 \ln 2$

$$37. \begin{cases} x_1(t) = 15e^{-t/5} \\ x_2(t) = -40e^{-t/5} + 40e^{-t/8} \end{cases}$$



No salt in tank 1 tank 2 = 6.85 lb

$$38. \begin{cases} x_1(t) = 10 + 5e^{-0.6t} \\ x_2(t) = 5 - 5e^{-0.6t} \end{cases}$$



39.
$$\begin{cases} x_1(t) = \frac{15}{13}(5 + 8e^{-0.6t}) \\ x_2(t) = \frac{120}{13}(1 - e^{-0.6t}) \end{cases}$$
40.
$$\begin{cases} x_1(t) = 27e^{-t} \\ x_2(t) = 27e^{-t} - 27e^{-2t} \\ x_3(t) = 27e^{-t} - 54e^{-2t} + 27e^{-3t} \end{cases} \quad \text{tank 2} = \frac{27}{4} \text{ lbs} \quad \text{tank 3} = 4 \text{ lbs}$$
41.
$$\begin{cases} x_1(t) = 45e^{-3t} \\ x_2(t) = -135e^{-3t} + 135e^{-2t} \\ x_3(t) = 135e^{-3t} - 270e^{-2t} + 135e^{-t} \end{cases} \quad \text{tank 2} = 20 \text{ lbs} \quad \text{tank 3} = 20 \text{ lbs}$$
42.
$$\begin{cases} x_1(t) = 45e^{-4t} \\ x_2(t) = 90e^{-4t} - 90e^{-6t} \\ x_3(t) = -270e^{-4t} + 135e^{-6t} + 135e^{-2t} \end{cases} \quad \text{tank 2} = 13.3 \text{ lbs} \quad \text{tank 3} = 20 \text{ lbs}$$
43.
$$\begin{cases} x_1(t) = 15e^{-.5t} \\ x_2(t) = -30e^{-.5t} + 30e^{-.25t} \\ x_3(t) = 25e^{-.5t} - 150e^{-.25t} + 125e^{-.2t} \end{cases}$$
44.
$$\begin{cases} x_1(t) = 2C_1 + (C_2 \cos 0.2t - C_3 \sin 0.2t)e^{-.4t} \\ x_2(t) = C_1 + (C_2 \sin 0.2t + C_3 \cos 0.2t)e^{-.4t} \\ x_3(t) = 2C_1 + ((-C_2 - C_3)\cos 0.2t + (C_3 - C_2)\sin 0.2t)e^{-.4t} \end{cases}$$
45.
$$\begin{cases} x(t) = -\frac{10}{\sqrt{7}}(2 + \sqrt{7})e^{\frac{-5-\sqrt{7}}{100}t} + \frac{10}{\sqrt{7}}(2 - \sqrt{7})e^{\frac{-5+\sqrt{7}}{100}t} + 20 \\ y(t) = \frac{30}{\sqrt{7}}e^{\frac{-5-\sqrt{7}}{100}t} - \frac{30}{\sqrt{7}}e^{\frac{-5+\sqrt{7}}{100}t} + 20 \end{cases}$$
46.
$$\begin{cases} x(t) = -\frac{20+10\sqrt{19}}{\sqrt{19}}e^{\frac{-7-\sqrt{19}}{100}t} + \frac{-20+10\sqrt{19}}{\sqrt{19}}e^{\frac{-7+\sqrt{19}}{100}t} + 20 \\ y(t) = \frac{50}{\sqrt{19}}e^{\frac{-7-\sqrt{19}}{100}t} - \frac{50}{\sqrt{19}}e^{\frac{-7+\sqrt{19}}{100}t} + 20 \end{cases}$$
47.
$$\begin{cases} x(t) = -\frac{5}{4}e^{-t/2} + \frac{5}{4}e^{-t/6} \\ y(t) = \frac{5}{2}e^{-t/2} + \frac{5}{2}e^{-t/6} \end{cases}$$

$$48. \begin{cases} x(t) = -\frac{70+10\sqrt{17}}{\sqrt{17}}e^{\frac{-9-\sqrt{17}}{200}t} + \frac{70-10\sqrt{17}}{\sqrt{17}}C_2e^{\frac{-9+\sqrt{17}}{200}t} + 20 \\ y(t) = \frac{25+15\sqrt{17}}{2\sqrt{17}}e^{\frac{-9-\sqrt{17}}{200}t} + \frac{-25+15\sqrt{17}}{2\sqrt{17}}e^{\frac{-9+\sqrt{17}}{200}t} + 5 \end{cases}$$

$$49. \begin{cases} x(t) = -\frac{10}{33}(207+29\sqrt{33})e^{\frac{-11-\sqrt{33}}{800}t} + \frac{10}{33}(-207+29\sqrt{33})e^{\frac{-11+\sqrt{33}}{800}t} + \frac{1600}{11} \\ y(t) = \frac{1}{11}(50\sqrt{33}+140)e^{\frac{-11-\sqrt{33}}{800}t} + \frac{1}{11}(-50\sqrt{33}+140)e^{\frac{-11+\sqrt{33}}{800}t} + \frac{600}{11} \end{cases}$$

$$50. a) \frac{dV_1}{dt} = \alpha(V - V_2(t)) - KV_1(t) \quad \frac{dV_2}{dt} = KV_1(t) - R_3$$

$$b) \begin{cases} V_1(t) = (5C_1 \cos 3t + 5C_2 \sin 3t)e^{-t} + 5 \\ V_2(t) = ((3C_2 - C_1)\cos 3t - (3C_1 + C_2)\sin 3t)e^{-t} + 18 \end{cases}$$

$$c) \lim_{t \rightarrow \infty} V_1(t) = 5 \text{ L} \quad \lim_{t \rightarrow \infty} V_2(t) = 18 \text{ L}$$

$$51. a) \begin{cases} i_2' = -\frac{R_1}{L_1}i_2 - \frac{R_1}{L_1}i_3 + \frac{1}{L_1}E(t) \\ i_3' = -\frac{R_1}{L_2}i_2 - \frac{1}{L_2}(R_1 + R_2)i_3 + \frac{1}{L_2}E(t) \end{cases}$$

$$b) \begin{cases} i_2(t) = -24e^{-t} - 6e^{-6t} + 30 \\ i_3(t) = 12e^{-t} - 12e^{-6t} \end{cases} \quad c) i_1(t) = -12e^{-t} - 18e^{-6t} + 30$$

$$52. a) \begin{cases} i_1' = -\frac{1}{L_2}(R_1 + R_2)i_1 + \frac{R_2}{L_2}i_2 + \frac{E}{L_2} \\ i_2' = \frac{R_2}{L_1}i_1 - \frac{R_2}{L_1}i_2 \end{cases}$$

$$b) \begin{cases} i_1(t) = 2e^{-2t} + \frac{18}{29}e^{-12t} + \frac{332}{29}\sin t - \frac{76}{29}\cos t \\ i_2(t) = 6e^{-2t} - \frac{6}{29}e^{-12t} + \frac{276}{29}\sin t - \frac{168}{29}\cos t \end{cases} \quad c) i_3(t) = -4e^{-2t} + \frac{24}{29}e^{-12t} + \frac{56}{29}\sin t - \frac{92}{29}\cos t$$

$$53. \begin{cases} I_1(t) = -e^{-20t} - 2e^{-5t} + 3 \\ I_2(t) = -2e^{-20t} + 2e^{-5t} \\ I_3(t) = e^{-20t} - 4e^{-5t} + 3 \end{cases}$$

54.
$$\begin{cases} I_1(t) = -\frac{33+\sqrt{33}}{22}e^{-\frac{5}{2}(7+\sqrt{33})t} + \frac{-33+\sqrt{33}}{22}e^{-\frac{5}{2}(7-\sqrt{33})t} + 3 \\ I_2(t) = -\frac{4}{11}e^{-\frac{5}{2}(7+\sqrt{33})t} + \frac{4}{11}e^{-\frac{5}{2}(7-\sqrt{33})t} \\ I_3(t) = \frac{-33+7\sqrt{33}}{22}e^{-\frac{5}{2}(7+\sqrt{33})t} + \frac{-33-7\sqrt{33}}{22}e^{-\frac{5}{2}(7-\sqrt{33})t} + 3 \end{cases}$$
55.
$$\begin{cases} i_1(t) = -\frac{5}{6}e^{-4000t} - \frac{20}{3}e^{-1000t} + \frac{15}{2} \\ i_2(t) = -\frac{5}{3}e^{-4000t} - \frac{10}{3}e^{-1000t} + 5 \\ i_3(t) = \frac{5}{6}e^{-4000t} - \frac{10}{3}e^{-1000t} + \frac{5}{2} \end{cases}$$
56.
$$\begin{cases} i_1(t) = -\frac{1}{4}e^{-2t} - \frac{9}{4}e^{-2t/3} + \frac{5}{2} \\ i_2(t) = -\frac{1}{2}e^{-2t} - \frac{3}{2}e^{-2t/3} + 2 \\ i_3(t) = \frac{1}{4}e^{-2t} - \frac{3}{4}e^{-2t/3} + \frac{1}{2} \end{cases}$$
57.
$$q(t) = e^{-4t} \left(2\cos 12t + \frac{2}{3}\sin 12t \right) \begin{cases} I_1(t) = \frac{1}{4} - \frac{16}{3}e^{-4t}\sin 12t - 16e^{-4t}\cos 12t \\ I_2(t) = \frac{1}{4} + \frac{64}{3}e^{-4t}\sin 12t - 16e^{-4t}\cos 12t \\ I_3(t) = -\frac{80}{3}e^{-4t}\sin 12t \end{cases}$$
58.
$$I_1(t) = 1 - e^{-900t} \quad I_2(t) = \frac{5}{9} - \frac{5}{9}e^{-900t} \quad I_3(t) = \frac{4}{9} - \frac{4}{9}e^{-900t}$$
59.
$$I_1(t) = 2 - 2e^{-\frac{27}{5}t} \quad I_2(t) = \frac{50}{27} - \frac{50}{27}e^{-\frac{27}{5}t} \quad I_3(t) = \frac{4}{27} - \frac{4}{27}e^{-\frac{27}{5}t}$$
60.
$$q(t) = -\frac{4}{15}e^{-3t/2} + \frac{3}{5}e^{-2t/3} \begin{cases} I_1(t) = 1 + \frac{3}{5}e^{-3t/2} - \frac{8}{5}e^{-2t/3} \\ I_2(t) = 1 + \frac{1}{5}e^{-3t/2} - \frac{6}{5}e^{-2t/3} \\ I_3(t) = \frac{2}{5}e^{-3t/2} - \frac{2}{5}e^{-2t/3} \end{cases}$$
61.
$$\begin{cases} I_1(t) = \left(-\frac{3\sqrt{3}}{61}\sin\sqrt{3}t - \frac{81}{61}\cos\sqrt{3}t \right) e^{-t} - \frac{14}{61}\sin 3t + \frac{81}{61}\cos 3t \\ I_2(t) = \left(\frac{39\sqrt{3}}{61}\sin\sqrt{3}t - \frac{45}{61}\cos\sqrt{3}t \right) e^{-t} - \frac{54}{61}\sin 3t + \frac{45}{61}\cos 3t \\ I_3(t) = \left(-\frac{36}{61}\cos\sqrt{3}t - \frac{42\sqrt{3}}{61}\sin\sqrt{3}t \right) e^{-t} + \frac{36}{61}\cos 3t + \frac{30}{61}\sin 3t \end{cases}$$
62.
$$\begin{cases} I_1(t) = \left((C_1 - C_2)\sin t - (C_1 + C_2)\cos t \right) e^{-t} \\ I_2(t) = (-C_1\sin t + C_2\cos t) e^{-t} \\ I_3(t) = (C_1\cos t + C_2\sin t) e^{-t} \end{cases}$$
63.
$$\begin{cases} x(t) = 2\cos t + \cos 2t \\ y(t) = 4\cos t - \cos 2t \end{cases}$$

$$64. \quad a) \begin{cases} x(t) = C_1 \cos(\omega\sqrt{3})t + C_2 \sin(\omega\sqrt{3})t + C_3 \cos \omega t + C_4 \sin \omega t \\ y(t) = -C_1 \cos(\omega\sqrt{3})t - C_2 \sin(\omega\sqrt{3})t + C_3 \cos \omega t + C_4 \sin \omega t \end{cases}$$

$$b) \begin{cases} x(t) = \frac{3}{2} \cos t \\ y(t) = \frac{3}{2} \cos t \end{cases} \quad c) \begin{cases} x(t) = \cos(\sqrt{3})t \\ y(t) = -\cos(\sqrt{3})t \end{cases} \quad d) \begin{cases} x(t) = -\frac{1}{2} \cos(\sqrt{3})t + \frac{3}{2} \cos t \\ y(t) = \frac{1}{2} \cos(\sqrt{3})t + \frac{3}{2} \cos t \end{cases}$$

$$65. \quad \begin{aligned} x(t) &= C_1 \cos \omega\sqrt{2}t + C_2 \sin \omega\sqrt{2}t + C_3 \cos \omega\sqrt{2+\sqrt{2}}t + C_4 \sin \omega\sqrt{2+\sqrt{2}}t \\ &\quad + C_5 \cos \omega\sqrt{2-\sqrt{2}}t + C_6 \sin \omega\sqrt{2-\sqrt{2}}t \\ y(t) &= -\sqrt{2}C_3 \cos \omega\sqrt{2+\sqrt{2}}t - \sqrt{2}C_4 \sin \omega\sqrt{2+\sqrt{2}}t + \sqrt{2}C_5 \cos \omega\sqrt{2-\sqrt{2}}t + \sqrt{2}C_6 \sin \omega\sqrt{2-\sqrt{2}}t \\ z(t) &= -C_1 \cos \omega\sqrt{2}t - C_2 \sin \omega\sqrt{2}t + C_3 \cos \omega\sqrt{2+\sqrt{2}}t + C_4 \sin \omega\sqrt{2+\sqrt{2}}t \\ &\quad + C_5 \cos \omega\sqrt{2-\sqrt{2}}t + C_6 \sin \omega\sqrt{2-\sqrt{2}}t \end{aligned}$$

$$66. \quad a) \frac{\omega\sqrt{2}}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{2k}{m}} \quad b) \frac{\omega\sqrt{2+\sqrt{2}}}{2\pi} = \frac{1}{2\pi} \sqrt{(2+\sqrt{2})\frac{k}{m}} \quad c) \frac{\omega\sqrt{2-\sqrt{2}}}{2\pi} = \frac{1}{2\pi} \sqrt{(2-\sqrt{2})\frac{k}{m}}$$

$$67. \quad a) \begin{cases} x(t) = -\frac{9}{17} \cos \sqrt{\frac{20}{3}}t - \frac{8}{17} \cos t \\ y(t) = \frac{6}{17} \cos \sqrt{\frac{20}{3}}t - \frac{6}{17} \cos t \end{cases} \quad b) \begin{cases} x(t) = \frac{1-\sqrt{10}}{2\sqrt{10}} \cos(4+\sqrt{10})t - \frac{1+\sqrt{10}}{2\sqrt{10}} \cos(4-\sqrt{10})t \\ y(t) = \frac{3}{2\sqrt{10}} \cos(4+\sqrt{10})t - \frac{3}{2\sqrt{10}} \cos(4-\sqrt{10})t \end{cases}$$

$$68. \quad x_1(t) = x_2(t) = \frac{3\pi}{16} v_0 \quad x_3(t) = v_0 t - \frac{5\pi}{16} v_0$$

$$69. \quad \begin{cases} \vec{x}_1(t) = a_1 + b_1 t + a_2 \cos 2t + b_2 \sin 2t \\ \vec{x}_2(t) = a_1 + b_1 t - a_2 \cos 2t - b_2 \sin 2t \end{cases}$$

$$70. \quad \begin{cases} \vec{x}_1(t) = a_1 \cos t + b_1 \sin t + a_2 \cos t\sqrt{5} + b_2 \sin t\sqrt{5} \\ \vec{x}_2(t) = a_1 \cos t + b_1 \sin t - a_2 \cos t\sqrt{5} - b_2 \sin t\sqrt{5} \end{cases}$$

$$71. \quad \begin{cases} \vec{x}_1(t) = a_1 \cos t\sqrt{2} + b_1 \sin t\sqrt{2} + a_2 \cos 2t + b_2 \sin 2t \\ \vec{x}_2(t) = a_1 \cos t\sqrt{2} + b_1 \sin t\sqrt{2} - a_2 \cos 2t - b_2 \sin 2t \end{cases}$$

$$72. \quad \begin{cases} \vec{x}_1(t) = a_1 \cos t\sqrt{2} + b_1 \sin t\sqrt{2} + 2a_2 \cos t\sqrt{8} + 2b_2 \sin t\sqrt{8} \\ \vec{x}_2(t) = a_1 \cos t\sqrt{2} + b_1 \sin t\sqrt{2} - a_2 \cos t\sqrt{8} - b_2 \sin t\sqrt{8} \end{cases}$$

$$73. \quad \begin{cases} \vec{x}_1(t) = 2 \cos t + 3 \cos 3t - 5 \cos 5t \\ \vec{x}_2(t) = 2 \cos t - 3 \cos 3t + \cos 5t \end{cases}$$

$$74. \quad \begin{cases} \vec{x}_1(t) = 5 \cos t - 8 \cos 2t + 3 \cos 3t \\ \vec{x}_2(t) = 5 \cos t + 4 \cos 2t - 9 \cos 3t \end{cases}$$

75.
$$\begin{cases} \bar{x}_1(t) = \cos 2t - 15 \cos 3t + 14 \cos t \\ \bar{x}_2(t) = \cos 2t + 10 \cos 3t + 16 \cos t \end{cases}$$
76. a)
$$\begin{cases} \bar{x}(t) = 2a_1 \cos 6t + 2b_1 \sin 6t + a_2 \cos 8t + b_2 \sin 8t \\ \bar{y}(t) = a_1 \cos 6t + b_1 \sin 6t - 3a_2 \cos 8t - 3b_2 \sin 8t \end{cases}$$
 b)
$$\begin{cases} x(t) = 2 \sin 6t + 19 \cos 7t \\ y(t) = \sin 6t + 3 \cos 7t \end{cases}$$
77. $m_2 \approx 0.1 \text{ slug} \quad c_1 = 0 \quad c_2 \approx 1.15$
78.
$$\begin{cases} x_1(t) = \frac{1}{3} \cos 5t + \cos 5t \sqrt{3} - \frac{4}{3} \cos 10t \\ x_2(t) = \frac{2}{3} \cos 5t - 2 \cos 5t \sqrt{3} + \frac{4}{3} \cos 10t \end{cases}$$
79.
$$\begin{cases} m_1 x'' = -(k_1 + k_2)x + k_2 y \\ m_2 y'' = k_2 x - k_2 y - cy' \end{cases}$$
80.
$$\begin{cases} x(t) = -(1+t)e^{-t} - \cos t \\ y(t) = (t+1)e^{-t} - \cos t \end{cases}$$
81. a) $\omega_1 \approx 6.4675 \quad \omega_2 \approx 11.2918$ b) $v_1 \approx 41 \quad v_2 \approx 72$
82. a) $\omega_1 \approx 6.325 \text{ rad/sec} \quad \omega_2 \approx 11.180 \text{ rad/sec}$ b) $v_1 \approx 40.26 \text{ ft/sec} \quad v_2 \approx 71.18 \text{ ft/sec}$
83. a) $\omega_1 \approx 6.131 \text{ rad/sec} \quad \omega_2 \approx 10.315 \text{ rad/sec}$ b) $v_1 \approx 39.03 \text{ ft/sec} \quad v_2 \approx 65.67 \text{ ft/sec}$
84. a) $\omega_1 \approx 5.0424 \text{ rad/sec} \quad \omega_2 \approx 9.9158 \text{ rad/sec}$ b) $v_1 \approx 32.1 \text{ ft/sec} \quad v_2 \approx 63.13 \text{ ft/sec}$
85.
$$\begin{cases} \theta_1(t) = \frac{\pi}{12} \cos\left(\sqrt{\frac{9.8}{15}(5+\sqrt{10})}t\right) + \frac{\pi}{12} \cos\left(\sqrt{\frac{9.8}{15}(5-\sqrt{10})}t\right) \\ \theta_2(t) = -\frac{\pi\sqrt{10}}{12} \cos\left(\sqrt{\frac{9.8}{15}(5+\sqrt{10})}t\right) + \frac{\pi\sqrt{10}}{12} \cos\left(\sqrt{\frac{9.8}{15}(5-\sqrt{10})}t\right) \end{cases}$$
86. $\omega_1 = \frac{1}{2\pi} \sqrt{\frac{g}{\ell}} \quad \omega_2 = \frac{1}{2\pi} \sqrt{\frac{g}{\ell} + \frac{2k}{m}}$
87. a)
$$\begin{cases} x'' = -3x + y \\ y'' = 2x - 2y + 37 \cos 3t \end{cases}$$
 b) $x^{(4)}(t) + 5x''(t) + 4x(t) = 37 \cos 3t$
- c) $x(t) = C_1 \cos t + C_2 \sin t + C_3 \cos 2t + C_4 \sin 2t + \frac{37}{40} \cos 3t$
- d) $y(t) = 2C_1 \cos t + 2C_2 \sin t - C_3 \cos 2t - C_4 \sin 2t - \frac{111}{20} \cos 3t$
- e) $x(t) = \frac{23}{8} \cos t - \frac{9}{5} \cos 2t + \frac{37}{40} \cos 3t \quad y(t) = \frac{23}{4} \cos t + \frac{9}{5} \cos 2t - \frac{111}{20} \cos 3t$

Lecture 4

Section 4.1

1. Radius 1, centre 0, $(-1, 1)$, does not converge
 2. Radius 1, centre -1 , $(-2, 0)$, converges
 3. Radius 4, centre 0, $(-4, 4)$, converges
 4. Radius $\frac{1}{e}$, centre 4, $\left[4 - \frac{1}{e}, 4 + \frac{1}{e}\right]$, converges
 5. Radius ∞ , centre $\frac{1}{4}$, converges $(-\infty, \infty)$
 6. Radius ∞ , centre 0, converges $(-\infty, \infty)$
 7. $1 + 2x + 2x^2 + \frac{4}{3}x^3$
 8. $x - \frac{1}{6}x^3$
 9. $P_3(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3$
 10. $P_3(x) = \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3$
 11. $P_3(x) = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$
 12. $P_3(x) = 1 + 3(x-1) + 3(x-1)^2 + (x-1)^3$
13. $P_0(x) = 8$;
 $P_1(x) = 8 + 4(x-1)$;
 $P_2(x) = 8 + 4(x-1) - (x-1)^2$;
 $P_3(x) = 8 + 4(x-1) - (x-1)^2 + 3(x-1)^3$
14. $P_0(x) = \frac{\sqrt{2}}{2}$;
 $P_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$;
 $P_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$;
 $P_3(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3$
15. $P_0(x) = \frac{\sqrt{3}}{2}$;
 $P_1(x) = \frac{\sqrt{2}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right)$;
 $P_2(x) = \frac{\sqrt{2}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)^2$;
 $P_3(x) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)^2 + \frac{1}{12}\left(x - \frac{\pi}{6}\right)^3$
16. $P_0(x) = 3$;
 $P_1(x) = 3 + \frac{1}{6}(x-9)$;
 $P_2(x) = 3 + \frac{1}{2 \cdot 3}(x-9) - \frac{1}{2^3 \cdot 3^3}(x-9)^2$;
 $P_3(x) = 3 + \frac{1}{2 \cdot 3}(x-9) - \frac{1}{2^2 \cdot 3^3}(x-9)^2 + \frac{1}{2^4 \cdot 3^5}(x-9)^3$
17. $P_0(x) = 2$;

$$P_1(x) = 2 + \frac{1}{2^2 \cdot 3}(x-8);$$

$$P_2(x) = 2 + \frac{1}{2^2 \cdot 3}(x-8) - \frac{1}{2^5 \cdot 3^2}(x-8)^2;$$

$$P_3(x) = 2 + \frac{1}{2^2 \cdot 3}(x-8) - \frac{1}{2^5 \cdot 3^2}(x-8)^2 + \frac{1}{2^8 \cdot 3^4}(x-8)^3$$

18. $P_0(x) = 1;$

$$P_1(x) = 1 + \frac{1}{e}(x-e);$$

$$P_2(x) = 1 + \frac{1}{e}(x-e) - \frac{1}{2e^2}(x-e)^2;$$

$$P_3(x) = 1 + \frac{1}{e}(x-e) - \frac{1}{2e^2}(x-e)^2 + \frac{1}{3e^3}(x-e)^3$$

19. $P_0(x) = \sqrt[4]{8};$

$$P_1(x) = \sqrt[4]{8} + \frac{1}{2^4 \cdot \sqrt[4]{2}}(x-8);$$

$$P_2(x) = \sqrt[4]{8} + \frac{1}{2^4 \cdot \sqrt[4]{2}}(x-8) - \frac{3}{2^{10} \cdot \sqrt[4]{2}}(x-8)^2;$$

$$P_3(x) = \sqrt[4]{8} + \frac{1}{2^4 \cdot \sqrt[4]{2}}(x-8) - \frac{3}{2^{10} \cdot \sqrt[4]{2}}(x-8)^2 + \frac{7}{2^{15} \cdot \sqrt[4]{2}}(x-8)^3$$

20. $P_0(x) = \frac{\pi}{4} + 2;$

$$P_1(x) = \frac{\pi}{4} + 2 + \frac{5}{2}(x-1);$$

$$P_2(x) = \frac{\pi}{4} + 2 + \frac{5}{2}(x-1) - \frac{3}{4}(x-1)^2;$$

$$P_3(x) = \frac{\pi}{4} + 2 + \frac{5}{2}(x-1) - \frac{3}{4}(x-1)^2$$

21. $P_0(x) = 2;$

$$P_1(x) = 2 + 2(x - \ln 2);$$

$$P_2(x) = 2 + 2(x - \ln 2) + (x - \ln 2)^2;$$

$$P_3(x) = 2 + 2(x - \ln 2) + (x - \ln 2)^2 + \frac{1}{3}(x - \ln 2)^3$$

22. $P_4(x) = 1 + 4x + 8x^2 + \frac{32}{3}x^3 + \frac{32}{3}x^4$

23. $P_5(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5$

24. $P_4(x) = 1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{48}x^3 + \frac{1}{384}x^4$

25. $P_4(x) = 1 + \frac{1}{3}x + \frac{1}{18}x^2 + \frac{1}{162}x^3 + \frac{1}{1944}x^4$

$$26. P_5(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

$$27. P_4(x) = 1 - \frac{\pi^2}{2}x^2 + \frac{\pi^4}{24}x^4$$

$$28. P_4(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$$

$$29. P_4(x) = x^2 - x^3 + \frac{1}{2}x^4$$

$$30. P_5(x) = 1 - x + x^2 - x^3 + x^4 - x^5$$

$$31. P_4(x) = x - x^2 + x^3 - x^4$$

$$32. P_2(x) = 1 + \frac{1}{2}x^2$$

$$33. P_4(x) = x + \frac{1}{3}x^3$$

$$34. \sum_{n=0}^{\infty} \frac{1}{(n-1)!} x^n$$

$$35. 5 \sum_{n=0}^{\infty} \frac{(-1)^n (\pi x)^{2n}}{(2n)!}$$

$$36. \sum_{n=2}^{\infty} (-1)^n x^n$$

$$37. = \sum_{n=0}^{\infty} \frac{e 3^n x^n}{n!}$$

$$38. \sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{(2n)!} x^{6n}$$

$$39. \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 4^n}{(2n)!} x^{2n}$$

$$40. \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{3^{2n+1} (2n+1)!}$$

$$41. 1 + \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$42. \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n}}{(2n+1)!} x^{2n+1}$$

$$43. \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1}}{2n+1} x^{4n+2}$$

$$44. \ln 2 + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n} \frac{x^{2n}}{2^n}$$

$$45. 1 - x^2 + \sum_{n=2}^{\infty} (-1)^n (x^{2n-1} + x^{2n})$$

$$46. 2 \sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}$$

$$47. \sum_{n=0}^{\infty} \frac{2^{n+1}}{(n+1)!} x^{2n}$$

$$48. 2 \sum_{n=0}^{\infty} \frac{x^{4n+2}}{(4n+2)!}$$

$$49. 2 \sum_{n=0}^{\infty} \frac{x^{4n+3}}{(4n+3)!}$$

$$50. 8 + 10(x-2) + 6(x-2)^2 + (x-2)^3$$

$$51. -2 + 11(x-1) + 7(x-1)^2 + 2(x-1)^3$$

$$52. -7 + 23(x+1) - 41(x+1)^2 + 36(x+1)^3 - 16(x+1)^4 + 3(x+1)^5$$

$$53. \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!} \left(x - \frac{\pi}{4}\right)^{2n}$$

Section 4.2

1. $y(x) = a_0 e^{3x}$
2. $y(x) = a_0 e^{4x}$
3. $y(x) = a_0 e^{x^3/3} \quad P = \infty$
4. $y(x) = a_0 e^{-x^2} \quad P = \infty$
5. $y(x) = \frac{2a_0}{2-x}$
6. $y(x) = \frac{a_0}{1-2x}$
7. $y(x) = a_0 \left(1 - \frac{3}{2}x + \frac{3}{8}x^2 + \frac{1}{16}x^3 + \frac{3}{128}x^4 + \dots\right)$
8. $y = a_0 (x+1)$
9. $y = \frac{1}{4}a_0 (2-x)^2$
10. $y(x) = \frac{-4a_0}{x-4}$
11. $y(x) = 1 + x + x^2 + 2!x^3 + 3!x^4 + \dots$
12. $y(x) = \frac{1}{(3-x)^2}$
13. $y(x) \equiv 0$
14. $y(x) \equiv 0$
15. $y(x) = a_0 \cosh 2x + a_1 \sinh 2x$
16. $y(x) = a_0 \left(1 + \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} + \dots\right) + a_1 \left(x + \frac{3^2 x^3}{3!} + \frac{3^4 x^5}{5!} + \dots\right)$
17. $y(x) = a_0 \cos x + a_1 \sin x$
18. $y(x) = a_0 \cosh x + a_1 \sinh x$
19. $y(x) = x + a_0 \cos x + (a_1 - 1) \sin x$
20. $y(x) = \left(1 + \frac{1}{6}x^3 + \frac{1}{180}x^6 + \dots\right)a_0 + \left(x + \frac{1}{12}x^4 + \frac{1}{504}x^7 + \dots\right)a_1$
21. $y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{180}x^6 - \frac{1}{12,960}x^9 + \dots\right)a_0 + \left(x - \frac{1}{12}x^4 + \frac{1}{504}x^7 - \dots\right)a_1$
22. $y(x) = a_0 \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6 + \dots\right) + a_1 \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5 - \frac{1}{105}x^7 + \dots\right)$
23. $y(x) = a_0 \left(1 + \frac{x^2}{2} + \frac{x^4}{2^2 \cdot 2} + \dots + \frac{x^{2n}}{2^n \cdot n!}\right) + a_1 \left(x + \frac{x^3}{3} + \frac{x^5}{5 \cdot 3} + \dots + \frac{2^n \cdot n!}{(2n+1)!!}x^{2n+1}\right)$
24. $y(x) = \left(1 - \frac{1}{12}x^4 + \frac{1}{672}x^8 - \dots\right)a_0 + \left(x - \frac{1}{20}x^5 + \frac{1}{1,440}x^9 + \dots\right)a_1$
25. $y(x) = a_0 \left(1 - \frac{k^2}{3 \cdot 4}x^4 + -\frac{k^4}{3 \cdot 4 \cdot 7 \cdot 8}x^8 - \frac{k^6}{3 \cdot 4 \cdot 7 \cdot 8 \cdot 11 \cdot 12}x^{12} + \dots\right) + a_1 \left(x - \frac{k^2}{4 \cdot 5}x^5 + -\frac{k^4}{4 \cdot 5 \cdot 8 \cdot 9}x^9 - \frac{k^6}{4 \cdot 5 \cdot 8 \cdot 9 \cdot 12 \cdot 13}x^{13} + \dots\right)$
26. $y(x) = a_0 \left(1 - \frac{3}{2}x^2 + \frac{9}{8}x^4 - \frac{27}{56}x^6 + \dots\right) + a_1 \left(x - x^3 + \frac{3^2}{3 \cdot 5}x^5 - \frac{27}{3 \cdot 5 \cdot 7}x^7 + \dots\right)$
27. $y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{7}{240}x^6 - \dots\right)a_0 + \left(x + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \frac{1}{112}x^7 + \dots\right)a_1$
28. $y(x) = 1 - x^2 + \left(x - \frac{1}{6}x^3 + \frac{1}{5!}x^5 + \frac{3}{7!}x^7 + \dots\right)a_1$
29. $y(x) = a_0 \left(1 + \frac{1}{12}x^2 + \frac{1}{90}x^4 + \frac{3}{1120}x^6 + \dots\right) + a_1 \left(x + \frac{1}{12}x^3 + \frac{1}{72}x^5 + \dots\right)$
30. $y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{45}x^6 - \frac{7}{3,240}x^9 + \dots\right)a_0 + \left(x - \frac{1}{6}x^4 + \frac{5}{252}x^7 - \dots\right)a_1$

31. $y(x) = a_0 \left(1 - \frac{1}{3}x^3 + \frac{1}{18}x^6 - \frac{1}{162}x^9 + \dots\right) + a_1 \left(x - \frac{1}{4}x^4 + \frac{1}{28}x^7 - \frac{1}{280}x^{10} + \dots\right)$
32. $y(x) = a_0 \left(1 + \frac{1}{2}x^3 + \frac{1}{10}x^6 + \frac{1}{80}x^9 + \dots\right) + a_1 \left(x + \frac{1}{3}x^4 + \frac{1}{18}x^7 + \frac{1}{162}x^{10} + \dots\right)$
33. $y(x) = \left(1 - x^2 + \frac{1}{2!}x^4 - \frac{1}{3!}x^6 + \frac{1}{4!}x^8 - \dots\right)a_0 + \left(x - \frac{2}{3}x^3 + \frac{4}{15}x^5 - \frac{8}{105}x^7 + \dots\right)a_1$
34. $y(x) = a_0 \left(1 - \frac{1}{4}x^2 + \frac{1}{32}x^4 - \dots\right) + a_1 \left(x - \frac{1}{6}x^3 + \frac{1}{60}x^5 - \dots\right)$
35. $y(x) = a_0 \left(1 + \frac{2}{3}x^2 + \frac{1}{27}x^4\right) + a_1 \left(x + \frac{1}{6}x^3 + \frac{1}{360}x^5 - \frac{1}{45,360}x^7 + \dots\right)$
36. $y(x) = a_0 \left(1 - x^2 + \frac{1}{10}x^4 - \frac{1}{750}x^6 - \frac{1}{105,000}x^8 - \dots\right) + a_1 \left(x - \frac{4}{15}x^3 + \frac{4}{375}x^5\right)$
37. $y(x) = a_0 + \left(x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \dots\right)a_1$
38. $y(x) = \left(1 + \frac{1}{4}x^2 - \frac{1}{24}x^3 + \frac{1}{480}x^5 - \dots\right)a_0 + a_1$
39. $y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{30}x^5 + \frac{1}{45}x^6 + \dots\right)a_0 + \left(x + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{120}x^5 + \frac{7}{180}x^6 + \dots\right)a_1$
40. $y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{6}x^4 + \dots\right)a_0 + \left(x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \dots\right)a_1$
41. $y(x) = \left(1 + 3x^2 + x^4 - \frac{1}{5}x^6 + \dots\right)a_0 + \left(x + x^3\right)a_1$
42. $y(x) = \left(1 + \frac{1}{4}x^2 - \frac{7}{96}x^4 + \frac{161}{5760}x^6 - \dots\right)a_0 + \left(1 - \frac{1}{6}x^3 + \frac{7}{120}x^5 - \frac{17}{720}x^7 + \dots\right)a_1$
43. $y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 - \dots\right)a_0 + a_1x$
44. $y(x) = a_0 \left(1 + \frac{1}{2}x^2 - \frac{1}{2^2 \cdot 2!}x^4 + \frac{1 \cdot 3}{2^3 \cdot 3!}x^6 - \frac{1 \cdot 3 \cdot 5}{2^4 \cdot 4!}x^8 + \dots\right) + a_1x$
45. $y(x) = a_0 \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{9}{6!}x^6 - \frac{1 \cdot 3^2 \cdot 5^2}{8!}x^8 - \dots\right) + a_1x$
46. $y(x) = a_0 \left(1 + 2x^2 + 3x^4 + 4x^6 + \dots\right) + a_1 \left(x + \frac{5}{3}x^3 + \frac{7}{3}x^5 + \frac{11}{3}x^7 + \dots\right)$
47. $y(x) = a_0 \left(1 + \frac{2}{3}(x-1)^3 + \frac{1}{45}(x-1)^6 - \frac{1}{1,620}(x-1)^9 + \dots\right) + a_1 \left(x - 1 + \frac{1}{4}(x-1)^4\right)$
48. $y(x) = a_0 \left(1 - 4x^2 + 2x^4\right) + a_1 \left(x - \frac{5}{4}x^3 + \frac{7}{32}x^5 + \frac{9}{320}x^7 + \dots\right)$
49. $y(x) = a_0 \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{3}{40}x^5 - \frac{17}{720}x^6 + \dots\right) + a_1 \left(x - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{40}x^5 + \frac{1}{20}x^6 + \dots\right)$
50. $y(x) = a_0 \left(1 - 2x^2 + 3x^4 - 4x^6 + \dots\right) + \frac{1}{3}a_1 \left(x - \frac{5}{3}x^3 + \frac{7}{3}x^5 - \frac{9}{3}x^7 + \dots\right)$
51. $y(x) = a_0 \left(1 + 6x^2 + x^4\right) + a_1 \left(x + x^3\right)$
52. $y(x) = a_0 \left(1 + 6x^2 + 15x^4 + 28x^6 + \dots\right) + a_1 \left(x + \frac{10}{3}x^3 + 7x^5 + 12x^7 + \dots\right)$
53. $y(x) = a_0 \left(1 + x^2 + x^4 + x^6 + \dots\right) + a_1 \left(x + x^3 + x^5 + x^7 + \dots\right)$

54. $y(x) = a_0 \left(1 - 3x^2\right) + a_1 \left(x - \frac{1}{3}x^3\right)$
55. $y(x) = a_0 \left(1 - \frac{1}{2}x^2 + \frac{1}{4}x^4 - \frac{1}{8}x^6 + \dots\right) + a_1 \left(x - \frac{1}{2}x^3 + \frac{1}{4}x^5 - \frac{1}{8}x^7 + \dots\right)$
56. $y(x) = a_0 + a_1 \left(x + \frac{1}{9}x^3 + \frac{1}{45}x^5 + \frac{1}{189}x^7 + \dots\right)$
57. $y(x) = a_0 \left(1 - \frac{8}{3}x^2 + \frac{8}{27}x^4\right) + a_1 \left(x - \frac{1}{2}x^3 + \frac{1}{120}x^5 + \frac{1}{15,120}x^7 + \dots\right)$
58. $y(x) = \frac{3}{2} \sin 2x$
59. $y(x) = 1 - \frac{1}{12}x^4 + \frac{1}{672}x^8 - \frac{1}{88,704}x^{12} + \dots$
60. $y(x) = 3 - 12x^2 + 4x^4$
61. $y(x) = 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5 + \frac{4}{45}x^6 - \frac{8}{315}x^7 + \dots = e^{-2x}$
62. $y(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5 + \frac{1}{120}x^6 + \frac{1}{720}x^7 + \dots = xe^x$
63. $y(x) = 1 - \frac{1}{2}x^2 + \frac{1}{2^2 2!}x^4 - \frac{1}{2^3 3!}x^5 + \dots$
64. $y(x) = 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{15}x^5 + \frac{1}{24}x^6 + \dots$
65. $y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \frac{1}{48}x^6 + \dots$
66. $y(x) = 1 + x^2$
67. $y(x) = 2 - (x-1)^2 + \frac{1}{4}(x-1)^4 - \frac{1}{24}(x-1)^6 + \dots$
68. $y(x) = 8x - 2e^x$
69. $y(x) = 2 - x - 2x^2 + x^3 + \frac{1}{2}x^4 - \frac{3}{10}x^5 + \dots$
70. $y(x) = x + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{24}x^5 + \frac{1}{45}x^6 + \frac{13}{1008}x^7 + \dots$
71. $y(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots$
72. $y(x) = 4 + 6x + \frac{11}{3}x^3 + \frac{1}{2}x^4 + \frac{11}{4}x^5 + \frac{47}{90}x^6 + \dots$
73. $y(x) = -1 + 3x + x^2 - \frac{3}{4}x^3 - \frac{1}{6}x^4 + \frac{21}{160}x^5 + \dots$
74. $y(x) = 1 - x^2$
75. $y(x) = x - \frac{1}{12}x^3 - \frac{1}{240}x^5 - \frac{1}{2,240}x^7 - \frac{1}{16,128}x^9 - \dots$
76. $y(x) = 4 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{3}{32}x^4 + \frac{1}{30}x^5 + \frac{5}{256}x^6 + \frac{1}{140}x^7 + \dots$
77. $y(x) = a_1 x + a_0 \left(1 + x^2 - \frac{1}{3}x^4 + \frac{1}{5}x^6 - \frac{1}{7}x^8 + \dots\right) = a_1 x + a_0 \left(1 + x \tan^{-1} x\right)$
78. $y(x) = (x-1) + \frac{5}{3}(x-1)^3 + \frac{7}{3}(x-1)^5 + 3(x-1)^7 + \frac{11}{3}(x-1)^9 + \dots$
79. $y(x) = 2 - 6(x-3)^2$

80. $y(x) = 1 + 4(x+2)^2$

81. $y(x) = 2x + 6$

82. $y(x) = a_0 \left(1 - (x-2)^2 \right) + a_1 \left((x-2) - \frac{1}{6}(x-2)^3 - \frac{1}{120}(x-2)^5 - \frac{1}{1680}(x-2)^7 - \dots \right)$

83. $y(x) = a_0 \left(1 + \frac{2}{3}(x-1)^3 + \frac{1}{45}(x-1)^6 - \frac{1}{1,620}(x-1)^9 + \dots \right) + a_1 \left((x-1) + \frac{1}{4}(x-1)^4 \right)$

84. $y(x) = e \left(\frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 + \frac{1}{24}(x-1)^4 - \frac{1}{60}(x-1)^5 + \dots \right) + a_0 \left(1 - \frac{1}{6}(x-1)^3 + \dots \right) + a_1 \left((x-1) - \frac{1}{12}(x-1)^4 + \dots \right)$

85. $y(x) = 2 - 2(x+1) + 3(x+1)^2 - \frac{1}{3}(x+1)^3 + \frac{1}{3}(x+1)^4 + \dots$

86. $x(t) = 3 - \frac{9}{2}t^2 + t^3 + t^4 + \dots$

Section 4.3

4. $y(x) = a_0 \left(1 - \frac{1}{2 \cdot 3}x^3 + \frac{1}{2 \cdot 3 \cdot 5 \cdot 6}x^6 - \dots \right) + a_1 \left(x - \frac{1}{3 \cdot 4}x^4 + \frac{1}{3 \cdot 4 \cdot 6 \cdot 7}x^7 - \dots \right)$

5. $y_1(x) = 1 + \sum_{m=1}^{\infty} (-1)^m \frac{2^m \alpha(\alpha-2) \cdots (\alpha-2m+2)}{(2m)!} x^{2m}$

$$y_2(x) = x + \sum_{m=1}^{\infty} (-1)^m \frac{2^m (\alpha-1)(\alpha-3) \cdots (\alpha-2m+1)}{(2m+1)!} x^{2m+1}$$

6. a) $y_0(x) = 1$; $y_1(x) = x$; $y_2(x) = 1 - 2x^2$; $y_3(x) = x - \frac{2}{3}x^3$; $y_4(x) = 1 - 4x^2 + \frac{4}{3}x^4$;

$$y_5(x) = x - \frac{4}{3}x^3 + \frac{4}{15}x^5$$

b) $H_0(x) = 1$; $H_1(x) = 2x$; $H_2(x) = 4x^2 - 2$; $H_3(x) = 8x^3 - 12x$;

$$H_4(x) = 16x^4 - 48x^2 + 12$$
; $H_5(x) = 32x^5 - 160x^3 + 120x$

Section 4.4

1. The singular point is: $x = 0$ 2. The singular points are: $x = 0, -1$ 3. The singular point is: $x = -1$

4. No singular points.

5. $x \leq 1, x = 2$ 6. $x = 1$ 7. $x = 0, -3$ 8. Regular $x = -3$, Irregular $x = 3$ 9. Regular $x = 0$, Irregular $x = 1$ 10. Regular $x = 0, \pm 2i$ 11. Regular $x = 0$, Irregular $x = 5$ 12. Regular $x = -3, 2$ 13. Regular $x = 0, \pm i$ 14. Regular $x = 2, \pm 5$, Irregular $x = 0$ 15. Regular $x = 0, 3$, Irregular $x = -1$ 16. Regular $x = 0, \pm 1, \pm \frac{\pi}{2}$ 17. Regular $x = 0, -2$, Irregular $x = 1$ 18. Regular $x = 0, \pm i$, Irregular $x = 0$ 19. $x = 0$ is an ordinary point20. $x = 0$ is a regular singular point

21. $y(x) = C_1 \left(1 + \sum_{n=0}^{\infty} \frac{4}{n! \cdot 1 \cdot 3 \cdot 5 \cdots (2n-1)} x^n \right) + C_2 x^{1/2} \left(1 + \sum_{n=0}^{\infty} \frac{4^n}{n! \cdot 3 \cdot 5 \cdots (2n+1)} x^n \right)$
22. $y(x) = a_0 \left(x^{1/2} - \frac{1}{6} x^{5/2} + \frac{1}{168} x^{9/2} - \frac{1}{11,088} x^{13/2} + \cdots \right) + b_0 \left(x - \frac{1}{10} x^3 + \frac{1}{360} x^5 - \frac{1}{28,080} x^7 + \cdots \right)$
23. $y(x) = a_0 \frac{\sin x}{x} + b_0 \frac{\cos x}{x}$
 $y(x) = a_0 \left(1 + 2x - 2x^2 + \frac{4}{9} x^3 - \frac{2}{45} x^4 + \cdots \right) + b_0 \sqrt{x} \left(x - \frac{2}{5} x^2 + \frac{2}{35} x^3 - \frac{4}{945} x^4 + \frac{2}{20,790} x^5 - \cdots \right)$
24. $y(x) = a_0 \left(1 - \frac{1}{14} x^2 + \frac{1}{616} x^4 - \cdots \right) + b_0 \left(x^{-3/2} - \frac{1}{2} x^{1/2} + \frac{1}{40} x^{3/2} - \cdots \right)$
25. $y(x) = a_0 \left(1 - 2x + \frac{2}{9} x^2 - \frac{4}{459} x^3 + \cdots \right) + b_0 x^{7/8} \left(1 - \frac{2}{15} x + \frac{2}{345} x^2 - \frac{4}{32,085} x^3 + \cdots \right)$
26. $y(x) = a_0 \left(x - \frac{1}{10} x^3 + \frac{1}{360} x^5 - \cdots \right) + b_0 x^{1/2} \left(1 - \frac{1}{6} x^2 + \frac{1}{168} x^4 - \cdots \right)$
27. $y(x) = a_0 \left(1 + \frac{1}{2} x + \frac{1}{10} x^2 + \frac{1}{80} x^3 + \cdots \right) + b_0 x^{1/3} \left(1 + \frac{1}{3} x + \frac{1}{18} x^2 + \frac{1}{162} x^3 + \cdots \right)$
28. $y(x) = a_0 \left(1 + \frac{1}{3} x - \frac{1}{6} x^2 - \frac{1}{6} x^3 - \frac{5}{72} x^4 - \cdots \right) + b_0 x^{5/2} \left(1 + \frac{4}{7} x + \frac{4}{21} x^2 + \frac{32}{693} x^3 + \cdots \right)$
29. $y(x) = a_0 \left(1 - \frac{1}{2} x + \frac{1}{10} x^2 - \frac{1}{120} x^3 \right) + b_0 x^7 \left(1 - \frac{1}{2} x + \frac{5}{36} x^2 - \frac{1}{36} x^3 + \frac{7}{1,584} x^4 - \cdots \right)$
30. $y(x) = a_0 \left(1 + \frac{2}{3} x + \frac{1}{3} x^2 \right) + b_0 \left(x^4 + 2x^5 + 3x^6 + 4x^7 + 5x^8 + \cdots \right)$
31. $y(x) = a_0 x^{1/3} \left(1 + \frac{3}{2} x + \frac{9}{20} x^2 + \frac{9}{160} x^3 + \cdots \right) + b_0 x^{2/3} \left(1 + \frac{3}{4} x + \frac{9}{56} x^2 + \frac{9}{560} x^3 + \cdots \right)$
32. $y(x) = a_0 x \left(1 - \frac{1}{5} x + \frac{1}{30} x^2 - \frac{1}{210} x^3 + \frac{1}{1,680} x^4 - \cdots \right) + b_0 x^{-3} \left(1 - x + \frac{1}{2} x^2 - \frac{1}{6} x^3 \right)$
33. $y(x) = a_0 x \left(1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots \right) + b_0 x \ln x \left(-x + x^2 - \frac{x^3}{2} + \cdots \right) + x \ln x \left(1 - x + \frac{x^3}{4} - \frac{5}{36} x^4 + \cdots \right)$
34. $y(x) = a_0 x \left(1 - \frac{x}{4} + \frac{x^2}{20} - \frac{x^3}{120} + \frac{x^4}{840} - \cdots \right) + b_0 x^2 \left(1 - \frac{x}{5} + \frac{x^2}{30} - \frac{x^3}{210} + \frac{x^4}{1,680} - \cdots \right)$
35. $y(x) = a_0 \left(1 + \frac{1}{3} x + \frac{1}{30} x^2 + \frac{1}{630} x^3 + \frac{1}{22,680} x^4 + \cdots \right) + b_0 x^{-1/2} \left(1 + x + \frac{1}{6} x^2 + \frac{1}{90} x^3 + \frac{1}{2,520} x^4 + \cdots \right)$
36. $y(x) = a_0 \left(1 - x - \frac{1}{2} x^2 - \frac{1}{18} x^3 - \frac{1}{360} x^4 + \cdots \right) + b_0 x^{3/2} \left(1 + \frac{1}{5} x + \frac{1}{70} x^2 + \frac{1}{1,890} x^3 + \frac{1}{83,160} x^4 - \cdots \right)$
37. $y(x) = a_0 \left(1 - x + \frac{1}{3} x^2 - \frac{1}{15} x^3 + \frac{1}{105} x^4 - \cdots \right) + b_0 x^{1/2} \left(1 - \frac{1}{2} x + \frac{1}{8} x^2 - \frac{1}{48} x^3 + \frac{1}{384} x^4 + \cdots \right)$
38. $y(x) = a_0 \left(1 + \frac{2}{3} x^2 + \frac{4}{21} x^4 + \frac{8}{231} x^6 + \frac{16}{3465} x^8 + \cdots \right) + b_0 x^{1/2} \left(1 + \frac{1}{2} x^2 + \frac{1}{8} x^4 + \frac{1}{48} x^6 + \frac{1}{384} x^8 + \cdots \right)$
39. $y(x) = a_0 x \left(1 + \frac{1}{7} x^2 + \frac{1}{154} x^4 + \frac{1}{6,930} x^6 + \cdots \right) + b_0 x^{-1/2} \left(1 + x^2 + \frac{1}{10} x^4 + \frac{1}{270} x^6 + \cdots \right)$
40. $y(x) = a_0 x^{-1} \left(1 + x^2 - \frac{1}{6} x^4 + \frac{1}{126} x^6 - \cdots \right) + b_0 x^{3/2} \left(1 - \frac{1}{9} x^2 + \frac{1}{234} x^4 - \frac{1}{11,934} x^6 + \cdots \right)$
41. $y(x) = a_0 \left(1 - x + \frac{1}{5} x^2 - \frac{1}{60} x^3 + \frac{1}{1320} x^4 - \cdots \right) + b_0 x^{1/3} \left(1 - \frac{1}{2} x + \frac{1}{14} x^2 - \frac{1}{210} x^3 + \frac{1}{5460} x^4 - \cdots \right)$
42. $y(x) = a_0 \left(1 - \frac{1}{10} x^2 + \frac{1}{440} x^4 - \frac{1}{44,880} x^6 + \cdots \right) + b_0 x^{1/3} \left(1 - \frac{1}{14} x^2 + \frac{1}{728} x^4 - \frac{1}{82,992} x^6 + \cdots \right)$

44. $y(x) = a_0 x + b_0 x^{1/3}$
45. $y(x) = a_0 \left(1 - \frac{1}{2}x + \frac{1}{24}x^2 - \frac{1}{720}x^3 + \frac{1}{30,240}x^4 - \dots \right) + b_0 x^{1/2} \left(1 - \frac{1}{6}x + \frac{1}{120}x^2 - \frac{1}{5,040}x^3 + \dots \right)$
46. $y(x) = a_0 x^{1/2} \left(1 + \frac{x^2}{38} + \frac{x^4}{4,712} + \frac{x^6}{1,215,696} + \dots \right) + b_0 x^{-2/3} \left(1 + \frac{x^2}{10} + \frac{x^4}{680} + \frac{x^6}{118,320} + \dots \right)$
47. $y_1(x) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{(n!)^2} x^n$ $y_2(x) = y_1(x) \ln|x| + 4x - 3x^2 + \frac{22}{27}x^3 - \dots$
48. $y_1(x) = \sum_{n=1}^{\infty} \frac{1}{n!(n+1)!} x^{n+1}$ $y_2(x) = y_1(x) \ln x + 1 - \frac{3}{4}x^2 - \frac{7}{36}x^3 - \dots$
49. $y(x) = a_0(1+x) + b_0\sqrt{x}$
50. $y(x) = a_0 \left(1 - 2x + \frac{4}{3}x^2 - \frac{8}{15}x^3 + \frac{16}{105}x^4 - \dots \right) + b_0\sqrt{x} \left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \dots \right)$
51. $y(x) = a_0 \frac{1}{x^{1/6}} \left(1 + \frac{7}{6}x + \frac{35}{216}x^2 + \frac{7}{1296}x^3 + \dots \right) + b_0 x^{1/3} \left(1 + \frac{1}{3}x + \frac{1}{45}x^2 + \frac{1}{2835}x^3 \right)$
52. $y(x) = a_0 \frac{1}{x^3} \left(1 - \frac{21}{5}x + \frac{49}{5}x^2 - \frac{343}{15}x^3 \right) + b_0\sqrt{x} \left(1 - \frac{7}{18}x + \frac{49}{264}x^2 - \frac{1,715}{20,592}x^3 + \dots \right)$
53. $y(x) = a_0 \left(1 + \sum_{n=0}^{\infty} \frac{a(a+1)\cdots(a+n-1) \cdot b(b+1)\cdots(b+n-1)}{n! \cdot c(c+1)\cdots(c+n-1)} x^n \right)$
 $+ b_0 x^{1-c} \left(1 + \sum_{n=0}^{\infty} \frac{((n-c)(n-c+a+b)+ab)\cdots((1-c)(1-c+a+b)+ab)}{2(2-c)(3-c)\cdots(n+1-c)} x^n \right)$

Section 4.5

1. $y(x) = c_1 J_{1/3}(x) + c_2 J_{-1/3}(x)$
2. $y(x) = c_1 J_1(x) + c_2 Y_1(x)$
3. $y(x) = c_1 J_{5/2}(x) + c_2 J_{-5/2}(x)$
4. $y(x) = c_1 J_{1/4}(x) + c_2 J_{-1/4}(x)$
5. $y(x) = c_1 J_0(x) + c_2 Y_0(x)$
6. $y(x) = c_1 J_2(x) + c_2 Y_2(x)$
7. $y(x) = c_1 J_2(3x) + c_2 Y_2(3x)$
8. $y(x) = c_1 J_{1/2}(6x) + c_2 J_{-1/2}(6x)$
9. $y(x) = c_1 J_{2/3}(5x) + c_2 J_{-2/3}(5x)$
10. $y(x) = c_1 J_8(\sqrt{2}x) + c_2 Y_8(\sqrt{2}x)$
11. $y(x) = x^{-3/2} \left(C_1 \sqrt{\frac{2}{\pi}} \sin \frac{x^2}{4} + C_2 \sqrt{\frac{2}{\pi}} \cos \frac{x^2}{4} \right)$
12. $y(x) = x^{1/2} \left[c_1 J_{1/3}(2x^{3/2}) + c_2 J_{-1/3}(2x^{3/2}) \right]$
13. $y(x) = x^2 [c_1 Y_2(x) + c_2 J_2(x)]$
14. $y(x) = C_1 \sin x^2 + C_2 \cos x^2$
15. $y(x) = c_1 \sqrt{\frac{2}{\pi x}} \sinh x + c_2 \sqrt{\frac{2}{\pi x}} \cosh x$
16. $y(x) = (c_1 J_0(x) + c_2 Y_0(x)) e^{-x}$
17. $y(x) = x(c_1 I_1(x) + c_2 K_1(x))$
18. $y(x) = x \left(C_1 \sin \frac{a}{x} + C_2 \cos \frac{a}{x} \right)$

$$19. y(x) = \sqrt{x} \left(c_1 I_{\frac{1}{4}} \left(\frac{x^2}{2} \right) + c_2 I_{-\frac{1}{4}} \left(\frac{x^2}{2} \right) \right)$$

$$20. y(x) = x \left[c_1 J_0(x) + c_2 Y_0(x) \right]$$

$$21. y(x) = x^{-1} \left[c_1 J_1(x) + c_2 Y_1(x) \right]$$

$$22. y(x) = x \left[c_1 J_{1/2}(3x^2) + c_2 J_{-1/2}(3x^2) \right]$$

$$23. y(x) = x^3 \left[c_1 J_2(2x^{1/2}) + c_2 Y_2(2x^{1/2}) \right]$$

$$24. y(x) = x^{-1/3} \left[c_1 J_{1/3} \left(\frac{1}{3} x^{3/2} \right) + c_2 J_{-1/3} \left(\frac{1}{3} x^{3/2} \right) \right]$$

$$25. y(x) = x^{-1/4} \left[c_1 J_0(2x^{3/2}) + c_2 Y_0(2x^{3/2}) \right]$$

$$26. y(x) = x^{-1} \left[c_1 J_0(x) + c_2 Y_0(x) \right]$$

$$27. y(x) = x^2 \left[c_1 J_1(4x^{1/2}) + c_2 Y_1(4x^{1/2}) \right]$$

$$28. y(x) = x^{1/2} \left[c_1 J_{1/2}(2x^{3/2}) + c_2 J_{-1/2}(2x^{3/2}) \right]$$

$$29. y(x) = x^{-1/4} \left[c_1 J_{3/2} \left(\frac{2}{5} x^{5/2} \right) + c_2 J_{-3/2} \left(\frac{2}{5} x^{5/2} \right) \right]$$

$$30. y(x) = x^{1/2} \left[c_1 J_{1/6} \left(\frac{1}{3} x^3 \right) + c_2 J_{-1/6} \left(\frac{1}{3} x^3 \right) \right]$$

$$31. y(x) = x^{1/2} \left[c_1 J_{1/5} \left(\frac{4}{5} x^{5/2} \right) + c_2 J_{-1/5} \left(\frac{4}{5} x^{5/2} \right) \right]$$

$$32. J_0(x) = \frac{(-1)^n x^{2n}}{2^{2n} \cdot (n!)^2} = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \dots$$

$$33. a) y(x) = x \left(A \cos \left(\frac{\mu}{x} \right) + B \sin \left(\frac{\mu}{x} \right) \right) \quad b) P = \frac{n^2 \pi^2}{L^2} (EI_0) \left(\frac{a}{b} \right)^2$$

$$34. y(x) = x \left(A \cos \left(\frac{\sqrt{\lambda}}{x} \right) + B \sin \left(\frac{\sqrt{\lambda}}{x} \right) \right) \quad P_n = \frac{n^2 \pi^2}{L^2} (EI_0) \left(\frac{a}{b} \right)^2 \quad y(x) = Cx \sin \sqrt{\lambda} \left(\frac{1}{x} - \frac{1}{a} \right)$$

$$y_1(x) = x \sin \frac{11\pi}{10} \left(1 - \frac{1}{x} \right)$$

$$35. z = 1.8664 \quad L \approx 1.986352 \left(\frac{EI}{g\rho} \right)^{1/3}$$