Determinant

$$\left|\begin{array}{c|c} A & 0 \\ \hline 0 & B \end{array}\right| = |A| \cdot |B|$$

Proof

$$\left|\begin{array}{cc|cc} a & b & 0 & 0 \\ c & d & 0 & 0 \\ \hline 0 & 0 & e & f \\ 0 & 0 & g & h \end{array}\right| = a\begin{vmatrix} d & 0 & 0 \\ 0 & e & f \\ 0 & g & h \end{vmatrix} - b\begin{vmatrix} c & 0 & 0 \\ 0 & e & f \\ 0 & g & h \end{vmatrix}$$

$$= ad\begin{vmatrix} e & f \\ g & h \end{vmatrix} - bc\begin{vmatrix} e & f \\ g & h \end{vmatrix}$$

$$= (ad - bc)\begin{vmatrix} e & f \\ g & h \end{vmatrix}$$

$$= \begin{vmatrix} a & b \\ c & d \end{vmatrix}\begin{vmatrix} e & f \\ g & h \end{vmatrix}$$

$$= |A| \cdot |B|$$

Example

$$\begin{vmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & -2 & 5 \\ 0 & 0 & -2 & 7 \end{vmatrix} = 8 \qquad A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2 \qquad B = \begin{vmatrix} -2 & 5 \\ -2 & 7 \end{vmatrix} = -4$$

$$\boxed{8 = (-2)(-4)}$$

Example

$$\left|\begin{array}{ccc|cc} 1 & 2 & 2 & 0 & 0 \\ 3 & 4 & 5 & 0 & 0 \\ 6 & 7 & 8 & 0 & 0 \\ \hline 0 & 0 & 0 & -5 & 7 \\ 0 & 0 & 0 & 3 & 4 \end{array}\right| = -123 \qquad A = \begin{vmatrix} 1 & 2 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{vmatrix} = 3 \qquad B = \begin{vmatrix} -5 & 7 \\ 3 & 4 \end{vmatrix} = -41$$

$$\boxed{-123 = (3)(-41)}$$