5 GRAPHING AND OPTIMIZATION

EXERCISE 5-1

2. (b, c); (c, d); (f, g)

4. (a, b); (d, f); (g, h)

6. $x = b, g$

8. $x = d, g$

10. f has a local maximum at $x = d$, and a local minimum at $x = b$; f does not have a local extremum at $x = a$ or at $x = c$.

12. (b) **14.** (h) **16.** (g) **18.** (a)

20. (A) $f'(x) = 3x^2 - 27 = 3(x^2 - 9) = 3(x-3)(x+3)$

(B) Since $f'(x) = 0$ at $x = -3$ or 3, –3, 3 are critical values of f.

(C) –3, 3 are partition numbers of f.

22. (A) $f'(x) = \frac{2}{3}(x-9)^{-1/3} = \frac{2}{3(x-9)^{1/3}}$

(B) Since $f(x)$ is defined and $f'(x)$ is undefined at $x = 9$, 9 is a critical value of f.

(C) 9 is a partition number of f.

24. (A) $f'(x) = -5(x-4)^{-2} = \frac{5}{(x-4)^2}$

(B) Since $f(x)$ and $f'(x)$ are both undefined at $x = 4$, there are no critical values of f.

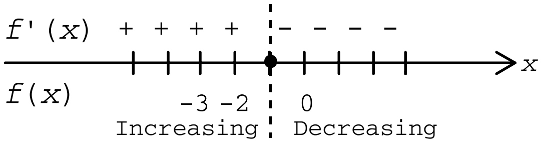
(C) 4 is a partition number of f.

26. (A) $f'(x) = \begin{cases} -1 & \text{if} \quad x < -3 \\ 1 & \text{if} \quad x > -3 \end{cases}$

(B) Since $f(x)$ is defined and $f'(x)$ is undefined at $x = -3$, –3 is a critical value of f.

(C) –3 is a partition numbers of f.

28. $f(x) = -3x^2 - 12x$
$f'(x) = -6x - 12$
f is continuous for all x and
$f'(x) = -6x - 12 = 0$
$x = -2$
Thus, $x = -2$ is a partition number for f':
Next we construct a sign chart for f'.



Test Numbers

x	$f'(x)$
0	$-12(-)$
-3	$6(+)$

Therefore, f is increasing on $(-\infty, -2)$ and decreasing on $(-2, \infty)$; f has a local maximum at $x = -2$, which is $f(-2) = 12$.

30. $f(x) = -3x^2 + 12x - 5$

$f'(x) = -6x + 12$

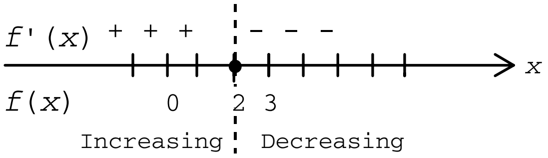
f is continuous for all x and

$f'(x) = -6x + 12 = 0$

$x = 2$

Thus, $x = 2$ is a partition number for f'.

Next we construct a sign chart for f'.



Test Numbers

x	$f'(x)$
0	12(+)
3	−6(−)

Therefore, f is increasing on $(-\infty, 2)$ and increasing on $(2, \infty)$;
f has a local maximum of 7 at $x = 2$.

32. $f(x) = -x^3 - 4x + 8$

$f'(x) = -3x^2 - 4$

f is continuous for all x and $f'(x) = -(3x^2 + 4) < 0$.

Thus, f is decreasing for all x; no local extrema.

34. $f(x) = -2x^3 + 3x^2 + 120x$

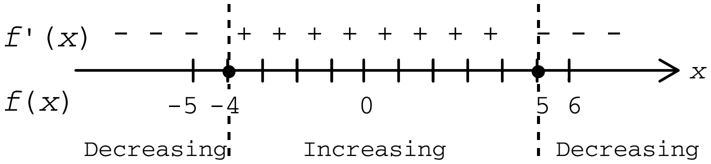
$f'(x) = -6x^2 + 6x + 120$ which is continuous for all x.

$f'(x) = -6(x^2 - x - 20) \qquad = -6(x + 4)(x - 5) = 0$

$x \qquad = -4, 5$

Thus, $x = -4$ and $x = 5$ are partition numbers for f'.

Next, we construct a sign chart for f':



Test Numbers

x	$f'(x)$
−5	−60(−)
0	120(+)
6	−60(−)

Therefore, f is decreasing on $(-\infty, -4)$ and $(5, \infty)$; increasing on $(-4, 5)$; f has a local minimum of –304 at $x = -4$ and a local maximum of 425 at $x = 5$.

36. $f(x) = x^4 + 2x^3 + 5$

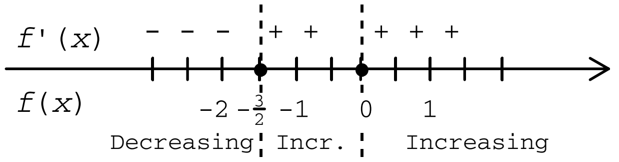
$f'(x) = 4x^3 + 6x^2$ which is continuous for all x.

$f'(x) = 4x^3 + 6x^2 \qquad = 2x^2(2x + 3) = 0$

$x \qquad = -\frac{3}{2}, 0$

Thus, $x = -\frac{3}{2}$ and $x = 0$ are partition numbers for f'.

Next, we construct a sign chart for f:



Test Numbers

x	$f'(x)$
–2	–8(–)
–1	2(+)
1	10(+)

Therefore, f is decreasing on $\left(-\infty, -\frac{3}{2}\right)$; increasing on $\left(-\frac{3}{2}, \infty\right)$;

$f(-1.5) = 3.3125$ is a local minimum.

38. $f(x) = x \ln x - x, x > 0$

$f'(x) = (1)\ln x + x\left(\frac{1}{x}\right) - 1 = \ln x + 1 - 1 = \ln x$

$f'(x) = \ln x = 0$ for $x = 1$.

$f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$.

Therefore, f is decreasing on (0, 1) and increasing on $(1, \infty)$. f has a local minimum of –1 at $x = 1$.

40. $f(x) = (x^2 - 9)^{2/3}$

$f'(x) = \frac{2}{3}(x^2 - 9)^{-1/3}(2x) = \frac{4x}{3(x^2 - 9)^{1/3}}$

$f'(x) = 0$ at $x = 0$. We have to determine the sign of $f'(x)$ in the intervals $(-\infty, -3)$, $(-3, 0)$, $(0, 3)$ and $(0, \infty)$.

For $x < -3$, $x^2 > 9$ and hence $(x^2 - 9)^{1/3} > 0$.

Thus, $f'(x) < 0$ on the interval $(-\infty, -3)$.

For $-3 < x < 0$, $9 > x^2 > 0$ and hence $(x^2 - 9)^{1/3} < 0$.

Thus, $f'(x) > 0$ on the interval $(-3, 0)$.

For $0 < x < 3$, $0 < x^2 < 9$ and hence $(x^2 - 9)^{1/3} < 0$.

Thus, $f'(x) < 0$ on the interval $(0, 3)$.

For $x > 3$, $x^2 > 9$ and hence $(x^2 - 9)^{1/3} > 0$.

Thus, $f'(x) > 0$ on the interval $(3, \infty)$.

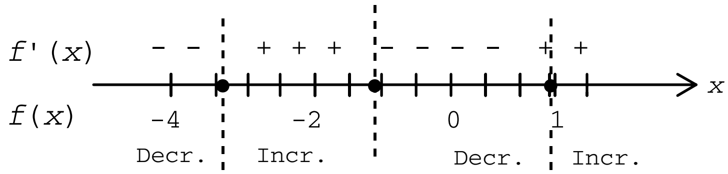
Summary: f is decreasing on $(-\infty, -3)$ and $(0, 3)$; increasing on $(-3, 0)$ and $(3, \infty)$. f has a local maximum of $(-9)^{2/3}$ at $x = 0$ and local minima of 0 at $x = -3$ and $x = 3$.

42. $f(x) = x^4 + 5x^3 - 15x$ is a polynomial.

$f'(x) = 4x^3 + 15x^2 - 15$

Using a graphing utility to approximate the zeros of $f'(x)$ we have:
$f'(x) = 0$ for $x_1 \approx -3.43$, $x_2 \approx -1.22$, $x_3 \approx 0.90$.

Sign chart for f':



Test Numbers

x	$f'(x)$
–4	–131(–)
–2	13(+)
0	–15(–)
1	4(+)

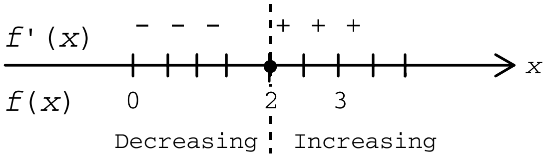
Therefore, f is decreasing on $(-\infty, -3.43)$ and $(-1.22, 0.90)$; increasing on $(-3.43, -1.22)$ and $(0.90, \infty)$; local minima at $x = -3.43$ and $x = 0.90$; local maximum at $x = -1.22$.

44. Critical values: $x = -2.83, -0.20$. f is decreasing on $(-2.83, -0.20)$ and on $(-0.20, \infty)$; it is increasing on $(-2.83, 0.20)$. f has a local minimum at $x = -2.83$ and a local maximum at $x = -0.20$.

46. Critical values: $x = 0.63$, $x = 2.49$. f is increasing on $(0, 0.63)$ and on $(2.49, \infty)$; it is decreasing on $(0.63, 2.49)$. f has local maximum at $x = 0.63$ and a local minimum at $x = 2.49$.

48. $f(x) = 2x^2 - 8x + 9$
$f'(x) = 4x - 8$
f is continuous for all x and
$f'(x) = 4x - 8 = 0$
$x = 2$
Thus, $x = 2$ is a partition number for f'.

The sign chart for f' is:



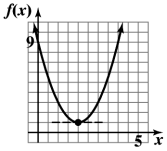
Test Numbers

x	$f'(x)$
0	-8 (-)
3	4 (+)

Therefore, f is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$; f has a local minimum at $x = 2$.

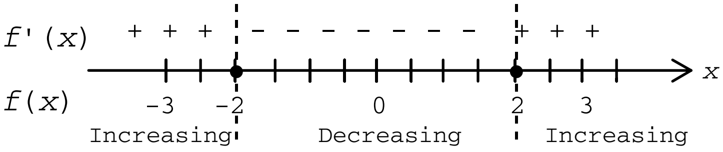
x	$f'(x)$	f	GRAPH OF f
$(-\infty, 2)$	-	Decreasing	Falling
$x = 2$	0	Local minimum	Horizontal tangent
$(2, \infty)$	+	Increasing	Rising

x	$f(x)$
0	9
2	1



50. $f(x) = x^3 - 12x + 2$
$f'(x) = 3x^2 - 12$
f is continuous for all x and
$f'(x) = 3x^2 - 12 = 0$
$x = -2, 2$
Thus, $x = -2$ and $x = 2$ are partition numbers for f'.

The sign chart for f' is:



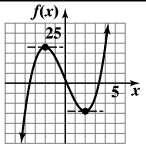
Test Numbers

x	$f'(x)$
-3	15 (+)
0	-12 (-)
3	15 (+)

Therefore, f is increasing on $(-\infty, -2)$ and on $(2, \infty)$, f is decreasing on $(-2, 2)$; f has a local maximum at $x = -2$ and a local minimum at
$x = 2$.

x	$f'(x)$	f	GRAPH OF f
$(-\infty, -2)$	+	Increasing	Rising
$x = -2$	0	Local maximum	Horizontal tangent
$(-2, 2)$	-	Decreasing	Falling
$x = 2$	0	Local minimum	Horizontal tangent
$(2, \infty)$	+	Increasing	Rising

x	$f(x)$
–2	18
0	2
2	–12



52. $f(x) = x^3 + 3x^2 + 3x$

$f'(x) = 3x^2 + 6x + 3$

f is continuous for all x and

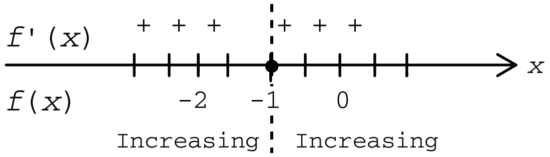
$f'(x) = 3x^2 + 6x + 3$

$= 3(x^2 + 2x + 1) = 3(x + 1)^2 = 0$

$x = -1$

Thus, $x = -1$ is a partition number for f'.

The sign chart for f' is:



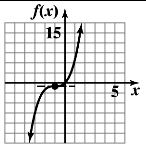
Test Numbers

x	$f'(x)$
-2	3 (+)
0	3 (+)

Therefore, f is increasing for all x, i.e., on $(-\infty, \infty)$, and there is a horizontal tangent line at $x = -1$.

x	$f'(x)$	f	GRAPH of f
$(-\infty, -1)$	+	Increasing	Rising
-1	0		Horizontal tangent
$(-1, \infty)$	+	Increasing	Rising

$xf(x)$	
-1	-1
0	0



54. $f(x) = -x^4 + 50x^2$

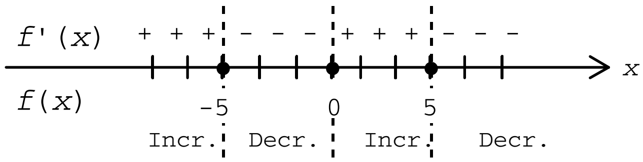
$f'(x) = -4x^3 + 100x$ which is continuous for all x.

$f'(x) = -4x^3 + 100x = -4x(x - 5)(x + 5) = 0$

$x = -5, 0, 5$

Thus, $x = -5$, $x = 0$, and $x = 5$ are partition numbers.

The sign chart for *f*′ is:



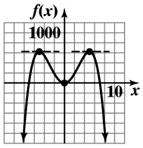
Test Numbers

x	$f'(x)$
-6	$264(+)$
-1	$-96(-)$
1	$96(+)$
6	$-264(-)$

Therefore, *f* is increasing on (-∞, -5) and (0, 5); decreasing on (-5, 0) and (5, ∞). There are 3 horizontal tangent lines at x = -5, x = 0, and x = 5.

x	$f'(x)$	f	GRAPH of f
(-∞, -5)	+	Increasing	Rising
-5	0		Horizontal tangent
(-5, 0)	-	Decreasing	Falling
0	0		Horizontal tangent
(0, 5)	+	Increasing	Rising
5	0		Horizontal tangent
(5, ∞)	-	Decreasing	Falling

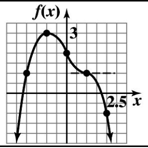
x	$f(x)$
-5	625
0	0
5	625



56.

x	$f'(x)$	$f(x)$	GRAPH of f
(-∞, -1)	+	Increasing	Rising
x = -1	0	Local maximum	Horizontal tangent
(-1, 1)	-	Decreasing	Falling
x = 1	0	Neither local maximum nor local minimum	Horizontal tangent
(1, ∞)	-	Decreasing	Falling

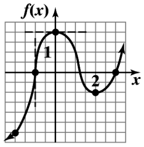
Using this information together with the points (-2, 1), (-1, 3), (0, 2), (1, 1), (2, -1) on the graph, we have



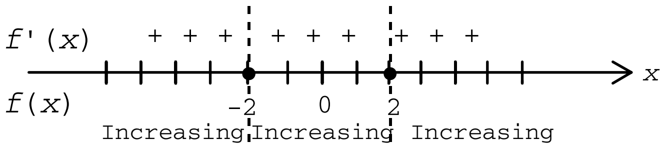
58.

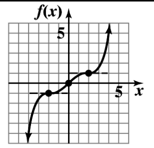
x	$f'(x)$	$f(x)$	Graph of $f(x)$
(-∞, -1)	+	Increasing	Rising
x = -1	Not defined	Neither local maximum nor local minimum	Vertical tangent
(-1, 0)	+	Increasing	Rising
x = 0	0	Local maximum	Horizontal tangent
(0, 2)	-	Decreasing	Falling
x = 2	0	Local minimum	Horizontal tangent
(2, ∞)	+	Increasing	Rising

Using this information together with the points (-2, -3), (-1, 0), (0, 2), (2, -1), (3, 0) on the graph, we have



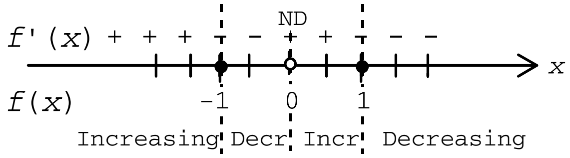
60.

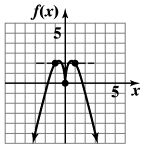




x	–2	0	2
$f(x)$	–1	0	1

62.





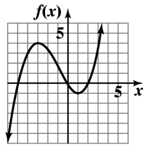
x	–1	0	1
$f(x)$	2	0	2

64. $f_2' = g_1$

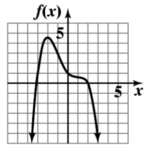
66. $f_4' = g_3$

68. $f_6' = g_5$

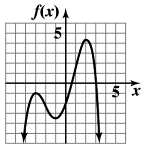
70. Increasing on (-∞, -3) and (1, ∞); decreasing on (-3, 1); local maximum at $x = -3$; local minimum at $x = 1$.



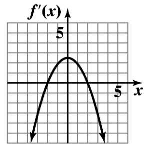
72. Increasing on (-∞, -2); decreasing on (-2,1)and (1, ∞);
local maximum at $x = -2$.



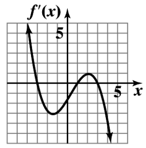
74. Increasing on (-∞, -3) and (-1, 2); decreasing on (-3, -1) and (2, ∞); local maxima at
$x = -3$ and $x = 2$;
local minimum at $x = -1$.



76. $f'(x) > 0$ on (-2, 2);
$f'(x) < 0$ on (-∞, -2) and (2, ∞); $f'(x) = 0$ at $x = -2$ and $x = 2$.



78. $f'(x) > 0$ on (-∞, -3) and (1, 3); $f'(x) < 0$ on (-3, 1) and (3, ∞); $f'(x) = 0$ at $x = -3$, $x = 1$, and $x = 3$.



80. $f(x) = \frac{9}{x} + x$ [<u>Note</u>: f is not defined at $x = 0$.]

$f'(x) = -\frac{9}{x^2} + 1$

Critical values: $x = 0$ is *not* a critical value of f since 0 is not in the domain of f, but $x = 0$ is a partition number for f'.

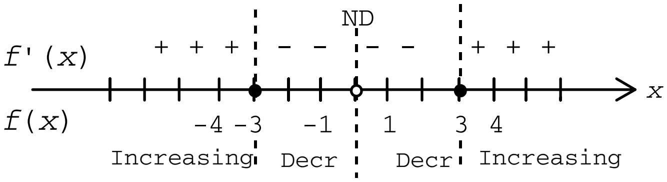
$$f'(x) = -\frac{9}{x^2} + 1 = 0$$

$$-9 + x^2 = 0$$

$$(x + 3)(x - 3) = 0$$

Thus, the critical values are $x = -3$ and $x = 3$; $x = -3$ and $x = 3$ are also partition numbers for f'.

The sign chart for f' is:



Test Numbers

x	$f'(x)$
-4	$\frac{7}{16}(+)$
-1	$-8(-)$
1	$-8(-)$
4	$\frac{7}{16}(+)$

Therefore, f is increasing on $(-\infty, -3)$ and on $(3, \infty)$, f is decreasing on $(-3, 0)$ and on $(0, 3)$; f has a local maximum at $x = -3$ and a local minimum at $x = 3$.

82. $f(x) = 3 - \frac{4}{x} - \frac{2}{x^2}$ [<u>Note</u>: f is not defined at $x = 0$.]

$f'(x) = \frac{4}{x^2} + \frac{4}{x^3}$

Critical values: $x = 0$ is not a critical value of f since 0 is not in the domain of f; $x = 0$ is a partition number for f'.

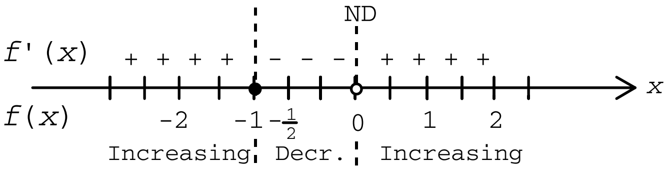
$$f'(x) = \frac{4}{x^2} + \frac{4}{x^3} = 0$$

$$x + 1 = 0$$

$$x = -1$$

Thus, the critical value is $x = -1$; -1 is also a partition number for f'.

The sign chart for f' is:



Test Numbers

x	$f'(x)$
-2	$\frac{1}{2}(+)$
$-\frac{1}{2}$	$-16(-)$
1	$8(+)$

Therefore, f is increasing on $(-\infty, -1)$ and $(0, \infty)$; decreasing on $(-1, 0)$; local maximum at $x = -1$.

84. $f(x) = \dfrac{x^2}{x+1}$ [Note: f is not defined at $x = -1$.]

$$f'(x) = \frac{2x(x+1)-x^2}{(x+1)^2} = \frac{2x^2+2x-x^2}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$$

Critical values: $x = -1$ is *not* a critical value of f since -1 is not in the domain of f; $x = -1$ is a partition number for f'.

$$f'(x) = \frac{x^2+2x}{(x+1)^2} = 0$$

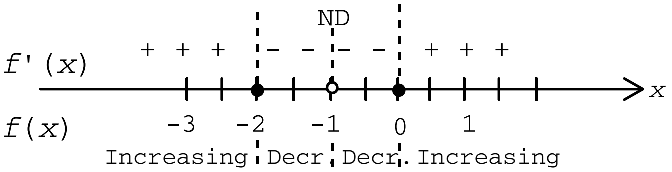
$$x^2 + 2x = 0$$

$$x(x + 2) = 0$$

$$x = 0, -2$$

Thus, the critical values are $x = -2$ and $x = 0$; -2 and 0 are also partition numbers for f'.

The sign chart for f' is:



Test Numbers

x	$f'(x)$
-3	$\frac{3}{4}(+)$
$-\frac{3}{2}$	$-3(-)$
$-\frac{1}{2}$	$-3(-)$
1	$\frac{3}{4}(+)$

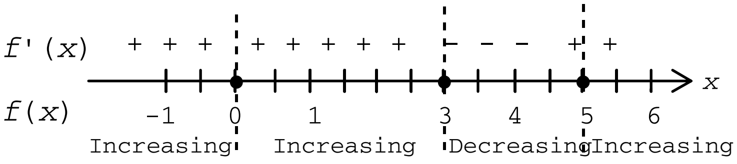
Therefore, f is increasing on $(-\infty, -2)$ and on $(0, \infty)$, f is decreasing on $(-2, -1)$ and on $(-1, 0)$; f has a local maximum at $x = -2$ and a local minimum at $x = 0$.

86. $f(x) = x^3(x - 5)^2$

$$f'(x) = 3x^2(x - 5)^2 + 2(x - 5)x^3$$
$$= x^2(x - 5)[3(x - 5) + 2x]$$
$$= x^2(x - 5)[3x - 15 + 2x] \qquad = x^2(x - 5)(5x - 15)$$
$$= 5x^2(x - 5)(x - 3)$$

Thus, the critical values of f are $x = 0$, $x = 3$, and $x = 5$.

Now we construct the sign chart for f' ($x = 0$, $x = 3$, $x = 5$ are partition numbers).



Test Numbers

x	$f'(x)$
-1	$120(+)$
1	$40(+)$
4	$-80(-)$
6	$540(+)$

Therefore, f is increasing on $(-\infty, 0)$, $(0, 3)$ and $(5, \infty)$, decreasing on $(3, 5)$; local maximum at $x = 3$; local minimum at $x = 5$.

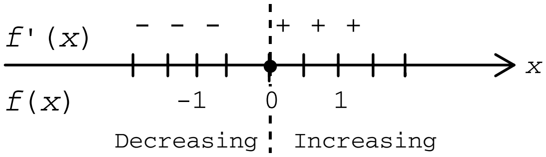
88. Let $f(x) = x^4 + kx^2$

(A) $k > 0$

$f'(x) = 4x^3 + 2kx = 2x(2x^2 + k) = 0$

$x = 0$ [<u>Note</u>: $2x^2 + k > 0$]

The only critical value is $x = 0$.



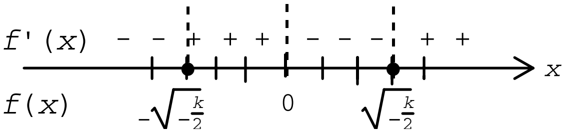
f is decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$;
local minimum at $x = 0$.

(B) $k < 0$

$f'(x) = 2x(2x^2 + k) = 0$

$x = 0, \pm\sqrt{-\frac{k}{2}}$

Critical values: $x = -\sqrt{-\frac{k}{2}}$, $x = 0$, $x = \sqrt{-\frac{k}{2}}$;



f is decreasing on $\left(-\infty, -\sqrt{-\frac{k}{2}}\right)$ and $\left(0, \sqrt{-\frac{k}{2}}\right)$; f is increasing on $\left(-\sqrt{-\frac{k}{2}}, 0\right)$ and $\left(\sqrt{-\frac{k}{2}}, \infty\right)$; f has local minima at $x = -\sqrt{-\frac{k}{2}}$ and $x = \sqrt{-\frac{k}{2}}$; local maximum at $x = 0$.

(C) $k = 0$

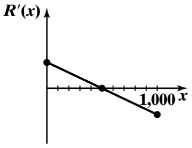
$f'(x) = 4x^3$

$f'(x) < 0$ for $x < 0$; $f'(x) = 0$ for $x = 0$; $f'(x) > 0$ for $x > 0$.

Thus, the only critical value is $x = 0$. The function is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$; local minimum at $x = 0$.

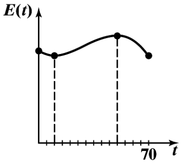
90. (A) The marginal revenue function, R', is positive on (0, 500), zero at $x = 500$, and negative on (500, 1,000).

(B)



92. (A) The price function, $E(t)$, decreases for the first 10 months to a local minimum, increases for the next 40 months to a local maximum, and then decreases for the remaining 20 months.

(B)



94. $C(x) = 0.08x^2 + 30x + 450$

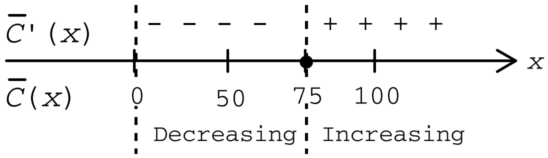
(A) $\overline{C}(x) = \dfrac{C(x)}{x} = 0.08x + 30 + \dfrac{450}{x}$

(B) Critical values:

$$\overline{C}'(x) = 0.08 - \frac{450}{x^2} = 0$$
$$0.08x^2 - 450 = 0$$
$$x^2 - 5625 = 0$$
$$(x - 75)(x + 75) = 0$$

Thus, the critical value of $\overline{C}$ on the interval (0, 200) is $x = 75$. $x = 75$ is a partition number for $\overline{C}'$.

Sign chart for $\overline{C}'$ is:



Test Numbers

x	$\overline{C}'(x)$
50	−0.1(−)
100	0.026(+)

Therefore, $\overline{C}$ is decreasing for $0 < x < 75$ and increasing for $75 < x < 200$; $\overline{C}$ has local minimum at $x = 75$.

96. $P(x) = R(x) - C(x)$

$P'(x) = R'(x) - C'(x)$

Thus, if $R'(x) < C'(x)$ on the interval (a, b),

then $P'(x) = R'(x) - C'(x) < 0$ on this interval and P is decreasing.

98. $C(t) = \dfrac{0.3t}{t^2 + 6t + 9}, \quad 0 < t < 12$

$$C'(t) = \frac{0.3(t^2 + 6t + 9) - (2t + 6)(0.3t)}{(t^2 + 6t + 9)^2}$$
$$= \frac{0.3(t + 3)^2 - 2(t + 3)(0.3t)}{(t + 3)^4}$$
$$= \frac{0.3(t + 3)[(t + 3) - 2t]}{(t + 3)^4}$$
$$= \frac{0.3(3 - t)}{(t + 3)^3}$$

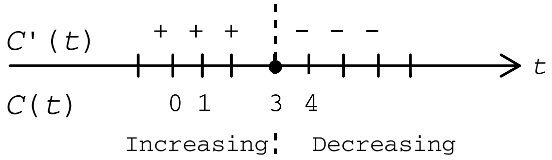
which is continuous for all t in the interval (0, 12)

$= 0$

$t = 3$

Thus, the critical value for C' on the interval (0, 12) is $t = 3$.
$t = 3$ is a partition number for C'.

Sign chart for C' is:



Test Numbers

t	$C'(t)$
1	$\frac{3}{320}(+)$
4	$\frac{-3}{3430}(-)$

Therefore, C is increasing on (0, 3) and decreasing on (3, 12); C has a local minimum at $t = 3$ ($C(3) = 0.025$).

EXERCISE 5-2

2. (A); (b, d), (e, f), (f, h)
(B); (a, b), (d, e)
(C); (a, b), (d, e)
(D); (b, d), (e, f), (f, h)
(E); (c, f) and (g, h)
(F); (a, c) and (f, g)
(G;) b, d, e
(H); b, d, e

4. $f'(x) > 0,\ f''(x) < 0$; (a)

6. $f'(x) < 0,\ f''(x) < 0$; (b)

8. $g(x) = -x^3 + 2x^2 - 3x + 9$

$g'(x) = -3x^2 + 4x - 3$

$g''(x) = -6x + 4$

10. $k(x) = -6x^{-2} + 12x^{-3}$

$k'(x) = 12x^{-3} - 36x^{-4}$

$k'(x) = -36x^{-4} + 144x^{-5}$

12. $y = x^3 - 24x^{1/3}$

$\frac{dy}{dx} = 3x^2 - 8x^{-2/3}$

$\frac{d^2y}{dx^2} = 6x + \frac{16}{3}x^{-5/3}$

14. $y = (x^2 - 16)^5$

$y' = 5(2x)(x^2 - 16)^4 = 10x(x^2 - 16)^4$

$y'' = 10(x^2 - 16)^4 + 4(2x)(x^2 - 16)^3(10x)$

$= 10(x^2 - 16)3[\quad (x^2 - 16) + 8x^2]$

$= 10(x^2 - 16)^3(9x^2 - 16)$

16. $f(x) = xe^{-x}$

$f'(x) = (x)\frac{d}{dx}(e^{-x}) + \frac{d}{dx}(x)e^{-x}$

$= x(-e^{-x}) + (1)e^{-x} = (1 - x)e^{-x}$

$f''(x) = (1 - x)\frac{d}{dx}(e^{-x}) + \frac{d}{dx}(1 - x)e^{-x}$

$= (1 - x)(-e^{-x}) + (-1)e^{-x} = (-(1 - x) - 1)e^{-x}$

$= (-1 + x - 1)e^{-x}$

$= (x - 2)e^{-x}$

18. $y = x^2 \ln x$

$$y' = (x^2)\frac{d}{dx}(\ln x) + \frac{d}{dx}(x^2)\ln x = x^2\left(\frac{1}{x}\right) + (2x)\ln x$$

$$= x + 2x \ln x$$

$$y'' = \frac{d}{dx}(x) + 2[(x)\frac{d}{dx}(\ln x) + \frac{d}{dx}(x)\ln x] = 1 + 2\left[x\left(\frac{1}{x}\right) + (1)\ln x\right]$$

$$= 1 + 2[1 + \ln x] = 1 + 2 + 2\ln x = 2\ln x + 3$$

20. $f(x) = x^3 - 24x^2$

$f'(x) = 3x^2 - 48x$

$f''(x) = 6x - 48$

Now, $f''(x) = 6x - 48 = 0$ at $x = 8$, $f''(x) < 0$ for $x < 8$ and $f''(x) > 0$ for $x > 8$, so $f(x)$ has an inflection point at $(8, f(8))$ or $(8,\ 1{,}024)$.

22. $f(x) = 5 - x^{4/3}$

$f'(x) = -\frac{4}{3}x^{1/3}$

$f''(x) = -\frac{4}{9}x^{-2/3}$

Now, $f''(x) = -\frac{4}{9}x^{-2/3} = 0$ at $x = 0$, $f''(x) < 0$ for $x < 0$ and $f''(x) < 0$ for $x > 0$, so $f(x)$ does not have an inflection point.

24. $f(x) = x^{3/5} - 6x + 7$

$f'(x) = \frac{3}{5}x^{-2/5} - 6$

$f''(x) = -\frac{6}{25}x^{-7/5}$

Now, $f''(x) = -\frac{6}{25}x^{-7/5} = 0$ at $x = 0$, $f''(x) > 0$ for $x < 0$ and $f''(x) < 0$ for $x > 0$, so $f(x)$ has an inflection point at $(0, f(0))$ or $(0, 7)$.

26. $f(x) = x^4 + 6x$

$f'(x) = 4x^3 + 6$

$f''(x) = 12x^2 \geq 0$ for all x.

Thus, f is concave upward for all x; there are no inflection points.

28. $f(x) = -x^3 - 5x^2 + 4x - 3$

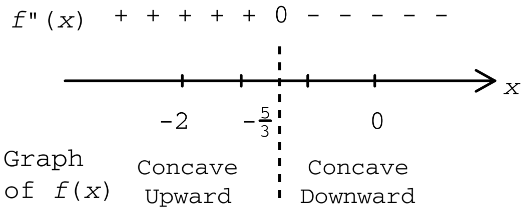
$f'(x) = -3x^2 - 10x + 4$

$f''(x) = -6x - 10 = -2(3x + 5)$

Now, $f''(x) = -2(3x + 5) = 0$

$$x = -\frac{5}{3}$$

Sign chart for f'' $\left(\text{partition number for } f''\text{ is} -\frac{5}{3}\right)$ is:



Test Numbers

x	$f''(x)$
-2	$2(+)$
0	$-10(-)$

Therefore, the graph of f is concave upward on $\left(-\infty, -\frac{5}{3}\right)$ and concave downward on $\left(-\frac{5}{3}, \infty\right)$; there is an inflection point at $x = -\frac{5}{3}$

30. $f(x) = x^4 - 2x^3 - 36x + 12$

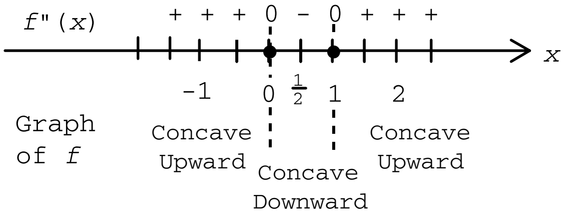
$f'(x) = 4x^3 - 6x^2 - 36$

$f''(x) = 12x^2 - 12x = 12x(x - 1)$

Now, $f''(x) = 12x(x - 1) = 0$

$$x = 0, 1$$

The sign chart for f'' (0 and 1 are partition numbers for f'') is:



Test Numbers

x	$f''(x)$
-1	$24(+)$
$\frac{1}{2}$	$-3(-)$
2	$24(+)$

Therefore, the graph of f is concave upward on $(-\infty, 0)$ and $(1, \infty)$; concave downward on $(0, 1)$; there are inflection points at $x = 0$ and
$x = 1$.

32. $f(x) = \ln(x^2 + 6x + 13)$

$$f'(x) = \frac{\frac{d}{dx}(x^2+6x+13)}{x^2+6x+13} = \frac{2x+6}{(x^2+6x+13)}$$

$$f''(x) = \frac{(2x+6)'(x^2+6x+13)-(x^2+6x+13)'(2x+6)}{(x^2+6x+13)^2}$$

$$= \frac{2(x^2+6x+13)-(2x+6)(2x+6)}{(x^2+6x+13)^2}$$

$$= \frac{2x^2+12x+26-4x^2-24x-36}{(x^2+6x+13)^2}$$

$$= \frac{-2x^2-12x-10}{(x^2+6x+13)^2} = \frac{-2(x^2+6x+5)}{(x^2+6x+13)^2}$$

$$= \frac{-2(x+1)(x+5)}{(x^2+6x+13)^2} \quad (x = \text{-5 and } x = \text{-1 are zeros of } f''(x))$$

$f''(x) < 0$ for $x < -5$ and for $x > -1$.

$f''(x) > 0$ for $-5 < x < -1$.

Summary: The graph of *f* is concave downward on $(-\infty, -5)$ and $(-1, \infty)$; it is concave upward on $(-5, -1)$. Inflection points are at $x = -5$ and $x = -1$.

34. $f(x) = e^{3x} - 9e^{x}$

$f'(x) = 3e^{3x} - 9e^{x}$

$f''(x) = 9e^{3x} - 9e^{x}$

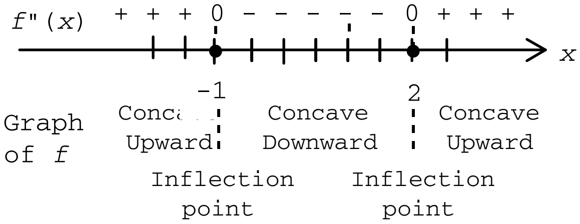
$= 9e^{x}(e^{2x} - 1)$

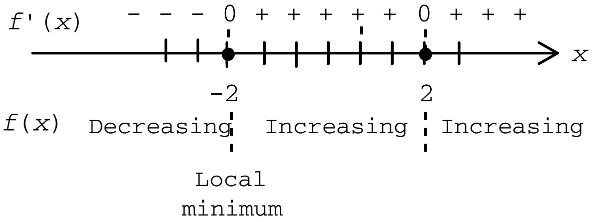
$f''(x) = 0$ for $x = 0$.

$f''(x) > 0$ for $x > 0$ and $f''(x) < 0$ for $x < 0$.

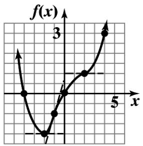
Summary: The graph of *f* is concave downward on $(-\infty, 0)$ and concave upward on $(0, \infty)$. *f* has an inflection point at $x = 0$.

36.

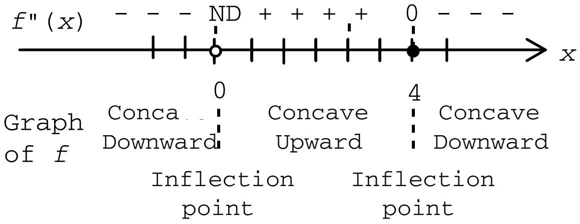


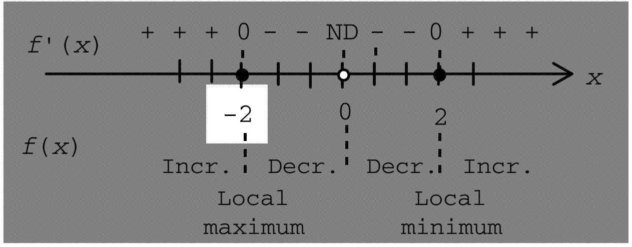


Using this information together with the points (-4, 0), (-2, -2), (-1, -1), (0, 0), (2, 1), (4, 3) on the graph, we have

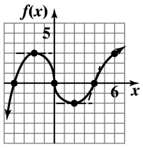


38.

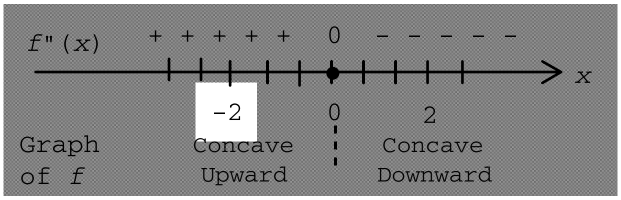


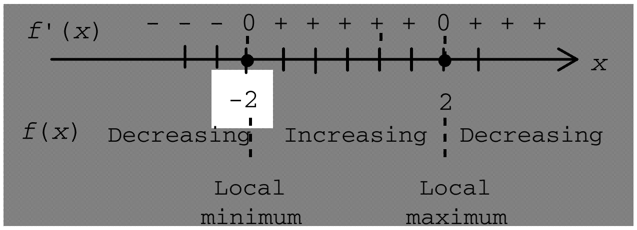


Using this information together with the points (-4, 0), (-2, 3), (0, 0), (2, -2), (4, 0), (6, 3) on the graph, we have

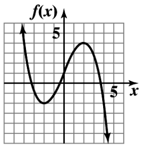


40.

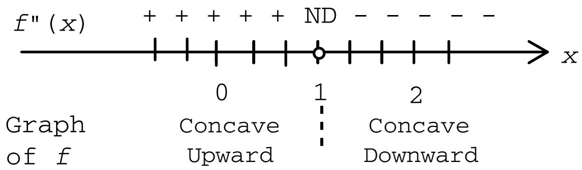


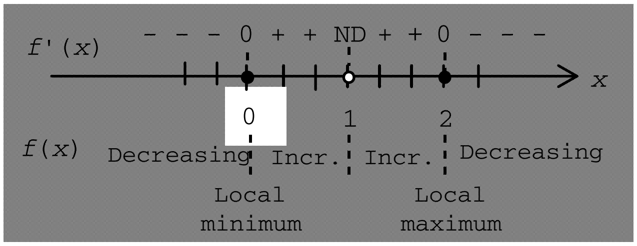


x	-2	0	2
$f(x)$	-2	1	4

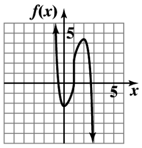


42.





x	0	1	2
$f(x)$	-2	0	4



44. $f(x) = (x - 3)(x^2 - 6x - 3)$

Domain: All real numbers

y intercept: $f(0) = (0 - 3)(0 - 0 - 3) = 9$

$f(x) = 0$ for $x = 3$, $x = 3 - 2\sqrt{3}$, $x = 3 + 2\sqrt{3}$,

so x intercepts: $3 - 2\sqrt{3}$, 3, $3 + 2\sqrt{3}$

$$f'(x) = (x^2 - 6x - 3) + (x - 3)(2x - 6)$$
$$= x^2 - 6x - 3 + 2x^2 - 12x + 18$$
$$= 3x^2 - 18x + 15 = 3(x - 1)(x - 5)$$

$f'(x) = 0$ for $x = 1$ and $x = 5$

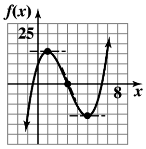
$f'(x) > 0$ on $(-\infty, 1)$ and $(5, \infty)$; $f'(x) < 0$ on $(1, 5)$.

Thus f is increasing on $(-\infty, 1)$ and $(5, \infty)$; it is decreasing on $(1, 5)$. f has a local maximum at $x = 1$ and a local minimum at $x = 5$.

$f''(x) = 6x - 18 = 6(x - 3)$ and $f''(3) = 0$.

$f''(x) < 0$ for $x < 3$ and $f''(x) > 0$ for $x > 3$.

Thus the graph of f is concave downward on $(-\infty, 3)$ and concave upward on $(3, \infty)$. f has an inflection point at $x = 3$.



46. $f(x) = (1 - x)(x^2 + x + 4)$

Domain: all real numbers

y intercept: $f(0) = 4$

$f(x) = (1 - x)(x^2 + x + 4) = 0$ only when $x = 1$, so

x intercept: 1

$f'(x) = -(x^2 + x + 4) + (1 - x)(2x + 1)$

$= -x^2 - x - 4 + 2x + 1 - 2x^2 - x = -3x^2 - 3$

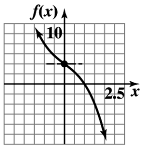
$= -3(x^2 + 1) < 0$ for all x

so f is decreasing on $(-\infty, \infty)$.

$f''(x) = -6x$ and $f''(0) = 0$

$f''(x) < 0$ for $x > 0$ and $f''(x) > 0$ for $x < 0$.

Thus, the graph of f is concave upward on $(-\infty, 0)$ and concave downward on $(0, \infty)$. f has an inflection point at $x = 0$.



48. $f(x) = 0.25x^4 - 2x^3$

$f'(x) = x^3 - 6x^2 = x^2(x - 6)$

$f''(x) = 3x^2 - 12x = 3x(x - 4)$

Critical values: $x = 0, 6$

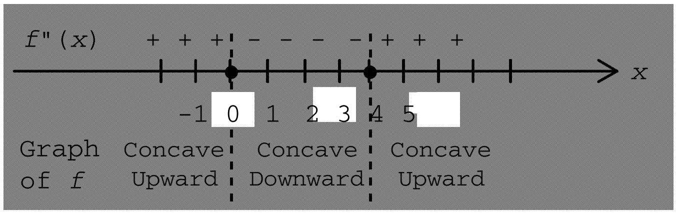
$f''(0) = 0$. Therefore, f may have an inflection point at $x = 0$.

$f''(6) = 36 > 0$. Therefore, f has a local minimum at $x = 6$.

$f''(x) = 3x(x - 4) = 0$

$x = 0, 4$ (partition numbers for f'')

The sign chart for f'' is:



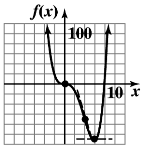
Test Numbers

x	$f''(x)$
-1	$9(+)$
1	$-3(-)$
5	$15(+)$

The graph of f has two inflection points at $x = 0$ and $x = 4$.

The graph of f is:

x	$f(x)$
-1	2.25
0	0
1	-1.75
2	-12
8	0



50. $f(x) = -4x(x + 2)^3$; Domain: All real numbers; y-intercept: 0; x-intercepts: 0, -2.

$$f'(x) = -4(x + 2)^3 - 12x(x + 2)^2 = -4(x + 2)^2[(x + 2) + 3x]$$
$$= -4(x + 2)^2(4x + 2)$$
$$= -8(x + 2)^2(2x + 1)$$

$$f''(x) = -8\{2(x + 2)(2x + 1) + 2(x + 2)^2\} = -16(x + 2)[(2x + 1) + (x + 2)]$$
$$= -16(x + 2)[3x + 3]$$
$$= -48(x + 2)(x + 1)$$

Critical values: $x = -2, -\frac{1}{2}$;

f increasing on $(-\infty, -0.5)$ and decreasing on $(-0.5, \infty)$.

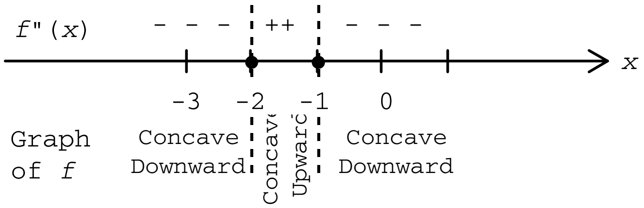
$f''(-2) = 0$. Therefore, f may have an inflection point at $x = -2$.

$f''\left(\frac{1}{2}\right) = -36 < 0$. Therefore, f has a local maximum at $x = -\frac{1}{2}$.

$f''(x) = -48(x + 2)(x + 1) = 0$

$x = -2, -1$ (partition numbers for f'')

The sign chart for f'' is:

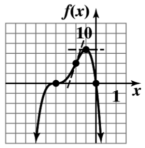


Test Numbers

x	$f''(x)$
-3	$-96(-)$
$-\frac{3}{2}$	$12(+)$
0	$-96(-)$

The graph of f has two inflection points at $x = -2$ and $x = -1$. The graph of f is:

x	$f(x)$
-3	-12
-2	0
-1	4
0	0



52. $f(x) = (x^2 + 3)(x^2 - 1)$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, $(-\infty, \infty)$.

(B) Intercepts: y-intercept: $f(0) = -3$

x-intercepts: $(x^2 + 3)(x^2 - 1) = 0$

$(x^2 + 3)(x - 1)(x + 1) = 0$

$x = -1, 1$

(C) Asymptotes: There are no asymptotes.

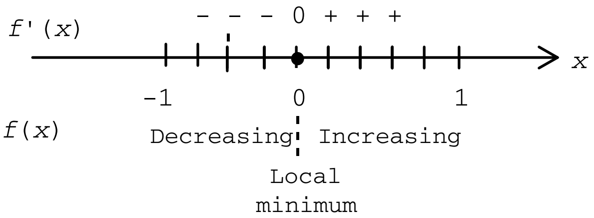
Step 2. Analyze $f'(x)$:

$$f'(x) = 2x(x^2 - 1) + 2x(x^2 + 3)$$
$$= 2x(x^2 - 1 + x^2 + 3) = 2x(2x^2 + 2) = 4x(x^2 + 1) = 0$$
$$x = 0$$

Critical values: $x = 0$

Partition numbers: $x = 0$

Sign chart for f':



Test Numbers

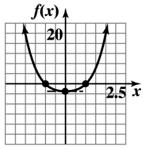
x	$f'(x)$
-1	-8 (-)
1	8 (+)

Step 3. Analyze $f''(x)$:

$f''(x) = 12x^2 + 4 > 0$ for all x. Thus, the graph of f is concave upward on $(-\infty, \infty)$.

Step 4. Sketch the graph of f:

x	$f(x)$
–1	0
0	–3
1	0



54. $f(x) = (x^2 - 1)(x^2 - 5)$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, $(-\infty, \infty)$.

(B) Intercepts: y-intercept: $f(0) = 5$

x-intercepts: $(x^2 - 1)(x^2 - 5) = 0$

$$(x - 1)(x + 1)(x - \sqrt{5})(x + \sqrt{5}) = 0$$
$$x = -\sqrt{5}, -1, 1, \sqrt{5}$$

(C) Asymptotes: There are no asymptotes.

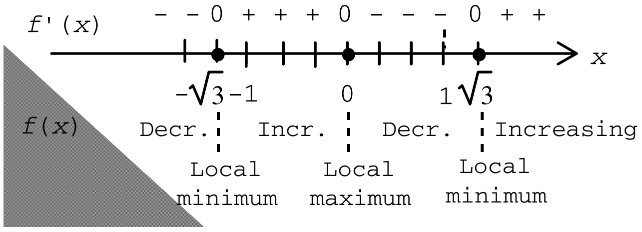
Step 2. Analyze $f'(x)$:

$$f'(x) = 2x(x^2 - 5) + 2x(x^2 - 1)$$
$$= 2x(x^2 - 5 + x^2 - 1) = 4x(x^2 - 3) = 0$$
$$x = 0, \pm\sqrt{3}$$

Critical values: $x = -\sqrt{3}, 0, \sqrt{3}$

Partition numbers: Same as critical values.

Sign chart for f':



Test Numbers

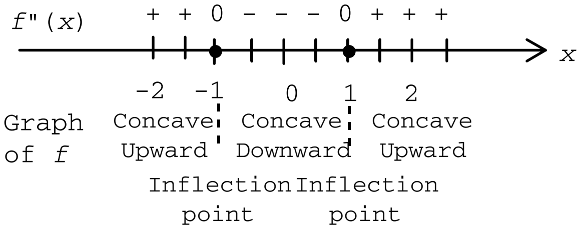
x	$f'(x)$
–2	–8(–)
–1	8(+)
1	–8(–)
2	8(+)

Step 3. Analyze $f''(x)$:

$f''(x) = 12x^2 - 12 = 12(x - 1)(x + 1)$

Partition numbers for f'' : $x = -1$, $x = 1$

Sign chart for f'' :



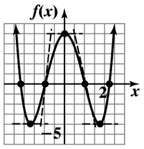
Test Numbers

x	$f''(x)$
−2	36(+)
0	−12(−)
2	36(+)

Thus, the graph of f is concave upward on (-∞, -1) and on (1, ∞); the graph of f is concave downward on (-1, 1); the graph has inflection points at $x = -1$ and $x = 1$.

Step 4. Sketch the graph of f:

x	$f(x)$
$-\sqrt{5}$	0
$-\sqrt{3}$	−4
−1	0
1	0
$\sqrt{3}$	−4
$\sqrt{5}$	0



56. $f(x) = 3x^5 - 5x^4$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, (-∞, ∞).

(B) Intercepts: y-intercept: $f(0) = 0$

x-intercepts: $3x^5 - 5x^4 = x^4(3x - 5) = 0$

$x = 0, \frac{5}{3}$

(C) Asymptotes: There are no asymptotes.

Step 2. Analyze $f'(x)$:

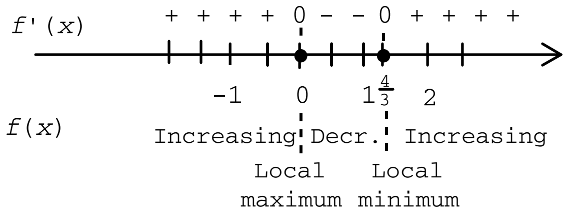
$f'(x) = 15x^4 - 20x^3 = 5x^3(3x - 4) = 0$

$x = 0, \frac{4}{3}$

Critical values: $x = 0$, $x = \frac{4}{3}$

Partition numbers: Same as critical values.

Sign chart for f':



Test Numbers

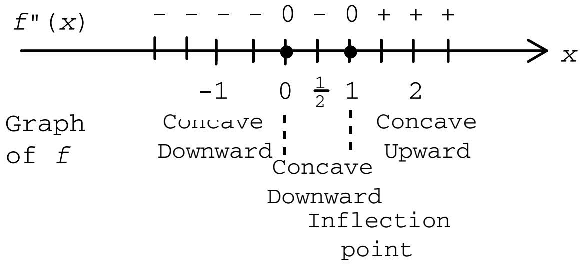
x	$f'(x)$
-1	$35(+)$
1	$-5(-)$
2	$80(+)$

Step 3. Analyze $f''(x)$:

$f''(x) = 60x^3 - 60x^2 = 60x^2(x - 1)$

Partition numbers for f'': $x = 0$, $x = 1$

Sign chart for f'':



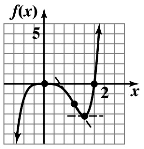
Test Numbers

x	$f''(x)$
-1	$-120(+)$
$\frac{1}{2}$	$-15(-)$
2	$60(+)$

Thus, the graph of f is concave downward on $(-\infty, 0)$ and on $(0, 1)$; concave upward on $(2, \infty)$; there is an inflection point at $x = 1$.

Step 4. Sketch the graph of f:

x	$f(x)$
0	0
1	-2
$\frac{4}{3}$	-3.16
$\frac{5}{3}$	0



58. $f(x) = 2 - 3e^{-2x}$

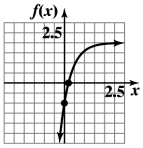
Domain: all real numbers

y intercept: $f(0) = 2 - 3 = -1$

$f(x) = 2 - 3e^{-2x} = 0$ implies

$3e^{-2x} = 2$ or $e^{-2x} = \frac{2}{3}$ or $-2x = \ln\left(\frac{2}{3}\right)$

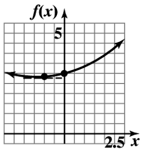
or $x = \frac{1}{2}\ln\left(\frac{3}{2}\right)$, so x intercept: $\frac{1}{2}\ln\left(\frac{3}{2}\right)$



$f'(x) = 6e^{-2x} > 0$ for all x in $(-\infty, \infty)$, thus f is increasing on $(-\infty, \infty)$.

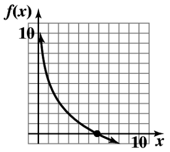
$f'(x) = -12e^{-2x} < 0$ on $(-\infty, \infty)$, therefore the graph of f is concave downward on $(-\infty, \infty)$.

60. $f(x) = 2e^{0.5x} + e^{-0.5x}$
Domain: all real numbers
y intercept: $f(0) = 2 + 1 = 3$
$f(x) > 0$ for all x, so there are no x intercepts
$f'(x) = 2(0.5)e^{0.5x} + (-0.5)e^{-0.5x}$
$= e^{0.5x} - 0.5e^{-0.5x}$



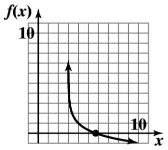
$f'(x) > 0$ if $e^{0.5x} > 0.5e^{-0.5x}$ or $e^{x} > 0.5$ or $x > \ln(0.5)$
so, $f'(x) < 0$ on $(-\infty, \ln(0.5))$ and $f'(x) > 0$ on $(\ln(0.5), \infty)$.
Thus, f is decreasing on $(-\infty,\ln(0.5))$ and increasing on $(\ln(0.5), \infty)$.
f has a local minimum at $x = \ln(0.5)$.
$f'(x) = 0.5e^{0.5x} + 0.25e^{-0.5x} > 0$ on $(-\infty, \infty)$.
Therefore, the graph of f is concave upward on $(-\infty, \infty)$.

62. $f(x) = 5 - 3 \ln x$
Domain: $(0, \infty)$
$f(x) = 5 - 3 \ln x = 0$ if $\ln x = \frac{5}{3}$ or $x = e^{5/3}$, so
x intercept: $e^{5/3}$



$f'(x) = -\frac{3}{x}$ and $f'(x) < 0$ on $(0, \infty)$, therefore f is decreasing on $(0, \infty)$.
$f'(x) = \frac{3}{x^2} > 0$ on $(0, \infty)$, thus the graph of f is concave upward on $(0, \infty)$.

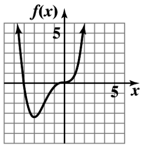
64. $f(x) = 1 - \ln(x - 3)$
since ln is defined for positive numbers $x - 3 > 0$ or $x > 3$. Thus
Domain: $(3, \infty)$
$f(x) = 1 - \ln(x - 3) = 0$ if $\ln(x - 3) = 1$ or
$x - 3 = e$ or $x = 3 + e$, so
x intercept: $3 + e$



$f'(x) = -\frac{1}{x-3} < 0$ on $(3, \infty)$, hence f is decreasing on $(3, \infty)$.
$f'(x) = \frac{1}{(x-3)^2} > 0$ on $(3, \infty)$, thus the graph of f is concave upward on $(3, \infty)$.

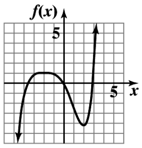
66.

x	$f'(x)$	$f(x)$
$-\infty < x < -3$	Negative and increasing	Decreasing and concave upward
$x = -3$	x-intercept	Local minimum
$-3 < x < -2$	Positive and increasing	Increasing and concave upward
$x = -2$	Local maximum	Inflection point
$-2 < x < 0$	Positive and decreasing	Increasing and concave downward
$x = 0$	Local minimum	Inflection point
$0 < x < \infty$	Positive and increasing	Increasing and concave upward



68.

x	$f'(x)$	$f(x)$
$-\infty < x < -2$	Positive and decreasing	Increasing and concave downward
$x = -2$	x-intercept	Local maximum
$-2 < x < 12$	Negative and decreasing	Decreasing and concave downward
$x = 1$	Local minimum	Inflection point
$1 < x < 2$	Negative and increasing	Decreasing and concave upward
$x = 2$	x-intercept	Local minimum
$2 < x < \infty$	Positive and increasing	Increasing and concave upward



70. $f(x) = x^4 + 2x^3 - 5x^2 - 4x + 4$

<u>Step 1. Analyze $f(x)$</u>:

(A) Domain: all real numbers

(B) Intercepts: y-intercept: $f(0) = 4$
x-intercepts: $x \approx -3.07, -1.22, 0.65, 1.64$

(C) Asymptotes: None

<u>Step 2. Analyze $f'(x)$</u>: $f'(x) = 4x^3 + 6x^2 - 10x - 4$
Critical values: $x = -2.38, -0.35, 1.22$
f is decreasing on $(-\infty, -2.38)$ and $(-0.35, 1.22)$; f is increasing on $(-2.38, -0.35)$ and $(1.22, \infty)$; f has local minima at $x = -2.38$ and 1.22; f has a local maximum at $x = -0.35$.

<u>Step 3. Analyze $f''(x)$</u>: $f''(x) = 12x^2 + 12x - 10$
The graph of f is concave upward on $(-\infty, -1.54)$ and $(0.54, \infty)$; the graph of f is concave downward on $(-1.54, 0.54)$; the graph has inflection points at $x = -1.54$ and 0.54.

72. $f(x) = x^4 - 12x^3 + 28x^2 + 76x - 50$

<u>Step 1. Analyze $f(x)$</u>:

(A) Domain: All real numbers

(B) Intercepts: y-intercept: $f(0) = -50$
x-intercepts: $x \approx -1.89, 0.57$

(C) Asymptotes: None

<u>Step 2. Analyze $f'(x)$</u>: $f'(x) = 4x^3 - 36x^2 + 56x + 76$
Critical values: $x \approx -0.85, 3.55, 6.30$

f is decreasing on $(-\infty, -0.85)$ and $(3.55, 6.30)$;
f is increasing on $(-0.85, 3.55)$ and $(6.30, \infty)$;
f has local minima at $x = -0.85$ and 6.30; f has a local maximum at $x = 3.55$.

<u>Step 3. Analyze $f''(x)$</u>: $f''(x) = 12x^2 - 72x + 56$
The graph of f is concave upward on $(-\infty, 0.92)$ and $(5.08, \infty)$;
the graph of f is concave downward on $(0.92, 5.08)$;
the graph has inflection points at $x = 0.92$ and $x = 5.08$.

74. $f(x) = -x^4 + x^3 + x^2 + 6$

<u>Step 1. Analyze $f(x)$</u>:

(A) Domain: all real numbers

(B) Intercepts: y-intercept: $f(0) = 6$
x-intercepts: $x \approx -1.49, 2.11$

(C) Asymptotes: None

Step 2. Analyze $f'(x)$: $f'(x) = -4x^3 + 3x^2 + 2x$
Critical values: $x \approx -0.43, 0, 1.18$
f is increasing on $(-\infty, -0.43)$ and $(0, 1.18)$; f is decreasing on $(-0.43, 0)$ and $(1.18, \infty)$; f has local maxima at $x = -0.43$ and 1.18; f has a local minimum at $x = 0$.

Step 3. Analyze $f''(x)$: $f''(x) = -12x^2 + 6x + 2$
The graph of f is concave downward on $(-\infty, -0.23)$ and $(0.73, \infty)$; the graph of f is concave upward on $(-0.23, 0.73)$; the graph has inflection points at $x = -0.23$ and $x = 0.73$.

76. $f(x) = x^5 + 4x^4 - 7x^3 - 20x^2 + 20x - 20$

Step 1. Analyze $f(x)$:

(A) Domain: all real numbers

(B) Intercepts: y-intercept: $f(0) = -20$
x-intercepts: $x \approx -4.20, -2.87, 2.22$

(C) Asymptotes: None

Step 2. Analyze $f'(x)$:
$f'(x) = 5x^4 + 16x^3 - 21x^2 - 40x + 20$
Critical values: $x \approx -3.67, -1.56, 0.44, 1.59$
f is increasing on $(-\infty, -3.67)$, $(-1.56, 0.44)$, and $(1.59, \infty)$; f is decreasing on $(-3.67, -1.56)$ and $(0.44, 1.59)$; f has local maxima at $x = -3.67$ and 0.44; f has local minima at $x = -1.56$ and 1.59.

Step 3. Analyze $f''(x)$:
The graph of f is concave downward on $(-\infty, -2.89)$ and $(-0.62, 1.11)$; the graph of f is concave upward on $(-2.89, -0.62)$ and $(1.11, \infty)$; the graph has inflection points at $x = -2.89$, -0.61 and 1.11.

78. If the graph of $y = f(x)$ is concave up on an interval, then $f'(x)$ is increasing on that interval. And if the graph of $y = f(x)$ is concave down on an interval, then $f'(x)$ is decreasing on that interval.

80. Let $x = c$ be an *x-intercept* for the graph of $y = f'(x)$. If $f'(x)$ is decreasing on an open interval containing c, then $f(x)$ has a local maximum at $x = c$. If $f'(x)$ is increasing on an open interval containing c, then $f(x)$ has a local minimum at $x = c$.

82. The graph of the PPI is concave downward.

84. The graph of $C'(x)$ is positive and increasing. Since the marginal costs are increasing, the production process is becoming less efficient as production increases. Initially, marginal costs are higher at Plant A, but as production levels increase, marginal costs at Plant B eventually exceed those at Plant A.

86. $P(x) = R(x) - C(x)$

$= 1{,}296x - 0.12x^3 - (830 + 396x)$

$= 1{,}296x - 0.12x^3 - 830 - 396x$

$= 900x - 0.12x^3 - 830$

$P'(x) = 900 - 0.36x^2 \qquad = 0.36(2{,}500 - x^2)$

$= 0.36(50 - x)(50 + x)$

$P''(x) = -0.72x$

Critical values: $P'(x) = 0$, $x = \pm 50$
Thus, $x = 50$ is the only critical value in the interval (0, 80).
$P''(50) = -0.72(50) = -36 < 0$

(A) P has a local maximum at $x = 50$.
(B) Since $P''(x) = -0.72x < 0$ for $0 < x < 80$, P is concave downward on the whole interval (0, 80).

88. $p = 8 - 2 \ln x$, $5 \le x \le 50$

The revenue function is:

$R(x) = xp(x) = x(8 - 2 \ln x) = 8x - 2x \ln x$, $5 \le x \le 50$.

$$R'(x) = 8 - 2\left[\ln x + x\left(\frac{1}{x}\right)\right]$$

$= 8 - 2[\ln x + 1] = 8 - 2 \ln x - 2 = 6 - 2 \ln x$

$R'(x) = 0$ if $6 - 2 \ln x = 0$ or $\ln x = 3$ or $x = e^3$.

$R'(x) < 0$ if $x > e^3$ and $R'(x) > 0$ if $x < e^3$, thus R is increasing on $(5, e^3)$ and decreasing on $(e^3, 50]$. Thus:

(A) R has a local maximum at $x = e^3$.

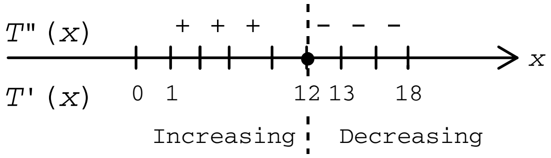
(B) $R''(x) = -\dfrac{2}{x} < 0$ on (5, 50), therefore the graph of R is concave downward on (5, 50).

90. $T(x) = -0.25x^4 + 6x^3$, $0 \le x \le 18$

$T'(x) = -x^3 + 18x^2 = -x^2(x - 18)$

$T''(x) = -3x^2 + 36x = -3x(x - 12)$

The sign chart for T'' (partition number is 12):

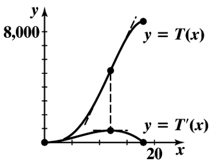


Test Numbers

x	$T''(x)$
1	33(+)
13	−39(−)

Thus, T is increasing on (0, 12) and decreasing on (12, 18).
$T''(x) = -3x(x - 12)$.

The point of diminishing returns is $x = 12$ and the maximum rate of change is $T'(12) = 864$. Note that $T'(x)$ has a local maximum and $T(x)$ has an inflection point at $x = 12$.

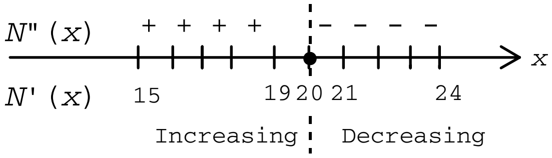


92. $N(x) = -0.25x^4 + 13x^3 - 180x^2 + 10{,}000,\ 15 \le x \le 24$

$N'(x) = -x^3 + 39x^2 - 360x$

$N''(x) = -3x^2 + 78x - 360 = -3(x^2 - 26x + 120) = -3(x - 6)(x - 20)$

The sign chart for N'' (partition number is 20):



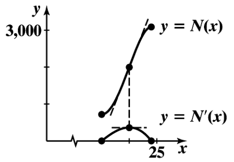
Test Numbers

x	$N''(x)$
19	39(+)
21	−45(−)

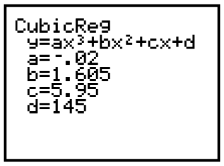
Thus, N' is increasing on (15, 20) and decreasing on (20, 24).

$N''(x) = -3(x - 6)(x - 20)$.

The point of diminishing returns is $x = 20$ and the maximum rate of change is $N'(20) = 400$. Note that $N'(x)$ has a local maximum and $N(x)$ has an inflection point at $x = 20$.



94. (A)



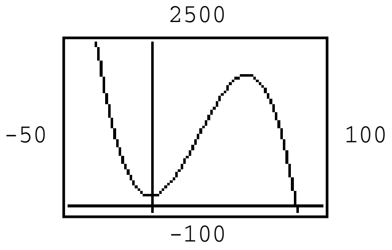
```
CubicReg
 y=ax³+bx²+cx+d
 a=-.02
 b=1.605
 c=5.95
 d=145
```

(B) From part (A),

$y(x) = -0.02x^3 + 1.605x^2 + 5.95x + 145$

so, $y'(x) = -0.06x^2 + 3.210x + 5.95$

The graph of $y'(x)$ is shown at the right and the maximum value of y' occurs at $x \approx 27$; and $y(27) \approx 1{,}082$.



The manager should place 27 ads each month to maximize the rate of change of sales; the manager can expect to sell 1,082 golf clubs.

96. $T(x) = x^2\left(1 - \frac{x}{9}\right),\ 0 \le x \le 6$

$$T'(x) = 2x\left(1 - \frac{x}{9}\right) + x^2\left(-\frac{1}{9}\right) = 2x - \frac{2x^2}{9} - \frac{x^2}{9} = 2x - \frac{1}{3}x^2$$

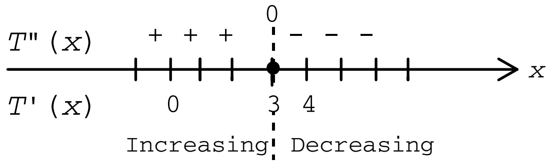
$$= x\left(2 - \frac{x}{3}\right)$$

$$= \frac{1}{3}x(6 - x)$$

$$T''(x) = 2 - \frac{2}{3}x = 0$$

$$x = 3$$

(A) The sign chart for T'' (partition number is 3) is:



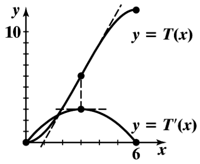
Test Numbers

x	$T''(x)$
0	+
4	–

Thus, T is increasing on (0, 3) and decreasing on (3, 6).

(B) From the results in (A), the graph of T has an inflection point at $x = 3$.

(C)



(D) Using the result in (A), T has a local maximum at $x = 3$:

$$T(3) = \frac{1}{3}(3)(6 - 3) = 3$$

EXERCISE 5-3

2. $\lim\limits_{x\to1} \dfrac{x^6-1}{x^5-1}$

Step 1: Check to see if L'Hôpital's rule applies.

$\lim\limits_{x\to1}(x^6 - 1) = 1 - 1 = 0$ and $\lim\limits_{x\to1}(x^5 - 1) = 1 - 1 = 0$.

Thus, L'Hôpital's rule applies.

Step 2: Apply L'Hôpital's rule.

$$\lim_{x\to1}\frac{D_x(x^6-1)}{D_x(x^5-1)} = \lim_{x\to1}\frac{6x^5}{5x^4} = \frac{6\cdot1^5}{5\cdot1^4} = \frac{6}{5}.$$

Therefore, $\lim\limits_{x\to1}\dfrac{x^6-1}{x^5-1} = \dfrac{6}{5}$.

4. $\lim\limits_{x\to4} \dfrac{x^2-8x+16}{x^2-5x+4}$

Step 1. Check to see if L'Hôpital's rule applies.

$\lim\limits_{x\to4}(x^2 - 8x + 16) = 4^2 - 8\cdot4 + 16 = 16 - 32 + 16 = 0$ and

$\lim\limits_{x\to4}(x^2 - 5x + 4) = 4^2 - 5\cdot4 + 4 = 16 - 20 + 4 = 0$

Thus, L'Hôpital's rule applies.

Step 2. Apply L'Hôpital's rule.

$$\lim_{x\to4}\frac{D_x(x^2-8x+16)}{D_x(x^2-5x+4)} = \lim_{x\to4}\frac{2x-8}{2x-5} = \frac{2(4)-8}{2(4)-5} = \frac{0}{3} = 0$$

Therefore, $\lim\limits_{x\to4}\dfrac{x^2-8x+16}{x^2-5x+4} = 0$

Note: This result could have been obtained algebraically.

$$\lim_{x\to 4}\frac{x^2-8x+16}{x^2-5x+4} = \lim_{x\to 4}\frac{(x-4)^2}{(x-4)(x-1)}$$

$$= \lim_{x\to 4}\frac{x-4}{x-1} = \frac{4-4}{4-1} = \frac{0}{3} = 0.$$

6. $\lim\limits_{x\to 1}\dfrac{\ln x}{x-1}$

<u>Step 1</u>. Check to see L'Hôpital's rule applies.

$\lim\limits_{x\to 1}\ln x = \ln 1 = 0$ and $\lim\limits_{x\to 1}(x - 1) = 1 - 1 = 0$.

Thus, L'Hôpital's rule applies.

<u>Step 2</u>. Apply L'Hôpital's rule.

$$\lim_{x\to 1}\frac{D_x(\ln x)}{D_x(x-1)} = \lim_{x\to 1}\frac{\frac{1}{x}}{1} = \lim_{x\to 1}\frac{1}{x} = \frac{1}{1} = 1.$$

Therefore, $\lim\limits_{x\to 1}\dfrac{\ln x}{x-1} = 1$.

8. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x}$

<u>Step 1</u>. Check to see if L'Hôpital's rule applies.

$\lim\limits_{x\to 0}(e^{2x} - 1) = e^{2(0)} - 1 = 1 - 1 = 0$ and $\lim\limits_{x\to 0} x = 0$.

Thus, L'Hôpital's rule applies.

<u>Step 2</u>. Apply L'Hôpital's rule.

$$\lim_{x\to 0}\frac{D_x(e^{2x}-1)}{D_x(x)} = \lim_{x\to 0}\frac{2e^{2x}}{1} = 2e^{2(0)} = 2e^{0} = 2.$$

Therefore, $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x} = 2$.

10. $\lim\limits_{x\to\infty}\dfrac{3x^4+6}{2x^2+5}$

<u>Step 1</u>. Check to see if L'Hôpital's rule applies.

$\lim\limits_{x\to\infty}(3x^4 + 6) = \infty$ and $\lim\limits_{x\to\infty}(2x^2 + 5) = \infty$

Thus, L'Hôpital's rule applies.

<u>Step 2</u>. Apply L'Hôpital's rule.

$$\lim_{x\to\infty}\frac{D_x(3x^4+6)}{D_x(2x^2+5)} = \lim_{x\to\infty}\frac{12x^3}{4x} = \lim_{x\to\infty}3x^2 = \infty.$$

Therefore, $\lim\limits_{x\to\infty}\dfrac{3x^4+6}{2x^2+5} = \infty$.

12. $\lim\limits_{x\to\infty}\dfrac{x}{e^{4x}}$

<u>Step 1</u>. Check to see if L'Hôpital's rule applies.

$\lim\limits_{x\to\infty} x = \infty$ and $\lim\limits_{x\to\infty} e^{4x} = \infty$. Thus, L'Hôpital's rule applies.

Step 2. Apply L'Hôpital's rule.

$$\lim_{x\to\infty} \frac{D_x(x)}{D_x(e^{4x})} = \lim_{x\to\infty} \frac{1}{4e^x} = 0.$$

Therefore, $\lim_{x\to\infty} \frac{x}{e^{4x}} = 0.$

14. $\lim_{x\to\infty} \frac{\ln x}{x^4}$

Step 1. Check to see if L'Hôpital's rule applies.

$\lim_{x\to\infty} \ln x = \infty$ and $\lim_{x\to\infty} x^4 = \infty$. Thus, L'Hôpital's rule applies.

Step 2. Apply L'Hôpital's rule.

$$\lim_{x\to\infty} \frac{D_x(\ln x)}{D_x(x^4)} = \lim_{x\to\infty} \frac{\frac{1}{x}}{4x^3} = \lim_{x\to\infty} \frac{1}{4x^4} = 0.$$

Therefore, $\lim_{x\to\infty} \frac{\ln x}{x^4} = 0.$

16. $\lim_{x\to\infty} \frac{e^{-x}}{\ln x}$; $\lim_{x\to\infty} e^{-x} = 0$ and $\lim_{x\to\infty} \ln x = \infty$.

Therefore, L'Hôpital's rule does not apply.

However, if we write $\frac{e^{-x}}{\ln x}$ as $\frac{1}{e^x \ln x}$, then as $x \to \infty$, $e^x \ln x \to \infty$ and hence $\lim_{x\to\infty} \frac{e^{-x}}{\ln x} = 0.$

18. $\lim_{x\to -3} \frac{x^2}{(x+3)^2}$; $\lim_{x\to -3} x^2 = (-3)^2 = 9$ and $\lim_{x\to -3} (x + 3)^2 = (-3 + 3)^2 = 0$. Therefore, L'Hôpital's rule does not apply.

As $x \to -3$, $x^2 \to 9$ and $(x + 3)^2 \to 0$, therefore, $\lim_{x\to -3} \frac{x^2}{(x+3)^2}$ does not exist.

20. L'Hôpital's rule: $\lim_{x\to 5} \frac{x-5}{x^2-25} = \lim_{x\to 5} \frac{\frac{d}{dx}(x-5)}{\frac{d}{dx}(x^2-25)} = \lim_{x\to 5} \frac{1}{2x} = \frac{1}{2(5)} = \frac{1}{10}$

Algebraic simplification: $\lim_{x\to 5} \frac{x-5}{x^2-25} = \lim_{x\to 5} \frac{x-5}{(x-5)(x+5)} = \lim_{x\to 5} \frac{1}{x+5} = \frac{1}{(5)+5} = \frac{1}{10}$

22. L'Hôpital's rule: $\lim_{x\to 7} \frac{x-7}{x^2+6x-91} = \lim_{x\to 7} \frac{\frac{d}{dx}(x-7)}{\frac{d}{dx}(x^2+6x-91)} = \lim_{x\to 7} \frac{1}{2x+6} = \frac{1}{2(7)+6} = \frac{1}{20}$

Algebraic simplification: $\lim_{x\to 7} \frac{x-7}{x^2+6x-91} = \lim_{x\to 7} \frac{x-7}{(x+13)(x-7)} = \lim_{x\to 7} \frac{1}{x+13} = \frac{1}{(7)+13} = \frac{1}{20}$

24. $\lim_{x\to 0} \frac{3x+1-e^{3x}}{x^2}$

Step 1. $\lim_{x\to 0} (3x + 1 - e^{3x}) = 3(0) + 1 - e^{3(0)} = 1 - 1 = 0$ and $\lim_{x\to 0} x^2 = 0$.

Thus, L'Hôpital's rule applies.

Step 2. $\lim\limits_{x\to 0} \dfrac{D_x(3x+1-e^{3x})}{D_x(x^2)} = \lim\limits_{x\to 0} \dfrac{3-3e^{3x}}{2x}$.

Since $\lim\limits_{x\to 0}(3 - 3e^{3x}) = 3 - 3e^{3(0)} = 3 - 3 = 0$ and $\lim\limits_{x\to 0}(2x) = 0$,

$\lim\limits_{x\to 0} \dfrac{3-3e^{3x}}{2x}$ is a 0/0 indeterminate form.

Step 3. Apply L'Hôpital's rule again.

$$\lim_{x\to 0} \frac{D_x(3-3e^{3x})}{D_x(2x)} = \lim_{x\to 0} \frac{-9e^{3x}}{2} = \frac{-9e^{3(0)}}{2} = -\frac{9}{2} = -4.5.$$

Therefore, $\lim\limits_{x\to 0} \dfrac{3x+1-e^{3x}}{x^2} = \lim\limits_{x\to 0} \dfrac{3-3e^{3x}}{2x} = \lim\limits_{x\to 0}\left(-\dfrac{9e^{3x}}{2}\right) = -\dfrac{9}{2}$.

26. $\lim\limits_{x\to -1} \dfrac{\ln(x+2)}{x+2}$

Since $\lim\limits_{x\to -1} \ln(x + 2) = \ln(-1 + 2) = \ln(1) = 0$ and

$\lim\limits_{x\to -1}(x + 2) = (-1 + 2) = 1$. Thus, L'Hôpital's rule does not apply. However, using rule for the limit of a quotient:

$$\lim_{x\to -1} \frac{\ln(x+2)}{x+2} = \frac{\lim\limits_{x\to -1} \ln(x+2)}{\lim\limits_{x\to -1}(x+2)} = \frac{0}{1} = 0.$$

28. $\lim\limits_{x\to 0^-} \dfrac{\ln(1+2x)}{x^2}$

Step 1. $\lim\limits_{x\to 0^-}(1 + 2x) = \ln 1 = 0$ and $\lim\limits_{x\to 0^-} x^2 = 0$. Thus, L'Hôpital's rule applies.

Step 2. $\lim\limits_{x\to 0^-} \dfrac{D_x(\ln(1+2x))}{D_x(x^2)} = \lim\limits_{x\to 0^-} \dfrac{\frac{2}{1+2x}}{2x}$

Therefore, $\lim\limits_{x\to -1} \dfrac{\ln(x+2)}{x+2} = \lim\limits_{x\to 0^-} \dfrac{1}{x(1+2x)} = -\infty$

30. $\lim\limits_{x\to 0^+} \dfrac{\ln(1+x)}{\sqrt{x}}$

Step 1. $\lim\limits_{x\to 0^+} \ln(1 + x) = \ln 1 = 0$ and $\lim\limits_{x\to 0^+} \sqrt{x} = 0$.

Thus, L'Hôpital's rule applies.

Step 2.
$$\lim_{x\to 0^+} \frac{D_x(\ln(1+x))}{D_x(\sqrt{x})} = \lim_{x\to 0^+} \frac{\frac{1}{1+x}}{\frac{1}{2\sqrt{x}}}$$
$$= \lim_{x\to 0^+} \frac{2\sqrt{x}}{1+x} = \frac{0}{1} = 0$$

Therefore, $\lim\limits_{x\to 0^+} \dfrac{\ln(1+x)}{\sqrt{x}} = 0$.

32. $\lim\limits_{x\to 1} \dfrac{2x^3 - 3x^2 + 1}{x^3 - 3x + 2}$

Step 1. $\lim\limits_{x\to 1} (2x^3 - 3x^2 + 1) = 2(1)^3 - 3(1)^2 + 1 = 0$ and

$\lim\limits_{x\to 1} (x^3 - 3x + 2) = (1)^3 - 3(1) + 2 = 0.$

Thus, L'Hôpital's rule applies.

Step 2. $\lim\limits_{x\to 1} \dfrac{D_x(2x^3 - 3x^2 + 1)}{D_x(x^3 - 3x + 2)} = \lim\limits_{x\to 1} \dfrac{6x^2 - 6x}{3x^2 - 3} = \dfrac{0}{0}.$

Step 3. Use L'Hôpital's rule again.

$\lim\limits_{x\to 1} \dfrac{D_x(6x^2 - 6x)}{D_x(3x^2 - 3)} = \lim\limits_{x\to 1} \dfrac{12x - 6}{6x} = \dfrac{6}{6} = 1.$

Therefore, $\lim\limits_{x\to 1} \dfrac{2x^3 - 3x^2 + 1}{x^3 - 3x + 2} = \lim\limits_{x\to 1} \dfrac{6x^2 - 6x}{3x^2 - 3} = \lim\limits_{x\to 1} \dfrac{12x - 6}{6x} = 1.$

34. $\lim\limits_{x\to 3} \dfrac{x^3 + 3x^2 - x - 3}{x^2 + 6x + 9}$

$\lim\limits_{x\to 3} (x^3 + 3x^2 - x - 3) = 3^3 + 3(3)^2 - 3 - 3 = 48$ and

$\lim\limits_{x\to 3} (x^2 + 6x + 9) = 3^2 + 6(3) + 9 = 36.$

Using quotient rule we have $\lim\limits_{x\to 3} \dfrac{x^3 + 3x^2 - x - 3}{x^2 + 6x + 9} = \dfrac{48}{36} = \dfrac{4}{3}.$

36. $\lim\limits_{x\to 1^+} \dfrac{x^3 + x^2 - x + 1}{x^3 + 3x^2 + 3x - 1}$

$\lim\limits_{x\to 1^+} (x^3 + x^2 - x + 1) = 1^3 + 1^2 - 1 + 1 = 2$ and

$\lim\limits_{x\to 1^+} (x^3 + 3x^2 + 3x - 1) = 1^3 + 3 \cdot 1^2 + 3 \cdot 1 - 1 = 6.$

Using quotient rule we have $\lim\limits_{x\to 1^+} \dfrac{x^3 + x^2 - x + 1}{x^3 + 3x^2 + 3x - 1} = \dfrac{2}{6} = \dfrac{1}{3}$

38. $\lim\limits_{x\to \infty} \dfrac{4x^2 + 9x}{5x^2 + 8}$

Step 1. $\lim\limits_{x\to \infty} (4x^2 + 9x) = \infty$ and $\lim\limits_{x\to \infty} (5x^2 + 8) = \infty.$

Thus, L'Hôpital's rule applies.

Step 2. $\lim_{x\to\infty} \frac{D_x(4x^2+9x)}{D_x(5x^2+8)} = \lim_{x\to\infty} \frac{8x+9}{10x} = \frac{\infty}{\infty}$.

Step 3. Apply L'Hôpital's rule again.

$\lim_{x\to\infty} \frac{D_x(8x+9)}{D_x(10x)} = \lim_{x\to\infty} \frac{8}{10} = \frac{4}{5}$.

Therefore, $\lim_{x\to\infty} \frac{4x^2+9x}{5x^2+8} = \lim_{x\to\infty} \frac{8x+9}{10x} = \lim_{x\to\infty} \frac{8}{10} = \frac{4}{5}$.

40. $\lim_{x\to\infty} \frac{e^{3x}}{x^3}$

Step 1. $\lim_{x\to\infty} e^{3x} = \infty$ and $\lim_{x\to\infty} x^3 = \infty$. Thus, L'Hôpital's rule applies.

Step 2. $\lim_{x\to\infty} \frac{D_x(e^{3x})}{D_x(x^3)} = \lim_{x\to\infty} \frac{3e^{3x}}{3x^2} = \lim_{x\to\infty} \frac{e^{3x}}{x^2} = \frac{\infty}{\infty}$.

Step 3. Apply L'Hôpital's rule again.

$\lim_{x\to\infty} \frac{D_x(e^{3x})}{D_x(x^2)} = \lim_{x\to\infty} \frac{3e^{3x}}{2x} = \frac{\infty}{\infty}$.

Step 4. Apply L'Hôpital's rule again.

$\lim_{x\to\infty} \frac{D_x(3e^{3x})}{D_x(2x)} = \lim_{x\to\infty} \frac{9e^{3x}}{2} = \infty$.

Therefore, $\lim_{x\to\infty} \frac{e^{3x}}{x^3} = \lim_{x\to\infty} \frac{e^{3x}}{x^2} = \lim_{x\to\infty} \frac{3e^{3x}}{2x} = \lim_{x\to\infty} \frac{9e^{3x}}{2} = \infty$.

42. $\lim_{x\to-\infty} \frac{1+e^{-x}}{1+x^2}$

Step 1. $\lim_{x\to-\infty} (1+e^{-x}) = \infty$ and $\lim_{x\to-\infty} (1+x^2) = \infty$.

Thus, L'Hôpital's rule applies.

Step 2. $\lim_{x\to-\infty} \frac{D_x(1+e^{-x})}{D_x(1+x^2)} = \lim_{x\to-\infty} \frac{-e^{-x}}{2x} = \frac{-\infty}{-\infty}$.

Step 3. Apply L'Hôpital's rule again.

$\lim_{x\to-\infty} \frac{D_x(-e^{-x})}{D_x(2x)} = \lim_{x\to-\infty} \frac{e^{-x}}{2} = \infty$

Therefore, $\lim_{x\to-\infty} \frac{1+e^{-x}}{1+x^2} = \lim_{x\to-\infty} \frac{-e^{-x}}{2x} = \lim_{x\to-\infty} \frac{e^{-x}}{2} = \infty$.

44. $\lim_{x\to\infty} \frac{\ln(1+2e^{-x})}{\ln(1+e^{-x})}$

Step 1. $\lim_{x\to\infty} \ln(1+2e^{-x}) = \ln 1 = 0$ and $\lim_{x\to\infty} \ln(1+e^{-x}) = \ln 1 = 0$.

Thus, L'Hôpital's rule applies.

Step 2. $$\lim_{x\to\infty} \frac{D_x(\ln(1+2e^{-x}))}{D_x(\ln(1+e^{-x}))} = \lim_{x\to\infty} \frac{\frac{-2e^{-x}}{1+2e^{-x}}}{\frac{-e^{-x}}{1+e^{-x}}} = \lim_{x\to\infty} \frac{2(1+e^{-x})}{1+2e^{-x}} = 2.$$

Therefore, $\lim_{x\to\infty} \frac{\ln(1+2e^{-x})}{\ln(1+e^{-x})} = 2.$

46. $\lim_{x\to 0} \frac{e^{2x}-1-2x-2x^2}{x^3}$

Step 1. $\lim_{x\to 0} (e^{2x} - 1 - 2x - 2x^2) = e^{2(0)} - 1 - 2(0) - 2(0)^2 = 0$ and $\lim_{x\to 0} x^3 = 0$. Thus, L'Hôpital's rule applies.

Step 2. $$\lim_{x\to 0} \frac{D_x(e^{2x}-1-2x-2x^2)}{D_x(x^3)} = \lim_{x\to 0} \frac{2e^{2x}-2-4x}{3x^2} = \frac{0}{0}.$$

Step 3. Apply L'Hôpital's rule again.

$$\lim_{x\to 0} \frac{D_x(2e^{2x}-2-4x)}{D_x(3x^2)} = \lim_{x\to 0} \frac{4e^{2x}-4}{6x} = \frac{0}{0}.$$

Step 4. Apply L'Hôpital's rule again.

$$\lim_{x\to 0} \frac{D_x(4e^{2x}-4)}{D_x(6x)} = \lim_{x\to 0} \frac{8e^{2x}}{6} = \frac{8}{6} = \frac{4}{3}$$

Therefore, $$\lim_{x\to 0} \frac{e^{2x}-1-2x-2x^2}{x^3} = \lim_{x\to 0} \frac{2e^{2x}-2-4x}{3x^2}$$
$$= \lim_{x\to 0} \frac{4e^{2x}-4}{6x} = \lim_{x\to 0} \frac{8e^{2x}}{6} = \frac{8}{6} = \frac{4}{3}.$$

48. $\lim_{x\to 0^+} (\sqrt{x}\ \ln x) = \lim_{x\to 0^+} \frac{\ln x}{x^{-1/2}}$

Step 1. $\lim_{x\to 0^+} (\ln x) = -\infty$ and $\lim_{x\to 0^+} x^{-1/2} = \lim_{x\to 0^+} \frac{1}{\sqrt{x}} = \infty.$

Thus, L'Hôpital's rule applies.

Step 2. $$\lim_{x\to 0^+} \frac{D_x(\ln x)}{D_x(x^{-1/2})} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}} = \lim_{x\to 0^+} (-2x^{1/2}) = 0$$

Therefore, $\lim_{x\to 0^+} (\sqrt{x}\ \ln x) = 0.$

50. $\lim_{x\to\infty} \frac{x^n}{\ln x}$

Step 1. $\lim_{x\to\infty} x^n = \infty$ and $\lim_{x\to\infty} \ln x = \infty$. Thus, L'Hôpital's rule applies.

Step 2. $\lim_{x\to\infty} \frac{D_x(x^n)}{D_x(\ln x)} = \lim_{x\to\infty} \frac{nx^{n-1}}{\frac{1}{x}} = n \lim_{x\to\infty} x^n = \infty$

Therefore, $\lim_{x\to\infty} \frac{x^n}{\ln x} = \infty$.

52. $\lim_{x\to\infty} \frac{x^n}{e^x}$

Step 1. $\lim_{x\to\infty} x^n = \infty$ and $\lim_{x\to\infty} e^x = \infty$. Thus, L'Hôpital's rule applies.

Step 2. $\lim_{x\to\infty} \frac{D_x(x^n)}{D_x(e^x)} = \lim_{x\to\infty} \frac{nx^{n-1}}{e^x} = \frac{\infty}{\infty}$.

Now applying L'Hôpital's rule n - 1 more time, we have:

$$\lim_{x\to\infty} \frac{x^n}{e^x} = \lim_{x\to\infty} \frac{nx^{n-1}}{e^x} = \lim_{x\to\infty} \frac{n(n-1)x^{n-2}}{e^x} = \ldots = \lim_{x\to\infty} \frac{n!}{e^x} = 0.$$

54. $\lim_{x\to-\infty} \frac{x}{\sqrt{4+x^2}}$

Observe that $\lim_{x\to-\infty} \frac{x}{\sqrt{4+x^2}} = \frac{-\infty}{\infty}$, thus using L'Hôpital's rule we have:

$$\lim_{x\to-\infty} \frac{x}{\sqrt{4+x^2}} = \lim_{x\to-\infty} \frac{1}{x(4+x^2)^{-1/2}} = \lim_{x\to-\infty} \frac{\sqrt{4+x^2}}{x} = \frac{\infty}{-\infty}$$

Again, using L'Hôpital's rule we arrive at

$$\lim_{x\to-\infty} \frac{x}{\sqrt{4+x^2}} = \lim_{x\to-\infty} \frac{\sqrt{4+x^2}}{x} = \lim_{x\to-\infty} \frac{x(4+x^2)^{-1/2}}{1} = \lim_{x\to-\infty} \frac{x}{\sqrt{4+x^2}},$$

which is what we started with. So repeated application of L'Hôpital's rule will not provide us with an answer.

Now we use algebraic manipulation to evaluate the limit. Divide numerator and denominator by (-x). For x < 0:

$$\lim_{x\to-\infty} \frac{x}{\sqrt{4+x^2}} = \lim_{x\to-\infty} \frac{-1}{\frac{1}{(-x)}\sqrt{4+x^2}} = \lim_{x\to-\infty} \frac{-1}{\frac{1}{|x|}\sqrt{4+x^2}}$$

$$= -\lim_{x\to-\infty} \frac{1}{\sqrt{\frac{4}{x^2}+1}} = -\frac{1}{\sqrt{0+1}} = -\frac{1}{1} = -1$$

56. $\lim\limits_{x\to\infty} \dfrac{x^2}{\sqrt[3]{(x^3+1)^2}}$

Observe that $\lim\limits_{x\to\infty} \dfrac{x^2}{\sqrt[3]{(x^3+1)^2}} = \dfrac{\infty}{\infty}$, thus using L'Hôpital's rule we have:

$$\lim_{x\to\infty} \frac{x^2}{\sqrt[3]{(x^3+1)^2}} = \lim_{x\to\infty} \frac{2x}{\frac{2}{3}(3x^2)(x^3+1)^{-1/3}} = \lim_{x\to\infty} \frac{(x^3+1)^{1/3}}{x} = \frac{\infty}{\infty}.$$

Again, using L'Hôpital's rule we arrive at

$$\lim_{x\to\infty} \frac{x^2}{\sqrt[3]{(x^3+1)^2}} = \lim_{x\to\infty} \frac{(x^3+1)^{1/3}}{x} = \lim_{x\to\infty} \frac{\frac{1}{3}(3x^2)(x^3+1)^{-2/3}}{1} = \lim_{x\to\infty} \frac{x^2}{\sqrt[3]{(x^3+1)^2}},$$

which is what we started with. So repeated application of L'Hôpital's rule will not provide us with an answer.

Now, we use algebraic manipulation to evaluate the limit. Divide numerator and denominator by x^2:

$$\lim_{x\to\infty} \frac{x^2}{\sqrt[3]{(x^3+1)^2}} = \lim_{x\to\infty} \frac{1}{\frac{1}{x^2}\sqrt[3]{(x^3+1)^2}} = \lim_{x\to\infty} \frac{1}{\frac{1}{\left(x^3\right)^{2/3}}(x^3+1)^{2/3}}$$

$$= \lim_{x\to\infty} \frac{1}{\left(\frac{x^3+1}{x^3}\right)^{2/3}} = \lim_{x\to\infty} \frac{1}{\left(1+\frac{1}{x^3}\right)^{2/3}} = \frac{1}{(1)^{2/3}} = 1.$$

EXERCISE 5-4

2. (A) (d, ∞)

(B) (-∞, a), (a, c), (c, d)

(C) (-∞, a), (a, c), (c, d)

(D) (d, ∞)

(E) $x = d$

(F) no local minima

(G) (a, b), (c, e)

(H) (-∞, a), (b, c), (e, ∞)

(I) (-∞, a), (b, c), (e, ∞)

(J) (a, b), (c, e)

(K) $x = b$, $x = e$

(L) $y = L$

(M) $x = a$, $x = c$

4. Step 1. Analyze $f(x)$:

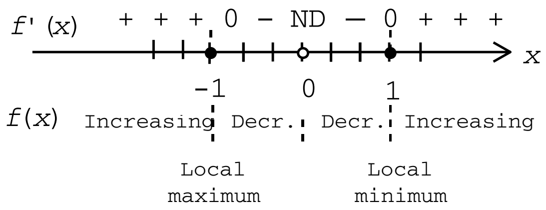
(A) Domain: All real numbers, (-∞, ∞)

(B) Intercepts: y-intercept: $f(0) = 0$

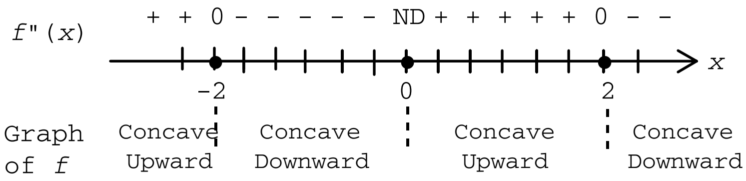
x-intercepts: -2, 0, 2

(C) Asymptotes: Horizontal asymptote: $y = -3$ and $y = 3$

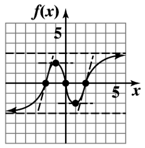
Step 2. Analyze $f'(x)$:



Step 3. Analyze $f''(x)$:



Step 4. Sketch the graph of f:



6. Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = 1$

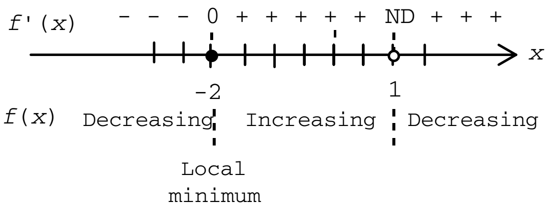
(B) Intercepts: y-intercept: 0

x-intercepts: -4, 0, 2

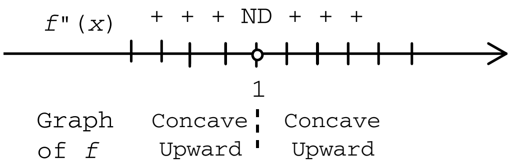
(C) Asymptotes: Horizontal asymptote: $y = -2$

Vertical asymptote: $x = 1$

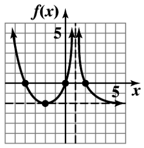
Step 2. Analyze $f'(x)$:



Step 3. Analyze $f''(x)$:



Step 4. Sketch the graph of f:



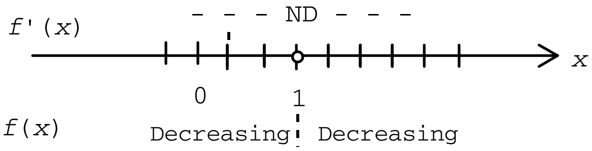
8. Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = 1$

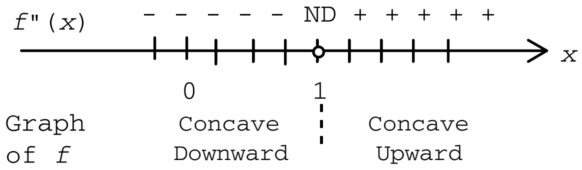
(B) Intercepts: y-intercept: -2
x-intercept: 2

(C) Asymptotes: Horizontal asymptote: $y = -1$
Vertical asymptote: $x = 1$

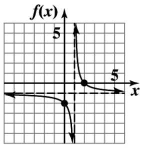
Step 2. Analyze $f'(x)$:



Step 3. Analyze $f''(x)$:



Step 4. Sketch the graph of f:



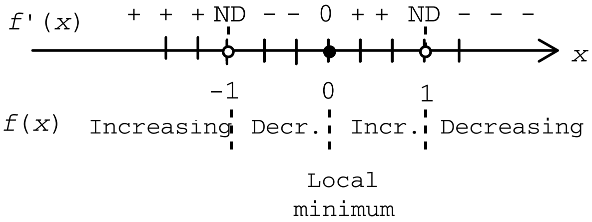
10. Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -1$, $x = 1$

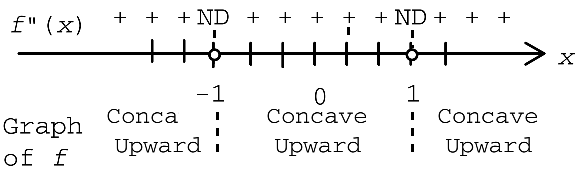
(B) Intercepts: y-intercept: 0
x-intercept: 0

(C) Asymptotes: Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -1$, $x = 1$

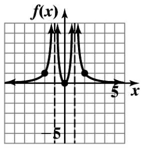
Step 2. Analyze $f'(x)$:



Step 3. Analyze $f''(x)$:



Step 4. Sketch the graph of f:



12. Domain: $[-25, \infty)$; y intercept: 5; x intercept: –25.

14. Domain: $(-\infty, \infty)$; y intercept: 52; x intercept: –13.

16. Domain: All real numbers except –3; y intercept: 34; no x intercept.

18. $f(x) = \frac{2x-4}{x+2}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -2$.

(B) Intercepts: y-intercept: $f(0) = -2$

$$x\text{-intercepts: } \frac{2x-4}{x+2} = 0$$
$$2x - 4 = 0$$
$$x = 2$$

(C) Asymptotes:

Horizontal asymptote: $\lim_{x\to\infty} \frac{2x-4}{x+2} = \lim_{x\to\infty} \frac{2x}{x} = 2$ and the line $y = 2$ is a horizontal asymptote.

Vertical asymptote: $D(x) = x + 2 = 0$, $D(-2) = 0$, but $N(-2) \neq 0$, so the line $x = -2$ is a vertical asymptote.

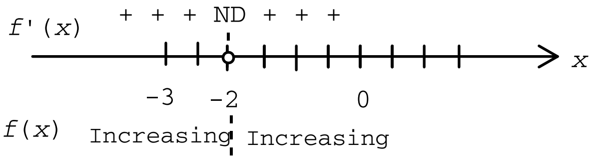
Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{2(x+2)-(2x-4)}{(x+2)^2} = \frac{2x+4-2x+4}{(x+2)^2} = \frac{8}{(x+2)^2} = 8(x+2)^{-2}$$

Critical values: None

Partition number: $x = -2$

Sign chart for f':



Test Numbers

x	$f'(x)$
-3	8 (+)
0	2 (+)

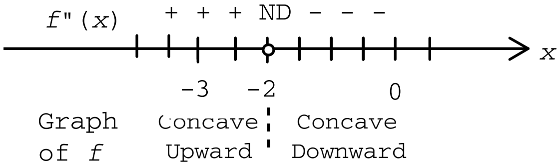
Thus, f is increasing on $(-\infty, -2)$ and on $(-2, \infty)$; there are no local extrema.

Step 3. Analyze $f''(x)$:

$$f''(x) = -16(x+2)^{-3} = -\frac{16}{(x+2)^3}$$

Partition number for f'': $x = -2$

Sign chart for f'':



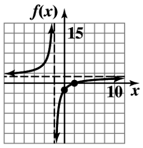
Test Numbers

x	$f\,'(x)$
-3	16 (+)
0	-2 (-)

Thus, the graph of f is concave upward on (-∞, -2) and concave downward on (-2, ∞).

Step 4. Sketch the graph of f:

x	$f(x)$
−3	10
0	−2
2	0



20. $f(x) = \dfrac{2+x}{3-x}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = 3$.

(B) Intercepts: y-intercept: $f(0) = \dfrac{2}{3}$

$$x\text{-intercepts: } \frac{2+x}{3-x} = 0$$
$$2 + x = 0$$
$$x = -2$$

(C) Asymptotes:

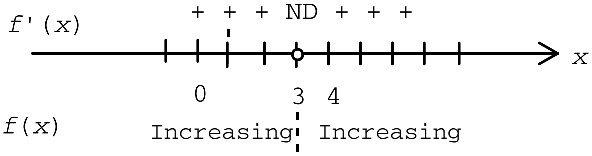
Horizontal asymptote: $\lim\limits_{x\to\infty} \dfrac{2+x}{3-x} = \dfrac{1}{-1} = -1$ and the line $y = -1$ is a horizontal asymptote.

Vertical asymptote: $D(x) = 3 - x = 0$, $x = 3$; i.e. $D(3) = 0$, $N(3) \neq 0$, so the line $x = 3$ is a vertical asymptote.

Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{(3-x)+(2+x)}{(3-x)^2} = \frac{3-x+2+x}{(3-x)^2} = \frac{5}{(3-x)^2} = 5(3 - x)^{-2}$$

Critical values: None
Partition number: $x = 3$
Sign chart for f':



Test Numbers

x	$f(x)$
0	$\frac{5}{9}$(+)
4	5(+)

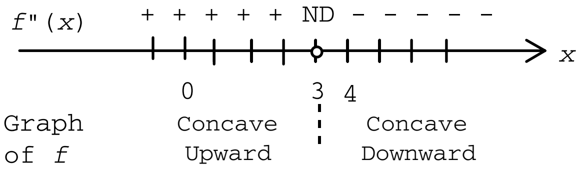
Thus, f is increasing on (-∞, 3) and on (3, ∞); there are no local extrema.

Step 3. Analyze $f''(x)$:

$$f''(x) = 10(3 - x)^{-3} = \frac{10}{(3-x)^3}$$

Partition number for f'': $x = 3$

Sign chart for f'':



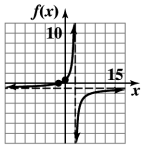
Test Numbers

x	$f''(x)$
0	$\frac{10}{27}(+)$
4	$-10(-)$

Thus, the graph of f is concave upward on $(-\infty, 3)$ and concave downward on $(3, \infty)$.

Step 4. Sketch the graph of f:

x	$f(x)$
-2	0
0	$\frac{2}{3}$
4	-6



22. $f(x) = 3 + 7e^{-0.2x}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, $(-\infty, \infty)$.

(B) Intercepts: y-intercept: 10
x-intercept: None, $f(x) > 3$ for all x.

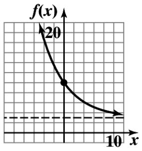
(C) Asymptotes:
Horizontal asymptote: $\lim_{x\to\infty} f(x) = 3 + 7(0) = 3$, thus $y = 3$ is a horizontal asymptote.

Vertical asymptotes: None

Step 2. Analyze $f'(x)$:
$f'(x) = -1.4e^{-0.2x} < 0$ for all x and hence $f(x)$ is decreasing on $(-\infty, \infty)$.

Step 3. Analyze $f''(x)$:
$f''(x) = 0.28e^{-0.2x} > 0$ for all x and hence $f(x)$ is concave upward on $(-\infty, \infty)$.

Step 4. Sketch the graph of f:



x	$f(x)$
−1	11.55
0	10
1	8.73

24. $f(x) = 10xe^{-0.1x}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, $(-\infty, \infty)$

(B) Intercepts: y-intercept: 0
x-intercept: 0

(C) Asymptotes:

Horizontal asymptote: $\lim_{x\to\infty} f(x) = 0$, thus $y = 0$ (x axis) is a horizontal asymptote.

Vertical asymptotes: None

Step 2. Analyze $f'(x)$:

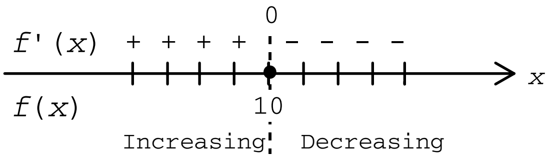
$f'(x) = 10e^{-0.1x} - xe^{-0.1x} = (10 - x)e^{-0.1x}$

$f'(x) = 0$ if $x = 10$

Thus, the only critical value of f is $x = 10$.

$x = 10$ is also a partition number for f'.

Sign chart for f':



Test Numbers

x	$f'(x)$
0	$10(+)$
11	$-e^{-1.1}(-)$

Thus, f has a local maximum at $x = 10$;

$f(10) = 10(10)e^{-0.1(10)} = 100e^{-1} \approx 36.79$ is absolute maximum value of f.

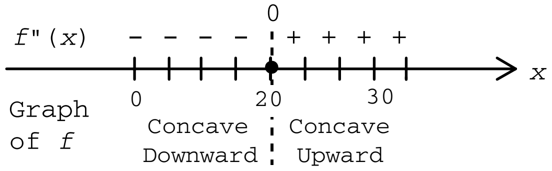
Step 3. Analyze $f''(x)$:

$f''(x) = 10(-0.1)e^{-0.1x} - e^{-0.1x} - x(-0.1)e^{-0.1x}$

$= -e^{-0.1x} - e^{-0.1x} + 0.1xe^{-0.1x} = (0.1x - 2)e^{-0.1x}$

Partition number for f'': $x = 20$

Sign chart for f'':



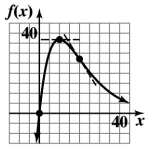
Test Numbers

x	$f''(x)$
0	$-2(-)$
30	$e^{-3}(+)$

Thus, the graph of f is concave downward on $(-\infty, 20)$ and concave upward on $(20, \infty)$; the graph of f has an inflection point at $x = 20$.

Step 4. Sketch the graph of f:

x	$f(x)$
-10	$-100e^{1}$
0	0
10	$100e^{1}$



26. $f(x) = \ln(2x + 4)$

Step 1. Analyze $f(x)$:

(A) Domain: $2x + 4 > 0$ or $x > -2$, $(-2, \infty)$

(B) Intercepts: y-intercept: $f(0) = \ln 4 \approx 1.386$

x-intercepts: $\ln(2x + 4) = 0$

$2x + 4 = 1$

$2x = -3$

$x = -1.5$

(C) Asymptotes:

Horizontal asymptote: $\lim\limits_{x\to\infty} \ln(2x+4)$ does not exist.
Thus, there are no horizontal asymptotes.

Vertical asymptote: $\lim\limits_{x\to -2^+} \ln(2x+4) = -\infty$, so $x = -2$ is a vertical asymptote.

Step 2. Analyze $f'(x)$:

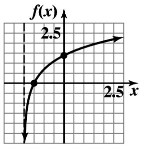
$$f'(x) = \frac{1}{2x+4}(2) = \frac{1}{x+2} > 0 \text{ for } x > -2$$

Therefore, f is increasing on $(-2, \infty)$.

Step 3. Analyze $f''(x)$:

$f''(x) = -\dfrac{1}{(x+2)^2} < 0$ and hence the graph of f is concave downward on $(-2, \infty)$; there are no inflection points.

Step 4. Sketch the graph of f:



x	$f(x)$
0	≈ 1.386
-1.5	0
1	≈ 1.792

28. $f(x) = \ln(x^2 + 4)$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, $(-\infty, \infty)$.

(B) Intercepts: y-intercept: $f(0) = \ln 4 \approx 1.386$
x-intercepts: None, since $x^2 + 4$ cannot be equal to 1.

(C) Asymptotes:

Horizontal asymptote: None; $\lim\limits_{x\to\pm\infty} \ln(x^2+4)$ does not exist.

Step 2. Analyze $f(x)$:

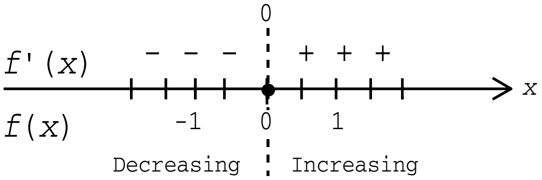
$$f(x) = \frac{1}{x^2+4}(2x) = \frac{2x}{x^2+4}$$

Critical value: $f(x) = \dfrac{2x}{x^2+4} = 0$

$x = 0$

Partition numbers: $x = 0$

Sign chart for f':



Test Numbers

x	$f'(x)$
-1	$-\frac{2}{5}(-)$
1	$\frac{2}{5}(+)$

Thus, f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$; f has a local minimum at $x = 0$.

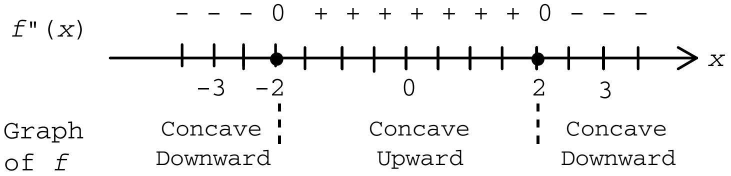
Step 3. Analyze $f''(x)$:

$$f''(x) = \frac{2(x^2+4)-2x(2x)}{(x^2+4)^2} = \frac{2x^2+8-4x^2}{(x^2+4)^2} = \frac{2(4-x^2)}{(x^2+4)^2}$$

Partition numbers for f'': $4 - x^2 = (2 - x)(2 + x) = 0$

$x = -2$ and $x = 2$

Sign chart for f'':



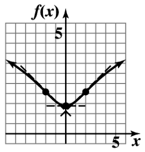
Test Numbers

x	$f''(x)$
−3	$-\frac{10}{169}$(−)
0	$\frac{1}{2}$(+)
3	$-\frac{10}{169}$(−)

Thus, the graph of f is concave downward on (-∞, -2) and on (2, ∞); the graph of f is concave upward on (-2, 2); the graph has inflection points at $x = -2$ and at $x = 2$.

Step 4. Sketch the graph of f:

$xf(x)$	
0	≈ 1.386
-2	≈ 2.079
2	≈ 2.079



30. $f(x) = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -2$, $x = 2$.

(B) Intercepts: y-intercept: $f(0) = -\frac{1}{4}$

x-intercepts: None

(C) Asymptotes:

Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = -2$, $x = 2$

Step 2. Analyze $f'(x)$:

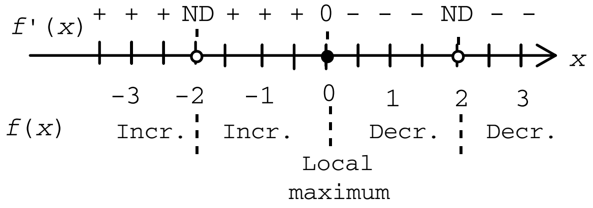
$$f'(x) = \frac{-2x}{(x^2-4)^2} = \frac{-2x}{(x-2)^2(x+2)^2}$$

$f'(x) = 0$ if $x = 0$

Critical values: $x = 0$

Partition numbers: $x = -2$, $x = 0$, $x = 2$

Sign chart for f':



Test Numbers

x	$f''(x)$
-3	$\frac{6}{25}$ (+)
-1	$\frac{2}{9}$ (+)
1	$-\frac{2}{9}$ (-)
3	$-\frac{6}{25}$ (-)

Thus, f is increasing on (-∞, -2) and (-2, 0); decreasing on (0, 2) and (2, ∞); f has a local maximum at $x = 0$.

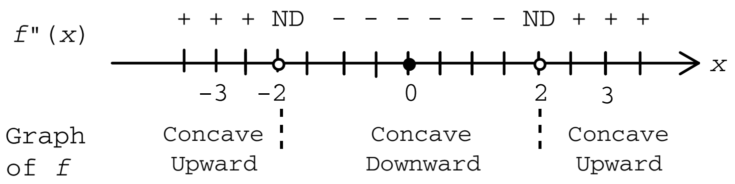
Step 3. Analyze $f''(x)$:

$$f''(x) = -2(x^2 - 4)^{-2} - 2x(-4x)(x^2 - 4)^{-3}$$

$$= \frac{-2}{(x^2-4)^2} + \frac{8x^2}{(x^2-4)^3} = \frac{-2(x^2-4)+8x^2}{(x^2-4)^3} = \frac{6x^2+8}{(x-2)^3(x+2)^3}$$

Partition numbers for f'': $x = -2$, $x = 2$

Sign chart for f'':



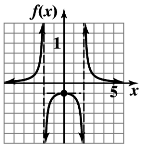
Test Numbers

x	$f''(x)$
-3	$\frac{62}{125}$ (+)
0	$-\frac{1}{8}$ (-)
3	$\frac{62}{125}$ (+)

Thus, the graph of f is concave upward on $(-\infty, -2)$ and on $(2, \infty)$; concave downward on $(-2, 2)$; there are no inflection points.

Step 4. Sketch the graph of f:

x	$f(x)$
-3	$\frac{1}{5}$
-1	$-\frac{1}{3}$
0	$-\frac{1}{4}$
1	$-\frac{1}{3}$
3	$\frac{1}{5}$



32. $f(x) = \frac{x^2}{1+x^2}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, $(-\infty, \infty)$

(B) Intercepts: y-intercept: $f(0) = 0$

x-intercepts: $\frac{x^2}{1+x^2} = 0$

$x = 0$

(C) Asymptotes:

Horizontal asymptote: $y = 1$

Vertical asymptotes: None

Step 2. Analyze $f'(x)$:

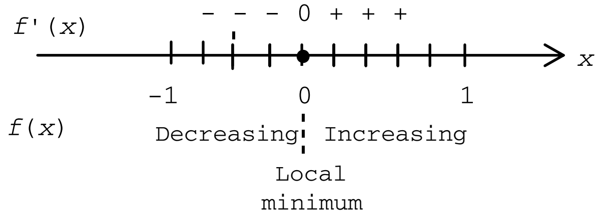
$$f'(x) = \frac{(2x)(1+x^2)-2x(x^2)}{(1+x^2)^2} = \frac{2x+2x^3-2x^3}{(1+x^2)^2} = \frac{2x}{(1+x^2)^2} = 0$$

$$x = 0$$

Critical value: $x = 0$

Partition number: $x = 0$

Sign chart for f':



Test Numbers

x	$f'(x)$
-1	$-\frac{1}{2}$ (-)
1	$\frac{1}{2}$ (+)

Thus, f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$; has a local minimum at $x = 0$.

Step 3. Analyze $f''(x)$:

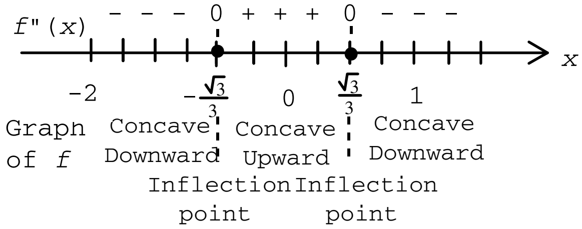
$$f''(x) = \frac{2(1+x^2)^2 - 4x(1+x^2)(2x)}{(1+x^2)^4}$$

$$= \frac{2(1+x^2)[(1+x^2)-4x^2]}{(1+x^2)^4} = \frac{2(1-3x^2)}{(1+x^2)^3}$$

$f''(x) = 0$ or $1 - 3x^2 = 0$, $x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

Partition numbers for f'': $x = -\frac{\sqrt{3}}{3}$, $x = \frac{\sqrt{3}}{3}$

Sign chart for f'':



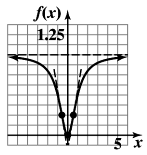
Test Numbers

x	$f''(x)$
−1	$-\frac{1}{2}(-)$
0	$2(+)$
1	$-\frac{1}{2}(-)$

Thus, the graph of f is concave downward on $\left(-\infty, -\frac{\sqrt{3}}{3}\right)$ and on $\left(\frac{\sqrt{3}}{3}, \infty\right)$; concave upward on $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$; f has inflection points at $x = -\frac{\sqrt{3}}{3}$ and $x = \frac{\sqrt{3}}{3}$.

Step 4. Sketch the graph of f:

x	$f(x)$
−1	$\frac{1}{2}$
$-\frac{\sqrt{3}}{3}$	$\frac{1}{4}$
0	0
$\frac{\sqrt{3}}{3}$	$\frac{1}{4}$
1	$\frac{1}{2}$



34. $f(x) = \frac{2x}{x^2 - 9}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -3$, $x = 3$.

(B) Intercepts: y-intercept: $f(0) = 0$

x-intercepts: $\frac{2x}{x^2-9} = 0$

$x = 0$

(C) Asymptotes:

Horizontal asymptote: $\lim_{x\to\infty} \frac{2x}{x^2-9} = 0$ and the x axis is a horizontal asymptote.

Vertical asymptotes: $D(x) = x^2 - 9 = 0$, $x = -3$, $x = 3$, $N(-3) \neq 0$, $N(3) \neq 0$, so the lines $x = -3$ and $x = 3$ are two vertical asymptotes.

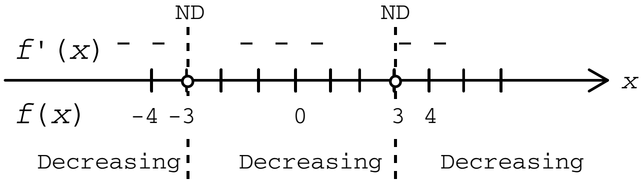
Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{2(x^2-9)-2x(2x)}{(x^2-9)^2} = \frac{2x^2-18-4x^2}{(x^2-9)^2} = \frac{-2(x^2+9)}{(x^2-9)^2}$$

Critical values: None

Partition numbers: $x = -3$, $x = 3$

Sign chart for f':



Test Numbers

x	$f'(x)$
-4	$-\frac{50}{49}(-)$
0	$-\frac{2}{9}(-)$
4	$-\frac{50}{49}(-)$

Thus, f is decreasing on $(-\infty, -3)$, on $(-3, 3)$ and on $(3, \infty)$; there are no local extrema.

Step 3. Analyze $f''(x)$:

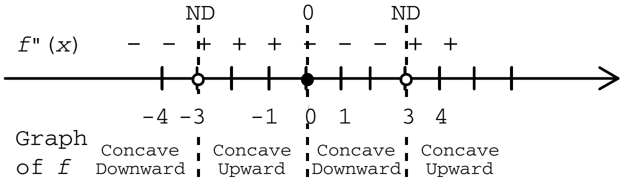
$$f''(x) = -2\{2x(x^2 - 9)^{-2} - 4x(x^2 - 9)^{-3}(x^2 + 9)\}$$

$$= -2\left\{\frac{2x}{(x^2-9)^2} - \frac{4x(x^2+9)}{(x^2-9)^3}\right\} = -2\left(\frac{2x(x^2-9)-4x(x^2+9)}{(x^2-9)^3}\right)$$

$$= -2\left(\frac{2x^3-18x-4x^3-36x}{(x^2-9)^3}\right) = \frac{4x^3+108x}{(x^2-9)^3}$$

Partition numbers for $f''(x)$: $x = -3$, $x = 0$, $x = 3$

Sign chart for f'':



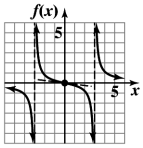
Test Numbers

x	$f''(x)$
-4	$-\frac{432}{343}(-)$
-1	$\frac{27}{128}(+)$
1	$-\frac{27}{128}(-)$
4	$\frac{432}{343}(+)$

Thus, the graph of f is concave downward on $(-\infty, -3)$ and on $(0, 3)$ and is concave upward on $(-3, 0)$ and on $(3, \infty)$; there is an inflection point at $x = 0$.

Step 4. Sketch the graph of f:

x	$f(x)$
-4	$-\frac{8}{7}$
0	0
4	$\frac{8}{7}$



36. $f(x) = \dfrac{x}{(x-2)^2}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except 2.

(B) Intercepts: y-intercept: $f(0) = 0$

x-intercepts: $\dfrac{x}{(x-2)^2} = 0$

$x = 0$

(C) Asymptotes:

Horizontal asymptote: $\lim\limits_{x\to\infty} \dfrac{x}{(x-2)^2} = 0$ and the x axis is a horizontal asymptote.

Vertical asymptote: $D(x) = (x - 2)^2 = 0$, $x = 2$; i.e. $D(2) = 0$, $N(2) \neq 0$, so the line $x = 2$ is a vertical asymptote.

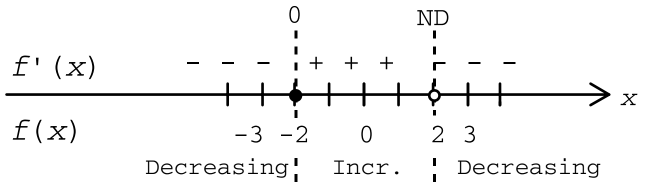
Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{(x-2)^2 - 2x(x-2)}{(x-2)^4} = \frac{x-2-2x}{(x-2)^3} = \frac{-(x+2)}{(x-2)^3}$$

Critical values: $x = -2$

Partition numbers: $x = -2$, $x = 2$

Sign chart for f':



Test Numbers

x	$f'(x)$
-3	$-\frac{1}{125}(-)$
0	$\frac{1}{4}(+)$
3	$-5(-)$

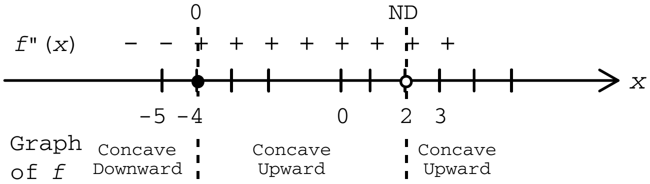
Thus, f is decreasing on $(-\infty, -2)$ and on $(2, \infty)$ and is increasing on $(-2, 2)$; f has a local minimum at $x = -2$.

Step 3. Analyze $f''(x)$:

$$f''(x) = -(x - 2)^{-3} + 3(x + 2)(x - 2)^{-4} = \frac{-(x-2)+3(x+2)}{(x-2)^4} = \frac{2(x+4)}{(x-2)^4}$$

Partition numbers for f'': $x = -4$, $x = 2$

Sign chart for f'':



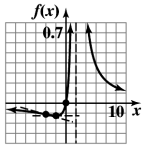
Test Numbers

x	$f''(x)$
-5	$-\frac{2}{2401}(-)$
0	$\frac{1}{2}(+)$
3	$14(+)$

Thus, the graph of f is concave downward on (-∞, -4), and concave upward on (-4, 2) and on (2, ∞); there is an inflection point at
$x = -4$.

Step 4. Sketch the graph of f:

x	$f(x)$
-4	$-\frac{1}{9}$
0	0
1	1
3	3



38. $f(x) = \dfrac{x^2 - 5x - 6}{x^2}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = 0$.

(B) Intercepts: y-intercept: $f(x)$ is not defined at 0.

x-intercept: $\dfrac{x^2 - 5x - 6}{x^2} = 0$

$x^2 - 5x - 6 = (x + 1)(x - 6) = 0$

$x = -1, x = 6$

(C) Asymptotes:

Horizontal asymptote: $\lim\limits_{x \to \infty} \dfrac{x^2 - 5x - 6}{x^2} = 1$ and the line $y = 1$ is a horizontal asymptote.

Vertical asymptote: $D(x) = x^2 = 0$, $x = 0$; i.e. $D(0) = 0$, $N(0) \neq 0$,
so the line $x = 0$ (y axis) is a vertical asymptote.

Step 2. Analyze $f(x)$:

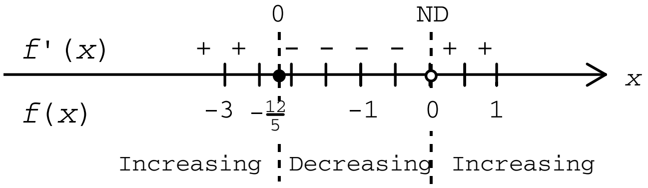
$$f(x) = \frac{(2x-5)x^2 - 2x(x^2 - 5x - 6)}{x^4} = \frac{2x^3 - 5x^2 - 2x^3 + 10x^2 + 12x}{x^4}$$

$$= \frac{5x^2 + 12x}{x^4} = \frac{5x + 12}{x^3}$$

Critical values: $x = -\dfrac{12}{5}$

Partition numbers: $x = -\dfrac{12}{5}$, $x = 0$

Sign chart for f':



Test Numbers

x	$f'(x)$
–3	$\frac{1}{9}(+)$
–1	–7(–)
1	17(+)

Thus, f is increasing on $\left(-\infty, -\frac{12}{5}\right)$ and on (0, ∞) and decreasing on $\left(-\frac{12}{5}, 0\right)$; f has a local maximum at $x = -\frac{12}{5}$.

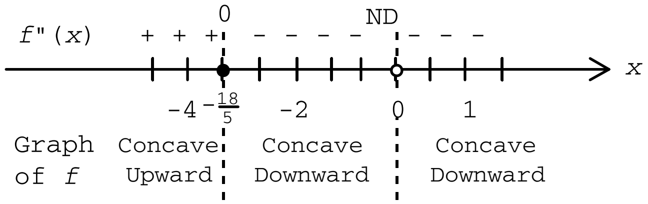
Step 3. Analyze $f''(x)$:

$$f''(x) = \frac{5x^3 - 3x^2(5x+12)}{x^6} = \frac{5x - 3(5x+12)}{x^4} \qquad = \frac{5x - 15x - 36}{x^4}$$

$$= \frac{-2(5x+18)}{x^4}$$

Partition numbers for f'': $x = -\frac{18}{5}$, $x = 0$.

Sign chart for f'':



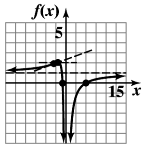
Test Numbers

x	$f''(x)$
–4	$\frac{1}{64}(+)$
–2	–1(–)
1	–46(–)

Thus, the graph of f is concave upward on $\left(-\infty, -\frac{18}{5}\right)$ and downward on $\left(-\frac{18}{5}, 0\right)$ and on (0, ∞); there is an inflection point at $x = -\frac{18}{5}$.

Sketch the graph of f:

x	$f(x)$
–4	$\frac{15}{8}$
–2	8
1	–10
6	0



40. $f(x) = \frac{x^2}{2+x}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -2$.

(B) Intercepts: y-intercept: $f(0) = 0$

$$x\text{-intercepts: } \frac{x^2}{2+x} = 0$$
$$x = 0$$

(C) Asymptotes:

Horizontal asymptote: $\lim_{x\to\infty} \frac{x^2}{2+x} = \infty$, so there is no horizontal asymptote.

Vertical asymptote: $D(x) = 2 + x = 0$, $x = -2$; i.e. $D(-2) = 0$, $N(-2) \neq 0$, so the line $x = -2$ is a vertical asymptote.

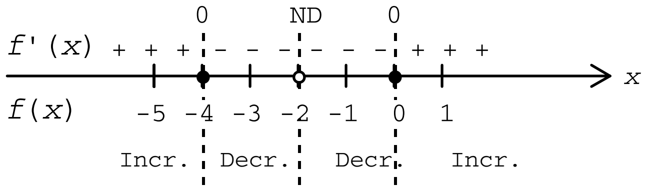
Step 2. Analyze $f(x)$:

$$f(x) = \frac{2x(2+x)-x^2}{(2+x)^2} = \frac{4x+2x^2-x^2}{(2+x)^2} = \frac{x(4+x)}{(2+x)^2}$$

Critical values: $x = -4$, $x = 0$

Partition numbers: $x = -4$, $x = -2$, $x = 0$

Sign chart for f:



Test Numbers

x	$f'(x)$
-5	$\frac{5}{9}(+)$
-3	$-3(-)$
-1	$-3(-)$
1	$\frac{5}{9}(+)$

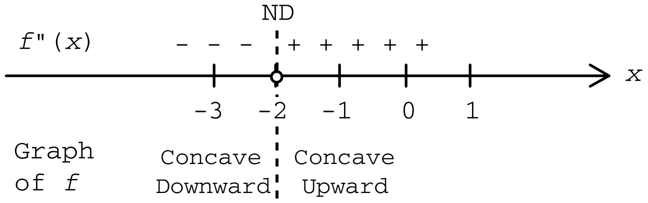
Thus, f is increasing on $(-\infty, -4)$ and on $(0, \infty)$ and decreasing on $(-4, -2)$ and on $(-2, 0)$; f has a local maximum at $x = -4$ and a local minimum at $x = 0$.

Step 3. Analyze $f''(x)$:

$$f''(x) = \frac{(4+2x)(2+x)^2 - 2(2+x)x(4+x)}{(2+x)^4} = \frac{(4+2x)(2+x) - 2x(4+x)}{(2+x)^3}$$
$$= \frac{8+8x+2x^2-8x-2x^2}{(2+x)^3}$$
$$= \frac{8}{(2+x)^3}$$

Partition numbers for f'': $x = -2$

Sign chart for f'':



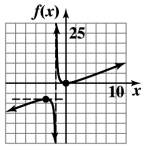
Test Numbers

x	$f''(x)$
-3	$-8(-)$
1	$\frac{8}{27}(+)$

Thus, the graph of f is concave downward on $(-\infty, -2)$ and concave upward on $(-2, \infty)$; there are no inflection points.

Step 4. Sketch the graph of f:

x	$f(x)$
-3	-9
0	0
1	$\frac{1}{3}$



42. $f(x) = \dfrac{2x^2+5}{4-x^2}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -2$, $x = 2$.

(B) Intercepts: y-intercept: $f(0) = \dfrac{5}{4}$

x-intercepts: None $(2x^2 + 5 > 0)$

(C) Asymptotes:

Horizontal asymptote: $\lim\limits_{x\to\infty} \dfrac{2x^2+5}{4-x^2} = -2$ and the line $y = -2$ is a horizontal asymptote.

Vertical asymptote: $D(x) = 4 - x^2 = 0$, $x = -2$, $x = 2$; i.e. $D(-2) = D(2) = 0$, $N(-2) \neq 0$, $N(2) \neq 0$, so the lines $x = -2$ and $x = 2$ are two vertical asymptotes.

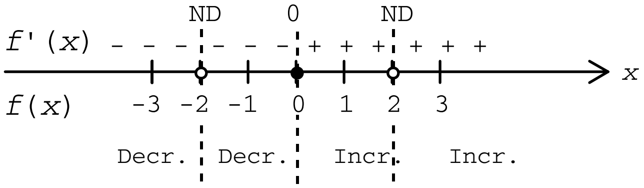
Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{4x(4-x^2)+2x(2x^2+5)}{(4-x^2)^2} = \frac{16x-4x^3+4x^3+10x}{(4-x^2)^2} = \frac{26x}{(4-x^2)^2}$$

Critical values: $x = 0$

Partition numbers: $x = -2$, $x = 0$, $x = 2$

Sign chart for f':



Test Numbers

x	$f'(x)$
-3	$-\frac{78}{25}(-)$
-1	$-\frac{26}{9}(-)$
1	$\frac{26}{9}(+)$
3	$\frac{78}{25}(+)$

Thus, f is decreasing on $(-\infty, -2)$ and on $(-2, 0)$ and is increasing on $(0, 2)$ and on $(2, \infty)$; f has a local minimum at $x = 0$.

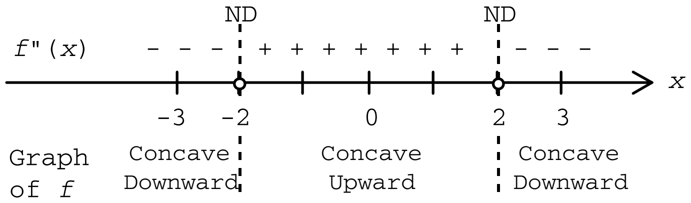
Step 3. Analyze $f''(x)$:

$$f''(x) = 26(4 - x^2)^{-2} + 4x(4 - 2x^2)^{-3}(26x)$$

$$= \frac{26(4-x^2)+4x(26x)}{(4-x^2)^3} = \frac{104-26x^2+104x^2}{(4-x^2)^3} = \frac{78x^2+104}{(4-x^2)^3}$$

Partition numbers for f'': $x = -2$, $x = 2$

Sign chart for f'':



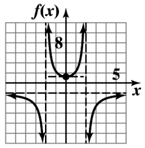
Test Numbers

x	$f''(x)$
-3	$-\frac{806}{125}(-)$
0	$\frac{104}{64}(+)$
3	$-\frac{806}{125}(-)$

Thus, the graph of *f* is concave downward on (-∞, -2) and on (2, ∞) and concave upward on (-2, 2); there are no inflection points.

Step 4. Sketch the graph of *f*:

x	$f(x)$
-3	$-\frac{23}{5}$
0	$\frac{5}{4}$
3	$-\frac{23}{5}$



44. $f(x) = \dfrac{x^3}{4-x}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = 4$.

(B) Intercepts: y-intercept: $f(0) = 0$

x-intercepts: $x^3 = 0$

$x = 0$

(C) Asymptotes:

Horizontal asymptote: $\lim\limits_{x\to\infty} \dfrac{x^3}{4-x}$ = -∞, so there is no horizontal asymptote.

Vertical asymptote: $D(x) = 4 - x = 0$, $x = 4$; i.e. $D(4) = 0$, $N(4) \neq 0$, so the line $x = 4$ is a vertical asymptote.

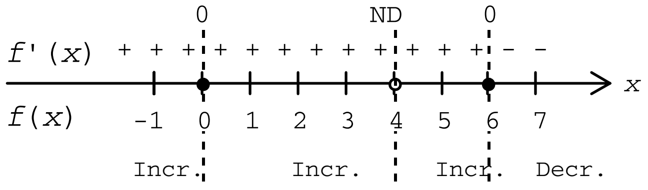
Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{3x^2(4-x)+x^3}{(4-x)^2} = \frac{12x^2-3x^3+x^3}{(4-x)^2} = \frac{2x^2(6-x)}{(4-x)^2}$$

Critical values: $x = 0$, $x = 6$

Partition numbers: $x = 0$, $x = 4$, $x = 6$

Sign chart for f':



Test Numbers

x	$f'(x)$
-1	$\frac{14}{25}(+)$
1	$\frac{10}{9}(+)$
5	$50(+)$
7	$-\frac{98}{9}(-)$

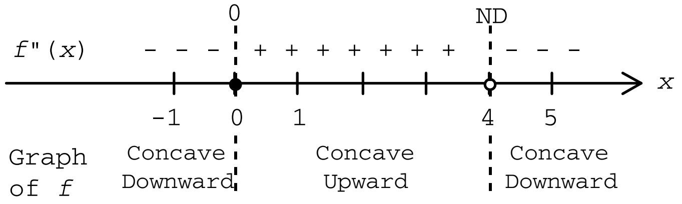
Thus, *f* is increasing on (-∞, 4) and on (4, 6) and decreasing on (6, ∞); *f* has a local maximum at $x = 6$.

Step 3. Analyze $f''(x)$:

$$f''(x) = (24x - 6x^2)(4 - x)^{-2} + 2(4 - x)^{-3}(12x^2 - 2x^3)$$

$$= \frac{(24x-6x^2)(4-x)+2(12x^2-2x^3)}{(4-x)^3} = \frac{2x(x^2-12x+48)}{(4-x)^3}$$

Partition numbers for f'':



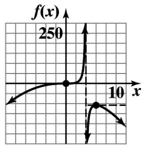
Test Numbers

x	$f''(x)$
-1	$-\frac{122}{125}(-)$
1	$\frac{74}{27}(+)$
5	$-26(-)$

Thus, the graph of f is concave downward on $(-\infty, 0)$ and on $(4, \infty)$ and concave upward on $(0, 4)$; there is an inflection point at $x = 0$.

Step 4. Sketch the graph of f:

x	$f(x)$
-1	$-\frac{1}{5}$
1	$\frac{1}{3}$
5	-125



46. $f(x) = (x - 2)e^x$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, $(-\infty, \infty)$.

(B) Intercepts: y-intercept: $f(0) = (0 - 2)e^0 = -2$

x-intercept: $(x - 2)e^x = 0$

$x - 2 = 0$

$x = 2$

(C) Asymptotes:

Horizontal asymptote: $\lim_{x\to-\infty} (x - 2)e^x = 0$

$\lim_{x\to\infty} (x - 2)e^x = \infty$;

Thus, $y = 0$ is a horizontal asymptote.

Vertical asymptotes: There are no vertical asymptotes.

Step 2. Analyze $f'(x)$:

$f'(x) = e^x + (x - 2)e^x = (x - 1)e^x$

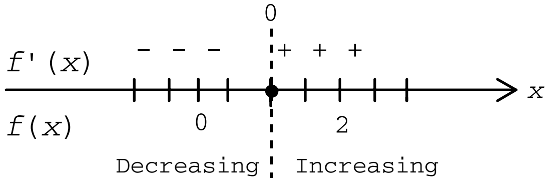
Critical values: $f'(x) = (x - 1)e^x = 0$

$x - 1 = 0$

$x = 1$

Thus, the only critical value of f is $x = 1$.
$x = 1$ is also a partition number for f'.

Sign chart for f':



Test Numbers

x	$f'(x)$
0	-1 (-)
2	e^2 (+)

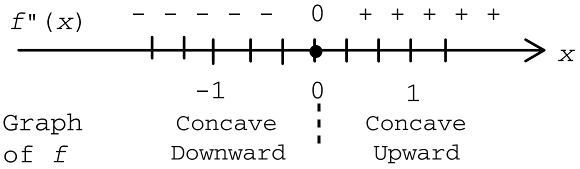
Thus, f has a minimum value at $x = 1$; $f(1) = (1 - 2)e^1 = -e$ is the absolute minimum of f.

Step 3. Analyze $f''(x)$:

$f''(x) = e^x + (x - 1)e^x = xe^x$

Partition number for f'': $x = 0$

Sign chart for f'':



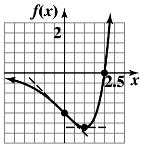
Test Numbers

x	$f''(x)$
-1	$-e^{-1}$ (-)
1	e (+)

Thus, the graph of f is concave downward on $(-\infty, 0)$ and concave upward on $(0, \infty)$; the graph has an inflection point at $x = 0$.

Step 4. Sketch the graph of f:

x	$f(x)$
-1	$-3e^{-1}$
0	-2
2	0
3	e^3



48. $f(x) = e^{-2x^2}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers, $(-\infty, \infty)$.

(B) Intercepts: y-intercept: 1
x-intercepts: None, $f(x) > 0$ for all x

(C) Asymptotes:

Horizontal asymptote: $\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(x) = 0$, thus $y = 0$ is a horizontal asymptote.

Vertical asymptotes: None

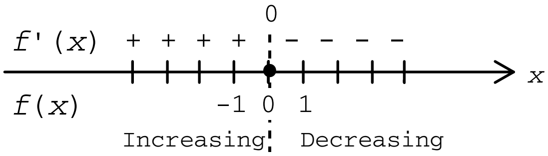
Step 2. Analyze $f'(x)$:

$f'(x) = -4xe^{-2x^2}$

Critical values: $f'(x) = -4xe^{-2x^2} = 0$
$x = 0$

Thus, the only critical value of f is $x = 0$. $x = 0$ is a partition number of f'.

Sign chart for $f'(x) = -4xe^{-2x^2}$



Test Numbers

x	$f'(x)$
-1	$4e^{-2}$ (+)
1	$-4e^{-2}$ (–)

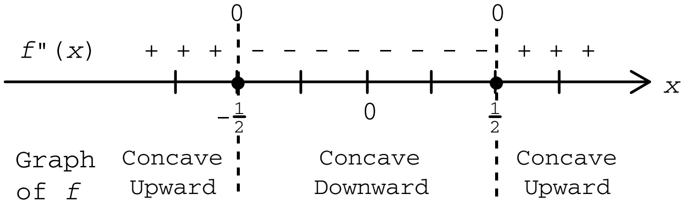
Thus, f has a maximum value at $x = 0$; $f(0) = 1$ is the absolute maximum of f.

Step 3. Analyze $f''(x)$:

$$f''(x) = -4e^{-2x^2} - 4x(-4x)e^{-2x^2} = (16x^2 - 4)e^{-2x^2}$$

Partition numbers for f'': $x = -\frac{1}{2}$, $x = \frac{1}{2}$

Sign chart for f'':



Test Numbers

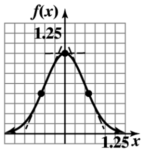
x	$f''(x)$
-1	$12e^{-2}$ (+)
0	-4 (–)
1	$12e^{-2}$ (+)

Thus, the graph of f is concave upward on $(-\infty, -0.5)$ and on $(0.5, \infty)$ and concave downward on $(-0.5, 0.5)$; the graph of f has inflection

points at $x = -0.5$ and $x = 0.5$.

Step 4. Sketch the graph of f:

x	$f(x)$
-1	e^{-2}
0	1
1	e^{-2}



50. $f(x) = \frac{\ln x}{x}$.

Step 1. Analyze $f(x)$:

(A) Domain: All positive real numbers, $(0, \infty)$.

(B) Intercepts: y-intercept: Does not exist;
0 is not in the domain of f.

x-intercept: $f(x) = \frac{\ln x}{x} = 0$

$\ln x = 0$

$x = 1$

(C) Asymptotes:

Horizontal asymptote: $\lim_{x\to\infty} \frac{\ln x}{x} = 0$;

thus, $y = 0$ is a horizontal asymptote.

Vertical asymptote: $\lim_{x\to 0^+} \frac{\ln x}{x} = -\infty$;

thus, $x = 0$ is a vertical asymptote.

Step 2. Analyze $f'(x)$:

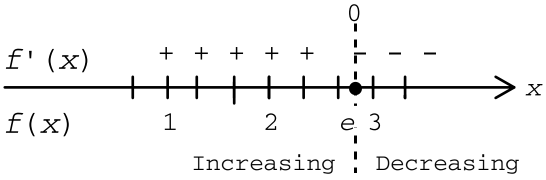
$$f'(x) = \frac{\frac{1}{x}(x) - (1)\ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

Critical values: $f'(x) = \dfrac{1-\ln x}{x^2} = 0$

$$\begin{aligned} 1 - \ln x &= 0 \\ \ln x &= 1 \\ x &= e \end{aligned}$$

Thus, $x = e$ is the only critical value of f.
$x = e$ is also a partition number for f'.

Sign chart for f':



Test Numbers

x	$f'(x)$
1	$1(+)$
3	$\frac{1-\ln 3}{9}(-)$

Thus, f has a maximum value at $x = e$;

$f(e) = \dfrac{\ln e}{e} = \dfrac{1}{e} = e^{-1}$ is the absolute maximum of f.

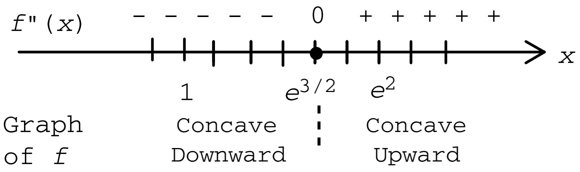
Step 3. Analyze $f''(x)$:

$$f''(x) = \frac{-\frac{1}{x}(x^2) - 2x(1 - \ln x)}{x^4} = \frac{-x - 2x + 2x\ln x}{x^4} = \frac{2\ln x - 3}{x^3}$$

$f''(x) = 0$, $2 \ln x - 3 = 0$, $\ln x = \dfrac{3}{2}$, $x = e^{3/2}$

Thus, $x = e^{3/2}$ is a partition number for f''.

Sign chart for f'':



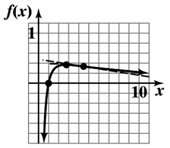
Test Numbers

x	$f''(x)$
1	$-3(-)$
e^2	$\frac{1}{e^6}(+)$

Thus, the graph of f is concave downward on $(0, e^{3/2})$ and concave upward on $(e^{3/2}, \infty)$; the graph has an inflection point at $x = e^{3/2}$.

Step 4. Sketch the graph of f:

x	$f(x)$
$\frac{1}{2}$	$-2\ln 2$
1	0
e	e^{-1}
e^2	$2e^{-2}$



52. $f(x) = \dfrac{x}{\ln x}$

Step 1. Analyze $f(x)$:

(A) Domain: All positive real numbers, except $x = 1$.

(B) Intercepts: y-intercept: None
x-intercept: None

(C) Asymptotes:
Horizontal asymptotes: None

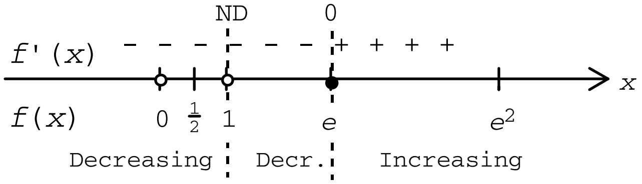
Vertical asymptotes: $x = 1$ (zero of the denominator)

Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{\left(\frac{d}{dx}x\right)\ln x - x\frac{d}{dx}(\ln x)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

Critical value: $f'(x) = 0$, $\ln x = 1$, $x = e$.
Partition numbers for f': $x = 1$, $x = e$.

Sign chart for f':



Test Numbers

x	$f'(x)$
$\frac{1}{2}$	(−)
2	(−)
e^2	(+)

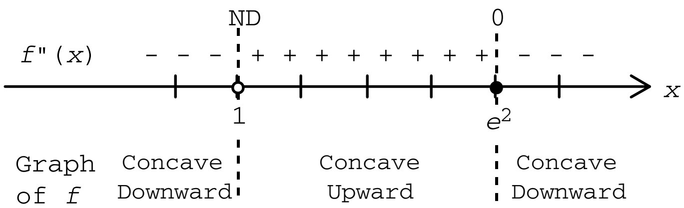
Thus, f is decreasing on $(0, 1)$ and on $(1, e)$ and increasing on (e, ∞); f has a local minimum at $x = e$.

Step 3. Analyze $f''(x)$:

$$f''(x) = \frac{\left(\frac{d}{dx}(\ln x - 1)\right)(\ln x)^2 - (\ln x - 1)\left(\frac{d}{dx}(\ln x)^2\right)}{(\ln x)^4}$$

$$= \frac{\frac{1}{x}(\ln x) - (\ln x - 1)\left(\frac{2}{x}\right)}{(\ln x)^3} = \frac{2 - \ln x}{x(\ln x)^3}$$

$f''(x) = 0$ if $x = e^2$ and hence 1 and e^2 are partition numbers for f'' on $(0, \infty)$.



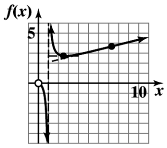
Test Numbers

x	$f''(x)$
0.5	−
e	+
e^3	−

Thus, the graph of *f* is concave downward on (0, 1) and on (e^2, ∞) and concave upward on $(1, e^2)$; the graph of *f* has an inflection point at $x = e^2$.

<u>Step 4. Sketch the graph of *f*</u>:

x	$f(x)$
0.5	–0.72
e	2.718
e^2	3.694



54. $f(x) = \dfrac{1}{3-2x-x^2} = \dfrac{1}{(3+x)(1-x)}$

<u>Step 1. Analyze *f*(*x*)</u>:

(A) Domain: All real numbers except *x* = -3, *x* = 1.

(B) Intercepts: *y*-intercept: $f(0) = \dfrac{1}{3}$

x-intercept: None

(C) Asymptotes:

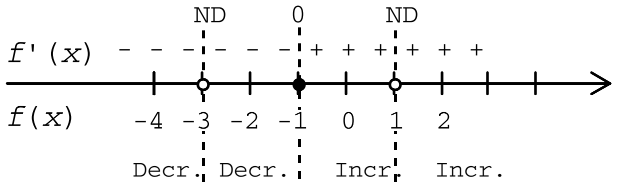
<u>Horizontal asymptote</u>: $\lim\limits_{x\to\infty} \dfrac{1}{3-2x-x^2} = 0$, and *x* axis is a horizontal asymptote.

<u>Vertical asymptote</u>: *D*(*x*) = (3 + *x*)(1 - *x*) = 0, *x* = -3, *x* = 1, i.e. *D*(-3) = *D*(1) = 0, *N*(-3) ≠ 0, *N*(1) ≠ 0, so the lines *x* = -3 and *x* = 1 are two vertical asymptotes.

<u>Step 2. Analyze *f*′(*x*)</u>:

$$f'(x) = \frac{2+2x}{(3-2x-x^2)^2} = \frac{2(x+1)}{(3-2x-x^2)^2}$$

Critical values: *x* = -1
Partition numbers: *x* = -3, *x* = -1, *x* = 1
Sign chart for *f*′:



Test Numbers

x	$f'(x)$
–4	$-\frac{6}{25}(-)$
–2	$-\frac{2}{9}(-)$
0	$\frac{2}{9}(+)$
2	$\frac{6}{25}(+)$

Thus, *f* is decreasing on (-∞, -3) and on (-3, -1) and increasing on (-1, 1) and on (1, ∞); *f* has a local minimum at *x* = -1.

Step 3. Analyze $f''(x)$:

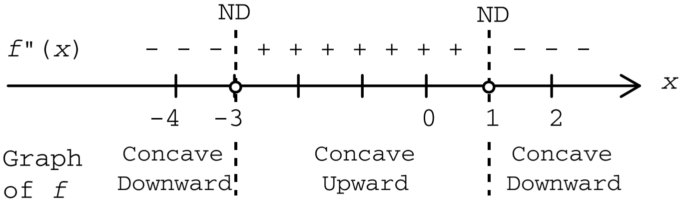
$$f''(x) = \frac{2(3-2x-x^2)^2 - 2(3-2x-x^2)(-2-2x)[2(x+1)]}{(3-2x-x^2)^4}$$

$$= \frac{2(3-2x-x^2)+8(x+1)^2}{(3-2x-x^2)^3} \qquad = \frac{6-4x-2x^2+8x^2+16x+8}{(3-2x-x^2)^3}$$

$$= \frac{2(3x^2+6x+7)}{(3-2x-x^2)^3}$$

Partition numbers for f'': $x = -3$, $x = 1$

Sign chart for f'':



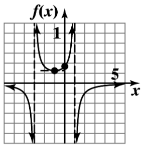
Test Numbers

x	$f''(x)$
-4	$-\frac{62}{125}(-)$
0	$\frac{14}{27}(+)$
2	$-\frac{62}{125}(-)$

Thus, the graph of f is concave upward on (-3, 1); concave downward on (-∞, -3) and on (1, ∞); there are no inflection points.

Step 4. Sketch the graph of f:

x	$f(x)$
-4	$-\frac{1}{5}$
0	$\frac{1}{3}$
-2	$\frac{1}{3}$



56. $f(x) = \dfrac{x^3}{x^2-12} = \dfrac{x^3}{\left(x+2\sqrt{3}\right)\left(x-2\sqrt{3}\right)}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -2\sqrt{3}$, $x = 2\sqrt{3}$.

(B) Intercepts: y-intercept: $f(0) = 0$

x-intercept: $x^3 = 0$

$x = 0$

(C) Asymptotes:

Horizontal asymptote: $\lim\limits_{x\to\infty} \dfrac{x^3}{x^2-12} = \infty$, so there are no horizontal asymptotes.

Vertical asymptote: $D(x) = x^2 - 12 = 0$, $x = -2\sqrt{3}$, $x = 2\sqrt{3}$,

i.e. $D(-2\sqrt{3}) = D(2\sqrt{3}) = 0$, $N(-2\sqrt{3}) \neq 0$, $N(2\sqrt{3}) \neq 0$, so the lines $x = -2\sqrt{3}$ and $x = 2\sqrt{3}$ are two vertical asymptotes.

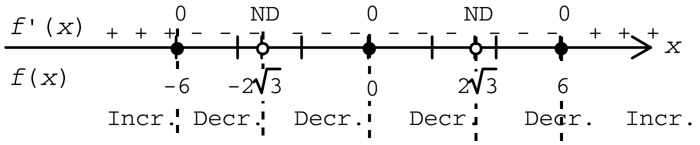
Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{3x^2(x^2-12)-2x(x^3)}{(x^2-12)^2} = \frac{3x^4-36x^2-2x^4}{(x^2-12)^2} = \frac{x^2(x+6)(x-6)}{(x^2-12)^2}$$

Critical values: $x = -6$, $x = 0$, $x = 6$

Partition numbers: $x = -6$, $x = -2\sqrt{3}$, $x = 0$, $x = 2\sqrt{3}$, $x = 6$

Sign chart for f':



Thus, f is increasing on (-∞, -6) and on (6, ∞) and is decreasing on (-6, $-2\sqrt{3}$), on ($-2\sqrt{3}$, $2\sqrt{3}$) and on ($2\sqrt{3}$, 6);

f has a local maximum at $x = -6$ and a local minimum at $x = 6$.

Step 3. Analyze $f''(x)$:

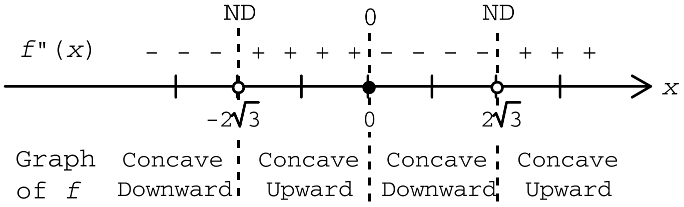
$f'(x) = (x^4 - 36x^2)(x^2 - 12)^{-2}$

$f''(x) = (4x^3 - 72x)(x^2 - 12)^{-2} - 4x(x^2 - 12)^{-3}(x^4 - 36x^2)$

$$= \frac{(4x^3-72x)(x^2-12)-4x(x^4-36x^2)}{(x^2-12)^3} = \frac{24x(x^2+36)}{(x^2-12)^3}$$

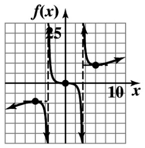
Partition numbers for f'': $x = -2\sqrt{3}$, $x = 0$, $x = 2\sqrt{3}$

Sign chart for f'':



Thus, the graph of f is concave downward on (-∞, $-2\sqrt{3}$), and on (0, $2\sqrt{3}$) and is concave upward on ($-2\sqrt{3}$, 0) and on ($2\sqrt{3}$, ∞); there is an inflection point at $x = 0$.

Step 4. Sketch the graph of f:



58. $f(x) = x - \dfrac{9}{x}$

For $|x|$ very large, $f(x) = x - \dfrac{9}{x} \approx x$. Thus, the line $y = x$ is an oblique asymptote.

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except 0.

(B) Intercepts: y-intercept: There is no y-intercept since f is not defined at $x = 0$.

x-intercepts: $x - \dfrac{9}{x} = 0$

$x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$x = -3, 3$

(C) Asymptotes:

Oblique asymptote: $y = x$

Vertical asymptote: $D(x) = x = 0$, $N(0) \neq 0$, so the line $x = 0$ is a vertical asymptote.

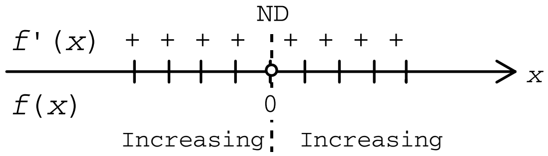
Step 2. Analyze $f'(x)$:

$$f'(x) = 1 + \frac{9}{x^2} = \frac{x^2 + 9}{x^2}$$

Critical values: None, since $f'(x) > 1$ for all x

Partition numbers: $x = 0$

Sign chart for f':



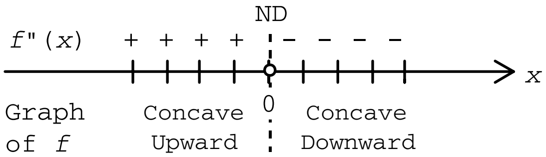
Thus, f is increasing on $(-\infty, 0)$ and on $(0, \infty)$; f has no extrema.

Step 3. Analyze $f''(x)$:

$$f''(x) = -\frac{18}{x^3}$$

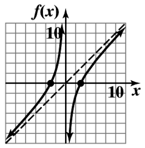
Partition numbers for f'': $x = 0$

Sign chart for f'':



Thus, the graph of f is concave upward on $(-\infty, 0)$ and is concave downward on $(0, \infty)$; there are no inflection points.

Step 4. Sketch the graph of f:



60. $f(x) = x + \dfrac{32}{x^2}$

For $|x|$ very large, $f(x) = x + \dfrac{32}{x^2} \approx x$. Thus, the line $y = x$ is an oblique asymptote.

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except 0.

(B) Intercepts: y-intercept: No y-intercept since f is not defined at 0.

x-intercepts: $x + \dfrac{32}{x^2} = 0$

$x^3 + 32 = 0$

$x = -\sqrt[3]{32}$

(C) Asymptotes:

Oblique asymptote: $y = x$

Vertical asymptote: $D(x) = x^2 = 0$, $x = 0$, $N(0) \neq 0$, so the line $x = 0$ is a vertical asymptote.

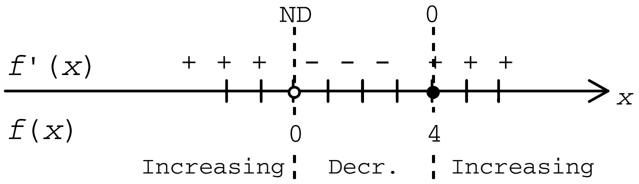
Step 2. Analyze $f'(x)$:

$$f'(x) = 1 - \frac{64}{x^3} = \frac{x^3 - 64}{x^3} = \frac{(x-4)(x^2+4x+16)}{x^3}$$

Critical values: $x = 4$

Partition numbers: $x = 4$, $x = 0$

Sign chart for f':



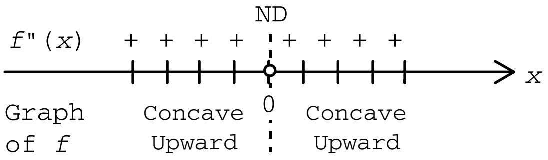
Thus, f is increasing on $(-\infty, 0)$ and on $(4, \infty)$, and is decreasing on $(0, 4)$; f has a local minimum at $x = 4$.

Step 3. Analyze $f''(x)$:

$$f''(x) = \frac{192}{x^4}$$

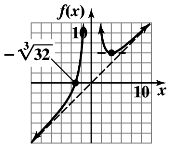
Partition numbers for f'': $x = 0$

Sign chart for f'':



Thus, the graph of f is concave upward on $(-\infty, 0)$ and on $(0, \infty)$; there are no inflection points.

Step 4. Sketch the graph of f:



62. $f(x) = x + \dfrac{27}{x^3}$

For $|x|$ very large, $f(x) = x + \dfrac{27}{x^3} \approx x$. Thus, the line $y = x$ is an oblique asymptote.

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except 0.

(B) Intercepts: y-intercept: No y-intercept since f is not defined at 0.

x-intercept: $x + \dfrac{27}{x^3} = \dfrac{x^4 + 27}{x^3}$, so none

(C) Asymptotes:

Oblique asymptote: $y = x$

Vertical asymptote: $D(x) = x^3 = 0$, $x = 0$, $N(0) \neq 0$, so the line $x = 0$ is a vertical asymptote.

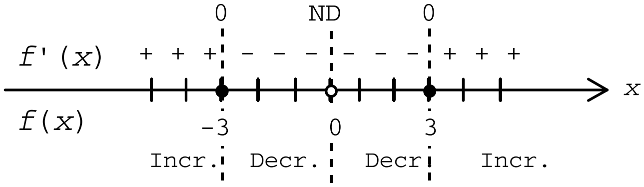
Step 2. Analyze $f'(x)$:

$$f'(x) = 1 - \frac{81}{x^4} = \frac{x^4 - 81}{x^4} = \frac{(x-3)(x+3)(x^2+9)}{x^4}$$

Critical values: $x = -3$, $x = 3$

Partition numbers: $x = -3$, $x = 0$, $x = 3$

Sign chart for f':



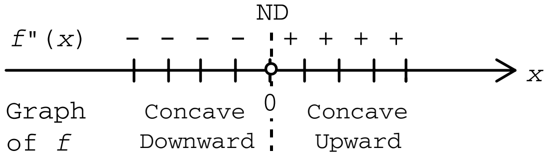
Thus, f is increasing on $(-\infty, -3)$ and on $(3, \infty)$ and is decreasing on $(-3, 0)$ and on $(0, 3)$; f has a local maximum at $x = -3$ and a local minimum at $x = 3$.

Step 3. Analyze $f''(x)$:

$$f''(x) = \frac{324}{x^5}$$

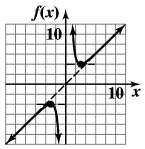
Partition numbers for f'': $x = 0$

Sign chart for f'':



Thus, the graph of f is concave downward on $(-\infty, 0)$ and concave upward on $(0, \infty)$; there are no inflection points.

Step 4. Sketch the graph of f:



64. $f(x) = x - \dfrac{16}{x^3}$

For $|x|$ very large, $f(x) = x - \dfrac{16}{x^3} \approx x$. Thus, the line $y = x$ is an oblique asymptote.

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except 0.

(B) Intercepts: y-intercept: No y-intercept since f is not defined at 0.

$$x\text{-intercept: } \frac{x^4 - 16}{x^3} = 0, x = -2, x = 2$$

(C) Asymptotes:

Oblique asymptote: $y = x$

Vertical asymptote: $D(x) = x^3 = 0$, $x = 0$; $N(0) \neq 0$, so the line $x = 0$ is a vertical asymptote.

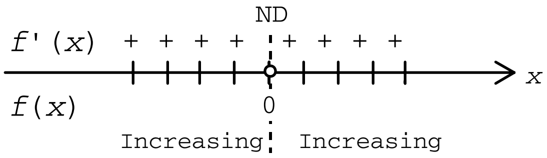
Step 2. Analyze $f'(x)$:

$f'(x) = 1 + \dfrac{48}{x^4} = \dfrac{x^4 + 48}{x^4}$

Critical values: none

Partition number: $x = 0$

Sign chart for f':



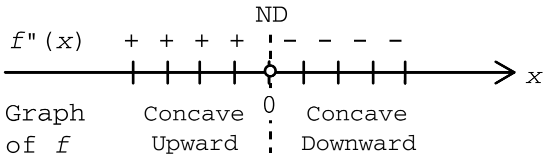
Thus, f is increasing on $(-\infty, 0)$ and on $(0, \infty)$; f has no local extrema.

Step 3. Analyze $f''(x)$:

$f''(x) = -\dfrac{192}{x^5}$

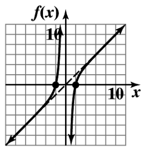
Partition numbers for f'': $x = 0$

Sign chart for f'':



Thus, the graph of f is concave upward on $(-\infty, 0)$ and is concave downward on $(0, \infty)$; there are no inflection points.

Step 4. Sketch the graph of f:



66. $f(x) = \dfrac{x^2+x-6}{x^2-x-12} = \dfrac{(x+3)(x-2)}{(x-4)(x+3)}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -3$, $x = 4$.

(B) Intercepts: y-intercept: $f(0) = \dfrac{-6}{-12} = \dfrac{1}{2}$

x-intercept: $x = 2$

(C) Asymptotes:

Horizontal asymptote: $\lim\limits_{x\to\infty} \dfrac{x^2+x-6}{x^2-x-12} = 1$, so the line $y = 1$ is a horizontal asymptote.

Vertical asymptote: $D(x) = (x - 4)(x + 3) = 0$, $x = -3$, $x = 4$, $D(-3) = D(4) = 0$, $N(-3) = 0$, $N(4) \neq 0$. So the line $x = 4$ is a vertical asymptote and we have to investigate the behavior of the function at $x = -3$.

$$\lim_{x\to-3} f(x) = \lim_{x\to-3} \frac{(x+3)(x-2)}{(x-4)(x+3)} = \lim_{x\to-3} \frac{x-2}{x-4} = \frac{5}{7}$$

Since the limit exists as x approaches -3, f does not have a vertical asymptote at $x = -3$.

Step 2. Analyze $f'(x)$:

$$f'(x) = \frac{(2x+1)(x^2-x-12)-(2x-1)(x^2+x-6)}{(x^2-x-12)^2} = \frac{-2(x+3)^2}{(x+3)^2(x-4)^2}$$

Thus, f is decreasing on $(-\infty, 4)$ and $(4, \infty)$.

Step 3. Analyze $f''(x)$:

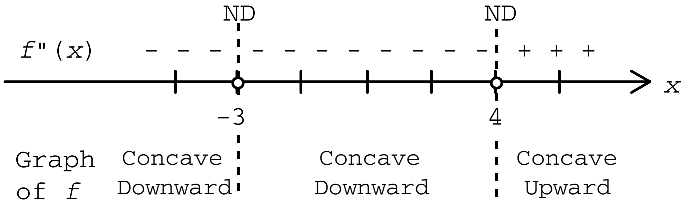
$$f''(x) = \frac{-4(x+3)[(x+3)^2(x-4)^2]-[2(x+3)(x-4)^2+2(x+3)^2(x-4)](-2(x+3)^2)}{(x+3)^4(x-4)^4}$$

$$= \frac{-4(x+3)^3(x-4)^2+4(x+3)^3(x-4)^2+4(x+3)^4(x-4)}{(x+3)^4(x-4)^4}$$

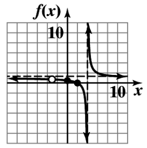
$$= \frac{4(x+3)^4(x-4)}{(x+3)^4(x-4)^4}$$

Partition numbers for f'': $x = -4$

Sign chart for f'':



Thus, the graph of f is concave downward on $(-\infty, 4)$ and concave upward on $(4, \infty)$; there are no inflection points.



<u>Step 4. Sketch the graph of *f*</u>:

<u>NOTE</u>: We may express *f* as $f(x) = \frac{x-2}{x-4}$ for $x \neq -3$ and $x \neq 4$. Then $f'(x) = -\frac{2}{(x-4)^2}$ for $x \neq -3$, $x \neq 4$ and $f''(x) = \frac{4}{(x-4)^3}$ for $x \neq -3$, $x \neq 4$.

68. $f(x) = \frac{2x^2+11x+14}{x^2-4} = \frac{(2x+7)(x+2)}{(x-2)(x+2)}$

<u>Step 1. Analyze *f*(*x*)</u>:

(A) Domain: All real numbers except $x = -2$, $x = 2$.

(B) Intercepts: *y*-intercept: $f(0) = -\frac{7}{2}$

x-intercept: $x = -\frac{7}{2}$

(C) Asymptotes:

<u>Horizontal asymptote</u>: $\lim_{x\to\infty} \frac{2x^2+11x+14}{x^2-4} = 2$, so the line $y = 2$ is a horizontal asymptote.

<u>Vertical asymptote</u>: $D(x) = (x-2)(x+2) = 0$, $x = 2$, $x = -2$, $D(2) = D(-2) = 0$, $N(2) \neq 0$, $N(-2) = 0$. So, the line $x = 2$ is a vertical asymptote and we have to investigate the behavior of the function at $x = -2$.

$$\lim_{x\to-2} f(x) = \lim_{x\to-2} \frac{(2x+7)(x+2)}{(x-2)(x+2)} = \lim_{x\to-2} \frac{2x+7}{x-2} = -\frac{3}{4}$$

Since the limit exists as *x* approaches -2, *f* does not have a vertical asymptote at $x = -2$.

<u>Step 2. Analyze *f*'(*x*)</u>:

We can express *f*(*x*) for $x \neq -2$ and $x \neq 2$ as $f(x) = \frac{2x+7}{x-2}$. Thus,

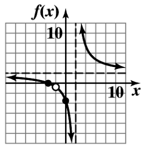
$$f'(x) = \frac{2(x-2)-(2x+7)}{(x-2)^2} = \frac{2x-4-2x-7}{(x-2)^2} = \frac{-11}{(x-2)^2} < 0$$

for all real numbers except $x = -2$ and $x = 2$. Thus, *f* is decreasing on (-∞, 2) and on (2, ∞).

<u>Step 3. Analyze *f*''(*x*)</u>:

$f''(x) = \frac{22}{(x-2)^3}$ for $x \neq -2$ and $x \neq 2$. Thus, $f''(x) < 0$ on (-∞, 2) and $f''(x) > 0$ on (2, ∞). The function *f* is concave downward on (-∞, 2) and concave upward on (2, ∞).

<u>Step 4. Sketch the graph of *f*</u>:



70. $f(x) = \frac{x^3-5x^2-6x}{x^2+3x+2} = \frac{x(x^2-5x-6)}{(x+1)(x+2)} = \frac{x(x-6)(x+1)}{(x+1)(x+2)}$

<u>Step 1. Analyze *f*(*x*)</u>:

(A) Domain: All real numbers except $x = -2$, $x = -1$.

(B) Intercepts: y-intercept: $f(0) = 0$
x-intercepts: $x = 0, x = 6$

(C) Asymptotes:
Horizontal asymptotes: None

Vertical asymptotes: $D(x) = (x + 1)(x + 2) = 0$, $x = -1$, $x = -2$, $D(-1) = 0$, $D(-2) = 0$, $N(-1) = 0$, $N(-2) \neq 0$. So the line $x = -2$ is a vertical asymptote. Since

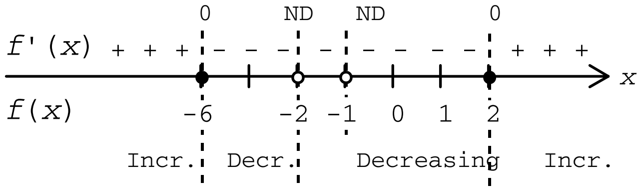
$$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{x(x-6)}{x+2} = 7,$$ f does not have a vertical asymptote at $x = -1$.

Step 2. Analyze $f'(x)$:

We may express $f(x)$ for $x \neq -2$ and $x \neq -1$ as $f(x) = \frac{x(x-6)}{x+2}$. Thus,

$$f'(x) = \frac{(2x-6)(x+2)-x(x-6)}{(x+2)^2} = \frac{2x^2-2x-12-x^2+6x}{(x+2)^2} = \frac{(x+6)(x-2)}{(x+2)^2}$$

Sign chart for $f'(x)$:



Thus, f is increasing on (-∞, -6) and (2, ∞) and is decreasing on (-6, -2) and (-2, 2). The function f has a local maximum at $x = -6$ and a local minimum at $x = 2$.

Step 3. Analyze $f''(x)$:

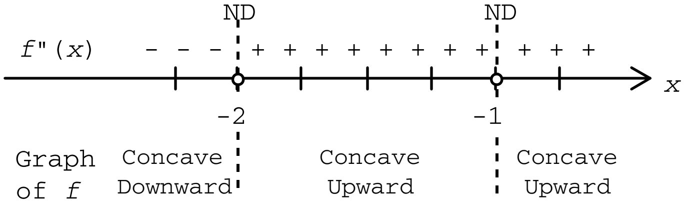
$$f''(x) = \frac{(2x+4)(x+2)^2 - 2(x+2)(x+6)(x-2)}{(x+2)^4}$$

$$= \frac{(2x+4)(x+2)-2(x+6)(x-2)}{(x+2)^3} = \frac{2x^2+8x+8-2x^2-8x+24}{(x+2)^3}$$

$$= \frac{32}{(x+2)^3} \text{ for } x \neq -2, x \neq -1$$

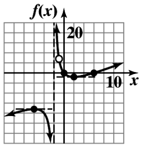
Partition number for f'': $x = -2$

Sign chart for f'':



Thus, the graph of f is concave downward on (-∞, -2) and concave upward on (-2, ∞). There are no inflection points.

Step 4. Sketch the graph of f:



72. $f(x) = \dfrac{x^2+x-2}{x^2+4x+4} = \dfrac{(x+2)(x-1)}{(x+2)^2}$

Step 1. Analyze $f(x)$:

(A) Domain: All real numbers except $x = -2$.

(B) Intercepts: y-intercept $f(0) = -\dfrac{1}{2}$

x-intercepts: $x = 1$

(C) Asymptotes:

Horizontal asymptote: $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^2+x-2}{x^2+4x+4} = 1$, thus, the line $y = 1$ is a horizontal asymptote.

Vertical asymptotes: $D(x) = (x + 2)^2 = 0$, $x = -2$, $D(-2) = 0$, $N(-2) = 0$. Since $\lim\limits_{x\to-2} f(x) = \lim\limits_{x\to-2} \dfrac{(x+2)(x-1)}{(x+2)^2} = \lim\limits_{x\to-2} \dfrac{x-1}{x+2} = \pm\infty$

Thus, the line $x = -2$ is a vertical asymptote.

Step 2. Analyze $f'(x)$:

We may express $f(x)$ for $x \neq -2$ as

$$f(x) = \frac{x-1}{x+2}$$

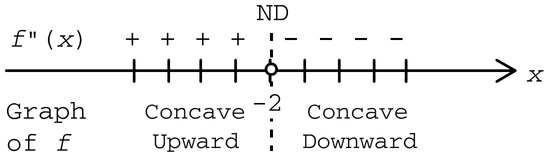
$$f'(x) = \frac{x+2-(x-1)}{(x+2)^2} = \frac{4}{(x+2)^2}$$ > 0 for all real numbers except $x = -2$. Thus, f is increasing on $(-\infty, -2)$ and $(-2, \infty)$.

Step 3. Analyze $f''(x)$:

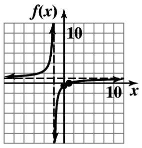
$$f''(x) = \frac{-8}{(x+2)^3}$$

Partition point for f'': $x = -2$

Sign chart for f'':



Step 4. Sketch the graph of f:



74. (A) $P(x) = R(x) - C(x) = 1{,}296x - 0.12x^3 - (830 + 396x)$

$= 900x - 0.12x^3 - 830,\ 0 \le x \le 80$

(B) Step 1. Analyze $P(x)$:

(A) Domain: [0, 80]

(B) Intercepts: There are no intercepts on [0, 80].

(C) Asymptotes: $P(x)$ is a polynomial and hence no asymptotes.

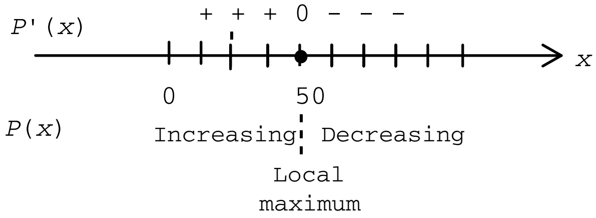
Step 2. Analyze $P'(x)$:

$P'(x) = -0.36x^2 + 900$

Critical values: [on [0, 80]]: $x = 50$

Partition number: $x = 50$

Sign chart for P':



Test Numbers

x	$P'(x)$
1	899.64(+)
51	−36.36(−)

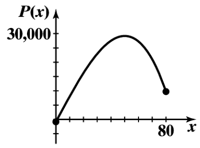
Thus, $P(x)$ is increasing on (0, 50) and decreasing on (50, 80), P has a local maximum at $x = 50$.

Step 3. Analyze $P''(x)$:

$P''(x) = -0.72x < 0$ for $0 < x < 80$

Thus, the graph of P is concave downward on (0, 80).

Step 4. Sketch the graph of P:



76. $N(t) = \frac{100t}{t+9}, t \geq 0$

(A) $N'(t) = \frac{100(t+9) - 100t}{(t+9)^2} = \frac{900}{(t+9)^2}$

$N'(t) > 0$ for $t \geq 0$. Thus, N is increasing on $(0, \infty)$.

(B) From (A), $N'(t) = 900(t + 9)^{-2}$. Thus,

$N''(t) = -1{,}800(t + 9)^{-3} = \frac{1{,}800}{(t+9)^3}$.

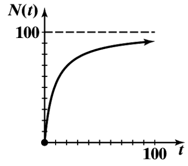
$N''(t) < 0$ for $t \geq 0$, and the graph of N is concave downward on $(0, \infty)$.

(C) $\lim_{t\to\infty} N(t) = 100$, thus $y = 100$ is a horizontal asymptote.

There are no vertical asymptotes on $(0, \infty)$.

(D) y-intercept: $N(0) = 0$
x-intercept: $N(t) = 0$, $t = 0$
Thus, (0, 0) is both an x and a y-intercept of the graph.

(E) The graph is:



78. $L(x) = 2x + \dfrac{20,000}{x}, x > 0$

(A) Step 1. Analyze $L(x)$:

(A_1) Domain: $x > 0$ or $(0, \infty)$

(B_1) Intercepts: No y-intercept

No x-intercepts

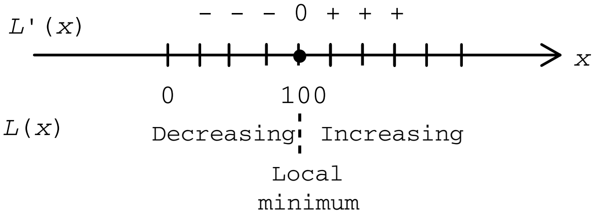
(C_1) Asymptotes: For very large x, $L(x) \approx 2x$. Thus, $y = 2x$ is an oblique asymptote. As $x \to 0$, $L(x) \to \infty$; thus, $x = 0$ is a vertical asymptote.

Step 2. Analyze $L'(x)$:

$L'(X) = 2 - \dfrac{20,000}{x^2} = \dfrac{2x^2 - 20,000}{x^2}$

Critical value: $x = 100$

Sign chart for $L'(x)$:



Test Numbers

x	$L'(x)$
99	$-398(-)$
101	$402(+)$

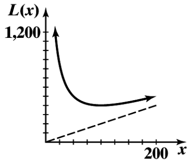
Thus, L is decreasing on $(0, 100)$ and increasing on $(100, \infty)$; $x = 100$ is a local minimum.

Step 3. Analyze $L''(x)$:

$L''(x) = \dfrac{40,000}{x^3}, x > 0$

$L''(x) > 0$ on $(0, \infty)$. Thus, the graph of L is concave upward on $(0, \infty)$.

Step 4. Sketch the graph of L:



(B) 100' by 200', with the longest side parallel to the building.

80. $C(x) = 500 + 2x + 0.2x^2, 0 < x < \infty$.

(A) The average cost function is: $\overline{C}(x) = \dfrac{500}{x} + 2 + 0.2x$.

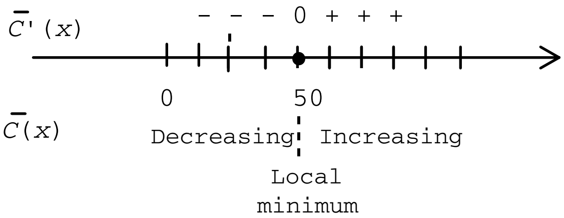
Now, $\overline{C}'(x) = -\dfrac{500}{x^2} + 0.2 = \dfrac{-500 + 0.2x^2}{x^2}$

$$= \frac{0.2(x^2 - 2,500)}{x^2}$$

$$= \frac{0.2(x - 50)(x + 50)}{x^2}$$

Critical value: $x = 50$ [in $(0, \infty)$]

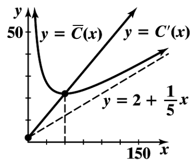
Sign chart for $\overline{C}\,'$:



Test Numbers

x	$\overline{C}\,'(x)$
49	(–)
51	(+)

Thus, $\overline{C}$ is decreasing on (0, 50) and increasing on (50, ∞); $\overline{C}$ has a minimum at $x = 50$.

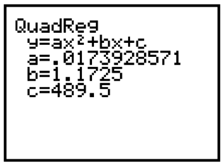


Since $\overline{C}\,''(x) = \dfrac{1{,}000}{x^3} > 0$ for $0 < x < \infty$, the graph of $\overline{C}$ is concave upward on (0, ∞). The line $x = 0$ is a vertical asymptote and the line $y = 2 + 0.2x$ is an oblique asymptote for the graph of $\overline{C}$.

(B) The minimum average cost is:

$$\overline{C}\,(50) = \frac{500}{50} + 2 + (0.2) = 22$$

82. (A)



```
QuadReg
 y=ax²+bx+c
 a=.0173928571
 b=1.1725
 c=489.5
```

(B) The average cost function $\overline{y}\,(x) = \dfrac{y(x)}{x}$ where $y(x)$ is the regression equation found in part (A). The minimum average cost is \$7.01 when 168 pizzas are produced daily.

84. $S(w) = \dfrac{26 + 0.06w}{w}$, $w \geq 5$

Step 1. Analyze $S(w)$:

(A) Domain: $w \geq 5$, i.e., [5, ∞)

(B) Intercepts: None on [5, ∞)

(C) Asymptotes: Vertical asymptote: $w = 0$ (y axis)
Horizontal asymptote: If the domain was not restricted, the horizontal asymptote would be: $S = 0.06$, but since $w \geq 5$, the graph of $S(w)$ has no horizontal asymptote.

Step 2. Analyze $S'(w)$:

$$S(w) = \frac{26}{w} + 0.06 = 26w^{-1} + 0.06$$

$$S'(w) = -26w^{-2} = -\frac{26}{w^2} < 0 \text{ for } w \geq 5$$

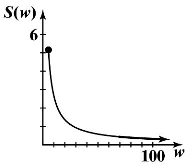
Thus, $S(w)$ is decreasing on [5, ∞).

Step 3. Analyze $S''(w)$:

$$S''(w) \quad = 52w^{-3} = \frac{52}{w^3} > 0 \text{ for } w \geq 5$$

Thus, S is concave upward on [5, ∞).

Step 4. Sketch the graph of S:



EXERCISE 5-5

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2. Interval [2, 8]; absolute minimum: $f(7) = 5$;
absolute maximum: $f(3) = 9$

4. Interval [2, 10]; absolute minimum: $f(7) = 5$;
absolute maximum: $f(10) = 14$

6. Interval [0, 9]; absolute minimum: $f(0) = 0$;
absolute maximum: $f(3) = f(9) = 9$

8. Interval [0, 2]; absolute minimum: $f(0) = 0$;
absolute maximum: $f(2) = 8$

10. Interval [5, 8]; absolute minimum: $f(7) = 5$
absolute maximum: $f(5) = 7$

12. $f(x) = -3x + 20,\ I = [-2, 6]$

$f'(x) = -3$, so there are no critical values on I.

$f(-2) = 26$ and $f(6) = 2$. Therefore $f(6) = 2$ is the absolute minimum and $f(-2) = 26$ is the absolution maximum.

14. $f(x) = \ln x,\ I = [1, 2]$

$f'(x) = \frac{1}{x}$, so there are no critical values on I.

$f(1) = \ln 1 = 0$ and $f(2) = \ln 2 \approx 0.693$. Therefore $f(1) = 0$ is the absolute minimum and $f(2) \approx 0.693$ is the absolution maximum.

16. $f(x) = x^2 - 6x + 7,\ I = [0, 10]$

$f'(x) = 2x - 6$, so $x = 3$ is the only critical value on I.

$f(0) = 7,\ f(3) = -2$ and $f(10) = 47$. Therefore $f(3) = -2$ is the absolute minimum and $f(10) = 47$ is the absolution maximum.

18. $f(x) = x^2 + 4x - 3$, $I = (-\infty, \infty)$
$f'(x) = 2x + 4 = 2(x + 2)$
$x = -2$ is the only critical value on I and $f(-2) = -7$.
$f''(x) = 2$
$f''(-7) = 2 > 0$. Therefore, $f(-2) = -7$ is the absolute minimum. The function does not have an absolute maximum.

20. $f(x) = -x^2 + 2x + 4$, $I = (-\infty, \infty)$
$f'(x) = -2x + 2 = -2(x - 1)$
$x = 1$ is the only critical value on I and $f(1) = 5$.
$f''(x) = -2$
$f''(1) = -2 < 0$. Thus, $f(1) = 5$ is the absolute maximum. The function does not have an absolute minimum.

22. $f(x) = -x^3 - 2x$, $I = (-\infty, \infty)$
$f'(x) = -3x^2 - 2 < 0$ for all x, i.e. no critical values.
Therefore, the function f is decreasing on I, and there are no extrema.

24. $f(x) = x^4 - 4x^3$, $I = (-\infty, \infty)$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$

$x = 0$ and $x = 3$ are critical values on I and $f(0) = 0$,

$f(3) = -27$

$f''(x) = 12x^2 - 24x = 12x(x - 2)$

$f''(0) = 0, f''(3) = 36 > 0$. Therefore, $f(3) = -27$ is the absolute minimum. The function does not have an absolute maximum.

26. $f(x) = x + \dfrac{25}{x}$, $I = (-\infty, 0) \cup (0, \infty)$

$$f'(x) = 1 - \frac{25}{x^2} = \frac{x^2 - 25}{x^2} = \frac{(x-5)(x+5)}{x^2}$$

Critical values: $x = -5$, $x = 5$, and $f(-5) = -10, f(5) = 10$

The function does not have an absolute minimum or absolute maximum on I.

28. $f(x) = \dfrac{1}{x^2+1}$, $I = (-\infty, \infty)$

$$f'(x) = -\frac{2x}{(x^2+1)^2}$$

$x = 0$ is the only critical value on I and $f(0) = 1$.

$$f''(x) = \frac{-2(x^2+1)^2 + 8x^2(x^2+1)}{(x^2+1)^4} = \frac{-2(x^2+1) + 8x^2}{(x^2+1)^3} = \frac{6x^2 - 2}{(x^2+1)^3}$$

$f''(0) = -2 < 0$. Therefore, $f(0) = 1$ is the absolute maximum. The function does not have an absolute minimum.

30. $f(x) = \dfrac{-8x}{x^2+4}$, $I = (-\infty, \infty)$

$$f'(x) = \frac{-8(x^2+4) + 16x^2}{(x^2+4)^2} = \frac{8(x^2-4)}{(x^2+4)^2} = \frac{8(x-2)(x+2)}{(x^2+4)^2}$$

Critical values: $x = -2$, $x = 2$, and $f(-2) = 2, f(2) = -2$.

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to\infty} f(x) = 0$, therefore, $f(-2) = 2$ is the absolute maximum and $f(2) = -2$ is the absolute minimum for f.

32. $f(x) = \dfrac{9 - x^2}{x^2+4}$, $I = (-\infty, \infty)$

$$f'(x) = \frac{-2x(x^2+4) - 2x(9-x^2)}{(x^2+4)^2} = \frac{-2x^3 - 8x - 18x + 2x^3}{(x^2+4)^2} = \frac{-26x}{(x^2+4)^2}$$

$x = 0$ is the only critical value on I and $f(0) = \dfrac{9}{4}$.

$f''(x) = \dfrac{26(3x^2-4)}{(x^2+4)^3}$ and $f''(0) = -\dfrac{13}{8} < 0$.

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to\infty} f(x) = -1$, therefore, $f(0) = \dfrac{9}{4}$ is the absolute maximum.

34. $f(x) = 6x - x^2 + 4, I = [0, \infty)$
$f'(x) = 6 - 2x = 2(3 - x)$
$x = 3$ is the only critical value on I and $f(3) = 13$. Also, $f(0) = 4$.
$f''(x) = -2$
$f''(3) = -2 < 0$. Therefore, $f(3) = 13$ is the absolute maximum.

36. $f(x) = x^3 - 6x^2, I = [0, \infty)$
$f'(x) = 3x^2 - 12x = 3x(x - 4)$
Critical values: $x = 0, x = 4$ and $f(0) = 0, f(4) = -32$.
$f''(x) = 6x - 12 = 6(x - 2)$
$f''(0) = -12 < 0, f''(4) = 12 > 0$. Therefore, $f(4) = -32$ is the absolute minimum.

38. $f(x) = (2 - x)(x + 1)^2, I = [0, \infty)$
$$\begin{aligned} f'(x) = -(x + 1)^2 + 2(2 - x)(x + 1) &= (x + 1)[2(2 - x) - (x + 1)] \\ &= (x + 1)[3(1 - x)] \\ &= 3(x + 1)(1 - x) \end{aligned}$$
Critical values: $x = -1, x = 1$ and $f(-1) = 0, f(1) = 4$.
Also, $f(0) = 2$ and $\lim_{x\to\infty} f(x) = -\infty$. Therefore, f does not have an absolute minimum.

40. $f(x) = 4x^3 - 8x^4, I = (0, \infty)$
$f'(x) = 12x^2 - 32x^3 = 4x^2(3 - 8x)$
Critical value: $x = \frac{3}{8}$ [Note: $x = 0$ is not a critical value, since the domain of f is $x > 0$.]
$$\begin{aligned} f''(x) &= 24x - 96x^2 \\ &= 24x(1 - 4x) \end{aligned}$$
$f''\left(\frac{3}{8}\right) = -\frac{9}{2} < 0$. Therefore, the absolute maximum of f is $f\left(\frac{3}{8}\right) = \frac{27}{512}$.

42. $f(x) = 4 + x + \frac{9}{x}, x > 0$
$f'(x) = 1 - \frac{9}{x^2} = \frac{x^2 - 9}{x^2} = \frac{(x-3)(x+3)}{x^2}$
critical value: $x = 3$ [Note: $x = -3$ is not a critical value, since the domain of f is $x > 0$.]
$f''(x) = \frac{18}{x^3} > 0$. Thus, the absolute minimum of f is $f(3) = 10$.

44. $f(x) = 20 - 4x - \frac{250}{x^2}, x > 0$
$f'(x) = -4 + \frac{500}{x^3} = \frac{-4x^3 + 500}{x^3} = \frac{-4(x^3 - 125)}{x^3}$
critical value: $x = 5$
$f''(x) = -\frac{1{,}500}{x^4} < 0$. Thus, the absolute maximum of f is $f(5) = -10$.

46. $f(x) = 2x - \frac{5}{x} + \frac{4}{x^3}$, $I = (0, \infty)$

$$f'(x) = 2 + \frac{5}{x^2} - \frac{12}{x^4} = \frac{2x^4 + 5x^2 - 12}{x^4} = \frac{2(x^2+4)\left(x^2 - \frac{3}{2}\right)}{x^4} = \frac{2(x^2+4)\left(x - \sqrt{\frac{3}{2}}\right)\left(x + \sqrt{\frac{3}{2}}\right)}{x^4}$$

Critical value: $x = \sqrt{\frac{3}{2}}$ $\left(x = -\sqrt{\frac{3}{2}} \text{ is not in the domain of } f.\right)$

$$f''(x) = -\frac{10}{x^3} + \frac{48}{x^5}$$

$$f''\left(\sqrt{\frac{3}{2}}\right) = -\frac{10}{\frac{3}{2}\sqrt{\frac{3}{2}}} + \frac{48}{\frac{9}{4}\sqrt{\frac{3}{2}}} = -\frac{40}{9}\sqrt{\frac{3}{2}} + \frac{128}{9}\sqrt{\frac{3}{2}} = \frac{88}{9}\sqrt{\frac{3}{2}} > 0.$$

Thus, the absolute minimum of f is $f\left(\sqrt{\frac{3}{2}}\right) = \frac{4}{9}\sqrt{\frac{3}{2}} \approx 0.544$

48. $f(x) = \frac{x^4}{e^x}$

$$f'(x) = \frac{\left(\frac{d}{dx}x^4\right)e^x - \left(\frac{d}{dx}e^x\right)x^4}{(e^x)^2} = \frac{4x^3e^x - x^4e^x}{(e^x)^2} = \frac{4x^3 - x^4}{e^x} = \frac{x^3(4-x)}{e^x}$$

$f'(x) = 0$ if $x = 4$ (since $x > 0$); $f'(x) > 0$ on $(0, 4)$ and $f'(x) < 0$ on $(4, \infty)$. Thus f is increasing on $(0, 4)$ and decreasing on $(4, \infty)$. Therefore, maximum $f(x) = f(4) = \frac{4^4}{e^4} \approx 4.689$.

<u>Note</u>: $f''(x) = \frac{(12x^2 - 4x^3)e^x - (4x^3 - x^4)e^x}{(e^x)^2} = \frac{x^4 - 8x^3 + 12x^2}{e^x}$

and $f''(4) = -\frac{64}{e^4} < 0$. Since $x = 4$ is the only critical value in

$(0, \infty)$, and $f''(4) < 0$, the function f has absolute maximum at $x = 4$.

50. $f(x) = \frac{e^x}{x}$

$$f'(x) = \frac{\left(\frac{d}{dx}e^x\right)x - \left(\frac{d}{dx}x\right)e^x}{x^2} = \frac{xe^x - e^x}{x^2} = \frac{(x-1)e^x}{x^2}$$

$f'(x) = 0$ if $x = 1$.

$f''(x) = \frac{(e^x + (x-1)e^x)x^2 - 2x(x-1)e^x}{x^4}$ and $f''(1) = e > 0$.

Thus, $f(x)$ has an absolute minimum at $x = 1$, $f(1) = \frac{e}{1} = e \approx 2.718$.

52. $f(x) = 4x \ln x - 7x$

$$f'(x) = 4 \ln x + 4x \frac{d}{dx} \ln x - 7$$

$$= 4 \ln x + 4x\left(\frac{1}{x}\right) - 7 = 4 \ln x + 4 - 7 = 4 \ln x - 3, x > 0$$

Critical values: $f'(x) =$ $4 \ln x - 3 = 0$

$$\ln x = \frac{3}{4} = 0.75$$

$$x = e^{0.75}$$

Thus, $x = e^{0.75}$ is the only critical value of f on $(0, \infty)$.

Now, $f''(x) = \frac{d}{dx}(4 \ln x - 3) = \frac{4}{x}$ and

$f''(e^{0.75}) = \frac{4}{e^{0.75}} > 0.$

Therefore, f has a minimum value at $x = e^{0.75}$,

and $f(e^{0.75})$ $= 4(e^{0.75})\ln e^{0.75} - 7(e^{0.75})$

$$= 4(e^{0.75})(0.75) - 7e^{0.75}$$

$$= 3e^{0.75} - 7e^{0.75} = -4e^{0.75} \approx -8.468$$

54. $f(x) = x^3(\ln x - 2), x > 0$

$$f'(x) = \left(\frac{d}{dx}(x^3)\right)(\ln x - 2) + x^3\left(\frac{d}{dx}(\ln x - 2)\right)$$

$$= 3x^2(\ln x - 2) + x^3\left(\frac{1}{x}\right)$$

$$= 3x^2(\ln x - 2) + x^2$$

$$= x^2[3 \ln x - 6 + 1] = x^2(3 \ln x - 5)$$

Critical values: $f'(x) = x^2(3 \ln x - 5) = 0$

$$3 \ln x - 5 = 0$$

$$\ln x = \frac{5}{3}$$

$$x = e^{5/3}$$

Thus, $x = e^{5/3}$ is the only critical value of f on $(0, \infty)$.

Now, $f''(x) = \left(\frac{d}{dx}x^2\right)(3 \ln x - 5) + x^2\left(\frac{d}{dx}(3\ln x - 5)\right)$

$$= 2x(3 \ln x - 5) + x^2\left(\frac{3}{x}\right)$$

$$= 2x(3 \ln x - 5) + 3x$$

$$= x(6 \ln x - 10 + 3)$$

$$= x(6 \ln x - 7), x > 0$$

$$f''(e^{5/3}) = e^{5/3}(6\ln(e^{5/3}) - 7)$$
$$= e^{5/3}\left(6\left(\frac{5}{3}\right)-7\right)$$
$$= e^{5/3}(10 - 7) = 3e^{5/3} > 0$$

Therefore, f has a minimum value at

$x = e^{5/3}$ and $f(e^{5/3}) = (e^{5/3})^3(\ln(e^{5/3}) - 2)$
$$= e^5\left(\frac{5}{3}-2\right) = -\frac{e^5}{3} \approx -49.471$$

is the absolute minimum of f.

56. $f(x) = \ln(x^2e^{-x})$

$$f'(x) = \frac{\frac{d}{dx}(x^2e^{-x})}{x^2e^{-x}} = \frac{2xe^{-x} - x^2e^{-x}}{x^2e^{-x}} = \frac{2-x}{x} = \frac{2}{x} - 1$$

$f'(x) = 0$ if $x = 2$. This is the only critical value on $(0, \infty)$.

$f''(x) = -\frac{2}{x^2} < 0$ for all x in $(0, \infty)$.

Thus, the absolute maximum of f is

$f(2) = \ln(4e^{-2}) = \ln 4 - 2 \approx -0.6137$.

58. $f(x) = 2x^3 - 3x^2 - 12x + 24$

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1)$

critical values: $x = -1, 2$

(A) On the interval [-3, 4]:

$f(-3) = -21$
$f(-1) = 31$
$f(2) = 4$
$f(4) = 56$

Thus, the absolute maximum of f is $f(4) = 56$, and the absolute minimum of f is $f(-3) = -21$.

(B) On the interval [-2, 3]:

$f(-2) = 20$
$f(-1) = 31$
$f(2) = 4$
$f(3) = 15$

Absolute maximum of f: $f(-1) = 31$
Absolute minimum of f: $f(2) = 4$

(C) On the interval [-2, 1]:

$f(-2) = 20$
$f(-1) = 31$
$f(1) = 11$

Absolute maximum of f: $f(-1) = 31$
Absolute minimum of f: $f(1) = 11$

60. $f(x) = x^4 - 8x^2 + 16$

$f'(x) = 4x^3 - 16x = 4x(x^2 - 4)$
$= 4x(x - 2)(x + 2)$

critical values: $x = -2, 0, 2$

(A) On the interval [-1, 3]:

$f(-1) = 9$
$f(0) = 16$
$f(2) = 0$
$f(3) = 25$

Absolute maximum of f: $f(3) = 25$
Absolute minimum of f: $f(2) = 0$

(B) On the interval [0, 2]:

$f(0) = 16$
$f(2) = 0$

Absolute maximum of f: $f(0) = 16$
Absolute minimum of f: $f(2) = 0$

(C) On the interval [-3, 4]:

$f(-3) = 25$
$f(-2) = 0$
$f(0) = 16$
$f(2) = 0$
$f(4) = 144$

Absolute maximum of f: $f(4) = 144$
Absolute minimum of f: $f(-2) = f(2) = 0$

62. $f(x) = x^4 - 18x^2 + 32$

$f'(x) = 4x^3 - 36x \quad = 4x(x^2 - 9)$
$\quad = 4x(x - 3)(x + 3)$

Critical values: $x = -3$, $x = 0$, $x = 3$

(A) On the interval [-4, 4]:

$f(-4) = 0$
$f(-3) = -49$
$f(0) = 32$
$f(3) = -49$
$f(4) = 0$

Absolute maximum of f: $f(0) = 32$
Absolute minimum of f: $f(-3) = f(3) = -49$

(B) On the interval [-1, 1]:

$f(-1) = 15$
$f(0) = 32$
$f(1) = 15$

Absolute maximum of f: $f(0) = 32$
Absolute minimum of f: $f(-1) = f(1) = 15$

(C) On the interval [1, 3]:

$f(1) = 15$
$f(3) = -49$

Absolute maximum of f: $f(1) = 15$
Absolute minimum of f: $f(3) = -49$

64. Neither a local maximum nor a local minimum at $x = 4$; $x = 4$ is not a critical value.

66. f has a local maximum at $x = -1$.

68. Unable to determine from the information given.

70. f has a local minimum at $x = 1$.

EXERCISE 5-6

2. Let one number = x. Then the other number = $21 - x$

Let $f(x) = x(21 - x) = 21x - x^2$
$f'(x) = 21 - 2x = 2(10.5 - x)$
critical value: $x = 10.5$
$f''(x) = -2 < 0$
Thus, the function f has its maximum at $x = 10.5$, so each number must be 10.5.

4. Let one number = x. Then the other number = $x + 21$.

Let $f(x) = x(x + 21) = x^2 + 21x$
$f'(x) = 2x^2 + 21 = 2(x + 10.5)$
critical value: $x = -10.5$
$f''(x) = 2 > 0$
Thus, the function f has its minimum at $x = -10.5$, so one number is -10.5 and the other is 10.5.

6. Let x be the first number and y be the second. Then $xy = 21$, $y = \frac{21}{x}$, and the sum of the numbers is $x + \frac{21}{x}$.

Let $f(x) = x + \frac{21}{x}$ and minimize f.

$f'(x) = 1 - \frac{21}{x^2} = \frac{x^2 - 21}{x^2}$

The only critical value: $x = \sqrt{21}$, since the numbers must be positive

$f''(x) = \frac{42}{x^3} > 0$ for $x > 0$.

Thus, the minimum product occurs when $x = \sqrt{21}$ and $y = \frac{21}{\sqrt{21}} = \sqrt{21}$ and is $\left(\sqrt{21}\right)\left(\sqrt{21}\right) = 21$.

8. Let x be the length and y be the width. Then $A = xy = 108$ and $y = \frac{108}{x}$,

and the perimeter is $2(x + y) = 2\left(x + \frac{108}{x}\right)$.

Let $f(x) = 2\left(x + \frac{108}{x}\right)$ and minimize f.

$f'(x) = 2 - \frac{216}{x^2} = \frac{2x^2 - 216}{x^2} = \frac{2(x^2 - 108)}{x^2} = \frac{2(x - \sqrt{108})(x + \sqrt{108})}{x^2} = \frac{2(x - 6\sqrt{3})(x + 6\sqrt{3})}{x^2}$
critical value: $x = 6\sqrt{3}$

$f''(x) = \frac{432}{x^3} > 0$ for $x > 0$.

Thus, the least perimeter occurs when $x = 6\sqrt{3}$ and $y = \frac{108}{6\sqrt{3}} = \frac{108 \cdot \sqrt{3}}{6 \cdot 3} = 6\sqrt{3}$.

Therefore, we will have a square with sides of $6\sqrt{3}$ feet; minimum perimeter = $24\sqrt{3}$ feet.

10. Let x be the length and y be the width. Then $P = 2x + 2y = 76$ so $y = 38 - x$,

and the area is $xy = x(38 - x)$.

Let $f(x) = x(38 - x) = 38x - x^2$ and minimize f.
$f'(x) = 38 - 2x$
critical value: $x = 19$
$f''(x) = -2 < 0$ for $x > 0$.

Thus, the greatest area occurs when $x = 19$ and $y = 19$.
Therefore, we will have a square with sides of 19 feet; maximum area = 361 square feet.

12. (A) Revenue $R(x) = x \cdot p(x) = x(400 - 0.4x) = 400x - 0.4x^2$
$R'(x) = 400 - 0.8x = 0$ implies $x = 500$.
$R''(x) = -0.8 < 0$
Thus, $R''(500) = -0.8 < 0$ and we conclude that R has an absolute maximum at $x = 500$. The maximum revenue is

$R(500) = 400(500) - 0.4(500)^2 = \$100,000$
when 500 cameras are produced and sold for \$200 each.

(B) Profit $P(x) = R(x) - C(x) = 400x - 0.4x^2 - (2,000 + 160x)$
$= 240x - 0.4x^2 - 2,000$

$P'(x) = 240 - 0.8x$
Now, $240 - 0.8x = 0$ implies $x = 300$.

$P''(x) = -0.8$ and $P''(300) = -0.8 < 0$. Thus, the maximum profit occurs when 300 cameras are produced weekly. The maximum profit is

$P(300) = 240(300) - 0.4(300)^2 - 2,000 = \$34,000$. The price that the company should charge is $p(300) = 400 - 0.4(300) = \$280$ for each camera.

14. (A) Revenue $R(x) = x{\cdot}p(x) = x\left(200 - \frac{x}{50}\right) = 200x - \frac{x^2}{50}$, $0 \le x \le 10,000$

$R'(x) = 200 - \frac{x}{25}$

Now $R'(x) = 200 - \frac{x}{25} = 0$ implies $x = 5,000$.

$R''(x) = -\frac{1}{25} < 0$.

Thus, $R''(5,000) = -\frac{1}{25} < 0$ and we conclude that R has an absolute maximum at $x = 5,000$.

The maximum revenue is

$$R(5,000) = 200(5,000) - \frac{(5,000)^2}{50} = \$500,000$$

(B) Profit $P(x) = R(x) - C(x) = 200x - \frac{x^2}{50} - (60,000 + 60x)$

$$= 140x - \frac{x^2}{50} - 60,000$$

$P'(x) = 140 - \frac{x}{25}$

Now $140 - \frac{x}{25} = 0$ implies $x = 3,500$.

$P''(x) = -\frac{1}{25}$ and $P''(3,500) = -\frac{1}{25} < 0$. Thus, the maximum profit occurs when 3,500 television sets are produced. The maximum profit is

$$P(3,500) = 140(3,500) - \frac{(3,500)^2}{50} - 60,000 = \$185,000$$

the price that the company should charge is

$$p(3,500) = 200 - \frac{3,500}{50} = \$130 \text{ for each set.}$$

(C) If the government taxes the company \$5 for each set, then the profit $P(x)$ is given by

$$P(x) = 200x - \frac{x^2}{50} - (60{,}000 + 60x) - 5x$$

$$= 135x - \frac{x^2}{50} - 60{,}000.$$

$$P'(x) = 135 - \frac{x}{25}.$$

Now $135 - \frac{x}{25} = 0$ implies $x = 3{,}375$.

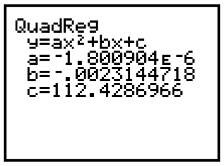
$P''(x) = -\frac{1}{25}$ and $P''(3{,}375) = -\frac{1}{25} < 0$. Thus, the maximum profit in this case occurs when 3,375 television sets are produced.
The maximum profit is

$$P(3{,}375) = 135(3{,}375) - \frac{(3{,}375)^2}{50} - 60{,}000$$

$$= \$167{,}812.50$$

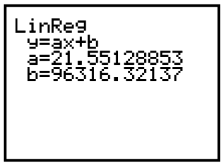
and the company should charge $p(3{,}375) = 200 - \frac{3{,}375}{50} = \$132.50$/set.

16. (A)



```
QuadReg
 y=ax²+bx+c
 a=⁻1.800904ᴇ⁻6
 b=⁻.0023144718
 c=112.4286966
```

(B)



```
LinReg
 y=ax+b
 a=21.55128853
 b=96316.32137
```

(C) The revenue at the demand level x is:
$R(x) = xp(x)$
where $p(x)$ is the quadratic regression equation in (A).

The cost at the demand level x is $C(x)$ given by the linear regression equation in (B). The profit $P(x) = R(x) - C(x)$.

The maximum profit is \$117,024 when the price per regular sleeping bag is \$79.

18. (A) Let x be the number of price reductions, then the price of a cup of coffee will be $p = 2.40 - 0.05x$ and the number of cups sold will be $1{,}600 + 50x$.
The revenue function is:

$R(x) = (1{,}600 + 50x)(2.40 - 0.05x) = 3{,}840 + 40x - 2.5x^2$
$R'(x) = 40 - 5x = 0$ implies $x = 8$.
$R''(x) = -5 < 0$.
Thus, $R''(8) = -5 < 0$ and we conclude that R has an absolute maximum at $x = 8$. The maximum revenue is $R(8) = 3{,}840 + 40(8) - 2.5(8^2) = \$4{,}000$, when 1,600 cups of coffee sold at the price of $p = 2.40 - 0.05(8) = \$2.00$ a cup.

(B) In this case
$R(x) = (1{,}600 + 60x)(2.40 - 0.10x)$
$= 3{,}840 - 16x - 6x^2$
$R'(x) = -16 - 12x = -4(4 + 3x) < 0$ for $x \geq 0$.

Thus, $R(x)$ is decreasing and its maximum occurs at $x = 0$, i.e. charge \$2.40 per cup.

20. Let x = number of dollar increases in the rate per night.
Then $300-3x$ = total number of rooms rented and $80+x$ = rate/night.
Total income = (total number of rooms rented)(rate - 10)

$$\begin{aligned} y(x) &= (300 - 3x)(80 + x - 10),\ 0 \le x \le 100 \\ y(x) &= (300 - 3x)(70 + x) \\ y'(x) &= -3(70 + x) + (300 - 3x) \\ &= -210 - 3x + 300 - 3x \\ &= 90 - 6x \\ &= 6(15 - x) \end{aligned}$$

Thus, $x = 15$ is the only critical value and

$y(15) = (300 - 45)(70 + 15) = 21,675$.
$y''(x) = -6$
$y''(15) = -6 < 0$

Therefore, the absolute maximum income is $y(15) = \$21,675$ when the rate is \$95 per night.

22. Let x = number of additional weeks the grower waits to pick the pears. Then $60 + 6x$ = the yield of a pear tree in pounds. The return will be:
$f(x) = (60 + 6x)(0.30 - 0.02x),\ 0 \le x \le 4$
since every week the price is dropped 2¢ per pound.

Now we want to maximize $f(x)$ over $[0, 4]$.

$$\begin{aligned} f'(x) &= 6(0.30 - 0.02x) - 0.02(60 + 6x) \\ &= 1.8 - 0.12x - 1.2 - 0.12x \\ &= 0.6 - 0.24x = 0,\ x = 2.5 \end{aligned}$$

critical value: $x = 2.5$

$f''(x) = -0.24 < 0$

$f''(2.5) = -0.24 < 0$. Thus, f has an absolute maximum of $f(2.5) = \$18.75$ at $x = 2.5$ weeks.

24. (A) Let x = length of the side of the square.
Then $L + 4x = 108$ or $L = 108 - 4x$.

The volume of the box $= Lx^2 = x^2(108 - 4x)$.

Let $y(x) = x^2(108 - 4x),\ 0 \le x \le 27$.

Would like to maximize $y(x)$ on $(0, 27)$.

$$\begin{aligned} y'(x) &= 2x(108 - 4x) - 4x^2 \\ &= 216x - 8x^2 - 4x^2 \\ &= 216x - 12x^2 \\ &= 12x(18 - x) \end{aligned}$$

Critical value: $x = 18$ [Note: $x = 0$ is not in the domain of $y(x)$]
$y''(x) = 216 - 24x$
$y''(18) = 216 - 24(18) = -216 < 0$. Thus, $y(x)$ has an absolute maximum at $x = 18$.

The maximum value $= y(18) = 11,664$. Note that $L = 108 - 4(18) = 36$. Therefore, volume is maximum at 11,644 in^3 for an 18" by 18" by 36" container.

(B) Let x be the radius and L the height of the cylindrical container. Then its volume $= \pi x^2 L$. Since $L + 2\pi x = 108$, then $L = 108 - 2\pi x$ and consequently $y(x) = \pi x^2 L = \pi x^2(108 - 2\pi x),\ 0 < x < \dfrac{54}{\pi}$.

We want to maximize $y(x)$ on $\left(0, \dfrac{54}{\pi}\right)$.

$y'(x) = 2\pi x(108 - 2\pi x) - 2\pi^2 x^2$

$= 216\pi x - 4\pi^2x^2 - 2\pi^2x^2$
$= 216\pi x - 6\pi^2x^2$
$= 6\pi x(36 - \pi x)$

critical value: $x = \frac{36}{\pi}$

$y''(x) = 216\pi - 12\pi^2 x$

$y''\left(\frac{36}{\pi}\right) = 216\pi - 12\pi^2\left(\frac{36}{\pi}\right) = 216\pi - 432\pi = -216\pi < 0$

Thus, $y(x)$ has an absolute maximum at $x = \frac{36}{\pi}$.

Also, $L = 108 - 2\pi x = 36$.

Therefore, volume is maximum at

$y\left(\frac{36}{\pi}\right) = \pi\left(\frac{36}{\pi}\right)^2\left(108 - 2\pi\left(\frac{36}{\pi}\right)\right) = 14{,}851 \text{ in}^3$ for a container of radius $\frac{36}{\pi}$ inches and height of 36 inches.

26. (A) Let x and y be the width and the length of the rectangle respectively. Then we have $2x + y + y - 100 = 240$ or $x + y = 170$ where $100 \le y \le 170$. The Area $= xy = (170 - y)y$.

Let $f(y) = (170 - y)y$, $100 \le y \le 170$
$f'(y) = 170 - 2y$ and $f''(y) = -2 < 0$.
$f'(y) = 0$ implies $y = 85$ which is not in the domain of f. We note that $f(100) = 7{,}000$, $f(170) = 0$. Thus, the maximum of f occurs when
$y = 100$ and hence $x = 170 - y = 70$.

(B) In this case, $2x + 2y - 100 = 400$ or $x + y = 250$,
and $f(y) = (250 - y)y$, $100 \le y \le 250$
$f'(y) = 250 - 2y$
$f'(y) = 0$ implies $y = 125$
$f''(y) = -2 < 0$

Thus, f has an absolute maximum at $y = 125$. So $x = 250 - y = 125$ and hence 125 feet by 125 feet.

28. Let x be the number of times the pharmacy places order and let y be the number of demands. Then the cost will be

$$C = 40x + 10\left(\frac{y}{2}\right) = 40x + 5y$$

We also have $xy = 200$. From this equation we find $y = \frac{200}{x}$ and substitute for y in the cost function to obtain:

$$C(x) = 40x + \frac{1000}{x}$$

$$C'(x) = 40 - \frac{1000}{x^2} = \frac{40x^2 - 1000}{x^2} = \frac{40(x^2 - 25)}{x^2} = \frac{40(x-5)(x+5)}{x^2}$$

$C'(x) = 0$ implies $x = -5$, $x = 5$ of which $x = 5$ is the only critical value (since $x > 0$).

$C''(x) = \dfrac{2000}{x^3} > 0$ for $x > 0$.

Thus, $C''(5) = \dfrac{2000}{125} > 0$ and hence $C(5) = 400$ is the minimum cost.

30. Let x = number of hours it takes the train to travel 360 miles.

Then $360 = xv$ or $x = \dfrac{360}{v}$.

$$\text{Cost} = \left(300+\frac{v^2}{4}\right)x = \left(300+\frac{v^2}{4}\right)\left(\frac{360}{v}\right)$$

$$= \frac{108{,}000}{v} + 90v$$

Let $C(v) = \dfrac{108{,}000}{v} + 90v$, $v > 0$.

We want to minimize $C(v)$.

$$C'(v) = -\frac{108{,}000}{v^2} + 90 = \frac{-108{,}000+90v^2}{v^2}$$

$C'(v) = 0$ implies $90v^2 = 108{,}000$ or $v = 34.64$

$C''(v) = \dfrac{108{,}000}{v^3} > 0$ for $v > 0$

So, $C(v)$ has an absolute minimum at $v = 34.64$ miles per hour.

32. $C(t) = \dfrac{0.16t}{t^2+4t+4} = \dfrac{0.16t}{(t+2)^2}$

$$C'(t) = \frac{0.16(t+2)^2 - 2(t+2)(0.16t)}{(t+2)^4}$$

$$= \frac{0.16(t+2)-0.32t}{(t+2)^3}$$

$$= \frac{0.16t+0.32-0.32t}{(t+2)^3} = \frac{0.32-0.16t}{(t+2)^3}$$

$C'(t) = 0$ implies $t = 2$

$$C''(t) = \frac{-0.16(t+2)^3 - 3(t+2)^2(0.32-0.16t)}{(t+2)^6}$$

$$= \frac{-0.16(t+2)-3(0.32-0.16t)}{(t+2)^4}$$

$$= \frac{-0.16t-0.32-0.96+0.48t}{(t+2)^4}$$

$$= \frac{0.32t-1.28}{(t+2)^4}$$

$$C''(2) = \frac{0.64-1.28}{4^4} < 0$$

Thus, $C(t)$ has an absolute maximum at $t = 2$ hours, and the maximum value is $C(2) = \dfrac{.32}{4^2} = .02\text{ mg/cm}^3$

34. (A) Let the energy to fly over land be 1 unit; then the energy to fly over the water is 1.4 units.

$E(x)$ = total energy = $(1.4)\sqrt{x^2+25} + (1)(10 - x)$

$E(x) = 1.4(x^2 + 25)^{1/2} + 10 - x,\ 0 \le x \le 10$

$E'(x) = (1.4)\frac{1}{2}(x^2 + 25)^{-1/2}(2x) - 1$

$= 1.4x(x^2 + 25)^{-1/2} - 1$

$= \dfrac{1.4x - \sqrt{x^2+25}}{\sqrt{x^2+25}}$

$E'(x) = 0$ when $1.4 - \sqrt{x^2+25} = 0$ or

$1.96x^2 = x^2 + 25$

$.96x^2 = 25$

$x^2 = \dfrac{25}{0.96} = 26.04$

$x = \pm 5.1$

Thus, the critical value is $x = 5.1$.

$E''(x) = 1.4(x^2 + 25)^{-1/2} + 1.4x\left(-\frac{1}{2}\right)(x^2 + 25)^{-3/2}(2x)$

$= \dfrac{1.4}{(x^2+25)^{1/2}} - \dfrac{1.4x^2}{(x^2+25)^{3/2}} = \dfrac{35}{(x^2+25)^{3/2}}$

$E''(5.1) = \dfrac{35}{[(5.1)^2+25]^{3/2}} > 0$

Thus, the energy will be minimum when $x = 5.1$.

Note that: $E(0) = (1.4)\sqrt{25} + 10 = 17$

$E(5.1) = (1.4)\sqrt{51.01} + (10 - 5.1) = 14.9$

$E(10) = (1.4)\sqrt{125} = 15.65$

Thus, the absolute minimum occurs when $x = 5.1$ miles.

(B) $E(x) = 1.1\sqrt{x^2+25} + (1)(10 - x),\ 0 \le x \le 10$

$E'(x) = \dfrac{1.1x - \sqrt{x^2+25}}{\sqrt{x^2+25}}$

$E'(x) = 0$ when $1.1x - \sqrt{x^2+25} = 0$ or

$1.21x^2 = x^2 + 25$

$x^2 = \dfrac{25}{.21} = 119.05$

$x = \pm 10.91$

critical value: $x = 10.91 > 10$, i.e., there are no critical values on the interval [0, 10].

Now, $E(0) = 1.1\sqrt{25} + 10 = 15.5$,

$E(10) = 1.1\sqrt{125} \approx 12.30$

Therefore, the absolute minimum occurs when $x = 10$ miles.

36. $C(x) = \frac{8k}{x^2} + \frac{k}{(10-x)^2}$, $0.5 \le x \le 9.5$, $k > 0$

$C'(x) = -\frac{16k}{x^3} + \frac{2k}{(10-x)^3}$

$= \frac{-16k(10-x)^3 + 2kx^3}{x^3(10-x)^3}$

$C'(x) = 0$ when $-16k(10 - x)^3 + 2kx^3 = 0$ or

$$\begin{aligned} 2kx^3 &= 16k(10 - x)^3 \\ x^3 &= 8(10 - x)^3 \\ x &= 2(10 - x) \\ 3x &= 20 \\ x &= \frac{20}{3} \end{aligned}$$

critical value: $x = \frac{20}{3}$

$C''(x) = \frac{48k}{x^4} + \frac{6k}{(10-x)^4} > 0$ for all x.

Thus, $C''\left(\frac{20}{3}\right) > 0$ and $C(x)$ has a local minimum at $x = \frac{20}{3}$.

$C(0.5) = \frac{8k}{(0.5)^2} + \frac{k}{(9.5)^2} = k(32.01) = 32.01k$

$C\left(\frac{20}{3}\right) = \frac{8k}{\left(\frac{20}{3}\right)^2} + \frac{k}{\left(10-\frac{20}{3}\right)^2} = k(0.27) = 0.27k$

$C(9.5) = \frac{8k}{(9.5)^2} + \frac{k}{(0.5)^2} = k(4.09) = 4.09k$

Therefore, $C(x)$ has an absolute minimum at $x = \frac{20}{3}$ miles.

38. $P(x) = 96x - 24x^2$, $0 \le x \le 3$

$P'(x) = 96 - 48x$

$P'(x) = 0$ implies $96 - 48x = 0$ or $x = 2$

critical value: $x = 2$

$P''(x) = -48 < 0$

$P''(2) = -48 < 0$. Thus, $P(x)$ has a local maximum at $x = 2$.

$P(0) = 0$

$P(2) = 96(2) - 24(2)^2 = 96$

$P(3) = 96(3) - 24(3)^2 = 72$

Therefore, $P(x)$ has its absolute maximum at $x = 2$ and $P(2) = 96$ is the maximum percentage.