$$\frac{dy}{dx} = y^2 - 4$$

$$\int \frac{dy}{y^2-4} = \int dx \longrightarrow y = \pm 2 \quad \text{C.N}$$

$$\int \frac{dy}{y+a} = \int \frac{d(y+a)}{y+a}$$

$$\int \left( \frac{1/4}{y-2} - \frac{1/4}{y+2} \right) dy = \int dx$$

$$\frac{1}{4}\left( \ln|y-2| - \ln|y+2| \right) = x + C_1$$

$$\ln\left|\frac{y-2}{y+2}\right| = 4x + 4C_1 \qquad C_2 = 4C_1$$

$$\frac{y-2}{y+2} = e^{4x+C_2} = e^{4x}\underbrace{e^{C_2}}_{\text{constant}} = C$$

$$= Ce^{4x}$$

$$y - 2 = Ce^{4x}y + 2Ce^{4x}$$

$$y(1 - Ce^{4x}) = 2 + 2Ce^{4x}$$

$$y(x) = \frac{2 + 2Ce^{4x}}{1 - Ce^{4x}}$$

$$y = 2 \Rightarrow 2 = \frac{2(1 + Ce^{4x})}{1 - Ce^{4x}}$$

$$2 - 2Ce^{4x} = 2 + 2Ce^{4x}$$

$$-2Ce^{4x} = 2Ce^{4x} \quad \text{if } C = 0 \checkmark$$

$$y = -2 = \frac{2 + 2Ce^{4x}}{1 - Ce^{4x}}$$

$$-2 + 2Ce^{4x} = 2 + 2Ce^{4x}$$

$$-2 \neq 2 \qquad y = -2 \text{ impossible}$$

Air Resistance.

$$R(x,v) = -r(x,y)\cdot v$$

$$F = -mg + R(x,v)$$

$$m\frac{dv}{dt} = -mg - rv$$

$$\frac{dv}{dt} = -g - \frac{r}{m}v \qquad -\left(g + \frac{r}{m}v\right)$$

$$\int \frac{m\,dv}{mg+rv} = -\int dt$$

$$\frac{m}{r}\int \frac{d(mg+rv)}{mg+rv} = -t + C_1$$

$$\frac{m}{r}\ln(mg+rv) = -t + C_1$$

$$\ln(mg+rv) = -\frac{r}{m}t + C_2$$

$$mg + rv = e^{-\frac{r}{m}t + C_2}$$

$$rv = C_3 e^{-rt/m} - mg$$

$$\boxed{v = C e^{-rt/m} - \frac{mg}{r}}$$

# Linear Eqns

$$y' + p(x)y = f(x)$$

If $f(x) = 0 \Rightarrow y' + p(x)y = 0$ Homogeneous

→ use separable

If $f(x) \neq 0$ non homogeneous
inhomogeneous

$$y' + py = 0$$

$$\frac{dy}{dx} = y' = -py$$

$$\int \frac{dy}{y} = -\int p(x)\,dx$$

$$\ln y = -\int p\,dx$$

$$y = e^{-\int p(x)\,dx}$$ sol'n of an homogeneous.

$$y' + p(x)y = f(x)$$

$$y(x) = y_h + y_p. \qquad y_h = e^{-\int p(x)\,dx}$$

$$y_p' + p y_p = f \qquad y_p = u(x)\, y_h$$

$$(u y_h)' + p(u y_h) = f \qquad y_h' + p y_h = 0$$

$$u' y_h + \underbrace{u y_h' + p u y_h} = f$$

$$u' y_h + u(y_h' + p y_h) = f$$

$$u' y_h = f$$

$$\frac{du}{dx} y_h = f$$

$$\frac{du}{dx} = \frac{f}{y_h} = \frac{f}{e^{-\int p\,dx}}$$

$$\int du = \int \frac{f(x)}{e^{-\int p(x)dx}}\,dx$$

$$u = \int e^{\int p(x)dx} f(x)\,dx$$

$$y_p = u\, e^{-\int p\,dx}$$

$$= e^{-\int p\,dx} \int f(x)\, e^{\int p\,dx}\,dx$$

$$y(x) = y_h + y_p$$

$$= ce^{-\int p\,dx} + e^{-\int p\,dx} \int f\, e^{\int p\,dx}$$

$$= e^{-\int p\,dx}\left(C + \int f(x)\, e^{\int p(x)dx}\right)$$

Ex $x' = x\sin t + 2t\,e^{-\cos t}$ $\qquad x(0) = 1$

$x' - (\sin t)\,x = \boxed{2t\,e^{-\cos t}}$

(H) $e^{\int -\sin t\,dt} = e^{\cos t}$

$$\int e^{\cos t}\left(2t\,e^{-\cos t}\right)dt = \int 2t\,dt = t^2$$

$$\boxed{x(t) = \frac{1}{e^{\cos t}}\left(t^2 + c\right)}$$

$x(0) = 1$

$$1 = \frac{1}{e}(0 + c) \Rightarrow \boxed{c = e}$$

$$\boxed{x(t) = e^{-\cos t}\left(t^2 + e\right)}$$

---

Ex $x' = x\tan t + \sin t$

$x' - (\tan t)\,x = \sin t$

$e^{-\int \tan t\,dt} = e^{\ln \cos t} = \cos t$

$$\int \cos t \sin t\,dt = -\int \cos t\,d(\cos t) = -\frac{1}{2}\cos^2 t$$

$$x(t) = \frac{1}{\cos t}\left(-\frac{1}{2}\cos^2 t + c\right)$$

$$= -\frac{1}{2}\cos t + \frac{c}{\cos t}$$

$x(0) = 2$ $\qquad \cos 0 = 1$

$2 = -\frac{1}{2} + c$

$c = 5/2$

$$\boxed{x(t) = -\frac{1}{2}\cos t + \frac{5}{2}\sec t}$$

29/ $y' - 2y = t^2 e^{2t}$

$$e^{\int -2\,dt} = e^{-2t}$$

$$\int e^{-2t} t^2 e^{2t}\,dt = \int t^2\,dt = \frac{1}{3}t^3$$

$$y(t) = \underbrace{e^{2t}}_{\frac{1}{e^{-2t}}} \left(\frac{1}{3}t^3 + C\right)$$