

Jose's *Method* Integration by Part

Evaluate $\int e^{ax} \cos bx \, dx$

$$\int e^{ax} \cos bx \, dx = \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx \, dx$$

$$\left(1 + \frac{a^2}{b^2}\right) \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{b^2} (b \sin bx + a \cos bx)$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (b \sin bx + a \cos bx) + C$$

		$\int \cos bx \, dx$
+	e^{ax}	$\frac{1}{b} \sin bx$
-	ae^{ax}	$-\frac{1}{b^2} \cos bx$
+	$a^2 e^{ax}$	$-\frac{1}{b^2} \int \cos bx \, dx$

Proof

Find $\int e^{ax} \cos bx \, dx$

Solution

Let: $u = e^{ax} \quad dv = \cos bx \, dx$
 $du = ae^{ax} \, dx \quad v = \int \cos bx \, dx = \frac{1}{b} \sin bx$

$$\int e^{ax} \cos bx \, dx = \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx \, dx \qquad \int u \, dv = uv - \int v \, du$$

Let: $u = e^{ax} \quad dv = \sin bx \, dx$
 $du = ae^{ax} \, dx \quad v = \int \sin bx \, dx = -\frac{1}{b} \cos bx$

$$\begin{aligned} \int e^{ax} \cos bx \, dx &= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \left[-\frac{1}{b} e^{ax} \cos bx + \frac{a}{b} \int e^{ax} \cos bx \, dx \right] \\ &= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx \, dx \end{aligned}$$

$$\int e^{ax} \cos bx \, dx + \frac{a^2}{b^2} \int e^{ax} \cos bx \, dx = \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx + C_1$$

$$\frac{a^2 + b^2}{b^2} \int e^{ax} \cos bx \, dx = \frac{1}{b^2} e^{ax} (b \sin bx + a \cos bx) + C_1$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (b \sin bx + a \cos bx) + C$$