Solution *Section* 3.7 – Implicit Differentiation and related Rates

Exercise

Find dy/dx for the equation $y^2+x^2-2y-4x=4$

Solution

$$\frac{d}{dx}[y^2+x^2-2y-4x]=\frac{d}{dx}[4]$$

$$\frac{d}{dx}\left[y^2\right]+\frac{d}{dx}\left[x^2\right]-\frac{d}{dx}[2y]-\frac{d}{dx}[4x]=\frac{d}{dx}[4]$$

$$2y\frac{dy}{dx}+2x-2\frac{dy}{dx}-4=0$$

$$2(y-1)\frac{dy}{dx}=4-2x$$

$$(y-1)\frac{dy}{dx}=2-x$$

$$\frac{dy}{dx}=\frac{2-x}{y-1}$$

Exercise

Find dy/dx: $x^2y^2-2x=3$

Solution

$$2xy^2+2x^2yy'-2=0$$

$$2x^2yy'=2-2xy^2$$

$$y'=\frac{2\left(1-xy^2\right)}{2x^2y}$$

$$\frac{dy}{dx}=\frac{1-xy^2}{x^2y}$$

Exercise

Find dy/dx: $x^2 - xy + y^2 = 4$ and evaluate the derivative at the given point $(0, \ -2)$

Solution

$$2x - (y + xy') + 2yy' = 0$$

$$-y - xy' + 2yy' = -2x$$

$$(2y - x)y' = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

$$@(0, -2) \rightarrow \frac{dy}{dx} = \frac{-2 - 2(0)}{2(-2) - (0)}$$

$$= \frac{-2}{-4}$$

$$= \frac{1}{2}$$

Exercise

Find the rate of change of x with respect to p. $p = \sqrt{\frac{200 - x}{2x}}$, $\quad 0 < x \le 200$

Solution

$$p^2 = \frac{200 - x}{2x}$$

$$2xp^2 = 200 - x$$

$$2p^2 \frac{dx}{dp} + 4xp = -\frac{dx}{dp}$$

$$2p^2 \frac{dx}{dp} + \frac{dx}{dp} = -4xp$$

$$\frac{dx}{dp}\left(2p^2 + 1\right) = -4xp$$

$$\frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}$$

Exercise

Implicit Differentiation: Find $\frac{dy}{dx}$, $\quad e^{xy} + x^2 - y^2 = 10$

Solution

$$\frac{d}{dx}\left(e^{xy}\right) + \frac{d}{dx}\left(x^2\right) - \frac{d}{dx}\left(y^2\right) = \frac{d}{dx}(10)$$

$$e^{xy}\frac{d}{dx}(xy) + 2x - 2yy' = 0$$

$$e^{xy}(xy' + y) + 2x - 2yy' = 0$$

$$e^{xy}xy' + e^{xy}y + 2x - 2yy' = 0$$

$$y'\left(xe^{xy} - 2y\right) = -2x - e^{xy}y$$

$$y' = \frac{-2x - e^{xy}y}{xe^{xy} - 2y}$$

Exercise

Find the slope of the tangent line to the circle $x^2 - 9y^2 = 16$ at the point (5, 1)

Solution

$$2x - 18y\frac{dy}{dx} = 0$$

$$-18y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{-18y} = \frac{x}{9y}$$

$$@(5,1) \to \frac{dy}{dx} = \frac{5}{9(1)} = \frac{5}{9}$$

Exercise

The demand function for a product is given by $P = \frac{2}{0.001x^2+x+1}$. Find dx/dp implicitly.

Solution

$$0.001x^2 + x + 1 = \frac{2}{p}$$

$$\frac{d}{dp}\left[0.001x^2 + x + 1\right] = \frac{d}{dp}\left[2p^{-1}\right]$$

$$0.002x\frac{dx}{dp} + \frac{dx}{dp} = -2p^{-2}$$

$$(0.002x + 1)\frac{dx}{dp} = -\frac{2}{p^2}$$

$$\frac{dx}{dp} = -\frac{2}{(0.002x+1)p^2}$$