techniques

2.1 Integration By Part

$$\int u\,dv = uv - \int v\,du$$

Ex $\int \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv}$

$u = \ln x$ $\quad du = \frac{dx}{x}$

$dv = dx$ $\quad v = \int dx = x$

$$\int \ln x\,dx = x\ln x - \int x\,\frac{dx}{x}$$
$$= x\ln x - \int dx$$
$$= x\ln x - x + C$$

$x(\ln x - 1)$

$\int x^n e^{ax}\,dx$

$\int x^n \cos bx\,dx$

$\int x^n \sin cx\,dx$

$\int e^{ax}\cos bx\,dx$

$\int e^{ax}\sin bx\,dx$

dv	$\int$	e^{ax} / $\cos bx$ / $\sin bx$
x^n		
$\downarrow$		
0		

Ex $\int x\cos x\,dx = x\sin x + \cos x + C$

	$\int \cos x\,dx$
$+$ x	$\sin x$
$-$ 1	$-\cos x$

Ex $\int x^2 e^x\,dx = x^2e^x - 2xe^x + 2e^x + C$
$= e^x(x^2 - 2x + 2) + C$

	$\int e^x dx$
$+$ x^2	e^x
$-$ $2x$	e^x
$+$ 2	e^x

Ex $\int x^3 \sin x\,dx$:

$= -x^3\cos x + 3x^2\sin x + 6\cos x - 6\sin x + C$

	$\int \sin x$
$+$ x^3	$-\cos x$
$-$ $3x^2$	$-\sin x$
$+$ $6x$	$\cos x$
$-$ 6	$\sin x$

Ex $\int e^x \cos x\,dx = \sin x e^x + (\cos x)e^x - \int e^x \cos x\,dx$

$2\int e^x \cos x\,dx = (\sin x + \cos x)e^x$

$\int e^x \cos x\,dx = \frac{1}{2}(\sin x + \cos x)e^x + C$

	$\int \cos x\,dx$
$+$ e^x	$\sin x$
$-$ e^x	$-\cos x$
$+$ e^x	$-\int \cos x\,dx$

Ex ?? $y = x e^{-x}$ $x: 0 \to 4$ $0 \le x \le 4$

$A = \int_0^4 x e^{-x}\,dx = e^{-x}(-x-1)\Big|_0^4$

$= -5e^{-4} + 1$ unit^2

	$\int e^{-x}\,dx$
$+$ x	$-e^{-x}$
$-$ 1	e^{-x}

$\int x^n e^{ax}\,dx = e^{ax}\left(\frac{x^n}{a} - \frac{n x^{n-1}}{a^2} + \frac{n(n-1)x^{n-2}}{a^3} - \cdots - \frac{n!}{a^n}\right)$

$\int x^3 e^{2x}\,dx = e^{2x}\left(\frac{x^3}{2} - \frac{3}{4}x^2 + \frac{3}{4}x - \frac{3}{8}\right) + C$