Solution *Section* 3.5 – Probability

Exercise

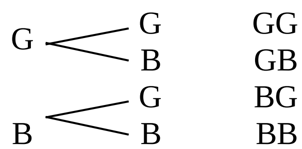
An experiment consists of recording the boy-girl composition of a two-child family.

a) What is an appropriate sample space if we are interested in the genders of the children in the order of their births? Draw a tree diagram.

b) What is an appropriate sample space if we are interested only in the *number* of the girls in a family?

Solution

a) What is an appropriate sample space if we are interested in the genders of the children in the order of their births? Draw a tree diagram.



$$S_1 = \{GG, GB, BG, BB\}$$

b) What is an appropriate sample space if we are interested only in the *number* of the girls in a family?

$$S_2 = \{0, 1, 2\}$$

Exercise

Given $S = \{1, 2, 3, \cdots, 17, 18\}$

a) The outcome is a number divisible by 12

b) The outcome is an even number greater than 15

c) Is divisible by 4

d) Is divisible by 5

Solution

a) $E = \{12\}$

b) $E = \{16, 18\}$

c) $E = \{4, 8, 12, 16\}$ $\quad$ *E*: Compound event

d) $F = \{5, 10, 15\}$

Exercise

Consider rolling 2 Dice.

a) What is the event that a sum of 5 turns up

b) What is the event that a sum that is a prime number greater than 7 turns up

Solution

a) Sum of 5: {(4, 1), (3, 2), (2, 3), (1, 4)}

b) {(6, 5), (5, 6)}

Exercise

A single fair die is rolled. Find the probability of each event

a) Getting a 2

b) Getting an odd number

c) Getting a number less than 5

d) Getting a number greater than 2

e) Getting a 3 or a 4

f) Getting any number except 3

Solution

a) $P = \frac{1}{6}$

b) $P(Odd) = \frac{3}{6} = \frac{1}{2}$

c) $P(<5) = \frac{4}{6} = \frac{2}{3}$

d) $P(>2) = \frac{4}{6} = \frac{2}{3}$

e) $P(3 \text{ or } 4) = \frac{2}{6} = \frac{1}{3}$

f) $P(no\ 3) = \frac{5}{6}$

Exercise

A card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing the following

a) A 9

b) A black card

c) A black 9

d) A heart

e) The 9 of hearts

f) A face card

g) A 2 or a queen

h) A black 7 or red 8
i) A red card or a 10
j) A spade or a king

Solution

a) $P(9)=\frac{4}{52}\ \boxed{=\frac{1}{13}}$

b) $P(black)=\frac{26}{52}\ \boxed{=\frac{1}{2}}$

c) $P(black-9)=\frac{2}{52}\ \boxed{=\frac{1}{26}}$

d) $P(heart)=\frac{13}{52}\ \boxed{=\frac{1}{4}}$

e) $P(9-heart)\ \boxed{=\frac{1}{52}}$

f) $P(face)=\frac{12}{52}\ \boxed{=\frac{3}{13}}$

g) $P(2\ or\ queen)=\frac{8}{52}\ \boxed{=\frac{2}{13}}$

h) $P(black-7\ or\ red-8)=\frac{4}{52}\ \boxed{=\frac{1}{13}}$

i) $P(red\ or\ 10)=\frac{28}{52}\ \boxed{=\frac{7}{13}}$

j) $P(spade\ or\ king)=\frac{16}{52}\ \boxed{=\frac{4}{13}}$

Exercise

The student sitting next to you in class concludes that the probability of the ceiling falling down on both of you before class ends is 1/2, because there are two possible outcomes - the ceiling will fall or not fall. What is wrong with this reasoning?

Solution

The outcomes are not equally likely.

Exercise

A jar contains 3 white, 4 orange, 5 yellow, and 8 black marbles. If a marble is drawn at random, find the probability that it is the following.

a) White
b) Orange
c) Yellow
d) Black
e) Not black

f) Orange or Yellow

Solution

a) $P(white) \boxed{=\frac{3}{20}}$

b) $P(orange)=\frac{4}{20} \boxed{=\frac{1}{5}}$

c) $P(yellow)=\frac{5}{20} \boxed{=\frac{1}{4}}$

d) $P(black)=\frac{8}{20} \boxed{=\frac{2}{5}}$

e) $P(no\ black)=\frac{12}{20} \boxed{=\frac{3}{5}}$ $\quad 1-P(black)$

f) $P(orange\ \ or\ yellow) \boxed{=\frac{9}{20}}$

Exercise

Let consider rolling 2 dice. Find the probabilities of the following events

a) E = Sum of 5 turns up

b) F = a sum that is a prime number greater than 7 turns up

Solution

a) $P(E)=\frac{4}{36} \boxed{=\frac{1}{9}}$

b) $P(F)=\frac{2}{36} \boxed{=\frac{1}{18}}$

1 – 1	1 – 2	1 – 3	1 – 4	1 – 5	1 – 6
2 – 1	2 – 2	2 – 3	2 – 4	2 – 5	2 – 6
3 – 1	3 – 2	3 – 3	3 – 4	3 – 5	3 – 6
4 – 1	4 – 2	4 – 3	4 – 4	4 – 5	4 – 6
5 – 1	5 – 2	5 – 3	5 – 4	5 – 5	5 – 6
6 – 1	6 – 2	6 – 3	6 – 4	6 – 5	6 – 6

Exercise

The board of regents of a university is made up of 12 men and 16 women. If a committee of 6 chosen at random, what is the probability that it will contain 4 men and 2 women?

Solution

S: the set of 6 out of 28 $n(S)=C_{28,6}\ =376,740$

4 men out of 12 men: $C_{12,4}$

2 women out of 16 men: $C_{16,2}$

$$P(E)=\frac{C_{12,4}.C_{16,2}}{C_{28,6}} \boxed{\approx 0.158}$$

Exercise

In drawing 7 cards from a 52-card deck without replacement, what is the probability of getting 7 hearts.

Solution

$$\mathrm{P}(7\ hearts) = \frac{C_{13,7}}{C_{52,7}} \approx 0.00013$$

Exercise

A committee of 4 people is to be chosen from a group of 5 men and 6 women. What is the probability that the committee will consist of 2 men and 2 women?

E is the set of all possible ways to select 2 men and 2 women

S is the set of all possible ways to select 4 people from 11

Solution

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}_5C_2 \bullet {}_6C_2}{{}_{11}C_4} = 0.4545$$

Exercise

A department store receives a shipment of 27 new portable radios. There are 4 defective radios in the shipment. If 6 radios are selected for display, what is the probability that 2 of them are defective?

E is the set of all possible ways to have 2 defective and 4 not defective.

S is the set of all possible ways to select 6 radios from 27.

Solution

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}_4C_2 \cdot {}_{23}C_4}{{}_{27}C_6} = 0.1795$$

Exercise

Eight cards are drawn from a standard deck of cards. What is the probability that there are 4 face cards and 4 non-face cards?

E is the set of all possible ways to have 4 faces and 4 nonfaces.

S is the set of all possible ways to select 8 cards from 52.

Solution

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}_{12}C_4 \cdot {}_{40}C_4}{{}_{52}C_8} = 0.0601$$

Exercise

Five cards are drawn from a standard deck of cards. What is the probability that there are exactly 3 hearts?

Solution

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}_{13}C_3 \cdot {}_{39}C_2}{{}_{52}C_5} \underline{= 0.0815|}$$

Exercise

A poll was conducted preceding an election to determine the relationship between voter persuasion concerning a controversial issue and the area of the city in which the voter lives. Five hundred registered voters were interviewed from three areas of the city. The data are shown below. Compute the probability of having no opinion on the issue or living in the inner city.

Area of city	***Favor***	***Oppose***	***No Opinion***
East	30	40	55
North	25	45	50
Inner	95	65	85

Solution

$$\Pr = \frac{Total\ Inner + No\ Opinion\ East + No\ Opinion\ North}{500}$$

$$= \frac{95+65+85+55+50}{500}$$

$$= \frac{350}{500}$$

$$\underline{\approx 0.7|}$$

Exercise

There are 11 members on the board of directors for the Coca Cola Company.

a) If they must select a chairperson, first vice chairperson, second vice chairperson, and secretary, how many different slates of candidates are possible?

b) If they must form an ethics subcommittee of 4 members, how many different subcommittees are possible?

Solution

a) Since order makes a difference, there are 4 different offices ${}_{11}P_4 = \frac{11!}{7!} \underline{= 7920|}$

b) Since the order in which the 4 are picked makes no differences ${}_{11}C_4 = \frac{11!}{7!4!} \underline{= 330|}$

Exercise

When testing for current in a cable with five color-coded wires, the author used a meter to test two wires at a time. How many different tests are required for every possible pairing of two wires?

Solution

Since the order of the 2 wires being tested is irrelevant:

$$_5C_2 = \frac{5!}{3!\cdot 2!} \boxed{= 10 \ \textit{different tests}}$$

Exercise

Identity theft often begins by someone discovering your 9-digit social security number. Answer each of the following. Express probabilities as fractions.

a) What is the probability of randomly generating 9 digits and getting your social security number?

b) In the past, many teachers posted grades along with the last 4 digits of your social security number, what is the probability that if they randomly generated the order digits, they would match yours? Is that something to worry about?

Solution

a) Let G = generating a given social security number in a single trial.

Total number of possible sequences $= 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10$

$= 1{,}000{,}000{,}000$

Since only one sequence is correct: $P(G) \boxed{= \frac{1}{1{,}000{,}000{,}000}}$

b) Let F = generating first 5 digits of a given social security number in a single trial.

Total number of possible sequences $= 10\cdot 10\cdot 10\cdot 10\cdot 10$

$= 100{,}000$

Since only one sequence is correct: $P(F) \boxed{= \frac{1}{100{,}000}}$

Since this probability is so small, need not worry about the given scenario

Exercise

You become suspicious when a genetics researcher randomly selects groups of 20 newborn babies and seems to consistently get 10 girls and 10 boys. The researchers claims that it is common to get 10 girls and 10 boys in such cases,

a) If 20 newborn babies are randomly selected, how many different gender sequences are possible?

b) How many different ways can 10 girls and 10 boys be arranged in sequence?

c) What is the probability of getting 10 girls and 10 boys when 10 babies are born?

Solution

a) There are 20 tasks to perform, and each task can be performed in either of 2 ways

Total number of possible sequences is $2\cdot 2\cdot 2\cdot \cdots \cdot 2 = 2^{20}$

$$\boxed{=1,048,576 \;\; possibilities}$$

b) The number of possible sequences of n objects is when some are alike is

$$\frac{n!}{n_1! \cdot n_2! \cdots n_k!} : \frac{20!}{10!10!} \; \boxed{=184,756 \;\; possibilities}$$

c) $P(10G,\ 10B) = \dfrac{184,756}{1,048,576} \; \boxed{=0.176}$

Exercise

Two dice are rolled. Find the probabilities of the following events.

a) The first die is 3 or the sum is 8

b) The second die is 5 or the sum is 10.

Solution

a)
$$\begin{aligned} P(3 \text{ or sum is } 8) &= P(3) + P(\text{sum } 8) - P(3 \text{ and sum } 8) \\ &= \frac{6}{36} + \frac{5}{36} - \frac{1}{36} \\ &= \frac{10}{36} \\ &\boxed{= \frac{5}{18}} \end{aligned}$$

b)
$$\begin{aligned} P(5 \text{ or sum } 10) &= P(5) + P(\text{sum } 10) - P(5 \text{ and sum } 10) \\ &= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} \\ &= \frac{8}{36} \\ &\boxed{= \frac{2}{9}} \end{aligned}$$

Exercise

One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) A 9 or 10

b) A red card or a 3

c) A 9 or a black 10

d) A heart or a black card

e) A face card or a diamond

Solution

a) $P(9 \text{ or } 10) = \dfrac{8}{52} \; \boxed{= \dfrac{2}{13}}$

b) $P(red\ or\ 3)=\dfrac{28}{52}\ \boxed{=\dfrac{7}{13}}$

c) $P(9\ or\ black\text{-}10)=\dfrac{6}{52}\ \boxed{=\dfrac{3}{26}}$

d) $P(heart\ or\ black)=\dfrac{39}{52}\ \boxed{=\dfrac{3}{4}}$

e) $P(face\ or\ diamond)=\dfrac{22}{52}\ \boxed{=\dfrac{11}{26}}$

Exercise

One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) Less than a 4 (count aces as ones)

b) A diamond or a 7

c) A black card or an ace

d) A heart or a jack

e) A red card or a face card

Solution

a) $P(<4)=P\left(ace,2,3\right)=\dfrac{12}{52}\ \boxed{=\dfrac{3}{13}}$

b) $P(diamond\ or\ 7)=\dfrac{16}{52}\ \boxed{=\dfrac{4}{13}}$

c) $P(black\ or\ ace)=\dfrac{28}{52}\ \boxed{=\dfrac{7}{13}}$

d) $P(heart\ or\ jack)=\dfrac{16}{52}\ \boxed{=\dfrac{4}{13}}$

e) $P(red\ or\ face)=\dfrac{32}{52}\ \boxed{=\dfrac{8}{13}}$

Exercise

Pam invites relatives to a party: her mother, 2 aunts, 3 uncles, 2 brothers, 1 male cousin, and 4 female cousins. If the chances of any one guest first equally likely, find the probabilities that the first guest to arrive is as follows.

a) A brother or an uncle

b) A brother or a cousin

c) A brother or her mother

d) An uncle or a cousin

e) A male or a cousin

f) A female or a cousin

Solution

a) $P(brother\ or\ uncle) = \boxed{\dfrac{5}{13}}$

b) $P(brother\ or\ cousin) = \boxed{\dfrac{7}{13}}$

c) $P(brother\ or\ mother) = \boxed{\dfrac{3}{13}}$

d) $P(uncle\ or\ cousin) = \boxed{\dfrac{8}{13}}$

e) $P(male\ or\ cousin) = \boxed{\dfrac{10}{13}}$

f) $P(brother\ or\ cousin) = \boxed{\dfrac{8}{13}}$

Exercise

The numbers $\{1, 2, 3, 4, and\ \ 5\}$ are written on slips of paper, and 2 slips are drawn at random one at a time without replacement. Find the probabilities:

a) The sum of the numbers is 9.
b) The sum of the numbers is 5 or less.
c) The first number is 2 or the sum is 6
d) Both numbers are even.
e) One of the numbers is even or greater than 3.
f) The sum is 5 or the second number is 2.

Solution

a) $P(sum = 9) = \dfrac{2}{20} \ \boxed{= \dfrac{1}{10}}$

b) $P(sum \le 5) = \dfrac{8}{20} \ \boxed{= \dfrac{2}{5}}$

c) $P(2\ or\ \ sum\ \ 6) = \boxed{\dfrac{7}{20}}$

d) $P(even) = \dfrac{2}{20} \ \boxed{= \dfrac{1}{10}}$

e) $P(even\ or\ \ > 3) = \dfrac{18}{20} \ \boxed{= \dfrac{9}{10}}$

f) $P(sum 5\ or\ \ 2nd\,\#2) \ \boxed{= \dfrac{7}{20}}$

Exercise

Suppose $P(E)=0.26$, $P(F)=0.41$, and $P(E\cap F)=0.16$. Find the following

a) $P(E\cup F)$

b) $P(E'\cap F)$

c) $P(E\cap F')$

d) $P(E'\cup F')$

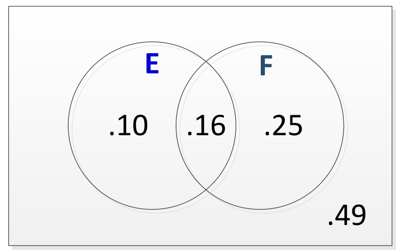
Solution

a) $P(E\cup F)=.1+.16+.25=.51$

b) $P(E'\cap F)=.25$

c) $P(E\cap F')=.10$

d) $P(E'\cup F')=.74+.59-.49=.84$



Exercise

Suppose $P(E)=0.42$, $P(F)=0.35$, and $P(E\cup F)=0.59$. Find the following

a) $P(E'\cap F')$

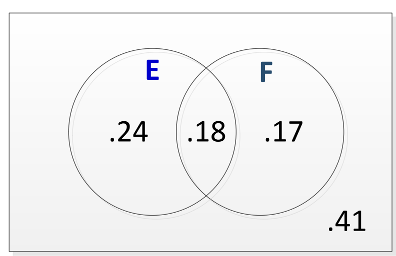
b) $P(E'\cup F')$

c) $P(E'\cup F)$

d) $P(E\cap F')$

Solution

$$\begin{aligned}P(E\cap F)&=P(E)+P(F)-P(E\cup F)\\&=.42+.35-.59\\&=.18\end{aligned}$$



a) $P(E'\cap F')=.41$

b) $P(E'\cup F')=1-.18=.82$

c) $P(E'\cup F)=.17+.41+.18=.76$

d) $P(E\cap F')=.24$

Exercise

A single fair die is rolled. Find the odds in favor of getting the results

a) 3

b) 4, 5, or 6

c) 2, 3, 4, or 5

d) Some number less than 6

Solution

a) $P(E)=\frac{1}{6} \quad \to P\left(E'\right)=\frac{5}{6}$

$$\Rightarrow odds=\frac{1}{5} \quad \to \boxed{1:5}$$

b) $P(E)=\frac{3}{6}=\frac{1}{2} \quad \to P\left(E'\right)=\frac{1}{2}$

$$\Rightarrow odds=\frac{1/2}{1/2}=1 \quad \to \boxed{1:1}$$

c) $P(E)=\frac{4}{6}=\frac{2}{3} \quad \to P\left(E'\right)=\frac{1}{3}$

$$\Rightarrow odds=\frac{2}{1} \quad \to \boxed{2:1}$$

d) $P(E)=\frac{5}{6} \quad \to P\left(E'\right)=\frac{1}{6}$

$$\Rightarrow odds=\frac{5}{1} \quad \to \boxed{5:1}$$

Exercise

If in repeated rolls of two fair dice the odds against rolling a 6 before rolling a 7 are 6 to 5, what is the probability of rolling a 6 before rolling 7?

Solution

$$odds\ \ 6:5 \to \frac{6}{5}$$

$$\Rightarrow Against \to \quad =\frac{5}{6}$$

$$P(E)=\frac{5}{5+6}=\frac{5}{11}$$

Exercise

From survey involving 1,000 people in the certain city, it was found that 500 people had tried a certain brand of diet cola, 600 had tried a certain brand of regular cola, and 200 had tried both types of cola. If a resident of the city is selected at random, what is the empirical probability that

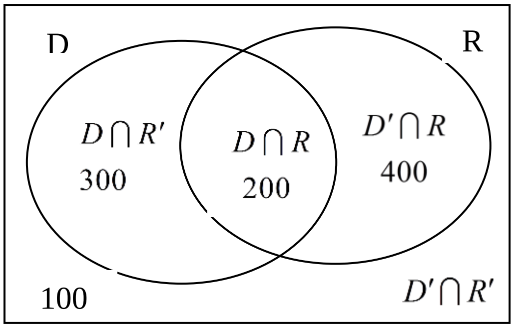
a) The resident has not tried either cola? What are the empirical odds for this event?

b) The resident has tried the diet or has not tried the regular cola? What are the empirical odds against this event?

Solution

a) $n(\text{S}) = 1000$ $\quad D \cap R = 200$

$D \cap R' = 300 \quad D' \cap R = 400$



$$P(\textit{neither}\ \text{D}\ \textit{or}\ R) = P(D' \cap R')$$

$$= \frac{100}{1000}$$

$$\underline{= .1|}$$

$$P(E') = 1 - .1 = 0.9$$

Odds for $\boldsymbol{E}$: $\dfrac{P(E)}{P(E')} = \dfrac{.1}{.9} = \dfrac{1}{9}$ *or* $\boxed{1:9}$

b) $P(E) = P(D \cup R')$

$$= P(D) + P(R') - P(D \cap R')$$

$$= \frac{500}{1000} + \frac{400}{1000} - \frac{300}{1000}$$

$$\underline{= .6|}$$

$$\Rightarrow P(E') = 1 - .6 = .4$$

Against Odds for P(E): $\dfrac{P(E)}{P(E')} = \dfrac{.4}{.6} = \dfrac{2}{3}$ *or* $\boxed{2:3}$

Exercise

The odds in favor of a particular horse winning a race are 4:5.

a) Find the probability of the horse winning.

b) Find the odds against the horse winning.

Solution

a) $P(E) = \dfrac{a}{a+b} = \dfrac{4}{4+5} \underline{= \dfrac{4}{9}|}$

b) The odds against the horse winning $\boxed{5:4}$

Exercise

Consider the sample space of equally likely events for the rolling of a single fair die.

a) What is the probability of rolling an odd number **and** a prime number?

b) What is the probability of rolling an odd number **or** a prime number?

Solution

a) $odd = \{1,\ 3,\ 5\} \quad prime = \{3,\ 5\}$

$$P(odd \cap prime) = \frac{n(A \cap B)}{n(S)} = \frac{2}{6} \ \boxed{= \frac{1}{3}}$$

b) $P(odd \cup prime) = \frac{n(A \cup B)}{n(S)} = \frac{4}{6} \ \boxed{= \frac{2}{3}}$

Exercise

Suppose that 2 fair Dice are rolled

a) What is the probability of that a sum of 2 or 3 turns up?

b) What is the probability of that both dice turn up the same or that a sum greater than 8 turns up?

Solution

a)
$$\begin{aligned} P(\Sigma{=}2 \text{ or } 3 \text{ turns up}) &= P(\Sigma{=}2) + P(\Sigma{=}3) \\ &= \frac{1}{36} + \frac{2}{36} \\ &= \frac{3}{36} \\ &\boxed{= \frac{1}{12}} \end{aligned}$$

b)
$$\begin{aligned} P(same \text{ or } \textstyle\sum > 8) &= P(same \cup \textstyle\sum > 8) \\ &= P(same) + P(\textstyle\sum > 8) - P(same \cap \textstyle\sum > 8) \\ &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\ &\boxed{= \frac{7}{18}} \end{aligned}$$

Exercise

A single card is drawn from an ordinary of 52 cards. Calculate the probabilities of and odds for each event

a) A face card or a club is drawn

b) A king or a heart is drawn

c) A black card or an ace is drawn

d) A heart or a number less than 7 (count an ace as 1) is drawn.

Solution

a) $\Pr(Face \text{ or } Club) = \Pr(F \cup C)$

$$= P(F) + P(C) - P(F \cap C)$$

$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52}$$

$$= \frac{11}{26}$$

$$P\left[(F \cup C)'\right] = 1 - \frac{11}{26} = \frac{15}{26}$$

Odds for $F \cup C = \dfrac{\frac{11}{26}}{\frac{15}{26}} = \dfrac{11}{15}$ $\boxed{11:15}$

b) $\Pr(\textit{King or Heart}) = \mathrm{P}(K \cup H)$

$$= P(K) + P(H) - P(K \cap H)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

$$P\left[(K \cup H)'\right] = 1 - \frac{4}{13} = \frac{9}{13}$$

Odds for $K \cup H = \dfrac{\frac{4}{13}}{\frac{9}{13}} = \dfrac{4}{9}$ $\boxed{4:9}$

c) $\Pr(\textit{Black card or Ace}) = \mathrm{P}(B \cup A)$

$$= P(B) + P(A) - P(B \cap A)$$

$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$$

$$= \frac{28}{52}$$

$$= \frac{7}{13}$$

$$P\left[(B \cup A)'\right] = 1 - \frac{7}{13} = \frac{6}{13}$$

Odds for $B \cup A = \dfrac{\frac{7}{13}}{\frac{6}{13}} = \dfrac{7}{6}$ $\boxed{7:6}$

d) $\Pr(\textit{Heart or } < 7) = \mathrm{P}(H \cup < 7)$

$$= P(B) + P(A) - P(B \cap A)$$

$$= \frac{13}{52} + \frac{6*4}{52} - \frac{6}{52}$$

$$= \frac{31}{52}$$

$$P\left[(H \cup \# < 7)'\right] = 1 - \frac{31}{52} = \frac{21}{52}$$

$$\text{Odds for } B \cup A = \frac{\frac{31}{52}}{\frac{21}{52}} = \frac{31}{21} \qquad \boxed{31:21}$$

Exercise

What is the probability of getting at least 1 black card in a 7-card hand dealt from a standard 52-card deck?

Solution

There are 26 black cards.

Let A = "at least 1 black card in a 7-card hand dealt"

A' = "0 black cards in a 7-card hand dealt"

$$n(A') = C_{26,7}$$

$$n(S) = C_{52,7}$$

$$P(A) = 1 - P(A')$$

$$= 1 - \frac{n(A')}{n(S)}$$

$$= 1 - \frac{C_{26,7}}{C_{52,7}}$$

$$= 1 - .005$$

$$= .995$$

Exercise

What is the probability that a number selected at random from the first 600 positive integers is (exactly) divisible by 6 or 9?

Solution

Let A = "Number divisible by 6"
B = "Number divisible by 9"

A number divisible by 6 $\Rightarrow n(A) = \frac{600}{6} = 100$

A number divisible by 9 $\Rightarrow n(B) = \frac{600}{9} \approx 66$

A number divisible by 6 and by 9 $\rightarrow 18k \quad \Rightarrow n(A \cap B) = \frac{600}{18} \approx 33$

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
&= \frac{100}{600} + \frac{66}{600} - \frac{33}{600} \\
&= \frac{133}{600} \\
&\approx 0.2217
\end{aligned}
$$

Exercise

What is the probability that a number selected at random from the first 1,000 positive integers is (exactly) divisible by 6 or 8?

Solution

Let A = "Number divisible by 6"
B = "Number divisible by 8"

A number divisible by 6 $\Rightarrow n(A) = \frac{1,000}{6} = 166$

A number divisible by 8 $\Rightarrow n(B) = \frac{1,000}{8} \approx 125$

A number divisible by 6 and by 8 $\rightarrow 24k \quad \Rightarrow n(A \cap B) = \frac{1,000}{24} \approx 41$

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
&= \frac{166}{1000} + \frac{125}{1000} - \frac{41}{1000} \\
&= \frac{250}{1000} \\
&\approx 0.25
\end{aligned}
$$

Exercise

From a survey involving 1,000 students at a large university, a market research company found that 750 students owned stereos, 450 owned cars, and 350 owned cars and stereos. If a student at the university is selected at random, what is the (empirical) probability that

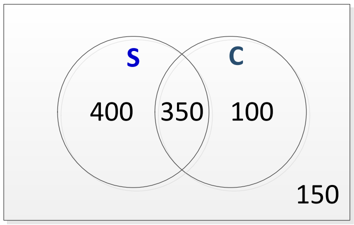
a) The student owns either a car or a stereo?

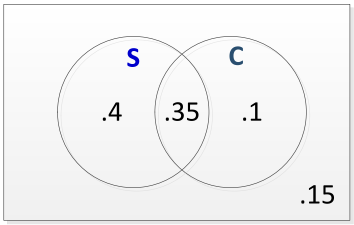
b) The student owns neither a car nor a stereo?

Solution

Let $S =$ "Number of stereos"

$C =$ "Number of cars"





a) $P(S\cup C) = P(S) + P(C) - P(S\cap C)$

$$= \frac{750}{1000} + \frac{450}{1000} - \frac{350}{1000}$$

$$= \frac{850}{1000}$$

$$\approx 0.85$$

b) $P(S'\cap C') = .15$

Exercise

In order to test a new car, an automobile manufacturer wants to select 4 employees to test drive the car for 1 year. If 12 management and 8 union employees volunteer to be test drivers and the selection is made at random, what is the probability that at least 1 union employee is selected.

Solution

Let $A =$ "at least 1 union employee is selected"

$A' =$ "no union employee is selected"

$$\Rightarrow n(A') = C_{12,4}, \quad n(S) = C_{20,4}$$

$$P(A) = 1 - P(A')$$

$$= 1 - \frac{C_{12,4}}{C_{20,4}}$$

$$\approx 0.90$$

Exercise

A shipment of 60 inexpensive digital watches, including 9 that are defective, is sent to a department store. The receiving department selects 10 at random for testing and rejects the whole shipment if 1 or more in the sample are found defective. What is the probability that the shipment will be rejected?

Solution

The sample space: $S = C_{60,10}$

Let E = "Event that contains at least 1 defective watch".

E' = "Event that contains no defective watches".

$n(E') = C_{51,10}$

Probability that the shipment will be rejected:

$$
\begin{aligned}
P(E) &= 1 - P(E') \\
&= 1 - \frac{n(E')}{n(S)} \\
&= 1 - \frac{C_{51,10}}{C_{60,10}} \\
&= .83
\end{aligned}
$$

Exercise

If you bet \$5 on the number 13 in roulette, your probability of winning is $\frac{1}{38}$ and the payoff odds are given by the casino as 35:1.

a) Find the actual odds against the outcome of 13.

b) How much net profit would you make if you win by betting on 13?

c) If the casino was not operating for profit, and the payoff odds were changed to match the actual odds against 13, how much would you win if the outcome were 13?

Solution

a) With odds: $P(13) = \frac{1}{38}$ and $P(not\ 13) = \frac{37}{38}$

Actual odds against 13 $\frac{P(not\ 13)}{P(13)} = \frac{37/38}{1/38} = \frac{37}{1}$ or 37:1

b) Because the payoffs odds against 13 are 35:1, we have:

35:1 = (*net profit*) : (*amount bet*)

So there is a \$35 profit for each \$1 bet. For \$5 bet, the net profit is $5 \times 35 = \$175$.
The winning bettor would collect \$175 plus the original \$5 bet. That is, the total amount collected would be \$180, for the net profit of \$175.

c) If the casino were not operating for profit, the payoff odds would be equal to the actual odds against the outcome of 13, or 37:1. So there is a net profit of $37 for each $1 bet.
For a $5 bet the net profit would be $185. (The casino makes its profit by paying only $175 instead of the $185 that would be paid with a roulette game that is fair instead of favoring the casino.)