

Homework Sec 2.5

1. Verify that $W = \{(x_1, x_2, x_3, 0) : x_1, x_2, x_3 \in \mathbb{R}\}$ is a subspace of $V = \mathbb{R}^4$
2. Verify that W is the set of all 2×2 matrices of the form $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$ is a subspace of $V = M_{2,2}$
3. Verify that W is the set of all vectors in \mathbb{R}^3 whose third component is -1 is a subspace in \mathbb{R}^3
4. Verify that W is the set of all 3×3 matrices of the form $\begin{bmatrix} 1 & a & b \\ c & 1 & d \\ e & f & 0 \end{bmatrix}$ is a subspace of $V = M_{3,3}$
5. Verify that W is the set of all positive functions: $f(x) > 0$ is a subspace of $C(-\infty, \infty)$
6. Verify that W is the set of all $n \times n$ matrices with integer entries is a subspace of $M_{n,n}$
7. Verify that $W = \{(a, a - 3b, b) : a, b \in \mathbb{R}\}$ is a subspace of $V = \mathbb{R}^3$
8. Verify that $W = \{(x_1, x_2, x_1 x_2) : x_1, x_2 \in \mathbb{R}\}$ is a subspace of $V = \mathbb{R}^3$
9. Write each vector as a linear combination of the vectors in S (if possible)
 $S = \{(2, -1, 3), (5, 0, 4)\}$
 $a) \vec{z} = (-1, -2, 2) \quad \left| \quad b) \vec{v} = \left(8, -\frac{1}{4}, \frac{27}{4}\right) \quad \left| \quad c) \vec{w} = (1, -8, 12) \quad \left| \quad d) \vec{u} = (1, 1, -1)\right.\right.$
10. Write each vector as a linear combination of the vectors in S (if possible)
 $S = \{(2, 0, 7), (2, 4, 5), (2, -12, 13)\}$
 $a) \vec{u} = (-1, 5, -6) \quad \left| \quad b) \vec{v} = (-3, 15, 18) \quad \left| \quad c) \vec{w} = \left(\frac{1}{3}, \frac{4}{3}, \frac{1}{2}\right) \quad \left| \quad d) \vec{z} = (2, 20, -3)\right.\right.$
11. Determine whether the set $S = \{(2, 1), (-1, 2)\}$ spans \mathbb{R}^2
12. Determine whether the set $S = \{(-3, 5)\}$ spans \mathbb{R}^2
13. Determine whether the set $S = \{(1, 3), (-2, -6), (4, 12)\}$ spans \mathbb{R}^2
14. Determine whether the set $S = \{(4, 7, 3), (-1, 2, 6), (2, -3, 5)\}$ spans \mathbb{R}^3