Cramer's Rule

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases}$$

$|A|$ determinant $\det(A)$ — $[\ ]$ $(\ )$ ↓ matrix

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - b_1 a_2$$

Δ

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

α ω

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

If $D \neq 0 \Rightarrow D_x, D_y$; if $D_x = 0$, $D_y = 0$ $(x(y), y)$; $D_y \neq 0$ No soln

Ex

$$\begin{cases} 5x + 7y = -1 \\ 6x + 8y = 1 \end{cases}$$

$$D = \begin{vmatrix} 5 & 7 \\ 6 & 8 \end{vmatrix}$$

$$= 5(8) - 7(6)$$
$$= 40 - 42$$
$$= -2$$

$$D_x = \begin{vmatrix} -1 & 7 \\ 1 & 8 \end{vmatrix} = -8 - 7 = -15$$

$$D_y = \begin{vmatrix} 5 & -1 \\ 6 & 1 \end{vmatrix} = 5 + 6 = 11$$

$$\therefore \left(\frac{-15}{-2}, \frac{11}{-2} \right) \qquad \left(\frac{15}{2}, -\frac{11}{2} \right)$$

Q1. $x - 3y + 2z = 13$

$x + y + z = 1$

$x - 2y + 3z = 4$

$$D = \begin{vmatrix} 1 & -3 & 2 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} \begin{matrix} 1 & -3 \\ 1 & 1 \\ 1 & -2 \end{matrix}$$

$= 3 - 3 - 14 - 2 + 2 + 9$

$= -10$

$$D_x = \begin{vmatrix} 13 & -3 & 2 \\ 1 & 1 & 1 \\ 4 & -2 & 3 \end{vmatrix} \begin{matrix} 13 & -3 \\ 1 & 1 \\ 4 & -2 \end{matrix}$$

$= 39 - 12 - 14 - 28 + 26 + 9$

$= 20$

$$D_y = \begin{vmatrix} 1 & 13 & 2 \\ 1 & 1 & 1 \\ 1 & 4 & 3 \end{vmatrix} \begin{matrix} 1 & 13 \\ 1 & 1 \\ 1 & 4 \end{matrix}$$

+	−
3	2
13	4
21	39
44	50

$= -6$

$$D_z = \begin{vmatrix} 1 & -3 & 13 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix} \begin{matrix} 1 & -3 \\ 1 & 1 \\ 1 & -2 \end{matrix}$$

+	−
4	3
2	26
12	13
18	42

$= -24$

$$\left(\frac{20}{-10}, \frac{-6}{-10}, \frac{-24}{-10} \right)$$

$$\therefore \left(-2, \frac{3}{5}, \frac{12}{5} \right)$$

#1) $\begin{cases} 3x+2y=-4 \\ 2x-y=-5 \end{cases}$

$D=\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}$ $D_x=\begin{vmatrix} -4 & 2 \\ -5 & -1 \end{vmatrix}$ $D_y=\begin{vmatrix} 3 & -4 \\ 2 & -5 \end{vmatrix}$

$=-3-4$ $=4+10$ $=-15+8$

$=-7$ $=14$ $=-7$

$\therefore (-2, 1)$ $\frac{14}{-7}=-2$

#3) $\begin{cases} 3x+2y-z=4 \\ 3x-2y+z=5 \\ 4x-5y-z=-1 \end{cases}$

$D=\begin{vmatrix} 3 & 2 & -1 \\ 3 & -2 & 1 \\ 4 & -5 & -1 \end{vmatrix}\begin{matrix} 3 & 2 \\ 3 & -2 \\ 4 & -5 \end{matrix}$ $\begin{matrix} + & - \\ 6 & -8 \\ 8 & 15 \\ 15 & 6 \end{matrix}$

$=0$

$D_z=\begin{vmatrix} 3 & 2 & 4 \\ 3 & -2 & 5 \\ 4 & -5 & -1 \end{vmatrix}\begin{matrix} 3 & 2 \\ 3 & -2 \\ 4 & -5 \end{matrix}$ $\begin{matrix} + & - \\ 6 & 60 \\ 40 & \\ 6 & \\ 75 & \\ 32 & \\ 62 & \end{matrix}$

$=99 \neq 0$

$\therefore$ No solution

$D_z=0 \Rightarrow (x(z), y(z), z)$

Unique infinite solution (!)

Read: 5.2, 5.3 } 7/14