Formulas

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\quad\quad\ \ -\qquad\qquad\qquad\qquad +$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

3, 4 → 5   7, 24 → 25
5, 12 → 13   20, 21 → 29 ←
8, 15 → 17

$\cos^2 x + \sin^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$
$\cos^2 x = 1 - \sin^2 x$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\boxed{\begin{aligned} \cos^2 A &= \tfrac{1}{2}(1 + \cos 2A) \\ \sin^2 A &= \tfrac{1}{2}(1 - \cos 2A) \end{aligned}}$$

Similar
cos → +
sin → −

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

3/ $\sin A = \frac{4}{5}$ $A \in Q_{II}$ $\cos B = -\frac{5}{13}$ $B \in Q_{III}$

$\cos A = -\frac{3}{5}$ $\sin B = -\frac{12}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{-3}{5}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{-20+36}{65}$$

sign

$$= \boxed{\frac{16}{65}}$$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{15+48}{65}$$

sign

$$= \boxed{\frac{63}{65}}$$

c) $\tan(A+B$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= \frac{-20-36}{65}$$

$$= \boxed{-\frac{56}{65}}$$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$= \frac{15-48}{65} = \boxed{-\frac{33}{65}}$$

$$\tan(A+B) = \boxed{\frac{56}{33}}$$ ✓

14 points

Ex $$\frac{\cos(x-y)}{\sin x \sin y} = \cot x \cot y + 1$$

$$\frac{\cos(x-y)}{\sin x \sin y} = \frac{\cos x \cos y + \sin x \sin y}{\sin x \sin y}$$

$$= \boxed{\frac{\cos x}{\sin x}}\boxed{\frac{\cos y}{\sin y}} + \frac{\sin x \sin y}{\sin x \sin y}$$

$$= \cot x \cot y + 1$$ ✓

Ex $$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

$$\cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)}$$

avoid

$$= \frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y}$$

$$= \frac{\boxed{\frac{\cos x}{\sin x}}\boxed{\frac{\cos y}{\sin y}} - \frac{\sin x \sin y}{\sin x \sin y}}{\frac{\cancel{\sin x}\cos y}{\cancel{\sin x}\sin y} + \frac{\cos x \cancel{\sin y}}{\sin x \cancel{\sin y}}}$$

$= 1$

$$= \frac{\cot x \cot y - 1}{\cot y + \cot x}$$ ✓

15, 16, 20, 26, 30

$$\sec(x-y) = \frac{\cos x \cos y \quad \sin x \sin y}{\cos^2 x - \sin^2 y}$$

$$\sec(x-y) = \frac{1}{\cos(x-y)} \cdot \frac{\overbrace{\cos x \cos y - \sin x \sin y}^{\cos(x+y)}}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\cos(x+y)}{(\cos x \cos y + \sin x \sin y)(\cos x \cos y - \sin x \sin y)}$$

$$= \frac{\cos(x+y)}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}$$

$$= \frac{\cos(x+y)}{\cos^2 x (1-\sin^2 y) - (1-\cos^2 x)\sin^2 y}$$

$$= \frac{\cos(x+y)}{\cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \sin^2 y}$$

13, 27, 28,

#12/ $$\frac{\sin(A-B)}{\cos A\cos B}=\tan A-\tan B$$

$$\frac{\sin(A-B)}{\cos A\cos B}=\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}$$

$$=\frac{\sin A\cancel{\cos B}}{\cos A\cancel{\cos B}}-\frac{\cancel{\cos A}\sin B}{\cancel{\cos A}\cos B}$$

$$=\tan A-\tan B \checkmark$$

15/ $$\frac{\cos(x+y)}{\cos(x-y)}=\frac{\cot y-\tan x}{\cot y+\tan x}$$

$$\frac{\cos(x+y)}{\cos(x-y)}=\frac{\dfrac{\cos x\cos y}{\cancel{\cos x}\sin y}-\dfrac{\sin x\sin y}{\cos x\cancel{\sin y}}}{\dfrac{\cos x\cos y}{\cancel{\cos x}\sin y}+\dfrac{\sin x\sin y}{\cos x\cancel{\sin y}}}$$

$$=\frac{\cot y-\tan x}{\cot y+\tan x} \checkmark$$

13/ $\sec(A+B) = \dfrac{\cos(A-B)}{\cos^2 A - \sin^2 B}$

$$\sec(A+B) = \frac{1}{\cos(A+B)} \cdot \frac{\cos(A-B)}{\cos(A-B)}$$

$$= \frac{\cos(A-B)}{(\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)}$$

$$= \frac{\cos(A-B)}{\cos^2 A\cos^2 B - \sin^2 A\sin^2 B}$$

$$= \frac{\cos(A-B)}{\cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B}$$

$$= \frac{\cos(A-B)}{\cos^2 A - \underline{\cos^2 A\sin^2 B} - \sin^2 B + \underline{\cos^2 A\sin^2 B}}$$

$$= \frac{\cos(A-B)}{\cos^2 A - \sin^2 B} \quad ✓$$

8.3

$\sin 2A$ double angle

$\sin(2A)$

$\sin^2 A = (\sin A)^2$ square

$\sin A^2 = \sin(A^2)$

$\sin 2A \neq 2\sin A$

$$\sin 2A = \sin(A+A)$$
$$= \sin A\cos A + \cos A\sin A$$
$$= \boxed{2\sin A\cos A}$$

$$\cos 2A = \cos(A+A)$$
$$= \cos A\cos A - \sin A\sin A$$
$$= \boxed{\cos^2 A - \sin^2 A}$$

$$= 1 - \sin^2 A - \sin^2 A$$
$$= \boxed{1 - 2\sin^2 A}$$

$$= \cos^2 A - (1 - \cos^2 A)$$
$$= \cos^2 A - 1 + \cos^2 A$$
$$= \boxed{2\cos^2 A - 1}$$

$$\cos A = 2\cos^2 \frac{A}{2} - 1$$

$$2\cos^2 \frac{A}{2} = 1 + \cos A$$

$$\cos^2 \frac{A}{2} = \frac{1 + \cos A}{2}$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\cos \frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

which Q

$$\sin \frac{A}{2} = \pm\sqrt{\frac{1}{2}(1-\cos A)}$$

$$\tan \frac{A}{2} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$$

$\sin A = \frac{3}{5}$ $\quad A \in Q\,II$

$\cos A = -\frac{4}{5}$

$$\sin 2A = 2\sin A\cos A$$

$$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$$

$$= -\frac{24}{25}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{25}$$

$$\frac{16-9}{25}$$