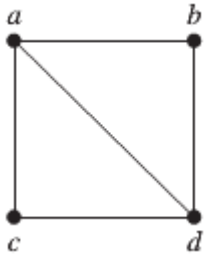
SOLUTION *Section* 4.7 – Representing Graphs and Graph Isomorphism

Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



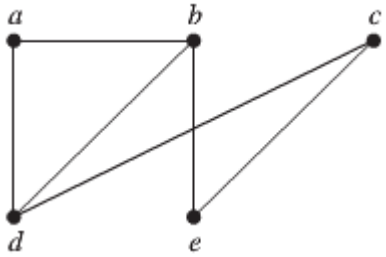
Solution

Vertex	*Adjacent Vertices*
a	*b, c*
b	*a, d*
c	*a, d*
d	*a, b, c*

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



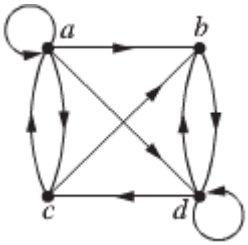
Solution

Vertex	*Adjacent Vertices*
a	*b, d*
b	*a, d, e*
c	*d, e*
d	*a, b, c*
e	*b, c*

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



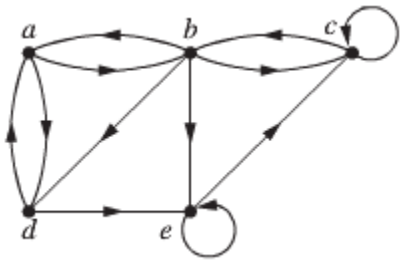
Solution

Initial Vertex	*Terminal Vertices*
a	*a, b , c, d*
b	*d*
c	*a, b*
d	*b, c, d*

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



Solution

Initial Vertex	*Terminal Vertices*
a	*b, d*
b	*a, c, d, e*
c	*b, c*
d	*a, e*
e	*c , e*

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

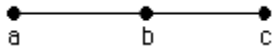
Exercise

Draw a graph with the given adjacency

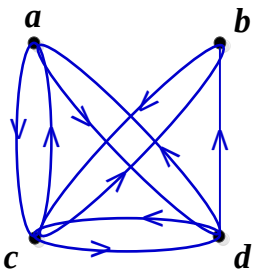
a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ *b*) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ *c*) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

Solution

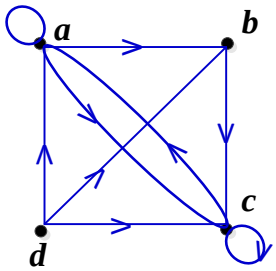
a)



b)



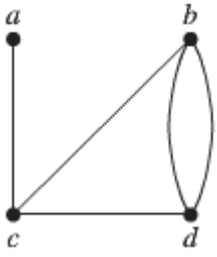
c)



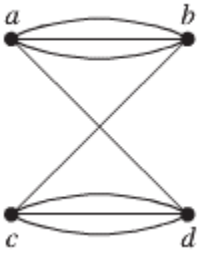
Exercise

Represent the given graph using adjacency matrix

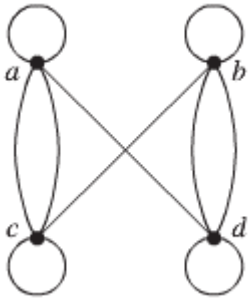
a)



b)



c)



Solution

a) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

Exercise

Draw an undirected graph represented by the given adjacency

a) $\begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix}$

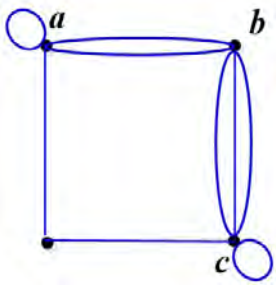
b) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 3 & 0 & 4 \\ 1 & 2 & 1 & 3 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$

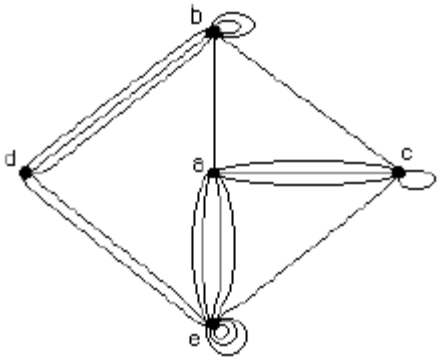
Solution

a)

b)



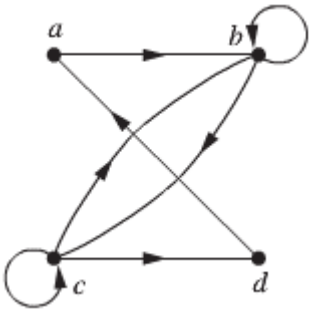
c)



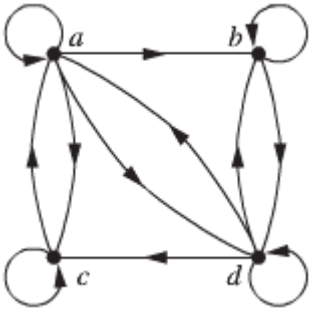
Exercise

Find the adjacency matrix of the given directed multigraph with respect to the vertices listed in alphabetic order.

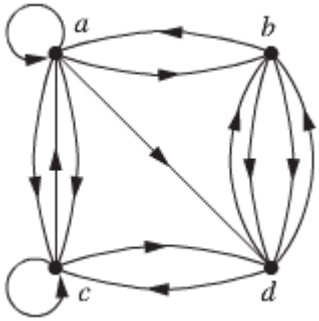
a)



b)



c)



Solution

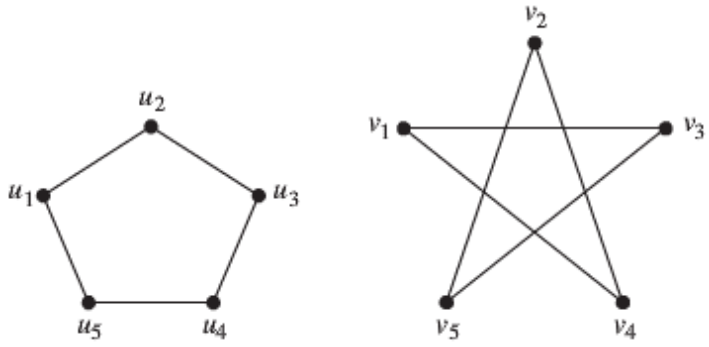
a) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

Exercise

Determine whether the given pair of graphs is isomorphic.
Exhibit an isomorphism or provide a rigorous argument that none exists.



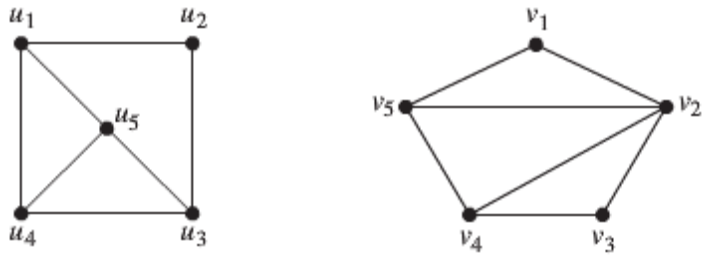
Solution

Both graphs have 5 vertices and 5 edges.

However, each vertex in the second graph has of degree 2, whereas the first does not.

Exercise

Determine whether the given pair of graphs is isomorphic.
Exhibit an isomorphism or provide a rigorous argument that none exists.

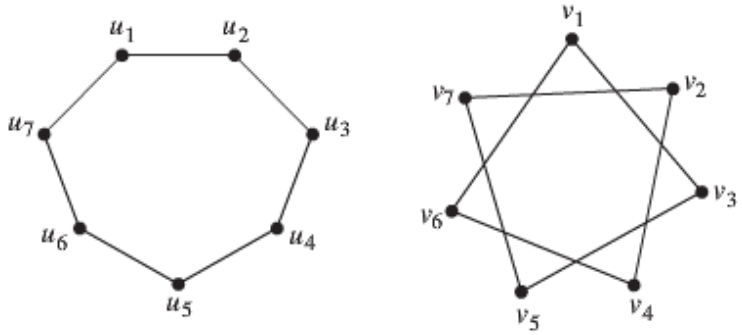


Solution

Both graphs have 5 vertices and 7 edges.
However, the second graph has a vertex of degree 4, whereas the first does not.

Exercise

Determine whether the given pair of graphs is isomorphic.
Exhibit an isomorphism or provide a rigorous argument that none exists.



Solution

Both graphs have 7 vertices and 7 edges.

$f(u_1) = v_1$

$f(u_2) = v_3$

$f(u_3) = v_5$

$f(u_4) = v_7$

$f(u_5) = v_2$

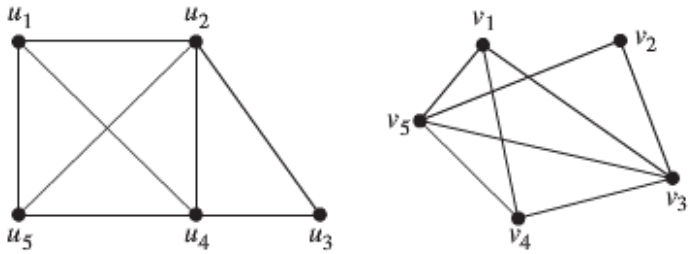
$f(u_6) = v_4$

and $f(u_7) = v_6$

$\therefore$ The graphs are isomorphic.

Exercise

Determine whether the given pair of graphs is isomorphic.
Exhibit an isomorphism or provide a rigorous argument that none exists.



Solution

Both graphs have 5 vertices and 8 edges.

$f(u_1) = v_1$

$f(u_2) = v_3$

$f(u_3) = v_2$

$f(u_4) = v_5$

and $f(u_5) = v_4$

$\therefore$ The graphs are isomorphic.