Solution Section 1.3 – Quadratic Functions

Exercise

For the function $f(x) = x^2 + 6x + 3$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function *increasing? decreasing?*

Solution

a)
$$x = -\frac{6}{2(1)} = -3$$

 $y = f(-3) = (-3)^2 + 6(-3) + 3 = -6$ Vertex point $(-3, -6)$

- **b**) Line of symmetry: x = -3
- c) Minimum point, value (-3, -6)

d)
$$x = \frac{-6 \pm \sqrt{36 - 12}}{2} = \frac{-6 \pm \sqrt{24}}{2} = \frac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$$

$$x = \begin{cases} -3 + \sqrt{6} = -0.5 \\ -3 - \sqrt{6} = -5.45 \end{cases}$$

- e) y-intercept y = 3
- f) Range: $[-6, \infty)$ Domain: $(-\infty, \infty)$

Symmetry: x = -3Symmetry: x = -3 $f(x) = x^2 + 6x + 3$ Vertex Point / Min (-3, -6)

h) Decreasing: $(-\infty, -3)$ Increasing: $(-3, \infty)$

For the function $f(x) = x^2 + 6x + 5$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{6}{2}$$
 $x = -\frac{b}{2a}$

$$y = f(-3) = (-3)^{2} + 6(-3) + 5$$
$$= -4$$

Vertex point: (-3,-4)

- **b**) Axis of symmetry: x = -3
- c) Minimum point @ (-3,-4)

d)
$$x^2 + 6x + 5 = 0$$

 $x = -5, -1$

- $e) \quad x = 0 \quad \to \quad \underline{y = 5}$
- f) Domain: \mathbb{R} Range: $[-4, \infty)$

g)

4-y
311x
-6 -5 -4 -3 -2 -1
-1-2-3-4-5-

- **h**) Increasing: $(-3, \infty)$
- Decreasing: $(-\infty, -3)$

For the function $f(x) = -x^2 - 6x - 5$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{-6}{-2}$$
 $x = -\frac{b}{2a}$
 $= -3$ $y = f(-3) = -9 + 18 - 5$

Vertex point: (-3, 4)

- **b**) Axis of symmetry: x = -3
- c) Maximum point @ (-3, 4)

d)
$$-(x^2 + 6x + 5) = 0$$

 $x = -5, -1$

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = -5}$
- f) Domain: \mathbb{R} Range: $(-\infty, 4]$
- g)

 5-y

 43211x

 -6 -5 -4 -3 -2 -1
 -1-2-3-
- **h**) Increasing: $(-\infty, -3)$ Decreasing: $(-3, \infty)$

For the function $f(x) = x^2 - 4x + 2$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{-4}{2}$$

$$= 2 \mid$$

$$f(2) = 4 - 8 + 2$$

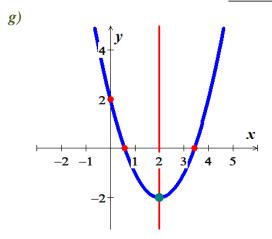
$$= -2 \mid$$

Vertex point: (2, -2)

- **b**) Axis of symmetry: x = 2
- c) Minimum point @ (2, -2)

d)
$$x^{2} - 4x + 2 = 0$$
$$x = \frac{4 \pm \sqrt{8}}{2}$$
$$x = 2 \pm \sqrt{2}$$

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = 2}$
- f) Domain: \mathbb{R} Range: $[-2, \infty)$



h) Increasing: $(2, \infty)$ Decreasing: $(-\infty, 2)$

For the function $f(x) = -2x^2 + 16x - 26$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{16}{-4}$$
 $x = -\frac{b}{2a}$
 $= 4$ $f(4) = -32 + 64 - 26$ $= 6$

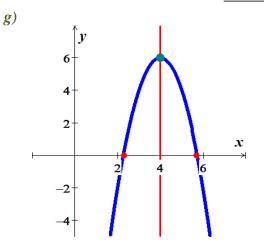
Vertex point: (4, 6)

- **b**) Axis of symmetry: x = 4
- c) Maximum point (4, 6)

d)
$$-2x^2 + 16x - 26 = 0$$

 $x = \frac{-16 \pm \sqrt{128}}{-4}$
 $x = 4 \pm 2\sqrt{2}$

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = -26}$
- f) Domain: \mathbb{R} Range: $(-\infty, 6]$



h) Increasing: $(-\infty, 4)$ Decreasing: $(4, \infty)$

For the function $f(x) = x^2 + 4x + 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{4}{2}$$

$$= -2$$

$$f(-2) = 4 - 8 + 1$$

$$= -3$$

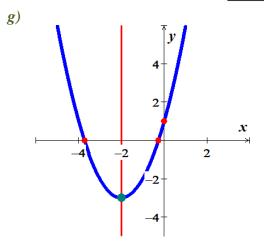
Vertex point: (-2, -3)

- **b**) Axis of symmetry: x = -2
- c) Minimum point @ (-2, -3)

d)
$$x^2 + 4x + 1 = 0$$

 $x = \frac{-4 \pm \sqrt{12}}{2}$
 $x = -2 \pm \sqrt{3}$

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = 1}$
- f) Domain: \mathbb{R} Range: $[-3, \infty)$



h) Increasing: $(-2, \infty)$ Decreasing: $(-\infty, -2)$

For the function $f(x) = x^2 - 8x + 5$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

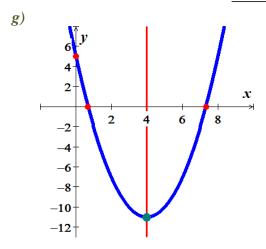
a)
$$x = -\frac{-8}{2}$$
 $x = -\frac{b}{2a}$
 $= 4$
 $f(4) = 16 - 32 + 5$
 $= -11$

Vertex point: (4, -11)

- **b**) Axis of symmetry: x = 4
- c) Minimum point @ (4, -11)

$$d) \quad x^2 - 8x + 5 = 0$$
$$x = \frac{8 \pm \sqrt{44}}{2}$$
$$x = 4 \pm \sqrt{11}$$

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = 5}$
- f) Domain: \mathbb{R} Range: $[-11, \infty)$



h) Increasing: $(4, \infty)$ Decreasing: $(-\infty, 4)$

For the function $f(x) = x^2 + 6x - 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{6}{2}$$

$$= -3$$

$$f(-3) = 9 - 18 - 1$$

$$= -10$$

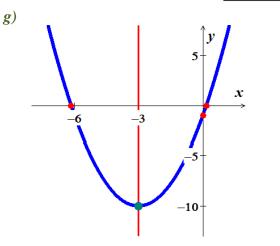
Vertex point: (-3, -10)

- **b**) Axis of symmetry: x = -3
- c) Minimum point @ (-3, -10)

d)
$$x^2 + 6x - 1 = 0$$

 $x = \frac{-6 \pm \sqrt{40}}{2}$
 $x = -3 \pm \sqrt{10}$

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = -1}$
- f) Domain: \mathbb{R} Range: $[-10, \infty)$



h) Increasing: $(-3, \infty)$ Decreasing: $(-\infty, -3)$

For the function $f(x) = x^2 + 6x + 3$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{6}{2}$$

$$= -3$$

$$f(-3) = 9 - 18 + 3$$

$$= -6$$

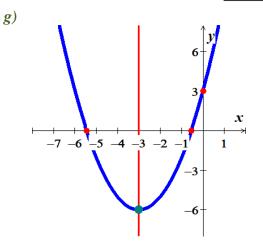
Vertex point: (-3, -6)

- **b**) Axis of symmetry: x = -3
- c) Minimum point @ (-3, -6)

d)
$$x^2 + 6x + 3 = 0$$

 $x = \frac{-6 \pm \sqrt{24}}{2}$
 $x = -3 \pm \sqrt{6}$

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = 3}$
- f) Domain: \mathbb{R} Range: $[-6, \infty)$



h) Increasing: $(-3, \infty)$ Decreasing: $(-\infty, -3)$

For the function $f(x) = x^2 - 10x + 3$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{-10}{2}$$
 $x = -\frac{b}{2a}$
 $= 5$ $f(5) = 25 - 50 + 3$ $= -22$

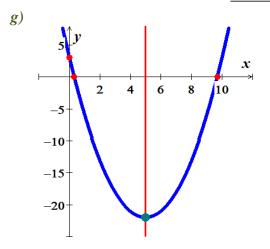
Vertex point: (5, -22)

- **b**) Axis of symmetry: x = 5
- c) Minimum point @ (5, -22)

d)
$$x^2 - 10x + 3 = 0$$

 $x = \frac{10 \pm \sqrt{88}}{2}$
 $x = 5 \pm \sqrt{22}$

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = 3}$
- f) Domain: \mathbb{R} Range: $[-22, \infty)$



h) Increasing: $(5, \infty)$ Decreasing: $(-\infty, 5)$

For the function $f(x) = x^2 - 3x + 4$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = \frac{3}{2}$$

$$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} + 4$$

$$= \frac{7}{4}$$

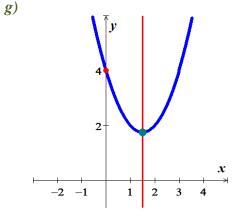
Vertex point: $\left(\frac{3}{2}, \frac{7}{4}\right)$

- **b**) Axis of symmetry: $x = \frac{3}{2}$
- c) Minimum point @ $\left(\frac{3}{2}, \frac{7}{4}\right)$

d)
$$x^2 - 3x + 4 = 0$$

 $x = \frac{3 \pm \sqrt{-7}}{2}$ \mathbb{C}

- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = 4}$
- f) Domain: \mathbb{R} Range: $\left[\frac{7}{4}, \infty\right)$



h) Increasing: $\left(\frac{3}{2}, \infty\right)$ Decreasing: $\left(-\infty, \frac{3}{2}\right)$

For the function $f(x) = x^2 - 3x - 4$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function *increasing? decreasing?*

Solution

$$x = \frac{3}{2}$$

$$x = -\frac{b}{2a}$$

$$x = -\frac{b}{2a}$$

$$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} - 4$$

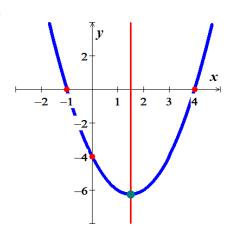
$$=-\frac{25}{4}$$

Vertex point: $\left(\frac{3}{2}, -\frac{25}{4}\right)$

- **b**) Axis of symmetry: $x = \frac{3}{2}$
- c) Minimum point @ $\left(\frac{3}{2}, -\frac{25}{4}\right)$
- **d**) $x^2 3x 4 = 0$ x = -1, 4
- $e) \quad x = 0 \quad \to \quad \underline{y = -4}$

f) Domain: \mathbb{R} Range: $\left|-\frac{25}{4}, \infty\right|$

g)



h) Increasing: $\left(\frac{3}{2}, \infty\right)$

Decreasing:

 $f(x) = x^2 - 4x - 5$ For the function

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$\underline{x=2}$$

$$x = -\frac{b}{2a}$$

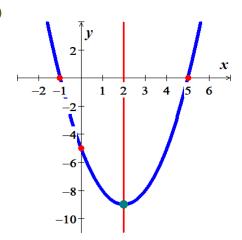
$$f(2) = 4 - 8 - 3$$

f(2) = 4 - 8 - 5 = -9Vertex point: (2, -9)

- **b**) Axis of symmetry: $\underline{x} = 2$
- c) Minimum point @ (2, -9)
- d) $x^2 4x 5 = 0$ x = -1, 5
- $e) \quad x = 0 \quad \rightarrow \quad y = -5 \mid$
- f) Domain: \mathbb{R}

Range: $[-9, \infty)$

g)



h) Increasing: $(2, \infty)$

Decreasing: $(-\infty, 2)$

For the function $f(x) = 2x^2 - 3x + 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function *increasing? decreasing?*

Solution

$$x = \frac{3}{4}$$

$$x = -\frac{b}{2a}$$

$$x = -\frac{b}{2a}$$

$$f\left(\frac{3}{4}\right) = \frac{9}{8} - \frac{9}{4} + 1$$

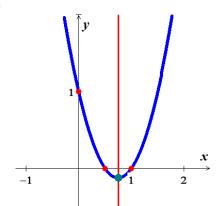
$$=-\frac{1}{8}$$

Vertex point: $\left(\frac{3}{4}, -\frac{1}{8}\right)$

- **b**) Axis of symmetry: $x = \frac{3}{4}$
- c) Minimum point @ $\left(\frac{3}{4}, -\frac{1}{8}\right)$
- d) $2x^2 3x + 1 = 0$ $x = 1, \frac{1}{2}$
- $e) \quad x = 0 \quad \to \quad \underline{y = 1}$

f) Domain: \mathbb{R} Range: $\left[-\frac{1}{8}, \infty\right)$

g)



h) Increasing: $\left(\frac{3}{4}, \infty\right)$ Decreasing: $\left(-\infty, \frac{3}{4}\right)$

For the function $f(x) = -x^2 - 3x + 4$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

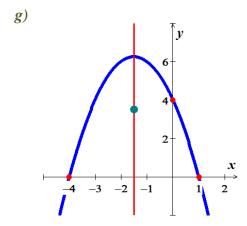
a)
$$x = -\frac{3}{2}$$

$$f\left(-\frac{3}{2}\right) = -\frac{9}{4} + \frac{9}{2} + 4$$

$$= \frac{7}{2}$$

Vertex point:
$$\left(-\frac{3}{2}, \frac{7}{2}\right)$$

- **b**) Axis of symmetry: $x = -\frac{3}{2}$
- c) Maximum point @ $\left(-\frac{3}{2}, \frac{7}{2}\right)$
- d) $-x^2 3x + 4 = 0$ x = 1, -4
- $e) \quad x = 0 \quad \rightarrow \quad \underline{y = 4}$
- f) Domain: \mathbb{R} Range: $\left(-\infty, \frac{7}{2}\right]$



h) Increasing: $\left(-\infty, -\frac{3}{2}\right)$ Decreasing: $\left(-\frac{3}{2}, \infty\right)$

For the function $f(x) = -2x^2 + 3x - 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function *increasing? decreasing?*

Solution

a)
$$x = \frac{3}{4}$$
 $x = -\frac{b}{2a}$

$$f\left(\frac{3}{4}\right) = -\frac{9}{8} + \frac{9}{4} - 1$$
$$= \frac{1}{8}$$

Vertex point: $\left(\frac{3}{4}, \frac{1}{8}\right)$

- **b**) Axis of symmetry: $x = \frac{3}{4}$
- c) Maximum point @ $\left(\frac{3}{4}, \frac{1}{8}\right)$
- d) $-2x^2 + 3x 1 = 0$ $x = 1, \frac{1}{2}$
- $e) \quad x = 0 \quad \to \quad \underline{y = -1}$
- f) Domain: \mathbb{R} | Range: $\left(-\infty, \frac{1}{8}\right]$

9) 0.5 v -0.5 1.0 1.5

h) Increasing: $\left(-\infty, \frac{3}{4}\right)$ Decreasing: $\left(\frac{3}{4}, \infty\right)$

For the function $f(x) = -2x^2 - 3x - 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$x = -\frac{3}{4}$$

$$f\left(-\frac{3}{4}\right) = -\frac{9}{8} + \frac{9}{4} - 1$$

$$= \frac{1}{8}$$

Vertex point: $\left(-\frac{3}{4}, \frac{1}{8}\right)$

- **b)** Axis of symmetry: $x = -\frac{3}{4}$
- c) Maximum point @ $\left(-\frac{3}{4}, \frac{1}{8}\right)$
- d) $-2x^2 3x 1 = 0$ $x = -1, -\frac{1}{2}$
- $e) \quad x = 0 \quad \to \quad \underline{y = -1}$
- f) Domain: \mathbb{R} Range: $\left(-\infty, \frac{1}{8}\right]$
- **h)** Increasing: $\left(-\infty, -\frac{3}{4}\right)$ Decreasing: $\left(-\frac{3}{4}, \infty\right)$

For the function $f(x) = -x^2 - 4x + 5$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function *increasing? decreasing?*

Solution

a)
$$\underline{x = -2}$$

$$a) \quad \underline{x = -2} \qquad \qquad x = -\frac{b}{2a}$$

$$f\left(\frac{-2}{-2}\right) = -4 + 8 + 5$$

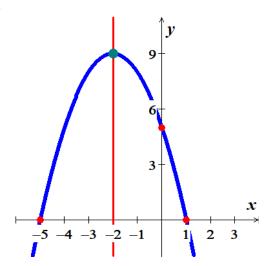
$$= 9$$
Vertex point: $\left(\frac{-2}{9}\right)$

- **b**) Axis of symmetry: x = -2
- c) Maximum point @ (-2, 9)
- d) $-x^2 4x + 5 = 0$ x = 1, -5

$$e) \quad x = 0 \quad \rightarrow \quad y = 5$$

f) Domain: \mathbb{R} Range: $(-\infty, 9]$

g)



h) Increasing: $(-\infty, -2)$

Decreasing: $(-2, \infty)$

 $f(x) = -x^2 + 4x + 2$ For the function

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

a)
$$\underline{x=2}$$

$$x = -\frac{b}{2a}$$

$$f\left(\frac{2}{2}\right) = -4 + 8 + 2$$

$$= 6$$
Vertex point: $(2, 6)$

- **b**) Axis of symmetry: x = 2
- (2, 6)c) Maximum point @

$$d) -x^2 + 4x + 2 = 0$$

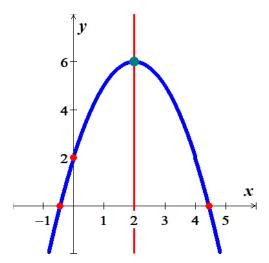
$$x = \frac{-4 \pm \sqrt{16 + 8}}{-2}$$

$$x = 2 \pm \sqrt{6}$$

- e) $x = 0 \rightarrow y = 2$
- f) Domain: \mathbb{R}

Range: $\left(-\infty, \underline{6}\right]$

g)



h) Increasing: $(-\infty, 2)$

Decreasing:

For the function $f(x) = -3x^2 + 3x + 7$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- f) Find the range and the domain of the function.
- g) Graph the function
- h) On what intervals is the function *increasing? decreasing?*

Solution

a)
$$x = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = -\frac{3}{4} + \frac{3}{2} + 7 \qquad \underline{=} \frac{31}{4}$$

Vertex point: $\left(\frac{1}{2}, \frac{31}{4}\right)$

- **b**) Axis of symmetry: $x = \frac{1}{2}$
- c) Maximum point @ $\left(\frac{1}{2}, \frac{31}{4}\right)$
- d) $-3x^2 + 3x + 7 = 0$ $x = \frac{-3 \pm \sqrt{93}}{-6}$ $x = \frac{3 \pm \sqrt{93}}{6}$
- $e) \quad x = 0 \quad \rightarrow \quad y = 7$
- f) Domain: \mathbb{R} Range: $\left(-\infty, \frac{31}{4}\right]$
- **h)** Increasing: $\left(-\infty, \frac{1}{2}\right)$ Decreasing: $\left(\frac{1}{2}, \infty\right)$

 $f(x) = -x^2 + 2x - 2$ For the function

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a maximum or minimum value and find that value
- d) Find the zeros of f(x)
- e) Find the y-intercept
- Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function increasing? decreasing?

Solution

$$a)$$
 $\underline{x=1}$

$$x = -\frac{b}{2a}$$

$$f(1) = -1 + 2 - 2$$

$$= -1$$

$$Vertex point: (1, -1)$$

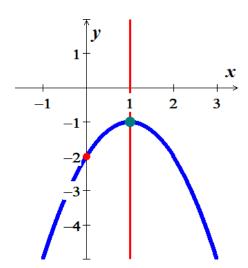
- **b**) Axis of symmetry: x = 1
- c) Maximum point @ (1, -1)
- $d) -x^2 + 2x 2 = 0$

$$x = \frac{-2 \pm \sqrt{-4}}{-2} \quad \mathbb{C}$$

- $e) \quad x = 0 \quad \to \quad \underline{y = -2}$

f) Domain: \mathbb{R} Range: $(-\infty, -1]$

g)



h) Increasing: $(-\infty, 1)$

Decreasing: $(1, \infty)$