8: 3. C $\sin A = -\frac{3}{5}$ $A \in$ QIII

$\cos A = -\frac{4}{5}$ $\frac{180°}{2} < \frac{A}{2} < \frac{270°}{2}$

$\frac{A}{2} \in$ QII

a) $\sin 2A = 2 \sin A \cos A$

$= 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= \frac{24}{25}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

c) $\tan 2A = \frac{24}{7}$

d) $\sin \frac{A}{2} = \sqrt{\frac{1}{2}(1 - \cos A)}$ $\frac{A}{2} \in$ QII

$= \sqrt{\frac{1}{2}\left(1 + \frac{4}{5}\right)}$

$= \sqrt{\frac{9}{10}}$

$= \frac{3}{\sqrt{10}}$

e) $\cos \frac{A}{2} = -\sqrt{\frac{1}{2}(1 + \cos A)}$

$= -\sqrt{\frac{1}{2}\left(1 - \frac{4}{5}\right)}$

$= -\frac{1}{\sqrt{10}}$

f) $\tan A/2 = -3$

P. 434

$\cos 2\theta + 3\sin\theta - 2 = 0$

$1 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$-2\sin^2\theta + 3\sin\theta - 1 = 0$

$\sin\theta = 1 \qquad \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin\theta \tan\theta = \sin\theta$

$\sin\theta \tan\theta - \sin\theta = 0$

$\sin\theta(\tan\theta - 1) = 0$

$\tan\theta - 1 = 0$

$\tan\theta = 1$

$\sin\theta = 0 \qquad \tan\theta = 1$

$\theta = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$

$2\tan x \csc x + 2\csc x + \tan x + 1 = 0$

$2\csc x(\tan x + 1) + (\tan x + 1) = 0$

$(\tan x + 1)(2\csc x + 1) = 0$

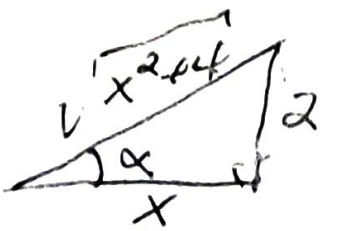
$\tan x = -1 \qquad \csc x = -\frac{1}{2} = \frac{1}{\sin x}$

$\sin x = -2$ #

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

7.5 $\sin\left(\cos^{-1}\frac{x}{\sqrt{x^2+4}}\right)$

$\cos\alpha = \frac{x}{\sqrt{x^2+4}}$



$\sin\alpha = \frac{2}{\sqrt{x^2+4}}$

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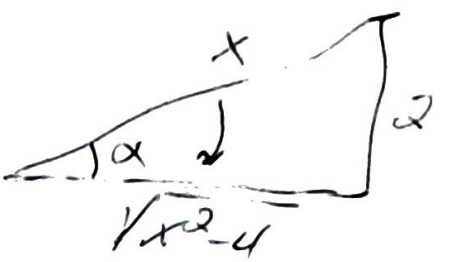
39. $\sec\left(\tan^{-1}\frac{2}{\sqrt{x^2-4}}\right)$

$\tan\alpha = \frac{2}{\sqrt{x^2-4}}$



$\sec\alpha = \frac{x}{\sqrt{x^2-4}}$

8.6 6 $(4\sqrt{3}, -\frac{\pi}{6})$

$x = r\cos\theta$
$= 4\sqrt{3}\cos(-\frac{\pi}{6})$
$= 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$
$= 6$

$y = r\sin\theta$
$= 4\sqrt{3}\sin(-\frac{\pi}{6})$
$= -4\sqrt{3}\left(\frac{1}{2}\right)$
$= -2\sqrt{3}$

$(x, y) = (6, -2\sqrt{3})$

16 $(-1, -\sqrt{3})$

$r = \sqrt{x^2+y^2}$

$= \sqrt{1+3}$

$= 2$

$\theta = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$ $\frac{\sqrt{3}/2}{1/2}$

$= \frac{\pi}{3}$

Ⅲ ⊆ G Ⅲ $\theta = \frac{4\pi}{3}$

$(r, \theta) = \left(2, \frac{4\pi}{3}\right)$

32) $r = 8\sin\theta - 2\cos\theta$

$r^2(r) = 8r\sin\theta - 2r\cos\theta$

$x^2 + y^2 = 8y - 2x$

43) $x + y = 4$

$r\cos\theta + r\sin\theta = 4$

$r(\cos\theta + \sin\theta) = 4$

$r = \frac{4}{\cos\theta + \sin\theta}$

22, §7.1

$$\cos^4 x - \sin^4 x = \cos 2x$$

$$\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$$

$$= \cos 2x \quad (1)$$

$$= \cos 2x \quad \checkmark$$

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51 $\dfrac{1-\cos^2 x}{1+\cos x} = \dfrac{\sec x - 1}{\sec x}$

$$\frac{\sec x - 1}{\sec x} = \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x}}$$

$$= \frac{1-\cos x}{\cos x} \cdot \frac{\cos x}{1}$$

$$= 1-\cos x \quad \frac{1+\cos x}{1+\cos x}$$

$$= \frac{1-\cos^2 x}{1+\cos x} \quad \checkmark$$

63 $\csc x - \sin x = \cos x \cot x$

$$\csc x - \sin x = \frac{1}{\sin x} - \sin x$$

$$= \frac{1-\sin^2 x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x}$$

$$= \cos x \frac{\cos x}{\sin x}$$

$$= \cos x \cot x \quad \checkmark$$

8. 2. $\sec(A+B) = \dfrac{\cos(A-B)}{\cos^2 A - \sin^2 B}$

$$\sec(A+B) = \frac{1}{\cos(A+B)} \cdot \frac{\cos(A-B)}{\cos(A-B)}$$

$$= \frac{\cos(A-B)}{(\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)}$$

$$= \frac{\cos(A-B)}{\cos^2 A\cos^2 B - \sin^2 A \sin^2 B}$$

$$= \frac{\cos(A-B)}{\cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B}$$

$$= \frac{\cos(A-B)}{\cos^2 A - \cos^2 A\sin^2 B - \sin^2 B + \cos^2 A \sin^2 B}$$

$$= \frac{\cos(A-B)}{\cos^2 A - \sin^2 B} \checkmark$$

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16 $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{\cot y + \cot x}{\cot y - \cot x}$

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\frac{\sin x\cos y}{\sin x\sin y} + \frac{\cos x\sin y}{\sin x\sin y}}{\frac{\sin x\cos y}{\sin x\sin y} - \frac{\cos x\sin y}{\sin x\sin y}}$$

$$= \frac{\cot y + \cot x}{\cot y - \cot x} \checkmark$$

11-17 $\frac{\sin(x-y)}{\sin x \cos y} = 1 - \cot x \tan y$

$$\frac{\sin(x-y)}{\sin x \cos y} = \frac{\sin x \cos y - \cos x \sin y}{\sin x \cos y}$$

$$= \frac{\sin x \cos y}{\sin x \cos y} - \frac{\cos x \sin y}{\sin x \cos y}$$

$$= 1 - \cot x \tan y.$$

5.3 (27) $\frac{\cos 2x}{\cos^2 x} = \sec^2 x - 2\tan^2 x$

$$\frac{\cos 2x}{\cos^2 x} = \frac{1 - 2\sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - 2\frac{\sin^2 x}{\cos^2 x}$$

$$= \sec^2 x - 2\tan^2 x$$

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42 $2\sin^2 \frac{x}{2} = \frac{\sin^2 x}{1+\cos x}$

$$2\sin^2 \frac{x}{2} = 2\,\frac{1-\cos x}{2}$$

$$= (1-\cos x)\frac{1+\cos x}{1+\cos x}$$

$$= \frac{1-\cos^2 x}{1+\cos x}$$

$\cos^2 x + \sin^2 x = 1$

$$= \frac{\sin^2 x}{1+\cos x}$$

40. $\sin^2\frac{x}{2}\cos^2\frac{x}{2} = \frac{\sin^2 x}{4}$

$$\sin^2\frac{x}{2}\cos^2\frac{x}{2} = \left(\sin\frac{x}{2}\cos\frac{x}{2}\right)^2$$

$$= \left(\frac{1}{2}\sin x\right)^2 \qquad \left(\sin 2\left(\frac{x}{2}\right)\right)$$

$$= \frac{1}{4}\sin^2 x$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$\sin^2\frac{x}{2}\cos^2\frac{x}{2} = \frac{1-\cos x}{2}\cdot\frac{1+\cos x}{2}$$

$$= \frac{1}{4}(1-\cos^2 x)$$

$$= \frac{1}{4}\sin^2 x$$

8.4

12 $1-\sin x = \sqrt{3}\cos x \qquad (\ )^2 = (\ )^2$

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{1}{2}$$

$\sin\frac{\pi}{6}\sin x + \cos\frac{\pi}{6}\cos x = \cos\frac{\pi}{3}\ ;\ \frac{5\pi}{3}$

$\cos\left(x-\frac{\pi}{6}\right) = \cos\frac{\pi}{3}\ ,\ \frac{5\pi}{3}$

$x-\frac{\pi}{6} = \frac{\pi}{3}$ $\qquad$ $x-\frac{\pi}{6} = \frac{5\pi}{3}$

$x = \frac{\pi}{3}+\frac{\pi}{6}$ $\qquad$ $x = \frac{5\pi}{3}+\frac{\pi}{6}$

$= \frac{\pi}{2}$ $\qquad$ $= \frac{11\pi}{6}$

20 $\tan x + \sqrt{3} = \sec x$

$$\frac{\sin x}{\cos x} + \sqrt{3} = \frac{1}{\cos x}$$

$\cos x \neq 0$

$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

$$\sin x + \sqrt{3}\cos x = 1$$

$x = \frac{\pi}{2}, \frac{11\pi}{6}$