Lecture Six – Trigonometric

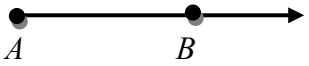
Section 6.1 – Introduction

Basic Terminology

Two distinct points determine line AB.

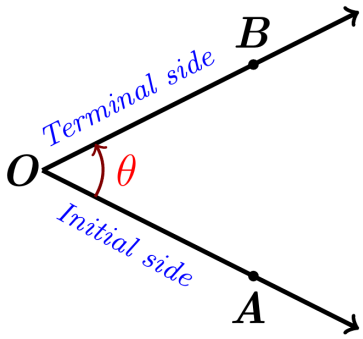
Line segment AB: portion of the line between A and B.

Ray AB: portion of the line AB starts at A and continues through B, and past B.



Angles in General

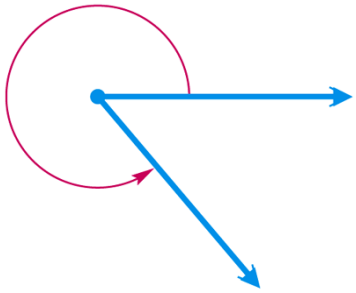
An angle is formed by 2 rays with the same end point.



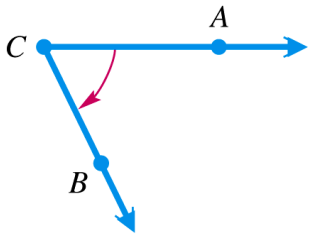
The two rays are the sides of the angle, angle $\theta = AOB$
O is the common endpoint and it is called ***vertex*** of the angle**.**

An angle is in a Counterclockwise (***CCW***) direction: positive angle.

An angle is in a Clockwise (***CW***) direction: negative angle.

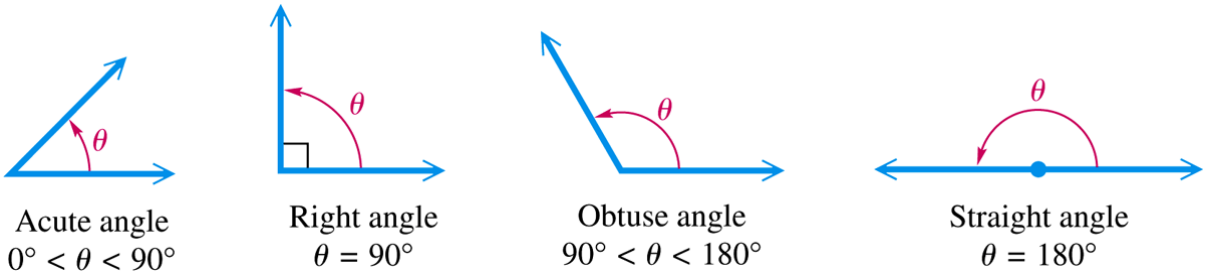


Positive angle



Negative angle

Type of Angles: *Degree*



Acute angle $0° < \theta < 90°$ | Right angle $\theta = 90°$ | Obtuse angle $90° < \theta < 180°$ | Straight angle $\theta = 180°$

Complementary angles: $\alpha + \beta = 90°$

Supplementary angles: $\alpha + \beta = 180°$

Example

Give the complement and the supplement of each angle: 40° 110°

Solution

a. 40° Complement: $90° - 40° = 50°$ Supplement: $180° - 40° = 140°$

b. 110° Complement: $90° - 110° = -20°$ Supplement: $180° - 110° = 70°$

Degrees, Minutes, Seconds

1°: 1 *degree*
1′: 1 *minute*
1″: 1 *second*

1 *full Rotation* or ***Revolution* = 360°** $1° = 60' = 3600''$ $1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)°$

Example

Change 27.25° to degrees and minutes

Solution

$27.25° = 27° + .25°$

$= 27° + .25(\mathbf{60'})$

$= 27° + 15'$

$= 27° \quad 15'$

Example

Add $48^\circ\ 49'$ and $72^\circ\ 26'$

Solution

$$\begin{array}{r} 48^\circ \quad 49' \\ +\ 72^\circ \quad 26' \\ \hline 120^\circ \quad 75' \end{array}$$

$$120^\circ\ 75' = 120^\circ\ 60' + 15'$$
$$= 121^\circ\ 15'$$

Example

Subtract $24^\circ\ 14'$ and 90°

Solution

$$\begin{array}{r} 90^\circ \\ -\ 24^\circ \quad 14' \\ \hline \end{array} = \begin{array}{r} 89^\circ \quad 60' \\ -\ 24^\circ \quad 14' \\ \hline 65^\circ \quad 46' \end{array}$$

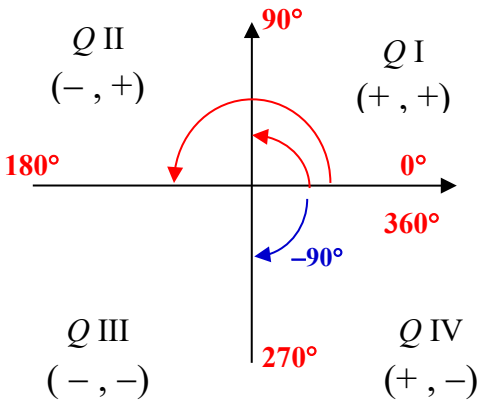
Angles in Standard Position

An angle is said to be in standard position if its initial side is along the positive x-axis and its vertex is at the origin. If angle θ is in standard position and the terminal side of θ lies in quadrant I, then we say θ lies in QI

$$\theta \in QI$$

If the terminal side of an angle in standard position lies along one of the axes (x-axis or y-axis), such as angles with measures 90°, 180°, 270°, then that called a ***quadrantal*** *angle*.

Two angles in standard position with the same terminal side are called ***coterminal*** *angles*.



Example

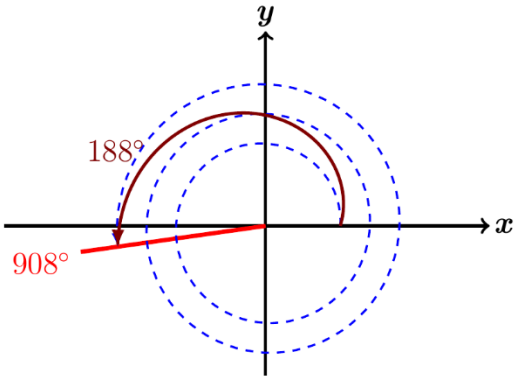
Find all angles that are coterminal with 120°.

Solution

$120° + 360°k$

Example

Find the angle of least possible positive measure coterminal with an angle of 908°.

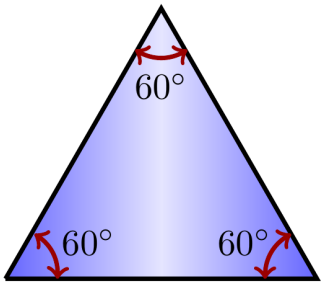
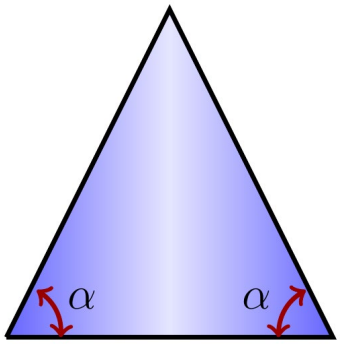
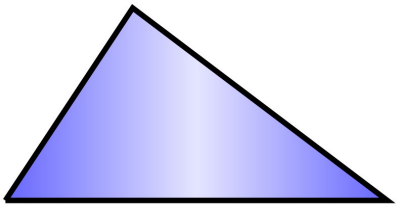
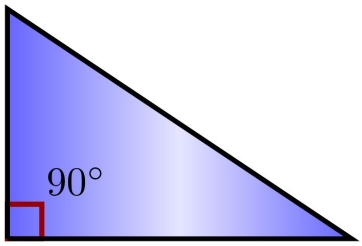
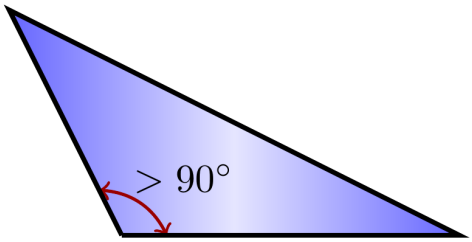
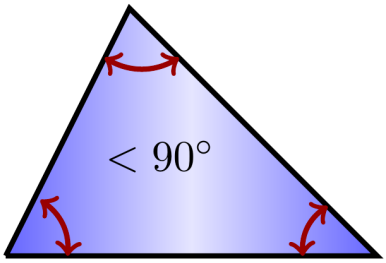


Solution

$908° - 2.360° = 188°$

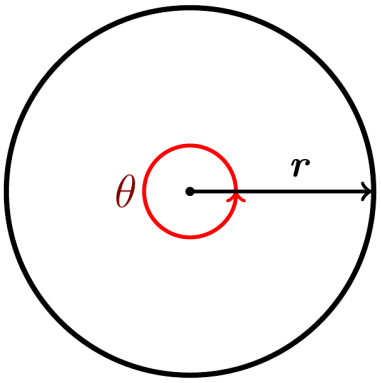
An angle of 908° is coterminal with an angle of 188°

Triangles

Equilateral – All angles always equal to 60°& all sides are equals 	***Isosceles***: 2 sides and angles are equals 
Scalene: No equal sides or angles 	***Right***: Has a right angle 90°. 
Obtuse: Has an angle more than 90°. 	***Acute***: All angles are less than 90°. 

Radians

Degrees - Radians



θ measures one full rotation $\theta = 2\pi$ *The measure of θ in radians is* 2π

$$\boxed{1 = 1\ rad}$$

$$1^\circ = 1\ degree$$

If no unit of angle measure is specified, then the angle is to be measured in radians.

Full Rotation: $360^\circ = 2\pi\ rad$

$180^\circ = \pi\ rad$

Converting from Degrees to Radians

$$\frac{180^\circ}{180} = \frac{\pi}{180}\ rad \qquad \Rightarrow 1^\circ = \frac{\pi}{180}\ rad$$

Multiply a degree measure by $\frac{\pi}{180}\ rad$ and simplify to convert to radians.

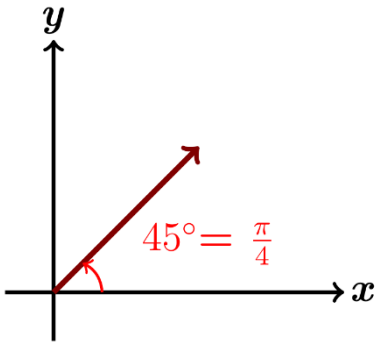
Example

Convert 45° to radians

Solution

$$45^\circ = 45\left(\frac{\pi}{180}\right) rad$$

$$= \frac{\pi}{4}\ rad$$



Example

Convert -450° to radians

Solution

$$-450° = -450\left(\frac{\pi}{180}\right) rad$$

$$= -\frac{5\pi}{2} \ rad$$

Example

Convert 249.8° to radians

Solution

$$249.8° = \frac{2498}{10}\left(\frac{\pi}{180}\right) rad$$

$$= \frac{1,249\pi}{900} \ rad$$

$$\approx 4.360 \ rad$$

Converting from Radians to Degrees

Multiply a radian measure by $\frac{180°}{\pi}$ radian and simplify to convert to degrees.

$$\frac{180°}{\pi} = \frac{\pi}{\pi} \ rad$$

$$\frac{180^\circ}{\pi} = 1 \ rad$$

Example

Convert 1 to *degrees*

Solution

$$1 \ rad = 1\left(\frac{180^\circ}{\pi}\right)$$

$$= 1\left(\frac{180^\circ}{3.14}\right)$$

$$= 57.3^\circ$$

Example

Convert $\frac{4\pi}{3}$ to *degrees*

Solution

$$\frac{4\pi}{3} = \frac{4\pi}{3}\left(\frac{180^\circ}{\pi}\right)$$

$$= 240^\circ$$

Example

Convert –4.5 to *degrees*

Solution

$$-4.5 = -4.5\left(\frac{180^\circ}{\pi}\right)$$

$$\approx -257.8^\circ$$

Exercises *Section* 6.1– Introduction

1. Indicate the angle if it is an acute or obtuse. Then give the complement and the supplement of each angle.

a) 10° *b*) 52° *c*) 90° *d*) 120° *e*) 150°

2. Change to decimal degrees.

a) 10° 45′ *c)* 274° 18′ 59″ *e)* 98° 22′ 45″ *g)* 1° 2′ 3″

b) 34° 51′ 35″ *d)* 74° 8′ 14″ *f)* 9° 9′ 9″ *h)* 73° 40′ 40″

3. Convert to degrees, minutes, and seconds.

a) 89.9004° *c)* 122.6853° *e)* 44.01° *g)* 29.411°

b) 34.817° *d)* 178.5994° *f)* 19.99° *h)* 18.255°

4. Perform each calculation

a) $51° 29′ + 32° 46′$ *b)* $90° - 73°12′$ *c)* $90° - 36° 18′ 47″$ *d)* $75° 15′ + 83° 32′$

5. Find the angle of least possible positive measure coterminal with an angle of

a) −75° *b)* −800° *c)* 270°

6. Convert to radians

a) 256° 20′ *b)* −78.4° *c)* 330° *d)* −60° *e)* −225°

7. Convert to degrees

a) $\frac{11\pi}{6}$ *c)* $\frac{\pi}{6}$ *e)* $\frac{\pi}{3}$ *g)* -4π

b) $-\frac{5\pi}{3}$ *d)* 2.4 *f)* $-\frac{5\pi}{12}$ *h)* $\frac{7\pi}{13}$

8. A vertical rise of the Forest Double chair lift 1,170 *feet* and the length of the chair lift as 5,570 *feet*. To the nearest foot, find the horizontal distance covered by a person riding this lift.

9. A tire is rotating 600 times per *minute*. Through how many degrees does a point of the edge of the tire move in $\frac{1}{2}$ second?

10. A windmill makes 90 *revolutions* per *minute*. How many revolutions does it make per second?