Series

$$\sum_{k=1}^{n} c = nc \qquad \sum_{k=m}^{n} c = (n-m+1)c$$

Arithmetic

$$\sum_{k=1}^{n} k = 1+2+3+\cdots+n = \frac{1}{2}n(n+1) \qquad \sum_{k=1}^{n}(2k-1) = 1+3+5+\cdots+(2n-1) = n^2$$

Geometric

$$\sum_{k=1}^{n} ar^k = a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a\left(1-r^n\right)}{1-r}$$

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r} \quad \textit{if } -1 < \boldsymbol{r} < 1$$

$$\sum_{k=1}^{\infty} ar^{-k} = \frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \ldots = \frac{a}{r-1}$$

$$\sum_{k=1}^{n} r^k = r + r^2 + r^3 + \ldots + r^n = \frac{r\left(1-r^n\right)}{1-r} \qquad \sum_{k=1}^{\infty} r^k = r + r^2 + r^3 + \ldots = \frac{r}{1-r}$$

$$\sum_{k=0}^{n}(a+kd)r^k = a + (a+d)r + (a+2d)r^2 + \ldots + \left(a+(n-1)d\right)r^{n-1}$$

$$= \frac{a\left(1-r^n\right)}{1-r} + \frac{rd\left[1-nr^{n-1}+(n-1)r^n\right]}{(1-r)^2}$$

$$\sum_{k=0}^{\infty}(a+kd)r^k = a + (a+d)r + (a+2d)r^2 + \ldots = \frac{a}{1-r} + \frac{rd}{(1-r)^2} \quad \textit{if } -1 < r < 1$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4} = (1+2+3+\cdots+n)^2$$

$$\sum_{k=1}^{n} k^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)\left(3n^2+3n-1\right)}{30}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\cos^{-1} x = \frac{\pi}{2} - x - \frac{1}{6}x^3 - \frac{3}{40}x^5 - \frac{5}{112}x^7 \qquad -1 \le x \le 1$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\sin^{-1} x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9 + \cdots$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \cdots$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}, \quad |x| \le 1$$

$$\cot x = x^{-1} - \frac{1}{3}x - \frac{1}{45}x^3 - \frac{2}{945}x^5 - \frac{1}{4{,}725}x^7 - \cdots$$

$$\cot^{-1} x = \frac{\pi}{2} + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1} x^{2n+1} \quad = \frac{\pi}{2} - x + \frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}x^7 - \frac{1}{9}x^9 + \cdots$$

$$\cot^{-1}\left(\frac{1}{x}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \quad = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 + \cdots$$

$$\sec x = 1 + \frac{1}{2}x^2 + \frac{5}{25}x^4 + \frac{61}{720}x^6 + \frac{277}{8{,}064}x^8 + \cdots$$

$$\csc^{-1} x = \sum_{n=0}^{\infty} \frac{(2n)!}{n!n!} \frac{1}{4^n(2n+1)} x^{-(2n+1)} \quad = x^{-1} + \frac{1}{6}x + \frac{7}{360}x^3 + \frac{31}{15{,}120}x^5 + \cdots$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

$$\ln x = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}(x-1)^{n+1} \qquad = x-1-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\cdots \qquad 0 < x \le 2$$

$$\ln x = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}(x-1)^{n+1} \qquad = 2\left[\frac{x-1}{x+1}+\frac{1}{3}\left(\frac{x-1}{x+1}\right)^3+\frac{1}{5}\left(\frac{x-1}{x+1}\right)^5+\cdots\right] \qquad x > 2$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}x^{n+1} = x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots$$

$$\ln(1-x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\cdots \qquad -1 \le x < 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\cdots \qquad |x| < 1$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1-x+x^2-x^3+\cdots \qquad |x| < 1$$

$$(1+x)^{\alpha} = 1+\alpha x+\alpha(\alpha-1)\frac{x^2}{2!}++\alpha(\alpha-1)(\alpha-2)\frac{x^3}{3!}+\cdots$$

Legendre Polynomial

$$\left(1-x^2\right)y''-2xy'+n(n+1)y=0 \qquad P_n(x) = \frac{1}{2^n}\sum_{k=0}^{n/2}\frac{(-1)^k(2n-2k)!}{k!(n-k)!(n-2k)!}x^{n-2k}$$

The nth term of an arithmetic sequence: $a_n = a_1 + (n-1)d$

$$n = \frac{a_n - a_1}{d} + 1$$

Sum of a certain number of terms of an arithmetic sequence $S_n = \frac{n}{2}(a_1 + a_n)$

The ***arithmetic mean*** of two numbers a and b is defined as $\frac{a+b}{2}$

The nth term of a geometric sequence: $a_n = a_1 r^{n-1}$ $\qquad r = \frac{a_n}{a_{n-1}}$

Sum of nth term of a geometric sequence: $S_n = a_1 \frac{1-r^n}{1-r}$

Sum of infinite term of a geometric sequence: $S_n = \frac{a_1}{1-r}$