Solution *Section* 1.1 – Functions

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(1,\ 2),\ (3,\ 4),\ (5,\ 6),\ (5,\ 8)\}$

Solution

Not a function

Domain: {1, 3, 5}

Range: {2, 4, 6, 8}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(1,\ 2),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5)\}$

Solution

It is a Function

Domain: {1, 3, 6, 8}

Range: {2, 4, 5}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(9,\ -5),\ (9,\ 5),\ (2,\ 4)\}$

Solution

It is ***not*** a function

Domain = {2, 9}
Range = {–5, 5, 4}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(-2,\ 5),\ (5,\ 7),\ (0,\ 1),\ (4,\ -2)\}$

Solution

It is a function
Domain = {–2, 0, 4, 5}
Range = {–2, 1, 5, 7}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(-5,\ 3),\ (0,\ 3),\ (6,\ 3)\}$

Solution

It is a function

Domain = {–5, 0, 6}

Range = {3}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(1,\ 2),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5),\ (1,\ 5)\}$

Solution

It is ***not*** a function

Domain = $\{1,\ 3,\ 6,\ 8\}$

Range = $\{2,\ 4,\ 5\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(-1,\ 3),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5),\ (1,\ 5)\}$

Solution

It is a function

Domain = $\{-1,\ 1,\ 3,\ 6,\ 8\}$

Range = $\{3,\ 4,\ 5\}$

Exercise

Find the domain and the range of the relation:

$\{(5,\ 12.8),\ (10,\ 16.2),\ (15,\ 18.9),\ (20,\ 20.7),\ (25,\ 21.81)\}$

Solution

Domain: {5, 10, 15, 20, 25}

Range: {12.8, 16.2, 18.9, 20.7, 21.81}

Exercise

Let $f(x) = -3x + 4$, find $f(0)$

Solution

$$f(0) = -3(0) + 4$$
$$\underline{= 4}$$

Exercise

Let $g(x) = -x^2 + 4x - 1$, find $g(-x)$

Solution

$$g(-x) = -(-x)^2 + 4(-x) - 1$$
$$\underline{= -x^2 - 4x - 1}$$

Exercise

Let $f(x) = -3x + 4$, find $f(a+4)$

Solution

$$f(a+4) = -3(a+4) + 4$$
$$= -3a - 12 + 4$$
$$\underline{= -3a - 8}$$

Exercise

Given: $f(x) = 2|x| + 3x$, find $f(2-h)$.

Solution

$$f(2-h) = 2|2-h| + 3(2-h)$$
$$\underline{= 2|2-h| + 6 - 3h}$$

Exercise

Given: $g(x) = \dfrac{x-4}{x+3}$, find $g(x+h)$

Solution

$$g(x+h) \underline{= \dfrac{x+h-4}{x+h+3}}$$

Exercise

Given: $g(x)=\dfrac{x}{\sqrt{1-x^2}}$, find $g(0)$ *and* $g(-1)$

Solution

$$g(0)=\frac{0}{\sqrt{1-0^2}}$$
$$\underline{=0}$$

$$g(-1)=\frac{-1}{\sqrt{1-(-1)^2}}$$
$$\underline{=\frac{-1}{0} \quad \textit{undefined}}$$

Exercise

Given that $g(x)=2x^2+2x+3$. Find $g(p+3)$

Solution

$$\begin{aligned} g(p+3) &= 2(p+3)^2+2(p+3)+3 \\ &= 2\left(p^2+2(p)(3)+3^2\right)+2p+6+3 \\ &= 2\left(p^2+6p+9\right)+2p+9 \\ &= 2p^2+12p+18+2p+9 \\ &= 2p^2+14p+27 \end{aligned}$$

$(a+b)^2=a^2+2ab+b^2$

Exercise

If $f(x)=x^2-2x+7$, evaluate each of the following: $f(-5)$, $f(x+4)$, $f(-x)$

Solution

$$\begin{aligned} f(-5) &= (-5)^2-2(-5)+7 \\ &= 25+10+7 \\ &= 42 \end{aligned}$$

$$\begin{aligned} f(x+4) &= (x+4)^2-2(x+4)+7 \\ &= x^2+2(4)x+4^2-2x-8+7 \\ &= x^2+8x+16-2x-1 \\ &= x^2+6x+15 \end{aligned}$$

$(a+b)^2=a^2+2ab+b^2$

$$\underline{=x^2+2x+7}$$

Exercise

Find $g(0),\ g(-4),\ g(7),\ and\ g\left(\frac{3}{2}\right)$ *for* $g(x)=\dfrac{x}{\sqrt{16-x^2}}$

Solution

$$g(0)=\frac{0}{\sqrt{16-0^2}}$$

$$=\frac{0}{\sqrt{16}}$$

$$\boxed{=0}$$

$$g(-4)=\frac{-4}{\sqrt{16-(-4)^2}}$$

$$=\frac{-4}{\sqrt{16-16}}$$

$$\boxed{=\frac{-4}{0}\quad \nexists}$$

$$g(7)=\frac{7}{\sqrt{16-7^2}}$$

$$=\frac{7}{\sqrt{16-49}}$$

$$\boxed{=\frac{7}{\sqrt{-33}}\quad \textit{doesn't exist in real number}}$$

$$g\left(\frac{3}{2}\right)=\frac{\frac{3}{2}}{\sqrt{16-\left(\frac{3}{2}\right)^2}}$$

$$=\frac{\frac{3}{2}}{\sqrt{16-\frac{9}{4}}}$$

$$=\frac{\frac{3}{2}}{\sqrt{\frac{4(16)-9}{4}}}$$

$$=\frac{\frac{3}{2}}{\frac{\sqrt{55}}{2}}$$

$$=\frac{3}{\sqrt{55}}$$

$$\boxed{=\frac{3\sqrt{55}}{55}}$$

Exercise

$f(x) = 3x - 4$

a) $f(0)$ *b)* $f\left(\frac{5}{3}\right)$ *c)* $f(-2a)$ *d)* $f(x+h)$

Solution

a) $f(0) \underline{= -4}$

b) $f\left(\frac{5}{3}\right) = 3\frac{5}{3} - 4$

$= 5 - 1$

$\underline{= 1}$

c) $f(-2a) = 3(-2a) - 4$

$\underline{= -6a - 4}$

d) $f(x+h) = 3(x+h) - 4$

$\underline{= 3x + 3h - 4}$

Exercise

$f(x) = 3x^2 + 3x - 1$

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(2)$ *d)* $f(h)$

Solution

a) $f(0) \underline{= -1}$

b) $f(x+h) = 3(x+h)^2 + 3(x+h) - 1$

$= 3\left(x^2 + 2hx + h^2\right) + 3x + 3h - 1$

$\underline{= 3x^2 + 6hx + 3h^2 + 3x + 3h - 1}$

c) $f(2) = 12 + 6 - 1$

$\underline{= 17}$

d) $\underline{f(h) = 3h^2 + 3h - 1}$

Exercise

$f(x) = 2x^2 - 4$

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(2)$ *d)* $f(2) - f(-3)$

Solution

a) $f(0) \underline{= -4}$

b) $f(x+h) = 2(x+h)^2 - 4$

$= 2\left(x^2 + 2hx + h^2\right) - 4$

$\underline{= 2x^2 + 4hx + 2h^2 - 4}$

c) $f(2) = 8 - 4$

$\underline{= 4}$

d) $f(2) - f(-3) = 8 - 4 - (18 - 4)$

$= 4 - 14$

$\underline{= -10}$

Exercise

$f(x) = 3x^2 + 4x - 2$

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(3)$ *d)* $f(-5)$

Solution

a) $f(0) \underline{= -2}$

b) $f(x+h) = 3(x+h)^2 + 4(x+h) - 2$

$= 3\left(x^2 + 2hx + h^2\right) + 4x + 4h - 2$

$\underline{= 3x^2 + 6hx + 3h^2 + 4x + 4h - 2}$

c) $f(3) = 27 + 12 - 2$

$\underline{= 37}$

d) $f(-5) = 75 - 20 - 2$

$\underline{= 53}$

Exercise

$f(x) = -x^3 - x^2 - x + 10$

a) $f(0)$ *b)* $f(-1)$ *c)* $f(2)$ *d)* $f(1) - f(-2)$

Solution

a) $f(0) \underline{= 10}$

b) $f(-1) = 1 - 1 + 1 + 10$

$\underline{= 11}$

c) $f(2) = -8 - 4 - 2 + 10$

$\underline{= -4}$

d) $f(1) - f(-2) = -1 - 1 - 1 + 10 - (8 - 4 + 2 + 10)$

$= 7 - 16$

$\underline{= -9}$

Exercise

For $\frac{1}{10}x^{10} - \frac{1}{2}x^6 + \frac{2}{3}x^3 - 10x$, determine

a) $f(2) - f(-2)$ *b)* $f(1) - f(-1)$ *c)* $f(2) - f(0)$

Solution

a) $f(2) - f(-2) = \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2}{3}2^3 - 20 - \left(\frac{2^{10}}{10} - \frac{2^6}{2} - \frac{2}{3}2^3 + 20\right)$

$= \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2^4}{3} - 20 - \frac{2^{10}}{10} + \frac{2^6}{2} + \frac{2^4}{3} - 20$

$= \frac{2^5}{3} - 40$

$= \frac{32}{3} - 40$

$\underline{= -\frac{88}{3}}$

b) $f(1) - f(-1) = \frac{1}{10} - \frac{1}{2} + \frac{2}{3} - 10 - \left(\frac{1}{10} - \frac{1}{2} - \frac{2}{3} + 10\right)$

$= \frac{1}{10} - \frac{1}{2} + \frac{2}{3} - 10 - \frac{1}{10} + \frac{1}{2} + \frac{2}{3} - 10$

$= \frac{4}{3} - 20$

$\underline{= -\frac{56}{3}}$

c) $f(2)-f(0)=\frac{2^{10}}{10}-\frac{2^6}{2}+\frac{2}{3}2^3-20-(0)$

$$=\frac{2^9}{5}-2^5+\frac{2^4}{3}-5\left(2^2\right)$$

$$=2^2\left(\frac{128}{5}-8+\frac{4}{3}-5\right)$$

$$=4\left(\frac{384+20-195}{15}\right)$$

$$=4\left(\frac{209}{15}\right)$$

$$=\frac{836}{15}$$

Exercise

For $f(x)=3x^4+x^2-4$, determine

a) $f(2)-f(-2)$ *b)* $f(1)-f(-1)$ *c)* $f(2)-f(0)$

Solution

a) $f(2)-f(-2)=3(16)+4-4-(3(16)+4-4)$

$$=48+4-4-48-4+4$$

$$=0$$

b) $f(1)-f(-1)=3+1-4-(3+1-4)$

$$=0$$

c) $f(2)-f(0)=3(16)+4-4-(0)$

$$=48$$

Exercise

For $f(x)=-\frac{2}{3}x^3+4x$, determine

a) $f(2)-f(-2)$ *b)* $f(1)-f(-1)$ *c)* $f(2)-f(0)$

Solution

a) $f(2)-f(-2)=-\frac{2}{3}\left(2^3\right)+8-\left(-\frac{2}{3}(-2)^3-8\right)$

$$=-\frac{16}{3}+8-\frac{16}{3}+8$$

$$=2\left(-\frac{16}{3}+8\right)$$

$$= 16\left(-\frac{1}{3}+1\right)$$

$$= 16\left(\frac{2}{3}\right)$$

$$\boxed{= \frac{32}{3}}$$

b) $f(1) - f(-1) = -\frac{2}{3} + 4 - \left(\frac{2}{3} - 4\right)$

$$= 2\left(-\frac{2}{3}+4\right)$$

$$\boxed{= \frac{20}{3}}$$

c) $f(2) - f(0) = -\frac{16}{3} + 8 - (0)$

$$\boxed{= \frac{8}{3}}$$

Exercise

For $f(x) = \frac{2x-3}{x-4}$, determine

a) $f(0)$ *b)* $f(3)$ *c)* $f(x+h)$ *d)* $f(-4)$

Solution

a) $f(0) \boxed{= \frac{3}{4}}$

b) $f(3) = \frac{6-3}{3-4}$

$$\boxed{= -3}$$

c) $f(x+h) = \frac{2(x+h)-3}{x+h-4}$

$$\boxed{= \frac{2x+2h-3}{x+h-4}}$$

d) $f(-4) = \frac{-8-3}{-4-4}$

$$\boxed{= \frac{11}{8}}$$

Exercise

For $f(x)=\dfrac{3x-1}{x-5}$, determine

a) $f(0)$ *b)* $f(3)$ *c)* $f(x+h)$ *d)* $f(-5)$

Solution

a) $f(0)=\boxed{\dfrac{1}{5}}$

b) $f(3)=\dfrac{9-1}{3-5}$

$=\boxed{-4}$

c) $f(x+h)=\dfrac{3(x+h)-1}{x+h-5}$

$=\boxed{\dfrac{3x+3h-1}{x+h-5}}$

d) $f(-5)=\dfrac{-12-1}{-4-5}$

$=\boxed{\dfrac{13}{9}}$