Exercise 1

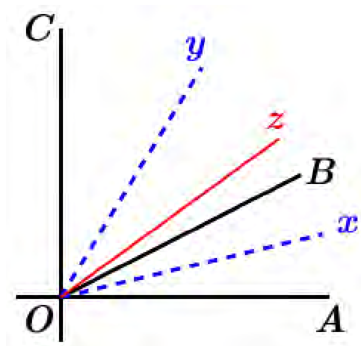
Ox and Oy are bisector to 2 adjacent acute angles, $\widehat{AOB}$ and $\widehat{BOC}$ where the difference is $36°$ and $\widehat{AOC} = 90°$. Oz is the bisector of the angle $\widehat{xOy}$. Determine the angle $\widehat{BOz}$

Solution

$$\widehat{BOC} - \widehat{AOB} = 36°$$
$$\underline{\widehat{BOC} + \widehat{AOB} = 90°}$$
$$2\ \widehat{BOC} = 126°$$

$$\widehat{BOC} = 63°$$

$$\widehat{AOB} = 27°$$



$$\widehat{xOB} = \frac{1}{2}\widehat{AOB}$$
$$= \frac{27}{2}^{\circ}$$

$$\widehat{BOy} = \frac{63}{2}^{\circ}$$

$$\widehat{xOy} = \widehat{xOB} + \widehat{BOy}$$
$$= \frac{1}{2}(63° + 27°)$$
$$= 45°$$

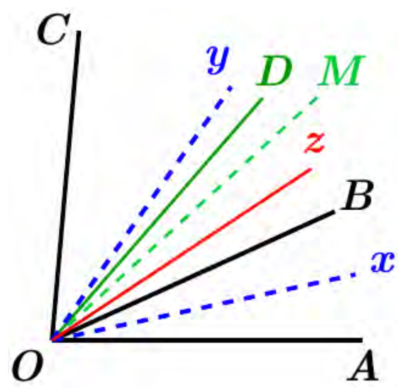
$$\widehat{xOz} = \frac{45}{2}^{\circ}$$

$$\widehat{BOZ} = \widehat{xOz} - \widehat{xOB}$$
$$= \frac{1}{2}(45° - 27°)$$
$$= 9°$$

Exercise 2

Ox and Oy are bisector to 2 adjacent acute angles, $\widehat{AOB}$ and $\widehat{BOC}$ where the difference is $36°$. Oz is the bisector of the angle $\widehat{xOy}$. Determine the angle $\widehat{BOz}$

Solution



Ox is the bisector $\widehat{AOB}$ $\quad(1)$

OB is the bisector $\widehat{AOD}$ $\quad(2)$

OM is the bisector $\widehat{AOC}$ $\quad(3)$

Oz is the bisector $\widehat{xOy}$ $\quad(4)$

Oy is the bisector $\widehat{BOC}$ $\quad(5)$

$$\widehat{BOC}-\widehat{AOB}=36°$$
$$\widehat{BOC}-\widehat{BOD}=36°$$
$$\widehat{DOC}=36°$$

$$(3)\rightarrow \quad \widehat{AOM}=\frac{1}{2}\widehat{AOC}$$
$$=\frac{1}{2}\left(2\widehat{AOB}+\widehat{DOC}\right)$$
$$=\frac{1}{2}\left(2\widehat{AOB}+36°\right)$$
$$=\widehat{AOB}+18°$$

$$\widehat{BOM}=\widehat{AOM}-\widehat{AOB}$$
$$=\widehat{AOB}+18°-\widehat{AOB}$$
$$=18°$$

$$(1)\rightarrow \quad \widehat{BOx}=\frac{1}{2}\widehat{AOB}$$

$$(4)\rightarrow \quad \widehat{BOy}=\frac{1}{2}\widehat{BOC}$$

$$(1)+(4)\rightarrow \quad \widehat{xOy}=\frac{1}{2}\widehat{AOC}$$

$$(3)\rightarrow \quad \widehat{AOM}=\frac{1}{2}\widehat{AOC}=\widehat{xOy}$$

$$\widehat{BOz}=\widehat{xOz}-\widehat{xOB}$$
$$=\frac{1}{2}\left(\widehat{xOy}-\widehat{AOB}\right)$$
$$=\frac{1}{2}\left(\widehat{AOM}-\widehat{AOB}\right)$$
$$=\frac{1}{2}\widehat{BOM}$$
$$=9°$$

Exercise 3

Four consecutive half-lines (segments): *OA*, *OB*, *OC*, and *OD* formed angles such as

$$\widehat{DOA} = \widehat{COB} = 2\widehat{AOB} \qquad \text{and} \qquad \widehat{COD} = 3\widehat{AOB}$$

Calculate the angles to demonstrate that the bisectors of $\widehat{AOB}$ and $\widehat{COD}$ are in a straight line.

Solution

$\widehat{AOB} + \widehat{BOC} + \widehat{COD} + \widehat{DOA} = 360°$

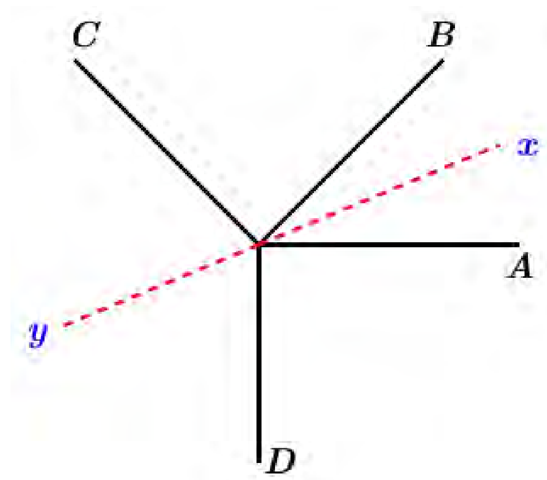
$\widehat{AOB} + 2\widehat{AOB} + 3\widehat{AOB} + 2\widehat{AOB} = 360°$

$8\widehat{AOB} = 360°$

$\boxed{\widehat{AOB} = 45°}$

$\boxed{\widehat{DOA} = \widehat{COB} = 90°}$

$\boxed{\widehat{COD} = 135°}$



Let:

Ox is the bisector $\widehat{AOB}$

Oy is the bisector $\widehat{COD}$

$$\begin{aligned} \widehat{xOy} &= \widehat{xOB} + \widehat{BOC} + \widehat{COy} \\ &= \frac{1}{2}\widehat{AOB} + 90° + \frac{1}{2}\widehat{COD} \\ &= \frac{1}{2}(45° + 135°) + 90° \\ &= \boxed{180°} \end{aligned}$$

Therefore; the bisectors of $\widehat{AOB}$ and $\widehat{COD}$ are in a straight line

Exercise 4

The segments OA and OB formed with OX the angles α and β.

a) Demonstrate that the bisector OC of the angle $\widehat{AOB}$ made with OX an angle $\dfrac{\alpha+\beta}{2}$.

b) Examine the cases where

  i. $\alpha+\beta=90^\circ$

  ii. $\alpha+\beta=180^\circ$

Solution

Given:

$\widehat{XOA}=\alpha \quad \& \quad \widehat{XOB}=\beta$

$$\widehat{AOC}=\frac{1}{2}\widehat{AOB}$$
$$=\frac{\beta-\alpha}{2}$$

a) $\widehat{XOC}=\widehat{XOA}+\widehat{AOC}$

$$=\alpha+\frac{\beta-\alpha}{2}$$
$$\boxed{=\frac{\alpha+\beta}{2}}$$

b) ***i.*** If $\alpha+\beta=90^\circ$, then

$\widehat{XOC}=45^\circ$

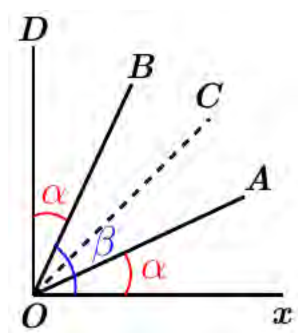
Let: $\widehat{XOD}=90^\circ$ that implies OC is the bisector of $\widehat{XOD}$

Since OC is the bisector of $\widehat{AOB}$, then

$\widehat{BOD}=90^\circ-\beta \qquad \beta=90^\circ-\alpha$

$=90^\circ-90^\circ+\alpha$

$\boxed{=\alpha}$



ii. If $\alpha+\beta=180^\circ$, then

$\widehat{XOC}=90^\circ$

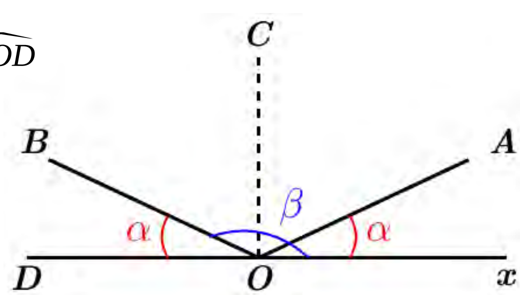
Let: $\widehat{XOD}=180^\circ$ that implies OC is the bisector of $\widehat{XOD}$

Since OC is the bisector of $\widehat{AOB}$, then

$\widehat{BOD}=180^\circ-\beta \qquad \beta=180^\circ-\alpha$

$=180^\circ-180^\circ+\alpha$

$\boxed{=\alpha}$

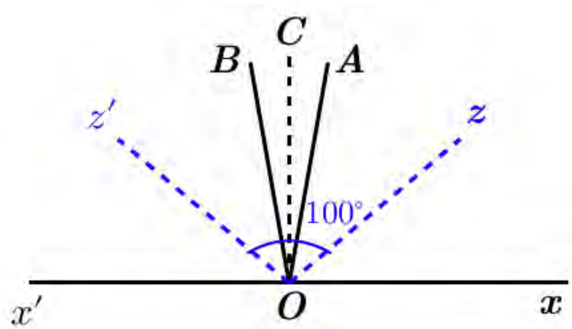


Exercise 5

A point O takes on an infinite right $x'Ox$ be conducted the same side half-lines OA and OB, as well as the bisectors of angles $\widehat{xOA}$, $\widehat{AOB}$, and $\widehat{BOx'}$.

Calculate the angles of the figure such that the bisector of the angle $\widehat{AOB}$ is perpendicular to $x'Ox$ and the bisectors of the extreme angles formed an angle of $100°$.

Solution



Given: $\widehat{zOz'} = 100°$

$\widehat{xOC} = 90°$

OC is the bisector $\widehat{AOB}$

$\widehat{AOC} = \widehat{COB}$

Oz is the bisector $\widehat{xOA}$

$\widehat{xOz} = \widehat{zOA}$

Oz' is the bisector $\widehat{x'OB}$

$\widehat{x'Oz'} = \widehat{z'OB}$

$$\widehat{xOz} = \frac{180° - 100°}{2}$$

$$= 40°$$

$$\widehat{AOB} = 2\widehat{AOC}$$

$$= 2\left(90° - 2\widehat{xOz}\right)$$

$$= 2\left(90° - 80°\right)$$

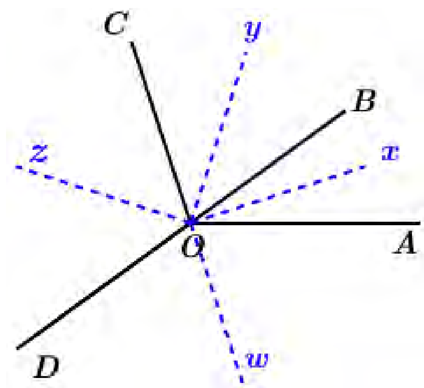
$$= 20°$$

Exercise 6

Four consecutive half-lines *OA*, *OB*, *OC*, and *OD* formed four adjacent consecutive angles which are between them like 1, 2, 3, 4.
Calculate the angles and the adjacent consecutive angles formed by their bisectors.

Solution

$$\widehat{AOB}+\widehat{BOC}+\widehat{COD}+\widehat{DOA}=360°$$
$$\widehat{AOB}+2\widehat{AOB}+3\widehat{AOB}+4\widehat{AOB}=360°$$
$$10\widehat{AOB}=360°$$
$$\widehat{AOB}=36°$$
$$\widehat{BOC}=72°$$
$$\widehat{COD}=108°$$
$$\widehat{DOA}=144°$$



$$\begin{aligned}\widehat{xOy}&=\frac{1}{2}\widehat{AOB}+\frac{1}{2}\widehat{BOC}\\&=\frac{1}{2}36°+\frac{1}{2}72°\\&=18°+36°\\&=54°\end{aligned}$$

$$\begin{aligned}\widehat{yOz}&=\frac{1}{2}\widehat{BOC}+\frac{1}{2}\widehat{COD}\\&=\frac{1}{2}72°+\frac{1}{2}108°\\&=36°+54°\\&=90°\end{aligned}$$

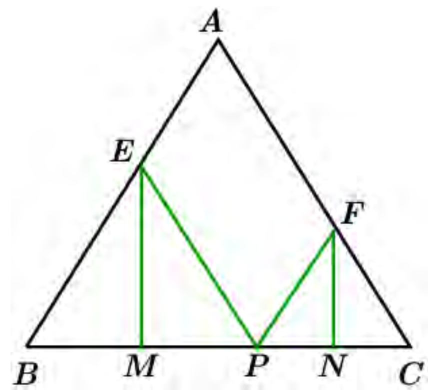
$$\begin{aligned}\widehat{zOw}&=\frac{1}{2}\widehat{COD}+\frac{1}{2}\widehat{DOA}\\&=\frac{1}{2}108°+\frac{1}{2}144°\\&=54°+72°\\&=126°\end{aligned}$$

$$\begin{aligned}\widehat{wOx}&=\frac{1}{2}\widehat{DOA}+\frac{1}{2}\widehat{AOB}\\&=\frac{1}{2}144°+\frac{1}{2}36°\\&=72°+18°\\&=90°\end{aligned}$$

Exercise 7

A point P is on the base BC of an isosceles triangle ABC. The two points M and N are the middle points of the segments PB and PC, respectively, which lead the perpendicular to the base BC; these perpendiculars meet AB in E, AC in F.
Demonstrate that the angle EPF is equal to A.

Solution



$$\widehat{BAC} = 180° - \widehat{ABC} - \widehat{ACB}$$

M is the middle of the segment BP and $EM \perp$ to BP, therefore

$$EB = EP \quad \& \quad \widehat{EBP} = \widehat{EPB}$$

N is the middle of the segment CP and $FN \perp$ to CP, therefore

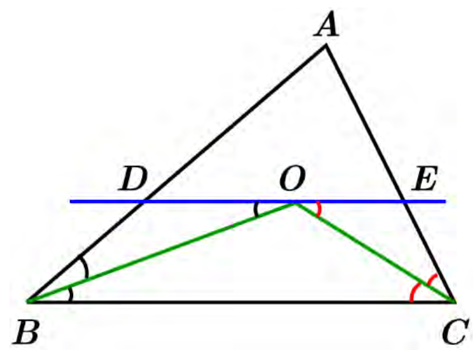
$$FP = FP \quad \& \quad \widehat{FPC} = \widehat{FCP}$$

$$\begin{aligned} \widehat{EPF} &= 180° - \widehat{CPF} - \widehat{BPE} \\ &= 180° - \widehat{PFC} - \widehat{PBE} \\ &= 180° - \widehat{ABC} - \widehat{ACB} \\ &= \widehat{A} \quad \checkmark \end{aligned}$$

Exercise 8

Given the triangle ABC and the bisectors BO and CO of the angles of the base, where the point O is the intersection of the 2 bisectors. A line DOE passes through the point O parallel to base BC.
Prove that $DE = DB + CE$

Solution



CO is the bisector of $\widehat{BCE} \Rightarrow \widehat{BCO} = \widehat{OCE}$

$OE \parallel BC \Rightarrow \widehat{COE} = \widehat{BOC}$

$\therefore \widehat{EOC} = \widehat{OCE} \rightarrow \underline{OE = EC}$

Similar; BO is the bisector of $\widehat{DBC} \Rightarrow \widehat{DBO} = \widehat{OBC}$

$DO \parallel BC \Rightarrow \widehat{DOB} = \widehat{OBC}$

$\therefore \widehat{DOB} = \widehat{OBC} \rightarrow \underline{DO = DB}$

$DE = DO + OE$

$\quad \underline{= DB + CE}$

Exercise 9

A right triangle *ABC* at *A* with a height *AH*. We drop perpendiculars *HE* and *HD* from *H* to sides *AB* and *AC* respectively.

a) Prove that $DE = AH$

b) Prove that *AM* is perpendicular to *DE*, where *M* is the middle point of *BC*.

c) Prove that *MN* (*N* is the middle point of *AB*) and the segment *Bx* (parallel to *DE*) are intersect on *AH*.

d) Prove that *AM* and *HD* are intersect on *Bx*.

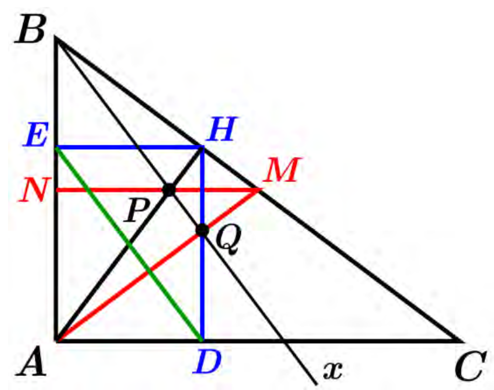
Solution

a) The triangles *AEH* and *ADH* are right triangles and angle *A* is right angle.
Then *AEHD* is a rectangle.
Therefore, $DE = AH$

b) A middle point of a hypotenuse of a right triangle is the center of the circle of that triangle.
Therefore, $MC = MA = MB$

That implies to: $\widehat{MAC} = \widehat{MCA}$

From the rectangle *ADHE*: $\widehat{EAH} = \widehat{EDH}$



$\widehat{EAH} + \widehat{HAM} + \widehat{MAC} = 90°$ $\qquad$ $\widehat{HAM} + \widehat{MAC} = \widehat{HAC}$

$\widehat{EAH} + \widehat{HAC} = 90°$

$\widehat{EAH} + 90° - \widehat{MCA} = 90°$

$\widehat{EAH} = \widehat{MCA} = \widehat{EDH} = \widehat{MAC}$

$\widehat{ADE} + \widehat{EDH} = 90°$

$\widehat{ADE} + \widehat{MAD} = 90°$

Therefore, *AM* is perpendicular to *DE*.

c) *N* is the middle point of *AB* $\Rightarrow$ $NA = NB$

Bx parallel to *DE* $\Rightarrow$ $\widehat{ABx} = \widehat{AED} = \widehat{EDH} = \widehat{EAH}$

Let point *P* the intersection of B*x* and *AH*. Since $\widehat{ABP} = \widehat{BAP}$, then the triangle *BPA* is an isosceles. *PN* is the perpendicular to *AB* as well *MN*. Which gives us that points *M*, *P*, *N* are on the same line.
Therefore, segment *MN* and *AH* intersect at point *P*.

d) Let Point *Q* be the intersection of *AM* and *Bx*.

$\widehat{ABQ} = \widehat{BAH}$ $\quad$ & $\quad$ $\widehat{BAQ} = \widehat{ABH}$

Then, the triangles *BHA* and *BQA* are equivalent, therefore $AQ \perp BQ$ with hypotenuse *AB*.

$HQ \parallel AB$, line *HQ* has to be perpendicular to *AC*.

AM and *HD* are intersecting on *Bx* at *Q*.

***Exercise* 10**

Given an isosceles triangle *ABC* with a peak at *A*. Extend base *BC* the length $CD = AB$, then extend *AB* of a length $BE = \frac{1}{2}BC$, at the end draw a line *EHF*, *H* is the middle point of *BC* and *F* is located on *AD*.

a) Prove that $\widehat{ADB} = \frac{1}{2}\widehat{ABC}$

b) Prove that $EA = HD$

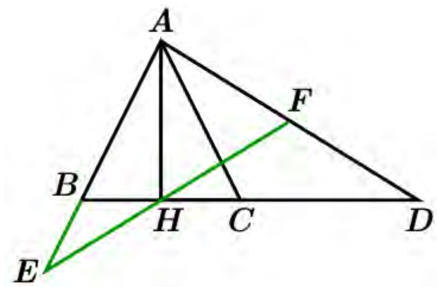
c) Prove that $FA = FD = FH$

d) Calculate the value of the angles $\widehat{AFH}$ and $\widehat{ADB}$ where $\widehat{BAC} = 58°$.

Solution

a) Triangle *ABC* is isosceles, then $\widehat{ABC} = \widehat{ACB}$

Since, $CD = AB = AC$, then $\widehat{CAD} = \widehat{ADC}$



$$2\widehat{ADC} = 180° - \widehat{ACD}$$

$$2\widehat{ADC} = 180° - \left(180° - \widehat{ACB}\right)$$

$$2\widehat{ADC} = \widehat{ACB}$$

$$\widehat{ADB} = \frac{1}{2}\widehat{ACB}$$

$$= \frac{1}{2}\widehat{ABC}$$

b) $BE = \frac{1}{2}BC$ *H* the middle point of *BC*

$= HC$

$CD = AB$

$HC + CD = BE + AB$

$EA = HD$ ✓

c) $\widehat{ADH} = \frac{1}{2}\widehat{ABD}$

$$= \frac{1}{2}\left(180° - \widehat{HBE}\right)$$

$$= \frac{1}{2}\left(180° - 180° + 2\widehat{BHE}\right)$$

$$= \widehat{BHE}$$

$\Rightarrow$ $FD = FH$

$\widehat{AHF} = 90° - \widehat{FHD}$

$= 90° - \widehat{ADH}$ $(\Delta\ HDA)$

$= 90° - \left(90° - \widehat{HAF}\right)$

$= \widehat{HAF}$

$\Rightarrow \quad \underline{FA = FH}$

$FA = FD = FH$ ✓

d) $\widehat{BAC} = 58°$

$$\widehat{ADB} = \frac{1}{2}\widehat{ACB}$$

$$= \frac{1}{2}\left(\frac{1}{2}\left(180° - \widehat{BAC}\right)\right)$$

$$= \frac{1}{4}(180° - 58°)$$

$$= \frac{122}{4}^{\circ}$$

$$\underline{= \frac{61}{2}^{\circ}} \qquad \underline{= 30.5°}$$

Triangle *AFH* is isosceles then,

$$\widehat{AFH} = 180° - \widehat{HFD}$$

$$= 180° - \left(180° - 2\widehat{FDH}\right)$$

$$= 2\widehat{FDH}$$

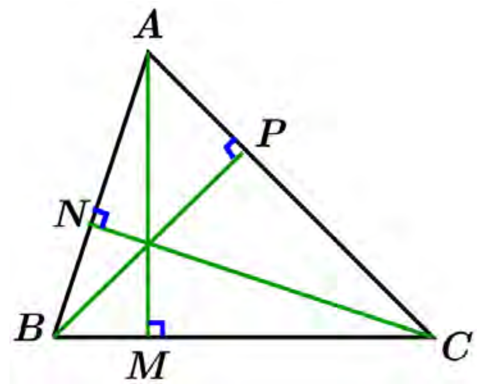
$$= 2\frac{61°}{2}$$

$$\underline{= 61°}$$

Exercise 11

Demonstrate that the heights of a triangle share the angles of triangle that equal to each other.

Solution



Consider the 2 right triangles *APB* and *ANC*, which they have the same angle *A*.

Therefore, $\widehat{ABP} = \widehat{ACN}$.

Similar, consider the 2 right triangles *BPC* and *AMC*, which they have the same angle *C*.

Therefore, $\widehat{MAC} = \widehat{CBP}$.

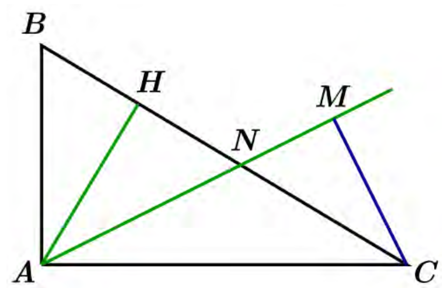
Similar, consider the 2 right triangles *BNC* and *AMB*, which they have the same angle *B*.

Therefore, $\widehat{BCN} = \widehat{BAM}$.

Exercise 12

A right triangle ABC at A where $AB < AC$, drop a perpendicular AH from A to the hypotenuse BC where $HN = HB$. From C drops a perpendicular CM at AN. Demonstrate that BC is the bisector of the angle $\widehat{ACM}$.

Solution



Consider the 2 right triangles ABC and ABH with a common angle B, then

$\widehat{BAH} = \widehat{ACB}$

Given: $HN = HB$, then $\widehat{HAN} = \widehat{BAH} = \widehat{ACB}$

$$\begin{aligned}\widehat{NAC} &= 90° - \widehat{HAB} - \widehat{HAN} \\ &= 90° - 2\widehat{HCA}\end{aligned}$$

Consider the 2 right triangles AHN and CMC, where $\widehat{HNA} = \widehat{MNC}$

Therefore, $\widehat{HAN} = \widehat{NCM}$

Since $\widehat{HAN} = \widehat{ACB}$

Then $\widehat{ACB} = \widehat{MCB}$

Therefore, BC is the bisector of the angle $\widehat{ACM}$

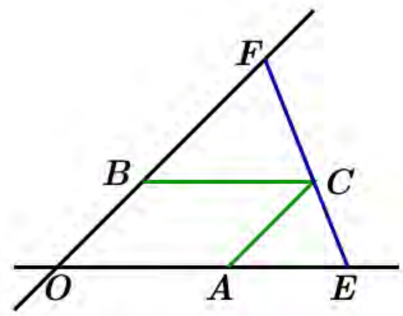
Exercise 13

On the sides of an angle that it takes the length *OA* and *OB*, so that $OA+OB=\ell$ (is given) and construct a parallelogram *OABC*. What is the place of the summit *C* of parallelogram?

Solution

Let segment *OE* extension of segment *OA* such that $OE=\ell$
Let segment *OF* extension of segment *OB* such that $OF=\ell$

Then, the triangle *OEF* is an isosceles.



$$\widehat{OEF}=\widehat{OFE}=90^\circ-\frac{1}{2}\widehat{EOF}$$

$$OA+OB=\ell$$

$$\begin{cases} OA+AC=\ell \\ OA+AE=\ell \end{cases} \Rightarrow \quad AC=AE$$

$$\begin{cases} OB+BC=\ell \\ OB+BF=\ell \end{cases} \Rightarrow \quad BC=BF$$

$$\widehat{OEF}=\widehat{OFE}=\widehat{FCB}=\widehat{ACE}$$

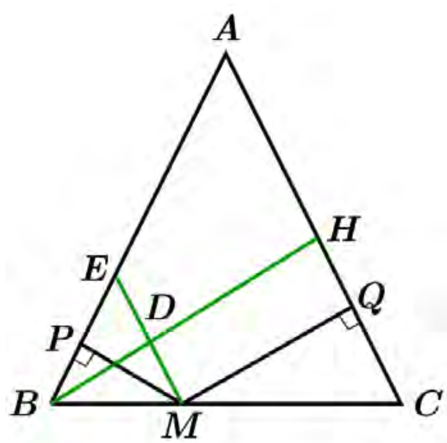
Therefore, the point *C*, *E*, and *F* are aligned.

Exercise 14

Demonstrate that the sum of distances from a point *M* on the base *BC* of an isosceles triangle *ABC* to the sides equal a constant.

Solution

The shortest distance from a point to a line is the perpendicular from that point to the line. Therefore, let:



$MP \perp AB$
$MQ \perp AC$

Let $BH \perp AC$ (Shortest distance from *B* to side *AC*.)

Let *D* be the point of intersection *ME* and *BH*.

Let $ME \parallel AC$
Where the point *E* is the intersection of the lines *MD* and *AB*.

Since $MD \parallel AC$ then $\widehat{DMB} = \widehat{ACB}$

Since triangle *ABC* is an isosceles
$\widehat{DMB} = \widehat{ACB} = \widehat{PBM}$

The right triangles *BPM* and *BDM* at *P* & *D* and have the same hypotenuse, then
$\Rightarrow \quad |MP| = |BD|$

$MD \parallel HQ$ and DH & $MQ \perp HQ$
$\Rightarrow \quad |MQ| = |DH|$

$$\begin{aligned} |MP| + |MQ| &= |BD| + |DH| \\ &= |BH| \\ &= \textbf{\textit{constant}} \end{aligned}$$

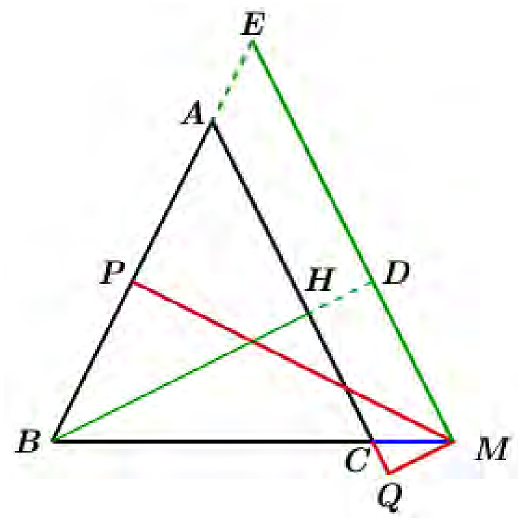
Therefore; the sum of distances from a point *M* on the base *BC* of an isosceles triangle *ABC* to the sides equal a constant.

Exercise 15

Demonstrate that the difference of distances from a point *M* taken on the extension of the base *BC* of an isosceles triangle *ABC* to the sides equal a constant.

Solution

The shortest distance from a point to a line is the perpendicular from that point to the line. Therefore, let:



$MP \perp AB$
$MQ \perp AC$

Let $BH \perp AC$ (Shortest distance from *B* to side *AC*.)

Let *D* be the point of intersection *ME* and *BH*.

Let $ME \,//\, AC$
Where the point *E* is the intersection of the extensions of the lines *MD* and *AB*.

Since $MD \,//\, AC$ then $\widehat{DMB} = \widehat{ACB}$

Since triangle *ABC* is an isosceles
$\widehat{DMB} = \widehat{ACB} = \widehat{PBM}$

The right triangles *BPM* and *BDM* at *P* & *D* and have the same hypotenuse, then
$\Rightarrow \quad |MP| = |BD|$

$MD \,//\, HQ$ and DH & $MQ \perp HQ$
$\Rightarrow \quad |MQ| = |DH|$

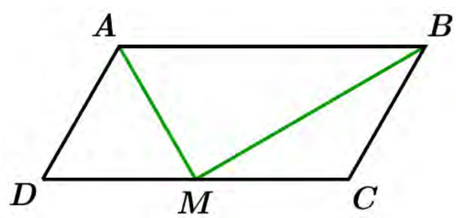
$$\begin{aligned} |MP| - |MQ| &= |BD| - |DH| \\ &= |BH| \\ &= \textit{constant} \end{aligned}$$

Therefore; the difference of distances from a point *M* taken on the extension of the base *BC* of an isosceles triangle *ABC* to the sides equal a constant.

Exercise 16

Consider a parallelogram *ABCD* in which $CD = 2AD$. In the joint *A* and *B* the middle *M* of *BC*. Prove that the angle $\widehat{AMB}$ is a right angle.

Solution



Since the point *M* is the middle of side *BC*, then

$$MD = MC = \frac{1}{2}CD$$

$$\Rightarrow \quad MD = AD = BC$$

Therefore; the triangles *ADM* and *BCM* are isosceles at *D* and *C* respectively.
Which implies that $MA = MB$

Let *O* be the middle point of the side *AB*, and $OA = OB = AD$

O and *M* are middle of the parallelogram *ABCD*, that implies

$$OM = BC = AD$$

$$\Rightarrow \quad OA = OB = OM$$

The triangle *MAB* inscribed in a circle with center at *O* and diameter *AB*, that will imply that is a right triangle at the point *M*.

Or

$$\widehat{AMD} = \frac{1}{2}\left(180^\circ - \widehat{MDA}\right)$$

$$\widehat{BMC} = \frac{1}{2}\left(180^\circ - \widehat{MCB}\right)$$

$$\widehat{ADM} + \widehat{MCB} = 180^\circ$$

$$\widehat{DMA} + \widehat{AMB} + \widehat{BMC} = 180^\circ$$

$$\begin{aligned}\widehat{AMB} &= 180^\circ - \left(\widehat{BMC} + \widehat{DMA}\right)\\ &= 180^\circ - \left(90^\circ - \frac{1}{2}\widehat{MDA} + 90^\circ - \frac{1}{2}\widehat{MCB}\right)\\ &= \frac{1}{2}\left(\widehat{MDA} + \widehat{MCB}\right)\\ &= \frac{1}{2}(180^\circ)\\ &= 90^\circ\end{aligned}$$

Exercise 17

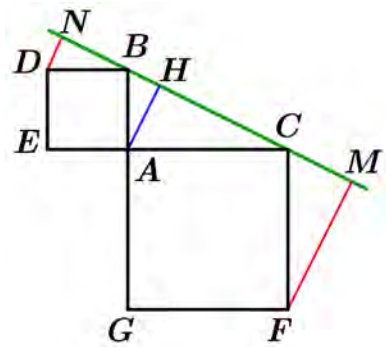
From the sides *AB* and *AC* of a right triangle *ABC* at *A*, draw two squares *ABDE* and *ACFG*. Then lead *DN* and *FM* perpendicular to the line *BC*.

a) Prove that $DN + FM = BC$

b) Prove that the points *D, A, F* on a straight line.

c) Prove that the lines *DE* and *FG* contribute on the extension of the height *AH*.

Solution



a) Let consider the 2 right triangles *DNB* & *BHA* at points *N* & *H* respectively, with $DB = AB$. Then

$$\widehat{HAB} = 90° - \widehat{ABH}$$
$$= 90° - \left(90° - \widehat{NBD}\right)$$
$$= \widehat{NBD}$$
$$\Rightarrow \quad \widehat{BDN} = \widehat{ABH}$$

$\therefore$ The 2 triangles are equals, which implies that $DN = BH$

Similar, for the 2 right triangles *CMF* & *AHC* at points *M* & *H* respectively, with $AC = CF$. Then

$$\widehat{HAC} = 90° - \widehat{ACH}$$
$$= 90° - \left(90° - \widehat{MCF}\right)$$
$$= \widehat{MCF}$$
$$\Rightarrow \quad \widehat{ACH} = \widehat{CFM}$$

$\therefore$ The 2 triangles are equals, which implies that $FM = HC$

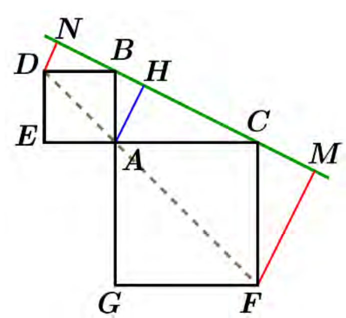
$$DN + FM = BH + HC$$
$$= BC \quad \checkmark$$

b) Since *ABDE* is a square, then $\widehat{BAD} = 45°$

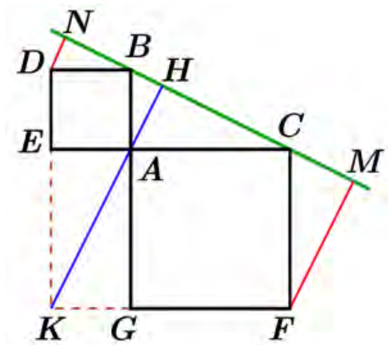
And *ACFG* is a square, then $\widehat{CAF} = 45°$

$$\widehat{DAF} = \widehat{DAB} + \widehat{BAC} + \widehat{CAF}$$
$$= 45° + 90° + 45°$$
$$= 180°$$

$\therefore$ The points *D, A,* & *F* are on a straight line.



c) Let the point K be the intersection of the extension of the sides DE and FG.
Which will result of $GKEA$ is a rectangle with $AE = GK$ & $EK = AG$



Consider the 2 right triangles BAC & KGA at points A & G respectively with $AE = AB = GK$

$\widehat{ACB} = \widehat{GAK} = \widehat{ACH}$

From the right triangle AHC:

$\widehat{HAC} + \widehat{ACH} = 90°$

$\rightarrow \quad \widehat{HAC} + \widehat{KAG} = 90°$

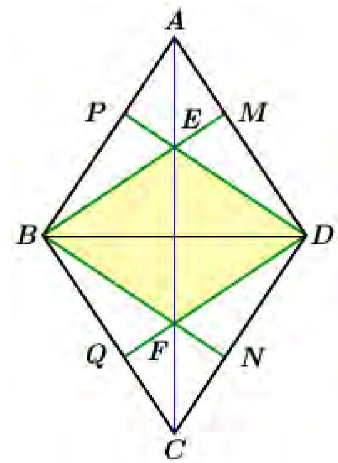
$$\begin{aligned}\widehat{HAC} + \widehat{CAG} + \widehat{KAG} &= \left(\widehat{HAC} + \widehat{KAG}\right) + \widehat{CAG} \\ &= 90° + 90° \\ &= 180°\end{aligned}$$

∴ The points K, A, & H are on a straight line.

Exercise 18

Given a diamond *ABCD*; the peak *B* and *D*, the same the perpendiculars *BM*, *BN*, *DP*, *DQ* on opposite sides. These perpendiculars are intersected at *E* and *F*.

Demonstrate that the angles of the quadrilateral *BFDE* are equals to the diamond and which is a diamond itself.



Solution

From the right triangles *BPD* & *BMD*, that implies $\widehat{MBD} = \widehat{PDB}$

$\Rightarrow \quad \widehat{EBD} = \widehat{EDB}$

Similar, from the right triangles *BND* & *BQD*, that implies $\widehat{NBD} = \widehat{QDB}$

$\Rightarrow \quad \widehat{FBD} = \widehat{FDB}$

$\widehat{EBD} + \widehat{DBF} = \widehat{EDB} + \widehat{BDF}$

$\widehat{EBF} = \widehat{EDF}$

Since, $AC \perp BD$, then $EF \perp BD$

The 2 triangles *EBF* & *EDF* have *EF* as a common side and $\widehat{EBF} = \widehat{EDF}$, then

$\widehat{BEF} = \widehat{DEF} = \widehat{BFE} = \widehat{DFE}$

$\widehat{BED} = \widehat{BFD}$

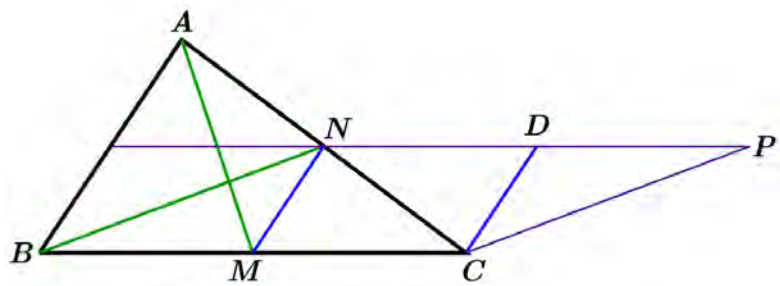
Therefore; the angles of the quadrilateral *BFDE* are equals to the diamond and which is a diamond itself.

Exercise 19

In a triangle *ABC*, we trace the median *AM* and *BN* and from *N* a parallel to *BC*, from *C* a parallel to *BN*; that the two sides intersect at a point *P*. Let *D* be the middle point of the segment *PN*.

Prove that *CD* is parallel to *MN*.

Solution



Since the points *M* & *N* are middle of the sides *BC* & *AC* of the triangle *ABC*, then $MN \parallel AB$

Given: $NP \parallel MC$

$BN \parallel CP$

Since *M* & *D* are the middle points of the segments *BC* and *NP* respectively, then $BN \parallel CP \parallel MD$

Therefore, *BNPC* is a parallelogram, and $MC = ND$.

Since $MC = ND$ & $MN = CD$

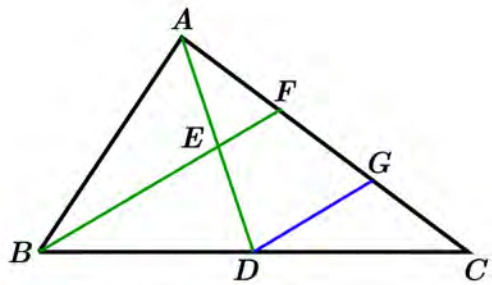
Therefore; *MCDC* is a parallelogram which implies to *CD* parallel to *MN*

Exercise 20

The median *AD* of a given triangle *ABC* to the side *BC*. The same the median *BE* to the side *AD* which intersect *AC* at a point *F*.

Prove that where $AF = \frac{1}{3}AC$

Solution



Let *DG* be parallel to segment *BEF*.

Given: *E* is the middle point of the segment *AD* $\Rightarrow \quad AE = ED$

D is the middle point of the segment *BC* $\Rightarrow \quad BD = DC$

Since $EF \,//\, DG$, and $AE = ED$, that implies $AF = FG$

Consider the triangles CDG and CBF:

$EF \,//\, DG$, and $CD = DB$, that implies $GC = FG$

That will imply to: $AF = FG = GC$

$$\begin{aligned} AC &= AF + FG + GC \\ &= 3AF \end{aligned}$$

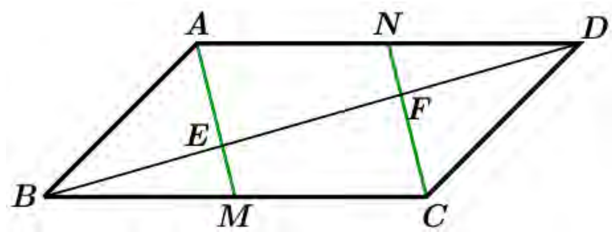
Therefore; $\boxed{AF = \frac{1}{3}AC}$

Exercise 21

In a parallelogram *ABCD*, from the points peak *A* and *C* joint the middle of opposite sides at *M* and *N* respectively.

Prove that the diagonal *BD* is divided in three equal parts.

Solution



M is the middle point of the segment *BC*, then $BM = CM$

N is the middle point of the segment *AD*, then $NA = ND$

From these, implies that $AM \parallel CN$.

From the triangles *BEM* & *BCF*, and since $ME \parallel CF$

It will give us that $BE = EF$

From the triangles *DFN* & *DEA*, and since $AE \parallel FN$

It will give us that $\Rightarrow \quad DF = EF$

Therefore, $BE = EF = DF$

$$BD = BE + EF + FD$$
$$= 3BE$$

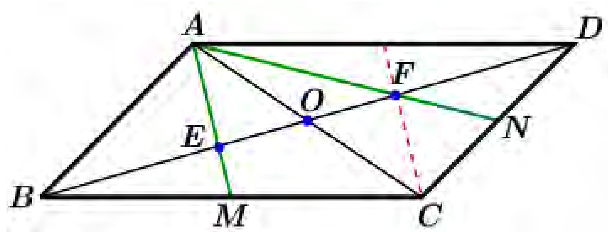
Therefore; the diagonal *BD* is divided in three equal parts

Exercise 22

In a parallelogram $ABCD$, from the point peak A, extend to the middle of sides BC and DC at M and N respectively.

Prove that the diagonal BD is divided in three equal parts.

Solution



Let a point E be the intersection of the segments AM & BD.
Let a point F be the intersection of the segments AN & BD.

Le O be the intersection of the both diagonal AC & BD.
From the triangles BEM & BCF, and since $ME \parallel CF$

$$\Rightarrow \quad BE = EF$$

Similar, $\Rightarrow \quad DF = EF$

$BO = OF \quad \rightarrow \quad OE = OF$

$$\begin{aligned} BO &= BE + EO \\ &= BE + \frac{1}{2}BE \\ &= \frac{3}{2}BE \end{aligned}$$

$$\begin{aligned} BE &= \frac{2}{3}BO \\ &= \frac{2}{3}\left(\frac{1}{2}BD\right) \\ &= \frac{1}{3}BD \end{aligned}$$

$$\begin{aligned} DF &= \frac{2}{3}DO \\ &= \frac{2}{3}\left(\frac{1}{2}BD\right) \\ &= \frac{1}{3}BD \end{aligned}$$

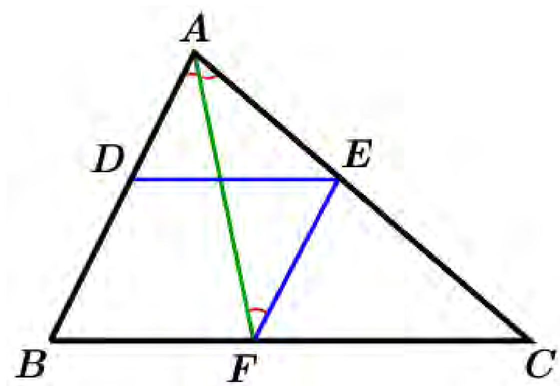
Therefore; the diagonal BD is divided in three equal parts

Exercise 23

Consider a triangle *ABC* with a bisector *AF* of the angle *A*. by *F*, we lead *FE* parallel to *AB*, and by *E* we lead *ED* parallel to *BC*.

Prove that $AE = BD$

Solution



Given: $\widehat{EAF} = \widehat{FAB}$

Since $FE \mathbin{/\mkern-6mu/} AB$, then

$$\widehat{FEC} = \widehat{BAE} = 2\widehat{EAF}$$

$$\widehat{AEF} = 180° - \widehat{FEC}$$
$$= 180° - 2\widehat{EAF}$$

Consider the triangle *AEF*:

$$\widehat{EAF} + \widehat{EFA} + \widehat{AEF} = 180°$$
$$\widehat{EAF} + \widehat{EFA} + 180° - 2\widehat{EAF} = 180°$$
$$\widehat{EFA} - \widehat{EAF} = 0°$$
$$\widehat{EFA} = \widehat{EAF}$$

∴ Triangle *AEF* is isosceles

$\Rightarrow$ $\underline{AE = EF}$

Given $DE \mathbin{/\mkern-6mu/} BF$ & $FE \mathbin{/\mkern-6mu/} DB$

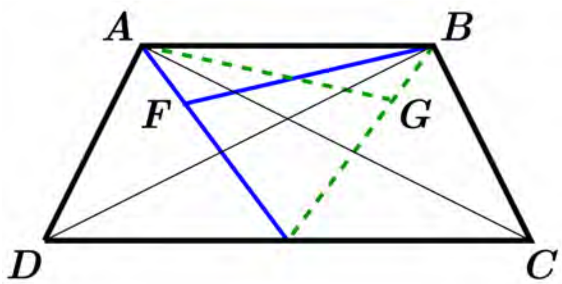
FEDB is a parallelogram;

Then, $\underline{EF = DB = AE}$

Exercise 24

Given an isosceles trapezoid $ABCD\ (AD = BC)$ with diagonals AC and BD. The bisector of angles $\widehat{DAB}$ and $\widehat{DBA}$ intersect in F, and the bisector of angles $\widehat{CBA}$ and $\widehat{CAB}$ intersect in G.
Demonstrate that FG is parallel to AB

Solution



Consider the 2 triangles ABD & ABC:

- Both has the AB as common
- $AD = BC$

That implies to: $\widehat{ABD} = \widehat{CAB}$

Since BF is the bisector of the angle $\widehat{ABD}$

$\widehat{ABF} = \widehat{FBD}$

$$\Rightarrow \quad \widehat{ABF} = \frac{1}{2}\widehat{ABD}$$
$$= \frac{1}{2}\widehat{CAB}$$
$$= \frac{1}{2}\left(2\widehat{BAG}\right)$$
$$= \widehat{BAG}$$

$\underline{\widehat{ABF} = \widehat{BAG}}$

From the 2 triangles AFB & AGB

- Both has the AB as common
- $\widehat{ABF} = \widehat{BAG}$

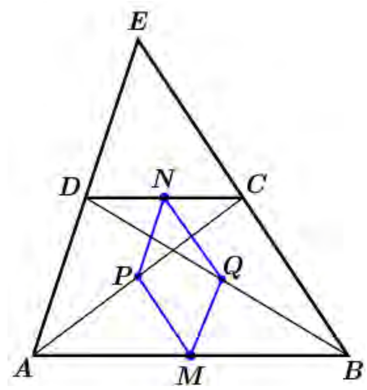
$\underline{FG \parallel AB}$

Exercise 25

Let M and N be the middle points of the bases AB and CD of a trapezoid $ABCD$.
Let P and Q be the middle points of the diagonals AC and BD respectively.

Demonstrate that the angles $\widehat{M}$ and $\widehat{N}$ of quadrilateral $MNPQ$ are equals to the angle formed by extending the sides not parallel to BC and AD, where intersect at point E.

Solution



Since N is the mid-point of the side DC, and P is the mid-point of the side AC, then

$$\Rightarrow \quad NP \,//\, AD$$

Since M is the mid-point of the side AB, and Q is the mid-point of the side DB, then

$$\Rightarrow \quad QM \,//\, AD$$

$$\therefore \quad NP \,//\, QM \,//\, AE$$

Since N is the mid-point of the side DC, and Q is the mid-point of the side DB, then

$$\Rightarrow \quad NQ \,//\, CB$$

Since M is the mid-point of the side AB, and P is the mid-point of the side AC, then

$$\Rightarrow \quad MP \,//\, CB$$

$$\therefore \quad NQ \,//\, MP \,//\, BE$$

$$\rightarrow \begin{cases} NP \,//\, QM \\ NQ \,//\, PM \end{cases}$$

$$\therefore \quad \boxed{\widehat{PMQ} = \widehat{PNQ}}$$

Exercise 26

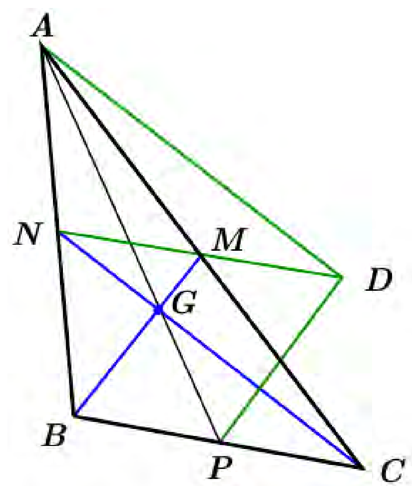
In a triangle *ABC*, the medians segment *BM* and *CN* intersect in right angles and the measurement are 3 and 6 units respectively.

1. Construct a geometrical to the triangle *ABC*.
2. In the trace of third median *AP* which leads *MN* extension such the distance $MD = MN$, which lead to the segments *AD* and *PD*. Calculate *AD* and *DP*.
3. What is the natural of the triangle APD?

Solution

1. Since *M* and *N* are the middle point of the sides *AC* & *AB*, then

$$BG = \frac{2}{3}BM$$
$$= \frac{2}{3}(3)$$
$$= 2 \quad units$$



Similar,

$$CG = \frac{2}{3}CN$$
$$= \frac{2}{3}(6)$$
$$= 4 \quad units$$

Wish, we lead to: $GM = 1$ & $GN = 2$

We can construct 2 perpendicular lines intersect at a point *G*, then we use to measure the distance from the point *G* to get the points *B*, *C*, *M*, & *N*.

By extending the segment *BN* and *CM* with equal distance and which it will intersect at point *A*.

2. Since $ND \,//\, BC$ & $MD = MN$

The parallelogram *BPDM*, $BP = MD = MN$

Then $PD = MB = 3 \;\; units$

$AD \,//\, CN$ and *M* is the intersection of the diagonals of the parallelogram *ADCN*, then

$AD = CN = 6 \;\; units$

3. $PD \,//\, BN$ & $MB \perp CN$, then $PD \perp CN$

$AD \,//\, CN$ & $PD \perp CN$, then $AD \perp PD$

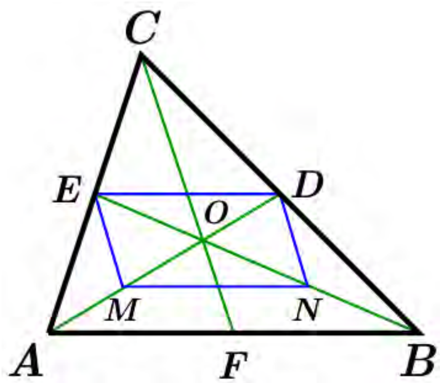
Therefore; the triangle *ADP* is right triangle at point *D*.

Exercise 27

Inside the triangle *ABC*, the median *AD*, *BE*, and *CF* intersect at a point *O*. We take *M* the middle point of the segment *OA*, *N* the middle point of segment *OB*.

Show that *DEMN* is a parallelogram.

Solution



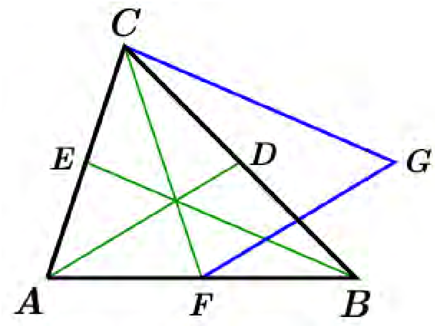
DE & *MN* are parallel to *AB* and equals to $\frac{1}{2}|AB|$

That implies to $ME \parallel DN$.

Therefore; *DEMN* is a parallelogram

Exercise 28

Inside the triangle *ABC*, the median *AD*, *BE*, and *CF* intersect at a point *O*. From the point *F*, draw *FG* parallel to *AD* and are equals, then joint *A* to *G*.



Show that $CG = BE$.

Solution

Given: $FG \parallel AD$ & $FG = AD$

Then, the quadrilateral *AFGD* is a parallelogram which it results to $DG \parallel AF$ & $DG = AF$.

$\rightarrow$ $DG \parallel BF$

Since *F* is the mid-point of the side *AB*, then $AF = DG = FB$.

Then, the quadrilateral *BFDG* is a parallelogram which it results to $FD \parallel BG$ & $DF = GB$.

So, $FD \parallel BG \parallel CE$

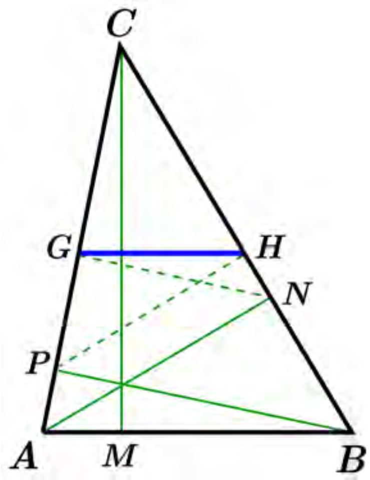
Given that D & F are midpoints, then $DF = \frac{1}{2} AC = CE$

And $CE \parallel BG$ & $DF = CE$, then *BGCE* is a parallelogram.

Therefore, $CG = BE$

Exercise 29

The height of a triangle *ABC* (each of the three lines passing through a vertex of the triangle and perpendicular to the opposite side to this vertex) are *AN*, *BP*, *CM*.

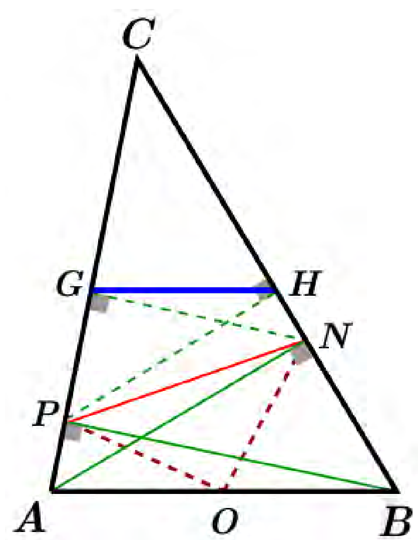


From *P*, let *PH* perpendicular to *BC*, same from *N*, let *NG* perpendicular to *AC*.

Show that *GH* is parallel to *AB*.

Solution

Let the point *O* be the middle of the segment *AB*.
Then *O* is the center of the 2 triangles *ANB* & *APB*.



The triangle *OBN* is isosceles, implies to $\widehat{ONB} = \widehat{OBN}$

The triangle *OPA* is isosceles, implies to $\widehat{OPA} = \widehat{OAP}$

$$\widehat{PON} = 180° - \left(\widehat{NOB} + \widehat{POA}\right)$$

$$= 180° - \left(180° - 2\widehat{NBO} + 180° - 2\widehat{OAP}\right)$$

$$= 2\widehat{B} + 2\widehat{A} - 180°$$

Consider the triangle *PON* with $OP = ON$, then

$$\widehat{OPN} = \widehat{ONP}$$

$$\widehat{OPN} = \frac{1}{2}\left(180° - \widehat{PON}\right)$$

$$= \frac{1}{2}\left(180° - 2\widehat{B} - 2\widehat{A} + 180°\right)$$

$$= 180° - \widehat{B} - \widehat{A}$$

$$\widehat{APN} = \widehat{APO} + \widehat{OPN}$$

$$= \widehat{A} + 180° - \widehat{B} - \widehat{A}$$

$$= 180° - \widehat{B}$$

$$\widehat{CPN} = 180° - \widehat{APN}$$

$$= 180° - 180° + \widehat{B}$$

$$= \widehat{B}$$

From the 2 right triangles *CHP* & *CGN*

$$\widehat{HPC} = \widehat{GNC}$$

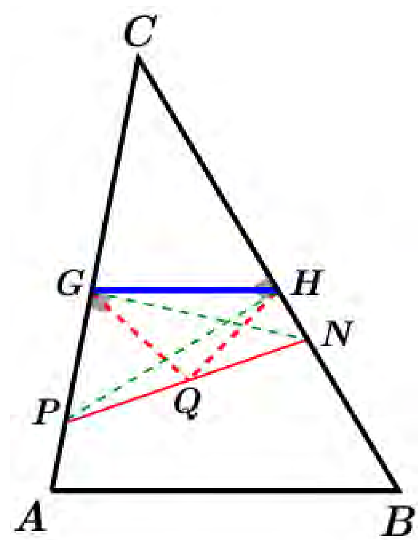
$$\widehat{GHN} = 180° - \widehat{HGN} - \widehat{HNG}$$

$$= 180° - \widehat{HGN} - \widehat{CPH}$$

$$180° - \widehat{GHC} = 180° - \widehat{HGN} - \widehat{CPH}$$

$$\widehat{GHC} = \widehat{HGN} + \widehat{CPH}$$

Let *Q* be the middle point of the segment *PN*.
Since *PGN* & *PHN* are right triangle with the same hypothesis.
Then, the triangles *HQN* & *GQN* are isosceles.



$$\widehat{H} = \widehat{N} \quad \& \quad \widehat{G} = \widehat{P}$$

$$\widehat{GQH} = 180° - \left(180° - 2\widehat{P} + 180° - 2\widehat{N}\right)$$

$$= 2\widehat{P} + 2\widehat{N} - 180°$$

Since $QG = QH \quad \Rightarrow \quad \widehat{QGH} = \widehat{QHG}$

$$\widehat{QGH} = \frac{1}{2}\left(180° - \widehat{GQH}\right)$$

$$= \frac{1}{2}\left(180° - 2\widehat{P} - 2\widehat{N} + 180°\right)$$

$$= 180° - \widehat{P} - \widehat{N}$$

$$\widehat{HGN} = \widehat{QGH} - \widehat{QGN}$$

$$= 180° - \widehat{P} - \widehat{N} - 90° + \widehat{QGP}$$

$$= 90° - \widehat{P} - \widehat{N} + \widehat{P}$$

$$= 90° - \widehat{N}$$

$$= \widehat{NPH}$$

$$\widehat{CHG} = \widehat{HGN} + \widehat{CPH}$$

$$= \widehat{NPH} + \widehat{CPH}$$

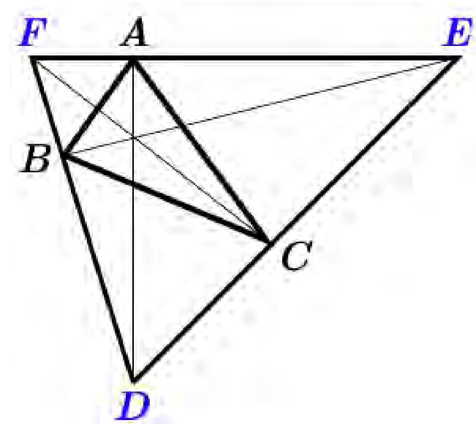
$$= \widehat{NPC}$$

$$= \widehat{B}$$

Therefore, $GH \parallel AB$

Exercise 30

From the top of a triangle, we lead the external bisectors of angles such that formed an outside triangle such that the top of the first are the feet of the second heights.



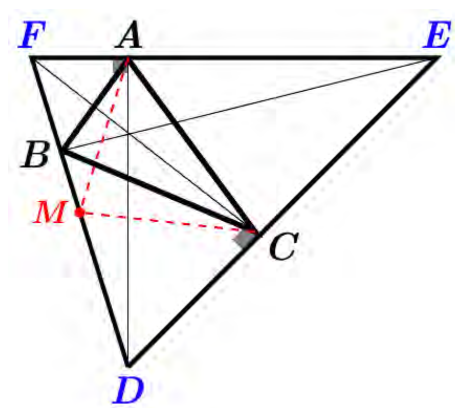
Solution

Let the triangle *DEF* where *DA*, *BE*, and *FC* are heights (perpendicular to sides).

Let the point *M* be the middle points of the same hypothenuse of the 2 right triangles *FAD* & *FCD*. Then, the 2 triangles inscribed the same circle with the center at point *M*.

$MF = MA = MC = MD$

$\widehat{MFA} = \widehat{MAF} \quad \& \quad \widehat{MCD} = \widehat{MDC}$



Therefore, the triangle *AMC* is isosceles.

$MA = MC \quad \& \quad \widehat{MAC} = \widehat{ACM}$

$$\begin{aligned}\widehat{AMC} &= 180° - \left(\widehat{FMA} + \widehat{CMD}\right)\\ &= 180° - \left(180° - 2\widehat{F} + 180° - 2\widehat{D}\right)\\ &= 2\widehat{F} + 2\widehat{D} - 180°\end{aligned}$$

$$\begin{aligned}\widehat{ACM} &= \frac{1}{2}\left(180° - \widehat{AMC}\right)\\ &= \frac{1}{2}\left(180° + 180° - 2\widehat{F} - 2\widehat{D}\right)\\ &= 180° - \widehat{F} - \widehat{D}\end{aligned}$$

$$\begin{aligned}\widehat{DCA} &= \widehat{DCM} + \widehat{MCA}\\ &= \widehat{D} + 180° - \widehat{F} - \widehat{D}\end{aligned}$$

$180° - \widehat{ACE} = 180° - \widehat{F}$

$\widehat{ACE} = \widehat{F}$

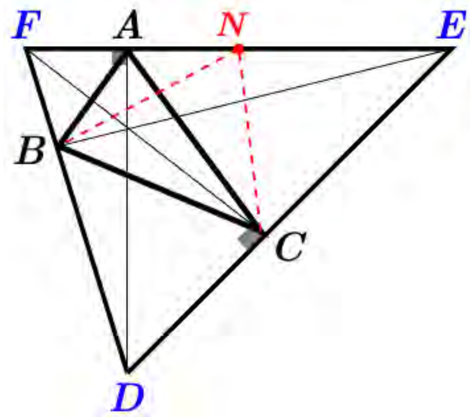
Similar,

Let the point *N* be the middle points of the same hypothenuse of the 2 right triangles *FCE* & *FBE*.

Then, the 2 triangles inscribed the same circle with the center at point *N*.

$NF = NB = NC = NE$

$\widehat{NBF} = \widehat{BFN} \quad \& \quad \widehat{NEC} = \widehat{NCE}$



Therefore, the triangle *NBC* is isosceles.

$MA = MC \quad \& \quad \widehat{MAC} = \widehat{ACM}$

$$\widehat{BNC} = 180° - \left(\widehat{FNB} + \widehat{CNE}\right)$$

$$= 180° - \left(180° - 2\widehat{F} + 180° - 2\widehat{E}\right)$$

$$\underline{= 2\widehat{F} + 2\widehat{E} - 180°}\,\Big|$$

$$\widehat{BCN} = \frac{1}{2}\left(180° - \widehat{BNC}\right)$$

$$= \frac{1}{2}\left(180° + 180° \; - 2\widehat{F} - 2\widehat{E}\right)$$

$$\underline{= 180° \; - \widehat{F} - \widehat{E}}\,\Big|$$

$$\widehat{BCE} = \widehat{BCN} + \widehat{NCE}$$

$$= \widehat{E} + 180° - \widehat{F} - \widehat{E}$$

$$180° - \widehat{BCD} = 180° - \widehat{F}$$

$$\underline{\widehat{BCD} = \widehat{F}}\,\Big|$$

Then, $\widehat{ACE} = \widehat{F} = \widehat{BCD}$

Therefore; *CF* is the interior bisector of $\widehat{BCA}$ and *DCE* is the exterior bisector.

$$\widehat{MAC} = \frac{1}{2}\left(180° - \widehat{AMC}\right)$$

$$= \frac{1}{2}\left(180° + 180° \; - 2\widehat{F} - 2\widehat{D}\right)$$

$$\underline{= 180° \; - \widehat{F} - \widehat{D}}\,\Big|$$

$$\widehat{FAC} = \widehat{FAM} + \widehat{MAC}$$

$$= \widehat{F} + 180 - \widehat{F} - \widehat{D}$$

$$180° - \widehat{CAE} = 180° - \widehat{D}$$

$$\underline{\widehat{CAE} = \widehat{D}}\,\Big|$$

Let the point *P* be the middle points of the same hypothenuse of the 2 right triangles *DAE* & *BDE*. Then, the 2 triangles inscribed the same circle with the center at point *P*.

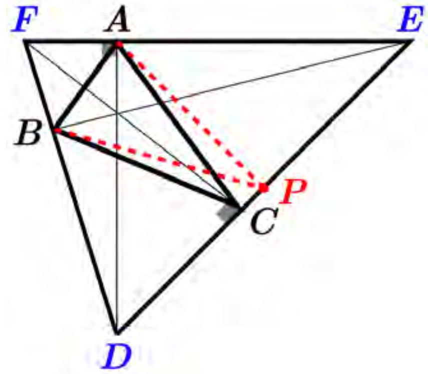
$PE = PA = PB = PD$

$\widehat{PAE} = \widehat{PAE} \quad \& \quad \widehat{PBD} = \widehat{PDB}$

Therefore, the triangle APB is isosceles.

$PA = PB \quad \& \quad \widehat{PAB} = \widehat{PBA}$

$$\widehat{APB} = 180° - \left(\widehat{DPB} + \widehat{APE}\right)$$
$$= 180° - \left(180° - 2\widehat{D} + 180° - 2\widehat{E}\right)$$
$$= 2\widehat{D} + 2\widehat{E} - 180°$$



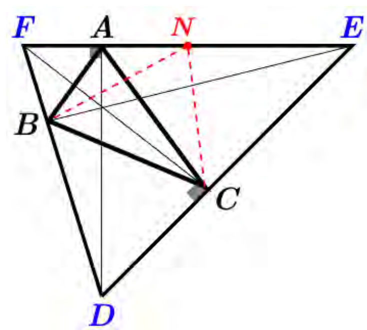
$$\widehat{PAB} = \frac{1}{2}\left(180° - \widehat{APB}\right)$$
$$= \frac{1}{2}\left(180° + 180° - 2\widehat{D} - 2\widehat{E}\right)$$
$$= 180° - \widehat{D} - \widehat{E}$$

$$\widehat{BAE} = \widehat{PAE} + \widehat{PAB}$$
$$= \widehat{E} + 180° - \widehat{D} - \widehat{E}$$
$$180° - \widehat{FAB} = 180° - \widehat{D}$$
$$\widehat{FAB} = \widehat{D}$$

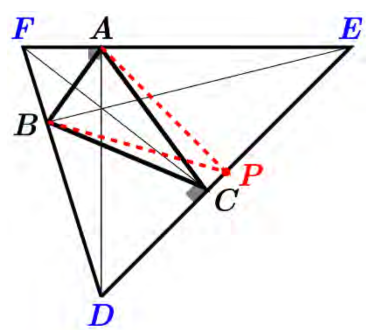
Then, $\widehat{FAB} = \widehat{D} = \widehat{CAE}$

Therefore; AD is the interior bisector of $\widehat{BAC}$ and FAE is the exterior bisector.

$$\widehat{NBC} = \frac{1}{2}\left(180° - \widehat{BNC}\right)$$
$$= \frac{1}{2}\left(180° + 180° - 2\widehat{F} - 2\widehat{E}\right)$$
$$= 180° - \widehat{F} - \widehat{E}$$



$$\widehat{CBF} = \widehat{CBN} + \widehat{NBF}$$
$$= 180° - \widehat{F} - \widehat{E} + \widehat{F}$$
$$180° - \widehat{CBD} = 180° - \widehat{E}$$
$$\widehat{CBD} = \widehat{E}$$



$$\widehat{PBA} = \frac{1}{2}\left(180° - \widehat{APB}\right)$$
$$= \frac{1}{2}\left(180° + 180° - 2\widehat{D} - 2\widehat{E}\right)$$
$$= 180° - \widehat{D} - \widehat{E}$$

$$\widehat{ABD} = \widehat{DBP} + \widehat{PBA}$$
$$= \widehat{D} + 180° - \widehat{D} - \widehat{E}$$

$180° - \widehat{ABF} = 180° - \widehat{E}$

$\widehat{ABF} = \widehat{E}$

Then, $\widehat{CBD} = \widehat{E} = \widehat{ABF}$

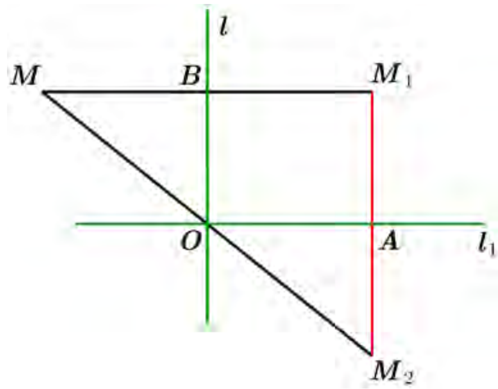
Therefore; BE is the interior bisector of $\widehat{ABC}$ and DBF is the exterior bisector.

Exercise 31

Consider a point *O* on a vertical line ℓ, a point *M* outside the line ℓ. We take the symmetries M_1 and M_2 from *M* across the line ℓ and the point *O*, respectively.

Demonstrate that the points M_1 and M_2 are symmetries with regard to a line perpendicular to the line ℓ_1 passing through the point *O*.

Solution



Since M_1 is the symmetry of *M* across the line ℓ, let *B* the middle point of the segment MM_1.

That implies to: $BM = BM_1$.

Similarly, the point *O* is the middle point of the segment MM_2.

That implies to: $OM = OM_2$.

Let *A* be the point intersection of the segment M_1M_2 and line ℓ_1.

Since $MM_1 \perp \ell$ and $\ell \perp \ell_1 \Rightarrow MM_1 // \ell_1$

From the right triangle MM_1M_2 Since O is the middle of MM_2 and $OA \perp MM_1$.

Therefore, the point *A* the middle point of the segment M_1M_2

Exercise 32

In a quadrilateral *ABCD* (*Kite*), the sides $AB = AD$, $\angle A = 135°$ and $\angle B = \angle D = 90°$.

1. Prove the symmetry in the figure and prove that the middles of the sides are the top of rectangle.
2. Prove there exists an interior of the given quadrilateral a point equidistant of 4 sides; determine these points.
3. On the same exterior bisector of angles *A*, *B*, *C*, *D*; they formed a quadrilateral

Solution