

Solution **Section 3.5 – Language of Hypothesis Testing**

Exercise

Bottles of Bayer aspirin are labeled with a statement that the tablets each contain 325 mg of aspirin. A quality control manager claims that a large sample of data can be used to support the claim that the mean amount of aspirin in the tablets is equal to 325 mg, as the label indicates. Can a hypothesis test be used to support that claim? Why or Why not?

Solution

No. Since the claim that the mean is equal to a specific value must be the null hypothesis, the only possible conclusions are to reject that claim or to fail to reject that claim, Hypothesis testing cannot be used to support a claim that a parameter is equal to a particular value.

Exercise

In the preliminary results from couples using the Gender Choice method of gender selection to increase the likelihood of having a baby girl, 20 couples used the Gender Choice method with the result that 8 of them had baby girls and 12 had baby boys. Given that the sample proportion of girls is $\frac{8}{20}$ or 0.4, can the sample data support the claim that the proportion of girls is greater than 0.5? Can any sample proportion less than 0.5 be used to support a claim that the population proportion is greater than 0.5?

Solution

No. Sample data that is not consistent with a claim can't be used to support that claim. In particular, no sample proportion less than 0.5 can ever be used to support a claim that the population proportion is greater than 0.5.

Exercise

Express the null hypothesis H_0 and alternative hypothesis H_1 in symbolic form. Be sure to use the correct symbol (μ , p , σ) for indicated parameter

- a) The mean annual income of employees who took a statistics course is greater than \$60,000.
- b) The proportion of people aged 18 to 25 who currently use illicit drugs is equal to 0.20 (or 20%).
- c) The standard deviation of human body temperatures is equal to 0.62°F.
- d) The majority of college students have credit cards.
- e) The proportion of homes with fire extinguishers is 0.80.
- f) The mean weight of plastic discarded by households in one week is less than 1 kg.

Solution

- a) Original claim: $\mu > \$60,000$

$$H_0 : \mu = \$60,000 \quad H_1 : \mu > \$60,000$$

- b) Original claim: $p = 0.20$
 $H_0 : p = 0.20 \quad H_1 : p \neq 0.20$
- c) Original claim: $p = 0.20$
 $H_0 : \sigma = 0.62^\circ F \quad H_1 : p \neq 0.62^\circ F$
- d) Original claim: $p > 0.5$
 $H_0 : p = 0.5 \quad H_1 : p > 0.5$
- e) Original claim: $p = 0.80$
 $H_0 : p = 0.80 \quad H_1 : p \neq 0.80$
- f) Original claim: $\mu < 1 \text{ kg}$
 $H_0 : \mu = 1 \text{ kg} \quad H_1 : \mu < 1 \text{ kg}$

Exercise

Assume that the normal distribution applies and find the critical z values.

- a) Two-tailed test: $\alpha = 0.01$.
b) Right-tailed test: $\alpha = 0.02$.
c) Left-tailed test: $\alpha = 0.10$.
d) $\alpha = 0.05$; H_1 is $p \neq 0.4$
e) $\alpha = 0.01$; H_1 is $p > 0.5$
- f) $\alpha = 0.005$; H_1 is $p < 0.8$
g) $\alpha = 0.05$ for two-tailed test
h) $\alpha = 0.05$ for left-tailed test
i) $\alpha = 0.08$; H_1 is $\mu \neq 3.25$

Solution

- a) Two-tailed test; place $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$ in each tail.

$$A = 1 - \frac{\alpha}{2} = 0.995$$

$$\text{Critical value: } \pm z_{\alpha/2} = \pm z_{0.005} = \pm 2.575$$

z score	Area
1.645	0.9500
2.575	0.9950

- b) Right-tailed test; place $\alpha = 0.02$ in the upper tail. $\Rightarrow A = 1 - \alpha = 0.98$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

$$\text{Critical value: } z_{\alpha/2} = z_{0.02} = 2.05$$

- c) Left-tailed test; place $\alpha = 0.10$ in the lower tail. $\Rightarrow A = \alpha = 0.1$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985

$$\text{Critical value: } z_{\alpha} = z_{0.1} = -1.28$$

- d) Two-tailed test; place $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$ in each tail. $\rightarrow A = 1 - \frac{\alpha}{2} = 0.975$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

Critical value: $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$

- e) Right-tailed test; place $\alpha = 0.01$ in the upper tail. $A = 1 - \alpha = 0.99$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916

Critical value: $z_{\alpha} = z_{0.01} = 2.33$

- f) Left-tailed test; place $\alpha = 0.005$ in the lower tail. $\Rightarrow A = 1 - \alpha = 0.995$

Critical value: $-z_{\alpha} = -z_{0.005} = -2.575$

z score	Area
1.645	0.9500
2.575	0.9950

- g) Two-tailed test; place $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$ in each tail.

$$A = 1 - \frac{\alpha}{2} = 0.975$$

Critical value: $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$

- h) Left-tailed test; $\alpha = 0.05 \Rightarrow A = 1 - \alpha = 0.95$

Critical value: $z_{\alpha} = z_{0.05} = -1.645$

- i) Two-tailed test; place $\frac{\alpha}{2} = \frac{0.08}{2} = 0.04$ in each tail. $A = \alpha = 0.04$

Critical value: $\pm z_{\alpha/2} = \pm z_{0.04} = \pm 1.75$

Exercise

The claim is that the proportion of peas with yellow pods is equal to 0.25 (or 25%). The sample statistics from one of Mendel's experiments include 580 peas with 152 of them having yellow pods. Find the value

of the test statistic z using $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$

Solution

$$\hat{p} = \frac{x}{n} = \frac{152}{580} = 0.262$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.262 - 0.250}{\sqrt{\frac{(0.25)(.75)}{580}}} = 0.67$$

Exercise

The claim is that less than $\frac{1}{2}$ of adults in U.S. have carbon monoxide detectors. A KRC Research survey of 1005 adults resulted in 462 who have carbon monoxide detectors. Find the value of the test statistic z

using $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$

Solution

$$\hat{p} = \frac{x}{n} = \frac{462}{1005} = 0.460$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.46 - 0.5}{\sqrt{\frac{(0.5)(.5)}{1005}}} = -2.56$$

Exercise

The claim is that more than 25% of adults prefer Italian food as their favorite ethnic food. A Harris Interactive survey of 1122 adults resulted in 314 who say that Italian food is their favorite ethnic food.

Find the value of the test statistic z using $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$

Solution

$$\hat{p} = \frac{x}{n} = \frac{314}{1122} = 0.28$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.28 - 0.25}{\sqrt{\frac{(0.25)(.75)}{1122}}} = 2.31$$

Exercise

Find P -value by using a 0.05 significance level and state the conclusion about the null hypothesis. (Reject the null hypothesis or fail to reject the null hypothesis)

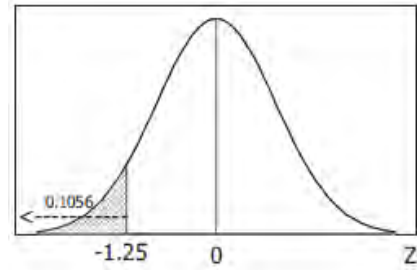
- a) The test statistic in a left-tailed test is $z = -1.25$
- b) The test statistic in a right-tailed test is $z = 2.50$
- c) The test statistic in a two-tailed test is $z = 1.75$
- d) With $H_1 : p \neq 0.707$, the test statistic is $z = -2.75$
- e) With $H_1 : p > \frac{1}{4}$, the test statistic is $z = 2.30$
- f) With $H_1 : p < 0.777$, the test statistic is $z = -2.95$

Solution

a) $P\text{-value} = P(z < -1.25)$
 $= 0.1056$

z	.00	.01	.02	.03	.04	.05
-1.2	.1151	.1131	.1112	.1093	.1075	.1056

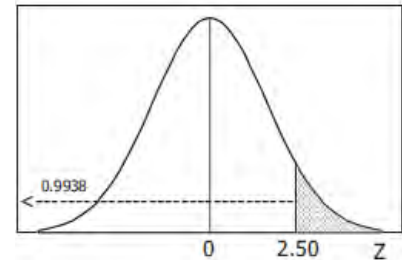
Since $0.1056 > 0.05$, fail to reject H_0



b) $P\text{-value} = P(z > 2.5)$
 $= 1 - 0.9938$
 $= 0.0062$

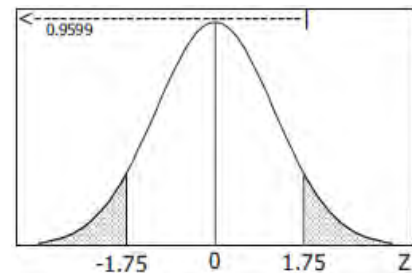
z	.00	.01
2.5	.9938	

Since $0.0062 < 0.05$, reject H_0



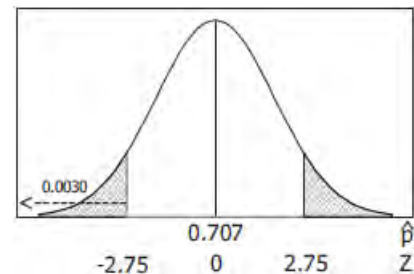
c) $P\text{-value} = 2 \cdot P(z > 1.75)$
 $= 2(1 - 0.9599)$
 $= 0.0802$

Since $0.0802 > 0.05$, fail to reject H_0



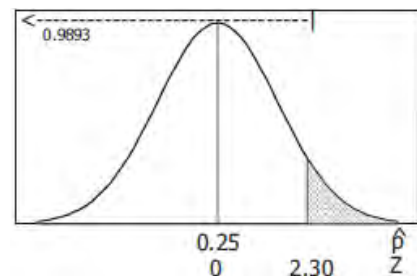
d) $P\text{-value} = 2 \cdot P(z < -2.75)$
 $= 2(0.003)$
 $= 0.006$

Since $0.006 > 0.05$, reject H_0



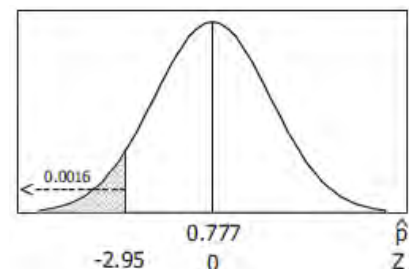
e) $P\text{-value} = P(z > 2.3)$
 $= 1 - 0.9893$
 $= 0.0107$

Since $0.0107 < 0.05$, reject H_0



f) $P\text{-value} = P(z < -2.95)$
 $= 0.0016$

Since $0.0016 < 0.05$, reject H_0



Exercise

The percentage of nonsmokers exposed to secondhand smoke is equal to 41%. Identify the type I error and type II error.

Solution

Original claim: $p = 0.41$

$$H_0 : p = 0.41$$

Type I error: rejecting H_0 when H_0 is actually true rejecting the claim that the percentage of non-smokers exposed to secondhand smoke is 41% when that percentage actually is 41%

Type II error: failing to reject H_0 when H_1 is actually true failing to reject the claim that the percentage is actually different from 41%

Exercise

The percentage of Americans who believe that life exists only on earth is equal to 20%. Identify the type I error and type II error.

Solution

Original claim: $p = 0.20$ $H_0 : p = 0.20$

Type I error: rejecting H_0 when H_0 is actually true rejecting the claim that the percentage of Americans who believe that life exists only on earth is 20% when that percentage actually is 20%

Type II error: failing to reject H_0 when H_1 is actually true failing to reject the claim that the percentage of Americans who believe that life exists only on earth is 20% when that percentage is actually different from 20%

Exercise

The percentage of college students who consume alcohol is greater than 70%. Identify the type I error and type II error.

Solution

Original claim: $p > 0.70$ $H_0 : p = 0.70$

Type I error: rejecting H_0 when H_0 is actually true rejecting the claim that the percentage of college students who use alcohol is 70% when that percentage actually is 70%.

Type II error: failing to reject H_0 when H_1 is actually true failing to reject the claim that the percentage of college students who use alcohol is 70% when that percentage actually is actually greater than 70%

Exercise

An entomologist writes an article in a scientific journal which claims that fewer than 13 in 10,000 male fireflies are unable to produce light due to a genetic mutation. Use the parameter p , the true proportion of fireflies unable to produce light. Express the null hypothesis and the alternative hypothesis in symbolic form. (μ , p , σ)

Solution

$$p = \frac{13}{10,000} = 0.0013$$

Since the claims are fewer than it will be “<”

$$H_0 : p = 0.0013$$

$$H_1 : p < 0.0013$$