$90°$ $\frac{\pi}{2}$

$(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$(\frac{\sqrt{3}}{2}, \frac{1}{2})$

$60°$ $45°$ $30°$ $\frac{\pi}{6}$

$0°$ $0°$

O

A

b   c

C   a   B

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

$$\csc A = \frac{c}{a} \qquad \sec A = \frac{c}{b} \qquad \cot A = \frac{b}{a}$$

$$\sin B = \frac{b}{c} \qquad \cos B = \frac{a}{c} \qquad \tan B = \frac{b}{a}$$

$$\csc B = \frac{c}{b} \qquad \sec B = \frac{c}{a} \qquad \cot B = \frac{a}{b}$$

sine
cosine $\}$ hyp + side   tangent   2 sides   (No hyp)

$\frac{opp}{adj}$

<u>Ex.</u> $C = 90°$ $A = 40°$ $C = 12$
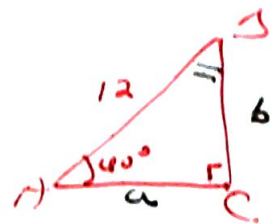
$B = 90° - 40°$
$\quad = 50°$

$\cos 40° = \dfrac{a}{12}$

$\quad a = 12 \cos 40° \quad \approx 7.7$

$\sin 40° = \dfrac{b}{12}$

$\quad b = 12 \sin 40°$

<u>Ex.</u>

$\sin 35° = \dfrac{18}{x + 18}$

$x + 18 = \dfrac{18}{\sin 35°}$

$x = \dfrac{18}{\sin 35°} - 18$

↗ angle of Elevation

↓ angle of depression
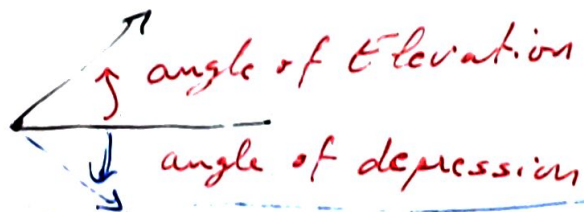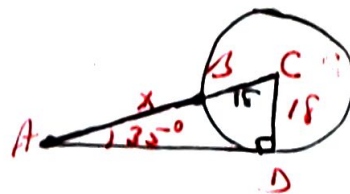
<u>Ex</u> $\sin 52.6° = \dfrac{h}{213}$

$\quad h = 213 \sin 52.6°$

Ex. $\angle CB$ & DCB $\angle B = h$

ΔACB:
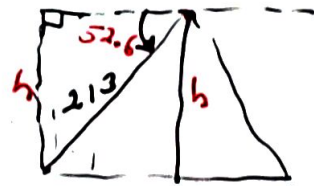
$$\tan 36.7° = \frac{h}{x}$$

$$h = x \tan 36.7° \quad ①$$

ΔDCB: $\tan 22.2° = \frac{h}{x+50}$

$$h = (x+50) \tan 22.2° \quad ②$$

① $=$ ② $x \tan 36.7° = x \tan 22.2° + 50 \tan 22.2°$

$x(\tan 36.7° - \tan 22.2°) = 50 \tan 22.2°$

$$x = \frac{50 \tan 22.2°}{\tan 36.7° - \tan 22.2°}$$

$$h = \frac{50 \tan 22.2° \cdot \tan 36.7°}{\tan 36.7° - \tan 22.2°}$$
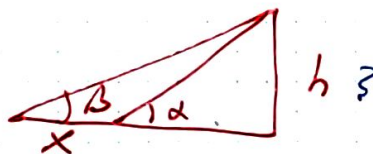
$$h = \frac{x \tan \alpha \cdot \tan \beta}{\tan \alpha - \tan \beta}$$
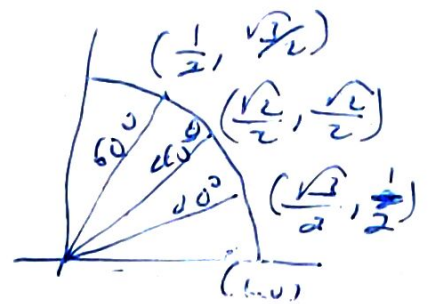
large small
angle

20

h = 100, angles 30°, 60°, height h
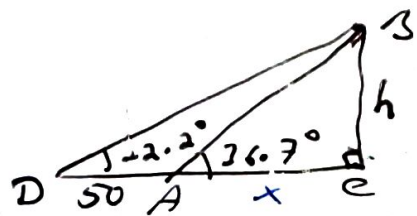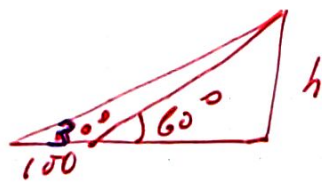
$$h = \frac{100 \tan 30° \tan 60°}{\tan 60° - \tan 30°}$$

$$= \frac{100 \left(\frac{1}{\sqrt{3}}\right)(\sqrt{3})}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$
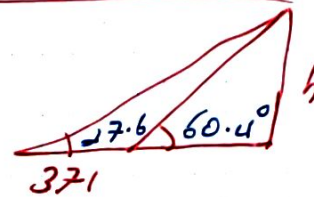
$$= \frac{100}{\frac{3-1}{\sqrt{3}}}$$

$$= 100 \cdot \frac{\sqrt{3}}{2}$$

$$= 50\sqrt{3}$$

Unit circle diagram with angles 60°, 45°, 30° and coordinates $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $(1,0)$

17

$$h = \frac{371 \tan 60.4° \tan 22.6°}{\tan 60.4° - \tan 22.6°}$$

Triangle with base 371, angles 27.6° and 60.4°, height h

EX

$|AB| = 25$

$y \overset{?}{=} 3.5$

Triangle ABC with A, F, B on top line; x from A to F, $25-x$ from F to B; angle $13°$ at A, angle $19°$ near B; C below with height y; $6 \cdot 2$ on left.

$\triangle AFC : \tan 13° = \frac{y}{x}$ ①

$\triangle FBC : \tan 19° = \frac{y}{25-x}$ ②

① → $y = x \tan 13°$ ③

② → $y = (25-x) \tan 19°$ }

$x \tan 13° = 25 \tan 19° - x \tan 19°$

$x(\tan 13° + \tan 19°) = 25 \tan 19°$

$x = \frac{25 \tan 19°}{\tan 13° + \tan 19°}$

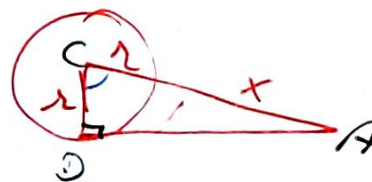$y = \frac{25 \tan 19°}{\tan 13° + \tan 19°} \cdot \tan 13°$
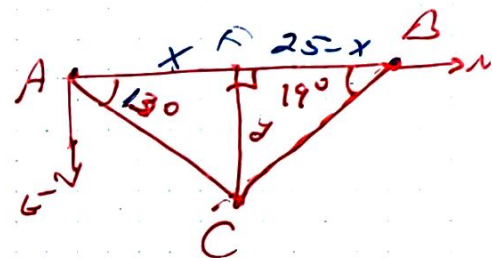
#6 $C = 30°$ $r = 15$ $x ??$

$$\cos 30° = \frac{15}{x+15} = \frac{\sqrt{3}}{2}$$

$$30 = \sqrt{3} (x+15)$$

$$x+15 = \frac{30}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \to$$

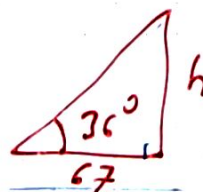$$\underline{x = 10\sqrt{3} - 15}$$

$$\frac{30\sqrt{3}}{3} \qquad \frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{x}$$
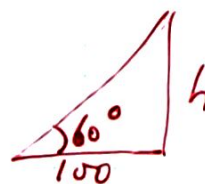

#10 $\tan 36° = \frac{h}{.67}$

$h = 67 \tan 36°$

#11 $\tan 60° = \frac{h}{100}$ $h = 100 \tan 60°$

$\underline{h = 100\sqrt{3}}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$\frac{\sqrt{3}}{\frac{2}{\frac{1}{2}}}$$

#30 $\tan B = \frac{100}{100} = 1$

$\underline{B = 45°}$
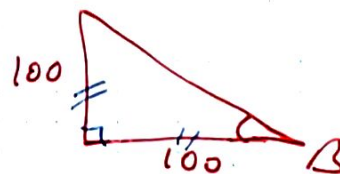
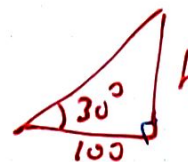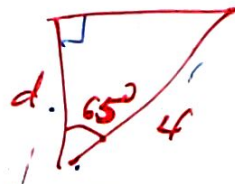
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

#37 $\tan 30° = \frac{h}{100}$
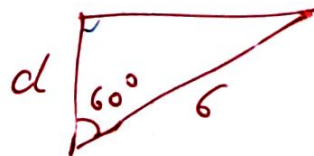
$h = 100 \cdot \frac{1}{\sqrt{3}}$

$= \frac{100}{\sqrt{3}}$ or $\frac{100\sqrt{3}}{3}$

39 $\cos 65^\circ = \dfrac{d}{4}$
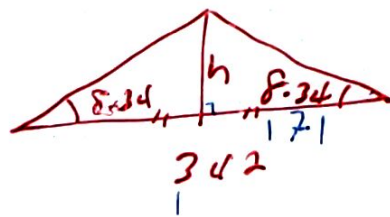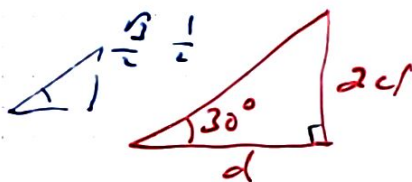
$d = 4\cos 65^\circ$

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40 $\cos 60^\circ = \dfrac{d}{6}$

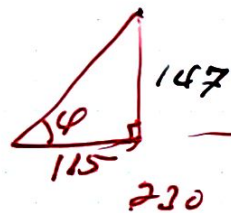$d = 6 \cdot \dfrac{1}{2}$

$= 3 \text{ mi}$

#41 $\tan 8.34^\circ = \dfrac{h}{171}$

$h = 171 \tan 8.34^\circ$

#40 $\tan 30^\circ = \dfrac{2d}{d}$

$d = \dfrac{2d}{\frac{1}{\sqrt{3}}}$
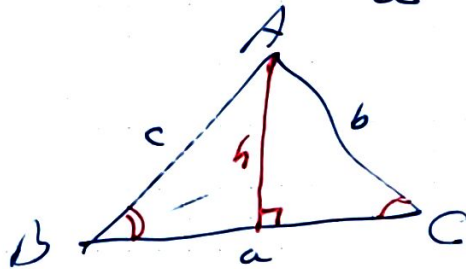
$= 2d\sqrt{3} \text{ in}$

#81 $\tan \varphi = \dfrac{147}{115}$

$\varphi = \tan^{-1} \dfrac{147}{115}$

(No right triangle)

Law of Sine : $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$A + B + C = 180°$

$$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$$

$\sin B = \dfrac{h}{c}$ $\qquad \sin C = \dfrac{h}{b}$

$h = c \sin B \quad = \quad h = b \sin C$

$c \sin B = b \sin C$

$\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Ex $\qquad A = 30° \qquad B = 70° \qquad a = 8 \qquad c?$

$\angle C = 180° - 30° - 70°$
$\qquad = 80°$

$\dfrac{c}{\sin C} \longrightarrow \dfrac{a}{\sin A}$

$c = \dfrac{8 \sin 80°}{\sin 30°}$

$\quad = 16 \sin 80°$

$\quad \approx 16$

Ex Given $A = 32°$ $C = 81.8°$ $a = 42.9$ cm

$$B = 180° - 32° - 81.8°$$
$$= 66.2°$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 66.2°} = \frac{42.9}{\sin 32°}$$

$$b = \frac{42.9 \sin 66.2°}{\sin 32°}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{42.9 \sin 81.8°}{\sin 32°}$$

Ex $C = 112.90°$ $A = 31.10°$

$$b = 347.6 \text{ ft}$$

$$B = 180° - 31.1° - 112.9°$$
$$= 36°$$

$$a = \frac{b \sin A}{\sin B}$$

$$= \frac{347.6 \sin 31.10°}{\sin 36°}$$
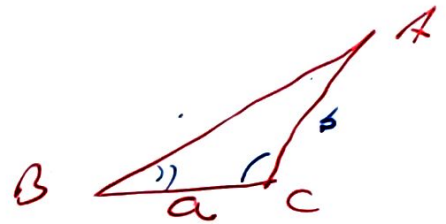
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Ex $a = 562$ $B = 5.7°$ $A = 85.3°$

$$\frac{x}{\sin B} = \frac{a}{\sin A}$$

$$X = \frac{562 \sin 5.7°}{\sin 85.3°}$$

① $0 \leq \sin \leq 1$

② $\sin (+)$ QI & QII

Ex   B? $a=2$, $b=6$ $A=30°$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{6 \cdot \sin 30°}{2}$$

$$= 3\left(\frac{1}{2}\right)$$

$$= \frac{3}{2} > 1$$

No Triangle.

Ex $C = 35.4°$ $a = 205$ $c = 314$ ft

$$\sin A = \frac{a \sin C}{c}$$

$$= \frac{205 \sin 35.4°}{314}$$

$$A = \sin^{-1}\left(\frac{205 \sin 35.4°}{314}\right)$$

$$\approx 22.2°$$

$A = 180° - 22.2°$
$= 157.8°$

$B \approx 180° - 35.4° - 22.2°$

$\approx 122.4°$

$B_1 = 180° - 157.8°$
$\quad - 35.4°$
$\quad = -$ #

$$b = \frac{c \sin B}{\sin C}$$

$$= \frac{314 \sin 122.4°}{\sin 35.4°}.$$

$$\approx 458 ft$$

Ex $a = 54$ $b = 62$ $A = 40°$

$$\sin B = \frac{b \sin A}{a}$$

$$= \frac{62 \sin 40°}{54}$$

$$B = \sin^{-1}\left(\frac{62 \sin 40°}{54}\right)$$

$$\approx 48° \rfloor$$

$B = 180° - 48°$
$= 132° \rfloor$

$$C = 180° - 40° - 48°$$
$$= 92°$$

$$C = 180° - 40° - 132°$$
$$= 8° \rfloor$$

$$c = \frac{54 \sin 92°}{\sin 40°}$$

$$c_1 = \frac{a \sin C}{\sin A}$$

$$\approx 85 \text{ cm} \rfloor$$

$$= \frac{54 \sin 8°}{\sin 40°}$$

$$\approx 12 \text{ cm} \rfloor$$

---

2 side $\overset{an}{\text{--}}$ Angle   (3 letters)

(Area)

$$K = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} ac \sin B$$

#ll) $b = 4$ $c = 1$ $A = 120°$

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} (4)(1) \sin 120°$$ $\sin 60°$

$$= 2 \left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3} \text{ unit}^2 \rfloor$$

$180°$ $>90°$

$90° <$ $< 180°$

$180° - 120° = 60°$

44) $a = 8$ $b = 6$. $C = 30°$

Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} (8)(6) \sin 30°$

$= 24 \left(\frac{1}{2}\right)$

$= 12 \text{ unit}^2$

$30°$ $45°$ $60°$ Q I

$150°$ $135°$ $120°$ Q II

$\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$