Polynomials

Adding and Subtracting Polynomials

Properties of Real numbers

For all real numbers a, b, and c:

$a+b=b+a$ — ***Commutative properties***

$ab=ba$

$(a+b)+c=a+(b+c)$ — ***Associative properties***

$(ab)c=a(bc)$

$a(b+c)=ab+ac$ — ***Distributive properties***

Add or subtract as indicated

a) $\left(8x^3-4x^2+6x\right)+\left(3x^3+5x^2-9x+8\right)$

$$=8x^3-4x^2+6x+3x^3+5x^2-9x+8$$

$$=\left(8x^3+3x^3\right)+\left(-4x^2+5x^2\right)+\left(6x-9x\right)+8$$

$$=11x^3+x^2-3x+8$$

b) $\left(-4x^4+6x^3-9x^2-12\right)+\left(-3x^3+8x^2-11x+7\right)$

$$=-4x^4+6x^3-3x^3-9x^2+8x^2-11x-12+7$$

$$=-4x^4+3x^3-x^2-11x-5$$

c) $\left(2x^2-11x+8\right)-\left(7x^2-6x+2\right)$

$$=2x^2-11x+8-7x^2+6x-2$$

$$=-5x^2-5x+6$$

Multiply

a) $8x(6x-4)$

$$\begin{aligned}8x(6x-4) &= 8x(6x)-8x(4)\\ &= 48x^2-32x\end{aligned}$$

b) $(3p-2)\left(p^2+5p-1\right)$

$$\begin{aligned}(3p-2)\left(p^2+5p-1\right) &= 3p^3+15p^2-3p-2p^2-10p+2\\ &= 3p^3+13p^2-13p+2\end{aligned}$$

c) $(x+2)(x+3)(x-4)$

$$\begin{aligned}(x+2)(x+3)(x-4) &= \left(x^2+3x+2x+6\right)(x-4)\\ &= \left(x^2+5x+6\right)(x-4)\\ &= x^3+5x^2+6x-4x^2-20x-24\\ &= x^3+x^2-14x-24\end{aligned}$$

Find $(2m-5)(m+4)$

$$\begin{aligned}(2m-5)(m+4) &= 2mm+2m(4)-5m-5(4)\\ &= 2m^2+8m-5m-20\\ &= 2m^2+3m-20\end{aligned}$$

Find $(2k-5)^2$

$$\begin{aligned}(2k-5)^2 &= (2k-5)(2k-5)\\ &= 4k^2-10k-10k+25\\ &= 4k^2-20k+25\end{aligned}$$

$$(a-b)^2 = a^2-2ab+b^2$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$(a-b)(a+b) = a^2-b^2$$

Perform the indicated operations: $2\left(3x^2+4x+2\right)-3\left(-x^2+4x-5\right)$

$$=6x^2+8x+4+3x^2-12x+15$$
$$=9x^2-4x+19$$

Perform the indicated operations: $(3t-2y)(3t+5y)$

$$=9t^2+15ty-6yt-10y^2$$
$$=9t^2+9yt-10y^2$$

Perform the indicated operations: $(2a-4b)^2$

$$(2a-4b)^2=(2a)^2-2(2a)(4b)+(4b)^2$$
$$=4a^2-16ab+16b^2$$

$$(a-b)^2=a^2-2ab+b^2$$

Factoring

Prime Factorization

A process that allows us to write a composite number as a product of two or more prime numbers.

Tree

$$\begin{array}{ccc} & 10 & \\ 2 & & 5 \end{array} \qquad 10 = 2 \times 5$$

$$\begin{aligned} 72 &= 2.\ 36 \\ &= 2.\ 6.\ 6 \\ &= 2.\ 2.\ 3.\ 2.\ 3 \\ &= 2^3\ 3^2 \end{aligned}$$

The Greatest Common Factor (GCF)

The largest factor that two or more numbers (or terms) have in common

Find GCF (18, 36)

18: 2. 9
2. 3. 3

36: 2. 18
2. 2. 3. 3

18: $2\ 3^2 \rightarrow 1, 2, 3, 6, 9, \underline{\mathbf{18}}$

36: $2^2\ 3^2 \rightarrow 1, 2, 3, 4, 6, 9, 12, \underline{\mathbf{18}}, 36$

GCF (18, 36) = 18 (is the greatest common factor)

Find GCF (27, 45)

$$\begin{aligned} 27 &= 3^3 \\ 45 &= \underline{3^2\ 5} \\ &\ \ \ 3^2 \end{aligned}$$

GCF (27, 45) = 9

Find GCF (40, 56)

$$\begin{aligned} 40 &= 2^3\ 5 \\ 56 &= \underline{2^3\ 7} \\ &\ \ \ 2^3 \end{aligned}$$

GCF (40, 56) = 8

Find GCF (80, 60)

$$\begin{aligned} 80 &= 2^4\ \ \ \ 5 \\ 60 &= \underline{2^2\ 3\ 5} \\ &\ \ \ 2^2\ \ \ \ 5 \end{aligned}$$

GCF (80, 60) = 20

Factor out the greatest common factor

12	2 . 2 . 3
18	2 . . 3 . 3
	2 . 3

a) $12p - 18q$

$$12p - 18q = 6(2p - 3q)$$

b) $8x^3 - 9x^2 + 15x$

$$8x^3 - 9x^2 + 15x = x\left(8x^2 - 9x + 15\right)$$

Factoring Trinomial

Factor $y^2 + 8y + 15$

Product 15	***Sum*** 8
15 *x* 1	15 + 1
3 *x* 5	3 + 5

$$y^2 + 8y + 15 = (y + 3)(y + 5)$$

Factor $4x^2 + 8xy - 5y^2$

$$4x^2 + 8xy - 5y^2 = (2x - y)(2x + 5y)$$

Special Factorization

$$a^2 - b^2 = (a - b)(a + b)$$
$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$$
$$a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$$

Factor

a) $64p^2-49q^2$

$$64p^2-49q^2=(8p)^2-(7q)^2$$
$$=(8p-7q)(8p+7q)$$

b) x^2+36

x^2+36 can't be factored (in real number) it is prime.

c) $x^2+12x+36$

$$x^2+12x+36=(x+6)^2$$

d) $9y^2-24yz+16z^2$

$$9y^2-24yz+16z^2=(3y)^2-2(3y)(4z)+(4z)^2$$
$$=(3y-4z)^2$$

e) y^3-8

$$y^3-8=y^3-2^3$$
$$=(y-2)\left(y^2+2y+4\right)$$

f) m^3+125

$$m^3+125=(m+5)\left(m^2-5m+25\right)$$

g) $8k^3-27z^3$

$$8k^3-27z^3=(2k)^3-(3z)^3$$
$$=(2k-3z)\left((2k)^2+6kz+(3z)^2\right)$$
$$=(2k-3z)\left(4k^2+6kz+9z^2\right)$$

h) p^4-1

$$p^4-1=\left(p^2\right)^2-(1)^2$$
$$=\left(p^2-1\right)\left(p^2+1\right)$$
$$=(p-1)(p+1)\left(p^2+1\right)$$

Factor: $60m^4 - 120m^3n + 50m^2n^2$

$$= 10m^2\left(6m^2 - 12mn + 5n^2\right)$$

Factor: $y^2 - 4yz - 21z^2$

$$= (y + 3z)(y - 7z)$$

Factor: $4a^2 + 10a + 6$

$$= 2\left(2a^2 + 5a + 3\right)$$

$$= 2(2a + 3)(a + 1)$$

Factor: $16a^4 - 81b^4$

$$= \left(4a^2\right)^2 - \left(9b^2\right)^2$$

$$= \left(4a^2 - 9b^2\right)\left(4a^2 + 9b^2\right)$$

$$= \left((2a)^2 - (3b)^2\right)\left(4a^2 + 9b^2\right)$$

$$= (2a - 3b)(2a + 3b)\left(4a^2 + 9b^2\right)$$

Fraction

$$\frac{a}{b} = \frac{numerator}{denominator}$$

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad ad = bc \qquad \textit{Cross multiplication}$$

$$\frac{a}{b} = \frac{na}{nb} = \frac{an}{bn}$$

a) $\frac{5}{6} = \frac{25}{30}$?

$$\frac{5}{6} = \frac{5}{6}\frac{5}{5} = \frac{25}{30}$$

b) $\frac{16}{48} = \frac{1}{3}$

$$\frac{16}{48} = \frac{1}{3} \Leftrightarrow (16)(3) = (1)(48)$$

$$48 = 48$$

Simplify: $\frac{12}{18} = \frac{2.6}{2.9}$

$$= \frac{2.2.3}{2.3.3}$$

$$= \frac{2}{3}$$

Simplify: $\frac{36}{56} = \frac{2.18}{2.28}$

$$= \frac{18}{28}$$

$$= \frac{2.9}{2.14}$$

$$= \frac{9}{14}$$

If the denominators are the same $\Rightarrow$ add the numerators

$$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5}$$

If the denominators are the same $\Rightarrow$ subtract the numerators

$$\frac{4}{9} - \frac{2}{9} = \frac{4-2}{9} = \frac{2}{9}$$

If the denominators are not the same

$\Rightarrow$ Find Least Common Denominator (LCD) and convert so that the fractions have the same denominators

LCD: is the smallest whole number that is a multiple of each

$\frac{5}{8}+\frac{1}{12}$ LCD (8, 12)

$8 = 2^3$

$12 = \underline{2^2\ 3}$

$2^3\ 3 = 24$ LCD (8, 12) = 24

$$\frac{5}{8}+\frac{1}{12}=\frac{5}{8}\frac{3}{3}+\frac{1}{12}\frac{2}{2}$$
$$=\frac{15}{24}+\frac{2}{24}$$
$$=\frac{15+2}{24}$$
$$=\frac{17}{24}$$

$\frac{69}{75}-\frac{1}{50}$

LCD (75, 50) $75 = 5^3$

$50 = \underline{2\ 5^2}$

$2\ 5^3 = 150$ LCD (75, 50) = 150

$$\frac{69}{75}-\frac{1}{50}=\frac{(69)(2)-(1)(3)}{150}$$
$$=\frac{138-3}{150}$$
$$=\frac{135}{150}$$
$$=\frac{9}{10}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

$$\frac{2}{7}+\frac{3}{5}=\frac{2(5)+3(7)}{7(5)}$$
$$=\frac{10+21}{35}$$
$$=\frac{31}{35}$$

or $\frac{2}{7}\frac{5}{5}+\frac{3}{5}\frac{7}{7}=\frac{10}{35}+\frac{21}{35}$
$$=\frac{10+21}{35}$$
$$=\frac{31}{35}$$

$$\frac{5}{9}+\frac{3}{4}=\frac{5(4)+3(9)}{9(4)}$$

$$=\frac{20+27}{36}$$

$$=\frac{47}{36}$$

$$\frac{17}{15}+\frac{5}{12}=\frac{17(12)+5(15)}{15(12)}$$

$$=\frac{204+75}{180}$$

$$=\frac{279}{180}$$

$$=\frac{31(9)}{20(9)}$$

$$=\frac{31}{20}$$

$$\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}=\frac{5(7)(9)+(3)(7)(9)+(3)(5)(9)+(3)(5)(7)}{(3)(5)(7)(9)}$$

$$=\frac{315+189+135+105}{945}$$

$$=\frac{744}{945}$$

$$=\frac{248}{315}\frac{3}{3}$$

$$=\frac{248}{315}$$

$$\frac{8}{9}+\frac{1}{12}+\frac{3}{16}$$

$$\frac{8}{9}+\frac{1}{12}+\frac{3}{16}=\frac{8(16)+1(12)+3(9)}{144}$$

$$\left\{\begin{array}{lll} 9 = & & 3^2 \\ 12 = & 2^2 & 3 \\ 16 = & 2^4 & \\ \hline \end{array}\right.$$

$$\text{LCD}\quad 2^4\quad 3^2 = 144$$

$$=\frac{128+12+27}{144}$$

$$=\frac{167}{144}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

$$\frac{2}{7}-\frac{3}{5}=\frac{2(5)-3(7)}{7(5)}=\frac{10-21}{35}=-\frac{11}{35}$$

$$\frac{a}{c}\frac{b}{d}=\frac{ab}{cd}$$

$$\frac{2}{7}\frac{3}{5}=\frac{6}{35}$$

$$\frac{a}{c}\div\frac{b}{d}=\frac{a}{c}\times\frac{d}{b}=\frac{ad}{cb}$$

$$\frac{2}{7}\div\frac{3}{5}=\frac{2}{7}\frac{5}{3}=\frac{10}{21}$$

$$\frac{\frac{a}{b}}{\frac{c}{b}}=\frac{a}{c}$$

$$\frac{\frac{a}{b}}{\frac{a}{c}}=\frac{c}{b}$$

Find:

1. $\frac{13}{21}+\frac{5}{21}=$

2. $\frac{7}{12}-\frac{4}{15}=$

3. $\frac{5}{8}+\frac{1}{2}=$

4. $\frac{5}{8}+\frac{1}{2}+\frac{2}{3}=$

5. $\frac{7}{8}-\frac{1}{10}=$

6. $\frac{11}{5}-\frac{31}{7}=$

7. $\frac{3}{4}\bullet\frac{3}{2}=$

8. $\frac{3}{4}\bullet\frac{4}{3}\bullet\frac{2}{3}=$

9. $\frac{3}{4}\div\frac{3}{2}=$

10. $\frac{14}{15}\div\frac{14}{3}=$

Solution

1. $\frac{13}{21}+\frac{5}{21}=\frac{13+5}{21}=\frac{6}{7}$

2. $\frac{7}{12}-\frac{4}{15}=\frac{7(5)-4(4)}{60}=\frac{35-16}{60}=\frac{19}{60}$

3. $\frac{5}{8}+\frac{1}{2}=\frac{5+4}{8}=\frac{9}{8}$

4. $\frac{5}{8}+\frac{1}{2}+\frac{2}{3}=\frac{5(3)+1(12)+2(8)}{24}=\frac{43}{24}$

5. $\frac{7}{8}-\frac{1}{10}=\frac{7(5)-1(4)}{40}=\frac{31}{40}$

6. $\frac{11}{5}-\frac{31}{7}=-\frac{78}{35}$

7. $\frac{3}{4}\bullet\frac{3}{2}=\frac{9}{8}$

8. $\frac{3}{4}\bullet\frac{4}{3}\bullet\frac{2}{3}=\frac{2}{3}$

9. $\frac{3}{4}\div\frac{3}{2}=\frac{3}{4}\bullet\frac{2}{3}=\frac{2}{4}=\frac{1}{2}$

10. $\frac{14}{15}\div\frac{14}{3}=\frac{14}{15}\bullet\frac{3}{14}=\frac{1}{5}$

Exponents

Integer Exponents

Definition of exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n-times}$$

a appears as a factor n times

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

$$a^m \cdot a^n = a^{m+n} \qquad \left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

$$\left(a^m\right)^n = a^{mn} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad (ab)^m = a^m b^m$$

a) 6^0

$$6^0 = 1$$

b) $(-9)^0$

$$(-9)^0 = 1$$

c) 3^{-2}

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

d) $\left(\frac{3}{4}\right)^{-1}$

$$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$$

a) $7^4.7^6$

$$7^4.7^6 = 7^{4+6} = 7^{10}$$

b) $\dfrac{9^{14}}{9^6}$

$$\frac{9^{14}}{9^6} = 9^{14-6} = 9^8$$

c) $\dfrac{r^9}{r^{17}}$

$$\frac{r^9}{r^{17}} = \frac{1}{r^{17-9}} = \frac{1}{r^8}$$

d) $\left(2m^3\right)^4$

$$\left(2m^3\right)^4 = (2)^4\left(m^3\right)^4$$

$$= 16m^{12}$$

e) $\left(\dfrac{x^2}{y^3}\right)^6$

$$\left(\frac{x^2}{y^3}\right)^6 = \frac{\left(x^2\right)^6}{\left(y^3\right)^6}$$

$$= \frac{x^{2.6}}{y^{3.6}}$$

$$= \frac{x^{12}}{y^{18}}$$

f) $\dfrac{a^{-3}b^5}{a^4b^{-7}}$

$$\frac{a^{-3}b^5}{a^4b^{-7}} = \frac{b^5b^7}{a^3a^4}$$

$$= \frac{b^{5+7}}{a^{4+3}}$$

$$= \frac{b^{12}}{a^7}$$

g) $p^{-1}+q^{-1}$

$$p^{-1}+q^{-1}=\frac{1}{p}+\frac{1}{q}$$

$$=\frac{1}{p}\frac{q}{q}+\frac{1}{q}\frac{p}{p}$$

$$=\frac{q+p}{pq}$$

h) $\frac{x^{-2}-y^{-2}}{x^{-1}-y^{-1}}$

$$\frac{x^{-2}-y^{-2}}{x^{-1}-y^{-1}}=\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}-\frac{1}{y}}$$

$$=\frac{\frac{y^2-x^2}{x^2y^2}}{\frac{y-x}{xy}}$$

$$=\frac{y^2-x^2}{x^2y^2}\cdot\frac{xy}{y-x}$$

$$=\frac{(y-x)(y+x)}{(xy)^2}\cdot\frac{xy}{y-x}$$

$$=\frac{y+x}{xy}$$

Calculations with exponents

a) $121^{1/2}=11$

b) $625^{1/4}=5$

c) $(-32)^{1/5}=-2$

d) $(-49)^{1/2}$ *is not a real number*

Rational Exponents

$$a^{m/n} = \left(a^{1/n}\right)^m$$

Calculations with Exponents

a) $27^{2/3}$ 27^(2/3)

$$27^{2/3} = \left(27^{1/3}\right)^2$$
$$= \left(\left(3^3\right)^{1/3}\right)^2$$
$$= \left(3^{3.\frac{1}{3}}\right)^2$$
$$= (3)^2$$
$$= 9$$

b) $32^{2/5}$ 32^(2/5)

$$32^{2/5} = \left(\left(2^5\right)^{1/5}\right)^2$$
$$= 2^2$$
$$= 4$$

c) $64^{4/3}$ 64^(4/3)

$$64^{4/3} = \left(\left(4^3\right)^{1/3}\right)^4$$
$$= (4)^4$$
$$= 256$$

Simplify

a) $\dfrac{y^{1/3}y^{5/3}}{y^3}$

$$\begin{aligned}\frac{y^{1/3}y^{5/3}}{y^3} &= \frac{y^{\frac{1}{3}+\frac{5}{3}}}{y^3}\\ &= \frac{y^{\frac{6}{3}}}{y^3}\\ &= \frac{y^2}{y^3}\\ &= \frac{1}{y^{3-2}}\\ &= \frac{1}{y}\end{aligned}$$

b) $m^{2/3}\left(m^{7/3}+7m^{1/3}\right)$

$$\begin{aligned}m^{2/3}\left(m^{7/3}+7m^{1/3}\right) &= m^{2/3}m^{7/3}+7m^{2/3}m^{1/3}\\ &= m^{\frac{2}{3}+\frac{7}{3}}+7m^{\frac{2}{3}+\frac{1}{3}}\\ &= m^{\frac{9}{3}}+7m^{\frac{3}{3}}\\ &= m^3+7m\end{aligned}$$

c) $\left(\dfrac{m^7n^{-2}}{m^{-5}n^2}\right)^{1/4}$

$$\begin{aligned}\left(\frac{m^7n^{-2}}{m^{-5}n^2}\right)^{1/4} &= \left(\frac{m^{7+5}}{n^{2+2}}\right)^{1/4}\\ &= \left(\frac{m^{12}}{n^4}\right)^{1/4}\\ &= \frac{\left(m^{12}\right)^{1/4}}{\left(n^4\right)^{1/4}}\\ &= \frac{m^{12/4}}{n^{4/4}}\\ &= \frac{m^3}{n}\end{aligned}$$

Simplify

a) $4m^{1/2}+3m^{3/2}$

$$4m^{1/2}+3m^{3/2}=m^{1/2}\left(4m^{1/2-1/2}+3m^{3/2-1/2}\right)$$
$$=m^{1/2}(4+3m)$$

b) $9x^{-2}-6x^{-3}$

$$9x^{-2}-6x^{-3}=3x^{-3}(3x-2)$$

c) $2\left(x^2+5\right)(3x-1)^{-1/2}+(3x-1)^{1/2}(2x)$

$$2\left(x^2+5\right)(3x-1)^{-1/2}+(3x-1)^{1/2}(2x)=2(3x-1)^{-1/2}\left[x^2+5+x(3x-1)\right]$$
$$=2(3x-1)^{-1/2}\left[x^2+5+3x^2-x\right]$$
$$=2(3x-1)^{-1/2}\left(4x^2-x+5\right)$$

Radicals

$$a^{1/n}=\sqrt[n]{a}$$

a) $\sqrt[4]{16}$

$$\sqrt[4]{16}=16^{1/4}=2$$

b) $\sqrt[5]{-32}=-2$

c) $\sqrt[3]{1000}$

$$\sqrt[3]{1000}=1000^{1/3}=10$$

d) $\sqrt[6]{\frac{64}{729}}$

$$\sqrt[6]{\frac{64}{729}}=\frac{\sqrt[6]{64}}{\sqrt[6]{729}}=\frac{2}{3}$$

Properties

$$\left(\sqrt[n]{a}\right)^n = a$$

$$\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n \text{ is even} \\ a & \text{if } n \text{ is odd} \end{cases}$$

$$\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

Simplify

a) $\sqrt{1000}$

$$\sqrt{1000} = \sqrt{100(10)}$$

$$= \sqrt{100}\sqrt{10}$$

$$= 10\sqrt{10}$$

b) $\sqrt{128}$

$$\sqrt{128} = \sqrt{64(2)}$$

$$= 8\sqrt{2}$$

c) $\sqrt{2}\sqrt{18}$

$$\sqrt{2}\sqrt{18} = \sqrt{2(18)}$$

$$= \sqrt{36}$$

$$= 6$$

d) $\sqrt[3]{54}$

$$\sqrt[3]{54} = \sqrt[3]{27(2)}$$

$$= 3\sqrt[3]{2}$$

e) $\sqrt{288m^5}$

$$\sqrt{288m^5} = \sqrt{144(2)m^4 m}$$

$$= 12m^2\sqrt{2m}$$

f) $2\sqrt{18}-5\sqrt{32}$

$$\begin{aligned}2\sqrt{18}-5\sqrt{32} &= 2\sqrt{9(2)}-5\sqrt{16(2)}\\ &= 6\sqrt{2}-20\sqrt{2}\\ &= -14\sqrt{2}\end{aligned}$$

Rationalizing Denominators

Simplify by rationalizing the denominator

a) $\dfrac{4}{\sqrt{3}}=\dfrac{4}{\sqrt{3}}\dfrac{\sqrt{3}}{\sqrt{3}}$

$$=\frac{4\sqrt{3}}{3}$$

b) $\dfrac{2}{\sqrt[3]{x}}=\dfrac{2}{\sqrt[3]{x}}\dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

$$=\frac{2\sqrt[3]{x^2}}{x}$$

c) $\dfrac{1}{1-\sqrt{2}}=\dfrac{1}{1-\sqrt{2}}\dfrac{1+\sqrt{2}}{1+\sqrt{2}}$

$$\begin{aligned}&=\frac{1+\sqrt{2}}{1-2}\\ &=\frac{1+\sqrt{2}}{-1}\\ &=-1-\sqrt{2}\end{aligned}$$

Simplify $\sqrt{27}\sqrt{3}$

$$\begin{aligned}\sqrt{27}\sqrt{3} &= \sqrt{27(3)}\\ &= \sqrt{81}\\ &= 9\end{aligned}$$

Simplify $\sqrt[4]{x^8y^7z^{11}}$

$$\sqrt[4]{x^8y^7z^{11}}=x^2yz^2\ \sqrt[4]{y^3z^3}$$

Simplify $\dfrac{5}{\sqrt{10}}$

$$\frac{5}{\sqrt{10}} = \frac{5}{\sqrt{10}}\,\frac{\sqrt{10}}{\sqrt{10}}$$

$$= \frac{5\sqrt{10}}{10}$$

$$= \frac{\sqrt{10}}{2}$$

Simplify $\dfrac{5}{2-\sqrt{6}}$

$$\frac{5}{2-\sqrt{6}} = \frac{5}{2-\sqrt{6}}\,\frac{2+\sqrt{6}}{2+\sqrt{6}}$$

$$= \frac{5\left(2+\sqrt{6}\right)}{4-6}$$

$$= -\frac{5}{2}\left(2+\sqrt{6}\right)$$

Simplify $\dfrac{1}{\sqrt{r}-\sqrt{3}}$

$$\frac{1}{\sqrt{r}-\sqrt{3}} = \frac{1}{\sqrt{r}-\sqrt{3}}\,\frac{\sqrt{r}+\sqrt{3}}{\sqrt{r}+\sqrt{3}}$$

$$= \frac{\sqrt{r}+\sqrt{3}}{r-3}$$