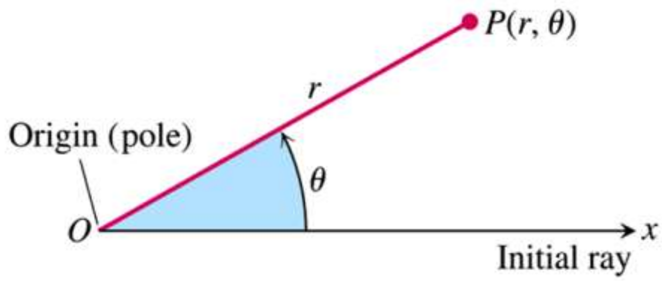
Section 4.5 – Polar Coordinates

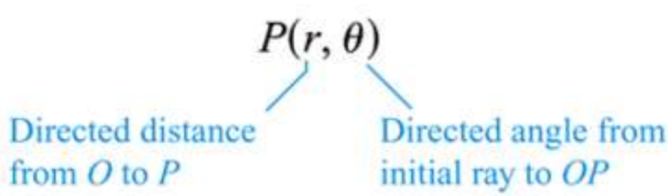
To reach the point whose address is (2, 1), we start from origin and travel 2 units right and then 1 unit up. Another way to get to that point, we can travel $\sqrt{5}$ units on the terminal side of an angle in standard position and this type is called *Polar Coordinates*.

Definition of Polar Coordinates

To define polar coordinates, let an ***origin*** *O* (called the ***pole***) and an ***initial ray*** from *O*. Then each point *P* can be located by assigning to it a ***polar coordinate pair*** $(r,\ \theta)$ in which r gives the directed from *O* to *P* and θ gives the directed angle from the initial ray to yay *OP*.



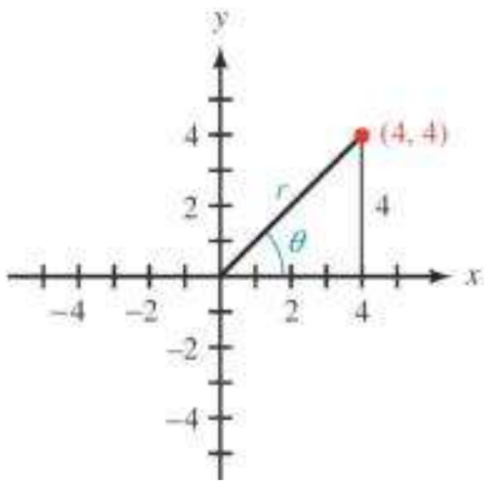
Polar Coordinates



Example

A point lies at (4, 4) on a rectangular coordinate system. Give its address in polar coordinates $(r,\ \theta)$

<u>*Solution*</u>



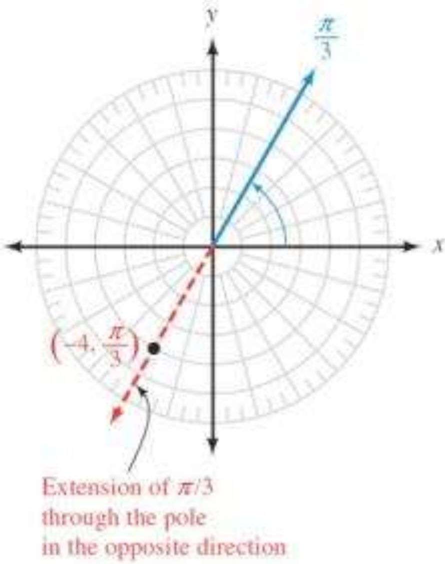
$$r = \sqrt{4^2 + 4^2}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$

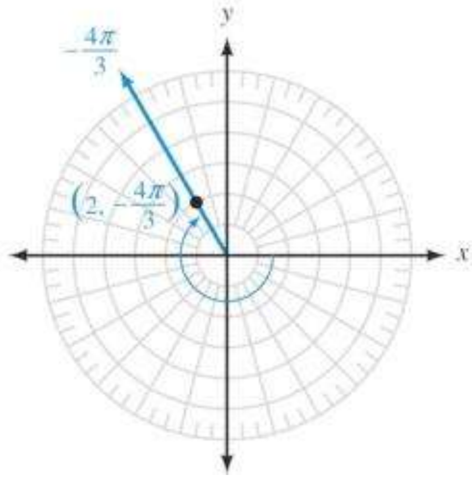
$$\theta = \tan^{-1}\left(\frac{4}{4}\right)$$
$$= \tan^{-1}(1)$$
$$= 45^\circ$$

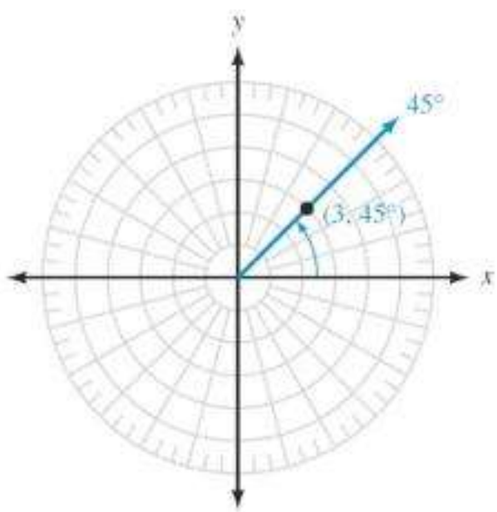
The address is $\left(4\sqrt{2}, 45^\circ\right)$

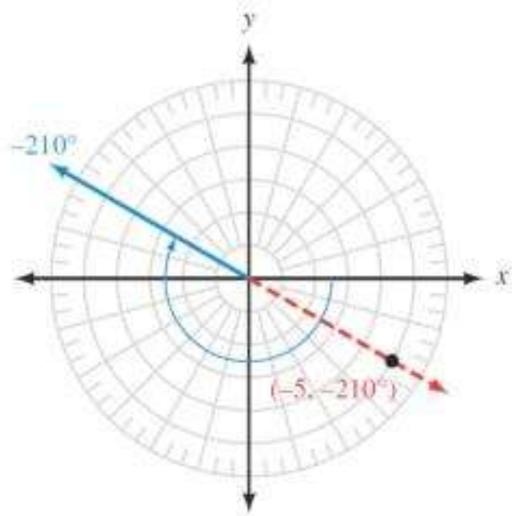
Example

Graph the points $(3, 45°)$, $\left(2,\ -\frac{4\pi}{3}\right)$, $\left(-4,\ \frac{\pi}{3}\right)$, and $(-5,\ -210°)$ on a polar coordinate system



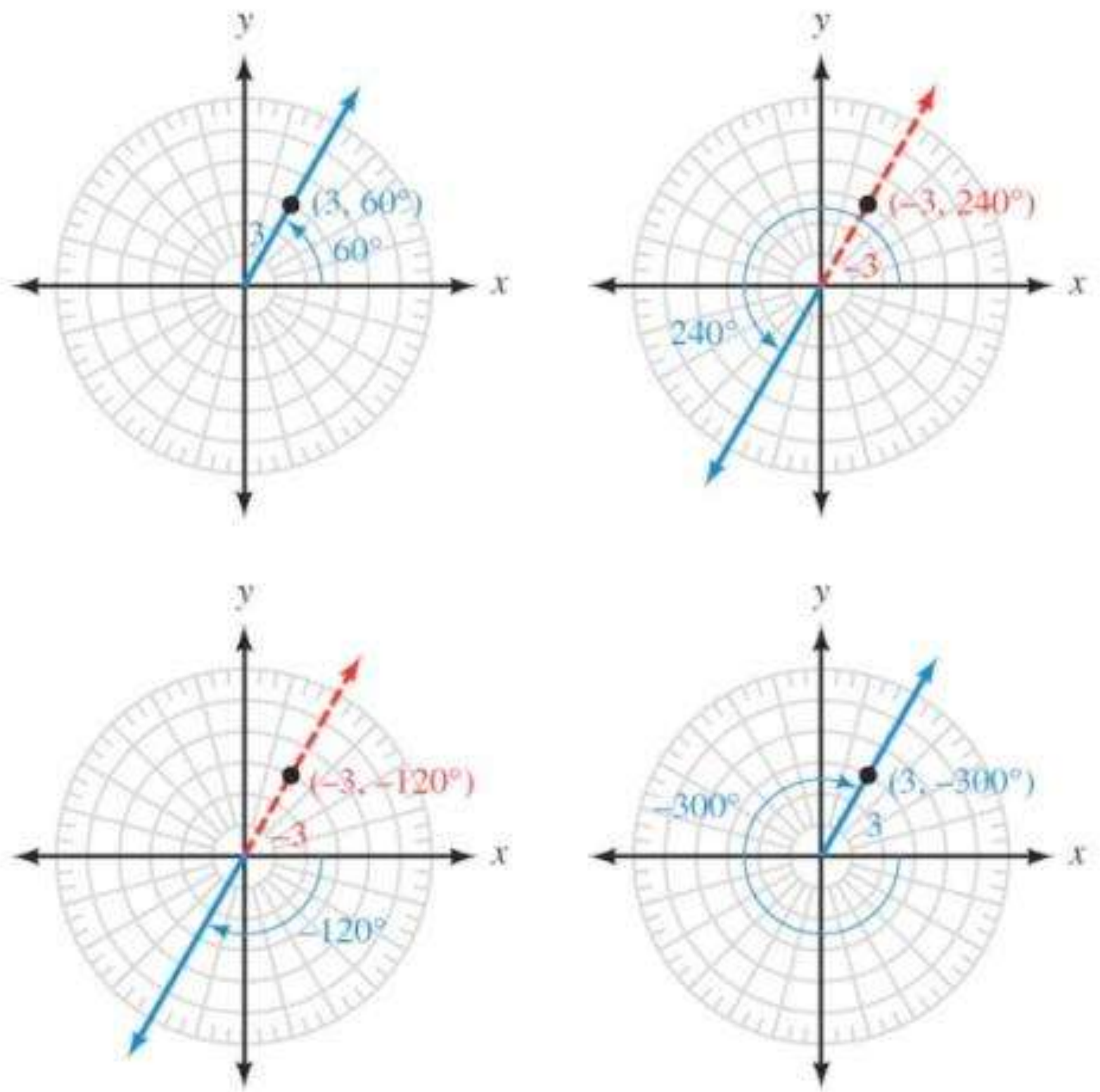






Example

Give three other order pairs that name the same point as $(3,\ 60°)$



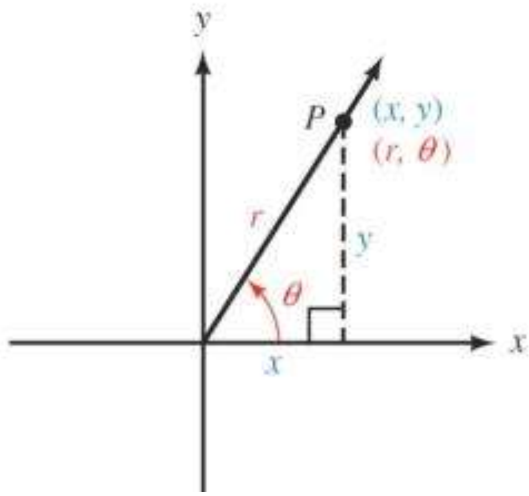
$(3,\ 60°),(-3,\ 240°),(-3,\ -120°),(3,\ -300°)$

Polar Coordinates and Rectangular Coordinates

To Convert Rectangular Coordinates to Polar Coordinates

Let $r = \pm\sqrt{x^2 + y^2}$ and $\tan\theta = \dfrac{y}{x}$

Where the sign of r and the choice of θ place the point $(r,\ \theta)$ in the same quadrant as (x, y)



To Convert Polar Coordinates to Rectangular Coordinates

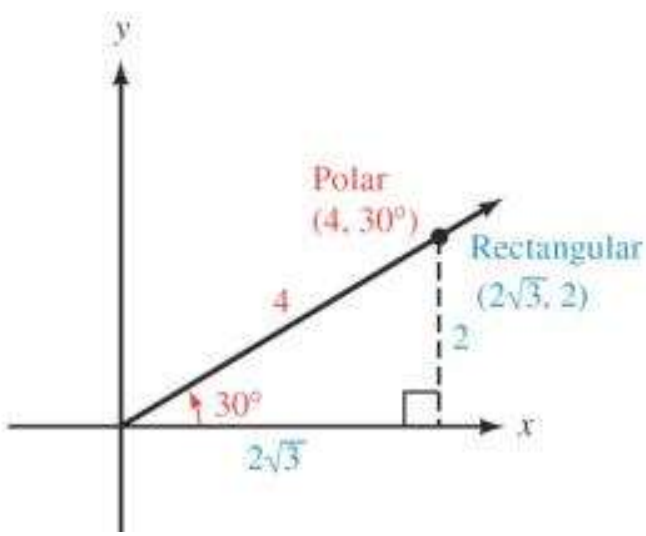
Let $x = r\cos\theta$ *and* $y = r\sin\theta$

Example

Convert to rectangular coordinates. $(4,\ 30°)$

<u>Solution</u>

$$\begin{aligned} x &= r\cos\theta \\ &= 4\cos 30° \\ &= 4\left(\frac{\sqrt{3}}{2}\right) \\ &= 2\sqrt{3} \end{aligned}$$



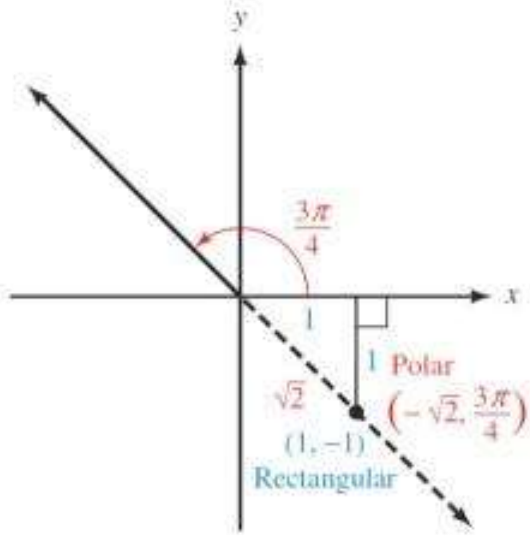
$$\begin{aligned} y &= r\sin\theta \\ &= 4\sin 30° \\ &= 4\left(\frac{1}{2}\right) \\ &= 2 \end{aligned}$$

The point $(2\sqrt{3},\ 2)$ in rectangular coordinates is equivalent to $(4,\ 30°)$ in polar coordinates.

Example

Convert to rectangular coordinates $\left(-\sqrt{2},\ \frac{3\pi}{4}\right)$.

<u>*Solution*</u>



$$x = -\sqrt{2}\cos\frac{3\pi}{4}$$

$$= -\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= 1$$

$$y = -\sqrt{2}\sin\frac{3\pi}{4}$$

$$= -\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$

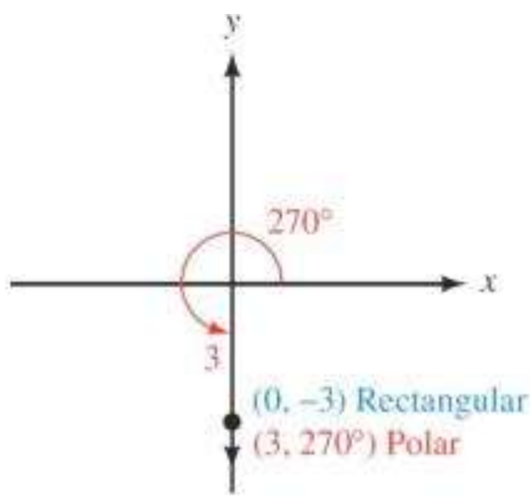
$$= -1$$

The point $(1,\ -1)$ in rectangular coordinates is equivalent to $\left(-\sqrt{2},\ \frac{3\pi}{4}\right)$ in polar coordinates.

Example

Convert to rectangular coordinates $(3,\ 270°)$.

<u>*Solution*</u>



$$x = 3\cos 270°$$

$$= 3(0)$$

$$= 0$$

$$y = 3\sin 270°$$

$$= 3(-1)$$

$$= -3$$

The point $(0,\ -3)$ in rectangular coordinates is equivalent to $(3,\ 270°)$ in polar coordinates.

Example

Convert to polar coordinates $(3,\ 3)$.

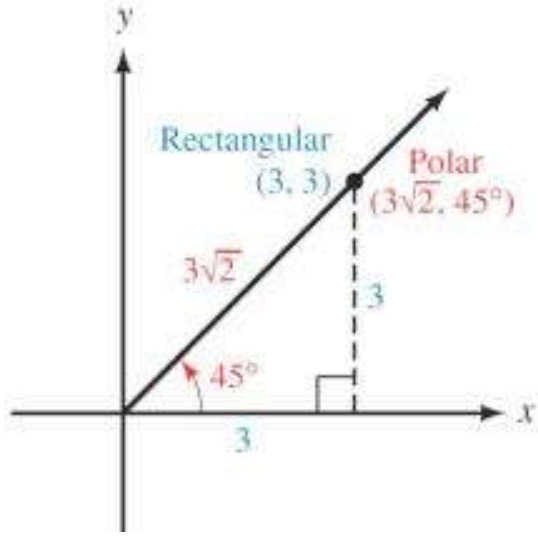
<u>*Solution*</u>

$$r = \pm\sqrt{3^2 + 3^2}$$
$$= \pm\sqrt{9+9}$$
$$= \pm 3\sqrt{2}$$

$$\tan\theta = \frac{3}{3} = 1$$

$$\theta = \tan^{-1} 1$$
$$= 45°$$

The point $\left(3\sqrt{2}, 45°\right)$ is just one.



Example

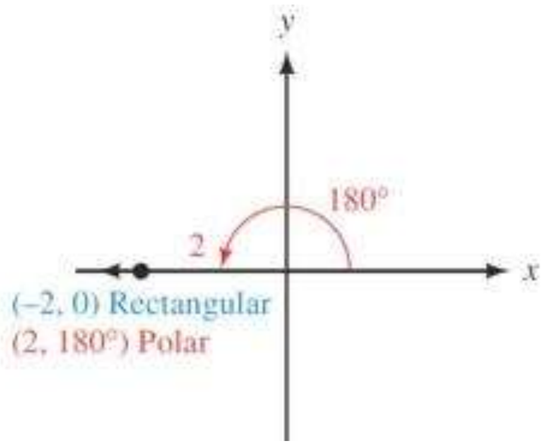
Convert to polar coordinates $(-2,\ 0)$.

<u>*Solution*</u>

$$r = \pm\sqrt{4+0}$$
$$= \pm\ 2$$

$$\theta = \tan^{-1}\frac{0}{-2}$$
$$= 0°$$

The point $r = 2, \quad \theta = 180°$



Example

Convert to polar coordinates $(-1,\ \sqrt{3})$.

<u>*Solution*</u>

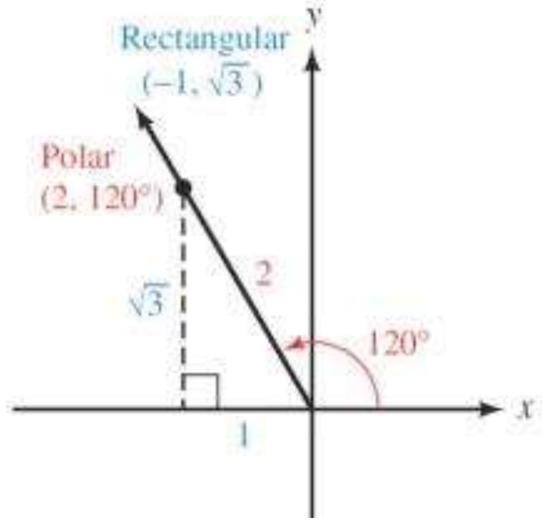
$$r = \pm\sqrt{1+3}$$

$$= \pm\ 2$$

$$\theta = \tan^{-1}\frac{\sqrt{3}}{-1}$$

$$= 120°$$

The point $r = 2,\quad \theta = 120°$



Example

Write the equation in rectangular coordinates $\quad r^2 = 4\sin 2\theta$

<u>*Solution*</u>

$r^2 = 4\sin 2\theta$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$= 4(2\sin\theta\cos\theta)$ $\qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$

$= 8\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$

$= 8\frac{xy}{r^2}$

$r^4 = 8xy$ $\qquad r^2 = x^2 + y^2$

$\left(x^2 + y^2\right)^2 = 8xy$

Example

Write the equation in polar coordinates $\quad x + y = 4$

<u>*Solution*</u>

$r\cos\theta + r\sin\theta = 4$ $\qquad x = r\cos\theta \quad y = r\sin\theta$

$r(\cos\theta + \sin\theta) = 4$

$$r = \frac{4}{\cos\theta + \sin\theta}$$

Exercises *Section* 4.5 – Polar Coordinates

1. Convert to rectangular coordinates $(2,\ 60^\circ)$

2. Convert to rectangular coordinates $\left(\sqrt{2},\ -225^\circ\right)$

3. Convert to rectangular coordinates $\left(4\sqrt{3},\ -\frac{\pi}{6}\right)$

4. Convert to polar coordinates $(-3,\ -3)$ $r \ge 0$ $0^\circ \le \theta < 360^\circ$

5. Convert to polar coordinates $\left(2,\ -2\sqrt{3}\right)$ $r \ge 0$ $0^\circ \le \theta < 360^\circ$

6. Convert to polar coordinates $(-2,\ 0)$ $r \ge 0$ $0 \le \theta < 2\pi$

7. Convert to polar coordinates $\left(-1,\ -\sqrt{3}\right)$ $r \ge 0$ $0 \le \theta < 2\pi$

8. Write the equation in rectangular coordinates $r^2 = 4$

9. Write the equation in rectangular coordinates $r = 6\cos\theta$

10. Write the equation in rectangular coordinates $r^2 = 4\cos 2\theta$

11. Write the equation in rectangular coordinates $r(\cos\theta - \sin\theta) = 2$

12. Write the equation in polar coordinates $x + y = 5$

13. Write the equation in polar coordinates $x^2 + y^2 = 9$

14. Write the equation in polar coordinates $x^2 + y^2 = 4x$

15. Write the equation in polar coordinates $y = -x$