

## ***Solution***      **Section 5.6 – Arithmetic and Geometric Sequences**

### ***Exercise***

Show that the sequence  $-6, -2, 2, \dots, 4n-10, \dots$  is arithmetic, and find the common difference.

### **Solution**

We to show that  $a_{k+1} - a_k$  equals to a constant.

$$\begin{aligned}a_{k+1} - a_k &= 4(k+1) - 10 - (4k - 10) \\&= 4k + 4 - 10 - 4k + 10 \\&= 4\end{aligned}$$

### ***Exercise***

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $2, 6, 10, 14, \dots$

### **Solution**

$$\begin{aligned}d &= 6 - 2 \\&= 4\end{aligned}$$

$$\begin{aligned}a_n &= 2 + (n-1)4 \\&= 2 + 4n - 4 \\&= 4n - 2\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 4(10) - 2 \\&= 38\end{aligned}$$

### ***Exercise***

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $3, 2.7, 2.4, 2.1, \dots$

### **Solution**

$$\begin{aligned}d &= 2.7 - 3 = -0.3 \\&= -0.3\end{aligned}$$

$$\begin{aligned}a_n &= 3 + (n-1)(-0.3) \\&= 3 - 0.3n + 0.3 \\&= 3.3 - 0.3n\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 3.3 - 0.3(10) \\&= 0.3\end{aligned}$$

### Exercise

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $-6, -4.5, -3, -1.5, \dots$

#### Solution

$$\begin{aligned}d &= -4.5 - (-6) \\ &= 1.5\end{aligned}$$

$$\begin{aligned}a_n &= -6 + (n-1)(1.5) \\ &= -6 + 1.5n - 1.5 \\ &= 1.5n - 7.5\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 1.5(10) - 7.5 \\ &= 7.5\end{aligned}$$

### Exercise

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $\ln 3, \ln 9, \ln 27, \ln 81, \dots$

#### Solution

$$\begin{aligned}\ln 3, \ln 3^2, \ln 3^3, \ln 3^4, \dots \\ \ln 3, 2\ln 3, 3\ln 3, 4\ln 3, \dots\end{aligned}$$

$$\begin{aligned}d &= 2\ln 3 - \ln 3 \\ &= \ln 3\end{aligned}$$

$$\begin{aligned}a_n &= \ln 3 + (n-1)\ln 3 \\ &= \ln 3 + n\ln 3 - \ln 3 \\ &= n\ln 3\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 10\ln 3 \\ &= \ln 3^{10}\end{aligned}$$

### Exercise

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = 2, \quad d = 3$

#### Solution

$$\begin{aligned}a_n &= 2 + 3(n-1) \\ &= 2 + 3n - 3 \\ &= 3n - 1\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$a_{10} = 29$$

### ***Exercise***

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = 5$ ,  $d = -3$

#### **Solution**

$$\begin{aligned}a_n &= 5 + (n-1)(-3) \\&= 5 - 3n + 3 \\&= \underline{8 - 3n} \quad | \end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 8 - 30 \\&= \underline{-22} \quad | \end{aligned}$$

### ***Exercise***

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = 1$ ,  $d = -\frac{1}{2}$

#### **Solution**

$$\begin{aligned}a_n &= 1 + (n-1)\left(-\frac{1}{2}\right) \\&= 1 - \frac{1}{2}n + \frac{1}{2} \\&= \underline{\frac{3}{2} - \frac{1}{2}n} \quad | \end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= \frac{3}{2} - \frac{1}{2}(10) \\&= \underline{-\frac{7}{2}} \quad | \end{aligned}$$

### ***Exercise***

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = -2$ ,  $d = 4$

#### **Solution**

$$\begin{aligned}a_n &= -2 + (n-1)(4) \\&= -2 + 4n - 4 \\&= \underline{4n - 6} \quad | \end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 4(10) - 6 \\&= \underline{36} \quad | \end{aligned}$$

### Exercise

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = \sqrt{2}$ ,  $d = \sqrt{2}$

#### Solution

$$\begin{aligned}a_n &= \sqrt{2} + (n-1)\sqrt{2} \\&= \sqrt{2} + \sqrt{2}n - \sqrt{2} \\&= \sqrt{2}n\end{aligned}$$

$$a_{10} = 10\sqrt{2}$$

$$a_n = a_1 + (n-1)d$$

### Exercise

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = 0$ ,  $d = \pi$

#### Solution

$$\begin{aligned}a_n &= 0 + (n-1)(\pi) \\&= \pi n - \pi\end{aligned}$$

$$\begin{aligned}a_{10} &= 10\pi - \pi \\&= 9\pi\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

### Exercise

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = 13$ ,  $d = 4$

#### Solution

$$\begin{aligned}a_n &= 13 + (n-1)(4) \\&= 4n + 9\end{aligned}$$

$$\begin{aligned}a_{10} &= 4(10) + 9 \\&= 49\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

### Exercise

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = -40$ ,  $d = 5$

#### Solution

$$\begin{aligned}a_n &= -40 + (n-1)(5) \\&= 5n - 45\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$a_{10} = 4(10) - 45$$

$$\underline{= -5}$$

### Exercise

Find the  $n$ th term, and the tenth term of the arithmetic sequence:  $a_1 = -32$ ,  $d = 4$

#### Solution

$$a_n = -32 + (n-1)(4)$$

$$\underline{= 4n - 36}$$

$$a_n = a_1 + (n-1)d$$

$$a_{10} = 4(10) - 36$$

$$\underline{= 4}$$

### Exercise

Find the common difference for the arithmetic sequence with the specified terms:  $a_4 = 14$ ,  $a_{11} = 35$

#### Solution

$$a_n = a_1 + (n-1)d$$

$$a_{11} = a_1 + 10d \rightarrow 35 = a_1 + 10d$$

$$a_4 = a_1 + 3d \rightarrow \frac{14 = a_1 + 3d}{21 = 7d}$$

$$\underline{d = 3}$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_{12}$ ;  $a_1 = 9.1$ ,  $a_2 = 7.5$

#### Solution

$$d = a_2 - a_1$$

$$= 7.5 - 9.1$$

$$\underline{= -1.6}$$

$$a_n = a_1 + (n-1)d$$

$$a_{12} = 9.1 + (11)(-1.6)$$

$$\underline{= -8.5}$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_1$ ;  $a_8 = 47$ ,  $a_9 = 53$

#### Solution

$$\begin{aligned}d &= a_9 - a_8 \\&= 53 - 47 \\&= 6\end{aligned}$$

$$a_8 = a_1 + (7)(6)$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_1 &= 47 - 42 \\&= 5\end{aligned}$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_{10}$ ;  $a_2 = 1$ ,  $a_{18} = 49$

#### Solution

$$\begin{aligned}a_2 &= a_1 + d \\a_1 &= a_2 - d \\a_{18} &= a_1 + (17)d \\&= a_2 - d + 17d \\&= a_2 + 16d\end{aligned}$$

$$49 = 1 + 16d$$

$$16d = 48$$

$$d = \frac{48}{16} = 3$$

$$\begin{aligned}a_1 &= a_2 - d \\&= 1 - 3 \\&= -2\end{aligned}$$

$$\begin{aligned}a_{10} &= -2 + 9(3) \\&= 25\end{aligned}$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_{10}$ ;  $a_8 = 8$ ,  $a_{20} = 44$

#### Solution

$$\begin{aligned}
 d &= \frac{44-8}{20-8} \\
 &= \frac{36}{12} \\
 &= 3
 \end{aligned}$$

$$a_8 = a_1 + (8-1)(3)$$

$$8 = a_1 + 21$$

$$a_1 = -13$$

$$\begin{aligned}
 a_{10} &= -13 + 9(3) \\
 &= 14
 \end{aligned}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_n = a_1 + (n-1)d$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_{12}$ ;  $a_8 = 4$ ,  $a_{18} = -96$

### Solution

$$\begin{aligned}
 d &= \frac{-96-4}{18-8} \\
 &= \frac{-100}{10} \\
 &= -10
 \end{aligned}$$

$$a_8 = a_1 + (8-1)(-10)$$

$$4 = a_1 - 70$$

$$a_1 = 74$$

$$\begin{aligned}
 a_{12} &= 74 + (11)(-10) \\
 &= -36
 \end{aligned}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_n = a_1 + (n-1)d$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_8$ ;  $a_{15} = 0$ ,  $a_{40} = -50$

### Solution

$$\begin{aligned}
 d &= \frac{-50-0}{40-15} \\
 &= \frac{-50}{25} \\
 &= -2
 \end{aligned}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_{15} = a_1 + (15-1)(-2)$$

$$0 = a_1 - 28$$

$$\underline{a_1 = 28}$$

$$a_8 = 28 + (7)(-2)$$

$$\underline{= 14}$$

$$a_n = a_1 + (n-1)d$$

### ***Exercise***

Find the specified term of the arithmetic sequence that has two given terms:  $a_{20}$ ;  $a_9 = -5$ ,  $a_{15} = 31$

### **Solution**

$$d = \frac{31+5}{15-9}$$

$$= \frac{36}{6}$$

$$\underline{= 6}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_9 = a_1 + (9-1)(6)$$

$$-5 = a_1 + 42$$

$$\underline{a_1 = -47}$$

$$a_{20} = -47 + (19)(6)$$

$$\underline{= 67}$$

$$a_n = a_1 + (n-1)d$$

### ***Exercise***

Find the specified term of the arithmetic sequence that has two given terms:  $a_n$ ;  $a_8 = 8$ ,  $a_{20} = 44$

### **Solution**

$$d = \frac{44-8}{20-8}$$

$$= \frac{36}{12}$$

$$\underline{= 3}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_8 = a_1 + 3(8-1)$$

$$8 = a_1 + 21$$

$$\underline{a_1 = -13}$$

$$a_n = a_1 + (n-1)d$$



$$a_n = -13 + 3(n-1)$$

$$\underline{= 3n - 16}$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_n$ ;  $a_8 = 4$ ,  $a_{18} = -96$

### Solution

$$d = \frac{-96 - 4}{18 - 8}$$

$$\underline{= -10}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_8 = a_1 - 10(8 - 1)$$

$$a_n = a_1 + (n - 1)d$$

$$4 = a_1 - 70$$

$$\underline{a_1 = 74}$$

$$a_n = 74 - 10(n - 1)$$

$$\underline{= -10n + 84}$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_n$ ;  $a_{14} = -1$ ,  $a_{15} = 31$

### Solution

$$d = \frac{31 - (-1)}{15 - 14}$$

$$\underline{= 32}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_{14} = a_1 + 32(14 - 1)$$

$$a_n = a_1 + (n - 1)d$$

$$-1 = a_1 + 416$$

$$\underline{a_1 = -417}$$

$$a_n = -417 + 32(n - 1)$$

$$\underline{= 32n - 449}$$

### Exercise

Find the specified term of the arithmetic sequence that has two given terms:  $a_n$ ;  $a_9 = -5$ ,  $a_{15} = 31$

#### Solution

$$\begin{aligned}d &= \frac{31+5}{15-9} \\&= 6\end{aligned}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_9 = a_1 + 6(9-1)$$

$$a_n = a_1 + (n-1)d$$

$$-5 = a_1 + 48$$

$$\underline{a_1 = -53}$$

$$a_n = -53 + 6(n-1)$$

$$\underline{= 6n - 59}$$

### Exercise

Find the sum  $S_n$  of the arithmetic sequence that satisfies the conditions:  $a_1 = 40$ ,  $d = -3$ ,  $n = 30$

#### Solution

$$\begin{aligned}S_n &= \frac{30}{2} [2(40) + (30-1)(-3)] \\&= -105\end{aligned}$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

### Exercise

Find the sum  $S_n$  of the arithmetic sequence that satisfies the conditions:  $a_7 = \frac{7}{3}$ ,  $d = -\frac{2}{3}$ ,  $n = 15$

#### Solution

$$a_7 = a_1 + (6)\left(-\frac{2}{3}\right) = \frac{7}{3}$$

$$\frac{7}{3} = a_1 - 4$$

$$a_1 = \frac{7}{3} + 4$$

$$\underline{= \frac{19}{3}}$$

$$\begin{aligned}S_n &= \frac{15}{2} \left[ 2\left(\frac{19}{3}\right) + (15-1)\left(-\frac{2}{3}\right) \right] \\&= 25\end{aligned}$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

### Exercise

Find the number of integers between 32 and 390 that are divisible by 6, find their sum

#### Solution

$$\text{Number of terms: } n = \frac{390-36}{6} + 1 = 60$$

$$S_n = \frac{60}{2}(36+390) \\ = 12780$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find the number of terms in the arithmetic sequence with the given conditions:

$$a_1 = -2, \quad d = \frac{1}{4}, \quad S = 21$$

#### Solution

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$21 = \frac{n}{2}[2(-2) + (n-1)\frac{1}{4}]$$

$$21 = -2n + \frac{1}{8}n(n-1)$$

$$(8)21 = -2n(8) + \frac{1}{8}n(n-1)(8)$$

$$168 = -16n + (n^2 - n)$$

$$0 = n^2 - 17n - 168$$

$$n = 24 \quad n = -7$$

### Exercise

Express the sum in terms of summation notation and find the sum  $2 + 11 + 20 + \dots + 16,058$ .

#### Solution

Difference in terms:

$$d = 11 - 2 = 9$$

Number of terms:

$$n = \frac{16058-2}{9} + 1 = 1785$$

$$a_n = 2 + (n-1)(9)$$

$$= 2 + 9n - 9$$

$$= 9n - 7$$

$$a_n = a_1 + (n-1)d$$

Hence the  $n$ th term is:  $\sum_{n=1}^{1785} (9n - 7)$

$$S_{1785} = \frac{1789}{2}(2 + 16058) \\ = \underline{14,333,550}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Express the sum in terms of summation notation and find the sum  $60 + 64 + 68 + 72 + \cdots + 120$ .

### Solution

Difference in terms:

$$d = 64 - 60 = 4$$

Number of terms:

$$n = \frac{120 - 60}{4} + 1 = \underline{16}$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$a_n = 60 + (n - 1)(4) \\ = \underline{4n - 54}$$

Hence the  $n$ th term is:  $\sum_{n=1}^{16} (4n - 54)$

$$S = \frac{16}{2}(60 + 120) \\ = \underline{1440}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $1 + 3 + 5 + \cdots + (2n - 1)$

### Solution

Difference in terms:

$$d = 3 - 1 = 2$$

Number of terms:

$$n = \frac{(2n - 1) - 1}{2} + 1 \\ = \frac{2n - 2}{2} + 1 \\ = n - 1 + 1 \\ = \underline{n}$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$\begin{aligned}
 S &= \frac{n}{2}(1 + (2n - 1)) \\
 &= \frac{n}{2}(2n) \\
 &= \underline{n^2}
 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $2 + 4 + 6 + \cdots + 2n$

### Solution

Difference in terms:  $d = 4 - 2 = 2$

Number of terms:

$$\begin{aligned}
 n &= \frac{2n - 2}{2} + 1 \\
 &= n - 1 + 1 \\
 &= \underline{n}
 \end{aligned}$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$\begin{aligned}
 S &= \frac{n}{2}(2 + 2n) \\
 &= n(n + 1) \\
 &= \underline{n^2 + n}
 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $2 + 5 + 8 + \cdots + 41$

### Solution

Difference in terms:

$$d = 5 - 2 = 3$$

Number of terms:

$$\begin{aligned}
 n &= \frac{41 - 2}{3} + 1 \\
 &= \underline{14}
 \end{aligned}$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$\begin{aligned}
 S &= \frac{14}{2}(2 + 41) \\
 &= \underline{301}
 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $7 + 12 + 17 + \cdots + (2 + 5n)$

#### Solution

Difference in terms:

$$d = 12 - 7 = 5$$

Number of terms:

$$n = \frac{2 + 5n - 7}{5} + 1$$

$$= \frac{5n - 5}{5} + 1$$

$$= \frac{5n}{5} - \frac{5}{5} + 1$$

$$= n$$

$$S = \frac{n}{2}(7 + 2 + 5n)$$

$$= \frac{n}{2}(9 + 5n)$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $73 + 78 + 83 + 88 + \cdots + 558$

#### Solution

Difference in terms:

$$d = 78 - 73 = 5$$

Number of terms:

$$n = \frac{558 - 73}{5} + 1$$

$$= 98$$

$$S = \frac{98}{2}(73 + 558)$$

$$= 30,919$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $7 + 1 - 5 - 11 - \cdots - 299$

#### Solution

Difference in terms:

$$d = 1 - 7 = -6$$

Number of terms:

$$n = \frac{-299 - 7}{-6} + 1$$
$$= 52$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$S = \frac{52}{2}(7 - 299)$$
$$= -7592$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $-1 + 2 + 7 + \cdots + (4n - 5)$

#### Solution

$$S = \frac{n}{2}(-1 + 4n - 5)$$
$$= n(2n - 3)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $5 + 9 + 13 + \cdots + 49$

#### Solution

Difference in terms:  $d = 9 - 5 = 4$

Number of terms:  $n = \frac{49 - 5 + 4}{4} = 12$

$$S = \frac{12}{2}(5 + 49)$$
$$= 324$$

$$n = \frac{a_n - a_1 + d}{d}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find each arithmetic sum  $2 + 4 + 6 + \cdots + 70$

#### Solution

Difference in terms:  $d = 4 - 2 = 2$

Number of terms:

$$n = \frac{70 - 2 + 2}{2}$$
$$= 35$$

$$n = \frac{a_n - a_1 + d}{d}$$

$$S = \frac{35}{2}(70 + 2)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

=1,260 |



### Exercise

Find each arithmetic sum  $1 + 3 + 5 + \cdots + 59$

#### Solution

Difference in terms:  $d = 3 - 1 = 2$

Number of terms:

$$n = \frac{59 - 1 + 2}{2} \qquad n = \frac{a_n - a_1 + d}{d}$$
$$= 30|$$

$$S = \frac{30}{2}(59 + 1) \qquad S_n = \frac{n}{2}(a_1 + a_n)$$
$$= 900|$$

### Exercise

Find each arithmetic sum  $4 + 4.5 + 5 + 5.5 + \cdots + 100$

#### Solution

Difference in terms:  $d = 4.5 - 4 = 0.5$

Number of terms:

$$n = \frac{100 - 4 + 0.5}{0.5} \qquad n = \frac{a_n - a_1 + d}{d}$$
$$= 193|$$

$$S = \frac{193}{2}(4 + 100) \qquad S_n = \frac{n}{2}(a_1 + a_n)$$
$$= 10,036|$$

### Exercise

Find each arithmetic sum  $8 + 8\frac{1}{4} + 8\frac{1}{2} + 8\frac{3}{4} + 9 + \cdots + 50$

#### Solution

Difference in terms:  $d = 8\frac{1}{4} - 8 = \frac{1}{4}$

Number of terms:

$$n = \frac{50 - 8 + 0.25}{0.25} \qquad n = \frac{a_n - a_1 + d}{d}$$
$$= 169|$$

$$S = \frac{169}{2}(8 + 50) \qquad S_n = \frac{n}{2}(a_1 + a_n)$$
$$= 4,901|$$

### Exercise

Show that the given sequence is geometric, and find the common ratio

$$5, -\frac{5}{4}, \frac{5}{16}, \dots, 5\left(-\frac{1}{4}\right)^{n-1}, \dots$$

### Solution

To be geometric, we must show that  $\frac{a_{k+1}}{a_k} = r$  is equal to some constant, which is the common ratio.

The common ratio:

$$\begin{aligned} r &= \frac{a_2}{a_1} & r &= \frac{a_{k+1}}{a_k} \\ &= \frac{-\frac{5}{4}}{5} \\ &= -\frac{1}{4} \end{aligned}$$

### Exercise

Find the *n*th term, the *fifth* term, and the *eighth* term of the geometric sequence 8, 4, 2, 1, ...

### Solution

$$\text{Given: } a_1 = 8, \quad r = \frac{4}{8} = \frac{1}{2}$$

$$\begin{aligned} a_n &= a_1 r^{n-1} = 8\left(\frac{1}{2}\right)^{n-1} \\ &= 2^3 \left(2^{-1}\right)^{n-1} \\ &= 2^3 2^{-n+1} \\ &= 2^{4-n} \end{aligned}$$

$$\begin{aligned} a_5 &= 2^{4-5} \\ &= 2^{-1} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} a_8 &= 2^{4-8} \\ &= 2^{-4} \\ &= \frac{1}{16} \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  
300, -30, 3, -0.3, ...

### Solution

$$\text{Given: } a_1 = 300, \quad r = \frac{-30}{300} = -0.1$$

$$\begin{aligned} a_n &= a_1 r^{n-1} = 300(-0.1)^{n-1} \\ &= 3(10^2)(-10^{-1})^{n-1} = 3(10)^2(-10)^{-n+1} = 3(-10)^{-n+3} \end{aligned}$$

$$\begin{aligned} a_5 &= 300(-0.1)^{5-1} \\ &= 300(-10^{-1})^4 \\ &= \underline{0.03} \end{aligned}$$

$$\begin{aligned} a_8 &= 3(-10)^{-8+3} \\ &= 3(-10)^{-5} \\ &= \underline{-0.00003} \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence 1,  $-\sqrt{3}$ , 3,  $-3\sqrt{3}$ , ...

### Solution

$$\text{Given: } a_1 = 1, \quad r = \frac{a_2}{a_1} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

$$\begin{aligned} a_n &= 1(-\sqrt{3})^{n-1} & a_n &= a_1 r^{n-1} \\ &= a_1 r^{n-1} \end{aligned}$$

$$\begin{aligned} a_5 &= 1(-\sqrt{3})^{5-1} \\ &= \underline{9} \end{aligned}$$

$$\begin{aligned} a_8 &= 1(-\sqrt{3})^{8-1} \\ &= (-\sqrt{3})^7 \\ &= \underline{-27\sqrt{3}} \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence 4, -6, 9, -13.5, ...

### Solution

$$\text{Given: } a_1 = 4, \quad r = \frac{a_2}{a_1} = \frac{-6}{4} = -\frac{3}{2}$$

$$a_n = 4\left(-\frac{3}{2}\right)^{n-1} \qquad a_n = a_1 r^{n-1}$$

$$\begin{aligned} a_5 &= 4\left(-\frac{3}{2}\right)^{5-1} \\ &= 4\left(-\frac{3}{2}\right)^4 \\ &= 4\left(\frac{3^4}{2^4}\right) \\ &= \frac{81}{4} \end{aligned}$$

$$\begin{aligned} a_8 &= 4\left(-\frac{3}{2}\right)^7 \\ &= -4\left(\frac{3^7}{2^7}\right) \\ &= -\frac{2187}{32} \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence 1,  $-x^2$ ,  $x^4$ ,  $-x^6$ , ...

### Solution

$$\text{Given: } a_1 = 1, \quad r = \frac{a_2}{a_1} = \frac{-x^2}{1} = -x^2$$

$$a_n = \left(-x^2\right)^{n-1} \qquad a_n = a_1 r^{n-1}$$

$$\begin{aligned} a_5 &= \left(-x^2\right)^4 \\ &= x^8 \end{aligned}$$

$$a_8 = \left(-x^2\right)^7$$

$$\underline{= -x^{14} \mid}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence

$$10, 10^{2x-1}, 10^{4x-3}, 10^{6x-5}, \dots$$

### Solution

**Given:**  $a_1 = 10$

$$r = \frac{a_2}{a_1} = \frac{10^{2x-1}}{10} = 10^{2x-1-1} = 10^{2x-2}$$

$$a_n = 10 \left( 10^{2x-2} \right)^{n-1} \qquad a_n = a_1 r^{n-1}$$

$$= 10 \left( 10^{(2x-2)(n-1)} \right)$$

$$= 10 \left( 10^{(2x-2)n-2x+2} \right)$$

$$= 10^{2nx-2n-2x+2+1}$$

$$\underline{= 10^{2(n-1)x-2n+3} \mid}$$

$$a_5 = 10^{2(5-1)x-2(5)+3}$$

$$\underline{= 10^{8x-7} \mid}$$

$$a_8 = 10^{2(8-1)x-2(8)+3}$$

$$\underline{= 10^{14x-13} \mid}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  $a_1 = 2, \quad r = 3$

### Solution

**Given:**  $a_1 = 2, \quad r = 3$

$$\underline{a_n = 2 \cdot 3^{n-1} \mid}$$

$$a_n = a_1 r^{n-1}$$

$$a_5 = 2 \cdot 3^4$$

$$\underline{= 162 \mid}$$

$$a_8 = 2 \cdot 3^7$$

$$\underline{= 4374 \mid}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  $a_1 = 1, \quad r = -\frac{1}{2}$

#### Solution

**Given:**  $a_1 = 1, \quad r = -\frac{1}{2}$

$$\underline{a_n = \left(-\frac{1}{2}\right)^{n-1}} \qquad a_n = a_1 r^{n-1}$$

$$\begin{aligned} a_5 &= \left(-\frac{1}{2}\right)^4 \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} a_8 &= \left(-\frac{1}{2}\right)^7 \\ &= -\frac{1}{128} \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  $a_1 = -2, \quad r = 4$

#### Solution

**Given:**  $a_1 = -2, \quad r = 4$

$$\underline{a_n = -2 \cdot (4)^{n-1}} \qquad a_n = a_1 r^{n-1}$$

$$\begin{aligned} a_5 &= \left(-\frac{1}{2}\right)^4 \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} a_8 &= \left(-\frac{1}{2}\right)^7 \\ &= -\frac{1}{128} \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  $a_1 = \sqrt{2}, \quad r = \sqrt{2}$

#### Solution

**Given:**  $a_1 = \sqrt{2}, \quad r = \sqrt{2}$

$$a_n = \sqrt{2}(\sqrt{2})^{n-1}$$

$$a_n = a_1 r^{n-1}$$

$$= (\sqrt{2})^n$$

$$a_5 = (\sqrt{2})^5$$

$$= 4\sqrt{2}$$

$$a_8 = (\sqrt{2})^8$$

$$= 16$$

### Exercise

Find the *n*th term, the *fifth* term, and the *eighth* term of the geometric sequence  $a_1 = 0, \quad r = \pi$

### Solution

**Given:**  $a_1 = 0, \quad r = \pi$

$$a_n = 0(\pi)^{n-1}$$

$$a_n = a_1 r^{n-1}$$

$$= 0$$

$$a_5 = 0^5$$

$$= 0$$

$$a_8 = 0^8$$

$$= 0$$

### Exercise

Find the *n*th term, the *fifth* term, and the *eighth* term of the geometric sequence  $\{s_n\} = \{3^n\}$

### Solution

$$a_n = 3^n$$

$$a_5 = 3^5$$

$$a_8 = 3^8$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  $\{s_n\} = \{(-5)^n\}$

### Solution

$$\underline{a_n = 3^n}$$

$$\begin{aligned} a_5 &= (-5)^5 \\ &= -5^5 \end{aligned}$$

$$\begin{aligned} a_8 &= (-5)^8 \\ &= 5^8 \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  $\{s_n\} = \left\{-3\left(\frac{1}{2}\right)^n\right\}$

### Solution

$$\underline{a_n = -3\left(\frac{1}{2}\right)^n}$$

$$\begin{aligned} a_5 &= -3\left(\frac{1}{2}\right)^5 \\ &= -\frac{3}{32} \end{aligned}$$

$$\begin{aligned} a_8 &= -3\left(\frac{1}{2}\right)^8 \\ &= -\frac{3}{256} \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  $\{u_n\} = \left\{\frac{3^{n-1}}{2^n}\right\}$

### Solution

$$\underline{a_n = \frac{3^{n-1}}{2^n}}$$

$$a_5 = \frac{3^4}{2^5}$$



$$\begin{aligned}
 &= \frac{81}{32} \\
 a_8 &= \frac{3^7}{2^8} \\
 &= \frac{3^7}{256}
 \end{aligned}$$

### Exercise

Find the  $n$ th term, the *fifth* term, and the *eighth* term of the geometric sequence  $\{u_n\} = \left\{ \frac{2^n}{3^{n-1}} \right\}$

### Solution

$$\begin{aligned}
 a_n &= \frac{2^n}{3^{n-1}} \\
 a_5 &= \frac{2^5}{3^4} \\
 &= \frac{32}{81} \\
 a_8 &= \frac{2^8}{3^7} \\
 &= \frac{256}{3^7}
 \end{aligned}$$

### Exercise

Find all possible values of  $r$  for a geometric sequence with the two given terms  $a_4 = 3$ ,  $a_6 = 9$

### Solution

$$\begin{aligned}
 \frac{a_6}{a_4} &= \frac{a_1 r^5}{a_1 r^3} \\
 \frac{9}{3} &= r^2 \\
 r^2 &= 3 \\
 r &= \pm\sqrt{3}
 \end{aligned}$$

### Exercise

Find the *sixth* term of the geometric sequence whose first two terms are 4 and 6

### Solution

$$\text{Given: } a_1 = 4, \quad a_2 = 6$$

$$r = \frac{a_2}{a_1}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

$$a_6 = a_1 r^{n-1}$$

$$= 4 \left( \frac{3}{2} \right)^5$$

$$= \frac{243}{8}$$

### Exercise

Given a geometric sequence with  $a_4 = 4$ ,  $a_7 = 12$ , find  $r$  and  $a_{10}$

### Solution

$$r = \left( \frac{12}{4} \right)^{1/(7-4)}$$

$$= 3^{1/3}$$

$$= \sqrt[3]{3}$$

$$r = \left( \frac{a_y}{a_x} \right)^{1/(y-x)}$$

$$a_1 = \frac{a_4}{r^3}$$

$$= \frac{4}{3}$$

$$a_{10} = \frac{4}{3} \left( \sqrt[3]{3} \right)^9$$

$$= 36$$

$$a_4 = a_1 r^{n-1}$$

$$a_{10} = a_1 r^{n-1}$$

**Exercise**

Find the specified term of the geometric sequence  $a_6$ ;  $a_1 = 4$ ,  $a_2 = 6$

**Solution**

$$r = \left(\frac{6}{4}\right)^{1/(2-1)}$$

$$= \frac{3}{2}$$

$$r = \left(\frac{a_y}{a_x}\right)^{1/(y-x)}$$

$$a_6 = 4\left(\frac{3}{2}\right)^5$$

$$= \frac{3^5}{8}$$

$$a_n = a_1 r^{n-1}$$

**Exercise**

Find the specified term of the geometric sequence  $a_7$ ;  $a_2 = 3$ ,  $a_3 = -\sqrt{3}$

**Solution**

$$r = \left(\frac{-\sqrt{3}}{3}\right)^{1/(3-2)}$$

$$= -\frac{\sqrt{3}}{3}$$

$$r = \left(\frac{a_y}{a_x}\right)^{1/(y-x)}$$

$$a_2 = a_1 \left(-\frac{\sqrt{3}}{3}\right)^1$$

$$a_n = a_1 r^{n-1}$$

$$3 = -\frac{\sqrt{3}}{3} a_1$$

$$a_1 = -\frac{9}{\sqrt{3}}$$

$$= -3\sqrt{3}$$

$$a_7 = -3\sqrt{3} \left(-\frac{\sqrt{3}}{3}\right)^6$$

$$= -3\sqrt{3} \frac{3^3}{3^6}$$

$$= -\frac{\sqrt{3}}{9}$$

### Exercise

Find the specified term of the geometric sequence  $a_6$ ;  $a_2 = 3$ ,  $a_3 = -\sqrt{2}$

### Solution

$$r = \left( \frac{-\sqrt{2}}{3} \right)^{1/(3-2)}$$
$$= -\frac{\sqrt{2}}{3}$$

$$r = \left( \frac{a_y}{a_x} \right)^{1/(y-x)}$$

$$a_2 = a_1 \left( -\frac{\sqrt{2}}{3} \right)^1$$

$$a_n = a_1 r^{n-1}$$

$$3 = -\frac{\sqrt{2}}{3} a_1$$

$$a_1 = -\frac{9}{\sqrt{2}}$$

$$a_6 = -\frac{9}{\sqrt{2}} \left( -\frac{\sqrt{2}}{3} \right)^5$$
$$= 9 \frac{\sqrt{2}^4}{3^5}$$
$$= \frac{4}{27}$$

### Exercise

Find the specified term of the geometric sequence  $a_5$ ;  $a_1 = 4$ ,  $a_2 = 7$

### Solution

$$r = \frac{7}{4}$$

$$r = \left( \frac{a_y}{a_x} \right)^{1/(y-x)}$$

$$a_5 = 4 \left( \frac{7}{4} \right)^4$$
$$= \frac{7^4}{64}$$

$$a_n = a_1 r^{n-1}$$

**Exercise**

Find the specified term of the geometric sequence  $a_9$ ;  $a_2 = 3$ ,  $a_5 = -81$

**Solution**

$$r = \left( \frac{-81}{3} \right)^{1/(5-2)}$$

$$= (-27)^{1/3}$$

$$= \underline{-3}$$

$$r = \left( \frac{a_y}{a_x} \right)^{1/(y-x)}$$

$$a_2 = a_1 (-3)^3$$

$$3 = -27a_1$$

$$a_1 = \underline{-\frac{1}{9}}$$

$$a_9 = -\frac{1}{9}(-3)^8$$

$$= \underline{-3^6}$$

$$a_n = a_1 r^{n-1}$$

**Exercise**

Find the specified term of the geometric sequence  $a_7$ ;  $a_1 = -4$ ,  $a_3 = -1$

**Solution**

$$r = \left( \frac{-1}{-4} \right)^{1/(3-1)}$$

$$= \left( \frac{1}{4} \right)^{1/2}$$

$$= \underline{\frac{1}{2}}$$

$$r = \left( \frac{a_y}{a_x} \right)^{1/(y-x)}$$

$$a_7 = -4 \left( \frac{1}{2} \right)^6$$

$$= \underline{-\frac{1}{16}}$$

$$a_n = a_1 r^{n-1}$$

### Exercise

Find the specified term of the geometric sequence  $a_8$ ;  $a_2 = 3$ ,  $a_4 = 6$

### Solution

$$\begin{aligned} r &= \left( \frac{-81}{3} \right)^{1/(5-2)} & r &= \left( \frac{a_y}{a_x} \right)^{1/(y-x)} \\ &= (-27)^{1/3} \\ &= \underline{-3} \end{aligned}$$

$$\begin{aligned} a_2 &= a_1 (-3)^3 & a_n &= a_1 r^{n-1} \\ 3 &= -81a_1 \\ a_1 &= \underline{-\frac{1}{9}} \end{aligned}$$

$$\begin{aligned} a_8 &= -\frac{1}{9}(-3)^7 \\ &= \underline{3^5} \end{aligned}$$

### Exercise

Express the sum in terms of summation notation:  $4 + 11 + 18 + 25 + 32$ . (Answers are not unique)

### Solution

$$\begin{aligned} n &= 5 \\ d &= 11 - 4 = 7 \\ a_n &= 4 + (n-1)7 & a_n &= a_1 + (n-1)d \\ &= 4 + 7n - 7 \\ &= \underline{7n - 3} \end{aligned}$$

$$4 + 11 + 18 + 25 + 32 = \sum_{n=1}^5 (7n - 3)$$

### Exercise

Express the sum in terms of summation notation:  $4 + 11 + 18 + \dots + 466$ . (Answers are not unique)

### Solution

Difference in terms:  $d = 11 - 4 = 7$

$$\text{Number of terms: } n = \frac{466-4}{7} + 1 = \underline{67}$$

$$\begin{aligned} a_n &= 4 + (n-1)7 \\ &= 4 + 7n - 7 \\ &= \underline{7n - 3} \end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$4 + 11 + 18 + \dots + 466 = \sum_{n=1}^{67} (7n - 3)$$

### Exercise

Express the sum in terms of summation notation (*Answers are not unique*)  $2 + 4 + 8 + 16 + 32 + 64 + 128$

#### Solution

$$\begin{aligned} 2 + 4 + 8 + 16 + 32 + 64 + 128 &= 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 \\ &= \sum_{n=1}^7 2^n \end{aligned}$$

### Exercise

Express the sum in terms of summation notation (*Answers are not unique*)  $2 - 4 + 8 - 16 + 32 - 64$

#### Solution

$$r = \frac{-4}{2} = -2$$

$$r = \frac{a_2}{a_1}$$

$$\begin{aligned} a_n &= 2(-2)^{n-1} \\ &= (-1)^{n-1} 2^n \end{aligned}$$

$$a_n = a_1 r^{n-1}$$

$$2 - 4 + 8 - 16 + 32 - 64 = \sum_{n=1}^6 (-1)^{n-1} 2^n$$

### Exercise

Express the sum in terms of summation notation (*Answers are not unique*)  $3 + 8 + 13 + 18 + 23$

#### Solution

$$\begin{aligned} d &= 8 - 3 \\ &= \underline{5} \end{aligned}$$

$$d = a_2 - a_1$$

$$a_n = 3 + 5(n-1)$$

$$a_n = a_1 + (n-1)d$$

$$= 5n - 2$$

$$3 + 8 + 13 + 18 + 23 = \sum_{n=1}^5 (5n - 2)$$

### Exercise

Express the sum in terms of summation notation (*Answers are not unique*)  $256 + 192 + 144 + 108 + \dots$

### Solution

$$r = \frac{192}{256}$$

$$= \frac{3}{4}$$

$$a_n = 256 \left( \frac{3}{4} \right)^{n-1}$$

$$r = \frac{a_2}{a_1}$$

$$a_n = a_1 r^{n-1}$$

$$256 + 192 + 144 + 108 + \dots = \sum_{n=1}^{\infty} 256 \left( \frac{3}{4} \right)^{n-1}$$

### Exercise

Express the sum in terms of summation notation (*Answers are not unique*):  $\frac{5}{13} + \frac{10}{11} + \frac{15}{9} + \frac{20}{7}$

### Solution

Number of terms:  $n = 4$

Numerators : 5, 10, 15, 20    common difference 5

Denominators : 13, 11, 9, 7    common difference -2

#### Numerator:

$$a_n = 5 + (n-1)5$$

$$= 5 + 5n - 5$$

$$= 5n$$

$$a_n = a_1 + (n-1)d$$

#### Denominator:

$$a_n = 13 + (n-1)(-2)$$

$$= 13 - 2n + 2$$

$$= 15 - 2n$$

Hence the  $n$ th term is:  $\frac{5}{13} + \frac{10}{11} + \frac{15}{9} + \frac{20}{7} = \sum_{n=1}^4 \frac{5n}{15-2n}$



### Exercise

Express the sum in terms of summation notation (Answers are not unique.)  $\frac{1}{4} - \frac{1}{12} + \frac{1}{36} - \frac{1}{108}$

### Solution

$$\begin{aligned}\frac{1}{4} - \frac{1}{12} + \frac{1}{36} - \frac{1}{108} &= \frac{1}{4} - \frac{1}{4} \frac{1}{3^1} + \frac{1}{4} \frac{1}{3^2} - \frac{1}{4} \frac{1}{3^3} \\ &= \sum_{n=1}^4 (-1)^{n+1} \frac{1}{4} \left(\frac{1}{3}\right)^{n-1}\end{aligned}$$

### Exercise

Express the sum in terms of summation notation (Answers are not unique.)  $3 + \frac{3}{5} + \frac{3}{25} + \frac{3}{125} + \frac{3}{625}$

### Solution

$$\begin{aligned}3 + \frac{3}{5} + \frac{3}{25} + \frac{3}{125} + \frac{3}{625} &= \frac{3}{5^0} + \frac{3}{5^1} + \frac{3}{5^2} + \frac{3}{5^3} + \frac{3}{5^4} \\ &= \sum_{n=0}^4 \frac{3}{5^n}\end{aligned}$$

### Exercise

Express the sum in terms of summation notation (*Answers are not unique*):  $\frac{3}{7} + \frac{6}{11} + \frac{9}{15} + \frac{12}{19} + \frac{15}{23} + \frac{18}{27}$

### Solution

Numerators : 3, 6, 9, 12, 15, 18    *common difference 3*

Denominators : 7, 11, 15, 19, 23, 27    *common difference 4*

**Numerator:**

$$\begin{aligned}a_n &= 3 + 3(n-1) & a_n &= a_1 + (n-1)d \\ &= \underline{3n}\end{aligned}$$

**Denominator:**

$$\begin{aligned}a_n &= 7 + 4(n-1) \\ &= \underline{4n+3}\end{aligned}$$

$$\frac{3}{7} + \frac{6}{11} + \frac{9}{15} + \frac{12}{19} + \frac{15}{23} + \frac{18}{27} = \sum_{n=1}^6 \frac{3n}{4n+3}$$

### Exercise

Express the sum in terms of summation notation (*Answers are not unique.*)  $\frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \dots, \quad |x| < 3$

### Solution

$$\begin{aligned}\frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \dots &= \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \dots \\ &= \sum_{n=1}^{\infty} \left(\frac{x}{3}\right)^n\end{aligned}$$

### Exercise

Express the sum in terms of summation notation (*Answers are not unique.*)  $2x + 4x^2 + 8x^3 + \dots, \quad |x| < \frac{1}{2}$

### Solution

$$\begin{aligned}2x + 4x^2 + 8x^3 + \dots &= 2x + (2x)^2 + (2x)^3 + \dots \\ &= \sum_{n=1}^{\infty} (2x)^n\end{aligned}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

### Solution

$$\begin{aligned}a_1 &= 1, \quad r = -\frac{1}{2} \\ S &= \frac{1}{1 + \frac{1}{2}} & S &= \frac{a_1}{1 - r} \\ &= \frac{1}{\frac{3}{2}} \\ &= \frac{2}{3}\end{aligned}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $1.5 + 0.015 + 0.00015 + \dots$

### Solution

$$a_1 = 0.015$$

$$a_2 = .00015$$

$$r = \frac{.00015}{.015}$$

$$= .01$$

$$S = 1.5 + \frac{a_1}{1-r}$$

$$= 1.5 + \frac{.015}{1-.01}$$

$$= \frac{15}{10} + \frac{.015}{.99}$$

$$= \frac{15}{10} + \frac{15}{990}$$

$$= \frac{15}{10} + \frac{15}{990}$$

$$= \frac{1500}{990}$$

$$= \frac{50}{33}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $\sqrt{2} - 2 + \sqrt{8} - 4 + \dots$

### Solution

$$r = \frac{-2}{\sqrt{2}}$$

$$= -\sqrt{2}$$

$$r = \frac{a_2}{a_1}$$

$$|r| = \sqrt{2} > 1 \Rightarrow \text{The sum *doesn't exist* .}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $256 + 192 + 144 + 108 + \dots$

### Solution

$$r = \frac{192}{256}$$

$$= \frac{3}{4}$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{256}{1 - \frac{3}{4}}$$

$$= 1024$$

$$S = \frac{a_1}{1-r}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $\frac{1}{4} + \frac{2}{4} + \frac{2^2}{4} + \frac{2^3}{4} + \dots + \frac{2^{n-1}}{4}$

### Solution

$$r = \frac{\frac{2}{4}}{\frac{1}{4}}$$

$$= 2$$

$$S_n = \frac{1}{4} \left( \frac{1-2^n}{1-2} \right)$$
$$= -\frac{1}{4} (1-2^n)$$

$$r = \frac{a_2}{a_1}$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $\frac{3}{9} + \frac{3^2}{9} + \frac{3^3}{9} + \dots + \frac{3^n}{9}$

### Solution

$$r = \frac{\frac{3^2}{9}}{\frac{3}{9}}$$

$$= 3$$

$$S_n = \frac{3}{9} \left( \frac{1-3^n}{1-3} \right)$$
$$= -\frac{1}{6} (1-3^n)$$

$$r = \frac{a_2}{a_1}$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $-1 - 2 - 4 - 8 - \dots - 2^{n-1}$

### Solution

$$r = \frac{-2}{-1}$$

$$= 2$$

$$S_n = -1 \left( \frac{1-2^n}{1-2} \right)$$
$$= 1-2^n$$

$$r = \frac{a_2}{a_1}$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $2 + \frac{6}{5} + \frac{18}{25} + \dots + 2\left(\frac{3}{5}\right)^{n-1}$

#### Solution

$$r = \frac{\frac{6}{5}}{2}$$

$$= \frac{3}{5} < 1$$

$$r = \frac{a_2}{a_1}$$

$$S_n = 2 \cdot \frac{1 - \left(\frac{3}{5}\right)^n}{1 - \frac{3}{5}}$$

$$= 2 \cdot \frac{1 - \left(\frac{3}{5}\right)^n}{\frac{2}{5}}$$

$$= \underline{5 \left(1 - \left(\frac{3}{5}\right)^n\right)}$$

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

#### Solution

$$r = \frac{1}{3} < 1$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{1}{1 - \frac{1}{3}}$$

$$S = \frac{a_1}{1 - r}$$

$$= \underline{\frac{3}{2}} \quad \text{The series *converges*}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \dots$

#### Solution

$$r = \frac{\frac{4}{3}}{2}$$

$$r = \frac{a_2}{a_1}$$

$$= \frac{2}{3} < 1$$

$$S = \frac{2}{1 - \frac{2}{3}}$$

$$S = \frac{a_1}{1 - r}$$

$$= 6 \quad \text{The series *converges*}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \dots$

### Solution

$$a_1 = 2$$

$$|r| = \left| -\frac{1}{4} \right| < 1$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{2}{1 + \frac{1}{4}}$$

$$S = \frac{a_1}{1 - r}$$

$$= \frac{8}{5} \quad \text{The series *converges*}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \dots$

### Solution

$$a_1 = 1$$

$$|r| = \left| -\frac{3}{4} \right| < 1$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{1}{1 + \frac{3}{4}}$$

$$S = \frac{a_1}{1 - r}$$

$$= \frac{4}{7} \quad \text{The series *converges*}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $9 + 12 + 16 + \frac{64}{3} + \dots$

#### Solution

$$a_1 = 9$$

$$|r| = \left| \frac{4}{3} \right| > 1 \quad \text{The series *diverges*}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $8 + 12 + 18 + 27 + \dots$

#### Solution

$$a_1 = 8$$

$$r = \frac{12}{8} = \frac{3}{2} > 1 \quad \text{The series *diverges*}$$

$$r = \frac{a_2}{a_1}$$

### Exercise

Find the sum of the infinite geometric series if it exists:  $6 + 2 + \frac{2}{3} + \frac{2}{9} + \dots$

#### Solution

$$a_1 = 6$$

$$|r| = \frac{2}{6}$$

$$= \frac{1}{3} < 1$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{6}{1 - \frac{1}{3}}$$

$$S = \frac{a_1}{1 - r}$$

$$= \frac{6}{\frac{2}{3}}$$

$$= 9 \quad \text{The series *converges*}$$

### Exercise

Find the sum:  $\sum_{k=1}^{20} (3k - 5)$

### Solution

$$a_1 = 3(1) - 5 = \underline{-2}$$

$$a_{20} = 3(20) - 5 = \underline{55}$$

$$\begin{aligned} \sum_{k=1}^{20} (3k - 5) &= \frac{20}{2}(-2 + 55) \\ &= \underline{530} \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find the sum:  $\sum_{k=1}^{18} \left(\frac{1}{2}k + 7\right)$

### Solution

$$a_1 = \frac{1}{2}(1) + 7 = \underline{\frac{15}{2}}$$

$$a_{18} = \frac{1}{2}(18) + 7 = \underline{16}$$

$$\begin{aligned} \sum_{k=1}^{18} \left(\frac{1}{2}k + 7\right) &= \frac{18}{2}\left(\frac{15}{2} + 16\right) \\ &= \underline{\frac{423}{2}} \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find the sum:  $\sum_{k=1}^{80} (2k - 5)$

### Solution

$$a_1 = 2(1) - 5 = \underline{-3}$$

$$a_{80} = 2(80) - 5 = \underline{155}$$



$$\begin{aligned}\sum_{k=1}^{80} (2k-5) &= \frac{80}{2}(-3+155) \\ &= 40(152) \\ &= \underline{6080}\end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### ***Exercise***

Find the sum:  $\sum_{n=1}^{90} (3-2n)$

### **Solution**

$$a_1 = 3 - 2(1) = \underline{1}$$

$$a_{90} = 3 - 2(90) = \underline{-177}$$

$$\begin{aligned}\sum_{n=1}^{90} (3-2n) &= \frac{90}{2}(1-177) \\ &= 45(-176) \\ &= \underline{-7920}\end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### ***Exercise***

Find the sum:  $\sum_{n=1}^{100} \left(6 - \frac{1}{2}n\right)$

### **Solution**

$$a_1 = 6 - \frac{1}{2}(1) = \underline{\frac{11}{2}}$$

$$a_{100} = 6 - \frac{1}{2}(100) = \underline{-44}$$

$$\begin{aligned}\sum_{n=1}^{100} \left(6 - \frac{1}{2}n\right) &= \frac{100}{2}\left(\frac{11}{2} - 44\right) \\ &= 50\left(-\frac{77}{2}\right) \\ &= \underline{-1925}\end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Exercise

Find the sum:  $\sum_{n=1}^{80} \left( \frac{1}{3}n + \frac{1}{2} \right)$

### Solution

$$a_1 = \frac{1}{3}(1) + \frac{1}{2} = \underline{\frac{5}{6}}$$

$$a_{80} = \frac{1}{3}(80) + \frac{1}{2} = \underline{\frac{163}{6}}$$

$$\begin{aligned} \sum_{n=1}^{80} \left( \frac{1}{3}n + \frac{1}{2} \right) &= \frac{80}{2} \left( \frac{5}{6} + \frac{163}{6} \right) \\ &= 40 \left( \frac{168}{6} \right) \\ &= \underline{1,120} \end{aligned}$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

### Exercise

Find the sum:  $\sum_{k=1}^{10} 3^k$

### Solution

$$\begin{aligned} \sum_{k=1}^{10} 3^k &= 3 \frac{1-3^{10}}{1-3} \\ &= 3 \frac{-59048}{-2} \\ &= \underline{88,572} \end{aligned}$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

### Exercise

Find the sum:  $\sum_{k=1}^9 (-\sqrt{5})^k$

### Solution

$$\begin{cases} a_1 = -\sqrt{5} \\ a_2 = (-\sqrt{5})^2 = 5 \end{cases}$$

$$r = \frac{5}{-\sqrt{5}}$$

$$r = \frac{a_2}{a_1}$$

$$= -\sqrt{5}$$

$$\sum_{k=1}^9 (-\sqrt{5})^k = (-\sqrt{5}) \frac{1 - (-\sqrt{5})^9}{1 - (-\sqrt{5})}$$

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

$$= \frac{(-\sqrt{5})(1 + 625\sqrt{5})}{1 + \sqrt{5}} \cdot \frac{1 - \sqrt{5}}{1 - \sqrt{5}}$$

$$= \frac{3124\sqrt{5} - 3120}{-4}$$

$$= 780 - 781\sqrt{5}$$

### Exercise

Find the sum:  $\sum_{k=0}^9 \left(-\frac{1}{2}\right)^{k+1}$

### Solution

$$\sum_{k=0}^9 \left(-\frac{1}{2}\right)^{k+1} = \left(-\frac{1}{2}\right) \frac{1 - \left(-\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}}$$

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

$$= -\frac{1}{2} \frac{1 - \frac{1}{2^{10}}}{\frac{3}{2}}$$

$$= -\frac{\frac{1024-1}{1024}}{3}$$

$$= -\frac{1023}{3072}$$

$$= -\frac{341}{1024}$$

### Exercise

Find the sum :  $\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1}$

### Solution

$$|r| = \frac{2}{3} < 1$$

$$\begin{aligned} \sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1} &= \frac{2}{1 - \frac{2}{3}} & S &= \frac{a_1}{1-r} \\ &= \frac{2}{\frac{1}{3}} \\ &= \underline{6}, \quad \text{the series } \textit{converges} \end{aligned}$$

### Exercise

Find the sum:  $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

### Solution

$$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \sum_{n=1}^{\infty} \frac{2}{3} \left(\frac{2}{3}\right)^{n-1}$$

$$|r| = \frac{2}{3} < 1$$

$$\begin{aligned} \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n &= \frac{2}{3} \frac{1}{1 - \frac{2}{3}} & S &= \frac{a_1}{1-r} \\ &= \frac{2}{3}(3) \\ &= \underline{2} \quad \text{The series } \textit{converges} \end{aligned}$$

### Exercise

Find the sum:  $\sum_{n=1}^{\infty} 3\left(\frac{3}{2}\right)^n$

### Solution

Since  $|r| = \frac{3}{2} > 1$ , the series *diverges*

### Exercise

Find the sum:  $\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1}$

### Solution

$$|r| = \frac{1}{4} < 1$$

$$a_1 = 5$$

$$\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1} = \frac{5}{1 - \frac{1}{4}} \\ = \underline{\underline{\frac{20}{3}}}$$

$$S = \frac{a_1}{1 - r}$$

The series *converges*

### Exercise

Find the sum:  $\sum_{n=1}^{\infty} 8\left(\frac{1}{3}\right)^{n-1}$

### Solution

$$|r| = \frac{1}{3} < 1$$

$$a_1 = 8$$

$$\sum_{n=1}^{\infty} 8\left(\frac{1}{3}\right)^{n-1} = \frac{8}{1 - \frac{1}{3}} \\ = \underline{\underline{12}}$$

$$S = \frac{a_1}{1 - r}$$

The series *converges*

### Exercise

Find the sum:  $\sum_{k=1}^{\infty} \frac{1}{2} \cdot 3^{k-1}$

### Solution

Since  $|r| = 3 > 1$ , the series *diverges*

### Exercise

Find the sum:  $\sum_{k=1}^{\infty} 6\left(-\frac{2}{3}\right)^{k-1}$

### Solution

$$|r| = \frac{2}{3} < 1$$

$$a_1 = 6$$

$$\sum_{k=1}^{\infty} 6\left(-\frac{2}{3}\right)^{k-1} = \frac{6}{1 + \frac{2}{3}} = \frac{18}{5}$$

$$S = \frac{a_1}{1-r}$$

The series *converges*

### Exercise

Find the sum:  $\sum_{k=1}^{\infty} 4\left(-\frac{1}{2}\right)^{k-1}$

### Solution

$$|r| = \frac{1}{2} < 1$$

$$a_1 = 4$$

$$\sum_{k=1}^{\infty} 4\left(-\frac{1}{2}\right)^{k-1} = \frac{4}{1 + \frac{1}{2}} = \frac{8}{3}$$

$$S = \frac{a_1}{1-r}$$

The series *converges*

### Exercise

Find the sum:  $\sum_{k=8}^{14} (3^{k-7} + 2j^2)$

### Solution

$$a_n = 3^{n-7} \rightarrow a_1 = 3^{-6}$$

$$r = 3$$

$$n = 14 - 8 + 1 = 7$$

$$\begin{aligned}
\sum_{k=8}^{14} (3^{k-7} + 2j^2) &= \sum_{k=8}^{14} 3^{k-7} + 2 \sum_{k=8}^{14} j^2 \\
&= 3^{-6} \cdot \frac{1-3^7}{1-3} + 2(7)j^2 \\
&= -\frac{1}{2} \left( \frac{1-3^7}{3^6} \right) + 14j^2 \\
&= -\frac{1}{2} \left( \frac{-2,186}{729} \right) + 14j^2 \\
&= \frac{1,093}{729} + 14j^2
\end{aligned}$$

### Exercise

Find the sum of the first 120 terms of: 14, 16, 18, 20, ...

### Solution

$$n = 120$$

$$a_1 = 14$$

$$d = 16 - 14 = 2$$

$$\begin{aligned}
S_{120} &= \frac{120}{2} [2(14) + 2(120-1)] \\
&= 60(48 + 238) \\
&= 17,160
\end{aligned}$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

### Exercise

Find the sum of the first 46 terms of 2, -1, -4, -7, ...

### Solution

$$n = 46$$

$$a_1 = 2$$

$$d = -1 - 2 = -3$$

$$\begin{aligned}
S_{46} &= \frac{46}{2} [2(2) - 3(46-1)] \\
&= 23(4 - 135) \\
&= -3,013
\end{aligned}$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

### Exercise

Find the rational number represented by the repeating decimal  $0.\overline{23}$

#### Solution

$$0.\overline{23} = 0.23 + 0.0023 + .000023 + \dots$$

$$a_1 = 0.23$$

$$r = \frac{.0023}{.23} = 0.01$$

$$S = \frac{0.23}{1 - 0.01}$$

$$= \frac{0.23}{0.99}$$

$$= \frac{23}{99}$$

$$S = \frac{a_1}{1-r}$$

### Exercise

Find the rational number represented by the repeating decimal  $0.0\overline{71}$

#### Solution

$$0.0\overline{71} = 0.071 + 0.00071 + .0000071 + \dots$$

$$a_1 = 0.071$$

$$r = \frac{.00071}{.071} = 0.01$$

$$S = \frac{0.071}{1 - 0.01}$$

$$= \frac{0.071}{0.990}$$

$$= \frac{71}{990}$$

$$S = \frac{a_1}{1-r}$$

### Exercise

Find the rational number represented by the repeating decimal  $2.4\overline{17}$

#### Solution

$$2.4\overline{17} = 2.4 + 0.017 + 0.00017 + .0000017 + \dots$$

$$a_1 = 0.017$$

$$r = \frac{.00017}{.017} = 0.01$$



$$\begin{aligned}
 S &= 2.4 + \frac{0.017}{1 - 0.01} \\
 &= \frac{24}{10} + \frac{0.017}{0.990} \\
 &= \frac{24}{10} + \frac{17}{990} \\
 &= \frac{240 + 17}{990} \\
 &= \frac{2,393}{990}
 \end{aligned}$$

$$S = \frac{a_1}{1-r}$$

### Exercise

Find the rational number represented by the repeating decimal  $10.\overline{5}$

#### Solution

$$10.\overline{5} = 10 + 0.5 + 0.05 + .005 + \dots$$

$$a_1 = 0.5$$

$$r = \frac{0.05}{0.5} = 0.1$$

$$\begin{aligned}
 S &= 10 + \frac{0.5}{1 - 0.1} \\
 &= 10 + \frac{0.5}{0.9} \\
 &= 10 + \frac{5}{9} \\
 &= \frac{95}{9}
 \end{aligned}$$

$$S = \frac{a_1}{1-r}$$

### Exercise

Find the rational number represented by the repeating decimal  $5.\overline{146}$

#### Solution

$$5.\overline{146} = 5 + 0.146 + 0.000146 + .000000146 + \dots$$

$$a_1 = 0.146$$

$$r = \frac{0.000146}{0.146} = 0.001$$

$$\begin{aligned}
 S &= 5 + \frac{0.146}{1 - 0.001} \\
 &= 5 + \frac{0.146}{0.999}
 \end{aligned}$$

$$S = \frac{a_1}{1-r}$$

$$= 5 + \frac{146}{999}$$

$$= \frac{5,141}{999}$$

### Exercise

Find the rational number represented by the repeating decimal  $3.\overline{2394}$

### Solution

$$3.\overline{2394} = 3.2 + 0.0394 + 0.0000394 + \dots$$

$$a_1 = 0.0394$$

$$r = \frac{0.0000394}{0.0394} = 0.001$$

$$S = 3.2 + \frac{0.0394}{1 - 0.001}$$

$$S = \frac{a_1}{1 - r}$$

$$= \frac{32}{10} + \frac{0.0394}{0.9990}$$

$$= \frac{32}{10} + \frac{394}{9990}$$

$$= \frac{31968 + 394}{9990}$$

$$= \frac{32,362}{9,990}$$

$$= \frac{16,181}{4,995}$$

### Exercise

Find the rational number represented by the repeating decimal  $1.\overline{6124}$

### Solution

$$1.\overline{6124} = 1 + 0.6124 + 0.00006124 + \dots$$

$$a_1 = 0.6124$$

$$r = \frac{0.00006124}{0.6124} = 0.0001$$

$$S = 1 + \frac{0.6124}{1 - 0.0001}$$

$$S = \frac{a_1}{1 - r}$$

$$= 1 + \frac{0.6124}{0.9999}$$

$$= 1 + \frac{6124}{9999}$$

$$= \frac{16,123}{9,999}$$

### ***Exercise***

Find  $x$  so that  $x + 3$ ,  $2x + 1$ , and  $5x + 2$  are consecutive terms of an arithmetic sequence.

### ***Solution***

$$d = 2x + 1 - (x + 3)$$

$$= x - 2$$

$$d = 5x + 2 - (2x + 1)$$

$$= 3x + 1$$

$$d = 3x + 1 = x - 2$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

### ***Exercise***

Find  $x$  so that  $2x$ ,  $3x + 2$ , and  $5x + 3$  are consecutive terms of an arithmetic sequence.

### ***Solution***

$$d = 3x + 2 - 2x$$

$$= x + 2$$

$$d = 5x + 3 - (3x + 2)$$

$$= 2x + 1$$

$$d = 2x + 1 = x + 2$$

$$x = 1$$

### ***Exercise***

Find  $x$  so that  $x$ ,  $x + 2$ , and  $x + 3$  are consecutive terms of a geometric sequence.

### ***Solution***

$$r = \frac{x + 2}{x}$$

$$r = \frac{x + 3}{x + 2}$$

$$r = \frac{x+2}{x} = \frac{x+3}{x+2}$$

$$(x+2)^2 = x^2 + 3x$$

$$x^2 + 4x + 4 - x^2 - 3x = 0$$

$$x + 4 = 0$$

$$\underline{x = -4}$$

### Exercise

Find  $x$  so that  $x-1$ ,  $x$  and  $x+2$  are consecutive terms of a geometric sequence.

#### Solution

$$r = \frac{x}{x-1} = \frac{x+2}{x}$$

$$x^2 = x^2 + x - 2$$

$$x - 2 = 0$$

$$\underline{x = 2}$$

### Exercise

How many terms must be added in an arithmetic sequence whose first term is 11 and whose common difference is 3 to obtain a sum of 1092?

#### Solution

**Given:**  $a_1 = 11$ ;  $d = 3$ ;  $S = 1092$

$$1092 = \frac{n}{2}(22 + 3(n-1))$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$n(3n+19) = 2184$$

$$3n^2 + 19n - 2184 = 0$$

$$n = \frac{-19 \pm \sqrt{361 + 26208}}{6}$$

$$= \frac{-19 \pm 163}{6}$$

$$\underline{n = 24} \quad \& \quad \cancel{n = \frac{91}{3}}$$

### Exercise

How many terms must be added in an arithmetic sequence whose first term is 78 and whose common difference is  $-4$  to obtain a sum of 702?

### Solution

**Given:**  $a_1 = 78$ ;  $d = -4$ ;  $S = 702$

$$702 = \frac{n}{2}(2(78) - 4(n-1)) \quad S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$n(160 - 4n) = 1404$$

$$-4n^2 + 160n - 1404 = 0$$

$$\begin{aligned} n &= \frac{-160 \pm \sqrt{25,600 - 22464}}{-8} \\ &= \frac{160 \pm 56}{8} \end{aligned}$$

$$\underline{n = 13} \quad \& \quad \underline{n = 27}$$

### Exercise

The first ten rows of seating in a certain section of a stadium have 30 seats, 32 seats, 34 seats, and so on. The eleventh through the twentieth rows each contain 50 seats. Find the total number of seats in the section.

### Solution

**Given:**  $a_1 = 30$ ;  $d = 2$

$$S = S_{10} + 50(20 - 11 + 1) \quad S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

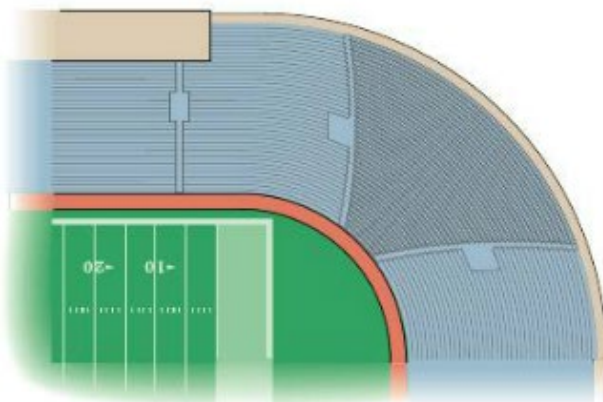
$$= \frac{10}{2}(2(30) + 2(9)) + 50(10)$$

$$= 5(78) + 500$$

$$\underline{= 890 \text{ seats}}$$

### Exercise

The corner section of a football stadium has 15 seats in the first row and 40 rows in all. Each successive row contains two additional seats. How many seats are in this section?



### Solution

**Given:**  $a_1 = 15$ ;  $d = 2$ ;  $n = 40$

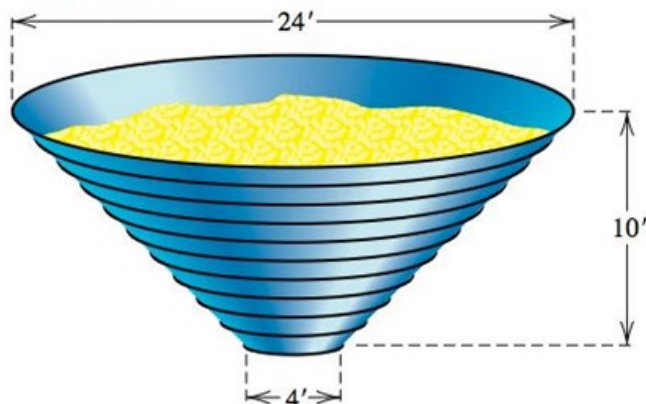
$$\begin{aligned} S_{40} &= \frac{40}{2}(30 + 2(40 - 1)) \\ &= 20(30 + 78) \\ &= 20(108) \\ &= \underline{2,160} \end{aligned}$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

The corner section has 2,160 seats.

### Exercise

A grain bin is to be constructed in the shape of a frustum of a cone.



The bin is to be 10 feet tall with 11 metal rings positioned uniformly around it, from the 4-foot opening at the bottom to the 24-foot opening at the top. Find the total length of metal needed to make the rings.

### Solution

The circumference of each ring is  $\pi D$ .

$$a_1 = 4\pi; \quad a_{11} = 24\pi$$

$$24 = 4 + (11 - 1)d$$

$$a_n = a_1 + (n - 1)d$$

$$10d = 20$$

$$\underline{d = 2}$$

$$S_{11} = \frac{11}{2}(4\pi + 24\pi)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$\underline{= 154\pi \text{ ft}}$$

### ***Exercise***

A bicycle rider coasts downhill, traveling 4 *feet* the first second. In each succeeding second, the rider travels 5 *feet* farther than in the preceding second. If the rider reaches the bottom of the hill in 11 *seconds*, find the total distance traveled.

### **Solution**

**Given:**  $a_1 = 4 \text{ ft}$  &  $d = 5 \text{ ft}$

$$S_{11} = \frac{11}{2}(8 + 5(10))$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$\underline{= 319 \text{ ft}}$$

∴ the total distance traveled 319 *feet*.

### ***Exercise***

A contest will have five each prizes totaling \$5,000, and there will be a \$100 difference between successive prizes. Find the first prize.

### **Solution**

**Given:**  $n = 5$   $S_5 = 5000$   $d = -100$

$$5,000 = \frac{5}{2}[2a_1 + 4(-100)]$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$2,000 = 2a_1 - 400$$

$$\underline{a_1 = \$1,200}$$

### ***Exercise***

A Company is to distribute \$46,000 in bonuses to its top ten salespeople. The tenth salesperson on the list will receive \$1,000, and the difference in bonus money between successively ranked salesperson is to be constant. Find the bonus for each salesperson.

### **Solution**

**Given:**  $n = 10$     $S_{10} = 46,000$     $a_{10} = 1,000$

$$46,000 = \frac{10}{2}(a_1 + 1000) \qquad S_n = \frac{n}{2}(a_1 + a_n)$$

$$9,200 = a_1 + 1000$$

$$a_1 = 8,200$$

$$d = \frac{1,000 - 8,200}{9} \qquad a_n = a_1 + (n-1)d$$

$$= -800 \quad |$$

$$\text{\$8,200} \quad \text{\$7,400} \quad \text{\$6,600} \quad \text{\$5,800} \quad \text{\$5,000} \quad \text{\$4,200} \quad \text{\$3,400} \quad \text{\$2,600} \quad \text{\$1,800} \quad \text{\$1,000}$$

### Exercise

Assuming air resistance is negligible, a small object that is dropped from a hot air balloon falls 16 *feet* during the first second, 48 *feet* during the second second, 80 *feet* during the third second, 112 *feet* during the fourth second, and so on. Find an expression for the distance the object falls in  $n$  seconds.

### Solution

**Given** the sequence: 16, 48, 80, 112, ...

This is an arithmetic sequence with:

$$a_1 = 16 \quad \& \quad d = 48 - 16 = 32$$

$$S_n = \frac{n}{2}(32 + 32(n-1)) \qquad S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$= \frac{n}{2}(32n)$$

$$= 16n^2 \quad |$$

### Exercise

A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two fewer bricks than the prior step.

- How many bricks are required for the top step?
- How many bricks are required to build the staircase?

### Solution

a) **Given:**  $n = 30$     $a_1 = 100$     $d = -2$

$$a_n = 100 - 2(n-1) \qquad a_n = a_1 + (n-1)d$$

$$= -2n + 102 \quad |$$

$$a_{30} = 102 - 60$$

$$= 42 \quad |$$



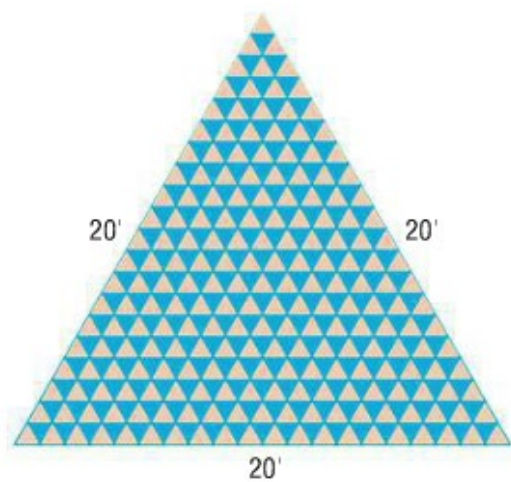
$$\begin{aligned} b) \quad S_{30} &= 15(100 + 42) \\ &= 2,130 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

It required 2130 *bricks* to build the staircase.

### Exercise

A mosaic is designed in the shape of an equilateral triangle, 20 *feet* on each side. Each tile in the mosaic is in the shape of an equilateral triangle, 12 *inches* to a side. The tiles are to alternate in color as shown below.



How many tiles of each color will be required?

### Solution

Bottom row has 20 lighter colored tiles.

Top row has 1 lighter colored tile.

The number decreases by 1 as we move up the triangle.

∴ This is an arithmetic sequence with:  $a_1 = 20$ ;  $d = -1$ ;  $n = 20$

$$\begin{aligned} S_{20} &= \frac{20}{2}(40 + (-1)(20 - 1)) \\ &= 10(40 - 19) \\ &= 10(21) \\ &= 210 \end{aligned}$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

∴ There are 210 *lighter colored* tiles.

Bottom row has 19 darker colored tiles.

Top row has 1 darker colored tile.

∴ This is an arithmetic sequence with:  $a_1 = 1$ ;  $d = -1$ ;  $n = 19$

$$S_{19} = \frac{19}{2}(2(19) + (-1)(19 - 1))$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$= \frac{19}{2}(38 - 18)$$

$$= \underline{190}$$

∴ There are 190 *darker colored* tiles.