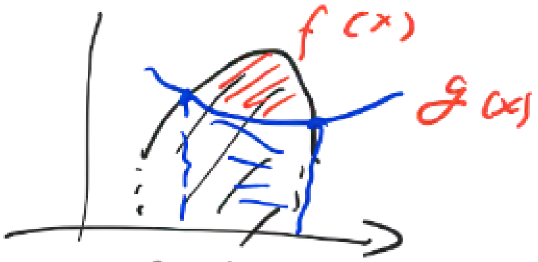
1.2 Area between 2 curves.



To do so: for 2 fctns
intersection letting $y = f(x) = g(x)$
Solve for the variable

Ex | Given: $y = 2 - x^2$ ($x = 0$) $y = -x$ ($x = 0$)   A?
Find: A?

Soln

$y = 2 - x^2 = -x$

$x^2 - x - 2 = 0$

$x = -1, 2$

$a + b + c = 0$
$x = 1, c/a$
$a - b + c = 0$
$x = -1, -c/a$

$\sqrt{b^2 - 4ac}$

$\frac{1 \pm 3}{2}$

$A = \int_{-1}^{2} (2 - x^2 + x)\,dx$

$= 2x - \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_{-1}^{2}$

$= 4 - \frac{8}{3} + 2 - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$

$= 8 - 3 - \frac{1}{2}$

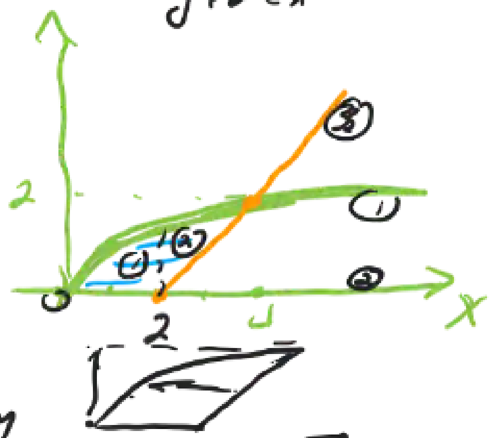
$= \frac{9}{2}$ unit^2

$5 - \frac{1}{2} = \frac{9}{2}$

Ex #1 A? $y=\sqrt{x}$ ①   x-axis $y=0$ ② $y=x-2$ ③

$y+2=x$

Soln



① ∩ ③ $(4,2)$

$(\sqrt{x})^2=(x-2)^2$

$x=\cancel{1},4$

$$A=\int_0^2 (y+2-y^2)\,dy$$

$y=\sqrt{x}$
$x=y^2$

$$=\frac{1}{2}y^2+2y-\frac{1}{3}y^3\Big|_0^2$$

$$=2+4-\frac{8}{3}$$

$$=\frac{10}{3}\ \text{unit}^2$$

Ex #2 $y=7-2x^2$ $y=x^2+4$ A?

$y: 7-2x^2=x^2+4$

$-(x^2+4)$

$-3x^2=-3$

$x^2=1 \to x=\pm 1$

$$A=\int_{-1}^{1}(7-2x^2-x^2-4)\,dx$$

even fctn

$$=\int_{-1}^{1}(3-3x^2)\,dx$$

$\int 3x^2$ x^3 $\frac{3}{3}$

$$=3x-x^3\Big|_{-1}^{1}$$

$2(3x-x^3\Big|_0^1$

$$=3-1-(-3+1)$$

$$=2+2$$

$$=4\ \text{unit}^2$$

Hwk 1.2

name

① Given #1

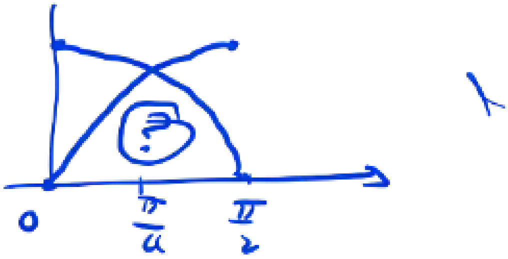
$f(x) = \cos x \quad g(x) = \sin x \quad 0 \le x \le \frac{\pi}{2} \quad A?$

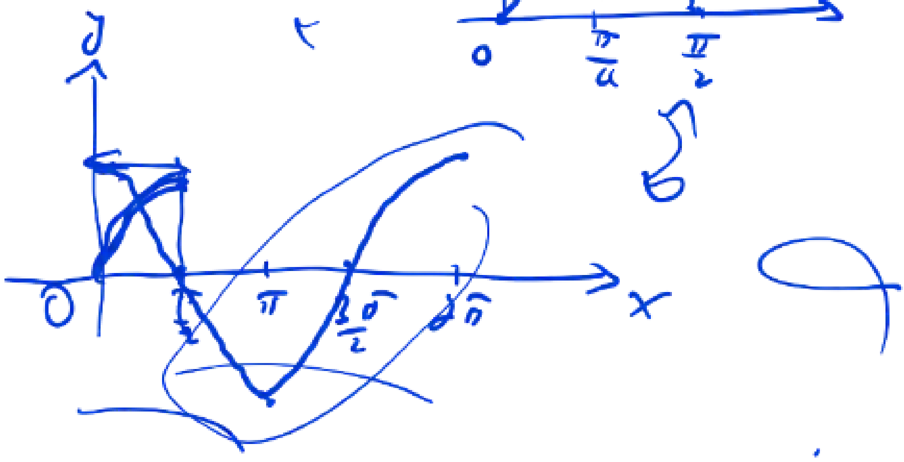
$\cos x = \sin x$

$x = ?$

$A = \int \quad +$

$y = \cos x$



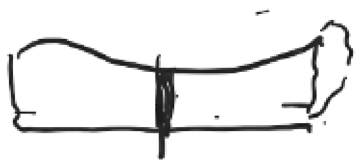


1.3 Volume

Volume = Area x height

$V = A \cdot h$

$V = \int^b A(x)\,dx$



$\int_a$

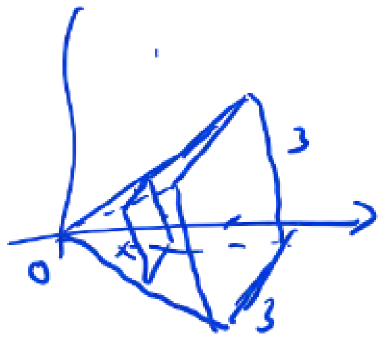
Ex $3\,m$ sq: $3\,m$ (side)

$A(x) = x^2$

$$V = \int_0^3 x^2\,dx$$

$$= \frac{1}{3}x^3 \Big|_0^3$$

$$= 9\ m^3$$

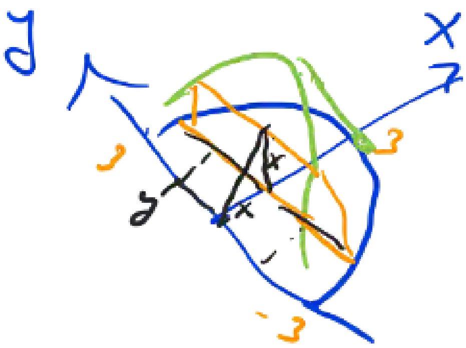


Ex $\theta = 45°$

$x^2 + y^2 = 9$

$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$



$A = x(2y)$

$A(x) = 2x\sqrt{9 - x^2}$

$$V = \frac{2}{2}\int_0^3 \frac{x}{2}(9 - x^2)^{1/2}\,dx$$

$d(9 - x^2) = -2x\,dx$

$$= -\int_0^3 (9 - x^2)^{1/2}\,d(9 - x^2)$$

$$= -\frac{2}{3}(9 - x^2)^{3/2}\Big|_0^3$$

$9^{3/2}$

$(3^2)^{3/2}$

3^3

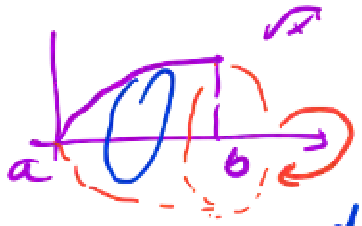
$$= \frac{-2}{3}\left[0 - 27\right]$$

$$= 18\ \text{unit}^3$$

Disk Method

solid

revolution



$$V = \pi \int_a^b [R(x)]^2\, dx \quad = \pi \int_c^d R(y)^2\, dy$$

Ex V? $y = \sqrt{x}$ $0 \le x \le 4$ rev x-axis

$$V = \pi \int_0^4 (\sqrt{x})^2\, dx$$

setup

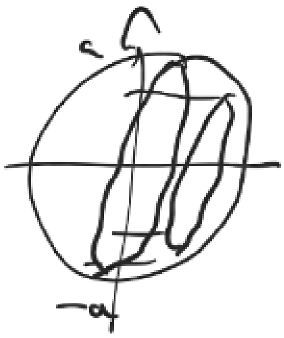
$$= \pi \int_0^4 x\, dx$$

$$= \frac{\pi}{2} x^2 \Big|_0^4$$

$$= 8\pi \text{ unit}^3$$

Ex Volume of a sphere w/ $r = a$

$$\boxed{\begin{aligned} A &= \pi r^2 \\ &= \pi a^2 \end{aligned}}$$



$$x^2 + y^2 = a^2$$

$$A(x) = \pi y^2$$
$$= \pi (a^2 - x^2)$$

$$V = \pi \int_{-a}^{a} (a^2 - x^2)\, dx$$

→ even fctn

$$= 2\pi \left(a^2 x - \frac{1}{3} x^3\right) \Big|_0^a$$

$= 2\pi\left(a^3 - \frac{1}{3}a^3\right)$

$= \frac{4\pi}{3} a^3 \text{ unit}^3$

Ex V? $x=0$, $\left(x = \frac{2}{y}\right)^2$ $1 \le y \le 4$

about y-axis

Soln

$$V = \pi \int_1^4 \frac{4}{y^2}\, dy \qquad \left(\frac{2}{y}\right)^2$$

$$\int \frac{dx}{x^2} = \frac{-1}{x}$$

$$\int x^{-2}\, dx$$

$$= -4\pi \left.\frac{1}{y}\right|_1^4$$

$$= -4\pi\left(\frac{1}{4} - 1\right)$$

$$= 3\pi \text{ unit}^3$$

revolve about line $\begin{cases} x = a \\ y = b \end{cases}$

Ex V? $x = y^2+1$, $x=3$ about $x=3$

dy

Soln

$y^2 + 1 = 3$

$y^2 = 2 \Rightarrow y = \pm\sqrt{2}$

$$V = \pi \int_{-\sqrt{2}}^{\sqrt{2}} (3 - y^2 - 1)^2\, dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (2 - y^2)^2\, dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (4 - 4y^2 + y^4)\, dy$$

$$= 2\pi \left(4y - \frac{4}{3}y^3 + \frac{1}{5}y^5\right)\Big|_0^{\sqrt{2}}$$

$$= 2\pi \left(4\sqrt{2} - \frac{8}{3}\sqrt{2} + \frac{4}{5}\sqrt{2}\right)$$

$$= 2\pi\sqrt{2}\left(4 - \frac{8}{3} + \frac{4}{5}\right)$$

$$= 2\pi\sqrt{2}\left(\frac{60 - 40 + 12}{15}\right)$$

$$= \boxed{\frac{64\pi\sqrt{2}}{15} \text{ unit}^3}$$

Washer Method

$$V = \pi \int_a^b \left(R_{(x)}^2 - r_{(x)}^2\right) dx \leftarrow \text{rev } x\text{-axis}$$

$$= \pi \int_c^d \left([R(y)]^2 - [r(y)]^2\right) dy$$

Ex QI $\begin{cases} x = y^3 \leftarrow (1) \\ x = 4y \leftarrow (2) \end{cases}$ rev: x-axis, y-axis

Soln boundary?

$x = y^3 = 4y$ $\quad y = 0 \quad y^2 = 4$

$y^3 - 4y = 0$

$y(y^2 - 4) = 0 \quad y^2 = 4$

$y = 0, \pm 2)$ — $\quad (1) \to y = x^{1/3}$

$y = 0, 2$ $\qquad$ $x = 0, 8$ $\quad$ ① $\to y = \frac{x}{4}$

$$V = \pi \int_0^8 \left[x^{2/3} - \frac{x^2}{16} \right] dx$$

$$= \pi \left[\frac{3}{5} x^{5/3} - \frac{1}{48} x^3 \right|_0^8$$

$$= \pi \left[\frac{3}{5} 2^5 - \frac{32}{3} \right)$$

$$= \pi \left(\frac{96}{5} - \frac{32}{3} \right)$$

$$= 32\pi \left(\frac{3}{5} - \frac{1}{3} \right)$$

$$= \boxed{\frac{128\pi}{15} \text{ unit}^3}$$

$(2^3)^{5/3}$

$\frac{8^2}{2}$

$$V = \pi \int_0^2 \left(16y^2 - y^6 \right) dy$$

$$= \pi \left(\frac{16}{3} y^3 - \frac{1}{7} y^7 \right|_0^2$$

$$= \pi \left(\frac{128}{3} - \frac{128}{7} \right)$$

$\frac{1}{3} - \frac{1}{7}$

$$= 128\pi \left(\frac{4}{21} \right)$$

$$= \boxed{\frac{512\pi}{21} \text{ unit}^3}$$

$$\frac{\cancel{128}^{1}}{\cancel{15}_{5}} \; ? \; \frac{\cancel{512}^{4}}{\cancel{21}_{7}}$$

$$7 \quad ? \quad 20$$

x-axis $\qquad$ y-axis

Washer → shell

2 fetn →

1x ⇒ ✓ ?

Pg 35 ✓cut
tobe