# Cramer's Rule

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases}$$

$|A|$ determinant $\det(A)$ — $[\ ]$ $(\ )$ ↓ matrix

$$D = \Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - b_1 a_2$$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$\alpha$ alpha

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

If $D \neq 0 \Rightarrow D_x, D_y$ ; if $D_y = 0$ $(x(y), y)$ ; $D_y \neq 0$ No soln

Ex

$$\begin{cases} 5x + 7y = -1 \\ 6x + 8y = 1 \end{cases}$$

$$D = \begin{vmatrix} 5 & 7 \\ 6 & 8 \end{vmatrix}$$

$$= 5(8) - 7(6)$$

$$= 40 - 42$$

$$= -2$$

$$D_x = \begin{vmatrix} -1 & 7 \\ 1 & 8 \end{vmatrix} = -8 - 7 = -15$$

$$D_y = \begin{vmatrix} 5 & -1 \\ 6 & 1 \end{vmatrix} = 5 + 6 = 11$$

$$\therefore \left( \frac{-15}{-2}, \frac{11}{-2} \right) \qquad \left( \frac{15}{2}, -\frac{11}{2} \right)$$

Q1. $x - 3y + 7z = 13$

$x + y + z = 1$

$x - 2y + 3z = 4$

$$D = \begin{vmatrix} 1 & -3 & 7 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} \begin{matrix} 1 & -3 \\ 1 & 1 \\ 1 & -2 \end{matrix}$$

$= 3 - 3 - 14 - 7 + 2 + 9$

$= -10$

$$D_x = \begin{vmatrix} 13 & -3 & 7 \\ 1 & 1 & 1 \\ 4 & -2 & 3 \end{vmatrix} \begin{matrix} 13 & -3 \\ 1 & 1 \\ 4 & -2 \end{matrix}$$

$= 39 - 12 - 14 - 28 + 26 + 9$

$= 20$

$$D_y = \begin{vmatrix} 1 & 13 & 7 \\ 1 & 1 & 1 \\ 1 & 4 & 3 \end{vmatrix} \begin{matrix} 1 & 13 \\ 1 & 1 \\ 1 & 4 \end{matrix}$$

| + | − |
|---|---|
| 3 | 7 |
| 13 | 4 |
| 21 | 39 |
| 44 | 50 |

$= -6$

$$D_z = \begin{vmatrix} 1 & -3 & 13 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix} \begin{matrix} 1 & -3 \\ 1 & 1 \\ 1 & -2 \end{matrix}$$

| + | − |
|---|---|
| 4 | 3 |
| 2 | 26 |
| 12 | 13 |
| 18 | 42 |

$= -24$

$$\left( \frac{20}{-10}, \frac{-6}{-10}, \frac{-24}{-10} \right)$$

$$\therefore \left( -2, \frac{3}{5}, \frac{12}{5} \right)$$

#1) $\begin{cases} 3x+2y=-4 \\ 2x-y=-5 \end{cases}$

$D = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}$ $= -3-4$ $= -7$

$D_x = \begin{vmatrix} -4 & 2 \\ -5 & -1 \end{vmatrix}$ $= 4+10$ $= 14$

$D_y = \begin{vmatrix} 3 & -4 \\ 2 & -5 \end{vmatrix}$ $= -15+8$ $= -7$

$\therefore (-2, 1)$ $\quad \frac{14}{-7} = -2$

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#3) $\begin{cases} 3x+2y-z=4 \\ 3x-2y+z=5 \\ 4x-5y-z=-1 \end{cases}$

$D = \begin{vmatrix} 3 & 2 & -1 \\ 3 & -2 & 1 \\ 4 & -5 & -1 \end{vmatrix} \begin{matrix} 3 & 2 \\ 3 & -2 \\ 4 & -5 \end{matrix}$

$+ \quad -$
$6 \quad -8$
$8 \quad -15$
$15 \quad 6$

$= 0$

$D_z = \begin{vmatrix} 3 & 2 & 4 \\ 3 & -2 & 5 \\ 4 & -5 & -1 \end{vmatrix} \begin{matrix} 3 & 2 \\ 3 & -2 \\ 4 & -5 \end{matrix}$

$+ \quad -$
$6 \quad -60$
$40$
$6$
$75$
$32$
$62$

$= 99 \neq 0$

$\therefore$ No solution

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$D_z = 0 \Rightarrow (x(z), y(z), z)$

Unique infinite solution (!)

Read: 5.2, 5.3 } 7/14