

4.1 #9

$$x = \frac{t}{t-1}$$

$$y = \frac{t-2}{t+1}$$

$$-1 < t < 1$$

$$y = tx - x$$

$$t(1-x) = -x$$

$$t = \frac{-x}{x-1}; x \neq 1$$

$$\frac{-2.9}{.1}$$

$$\frac{[-.9]}{[.1.9]} = \frac{9}{19}$$

$$y = \frac{\frac{x}{x-1} - 2}{\frac{x}{x-1} + 1}$$

$$= \frac{x - 2x + 2}{x + x - 1}$$

$$= \frac{-x + 2}{2x - 1}$$

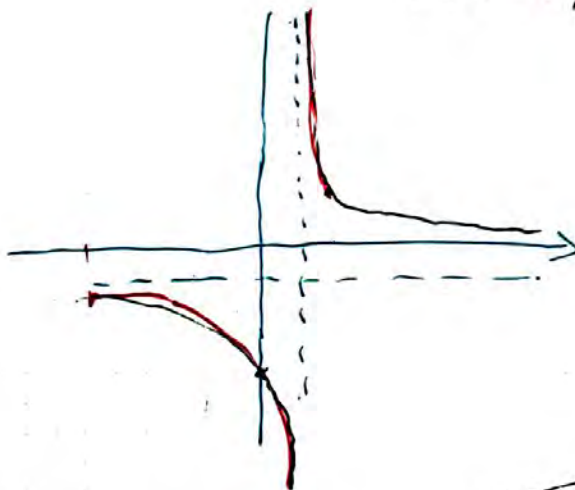
t	x	y
0	0	-2
-.9	4/19	-19
.9	9	1/19

$$\frac{-y}{-.1}$$

$$t = -1 \Rightarrow x = \frac{1}{2}$$

$$t = 1 \Rightarrow y = -\frac{1}{2}$$

$$\frac{[.1]}{[.9]}$$



$$x = \lim_{t \rightarrow +1} \frac{t}{t-1} = +\infty$$

$$\Rightarrow \lim_{t \rightarrow +1} \frac{t}{t-1}$$

$$x = \lim_{t \rightarrow -1} \frac{t}{t-1} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = \lim_{t \rightarrow -1} \frac{t-2}{t+1} = \frac{-3}{0} = -\infty$$

$$\frac{t-2}{t+1} \xrightarrow{t \rightarrow 1} -\frac{1}{2}$$

$$4.2 \neq 2\theta$$

$$\begin{cases} x = \theta - \sin \theta = t - \sin t \\ y = 1 - \cos \theta = 1 - \cos t \end{cases} \quad \theta = \pi$$

Soln $\frac{d^2 y}{dx^2} ?$

$$\frac{dx}{d\theta} = 1 - \cos \theta \quad \frac{dy}{d\theta} = \sin \theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\sin \theta}{1 - \cos \theta} \quad \begin{matrix} \sin \theta & 0 \\ -\cos \theta & 1 \end{matrix}$$

$$\frac{dy'}{d\theta} = \frac{\cos \theta - \cos^2 \theta - \sin^2 \theta}{(1 - \cos \theta)^2} =$$

$$= \frac{\cos \theta - 1}{(1 - \cos \theta)^2} = \left| - \frac{(1 - \cos \theta)}{()^2} \right|$$

$$= - \frac{1}{1 - \cos \theta}$$

$$\frac{d^2 y}{dx^2} = \frac{dy'/d\theta}{dx/d\theta}$$

$$= - \frac{1}{1 - \cos \theta} \cdot \frac{1}{1 - \cos \theta}$$

$$= \frac{-1}{(1 - \cos \theta)^2} \Big|_{\theta = \pi}$$

$$= \left[-\frac{1}{4} \right]$$

Q.2 #35

$$\begin{cases} x = 2 - \pi \cos t \\ y = 2t - \pi \sin t \end{cases}$$

Soln

$$\frac{dx}{dt} = \pi \sin t$$

$$\frac{dy}{dt} = 2 - \pi \cos t = x$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2 - \pi \cos t}{\pi \sin t}$$

$$\sin t \neq 0 \quad (t \neq 0, \pi)$$

$$x = y \Rightarrow 2 - \pi \cos t = 2t - \pi \sin t$$

$$\begin{cases} x = 2 - \pi \cos t = 0 \rightarrow \cos t = \frac{2}{\pi} \end{cases}$$

$$\begin{cases} y = 2t - \pi \sin t = 0 \rightarrow 2t = -\pi \sin t \end{cases}$$

$$x = 2 = 2 - \pi \cos t \Rightarrow \cos t = 0 \Rightarrow t = \pm \frac{\pi}{2}$$

4.2 #50 A? arc: $x = a(t - \sin t)$ $y = a(1 - \cos t)$

Area = $\int y dx$ $dx = d(at - a \sin t)$
 $= a(1 - \cos t) dt$

$= a^2 \int_0^{2\pi} (1 - \cos t)^2 dt$

$= a^2 \int_0^{2\pi} (1 - 2\cos t + \cos^2 t) dt$

$= a^2 \int_0^{2\pi} (\frac{3}{2} - 2\cos t + \frac{1}{2} \cos 2t) dt$

$= a^2 (\frac{3}{2}t - 2\sin t + \frac{1}{4}\sin 2t) \Big|_0^{2\pi}$

$= 3\pi a^2$

4.2 #61 L? $\begin{cases} x = 8\cos t + 8t\sin t \\ y = 8\sin t - 8t\cos t \end{cases} \quad 0 \leq t \leq \frac{\pi}{2}$

Soln

$\frac{dx}{dt} = -8\sin t + 8\sin t + 8t\cos t = 8t\cos t$

$\frac{dy}{dt} = 8\cos t - 8\cos t + 8t\sin t = 8t\sin t$

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{64t^2\cos^2 t + 64t^2\sin^2 t}$
 $= 8t$

$L = \int_0^{\pi/2} 8t dt$

$= 4t^2 \Big|_0^{\pi/2}$

$= \pi^2 \text{ unit}^2$

4.2 # 71

$$\begin{cases} x = t + \sqrt{2} \\ y = \frac{1}{2}t^2 + \sqrt{2}t \end{cases}$$

$$-\sqrt{2} \leq t \leq \sqrt{2}$$

Soln $\frac{dx}{dt} = 1$ $\frac{dy}{dt} = t + \sqrt{2}$

$$\begin{aligned} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} &= \sqrt{1 + t^2 + 2\sqrt{2}t + 2} \\ &= \sqrt{t^2 + 2\sqrt{2}t + 3} \end{aligned}$$

$$A = 2\pi \int_{-\sqrt{2}}^{\sqrt{2}} (t + \sqrt{2}) (t^2 + 2\sqrt{2}t + 3)^{1/2} dt.$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (t^2 + 2\sqrt{2}t + 3)^{1/2} d(t^2 + 2\sqrt{2}t + 3)$$

$$= \frac{2\pi}{3} (t^2 + 2\sqrt{2}t + 3)^{3/2} \Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= \frac{2\pi}{3} [27 - (1)]$$

$$= \frac{52\pi}{3} \text{ unit}^2$$

4.4 # 20

$$r = 2 \cos 2\theta \quad \text{even } 0 \leq \theta \leq \pi$$

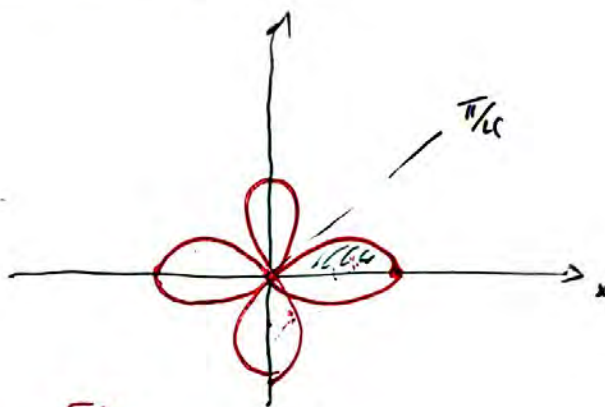
$$A = 2 \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta \quad 2 \rightarrow (\text{twice})$$

$$= 2 \frac{1}{2} \int_0^{\pi} 4 \cos^2 2\theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} (1 + \cos 4\theta) d\theta$$

$$= 2 \left(\theta + \frac{1}{4} \sin 4\theta \right) \Big|_0^{\pi}$$

$$= 2\pi \text{ unit}^2$$



$$A = 8 \int_0^{\pi/4} \frac{1}{2} r^2 d\theta$$

$$= 4 \left(\theta + \frac{1}{4} \sin 4\theta \right) \Big|_0^{\pi/4}$$

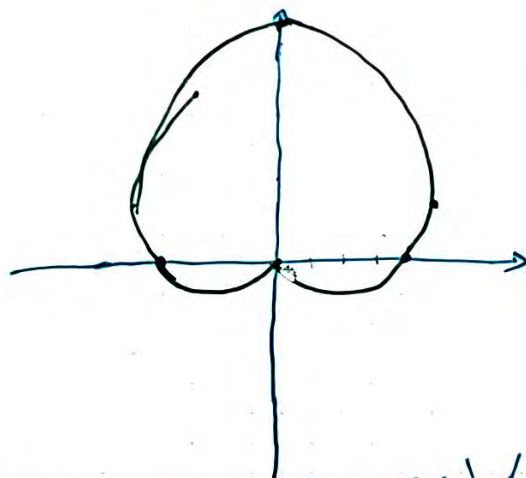
$$= 4 \left(\frac{\pi}{4} \right)$$

$$= \pi$$

4.4 #13

$$r = 4 + 4 \sin \theta = 4(1 + \sin \theta)$$

$$\begin{aligned} A &= \frac{1}{2} \int_0^{2\pi} 16 (1 + \sin \theta)^2 d\theta \\ &= 8 \int_0^{2\pi} (1 + 2 \sin \theta + \underbrace{\sin^2 \theta}_{\frac{1 - \cos 2\theta}{2}}) d\theta \\ &= 8 \int_0^{2\pi} \left(\frac{3}{2} + 2 \sin \theta - \frac{1}{2} \cos 2\theta \right) d\theta \\ &= 8 \left(\frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta \right) \Big|_0^{2\pi} \\ &= 8(3\pi - 2 + 2) \\ &= 24\pi \text{ unit}^2 \end{aligned}$$



$$A = 2 \cdot \frac{1}{2} \int_{-\pi/2}^{\pi/2} 16 (1 + \sin \theta)^2 d\theta$$

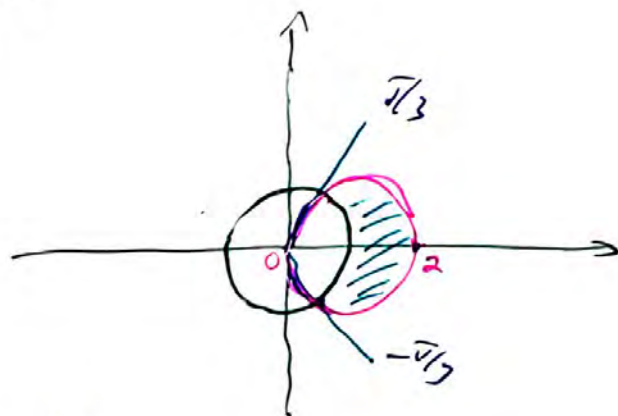
$$\begin{aligned} A &= 16 \int_{-\pi/2}^{\pi/2} \left(\frac{3}{2} + 2 \sin \theta - \frac{1}{2} \cos 2\theta \right) d\theta \\ &= 16 \left(\frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta \right) \Big|_{-\pi/2}^{\pi/2} \\ &= 16 \left(\frac{3\pi}{4} + \frac{3\pi}{4} \right) \\ &= 24\pi \text{ unit}^2 \end{aligned}$$

4.4 # 4c Inside $r = 2\cos\theta$
outside $r = 1$

$$r = 2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$$



$$\begin{aligned} A &= \frac{1}{2} \int_{-\pi/3}^{\pi/3} ((2\cos\theta)^2 - 1) d\theta \\ &= 2 \left(\frac{1}{2} \right) \int_0^{\pi/3} \left(4 \frac{1+\cos 2\theta}{2} - 1 \right) d\theta \\ &= \int_0^{\pi/3} (1 + 2\cos 2\theta) d\theta \\ &= \theta + \sin 2\theta \Big|_0^{\pi/3} \\ &= \frac{\pi}{3} + \frac{\sqrt{3}}{2} \text{ unit}^2 \end{aligned}$$

4.4 # 49

Common interior

$$\begin{cases} r = 2(1 + \cos \theta) \\ r = 2(1 - \cos \theta) \end{cases}$$

$$A = 4\left(\frac{1}{2}\right) \int_0^{\pi/2} 4(1 - \cos \theta)^2 d\theta$$

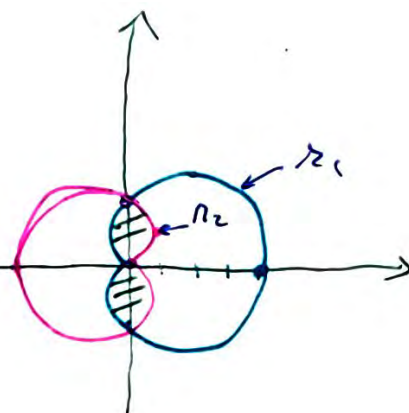
$$= 8 \int_0^{\pi/2} \left(1 - 2\cos \theta + \frac{1}{2} + \frac{1}{2} \overbrace{\cos^2 \theta}^{\cos^2 \theta}\right) d\theta$$

$$= 8 \int_0^{\pi/2} \left(\frac{3}{2} - 2\cos \theta + \frac{1}{2} \cos 2\theta\right) d\theta$$

$$= 8 \left(\frac{3}{2} \theta - 2\sin \theta + \frac{1}{4} \sin 4\theta \right) \Big|_0^{\pi/2}$$

$$= 8 \left(\frac{3\pi}{4} - 2 \right)$$

$$= \underline{6\pi - 16 \text{ unit}^2}$$



4.4 #68

$$r = \frac{1}{\sqrt{2}} e^{\theta} \quad 0 \leq \theta \leq \pi \quad L?$$

$$\begin{aligned} \sqrt{r^2 + (r')^2} &= \sqrt{\frac{1}{2} e^{2\theta} + \frac{1}{2} e^{2\theta}} \\ &= e^{\theta} \end{aligned}$$

$$\begin{aligned} L &= \int_0^{\pi} e^{\theta} d\theta \\ &= e^{\theta} \Big|_0^{\pi} \\ &= e^{\pi} - 1 \quad \text{unit} \end{aligned}$$

#4.4 #80

$$r = \sec \theta \quad 0 \leq \theta \leq \pi/3$$

$$\begin{aligned} \sqrt{r^2 + (r')^2} &= \sqrt{\sec^2 \theta + \sec^2 \theta \tan^2 \theta} \\ &= \sec \theta \sqrt{1 + \tan^2 \theta} \rightarrow \sec^2 \theta \\ &= \sec^2 \theta \end{aligned}$$

$$\begin{aligned} L &= \int_0^{\pi/3} \sec^2 \theta d\theta \\ &= \tan \theta \Big|_0^{\pi/3} \\ &= \sqrt{3} \quad \text{unit} \end{aligned}$$

4.4 # 44

$$r = 1 + 4 \cos \theta$$

$$0 \leq \theta \leq \frac{\pi}{2}$$

Polar axis

$$P = 2\pi \int_a^b f(\theta) \sin \theta \sqrt{f^2 + (f')^2} d\theta$$

$$\sqrt{(1+4\cos\theta)^2 + 16\sin^2\theta} = \sqrt{1+8\cos\theta + \underbrace{16\cos^2\theta + 16\sin^2\theta}_{=16}} \\ = \sqrt{17+8\cos\theta}$$

$$S = 2\pi \int_0^{\pi/2} (1+4\cos\theta) \sin \theta \sqrt{17+8\cos\theta} d\theta$$

$$= 2\pi \int_0^{\pi/2} \sin \theta (17+8\cos\theta)^{1/2} d\theta + 8\pi \int_0^{\pi/2} \cos \theta \sin \theta (17+8\cos\theta)^{1/2} d\theta$$

$$= \frac{-2\pi}{8} \int_0^{\pi/2} (17+8\cos\theta)^{1/2} d(17+8\cos\theta) + S_1$$

$$u = 17+8\cos\theta \Rightarrow \cos\theta = \frac{u-17}{8}$$

$$du = -8 \sin \theta d\theta$$

$$S_1 = 8\pi \int_0^{\pi/2} \frac{1}{8} (u-17) u^{1/2} \left(-\frac{1}{8} du \right)$$

$$= -\frac{\pi}{8} \int_0^{\pi/2} (u^{3/2} - 17u^{1/2}) du$$

$$= -\frac{\pi}{8} \left(\frac{2}{5} u^{5/2} - \frac{34}{3} u^{3/2} \right) \Big|_0^{\pi/2}$$

$$= -\frac{\pi}{8} \left(\frac{2}{5} (17+8\cos\theta)^{5/2} - \frac{34}{3} (17+8\cos\theta)^{3/2} \right) \Big|_0^{\pi/2}$$

$$= -\frac{\pi}{8} \left(\frac{2}{5} (17)^{5/2} - \frac{34}{3} (17)^{3/2} - \frac{2}{5} 5^5 + \frac{34}{3} 5^3 \right)$$

$$= -\frac{2\pi}{8} \frac{2}{3} (17+8\cos\theta)^{3/2} \Big|_0^{\pi/2}$$

$$= -\frac{\pi}{6} (17^{3/2} - 5^3)$$

$$S = -\frac{17\pi}{6} \sqrt{17} + \frac{125\pi}{6} - \frac{\pi}{20} (17)^2 \sqrt{17} + \frac{\pi}{12} (17)^2 \sqrt{17} - 2(5)^4 + \frac{34}{3} 5^3$$

4.4 # 95

$$r = 2 \sin \theta \quad 0 \leq \theta \leq \frac{\pi}{2} \quad \theta = \frac{\pi}{2}$$

$$\sqrt{r^2 + r'^2} = \sqrt{4 \sin^2 \theta + 4 \cos^2 \theta}$$

$$= 2$$

$$S = 2\pi \int_0^{\pi/2} \underbrace{2 \sin \theta \cos \theta}_{\sin 2\theta} (2) d\theta$$

$$= 4 \left(-\frac{1}{2}\right) \cos 2\theta \Big|_0^{\pi/2}$$

$$= -2(-1 - 1)$$

$$= 4 \text{ unit}^2$$