

Review

$$A = \begin{pmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & 7 & -1 \\ 0 & 1-\lambda & 0 \\ 0 & 15 & -2-\lambda \end{vmatrix}$$

$$= (-1-\lambda)(1-\lambda)(-2-\lambda) = 0$$

$$\lambda_{1,2,3} = \underline{-1, 1, -2}$$

$$\text{For } \lambda_1 = -1 \Rightarrow (A - \lambda_1 I) V_1 = 0$$

$$\begin{pmatrix} 0 & 7 & -1 \\ 0 & 2 & 0 \\ 0 & 15 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{matrix} \textcircled{1} \\ 2y_1 = 0 \\ \underline{y_1 = 0} \end{matrix}$$

$$x_1 = 1$$

$$-z_1 = -7y_1 = \underline{0}$$

$$\underline{V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$$

$$\text{For } \lambda_2 = 1 \Rightarrow (A - \lambda_2 I) V_2 = 0$$

$$\begin{pmatrix} -2 & 7 & -1 \\ 0 & 0 & 0 \\ 0 & 15 & -3 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{matrix} \textcircled{2} & 15y_2 = 3z_2 & \rightarrow & \underline{5y_2 = z_2} \\ \textcircled{1} & -2x_2 = z_2 - 7y_2 & \rightarrow & \underline{x_2 = y_2} \end{matrix}$$

$$V_2 = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$$

$$\text{For } \lambda_3 = -2 \Rightarrow (A - \lambda_3 I) V_3 = 0$$

$$\begin{pmatrix} 1 & 7 & -1 \\ 0 & 3 & 0 \\ 0 & 15 & 0 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \underline{y_3 = 0}$$

$$x_3 - z_3 = 0 \Rightarrow x_3 = z_3$$

$$V_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$ax + by + cz = 0$$

$$\begin{pmatrix} -b \\ a \\ 0 \end{pmatrix}$$

$$ax = -by$$

$$\begin{pmatrix} -c \\ 0 \\ a \end{pmatrix}$$

$$ax = -cz$$

$$\begin{pmatrix} 0 \\ -c \\ b \end{pmatrix}$$

$$by = -cz$$

$$\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = A$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)^2 = 0$$

Eigen values : $\lambda_{1,2} = 2$

since it mult A is not diagonalizable.

$P^{-1}AP$?

$$A = \begin{pmatrix} -14 & 12 \\ -20 & 17 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -14-\lambda & 12 \\ -20 & 17-\lambda \end{vmatrix}$$

$$= \lambda^2 - 3\lambda + 2 = 0$$

$$\lambda_{1,2} = 1, 2$$

For $\lambda_1 = 1 \Rightarrow (A - \lambda_1 I) V_1 = 0$

$$\begin{pmatrix} -15 & 12 \\ -20 & 16 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow$$

$$\begin{aligned} 15x_1 &= 12y_1 \\ 5x_1 &= 4y_1 \end{aligned}$$

$$\underline{V_1 = \begin{pmatrix} 4 \\ 5 \end{pmatrix}}$$

$$\text{For } \lambda_2 = 2 \Rightarrow (A - \lambda_2 I) V_2 = 0$$

$$\begin{pmatrix} -16 & 12 \\ -20 & 15 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow 4x_2 = 3y_2$$

$$V_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$P = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix} \Rightarrow P^{-1} = \begin{pmatrix} 4 & -3 \\ -5 & 4 \end{pmatrix}$$

$$P^{-1} A P = \begin{pmatrix} 4 & -3 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} -16 & 12 \\ -20 & 12 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -3 \\ -10 & 7 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \checkmark$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}$$

$$|A| = 1$$

$$|B| = -2$$

Since $|A| \neq |B|$

$\therefore A$ & B are not similar

$$\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$$

$$11 = 3$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$11 = 3$$

$$\begin{pmatrix} 3-\lambda & 1 \\ 0 & 3-\lambda \end{pmatrix}$$

$$(3-\lambda)^2 = 0$$

$$\begin{vmatrix} 3-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 = 0$$

$$\lambda_1, \lambda_2 = 3$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ line}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$y_2 = 1$$

$$V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ plane}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$0 = 1$$