Exercise

Find the distance between $P(8,\ 2)$ and $Q(3,\ 5)$

Solution

$$d = \sqrt{(3-8)^2 + (5-2)^2}$$

$$= \sqrt{(-5)^2 + (3)^2}$$

$$= \sqrt{25+9}$$

$$= \sqrt{34}$$

Exercise

Find the distance between $P(-4,\ 3)$ and $Q(2,\ -5)$

Solution

$$d = \sqrt{(-4-2)^2 + (3+5)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100}$$

$$= 10$$

Exercise

Find the distance between $(-4,-1)$ and $(2,-3)$

Solution

$$d = \sqrt{(2+4)^2 + (-3+1)^2}$$

$$= \sqrt{(6)^2 + (-2)^2}$$

$$= \sqrt{36+4}$$

$$= \sqrt{40}$$

$$= 2\sqrt{10}$$

Exercise

Find the distance between $\left(2\sqrt{3},\sqrt{6}\right)$ and $\left(-\sqrt{3},5\sqrt{6}\right)$

Solution

$$d=\sqrt{\left(2\sqrt{3}+\sqrt{3}\right)^2+\left(\sqrt{6}-5\sqrt{6}\right)^2}$$

$$=\sqrt{\left(3\sqrt{3}\right)^2+\left(-4\sqrt{6}\right)^2}$$

$$=\sqrt{9(3)+16(6)}$$

$$=\sqrt{123}$$

Exercise

Find the distance between (−1, −5) and (−1, 2)

Solution

$$d=\sqrt{(-1-(-1))^2+(-5-2)^2}$$

$$=\sqrt{0^2+(-7)^2}$$

$$=\sqrt{49}$$

$$=7$$

Exercise

Find the distance between (−4, 9) and (1, −3)

Solution

$$d=\sqrt{(1+4)^2+(-3-9)^2}$$

$$=\sqrt{25+144}$$

$$=\sqrt{169}$$

$$=13$$

Exercise

Find the distance between (–2, 2) and (3, –6)

Solution

$$d = \sqrt{(3-(-2))^2 + (-6-2)^2}$$

$$= \sqrt{5^2 + (-8)^2}$$

$$= \sqrt{25+64}$$

$$= \sqrt{89}$$

Exercise

Find the distance between the points $\left(\sqrt{5}, -\sqrt{3}\right)$ and $\left(-\sqrt{7}, \sqrt{7}\right)$. Give an exact answer and an approximation to three decimal places.

Solution

$$d = \sqrt{\left(-\sqrt{7}-\sqrt{5}\right)^2 + \left(\sqrt{7}+\sqrt{3}\right)^2} \qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$= \sqrt{\left(\sqrt{7}\right)^2 + 2\sqrt{7}\sqrt{5} + \left(\sqrt{5}\right)^2 + \left(\sqrt{7}\right)^2 + 2\sqrt{7}\sqrt{3} + \left(\sqrt{3}\right)^2}$$

$$= \sqrt{7 + 2\sqrt{35} + 5 + 7 + 2\sqrt{21} + 3}$$

$$= \sqrt{22 + 2\sqrt{35} + 2\sqrt{21}}$$

Exercise

Find the distance between the points. Give an exact answer and an approximation to three decimal places.
$\left(\sqrt{7}, -\sqrt{2}\right)$ and $\left(-\sqrt{3}, \sqrt{3}\right)$

Solution

$$d = \sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2 + \left(-\sqrt{3}-\sqrt{7}\right)^2} \qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$= \sqrt{\left(\sqrt{3}\right)^2 + 2\sqrt{3}\sqrt{2} + \left(\sqrt{2}\right)^2 + \left(-\sqrt{3}\right)^2 + 2\left(-\sqrt{3}\right)\left(-\sqrt{7}\right) + \left(-\sqrt{7}\right)^2}$$

$$= \sqrt{3 + 2\sqrt{6} + 2 + 3 + 2\sqrt{21} + 7}$$

$$= \sqrt{15 + 2\sqrt{21} + 2\sqrt{6}}$$

Exercise

Find the distance between the points. Give an exact answer and an approximation to three decimal places.
$\left(\sqrt{7}, -\sqrt{2}\right)$ and $\left(-\sqrt{3}, \sqrt{7}\right)$

Solution

$$d = \sqrt{\left(\sqrt{7} - \left(-\sqrt{2}\right)\right)^2 + \left(-\sqrt{3} - \sqrt{7}\right)^2}$$

$$= \sqrt{\left(\sqrt{7} - \sqrt{2}\right)^2 + \left(-\sqrt{3} - \sqrt{7}\right)^2} \qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$= \sqrt{\left(\sqrt{7}\right)^2 + 2\sqrt{7}\sqrt{2} + \left(-\sqrt{2}\right)^2 + \left(-\sqrt{3}\right)^2 + 2\left(-\sqrt{3}\right)\left(-\sqrt{7}\right) + \left(-\sqrt{7}\right)^2}$$

$$= \sqrt{7 + 2\sqrt{14} + 2 + 3 + 2\sqrt{21} + 7}$$

$$= \sqrt{19 + 2\sqrt{21} + 2\sqrt{14}}$$

Exercise

Find the midpoint of the line segment with endpoints (1, 2) and (7, –3)

Solution

$$\left(\frac{1+7}{2}, \frac{2-3}{2}\right)$$

$$\rightarrow \left(\frac{8}{2}, \frac{-1}{2}\right)$$

$$\left(4, \frac{-1}{2}\right)$$

Exercise

Find the midpoint of the segment with endpoints $P(8,\ 2)$ and $Q(3,\ 5)$

Solution

$$M\left(\frac{8+3}{2}, \frac{2+5}{2}\right)$$

$$M\left(\frac{11}{2}, \frac{7}{2}\right)$$

Exercise

Find the midpoint of the segment with endpoints $P(-4,\ 3)$ and $Q(2,\ -5)$

Solution

$$M\left(\frac{-4+2}{2},\frac{3-5}{2}\right)$$

$$M\ (-1,\ -1)$$

Exercise

Find the midpoint of the segment whose endpoints are (4, –9) and (–12, –3).

Solution

$$\left(\frac{4-12}{2},\frac{-9-3}{2}\right)$$

$$\left(\frac{-8}{2},\frac{-12}{2}\right)$$

$$(-4,\ -6)$$

Exercise

Find the midpoint of the segment whose endpoints are (7, –2) and (9, 5).

Solution

$$\left(\frac{7+9}{2},\frac{-2+5}{2}\right)$$

$$\left(8,\ \frac{3}{2}\right)$$

Exercise

Find the midpoint of the line segment with endpoints $(-2,-1)$ and $(-8,6)$

Solution

$$\left(\frac{-2-8}{2},\frac{-1+6}{2}\right)$$

$$\left(-5,\ \frac{5}{2}\right)$$

Exercise

Find the midpoint of the line segment with endpoints $\left(7\sqrt{3},-6\right)$ and $\left(3\sqrt{3},-2\right)$

Solution

$$\left(\frac{7\sqrt{3}+3\sqrt{3}}{2},\frac{-6-2}{2}\right)$$

$$\boxed{\left(5\sqrt{3},\ -4\right)}$$

Exercise

Find the center-radius form of the equation of center $\left(-\sqrt{3},-\sqrt{3}\right)$, radius $\sqrt{3}$

Solution

$$\left(x+\sqrt{3}\right)^2+\left(y+\sqrt{3}\right)^2=\left(\sqrt{3}\right)^2$$

$$\boxed{\left(x+\sqrt{3}\right)^2+\left(y+\sqrt{3}\right)^2=3}$$

Exercise

Write the standard form of the equation of the circle with center (0, 0) and radius 4.

Solution

$$(x-0)^2+(y-0)^2=4^2$$

$$\boxed{x^2+y^2=16}$$

Exercise

Write the standard form of the equation of the circle with center (5, –6) and radius 10.

Solution

$$(x-5)^2+(y+6)^2=10^2$$

$$\boxed{(x-5)^2+(y+6)^2=100}$$

Exercise

Write the standard form of the equation of the circle with center $(2,-1)$ and $r=4$.

Solution

$$\boxed{(x-2)^2+(y+1)^2=16}$$

Exercise

Write the standard form of the equation of the circle with center $(-5,-3)$ and $r=\sqrt{5}$.

Solution

$$\boxed{(x+5)^2+(y+3)^2=5}$$

Exercise

Find an equation of the circle having radius 5 and center (3, -7)

Solution

$$(x-3)^2+(y-(-7))^2=5^2$$

$$\boxed{(x-3)^2+(y+7)^2=25}$$

Exercise

Find the equation for the circle with center (–5, 2) passing through (–1, 5)

Solution

$$\begin{aligned}\text{Radius} &= \sqrt{(-1-(-5))^2+(2-5)^2}\\ &= \sqrt{(-1+5)^2+(-3)^2}\\ &= \sqrt{(4)^2+(-3)^2}\\ &= \sqrt{16+9}\\ &= \sqrt{25}\end{aligned}$$

Equation of the circle: $\boxed{(x+5)^2+(y-2)^2=25}$ $\qquad (x-h)^2+(y-k)^2=r^2$

Exercise

Find the equation for the circle with a diameter whose endpoints are (4, 4) and (–2, 3)

Solution

Center = midpoint of the endpoints

$$= \left(\frac{4-2}{2}, \frac{4+3}{2}\right)$$

$$= \left(1, \frac{7}{2}\right)$$

$$\text{Radius} = \sqrt{(1-4)^2 + \left(\frac{7}{2}-4\right)^2}$$

$$= \sqrt{(-3)^2 + \left(-\frac{1}{2}\right)^2}$$

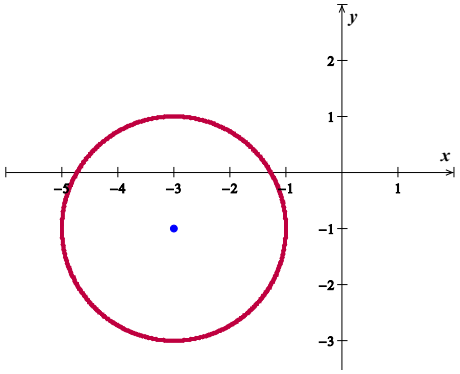
$$= \sqrt{9 + \frac{1}{4}} \qquad 9+\frac{1}{4}=\frac{4(9)+1}{4}=\frac{37}{4} \qquad 9 + \frac{1}{4} = 9\frac{4}{4} + \frac{1}{4} = \frac{4(9)+1}{4} = \frac{37}{4}$$

$$= \sqrt{\frac{37}{4}}$$

Equation of the circle: $\boxed{(x-1)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{37}{4}}$ $\qquad (x-h)^2 + (y-k)^2 = r^2$

Exercise

Find the center and the radius of $x^2 + y^2 + 6x + 2y + 6 = 0$



Solution

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 + 2y + \left(\frac{2}{2}\right)^2 = -6 + 9 + 1$$

$$(x+3)^2 + (y+1)^2 = 4$$

Center (–3, –1) and $r = 2$

Exercise

Find the center and the radius of $x^2 + y^2 + 8x + 4y + 16 = 0$

Solution

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 + y^2 + 4y + \left(\frac{4}{2}\right)^2 = -16 + 16 + 4$$

$$(x+4)^2 + (y+2)^2 = 4$$

Center (–4, –2) and $r = 2$

Exercise

Find the center and the radius of $x^2 + y^2 - 10x - 6y - 30 = 0$

Solution

$$x^2 - 10x + \left(\frac{-10}{2}\right)^2 + y^2 - 6y + \left(\frac{-6}{2}\right)^2 = 30 + 25 + 9$$

$$(x-5)^2 + (y-3)^2 = 64$$

Center (5, 3) and $r = 8$

Exercise

Find the center and the radius of $x^2 - 6x + y^2 + 10y + 25 = 0$

Solution

$$x^2 - 6x \quad + y^2 + 10y \quad = -25$$

$$x^2 - 6x + \left(\tfrac{1}{2}(-6)\right)^2 + y^2 + 10y + \left(\tfrac{1}{2}(10)\right)^2 = -25 + \left(\tfrac{1}{2}(-6)\right)^2 + \left(\tfrac{1}{2}(10)\right)^2$$

$$(x-3)^2 + (y+5)^2 = -25 + 9 + 25$$

$$(x-3)^2 + (y+5)^2 = 9$$

The equation represents a circle with ***center*** at (3, –5) and ***radius*** 3