1/ $\int_{-2}^{2}(3x^4-2x+1)\,dx = \frac{3}{5}x^5 - x^2 + x \Big|_{-2}^{2}$

$= \frac{96}{5} - 4 + 2 - \left(-\frac{96}{5} - 4 - 2\right)$

$= \frac{182}{5} + 4$

$= \boxed{\frac{202}{5}}$

2/ $\int_{0}^{1}(4x^{21} - 2x^{16} + 1)\,dx = \frac{2}{11}x^{22} - \frac{2}{17}x^{17} + x \Big|_{0}^{1}$

$= \frac{2}{11} - \frac{2}{17} + 1$

$= \boxed{\frac{199}{187}}$

3/ $f(x) = 16 - x^2 = 0 \qquad x \in [-4, 4]$

$\boxed{x = \pm 4}$

$A = \int_{-4}^{4}(16 - x^2)\,dx$

$= 16x - \frac{1}{3}x^3 \Big|_{-4}^{4}$

$= 4^3 - \frac{1}{3}4^3 - \left(-4^3 + \frac{4^3}{3}\right)$

$= 64\left(1 - \frac{1}{3} + 1 - \frac{1}{3}\right)$

$= 64\left(2 - \frac{2}{3}\right)$

$= \boxed{\frac{256}{3}}\ \boxed{\text{unit}^2}$

4/ $f(x) = x^3 - x \qquad x \in [-1, 0]$

$x(x^2-1) \stackrel{\text{set}}{=} 0 \Rightarrow x = 0, \pm 1$

$$A = \int_{-1}^{0} (x^3 - x)\,dx$$

$$= \frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_{-1}^{0}$$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4} \text{ unit}^2$$

5/ $f(x) = x^2 - x = 0 \qquad x \in [0, 3]$

$x = 0, 1$

$$A = -\int_0^1 (x^2 - x)\,dx + \int_1^3 (x^2 - x)\,dx$$

$$= -\left(\frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_0^1 + \left(\frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_1^3\right.\right.$$

$$= -\left(\frac{1}{3} - \frac{1}{2}\right) + 9 - \frac{9}{2} - \frac{1}{3} + \frac{1}{2}$$

$$= \frac{1}{6} - \frac{1}{3} + 5$$

$$= \frac{27}{6}$$

$$= \frac{9}{2} \text{ unit}^2$$

6/ $f(x) = x^4 - x^2 = 0$ $[-1, 1]$

$x = \underbrace{0, 0}, \pm 1$ $x^2(x^2-1)$

below x-axis.

$$A = \left| \int_{-1}^{1} (x^4 - x^2)\,dx \right|$$

$$= \left| \frac{1}{5}x^5 - \frac{1}{3}x^3 \Big|_{-1}^{1} \right|$$

$$= \left| \frac{1}{5} - \frac{1}{3} + \frac{1}{5} - \frac{1}{3} \right|$$

$$= \frac{4}{15} \text{ unit}^2$$