Exercise

Determine whether $f(x)$ is continuous on the entire number line. Explain your reasoning.

$$f(x)=\frac{x}{x^2-1}$$

Solution

$x^2-1=0\rightarrow x^2=1\Rightarrow \boxed{x=\pm 1}$

The function is continuous on $(-\infty,\ -1)\cup(-1,\ 1)\cup(1,\ \infty)$

Exercise

Determine whether $f(x)$ is continuous on the entire number line. Explain your reasoning.

$$f(x)=\frac{x-5}{x^2-9x+20}$$

Solution

$x^2-9x+20=0\Rightarrow \boxed{x=4,\ 5}$

The function is continuous on $(-\infty,\ 4)\cup(4,\ 5)\cup(5,\ \infty)$

Exercise

Find the slope of the graph of $f(x) = 2x + 5$

Solution

$$m=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{2(x+\Delta x)+5-(2x+5)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{2x+2\Delta x+5-2x-5}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{2\Delta x}{\Delta x}$$

$$=\lim_{\Delta x\to 0}2$$

$= 2$

Exercise

Find the slope of the graph of $f(x) = \sqrt{x}$

Solution

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

$$m = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$