3.1

Ex $z = 4 - x - y$ $\quad R: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le 1 \end{cases}$

$$V = \int_0^1 \int_0^2 (4 - x - y)\, dx\, dy$$

$$= \int_0^1 \left(4x - \frac{1}{2}x^2 - yx\right)\Big|_0^2 dy$$

$$= \int_0^1 (8 - 2 - 2y)\, dy$$

$$= 6y - y^2 \Big|_0^1$$

$$= 6 - 1$$

$$= 5 \text{ unit}^3$$

Ex $f(x,y) = 100 - 6x^2y$

$R: \quad 0 \le x \le 2 \qquad -1 \le y \le 1$

$$\int_{-1}^{1}\int_{0}^{2} (100 - 6x^2y)\,dx\,dy$$

$$= \int_{-1}^{1} \left(100x - 2x^3y\right)\Big|_0^2 dy$$

$$= \int_{-1}^{1} (200 - 16y)\,dy$$

$$= 200y - 8y^2\Big|_{-1}^{1}$$

$$= 200 - 8 + 200 + 8$$

$$= 400$$

$$\iint e^{4x} y^3\,dy\,dx = \int e^{4x}\,dx \int y^3\,dy$$

$$= \frac{1}{16} e^{4x} y^4 + C$$

Ex V? $z = 10 + x^2 + 3y^2$ $\begin{cases} 0 \le x \le 1 \\ 0 \le y \le 2 \end{cases}$

$$V = \int_0^1 \int_0^2 (10 + x^2 + 3y^2)\, dy\, dx$$

$$= \int_0^1 \left(10y + x^2 y + y^3 \Big|_0^2\right) dx$$

$$= \int_0^1 (20 + 2x^2 + 8)\, dx$$

$$= 28x + \frac{2}{3}x^3 \Big|_0^1$$

$$= 28 + \frac{2}{3}$$

$$= \frac{83}{3} \text{ unit}^3$$

Ex $\int_0^1 \int_y^1 y e^{-x^3} dx\, dy$

$1 \le x \le y$

$x = 1$

$x = y$ [$y=0 \to x=0$, $y=1 \to x=1$

$y = x$

$0 \le y \le 1$

$y = 0$

$y = 1$

$0 \le x \le 1$ $\quad$ $0 \le y \le x$

$$\int_0^1 \int_y^1 y e^{-x^3} dx\, dy = \int_0^1 \int_0^x y e^{-x^3} dy\, dx$$

$$= \frac{1}{2} \int_0^1 y^2 e^{-x^3} \Big|_0^x dx$$

$$= \frac{1}{2} \int_0^1 x^2 e^{-x^3} dx$$

$$= -\frac{1}{6} \int_0^1 e^{-x^3} d(-x^3)$$

$$= -\frac{1}{6} e^{-x^3} \Big|_0^1$$

$$= -\frac{1}{6} (e^{-1} - 1)$$

$$= \frac{1}{6} - \frac{1}{6e}$$

$$= \frac{1}{6}\left(1 - \frac{1}{e}\right)$$

$\frac{1}{6}\left(1 - \frac{1}{e}\right)$