Solution *Section* 2.1 – Graphs and Level Curves

Exercise

Find the specific values for $f(x,y,z)=\dfrac{x-y}{y^2+z^2}$

a) $f(3,-1,2)$ *b)* $f\left(1,\ \frac{1}{2},\ -\frac{1}{4}\right)$ *c)* $f\left(0,\ -\frac{1}{3},\ 0\right)$ *d)* $f(2,\ 2,\ 100)$

Solution

a) $$f(3,-1,2)=\frac{3-(-1)}{(-1)^2+2^2}$$
$$=\frac{4}{5}$$

b) $$f\left(1,\ \frac{1}{2},\ -\frac{1}{4}\right)=\frac{1-\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2+\left(-\frac{1}{4}\right)^2}$$
$$=\frac{\frac{1}{2}}{\frac{1}{4}+\frac{1}{16}}$$
$$=\frac{\frac{1}{2}}{\frac{5}{16}}$$
$$=\frac{8}{5}$$

c) $$f\left(0,\ -\frac{1}{3},\ 0\right)=\frac{0-\left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)^2+0^2}$$
$$=\frac{\frac{1}{3}}{\frac{1}{9}}$$
$$=3$$

d) $$f(2,\ 2,\ 100)=\frac{2-(2)}{(2)^2+100^2}$$
$$=0$$

Exercise

Find the specific values for $f(x,y,z)=\sqrt{49-x^2-y^2-z^2}$

a) $f(0,\ 0,\ 0)$ b) $f(2,\ -3,\ 6)$ c) $f(-1,\ 2,\ 3)$ d) $f\left(\frac{4}{\sqrt{2}},\ \frac{5}{\sqrt{2}},\ \frac{6}{\sqrt{2}}\right)$

Solution

a) $f(0,\ 0,\ 0)=\sqrt{49-0^2-0^2-0^2}$

$$=7$$

b) $f(2,\ -3,\ 6)=\sqrt{49-2^2-(-3)^2-6^2}$

$$=0$$

c) $f(-1,\ 2,\ 3)=\sqrt{49-(-1)^2-2^2-3^2}$

$$=\sqrt{35}$$

d) $f\left(\frac{4}{\sqrt{2}},\ \frac{5}{\sqrt{2}},\ \frac{6}{\sqrt{2}}\right)=\sqrt{49-\left(\frac{4}{\sqrt{2}}\right)^2-\left(\frac{5}{\sqrt{2}}\right)^2-\left(\frac{6}{\sqrt{2}}\right)^2}$

$$=\sqrt{49-\frac{16}{2}-\frac{25}{2}-\frac{36}{2}}$$

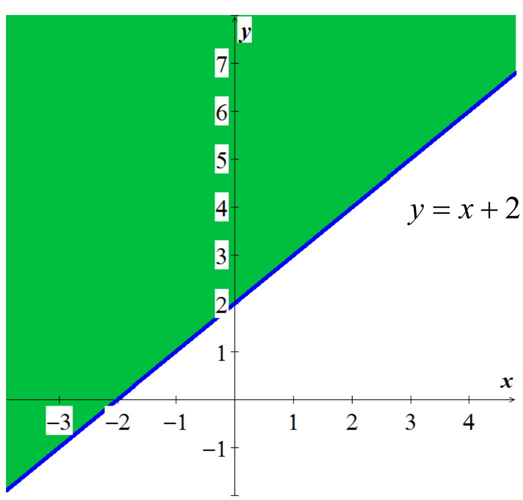
$$=\sqrt{\frac{21}{2}}$$

Exercise

Find and sketch the domain for function $f(x,y)=\sqrt{y-x-2}$

Solution

$y-x-2\ge 0 \quad\Rightarrow\quad y\ge x+2$



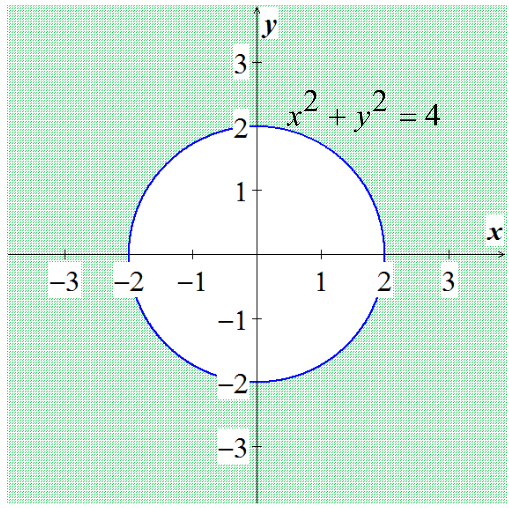
Exercise

Find and sketch the domain for function $f(x,y)=\ln\left(x^2+y^2-4\right)$

Solution

$$x^2+y^2-4>0 \quad\Rightarrow\quad x^2+y^2>4$$

Domain: All points (x,y) outside the circle



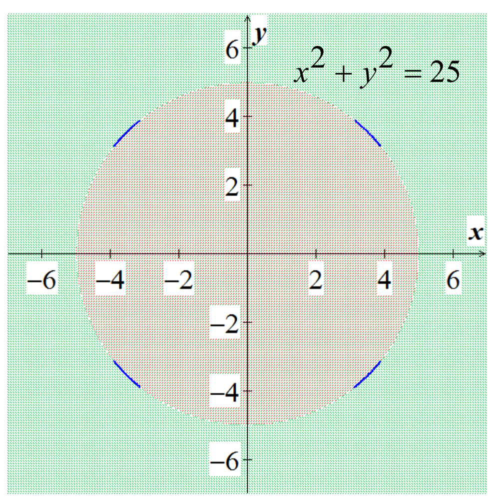
Exercise

Find and sketch the domain for function $f(x,y)=\dfrac{\sin(xy)}{x^2+y^2-25}$

Solution

$$x^2+y^2-25\neq 0 \quad\Rightarrow\quad x^2+y^2\neq 25$$

Domain: All points (x,y) not lying on the circle $x^2+y^2=25$



Exercise

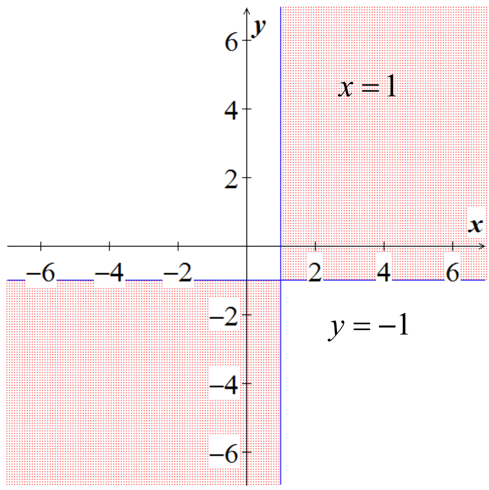
Find and sketch the domain for function $f(x,y) = \ln(xy + x - y - 1)$

Solution

$$xy + x - y - 1 > 0 \quad \Rightarrow \quad x(y+1) - (y+1) > 0$$

$$(x-1)(y+1) > 0$$

Domain: All points (x, y) satisfying $(x-1)(y+1) > 0$



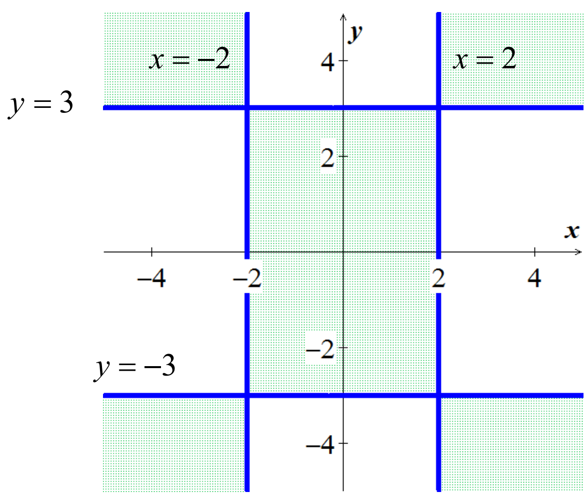
Exercise

Find and sketch the domain for function $f(x,y) = \sqrt{(x^2-4)(y^2-9)}$

Solution

$$(x^2-4)(y^2-9) \geq 0 \quad \Rightarrow \quad (x-2)(x+2)(y-3)(y+3) \geq 0$$

Domain: All points (x, y) satisfying $(x-2)(x+2)(y-3)(y+3) \geq 0$

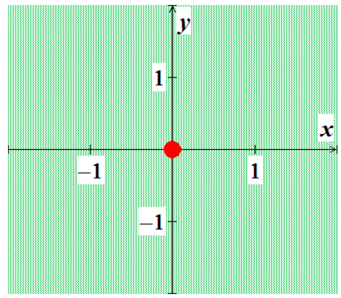


Exercise

Find and sketch the domain for function $f(x, y) = \dfrac{1}{x^2 + y^2}$

Solution

$$Domain = \{\ (x, y) \mid\ (x, y) \neq (0,\ 0)\}$$

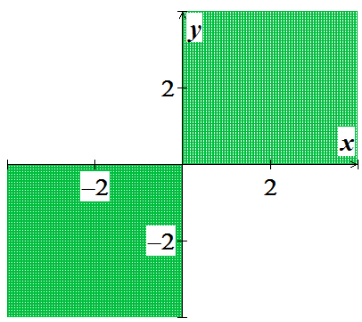


Exercise

Find and sketch the domain for function $f(x, y) = \ln xy$

Solution

$$Domain = \{\ (x, y) \mid\ xy > 0\}$$

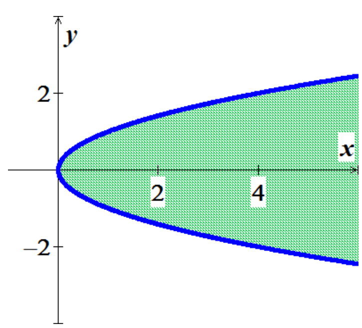


Exercise

Find and sketch the domain for function $f(x, y) = \sqrt{x - y^2}$

Solution

$$Domain = \left\{\ (x, y) \mid\ x > y^2\right\}$$

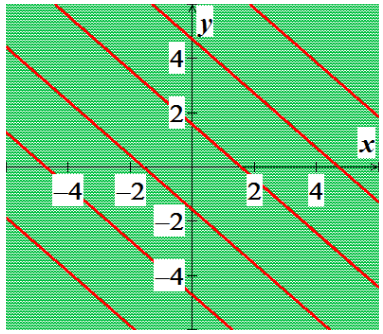


Exercise

Find and sketch the domain for function $f(x, y) = \tan(x + y)$

Solution

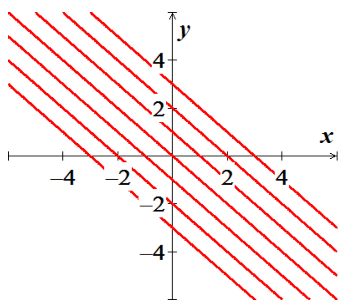
$$Domain = \left\{ (x, y) \mid x + y \neq \frac{\pi}{2} + k\pi \right\} \qquad (k \in \mathbb{Z})$$



Exercise

Find and sketch the level curves $f(x,y) = c$ on the same set of coordinate axes for the given values of c, we refer to these level curves as a contour map. $f(x,y) = x + y - 1, \quad c = -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3$

Solution

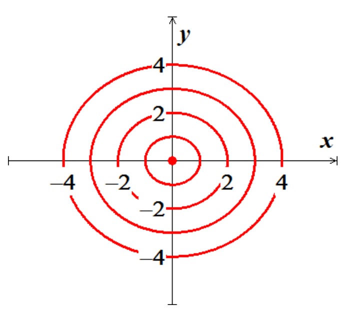


Exercise

Find and sketch the level curves $f(x,y) = c$ on the same set of coordinate axes for the given values of c, we refer to these level curves as a contour map.

$f(x,y) = x^2 + y^2, \quad c = 0,\ 1,\ 4,\ 9,\ 16,\ 25$

Solution



Exercise

For the function: $f(x,y) = 4x^2 + 9y^2$:

a) Find the function's domain
b) Find the function's range
c) Find the function's level curves
d) Find the boundary of the function's domain
e) Determine if the domain is an open region, a closed region, or neither
f) Decide if the domain is bounded or unbounded

Solution

a) *Domain*: all points in the *xy*-plane

b) *Range*: $z \ge 0$

c) Level curves: For $f(x,y) = 0 \quad \to \quad Origin$

For $f(x,y) = c > 0 \quad \to \quad$ *ellipses* with center (0, 0) and major and minor axes along the *x*- and *y*-axes, respectively

d) No boundary points

e) Both open and closed

f) Unbounded

Exercise

For the function: $f(x,y) = xy$:

a) Find the function's domain
b) Find the function's range
c) Find the function's level curves
d) Find the boundary of the function's domain
e) Determine if the domain is an open region, a closed region, or neither
f) Decide if the domain is bounded or unbounded

Solution

a) *Domain*: all points in the *xy*-plane

b) *Range*: $\mathbb{R}$

c) Level curves: Hyperbolas with the *x*- and *y*-axes as asymptotes when $f(x,y) \ne 0$ and the *x*- and *y*-axes when $f(x,y) = 0$

d) No boundary points

e) Both open and closed

f) Unbounded

Exercise

For the function: $f(x,y) = e^{-\left(x^2+y^2\right)}$

a) Find the function's domain
b) Find the function's range
c) Find the function's level curves
d) Find the boundary of the function's domain
e) Determine if the domain is an open region, a closed region, or neither
f) Decide if the domain is bounded or unbounded

Solution

a) *Domain*: all points in the *xy*-plane

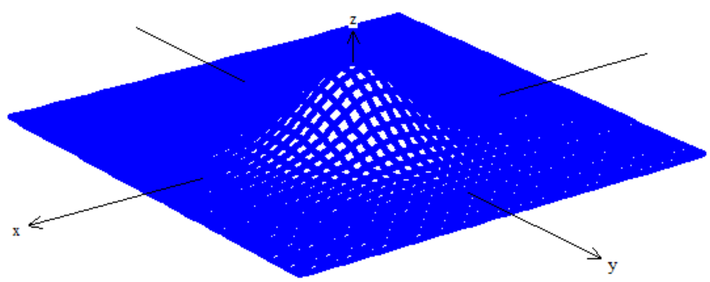
b) *Range*: $0 < z \le 1$

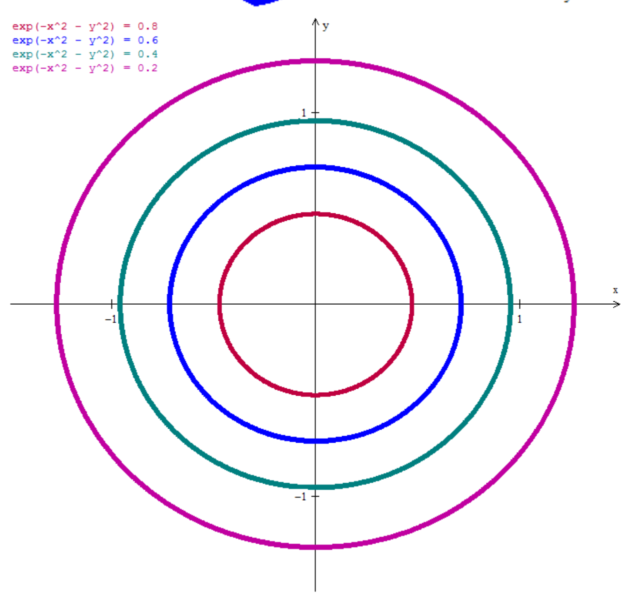
c) Level curves are the origin itself and the circles with center (0, 0) and radii $r > 0$

d) No boundary points

e) Both open and closed

f) Unbounded





Exercise

For the function: $f(x,y) = \ln\left(9 - x^2 - y^2\right)$

a) Find the function's domain
b) Find the function's range
c) Find the function's level curves
d) Find the boundary of the function's domain
e) Determine if the domain is an open region, a closed region, or neither
f) Decide if the domain is bounded or unbounded

Solution

$$9 - x^2 - y^2 > 0 \quad \rightarrow \quad x^2 + y^2 < 9$$

a) *Domain*: all points inside the circle $x^2 + y^2 = 9$

b) *Range*: $z < \ln 9$

c) Level curves are circles centered at the origin and radii $r < 9$

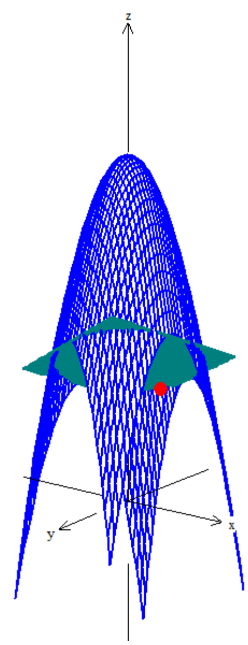
d) Boundary: the circle $x^2 + y^2 = 9$

e) Open

f) Bounded

Exercise

Find an equation for $f(x,y) = 16 - x^2 - y^2$ and sketch the graph of the level curve of the function $f(x,y)$ that passes through the point $\left(2\sqrt{2},\ \sqrt{2}\right)$



Solution

$$z = \left(16 - x^2 - y^2\right)_{\left(2\sqrt{2},\ \sqrt{2}\right)}$$

$$= 16 - \left(2\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2$$

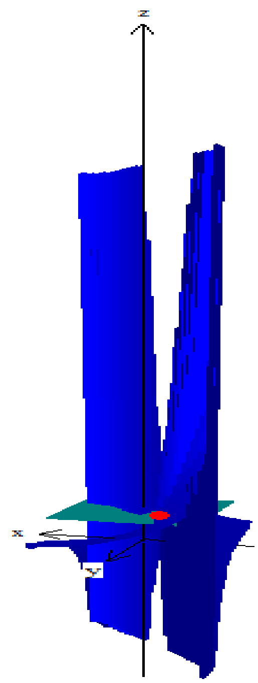
$$= 6$$

$$6 = 16 - x^2 - y^2$$

$$\underline{x^2 + y^2 = 10}$$

Exercise

Find an equation for $f(x,y)=\dfrac{2y-x}{x+y+1}$ and sketch the graph of the level curve of the function $f(x,y)$ that passes through the point $(-1,\ 1)$



Solution

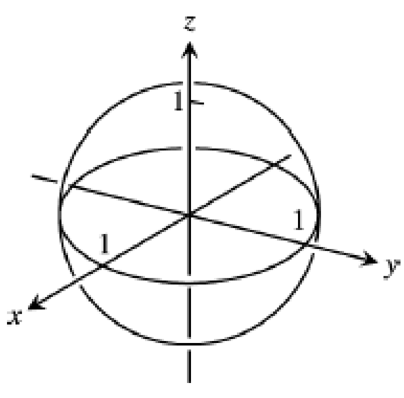
$$z=\left(\frac{2y-x}{x+y+1}\right)_{(-1,1)}$$
$$=\frac{2(1)-(-1)}{-1+1+1}$$
$$=3$$

$$3=\frac{2y-x}{x+y+1}$$
$$3x+3y+3=2y-x$$
$$\underline{y=-4x-3}$$

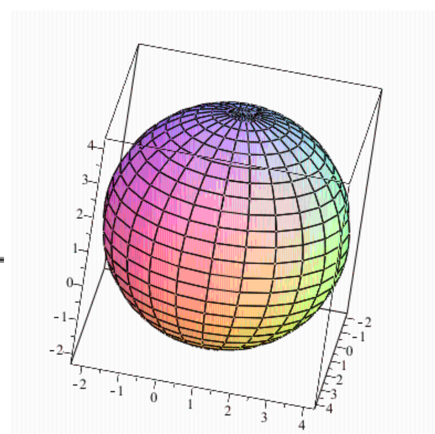
Exercise

Sketch a typical level surface for the function $f(x,y,z)=x^2+y^2+z^2$

Solution



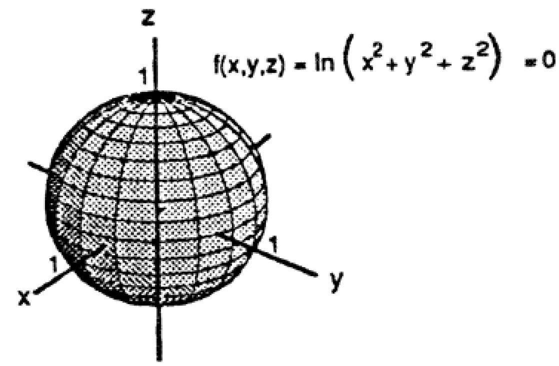
$f(x,y,z)=x^2+y^2+z^2=1$



Exercise

Sketch a typical level surface for the function $f(x,y,z)=\ln\left(x^2+y^2+z^2\right)$

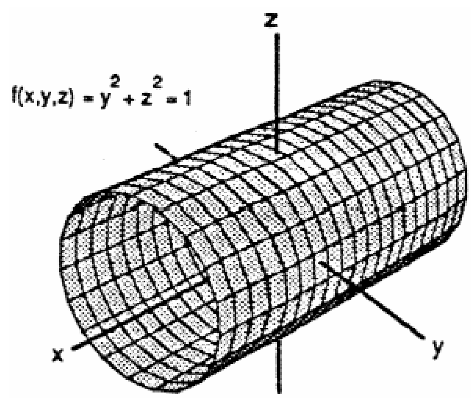
Solution



Exercise

Sketch a typical level surface for the function $f(x,y,z) = y^2 + z^2$

Solution



Exercise

Sketch a typical level surface for the function $f(x,y,z) = z - x^2 - y^2$

Solution