Solution *Section* 3.3 – Counting; Multiplication Principle

Exercise

How many different types of homes are available if a builder offers a choice of 6 basic plans, 3 roof styles, and 2 exterior finishes?

Solution

$$6.3.2 = 36 \ \textit{different homes types}$$

Exercise

A menu offers a choice of 3 salads, 8 main dishes, and 7 desserts. How many different meals consisting of one salad, one main dish, and one dessert are possible?

Solution

$$3.8.7 = 168 \ \textit{different meals}$$

Exercise

A couple has narrowed down the choice of a name for their new baby to 4 first names and 5 middle names. How many different first- and middle-name arrangements are possible?

Solution

$$4.5 = 20 \ \textit{possible arrangements}$$

Exercise

An automobile manufacturer produces 8 models, each available in 7 different exterior colors, with 4 different upholstery fabrics and 5 interior colors. How many varieties of automobile are available?

Solution

$$8.7.4.5 = \underline{1120}$$

Exercise

A biologist is attempting to classify 52,000 species of insects by assigning 3 initials to each species. Is it possible to classify all the species in this way? If not, how many initials should be used?

Solution

$26^3 = 17,576$ *This would not be enough.*

$26^4 = 456,976$ *Which is more than enough*

Exercise

How many 4-letter code words are possible using the first 10 letters of the alphabet under:

a) No letter can be repeated
b) Letters can be repeated
c) Adjacent can't be alike

Solution

a) 10.9.8.7 = 5040

b) 10.10.10.10 = 10,000

c) 10.9.9.9 = 7290

Exercise

3 letters license plate without repeats:

Solution

26.25.24 = 15600 possible

Exercise

How many ways can 2 coins turn up heads, H, or tails, T – if the combined outcome (H, T) is to be distinguished from the outcome (T, H)?

Solution

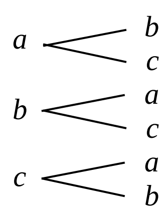
2x2 = 4 ***outcomes***

Exercise

How many 2-letter code words can be formed from the first 3 letters of the alphabet if no letter can be used more than once?

Solution

3x2 = 6 ***outcomes***



Exercise

A coin is tossed with possible outcomes heads, H, or tails, T. Then a single die is tossed with possible outcomes 1, 2, 3, 4, 5, or 6. How many combined outcomes are there?

Solution

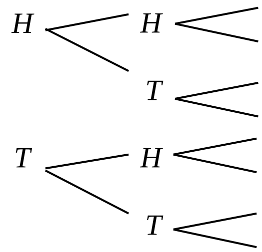
2x6 = 12 ***outcomes***

Exercise

In how many ways can 3 coins turn up heads, H, or tails, T – if combined outcomes such as (H,T,H), (H, H, T), and (T, H, H) are to be considered different?

Solution

$2.2.2 = 8$



Exercise

An entertainment guide recommends 6 restaurants and 3 plays that appeal to a couple.

a) If the couple goes to dinner or to a play, how many selections are possible?

b) If the couple goes to dinner and then to a play, how many combined selections are possible?

Solution

a) $3 + 6 = 9$

b) $6.3 = 18$

Exercise

There are 18 mathematics majors and 325 computer science majors at a college

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

b) In how many ways can one representative be picked who either a mathematics major or a computer science major?

Solution

a) $18 \cdot 325 \boxed{= 5850 \;\; \textit{ways}}$

b) $18 + 325 \boxed{= 343 \;\; \textit{ways}}$

Exercise

An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

Solution

Using the product rule: there are $27 \cdot 37 \boxed{= 999 \;\; \textit{offices}}$

Exercise

A multiple-choice test contains 10 questions. There are four possible answers for each question

a) In how many ways can a student answer the questions on the test if the student answers every question?

b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

Solution

a) $4\cdot 4\cdot 4\cdots 4 = 4^{10} = \boxed{1{,}048{,}576\ ways}$

b) There are 5 ways to answer each question 0 give any if the 4 answers or give no answer at all

$5^{10} = \boxed{9{,}765{,}625\ ways}$

Exercise

A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of the shirts are made?

Solution

$12\cdot 2\cdot 3 = \boxed{72}$ different types of shirt.

Exercise

How many different three-letter initials can people have?

Solution

$26\cdot 26\cdot 26 = \boxed{17{,}576}$ different initials.

Exercise

How many different three-letter initials with none of the letters repeated can people have?

Solution

$26\cdot 25\cdot 24 = \boxed{15{,}600\ \ ways}$

Exercise

How many different three-letter initials are there that begin with an A?

Solution

$1\cdot 26\cdot 26 = \boxed{676\ \ ways}$

Exercise

How many strings are there of four lowercase letters that have the letter x in them?

Solution

Number of strings of length of 4 lowercase: 26^4

Number of strings of length of 4 lowercase other than $\boldsymbol{x}$: 25^4

$26^4 - 25^4 = \underline{66{,}351 \ \ strings}$

Exercise

How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

Solution

$10^3 \cdot 26^3 + 26^3 \cdot 10^3 = \underline{35{,}152{,}000 \ \ license\ plates}$

Exercise

How many license plates can be made using either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters?

Solution

Letters		***Digits***			
L	L	D	D	D	D
26	26	10	10	10	10

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 6{,}760{,}000$

Digits		***Letters***			
D	D	L	L	L	L
10	10	26	26	26	26

$10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 45{,}697{,}600$

Therefore: $6{,}760{,}000 + 45{,}697{,}600 = \underline{52{,}457{,}600 \ \ license\ plates}$

Exercise

How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

Solution

$26^3 \cdot 10^3 + 26^4 \cdot 10^2 = \underline{63{,}273{,}600 \ \ license\ plates}$

Exercise

How many strings of eight English letter are there

a) That contains no vowels, if letters can be repeated?

b) That contains no vowels, if letters cannot be repeated?

c) That starts with a vowel, if letters can be repeated?

d) That starts with a vowel, if letters cannot be repeated?

e) That contains at least one vowel, if letters can be repeated?
f) That contains at least one vowel, if letters cannot be repeated?

Solution

a) There are 8 slots which can be filled with $26-5=21$ non-vowels.

By the product rule: $21^8 = \boxed{37,822,859,361 \ \ strings}$

b) $21\cdot 20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15\cdot 14 = \boxed{8,204,716,800 \ \ strings}$

c) $5\cdot 26^7 = \boxed{40,159,050,880 \ \ strings}$

d) $5\cdot 25\cdot 24\cdot 23\cdot 22\cdot 21\cdot 20\cdot 19 = \boxed{12,113,640,000 \ \ strings}$

e) By the product rule: $26^8 - 21^8 = \boxed{171,004,205,215 \ \ strings}$

f) $8\cdot 5\cdot 21^7 = \boxed{72,043,541,640 \ \ strings}$

	1	2	3	4	5	6	7	8
	NV	**NV**	**NV**	**NV**	**NV**	**NV**	**NV**	**NV**
a	21	21	21	21	21	21	21	21
b	21	20	19	18	17	16	15	14
	V	**L**	**L**	**L**	**L**	**L**	**L**	**L**
c	5	26	26	26	26	26	26	26
d	5	25	24	23	22	21	20	19

Exercise

In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) The bride must be in the picture?
b) Both the bride and groom must be in the picture?
c) Exactly one of the bride and the groom is in the picture?

Solution

a) The bride is in any of the 6 positions.
Then, it will leave us with 5 remaining positions.
This can be done in $9\cdot 8\cdot 7\cdot 6\cdot 5 = 15120$ ways.
Therefore $6\cdot 15120 = \boxed{90,720 \ \ ways}$

1	2	3	4	5	6
B	***P***	***P***	***P***	***P***	***P***
1	9	8	7	6	5

b) The bride is in any of the 6 positions.
Then place the groom in any of the 5 remaining positions.
Then, it will leave us with 4 remaining positions in the picture.
This can be done in $8\cdot 7\cdot 6\cdot 5 = 1680$ ways.
Therefore $6\cdot 5\cdot 1680 = \boxed{50,400 \ \ ways}$

1	2	3	4	5	6
B	***G***	***P***	***P***	***P***	***P***
1	1	8	7	6	5

c) For the just the bride to be in the picture: $90720 - 50400 = 40,320$ ways. There are 40,320 ways for just the groom to be in the picture. Therefore $40320 + 40320 = \boxed{80,640 \ \ ways}$