Solution Section 4.5 – Polar Coordinates

Exercise

Convert to rectangular coordinates $\left(2,\ 60°\right)$

Solution

$$\left(2,\ 60°\right)=2\left(\cos 60°+i\sin 60°\right)$$

$$=2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$$

$$=1+i\sqrt{3}$$

Exercise

Convert to rectangular coordinates $\left(\sqrt{2},\ -225°\right)$

Solution

$$\left(\sqrt{2},\ -225°\right)=\sqrt{2}\left(\cos(-225°)+i\sin(-225°)\right)$$

$$=\sqrt{2}\left(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)$$

$$=-1+i$$

Exercise

Convert to rectangular coordinates $\left(4\sqrt{3},\ -\frac{\pi}{6}\right)$

Solution

$$\left(4\sqrt{3},\ -\frac{\pi}{6}\right)=4\sqrt{3}\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$$

$$=4\sqrt{3}\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)$$

$$=6-2\sqrt{3}$$

Exercise

Convert to polar coordinates $\qquad (-3,\ -3) \quad r \ge 0 \quad 0° \le \theta < 360°$

Solution

$$(-3,\ -3) \to \begin{cases} r = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2} \\ \hat{\theta} = \tan^{-1}\left(\frac{3}{3}\right) = \tan^{-1}(1) = 45° \end{cases}$$

The angle is in quadrant III; therefore, $\theta = 180° + 45° = 225°$

$$\boxed{(-3,\ -3) \ = \ \left(3\sqrt{2},\ 225°\right)}$$

Exercise

Convert to polar coordinates $\qquad \left(2,\ -2\sqrt{3}\right) \quad r \ge 0 \quad 0° \le \theta < 360°$

Solution

$$\left(2,\ -2\sqrt{3}\right) \to \begin{cases} r = \sqrt{2^2 + \left(-2\sqrt{3}\right)^2} = 4 \\ \hat{\theta} = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = \tan^{-1}\left(\sqrt{3}\right) = 60° \end{cases}$$

The angle is in quadrant IV; therefore, $\theta = 360° - 60° = 300°$

$$\boxed{\left(2,\ -2\sqrt{3}\right) \ = \ (4,\ 300°)}$$

Exercise

Convert to polar coordinates $\qquad (-2,\ 0) \quad r \ge 0 \quad 0 \le \theta < 2\pi$

Solution

$$(-2,\ 0) \to \begin{cases} r = \sqrt{(-2)^2 + 0^2} = 2 \\ \hat{\theta} = \tan^{-1}\left(\frac{0}{2}\right) = 0 \Rightarrow \theta = \pi \end{cases}$$

$$\boxed{(-2,\ 0) \ = \ (2,\ \pi)}$$

Exercise

Convert to polar coordinates $\left(-1,\ -\sqrt{3}\right)$ $r \ge 0 \quad 0 \le \theta < 2\pi$

Solution

$$\left(-1,\ -\sqrt{3}\right) \to \begin{cases} r = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} = 2 \\ \hat{\theta} = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3} \end{cases}$$

The angle is in quadrant III; therefore, $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$$\boxed{\left(-1,\ -\sqrt{3}\right) = \left(2,\ \frac{4\pi}{3}\right)}$$

Exercise

Write the equation in rectangular coordinates $r^2 = 4$

Solution

$r^2 = 4$

$x^2 + y^2 = 4$

Exercise

Write the equation in rectangular coordinates $r = 6\cos\theta$

Solution

$r = 6\cos\theta$

$r = 6\frac{x}{r}$

$r^2 = 6x$

$x^2 + y^2 = 6x$

Exercise

Write the equation in rectangular coordinates $r^2 = 4\cos 2\theta$

Solution

$$r^2 = 4\left(\cos^2\theta - \sin^2\theta\right) \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$= 4\left(\left(\frac{x}{r}\right)^2 - \left(\frac{y}{r}\right)^2\right)$$

$$= 4\left(\frac{x^2}{r^2} - \frac{y^2}{r^2}\right)$$

$$= 4\left(\frac{x^2 - y^2}{r^2}\right)$$

$$r^4 = 4\left(x^2 - y^2\right) \qquad r^2 = x^2 + y^2$$

$$\left(x^2 + y^2\right)^4 = 4x^2 - 4y^2$$

Exercise

Write the equation in rectangular coordinates $r(\cos\theta - \sin\theta) = 2$

Solution

$$r(\cos\theta - \sin\theta) = 2 \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$r\left(\frac{x}{r} - \frac{y}{r}\right) = 2$$

$$r\left(\frac{x - y}{r}\right) = 2$$

$$x - y = 2$$

Exercise

Write the equation in polar coordinates $x + y = 5$

Solution

$$r\cos\theta + r\sin\theta = 5 \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$r(\cos\theta + \sin\theta) = 5$$

$$r = \frac{5}{\cos\theta + \sin\theta}$$

Exercise

Write the equation in polar coordinates $x^2 + y^2 = 9$

Solution

$$x^2 + y^2 = 9 \qquad r^2 = x^2 + y^2$$

$$r^2 = 9$$

Exercise

Write the equation in polar coordinates $x^2 + y^2 = 4x$

Solution

$$r^2 = 4r\cos\theta$$

$$\frac{r^2}{r} = \frac{4r\cos\theta}{r}$$

$$r = 4\cos\theta$$

Exercise

Write the equation in polar coordinates $y = -x$

Solution

$$y = -x \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$r\sin\theta = -r\cos\theta$$

$$\sin\theta = -\cos\theta$$