Math 2318 – Linear Algebra **Exam 3** **Review**

Professor: Fred Khoury

1. Let $\langle \boldsymbol{u}, \boldsymbol{v} \rangle$ be the Euclidean inner product on R^2, and let $\boldsymbol{u} = (3, \ -2)$, $\boldsymbol{v} = (4, \ 5)$, and $k = 4$.

Verify the following for the weighted Euclidean inner product $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = 3u_1v_1 + 4u_2v_2$

a) $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = \langle \boldsymbol{v}, \boldsymbol{u} \rangle$ *b)* $\langle k\boldsymbol{u}, \boldsymbol{v} \rangle = k\langle \boldsymbol{u}, \boldsymbol{v} \rangle$

2. Which of the following form orthonormal sets?

a) $\left(\frac{1}{\sqrt{3}}, \ -\frac{1}{\sqrt{3}}, \ \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{6}}, \ \frac{2}{\sqrt{6}}, \ \frac{1}{\sqrt{6}}\right)$ in $\boldsymbol{R}^3$

b) $\left(\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right), \left(0,\frac{\sqrt{6}}{3},\frac{1}{\sqrt{6}},-\frac{1}{\sqrt{6}}\right), \left(\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{6},\frac{\sqrt{3}}{6},-\frac{\sqrt{3}}{6}\right), \left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

3. Use the Gram-Schmidt process to find an orthonormal basis for the subspaces of $\boldsymbol{R}^m$.

a) $x_1 = (1, \ 1), \quad x_2 = (1, \ 2)$

b) $x_1 = (1, \ 2), \quad x_2 = (1, \ 3)$

c) $x_1 = (1, \ 2, \ 2), \quad x_2 = (2, \ 1, \ 3)$

d) $v_1 = (1,-1,-1,1), \quad v_2 = (2,1,0,1), \quad v_3 = (2,2,1,2)$

4. Find the ***QR***-decomposition of

a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ *b)* $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

5. Determine if the matrix is orthogonal. For those that is orthogonal find the inverse

a) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ *b)* $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

c) $\begin{pmatrix} \cos\theta\sin\theta & -\cos\theta & -\sin^2\theta \\ \cos^2\theta & \sin\theta & -\cos\theta\sin\theta \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$

Solution

1.

2. *a*) *Orthonormal* *b*) *Orthonormal*

3. *a*) $v_1 = (1,1),\ v_1 = \left(-\frac{1}{2}, \frac{1}{2}\right);\quad q_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right),\ q_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

b) $q_1 = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right),\ q_2 = \left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

c) $q_1 = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right),\quad q_2 = \left(\frac{8}{3\sqrt{26}},\ -\frac{11}{3\sqrt{26}},\ \frac{7}{3\sqrt{26}}\right)$

d) $q_1 = \left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right),\quad q_2 = \left(\frac{3\sqrt{5}}{10}, \frac{3\sqrt{5}}{10}, \frac{\sqrt{5}}{10}, \frac{\sqrt{5}}{10}\right)\quad q_3 = \left(-\frac{\sqrt{6}}{6}, 0, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}\right)$

4. *a*) $Q = \begin{pmatrix} \frac{1}{2} & -\frac{3}{\sqrt{12}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{6}} \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{6}} \end{pmatrix}\quad R = \begin{pmatrix} 2 & \frac{3}{2} & 1 \\ 0 & \frac{3}{\sqrt{12}} & \frac{2}{\sqrt{12}} \\ 0 & 0 & \frac{2}{\sqrt{6}} \end{pmatrix}$

b) $Q = \begin{bmatrix} \frac{1}{2} & \frac{3\sqrt{5}}{10} & -\frac{\sqrt{6}}{6} \\ -\frac{1}{2} & \frac{3\sqrt{5}}{10} & 0 \\ -\frac{1}{2} & \frac{\sqrt{5}}{10} & \frac{\sqrt{6}}{6} \\ \frac{1}{2} & \frac{\sqrt{5}}{10} & \frac{\sqrt{6}}{3} \end{bmatrix}\quad R = \begin{pmatrix} 2 & 1 & \frac{1}{2} \\ 0 & \sqrt{5} & \frac{3\sqrt{5}}{2} \\ 0 & 0 & \frac{\sqrt{6}}{2} \end{pmatrix}$

5. *a*) *Orthogonal* $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

b) *Orthogonal* $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

c) *Orthogonal* $\begin{pmatrix} \cos\theta\sin\theta & \cos^2\theta & \sin\theta \\ -\cos\theta & \sin\theta & 0 \\ -\sin^2\theta & -\cos\theta\sin\theta & \cos\theta \end{pmatrix}$