

Solution

Section 1.5 – Other Types of Equations

Exercise

Solve $3x^3 + 2x^2 = 12x + 8$

Solution

$$3x^3 + 2x^2 - (12x + 8) = 0$$

$$x^2(3x + 2) - 4(3x + 2) = 0$$

$$(3x + 2)(x^2 - 4) = 0$$

$$3x + 2 = 0$$

$$x^2 - 4 = 0$$

$$3x = -2$$

$$x^2 = 4$$

$$\underline{x = -\frac{2}{3} \mid}$$

$$\underline{x = \pm 2 \mid}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{2}{3}, \pm 2 \mid}$$

Exercise

Solve: $x^3 + x^2 - 4x - 4 = 0$

Solution

$$x^2(x + 1) - 4(x + 1) = 0$$

$$(x + 1)(x^2 - 4) = 0$$

$$x + 1 = 0$$

$$\underline{x = -1 \mid}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\underline{x = \pm 2 \mid}$$

$$\therefore \text{Solutions: } \underline{x = -1, \pm 2 \mid}$$

Exercise

Solve: $x^3 + x^2 + 4x + 4 = 0$

Solution

$$x^2(x + 1) + 4(x + 1) = 0$$

$$(x+1)(x^2+4)=0$$

$$x+1=0$$

$$\underline{x=-1}$$

$$x^2+4=0$$

$$x^2=-4$$

$$\underline{x=\pm 2i}$$

$$\therefore \text{Solutions: } \underline{x=-1, \pm 2i}$$

Exercise

$$\text{Solve: } x^3+4x^2-25x-100=0$$

Solution

$$x^2(x+4)-25(x+4)=0$$

$$(x+4)(x^2-25)=0$$

$$x+4=0$$

$$\underline{x=-4}$$

$$x^2-25=0$$

$$x^2=25$$

$$\underline{x=\pm 5}$$

$$\therefore \text{Solutions: } \underline{x=-4, \pm 5}$$

Exercise

$$\text{Solve: } x^3-2x^2-x+2=0$$

Solution

$$x^2(x-2)-(x-2)=0$$

$$(x-2)(x^2-1)=0$$

$$x-2=0$$

$$\underline{x=2}$$

$$x^2-1=0$$

$$x^2=1$$

$$\underline{x=\pm 1}$$

$$\therefore \text{Solutions: } \underline{x=2, \pm 1}$$

Exercise

Solve: $x^3 - x^2 - 25x + 25 = 0$

Solution

$$x^2(x-1) - 25(x-1) = 0$$

$$(x-1)(x^2 - 25) = 0$$

$$x-1=0$$

$$\underline{x=1}$$

$$x^2 - 25 = 0$$

$$x^2 = 25$$

$$\underline{x = \pm 5}$$

$$\therefore \text{Solutions: } \underline{x = 1, \pm 5}$$

Exercise

Solve: $x^3 - x^2 = 16x - 16$

Solution

$$x^3 - x^2 - 16x + 16 = 0$$

$$x^2(x-1) - 16(x-1) = 0$$

$$(x-1)(x^2 - 16) = 0$$

$$x-1=0$$

$$\underline{x=1}$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$\underline{x = \pm 4}$$

$$\therefore \text{Solutions: } \underline{x = 1, \pm 4}$$

Exercise

Solve: $x^3 + x^2 + 25x + 25 = 0$

Solution

$$x^2(x+1) + 25(x+1) = 0$$

$$(x+1)(x^2 + 25) = 0$$

$$x+1=0$$

$$\underline{x = -1}$$

$$x^2 + 25 = 0$$

$$x^2 = -25$$

$$\underline{x = \pm 5i}$$

∴ Solutions: $x = -1, \pm 5i$

Exercise

Solve: $x^3 + 2x^2 = 16x + 32$

Solution

$$x^3 + 2x^2 - 16x - 32 = 0$$

$$x^2(x+2) - 16(x+2) = 0$$

$$(x+2)(x^2 - 16) = 0$$

$$x + 2 = 0$$

$$\underline{x = -2}$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$\underline{x = \pm 4}$$

∴ Solutions: $x = -2, \pm 4$

Exercise

Solve: $2x^3 + 3x^2 - 6x - 9 = 0$

Solution

$$x^2(2x+3) - 3(2x+3) = 0$$

$$(2x+3)(x^2 - 3) = 0$$

$$2x + 3 = 0$$

$$\underline{x = -\frac{3}{2}}$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$\underline{x = \pm\sqrt{3}}$$

∴ Solutions: $x = -\frac{3}{2}, \pm\sqrt{3}$

Exercise

Solve: $2x^3 + x^2 - 8x - 4 = 0$

Solution

$$x^2(2x+1) - 4(2x+1) = 0$$

$$(2x+1)(x^2 - 4) = 0$$

$$\begin{array}{l|l}
 2x+1=0 & x^2-4=0 \\
 \hline x=-\frac{1}{2} & x^2=4 \\
 & \hline x=\pm 2
 \end{array}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{1}{2}, \pm 2}$$

Exercise

Solve: $2x^3 + 16x^2 + 30x = 0$

Solution

$$2x(x^2 + 8x + 15) = 0$$

$$2x(x+3)(x+5) = 0$$

$$\therefore \text{Solutions: } \underline{x = 0, -3, -5}$$

Exercise

Solve: $3x^3 - 9x^2 - 30x = 0$

Solution

$$3x(x^2 - 3x - 10) = 0$$

$$3x(x+2)(x-5) = 0$$

$$\therefore \text{Solutions: } \underline{x = 0, -2, 5}$$

Exercise

Solve $x^4 + 3x^2 = 10$

Solution

$$x^4 + 3x^2 - 10 = 0$$

$$(x^2 + 5)(x^2 - 2) = 0$$

$$\begin{array}{l|l}
 x^2 + 5 = 0 & x^2 - 2 = 0 \\
 \hline x^2 = -5 & x^2 = 2 \\
 \hline x = \pm i\sqrt{5} & \hline x = \pm \sqrt{2}
 \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm i\sqrt{5}, \pm \sqrt{2}}$$

Exercise

Solve: $5x^4 = 40x$

Solution

$$5x^4 - 40x = 0$$

$$5x(x^3 - 8) = 0$$

$$5x(x^3 - 2^3) = 0$$

$$5x(x-2)(x^2 + 2x + 2^2) = 0$$

$$5x(x-2)(x^2 + 2x + 4) = 0$$

$$\underline{x = 0}$$

$$\underline{x = 2}$$

$$x^2 + 2x + 4 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$= \underline{-1 \pm i\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = 0, 2, -1 \pm i\sqrt{3}}$$

Exercise

Solve $9x^4 - 9x^2 + 2 = 0$

Solution

$$x^2 = \frac{9 \pm \sqrt{81 - 72}}{18}$$

$$= \frac{9 \pm \sqrt{9}}{18}$$

$$= \frac{9 \pm 3}{18}$$

$$= \begin{cases} \frac{9-3}{18} = \frac{6}{18} = \frac{1}{3} \\ \frac{9+3}{18} = \frac{12}{18} = \frac{2}{3} \end{cases}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{cases} x^2 = \frac{1}{3} \Rightarrow x = \pm \frac{1}{\sqrt{3}} \\ x^2 = \frac{2}{3} \Rightarrow x = \pm \frac{\sqrt{2}}{\sqrt{3}} \end{cases}$$

$$\begin{cases} x = \pm \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3} \\ x = \pm \frac{\sqrt{2}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{6}}{3} \end{cases}$$

$$\therefore \text{Solutions: } x = \pm \frac{\sqrt{3}}{3}, \pm \frac{\sqrt{6}}{3}$$

Exercise

Solve: $x^4 + 720 = 89x^2$

Solution

$$x^4 - 89x^2 + 720 = 0$$

$$x^2 = \frac{89 \pm \sqrt{7,921 - 2,880}}{2}$$

$$= \frac{89 \pm \sqrt{5041}}{2}$$

$$= \frac{89 \pm 71}{2}$$

$$= \begin{cases} \frac{89 - 71}{2} = 9 \\ \frac{89 + 71}{2} = 80 \end{cases}$$

$$x^2 = 9 \Rightarrow x = \pm 3$$

$$x^2 = 80 \Rightarrow x = \pm \sqrt{80}$$

$$= \pm \sqrt{(16)(5)}$$

$$= \pm 4\sqrt{5}$$

$$\therefore \text{Solutions: } x = \pm 3, \pm 4\sqrt{5}$$

Exercise

Solve $12x^4 - 11x^2 + 2 = 0$

Solution

$$x^2 = \frac{11 \pm \sqrt{121 - 96}}{24}$$

$$= \frac{11 \pm \sqrt{25}}{24}$$

$$= \frac{11 \pm 5}{24}$$

$$= \begin{cases} \frac{11-5}{24} = \frac{1}{4} \\ \frac{11+5}{24} = \frac{2}{3} \end{cases}$$

$$x^2 = \frac{1}{4}$$

$$x = \pm \frac{1}{2}$$

$$x^2 = \frac{2}{3}$$

$$x = \pm \sqrt{\frac{2}{3}}$$

$$x = \pm \frac{\sqrt{6}}{3}$$

$$\therefore \text{Solutions: } \left\{ \pm \frac{\sqrt{6}}{3}, \pm \frac{1}{2} \right\}$$

Exercise

Solve $2x^4 - 7x^2 + 5 = 0$

Solution

$$(2x^2 - 5)(x^2 - 1) = 0$$

$$2x^2 - 5 = 0$$

$$x^2 = \frac{5}{2}$$

$$x = \pm \frac{\sqrt{5}}{\sqrt{2}}$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$\therefore \text{Solutions: } \left\{ \pm \frac{\sqrt{10}}{2}, \pm 1 \right\}$$

Exercise

Solve $x^4 - 5x^2 + 4 = 0$

Solution

$$1 - 5 + 4 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, 4} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$\begin{array}{c} x^2 = 1 \\ \underline{x = \pm 1} \end{array} \quad \begin{array}{c} x^2 = 4 \\ \underline{x = \pm 2} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm 2}$$

Exercise

Solve $x^4 + 3x^2 = 10$

Solution

$$x^4 + 3x^2 - 10 = 0$$
$$(x^2 + 5)(x^2 - 2) = 0$$

$$\begin{array}{c} x^2 + 5 = 0 \\ x^2 = -5 \\ x = \pm\sqrt{-5} \\ \underline{= \pm i\sqrt{5}} \end{array} \quad \begin{array}{c} x^2 - 2 = 0 \\ x^2 = 2 \\ x = \pm\sqrt{2} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm i\sqrt{5}, \pm\sqrt{2}}$$

Exercise

Solve: $3x^4 - 48x^2 = 0$

Solution

$$3x^2(x^2 - 16) = 0$$
$$\begin{array}{c} x^2 = 0 \\ \underline{x = 0, 0} \end{array} \quad \begin{array}{c} x^2 - 16 = 0 \\ x^2 = 16 \\ \underline{x = \pm 4} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, \pm 4}$$

Exercise

Solve: $5x^4 - 20x^2 = 0$

Solution

$$5x^2(x^2 - 4) = 0$$

$$\begin{array}{l} x^2 = 0 \\ \underline{x = 0, 0} \end{array}$$

$$\begin{array}{l} x^2 - 4 = 0 \\ x^2 = 4 \\ \underline{x = \pm 2} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, \pm 2}$$

Exercise

Solve: $x^4 - 4x^3 - 4x^2 = 0$

Solution

$$x^2(x^2 - 4x - 4) = 0$$

$$\begin{array}{l} x^2 = 0 \\ \underline{x = 0, 0} \end{array}$$

$$\begin{array}{l} x^2 - 4x - 4 = 0 \\ x = \frac{4 \pm \sqrt{16 + 16}}{2} \\ = \frac{4 \pm 4\sqrt{2}}{2} \\ \underline{x = 2 \pm 2\sqrt{2}} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 2 \pm 2\sqrt{2}}$$

Exercise

Solve: $x^4 - 6x^3 + 9x^2 = 0$

Solution

$$x^2(x^2 - 6x + 9) = 0$$

$$\begin{array}{l} x^2 = 0 \\ \underline{x = 0, 0} \end{array}$$

$$\begin{array}{l} x^2 - 6x + 9 = 0 \\ (x - 3)^2 = 0 \\ \underline{x = 3, 3} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 3, 3}$$

Exercise

Solve: $x^4 - 4x^3 + 3x^2 = 0$

Solution

$$x^2(x^2 - 4x + 3) = 0$$

$$x^2 = 0$$

$$\underline{x = 0, 0}$$

$$x^2 - 4x + 3 = 0$$

$$\underline{x = 1, 3}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 1, 3}$$

Exercise

Solve: $x^4 - 4x^2 + 3 = 0$

Solution

$$1 - 4 + 3 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, 3}$$

$$x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$x^2 = 3$$

$$\underline{x = \pm\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm\sqrt{3}}$$

Exercise

Solve: $x^4 + 4x^2 + 3 = 0$

Solution

$$1 - 4 + 3 = 0 \quad a - b + c = 0$$

$$\underline{x^2 = -1, -3}$$

$$x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = -1$$

$$\underline{x = \pm i}$$

$$x^2 = -3$$

$$\underline{x = \pm i\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm i, \pm i\sqrt{3}}$$

Exercise

Solve: $x^4 + 6x^2 - 7 = 0$

Solution

$$1 + 6 - 7 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, -7} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$x^2 = -7$$

$$\underline{x = \pm i\sqrt{7}}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm i\sqrt{7}}$$

Exercise

Solve: $x^4 - 6x^2 - 7 = 0$

Solution

$$1 - (-6) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x^2 = -1, 7} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = -1$$

$$\underline{x = \pm i}$$

$$x^2 = 7$$

$$\underline{x = \pm \sqrt{7}}$$

$$\therefore \text{Solutions: } \underline{x = \pm i, \pm \sqrt{7}}$$

Exercise

Solve: $3x^4 + 4x^2 - 7 = 0$

Solution

$$3 + 4 - 7 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, -\frac{7}{3}} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$x^2 = -\frac{7}{3}$$

$$x = \pm i \sqrt{\frac{7}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$\underline{= \pm i \frac{\sqrt{21}}{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm i \frac{\sqrt{21}}{3}}$$

Exercise

Solve: $3x^4 - 4x^2 - 7 = 0$

Solution

$$3 - (-4) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x^2 = -1, \frac{7}{3}} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = -1$$

$$\underline{x = \pm i}$$

$$x^2 = \frac{7}{3}$$

$$x = \pm \sqrt{\frac{7}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$\underline{= \pm \frac{\sqrt{21}}{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm i, \pm \frac{\sqrt{21}}{3}}$$

Exercise

Solve: $3x^4 - x^2 - 2 = 0$

Solution

$$3 - 1 - 2 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, -\frac{2}{3}} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$x^2 = -\frac{2}{3}$$

$$x = \pm i \sqrt{\frac{2}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$\underline{= \pm i \frac{\sqrt{6}}{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm i \frac{\sqrt{6}}{3}}$$

Exercise

Solve: $3x^4 + x^2 - 2 = 0$

Solution

$$3 - 1 - 2 = 0 \quad a - b + c = 0$$

$$\boxed{x^2 = -1, \frac{2}{3}}$$

$$x^2 = -1$$

$$\boxed{x = \pm i}$$

$$x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = \frac{2}{3}$$

$$x = \pm \sqrt{\frac{2}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{= \pm \frac{\sqrt{6}}{3}}$$

$$\therefore \text{Solutions: } \boxed{x = \pm i, \pm \frac{\sqrt{6}}{3}}$$

Exercise

Solve $x - 3\sqrt{x} - 4 = 0$

Solution

$$(\sqrt{x} - 4)(\sqrt{x} + 1) = 0$$

$$\sqrt{x} - 4 = 0$$

$$\sqrt{x} + 1 = 0$$

$$\sqrt{x} = 4$$

$$\sqrt{x} = -1 \quad \text{Impossible}$$

$$x = 16$$

$$\therefore \text{Solution: } \boxed{x = 16}$$

Exercise

Solve $(5x^2 - 6)^{1/4} = x$

Solution

$$\left[(5x^2 - 6)^{1/4} \right]^4 = x^4$$

$$5x^2 - 6 = x^4$$

$$x^4 - 5x^2 + 6 = 0$$

$$(x^2 - 3)(x^2 - 2) = 0$$

$$x^2 = 3$$

$$x^2 = 2$$

$$x = \pm\sqrt{3}$$

$$x = \pm\sqrt{2}$$

$$\therefore \text{Solutions: } \boxed{x = \pm\sqrt{3}, \pm\sqrt{2}}$$

Exercise

Solve $\left(x^2 + 24x\right)^{1/4} = 3$

Solution

$$\left[\left(x^2 + 24x\right)^{1/4}\right]^4 = 3^4$$

$$x^2 + 24x = 81$$

$$x^2 + 24x - 81 = 0$$

$$(x + 27)(x - 3) = 0$$

$$x + 27 = 0 \quad x - 3 = 0$$

$$x = -27 \quad x = 3$$

$$\therefore \text{Solutions: } \underline{x = -27, 3}$$

Exercise

Solve: $x^{5/2} = 32$

Solution

$$x = 32^{2/5} \quad \text{Reciprocal}$$

$$= \left(2^5\right)^{2/5}$$

$$\underline{= 4}$$

Exercise

Solve: $\sqrt[3]{2x+11} = 3$

Solution

$$2x + 11 = 3^3$$

$$2x = 27 - 11$$

$$x = \frac{16}{2}$$

$$\underline{= 8}$$

Exercise

Solve: $\sqrt[3]{6x-3} = 3$

Solution

$$6x - 3 = 3^3$$

$$6x = 27 + 3$$

$$x = \frac{30}{6}$$

$$= 5$$

Exercise

Solve: $\sqrt[3]{2x-6} = 4$

Solution

$$2x - 6 = 4^3$$

$$2x = 64 + 6$$

$$x = \frac{70}{2}$$

$$= 35$$

Exercise

Solve: $\sqrt[3]{4x-3} - 5 = 0$

Solution

$$\sqrt[3]{4x-3} = 5$$

$$4x - 3 = 5^3$$

$$4x = 125 + 3$$

$$x = \frac{128}{4}$$

$$= 32$$

Exercise

Solve: $(3x-1)^{1/3} + 4 = 0$

Solution

$$(3x-1)^{1/3} = -4$$

$$3x - 1 = (-4)^3$$

$$3x = 1 - 64$$

$$x = -\frac{63}{3}$$

$$\underline{= -21}$$

Exercise

Solve: $(2x + 3)^{1/3} + 4 = 6$

Solution

$$(2x + 3)^{1/3} = 2$$

$$2x + 3 = 2^3$$

$$2x = 8 - 3$$

$$\underline{x = \frac{5}{2}}$$

Exercise

Solve: $(3x - 6)^{1/3} + 5 = 8$

Solution

$$(3x - 6)^{1/3} = 3$$

$$3x - 6 = 3^3$$

$$3x = 27 + 6$$

$$x = \frac{33}{3}$$

$$\underline{= 11}$$

Exercise

Solve: $(3x + 1)^{1/4} + 7 = 9$

Solution

$$(3x + 1)^{1/4} = 2$$

$$3x + 1 = 2^4$$

$$3x = 16 - 1$$

$$x = \frac{15}{3}$$

$$= 5$$

∴ Solution set is: $\{5\}$

Exercise

Solve: $(2x + 3)^{1/4} + 7 = 10$

Solution

$$(2x + 3)^{1/4} = 3$$

$$2x + 3 = 3^4$$

$$2x = 81 - 3$$

$$x = \frac{78}{2}$$

$$= 39$$

∴ Solution set is: $\{39\}$

Exercise

Solve: $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$

Solution

$$\left(\sqrt[3]{4x^2 - 4x + 1}\right)^3 = \left(\sqrt[3]{x}\right)^3$$

$$4x^2 - 4x + 1 = x$$

$$4x^2 - 5x + 1 = 0$$

$$(4x - 1)(x - 1) = 0$$

$$4x - 1 = 0$$

$$x = \frac{1}{4}$$

$$x - 1 = 0$$

$$x = 1$$

Check

$$x = \frac{1}{4}$$

$$\sqrt[3]{4\left(\frac{1}{4}\right)^2 - 4\frac{1}{4} + 1} - \sqrt[3]{\frac{1}{4}} = 0$$

$$\sqrt[3]{\frac{1}{4}} - \sqrt[3]{\frac{1}{4}} = 0$$

$$0 = 0 \quad (\text{true})$$

$$x = 1$$

$$\sqrt[3]{4(1)^2 - 4(1) + 1} - \sqrt[3]{1} = 0$$

$$\sqrt[3]{4 - 4 + 1} - \sqrt[3]{1} = 0$$

$$0 = 0 \quad (\text{true})$$

\therefore Solution set is: $\left\{-1, \frac{1}{4}\right\}$

Exercise

Solve: $\sqrt{2x+3}=5$

Solution

$$2x+3=5^2$$

$$\sqrt[n]{u}=a \rightarrow u=a^n$$

$$2x=25-3$$

$$x=\frac{22}{2}$$

$$=11$$

Check: $\sqrt{2(\textcolor{red}{11})+3} \stackrel{?}{=} 5$
 $\sqrt{25}=5$ ✓

\therefore Solution set is: $\left\{\textcolor{blue}{11}\right\}$

Exercise

Solve: $\sqrt{x-3}+6=5$

Solution

$$\sqrt{x-3}=-1 \times$$

\therefore **No** solution.

Exercise

Solve: $\sqrt{3x-2}=4$

Solution

$$3x-2=4^2$$

$$\sqrt[n]{u}=a \rightarrow u=a^n$$

$$3x=16+2$$

$$x=\frac{18}{3}$$

$$=6$$

Check: $\sqrt{3(\textcolor{red}{6})-2} \stackrel{?}{=} 4$
 $\sqrt{16}=4$ ✓

\therefore Solution set is: $\left\{\textcolor{blue}{6}\right\}$

Exercise

Solve: $\sqrt{5x-4} = 9$

Solution

$$5x - 4 = 9^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$5x = 81 + 4$$

$$x = \frac{85}{5}$$

$$= 17$$

Check: $\sqrt{5(17)-4} = 9$?

$$\sqrt{81} = 9 \quad \checkmark$$

\therefore Solution set is: $\{17\}$

Exercise

Solve: $\sqrt{5x-1} = 8$

Solution

$$5x - 1 = 8^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$5x = 64 + 1$$

$$x = \frac{65}{5}$$

$$= 13$$

Check: $\sqrt{5(13)-1} = 8$?

$$\sqrt{64} = 8 \quad \checkmark$$

\therefore Solution set is: $\{13\}$

Exercise

Solve: $\sqrt{3x-2} - 5 = 0$

Solution

$$\sqrt{3x-2} = 5$$

$$3x - 2 = 5^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$3x = 25 + 2$$

$$x = \frac{27}{3}$$

$$\underline{=9}$$

Check: $\sqrt{3(\overset{?}{9})} - 2 - 5 = 0$
 $5 - 5 = 0 \quad \checkmark$

\therefore Solution set is: $\underline{\{9\}}$

Exercise

Solve: $\sqrt{2x+5} + 11 = 6$

Solution

$$\sqrt{2x+5} = -5 \quad \times$$

\therefore **No** solution.

Exercise

Solve: $\sqrt{3x+7} + 10 = 4$

Solution

$$\sqrt{3x+7} = -6 \quad \times$$

\therefore **No** solution.

Exercise

Solve: $x = \sqrt{7x+8}$

Solution

$$x^2 = 7x + 8$$

$$x^2 - 7x - 8 = 0$$

$$1 - (-7) - 8 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, 8} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Check:

$$x = -1$$

$$-1 \neq \sqrt{7(-1)+8}$$

$$x = 8$$

$$\overset{?}{8} = \sqrt{7(\overset{?}{8})+8}$$

$$8 = \sqrt{64} \quad \checkmark$$

\therefore Solution set is: $\underline{\{8\}}$

Exercise

Solve: $x = \sqrt{6x+7}$

Solution

$$x^2 = 6x + 7$$

$$x^2 - 6x - 7 = 0$$

$$1 - (-6) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, 7}$$

$$x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Check:

$$\begin{array}{l|l} x = -1 & x = 7 \\ -1 \neq \sqrt{} & 7 = \sqrt{6(\textcolor{red}{7})+7} \\ & 7 = \sqrt{49} \quad \checkmark \end{array}$$

\therefore Solution is: $\underline{x = \textcolor{blue}{7}}$

Exercise

Solve: $\sqrt{5x+1} = x+1$

Solution

$$5x+1 = (x+1)^2$$

$$5x+1 = x^2 + 2x + 1$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$\underline{x = 0, 3}$$

Check:

$$\begin{array}{l|l} x = 0 & x = 3 \\ \sqrt{1} = 1 \quad \checkmark & \sqrt{5(\textcolor{red}{3})+1} = \textcolor{red}{3} + 1 \\ & \sqrt{16} = 4 \quad \checkmark \end{array}$$

\therefore Solutions are: $\underline{x = \textcolor{blue}{0}, \textcolor{blue}{3}}$

Exercise

Solve: $x = \sqrt{2x-2} + 1$

Solution

$$x-1=\sqrt{2x-2}$$

$$(x-1)^2=2x-2$$

$$x^2-2x+1=2x-2$$

$$x^2-4x+3=0$$

$$1-4+3=0 \quad a+b+c=0$$

$$\underline{x=1, 3} \quad x_1=1, \quad x_2=\frac{c}{a}$$

Check:

$$\begin{array}{l|l} x=1 & x=3 \\ 1=1 \quad \checkmark & \begin{array}{l} ? \\ 3=\sqrt{4}+1 \\ 3=3 \quad \checkmark \end{array} \end{array}$$

\therefore Solutions are: $\underline{x=1, 3}$

Exercise

Solve: $x-2\sqrt{x-3}=3$

Solution

$$x-3=2\sqrt{x-3}$$

$$(x-3)^2=(2\sqrt{x-3})^2$$

$$x^2-6x+9=4(x-3)$$

$$x^2-6x+9=4x-12$$

$$x^2-10x+21=0$$

$$\underline{x=3, 7}$$

Check:

$$\begin{array}{l|l} x=3 & x=7 \\ 3-2(0)=3 \quad ? & \begin{array}{l} 7-2\sqrt{4}=3 \quad ? \\ 7-4=3 \quad \checkmark \end{array} \\ 3=3 \quad \checkmark & \end{array}$$

\therefore Solution set is: $\underline{\{3, 7\}}$

Exercise

Solve: $x+\sqrt{26-11x}=4$

Solution

$$\sqrt{26-11x} = 4-x$$

$$26-11x = (4-x)^2$$

$$26-11x = 16-8x+x^2$$

$$x^2 + 3x - 10 = 0$$

$$(x-2)(x+5) = 0$$

$$\underline{x = 2, -5}$$

Check:

$x = 2$	$x = -5$
$2 + \sqrt{26-22} \stackrel{?}{=} 4$	$-5 + \sqrt{26+55} \stackrel{?}{=} 4$
$2 + 2 = 4 \quad \checkmark$	$-5 + 9 = 4 \quad \checkmark$

$$\therefore \text{Solutions are: } \underline{x = 2, -5}$$

Exercise

Solve $x - \sqrt{2x+3} = 0$

Solution

$$x = \sqrt{2x+3}$$

$$(x)^2 = (\sqrt{2x+3})^2$$

$$x^2 = 2x+3$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x+1=0 \quad x-3=0$$

$$x=-1 \quad x=3$$

Check

$$x = -1$$

$$(-1) - \sqrt{2(-1)+3} = 0$$

$$-1 - \sqrt{1} = 0$$

False

$$x = 3$$

$$(3) - \sqrt{2(3)+3} = 0$$

$$3 - \sqrt{9} = 0$$

True

$$\therefore \text{Solution set is: } \underline{\{3\}}$$

Exercise

Solve: $\sqrt{x+3} + 3 = x$

Solution

$$\sqrt{x+3} = x-3$$

$$(\sqrt{x+3})^2 = (x-3)^2$$

$$x+3 = x^2 - 6x + 9$$

$$x^2 - 7x + 6 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{25}}{2}$$

$$= \frac{7 \pm 5}{2} = \begin{cases} \frac{7+5}{2} = \frac{12}{2} = 6 \\ \frac{7-5}{2} = \frac{2}{2} = 1 \end{cases} \Rightarrow x = 1, 6$$

Check:

$$x = 1 \Rightarrow \sqrt{1+3} + 3 = 1 \Rightarrow 5 = 1 \text{ (Not a solution)}$$

$$x = 6 \Rightarrow \sqrt{6+3} + 3 = 6 \Rightarrow 6 = 6 \rightarrow x = 6 \text{ is the only solution}$$

Exercise

Solve $x - \sqrt{x+11} = 1$

Solution

$$-\sqrt{x+11} = 1 - x$$

Square both side

$$(-\sqrt{x+11})^2 = (1-x)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$x+11 = 1 - 2x + x^2$$

$$0 = x^2 - 2x + 1 - x - 11$$

$$0 = x^2 - 3x - 10$$

$$x^2 - 3x - 10 = 0$$

Solve for x

$$x = 5, -2$$

Check:

$$x = 5 \Rightarrow 5 - \sqrt{5+11} = 1 \Rightarrow 5 - \sqrt{16} = 1 \Rightarrow 5 - 4 = 1 \Rightarrow 1 = 1$$

$$x = -2 \Rightarrow -2 - \sqrt{-2+11} = 1 \Rightarrow -2 - \sqrt{9} = 1 \Rightarrow -2 - 3 = 1 \Rightarrow -5 = 1 \text{ (False)}$$

\therefore Solution set is: $\boxed{\{5\}}$

Exercise

Solve: $\sqrt{x-7} = 7 - \sqrt{x}$

Solution

$$(\sqrt{x-7})^2 = (7 - \sqrt{x})^2$$

$$x - 7 = 49 - 14\sqrt{x} + x$$

$$14\sqrt{x} = 56$$

$$\sqrt{x} = \frac{56}{14}$$

$$= 4$$

$$\underline{x = 16}$$

Check:

$$x = 16$$

$$\sqrt{16-7} \stackrel{?}{=} 7-4$$

$$3 = 3 \quad \checkmark$$

\therefore Solution set is: $\underline{\{16\}}$

Exercise

Solve: $\sqrt{x-8} = \sqrt{x} - 2$

Solution

$$(\sqrt{x-8})^2 = (\sqrt{x} - 2)^2$$

$$x - 8 = x - 4\sqrt{x} + 4$$

$$4\sqrt{x} = 12$$

$$\sqrt{x} = 3$$

$$\underline{x = 9}$$

Check:

$$x = 9$$

$$\sqrt{9-8} \stackrel{?}{=} \sqrt{9} - 2$$

$$1 = 3 - 2 \quad \checkmark$$

\therefore Solution set is: $\underline{\{9\}}$

Exercise

Solve: $\sqrt{2x-5} = \sqrt{x+4}$

Solution

$$\left(\sqrt{2x-5}\right)^2 = \left(\sqrt{x+4}\right)^2$$

$$2x - 5 = x + 4$$

$$\underline{x = 9}$$

Check:

$$x = 9$$

$$\sqrt{18-5} \stackrel{?}{=} \sqrt{9+4}$$

$$\sqrt{13} = \sqrt{13} \quad \checkmark$$

\therefore Solution set is: $\underline{\{9\}}$

Exercise

Solve: $\sqrt{6x+2} = \sqrt{5x+3}$

Solution

$$\left(\sqrt{6x+2}\right)^2 = \left(\sqrt{5x+3}\right)^2$$

$$6x + 2 = 5x + 3$$

$$\underline{x = 1}$$

Check:

$$x = 1$$

$$\sqrt{6+2} \stackrel{?}{=} \sqrt{5+3}$$

$$\sqrt{8} = \sqrt{8} \quad \checkmark$$

\therefore Solution set is: $\underline{\{1\}}$

Exercise

Solve: $\sqrt{3x+1} - \sqrt{x+4} = 1$

Solution

$$\sqrt{3x+1} = 1 + \sqrt{x+4}$$

$$\left(\sqrt{3x+1}\right)^2 = \left(1 + \sqrt{x+4}\right)^2$$

$$3x+1=1+2\sqrt{x+4}+x+4$$

$$2x-4=2\sqrt{x+4}$$

$$x-2=\sqrt{x+4}$$

$$(x-2)^2=x+4$$

$$x^2-4x+4=x+4$$

$$x^2-5x=0$$

$$x(x-5)=0$$

$$\underline{x=0, 5}$$

Check:

$$x=0$$

$$1-\sqrt{4} \stackrel{?}{=} 1$$

$$-1 \neq 1 \quad \times$$

$$x=5$$

$$\sqrt{15+1}-\sqrt{5+4} \stackrel{?}{=} 1$$

$$4-3=1 \quad \checkmark$$

\therefore Solution is: $\underline{x=5}$

Exercise

Solve: $\sqrt{x+2}+\sqrt{x-1}=3$

Solution

$$\sqrt{x+2}=3-\sqrt{x-1}$$

$$x+2=\left(3-\sqrt{x-1}\right)^2$$

$$x+2=9-6\sqrt{x-1}+x-1$$

$$6\sqrt{x-1}=6$$

$$\sqrt{x-1}=1$$

$$x-1=1^2$$

$$\underline{x=2}$$

Check:

$$x=2$$

$$\sqrt{4}+1 \stackrel{?}{=} 3$$

$$2+1=3 \quad \checkmark$$

\therefore Solution is: $\underline{x=2}$

Exercise

Solve: $\sqrt{x-4} + \sqrt{x+4} = 4$

Solution

$$\sqrt{x-4} = 4 - \sqrt{x+4}$$

$$x-4 = (4 - \sqrt{x+4})^2$$

$$x-4 = 16 - 8\sqrt{x+4} + x+4$$

$$8\sqrt{x+4} = 24$$

$$\sqrt{x+4} = 3$$

$$x+4 = 9$$

$$\underline{x = 5}$$

Check:

$$x = 5$$

$$\sqrt{1} + \sqrt{9} \stackrel{?}{=} 4$$

$$1 + 3 = 4 \quad \checkmark$$

$$\therefore \text{Solution is: } \underline{x = 5}$$

Exercise

Solve: $\sqrt{2x-3} - \sqrt{x-2} = 1$

Solution

$$\sqrt{2x-3} = 1 + \sqrt{x-2}$$

$$2x-3 = (1 + \sqrt{x-2})^2$$

$$2x-3 = 1 + 2\sqrt{x-2} + x-2$$

$$x-2 = 2\sqrt{x-2}$$

$$(x-2)^2 = (2\sqrt{x-2})^2$$

$$x^2 - 4x + 4 = 4(x-2)$$

$$x^2 - 4x + 4 = 4x - 8$$

$$x^2 - 8x + 12 = 0$$

$$\underline{x = 2, 6}$$

Check:

$$x = 2 \quad \bigg| \quad x = 6$$

$$\left. \begin{array}{l} \sqrt{4-3} - \sqrt{2-2} \stackrel{?}{=} 1 \\ 1 = 1 \quad \checkmark \end{array} \right| \left. \begin{array}{l} \sqrt{12-3} - \sqrt{6-2} \stackrel{?}{=} 1 \\ 3 - 2 = 1 \quad \checkmark \end{array} \right|$$

\therefore Solution is: $\underline{x = 2, 6}$

Exercise

Solve: $\sqrt{x+2} + \sqrt{3x+7} = 1$

Solution

$$\begin{aligned} \sqrt{x+2} &= 1 - \sqrt{3x+7} \\ x+2 &= (1 - \sqrt{3x+7})^2 \\ x+2 &= 1 - 2\sqrt{3x+7} + 3x+7 \\ 2\sqrt{3x+7} &= 2x+6 \\ \sqrt{3x+7} &= x+3 \\ 3x+7 &= (x+3)^2 \\ 3x+7 &= x^2 + 6x+9 \\ x^2 + 3x+2 &= 0 \\ x &= -1, -2 \end{aligned}$$

Check:

$$\left. \begin{array}{l} x = -1 \\ \sqrt{-1+2} \stackrel{?}{=} 1 - \sqrt{-3+7} \\ 1 \neq 1 - 2 \quad \times \end{array} \right| \left. \begin{array}{l} x = -2 \\ \sqrt{-2+2} \stackrel{?}{=} 1 - \sqrt{-6+7} \\ 0 = 1 - 1 \quad \checkmark \end{array} \right|$$

\therefore Solution is: $\underline{x = -2}$

Exercise

Solve: $2\sqrt{4x+1} - 9 = x - 5$

Solution

$$\begin{aligned} 2\sqrt{4x+1} &= x+4 \\ (2\sqrt{4x+1})^2 &= (x+4)^2 \\ 4(4x+1) &= x^2 + 8x+16 \\ 16x+4 &= x^2 + 8x+16 \\ x^2 - 8x+12 &= 0 \end{aligned}$$

$$\underline{x = 2, 6}$$

Check:

$$\begin{array}{l|l} x = 2 & x = 6 \\ 2\sqrt{8+1}-9 \stackrel{?}{=} 2-5 & 2\sqrt{24+1}-9 \stackrel{?}{=} 6-5 \\ 6-9 = -3 \quad \checkmark & 10-9 = 1 \quad \checkmark \end{array}$$

\therefore Solution is: $\underline{x = 2, 6}$

Exercise

Solve: $x\sqrt{x-3} + 4 = x + 1$

Solution

$$x\sqrt{x-3} = x - 3$$

$$(x\sqrt{x-3})^2 = (x-3)^2$$

$$x^2(x-3) - (x-3)^2 = 0$$

$$(x-3)(x^2 - x + 3) = 0$$

$$x^2 - x + 3 = 0 \rightarrow x = \frac{1 \pm \sqrt{1-12}}{2} \in \mathbb{C}$$

$$\underline{x = 3} \quad \text{Only result.}$$

Check:

$$\begin{array}{l} x = 3 \\ 3\sqrt{3-3} + 4 \stackrel{?}{=} 3 + 1 \\ 4 = 4 \quad \checkmark \end{array}$$

\therefore Solution is: $\underline{x = 3}$

Exercise

Solve: $\sqrt{2x-3} + \sqrt{x-2} = 1$

Solution

$$\sqrt{2x-3} = 1 - \sqrt{x-2}$$

$$(\sqrt{2x-3})^2 = (1 - \sqrt{x-2})^2$$

$$2x-3 = 1 - 2\sqrt{x-2} + (\sqrt{x-2})^2$$

$$2x-3-1 = -2\sqrt{x-2} + x-2$$

$$2x - 4 - x + 2 = -2\sqrt{x-2}$$

$$x - 2 = -2\sqrt{x-2}$$

$$(x-2)^2 = (-2\sqrt{x-2})^2$$

$$x^2 - 4x + 4 = 4(x-2)$$

$$x^2 - 4x + 4 = 4x - 8$$

$$x^2 - 4x + 4 - 4x + 8 = 0$$

$$x^2 - 8x + 12 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$$

$$x = 2, 6$$

Check

$$x = 2 \Rightarrow \sqrt{2(2)-3} + \sqrt{2-2} = 1 \Rightarrow 1 + 0 = 1$$

$$x = 6 \Rightarrow \sqrt{2(6)-3} + \sqrt{6-2} = 1 \Rightarrow 3 + 2 = 1 \Rightarrow 5 \neq 1$$

\therefore Solution is: $x = 2$

Exercise

Solve: $\sqrt{x+5} - \sqrt{x-3} = 2$

Solution

$$\sqrt{x+5} = 2 + \sqrt{x-3}$$

$$(\sqrt{x+5})^2 = (2 + \sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + (\sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + x-3$$

$$x-x+5-4-3 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$$1 = \sqrt{x-3}$$

$$1 = x-3$$

$$\Rightarrow x = 4$$

Check: $\sqrt{4+5} - \sqrt{4-3} = 2$

$$3 - 1 = 2 \text{ (True statement)}$$

\therefore Solution is: $x = 4$

Exercise

Solve: $\sqrt{2x+3} = 1 + \sqrt{x+1}$

Solution

$$(\sqrt{2x+3})^2 = (1 + \sqrt{x+1})^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$2x+3 = 1 + 2\sqrt{x+1} + x+1$$

$$2x+3 = 2\sqrt{x+1} + x+2$$

$$x+1 = 2\sqrt{x+1}$$

$$(x+1)^2 = (2\sqrt{x+1})^2$$

$$x^2 + 2x + 1 = 4(x+1)$$

$$x^2 + 2x + 1 = 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x-3=0$$

$$x+1=0$$

$$x=3$$

$$x=-1$$

Check

$$x=3$$

$$\sqrt{2(3)+3} = 1 + \sqrt{(3)+1}$$

$$\sqrt{9} = 1 + \sqrt{4}$$

$$3 = 3 \quad (\text{true})$$

$$x=-1$$

$$\sqrt{2(-1)+3} = 1 + \sqrt{(-1)+1}$$

$$\sqrt{1} = 1 + \sqrt{0}$$

$$1 = 1 \quad (\text{true})$$

$$\therefore \text{Solution: } \underline{x = -1, 3}$$

Exercise

Solve: $\sqrt{x+5} - \sqrt{x-3} = 2$

Solution

$$\sqrt{x+5} = 2 + \sqrt{x-3}$$

$$(\sqrt{x+5})^2 = (2 + \sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + (\sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + x-3$$

$$x-x+5-4-3 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$$1 = \sqrt{x-3}$$

$$1 = x - 3$$

$$\Rightarrow x = 4$$

Check: $\sqrt{4+5} - \sqrt{4-3} = 2$

$$3 - 1 = 2 \text{ (True statement)}$$

\therefore Solution is: $\underline{x = 4}$

Exercise

Solve: $|x| = -9$

Solution

$|x| = -9$ Not True

\therefore **No** Solution

Exercise

Solve: $|x| = 9$

Solution

\therefore Solutions: $\underline{x = \pm 9}$

Exercise

Solve: $|x-2| = 7$

Solution

$$x - 2 = 7 \qquad x - 2 = -7$$

$$\underline{x = 9} \qquad \underline{x = -5}$$

\therefore Solutions: $\underline{x = -5, 9}$

Exercise

Solve: $|x-2| = 0$

Solution

$$x - 2 = 0$$

\therefore Solution: $\underline{x = 2}$

Exercise

Solve: $|2x - 3| = 6$

Solution

$$2x - 3 = 6 \quad 2x - 3 = -6$$

$$2x = 9 \quad 2x = -3$$

$$\underline{x = \frac{9}{2} \quad x = -\frac{3}{2}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{3}{2}, \frac{9}{2}}$$

Exercise

Solve: $|2x - 1| = 11$

Solution

$$2x - 1 = 11 \quad 2x - 1 = -11$$

$$2x = 12 \quad 2x = -10$$

$$\underline{x = 6 \quad x = -5}$$

$$\therefore \text{Solutions: } \underline{x = -5, 6}$$

Exercise

Solve $7|5x| + 2 = 16$

Solution

$$7|5x| = 16 - 2$$

$$7|5x| = 14$$

$$|5x| = \frac{14}{7}$$

$$|5x| = 2$$

$$5x = 2 \quad 5x = -2$$

$$x = \frac{2}{5} \quad x = -\frac{2}{5}$$

$$\therefore \text{Solution: } \underline{x = \pm \frac{2}{5}}$$

Exercise

Solve $4\left|1 - \frac{3}{4}x\right| + 7 = 10$

Solution

$$4\left|1 - \frac{3}{4}x\right| = 10 - 7$$

$$|4 - 3x| = 10 - 7 \quad \text{Distribute 4}$$

$$4\left|1 - \frac{3}{4}x\right| = 3$$

$$|4 - 3x| = 3$$

$$\left|1 - \frac{3}{4}x\right| = \frac{3}{4}$$

$$\begin{array}{l|l} 1 - \frac{3}{4}x = \frac{3}{4} & 1 - \frac{3}{4}x = -\frac{3}{4} \\ -\frac{3}{4}x = \frac{3}{4} - 1 & -\frac{3}{4}x = -\frac{3}{4} - 1 \\ -\frac{3}{4}x = -\frac{1}{4} & -\frac{3}{4}x = -\frac{7}{4} \\ x = -\frac{1}{4}\left(-\frac{4}{3}\right) & x = -\frac{7}{4}\left(-\frac{4}{3}\right) \\ x = \frac{1}{3} & x = \frac{7}{3} \end{array}$$

$$\begin{array}{l|l} 4 - 3x = -3 & 4 - 3x = 3 \\ -3x = -7 & -3x = -1 \\ x = \frac{7}{3} & x = \frac{1}{3} \end{array}$$

$$\therefore \text{Solutions: } x = \underline{\frac{1}{3}, \frac{7}{3}}$$

Exercise

Solve $|x + 7| + 6 = 2$

Solution

$$|x + 7| = 2 - 6$$

$$|x + 7| = -4$$

\Rightarrow No solution or \emptyset , since the absolute value can't be equal to a negative.

Exercise

Solve equation: $|5 - 3x| = 12$

Solution

$$\begin{array}{ll} 5 - 3x = 12 & 5 - 3x = -12 \\ 5 - 3x - 5 = 12 - 5 & 5 - 3x - 5 = -12 - 5 \\ -3x = 7 & -3x = -17 \\ x = -\frac{7}{3} & x = \frac{17}{3} \end{array}$$

$$\therefore \text{Solutions: } x = \underline{\frac{17}{3}, -\frac{7}{3}}$$

Exercise

Solve equation: $|4x + 2| = 5$

Solution

$$4x + 2 = -5 \qquad 4x + 2 = 5$$

$$4x = -7 \qquad 4x = 3$$

$$x = -\frac{7}{4} \qquad x = \frac{3}{4}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{7}{4}, \frac{3}{4}}$$

Exercise

Solve: $3|x + 5| = 12$

Solution

$$|x + 5| = 4$$

$$x + 5 = 4 \qquad x + 5 = -4$$

$$\underline{x = -1} \qquad \underline{x = -9}$$

$$\therefore \text{Solutions: } \underline{x = -9, -1}$$

Exercise

Solve: $2|x - 6| = 8$

Solution

$$x - 6 = 4$$

$$x - 6 = 4 \qquad x - 6 = -4$$

$$\underline{x = 10} \qquad \underline{x = 2}$$

$$\therefore \text{Solutions: } \underline{x = 2, 10}$$

Exercise

Solve: $3|2x - 1| = 21$

Solution

$$|2x - 1| = 7$$

$$2x - 1 = 7 \qquad 2x - 1 = -7$$

$$2x = 8 \qquad 2x = -6$$

$$\underline{x = 4} \qquad \underline{x = -3}$$

$$\therefore \text{Solutions: } \underline{x = -3, 4}$$

Exercise

$$\text{Solve: } 2|3x - 2| = 14$$

Solution

$$|3x - 2| = 7$$

$$3x - 2 = 7 \qquad 3x - 2 = -7$$

$$3x = 9 \qquad 3x = -5$$

$$\underline{x = 3} \qquad \underline{x = -\frac{5}{3}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{5}{3}, 3}$$

Exercise

$$\text{Solve: } |3x - 1| + 2 = 16$$

Solution

$$|3x - 1| = 14$$

$$3x - 1 = 14 \qquad 3x - 1 = -14$$

$$3x = 15 \qquad 3x = -13$$

$$\underline{x = 5} \qquad \underline{x = -\frac{13}{3}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{13}{3}, 5}$$

Exercise

$$\text{Solve: } |6x - 2| + 4 = 32$$

Solution

$$|6x - 2| = 28$$

$$6x - 2 = 28 \qquad 6x - 2 = -28$$

$$6x = 30 \qquad 6x = -26$$

$$\underline{x = 5}$$

$$\begin{array}{l} x = -\frac{13}{3} \\ \hline \therefore \text{Solutions: } x = -\frac{13}{3}, 5 \end{array}$$

Exercise

Solve: $7|5x| + 2 = 16$

Solution

$$7|5x| = 14$$

$$|5x| = 2$$

$$5x = 2$$

$$5x = -2$$

$$x = \frac{2}{5}$$

$$x = -\frac{2}{5}$$

$$\therefore \text{Solutions: } x = -\frac{2}{5}, \frac{2}{5}$$

Exercise

Solve: $|4x + 1| + 10 = 4$

Solution

$$|4x + 1| = -6$$

\therefore **No Solution**

Exercise

Solve: $|4x + 1| + 4 = 10$

Solution

$$|4x + 1| = 6$$

$$4x + 1 = 6$$

$$4x + 1 = -6$$

$$4x = 5$$

$$4x = -7$$

$$x = \frac{5}{4}$$

$$x = -\frac{7}{4}$$

$$\therefore \text{Solutions: } x = -\frac{7}{4}, \frac{5}{4}$$

Exercise

Solve: $|3x - 2| + 8 = 1$

Solution

$$|3x - 2| = -7$$

\therefore **No** Solution

Exercise

Solve: $|3x - 2| + 1 = 8$

Solution

$$|3x - 2| = 7$$

$$4x + 1 = 6 \quad 4x + 1 = -6$$

$$4x = 5 \quad 4x = -7$$

$$\underline{x = \frac{5}{4} \quad \quad x = -\frac{7}{4}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{7}{4}, \frac{5}{4}}$$

Exercise

Solve equation: $\left| \frac{6x+1}{x-1} \right| = 3$

Solution

$$\frac{6x+1}{x-1} = -3$$

$$(x-1) \frac{6x+1}{x-1} = -3(x-1)$$

$$6x+1 = -3x+3$$

$$6x+1 + 3x = -3x+3 + 3x$$

$$9x+1 = 3$$

$$9x = 2$$

$$x = \frac{2}{9}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{4}{3}, \frac{2}{9}}$$

$$\frac{6x+1}{x-1} = 3$$

$$6x+1 = 3(x-1)$$

$$6x+1 = 3x-3$$

$$6x+1 - 3x = 3x-3 - 3x$$

$$3x+1 = -3$$

$$3x = -4$$

$$x = -\frac{4}{3}$$

Exercise

Solve equation: $|x+1| = |1-3x|$

Solution

$$x+1 = -(1-3x) \qquad x+1 = 1-3x$$

$$x+1 = -1+3x \qquad x+3x = 1-1$$

$$x-3x = -1-1 \qquad 4x = 0$$

$$-2x = -2 \qquad x = 0$$

$$x = 1$$

\therefore Solutions: $\underline{x = 0, 1}$

Exercise

Solve: $|3x-1| = |x+5|$

Solution

$$3x-1 = x+5 \qquad 3x-1 = -(x+5)$$

$$2x = 6 \qquad 3x-1 = -x-5$$

$$\underline{x = 3} \qquad 4x = -4$$

$$\underline{x = -1}$$

\therefore Solutions: $\underline{x = -1, 3}$

Exercise

Solve: $|5x-8| = |3x+2|$

Solution

$$5x-8 = 3x+2 \qquad 5x-8 = -(3x+2)$$

$$2x = 10 \qquad 5x-8 = -3x-2$$

$$\underline{x = 5} \qquad 8x = 6$$

$$\underline{x = \frac{3}{4}}$$

\therefore Solutions: $\underline{x = \frac{3}{4}, 5}$

Exercise

Solve: $|4x - 9| = |2x + 1|$

Solution

$$4x - 9 = 2x + 1 \quad 4x - 9 = -(2x + 1)$$

$$2x = 10$$

$$5x - 8 = -3x - 2$$

$$\underline{x = 5}$$

$$8x = 6$$

$$\underline{x = \frac{3}{4}}$$

$$\therefore \text{Solutions: } \underline{x = \frac{3}{4}, 5}$$

Exercise

Solve: $|2x - 4| = |x - 1|$

Solution

$$2x - 4 = x - 1$$

$$2x - 4 = -x + 1$$

$$\underline{x = 3}$$

$$3x = -5$$

$$\underline{x = -\frac{5}{3}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{5}{3}, 3}$$

Exercise

Solve: $|3x - 4| = |3x + 4|$

Solution

$$3x - 4 = 3x + 4$$

$$3x - 4 = -3x + 4$$

$$-4 \neq 4$$

$$6x = 8$$

$$\underline{x = \frac{4}{3}}$$

$$\therefore \text{Solution: } \underline{x = \frac{4}{3}}$$

Exercise

Solve: $|3x - 5| = |3x + 5|$

Solution

$$3x - 5 = 3x + 5 \quad 3x - 5 = -3x + 5$$

$$-5 \neq 5 \quad 6x = 10$$

$$\underline{x = \frac{5}{3} \mid}$$

$$\therefore \text{Solution: } \underline{x = \frac{5}{3} \mid}$$

Exercise

Solve: $|x - 3| = |5 - x|$

Solution

$$x - 3 = 5 - x \quad x - 3 = -5 + x$$

$$2x = 8 \quad -3 \neq -5$$

$$\underline{x = 4 \mid}$$

$$\therefore \text{Solutions: } \underline{x = 4 \mid}$$

Exercise

Solve: $|x - 3| = |6 - x|$

Solution

$$x - 3 = 6 - x \quad x - 3 = -6 + x$$

$$2x = 9 \quad -3 \neq -6$$

$$\underline{x = \frac{9}{2} \mid}$$

$$\therefore \text{Solutions: } \underline{x = \frac{9}{2} \mid}$$

Exercise

Solve: $\left| \frac{2}{3}x - 2 \right| = \left| \frac{1}{3}x + 3 \right|$

Solution

$$\frac{2}{3}x - 2 = \frac{1}{3}x + 3 \quad \frac{2}{3}x - 2 = -\frac{1}{3}x - 3$$

$$\frac{2}{3}x - \frac{1}{3}x = 3 + 2 \quad \frac{2}{3}x + \frac{1}{3}x = -3 + 2$$

$$\frac{1}{3}x = 5 \quad \underline{x = -1}$$

$$\underline{x = 15}$$

∴ Solutions: $\underline{x = -1, 15}$

Exercise

Solve: $\left| \frac{1}{2}x - 2 \right| = \left| x - \frac{1}{2} \right|$

Solution

$$\frac{1}{2}x - 2 = x - \frac{1}{2} \quad \frac{1}{2}x - 2 = -x + \frac{1}{2}$$

$$\frac{1}{2}x - x = 2 - \frac{1}{2} \quad \frac{1}{2}x + x = 2 + \frac{1}{2}$$

$$\frac{1}{2}x = \frac{3}{2} \quad \frac{3}{2}x = \frac{5}{2}$$

$$\underline{x = 3} \quad \underline{x = \frac{5}{3}}$$

∴ Solutions: $\underline{x = 3, \frac{5}{3}}$