

## Section 4.3 – Multiplicative Inverses of Matrices

### Identity Matrix

The  $n \times n$  identity matrix with 1's on the main diagonal and 0's elsewhere and is denoted by

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$I = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

### The Multiplicative Identity Matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Then  $AI = IA = A$

### Example

$$A = \begin{bmatrix} 4 & -7 \\ -3 & 2 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

### Solution

$$\begin{aligned} AI &= \begin{bmatrix} 4 & -7 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4(1) - 7(0) & 4(0) - 7(1) \\ -3(1) + 2(0) & -3(0) + 2(1) \end{bmatrix} \\ &= \begin{bmatrix} 4 & -7 \\ -3 & 2 \end{bmatrix} = A \\ &= A \end{aligned}$$

## Multiplicative inverse of a matrix

Multiplicative inverse of a matrix  $A_{n \times n}$  and  $A^{-1}_{n \times n}$  if exists, then:

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

### *Example*

Show that  $B$  is Multiplicative inverse of  $A$ .

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

### *Solution*

$$\begin{aligned} A \cdot B &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2(1) - 1(1) & 2(-1) + 1(2) \\ 1(1) + 1(-1) & 1(-1) + 1(2) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= I \end{aligned}$$

$$\begin{aligned} BA &= \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= I \end{aligned}$$

$\therefore B$  is multiplicative inverse of a matrix  $A$ :  $B = A^{-1}$

## ***Finding Inverse matrix***

To find inverse matrix using Gauss-Jordan method:

$$\left[ A | I \right] \rightarrow \left[ I | A^{-1} \right] \quad \text{where } A^{-1} \text{ read as "A inverse"}$$

**For 2 by 2 matrices (*only*)**

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{aligned} A^{-1} &= \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \begin{bmatrix} \frac{d}{ad - bc} & \frac{-b}{ad - bc} \\ \frac{-c}{ad - bc} & \frac{a}{ad - bc} \end{bmatrix} \end{aligned}$$

If  $ad - bc = 0$ , then  $A^{-1}$  doesn't exist

## ***Example***

$$A = \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix} \Rightarrow A^{-1} = ?$$

## **Solution**

$$\begin{aligned} A^{-1} &= \frac{1}{(-1)(4) - (-2)(3)} \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} \end{aligned}$$

***Example***

$$A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \Rightarrow A^{-1} = ?$$

**Solution**

$$\begin{aligned} A^{-1} &= \frac{1}{(3)(1) - (-2)(-1)} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \end{aligned}$$

To find inverse matrix using Gauss-Jordan method:

$$\left[ A | I \right] \rightarrow \left[ I | A^{-1} \right] \quad \text{where } A^{-1} \text{ read as "A inverse"}$$

### Example

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \quad \text{Find } A^{-1}$$

### Solution

$$\begin{array}{l} \left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \\ R_2 + R_1 \\ R_3 - R_1 \end{array} \quad \begin{array}{ccc|ccc} -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \end{array} \quad \begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & -1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array} \\ \\ \left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array} \right] 2R_3 + R_2 \quad \begin{array}{ccc|ccc} 0 & -2 & -4 & -2 & 0 & 2 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array} \\ \\ \left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array} \right] \begin{array}{l} R_1 - 2R_3 \\ R_2 - 5R_3 \end{array} \quad \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 & -2 & -4 \\ 0 & 0 & 0 & 3 & -2 & -4 \end{array} \quad \begin{array}{ccc|ccc} 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & 0 & -5 & 5 & -5 & -10 \\ 0 & 2 & 0 & 6 & -4 & -10 \end{array} \\ \\ \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -4 \\ 0 & 2 & 0 & 6 & -4 & -10 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array} \right] \frac{1}{2}R_2 \\ \\ \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -4 \\ 0 & 1 & 0 & 3 & -2 & -5 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array} \right] \\ \\ \Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \end{array}$$

### Example

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \quad \text{Find } A^{-1}$$

### Solution

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \\ R_2 + R_1 \\ R_3 - R_1 \end{array}$$

$$\begin{array}{cccccc|cccccc} -1 & 2 & 3 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 2 & 1 & 0 & 0 & -1 & 0 & -2 & -1 & 0 & 0 \\ \hline 0 & 2 & 5 & 1 & 1 & 0 & 0 & -1 & -2 & -1 & 0 & 1 \end{array}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array} \right] \frac{1}{2}R_2$$

$$0 \quad 1 \quad \frac{5}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad 0$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array} \right] R_3 + R_2$$

$$\begin{array}{cccccc} 0 & -1 & -2 & -1 & 0 & 1 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ \hline 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 \end{array}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 \end{array} \right] 2R_3$$

$$0 \quad 0 \quad 1 \quad -1 \quad 1 \quad 2$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array} \right] \begin{array}{l} R_1 - 2R_3 \\ R_2 - \frac{5}{2}R_3 \end{array}$$

$$\begin{array}{cccccc|cccccc} 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -\frac{5}{2} & \frac{5}{2} & -\frac{5}{2} & -5 & 0 & 0 & -2 & 2 & -2 & -4 \\ \hline 0 & 1 & 0 & 3 & -2 & -5 & 1 & 0 & 0 & 3 & -2 & -4 \end{array}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -4 \\ 0 & 1 & 0 & 3 & -2 & -5 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array} \right]$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

## Solving a System Using $A^{-1}$

To solve the matrix equation  $AX = B$ .

- $X$ : matrix of the variables
- $A$ : Coefficient matrix
- $B$ : Constant matrix

$$AX = B$$

$$A^{-1}(AX) = A^{-1}B \quad \text{Multiply both side by } A^{-1}$$

$$(A^{-1}A)X = A^{-1}B \quad \text{Associate property}$$

$$IX = A^{-1}B \quad \text{Multiplicative inverse property}$$

$$X = A^{-1}B \quad \text{Identity property}$$

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### Example

Solve the system using  $A^{-1}$

$$x + 2z = 6$$

$$-x + 2y + 3z = -5$$

$$x - y = 6$$

$$\text{Given } A^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

### Solution

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 6 \end{bmatrix}$$

$A \quad X = B$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ -5 \\ 6 \end{bmatrix} = \begin{bmatrix} 3(6)-2(-5)-4(6) \\ 3(6)-2(-5)-5(6) \\ -1(6)+1(-5)+2(6) \end{bmatrix} = \begin{bmatrix} 18+10-24 \\ 18+10-30 \\ -6-5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

**Solution:**  $\{(4, -2, 1)\}$

### ***Example***

Use the inverse of the coefficient matrix to solve the linear system

$$2x - 3y = 4$$

$$x + 5y = 2$$

### **Solution**

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{5}{13} & \frac{3}{13} \\ -\frac{1}{13} & \frac{2}{13} \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{5}{13} & \frac{3}{13} \\ -\frac{1}{13} & \frac{2}{13} \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The solution of the system is (2,0)



## Exercise

## Section 4.3 – Multiplicative Inverses of Matrices

Show that  $B$  is Multiplicative inverse of  $A$

1.  $A = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

2.  $A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \quad \& \quad B = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$

Find the inverse, if exists, of

3.  $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

14.  $A = \begin{pmatrix} 4 & 6 \\ 2 & 3 \end{pmatrix}$

25.  $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{bmatrix}$

4.  $A = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

15.  $A = \begin{pmatrix} -6 & 9 \\ 2 & -3 \end{pmatrix}$

26.  $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & 3 \\ 2 & 4 & 3 \end{bmatrix}$

5.  $A = \begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix}$

16.  $A = \begin{pmatrix} -2 & 7 \\ 0 & 2 \end{pmatrix}$

27.  $A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

6.  $A = \begin{bmatrix} a & b \\ 3 & 3 \end{bmatrix}$

17.  $A = \begin{pmatrix} 4 & -16 \\ 1 & -4 \end{pmatrix}$

28.  $A = \begin{bmatrix} -2 & 5 & 3 \\ 4 & -1 & 3 \\ 7 & -2 & 5 \end{bmatrix}$

7.  $A = \begin{bmatrix} -2 & a \\ 4 & a \end{bmatrix}$

18.  $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

29.  $A = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{pmatrix}$

8.  $A = \begin{bmatrix} 4 & 4 \\ b & a \end{bmatrix}$

19.  $A = \begin{pmatrix} 2 & 1 \\ a & a \end{pmatrix}$

30.  $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{pmatrix}$

9.  $A = \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$

20.  $A = \begin{pmatrix} b & 3 \\ b & 2 \end{pmatrix}$

31.  $A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix}$

10.  $A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$

21.  $A = \begin{pmatrix} 1 & a \\ 3 & a \end{pmatrix}$

32.  $A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 2 & -1 \\ 3 & 1 & 2 \end{pmatrix}$

11.  $A = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$

22.  $A = \begin{pmatrix} a & 2 \\ 2 & a \end{pmatrix}$

12.  $A = \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$

23.  $A = \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}$

13.  $A = \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}$

24.  $A = \begin{pmatrix} -3 & \frac{1}{2} \\ 6 & -1 \end{pmatrix}$

$$33. \quad A = \begin{pmatrix} 3 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

$$34. \quad A = \begin{pmatrix} -3 & 1 & -1 \\ 1 & -4 & -7 \\ 1 & 2 & 5 \end{pmatrix}$$

$$35. \quad A = \begin{pmatrix} 1 & 1 & -3 \\ 2 & -4 & 1 \\ -5 & 7 & 1 \end{pmatrix}$$

$$36. \quad A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 5 & 3 \\ 2 & 4 & 3 \end{pmatrix}$$

$$37. \quad A = \begin{bmatrix} -2 & -3 & 4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 4 & -6 & 1 \\ -2 & -2 & 5 & 1 \end{bmatrix}$$

$$38. \quad A = \begin{bmatrix} 1 & -14 & 7 & 38 \\ -1 & 2 & 1 & -2 \\ 1 & 2 & -1 & -6 \\ 1 & -2 & 3 & 6 \end{bmatrix}$$

$$39. \quad A = \begin{bmatrix} 10 & 20 & -30 & 15 \\ 3 & -7 & 14 & -8 \\ -7 & -2 & -1 & 2 \\ 4 & 4 & -3 & 1 \end{bmatrix}$$

State the conditions under which  $A^{-1}$  exists. Then find a formula for  $A^{-1}$

$$40. \quad A = [x]$$

$$41. \quad A = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$$

$$42. \quad A = \begin{bmatrix} 0 & 0 & x \\ 0 & y & 0 \\ z & 0 & 0 \end{bmatrix}$$

$$43. \quad A = \begin{bmatrix} x & 1 & 1 & 1 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & w \end{bmatrix}$$

$$44. \quad \text{Solve the system using } A^{-1} \quad \begin{cases} x & +2z = 6 \\ -x+2y+3z = -5 \\ x-y & = 6 \end{cases} \quad \text{Given } A^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

45. Solve the system using  $A^{-1}$

$$\begin{cases} x+2y+5z = 2 \\ 2x+3y+8z = 3 \\ -x+y+2z = 3 \end{cases}$$

a) Write the linear system as a matrix equation in the form  $AX = B$

b) Solve the system using the inverse that is given for the coefficient matrix

$$\text{the inverse of } \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 8 \\ -1 & 1 & 2 \end{bmatrix} \text{ is } \begin{bmatrix} 2 & -1 & -1 \\ 12 & -7 & -2 \\ -5 & 3 & 1 \end{bmatrix}$$

46. Solve the system using  $A^{-1}$

$$\begin{cases} x - y + z = 8 \\ 2y - z = -7 \\ 2x + 3y = 1 \end{cases}$$

a) Write the linear system as a matrix equation in the form  $AX = B$

b) Solve the system using the inverse that is given for the coefficient matrix

$$\text{the inverse is } \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

(47–75) Use the *inverse* of the coefficient matrix to solve the linear system

47.  $\begin{cases} 3x + 2y = -4 \\ 2x - y = -5 \end{cases}$

58.  $\begin{cases} x + 2y = -1 \\ 4x - 2y = 6 \end{cases}$

67.  $\begin{cases} 3y - z = -1 \\ x + 5y - z = -4 \\ -3x + 6y + 2z = 11 \end{cases}$

48.  $\begin{cases} 2x + 5y = 7 \\ 5x - 2y = -3 \end{cases}$

59.  $\begin{cases} x - 2y = 5 \\ -10x + 2y = 4 \end{cases}$

68.  $\begin{cases} x + 3y + 4z = 14 \\ 2x - 3y + 2z = 10 \\ 3x - y + z = 9 \end{cases}$

49.  $\begin{cases} 4x - 7y = -16 \\ 2x + 5y = 9 \end{cases}$

60.  $\begin{cases} 12x + 15y = -27 \\ 30x - 15y = -15 \end{cases}$

50.  $\begin{cases} 3x + 2y = 4 \\ 2x + y = 1 \end{cases}$

61.  $\begin{cases} 4x - 4y = -12 \\ 4x + 4y = -20 \end{cases}$

69.  $\begin{cases} x + 4y - z = 20 \\ 3x + 2y + z = 8 \\ 2x - 3y + 2z = -16 \end{cases}$

51.  $\begin{cases} 3x + 4y = 2 \\ 2x + 5y = -1 \end{cases}$

62.  $\begin{cases} -2x + 3y = 4 \\ -3x + 4y = 5 \end{cases}$

70.  $\begin{cases} 2y - z = 7 \\ x + 2y + z = 17 \\ 2x - 3y + 2z = -1 \end{cases}$

52.  $\begin{cases} 5x - 2y = 4 \\ -10x + 4y = 7 \end{cases}$

63.  $\begin{cases} x - 2y = 6 \\ 4x + 3y = 2 \end{cases}$

53.  $\begin{cases} x - 4y = -8 \\ 5x - 20y = -40 \end{cases}$

64.  $\begin{cases} 2x - 3y = 7 \\ 4x + y = -7 \end{cases}$

71.  $\begin{cases} -2x + 6y + 7z = 3 \\ -4x + 5y + 3z = 7 \\ -6x + 3y + 5z = -4 \end{cases}$

54.  $\begin{cases} 2x + y = 3 \\ x - y = 3 \end{cases}$

65.  $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$

72.  $\begin{cases} 2x - y + z = 1 \\ 3x - 3y + 4z = 5 \\ 4x - 2y + 3z = 4 \end{cases}$

55.  $\begin{cases} 2x + 10y = -14 \\ 7x - 2y = -16 \end{cases}$

66.  $\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \\ 3x - y + z = 9 \end{cases}$

73.  $\begin{cases} x - 2y - z = 2 \\ 2x - y + z = 4 \\ -x + y + z = 4 \end{cases}$

57.  $\begin{cases} 4x + 2y = 12 \\ 3x - 2y = 16 \end{cases}$