

Solution

Section 5.1– Introduction

Exercise

Indicate the angle if it is an acute or obtuse. Then give the complement and the supplement of each angle.

- a) 10° b) 52° c) 90° d) 120° e) 150°

Solution

- a) Acute;
Complement is $90^\circ - 10^\circ = 80^\circ$;
Supplement is $180^\circ - 10^\circ = 170^\circ$.
- b) Acute;
Complement is $90^\circ - 52^\circ = 38^\circ$;
Supplement is $180^\circ - 52^\circ = 128^\circ$.
- c) Neither (*right angle*);
Complement is $90^\circ - 90^\circ = 0^\circ$;
Supplement is $180^\circ - 90^\circ = 90^\circ$.
- d) Obtuse;
Complement is $90^\circ - 120^\circ = -30^\circ$;
Supplement is $180^\circ - 120^\circ = 60^\circ$.
- e) Obtuse;
Complement is $90^\circ - 150^\circ = -60^\circ$;
Supplement is $180^\circ - 150^\circ = 30^\circ$.

Exercise

Change to decimal degrees

- a) $10^\circ 45'$ c) $274^\circ 18' 59''$ e) $98^\circ 22' 45''$ g) $1^\circ 2' 3''$
b) $34^\circ 51' 35''$ d) $74^\circ 8' 14''$ f) $9^\circ 9' 9''$ h) $73^\circ 40' 40''$

Solution

- a) $10^\circ 45' = 10^\circ + 45'$
 $= 10^\circ + 45' \cdot \frac{1^\circ}{60'}$
 $= 10^\circ + 0.75^\circ$
 $= \underline{10.75^\circ}$
- b) $34^\circ 51' 35'' = 34^\circ + 51' + 35''$
 $= 34^\circ + 51' \cdot \frac{1^\circ}{60'} + 35'' \cdot \frac{1^\circ}{3600''}$
 $= 34^\circ + 0.85^\circ + 0.00972^\circ$

$$= \underline{34.85972^\circ}$$

$$\begin{aligned} c) \quad 274^\circ 18' 59'' &= 274^\circ + 18' + 59'' \\ &= 274^\circ + 18' \cdot \frac{1^\circ}{60'} + 59'' \cdot \frac{1^\circ}{3600''} \\ &= 274^\circ + 0.3^\circ + 0.016389^\circ \\ &= \underline{274.316389^\circ} \end{aligned}$$

$$\begin{aligned} d) \quad 74^\circ 8' 14'' &= 74^\circ + \frac{8^\circ}{60} + \frac{14^\circ}{3600} \\ &= 74^\circ + 0.1333^\circ + 0.0039^\circ \\ &= \underline{74.137^\circ} \end{aligned}$$

$$\begin{aligned} e) \quad 98^\circ 22' 45'' &= 98^\circ + 22' + 45'' \\ &= 98^\circ + 22' \cdot \frac{1^\circ}{60'} + 45'' \cdot \frac{1^\circ}{3600''} \\ &= 98^\circ + 0.36667^\circ + 0.0125^\circ \\ &= \underline{98.37917^\circ} \end{aligned}$$

$$\begin{aligned} f) \quad 9^\circ 9' 9'' &= 9^\circ + 9' + 9'' \\ &= 9^\circ + 9' \cdot \frac{1^\circ}{60'} + 9'' \cdot \frac{1^\circ}{3600''} \\ &= 9^\circ + 0.15^\circ + 0.0025^\circ \\ &= \underline{9.1525^\circ} \end{aligned}$$

$$\begin{aligned} g) \quad 1^\circ 2' 3'' &= 1^\circ + 2' + 3'' \\ &= 1^\circ + 2' \cdot \frac{1^\circ}{60'} + 3'' \cdot \frac{1^\circ}{3600''} \\ &= 1^\circ + 0.03333^\circ + 0.000833^\circ \\ &= \underline{1.034163^\circ} \end{aligned}$$

$$\begin{aligned} h) \quad 73^\circ 40' 40'' &= 73^\circ + 40' + 40'' \\ &= 73^\circ + 40' \cdot \frac{1^\circ}{60'} + 40'' \cdot \frac{1^\circ}{3600''} \\ &= 73^\circ + 0.6667^\circ + 0.0111^\circ \\ &= \underline{73.67778^\circ} \end{aligned}$$

Exercise

Convert to degrees, minutes, and seconds.

a) 89.9004°

c) 122.6853°

e) 44.01°

g) 29.411°

b) 34.817°

d) 178.5994°

f) 19.99°

h) 18.255°

Solution

$$\begin{aligned} a) \quad 89.9004^\circ &= 89^\circ + 0.9004^\circ \\ &= 89^\circ + 0.9004^\circ \cdot (60') \\ &= 89^\circ \quad 54.024' \\ &= 89^\circ \quad 54' + 0.024' \\ &= 89^\circ \quad 54' \quad 0.024' \cdot (60'') \\ &= 89^\circ \quad 54' \quad 1.44'' \end{aligned}$$

$$\begin{aligned} b) \quad 34.817^\circ &= 34^\circ + 0.817^\circ \\ &= 34^\circ + 0.817^\circ (60') \\ &= 34^\circ + 49.02' \\ &= 34^\circ + 49' + .02(60'') \\ &= 34^\circ + 49' + 1.2'' \\ &= 34^\circ \quad 49' \quad 1.2'' \end{aligned}$$

$$\begin{aligned} c) \quad 122.6853^\circ &= 122^\circ + .6853^\circ \\ &= 122^\circ + 0.6853^\circ \cdot (60') \\ &= 122^\circ \quad 41.118' \\ &= 122^\circ \quad 41' + 0.118' \\ &= 122^\circ \quad 41' \quad 0.118^\circ \cdot (60'') \\ &= 122^\circ \quad 41' \quad 7.1'' \end{aligned}$$

$$\begin{aligned} d) \quad 178.5994^\circ &= 178^\circ + .5994^\circ \\ &= 178^\circ + .5994^\circ \cdot (60') \\ &= 178^\circ \quad 35.964' \\ &= 178^\circ \quad 35' + .964' \\ &= 178^\circ \quad 35' \quad 0.964^\circ \cdot (60'') \\ &= 178^\circ \quad 35' \quad 57.84'' \end{aligned}$$

$$\begin{aligned} e) \quad 44.01^\circ &= 44^\circ + .01^\circ \\ &= 44^\circ + .01^\circ \cdot (60') \\ &= 44^\circ \quad 0.6' \end{aligned}$$

$$= 44^\circ \quad 0.6 \cdot (60'') \\ = 44^\circ \quad 36''$$

$$\begin{aligned} f) \quad 19.99^\circ &= 19^\circ + .99^\circ \\ &= 19^\circ + .99 \cdot (60') \\ &= 19^\circ \quad 59.4' \\ &= 19^\circ \quad 59' + 0.4' \\ &= 19^\circ \quad 59' \quad 0.4 \cdot (60'') \\ &= 19^\circ \quad 59' \quad 24'' \end{aligned}$$

$$\begin{aligned} g) \quad 29.411^\circ &= 29^\circ + 0.411^\circ \\ &= 29^\circ + 0.411 \cdot (60') \\ &= 29^\circ \quad 24.66' \\ &= 29^\circ \quad 24' + 0.66' \\ &= 29^\circ \quad 24' \quad 0.66 \cdot (60'') \\ &= 29^\circ \quad 24' \quad 39.6'' \end{aligned}$$

$$\begin{aligned} h) \quad 18.255^\circ &= 18^\circ + 0.255^\circ \\ &= 18^\circ + 0.255 \cdot (60') \\ &= 18^\circ \quad 15.3' \\ &= 18^\circ \quad 15' + 0.3' \\ &= 18^\circ \quad 15' \quad 0.3 \cdot (60'') \\ &= 18^\circ \quad 15' \quad 18'' \end{aligned}$$

Exercise

Perform each calculation

$$a) \quad 51^\circ 29' + 32^\circ 46' \quad b) \quad 90^\circ - 73^\circ 12' \quad c) \quad 90^\circ - 36^\circ 18' 47'' \quad d) \quad 75^\circ 15' + 83^\circ 32'$$

Solution

$$\begin{aligned} a) \quad &51^\circ 29' + 32^\circ 46' \\ &\begin{array}{r} 51^\circ \quad 29' \\ + 32^\circ \quad 46' \\ \hline 83^\circ \quad 75' \end{array} \\ &83^\circ \quad 75' = 1^\circ 15' \quad \boxed{84^\circ \quad 15'} \end{aligned}$$

$$b) \quad 90^\circ - 73^\circ 12'$$

$$\begin{array}{r} 89^\circ \ 60' \\ -73^\circ \ 12' \\ \hline 16^\circ \ 48' \end{array}$$

$$c) \ 90^\circ - 36^\circ 18' 47''$$

$$\begin{array}{r} 90^\circ \\ -36^\circ \ 18' \ 47'' \\ \hline \end{array} \Rightarrow \begin{array}{r} 89^\circ \ 59' \ 60'' \\ -36^\circ \ 18' \ 47'' \\ \hline 53^\circ \ 41' \ 13'' \end{array}$$

$$d) \ 75^\circ 15' + 83^\circ 32'$$

$$\begin{array}{r} 75^\circ \ 15' \\ 83^\circ \ 32' \\ \hline 158^\circ \ 47' \end{array}$$

Exercise

Find the angle of least possible positive measure coterminal with an angle of

$$a) \ -75^\circ$$

$$b) \ -800^\circ$$

$$c) \ 270^\circ$$

Solution

$$a) \ 360^\circ - 75^\circ = \underline{285^\circ}$$

$$b) \ 3(360^\circ) - 800^\circ = \underline{280^\circ}$$

$$c) \ 360^\circ + 270^\circ = \underline{630^\circ}$$

Exercise

A vertical rise of the Forest Double chair lift 1,170 *feet* and the length of the chair lift as 5,570 *feet*. To the nearest foot, find the horizontal distance covered by a person riding this lift.

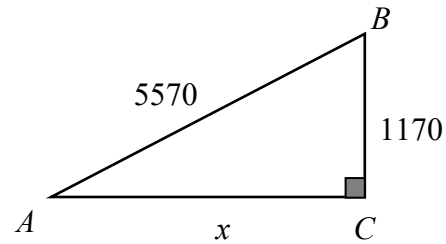
Solution

$$x^2 + 1170^2 = 5570^2$$

$$x^2 = 5570^2 - 1170^2$$

$$x = \sqrt{5570^2 - 1170^2}$$

$$x = 5,445.73 \text{ ft}$$



Exercise

A tire is rotating 600 times per minute. Through how many degrees does a point of the edge of the tire move in $\frac{1}{2}$ second?

Solution

$$\frac{1}{2} 600 \frac{\text{rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{360^\circ}{1 \text{ rev}} = 1800 \text{ deg / sec}$$

Exercise

A windmill makes 90 revolutions per minute. How many revolutions does it make per second?

Solution

$$90 \frac{\text{rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 1.5 \text{ rev / sec}$$

Exercise

Convert to radians

- a) $256^\circ 20'$ b) -78.4° c) 330° d) -60° e) -225°

Solution

$$\begin{aligned} \text{a) } 256^\circ 20' &= 256^\circ + \frac{20^\circ}{60} \\ &= 256^\circ + \frac{2^\circ}{6} \\ &= \frac{1538^\circ}{6} = \left(\frac{769}{3} \right)^\circ \end{aligned}$$

$$\frac{769^\circ}{3} \cdot \frac{\pi}{180^\circ} = \frac{769\pi}{540} \text{ rad} \approx 4.47 \text{ rad}$$

$$b) -78.4^\circ = -78.4^\circ \left(\frac{\pi}{180^\circ} \right) rad$$

$$\underline{\approx -1.37 rad}$$

$$c) 330^\circ = 330^\circ \left(\frac{\pi}{180^\circ} \right) rad$$

$$\underline{= \frac{11\pi}{6} rad}$$

$$d) -60^\circ = -60^\circ \left(\frac{\pi}{180^\circ} \right) rad$$

$$\underline{= -\frac{\pi}{3} rad}$$

$$e) -225^\circ = -225^\circ \left(\frac{\pi}{180^\circ} \right) rad$$

$$\underline{= -\frac{5\pi}{4} rad}$$

Exercise

Convert to degrees

$$a) \frac{11\pi}{6}$$

$$c) \frac{\pi}{6}$$

$$e) \frac{\pi}{3}$$

$$g) -4\pi$$

$$b) -\frac{5\pi}{3}$$

$$d) 2.4$$

$$f) -\frac{5\pi}{12}$$

$$h) \frac{7\pi}{13}$$

Solution

$$a) \frac{11\pi}{6} (rad) = \frac{11\pi}{6} \cdot \frac{180^\circ}{\pi}$$

$$\underline{= 330^\circ}$$

$$b) -\frac{5\pi}{3} (rad) = -\frac{5\pi}{3} \cdot \frac{180^\circ}{\pi}$$

$$\underline{= -300^\circ}$$

$$c) \frac{\pi}{6} (rad) = \frac{\pi}{6} \left(\frac{180^\circ}{\pi} \right)$$

$$\underline{= 30^\circ}$$

$$d) 2.4 rad = 2.4 \cdot \frac{180^\circ}{\pi}$$

$$= \frac{432^\circ}{\pi}$$

$$\underline{\approx 137.5^\circ}$$

$$e) \quad \frac{\pi}{3}(\text{rad}) = \frac{\pi}{3} \left(\frac{180}{\pi} \right)^{\circ}$$

$$\underline{= 60^{\circ}} \quad |$$

$$f) \quad -\frac{5\pi}{12}(\text{rad}) = -\frac{5\pi}{12} \left(\frac{180}{\pi} \right)^{\circ}$$

$$\underline{= -75^{\circ}} \quad |$$

$$g) \quad -4\pi(\text{rad}) = -4\pi \left(\frac{180}{\pi} \right)^{\circ}$$

$$\underline{= -720^{\circ}} \quad |$$

$$h) \quad \frac{7\pi}{13}(\text{rad}) = \frac{7\pi}{13} \left(\frac{180}{\pi} \right)^{\circ}$$

$$\underline{\approx 96.923^{\circ}} \quad |$$