Solution Section 2.2 – Function Operations

Exercise

Find the domain: $f(x) = 7x + 4$

Solution

Domain: $(-\infty, \infty)$

Exercise

Find the domain: $f(x) = |3x - 2|$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = 3x + \pi$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = \sqrt{7}x + \frac{1}{2}$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = -2x^2 + 3x - 5$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = x^3 - 2x^2 + x - 3$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = x^2 - 2x - 15$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain $f(x) = 4 - \frac{2}{x}$

Solution

Domain: $x \neq 0$

Exercise

Find the domain $f(x) = \frac{1}{x^4}$

Solution

Domain: $x \neq 0$

Exercise

Find the domain: $g(x) = \frac{3}{x-4}$

Solution

Domain: $x \neq 4$

Exercise

Find the domain $y = \frac{2}{x-3}$

Solution

Domain: $x \neq 3$

Exercise

Find the domain $y = \frac{-7}{x-5}$

Solution

Domain: $\underline{x \ne 5}$

Exercise

Find the domain $f(x) = \frac{x+5}{2-x}$

Solution

$2 - x \ne 0$

Domain: $\underline{x \ne 2}$

Exercise

Find the domain $f(x) = \frac{8}{x+4}$

Solution

$x + 4 \ne 0$

Domain: $\underline{x \ne -4}$

Exercise

Find the domain $f(x) = \frac{1}{x+4}$

Solution

Domain: $\underline{x \ne -4}$

Exercise

Find the domain $f(x) = \frac{1}{x-4}$

Solution

Domain: $\underline{x \ne 4}$

Exercise

Find the domain $f(x) = \dfrac{3x}{x+2}$

Solution

Domain: $\underline{x \neq -2}$

Exercise

Find the domain $f(x) = x - \dfrac{2}{x-3}$

Solution

Domain: $\underline{x \neq 3}$

Exercise

Find the domain $f(x) = x + \dfrac{3}{x-5}$

Solution

Domain: $\underline{x \neq 5}$

Exercise

Find the domain $f(x) = \dfrac{1}{2}x - \dfrac{8}{x+7}$

Solution

Domain: $\underline{x \neq -7}$

Exercise

Find the domain $f(x) = \dfrac{1}{x-3} - \dfrac{8}{x+7}$

Solution

Domain: $\underline{x \neq -7,\ 3}$

Exercise

Find the domain $f(x) = \dfrac{1}{x+4} - \dfrac{2x}{x-4}$

Solution

Domain: $\underline{x \neq \pm 4}$

Exercise

Fib+cnd the domain $f(x)=\frac{3x^2}{x+3}-\frac{4x}{x-2}$

Solution

Domain: $\boxed{x \neq -3,\ 2}$

Exercise

Find the domain $f(x)=\frac{1}{x^2-2x+1}$

Solution

$x^2-2x+1 \neq 0 \qquad a+b+c=0 \ \rightarrow\ x=1,\ \frac{c}{a}$

Domain: $\boxed{x \neq 1}$

Exercise

Find the domain $f(x)=\frac{x}{x^2+3x+2}$

Solution

$x^2+3x+2 \neq 0 \qquad a-b+c=0 \ \rightarrow\ x=-1,\ -\frac{c}{a}$

Domain: $\boxed{x \neq -1,\ -2}$

Exercise

Find the domain $f(x)=\frac{x^2}{x^2-5x+4}$

Solution

$x^2-5x+4 \neq 0 \qquad a+b+c=0 \ \rightarrow\ x=1,\ \frac{c}{a}$

Domain: $\boxed{x \neq 1,\ 4}$

Exercise

Find the domain $f(x)=\frac{1}{x^2-4x-5}$

Solution

$x^2-4x-5 \neq 0 \qquad a-b+c=0 \ \rightarrow\ x=-1,\ -\frac{c}{a}$

Domain: $\boxed{x \neq -1,\ 5}$

Exercise

Find the domain $g(x) = \dfrac{2}{x^2 + x - 12}$

Solution

$$x^2 + x - 12 \neq 0$$
$$(x+4)(x-3) \neq 0$$

Domain: $\underline{x \neq -4,\ 3 \,|}$ $\quad \underline{(-\infty,-4)\cup(-4,3)\cup(3,\infty)|}$

Exercise

Find the domain $h(x) = \dfrac{5}{\frac{4}{x} - 1}$

Solution

$x \neq 0 \qquad \dfrac{4}{x} - 1 \neq 0$

$$\frac{4-x}{x} \neq 0$$
$$4 - x \neq 0$$
$$x \neq 4$$

Domain: $\underline{x \neq 0,\ 4 \,|}$ $\quad \underline{(-\infty,0)\cup(0,4)\cup(4,\infty)|}$

Exercise

Find the domain $y = \sqrt{x}$

Solution

$$x \geq 0$$

Domain: $\underline{x \geq 0 \,|}$ $\ \underline{[0,\ \infty)|}$

Exercise

Find the domain $f(x) = \sqrt{8 - 3x}$

Solution

$$8 - 3x \geq 0$$
$$8 \geq 3x$$

Domain: $\underline{x \leq \frac{8}{3} \,|}$ $\quad \underline{\left(-\infty,\ \frac{8}{3}\right] |}$

Exercise

Find the domain $y = \sqrt{4x+1}$

Solution

$$4x+1 \ge 0 \Rightarrow x \ge -\frac{1}{4}$$

Domain: $\boxed{x \ge -\frac{1}{4}}$ $\left[-\frac{1}{4}, \infty\right)$

Exercise

Find the domain $y = \sqrt{7-2x}$

Solution

$$7-2x \ge 0$$

$$-2x \ge -7$$

Domain: $\boxed{x \le \frac{7}{2}}$ $\left(-\infty, \frac{7}{2}\right]$

Exercise

Find the domain $f(x) = \sqrt{8-x}$

Solution

$$8-x \ge 0$$

Domain: $\boxed{x \le -8}$ $(-\infty,\ 8]$

Exercise

Find the domain $f(x) = \sqrt{3-2x}$

Solution

Domain: $\boxed{x \le \frac{3}{2}}$

Exercise

Find the domain $f(x) = \sqrt{3+2x}$

Solution

Domain: $\boxed{x \ge -\frac{3}{2}}$

Exercise

Find the domain $f(x)=\sqrt{5-x}$

Solution

Domain: $\underline{x \le 5}$

Exercise

Find the domain $f(x)=\sqrt{x-5}$

Solution

Domain: $\underline{x \ge 5}$

Exercise

Find the domain $f(x)=\sqrt{6-3x}$

Solution

Domain: $\underline{x \le 2}$

Exercise

Find the domain $f(x)=\sqrt{3x-6}$

Solution

Domain: $\underline{x \ge 2}$

Exercise

Find the domain $f(x)=\sqrt{2x+7}$

Solution

Domain: $\underline{x \ge -\frac{7}{2}}$

Exercise

Find the domain $f(x)=\sqrt{x^2-16}$

Solution

$x^2-16=0$

$x^2 = 16$

$x = \pm 4$

Domain: $\underline{x \le -4 \quad x \ge 4}$

Exercise

Find the domain $f(x) = \sqrt{16 - x^2}$

Solution

$x = \pm 4$

Domain: $\underline{-4 \le x \le 4}$

Exercise

Find the domain $f(x) = \sqrt{9 - x^2}$

Solution

$x = \pm 3$

Domain: $\underline{-3 \le x \le 3}$

Exercise

Find the domain $f(x) = \sqrt{x^2 - 25}$

Solution

$x = \pm 5$

Domain: $\underline{-5 \le x \le 5}$

Exercise

Find the domain $f(x) = \sqrt{x^2 - 5x + 4}$

Solution

$x^2 - 5x + 4$ $\qquad a + b + c = 0 \quad \rightarrow \quad x = 1, \ \frac{c}{a}$

$x = 1, \ 4$

Domain: $\underline{x \le 1 \quad x \ge 4}$

Exercise

Find the domain $\quad f(x)=\sqrt{x^2+5x+4}$

Solution

$x^2+5x+4 \qquad a-b+c=0 \rightarrow x=-1, \ -\frac{c}{a}$

$x=-1, \ -4$

Domain: $\underline{x\le -4 \quad x\ge -1}$

Exercise

Find the domain $\quad f(x)=\sqrt{x^2+3x+2}$

Solution

$x^2+3x+2 \qquad a-b+c=0 \rightarrow x=-1, \ -\frac{c}{a}$

$x=-1, \ -2$

Domain: $\underline{x\le -2 \quad x\ge -1}$

Exercise

Find the domain $\quad f(x)=\sqrt{x^2-3x+2}$

Solution

$x^2-3x+2 \qquad a+b+c=0 \rightarrow x=1, \ \frac{c}{a}$

$x=1, \ 2$

Domain: $\underline{x\le 1 \quad x\ge 2}$

Exercise

Find the domain $\quad f(x)=\sqrt{x-4}+\sqrt{x+1}$

Solution

$x\ge 4 \quad x\ge -1$

Domain: $\underline{x\ge 4}$

Exercise

Find the domain $\qquad f(x)=\sqrt{3-x}+\sqrt{x-2}$

Solution

$x \le 3 \quad x \ge 2$

Domain: $\underline{2 \le x \le 3}$

Exercise

Find the domain $\qquad f(x)=\sqrt{1-x}+\sqrt{4-x}$

Solution

$x \le 1 \quad x \le 4$

Domain: $\underline{x \le 1}$

Exercise

Find the domain $\qquad f(x)=\sqrt{1-x}-\sqrt{x-3}$

Solution

$x \le 1 \quad x \ge 3$

Domain: $\underline{\varnothing}$

Exercise

Find the domain $\qquad f(x)=\sqrt{x+4}-\sqrt{x-1}$

Solution

$x \ge -4 \quad x \ge 1$

Domain: $\underline{x \ge 1}$

Exercise

Find the domain $\qquad f(x)=\dfrac{\sqrt{x+1}}{x}$

Solution

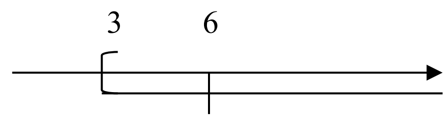
$x+1 \ge 0 \qquad\qquad x \ne 0$

$x \ge -1$

Domain: $\underline{x \ge -1 \quad x \ne 0} \qquad \underline{[-1,\ 0)\cup(0,\ \infty)}$

Exercise

Find the domain $\qquad g(x)=\dfrac{\sqrt{x-3}}{x-6}$



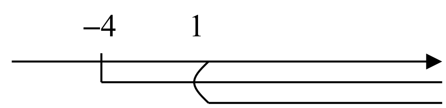
Solution

$$\to \begin{cases} x \ge 3 \\ x \ne 6 \end{cases}$$

Domain: $\boxed{x \ge 3 \quad x \ne 6}$ $\qquad \boxed{[3,\ 6)\cup(6,\ \infty)}$

Exercise

Find the domain $\qquad f(x)=\dfrac{\sqrt{x+4}}{\sqrt{x-1}}$



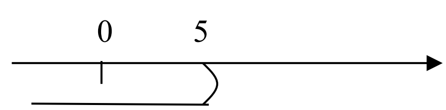
Solution

$$\to \begin{cases} x \ge -4 \\ x > 1 \end{cases}$$

Domain: $\boxed{x > 1}$ $\quad \boxed{(1,\ \infty)}$

Exercise

Find the domain $\qquad f(x)=\dfrac{\sqrt{5-x}}{x}$



Solution

$x \le 5 \quad x \ne 0$

Domain: $\boxed{x \le 5 \quad x \ne 0}$ $\quad \boxed{(-\infty,\ 0)\cup(0,\ 5]}$

Exercise

Find the domain $\qquad f(x)=\dfrac{x}{\sqrt{5-x}}$

Solution

Domain: $\boxed{x < 5}$ $\quad \boxed{(-\infty,\ 5)}$

Exercise

Find the domain $f(x)=\dfrac{1}{x\sqrt{5-x}}$

Solution

$x<5 \quad x\neq 0$

Domain: $\underline{x<5 \quad x\neq 0}$ $\underline{(-\infty,\ 0)\cup(0,\ 5)}$

Exercise

Find the domain $f(x)=\dfrac{x+1}{x^3-4x}$

Solution

$x^3-4x\neq 0$

$x\left(x^2-4\right)\neq 0$

Domain: $\underline{x\neq 0,\ \ \pm 2}$ $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$

Exercise

Find the domain $f(x)=\dfrac{\sqrt{x+5}}{x}$

Solution

$x\geq -5 \quad x\neq 0$

Domain: $\underline{x\geq -5 \quad x\neq 0}$

Exercise

Find the domain $f(x)=\dfrac{x}{\sqrt{x+5}}$

Solution

$x>-5$

Domain: $\underline{x>-5}$

Exercise

Find the domain $f(x)=\dfrac{1}{x\sqrt{x+5}}$

Solution

$x>-5 \quad x\neq 0$

Domain: $\underline{x>-5 \quad x\neq 0}$

Exercise

Find the domain $f(x)=\dfrac{x+3}{\sqrt{x-3}}$

Solution

Domain: $\underline{x>3}$

Exercise

Find the domain $f(x)=\dfrac{\sqrt{x+3}}{\sqrt{x-3}}$

Solution

$x\geq -3 \quad x>3$

Domain: $\underline{x>3}$

Exercise

Find the domain $f(x)=\dfrac{\sqrt{x-2}}{\sqrt{x+2}}$

Solution

$x\geq 2 \quad x>-2$

Domain: $\underline{x\geq 2}$

Exercise

Find the domain $f(x)=\dfrac{\sqrt{2-x}}{\sqrt{x+2}}$

Solution

$x\leq 2 \quad x>-2$

Domain: $\underline{-2<x\leq 2}$

Exercise

Find the domain $f(x)=\dfrac{x-4}{\sqrt{x-2}}$

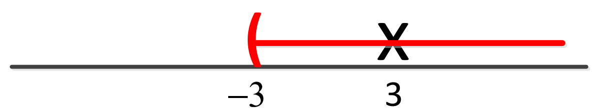
Solution

Domain: $x>2$

Exercise

Find the domain of $f(x)=\dfrac{1}{(x-3)\sqrt{x+3}}$

Solution



$x-3\neq 0 \qquad x+3>0$

$x\neq 3 \qquad x>-3$

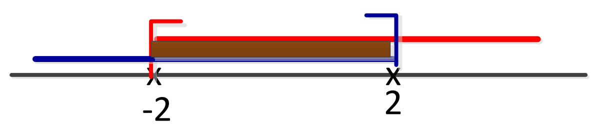
Domain: $\{x\mid x>-3 \;\; and \;\; x\neq 3\}$

$(-3,\ 3)\cup(3,\ \infty)$

Exercise

Find the domain of $f(x)=\sqrt{x+2}+\sqrt{2-x}$

Solution



$x+2\geq 0 \qquad 2-x\geq 0$

$x\geq -2 \qquad -x\geq -2\to x\leq 2$

Domain: $\{x\mid -2\leq x\leq 2\}$

Exercise

Find the domain of $f(x)=\sqrt{(x-2)(x-6)}$

Solution

$x-2\geq 0 \quad x-6\geq 0$

$x\geq 2 \quad x\geq 6$

Domain: $\{x\mid x\leq 2,\ x\geq 6\}$

2 6

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−	−	+
+	−	+

Exercise

Find the domain of $f(x)=\sqrt{x+3}-\sqrt{4-x}$

Solution

$x \ge -3 \quad x \le 4$

Domain: $-3 \le x \le 4$

Exercise

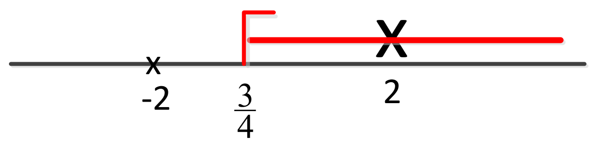
Find the domain of $f(x)=\dfrac{\sqrt{4x-3}}{x^2-4}$

Solution

$4x-3 \ge 0 \qquad x^2-4 \ne 0$

$4x \ge 3 \qquad x \ne \pm 2$

$x \ge \dfrac{3}{4}$



Domain: $\left[\frac{3}{4},\ 2\right)\cup(2,\ \infty)$

Exercise

Find the domain of $f(x)=\dfrac{4x}{6x^2+13x-5}$

Solution

$6x^2+13x-5 \ne 0$

$$x=\frac{-13\pm\sqrt{169+120}}{12}$$

$$=\begin{cases}\dfrac{-13-17}{12}=-\dfrac{5}{2}\\[2mm] \dfrac{-13+17}{12}=\dfrac{1}{3}\end{cases}$$

Domain: $x \ne -\frac{5}{2},\ \frac{1}{3}$

Exercise

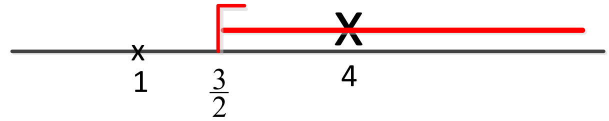
Find the domain of $f(x)=\dfrac{\sqrt{2x-3}}{x^2-5x+4}$

Solution

$2x-3 \ge 0 \qquad x^2-5x+4 \ne 0$

$2x \ge 3 \qquad\qquad x \ne 1,\ 4$

$x \ge \frac{3}{2}$



Domain: $\underline{x \ge \frac{3}{2},\ \ x \ne 4}\Big|$ $\qquad \left[\frac{3}{2},\ 4\right) \cup (4,\ \infty)$

Exercise

Find the domain of $\quad f(x) = \dfrac{x^2}{\sqrt{x^2 - 5x + 4}}$

Solution

$x^2 - 5x + 4 \qquad\qquad a + b + c = 0 \quad \to \quad x = 1,\ \frac{c}{a}$

$x = 1,\ \ 4$

Domain: $\underline{x < 1 \quad x > 4}\Big|$

Exercise

Find the domain of $\quad f(x) = \dfrac{x+2}{\sqrt{x^2 + 5x + 4}}$

Solution

$x^2 + 5x + 4 \qquad\qquad a - b + c = 0 \quad \to \quad x = -1,\ -\frac{c}{a}$

$x = -1,\ \ -4$

Domain: $\underline{x < -4 \quad x > -1}\Big|$

Exercise

Find the domain of $\quad f(x) = \dfrac{\sqrt{x+2}}{\sqrt{x^2 + 3x + 2}}$

Solution

$x^2 + 3x + 2 \qquad\qquad a - b + c = 0 \quad \to \quad x = -1,\ -\frac{c}{a}$

$x < -2 \quad x > -1$

$\sqrt{x+2} \quad \to \quad x \ge -2$

Domain: $\underline{x > -1}\Big|$

Exercise

Find the domain of $f(x)=\dfrac{\sqrt{2x+3}}{x^2-6x+5}$

Solution

$x^2-6x+5 \qquad a+b+c=0 \quad \rightarrow \quad x=1,\ \frac{c}{a}$

$x \neq 1,\ 5$

$\sqrt{2x+3} \quad \rightarrow \quad x \geq -\frac{3}{2}$

Domain: $\underline{x \geq -\frac{3}{2} \quad x \neq 1,\ 5}$

Exercise

Let $f(x)=4x-3$ and $g(x)=5x+7$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f+g)(x)=4x-3+5x+7$

$\qquad\qquad \underline{=9x+4}$

Domain: $\underline{\mathbb{R}}$

b) $(f-g)(x)=4x-3-(5x+7)$

$\qquad\qquad =4x-3-5x-7$

$\qquad\qquad \underline{=-x-10}$

Domain: $\underline{\mathbb{R}}$

c) $(fg)(x)=(4x-3)(5x+7)$

$\qquad\qquad \underline{=20x^2+13x-21}$

Domain: $\underline{\mathbb{R}}$

d) $\underline{\left(\frac{f}{g}\right)(x)=\frac{4x-3}{5x+7}}$

Domain: $\underline{x \neq -\frac{7}{5}}$

Exercise

Let $f(x) = 2x^2 + 3$ and $g(x) = 3x - 4$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f+g)(x) = 2x^2 + 3 + 3x - 4$

$$= 2x^2 + 3x - 1$$

Domain: $\mathbb{R}$

b) $(f-g)(x) = 2x^2 + 3 - (3x - 4)$

$$= 2x^2 + 3 - 3x + 4$$

$$= 2x^2 - x + 7$$

Domain: $\mathbb{R}$

c) $(fg)(x) = \left(2x^2 + 3\right)(3x - 4)$

$$= 6x^2 + x - 12$$

Domain: $\mathbb{R}$

d) $\left(\frac{f}{g}\right)(x) = \dfrac{2x^2 + 3}{3x - 4}$

Domain: $x \neq -\frac{4}{3}$

Exercise

Let $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 + 3x - 2$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f+g)(x) = x^2 - 2x - 3 + x^2 + 3x - 2$

$$= 2x^2 + x - 5$$

Domain: $\mathbb{R}$

b) $(f-g)(x) = x^2 - 2x - 3 - x^2 - 3x + 2$

$$= -5x - 1$$

Domain: $\boxed{\mathbb{R}}$

c) $(fg)(x) = \left(x^2 - 2x - 3\right)\left(x^2 + 3x - 2\right)$

$$= x^4 + 3x^3 - 2x^2 - 2x^3 - 6x^2 + 4x - 3x^2 - 9x + 6$$

$$\boxed{= x^4 + x^3 - 11x^2 - 5x + 6}$$

Domain: $\boxed{\mathbb{R}}$

d) $\boxed{\left(\frac{f}{g}\right)(x) = \frac{x^2 - 2x - 3}{x^2 + 3x - 2}}$

Domain: $\boxed{x \neq \frac{-3 \pm \sqrt{17}}{2}}$

Exercise

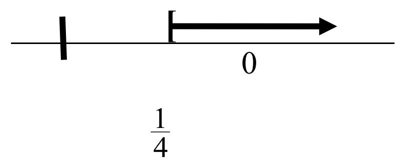
Let $f(x) = \sqrt{4x-1}$ and $g(x) = \frac{1}{x}$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f+g)(x)$

$(f+g)(x) = \sqrt{4x-1} + \frac{1}{x}$



$4x - 1 \geq 0 \qquad x \neq 0$

$x \geq \frac{1}{4}$

Domain: $\left[\frac{1}{4}, \infty\right)$

b) $(f-g)(x)$

$(f-g)(x) = \sqrt{4x-1} - \frac{1}{x}$

$4x - 1 \geq 0 \qquad x \neq 0$

$x \geq \frac{1}{4}$

Domain: $\left[\frac{1}{4}, \infty\right)$

c) $(fg)(x) = \sqrt{4x-1}\left(\frac{1}{x}\right)$

$$= \frac{\sqrt{4x-1}}{x}$$

$4x-1\geq 0 \qquad\qquad x\neq 0$

$x\geq \frac{1}{4}$

Domain: $\left[\frac{1}{4},\infty\right)$

d) $\left(\frac{f}{g}\right)(x)=\dfrac{\sqrt{4x-1}}{\frac{1}{x}}$ *Domain:* $x\neq 0$

$=x\sqrt{4x-1}$

$4x-1\geq 0$

$x\geq \frac{1}{4}$

Domain: $\left[\frac{1}{4},\infty\right)$

Exercise

Given that $f(x)=x+1$ and $g(x)=\sqrt{x+3}$

a) Find $(f+g)(x)$

b) Find the domain of $(f+g)(x)$

c) Find: $(f+g)(6)$

Solution

a) $(f+g)(x)=f(x)+g(x)$

$=x+1+\sqrt{x+3}$

b) $x+3\geq 0\rightarrow x\geq -3$

Domain $=[-3,\ \infty)$

c) $(f+g)(6)=6+1+\sqrt{6+3}$

$=10$

Exercise

Given that $f(x)=x^2-4$ and $g(x)=x+2$

a) Find $(f+g)(x)$ and its domain

b) Find $(f/g)(x)$ and its domain

Solution

a) $(f+g)(x) = x^2 - 4 + x + 2$

$= x^2 + x - 2$

Domain: $\underline{\mathbb{R}}$

b) $\dfrac{f(x)}{g(x)} = \dfrac{x^2-4}{x+2}$

$x \neq -2$

Domain: $\underline{(-\infty,-2)\cup(-2,\infty)}$

Exercise

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find $(f+g)(1)$, $(f-g)(-3)$, $(fg)(5)$, and $\left(\dfrac{f}{g}\right)(0)$

Solution

a) $(f+g)(1) = f(1) + g(1)$

$= 1^2 + 1 + 3(1) + 5$

$\underline{= 10}$

b) $(f-g)(-3) = f(-3) - g(-3)$

$= (-3)^2 + 1 - (3(-3) + 5)$

$\underline{= 10}$

c) $(fg)(5) = f(5) \cdot g(5)$

$= \left(5^2 + 1\right) \cdot (3(5) + 5)$

$= (26) \cdot (20)$

$\underline{= 520}$

d) $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)}$

$= \dfrac{0^2 + 1}{3(0) + 5}$

$\underline{= \dfrac{1}{5}}$

Exercise

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, *and* $(f/g)(x)$ and the domain of

$f(x)=\sqrt{3-2x}$, $g(x)=\sqrt{x+4}$

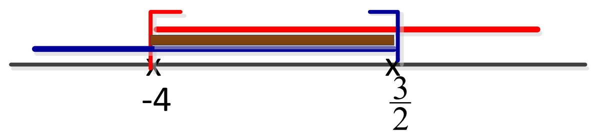
Solution

$$f(x)+g(x)=\sqrt{3-2x}+\sqrt{x+4}$$

$3-2x\ge 0 \qquad x+4\ge 0$

$-2x\ge -3 \qquad x\ge -4$

$x\le \frac{3}{2}$



Domain: $\left\{x \mid -4\le x\le \frac{3}{2}\right\}$

$$f(x)-g(x)=\sqrt{3-2x}-\sqrt{x+4}$$

$3-2x\ge 0 \qquad x+4\ge 0$

$-2x\ge -3 \qquad x\ge -4$

$x\le \frac{3}{2}$

Domain: $\left\{x \mid -4\le x\le \frac{3}{2}\right\}$

$$\begin{aligned}(f\cdot g)(x)&=\left(\sqrt{3-2x}\right)\left(\sqrt{x+4}\right)\\&=\sqrt{(3-2x)(x+4)}\\&=\sqrt{-2x^2-5x+12}\end{aligned}$$

$3-2x\ge 0 \qquad x+4\ge 0$

$-2x\ge -3 \qquad x\ge -4$

$x\le \frac{3}{2}$

Domain: $\left\{x \mid -4\le x\le \frac{3}{2}\right\}$

$$\begin{aligned}(f/g)(x)&=\frac{\sqrt{3-2x}}{\sqrt{x+4}}\,\frac{\sqrt{x+4}}{\sqrt{x+4}}\\&=\frac{\sqrt{-2x^2-5x+12}}{x+4}\end{aligned}$$

$3-2x\ge 0 \qquad x+4> 0$

$-2x\ge -3 \qquad x> -4$

$x\le \frac{3}{2}$

Domain: $\left\{x \mid -4< x\le \frac{3}{2}\right\}$ $\qquad \left(-4,\ \frac{3}{2}\right]$

Exercise

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, *and* $(f/g)(x)$ and the domain of

$f(x)=\frac{2x}{x-4}$, $g(x)=\frac{x}{x+5}$

Solution

$$\begin{aligned}(f+g)(x)&=\frac{2x}{x-4}+\frac{x}{x+5}\\&=\frac{2x(x+5)+x(x-4)}{(x-4)(x+5)}\\&=\frac{2x^2+10x+x^2-4x}{(x-4)(x+5)}\\&=\frac{3x^2+6x}{(x-4)(x+5)}\end{aligned}$$

$x-4\neq 0 \qquad x+5\neq 0$

$x\neq 4 \qquad x\neq -5$

Domain: $\{x \mid x\neq -5,4\}$ $\quad (-\infty,\ -5)\cup(-5,\ 4)\cup(4,\ \infty)$

$$\begin{aligned}(f-g)(x)&=\frac{2x}{x-4}-\frac{x}{x+5}\\&=\frac{2x(x+5)-x(x-4)}{(x-4)(x+5)}\\&=\frac{2x^2+10x-x^2+4x}{(x-4)(x+5)}\\&=\frac{x^2+14x}{(x-4)(x+5)}\end{aligned}$$

$x\neq 4 \qquad x\neq -5$

Domain: $\{x \mid x\neq -5,4\}$

$$\begin{aligned}(f\bullet g)(x)&=\frac{2x}{x-4}\frac{x}{x+5}\\&=\frac{2x^2}{(x-4)(x+5)}\end{aligned}$$

$x\neq 4 \qquad x\neq -5$

Domain: $\{x \mid x\neq -5,4\}$

$$\begin{aligned}(f/g)(x)&=\frac{2x}{x-4}\div\frac{x}{x+5}\\&=\frac{2x}{x-4}\frac{x+5}{x}\end{aligned}$$

$$= 2\frac{x+5}{x-4}$$

$x \neq 4 \qquad x \neq -5$

Domain: $\{x \mid x \neq -5, 4\}$

Exercise

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, *and* $(f/g)(x)$ of $f(x) = x-5$ and $g(x) = x^2 - 1$

Solution

a) $(f+g)(x) = f(x) + g(x)$

$$= x - 5 + x^2 - 1$$

$$= x^2 + x - 6$$

b) $(f-g)(x) = f(x) - g(x)$

$$= x - 5 - (x^2 - 1)$$

$$= x - 5 - x^2 + 1$$

$$= -x^2 + x - 4$$

c) $(fg)(x) = f(x)g(x)$

$$= (x-5)(x^2-1)$$

$$= x^3 - x - 5x^2 + 5$$

$$= x^3 - 5x^2 - x + 5$$

d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$$= \frac{x-5}{x^2-1}$$

Exercise

For the function *f* given by $f(x) = 9x + 5$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$f(x+h) = 9(x+h) + 5 = 9x + 9h + 5$

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{9x + 9h + 5}^{f(x+h)} - \overbrace{(9x+5)}^{f(x)}}{h}$$

$$= \frac{9x + 9h + 5 - 9x - 5}{h}$$

$$= \frac{9h}{h}$$
$$\underline{= 9}$$

Exercise

For the function *f* given by $f(x) = 6x + 2$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{6(x+h) + 2 - (6x+2)}{h}$$
$$= \frac{6x + 6h + 2 - 6x - 2}{h}$$
$$= \frac{6h}{h}$$
$$\underline{= 6}$$

Exercise

For the function *f* given by $f(x) = 4x + 11$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{4(x+h) + 11 - (4x+11)}{h}$$
$$= \frac{4x + 4h + 11 - 4x - 11}{h}$$
$$= \frac{4h}{h}$$
$$\underline{= 4}$$

Exercise

For the function *f* given by $f(x) = 3x - 5$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h) - 5 - 3x + 5}{h}$$
$$= \frac{3x + 3h - 5 - 3x + 5}{h}$$
$$= \frac{3h}{h}$$
$$\underline{= 3}$$

Exercise

For the function *f* given by $f(x) = -2x - 3$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{-2(x+h)-3+2x+3}{h} \\
&= \frac{-2x-2h-3+2x+3}{h} \\
&= \frac{-2h}{h} \\
&= -2
\end{aligned}$$

Exercise

For the function *f* given by $f(x) = -4x + 3$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{-4(x+h)+3+4x-3}{h} \\
&= \frac{-4x-4h+3+4x-3}{h} \\
&= \frac{-4h}{h} \\
&= -4
\end{aligned}$$

Exercise

For the function *f* given by $f(x) = 3x - 6$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)-6-3x+6}{h} \\
&= \frac{3x+3h-6-3x+6}{h} \\
&= \frac{3h}{h} \\
&= 3
\end{aligned}$$

Exercise

For the function *f* given by $f(x) = -5x - 7$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{-5(x+h) - 7 + 5x + 7}{h} \\
&= \frac{-5x - 5h - 7 + 5x + 7}{h} \\
&= \frac{-5h}{h} \\
&= -5
\end{aligned}$$

Exercise

Given the function: $f(x) = 2x^2$. Find and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}
f(x+h) &= 2(x+h)^2 \\
&= 2\left(x^2 + 2hx + h^2\right) \\
&= 2x^2 + 4hx + 2h^2
\end{aligned}$$

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{2x^2 + 4hx + 2h^2 - 2x^2}{h} \\
&= \frac{4hx + 2h^2}{h} \\
&= \frac{4hx}{h} + \frac{2h^2}{h} \\
&= 4x + 2h
\end{aligned}$$

Exercise

For the function *f* given by $f(x) = 5x^2$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{5(x+h)^2 - 5x^2}{h} \\
&= \frac{5\left(x^2 + 2hx + h^2\right) - 5x^2}{h}
\end{aligned}$$

$$= \frac{5x^2 + 10hx + 5h^2 - 5x^2}{h}$$

$$= \frac{10hx + 5h^2}{h}$$

$$\underline{= 10x + 5h}$$

Exercise

For the function *f* given by $f(x) = 3x^2 - 4x$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 4x - 3x^2 + 4x}{h}$$

$$= \frac{3\left(x^2 + 2hx + h^2\right) - 4(x+h) - 3x^2 + 4x}{h}$$

$$= \frac{3x^2 + 6hx + 3h^2 - 4x - 4h - 3x^2 + 4x}{h}$$

$$= \frac{6hx + 3h^2 - 4h}{h}$$

$$\underline{= 6x + 3h - 4}$$

Exercise

For the function *f* given by $f(x) = 2x^2 - 3x$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$f(x+h) = 2(\text{—})^2 - 3(\text{—})$$

$$= 2(x+h)^2 - 3(x+h) \qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$= 2\left(x^2 + 2xh + h^2\right) - 3x - 3h$$

$$= 2x^2 + 4xh + 2h^2 - 3x - 3h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{2x^2 + 4xh + 2h^2 - 3x - 3h}^{f(x+h)} - \overbrace{(2x^2 - 3x)}^{f(x)}}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \frac{4xh + 2h^2 - 3h}{h}$$

$$= \frac{4xh}{h} + \frac{2h^2}{h} - \frac{3h}{h}$$

$$\underline{= 4x + 2h - 3}$$

Exercise

For the function f given by $f(x) = 2x^2 - x - 3$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$f(x+h) = 2(x+h)^2 - (x+h) - 3$$

$$= 2\left(x^2 + 2hx + h^2\right) - x - h - 3$$

$$= 2x^2 + 4hx + 2h^2 - x - h - 3$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2x^2 + 2h^2 + 4hx - x - h - 3 - \left(2x^2 - x - 3\right)}{h}$$

$$= \frac{2x^2 + 2h^2 + 4hx - x - h - 3 - 2x^2 + x + 3}{h}$$

$$= \frac{2h^2 + 4hx - h}{h}$$

$$= \frac{2h^2}{h} + \frac{4hx}{h} - \frac{h}{h}$$

$$\underline{= 2h + 4x - 1}$$

Exercise

For the given function $f(x) = 2x^2 - x - 3$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - (x+h) - 3 - 2x^2 + x + 3}{h}$$

$$= \frac{2\left(x^2 + 2hx + h^2\right) - x - h - 2x^2 + x}{h}$$

$$= \frac{2x^2 + 4hx + 2h^2 - h - 2x^2}{h}$$

$$= \frac{4hx + 2h^2 - h}{h}$$

$$\underline{= 4x + 2h - 1}$$

Exercise

For the given function $f(x) = x^2 - 2x + 5$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 5 - x^2 + 2x - 5}{h}$$

$$= \frac{x^2 + 2hx + h^2 - 2x - 2h - x^2 + 2x}{h}$$

$$= \frac{2hx + h^2 - 2h}{h}$$

$$= 2x + h - 2$$

Exercise

For the given function $f(x) = 3x^2 - 2x + 5$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 2(x+h) + 5 - 3x^2 + 2x - 5}{h}$$

$$= \frac{3\left(x^2 + 2hx + h^2\right) - 2x - 2h - 3x^2 + 2x}{h}$$

$$= \frac{3x^2 + 6hx + 3h^2 - 2h - 3x^2}{h}$$

$$= \frac{6hx + 3h^2 - 2h}{h}$$

$$= 6x + 3h - 2$$

Exercise

For the given function $f(x) = -2x^2 - 3x + 7$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{-2(x+h)^2 - 3(x+h) + 7 + 2x^2 + 3x - 7}{h}$$

$$= \frac{-2\left(x^2 + 2hx + h^2\right) - 3x - 3h + 2x^2 + 3x}{h}$$

$$= \frac{-2x^2 - 4hx - 2h^2 - 3h + 2x^2}{h}$$

$$= \frac{-4hx - 2h^2 - 3h}{h}$$

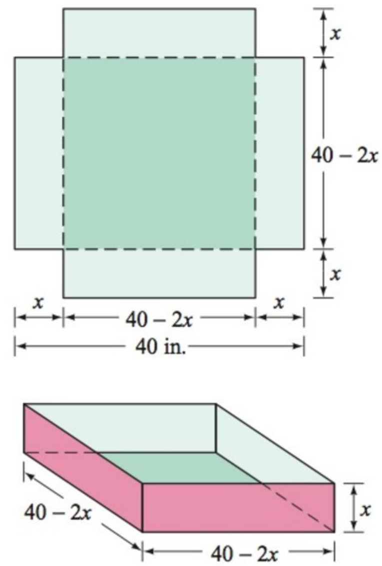
$$\boxed{= -4x - 2h - 3}$$

Exercise

An open box is to be made from a square piece of cardboard that measures 40 *inches* on each side, to construct the box, squares that measure *x inches* on each side are cut from each corner of the cardboard.

a) Express the volume *V* of the box as a function of *x*.

b) Determine the domain of *V*.



Solution

a) $V(x) = x(40 - 2x)^2$

$$= x\left(1600 - 160x + 4x^2\right)$$

$$\boxed{= 4x^3 - 160x^2 + 1600x}$$

b) $40 - 2x = 0$

$\boxed{x = 20}$

Domain: $\{x \mid 0 < x < 20\}$

Exercise

A child 4 *feet* tall is standing near a street lamp that is 12 *feet* high. The light from the lamp casts a shadow.

a) Find the length *l* of the shadow as a function of the distance *d* of the child from the lamppost.

b) What is the domain of the function?

c) What is the length of the shadow when the child is 8 *feet* from the base of the lamppost?

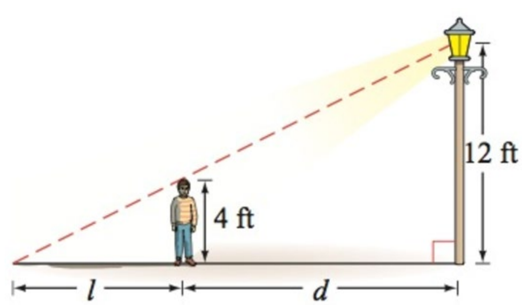
Solution

a) $\frac{l+d}{12} = \frac{l}{4}$

$l + d = 3l$

$2l = d$

$\boxed{l(d) = \frac{1}{2}d}$



b) ***Domain***: $\{x \mid 0 \le d < \infty\}$

c) ***Given***: $d = 8$

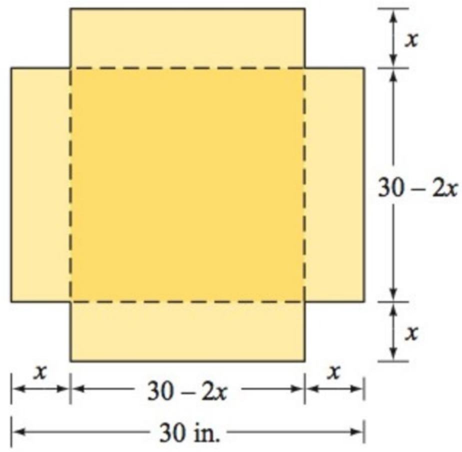
$\boxed{l = 4 \text{ feet}}$

Exercise

An open box is to be made from a square piece of cardboard with the dimensions 30 *inches* by 30 *inches* by cutting out squares of area x^2 from each corner.

a) Express the volume *V* of the box as a function of *x*.

b) Determine the domain of *V*.



Solution

a) $V(x) = x(30-2x)^2$

$$= x\left(900 - 120x + 4x^2\right)$$

$$= 4x^3 - 120x^2 + 900x$$

b) $30 - 2x = 0$

$x = 15$

Domain: $\{x \mid 0 < x < 15\}$

Exercise

Two guy wires are attached to utility poles that are 40 *feet* apart.

a) Find the total length of the two guy wires as a function of *x*.

b) What is the domain of this function?

Solution

a) $\ell_1 = \sqrt{(40-x)^2 + 20^2}$

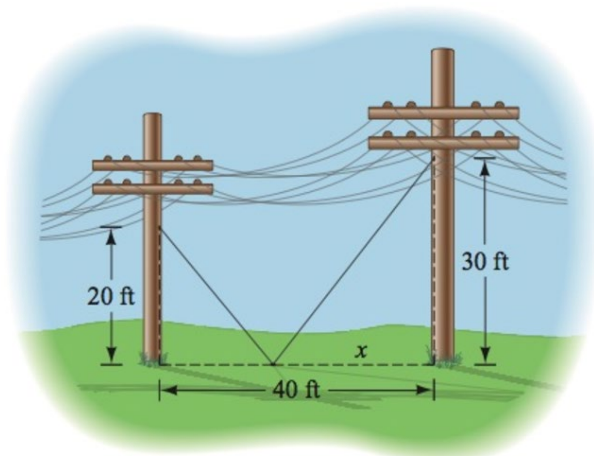
$$= \sqrt{1,600 - 80x + x^2 + 400}$$

$$= \sqrt{2,000 - 80x + x^2}$$

$$\ell_2 = \sqrt{x^2 + 30^2}$$

$$= \sqrt{x^2 + 900}$$

$$\ell(x) = \sqrt{2,000 - 80x + x^2} + \sqrt{x^2 + 900}$$



b) ***Domain***: $[0,\ 40]$

Exercise

A rancher has 360 *yards* of fencing with which to enclose two adjacent rectangular corrals, one for sheep and one for cattle. A river forms one side of the corrals. Suppose the width of each corral is *x* yards.

a) Express the total area of the two corrals as a function of *x*.

b) Find the domain of the function.

Solution

a) $P = 3x + l = 360$

$l = 360 - 3x$

$A = xl$

$= x(360 - 3x)$

$\underline{A(x) = 360x - 3x^2}$

b) $x(360 - 3x) = 0$

$\underline{x = 0}$

$360 - 3x = 0$

$3x = 360$

$\Rightarrow \underline{x = 120}$

Domain: $\underline{0 < x < 120}$

Exercise

A rectangle is bounded by the *x*- and *y-axis* of $y = -\frac{1}{2}x + 4$

a) Find the area of the rectangle as a function of *x*.

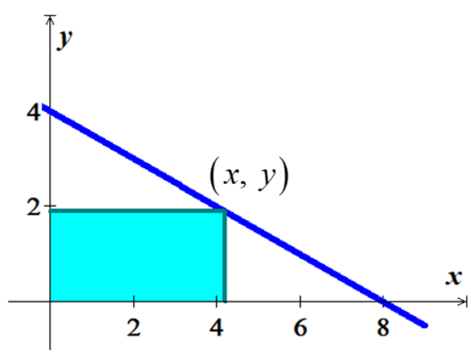
b) What is the domain of this function.

Solution

a) *Area* $= xy$

$A(x) = x\left(-\frac{1}{2}x + 4\right)$

$\underline{= -\frac{1}{2}x^2 + 4x}$



b) $x\left(-\frac{1}{2}x + 4\right) = 0$

$x = 0 \quad x = 8$

Domain: $\underline{0 < x < 8}$