Math 2312: **Pre-Calculus** ***Solution*** ***Sec* 5.2**

Professor: Fred Khoury

1. $\dfrac{2s-1}{(s+1)(s-2)} = \dfrac{A}{s+1} + \dfrac{B}{s-2}$

$2s-1 = (A+B)s - 2A + B$

$$\begin{cases} A+B=2 \\ -2A+B=-1 \end{cases} \Rightarrow A=B=1$$

$$\boxed{\dfrac{2s-1}{(s+1)(s-2)} = \dfrac{1}{s+1} + \dfrac{1}{s-2}}$$

2. $\dfrac{2s-2}{(s-4)(s+2)} = \dfrac{A}{s-4} + \dfrac{B}{s+2}$

$2s-2 = (A+B)s + 2A - 4B$

$$\begin{cases} A+B=2 \\ 2A-4B=-2 \end{cases} \Rightarrow A=B=1$$

$$\boxed{\dfrac{2s-2}{(s-4)(s+2)} = \dfrac{1}{s-4} + \dfrac{1}{s+2}}$$

3. $\dfrac{7s^2+3s+16}{(s+1)(s^2+4)} = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+4}$

$7s^2+3s+16 = (A+B)s^2 + (B+C)s + 4A + C$

$$\begin{cases} A+B=7 \\ B+C=3 \\ 4A+C=16 \end{cases} \rightarrow A-C=4 \quad 4A+C=16 \Rightarrow 5A=20 \rightarrow A=4 \quad B=3 \quad C=0$$

$$\boxed{\dfrac{7s^2+3s+16}{(s+1)(s^2+4)} = \dfrac{4}{s+1} + \dfrac{3s}{s^2+4}}$$

4. $\dfrac{1}{(s+2)^2(s^2+9)} = \dfrac{A}{s+2} + \dfrac{B}{(s+2)^2} + \dfrac{Cs+D}{s^2+9}$

$1 = (A+C)s^3 + (2A+B+4C+D)s^2 + (9A+4C+4D)s + 18A + 9B + 4D$

$$\begin{cases} A+C=0 \\ 2A+B+4C+D=0 \\ 9A+4C+4D=0 \\ 18A+9B+4D=1 \end{cases} \Rightarrow A=\frac{4}{169} \quad B=\frac{1}{13} \quad C=-\frac{4}{169} \quad D=-\frac{5}{169}$$

$$\boxed{\frac{1}{(s+2)^2\left(s^2+9\right)}=\frac{4}{169}\frac{1}{s+2}+\frac{1}{13}\frac{1}{(s+2)^2}-\frac{1}{169}\frac{4s+5}{s^2+9}}$$

5. $$\frac{s}{(s+2)^2\left(s^2+9\right)}=\frac{A}{s+2}+\frac{B}{(s+2)^2}+\frac{Cs+D}{s^2+9}$$

$$s=\left(A+C\right)s^3+\left(2A+B+4C+D\right)s^2+\left(9A+4C+4D\right)s+18A+9B+4D$$

$$\begin{cases} A+C=0 \\ 2A+B+4C+D=0 \\ 9A+4C+4D=1 \\ 18A+9B+4D=0 \end{cases} \Rightarrow A=\frac{5}{169} \quad B=-\frac{2}{13} \quad C=-\frac{5}{169} \quad D=\frac{36}{169}$$

$$\boxed{\frac{1}{(s+2)^2\left(s^2+9\right)}=\frac{5}{169}\frac{1}{s+2}-\frac{2}{13}\frac{1}{(s+2)^2}-\frac{1}{169}\frac{5s+36}{s^2+9}}$$

6. $$\frac{1}{(s+1)^2\left(s^2-4\right)}=\frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{Cs+D}{s^2-4}$$

$$1=As^3-4As+As^2-4A+Bs^2-4B+Cs^3+2Cs^2+Cs+Ds^2+2Ds+D$$

$$\begin{matrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{matrix}\begin{cases} A+C=0 \\ A+B+2C+D=0 \\ -4A+C+2D=0 \\ -4A-4B+D=1 \end{cases} \qquad \begin{matrix} A=-\frac{2}{15} & B=\frac{1}{5} \\ C=\frac{2}{15} & D=-\frac{1}{3} \end{matrix}$$

$$\boxed{\frac{1}{(s+1)^2\left(s^2-4\right)}=-\frac{2}{15}\frac{1}{s+1}+\frac{1}{5}\frac{1}{(s+1)^2}+\frac{\frac{2}{15}s-\frac{1}{3}}{s^2-4}}$$

7. $$\frac{7s^2+20s+53}{(s-1)\left(s^2+2s+5\right)}=\frac{A}{s-1}+\frac{Bs+C}{s^2+2s+5}$$

$$7s^2+20s+53=As^2+2As+5A+Bs^2-Bs+Cs-C$$

$$\begin{matrix} s^2 \\ s^1 \\ s^0 \end{cases} \begin{matrix} A+B=7 \\ 2A-B+C=20 \\ 5A-C=53 \end{matrix}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 5 & 0 & -1 \end{vmatrix} = 8 \qquad \Delta_A = \begin{vmatrix} 7 & 1 & 0 \\ 20 & -1 & 1 \\ 53 & 0 & -1 \end{vmatrix} = 80$$

$$\underline{A = 10}$$

$$\begin{cases} \underline{B = 7 - A = -3} \\ \underline{C = 5A - 53 = -3} \end{cases}$$

$$\underline{\frac{7s^2+20s+53}{(s-1)\left(s^2+2s+5\right)} = \frac{10}{s-1} + \frac{-3s-3}{s^2+2s+5}}$$

8. $\dfrac{s^2+1}{s^3-2s^2-8s} = \dfrac{A}{s} + \dfrac{B}{s-4} + \dfrac{C}{s+2}$

$$s^2+1 = As^2 - 2As - 8A + Bs^2 + 2Bs + Cs^2 - 4Cs$$

$$\begin{matrix} s^2 \\ s^1 \\ s^0 \end{cases} \begin{matrix} A+B+C=1 \\ -2A+2B-4C=0 \\ -8A=1 \end{matrix} \Rightarrow \qquad \underline{A = -\frac{1}{8}}$$

$$\begin{cases} B + C = \dfrac{9}{8} \\ 2B - 4C = -\dfrac{1}{4} \end{cases}$$

$$B = \frac{\begin{vmatrix} \frac{9}{8} & 1 \\ -\frac{1}{4} & -4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix}} = \frac{-\frac{17}{4}}{-6} = \underline{\frac{17}{24}} \qquad C = \frac{\begin{vmatrix} 1 & \frac{9}{8} \\ 2 & -\frac{1}{4} \end{vmatrix}}{-6} = \underline{\frac{5}{12}}$$

$$\underline{\frac{s^2+1}{s^3-2s^2-8s} = -\frac{1}{8}\frac{1}{s} + \frac{17}{24}\frac{1}{s-4} + \frac{5}{12}\frac{1}{s+2}}$$

9. $\dfrac{1}{x^2+2x} = \dfrac{A}{x} + \dfrac{B}{x+2}$

$1 = Ax + 2A + Bx$

$x \qquad 2A = 1 \qquad \rightarrow A = \dfrac{1}{2}$

$x^0 \qquad A + B = 0 \quad \rightarrow B = -\dfrac{1}{2}$

$$\underline{\dfrac{1}{x^2+2x} = \dfrac{1}{2}\dfrac{1}{x} - \dfrac{1}{2}\dfrac{1}{x+2}}$$

10. $\dfrac{2x+1}{x^2-7x+12} = \dfrac{A}{x-4} + \dfrac{B}{x-3}$

$2x + 1 = Ax - 3A + Bx - 4B$

$x \qquad A + B = 2$

$x^0 \qquad -3A - 4B = 1$

$$A = \frac{\begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & -4 \end{vmatrix}} = \frac{-9}{-1} = \underline{9} \qquad B = \frac{\begin{vmatrix} 1 & 2 \\ -3 & 1 \end{vmatrix}}{-1} = \frac{7}{-1} = \underline{-7}$$

$$\underline{\dfrac{2x+1}{x^2-7x+12} = \dfrac{9}{x-4} - \dfrac{7}{x-3}}$$