Solution Section 3.6 – Solving Trigonometry Equations

Exercise

Solve $2\cos\theta + \sqrt{3} = 0 \qquad if \;\; 0^\circ \le \theta < 360^\circ$

Solution

$$2\cos\theta = -\sqrt{3}$$

$$\cos\theta = -\frac{\sqrt{3}}{2} \quad \Rightarrow \quad \hat{\theta} = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\theta = 150^\circ,\ 210^\circ}$$

Exercise

Solve $5\cos t + \sqrt{12} = \cos t \qquad if \;\; 0 \le t < 2\pi$

Solution

$$5\cos t - \cos t = -\sqrt{12} \qquad\qquad \sqrt{12} = \sqrt{4.3} = 2\sqrt{3}$$

$$4\cos t = -2\sqrt{3}$$

$$4\cos t = -2\sqrt{3}$$

$$\cos t = -\frac{2\sqrt{3}}{4}$$

$$\cos t = -\frac{\sqrt{3}}{2} \quad \Rightarrow \quad t = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\theta = \frac{5\pi}{6},\ \frac{7\pi}{6}}$$

Exercise

Solve $\tan\theta - 2\cos\theta\tan\theta = 0 \qquad if \;\; 0^\circ \le \theta < 360^\circ$

Solution

$$\tan\theta(1 - 2\cos\theta) = 0$$

$$\tan\theta = 0 \qquad\qquad 1 - 2\cos\theta = 0$$

$$\theta = 0^\circ,\ 180^\circ \qquad\qquad 1 = 2\cos\theta$$

$$\cos\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 60^\circ,\ 300^\circ$$

$$\boxed{\theta = 0^\circ,\ 60^\circ,\ 180^\circ,\ 300^\circ}$$

Exercise

Solve $2\sin^2\theta - 2\sin\theta - 1 = 0 \qquad if \ \ 0° \le \theta < 360°$

Solution

$$\begin{aligned}\sin\theta &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{2 \pm \sqrt{12}}{4} \\ &= \frac{2 \pm 2\sqrt{3}}{4} \\ &= \frac{1 \pm \sqrt{3}}{2}\end{aligned}$$

$\hat{\theta} = \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right) = -21.47°$ $\qquad \sin\theta = \frac{1+\sqrt{3}}{2} = 1.366 > 1$

$\theta = 360° - 21.47° \ \underline{= 338.53°}$

$\theta = 180° + 21.47° \ \underline{= 201.47°}$

Exercise

Solve $\cos\left(A - \frac{\pi}{9}\right) = -\frac{1}{2}$

Solution

$-\frac{1}{2}$ *is negative* $\rightarrow$ *cosine is in QII or QIII.*

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\cos\left(A - \frac{\pi}{9}\right) = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$A - \frac{\pi}{9} = \frac{2\pi}{3} + 2\pi k$

$A = \frac{2\pi}{3} + \frac{\pi}{9} + 2\pi k$

$\boxed{A = \frac{7\pi}{9} + 2\pi k}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{4\pi}{3}$

$\cos\left(A - \frac{\pi}{9}\right) = -\frac{1}{2} = \cos\frac{4\pi}{3}$

$A - \frac{\pi}{9} = \frac{4\pi}{3} + 2\pi k$

$A = \frac{4\pi}{3} + \frac{\pi}{9} + 2\pi k$

$\boxed{A = \frac{13\pi}{9} + 2\pi k}$

Exercise

Solve: $4\cos\theta - 3\sec\theta = 0$ *if* $0° \le \theta < 360°$

Solution

$$4\cos\theta - 3\frac{1}{\cos\theta} = 0 \qquad \boxed{\cos\theta \ne 0}$$

$$4\cos\theta\cos\theta - 3\frac{1}{\cos\theta}\cos\theta = 0$$

$$4\cos^2\theta - 3 = 0$$

$$4\cos^2\theta = 3$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = \cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$$

The solutions are: $\boxed{\theta = 30°,\ 150°,\ 210°,\ 330°}$

Exercise

Solve: $2\sin^2 x - \cos x - 1 = 0$ *if* $0 \le x < 2\pi$

Solution

$$2\left(1 - \cos^2 x\right) - \cos x - 1 = 0$$

$$2 - 2\cos^2 x - \cos x - 1 = 0$$

$$-2\cos^2 x - \cos x + 1 = 0$$

$$\cos x = -1 \qquad \cos x = \frac{1}{2}$$

$$x = \pi \qquad x = \frac{\pi}{3},\ \frac{5\pi}{3}$$

The solutions are: $\boxed{x = \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}}$

Exercise

Solve: $\sin\theta - \sqrt{3}\cos\theta = 1$ *if* $0° \le \theta < 360°$

Solution

$$\sin\theta - 1 = -\sqrt{3}\cos\theta$$

$$(\sin\theta - 1)^2 = \left(-\sqrt{3}\cos\theta\right)^2$$

$$\sin^2\theta - 2\sin\theta + 1 = 3\cos^2\theta \qquad \cos^2\theta = 1 - \sin^2\theta$$

$$\sin^2\theta - 2\sin\theta + 1 = 3\left(1 - \sin^2\theta\right)$$

$$\sin^2\theta - 2\sin\theta + 1 = 3 - 3\sin^2\theta$$

$$\sin^2\theta - 2\sin\theta + 1 - 3 + 3\sin^2\theta = 0$$

$$4\sin^2\theta - 2\sin\theta - 2 = 0$$

$\sin\theta = 1$ $\qquad$ $\sin\theta = -\frac{1}{2}$

$\theta = 90°$ $\qquad$ $\theta = 210°,\ 330°$

Check

$\theta = 90°$	$\theta = 210°$	$\theta = 330°$
$\sin 90° - \sqrt{3}\cos 90° \overset{?}{=} 1$ $1 - \sqrt{3}(0) \overset{?}{=} 1$ $1 = 1$	$\sin 210° - \sqrt{3}\cos 210° \overset{?}{=} 1$ $-\frac{1}{2} - \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) \overset{?}{=} 1$ $-\frac{1}{2} + \frac{3}{2} \overset{?}{=} 1$ $1 = 1$	$\sin 330° - \sqrt{3}\cos 330° \overset{?}{=} 1$ $-\frac{1}{2} - \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) \overset{?}{=} 1$ $-\frac{1}{2} - \frac{3}{2} \overset{?}{=} 1$ $-2 \ne 1$ **(*False statement*)**

The solutions are: $\boxed{90°,\ 210°}$

Exercise

Solve: $7\sin^2\theta - 9\cos 2\theta = 0 \qquad if \;\; 0° \le \theta < 360°$

Solution

$$7\sin^2\theta - 9\left(1 - 2\sin^2\theta\right) = 0 \qquad\qquad \cos^2\theta = 1 - 2\sin^2\theta$$

$$7\sin^2\theta - 9 + 18\sin^2\theta = 0$$

$$25\sin^2\theta - 9 = 0$$

$$25\sin^2\theta = 9$$

$$\sin^2\theta = \frac{9}{25} \quad \Rightarrow \quad \sin\theta = \pm\frac{3}{5}$$

$$\hat{\theta} = \sin^{-1}\left(\frac{3}{5}\right) \approx 36.87°$$

$$\theta \approx 36.87° \qquad \theta \approx 180° - 36.87° \approx 143.13°$$

$$\theta \approx 180° + 36.87° \approx 216.87° \qquad \theta \approx 360° - 36.87° \approx 323.13°$$

The solutions are: $\boxed{36.87°,\ 143.13°,\ 216.87°,\ 323.13°}$

Exercise

Solve: $2\cos^2 t - 9\cos t = 5 \qquad if \;\; 0 \le t < 2\pi$

Solution

$$2\cos^2 t - 9\cos t - 5 = 0$$

$$(2\cos t + 1)(\cos t - 5) = 0$$

$$2\cos t + 1 = 0 \qquad \cos t - 5 = 0$$

$$\cos t = -\frac{1}{2} \qquad \cos t = 5$$

$$\cos t = -\frac{1}{2} \qquad \cos t = 5$$

$$\hat{t} = \cos^{-1}\left(-\frac{1}{2}\right) \qquad \textit{No solution}$$

$$\hat{t} = \frac{\pi}{3}$$

Negative sign → cosine is in QII or QIII

$$t = \pi - \frac{\pi}{3} \qquad t = \pi + \frac{\pi}{3}$$

$$t = \frac{2\pi}{3} \qquad t = \frac{4\pi}{3}$$

The solutions are: $\boxed{\frac{2\pi}{3},\ \frac{4\pi}{3}}$

Exercise

Solve $\sin\theta\tan\theta = \sin\theta \quad if \;\; 0° \le \theta < 360°$

Solution

$$\sin\theta\tan\theta - \sin\theta = 0$$

$$\sin\theta(\tan\theta - 1) = 0$$

$\sin\theta = 0$ $\qquad\qquad$ $\tan\theta - 1 = 0$

$\theta = 0°,\ 180°$ $\qquad\qquad$ $\tan\theta = 1$

$\theta = 45°,\ 225°$

The solutions are: $\boxed{0°,\ 45°,\ 180°,\ 225°}$

Exercise

Solve $\tan^2 x + \tan x - 2 = 0 \quad if \;\; 0 \le x < 2\pi$

Solution

$$\tan^2 x + \tan x - 2 = 0$$

$\tan x = 1$ $\qquad\qquad$ $\tan x = -2$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ $\qquad\qquad$ $\hat{x} = \tan^{-1}(2) \approx 1.107$ $\qquad\qquad$ $x \in QII,\ QIV$

$x = 2.034,\ 5.176$

The solutions are: $\boxed{\frac{\pi}{4},\ \frac{5\pi}{4},\ 2.034,\ 5.176}$

Exercise

Solve $\tan x + \sqrt{3} = \sec x$ $\quad$ *if* $0 \le x < 2\pi$

Solution

$$\left(\tan x + \sqrt{3}\right)^2 = (\sec x)^2$$

$$\tan^2 x + 2\sqrt{3}\tan x + 3 = \sec^2 x$$

$$\tan^2 x + 2\sqrt{3}\tan x + 3 = 1 + \tan^2 x$$

$$\tan^2 x + 2\sqrt{3}\tan x + 3 - 1 - \tan^2 x = 0$$

$$2\sqrt{3}\tan x + 2 = 0$$

$$2\sqrt{3}\tan x = -2$$

$$\tan x = -\frac{2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$$x = \frac{5\pi}{6} \quad or \quad x = \frac{11\pi}{6}$$

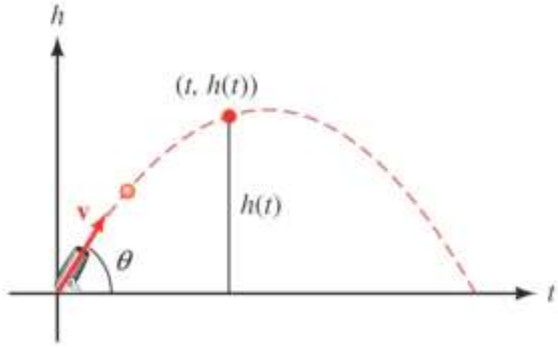
$\tan\frac{5\pi}{6} + \sqrt{3} \stackrel{?}{=} \sec\frac{5\pi}{6}$ $-\frac{\sqrt{3}}{3} + \sqrt{3} \stackrel{?}{=} -\frac{2\sqrt{3}}{3}$ $\frac{2\sqrt{3}}{3} \ne -\frac{2\sqrt{3}}{3}$ ***False***	$\tan\frac{11\pi}{6} + \sqrt{3} \stackrel{?}{=} \sec\frac{11\pi}{6}$ $-\frac{\sqrt{3}}{3} + \sqrt{3} \stackrel{?}{=} \frac{2\sqrt{3}}{3}$ $\frac{2\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$

The solutions are: $\boxed{\frac{11\pi}{6}}$

Exercise

If a projectile (such as a bullet) is fired into the air with an initial velocity $\boldsymbol{v}$ at an angle of elevation θ, then the height h of the projectile at time t is given by:

$$h(t) = -16t^2 + vt\sin\theta$$



a) Give the equation for the height, if $\boldsymbol{v}$ is 600 *ft./sec* and $\theta = 45°$.

b) Use the equation in part (*a*) to find the height of the object after $\sqrt{3}$ seconds.

c) Find the angle of elevation of θ of a rifle barrel, if a bullet fired at 1,500 ft./sec takes 3 seconds to reach a height of 750 feet. Give your answer in the nearest of a degree.

Solution

a) $h(t) = -16t^2 + 600t\sin 45°$

$$= -16t^2 + 600t\frac{\sqrt{2}}{2}$$

$$= -16t^2 + 300\sqrt{2}\,t$$

b) $h\left(t = \sqrt{3}\right) = -16\left(\sqrt{3}\right)^2 + 300\sqrt{2}\,\sqrt{3}$

$$\approx 686.8\ ft$$

c) $h(t) = -16t^2 + vt\sin\theta$

$$750 = -16(3)^2 + 1500(3)\sin\theta$$

$$750 = -144 + 4500\sin\theta$$

$$750 + 144 = 4500\sin\theta$$

$$\frac{894}{4500} = \sin\theta$$

$$\theta = \sin^{-1}\left(\frac{894}{4500}\right) \approx 11.5°$$