Math 2312 – Pre-Calculus **_Exam 4_** **_Review_**

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1. Find the first four terms and the seventh term of the sequence:

a) $\left\{\frac{2^n}{(n+1)(n+2)(n+3)}\right\}$ *b)* $\left\{(-1)^{n+1}-(0.1)^n\right\}$

2. Find the specified term of the arithmetic sequence that has two given terms:

a) a_{11}: $a_1=2+\sqrt{2},\quad a_2=3$ *b)* a_{15}; $a_3=7,\quad a_{20}=43$

3. Express the sum in terms of summation notation:

a) $4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$

b) $\frac{1}{1\bullet 2}+\frac{1}{2\bullet 3}+\frac{1}{3\bullet 4}+\cdots+\frac{1}{99\bullet 100}$

c) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}$

d) $1+x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots+\frac{x^n}{n}$

e) $1-\frac{x^2}{2}+\frac{x^4}{4}-\frac{x^6}{6}+\cdots+(-1)^n\frac{x^{2n}}{2n}$

4. Find the *n*th term, and the tenth term of the sequence:

a) $\log 1000,\ \log 100,\ \log 10,\ \log 1,\ \cdots$

b) $x-8,\ x-3,\ x+2,\ x+7,\ \cdots$

c) $1,\ -\frac{x}{3},\ \frac{x^2}{9},\ -\frac{x^3}{27},\ \cdots$

d) $2,\ 2^{x+1},\ 2^{2x+1},\ 2^{3x+1},\ \ldots$

5. Evaluate:

a) $\sum_{k=2}^{6}\frac{2k-8}{k-1}$ *b)* $\sum_{k=1}^{4}\left(2^k-10\right)$ *c)* $\sum_{k=1}^{7}\left(3^{-k}\right)$

6. Find the sum of the infinite geometric series if it exists:

a) $2+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\ldots$ *b)* $1-0.1+0.01-0.001+\cdots$ *c)* $250-100+40-16$

7. Use the binomial theorem to expand and simplify

a) $\left(4x^2-y\right)^6$ *b)* $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^5$ *c)* $\left(\sqrt{2}x+\frac{1}{y}\right)^4$ *d)* $(7x+2y)^4$

8. Find the center and the radius of

a) $x^2+y^2+6x+8y+9=0$

b) $x^2+y^2+8x-6y+16=0$

c) $x^2+y^2-4x+12y=-4$

d) $4x^2+4y^2+4x-16y-19=0$

9. Find the vertex, focus, and directrix of the parabola.

a) $y^2=-16x$

b) $x^2=\frac{1}{9}y$

c) $(x+2)^2=\frac{1}{2}(y+3)$

d) $(y-2)^2=-8(x+1)$

10. Find an equation of a parabola that satisfies the given conditions

a) $Vertex: V(0,\ 1)\quad focus: F(0,\ 2)$

b) $Focus: F(1,\ 1)\quad directrix: x=-1$

c) $Vertex: V(1,\ 1)\quad directrix: y=4$

11. Find the vertices, minors and foci of the ellipse, and then sketch the graph of

a) $\frac{x^2}{36}+\frac{y^2}{16}=1$

b) $9x^2+6y^2=54$

c) $\frac{25y^2}{36}+\frac{64x^2}{9}=1$

d) $9x^2+4y^2+18x-8y-23=0$

e) $\frac{(x-3)^2}{25}+\frac{(y+2)^2}{16}=1$

12. Find the vertices, the endpoints, the foci, and the equations of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes.

a) $49x^2-36y^2=1764$

b) $\frac{4x^2}{9}-\frac{25y^2}{16}=1$

c) $25y^2-9x^2=1$

d) $4x^2+16x-9y^2+18y=29$

e) $16(x+5)^2-(y-3)^2=1$

13. Find the partial fraction decomposition

a) $\frac{x+34}{x^2-4x-12}$

b) $\frac{4x^2-5x-15}{x^3-4x^2-5x}$

c) $\frac{2x^2+x}{(x-1)^2(x+1)^2}$

d) $\frac{x^2+19x+20}{x(x+2)(x-5)}$

e) $\frac{4x^3+3x^2+5x-2}{x^3(x+2)}$

f) $\frac{x^5-5x^4+7x^3-x^2-4x+12}{x^3-3x^2}$

g) $\frac{2x}{(x-1)^3}$

h) $\frac{x^2+3x-1}{(x+1)(x^2+2)}$

i) $\frac{2x}{(x-1)(x^2+1)^2}$

j) $\frac{4x+2}{(x+2)(2x-1)}$

k) $\frac{x}{x^2+4x-5}$

l) $\frac{2x^5+3x^4-3x^3-2x^2+x}{2x^2+4x+2}$

Solution

1. ***a***) $\left\{\frac{1}{12}, \frac{1}{15}, \frac{1}{15}, \frac{8}{105}, \frac{8}{45}\right\}$ ***b***) $\{0.9,\ -1.01,\ 0.999,\ -1.0001,\ 0.9999999\}$

2. ***a***) $d = 1-\sqrt{2}\quad a_{11} = 12-9\sqrt{2}$ ***b***) $d = \frac{36}{17};\quad a_{15} = \frac{551}{17}$

3. ***a***) $\sum_{n=1}^{6} 2^{3-n}$ ***b***) $\sum_{n=1}^{99} \frac{1}{n(n+1)}$ ***c***) $\sum_{n=1}^{7} (-1)^{n-1}\frac{1}{n}$ ***d***) $1+\sum_{k=1}^{n} \frac{x^k}{k}$

e) $1+\sum_{k=1}^{n} (-1)^k \frac{x^{2k}}{2k}$

4. *a*) $d=-1;\quad a_n = -n+4;\quad a_{10} = -6$

b) $d=5;\quad a_n = x+5n-13;\quad a_{10} = x+37$

c) $r=-\frac{x}{3};\quad a_n = (-1)^{n-1}\left(\frac{x}{3}\right)^{n-1};\quad a_{10} = -\frac{x^9}{19683}$

d) $r=2^x;\quad a_n = 2^{(n-1)x+1};\quad a_{10} = 2^{9x+1}$

5. *a*) $-\frac{37}{10}$ *b*) -10 *c*) $\frac{1093}{2187}$

6. *a*) $S=3$ *b*) $S=\frac{10}{11}$ *c*) $S=\frac{1250}{7}$

7. *a*) $4096x^{12}-6144x^{10}y+3840x^8y^2-1280x^6y^3+240x^4y^4-24x^2y^5+y^6$

b) $x^{5/2}-5x^{3/2}+10x^{1/2}-10x^{-1/2}+5x^{-3/2}-x^{-5/2}$

b) $4x^4+\frac{8\sqrt{2}x^3}{y}+\frac{12x^2}{y^2}+\frac{4\sqrt{2}x}{y^3}+\frac{1}{y^4}$

b) $2401x^4+2744x^3y+1176x^2y^2+224xy^3+16y^4$

8. ***a***) Center $(-3,-4)$; radius: 4 ***b***) Center $(-4,3)$; radius: 3

c) Center $(2,-6)$; radius: 6 ***d***) Center $\left(-\frac{1}{2},2\right)$; radius: 3

9. ***a***) $V(0,0)\quad F(-4,0)\quad \textit{directrix}: x=4$

b) $V(0,0)\quad F\left(0,\frac{1}{36}\right)\quad \textit{directrix}: y=-\frac{1}{36}$

c) $V(-2,-3)\quad F\left(-2,-\frac{23}{8}\right)\quad \textit{directrix}: y=-\frac{25}{8}$

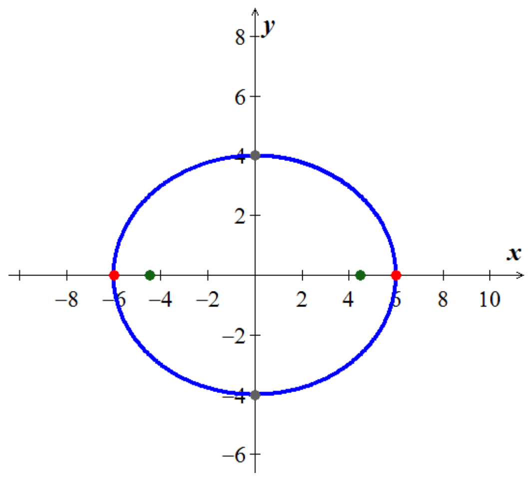
$\boldsymbol{d})\ V(-1,2)\quad F(-3,2)\quad directrix: x=1$

10. $\boldsymbol{a})\ x^2=4(y-1)$

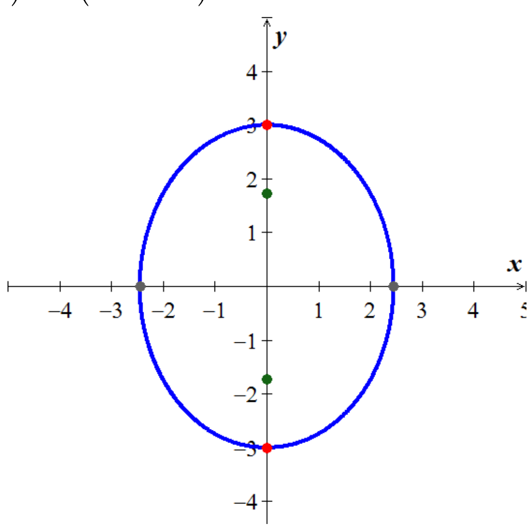
$\boldsymbol{b})\ (y-1)^2=4x$

$\boldsymbol{c})\ (x-1)^2=-12(y-1)$

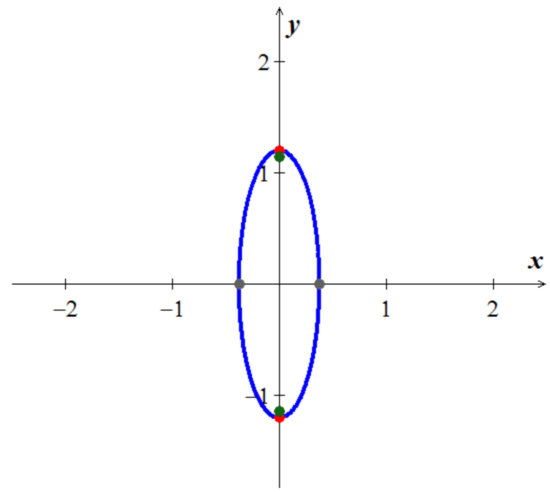
11. $a)\ V(\pm 6,\ 0)\quad M(0,\ \pm 4)\quad F\left(\pm 2\sqrt{5},\ 0\right)$



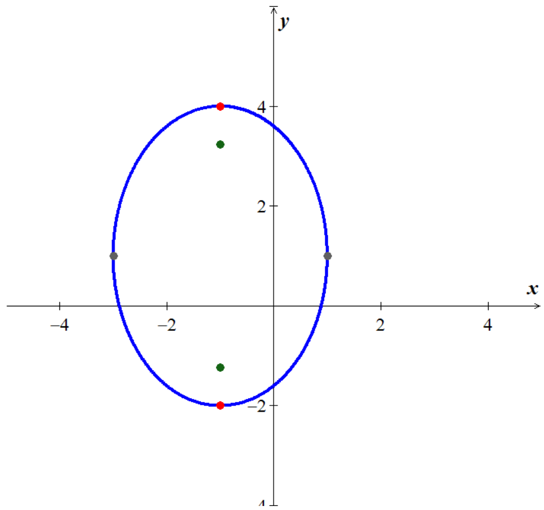
$b)\ V(0,\ \pm 3)\quad M\left(\pm\sqrt{6},\ 0\right)\quad F\left(0,\ \pm\sqrt{3}\right)$



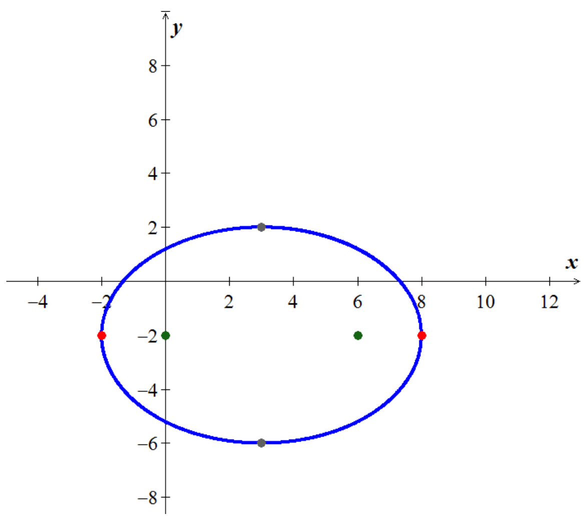
$c)\ V\left(0,\ \pm\frac{6}{5}\right)\quad M\left(\pm\frac{3}{8},\ 0\right)\quad F\left(0,\ \pm\frac{3\sqrt{231}}{40}\right)$



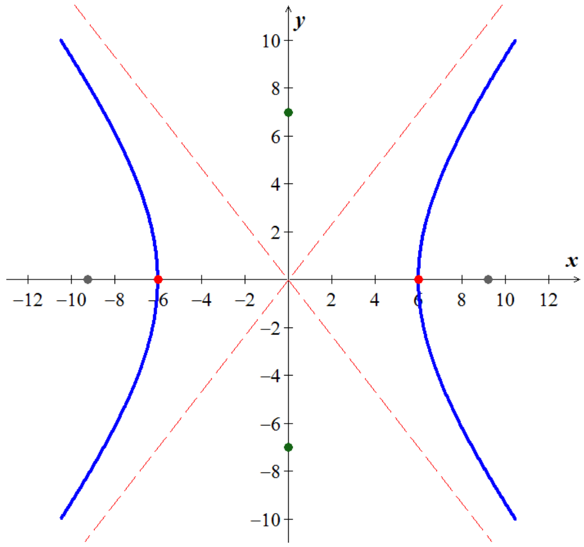
d) $center(-1,\ 1)\quad V(-1,\ -2),(-1,\ 4)\quad F\left(-1,\ 1\pm\sqrt{5}\right)\quad M(-3,\ 1),(1,\ 1)$



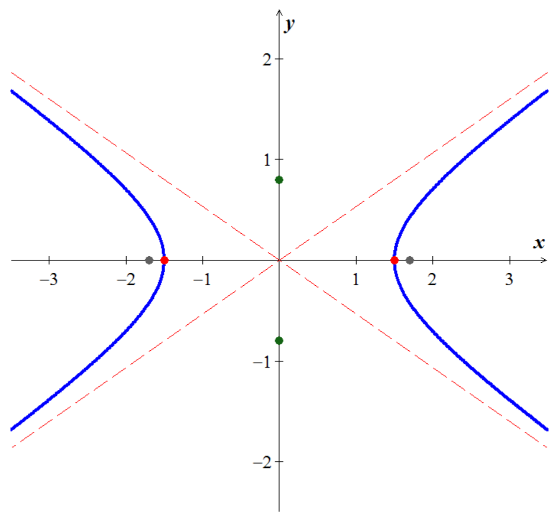
e) $center(3,\ -2)\quad V(-2,\ -2),(8,\ -2)\quad F(0,\ -2),(6,\ -2)\quad M(3,\ -6),(3,\ 2)$



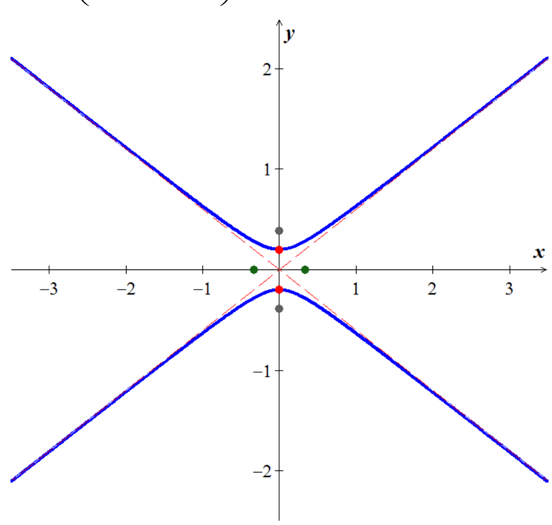
12. *a*) $V(\pm 6,\ 0)$ $W(0,\ \pm 7)$ $F\left(\pm\sqrt{85},\ 0\right)$ $y=\pm\frac{7}{6}x$



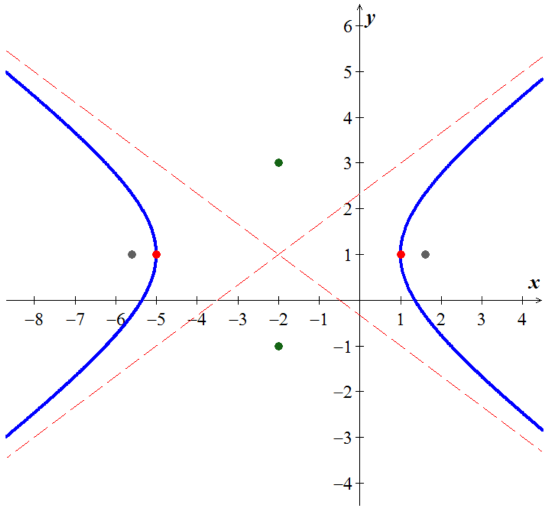
b) $V\left(\pm\frac{3}{2},\ 0\right)$ $W\left(0,\ \pm\frac{4}{5}\right)$ $F\left(\pm\frac{\sqrt{289}}{10},\ 0\right)$ $y=\pm\frac{8}{15}x$



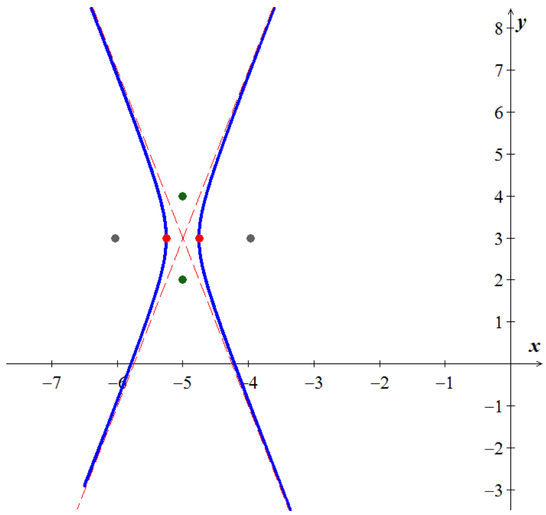
c) $V\left(0,\ \pm\frac{1}{5}\right)$ $W\left(\pm\frac{1}{3},\ 0\right)$ $F\left(0,\ \pm\frac{\sqrt{34}}{15}\right)$ $y=\pm\frac{3}{5}x$



$d)$ $center(-2,\ 1)$ $V(-5,\ 1),(1,\ 1)$ $W(-2,\ -1),(-2,\ 3)$ $F\left(-2\pm\sqrt{13},\ 1\right)$ $y=\pm\frac{2}{3}(x+2)+1$



$e)$ $center(-5,\ 3)$ $V\left(-\frac{21}{4},\ 3\right),\left(-\frac{19}{4},\ 3\right)$ $W(-5,\ 2),(-5,\ 4)$ $F\left(\pm\frac{\sqrt{17}}{4},\ 3\right)$ $y=\pm4(x+5)+3$



13. $a)$ $\dfrac{5}{x-6}-\dfrac{4}{x+2}$ $b)$ $\dfrac{3}{x}+\dfrac{2}{x-5}-\dfrac{1}{x+1}$ $c)$ $\dfrac{\frac{1}{2}}{x-1}+\dfrac{\frac{3}{4}}{(x-1)^2}-\dfrac{\frac{1}{2}}{x+1}+\dfrac{\frac{1}{4}}{(x+1)^2}$

$d)$ $-\dfrac{2}{x}-\dfrac{1}{x+2}+\dfrac{4}{x-5}$ $e)$ $\dfrac{3}{x^2}-\dfrac{1}{x^3}+\dfrac{4}{x+2}$ $f)$ $x^2-2x+1-\dfrac{4}{x^2}+\dfrac{2}{x-3}$

$g)$ $\dfrac{2}{(x-1)^2}+\dfrac{2}{(x-1)^3}$ $h)$ $\dfrac{-1}{x+1}+\dfrac{2x+1}{x^2+2}$ $i)$ $\dfrac{\frac{1}{2}}{x-1}+\dfrac{-\frac{1}{2}x-\frac{1}{2}}{x^2+1}+\dfrac{-x+1}{\left(x^2+1\right)^2}$

$j)$ $\dfrac{\frac{6}{5}}{x+2}+\dfrac{\frac{8}{5}}{2x-1}=\dfrac{6}{5(x+2)}+\dfrac{8}{5(2x-1)}$

$k)$ $\dfrac{5}{6(x+5)}+\dfrac{1}{6(x-1)}$ $l)$ $x^3-x^2+\dfrac{-1}{3(2x+1)}+\dfrac{2}{3(x+2)}$