

Solution ***Section R.1 – Derivative***

Exercise

Find the derivative of $f(t) = -3t^2 + 2t - 4$

Solution

$$f'(t) = \underline{-6t + 2}$$

Exercise

Find the derivative of $g(x) = 4\sqrt[3]{x} + 2$

Solution

$$g(x) = 4x^{1/3} + 2$$

$$g'(x) = \frac{4}{3}x^{-2/3}$$

$$= \frac{4}{3x^{2/3}}$$

$$= \underline{\frac{4}{3\sqrt[3]{x^2}}}$$

Exercise

Find the derivative of $f(x) = x(x^2 + 1)$

Solution

$$f(x) = x^3 + x$$

$$f'(x) = \underline{3x^2 + 1}$$

Exercise

Find the derivative of $f(x) = \frac{2x^2 - 3x + 1}{x}$

Solution

$$f(x) = \frac{2x^2}{x} - \frac{3x}{x} + \frac{1}{x}$$

$$= 2x - 3 + \frac{1}{x}$$

$$f'(x) = \underline{2 - \frac{1}{x^2}}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

Exercise

Find the derivative of $f(x) = \frac{4x^3 - 3x^2 + 2x + 5}{x^2}$

Solution

$$f(x) = 4x - 3 + \frac{2}{x} + 5x^{-2} \qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\begin{aligned} f'(x) &= 4 - \frac{2}{x^2} - 10x^{-3} \\ &= 4 - \frac{2}{x^2} - \frac{10}{x^3} \end{aligned}$$

Exercise

Find the derivative of $f(x) = \frac{-6x^3 + 3x^2 - 2x + 1}{x}$

Solution

$$f(x) = -6x^2 + 3x - 2 + \frac{1}{x} \qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$f'(x) = -12x + 3 - \frac{1}{x^2}$$

Exercise

Find the derivative of $f(x) = x\left(1 - \frac{2}{x+1}\right)$

Solution

$$f(x) = x - \frac{2x}{x+1}$$

$$\left(\frac{2x}{x+1}\right)' \Rightarrow \begin{array}{ll} f = 2x & f' = 2 \\ g = x+1 & g' = 1 \end{array}$$

$$\begin{aligned} f'(x) &= 1 - \frac{2(x+1) - 2x}{(x+1)^2} \\ &= 1 - \frac{2x + 2 - 2x}{(x+1)^2} \\ &= 1 - \frac{2}{(x+1)^2} \end{aligned}$$

Exercise

Find the derivative of $g(s) = \frac{s^2 - 2s + 5}{\sqrt{s}}$

Solution

$$\begin{aligned}g(s) &= \frac{s^2}{s^{1/2}} - 2\frac{s}{s^{1/2}} + \frac{5}{s^{1/2}} \\&= s^{3/2} - 2s^{1/2} + 5s^{-1/2} \\g'(s) &= \frac{3}{2}s^{1/2} - 2\frac{1}{2}s^{-1/2} + 5\left(-\frac{1}{2}\right)s^{-3/2} \\&= \frac{3}{2}s^{1/2} - s^{-1/2} - \frac{5}{2}s^{-3/2} \\&= \frac{3}{2}\sqrt{s} - \frac{1}{\sqrt{s}} - \frac{5}{2s^{3/2}} \\&= \frac{\frac{3}{2}\sqrt{s} - \frac{1}{\sqrt{s}} - \frac{5}{2s\sqrt{s}}}{\quad}\end{aligned}$$

Exercise

Find the derivative of $f(x) = \frac{x+1}{\sqrt{x}}$

Solution

$$\begin{aligned}f(x) &= \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}} \\&= x^{1/2} + x^{-1/2} \\f'(x) &= \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} \\&= \frac{\frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}}{\quad}\end{aligned}$$

Exercise

Find the derivative to the following functions $y = 3x(2x^2 + 5x)$

Solution

$$\begin{aligned}y &= 6x^3 + 15x^2 \\&\Rightarrow \underline{y' = 18x^2 + 30x}\end{aligned}$$

Exercise

Find the derivative to the following functions $y = 3(2x^2 + 5x)$

Solution

$$y = 6x^2 + 15x$$

$$\Rightarrow \boxed{y' = 12x + 15}$$

Exercise

Find the derivative to the following functions $y = \frac{x^2 + 4x}{5}$

Solution

$$y = \frac{1}{5} [x^2 + 4x]$$

$$\boxed{y' = \frac{1}{5} (2x + 4)}$$

Exercise

Find the derivative to the following functions $y = \frac{3x^4}{5}$

Solution

$$y = \frac{3}{5} x^4$$

$$\underline{\underline{y' = \frac{12}{5} x^3}}$$

Exercise

Find the derivative to the following functions $y = \frac{x^2 - 4}{2x + 5}$

Solution

$$y' = \frac{(2x + 5)(2x) - (x^2 - 4)(2)}{(2x + 5)^2}$$

$$= \frac{4x^2 + 10x - 2x^2 + 8}{(2x + 5)^2}$$

$$\underline{\underline{= \frac{2x^2 + 10x + 8}{(2x + 5)^2}}}$$

Exercise

Find the derivative to the following functions $y = \frac{(1+x)(2x-1)}{x-1}$

Solution

$$\begin{aligned}y &= \frac{(1+x)(2x-1)}{x-1} \\&= \frac{2x-1+2x^2-x}{x-1} \\&= \frac{2x^2+x-1}{x-1} \\y' &= \frac{(x-1)(4x+1) - (2x^2+x-1)(1)}{(x-1)^2} \\&= \frac{4x^2+x-4x-1-2x^2-x+1}{(x-1)^2} \\&= \frac{2x^2-4x}{(x-1)^2} \Bigg| \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{4}{2x+1}$

Solution

$$\begin{aligned}y &= 4(2x+1)^{-1} \\y' &= -4(2x+1)^{-2}(2) \\&= -8(2x+1)^{-2} \\&= -\frac{8}{(2x+1)^2} \Bigg| \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{2}{(x-1)^3} = 2(x-1)^{-3}$

Solution

$$\begin{aligned}y &= 2(x-1)^{-3} \\y' &= -\frac{6}{(x-1)^4} \Bigg| \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \sqrt[3]{(x+4)^2}$

Solution

$$\begin{aligned} y &= (x+4)^{2/3} \\ y' &= \frac{2}{3}(x+4)^{-1/3} \\ &= \frac{2}{3} \frac{1}{(x+4)^{1/3}} \\ &= \frac{2}{3 \sqrt[3]{x+4}} \end{aligned}$$

Exercise

Find the derivative of $f(x) = \sqrt{2t^2 + 5t + 2}$

Solution

$$\begin{aligned} f(t) &= (2t^2 + 5t + 2)^{1/2} & U &= 2t^2 + 5t + 2 \rightarrow U' = 4t + 5 \\ f'(t) &= \frac{1}{2}(4t + 5)(2t^2 + 5t + 2)^{-1/2} & (U^n)' &= nU'U^{n-1} \\ &= \frac{1}{2} \frac{4t + 5}{\sqrt{2t^2 + 5t + 2}} \end{aligned}$$

Exercise

Find the derivative of $f(x) = \frac{1}{(x^2 - 3x)^2}$

Solution

$$\begin{aligned} f(x) &= (x^2 - 3x)^{-2} \\ f'(x) &= -2(2x - 3)(x^2 - 3x)^{-3} \\ &= -\frac{2(2x - 3)}{(x^2 - 3x)^3} \end{aligned}$$

Exercise

Find the derivative of $y = t^2\sqrt{t-2}$

Solution

$$y' = 2t\sqrt{t-2} + t^2 \frac{1}{2}(t-2)^{-1/2}$$

$$= \left[2t(t-2)^{1/2} + t^2 \frac{1}{2}(t-2)^{-1/2} \right] \frac{2(t-2)^{1/2}}{2(t-2)^{1/2}}$$

$$= \frac{4t(t-2) + t^2}{2(t-2)^{1/2}}$$

$$= \frac{4t^2 - 8t + t^2}{2\sqrt{t-2}}$$

$$= \frac{5t^2 - 8t}{2\sqrt{t-2}}$$

$$f = t^2$$

$$f' = 2t$$

$$g = (t-2)^{1/2} \quad g' = \frac{1}{2}(t-2)^{-1/2}$$

Exercise

Find the derivative of $y = \left(\frac{6-5x}{x^2-1} \right)^2$

Solution

$$f = 6-5x \quad g = x^2-1$$

$$f' = -5 \quad g' = 2x$$

$$y' = 2 \frac{-5(x^2-1) - 2x(6-5x)}{(x^2-1)^2} \left(\frac{6-5x}{x^2-1} \right)$$

$$= 2 \frac{-5x^2 + 5 - 12x + 10x^2}{(x^2-1)^3} (6-5x)$$

$$= \frac{2(5x^2 - 12x + 5)(6-5x)}{(x^2-1)^3}$$

$$(U^n)' = nU'U^{n-1}$$

Exercise

Find the derivative to the following functions $y = x^2\sqrt{x^2+1}$

Solution

$$y = x^2 (x^2 + 1)^{1/2}$$

$$\begin{aligned}
 y' &= x^2 \left[\frac{1}{2} (x^2 + 1)^{-1/2} (2x) \right] + (x^2 + 1)^{1/2} [2x] \\
 &= x^3 (x^2 + 1)^{-1/2} + 2x (x^2 + 1)^{1/2} \\
 &= \frac{(x^2 + 1)^{1/2}}{(x^2 + 1)^{1/2}} \left[x^3 (x^2 + 1)^{-1/2} + 2x (x^2 + 1)^{1/2} \right] \\
 &= \frac{x^3 (x^2 + 1)^{-1/2} (x^2 + 1)^{1/2} + 2x (x^2 + 1)^{1/2} (x^2 + 1)^{1/2}}{(x^2 + 1)^{1/2}} \\
 &= \frac{x^3 + 2x(x^2 + 1)}{(x^2 + 1)^{1/2}} \\
 &= \frac{x^3 + 2x^3 + 2x}{\sqrt{x^2 + 1}} \\
 &= \frac{3x^3 + 2x}{\sqrt{x^2 + 1}} \\
 &= \frac{x(3x^2 + 2)}{\sqrt{x^2 + 1}}
 \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \left(\frac{x+1}{x-5} \right)^2$

Solution

$$\begin{aligned}
 y' &= 2 \left(\frac{x+1}{x-5} \right) \frac{d}{dx} \left[\frac{x+1}{x-5} \right] \\
 &= 2 \left(\frac{x+1}{x-5} \right) \left[\frac{(1)(x-5) - (1)(x+1)}{(x-5)^2} \right] \\
 &= 2 \left(\frac{x+1}{x-5} \right) \left(\frac{x-5-x-1}{(x-5)^2} \right) \\
 &= 2 \left(\frac{x+1}{x-5} \right) \left(\frac{-6}{(x-5)^2} \right) \\
 &= -\frac{12(x+1)}{(x-5)^3}
 \end{aligned}$$

Exercise

Find the derivative to the following functions $y = x^2 \sin x$

Solution

$$y' = \underline{2x \sin x + x^2 \cos x}$$

$$\begin{aligned} u &= x^2 & v &= \sin x \\ u' &= 2x & v' &= \cos x \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{\sin x}{x}$

Solution

$$y' = \underline{\frac{x \cos x - \sin x}{x^2}}$$

$$\begin{aligned} u &= \sin x & v &= x \\ u' &= \cos x & v' &= 1 \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{\cot x}{1 + \cot x}$

Solution

$$\begin{aligned} y' &= \frac{-\csc^2 x (1 + \cot x) + \csc^2 x \cot x}{(1 + \cot x)^2} \\ &= \frac{-\csc^2 x - \csc^2 x \cot x + \csc^2 x \cot x}{(1 + \cot x)^2} \\ &= \underline{\frac{-\csc^2 x}{(1 + \cot x)^2}} \end{aligned}$$

$$\begin{aligned} u &= \cot x & v &= 1 + \cot x \\ u' &= -\csc^2 x & v' &= -\csc^2 x \end{aligned}$$

Exercise

Find the derivative to the following functions $y = x^2 \sin x + 2x \cos x - 2 \sin x$

Solution

$$\begin{aligned} y' &= 2x \sin x + x^2 \cos x + 2 \cos x - 2x \sin x - 2 \cos x \\ &= \underline{x^2 \cos x} \end{aligned}$$

Exercise

Find the derivative to the following functions $y = x^3 \sin x \cos x$

Solution

$$y' = (x^3)' \sin x \cos x + x^3 (\sin x)' \cos x + x^3 \sin x (\cos x)'$$

$$= \underline{3x^2 \sin x \cos x + x^3 \cos^2 x - x^3 \sin^2 x}$$

Exercise

Find the derivative to the following functions $y = \frac{4}{\cos x} + \frac{1}{\tan x}$

Solution

$$y' = \frac{-4 \sin x}{\cos^2 x} - \frac{\sec^2 x}{\tan^2 x} \quad \left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$= -4 \frac{\sin x}{\cos x} \frac{1}{\cos x} - \frac{1}{\cos^2 x} \frac{\cos^2 x}{\sin^2 x}$$

$$= \underline{-4 \tan x \sec x - \csc^2 x}$$

Exercise

Find the derivative of $f(x) = \frac{(x^2 - 6x)^5}{(3x^2 + 5x - 2)^4}$

Solution

$$f(x) = (x^2 - 6x)^5 (3x^2 + 5x - 2)^{-4} \quad (U^m V^n)' = U^{m-1} V^{n-1} (mU'V + nUV')$$

$$f'(x) = (x^2 - 6x)^4 (3x^2 + 5x - 2)^{-5} \left[5(2x - 6)(3x^2 + 5x - 2) - 4(x^2 - 6x)(6x + 5) \right]$$

$$= (x^2 - 6x)^4 (3x^2 + 5x - 2)^{-5} \left[(10x - 30)(3x^2 + 5x - 2) - 4(6x^3 - 31x^2 - 30x) \right]$$

$$= (x^2 - 6x)^4 (3x^2 + 5x - 2)^{-5}$$

$$\begin{bmatrix} x^3 & 30 - 24 \\ x^2 & 50 - 90 + 124 \\ x & -20 - 150 + 120 \\ x^0 & 60 \end{bmatrix}$$

$$= \underline{\frac{(x^2 - 6x)^4 (6x^3 + 84x^2 - 50x + 60)}{(3x^2 + 5x - 2)^5}}$$

Exercise

Find the derivative of $y = \ln \sqrt{x+5}$

Solution

$$y = \ln(x+5)^{1/2} = \frac{1}{2} \ln(x+5)$$

$$\underline{y' = \frac{1}{2(x+5)}}|$$

Exercise

Find the Derivatives of $y = (3x+7)\ln(2x-1)$

Solution

$$f = 3x+7 \quad f' = 3$$

$$g = \ln(2x-1) \quad g' = \frac{2}{2x-1}$$

$$\underline{y' = 3x\ln(2x-1) + \frac{2(3x+7)}{2x-1}}|$$

Exercise

Find the Derivatives of $f(x) = \ln \sqrt[3]{x+1}$

Solution

$$\begin{aligned} f(x) &= \ln(x+1)^{1/3} \\ &= \frac{1}{3} \ln(x+1) \end{aligned}$$

$$u = x+1 \Rightarrow \frac{du}{dx} = 1$$

$$\begin{aligned} f'(x) &= \frac{1}{3} \frac{1}{x+1} \\ &= \frac{1}{3(x+1)} \end{aligned} \quad |$$

Exercise

Find the Derivatives of $f(x) = \ln \left[x^2 \sqrt{x^2+1} \right]$

Solution

$$\begin{aligned} f(x) &= \ln(x^2) + \ln \sqrt{x^2+1} \\ &= \ln(x^2) + \ln(x^2+1)^{1/2} \end{aligned}$$

Product Property

$$= 2 \ln x + \frac{1}{2} \ln(x^2 + 1) \quad \text{Power Property}$$

$$f'(x) = 2 \frac{1}{x} + \frac{1}{2} \frac{2x}{x^2 + 1}$$

$$\underline{= \frac{2}{x} + \frac{x}{x^2 + 1}}$$

Differentiate

Exercise

Find the Derivatives of $y = \ln \frac{x^2}{x^2 + 1}$

Solution

$$y = \ln x^2 - \ln x^2 + 1$$

$$y' = \frac{2x}{x^2} - \frac{2x}{x^2 + 1} \quad \underline{= \frac{2}{x} - \frac{2x}{x^2 + 1}}$$

Exercise

Find the derivative of $f(x) = e^{-2x^3}$

Solution

$$f'(x) = e^{-2x^3} \left(-6x^2 \right) \quad \underline{= -\frac{6x^2}{e^{2x^3}}}$$

Exercise

Find the derivative of $f(x) = 4e^{x^2}$

Solution

$$f'(x) = 4e^{x^2} (2x) \quad \underline{= 8xe^{x^2}}$$

Exercise

Find the derivative of $f(x) = x^2 e^x$

Solution

$$f'(x) = e^x \frac{d}{dx}[x^2] + x^2 \frac{d}{dx}[e^x]$$

$$= e^x (2x) + x^2 e^x$$

$$\underline{= xe^x (2 + x)}$$

Exercise

Find the derivative $f(x) = 2x^3 e^x$

Solution

$$f'(x) = 6x^2 e^x + 2x^3 e^x$$

$$u = 2x^3 \quad v = e^x$$

$$u' = 6x^2 \quad v' = e^x$$

$$= 2x^2 e^x (3 + x)$$

Exercise

Find the derivative $f(x) = \frac{3e^x}{1 + e^x}$

Solution

$$f'(x) = \frac{3e^x(1 + e^x) - 3e^x e^x}{(1 + e^x)^2}$$

$$u = 3e^x \quad v = 1 + e^x$$

$$u' = 3e^x \quad v' = e^x$$

$$= \frac{3e^x + 3e^{2x} - 3e^{2x}}{(1 + e^x)^2}$$

$$= \frac{3e^x}{(1 + e^x)^2}$$

Exercise

Find the derivative $f(x) = 5e^x + 3x + 1$

Solution

$$f'(x) = 5e^x + 3$$

Exercise

Find the derivative of $f(x) = \frac{e^x + e^{-x}}{2}$

Solution

$$f(x) = \frac{1}{2}(e^x + e^{-x})$$

$$f'(x) = \frac{1}{2}(e^x - e^{-x})$$

Exercise

Find the derivative of $f(x) = \frac{e^x}{x^2}$

Solution

$$\begin{aligned} f'(x) &= \frac{x^2 e^x - 2x e^x}{x^4} \\ &= \frac{x e^x (x-2)}{x^4} \\ &= \frac{e^x (x-2)}{x^3} \end{aligned}$$

Exercise

Find the derivative of $f(x) = x^2 e^x - e^x$

Solution

$$\begin{aligned} f'(x) &= e^x \frac{d}{dx}[x^2] + x^2 \frac{d}{dx}[e^x] - \frac{d}{dx}[e^x] \\ &= e^x(2x) + x^2 e^x - e^x \\ &= e^x(x^2 + 2x - 1) \end{aligned}$$

Exercise

Find the derivative of $f(x) = (1 + 2x)e^{4x}$

Solution

$$\begin{aligned} f'(x) &= (2)e^{4x} + (1 + 2x)(4e^{4x}) \\ &= 2e^{4x} + (1 + 2x)(4e^{4x}) \\ &= 2e^{4x}(1 + 2(1 + 2x)) \\ &= 2e^{4x}(1 + 2 + 4x) \\ &= 2e^{4x}(3 + 4x) \end{aligned}$$

Exercise

Find the derivative of $y = x^2 e^{5x}$

Solution

$$y' = x^2(5e^{5x}) + 2x(e^{5x})$$

$$= \underline{xe^{5x}(5x+2)}$$

Exercise

Find the derivative of $y = x^2 e^{-2x}$

Solution

$$y' = 2xe^{-2x} - 2x^3 e^{-2x}$$

$$= \underline{2xe^{-2x}(1-x^2)}$$

Exercise

Find the derivative $f(x) = \frac{e^x}{x^2 + 1}$

Solution

$$f'(x) = \frac{e^x(x^2 + 1) - 2xe^x}{(x^2 + 1)^2}$$

$$= \underline{\frac{(x^2 + 1 - 2x)e^x}{(x^2 + 1)^2}}$$

$$u = e^x \quad v = x^2 + 1$$

$$u' = e^x \quad v' = 2x$$

Exercise

Find the derivative $f(x) = \frac{1 - e^x}{1 + e^x}$

Solution

$$f'(x) = \frac{-e^x(1 + e^x) - e^x(1 - e^x)}{(1 + e^x)^2}$$

$$= \frac{-e^x - e^{2x} - e^x + e^{2x}}{(1 + e^x)^2}$$

$$= \underline{-\frac{2e^x}{(1 + e^x)^2}}$$

$$u = 1 - e^x \quad v = 1 + e^x$$

$$u' = -e^x \quad v' = e^x$$

Exercise

Find the Derivatives of $y = \frac{\ln x}{e^{2x}}$

Solution

$$\begin{aligned} y' &= \frac{e^{2x}(1/x) - \ln x(2e^{2x})}{e^{4x}} \\ &= \frac{e^{2x} - 2x \ln x(e^{2x})}{e^{4x}} \\ &= \frac{e^{2x}(1 - 2x \ln x)}{e^{4x}} \end{aligned}$$

Exercise

Find the Derivatives of $f(x) = e^{2x} \ln(xe^x + 1)$

Solution

$$\begin{aligned} f'(x) &= 2e^{2x} \ln(xe^x + 1) + e^{2x} \frac{e^x + xe^x}{xe^x + 1} \\ &= e^{2x} \left[2 \ln(xe^x + 1) + \frac{e^x(1+x)}{xe^x + 1} \right] \end{aligned}$$

$$\begin{aligned} f &= e^{2x} & U &= 2x \rightarrow U' = 2 & f' &= 2e^{2x} \\ g &= \ln(xe^x + 1) & U &= xe^x + 1 \rightarrow U' = e^x + xe^x & g' &= \frac{e^x + xe^x}{xe^x + 1} \end{aligned}$$

Exercise

Find the Derivatives of $f(x) = \frac{xe^x}{\ln(x^2 + 1)}$

Solution

$$\begin{aligned} f'(x) &= \frac{e^x(1+x) \ln(x^2 + 1) - \frac{2x}{x^2 + 1} xe^x}{\left[\ln(x^2 + 1) \right]^2} \\ &= \frac{e^x \left[(1+x) \ln(x^2 + 1) - \frac{2x^2}{x^2 + 1} \right]}{\left[\ln(x^2 + 1) \right]^2} \\ &= \frac{e^x \left[\frac{(x^2 + 1)(1+x) \ln(x^2 + 1) - 2x^2}{x^2 + 1} \right]}{\left[\ln(x^2 + 1) \right]^2} \end{aligned}$$

$$\begin{aligned} u &= xe^x & u' &= e^x + xe^x \\ v &= \ln(x^2 + 1) & v' &= \frac{2x}{x^2 + 1} \end{aligned}$$

$$= \frac{e^x \left[(x^2 + 1)(1 + x) \ln(x^2 + 1) - 2x^2 \right]}{(x^2 + 1) \left[\ln(x^2 + 1) \right]^2}$$

Exercise

Find the derivative $y = \cos^{-1}\left(\frac{1}{x}\right)$

Solution

$$y = \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(x)$$

$$y' = \frac{1}{|x| \cdot \sqrt{x^2 - 1}}$$

Exercise

Find the derivative $y = \sin^{-1}\sqrt{2t}$

Solution

$$y' = \frac{\sqrt{2}}{\sqrt{1 - (\sqrt{2t})^2}}$$

$$= \frac{\sqrt{2}}{\sqrt{1 - 2t^2}}$$

Exercise

Find the derivative $y = \sec^{-1}(5s)$

Solution

$$y' = \frac{5s}{|5s| \sqrt{(5s)^2 - 1}}$$

$$= \frac{s}{|s| \sqrt{25s^2 - 1}}$$

Exercise

Find the derivative $y = \cot^{-1} \sqrt{t-1}$

Solution

$$\begin{aligned} y' &= -\frac{\frac{1}{2}(t-1)^{-1/2}}{1 + \left[(t-1)^{1/2}\right]^2} \\ &= -\frac{1}{2(t-1)^{1/2}(1+t-1)} \\ &= -\frac{1}{2t\sqrt{t-1}} \end{aligned}$$

Exercise

Find the derivative $y = \ln(\tan^{-1} x)$

Solution

$$y' = \frac{\frac{1}{1+x^2}}{\tan^{-1} x} = \frac{1}{(1+x^2)\tan^{-1} x}$$

Exercise

Find the derivative $y = \tan^{-1}(\ln x)$

Solution

$$\begin{aligned} y' &= \frac{\frac{1}{x}}{1 + (\ln x)^2} \\ &= \frac{1}{x[1 + (\ln x)^2]} \end{aligned}$$

$$\left(\tan^{-1} u\right)' = \frac{u'}{1+u^2}$$