Section 2.2 – Homogeneous Equations with Constant Coefficients

The homogeneous equations of the form:

$$y'' + py' + qy = 0 \qquad (H: \ Homogeneous)$$

Where p and q are continuous functions on some interval I.

The zero function, $y(x) = 0$ for all $x \in I$, $(y \equiv 0)$ is a solution of the equation (given above).
The zero solution is called ***trivial solution***. Any other solution is a ***nontrivial*** solution.
This is a class of equations that we can solve easily.

Theorem

If $y = y(x)$ is a solution of $y'' + py' + qy = 0$ and if C is any real number, then $u(x) = Cy(x)$ is also solution of $y'' + py' + qy = 0$.
Any constant multiple of a solution of $y'' + py' + qy = 0$ is also a solution of $y'' + py' + qy = 0$

Proof

$$u'(x) = Cy'(x)$$

$$u''(x) = Cy''(x)$$

$$u'' + pu' + qu \overset{?}{=} 0$$

$$Cy'' + pCy' + qCy \overset{?}{=} 0$$

$$\Rightarrow \ C(y'' + py' + qy) = 0$$

Theorem

If $y = y_1(x)$ and $y = y_2(x)$ are any two solutions of $y'' + py' + qy = 0$ the $u(x) = y_1(x) + y_2(x)$ is also a solution of $y'' + py' + qy = 0$.

Any linear combination of solutions of $y'' + py' + qy = 0$ is also a solution of $y'' + py' + qy = 0$.

Definition

Let $f = f(x)$ *and* $g = g(x)$ be functions defined on some interval I, and let C_1 and C_2 be real numbers. The expression

$$C_1 f(x) + C_2 g(x)$$

Is called a linear combination of f and g.

Definition

Let $y = y_1(x)$ and $y = y_2(x)$ are any two solutions of $y'' + py' + qy = 0 \quad (H)$. The function W defined by

$$W\left[y_1, y_2\right] = y_1(x)y_2'(x) - y_2(x)y_1'(x)$$

Is called the Wronskian of y_1, y_2

Determinant notation:

$$\begin{aligned} W(x) &= y_1(x)y_2'(x) - y_2(x)y_1'(x) \\ &= \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix} \end{aligned}$$

Theorem

Let $y = y_1(x)$ and $y = y_2(x)$ be solutions of equation (*H*), and let $W(x)$ be their Wronskian. Exactly one of the following holds

1. $W(x) = 0$ for all $x \in I$ and y_1 is a constant multiple of y_2

Example:

$$W(x) = \begin{vmatrix} x^2 & 2x^2 \\ 2x & 4x \end{vmatrix} = 4x^3 - 4x^3 \underline{= 0}$$

2. $W(x) \neq 0$ for all $x \in I$ and $y = C_1y_1(x) + C_2y_2(x)$ is the general solution of (*H*)

Definition **Fundamental Set – Solution basis**

A pair of solutions $y = y_1(x)$ and $y = y_2(x)$ of equation (*H*) forms a ***fundamental set of solutions*** (also called a ***solution basis***) if $W(x) \neq 0$ for all $x \in I$.

The analogous first-order, linear, homogeneous equation:

$$y' + p(t)y = 0$$

It is separable and easily solved, its general solution is

$$y(t) = Ce^{-pt}$$

Let look for a solution of the type

$$y(t) = e^{\lambda t}$$

$$y' = \lambda e^{\lambda t}$$
$$y'' = \lambda^2 e^{\lambda t}$$

$$\begin{aligned} y'' + py' + qy &= \lambda^2 e^{\lambda t} + p\lambda e^{\lambda t} + qe^{\lambda t} \\ &= \left(\lambda^2 + p\lambda + q\right)e^{\lambda t} \\ &= 0 \end{aligned}$$

$\lambda^2 + p\lambda + q = 0$ *This is called the* ***characteristic equation***

We can rewrite the differential equation and its characteristic equations

$$y'' + py' + qy = 0$$

$$\lambda^2 + p\lambda + q = 0$$

The roots are: $\lambda_{1,2} = \dfrac{-p \pm \sqrt{p^2 - 4q}}{2}$

$$\begin{aligned} &If \ \ p^2 - 4q > 0 \ \Rightarrow \ \ Two\ distinct\ real\ roots \\ &If \ \ p^2 - 4q < 0 \ \Rightarrow \ \ Two\ distinct\ complex\ roots \\ &If \ \ p^2 - 4q = 0 \ \Rightarrow \ \ \text{One}\ repeated\ real\ root \end{aligned}$$

Case 1: Distinct Real Root

If $\lambda^2 + p\lambda + q = 0$ has two distinct real roots: $\lambda_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$

Then:

$$y_1 = C_1 e^{\lambda_1 t} \text{ and } y_2 = C_2 e^{\lambda_2 t} \text{ are both solutions.}$$

Proposition

If the characteristic equations $\lambda^2 + p\lambda + q = 0$ has two distinct real roots λ_1 and λ_2, then the ***general solution*** to $y'' + py' + qy = 0$ is

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Where C_1 and C_2 are arbitrary constants.

Example

Find the general solution to the equation $y'' - 3y' + 2y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = 1$

Solution

The characteristic equation:

$$y'' - 3y' + 2y = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

The solution: $\lambda_{1,2} = 1,\ 2$

The general solution

$$y(t) = C_1 e^t + C_2 e^{2t}$$

$$y' = C_1 e^t + 2C_2 e^{2t}$$

$y(0) = 2 \quad y(0) = C_1 e^0 + C_2 e^{2(0)}$

$$2 = C_1 + C_2$$

$y'(0) = 1 \quad y'(0) = C_1 e^0 + 2C_2 e^{2(0)}$

$$1 = C_1 + 2C_2$$

$$\begin{matrix} C_1 + C_2 = 2 \\ C_1 + 2C_2 = 1 \end{matrix} \Rightarrow C_2 = -1 \quad C_1 = 3$$

The unique solution is: $\boxed{y(t) = 3e^t - e^{2t}}$

Case 2: Complex Roots

Proposition

If the characteristic equations $\lambda^2 + p\lambda + q = 0$ has two complex conjugate roots $\lambda = a + ib$ and $\bar{\lambda} = a - ib$.

1. The functions

$$z = e^{(a+ib)t} \text{ and } \bar{z} = e^{(a-ib)t}$$

So, the general solution is

$$w(t) = C_1 e^{(a+ib)t} + C_2 e^{(a-ib)t}$$

Where C_1 and C_2 are arbitrary complex constants.

2. The functions

$$y_1(t) = e^{at}\cos(bt) \text{ and } y_2(t) = e^{at}\sin(bt)$$

So, the general solution is

$$y(t) = e^{at}\left(A_1 \cos bt + A_2 \sin bt\right)$$

Where A_1 and A_2 are constants.

Example

Find the general solution to the equation $y'' + 2y' + 2y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = 3$

Solution

The characteristic equation: $\lambda^2 + 2\lambda + 2 = 0$

The solution: $\lambda_{1,2} = -1 \pm i = a \pm ib$

$a = -1; \quad b = 1$

The general solution

$$y(t) = e^{-t}\left(C_1 \cos t + C_2 \sin t\right)$$

$$y(0) = e^{-(0)}\left(C_1 \cos(0) + C_2 \sin(0)\right)$$

$$2 = 1\left(C_1 + C_2(0)\right) \Rightarrow \boxed{C_1 = 2}$$

$$y' = -e^{-t}\left(C_1 \cos t + C_2 \sin t\right) + e^{-t}\left(-C_1 \sin t + C_2 \cos t\right)$$

$$y'(0) = -e^{-(0)}\left(C_1 \cos(0) + C_2 \sin(0)\right) + e^{-(0)}\left(-C_1 \sin(0) + C_2 \cos(0)\right)$$

$$3 = -(C_1) + (C_2)$$

$$C_2 - C_1 = 3$$

$$\boxed{C_2 = 3 + 2 = 5}$$

$$\boxed{y(t) = e^{-t}(2\cos t + 5\sin t)}$$

Example

Find the general solution to the equation $y'' - 4y' + 13y = 0$

Solution

The characteristic equation: $\lambda^2 - 4\lambda + 13 = 0$

The solutions: $\lambda_{1,2} = \dfrac{4 \pm \sqrt{16-52}}{2} = \dfrac{4 \pm \sqrt{-36}}{2} = \dfrac{4 \pm 6i}{2} \ \boxed{= 2 \pm 3i}$

$a = 2;\ \ b = 3$

The general solution: $\boxed{y(x) = e^{2x}(C_1 \cos 3x + C_2 \sin 3x)}$

Case 3: Repeated Roots

If the roots of the characteristic equations are repeated and the solutions are independent.

$$\lambda^2 + p\lambda + q = 0$$

$$\left(\lambda - \lambda_1\right)^2 = 0$$

$$\lambda_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$$

$$p^2 - 4q = 0 \;\Rightarrow\; q = \frac{p^2}{4}$$

$$\lambda_{1,2} = -\frac{p}{2}$$

$$\begin{aligned} y_1 &= C_1 e^{\lambda_1 t} \\ &= C_1 e^{-pt/2} \end{aligned}$$

$$\begin{aligned} y_2 &= v(t) y_1(t) \\ &= v(t) e^{-pt/2} \end{aligned}$$

$$y'' + py' + qy = 0$$

$$y'' + py' + \frac{p^2}{4} y = 0$$

$$y_2{}' = v' e^{-pt/2} - \frac{P}{2} v e^{-pt/2}$$

$$y_2{}'' = v'' e^{-pt/2} - \frac{P}{2} v' e^{-pt/2} - \frac{P}{2} v' e^{-pt/2} + \frac{P^2}{4} v e^{-pt/2}$$

$$v'' e^{-pt/2} - \frac{P}{2} v' e^{-pt/2} - \frac{P}{2} v' e^{-pt/2} + \frac{P^2}{4} v e^{-pt/2} + p\left(v' e^{-pt/2} - \frac{P}{2} v e^{-pt/2}\right) + \frac{P^2}{4} v e^{-pt/2} = 0$$

$$v'' e^{-pt/2} = 0$$

$$\begin{aligned} &v'' = 0 \\ &\Rightarrow v' = a \\ &\Rightarrow v = at + b \\ &v = t \end{aligned}$$

$$\boxed{y_2 = t e^{-pt/2}}$$

Proposition

If the characteristic equations $\lambda^2 + p\lambda + q = 0$ has one double root λ_1, then the ***general solution*** to $y'' + py' + qy = 0$ is

$$y(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t}$$
$$= \left(C_1 + C_2 t\right) e^{\lambda_1 t}$$

Where C_1 and C_2 are arbitrary constants.

Example

Find the general solution to the equation $y'' - 2y' + y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = -1$

Solution

The characteristic equation: $\lambda^2 - 2\lambda + 1 = 0$

The solutions are: $\lambda_{1,2} = 1$

$$y(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t}$$
$$= C_1 e^t + C_2 t e^t$$

$$y(0) = C_1 e^{(0)} + C_2 (0) e^{(0)} \quad \Rightarrow \quad 2 = C_1$$

$$y' = C_1 e^t + C_2 e^t + C_2 t e^t$$

$$y'(0) = 2e^{(0)} + C_2 e^{(0)} + C_2 (0) e^{(0)}$$

$$-1 = 2 + C_2 \quad \Rightarrow \quad \boxed{C_2 = -3}$$

$$\boxed{y(t) = 2e^t - 3te^t}$$

Example

Find the general solution to the equation $y'' - 10y' + 25y = 0$

Solution

The characteristic equation: $\lambda^2 - 10\lambda + 25 = (\lambda - 5)^2 = 0$

The solutions are: $\lambda_{1,2} = 5$

The general solution: $\boxed{y(t) = C_1 e^{5t} + C_2 t e^{5t}}$

Higher-Order Equations

In general, to solve an *n*th-order differential equation, we must solve an *n*th degree characteristic polynomial equation

$$a_n\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

If all roots are real and distinct, then the general solution is

$$y(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + \cdots + C_n e^{\lambda_n x}$$

If all roots are equal to λ, then the general solution is

$$y(x) = C_1 e^{\lambda x} + C_2 x e^{\lambda x} + C_3 x^2 e^{\lambda x} + \cdots + C_n x^{n-1} e^{\lambda x}$$

Example

Find the general solution of $\quad y''' + 3y'' - 4y = 0$

Solution

$\lambda^3 + 3\lambda^2 - 4 = 0$ ***Solve for λ***

$\lambda_1 = 1, \quad \lambda_{2,\,3} = -2$

$$\boxed{y(x) = C_1 e^{x} + \left(C_2 + C_3 x\right)e^{-2x}}$$

Rational zero theorem: $\pm\left\{\frac{4}{1}\right\} = \pm\{1,\ 2,\ 4\}$

$\lambda_1 = 1, \quad \lambda_2 = -2$

$(\lambda-1)(\lambda+2)(\lambda-a) = 0$

$(-1)(2)(-a) = -4 \quad \Rightarrow \quad a = -2$

Example

Find the general solution of $\quad \lambda^4(\lambda+1)(\lambda+2)^2\left(\lambda^2+4\right) = 0$

Solution

$\lambda^2 + 4 = 0 \quad \Rightarrow \quad \lambda^2 = -4 \ \to \ \lambda = \pm 2i$

The solution: $\lambda = 0,\ 0,\ 0,\ 0,\ -1,\ -2,\ -2,\ \pm 2i$

$$\boxed{y(x) = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + C_5 e^{-x} + \left(C_6 + C_7 x\right)e^{-2x} + C_8 \cos 2x + C_9 \sin 2x}$$

Summary

The equation: $y'' + py' + qy = 0$

The characteristic equations $\lambda^2 + p\lambda + q = 0$

If $p^2 - 4q > 0$	$y_1(t) = C_1 e^{\lambda_1 t}$ *and* $y_1(t) = C_2 e^{\lambda_2 t}$	$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$
If $p^2 - 4q < 0$	$y_1(t) = e^{at}\cos bt$ *and* $y_1(t) = e^{at}\sin bt$	$y(t) = e^{at}\left(A_1 \cos bt + A_2 \sin bt\right)$
If $p^2 - 4q = 0$	$y_1 = e^{\lambda t}$ *and* $y_1 = te^{\lambda t}$	$y(t) = \left(C_1 + C_2 t\right)e^{\lambda_1 t}$

Exercises *Section* 2.2 – Linear, Homogeneous Equations with Constant Coefficients

(*Exercises* 1- 4) Decide whether the equation is linear or nonlinear. For the linear equation, state whether the equation is homogenous or inhomogeneous.

35. $t^2 y'' = 4y' - \sin t$

36. $ty'' + (\sin t) y' = 4y - \cos 5t$

37. $t^2 y'' + 4yy' = 0$

38. $y'' + 4y' + 7y = 3e^{-t}\sin t$

(*Exercises* 5- 6) Show by direct substitution that the given functions $y_1(t)$ and $y_2(t)$ are solutions of the given differential equation. Then verify by direct substitution, that any linear combination $C_1 y_1(t) + C_2 y_2(t)$ of the 2 given solutions is also a solution.

39. $y'' + 4y = 0; \quad y_1(t) = \cos 2t \quad y_2(t) = \sin 2t$

40. $y'' - 2y' + 2y = 0; \quad y_1(t) = e^t \cos t \quad y_2(t) = e^t \sin t$

41. Explain why $y_1(t)$ and $y_2(t)$ are linearly independent solutions. Calculate Wronskian and use it to explain the independence of the given solutions.

$$y'' + 9y = 0; \quad y_1(t) = \cos 3t \quad y_2(t) = \sin 3t$$

42. Show that $y_1(t) = e^t$ and $y_2(t) = e^{-3t}$ form a fundamental set of solutions for $y'' + 2y' - 3y = 0,$ then find a solution satisfying $y(0) = 1$ and $y'(0) = -2$.

Find the general solution of the second order differential equation

43. $y'' + y' = 0$

44. $y'' - 4y = 0$

45. $y'' + 8y = 0$

46. $y'' - 36y = 0$

47. $y'' + 9y = 0$

48. $y'' - 9y = 0$

49. $y'' + 16y = 0$

50. $y'' + 25y = 0$

51. $y'' - 64y = 0$

52. $y'' + y' + y = 0$

53. $y'' + y' - y = 0$

54. $y'' - y' - 2y = 0$

55. $y'' - y' - 6y = 0$

56. $y'' + y' - 6y = 0$

57. $y'' - y' - 11y = 0$

58. $y'' - y' - 12y = 0$

59. $y'' + 2y' + y = 0$

60. $y'' + 2y' + 3y = 0$

61. $y'' + 2y' - 3y = 0$

62. $y'' - 2y' - 3y = 0$

63. $y'' - 2y' + 3y = 0$

64. $y'' + 2y' + 4y = 0$

65. $y'' - 2y' + 5y = 0$

66. $y'' + 2y' - 15y = 0$

67. $y'' + 2y' + 17y = 0$

68. $y'' - 3y' + 2y = 0$

69. $y'' + 3y' - 4y = 0$

70. $y'' + 4y' - y = 0$

71. $y'' - 4y' + 4y = 0$

72. $y'' + 4y' + 4y = 0$

73. $y'' - 4y' + 5y = 0$

74. $y'' + 4y' + 5y = 0$

75. $y'' + 4y' - 5y = 0$

76. $y'' + 4y' + 7y = 0$

77. $y'' + 4y' + 9y = 0$

78. $y'' + 5y' = 0$

79. $y'' + 5y' + 6y = 0$

80. $y'' + 6y' + 9y = 0$

81. $y'' - 6y' + 9y = 0$

82. $y'' - 6y' + 25y = 0$
83. $y'' + 8y' + 16y = 0$
84. $y'' + 8y' - 16y = 0$
85. $y'' - 9y' + 20y = 0$
86. $y'' - 10y' + 25y = 0$
87. $y'' + 14y' + 49y = 0$
88. $2y'' - y' - 3y = 0$
89. $2y'' + y' - y = 0$
90. $2y'' + 2y' + y = 0$
91. $2y'' + 2y' + 3y = 0$
92. $2y'' - 3y' - 2y = 0$
93. $2y'' - 3y' + 4y = 0$
94. $2y'' - 4y' + 8y = 0$
95. $2y'' + 5y' = 0$
96. $2y'' - 5y' - 3y = 0$
97. $2y'' + 7y' - 4y = 0$
98. $3y'' + y = 0$
99. $3y'' - y' = 0$
100. $3y'' + 2y' + y = 0$
101. $3y'' + 11y' - 7y = 0$
102. $3y'' - 20y' + 12y = 0$
103. $4y'' + y' = 0$
104. $4y'' + 4y' + y = 0$
105. $4y'' - 4y' + y = 0$
106. $4y'' + 4y' + 2y = 0$
107. $4y'' - 4y' + 13y = 0$
108. $4y'' - 8y' + 7y = 0$
109. $4y'' - 12y' + 9y = 0$
110. $4y'' + 20y' + 25y = 0$
111. $6y'' + 5y' - 6y = 0$
112. $6y'' + y' - 2y = 0$
113. $6y'' - 7y' - 20y = 0$
114. $6y'' + 13y' - 5y = 0$
115. $6y'' + 13y' + 7y = 0$
116. $6y'' - 13y' + 7y = 0$
117. $8y'' - 10y' - 3y = 0$
118. $9y'' - y = 0$
119. $9y'' + 6y' + y = 0$
120. $9y'' - 12y' + 4y = 0$
121. $9y'' + 24y' + 16y = 0$
122. $12y'' - 5y' - 2y = 0$
123. $16y'' - 8y' + 7y = 0$
124. $16y'' - 12y' - 4y = 0$
125. $16y'' - 24y' + 9y = 0$
126. $25y'' + 10y' + y = 0$
127. $25y'' - 10y' + y = 0$
128. $35y'' - y' - 12y = 0$

Find the general solution of the given higher-order differential equation

129. $y''' + 3y'' + 3y' + y = 0$
130. $y''' + 3y'' - y' - 3y = 0$
131. $y^{(3)} + 3y'' - 4y = 0$
132. $3y''' - 19y'' + 36y' - 10y = 0$
133. $y''' - 6y'' + 12y' - 8y = 0$
134. $y''' + 5y'' + 7y' + 3y = 0$
135. $y^{(3)} + y' - 10y = 0$
136. $y''' + y'' - 6y' + 4y = 0$
137. $y''' - 6y'' - y' + 6y = 0$
138. $y''' + 2y'' - 4y' - 8y = 0$
139. $y''' - 7y'' + 7y' + 15y = 0$
140. $y''' + 3y'' - 4y' - 12y = 0$
141. $y''' - 4y'' - 5y' = 0$
142. $y''' - y = 0$
143. $y''' - 5y'' + 3y' + 9y = 0$
144. $y''' + 3y'' - 4y' - 12y = 0$
145. $y''' + y'' - 2y = 0$
146. $y''' - y'' - 4y = 0$
147. $y''' + 3y'' + 3y' + y = 0$
148. $y''' - 6y'' + 12y' - 8y = 0$
149. $y^{(4)} + y''' + y'' = 0$
150. $y^{(4)} - 2y'' + y = 0$
151. $16y^{(4)} + 24y'' + 9y = 0$
152. $y^{(4)} - 7y'' - 18y = 0$
153. $y^{(4)} + 2y'' + y = 0$
154. $y^{(4)} + y''' + y'' = 0$
155. $y^{(4)} + 4y = 0$
156. $y^{(4)} + 2y''' + 9y'' - 2y' - 10y = 0$
157. $x^{(4)} - 4x^{(3)} + 7x'' - 4x' + 6x = 0$
158. $x^{(4)} + 8x^{(3)} + 24x'' + 32x' + 16x = 0$
159. $x^{(4)} - 4x'' + 16x' + 32x = 0$
160. $x^{(4)} + 4x^{(3)} + 6x'' + 4x' + x = 0$
161. $y^{(4)} - y^{(3)} + y'' - 3y' - 6y = 0$
162. $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$
163. $x^{(5)} - x^{(4)} - 2x^{(3)} + 2x'' + x' - x = 0$
164. $x^{(5)} + 5x^{(4)} + 10x^{(3)} + 10x'' + 5x' + x = 0$

165. $y^{(5)}+5y^{(4)}-2y'''-10y''+y'+5y=0$

166. $2y^{(5)}-7y^{(4)}+12y'''+8y''=0$

167. $y^{(5)}-2y^{(4)}+17y'''=0$

168. $x^{(6)}-5x^{(4)}+16x^{(3)}+36x''-16x'-32x=0$

169. $\left(D^2+6D+13\right)^2 y=0$

170. $\lambda^3(\lambda-1)(\lambda-2)^3\left(\lambda^2+9\right)=0$

Find the solution of the given initial value problem.

171. $y''+y=0$; $y\left(\frac{\pi}{3}\right)=0,\quad y'\left(\frac{\pi}{3}\right)=2$

172. $y''+y=0$; $y(0)=0,\quad y'\left(\frac{\pi}{2}\right)=0$

173. $y''+y'=0;\quad y(0)=2,\quad y'(0)=1$

174. $y''-y'-2y=0;\quad y(0)=-1,\quad y'(0)=2$

175. $y''+y'+2y=0;\quad y(0)=0,\quad y'(0)=0$

176. $y''+2y'+y=0;\quad y(0)=1,\quad y'(0)=-3$

177. $y''-2y'+y=0,\quad y(0)=5,\quad y'(0)=10$

178. $y''-2y'-2y=0$; $y(0)=0,\quad y'(0)=3$

179. $y''-2y'+2y=0$; $y(0)=1,\quad y(\pi)=1$

180. $y''-2y'-3y=0$; $y(0)=2,\quad y'(0)=-3$

181. $y''+2y'-8y=0$; $y(0)=3,\quad y'(0)=-12$

182. $y''-2y'+17y=0$; $y(0)=-2,\quad y'(0)=3$

183. $y''+2\sqrt{2}y'+2y=0$; $y(0)=1,\quad y'(0)=0$

184. $y''+3y'-10y=0;\quad y(0)=4,\quad y'(0)=-2$

185. $y''+4y=0$; $y(0)=0,\quad y(\pi)=0$

186. $y''+4y=0$; $y\left(\frac{\pi}{4}\right)=-2,\quad y\left(\frac{\pi}{4}\right)=1$

187. $y''+4y'+2y=0$; $y(0)=-1,\quad y'(0)=2$

188. $y''-4y'+3y=0$; $y(0)=1,\quad y'(0)=\frac{1}{3}$

189. $y''-4y'+4y=0,\quad y(1)=1,\quad y'(1)=1$

190. $y''+4y'+4y=0,\quad y(0)=1,\quad y'(0)=3$

191. $y''-4y'+5y=0;\quad y(0)=1,\quad y'(0)=5$

192. $y''+4y'+5y=0;\quad y(0)=1,\quad y'(0)=0$

193. $y''+4y'+5y=0;\quad y\left(\frac{\pi}{2}\right)=\frac{1}{2},\quad y'\left(\frac{\pi}{2}\right)=-2$

194. $y''-4y'-5y=0,\quad y(1)=0,\quad y'(1)=2$

195. $y''-4y'-5y=0,\quad y(-1)=3,\quad y'(-1)=9$

196. $y''-4y'+9y=0,\quad y(0)=0,\quad y'(0)=-8$

197. $y''-4y'+13y=0;\quad y(0)=-1,\quad y'(0)=2$

198. $y''-5y'+6y=0;\quad y(1)=e^2,\quad y'(1)=3e^2$

199. $y''+6y'+9y=0;\quad y(0)=2,\quad y'(0)=-2$

200. $y''+6y'+5y=0,\quad y(1)=0,\quad y'(0)=3$

201. $y''-6y'+5y=0;\quad y(0)=3,\quad y'(0)=11$

202. $y''-6y'+9y=0,\quad y(0)=2,\quad y'(0)=\frac{25}{3}$

203. $y''-6y'+9y=0;\quad y(0)=0,\quad y'(0)=5$

204. $y''+8y'-9y=0;\quad y(1)=2,\quad y'(1)=0$

205. $y''-8y'+17y=0;\quad y(0)=4,\quad y'(0)=-1$

206. $y''-9y=0,\quad y(0)=2,\quad y'(0)=-1$

207. $y''-10y'+25y=0,\quad y(0)=1,\quad y'(1)=0$

208. $y''+10y'+25y=0;\quad y(0)=2,\quad y'(0)=-1$

209. $y''+11y'+24y=0;\quad y(0)=0,\quad y'(0)=-7$

210. $y''+12y=0,\quad y(0)=0,\quad y'(0)=1$

211. $y''+16y=0,\quad y(0)=2,\quad y'(0)=-2$

212. $y''+16y=0,\quad y(\pi)=2,\quad y'(0)=-2$

213. $y''+16y=0,\quad y\left(\frac{\pi}{2}\right)=-10,\quad y'\left(\frac{\pi}{2}\right)=3$

214. $y''+25y=0;\quad y(0)=1,\quad y'(0)=-1$

215. $2y''-2y'+y=0;\quad y(-\pi)=1,\quad y'(-\pi)=-1$

216. $3y''+y'-14y=0,\quad y(0)=2,\quad y'(0)=-1$

217. $3y''+2y'-8y=0,\quad y(0)=-6,\quad y'(0)=-18$

218. $4y''-4y'+y=0,\quad y(0)=4,\quad y'(0)=4$

219. $4y''-4y'+y=0,\quad y(1)=-4,\quad y'(1)=0$

220. $4y'' - 4y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 5$

221. $4y'' + 4y' + 5y = 0, \quad y(\pi) = 1, \quad y'(\pi) = 0$

222. $4y'' + 4y' + 17y = 0, \quad y(0) = -1, \quad y'(0) = 2$

223. $4y'' - 5y' = 0, \quad y(-2) = 0, \quad y'(-2) = 7$

224. $4y'' + 12y' + 9y = 0, \quad y(0) = 2, \quad y'(0) = 1$

225. $4y'' + 24y' + 37y = 0, \quad y(\pi) = 1, \quad y'(\pi) = 0$

226. $9y'' + y = 0; \quad y\left(\frac{\pi}{2}\right) = 4, \quad y'\left(\frac{\pi}{2}\right) = 0$

227. $9y'' + \pi^2 y = 0; \quad y(3) = 2, \quad y'(3) = -\pi$

228. $9y'' - 6y' + y = 0; \quad y(3) = -2, \quad y'(3) = -\frac{5}{3}$

229. $9y'' + 6y' + 2y = 0\ ; \quad y(3\pi) = 0, \quad y'(3\pi) = \frac{1}{3}$

230. $9y'' - 12y' + 4y = 0, \quad y(0) = -1, \quad y'(0) = 1$

231. $12y'' + 5y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = -1$

232. $16y'' - 8y' + y = 0\ ; \quad y(0) = -4, \quad y'(0) = 3$

233. $25y'' + 20y' + 4y = 0\ ; \quad y(5) = 4e^{-2}, \quad y'(5) = -\frac{3}{5}e^{-2}$

234. $y''' + 12y'' + 36y' = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -7$

235. $y''' + 2y'' - 5y' - 6y = 0, \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 1$

236. The roots of the characteristic equation of a certain differential equation are:

$$3, -5, 0, 0, 0, 0, -5, \ 2 \pm 3i \ \text{ and } \ 2 \pm 3i$$

Write a general solution of this homogeneous differential equation.

237. $y(x) = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos 2x + C_4 \sin 2x$ is the general solution of a homogeneous equation. What is the equation?

238. Show that the second differential equation $y'' + 4y = 0$

a) Has no solution to the boundary value $y(0) = 0, \quad y(\pi) = 1$

b) There are infinitely many solutions to the boundary value $y(0) = 0, \quad y(\pi) = 0$

239. Show that the general solution of the equation

$$y'' + Py' + Qy = 0$$

(where P and Q are constant) approaches 0 as $x \to \infty$ if and only if P and Q are both positive.