

Distance Formula: $|P_1 P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$

Standard Equation for the Sphere $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$

Magnitude: $|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Direction / Unit vector: $\frac{\mathbf{v}}{|\mathbf{v}|}$

Angle between Vectors: $\theta = \cos^{-1} \left(\frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{|\mathbf{u}| |\mathbf{v}|} \right)$

Dot Product: $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

Vector Projection: $\text{proj}_{\mathbf{v}} \mathbf{u} = (|\mathbf{u}| \cos \theta) \frac{\mathbf{u}}{|\mathbf{v}|} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} \right) \mathbf{v}$

The scalar component of \mathbf{u} in the direction of \mathbf{v} is the scalar $|\mathbf{u}| \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|} = \mathbf{u} \cdot \frac{\mathbf{v}}{|\mathbf{v}|}$

Work: $W = \mathbf{F} \cdot \mathbf{D} = |\mathbf{F}| |\mathbf{D}| \cos \theta$

Cross Product: $\mathbf{u} \times \mathbf{v} = (|\mathbf{u}| |\mathbf{v}| \sin \theta) \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

Magnitude of torque vector: $|\mathbf{r}| |\mathbf{F}| \sin \theta$

Torque vector: $(|\mathbf{r}| |\mathbf{F}| \sin \theta) \mathbf{n}$

Triple scalar product: $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = |\mathbf{u} \times \mathbf{v}| |\mathbf{w}| \cos \theta$

Volume: $V = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \det \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix}$

Vector equation for the line L : $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}, \quad -\infty < t < \infty$

The distance from a Point to a line: $d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|}$

The distance from a Point to a Plane: $d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$

Angle between the planes: $\theta = \cos^{-1} \left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1| |\mathbf{n}_2|} \right)$

Arc Length: $s(t) = \int_{t_0}^t \sqrt{[x'(\tau)]^2 + [y'(\tau)]^2 + [z'(\tau)]^2} d\tau = \int_{t_0}^t |\mathbf{v}(\tau)| d\tau$

Maximum height: $y_{\max} = \frac{(v_0 \sin \alpha)^2}{2g}$

Maximum time: $t = \frac{v_0 \sin \alpha}{g}$

Flight time: $t = \frac{2v_0 \sin \alpha}{g}$

Range: $R = \frac{v_0^2}{g} \sin 2\alpha$

Unit tangent vector: $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$

Principal unit normal vector: $\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} = \frac{1}{\kappa} \frac{d\mathbf{T}}{ds}$

Binormal vector: $\mathbf{B} = \mathbf{T} \times \mathbf{N}$

Curvature: $\kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \frac{1}{|\mathbf{v}|} \left| \frac{d\mathbf{T}}{dt} \right|$

Torsion: $\tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N} = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \ddot{\ddot{x}} & \ddot{\ddot{y}} & \ddot{\ddot{z}} \end{vmatrix}}{|\mathbf{v} \times \mathbf{a}|^2}$

Acceleration vector: $\mathbf{a} = a_T \mathbf{T} + a_N \mathbf{N}$

Tangential acceleration: $a_T = \frac{d}{dt} |\mathbf{v}|$

Normal acceleration: $a_N = \kappa |\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$

$$\frac{\partial^2 f}{\partial y \partial x} \quad \text{or} \quad f_{xy} = (f_x)_y$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt}$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}, \quad \frac{dz}{dx} = -\frac{F_x}{F_z}, \quad \frac{dz}{dy} = -\frac{F_y}{F_z}$$

$$\text{Gradient Vector:} \quad \nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$

$$\left(\frac{df}{ds} \right)_{\mathbf{u}, P_0} = (\nabla f)_{P_0} \cdot \mathbf{u}$$

$$\text{Directional Derivative:} \quad D_{\mathbf{u}} f = \nabla f \cdot \mathbf{u} = |\nabla f| |\mathbf{u}| \cos \theta = |\nabla f| \cos \theta$$

$$\text{Tangent Plane:} \quad f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$$

$$\text{Normal Line:} \quad x = x_0 + f_x(P_0)t, \quad y = y_0 + f_y(P_0)t, \quad z = z_0 + f_z(P_0)t$$

$$\text{Linearization:} \quad L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

f has a local maximum at (a, b) if $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b) .

f has a local minimum at (a, b) if $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b) .

f has a saddle point at (a, b) if $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a, b) .

The test is inconclusive at (a, b) if $f_{xx}f_{yy} - f_{xy}^2 = 0$ at (a, b) .

$$\text{Lagrange Multipliers:} \quad \nabla f = \lambda \nabla g \quad \text{and} \quad g(x, y, z) = 0$$

Volume:
$$V = \int_a^b A(x) dx = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx$$

$$V = \int_{x=a}^{x=b} \int_{y=g_1(x)}^{y=g_2(x)} \int_{z=f_1(x,y)}^{z=f_2(x,y)} F(x, y, z) dz dy dx$$

Average values of f over $R = \frac{1}{\text{area of } R} \iint_R f dA$

Average value of F over $D = \frac{1}{\text{volume of } D} \iiint_D F dV$

$$A = \iint_R r dr d\theta$$

Cartesian Integrals into Polar:
$$\iint_R f(x, y) dx dy = \iint_G f(r \cos \theta, r \sin \theta) r dr d\theta$$

$$r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x}$$

$$r = \rho \sin \phi, \quad x = r \cos \theta = \rho \sin \phi \cos \theta, \quad \rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$

$$z = \rho \cos \phi, \quad y = r \sin \theta = \rho \sin \phi \sin \theta,$$

$$dV = \rho^2 \sin \phi d\rho d\phi d\theta$$

$$\iiint_D f(\rho, \phi, \theta) dV = \int_{\theta=\alpha}^{\theta=\beta} \int_{\phi=\phi_{\min}}^{\phi=\phi_{\max}} \int_{\rho=g_1(\phi, \theta)}^{\rho=g_2(\phi, \theta)} f(\rho, \phi, \theta) \rho^2 \sin \phi d\rho d\phi d\theta$$

$$\iiint_R f(x, y) dx dy = \iiint_R H(u, v, w) |J(u, v, w)| du dv dw$$

Jacobian:
$$J(u, v, w) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \frac{\partial(x, y, z)}{\partial(u, v, w)}$$

$$W = \int_C \mathbf{F} \cdot \mathbf{T} ds = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \frac{d\mathbf{r}}{dt} dt$$

$$\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$$

$$\text{Flux of } \mathbf{F} \text{ across } C = \int_C \mathbf{F} \cdot \mathbf{n} \, ds$$

$$\mathbf{F} \cdot \mathbf{n} = M(x, y) \frac{dy}{ds} - N(x, y) \frac{dx}{ds}$$

$$(\text{Flux of } \mathbf{F} = M\mathbf{i} + N\mathbf{j} \text{ across } C) = \oint_C Mdy - Ndx$$

$$\text{Divergence} \quad \text{div} \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$$

$$\oint_C \mathbf{F} \cdot \mathbf{N} ds = \oint_C Mdy - Ndx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy$$

Outward flux *Divergence integral*

$$\oint_C \mathbf{F} \cdot \mathbf{T} ds = \oint_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

Counterclockwise circulation *Curl integral*

$$A = \iint_R |\mathbf{r}_u \times \mathbf{r}_v| \, dA = \int_c^d \int_a^b |\mathbf{r}_u \times \mathbf{r}_v| \, du dv$$

$$\text{Surface area} = \iint_R \frac{|\nabla F|}{|\nabla F \cdot \mathbf{p}|} \, dA \quad \mathbf{p} = \mathbf{i}, \mathbf{j}, \text{ or } \mathbf{k}$$

$$A = \iint_R \sqrt{f_x^2 + f_y^2 + 1} \, dx dy$$