

## Section 3.5 – Exponential and logarithmic Equations

### Exponential Equations

$$b^{\textcolor{red}{M}} = b^{\textcolor{blue}{N}} \leftrightarrow \textcolor{red}{M} = \textcolor{blue}{N} \text{ for any } b > 0, \neq 1$$

#### *Example*

Solve  $5^{3x-6} = 125$

#### *Solution*

$$\textcolor{red}{5}^{3x-6} = \textcolor{red}{5}^3$$

$$3\textcolor{blue}{x} - 6 = 3$$

$$3\textcolor{blue}{x} = 9$$

$$\underline{\textcolor{blue}{x} = 3}$$

#### *Example*

Solve  $8^{x+2} = 4^{x-3}$

#### *Solution*

$$\left(\textcolor{red}{2}^3\right)^{x+2} = \left(\textcolor{red}{2}^2\right)^{x-3}$$

$$2^{3(x+2)} = 2^{2(x-3)}$$

$$3(x+2) = 2(x-3)$$

$$3x + 6 = 2x - 6$$

$$3x - 2x = -6 - 6$$

$$\underline{\textcolor{blue}{x} = -12}$$

## Using *Natural Logarithms*

1. Isolate the exponential expression
2. Take the natural logarithm on both sides of the equation
3. Simplify using one of the following properties:  $\ln b^x = x \ln b$  or  $\ln e^x = x$
4. Solve for the variable

### ***Example***

Solve:  $7e^{2x} - 5 = 58$

#### **Solution**

$$7e^{2x} - 5 = 58$$

*Isolate the exponential expression*

$$7e^{2x} = 63$$

*Divide by 7 both sides*

$$e^{2x} = 9$$

*Natural logarithm on both sides*

$$\ln e^{2x} = \ln 9$$

*Use inverse Property*

$$2x = \ln 9$$

$$x = \frac{\ln 9}{2} \approx 1.0986$$

### ***Example***

Solve:  $3^{2x-1} = 7^{x+1}$

#### **Solution**

$$\ln 3^{2x-1} = \ln 7^{x+1}$$

*Natural logarithm on both sides*

$$(2x-1)\ln 3 = (x+1)\ln 7$$

*Power Rule*

$$2x\ln 3 - \ln 3 = x\ln 7 + \ln 7$$

$$2x\ln 3 - x\ln 7 = \ln 3 + \ln 7$$

$$x(2\ln 3 - \ln 7) = \ln 3 + \ln 7$$

$$x = \frac{\ln 3 + \ln 7}{2\ln 3 - \ln 7} \approx 12.1143$$

## Logarithmic Equations

1. Express the equation in the form  $\log_b M = c$
2. Use the definition of a logarithm to rewrite the equation in exponential form:

$$\log_b M = c \Rightarrow b^c = M$$

3. Solve for the variable
4. Check proposed solution in the original equation. Include only the set for  $M > 0$

### Example

Solve:  $\log(x) + \log(x-3) = 1$

#### Solution

$$\log(x(x-3)) = 1$$

*Product Rule*

$$x(x-3) = 10^1$$

*Convert to exponential form*

$$x^2 - 3x = 10$$

$$x^2 - 3x - 10 = 0$$

*Solve for x*

$$x = -2, 5$$

**Check:**  $x = -2 \Rightarrow \log(-2) + \log(-2-3) = 1$

$$x = 5 \Rightarrow \log(5) + \log(5-3) = 1$$

$\therefore$  **Solution:**  $x = 5$

### Example

Solve:  $\log_6(3x+2) + \log_6(x-1) = 1$

#### Solution

$$\log_6[(3x+2)(x-1)] = 1$$

*Product Rule*

$$(3x+2)(x-1) = 6^1$$

*Convert to exponential form*

$$3x^2 - x - 2 = 6$$

$$3x^2 - x - 8 = 0$$

*Solve for x*

$$x = \frac{1-\sqrt{97}}{6} < 0 \quad x = \frac{1+\sqrt{97}}{6} > 1$$

$\therefore$  **Solution:**  $x = \frac{1+\sqrt{97}}{6}$

## Property of Logarithmic Equality

For any  $M > 0, N > 0, b > 0, \neq 1$

$$\log_b M = \log_b N \Rightarrow M = N$$

### Example

Solve:  $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

#### Solution

$$\ln(x-3) = \ln\left(\frac{7x-23}{x+1}\right) \quad \text{Quotient Rule}$$

$$x-3 = \frac{7x-23}{x+1}$$

$$(x-3)(x+1) = 7x-23$$

$$x^2 - 2x - 3 = 7x - 23$$

$$x^2 - 9x + 20 = 0$$

$$x = 4, 5$$

**Check:**  $x = 4 \Rightarrow \ln(4-3) = \ln(7(4)-23) - \ln(4+1)$

$x = 5 \Rightarrow \ln(5-3) = \ln(7(5)-23) - \ln(5+1)$

$\therefore$  **Solution:**  $x = 4, 5$

### Example

Solve:  $\log(x+6) - \log(x+2) = \log x$

#### Solution

$$\log \frac{x+6}{x+2} = \log x \quad \text{Quotient Rule}$$

$$\frac{x+6}{x+2} = x \quad \text{Multiply by } x+2$$

$$x+6 = x(x+2)$$

$$x+6 = x^2 + 2x$$

$$x^2 + x - 6 = 0 \quad \text{Solve for } x$$

$$x = -3, 2$$

**Check:**  $x = -3 \rightarrow \log(-3+6) - \log(-3+2) = \log(-3)$

$x = 2 \rightarrow \log(2+6) - \log(2+2) = \log(2)$

**Or Domain**

$\therefore$  **Solution:**  $x = 2$

## **Exercises**      **Section 3.5 – Exponential and logarithmic Equations**

(1 – 105) Solve the equations

1.  $2^x = 128$

2.  $3^x = 243$

3.  $5^x = 70$

4.  $6^x = 50$

5.  $5^x = 134$

6.  $7^x = 12$

7.  $9^x = \frac{1}{\sqrt[3]{3}}$

8.  $49^x = \frac{1}{343}$

9.  $2^{5x+3} = \frac{1}{16}$

10.  $\left(\frac{2}{5}\right)^x = \frac{8}{125}$

11.  $2^{3x-7} = 32$

12.  $4^{2x-1} = 64$

13.  $3^{1-x} = \frac{1}{27}$

14.  $2^{-x^2} = 5$

15.  $2^{-x} = 8$

16.  $\left(\frac{1}{3}\right)^x = 81$

17.  $3^{-x} = 120$

18.  $27 = 3^{5x} 9^{x^2}$

19.  $4^{x+3} = 3^{-x}$

20.  $2^{x+4} = 8^{x-6}$

21.  $8^{x+2} = 4^{x-3}$

22.  $7^x = 12$

23.  $5^{x+4} = 4^{x+5}$

24.  $5^{x+2} = 4^{1-x}$

25.  $3^{2x-1} = 0.4^{x+2}$

26.  $4^{3x-5} = 16$

27.  $4^{x+3} = 3^{-x}$

28.  $7^{2x+1} = 3^{x+2}$

29.  $3^{x-1} = 7^{2x+5}$

30.  $4^{x-2} = 2^{3x+3}$

31.  $3^{5x-8} = 9^{x+2}$

32.  $3^{x+4} = 2^{1-3x}$

33.  $3^{2-3x} = 4^{2x+1}$

34.  $4^{x+3} = 3^{-x}$

35.  $7^{x+6} = 7^{3x-4}$

36.  $2^{-100x} = (0.5)^{x-4}$

37.  $4^x \left(\frac{1}{2}\right)^{3-2x} = 8 \cdot (2^x)^2$

38.  $5^x + 125(5^{-x}) = 30$

39.  $4^x - 3(4^{-x}) = 8$

40.  $5^{3x-6} = 125$

41.  $e^x = 15$

42.  $e^{x+1} = 20$

43.  $9e^x = 107$

44.  $e^{x \ln 3} = 27$

45.  $e^{x^2} = e^{7x-12}$

46.  $f(x) = xe^x + e^x$
47.  $f(x) = x^3(4e^{4x}) + 3x^2e^{4x}$
48.  $e^{2x} - 2e^x - 3 = 0$
49.  $e^{0.08t} = 2500$
50.  $e^{x^2} = 200$
51.  $e^{2x+1} \cdot e^{-4x} = 3e$
52.  $e^{2x} - 8e^x + 7 = 0$
53.  $e^{2x} + 2e^x - 15 = 0$
54.  $e^x + e^{-x} - 6 = 0$
55.  $e^{1-3x} \cdot e^{5x} = 2e$
56.  $6\ln(2x) = 30$
57.  $\log_5(x-7) = 2$
58.  $\log_4(5+x) = 3$
59.  $\log(4x-18) = 1$
60.  $\log_3 x = -2$
61.  $\log(x^2 + 19) = 2$
62.  $\ln(x^2 - 12) = \ln x$
63.  $\log(2x^2 + 3x) = \log(10x + 30)$
64.  $\log_5(2x+3) = \log_5 11 + \log_5 3$
65.  $\log_3 x - \log_9(x+42) = 0$
66.  $\log_5 x + \log_5(4x-1) = 1$
67.  $\log x - \log(x+3) = 1$
68.  $\log x + \log(x-9) = 1$
69.  $\log_2(x+1) + \log_2(x-1) = 3$
70.  $\log_8(x+1) - \log_8 x = 2$
71.  $\ln(x+8) + \ln(x-1) = 2\ln x$
72.  $\ln(4x+6) - \ln(x+5) = \ln x$
73.  $\ln(5+4x) - \ln(x+3) = \ln 3$
74.  $\ln \sqrt[4]{x} = \sqrt{\ln x}$
75.  $\sqrt{\ln x} = \ln \sqrt{x}$
76.  $\log x^2 = (\log x)^2$
77.  $\log x^3 = (\log x)^2$
78.  $\log(\log x) = 1$
79.  $\log(\log x) = 2$
80.  $\ln(\ln x) = 2$
81.  $\ln\left(e^{x^2}\right) = 64$
82.  $e^{\ln(x-1)} = 4$
83.  $10^{\log(2x+5)} = 9$
84.  $\log \sqrt{x^3 - 9} = 2$
85.  $\log \sqrt{x^3 - 17} = \frac{1}{2}$
86.  $\log_4 x = \log_4(8-x)$
87.  $\log_7(x-5) = \log_7(6x)$
88.  $\ln x^2 = \ln(12-x)$
89.  $\log_2(x+7) + \log_2 x = 3$
90.  $\ln x = 1 - \ln(x+2)$
91.  $\ln x = 1 + \ln(x+1)$
92.  $\log_6(2x-3) = \log_6 12 - \log_6 3$
93.  $\log(3x+2) + \log(x-1) = 1$
94.  $\log_5(x+2) + \log_5(x-2) = 1$

$$95. \log_2 x + \log_2 (x-4) = 2$$

$$98. \ln x = \frac{1}{2} \ln \left( 2x + \frac{5}{2} \right) + \frac{1}{2} \ln 2$$

$$96. \log_3 x + \log_3 (x+6) = 3$$

$$99. \ln(-4-x) + \ln 3 = \ln(2-x)$$

$$97. \log_3 (x+3) + \log_3 (x+5) = 1$$

$$100. \log_4 x + \log_4 (x-2) = \log_4 (15)$$

$$101. \ln(x-5) - \ln(x+4) = \ln(x-1) - \ln(x+2)$$

$$102. \ln(4-x) = \ln(x+8) + \ln(2x+13)$$

$$103. \log(x^2 + 4) - \log(x+2) = 2 + \log(x-2)$$

$$104. \log_3 (x-2) = \log_3 27 - \log_3 (x-4) - 5^{\log_5 1}$$

$$105. \log_2 (x+3) = \log_2 (x-3) + \log_3 9 + 4^{\log_4 3}$$

$$106. \text{Solve for } t \text{ using logarithms with base } a: 2a^{t/3} = 5$$

$$107. \text{Solve for } t \text{ using logarithms with base } a: K = H - Ca^t$$