Solution   *Section* 3.4 – Properties of Logarithms

Exercise

Express the following in terms of sums and differences of logarithms: $\log_3(ab)$

Solution

$$\boxed{\log_3(ab) = \log_3 a + \log_3 b}$$

Exercise

Express the following in terms of sums and differences of logarithms: $\log_7(7x)$

Solution

$$\begin{aligned}\log_7(7x) &= \log_7 7 + \log_7 x\\ &\boxed{= 1 + \log_7 x}\end{aligned}$$

Exercise

Express the following in terms of sums and differences of logarithms: $\log \frac{x}{1000}$

Solution

$$\begin{aligned}\log \frac{x}{1000} &= \log x - \log 10^3\\ &\boxed{= \log x - 3}\end{aligned}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_5 \left(\frac{125}{y}\right)$

Solution

$$\begin{aligned}\log_5 \left(\frac{125}{y}\right) &= \log_5 5^3 - \log_5 y\\ &\boxed{= 3 - \log_5 y}\end{aligned}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b x^7$

Solution

$$\boxed{\log_b x^7 = 7\log_b x}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln \sqrt[7]{x}$

Solution

$$\ln \sqrt[7]{x} = \ln x^{1/7}$$

$$\boxed{= \frac{1}{7}\ln x}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \frac{x^2 y}{z^4}$

Solution

$$\log_a \frac{x^2 y}{z^4} = \log_a x^2 y - \log_a z^4 \qquad \textit{Quotient Rule}$$

$$= \log_a x^2 + \log_a y - \log_a z^4 \qquad \textit{Product Rule}$$

$$\boxed{= 2\log_a x + \log_a y - 4\log_a z} \qquad \textit{Power Rule}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \frac{x^2 y}{b^3}$

Solution

$$\log_b\left(\frac{x^2 y}{b^3}\right) = \log_b x^2 y - \log_b b^3$$

$$= \log_b x^2 + \log_b y - \log_b b^3$$

$$= 2\log_b x + \log_b y - 3\log_b b$$

$$\boxed{= 2\log_b x + \log_b y - 3}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b\left(\frac{x^3y}{z^2}\right)$

Solution

$$\log_b\left(\frac{x^3y}{z^2}\right)=\log_b\left(x^3y\right)-\log_b z^2$$

$$=\log_b x^3+\log_b y-\log_b z^2$$

$$=3\log_b x+\log_b y-2\log_b z$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b\left(\frac{\sqrt[3]{x}\,y^4}{z^5}\right)$

Solution

$$\log_b\left(\frac{\sqrt[3]{x}\,y^4}{z^5}\right)=\log_b\left(\sqrt[3]{x}\,y^4\right)-\log_b\left(z^5\right)$$

$$=\log_b\left(x^{1/3}\right)+\log_b\left(y^4\right)-\log_b\left(z^5\right)$$

Exercise

Express the following in terms of sums and differences of logarithms $\log\left(\frac{100x^3\ \sqrt[3]{5-x}}{3(x+7)^2}\right)$

Solution

$$\log\left(\frac{100x^3\ \sqrt[3]{5-x}}{3(x+7)^2}\right)=\log\left(100x^3\ \sqrt[3]{5-x}\right)-\log\left(3(x+7)^2\right)$$

$$=\log 10^2+\log x^3+\log(5-x)^{1/3}-\left[\log 3+\log\left((x+7)^2\right)\right]$$

$$=2\log 10+3\log x+\tfrac{1}{3}\log(5-x)-\log 3-2\log(x+7)$$

$$=2+3\log x+\frac{1}{3}\log(5-x)-\log 3-2\log(x+7)$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \sqrt[4]{\dfrac{m^8 n^{12}}{a^3 b^5}}$

Solution

$$\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}} = \log_a \left(\frac{m^8 n^{12}}{a^3 b^5}\right)^{1/4} \qquad \textit{Power Rule}$$

$$= \frac{1}{4}\log_a \left(\frac{m^8 n^{12}}{a^3 b^5}\right) \qquad \textit{Quotient Rule}$$

$$= \frac{1}{4}\left[\log_a m^8 n^{12} - \log_a a^3 b^5\right] \qquad \textit{Product Rule}$$

$$= \frac{1}{4}\left[\log_a m^8 + \log_a n^{12} - \left(\log_a a^3 + \log_a b^5\right)\right] \qquad \textit{Power Rule}$$

$$= \frac{1}{4}\left[8\log_a m + 12\log_a n - 3 - 5\log_a b\right]$$

$$= 2\log_a m + 3\log_a n - \frac{3}{4} - \frac{5}{4}\log_a b$$

Exercise

Use the properties of logarithms to rewrite: $\log_p \sqrt[3]{\dfrac{m^5 n^4}{t^2}}$

Solution

$$\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}} = \log_p \left(\frac{m^5 n^4}{t^2}\right)^{1/3} \qquad \textit{Power Rule}$$

$$= \frac{1}{3}\log_p \left(\frac{m^5 n^4}{t^2}\right) \qquad \textit{Quotient Rule}$$

$$= \frac{1}{3}\left(\log_p m^5 n^4 - \log_p t^2\right) \qquad \textit{Product Rule}$$

$$= \frac{1}{3}\left(\log_p m^5 + \log_p n^4 - \log_p t^2\right) \qquad \textit{Power Rule}$$

$$= \frac{1}{3}\left(5\log_p m + 4\log_p n - 2\log_p t\right)$$

$$= \frac{5}{3}\log_p m + \frac{4}{3}\log_p n - \frac{2}{3}\log_p t$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \sqrt[n]{\dfrac{x^3y^5}{z^m}}$

Solution

$$\log_b \sqrt[n]{\frac{x^3y^5}{z^m}} = \log_b \left(\frac{x^3y^5}{z^m}\right)^{1/n}$$

$$= \frac{1}{n}\log_b \left(\frac{x^3y^5}{z^m}\right) \qquad \textit{Power Rule}$$

$$= \frac{1}{n}\left(\log_b x^3y^5 - \log_b z^m\right) \qquad \textit{Quotient Rule}$$

$$= \frac{1}{n}\left(\log_b x^3 + \log_b y^5 - \log_b z^m\right) \qquad \textit{Product Rule}$$

$$= \frac{1}{n}\left(3\log_b x + 5\log_b y - m\log_b z\right) \qquad \textit{Power Rule}$$

$$= \frac{3}{n}\log_b x + \frac{5}{n}\log_b y - \frac{m}{n}\log_b z$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \sqrt[3]{\dfrac{a^2\, b}{c^5}}$

Solution

$$\log_a \sqrt[3]{\frac{a^2\, b}{c^5}} = \log_a \left(\frac{a^2\, b}{c^5}\right)^{1/3} \qquad \textbf{\textit{Convert the radical to power}}$$

$$= \frac{1}{3}\log_a \left(\frac{a^2\, b}{c^5}\right) \qquad \textbf{\textit{Power Rule}}$$

$$= \frac{1}{3}\left[\log_a a^2\, b - \log_a c^5\right] \qquad \textbf{\textit{Quotient Rule}}$$

$$= \frac{1}{3}\left[\log_a a^2 + \log_a b - \log_a c^5\right] \qquad \textbf{\textit{Product Rule}}$$

$$= \frac{1}{3}\left[2\log_a a + \log_a b - 5\log_a c\right] \qquad \textbf{\textit{Power Rule}}$$

$$= \frac{2}{3}\log_a a + \frac{1}{3}\log_a b - \frac{5}{3}\log_a c$$

$$= \frac{2}{3} + \frac{1}{3}\log_a b - \frac{5}{3}\log_a c$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b\left(x^4\sqrt[3]{y}\right)$

Solution

$$\log_b\left(x^4\sqrt[3]{y}\right)=\log_b\left(x^4\right)+\log_b\left(\sqrt[3]{y}\right)$$

$$=\log_b\left(x^4\right)+\log_b\left(y^{1/3}\right)$$

$$=4\log_b x+\frac{1}{3}\log_b y$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_5\left(\frac{\sqrt{x}}{25y^3}\right)$

Solution

$$\log_5\left(\frac{\sqrt{x}}{25y^3}\right)=\log_5\left(x^{1/2}\right)-\log_5\left(25y^3\right)$$

$$=\log_5\left(x^{1/2}\right)-\left[\log_5\left(5^2\right)+\log_5\left(y^3\right)\right]$$

$$=\log_5\left(x^{1/2}\right)-\log_5\left(5^2\right)-\log_5\left(y^3\right)$$

$$=\frac{1}{2}\log_5(x)-2\log_5(5)-3\log_5(y)$$

$$=\frac{1}{2}\log_5(x)-2-3\log_5(y)$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a\frac{x^3w}{y^2z^4}$

Solution

$$\log_a\frac{x^3w}{y^2z^4}=\log_a x^3w-\log_a y^2z^4 \qquad \textit{Quotient rule}$$

$$=\log_a x^3+\log_a w-\left(\log_a y^2+\log_a z^4\right) \qquad \textit{Product rule}$$

$$=\log_a x^3+\log_a w-\log_a y^2-\log_a z^4 \qquad \textit{Distribute minus}$$

$$=3\log_a x+\log_a w-2\log_a y-4\log_a z \qquad \textit{Power rule}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \dfrac{\sqrt{y}}{x^4\sqrt[3]{z}}$

Solution

$$\log_a \frac{\sqrt{y}}{x^4\sqrt[3]{z}} = \log_a y^{1/2} - \log_a x^4 z^{1/3} \qquad \textit{Quotient rule}$$

$$= \log_a y^{1/2} - \left(\log_a x^4 + \log_a z^{1/3}\right) \qquad \textit{Product rule}$$

$$= \log_a y^{1/2} - \log_a x^4 - \log_a z^{1/3} \qquad \textit{Distribute minus}$$

$$= \frac{1}{2}\log_a y - 4\log_a x - \frac{1}{3}\log_a z \qquad \textit{Power rule}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln \sqrt[4]{\dfrac{x^7}{y^5 z}}$

Solution

$$\ln \sqrt[4]{\frac{x^7}{y^5 z}} = \ln\left(\frac{x^7}{y^5 z}\right)^{1/4}$$

$$= \frac{1}{4}\ln\left(\frac{x^7}{y^5 z}\right) \qquad \textit{Power rule}$$

$$= \frac{1}{4}\left(\ln x^7 - \ln y^5 z\right) \qquad \textit{Quotient rule}$$

$$= \frac{1}{4}\left(\ln x^7 - \left(\ln y^5 + \ln z\right)\right) \qquad \textit{Product rule}$$

$$= \frac{1}{4}\left(\ln x^7 - \ln y^5 - \ln z\right)$$

$$= \frac{1}{4}(7\ln x - 5\ln y - \ln z) \qquad \textit{Power rule}$$

$$= \frac{7}{4}\ln x - \frac{5}{4}\ln y - \ln z$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln x\sqrt[3]{\dfrac{y^4}{z^5}}$

Solution

$$\ln x\sqrt[3]{\frac{y^4}{z^5}} = \ln x + \ln\left(\frac{y^4}{z^5}\right)^{1/3} \qquad \textit{Product rule}$$

$$= \ln x + \ln\left(\frac{y^{4/3}}{z^{5/3}}\right)$$

$$= \ln x + \ln y^{4/3} - \ln z^{5/3}$$ ***Quotient rule***

$$= \ln x + \frac{4}{3}\ln y - \frac{5}{3}\ln z$$ ***Power rule***

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \sqrt[5]{\frac{m^4n^5}{x^2ab^{10}}}$

Solution

$$\log_b \sqrt[5]{\frac{m^4n^5}{x^2ab^{10}}} = \log_b \left(\frac{m^4n^5}{x^2ab^{10}}\right)^{1/5}$$

$$= \frac{1}{5}\log_b \left(\frac{m^4n^5}{x^2ab^{10}}\right)$$

$$= \frac{1}{5}\left(\log_b \left(m^4n^5\right) - \log_b \left(x^2ab^{10}\right)\right)$$

$$= \frac{1}{5}\left(\left(\log_b \left(m^4\right) + \log_b \left(n^5\right)\right) - \left(\log_b \left(x^2\right) + \log_b (a) + \log_b \left(b^{10}\right)\right)\right)$$

$$= \frac{1}{5}\left(4\log_b m + 5\log_b n - 2\log_b x - \log_b a - 10\right)$$

$$= \frac{4}{5}\log_b m + \log_b n - \frac{2}{5}\log_b x - \frac{1}{5}\log_b (a) - 2$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \frac{a^5b^{10}}{c^2\sqrt[4]{d^3}}$

Solution

$$\log_b \frac{a^5b^{10}}{c^2\sqrt[4]{d^3}} = \log_b \left(a^5b^{10}\right) - \log_b \left(c^2 d^{3/4}\right)$$

$$= \log_b \left(a^5\right) + \log_b \left(b^{10}\right) - \left(\log_b \left(c^2\right) + \log_b \left(d^{3/4}\right)\right)$$

$$= 5\log_b a + 10 - 2\log_b c - \frac{3}{4}\log_b d$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\left(x^2\sqrt{x^2+1}\right)$

Solution

$$\ln\left(x^2\sqrt{x^2+1}\right) = \ln x^2 + \ln\left(x^2+1\right)^{1/2}$$

$$\underline{= 2\ln x + \tfrac{1}{2}\ln\left(x^2+1\right)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\dfrac{x^2}{x^2+1}$

Solution

$$\ln\frac{x^2}{x^2+1} = \ln x^2 - \ln\left(x^2+1\right)$$

$$\underline{= 2\ln x - \ln\left(x^2+1\right)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\left(\dfrac{x^2(x+1)^3}{(x+3)^{1/2}}\right)$

Solution

$$\ln\left(\frac{x^2(x+1)^3}{(x+3)^{1/2}}\right) = \ln\left(x^2(x+1)^3\right) - \ln(x+3)^{1/2}$$

$$= \ln x^2 + \ln(x+1)^3 - \tfrac{1}{2}\ln(x+3)$$

$$\underline{= 2\ln x + 3\ln(x+1) - \tfrac{1}{2}\ln(x+3)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\sqrt{\dfrac{(x+1)^5}{(x+2)^{20}}}$

Solution

$$\ln\sqrt{\frac{(x+1)^5}{(x+2)^{20}}} = \ln\left(\frac{(x+1)^5}{(x+2)^{20}}\right)^{1/2}$$

$$= \frac{1}{2}\ln\left(\frac{(x+1)^5}{(x+2)^{20}}\right)$$

$$= \frac{1}{2}\left(\ln(x+1)^5 - \ln(x+2)^{20}\right)$$

$$= \frac{1}{2}\left(5\ln(x+1) - 20\ln(x+2)\right)$$

$$= \frac{5}{2}\ln(x+1) - 10\ln(x+2)$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\frac{\left(x^2+1\right)^5}{\sqrt{1-x}}$

Solution

$$\ln\frac{\left(x^2+1\right)^5}{\sqrt{1-x}} = \ln\left(x^2+1\right)^5 - \ln(1-x)^{1/2}$$

$$= 5\ln\left(x^2+1\right) - \frac{1}{2}\ln(1-x)$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\left(\sqrt[3]{\frac{x(x+1)(x-2)}{\left(x^2+1\right)(2x+3)}}\right)$

Solution

$$\ln\left(\sqrt[3]{\frac{x(x+1)(x-2)}{\left(x^2+1\right)(2x+3)}}\right) = \ln\left(\frac{x(x+1)(x-2)}{\left(x^2+1\right)(2x+3)}\right)^{1/3}$$

$$= \frac{1}{3}\ln\left(\frac{x(x+1)(x-2)}{\left(x^2+1\right)(2x+3)}\right)$$

$$= \frac{1}{3}\left(\ln\left(x(x+1)(x-2)\right) - \ln\left(\left(x^2+1\right)(2x+3)\right)\right)$$

$$= \frac{1}{3}\left(\ln x + \ln(x+1) + \ln(x-2) - \left(\ln\left(x^2+1\right) + \ln(2x+3)\right)\right)$$

$$=\frac{1}{3}\left(\ln x+\ln(x+1)+\ln(x-2)-\ln\left(x^2+1\right)-\ln(2x+3)\right)$$

$$\boxed{=\frac{1}{3}\ln x+\frac{1}{3}\ln(x+1)+\frac{1}{3}\ln(x-2)-\frac{1}{3}\ln\left(x^2+1\right)-\frac{1}{3}\ln(2x+3)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\left(\sqrt{\frac{1}{x(x+1)}}\right)$

Solution

$$\ln\left(\sqrt{\frac{1}{x(x+1)}}\right)=\ln\left(\frac{1}{x(x+1)}\right)^{1/2}$$

$$=\frac{1}{2}\left(\ln 1-\ln\left(x(x+1)\right)\right)$$

$$=-\frac{1}{2}\left(\ln x+\ln(x+1)\right)$$

$$\boxed{=-\frac{1}{2}\ln x-\frac{1}{2}\ln(x+1)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\left(\sqrt{\left(x^2+1\right)(x-1)^2}\right)$

Solution

$$\ln\left(\sqrt{\left(x^2+1\right)(x-1)^2}\right)=\ln\left(\left(x^2+1\right)(x-1)^2\right)^{1/2}$$

$$=\frac{1}{2}\ln\left(\left(x^2+1\right)(x-1)^2\right)$$

$$=\frac{1}{2}\left(\ln\left(x^2+1\right)+\ln(x-1)^2\right)$$

$$=\frac{1}{2}\left(\ln\left(x^2+1\right)+2\ln(x-1)\right)$$

$$\boxed{=\frac{1}{2}\ln\left(x^2+1\right)+\ln(x-1)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\log(x+5)+2\log x$

Solution

$$\log(x+5)+2\log x=\log(x+5)+\log x^2$$
$$=\log\left(x^2(x+5)\right)$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $3\log_b x-\frac{1}{3}\log_b y+4\log_b z$

Solution

$$3\log_b x-\tfrac{1}{3}\log_b y+4\log_b z=\log_b x^3+\log_b z^4-\log_b y^{1/3}$$
$$=\log_b\left(x^3z^4\right)-\log_b\sqrt[3]{y}$$
$$=\log_b\left(\frac{x^3z^4}{\sqrt[3]{y}}\right)$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\frac{1}{2}\log_b(x+5)-5\log_b y$

Solution

$$\tfrac{1}{2}\log_b(x+5)-5\log_b y=\log_b(x+5)^{1/2}-\log_b y^5$$
$$=\log_b\left(\frac{\sqrt{x+5}}{y^5}\right)$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\ln\left(x^2-y^2\right)-\ln(x-y)$

Solution

$$\ln\left(x^2-y^2\right)-\ln(x-y)=\ln\frac{x^2-y^2}{x-y}$$
$$=\ln\frac{(x-y)(x+y)}{x-y}$$
$$=\ln(x+y)$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\ln(xz) - \ln\left(x\sqrt{y}\right) + 2\ln\frac{y}{z}$

Solution

$$\begin{aligned} \ln(xz) - \ln\left(x\sqrt{y}\right) + 2\ln\frac{y}{z} &= \ln(xz) + \ln\left(\frac{y}{z}\right)^2 - \ln\left(x\sqrt{y}\right) \\ &= \ln\left(\frac{xzy^2}{z^2}\right) - \ln\left(x\sqrt{y}\right) \\ &= \ln\left(\frac{xy^2}{z}\frac{1}{x\sqrt{y}}\right) \\ &= \ln\left(\frac{y^{3/2}}{z}\right) \end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\log\left(x^2y\right) - \log z$

Solution

$$\log\left(x^2y\right) - \log z = \log\left(\frac{x^2y}{z}\right)$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\log\left(z^2\sqrt{y}\right) - \log z^{1/2}$

Solution

$$\begin{aligned} \log\left(z^2\sqrt{y}\right) - \log z^{1/2} &= \log\left(\frac{z^2\sqrt{y}}{z^{1/2}}\right) \\ &= \log\left(z^{3/2}\sqrt{y}\right) \\ &= \log\left(\sqrt{z^3y}\right) \end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$2\log_a x + \frac{1}{3}\log_a(x-2) - 5\log_a(2x+3)$$

Solution

$$\begin{aligned} 2\log_a x + \tfrac{1}{3}\log_a(x-2) - 5\log_a(2x+3) &= \log_a x^2 + \log_a(x-2)^{1/3} - \log_a(2x+3)^5 \\ &= \log_a x^2(x-2)^{1/3} - \log_a(2x+3)^5 \\ &= \log_a \frac{x^2(x-2)^{1/3}}{(2x+3)^5} \end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$5\log_a x - \frac{1}{2}\log_a(3x-4) - 3\log_a(5x+1)$$

Solution

$$\begin{aligned} 5\log_a x - \tfrac{1}{2}\log_a(3x-4) - 3\log_a(5x+1) &= \log_a x^5 - \log_a(3x-4)^{1/2} - \log_a(5x+1)^3 \\ &= \log_a x^5 - \left[\log_a(3x-4)^{1/2} + \log_a(5x+1)^3\right] \\ &= \log_a x^5 - \left[\log_a(3x-4)^{1/2}(5x+1)^3\right] \\ &= \log_a \frac{x^5}{(3x-4)^{1/2}(5x+1)^3} \end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\log\left(x^3y^2\right) - 2\log\left(x\sqrt[3]{y}\right) - 3\log\left(\frac{x}{y}\right)$$

Solution

$$\begin{aligned} \log\left(x^3y^2\right) - 2\log\left(x\sqrt[3]{y}\right) - 3\log\left(\frac{x}{y}\right) &= \log\left(x^3y^2\right) - \log\left(xy^{1/3}\right)^2 - \log\left(xy^{-1}\right)^3 \\ &= \log\left(x^3y^2\right) - \left[\log\left(x^2y^{2/3}\right) + \log\left(x^3y^{-3}\right)\right] \\ &= \log\left(x^3y^2\right) - \log\left(x^2y^{2/3}x^3y^{-3}\right) \end{aligned}$$

$$= \log\left(x^3y^2\right) - \log\left(x^5y^{-7/3}\right)$$

$$= \log\left(\frac{x^3y^2}{x^5y^{-7/3}}\right)$$

$$= \log\left(\frac{y^2y^{7/3}}{x^2}\right)$$

$$= \log\left(\frac{y^{13/3}}{x^2}\right)$$

$$= \log\left(\frac{\sqrt[3]{y^{13}}}{x^2}\right)$$

$$\underline{= \log\left(\frac{y^4\sqrt[3]{y}}{x^2}\right)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\ln y^3 + \frac{1}{3}\ln\left(x^3y^6\right) - 5\ln y$$

Solution

$$\ln y^3 + \frac{1}{3}\ln\left(x^3y^6\right) - 5\ln y = \ln y^3 + \ln\left(x^3y^6\right)^{1/3} - \ln y^5$$

$$= \ln y^3 + \ln\left(x^{3/3}y^{6/3}\right) - \ln y^5$$

$$= \ln y^3 + \ln\left(xy^2\right) - \ln y^5$$

$$= \ln\left(y^3xy^2\right) - \ln y^5$$

$$= \ln\left(\frac{y^5x}{y^5}\right)$$

$$\underline{= \ln x}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$2\ln x - 4\ln\left(\frac{1}{y}\right) - 3\ln(xy)$

Solution

$$\begin{aligned}
2\ln x - 4\ln\left(\frac{1}{y}\right) - 3\ln(xy) &= \ln x^2 - \ln\left(\frac{1}{y}\right)^4 - \ln(xy)^3 \\
&= \ln x^2 - \left[\ln\left(y^{-4}\right) + \ln\left(x^3y^3\right)\right] \\
&= \ln x^2 - \ln\left(y^{-4}x^3y^3\right) \\
&= \ln x^2 - \ln\left(y^{-1}x^3\right) \\
&= \ln\frac{x^2}{y^{-1}x^3} \\
&= \boxed{\ln\frac{y}{x}}
\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$4\ln x + 7\ln y - 3\ln z$

Solution

$$\begin{aligned}
4\ln x + 7\ln y - 3\ln z &= \ln x^4 + \ln y^7 - \ln z^3 \\
&= \ln\left(x^4y^7\right) - \ln z^3 \\
&= \boxed{\ln\left(\frac{x^4y^7}{z^3}\right)}
\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{1}{3}\left[5\ln(x+6) - \ln x - \ln(x^2-25)\right]$

Solution

$$\frac{1}{3}\left[5\ln(x+6) - \ln x - \ln(x^2-25)\right] = \frac{1}{3}\left[5\ln(x+6) - \left(\ln x + \ln(x^2-25)\right)\right]$$

$$= \frac{1}{3}\left[\ln(x+6)^5 - \ln x(x^2-25)\right]$$

$$= \frac{1}{3}\left[\ln \frac{(x+6)^5}{x(x^2-25)}\right]$$

$$\underline{= \ln\left(\frac{(x+6)^5}{x(x^2-25)}\right)^{1/3}}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\frac{2}{3}\left[\ln\left(x^2-4\right) - \ln\left(x+2\right)\right] + \ln(x+y)$$

Solution

$$\begin{aligned}
\frac{2}{3}\left[\ln\left(x^2-4\right) - \ln\left(x+2\right)\right] + \ln(x+y) &= \frac{2}{3}\left[\ln \frac{x^2-4}{x+2}\right] + \ln(x+y) \\
&= \frac{2}{3}\left[\ln \frac{(x+2)(x-2)}{x+2}\right] + \ln(x+y) \\
&= \frac{2}{3}\ln(x-2) + \ln(x+y) \\
&= \ln(x-2)^{2/3} + \ln(x+y) \\
&= \ln(x-2)^{2/3}(x+y) \\
&\underline{= \ln(x+y)\sqrt[3]{(x-2)^2}}
\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\frac{1}{2}\log_b m + \frac{3}{2}\log_b 2n - \log_b m^2 n$$

Solution

$$\begin{aligned}
\frac{1}{2}\log_b m + \frac{3}{2}\log_b 2n - \log_b m^2 n &= \log_b m^{1/2} + \log_b (2n)^{3/2} - \log_b m^2 n \\
&= \log_b \left(m^{1/2}(2n)^{3/2}\right) - \log_b m^2 n \\
&= \log_b \frac{m^{1/2} 2^{3/2} n^{3/2}}{m^2 n}
\end{aligned}$$

$$= \log_b \frac{2^{3/2} n^{1/2}}{m^{3/2}}$$

$$= \log_b \left(\frac{2^3 n}{m^3}\right)^{1/2}$$

$$= \log_b \sqrt{\frac{8n}{m^3}}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{1}{2}\log_y p^3q^4 - \frac{2}{3}\log_y p^4q^3$

Solution

$$\frac{1}{2}\log_y p^3q^4 - \frac{2}{3}\log_y p^4q^3 = \log_y \left(p^3q^4\right)^{1/2} - \log_y \left(p^4q^3\right)^{2/3}$$

$$= \log_y \frac{\left(p^3q^4\right)^{1/2}}{\left(p^4q^3\right)^{2/3}}$$

$$= \log_y \frac{\left(p^3\right)^{1/2}\left(q^4\right)^{1/2}}{\left(p^4\right)^{2/3}\left(q^3\right)^{2/3}}$$

$$= \log_y \frac{p^{3/2}q^2}{p^{8/3}q^2}$$

$$= \log_y \frac{p^{3/2}}{p^{8/3}}$$

$$= \log_y \frac{1}{p^{8/3-3/2}}$$

$$= \log_y \frac{1}{p^{7/6}}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x$

Solution

$$\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x = 4\log_a y - \frac{5}{2}\log_a x$$

$$= \log_a y^4 - \log_a x^{5/2}$$

$$\boxed{= \log_a \frac{y^4}{\sqrt{x^5}}}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\frac{2}{3}\left[\ln\left(x^2-9\right)-\ln(x+3)\right]+\ln(x+y)$$

Solution

$$\begin{aligned}
\frac{2}{3}\left[\ln\left(x^2-9\right)-\ln(x+3)\right]+\ln(x+y) &= \frac{2}{3}\ln\frac{x^2-9}{x+3}+\ln(x+y) \\
&= \frac{2}{3}\ln\frac{(x+3)(x-3)}{x+3}+\ln(x+y) \\
&= \frac{2}{3}\ln(x-3)+\ln(x+y) \\
&= \ln(x-3)^{2/3}+\ln(x+y) \\
&= \ln\left((x-3)^{2/3}(x+y)\right) \\
&\boxed{= \ln\left((x+y)\sqrt[3]{(x-3)^2}\right)}
\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\frac{1}{4}\log_b x - 2\log_b 5 - 10\log_b y$$

Solution

$$\begin{aligned}
\frac{1}{4}\log_b x - 2\log_b 5 - 10\log_b y &= \log_b x^{1/4} - \log_b 5^2 - \log_b y^{10} \\
&= \log_b x^{1/4} - \left[\log_b 5^2 + \log_b y^{10}\right] \\
&= \log_b x^{1/4} - \log_b \left(5^2 y^{10}\right) \\
&\boxed{= \log_b \frac{\sqrt[4]{x}}{25y^{10}}}
\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$2\ln(x+4)-\ln x-\ln\left(x^2-3\right)$$

Solution

$$\begin{aligned}2\ln(x+4)-\ln x-\ln\left(x^2-3\right)&=\ln(x+4)^2-\left(\ln x+\ln\left(x^2-3\right)\right)\\&=\ln(x+4)^2-\ln\left(x\left(x^2-3\right)\right)\\&=\ln\frac{(x+4)^2}{x\left(x^2-3\right)}\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\ln x+\ln(y+3)+\ln(y+2)-\ln\left(y^2+5y+6\right)$$

Solution

$$\begin{aligned}\ln x+\ln(y+3)+\ln(y+2)-\ln\left(y^2+5y+6\right)&=\ln\left(x(y+3)(y+2)\right)-\ln\left((y+3)(y+2)\right)\\&=\ln\left(\frac{x(y+3)(y+2)}{(y+3)(y+2)}\right)\\&=\ln x\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\ln x+\ln(x+4)+\ln(x+1)-\ln\left(x^2+5x+4\right)$$

Solution

$$\begin{aligned}\ln x+\ln(x+4)+\ln(x+1)-\ln\left(x^2+5x+4\right)&=\ln\left(x(x+4)(x+1)\right)-\ln\left((x+4)(x+1)\right)\\&=\ln\left(\frac{x(x+4)(x+1)}{(x+4)(x+1)}\right)\\&=\ln x\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\ln\left(x^2-25\right)-2\ln(x+5)+\ln(x-5)$

Solution

$$\ln\left(x^2-25\right)-2\ln(x+5)+\ln(x-5)=\ln\left(x^2-25\right)+\ln(x-5)-\ln(x+5)^2$$

$$=\ln\frac{(x-5)(x+5)(x-5)}{(x+5)^2}$$

$$\boxed{=\ln\left(\frac{(x-5)^2}{x+5}\right)}$$

Exercise

Assume that $\log_{10}2=.3010$. Find each logarithm $\log_{10}4$, $\log_{10}5$

Solution

a) $\log_{10}4=\log_{10}2^2$

$=2\log_{10}2$

$=2(.301)$

$\boxed{=.6020}$

b) $\log_{10}5=\log_{10}\frac{10}{2}$

$=\log_{10}10-\log_{10}2$

$=1-.03010$

$\boxed{=.6990}$

Exercise

Given that: $\log_a 2\approx 0.301, \log_a 7\approx 0.845$, and $\log_a 11\approx 1.041$ find each of the following:

a) $\log_a\frac{2}{11}$ *c)* $\log_a 98$ *e)* $\log_a 9$

b) $\log_a 14$ *d)* $\log_a\frac{1}{7}$ *f)* $\log_a\frac{77}{8}$

Solution

a) $\log_a\frac{2}{11}=\log_a 2-\log_a 11$

$=0.301-1.041$

≈ 1.342

b) $\log_a 14 = \log_a 2(7)$

$= \log_a 2 + \log_a 7$

$= 0.301 + 0.845$

≈ 1.146

c) $\log_a 98 = \log_a 2\left(7^2\right)$

$= \log_a 2 + \log_a 7^2$

$= \log_a 2 + 2\log_a 7$

$= 0.301 + 2(0.845)$

≈ 1.991

d) $\log_a \frac{1}{7} = \log_a 1 - \log_a 7$

$\approx 0 - 0.845$

≈ -0.845

e) $\log_a 9$ Can't be found from the given information

f) $\log_a \frac{77}{8} = \log_a 77 - \log_a 8$

$= \log_a (7 \times 11) - \log_a 2^3$

$= \log_a 7 + \log_a 11 - 3\log_a 2$

$\approx 0.845 + 1.041 - 3(0.301)$

$\approx 1.886 - 0.903$

≈ 0.983