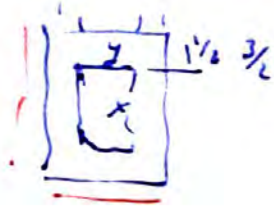
22 Given $A = 24 \text{ in}^2$

$A_p = xy = 24$ ① Find: paper?

$A = (y+2)(x+3)$ ②



① $\to y = \frac{24}{x}$ ③

② $A = \left(\frac{24}{x} + 2\right)(x+3)$

$= 24 + \frac{72}{x} + 2x + 6$

$A(x) = 2x + \frac{72}{x} + 30$

$\frac{dA}{dx} = 2 - \frac{72}{x^2} = 0$

$\frac{72}{x^2} = 2 \Rightarrow x^2 = 36$

CN: $x = 6$

$x = 6 \to$ ③ $\to y = 4$

Dimension: $x + 3 = 9$

$y + 2 = 6$

9×6 in

$x^2 + y^2 = 9$

$x^2 = 9 - y^2$

$V = \frac{1}{3}\pi r^2 h$

$r = \sqrt{9 - y^2}$

$h = y + 3$

$V = \frac{\pi}{3}(9 - y^2)(y + 3)$

$= \frac{\pi}{3}(9y + 27 - y^3 - 3y^2)$

$V' = \frac{\pi}{3}(9 - 3y^2 - 6y) = 0$

$y^2 + 2y - 3 = 0$

$y = 1, \cancel{-3}$

$V(1) = \frac{\pi}{3}(9 + 27 - 1 - 3)$

$= \frac{32\pi}{3}$ unit^3

3.4 L'Hôpital's Rule

$\frac{0}{0}$ $\qquad \frac{0}{\neq 0} = 0, \quad \frac{\neq 0}{0} = \infty$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Ex $\lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{\cos x}{1} \quad \begin{matrix} \to (\sin x)' \\ \leftarrow (x)' \end{matrix}$$

$$= 1$$

Ex $\lim_{x \to 0} \frac{3x - \sin x}{x} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{3 - \cos x}{1}$$

$$= 3 - 1$$

$$= 2$$

Ex $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{\frac{1}{2\sqrt{1+x}}}{1} \to \left.\frac{1}{2\sqrt{1+x}}\right|_{x=0}$$

$$= \frac{1}{2}$$

Ex $\lim_{x \to 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2} = \frac{1-1}{0} = \frac{0}{0}$ $\qquad (1+x)^{-1/2}$

$$= \lim_{x \to 0} \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2}}{2x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\frac{1}{2}\left(-\frac{1}{2}\right)(1+x)^{-3/2}}{2} \qquad \frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\right)(1+x)^{-3/2}\right)$$

$$= \frac{-\frac{1}{4}}{2}$$

$$= -\frac{1}{8}$$

$$\lim_{x \to 0} \frac{x - \sin x}{x^3} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{3x^2} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\sin x}{6x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\cos x}{6}$$

$$= \frac{1}{6}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x + x^2} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\sin x}{1 + 2x}$$

$$= \frac{0}{1}$$

$$= 0$$

$$\lim_{x \to 0} \frac{\sin x}{x^2} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\cos x}{2x}$$

$$= \frac{1}{0}$$

$$= \infty$$

$x \to a$ $\quad \frac{\infty}{\infty}$ $\quad \infty - \infty$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sec x}{1 + \tan x} = \frac{\infty}{\infty}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sec x \tan x}{\sec^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\tan x}{\sec x} \qquad \frac{1}{\sec x} \quad \cos x$$

$$= \lim_{x \to \frac{\pi}{2}} \left(\frac{\sin x}{\cos x}\right)\left(\cos x\right)$$

$$= \lim_{x \to \frac{\pi}{2}} \sin x$$

$$= 1$$

$$\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}} = \frac{\infty}{\infty}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}}{x}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{x}}$$

$$= 0$$

$$\lim_{x \to \infty} \frac{e^x}{x^2} = \frac{\infty}{\infty}$$

$$= \lim_{x \to \infty} \frac{e^x}{2x} = \frac{\infty}{\infty}$$

$$= \lim_{x \to \infty} \frac{e^x}{2}$$

$$= \infty$$

$0 \cdot \infty$?

$$\lim_{x\to\infty}\left(x \sin\frac{1}{x}\right) = \infty \cdot 0$$

$$= \lim_{x\to\infty} \frac{\sin(1/x)}{\frac{1}{x}} \qquad \frac{1}{x} \to 0$$

$$h = \frac{1}{x}$$

$$= \lim_{h\to 0} \frac{\sin h}{h}$$

$$= 1$$

$$= \lim_{\frac{1}{x}\to 0} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1$$

$$\lim_{x\to 0^+} \sqrt{x}\,\ln x = 0 \cdot (-\infty)$$

$$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}} = \frac{\infty}{\infty}$$

$$\infty \quad (x)^{-1/2}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{1}{2x^{3/2}}}$$

$$= \lim_{x\to 0^+} \frac{2x^{3/2}}{x}$$

$$= 2 \lim_{x\to 0^+} x^{1/2}$$

$$= 0$$

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = \infty - \infty$$

$$= \lim_{x \to 0} \left(\frac{x - \sin x}{x \sin x} \right) = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{\sin x + x \cos x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\sin x}{\cos x + \cos x - x \sin x}$$

$$= \frac{0}{2}$$

$$= 0$$

$$\lim_{x \to a} \ln f(x) = L$$

$$\lim_{x \to a} f(x) = e^{L}$$

$$\lim_{x \to 0^+} (1+x)^{1/x} = e \ ?$$

$$\lim_{x \to 0^+} (1+x)^{1/x} = 1^{\infty}$$

$$\lim_{x \to 0^+} \ln (1+x)^{1/x} = \lim_{x \to 0^+} \frac{1}{x} \ln (1+x)$$

$$= \lim_{x \to 0^+} \frac{\ln (x+1)}{x} = \frac{0}{0}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x+1}}{1}$$

$$= \lim_{x \to 0^+} \frac{1}{x+1}$$

$$= 1$$

$$\lim_{x \to 0^+} (1+x)^{1/x} = e^{1} = e$$

$$\lim_{x\to\infty} x^{1/x} = \infty^0$$

$$\lim_{x\to\infty} \ln x^{1/x} = \lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1}$$

$$= \frac{1}{\infty}$$

$$= 0$$

$$\lim_{x\to\infty} x^{1/x} = e^0$$

$$= 1$$

\# 115 $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^x = 1^\infty$

$$\lim_{x\to\infty} \ln\left(1+\frac{a}{x}\right)^x = \lim_{x\to\infty} x \ln\left(1+\frac{a}{x}\right)$$

$$= \lim_{x\to\infty} \frac{\ln\left(\frac{x+a}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$$

$$= \lim_{x\to\infty} \frac{\frac{\frac{-a}{x^2}}{\frac{x+a}{x}}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \left(\frac{-a}{x^2}\right)\left(\frac{x}{x+a}\right)(-x^2)$$

$$= \lim_{x\to\infty} \frac{ax}{x+a}$$

$$= a$$

$$\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^x = e^a$$

#116. $\lim\limits_{x \to 0} (e^{5x} + x)^{1/x} = 1^{\infty}$

$$\lim_{x \to 0} \ln (e^{5x} + x)^{1/x} = \lim_{x \to 0} \frac{\ln(e^{5x}+x)}{x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\frac{5e^{5x}+1}{e^{5x}+x}}{1}$$

$$= \lim_{x \to 0} \frac{5e^{5x}+1}{e^{5x}+x}$$

$$= \frac{5+1}{1}$$

$$= 6$$

$$\lim_{x \to 0} (e^{5x} + x)^{1/x} = \boxed{e^6}$$

#112 $\lim\limits_{\theta \to \frac{\pi}{2}^-} \ln(\tan\theta)^{\cos\theta} = \ln(\infty)^{\infty}$

$$= \lim_{\theta \to \frac{\pi}{2}^-} \cos\theta \ln(\tan\theta)$$

$$= \lim_{\theta \to \frac{\pi}{2}^-} \frac{\ln(\tan\theta)}{\sec\theta} = \frac{\infty}{\infty}$$

$$= \lim_{\theta \to \frac{\pi}{2}^-} \frac{\frac{\sec^2\theta}{\tan\theta}}{\sec\theta\tan\theta}$$

$$= \lim_{\theta \to \frac{\pi}{2}^-} \frac{\sec^2\theta}{\tan\theta} \frac{1}{\sec\theta\tan\theta}$$

$$= \lim_{\theta \to \frac{\pi}{2}^-} \frac{\sec\theta}{\tan^2\theta} = \frac{\infty}{\infty}$$

$$= \lim_{\theta \to \frac{\pi}{2}^-} \frac{1}{\cos\theta} \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \lim_{\theta \to \frac{\pi}{2}^-} \frac{\cos\theta}{\sin^2\theta}$$

$$= \frac{0}{1}$$

$$= \boxed{0}$$

5. $\lim\limits_{x\to\infty} \dfrac{e^{1/x}-1}{\sin\frac{1}{x}} = \dfrac{1-1}{0} = \dfrac{0}{0}$

$$= \lim_{x\to\infty} \frac{-\frac{1}{x^2}e^{1/x}}{-\frac{1}{x^2}\cos\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{e^{1/x}}{\cos\frac{1}{x}}$$

$$= \frac{1}{1}$$

$$= 1$$

$f(x) = \sqrt{x}$

$f'(x) = \dfrac{1}{2\sqrt{x}}\Big|_{x=1} = \dfrac{1}{2}$

$y = y_0 + \frac{1}{2}(x - x_0)$

$L = 1 + \frac{1}{2}(x-1) = \frac{1}{2}x + \frac{1}{2}$

$L(3) = \frac{3}{2} + \frac{1}{2}$

$= 2$ $\qquad\qquad \sqrt{3} \approx 1.7$