

Section 1.3 – Quadratic Functions

Basic Complex Number

$$i^2 = -1 \quad \Rightarrow \quad i = \sqrt{-1} \quad \Rightarrow \quad \sqrt{-1} = i$$

The number i is called the **imaginary unit**.

Example

$$\sqrt{-8} = 2i \sqrt{2}$$

$$\begin{aligned}\sqrt{-7}\sqrt{-7} &= i\sqrt{7} \ i\sqrt{7} \\ &= i^2 (\sqrt{7})^2 \\ &= -7\end{aligned}$$

Complex number is written in a form: $z = a + ib$

a is the real part

b is the imaginary part

Conjugate of a complex number $a + bi$ is $a - bi$

A **quadratic equation** in x is an equation that can be written in the general form:

$$ax^2 + bx + c = 0 \quad \text{where } a, b, \text{ and } c \text{ are real numbers,}$$
$$4x^2 - 3x + 2 = 0 \quad a = 4 \quad b = -3 \quad c = 2$$

Solving Quadratic Equations by *Factoring*

The Zero-Product Principle

If $AB = 0$ then $A = 0$ or $B = 0$.

Example

Solve $6x^2 + 7x - 3 = 0$

Solution

$$\begin{array}{ll}(3x - 1)(2x + 3) = 0 \\ \begin{array}{l} 3x - 1 = 0 \\ \underline{x = \frac{1}{3}} \end{array} & \begin{array}{l} 2x + 3 = 0 \\ \underline{x = -\frac{3}{2}} \end{array}\end{array}$$

The Square Root Property

If u is an algebraic expression and d is a nonzero real number, then $u^2 = d$ has exactly two solutions:

$$\text{If } u^2 = d, \text{ then } u = \sqrt{d} \text{ or } u = -\sqrt{d}$$

Equivalently,

$$\text{If } u^2 = d \Rightarrow u = \pm\sqrt{d}.$$

Example

Solve $3x^2 - 21 = 0$

Solution

$$3x^2 = 21$$

$$x^2 = 7$$

$$x = \pm\sqrt{7}$$

Example

Solve $5x^2 + 45 = 0$

Solution

$$5x^2 = -45$$

$$x^2 = -9$$

$$x = \pm\sqrt{-9}$$

$$x = \pm 3i$$

Example

Solve $(x + 5)^2 = 11$

Solution

$$x + 5 = \pm\sqrt{11}$$

$$x = -5 \pm \sqrt{11}$$

Completing the Square

If $x^2 + bx$ is a binomial, then by **adding** $\left(\frac{b}{2}\right)^2$ which is the square of half the coefficient of x , a perfect square trinomial will result. That is,

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 \qquad x^2 + bx + \left(\frac{1}{2}b\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example

Solve: $x^2 + 4x - 1 = 0$

Solution

$$x^2 + 4x = 1$$

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 1 + \left(\frac{4}{2}\right)^2$$

$$x^2 + 4x + (2)^2 = 1 + 4$$

$$(x + 2)^2 = 5$$

$$x + 2 = \pm\sqrt{5}$$

$$\underline{x = -2 \pm \sqrt{5}}$$

Quadratic Formula

(Using Completing the Square)

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{1}{2}\frac{b}{a}\right)^2 = -\frac{c}{a} + \left(\frac{1}{2}\frac{b}{a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$*** \text{ } ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{cases} b^2 - 4ac > 0 \rightarrow 2 \text{ Real numbers} \\ b^2 - 4ac < 0 \rightarrow 2 \text{ Complex numbers} \\ b^2 - 4ac = 0 \rightarrow \text{One solution (repeated)} \end{cases}$$

Example

Solve: $2x^2 + 2x - 1 = 0$

Solution

$$\Rightarrow a = 2 \quad b = 2 \quad c = -1$$

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{-2 \pm \sqrt{4+8}}{4} \\ &= -\frac{2}{4} \pm \frac{\sqrt{12}}{4} \\ &= -\frac{1}{2} \pm \frac{2\sqrt{3}}{4} \\ &= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

or

$$\begin{aligned} &= \frac{-2 \pm \sqrt{12}}{4} \\ &= \frac{-2 \pm 2\sqrt{3}}{4} \\ &= \frac{2(-1 \pm \sqrt{3})}{4} \\ &= \frac{-1 \pm \sqrt{3}}{2} \end{aligned}$$

Example

Solve $x^2 - 4x = -2$

Solution

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} \\ &= \frac{4 \pm \sqrt{16-8}}{2} \\ &= \frac{4 \pm \sqrt{8}}{2} \\ &= \frac{4 \pm 2\sqrt{2}}{2} \\ &= \frac{2(2 \pm \sqrt{2})}{2} \\ &= 2 \pm \sqrt{2} \end{aligned}$$

$$\Rightarrow a = 1 \quad b = -4 \quad c = 2 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Solve: $x^2 - 2x + 2 = 0$

Solution

$$\Rightarrow a = 1 \quad b = -2 \quad c = 2$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2}{2} \pm \frac{\sqrt{-4}}{2}$$

$$= 1 \pm \frac{2i}{2}$$

$$= \underline{1 \pm i}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= \frac{2(1 \pm i)}{2}$$

$$= \underline{1 \pm i}$$

$$ax^2 + bx + c = 0$$

$$\text{If } a + b + c = 0 \Rightarrow x = 1, \frac{c}{a}$$

Example

$$2x^2 + x - 3 = 0$$

$$2 + 1 - 3 = 0$$

$$\Rightarrow \underline{x = 1, -\frac{3}{2}}$$

$$\text{If } a - b + c = 0 \Rightarrow x = -1, -\frac{c}{a}$$

Example

$$2x^2 - x - 3 = 0$$

$$2 - (-1) - 3 = 0$$

$$\Rightarrow \underline{x = -1, \frac{3}{2}}$$

Exercises Section 1.3 – Quadratic Functions

(1 – 48) Solve

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|-----------------------|---------------------------|-------------------------------------|
| 1. $x^2 = -25$ | 17. $x^2 + 8x + 15 = 0$ | 34. $x^2 + 2x + 29 = 0$ |
| 2. $x^2 = 49$ | 18. $x^2 + 5x + 2 = 0$ | 35. $4x^2 + 4x + 13 = 0$ |
| 3. $9x^2 = 100$ | 19. $x^2 + x - 12 = 0$ | 36. $x^2 - 2x + 26 = 0$ |
| 4. $4x^2 + 25 = 0$ | 20. $x^2 - 2x - 15 = 0$ | 37. $9x^2 - 4x + 20 = 0$ |
| 5. $5x^2 + 35 = 0$ | 21. $x^2 - 4x - 45 = 0$ | 38. $x^2 + 6x + 21 = 0$ |
| 6. $5x^2 - 45 = 0$ | 22. $x^2 - 6x - 10 = 0$ | 39. $9x^2 - 12x - 49 = 0$ |
| 7. $(x - 4)^2 = 12$ | 23. $2x^2 + 3x - 4 = 0$ | 40. $x(x - 3) = 18$ |
| 8. $(x + 3)^2 = -16$ | 24. $x^2 - x + 8 = 0$ | 41. $x(x - 4) - 21 = 0$ |
| 9. $(x - 2)^2 = -20$ | 25. $2x^2 - 13x = 1$ | 42. $(x - 1)(x + 4) = 14$ |
| 10. $(4x + 1)^2 = 20$ | 26. $r^2 + 3r - 3 = 0$ | 43. $(x - 3)(x + 8) = -30$ |
| 11. $x^2 - 6x = -7$ | 27. $x^3 + 8 = 0$ | 44. $x(x + 8) = 16(x - 1)$ |
| 12. $-6x^2 = 3x + 2$ | 28. $4x^2 - 12x + 9 = 0$ | 45. $x(x + 9) = 4(2x + 5)$ |
| 13. $3x^2 + 2x = 7$ | 29. $9x^2 - 30x + 25 = 0$ | 46. $(x + 1)^2 = 2(x + 3)$ |
| 14. $3x^2 + 6 = 10x$ | 30. $x^2 - 14x + 49 = 0$ | 47. $(x + 1)^2 - 5(x + 2) = 3x + 7$ |
| 15. $5x^2 + 2 = x$ | 31. $x^2 - 8x + 16 = 0$ | 48. $x(8x + 1) = 3x^2 - 2x + 2$ |
| 16. $5x^2 = 2x - 3$ | 32. $x^2 + 6x + 13 = 0$ | |
| | 33. $2x^2 - 2x + 13 = 0$ | |

(49 – 60) Solve using formula

- | | | |
|-------------------------|-------------------------|------------------------|
| 49. $x^2 + 6x - 7 = 0$ | 53. $3x^2 - x - 2 = 0$ | 57. $x^2 - 3x - 4 = 0$ |
| 50. $x^2 - 6x - 7 = 0$ | 54. $3x^2 + x - 2 = 0$ | 58. $x^2 + 3x - 4 = 0$ |
| 51. $3x^2 + 4x - 7 = 0$ | 55. $2x^2 + 3x - 5 = 0$ | 59. $x^2 + 2x + 1 = 0$ |
| 52. $3x^2 - 4x - 7 = 0$ | 56. $2x^2 - 3x - 5 = 0$ | 60. $4x^2 - x - 5 = 0$ |

61. Solve for the specified variable $A = \frac{\pi d^2}{4}$, for d

62. Solve for the specified variable $rt^2 - st - k = 0$ ($r \neq 0$), for t