Lecture 3 Review

1/ b) $G = \{(-7,3), (-2,1), (-2,4), (0,7)\}$

inverse relation:

$G^{-1} = \{(3,-7), (1,-2), (4,-2), (7,0)\}$

2/ $f(x) = \frac{2x+3}{4x-5}$ (1-1)

a) $y = \frac{2x+3}{4x-5}$

$x = \frac{2y+3}{4y-5}$

$x(4y-5) = 2y+3$ *

$4xy - 5x = 2y + 3$

$4xy - 2y = 5x + 3$

$(4x-2)y = 5x+3$

$y = \frac{5x+3}{4x-2} = f^{-1}(x)$

b) Domain of $f(x)$ = Range $f^{-1}(x)$: $\mathbb{R} - \{\frac{5}{4}\}$

" $f^{-1}(x)$ = " $f(x)$: $\mathbb{R} - \{\frac{1}{2}\}$

#3/ c) $e^{-1} = .368 \iff -1 = \ln(.368)$

d) $15^{0.457} = 3 \iff .457 = \log_{15} 3$

e) $e^x = 2 \iff x = \ln 2$ $\log_e$

#4/ c) $y = \ln 2^\pi \iff 2^\pi = e^y$

d) $\ln x = 6.2 \iff x = e^{6.2}$

e) $\log y = x \iff y = 10^x$

f) $\log_3 x = \frac{1}{3} \iff x = 3^{1/3}$

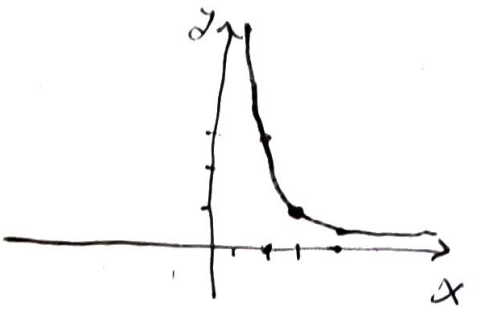
#5 $f(x) = \left(\frac{1}{3}\right)^{x-3}$

Asymptote: $y = 0$

Domain: $\mathbb{R}$

Range: $(0, \infty)$

x	y
2	3
3	1
4	$\frac{1}{3}$



$\left(\left(\frac{1}{3}\right)^{-2}(3^{-1})^{-2}\right)$

#6

	Asymptote	Domain	Range
a) $f(x) = 2 + \ln(2x-4)$	$x = 2$	$x > 2$	$\mathbb{R}$
b) $f(x) = \ln(7-x)$	$x = 7$	$x < 7$	$\mathbb{R}$
c) $f(x) = 5 + e^{2x+3}$	$y = 5$	$\mathbb{R}$	$y > 5$
d) $f(x) = 2 - 3e^{x+1}$	$y = 2$	$\mathbb{R}$	$y < 2$
h) $f(x) = 2^{3x+1}$	$y = 0$	$\mathbb{R}$	$y > 0$

#7 a) $\log_3 \frac{x^3y^2}{z} = \log_3 x^3y^2 - \log_3 z$

$= \log_3 x^3 + \log_3 y^2 - \log_3 z$

$= 3\log_3 x + 2\log_3 y - \log_3 z$

c) $\log_6\left(\frac{x^3y^2}{a^4b^5}\right) = \log_a(x^3y^2) - \log_b(a^4b^5)$

$= \log_a x^3 + \log_a y^2 - (\log_b a^4 + \log_b b^5)$

$= 3\log_a x + 2\log_a y - 4\log_b a - 5$

#8 b) $2\ln(x-3) - \frac{1}{2}\ln(x+2) + 4\ln x - \ln y$

$= \ln(x-3)^2 + \ln x^4 - (\ln(x+2)^{1/2} + \ln y)$

$= \ln((x-3)^2 x^4) - \ln((x+2)^{1/2} y)$

$= \ln \frac{x^4(x-3)^2}{y\sqrt{x+2}}$

c) $\frac{2}{3}[\ln(x^2-4) - \ln(x+2)] + \ln(x+y)$

$= \frac{2}{3}\ln\frac{x^2-4}{x+2} + \ln(x+y)$

$= \frac{2}{3}\ln\frac{(x-2)(x+2)}{x+2} + \ln(x+y)$

$= \ln(x-2)^{2/3}(x+y)$

4-10 Solve

9 b) $5^{x+3} = 25^{x-5}$

$= (5^2)^{x-5}$

$= 5^{2x-10}$

$x+3 = 2x-10$

$13 = x$

c) $3^{x+4} = 2^{2x+5}$

$\ln 3^{x+4} = \ln 2^{2x+5}$

$(x+4)\ln 3 = (2x+5)\ln 2$

$x\ln 3 + 4\ln 3 = 2x\ln 2 + 5\ln 2$

$x\ln 3 - 2x\ln 2 = 5\ln 2 - 4\ln 3$

$x(\ln 3 - 2\ln 2) = 5\ln 2 - 4\ln 3$

$x = \frac{5\ln 2 - 4\ln 3}{\ln 3 - 2\ln 2}$

#10 b) $\ln\sqrt{x+4} = 1$ $\quad \ln e = 1$

$\sqrt{x+4} = e$

$x+4 = e^2$

$\boxed{x = e^2 - 4}$

c) $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

$\ln(x-3) = \ln\frac{7x-23}{x+1}$

$x-3 = \frac{7x-23}{x+1}$

$x^2 + x - 3x - 3 = 7x - 23$

$x^2 - 9x + 20 = 0$

$\boxed{x = 4, 5}$

d) $\log_3(x+2) + \log_3 x = 1$

$\log_3((x+2)x) = 1$

$x^2 - 2x = 3^1 = 3$

$x^2 - 2x - 3 = 0$

$\boxed{x = -1, 3}$

(3) Simplify

a) $\log_5 1 = 0$

b) $\log_7 7^2 = 2$

c) $3^{\log_3 8} = 8$

d) $10^{\log_{10} 3} = 3$

$\ln = \log_e$

e) $e^{2+\ln 3} = e^2 e^{\ln 3} = 3e^2$

f) $\ln e^{-3} = -3$ $\boxed{\ln e = 1}$

g) $\ln e^{x-5} = x-5$

h) $\log_6 6^n = n$

$\log_b b = 1$

i) $\ln e^{x^2+3x} = x^2+3x$