

3.1

Maxima & Minima  
(Maximum) Min  
MAX

extreme points

Crit. point  $\rightarrow$  Max. pt. or Min. pt.  
more than 1  $\rightarrow$   $\begin{cases} RMAX & RMIN \\ \text{or} \\ LMAX & LMIN \end{cases}$

Absolute extreme

abs. Max, abs Min

$[a, b]$

Critical number. CN  
 $x, t$

Critical point CP  
 $(x, y)$

How to find CN by forcing 1<sup>st</sup> derivative to be  
equal 0.  
solve for the variable

Ex  $f(x) = x^2$   $[-2, 1]$

$$f'(x) = 2x = 0 \Rightarrow \underline{CN: x=0}$$

$x$	$f(x)$	
-2	4	$\rightarrow$ abs. Max: $(-2, 4)$
0	0	$\rightarrow$ abs. Min: $(0, 0)$
1	1	

Ex  $g(t) = 8t - t^4$   $[-2, 1]$

$$g'(t) = 8 - 4t^3 = 0$$

$$t^3 = 2 \Rightarrow \underline{t = \sqrt[3]{2} \text{ CN}}$$

t	g(t)
-2	-32
1	7

$\rightarrow$  abs Min @  $(-2, -32)$   
 $\rightarrow$  abs Max @  $(1, 7)$

Ex  $f(x) = x^{2/3}$   $[-2, 3]$

$$f'(x) = \frac{2}{3} x^{-1/3}$$

$$= \frac{2}{3} \frac{1}{x^{1/3}} \neq 0$$

CN:  $x = 0$

x	f(x)
-2	$\sqrt[3]{4}$
0	0
3	$\sqrt[3]{9}$

$\rightarrow$  abs. Min  $(0, 0)$   
 $\rightarrow$  abs. Max  $(3, \sqrt[3]{9})$

$f(\theta) = \sin \theta$   $-\frac{\pi}{2} \leq \theta \leq \frac{5\pi}{6}$

$$f'(\theta) = \cos \theta = 0$$

CN:  $\theta = \pm \frac{\pi}{2}$

$\theta$	f( $\theta$ )
$-\frac{\pi}{2}$	-1
$\frac{\pi}{2}$	1
$\frac{5\pi}{6}$	$\frac{1}{2}$

$\rightarrow$  abs. Min @  $(-\frac{\pi}{2}, -1)$   
 $\rightarrow$  abs Max @  $(\frac{\pi}{2}, 1)$