$$\int_0^{\pi/2} \cos^{12} x\,dx = \frac{\pi}{2}\,\frac{11}{\cancel{12}}\,\frac{\overset{3}{9}}{\cancel{10}}\,\frac{7}{8}\,\frac{\cancel{5}}{\cancel{6}}\,\frac{\cancel{3}}{4}\,\frac{1}{2}$$

$$= \frac{231\,\pi}{2^{11}}$$

$$\int_0^{\pi/2} \cos^{15} x\,dx = \frac{\overset{2}{14}}{15}\,\frac{\overset{4}{\cancel{12}}}{13}\,\frac{\overset{2}{\cancel{10}}}{11}\,\frac{8}{\cancel{9}_{3}}\,\frac{\overset{2}{\cancel{6}}}{\cancel{7}}\,\frac{4}{\cancel{5}}\,\frac{2}{\cancel{3}}$$

$$= \frac{2^4}{6,435}$$

2.4. Partial Fractions

Ex $\int \frac{5x-3}{x^2-2x-3}\,dx$

$$\frac{5x-3}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3} \qquad A(x-3)$$

$$5x-3 = Ax-3A + Bx + B$$

x^1 $A + B = 5 \rightarrow B = 3$ $\quad 5-2$

x^0 $-3A + B = -3$

$4A = 8 \Rightarrow A = 2$

$$\int \frac{5x-3}{x^2-2x-3}\,dx = \int \left(\frac{2}{x+1} + \frac{3}{x-3}\right) dx \qquad d(x+a) = \frac{dx}{x+a}$$

$$= 2\ln|x+1| + 3\ln|x-3| + C$$

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx$$

$$\frac{x^2+4x+1}{(x-1)(x+1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3}$$

$$x^2+4x+1 = A(x+1)(x+3) + B(x-1)(x+3) + C(x^2-1)$$

$$\begin{array}{ll} x^2 & A + B + C = 1 \\ x^1 & 4A + 2B \quad = 4 \\ x^0 & 3A - 3B - C = 1 \end{array}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 0 \\ 3 & -3 & -1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 4 & 2 \\ 3 & -3 \end{matrix} = -16$$

$$\Delta_A = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 0 \\ 1 & -3 & -1 \end{vmatrix} = -12 \qquad A = \frac{-12}{-16} = \frac{3}{4}$$

$$\Delta_B = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 0 \\ 3 & 1 & -1 \end{vmatrix} = -8 \qquad B = \frac{1}{2}$$

$$C = -\frac{1}{4}$$

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx = \frac{3}{4}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{dx}{x+3}$$

$$= \frac{3}{4}\ln|x-1| + \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln|x+3| + K$$

$\overline{(x+a)^n}$ $(x+a)$ $(x+a)^2$ ... $(x+a)^n$

Ex $\int \frac{6x+7}{(x+2)^2}\, dx$

$$\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

$$6x+7 = A(x+2) + B$$

x^1 $A = 6$

x^0 $2A + B = 7 \rightarrow B = -5$

$$\int \frac{6x+7}{(x+2)^2}\, dx = 6\int \frac{dx}{x+2} - 5\int \frac{d(x+2)}{(x+2)^2}$$ $d(x+2) = dx$

$$= 6\ln|x+2| + \frac{5}{x+2} + C$$

$$\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx$$

$$\begin{array}{r} 2x \\ x^2-2x-3 \,\big)\, \overline{2x^3-4x^2-x-3} \\ \underline{-2x^3+4x^2+6x} \\ 5x-3 \end{array}$$

$$\frac{2x^3-4x^2-x-3}{x^2-2x-3} = 2x + \underbrace{\frac{5x-3}{x^2-2x-3}}$$

$$\frac{5x-3}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$5x-3 = A(x-3) + B(x+1)$$

x^1 $\quad A + B = 5 \rightarrow B = 3$

x^0 $\quad -3A + B = -3$

$$4A = 8 \Rightarrow A = 2$$

$$\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx = \int 2x\,dx + 2\int \frac{dx}{x+1} + 3\int \frac{dx}{x-3}$$

$$= x^2 + 2\ln|x+1| + 3\ln|x-3| + C$$

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$$

$$\frac{-2x+4}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$-2x+4 = (Ax+B)(x^2-2x+1) + C(x-1)(x^2+1) + D(x^2+1)$$

x^3: $A + C = 0 \to C = -A$

x^2: $-2A + B - C + D = 0$ ①

x^1: $A - 2B + C = -2 \to -2B = -2 \Rightarrow B = +1$

x^0: $B - C + D = 4$ ②

① $\to$ $2C + 1 - C + D = 0$

② $\to$ $1 - C + D = 4$

$$\begin{cases} C + D = -1 \\ -C + D = 3 \end{cases} \to C = -2,\quad A = 2$$

$2D = 2 \Rightarrow D = 1$

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx = \int \frac{2x+1}{x^2+1}\,dx - 2\int\frac{dx}{x-1} + \int \frac{d(x-1)}{(x-1)^2}$$

$$= \int \frac{2x\,dx}{x^2+1} + \int \frac{dx}{x^2+1} - 2\ln|x-1| - \frac{1}{x-1}$$

$$= \int \frac{d(x^2+1)}{x^2+1} + \tan^{-1}x - 2\ln|x-1| - \frac{1}{x-1}$$

$$= \boxed{\ln(x^2+1) + \tan^{-1}x - 2\ln|x-1| - \frac{1}{x-1} + K}$$

$x(x^2+1)^2$

$$\frac{1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$1 = A(x^4+2x^2+1) + (Bx+C)(x^3+x) + Dx^2 + Ex$$

x^4 $\quad A + B = 0 \rightarrow B = -1$

x^3 $\quad C = 0$

x^2 $\quad 2A + B + D = 0 \rightarrow D = -2 + 1 = -1$

x^1 $\quad C + E = 0 \rightarrow E = 0$

x^0 $\quad A = 1$

$$\int \frac{dx}{x(x^2+1)^2} = \int \frac{dx}{x} - \int \frac{x\,dx}{x^2+1} - \int \frac{x\,dx}{(x^2+1)^2}$$

$$= \ln|x| - \frac{1}{2}\int \frac{d(x^2+1)}{x^2+1} - \frac{1}{2}\int \frac{d(x^2+1)}{(x^2+1)^2}$$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + \frac{1}{2}\cdot\frac{1}{x^2+1} + C$$

$\frac{1}{\infty} = 0 \qquad e^{-\infty} = 0 \quad \frac{1}{e^{\infty}} = \frac{1}{\infty}$

$y = \frac{\ln x}{x^2} \qquad x = 1 \to \infty$

$I = \int_1^{\infty} \frac{\ln x}{x^2}\,dx \qquad u = \ln x \qquad v = \int \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx \qquad = -\frac{1}{x}$

$= -\frac{\ln x}{x}\Big|_1^{\infty} + \int_1^{\infty} \frac{dx}{x^2}$

$= -(0-0) - \frac{1}{x}\Big|_1^{\infty}$

$= +1$ unit^2

$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \tan^{-1} x \Big|_{-\infty}^{\infty}$

$\frac{1}{0} = \infty$

$\frac{\pi}{2}$

$= \tan^{-1}\infty - \tan^{-1}(-\infty)$

$= \frac{\pi}{2} + \frac{\pi}{2}$

$= \pi$

$$\int_1^\infty \frac{dx}{x^p}$$

If $p \neq 1$

$$\int_1^\infty \frac{dx}{x^p} = \int_1^\infty x^{-p}\,dx$$

$$= \frac{x^{-p+1}}{1-p}\Big|_1^\infty$$

$$= \frac{1}{1-p}\left((\infty)^{1-p} - 1\right)$$

$$= \begin{cases} \frac{1}{p-1} & p > 1 \\ \infty & p \leq 1 \end{cases}$$

$\frac{1}{\infty} = 0$

$\infty = \infty$

$(x)^{-}$

$\frac{1}{x^{+}}$

$1-p<0$

$p>1$

If $p = 1$

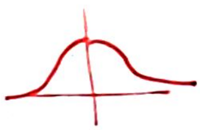
$$\int_1^\infty \frac{dx}{x} = \ln x \Big|_1^\infty$$

$$= \underline{\infty}$$

x^p converges when $p > 1$

$p \leq 1 \to$ diverges

$\int e^{x^2} dx$?? (1) e^{x^2} $u = e^{x^2}$, $du = 2x e^{x^2}$ $\int dx = x$



$$\int_0^1 \frac{dx}{1-x} = -\int_0^1 \frac{d(1-x)}{1-x}$$
$$= -\ln|1-x| \Big|_0^1$$
$$= -(\ln 0^+ - \ln 1)$$
$$= -(-\infty - 0)$$
$$= \underline{\infty}$$

$$\int_0^3 \frac{dx}{(1-x)^{2/3}} = -\int_0^3 (1-x)^{-2/3}\, d(1-x) \qquad d(1-x) = -dx$$
$$= -3(1-x)^{1/3} \Big|_0^3$$
$$= -3\left[(-2)^{1/3} - 1\right]$$
$$= -3\left(-\sqrt[3]{2} - 1\right)$$
$$= \underline{3 + 3\sqrt[3]{2}}$$

Direct Comparison Test $0 \le f(x) \le g(x)$

$\int_a^\infty f(x)\,dx$ converges if $\int g(x)\,dx$ converges

if f(x) diverges → g(x) diverges

$f(x) = x^{-1} \qquad x \geq 1$

$$V = \pi \int_1^\infty (x^{-1})^2\,dx \qquad \int_1^\infty \frac{dx}{x^2}$$

$$= -\pi \left.\frac{1}{x}\right|_1^\infty$$

$$= -\pi(0 - 1)$$

$$= \pi \text{ unit}^3$$

$$S = 2\pi \int_1^\infty \frac{1}{x}\cdot\sqrt{1+\frac{1}{x^4}}\,dx$$

$$= 2\pi \int_1^\infty \frac{1}{x}\,\frac{\sqrt{x^4+1}}{x^2}\,dx$$

$$= 2\pi \int_1^\infty \frac{\sqrt{x^4+1}}{x^3}\,dx \qquad \sqrt{x^4+1} > \sqrt{x^4} = x^2$$

$$> 2\pi \int_1^\infty \frac{x^2}{x^3}\,dx$$

$$= 2\pi \left.\ln x\right|_1^\infty$$

$$= \infty$$