

## ***Solution***

## **Section 4.2 – Matrix operations and Their Applications**

### ***Exercise***

Find values for the variables so that the matrices are equal.  $\begin{bmatrix} w & x \\ 8 & -12 \end{bmatrix} = \begin{bmatrix} 9 & 17 \\ y & z \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} w & x \\ 8 & -12 \end{bmatrix} = \begin{bmatrix} 9 & 17 \\ y & z \end{bmatrix}$$

$$\Rightarrow \begin{cases} w = 9 & x = 17 \\ y = 8 & z = -12 \end{cases}$$

### ***Exercise***

Find values for the variables so that the matrices are equal.  $\begin{bmatrix} x & y+3 \\ 2z & 8 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 6 & 8 \end{bmatrix}$

### **Solution**

$$\begin{cases} x = 12 \\ y + 3 = 5 \rightarrow y = 2 \\ 2z = 6 \rightarrow z = 3 \end{cases}$$

### ***Exercise***

Find values for the variables so that the matrices are equal.  $\begin{bmatrix} 5 & x-4 & 9 \\ 2 & -3 & 8 \\ 6 & 0 & 5 \end{bmatrix} = \begin{bmatrix} y+3 & 2 & 9 \\ z+4 & -3 & 8 \\ 6 & 0 & w \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} 5 = y + 3 & x - 4 = 2 & 9 = 9 \\ 2 = z + 4 & -3 = -3 & 8 = 8 \\ 6 = 6 & 0 = 0 & 5 = w \end{bmatrix}$$

$$\rightarrow \begin{cases} y = 2 & z = -2 \\ x = 6 & w = 5 \end{cases}$$

### Exercise

Find values for the variables so that the matrices are equal.

$$\begin{bmatrix} a+2 & 3b & 4c \\ d & 7f & 8 \end{bmatrix} + \begin{bmatrix} -7 & 2b & 6 \\ -3d & -6 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 25 & 6 \\ -8 & 1 & 6 \end{bmatrix}$$

### Solution

$$\begin{bmatrix} a-5 & 5b & 4c+6 \\ -2d & 7f-6 & 6 \end{bmatrix} = \begin{bmatrix} 15 & 25 & 6 \\ -8 & 1 & 6 \end{bmatrix}$$

$$\begin{cases} a-5=15 & \rightarrow & a=20 \\ 5b=25 & \rightarrow & b=5 \\ 4c+6=6 & \rightarrow & 4c=0 \rightarrow c=0 \\ -2d=-8 & \rightarrow & d=4 \\ 7f-6=1 & \rightarrow & 7f=7 \rightarrow f=1 \end{cases}$$

### Exercise

Find values for the variables so that the matrices are equal.

$$\begin{bmatrix} a+11 & 12z+1 & 5m \\ 11k & 3 & 1 \end{bmatrix} + \begin{bmatrix} 9a & 9z & 4m \\ 12k & 5 & 3 \end{bmatrix} = \begin{bmatrix} 41 & -62 & 72 \\ 92 & 8 & 4 \end{bmatrix}$$

### Solution

$$\begin{bmatrix} a+11+9a & 12z+1+9z & 5m+4m \\ 11k+12k & 3+5 & 1+3 \end{bmatrix} = \begin{bmatrix} 41 & -62 & 72 \\ 92 & 8 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 10a+11 & 21z+1 & 9m \\ 23k & 8 & 4 \end{bmatrix} = \begin{bmatrix} 41 & -62 & 72 \\ 92 & 8 & 4 \end{bmatrix}$$

$$10a+11=41 \rightarrow 10a=30$$

$$\underline{a=3}$$

$$21z+1=-62 \rightarrow 21z=-63$$

$$\underline{z=-3}$$

$$9m=72 \rightarrow \underline{m=8}$$

$$23k=92 \rightarrow \underline{k=\frac{92}{23}=4}$$

### Exercise

Find values for the variables so that the matrices are equal.

$$\begin{bmatrix} x+2 & 3y+1 & 5z \\ 8w & 2 & 3 \end{bmatrix} + \begin{bmatrix} 3x & 2y & 5z \\ 2w & 5 & -5 \end{bmatrix} = \begin{bmatrix} 10 & -14 & 80 \\ 10 & 7 & -2 \end{bmatrix}$$

### Solution

$$\begin{bmatrix} 4x+2 & 5y+1 & 10z \\ 10w & 7 & -2 \end{bmatrix} = \begin{bmatrix} 10 & -14 & 80 \\ 10 & 7 & -2 \end{bmatrix}$$

$$\begin{cases} 4x+2=10 & \rightarrow \underline{x=2} \\ 5y+1=-14 & \rightarrow \underline{y=-3} \\ 10z=80 & \rightarrow \underline{z=8} \\ 10w=10 & \rightarrow \underline{w=1} \end{cases}$$

### Exercise

Find values for the variables so that the matrices are equal.

$$\begin{bmatrix} 2x-3 & y-2 & 2z+1 \\ 5 & 2w & 7 \end{bmatrix} + \begin{bmatrix} 3x-3 & y+2 & z-1 \\ -5 & 5w+1 & 3 \end{bmatrix} = \begin{bmatrix} 20 & 8 & 9 \\ 0 & 8 & 10 \end{bmatrix}$$

### Solution

$$\begin{bmatrix} 5x-6 & 2y & 3z \\ 0 & 7w+1 & 10 \end{bmatrix} = \begin{bmatrix} 20 & 8 & 9 \\ 0 & 8 & 10 \end{bmatrix}$$

$$\begin{cases} 5x-6=20 & \rightarrow \underline{x=\frac{26}{5}} \\ 2y=8 & \rightarrow \underline{y=4} \\ 3z=9 & \rightarrow \underline{z=3} \\ 7w+1=8 & \rightarrow \underline{w=1} \end{cases}$$

### Exercise

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix}$$

### Solution

$$\begin{aligned} A - B &= \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 4 & -5 \\ 2 & 1 & 9 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 3A + 2B &= 3 \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix} + 2 \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix} \\
 &= \begin{bmatrix} 9 & 3 & 3 \\ -3 & 6 & 15 \end{bmatrix} + \begin{bmatrix} 4 & -6 & 12 \\ -6 & 2 & -8 \end{bmatrix} \\
 &= \begin{bmatrix} 13 & -3 & 15 \\ -9 & 8 & 7 \end{bmatrix}
 \end{aligned}$$

### ***Exercise***

Given  $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$      $F = \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$     Find     $3F + 2A$

### **Solution**

$$\begin{aligned}
 3F + 2A &= 3 \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 3(3) & 3(3) \\ 3(-1) & 3(-1) \end{bmatrix} + \begin{bmatrix} 2(1) & 2(2) \\ 2(4) & 2(3) \end{bmatrix} \\
 &= \begin{bmatrix} 9 & 9 \\ -3 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} 9+2 & 9+4 \\ -3+8 & -3+6 \end{bmatrix} \\
 &= \begin{bmatrix} 11 & 13 \\ 5 & 3 \end{bmatrix}
 \end{aligned}$$

### ***Exercise***

Evaluate     $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 12 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 12 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \\ 20 \end{bmatrix}$$

### ***Exercise***

Evaluate  $\begin{bmatrix} 5 & 8 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 & 1 \\ 4 & 2 & 5 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} 5 & 8 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 & 1 \\ 4 & 2 & 5 \end{bmatrix}$$

It is **impossible**;  $2 \times 2$  and  $2 \times 3$  are not the same size.

### ***Exercise***

Evaluate  $\begin{bmatrix} -5 & 0 \\ 4 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} -5 & 0 \\ 4 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -5+6 & 0+(-3) \\ 4+2 & \frac{1}{2}+3 \end{bmatrix} \\ = \begin{bmatrix} 1 & -3 \\ 6 & \frac{7}{2} \end{bmatrix}$$

### ***Exercise***

Evaluate  $\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 5-4 & -6+6 \\ 8+8 & 9-3 \end{bmatrix} \\ = \begin{bmatrix} 1 & 0 \\ 16 & 6 \end{bmatrix}$$

### ***Exercise***

Evaluate  $\begin{bmatrix} -5 & 6 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 5 & -8 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} -5 & 6 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 5 & -8 \end{bmatrix} = \begin{bmatrix} -5 - (-3) & 6 - 2 \\ 2 - 5 & 4 - (-8) \end{bmatrix} \\ = \begin{bmatrix} -2 & 4 \\ -3 & 12 \end{bmatrix}$$

### ***Exercise***

Evaluate  $\begin{bmatrix} 8 & 6 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -8 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} 8 & 6 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -8 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 4 \end{bmatrix}$$

### ***Exercise***

Evaluate  $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(4) + 3(1) & 1(6) + 3(0) \\ 2(4) + 5(1) & 2(6) + 5(0) \end{bmatrix} \\ = \begin{bmatrix} 7 & 6 \\ 13 & 12 \end{bmatrix}$$

### ***Exercise***

Evaluate  $\begin{bmatrix} -3 & 4 & 2 \\ 5 & 0 & 4 \end{bmatrix} \begin{bmatrix} -6 & 4 \\ 2 & 3 \\ 3 & -2 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} -3 & 4 & 2 \\ 5 & 0 & 4 \end{bmatrix} \begin{bmatrix} -6 & 4 \\ 2 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} -3(-6) + 4(2) + 2(3) & -3(4) + 4(3) + 2(-2) \\ 5(-6) + 0(2) + 4(3) & 5(4) + 0(3) + 4(-2) \end{bmatrix} \\ = \begin{bmatrix} 32 & -4 \\ -18 & 12 \end{bmatrix}$$

**Exercise**

Evaluate  $\begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

**Solution**

$$\begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1(1) - 1(1) + 4(1) & 1(1) - 1(2) + 4(-1) & 1(0) - 1(4) + 4(3) \\ 4(1) - 1(1) + 3(1) & 4(1) - 1(2) + 3(-1) & 4(0) - 1(4) + 3(3) \\ 2(1) + 0(1) - 2(1) & 2(1) + 0(2) - 2(-1) & 2(0) + 0(4) - 2(3) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -5 & 8 \\ 6 & -1 & 5 \\ 0 & 4 & -6 \end{bmatrix}$$

**Exercise**

Evaluate  $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}$

**Solution**

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1(1) + 1(4) + 0(2) & 1(-1) + 1(-1) + 0(0) & 1(4) + 1(3) + 0(-2) \\ 1(1) + 2(4) + 4(2) & 1(-1) + 2(-1) + 4(0) & 1(4) + 2(3) + 4(-2) \\ 1(1) - 1(4) + 3(2) & 1(-1) - 1(-1) + 3(0) & 1(4) - 1(3) + 3(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 & 7 \\ 17 & -3 & 2 \\ 3 & 0 & -5 \end{bmatrix}$$

**Exercise**

Evaluate  $\begin{bmatrix} -2 & -3 & -4 \\ 2 & -1 & 0 \\ 4 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix}$

**Solution**

$$\begin{bmatrix} -2 & -3 & -4 \\ 2 & -1 & 0 \\ 4 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -3 - 12 & -2 - 6 - 8 & -8 + 3 + 8 \\ -1 & 2 - 2 & 8 + 1 \\ -2 + 9 & 4 - 4 + 6 & 16 + 2 - 6 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & -16 & 3 \\ -1 & 0 & 9 \\ 7 & 6 & 12 \end{bmatrix}$$

### ***Exercise***

Evaluate  $\begin{bmatrix} \sqrt{2} & \sqrt{2} & -\sqrt{18} \\ \sqrt{3} & \sqrt{27} & 0 \end{bmatrix} \begin{bmatrix} 8 & -10 \\ 9 & 12 \\ 0 & 2 \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} \sqrt{2} & \sqrt{2} & -\sqrt{18} \\ \sqrt{3} & \sqrt{27} & 0 \end{bmatrix} \begin{bmatrix} 8 & -10 \\ 9 & 12 \\ 0 & 2 \end{bmatrix} = \begin{pmatrix} 17\sqrt{2} & -4\sqrt{2} \\ 35\sqrt{3} & 26\sqrt{3} \end{pmatrix}$$

### ***Exercise***

Evaluate  $\begin{bmatrix} x & 2x+1 & 4 \\ 5 & x-1 & 8 \\ -2 & 3x & 2x+1 \end{bmatrix} + \begin{bmatrix} 2x-1 & -2x-1 & 4x \\ -5 & 6 & x+1 \\ -5 & 2 & -2x \end{bmatrix}$

### **Solution**

$$\begin{bmatrix} x & 2x+1 & 4 \\ 5 & x-1 & 8 \\ -2 & 3x & 2x+1 \end{bmatrix} + \begin{bmatrix} 2x-1 & -2x-1 & 4x \\ -5 & 6 & x+1 \\ -5 & 2 & -2x \end{bmatrix} = \begin{bmatrix} 3x-1 & 0 & 4x+4 \\ 0 & x+5 & x+9 \\ -7 & 3x+2 & 1 \end{bmatrix}$$

### ***Exercise***

Given  $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & 7 \\ 0 & 2 \end{bmatrix}$ . Find  $AB$  and  $BA$ .

### **Solution**

$$AB = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} -2 & 7 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 13 \\ 4 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 7 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -16 & 29 \\ -4 & 10 \end{bmatrix}$$

**Note:**  $AB \neq BA$



**Exercise**

Given  $A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$   $B = \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

**Solution**

$$\begin{aligned} AB &= \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -6 & 17 \\ 6 & -8 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 14 \\ 1 & -20 \end{pmatrix} \end{aligned}$$

**Exercise**

Given  $A = \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix}$   $B = \begin{pmatrix} -1 & -1 \\ 0 & 4 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

**Solution**

$$\begin{aligned} AB &= \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & 4 \end{pmatrix} \\ &= \begin{pmatrix} -3 & -11 \\ 4 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} -1 & -1 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -7 & 1 \\ 16 & 4 \end{pmatrix} \end{aligned}$$

**Exercise**

Given  $A = \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}$   $B = \begin{pmatrix} 4 & 1 \\ 2 & -3 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

**Solution**

$$\begin{aligned} AB &= \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 2 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 6 \\ 14 & -7 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{pmatrix} 4 & 1 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \\
 &= \begin{pmatrix} 14 & -1 \\ 0 & -11 \end{pmatrix}
 \end{aligned}$$

### ***Exercise***

Given  $A = \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix}$   $B = \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

### **Solution**

$$\begin{aligned}
 AB &= \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} -4 & 2 \\ 4 & -4 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix} \\
 &= \begin{pmatrix} 4 & -4 \\ 14 & -12 \end{pmatrix}
 \end{aligned}$$

### ***Exercise***

Given  $A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & -2 \end{pmatrix}$   $B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

### **Solution**

$$\begin{aligned}
 AB &= \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & -4 & 5 \\ 6 & 0 & 3 \\ -3 & -2 & 1 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & -2 \end{pmatrix} \\
 &= \begin{pmatrix} 5 & -13 \\ 3 & -4 \end{pmatrix}
 \end{aligned}$$

**Exercise**

Given  $A = \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix}$   $B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

**Solution**

$$AB = \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -1 & 5 & 4 \\ 2 & -3 & 8 \\ -3 & 8 & -5 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -14 & 7 \\ -4 & 5 \end{pmatrix}$$

**Exercise**

Given  $A = \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ -3 & 2 \end{pmatrix}$   $B = \begin{pmatrix} 3 & 0 & -2 \\ -2 & 6 & 2 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

**Solution**

$$AB = \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 3 & 0 & -2 \\ -2 & 6 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -2 & 24 & 4 \\ 2 & -6 & -2 \\ -13 & 12 & 10 \end{pmatrix}$$

$$BA = \begin{pmatrix} 3 & 0 & -2 \\ -2 & 6 & 2 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ -3 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 12 & 8 \\ -10 & 10 \end{pmatrix}$$

**Exercise**

Given  $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$   $B = \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

**Solution**

$$AB = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & -2 & -6 \\ 0 & -1 & 2 \\ 5 & -3 & -4 \end{pmatrix}$$

$$BA = \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix} \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -3 & 9 \\ 2 & -3 & 4 \\ 4 & -6 & 3 \end{pmatrix}$$

**Exercise**

Given  $A = \begin{pmatrix} -1 & 2 & 0 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix}$   $B = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

**Solution**

$$AB = \begin{pmatrix} -1 & 2 & 0 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 8 & -2 \\ 3 & -8 & 4 \\ -2 & 13 & -5 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} -1 & 2 & 0 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 5 & -1 \\ 11 & -5 & 6 \\ -8 & 7 & -4 \end{pmatrix}$$

### Exercise

Given  $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & -2 & -1 \end{pmatrix}$   $B = \begin{pmatrix} -3 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix}$ . Find  $AB$  and  $BA$ .

### Solution

$$AB = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & -2 & -1 \end{pmatrix} \begin{pmatrix} -3 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -5 & -7 & 2 \\ -6 & 2 & 2 \\ -8 & -6 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} -3 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & -2 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} -1 & 6 & 1 \\ 7 & 0 & 5 \\ 4 & -4 & -2 \end{pmatrix}$$

### Exercise

Given  $A = \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix}$   $B = \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}$ , Find

a)  $A + B$

c)  $3A$

e)  $2A + 3B$

g)  $AB$

b)  $A - B$

d)  $-2B$

f)  $A^2$

h)  $BA$

### Solution

$$a) \quad A + B = \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 5 \\ 3 & -5 \\ 2 & -4 \end{bmatrix}$$

$$b) \quad A - B = \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 3 \\ 1 & -1 \\ -4 & 4 \end{bmatrix}$$

$$c) \quad 3A = 3 \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 12 \\ 6 & -9 \\ -3 & 0 \end{bmatrix}$$

$$d) \quad -2B = -2 \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -2 \\ -2 & 4 \\ -6 & 8 \end{bmatrix}$$

$$e) \quad 2A + 3B = 2 \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} + 3 \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 8 \\ 4 & -6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 12 & 3 \\ 3 & -6 \\ 9 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 11 \\ 7 & -12 \\ 7 & -12 \end{bmatrix}$$

$$f) \quad A^2 = \text{doesn't exist} \quad (\text{not a square matrix})$$

$$g) \quad AB = \cancel{\exists} \quad (2 \times 3 \quad 2 \times 3) \text{ the inner not equal}$$

$$h) \quad BA = \cancel{\exists} \quad (2 \times 3 \quad 2 \times 3) \text{ the inner not equal}$$

### Exercise

Given  $A = \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix}$   $B = \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix}$ , Find

a)  $A + B$

c)  $3A$

e)  $2A + 3B$

g)  $AB$

b)  $A - B$

d)  $-2B$

f)  $A^2$

h)  $BA$

### Solution

$$\begin{aligned} a) \quad A + B &= \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 6 \\ 5 & 2 \\ -3 & 3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} b) \quad A - B &= \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -10 \\ 1 & 6 \\ 5 & -3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} c) \quad 3A &= 3 \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 6 & -6 \\ 9 & 12 \\ 3 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} d) \quad -2B &= -2 \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -16 \\ -4 & 4 \\ 8 & -6 \end{bmatrix} \end{aligned}$$

$$e) \quad 2A + 3B = 2 \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix} + 3 \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ 6 & 8 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 24 \\ 6 & -6 \\ -12 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 20 \\ 12 & 2 \\ -10 & 9 \end{bmatrix}$$

**f)**  $A^2 = \text{doesn't exist}$  (not a square matrix)

**g)**  $AB = \nexists$  ( $2 \times 3$   $2 \times 3$ ) the inner not equal

**h)**  $BA = \nexists$  ( $2 \times 3$   $2 \times 3$ ) the inner not equal

### Exercise

Given  $A = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix}$   $B = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$ , Find

**a)**  $A + B$

**c)**  $3A$

**e)**  $2A + 3B$

**g)**  $AB$

**b)**  $A - B$

**d)**  $-2B$

**f)**  $A^2$

**h)**  $BA$

### Solution

**a)**  $A + B = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 1 & -1 \\ 2 & 2 & 1 \\ -1 & 2 & 5 \end{bmatrix}$$

**b)**  $A - B = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 5 & -1 \\ -2 & -4 & 3 \\ -7 & 4 & 1 \end{bmatrix}$$

**c)**  $3A = 3 \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix}$



$$= \begin{bmatrix} -6 & 9 & -3 \\ 0 & -3 & 6 \\ -12 & 9 & 9 \end{bmatrix}$$

$$\begin{aligned} d) \quad -2B &= -2 \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 4 & 0 \\ -4 & -6 & 2 \\ -6 & 2 & -4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} e) \quad 2A + 3B &= 2 \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} + 3 \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -4 & 6 & -2 \\ 0 & -2 & 4 \\ -8 & 6 & 6 \end{bmatrix} + \begin{bmatrix} 3 & -6 & 0 \\ 6 & 9 & -3 \\ 9 & -3 & 6 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & -2 \\ 6 & 7 & 1 \\ 1 & 3 & 12 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} f) \quad A^2 &= \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 4+4 & -6-3-3 & 2+6-3 \\ -8 & 1+6 & -2+6 \\ 8-12 & -12-3+9 & 4+6+9 \end{bmatrix} \\ &= \begin{bmatrix} 8 & -12 & 5 \\ -8 & 7 & 4 \\ -4 & -6 & 19 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} g) \quad AB &= \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -2+6-3 & 4+9+1 & -3-2 \\ -2+6 & -3-2 & 1+4 \\ -4+6+9 & 8+9-3 & -3+6 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 1 & 14 & -5 \\ 4 & -5 & 5 \\ 11 & 14 & 3 \end{bmatrix}$$

$$\begin{aligned} h) \quad BA &= \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 3+2 & -1-4 \\ -4+4 & 6-3-3 & -2+6-3 \\ -6-8 & 9+1+6 & -3-2+6 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 5 & -5 \\ 0 & 0 & 1 \\ -14 & 16 & 1 \end{bmatrix} \end{aligned}$$

### ***Exercise***

Given  $A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix}$   $B = \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix}$ , Find

a)  $A + B$

c)  $3A$

e)  $2A + 3B$

g)  $AB$

b)  $A - B$

d)  $-2B$

f)  $A^2$

h)  $BA$

### **Solution**

$$\begin{aligned} a) \quad A + B &= \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 4 & 4 \\ 4 & 0 & 1 \\ 1 & 8 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} b) \quad A - B &= \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & -4 \\ -2 & -6 & 5 \\ 9 & 0 & -5 \end{bmatrix} \end{aligned}$$

$$c) \quad 3A = 3 \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 6 & 0 \\ 3 & -9 & 9 \\ 15 & 12 & -6 \end{bmatrix}$$

$$\begin{aligned} d) \quad -2B &= -2 \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -4 & -8 \\ -6 & -6 & 4 \\ 8 & -8 & -6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} e) \quad 2A + 3B &= 2 \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} + 3 \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 4 & 0 \\ 2 & -6 & 6 \\ 10 & 8 & -4 \end{bmatrix} + \begin{bmatrix} -3 & 6 & 12 \\ 9 & 9 & -6 \\ -12 & 12 & 9 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 10 & 12 \\ 11 & 3 & 0 \\ -2 & 20 & 5 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} f) \quad A^2 &= \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -6 & 6 \\ -3+15 & 2+9+12 & -9-6 \\ 4-10 & 10-12-8 & 12+4 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -6 & 6 \\ 12 & 23 & -15 \\ -6 & -10 & 16 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} g) \quad AB &= \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 6 & -4 \\ -1-9-12 & 2-9+12 & 4+6+9 \\ -5+12+8 & 10+12-8 & 20-8-6 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 6 & 6 & -4 \\ -22 & 5 & 19 \\ 15 & 14 & 6 \end{bmatrix}$$

$$\begin{aligned} h) \quad BA &= \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2+10 & -2-6+16 & 6-8 \\ 3-10 & 6-9-8 & 9+4 \\ 4+15 & -8-12+12 & 12-6 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 8 & -2 \\ -7 & -11 & 13 \\ 19 & -8 & 6 \end{bmatrix} \end{aligned}$$

### Exercise

Given  $A = \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$   $B = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$   $C = \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix}$   $D = \begin{pmatrix} -2 & 3 \\ 2 & -1 \\ 3 & 2 \end{pmatrix}$ , Find

a)  $4A - 2B$

d)  $2A - 3B$

g)  $A^2$

j)  $CA$

b)  $3A + C$

e)  $AB$

h)  $B^3$

k)  $CD$

c)  $3A + B$

f)  $BA$

i)  $AC$

l)  $DC$

### Solution

$$\begin{aligned} a) \quad 4A - 2B &= 4 \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} - 2 \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -4 & 8 \\ -8 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -4 \\ 4 & -2 \end{pmatrix} \\ &= \begin{pmatrix} -6 & 12 \\ -12 & 6 \end{pmatrix} \end{aligned}$$

b)  $3A + C = \text{not possible}$

They are not the same order.

$$\begin{aligned} c) \quad 3A + B &= 3 \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -3 & 6 \\ -6 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -2 & 4 \\ -4 & 2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
 d) \quad 2A - 3B &= 2 \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} - 3 \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} -2 & 4 \\ -4 & 2 \end{pmatrix} - \begin{pmatrix} 3 & -6 \\ 6 & -3 \end{pmatrix} \\
 &= \begin{pmatrix} -5 & 10 \\ -10 & 5 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad AB &= \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad BA &= \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 g) \quad A^2 &= \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 h) \quad B^3 &= \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} -3 & 6 \\ -6 & 3 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 i) \quad AC &= \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix} & 2 \times 2 \quad 2 \times 3 \quad \rightarrow 2 \times 3 \\
 &= \begin{pmatrix} -6 & 1 & 0 \\ -9 & -4 & -3 \end{pmatrix}
 \end{aligned}$$

$$j) \quad CB = \text{not defined} \quad 2 \times 3 \quad 2 \times 2$$

$C$  and  $B$  are not the same order.

$$\begin{aligned}
 k) \quad CD &= \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 2 & -1 \\ 3 & 2 \end{pmatrix} & 2 \times 3 \quad 3 \times 2 \quad \rightarrow 2 \times 2 \\
 &= \begin{pmatrix} -8 + 6 + 6 & 12 - 3 + 4 \\ 2 + 4 + 3 & -3 + 2 + 2 \end{pmatrix}
 \end{aligned}$$

$$= \begin{pmatrix} 4 & 13 \\ 9 & 1 \end{pmatrix}$$

$$l) \quad DC = \begin{pmatrix} -2 & 3 \\ 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix} \quad 3 \times 2 \quad 2 \times 3 \rightarrow 3 \times 3$$

$$= \begin{pmatrix} -8-3 & -6+6 & -4+3 \\ 8+1 & 6-2 & 4-1 \\ 12-2 & 9+4 & 6+2 \end{pmatrix}$$

$$= \begin{pmatrix} -11 & 0 & -1 \\ 9 & 4 & 3 \\ 10 & 13 & 8 \end{pmatrix}$$

### Exercise

Given  $A = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$   $B = \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$   $C = \begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix}$   $D = \begin{pmatrix} 2 & 4 & -2 \\ 0 & 3 & 5 \\ -3 & 1 & 1 \end{pmatrix}$ , Find

a)  $4A - 2B$

d)  $2A - 3B$

g)  $A^2$

j)  $CB$

b)  $3A + C$

e)  $AB$

h)  $B^3$

k)  $CD$

c)  $3A + B$

f)  $BA$

i)  $AC$

l)  $DC$

### Solution

$$\begin{aligned} a) \quad 4A - 2B &= 4 \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} - 2 \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 16 \\ 12 & -4 \end{pmatrix} - \begin{pmatrix} -2 & 6 \\ 4 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 10 \\ 8 & -2 \end{pmatrix} \end{aligned}$$

b)  $3A + C = \cancel{A}$

They are not the same order.

$$\begin{aligned} c) \quad 3A + B &= 3 \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 6 & 12 \\ 9 & -3 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 15 \\ 11 & -4 \end{pmatrix} \end{aligned}$$

$$d) \quad 2A - 3B = 2 \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} - 3 \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 8 \\ 6 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 9 \\ 6 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -1 \\ 0 & 1 \end{pmatrix}$$

$$e) \quad AB = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 2 \\ -5 & 10 \end{pmatrix}$$

$$f) \quad BA = \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -7 \\ 1 & 9 \end{pmatrix}$$

$$g) \quad A^2 = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 4 \\ 3 & 13 \end{pmatrix}$$

$$h) \quad B^3 = \left( \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \right) \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -6 \\ -4 & 7 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -19 & 27 \\ 18 & -19 \end{pmatrix}$$

$$i) \quad AC = \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix} \quad 2 \times 2 \quad 2 \times 3 \quad \rightarrow 2 \times 3$$

$$= \begin{pmatrix} -6 & 1 & 0 \\ -9 & -4 & -3 \end{pmatrix}$$

$$j) \quad CB = \text{not possible} \quad 2 \times 3 \quad 2 \times 2$$

$C$  and  $B$  are not the same order.

$$k) \quad CD = \begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 4 & -2 \\ 0 & 3 & 5 \\ -3 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -12 & 21 & 13 \\ -16 & 5 & 23 \\ 4 & -6 & 0 \end{pmatrix}$$

$$\begin{aligned}
 l) \quad DC &= \begin{pmatrix} 2 & 4 & -2 \\ 0 & 3 & 5 \\ -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix} \\
 &= \begin{pmatrix} 2-8+2 & 8+12 & 10+16+4 \\ -6-5 & 9 & 12-10 \\ -3-2-1 & -12+3 & -15+4-2 \end{pmatrix} \\
 &= \begin{pmatrix} -4 & 20 & 30 \\ -11 & 9 & 2 \\ -6 & -9 & -12 \end{pmatrix}
 \end{aligned}$$

### Exercise

A contractor builds three kinds of houses, models  $A$ ,  $B$ , and  $C$ , with a choice of two styles, Spanish and contemporary. Matrix  $P$  shows the number of each kind of house planned for a new 100-home subdivision. The amounts for each of the exterior materials depend primarily on the style of the house. These amounts are shown in matrix  $Q$ . (concrete is in cubic yards, lumber in units of 1000 board feet, brick in 1000s, and shingles in units of  $100 \text{ ft}^2$ .) Matrix  $R$  gives the cost in dollars for each kind of material.

- What is the total cost of these materials for each model?
- How much of each of four kinds of material must be ordered
- What is the total cost for exterior materials?

### Solution

$$\begin{array}{cc}
 \text{Spanish} & \text{Contemporary} \\
 \begin{array}{l} \text{Model A} \\ \text{Model B} \\ \text{Model C} \end{array} & \begin{bmatrix} 0 & 30 \\ 10 & 20 \\ 20 & 20 \end{bmatrix} = P
 \end{array}$$
  

$$\begin{array}{ccccc}
 & \text{Concrete} & \text{Lumber} & \text{Brick} & \text{Shingles} \\
 \begin{array}{l} \text{Spanish} \\ \text{Contemporary} \end{array} & \begin{bmatrix} 10 & 2 & 0 & 2 \\ 50 & 1 & 20 & 2 \end{bmatrix} = Q
 \end{array}$$
  

$$\begin{array}{cc}
 \text{Cost per unit} \\
 \begin{array}{l} \text{Concrete} \\ \text{Lumber} \\ \text{Brick} \\ \text{Shingles} \end{array} & \begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix} = R
 \end{array}$$

- What is the total cost of these materials for each model?



$$PQ = \begin{bmatrix} 0 & 30 \\ 10 & 20 \\ 20 & 20 \end{bmatrix} \begin{bmatrix} 10 & 2 & 0 & 2 \\ 50 & 1 & 20 & 2 \end{bmatrix}$$

$$= \begin{array}{cccc} \text{Concrete} & \text{Lumber} & \text{Brick} & \text{Shingles} \\ \begin{bmatrix} 1500 & 30 & 600 & 60 \\ 100 & 40 & 400 & 60 \\ 1200 & 60 & 400 & 80 \end{bmatrix} & \text{Model A} & \text{Model B} & \text{Model C} \end{array}$$

$$(PQ)R = \begin{bmatrix} 1500 & 30 & 600 & 60 \\ 100 & 40 & 400 & 60 \\ 1200 & 60 & 400 & 80 \end{bmatrix} \begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix}$$

$$= \begin{array}{l} \begin{bmatrix} 72,900 \\ 54,700 \\ 60,800 \end{bmatrix} \\ \text{Model A} \\ \text{Model B} \\ \text{Model C} \end{array}$$

The total cost of materials is \$72,900 for model A, \$54,700 for model B, \$60,800 for model C.

**b)** How much of each of four kinds of material must be ordered

$$\begin{array}{cccc} \begin{bmatrix} 1500 & 30 & 600 & 60 \\ 100 & 40 & 400 & 60 \\ 1200 & 60 & 400 & 80 \end{bmatrix} \\ \hline 3800 & 130 & 1400 & 200 \end{array}$$

$$T = [3800 \quad 130 \quad 1400 \quad 200]$$

3800  $yd^3$  of concrete, 130,000 board feet of lumber, 1,400,000 bricks, and 20,000  $ft^2$  of shingles are needed.

**c)** What is the total cost for exterior materials?

$$TR = [3800 \quad 130 \quad 1400 \quad 200] \begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix}$$

$$= [188,400]$$

The total cost for exterior materials is \$188,400

### Exercise

Mitchell Fabricators manufactures three styles of bicycle frames in its two plants. The following table shows the number of each style produced at each plant

	<i>Mountain Bike</i>	<i>Racing Bike</i>	<i>Touring Bike</i>
<i>North Plant</i>	150	120	100
<i>South Plant</i>	180	90	130

- a) Write a  $2 \times 3$  matrix  $A$  that represents the information in the table
- b) The manufacturer increased production of each style by 20%. Find a Matrix  $M$  that represents the increased production figures.
- c) Find the matrix  $A + M$  and tell what it represents

### Solution

$$a) \quad A = \begin{bmatrix} 150 & 120 & 100 \\ 180 & 90 & 130 \end{bmatrix}$$

- b) The 20% production will represent

$$A + 20\%(A)$$

$$\rightarrow A + .2A = 1.2A$$

$$M = (1.2) \begin{bmatrix} 150 & 120 & 100 \\ 180 & 90 & 130 \end{bmatrix}$$

$$= \begin{bmatrix} 180 & 144 & 120 \\ 216 & 108 & 156 \end{bmatrix}$$

$$c) \quad A + M = \begin{bmatrix} 150 & 120 & 100 \\ 180 & 90 & 130 \end{bmatrix} + \begin{bmatrix} 180 & 144 & 120 \\ 216 & 108 & 156 \end{bmatrix}$$
$$= \begin{bmatrix} 330 & 264 & 220 \\ 396 & 198 & 286 \end{bmatrix}$$

The matrix  $A + M$  represents the total production of each style at each plant for the time period (2 months)

### Exercise

Sal's Shoes and Fred's Footwear both have outlets in California and Arizona. Sal's sells shoes for \$80, sandals for \$40, and boots for \$120. Fred's prices are \$60, \$30, and \$150 for shoes, sandals and boots, respectively. Half of all sales in California stores are shoes, 1/4 are *sandals*, and 1/4 are *boots*. In Arizona the fractions are 1/5 *shoes*, 1/5 are *sandals*, and 3/5 are *boots*.

- Write a  $2 \times 3$  matrix called  $P$  representing prices for the two stores and three types of footwear.
- Write a  $2 \times 3$  matrix called  $F$  representing fraction of each type of footwear sold in each state.
- Only one of the two products  $PF$  and  $FP$  is meaningful. Determine which one it is, calculate the product, and describe what the entries represent.

### Solution

- Write a  $2 \times 3$  matrix called  $P$  representing prices for the two stores and three types of footwear.

$$P = \begin{bmatrix} 80 & 40 & 120 \\ 60 & 30 & 150 \end{bmatrix} \quad \begin{array}{l} \text{Sal's} \\ \text{Fred's} \end{array}$$

- Write a  $2 \times 3$  matrix called  $F$  representing fraction of each type of footwear sold in each state.

$$F = \begin{array}{c} \text{CA} \quad \text{AR} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{3}{5} \end{bmatrix} \end{array}$$

$$\begin{aligned} c) \quad PF &= \begin{bmatrix} 80 & 40 & 120 \\ 60 & 30 & 150 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{3}{5} \end{bmatrix} \\ &= \begin{bmatrix} 80\frac{1}{2} + 40\frac{1}{4} + 120\frac{1}{4} & 80\frac{1}{5} + 40\frac{1}{5} + 120\frac{3}{5} \\ 60\frac{1}{2} + 30\frac{1}{4} + 150\frac{1}{4} & 60\frac{1}{5} + 30\frac{1}{5} + 150\frac{3}{5} \end{bmatrix} \\ &= \begin{bmatrix} 80 & 96 \\ 75 & 108 \end{bmatrix} \end{aligned}$$