Solution Section 2.9 – Inverse Laplace Transform

Exercise

Find the inverse Laplace Transform of $Y(s)=\frac{1}{3s+2}$

Solution

$$Y(s)=\frac{1}{3}\frac{1}{s+2/3} \qquad \textit{Factor}$$

$$y(t)=\mathcal{L}^{-1}\left\{\frac{1}{3}\frac{1}{s+2/3}\right\}$$

$$=\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+2/3}\right\}$$

$$=\frac{1}{3}e^{-(2/3)t}$$

Exercise

Find the inverse Laplace Transform of $Y(s)=\frac{2}{3-5s}$

Solution

$$Y(s)=-2\frac{1}{5s-3}$$

$$=-\frac{2}{5}\frac{1}{s-\frac{3}{5}}$$

Thus, by linearity;

$$y(t)=-\frac{2}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{3}{5}}\right\}$$

$$=-\frac{2}{5}e^{(3/5)t}$$

Exercise

Find the inverse Laplace Transform of $Y(s)=\frac{1}{s^2+4}$

Solution

$$Y(s)=\frac{1}{2}\frac{2}{s^2+4}$$

$$y(t)=\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$=\frac{1}{2}\sin 2t$$

Exercise

Find the inverse Laplace Transform of $Y(s) = \frac{3}{s^2}$

Solution

$$y(t) = 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$$

$$= 3t$$

Exercise

Find the inverse Laplace Transform of $Y(s) = \frac{3s+2}{s^2+25}$

Solution

$$Y(s) = \frac{3s}{s^2+25} + \frac{2}{s^2+25}$$

$$= 3\frac{s}{s^2+25} + \frac{2}{5}\frac{5}{s^2+25}$$

$$y(t) = 3\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+25}\right\} + \frac{2}{5}\,\mathcal{L}^{-1}\left\{\frac{5}{s^2+25}\right\}$$

$$= 3\cos 5t + \frac{2}{5}\sin 5t$$

Exercise

Find the inverse Laplace Transform of $Y(s) = \frac{2-5s}{s^2+9}$

Solution

$$Y(s) = \frac{2}{s^2+9} - \frac{5s}{s^2+25}$$

$$= \frac{2}{3}\frac{3}{s^2+9} - 5\frac{s}{s^2+9}$$

$$y(t) = \frac{2}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\} - 5\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}$$

$$= \frac{2}{3}\sin 3t - 5\cos 3t$$

Exercise

Find the inverse Laplace Transform of $Y(s) = \frac{5}{(s+2)^3}$

Solution

$$\mathcal{L}^{-1}\left\{\frac{n!}{(s+a)^{n+1}}\right\}=t^n e^{-at} \qquad n=2 \quad a=2$$

$$Y(s)=\frac{5}{2!}\frac{2!}{(s+2)^3}$$

$$y(t)=\mathcal{L}^{-1}\left\{\frac{5}{(s+2)^3}\right\}$$

$$\boxed{=\frac{5}{2}t^2e^{-2t}}$$

Exercise

Find the inverse Laplace Transform of $Y(s)=\dfrac{1}{(s-1)^6}$

Solution

$$\mathcal{L}^{-1}\left\{\frac{n!}{(s+a)^{n+1}}\right\}=t^n e^{-at} \qquad n=5 \quad a=-1$$

$$Y(s)=\frac{1}{5!}\frac{5!}{(s-1)^6}$$

$$y(t)=\mathcal{L}^{-1}\left\{\frac{1}{5!}\frac{5!}{(s-1)^6}\right\}$$

$$\boxed{=\frac{1}{120}t^5e^{t}}$$

Exercise

Find the inverse Laplace Transform of $\qquad Y(s)=\dfrac{4(s-1)}{(s-1)^2+4}$

Solution

$$\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\}=e^{-at}\cos\omega t \qquad a=-1 \quad \omega=2$$

$$Y(s)=4\frac{s-1}{(s-1)^2+4}$$

$$y(t)=\mathcal{L}^{-1}\left\{4\frac{s-1}{(s-1)^2+4}\right\}$$

$$\boxed{=4e^{t}\cos 2t}$$

Exercise

Find the inverse Laplace Transform of $\qquad Y(s)=\dfrac{2s-3}{(s-1)^2+5}$

Solution

$$y(t)=\mathcal{L}^{-1}\left\{\frac{2s-3}{(s-1)^2+5}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{2s-2-1}{(s-1)^2+5}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{2(s-1)}{(s-1)^2+5}-\frac{1}{(s-1)^2+5}\right\}$$

$$=\mathcal{L}^{-1}\left\{2\frac{s-1}{(s-1)^2+5}-\frac{1}{\sqrt{5}}\frac{\sqrt{5}}{(s-1)^2+5}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\}=e^{-at}\cos\omega t$$

$$\mathcal{L}^{-1}\left\{\frac{\omega}{(s+a)^2+\omega^2}\right\}=e^{-at}\sin\omega t$$

$$=2e^{t}\cos\sqrt{5}t-\frac{1}{\sqrt{5}}e^{t}\sin\sqrt{5}t$$

$$\underline{=e^{t}\left(2\cos\sqrt{5}t-\frac{\sqrt{5}}{5}\sin\sqrt{5}t\right)}$$

Exercise

Find the inverse Laplace Transform of $\qquad Y(s)=\dfrac{2s-1}{(s+1)(s-2)}$

Solution

Use partial fraction $\qquad \dfrac{2s-1}{(s+1)(s-2)}=\dfrac{A}{s+1}+\dfrac{B}{s-2}$

$$=\frac{As-2A+Bs+B}{(s+1)(s-2)}$$

$$2s-1=(A+B)s-2A+B$$

$$\begin{cases} A+B=2 \\ -2A+B=-1 \end{cases} \Rightarrow A=B=1$$

$$y(t)=\mathcal{L}^{-1}\left\{\frac{2s-1}{(s+1)(s-2)}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

$$\underline{=e^{-t}+e^{2t}}$$

Exercise

Find the inverse Laplace Transform of $Y(s) = \dfrac{2s-2}{(s-4)(s+2)}$

Solution

$$\frac{2s-2}{(s-4)(s+2)} = \frac{A}{s-4} + \frac{B}{s+2}$$

$$= \frac{As+2A+Bs-4B}{(s-4)(s+2)}$$

$$2s-2 = (A+B)s + 2A - 4B$$

$$\begin{cases} A+B=2 \\ 2A-4B=-2 \end{cases} \Rightarrow A = B = 1$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{2s-2}{(s-4)(s+2)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s-4} + \frac{1}{s+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$$

$$= e^{4t} + e^{-2t}$$

Exercise

Find the inverse Laplace Transform of $Y(s) = \dfrac{7s^2+3s+16}{(s+1)\left(s^2+4\right)}$

Solution

$$\frac{7s^2+3s+16}{(s+1)\left(s^2+4\right)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4}$$

$$= \frac{As^2+4A+Bs^2+Bs+Cs+C}{(s+1)\left(s^2+4\right)}$$

$$= \frac{(A+B)s^2+(B+C)s+4A+C}{(s+1)\left(s^2+4\right)}$$

$$7s^2+3s+16 = (A+B)s^2 + (B+C)s + 4A + C$$

$$\begin{cases} A+B=7 \\ B+C=3 \\ 4A+C=16 \end{cases} \begin{matrix} \to A-C=4 \\ \\ 4A+C=16 \end{matrix} \Rightarrow 5A = 20 \to A = 4 \quad B = 3 \quad C = 0$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{7s^2+3s+16}{(s+1)\left(s^2+4\right)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{4}{s+1}+\frac{3s}{s^2+4}\right\}$$

$$= 4\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}+3\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}$$

$$\underline{= 4e^{-t}+3\cos 2t}$$

Exercise

Find the inverse Laplace Transform of $\quad Y(s) = \dfrac{1}{(s+2)^2\left(s^2+9\right)}$

Solution

$$\frac{1}{(s+2)^2\left(s^2+9\right)} = \frac{A}{s+2}+\frac{B}{(s+2)^2}+\frac{Cs+D}{s^2+9}$$

$$= \frac{A(s+2)\left(s^2+9\right)+Bs^2+9B+(Cs+D)(s^2+4s+4)}{(s+2)^2\left(s^2+9\right)}$$

$$= \frac{As^3+2As^2+9As+18A+Bs^2+9B+Cs^3+4Cs^2+4Cs+Ds^2+4Ds+4D}{(s+2)^2\left(s^2+9\right)}$$

$$1 = (A+C)s^3+(2A+B+4C+D)s^2+(9A+4C+4D)s+18A+9B+4D$$

$$\begin{cases} A+C=0 \\ 2A+B+4C+D=0 \\ 9A+4C+4D=0 \\ 18A+9B+4D=1 \end{cases} \Rightarrow A=\frac{4}{169} \quad B=\frac{1}{13} \quad C=-\frac{4}{169} \quad D=-\frac{5}{169}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2\left(s^2+9\right)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{4}{169}\frac{1}{s+2}+\frac{1}{13}\frac{1}{(s+2)^2}-\frac{1}{169}\frac{4s+5}{s^2+9}\right\}$$

$$= \frac{4}{169}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}+\frac{1}{13}\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\}-\frac{4}{169}\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}-\frac{5}{169}\mathcal{L}^{-1}\left\{\frac{1}{3}\frac{3}{s^2+9}\right\}$$

$$\underline{= \frac{4}{169}e^{-2t}+\frac{1}{13}te^{-2t}-\frac{4}{169}\cos 3t-\frac{5}{507}\sin 3t}$$

Exercise

Find the inverse Laplace Transform of $\quad Y(s)=\dfrac{s}{(s+2)^2\left(s^2+9\right)}$

Solution

$$\frac{s}{(s+2)^2\left(s^2+9\right)}=\frac{A}{s+2}+\frac{B}{(s+2)^2}+\frac{Cs+D}{s^2+9}$$

$$=\frac{As^3+2As^2+9As+18A+Bs^2+9B+Cs^3+4Cs^2+4Cs+Ds^2+4Ds+4D}{(s+1)^2\left(s^2+9\right)}$$

$$s=(A+C)s^3+(2A+B+4C+D)s^2+(9A+4C+4D)s+18A+9B+4D$$

$$\begin{cases} A+C=0 \\ 2A+B+4C+D=0 \\ 9A+4C+4D=1 \\ 18A+9B+4D=0 \end{cases} \Rightarrow A=\frac{5}{169} \quad B=-\frac{2}{13} \quad C=-\frac{5}{169} \quad D=\frac{36}{169}$$

$$y(t)=\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2\left(s^2+9\right)}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{5}{169}\frac{1}{s+2}-\frac{2}{13}\frac{1}{(s+2)^2}-\frac{1}{169}\frac{5s+36}{s^2+9}\right\}$$

$$=\frac{5}{169}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}-\frac{2}{13}\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\}-\frac{5}{169}\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}-\frac{36}{169}\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$=\frac{5}{169}e^{-2t}-\frac{2}{13}te^{-2t}-\frac{5}{169}\cos 3t+\frac{12}{169}\sin 3t$$

Exercise

Find the inverse Laplace Transform of $\quad Y(s)=\dfrac{1}{(s+1)^2\left(s^2-4\right)}$

Solution

$$\frac{1}{(s+1)^2\left(s^2-4\right)}=\frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{Cs+D}{s^2-4}$$

$$=\frac{A(s+1)\left(s^2-4\right)+B\left(s^2-4\right)+(Cs+D)(s+1)^2}{(s+1)^2\left(s^2-4\right)}$$

$$1=As^3-4As+As^2-4A+Bs^2-4B+Cs^3+2Cs^2+Cs+Ds^2+2Ds+D$$

$$=(A+C)s^3+(A+B+2C+D)s^2+(-4A+C+2D)s-4A-4B+D$$

$$\begin{array}{l} s^3 \\ s^2 \\ s^1 \\ s^0 \end{array}\begin{cases} A+C=0 \\ A+B+2C+D=0 \\ -4A+C+2D=0 \\ -4A-4B+D=1 \end{cases} \qquad \begin{array}{ll} A=-\frac{2}{15} & B=\frac{1}{5} \\ C=\frac{2}{15} & D=-\frac{1}{3} \end{array}$$

$$Y(s)=-\frac{2}{15}\frac{1}{s+1}+\frac{1}{5}\frac{1}{(s+1)^2}+\frac{\frac{2}{15}s-\frac{1}{3}}{s^2-4}$$

$$=-\frac{2}{15}\frac{1}{s+1}+\frac{1}{5}\frac{1}{(s+1)^2}+\frac{2}{15}\frac{s}{s^2-4}-\frac{1}{3}\frac{1}{s^2-4}$$

$$y(t)=-\frac{2}{15}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}+\frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}+\frac{2}{15}\mathcal{L}^{-1}\left\{\frac{s}{s^2-4}\right\}-\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2-4}\right\}$$

$$=-\frac{2}{15}e^{-t}+\frac{1}{5}te^{-t}+\frac{2}{15}\cosh 2t-\frac{1}{6}\sinh 2t$$

$$=-\frac{2}{15}e^{-t}+\frac{1}{5}te^{-t}+\frac{2}{15}\frac{e^{2t}+e^{-2t}}{2}-\frac{1}{6}\frac{e^{2t}-e^{-2t}}{2}$$

$$=-\frac{2}{15}e^{-t}+\frac{1}{5}te^{-t}+\frac{1}{15}e^{2t}+\frac{1}{15}e^{-2t}-\frac{1}{12}e^{2t}+\frac{1}{12}e^{-2t}$$

$$=-\frac{2}{15}e^{-t}+\frac{1}{5}te^{-t}-\frac{1}{60}e^{2t}+\frac{3}{20}e^{-2t}$$

Exercise

Find the inverse Laplace Transform of $\quad Y(s)=\dfrac{7s^2+20s+53}{(s-1)\left(s^2+2s+5\right)}$

Solution

$$\frac{7s^2+20s+53}{(s-1)\left(s^2+2s+5\right)}=\frac{A}{s-1}+\frac{Bs+C}{s^2+2s+5}$$

$$7s^2+20s+53=As^2+2As+5A+Bs^2-Bs+Cs-C$$

$$\begin{array}{l} s^2 \\ s^1 \\ s^0 \end{array}\begin{cases} A+B=7 \\ 2A-B+C=20 \\ 5A-C=53 \end{cases} \Rightarrow \begin{cases} A=10 \\ B=-3 \\ C=-3 \end{cases}$$

$$Y(x)=\frac{10}{s-1}+\frac{-3s-3}{s^2+2s+5}$$

$$=\frac{10}{s-1}-3\frac{s+1}{s^2+2s+5}$$

$$y(t)=10\mathcal{L}^{-1}\left\{\frac{10}{s-1}\right\}-3\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\}$$

$$=10e^{t}-3e^{-t}\cos 2t$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{1}{s^3}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$$

$$=\frac{1}{2}t^2$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{1}{s^4}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$$

$$=\frac{1}{6}t^3$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{1}{s^2}-\frac{48}{s^5}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{s^2}-\frac{48}{s^5}\right\}$$

$$=t-2t^4$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s-2}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s-2}\right\}$$

$$=t-1+e^{2t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{4}{s}+\frac{4}{s^5}+\frac{1}{s-8}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{4}{s}+\frac{4}{s^5}+\frac{1}{s-8}\right\}$$

$$\boxed{=4+\frac{1}{6}t^4+e^{8t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{1}{4s+1}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{4}\frac{1}{s+\frac{1}{4}}\right\}$$

$$\boxed{=\frac{1}{4}e^{-t/2}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{1}{5s-2}$

Solution

$$f(t)=\frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{2}{5}}\right\}$$

$$\boxed{=\frac{1}{5}e^{-2t/5}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{s+1}{s^2+2}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s}{s^2+2}+\frac{1}{s^2+2}\right\}$$

$$\boxed{=\cos\sqrt{2}t+\frac{1}{2}\sin\sqrt{2}t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{2s-6}{s^2+9}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{2s}{s^2+9}-\frac{6}{s^2+9}\right\}$$

$$\underline{=2\cos 3t-2\sin 3t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{10s}{s^2+16}$

Solution

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{\frac{10s}{s^2+16}\right\}$$

$$\underline{f(t)=10\cos 4t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\left(\dfrac{2}{s}-\dfrac{1}{s^3}\right)^2$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{4}{s^2}-\frac{4}{s^4}+\frac{1}{s^6}\right\}$$

$$\underline{=4t-\frac{2}{3}t^3+\frac{1}{5!}t^5}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{(s+1)^3}{s^4}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s^3+3s^2+3s+1}{s^4}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{s}+\frac{3}{s^2}+\frac{3}{s^3}+\frac{1}{s^4}\right\}$$

$$\underline{=1+3t+\frac{3}{2}t^2+\frac{1}{6}t^3}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{(s+2)^2}{s^3}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s^2+4s+4}{s^3}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{s}+\frac{4}{s^2}+\frac{4}{s^3}\right\}$$

$$=1+4t+2t^2$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{1}{s^4-9}$

Solution

$$F(s)=\frac{1}{s^4-9}=\frac{A}{s-\sqrt{3}}+\frac{B}{s+\sqrt{3}}+\frac{Cs+D}{s^2+3} \qquad s^4-9=\left(s^2-3\right)\left(s^2+3\right)=\left(s-\sqrt{3}\right)\left(s+\sqrt{3}\right)\left(s^2+3\right)$$

$$A\left(s+\sqrt{3}\right)\left(s^2+3\right)+B\left(s-\sqrt{3}\right)\left(s^2+3\right)+Cs^3-3Cs+Ds^2-3D=1$$

$$As^3+3As+As^2\sqrt{3}+3A\sqrt{3}+Bs^3+3Bs-Bs^2\sqrt{3}-3B\sqrt{3}+Cs^3-3Cs+Ds^2-3D=1$$

$$\begin{array}{ll} s^3 & A+B+C=0 \\ s^2 & \sqrt{3}A-\sqrt{3}B+D=0 \\ s^1 & 3A+3B-3C=0 \\ s^0 & 3\sqrt{3}A-3\sqrt{3}B-3D=1 \end{array} \quad \to \begin{cases} A+B+C=0 \\ A+B-C=0 \\ \sqrt{3}A-\sqrt{3}B+D=0 \\ \sqrt{3}A-\sqrt{3}B-D=1 \end{cases} \quad \to \begin{cases} A+B=0 \\ 2\sqrt{3}A-2\sqrt{3}B=1 \end{cases}$$

$$A=\frac{1}{4\sqrt{3}}=\frac{\sqrt{3}}{12} \qquad B=-\frac{\sqrt{3}}{12}$$

$$C=A+B=0 \qquad D=\sqrt{3}B-\sqrt{3}A=-\frac{1}{4}$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{\frac{\sqrt{3}}{12}\frac{1}{s-\sqrt{3}}-\frac{\sqrt{3}}{12}\frac{1}{s+\sqrt{3}}-\frac{1}{4}\frac{\sqrt{3}}{\sqrt{3}}\frac{1}{s^2+3}\right\}$$

$$f(t)=\frac{\sqrt{3}}{12}\left(e^{\sqrt{3}t}-e^{-\sqrt{3}t}-\sin\sqrt{3}t\right)$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{1}{s^3+5s}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{s\left(s^2+5\right)}\right\}$$

$$\frac{1}{s\left(s^2+5\right)}=\frac{A}{s}+\frac{Bs+C}{s^2+5}$$

$$1=As^2+5A+Bs^2+Cs$$

$$s^2 \quad A+B=0 \quad \boxed{B=-\frac{1}{5}}$$

$$s \quad \boxed{C=0}$$

$$s^0 \quad 5A=1 \quad \boxed{A=\frac{1}{5}}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{5}\frac{1}{s}-\frac{1}{5}\frac{s}{s^2+5}\right\}$$

$$\boxed{=\frac{t}{5}+\frac{1}{5}\cos\sqrt{5}t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{5}{s^2+36}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{5}{s^2+36}\right\} \qquad \mathcal{L}^{-1}\left\{\frac{\omega}{s^2+\omega^2}\right\}=\sin\omega t$$

$$\boxed{=\frac{5}{6}\sin 6t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\frac{10s}{s^2+16}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{10s}{s^2+16}\right\} \qquad \mathcal{L}^{-1}\left\{\frac{s}{s^2+\omega^2}\right\}=\cos\omega t$$

$$\boxed{=10\cos 4t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{4s}{4s^2+1}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{s}{s^2+\frac{1}{4}}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+\omega^2}\right\}=\cos\omega t$$

$$=\cos\frac{1}{2}t$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{1}{4s^2+1}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{4s^2+1}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{4}\frac{s}{s^2+\frac{1}{4}}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+\omega^2}\right\}=\cos\omega t$$

$$=\frac{1}{4}\cos\frac{1}{2}t$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{1}{s^2+3s}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{s(s+3)}\right\}$$

$$\frac{1}{s(s+3)}=\frac{A}{s}+\frac{B}{s+3}$$

$$1=As+3A+Bs$$

$$s\qquad A+B=0\qquad B=-\frac{1}{3}$$

$$s^0\qquad 3A=1\qquad A=\frac{1}{3}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{3}\frac{1}{s}-\frac{1}{3}\frac{1}{s+3}\right\}$$

$$=\frac{1}{3}t-\frac{1}{3}e^{-3t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s+1}{s^2-4s}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s+1}{s(s-4)}\right\}$$

$$\frac{s+1}{s(s-4)}=\frac{A}{s}+\frac{B}{s-4}$$

$$s+1=As-4A+Bs$$

$$s \qquad A+B=1 \qquad B=\frac{5}{4}$$

$$s^0 \qquad -4A=1 \qquad A=-\frac{1}{4}$$

$$=\mathcal{L}^{-1}\left\{-\frac{1}{4}\frac{1}{s}+\frac{5}{4}\frac{1}{s-4}\right\}$$

$$=-\frac{1}{4}t+\frac{5}{4}e^{4t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{1}{s^3+5s}$

Solution

$$F(s)=\frac{1}{s^3+5s}=\frac{A}{s}+\frac{Bs+C}{s^2+5}$$

$$As^2+5A+Bs^2+Cs=1$$

$$s^2 \qquad A+B=0 \qquad B=-\frac{1}{5}$$

$$s^1 \qquad C=0$$

$$s^0 \qquad 5A=1 \qquad A=\frac{1}{5}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{5}\frac{1}{s}-\frac{1}{5}\frac{s}{s^2+5}\right\}$$

$$=\frac{1}{5}\left(t-\cos\sqrt{5}t\right)$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{3}{s^2+9}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{\omega}{s^2+\omega^2}\right\}=\sin\omega t$$

$$=\sin 3t$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{2}{s^2+4}$

Solution

$$f(t) = \mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\} \qquad\qquad \mathcal{L}^{-1}\left\{\frac{\omega}{s^2+\omega^2}\right\} = \sin\omega t$$

$$\underline{= \sin 2t \,|}$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{3}{(2s+5)^3}$

Solution

$$f(t) = \mathcal{L}^{-1}\left\{\frac{3}{(2s+5)^3}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{3}{2^3\left(s+\frac{5}{8}\right)^3}\right\} \qquad\qquad \mathcal{L}^{-1}\left\{\frac{n!}{(s+a)^{n+1}}\right\} = t^n e^{-at}$$

$$\underline{= \frac{3}{16}t^2 e^{-5t/8} \,|}$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{6}{(s-1)^4}$

Solution

$$f(t) = \mathcal{L}^{-1}\left\{\frac{6}{(s-1)^4}\right\} \qquad\qquad \mathcal{L}^{-1}\left\{\frac{n!}{(s+a)^{n+1}}\right\} = t^n e^{-at}$$

$$\underline{= t^3 e^t \,|}$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{5}{(s+2)^4}$

Solution

$$f(t) = \mathcal{L}^{-1}\left\{\frac{5}{(s+2)^4}\right\} \qquad\qquad \mathcal{L}^{-1}\left\{\frac{n!}{(s+a)^{n+1}}\right\} = t^n e^{-at}$$

$$= \frac{5}{3!}\mathcal{L}^{-1}\left\{\frac{3!}{(s+2)^4}\right\}$$

$$=\boxed{\frac{5}{6}t^3e^{-2t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s-1}{s^2-2s+5}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s-1}{s^2-2s+5}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\} \qquad \mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\}=e^{-at}\cos\omega t$$

$$=\boxed{e^t\cos 2t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{3s+2}{s^2+2s+10}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{3s+2}{s^2+2s+10}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{3s+3-1}{(s+1)^2+9}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{3(s+1)}{(s+1)^2+9}-\frac{1}{(s+1)^2+9}\right\} \qquad \mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\}=e^{-at}\cos\omega t$$

$$=\boxed{3e^t\cos 3t-\frac{1}{3}e^{-t}\sin 3t} \qquad \mathcal{L}^{-1}\left\{\frac{\omega}{(s+a)^2+\omega^2}\right\}=e^{-at}\sin\omega t$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s}{s^2+2s-3}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s}{(s-1)(s+3)}\right\}$$

$$\frac{s}{(s-1)(s+3)}=\frac{A}{s-1}+\frac{B}{s+3}$$

$$s=As+3A+Bs-B$$

$$s \qquad A+B=1 \qquad \boxed{B=\frac{3}{4}}$$

$$s^0 \qquad 3A-B=0 \qquad \boxed{A=\frac{1}{4}}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{4}\frac{1}{s-1}+\frac{3}{4}\frac{1}{s+3}\right\}$$

$$=\boxed{-\frac{1}{4}e^t+\frac{3}{4}e^{-3t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{1}{s^2+2s-20}$

Solution

$$s^2+2s-20=0 \quad\to\quad \boxed{s_{1,2}=-1\pm 2\sqrt{21}}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{\left(s+1+2\sqrt{21}\right)\left(s+1-2\sqrt{21}\right)}\right\}$$

$$\frac{1}{\left(s+1+2\sqrt{21}\right)\left(s+1-2\sqrt{21}\right)}=\frac{A}{s+1+2\sqrt{21}}+\frac{B}{s+1-2\sqrt{21}}$$

$$1=sA+\left(1-2\sqrt{21}\right)A+sB+\left(1+2\sqrt{21}\right)B$$

$$s \qquad A+B=0 \qquad A=-B$$

$$s^0 \qquad \left(1-2\sqrt{21}\right)A+\left(1+2\sqrt{21}\right)B=1 \quad\to\left(1-2\sqrt{21}-1-2\sqrt{21}\right)A=1$$

$$\boxed{A=-\frac{1}{4\sqrt{21}} \qquad B=\frac{1}{4\sqrt{21}}}$$

$$=\mathcal{L}^{-1}\left\{-\frac{1}{4\sqrt{21}}\frac{1}{s+1+2\sqrt{21}}+\frac{1}{4\sqrt{21}}\frac{1}{s+1-2\sqrt{21}}\right\}$$

$$\boxed{=-\frac{1}{4\sqrt{21}}e^{\left(-1-2\sqrt{21}\right)t}+\frac{1}{4\sqrt{21}}e^{\left(-1+2\sqrt{21}\right)t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s+1}{s^2+2s+10}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s+10}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+9}\right\} \qquad\qquad \mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\}=e^{-at}\cos\omega t$$

$$\boxed{=e^{-t}\cos 3t}$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{1}{s^2 + 4s + 8}$

Solution

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2 + 4s + 8}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2 + 4}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{\omega}{(s+a)^2 + \omega^2}\right\} = e^{-at}\sin \omega t$$

$$= \frac{1}{2}e^{-2t}\sin 2t$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{2s + 16}{s^2 + 4s + 13}$

Solution

$$f(t) = \mathcal{L}^{-1}\left\{\frac{2s + 16}{s^2 + 4s + 13}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2) + 12}{(s+2)^2 + 9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2)}{(s+2)^2 + 3^2} + \frac{4(3)}{(s+2)^2 + 3^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{\omega}{(s+a)^2 + \omega^2}\right\} = e^{-at}\sin \omega t$$

$$\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2 + \omega^2}\right\} = e^{-at}\cos \omega t$$

$$= \frac{1}{2}e^{-2t}\sin 2t$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{s - 1}{2s^2 + s + 6}$

Solution

$$\frac{s-1}{2s^2 + s + 6} = \frac{s-1}{2\left(s^2 + \frac{1}{2}s + 3\right)}$$

$$= \frac{1}{2}\frac{s-1}{\left(s + \frac{1}{4}\right)^2 + \frac{47}{16}}$$

$$=\frac{1}{2}\frac{s+\frac{1}{4}-\frac{5}{4}}{\left(s+\frac{1}{4}\right)^2+\frac{47}{16}}$$

$$=\frac{1}{2}\left[\frac{s+\frac{1}{4}}{\left(s+\frac{1}{4}\right)^2+\frac{47}{16}}-\frac{5}{4}\frac{1}{\left(s+\frac{1}{4}\right)^2+\frac{47}{16}}\right]$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s-1}{2s^2+s+6}\right\}$$

$$=\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+\frac{1}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{47}}{4}\right)^2}-\frac{5}{4}\frac{4}{\sqrt{47}}\frac{\frac{\sqrt{47}}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{47}}{4}\right)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{\omega}{(s+a)^2+\omega^2}\right\}=e^{-at}\sin\omega t \qquad \mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\}=e^{-at}\cos\omega t$$

$$=\frac{1}{2}e^{-t/4}\cos\left(\frac{\sqrt{47}}{4}t\right)-\frac{5}{2\sqrt{47}}e^{-t/4}\sin\left(\frac{\sqrt{47}}{4}t\right)$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s^2+1}{s^3-2s^2-8s}$

Solution

$$\frac{s^2+1}{s^3-2s^2-8s}=\frac{A}{s}+\frac{B}{s-4}+\frac{C}{s+2}$$

$$s^2+1=As^2-2As-8A+Bs^2+2Bs+Cs^2-4Cs$$

$$\begin{array}{l} s^2 \\ s^1 \\ s^0 \end{array}\begin{cases} A+B+C=1 \\ -2A+2B-4C=0 \\ -8A=1 \end{cases} \Rightarrow A=-\frac{1}{8}\quad B=\frac{17}{24}\quad C=\frac{5}{12}$$

$$F(s)=-\frac{1}{8}\frac{1}{s}+\frac{17}{24}\frac{1}{s-4}+\frac{5}{12}\frac{1}{s+2}$$

$$f(t)=\mathcal{L}^{-1}\left\{-\frac{1}{8}\frac{1}{s}+\frac{17}{24}\frac{1}{s-4}+\frac{5}{12}\frac{1}{s+2}\right\}$$

$$=-\frac{1}{8}t+\frac{17}{24}e^{4t}+\frac{5}{12}e^{-2t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{6s+3}{s^4+5s^2+4}$

Solution

$$F(s)=\frac{6s+3}{s^4+5s^2+4}=\frac{As+B}{s^2+1}+\frac{Cs+D}{s^2+4}$$

$$6s+3=As^3+4As+Bs^2+4B+Cs^3+Cs+Ds^2+D$$

$$\begin{array}{lll} s^3 & A+C=0 & \\ s^2 & B+D=0 & A=2 \quad B=1 \\ s^1 & 4A+C=6 & C=-2 \quad D=-1 \\ s^0 & 4B+D=3 & \end{array}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{2s}{s^2+1}+\frac{1}{s^2+1}-\frac{2s}{s^2+4}-\frac{1}{s^2+4}\right\}$$

$$=2\cos t+\sin t-2\cos 2t-\frac{1}{2}\sin 2t$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s-3}{\left(s-\sqrt{3}\right)\left(s+\sqrt{3}\right)}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s-3}{\left(s-\sqrt{3}\right)\left(s+\sqrt{3}\right)}\right\}$$

$$\frac{s-3}{\left(s-\sqrt{3}\right)\left(s+\sqrt{3}\right)}=\frac{A}{s-\sqrt{3}}+\frac{B}{s+\sqrt{3}}$$

$$s-3=sA+\sqrt{3}A+sB-\sqrt{3}B$$

$$\begin{array}{ll} s & A+B=1 \\ s^0 & \sqrt{3}A-\sqrt{3}B=-3 \end{array}$$

$$\Delta=\begin{vmatrix}1 & 1\\ \sqrt{3} & -\sqrt{3}\end{vmatrix}=-2\sqrt{3} \quad \Delta_A=\begin{vmatrix}1 & 1\\ -3 & -\sqrt{3}\end{vmatrix}=3-\sqrt{3} \quad \Delta_B=\begin{vmatrix}1 & 1\\ \sqrt{3} & -3\end{vmatrix}=-3-\sqrt{3}$$

$$A=\frac{-3+\sqrt{3}}{2\sqrt{3}}=\frac{1-\sqrt{3}}{2} \qquad B=\frac{3+\sqrt{3}}{2\sqrt{3}}=\frac{\sqrt{3}+1}{2}$$

$$=\mathcal{L}^{-1}\left\{\frac{1-\sqrt{3}}{2}\frac{1}{s-\sqrt{3}}+\frac{1+\sqrt{3}}{2}\frac{1}{s+\sqrt{3}}\right\}$$

$$=\frac{1-\sqrt{3}}{2}e^{\sqrt{3}t}+\frac{1+\sqrt{3}}{2}e^{\sqrt{3}t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{1}{\left(s^2+1\right)\left(s^2+4\right)}$

Solution

$$F(s)=\frac{1}{\left(s^2+1\right)\left(s^2+4\right)}=\frac{A}{s^2+1}+\frac{B}{s^2+4}$$

$$(A+B)s^2+4A+B=1$$

$$\begin{cases} A+B=0 \\ 4A+B=1 \end{cases} \rightarrow \quad A=\frac{1}{3};\ B=-\frac{1}{3}$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{\frac{1}{3}\frac{1}{s^2+1}-\frac{1}{3}\frac{2}{2}\frac{1}{s^2+4}\right\}$$

$$f(t)=\frac{1}{3}\sin t-\frac{1}{6}\sin 2t$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{2s-4}{\left(s^2+s\right)\left(s^2+1\right)}$

Solution

$$F(s)=\frac{2s-4}{\left(s^2+s\right)\left(s^2+1\right)}=\frac{A}{s}+\frac{B}{s+1}+\frac{Cs+D}{s^2+1}$$

$$As^3+4As+As^2+4A+Bs^3+Bs+Cs^3+Cs^2+Ds^2+Ds=2s-4$$

$s^3 \qquad A+B+C=0$

$s^2 \qquad A+C+D=0$

$s^1 \qquad 4A+B+D=2$

$s^0 \qquad 4A=-4\rightarrow A=-1$

$$\rightarrow\begin{cases} B+C=1 & & \rightarrow & 6-D+1-D=1\Rightarrow D=3 \\ C+D=1 & \rightarrow & C=1-D & \rightarrow C=-2 \\ B+D=6 & \rightarrow & B=6-D & \rightarrow B=3 \end{cases}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{-1}{s}+\frac{3}{s+1}-\frac{2s}{s^2+1}+\frac{3}{s^2+1}\right\}$$

$$=-t+3e^{-t}-2\cos t+3\sin t$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s}{(s+2)\left(s^2+4\right)}$

Solution

$$F(s)=\frac{s}{(s+2)\left(s^2+4\right)}=\frac{A}{s+2}+\frac{Bs+C}{s^2+4}$$

$$As^2+4A+Bs^2+2Bs+Cs+2C=s$$

$$\begin{array}{llll} s^2 & A+B=0 & \to A=-B & \boxed{A=-\frac{1}{4}} \\ s^1 & 2B+C=1 & \to C=1-2B & \boxed{C=\frac{1}{2}} \\ s^0 & 4A+2C=0 & \Rightarrow \quad -4B+2-4B=0 \Rightarrow & \boxed{B=\frac{1}{4}} \end{array}$$

$$f(t)=\mathcal{L}^{-1}\left\{-\frac{1}{4}\frac{1}{s+2}+\frac{1}{4}\frac{s}{s^2+4}+\frac{1}{2}\frac{2}{2}\frac{1}{s^2+4}\right\}$$

$$\boxed{=-\frac{1}{4}e^{-2t}+\frac{1}{4}\cos 2t+\frac{1}{4}\sin 2t}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s^2+1}{s(s-1)(s+1)(s-2)}$

Solution

$$F(s)=\frac{s^2+1}{s(s-1)(s+1)(s-2)}=\frac{A}{s}+\frac{B}{s-1}+\frac{C}{s+1}+\frac{D}{s-2}$$

$$A\left(s^2-1\right)(s-2)+Bs(s+1)(s-2)+Cs(s-1)(s-2)+Ds\left(s^2-1\right)=s^2+1$$

$$As^3-2As^2-As+2A+Bs^3-Bs^2-2Bs+Cs^3-3Cs^2+2Cs+Ds^3-Ds=s^2+1$$

$$\begin{array}{ll} s^3 & A+B+C+D=0 \\ s^2 & -2A-B-3C=1 \\ s^1 & -A-2B+2C-D=0 \\ s^0 & 2A=1\to A=\frac{1}{2} \end{array} \to \begin{cases} B+C+D=-\frac{1}{2} & B=-1 \\ B+3C=-2 & \to C=-\frac{1}{3} \\ -2B+2C-D=\frac{1}{2} & D=\frac{5}{6} \end{cases}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{2}\frac{1}{s}-\frac{1}{s-1}-\frac{1}{3}\frac{1}{s+1}+\frac{5}{6}\frac{1}{s-2}\right\}$$

$$\boxed{=\frac{1}{2}t-e^{t}-\frac{1}{3}e^{-t}+\frac{5}{6}e^{2t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s}{(s-2)(s-3)(s-6)}$

Solution

$$F(s)=\frac{s}{(s-2)(s-3)(s-6)}=\frac{A}{s-2}+\frac{B}{s-3}+\frac{C}{s-6}$$

$$As^2-9As+18A+Bs^2-8Bs+12B+Cs^2-5Cs+6C=s$$

$$\begin{array}{ll} s^2 & A+B+C=0 \\ s^1 & -9A-8B-5C=1 \quad \rightarrow \quad \underline{A=\frac{1}{2} \quad B=-1 \quad C=\frac{1}{2}} \\ s^0 & 18A+12B+6C=0 \end{array}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{1}{2}\frac{1}{s-2}-\frac{1}{s-3}+\frac{1}{2}\frac{1}{s-6}\right\}$$

$$\underline{=\frac{1}{2}e^{2t}-e^{3t}+\frac{1}{2}e^{6t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{7s-1}{(s+1)(s+2)(s-3)}$

Solution

$$f(t)=\mathcal{L}^{-1}\left\{\frac{7s-1}{(s+1)(s+2)(s-3)}\right\}$$

$$\frac{7s-1}{(s+1)(s+2)(s-3)}=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s-3}$$

$$7s-1=As^2-As-6A+Bs^2-2Bs-3B+Cs^2+3Cs+2C$$

$$\begin{array}{ll} s^2 & A+B+C=0 \\ s & -A-2B+3C=7 \\ s^0 & -6A-3B+2C=-1 \end{array}$$

$$\Delta=\begin{vmatrix}1&1&1\\-1&-2&3\\-6&-3&2\end{vmatrix}=-20 \quad \Delta_A=\begin{vmatrix}0&1&1\\7&-2&3\\-1&-3&2\end{vmatrix}=-40 \quad \Delta_B=\begin{vmatrix}1&0&1\\-1&7&3\\-6&-1&2\end{vmatrix}=60$$

$$\underline{A=2, \quad B=-3, \quad C=1}$$

$$=\mathcal{L}^{-1}\left\{\frac{2}{s+1}-\frac{3}{s+2}+\frac{1}{s-3}\right\}$$

$$\underline{=2e^{-t}-3e^{-2t}+e^{3t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{s^2+9s+2}{(s-1)^2(s+3)}$

Solution

$$\frac{s^2+9s+2}{(s-1)^2(s+3)}=\frac{A}{s-1}+\frac{B}{(s-1)^2}+\frac{C}{s+3}$$

$$s^2+9s+2=As^2+2As-3A+Bs+3B+Cs^2-2Cs+C$$

$s^2 \quad A+C=1$

$s \quad 2A+B-2C=9$

$s^0 \quad -3A+3B+C=2$

$$\Delta=\begin{vmatrix}1&0&1\\2&1&-2\\-3&3&1\end{vmatrix}=16 \quad \Delta_A=\begin{vmatrix}1&0&1\\9&1&-2\\2&3&1\end{vmatrix}=32 \quad \Delta_B=\begin{vmatrix}1&1&1\\2&9&-2\\-3&2&1\end{vmatrix}=48$$

$\underline{A=2,\quad B=3,\quad C=-1}$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{s^2+9s+2}{(s-1)^2(s+3)}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{2}{s-1}+\frac{3}{(s-1)^2}-\frac{1}{s+3}\right\}$$

$$\underline{=2e^t+3te^t-e^{-3t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{2s^2+10s}{\left(s^2-2s+5\right)(s+1)}$

Solution

$$\frac{2s^2+10s}{\left(s^2-2s+5\right)(s+1)}=\frac{2s^2+10s}{\left((s-1)^2+4\right)(s+1)}=\frac{A(s-1)+B}{(s-1)^2+4}+\frac{C}{s+1}$$

$$2s^2+10s=As^2-A+Bs+B+Cs^2-2Cs+5C$$

$s^2 \quad A+C=2$

$s \quad B-2C=10$

$s^0 \quad -A+B+5C=0$

$$\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 5 \end{vmatrix} = 8 \quad \Delta_A = \begin{vmatrix} 2 & 0 & 1 \\ 10 & 1 & -2 \\ 0 & 1 & 5 \end{vmatrix} = 24 \quad \Delta_B = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 10 & -2 \\ -1 & 0 & 5 \end{vmatrix} = 64$$

$$\underline{A = 3, \quad B = 8, \quad C = -1}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{2s^2 + 10s}{\left(s^2 - 2s + 5\right)(s+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{3(s-1)}{(s-1)^2 + 4} + \frac{4(2)}{(s-1)^2 + 4} - \frac{1}{s+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2 + \omega^2}\right\} = e^{-at}\cos \omega t \qquad \mathcal{L}^{-1}\left\{\frac{\omega}{(s+a)^2 + \omega^2}\right\} = e^{-at}\sin \omega t$$

$$\underline{= 3e^t \cos 2t + 4e^t \sin 2t - e^{-t}}$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{s^2 - 26s - 47}{(s-1)(s+2)(s+5)}$

Solution

$$\frac{s^2 - 26s - 47}{(s-1)(s+2)(s+5)} = \frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$s^2 - 26s - 47 = As^2 + 7As + 10A + Bs^2 + 4Bs - 5B + Cs^2 + Cs - 2C$$

$s^2 \qquad A + B + C = 1$

$s \qquad 7A + 4B + C = -26$

$s^0 \qquad 10A - 5B - 2C = -47$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 7 & 4 & 1 \\ 10 & -5 & -2 \end{vmatrix} = -54 \quad \Delta_A = \begin{vmatrix} 1 & 1 & 1 \\ -26 & 4 & 1 \\ -47 & -5 & -2 \end{vmatrix} = 216 \quad \Delta_B = \begin{vmatrix} 1 & 1 & 1 \\ 7 & -26 & 1 \\ 10 & -47 & -2 \end{vmatrix} = 54$$

$$\underline{A = -4, \quad B = -1, \quad C = 6}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{s^2 - 26s - 47}{(s-1)(s+2)(s+5)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-4}{s-1} - \frac{1}{s+2} + \frac{6}{s+5}\right\}$$

$$\underline{= -4e^t - e^{-2t} + 6e^{-5t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{-s-7}{(s-1)(s+2)}$

Solution

$$\frac{-s-7}{(s-1)(s+2)}=\frac{A}{s-1}+\frac{B}{s+2}$$

$$-s-7=As+2A+Bs-B$$

$$s \qquad A+B=-1$$

$$s^0 \qquad 2A-B=-7$$

$$\boxed{A=-\frac{8}{3},\quad B=\frac{5}{3}}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{-s-7}{(s-1)(s+2)}\right\}$$

$$=\mathcal{L}^{-1}\left\{-\frac{8}{3}\frac{1}{s-1}+\frac{5}{3}\frac{1}{s+2}\right\}$$

$$\boxed{=-\frac{8}{3}e^{t}+\frac{5}{3}e^{-2t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{-8s^2-5s+9}{\left(s^2-3s+2\right)(s+1)}$

Solution

$$\frac{-8s^2-5s+9}{\left(s^2-3s+2\right)(s+1)}=\frac{-8s^2-5s+9}{(s-1)(s-2)(s+1)}$$

$$=\frac{A}{s-1}+\frac{B}{s-2}+\frac{C}{s+1}$$

$$-8s^2-5s+9=As^2-As-2A+Bs^2-B+Cs^2-3Cs+2C$$

$$s^2 \qquad A+B+C=-8$$

$$s \qquad -A-3C=-5$$

$$s^0 \qquad -2A-B+2C=9$$

$$\Delta=\begin{vmatrix}1&1&1\\-1&0&-3\\-2&-1&2\end{vmatrix}=6 \quad \Delta_A=\begin{vmatrix}-8&1&1\\-5&0&-3\\9&-1&2\end{vmatrix}=12 \quad \Delta_B=\begin{vmatrix}1&-8&1\\-1&-5&-3\\-2&9&2\end{vmatrix}=-66$$

$$\boxed{A=2,\quad B=-11,\quad C=1}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{-8s^2-5s+9}{\left(s^2-3s+2\right)(s+1)}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{2}{s-1}-\frac{11}{s-2}+\frac{1}{s+1}\right\}$$

$$\underline{=2e^t-11e^{2t}+e^{-t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{-2s^2+8s-14}{(s+1)\left(s^2-2s+5\right)}$

Solution

$$\frac{-2s^2+8s-14}{(s+1)\left(s^2-2s+5\right)}=\frac{-2s^2+8s-14}{(s-1)\left((s-1)^2+4\right)}$$

$$=\frac{A}{s-1}+\frac{B(s-1)+C}{(s-1)^2+4}$$

$$-2s^2+8s-14=As^2-2As+5A+Bs^2-2Bs+B+Cs-C$$

$s^2 \quad A+B=-2$

$s \quad -2A-2B+C=8$

$s^0 \quad 5A+B-C=-14$

$$\Delta=\begin{vmatrix}1&1&0\\-2&-2&1\\5&1&-1\end{vmatrix}=4 \quad \Delta_A=\begin{vmatrix}-2&1&0\\8&-2&1\\-14&1&-1\end{vmatrix}=-8 \quad \Delta_B=\begin{vmatrix}1&-2&0\\-2&8&1\\5&-14&-1\end{vmatrix}=0 \quad \Delta_C=\begin{vmatrix}1&1&-2\\-2&-2&8\\5&1&-14\end{vmatrix}=16$$

$$\underline{A=-2,\quad B=0,\quad C=4}$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{-2s^2+8s-14}{(s+1)\left(s^2-2s+5\right)}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{-2}{s-1}+\frac{4}{(s-1)^2+2^2}\right\} \qquad \mathcal{L}^{-1}\left\{\frac{\omega}{(s+a)^2+\omega^2}\right\}=e^{-at}\sin\omega t$$

$$\underline{=-2e^t+2e^t\sin 2t}$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{-5s-36}{(s+2)\left(s^2+9\right)}$

Solution

$$\frac{-5s-36}{(s+2)\left(s^2+9\right)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+9}$$

$$-5s-36 = As^2 + 9A + Bs^2 + 2Bs + Cs + 2C$$

$$s^2 \qquad A+B=0$$

$$s \qquad 2B+C=-5$$

$$s^0 \qquad 9A+2C=-6$$

$$\Delta = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 9 & 0 & 2 \end{vmatrix} = 13 \qquad \Delta_A = \begin{vmatrix} 0 & 1 & 0 \\ -5 & 2 & 1 \\ -36 & 0 & 2 \end{vmatrix} = -26$$

$$\underline{A=-2, \quad B=2, \quad C=-9}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{-2}{s+2} + \frac{2s}{s^2+3^2} - \frac{9}{s^2+3^2}\right\}$$

$$\underline{= -2e^{-2t} + 2\cos 3t - 3\sin 3t}$$

Exercise

Find the inverse Laplace Transform of $F(s) = \dfrac{3s^2+5s+3}{s^4+s^3}$

Solution

$$\frac{3s^2+5s+3}{s^4+s^3} = \frac{3s^2+5s+3}{s^3(s+1)}$$

$$= \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{s+1}$$

$$3s^2+5s+3 = As + A + Bs^2 + Bs + Cs^3 + Cs^2 + Ds^3$$

$$s^3 \qquad C+D=0 \qquad \underline{D=-1}$$

$$s^2 \qquad B+C=3 \qquad \underline{C=1}$$

$$s \qquad A+B=5 \qquad \underline{B=2}$$

$$s^0 \qquad \underline{A=3}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{3s^2+5s+3}{s^4+s^3}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{3}{s^3}+\frac{2}{s^2}+\frac{1}{s}-\frac{1}{s+1}\right\}$$

$$\boxed{= \frac{3}{2}t^2+2t+1-e^{-t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{7s^3-2s^2-3s+6}{s^3(s-2)}$

Solution

$$\frac{7s^3-2s^2-3s+6}{s^3(s-2)}=\frac{A}{s^3}+\frac{B}{s^2}+\frac{C}{s}+\frac{D}{s-2}$$

$$7s^3-2s^2-3s+6=As-2A+Bs^2-2Bs+Cs^3-2Cs^2+Ds^3$$

s^3	$C+D=7$	$\underline{D=6}$
s^2	$B-2C=-2$	$\underline{C=1}$
s	$A-2B=-3$	$\underline{B=0}$
s^0	$-2A=6$	$\underline{A=-3}$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{7s^3-2s^2-3s+6}{s^3(s-2)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-3}{s^3}+\frac{1}{s}+\frac{6}{s-2}\right\}$$

$$\boxed{= -\frac{3}{2}t^2+1+6e^{2t}}$$

Exercise

Find the inverse Laplace Transform of $F(s)=\dfrac{7s^2-41s+84}{(s-1)\left(s^2-4s+13\right)}$

Solution

$$\frac{7s^2-41s+84}{(s-1)\left(s^2-4s+13\right)}=\frac{A}{s-1}+\frac{B(s-2)+C}{(s-2)^2+9}$$

$$7s^2-41s+84=As^2-4As+13A+Bs^2-3Bs+2B+Cs-C$$

s^2	$A+B=7$
s	$-4A-3B+C=-41$
s^0	$13A+2B-C=84$

$$\Delta = \begin{vmatrix} 1 & 1 & 0 \\ -4 & -3 & 1 \\ 13 & 2 & -1 \end{vmatrix} = 10 \quad \Delta_A = \begin{vmatrix} 7 & 1 & 0 \\ -41 & -3 & 1 \\ 84 & 2 & -1 \end{vmatrix} = 50 \quad \Delta_B = \begin{vmatrix} 1 & 7 & 0 \\ -4 & -41 & 1 \\ 13 & 84 & -1 \end{vmatrix} = 20$$

$$\underline{A = 5 \quad B = 2 \quad C = -15}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{7s^2 - 41s + 84}{(s-1)\left(s^2 - 4s + 13\right)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{5}{s-1} + \frac{2(s-2)}{(s-2)^2 + 3^2} - \frac{5(3)}{(s-2)^2 + 3^2}\right\}$$

$$\underline{= 5e^t + 2e^{2t}\cos 3t - 5e^{2t}\sin 3t}$$

Exercise

Find the inverse Laplace transform of $F(s) = \dfrac{6s - 5}{s^2 + 7}$

Solution

$$F(s) = \frac{6s}{s^2 + 7} - \frac{5}{s^2 + 7}$$

$$= \frac{6s}{s^2 + 7} - \frac{\sqrt{7}}{\sqrt{7}}\frac{5}{s^2 + 7}$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{6s}{s^2 + 7} - \frac{5}{\sqrt{7}}\frac{\sqrt{7}}{s^2 + 7}\right\}$$

$$\underline{f(t) = 6\cos\sqrt{7}t - \frac{5}{\sqrt{7}}\sin\sqrt{7}t}$$

Exercise

Find the inverse Laplace transform of $F(s) = \dfrac{1 - 3s}{s^2 + 8s + 21}$

Solution

$$s^2 + 8s + 21 = s^2 + 8s + 16 - 16 + 21$$

$$= (s+4)^2 + 5$$

$$F(s) = \frac{1 - 3s}{s^2 + 8s + 21}$$

$$= \frac{1 - 3(s+4) + 12}{(s+4)^2 + 5}$$

$$= \frac{13}{(s+4)^2+5} - 3\frac{s+4}{(s+4)^2+5}$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{13}{(s+4)^2+5} - 3\frac{s+4}{(s+4)^2+5}\right\}$$

$$\boxed{f(t) = \frac{13}{\sqrt{5}}e^{-4t}\sin\sqrt{5}t - 3e^{-4t}\cos\sqrt{5}t}$$

Exercise

Find the inverse Laplace transform of $F(s) = \dfrac{3s-2}{2s^2-6s-2}$

Solution

$$2s^2 - 6s - 2 = 2\left(s^2 - 3s - 1\right)$$

$$= 2\left(s - \frac{3-\sqrt{13}}{2}\right)\left(s - \frac{3+\sqrt{13}}{2}\right)$$

$$= 2\left[\left(s-\frac{3}{2}\right)^2 - \frac{9}{4} - 1\right]$$

$$= 2\left(\left(s-\frac{3}{2}\right)^2 - \frac{13}{4}\right)$$

$$F(s) = \frac{1}{2}\frac{3\left(s-\frac{3}{2}\right) + \frac{9}{2} - 2}{\left(s-\frac{3}{2}\right)^2 - \frac{13}{4}}$$

$$= \frac{3}{2}\frac{s-\frac{3}{2}}{\left(s-\frac{3}{2}\right)^2 - \frac{13}{4}} + \frac{5}{4}\frac{1}{\left(s-\frac{3}{2}\right)^2 - \frac{13}{4}}$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{3}{2}\frac{s-\frac{3}{2}}{\left(s-\frac{3}{2}\right)^2 - \frac{13}{4}} + \frac{5}{2\sqrt{13}}\frac{\frac{\sqrt{13}}{2}}{\left(s-\frac{3}{2}\right)^2 - \frac{13}{4}}\right\}$$

$$\boxed{f(t) = \frac{3}{2}e^{3t/2}\cosh\left(\frac{\sqrt{13}}{2}t\right) + \frac{5}{2\sqrt{13}}e^{3t/2}\sinh\left(\frac{\sqrt{13}}{2}t\right)}$$

Exercise

Find the inverse Laplace transform of $F(s)=\dfrac{s+7}{s^2-3s-10}$

Solution

$$F(s)=\frac{s+7}{(s+2)(s-5)}=\frac{A}{s+2}+\frac{B}{s-5}$$

$$s+7=As-5A+Bs+2B$$

$$\begin{cases} A+B=1 \\ -5A+2B=7 \end{cases} \rightarrow \quad A=-\frac{5}{7},\;\; B=\frac{12}{7}$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{-\frac{5}{7}\frac{1}{s+2}+\frac{12}{7}\frac{1}{s-5}\right\}$$

$$f(t)=-\frac{5}{7}e^{-2t}+\frac{12}{7}e^{5t}$$

Exercise

Find the inverse Laplace transform of $F(s)=\dfrac{86s-78}{(s+3)(s-4)(5s-1)}$

Solution

$$F(s)=\frac{86s-78}{(s+3)(s-4)(5s-1)}=\frac{A}{s+3}+\frac{B}{s-4}+\frac{C}{5s-1}$$

$$86s-78=A(s-4)(5s-1)+B(s+3)(5s-1)+C(s+3)(s-4)$$

$$\begin{cases} s^2 & 5A+5B+C=0 \\ s & -21A+14B-C=86 \\ s^0 & 4A-3B-12C=-78 \end{cases}$$

$$\Delta=\begin{vmatrix} 5 & 5 & 1 \\ -21 & 14 & -1 \\ 4 & -3 & -12 \end{vmatrix}=-2128 \quad \Delta_A=\begin{vmatrix} 0 & 5 & 1 \\ 86 & 14 & -1 \\ -78 & -3 & -12 \end{vmatrix}=6384 \quad \Delta_B=\begin{vmatrix} 5 & 0 & 1 \\ -21 & 86 & -1 \\ 4 & -78 & -12 \end{vmatrix}=-4256$$

$$A=-\frac{6384}{2128}=-3, \quad B=\frac{4256}{2128}=2, \quad C=5$$

$$F(s)=-\frac{3}{s+3}+\frac{2}{s-4}+\frac{5}{5s-1}$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{-\frac{3}{s+3}+\frac{2}{s-4}+\frac{1}{s-\frac{1}{5}}\right\}$$

$$f(t)=-3e^{-3t}+2e^{4t}+e^{t/5}$$

Exercise

Find the inverse Laplace transform of $F(s)=\dfrac{2-5s}{(s-6)\left(s^2+11\right)}$

Solution

$$F(s)=\frac{2-5s}{(s-6)\left(s^2+11\right)}=\frac{A}{s-6}+\frac{Bs+C}{s^2+11}$$

$$2-5s=As^2+11A+Bs^2-6Bs+Cs-6C$$

$$\begin{cases} s^2 & A+B=0 \\ s & -6B+C=-5 \\ s^0 & 11A-6C=2 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -6 & 1 & -5 \\ 11 & 0 & -6 & 2 \end{array}\right)$$

$$A=-\frac{28}{47},\quad B=\frac{28}{47},\quad C=-\frac{67}{47}$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{-\frac{28}{47}\frac{1}{s-6}+\frac{28}{47}\frac{s}{s^2+11}-\frac{67}{47}\frac{1}{s^2+11}\frac{\sqrt{11}}{\sqrt{11}}\right\}$$

$$f(t)=-\frac{28}{47}e^{6t}+\frac{28}{47}\cos\sqrt{11}t-\frac{67}{47\sqrt{11}}\sin\sqrt{11}t$$

Exercise

Find the inverse Laplace transform of $F(s)=\dfrac{25}{s^3\left(s^2+4s+5\right)}$

Solution

$$F(s)=\frac{25}{s^3\left(s^2+4s+5\right)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{Ds+E}{s^2+4s+5}$$

$$25=As^4+4As^3+5As^2+Bs^3+4Bs^2+5Bs+Cs^2+4Cs+5C+Ds^4+Es^3$$

$$\begin{cases} s^4 & A+D=0 & \rightarrow D=-\frac{11}{5} \\ s^3 & 4A+B+E=0 & \rightarrow E=-\frac{24}{5} \\ s^2 & 5A+4B+C=0 & \rightarrow A=\frac{11}{5} \\ s & 5B+4C=0 & \rightarrow B=-4 \\ s^0 & 5C=25 & \rightarrow C=5 \end{cases}$$

$$F(s)=\frac{11}{5}\frac{1}{s}-\frac{4}{s^2}+\frac{5}{s^3}-\frac{1}{5}\frac{11s+24}{(s+2)^2-4+5}$$

$$= \frac{11}{5}\frac{1}{s} - \frac{4}{s^2} + \frac{5}{s^3} - \frac{1}{5}\frac{11(s+2)-22+24}{(s+2)^2+1}$$

$$= \frac{11}{5}\frac{1}{s} - \frac{4}{s^2} + \frac{5}{s^3} - \frac{1}{5}\frac{11(s+2)+2}{(s+2)^2+1}$$

$$= \frac{11}{5}\frac{1}{s} - \frac{4}{s^2} + \frac{5}{s^3} - \frac{11}{5}\frac{s+2}{(s+2)^2+1} - \frac{2}{5}\frac{1}{(s+2)^2+1}$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{11}{5}\frac{1}{s} - \frac{4}{s^2} + \frac{5}{s^3} - \frac{11}{5}\frac{s+2}{(s+2)^2+1} - \frac{2}{5}\frac{1}{(s+2)^2+1}\right\}$$

$$f(t) = \frac{11}{5} - 4t + \frac{5}{2}t^2 - \frac{11}{5}e^{-2t}\cos t - \frac{2}{5}\sin t$$

Exercise

Find the inverse Laplace transform of $F(s) = \dfrac{5e^{-6s} - 3e^{-11s}}{(s+2)(s^2+9)}$

Solution

$$F(s) = \left(5e^{-6s} - 3e^{-11s}\right)\frac{1}{(s+2)(s^2+9)}$$

$$= \left(5e^{-6s} - 3e^{-11s}\right)G(s)$$

$$G(s) = \frac{1}{(s+2)(s^2+9)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+9}$$

$$1 = As^2 + 9A + Bs^2 + 2Bs + Cs + 2C$$

$$\begin{cases} s^2 & A+B=0 \\ s & 2B+C=0 \\ s^0 & 9A+2C=1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 9 & 0 & 2 & 1 \end{array}\right)$$

$$A = \frac{1}{13}, \quad B = -\frac{1}{13}, \quad C = \frac{2}{13}$$

$$\mathcal{L}^{-1}\{G(s)\} = \frac{1}{13}\mathcal{L}^{-1}\left\{\frac{1}{s+2} - \frac{s}{s^2+9} + \frac{2}{s^2+9}\right\}$$

$$g(t) = \frac{1}{13}\left(e^{-2t} - \cos 3t + \frac{2}{3}\sin 3t\right)$$

$$f(t) = 5u_6(t)g(t-6) - 3u_{11}(t)g(t-11)$$