Solution Section 4.7 – Trigonometric Form

Exercise

Write $-\sqrt{3} + i$ in trigonometric form. (Use radian measure)

Solution

$$-\sqrt{3} + i \implies \begin{cases} x = -\sqrt{3} \\ y = 1 \end{cases}$$

$$r = \sqrt{\left(-\sqrt{3}\right)^2 + 1^2}$$

$$= 2$$

$$\hat{\theta} = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$\theta = \frac{\pi}{6}$$

The angle is in quadrant *II*, therefore;

$$\theta = \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

$$-\sqrt{3} + i = 2 \operatorname{cis} \frac{5\pi}{6}$$

Exercise

Write 3-4i in trigonometric form.

Solution

$$3-4i \Rightarrow \begin{cases} x=3\\ y=-4 \end{cases}$$

$$r = \sqrt{3^2 + (-4)^2}$$

$$= 2 \mid$$

$$\hat{\theta} = \tan^{-1} \left(\frac{4}{3}\right)$$

$$\approx 53^{\circ} \mid$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$\theta = 180^{\circ} - 53^{\circ}$$

$$= 127^{\circ} \rfloor$$

$$3 - 4i = 5 \text{ cis} 127^{\circ} \rfloor$$

Write -21-20i in trigonometric form.

Solution

$$-21-20i \implies \begin{cases} x = -21 \\ y = -20 \end{cases}$$

$$r = \sqrt{(-21)^2 + (-20)^2}$$

$$= 29 \rfloor$$

$$\hat{\theta} = \tan^{-1}\left(\frac{20}{21}\right) \qquad \theta = \tan^{-1}\frac{y}{x}$$

$$\approx 43.6^{\circ} \rfloor$$

The angle is in quadrant III, therefore;

$$\theta = 180^{\circ} + 43.6^{\circ}$$

= 223.6° \rightarrow \frac{-21 - 20i = 29 \cis 223.6^{\circ}}

Exercise

Write 11+2i in trigonometric form.

$$11+2i \implies \begin{cases} x=11 \\ y=2 \end{cases}$$

$$r = \sqrt{11^2 + 2^2}$$

$$= \sqrt{125}$$

$$= 5\sqrt{5}$$

$$\theta = \tan^{-1}\left(\frac{2}{11}\right)$$

$$\approx 10.3^{\circ}$$

$$11+2i = 5\sqrt{5} cis10.3^{\circ}$$

Write $\sqrt{3} - i$ in trigonometric form.

Solution

$$\sqrt{3} - i \implies \begin{cases} x = \sqrt{3} \\ y = -1 \end{cases}$$

$$r = \sqrt{3 + 1}$$

$$= 2 \rfloor$$

$$\hat{\theta} = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\approx 30^{\circ} \rfloor$$

$$\theta = \tan^{-1} \frac{y}{x}$$

The angle is in quadrant IV, therefore;

$$\theta = 360^{\circ} - 30^{\circ}$$

$$= 330^{\circ}$$

$$\sqrt{3} - i = 2 \text{ cis} 330^{\circ}$$

Exercise

Write $1-\sqrt{3}i$ in trigonometric form.

Solution

$$1 - \sqrt{3}i \implies \begin{cases} x = 1 \\ y = -\sqrt{3} \end{cases}$$

$$r = \sqrt{1+3}$$

$$= 2 \rfloor$$

$$\hat{\theta} = \tan^{-1} \left(\frac{\sqrt{3}}{1} \right)$$

$$= 60^{\circ} \rfloor$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$\theta = 360^{\circ} - 60^{\circ}$$

$$= 300^{\circ}$$

$$1 - \sqrt{3}i = 2 \ cis300^{\circ}$$

Write $9\sqrt{3} + 9i$ in trigonometric form.

Solution

$$9\sqrt{3} + 9i \implies \begin{cases} x = 9\sqrt{3} \\ y = 9 \end{cases}$$

$$r = 9\sqrt{3 + 1}$$

$$= 18 \rfloor$$

$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{3}}\right)$$

$$= 30^{\circ} \rfloor$$

$$9\sqrt{3} + 9i = 18 \text{ cis} 30^{\circ} \rfloor$$

Exercise

Write -2+3i in trigonometric form.

Solution

$$-2+3i \implies \begin{cases} x = -2 \\ y = 3 \end{cases}$$

$$r = \sqrt{4+9}$$

$$= \sqrt{13}$$

$$\hat{\theta} = \tan^{-1} \left(\frac{3}{2}\right)$$

$$\approx 56.31^{\circ}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

The angle is in quadrant *II*, therefore;

$$\theta = 180^{\circ} - 56.31^{\circ}$$
 $\approx 123.69^{\circ}$

$$-2 + 3i = \sqrt{13} \ cis123.69^{\circ}$$

Exercise

Write $4(\cos 30^{\circ} + i \sin 30^{\circ})$ in standard form.

$$4(\cos 30^\circ + i\sin 30^\circ) = 4\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$
$$= 2\sqrt{3} + 2i$$

Write $\sqrt{2} cis \frac{7\pi}{4}$ in standard form.

Solution

$$\sqrt{2} cis \frac{7\pi}{4} = \sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$
$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right)$$
$$= 1 - i$$

Exercise

Write 3cis210° in standard form.

Solution

$$3cis210^{\circ} = 3(\cos 210^{\circ} + i \sin 210^{\circ})$$
$$= -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$$

Exercise

Write $4\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ in standard form.

Solution

$$4\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = 4\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$$
$$= 2\sqrt{2} - 2i\sqrt{2}$$

Exercise

Write $4cis\frac{\pi}{2}$ in standard form.

Solution

$$4cis\frac{\pi}{2} = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$
$$= 4i$$

Exercise

Find the quotient $\frac{20cis(75^\circ)}{4cis(40^\circ)}$. Write the result in rectangular form.

$$\frac{20cis(75^\circ)}{4cis(40^\circ)} = \frac{20}{4}cis(75^\circ - 40^\circ)$$

$$= 5cis(35^\circ)$$

$$= 5(\cos 35^\circ + i\sin 35^\circ)$$

$$= 4.1 + 2.87i$$

Divide $z_1 = 1 + i\sqrt{3}$ by $z_2 = \sqrt{3} + i$. Write the result in rectangular form.

Solution

$$\frac{z_1}{z_2} = \frac{1+i\sqrt{3}}{\sqrt{3}+i}$$

$$= \frac{1+i\sqrt{3}}{\sqrt{3}+i} \frac{\sqrt{3}-i}{\sqrt{3}-i}$$

$$= \frac{\sqrt{3}-i+3i-\sqrt{3}}{3+1}$$

$$= \frac{2\sqrt{3}+2i}{4}$$

$$= \frac{2\sqrt{3}}{4} + \frac{2i}{4}$$

$$= \frac{\sqrt{3}}{2} + \frac{i}{2}$$

or

$$1+i\sqrt{3} \rightarrow \begin{cases} r = \sqrt{1^2 + (\sqrt{3})^2} \\ \theta = \tan^{-1} \frac{\sqrt{3}}{1} = \frac{\pi}{3} \end{cases}$$

$$\sqrt{3} + i \rightarrow \begin{cases} r = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} \\ \theta = \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{cases}$$

$$\frac{z_1}{z_2} = \frac{2cis\frac{\pi}{3}}{2cis\frac{\pi}{6}}$$
$$= \frac{2}{2}cis\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$
$$= \frac{2}{2}cis\left(\frac{\pi}{6}\right)$$

$$= cis\left(\frac{\pi}{6}\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{i}{2}$$

Find $(1+i)^8$ and express the result in rectangular form.

Solution

$$1+i \Rightarrow \begin{cases} x=1\\ y=1 \end{cases}$$

$$r = \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$\theta = \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

$$1+i = \sqrt{2}cis\frac{\pi}{4}$$

$$(1+i)^8 = \left(\sqrt{2}cis\frac{\pi}{4}\right)^8$$

$$= \left(\sqrt{2}\right)^8 cis\left[8\left(\frac{\pi}{4}\right)\right]$$

$$= 16cis2\pi$$

$$= 16\left(\cos 2\pi + i\sin 2\pi\right)$$

$$= 16\left(1+i0\right)$$

$$= 16$$

Exercise

Find $(1+i)^{10}$ and express the result in rectangular form.

$$1+i \implies \begin{cases} x=1 \\ y=1 \end{cases}$$

$$r = \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$\theta = \tan^{-1}(1)$$

$$\frac{=\frac{\pi}{4}}{4}$$

$$1+i=\sqrt{2}cis\frac{\pi}{4}$$

$$(1+i)^{10} = \left(\sqrt{2}cis\frac{\pi}{4}\right)^{10}$$

$$= \left(\sqrt{2}\right)^{10}cis\left[10\left(\frac{\pi}{4}\right)\right]$$

$$= 32cis\frac{5\pi}{2}$$

$$= 32cis\frac{\pi}{2}$$

$$= 32\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$= 32(0+i)$$

$$= 32i$$

Find and express the result in rectangular form $(1-i)^5$

Solution

$$1-i \implies \begin{cases} x=1 \\ y=-1 \end{cases}$$

$$r = \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$\hat{\theta} = \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

$$\theta = 2\pi - \frac{\pi}{4}$$

$$= \frac{7\pi}{4}$$

$$1 - i = \sqrt{2} \operatorname{cis} \frac{7\pi}{4}$$

$$(1 - i)^5 = \left(\sqrt{2} \operatorname{cis} \frac{7\pi}{4}\right)^5$$

$$= 4\sqrt{2} \left(\operatorname{cis} \left(5 \times \frac{7\pi}{4}\right)\right)$$

$$= 4\sqrt{2} \left(cis \frac{35\pi}{4} \right)$$

$$= 4\sqrt{2} \left(cos \frac{3\pi}{4} + i sin \frac{3\pi}{4} \right)$$

$$= 4\sqrt{2} \left(-\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)$$

$$= -4 + 4i$$

Find and express the result in rectangular form $(1-\sqrt{5}i)^8$

Solution

$$1 - \sqrt{5}i \implies \begin{cases} x = 1 \\ y = -\sqrt{5} \end{cases}$$

$$r = \sqrt{1+5}$$

$$= \sqrt{6}$$

$$\hat{\theta} = \tan^{-1} \left(\frac{\sqrt{5}}{1} \right)$$

$$\approx 66^{\circ}$$

$$\theta = 360^{\circ} - 66^{\circ}$$

$$= 294^{\circ} \rfloor$$

$$1 - \sqrt{5}i = \sqrt{6} cis294^{\circ} \rfloor$$

$$(1 - \sqrt{5}i)^{8} = (\sqrt{6}cis294^{\circ})^{8}$$

$$= (\sqrt{6})^{8} (cis2352^{\circ})$$

$$= 1296(\cos 192^{\circ} + i \sin 192^{\circ})$$

$$= 1296(-.978 - 0.208i)$$

$$= -1267.488 - 269.568 i \rfloor$$

Find and express the result in rectangular form $(3cis80^\circ)^3$

Solution

$$(3cis80^{\circ})^{3} = 3^{3} (cis240^{\circ})$$

$$= 27 (\cos 240^{\circ} + i \sin 240^{\circ})$$

$$= 27 \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)$$

$$= -\frac{27}{2} - i \frac{27\sqrt{3}}{2}$$

Exercise

Find and express the result in rectangular form $(\sqrt{3}cis10^\circ)^6$

Solution

$$(\sqrt{3}cis10^{\circ})^{6} = 27(cis60^{\circ})$$
$$= 27(\cos 60^{\circ} + i\sin 60^{\circ})$$
$$= \frac{27}{2} + i\frac{27\sqrt{3}}{2}$$

Exercise

Find and express the result in rectangular form $(\sqrt{2}-i)^6$

Solution

$$\sqrt{2} - i \implies \begin{cases} x = \sqrt{2} \\ y = -1 \end{cases}$$

$$r = \sqrt{2 + 1}$$

$$= \sqrt{3}$$

$$\hat{\theta} = \tan^{-1} \left(\frac{1}{\sqrt{2}} \right)$$

$$\approx 35.26^{\circ}$$

$$\theta = 360^{\circ} - 35.26^{\circ}$$

= 324.74° \rfloor
 $\sqrt{2} - i = \sqrt{3} \ cis324.74^{\circ}$

$$(\sqrt{2} - i)^{6} = (\sqrt{3} cis324.74^{\circ})^{6}$$

$$= 27(cis1948.44^{\circ})$$

$$= 27(\cos 148.44^{\circ} + i \sin 148.44^{\circ})$$

$$= -23 + 14.142i$$

Find and express the result in rectangular form $(4cis40^\circ)^6$

Solution

$$(4cis40^{\circ})^{6} = 4^{6} \left(cis (6 \times 40^{\circ}) \right)$$

$$= 4^{6} \left(\cos 240^{\circ} + i \sin 240^{\circ} \right)$$

$$= 4096 \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)$$

$$= -2048 + 2048 i \sqrt{3}$$

Exercise

Find and express the result in rectangular form $(2cis30^\circ)^5$

Solution

$$(2cis30^{\circ})^{5} = 2^{5}cis(5(30^{\circ}))$$

$$= 32(\cos 150^{\circ} + i\sin 150^{\circ})$$

$$= 32\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= -16\sqrt{3} + 16i$$

Exercise

Find and express the result in rectangular form $\left(\frac{1}{2}cis72^{\circ}\right)^{3}$

$$\left(\frac{1}{2}cis72^{\circ}\right)^{5} = \frac{1}{2^{5}}cis(5 \times 72^{\circ})$$

$$= \frac{1}{32}cis(\cos 360^{\circ} + i\sin 360^{\circ})$$

$$= \frac{1}{32} \left|$$

Find *fifth* roots of $z = 1 + i\sqrt{3}$ and express the result in rectangular form.

Solution

$$1+i\sqrt{3} \implies \begin{cases} x=1\\ y=\sqrt{3} \end{cases}$$

$$r = \sqrt{1+3}$$

$$= 2 \rfloor$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{3}}{1}\right)$$

$$= 60^{\circ} \rfloor$$

$$1+i\sqrt{3} = 2 \operatorname{cis} 60^{\circ} \rfloor$$

$$(1+i\sqrt{3})^{1/5} = (2 \operatorname{cis} 60^{\circ})^{1/5}$$

$$= \sqrt[5]{2} \left(\operatorname{cis} \frac{60^{\circ}}{5} + \frac{360^{\circ}k}{5}\right)$$

$$= \sqrt[5]{2} \operatorname{cis} (12^{\circ} + 72^{\circ}k) \rfloor \qquad k = 0, 1, 2, 3, 4$$
For $k = 0$

$$\sqrt[5]{2} \operatorname{cis} (12^{\circ} + 72^{\circ}0) = \sqrt[5]{2} \operatorname{cis} 12^{\circ} \rfloor$$
For $k = 1$

$$\sqrt[5]{2} \operatorname{cis} (12^{\circ} + 72^{\circ}.(1)) = \sqrt[5]{2} \operatorname{cis} 84^{\circ} \rfloor$$
For $k = 2$

$$\sqrt[5]{2} \operatorname{cis} (12^{\circ} + 72^{\circ}.(2)) = \sqrt[5]{2} \operatorname{cis} 156^{\circ} \rfloor$$
For $k = 3$

$$\sqrt[5]{2} \operatorname{cis} (12^{\circ} + 72^{\circ}.(3)) = \sqrt[5]{2} \operatorname{cis} 228^{\circ} \rfloor$$
For $k = 4$

 $\sqrt[5]{2} cis(12^{\circ} + 72^{\circ}.(4)) = \sqrt[5]{2} cis300^{\circ}$

Find the *fourth* roots of $z = 16cis60^{\circ}$

Solution

$$\sqrt[4]{z} = \sqrt[4]{16} \ cis\left(\frac{60^{\circ}}{4} + \frac{360^{\circ}}{4}k\right)$$

$$= 2cis\left(15^{\circ} + 90^{\circ}k\right) \qquad k = 0, 1, 2, 3$$
For $k = 0$

$$2 \ cis\left(15^{\circ} + 90^{\circ}(0)\right) = 2cis15^{\circ}$$
For $k = 1$

$$2 \ cis\left(15^{\circ} + 90^{\circ}(1)\right) = 2cis105^{\circ}$$
For $k = 2$

$$2 \ cis\left(15^{\circ} + 90^{\circ}(2)\right) = 2cis195^{\circ}$$
For $k = 3$

$$2 \ cis\left(15^{\circ} + 90^{\circ}(3)\right) = 2cis285^{\circ}$$

Exercise

Find the *fourth* roots of $\sqrt{3} - i$

Solution

$$\sqrt{3} - i \implies \begin{cases} x = \sqrt{3} \\ y = -1 \end{cases}$$

$$r = \sqrt{3+1}$$

$$= 2 \rfloor$$

$$\hat{\theta} = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$= \frac{\pi}{6} \rfloor$$

$$\theta = 2\pi - \frac{\pi}{6}$$

$$= \frac{11\pi}{6}$$

$$\sqrt{3} - i = 2 \operatorname{cis} \frac{11\pi}{6}$$

$$\sqrt[4]{\sqrt{3} - i} = \sqrt[4]{2 \operatorname{cis} \frac{11\pi}{6}}$$

$$= \sqrt[4]{2} cis\left(\frac{1}{4}\frac{11\pi}{6} + \frac{2\pi k}{4}\right)$$
$$= \sqrt[4]{2} cis\left(\frac{11\pi}{24} + \frac{\pi k}{2}\right) \qquad k = 0, 1, 2, 3$$

For
$$k = 0$$

$$\sqrt[4]{2} \ cis\left(\frac{11\pi}{24} + \frac{0}{0}\right) = \sqrt[4]{2} \ cis\frac{11\pi}{24}$$

For
$$k=1$$

$$\sqrt[4]{2} cis\left(\frac{11\pi}{24} + \frac{\pi}{2}\right) = \sqrt[4]{2} cis\frac{23\pi}{24}$$

For
$$k = 2$$

$$\sqrt[4]{2} \operatorname{cis}\left(\frac{11\pi}{24} + \pi\right) = \sqrt[4]{2} \operatorname{cis}\frac{35\pi}{24}$$

For
$$k = 3$$

$$\sqrt[4]{2} cis\left(\frac{11\pi}{24} + \frac{3\pi}{2}\right) = \sqrt[4]{2} cis\frac{47\pi}{24}$$

Find the *fourth* roots of $4-4\sqrt{3}i$

Solution

$$4 - 4\sqrt{3}i \implies \begin{cases} x = 4 \\ y = -4\sqrt{3} \end{cases}$$
$$r = 4\sqrt{3+1}$$
$$= 8 \mid$$

$$\hat{\theta} = \tan^{-1} \left(\frac{4\sqrt{3}}{4} \right)$$

$$=\tan^{-1}\left(\sqrt{3}\right)$$

$$=\frac{\pi}{3}$$

$$\theta = 2\pi - \frac{\pi}{3}$$
$$= \frac{5\pi}{3}$$

$$4 - 4\sqrt{3}i = 8 \operatorname{cis} \frac{5\pi}{3}$$

$$\sqrt[4]{4 - 4\sqrt{3}i} = \sqrt[4]{8 \ cis \frac{5\pi}{3}}$$

$$= \frac{4\sqrt{8} \operatorname{cis}\left(\frac{5\pi}{12} + \frac{\pi k}{2}\right)}{k = 0, 1, 2, 3}$$
For $k = 0$

$$\sqrt[4]{8} \operatorname{cis}\left(\frac{5\pi}{12} + 0\right) = \sqrt[4]{8} \operatorname{cis}\frac{5\pi}{12}$$
For $k = 1$

$$\sqrt[4]{8} \operatorname{cis}\left(\frac{5\pi}{12} + \frac{\pi}{2}\right) = \sqrt[4]{8} \operatorname{cis}\frac{11\pi}{12}$$
For $k = 2$

For
$$k = 2$$
 $\sqrt[4]{8} cis(\frac{5\pi}{12} + \pi) = \sqrt[4]{8} cis(\frac{17\pi}{12})$

For
$$k = 3$$

 $\sqrt[4]{8} cis\left(\frac{5\pi}{12} + \frac{3\pi}{2}\right) = \sqrt[4]{8} cis\frac{23\pi}{12}$

Find the *fourth* roots of -16i

$$-16i \implies \begin{cases} x = 0 \\ y = -16 \end{cases}$$

$$\frac{r = 16 \mid}{\theta = \frac{3\pi}{2} \mid}$$

$$\frac{-16i = 16 \text{ } cis \frac{3\pi}{2} \mid}{\sqrt[4]{-16i}} = \sqrt[4]{16 \text{ } cis \frac{3\pi}{2}}$$

$$= 2 \text{ } cis \left(\frac{3\pi}{8} + \frac{\pi k}{2}\right) \mid k = 0, 1, 2, 3$$

For
$$k = 0$$

$$2 \operatorname{cis} \left(\frac{3\pi}{8} + 0 \right) = 2 \operatorname{cis} \frac{3\pi}{8}$$

For
$$k=1$$

$$2 cis \left(\frac{3\pi}{8} + \frac{\pi}{2}\right) = 2 cis \frac{7\pi}{8}$$

For
$$k = 2$$

$$2 \operatorname{cis} \left(\frac{3\pi}{8} + \pi \right) = 2 \operatorname{cis} \frac{11\pi}{8}$$

For
$$k = 3$$

$$2 \operatorname{cis} \left(\frac{3\pi}{8} + \frac{3\pi}{2} \right) = 2 \operatorname{cis} \frac{15\pi}{8}$$

Find the *cube* roots of 27.

Solution

$$\sqrt[3]{27} = (27cis0^{\circ})^{1/3}$$

$$= \sqrt[3]{27} cis(\frac{0^{\circ}}{3} + \frac{360^{\circ}}{3}k)$$

$$= 3 cis(0^{\circ} + 120^{\circ}k) \qquad k = 0, 1, 2$$
For $k = 0$

$$z = 3 cis(0^{\circ} + 120^{\circ}(0)) = 2 cis0^{\circ}$$
For $k = 1$

$$z = 3 cis(0^{\circ} + 120^{\circ}(1)) = 2 cis120^{\circ}$$
For $k = 2$

$$z = 3 cis(0^{\circ} + 120^{\circ}(2)) = 2 cis240^{\circ}$$

Exercise

Find the *cube* roots of 8-8i

Solution

$$8-8i \implies \begin{cases} x=8 \\ y=-8 \end{cases}$$

$$r = 8\sqrt{1+1}$$

$$= 8\sqrt{2} \mid$$

$$\hat{\theta} = \tan^{-1} \left(\frac{8}{8}\right)$$

$$= \tan^{-1} (1)$$

$$= \frac{\pi}{4} \mid$$

$$\theta = 2\pi - \frac{\pi}{4}$$

$$= \frac{7\pi}{4}$$

$$\sqrt[3]{8-8i} = \sqrt[3]{8\sqrt{2} cis \frac{7\pi}{4}}$$

$$= 2\sqrt[3]{2} cis \left(\frac{7\pi}{12} + \frac{2\pi k}{3}\right) \qquad k = 0, 1, 2$$

For
$$k = 0$$

 $z = 2\sqrt[3]{2} cis(\frac{7\pi}{12} + 0) = 2\sqrt[3]{2} cis(\frac{7\pi}{12})$

For
$$k = 1$$

 $z = 2\sqrt[3]{2} cis\left(\frac{7\pi}{12} + \frac{2\pi}{3}\right) = 2\sqrt[3]{2} cis\frac{15\pi}{12}$

For
$$k = 2$$

 $z = 2\sqrt[3]{2} cis\left(\frac{7\pi}{12} + \frac{4\pi}{3}\right) = 2\sqrt[3]{2} cis\frac{23\pi}{12}$

Find the *cube* roots of -8

$$\frac{r=8}{\theta = \frac{3\pi}{2}}$$

$$\sqrt[3]{-8} = \sqrt[3]{8 \ cis \frac{3\pi}{2}}$$

$$= 2 \ cis \left(\frac{\pi}{2} + \frac{2\pi k}{3}\right) \left| k = 0, 1, 2 \right|$$

For
$$k = 0$$

$$z = 2 \operatorname{cis}\left(\frac{\pi}{2} + 0\right) = 2 \operatorname{cis}\frac{\pi}{2}$$

For
$$k=1$$

$$z = 2 \operatorname{cis}\left(\frac{\pi}{2} + \frac{2\pi}{3}\right) = 2 \operatorname{cis}\frac{7\pi}{6}$$

For
$$k = 2$$

 $z = 2 cis(\frac{\pi}{2} + \frac{4\pi}{3}) = 2 cis \frac{11\pi}{6}$

Find all complex number solutions of $x^3 + 1 = 0$.

$$x^{3} + 1 = 0$$

$$x^{3} = -1$$

$$\underline{r} = 1$$

$$\theta = \tan^{-1}\left(\frac{0}{-1}\right)$$

$$\underline{=} \pi$$

$$x^{3} = 1 \operatorname{cis}(\pi)$$

$$x = (1 \operatorname{cis}\pi)^{1/3}$$

$$= (1)^{1/3} \operatorname{cis}\left(\frac{\pi}{3} + \frac{2\pi}{3}k\right)$$

$$= \operatorname{cis}\left(\frac{\pi}{3} + \frac{2\pi}{3}k\right)$$

$$= \operatorname{cis}\left(\frac{\pi}{3} + \frac{2\pi}{3}(0)\right)$$

$$= \operatorname{cis}\frac{\pi}{3}$$

$$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$$

$$= \frac{1}{2} + i\frac{\sqrt{3}}{2}$$
For $k = 1$

$$x = \operatorname{cis}\left(\frac{\pi}{3} + \frac{2\pi}{3}(1)\right)$$

$$= \operatorname{cis}\left(\frac{3\pi}{3}\right)$$

$$= cis\pi$$

$$=\cos \pi + i\sin \pi$$

$$= -1$$

For
$$k = 2$$

$$x = cis\left(\frac{\pi}{3} + \frac{2\pi}{3}(2)\right)$$
$$= cis\frac{5\pi}{3}$$

$$= \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$$
$$= \frac{1}{2} - i\frac{\sqrt{3}}{2}$$