

$$\text{Rate of Change} = \frac{dx}{dt}$$

$$= \text{Rate in} - \text{Rate out.}$$

$$\text{Rate} = \text{Vol} \overset{\text{Rate}}{\frac{\text{gal}}{\text{min}}} \times \text{concentration} \frac{\text{lb}}{\text{gal}}$$

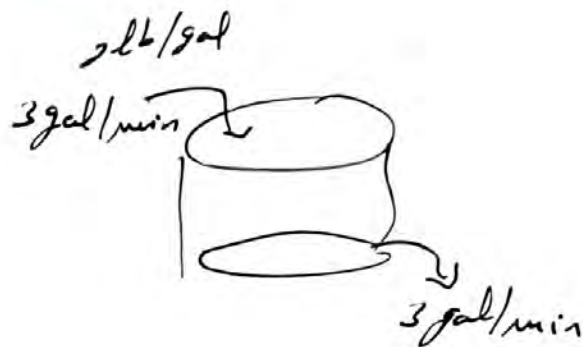
Ex $x(t)$: lb

$$\text{Volume} = 100 + (3 - 3)t$$

$$= 100$$

$$\text{Concentration: } C(t) = \frac{x(t)}{V(t)}$$

$$= \frac{x(t)}{100}$$



$$\text{Rate in} = 2 \frac{\text{lb}}{\text{gal}} \times 3 \frac{\text{gal}}{\text{min}}$$

$$= 6 \text{ lb/min}$$

$$\text{Rate out} = 3 \frac{\text{gal}}{\text{min}} \times \frac{x(t)}{100} = \frac{3}{100} x(t)$$

$$\frac{dx}{dt} = R_{\text{in}} - R_{\text{out}}$$

$$= 6 - \frac{3}{100}x$$

$$x' + \frac{3}{100}x = 6$$

$$e^{\int \frac{3}{100} dt} = e^{\frac{3t}{100}}$$

$$\int 6 e^{\frac{3t}{100}} dt = 200 e^{\frac{3t}{100}}$$

$$x(t) = \frac{1}{e^{\frac{3t}{100}}} (200 e^{\frac{3t}{100}} + C)$$

$$= 200 + \frac{C}{e^{\frac{3t}{100}}}$$

$$t=0 \Rightarrow x(t)=0$$

$$0 = 200 + \frac{C}{1} \Rightarrow \underline{C = -200}$$

$$X(t) = 200 - \frac{200}{e^{3t/100}} \quad \frac{169}{100}$$

$$X(60) = 200(1 - e^{-14.5})$$

$\approx 167 \text{ lb}$

600 Tank w/ 300 ($X(0) = 0$)

In: $1.5 \text{ lb/gal} = \frac{3}{2}$, 3 gal/min

Out: 1 gal/min

Soln

$$\begin{aligned} V'(t) &= 300 + (3-1)t \\ &= 300 + 2t = 600 \\ t &= 150 \text{ min} \end{aligned} \quad \text{Full.}$$

$$C(t) = \frac{X}{300+2t}$$

$$R_{in} = \frac{3}{2} \times 3 = \frac{9}{2}$$

$$R_{out} = 1 \times \frac{X}{300+2t} = \frac{X}{300+2t}$$

$$X' = \frac{9}{2} - \frac{X}{300+2t}$$

$$X' + \frac{1}{300+2t} X = \frac{9}{2}$$

$$\begin{aligned} e^{\int \frac{dt}{300+2t}} &= e^{\frac{1}{2} \int \frac{d(300+2t)}{300+2t}} \\ &= e^{\left(\frac{1}{2}\right) \ln(300+2t)} \\ &= e^{\ln(300+2t)^{1/2}} \\ &= \sqrt{300+2t} \end{aligned}$$

$$\frac{1}{2} \frac{9}{2} \int (300+2t)^{1/2} d(300+2t)$$

$$= \frac{3}{2} (300+2t)^{3/2}$$

$$x(t) = \frac{1}{\sqrt{300+2t}} \left(\frac{3}{2} (300+2t)^{3/2} + C \right)$$

$$= \frac{3}{2} (300+2t) + \frac{C}{\sqrt{300+2t}}$$

$$= 450 + 3t + \frac{C}{\sqrt{300+2t}}$$

$$0 = 450 + \frac{C}{\sqrt{300}}$$

$$C = -4500\sqrt{3}$$

$$x(t) = 450 + 3t - \frac{4500\sqrt{3}}{\sqrt{300+2t}}$$

1.6

Exact Differential Eqn.

$$M(x,y)dx + N(x,y)dy = 0$$

$$M(x,y) + N(x,y) \frac{dy}{dx} = 0$$

$$M + N'y' = 0$$

$$\text{If exact, } M_y(x,y) = N'_x(x,y)$$

$$\psi'_x = M(x,y) \quad \psi'_y = N(x,y)$$

$$\psi(x,y) = \int M(x,y) dx$$

Ex $2x + y^2 + 2xy' = 0$

$$M = 2x + y^2$$

$$N = 2xy$$

$$M_y = 2y$$

$$N'_x = 2y$$

$$\Rightarrow M_y = N'_x = 2y$$

$$\psi = \int M dx$$

$$= \int (2x + y^2) dx$$

$$= x^2 + y^2 x + h(y)$$

$$\psi_y = N$$

$$2xy + h'(y) = 2xy$$

$$h'(y) = 0 \Rightarrow h(y) = \int 0 dy = C$$

$$\psi(x,y) = x^2 + y^2 x + C = 0$$

$$\underline{x^2 + xy^2 = C}$$

$$\underline{\text{Ex.}} \quad \underbrace{y \cos x + 2xe^x}_M + \underbrace{(\sin x + x^2 e^x - 1)}_N y' = 0$$

$$M_y = \cos x + 2xe^x$$

$$N'_x = \cos x + 2xe^x$$

$$M_y = N'_x \quad \checkmark$$

$$U = \int (y \cos x + 2xe^x) dx$$

$$= y \sin x + x^2 e^x + h(y)$$

$$\left(\begin{aligned} U'_y &= \sin x + x^2 e^x + h'(y) = N \\ &= \sin x + x^2 e^x - 1 \end{aligned} \right.$$

$$h'(y) = -1 \Rightarrow h(y) = -\int dy = -y$$

$$\underline{y \sin x + x^2 e^x - y = C}$$

$$\underline{\text{Ex}} \quad \underbrace{(3xy + y^2)}_M + \underbrace{(x^2 + xy)}_N y' = 0$$

$$M_y = 3x + 2y \quad N_x = 2x + y$$

$$M_y \neq N_x$$

$$\frac{M_y - N_x}{N} = \frac{3x + 2y - 2x - y}{x^2 + xy}$$

$$= \frac{x + y}{x(x + y)}$$

$$= \frac{1}{x}$$

No need

$$\frac{d\mu}{dx} = \frac{1}{x} \mu$$

$$\frac{d\mu}{\mu} = \frac{dx}{x}$$

$$\ln \mu = \ln x$$

$$\mu = x$$

Given

$$x(3xy + y^2) + x(x^2 + xy)y' = 0$$

$$\underbrace{(3x^2y + xy^2)}_M + \underbrace{(x^3 + x^2y)}_N y' = 0$$

$$M_y = 3x^2 + 2xy$$

$$N_x = 3x^2 + 2xy$$

$$M_y = N_x$$

$$\psi = \int (3x^2y + xy^2) dx$$

$$= x^3y + \frac{1}{2}x^2y^2 + h(y)$$

$$\psi_y = x^3 + x^2y + h'(y) = x^3 + x^2y$$

N

$$h'(y)=0 \Rightarrow h(y)=C$$

$$\underline{x^3 y + \frac{1}{2} x^2 y^2 = C}$$