Math 2414 – Calculus II ***Exam* 1** ***Review***

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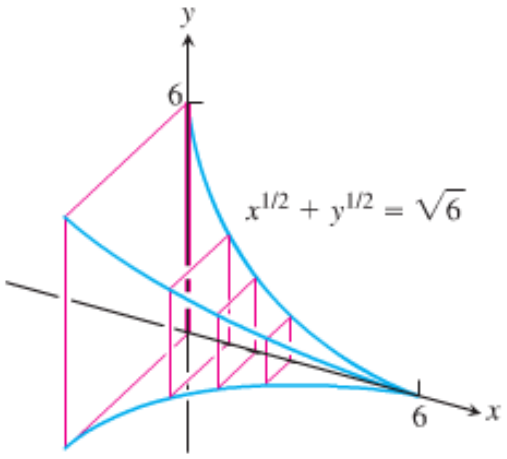
1. The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 1$. The cross-sections perpendicular to the x-axis between these planes are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = \sqrt{x}$. Find the volume of the solid.

2. The base of the solid is the region in the first quadrant between the line $y = x$ and the parabola $y = 2\sqrt{x}$. The cross-sections of the solid perpendicular to the x-axis are equilateral triangles whose bases stretch from the line to the curve. Find the volume of the solid.

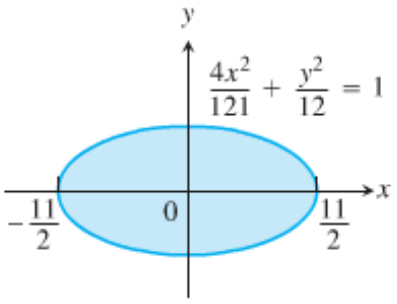
3. The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 6$. The cross-sections between these planes are squares whose bases run from the x-axis up to the curve $x^{1/2} + y^{1/2} = \sqrt{6}$. Find the volume of the solid.



4. Find the volume of the solid generated by revolving the region bounded by $y = \dfrac{4}{x^3}$ and the lines $x = 1$, *and* $y = \dfrac{1}{2}$ about (***a***) the x-axis; (***b***) the y-axis; (***c***) the line $x = 2$; (***d***) the line $y = 4$.

5. Find the volume of the solid generated by revolving the region bounded by $y = \sin x$ and the lines $x = 0$, $x = \pi$, *and* $y = 2$ about the line $y = 2$.

6. The profile of a football resembles the ellipse. Find the football's volume to the nearest cubic inch.



7. The region in the first quadrant that is bounded by the curve $y = \frac{1}{\sqrt{x}}$, on the left by the line $x = \frac{1}{4}$, and below by the line $y = 1$ is revolved about the y-axis to generate a solid. Find the volume of the solid by

a) The *shell* method *b)* The *washer* method

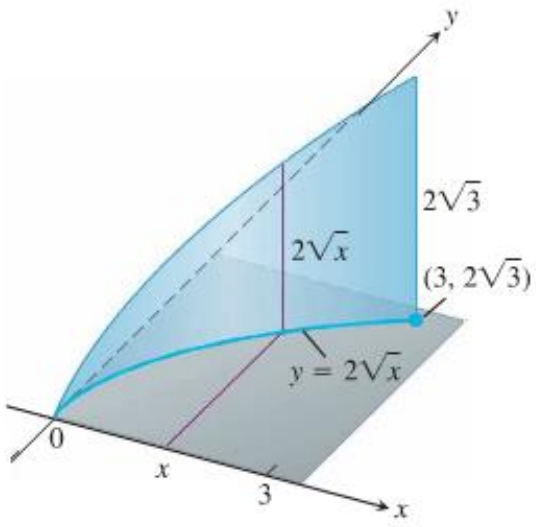
8. Find the length of the curve $y = x^{1/2} - \frac{1}{3}x^{3/2}$ from $x = 1$ to $x = 4$

9. Find the length of the curve $x = y^{2/3}, \quad 1 \le y \le 8$

10. Find the area of the surface generated by $y = \frac{1}{3}x^3, \quad 0 \le x \le 1, \quad x - axis$

11. Find the area of the surface generated by $x = \sqrt{4y - y^2}, \quad 1 \le y \le 2; \quad y - axis$

12. At points on the curve $y = 2\sqrt{x}$, line segments of length $h = y$ are drawn perpendicular to the *xy*-plane. Find the area of the surface formed by these perpendiculars from $(0,\ 0)$ *to* $(3,\ 2\sqrt{3})$



13. A rock climber is about to haul up 100 *N* (about 22.5 *lb.*) of equipment that has been hanging beneath her on 40 *m* rope that weighs 0.8 *N/m*. How much work will it take? (*Hint*: Solve for the rope and equipment separately, then add)

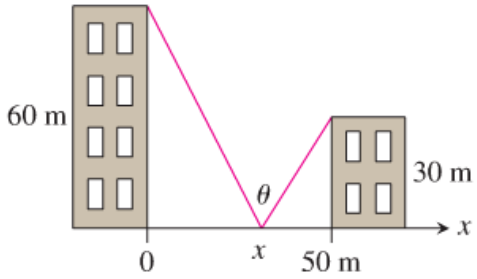
14. You drove an 800-*gal* tank truck of water from the base of a mountain to the summit and discovered on arrival that the tank was only half full. You started with a full tank, climbed at a steady rate, and accomplished the 4750-*ft* elevation change in 50 *min*. Assuming that the water leaked out at a steady rate, how much work was spent in carrying water to the top? Do not count the work done in getting yourself and the truck there. Water weighs 8 *lb/gal*.

15. A force of 200 *N* will stretch a garage door spring 0.8 *m* beyond its unstressed strength. How far will a 300-*N* force stretch the spring? How much work does it take to stretch the spring this far from its unstressed length?

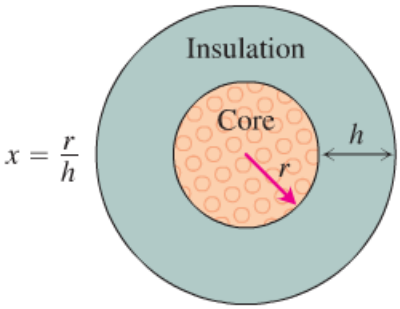
16. You are under contract to build a solar station at ground level on the east-west line between the two buildings. How far from the taller building should you place the station to maximize the number of hours it will be in the sun on a day when passes directly overhead? Begin by observing that

$$\theta = \pi - \cot^{-1}\left(\frac{x}{60}\right) - \cot^{-1}\left(\frac{50-x}{30}\right)$$

Then find the value of x that maximizes θ.



17. A round underwater transmission cable consists of a core of copper wires surrounded by nonconducting insulation. If x denotes the ratio of the radius of the core to the thickness of the insulation, it is known that the speed of the transmission signal is given by the equation $v = x^2 \ln\left(\frac{1}{x}\right)$. If the radius of the core is 1 *cm*, what insulation thickness h will allow the greatest transmission speed?

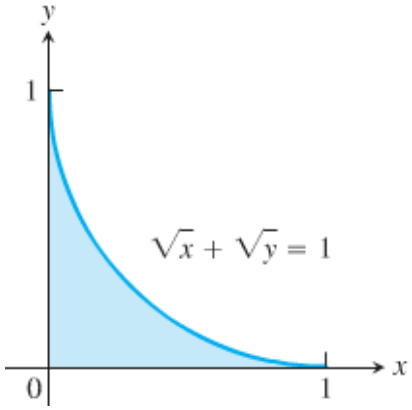


18. Find the area of the triangular region bounded on the left by $x + y = 2$, on the right by $y = x^2$, and above by $y = 2$

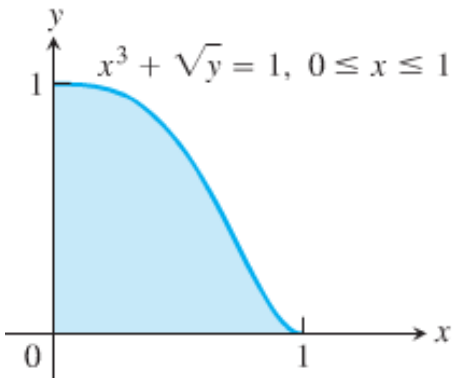
19. Find the extreme values of $f(x) = x^3 - 3x^2$ and find the area of the region enclosed by the graph of f and the x-axis.

20. Find the area of the region bounded by the curves and line

a) $\sqrt{x} + \sqrt{y} = 1, \quad x = 0, \quad y = 0$



b) $x^3+\sqrt{y}=1, \quad x=0, \quad y=0, \quad$ *for* $\quad 0\le x\le 1$



c) $y=8\cos x, \quad y=\sec^2 x, \quad -\frac{\pi}{3}\le x\le \frac{\pi}{3}$

d) $y=2\sin x, \quad y=\sin 2x, \quad 0\le x\le \pi$

e) $y^2=4x+4, \quad y=4x-16$

f) $x=2y^2, \quad x=0, \quad y=3$

Answer

1. $V = \frac{9\pi}{280}$

2. $V = \frac{8\sqrt{3}}{15}$

3. $V = \frac{72}{5}$

4. ***a***) *Washer Method:* $V = \frac{57\pi}{20}$ ***b***) *Shell Method:* $V = \frac{5\pi}{2}$

c) *Shell Method:* $V = \frac{3\pi}{2}$ ***d***) *Washer Method:* $V = \frac{103\pi}{20}$

5. $V = \frac{\pi}{2}(9\pi - 16)$

6. $V \approx 276\ in^3$

7. ***a***) *Shell Method:* $V = \frac{11\pi}{48}$ ***b***) *Washer Method:* $V = \frac{11\pi}{48}$

8. $L = \frac{10}{3}$

9. $L \approx 7.634$

10. $S = \frac{\pi}{9}\left(2\sqrt{2} - 1\right)$

11. $S = 4\pi$

12. $A = \frac{28}{3}$

13. $W = 4640\ J$

14. $W = 22,800,000\ ft.lb$

15. $W = 180\ J$

16. ***a***) $\frac{a}{b}$ ***b***) 1 ***c***) $\frac{m}{n}$ ***d***) 0 ***e***) $-\ln 2$ ***f***) $2\pi^2$ ***g***) 1

17. $x \approx 17.54\ m$

18. $h = \sqrt{e} \approx 1.65\ cm$

19. $\frac{8\sqrt{2} - 7}{6}$

20. $\frac{27}{4}$

21. ***a***) $\frac{1}{6}$ ***b***) $\frac{9}{14}$ ***c***) $6\sqrt{3}$

d) 4 ***e***) $\frac{243}{8}$ ***f***) 18