# Review

## *Review* R.1

## *Polynomials*

### Adding and Subtracting Polynomials

**Properties of Real numbers**

For all real numbers $a$, $b$, and $c$:

| | |
|---|---|
| $a+b=b+a$<br>$ab=ba$ | ***Commutative properties*** |
| $(a+b)+c=a+(b+c)$<br>$(ab)c=a(bc)$ | ***Associative properties*** |
| $a(b+c)=ab+ac$ | ***Distributive properties*** |

***Add or subtract as indicated***

*a)* $\left(8x^3-4x^2+6x\right)+\left(3x^3+5x^2-9x+8\right)$

$$=8x^3-4x^2+6x+3x^3+5x^2-9x+8$$

$$=\left(8x^3+3x^3\right)+\left(-4x^2+5x^2\right)+(6x-9x)+8$$

$$=11x^3+x^2-3x+8$$

*b)* $\left(-4x^4+6x^3-9x^2-12\right)+\left(-3x^3+8x^2-11x+7\right)$

$$=-4x^4+6x^3-3x^3-9x^2+8x^2-11x-12+7$$

$$=-4x^4+3x^3-x^2-11x-5$$

*c)* $\left(2x^2-11x+8\right)-\left(7x^2-6x+2\right)$

$$=2x^2-11x+8-7x^2+6x-2$$

$$=-5x^2-5x+6$$

***Multiply***

*a)* $8x(6x-4)$

$$8x(6x-4) = 8x(6x) - 8x(4)$$
$$= 48x^2 - 32x$$

*b)* $(3p-2)(p^2+5p-1) = 3p^3 + 15p^2 - 3p - 2p^2 - 10p + 2$

$$\underline{= 3p^3 + 13p^2 - 13p + 2 \,|}$$

*c)* $(x+2)(x+3)(x-4) = (x^2+3x+2x+6)(x-4)$

$$= (x^2+5x+6)(x-4)$$
$$= x^3 + 5x^2 + 6x - 4x^2 - 20x - 24$$
$$\underline{= x^3 + x^2 - 14x - 24 \,|}$$

*d)* $(2k-5)^2 = (2k-5)(2k-5)$

$$= 4k^2 - 10k - 10k + 25$$
$$\underline{= 4k^2 - 20k + 25 \,|}$$

***Perform the indicated operations:***

$$2(3x^2+4x+2) - 3(-x^2+4x-5) = 6x^2 + 8x + 4 + 3x^2 - 12x + 15$$
$$\underline{= 9x^2 - 4x + 19 \,|}$$

***Perform the indicated operations:***

$$(3t-2y)(3t+5y) = 9t^2 + 15ty - 6yt - 10y^2$$
$$\underline{= 9t^2 + 9yt - 10y^2 \,|}$$

***Perform the indicated operations:*** $(2a-4b)^2$

$$(2a-4b)^2 = (2a)^2 - 2(2a)(4b) + (4b)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$= 4a^2 - 16ab + 16b^2$$

# *Factoring*

**Prime Factorization**

A process that allows us to write a composite number as a product of two or more prime numbers.

Tree

$$\begin{array}{ccc} & 10 & \\ 2 & & 5 \end{array} \qquad 10 = 2 \times 5$$

$72 = 2.\ 36$

$= 2.\ 6.\ 6$

$= 2.\ 2.\ 3.\ 2.\ 3$

$= 2^3\ 3^2$

## The Greatest Common Factor (*GCF*)

The largest factor that two or more numbers (or terms) have in common

***Find GCF*** (18, 36)

18: 2. 9 | 36: 2. 18

2. 3. 3 | 2. 2. 3. 3

18: $2\ 3^2 \rightarrow 1, 2, 3, 6, 9,$ **<u>18</u>**

36: $2^2\ 3^2 \rightarrow 1, 2, 3, 4, 6, 9, 12,$ **<u>18</u>**, 36 ***GCF*** (18, 36) = 18 (is the greatest common factor)

***Find GCF*** (27, 45)

$27 = 3^3$

$45 = \underline{3^2\ 5}$

$3^2$ ***GCF*** (27, 45) = 9

***Find*** GCF (40, 56)

$40 = 2^3\ 5$

$56 = \underline{2^3\ 7}$

$2^3$ ***GCF*** (40, 56) = 8

***Find GCF*** (80, 60)

$80 = 2^4 \quad 5$

$60 = \underline{2^2\ 3\ \ 5}$

$\quad 2^2 \quad 5$ ***GCF*** (80, 60) = 20

***Factor out the greatest common factor***

*a)* $12p - 18q$

| **12** | 2 . 2 . 3 |
|---|---|
| **18** | 2 . . 3 . 3 |
| | 2 . 3 |

$$12p - 18q = 6(2p - 3q)$$

*b)* $8x^3 - 9x^2 + 15x$

$$8x^3 - 9x^2 + 15x = x\left(8x^2 - 9x + 15\right)$$

## Factoring Trinomial

***Factor*** $y^2 + 8y + 15$

| ***Product*** 15 | ***Sum*** 8 |
|---|---|
| 15 *x* 1 | 15 + 1 |
| 3 *x* 5 | 3 + 5 |

$$y^2 + 8y + 15 = (y+3)(y+5)$$

***Factor*** $4x^2 + 8xy - 5y^2 = (2x - y)(2x + 5y)$

## Special Factorization

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$

$$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

***Factor***

*a)* $64p^2 - 49q^2 = (8p)^2 - (7q)^2$

$$= (8p - 7q)(8p + 7q)$$

*b)* $x^2 + 36$

Can't be factored (in real number) it is prime.

*c)* $x^2 + 12x + 36 = (x+6)^2$

*d)* $9y^2 - 24yz + 16z^2 = (3y)^2 - 2(3y)(4z) + (4z)^2$

$$= (3y - 4z)^2$$

*e)* $y^3 - 8 = y^3 - 2^3$

$$= (y-2)\left(y^2 + 2y + 4\right)$$

*f)* $m^3 + 125 = (m+5)\left(m^2 - 5m + 25\right)$

*g)* $8k^3 - 27z^3 = (2k)^3 - (3z)^3$

$$= (2k - 3z)\left((2k)^2 + 6kz + (3z)^2\right)$$

$$= (2k - 3z)\left(4k^2 + 6kz + 9z^2\right)$$

*h)* $p^4 - 1 = \left(p^2\right)^2 - (1)^2$

$$= \left(p^2 - 1\right)\left(p^2 + 1\right)$$

$$= (p-1)(p+1)\left(p^2 + 1\right)$$

*i)* $60m^4 - 120m^3n + 50m^2n^2 = 10m^2\left(6m^2 - 12mn + 5n^2\right)$

*j)* $y^2 - 4yz - 21z^2 = (y + 3z)(y - 7z)$

*k)* $4a^2 + 10a + 6 = 2\left(2a^2 + 5a + 3\right)$

$$= 2(2a+3)(a+1)$$

*l)* $16a^4 - 81b^4 = \left(4a^2\right)^2 - \left(9b^2\right)^2$

$$= \left(4a^2 - 9b^2\right)\left(4a^2 + 9b^2\right)$$

$$= \left((2a)^2 - (3b)^2\right)\left(4a^2 + 9b^2\right)$$

$$= (2a - 3b)(2a + 3b)\left(4a^2 + 9b^2\right)$$

# Fraction

$$\frac{a}{b} = \frac{numerator}{denominator}$$

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad ad = bc \qquad \textit{Cross multiplication}$$

$$\frac{a}{b} = \frac{na}{nb} = \frac{an}{bn}$$

***a)*** $\frac{5}{6} = \frac{25}{30}?$

$$\frac{5}{6} = \frac{5}{6}\frac{5}{5} = \frac{25}{30}$$

***b)*** $\frac{16}{48} = \frac{1}{3}$

$$\frac{16}{48} = \frac{1}{3} \Leftrightarrow (16)(3) = (1)(48)$$

$$48 = 48$$

Simplify: $\frac{12}{18} = \frac{2.6}{2.9}$

$$= \frac{2.2.3}{2.3.3}$$

$$= \frac{2}{3}$$

Simplify: $\frac{36}{56} = \frac{2.18}{2.28}$

$$= \frac{18}{28}$$

$$= \frac{2.9}{2.14}$$

$$= \frac{9}{14}$$

If the denominators are the same $\Rightarrow$ add the numerators

$$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5}$$

If the denominators are the same $\Rightarrow$ subtract the numerators

$$\frac{4}{9} - \frac{2}{9} = \frac{4-2}{9} = \frac{2}{9}$$

If the denominators are not the same

$\Rightarrow$ Find Least Common Denominator (***LCD***) and convert so that the fractions have the same denominators

***LCD:*** is the smallest whole number that is a multiple of each

$\frac{5}{8}+\frac{1}{12}$ ***LCD*** (8, 12)

$$8 = 2^3$$
$$12 = \underline{2^2\ 3}$$
$$2^3\ 3 = 24$$

***LCD*** (8, 12) = 24

$$\frac{5}{8}+\frac{1}{12}=\frac{5}{8}\frac{3}{3}+\frac{1}{12}\frac{2}{2}$$
$$=\frac{15}{24}+\frac{2}{24}$$
$$=\frac{15+2}{24}$$
$$=\frac{17}{24}$$

$\frac{69}{75}-\frac{1}{50}$

***LCD*** (75, 50)

$$75 = 5^3$$
$$50 = \underline{2\ 5^2}$$
$$2\ 5^3 = 150$$

***LCD*** (75, 50) = 150

$$\frac{69}{75}-\frac{1}{50}=\frac{(69)(2)-(1)(3)}{150}$$
$$=\frac{138-3}{150}$$
$$=\frac{135}{150}$$
$$=\frac{9}{10}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

$$\frac{2}{7}+\frac{3}{5}=\frac{2(5)+3(7)}{7(5)}$$
$$=\frac{10+21}{35}$$

$$= \frac{31}{35}$$

*or* $$\frac{2}{7}\frac{5}{5} + \frac{3}{5}\frac{7}{7} = \frac{10}{35} + \frac{21}{35}$$
$$= \frac{10+21}{35}$$
$$= \frac{31}{35}$$

$$\frac{5}{9} + \frac{3}{4} = \frac{5(4)+3(9)}{9(4)}$$
$$= \frac{20+27}{36}$$
$$\boxed{= \frac{47}{36}}$$

$$\frac{17}{15} + \frac{5}{12} = \frac{17(12)+5(15)}{15(12)}$$
$$= \frac{204+75}{180}$$
$$= \frac{279}{180}$$
$$= \frac{31(9)}{20(9)}$$
$$\boxed{= \frac{31}{20}}$$

$$\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} = \frac{5(7)(9)+(3)(7)(9)+(3)(5)(9)+(3)(5)(7)}{(3)(5)(7)(9)}$$
$$= \frac{315+189+135+105}{945}$$
$$= \frac{744}{945}$$
$$= \frac{248}{315}\frac{3}{3}$$
$$\boxed{= \frac{248}{315}}$$

$$\frac{8}{9} + \frac{1}{12} + \frac{3}{16} = \frac{8(16)+1(12)+3(9)}{144} \qquad \left\{\begin{array}{lll} 9 = & & 3^2 \\ 12 = & 2^2 & 3 \\ 16 = & 2^4 & \end{array}\right.$$

$$LCD \quad 2^4 \quad 3^2 = 144$$

$$= \frac{128 + 12 + 27}{144}$$

$$\boxed{= \frac{167}{144}}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{2}{7} - \frac{3}{5} = \frac{2(5) - 3(7)}{7(5)} = \frac{10 - 21}{35} = -\frac{11}{35}$$

$$\frac{a}{c}\frac{b}{d} = \frac{ab}{cd}$$

$$\frac{2}{7}\frac{3}{5} = \frac{6}{35}$$

$$\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \times \frac{d}{b} = \frac{ad}{cb}$$

$$\frac{2}{7} \div \frac{3}{5} = \frac{2}{7}\frac{5}{3} = \frac{10}{21}$$

$$\frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{c}$$

$$\frac{\frac{a}{b}}{\frac{a}{c}} = \frac{c}{b}$$

# *Exponents*

## Integer Exponents

*Definition of exponent*

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n-times}$$

***a*** appears as a factor $n$ times

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

$$a^m \cdot a^n = a^{m+n} \qquad \left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

$$\left(a^m\right)^n = a^{mn} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad (ab)^m = a^m b^m$$

***a)*** $6^0$

$$6^0 = 1$$

***b)*** $(-9)^0$

$$(-9)^0 = 1$$

***c)*** $3^{-2}$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

***d)*** $\left(\frac{3}{4}\right)^{-1}$

$$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$$

***e)*** $7^4.7^6 = 7^{4+6} = 7^{10}$

***f)*** $\dfrac{9^{14}}{9^6} = 9^{14-6} = 9^8$

***g)*** $\dfrac{r^9}{r^{17}} = \dfrac{1}{r^{17-9}} = \dfrac{1}{r^8}$

***h)*** $\left(2m^3\right)^4 = (2)^4\left(m^3\right)^4$

$= 16m^{12}$

***i)*** $\left(\frac{x^2}{y^3}\right)^6 = \frac{\left(x^2\right)^6}{\left(y^3\right)^6}$

$= \frac{x^{2.6}}{y^{3.6}}$

$= \frac{x^{12}}{y^{18}}$

***j)*** $\frac{a^{-3}b^5}{a^4b^{-7}} = \frac{b^5b^7}{a^3a^4}$

$= \frac{b^{5+7}}{a^{4+3}}$

$= \frac{b^{12}}{a^7}$

***k)*** $p^{-1} + q^{-1} = \frac{1}{p} + \frac{1}{q}$

$= \frac{1}{p}\frac{q}{q} + \frac{1}{q}\frac{p}{p}$

$= \frac{q+p}{pq}$

***l)*** $\frac{x^{-2} - y^{-2}}{x^{-1} - y^{-1}} = \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} - \frac{1}{y}}$

$= \frac{\frac{y^2 - x^2}{x^2y^2}}{\frac{y-x}{xy}}$

$= \frac{y^2 - x^2}{x^2y^2} \cdot \frac{xy}{y-x}$

$$= \frac{(y-x)(y+x)}{(xy)^2} \cdot \frac{xy}{y-x}$$

$$= \frac{y+x}{xy}$$

***Calculations with exponents***

*a)* $121^{1/2} = 11$

*b)* $625^{1/4} = 5$

*c)* $(-32)^{1/5} = -2$

*d)* $(-49)^{1/2}$ *is not a real number*

## *Rational Exponents*

$$a^{m/n} = \left(a^{1/n}\right)^m$$

### *Calculations with Exponents*

*a)* $27^{2/3} = \left(27^{1/3}\right)^2$ 27^(2/3)

$$= \left(\left(3^3\right)^{1/3}\right)^2$$

$$= \left(3^{3 \cdot \frac{1}{3}}\right)^2$$

$$= (3)^2$$

$$= 9$$

*b)* $32^{2/5} = \left(\left(2^5\right)^{1/5}\right)^2$ 32 ^ (2/5)

$$= 2^2$$

$$= 4$$

*c)* $64^{4/3} = \left(\left(4^3\right)^{1/3}\right)^4$ 64 ^ (4/3)

$$= (4)^4$$

$$= 256$$

*d)* $\dfrac{y^{1/3}y^{5/3}}{y^3} = \dfrac{y^{\frac{1}{3}+\frac{5}{3}}}{y^3}$

$$= \frac{y^{\frac{6}{3}}}{y^3}$$

$$= \frac{y^2}{y^3}$$

$$= \frac{1}{y^{3-2}}$$

$$= \frac{1}{y}$$

*e)* $m^{2/3}\left(m^{7/3}+7m^{1/3}\right)=m^{2/3}m^{7/3}+7m^{2/3}m^{1/3}$

$$
\begin{aligned}
&= m^{\frac{2}{3}+\frac{7}{3}}+7m^{\frac{2}{3}+\frac{1}{3}}\\
&= m^{\frac{9}{3}}+7m^{\frac{3}{3}}\\
&= m^{3}+7m
\end{aligned}
$$

*f)* $\left(\dfrac{m^{7}n^{-2}}{m^{-5}n^{2}}\right)^{1/4}=\left(\dfrac{m^{7+5}}{n^{2+2}}\right)^{1/4}$

$$
\begin{aligned}
&= \left(\frac{m^{12}}{n^{4}}\right)^{1/4}\\
&= \frac{\left(m^{12}\right)^{1/4}}{\left(n^{4}\right)^{1/4}}\\
&= \frac{m^{12/4}}{n^{4/4}}\\
&= \frac{m^{3}}{n}
\end{aligned}
$$

*g)* $9x^{-2}-6x^{-3}=3x^{-3}\left(3x-2\right)$

*h)* $2\left(x^{2}+5\right)\left(3x-1\right)^{-1/2}+\left(3x-1\right)^{1/2}\left(2x\right)=2\left(3x-1\right)^{-1/2}\left[x^{2}+5+x\left(3x-1\right)\right]$

$$
\begin{aligned}
&= 2\left(3x-1\right)^{-1/2}\left[x^{2}+5+3x^{2}-x\right]\\
&= 2\left(3x-1\right)^{-1/2}\left(4x^{2}-x+5\right)
\end{aligned}
$$

## *Radicals*

$$a^{1/n} = \sqrt[n]{a}$$

*a)* $\sqrt[4]{16} = 16^{1/4} = 2$

*b)* $\sqrt[5]{-32} = -2$

*c)* $\sqrt[3]{1000} = 1000^{1/3} = 10$

*d)* $\sqrt[6]{\dfrac{64}{729}} = \dfrac{\sqrt[6]{64}}{\sqrt[6]{729}} = \dfrac{2}{3}$

## Properties

$$\left(\sqrt[n]{a}\right)^n = a$$

$$\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n \text{ is even} \\ a & \text{if } n \text{ is odd} \end{cases}$$

$$\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

Simplify

*a)* $\sqrt{1000} = \sqrt{100(10)}$
$= \sqrt{100}\sqrt{10}$
$= 10\sqrt{10}$

*b)* $\sqrt{128} = \sqrt{64(2)}$
$= 8\sqrt{2}$

*c)* $\sqrt{2}\sqrt{18} = \sqrt{2(18)}$

$= \sqrt{36}$

$= 6$

*d)* $\sqrt[3]{54} = \sqrt[3]{27(2)}$

$= 3\sqrt[3]{2}$

*e)* $\sqrt{288m^5} = \sqrt{144(2)m^4m}$

$= 12m^2\sqrt{2m}$

*f)* $2\sqrt{18} - 5\sqrt{32} = 2\sqrt{9(2)} - 5\sqrt{16(2)}$

$= 6\sqrt{2} - 20\sqrt{2}$

$= -14\sqrt{2}$

## Rationalizing Denominators

Simplify by rationalizing the denominator

*a)* $\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{4\sqrt{3}}{3}$

*b)* $\frac{2}{\sqrt[3]{x}} = \frac{2}{\sqrt[3]{x}}\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

$= \frac{2\sqrt[3]{x^2}}{x}$

*c)* $\frac{1}{1-\sqrt{2}} = \frac{1}{1-\sqrt{2}}\frac{1+\sqrt{2}}{1+\sqrt{2}}$

$= \frac{1+\sqrt{2}}{1-2}$

$= \frac{1+\sqrt{2}}{-1}$

$= -1-\sqrt{2}$

Simplify

$$\sqrt{27}\sqrt{3} = \sqrt{27(3)}$$
$$= \sqrt{81}$$
$$= 9$$

Simplify

$$\sqrt[4]{x^8y^7z^{11}} = x^2yz^2 \ \sqrt[4]{y^3z^3}$$

Simplify

$$\frac{5}{\sqrt{10}} = \frac{5}{\sqrt{10}}\frac{\sqrt{10}}{\sqrt{10}}$$
$$= \frac{5\sqrt{10}}{10}$$
$$= \frac{\sqrt{10}}{2}$$

Simplify

$$\frac{5}{2-\sqrt{6}} = \frac{5}{2-\sqrt{6}}\frac{2+\sqrt{6}}{2+\sqrt{6}}$$
$$= \frac{5\left(2+\sqrt{6}\right)}{4-6}$$
$$= -\frac{5}{2}\left(2+\sqrt{6}\right)$$

Simplify

$$\frac{1}{\sqrt{r}-\sqrt{3}} = \frac{1}{\sqrt{r}-\sqrt{3}}\frac{\sqrt{r}+\sqrt{3}}{\sqrt{r}+\sqrt{3}}$$
$$= \frac{\sqrt{r}+\sqrt{3}}{r-3}$$