

$$\begin{aligned}\int_{-\infty}^{\infty} x e^{-x^2} dx &= -\frac{1}{2} \int_{-\infty}^{\infty} e^{-x^2} d(-x^2) \\ &= -\frac{1}{2} e^{-x^2} \Big|_{-\infty}^{\infty} \\ &= -\frac{1}{2} (e^{-\infty} - e^{-\infty}) \\ &= -\frac{1}{2} (1 - 1) \\ &= 0\end{aligned}$$


---

$$\begin{aligned}\int_0^{\infty} \cos x dx &= \sin x \Big|_0^{\infty} \\ &= \sin \infty - \sin 0 \\ &= \text{doesn't exist} \Rightarrow \text{diverges}\end{aligned}$$


---

$$\begin{aligned}\int_2^{\infty} \frac{\cos(\frac{\pi}{x})}{x^2} dx &= -\frac{1}{\pi} \int_2^{\infty} \cos(\frac{\pi}{x}) d(\frac{\pi}{x}) & d(\frac{\pi}{x}) &= -\frac{\pi}{x^2} dx \\ &= -\frac{1}{\pi} \sin \frac{\pi}{x} \Big|_2^{\infty} & \frac{\pi}{x} &\rightarrow 0 \\ &= -\frac{1}{\pi} (\sin 0 - \sin \frac{\pi}{2}) \\ &= \frac{1}{\pi}\end{aligned}$$


---

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+5} &= \int_{-\infty}^{\infty} \frac{dx}{(x+1)^2+4} & x+1 &= 2 \tan \theta \\ & & dx &= 2 \sec^2 \theta d\theta \\ & & (x+1)^2+4 &= 4 \sec^2 \theta \\ &= \int_{-\infty}^{\infty} \frac{2 \sec^2 \theta d\theta}{4 \sec^2 \theta} \\ &= \frac{1}{2} \int_{-\infty}^{\infty} d\theta \\ &= \frac{1}{2} \theta \Big|_{-\infty}^{\infty} \\ &= \frac{1}{2} \tan^{-1} \frac{x+1}{2} \Big|_{-\infty}^{\infty} \\ &= \frac{1}{2} (\tan^{-1} \infty - \tan^{-1} (-\infty)) \\ &= \frac{1}{2} (\frac{\pi}{2} + \frac{\pi}{2}) \\ &= \frac{\pi}{2}\end{aligned}$$

$$\begin{aligned}\int_{-\infty}^a \sqrt{e^x} dx &= \int_{-\infty}^a e^{x/2} dx & a \in \mathbb{R} \\ &= 2 e^{x/2} \Big|_{-\infty}^a \\ &= 2 (e^{a/2} - 0) \\ &= \underline{2 e^{a/2}}\end{aligned}$$

$$\begin{aligned}\int_0^{\infty} \frac{e^u}{e^{2u} + 1} du &= \int_0^{\infty} \frac{d(e^u)}{(e^u)^2 + 1} & \int \frac{dx}{x^2 + 1} = \tan^{-1} x \\ &= \tan^{-1} e^u \Big|_0^{\infty} \\ &= \tan^{-1} \infty - \tan^{-1} 1 \\ &= \frac{\pi}{2} - \frac{\pi}{4} \\ &= \underline{\frac{\pi}{4}}\end{aligned}$$

$$\begin{aligned}\int_1^{\infty} \frac{dx}{x(x+1)} &= \int_1^{\infty} \left( \frac{1}{x} - \frac{1}{x+1} \right) dx \\ &= \ln|x| - \ln|x+1| \Big|_1^{\infty} \\ &= \ln \frac{x}{x+1} \Big|_1^{\infty} & \lim_{x \rightarrow \infty} \ln \frac{x}{x+1} = \ln 1 = 0 \\ &= 0 - \ln \frac{1}{2} \\ &= \underline{\ln 2}\end{aligned}$$

$$\begin{aligned}\int_1^{\infty} \frac{dx}{x^2(x+1)} &= \int_1^{\infty} \left( \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1} \right) dx \\ &= -\ln x - \frac{1}{x} + \ln(x+1) \Big|_1^{\infty} \\ &= \ln \frac{x+1}{x} - \frac{1}{x} \Big|_1^{\infty} \\ &= \ln 1 - 0 - \ln 2 + 1 \\ &= \underline{1 - \ln 2}\end{aligned}$$

$$\begin{aligned}\frac{1}{x^2(x+1)} &= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} \\ 1 &= Ax^2 + Ax + Bx + B + Cx^2 \\ \begin{cases} A+C=0 & \rightarrow C=1 \\ A+B=0 & \rightarrow A=-1 \\ B=1 \end{cases}\end{aligned}$$

$$\lim_{x \rightarrow \infty} \ln \frac{x+1}{x} = \ln 1$$

$$\begin{aligned}\int_1^{\infty} \frac{3x^2+1}{x^3+x} dx &= \int_1^{\infty} \frac{d(x^3+x)}{x^3+x} \\ &= \ln(x^3+x) \Big|_1^{\infty} \\ &= \infty - \ln 2 \\ &= \infty \text{ diverges.}\end{aligned}$$

$$\begin{aligned}\int_1^{\infty} \frac{1}{x^2} \sin \frac{\pi}{x} dx &= \frac{-1}{\pi} \int_1^{\infty} \sin\left(\frac{\pi}{x}\right) d\left(\frac{\pi}{x}\right) & d\left(\frac{\pi}{x}\right) &= -\frac{\pi}{x^2} dx \\ &= \frac{1}{\pi^2} \cos \frac{\pi}{x} \Big|_1^{\infty} \\ &= \frac{1}{\pi^2} (\cos 0 - \cos \pi) \\ &= \frac{2}{\pi^2}\end{aligned}$$

$$\begin{aligned}\int_2^{\infty} \frac{dx}{(x+2)^2} &= \int_2^{\infty} \frac{d(x+2)}{(x+2)^2} \\ &= -\frac{1}{x+2} \Big|_2^{\infty} \\ &= -\left(0 - \frac{1}{4}\right) \\ &= \frac{1}{4}\end{aligned}$$

$$\begin{aligned}\int_1^{\infty} \frac{\tan^{-1} x}{x^2+1} dx &= \int_1^{\infty} \tan^{-1} x d(\tan^{-1} x) \\ &= \frac{1}{2} (\tan^{-1} x)^2 \Big|_1^{\infty} \\ &= \frac{1}{2} \left[ \left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{4}\right)^2 \right] \\ &= \frac{1}{2} \left( \frac{\pi^2}{4} - \frac{\pi^2}{16} \right) \\ &= \frac{3\pi^2}{32}\end{aligned}$$

$$\begin{aligned}\int_0^8 \frac{dx}{\sqrt[3]{x}} &= \int_0^8 x^{-1/3} dx \\ &= \frac{3}{2} x^{2/3} \Big|_0^8 \\ &= \frac{3}{2} (4 - 0) \\ &= \underline{6}\end{aligned}$$

$$\begin{aligned}\int_{-3}^1 \frac{dx}{(2x+6)^{2/3}} &= \frac{1}{2} \int_{-3}^1 (2x+6)^{-2/3} d(2x+6) \\ &= \frac{3}{2} (2x+6)^{1/3} \Big|_{-3}^1 \\ &= \frac{3}{2} [2 - 0] \\ &= \underline{3}\end{aligned}$$

$$\begin{aligned}\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx &= 2 \int_0^1 e^{\sqrt{x}} d(\sqrt{x}) \quad d(\sqrt{x}) = \frac{1}{2\sqrt{x}} dx \\ &= 2 e^{\sqrt{x}} \Big|_0^1 \\ &= \underline{2(e-1)}\end{aligned}$$

$$\begin{aligned}\int_0^{\ln 3} \frac{e^x}{(e^x-1)^{2/3}} dx &= \int_0^{\ln 3} (e^x-1)^{-2/3} d(e^x-1) \\ &= 3 (e^x-1)^{1/3} \Big|_0^{\ln 3} \\ &= 3 (2^{1/3} - 0) \\ &= \underline{3 \sqrt[3]{2}}\end{aligned}$$