

$$3y'' + xy' - 4y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

$$3y'' + xy' - 4y = 0$$

$$3 \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + x \sum_{n=1}^{\infty} n a_n x^{n-1} - 4 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (3(n+1)(n+2) a_{n+2} - 4a_n) x^n + \sum_{n=0}^{\infty} n a_n x^n = 0$$

$$\sum_{n=0}^{\infty} [3(n+1)(n+2) a_{n+2} + (n-4) a_n] x^n = 0$$

$$a_{n+2} = - \frac{n-4}{3(n+1)(n+2)} a_n$$

$$n \quad a_0 \quad a_1$$

$$0 \quad a_2 = + \frac{-4}{6} a_0 = -\frac{2}{3} a_0$$

$$a_3 = + \frac{1}{6} a_1 = \frac{1}{3!} a_1$$

$$2 \quad a_4 = -\frac{2}{36} a_2 = \frac{1}{27} a_0 \quad 3 \quad a_5 = -\frac{1}{4 \cdot 5 \cdot 3} a_3 = -\frac{1}{5! \cdot 3} a_1$$

$$n=4 \quad a_6 = 0$$

$$5 \quad a_7 = -\frac{1}{3 \cdot 6 \cdot 7} a_5 = -\frac{1}{7! \cdot 3^2} a_1$$

$$7 \quad a_9 = -\frac{3}{3 \cdot 8 \cdot 9} a_7 = -\frac{3}{9! \cdot 3^3} a_1$$

$$y(x) = a_0 \left(1 + \frac{2}{3} x^2 + \frac{1}{27} x^4 \right)$$

$$+ a_1 \left(x + \frac{1}{3!} x^3 + \frac{1}{5! \cdot 3} x^5 + \dots \right)$$

$$y'' + x^2 y = 0 \quad y(0) = 1 \quad y'(0) = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n + x^2 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n + \sum_{n=0}^{\infty} a_n x^{n+2} = 0$$

$$2a_2 + 6a_3 x + \sum_{n=2}^{\infty} (n+1)(n+2) a_{n+2} x^n + \sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$$2a_2 + 6a_3 x + \sum_{n=2}^{\infty} [(n+1)(n+2) a_{n+2} + a_{n-2}] x^n = 0$$

$$2a_2 + 6a_3 x = 0$$

$$a_{n+2} = -\frac{1}{(n+1)(n+2)} a_{n-2}$$

$$a_2 = a_3 = 0$$

$$\text{Given } y(0) = 1 = a_0 \quad y'(0) = 0 = a_1$$

$$a_0 = 1$$

$$a_2 = a_3 = a_4 = 0$$

$$n=2 \quad a_4 = -\frac{1}{12}$$

$$n=3 \quad a_5 = -a_1 = 0$$

$$n=4 \quad a_6 = -a_2 = 0$$

$$n=5 \quad a_7 = -a_3 = 0$$

$$n=6 \quad a_8 = -\frac{1}{7 \cdot 8} a_4$$

$$= \frac{+1}{3 \cdot 4 \cdot 7 \cdot 8}$$

$$y(x) = 1 - \frac{1}{12} x^4 + \frac{1}{3 \cdot 4 \cdot 7 \cdot 8} x^8 - \dots$$

$$y'' = y'' + y'' = 0$$

$$y' = y' + y' = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} [n(n-1) a_n + (n+1) a_n] x^n + \sum_{n=0}^{\infty} n a_n x^n = 0$$

$$\sum_{n=0}^{\infty} [n(n-1) a_n + (n+1) a_n] x^n = 0$$

$$a_{n+2} = -\frac{1}{n(n+1)} a_n$$

$$a_0 = 1$$

$$a_1 = 0$$

$$n=0 \quad a_2 = -\frac{1}{2}$$

$$a_3 = a_1 = 0$$

$$n=2 \quad a_4 = -\frac{1}{4} a_2 = \frac{1}{8}$$

$$n=4 \quad a_6 = -\frac{1}{6} a_4 = -\frac{1}{2 \cdot 3}$$

$$y(x) = 1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 - \frac{1}{48} x^6 + \dots$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$y''' = \sum_{n=3}^{\infty} n(n-1)(n-2) a_n x^{n-3}$$

$$x^2 y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^n + \sum_{n=2}^{\infty} n(n-1) a_n x^n + \sum_{n=2}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^n + \sum_{n=2}^{\infty} n(n-1) a_n x^n + \sum_{n=2}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} [n(n-1) a_n + (n(n-1) + 1) a_n] x^n = 0$$

$$a_{n+1} = - \frac{n^2 - n + 1}{n+1} \cdot a_n$$

$$a_0$$

$$n=0 \quad a_1 = -a_0$$

$$n=1 \quad a_2 = -\frac{1}{2} a_1 = \frac{1}{2} a_0$$

$$n=2 \quad a_3 = -\frac{3}{3} a_2 = -\frac{1}{2} a_0$$

$$n=3 \quad a_4 = -\frac{7}{4} a_3 = \frac{7}{8} a_0$$

$$y(x) = a_0 \left(1 - x + \frac{1}{2} x^2 - \frac{1}{2} x^3 + \frac{7}{8} x^4 - \dots \right)$$