# **Section 8.4 – Solving Trigonometry Equations**

# Example

Find the solutions of the equation  $\sin \theta = \frac{1}{2}$  if

- a)  $\theta$  is in the interval  $[0, 2\pi)$
- b)  $\theta$  is any real number

#### Solution

$$a) \quad \theta = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

b) Since the sine function has period  $2\pi$ .

$$\theta = \frac{\pi}{6} + 2\pi n$$
 and  $\theta = \frac{5\pi}{6} + 2\pi n$ 

# **Example**

Solve the equation  $\sin x \tan x = \sin x$ 

# Solution

$$\sin x \, \tan x - \sin x = 0$$

$$\sin x (\tan x - 1) = 0$$

$$\sin x = 0 \qquad \tan x - 1 = 0$$

$$\tan x = 1$$

$$\hat{x} = \sin^{-1} 0 = 0$$
  $\hat{x} = \tan^{-1} 1 = \frac{\pi}{4}$ 

$$x = 0, \pm \pi, \pm 2\pi, \dots$$
  $x = \pm \frac{\pi}{4}, \pm \frac{5\pi}{4}, \dots$ 

$$x = \pi n \qquad \qquad x = \frac{\pi}{4} + \pi n$$

The solutions are:  $x = \pi n$  and  $x = \frac{\pi}{4} + \pi n$  for every integer n.

# Example

Solve the equation  $2\sin^2 t - \cos t - 1 = 0$ , and express the solutions both in radians and degrees.

#### Solution

$$2\sin^{2}t - \cos t - 1 = 0$$

$$2\left(1 - \cos^{2}t\right) - \cos t - 1 = 0$$

$$2 - 2\cos^{2}t - \cos t - 1 = 0$$

$$-2\cos^{2}t - \cos t + 1 = 0$$

$$2\cos^{2}t + \cos t - 1 = 0$$

$$2\cos^{2}t + \cos t - 1 = 0$$

$$2\cos^{2}t + \cos t - 1 = 0$$

$$2\cos t = 1$$

$$\cos t = \frac{1}{2}$$

$$t = \frac{\pi}{3} \quad or \quad t = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$t = \pi$$

$$t = \frac{\pi}{3} + 2\pi n, \quad t = \frac{5\pi}{3} + 2\pi n, \quad t = \pi + 2\pi n$$

$$t = 60^{\circ} + 360^{\circ}n, \quad 300^{\circ} + 360^{\circ}n, \quad and \quad 180^{\circ} + 360^{\circ}n$$

# **Example**

Solve the equation  $4\sin^2 x \tan x - \tan x = 0$  in the interval  $[0, 2\pi)$ .

### **Solution**

$$4\sin^2 x \tan x - \tan x = 0$$

$$\tan x \left( 4\sin^2 x - 1 \right) = 0$$

$$\tan x = 0$$

$$\tan x = 0$$

$$\sin^2 x - 1 = 0$$

$$\sin^2 x = \frac{1}{4}$$

$$\tan x = 0$$

$$\sin x = \frac{1}{2}$$

$$\sin x = -\frac{1}{2}$$

$$\underline{x} = 0, \pi$$

$$\underline{x} = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\underline{x} = \frac{7\pi}{6}, \frac{11\pi}{6}$$

### Example

Find the solutions of  $\csc^4 2u - 4 = 0$ 

#### Solution

$$(\csc^2 2u - 2)(\csc^2 2u + 2) = 0$$

$$\csc^2 2u - 2 = 0 \qquad \csc^2 2u + 2 = 0$$

$$\csc^2 2u = 2 \qquad \csc^2 2u = -2 \times 2$$

$$\csc 2u = \pm \sqrt{2}$$

$$\sin 2u = \pm \frac{1}{\sqrt{2}}$$

$$\sin 2u = \frac{\sqrt{2}}{2} \implies 2u = \frac{\pi}{4} + 2\pi n \implies u = \frac{\pi}{8} + \pi n$$

$$\Rightarrow 2u = \frac{3\pi}{4} + 2\pi n \implies u = \frac{3\pi}{8} + \pi n$$

$$\Rightarrow 2u = \frac{5\pi}{4} + 2\pi n \implies u = \frac{5\pi}{8} + \pi n$$

$$\Rightarrow 2u = \frac{7\pi}{4} + 2\pi n \implies u = \frac{7\pi}{8} + \pi n$$

#### **Example**

Approximate to the nearest degree, the solutions of the following equation in the interval [0°, 360°):

$$5\sin\theta\tan\theta - 10\tan\theta + 3\sin\theta - 6 = 0$$

#### **Solution**

$$\tan \theta (5\sin \theta - 10) + (3\sin \theta - 6) = 0$$

$$5\tan \theta (\sin \theta - 2) + 3(\sin \theta - 2) = 0$$

$$(\sin \theta - 2)(5\tan \theta + 3) = 0$$

$$\sin \theta - 2 = 0 \qquad 5\tan \theta + 3 = 0$$

$$\sin \theta = 2 > 1 \qquad \tan \theta = -\frac{3}{5} \qquad \theta \in \mathbf{QII}, \mathbf{QIV}$$

$$\hat{\theta} = \tan^{-1} \left(\frac{3}{5}\right) = 31^{\circ}$$

$$\begin{cases} \theta = 180^{\circ} - 31^{\circ} = 149^{\circ} \\ \theta = 360^{\circ} - 31^{\circ} = 329^{\circ} \end{cases}$$

# **Exercises** Section 8.4 – Trigonometric Equations

(1-9) Find all solutions of the equation

$$1. \qquad \sin x = \frac{\sqrt{2}}{2}$$

$$2. \qquad \cos x = -\frac{\pi}{3}$$

$$3. \qquad 2\cos\theta - \sqrt{3} = 0$$

4. 
$$\sqrt{3} \tan \frac{1}{3} x = 1$$

$$5. \qquad \cos\left(4x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$6. \qquad (\cos\theta - 1)(\sin\theta + 1) = 0$$

7. 
$$\cot^2 x - 3 = 0$$

8. 
$$\cos x + 1 = 2\sin^2 x$$

$$9. \quad \cos(\ln x) = 0$$

(10-24) Find the solutions of the equation that are in the interval  $[0, 2\pi)$ 

10. 
$$2\sin^2 x = 1 - \sin x$$

11. 
$$\tan^2 x \sin x = \sin x$$

12. 
$$1 - \sin x = \sqrt{3} \cos x$$

13. 
$$\sin x + \cos x \cot x = \csc x$$

14. 
$$2\sin^3 x + \sin^2 x - 2\sin x - 1 = 0$$

15. 
$$2 \tan x \csc x + 2 \csc x + \tan x + 1 = 0$$

$$16. \quad 5\cos t + \sqrt{12} = \cos t$$

17. 
$$2\sin^2 x - \cos x - 1 = 0$$

18. 
$$2\cos^2 t - 9\cos t = 5$$

19. 
$$\tan^2 x + \tan x - 2 = 0$$

**20.** 
$$\tan x + \sqrt{3} = \sec x$$

**21.** 
$$2\sin^2\theta + 2\sin\theta - 1 = 0$$

**22.** 
$$2\cos x - 1 = \sec x$$

23. 
$$4\cos^2 x + 4\sin x - 5 = 0$$

**24.** 
$$\sin \theta - \cos \theta = 1$$

(25 – 35) Find the solutions of the equation that are in the interval if  $0^{\circ} \le \theta < 360^{\circ}$ 

**25.** 
$$2\cos\theta + \sqrt{3} = 0$$

**26.** 
$$\tan \theta - 2\cos \theta \tan \theta = 0$$

**27.** 
$$2\sin^2\theta - 2\sin\theta - 1 = 0$$

$$28. \quad 4\cos\theta - 3\sec\theta = 0$$

$$29. \quad \sin\theta - \sqrt{3}\cos\theta = 1$$

$$30. \quad 7\sin^2\theta - 9\cos 2\theta = 0$$

31. 
$$\sin \theta \tan \theta = \sin \theta$$

**32.** 
$$2\sin\theta - 3 = 0$$

$$33. \quad 3\sin\theta - 2 = 7\sin\theta - 1$$

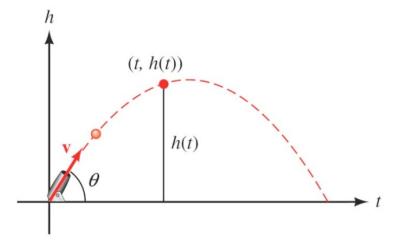
$$34. \quad \cos 2\theta + 3\sin \theta - 2 = 0$$

$$35. \quad \sin 2\theta + \sqrt{2}\cos \theta = 0$$

**36.** Solve 
$$\cos\left(A - \frac{\pi}{9}\right) = -\frac{1}{2}$$

37. Solve 
$$\cos(A - 25^\circ) = -\frac{1}{\sqrt{2}}$$

38. If a projectile (such as a bullet) is fired into the air with an initial velocity v at an angle of elevation  $\theta$ , then the height h of the projectile at time t is given by:  $h(t) = -16t^2 + vt\sin\theta$ 



- a) Give the equation for the height, if v is  $600 \, ft$ ./sec and  $\theta = 45^{\circ}$ .
- b) Use the equation in part (a) to find the height of the object after  $\sqrt{3}$  seconds.
- c) Find the angle of elevation of  $\theta$  of a rifle barrel, if a bullet fired at 1,500 ft./sec takes 3 seconds to reach a height of 750 feet. Give your answer in the nearest of a degree.