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8.3 Double angle

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1 \quad (1)$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2 \sin A \cos A$$

(1) $2\cos^2 A = 1 + \cos 2A$

$$\cos^2 A = \frac{1+\cos 2A}{2}$$

$$\cos^2 \frac{A}{2} = \frac{1+\cos A}{2}$$

$$\cos \frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \quad Q?$$

$$\sin \frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

Ex $\sin A = \frac{3}{5}$ $\quad A \in Q\text{II}$ $\quad 3, 4 \to 5$

$$\cos A = -\frac{4}{5}$$

a) $\sin 2A = 2 \sin A \cos A$

$$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$$

$$= -\frac{24}{25}$$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{25}$$

c) $\boxed{\tan 2A = -\frac{24}{7}}$

$\frac{90^\circ}{2} < \frac{A}{2} < \frac{180^\circ}{2}$

$45^\circ < \frac{A}{2} < 90^\circ \Rightarrow \boxed{\frac{A}{2} \in QI}$

d) $\sin\frac{A}{2} = \sqrt{\frac{1}{2}\left(1-\left(-\frac{4}{5}\right)\right)}$

$\boxed{\sqrt{\frac{1}{2}(1-\cos A)}}$

$= \sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)}$

$= \sqrt{\frac{1}{2}\,\frac{9}{5}}$

$= \boxed{\frac{3}{\sqrt{10}}}$

$\sqrt{\frac{1}{2}(1+\cos A)}$

e) $\cos\frac{A}{2} = \sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)}$

$= \sqrt{\frac{1}{2}\,\frac{1}{5}}$

$= \boxed{\frac{1}{\sqrt{10}}}$

f) $\boxed{\tan\frac{A}{2} = 3}$ $\qquad \frac{3}{1}$

Prove: $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= 1 + \sin 2\theta$ ✓

Prove

Prove

$$\sin 2x = \frac{2\cot x}{1+\cot^2 x}$$

$$\frac{2\cot x}{\underbrace{1+\cot^2 x}_{\csc^2 x}} = 2\,\frac{\frac{\cos x}{\sin x}}{1+\frac{\cos^2 x}{\sin^2 x}}$$

$$= 2\,\frac{\frac{\cos x}{\sin x}}{\frac{\sin^2 x+\cos^2 x}{\sin^2 x}} \quad \div$$

$$= 2\,\frac{\cos x}{\sin x}\,\frac{\sin^2 x}{1}$$

$$= 2\cos x \sin x$$

$$= \sin 2x$$

Prove: $\cos 4x = \cancel{x}\,8\cos^4 x - 8\cos^2 x + 1$

$$\cos 4x = \cos(2(2x)) \qquad \underline{\cos 2\alpha = 2\cos^2\alpha - 1}$$

$$= 2\cos^2(2x) - 1$$

$$= 2(\cos 2x)^2 - 1$$

$$= 2(2\cos^2 x - 1)^2 - 1$$

$$= 2(4\cos^4 x - 4\cos^2 x + 1) - 1$$

$$= 8\cos^4 x - 8\cos^2 x + 2 - 1$$

$$= 8\cos^4 x - 8\cos^2 x + 1$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} \qquad A+B \quad \frac{\tan A + \tan B}{1 - \tan \tan}$$

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$$

Prove: $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta}$

$$\frac{1-\cos 2A}{\sin 2\theta} = \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$$
$$= \frac{1-1+2\sin^2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta \checkmark$$

Prove: $\sin^2\frac{x}{2} = \dfrac{\tan x - \sin x}{2\tan x}$

$$\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)\cdot\frac{\tan x}{\tan x}$$
$$= \frac{\tan x - \cos x\tan x}{2\tan x}$$
$$= \frac{\tan x - \cos x\,\frac{\sin x}{\cos x}}{2\tan x}$$
$$= \frac{\tan x - \sin x}{2\tan x} \checkmark$$

II 20 Prove: $\sin 3x = \sin x(3\cos^2 x - \sin^2 x)$

$$\begin{aligned}
\sin 3x &= \sin(x+2x) \\
&= \sin x \cos 2x + \cos x \sin 2x \\
&= \sin x(\cos^2 x - \sin^2 x) + \cos x(2\sin x \cos x) \\
&= \sin x \cos^2 x - \sin^3 x + 2\sin x \cos^2 x \\
&= 3\sin x \cos^2 x - \sin^3 x \\
&= \sin x(3\cos^2 x - \sin^2 x) \checkmark
\end{aligned}$$

8.4 Solving Trig

$\boxed{\text{trig fctn}}$ ~~as~~ a variable

Cosine & Sine → 1 angle for ±1

$1 \to \cos 0,\ \sin\frac{\pi}{2}$

$-1 \to \cos\pi,\ \sin\frac{3\pi}{2}$

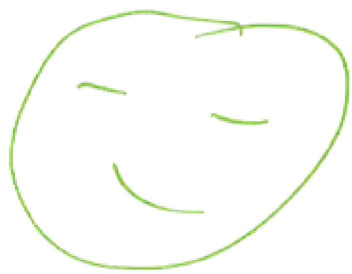
otherwise each value → 2 angles

$\frac{\pi}{n}$ Q I

$\frac{(n-1)\pi}{n}$ Q II

$\frac{(n+1)\pi}{n}$ Q III

$\frac{(2n-1)\pi}{n}$ Q IV



Ex $\sin\theta = \frac{1}{2}$ Q I, II

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \qquad [0, 2\pi)$$

$$\theta = \frac{\pi}{6} + 2n\pi, \quad \frac{5\pi}{6} + 2n\pi$$

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solve: $\sin x \tan x = \sin x \qquad [0, 2\pi)$

$$\sin x \tan x - \sin x = 0$$

$$\sin x (\tan x - 1) = 0$$

$$\sin x = 0 \qquad \tan x - 1 = 0 \leftarrow \text{I} \ \text{III}$$

$$\tan x = 1$$

$$x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$$

$$x = n\pi, \quad \frac{\pi}{4} + n\pi$$

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solve: $2\sin^2 t - \cos t - 1 = 0 \qquad [0, 2\pi]$

$$2(1 - \cos^2 t) - \cos t - 1 = 0$$

$$2 - 2\cos^2 t - \cos t - 1 = 0$$

$$-2\cos^2 t - \cos t + 1 = 0$$

$$\cos t = \frac{1 \pm \sqrt{1+8}}{2(-2)} = \frac{1 \pm 3}{-4}$$

$$\cos t = -1 \qquad \cos t = \frac{1}{2}$$

$$t = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$$

Solve $4\sin^2 x \tan x - \tan x = 0 \qquad [0, 2\pi]$

$\dots x \tan x - \tan x = 0 \quad [0, 2\pi)$

$$\tan x\,(4\sin^2 x - 1) = 0$$

$\tan x = 0$ $\quad \frac{\text{sine}}{\text{cosine}} \quad$ $4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$$x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

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solve $\csc^4 2u - 4 = 0$

$$(\csc^2 2u - 2)(\csc^2 2u + 2) = 0$$

$\csc^2 2u = 2$ $\qquad$ $\csc^2 2u = -2$ #

$\csc 2u = \pm\sqrt{2}$

$\frac{1}{\sin 2u} = \pm\sqrt{2}$

$\sin 2u = \pm\frac{1}{\sqrt{2}}$ $\qquad \frac{\pi}{4}$

$2u = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$u = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

$\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ ✓

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solve:

$$5\sin\theta\tan\theta - 10\tan\theta + 3\sin\theta - 6 = 0$$

$$5\tan\theta(\sin\theta - 2) + 3(\sin\theta - 2) = 0$$

$$(\sin\theta - 2)(5\tan\theta + 3) = 0$$

$\sin\theta = 2 > 1$ # $\tan\theta = -\frac{3}{5}$

$\theta' = \tan^{-1}\left(\frac{3}{5}\right)$

$(-) \; \text{II}, \text{IV}$ $\theta = \pi - \tan^{-1}\left(\frac{3}{5}\right)$

$\theta = 2\pi - \tan^{-1}\left(\frac{3}{5}\right)$

#10 $2\sin^2 x = 1 - \sin x$ $a - b + c = 0$, $x = -1, -\frac{c}{a}$

$2\sin^2 x + \sin x - 1 = 0$

$\sin x = -1$ $\sin x = \frac{1}{2}$

$$x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

$$ax^2 + bx + c = 0$$

$a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$

$a - b + c = 0 \quad x = -1, -c/a$

#11 $2\sin^3 x + \sin^2 x - 2\sin x - 1 = 0$

$\sin^2 x(2\sin x + 1) - (2\sin x + 1) = 0$

$(2\sin x + 1)(\sin^2 x - 1) = 0$

$\sin x = -\frac{1}{2}$ $\sin^2 x = 1$

$\sin x = \pm 1$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$$

24/ $\sin\theta - \cos\theta = 1$

$(\sin\theta)^2 = (1+\cos\theta)^2$ (Method/ check!!

$\frac{\sqrt{2}}{2}\sin\theta - \frac{\sqrt{2}}{2}\cos\theta = \frac{\sqrt{2}}{2}$

$\sin\frac{\pi}{4}\sin\theta - \cos\frac{\pi}{4}\cos\theta = \frac{\sqrt{2}}{2}$ $\Big\}$ $\cos\frac{\pi}{4}\sin\theta - \sin\theta$

$-(\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta) = \frac{\sqrt{2}}{2}$

$\cos(\theta + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$ $\Big\}$ $\cos(\frac{3\pi}{4})$, $\cos(\frac{5\pi}{4})$

$\theta + \frac{\pi}{4} = \frac{3\pi}{4}$

$\theta = \frac{3\pi}{4} - \frac{\pi}{4}$

$= \boxed{\frac{\pi}{2}}$

$\theta + \frac{\pi}{4} = \frac{5\pi}{4}$

$\theta = \frac{5\pi}{4} - \frac{\pi}{4}$

$= \boxed{\pi}$

- By grouping

- $\sin^2$, cosine^2 $\qquad \underbrace{\sin^2}_{1-\cos^2 x} + \cos$

$a\sin^2 x + b\sin x + c = 0$

$(\quad)(\quad) = 0$, $\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$[0, 2\pi)$

#27 $2\sin^2\theta - 2\sin\theta - 1 = 0$

$\sin\theta = \frac{2 \pm \sqrt{4+8}}{4}$ $\qquad 12 = 4\cdot 3$

$$= \frac{2 \pm 2\sqrt{3}}{4}$$

$$= \frac{1 \pm \sqrt{3}}{4}$$

$$\hat{\theta} = \sin^{-1} \frac{1 \pm \sqrt{3}}{4} \qquad \underline{4 \text{ Quad}}$$

$$\theta = \hat{\theta}$$

$$\theta = \pi - \hat{\theta}, \quad \pi + \hat{\theta}, \quad 2\pi - \hat{\theta}$$