

Solution ***Section 3.6 – Polar Coordinates***

Exercise

Convert to rectangular coordinates. $(4, 30^\circ)$

Solution

$$\begin{aligned}x &= r \cos \theta \\&= 4 \cos 30^\circ \\&= 4 \left(\frac{\sqrt{3}}{2} \right) \\&= \underline{2\sqrt{3}}\end{aligned}$$

$$\begin{aligned}y &= r \sin \theta \\&= 4 \sin 30^\circ \\&= 4 \left(\frac{1}{2} \right) \\&= \underline{2}\end{aligned}$$

\therefore The point $(2\sqrt{3}, 2)$ in rectangular coordinates is equivalent to $(4, 30^\circ)$ in polar coordinates.

Exercise

Convert to rectangular coordinates $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$.

Solution

$$\begin{aligned}x &= -\sqrt{2} \cos \frac{3\pi}{4} \\&= -\sqrt{2} \left(-\frac{1}{\sqrt{2}} \right) \\&= \underline{1}\end{aligned}$$

$$\begin{aligned}y &= -\sqrt{2} \sin \frac{3\pi}{4} \\&= -\sqrt{2} \left(\frac{1}{\sqrt{2}} \right) \\&= \underline{-1}\end{aligned}$$

\therefore The point $(1, -1)$ in rectangular coordinates is equivalent to $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$ in polar coordinates.

Exercise

Convert to rectangular coordinates $(3, 270^\circ)$.

Solution

$$x = 3 \cos 270^\circ$$

$$= 3(0)$$

$$= \underline{0}$$

$$y = 3 \sin 270^\circ$$

$$= 3(-1)$$

$$= \underline{-3}$$

\therefore The point $(3, 270^\circ)$ in polar coordinates is equivalent to $(0, -3)$ in rectangular coordinates.

Exercise

Convert to rectangular coordinates $(2, 60^\circ)$

Solution

$$x = 2 \cos 60^\circ$$

$$= 2\left(\frac{1}{2}\right)$$

$$= \underline{0}$$

$$y = 2 \sin 60^\circ$$

$$= 2 \frac{\sqrt{3}}{2}$$

$$= \underline{\sqrt{3}}$$

\therefore The point $(2, 60^\circ)$ in polar coordinates is equivalent to $(1, \sqrt{3})$ in rectangular coordinates.

Exercise

Convert to rectangular coordinates $(\sqrt{2}, -225^\circ)$

Solution

$$x = \sqrt{2} \cos(-225^\circ)$$

$$= \sqrt{2} \left(-\frac{1}{\sqrt{2}} \right)$$

$$= \underline{-1}$$

$$y = \sqrt{2} \sin(-225^\circ)$$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \right)$$

$$\underline{= 1}$$

∴ The point $(\sqrt{2}, -225^\circ)$ in polar coordinates is equivalent to $(-1, 1)$ in rectangular coordinates.

Exercise

Convert to rectangular coordinates $(4\sqrt{3}, -\frac{\pi}{6})$

Solution

$$x = 4\sqrt{3} \cos\left(-\frac{\pi}{6}\right)$$

$$= 4\sqrt{3} \left(\frac{\sqrt{3}}{2} \right)$$

$$\underline{= 6}$$

$$y = 4\sqrt{3} \sin\left(-\frac{\pi}{6}\right)$$

$$= 4\sqrt{3} \left(-\frac{1}{2} \right)$$

$$\underline{= -2\sqrt{3}}$$

∴ The point $(4\sqrt{3}, -\frac{\pi}{6})$ in polar coordinates is equivalent to $(6, -2\sqrt{3})$ in rectangular coordinates.

Exercise

Change the polar coordinates to rectangular coordinates $(-2, \frac{7\pi}{6})$

Solution

$$x = -2 \cos\left(\frac{7\pi}{6}\right)$$

$$= -2 \left(-\frac{\sqrt{3}}{2} \right)$$

$$\underline{= \sqrt{3}}$$

$$y = -2 \sin\left(\frac{7\pi}{6}\right)$$

$$= -2 \left(-\frac{1}{2} \right)$$

$$\underline{= 1}$$

∴ The point $(-2, \frac{7\pi}{6})$ in polar coordinates is equivalent to $(\sqrt{3}, 1)$ in rectangular coordinates.

Exercise

Change the polar coordinates to rectangular coordinates $\left(6, \arctan \frac{3}{4}\right)$

Solution

$$\arctan \frac{3}{4} = \beta \Rightarrow \tan \beta = \frac{3}{4}$$

$$x = 2 \cos \beta$$

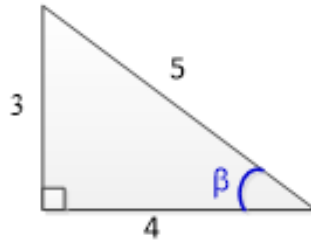
$$= 2 \left(\frac{4}{5} \right)$$

$$= \frac{8}{5}$$

$$y = 2 \sin \beta$$

$$= 2 \left(\frac{3}{5} \right)$$

$$= \frac{6}{5}$$



∴ The point $\left(6, \arctan \frac{3}{4}\right)$ in polar coordinates is equivalent to $\left(\frac{8}{5}, \frac{6}{5}\right)$ in rectangular coordinates.

Exercise

Change the polar coordinates to rectangular coordinates $\left(10, \arccos\left(-\frac{1}{3}\right)\right)$

Solution

$$\arccos\left(-\frac{1}{3}\right) = \alpha \Rightarrow \cos \alpha = -\frac{1}{3} \quad (QII)$$

$$x = 10 \cos \alpha$$

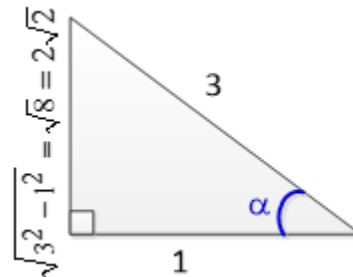
$$= 10 \left(-\frac{1}{3} \right)$$

$$= -\frac{10}{3}$$

$$y = 10 \sin \alpha$$

$$= 10 \left(\frac{2\sqrt{2}}{3} \right)$$

$$= \frac{20\sqrt{2}}{3}$$



∴ The point $\left(10, \arccos\left(-\frac{1}{3}\right)\right)$ in polar coordinates is equivalent to $\left(-\frac{10}{3}, \frac{20\sqrt{2}}{3}\right)$ in rectangular coordinates.

Exercise

Convert to polar coordinates $(3, 3)$.

Solution

$$\begin{aligned} r &= \sqrt{3^2 + 3^2} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{3}{3}\right) \\ &= \tan^{-1}(1) \\ &= 45^\circ \end{aligned}$$

\therefore The point $(3, 3)$ in rectangular coordinates is equivalent to $(3\sqrt{2}, 45^\circ)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-2, 0)$.

Solution

$$\begin{aligned} r &= \pm\sqrt{4+0} \\ &= \pm 2 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} \frac{0}{-2} \\ &= 0^\circ \end{aligned}$$

\therefore The point $(-2, 0)$ in rectangular coordinates is equivalent to $(-2, 0^\circ)$ $(2, 180^\circ)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-1, \sqrt{3})$.

Solution

$$\begin{aligned} r &= \pm\sqrt{1+3} \\ &= \pm 2 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) \\ &= 120^\circ \end{aligned}$$

\therefore The point $(-1, \sqrt{3})$ in rectangular coordinates is equivalent to $(2, 120^\circ)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-3, -3) \quad r \geq 0 \quad 0^\circ \leq \theta < 360^\circ$

Solution

$$r = \sqrt{(-3)^2 + (-3)^2}$$
$$= 3\sqrt{2}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{3}{3}\right)$$
$$= \tan^{-1}(1)$$
$$= 45^\circ$$

The angle is in quadrant III

$$\text{Therefore, } \theta = 180^\circ + 45^\circ$$
$$= 225^\circ$$

\therefore The point $(-3, -3)$ in rectangular coordinates is equivalent to $(3\sqrt{2}, 225^\circ)$ in polar coordinates.

Exercise

Convert to polar coordinates $(2, -2\sqrt{3}) \quad r \geq 0 \quad 0^\circ \leq \theta < 360^\circ$

Solution

$$r = \sqrt{2^2 + (-2\sqrt{3})^2}$$
$$= 4$$

$$\hat{\theta} = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)$$
$$= \tan^{-1}(\sqrt{3})$$
$$= 60^\circ$$

The angle is in quadrant IV

$$\text{Therefore, } \theta = 360^\circ - 60^\circ$$
$$= 300^\circ$$

\therefore The point $(2, -2\sqrt{3})$ in rectangular coordinates is equivalent to $(4, 300^\circ)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-2, 0) \quad r \geq 0 \quad 0 \leq \theta < 2\pi$

Solution

$$r = \sqrt{(-2)^2 + 0^2}$$
$$= 2$$

$$\hat{\theta} = \tan^{-1}\left(\frac{0}{-2}\right)$$
$$= 0$$
$$\theta = \pi$$

\therefore The point $(-2, 0)$ in rectangular coordinates is equivalent to $(2, \pi)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-1, -\sqrt{3}) \quad r \geq 0 \quad 0 \leq \theta < 2\pi$

Solution

$$r = \sqrt{(-1)^2 + (-\sqrt{3})^2}$$
$$= 2$$

$$\hat{\theta} = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right)$$
$$= \frac{\pi}{3}$$

The angle is in quadrant III

Therefore, $\theta = \pi + \frac{\pi}{3}$

$$= \frac{4\pi}{3}$$

\therefore The point $(-1, -\sqrt{3})$ in rectangular coordinates is equivalent to $(2, \frac{4\pi}{3})$ in polar coordinates.

Exercise

Change the rectangular coordinates to polar coordinates $(7, -7\sqrt{3}) \quad r > 0 \quad 0 \leq \theta < 2\pi$

Solution

$$r = \sqrt{(7)^2 + (-7\sqrt{3})^2}$$

$$= \sqrt{196}$$

$$= 14$$

$$\hat{\theta} = \tan^{-1}\left(\frac{7\sqrt{3}}{7}\right)$$

$$= \frac{\pi}{3}$$

The angle is in quadrant *IV*; therefore,

$$\theta = 2\pi - \frac{\pi}{3}$$

$$= \frac{5\pi}{3}$$

\therefore The point $(7, -7\sqrt{3})$ in rectangular coordinates is equivalent to $(14, \frac{5\pi}{3})$ in polar coordinates.

Exercise

Change the rectangular coordinates to polar coordinates $(-2\sqrt{2}, -2\sqrt{2})$ $r > 0$ $0 \leq \theta < 2\pi$

Solution

$$r = \sqrt{(-2\sqrt{2})^2 + (-2\sqrt{2})^2}$$

$$= 4$$

$$\hat{\theta} = \tan^{-1}\left(\frac{-2\sqrt{2}}{-2\sqrt{2}}\right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

The angle is in quadrant *III*; therefore,

$$\theta = \pi + \frac{\pi}{4}$$

$$= \frac{5\pi}{4}$$

\therefore The point $(-2\sqrt{2}, -2\sqrt{2})$ in rectangular coordinates is equivalent to $(4, \frac{5\pi}{4})$ in polar coordinates.

Exercise

The point $(0, -3)$ in rectangular coordinates is equivalent to $(3, 270^\circ)$ in polar coordinates.

Solution

$$r = \sqrt{0^2 + (-3)^2}$$
$$= 3$$

$$\hat{\theta} = \tan^{-1} \frac{0}{-3}$$
$$= 90^\circ$$

The polar point is $(3, 270^\circ)$

Exercise

The point $(1, -1)$ in rectangular coordinates is equivalent to $(\sqrt{2}, \frac{3\pi}{4})$ in polar coordinates.

Solution

$$r = \sqrt{(1)^2 + (-1)^2}$$
$$= \sqrt{2}$$

$$\hat{\theta} = \tan^{-1} \left(\frac{-1}{1} \right)$$
$$= \frac{3\pi}{4}$$

$$\theta \in QIV \rightarrow \theta = \frac{7\pi}{4}$$

$$\left(\sqrt{2}, \frac{7\pi}{4} \right) \Leftrightarrow \left(-\sqrt{2}, \frac{3\pi}{4} \right)$$

Exercise

A point lies at $(4, 4)$ on a rectangular coordinate system. Give its address in polar coordinates (r, θ)

Solution

$$r = \sqrt{4^2 + 4^2}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$

$$\theta = \tan^{-1} \left(\frac{4}{4} \right)$$
$$= \tan^{-1}(1)$$

$$= 45^\circ \mid$$

\therefore The point $(4, 4)$ in rectangular coordinates is equivalent to $(4\sqrt{2}, 45^\circ)$ in polar coordinates.

Exercise

Write the equation in rectangular coordinates $r^2 = 4$

Solution

$$r^2 = 4$$

$$\underline{x^2 + y^2 = 4 \mid}$$

Exercise

Write the equation in rectangular coordinates $r = 6 \cos \theta$

Solution

$$r = 6 \cos \theta$$

$$r = 6 \frac{x}{r}$$

$$r^2 = 6x$$

$$\underline{x^2 + y^2 = 6x \mid}$$

Exercise

Write the equation in rectangular coordinates $r^2 = 4 \cos 2\theta$

Solution

$$r^2 = 4 \cos 2\theta$$

$$= 4(\cos^2 \theta - \sin^2 \theta)$$

$$= 4\left(\frac{x^2}{r^2} - \frac{y^2}{r^2}\right)$$

$$= 4\left(\frac{x^2 - y^2}{r^2}\right)$$

$$r^4 = 4(x^2 - y^2)$$

$$\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$r^2 = x^2 + y^2$$

$$\underline{(x^2 + y^2)^2 = 4x^2 - 4y^2 \mid}$$

Exercise

Write the equation in rectangular coordinates $r(\cos \theta - \sin \theta) = 2$

Solution

$$r(\cos \theta - \sin \theta) = 2 \quad \cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$r\left(\frac{x}{r} - \frac{y}{r}\right) = 2$$

$$r\left(\frac{x-y}{r}\right) = 2$$

$$\underline{x - y = 2}$$

Exercise

Write the equation in rectangular coordinates $r^2 = 4 \sin 2\theta$

Solution

$$r^2 = 4 \sin 2\theta \quad \sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 4(2 \sin \theta \cos \theta) \quad \cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$= 8\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$$

$$= 8 \frac{xy}{r^2}$$

$$r^4 = 8xy \quad r^2 = x^2 + y^2$$

$$\underline{(x^2 + y^2)^2 = 8xy}$$

Exercise

Find an equation in x and y that has the same graph as polar equation. $r \sin \theta = -2$

Solution

$$r \sin \theta = -2 \quad y = r \sin \theta$$

$$\underline{y = -2}$$

Exercise

Find an equation in x and y that has the same graph as polar equation. $\theta = \frac{\pi}{4}$

Solution

$$\tan \theta = \tan \frac{\pi}{4}$$

$$\frac{y}{x} = 1$$

$$\underline{y = x}$$

Exercise

Find an equation in x and y that has the same graph as polar $r^2(4 \sin^2 \theta - 9 \cos^2 \theta) = 36$

Solution

$$r^2(4 \sin^2 \theta - 9 \cos^2 \theta) = 36$$

$$\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$r^2 \left(4 \frac{y^2}{r^2} - 9 \frac{x^2}{r^2} \right) = 36$$

$$r^2 \left(\frac{4y^2 - 9x^2}{r^2} \right) = 36$$

$$\underline{4y^2 - 9x^2 = 36}$$

Exercise

Find an equation in x and y that has the same graph as polar $r^2(\cos^2 \theta + 4 \sin^2 \theta) = 16$

Solution

$$r^2(\cos^2 \theta + 4 \sin^2 \theta) = 16$$

$$\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$r^2 \left(\frac{x^2}{r^2} + 4 \frac{y^2}{r^2} \right) = 16$$

$$r^2 \left(\frac{x^2 + 4y^2}{r^2} \right) = 16$$

$$\underline{x^2 + 4y^2 = 16}$$

Exercise

Find an equation in x and y that has the same graph as polar $r(\sin \theta - 2 \cos \theta) = 6$

Solution

$$r(\sin \theta - 2 \cos \theta) = 6$$

$$\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$r\left(\frac{y}{r} - 2\frac{x}{r}\right) = 6$$

$$r\left(\frac{y - 2x}{r}\right) = 6$$

$$\underline{y - 2x = 6}$$

Exercise

Find an equation in x and y that has the same graph as polar $r(\sin \theta + r \cos^2 \theta) = 1$

Solution

$$r(\sin \theta + r \cos^2 \theta) = 1$$

$$\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$r\left(\frac{y}{r} + r\frac{x^2}{r^2}\right) = 1$$

$$r\left(\frac{y}{r} + \frac{x^2}{r}\right) = 1$$

$$r\left(\frac{y + x^2}{r}\right) = 1$$

$$\underline{y + x^2 = 1}$$

Exercise

Find an equation in x and y that has the same graph as polar $r = 8 \sin \theta - 2 \cos \theta$

Solution

$$r = 8 \sin \theta - 2 \cos \theta$$

$$\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$r = 8\frac{y}{r} - 2\frac{x}{r}$$

$$r^2 = 8y - 2x$$

$$r^2 = x^2 + y^2$$

$$\underline{x^2 + y^2 = 8y - 2x}$$

Exercise

Find an equation in x and y that has the same graph as polar $r = \tan \theta$

Solution

$$r = \tan \theta$$

$$x^2 + y^2 = \frac{y^2}{x^2}$$

$$x^4 + x^2 y^2 = y^2$$

$$\sqrt{x^2 + y^2} = \frac{y}{x}$$

Exercise

Find a polar equation that has the same graph as the equation in x and y . $y^2 = 6x$

Solution

$$y^2 = 6x$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$(r \sin \theta)^2 = 6(r \cos \theta)$$

$$r^2 \sin^2 \theta = 6r \cos \theta$$

$$r = 6 \frac{\cos \theta}{\sin^2 \theta}$$

Exercise

Find a polar equation that has the same graph as the equation in x and y . $xy = 8$

Solution

$$xy = 8$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$(r \cos \theta)(r \sin \theta) = 8$$

$$r^2 = \frac{8}{\cos \theta \sin \theta}$$

Exercise

Find a polar equation that has the same graph as the equation in x and y . $(x+2)^2 + (y-3)^2 = 13$

Solution

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 13$$

$$x^2 + 4x + y^2 - 6y = 13 - 9 - 4$$

$$x^2 + 4x + y^2 - 6y = 0$$

$$x^2 + y^2 = 6y - 4x$$

$$r^2 = 6r \sin \theta - 4r \cos \theta$$

$$r^2 = r(6 \sin \theta - 4 \cos \theta)$$

$$x = r \cos \theta \quad y = r \sin \theta$$

Divide by r

$$\underline{r = 6 \sin \theta - 4 \cos \theta}$$

Exercise

Find a polar equation that has the same graph as the equation in x and y. $y^2 - x^2 = 4$

Solution

$$y^2 - x^2 = 4$$

$$r^2 \sin^2 \theta - r^2 \cos^2 \theta = 4$$

$$r^2 (\sin^2 \theta - \cos^2 \theta) = 4$$

$$r^2 (-\cos 2\theta) = 4$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\underline{r^2 = -\frac{4}{\cos 2\theta}}$$

Exercise

Write the equation in polar coordinates $x + y = 5$

Solution

$$x + y = 5$$

$$r \cos \theta + r \sin \theta = 5$$

$$r(\cos \theta + \sin \theta) = 5$$

$$\underline{r = \frac{5}{\cos \theta + \sin \theta}}$$

$$x = r \cos \theta \quad y = r \sin \theta$$

Exercise

Write the equation in polar coordinates $x^2 + y^2 = 9$

Solution

$$x^2 + y^2 = 9$$

$$r^2 = x^2 + y^2$$

$$\underline{r^2 = 9}$$

Exercise

Write the equation in polar coordinates $x^2 + y^2 = 4x$

Solution

$$x^2 + y^2 = 4x$$

$$r^2 = x^2 + y^2 \quad x = r \cos \theta$$

$$r^2 = 4r \cos \theta$$

$$\frac{r^2}{r} = \frac{4r \cos \theta}{r}$$

$$\underline{r = 4 \cos \theta} \quad |$$

Exercise

Write the equation in polar coordinates $y = -x$

Solution

$$y = -x$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$r \sin \theta = -r \cos \theta$$

$$\underline{\sin \theta = -\cos \theta} \quad |$$

Exercise

Write the equation in polar coordinates $x + y = 4$

Solution

$$x + y = 4$$

$$r \cos \theta + r \sin \theta = 4$$

$$x = r \cos \theta \quad y = r \sin \theta$$

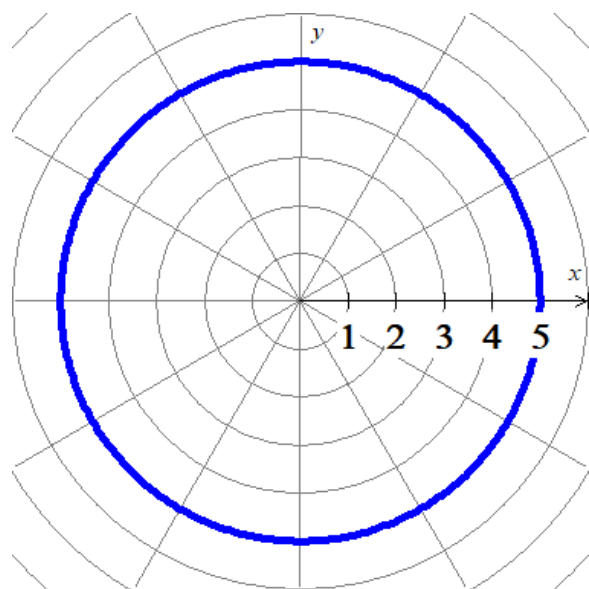
$$r(\cos \theta + \sin \theta) = 4$$

$$\underline{r = \frac{4}{\cos \theta + \sin \theta}} \quad |$$

Exercise

Sketch the graph of the polar equation $r = 5$

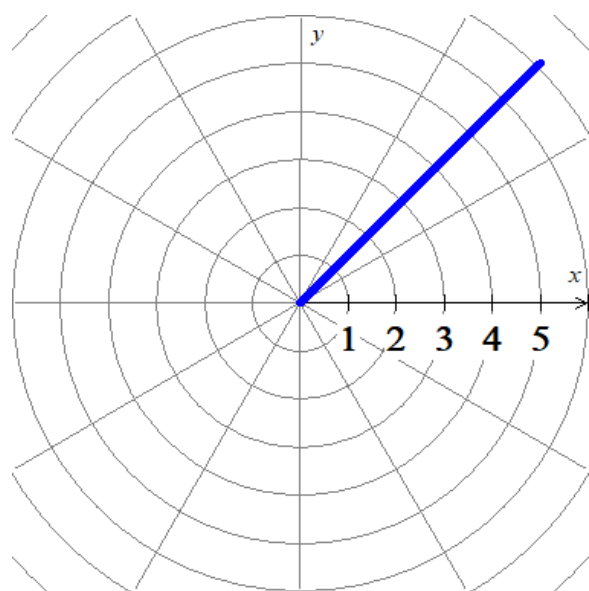
Solution



Exercise

Sketch the graph of the polar equation $\theta = \frac{\pi}{4}$

Solution

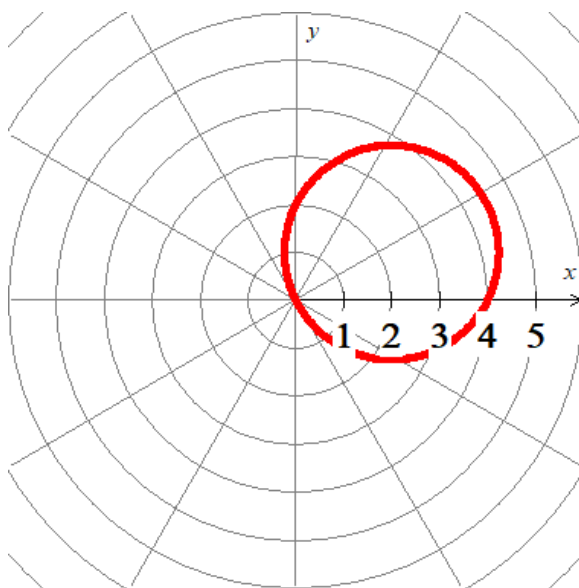


Exercise

Sketch graph $r = 4 \cos \theta + 2 \sin \theta$

Solution

θ	r
0	4
$\frac{\pi}{4}$	$3\sqrt{2}$
$\frac{\pi}{2}$	2
$\frac{3\pi}{4}$	$-\sqrt{2}$
π	-4
$\frac{3\pi}{2}$	-2

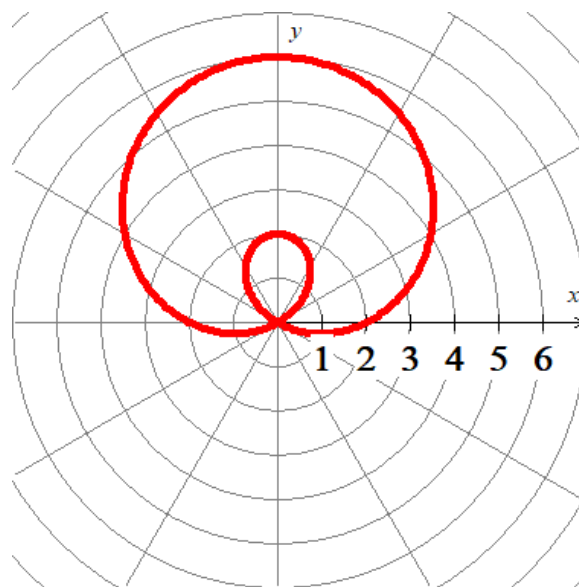


Exercise

Sketch the graph of the polar $r = 2 + 4 \sin \theta$

Solution

θ	r
0	2
$\frac{\pi}{6}$	4
$\frac{\pi}{4}$	$2 + 2\sqrt{2}$
$\frac{\pi}{2}$	6
$\frac{5\pi}{6}$	4
π	2
$\frac{7\pi}{6}$	0
$\frac{3\pi}{2}$	-2
$\frac{11\pi}{6}$	0

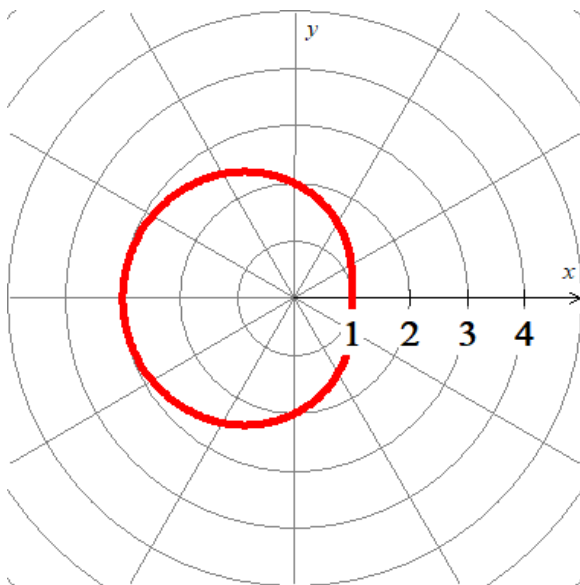


Exercise

Sketch the graph $r = 2 - \cos \theta$

Solution

θ	r
0	1
$\frac{\pi}{3}$	$\frac{3}{2}$
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	$\frac{5}{2}$
π	3
$\frac{4\pi}{3}$	$\frac{5}{2}$
$\frac{3\pi}{2}$	2
$\frac{5\pi}{3}$	$\frac{3}{2}$

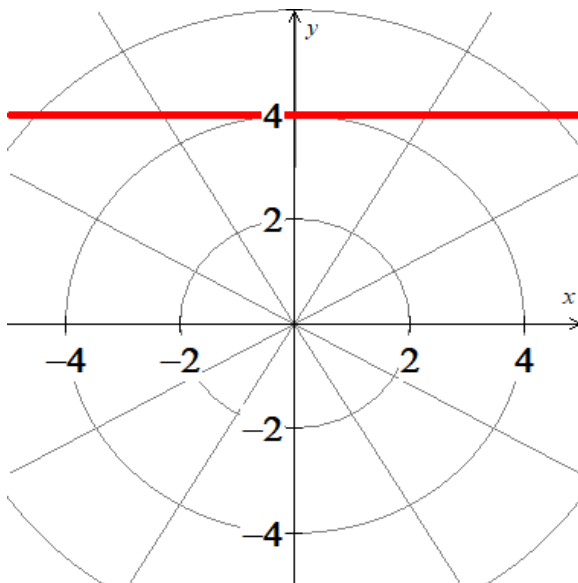


Exercise

Sketch the graph $r = 4 \csc \theta$

Solution

$$\begin{aligned} r &= 4 \csc \theta \\ &= \frac{4}{\sin \theta} \\ r \sin \theta &= \underline{4 = y} \end{aligned}$$



Exercise

Sketch the graph $r^2 = 4 \cos 2\theta$

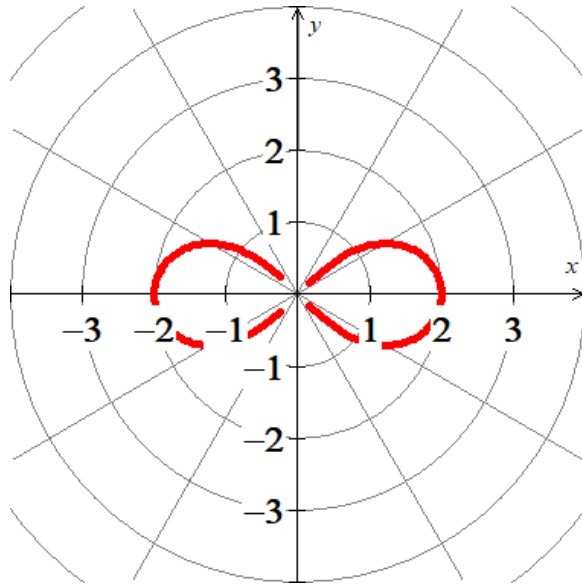
Solution

$$r^2 = 4 \cos 2\theta \geq 0$$

$$-\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$$

$$-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \quad \& \quad \frac{3\pi}{4} \leq \theta \leq \frac{5\pi}{4}$$

θ	r
0	2
$\frac{\pi}{6}$	$\sqrt{2}$
$\frac{\pi}{4}$	0
$\frac{3\pi}{4}$	0
π	2
$\frac{5\pi}{4}$	0
$\frac{7\pi}{4}$	0

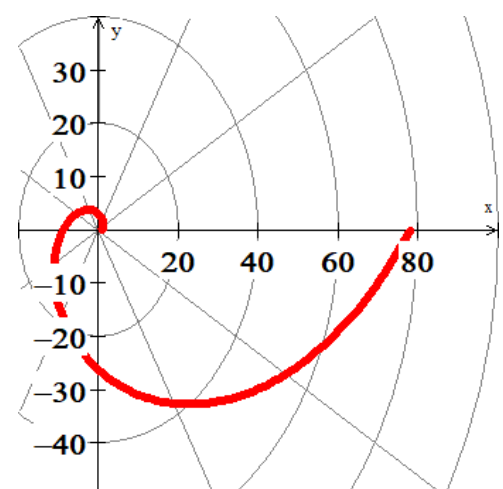
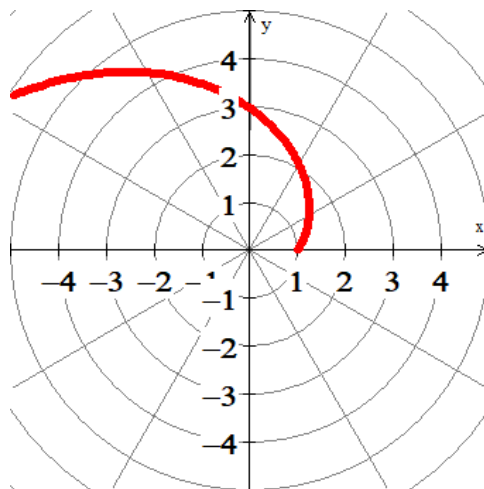


Exercise

Sketch the graph $r = 2^\theta \quad \theta \geq 0$

Solution

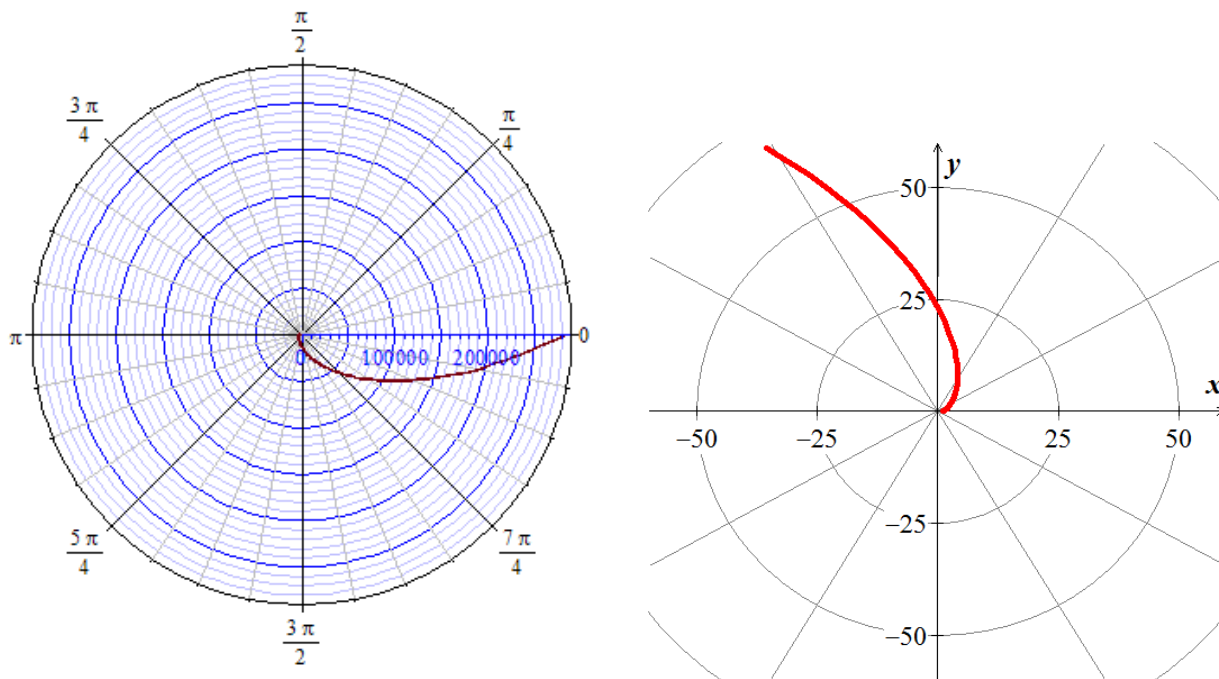
θ	r
0	1
$\frac{\pi}{2}$	$2^{\pi/2}$



Exercise

Sketch the graph of the polar equation $r = e^{2\theta}$ $\theta \geq 0$

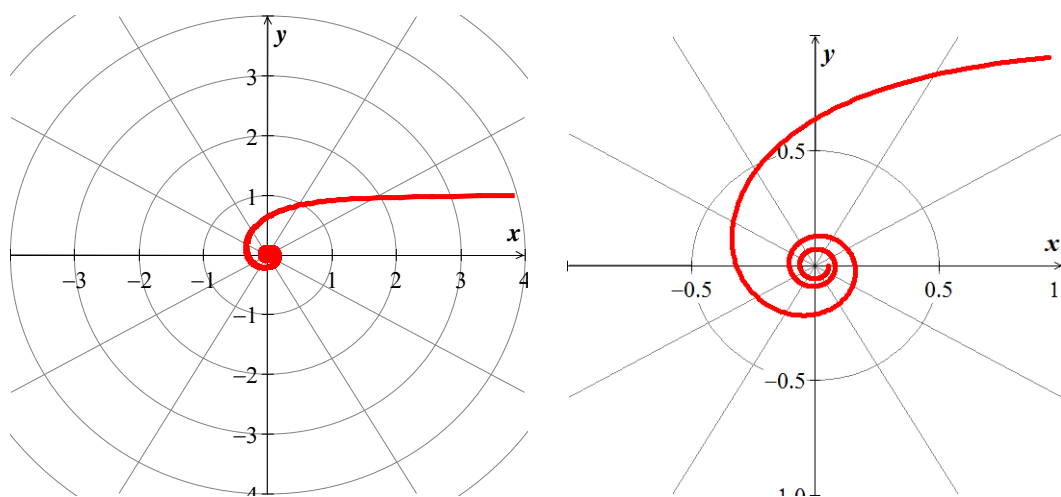
Solution



Exercise

Sketch the graph of the polar equation $r\theta = 1$ $\theta > 0$

Solution



Exercise

Sketch the graph of the polar equation $r = 2 + 2 \sec \theta$

Solution

