Section 1.2 – Solutions to Separable Equations

Separable Equation

Separable equation is an equation that can be written with its variables separated and then easily solved.

If f is independent of $y \Rightarrow f(x,y) = g(x) = \frac{dy}{dx}$

$$g(x)dx = dy$$

$$y = \int g(x)dx$$

Definition

A 1st order differential equation of the form $\frac{dy}{dx} = g(x)h(y)$ is said to be separable or to have separable variables.

$$\frac{dy}{h(y)} = g(x)dx$$

$$\frac{dy}{dx} = y^2 xe^{3x+4y} = \left(xe^{3x}\right)\left(y^2 e^{4y}\right)$$

$$\frac{dy}{dx} = y + \sin x \qquad \textit{not separable}$$

Example

At time t the sample contains $N(t)$ radioactive nuclei and is given by the differential equation:

$$N' = -\lambda N$$

This is called the ***exponential equation***.

$$N' = -\lambda N$$

$$\frac{dN}{dt} = -\lambda N \qquad \textit{Separable equation}$$

$$\frac{dN}{N} = -\lambda dt$$

$$\int \frac{dN}{N} = -\int \lambda dt$$

$$\ln|N| = -\lambda t + C$$

$$|N(t)| = e^{-\lambda t + C}$$

$$= e^{C}e^{-\lambda t}$$

$$N(t) = \begin{cases} e^{C}e^{-\lambda t} & if\ N > 0 \\ -e^{C}e^{-\lambda t} & if\ N < 0 \end{cases}$$

$$N(t) = Ae^{-\lambda t} \qquad A = \begin{cases} e^{C} & if\ N > 0 \\ -e^{C} & if\ N < 0 \end{cases}$$

Example

Solve the differential equation $y' = ty^2$

Solution

$$\frac{dy}{dt} = ty^2$$

$$\frac{dy}{y^2} = tdt$$

$$\int y^{-2}dy = \int tdt$$

$$-y^{-1} = \frac{1}{2}t^2 + C$$

$$-\frac{1}{y} = \frac{t^2+2C}{2}$$ *Cross multiplication*

$$-\frac{2}{t^2+2C} = y$$

$$y(t) = -\frac{2}{t^2+2C}$$

General Method

1. Separate the variables
2. Integrate both sides
3. Solve for the solution $y(t)$, if possible

Newton's Law of Cooling

Newton's Law of Cooling states that the rate of change of an object's temperature (***T***) is proportional to the difference between its temperature and the ambient temperature (***A***) (i.e. the temperature of its surroundings).

$$\frac{dT}{dt} = -k\left(T - A\right)$$

Example

A can of beer at 40° *F* is placed into a room when the temperature is 70° *F*. After 10 *minutes* the temperature of the beer is 50° *F*. What is the temperature of the beer as a function of time? What is the temperature of the beer 30 *minutes* after the beer was placed into the room?

Solution

By Newton's law of cooling: The rate of change of an object's temperature (***T***) is proportional to the difference between its temperature and the ambient temperature (***A***).

$$\frac{dT}{dt} = -k\left(T - A\right)$$

$$\frac{dT}{T - A} = -kdt$$

$$\int_{T_0}^{T} \frac{dT}{T - A} = -\int_0^t kdt$$

$$\ln\left|T - A\right| \Big|_{T_0}^{T} = -kt \Big|_0^t$$

$$\ln\left|T - A\right| - \ln\left|T_0 - A\right| = -kt$$

$$\ln\frac{\left|T - A\right|}{\left|T_0 - A\right|} = -kt \qquad \textit{Quotient Rule}$$

$$\frac{T - A}{T_0 - A} = e^{-kt} \qquad \Rightarrow \; T - A = \left(T_0 - A\right)e^{-kt}$$

Given: $T_0 = 40°F \quad \& \quad A = 70°F$

$$T(t) = 70 + \left(40 - 70\right)e^{-kt} = \boxed{70 - 30e^{-kt}}$$

$$T(t = 10) = 70 - 30e^{-10k} = 50$$

$$-30e^{-10k} = -20$$

$$e^{-10k} = \frac{2}{3}$$

$$-10k = \ln\frac{2}{3} \quad \rightarrow \; k = \frac{\ln\frac{2}{3}}{-10} \approx \boxed{0.0405}$$

$$T(t) = 70 - 30e^{-0.0405t}$$

$$T(t = 30) = 70 - 30e^{-0.0405(30)} \boxed{= 61.1 \;\; °F}$$

Losing a solution

When we use separate variables, the variable divisors could be zero at a point.

Example

Find a general solution to $\frac{dy}{dx} = y^2 - 4$

Solution

$$\frac{dy}{y^2 - 4} = dx$$

$$\left(\frac{1/4}{y-2} - \frac{1/4}{y+2}\right)dy = dx \qquad y = \pm 2 \text{ Critical points}$$

$$\frac{1}{4}\left(\int \frac{dy}{y-2} - \int \frac{dy}{y+2}\right) = \int dx$$

$$\frac{1}{4}\left[\ln|y-2| - \ln|y+2|\right] = x + c_1$$

$$\ln\left|\frac{y-2}{y+2}\right| = 4x + c_2$$

$$\left|\frac{y-2}{y+2}\right| = e^{4x+c_2}$$

$$\frac{y-2}{y+2} = \pm e^{c_2} e^{4x}$$

$$y - 2 = Ce^{4x}(y+2)$$

$$y - Ce^{4x}y = 2Ce^{4x} + 2$$

$$\left(1 - Ce^{4x}\right)y = 2\left(Ce^{4x} + 1\right)$$

$$\boxed{y = 2\frac{1+Ce^{4x}}{1-Ce^{4x}}}$$

If $y = -2 \Rightarrow -2 = 2\frac{1+Ce^{4x}}{1-Ce^{4x}}$

$$-1 = \frac{1+Ce^{4x}}{1-Ce^{4x}}$$

$$-1 + Ce^{4x} = 1 + Ce^{4x} \quad \Rightarrow \quad -1 = 1 \quad \textit{impossible}$$

If $y = 2 \quad \Rightarrow 2 = 2\frac{1+Ce^{4x}}{1-Ce^{4x}}$

$$1 - Ce^{4x} = 1 + Ce^{4x}$$

$$-Ce^{4x} = Ce^{4x} \quad \Rightarrow \quad -C = C$$

$$y = 2 \Rightarrow C = 0$$

Implicitly Defined Solutions

Example

Find the solutions of the equation $y' = \dfrac{e^x}{1+y}$, having initial conditions $y(0)=1$ and $y(0)=-4$

Solution

$$\frac{dy}{dx} = \frac{e^x}{1+y}$$

$$(1+y)\,dy = e^x dx$$

$$\int (1+y)\,dy = \int e^x dx$$

$$y + \tfrac{1}{2}y^2 = e^x + c$$

$$y^2 + 2y - 2\left(e^x + c\right) = 0$$

$$y(x) = \tfrac{1}{2}\left(-2 \pm \sqrt{4 + 8\left(e^x + c\right)}\right) \qquad \textit{Quadratic Formula}$$

$$= -1 \pm \sqrt{1 + 2\left(e^x + c\right)} \qquad \textit{Implicit}$$

$$y(0) = -1 + \sqrt{1 + 2\left(e^0 + c\right)} = 1$$

$$\sqrt{1 + 2(1+c)} = 2$$

$$1 + 2 + 2c = 4$$

$$2c = 1$$

$$\boxed{c = \frac{1}{2}}$$

$$y(0) = -1 - \sqrt{1 + 2\left(e^0 + c\right)} = -4$$

$$-\sqrt{1 + 2 + 2c} = -3$$

$$1 + 2 + 2c = 9$$

$$2c = 6$$

$$\boxed{c = 3}$$

$$\begin{cases} y(t) = -1 + \sqrt{2 + 2e^x} \\ y(t) = -1 - \sqrt{7 + 2e^x} \end{cases}$$

$\therefore y \ne -1$ *from* y', but it never it will be.

Explicit Solutions: $y = -1 + \sqrt{\ }$

Implicit solutions: $y^2 + by + c$

Example

Find the solutions to the differential equation $x' = \dfrac{2tx}{1+x}$, having $x(0) = 1,\ -2,\ 0$

Solution

$$\frac{dx}{dt} = \frac{2tx}{1+x}$$

$$\frac{1+x}{x}dx = 2tdt$$

$$\left(\frac{1}{x}+1\right)dx = 2tdt$$

$$\int\left(\frac{1}{x}+1\right)dx = \int 2tdt$$

$$\ln|x| + x = t^2 + c$$

For $x(0) = 1$

$$1 = 0^2 + c$$

$$c = 1$$

$$\ln|x| + x = t^2 + c \qquad x > 0$$

We can't solve for $x(t)$

$\Rightarrow$ This solution is defined as implicit.

For $x(0) = -2$

$$\ln|-2| + (-2) = 0^2 + c$$

$$c = -2 + \ln 2$$

$$\ln|x| + x = t^2 - 2 + \ln 2$$

Since the initial condition < 0, then:

$$x + \ln(-x) = t^2 - 2 + \ln 2$$

For $x(0) = 0$

$0 = 0^2 + c$ *True statement*

$y' = 0 \Rightarrow x(t) = 0$ is a solution

Exercises *Section* 1.2 – Solutions to Separable Equations

Find the general solution of the differential equation.

1. $y' = xy$
2. $xy' = 2y$
3. $y' = e^{x-y}$
4. $y' = \left(1+y^2\right)e^x$
5. $y' = xy + y$
6. $y' = ye^x - 2e^x + y - 2$
7. $y' = \dfrac{x}{y+2}$
8. $y' = \dfrac{xy}{x-1}$
9. $y' = \dfrac{y^2 + ty + t^2}{t^2}$
10. $\dfrac{dy}{dx} = \dfrac{4x - x^3}{4 + y^3}$
11. $y' = \dfrac{2xy + 2x}{x^2 - 1}$
12. $\dfrac{dy}{dx} = \sin 5x$
13. $\dfrac{dy}{dx} = (x+1)^2$
14. $dx + e^{3x}dy = 0$
15. $dy - (y-1)^2\, dx = 0$
16. $x\dfrac{dy}{dx} = 4y$
17. $\dfrac{dx}{dy} = y^2 - 1$
18. $\dfrac{dy}{dx} = e^{2y}$
19. $\dfrac{dy}{dx} + 2xy^2 = 0$
20. $\dfrac{dy}{dx} = e^{3x+2y}$
21. $e^x y\dfrac{dy}{dx} = e^{-y} + e^{-2x-y}$
22. $y\ln x\dfrac{dx}{dy} = \left(\dfrac{y+1}{x}\right)^2$
23. $\dfrac{dy}{dx} = \left(\dfrac{2y+3}{4x+5}\right)^2$
24. $\csc y dx + \sec^2 x dy = 0$
25. $\sin 3x dx + 2y\cos^3 3x dy = 0$
26. $\left(e^y + 1\right)^2 e^{-y}dx + \left(e^x + 1\right)^3 e^{-x}dy = 0$
27. $x\left(1+y^2\right)^{1/2} dx = y\left(1+x^2\right)^{1/2} dy$
28. $\dfrac{dy}{dx} = y\sin x$
29. $(1+x)\dfrac{dy}{dx} = 4y$
30. $2\sqrt{x}\dfrac{dy}{dx} = \sqrt{1-y^2}$
31. $\dfrac{dy}{dx} = 3\sqrt{xy}$
32. $\dfrac{dy}{dx} = (64xy)^{1/3}$
33. $\dfrac{dy}{dx} = 2x\sec y$
34. $\left(1-x^2\right)\dfrac{dy}{dx} = 2y$
35. $(1+x)^2\dfrac{dy}{dx} = (1+y)^2$
36. $\dfrac{dy}{dx} = xy^3$
37. $y\dfrac{dy}{dx} = x\left(y^2 + 1\right)$
38. $y^3\dfrac{dy}{dx} = \left(y^4 + 1\right)\cos x$
39. $\dfrac{dy}{dx} = \dfrac{1+\sqrt{x}}{1+\sqrt{y}}$
40. $\dfrac{dy}{dx} = \dfrac{(x-1)y^5}{x^2\left(2y^3 - y\right)}$
41. $\left(x^2 + 1\right)(\tan y)\,y' = x$
42. $x^2 y' = 1 - x^2 + y^2 - x^2y^2$

43. $xy' + 4y = 0$

44. $\left(x^2+1\right)y' + 2xy = 0$

45. $\dfrac{y'}{\left(x^2+1\right)y} = 3$

46. $y + e^x y' = 0$

47. $\dfrac{dx}{dt} = 3xt^2$

48. $x\dfrac{dy}{dx} = \dfrac{1}{y^3}$

49. $\dfrac{dy}{dx} = \dfrac{x}{y^2\sqrt{x+1}}$

50. $\dfrac{dx}{dt} - x^3 = x$

51. $\dfrac{dy}{dx} = \dfrac{x}{ye^{x+2y}}$

52. $\dfrac{dy}{dx} = \dfrac{\sec^2 y}{1+x^2}$

53. $x\dfrac{dv}{dx} = \dfrac{1-4v^2}{3v}$

54. $\dfrac{dy}{dx} = 3x^2\left(1+y^2\right)^{3/2}$

55. $\dfrac{1}{y}dy + ye^{\cos x}\sin x dx = 0$

56. $\left(x+xy^2\right)dx + e^{x^2}ydy = 0$

Find the exact solution of the initial value problem. Indicate the interval of existence.

57. $y' = \dfrac{y}{x}, \quad y(1) = -2$

58. $y' = -\dfrac{2t\left(1+y^2\right)}{y}, \quad y(0) = 1$

59. $y' = \dfrac{\sin x}{y}, \quad y\left(\frac{\pi}{2}\right) = 1$

60. $4tdy = \left(y^2 + ty^2\right)dt, \quad y(1) = 1$

61. $y' = \dfrac{1-2t}{y}, \quad y(1) = -2$

62. $y' = y^2 - 4, \quad y(0) = 0$

63. $\dfrac{dy}{dx} = \dfrac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1$

64. $y' = \dfrac{x}{1+2y}, \quad y(-1) = 0$

65. $\left(e^{2y} - y\right)\cos x\dfrac{dy}{dx} = e^y \sin 2x, \quad y(0) = 0$

66. $\dfrac{dy}{dx} = e^{-x^2}, \quad y(3) = 5$

67. $\dfrac{dy}{dx} + 2y = 1, \quad y(0) = \frac{5}{2}$

68. $\sqrt{1-y^2}dx - \sqrt{1-x^2}dy = 0, \quad y(0) = \frac{\sqrt{3}}{2}$

69. $\left(1+x^4\right)dy + x\left(1+4y^2\right)dx = 0, \quad y(1) = 0$

70. $\dfrac{1}{t^2}\dfrac{dy}{dt} = y, \quad y(0) = 1$

71. $\dfrac{dy}{dt} = -y^2e^{2t}; \quad y(0) = 1$

72. $\dfrac{dy}{dt} - (2t+1)y = 0; \quad y(0) = 2$

73. $\dfrac{dy}{dt} + 4ty^2 = 0; \quad y(0) = 1$

74. $\dfrac{dy}{dx} = ye^x; \quad y(0) = 2e$

75. $\dfrac{dy}{dx} = 3x^2\left(y^2+1\right); \quad y(0) = 1$

76. $2y\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x^2-16}}; \quad y(5) = 2$

77. $\dfrac{dy}{dx} = 4x^3y - y; \quad y(1) = -3$

78. $\dfrac{dy}{dx} + 1 = 2y; \quad y(1) = 1$

79. $(\tan x)\dfrac{dy}{dx} = y; \quad y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

80. $e^{-2t}\dfrac{dy}{dt} = \dfrac{1+e^{-2t}}{y}, \quad y(0) = 0$

81. $\dfrac{dy}{dt} = y\cos t + y, \quad y(0) = 2$

82. $\dfrac{dy}{dt} = \dfrac{t+2}{y}, \quad y(0) = 2$

83. $x\frac{dy}{dx} - y = 2x^2 y;\quad y(1) = 1$

84. $\frac{dy}{dx} = 2xy^2 + 3x^2y^2;\quad y(1) = -1$

85. $\frac{dy}{dx} = 6e^{2x-y};\quad y(0) = 0$

86. $2\sqrt{x}\frac{dy}{dx} = \cos^2 y;\quad y(4) = \frac{\pi}{4}$

87. $y' + 3y = 0;\quad y(0) = -3$

88. $2y' - y = 0;\quad y(-1) = 2$

89. $2xy - y' = 0;\quad y(1) = 3$

90. $y\frac{dy}{dx} - \sin x = 0;\quad y\left(\frac{\pi}{2}\right) = -2$

91. $\frac{dy}{dt} = \frac{1}{y^2};\quad y(1) = 2$

92. $y' + \frac{1}{y+1} = 0;\quad y(1) = 0$

93. $y' + e^y t = e^y \sin t;\quad y(0) = 0$

94. $y' - 2ty^2 = 0;\quad y(0) = -1$

95. $\frac{dy}{dx} = 1 + y^2;\quad y\left(\frac{\pi}{4}\right) = -1$

96. $\frac{dy}{dt} = t - ty^2;\quad y(0) = \frac{1}{2}$

97. $3y^2\frac{dy}{dt} + 2t = 1;\quad y(-1) = -1$

98. $e^x y' + (\cos y)^2 = 0;\quad y(0) = \frac{\pi}{4}$

99. $(2y - \sin y)y' + x = \sin x;\quad y(0) = 0$

100. $e^y y' + \frac{x}{y+1} = \frac{2}{y+1};\quad y(1) = 2$

101. $(\ln y)\,y' + x = 1;\quad y(3) = e$

102. $y' = x^3(1-y);\quad y(0) = 3$

103. $y' = \left(1 + y^2\right)\tan x;\quad y(0) = \sqrt{3}$

104. $\frac{1}{2}\frac{dy}{dx} = \sqrt{1+y}\ \cos x;\quad y(\pi) = 0$

105. $x^2\frac{dy}{dx} = \frac{4x^2 - x - 2}{(x+1)(y+1)};\quad y(1) = 1$

106. $\frac{1}{\theta}\frac{dy}{d\theta} = \frac{y\sin\theta}{y^2+1}\quad y(\pi) = 1$

107. $x^2dx + 2ydy = 0;\quad y(0) = 2$

108. $\frac{1}{t}\frac{dy}{dt} = 2\cos^2 y;\quad y(0) = \frac{\pi}{4}$

109. $\frac{dy}{dx} = 8x^3e^{-2y};\quad y(1) = 0$

110. $\frac{dy}{dx} = x^2(1+y);\quad y(0) = 3$

111. $\sqrt{y}dx + (1+x)dy = 0;\quad y(0) = 1$

112. $\frac{dy}{dx} = 6y^2x,\quad y(1) = \frac{1}{25}$

113. $\frac{dy}{dx} = \frac{3x^2 + 4x - 4}{2y - 4},\quad y(1) = 3$

114. $y' = e^{-y}(2x-4)\quad y(5) = 0$

115. $\frac{dr}{d\theta} = \frac{r^2}{\theta},\quad r(1) = 2$

116. $\frac{dy}{dt} = e^{y-t}\left(1+t^2\right)\sec y,\quad y(0) = 0$

117. A thermometer reading 100°*F* is placed in a medium having a constant temperature of 70°*F*. After 6 *min*, the thermometer reads 80°*F*. What is the reading after 20 *min*?

118. Blood plasma is stored at 40°*F*. Before the plasma can be used, it must be at 90°*F*. When the plasma is placed in an oven at 120°*F*, it takes 45 *min* for the plasma to warm to 90°*F*. How long will it take for the plasma to warm to 90°*F* if the oven temperature is set at:

a) 100°*F*.
b) 140°*F*.
c) 80°*F*.

119. A pot of boiling water at $100°C$ is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 *min*, the water temperature has decreased to $80°C$, and another 5 *min* later it has dropped to $65°C$. Assuming Newton's law for cooling, determine the (constant) temperature of the kitchen.

120. A murder victim is discovered at midnight and the temperature of the body is recorded at $31°C$. One hour later, the temperature of the body is $29°C$. Assume that the surrounding air temperature remains constant at $21°C$. Use Newton's law of cooling to calculate the victim's time of death. *Note*: The normal temperature of a living human being is approximately $37°C$.

121. Suppose a cold beer at $40°F$ is placed into a warn room at $70°F$. suppose 10 *minutes* later, the temperature of the beer is $48°F$. Use Newton's law of cooling to find the temperature 25 *minutes* after the beer was placed into the room.

122. A thermometer is removed from a room where the temperature is $70°\ F$ and is taken outside, where the air temperature is $10°\ F$. After one-half minute the thermometer reads $50°\ F$.

a) What is the reading of the thermometer at $t = 1\ min$?

b) How long will it take for the thermometer to reach $15°\ F$?

123. A thermometer is taken from an inside room to the outside, where the air temperature is $5°\ F$. After 1 *minute* the thermometer reads $55°\ F$, and after 5 *minutes* the thermometer reads $30°\ F$. What is the initial temperature of the inside room?

124. The temperature inside a house is $70°\ F$. A thermometer is taken outside after being inside the house for enough time for it to read $70°\ F$. The outside air temperature is $10°\ F$. After three *minutes* the thermometer reading is found to be $25°\ F$. Find the reading on the thermometer as a function of time.

125. A metal bar at a temperature of $100°\ F$ is placed in a room at a constant temperature of $0°\ F$. If after 20 minutes the temperature of the bar is $50°\ F$.

a) Find the time it will take the bar to reach a temperature of $25°\ F$

b) Find the temperature of the bar after 10 *minutes*.

126. A small metal bar, whose initial temperature was $20°\ C$, is dropped into a large container of boiling water.

a) How long will it take the bar to reach $90°\ C$ if it is known that its temperature increases $2°$ in 1 *second*?

b) How long will it take the bar to reach $98°\ C$

127. Two large containers ***A*** and ***B*** of the same size are filled with different fluids. The fluids in containers ***A*** and ***B*** are maintained at $0°\ C$ and $100°\ C$, respectively. A small metal bar, whose initial temperature is $100°\ C$, is lowered into container ***A***. After 1 *minute* the temperature of the bar is $90°\ C$. After 2 *minutes* the bar is removed and instantly transferred to the other container. After 1 *minute* in container ***B*** the temperature of the bar rises $10°\ C$. How long, measured from the start of the entire process, will it take the bar to reach $99.9°\ C$?

128. A thermometer reading $70°\ F$ is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer reads $110°\ F$ after $\frac{1}{2}$ *minute* and $145°\ F$ after 1 *minute*. How hot is the oven?

129. At $t = 0$ a sealed test tube containing a chemical is immersed in a liquid bath. The initial temperature of the chemical in the test tube is $80°\ F$. the liquid bath has a controlled temperature given by $T_m(t) = 100 - 40e^{-0.1t}$, $t \ge 0$, where t is measured in *minutes*.

a) Assume that $k = -0.1$, describe in words what you expect the temperature $T(t)$ of the chemical to be like in the short term. In the long term.

b) Solve the initial-value problem.

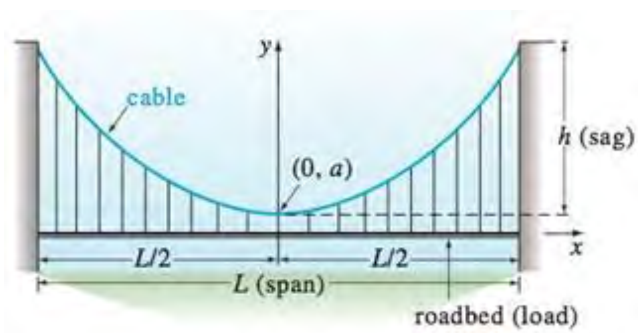
c) Graph $T(t)$.

130. The mathematical model for the shape of a flexible cable strung between two vertical supports is given by

$$\frac{dy}{dx} = \frac{W}{T_1}$$

Where W denotes the portion of the total vertical load between the points P_1 and P_2

The model is separable under the following conditions that describe a suspension bridge.



Let assume that the x-axis runs along the horizontal roadbed, and the y-axis passes through $(0,\ a)$, which is the lowest point on one cable over the span of the bridge, coinciding with the interval $\left[-\frac{L}{2},\ \frac{L}{2}\right]$.

In the case of a suspension bridge, the usual assumption is that the vertical load in the given equation is only a uniform roadbed, distributed along the horizontal axis. In other words, it is assumed that the weight of all cables is negligible in comparison to the weight of the roadbed and that the weight per unit length of the roadbed (lb/ft) is a constant ρ. Use this information to set up and solve an appropriate initial-value problem from which the shape (a curve with equation $y = \varphi(x)$) of each of the two cables in a suspension bridge is determined.

Express the solution of the IVP in terms of the sag h and span L.

131. The Brentano-Stevens Law in psychology models the way that a subject reacts to a stimulus. It states that if R represents the reaction to an amount S of stimulus, then the relative rates of increase are proportional:

$$\frac{1}{R}\frac{dR}{dt}=\frac{k}{S}\frac{dS}{dt}$$

Where k is a positive constant. Find R as a function of S.

132. Barbara weighs 60 *kg* and is on a diet of 1600 *calories* per day, of which 850 are used automatically by basal metabolism. She spends about 15 *cal/kg/day* times her weight doing exercises. If 1 *kg* of fat contains 10,000 *cal*. and we assume that the storage of calories in the form of fat is 100% efficient, formulate a differential equation and solve it to find her weight as a function of time. Does her weight ultimately approach an equilibrium weight?

133. When a chicken is removed from an oven, its temperature is measured at 300° *F*. Three minutes later its temperature is 200° *F*. How long will it take for the chicken to cool off to a room temperature of 70° *F*.

134. Suppose that a corpse was discovered in a motel room at midnight and its temperature was $80°\ F$. The temperature of the room is kept constant at $60°\ F$. Two hours later the temperature of the corpse dropped to $75°\ F$. Find the time of death.

135. Suppose that a corpse was discovered at 10 PM and its temperature was $85°\ F$. Two hours later, its temperature is $74°\ F$. If the ambient temperature is $68°\ F$. Estimate the time of death.