Definite Integral

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$

$$= F(b) - F(a)$$

1) Evalue $\int_a^b$ any $\mathbb{R}$.

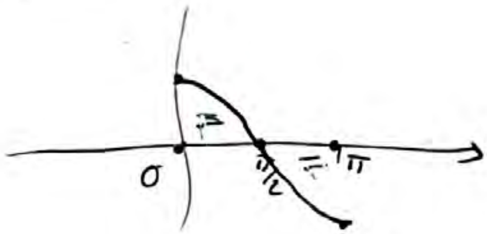
2) Area, Vol, L, S > 0 $\mathbb{R}^+$

Ex $\int_0^{\pi} \cos x\,dx = \sin x\Big|_0^{\pi}$

$$\boxed{\begin{aligned}&\sin x + C\Big|_0^{\pi}\\ &\sin\pi + \cancel{C} - \sin 0 - \cancel{C}\end{aligned}}$$

$= \sin\pi - \sin 0$

$= 0$



Ex $\int_{-\frac{\pi}{4}}^{0} \sec x\tan x\,dx = \sec x\Big|_{-\pi/4}^{0}$

$= \sec 0 - \sec(-\pi/4)$

$= 1 - \sqrt{2}$

$$\int_1^4 \left(\frac{3}{2}\sqrt{x} - \frac{4}{x^2}\right) dx = x^{3/2} + \frac{4}{x}\Big|_1^4$$

$$(2^2)^{3/2} \quad = 4^{3/2}$$

$$= 8 + 1 - (1 + 4)$$

$$= \boxed{4}$$

$$\cancel{8 + 1 - 1 - 4}$$

Ex $\quad v(t) = 160 - 32t \qquad v(t) = s'(t)$

$$s(t) = \int v(t)\,dt \qquad 0 \le t \le 8$$

$$s(t) = \int_0^8 (160 - 32t)\,dt$$

$$= 160t - 16t^2 \Big|_0^8$$

$$= 1280 - 1024$$

$$= \boxed{256}$$

$$\begin{array}{r} 64 \\ 16 \\ \hline 384 \\ 64 \end{array}$$

8

0

$$v(t) = 160 - 32t = 0 \Rightarrow \boxed{t = 5}$$

$$s(t) = \int_0^5 (160 - 32t)\,dt - \int_5^8 (160 - 32t)\,dt$$

$$= 160t - 16t^2 \Big|_0^5 - \left(160t - 16t^2 \Big|_5^8\right)$$

$$= 800 - 16(25) - \left(1280 - 1024 - (800 - 16(5^2))\right)$$

$$= 400 - 256 + 400$$

$$= \boxed{544}$$

Ex $f(x) = x^2 - 4$ $g(x) = 4 - x^2$

$[-2, 2]$

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{2} (x^2 - 4)\,dx$$

$$= \frac{1}{3}x^3 - 4x \Big|_{-2}^{2}$$

$$= \frac{8}{3} - 8 - \left(-\frac{8}{3} + 8\right)$$

$$= \frac{16}{3} - 16 \longrightarrow 16\left(\frac{1}{3} - 1\right)$$

$$= -\frac{32}{3}$$

$$\int_{-2}^{2} g(x)\,dx = \int_{-2}^{2} (4 - x^2)\,dx$$

$$= 4x - \frac{1}{3}x^3 \Big|_{-2}^{2}$$

$$= 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)$$

$$= 16 - \frac{16}{3}$$

$$= 16\left(1 - \frac{1}{3}\right) \rightarrow$$

$$= \frac{32}{3}$$

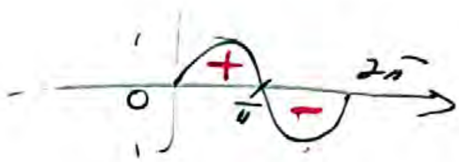
Area $f(x) = \left|-\frac{32}{3}\right| = \frac{32}{3}$

Ex $f(x) = \sin x$ $\quad x = 0$ & $x = 2\pi$

a) $$\int_0^{2\pi} \sin x \, dx = -\cos x \Big|_0^{2\pi}$$
$$= -(1 - 1)$$
$$= \underline{0}$$

b) area. $f(x) = \sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

$$\text{Area} = \int_0^{\pi} \sin x \, dx - \int_{\pi}^{2\pi} \sin x \, dx$$
$$= -\cos x \Big|_0^{\pi} + \cos x \Big|_{\pi}^{2\pi}$$
$$= -(-1-1) + 1 + 1$$
$$= \underline{4 \text{ unit}^2}$$



Ex $f(x) = x^3 - x^2 - 2x \qquad -1 \le x \le 2$

A?

$f(x) = x(x^2 - x - 2) = 0 \qquad x = 0, -1, 2$

$$\text{Area} = \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx - \int_{0}^{2} (x^3 - x^2 - 2x)\,dx$$

$$= \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 \Big|_{-1}^{0} - \left(\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\Big|_{0}^{2}\right.$$

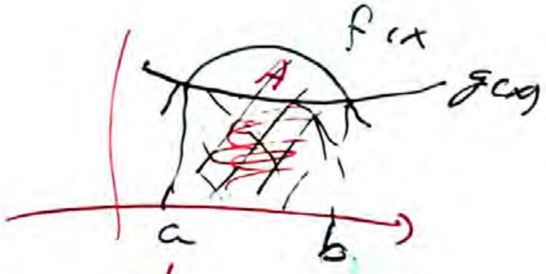
$$= -\left(\frac{1}{4} + \frac{1}{3} - 1\right) - \left(4 - \frac{8}{3} - 4\right)$$

$$= -\frac{7}{12} + 1 + \frac{8}{3}$$

$$= \frac{-7 + 12 + 32}{12}$$

$$= \frac{37}{12} \text{ unit}^2$$

Area Between curves.



$$A = \int_a^b (f(x) - g(x))\,dx$$

Ex A? $y = 2 - x^2$ + $y = -x$

Soln $y = 2 - x^2 = -x$

$x^2 - x - 2 = 0 \Rightarrow \boxed{x = -1, 2}$

$$\text{Area} = \int_{-1}^{2} (2 - x^2 + x)\,dx$$

$$= 2x - \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_{-1}^{2}$$

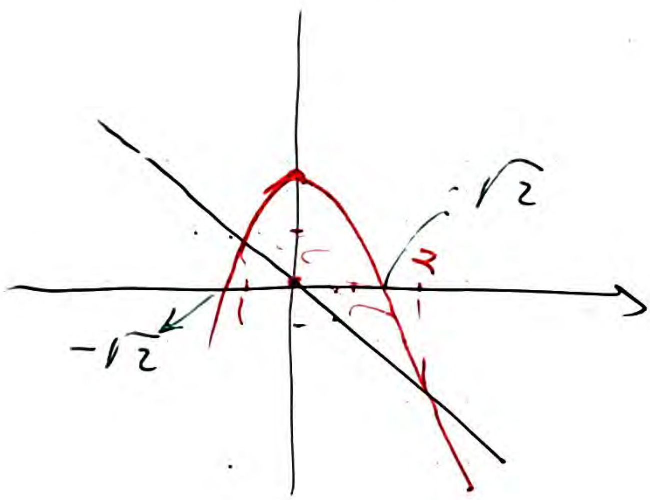
$$= 4 - \frac{8}{3} + 2 - \Big(-2 + \underbrace{\frac{1}{3} + \frac{1}{2}}_{5/6}\Big)$$

$$= \frac{10}{3} - \left(-\frac{7}{6}\right)$$

$$= \frac{10}{3} + \frac{7}{6}$$

$$= \frac{27}{6}$$

$$= \boxed{\frac{9}{2} \text{ unit}^2}$$



$y = 2 - x^2$

$-2x = 0$

Ex Area? QI $y=\sqrt{x}$, x-axis, $y=x-2$
(1) $y=0$ (2) (3)

$(1)\cap(2) \Rightarrow \boxed{(0,0)}$

$(1)\cap(3) \rightarrow \sqrt{x} = x-2$

$x = (x-2)^2$

$x = x^2-4x+4$ (QI)

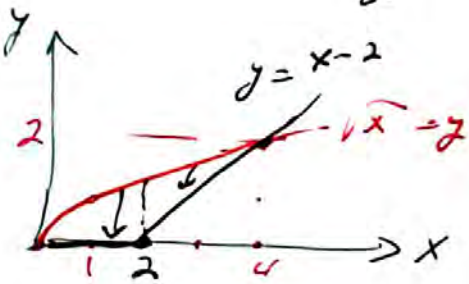
$x^2-5x+4 \Rightarrow x = \begin{cases} 1 \rightarrow y=-1 \ \# \\ 4 \Rightarrow y=2 \end{cases}$

$\boxed{(4,2)}$

$(2)\cap(3) \rightarrow x-2=0 \Rightarrow x=2 \Rightarrow (2,0)$

$x: 0 \rightarrow 2 \rightarrow 4$

$$\text{Area} = \int_0^2 x^{1/2}\,dx + \int_2^4 (x^{1/2}-x+2)\,dx$$

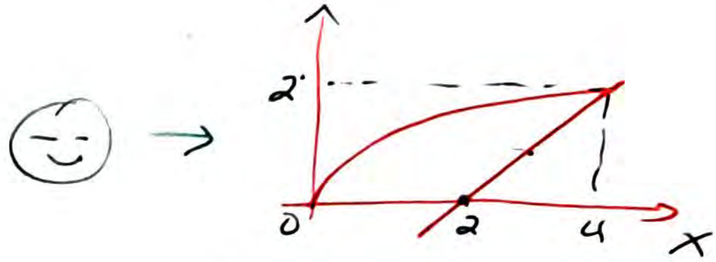


$$A = \frac{2}{3}x^{3/2}\Big|_0^2 + \left(\frac{2}{3}x^{3/2}-\frac{1}{2}x^2+2x\right)\Big|_2^4$$

$$= \frac{2}{3}2^{3/2} + \frac{16}{3} - 8 + 8 - \left(\frac{2}{3}2^{3/2} - 2 + 4\right)$$

$$= \frac{16}{3} - 2$$

$$= \frac{10}{3} \text{ unit}^2$$



$y = x - 2$

$x = y + 2$

$y = \sqrt{x}$

$x = y^2$

$$\text{Area} = \int_0^2 (y + 2 - y^2)\, dy$$

$$= \frac{1}{2} y^2 + 2y - \frac{1}{3} y^3 \Big|_0^2$$

$$= 2 + 4 - \frac{8}{3}$$

$$= \frac{10}{3} \text{ unit}^2$$

#1 $\int_0^3 (2x+1)\, dx = x^2 + x \Big|_0^3$

$= 9 + 3$

$= 12$

4^4

4^2

#3 $\int_0^4 \left(3x - \frac{1}{4}x^3\right) dx = \frac{3}{2}x^2 - \frac{1}{16}x^4 \Big|_0^4$

$= 24 - 16$

$= 8$

#21 $\int_1^7 \frac{dx}{x} = \ln x \Big|_1^7$ — $\ln|x|$

$= \ln 7 - \ln 1$

$= \ln 7$

$\ln 1 = 0$

$\ln e = 1$

#29 $\int_{-2}^{-1} \left(3e^{3x} + \frac{2}{x}\right) dx = e^{3x} + 2\ln|x| \Big|_{-2}^{-1}$

$= e^{-3} + 2\ln 1 - (e^{-6} + 2\ln 2)$

$= e^{-3} - e^{-6} + 2\ln 2$

$= \boxed{\frac{1}{e^3} - \frac{1}{e^6} + \ln 4}$

#31 A? $y = -x^2 - 2x \quad -3 \le x \le 2$

$y = -x(x-2) = 0 \Rightarrow \underline{x = 0, 2}$

$A = \int_{-3}^{0} (-x^2 - 2x)\,dx - \int_{0}^{2} (-x^2 - 2x)\,dx$

$= -\frac{1}{3}x^3 - x^2 \Big|_{-3}^{0} - \left(-\frac{1}{3}x^3 - x^2\right) \Big|_{0}^{2}$

$= -(9-9) - \left[-\frac{8}{3} - 4\right]$

$= \boxed{\frac{20}{3} \text{ unit}^2}$

#43 A? $f(x) = x^2 - 4x + 3 \quad 0 \le x \le 3$

$$x^2 - 4x + 3 = 0 \Rightarrow \boxed{x = 1, 3} \leftarrow$$

$$A = \int_0^1 (x^2 - 4x + 3)\,dx - \int_1^3 (x^2 - 4x + 3)\,dx$$

$$= \frac{1}{3}x^3 - 2x^2 + 3x \Big|_0^1 - \left(\frac{1}{3}x^3 - 2x^2 + 3x \Big|_1^3\right.$$

$$= \frac{1}{3} - 2 + 3 - \left[9 - 18 + 9 - \left(\frac{1}{3} - 2 + 3\right)\right.$$

$$= \frac{4}{3} + \frac{4}{3}$$

$$= \boxed{\frac{8}{3} \text{ unit}^2}$$

#48 $f(x) = 2x^2 + 4x + 2 \quad -1 \le x \le 1$

$$2x^2 + 4x + 2 = 0 \Rightarrow x = -1, -1$$

$$A = \int_{-1}^1 (2x^2 + 4x + 2)\,dx$$

$$= \frac{2}{3}x^3 + 2x^2 + 2x \Big|_{-1}^1$$

$$= \frac{2}{3} + 2 + 2 - \left(-\frac{2}{3} + 2 - 2\right.$$

$$= \frac{4}{3} + 4$$

$$= \boxed{\frac{16}{3} \text{ unit}^2}$$

function is odd or even

power odd

all power are even

$\mathbb{R} \rightarrow$ fctn

$f(x)$ even $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

odd $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$