

Review

Prove 5

$$\sec \theta \cot \theta - \sin \theta = \frac{\cos^2 \theta}{\sin \theta}$$

$$\begin{aligned}\sec \theta \cot \theta - \sin \theta &= \frac{1}{\cos \theta} \frac{\cos \theta}{\sin \theta} - \sin \theta \\&= \frac{1}{\sin \theta} - \sin \theta \\&= \frac{1 - \sin^2 \theta}{\sin \theta} \\&= \frac{\cos^2 \theta}{\sin \theta} \quad \checkmark\end{aligned}$$

$$\frac{1 + \cot^2 t}{1 + \cot t} = \csc^2 t - \cot t$$

$$\begin{aligned}\frac{1 + \cot^2 t}{1 + \cot t} &= \frac{(1 + \cot t)(1 - \cot t + \cot^2 t)}{1 + \cot t} \\&= 1 - \cot t + \cot^2 t \\&= 1 - \cot t + \csc^2 t - 1 \\&= \csc^2 t - \cot t \quad \checkmark\end{aligned}$$

$$\frac{\cos \alpha}{1 + \sin \alpha} = \sec \alpha - \tan \alpha$$

$$\begin{aligned} \frac{\cos \alpha}{1 + \sin \alpha} &= \frac{\cos \alpha}{1 + \sin \alpha} \cdot \frac{1 - \sin \alpha}{1 - \sin \alpha} \\ &= \frac{\cos \alpha (1 - \sin \alpha)}{1 - \sin^2 \alpha} \\ &= \frac{\cos \alpha (1 - \sin \alpha)}{\cos^2 \alpha} \\ &= \frac{1 - \sin \alpha}{\cos \alpha} \\ &= \frac{1}{\cos \alpha} - \frac{\sin \alpha}{\cos \alpha} \\ &= \sec \alpha - \tan \alpha \quad \checkmark \end{aligned}$$

$$\tan^2 x = \sec^2 x - \sin^2 x - \cos^2 x$$

$$\begin{aligned} \sec^2 x - \sin^2 x - \cos^2 x &= \sec^2 x - (\sin^2 x + \cos^2 x) \\ &= \sec^2 x - 1 \\ &= \tan^2 x \quad \checkmark \end{aligned}$$

$$\tan^2 x = \sec^2 x - 1$$

$$\begin{aligned} &= \sec^2 x - (\cos^2 x + \sin^2 x) \\ &= \sec^2 x - \cos^2 x - \sin^2 x \quad \checkmark \end{aligned}$$

$$10 \csc^2 x - 6 \cot^2 x = 4 \csc^2 x + 6$$

$$\begin{aligned} 10 \csc^2 x - 6 \cot^2 x &= 10 \csc^2 x - 6(\csc^2 x - 1) \\ &= 10 \csc^2 x - 6 \csc^2 x + 6 \\ &= 4 \csc^2 x + 6 \quad \checkmark \end{aligned}$$

$$\frac{\csc x - 1}{\csc x + 1} = \frac{\cot^2 x}{\csc^2 x + 2 \csc x + 1}$$

$$\begin{aligned} \frac{\csc x - 1}{\csc x + 1} &= \frac{\csc x - 1}{\csc x + 1} \cdot \frac{\csc x + 1}{\csc x + 1} \\ &= \frac{\csc^2 x - 1}{\csc^2 x + 2 \csc x + 1} \\ &= \frac{\cot^2 x}{\csc^2 x + 2 \csc x + 1} \quad \checkmark \end{aligned}$$

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 2 \sec x$$

$$\begin{aligned} \frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} &= \frac{\cos^2 x + (1 + \sin x)^2}{\cos x (1 + \sin x)} \\ &= \frac{\cos^2 x + 1 + 2 \sin x + \sin^2 x}{\cos x (1 + \sin x)} \\ &= \frac{2 + 2 \sin x}{\cos x (1 + \sin x)} \\ &= \frac{2(1 + \sin x)}{\cos x (1 + \sin x)} \\ &= 2 \sec x \quad \checkmark \end{aligned}$$

$$\frac{\cos(x+y)}{\cos(x-y)}$$

$$\frac{\cos(x+y)}{\cos(x-y)}$$

$$\frac{\cot y - \tan x}{\cot y + \tan x}$$

$$= \frac{\frac{\cos x \cos y}{\cos x \sin y} - \frac{\sin x \sin y}{\cos x \sin y}}{\frac{\cos x \cos y}{\cos x \sin y} + \frac{\sin x \sin y}{\cos x \sin y}}$$

$$= \frac{\cot y - \tan x}{\cot y + \tan x} \quad \checkmark$$

$$\frac{\sin(x-y)}{\sin x \sin y} = \cot y - \cot x$$

$$\frac{\sin(x-y)}{\sin x \sin y} =$$

$$\frac{\sin x \cos y - \cos x \sin y}{\sin x \sin y}$$

$$= \frac{\sin x \cos y}{\sin x \sin y} - \frac{\cos x \sin y}{\sin x \sin y}$$

$$= \cot y - \cot x \quad \checkmark$$

$$\csc(x-y) = \frac{\sin x \cos y + \cos x \sin y}{\sin^2 x - \sin^2 y}$$

$$\begin{aligned} \csc(x-y) &= \frac{1}{\sin(x-y)} \cdot \frac{\sin(x+y)}{\sin(x+y)} \\ &= \frac{\sin(x+y)}{(\sin x \cos y - \cos x \sin y)(\sin x \cos y + \cos x \sin y)} \\ &= \frac{\sin(x+y)}{\sin^2 x \cos^2 y - \cos^2 x \sin^2 y} \\ &= \frac{\sin(x+y)}{\sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y} \\ &= \frac{\sin(x+y)}{\sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y} \\ &= \frac{\sin x \cos y + \cos x \sin y}{\sin^2 x - \sin^2 y} \quad \checkmark \end{aligned}$$

$$\cos 3x = \cos^3 x - \underline{3\cos x \sin^2 x}$$

$$\cos 3x = \cos (2x + x)$$

$$= \cos 2x \cos x - \sin 2x \sin x$$

$$= (\cos^2 x - \sin^2 x) \cos x - 2\sin^2 x \cos x$$

$$= \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x$$

$$= \cos^3 x - 3\cos x \sin^2 x \checkmark$$

$$\sin 4x = (4 \sin x \cos x) (2\cos^2 x - 1)$$

$$\sin 4x = \sin 2(2x)$$

$$= 2 \sin 2x \cos 2x$$

$$= 4 \sin x \cos x (2\cos^2 x - 1) \checkmark$$

$$\cos 4x = \cos^4 x - 6 \sin^2 x \cos^2 x + \sin^4 x$$

$$\begin{aligned}\cos 4x &= \cos 2(2x) \\&= \cos^2 2x - \sin^2 2x \\&= (\cos^2 x - \sin^2 x)^2 - (2 \sin x \cos x)^2 \\&= \cos^4 x - 2 \cos^2 x \sin^2 x + \sin^4 x \\&\quad - 4 \sin^2 x \cos^2 x \\&= \cos^4 x - 6 \cos^2 x \sin^2 x + \sin^4 x \checkmark\end{aligned}$$

$$\frac{\cos 2x}{\sin^2 x} = 2 \cot^2 x - \csc^2 x$$

$$\frac{\cos 2x}{\sin^2 x} = \frac{2 \cos^2 x - 1}{\sin^2 x}$$

$$= 2 \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x}$$

$$= 2 \cot^2 x - \csc^2 x \checkmark$$

$$\sin^2 \frac{x}{2} \cos^2 \frac{x}{2} = \frac{\sin^2 x}{4}$$

$\sin 2x$

$$\begin{aligned} \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} &= \left(\sin \frac{x}{2} \cos \frac{x}{2} \right)^2 \\ &= \left(\frac{1}{2} \sin x \right)^2 \\ &= \frac{1}{4} \sin^2 x \quad \checkmark \end{aligned}$$

$$2 \sin^2 \frac{x}{2} = \frac{\sin^2 x}{1 + \cos x}$$

$$\sin^2 \frac{x}{2} = \frac{1}{2} (1 - \cos x)$$

$$\begin{aligned} 2 \sin^2 \frac{x}{2} &= (1 - \cos x) \frac{1 + \cos x}{1 + \cos x} \\ &= \frac{1 - \cos^2 x}{1 + \cos x} \\ &= \frac{\sin^2 x}{1 + \cos x} \quad \checkmark \end{aligned}$$

$$\tan^2 \frac{x}{2} = \frac{\sec x + \cos x - 2}{\sec x - \cos x}$$

$$\tan^2 \frac{x}{2} = \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}}$$

$$\frac{1}{\cos x} - \cos x$$

$$\begin{aligned} &= \frac{\frac{1}{2} (1 - \cos x)}{\frac{1}{2} (1 + \cos x)} \\ &= \frac{1 - \cos x}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x} \\ &= \frac{1 - 2 \cos x + \cos^2 x}{1 - \cos^2 x} \end{aligned}$$

$$\begin{aligned}
 &= \frac{\frac{1}{\cos x} - \frac{2\cos x}{\cos x} + \frac{\cos^2 x}{\cos x}}{\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}} \\
 &= \frac{\sec x - 2 + \cos x}{\sec x - \cos x} \quad \checkmark
 \end{aligned}$$

$$\frac{1}{4} \sin 4t = \cos^3 t \sin t - \sin^3 t \cos t$$

$$\begin{aligned}
 \frac{1}{4} \sin 4t &= \frac{1}{4} \sin 2(2t) \\
 &= \frac{1}{4} 2 \sin 2t \cos 2t \\
 &= \frac{1}{2} (2 \sin t \cos t) (\cos^2 t - \sin^2 t) \\
 &= \cos^3 t \sin t - \sin^3 t \cos t \quad \checkmark
 \end{aligned}$$

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$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\begin{aligned}\cos 2A &= 2\cos^2 A - 1 \\ &= \cos^2 A - \sin^2 A \\ &= 1 - 2\sin^2 A\end{aligned}$$