2.4 Functions

Calculus is an area of mathematics in which you can study functions of one or more real variables in a variety of ways. The topics below will help you to enter functions into your calculator and to analyze their values and graphs. First, make sure that your calculator is set to Function Mode, that is `Func` should be highlighted in the mode settings screen. (See Section 2.2 of this manual.)

2.4.1 Entering Functions

The TI-83+/84+ allows you to store ten functions in its memory. To store a function press the `Y=` key to access the `Y= Editor`. You can use the arrow keys to scroll up or down to select a function or to scroll left and right if you are editing a function. Use the `CLEAR` key to erase an entire line. In function mode, the `X,T,θ,n` key produces `X`, which is used as the independent variable; the sequence `VARS` `Y-VARS Function Y1` copies `Y1` onto the screen. You cannot use the alpha keys to recall the stored function `Y1`.

Figure 9 shows how to enter the functions $y_1 = 5x - 2$ and $y_2 = x^2 - y_1(x) = x^2 - 5x + 2$.

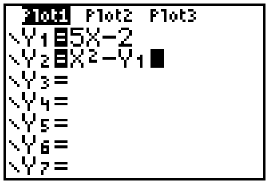

Figure 9: The `Y= Editor`

When you enter the first character of the function the '=' sign is highlighted indicating that the function is selected and that its graph will be shown in the graph window. If you wish to deselect the function, position the cursor over the '=' sign and press `ENTER`. One nice thing about the TI-83+/84+ is that you can use numbers, variables, matrices, lists, and other functions to define new functions, these features can be particularly useful when studying calculus.

2.4.2 Graph Style

Functions can be graphed in different styles. Two such styles and the necessary keystrokes to display them are described in this section. For additional information see the guidebook that came with your calculator.

The standard style for drawing graphs is called `Connected`. This is the default style setting. (You can change this default for all stored functions in the `MODE` menu.) In the `Connected` setting the calculator plots certain points of the graph and then joins them with tiny line segments, creating a continuous-looking graph. In `Dot` style, the calculator simply plots certain points on the graph of the function. In this setting, points are not joined together by line segments. To change the style of a specific graph you must be in the `Y= Editor`. Use the arrow keys to place the cursor in the extreme left position. Press the $\boxed{\texttt{ENTER}}$ key to toggle between different styles. A diagonal segment with three dots is the Dot style. Move the cursor away from the style marker and the new style will be selected. Figure 10 shows the function y_1 entered in `Dot` mode, while y_2 is entered in `Connected` mode.

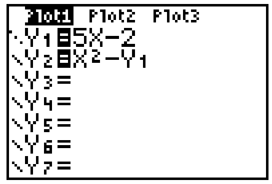

Figure 10: The `Dot` style selected

2.4.3 Viewing Window

The viewing window of your calculator only represents a portion of the Cartesian plane. The standard viewing window is within the bounds $-10 \leq x \leq 10$, and $-10 \leq y \leq 10$. In many cases you will need to draw graphs of functions that are outside this range, but this is not a problem if you are using a TI-83+/84+, since you can set the viewing window as needed. Press the $\boxed{\texttt{WINDOW}}$ key to access the viewing window feature (Figure 11).

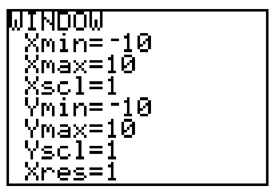

Figure 11: `WINDOW`

The values of `Xmin`, `Xmax`, `Ymin`, and `Ymax` determine the portion of the Cartesian plane that will be shown. You must enter values that satisfy `Xmin<Xmax`, and `Ymin<Ymax`. The numbers `Xscl` and `Yscl` determine the distance between tickmarks. Setting these numbers equal to ten will result in a tickmark at every ten units; setting these numbers equal to zero will result in no tickmarks. The number `Xres` sets pixel resolution, for our purposes we want `Xres=1`.

2.4.4 Graphing a Function

Press the $\boxed{\texttt{GRAPH}}$ key to display the graphs of the functions that you have selected. Your calculator allows you to analyze graphs in a variety of ways. The remainder of the section contains descriptions of several of the features connected to functions and their graphs. See Section 2.5 for topics that require knowledge of calculus.

2.4.5 `ZOOM`

The $\boxed{\texttt{ZOOM}}$ key allows you to change the viewing window in ten specific ways. Select the first item by highlighting `Zbox`. After the graph is drawn use the arrow keys to move the cursor to a position that you want to become one corner of the viewing window. Press $\boxed{\texttt{ENTER}}$, and move the cursor to the opposite corner of the window. Press $\boxed{\texttt{ENTER}}$, and the graph will be redrawn within the boundaries of the window you selected. The `Zoom In` and `Zoom Out` features allow you to look at a graph closer or further away, respectively. To select one of these items, highlight `Zoom In` or `Zoom Out` and press $\boxed{\text{ENTER}}$. A cursor will appear in the graph, which will determine the center of the new viewing window. Move the cursor to the desired center and press $\boxed{\text{ENTER}}$. The graph will be redrawn. The viewing window `Xmin=-10, Xmax=10, Xscl=1, Ymin=-10, Ymax=10, Yscl=1` is the default set at the factory. You can restore this window by selecting `ZStandard`. A square viewing area is sometimes necessary: `ZSquare` sets the dimensions of the viewing window so that a circle will look like a circle, not like an ellipse. When plotting statistical data points `ZoomStat` sets the viewing window so all data points are visible in the window. `ZoomFit` resizes the window, changing only the `Y` values in such a way that the graph is displayed within the prespecified values of `X`. The other items in the `Zoom` menu are discussed in the guidebook that came with your calculator.

2.4.6 `TRACE`

The $\boxed{\text{TRACE}}$ key allows you to move the cursor along the graph of a function as the calculator displays the values of the coordinates of the points on the graph. Press the $\boxed{\text{TRACE}}$ key, and you will see your graph displayed and the trace cursor will appear on the graph. Use the left and right arrow keys to move the cursor along the graph. You can also move the cursor to a specific point by entering the x-value of the point and pressing the $\boxed{\text{ENTER}}$ key. If the values of x and y are within the viewing window, the cursor will immediately move to the point on the graph that has the given x-coordinate and the calculator will display both coordinates. Figure 12 shows the cursor on the graph of the function $y_2 = x^2 - y_1(x) = x^2 - 5x + 2$ and the coordinates of the point where the cursor is positioned. Use the up and down arrows to move from function to function.

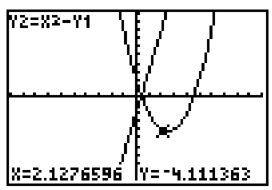

Figure 12: `TRACE`

2.4.7 `TABLE`

If you have entered a function into `Y1` (or any other dependent variable), the table feature will allow you to compute values for this function for many values of the independent variable. First, press $\boxed{\text{2nd}}$ `[TBLSET]` to set the starting value of `X`, `TblStart`, and the increment of `X`, `ΔTbl`. As in Figure 13, set `TblStart=-1` and `Tbl=.5`. Set both `Indpnt` and `Depend` to `AUTO`, press $\boxed{\text{ENTER}}$ to save the values. Press $\boxed{\text{2nd}}$ `[TABLE]` to view a table in which the values for `Y1` are computed automatically. Figure 14 displays a table of values for the function $y_1 = 5x - 2$. You can scroll through the table of values using the up and down arrow keys. When setting the options for the table, you can also set `Indpnt` to `ASK` and `Depend` to `AUTO`. Press $\boxed{\text{ENTER}}$ to save these options, then press $\boxed{\text{2nd}}$ `[TABLE]`. Enter a value for `X`, press $\boxed{\text{ENTER}}$ and the corresponding value for `Y1` will be computed. For more information on tables, see the guidebook that came with your calculator.

2.4.8 Finding Zeros of Functions

This section contains methods for finding zeros of functions, that is, points where the graph of the function crosses the x-axis. Your calculator has built-in algorithms that make use of graphs and

Figure 13: `TBLSET`

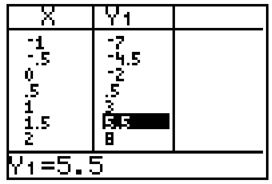

Figure 14: `TABLE`

tables for finding zeros of functions. The values obtained with these methods may be very rough approximations, depending on your calculator. (See Section 2.3 for other methods of finding zeros of functions.)

Trace. Enter and graph the function $y_1 = x^3 + 2.55x^2 - 2.655x - 5.13$ in the viewing window `Xmin=-3, Xmax=3, Xscl=1, Ymin=-4, Ymax=2, Yscl=1`. Press the `TRACE` key and use the arrow keys to move the cursor to the point where the graph meets the x-axis. Once you establish an x-value that gives you a y-value close to zero, you can experiment with the graph and zoom in to reach other x-values that may give a y-value closer to zero (Figure 15). In many cases, you may not be able to arrive at an x-value that lies exactly on the x-axis.

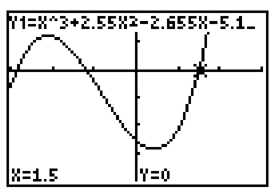

Figure 15: Finding zeros of a function with `TRACE`

Table. Enter the function $y_1 = x^3 + 2.55x^2 - 2.655x - 5.13$ and construct a table of values for the function (see Section 2.4.7). You might want to take a peek at the graph to see if there is a zero between 1 and 2. If this is the case, it's a good idea to set `TblStart=1` and Δ`Tbl=0.1`, and both `Indpnt` and `Depend` to `AUTO`. Scroll through the values in the table to find values of the dependent variable close to zero. Once you establish an x-value that gives you a y-value close to zero, you can experiment with other values of `TblStart` and Δ`Tbl` to see if you can achieve a y-value of closer to zero (Figure 16). In many cases, you may not be able to arrive at an x-value that yields a y-value of exactly zero.

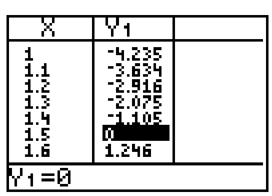

Figure 16: Finding zeros of a function using `TABLE`

Zero. Enter the function $y_1 = x^3 + 2.55x^2 - 2.655x - 5.13$ and graph it using the viewing window

Xmin=-3, Xmax=3, Xscl=1, Ymin=-8, Ymax=4, Yscl=2. Press 2nd [CALC] to access the CALCULATE menu, select zero by pressing 2. Use the arrow keys to move the cursor to select the left bound, the right bound, and a guess, as prompted by the calculator. Press ENTER to save each of your selections. The cursor will move to the zero of the function, and the calculator will display the values of x and y at that point (Figures 17–19).

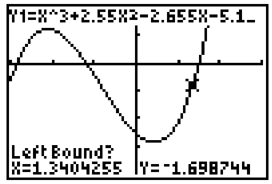

Figure 17: Left bound

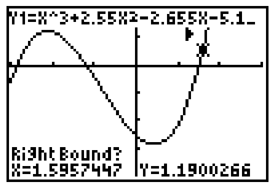

Figure 18: Right bound

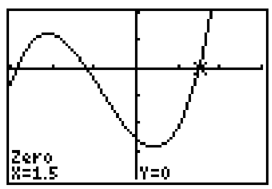

Figure 19: The zero

Intersection. Suppose you want to solve $e^{3x} - 5x - 7 = 0$ for x. This problem is equivalent to finding the x-value of the point where the graphs of $y_1 = e^{3x}$ and $y_2 = 5x + 7$ meet. Enter both functions into memory and graph them. Use the viewing window Xmin= -5, Xmax=5, Xscl=1, Ymin=-3, Ymax=15, Yscl=1. Press 2nd [CALC] to access the [CALCULATE] menu, select intersect by pressing 5. Use the arrow keys to move the cursor to select the first curve and the second curve, as prompted by the calculator. Press ENTER to save (Figures 20–21). Optionally you can enter a guess for the x-coordinate. The cursor will move to the point of intersection of the curves, and the calculator will display the values of x and y at that point (Figure 22).

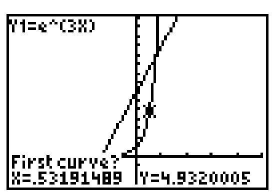

Figure 20: First curve

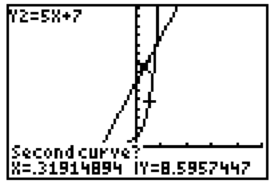

Figure 21: Second curve

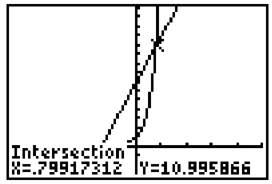

Figure 22: The intersection

2.4.9 Composition of Functions

Functions defined in the TI-83+/84+ can be combined to form new functions, one such combination is the composition of two functions. Enter the functions $y_1 = 1 - x$ and $y_2 = e^x$ into your calculator by pressing the `Y=` key to access the `Y= Editor`. Both functions have domain equal to the set of real numbers, therefore the compositions $y_1(y_2(x))$, and $y_2(y_1(x))$ can both be formed without restrictions.

Enter $\mathtt{Y}_3 = \mathtt{Y}_1(\mathtt{Y}_2(\mathtt{x}))$ as shown in Figure 23. Recall that the symbol $\mathtt{Y}_1$ ($\mathtt{Y}_2$, respectively) is obtained by means of the keystroke sequence `VARS` `Y-VARS Function Y1` (`VARS` `Y-VARS Function Y2`, respectively). This is the function $y_3 = 1 - e^x$; its graph is shown in Figure 24, using the viewing window `Xmin= -5, Xmax=5, Xscl=1, Ymin=-5, Ymax=5, Yscl=1`. Note that to create the graph in Figure 24, $\mathtt{Y}_1$ and $\mathtt{Y}_2$ have been de-selected by removing the highlighted equal signs from the definitions in the `Y= Editor` (See Section 2.4.1).

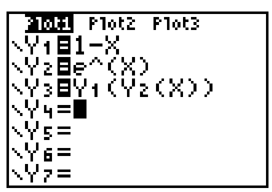

Figure 23: The function $y_3 = y_1(y_2(x))$ entered

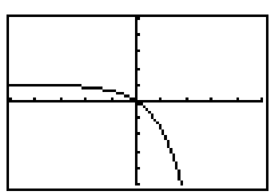

Figure 24: The graph of $y_3 = 1 - e^x$

Similarly, enter $\mathtt{Y}_4 = \mathtt{Y}_2(\mathtt{Y}_1(\mathtt{x}))$, this is the function $y_4 = e^{1-x}$; its graph is shown in Figure 25.

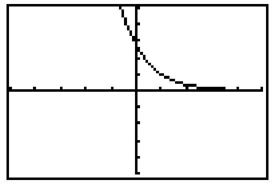

Figure 25: The graph of $y_4 = e^{1-x}$

2.4.10 Piecewise-defined functions

In many applications, functions cannot be given by one unique formula. Instead, functions related to applications are given in parts. Such functions are called *piecewise-defined functions*. The TI-83+/84+ allows you to enter and graph piecewise-defined functions. Consider the function

$$f(x) = \begin{cases} e^x + 1 & \text{if } -2 \leq x \leq 0, \\ x^2 - 2x + 2 & \text{if } 0 < x \leq \frac{3}{2} \end{cases}$$

In order to avoid any vertical lines, you must first change the Graph Style to `Dot`, (see Section 2.4.2) then enter the function as shown in Figure 26. The full function will be written out as:

$$(\mathtt{e} \wedge (\mathtt{X}) + 1) * (\mathtt{X} \geq -2) * (\mathtt{X} \leq 0) + (\mathtt{X}^2 - 2\mathtt{X} + 2) * (\mathtt{X} > 0) * (\mathtt{X} \leq 3/2)$$

The symbols '$\leq$', '$\geq$' and '$>$' are found in the $\boxed{\texttt{2nd}}$ `[TEST]` menu.

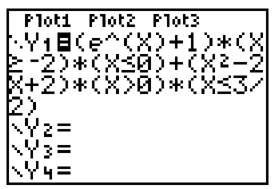

Figure 26: Entering a piecewise-defined function

Set the viewing window to `Xmin=-2.5, Xmax=2, Xscl=1, Ymin=-1, Ymax=4, Yscl=1`, and press $\boxed{\texttt{GRAPH}}$. The graph of the piece-wise defined function is shown in Figure 27. Notice that the graph is limited to the interval $[-2, \frac{3}{2}]$.

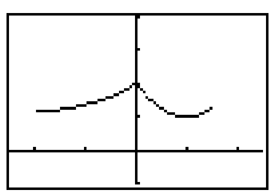

Figure 27: The graph of a piecewise-defined function

2.4.11 Polar Graphing

Polar graphing is used to plot graphs whose equation is given in polar coordinates, that is $r = r(\theta)$. Polar coordinates are used to describe some interesting classical geometric figures whose Cartesian representation would result in extremely complicated equations. Polar coordinates are also used in complex analysis and in engineering applications.

Polar graphing is illustrated below with the equation $r = 2\cos(\theta) - 1$. To use polar graphing on the TI-83+/84+, you need to change the mode settings. Press the $\boxed{\texttt{MODE}}$ key, then use the arrow keys to select `Pol` as shown in Figure 28, then press ENTER. Press the Y= key to access the `Y= Editor`, and enter the equations, Now the X,T,θ,n key produces θ (Figure 29).

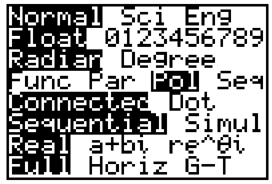

Figure 28: Mode Settings on the TI-83+/84+

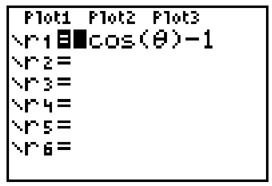

Figure 29: `Y= Editor` in polar mode

Press the WINDOW key to change window settings. In polar mode you also need to specify values for θ. Let $0 \leq \theta \leq 2\pi$, that is, θ`Min=0,` θ`Max=2`π, then set θ`step=`$\frac{\pi}{24}$`, Xmin=-1, Xmax=4, Xscl=1, Ymin=-2, Ymax=2, Yscl=1` (Figure 30). Press GRAPH to view the graph (Figure 31). You can use the `TABLE`, `ZOOM` and `TRACE` features when graphing in polar mode, in the latter case the values of θ, x and y are displayed on the screen.

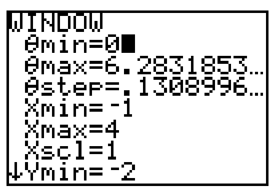

Figure 30: `WINDOW` in polar mode

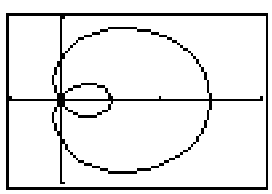

Figure 31: Polar graph

2.4.12 Parametric Graphing

Parametric graphing allows you to plot functions given by two equations $x = x(t)$, and $y = y(t)$, that is, both the x- and y-variables are given in terms of a *parameter* t. Parametric graphing arises in applications where the variables in question are dependent on time. One particularly useful example is when x and y represent the position of an object at time t.

Parametric graphing is illustrated below with the equations $x = \cos(t-1)$, and $y = \sin(t)$. For parametric graphing on the TI-83+/84+ you need to change mode settings. Press the `MODE` key and use the arrow keys to scroll up and across to select `Par` as shown in Figure 32. Press `ENTER`.

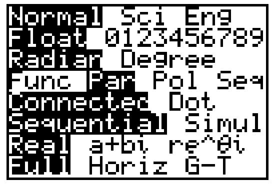

Figure 32: Mode Settings on the TI-83+/84+

Press the `Y=` key to access the `Y= Editor`, and enter the equations. In the `Par` setting, the `X,T,θ,n` key produces `T` which is used as the variable instead of t (Figure 33).

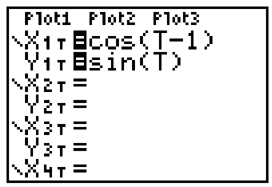

Figure 33: `Y= Editor` in parametric mode

Press `WINDOW` to change the window settings. In parametric mode you also need to specify values for `T`. When using trigonometric functions, you can let $0 \le t \le 2\pi$, that is `TMin=0, TMax=`2π. Then set `Tstep=`$\frac{\pi}{24}$`, Xmin=-2, Xmax=2, Xscl=1, Ymin=-2, Ymax=2, Yscl=1` (Figure 34). Press `GRAPH` to view the graph (Figure 35). You can use the `TABLE`, `ZOOM` and `TRACE` features when graphing in parametric mode, in the latter case, the values of `T`, x and y are displayed in the screen.

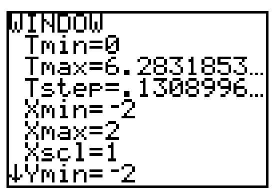

Figure 34: `WINDOW` in parametric mode

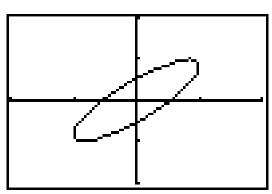

Figure 35: Parametric graph

2.4.13 Split Screen

The TI-83+/84+ allows you to view two screens at a time. For example, you can view the graph of a function while computing its values in an adjacent table. To display split screens, press the `MODE` key, scroll to the bottom row and select `G-T` for the Graph/Table mode. Press `ENTER`, then `2nd` `[QUIT]` to exit. Press `GRAPH`, and the calculator will display the split screen displaying both the graph of the function and a table of values for the function. To activate the right half of the screen press `2nd` `[TABLE]`. To set the table, press `2nd` `[Tblset]` as in Section 2.4.7. To move back to the left portion of the screen press `GRAPH`. See Figure 36. Split screens can be used when graphing in polar and parametric modes.

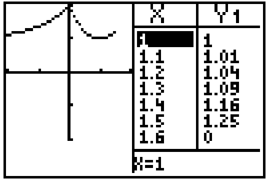

Figure 36: Split screen