Section 3.6 – Hypothesis Tests for a Population Proportion

A researcher obtains a random sample of 1000 people and finds that 534 are in favor of the banning cell phone use while driving, so $\hat{p}$ = 534/1000. Does this suggest that more than 50% of people favor the policy? Or is it possible that the true proportion of registered voters who favor the policy is some proportion less than 0.5 and we just happened to survey a majority in favor of the policy? In other words, would it be unusual to obtain a sample proportion of 0.534 or higher from a population whose proportion is 0.5? What is convincing, or statistically significant, evidence?

When observed results are unlikely under the assumption that the null hypothesis is true, we say the result is ***statistically significant***. When results are found to be statistically significant, we reject the null hypothesis.

Basic Methods of Testing Claims about a Population Proportion *p*

Notation

n = number of trials

$\hat{p} = \frac{x}{n}$ (***sample*** proportion)

p = population proportion (used in the null hypothesis)

$q = 1 - p$

Obtaining $\hat{p}$

$\hat{p}$ sometimes is given directly "10% of the observed sports cars are red" is expressed as $\underline{\hat{p} = 0.10}$

$\hat{p}$ sometimes must be calculated "96 surveyed households have cable TV and 54 do not" is calculated using

$$\hat{p} = \frac{x}{n} = \frac{96}{96+54} \underline{= 0.64}$$

Requirements for Testing Claims about a Population Proportion *p*

1. The sample observations are a simple random sample.
2. The conditions for a ***binomial distribution*** are satisfied.
3. The conditions $np \ge 5$ and $nq \ge 5$ are both satisfied, so the binomial distribution of sample proportions can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$. Note: p is the assumed proportion not the sample proportion.

Test Statistic for Testing a Claim about a Proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

***P*-values:** Use the standard Normal Distribution Table and refer to Figure

Critical Values: Use the standard Normal Distribution Table

***Caution*:** Don't confuse a P-value with a proportion p.

P-value = probability of getting a test statistic at least as extreme as the one representing sample data
p = population proportion

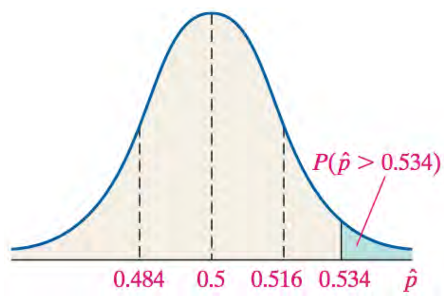
- When testing claims about a population proportion, the traditional method and the P-value method are equivalent and will yield the same result since they use the same standard deviation based on the ***claimed proportion*** p. However, the confidence interval uses an estimated standard deviation based upon the ***sample proportion*** $\hat{p}$. Consequently, it is possible that the traditional and P-value methods may yield a different conclusion than the confidence interval method.
  A good strategy is to use a confidence interval to estimate a population proportion, but use the P-value or traditional method for testing a claim about the proportion.

P–Value Method

If the sample proportion of getting a sample proportion as extreme or more extreme than the one obtained is small under the assumption the statement in the null hypothesis is true, reject the null hypothesis.

The Logic of the *P*-Value Approach

A second criterion we may use for testing hypotheses is to determine how likely it is to obtain a sample proportion of $\hat{p}$ = 534/1000 = 0.534 or higher from a population whose proportion is 0.5. If a sample proportion of 0.534 or higher is unlikely (or unusual), we have evidence against the statement in the null hypothesis. Otherwise, we do not have sufficient evidence against the statement in the null hypothesis.



We can compute the probability of obtaining a sample proportion of 0.534 or higher from a population whose proportion is 0.5 using the normal model

Example

A sample proportion of 0.534 (random sample of 1000 people out 534) or higher from a population whose proportion is 0.5.If a sample proportion of 0.534 or higher is unlikely (or unusual), Find P-value.

Solution

$$z = \frac{0.534 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{1000}}} = 2.15$$

$$P(\hat{p} > 0.534) = P(z > 2.15) = 1 - 0.9842 \underline{= 0.0158}$$

The value 0.0158 is called the P-value, which means about 2 samples in 100 will give a sample proportion as high or higher than the one we obtained if the population proportion really is 0.5. Because these results are unusual, we take this as evidence against the statement in the null hypothesis.

Example

The text refers to a study in which 57 out of 104 pregnant women correctly guessed the sex of their babies. Use these sample data to test the claim that the success rate of such guesses is no different from the 50% success rate expected with random chance guesses. Use a 0.05 significance level.

Solution

Requirements are satisfied: simple random sample; fixed number of trials (104) with two categories (guess correctly or do not)

$np = (104)(0.5) = 52 \ge 5$

$nq = (104)(0.5) = 52 \ge 5$

Step 1: Original claim is that the success rate is no different from 50%: $p = 0.50$

Step 2: Opposite of original claim is $p \ne 0.50$

Step 3: $p \ne 0.50$ does not contain equality so it is H_1.

$H_0 : p = 0.50$ null hypothesis and original claim

$H_1 : p \ne 0.50$ alternative hypothesis

Step 4: significance level is $\alpha = 0.50$

Step 5: sample involves proportion so the relevant statistic is the sample proportion, $\hat{p}$

Step 6: calculate z:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{57}{104} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{104}}}$$

$$\underline{= 0.98}$$

Two-tailed test, P-value is twice the area to the right of test statistic

From the Normal Distribution Table; $z = 0.98$ has an area of 0.8365 to its left, so area to the right is $1 - 0.8365 = 0.1635$, doubles yields 0.3270 (technology provides a more accurate P–value of 0.3268

Example

Suppose a geneticist claims that the XSORT method of gender selection. Among 726 babies born to couples using the XSORT method in an attempt to have a baby girl, 668 of the babies were girls and the others were boys. Use these results with a 0.05 significant level to test the claim that among babies born to couples using the XSORT method, the proportion of girls is greater than the value of 0.5 that is expected with no treatment. Here is a summary of the claim and the sample data:

Solution

Claim: With the XSORT method, the proportion of girls $p > 0.5$

Sample data: $n = 726 \quad and \quad \hat{p} = \frac{668}{726} = 0.920$

***Step* 1:** The original claim is symbolic is $p > 0.5$

Step 2: The opposite of the original claim is $p \le 0.5$

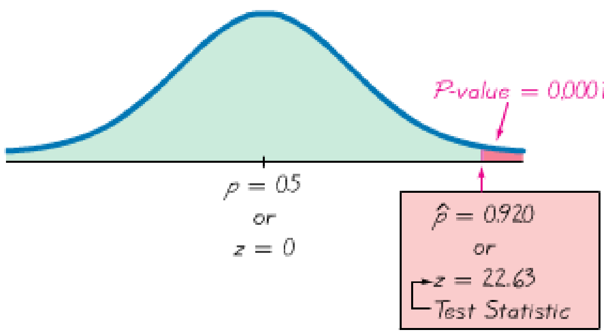
Step 3: $p > 0.5$ does not contain equality, so it becomes the alternative hypothesis. The null hypothesis is the statement that *p* equals the fixed value of 0.5.

$H_0 : p = 0.5$

$H_1 : p > 0.5$

Step 4: The significant level of $\alpha = 0.05$, which is a very common choice.

Step 5: Because we are testing a claim about a population proportion *p*, the sample statistic $\hat{p}$ is relevant to this test. The sampling distribution of sample proportion $\hat{p}$ can be approximated by a normal distribution.



Step 6: The test statistic:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.920 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{726}}} = 22.63$$

P–values are:

Left–tailed test: *P*–value = area to left of test statistic *z*

Right–tailed test: *P*–value = area to right of test statistic *z*

Two–tailed test: *P*–value = twice the area of the extreme region bounded by test statistic *z*

Because the hypothesis test we are considering is right-tailed with a test statistic of $z = 22.63$, the *P*-value is the area to the right of $z = 22.63$. Referring to Normal Distribution Table, for values of $z = 3.50$ and higher, we use 0.0001 for the cumulative area to the *right* of the test statistic. The *P*-value is therefore 0.0001.

Step 7: Because the *P*-value is 0.0001 is less than or equal to the significance level of $\alpha = 0.05$, we reject the null hypothesis

Step 8: **W**e conclude that there is sufficient sample evidence to support the claim that among babies born to couples using the XSORT method, the proportion of girls is greater than 0.5.

Testing Hypotheses Claims

To test hypotheses regarding the population proportion, we can use the steps that follow, provided that:

✓ The sample is obtained by simple random sampling.

✓ $np_0\left(1-p_0\right) \geq 0$

✓ The sampled values are independent of each other.

***Step* 1**: Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

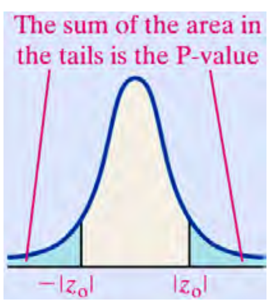
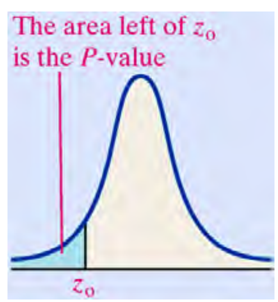
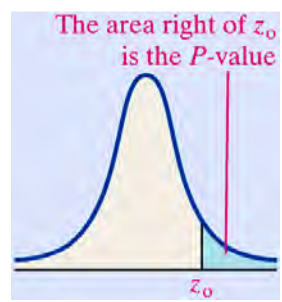
Two–Tailed	***Left–Tailed***	***Right–Tailed***
$H_0 : p = p_0$	$H_0 : p = p_0$	$H_0 : p = p_0$
$H_1 : p \neq p_0$	$H_1 : p < p_0$	$H_1 : p > p_0$

p_0 is assumed value of the population proportion.

***Step* 2:** Select a level of significance, α, based on the seriousness of making a Type I error.

Step* 3**: Compute the ***test statistic $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0\left(1-p_0\right)}{n}}}$

***Step* 4:** Compare the critical value with the test statistic:

Two–Tailed	***Left–Tailed***	***Right–Tailed***
$z_0 < -z_{\alpha/2}$ *or* $z_0 > z_{\alpha/2}$	$z_0 < -z_\alpha$	$z_0 > z_\alpha$
Reject the null hypothesis	*Reject the null hypothesis*	*Reject the null hypothesis*
		

***Step* 5:** If the *P*-value < α, reject the null hypothesis.

***Step* 6:** State the conclusion.

Exercises *Section* 3.6 – Hypothesis Tests for a Population Proportion

1. In a Harris poll, adults were asked if they are in favor of abolishing the penny. Among the responses, 1261 answered "no", and 491 answered "yes", and 384 had no opinion. What is the sample proportion of ***yes*** responses, and what notation is used to represent it?

2. A recent study showed that 53% of college applications were submitted online. Assume that this result is based on a simple random sample of 1000 college applications, with 530 submitted online. Use a 0.01 significance level to test the claim that among all college applications the percentage submitted online is equal to 50%

a) What is the test statistic?
b) What are the critical values?
c) What is the *P*-Value?
d) What is the conclusion?
e) Can a hypothesis test be used to "prove" that the percentage of college applications submitted online is equal to 50% as claimed?

3. In a survey, 1864 out of 2246 randomly selected adults in the U.S. said that texting while driving should be illegal. Consider a hypothesis test that uses a 0.05 significance level to test the claim that more than 80% of adults believe that testing while driving should be illegal

a) What is the test statistic?
b) What are the critical values?
c) What is the *P*-Value?
d) What is the conclusion?

4. In a Pew Research Center poll of 745 randomly selected adults, 589 said that it is morally wrong to not report all income on tax returns. Use a 0.01 significance level to test the claim that 75% of adults say that it is morally wrong to not report all income on tax returns.
Identify the null hypothesis, alternative hypothesis, test statistic, *P*-value or critical value(s), conclusion about the null hypothesis, and final conclusion that address the original claim.

5. 308 out of 611 voters surveyed said that they voted for the candidate who won. Use a 0.01 significance level to test the claim that among all voters, the percentage who believe that they voted for the winning candidate is equal to 43%, which is the actual percentage of votes for the winning candidate. What does the result suggest about voter perceptions?
Identify the null hypothesis, alternative hypothesis, test statistic, *P*-value or critical value(s), conclusion about the null hypothesis, and final conclusion that address the original claim.

6. The company Drug Test Success provides a "1-Panel-THC" test for marijuana usage. Among 300 tested subjects, results from 27 subjects were wrong (either a false positive or a false negative). Use a 0.05 significance level to test the claim that less than 10% of the test results are wrong. Does the test appear to be good for most purposes?

7. When testing gas pumps in Michigan for accuracy, fuel-quality enforcement specialists tested pumps and found that 1299 of them were not pumping accurately (within 3.3 oz. when 5 gal. is pumped), and 5686 pumps were accurate. Use a 0.01 significance level to test the claim of an industry representative that less than 20% of Michigan gas pumps are inaccurate. From the perspective of the consumer, does that rate appear to be low enough?

8. Trials in an experiment with a polygraph include 98 results that include 24 cases of wrong results and 74 cases of correct results. Use a 0.05 significance level to test the claim that such polygraph results are correct less than 80% of the time. Based on the results, should polygraph test results be prohibited as evidence in trials?

9. In recent years, the Town of Newport experienced an arrest rate of 25% for robberies. The new sheriff compiles records showing that among 30 recent robberies, the arrest rate is 30%, so she claims that her arrest rate is greater than the 25% rate in the past. Is there sufficient evidence to support her claim that the arrest rate is greater than 25%?

10. A survey showed that among 785 randomly selected subjects who completed 4 years of college, 18.3 % smoke and 81.7% do not smoke. Use a 0.01 significance level to test the claim that the rate of smoking among those with 4 years of college is less than the 27% rate for the general population. Why would college graduates smoke at a lower rate than others?

11. When 3011 adults were surveyed, 73% said that they use the Internet. Is it okay for a newspaper reporter to write that "3/4 of all adults use the internet"? Why or Why not?

12. A hypothesis test is performed to test the claim that a population proportion is greater than 0.7. Find the probability of a type II error, β, given that the true value of the population proportion is 0.72. The sample size is 50 and the significance level is 0.05.

13. In a sample of 88 children selected randomly from one town, it is found that 8 of them suffer asthma. Find the *P*-value for a test of the claim that the proportion of all children in the town who suffer from asthma is equal to 11%.

14. An airline claims that the no-show rate for passengers booked on its flights is less than 6%. Of 380 randomly selected reservation, 18 were no-shows. Find the P-value for a test of the airline's claim.

15. In 1997, 46% of Americans said they did not trust the media "when it comes to reporting the news fully, accurately and fairly". In a 2007 poll of 1010 adults nationwide, 525 stated they did not trust the media. At the $\alpha = 0.05$ level of significance, is there evidence to support the claim that the percentage of Americans that do not trust the media to report fully and accurately has increased since 1997?

16. In 2006, 10.5% of all live births in the United States were to mothers under 20 years of age. A sociologist claims that births to mothers under 20 years of age is decreasing. She conducts a simple random sample of 34 births and finds that 3 of them were to mothers under 20 years of age. Test the sociologist's claim at the $\alpha = 0.01$ level of significance.