

Homework

1. Show that B is the inverse of A . $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}; \quad B = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

2. Find the inverse of the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

3. Find the inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$

4. Find the inverse of the matrix $\begin{pmatrix} -1 & 1 \\ 3 & -3 \end{pmatrix}$

5. Find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{pmatrix}$

6. Find the inverse of the matrix $\begin{pmatrix} -4 & -6 \\ 2 & 3 \end{pmatrix}$

7. Find the inverse of the matrix $A = \begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$

8. Prove that if A is an invertible matrix, then $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$

9. Prove if C is an invertible matrix such that $CA = CB$, then $A = B$

10. Prove that if $A^2 = A$, then $I - 2A = (I - 2A)^{-1}$

11. Prove that the inverse of a symmetric nonsingular matrix is symmetric.

12. Prove that if A , B , and C are square symmetric matrices and $ABC = I$, then B is an invertible and $B^{-1} = CA$.

13. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

a) Show that $A^2 - 2A + 5I = 0$

b) Show that $A^2 = \frac{1}{5}(2I - A)$

c) Show that for any square matrix satisfying $A^2 - 2A + 5I = 0$, the inverse of A is

$$A^{-1} = \frac{1}{5}(2I - A)$$