

$$1/ y'' - 4y = 0$$

$$\lambda^2 - 4 = 0 \Rightarrow \lambda = \pm 2$$

$$y(t) = C_1 e^{-2t} + C_2 e^{2t}$$

$$2/ 2y'' - 3y' = 0$$

$$2\lambda^2 - 3\lambda = 0 \Rightarrow \lambda = 0, \frac{3}{2}$$

$$y(t) = C_1 + C_2 e^{3t/2}$$

$$3/ y'' + 3y' + 10y = 0$$

$$\lambda^2 + 3\lambda + 10 = 0 \Rightarrow \lambda = \frac{-3 \pm \sqrt{-31}}{2} = -\frac{3}{2} \pm i \frac{\sqrt{31}}{2}$$

$$y(t) = e^{-3t/2} \left(C_1 \cos \frac{\sqrt{31}}{2} t + C_2 \sin \frac{\sqrt{31}}{2} t \right)$$

$$4/ y'' + 6y' + 9y = 0$$

$$\lambda^2 + 6\lambda + 9 = 0 \Rightarrow \lambda = -3, -3$$

$$y(t) = (C_1 + C_2 t) e^{-3t}$$

$$5/ 4y'' - 12y' + 9y = 0$$

$$4\lambda^2 - 12\lambda + 9 = 0 \quad \lambda = \frac{12 \pm \sqrt{144 - 144}}{8} = \frac{3}{2}$$

$$y(t) = (C_1 + C_2 t) e^{3t/2}$$

$$6/ y'' + 8y' + 25y = 0$$

$$\lambda^2 + 8\lambda + 25 = 0 \quad \lambda_{1,2} = \frac{-8 \pm \sqrt{64 - 100}}{2} = -4 \pm 6i$$

$$y(t) = e^{-4t} (C_1 \cos 6t + C_2 \sin 6t)$$

$$7/ \quad 2y'' - 7y' + 3y = 0$$

$$2\lambda^2 - 7\lambda + 3 = 0$$

$$\lambda_{1,2} = \frac{7 \pm \sqrt{49 - 24}}{2} = 1, 6$$

$$y(t) = C_1 e^t + C_2 e^{6t}$$

$$8/ \quad y^{(3)} + y'' - y' - y = 0$$

$$\lambda^3 + \lambda^2 - \lambda - 1 = 0$$

$$\lambda^2(\lambda + 1) - (\lambda + 1) = 0$$

$$(\lambda + 1)(\lambda^2 - 1) = 0 \Rightarrow \lambda = -1, -1, 1$$

$$y(t) = (C_1 + C_2 t) e^{-t} + C_3 e^t$$

$$9/ \quad 9y^{(3)} + 12y'' + 4y' = 0$$

$$9\lambda^3 + 12\lambda^2 + 4\lambda = 0$$

$$\lambda(9\lambda^2 + 12\lambda + 4) = 0 \quad \lambda = -6, -6, 0$$

$$y = C_1 + (C_2 + C_3 t) e^{-6t}$$

$$10/ \quad 5y^{(4)} + 3y^{(3)} = 0$$

$$5\lambda^4 + 3\lambda^3 = 0$$

$$\lambda^3(5\lambda + 3) = 0 \rightarrow \lambda = 0, 0, 0, -\frac{3}{5}$$

$$y(t) = C_1 + C_2 t + C_3 t^2 + C_4 \cos \frac{\sqrt{15}}{5} t + C_5 \sin \frac{\sqrt{15}}{5} t$$

$$11/ \quad y^{(4)} - 8y^{(3)} + 16y'' = 0$$

$$\lambda^4 - 8\lambda^3 + 16\lambda^2 = 0$$

$$\lambda^2(\lambda^2 - 8\lambda + 16) = 0 \rightarrow \lambda = 0, 0, 4, 4$$

$$y(t) = C_1 + C_2 t + C_3 e^{4t} + C_4 t e^{4t}$$

$$12/ y^{(4)} - 3y^{(3)} + 3y'' - y' = 0$$

$$\lambda^4 - 3\lambda^3 + 3\lambda^2 - \lambda = 0$$

$$\lambda(\lambda^3 - 3\lambda^2 + 3\lambda - 1) = 0$$

$$\lambda(\lambda-1)^3 = 0 \Rightarrow \lambda = 0, 1, 1, 1$$

$$y(t) = C_1 + C_2 e^t + C_3 t e^t + C_4 t^2 e^t$$

$$13/ y^{(4)} + 3y'' - 4y = 0$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

$$(\lambda^2 - 1)(\lambda^2 + 4) = 0 \quad \lambda = \pm 1, \pm 2i$$

$$y(t) = C_1 e^{-t} + C_2 e^t + C_3 \cos 2t + C_4 \sin 2t$$

$$14/ y^{(4)} - 8y'' + 16y = 0$$

$$\lambda^4 - 8\lambda^2 + 16 = 0 \Rightarrow (\lambda - 4)^2 = 0 \quad \lambda = 4, 4$$

$$y(t) = (C_1 + C_2 t) e^{4t}$$

$$15/ 6y^{(4)} + 11y'' + 4y = 0$$

$$6\lambda^4 + 11\lambda^2 + 4 = 0$$

$$\lambda^2 = \frac{-11 \pm 5}{12} \quad \left. \begin{array}{l} \lambda^2 = \frac{-4}{3} \rightarrow \lambda = \pm \frac{2\sqrt{3}}{3}i \\ \lambda^2 = -\frac{1}{2} \rightarrow \lambda = \pm \frac{\sqrt{2}}{2}i \end{array} \right\}$$

$$y(t) = C_1 \cos \frac{2\sqrt{3}}{3}t + C_2 \sin \frac{2\sqrt{3}}{3}t + C_3 \cos \frac{\sqrt{2}}{2}t + C_4 \sin \frac{\sqrt{2}}{2}t$$

$$16/ y^{(4)} + 2y^{(3)} + 3y'' + 2y' + y = 0$$

$$\lambda^4 + 2\lambda^3 + 3\lambda^2 + 2\lambda + 1 = 0$$

$$\lambda^4 + \lambda^3 + \lambda^2 + \lambda^3 + \lambda^2 + \lambda + \lambda^2 + \lambda + 1 = 0$$

$$\lambda^2(\lambda^2 + \lambda + 1) + \lambda(\lambda^2 + \lambda + 1) + (\lambda^2 + \lambda + 1) = 0$$

$$(\lambda^2 + \lambda + 1)(\lambda^2 + \lambda + 1) = 0$$

$$(\lambda^2 + \lambda + 1)^2 = 0 \quad \lambda = \frac{-1 \pm i\sqrt{3}}{2}$$

$$y(t) = e^{-t/2} \left(C_1 \cos \frac{\sqrt{3}}{2}t + C_2 \sin \frac{\sqrt{3}}{2}t + C_3 t \cos \frac{\sqrt{3}}{2}t + C_4 t \sin \frac{\sqrt{3}}{2}t \right)$$

$$59) y'' - 4y' + 3y = 0; \quad y(0) = 7, \quad y'(0) = 11$$

$$\lambda^2 - 4\lambda + 3 = 0 \rightarrow \lambda = 1, 3$$

$$y(t) = C_1 e^t + C_2 e^{3t}$$

$$y' = C_1 e^t + 3C_2 e^{3t}$$

$$7 = C_1 + C_2$$

$$11 = C_1 + 3C_2$$

$$\begin{cases} C_1 + C_2 = 7 \\ C_1 + 3C_2 = 11 \end{cases}$$

$$[C_1 = \frac{10}{2} = 5] \quad [C_2 = 2]$$

$$y(t) = 5e^t + 2e^{3t}$$

$$60) 9y'' + 6y' + 4y = 0; \quad y(0) = 3, \quad y'(0) = 4$$

$$9\lambda^2 + 6\lambda + 4 = 0$$

$$\lambda = \frac{-6 \pm 6i\sqrt{3}}{18} = -\frac{1}{3} \pm i\frac{\sqrt{3}}{3}$$

$$y(t) = e^{-\frac{1}{3}t} (C_1 \cos\frac{\sqrt{3}}{3}t + C_2 \sin\frac{\sqrt{3}}{3}t)$$

$$y(0) = 3 = C_1$$

$$y'(t) = -\frac{1}{3}e^{-\frac{1}{3}t} (C_1 \cos\frac{\sqrt{3}}{3}t + C_2 \sin\frac{\sqrt{3}}{3}t) + e^{-\frac{1}{3}t} (-\frac{\sqrt{3}}{3}C_1 \sin\frac{\sqrt{3}}{3}t + \frac{\sqrt{3}}{3}C_2 \cos\frac{\sqrt{3}}{3}t)$$

$$y'(0) = 4 = -\frac{1}{3}C_1 + \frac{\sqrt{3}}{3}C_2 \Rightarrow \frac{\sqrt{3}}{3}C_2 = 5 \rightarrow C_2 = 5\sqrt{3}$$

$$y(t) = e^{-\frac{1}{3}t} (3 \cos\frac{\sqrt{3}}{3}t + 5\sqrt{3} \sin\frac{\sqrt{3}}{3}t)$$

$$61) y'' - 6y' + 25y = 0$$

$$y(0) = 3, \quad y'(0) = 1$$

$$\lambda^2 - 6\lambda + 25 = 0$$

$$\lambda = \frac{6 \pm 8i}{2} = 3 \pm 4i$$

$$y(t) = e^{3t} (C_1 \cos 4t + C_2 \sin 4t)$$

$$y(0) = C_1 = 3$$

$$y'(t) = 3e^{3t} (C_1 \cos 4t + C_2 \sin 4t) + e^{3t} (-4C_1 \sin 4t + 4C_2 \cos 4t)$$

$$y'(0) = 3C_1 + 4C_2 = 1 \Rightarrow C_2 = -2$$

$$y(t) = e^{3t} (3 \cos 4t - 2 \sin 4t)$$

$$62/ 2y''' - 3y'' - 2y' = 0$$

$$y(0) = 1, y'(0) = -1, y''(0) = 3$$

$$2\lambda^3 - 3\lambda^2 - 2\lambda = 0$$

$$\lambda(2\lambda^2 - 3\lambda - 2) = 0$$

$$\lambda = \frac{3 \pm 5}{4}$$

$$\lambda = 0, -\frac{1}{2}, 2$$

$$y(t) = C_1 + C_2 e^{-t/2} + C_3 e^{2t}$$

$$y(0) = C_1 = 1$$

$$y'(t) = -\frac{1}{2} C_2 e^{-t/2} + 2C_3 e^{2t} \rightarrow y'(0) = -\frac{1}{2} C_2 + 2C_3 = -1 \quad (\frac{1}{2})$$

$$y''(t) = \frac{1}{4} C_2 e^{-t/2} + 4C_3 e^{2t} \Rightarrow y''(0) = \frac{1}{4} C_2 + 4C_3 = 3$$

$$5C_3 = \frac{5}{2} \rightarrow C_3 = \frac{1}{2}$$

$$-\frac{1}{4} C_2 + C_3 = -\frac{1}{2}$$

$$C_2 = 4(3 - 2) = 8$$

$$y(t) = 1 + 8e^{-t/2} + \frac{1}{2} e^{2t}$$

$$63/ 3y''' + 2y'' = 0$$

$$y(0) = -1, y'(0) = 0, y''(0) = 1$$

$$3\lambda^3 + 2\lambda^2 = 0$$

$$\lambda^2(3\lambda + 2) = 0 \rightarrow \lambda = 0, 0, -2/3$$

$$y(t) = C_1 + C_2 t + C_3 e^{-2t/3}$$

$$y(0) = C_1 + C_3 = -1 \quad (1)$$

$$y'(t) = C_2 - \frac{2}{3} C_3 e^{-2t/3} \rightarrow y'(0) = C_2 - \frac{2}{3} C_3 = 0 \quad (2)$$

$$y''(t) = \frac{4}{9} C_3 e^{-2t/3} \rightarrow y''(0) = \frac{4}{9} C_3 = 1 \rightarrow C_3 = \frac{9}{4}$$

$$(1) \rightarrow C_1 = -\frac{13}{4}$$

$$(2) \rightarrow C_2 = \frac{3}{2}$$

$$y(t) = -\frac{13}{4} + \frac{3}{2}t + \frac{9}{4} e^{-2t/3}$$

$$64/ y''' + 10y'' + 25y' = 0; \quad y(0) = 3, \quad y'(0) = 4, \quad y''(0) = 5$$

$$\lambda^3 + 10\lambda^2 + 25\lambda = 0$$

$$\lambda(\lambda^2 + 10\lambda + 25) = 0$$

$$\lambda(\lambda + 5)^2 = 0 \rightarrow \lambda = 0, -5, -5$$

$$y(t) = C_1 + (C_2 + C_3 t) e^{-5t}$$

$$y(0) = C_1 + C_2 = 3 \quad (1)$$

$$y'(t) = -5C_2 e^{-5t} + C_3(1 - 5t)e^{-5t} \rightarrow y'(0) = -5C_2 + C_3 = 4 \quad (2)$$

$$y''(t) = 25C_2 e^{-5t} + C_3(-5 - 5 + 25t)e^{-5t} \rightarrow y''(0) = 25C_2 - 10C_3 = 5 \quad (3)$$

$$\begin{aligned} (2) &\rightarrow -5C_2 + C_3 = 4 \\ (3) &\rightarrow 25C_2 - 10C_3 = 5 \end{aligned} \rightarrow \underline{C_3 = -5} \quad C_2 = \underline{-\frac{9}{5}}$$

$$(1) \rightarrow C_1 = 3 + \frac{9}{5} = \frac{24}{5}$$

$$y(t) = \frac{24}{5} - \left(\frac{9}{5} + 5t\right)e^{-5t}$$