Section 1.3 – Fractions and Rationalization

Fraction (*Basic*)

$$\frac{a}{b}=\frac{numerator}{denominator}$$

$\frac{a}{b}=\frac{c}{d} \iff ad=bc$ *Cross multiplication*

$$\frac{a}{b}=\frac{na}{nb}=\frac{an}{bn}$$

a) $\frac{5}{6}=\frac{25}{30}?$

$$\frac{5}{6}=\frac{5}{6}\frac{5}{5}=\frac{25}{30}$$

b) $\frac{16}{48}=\frac{1}{3}$

$$\frac{16}{48}=\frac{1}{3} \iff (16)(3)=(1)(48)$$

$$48=48$$

Simplify: $\frac{12}{18}=\frac{2.6}{2.9}$

$$=\frac{2.2.3}{2.3.3}$$

$$=\frac{2}{3}$$

Simplify: $\frac{36}{56}=\frac{2.18}{2.28}$

$$=\frac{18}{28}$$

$$=\frac{2.9}{2.14}$$

$$=\frac{9}{14}$$

If the denominators are the same $\Rightarrow$ add the numerators

$$\frac{3}{5}+\frac{4}{5}=\frac{3+4}{5}=\frac{7}{5}$$

If the denominators are the same $\Rightarrow$ subtract the numerators

$$\frac{4}{9}-\frac{2}{9}=\frac{4-2}{9}=\frac{2}{9}$$

If the denominators are not the same

$\Rightarrow$ Find Least Common Denominator (LCD) and convert so that the fractions have the same denominators

LCD: is the smallest whole number that is a multiple of each

$\frac{5}{8}+\frac{1}{12}$ LCD (8, 12)

$$\begin{aligned} 8 &= 2^3 \\ 12 &= \underline{2^2\ 3} \\ & \quad 2^3\ 3 = 24 \end{aligned}$$

LCD (8, 12) = 24

$$\begin{aligned} \frac{5}{8}+\frac{1}{12} &= \frac{5}{8}\frac{3}{3}+\frac{1}{12}\frac{2}{2} \\ &= \frac{15}{24}+\frac{2}{24} \\ &= \frac{15+2}{24} \\ &= \frac{17}{24} \end{aligned}$$

$\frac{69}{75}-\frac{1}{50}$

LCD (75, 50)

$$\begin{aligned} 75 &= \ \ 5^3 \\ 50 &= \underline{2\ 5^2} \\ & \quad 2\ 5^3 = 150 \end{aligned}$$

LCD (75, 50) = 150

$$\begin{aligned} \frac{69}{75}-\frac{1}{50} &= \frac{(69)(2)-(1)(3)}{150} \\ &= \frac{138-3}{150} \\ &= \frac{135}{150} \\ &= \frac{9}{10} \end{aligned}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

$$\frac{2}{7}+\frac{3}{5}=\frac{2(5)+3(7)}{7(5)}$$

$$=\frac{10+21}{35}$$

$$=\frac{31}{35}$$

or $$\frac{2}{7}\frac{5}{5}+\frac{3}{5}\frac{7}{7}=\frac{10}{35}+\frac{21}{35}$$

$$=\frac{10+21}{35}$$

$$=\frac{31}{35}$$

$$\frac{5}{9}+\frac{3}{4}=\frac{5(4)+3(9)}{9(4)}$$

$$=\frac{20+27}{36}$$

$$=\frac{47}{36}$$

$$\frac{17}{15}+\frac{5}{12}=\frac{17(12)+5(15)}{15(12)}$$

$$=\frac{204+75}{180}$$

$$=\frac{279}{180}$$

$$=\frac{31(9)}{20(9)}$$

$$=\frac{31}{20}$$

$$\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}=\frac{5(7)(9)+(3)(7)(9)+(3)(5)(9)+(3)(5)(7)}{(3)(5)(7)(9)}$$

$$=\frac{315+189+135+105}{945}$$

$$=\frac{744}{945}$$

$$=\frac{248}{315}\frac{3}{3}$$

$$=\frac{248}{315}$$

$$\frac{8}{9}+\frac{1}{12}+\frac{3}{16}$$

$$\frac{8}{9}+\frac{1}{12}+\frac{3}{16}=\frac{8(16)+1(12)+3(9)}{144}$$

$$\begin{cases} 9 = \quad 3^2 \\ 12 = 2^2 \quad 3 \\ 16 = 2^4 \end{cases}$$

$$\text{LCD} \quad 2^4 \quad 3^2 = 144$$

$$=\frac{128+12+27}{144}$$

$$=\frac{167}{144}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

$$\frac{2}{7}-\frac{3}{5}=\frac{2(5)-3(7)}{7(5)}=\frac{10-21}{35}=-\frac{11}{35}$$

$$\frac{a}{c}\frac{b}{d}=\frac{ab}{cd}$$

$$\frac{2}{7}\frac{3}{5}=\frac{6}{35}$$

$$\frac{a}{c}\div\frac{b}{d}=\frac{a}{c}\times\frac{d}{b}=\frac{ad}{cb}$$

$$\frac{2}{7}\div\frac{3}{5}=\frac{2}{7}\frac{5}{3}=\frac{10}{21}$$

Find:

1. $\frac{13}{21}+\frac{5}{21}=\frac{13+5}{21}=\frac{6}{7}$

2. $\frac{7}{12}-\frac{4}{15}=\frac{7(5)-4(4)}{60}=\frac{35-16}{60}=\frac{19}{60}$

3. $\frac{5}{8}+\frac{1}{2}=\frac{5+4}{8}=\frac{9}{8}$

4. $\frac{5}{8}+\frac{1}{2}+\frac{2}{3}=\frac{5(3)+1(12)+2(8)}{24}=\frac{43}{24}$

5. $\frac{7}{8}-\frac{1}{10}=\frac{7(5)-1(4)}{40}=\frac{31}{40}$

6. $\frac{11}{5}-\frac{31}{7}=-\frac{78}{35}$

7. $\frac{3}{4}\bullet\frac{3}{2}=\frac{9}{8}$

8. $\frac{3}{4}\bullet\frac{4}{3}\bullet\frac{2}{3}=\frac{2}{3}$

9. $\frac{3}{4}\div\frac{3}{2}=\frac{3}{4}\bullet\frac{2}{3}=\frac{2}{4}=\frac{1}{2}$

10. $\frac{14}{15}\div\frac{14}{3}=\frac{14}{15}\bullet\frac{3}{14}=\frac{1}{5}$

Operations with Fractions

A rational expression is proper if the degree of numerator is less than the degree of denominator
A rational expression is improper if the degrees of numerator is greater than or equal the degree of denominator

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)=\frac{ac}{bd}$$

$$\frac{a/b}{c/d}=\frac{a}{b}\frac{d}{c}=\frac{ad}{bc}$$

$$\frac{a/b}{c}=\frac{a}{b}\frac{1}{c}=\frac{a}{bc}$$

$$\frac{ab}{ac}=\frac{b}{c}$$

$$\frac{ad+ac}{ad}=\frac{a(d+c)}{ad}=\frac{b+c}{d}$$

$$\frac{ab+cd}{ad} \quad stay$$

Example

Perform each indicated operation & simplify

a) $x+\frac{2}{x}=\frac{x^2+2}{x}$

b) $\frac{2}{x+1}-\frac{1}{2x+1}=\frac{2(2x+1)-1(x+1)}{(x+1)(2x+1)}$

$$=\frac{4x+2-x-1}{(x+1)(2x+1)}$$

$$=\frac{3x+1}{(x+1)(2x+1)}$$

Example

Perform each indicated operation & simplify

a) $\dfrac{x}{x^2-4}-\dfrac{1}{x-2}=\dfrac{x-1(x+2)}{(x-2)(x+2)}$ $\qquad x^2-4=(x-2)(x+2)$

$$=\frac{x-x-2}{(x-2)(x+2)}$$

$$=\frac{-2}{(x-2)(x+2)}$$

b) $\dfrac{1}{3(x^2+2x)}-\dfrac{1}{3x}=\dfrac{1-1(x+2)}{3x(x+2)}$ $\qquad 3(x^2+2x)=3x(x+2)$

$$=\frac{1-x-2}{3x(x+2)}$$

$$=\frac{-x-1}{3x(x+2)}$$

Example

Perform each indicated operation & simplify

a) $\dfrac{\sqrt{x+2}-\frac{x}{4\sqrt{x+2}}}{x+2}=\left(\sqrt{x+2}-\dfrac{x}{4\sqrt{x+2}}\right)\div(x+2)$

$$=\left(\frac{4\sqrt{x+2}\sqrt{x+2}-x}{4\sqrt{x+2}}\right)\left(\frac{1}{x+2}\right)$$

$$=\frac{4(x+2)-x}{4(x+2)\sqrt{x+2}}$$

$$=\frac{4x+8-x}{4(x+2)\sqrt{x+2}}$$

$$=\boxed{\frac{3x+8}{4(x+2)\sqrt{x+2}}}$$

b) $\left(\dfrac{1}{x+\sqrt{x^2+4}}\right)\left(1+\dfrac{x}{\sqrt{x^2+4}}\right)=\dfrac{1}{x+\sqrt{x^2+4}}\dfrac{\sqrt{x^2+4}+x}{\sqrt{x^2+4}}$

$$=\boxed{\frac{1}{\sqrt{x^2+4}}}$$

Example

Perform each indicated operation & simplify

$$\frac{-x\left(\frac{3x}{3\sqrt{x^2+4}}\right)+\sqrt{x^2+4}}{x^2}+\left(\frac{1}{x+\sqrt{x^2+4}}\right)\left(1+\frac{3x}{3\sqrt{x^2+4}}\right)$$

$$=\left(-\frac{3x^2}{3\sqrt{x^2+4}}+\sqrt{x^2+4}\right)\frac{1}{x^2}+\left(\frac{1}{x+\sqrt{x^2+4}}\right)\left(\frac{3\sqrt{x^2+4}+3x}{3\sqrt{x^2+4}}\right)$$

$$=\left(\frac{-3x^2+3\left(\sqrt{x^2+4}\right)^2}{3\sqrt{x^2+4}}\right)\frac{1}{x^2}+\left(\frac{1}{x+\sqrt{x^2+4}}\right)\left(\frac{3(\sqrt{x^2+4}+x)}{3\sqrt{x^2+4}}\right)$$

$$=\left(\frac{-3x^2+3(x^2+4)}{3\sqrt{x^2+4}}\right)\frac{1}{x^2}+\frac{3}{3\sqrt{x^2+4}}$$

$$=\frac{-3x^2+3x^2+12}{3\sqrt{x^2+4}}\frac{1}{x^2}+\frac{3}{3\sqrt{x^2+4}}$$

$$=\frac{12}{3\sqrt{x^2+4}}\frac{1}{x^2}+\frac{3}{3\sqrt{x^2+4}}$$

$$=\frac{12+3x^2}{3x^2\sqrt{x^2+4}}$$

$$=\frac{3(x^2+4)}{3x^2\left(x^2+4\right)^{1/2}}$$

$$=\frac{\sqrt{x^2+4}}{x^2}$$

Rationalization Techniques

1. If the denominator is $\sqrt{a}$, multiply by $\frac{\sqrt{a}}{\sqrt{a}}$

2. If the denominator is $\sqrt{a}-\sqrt{b}$, multiply by $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

3. If the denominator is $\sqrt{a}+\sqrt{b}$, multiply by $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

$$\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)=a-b$$

Example

Simplify by rationalizing the denominator

a) $\frac{4}{\sqrt{3}}=\frac{4}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{4\sqrt{3}}{3}$

b) $\frac{2}{\sqrt[3]{x}}=\frac{2}{\sqrt[3]{x}}\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

$=\frac{2\sqrt[3]{x^2}}{x}$

c) $\frac{1}{1-\sqrt{2}}=\frac{1}{1-\sqrt{2}}\frac{1+\sqrt{2}}{1+\sqrt{2}}$

$=\frac{1+\sqrt{2}}{1-2}$

$=\frac{1+\sqrt{2}}{-1}$

$=-1-\sqrt{2}$

Example

Simplify $\sqrt{27}\sqrt{3}$

$$\begin{aligned}\sqrt{27}\sqrt{3} &= \sqrt{27(3)}\\ &= \sqrt{81}\\ &= 9\end{aligned}$$

Example

Simplify $\sqrt[4]{x^8y^7z^{11}}$

$$\sqrt[4]{x^8y^7z^{11}} = x^2yz^2 \ \sqrt[4]{y^3z^3}$$

Example

Simplify $\dfrac{5}{\sqrt{10}}$

$$\begin{aligned}\frac{5}{\sqrt{10}} &= \frac{5}{\sqrt{10}}\frac{\sqrt{10}}{\sqrt{10}}\\ &= \frac{5\sqrt{10}}{10}\\ &= \frac{\sqrt{10}}{2}\end{aligned}$$

Example

Simplify $\dfrac{5}{2-\sqrt{6}}$

$$\begin{aligned}\frac{5}{2-\sqrt{6}} &= \frac{5}{2-\sqrt{6}}\frac{2+\sqrt{6}}{2+\sqrt{6}}\\ &= \frac{5\left(2+\sqrt{6}\right)}{4-6}\\ &= -\frac{5}{2}\left(2+\sqrt{6}\right)\end{aligned}$$

Example

Simplify $\dfrac{1}{\sqrt{r}-\sqrt{3}}$

$$\begin{aligned}\frac{1}{\sqrt{r}-\sqrt{3}} &= \frac{1}{\sqrt{r}-\sqrt{3}}\frac{\sqrt{r}+\sqrt{3}}{\sqrt{r}+\sqrt{3}}\\ &= \frac{\sqrt{r}+\sqrt{3}}{r-3}\end{aligned}$$

Example

Rationalize the denominator or numerator

a) $\dfrac{5}{\sqrt{8}}$

$$= \frac{5}{\sqrt{8}}\frac{\sqrt{8}}{\sqrt{8}}$$

$$= \frac{5\sqrt{8}}{8}$$

b) $\dfrac{1}{\sqrt{6}-\sqrt{3}}$

$$= \frac{1}{\sqrt{6}-\sqrt{3}}\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}$$

$$= \frac{\sqrt{6}+\sqrt{3}}{\left(\sqrt{6}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{6}+\sqrt{3}}{6-3} = \frac{\sqrt{6}+\sqrt{3}}{3}$$

$$\boxed{= \frac{\sqrt{6}+\sqrt{3}}{3}}$$

c) $\dfrac{1}{\sqrt{x}+\sqrt{x+2}}$

$$= \frac{1}{\sqrt{x}+\sqrt{x+2}}\frac{\sqrt{x}-\sqrt{x+2}}{\sqrt{x}-\sqrt{x+2}}$$

$$= \frac{\sqrt{x}-\sqrt{x+2}}{x-(x+2)}$$

$$= \frac{\sqrt{x}-\sqrt{x+2}}{x-x-2}$$

$$= \frac{\sqrt{x}-\sqrt{x+2}}{-2}$$

$$\boxed{= \frac{\sqrt{x+2}-\sqrt{x}}{2}}$$

Example

$$-\frac{\sqrt{x^2+1}}{x^2}-\frac{1}{\sqrt{x^2+1}}=\frac{-\sqrt{x^2+1}\sqrt{x^2+1}-x^2}{x^2\sqrt{x^2+1}}$$

$$-\frac{\sqrt{x^2+1}}{x^2}\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}\frac{x^2}{x^2}$$

$$=\frac{-\left(x^2+1\right)-x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{-x^2-1-x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{-2x^2-1}{x^2\sqrt{x^2+1}}$$

$$\boxed{=-\frac{2x^2+1}{x^2\sqrt{x^2+1}}}$$

Example

$$\left(\sqrt{x^2+1}-\frac{3x^3}{2\sqrt{x^2+1}}\right)\div\left(x^3+1\right)=\left(\frac{\sqrt{x^2+1}\left(2\sqrt{x^2+1}\right)-3x^3}{2\sqrt{x^2+1}}\right)\cdot\frac{1}{x^3+1}$$

$$=\frac{2\left(x^2+1\right)-3x^3}{2\left(x^3+1\right)\sqrt{x^2+1}}$$

$$\boxed{=\frac{-3x^3+2x^2+2}{2\left(x^3+1\right)\sqrt{x^2+1}}}$$

Exercises *Section* 1.3 – Fractions and Rationalization

1. Perform the operation and simplify $\dfrac{2}{x^2-4}-\dfrac{1}{x-2}$

2. Perform each indicated operation & simplify: $\dfrac{A}{x+1}-\dfrac{B}{x-1}+\dfrac{C}{x+2}$

3. Perform the operation and simplify: $-\dfrac{\sqrt{x^2+1}}{x^2}-\dfrac{1}{\sqrt{x^2+1}}$

4. Perform the operation and simplify: $\left(\sqrt{x^2+1}-\dfrac{3x^3}{2\sqrt{x^2+1}}\right)\div\left(x^3+1\right)$

5. Perform the operation and simplify: $\dfrac{6}{x(3x-2)}+\dfrac{5}{3x-2}-\dfrac{2}{x^2}$

6. Simplify the fraction: $\dfrac{\frac{2}{x+3}-\frac{2}{a+3}}{x-a}$

7. Simplify: $\dfrac{3x^2(2x+5)^{1/2}-x^3\left(\frac{1}{2}\right)(2x+5)^{-1/2}(2)}{\left[(2x+5)^{1/2}\right]^2}$

8. Simplify the expression: $\dfrac{\left(4x^2+9\right)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left[\left(4x^2+9\right)^{1/2}\right]^2}$

9. Simplify the expression: $\dfrac{\left(1-x^2\right)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)}{\left[\left(1-x^2\right)^{1/2}\right]^2}$

10. Simplify the expression: $\dfrac{\left(x^2+4\right)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)\left(x^2+4\right)^{-2/3}(2x)}{\left[\left(x^2+4\right)^{1/3}\right]^2}$

11. Simplify the expression: $\dfrac{\left(x^2-5\right)^4(3x^2)-x^3(4)\left(x^2-5\right)^3(2x)}{\left[\left(x^2-5\right)^4\right]^2}$

12. Simplify the expression: $\dfrac{(3x+2)^{1/2}\left(\frac{1}{3}\right)(2x+3)^{-2/3}(2)-(2x+3)^{1/3}\left(\frac{1}{2}\right)(3x+2)^{-1/2}(3)}{\left[(3x+2)^{1/2}\right]^2}$

13. Simplify the expression: $\dfrac{\left(x^2+2\right)^3(2x)-x^2(3)\left(x^2+2\right)^2(2x)}{\left[\left(x^2+2\right)^3\right]^2}$