

Section 1.6 – Other Types of Equations

The numbers of solutions to a polynomial with n degree, where n is Natural Number, are n solutions.

Solving a Polynomial Equation by factoring

Example

Solve: $4x^4 = 12x^2$

Solution

$$4x^4 - 12x^2 = 0$$

$$4x^2 (x^2 - 3) = 0$$

$$4x^2 = 0$$

$$x^2 = 0$$

$$\rightarrow \underline{x = 0, 0}$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$\underline{x = \pm\sqrt{3}}$$

Example

Solve: $2x^3 + 3x^2 = 8x + 12$

Solution

$$2x^3 + 3x^2 - 8x - 12 = 0$$

$$x^2(2x + 3) - 4(2x + 3) = 0$$

$$(2x + 3)(x^2 - 4) = 0$$

$$2x + 3 = 0$$

$$2x = -3$$

$$\underline{x = -\frac{3}{2}}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\underline{x = \pm\sqrt{4} = \pm 2}$$

Equations that Are Quadratic in Form

$$ax^2 + bx + c = 0$$

$$a(x)^2 + b(x)^1 + c = 0$$

$$a(u)^2 + b(u)^1 + c = 0$$

$$a(x^n)^2 + b(x^n)^1 + c = 0$$

$$au^2 + bu + c = 0$$

Example

Solve: $x^4 - 5x^2 + 6 = 0$

Solution

$$(x^2)^2 - 5(x^2) + 6 = 0$$

$$(U)^2 - 5(U) + 6 = 0$$

$$U^2 - 5U + 6 = 0$$

Solve for U

$$\Rightarrow U = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$= \frac{5 \pm \sqrt{1}}{2}$$

$$\rightarrow \begin{cases} U = \frac{5-1}{2} = 2 \\ U = \frac{5+1}{2} = 3 \end{cases}$$

$$x^2 = U \quad \rightarrow \begin{cases} x^2 = 2 \rightarrow \underline{x = \pm\sqrt{2}} \\ x^2 = 3 \rightarrow \underline{x = \pm\sqrt{3}} \end{cases}$$

$$\text{or} \quad (x^2 - 2)(x^2 - 3) = 0$$

$$x^2 - 2 = 0 \quad x^2 - 3 = 0$$

$$x^2 = 2 \quad x^2 = 3$$

$$x = \pm\sqrt{2} \quad x = \pm\sqrt{3}$$

Example

Solve: $(x+1)^{2/3} - (x+1)^{1/3} - 2 = 0$

Solution

$$u = (x+1)^{1/3}$$

$$u^2 - u - 2 = 0$$

$$(u-2)(u+1) = 0$$

$$u - 2 = 0$$

$$u = 2$$

$$u = (x+1)^{1/3} = 2$$

$$x+1 = 2^3$$

$$x+1 = 8$$

$$\underline{x = 7}$$

$$u + 1 = 0$$

$$u = -1$$

$$u = (x+1)^{1/3} = -1$$

$$x+1 = (-1)^3$$

$$x+1 = -1$$

$$\underline{x = -2}$$

$$\left((x+1)^{1/3} - 2\right)\left((x+1)^{1/3} + 1\right) = 0$$

Example

Solve: $3x^{2/3} - 11x^{1/3} - 4 = 0$

Solution

$$3\left(x^{1/3}\right)^2 - 11\left(x^{1/3}\right) - 4 = 0$$

$$x^{1/3} = \frac{11 \pm \sqrt{121 + 48}}{2(3)}$$

$$x^{1/3} = \frac{11-13}{6}$$

$$= -\frac{1}{3}$$

$$x = \left(-\frac{1}{3}\right)^3$$

$$\underline{= -\frac{1}{27}}$$

$$x^{1/3} = \frac{11+13}{6}$$

$$= 4$$

$$x = 4^3$$

$$\underline{= 64}$$

Or factor

$$\left(3x^{1/3} + 1\right)\left(x^{1/3} - 4\right) = 0$$

$$3x^{1/3} + 1 = 0$$

$$x^{1/3} - 4 = 0$$

Solving a *Radical* Equation

Power Property

If P and Q are algebraic expressions, then every solution of the equation $P = Q$ is also a solution of the equation $P^n = Q^n$; for any positive integer n .

Example

Solve $x - \sqrt{15 - 2x} = 0$

Solution

$$x = \sqrt{15 - 2x}$$

$$x^2 = (\sqrt{15 - 2x})^2$$

$$x^2 = 15 - 2x$$

$$x^2 + 2x - 15 = 0$$

$$(x - 3)(x + 5) = 0$$

$$x - 3 = 0 \quad x + 5 = 0$$

$$x = 3 \quad x = -5$$

Check

$$x = 3$$

$$3 - \sqrt{15 - 2(3)} = 0$$

$$3 - \sqrt{9} = 0$$

$$3 - 3 = 0 \quad (\text{true})$$

$$x = -5$$

$$-5 - \sqrt{15 - 2(-5)} = 0$$

$$-5 - \sqrt{25} = 0$$

$$-5 - 5 \neq 0 \quad (\text{false})$$

$x = 3$ is the only solution

Solving Radical Equations of the Form $x^{\frac{m}{n}} = k$

Assume that m and n are positive integers

$$\text{If } m \text{ is even: } x^{\frac{m}{n}} = k \Rightarrow \left(x^{\frac{m}{n}} \right)^{\frac{n}{m}} = k^{\frac{n}{m}} \Rightarrow x = \pm k^{\frac{n}{m}}$$

$$\text{If } m \text{ is odd: } x^{\frac{m}{n}} = k \Rightarrow \left(x^{\frac{m}{n}} \right)^{\frac{n}{m}} = k^{\frac{n}{m}} \Rightarrow x = k^{\frac{n}{m}}$$

Example

Solve: $5x^{3/2} - 25 = 0$

Solution

$$\begin{aligned} 5x^{3/2} &= 25 \\ x^{3/2} &= \frac{25}{5} = 5 \\ x &= 5^{\frac{2}{3}} \\ &= \sqrt[3]{5^2} \\ &= \sqrt[3]{25} \end{aligned}$$

Example

Solve: $x^{2/3} - 8 = -4$

Solution

$$\begin{aligned} x^{2/3} &= 4 \\ x &= \pm (4)^{3/2} \\ &= \pm (2^2)^{3/2} \\ &= \pm 2^3 \\ &= \pm 8 \end{aligned}$$

Solving an Absolute Value Equation

If c is a positive real number and X represents any algebraic expression, then $|X| = c$ is equivalent to $X = c$ or $X = -c$

$$|X| = c \rightarrow X = c \text{ or } X = -c$$

Properties of Absolute Value

1. For $b > 0$, $|a| = b$ if and only if (*iff*) $a = b$ or $a = -b$
2. $|a| = |b|$ *iff* $a = b$ or $a = -b$

For any positive number b :

3. $|a| < b$ *iff* $-b < a < b$
4. $|a| < b$ *iff* $a < -b$ or $a > b$

Example

Solve: $|2x - 1| = 5$

Solution

$$2x - 1 = 5$$

$$2x = 6$$

$$x = 3$$

$$2x - 1 = -5$$

$$2x = -4$$

$$x = -2$$

Solutions: $x = -2, 3$

Example

Solve: $4|1 - 2x| - 20 = 0$

Solution

$$4|1 - 2x| = 20$$

$$|1 - 2x| = 5$$

$$1 - 2x = 5$$

$$-2x = 4$$

$$x = -2$$

$$1 - 2x = -5$$

$$-2x = -6$$

$$x = 3$$

Solutions: $x = -2, 3$

Exercise Section 1.6 – Other Types of Equations

(1 – 112) Solve

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|---------------------------------|-----------------------------|-----------------------------|
| 1. $3x^3 + 2x^2 = 12x + 8$ | 13. $3x^3 - 9x^2 - 30x = 0$ | 24. $x^4 - 4x^3 - 4x^2 = 0$ |
| 2. $x^3 + x^2 - 4x - 4 = 0$ | 14. $x^4 + 3x^2 = 10$ | 25. $x^4 - 6x^3 + 9x^2 = 0$ |
| 3. $x^3 + x^2 + 4x + 4 = 0$ | 15. $5x^4 = 40x$ | 26. $x^4 - 4x^3 + 3x^2 = 0$ |
| 4. $x^3 + 4x^2 - 25x - 100 = 0$ | 16. $9x^4 - 9x^2 + 2 = 0$ | 27. $x^4 - 4x^2 + 3 = 0$ |
| 5. $x^3 - 2x^2 - x + 2 = 0$ | 17. $x^4 + 720 = 89x^2$ | 28. $x^4 + 4x^2 + 3 = 0$ |
| 6. $x^3 - x^2 - 25x + 25 = 0$ | 18. $12x^4 - 11x^2 + 2 = 0$ | 29. $x^4 + 6x^2 - 7 = 0$ |
| 7. $x^3 - x^2 = 16x - 16$ | 19. $2x^4 - 7x^2 + 5 = 0$ | 30. $x^4 - 6x^2 - 7 = 0$ |
| 8. $x^3 + x^2 + 25x + 25 = 0$ | 20. $x^4 - 5x^2 + 4 = 0$ | 31. $3x^4 + 4x^2 - 7 = 0$ |
| 9. $x^3 + 2x^2 = 16x + 32$ | 21. $x^4 + 3x^2 = 10$ | 32. $3x^4 - 4x^2 - 7 = 0$ |
| 10. $2x^3 + 3x^2 - 6x - 9 = 0$ | 22. $3x^4 - 48x^2 = 0$ | 33. $3x^4 - x^2 - 2 = 0$ |
| 11. $2x^3 + x^2 - 8x - 4 = 0$ | 23. $5x^4 - 20x^2 = 0$ | 34. $3x^4 + x^2 - 2 = 0$ |
| 12. $2x^3 + 16x^2 + 30x = 0$ | | |
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|-----------------------------|------------------------------|---|
| 35. $x - 3\sqrt{x} - 4 = 0$ | 40. $\sqrt[3]{6x-3} = 3$ | 45. $(3x-6)^{1/3} + 5 = 8$ |
| 36. $(5x^2 - 6)^{1/4} = x$ | 41. $\sqrt[3]{2x-6} = 4$ | 46. $(3x+1)^{1/4} + 7 = 9$ |
| 37. $(x^2 + 24x)^{1/4} = 3$ | 42. $\sqrt[3]{4x-3} - 5 = 0$ | 47. $(2x+3)^{1/4} + 7 = 10$ |
| 38. $x^{5/2} = 32$ | 43. $(3x-1)^{1/3} + 4 = 0$ | 48. $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$ |
| 39. $\sqrt[3]{2x+11} = 3$ | 44. $(2x+3)^{1/3} + 4 = 6$ | |
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|----------------------------|---------------------------------|------------------------------------|
| 49. $\sqrt{2x+3} = 5$ | 59. $\sqrt{5x+1} = x+1$ | 69. $\sqrt{6x+2} = \sqrt{5x+3}$ |
| 50. $\sqrt{x-3} + 6 = 5$ | 60. $x = \sqrt{2x-2} + 1$ | 70. $\sqrt{3x+1} - \sqrt{x+4} = 1$ |
| 51. $\sqrt{3x-2} = 4$ | 61. $x - 2\sqrt{x-3} = 3$ | 71. $\sqrt{x+2} + \sqrt{x-1} = 3$ |
| 52. $\sqrt{5x-4} = 9$ | 62. $x + \sqrt{26-11x} = 4$ | 72. $\sqrt{x-4} + \sqrt{x+4} = 4$ |
| 53. $\sqrt{5x-1} = 8$ | 63. $x - \sqrt{2x+3} = 0$ | 73. $\sqrt{2x-3} - \sqrt{x-2} = 1$ |
| 54. $\sqrt{3x-2} - 5 = 0$ | 64. $\sqrt{x+3} + 3 = x$ | 74. $\sqrt{x+2} + \sqrt{3x+7} = 1$ |
| 55. $\sqrt{2x+5} + 11 = 6$ | 65. $x - \sqrt{x+11} = 1$ | 75. $2\sqrt{4x+1} - 9 = x-5$ |
| 56. $\sqrt{3x+7} + 10 = 4$ | 66. $\sqrt{x-7} = 7 - \sqrt{x}$ | 76. $\sqrt{2x-3} + \sqrt{x-2} = 1$ |
| 57. $x = \sqrt{7x+8}$ | 67. $\sqrt{x-8} = \sqrt{x} - 2$ | 77. $\sqrt{2x+3} = 1 + \sqrt{x+1}$ |
| 58. $x = \sqrt{6x+7}$ | 68. $\sqrt{2x-5} = \sqrt{x+4}$ | 78. $\sqrt{x+5} - \sqrt{x-3} = 2$ |
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79. $|x| = -9$

80. $|x| = 9$

81. $|x - 2| = 7$

82. $|x - 2| = 0$

83. $|2x - 3| = 6$

84. $|2x - 1| = 11$

85. $7|5x| + 2 = 16$

86. $4\left|1 - \frac{3}{4}x\right| + 7 = 10$

87. $|x + 7| + 6 = 2$

88. $|5 - 3x| = 12$

89. $|4x + 2| = 5$

90. $3|x + 5| = 12$

91. $2|x - 6| = 8$

92. $3|2x - 1| = 21$

93. $2|3x - 2| = 14$

94. $|3x - 1| + 2 = 16$

95. $|6x - 2| + 4 = 32$

96. $7|5x| + 2 = 16$

97. $|4x + 1| + 10 = 4$

98. $|4x + 1| + 4 = 10$

99. $|3x - 2| + 8 = 1$

100. $|3x - 2| + 1 = 8$

101. $\left|\frac{6x+1}{x-1}\right| = 3$

102. $|x + 1| = |1 - 3x|$

103. $|3x - 1| = |x + 5|$

104. $|5x - 8| = |3x + 2|$

105. $|4x - 9| = |2x + 1|$

106. $|2x - 4| = |x - 1|$

107. $|3x - 4| = |3x + 4|$

108. $|3x - 5| = |3x + 5|$

109. $|x - 3| = |5 - x|$

110. $|x - 3| = |6 - x|$

111. $\left|\frac{2}{3}x - 2\right| = \left|\frac{1}{3}x + 3\right|$

112. $\left|\frac{1}{2}x - 2\right| = \left|x - \frac{1}{2}\right|$