Section 2.7 - Inverse Trigonometry Functions

Definition

A ***function*** is a rule or correspondence that pairs each element of the domain with exactly one element from the range. That is, a function is a set of ordered pairs in which no two different ordered pairs have the same first coordinate.

The ***inverse*** of function is found by interchanging the coordinates in each ordered pair that is an element of the function

Inverse Function Notation

if $y = f(x)$ is one-to-one function, then the inverse of f is also a function and can be denoted by

$$y = f^{-1}(x)$$

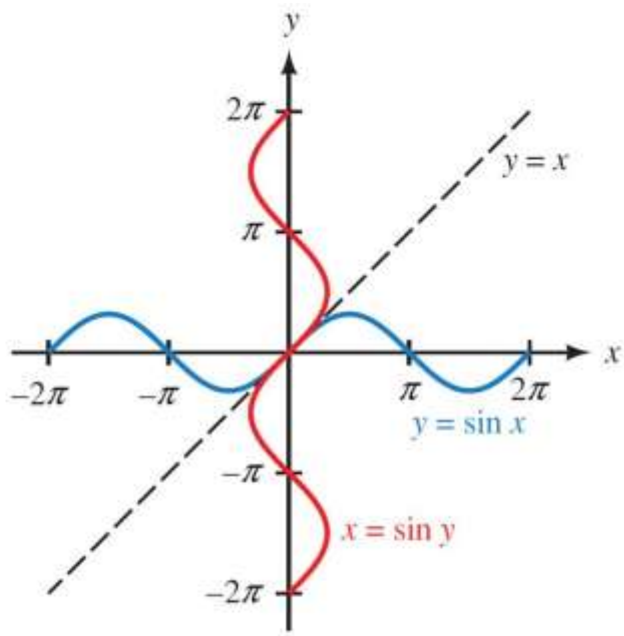
The inverse Sine Relation

To find the inverse of $y = \sin x$

1. Interchange x and y $\quad \rightarrow x = \sin y$

To graph $x = \sin y$

1. Graph $y = \sin x$
2. Draw the line $y = x$
3. Reflect $y = \sin x$ about the line $y = x$

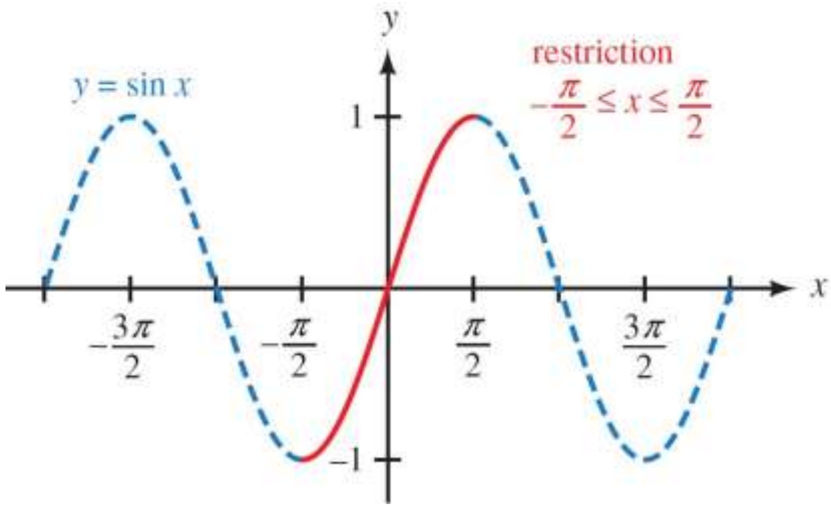


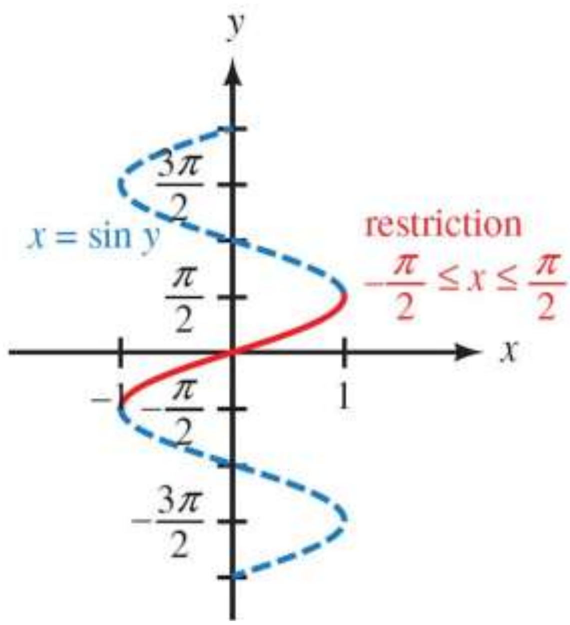
The Inverse *Sine* Function

Notation

The notation used to indicate the inverse *sine* function is as follow:

Notation	***Meaning***
$y=\sin^{-1}x$ *or* $y=\arcsin x$	$x=\sin y$ *and* $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$



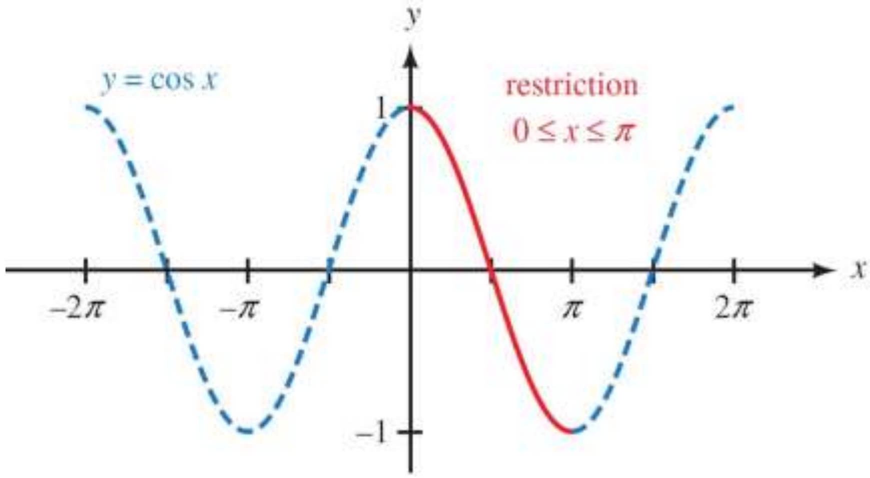


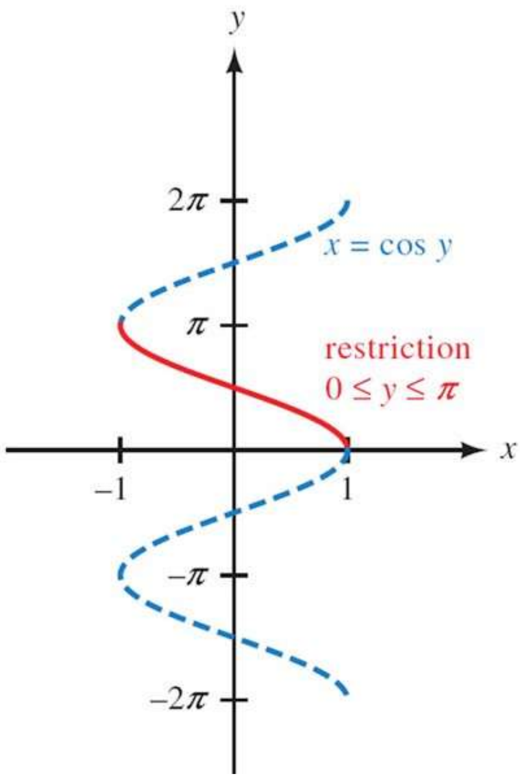
The Inverse *Cosine* Function

Notation

The notation used to indicate the inverse *cosine* function is as follow:

Notation	***Meaning***
$y = \cos^{-1} x \quad or \quad y = \arccos x$	$x = \cos y \quad and \quad 0 \le y \le \pi$



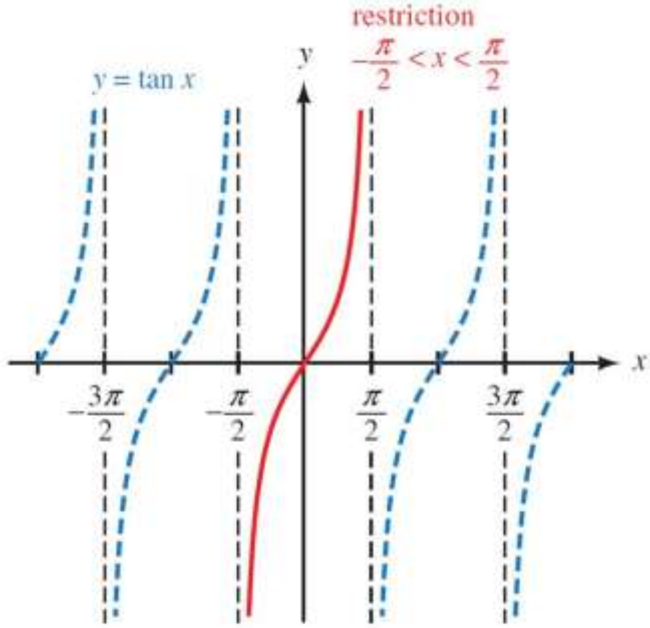


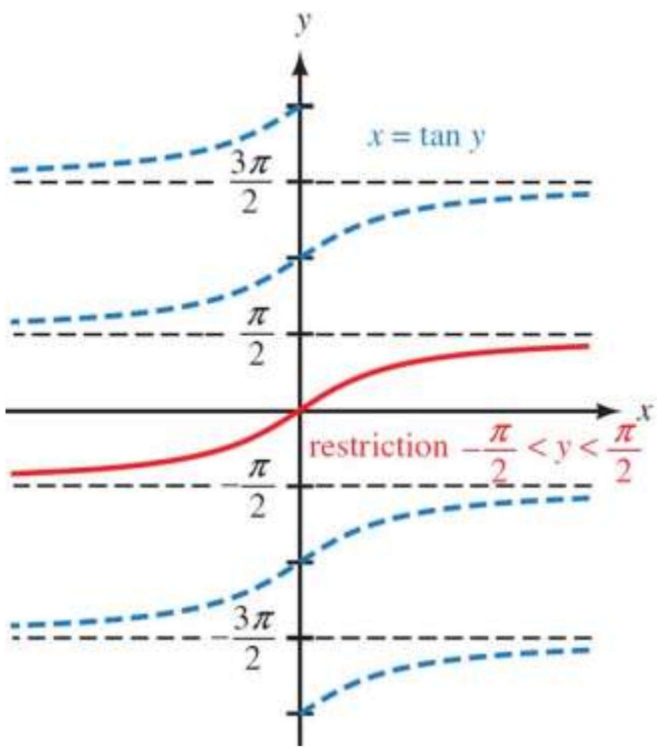
The Inverse *Tangent* Function

Notation

The notation used to indicate the inverse *tangent* function is as follow:

Notation	*Meaning*
$y = \tan^{-1} x$ *or* $y = \arctan x$	$x = \tan y$ *and* $-\frac{\pi}{2} < y < \frac{\pi}{2}$





Example

Evaluate in radians without using a calculator or tables.

a. $\sin^{-1}\frac{1}{2}$

$$-\frac{\pi}{2} \le angle \le \frac{\pi}{2} \Rightarrow \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

b. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

$$0 < angle < \pi \Rightarrow \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

c. $\tan^{-1}(-1)$

$$-\frac{\pi}{2} < angle < \frac{\pi}{2} \Rightarrow \tan\left(-\frac{\pi}{4}\right) = -1$$

$$\tan^{-1}(-1) = -\frac{\pi}{4}$$

Example

Use a calculator to evaluate each expression to the nearest tenth of a degree

a. $\arcsin(0.5075)$

$$\arcsin(0.5075) = 30.5°$$

b. $\arcsin(-0.5075)$

$$\arcsin(-0.5075) = -30.5°$$

c. $\cos^{-1}(0.6428)$

$$\cos^{-1}(0.6428) = 50.0°$$

d. $\cos^{-1}(-0.6428)$

$$\cos^{-1}(-0.6428) = 130.0°$$

e. $\arctan(4.474)$

$$\arctan(4.474) = 77.4°$$

f. $\arctan(-4.474)$

$$\arctan(-4.474) = -77.4°$$

Example

Simplify $3|\sec\theta|$ if $\theta = \tan^{-1}\frac{x}{3}$ for some real number x.

<u>*Solution*</u>

$$\theta = \tan^{-1}\frac{x}{3} \rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Since $-\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow \cos\theta > 0$

$$\Rightarrow \sec\theta > 0$$

$$3|\sec\theta| = 3\sec\theta$$

Example

Evaluate each expression

a. $\sin\left(\sin^{-1}\frac{1}{2}\right)$

$$\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin\left(\frac{\pi}{6}\right) \qquad\qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \rightarrow \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$= \frac{1}{2}$$

b. $\sin^{-1}\sin(135°)$

$$\sin(135°) = \sin(180° - 135°)$$

$$= \sin(45°)$$

$$= \frac{\sqrt{2}}{2}$$

$$\sin^{-1}\sin(135°) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= 45°$$

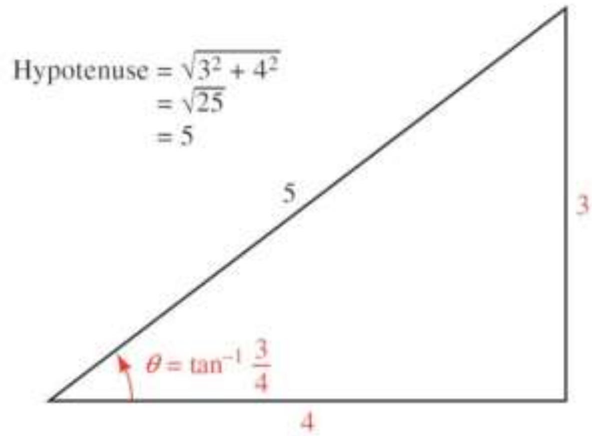
Example

Simplify $\tan^{-1}(\tan x)$ if $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$\tan^{-1}(\tan x) = x$$

Example

Evaluate $\sin\left(\tan^{-1}\frac{3}{4}\right)$ without using a calculator



Solution

$$\theta = \tan^{-1}\frac{3}{4} \Rightarrow \tan\theta = \frac{3}{4} \rightarrow 0° < \theta < 90°$$

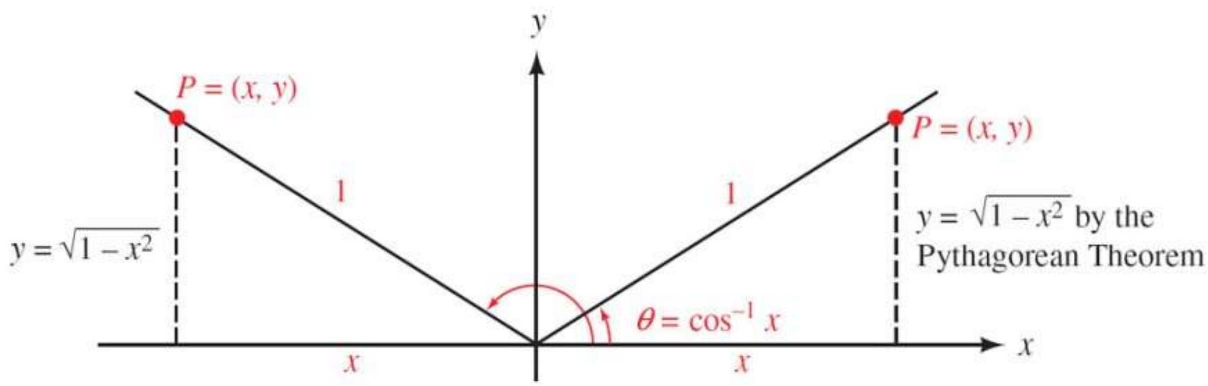
$$\sin\left(\tan^{-1}\frac{3}{4}\right) = \sin\theta$$

$$= \frac{3}{5}$$

Example

Evaluate $\sin(\cos^{-1}x)$ as an equivalent expression in x only

Solution



$$\sin(\theta) = \frac{y}{r}$$

$$= \frac{\sqrt{1-x^2}}{1}$$

$$= \sqrt{1-x^2}$$

$$\sin(\cos^{-1}x) = \sin\theta$$

$$= \sqrt{1-x^2}$$

Exercises *Section* 2.7 - Inverse Trigonometry Functions

1. Evaluate without using a calculator: $\cos\left(\cos^{-1}\frac{3}{5}\right)$

2. Evaluate without using a calculator: $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

3. Evaluate without using a calculator: $\tan\left(\cos^{-1}\frac{3}{5}\right)$

4. Evaluate without using a calculator: $\sin\left(\cos^{-1}\frac{1}{\sqrt{5}}\right)$

5. Evaluate without using a calculator: $\cos\left(\sin^{-1}\frac{1}{2}\right)$

6. Evaluate without using a calculator: $\sin\left(\sin^{-1}\frac{3}{5}\right)$

7. Evaluate without using a calculator: $\cos\left(\tan^{-1}\frac{3}{4}\right)$

8. Evaluate without using a calculator: $\tan\left(\sin^{-1}\frac{3}{5}\right)$

9. Evaluate without using a calculator: $\sec\left(\cos^{-1}\frac{1}{\sqrt{5}}\right)$

10. Evaluate without using a calculator: $\cot\left(\tan^{-1}\frac{1}{2}\right)$

11. Write an equivalent expression that involves x only for $\cos\left(\cos^{-1}x\right)$

12. Write an equivalent expression that involves x only for $\tan\left(\cos^{-1}x\right)$

13. Write an equivalent expression that involves x only for $\csc\left(\sin^{-1}\frac{1}{x}\right)$