Laplace Transform

Time Function	Laplace Transform	Time Function	Laplace Transform
Unit impulse $\delta(t)$	1	$u(t-a)$	$\frac{1}{s}e^{-as}$
Unit step $u(t)=1$	$\frac{1}{s}$	$u(t)-u(t-a)$	$\frac{1}{s}\left(1-e^{-as}\right)$
t	$\frac{1}{s^2}$	$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\frac{t^2}{2}$	$\frac{1}{s^3}$	$1-(at+1)e^{-at}$	$\frac{a^2}{s(s+a)^2}$
$t^{-\frac{1}{2}}$	$\sqrt{\frac{\pi}{s}}$	$at-1+e^{-at}$	$\frac{a^2}{s^2(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$	$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$
$t^n \quad n=1,2,\ldots$	$\frac{n!}{s^{n+1}}$	$1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt}$	$\frac{ab}{s(s+a)(s+b)}$
$t^{n-\frac{1}{2}} \quad n=1,2,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n\, s^{n+\frac{1}{2}}}$	$\frac{c}{ab}-\frac{(c-a)e^{-at}}{a(b-a)}+\frac{(c-b)e^{-bt}}{b(b-a)}$	$\frac{s+c}{s(s+a)(s+b)}$
e^{at}	$\frac{1}{s-a}$	$\frac{(c-a)e^{-at}-(c-b)e^{-bt}}{b-a}$	$\frac{s+c}{(s+a)(s+b)}$
e^{-at}	$\frac{1}{s+a}$	$b-be^{-at}+a(a-b)te^{-at}$	$\frac{a^2(s+b)}{s(s+a)^2}$
te^{at}	$\frac{1}{(s-a)^2}$	$-2+at+(2+at)e^{-at}$	$\frac{a^3}{s^2(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$		
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}=\frac{1/2j}{s-j\omega}-\frac{1/2j}{s+j\omega}$	$\cos \omega t$	$\frac{s}{s^2+\omega^2}=\frac{1/2}{s-j\omega}+\frac{1/2}{s+j\omega}$
$t\sin \omega t$	$\frac{2\omega s}{\left(s^2+\omega^2\right)^2}$	$t\cos \omega t$	$\frac{s^2-\omega^2}{\left(s^2+\omega^2\right)^2}$
$\sin^2 \omega t$	$\frac{2\omega^2}{s\left(s^2+4\omega^2\right)}$	$\cos^2 \omega t$	$\frac{s^2+2\omega^2}{s\left(s^2+4\omega^2\right)}$
$\sin(\omega t+\phi)$	$\frac{s\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2}$	$\cos(\omega t+\phi)$	$\frac{s\cos(\phi)-\omega\sin(\phi)}{s^2+\omega^2}$

$\sin(at) - at\cos(at)$	$\dfrac{2a^3}{\left(s^2+a^2\right)^2}$	$\cos(at) - at\sin(at)$	$\dfrac{s\left(s^2-a^2\right)}{\left(s^2+a^2\right)^2}$
$\sin(at) + at\cos(at)$	$\dfrac{2as^2}{\left(s^2+a^2\right)^2}$	$\cos(at) + at\sin(at)$	$\dfrac{s\left(s^2+3a^2\right)}{\left(s^2+a^2\right)^2}$
$e^{-at}\sin\omega t$	$\dfrac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$	$\dfrac{s+a}{(s+a)^2+\omega^2}$
$e^{at}\sin\omega t$	$\dfrac{\omega}{(s-a)^2+\omega^2} \quad (s>a)$	$e^{at}\cos\omega t$	$\dfrac{s-a}{(s-a)^2+\omega^2} \quad (s>a)$
$te^{-at}\sin\omega t$	$\dfrac{2\omega(s+a)}{\left((s+a)^2+\omega^2\right)^2}$	$e^{-\zeta\omega t}\sin\left(\omega t\sqrt{1-\zeta^2}\right)$	$\dfrac{\omega\sqrt{1-\zeta^2}}{s^2+2\zeta\omega s+\omega^2}$
$\dfrac{\sin\omega t}{t}$	$\arctan\dfrac{\omega}{s}$		
$\sinh\omega t$	$\dfrac{\omega}{s^2-\omega^2}$	$\cosh\omega t$	$\dfrac{s}{s^2-\omega^2}$
$e^{at}\sinh\omega t$	$\dfrac{a}{(s-a)^2-\omega^2}$	$e^{at}\cosh\omega t$	$\dfrac{s-a}{(s-a)^2-\omega^2}$
$t\sinh\omega t$	$\dfrac{2\omega s}{\left(s^2-\omega^2\right)^2}$	$t\cosh\omega t$	$\dfrac{s^2-\omega^2}{\left(s^2-\omega^2\right)^2}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$f(t)$	$F(s) = \int_0^\infty f(t)e^{-st}dt$
$tf(t)$	$-F'(s)$	$e^{ct}f(t)$	$F(s-c)$
$\dfrac{f(t)}{t}$	$\int_s^\infty F(s)ds$	$f'(t)$	$sF(s) - f(0)$
$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$

$\dfrac{(d-a)e^{-at}}{(b-a)(c-a)} + \dfrac{(d-b)e^{-bt}}{(c-b)(a-b)} + \dfrac{(d-c)e^{-ct}}{(a-c)(b-c)}$	$\dfrac{s+d}{(s+a)(s+b)(s+c)}$

$$\mathcal{L}\left\{f^{(n)}(t)\right\}(s) = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-b)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)} \longrightarrow \frac{}{(s+a)(s+b)(s+c)}$$

$$\frac{(\alpha-a)e^{-at}}{(b-a)(c-a)}+\frac{(\alpha-b)e^{-bt}}{(c-b)(a-b)}+\frac{(\alpha-c)e^{-ct}}{(a-c)(b-c)} \qquad \frac{s+\alpha}{(s+a)(s+b)(s+c)}$$

$$\frac{\alpha_0}{ab}+\frac{a^2-\alpha_1 a+\alpha_0}{a(a-b)}e^{-at}-\frac{b^2-\alpha_1 b+\alpha_0}{b(a-b)}e^{-bt} \qquad \frac{s^2+\alpha_1 s+\alpha_0}{s(s+a)(s+b)}$$

$$\frac{a_0}{c^2}+\frac{1}{bc}\sqrt{(a^2-b^2-\alpha_1 a+\alpha_0)^2+b^2(\alpha_1-2a)^2}\;e^{-at}\sin(bt+\phi)$$

$$\begin{cases}\phi = \arctan 2[b(\alpha_1-2a),\, a^2-b^2-\alpha_1 a+a+\alpha_0]-\arctan 2(b,-a)\\ c^2=a^2+b^2\end{cases}$$

$$\longrightarrow \frac{s^2+\alpha_1 s+\alpha_0}{s[(s+a)^2+b^2]}$$

$$\frac{\frac{1}{\omega}\sin(\omega t+\varphi_1)+\frac{1}{b}e^{-at}\sin(bt+\varphi_2)}{\sqrt{4a^2\omega^2+(a^2+b^2-\omega^2)^2}} \longrightarrow \frac{1}{(s^2+\omega^2)[(s+a)^2+b^2]}$$

$$\varphi_1 = \operatorname{atan} 2(-2a\omega,\ a^2+b^2-\omega^2)$$

$$\varphi_2 = \operatorname{atan} 2(2ab,\ a^2-b^2+\omega^2)$$

$$\frac{1}{\omega}\sqrt{\frac{\alpha^2+\omega^2}{c}}\sin(\omega t+\varphi_1)+\frac{1}{b}\sqrt{\frac{(\alpha-a)^2+b^2}{c}}\,e^{-at}\sin(bt+\varphi_2)$$

$$c=(2a\omega)^2+(a^2+b^2-\omega^2)^2 \qquad \longrightarrow \frac{s+\alpha}{(s^2+\omega^2)[(s+a)^2+b^2]}$$

$$\varphi_1 = \operatorname{atan} 2(\omega,\alpha)-\operatorname{atan} 2(2a\omega,\, a^2+b^2+\omega^2)$$

$$\varphi_2 = \operatorname{atan} 2(b,\alpha-a)+\operatorname{atan} 2(2ab,\, a^2-b^2-\omega^2)$$

$$\frac{s+\alpha}{s^2[(s+a)^2+b^2]} \longrightarrow \frac{1}{c}\left(\alpha t+1-\frac{2\alpha a}{c}\right)+\frac{\sqrt{b^2+(\alpha-a)^2}}{bc}e^{-at}\sin(bt+\phi)$$

$$\begin{cases}c=a^2+b^2\\ \phi = 2\operatorname{atan} 2(b,a)+\operatorname{atan} 2(b,\alpha-a)\end{cases}$$

$$\frac{s^2+\alpha_1 s+\alpha_0}{s^2(s+a)(s+b)} \longrightarrow \frac{\alpha_1+\alpha_0 t}{ab}-\frac{\alpha_0(a+b)}{(ab)^2}-\frac{1}{a-b}\left(1-\frac{\alpha_1}{a}+\frac{\alpha_0}{a^2}\right)e^{-at}$$

$$-\frac{1}{b-a}\left(1-\frac{\alpha_1}{b}+\frac{\alpha_0}{b^2}\right)e^{-bt}$$

$$\frac{s+\alpha}{s^2+2\zeta\omega_n s+\omega_n^2} \rightarrow \sqrt{\frac{\left(\frac{\alpha}{\omega_0}-\zeta\omega_n\right)^2}{1-\zeta^2}+1}\; e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$$

$$\phi = \operatorname{atan2}(\omega_n\sqrt{1-\zeta^2},\ \alpha-\zeta\omega_n)$$

$$\frac{1}{s[(s+a)^2+b^2]} \longrightarrow \frac{1}{a^2+b^2}+\frac{1}{b\sqrt{a^2+b^2}}\, e^{-at}\sin(bt-\phi)$$

$$\phi = \operatorname{atan2}(b,-a)$$

$$\frac{1}{s(s^2+2\zeta\omega_n s+\omega_n^2)} \rightarrow \frac{1}{\omega_n^2}-\frac{1}{\omega_n^2\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$$

$$\phi = \cos^{-1}\zeta$$

$$\frac{s+\alpha}{s[(s+a)^2+b^2]} \longrightarrow \frac{\alpha}{a^2+b^2}+\frac{1}{b}\sqrt{\frac{(\alpha-a)^2+b^2}{a^2+b^2}}\, e^{-at}\sin(bt+\phi)$$

$$\phi = \operatorname{atan2}(b,\alpha-a)-\operatorname{atan2}(b,-a)$$

$$\frac{s+\alpha}{s(s^2+2\zeta\omega_n s+\omega_n^2)} \rightarrow \frac{\alpha}{\omega_n^2}+\frac{1}{\omega_n\sqrt{1-\zeta^2}}\sqrt{\left(\frac{\alpha}{\omega_n}-\zeta\right)^2+(1-\zeta^2)}\cdot$$

$$e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$$

$$\phi = \operatorname{atan2}(\omega_n\sqrt{1-\zeta^2},\ \alpha-\omega_n\zeta)$$
$$-\operatorname{atan2}(\sqrt{1-\zeta^2},\ -\zeta)$$

$$\frac{1}{(s+c)[(s+a)^2+b^2]} \rightarrow \frac{e^{-ct}}{(c-a)^2+b^2}+\frac{e^{-at}\sin(bt-\phi)}{b\sqrt{(c-a)^2+b^2}}$$

$$\phi = \operatorname{atan2}(b,c-a).$$