2 Area between Curves

$$\text{Area} = \int_a^b \left(f(x) - g(x)\right)dx$$

Ex Find A? Given $y = 2 - x^2$ $y = -x$

$$y = 2 - x^2 = -x$$

$$x^2 - x - 2 = 0$$

$$x = -1,\ 2$$

$a - b + c = 0$ $x = 1, -c/a$

$a + b + c = 0$ $x = 1, c/a$

$$A = \int_{-1}^{2} (2 - x^2 + x)\,dx$$

$$= 2x - \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_{-1}^{2}$$

$$= 4 - \frac{8}{3} + 2 - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$$

$$= 8 - 3 - \frac{1}{2}$$

$$= \frac{9}{2} \text{ unit}^2$$

$-\frac{1}{2}$

[illegible]

$\therefore$ Area $= \int$ [illegible] $dx + \int$ [illegible] dx

$=$ [illegible]

$=$ [illegible] unit^2

$A = \int$ [illegible] dy

$=$ [illegible]

$=$ [illegible] unit^2

Ex: $y = 7 - 2x^2 = x^2 + 4$

$$3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow \underline{x = \pm 1}$$

$$A = \int_{-1}^{1} (7 - 2x^2 - x^2 - 4)\,dx$$

$$= \int_{-1}^{1} (3 - 3x^2)\,dx \qquad 2\left[3x - x^2\right|_0^1$$

$$= 3x - x^3 \Big|_{-1}^{1}$$

$$= 3 - 1 - (-3 + 1)$$

$$= \underline{4 \text{ unit}^2}$$

5/ $x = y^3 - y^2 \; : \; x = 2y$

$$y^3 - y^2 - 2y = 0 \qquad \underline{y = 0, -1, 2}$$

$$y^2 - y - 2 = 0 \qquad + \int_0^2 (2y - y^3 + y^2)\,dy$$

$$A = \int_{-1}^{0} (y^3 - y^2 - 2y)\,dy - \int_{0}^{2} (y^3 - y^2 - 2y)\,dy$$

$$= \frac{1}{4}y^4 - \frac{1}{3}y^3 - y^2 \Big|_{-1}^{0} - \left(\frac{1}{4}y^4 - \frac{1}{3}y^3 - y^2 \Big|_{0}^{2}\right.$$

$$= -\left(\frac{1}{4} + \frac{1}{3} - 1\right) - \left(4 - \frac{8}{3} - 4\right)$$

$$= -\frac{7}{12} + 1 + \frac{8}{3}$$

$$= \frac{-7 + 12 + 32}{12}$$

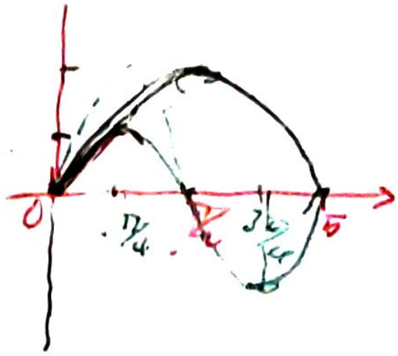
$$= \frac{37}{12} \text{ unit}^2$$

3/ $y = 2\sin x \qquad y = \sin 2x \qquad 0 \le x \le \pi$

$2\sin x = 2\sin x\cos x$

$\cos x = 1 \Rightarrow x = 0.$

$\sin x = 0 \Rightarrow x = 0, \pi$



$A = \int_0^{\pi} (2\sin x - \sin 2x)\,dx$

$= -2\cos x + \frac{1}{2}\cos 2x \Big|_0^{\pi}$

$= 2 + \frac{1}{2} - \left(-2 + \frac{1}{2}\right)$

$= 4$ unit^2

36/ $f(x) = 2\sin x \qquad g(x) = \tan x \qquad -\frac{\pi}{3} \le x \le \frac{\pi}{3}$

$2\sin x = \tan x = \frac{\sin x}{\cos x}$

$\begin{cases} \sin x = 0 \\ 2 = \frac{1}{\cos x} \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3} \end{cases}$

$A = \int_{-\pi/3}^{0} (\tan x - 2\sin x)\,dx + \int_0^{\pi/3} (2\sin x - \tan x)\,dx$

$-\ln|\cos x| \qquad\qquad +\ln|\cos x|$

$= \ln|\sec x| + 2\cos x \Big|_{-\pi/3}^{0} + \left(-2\cos x - \ln|\sec x|\right)\Big|_0^{\pi/3}$

$= 2 + \ln\frac{1}{2} - 1 + \left(-1 + \ln\frac{1}{2} + 2\right)$

$= 2 + 2\ln\frac{1}{2}$ unit^2 $\qquad \ln\frac{1}{2} = -\ln 2$

$= 2(1 - \ln 2)$