5.5

#1 $2^x = 128 \Rightarrow x = \log_2 128$

$2^x = 2^7$ $= \log_2 2^7$

$x = 7$ $= 7$

#12 $7^x = 12 \Rightarrow x = \log_7 12$

$a^{f(x)} = b^{g(x)}$

Eg $3^{2x-1} = 7^{x+1}$ Solve

$\ln 3^{2x-1} = \ln 7^{x+1}$

$(2x-1)\ln 3 = (x+1)\ln 7$

$2x\ln 3 - \ln 3 = x\ln 7 + \ln 7$

$2x\ln 3 - x\ln 7 = \ln 3 + \ln 7$

$x(2\ln 3 - \ln 7) = \ln 3 + \ln 7$

$x = \dfrac{\ln 3 + \ln 7}{2\ln 3 - \ln 7} = \dfrac{\ln 21}{\ln 9 - \ln 7} = \dfrac{\ln 21}{\ln \frac{9}{7}}$

$= \log_{9/7} 21$

Ex $7e^{2x} - 5 = 58$ solve x?

$7e^{2x} = 63$

$e^{2x} = 9 \rightarrow 2x = \ln 3^2$

$x = \frac{2}{2}\ln 3$

$= \ln 3$

$\log_b M = c \Rightarrow M = b^c$

Check the variable inside $\log > 0$

Ex $\log x + \log(x-3) = 1$

$\log x(x-3) = 1$

$x^2 - 3x = 10^1$

$x^2 - 3x - 10 = 0$

$x = -2$ # $\quad x = 5$

Ex $\log_6(3x+2) + \log_6(x-1) = 1$

$\log_6(3x+2)(x-1) = 1$

$3x^2 - x - 2 = 6^1$

$3x^2 - x - 8 = 0$

$x = \frac{1 \pm \sqrt{1+96}}{6}$

$x = \frac{1-\sqrt{96}}{6} < 0$ # $\quad x = \frac{1+\sqrt{96}}{6}$ ✓

$\log_b M = \log_b N \Rightarrow M = N$

Ex Solve: $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

$$\ln(x-3) = \ln\left(\frac{7x-23}{x+1}\right)$$

$$x-3 = \frac{7x-23}{x+1}$$

$$x^2 - 2x - 3 = 7x - 23$$

$$x^2 - 9x + 20 = 0$$

$$x = 4, 5$$ ✓ ✳

Ex Solve: $\log(x+6) - \log(x+2) = \log x$

$$\log\frac{x+6}{x+2} = \log x$$

$$\frac{x+6}{x+2} = x$$

$$x+6 = x^2 + 2x$$

$$x^2 + x - 6 = 0$$

$$x = \cancel{-3}, 2$$

#23 $5^{x+4} = 4^{x+5}$

$\ln 5^{x+4} = \ln 4^{x+5}$

$(x+4)\ln 5 = (x+5)\ln 4$

$x\ln 5 + 4\ln 5 = x\ln 4 + 5\ln 4$

$x\ln 5 - x\ln 4 = 5\ln 4 - 4\ln 5$

$x(\ln 5 - \ln 4) = 5\ln 4 - 4\ln 5$

$$x = \frac{5\ln 4 - 4\ln 5}{\ln 5 - \ln 4}$$

$ax = b$

$x = \frac{b}{a}$

#69 $\log_2(x+1) + \log_2(x-1) = 3$

$\log_2(x+1)(x-1) = 3$

$x^2 - 1 = 2^3 = 8$

$x^2 = 9$

$x = \cancel{-3}, 3$

#101 $\ln(x-5) - \ln(x+4) = \ln(x-1) - \ln(x+2)$

$$\ln\frac{x-5}{x+4} = \ln\frac{x-1}{x+2}$$

$\frac{x-5}{x+4} = \frac{x-1}{x+2}$

$(x-5)(x+2) = (x-1)(x+4)$

$x^2 - 3x - 10 = x^2 + 3x - 4$

$-6x = 6$

$x = -1$

No soln.

105

$\log_2(x+3) = \log_2(x-3) + \log_3 9 + 4\underbrace{\log_4 3}_{\frac{1}{2}}$

$\log_2(x+3) - \log_2(x-3) = \log_3 3^2 + 3$

$\log_2 \frac{x+3}{x-3} = 5$

$\frac{x+3}{x-3} = 2^5 = 32$

$x+3 = 32x - 96$

$99 = 31x$

$x = \frac{99}{31}$ (>3)

3.6 $A(t) = A_0 e^{kt}$

exponential fctn e^t ← e^{kt} → initial Value

k : Exponential Rate

$k > 0$: Growth

$k < 0$: Decay

Ex 1980 → 643M $t = 0 \to A_0$

2000 → 813M $t = 10 \to A(10)$

$A(t) = 643 e^{kt}$

$kt = \ln \frac{A}{A_0}$

$k = \frac{1}{10} \ln \frac{813}{643}$

$A(t) = 643^{(\ln \frac{813}{643})t/10}$

$kT = \ln \frac{A}{A_0}$

$A_0 \to 2A_0$

$\frac{2A_0}{A_0} = 0$

Doubling: $kT = \ln 2$

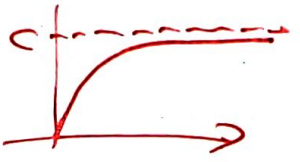
Half-life: $kT = \ln \frac{1}{2} = -\ln 2$

Ex $T_{1/2} = 25$

$kT = \ln \frac{1}{2}$ $\ln 1 - \ln 2$

$k = \frac{-1}{25} \ln 2$

Logistics $P(t) = \frac{C}{1 + ae^{-bt}}$



$A = P\left(1 + \frac{r}{n}\right)^{nt}$ daily: 360

$A = Pe^{rt}$

Review

#1 a) $\{(2,1), (-2,3), (3,4), (-3,2), (1,-5)\} = G$

$G^{-1} = \{(1,2), (3,-2), (4,3), (2,-3), (5,1)\}$

#2 b) $f(x) = \frac{x+4}{x-3}$ is 1-1

a) $f^{-1}(x)$?

$y = \frac{x+4}{x-3}$

$x = \frac{y+4}{y-3}$

$xy - 3x = y + 4$

$xy - y = 3x + 4$

$(x-1)y = 3x + 4$

$y = \frac{3x+4}{x-1} = f^{-1}(x)$

b) Domain $f(x)$ = Range $f^{-1}(x)$: $\mathbb{R} - \{3\}$

" $f^{-1}(x)$ = " $f(x)$: $\mathbb{R} - \{1\}$

#3 skip

#4 a) $5^{-3} = \frac{1}{125} \iff -3 = \log_5 \frac{1}{125}$

b) $4^{2y} = 24.5 \iff 2y = \log_4 24.5$

#5 a) $6 = \log_2 64 \iff 64 = 2^6$

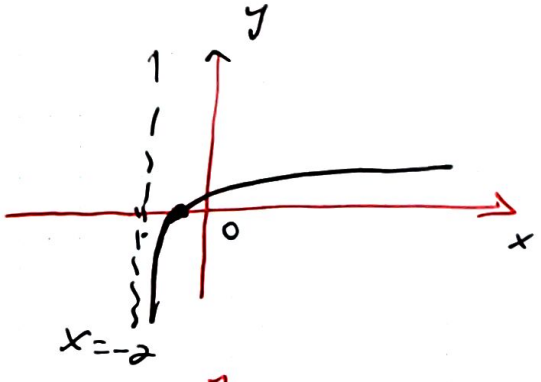
b) $2 = \log_3 x \iff x = 3^2$

#6 a) $f(x) = \log(x+2)$

Asymptote: $x = -2$

Domain: $x > -2$

Range: $\mathbb{R}$.

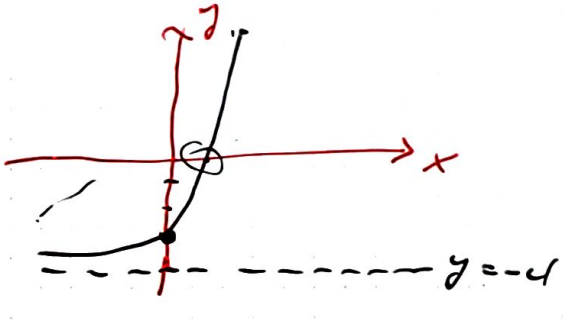


d) $f(x) = e^{2x} - 4$

Asymptote: $y = -4$

Domain: $\mathbb{R}$

Range: $(-4, \infty)$



0	-3
1	$e^2 - 4$

$e^{2x} - 4 = 0 \Rightarrow e^{2x} = 4$

$2x = \ln 4 = \ln 2^2$

$= 2\ln 2$

$x = \ln 2$

#7

		Asymptote	Domain	Range
7.a	$f(x) = 2 + \ln(2x-4)$	$x = 2$	$x > 2$	$\mathbb{R}$
7.c	$f(x) = 2 - 3^{x+1}$	$y = 2$	$\mathbb{R}$	$y < 2$

#8 b) $\log \dfrac{x^3 y^2}{\sqrt[3]{(z+1)^2}} = \log x^3 y^2 - \log (z+1)^{2/3}$

$= \log x^3 + \log y^2 - \dfrac{2}{3} \log (z+1)$

$= 3\log x + 2\log y - \dfrac{2}{3} \log (z+1)$