Solving Exponential Function with different bases

$$a^{mx+n} = b^{px+q} \implies x = \frac{q \ln b - n \ln a}{m \ln a - p \ln b}$$
 coefficient $\frac{no \ x's}{x's}$

Numerator: multiply q with $\ln b$ minus multiply n with $\ln a$ multiply n with $\ln a$ multiply p with $\ln b$

Proof

$$\ln a^{mx+n} = \ln b^{px+q}$$

$$(mx+n)\ln a = (px+q)\ln b$$

$$mx\ln a + n\ln a = px\ln b + q\ln b$$

$$mx\ln a - px\ln b = q\ln b - n\ln a$$

$$x(m\ln a - p\ln b) = q\ln b - n\ln a$$

$$x = \frac{q\ln b - n\ln a}{m\ln a - p\ln b}$$

 $mx \ln a + n \ln a = px \ln b + q \ln b$

Example

Solve:
$$\frac{2x-1}{3} = \frac{x+1}{7}$$

Solution

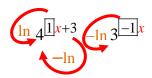
$$x = \frac{\ln 3 + \ln 7}{2\ln 3 - \ln 7}$$

Example

Solve:
$$4^{x+3} = 3^{-x}$$

Solution

$$x = \frac{-3\ln 4}{\ln 4 + \ln 3}$$



Example

Solve:
$$4^{-x} = 3^{x+3}$$

Solution

$$x = \frac{3\ln 3}{\ln 3 - \ln 4}$$

$$\underbrace{\ln 4^{-1}x}_{\ln 3}
\underbrace{-\ln 3^{1}x+3}_{\ln 3}$$