

## ***Solution***      **Section 5.3 – Trigonometric Functions**

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(-2, 3)$  is on the terminal side of  $\theta$ .

### **Solution**

$$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

$$\sin \theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$$

$$\cos \theta = \frac{x}{r} = -\frac{2}{\sqrt{13}}$$

$$\tan \theta = \frac{y}{x} = -\frac{3}{2}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{r}{x} = -\frac{\sqrt{13}}{2}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

$$\cot \theta = \frac{x}{y} = -\frac{2}{3}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(-3, -4)$  is on the terminal side of  $\theta$ .

### **Solution**

$$3, 4 \rightarrow 5$$

$$\sin \theta = -\frac{4}{5}$$

$$\cos \theta = -\frac{3}{5}$$

$$\tan \theta = \frac{-4}{-3} = \frac{4}{3}$$

$$\csc \theta = -\frac{5}{4}$$

$$\sec \theta = -\frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  in standard position with terminal side through the point  $(-3, 0)$ .

### **Solution**

$$r = \sqrt{(-3)^2 + 0^2} = 3$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin \theta = \frac{0}{3} = 0$$

$$\cos \theta = \frac{-3}{3} = -1$$

$$\tan \theta = \frac{0}{-3} = 0$$

$$\csc \theta = \frac{1}{0} \rightarrow \infty$$

$$\sec \theta = \frac{1}{-1} = -1$$

$$\cot \theta = \frac{1}{0} = \infty$$

### Exercise

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(12, -5)$  is on the terminal side of  $\theta$ .

#### Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{12^2 + (-5)^2} = \underline{13}$$

$$\sin \theta = -\frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$\tan \theta = -\frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = \frac{13}{12}$$

$$\cot \theta = -\frac{12}{5}$$

### Exercise

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(5, -12)$  is on the terminal side of  $\theta$ .

#### Solution

$$5 \quad 12 \rightarrow 13$$

$$\sin \theta = -\frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = -\frac{12}{5}$$

$$\csc \theta = -\frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = -\frac{5}{12}$$

### Exercise

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(9, -12)$  is on the terminal side of  $\theta$ .

#### Solution

$$(9, -12) = 3(3, -4) \Rightarrow 3 \quad 4 \rightarrow 5$$

$$\sin \theta = -\frac{4}{5}$$

$$\cos \theta = \frac{3}{5}$$

$$\tan \theta = -\frac{4}{3}$$

$$\csc \theta = -\frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = -\frac{3}{4}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(16, -12)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$(16, -12) = 4(4, -3) \Rightarrow 4 \quad 3 \rightarrow 5$$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = -\frac{3}{4}$$

$$\csc \theta = -\frac{5}{3}$$

$$\sec \theta = \frac{5}{4}$$

$$\cot \theta = -\frac{4}{3}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(15, -8)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$15 \quad 8 \rightarrow 17$$

$$\sin \theta = -\frac{8}{17}$$

$$\cos \theta = \frac{15}{17}$$

$$\tan \theta = -\frac{8}{15}$$

$$\csc \theta = -\frac{17}{8}$$

$$\sec \theta = \frac{17}{15}$$

$$\cot \theta = -\frac{15}{8}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(-6, 8)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$(-6, 8) = 2(-3, 4) \Rightarrow 3 \quad 4 \rightarrow 5$$

$$\sin \theta = \frac{4}{5}$$

$$\cos \theta = -\frac{3}{5}$$

$$\tan \theta = -\frac{4}{3}$$

$$\csc \theta = \frac{5}{4}$$

$$\sec \theta = -\frac{5}{3}$$

$$\cot \theta = -\frac{3}{4}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(-15, 8)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$15 \quad 8 \rightarrow 17$$

$$\sin \theta = \frac{8}{17}$$

$$\cos \theta = -\frac{15}{17}$$

$$\tan \theta = -\frac{8}{15}$$

$$\csc \theta = \frac{17}{8}$$

$$\sec \theta = -\frac{17}{15}$$

$$\cot \theta = -\frac{15}{8}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(-7, 24)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$7 \quad 24 \rightarrow 25$$

$$\sin \theta = \frac{24}{25}$$

$$\cos \theta = -\frac{7}{25}$$

$$\tan \theta = -\frac{24}{7}$$

$$\csc \theta = \frac{25}{24}$$

$$\sec \theta = -\frac{25}{7}$$

$$\cot \theta = -\frac{7}{24}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(10, -24)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$(10, -24) = 2(5, -12) \Rightarrow 5 \quad 12 \rightarrow 13$$

$$\sin \theta = -\frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = -\frac{12}{5}$$

$$\csc \theta = -\frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = -\frac{5}{12}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(7, 24)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$7 \quad 24 \rightarrow 25$$

$$\sin \theta = \frac{24}{25}$$

$$\cos \theta = \frac{7}{25}$$

$$\tan \theta = \frac{24}{7}$$

$$\csc \theta = \frac{25}{24}$$

$$\sec \theta = \frac{25}{7}$$

$$\cot \theta = \frac{7}{24}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(-7, -24)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$7 \quad 24 \rightarrow 25$$

$$\sin \theta = -\frac{24}{25}$$

$$\cos \theta = -\frac{7}{25}$$

$$\tan \theta = \frac{24}{7}$$

$$\csc \theta = -\frac{25}{24}$$

$$\sec \theta = -\frac{25}{7}$$

$$\cot \theta = \frac{7}{24}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(-24, -7)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$24 \quad 7 \rightarrow 25$$

$$\sin \theta = -\frac{7}{25}$$

$$\cos \theta = -\frac{24}{25}$$

$$\tan \theta = \frac{7}{24}$$

$$\csc \theta = -\frac{25}{7}$$

$$\sec \theta = -\frac{25}{24}$$

$$\cot \theta = \frac{24}{7}$$

### ***Exercise***

Find the six trigonometry functions of  $\theta$  if  $\theta$  is in the standard position and the point  $(24, -10)$  is on the terminal side of  $\theta$ .

#### **Solution**

$$(24, -10) = 2(12, -5) \Rightarrow 12 \ 5 \rightarrow 13$$

$$\sin \theta = -\frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$\tan \theta = -\frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = \frac{13}{12}$$

$$\cot \theta = -\frac{12}{5}$$

### ***Exercise***

Find the values of the six trigonometric functions for an angle of  $90^\circ$ .

#### **Solution**

$$\sin 90^\circ = 1$$

$$\tan 90^\circ = \infty$$

$$\csc 90^\circ = 1$$

$$\cos 90^\circ = 0$$

$$\cot 90^\circ = 0$$

$$\sec 90^\circ = \infty$$

### ***Exercise***

Indicate the two quadrants  $\theta$  could terminate in if  $\cos \theta = \frac{1}{2}$

#### **Solution**

$$\cos \theta = \frac{1}{2} \rightarrow \text{QI \& QIV}$$

### ***Exercise***

Indicate the two quadrants  $\theta$  could terminate in if  $\csc \theta = -2.45$

#### **Solution**

$$\csc \theta = -2.45$$

$$= \frac{1}{\sin \theta} \rightarrow \text{QIII \& QIV}$$

### Exercise

Find the remaining trigonometric function of  $\theta$  if  $\sin \theta = \frac{12}{13}$  and  $\theta$  terminates in **QI**

#### Solution

$$5 \quad 12 \rightarrow 13$$

$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = \frac{12}{5}$$

$$\csc \theta = \frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = \frac{5}{12}$$

### Exercise

Find the remaining trigonometric function of  $\theta$  if  $\cot \theta = -2$  and  $\theta$  terminates in **QII**.

#### Solution

$$\cot \theta = -2 = \frac{x}{y} \quad (\theta \in \text{QII})$$

$$x = -2, \quad y = 1$$

$$r = \sqrt{(-2)^2 + (1)^2} \\ = \sqrt{5}$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin \theta = \frac{1}{\sqrt{5}}$$

$$\cos \theta = -\frac{2}{\sqrt{5}}$$

$$\tan \theta = -\frac{1}{2}$$

$$\csc \theta = \sqrt{5}$$

$$\sec \theta = -\frac{\sqrt{5}}{2}$$

### Exercise

Find the remaining trigonometric function of  $\theta$  if  $\tan \theta = \frac{3}{4}$  and  $\theta$  terminates in **QIII**.

#### Solution

$$\tan \theta = \frac{3}{4} = \frac{y}{x} \quad (\theta \in \text{QIII})$$

$$x = -4, \quad y = -3$$

$$4 \quad 3 \rightarrow 5$$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = -\frac{4}{5}$$

$$\csc \theta = -\frac{5}{3}$$

$$\sec \theta = -\frac{5}{4}$$

$$\cot \theta = \frac{4}{3}$$

### Exercise

Find the remaining trigonometric function of  $\theta$  if  $\cos \theta = \frac{24}{25}$  and  $\theta$  terminates in **QIV**.

#### Solution

$$24 \quad 7 \rightarrow 25$$

$$\cos \theta = \frac{24}{25} \quad \theta \in QIV \Rightarrow y = -7$$

$$\sin \theta = -\frac{7}{25}$$

$$\cos \theta = \frac{24}{25}$$

$$\tan \theta = -\frac{7}{24}$$

$$\csc \theta = -\frac{25}{7}$$

$$\sec \theta = \frac{25}{24}$$

$$\cot \theta = -\frac{24}{7}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = \frac{\sqrt{3}}{2}$  and  $\theta$  is terminates in **QIV**.

#### Solution

$$\cos \theta = \frac{\sqrt{3}}{2} = \frac{x}{r} \Rightarrow x = \sqrt{3}, \quad r = 2$$

Since  $\theta$  is **QIV**

$$y = -\sqrt{2^2 - \sqrt{3}^2}$$

$$= -\sqrt{4 - 3}$$

$$= -1$$

$$\sqrt{3} \quad 1 \rightarrow 2$$

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{1}{\sqrt{3}}$$

$$\csc \theta = -2$$

$$\sec \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot \theta = -\sqrt{3}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\tan \theta = -\frac{1}{2}$  and  $\cos \theta > 0$ .

#### Solution

$$\tan \theta = \frac{\sin \theta}{\cos \theta} < 0 \quad \& \quad \cos \theta > 0$$

$$\sin \theta < 0 \Rightarrow \theta \text{ in } QIV$$



$$\Rightarrow y=1, \ x=2$$

$$r = \sqrt{1^2 + 2^2} \\ = \sqrt{5}$$

$$2 \quad -1 \rightarrow \sqrt{5}$$

$$\sin \theta = -\frac{1}{\sqrt{5}}$$

$$\cos \theta = \frac{2}{\sqrt{5}}$$

$$\tan \theta = -\frac{1}{2}$$

$$\csc \theta = -\frac{\sqrt{5}}{5}$$

$$\sec \theta = \frac{\sqrt{5}}{2}$$

$$\cot \theta = -2$$

### ***Exercise***

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = \frac{3}{5}$  &  $\theta \in QI$

#### **Solution**

$$3 \quad 4 \rightarrow 5$$

$$\sin \theta = \frac{4}{5}$$

$$\cos \theta = \frac{3}{5}$$

$$\tan \theta = \frac{4}{3}$$

$$\csc \theta = \frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

### ***Exercise***

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = -\frac{4}{5}$  &  $\theta \in QII$

#### **Solution**

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-4 \quad 3 \rightarrow 5$$

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = -\frac{4}{5}$$

$$\tan \theta = -\frac{3}{4}$$

$$\csc \theta = \frac{5}{3}$$

$$\sec \theta = -\frac{5}{4}$$

$$\cot \theta = -\frac{4}{3}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\sin \theta = -\frac{3}{5}$  &  $\theta \in QIII$

#### Solution

$$\theta \in QIII \quad \& \quad \cos \theta < 0$$

$$\text{red } -4 \quad \text{blue } -3 \rightarrow \text{blue } 5$$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = -\frac{4}{5}$$

$$\tan \theta = \frac{3}{4}$$

$$\csc \theta = -\frac{5}{3}$$

$$\sec \theta = -\frac{5}{4}$$

$$\cot \theta = \frac{4}{3}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\sin \theta = -\frac{3}{5}$  &  $\theta \in QIV$

#### Solution

$$\theta \in QIV \quad \& \quad \cos \theta > 0$$

$$\text{red } 4 \quad \text{blue } -3 \rightarrow \text{blue } 5$$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = -\frac{3}{4}$$

$$\csc \theta = -\frac{5}{3}$$

$$\sec \theta = \frac{5}{4}$$

$$\cot \theta = -\frac{4}{3}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = -\frac{12}{13}$  &  $\theta \in QIII$

#### Solution

$$\theta \in QIII \quad \& \quad \sin \theta < 0$$

$$\text{red } -12 \quad \text{blue } -5 \rightarrow \text{blue } 13$$

$$\sin \theta = -\frac{5}{13}$$

$$\cos \theta = -\frac{12}{13}$$

$$\tan \theta = \frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = -\frac{13}{12}$$

$$\cot \theta = \frac{12}{5}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = -\frac{5}{13}$  &  $\theta \in QII$

#### Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-5 \quad 12 \rightarrow 13$$

$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = -\frac{5}{13}$$

$$\tan \theta = -\frac{12}{5}$$

$$\csc \theta = \frac{13}{12}$$

$$\sec \theta = -\frac{13}{5}$$

$$\cot \theta = -\frac{5}{12}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = \frac{12}{13}$  &  $\theta \in QIV$

#### Solution

$$\theta \in QIV \quad \& \quad \sin \theta < 0$$

$$12 \quad -5 \rightarrow 13$$

$$\sin \theta = -\frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$\tan \theta = -\frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = \frac{13}{12}$$

$$\cot \theta = -\frac{12}{5}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\sin \theta = -\frac{8}{17}$  &  $\theta \in QIII$

#### Solution

$$\theta \in QIII \quad \& \quad \cos \theta < 0$$

$$-15 \quad -8 \rightarrow 17$$

$$\sin \theta = -\frac{8}{17}$$

$$\cos \theta = -\frac{15}{17}$$

$$\tan \theta = \frac{8}{15}$$

$$\csc \theta = -\frac{17}{8}$$

$$\sec \theta = -\frac{17}{15}$$

$$\cot \theta = \frac{15}{8}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = -\frac{15}{17}$  &  $\theta \in QII$

#### Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-15 \quad 8 \rightarrow 17$$

$$\sin \theta = \frac{8}{17}$$

$$\cos \theta = -\frac{15}{17}$$

$$\tan \theta = -\frac{8}{15}$$

$$\csc \theta = \frac{17}{8}$$

$$\sec \theta = -\frac{17}{15}$$

$$\cot \theta = -\frac{15}{8}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = -\frac{8}{17}$  &  $\theta \in QII$

#### Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-8 \quad 15 \rightarrow 17$$

$$\sin \theta = \frac{15}{17}$$

$$\cos \theta = -\frac{8}{17}$$

$$\tan \theta = -\frac{15}{8}$$

$$\csc \theta = \frac{17}{15}$$

$$\sec \theta = -\frac{17}{8}$$

$$\cot \theta = -\frac{8}{15}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\cos \theta = -\frac{7}{25}$  &  $\theta \in QII$

#### Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-7 \quad 24 \rightarrow 25$$

$$\sin \theta = \frac{24}{25}$$

$$\cos \theta = -\frac{7}{25}$$

$$\tan \theta = -\frac{24}{7}$$

$$\csc \theta = \frac{25}{24}$$

$$\sec \theta = -\frac{25}{7}$$

$$\cot \theta = -\frac{7}{24}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\sin \theta = -\frac{7}{25}$  &  $\theta \in QIII$

#### Solution

$$\theta \in QIII \quad \& \quad \cos \theta < 0$$

$$\text{red } -24 \quad \text{blue } -7 \quad \rightarrow \quad \text{blue } 25$$

$$\sin \theta = -\frac{7}{25} \qquad \cos \theta = -\frac{24}{25} \qquad \tan \theta = \frac{7}{24}$$

$$\csc \theta = -\frac{25}{7} \qquad \sec \theta = -\frac{25}{24} \qquad \cot \theta = \frac{24}{7}$$

### Exercise

Find the remaining trigonometric functions of  $\theta$  if  $\sin \theta = -\frac{24}{25}$  &  $\theta \in QIV$

#### Solution

$$\theta \in QIV \quad \& \quad \cos \theta > 0$$

$$\text{red } 7 \quad \text{blue } -24 \quad \rightarrow \quad \text{blue } 25$$

$$\sin \theta = -\frac{24}{25} \qquad \cos \theta = \frac{7}{25} \qquad \tan \theta = -\frac{24}{7}$$

$$\csc \theta = -\frac{25}{24} \qquad \sec \theta = \frac{25}{7} \qquad \cot \theta = -\frac{7}{24}$$

### Exercise

If  $\sin \theta = -\frac{5}{13}$ , and  $\theta$  is  $QIII$ , find  $\cos \theta$  and  $\tan \theta$ .

#### Solution

$$\sin \theta = -\frac{5}{13} = \frac{y}{r} \rightarrow y = -5, \quad r = 13$$

$$\Rightarrow x = \pm \sqrt{13^2 - 5^2} = \pm 12 \quad \text{Since } \theta \text{ is } QIII \Rightarrow x = -12 \qquad x = \pm \sqrt{r^2 - y^2}$$

$$\cos \theta = -\frac{12}{13}$$

$$\tan \theta = \frac{5}{12}$$

### ***Exercise***

If  $\cos \theta = \frac{3}{5}$ , and  $\theta$  is *QIV*, find  $\sin \theta$  and  $\tan \theta$ .

#### **Solution**

$$\cos \theta = \frac{3}{5} = \frac{x}{r} \quad (\theta \in QIV) \Rightarrow \boxed{x=3} \quad y = \boxed{-4}$$

$$\sin \theta = \boxed{-\frac{4}{5}}, \quad \tan \theta = \boxed{-\frac{4}{3}}$$

### ***Exercise***

Use the reciprocal identities if  $\cos \theta = \frac{\sqrt{3}}{2}$  find  $\sec \theta$

#### **Solution**

$$\begin{aligned} \sec \theta &= \frac{1}{\cos \theta} \\ &= \frac{2}{\sqrt{3}} \\ &= \boxed{\frac{2\sqrt{3}}{3}} \end{aligned}$$

### ***Exercise***

Find  $\cos \theta$ , given that  $\sec \theta = \frac{5}{3}$

#### **Solution**

$$\begin{aligned} \cos \theta &= \frac{1}{\sec \theta} \\ &= \frac{1}{\frac{5}{3}} \\ &= \boxed{\frac{3}{5}} \end{aligned}$$

### ***Exercise***

Find  $\sin \theta$ , given that  $\csc \theta = -\frac{\sqrt{12}}{2}$

#### **Solution**

$$\begin{aligned} \sin \theta &= \frac{1}{\csc \theta} \\ &= -\frac{2}{\sqrt{12}} \frac{\sqrt{12}}{\sqrt{12}} \end{aligned}$$

$$= -\frac{2\sqrt{12}}{12}$$

$$= -\frac{\sqrt{12}}{6}$$

### ***Exercise***

Use a ratio identity to find  $\tan \theta$  if  $\sin \theta = \frac{3}{5}$  and  $\cos \theta = -\frac{4}{5}$

### **Solution**

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{3}{5}}{-\frac{4}{5}}$$

$$= -\frac{3}{4}$$

### ***Exercise***

If  $\cos \theta = -\frac{1}{2}$  and  $\theta$  terminates in **QII**, find  $\sin \theta$

### **Solution**

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

### ***Exercise***

If  $\sin \theta = \frac{3}{5}$  and  $\theta$  terminated in **QII**, find  $\cos \theta$  and  $\tan \theta$ .

### **Solution**

$$\cos \theta = -\frac{4}{5} \quad (3, 4 \rightarrow 5)$$

$$\tan \theta = -\frac{3}{4}$$

### ***Exercise***

Find  $\tan \theta$  if  $\sin \theta = \frac{1}{3}$  and  $\theta$  terminates in QI

#### **Solution**

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$= \sqrt{1 - \frac{1}{9}}$$

$$= \sqrt{\frac{8}{9}}$$

$$= \frac{2\sqrt{2}}{3}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}$$

$$= \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}$$

### ***Exercise***

Find the remaining trigonometric ratios of  $\theta$ , if  $\sec \theta = -3$  and  $\theta \in QIII$

#### **Solution**

$$\sec \theta = \frac{1}{\cos \theta} = -3$$

$$\cos \theta = -\frac{1}{3}$$

$$\sin \theta = -\sqrt{1 - \cos^2 \theta}$$

$$= -\sqrt{1 - \frac{1}{9}}$$

$$= -\sqrt{\frac{8}{9}}$$

$$= -\frac{2\sqrt{2}}{3}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{2\sqrt{2}}$$



$$= \frac{\sqrt{2}}{4} \Big|$$

$$\csc \theta = -\frac{3}{2\sqrt{2}}$$

$$= -\frac{3\sqrt{2}}{4} \Big|$$

$$\csc \theta = \frac{1}{\sin \theta}$$

## Exercise

Using the calculator and rounding your answer to the nearest hundredth, find the remaining trigonometric ratios of  $\theta$  if  $\csc \theta = -2.45$  and  $\theta \in QIII$

### Solution

$$\sin \theta = \frac{1}{-2.45}$$

$$= -\frac{100}{245}$$

$$= -\frac{20}{49} \Big|$$

$$= -0.41 \Big|$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = -\sqrt{1 - \sin^2 \theta}$$

$$= -\sqrt{1 - .41^2}$$

$$= -0.91 \Big|$$

$$\tan \theta = \frac{-0.41}{-0.91}$$

$$= \frac{41}{91} \Big|$$

$$= 0.45 \Big|$$

$$\cot \theta = \frac{1}{0.45}$$

$$= \frac{100}{45}$$

$$= \frac{20}{9} \Big|$$

$$= 2.22 \Big|$$

$$\sec \theta = \frac{1}{-0.91}$$

$$= -\frac{100}{91} \Big|$$

$$= -1.1 \Big|$$

### ***Exercise***

Write  $\frac{\sec \theta}{\csc \theta}$  in terms of  $\sin \theta$  and  $\cos \theta$ , and then simplify if possible.

### ***Solution***

$$\begin{aligned}\frac{\sec \theta}{\csc \theta} &= \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta}} \\ &= \frac{1}{\cos \theta} \frac{\sin \theta}{1} \\ &= \frac{\sin \theta}{\cos \theta}\end{aligned}$$

### ***Exercise***

Write  $\cot \theta - \csc \theta$  in terms of  $\sin \theta$  and  $\cos \theta$ , and then simplify if possible.

### ***Solution***

$$\begin{aligned}\cot \theta - \csc \theta &= \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} \\ &= \frac{\cos \theta - 1}{\sin \theta}\end{aligned}$$

### ***Exercise***

Write  $\frac{\sin \theta}{\cos \theta} + \frac{1}{\sin \theta}$  in terms of  $\sin \theta$  and/or  $\cos \theta$ , and then simplify if possible.

### ***Solution***

$$\frac{\sin \theta}{\cos \theta} + \frac{1}{\sin \theta} = \frac{\sin^2 \theta + \cos \theta}{\cos \theta \sin \theta}$$

### ***Exercise***

Write  $\sin \theta \cot \theta + \cos \theta$  in terms of  $\sin \theta$  and  $\cos \theta$ , and then simplify if possible.

### ***Solution***

$$\begin{aligned}\sin \theta \cot \theta + \cos \theta &= \sin \theta \frac{\cos \theta}{\sin \theta} + \cos \theta \\ &= \cos \theta + \cos \theta \\ &= 2 \cos \theta\end{aligned}$$

### ***Exercise***

Multiply  $(1 - \cos \theta)(1 + \cos \theta)$

#### **Solution**

$$\begin{aligned}(1 - \cos \theta)(1 + \cos \theta) &= 1 - \cos^2 \theta \\ &= \sin^2 \theta\end{aligned}$$

### ***Exercise***

Multiply  $(\sin \theta + 2)(\sin \theta - 5)$

#### **Solution**

$$(\sin \theta + 2)(\sin \theta - 5) = \sin^2 \theta - 3\sin \theta - 10$$

### ***Exercise***

Simplify the expression  $\sqrt{25 - x^2}$  as much as possible after substituting  $5\sin \theta$  for  $x$ .

#### **Solution**

$$\begin{aligned}\sqrt{25 - x^2} &= \sqrt{25 - (5\sin \theta)^2} \\ &= \sqrt{25 - 25\sin^2 \theta} \\ &= \sqrt{25(1 - \sin^2 \theta)} \\ &= \sqrt{25}\sqrt{\cos^2 \theta} \\ &= 5\cos \theta\end{aligned}$$

### ***Exercise***

Simplify the expression  $\sqrt{4x^2 + 16}$  as much as possible after substituting  $2\tan \theta$  for  $x$

#### **Solution**

$$\begin{aligned}\sqrt{4x^2 + 16} &= \sqrt{4(2\tan \theta)^2 + 16} \\ &= \sqrt{16\tan^2 \theta + 16} \\ &= \sqrt{16(\tan^2 \theta + 1)} \\ &= 4\sqrt{\tan^2 \theta + 1}\end{aligned}$$

$$= 4\sqrt{\sec^2 \theta}$$

$$= \underline{4 \sec \theta}$$

### ***Exercise***

Simplify by using the table.  $5 \sin^2 30^\circ$

#### **Solution**

$$5 \sin^2 30^\circ = 5 \left( \frac{1}{2} \right)^2$$

$$= \underline{\frac{5}{4}}$$

### ***Exercise***

Simplify by using the table.  $\sin^2 60^\circ + \cos^2 60^\circ$

#### **Solution**

$$\sin^2 60^\circ + \cos^2 60^\circ = \left( \frac{\sqrt{3}}{2} \right)^2 + \left( \frac{1}{2} \right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \underline{1}$$

### ***Exercise***

Simplify by using the table.  $(\tan 45^\circ + \tan 60^\circ)^2$

#### **Solution**

$$(\tan 45^\circ + \tan 60^\circ)^2 = (1 + \sqrt{3})^2$$

$$= 1 + 3 + 2\sqrt{3}$$

$$= \underline{4 + 2\sqrt{3}}$$

### ***Exercise***

Find the exact value of  $\csc 300^\circ$

#### **Solution**

$$\hat{\theta} = 360^\circ - 300^\circ = 60^\circ \rightarrow 300^\circ \in QIV$$

$$\begin{aligned}\csc 300^\circ &= -\frac{1}{\sin 60^\circ} \\ &= -\frac{1}{\frac{\sqrt{3}}{2}} \\ &= -\frac{2}{\sqrt{3}}\end{aligned}$$

### ***Exercise***

Find  $\theta$  if  $\sin \theta = -\frac{1}{2}$  and  $\theta$  terminates in **QIII** with  $0^\circ \leq \theta \leq 360^\circ$ .

### **Solution**

$$\begin{aligned}\hat{\theta} &= \sin^{-1} \frac{1}{2} = 30^\circ \\ \theta &\in \mathbf{QIII} \\ \Rightarrow \theta &= 180^\circ + 30^\circ \\ &= 210^\circ\end{aligned}$$

### ***Exercise***

Find  $\theta$  to the nearest degree if  $\sec \theta = 3.8637$  and  $\theta$  terminates in **QIV** with  $0^\circ \leq \theta \leq 360^\circ$ .

### **Solution**

$$\begin{aligned}\sec \theta &= 3.8637 = \frac{1}{\cos \theta} \\ \cos \theta &= \frac{1}{3.8637}\end{aligned}$$

$$\begin{aligned}\hat{\theta} &= \cos^{-1} \frac{1}{3.8637} \\ &= 75^\circ\end{aligned}$$

$$\text{Calculator : } \cos^{-1}(1 / 3.8637)$$

$$\begin{aligned}\theta &\in \mathbf{QIV} \\ \Rightarrow \theta &= 360^\circ - 75^\circ \\ &= 285^\circ\end{aligned}$$

### ***Exercise***

Find the exact value of  $\cos 225^\circ$

#### **Solution**

$$\hat{\theta} = 225^\circ - 180^\circ = 45^\circ$$

$$\rightarrow 225^\circ \in QIII$$

$$\cos 225^\circ = -\cos 45^\circ$$

$$= -\frac{\sqrt{2}}{2}$$

### ***Exercise***

Find the exact value of  $\tan 315^\circ$

#### **Solution**

$$\hat{\theta} = 360^\circ - 315^\circ = 45^\circ \quad \rightarrow 315^\circ \in QIV$$

$$\tan 315^\circ = -\tan 45^\circ$$

$$= -1$$

### ***Exercise***

Find the exact value of  $\cos 420^\circ$

#### **Solution**

$$\hat{\theta} = 420^\circ - 360^\circ = 60^\circ \quad \rightarrow 420^\circ \in QI$$

$$\cos 420^\circ = \cos 60^\circ$$

$$= \frac{1}{2}$$

### ***Exercise***

Find the exact value of  $\cot 480^\circ$

#### **Solution**

$$\hat{\theta} = 480^\circ - 360^\circ = 120^\circ$$

$$\hat{\theta} = 180^\circ - 120^\circ = 60^\circ \quad \rightarrow 480^\circ \in QII$$

$$\cot 480^\circ = -\frac{\cos 60^\circ}{\sin 60^\circ}$$

$$= -\frac{1/2}{\sqrt{3}/2}$$

$$\underline{= -\frac{1}{\sqrt{3}}}$$

### ***Exercise***

Use the calculator to find the value of  $\csc 166.7^\circ$

### **Solution**

$$\csc 166.7^\circ = \frac{1}{\sin 166.7^\circ}$$

$$\underline{\approx 4.3469}$$

### ***Exercise***

Use the calculator to find the value of  $\sec 590.9^\circ$

### **Solution**

$$\sec 590.9^\circ = \frac{1}{\cos 590.9^\circ}$$

$$\underline{\approx -1.5856}$$

### ***Exercise***

Use the calculator to find the value of  $\tan 195^\circ 10'$

### **Solution**

$$\tan(195^\circ 10') = \tan\left(195^\circ + \frac{10}{60}\right)$$

$$= \tan 195.1667^\circ$$

$$\underline{\approx 0.271}$$

### ***Exercise***

Use the calculator to find  $\theta$  to the nearest degree if  $\sin \theta = -0.3090$  with  $\theta \in \text{QIV}$  with  $0^\circ \leq \theta \leq 360^\circ$

### **Solution**

$$\hat{\theta} = \sin^{-1}(0.3090) \quad \text{Since } \theta \in \text{QIV}$$

$$\underline{\approx 18.0^\circ}$$

$$\theta = 180^\circ + 40.0^\circ$$

$$\underline{= 220.0^\circ}$$

### Exercise

Use the calculator to find  $\theta$  to the nearest degree if  $\cos \theta = -0.7660$  with  $\theta \in \text{QIII}$  with  $0^\circ \leq \theta \leq 360^\circ$

### Solution

$$\hat{\theta} = \cos^{-1}(0.7660) \quad \text{Since } \theta \in \text{QIII}$$

$$\approx 40.0^\circ$$

$$\theta = 180^\circ + 40.0^\circ$$

$$= \underline{220.0^\circ}$$

### Exercise

Use the calculator to find  $\theta$  to the nearest degree if  $\sec \theta = -3.4159$  with  $\theta \in \text{QII}$  with  $0^\circ \leq \theta \leq 360^\circ$

### Solution

$$\sec \theta = -3.4159$$

$$\cos \theta = -\frac{1}{3.4159}$$

$$\hat{\theta} = \cos^{-1}\left(\frac{1}{3.4159}\right) \quad \text{Since } \theta \in \text{QII}$$

$$\approx 73.0^\circ$$

$$\theta \approx 180^\circ - 73.0^\circ$$

$$= \underline{107.0^\circ}$$

### Exercise

Find  $\theta$  to the nearest tenth of a degree if  $\tan \theta = -0.8541$  and  $\theta$  terminates in  $\text{QIV}$  with  $0^\circ \leq \theta \leq 360^\circ$ .

### Solution

$$\hat{\theta} = \tan^{-1} 0.8541 \quad \theta \in \text{QIV}$$

$$\approx 40.5^\circ$$

$$\Rightarrow \theta = 360^\circ - 40.5^\circ$$

$$\approx \underline{319.5^\circ}$$



### ***Exercise***

Use the calculator to find  $\theta$  to the nearest degree if  $\sin \theta = 0.49368329$  with  $\theta \in QII$  with  $0^\circ \leq \theta < 360^\circ$

### **Solution**

$$\begin{aligned}\hat{\theta} &= \sin^{-1} 0.49368329 & \theta &\in QII \\ &= \underline{29.6^\circ}\end{aligned}$$

$$\begin{aligned}\Rightarrow \theta &= 180^\circ - 29.6^\circ \\ &= \underline{150.4^\circ}\end{aligned}$$