Exercise

Determine all asymptotes of the function: $y = \dfrac{3x}{1-x}$

Solution

VA: $x = 1$ ***HA***: $y = -3$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $y = \dfrac{x^2}{x^2+9}$

Solution

VA: *n / a* $x^2 + 9 \neq 0$ ***HA***: $y = 1$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $y = \dfrac{x-2}{x^2-4x+3}$

Solution

$x^2 - 4x + 3 = 0 \Rightarrow x = 1,\ 3$

$y = \dfrac{x}{x^2} \to 0$

VA: $x = 1,\ x = 3$ ***HA***: $y = 0$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $y = \dfrac{3}{x-5}$

Solution

VA: $x = 5$ ***HA***: $y = 0$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $y = \dfrac{x^3 - 1}{x^2 + 1}$

Solution

VA: *none*

HA: *none*

Hole: n / a

Oblique asymptote: $y = x$

$$\begin{array}{r|l} & x \\ x^2+1 & x^3-1 \\ & \underline{-x^3 - x} \\ & -x-1 \end{array}$$

$$y = x - \frac{x+1}{x^2+1}$$

Exercise

Determine all asymptotes of the function: $y = \dfrac{x^3 + 3x^2 - 2}{x^2 - 4}$

Solution

VA: $x = \pm 2$

HA: n / a

Hole: n / a

Oblique asymptote: $y = x + 3$

$$\begin{array}{r|l} & x+3 \\ x^2-4 & x^3+3x^2-2 \\ & \underline{-x^3 + 4x} \\ & 3x^2+4x-2 \\ & \underline{-3x^2 \quad +12} \\ & 4x+10 \end{array}$$

$$y = x + 3 + \frac{4x+10}{x^2-4}$$

Exercise

Determine all asymptotes of the function: $y = \dfrac{3x^2 - 27}{(x+3)(2x+1)}$

Solution

$$y = \frac{3x^2-27}{(x+3)(2x+1)} = \frac{3\left(x^2-9\right)}{(x+3)(2x+1)} = \frac{3(x+3)(x-3)}{(x+3)(2x+1)} = \frac{3(x-3)}{(2x+1)}$$

VA: $x = -3, \ -\frac{1}{2}$ ***HA***: $y = \frac{3}{2}$

Hole: n / a ***Oblique asymptote***: n / a

Exercise

Determine all asymptotes of the function: $y = \dfrac{x-3}{x^2-9}$

Solution

$$x^2 - 9 = 0 \rightarrow \boxed{x = \pm 3}$$

$$y = \frac{x-3}{(x-3)(x+3)}$$

$$= \frac{1}{x+3}$$

VA: $x = 3$ ***HA***: $y = 0$

Hole: $x = 3 \rightarrow y = \frac{1}{6}$ ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $y = \dfrac{6}{\sqrt{x^2 - 4x}}$

Solution

$$x^2 - 4x = 0$$

$$\Rightarrow x(x-4) = 0 \rightarrow \boxed{x = 0, 4}$$

VA: $x = 0,\ x = 4$ ***HA***: $y = 0$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $y = \dfrac{5x-1}{1-3x}$

Solution

VA: $x = \frac{1}{3}$ ***HA***: $y = -\frac{5}{3}$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $f(x) = \dfrac{2x-11}{x^2+2x-8}$

Solution

VA: $x = 2,\ x = -4$ ***HA***: $y = 0$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $f(x)=\dfrac{x^2-4x}{x^3-x}$

Solution

$$f(x)=\frac{x(x-4)}{x(x^2-1)}$$
$$=\frac{x-4}{x^2-1}$$

VA: $x=-1,\ x=1$ ***HA***: $y=0$

Hole: $x=0\rightarrow y=4$ ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function: $f(x)=\dfrac{x-2}{x^3-5x}$

Solution

VA: $x=0,\ x=\pm\sqrt{5}$ ***HA***: $y=0$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{4x}{x^2+10x}$

Solution

$x^2+10x=0 \quad\rightarrow\quad x=0,\ -10$

Domain: $(-\infty,\ -10)\cup(-10,\ 0)\cup(0,\ \infty)$

$$f(x)=\frac{4x}{x(x+10)}$$
$$=\frac{4}{x+10}$$

VA: $x=-10$ ***HA***: $y=0$

Hole: $x=0 \quad\rightarrow\quad y=\frac{4}{10}\Rightarrow$ *hole* $\left(0,\ \frac{2}{5}\right)$ ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{3-x}{(x-4)(x+6)}$

Solution

VA: $x=-6$ *and* $x=4$ ***HA***: $y=0$

Hole: n/a ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3}{2x^3-x^2-3x}$

Solution

$$2x^3-x^2-3x=x\left(2x^2-x-3\right)=0 \quad \rightarrow \quad x=0,\ -1,\ \frac{3}{2}$$

$$f(x)=\frac{x^3}{2x^3-x^2-3x}$$

$$=\frac{x^3}{x\left(2x^2-x-3\right)}$$

$$=\frac{x^2}{2x^2-x-3}$$

VA: $x=-1$ *and* $x=\dfrac{3}{2}$ ***HA***: $y=\dfrac{1}{2}$

Hole: $x=0 \quad \rightarrow \quad y=0 \Rightarrow$ *hole* $(0,\ 0)$ ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{3x^2+5}{4x^2-3}$

Solution

$$4x^2-3=0 \quad \rightarrow \quad x=\pm\frac{\sqrt{3}}{2}$$

Domain: $\left(-\infty,\ -\dfrac{\sqrt{3}}{2}\right)\cup\left(-\dfrac{\sqrt{3}}{2},\ \dfrac{\sqrt{3}}{2}\right)\cup\left(\dfrac{\sqrt{3}}{2},\ \infty\right)$

VA: $x=-\dfrac{\sqrt{3}}{2}$ *and* $x=\dfrac{\sqrt{3}}{2}$ ***HA***: $y=\dfrac{3}{4}$

Hole: n/a ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x+6}{x^3+2x^2}$

Solution

$$x^3+2x^2=x^2(x+2)=0 \quad \rightarrow \quad x=0,-2$$

Domain: $(-\infty,\ -2)\cup(-2,\ 0)\cup(0,\ \infty)$

VA: $x=0$ *and* $x=2$ ***HA***: $y=0$

Hole: n/a ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2+4x-1}{x+3}$

Solution

VA: $x=-3$

HA: n/a

Hole: n/a

Oblique asymptote: $y=x+1$

$$
\begin{array}{r|l}
 & x+1 \\
\hline
x+3 & x^2+4x-1 \\
 & \underline{-x^2-3x} \\
 & \quad x-1 \\
 & \quad \underline{-x-3} \\
 & \qquad -4
\end{array}
$$

$$f(x)=\frac{x^2+4x-1}{x+3}=x+1-\frac{4}{x+3}$$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2-6x}{x-5}$

Solution

$x-5=0 \quad\rightarrow\quad x=5$

Domain: $(-\infty,\ 5)\cup(5,\ \infty)$

$$f(x)=\frac{x^2-6x}{x-5}$$
$$=x-1-\frac{5}{x-5}$$

$$
\begin{array}{r|l}
 & x-1 \\
\hline
x-5 & x^2-6x \\
 & \underline{-x^2+5x} \\
 & \quad -x \\
 & \quad \underline{x-5} \\
 & \qquad -5
\end{array}
$$

VA: $x=5$

HA: N/A

Hole: N/A

Oblique asymptote: $y=x-1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3-x^2+x-4}{x^2+2x-1}$

Solution

$x^2+2x-1=0 \quad \rightarrow \quad x=-1\pm\sqrt{2}$

Domain: $\left(-\infty,\ -1-\sqrt{2}\right)\cup\left(-1-\sqrt{2},\ -1+\sqrt{2}\right)\cup\left(-1+\sqrt{2},\ \infty\right)$

$$f(x)=\frac{x^3-x^2+x-4}{x^2+2x-1}$$
$$=x-3+\frac{8x-7}{x^2+2x-1}$$

VA: $x=-1\pm\sqrt{2}$

HA: n/a

Hole: n/a

Oblique asymptote: $y=x-3$

$$\begin{array}{r} x-3 \\ x^2+2x-1\overline{)\,x^3-x^2+x-4} \\ \underline{-x^3-2x^2+x} \\ -3x^2+2x-4 \\ \underline{3x^2+6x-3} \\ 8x-7 \end{array}$$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{4x}{x^2+10x}$

Solution

$x^2+10x=0 \quad \rightarrow \quad x=0,\ -10$ ***Domain***: $\left(-\infty,\ -10\right)\cup\left(-10,\ 0\right)\cup\left(0,\ \infty\right)$

$$f(x)=\frac{4x}{x(x+10)}=\frac{4}{x+10}$$

VA: $x=-10$ ***HA***: $y=0$

Hole: $x=0 \quad \rightarrow \quad y=\frac{4}{10}\Rightarrow$ *hole* $\left(0,\ \frac{2}{5}\right)$

Oblique asymptote: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{3-x}{(x-4)(x+6)}$

Solution

Domain: $\left(-\infty,\ -6\right)\cup\left(-6,\ 4\right)\cup\left(4,\ \infty\right)$

VA: $x=-6$ *and* $x=4$ ***HA***: $y=0$

Hole: n/a ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3}{2x^3-x^2-3x}$

Solution

$2x^3-x^2-3x=x\left(2x^2-x-3\right)=0 \quad \rightarrow \quad x=0,\ -1,\ \dfrac{3}{2}$

Domain: $(-\infty,\ -1)\cup(-1,\ 0)\cup\left(0,\ \dfrac{3}{2}\right)\cup\left(\dfrac{3}{2},\ \infty\right)$

$f(x)=\dfrac{x^3}{2x^3-x^2-3x}=\dfrac{x^3}{x\left(2x^2-x-3\right)}=\dfrac{x^2}{2x^2-x-3}$

VA: $x=-1$ *and* $x=\dfrac{3}{2}$ ***HA***: $y=\dfrac{1}{2}$

Hole: $x=0 \quad \rightarrow \quad y=0 \Rightarrow$ *hole* $(0,\ 0)$

Oblique asymptote: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{3x^2+5}{4x^2-3}$

Solution

$4x^2-3=0 \quad \rightarrow \quad x=\pm\dfrac{\sqrt{3}}{2}$ ***Domain***: $\left(-\infty,\ -\dfrac{\sqrt{3}}{2}\right)\cup\left(-\dfrac{\sqrt{3}}{2},\ \dfrac{\sqrt{3}}{2}\right)\cup\left(\dfrac{\sqrt{3}}{2},\ \infty\right)$

VA: $x=-\dfrac{\sqrt{3}}{2}$ *and* $x=\dfrac{\sqrt{3}}{2}$ ***HA***: $y=\dfrac{3}{4}$

Hole: n/a ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x+6}{x^3+2x^2}$

Solution

$x^3+2x^2=x^2(x+2)=0 \quad \rightarrow \quad x=0,-2$ ***Domain***: $(-\infty,\ -2)\cup(-2,\ 0)\cup(0,\ \infty)$

VA: $x=0$ *and* $x=2$ ***HA***: $y=0$

Hole: n/a ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2+4x-1}{x+3}$

Solution

$x+3=0 \quad \rightarrow \quad x=-3$ ***Domain***: $(-\infty,\ -3)\cup(-3,\ \infty)$

$$x+3\overline{)\,x^2+4x-1}\qquad x+1$$
$$-x^2-3x$$
$$x-1$$
$$-x-3$$
$$-4$$

$$f(x)=\frac{x^2+4x-1}{x+3}=x+1-\frac{4}{x+3}$$

VA: $x=-3$ ***HA***: n/a

Hole: n/a ***Oblique asymptote***: $y=x+1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2-6x}{x-5}$

Solution

$x-5=0 \quad \rightarrow \quad x=5$ ***Domain***: $(-\infty,\ 5)\cup(5,\ \infty)$

$$x-5\overline{)\,x^2-6x}\qquad x-1$$
$$-x^2+5x$$
$$-x$$
$$x-5$$
$$-5$$

$$f(x)=\frac{x^2-6x}{x-5}=x-1-\frac{5}{x-5}$$

VA: $x=5$

HA: N/A

Hole: N/A

Oblique asymptote: $y=x-1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3-x^2+x-4}{x^2+2x-1}$

Solution

$x^2+2x-1=0 \quad \rightarrow \quad x=-1\pm\sqrt{2}$

Domain: $(-\infty,\ -1-\sqrt{2})\cup(-1-\sqrt{2},\ -1+\sqrt{2})\cup(-1+\sqrt{2},\ \infty)$

$$x^2+2x-1\overline{)\,x^3-x^2+x-4}\qquad x-3$$

$$-x^3-2x^2+x$$

$$-3x^2+2x-4$$

$$3x^2+6x-3$$

$$8x-7$$

$$f(x)=\frac{x^3-x^2+x-4}{x^2+2x-1}$$

$$=x-3+\frac{8x-7}{x^2+2x-1}$$

VA: $x=-1\pm\sqrt{2}$

HA: N/A

Hole: N/A

Oblique asymptote: $y=x-3$

Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

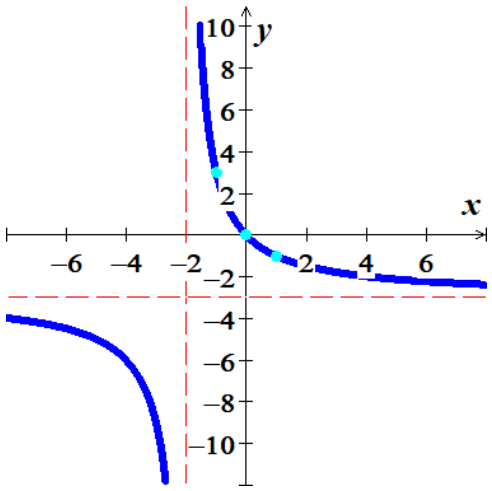
$$f(x)=\frac{-3x}{x+2}$$

Solution

VA: $x=-2$ ***HA***: $y=-3$

Hole: n/a ***OA***: n/a

x	y
0	0
1	−1
−1	3



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

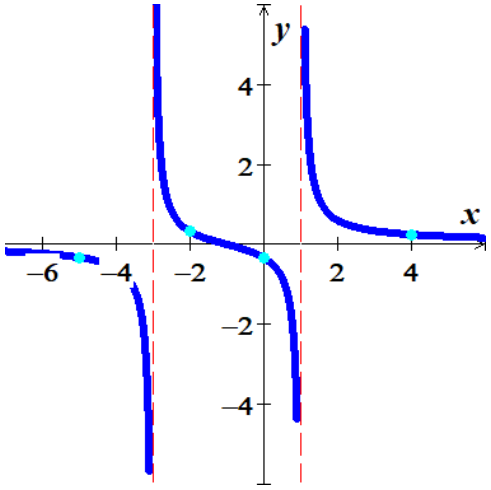
$$f(x) = \frac{x+1}{x^2+2x-3}$$

Solution

VA: $x = 1,\ x = -3$ ***HA***: $y = 0$

Hole: *n / a* ***Oblique asymptote***: *n / a*

x	y
−5	−0.33
−2	0.33
0	−1/3
4	0.24



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

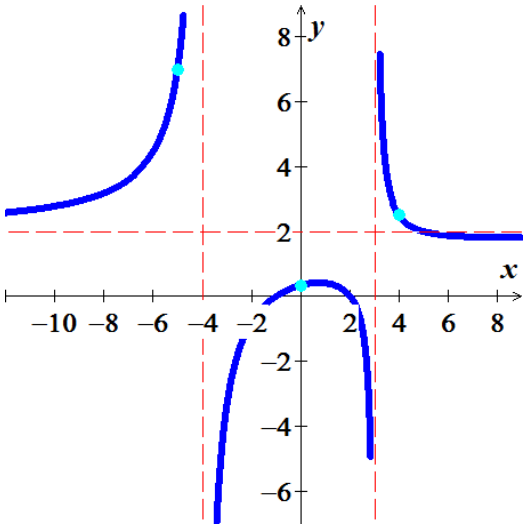
$$f(x) = \frac{2x^2-2x-4}{x^2+x-12}$$

Solution

VA: $x = -4,\ 3$ ***HA***: $y = 2$

Hole: *n / a* ***OA***: *n / a*

x	y
−5	7
−2	−0.8
0	1/3
4	2.5
5	2



Exercise

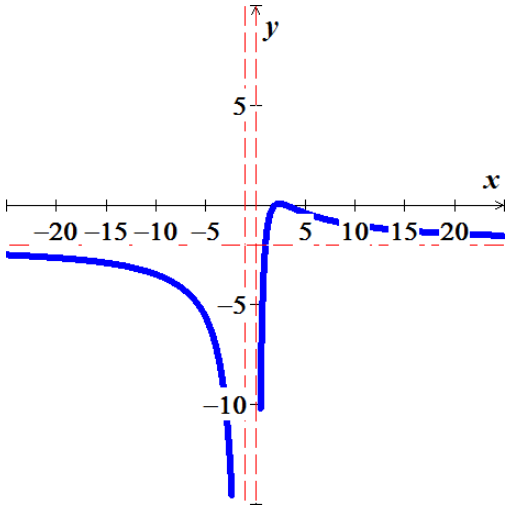
Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

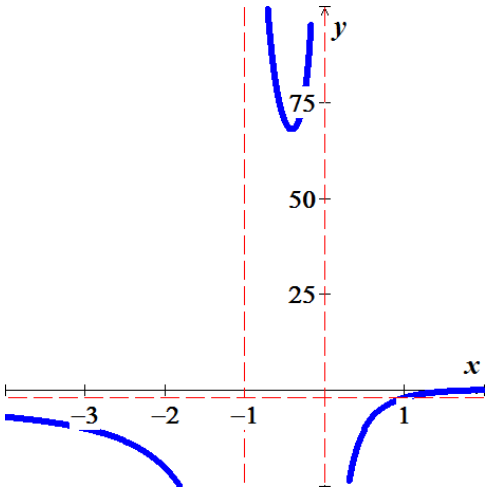
$$f(x)=\frac{-2x^2+10x-12}{x^2+x}$$

Solution

VA: $x=-1,\ 0$ ***HA***: $y=-2$

Hole: *n / a* ***OA***: *n / a*





Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^2-x-6}{x+1}$$

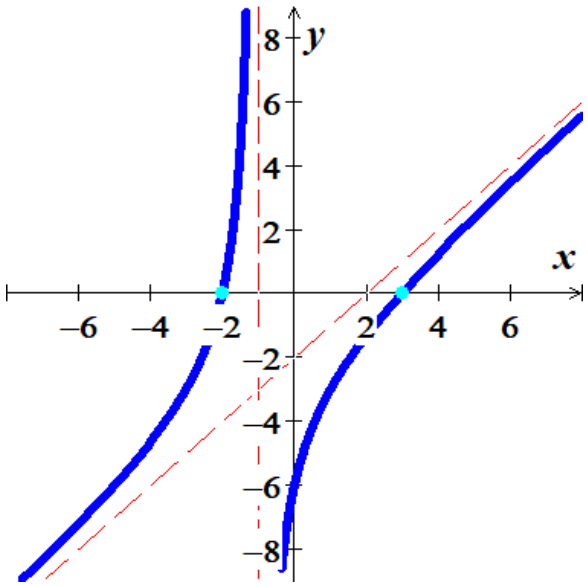
Solution

$$\begin{array}{r} x-2 \\ x+1\overline{)\,x^2-x-6} \\ \underline{x^2+x} \\ -2x-6 \\ \underline{-2x-2} \\ \boxed{-4} \end{array}$$

VA: $x=-1$ ***HA***: *n / a*

Hole: *n / a* ***OA***: $y=x-2$

x	*y*
2	0
−2	0
0	−6



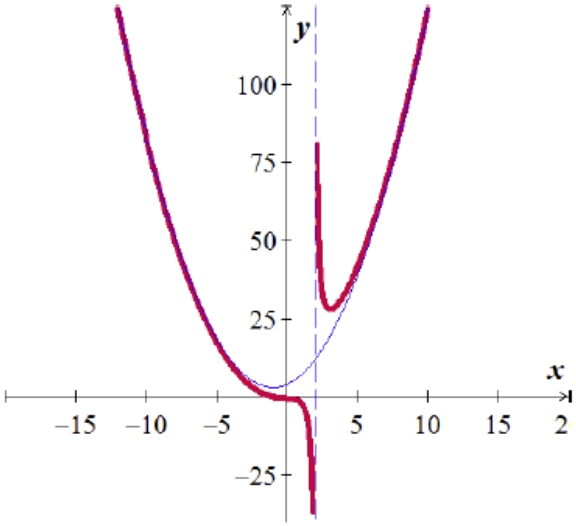
Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^3+1}{x-2}$$

Solution

$$\begin{array}{r} x^2+2x+4 \\ x-2\overline{)\,x^3-1} \\ \underline{x^3-2x^2} \\ 2x^2 \\ \underline{2x^2-4x} \\ 4x-1 \\ \underline{4x-8} \\ 7 \end{array}$$



VA: $x=2$ ***HA***: n/a

Hole: n/a ***OA***: $y=x^2+2x+4$

Exercise

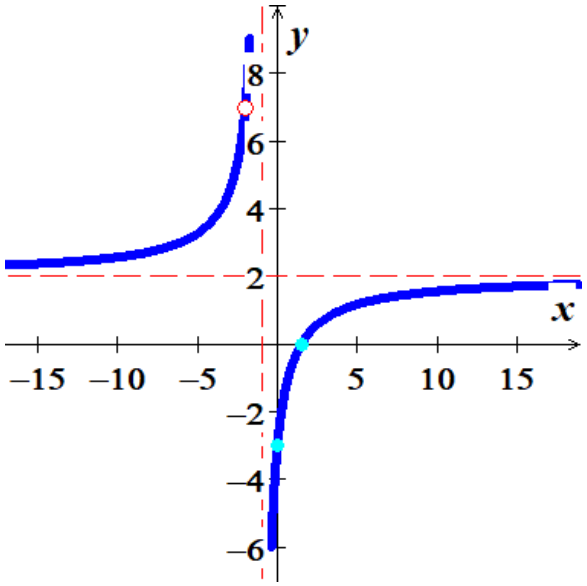
Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{2x^2+x-6}{x^2+3x+2}$$

Solution

$$f(x)=\frac{(2x-3)(x+2)}{(x+1)(x+2)}$$

$$=\frac{2x-3}{x+1}$$



VA: $x=-1$ ***HA***: $y=2$

Hole: $(-2,\ 7)$ ***OA***: n/a

x	y
0	−3
$-\frac{3}{2}$	0

Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x-1}{1-x^2}$$

Solution

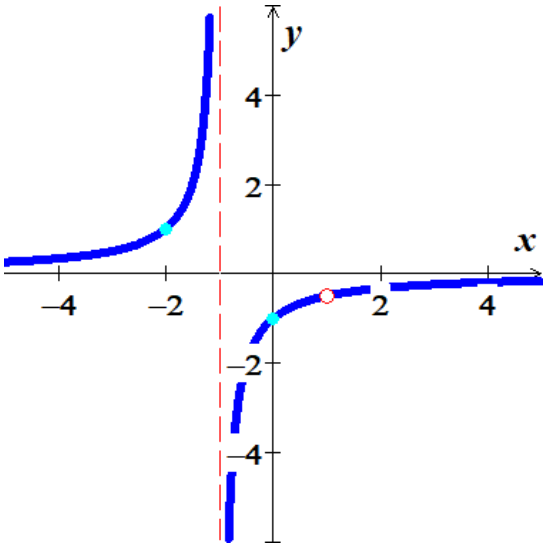
$$f(x)=\frac{x-1}{(x+1)(1-x)}$$

$$=-\frac{1}{x+1}$$

VA: $x=-1$ ***HA***: $y=0$

Hole: $\left(1,\ -\frac{1}{2}\right)$ ***OA***: *n / a*

x	*y*
0	–1
–2	1



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^2+x-2}{x+2}$$

Solution

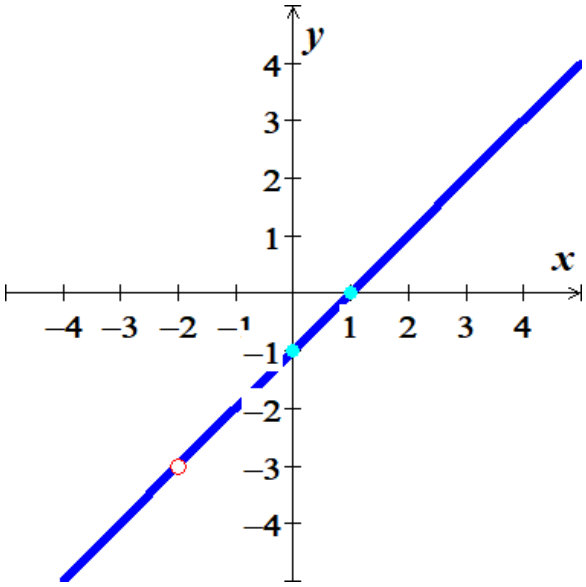
$$f(x)=\frac{(x+2)(x-1)}{x+2}$$

$$=x-1$$

VA: *n / a* ***HA***: *n / a*

Hole: $(-2,\ -3)$ ***OA***: *n / a*

x	*y*
0	–1
1	0



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^3-2x^2-4x+8}{x-2}$$

Solution

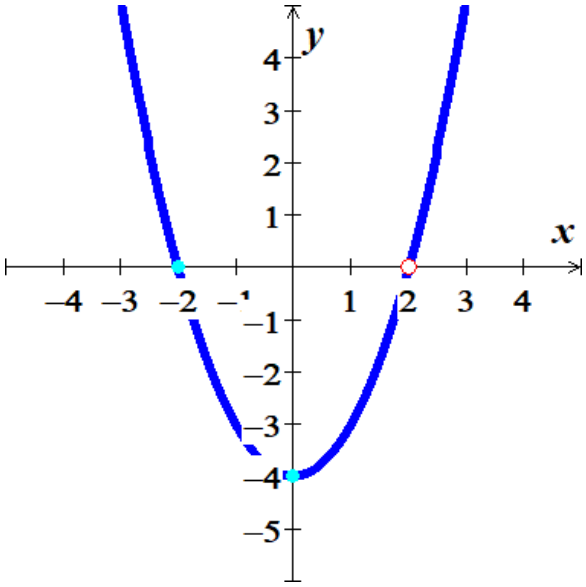
$$f(x)=\frac{\left(x^2-4\right)(x-2)}{x-2}$$

$$=x^2-4$$

VA: n/a ***HA***: n/a

Hole: $(2,\ 0)$ ***OA***: n/a

x	y
0	–4
–2	0



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{2x^2-3x-1}{x-2}$$

Solution

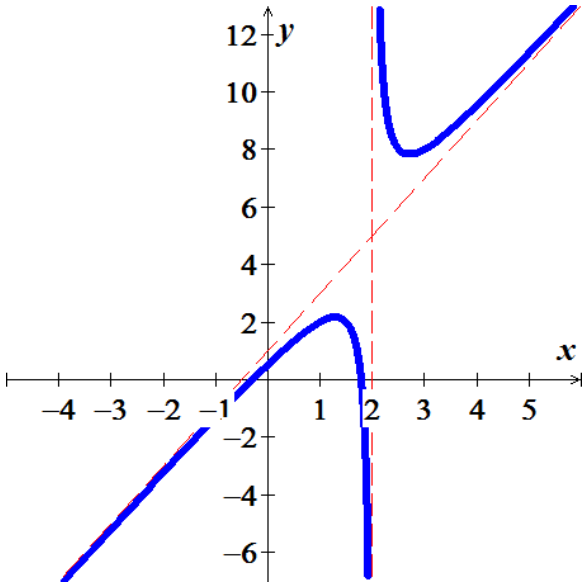
$$\begin{array}{r|l} x-2 & 2x^2-3x-1 \\ & \underline{-2x^2+4x} \\ & \quad x-1 \\ & \quad \underline{-x+2} \\ & \qquad 1 \end{array}$$

$$f(x)=\frac{2x^2-3x-1}{x-2}$$

$$=(2x+1)+\frac{1}{x-2}$$

VA: $x=2$ ***HA***: $y=1$

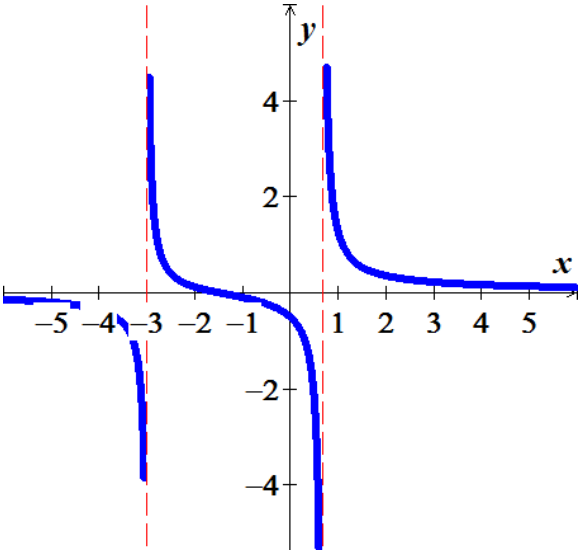
Hole: n/a ***OA***: $y=2x+1$



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{2x+3}{3x^2+7x-6}$$



Solution

$$3x^2+7x-6=0 \quad \Rightarrow \quad x=-3,\ \frac{2}{3}$$

VA: $x=-3$ *and* $x=\frac{2}{3}$

HA: $y=0$

Hole: *n / a*

OA: *n / a*

Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x)=\frac{x^2-1}{x^2+x-6}$$

Solution

$$x^2+x-6=0 \quad \Rightarrow \quad \underline{x=-3,\ 2}$$

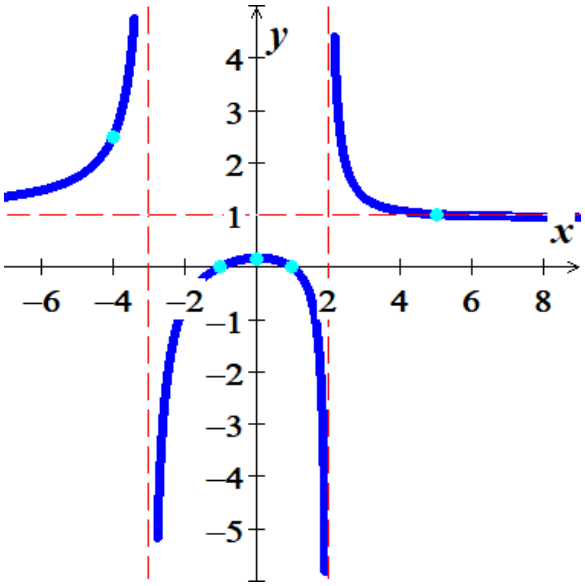
VA: $x=-3$ *and* $x=2$ ***HA***: $y=1$

Hole: *n / a* ***OA:*** *n / a*

$$1=\frac{x^2-1}{x^2+x-6}$$

$$x^2+x-6=x^2-1$$

$$\underline{x=5}$$



x	*y*
0	$\frac{1}{6}$
5	1
±1	0
−4	$\frac{5}{2}$

Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{-2x^2-x+15}{x^2-x-12}$$

Solution

$x^2-x-12=0 \quad \Rightarrow \quad \underline{x=-3,\ 4}$

Domain: $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

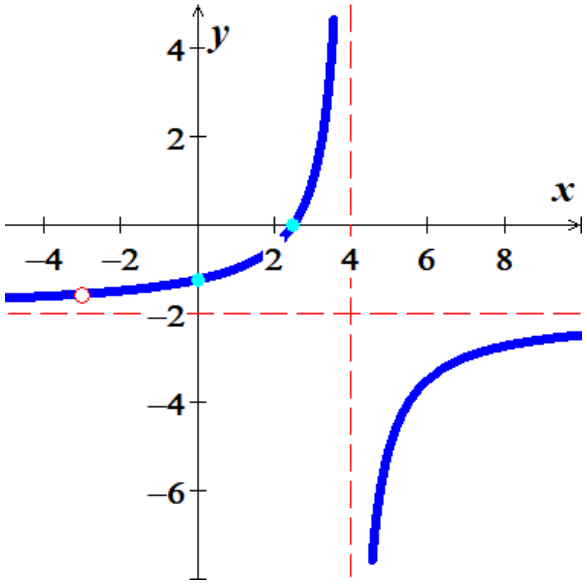
$$f(x)=\frac{(-2x+5)(x+3)}{(x-4)(x+3)}$$

$$=\frac{-2x+5}{x-4}$$

VA: $x=4$ ***HA***: $y=-2$

Hole: $\left(-3,\ -\frac{11}{7}\right)$ ***OA:*** *n / a*

x	*y*
0	$-\frac{5}{4}$
$\frac{5}{2}$	0



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

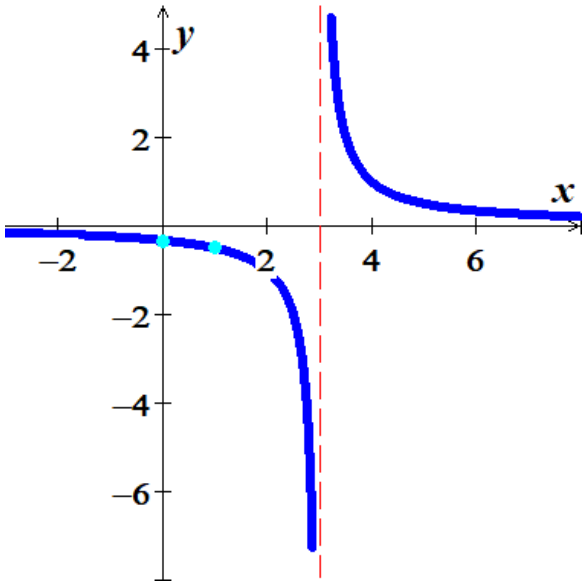
$$f(x)=\frac{1}{x-3}$$

Solution

VA: $x=3$ ***HA***: $y=0$

Hole: *n / a* ***OA:*** *n / a*

x	*y*
0	$-\frac{1}{3}$
1	$-\frac{1}{2}$



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

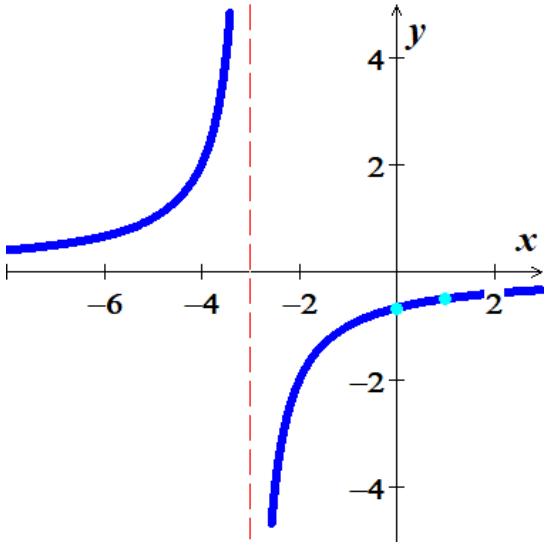
$$f(x)=\frac{-2}{x+3}$$

Solution

VA: $x=-3$ ***HA***: $y=0$

Hole: *n / a* ***OA:*** *n / a*

x	*y*
0	$-\frac{2}{3}$
1	$-\frac{1}{2}$



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

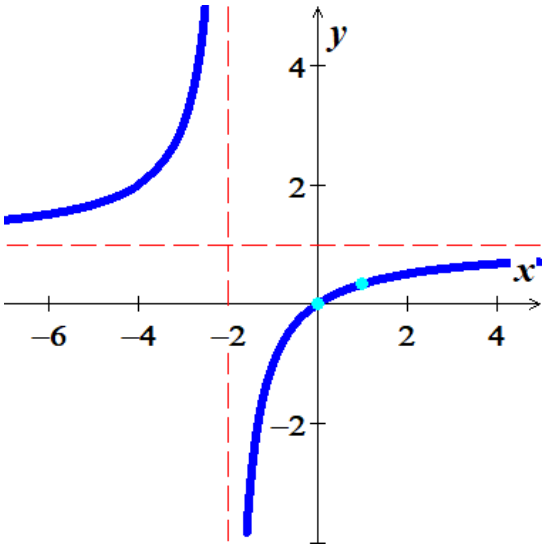
$$f(x)=\frac{x}{x+2}$$

Solution

VA: $x=-2$ ***HA***: $y=1$

Hole: *n / a* ***OA:*** *n / a*

x	*y*
0	0
1	$\frac{1}{3}$



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

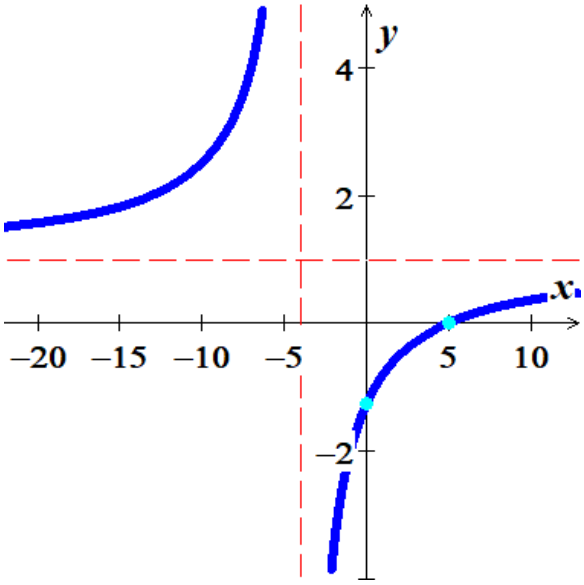
$$f(x)=\frac{x-5}{x+4}$$

Solution

VA: $x=-4$ ***HA***: $y=1$

Hole: *n / a* ***OA:*** *n / a*

x	y
0	$-\frac{5}{4}$
5	0



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{2x^2-2}{x^2-9}$$

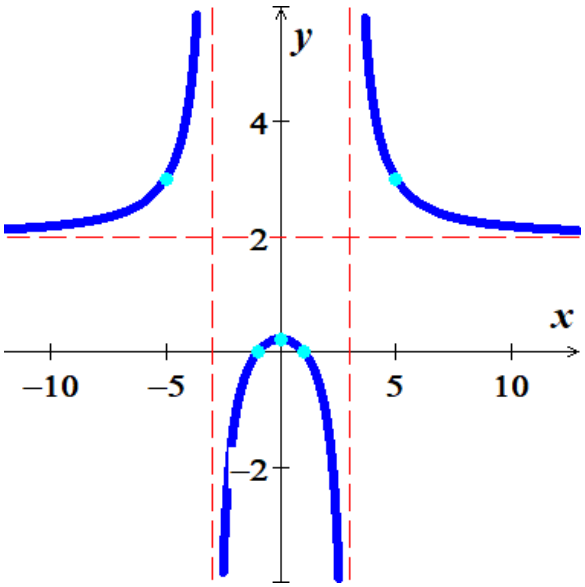
Solution

$x^2=9 \rightarrow \underline{x=\pm 3}$

VA: $x=\pm 3$ ***HA***: $y=2$

Hole: *n / a* ***OA:*** *n / a*

x	y
0	$\frac{2}{9}$
± 1	0
± 5	3



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

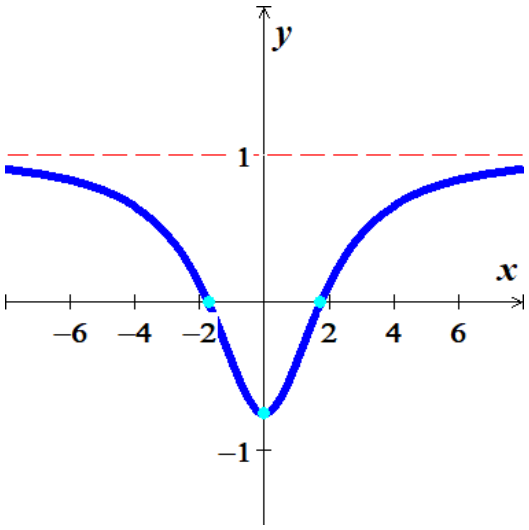
$$f(x)=\frac{x^2-3}{x^2+4}$$

Solution

VA: n/a ***HA***: $y=1$

Hole: n/a ***OA:*** n/a

x	y
0	$-\frac{3}{4}$
$\pm\sqrt{3}$	0



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2+4}{x^2-3}$$

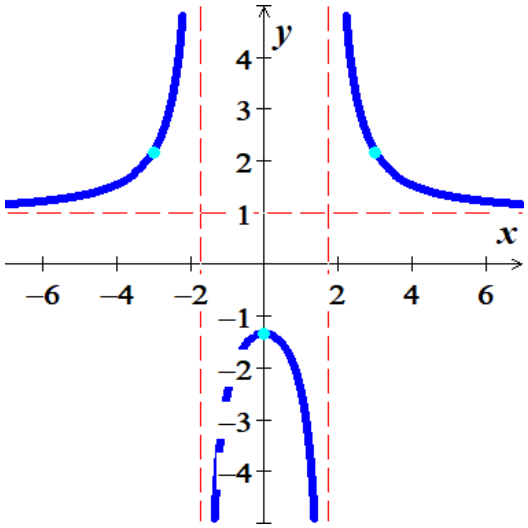
Solution

$x^2-3=0 \rightarrow x=\pm\sqrt{3}$

VA: $x=\pm\sqrt{3}$ ***HA***: $y=1$

Hole: n/a ***OA:*** n/a

x	y
0	$-\frac{4}{3}$
± 3	$\frac{13}{6}$



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2}{x^2-6x+9}$$

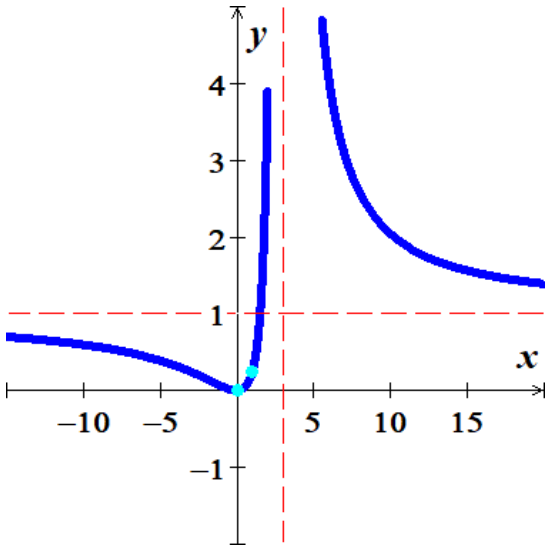
Solution

$x^2-6x+9=0 \quad \rightarrow \quad x=3$

VA: $x=3$ ***HA***: $y=1$

Hole: *n / a* ***OA:*** *n / a*

x	*y*
0	0
1	$\frac{1}{4}$



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2+x+4}{x^2+2x-1}$$

Solution

$x^2+2x-1=0$

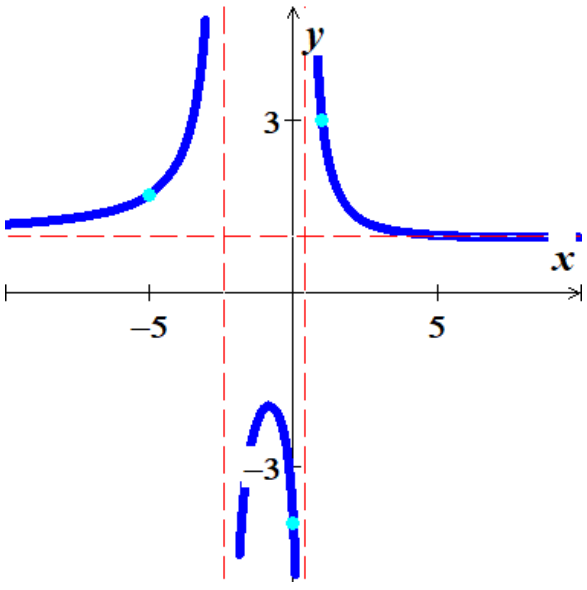
$x=\frac{-2\pm\sqrt{8}}{2}$

$=-1\pm\sqrt{2}$

VA: $x=-1\pm\sqrt{2}$ ***HA***: $y=1$

Hole: *n / a* ***OA:*** *n / a*

x	*y*
0	−4
1	3
−5	$\frac{12}{7}$



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

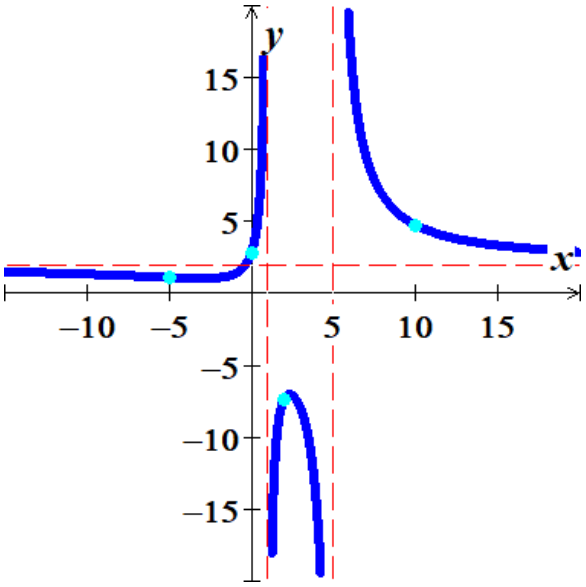
$$f(x)=\frac{2x^2+14}{x^2-6x+5}$$

Solution

VA: $x=1,\ 5$ ***HA***: $y=2$

Hole: n/a ***OA:*** n/a

x	y
0	$\frac{14}{5}$
2	$-\frac{22}{3}$
−5	$\frac{16}{15}$
10	$\frac{214}{45}$



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2-4x-5}{2x+5}$$

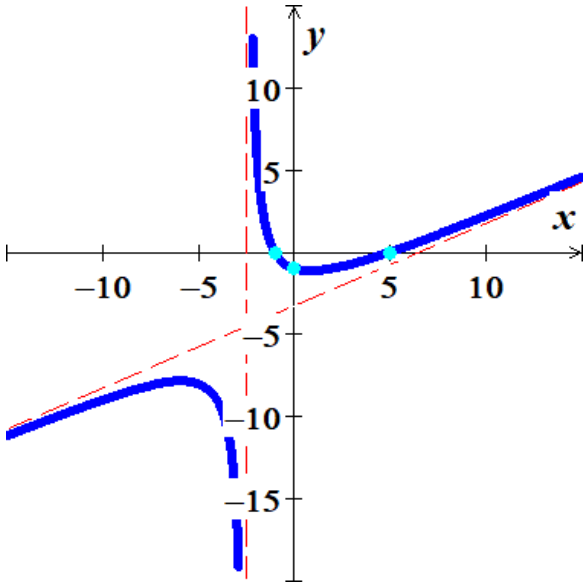
Solution

$$\begin{array}{r} \frac{1}{2}x-\frac{13}{4} \\ 2x+5\overline{)\,x^2-4x-5} \\ \underline{x^2+\frac{5}{2}x} \\ -\frac{13}{2}x-5 \end{array}$$

VA: $x=-\frac{5}{2}$ ***HA***: n/a

Hole: n/a ***OA:*** $y=\frac{1}{2}x-\frac{13}{2}$

x	y
0	−1
−1, 5	0



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x-3}{x^2-3x+2}$$

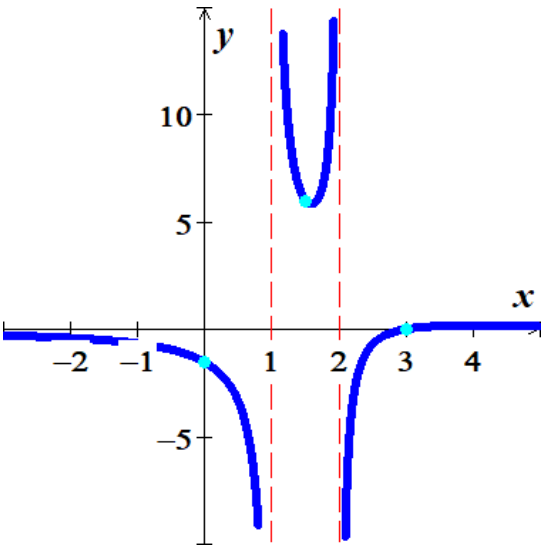
Solution

$x^2-3x+2 \rightarrow \underline{x=1,\ 2}$

VA: $x=1,\ 2$ ***HA***: $y=0$

Hole: n/a ***OA:*** n/a

x	y
0	$-\frac{3}{2}$
3	0
$\frac{3}{2}$	6



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2+2}{x^2+3x+2}$$

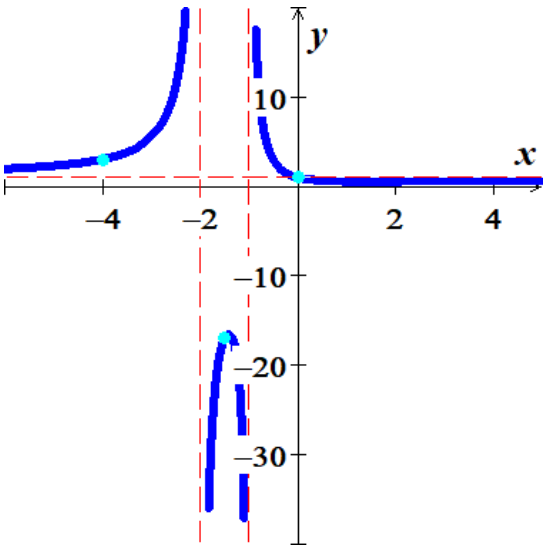
Solution

$x^2+3x+2 \rightarrow \underline{x=-1,\ -2}$

VA: $x=-1,\ -2$ ***HA***: $y=1$

Hole: n/a ***OA:*** n/a

x	y
0	1
$-\frac{3}{2}$	−17
−4	3



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x-2}{x^2-3x+2}$$

Solution

$$x^2-3x+2 \;\rightarrow\; \underline{x=1,\ 2}$$

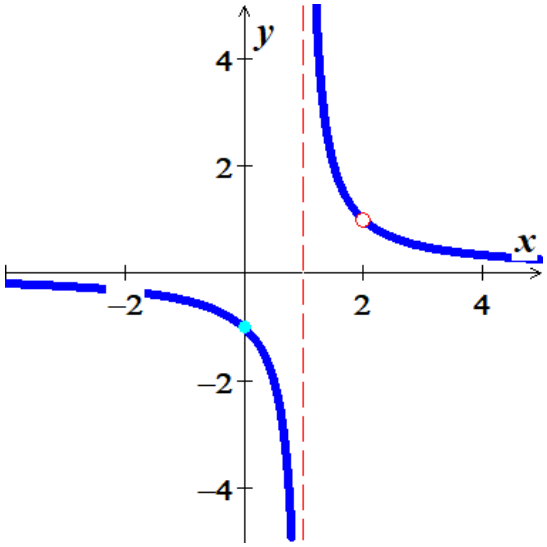
$$f(x)=\frac{x-2}{(x-2)(x-1)}$$

$$=\frac{1}{x-1}$$

VA: $x=1$ ***HA***: $y=0$

Hole: $(2,\ 1)$ ***OA:*** n/a

x	y
0	-1



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2+x}{x+1}$$

Solution

$$f(x)=\frac{x(x+1)}{x+1}$$

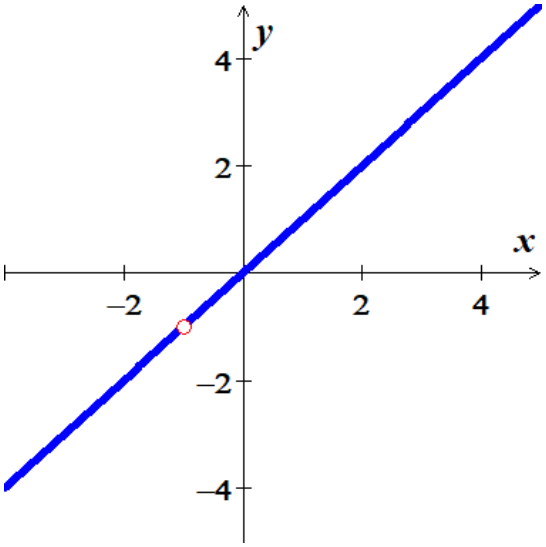
$$=x$$

VA: n/a ***HA***: n/a

Hole: $(-1,\ -1)$ ***OA:*** n/a

Hole: $\left(-3,\ -\frac{11}{7}\right)$ ***OA:*** n/a

x	y
0	0



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2-2x}{x-2}$$

Solution

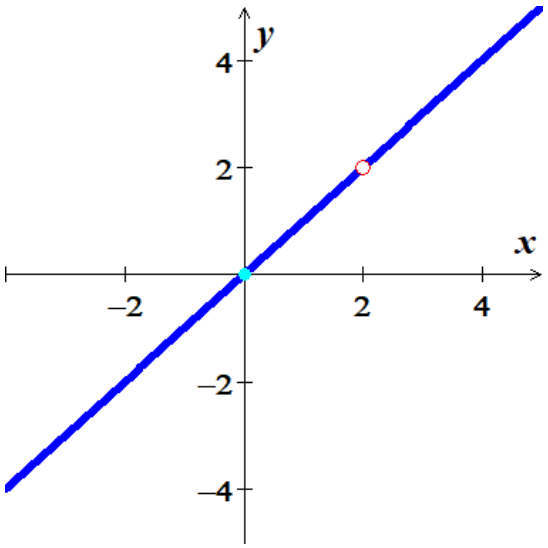
$$f(x)=\frac{x(x-2)}{x-2}$$
$$=x$$

VA: n/a ***HA***: n/a

Hole: $(2,\ 2)$ ***OA:*** n/a

Hole: $\left(-3,\ -\frac{11}{7}\right)$ ***OA:*** n/a

x	y
0	0



Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^2-3x}{x+3}$$

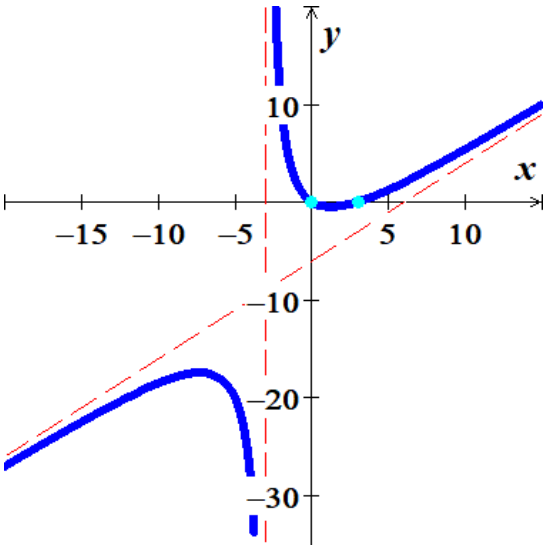
Solution

$$\begin{array}{r} x-6 \\ x+3\ \overline{)\ x^2-3x} \\ \underline{x^2+3x} \\ -6x-5 \end{array}$$

VA: $x=-3$ ***HA***: n/a

Hole: n/a ***OA:*** $y=x-6$

x	y
0	0
3	0



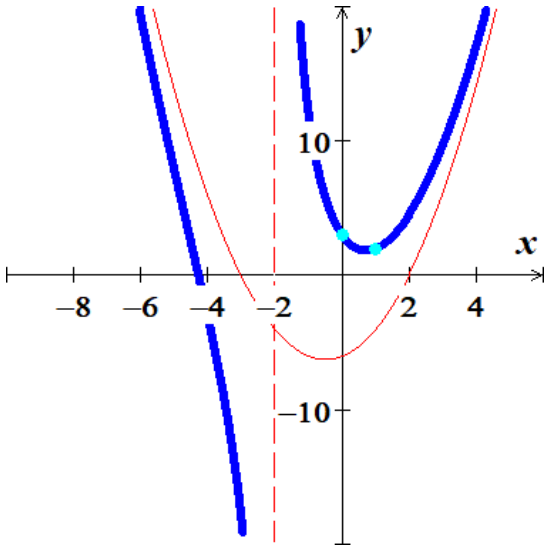
Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x)=\frac{x^3+3x^2-4x+6}{x+2}$$

Solution

$$\begin{array}{r|l} x+2 & x^2+x-6 \\ \hline & x^3+3x^2-4x+6 \\ & \underline{x^3+2x^2} \\ & x^2-4x \\ & \underline{x^2+2x} \\ & -6x+6 \end{array}$$



VA: $x=-2$ ***HA***: n/a

Hole: n/a ***OA:*** $y=x^2+x-6$

x	*y*
0	3
1	2

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=4 \\ \textit{horizontal asymptote}: \ y=-1 \\ x-\textit{intercept}: \ 3 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\dfrac{\quad}{x-4}$

Horizontal Asymptote: $f(x)=\dfrac{-x+a}{x-4}$

x-intercept: $f(x=3)=\dfrac{-3+a}{3-4}=0 \quad \Rightarrow \quad \underline{a=3}$

$$\underline{f(x)=\frac{-x+3}{x-4}}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} vertical\ asymptote: \ x=-4, x=5 \\ horizontal\ asymptote: \ y=\dfrac{3}{2} \\ x-intercept: \ -2 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\dfrac{}{(x+4)(x-5)}$

Horizontal Asymptote: $f(x)=\dfrac{3}{2}\dfrac{(x+a)(x+b)}{(x+4)(x-5)}$

x-intercept: $f(x=-2)=\dfrac{3}{2}\dfrac{(-2+a)(-2+b)}{}$

$$0=(-2+a)(-2+b)$$

$$\boxed{a=b=2}$$

$$f(x)=\frac{3}{2}\frac{(x-2)^2}{x^2-x-20}$$

$$=\underline{\left|\frac{3x^2-12x+12}{2x^2-2x-40}\right|}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} vertical\ asymptote: \ x=5 \\ horizontal\ asymptote: \ y=-1 \\ x-intercept: \ 2 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\dfrac{}{x-5}$

x-intercept : $f(x)=\dfrac{x-2}{x-5}$

Horizontal Asymptote: $f(x)=-\dfrac{x-2}{x-5}$

$$\underline{\left|f(x)=-\frac{x-2}{x-5}\right|}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=-2, \ x=0 \\ \textit{horizontal asymptote}: \ y=0 \\ x-\textit{intercept}: \ 2, \quad f(3)=1 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\dfrac{}{x(x+2)}$

x-intercept : $f(x)=\dfrac{x-2}{x(x+2)}$

Horizontal Asymptote: $f(x)=\dfrac{a(x-2)}{x(x+2)}$

$$f(3)=1 \quad \rightarrow \frac{a(1)}{(3)(5)}=1 \quad \Rightarrow \ \underline{a=15}$$

$$\underline{\boxed{f(x)=\frac{15x-30}{x^2+2x}}}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=-3, \ x=1 \\ \textit{horizontal asymptote}: \ y=0 \\ x-\textit{intercept}: \ -1, \quad f(0)=-2 \\ \textit{hole}: \ x=2 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\dfrac{}{(x+3)(x-1)}$

x-intercept : $f(x)=\dfrac{(x+1)}{(x+3)(x-1)}$

Horizontal Asymptote: $f(x)=\dfrac{a(x+1)}{(x+3)(x-1)}$

$$f(0)=-2 \quad \rightarrow \frac{a}{-3}=-2 \quad \Rightarrow \ \underline{a=6}$$

Hole at $x=2$**:** $f(x)=\dfrac{6(x+1)(x-2)}{\left(x^2+2x-3\right)(x-2)}$

$$\underline{\boxed{f(x)=\frac{6x^2-6x-12}{x^3-7x+6}}}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} vertical\ asymptote: \ x=-1, \ \ x=3 \\ horizontal\ asymptote: \ y=2 \\ x-intercept: \ \ -2, \ \ 1 \\ \ \ hole: \ \ x=0 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\dfrac{}{(x+1)(x-3)}$

Horizontal Asymptote: $f(x)=\dfrac{2}{(x+1)(x-3)}$

x-intercept : $f(x)=\dfrac{2(x+2)(x-1)}{(x+1)(x-3)}$

Hole at $x=0$**:** $f(x)=\dfrac{2x(x+2)(x-1)}{x(x+1)(x-3)}$

$$f(x)=\frac{2x^3+2x^2-4x}{x^3-2x^2-3x}$$