

Solution

Section 2.4 – Integer Representations and Algorithms

Exercise

Convert the decimal expansion of each of these integers to a binary expansion

- a) 321 b) 1023 c) 100632 d) 231 e) 4532

Solution

a)

321	160	80	40	20	10	5	2	1	
1	0	0	0	0	0	1	0	1	←

$$321 = \underline{(1\ 0100\ 0001)}_2$$

b) $1023 = 1024 - 1 = 2^{10} - 1$ 1 less than $(100\ 0000\ 0000)_2$

1023	511	255	127	63	31	15	7	3	1	
1	1	1	1	1	1	1	1	1	1	←

$$1023 = \underline{(11\ 1111\ 1111)}_2$$

c)

100632	50316	25158	12579	636289	3144	1572	786	393	196	98	49	24
	0	0	1	1	0	0	0	1	0	0	1	0

12	6	3	1	
0	0	1	1	←

$$100632 = \underline{(1\ 1000\ 1001\ 0001\ 1000)}_2$$

d)

231	115	57	28	14	7	3	1	
1	1	1	0	0	1	1	1	←

$$231 = \underline{(1110\ 0111)}_2$$

e)

4532	2266	1133	566	283	141	70	35	17	8	4	2	1	
0	0	1	0	1	1	0	1	1	0	0	0	1	←

$$4532 = \underline{(1\ 0001\ 1011\ 0100)}_2$$

Exercise

Convert binary the expansion of each of these integers to a decimal expansion

$$a) (11011)_2$$

$$b) (1010110101)_2$$

$$c) (1110111110)_2$$

$$d) (111110000011111)_2$$

$$e) (11111)_2$$

$$f) (1000000001)_2$$

$$g) (1001010101)_2$$

$$h) (110100100010000)_2$$

Solution

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

$$\begin{aligned} a) (11011)_2 &= 1 + 2^1 + 2^3 + 2^4 \\ &= 1 + 2 + 8 + 16 \\ &= \underline{27} \end{aligned}$$

$$\begin{aligned} b) (1010110101)_2 &= 1 + 2^2 + 2^4 + 2^5 + 2^7 + 2^9 \\ &= 1 + 4 + 16 + 32 + 128 + 512 \\ &= \underline{693} \end{aligned}$$

$$\begin{aligned} c) (1110111110)_2 &= 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^7 + 2^8 + 2^9 \\ &= \underline{958} \end{aligned}$$

$$\begin{aligned} d) (111110000011111)_2 &= 1 + 2^1 + 2^2 + 2^3 + 2^4 + 2^{10} + 2^{11} + 2^{12} + 2^{13} + 2^{14} \\ &= \underline{31775} \end{aligned}$$

$$\begin{aligned} e) (11111)_2 &= 1 + 2^1 + 2^2 + 2^3 + 2^4 \\ &= 1 + 2 + 8 + 16 \\ &= \underline{31} \end{aligned}$$

$$\begin{aligned} f) (1000000001)_2 &= 1 + 2^9 \\ &= 1 + 512 \\ &= \underline{513} \end{aligned}$$

$$g) (1001010101)_2 = 2^9 + 2^6 + 2^4 + 2^2 + 1 = \underline{597}$$

$$h) (110100100010000)_2 = 2^{14} + 2^{13} + 2^{11} + 2^8 + 2^4 = \underline{26896}$$

Exercise

Convert the binary expansion of each of these integers to an octal expansion

$$a) (1111\ 0111)_2$$

$$b) (1010\ 1010\ 1010)_2$$

$$c) (111\ 0111\ 0111\ 0111)_2$$

$$d) (101\ 0101\ 0101\ 0101)_2$$

Solution

$$a) (1111\ 0111)_2 = (11\ 110\ 111)_2 = \underline{(367)_8}$$

$$b) (1010\ 1010\ 1010)_2 = (101\ 010\ 101\ 010)_2 = \underline{(5252)_8}$$

$$c) (111\ 0111\ 0111\ 0111)_2 = (111\ 011\ 101\ 110\ 111)_2 = \underline{(73567)_8}$$

$$d) (101\ 0101\ 0101\ 0101)_2 = (101\ 010\ 101\ 010\ 101)_2 = \underline{(52525)_8}$$

Exercise

Convert the octal expansion of each of these integers to a binary expansion

a) $(572)_8$ b) $(1604)_8$ c) $(423)_8$ d) $(2417)_8$ e) $(73567)_8$ f) $(52525)_8$

Solution

Octal	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

$$a) \begin{array}{c|c|c} 5_8 & 7_8 & 2_8 \\ \hline 101_2 & 111_2 & 010_2 \end{array} \Rightarrow (572)_8 = \underline{(1\ 0111\ 1010)_2}$$

$$b) \begin{array}{c|c|c|c} 1_8 & 6_8 & 0_8 & 4_8 \\ \hline 1_2 & 110_2 & 000_2 & 100_2 \end{array} \Rightarrow (1604)_8 = \underline{(11\ 1000\ 0100)_2}$$

$$c) \begin{array}{c|c|c} 4_8 & 2_8 & 3_8 \\ \hline 100_2 & 010_2 & 011_2 \end{array} \Rightarrow (423)_8 = \underline{(1\ 0001\ 0011)_2}$$

$$d) \begin{array}{c|c|c|c|c} 7_8 & 3_8 & 5_8 & 6_8 & 7_8 \\ \hline 111_2 & 011_2 & 101_2 & 110_2 & 111_2 \end{array} \Rightarrow (73567)_8 = \underline{(111\ 0111\ 0111\ 0111)_2}$$

$$e) \begin{array}{c|c|c|c|c} 5_8 & 2_8 & 5_8 & 2_8 & 5_8 \\ \hline 101_2 & 010_2 & 101_2 & 010_2 & 101_2 \end{array} \Rightarrow (52525)_8 = \underline{(101\ 0101\ 0101\ 0101)_2}$$

Exercise

Convert the hexadecimal expansion of each of these integers to a binary expansion

a) $(80E)_{16}$ b) $(135AB)_{16}$ c) $(ABBA)_{16}$
d) $(DEFACED)_{16}$ e) $(BADFACED)_{16}$ f) $(ABCDEF)_{16}$

Solution

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

$$a) \begin{array}{c|c|c} 8_{16} & 0_{16} & E_{16} \\ \hline 1000_2 & 0000_2 & 1110_2 \end{array} \Rightarrow (80E)_{16} = \underline{(1000\ 0000\ 1110)_2}$$

$$b) \begin{array}{c|c|c|c|c} 1_{16} & 3_{16} & 5_{16} & A_{16} & B_{16} \\ \hline 0001_2 & 0011_2 & 0101_2 & 1010_2 & 1011_2 \end{array} \Rightarrow (135AB)_{16} = \underline{(0001\ 0011\ 0101\ 1010\ 1011)_2}$$

$$c) \begin{array}{c|c|c|c} A_{16} & B_{16} & B_{16} & A_{16} \\ \hline 1010_2 & 1011_2 & 1011_2 & 1010_2 \end{array} \Rightarrow (ABBA)_{16} = \underline{(1010 \ 1011 \ 1011 \ 1010)_2}$$

$$d) \begin{array}{c|c|c|c|c|c|c} D_{16} & E_{16} & F_{16} & A_{16} & C_{16} & E_{16} & D_{16} \\ \hline 1101_2 & 1110_2 & 1111_2 & 1010_2 & 1100_2 & 1110_2 & 1101_2 \end{array} \Rightarrow (DEFACED)_{16} = \underline{(1101 \ 1110 \ 1111 \ 1010 \ 1100 \ 1110 \ 1101)_2}$$

$$e) \begin{array}{c|c|c|c|c|c|c|c} B_{16} & A_{16} & D_{16} & F_{16} & A_{16} & C_{16} & E_{16} & D_{16} \\ \hline 1011_2 & 1010_2 & 1101_2 & 1111_2 & 1010_2 & 1100_2 & 1110_2 & 1101_2 \end{array} \Rightarrow (BADFACED)_{16} = \underline{(1011 \ 1010 \ 1101 \ 1111 \ 1010 \ 1100 \ 1110 \ 1101)_2}$$

$$f) \begin{array}{c|c|c|c|c|c} A_{16} & B_{16} & C_{16} & D_{16} & E_{16} & F_{16} \\ \hline 1010_2 & 1011_2 & 1100_2 & 1101_2 & 1110_2 & 1111_2 \end{array} \Rightarrow (ABCDEF)_{16} = \underline{(1010 \ 1011 \ 1100 \ 1101 \ 1110 \ 1111)_2}$$

Exercise

Show that the binary expansion of a positive integer can be obtained from its hexadecimal expansion by translating each hexadecimal digit into a block of four binary digits.

Solution

Let $(\dots h_2 h_1 h_0)_{16}$ be the hexadecimal expansion of a positive integer. The value of that integer is

$$h_0 + h_1 \cdot 16 + h_2 \cdot 16^2 + \dots = h_0 + h_1 \cdot 2^4 + h_2 \cdot 2^8 + \dots$$

If we replace each hexadecimal digit h_i by its binary expansion $(b_{i3} b_{i2} b_{i1} b_{i0})_2$, then

$$h_i = b_{i0} + 2b_{i1} + 4b_{i2} + 8b_{i3}$$

Therefore the value of the entire number is

$$\begin{aligned} & b_{00} + 2b_{01} + 4b_{02} + 8b_{03} + (b_{10} + 2b_{11} + 4b_{12} + 8b_{13}) \cdot 2^4 \\ & \quad + (b_{20} + 2b_{21} + 4b_{22} + 8b_{23}) \cdot 2^8 + \dots \\ & = b_{00} + 2b_{01} + 4b_{02} + 8b_{03} + 2^4 b_{10} + 2^5 b_{11} + 2^6 b_{12} + 2^7 b_{13} \\ & \quad + 2^8 b_{20} + 2^9 b_{21} + 2^{10} b_{22} + 2^{11} b_{23} + \dots \end{aligned}$$

Which is the value of the binary expansion $(\dots b_{23} b_{22} b_{21} b_{20} b_{13} b_{12} b_{11} b_{10} b_{03} b_{02} b_{01} b_{00})_2$

Exercise

Show that the binary expansion of a positive integer can be obtained from its octal expansion by translating each octal digit into a block of three binary digits.

Solution

Let $(\dots d_2 d_1 d_0)_8$ be the octal expansion of a positive integer. The value of that integer is

$$d_0 + d_1 \cdot 8 + d_2 \cdot 8^2 + \dots = d_0 + d_1 \cdot 2^2 + d_2 \cdot 2^6 + \dots$$

If we replace each octal digit d_i by its binary expansion $(b_{i2} b_{i1} b_{i0})_2$, then

$$d_i = b_{i0} + 2b_{i1} + 4b_{i2}$$

Therefore the value of the entire number is

$$\begin{aligned} & b_{00} + 2b_{01} + 4b_{02} + (b_{10} + 2b_{11} + 4b_{12}) \cdot 2^3 + (b_{20} + 2b_{21} + 4b_{22}) \cdot 2^6 + \dots \\ &= b_{00} + 2b_{01} + 4b_{02} + 2^3 b_{10} + 2^4 b_{11} + 2^5 b_{12} + 2^6 b_{20} + 2^7 b_{21} + 2^8 b_{22} + \dots \end{aligned}$$

Which is the value of the binary expansion $(\dots b_{22} b_{21} b_{20} b_{12} b_{11} b_{10} b_{02} b_{01} b_{00})_2$

Exercise

Explain how to convert from binary to base 64 expansions and from base 64 expansions to binary expansions and from octal to base 64 expansions and from base 64 expansions to octal expansions

Solution

$64 = 2^8 = 8^2$, in base 64 we need 64 symbols, from 0 to up to something representing 63.

Corresponding to each such symbol would be a binary string of 6 digits, from 000000 for 0 to 001010 for *a*, 100011 for *z*, 100100 for *A*, 111101 for *Z*, for 111110 for *@*, and 111111 for *\$*.

To translate from binary to base 64, we group the binary digits from the right in groups of 6 and use the list of correspondences to replace each 6 bits by one base-64 digits.

To convert from base 64 to binary, we just replace each base-64 digit by its corresponding 6 bits.

For conversion between octal and base 64, we change the binary strings in the table to octal strings, replacing each 6-bit string by its 2-digit octal equivalent, and then follow the same procedures as above, interchanging base-64 digits and 2-digits strings of octal digits.

Exercise

Find the sum and product of each of these pairs of numbers. Express your answers as a base 3 expansions

a) $(112)_3, (210)_3$

b) $(2112)_3, (12021)_3$

c) $(20001)_3, (1111)_3$

d) $(120021)_3, (2002)_3$

Solution

$$\begin{array}{r} a) \quad \begin{array}{r} 1 \ 1 \ 2 \\ + \ 2 \ 1 \ 0 \\ \hline 1 \ 0 \ 2 \ 2 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 1 \ 1 \ 2 \\ \times \ 2 \ 1 \ 0 \\ \hline 0 \ 0 \ 0 \\ 1 \ 1 \ 2 \\ 1 \ 0 \ 0 \ 1 \\ \hline 1 \ 0 \ 1 \ 2 \ 2 \ 0 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 1 \ 1 \\ \hline 1 \ 1 \ 2 \\ \times \quad \quad 2 \\ \hline 1 \ 0 \ 0 \ 1 \end{array} \end{array}$$

$$\begin{array}{r} b) \quad \begin{array}{r} 2 \ 1 \ 1 \ 2 \\ + \ 1 \ 2 \ 0 \ 2 \ 1 \\ \hline 2 \ 1 \ 2 \ 1 \ 0 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 2 \ 1 \ 1 \ 2 \\ \times \ 1 \ 2 \ 0 \ 2 \ 1 \\ \hline 2 \ 1 \ 1 \ 2 \\ 1 \ 2 \ 0 \ 0 \ 1 \\ 0 \\ 1 \ 2 \ 0 \ 0 \ 1 \\ 2 \ 1 \ 1 \ 2 \\ \hline 1 \ 1 \ 1 \ 0 \ 2 \ 0 \ 1 \ 2 \ 2 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 1 \ 1 \ 1 \ 1 \\ \hline 2 \ 1 \ 1 \ 2 \\ \times \quad \quad 0 \ 2 \\ \hline 1 \ 2 \ 0 \ 0 \ 1 \end{array} \end{array}$$

$$\begin{array}{r} c) \quad \begin{array}{r} 2 \ 0 \ 0 \ 0 \ 1 \\ + \ 1 \ 1 \ 1 \ 1 \\ \hline 2 \ 1 \ 1 \ 1 \ 2 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 2 \ 0 \ 0 \ 0 \ 1 \\ \times \quad \quad 1 \ 1 \ 1 \ 1 \\ \hline 2 \ 0 \ 0 \ 0 \ 1 \\ 2 \ 0 \ 0 \ 0 \ 1 \\ 2 \ 0 \ 0 \ 0 \ 1 \\ 2 \ 0 \ 0 \ 0 \ 1 \\ \hline 2 \ 2 \ 2 \ 2 \ 1 \ 1 \ 1 \ 1 \end{array} \end{array}$$

$$\begin{array}{r} d) \quad \begin{array}{r} \quad \quad 1 \ 1 \\ 1 \ 2 \ 0 \ 0 \ 2 \ 1 \\ + \quad \quad 2 \ 0 \ 0 \ 2 \\ \hline 1 \ 2 \ 2 \ 1 \ 0 \ 0 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 1 \ 2 \ 0 \ 0 \ 2 \ 1 \\ \times \quad \quad 2 \ 0 \ 0 \ 2 \\ \hline 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 2 \\ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 2 \\ \hline 1 \ 0 \ 1 \ 1 \ 1 \ 2 \ 2 \ 1 \ 1 \ 2 \end{array} \end{array}$$

Exercise

Find the sum and product of each of these pairs of numbers. Express your answers as an octal expansion.

a) $(763)_8, (147)_8$

b) $(6001)_8, (272)_8$

c) $(1111)_8, (777)_8$

d) $(54321)_8, (3456)_8$

Solution

$$\begin{array}{r} \text{a)} \quad \quad \quad \textcolor{red}{1} \textcolor{red}{1} \\ \quad \quad \quad 7 \ 6 \ 3 \\ + \quad \quad \quad 1 \ 4 \ 7 \\ \hline \textcolor{blue}{1} \ \textcolor{blue}{1} \ \textcolor{blue}{3} \ \textcolor{blue}{2} \end{array}$$

$$\begin{array}{r} \quad \quad \quad 7 \ 6 \ 3 \\ \times \quad \quad 1 \ 4 \ 7 \\ \hline \quad \quad 5 \ 4 \ 4 \ 5 \\ 3 \ 7 \ 1 \ 4 \\ 7 \ 6 \ 3 \\ \hline \textcolor{blue}{1} \ \textcolor{blue}{4} \ \textcolor{blue}{4} \ \textcolor{blue}{3} \ 0 \ 5 \end{array}$$

$$\begin{array}{r} \text{b)} \quad \quad \quad 6 \ 0 \ 0 \ 1 \\ + \quad \quad \quad 2 \ 7 \ 2 \\ \hline \textcolor{blue}{6} \ \textcolor{blue}{2} \ \textcolor{blue}{7} \ \textcolor{blue}{3} \end{array}$$

$$\underline{(6001)_8 + (272)_8 = \textcolor{blue}{6273} \mid}$$

$$6001 = 6 \cdot 8^3 + 1 = 3073$$

$$272 = 2 \cdot 8^2 + 7 \cdot 8 + 2 = 186$$

$$6001 \cdot 272 = 3073 \cdot 186 = 571,578$$

$$571,578 = 8 \times 71447 + 2$$

$$71447 = 8 \times 8930 + 7$$

$$8930 = 8 \times 1116 + 2$$

$$1116 = 8 \times 139 + 4$$

$$139 = 8 \times 17 + 3$$

$$17 = 8 \times 2 + 1$$

$$2$$

$$\underline{(6001)_8 \cdot (272)_8 = \textcolor{blue}{2,134,272} \mid}$$

$$\begin{array}{r} \text{c)} \quad \quad \quad \textcolor{red}{1} \ \textcolor{red}{1} \ \textcolor{red}{1} \\ \quad \quad \quad 1 \ 1 \ 1 \ 1 \\ + \quad \quad \quad 7 \ 7 \ 7 \\ \hline \textcolor{blue}{2} \ \textcolor{blue}{1} \ \textcolor{blue}{1} \ 0 \end{array}$$

$$\underline{(1111)_8 + (777)_8 = \textcolor{blue}{2110} \mid}$$

$$(1111)_8 = 1 \cdot 8^3 + 1 \cdot 8^2 + 1 \cdot 8 + 1 = 585$$

$$(777)_8 = 7 \cdot 8^2 + 7 \cdot 8 + 7 = 511$$

$$(1111)_8 \cdot (777)_8 = (585)(511) = 298,935$$

$$298935 = 8 \times 37366 + 7$$

$$37366 = 8 \times 4670 + 6$$

$$4670 = 8 \times 583 + 6$$

$$583 = 8 \times 72 + 7$$

$$72 = 8 \times 9 + 0$$

$$9 = 8 \times 1 + 1$$

$$1$$

$$\underline{(1111)_8 \cdot (777)_8 = \textcolor{blue}{1,107,667} \mid}$$

d)

$$\begin{array}{r}
 5 \ 4 \ 3 \ 2 \ 1 \\
 + \quad 3 \ 4 \ 5 \ 6 \\
 \hline
 5 \ 7 \ 7 \ 7 \ 7
 \end{array}$$

$$(54321)_8 + (3456)_8 = 57,777$$

$$(54321)_8 = 5 \cdot 8^4 + 4 \cdot 8^3 + 3 \cdot 8^2 + 2 \cdot 8 + 1 = 22,737$$

$$(3456)_8 = 3 \cdot 8^3 + 4 \cdot 8^2 + 5 \cdot 8 + 6 = 1838$$

$$(54321)_8 \cdot (3456)_8 = (22,737)(1838) = 41,790,606$$

$$41790606 = 8 \times 5223825 + 6$$

$$5223825 = 8 \times 652978 + 1$$

$$652978 = 8 \times 81622 + 2$$

$$81622 = 8 \times 10202 + 6$$

$$10202 = 8 \times 1275 + 2$$

$$1275 = 8 \times 159 + 3$$

$$159 = 8 \times 19 + 7$$

$$19 = 8 \times 2 + 3$$

$$2$$

$$(54321)_8 \cdot (3456)_8 = 237,326,216$$

Exercise

Find the sum and product of each of these pairs of numbers. Express your answers as a hexadecimal expansion.

a) $(1AB)_{16}, (BBC)_{16}$

b) $(20CBA)_{16}, (A01)_{16}$

c) $(ABCDE)_{16}, (1111)_{16}$

d) $(E0000E)_{16}, (BAAA)_{16}$

Solution

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

a) $1AB = 1 \cdot 16^2 + 10 \cdot 16 + 11 = 427$

$$BBC = 11 \cdot 16^2 + 11 \cdot 16 + 12 = 3004$$

$$\begin{aligned}
 1AB + BBC &= 427 + 3004 \\
 &= 3431
 \end{aligned}$$

$$3431 = 16 \times 214 + 7$$

$$214 = 16 \times 14 + 6$$

$$14$$

$$14 = D$$

$$1AB + BBC = D67$$

$$\begin{aligned}
 (1AB) \times (BBC) &= (427)(3004) \\
 &= 1,282,708
 \end{aligned}$$

$$1282708 = 16 \times 80169 + 4$$

$$80169 = 16 \times 5010 + 9$$

$$5010 = 16 \times 313 + 2$$

$$313 = 16 \times 19 + 9$$

$$19 = 16 \times 1 + 3$$

$$1$$

$$(1AB) \times (BBC) = 139,294$$

$$b) (20CBA)_{16} = 2 \cdot 16^4 + 0 \cdot 16^3 + 12 \cdot 16^2 + 11 \cdot 16 + 10 = 134,330$$

$$(A01)_{16} = 10 \cdot 16^2 + 0 \cdot 16 + 12 = 2,561$$

$$(20CBA)_{16} + (A01)_{16} = 134,330 + 2,561 \\ = 136,891$$

$$136891 = 16 \times 8555 + 11 \quad 11 = B$$

$$8555 = 16 \times 534 + 11 \quad 11 = B$$

$$534 = 16 \times 33 + 6$$

$$33 = 16 \times 2 + 1$$

$$2$$

$$(20CBA)_{16} + (A01)_{16} = \underline{21,6BB}$$

$$(20CBA)_{16} \times (A01)_{16} = (134,330)(2,561) \\ = 344,019,130$$

$$344019130 = 16 \times 21501195 + 10 \quad 10 = A$$

$$21501195 = 16 \times 1343824 + 11 \quad 11 = B$$

$$1343824 = 16 \times 83989 + 0$$

$$83989 = 16 \times 5249 + 5$$

$$5249 = 16 \times 328 + 1$$

$$328 = 16 \times 20 + 8$$

$$20 = 16 \times 1 + 4$$

$$1$$

$$(20CBA)_{16} \times (A01)_{16} = \underline{14,815,0BA}$$

$$c) (ABCDE)_{16} = 10 \cdot 16^4 + 11 \cdot 16^3 + 12 \cdot 16^2 + 13 \cdot 16 + 14 = 703,710$$

$$(1111)_{16} = 1 \cdot 16^3 + 1 \cdot 16^2 + 1 \cdot 16 + 1 = 4369$$

$$(ABCDE)_{16} + (1111)_{16} = 703,710 + 4369 \\ = 708,079$$

$$708079 = 16 \times 44254 + 15 \quad 15 = F$$

$$44254 = 16 \times 2765 + 14 \quad 14 = E$$

$$2765 = 16 \times 172 + 13 \quad 13 = D$$

$$172 = 16 \times 10 + 12 \quad 12 = C$$

$$10 \quad 10 = A$$

$$(ABCDE)_{16} + (1111)_{16} = \underline{AC,DEF}$$

$$(ABCDE)_{16} \times (1111)_{16} = (703,710)(4369) \\ = 3,074,508,990$$

$$3074508990 = 16 \times 192156811 + 14 \quad 14 = E$$

$$192156811 = 16 \times 12009800 + 11 \quad 11 = B$$

$$12009800 = 16 \times 750612 + 8$$

$$750612 = 16 \times 46913 + 4$$

$$46913 = 16 \times 2932 + 1$$

$$2932 = 16 \times 183 + 4$$

$$183 = 16 \times 11 + 7$$

$$11 \quad \quad \quad 11 = B$$

$$(ABCDE)_{16} \times (1111)_{16} = \underline{B7,414,8BE}$$

d) $(E0000E)_{16} = 14 * 16^5 + 14 = 14,680,078$

$$(BAAA)_{16} = 11 * 16^3 + 10 * 16^2 + 10 * 16 + 10 = 47,786$$

$$(E0000E)_{16} + (BAAA)_{16} = 14,680,078 + 47,786 \\ = 14,727,864$$

$$14727864 = 16 \times 920491 + 8$$

$$920491 = 16 \times 57530 + 11 \quad 11 = B$$

$$57530 = 16 \times 3595 + 10 \quad 10 = A$$

$$3595 = 16 \times 224 + 11 \quad 11 = B$$

$$224 = 16 \times 14 + 0$$

$$14 \quad \quad \quad 14 = E$$

$$(E0000E)_{16} + (BAAA)_{16} = \underline{E0B,AB8}$$

$$(E0000E)_{16} (BAAA)_{16} = (14,680,078)(47,786) \\ = 701,502,207,308$$

$$701502207308 = 16 \times 43843887956 + 12 \quad 12 = C$$

$$43843887956 = 16 \times 2740242997 + 4$$

$$2740242997 = 16 \times 171265187 + 5$$

$$171265187 = 16 \times 10704074 + 3$$

$$10704074 = 16 \times 669004 + 10 \quad 10 = A$$

$$669004 = 16 \times 41812 + 12 \quad 12 = C$$

$$41812 = 16 \times 2613 + 4$$

$$2613 = 16 \times 163 + 5$$

$$163 = 16 \times 10 + 3$$

$$10 \quad \quad \quad 10 = A$$

$$(E0000E)_{16} \times (BAAA)_{16} = \underline{A,354,CA3,54C}$$