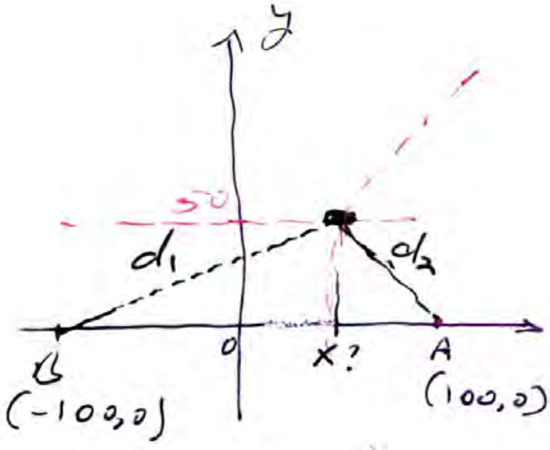
$v = 980 \ ft/\mu sec$

$t = 400 \ \mu sec$



$d_2 - d_1 = 980 \times 400$

$= 392 \times 10^3 = 2a$

$a = 196 \times 10^3 \ \frac{1 \ mi}{5280 ft}$

$\approx 37.\overline{12}$

$v = \frac{d}{t}$

$a^2 \approx 1378.$

$c = 100 \Rightarrow c^2 = 10^4$

$\} \ a^2 + b^2 = c^2$

$b^2 = c^2 - a^2$

$= 10^4 - 1378$

≈ 8622

$$\frac{x^2}{1378} - \frac{y^2}{8622} = 1$$

$$\frac{x^2}{1378} = 1 + \frac{50^2}{8622}$$

$$x = \sqrt{1378\left(1 + \frac{50^2}{8622}\right)}$$

≈ 42.16

$P(42, 50)$

27/ $625y^2 - 400x^2 = 250{,}000$

$2a$?

$$\frac{y^2}{400} - \frac{x^2}{625} = 1$$

$a^2 = 400$ $\qquad \sqrt{\underbrace{00}_{1/2}}$

$a = 20$

distance $= 2a = 40$

5.5

Sequence $\qquad$ Infinite

$$a_1, a_2, \dots, a_n, \dots$$

Ex 1st 4 terms & 10th term $\left\{\frac{n}{n+1}\right\}$

$n = 1 \rightarrow a_1 = \frac{1}{2}$

$n = 2 \rightarrow a_2 = \frac{2}{3}$

$n = 3 \rightarrow a_3 = \frac{3}{3+1} = \frac{3}{4}$

$n = 4 \rightarrow a_4 = \frac{4}{5}$

$n = 10 \rightarrow a_{10} = \frac{10}{11}$

$$\frac{1}{2}, \frac{2}{3}, \dots, \frac{n}{n+1}, \dots$$

Ex 1st 4 term a_{10} $\{2+(.1)^n\}$

$a_1 = 2 + .1 = 2.1$

$a_2 = 2 + (.1)^2 = 2 + .01 = 2.01$

$a_3 = 2 + (.1)^3 = 2 + .001 = 2.001$

$a_4 = 2 + (.1)^4 = 2 + .0001 = 2.0001$

$a_{10} = 2 + (.1)^{10} = 2.0000000001$

Ex $\left\{(-1)^{n+1} \dfrac{n^2}{3n-1}\right\}$

$a_1 = (-1)^2 \dfrac{1}{3-1} = \dfrac{1}{2}$

$a_2 = (-1)^3 \dfrac{4}{5} = -\dfrac{4}{5}$

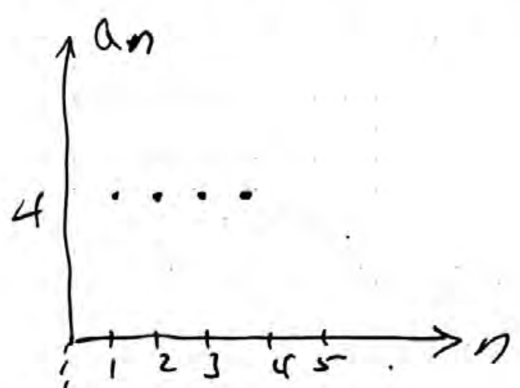
$a_3 = (-1)^4 \dfrac{9}{8} = \dfrac{9}{8}$

$a_4 = (-1)^5 \dfrac{16}{11} = -\dfrac{16}{11}$

$a_{10} = (-1)^{11} \dfrac{100}{29} = -\dfrac{100}{29}$

Ex $\{4\}$

$a_1 = 4$, $a_2 = 4$, $a_3 = 4$, $a_4 = 4$



$\underline{Ex}$ 1^{st} 4 $a_1 = 3$ $a_{n+1} = (n+1)a_n$

$\underline{a_1 = 3}$

$n=1$ $a_2 = 2a_1 = 2(3) = \underline{6}$

$n=2$ $a_3 = 3a_2 = 3(6) = \underline{18}$

$n=3$ $a_4 = 4a_3 = 4(18) = \underline{72}$

Summation Notation

$$a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$$

last value

a_n : formula

$$\sum_{n=1}^{5} (2n+3) = \overset{n=1}{(2+3)} + \overset{n=2}{7} + \overset{3}{9} + \overset{4}{11} + \overset{5}{13}$$

$$= \underline{45}$$

1st value

$$\sum_{k=1}^{4} k^2(k-3) = \overset{k=1}{1(-2)} + 4(-1) + 9(0) + 16(1)$$

$$= -2 - 4 + 16$$

$$= \underline{10}$$

$$\sum_{k=1}^{n} c = \overbrace{c + \cdots + c}^{n \text{ times}}$$

$$= nc$$

$$\sum_{k=m}^{n} c = c(n - m + 1)$$

$$\sum_{k=10}^{20} 5 = 5\,(20-10+1)$$
$$= 5\,(11)$$
$$= \underline{55}$$

$$2^1 + 2^2 + 2^3 + \cdots + 2^{16} = \sum_{n=1}^{16} 2^n$$

$$2^1 + 2^2 + \cdots + 2^n + \cdots = \sum_{n=1}^{\infty} 2^n = \sum 2^n$$

$$\sum a_n = \sum_{n=1}^{\infty} a_n$$

5.6

Arithmetic Sequence. $a_1, \ldots, a_n, \ldots$

$k \in \mathbb{Z}^+$

$$a_{k+1} = a_k + d \rightarrow \text{common difference}$$

__Ex__ $1, 4, 7, 10, \ldots, 3n-2, \ldots$

$$d = 4 - 1 = 3 \qquad a_2 - a_1$$

$$a_{k+1} = a_k + d$$

$$a_{k+1} - a_k = 3(k+1) - 2 - (3k-2)$$
$$= 3k + 3 - 2 - 3k + 2$$
$$= 3$$

Ex fourth term

Find

Given: $a_4 = 5$, $a_9 = 20$ a_6?

$a_n = a_1 + (n-1)d$

$a_4 = a_1 + 3d = 5$

$a_9 = a_1 + 8d = 20$

$5d = 15$

$\underline{d = 3}$

$\Rightarrow d = \frac{20-5}{9-4} = \frac{15}{5}$

$a_4 = a_1 + 3(3) = 5$

$\underline{a_1 = -4}$

$a_6 = -4 + 5(3)$

$\underline{= 11}$

#22 a_{20}: $a_9 = -5$ $a_{15} = 31$

$d = \frac{31+5}{15-9} = \underline{6}$ $\frac{36}{6}$

$a_9 = a_1 + 8(6) = -5$

$\Rightarrow \underline{a_1 = -53}$

$a_{20} = -53 + 19(6)$

$\underline{= 61}$

Theorem

$$S_n = \frac{1}{2} n (a_1 + a_n)$$
$$= \frac{n}{2} (2a_1 + (n-1)d)$$

$$S_n = a_1 + a_2 + \cdots + a_n$$
$$= a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + a_1 + (n-1)d$$
$$= \underbrace{a_1 + \cdots + a_1} + (d + 2d + \cdots + (n-1)d)$$
$$= n a_1 + d(1 + 2 + \cdots + (n-1))$$
$$= n a_1 + d \frac{(n-1)n}{2}$$
$$= \frac{n}{2} (2a_1 + (n-1)d)$$

Ex sum Even $2 \to 100$

$$S_n = \frac{50}{2} (2 + 100)$$
$$= 2550$$

$$\frac{1}{4} + \frac{2}{9} + \frac{3}{14} + \frac{4}{19} + \frac{5}{24} + \frac{6}{29} = \sum_{n=1}^{6} \frac{n}{5n-1}$$

$n \to 1, 2, 3, 4, 5, 6 \to n$

den $4, 9, 14, 19, 24, 29$

$d = 5$

$a_n = 4 + (n-1)5$

$= 5n - 5 + 4$

$= 5n - 1$

Geometric Seq.

common
$\to$ ratio

$$a_{k+1} = a_k r$$

$$r = \frac{a_{k+1}}{a_k}$$

$$a_n = a_1 r^{n-1} =$$

$a_1 = 3 \quad r = -\frac{1}{2}$ $\qquad a_n = 3\left(-\frac{1}{2}\right)^{n-1}$

$$a_1 = 3$$

$$a_2 = 3\left(-\frac{1}{2}\right) = -\frac{3}{2}$$

$$a_3 = 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$$

<u>Ex</u> $\quad a_3 = +5 \quad a_6 = -40 \quad a_8 = ?$

<u>Geom.</u>

$$a_3 = a_1 r^2 = 5$$

$$a_6 = a_1 r^5 = -40$$

$$\frac{a_1 r^5}{a_1 r^2} = \frac{-40}{5}$$

$$r^3 = -8 \qquad 2^3$$

$$r = -2$$

$$r = \left(-\frac{40}{5}\right)^{\frac{1}{6-3}} = (-8)^{1/3} = -2$$

$$a_1 (-2)^2 = 5 \Rightarrow a_1 = \frac{5}{4}$$

$$a_8 = \frac{5}{4}(-2)^7 = -5(2^5)$$

$$= -160$$