Solution Section 1.6 – Inequalities

Exercise

Find: $(-3, 0) \cap [-1, 2]$

Solution

 $(-3, 0) \cap [-1, 2] = [-1, 0)$

Exercise

Find: $(-3, 0) \cup [-1, 2]$

Solution

 $(-3, 0) \cup [-1, 2] = (-3, 2]$

Exercise

Find: $(-4, 0) \cap [-2, 1]$

Solution

 $(-4, 0) \cap [-2, 1] = [-2, 0)$

Exercise

Find: $(-4, 0) \cup [-2, 1]$

Solution

 $(-4, 0) \cup [-2, 1] = (-4, 1]$

Exercise

Find: $(-\infty, 5) \cap [1, 8)$

Solution

 $(-\infty, 5) \cap [1, 8) = [1, 5)$

Find: $(-\infty, 5) \cup [1, 8)$

Solution

 $(-\infty, 5) \cup [1, 8) = (-\infty, 8)$

Exercise

Solve -3x + 5 > -7

Solution

-3x > -7 - 5

$$-3x > -12$$

$$\frac{-3}{-3}x < \frac{-12}{-3}$$

 $\therefore Solution: \underline{x < 4} \qquad (-\infty, 4)$

Exercise

 $2 - 3x \le 5$ Solve

Solution

 $-3x \le 3$

Divide by -3 both sides

$$\frac{-3}{-3}x \ge \frac{3}{-3}$$

∴ Solution: $\underline{x \ge -1}$ or $[-1, \infty)$

Exercise

Solve $4-3x \le 7+2x$

Solution

 $4 - 3x - 4 \le 7 + 2x - 4$

$$-3x \le 3 + 2x$$

$$-3x - 2x \le 3 + 2x - 2x$$

$$-5x \le 3$$

 $\therefore Solution: \quad x \ge -\frac{3}{5} \quad or \quad \left[-\frac{3}{5}, \infty \right)$

Solve the inequality equation 5x + 11 < 26

Solution

 \therefore Solution: $\underline{x < 3}$ or $(-\infty, 3)$

Exercise

Solve the inequality equation $3x - 8 \ge 13$

Solution

$$3x \ge 21$$

 $\therefore Solution: \quad \underline{x \geq 7} \quad or \quad [7, \infty)$

Exercise

Solve the inequality equation $-9x \ge 36$

Solution

$$-\frac{9x}{9} \ge \frac{36}{9}$$

$$-x \ge 4$$

 $\therefore Solution: \quad \underline{x \leq -4} \quad or \quad (-\infty, -4]$

Exercise

Solve the inequality equation $-4x \le 64$

Solution

$$-\frac{4x}{4} \le \frac{64}{4}$$

$$-x \le 16$$

∴ Solution: $\underline{x \ge -16}$ or $[-16, \infty)$

Exercise

Solve the inequality equation $8x - 11 \le 3x - 13$

$$5x \le -2$$

$$\therefore Solution: \quad \underline{x \le -\frac{2}{5}} \quad or \quad \left(-\infty, -\frac{2}{5}\right]$$

Solve the inequality equation $18x + 45 \le 12x - 8$

Solution

$$6x \le -53$$

$$\therefore Solution: \quad \underline{x \le -\frac{53}{6}} \quad or \left(-\infty, -\frac{53}{6}\right]$$

Exercise

Solve the inequality equation $4(x+1)+2 \ge 3x+6$

Solution

$$4x + 4 + 2 \ge 3x + 6$$

 $\therefore Solution: \quad \underline{x \ge 0} \quad or \quad [0, \infty)$

Exercise

Solve the inequality equation 8x + 3 > 3(2x + 1) + x + 5

Solution

$$8x + 3 > 6x + 3 + x + 5$$

$$8x + 3 > 7x + 8$$

 $\therefore Solution: \quad \underline{x > 5} \quad or \quad (5, \infty)$

Exercise

Solve the inequality equation 2x-11 < -3(x+2)

Solution

$$2x-11 < -3x-6$$

∴ Solution: $\underline{x < 1}$ or $(-\infty, 1)$

Solve the inequality equation -4(x+2) > 3x + 20

Solution

$$-4x - 8 > 3x + 20$$

$$-7x > 28$$

$$-x > 4$$

∴ Solution: $\underline{x < -4}$ or $(-\infty, -4)$

Exercise

Solve the inequality equation $1-(x+3) \ge 4-2x$

Solution

$$1-x-3 \ge 4-2x$$

$$-x-2 \ge 4-2x$$

$$x \ge 6$$

$$\therefore Solution: \quad \underline{x \ge 6} \quad or \quad [6, \infty)$$

Exercise

Solve the inequality equation $5(3-x) \le 3x-1$

Solution

$$15 - 5x \le 3x - 1$$

$$-8x \le -16$$

$$-x \le -2$$

 $\therefore Solution: \quad \underline{x \ge 2} \quad or \quad [2, \infty)$

Exercise

Solve the inequality equation $\frac{x}{4} - \frac{1}{2} \le \frac{x}{2} + 1$

Solution

$$4 \times \frac{x}{4} - \frac{1}{2} \le \frac{x}{2} + 1$$

$$x - 2 \le 2x + 4$$

$$-x \le 6$$

 $\therefore Solution: \quad \underline{x \ge -6} \quad or \quad [-6, \infty)$

Solve the inequality equation $\frac{3x}{10} + 1 \ge \frac{1}{5} - \frac{x}{10}$

Solution

$$10 \times \quad \frac{3x}{10} + 1 \ge \frac{1}{5} - \frac{x}{10}$$

$$3x + 10 \ge 2 - x$$

$$4x \ge -8$$

∴ Solution:
$$\underline{x \ge -2}$$
 or $[-2, \infty)$

Exercise

Solve
$$6x - (2x + 3) \ge 4x - 5$$

Solution

$$6x - 2x - 3 \ge 4x - 5$$

$$4x-3 \ge 4x-5$$

$$4x - 4x \ge 3 - 5$$

$$0 \ge -2$$
 (true)

∴ Solution:
$$\mathbb{R}$$
 $(-\infty, \infty)$

Exercise

Solve
$$\frac{2x-5}{-8} \le 1-x$$

Solution

$$(-8)\frac{2x-5}{-8} \ge (-8)(1-x)$$

$$2x-5 \ge -8+8x$$

$$2x - 8x \ge -8 + 5$$

$$-6x \ge -3$$

$$\frac{-6}{-6}x \le \frac{-3}{-6}$$

$$\therefore Solution: \quad \underline{x \leq \frac{1}{2}} \qquad \left(-\infty, \ \frac{1}{2}\right]$$

Exercise

Solve the inequality equation $1 - \frac{x}{2} > 4$

$$2 \times 1 - \frac{x}{2} > 4$$

$$2 - x > 8$$

$$-x > 6$$

$$\therefore Solution: \quad \underline{x < -6} \quad or \quad (-\infty, -6)$$

Solve the inequality equation $7 - \frac{4}{5}x < \frac{3}{5}$

Solution

$$5 \times 7 - \frac{4}{5}x < \frac{3}{5}$$

$$35 - 4x < 3$$

$$-4x > -32$$

$$\therefore$$
 Solution: $x < 8$ or $(-\infty, 8)$

Exercise

Solve the inequality equation $\frac{x-4}{6} \ge \frac{x-2}{9} + \frac{5}{18}$

Solution

$$18 \times \frac{x-4}{6} \ge \frac{x-2}{9} + \frac{5}{18}$$

$$3x - 12 \ge 2x - 4 + 5$$

$$3x - 12 \ge 2x + 1$$

∴ Solution:
$$x \ge -13$$
 or $[-13, \infty)$

Exercise

Solve the inequality equation $\frac{4x-3}{6} + 2 \ge \frac{2x-1}{12}$

$$\frac{12}{6} \times \frac{4x-3}{6} + 2 \ge \frac{2x-1}{12}$$

$$8x - 6 + 24 \ge 2x - 1$$

$$8x + 18 \ge 2x - 1$$

$$6x \ge -19$$

$$\therefore Solution: \quad \underline{x \ge -\frac{19}{6}} \quad or \left[-\frac{19}{6}, \infty \right)$$

Solve the inequality equation

$$4(3x-2)-3x < 3(1+3x)-7$$

Solution

$$12x - 8 - 3x < 3 + 9x - 7$$

$$9x - 8 < 9x - 4$$

∴ Solution: \mathbb{R}

Exercise

Solve the inequality equation
$$3(x-8)-2(10-x)<5(x-1)$$

Solution

$$3x - 24 - 20 + 2x < 5x - 5$$

$$5x - 44 < 5x - 5$$

∴ Solution: \mathbb{R}

Exercise

Solve the inequality equation $8(x+1) \le 7(x+5) + x$

$$8(x+1) \le 7(x+5) + x$$

Solution

$$8x + 8 \le 7x + 35 + x$$

 \mathbb{R}

$$8x + 8 \le 8x + 35$$

$$8 \le 35$$

∴ Solution:

Exercise

Solve the inequality equation $4(x-1) \ge 3(x-2) + x$

$$4(x-1) \ge 3(x-2) + x$$

$$4x - 4 \ge 3x - 6 + x$$

$$4x - 4 \ge 4x - 6$$

$$-4 \ge -6$$
 True

Solve the inequality equation 7(x+4)-13>12+13(3+x)

Solution

$$7x + 28 - 13 > 12 + 39 + 13x$$

 $7x + 15 > 51 + 13x$
 $-6x > 36$

 \therefore Solution: $\underline{x < -9}$ or $(-\infty, -9)$

Exercise

Solve the inequality equation -2[7x-(2x-3)]<-2(x+1)

Solution

$$-2(7x-2x+3) < -2x-2$$

$$-2(5x+3) < -2x-2$$

$$-10x-6 < -2x-2$$

$$-8x < 4$$

 $\therefore Solution: x > \frac{1}{2} or \left(\frac{1}{2}, \infty\right)$

Exercise

Solve the inequality equation $6 - \frac{2}{3}(3x - 12) \le \frac{2}{5}(10x + 50)$

Solution

15 ×
$$6 - \frac{2}{3}(3x - 12) \le \frac{2}{5}(10x + 50)$$

$$90-10(3x-12) \le 6(10x+50)$$

$$90 - 30x + 120 \le 60x + 300$$

$$210 - 30x \le 60x + 300$$

$$-90x \le 90$$

∴ Solution: $x \ge -1$ or $[-1, \infty)$

Exercise

Solve the inequality equation $\frac{2}{7}(7-21x)-4<10-\frac{3}{11}(11x-11)$

$$77 \times \frac{2}{7}(7-21x)-4<10-\frac{3}{11}(11x-11)$$

$$22(7-21x)-308 < 770-21(11x-11)$$

$$154 - 462x - 308 < 770 - 231x + 231$$

$$-462x - 154 < -231x + 1,001$$

$$-231x < 1,155$$

$$-x < 5$$

$$\therefore$$
 Solution: $x > 5$ or $(5, \infty)$

Solve the inequality equation
$$3[3(x+5)+8x+7]+5[3(x-6)-2(3x-5)]<2(4x+3)$$

Solution

$$3(3x+15+8x+7)+5(3x-18-6x+10)<8x+6$$

$$3(11x+22)+5(-3x-8)<8x+6$$

$$33x + 66 - 15x - 40 < 8x + 6$$

$$18x + 26 < 8x + 6$$

$$10x < -20$$

∴ Solution:
$$\underline{x < -2}$$
 or $(-\infty, -2)$

Exercise

Solve the inequality equation
$$5[3(2-3x)-2(5-x)]-6[5(x-2)-2(4x-3)]<3x+19$$

Solution

$$5(6-9x-10+2x)-6(5x-10-8x+6)<3x+19$$

$$5(-7x-4)-6(-3x-4)<3x+19$$

$$-35x - 20 + 18x + 24 < 3x + 19$$

$$-17x + 4 < 3x + 19$$

$$-20x < 15$$

$$\therefore Solution: \qquad x > -\frac{3}{4} \quad or \quad \left(-\frac{3}{4}, \infty\right)$$

Exercise

Solve the inequality equation $0 \le 3x - 1 \le 10$

$$0 + 1 \le 3x - 1 + 1 \le 10 + 1$$

$$1 \le 3x \le 11$$

$$\therefore Solution: \qquad \frac{1}{3} \le x \le \frac{11}{3}$$

Solve the inequality equation $0 \le 1 - 3x \le 10$

Solution

$$-1 \le -3x \le 9$$

$$-\frac{1}{3} \le -x \le 3$$

$$\therefore Solution: \qquad -3 \le x \le \frac{1}{3}$$

Exercise

Solve the inequality equation $0 \le 2x + 6 \le 54$

Solution

$$-6 \le 2x \le 48$$

∴ *Solution*:
$$-3 \le x \le 24$$

Exercise

Solve the inequality equation $-3 \le \frac{2}{3}x - 5 \le -1$

$$-3 \le \frac{2}{3}x - 5 \le -1$$

$$-3+5 \le \frac{2}{3}x-5+5 \le -1+5$$

$$2 \le \frac{2}{3}x \le 4$$

$$2\frac{3}{2} \le \frac{3}{2} \frac{2}{3} x \le \frac{3}{2} 4$$

∴ Solution:
$$3 \le x \le 6$$

Solve $-6 \le 6x + 3 \le 21$

Solution

$$-6-3 \le 6x+3-3 \le 21-3$$

$$-9 \le 6x \le 18$$

$$-\frac{9}{6} \le \frac{6}{6}x \le \frac{18}{6}$$

$$\therefore Solution: \quad -\frac{3}{2} \le x \le 3 \qquad \left[-\frac{3}{2}, \ 3 \right]$$

Exercise

Solve the inequality equation: $1 \le 2x + 3 < 11$

Solution

$$1 - 3 \le 2x + 3 - 3 < 11 - 3$$

$$-2 \le 2x < 8$$

$$-\frac{2}{2} \le \frac{2}{2}x < \frac{8}{2}$$

∴ Solution:
$$-1 \le x < 4$$

Exercise

Solve the inequality equation |x| < 2

Solution

$$\therefore Solution: \quad \underline{-2 < x < 2}$$

Exercise

Solve the inequality equation $|x| \ge 2$

Solution

$$\therefore Solution: \underline{x \le -2 \quad x \ge 2}$$

Exercise

Solve the inequality equation |x-2| < 1

$$-1 < x - 2 < 1$$

 $\therefore Solution: \ \ \underline{1 < x < 3} \ \ |$

Exercise

Solve the inequality equation |x-1| < 4

Solution

$$-4 < x - 1 < 4$$

 $\therefore Solution: \quad \underline{-3 < x < 5}$

Exercise

Solve the inequality equation $|x+2| \ge 1$

Solution

$$x + 2 \le -1 \qquad \qquad x + 2 \ge 1$$

$$x + 2 \ge 1$$

$$x \le -3$$
 $x \ge -1$

$$x \ge -1$$

$$\therefore Solution: \quad \underline{x \le -3 \quad x \ge -1}$$

Exercise

 $|x+1| \ge 4$ Solve the inequality equation

Solution

$$x+1 \le -4 \qquad \qquad x+1 \ge 4$$

$$x+1 \ge 4$$

$$x \le -5$$

$$x \ge 3$$

$$\therefore Solution: \quad \underline{x \le -5} \quad x \ge 3$$

Exercise

Solve the inequality equation |3x + 5| < 17

$$-17 < 3x + 5 < 17$$

$$-22 < 3x < 12$$

$$\therefore Solution: \quad \underline{-11 < x < 4}$$

Solve the inequality equation |5x-2| < 13

Solution

$$-13 < 5x - 2 < 13$$

$$-11 < 5x < 15$$

$$\therefore Solution: \quad -\frac{11}{5} < x < 3$$

Exercise

Solve the inequality equation $|5x-2| \ge 13$

Solution

$$5x - 2 \le -13$$
 $5x - 2 \ge 13$

$$5x - 2 \ge 13$$

$$5x \le -11$$

$$5x \ge 15$$

$$3x \le -11 \qquad 3x \ge 1$$
$$x \le -\frac{11}{5} \qquad x \ge 3$$

$$x \ge 3$$

$$\therefore Solution: \quad x \le -\frac{11}{5} \quad x \ge 3$$

Exercise

Solve the inequality equation $|2(x-1)+4| \le 8$

Solution

$$-8 \le 2x - 2 + 4 \le 8$$

$$-8 \le 2x + 2 \le 8$$

$$-10 \le 2x \le 6$$

$$\therefore$$
 Solution: $-5 \le x \le 3$

Exercise

Solve the inequality equation $|3(x-1)+2| \le 20$

$$-20 \le 3x - 3 + 2 \le 20$$

$$-20 \le 3x - 1 \le 20$$

$$-19 \le 3x \le 21$$

$$\therefore Solution: \quad -\frac{19}{3} \le x \le 7$$

 $\left|\frac{2x+6}{3}\right| > 2$ Solve the inequality equation

Solution

$$|2x+6| > 6$$

$$2x + 6 < -6$$
 $2x + 6 > 6$

$$2x + 6 > 6$$

$$2x < -12$$

$$2x > 0$$
$$x > 0$$

$$\therefore Solution: \quad \underline{x < -6} \quad x > 0$$

Exercise

Solve the inequality equation

$$\left|\frac{3x-3}{4}\right| < 6$$

Solution

$$\frac{3}{4}|x-1| < 6$$

$$|x-1| < 8$$

$$-8 < x - 1 < 8$$

∴ Solution: -7 < x < 9

Exercise

Solve the inequality equation

$$\left|\frac{2x+2}{4}\right| \ge 2$$

Solution

$$\frac{1}{2}|x+1| \ge 2$$

$$|x+1| \ge 4$$

$$x+1 \le -4 \qquad \qquad x \ge 3$$

$$x \ge 3$$

$$\therefore Solution: \underline{x \le -4 \quad x \ge 3}$$

Exercise

Solve the inequality equation

$$\left|\frac{3x-3}{9}\right| \le 1$$

$$\frac{1}{3} \left| x - 1 \right| \le 1$$

$$|x-1| \le 3$$

$$-3 \le x - 1 \le 3$$

∴ Solution:
$$-2 \le x \le 4$$

Solve the inequality equation
$$\left| 3 - \frac{2x}{3} \right| > 5$$

Solution

$$\frac{1}{3}|9-2x|>5$$

$$|9 - 2x| > 15$$

$$9-2x < -15$$
 $9-2x > 15$
 $-2x < -24$ $-2x > 6$
 $x > 12$ $x < -3$

$$9 - 2x > 15$$

$$-2x < -24$$

$$-2x > 6$$

$$x < -3$$

$$\therefore Solution: \quad x < -3 \quad x > 12$$

Exercise

Solve the inequality equation
$$\left| 3 - \frac{3x}{4} \right| < 9$$

Solution

$$\frac{3}{4} \left| 4 - x \right| < 9$$

$$|4-x| < 12$$

$$-12 < 4 - x < 12$$

$$-16 < -x < 8$$

$$16 > x > -8$$

$$\therefore$$
 Solution: $-8 < x < 16$

Exercise

Solve the inequality equation |x-2| < -1

Solution

Impossible, since Absolute value can't be negative.

∴ No Solution

Solve the inequality equation |x+2| < -3

Solution

Impossible, since Absolute value can't be negative.

∴ No Solution

Exercise

Solve the inequality equation |x+6| > -10

Solution

∴ Solution: ℝ |

Exercise

Solve the inequality equation |x+2| > -8

Solution

∴ Solution: ℝ |

Exercise

Solve the inequality equation $|x+2| + 9 \le 16$

Solution

$$|x+2| \le 5$$

$$-5 \le x + 2 \le 5$$

∴ Solution: $-7 \le x \le 3$

Exercise

Solve the inequality equation $|x-2|+4 \ge 5$

Solution

$$|x-2| \ge 1$$

 $x-2 \le -1$ $x - 2 \ge 1$
 $x \le 1$ $x \ge 3$

 $\therefore Solution: \underline{x \le 1 \quad x \ge 3}$

Solve the inequality equation 2|2x-3|+10>12

Solution

$$2|2x-3|>2$$

$$|2x-3| > 1$$

$$2x-3 < -1$$
 $2x-3 > 1$ $2x < 2$ $2x > 4$

$$2x - 3 > 1$$

$$\therefore Solution: x < 1 x > 3$$

Exercise

Solve the inequality equation 3|2x-1|+2<8

$$3|2x-1|+2<8$$

Solution

$$3|2x-1| < 6$$

$$|2x-1| < 2$$

$$-2 < 2x - 1 < 2$$

$$-1<2x<3$$

$$\therefore Solution: -\frac{1}{2} < x < \frac{3}{2}$$

Exercise

Solve the inequality equation -4|1-x| < -16

$$-4|1-x|<-16$$

Solution

$$|1-x|>4$$

$$1 - x > 4$$

$$-x < -5$$

$$-x > 3$$

$$x < -3$$

$$\therefore Solution: \quad \underline{x < -3 \quad x > 5}$$

Exercise

Solve the inequality equation -2|5-x| < -6

$$-2|5-x|<-6$$

$$|5-x|>3$$

$$5-x < -3$$
 $5-x > 3$
 $-x < -8$ $-x > -2$
 $x > 8$ $x < 2$

$$5 - x > 3$$

$$-x < -8$$

$$-x > -2$$

$$\therefore$$
 Solution: $x < 2$ $x > 8$

Solve the inequality equation $3 \le |2x-1|$

Solution

$$|2x-1| \ge 3$$

$$2x-1 \le -3 \qquad 2x-1 \ge 3$$

$$2x - 1 \ge 3$$

$$2x \le -2 \qquad 2x \ge 4$$
$$x \le -1 \qquad x \ge 2$$

$$2x \ge 4$$

$$x \le -1$$

$$x \ge 2$$

$$\therefore Solution: \underline{x \le -1 \quad x \ge 2}$$

$$1 \quad x > 2$$

Exercise

Solve the inequality equation $9 \le |4x + 7|$

$$9 \le |4x + 7|$$

Solution

$$|4x+7| \ge 9$$

$$|4x + 1| \ge 9$$

$$4x + 7 \le -9 \qquad 4x + 7 \ge 9$$

$$4x+7 \ge 9$$

$$4x \le -16$$

$$4x \ge 2$$

$$x \le -4$$

$$4x \ge 2$$
$$x \ge \frac{1}{2}$$

$$\therefore Solution: \quad \underline{x \le -4 \quad x \ge \frac{1}{2}}$$

Exercise

Solve the inequality equation: $12 < \left| -2x + \frac{6}{7} \right| + \frac{3}{7}$

Solution

$$(7)12 < \left| -(7)2x + (7)\frac{6}{7} \right| + (7)\frac{3}{7}$$

$$84 < |-14x + 6| + 3$$

$$81 < |-14x + 6|$$

Multiply by 7 both sides

$$\left| -14x + 6 \right| > 81$$

$$-14x + 6 < -81 \qquad -14x + 6 > 81$$

$$-14x < -81 - 6 \qquad -14x > 81 - 6$$

$$-14x < -87 \qquad -14x > 75$$

$$x > \frac{87}{14} \qquad x < -\frac{75}{14}$$

$$\therefore Solution: \quad \underline{x < -\frac{75}{14} \quad x > \frac{87}{14}} \qquad \qquad \left(-\infty, -\frac{75}{14}\right) \cup \left(\frac{87}{14}, \infty\right)$$

Solve the inequality equation: $4 + \left| 3 - \frac{x}{3} \right| \ge 9$

Solution

$$\left|3 - \frac{x}{3}\right| \ge 9 - 4$$

$$\left|3 - \frac{x}{3}\right| \ge 5$$

$$\left| (3)3 - (3)\frac{x}{3} \right| \ge (3)5$$

$$|9-x| \ge 15$$

$$9-x \le -15$$
 $9-x \ge 15$
 $-x \le -24$ $-x \ge 6$
 $x \ge 24$ $x \le -6$

$$9 - x \ge 15$$

$$-x \le -24$$

$$-x \ge 6$$

$$x \le -6$$

$$\therefore Solution: \quad \underline{x \le -6 \quad x \ge 24} \quad \left(-\infty, -6\right] \cup \left[24, \infty\right)$$

Exercise

Solve the inequality equation: |x-2| < 5

Solution

$$-5 < x - 2 < 5$$

$$\therefore Solution: \quad \underline{-3 < x < 7}$$

Exercise

Solve the inequality equation: |2x+1| < 7

$$-7 < 2x + 1 < 7$$

$$-7-1 < 2x+1-1 < 7-1$$

$$-8 < 2x < 6$$

$$-\frac{8}{2} < \frac{2}{2}x < \frac{6}{2}$$

$$\therefore Solution: \quad \underline{-4 < x < 3}$$

Solve the inequality equation: |5x + 2| - 2 < 3

Solution

$$|5x+2| < 5$$

$$-5 < 5x + 2 < 5$$

$$-7 < 5x < 3$$

$$\therefore Solution: \qquad -\frac{7}{5} < x < \frac{3}{5} \qquad \left(-\frac{7}{5}, \frac{3}{5} \right)$$

Exercise

Solve the inequality equation: |2-7x|-1>4

Solution

$$|2-7x| > 5$$

$$2-7x < -5$$
 $2-7x > 5$
 $-7x < -7$ $-7x > 3$
 $x > 1$ $x < -\frac{3}{7}$

$$2 - 7x > 5$$

$$-7x < -7$$

$$-7x > 3$$

$$x < -\frac{3}{7}$$

$$x < -\frac{3}{7}$$
 $x > \frac{1}{14}$

$$\therefore Solution: \quad \underline{x < -\frac{3}{7} \quad x > \frac{1}{14}} \quad \left(-\infty, -\frac{3}{7}\right) \cup \left(1, \infty\right)$$

Exercise

Solve the inequality equation: |3x-4| < 2

$$-2 < 3x - 4 < 2$$

$$-2+4 < 3x-4+4 < 2+4$$

$$\frac{2}{3} < x < 2$$

$$\therefore Solution: \qquad \qquad \frac{2}{3} < x < 2 \qquad \qquad \left(\frac{2}{3}, 2\right)$$

Solve the inequality equation: $|2x+5| \ge 3$

Solution

$$2x+5 \le -3 \qquad 2x+5 \ge 3$$

$$2x + 5 \ge 3$$

$$2x \le -8$$

$$2x \ge -2$$

$$x \le -4$$

$$x \ge -1$$

∴ Solution:
$$\underline{x \le -4 \quad x \ge -1}$$
 $(-\infty, -4] \cup [-1, \infty)$

Exercise

$$|12-9x| \ge -12$$

Solution

 \therefore *Solution* set: $(-\infty, \infty)$ because the absolute value always greater than any negative number.

Exercise

$$|6-3x| < -11$$

Solution

: No solution, because the absolute value cannot be less than any negative number

Exercise

Solve the inequality equation |7 + 2x| < 0

$$|7+2x|<0$$

Solution

: No solution, because the absolute value cannot be any negative number

Exercise

Solve:
$$x^2 - 7x + 10 > 0$$

$$x^{2} - 7x + 10 > 0$$
$$(x - 5)(x - 2) > 0$$

$$x = 2, 5$$

$$x < 2$$
 $x > 5$

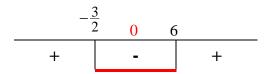
$$\therefore Solution: \quad x < 2 \quad x > 5 \quad (-\infty, 2) \cup (5, \infty)$$

Solve: $2x^2 - 9x \le 18$

Solution

$$2x^2 - 9x - 18 \le 0$$
$$(2x+3)(x-6) \le 0$$

$$\therefore Solution: \quad -\frac{3}{2} \le x \le 6 \quad \left[-\frac{3}{2}, 6 \right]$$



Exercise

Solve the inequality: $x^2 - 5x + 4 > 0$

Solution

$$x^2 - 5x + 4 > 0$$

x = 1, 4

$$\therefore Solution: \underline{x < 1 \quad x > 4} \qquad \underline{\left(-\infty, 1\right) \cup \left(4, \infty\right)}$$

Exercise

Solve $x^2 + x - 2 > 0$

Solution

$$x^2 + x - 2 = 0 \rightarrow x = -2,1$$

∴ Solution: x < -2 x > 1 $(-\infty, -2)$ $(1, \infty)$

Exercise

Solve
$$x^2 - 4x + 12 < 0$$

Solution

$$x^{2} - 4x + 12 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 48}}{2}$$

$$= \frac{4 \pm \sqrt{-32}}{2} \quad Complex \quad number$$

∴ No Solution

Solve the inequality equation $x^2 + 7x > 0$

Solution

$$x^2 + 7x = 0$$

$$x(x+7)=0$$

$$x = 0, -7$$

$$\therefore Solution: \quad \underline{x < -7 \quad x > 0}$$

Exercise

Solve the inequality equation $x^2 - 49 < 0$

Solution

$$x^2 - 49 = 0$$

$$x = \pm 7$$

∴ Solution:
$$-7 < x < 7$$

Exercise

Solve the inequality equation $x^2 - 5x \ge 0$

Solution

$$x^2 - 5x = 0$$

$$x(x-5)=0$$

$$x = 0, 5$$

$$\therefore Solution: \quad \underline{x \le 0 \quad x \ge 5}$$

Exercise

Solve the inequality equation $x^2 - 16 \le 0$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

∴ Solution:
$$-4 \le x \le 4$$

Solve the inequality equation $x^2 + 7x + 10 < 0$

$$x^2 + 7x + 10 < 0$$

Solution

$$x^{2} + 7x + 10 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{2}$$

$$= \frac{-7 \pm 3}{2}$$

$$= \begin{cases} \frac{-7 - 3}{2} = -5\\ \frac{-7 + 3}{2} = -2 \end{cases}$$

 $\therefore Solution: \quad \underline{-5 < x < 2}$

$$-5 < x < 2$$

Exercise

Solve the inequality equation $x^2 - 3x \ge 28$

$$x^2 - 3x \ge 28$$

Solution

$$x^{2} - 3x - 28 = 0$$

$$x = \frac{3 \pm \sqrt{121}}{2}$$

$$= \begin{cases} \frac{3 - 11}{2} = -4\\ \frac{3 + 11}{2} = 7 \end{cases}$$

$$\therefore Solution: \quad \underline{x \le -4 \quad x \ge 7}$$

Exercise

Solve the inequality equation $x^2 + 5x + 6 < 0$

$$x^2 + 5x + 6 < 0$$

Solution

$$x^{2} + 5x + 6 = 0$$

$$x = \frac{-5 \pm 1}{2}$$

$$= \begin{cases} \frac{-5 - 1}{2} = -3\\ \frac{-5 + 1}{2} = -2 \end{cases}$$

 $\therefore Solution: \quad \underline{-3 < x < -2}$

$$-3 < x < -2$$

Solve the inequality equation $x^2 < -x + 30$

Solution

$$x^{2} + x - 30 = 0$$

$$x = \frac{-1 \pm 11}{2}$$

$$= \begin{cases} \frac{-1 - 11}{2} = -6\\ \frac{-1 + 11}{2} = -5 \end{cases}$$

 $\therefore Solution: \quad \underline{-6 < x < -5}$

Exercise

Solve: $x^3 - 3x^2 - 9x + 27 < 0$

Solution

$$x^{3} - 3x^{2} - 9x + 27 = 0$$

$$x^{2}(x - 3) - 9(x - 3) = 0$$

$$(x - 3)(x^{2} - 9) = 0$$

$$\Rightarrow \begin{cases} x - 3 = 0 \Rightarrow \underline{x} = 3 \\ x^{2} - 9 = 0 \Rightarrow x^{2} = 9 \Rightarrow \underline{x} = \pm 3 \end{cases}$$

 $\therefore Solution: \underline{x < -3} \qquad \underline{\left(-\infty, -3\right)}$

Exercise

Solve $x^3 - x > 0$

Solution

$$x(x^{2}-1) = 0$$

$$\rightarrow \begin{cases} x = 0 \\ x^{2}-1 = 0 \rightarrow x^{2} = 1 \rightarrow x = \pm 1 \end{cases}$$

 $\therefore Solution: \quad -1 < x < 0 \quad x > 1$ (-1,0) \cup (1,\infty)

<u>-∞ -3 3 ∞</u>

$$x^3 + 3x^2 \le x + 3$$

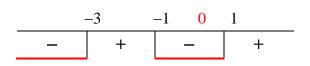
Solution

$$x^{3} + 3x^{2} - x - 3 = 0$$

$$x^{2}(x+3) - (x+3) = 0$$

$$(x+3)(x^{2} - 1) = 0$$

$$\begin{cases} x+3 = 0 \to x = -3 \\ x^{2} - 1 = 0 \to x^{2} = 1 \to x = \pm 1 \end{cases}$$



$$-1 < x < 0$$
 $x > 1$

$$\therefore Solution: \quad \underline{-1 < x < 0 \quad x > 1} \qquad \underline{\left(-\infty, -3\right] \cup \left[-1, 1\right]}$$

Exercise

Solve
$$x^3 + x^2 \ge 48x$$

Solution

$$x^{3} + x^{2} - 48x = 0$$
$$x(x^{2} + x - 48) = 0$$

$$x = 0$$

$$x^{2} + x - 48 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 192}}{2}$$

$$\frac{-1 - \sqrt{193}}{2} < x < 0 \quad x > \frac{-1 + \sqrt{193}}{2}$$

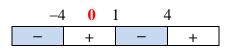
∴ Solution:
$$\frac{-1-\sqrt{193}}{2} < x < 0$$
 $x > \frac{-1+\sqrt{193}}{2}$ $\left[\frac{-1-\sqrt{193}}{2}, 0\right] \cup \left[\frac{-1+\sqrt{193}}{2}, \infty\right]$

Exercise

Solve the inequality equation $x^3 - x^2 - 16x + 16 < 0$

Solution

$$x^{2}(x-1)-16(x-1) = 0$$
$$(x-1)(x^{2}-16) = 0$$
$$x = 1, \pm 4$$



∴ Solution:

$$x < -4 \quad 1 < x < 4$$

Solve the inequality equation $x^3 + x^2 - 9x - 9 > 0$

$$x^3 + x^2 - 9x - 9 > 0$$

Solution

$$x^{2}(x+1)-9(x+1) = 0$$
$$(x+1)(x^{2}-9) = 0$$
$$x = -1, \pm 3$$

$$\therefore Solution: \qquad \underline{-3 < x < -1 \quad x > 3}$$

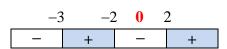
Exercise

Solve the inequality equation $x^3 + 3x^2 - 4x - 12 \ge 0$

$$x^3 + 3x^2 - 4x - 12 \ge 0$$

Solution

$$x^{2}(x+3)-4(x+3) = 0$$
$$(x+3)(x^{2}-4) = 0$$



 $x = -3, \pm 2$

$$\therefore Solution: \qquad -3 < x < -2 \quad x > 2$$

Exercise

Solve the inequality equation $x^4 - 20x^2 + 64 \le 0$

$$x^4 - 20x^2 + 64 \le 0$$

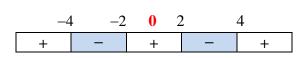
Solution

$$x^{4} - 20x^{2} + 64 = 0$$

$$x^{2} = \frac{20 \pm \sqrt{400 - 256}}{2}$$

$$= \begin{cases} \frac{20 - 12}{2} = 4\\ \frac{20 + 12}{2} = 16 \end{cases}$$

$$\begin{cases} x^{2} = 4 & \rightarrow x = \pm 2 \end{cases}$$



$$\begin{cases} x^2 = 4 & \rightarrow & \underline{x = \pm 2} \\ x^2 = 16 & \rightarrow & \underline{x = \pm 4} \end{cases}$$

∴ Solution: $-4 \le x \le -2$ $2 \le x \le 4$

Solve the inequality equation $x^4 - 10x^2 + 9 \ge 0$

$$x^4 - 10x^2 + 9 \ge 0$$

Solution

$$x^{4} - 10x^{2} + 9 = 0$$

$$\begin{cases} x^{2} = 1 & \rightarrow & \underline{x} = \pm 1 \\ x^{2} = 9 & \rightarrow & \underline{x} = \pm 3 \end{cases}$$

$$\therefore Solution: \quad \underline{x \le -3 \quad -1 \le x \le 1 \quad x \ge 3}$$

Exercise

Solve the inequality equation $\frac{x+4}{x-1} < 0$

Solution

Restriction: $x \neq 1$

$$\frac{x+4}{x-1} = 0$$

$$\underline{x} = -4$$

 $\therefore Solution: \quad \underline{-4 < x < 1}$



Exercise

Solve the inequality equation $\frac{x-2}{x+3} > 0$

Solution

Restriction: $x \neq -3$

$$\frac{x-2}{x+3} = 0$$

$$\underline{x} = 2$$

 $\therefore Solution: \quad x < -3 \quad x > 2$



Exercise

Solve the inequality equation $\frac{x-5}{x+8} \ge 3$

Solution

Restriction: $x \neq -8$

$$\frac{x-5}{x+8} - 3 = 0$$

$$x-5-3x-24 = 0$$

$$-2x = 29$$

$$x = -\frac{29}{2}$$

$$\therefore Solution: \quad x \le -\frac{29}{2} \quad x > -8$$

Solve the inequality equation
$$\frac{x-4}{x+6} \le 1$$

Solution

Restriction: $x \neq -6$

$$\frac{x-4}{x+6} - 1 = 0$$

$$x - 4 - x - 6 = 0$$

$$-10 = 0 \$$

 \therefore Solution: x > -6



Exercise

Solve the inequality equation $\frac{x}{2x+7} \ge 4$

Solution

Restriction: $x \neq -\frac{7}{2}$

$$\frac{x}{2x+7} - 4 = 0$$

$$x - 8x - 28 = 0$$

$$7x = -28$$

$$\underline{x} = -4$$

 $\therefore Solution: \quad \underline{x \le -4 \quad x > -\frac{7}{2}}$



Exercise

Solve the inequality equation $\frac{x}{3x-5} \le -5$

Restriction: $x \neq \frac{5}{3}$

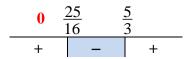
$$\frac{x}{3x-5} + 5 = 0$$

$$x + 15x - 25 = 0$$

$$16x = 25$$

$$x = \frac{25}{16}$$

$$\therefore Solution: \qquad \frac{25}{16} \le x < \frac{5}{3}$$



Exercise

Solve the inequality equation $\frac{x+2}{x-5} \le 2$

Solution

Restriction: $x \neq 5$

$$\frac{x+2}{x-5} - 2 = 0$$

$$x + 2 - 2x + 10 = 0$$

$$x = 12$$

∴ *Solution*: $5 \le x < 12$



Exercise

Solve the inequality equation $\frac{3x+1}{x-2} \ge 4$

Solution

Restriction: $x \neq 2$

$$\frac{3x+1}{x-2} - 4 = 0$$

$$3x + 1 - 4x + 8 = 0$$

$$x = 9$$

 $\therefore Solution: \quad \underline{2 \le x < 9}$



Exercise

Solve the inequality equation $\frac{x}{x-3} > 0$

Restriction:
$$x \neq 3$$

$$\frac{x}{x-3} = 0$$

$$x = 0$$

$$\therefore Solution: \underline{x < 0 \quad x > 3} \qquad \underline{\left(-\infty, 0\right) \cup \left(3, \infty\right)}$$

Solve the inequality equation
$$\frac{x-3}{x+2} \ge 0$$

Solution

Restriction:
$$x \neq -2$$

$$\frac{x-3}{x+2} = 0$$

$$x = 3$$

$$\therefore Solution: \quad \underline{x < -2 \quad x \ge 3}$$



Exercise

Solve the inequality equation
$$\frac{x-2}{x+2} \le 2$$

Solution

Restriction:
$$x \neq -2$$

$$\frac{x-2}{x+2}-2=0$$

$$x-2-2x-4=0$$

$$-x-6=0$$

$$\underline{x} = -6$$

$$\therefore Solution: \underline{x \le -6 \quad x > -2} \quad \underline{\left(-\infty, -6\right] \cup \left(-2, \infty\right)}$$

Exercise

Solve the inequality equation
$$\frac{x+2}{x-2} \ge 2$$

Restriction:
$$x \neq 2$$

$$\frac{x+2}{x-2} - 2 = 0$$

$$x + 2 - 2x + 4 = 0$$

$$x = 6$$

∴ Solution:
$$2 < x \le 6$$

Solve the inequality equation
$$\frac{x+2}{3+2x} \le 5$$

Solution

Restriction:
$$x \neq -\frac{3}{2}$$

$$\frac{x+2}{3+2x} - 5 = 0$$

$$x+2-15-10x = 0$$

$$-9x-13 = 0$$

$$x = -\frac{13}{9}$$

$$x < -\frac{3}{2}$$
 $x > -\frac{13}{9}$

$$\therefore Solution: \quad \underline{x < -\frac{3}{2} \quad x > -\frac{13}{9}} \quad \left(-\infty, -\frac{3}{2} \right) \cup \left[-\frac{13}{9}, \infty \right)$$

Exercise

Solve the inequality
$$\frac{x+6}{x-14} \ge 1$$

Solution

Restriction:
$$x-14 \neq 0 \Rightarrow \boxed{x \neq 14}$$

$$\frac{x+6}{x-14} - 1 = 0$$

$$x + 6 - x + 14 = 0$$

$$20 = 0$$
 (Implossible) No Solution

$$\therefore Solution: \qquad x > 14 \qquad (14, \infty)$$

$$(14, \infty)$$

Exercise

Solve:
$$\frac{x-3}{x+4} \ge \frac{x+2}{x-5}$$

Conditions:
$$x + 4 \neq 0 \rightarrow x \neq -4$$
 and $x - 5 \neq 0 \rightarrow x \neq 5$

and
$$x-5 \neq 0 \rightarrow x \neq 5$$

$$\frac{x-3}{x+4} - \frac{x+2}{x-5} = 0$$

$$\frac{0-3}{0+4} - \frac{0+2}{0-5} = \frac{-3}{4} - \frac{2}{-5} = \frac{-3}{4} + \frac{2}{5} = -$$

$$(x+4)(x-5)\left[\frac{x-3}{x+4} - \frac{x+2}{x-5}\right] = 0$$

$$(x-5)(x-3)-(x+4)(x+2)=0$$

$$x^{2}-3x-5x+15-(x^{2}+2x+4x+8)=0$$

$$x^2 - 3x - 5x + 15 - x^2 - 2x - 4x - 8 = 0$$

$$-14x + 7 = 0$$

$$-14x = -7$$

$$x = \frac{-7}{-14} = \frac{1}{2}$$

$$\therefore Solution: \quad x < -4 \quad \frac{1}{2} \le x < 5 \quad \left(-\infty, -4 \right) \cup \left[\frac{1}{2}, 5 \right)$$

Solve:
$$\frac{x-4}{x+3} - \frac{x+2}{x-1} \le 0$$

Solution

Conditions: $x \neq -3$ and $x \neq 1$

$$\frac{x-4}{x+3} - \frac{x+2}{x-1} = 0$$

$$\frac{0-4}{0+3} - \frac{0+2}{0-1} = -\frac{4}{3} + 2 > 0$$

$$(x+3)(x-1)\left[\frac{x-4}{x+3} - \frac{x+2}{x-1}\right] = 0$$

$$(x-1)(x-4) - (x+3)(x+2) = 0$$

$$x^2 - 5x + 4 - (x^2 + 5x + 6) = 0$$

$$x^2 - 5x + 4 - x^2 - 5x - 6 = 0$$

$$x = -\frac{1}{5}$$

-10x - 2 = 0

$$-3 < x \le -\frac{1}{5} \quad x > 1$$

$$\therefore Solution: \qquad -3 < x \le -\frac{1}{5} \quad x > 1 \qquad \left(-3, -\frac{1}{5} \right] \quad \bigcup \quad \left(1, \infty \right)$$

Exercise

Solve:
$$\frac{2x-1}{x+3} \ge \frac{x+1}{3x+1}$$

Conditions:
$$x \neq -3$$
 and $x \neq -\frac{1}{3}$

$$\frac{2x-1}{x+3} - \frac{x+1}{3x+1} \ge 0$$

$$(x+3)(3x+1)\frac{2x-1}{x+3} - (x+3)(3x+1)\frac{x+1}{3x+1} = 0$$

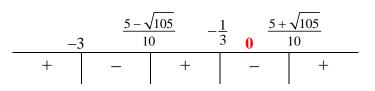
$$(3x+1)(2x-1)-(x+3)(x+1)=0$$

$$6x^2 - 3x + 2x - 1 - \left(x^2 + x + 3x + 3\right) = 0$$

$$6x^2 - x - 1 - x^2 - 4x - 3 = 0$$

$$5x^2 - 5x - 4 = 0$$

$$x = \frac{5 \pm \sqrt{105}}{10} \to -.5$$
 1.5



∴ *Solution*:
$$x < -3$$
 $\frac{5 - \sqrt{105}}{10} \le x < -\frac{1}{3}$ $x \ge \frac{5 + \sqrt{105}}{10}$

$$(-\infty, -3) \cup \left[\frac{5 - \sqrt{105}}{10}, -\frac{1}{3} \right] \cup \left[\frac{5 + \sqrt{105}}{10}, \infty \right]$$

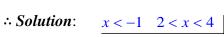
Solve the inequality equation
$$\frac{(x+1)(x-4)}{x-2} < 0$$

Solution

Restriction: $x \neq 2$

$$\frac{(x+1)(x-4)}{x-2} = 0$$

$$x = -1, 4$$





Exercise

Solve the inequality equation $\frac{x(x-4)}{x+5} > 0$

Solution

Restriction: $x \neq -5$

$$\frac{x(x-4)}{x+5} = 0$$

$$x = 0, 4$$

$$\therefore Solution: \quad \underline{-5 < x < 0 \quad x > 4}$$

Solve the inequality equation
$$\frac{6x^2 - 11x - 10}{r} > 0$$

$$\frac{6x^2 - 11x - 10}{x} > 0$$

Solution

Restriction: $x \neq 0$

$$6x^{2} - 11x - 10 = 0$$

$$x = \frac{11 \pm \sqrt{121 + 240}}{12}$$

$$= \begin{cases} \frac{11 - 19}{12} = -\frac{2}{3} \\ \frac{11 + 19}{12} = \frac{5}{2} \end{cases}$$

$$\therefore Solution: \quad -\frac{2}{3} < x < 0 \quad x > \frac{5}{2}$$

Exercise

Solve the inequality equation $\frac{3x^2 - 2x - 8}{x - 1} \ge 0$

$$\frac{3x^2 - 2x - 8}{x - 1} \ge 0$$

Solution

Restriction: $x \neq 1$

$$3x^{2} - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 96}}{6}$$

$$= \begin{cases} \frac{2 - 10}{6} = -\frac{4}{3} \\ \frac{2 + 10}{6} = 2 \end{cases}$$

$$\therefore Solution: \quad -\frac{4}{3} \le x < 1 \quad x \ge 2$$

Exercise

Solve the inequality equation $\frac{x^2 - 6x + 9}{x - 5} \le 0$

$$\frac{x^2 - 6x + 9}{x - 5} \le 0$$

Solution

Restriction: $x \neq 5$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3$$

 $\therefore Solution: \underline{x < 5}$

Exercise

Solve the inequality equation $\frac{x^2 + 10x + 25}{x+1} \le 0$

Solution

Restriction: $x \neq -1$

$$x^{2} + 10x + 25 = 0$$
$$(x+5)^{2} = 0$$

$$\underline{x} = -5$$

 $\therefore Solution: \underline{x < -1}$

