1. Find the inverse of the given relation?

a) $\{(2, 1), (-2, 3), (3, 4), (-3, 2), (1, 5)\}$

b) $\{(-7, 3), (-2, 1), (-2, 4), (0, 7)\}$

2. For the following functions:

a) $f(x) = \sqrt{x+5} + 1$

b) $f(x) = \frac{x+4}{x-3}$

i) Is $f(x)$ one-to-one?

ii) Find the inverse, if exists.

iii) Determine the Domain for the inverse function

3. Write each equation in its logarithmic form

a) $5^{-3} = \frac{1}{125}$

b) $4^{2y} = 24.5$

c) $e^{-1} = 0.368$

d) $15^{0.457} = 3$

e) $e^{x} = z$

4. Write each equation in its exponential form

a) $6 = \log_2 64$

b) $2 = \log_3 x$

c) $y = \ln 2^{\pi}$

d) $6.2 = \ln x$

e) $\log y = x$

f) $\log_3 x = \frac{1}{3}$

5. Graph and determine its ***asymptote*** (**label** the graph).

a) $f(x) = \log(x+2)$

b) $f(x) = \left(\frac{1}{3}\right)^{x-3}$

c) $f(x) = \ln(2x-4)$

d) $f(x) = e^{2x} - 4$

6. Find the ***domain***, ***range*** and the ***asymptote*** of each logarithmic function

a) $f(x) = 2 + \ln(2x-4)$

b) $f(x) = \ln(7-x)$

c) $f(x) = \ln\left(x^2 - 4x - 5\right)$

d) $f(x) = \ln(x-3)^2$

e) $f(x) = \log\left(\frac{x-7}{x+5}\right)$

f) $f(x) = 5 + e^{2x+3}$

g) $f(x) = 2 - 3e^{x+1}$

h) $f(x) = 2^{3x+1}$

7. Express in terms of sums and differences of logarithms

a) $\log_3\left(\dfrac{x^3y^2}{z}\right)$ *b)* $\log\left(\dfrac{x^3y^2}{\sqrt[3]{(z+1)^2}}\right)$ *c)* $\log_b\left(\dfrac{x^3y^2}{a^4b^5}\right)$

8. Write each expression as a single logarithm

a) $\frac{1}{3}\left(\log_4 x - \log_4 y\right)$

b) $2\ln(x-3) - \frac{1}{2}\ln(x+2) + 4\ln x - \ln y$

c) $\frac{2}{3}\left[\ln\left(x^2-4\right) - \ln(x+2)\right] + \ln(x+y)$

9. Solve the exponential equation

a) $2^{2x+1} = 64$

b) $5^{x+3} = 25^{x-5}$

c) $3^{x+4} = 2^{2x+5}$

d) $e^{1-8x} = 7957$

10. Solve the Logarithmic equation

a) $\log_3(x+2) + \log_3 x = 1$

b) $\ln\sqrt{\mathrm{x}+4} = 1$

c) $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

d) $\log_2 3x + \log_2 3 = \log_2(2x+15)$

11. The population of the United States is about 300 million. If it is growing at a rate of 2.1% per year, how long to the nearest tenth of a year, will it take for the population to triple?

12. An endangered species of fish has a population that is decreasing exponentially according to the equation $A(t) = 14000e^{kt}$ where A is the fish population t years after 1990. The fish population was 14,000 in 1990, and nine years later it was 12,000. Use this information to find $\boldsymbol{k}$ to 4 decimal places.

13. In 2000, the population of China was about 1.3 billion. In 2003, the population was 1.33 billion.

a) Find the exponential growth rate
b) Find the exponential growth function
c) Estimate the population in 2009
d) After how long will the population be double what it was in 2000?

Solution

1. *a*. {(1, 2), (2, –2), (4, 3), (2, –3), (5, 1)}

b. {(3, –7), (1, –2), (4, –2), (7, 0)}

2. *a*) $f(x) = \sqrt{x+5} + 1$

i) $f(a) = f(b)$

$\Rightarrow \sqrt{a+5} + 1 = \sqrt{b+5} + 1$

$\Rightarrow \sqrt{a+5} = \sqrt{b+5}$ (square both side)

$\Rightarrow a + 5 = b + 5$

$\Rightarrow a = \mathrm{b} \rightarrow f(x)$ is one-to-one

ii) $y = \sqrt{x+5} + 1$

$\Rightarrow x = \sqrt{y+5} + 1$

$\Rightarrow x - 1 = \sqrt{y+5}$

$\Rightarrow (x-1)^2 = y + 5$

$\Rightarrow y = (x-1)^2 - 5 = f^{-1}(x)$

iii) Domain: $x \geq 1$

b) $f(x) = \dfrac{x+4}{x-3}$

i) $f(a) = f(b)$

$\Rightarrow \frac{a+4}{a-3} = \frac{b+4}{b-3}$

$\Rightarrow (a + 4)(b - 3) = (a - 3)(b + 4)$

$\Rightarrow ab - 3a + 4b - 12 = ab + 4a - 3b - 12$

$\Rightarrow - 3a = 4a - 7b$

$\Rightarrow - 7a = - 7b$

$\Rightarrow a = b$

$\rightarrow f(x)$ is one-to-one

ii) $y = \frac{x+4}{x-3}$

$\Rightarrow x = \dfrac{y+4}{y-3}$

$\Rightarrow x(y - 3) = y + 4$

$\Rightarrow xy - 3x = y + 4$

$\Rightarrow xy - y = 3x + 4$

$\Rightarrow y(x - 1) = 3x + 4$

$\Rightarrow y = \dfrac{3x+4}{x-1} = f^{-1}(x)$

iii) Domain of $f^{-1}(x)$: $\{x|\, x \neq 1\}$

3. *a)* $\log_5 \frac{1}{125} = -3$ *b)* $2y = \log_4 24.5$ *c)* $\ln(0.3679) = -1$

d) $0.4057 = \log_{15} 3$ *e)* $x = \ln z$

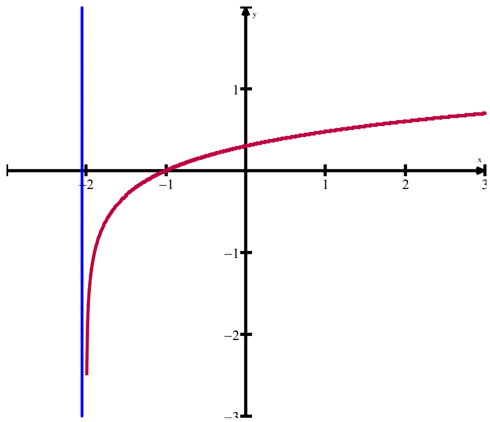
4. *a)* $2^6 = 64$ *b)* $3^2 = x$ *c)* $e^y = 2^\pi$ *d)* $e^{6.2} = x$ *e)* $y = 10^x$ *f)* $x = 3^{\frac{1}{3}}$

5. *a)* $f(x) = \log(x+2)$

Asymptote: $x = -2$

x	y
~~-2~~	
-1.5	-.3
-1	1
0	.3

Shifted

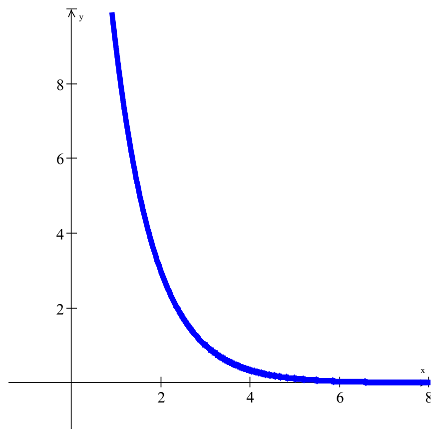


left 2 units

b) $f(x) = \left(\frac{1}{3}\right)^{x-3}$

Asymptote: $y = 0$

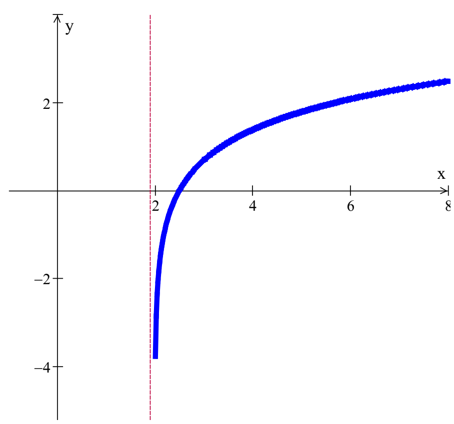
x	y
2	~~3~~
3	1
4	.33
5	.1



c) $f(x) = \ln(2x-4)$

Asymptote: $x = 2$

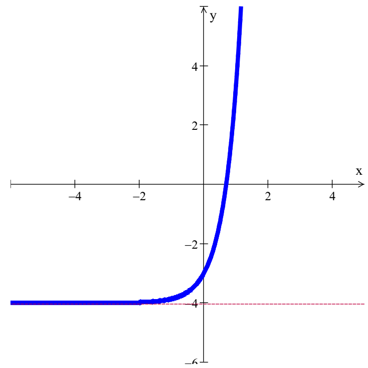
x	y
~~2~~	
2.5	0
3	.7
4	1.4



$a)\ \ f(x) = e^{2x} - 4$

Asymptote: $y = -4$

x	y
-1	-3.9
0	-3
1	3.4
2	51



6. $a)$ *Domain* $:(2,\ \infty)$; *Range* $:(-\infty,\ \infty)$; *Asymptote* $: x = 2$

$b)$ *Domain* $:(-\infty,\ 7)$; *Range* $:(-\infty,\ \infty)$; *Asymptote* $: x = 7$

$c)$ *Domain* $:(-\infty,\ -1)\cup(5,\ \infty)$; *Range* $:(-\infty,\ \infty)$; *Asymptote* $: x = -1,\ x = 5$

$d)$ *Domain* $:(-\infty,\ 3)\cup(3,\ \infty)$; *Range* $:(-\infty,\ \infty)$; *Asymptote* $: x = 3$

$e)$ *Domain* $:(-\infty,\ -5)\cup(7,\ \infty)$; *Range* $:(-\infty,\ 0)\cup(0,\ \infty)$; *Asymptote* $: x = -5,\ x = 7$

$f)$ *Domain* $:(-\infty,\ \infty)$; *Range* $:(5,\ \infty)$; *Asymptote* $: y = 5$

$g)$ *Domain* $:(-\infty,\ \infty)$; *Range* $:(-\infty,\ 2)$; *Asymptote* $: y = 2$

$h)$ *Domain* $:(-\infty,\ \infty)$; *Range* $:(0,\ \infty)$; *Asymptote* $: y = 0$

7. $a)\ 3\log_3 x + 2\log_3 y - \log_3 z$

$b)\ 3\log x + 2\log y - \frac{2}{3}\log(z+1)$

$c)\ 3\log_b x + 2\log_b y - 4\log_b a - 5$

8. $a)\ \log_4\left(\sqrt[3]{\frac{x}{y}}\right)$ $b)\ ln\left(\frac{x^4(x-3)^2}{y\sqrt{x+2}}\right)$ $c)\ ln(x-2)^{2/3}(x+y)$ or $\ln\sqrt[3]{(x-2)^2}(x+y)$

9. $a)\ \frac{5}{2}$ $b)\ x = 13$ $c)\ \frac{5\ln 2 - 4\ln 3}{\ln 3 - 2\ln 2}$ $d)\ \approx -0.9977$

10. $a)\ 1$ $b)\ 3.389$ $c)\ 4,\ 5$ $d)\ \frac{15}{7}$

11. $\frac{1000}{21}\ln 3$ *years*

12. $k = -0.0171$

13. $a)\ k \approx 0.0076$ $b)\ A(t) = 1.3e^{.0076t}$ $c)$ 1.392 billion $d)$ 91.2 years