

## ***Solution***

### **Section 1.4- Other Types of Equations**

#### ***Exercise***

Solve  $3x^3 + 2x^2 = 12x + 8$

#### **Solution**

$$3x^3 + 2x^2 - (12x + 8) = 0$$

$$x^2(3x + 2) - 4(3x + 2) = 0$$

$$(3x + 2)(x^2 - 4) = 0$$

$$3x + 2 = 0$$

$$x^2 - 4 = 0$$

$$3x = -2$$

$$x^2 = 4$$

$$\underline{x = -\frac{2}{3}}$$

$$\underline{x = \pm 2}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{2}{3}, \pm 2}$$

#### ***Exercise***

Solve:  $x^3 + x^2 - 4x - 4 = 0$

#### **Solution**

$$x^2(x + 1) - 4(x + 1) = 0$$

$$(x + 1)(x^2 - 4) = 0$$

$$x + 1 = 0$$

$$\underline{x = -1}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\underline{x = \pm 2}$$

$$\therefore \text{Solutions: } \underline{x = -1, \pm 2}$$

#### ***Exercise***

Solve:  $x^3 + x^2 + 4x + 4 = 0$

#### **Solution**

$$x^2(x + 1) + 4(x + 1) = 0$$

$$(x+1)(x^2+4)=0$$

$$x+1=0$$

$$\underline{x=-1}$$

$$x^2+4=0$$

$$x^2=-4$$

$$\underline{x=\pm 2i}$$

$$\therefore \text{Solutions: } \underline{x=-1, \pm 2i}$$

### ***Exercise***

$$\text{Solve: } x^3 + 4x^2 - 25x - 100 = 0$$

### **Solution**

$$x^2(x+4) - 25(x+4) = 0$$

$$(x+4)(x^2-25) = 0$$

$$x+4=0$$

$$\underline{x=-4}$$

$$x^2-25=0$$

$$x^2=25$$

$$\underline{x=\pm 5}$$

$$\therefore \text{Solutions: } \underline{x=-4, \pm 5}$$

### ***Exercise***

$$\text{Solve: } x^3 - 2x^2 - x + 2 = 0$$

### **Solution**

$$x^2(x-2) - (x-2) = 0$$

$$(x-2)(x^2-1) = 0$$

$$x-2=0$$

$$\underline{x=2}$$

$$x^2-1=0$$

$$x^2=1$$

$$\underline{x=\pm 1}$$

$$\therefore \text{Solutions: } \underline{x=2, \pm 1}$$

### Exercise

Solve:  $x^3 - x^2 - 25x + 25 = 0$

### Solution

$$x^2(x-1) - 25(x-1) = 0$$

$$(x-1)(x^2 - 25) = 0$$

$$x-1 = 0$$

$$\underline{x = 1}$$

$$x^2 - 25 = 0$$

$$x^2 = 25$$

$$\underline{x = \pm 5}$$

$$\therefore \text{Solutions: } \underline{x = 1, \pm 5}$$

### Exercise

Solve:  $x^3 - x^2 = 16x - 16$

### Solution

$$x^3 - x^2 - 16x + 16 = 0$$

$$x^2(x-1) - 16(x-1) = 0$$

$$(x-1)(x^2 - 16) = 0$$

$$x-1 = 0$$

$$\underline{x = 1}$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$\underline{x = \pm 4}$$

$$\therefore \text{Solutions: } \underline{x = 1, \pm 4}$$

### Exercise

Solve:  $x^3 + x^2 + 25x + 25 = 0$

### Solution

$$x^2(x+1) + 25(x+1) = 0$$

$$(x+1)(x^2 + 25) = 0$$

$$x+1 = 0$$

$$\underline{x = -1}$$

$$x^2 + 25 = 0$$

$$x^2 = -25$$

$$\underline{x = \pm 5i}$$

∴ Solutions:  $\underline{x = -1, \pm 5i}$

### ***Exercise***

Solve:  $x^3 + 2x^2 = 16x + 32$

#### **Solution**

$$x^3 + 2x^2 - 16x - 32 = 0$$

$$x^2(x + 2) - 16(x + 2) = 0$$

$$(x + 2)(x^2 - 16) = 0$$

$$x + 2 = 0$$

$$\underline{x = -2}$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$\underline{x = \pm 4}$$

∴ Solutions:  $\underline{x = -2, \pm 4}$

### ***Exercise***

Solve:  $2x^3 + 3x^2 - 6x - 9 = 0$

#### **Solution**

$$x^2(2x + 3) - 3(2x + 3) = 0$$

$$(2x + 3)(x^2 - 3) = 0$$

$$2x + 3 = 0$$

$$\underline{x = -\frac{3}{2}}$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$\underline{x = \pm\sqrt{3}}$$

∴ Solutions:  $\underline{x = -\frac{3}{2}, \pm\sqrt{3}}$

### ***Exercise***

Solve:  $2x^3 + x^2 - 8x - 4 = 0$

#### **Solution**

$$x^2(2x + 1) - 4(2x + 1) = 0$$

$$(2x + 1)(x^2 - 4) = 0$$

$$2x + 1 = 0$$

$$\underline{x = -\frac{1}{2} \mid}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\underline{x = \pm 2 \mid}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{1}{2}, \pm 2 \mid}$$

### ***Exercise***

$$\text{Solve: } 2x^3 + 16x^2 + 30x = 0$$

### **Solution**

$$2x(x^2 + 8x + 15) = 0$$

$$2x(x + 3)(x + 5) = 0$$

$$\therefore \text{Solutions: } \underline{x = 0, -3, -5 \mid}$$

### ***Exercise***

$$\text{Solve: } 3x^3 - 9x^2 - 30x = 0$$

### **Solution**

$$3x(x^2 - 3x - 10) = 0$$

$$3x(x + 2)(x - 5) = 0$$

$$\therefore \text{Solutions: } \underline{x = 0, -2, 5 \mid}$$

### ***Exercise***

$$\text{Solve } x^4 + 3x^2 = 10$$

### **Solution**

$$x^4 + 3x^2 - 10 = 0$$

$$(x^2 + 5)(x^2 - 2) = 0$$

$$x^2 + 5 = 0$$

$$x^2 = -5$$

$$\underline{x = \pm i\sqrt{5} \mid}$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$\underline{x = \pm\sqrt{2} \mid}$$

$$\therefore \text{Solutions: } \underline{x = \pm i\sqrt{5}, \pm\sqrt{2} \mid}$$

### Exercise

Solve:  $5x^4 = 40x$

### Solution

$$5x^4 - 40x = 0$$

$$5x(x^3 - 8) = 0$$

$$5x(x^3 - 2^3) = 0$$

$$5x(x-2)(x^2 + 2x + 2^2) = 0$$

$$5x(x-2)(x^2 + 2x + 4) = 0$$

$$\underline{x = 0}$$

$$\underline{x = 2}$$

$$x^2 + 2x + 4 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$= \underline{-1 \pm i\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = 0, 2, -1 \pm i\sqrt{3}}$$

### Exercise

Solve  $9x^4 - 9x^2 + 2 = 0$

### Solution

$$x^2 = \frac{9 \pm \sqrt{81 - 72}}{18}$$

$$= \frac{9 \pm \sqrt{9}}{18}$$

$$= \frac{9 \pm 3}{18}$$

$$= \begin{cases} \frac{9-3}{18} = \frac{6}{18} = \frac{1}{3} \\ \frac{9+3}{18} = \frac{12}{18} = \frac{2}{3} \end{cases}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left\{ \begin{array}{l} x^2 = \frac{1}{3} \Rightarrow x = \pm \frac{1}{\sqrt{3}} \\ x^2 = \frac{2}{3} \Rightarrow x = \pm \frac{\sqrt{2}}{\sqrt{3}} \end{array} \right|$$

$$\left\{ \begin{array}{l} x = \pm \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3} \\ x = \pm \frac{\sqrt{2}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{6}}{3} \end{array} \right|$$

$$\therefore \text{Solutions: } x = \pm \frac{\sqrt{3}}{3}, \pm \frac{\sqrt{6}}{3} \quad |$$

### ***Exercise***

Solve:  $x^4 + 720 = 89x^2$

### **Solution**

$$x^4 - 89x^2 + 720 = 0$$

$$x^2 = \frac{89 \pm \sqrt{7,921 - 2,880}}{2}$$

$$= \frac{89 \pm \sqrt{5041}}{2}$$

$$= \frac{89 \pm 71}{2}$$

$$= \left\{ \begin{array}{l} \frac{89 - 71}{2} = 9 \\ \frac{89 + 71}{2} = 80 \end{array} \right|$$

$$x^2 = 9 \Rightarrow x = \pm 3$$

$$x^2 = 80 \Rightarrow x = \pm \sqrt{80}$$

$$= \pm \sqrt{(16)(5)}$$

$$= \pm 4\sqrt{5}$$

$$\therefore \text{Solutions: } x = \pm 3, \pm 4\sqrt{5} \quad |$$

### Exercise

Solve  $12x^4 - 11x^2 + 2 = 0$

### Solution

$$x^2 = \frac{11 \pm \sqrt{121 - 96}}{24}$$

$$= \frac{11 \pm \sqrt{25}}{24}$$

$$= \frac{11 \pm 5}{24}$$

$$= \begin{cases} \frac{11-5}{24} = \frac{1}{4} \\ \frac{11+5}{24} = \frac{2}{3} \end{cases}$$

$$x^2 = \frac{1}{4}$$

$$x = \pm \frac{1}{2}$$

$$x^2 = \frac{2}{3}$$

$$x = \pm \sqrt{\frac{2}{3}}$$

$$x = \pm \frac{\sqrt{6}}{3}$$

$$\therefore \text{Solutions: } \left\{ \pm \frac{\sqrt{6}}{3}, \pm \frac{1}{2} \right\}$$

### Exercise

Solve  $2x^4 - 7x^2 + 5 = 0$

### Solution

$$(2x^2 - 5)(x^2 - 1) = 0$$

$$2x^2 - 5 = 0$$

$$x^2 = \frac{5}{2}$$

$$x = \pm \frac{\sqrt{5}}{\sqrt{2}}$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$\therefore \text{Solutions: } \left\{ \pm \frac{\sqrt{10}}{2}, \pm 1 \right\}$$



### Exercise

Solve  $x^4 - 5x^2 + 4 = 0$

#### Solution

$$1 - 5 + 4 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, 4} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$\begin{array}{l|l} x^2 = 1 & x^2 = 4 \\ \underline{x = \pm 1} & \underline{x = \pm 2} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm 2}$$

### Exercise

Solve  $x^4 + 3x^2 = 10$

#### Solution

$$x^4 + 3x^2 - 10 = 0$$

$$(x^2 + 5)(x^2 - 2) = 0$$

$$\begin{array}{l|l} x^2 + 5 = 0 & x^2 - 2 = 0 \\ x^2 = -5 & x^2 = 2 \\ x = \pm\sqrt{-5} & \underline{x = \pm\sqrt{2}} \\ \underline{= \pm i\sqrt{5}} & \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm i\sqrt{5}, \pm\sqrt{2}}$$

### Exercise

Solve:  $3x^4 - 48x^2 = 0$

#### Solution

$$3x^2(x^2 - 16) = 0$$

$$\begin{array}{l|l} x^2 = 0 & x^2 - 16 = 0 \\ \underline{x = 0, 0} & x^2 = 16 \\ & \underline{x = \pm 4} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, \pm 4}$$

### ***Exercise***

Solve:  $5x^4 - 20x^2 = 0$

### **Solution**

$$5x^2(x^2 - 4) = 0$$

$$x^2 = 0$$

$$\underline{x = 0, 0}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\underline{x = \pm 2}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, \pm 2}$$

### ***Exercise***

Solve:  $x^4 - 4x^3 - 4x^2 = 0$

### **Solution**

$$x^2(x^2 - 4x - 4) = 0$$

$$x^2 = 0$$

$$\underline{x = 0, 0}$$

$$x^2 - 4x - 4 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 16}}{2}$$

$$= \frac{4 \pm 4\sqrt{2}}{2}$$

$$\underline{x = 2 \pm 2\sqrt{2}}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 2 \pm 2\sqrt{2}}$$

### ***Exercise***

Solve:  $x^4 - 6x^3 + 9x^2 = 0$

### **Solution**

$$x^2(x^2 - 6x + 9) = 0$$

$$x^2 = 0$$

$$\underline{x = 0, 0}$$

$$x^2 - 6x + 9 = 0$$

$$(x - 3)^2 = 0$$

$$\underline{x = 3, 3}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 3, 3}$$

### Exercise

Solve:  $x^4 - 4x^3 + 3x^2 = 0$

#### Solution

$$x^2(x^2 - 4x + 3) = 0$$

$$x^2 = 0$$

$$\underline{x = 0, 0}$$

$$x^2 - 4x + 3 = 0$$

$$\underline{x = 1, 3}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 1, 3}$$

### Exercise

Solve:  $x^4 - 4x^2 + 3 = 0$

#### Solution

$$1 - 4 + 3 = 0$$

$$a + b + c = 0$$

$$\underline{x^2 = 1, 3}$$

$$x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$x^2 = 3$$

$$\underline{x = \pm\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm\sqrt{3}}$$

### Exercise

Solve:  $x^4 + 4x^2 + 3 = 0$

#### Solution

$$1 - 4 + 3 = 0$$

$$a - b + c = 0$$

$$\underline{x^2 = -1, -3}$$

$$x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = -1$$

$$\underline{x = \pm i}$$

$$x^2 = -3$$

$$\underline{x = \pm i\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm i, \pm i\sqrt{3}}$$

### Exercise

Solve:  $x^4 + 6x^2 - 7 = 0$

#### Solution

$$1 + 6 - 7 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, -7} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$\begin{array}{l} x^2 = 1 \\ x = \pm 1 \end{array} \quad \begin{array}{l} x^2 = -7 \\ x = \pm i\sqrt{7} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm i\sqrt{7}}$$

### Exercise

Solve:  $x^4 - 6x^2 - 7 = 0$

#### Solution

$$1 - (-6) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x^2 = -1, 7} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$\begin{array}{l} x^2 = -1 \\ x = \pm i \end{array} \quad \begin{array}{l} x^2 = 7 \\ x = \pm \sqrt{7} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm i, \pm \sqrt{7}}$$

### Exercise

Solve:  $3x^4 + 4x^2 - 7 = 0$

#### Solution

$$3 + 4 - 7 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, -\frac{7}{3}} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$\begin{array}{l} x^2 = 1 \\ x = \pm 1 \end{array} \quad \begin{array}{l} x^2 = -\frac{7}{3} \\ x = \pm i\sqrt{\frac{7}{3}} \frac{\sqrt{3}}{\sqrt{3}} \\ \quad \quad \quad = \pm i\frac{\sqrt{21}}{3} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm i\frac{\sqrt{21}}{3}}$$

### Exercise

Solve:  $3x^4 - 4x^2 - 7 = 0$

#### Solution

$$3 - (-4) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x^2 = -1, \frac{7}{3}} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = -1$$

$$\underline{x = \pm i}$$

$$x^2 = \frac{7}{3}$$

$$x = \pm \sqrt{\frac{7}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$\underline{= \pm \frac{\sqrt{21}}{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm i, \pm \frac{\sqrt{21}}{3}}$$

### Exercise

Solve:  $3x^4 - x^2 - 2 = 0$

#### Solution

$$3 - 1 - 2 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, -\frac{2}{3}} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$x^2 = -\frac{2}{3}$$

$$x = \pm i \sqrt{\frac{2}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$\underline{= \pm i \frac{\sqrt{6}}{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm i \frac{\sqrt{6}}{3}}$$

### Exercise

Solve:  $3x^4 + x^2 - 2 = 0$

#### Solution

$$3 - 1 - 2 = 0 \quad a - b + c = 0$$

$$\begin{array}{l|l}
 \left. \begin{array}{l} x^2 = -1, \frac{2}{3} \\ x^2 = -1 \\ \underline{x = \pm i} \end{array} \right\} & \begin{array}{l} x_1 = -1, \quad x_2 = -\frac{c}{a} \\ x^2 = \frac{2}{3} \\ x = \pm \sqrt{\frac{2}{3}} \frac{\sqrt{3}}{\sqrt{3}} \\ \underline{= \pm \frac{\sqrt{6}}{3}} \end{array} \\
 \hline
 \therefore \text{Solutions: } \underline{x = \pm i, \pm \frac{\sqrt{6}}{3}} & 
 \end{array}$$

### Exercise

Solve  $x - 3\sqrt{x} - 4 = 0$

#### Solution

$$(\sqrt{x} - 4)(\sqrt{x} + 1) = 0$$

$$\sqrt{x} - 4 = 0 \quad \sqrt{x} + 1 = 0$$

$$\sqrt{x} = 4 \quad \sqrt{x} = -1 \quad \text{Impossible}$$

$$x = 16$$

$$\therefore \text{Solution: } \underline{x = 16}$$

### Exercise

Solve  $(5x^2 - 6)^{1/4} = x$

#### Solution

$$\left[ (5x^2 - 6)^{1/4} \right]^4 = x^4$$

$$5x^2 - 6 = x^4$$

$$x^4 - 5x^2 + 6 = 0$$

$$(x^2 - 3)(x^2 - 2) = 0$$

$$x^2 = 3$$

$$x^2 = 2$$

$$x = \pm\sqrt{3}$$

$$x = \pm\sqrt{2}$$

$$\therefore \text{Solutions: } \underline{x = \pm\sqrt{3}, \pm\sqrt{2}}$$

### ***Exercise***

Solve  $\left(x^2 + 24x\right)^{1/4} = 3$

### ***Solution***

$$\left[\left(x^2 + 24x\right)^{1/4}\right]^4 = 3^4$$

$$x^2 + 24x = 81$$

$$x^2 + 24x - 81 = 0$$

$$(x + 27)(x - 3) = 0$$

$$x + 27 = 0 \quad x - 3 = 0$$

$$x = -27 \quad x = 3$$

$$\therefore \text{Solutions: } x = \underline{-27, 3}$$

### ***Exercise***

Solve:  $x^{5/2} = 32$

### ***Solution***

$$x = 32^{2/5} \quad \text{Reciprocal}$$

$$= \left(2^5\right)^{2/5}$$

$$= \underline{4}$$

### ***Exercise***

Solve:  $\sqrt[3]{2x+11} = 3$

### ***Solution***

$$2x + 11 = 3^3$$

$$2x = 27 - 11$$

$$x = \frac{16}{2}$$

$$= \underline{8}$$

***Exercise***

Solve:  $\sqrt[3]{6x-3} = 3$

**Solution**

$$6x - 3 = 3^3$$

$$6x = 27 + 3$$

$$x = \frac{30}{6}$$

$$\underline{= 5}$$

***Exercise***

Solve:  $\sqrt[3]{2x-6} = 4$

**Solution**

$$2x - 6 = 4^3$$

$$2x = 64 + 6$$

$$x = \frac{70}{2}$$

$$\underline{= 35}$$

***Exercise***

Solve:  $\sqrt[3]{4x-3} - 5 = 0$

**Solution**

$$\sqrt[3]{4x-3} = 5$$

$$4x - 3 = 5^3$$

$$4x = 125 + 3$$

$$x = \frac{128}{4}$$

$$\underline{= 32}$$

***Exercise***

Solve:  $(3x-1)^{1/3} + 4 = 0$

**Solution**

$$(3x-1)^{1/3} = -4$$



$$3x - 1 = (-4)^3$$

$$3x = 1 - 64$$

$$x = -\frac{63}{3}$$

$$\underline{= -21}$$

### ***Exercise***

Solve:  $(2x + 3)^{1/3} + 4 = 6$

### **Solution**

$$(2x + 3)^{1/3} = 2$$

$$2x + 3 = 2^3$$

$$2x = 8 - 3$$

$$\underline{x = \frac{5}{2}}$$

### ***Exercise***

Solve:  $(3x - 6)^{1/3} + 5 = 8$

### **Solution**

$$(3x - 6)^{1/3} = 3$$

$$3x - 6 = 3^3$$

$$3x = 27 + 6$$

$$x = \frac{33}{3}$$

$$\underline{= 11}$$

### ***Exercise***

Solve:  $(3x + 1)^{1/4} + 7 = 9$

### **Solution**

$$(3x + 1)^{1/4} = 2$$

$$3x + 1 = 2^4$$

$$3x = 16 - 1$$

$$x = \frac{15}{3}$$

$$= 5$$

$\therefore$  Solution set is:  $\{5\}$

### Exercise

Solve:  $(2x+3)^{1/4} + 7 = 10$

### Solution

$$(2x+3)^{1/4} = 3$$

$$2x+3 = 3^4$$

$$2x = 81 - 3$$

$$x = \frac{78}{2}$$

$$= 39$$

$\therefore$  Solution set is:  $\{39\}$

### Exercise

Solve:  $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$

### Solution

$$\left(\sqrt[3]{4x^2 - 4x + 1}\right)^3 = \left(\sqrt[3]{x}\right)^3$$

$$4x^2 - 4x + 1 = x$$

$$4x^2 - 5x + 1 = 0$$

$$(4x-1)(x-1) = 0$$

$$4x-1 = 0$$

$$x = \frac{1}{4}$$

$$x-1 = 0$$

$$x = 1$$

### Check

$$x = \frac{1}{4}$$

$$\sqrt[3]{4\left(\frac{1}{4}\right)^2 - 4\frac{1}{4} + 1} - \sqrt[3]{\frac{1}{4}} = 0$$

$$\sqrt[3]{\frac{1}{4}} - \sqrt[3]{\frac{1}{4}} = 0$$

$$0 = 0 \quad (\text{true})$$

$$x = 1$$

$$\sqrt[3]{4(1)^2 - 4(1) + 1} - \sqrt[3]{1} = 0$$

$$\sqrt[3]{4-4+1} - \sqrt[3]{1} = 0$$

$$0 = 0 \quad (\text{true})$$

∴ Solution set is:  $\left\{-1, \frac{1}{4}\right\}$

### Exercise

Solve:  $\sqrt{2x+3} = 5$

#### Solution

$$2x+3=5^2 \qquad \sqrt[n]{u}=a \rightarrow u=a^n$$

$$2x=25-3$$

$$x=\frac{22}{2}$$

$$=11$$

**Check:**  $\sqrt{2(11)+3} \stackrel{?}{=} 5$   
 $\sqrt{25} = 5 \quad \checkmark$

∴ Solution set is:  $\{11\}$

### Exercise

Solve:  $\sqrt{x-3} + 6 = 5$

#### Solution

$$\sqrt{x-3} = -1 \quad \times$$

∴ **No** solution.

### Exercise

Solve:  $\sqrt{3x-2} = 4$

#### Solution

$$3x-2=4^2 \qquad \sqrt[n]{u}=a \rightarrow u=a^n$$

$$3x=16+2$$

$$x=\frac{18}{3}$$

$$=6$$

**Check:**  $\sqrt{3(6)-2} \stackrel{?}{=} 4$   
 $\sqrt{16} = 4 \quad \checkmark$

∴ Solution set is:  $\{6\}$

### Exercise

Solve:  $\sqrt{5x-4} = 9$

#### Solution

$$5x - 4 = 9^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$5x = 81 + 4$$

$$x = \frac{85}{5}$$

$$= 17$$

**Check:**  $\sqrt{5(\textcolor{red}{17}) - 4} = 9$  <sup>?</sup>

$$\sqrt{81} = 9 \quad \checkmark$$

$\therefore$  Solution set is:  $\{17\}$

### Exercise

Solve:  $\sqrt{5x-1} = 8$

#### Solution

$$5x - 1 = 8^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$5x = 64 + 1$$

$$x = \frac{65}{5}$$

$$= 13$$

**Check:**  $\sqrt{5(\textcolor{red}{13}) - 1} = 8$  <sup>?</sup>

$$\sqrt{64} = 8 \quad \checkmark$$

$\therefore$  Solution set is:  $\{13\}$

### Exercise

Solve:  $\sqrt{3x-2} - 5 = 0$

#### Solution

$$\sqrt{3x-2} = 5$$

$$3x - 2 = 5^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$3x = 25 + 2$$

$$x = \frac{27}{3}$$

$$\underline{=9}$$

**Check:**  $\sqrt{3(9)-2}-5=0$  <sup>?</sup>  
 $5-5=0$  ✓

∴ Solution set is:  $\underline{\{9\}}$

### Exercise

Solve:  $\sqrt{2x+5}+11=6$

#### Solution

$$\sqrt{2x+5}=-5 \times$$

∴ **No** solution.

### Exercise

Solve:  $\sqrt{3x+7}+10=4$

#### Solution

$$\sqrt{3x+7}=-6 \times$$

∴ **No** solution.

### Exercise

Solve:  $x=\sqrt{7x+8}$

#### Solution

$$x^2=7x+8$$

$$x^2-7x-8=0$$

$$1-(-7)-8=0 \quad a-b+c=0$$

$$\underline{x=-1, 8} \quad x_1=-1, \quad x_2=-\frac{c}{a}$$

**Check:**

$$\begin{array}{l|l} x=-1 & x=8 \\ -1 \neq \sqrt{7(-1)+8} & 8 = \sqrt{7(8)+8} \\ & 8 = \sqrt{64} \quad \checkmark \end{array}$$

∴ Solution set is:  $\underline{\{8\}}$

### Exercise

Solve:  $x = \sqrt{6x+7}$

#### Solution

$$x^2 = 6x + 7$$

$$x^2 - 6x - 7 = 0$$

$$1 - (-6) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, 7} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

**Check:**

$$\left. \begin{array}{l} x = -1 \\ -1 \neq \sqrt{\phantom{x}} \end{array} \right| \begin{array}{l} x = 7 \\ 7 = \sqrt{6(\textcolor{red}{7})+7} \\ 7 = \sqrt{49} \quad \checkmark \end{array}$$

$\therefore$  Solution is:  $\underline{x = \textcolor{blue}{7}}$

### Exercise

Solve:  $\sqrt{5x+1} = x+1$

#### Solution

$$5x+1 = (x+1)^2$$

$$5x+1 = x^2 + 2x + 1$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$\underline{x = 0, 3}$$

**Check:**

$$\left. \begin{array}{l} x = 0 \\ \sqrt{1} = 1 \quad \checkmark \end{array} \right| \begin{array}{l} x = 3 \\ \sqrt{5(\textcolor{red}{3})+1} = \textcolor{red}{3} + 1 \\ \sqrt{16} = 4 \quad \checkmark \end{array}$$

$\therefore$  Solutions are:  $\underline{x = \textcolor{blue}{0}, \textcolor{blue}{3}}$

### Exercise

Solve:  $x = \sqrt{2x-2} + 1$

#### Solution

$$x-1=\sqrt{2x-2}$$

$$(x-1)^2=2x-2$$

$$x^2-2x+1=2x-2$$

$$x^2-4x+3=0$$

$$1-4+3=0 \quad a+b+c=0$$

$$\underline{x=1, 3} \quad x_1=1, \quad x_2=\frac{c}{a}$$

**Check:**

$$\begin{array}{l|l} x=1 & x=3 \\ 1=1 \quad \checkmark & \begin{array}{l} ? \\ 3=\sqrt{4}+1 \\ 3=3 \quad \checkmark \end{array} \end{array}$$

$\therefore$  Solutions are:  $\underline{x=1, 3}$

### Exercise

Solve:  $x-2\sqrt{x-3}=3$

**Solution**

$$x-3=2\sqrt{x-3}$$

$$(x-3)^2=(2\sqrt{x-3})^2$$

$$x^2-6x+9=4(x-3)$$

$$x^2-6x+9=4x-12$$

$$x^2-10x+21=0$$

$$\underline{x=3, 7}$$

**Check:**

$$\begin{array}{l|l} x=3 & x=7 \\ \begin{array}{l} ? \\ 3-2(0)=3 \\ 3=3 \quad \checkmark \end{array} & \begin{array}{l} ? \\ 7-2\sqrt{4}=3 \\ 7-4=3 \quad \checkmark \end{array} \end{array}$$

$\therefore$  Solution set is:  $\underline{\{3, 7\}}$

### Exercise

Solve:  $x+\sqrt{26-11x}=4$

**Solution**

$$\sqrt{26-11x} = 4-x$$

$$26-11x = (4-x)^2$$

$$26-11x = 16-8x+x^2$$

$$x^2+3x-10=0$$

$$(x-2)(x+5)=0$$

$$\underline{x=2, -5}$$

**Check:**

$$x=2$$

$$2 + \sqrt{26-22} \stackrel{?}{=} 4$$

$$2+2=4 \quad \checkmark$$

$$x=-5$$

$$-5 + \sqrt{26+55} \stackrel{?}{=} 4$$

$$-5+9=4 \quad \checkmark$$

$\therefore$  Solutions are:  $\underline{x=2, -5}$

### Exercise

Solve  $x - \sqrt{2x+3} = 0$

**Solution**

$$x = \sqrt{2x+3}$$

$$(x)^2 = (\sqrt{2x+3})^2$$

$$x^2 = 2x+3$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x+1=0$$

$$x=-1$$

$$x-3=0$$

$$x=3$$

**Check**

$$x=-1$$

$$(-1) - \sqrt{2(-1)+3} = 0$$

$$-1 - \sqrt{1} = 0$$

False

$$x=3$$

$$(3) - \sqrt{2(3)+3} = 0$$

$$3 - \sqrt{9} = 0$$

True

$\therefore$  Solution set is:  $\underline{\{3\}}$

### Exercise

Solve:  $\sqrt{x+3} + 3 = x$

**Solution**



$$\sqrt{x+3} = x-3$$

$$(\sqrt{x+3})^2 = (x-3)^2$$

$$x+3 = x^2 - 6x + 9$$

$$x^2 - 7x + 6 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{25}}{2}$$

$$= \frac{7 \pm 5}{2} = \begin{cases} \frac{7+5}{2} = \frac{12}{2} = 6 \\ \frac{7-5}{2} = \frac{2}{2} = 1 \end{cases} \Rightarrow x = 1, 6$$

**Check:**

$$x = 1 \Rightarrow \sqrt{1+3} + 3 = 1 \Rightarrow 5 = 1 \text{ (Not a solution)}$$

$$x = 6 \Rightarrow \sqrt{6+3} + 3 = 6 \Rightarrow 6 = 6 \rightarrow x = 6 \text{ is the only solution}$$

## Exercise

Solve  $x - \sqrt{x+11} = 1$

**Solution**

$$-\sqrt{x+11} = 1 - x$$

*Square both side*

$$(-\sqrt{x+11})^2 = (1-x)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$x+11 = 1 - 2x + x^2$$

$$0 = x^2 - 2x + 1 - x - 11$$

$$0 = x^2 - 3x - 10$$

$$x^2 - 3x - 10 = 0$$

*Solve for x*

$$x = 5, -2$$

**Check:**

$$x = 5 \Rightarrow 5 - \sqrt{5+11} = 1 \Rightarrow 5 - \sqrt{16} = 1 \Rightarrow 5 - 4 = 1 \Rightarrow 1 = 1$$

$$x = -2 \Rightarrow -2 - \sqrt{-2+11} = 1 \Rightarrow -2 - \sqrt{9} = 1 \Rightarrow -2 - 3 = 1 \Rightarrow -5 = 1 \text{ (False)}$$

$\therefore$  Solution set is:  $\boxed{\{5\}}$

### ***Exercise***

Solve:  $\sqrt{x-7} = 7 - \sqrt{x}$

### ***Solution***

$$(\sqrt{x-7})^2 = (7 - \sqrt{x})^2$$

$$x - 7 = 49 - 14\sqrt{x} + x$$

$$14\sqrt{x} = 56$$

$$\sqrt{x} = \frac{56}{14}$$

$$= 4$$

$$\underline{x = 16}$$

### ***Check:***

$$x = 16$$

$$\sqrt{16-7} \stackrel{?}{=} 7 - 4$$

$$3 = 3 \quad \checkmark$$

$\therefore$  Solution set is:  $\underline{\{16\}}$

### ***Exercise***

Solve:  $\sqrt{x-8} = \sqrt{x} - 2$

### ***Solution***

$$(\sqrt{x-8})^2 = (\sqrt{x} - 2)^2$$

$$x - 8 = x - 4\sqrt{x} + 4$$

$$4\sqrt{x} = 12$$

$$\sqrt{x} = 3$$

$$\underline{x = 9}$$

### ***Check:***

$$x = 9$$

$$\sqrt{9-8} \stackrel{?}{=} \sqrt{9} - 2$$

$$1 = 3 - 2 \quad \checkmark$$

$\therefore$  Solution set is:  $\underline{\{9\}}$

### ***Exercise***

Solve:  $\sqrt{2x-5} = \sqrt{x+4}$

#### **Solution**

$$(\sqrt{2x-5})^2 = (\sqrt{x+4})^2$$

$$2x - 5 = x + 4$$

$$\underline{x = 9}$$

***Check:***

$$x = 9$$

$$\sqrt{18-5} \stackrel{?}{=} \sqrt{9+4}$$

$$\sqrt{13} = \sqrt{13} \quad \checkmark$$

$\therefore$  Solution set is:  $\underline{\{9\}}$

### ***Exercise***

Solve:  $\sqrt{6x+2} = \sqrt{5x+3}$

#### **Solution**

$$(\sqrt{6x+2})^2 = (\sqrt{5x+3})^2$$

$$6x + 2 = 5x + 3$$

$$\underline{x = 1}$$

***Check:***

$$x = 1$$

$$\sqrt{6+2} \stackrel{?}{=} \sqrt{5+3}$$

$$\sqrt{8} = \sqrt{8} \quad \checkmark$$

$\therefore$  Solution set is:  $\underline{\{1\}}$

### ***Exercise***

Solve:  $\sqrt{3x+1} - \sqrt{x+4} = 1$

#### **Solution**

$$\sqrt{3x+1} = 1 + \sqrt{x+4}$$

$$(\sqrt{3x+1})^2 = (1 + \sqrt{x+4})^2$$

$$3x+1=1+2\sqrt{x+4}+x+4$$

$$2x-4=2\sqrt{x+4}$$

$$x-2=\sqrt{x+4}$$

$$(x-2)^2=x+4$$

$$x^2-4x+4=x+4$$

$$x^2-5x=0$$

$$x(x-5)=0$$

$$\underline{x=0, 5}$$

**Check:**

$$x=0$$

$$1-\sqrt{4} \stackrel{?}{=} 1$$

$$-1 \neq 1 \quad \times$$

$$x=5$$

$$\sqrt{15+1}-\sqrt{5+4} \stackrel{?}{=} 1$$

$$4-3=1 \quad \checkmark$$

$\therefore$  Solution is:  $\underline{x=5}$

### Exercise

Solve:  $\sqrt{x+2}+\sqrt{x-1}=3$

**Solution**

$$\sqrt{x+2}=3-\sqrt{x-1}$$

$$x+2=\left(3-\sqrt{x-1}\right)^2$$

$$x+2=9-6\sqrt{x-1}+x-1$$

$$6\sqrt{x-1}=6$$

$$\sqrt{x-1}=1$$

$$x-1=1^2$$

$$\underline{x=2}$$

**Check:**

$$x=2$$

$$\sqrt{4}+1 \stackrel{?}{=} 3$$

$$2+1=3 \quad \checkmark$$

$\therefore$  Solution is:  $\underline{x=2}$

### Exercise

Solve:  $\sqrt{x-4} + \sqrt{x+4} = 4$

#### Solution

$$\sqrt{x-4} = 4 - \sqrt{x+4}$$

$$x-4 = (4 - \sqrt{x+4})^2$$

$$x-4 = 16 - 8\sqrt{x+4} + x+4$$

$$8\sqrt{x+4} = 24$$

$$\sqrt{x+4} = 3$$

$$x+4 = 9$$

$$\underline{x = 5}$$

#### Check:

$$x = 5$$

$$\sqrt{1} + \sqrt{9} = 4$$

$$1 + 3 = 4 \quad \checkmark$$

$$\therefore \text{Solution is: } \underline{x = 5}$$

### Exercise

Solve:  $\sqrt{2x-3} - \sqrt{x-2} = 1$

#### Solution

$$\sqrt{2x-3} = 1 + \sqrt{x-2}$$

$$2x-3 = (1 + \sqrt{x-2})^2$$

$$2x-3 = 1 + 2\sqrt{x-2} + x-2$$

$$x-2 = 2\sqrt{x-2}$$

$$(x-2)^2 = (2\sqrt{x-2})^2$$

$$x^2 - 4x + 4 = 4(x-2)$$

$$x^2 - 4x + 4 = 4x - 8$$

$$x^2 - 8x + 12 = 0$$

$$\underline{x = 2, 6}$$

#### Check:

$$x = 2 \quad | \quad x = 6$$

$$\begin{array}{l|l} \sqrt{4-3} - \sqrt{2-2} \stackrel{?}{=} 1 & \sqrt{12-3} - \sqrt{6-2} \stackrel{?}{=} 1 \\ 1 = 1 \quad \checkmark & 3 - 2 = 1 \quad \checkmark \end{array}$$

$\therefore$  Solution is:  $\underline{x = 2, 6}$

### Exercise

Solve:  $\sqrt{x+2} + \sqrt{3x+7} = 1$

#### Solution

$$\sqrt{x+2} = 1 - \sqrt{3x+7}$$

$$x+2 = (1 - \sqrt{3x+7})^2$$

$$x+2 = 1 - 2\sqrt{3x+7} + 3x+7$$

$$2\sqrt{3x+7} = 2x+6$$

$$\sqrt{3x+7} = x+3$$

$$3x+7 = (x+3)^2$$

$$3x+7 = x^2 + 6x+9$$

$$x^2 + 3x + 2 = 0$$

$$\underline{x = -1, -2}$$

#### **Check:**

$$\begin{array}{l|l} x = -1 & x = -2 \\ \sqrt{-1+2} \stackrel{?}{=} 1 - \sqrt{-3+7} & \sqrt{-2+2} \stackrel{?}{=} 1 - \sqrt{-6+7} \\ 1 \neq 1 - 2 \quad \times & 0 = 1 - 1 \quad \checkmark \end{array}$$

$\therefore$  Solution is:  $\underline{x = -2}$

### Exercise

Solve:  $2\sqrt{4x+1} - 9 = x - 5$

#### Solution

$$2\sqrt{4x+1} = x+4$$

$$(2\sqrt{4x+1})^2 = (x+4)^2$$

$$4(4x+1) = x^2 + 8x + 16$$

$$16x + 4 = x^2 + 8x + 16$$

$$x^2 - 8x + 12 = 0$$

$$\underline{x = 2, 6}$$

**Check:**

$$\begin{array}{l|l} x = 2 & x = 6 \\ 2\sqrt{8+1}-9 \stackrel{?}{=} 2-5 & 2\sqrt{24+1}-9 \stackrel{?}{=} 6-5 \\ 6-9 = -3 \quad \checkmark & 10-9 = 1 \quad \checkmark \end{array}$$

$\therefore$  Solution is:  $\underline{x = 2, 6}$

### Exercise

Solve:  $x\sqrt{x-3} + 4 = x + 1$

**Solution**

$$x\sqrt{x-3} = x - 3$$

$$(x\sqrt{x-3})^2 = (x-3)^2$$

$$x^2(x-3) - (x-3)^2 = 0$$

$$(x-3)(x^2 - x + 3) = 0$$

$$x^2 - x + 3 = 0 \rightarrow x = \frac{1 \pm \sqrt{1-12}}{2} \in \mathbb{C}$$

$$\underline{x = 3} \quad \text{Only result.}$$

**Check:**

$$\begin{array}{l} x = 3 \\ 3\sqrt{3-3} + 4 \stackrel{?}{=} 3 + 1 \\ 4 = 4 \quad \checkmark \end{array}$$

$\therefore$  Solution is:  $\underline{x = 3}$

### Exercise

Solve:  $\sqrt{2x-3} + \sqrt{x-2} = 1$

**Solution**

$$\sqrt{2x-3} = 1 - \sqrt{x-2}$$

$$(\sqrt{2x-3})^2 = (1 - \sqrt{x-2})^2$$

$$2x - 3 = 1 - 2\sqrt{x-2} + (\sqrt{x-2})^2$$

$$2x - 3 - 1 = -2\sqrt{x-2} + x - 2$$

$$2x - 4 - x + 2 = -2\sqrt{x-2}$$

$$x - 2 = -2\sqrt{x-2}$$

$$(x-2)^2 = (-2\sqrt{x-2})^2$$

$$x^2 - 4x + 4 = 4(x-2)$$

$$x^2 - 4x + 4 = 4x - 8$$

$$x^2 - 4x + 4 - 4x + 8 = 0$$

$$x^2 - 8x + 12 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$$

$$x = 2, 6$$

**Check**

$$x = 2 \Rightarrow \sqrt{2(2) - 3} + \sqrt{2 - 2} = 1 \Rightarrow 1 + 0 = 1$$

$$x = 6 \Rightarrow \sqrt{2(6) - 3} + \sqrt{6 - 2} = 1 \Rightarrow 3 + 2 = 1 \Rightarrow 5 \neq 1$$

$\therefore$  Solution is:  $x = 2$

### ***Exercise***

Solve:  $\sqrt{x+5} - \sqrt{x-3} = 2$

**Solution**

$$\sqrt{x+5} = 2 + \sqrt{x-3}$$

$$(\sqrt{x+5})^2 = (2 + \sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + (\sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + x-3$$

$$x - x + 5 - 4 - 3 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$$1 = \sqrt{x-3}$$

$$1 = x - 3$$

$$\Rightarrow x = 4$$

***Check:***  $\sqrt{4+5} - \sqrt{4-3} = 2$

$$3 - 1 = 2 \text{ (True statement)}$$

$\therefore$  Solution is:  $x = 4$



### Exercise

Solve:  $\sqrt{2x+3} = 1 + \sqrt{x+1}$

### Solution

$$\left(\sqrt{2x+3}\right)^2 = \left(1 + \sqrt{x+1}\right)^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$2x+3 = 1 + 2\sqrt{x+1} + x+1$$

$$2x+3 = 2\sqrt{x+1} + x+2$$

$$x+1 = 2\sqrt{x+1}$$

$$(x+1)^2 = (2\sqrt{x+1})^2$$

$$x^2 + 2x + 1 = 4(x+1)$$

$$x^2 + 2x + 1 = 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x-3=0 \quad x+1=0$$

$$x=3 \quad x=-1$$

### Check

$$x=3$$

$$\sqrt{2(3)+3} = 1 + \sqrt{(3)+1}$$

$$\sqrt{9} = 1 + \sqrt{4}$$

$$3 = 3 \quad (\text{true})$$

$$x=-1$$

$$\sqrt{2(-1)+3} = 1 + \sqrt{(-1)+1}$$

$$\sqrt{1} = 1 + \sqrt{0}$$

$$1 = 1 \quad (\text{true})$$

$$\therefore \text{Solution: } \underline{x = -1, 3}$$

### Exercise

Solve:  $\sqrt{x+5} - \sqrt{x-3} = 2$

### Solution

$$\sqrt{x+5} = 2 + \sqrt{x-3}$$

$$\left(\sqrt{x+5}\right)^2 = \left(2 + \sqrt{x-3}\right)^2$$

$$x+5 = 4 + 4\sqrt{x-3} + (\sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + x-3$$

$$x-x+5-4-3 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$$1 = \sqrt{x-3}$$

$$1 = x - 3$$

$$\Rightarrow x = 4$$

**Check:**  $\sqrt{4+5} - \sqrt{4-3} = 2$

$$3 - 1 = 2 \text{ (True statement)}$$

$\therefore$  Solution is:  $\underline{x = 4}$

### ***Exercise***

Solve:  $|x| = -9$

#### **Solution**

$|x| = -9$       Not True

$\therefore$  **No** Solution

### ***Exercise***

Solve:  $|x| = 9$

#### **Solution**

$\therefore$  Solutions:  $\underline{x = \pm 9}$

### ***Exercise***

Solve:  $|x-2| = 7$

#### **Solution**

$$x - 2 = 7 \qquad x - 2 = -7$$

$$\underline{x = 9} \qquad \underline{x = -5}$$

$\therefore$  Solutions:  $\underline{x = -5, 9}$

### ***Exercise***

Solve:  $|x-2| = 0$

#### **Solution**

$$x - 2 = 0$$

$\therefore$  Solution:  $\underline{x = 2}$

### ***Exercise***

Solve:  $|2x - 3| = 6$

#### **Solution**

$$2x - 3 = 6 \qquad 2x - 3 = -6$$

$$2x = 9 \qquad 2x = -3$$

$$\underline{x = \frac{9}{2} \quad \quad x = -\frac{3}{2}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{3}{2}, \frac{9}{2}}$$

### ***Exercise***

Solve:  $|2x - 1| = 11$

#### **Solution**

$$2x - 1 = 11 \qquad 2x - 1 = -11$$

$$2x = 12 \qquad 2x = -10$$

$$\underline{x = 6 \quad \quad x = -5}$$

$$\therefore \text{Solutions: } \underline{x = -5, 6}$$

### ***Exercise***

Solve  $7|5x| + 2 = 16$

#### **Solution**

$$7|5x| = 16 - 2$$

$$7|5x| = 14$$

$$|5x| = \frac{14}{7}$$

$$|5x| = 2$$

$$\begin{array}{l|l} 5x = 2 & 5x = -2 \\ x = \frac{2}{5} & x = -\frac{2}{5} \end{array}$$

$$\therefore \text{Solution: } \underline{x = \pm \frac{2}{5}}$$

### Exercise

Solve  $4\left|1 - \frac{3}{4}x\right| + 7 = 10$

### Solution

$$4\left|1 - \frac{3}{4}x\right| = 10 - 7$$

$$|4 - 3x| = 10 - 7 \quad \text{Distribute 4}$$

$$4\left|1 - \frac{3}{4}x\right| = 3$$

$$|4 - 3x| = 3$$

$$\left|1 - \frac{3}{4}x\right| = \frac{3}{4}$$

$$\begin{array}{l|l} 1 - \frac{3}{4}x = \frac{3}{4} & 1 - \frac{3}{4}x = -\frac{3}{4} \\ -\frac{3}{4}x = \frac{3}{4} - 1 & -\frac{3}{4}x = -\frac{3}{4} - 1 \\ -\frac{3}{4}x = -\frac{1}{4} & -\frac{3}{4}x = -\frac{7}{4} \\ x = -\frac{1}{4}\left(-\frac{4}{3}\right) & x = -\frac{7}{4}\left(-\frac{4}{3}\right) \\ x = \frac{1}{3} & x = \frac{7}{3} \end{array}$$

$$\begin{array}{l|l} 4 - 3x = -3 & 4 - 3x = 3 \\ -3x = -7 & -3x = -1 \\ x = \frac{7}{3} & x = \frac{1}{3} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \frac{1}{3}, \frac{7}{3}}$$

### Exercise

Solve  $|x + 7| + 6 = 2$

### Solution

$$|x + 7| = 2 - 6$$

$$|x + 7| = -4$$

$\Rightarrow$  No solution or  $\emptyset$ , since the absolute value can't be equal to a negative.

### Exercise

Solve equation:  $|5 - 3x| = 12$

### Solution

$$\begin{array}{l|l} 5 - 3x = 12 & 5 - 3x = -12 \\ 5 - 3x - 5 = 12 - 5 & 5 - 3x - 5 = -12 - 5 \\ -3x = 7 & -3x = -17 \\ x = -\frac{7}{3} & x = \frac{17}{3} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \frac{17}{3}, -\frac{7}{3}}$$

### ***Exercise***

Solve equation:  $|4x + 2| = 5$

#### **Solution**

$$4x + 2 = -5 \qquad 4x + 2 = 5$$

$$4x = -7 \qquad 4x = 3$$

$$x = -\frac{7}{4} \qquad x = \frac{3}{4}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{7}{4}, \frac{3}{4}}$$

### ***Exercise***

Solve:  $3|x + 5| = 12$

#### **Solution**

$$|x + 5| = 4$$

$$x + 5 = 4 \qquad x + 5 = -4$$

$$\underline{x = -1} \qquad \underline{x = -9}$$

$$\therefore \text{Solutions: } \underline{x = -9, -1}$$

### ***Exercise***

Solve:  $2|x - 6| = 8$

#### **Solution**

$$x - 6 = 4$$

$$x - 6 = 4 \qquad x - 6 = -4$$

$$\underline{x = 10} \qquad \underline{x = 2}$$

$$\therefore \text{Solutions: } \underline{x = 2, 10}$$

### ***Exercise***

Solve:  $3|2x - 1| = 21$

#### **Solution**

$$|2x - 1| = 7$$

$$2x - 1 = 7 \qquad 2x - 1 = -7$$

$$2x = 8 \qquad 2x = -6$$

$$\underline{x = 4} \qquad \underline{x = -3}$$

$$\therefore \text{Solutions: } \underline{x = -3, 4}$$

### ***Exercise***

$$\text{Solve: } 2|3x - 2| = 14$$

### **Solution**

$$|3x - 2| = 7$$

$$3x - 2 = 7 \qquad 3x - 2 = -7$$

$$3x = 9 \qquad 3x = -5$$

$$\underline{x = 3} \qquad \underline{x = -\frac{5}{3}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{5}{3}, 3}$$

### ***Exercise***

$$\text{Solve: } |3x - 1| + 2 = 16$$

### **Solution**

$$|3x - 1| = 14$$

$$3x - 1 = 14 \qquad 3x - 1 = -14$$

$$3x = 15 \qquad 3x = -13$$

$$\underline{x = 5} \qquad \underline{x = -\frac{13}{3}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{13}{3}, 5}$$

### ***Exercise***

$$\text{Solve: } |6x - 2| + 4 = 32$$

### **Solution**

$$|6x - 2| = 28$$

$$6x - 2 = 28 \qquad 6x - 2 = -28$$

$$6x = 30 \qquad 6x = -26$$

$$\underline{x = 5}$$

$$\begin{array}{l} x = -\frac{13}{3} \\ \hline \therefore \text{Solutions: } x = -\frac{13}{3}, 5 \end{array}$$

### ***Exercise***

Solve:  $7|5x| + 2 = 16$

### **Solution**

$$7|5x| = 14$$

$$|5x| = 2$$

$$5x = 2$$

$$5x = -2$$

$$x = \frac{2}{5}$$

$$x = -\frac{2}{5}$$

$$\therefore \text{Solutions: } x = -\frac{2}{5}, \frac{2}{5}$$

### ***Exercise***

Solve:  $|4x + 1| + 10 = 4$

### **Solution**

$$|4x + 1| = -6$$

$\therefore$  **No Solution**

### ***Exercise***

Solve:  $|4x + 1| + 4 = 10$

### **Solution**

$$|4x + 1| = 6$$

$$4x + 1 = 6$$

$$4x + 1 = -6$$

$$4x = 5$$

$$4x = -7$$

$$x = \frac{5}{4}$$

$$x = -\frac{7}{4}$$

$$\therefore \text{Solutions: } x = -\frac{7}{4}, \frac{5}{4}$$

### ***Exercise***

Solve:  $|3x - 2| + 8 = 1$

#### **Solution**

$$|3x - 2| = -7$$

$\therefore$  **No Solution**

### ***Exercise***

Solve:  $|3x - 2| + 1 = 8$

#### **Solution**

$$|3x - 2| = 7$$

$$4x + 1 = 6 \quad 4x + 1 = -6$$

$$4x = 5 \quad 4x = -7$$

$$\underline{x = \frac{5}{4} \quad x = -\frac{7}{4}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{7}{4}, \frac{5}{4}}$$

### ***Exercise***

Solve equation:  $\left| \frac{6x+1}{x-1} \right| = 3$

#### **Solution**

$$\frac{6x+1}{x-1} = -3$$

$$(x-1) \frac{6x+1}{x-1} = -3(x-1)$$

$$6x+1 = -3x+3$$

$$6x+1+3x = -3x+3+3x$$

$$9x+1 = 3$$

$$9x = 2$$

$$x = \frac{2}{9}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{4}{3}, \frac{2}{9}}$$

$$\frac{6x+1}{x-1} = 3$$

$$6x+1 = 3(x-1)$$

$$6x+1 = 3x-3$$

$$6x+1-3x = 3x-3-3x$$

$$3x+1 = -3$$

$$3x = -4$$

$$x = -\frac{4}{3}$$



### ***Exercise***

Solve equation:  $|x+1| = |1-3x|$

#### **Solution**

$$\begin{array}{ll} x+1 = -(1-3x) & x+1 = 1-3x \\ x+1 = -1+3x & x+3x = 1-1 \\ x-3x = -1-1 & 4x = 0 \\ -2x = -2 & x = 0 \\ x = 1 & \end{array}$$

$\therefore$  Solutions:  $\underline{x = 0, 1}$

### ***Exercise***

Solve:  $|3x-1| = |x+5|$

#### **Solution**

$$\begin{array}{ll} 3x-1 = x+5 & 3x-1 = -(x+5) \\ 2x = 6 & 3x-1 = -x-5 \\ \underline{x = 3} & 4x = -4 \\ & \underline{x = -1} \end{array}$$

$\therefore$  Solutions:  $\underline{x = -1, 3}$

### ***Exercise***

Solve:  $|5x-8| = |3x+2|$

#### **Solution**

$$\begin{array}{ll} 5x-8 = 3x+2 & 5x-8 = -(3x+2) \\ 2x = 10 & 5x-8 = -3x-2 \\ \underline{x = 5} & 8x = 6 \\ & \underline{x = \frac{3}{4}} \end{array}$$

$\therefore$  Solutions:  $\underline{x = \frac{3}{4}, 5}$

### ***Exercise***

Solve:  $|4x - 9| = |2x + 1|$

#### **Solution**

$$4x - 9 = 2x + 1 \quad 4x - 9 = -(2x + 1)$$

$$2x = 10$$

$$5x - 8 = -3x - 2$$

$$\underline{x = 5}$$

$$8x = 6$$

$$\underline{x = \frac{3}{4}}$$

$$\therefore \text{Solutions: } \underline{x = \frac{3}{4}, 5}$$

### ***Exercise***

Solve:  $|2x - 4| = |x - 1|$

#### **Solution**

$$2x - 4 = x - 1$$

$$2x - 4 = -x + 1$$

$$\underline{x = 3}$$

$$3x = -5$$

$$\underline{x = -\frac{5}{3}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{5}{3}, 3}$$

### ***Exercise***

Solve:  $|3x - 4| = |3x + 4|$

#### **Solution**

$$3x - 4 = 3x + 4$$

$$3x - 4 = -3x + 4$$

$$-4 \neq 4$$

$$6x = 8$$

$$\underline{x = \frac{4}{3}}$$

$$\therefore \text{Solution: } \underline{x = \frac{4}{3}}$$

### ***Exercise***

Solve:  $|3x - 5| = |3x + 5|$

#### **Solution**

$$\begin{array}{ll}
 3x - 5 = 3x + 5 & 3x - 5 = -3x + 5 \\
 -5 \neq 5 & 6x = 10 \\
 & x = \frac{5}{3}
 \end{array}$$

$$\therefore \text{Solution: } x = \frac{5}{3}$$

### Exercise

$$\text{Solve: } |x - 3| = |5 - x|$$

#### Solution

$$\begin{array}{ll}
 x - 3 = 5 - x & x - 3 = -5 + x \\
 2x = 8 & -3 \neq -5 \\
 x = 4
 \end{array}$$

$$\therefore \text{Solutions: } x = 4$$

### Exercise

$$\text{Solve: } |x - 3| = |6 - x|$$

#### Solution

$$\begin{array}{ll}
 x - 3 = 6 - x & x - 3 = -6 + x \\
 2x = 9 & -3 \neq -6 \\
 x = \frac{9}{2}
 \end{array}$$

$$\therefore \text{Solutions: } x = \frac{9}{2}$$

### Exercise

$$\text{Solve: } \left| \frac{2}{3}x - 2 \right| = \left| \frac{1}{3}x + 3 \right|$$

#### Solution

$$\begin{array}{ll}
 \frac{2}{3}x - 2 = \frac{1}{3}x + 3 & \frac{2}{3}x - 2 = -\frac{1}{3}x - 3 \\
 \frac{2}{3}x - \frac{1}{3}x = 3 + 2 & \frac{2}{3}x + \frac{1}{3}x = -3 + 2 \\
 \frac{1}{3}x = 5 & x = -1 \\
 x = 15
 \end{array}$$

∴ Solutions:  $x = -1, 15$

### Exercise

Solve:  $\left| \frac{1}{2}x - 2 \right| = \left| x - \frac{1}{2} \right|$

### Solution

$$\frac{1}{2}x - 2 = x - \frac{1}{2} \quad \frac{1}{2}x - 2 = -x + \frac{1}{2}$$

$$\frac{1}{2}x - x = 2 - \frac{1}{2} \quad \frac{1}{2}x + x = 2 + \frac{1}{2}$$

$$\frac{1}{2}x = \frac{3}{2} \quad \frac{3}{2}x = \frac{5}{2}$$

$$\underline{x = 3} \quad \underline{x = \frac{5}{3}}$$

∴ Solutions:  $x = 3, \frac{5}{3}$

### Exercise

Two vertical poles of lengths 4 feet and 10 feet stand 15 feet apart. A cable reaches from the top of one pole to some point on the ground between the poles and then to the top of the other pole. Where should this point be located to use 24 feet of cable?

### Solution

$$l_1^2 = x^2 + 4^2 \quad l_1 = \sqrt{x^2 + 16}$$

$$l_2^2 = (15 - x)^2 + 10^2 \quad l_2 = \sqrt{(15 - x)^2 + 100}$$

$$l_1 + l_2 = 24$$

$$\sqrt{x^2 + 16} + \sqrt{(15 - x)^2 + 100} = 24$$

$$\sqrt{(15 - x)^2 + 100} = 24 - \sqrt{x^2 + 16}$$

$$\left( \sqrt{(15 - x)^2 + 100} \right)^2 = \left( 24 - \sqrt{x^2 + 16} \right)^2$$

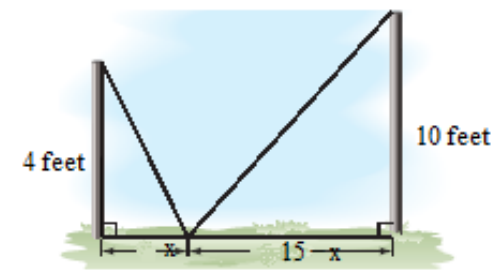
$$x^2 - 30x + 225 + 100 = 576 - 48\sqrt{x^2 + 16} + x^2 + 16$$

$$x^2 - 30x + 325 - x^2 - 576 - 16 = -48\sqrt{x^2 + 16}$$

$$-30x - 267 = -48\sqrt{x^2 + 16}$$

$$30x + 267 = 48\sqrt{x^2 + 16}$$

$$(30x + 267)^2 = 48^2(x^2 + 16)$$



$$900x^2 + 16020x + 71289 = 2304(x^2 + 16)$$

$$900x^2 + 16020x + 71289 = 2304x^2 + 36864$$

$$900x^2 + 16020x + 71289 - 2304x^2 - 36864 = 0$$

$$-1404x^2 + 16020x + 34425 = 0$$

$$x \approx 13.259$$

### Exercise

Towns **A** and **B** are located 6 miles and 3 miles, respectively, from a major expressway. The point on the expressway closet to town **A** is 12 miles from the point on the expressway closet to town **B**. Two new roads are to be built from **A** to the expressway and then to **B**.

- Express the combined lengths of the new road in terms of  $x$ .
- If the combined lengths of the new roads is 15 miles, what distance does  $x$  represent?

### Solution

$$a) \quad d_1^2 = x^2 + 6^2 \rightarrow d_1 = \sqrt{x^2 + 36}$$

$$d_2^2 = (12 - x)^2 + 3^2 \rightarrow d_2 = \sqrt{(12 - x)^2 + 9}$$

$$d_1 + d_2 = \sqrt{x^2 + 36} + \sqrt{(12 - x)^2 + 9}$$

$$b) \quad \sqrt{x^2 + 36} + \sqrt{(12 - x)^2 + 9} = 15$$

$$\sqrt{x^2 + 36} = 15 - \sqrt{144 - 24x + x^2 + 9}$$

$$\left(\sqrt{x^2 + 36}\right)^2 = \left(15 - \sqrt{x^2 - 24x + 153}\right)^2$$

$$x^2 + 36 = 225 - 30\sqrt{x^2 - 24x + 153} + x^2 - 24x + 153$$

$$30\sqrt{x^2 - 24x + 153} = -24x + 342$$

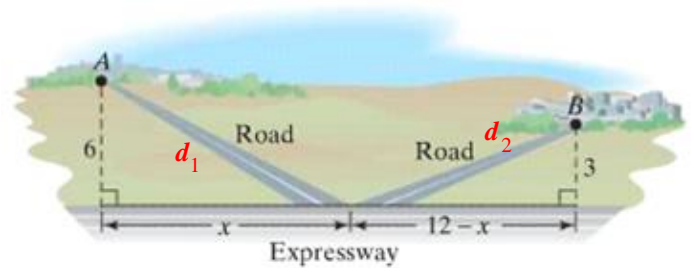
$$\left(30\sqrt{x^2 - 24x + 153}\right)^2 = (-24x + 342)^2$$

$$900(x^2 - 24x + 153) = 576x^2 - 16416x + 116964$$

$$900x^2 - 21600x + 137700 = 576x^2 - 16416x + 116964$$

$$324x^2 - 5184x + 20736 = 0 \quad \text{Solve for } x:$$

$$x = 8$$



### Exercise

A solid silver sphere has a diameter of 8 *millimeters*, and a second silver has a diameter of 12 *millimeters*. The spheres are melted down and recast to form a single cube. What is the length  $s$  of each edge of the cube?

### Solution

$$\begin{aligned} V &= \frac{4\pi}{3} \left(\frac{8}{2}\right)^3 + \frac{4\pi}{3} \left(\frac{12}{2}\right)^3 & V &= \frac{4\pi}{3} r^3 \\ &= \frac{4\pi}{3} (64 + 216) \\ &= \frac{4\pi}{3} (280) \\ &= \frac{1,120\pi}{3} \end{aligned}$$

$$\begin{aligned} V_{\text{cube}} &= s^3 = \frac{1,120\pi}{3} \\ s &= \sqrt[3]{\frac{1,120\pi}{3}} \text{ mm} \approx 10.5 \text{ mm} \end{aligned}$$

### Exercise

The period  $T$  of the pendulum is the time it takes the pendulum to complete one swing from left to right and back. For a pendulum near the surface of Earth

$$T = 2\pi \sqrt{\frac{L}{32}}$$

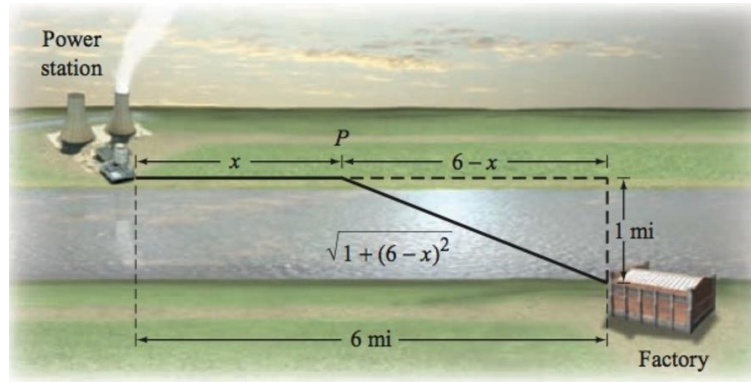
Where  $T$  is measured in *seconds* and  $L$  is the length of the pendulum in *feet*. Find the length of a pendulum that has a period of 4 *seconds*.

### Solution

$$\begin{aligned} T &= 2\pi \sqrt{\frac{L}{32}} = 4 \\ \sqrt{\frac{L}{32}} &= \frac{2}{\pi} \\ \frac{L}{32} &= \frac{4}{\pi^2} \\ L &= \frac{128}{\pi^2} \text{ feet} \\ &\approx 13 \text{ feet} \end{aligned}$$

### Exercise

A power station is on one side of a river that is 1 *mile* wide, and a factory is 6 *miles* down-stream on the other side of the river, the cost is \$0.125 *million* per *mile* to run power lines over land and \$0.2 *million* per *mile* to run power lines under water. How far over the land should the power line be run if the total cost of the project is to be \$1 *million*?



### Solution

Let  $x$  be the distance the power lines overland.

$\sqrt{1 + (6 - x)^2}$  the distance the power lines underwater.

The total cost is given:

$$0.125x + 0.2\sqrt{1 + (6 - x)^2} = 1$$

$$0.2\sqrt{1 + (6 - x)^2} = 1 - 0.125x$$

$$200\sqrt{1 + (6 - x)^2} = 1000 - 125x$$

$$8\sqrt{1 + 36 - 12x + x^2} = 40 - 5x$$

$$\left(8\sqrt{1 + 36 - 12x + x^2}\right)^2 = (40 - 5x)^2$$

$$64(37 - 12x + x^2) = 1600 - 400x + 25x^2$$

$$2,368 - 768x + 64x^2 = 1600 - 400x + 25x^2$$

$$39x^2 - 368x + 768 = 0$$

$$x = \frac{368 \pm \sqrt{15,616}}{78}$$

$$\approx \begin{cases} \frac{368 - 125}{78} \approx 3.11 \\ \frac{368 + 125}{78} \approx 6.32 > 6 \end{cases}$$

$\therefore$  Distance of the power lines overland is **3.11** km.

## Exercise

A cabin is located in a meadow at the end of a straight driveway 2 km long. A post office is located 5 km from the driveway along a straight road. A woman walks 2 km/hr through the meadow to point  $P$  and then 5 km/hr along the road to the post office. If it takes the woman 2.25 hours to reach the post office, what is the distance  $x$  of point  $P$  from the end of the driveway?

## Solution

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

$$\text{Time to walk from cabin to } P = \frac{\sqrt{4+x^2}}{2}$$

$$\text{Time to walk from } P \text{ to Post Office} = \frac{5-x}{5}$$

$$\frac{\sqrt{4+x^2}}{2} + \frac{5-x}{5} = 2.25$$

$$5\sqrt{4+x^2} + 10 - 2x = 22.5$$

$$5\sqrt{4+x^2} = 2x + 12.5$$

$$\left(5\sqrt{4+x^2}\right)^2 = (2x + 12.5)^2$$

$$25(4+x^2) = 4x^2 + 50x + 156.25$$

$$100 + 25x^2 = 4x^2 + 50x + 156.25$$

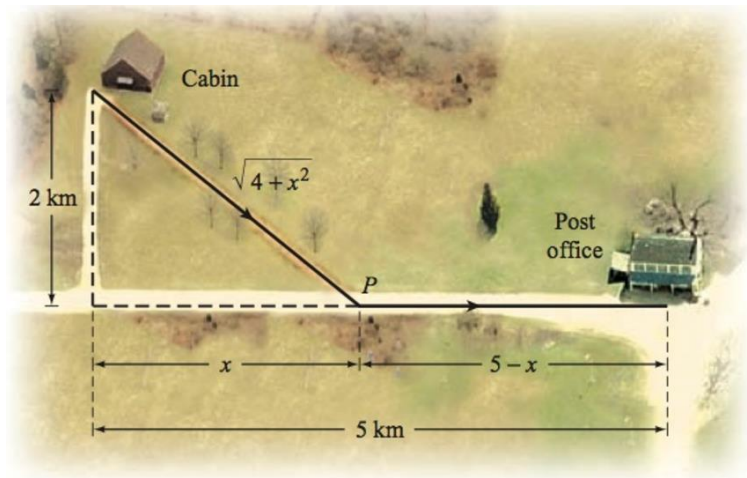
$$21x^2 - 50x - 56.25 = 0$$

$$x = \frac{50 \pm \sqrt{7,225}}{42}$$

$$= \frac{50 \pm 85}{42}$$

$$= \left\{ \begin{array}{l} \frac{50+85}{42} = \frac{45}{14} \\ \frac{50-85}{42} = -\frac{35}{42} \end{array} \right\} < 0$$

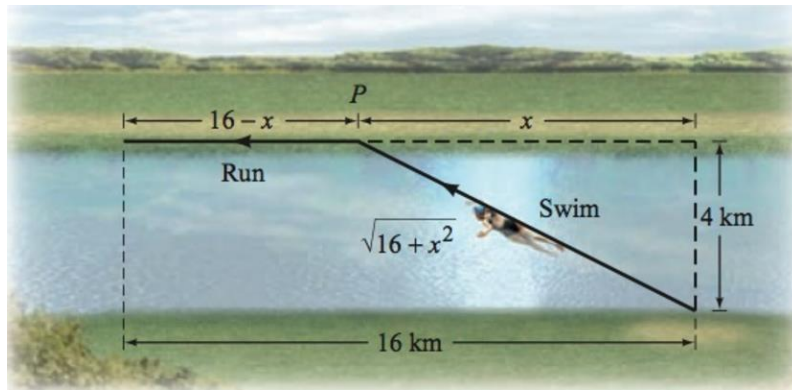
$\therefore$  Distance of point  $P$  from the end of the driveway is  $\frac{45}{14} \approx 3.21$  km.





## Exercise

To prepare for a triathlon, a person swims across a river to point  $P$  and then runs along a path.



The person swims at  $7 \text{ km/hr}$  and runs at  $22 \text{ km/hr}$ . For what distance  $x$  is the total time for swimming and running  $2 \text{ hours}$ ?

### Solution

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

$$\text{Time swimming} = \frac{\sqrt{16 + x^2}}{7}$$

$$\text{Time runs} = \frac{16 - x}{22}$$

$$\frac{\sqrt{16 + x^2}}{7} + \frac{16 - x}{22} = 2$$

$$22\sqrt{16 + x^2} + 7(16 - x) = 308$$

$$22\sqrt{16 + x^2} + 112 - 7x = 308$$

$$22\sqrt{16 + x^2} = 7x + 196$$

$$\left(22\sqrt{16 + x^2}\right)^2 = (7x + 196)^2$$

$$484(16 + x^2) = 49x^2 + 2,744x + 38,416$$

$$7,744 + 484x^2 = 49x^2 + 2,744x + 38,416$$

$$435x^2 - 2,744x - 30,672 = 0$$

$$x \approx \frac{2,744 \pm 7,803.77}{870}$$

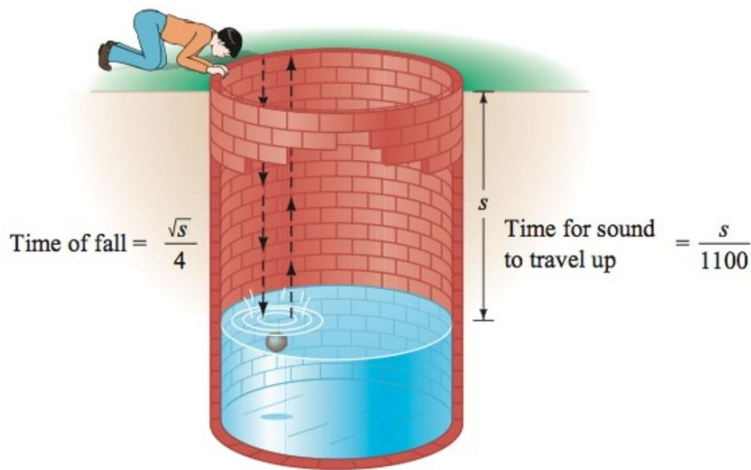
$$= \left\{ \begin{array}{l} \frac{2,744 + 7,803.77}{870} = 12.1 \\ \frac{2,744 - 7,803.77}{870} = - \end{array} \right\} < 0$$

$\therefore$  The total distance is  $12.1 \text{ km}$ .

### Exercise

The depth  $s$  from the opening of a well to the water below can be determined by measuring the total time between the instant you drop a stone and the moment you hear it hit the water. The time, in *seconds*, it takes the stone to hit the water is given by  $\frac{\sqrt{s}}{4}$ , where  $s$  is measured in *feet*. The time, also in seconds, required for the sound of the impact to travel up to your ears is given by  $\frac{s}{1,100}$ . Thus, the total time  $T$ , in *seconds*, between the instant you drop the stone and the moment you hear its impact is

$$T = \frac{\sqrt{s}}{4} + \frac{s}{1,100}$$



- One of the world's deepest water wells is 7,320 *feet* deep. Find the time between the instant you drop a stone and the time you hear it hit the water if the surface of the water is 7,100 *feet* below the opening of the well.
- Find the depth from the opening of a well to the water level if the time between the instant you drop a stone and the moment you hear its impact is 3 *seconds*.

### Solution

- a) **Given:**  $s = 7,100$

$$\begin{aligned} T &= \frac{\sqrt{7,100}}{4} + \frac{7,100}{1,100} \\ &= \frac{5\sqrt{71}}{2} + \frac{71}{11} \text{ sec} \\ &\approx 27.52 \text{ sec} \end{aligned}$$

- b) **Given:**  $T = 3$

$$\begin{aligned} T &= \frac{\sqrt{s}}{4} + \frac{s}{1,100} = 3 \\ \frac{\sqrt{s}}{4} &= 3 - \frac{s}{1,100} \end{aligned}$$

$$\frac{\sqrt{s}}{4} = \frac{3,300 - s}{1,100}$$

$$\sqrt{s} = \frac{3,300 - s}{275}$$

$$s = \left( \frac{3,300 - s}{275} \right)^2$$

$$275^2 s = 1,089 \times 10^4 - 6,600s + s^2$$

$$s^2 - 82,225s + 1,089 \times 10^4 = 0$$

$$s = \frac{82,225 \pm 1375\sqrt{3553}}{2}$$

$$= \begin{cases} \frac{82,225 + 1375\sqrt{3553}}{2} \approx 82,092.34 & \text{too large} \\ \frac{82,225 - 1375\sqrt{3553}}{2} \approx 132.66 \end{cases}$$

$\therefore$  The depth from the opening of a well to the water level is about **133 feet**.

### Exercise

On a ship, the distance  $d$  that you can see to the horizon is given by  $d = \sqrt{1.5h}$ , where  $h$  is the height of your eye measured in *feet* above the sea level and  $d$  is measured in *miles*. How high is the eye level of a navigator who can see 14 *miles* to the horizon?

### Solution

**Given:**  $d = 14$

$$d = \sqrt{1.5h} = 14$$

$$\frac{3}{2}h = (14)^2$$

$$h = \frac{2(196)}{3}$$

$$= \frac{392}{3} \text{ feet}$$

$$\approx 131 \text{ feet}$$