Exam 2 Review

#3 $g(x) = \begin{cases} -|x+2| & \text{(1) } x < -1 \\ -2x+3 & \text{(2) } x \geq -1 \end{cases}$

a) $f(-1) = -2(-1)+3 = \boxed{5}$

b) $g(2) = -2(2)+3 = \boxed{-1}$

$g(-2) = -|-2+2|$
$= \boxed{0}$

$f(x) = x^2 + \sqrt{5}\,x - \frac{3}{2}$ Domain: $\mathbb{R}$

$g(x) = \sqrt{3-x}$ Domain: $x \leq 3$

$h(x) = \frac{4}{x+5}$ Domain: $x \neq -5$

$p(x) = \frac{x}{\sqrt{4+x}}$ Domain: $x > -4$

8/ a) $f(x) = 4x-5$ $\frac{f(x+h)-f(x)}{h}$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left(4(x+h)-5-(4x-5)\right)$$
$$= \frac{1}{h}(4x+4h-5-4x+5)$$
$$= \boxed{4}$$

#7 $f(x) = \sqrt{x+1}$ $\quad g(x) = x^2 - 3$

a) $(f \circ g)(x) = f(g(x))$

$= f(x^2 - 3) \leftarrow \mathbb{R}$

$= \sqrt{x^2 - 3 + 1}$

$= \boxed{\sqrt{x^2 - 2}}$ $\quad x \le -\sqrt{2} \quad x \ge \sqrt{2}$

Domain: $\underline{x \le -\sqrt{2}\ ,\ x \ge \sqrt{2}}$

$(g \circ f)(2) = g(f(2))$

$= g(\sqrt{3})$ $\qquad \sqrt{2+1}$

$= \underline{0}$ $\qquad 3 - 3$

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10 a) $f(x) = x^3 + 3x^2 - 6x - 8$

$P: \pm\left\{\frac{8}{1}\right\} = \pm\{1, 2, 4, 8\}$

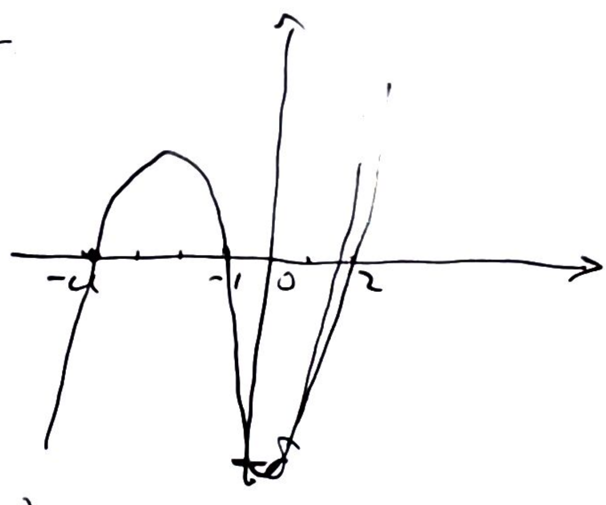
-1	1	3	-6	-8
		-1	-2	8
	1	2	-8	$\boxed{0}$

$x^2 + 2x - 8 = 0$

$(x - 2)(x + 4) = 0$

$\underline{x = -1, 2, -4}$

-4		-1	0	2	
$-$	$+$		$-$		$+$



$f(x) > 0$: $(-4, -1)$ $(2, \infty)$

$f(x) < 0$ $(-\infty, -4)$ $(-1, 2)$

#11. (2.6) $f(x) = \dfrac{2x-11}{x^2+2x-8}$

VA: $x = 2, -4$ HA: $y = 0$

hole: n/a OA: n/a

41) $f(x) = \dfrac{x^2-3}{x^2+4}$ $\cancel{x^2 = -4}$

VA: n/a HA: $y = 1$

hole: n/a OA: n/a

53) $f(x) = \dfrac{x^3+3x^2-4x+6}{x+2}$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -4 & 6 \\ & & -2 & -2 & 12 \\ \hline & 1 & 1 & -6 & \boxed{18} \end{array}$$

VA: $x = -2$ HA: n/a

hole: n/a OA: $y = x^2+x-6$

#50 $f(x) = \frac{x^2+x}{x+1} = \frac{x(x+1)}{x+1} = x$

VA: n/a HA: n/a

hole: $(-1,-1)$ OA: n/a

#49 $f(x) = \frac{x-2}{x^2-3x+2} = \frac{(x-2)}{(x-2)(x-1)} = \frac{1}{x-1}$

VA: $x=1,\cancel{2}$ HA: $y=0$

hole: $(2,1)$ OA: n/a.

#48 $f(x) = \frac{x-3}{x^2-3x+2}$

VA: $x = 1, 2$ HA: $y=0$

hole: n/a OA: n/a.

#46 $f(x) = \frac{x^2-4x-5}{2x+5}$

VA: $x = -\frac{5}{2}$ HA: n/a

hole: n/a OA: $y = \frac{1}{2}x - \frac{13}{4}$

$$\begin{array}{r|l} & \frac{1}{2}x - \frac{13}{4} \\ 2x+5 & x^2 - 4x - 5 \\ & -x^2 - \frac{5}{2}x \\ \hline & -\frac{13}{2}x - 5 \end{array}$$

$$\begin{array}{c|ccc} -\frac{5}{2} & 2 & -4 & -5 \\ & & -5 & \\ \hline & 2 & -9 & \end{array}$$

$y = 2x - 9$

#39 $f(x) = 2x^3 + 11x^2 - 7x - 6$

Possibilities: $\pm\left\{\frac{6}{2}\right\} = \pm\left\{\frac{1,2,3,6}{1,2}\right\}$

#56. $f(x) = 3x^3 - x^2 - 6x + 2$

Possibilities: $\pm\left\{\frac{2}{3}\right\} = \pm\left\{\frac{1,2}{1,3}\right\}$

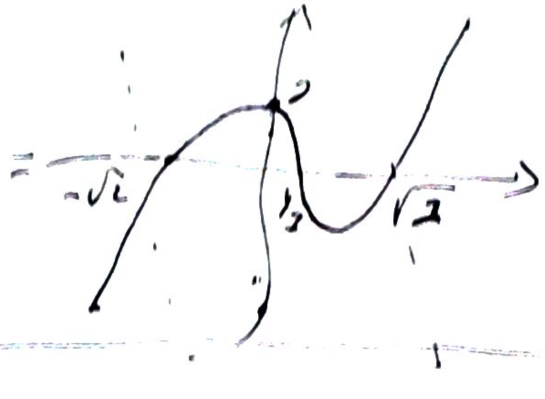
$= \pm\left\{1, 2, \frac{1}{3}, \frac{2}{3}\right\}$

$$
\begin{array}{c|cccc}
\frac{1}{3} & 3 & -1 & -6 & 2 \\
 & & 1 & 0 & -2 \\
\hline
 & 3 & 0 & -6 & \boxed{0}
\end{array}
$$

$3x^2 - 6 = 0 \Rightarrow x^2 = 2$

$x = \frac{1}{3}, \pm\sqrt{2}$

$$
\begin{array}{ccccccc}
 & -\sqrt{2} & & 0 \quad \frac{1}{3} & & \sqrt{2} & \\
- & | & + & | & - & | & +
\end{array}
$$



3.1 Inverse

relation $\{(x, y)\}$

inverse relation $\{(y, x)\}$

Ex $G = \{(4, 2), (3, -1), (-2, 0)\}$

inverse $G = \{(2, 4), (-1, 3), (0, -2)\}$

$\left.\begin{array}{l} x \to y \\ y \to x \end{array}\right\} \quad \underbrace{y = x}_{\text{symmetric}}$

Function has an inverse

One-to-One fctns

1-1 fctns

$\left\{\begin{array}{l} f(a) = f(b) \Rightarrow a = b \\ a \neq b \Rightarrow f(a) \neq f(b) \end{array}\right.$

$f(x) = x^2$

$-1 \neq 1$

$\left.\begin{array}{l} f(-1) = 1 \\ f(1) = 1 \end{array}\right\} \Rightarrow f(-1) = f(1)$

$\therefore$ Inverse fctn doesn't exist.

it's not 1-1 fctn

Ex $f(x) = 2x - 3$

$$f(a) = f(b)$$

$$2a - 3 = 2b - 3$$

$$2a = 2b$$

$$a = b \checkmark$$

$f(x)$ is 1-1.

$$a^2 = b^2 \Rightarrow a = \pm b \begin{cases} a = b \\ a = -b \end{cases} \#$$

Definition of Inverse fctn

$$\begin{tikzcd} x \arrow[r, "f"] \arrow[rr, bend right, "g(f(x))"'] & y = f(x) \arrow[r, "g"] & g(y) \end{tikzcd}$$

g((

Domain f

Range f^{-1}

$$\begin{tikzcd} x \arrow[r, bend left, "f"] & y \arrow[l, bend left, "g = f^{-1}"] \end{tikzcd}$$

1   1

Range f

domain y

input $\longrightarrow$ output

$$g(f(x)) = x$$

f^{-1} : f inverse

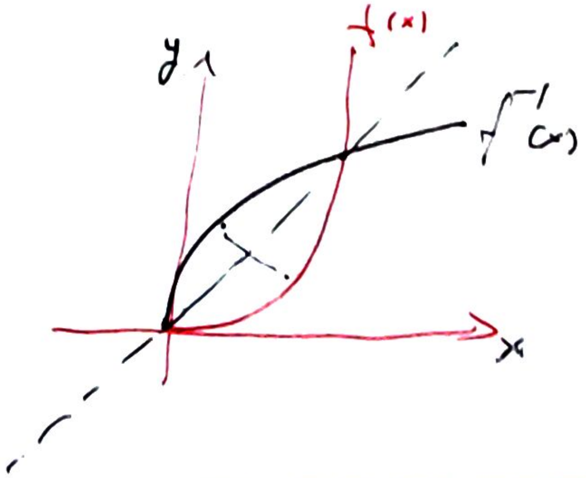
$$\text{Domain } f(x) = \text{Range } f^{-1}(x) = \mathbb{R} - \{\ \}$$

$$\text{''} \quad f^{-1}(x) = \quad \text{''} \quad f(x)$$

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

$f(x) = x^2$ is $\overset{not}{}$ 1-1.

$f(x) = x^2 \quad x \geq 0 \rightarrow$ restriction



$y = x$

Ex $\quad f(x) = x^3 - 1 \qquad g(x) = \sqrt[3]{x+1}$

g is inverse of f.

$f(g(x)) = f(\sqrt[3]{x+1}) \qquad (\sqrt[3]{x+1})^3$

$= x + 1 - 1$

$= x$ ✓

$g(f(x)) = g(x^3 - 1)$

$= \sqrt[3]{x^3 - 1 + 1}$

$= \sqrt[3]{x^3}$

$= x$

Finding inverse fctn

Ex

$f(x) = 2x+7$

Replace f(x) w/ y $\qquad y = 2x+7$

sub x + y (interchange $\qquad x = 2y+7$

solve for y $\qquad x-7 = 2y$

replace y w/ $f^{-1}(x)$ $\qquad \boxed{y = \frac{x-7}{2} = f^{-1}(x)}$

$f(x) = \frac{5x-3}{2x+1}$ $\qquad f^{-1}(x) = \frac{-x-3}{2x-5}$

$y = \frac{5x-3}{2x+1}$

$x = \frac{5y-3}{2y+1} \rightarrow x(2y+1) = 5y-3$

$2xy + x = 5y-3$

$2xy - 5y = -x-3$

$(2x-5)y = -x-3$

$\underline{y = \frac{-x-3}{2x-5} = f^{-1}(x)}$

$f(x) = \frac{ax+b}{cx+d} \Rightarrow f^{-1}(x) = \frac{-dx+b}{cx-a}$

$f(x) = \frac{2x+7}{1} \Rightarrow f^{-1}(x) = \frac{-x+7}{-2} = \frac{x-7}{2}$

* Ex) $f(x) = \frac{x+1}{x-1}$ $f^{-1}(x)$? d. R. f f^{-1}

a) $y = \frac{x+1}{x-1}$

$x = \frac{y+1}{y-1}$

$x(y-1) = y+1$

$xy - x = y+1$

$xy - y = x+1$

$(x-1)y = x+1$

$y = \frac{x+1}{x-1} = f^{-1}(x)$

b) domain $f(x)$ = Range $f^{-1}(x)$: $\mathbb{R} - \{1\}$

" $f^{-1}(x)$ = Range $f(x)$: $\mathbb{R} - \{1\}$

* $f(x) = \frac{ax+b}{cx+d}$ → domain.

Range

56. $f(x) = \dfrac{3x+2}{2x-5}$

a) $y = \dfrac{3x+2}{2x-5}$

$x = \dfrac{3y+2}{2y-5}$

$x(2y-5) = 3y+2$

$2xy - 5x = 3y + 2$

$2xy - 3y = 5x + 2$

$(2x-3)y = 5x+2$

$$\boxed{y = \frac{5x+2}{2x-3} = f^{-1}(x)}$$

b) Domain of $f(x)$ = Range $f^{-1}(x)$: $\mathbb{R} - \{\frac{5}{2}\}$

" $f^{-1}(x)$ = " $f(x)$: $\mathbb{R} - \{\frac{3}{2}\}$

Review

#4/ $f(x) = \sqrt{7}x + \frac{1}{2}$ Domain: $\mathbb{R}$

13/ $f(x) = \frac{x+5}{2-x}$ Domain $x \neq 2$

32/ $y = \sqrt{4x+1}$ Domain: $x \geq -\frac{1}{4}$

33/ $y = \sqrt{7-2x}$ Domain: $x \leq \frac{7}{2}$

58/ $f(x) = \frac{\sqrt{5-x}}{x}$ $\rightarrow x \leq 5$

$\rightarrow x \neq 0$

Domain: $x \leq 5,\ x \neq 0$

59/ $f(x) = \frac{x}{\sqrt{5-x}}$ Domain: $x < 5$

2.3

#18/ $f(x) = x^2 - 3x$ $g(x) = \sqrt{x+2}$

$f \circ g(x) = f(g(x))$

$= f(\sqrt{x+2})$ $x \geq -2$

$= x+2 - 3\sqrt{x+2}$

Domain. $x \geq -2$

$g \circ f(x) = g(f(x))$

$= g(x^2 - 3x)$

$= \sqrt{x^2 - 3x + 2}$

Domain: $x \leq 1,\ x \geq 2$

#11 $f(x) = x^2 - 2 \quad g(x) = 4x - 3$

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f(4x-3) \longrightarrow \mathbb{R} \\
&= (4x-3)^2 - 2 \\
&= 16x^2 - 24x + 9 - 2 \\
&= 16x^2 - 24 + 7 \rightarrow \mathbb{R}
\end{aligned}
$$

Domain: $\mathbb{R}$