Math 2415 – Calculus III **Assignment**

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1- Find:

a) $\lim\limits_{(x,y)\to(1,\ 2)} \dfrac{2x^2 - xy}{4x^2 - y^2}$

b) $\lim\limits_{(x,y)\to(0,\ 0)} \dfrac{\sin x + \sin y}{x + y}$

c) $\lim\limits_{(x,y)\to\left(0,\ \frac{\pi}{2}\right)} \dfrac{1 - \cos xy}{4x^2 y^3}$

d) $\lim\limits_{(x,y)\to(0,\ 0)} \dfrac{x^2 - 2xy + y^2}{x - y}$

2- Let $f(x,y,z) = x^2 y + 2xz^2 - 3y^2 z$

a) List all possible second partial derivatives that could be computed.

b) Find all second partial derivatives

c) Determine which second derivatives are equal

3- Given $f(x,y) = \frac{1}{2}\ln\left(x^2 + y^2\right) + \tan^{-1}\frac{y}{x}$, find f_x *and* f_y

4- The ***three-dimensional Laplace equation*** $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2} = 0$

Is satisfied by steady-state temperature distribution $T = f(x,y,z)$ in space, by gravitational potentials, and by electrostatic potentials. Show that the function satisfies a Laplace equation.

a) $f(x,\ y,\ z) = x^2 + y^2 + z^2$

b) $f(x,\ y,\ z) = 2z^3 - 3\left(x^2 + y^2\right)z$

c) $f(x,\ y,\ z) = e^{-2y}\cos 2x$

d) $f(x,\ y,\ z) = \ln\sqrt{x^2 + y^2}$

e) $f(x,\ y,\ z) = \tan^{-1}\dfrac{x}{y}$