Review

12/9 Exam 4

No class 12/7

$\cos A = \frac{15}{17}$ $A \in QI$ $\cos B = -\frac{12}{13}$ $\underline{B \in QII}$

$\sin A = \frac{8}{17}$ $\sin B = \frac{5}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \left(\frac{8}{17}\right)\left(-\frac{12}{13}\right) + \left(\frac{15}{17}\right)\left(\frac{5}{13}\right)$

$\frac{-96+75}{221}$

$\boxed{= -\frac{21}{221}}$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{15}{17}\left(\frac{-12}{13}\right) - \left(\frac{8}{17}\right)\frac{5}{13}$

$= \frac{-180-40}{221}$

$\boxed{= -\frac{220}{221}}$

c) $\tan(A+B) = \boxed{\frac{21}{220}}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= \frac{8}{17}\left(\frac{-12}{13}\right) - \frac{15}{17}\,\frac{5}{13}$$

$$= \frac{-96-75}{221}$$

$$= \boxed{-\frac{171}{221}}$$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$= \frac{15}{17}\left(\frac{-12}{13}\right) + \frac{8}{17}\,\frac{5}{13}$$

$$= \frac{-180+40}{221}$$

$$= \boxed{-\frac{140}{221}}$$

f) $\tan(A-B) = \boxed{\frac{171}{140}}$

#2/ $\sin A = \frac{3}{5}$ $A \in Q\ \mathrm{II}$ $\quad 90^\circ \le A \le 180^\circ$

$45^\circ \le \frac{A}{2} \le 90^\circ$

$\cos A = -\frac{4}{5}$

a) $\sin 2A = 2\sin A\cos A$

$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= \boxed{-\frac{24}{25}}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{16}{25} - \frac{9}{25}$

$= \boxed{\frac{7}{25}}$

c) $\tan 2A = -\frac{24}{7}$

d) $\sin\frac{A}{2} = \sqrt{\frac{1}{2}(1-\cos A)}$

$= \sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)}$

$= \boxed{\frac{3}{\sqrt{10}}}$

e) $\cos\frac{A}{2} = \sqrt{\frac{1}{2}(1+\cos A)}$

$= \sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)}$

$= \boxed{\frac{1}{\sqrt{10}}}$

f) $\mathrm{an}\frac{A}{2} = \boxed{3}$

$\cancel{\frac{3}{1}}$

$1-\sin x = \sqrt{3}\cos x \qquad [0, 2\pi)$

$$\sqrt{3}\cos x + \sin x = 1$$

$$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{1}{2}$$

$$\cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x = \frac{1}{2}$$

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

$x - \frac{\pi}{6} = \frac{\pi}{3}$

$x = \frac{\pi}{3} + \frac{\pi}{6}$

$\boxed{= \frac{\pi}{2}}$

$x - \frac{\pi}{6} = \frac{5\pi}{3}$

$x = \frac{5\pi}{3} + \frac{\pi}{6}$

$\boxed{= \frac{11\pi}{6}}$

$2\tan x \csc x + 2\csc x + \tan x + 1 = 0 \quad [0, 2\pi]$

$$2\csc x(\tan x + 1) + (\tan x + 1) = 0$$

$$(\tan x + 1)(2\csc x + 1) = 0$$

$\tan x = -1 \qquad \csc x = -\frac{1}{2}$

$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}} \qquad \sin x = -2 \ \#$

$2\sin^2 x - \cos x - 1 = 0 \qquad [0, 2\pi)$

$2(1-\cos^2 x) - \cos x - 1 = 0$

$2 - 2\cos^2 x - \cos x - 1 = 0$

$-2\cos^2 x - \cos x + 1 = 0$

$\cos x = -1 \qquad \cos x = \frac{1}{2}$

$$\boxed{x = \pi, \frac{\pi}{3}, \frac{5\pi}{3}}$$

$\sin x \tan x = \sin x \qquad [0, 2\pi)$

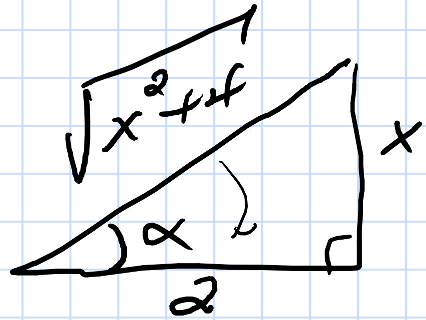
$\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0 \qquad \tan x = 1$

$$\boxed{x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}}$$

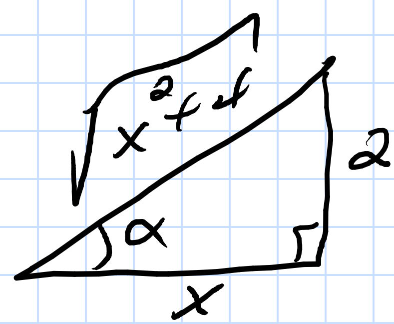
$$\sec\left(\underbrace{\sin^{-1}\frac{x}{\sqrt{x^2+4}}}_{\alpha}\right)$$



$$\sin\alpha = \frac{x}{\sqrt{x^2+4}}$$

$$\boxed{\sec\alpha = \frac{\sqrt{x^2+4}}{2}}$$

$$\sin\left(\underbrace{\cos^{-1}\frac{x}{\sqrt{x^2+4}}}_{\alpha}\right)$$



$$\cos\alpha = \frac{x}{\sqrt{x^2+4}}$$

$$\sin\alpha = \frac{2}{\sqrt{x^2+4}}$$

$(4, 30^\circ)$ $\qquad$ (x, y)?

$x = r\cos\theta$
$= 4\cos 30^\circ$
$= 4\left(\frac{\sqrt{3}}{2}\right)$
$= 2\sqrt{3}$

$y = r\sin\theta$
$= 4\sin 30^\circ$
$= 4\left(\frac{1}{2}\right)$
$= 2$

$(x, y) = (2\sqrt{3}, 2)$

4 all no need.

$(3, -3)$ $\qquad$ (r, θ)?

$r = \sqrt{x^2 + y^2}$
$= \sqrt{9 + 9}$
$= 3\sqrt{2}$

$\hat{\theta} = \tan^{-1}\left|\frac{3}{3}\right|$
$= \tan^{-1} 1$
$= \frac{\pi}{4}$

$(r, \theta) = \left(3\sqrt{2}, \frac{7\pi}{4}\right)$

$r(\sin\theta - 2\cos\theta) = 6$

$r\sin\theta - 2r\cos\theta = 6$

$$\boxed{y - 2x = 6}$$

$y^2 = x$

$(r\sin\theta)^2 = r\cos\theta$

$r^2\sin^2\theta = r\cos\theta \qquad (r \neq 0)$

$r\sin^2\theta = \cos\theta$

$$\boxed{r = \frac{\cos\theta}{\sin^2\theta}}$$

$-\sqrt{3} + i$

$x = -\sqrt{3}, \quad y = 1$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{3+1}$

$= \sqrt{4}$

$= \boxed{2}$

$\hat{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$$\boxed{-\sqrt{3} + i = 2 \text{ cis } \frac{5\pi}{6}}$$

$$\sqrt{2} \text{ cis } \frac{7\pi}{4} = \sqrt{2}\left(\cos\frac{7\pi}{4} + i \sin\frac{7\pi}{4}\right)$$

$$= \sqrt{2}\left(\frac{\sqrt{2}}{2} - i\,\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{1 - i}$$

$$\frac{20 \text{ cis }(75^\circ)}{4 \text{ cis }(40^\circ)} = 5 \text{ cis }(75^\circ - 40^\circ)$$

$$= \boxed{5 \text{ cis }(35^\circ)}$$

$$(20 \text{ cis } 75^\circ)(4 \text{ cis } 40^\circ) = 80 \text{ cis }(75^\circ + 40^\circ)$$

$$= 80 \text{ cis }(115^\circ)$$

$$a = \underline{b\cos C + c\cos B}$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

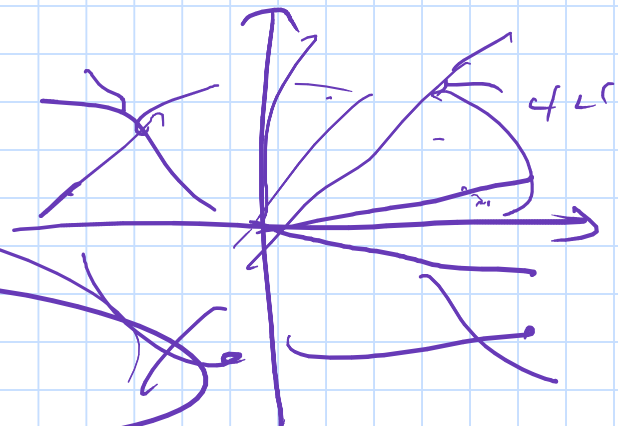
$$\cos B = \frac{a^2+\cancel{c^2}-b^2}{2a\cancel{c}}$$

$$\cos C = \frac{a^2+\cancel{b^2}-\cancel{c^2}}{2a\cancel{b}}$$

$$b\cos C + c\cos B =$$

$$= a$$

$(\sin 1^\circ)(\sin 359^\circ)$



$\sin 1^\circ + \ldots$

$-2 \ldots$

$a + \ldots + 2$

$-2 + \ldots -$

$\sin^2 1^\circ + \ldots + \sin^2 359^\circ = 180$

$\cos^2 1^\circ + \sin^2 1^\circ = 1$

$\underline{\sin(90^\circ - 1^\circ)} + \sin^2 1^\circ = 1$

$\sin 89^\circ + \sin 1^\circ = 1$

$\sin^2 45^\circ = \frac{1}{2}$

$$\cos 2x + \cos 4x = \cos x$$

$$2\cos(3x)\cos(-x) = \cos x$$

$$2\cos 3x \cos x - \cos x = 0$$

$$2\cos^2 x - 1 + 2\cos^2 2x - 1 = \cos x$$

$$2\cos^2 x - 2 + 2(2\cos^2 x - 1)^2 = \cos x$$

$$2\cos^2 x - \cancel{2} + 8\cos^4 x - 8\cos^2 x + \cancel{2} = \cos x$$

$$8\cos^4 x - 6\cos^2 x - \cos x = 0$$

$$\cos x = 0$$