# *Mathematica* Manual

## Notebook 13: Vectors and the Geometry of Space

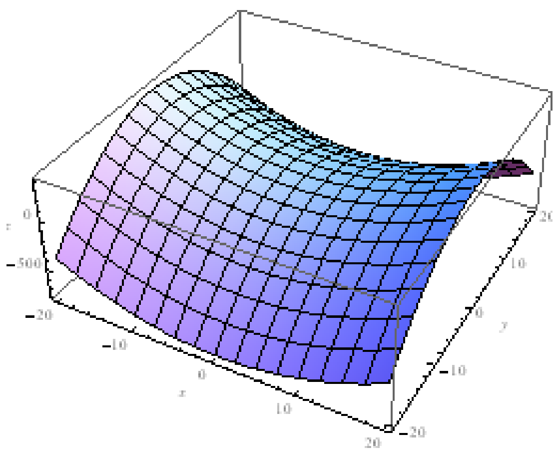
### Cylinders & Quadric Surfaces

#### Plotting Explicit Functions in Three Dimensions

Just as in two dimensions, we expressed functions in the form *y=f(x)*, in three dimensions, we express functions as *z=f(x, y)*. We domain for both x and y in order to get *Mathematica* to draw the surface. Following is an example.

In[1]:=
```
Clear[x, y, z]
Plot3D[x^2 - 2 y^2, {x, -20, 20}, {y, -20, 20}, AxesLabel → {x, y, z}]
```

Out[2]=



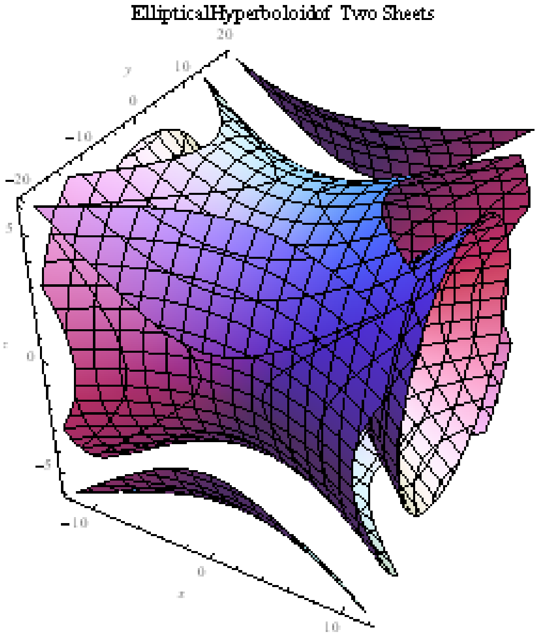
You will notice that the default is to put the surface in a box. You can change your viewpoint of the surface by left-clicking on the p mouse. The axes remain in place with their labels.

#### Plotting Implicit Functions in Three Dimensions

Begin by writing all terms of your equation on one side. That is the expression you will enter as your function. We will demonstrate You may change the terms in red to see other functions. Select a reasonable domain for each of the three variables.

In[3]:=
```
Clear[x, y, z]
ContourPlot3D[x^2/9 - y^2/16 - z^2/2 - 1, {x, -12, 12}, {y, -20, 20}, {z, -6, 6}, Axes → True,
 AxesLabel → {x, y, z}, Boxed → False, PlotLabel → "Elliptical Hyperboloid of Two Sheets"]
```

Out[4]=




This is some function! If you want to look at it from different angles, put your cursor on the picture and move it around.

### ■ Plotting Parametrically in Three Dimensions

Quadric surfaces can be plotted using a CAS such as *Mathematica* after first describing the equation in parametric form. In the quadric surface equations of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = f(z)$ are given where $f$ is a function of $z$. By letting $x = a\sqrt{f(z)}\,\cos(\theta)$ and $y =$

$$\frac{\left(a\sqrt{f(z)}\,\cos(\theta)\right)^2}{a^2} + \frac{\left(b\sqrt{f(z)}\,\sin(\theta)\right)^2}{b^2} = \frac{a^2\,f(z)\cos^2(\theta)}{a^2} + \frac{b^2\,f(z)\sin^2(\theta)}{b^2} = f(z)\left(\cos^2\theta + \sin^2\theta\right) = f(z)$$

and therefore the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = f(z)$ is satisfied. To display the graph of the quadric surface, use the *Mathematica* command

```
ParametricPlot3D[{a √f[z] Cos[θ], b √f[z] Sin[θ], z}, {θ, 0, 2π}, {z, zmin, zmax}].
```

**Example:** Use *Mathematica* to plot the following surfaces.

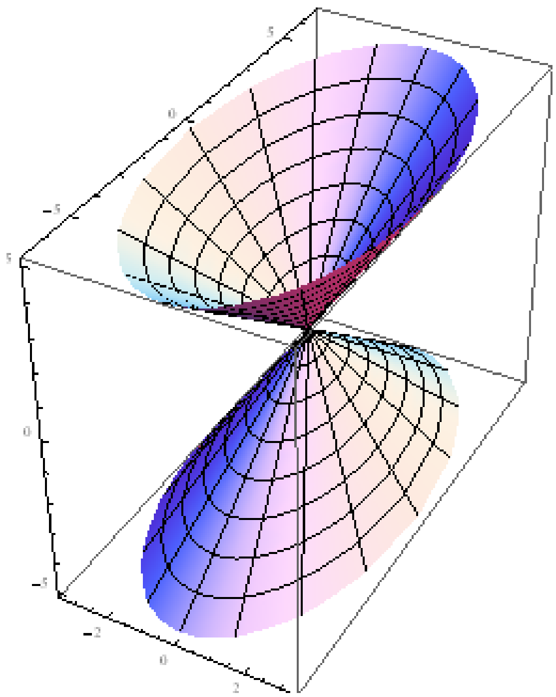
(a) $\frac{x^2}{4} + \frac{y^2}{16} = \frac{z^2}{9}$

(b) $\frac{y^2}{25} - \frac{z^2}{16} = \frac{x^2}{9} + 1$

**Part (a)** In this case, $a^2 = 4$, $b^2 = 16$ and $f(z) = \frac{z^2}{9}$. Therefore, let $x = 2\sqrt{\frac{z^2}{9}}\,\cos(\theta) = \frac{2z}{3}\cos(\theta)$ and let $y = 4\sqrt{\frac{z^2}{9}}\,\sin(\theta) = \frac{4z}{3}\sin(\theta)$

In[5]:=
```
ParametricPlot3D[{1/3 (2 z) Cos[θ], 1/3 (4 z) Sin[θ], z}, {θ, 0, 2 π}, {z, -5, 5}]
```

Out[5]=




As before, you can look at this from different angles.

**Part (b)** Rearrange the equation $\frac{y^2}{25} - \frac{z^2}{16} = \frac{x^2}{9} + 1$ to get $\frac{x^2}{9} + \frac{z^2}{16} = \frac{y^2}{25} - 1$. This is a hyperboloid of two sheets (why?). In this case, if $x = 3\cos(\theta)\sqrt{\frac{y^2}{25} - 1}$ , $z = 4\sin(\theta)\sqrt{\frac{y^2}{25} - 1}$ . Notice that $x$ and $z$ are defined only when $y \leq -5$ or $y \geq 5$ which corresponds tc graph. Study the following input commands. The option `BoxRatios→{1,1.5,1}` scales the plot so that the length in the y di long as the lengths in the other two directions.

In[6]:= 
```
backpart = ParametricPlot3D[{3 Cos[θ] Sqrt[y^2/25 - 1], y, 4 Sin[θ] Sqrt[y^2/25 - 1]}, {θ, 0, 2 π}, {y, 5, 15}];

frontpart = ParametricPlot3D[{3 Cos[θ] Sqrt[y^2/25 - 1], y, 4 Sin[θ] Sqrt[y^2/25 - 1]}, {θ, 0, 2 π}, {y, -15, -5}];

Show[{frontpart, backpart}, BoxRatios -> {1, 1.5, 1}, AxesLabel -> {x, y, z}]
```

Out[8]=