# *Section* 3.2 – Exponential Functions

## Definition

The exponential function $f$ with base $\boldsymbol{b}$ is defined by

$$f(x) = b^x \qquad \text{or} \qquad y = b^x$$

Base

where $b > 0$, $b \neq 1$ and $\boldsymbol{x}$ is any real number.

$$f(x) = 2^x \qquad f(x) = \left(\tfrac{1}{2}\right)^{2x+1} \qquad f(x) = 3^{-x} \qquad f(x) = (-2)^x$$

## *Example*

Given: $f(x) = 13.49\ (0.967)^x - 1$, find $f(60)$

***Solution***

$$f(\mathbf{60}) = 13.49\ (0.967)^{\mathbf{60}} - 1$$
$$= 0.8014$$

## *Example*

If $f(x) = 2^x$, find each of the following. $f(-1), f(3), f\left(\tfrac{5}{2}\right)$

***Solution***

*a)* $f(-1) = 2^{-1}$

$$= \frac{1}{2}$$

*b)* $f(3) = 2^3$

$$= 8$$

*c)* $f\left(\tfrac{5}{2}\right) = 2^{\frac{5}{2}}$

$$= 4\sqrt{2}$$
$$= 5.6569$$

## Graphing Exponential

*Example*

$f(x) = 3^x$

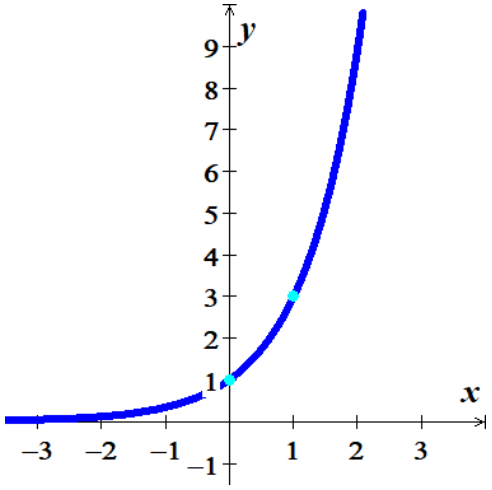
1. Define the Horizontal Asymptote $f(x) = b^x \pm d$

$y = 0 \pm d$

Asymptote: $y = 0$

*The exponential function always equals to 0*

$x \to \infty$ *or* $x \to -\infty \Rightarrow f(x) \to 0$




2. Define/Make a table

(Force your exponential to = 0, then solve for $x$)

| $x$ | $f(x)$ |
|---|---|
| $x - 2$ | |
| $x - 1$ | |
| $x$ | |
| $x + 1$ | |
| $x + 2$ | |

| $x$ | $f(x)$ |
|---|---|
| –2 | 1/9 |
| –1 | 1/3 |
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |

*Domain*: $(-\infty, \infty)$

*Range*: $(d, \infty)$

### *Example*

$f(x) = \left(\frac{1}{3}\right)^x$

$= \left(3^{-1}\right)^x$

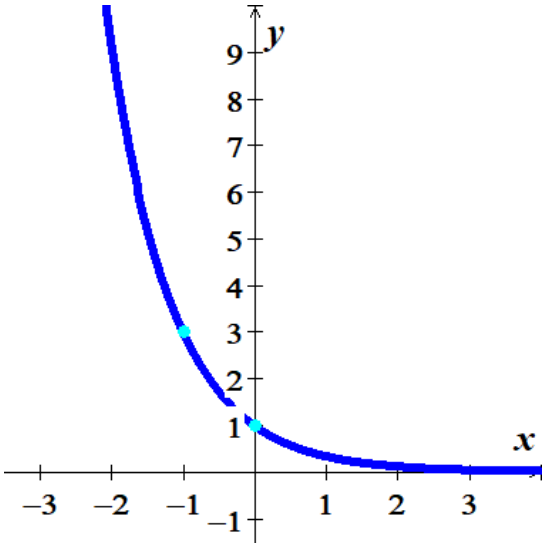
$= 3^{-x}$

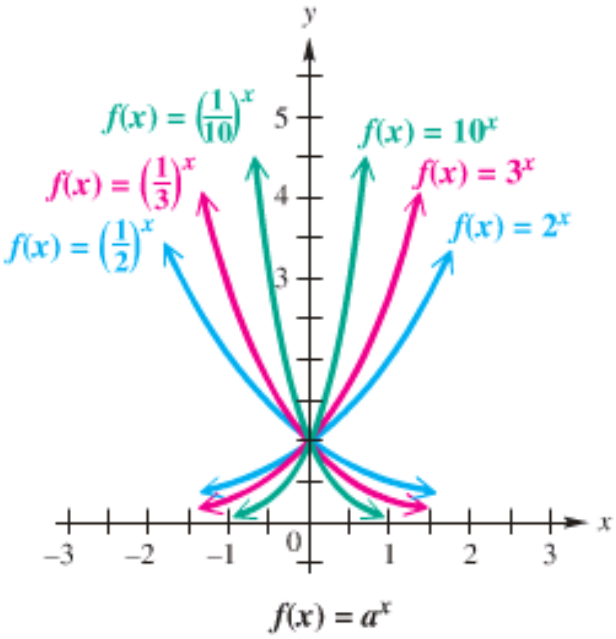
*Reflected across y-axis*

Asymptote: $y = 0$

*Domain*: $(-\infty, \infty)$

*Range*: $(0, \infty)$






$f(x) = a^x$

***Example***

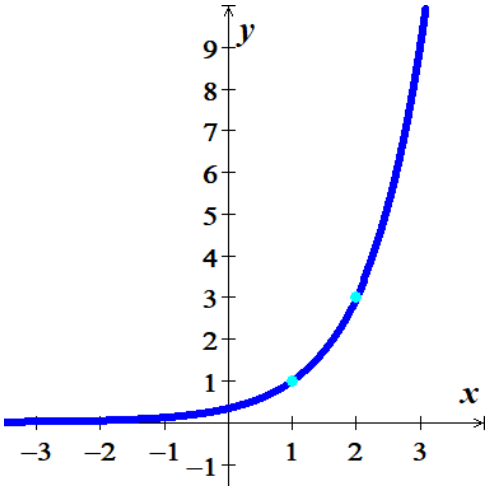
$f(x) = 3^{x-1}$

*Shift right 1 unit*

Asymptote: $y = 0$

*Domain*: $(-\infty, \infty)$

*Range*: $(0, \infty)$




***Example***

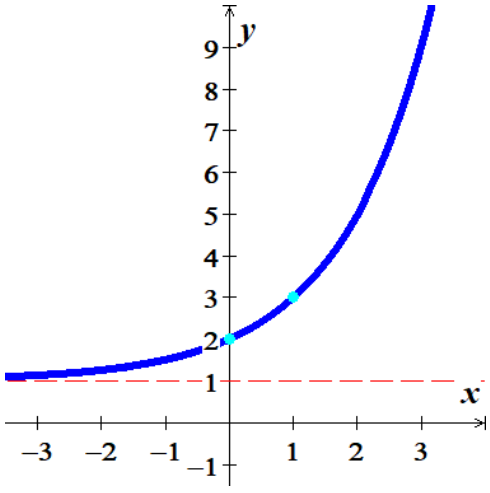
$f(x) = 2^x + 1$

*Shift up 1 unit*

Asymptote: $y = 1$

***Domain***: $(-\infty, \infty)$

***Range***: $(1, \infty)$



### *Example*

$f(x) = 5 - 2^{-x}$

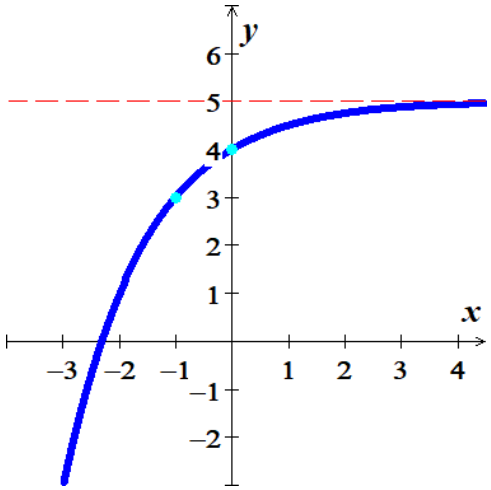
*Shifted up 5 units*

*Reflected across x-axis and y-axis*

***Asymptote***: $y = 5$

***Domain***: $(-\infty,\ \infty)$

***Range***: $(-\infty,\ 5)$




### *Example*

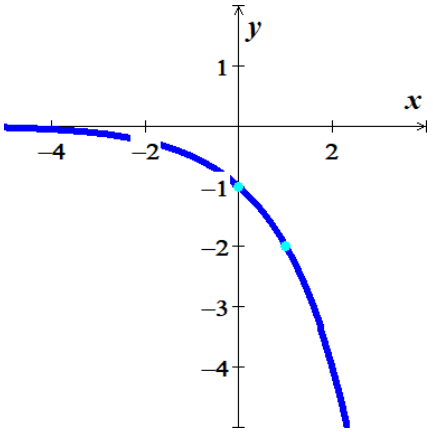
Give the *asymptote*, *domain* and *range*.

*a)* $f(x) = -2^{x}$

***Asymptote***: $y = 0$

***Domain***: $(-\infty,\ \infty)$

***Range***: $(-\infty,\ 0)$

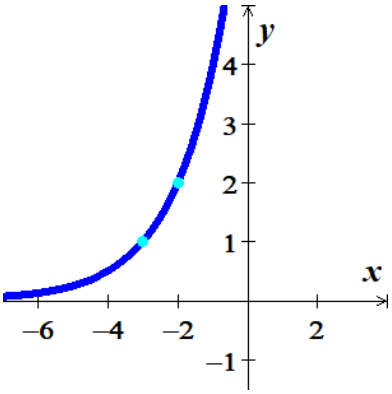



*b)* $f(x) = 2^{x+3}$

***Asymptote***: $y = 0$

***Domain***: $(-\infty,\ \infty)$

***Range***: $(0,\infty)$

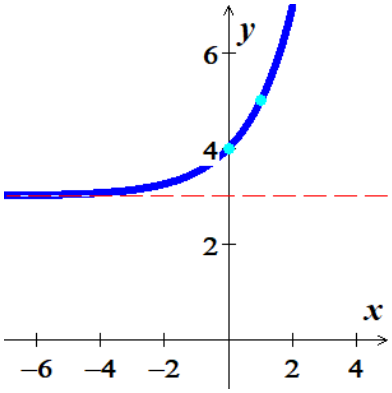



*c)* $f(x) = 2^{x} + 3$

***Asymptote***: $y = 3$

***Domain***: $(-\infty,\infty)$

***Range***: $(3,\infty)$



## Natural Base $e$

The irrational number $e$ is called natural base

$f(x) = e^x$ is called natural exponential function

$e^0 = 1$ $\quad$ $e \approx 2.7183$ $\quad$ $e^2 \approx 7.3891$ $\quad$ $e^{-1} \approx 0.3679$

### *Example*

The exponential function $f(x) = 1066e^{0.042x}$ models the gray wolf population of the Western Great Lakes, $f(x)$, in billions, $x$ *years* after 1978. Project the gray population in the recovery area in 2012.

***Solution***

$x = 2012 - 1978 = 34$

$f(x = 34) = 1066e^{0.042(34)}$

$= 4445.6$

$\approx 4446$

### *Example*

Graph $f(x) = e^x$

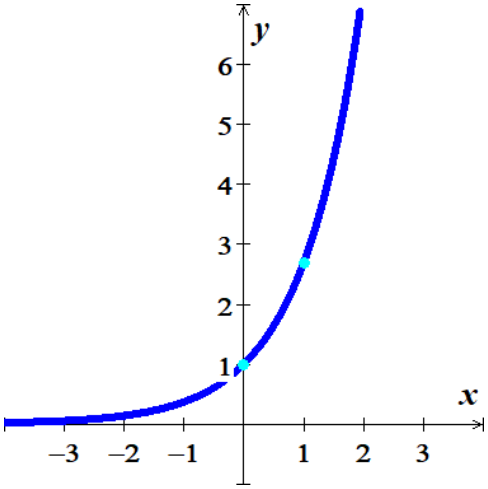
***Solution***

***Asymptote***: $y = 0$

| *x* | *f(x)* |
|---|---|
| –1 | .4 |
| 0 | 1 |
| 1 | 2.7 |

***Domain***: $(-\infty, \infty)$

***Range***: $(0, \infty)$



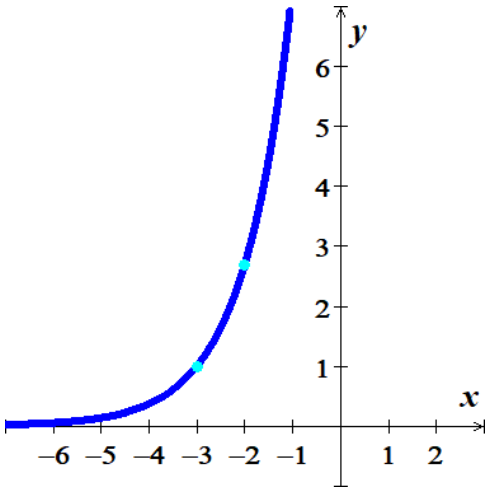

### *Example*

$f(x) = e^{x+3}$

***Solution***

*Shifted left 3 units*

***Asymptote***: $y = 0$



# *Exercises* *Section* 3.2 – Exponential Functions

(**1 – 8**) Evaluate to four decimal places using a calculator

**1.** $2^{3.4}$

**2.** $5^{\sqrt{3}}$

**3.** $6^{-1.2}$

**4.** $e^{-0.75}$

**5.** $e^{2.3}$

**6.** $e^{-0.95}$

**7.** $\pi^{\sqrt{\pi}}$

**8.** $e^{\sqrt{2}}$

(**9 – 20**) Find the ***asymptote***, ***domain***, and ***range*** of the given functions. Then, sketch the graph

**9.** $f(x) = 2^x + 3$

**10.** $f(x) = 2^{3-x}$

**11.** $f(x) = \left(\frac{2}{5}\right)^{-x}$

**12.** $f(x) = -\left(\frac{1}{2}\right)^x + 4$

**13.** $f(x) = 4^x$

**14.** $f(x) = 2 - 4^x$

**15.** $f(x) = -3 + 4^{x-1}$

**16.** $f(x) = 1 + \left(\frac{1}{4}\right)^{x+1}$

**17.** $f(x) = e^{x-2}$

**18.** $f(x) = 3 - e^{x-2}$

**19.** $f(x) = e^{x+4}$

**20.** $f(x) = 2 + e^{x-1}$

**21.** The exponential function $f(x) = 1066e^{0.042x}$ models the gray wolf population of the Western Great Lakes, $f(x)$, in *billions*, $x$ *years* after 1978. Project the gray population in the recovery area in 2012.

**22.** The function $f(x) = 6.4e^{0.0123x}$ describes world population, $f(x)$, in billions, $x$ years after 2004 subject to a growth rate of 1.23% *annually*. Use the function to predict world population in 2050.

**23.** A cup of coffee is heated to $160°F$ and placed in a room that maintains a temperature of $70°F$. The temperature $T$ of the coffee, in *degree Fahrenheit*, after $t$ minutes is given by

$$T(t) = 70 + 90e^{-0.0485t}$$

*a)* Find the temperature of the coffee 20 *minutes* after it is placed in the room
*b)* Determine when the temperature of the coffee will reach $90°F$

**24.** A cup of coffee is heated to $180°F$ and placed in a room that maintains a temperature of $65°F$. The temperature $T$ of the coffee, in *degree Fahrenheit*, after $t$ minutes is given by

$$T(t) = 65 + 115e^{-0.042t}$$

*a)* Find the temperature of the coffee 10 *minutes* after it is placed in the room
*b)* Determine when the temperature of the coffee will reach $100°F$

**25.** The percent $I(x)$ of the original intensity of light striking the surface of a lake that is available $x$ *feet* below the surface of the lake is given by the equation

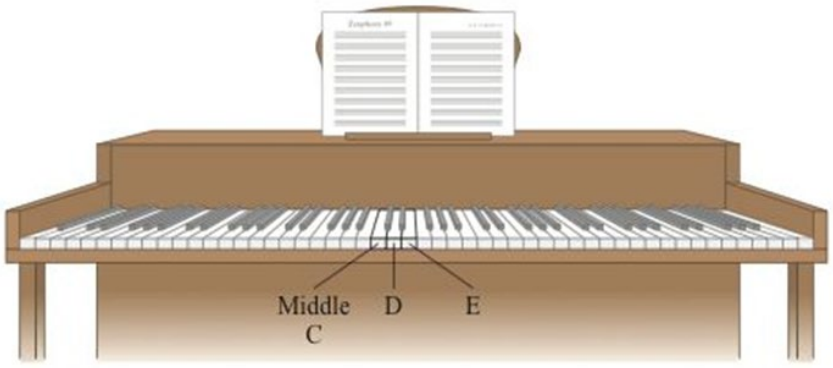
$$I(x) = 100e^{-.95x}$$

*a)* What percentage of the light is available 2 *feet* below the surface of the lake?

*b)* At what depth is the intensity of the light one-half the intensity at the surface?

**26.** Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

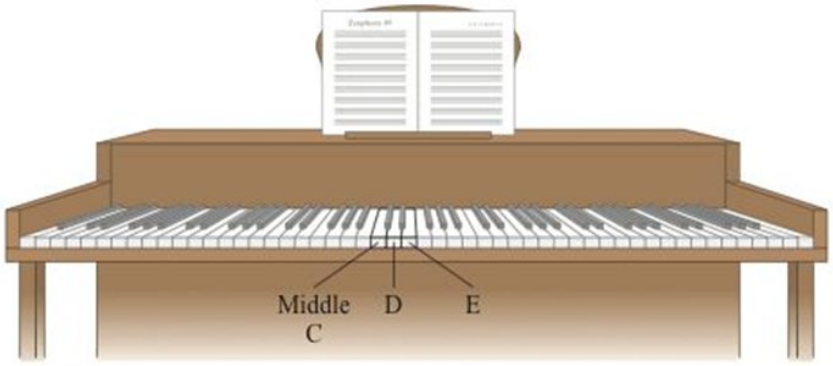
$$f(n) = (2.75)\ 2^{\frac{n-1}{12}}$$




*a)* Determine the frequency of middle *C*, key number 40 on an 88–*key* piano.
*b)* Is the difference in frequency between middle *C* (key number 40) and *D* (key number 42) the same as the difference in frequency between *D* (key number 42) and *E* (key number 44)?

**27.** Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

$$f(n) = (27.5)\ 2^{\frac{n-1}{12}}$$




*a)* Determine the frequency of middle *C*, key number 40 on an 88–*key* piano.
*b)* Is the difference in frequency between middle *C* (key number 40) and *D* (key number 42) the same as the difference in frequency between *D* (key number 42) and *E* (key number 44)?