3.2

$$\mathcal{L}(y')(s) = s\,\mathcal{L}(y(s)) - y(0)$$
$$= s\,Y(s) - y(0)$$

$$\mathcal{L}(y'')(s) = s^2 Y(s) - s\,y(0) - y'(0)$$

Ex

$$3\sin 2t - 4t + 5e^{3t}$$

$$\mathcal{L}\{3\sin 2t - 4t + 5e^{3t}\}$$
$$= \mathcal{L}\{3\sin 2t\} - 4\,\mathcal{L}\{t\} + 5\,\mathcal{L}\{e^{3t}\}$$
$$= 3\,\frac{2}{s^2+4} - 4\,\frac{1}{s^2} + \frac{5}{s-3}$$
$$= \frac{6}{s^2+4} - \frac{4}{s^2} + \frac{5}{s-3}$$

Ex. $y'' - y = e^{2t}$ $\quad y(0)=0$, $y'(0)=1$

$$\mathcal{L}\{y''-y\}(s) = \mathcal{L}\{e^{2t}\}(s)$$

$$s^2 Y(s) - s\,y(0) - y'(0) - Y(s) = \frac{1}{s-2}$$

$$(s^2-1)Y(s) = \frac{1}{s-2} + 1$$

$$= \frac{s-1}{s-2}$$

$$Y(s) = \frac{s-1}{(s-2)(s-1)(s+1)}$$

$$= \frac{1}{(s-2)(s+1)}$$

$y(t) = e^{2t}\sin 3t$

$$\mathcal{L}\{e^{2t}\sin 3t\}(s) = \frac{3}{(s-2)^2+9}$$

Ex $F(s) = \frac{1}{s-2} - \frac{16}{s^2+4}$

$y(t) = \mathcal{L}^{-1}\{F(s)\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \mathcal{L}^{-1}\left\{\frac{16}{s^2+4}\right\}$ $\quad 8 \cdot \frac{2}{s^2+4}$

$= e^{2t} - 8\sin 2t$

Ex $F(s) = \frac{1}{s^2-2s-3}$

$\frac{1}{s^2-2s-3} = \frac{A}{s+1} + \frac{B}{s-3}$

$\rightarrow 1 = A(s-3) + B(s+1)$

$s^1 \quad -A + B = 0$

$s^0 \quad -3A + B = 1$

$-4A = 1 \Rightarrow A = -\frac{1}{4} \rightarrow B = \frac{1}{4}$

$f(t) = \mathcal{L}^{-1}\left\{\frac{-1/4}{s+1} + \frac{1}{4}\frac{1}{s-3}\right\}$

$= -\frac{1}{4}e^{-t} + \frac{1}{4}e^{3t}$

$$F(s) = \frac{1}{s^2+4s+13} \rightarrow \textcircled{1} \rightarrow \left(\frac{1}{2}4\right)^2$$

$$s^2+4s+13 = s^2+4s+4+9$$
$$= (s+2)^2+9$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+9}\right\}$$

$$= \frac{1}{3} e^{-2t} \sin 3t$$

$$F(s) = \frac{2s^2+s+13}{(s-1)\left((s+1)^2+4\right)}$$

$$\frac{2s^2+s+13}{(s-1)((s+1)^2+4)} = \frac{A}{s-1} + \frac{Bs+C}{(s+1)^2+4}$$

$$As^2+2As+A+4A+Bs^2-Bs+Cs-C$$
$$= 2s^2+s+13$$

s^2 : $A+B = 2$

s^1 : $2A-B+C = 1$

s^0 : $5A-C = 13$

$$\Delta = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 5 & 0 & -1 \end{vmatrix} = 8 \qquad \Delta_A = \begin{vmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ 13 & 0 & -1 \end{vmatrix}$$

$$= 16$$

$A = 2 \rightarrow B = 0$

$C = 10-13 = -3$

$$f(t) = 2\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+4}\right\}$$

$$= 2e^t - \frac{3}{2} e^{-t} \sin 2t$$

$y'' - 2y' - 3y = 0 \quad , y(0) = 1 \quad , y'(0) = 0$

$s^2 Y(s) - s y(0) - y'(0) - 2[sY(s) - y(0)] - 3Y(s) = 0$

$Y(s)[s^2 - 2s - 3] = s - 2$

$Y(s) = \frac{s-2}{(s+1)(s-3)}$

$\frac{s-2}{(s+1)(s-3)} = \frac{A}{s+1} + \frac{B}{s-3}$

$s-2 = A(s-3) + B(s+1)$

$s^1 \quad A + B = 1$

$s^0 \quad -3A + B = -2$

$A = \frac{3}{4}, \quad B = \frac{1}{4}$

$y(t) = \frac{3}{4} \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{4} \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$

$= \frac{3}{4} e^{-t} + \frac{1}{4} e^{3t}$