Lecture One

Section 1.1 - Linear Equation and Slope

A function f is a linear function if it is in a form: $f(x) = mx + b$ and can be called an equation of a line

Slope = $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation of a line $y - y_1 = m\left(x - x_1\right)$ (**Given: slope and one point**)

Two slopes m_1 and m_2 are:

Parallel (//): $\boxed{m_1 = m_2}$

Perpendicular (⊥): $\boxed{m_1 \bullet m_2 = -1}$

Example

Find the slope of the line through the points $(-4, 8), (2, -3)$

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3 - 8}{2 - (-4)}$$

$$= \frac{-11}{6}$$

Example

Find the slope of the line through the points $(2, 7), (2, -4)$

Solution

$$m = \frac{-4 - 7}{2 - 2}$$

$$= \frac{-11}{0} \qquad \textit{Undefined}$$

Example

Write an equation in point-slope from for the line passing through $(-2,-1),(-1,-6)$

Solution:

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{-6-(-1)}{-1-(-2)} \\
&= \frac{-6+1}{-1+2} \\
&= \frac{-5}{1} \\
&= -5
\end{aligned}
$$

Equation of a line $\quad y - y_1 = m(x - x_1)$

$y-(-1)=-5(x-(-2))$

$y+1=-5(x+2)$

$y+1=-5x-10$

$y+1-1=-5x-10-1$

$\boxed{y=-5x-11}$

Example

a) Write a slope-intercept equation for a line passing through the point $(3,5)$ that is parallel to the line $2x+5y=4$

$$2x+5y=4$$
$$5y=-2x+4$$
$$y=-\frac{2}{5}x+\frac{4}{5} \quad \Rightarrow m_1=-\frac{2}{5}$$
$$\Rightarrow m_2 = m_1 = -\frac{2}{5}$$
$$y-y_1=m_2\left(x-x_1\right)$$
$$y-5=-\frac{2}{5}(x-3)$$
$$y-5=-\frac{2}{5}x+\frac{6}{5}$$
$$y=-\frac{2}{5}x+\frac{6}{5}+5$$
$$y=-\frac{2}{5}x+\frac{31}{5}$$

b) Write a slope-intercept equation for a line passing through the point $(3,5)$ that is perpendicular to the line $2x+5y=4$

$$\Rightarrow m=\frac{5}{2}$$
$$y-y_1=m\left(x-x_1\right)$$
$$y-5=\frac{5}{2}(x-3)$$
$$y-5=\frac{5}{2}x-\frac{15}{2}$$
$$y=\frac{5}{2}x-\frac{15}{2}+5$$
$$y=\frac{5}{2}x-\frac{5}{2}$$

Exercise *Section* 1.1 – Linear Equation and Slope

1. Find an equation of the line through $(-4,1)$ having slope -3.

2. Find an equation of the line containing the given pair of points (3, 2) and (9, 7)

3. Write a slope-intercept equation for a line passing through the point $(3,-2)$ that is parallel to the line $2x - y = 5$

4. Write a slope-intercept equation for a line passing through the point $(3,-2)$ that is perpendicular to the line $2x - y = 5$

5. Write a slope-intercept equation for a line passing through the point (3, 5) that is parallel to the line $y = \frac{2}{7}x + 1$

6. Write a slope-intercept equation for a line passing through the point (3, 5) that is perpendicular to the line $y = \frac{2}{7}x + 1$

7. Write a slope-intercept equation for a line passing through the point (3, -2) that is parallel to the line $3x + 4y = 5$

8. Write a slope-intercept equation for a line passing through the point (3, -2) that is perpendicular to the line $3x + 4y = 5$

Section 1.2 - Linear Equations and Rational Equations

Definition of a Linear Equation

A linear equation in one variable x is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real number, and $a \neq 0$

Addition and Multiplication Properties of Equalities

$$\text{If } a = b, \text{ then } a + c = b + c$$

$$\text{If } a = b, \text{ then } ac = bc$$

Example

Solve: $3(2x - 4) = 7 - (x + 5)$

Solution

$$6x - 12 = 7 - x - 5$$
$$6x - 12 + x = 2 - x + x$$
$$7x - 12 = 2$$
$$7x - 12 + 12 = 2 + 12$$
$$7x = 14$$
$$\frac{7}{7}x = \frac{14}{7}$$
$$\boxed{x = 2}$$

Example

Solve: $\frac{2x+4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$

Solution

$$(12)\frac{2x+4}{3} + (12)\frac{1}{2}x = (12)\frac{1}{4}x - (12)\frac{7}{3}$$
$$4(2x+4) + 6x = 3x - 28$$
$$8x + 16 + 6x = 3x - 28$$
$$14x + 16 = 3x - 28$$
$$14x - 3x = -28 - 16$$
$$11x = -44$$
$$\boxed{x = -4}$$

Example

Solve: $\frac{5}{2x}=\frac{17}{18}-\frac{1}{3x}$ *Restriction:* $x \neq 0$

Solution

$$(18x)\frac{5}{2x}=(18x)\frac{17}{18}-(18x)\frac{1}{3x}$$

$45 = 17x - 6$

$45 + 6 = 17x$

$17x = 51$

$\Rightarrow x = 3$

Example

Solve: $\frac{x}{x-2}=\frac{2}{x-2}-\frac{2}{3}$ *Restriction*: $x \neq 2$

Solution

$$3(x-2)\frac{x}{x-2}=3(x-2)\frac{2}{x-2}-3(x-2)\frac{2}{3}$$

$3x = 6 - 2(x - 2)$

$3x = 6 - 2x + 4$

$3x + 2x = 10$

$\Rightarrow 5x = 10$

$\Rightarrow x = 2$ *No Solution* ***or*** $\{\varnothing\}$

Identities, Conditional Equations, and Contradictions

Example

Solve: $-2(x+4)+3x=x-8$

Solution

$-2(x+4)+3x=x-8$

$-2x-8+3x=x-8$

$x-8=x-8$

$0=0$ *True statement*

Solution: All real numbers

Example

Solve: $5x-4=11$

Solution

$5x-4=11$

$5x=15$

$x=3$

This is a conditional equation, and its solution set is {3}

Example

Solve: $3(3x-1)=9x+7$

Solution

$3(3x-1)=9x+7$

$9x-3=9x+7$

$-3=7$ ***False statement***

This is a contradiction equation, and its solution set is empty set {∅} or null

Solving for a Specified Variable

Example

Solve

a) $I = \Pr t \quad for\ t$

$$\frac{I}{\Pr} = \frac{\Pr}{\Pr} t$$

$$\frac{I}{\Pr} = t$$

b) $A - P = Prt \quad for\ P$

$$A = Prt + P$$

$$A = P(rt + 1)$$

$$\frac{A}{rt+1} = P \qquad or\ \ P = \frac{A}{rt+1}$$

c) $3(2x - 5a) + 4b = 4x - 2 \qquad for\ \ x$

$$6x - 15a + 4b = 4x - 2$$

$$6x - 4x = 15a - 4b - 2$$

$$2x = 15a - 4b - 2$$

$$x = \frac{15a - 4b - 2}{2}$$

Example

Solve the formula $2l + 2w = P$ for $\boldsymbol{w}$

Solution

$$2\boldsymbol{w} = P - 2l$$

$$w = \frac{P - 2l}{2}$$

Example

Solve the formula $P = C + MC$ for C

Solution

$$P = C(1 + M)$$

$$\frac{P}{1+M} = \frac{C(1+M)}{1+M}$$

$$\frac{P}{1+M} = C$$

$$\boxed{C = \frac{P}{1+M}}$$

Exercises *Section* 1.2 - Linear Equations and Rational Equations

Solve

1. $\frac{1}{14}(3x-2)=\frac{x+10}{10}$

2. $4(x+7)=2(x+12)+2(x+1)$

3. $2x-\{x-[3x-(8x+6)]\}=2x-2$

4. $\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$

5. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

6. $\frac{x-8}{3}+\frac{x-3}{2}=0$

7. $\frac{5}{2x}-\frac{8}{9}=\frac{1}{18}-\frac{1}{3x}$

8. $\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{x^2-16}$

9. $\frac{3x-1}{3}-\frac{2x}{x-1}=x$

10. $\frac{x}{x-2}=\frac{2}{x-2}+2$

11. $\frac{x}{x-7}=\frac{7}{x-7}+8$

12. $\frac{3x+2}{x-2}+\frac{1}{x}=\frac{-2}{x^2-2x}$

13. $\frac{-4x}{x-1}+\frac{4}{x+1}=\frac{-8}{x^2-1}$

14. $\frac{4x+3}{x+1}+\frac{2}{x}=\frac{1}{x^2+x}$

15. $\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{x^2+x-6}$

16. $45-[4-2y-4(y+7)]=-4(1+3y)-[4-3(y+2)-2(2y-5)]$

Solve for the specific variable

17. $A=\frac{1}{2}h(B+b),\quad for\ \ B$

18. $A=\frac{1}{2}h(a+b),\quad for\quad a$

19. $A=\frac{1}{2}h(b_1+b_2),\ \ for\ h$

20. $A=\frac{1}{2}h(b_1+b_2),\ \ for\ b_2$

21. $S=P+\mathrm{Pr}t\quad fot\quad t$

22. $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}\quad for\quad R_1$

23. Solve the formula for the indicated variable $S=3\pi rk+3\pi r^2\quad for\quad k$

24. A sewage treatment plant has two inlet pipes to its settling pond. One can fill the pond in 10 *hrs*. the other in 12 *hrs*. If the first pipe is open for 5 *hrs*. and then the second pipe us opened, how long will it take to fill the pond?

Section 1.3 - Applications and Model

Solving an Applied Problem

1. ***Read*** the problem carefully until you understand what is given and what is to be found
2. ***Assign a variable*** to represent the unknown value.
3. ***Write an equation*** using the variable expression(s).
4. ***Solve*** the equation.
5. ***State the answer*** to the problem. Does it seem reasonable?
6. ***Check*** the answer.

Example

According to the US Department of Education (2007 data), there is a gap between teaching salaries for men and women at private colleges and universities. The average salary for men exceeds the average salary for women by \$14,037. Combined their average salaries are \$130,015. Determine the average teaching salaries at private colleges for women and for men.

Solution

The average salary for men exceeds the average salary for women by \$14,037

$$m = w + 14037 \qquad \textbf{(1)}$$

Combined their average salaries are \$130,015

$$m + w = 130015 \qquad \textbf{(2)}$$

$w + 14037 + w = 130015$ *Substitute* ***m*** *with equation* **(1)**

$2w + 14037 = 130015$

$2w = 130015 - 14037$

$2w = 115978$

$w = \dfrac{115978}{2} = \$57,989.00$

$m = 57989 + 14037$

$= \$\ 72,026.00$

Example

You are choosing between two long-distance telephone plans.
Plan A has a monthly fee of $15 with a charge of $0.08 per minute for all long distance calls.
Plan B has a monthly fee of $3 with a charge of $0.12 per minute for all long distance calls.
For how many minutes of long-distance calls will the costs for the two plans be the same?

Solution

Plan A has a monthly fee of $15 with a charge of $0.08 per minute for all long distance calls.

$A = 15 + .08x$

Plan B has a monthly fee of $3 with a charge of $0.12 per minute for all long distance calls.

$B = 3 + .12x$

Costs for the two plans be the same

$A = B$

$15 + .08\boldsymbol{x} = 3 + .12\boldsymbol{x}$

$.08\boldsymbol{x} - 0.12\boldsymbol{x} = 3 - 15$

$-0.04\boldsymbol{x} = -12$

$x = \frac{-12}{-0.04} = \underline{300 \ \ minutes}$

Example

You inherit $5000 with the stipulation that for the first year the money had to be invested in two funds paying 9% and 11% annual interest. How much did you invest at each rate if the total interest earned for the year was $487?

Solution

Amount	***Rate***	***Year***	***I = Prt***
$\boldsymbol{x}$	.09	1	$.09x$
$5000 - \boldsymbol{x}$	.11	1	$.11(5000 - \boldsymbol{x})$
$5000			$487

$.09\boldsymbol{x} + .11(5000 - \boldsymbol{x}) = 487$

$.09\boldsymbol{x} + 550 - .11\boldsymbol{x} = 487$

$-.02\boldsymbol{x} = 487 - 550$

$-.02\boldsymbol{x} = -63 \qquad \Rightarrow \underline{x = \frac{-63}{-.02} = \$3150.00}$ **9%**

for **11%**: $5000 - 3150 = \$1,850.00$

Example

After a 30% price reduction, you purchase a new computer for $840. What was the computer's price before the reduction?

Solution

$x - .3x = 840$ **Or 70 % = $840**

$.7\,\boldsymbol{x} = 840$

$x = \dfrac{840}{0.7}$

$\boldsymbol{x} = \$1,200.00$

Example

The length of a rectangular basketball court is 44 feet more than the width. If the perimeter of the basketball court is 288 feet. What are the dimensions?

Solution

Length of a rectangular basketball court is 44 feet more than the width

$l = w + 44$ **(1)**

$P = 288 = 2l + 2w$ ***Divide by 2 both sides***

$144 = l + w$

$l + w = 144$ **(2)**

From **(1)** $\rightarrow$ **(2)**: $w + 44 + w = 144$

$2w = 100$

$w = 50\ ft$

$\rightarrow l = 50 + 44 = 94\ ft$

Motion Problems

$$d = rt \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

d: *D*istance

r: *R*ate, speed, or velocity

t: *T*ime

Example

Maria and Eduardo are traveling to a business conference. The trip takes 2 hr for Maria and 2.5 hr for Eduardo, since he lives 40 mi farther away. Eduardo travels 5 mph faster than Maria. Find their average rates.

Solution

	r	***t***	***d***
Maria	x	2	$2x$
Eduardo	$x + 5$	2.5	$2.5(x + 5)$

He lives 40 mi farther away

$2.5(x+5) = 2x + 40$

$2.5x + 12.5 = 2x + 40$

$2.5x - 2x = 40 - 12.5$

$0.5x = 27.5$

$x = \frac{27.5}{0.5} = 55$

Maria's rate is 55 *mph*

Eduardo's rate is 55 + 5 = 60 *mph*

Exercises *Section* 1.3 - Applications and Model

1. When a number is decreased by 30% of itself, the result is 28. What is the number?
2. When 80% a number is added to the number, the result is 252. What is the number?
3. If the length of each side of a square is increased by 3 cm, the perimeter of the new square is 40 cm more than twice of each side of the original square. Find the dimensions of the origin square.
4. The length of a rectangular label is 2.5cm less than twice the width. The perimeter is 40.6 cm. Find the width.
5. An Automobile repair shop charged a customer $448, listing $63 for parts and the remainder for labor. If the cost of labor is $35 per hour, how many hours of labor did it take to repair the car?
6. In the morning, Margaret drove to a business appointment at 50 *mph*. Her average speed on the return trip in the afternoon was 40 *mph*. The return trip took $\frac{1}{4}\,hr$ longer because of heavy traffic. How far did she travel to the appointment?
7. One of the most effective ways of removing contaminants such as carbon monoxide and nitrogen dioxide from the air while cooking is to use a vented range hood. If a range hood removes contaminants at a rate of F liters of air per second, then the percent P of contaminants that are also removed from the surrounding air can be modeled by the linear equation

   $$P = 1.06F + 7.18$$

   Where $10 \le F \le 75$. What flow F must a range hood have to remove 50% of the contaminants from the air?
8. Latoya borrowed $5240 for new furniture. She will pay it off in 11 months at an annual simple interest rate of 4.5%. How much interest will she pay?
9. Americans spent about $511 billion dining out in 2006. This was a 5.1% increase over the amount spent in 2005. How much was spent dining out in 2005?
10. For households with at least one credit card, the average U.S. credit-card debt per household was $9312 in 2004. This was $6346 more than the average credit-card debt in 1990. What was the average credit-card debt per household in 1990?
11. Together, a dog owner and a cat owner spend an average of $376 annually for veterinary-related expenses. A dog owner spends $150 more per year than a cat owner. Find the average annual veterinary-related expenses of a dog owner and of a cat owner.
12. Morgan's Seeds has a rectangular test plot with a perimeter of 322 m. The length is 25 m more than the width. Find the dimensions of the plot?

13. America West Airlines fleet includes Boeing, each with a cruising speed of 500 mph, and Bombardier Dash each with a cruising speed of 302 mph. Suppose that a Dash takes off and travels at its cruising speed. One hour later, a Boeing takes off and follows the same route, traveling at its cruising speed. How long will it take the Boeing to overtake the Dash?

14. Jared's two student loans total $12,000. One loan is at 5% simple interest and the other is at 8% simple interest. After 1 yr. Jared owes $750 in interest. What is the amount of each loan?

15. You inherit $5000 with the stipulation that for the first year the money had to be invested in two funds paying 9% and 11% annual interest. How much did you invest at each rate if the total interest earned for the year was $487?

16. An artist has sold a painting for $410,000. He needs some of the money in 6 months and the rest in 1 yr. He can get a treasury bond for 6 months at 4.65% and for one year at 4.91%. His broker tells him the two investments will earn a total of $14,961. How much should be invested at each rate to obtain that amount of interest?

17. Cody wishes to sell a piece of property for $240,000. He wants the money to be paid off in two ways – a short-term note at 6% interest and a long-term note at 5%. Find the amount of each note if the total annual interest paid is $13,000.

18. The number of steps needed to burn off a <u>C</u>heeseburger exceeds the number needed to burn off a 12-ounce <u>S</u>oda by 4140. The number needed to burn off a <u>D</u>oughnut exceeds the number needed to burn off 12 ounce soda by 2300. If you chow down a cheeseburger, doughnut, and 12-ounce soda, a 16790 step walk is needed to burn off the calories (and perhaps alleviate the guilt).Determine the number of steps it takes to burn off a cheeseburger, a doughnut, and a 12-ounce soda.

19. Although organic milk accounts for only 12% of the market, consumption is increasing. In 2004, Americans purchased 40.7 million gallons of organic milk, increasing at a rate of 5.6 million gallons per year. If this trend continues, when will Americans purchase 79.9 million gallons of organic milk?

20. How many gallons of a 5% acid solution must be fixed with 5 gal of a 10% solution to obtain 7% solution?

21. In 1969, 88% of the women considered this objective essential or very important. Since then, this percentage has decreased by approximately 1.1 each year. If this trend continues, by which year will only 33% of female freshmen consider "developing a meaningful philosophy of life" essential or very important?

22. Charlotte is a chemist. She needs a 20% solution of alcohol. She has a 15% solution on hand, as well as a 30% solution. How many liters of the 15% solution should she add to 3 L of the 30% solution to obtain her 20% solution?

Section 1.4 - Quadratic Equations

Basic Complex Number

$$i^2 = -1 \qquad \Rightarrow\ i = \sqrt{-1} \qquad \Rightarrow\ \sqrt{-1} = i$$

The number i is called the ***imaginary unit***.

Example

$\sqrt{-8} = i2\sqrt{2} \qquad 2i\sqrt{2}$

$$\begin{aligned}\sqrt{-7}\sqrt{-7} &= i\sqrt{7}\ i\sqrt{7} \\ &= i^2\left(\sqrt{7}\right)^2 \\ &= -7\end{aligned}$$

Complex number is written in a form: $z = a + ib$

a is the real part

b is the imaginary part

Conjugate of a complex number $a + bi$ is $a - bi$

A ***quadratic equation*** in x is an equation that can be written in the general form:

$ax^2 + bx + c = 0$ where a, b, and c are real numbers,

$4x^2 - 3x + 2 = 0 \qquad a = 4 \quad b = -3 \quad c = 2$

Solving Quadratic Equations by Factoring

The Zero-Product Principle

If $AB = 0$ then $A = 0$ or $B = 0$.

Example

Solve $6x^2 + 7x - 3 = 0$

Solution

$(3x - 1)(2x + 3) = 0$

$$\begin{aligned}3x - 1 &= 0 & \qquad 2x + 3 &= 0 \\ 3x &= 1 & 2x &= -3 \\ x &= \frac{1}{3} & x &= -\frac{3}{2}\end{aligned}$$

The Square Root Property

If u is an algebraic expression and d is a nonzero real number, then $u^2 = d$ has exactly two solutions:

$$\text{If } u^2 = d, \text{ then } u = \sqrt{d} \text{ or } u = -\sqrt{d}$$

Equivalently,

$$\text{If } u^2 = d \Rightarrow u = \pm\sqrt{d}.$$

Example

a) $3x^2 - 21 = 0$

$3x^2 = 21$

$x^2 = 7 \qquad \Rightarrow \boxed{x = \pm\sqrt{7}}$

b) $5x^2 + 45 = 0$

$5x^2 = -45$

$x^2 = -9$

$x = \pm\sqrt{-9} \qquad \Rightarrow x = \pm 3i$

c) $(x+5)^2 = 11$

$x + 5 = \pm\sqrt{11} \qquad \Rightarrow \boxed{x = -5 \pm \sqrt{11}}$

Completing the Square

If $x^2 + bx$ is a binomial, then by ***adding*** $\left(\frac{b}{2}\right)^2$ which is the square of half the coefficient of x. a perfect square trinomial will result. That is.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 \qquad x^2 + bx + \left(\frac{1}{2}b\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example

Solve: $x^2 + 4x - 1 = 0$

$x^2 + 4x = 1$

$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 1 + \left(\frac{4}{2}\right)^2$

$x^2 + 4x + (2)^2 = 1 + 4$

$(x+2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

$\boxed{x = -2 \pm \sqrt{5}}$

Quadratic Formula

(*Using Completing the Square*)

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{1}{2}\frac{b}{a}\right)^2 = -\frac{c}{a} + \left(\frac{1}{2}\frac{b}{a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*** $ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{cases} b^2 - 4ac > 0 \rightarrow 2 \text{ Real numbers} \\ b^2 - 4ac < 0 \rightarrow 2 \text{ Complex numbers} \\ b^2 - 4ac = 0 \rightarrow \text{One solution (repeated)} \end{cases}$$

Example

Solve: $2x^2+2x-1=0$

Solution

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad \Rightarrow a=2 \quad b=2 \quad c=-1$$

$$=\frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}$$

$$=\frac{-2\pm\sqrt{4+8}}{4}$$

$$=-\frac{2}{4}\pm\frac{\sqrt{12}}{4}$$

$$=-\frac{1}{2}\pm\frac{2\sqrt{3}}{4}$$

$$=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}$$

$$=\left\{-\frac{1}{2}-\frac{\sqrt{3}}{2},\quad -\frac{1}{2}+\frac{\sqrt{3}}{2}\right\}$$

$$=\frac{-2\pm\sqrt{12}}{4}$$

$$=\frac{-2\pm2\sqrt{3}}{4}$$

$$=\frac{2\left(-1\pm\sqrt{3}\right)}{4}$$

$$=\frac{-1\pm\sqrt{3}}{2}$$

$$=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}$$

Example

Solve $x^2-4x=-2$

Solution

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad \Rightarrow a=1 \quad b=-4 \quad c=2$$

$$=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}$$

$$=\frac{4\pm\sqrt{16-8}}{2}$$

$$=\frac{4\pm\sqrt{8}}{2}$$

$$=\frac{4\pm2\sqrt{2}}{2}$$

$$=\frac{2\left(2\pm\sqrt{2}\right)}{2}$$

$$\boxed{=2\pm\sqrt{2}}$$

Example

Solve: $x^2 - 2x + 2 = 0$

Solution

$$\Rightarrow a = 1 \quad b = -2 \quad c = 2$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

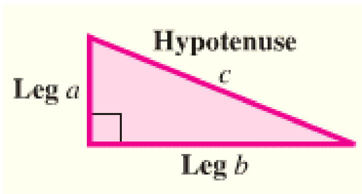
$$= \frac{2 \pm \sqrt{4-8}}{2} \qquad\qquad = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2}{2} \pm \frac{\sqrt{-4}}{2} \qquad\qquad = \frac{2 \pm 2i}{2}$$

$$= 1 \pm \frac{2i}{2} \qquad\qquad = \frac{2(1 \pm i)}{2}$$

$$\underline{= 1 \pm i} \qquad\qquad \underline{= 1 \pm i}$$

Pythagorean Theorem



The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If the legs have lengths a and b, and the hypotenuse has length c, then:

$$a^2 + b^2 = c^2$$

Example

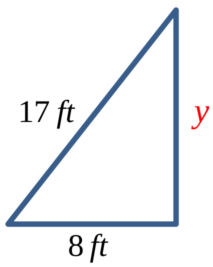
A ladder that is 17 feet long is 8 feet from the base of a wall. How far up the wall does the ladder reach?

Solution

$$8^2 + y^2 = 17^2$$

$$y^2 = 17^2 - 8^2$$

$$y = \sqrt{17^2 - 8^2} = \underline{15\ ft}$$



Height of a Projected Object *(Position Function)*

An object that is falling or vertically projected into the air has its height above the ground, $s(t)$, in feet, given by

$$s(t) = -16t^2 + v_0 t + s_0$$

v_0 is the original velocity (initial velocity) of the object, in feet per second

t is the time that the object is in motion, in second

s_0 is the original height (initial height) of the object, in feet

Example

If a projectile is shot vertically upward from the ground with an initial velocity of 100 ft per sec, neglecting air resistance, its height *s* (in feet) above the ground *t* seconds after projection is given by

$$s = -16t^2 + 100t$$

a) After how many seconds will it be 50 ft above the ground?

b) How long will it take for the projectile to return to the ground?

Solution

a) *After how many seconds will it be 50 ft above the ground?*

$$50 = -16t^2 + 100t$$

$$16t^2 - 100t + 50 = 0$$

$$8t^2 - 50t + 25 = 0$$

$$t = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(8)(25)}}{2(8)}$$

$$= \frac{50 \pm \sqrt{1700}}{16}$$

$$t = \frac{50 - \sqrt{1700}}{16} \approx 0.55 \qquad t = \frac{50 + \sqrt{1700}}{16} \approx 5.70$$

b) *How long will it take for the projectile to return to the ground?*

$$0 = -16t^2 + 100t$$

$$0 = -4t(4t - 25)$$

$$-4t = 0 \qquad 4t - 25 = 0$$

$$t = 0 \qquad 4t = 25$$

$$t = \frac{25}{4} = 6.25$$

Exercises *Section* 1.4 - Quadratic Equations

Solve

1. $x^2 = -25$
2. $5x^2 - 45 = 0$
3. $(x-4)^2 = 12$
4. $(4x+1)^2 = 20$
5. $x^2 - 6x = -7$
6. $-6x^2 = 3x + 2$
7. $3x^2 + 2x = 7$
8. $3x^2 + 6 = 10x$
9. $5x^2 + 2 = x$
10. $5x^2 = 2x - 3$
11. $x^2 + 8x + 15 = 0$
12. $x^2 + 5x + 2 = 0$
13. $x^2 - 6x - 10 = 0$
14. $2x^2 + 3x - 4 = 0$
15. $x^2 - x + 8 = 0$
16. $2x^2 - 13x = 1$
17. $r^2 + 3r - 3 = 0$
18. $x^3 + 8 = 0$

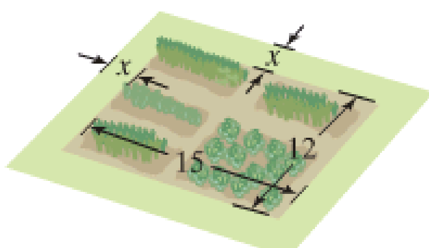
19. Solve for the specified variable $A = \frac{\pi d^2}{4}$, *for d*

20. Solve for the specified variable $rt^2 - st - k = 0 \;\; (r \neq 0)$, *for t*

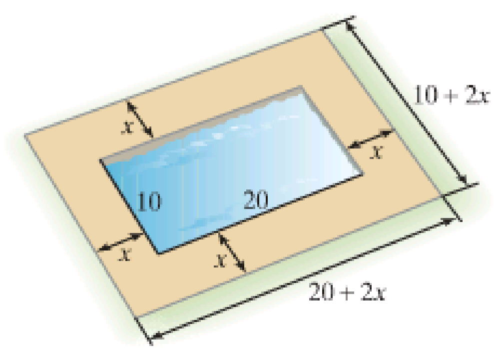
21. A rectangular park is 6 *miles* long and 2 *miles* wide. How long is a pedestrian route that runs diagonally across the park?

22. What is the width of a 25-*inch* television set whose height is 15 *inches*?

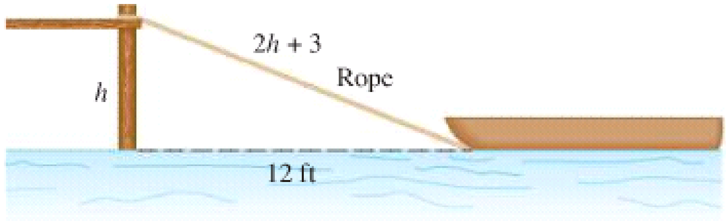
23. A vacant rectangular lot is being turned into a community vegetable garden measuring 15 *meters* by 12 *meters*. A path of uniform width is to surround the garden. If the area of the garden and path combined is 378 *square meters*, find the width of the path.



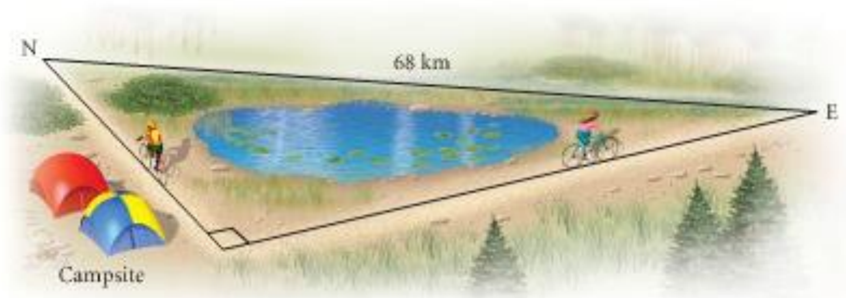
24. A pool measuring 10 m by 20 m is surrounded by a path of uniform width. If the area of the pool and the path combined is 600 m^2, what is the width of the path?



25. A boat is being pulled into a dock with a rope attached to the boat at water level. Where the boat is 12 ft from the dock, the length of the rope from the boat to the dock is 3 ft longer than twice the height of the dock above the water. Find the height of the dock.



26. Logan and Cassidy leave a campsite, Logan biking due north and Cassidy biking due east. Logan bikes 7 km/h slower than Cassidy. After 4 *hrs*, they are 68 km apart. Find the speed of each bicyclist.



27. Towers are 1482 ft. tall. How long would it take an object dropped from the top to reach the ground? Given $\boldsymbol{s} = 16\boldsymbol{t}^2$

28. The formula $P = 0.01A^2 + 0.05A + 107$ models a woman's normal Point systolic blood pressure, P, an age A. Use this formula to find the age, to the nearest year, of a woman whose normal systolic blood pressure is 115 mm Hg.

29. A rectangular piece of metal is 10 in. longer than it is wide. Squares with sides 2 in. long are cut from the four corners, and the flaps folded upward to form an open box. If the volume of the box is 832 in^3, what were the original dimensions of the piece of metal?

30. An astronaut on the moon throws a baseball upward. The astronaut is 6 ft., 6 in., tall, and the initial velocity of the ball is 30 ft. per sec. The height s of the ball in feet is given by the equation

$$s = -2.7t^2 + 30t + 6.5$$

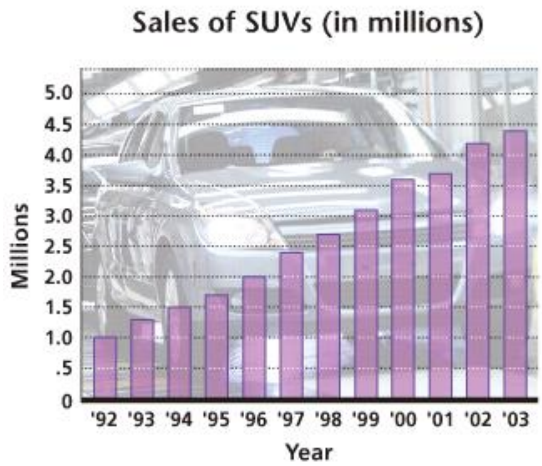
Where $\boldsymbol{t}$ is the number of seconds after the ball was thrown.

a) After how many seconds is the ball 12 ft above the moon's surface?

b) How many seconds will it take for the ball to return to the surface?

31. The bar graph shows of SUVs (sport utility vehicles0 in the US, in millions. The quadratic equation

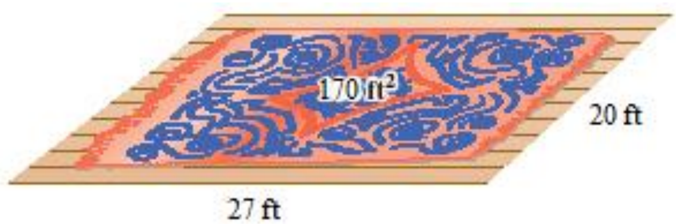
$S = .00579x^2 + .2579x + .9703$ models sales of SUVs from 1992 to 2003, where S represents sales in millions, and $x = 0$ represents 1992, $x = 1$ represents 1993 and so on.



a) Use the model to determine sales in 2002 and 2003. Compare the results to the actual figures of 4.2 million and 4.4 million from the graph.

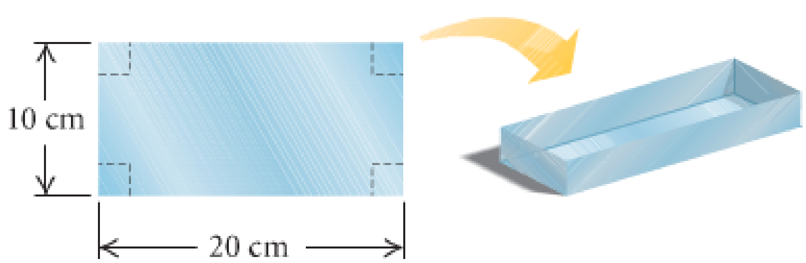
b) According to the model, in what year do sales reach 3.5 million? Is the result accurate?

32. Cynthia wants to buy a rug for a room that is 20 ft wide and 27 ft long. She wants to leave a uniform strip of floor around the rug. She can afford to buy 170 square feet of carpeting. What dimension should the rug have?



33. Erik finds a piece of property in the shape of a right triangle. He finds that the longer leg is 20 m longer than twice the length of the shorter leg. The hypotenuse is 10 m longer than the length of the longer leg. Find the lengths of the sides of the triangular lot.

34. An open box is made from a 10-cm by 20-cm piece of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2. What is the length of the sides of the squares?



Section 1.5 - Other Types of Equations

The numbers of solutions to a polynomial with n degree, where n is Natural Number, are n solutions.

Solving a Polynomial Equation by factoring

Example

Solve: $4x^4 = 12x^2$

Solution

$$4x^4 - 12x^2 = 0$$

$$4x^2(x^2 - 3) = 0$$

$$4x^2 = 0 \qquad\qquad x^2 - 3 = 0$$

$$x^2 = 0 \qquad\qquad x^2 = 3$$

$$\rightarrow\ x = 0, 0 \qquad\qquad x = \pm\sqrt{3}$$

Example

Solve: $2x^3 + 3x^2 = 8x + 12$

Solution

$$2x^3 + 3x^2 - 8x - 12 = 0$$

$$x^2(2x+3) - 4(2x+3) = 0$$

$$(2x+3)\left(x^2 - 4\right) = 0$$

$$2x + 3 = 0 \qquad\qquad x^2 - 4 = 0$$

$$2x = -3 \qquad\qquad x^2 = 4$$

$$x = -\frac{3}{2} \qquad\qquad x = \pm\sqrt{4} = \pm 2$$

Solving a *Radical* Equation

Power Property

If P and Q are algebraic expressions, then every solution of the equation $P = Q$ is also a solution of the equation $P^n = Q^n$; for any positive integer n.

Example

Solve $x - \sqrt{15 - 2x} = 0$

Solution

$$x = \sqrt{15 - 2x}$$

$$x^2 = \left(\sqrt{15 - 2x}\right)^2$$

$$x^2 = 15 - 2x$$

$$x^2 + 2x - 15 = 0$$

$$(x - 3)(x + 5) = 0$$

$$x - 3 = 0 \qquad x + 5 = 0$$

$$x = 3 \qquad x = -5$$

Check

$x = 3$	$x = -5$
$3 - \sqrt{15 - 2(3)} = 0$	$-5 - \sqrt{15 - 2(-5)} = 0$
$3 - \sqrt{9} = 0$	$-5 - \sqrt{25} = 0$
$3 - 3 = 0$ (*true*)	$-5 - 5 \neq 0$ (*false*)

$x = 3$ is the only solution

Solving Radical Equations of the Form $x^{\frac{m}{n}} = k$

Assume that *m* and *n* are positive integers

If m is even: $x^{\frac{m}{n}} = k \Rightarrow \left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = k^{\frac{n}{m}} \Rightarrow x = \pm k^{\frac{n}{m}}$

If m is odd: $x^{\frac{m}{n}} = k \Rightarrow \left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = k^{\frac{n}{m}} \Rightarrow x = k^{\frac{n}{m}}$

Example

Solve:

a) $5x^{\frac{3}{2}} - 25 = 0$

$$5x^{\frac{3}{2}} = 25$$

$$x^{\frac{3}{2}} = \frac{25}{5} = 5$$

$$x = 5^{\frac{2}{3}}$$

$$x = \sqrt[3]{5^2}$$

$$\underline{= \sqrt[3]{25}|}$$

b) $x^{\frac{2}{3}} - 8 = -4$

$$x^{\frac{2}{3}} = 4$$

$$x = \pm(4)^{3/2}$$

$$= \pm\sqrt{4^3}$$

$$= \pm 4\sqrt{4}$$

$$\underline{= \pm 8|}$$

Equations that Are Quadratic in Form

$$ax^2+bx+c=0$$
$$a(x)^2+b(x)^1+c=0$$
$$a(u)^2+b(u)^1+c=0$$
$$a\left(x^n\right)^2+b\left(x^n\right)^1+c=0$$
$$au^2+bu+c=0$$

Example

Solve: $x^4-5x^2+6=0$

Solution

$$\left(x^2\right)^2-5\left(x^2\right)+6=0$$
$$(U)^2-5(U)+6=0$$
$$U^2-5U+6=0$$

Solve for U

$$\Rightarrow U=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(6)}}{2(1)}$$
$$=\frac{5\pm\sqrt{25-24}}{2}$$
$$=\frac{5\pm\sqrt{1}}{2}$$
$$\rightarrow\begin{cases}U=\frac{5-1}{2}=2\\U=\frac{5+1}{2}=3\end{cases}$$
$$x^2=U\qquad\rightarrow\begin{cases}x^2=2\rightarrow \underline{x=\pm\sqrt{2}}\\x^2=3\rightarrow \underline{x=\pm\sqrt{3}}\end{cases}$$

or $\left(x^2-2\right)\left(x^2-3\right)=0$

$x^2-2=0 \qquad x^2-3=0$

$x^2=2 \qquad x^2=3$

$x=\pm\sqrt{2} \qquad x=\pm\sqrt{3}$

Example

Solve: $(x+1)^{2/3} - (x+1)^{1/3} - 2 = 0$

Solution

$u = (x+1)^{1/3}$

$\Big((x+1)^{1/3} - 2\Big)\Big((x+1)^{1/3} + 1\Big) = 0$

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u - 2 = 0 \qquad u + 1 = 0$

$u = 2 \qquad u = -1$

$u = (x+1)^{1/3} = 2 \qquad u = (x+1)^{1/3} = -1$

$x + 1 = 2^3 \qquad x + 1 = (-1)^3$

$x + 1 = 8 \qquad x + 1 = -1$

$\boxed{x = 7} \qquad \boxed{x = -2}$

Example

Solve: $3x^{2/3} - 11x^{1/3} - 4 = 0$

Solution

$3\left(x^{1/3}\right)^2 - 11\left(x^{1/3}\right) - 4 = 0$

$3(U)^2 - 11(U) - 4 = 0$

$3U^2 - 11U - 4 = 0$

Solve for U

$\Rightarrow U = \dfrac{-(-11) \pm \sqrt{11^2 - 4(3)(-4)}}{2(3)}$

$= \dfrac{11 \pm 13}{6}$

Or factor

$\left(3x^{1/3} + 1\right)\left(x^{1/3} - 4\right) = 0$

$3x^{1/3} + 1 = 0 \qquad x^{1/3} - 4 = 0$

$x^{1/3} = U \qquad x^{1/3} = \frac{11-13}{6} \qquad x^{1/3} = \frac{11+13}{6}$

$= -\frac{1}{3} \qquad = 4$

$\rightarrow x = \left(-\frac{1}{3}\right)^3 \qquad x = 4^3$

$\underline{= -\frac{1}{27}} \qquad \underline{= 64}$

Solving an Absolute Value Equation

If c is a positive real number and X represents any algebraic expression, then $|X| = c$ is equivalent to $X = c$ or $X = -c$

$$|X| = c \rightarrow X = c \text{ or } X = -c$$

Properties of Absolute Value

1. For b > 0, $|a| = b$ if and only if (*iff*) $a = b$ or $a = -b$
2. $|a| = |b|$ *iff* $a = b$ or $a = -b$

For any positive number b:

3. $|a| < b$ *iff* $-b < a < b$
4. $|a| < b$ *iff* $a < -b$ or $a > b$

Example

Solve: $|2x - 1| = 5$

$2x - 1 = 5$	$2x - 1 = -5$	
$2x = 6$	$2x = -4$	
$x = 3$	$x = -2$	*Solutions*: $x = -2, 3$

Example

Solve: $4|1 - 2x| - 20 = 0$

$4|1 - 2x| = 20$

$|1 - 2x| = 5$

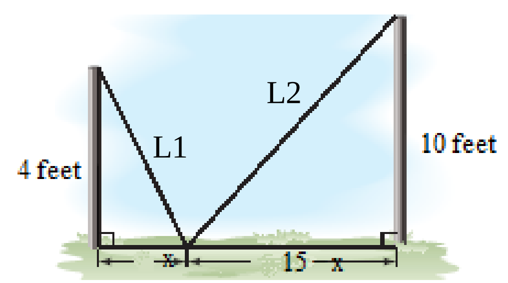
$1 - 2x = 5$	$1 - 2x = -5$	
$-2x = 4$	$-2x = -6$	
$x = -2$	$x = 3$	*Solutions*: $x = -2, 3$

Exercise Section 1.5 - Other Types of Equations

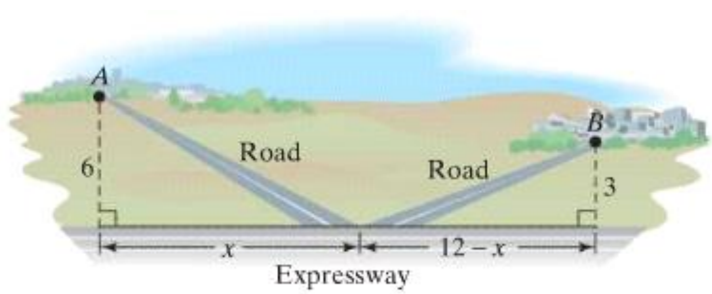
Solve

1. $3x^3+2x^2=12x+8$
2. $x^4+3x^2=10$
3. $5x^4=40x$
4. $9x^4-9x^2+2=0$
5. $x^4+720=89x^2$
6. $x-\sqrt{2x+3}=0$
7. $\sqrt{x+3}+3=x$
8. $x-\sqrt{x+11}=1$
9. $\sqrt{2x-3}+\sqrt{x-2}=1$
10. $\sqrt{2x+3}=1+\sqrt{x+1}$
11. $\sqrt{x+5}-\sqrt{x-3}=2$
12. $\sqrt[3]{4x^2-4x+1}-\sqrt[3]{x}=0$
13. $12x^4-11x^2+2=0$
14. $2x^4-7x^2+5=0$
15. $x^4-5x^2+4=0$
16. $x^4+3x^2=10$
17. $x-3\sqrt{x}-4=0$
18. $\left(5x^2-6\right)^{1/4}=x$
19. $\left(x^2+24x\right)^{1/4}=3$
20. $x^{5/2}=32$
21. $7\left|5x\right|+2=16$
22. $4\left|1-\frac{3}{4}x\right|+7=10$
23. $\left|x+7\right|+6=2$
24. $\left|5-3x\right|=12$
25. $\left|4x+2\right|=5$
26. $\left|\frac{6x+1}{x-1}\right|=3$
27. $\left|x+1\right|=\left|1-3x\right|$

28. Two vertical poles of lengths 4 feet and 10 feet stand 15 feet apart. A cable reaches from the top of one pole to some point on the ground between the poles and then to the top of the other pole. Where should this point be located to use 24 feet of cable?



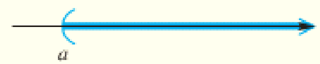
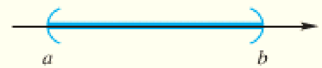
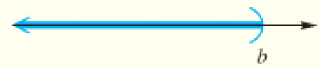
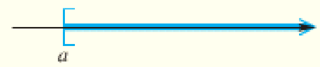
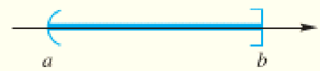
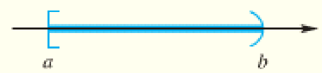
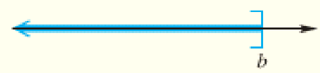
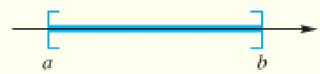
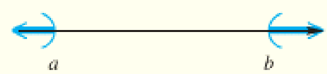
29. Towns A and B are located 6 miles and 3 miles, respectively, from a major expressway. The point on the expressway closet to town A is 12 miles from the point on the expressway closet to town B. Two new roads are to be built from A to the expressway and then to B.



a) Express the combined lengths of the new road in terms of *x*.
b) If the combined lengths of the new roads is 15 miles, what distance does *x* represent?

Section 1.6 – Inequalities

Notation

Type of Interval	Set	Interval Notation	Graph
Open interval	$\{x \mid x > a\}$	(a, ∞)	
	$\{x \mid a < x < b\}$	(a, b)	
	$\{x \mid x < b\}$	$(-\infty, b)$	
Other intervals	$\{x \mid x \geq a\}$	$[a, \infty)$	
	$\{x \mid a < x \leq b\}$	$(a, b]$	
	$\{x \mid a \leq x < b\}$	$[a, b)$	
	$\{x \mid x \leq b\}$	$(-\infty, b]$	
Closed interval	$\{x \mid a \leq x \leq b\}$	$[a, b]$	
Disjoint interval	$\{x \mid x < a \text{ or } x > b\}$	$(-\infty, a) \cup (b, \infty)$	
All real numbers	$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	

Properties of inequality

1. ***If a < b, then a + c < b + c***
2. ***If a < b and if c > 0, then ac < bc***
3. ***If a < b and if c < 0, then ac > bc***

Example

Solve $3x + 1 > 7x - 15$

Solution

$3x - 7x > -1 - 15$

$-4x > -16$ *Divide by – 4 both sides*

$x < 4$ *or* $(-\infty, 4)$ *or* $\{x \mid x < 4\}$

Example

Solve $\frac{x-4}{2} \ge \frac{x-2}{3} + \frac{5}{6}$ *LCD*: 2, 3, 6

Solution

$$(6)\frac{x-4}{2} \ge (6)\frac{x-2}{3} + (6)\frac{5}{6}$$

$$3(x-4) \ge 2(x-2) + 5$$

$$3x - 12 \ge 2x - 4 + 5$$

$$3x - 12 \ge 2x + 1$$

$$3x - 2x \ge 12 + 1$$

$$\underline{x \ge 13}$$

Example

a) $3(x + 1) > 3x + 2$

$3x + 3 > 3x + 2$

$3x - 3x > -3 + 2$

Sol.: $0 > -1$ (*True statement*)

$\{x \mid \textit{All Real numbers}\}$ or $(-\infty, \infty)$

b) $x + 1 \le x - 1$

$x - x \le -1 - 1$

$0 \le -2$ *Sol.*: $\varnothing$

Example

Solve $-2 < 5 + 3x < 20$ Give the solution set in interval notation and graph it.

Solution:

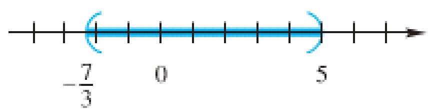
$$-2 - 5 < 5 + 3x - 5 < 20 - 5$$

$$-7 < 3x < 15$$

$$-\frac{7}{3} < \frac{3}{3}x < \frac{15}{3}$$

$$-\frac{7}{3} < x < 5$$

Solution: $\left(-\frac{7}{3}, 5\right)$

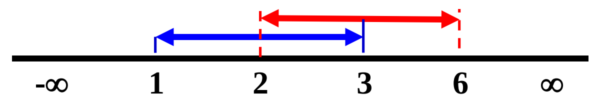


Intersections of Interval $\bigcap$

To find the intersection, take the portion of the number line that the two graphs have in ***common***

Example

$[1, 3] \cap (2, 6) = (2, 3]$

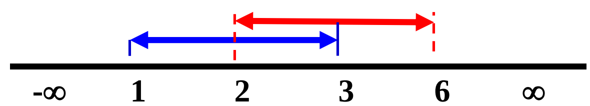


Unions of Interval $\bigcup$

To find the union, take the portion of the number line representing the total ***collection*** of numbers in the two graphs.

Example

$[1, 3] \cup (2, 6) = [1, 6)$



Solving an *Absolute Value* Inequality:

If X is an algebraic expression and c is a positive number,

1. The solutions of $|X| < c$ are the numbers that satisfy $-c < X < c$.
2. The solutions of $|X| > c$ are the numbers that satisfy $X < -c$ or $X > c$.

Example

Solve: $-3|5x-2|+20 \ge -19$

Solution

$-3|5x-2| \ge -39$

$-|5x-2| \ge -13$

$|5x-2| \le 13$

$-13 \le 5x-2 \le 13$

$-11 \le 5x \le 15$

$-\frac{11}{5} \le x \le 3$ or $[-\frac{11}{5}, 3]$

Example

Solve: $18 < |6-3x|$

Solution

$|6-3x| > 18$

$6-3x < -18$	$6-3x > 18$
$-3x < -18-6$	$-3x > 18-6$
$-3x < -24$	$-3x > 12$
$\frac{-3}{-3}x > -\frac{24}{-3}$	$\frac{-3}{-3}x < \frac{12}{-3}$
$x > 8$	$x < -4$

Solution: $(-\infty, -4) \cup (8, \infty)$

Special Cases

Example

Solve each equation or inequality

a) $|2-5x| \geq -4$

b) $|4x-7| < -3$

c) $|5x+15| = 0$

Solution

a) $|2-5x| \geq -4$

It is always ***true*** $\Rightarrow$ the solution set is: All real numbers $(-\infty, \infty)$

b) $|4x-7| < -3$

Any absolute value can't be less than any negative number. $\Rightarrow$ No solution or $\varnothing$

c) $|5x+15| = 0$

$5x+15=0$

$5x=-15$

$\frac{5}{5}x=-\frac{15}{5}$

$\underline{x=-3}$

Exercises *Section* 1.6 – Inequalities

1. Find: $(-3,0)\cap[-1,2]$

2. Find: $(-3,0)\cup[-1,2]$

3. Find: $(-4,0)\cap[-2,1]$

4. Find: $(-4,0)\cup[-2,1]$

5. Find: $(-\infty,5)\cap[1,8)$

6. Find: $(-\infty,5)\cup[1,8)$

7. Solve $-3x+5>-7$ Give the solution set in interval notation.

8. Solve $2-3x\le 5$

9. Solve $4-3x\le 7+2x$ Give the solution set in interval notation and graph it.

10. Solve $-3\le \frac{2}{3}x-5\le -1$

11. Solve $6x-(2x+3)\ge 4x-5$ Give the solution set in interval notation and graph it.

12. Solve $\frac{2x-5}{-8}\le 1-x$ Give the solution set in interval notation and graph it.

13. Solve $-6\le 6x+3\le 21$ Give the solution set in interval notation and graph it.

14. Solve the inequality equation: $1\le 2x+3<11$

15. Solve the inequality equation: $12<\left|-2x+\frac{6}{7}\right|+\frac{3}{7}$

16. Solve the inequality equation: $4+\left|3-\frac{x}{3}\right|\ge 9$

17. Solve the inequality equation: $|x-2|<5$

18. Solve the inequality equation: $|2x+1|<7$

19. Solve the inequality equation: $|5x+2|-2<3$

20. Solve the inequality equation: $|2-7x|-1>4$

21. Solve the inequality equation: $|3x-4|<2$

22. Solve the inequality equation: $|2x+5|\ge 3$

23. Solve $|12-9x|\ge -12$

24. Solve $|6-3x|<-11$

25. Solve $|7+2x|=0$