Math 2414 – Calculus II ***Exam* 4** ***Review***

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1. Give parametric equations and parameter intervals for the motion of a particle in the *xy*-plane. Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation.

a) $x = \frac{t}{2}, \quad y = t + 1, \quad -\infty < t < \infty$

b) $x = \sqrt{t}, \quad y = 1 - \sqrt{t}, \quad t \geq 0$

c) $x = -2\cos t, \quad y = 2\sin t, \quad 0 \leq t \leq \pi$

d) $x = -\cos t, \quad y = \cos^2 t, \quad 0 \leq t \leq \pi$

e) $x = 4\cos 2\pi t, \quad y = 4\sin 2\pi t, \quad 0 \leq t \leq 1$

f) $x = e^t, \quad y = e^{-2t}, \quad -1 \leq t \leq 1$

2. Find a parametric equations and a parameter interval for the motion of a particle in the *xy*-plane that traces the ellipse $16x^2 + 9y^2 = 144$ once counterclockwise.

3. Find a parametric equations and a parameter interval for the motion of a particle starting at the point $(-2,\ 0)$ and tracing the circle $x^2 + y^2 = 4$ three times clockwise.

4. Find the tangent to the curve at the point defined by the given value of *t*.

$x = t - \sin t, \quad y = 1 - \cos t, \quad t = \frac{\pi}{6}$ *and* $t = \frac{2\pi}{3}$

5. Find the tangent to the curve at the point defined by the given value of *t*. Also find the value of $\frac{d^2y}{dx^2}$ at this point

a) $x = 2\cos t, \quad y = 2\sin t, \quad t = \frac{\pi}{4}$

b) $x = -\sqrt{t+1}, \quad y = \sqrt{3t}, \quad t = 3$

6. Find the length of the curves

a) $x = 5\cos t - \cos 5t, \quad y = 5\sin t - \sin 5t, \quad 0 \leq t \leq \frac{\pi}{2}$

b) $x = t^3 - 6t^2, \quad y = t^3 + 6t^2, \quad 0 \leq t \leq 1$

c) $x = t^2, \quad y = \frac{1}{3}t^3 - t \quad -\sqrt{3} \leq t \leq \sqrt{3}$

7. Replace the polar equation with equivalent Cartesian equation

a) $r = 2\sec\theta$

b) $r = \left(3\sqrt{3}\right)\sec\theta$

c) $r = -4\sin\theta$

d) $r = \cot\theta\csc\theta$

e) $r\cos\theta = \sin 2\theta$

f) $r = \sin\theta\sec^2\theta$

g) $r = \dfrac{1}{2\cos\theta + 3\sin\theta}$

h) $r^2 = 4\cos 2\theta$

i) $r^2 = 4\sin 2\theta$

8. Replace the Cartesian equation with equivalent polar equation

a) $x^2 + y^2 + 5y = 0$

b) $x^2 + y^2 - 3x = 0$

c) $x^2 + y^2 + 4x = 0$

d) $y^2 = 6x$

e) $xy = 8$

f) $(x+2)^2 + (y-3)^2 = 13$

g) $y^2 - x^2 = 4$

9. Find the area of the region in the polar coordinate plane

a) Enclosed by the limaçon $r = 2 - \cos\theta$

b) Enclosed by one leaf of the three-leaved rose $r = \sin 3\theta$

c) Inside the cardioid $r = 2(1 + \sin\theta)$ and outside the circle $r = 2\sin\theta$

10. Find the length of the curve given by the polar coordinate equation

a) $r = -1 + \cos\theta$

b) $r = 2\sin\theta + 2\cos\theta, \quad 0 \le \theta \le \dfrac{\pi}{2}$

c) $r = 8\sin^3\left(\dfrac{\theta}{3}\right), \quad 0 \le \theta \le \dfrac{\pi}{4}$

d) $r = \sqrt{1 + \cos 2\theta}, \quad -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

11. Sketch the graph of the polar equation

a) $r = 2 + 4\sin\theta$

b) $r = 4\cos\theta + 2\sin\theta$

c) $r^2 = 4\cos 2\theta$

d) $r = 2 - \sin\theta$

Answers

1. *a*) $y = 2x + 1$ *b*) $y = 1 - x$ *c*) $x^2 + y^2 = 4$

d) $y = x^2$ *e*) $x^2 + y^2 = 16$ *f*) $y = \frac{1}{x^2}$

2. $x = 3\cos t, \quad y = 4\sin t, \quad 0 \le t \le 2\pi$

3. $x = -2\cos t, \quad y = 2\sin t, \quad 0 \le t \le 6\pi$

4. $t = \frac{\pi}{6} \Rightarrow y = \left(2 + \sqrt{3}\right)x + 2 - \frac{\pi}{3} - \frac{\pi\sqrt{3}}{6}$ $\qquad t = \frac{2\pi}{3} \Rightarrow y = \frac{1}{\sqrt{3}}x + 2 - \frac{2\pi}{3\sqrt{3}}$

5. *a*) $y = -x + 2\sqrt{2}; \quad \frac{d^2y}{dx^2} = -\frac{1}{2\sin^3 t}\Big|_{t=\frac{\pi}{4}} = -\sqrt{2}$

b) $y = -2x - 1; \quad \frac{d^2y}{dx^2} = -\frac{3}{t\sqrt{3t}}\Big|_{t=3} = -\frac{1}{3}$

6. *a*) $\mathrm{L} = 10$ *b*) $\mathrm{L} = 8.67$ *c*) $L = 4\sqrt{3}$

7. *a*) $x = 2$ *b*) $x = 3\sqrt{3}$ *c*) $x^2 + (y+2)^2 = 4$ *d*) $y^2 = x$

e) $x^2 + y^2 - 2y = 0$ *f*) $y = x^2$ *g*) $3y + 2x = 1$

h) $\left(x^2 + y^2\right)^2 = 4x^2 - 4y^2$ *i*) $\left(x^2 + y^2\right)^2 = 8xy$

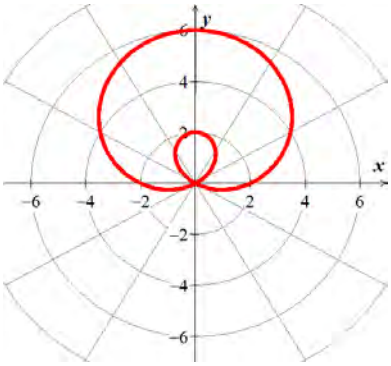
8. *a*) $r = -5\sin\theta$ *b*) $r = 3\cos\theta$ *c*) $r = -4\cos\theta$ *d*) $r = 6\frac{\cos\theta}{\sin^2\theta}$

e) $r^2 = \frac{8}{\cos\theta\sin\theta}$ *f*) $r = 6\sin\theta - 4\cos\theta$ *g*) $r^2 = -\frac{4}{\cos 2\theta}$

9. *a*) $A = \frac{9}{2}\pi$ *b*) $A = \frac{\pi}{12}$ *c*) $A = 5\pi$

10. *a*) $\mathrm{L} = 8$ *b*) $\mathrm{L} = \pi\sqrt{2}$ *c*) $L = \pi - 3$ *c*) $L = \pi\sqrt{2}$

11.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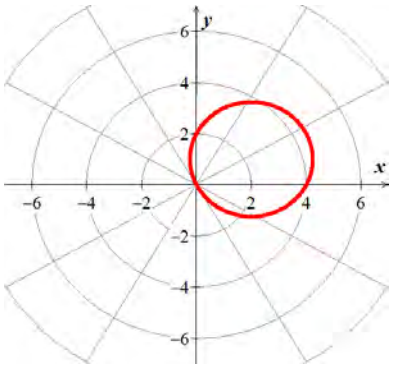
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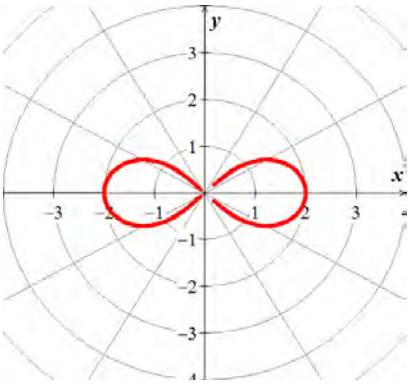
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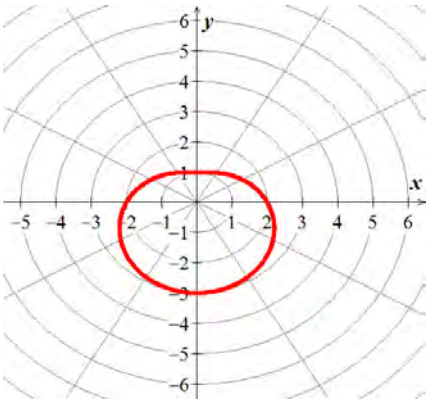
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