Exercise

Solve the simplex method:

Maximize: $P = 50x_1 + 80x_2$

Subject to
$$x_1 + 2x_2 \le 32$$
$$3x_1 + 4x_2 \le 84$$
$$x_1, x_2 \ge 0$$

Solution

The Initial Simplex Tableau

$$\begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 0 & 0 & 32 \\ 3 & 4 & 0 & 1 & 0 & 84 \\ \hline -50 & -80 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

So the basic feasible solution at this point is: $x_1 = 0,\ x_2 = 0,\ s_1 = 32,\ \ s_2 = 84,\ \ P = 0$

(-80) to identify column 2 $\{x_2\}$ as the pivot column.

$$\begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 0 & 0 & 32 \\ 3 & 4 & 0 & 1 & 0 & 84 \\ \hline -50 & -80 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \frac{1}{2}R_1 \\ \\ \\ \end{array} \qquad \begin{array}{l} \frac{32}{2} = 16 \leftarrow \\ \frac{84}{4} = 21 \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 & 16 \\ 3 & 4 & 0 & 1 & 0 & 84 \\ \hline -50 & -80 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ R_2 - 4R_1 \\ R_3 + 80R_1 \end{array}$$

$$\begin{array}{cccccc} 3 & 4 & 0 & 1 & 0 & 84 \\ -2 & -4 & -2 & 0 & 0 & -64 \\ \hline 1 & 0 & -2 & 1 & 0 & 20 \end{array} \qquad \begin{array}{cccccc} -50 & -80 & 0 & 0 & 1 & 0 \\ 40 & 80 & 40 & 0 & 0 & 1280 \\ \hline -10 & 0 & 40 & 0 & 1 & 1280 \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} .5 & 1 & .5 & 0 & 0 & 16 \\ 1 & 0 & -2 & 1 & 0 & 20 \\ \hline -10 & 0 & 40 & 0 & 1 & 1280 \end{array}\right] \end{array} \begin{array}{l} R_1 - .5R_2 \\ \\ R_3 + 10R_2 \end{array}$$

$$\begin{array}{cccccc} -10 & 0 & 40 & 0 & 1 & 1280 \\ 10 & 0 & -20 & 10 & 0 & 200 \\ \hline 0 & 0 & 20 & 10 & 1 & 1480 \end{array} \qquad \begin{array}{cccccc} .5 & 1 & .5 & 0 & 0 & 16 \\ -.5 & 0 & 1 & -.5 & 0 & -10 \\ \hline 0 & 1 & 1.5 & -.5 & 0 & 6 \end{array}$$

$$\begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & 1.5 & -.5 & 0 & 6 \\ 1 & 0 & -2 & 1 & 0 & 20 \\ \hline 0 & 0 & 20 & 10 & 1 & 1480 \end{array}\right] \end{array}$$

$$\boxed{x_1 = 20, \quad x_2 = 6, \quad s_1 = 0, \quad s_2 = 0, \quad P = 1480}$$

Exercise

Solve the simplex method:

Maximize: $P = 2x_1 + 3x_2$

Subject to: $\begin{cases} -3x_1 + 4x_2 \le 12 \\ x_2 \le 2 \\ x_1, x_2 \ge 0 \end{cases}$

Solution

$$\begin{array}{c} \\ s_1 \\ x_2 \\ P \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} -3 & 4 & 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 1 & 0 & 2 \\ \hline -2 & -3 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} R_1 - 4R_2 \\ \\ R_3 + 3R_2 \end{array}$$

$$\begin{array}{c} \\ s_1 \\ x_2 \\ P \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} -3 & 0 & 1 & -4 & 0 & 4 \\ 0 & 1 & 0 & .5 & 0 & 1 \\ \hline -2 & 0 & 0 & 3 & 1 & 6 \end{array}\right] \end{array} \begin{array}{l} -\frac{1}{3}R_1 \\ \\ \\ \end{array}$$

$$\begin{array}{c} \\ x_1 \\ x_2 \\ P \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 0 & -\frac{1}{3} & \frac{4}{3} & 0 & -\frac{4}{3} \\ 0 & 1 & 0 & .5 & 0 & 1 \\ \hline -2 & 0 & 0 & 3 & 1 & 6 \end{array}\right] \end{array}$$

No Optimal solution

Exercise

Solve the simplex method:

Maximize: $P = 2x_1 + x_2$

Subject to: $\begin{cases} 5x_1 + x_2 \le 9 \\ x_1 + x_2 \le 5 \\ x_1, x_2 \ge 0 \end{cases}$

Solution

$$\begin{array}{ccl} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} & \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 5 & 1 & 1 & 0 & 0 & 9 \\ 1 & 1 & 0 & 1 & 0 & 5 \\ \hline -2 & -1 & 0 & 0 & 1 & 0 \end{array}\right] & \begin{array}{l} \frac{1}{5}R_1 \\ \\ \\ \end{array} \end{array}$$

$$\begin{array}{ccl} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} & \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{9}{5} \\ 1 & 1 & 0 & 1 & 0 & 5 \\ \hline -2 & -1 & 0 & 0 & 1 & 0 \end{array}\right] & \begin{array}{l} \\ R_2 - R_1 \\ R_3 + 2R_1 \end{array} \end{array}$$

$$\begin{array}{ccl} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} & \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{9}{5} \\ 0 & \frac{4}{5} & -\frac{1}{5} & 1 & 0 & \frac{16}{5} \\ \hline 0 & -\frac{3}{5} & \frac{2}{5} & 0 & 1 & \frac{18}{5} \end{array}\right] & \begin{array}{l} \\ \frac{5}{4}R_2 \\ \\ \end{array} \end{array}$$

$$\begin{array}{ccl} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} & \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{9}{5} \\ 0 & 1 & -\frac{1}{4} & \frac{5}{4} & 0 & 4 \\ \hline 0 & -\frac{3}{5} & \frac{2}{5} & 0 & 1 & \frac{18}{5} \end{array}\right] & \begin{array}{l} R_1 - \frac{1}{5}R_2 \\ \\ R_3 + \frac{3}{5}R_2 \end{array} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & -\frac{1}{4} & 0 & 1 \\ 0 & 1 & -\frac{1}{4} & \frac{5}{4} & 0 & 4 \\ \hline 0 & 0 & \frac{1}{4} & \frac{3}{4} & 1 & 6 \end{array}\right] \end{array}$$

Optimal Solution : $\boxed{Max\ \ P = 6,\ when\ x_1 = 1\ and\ x_2 = 4}$

Second Method

$$\begin{array}{ccl} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} & \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 5 \\ 5 & 1 & 1 & 0 & 0 & 9 \\ \hline -2 & -1 & 0 & 0 & 1 & 0 \end{array}\right] & \begin{array}{l} \\ R_2 - 5R_1 \\ R_2 + 2R_1 \end{array} \end{array}$$

$$\begin{array}{ccl} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} & \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 5 \\ 0 & -4 & 1 & -5 & 0 & -16 \\ \hline 0 & 1 & 0 & 2 & 1 & 10 \end{array}\right] & \begin{array}{l} \\ -\frac{1}{4}R_2 \\ \\ \end{array} \end{array}$$

$$\begin{array}{ccl} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} & \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 5 \\ 0 & 1 & -\frac{1}{4} & \frac{5}{4} & 0 & 4 \\ \hline 0 & 1 & 0 & 2 & 1 & 10 \end{array}\right] & \begin{array}{l} R_1 - R_2 \\ \\ R_3 - R_2 \end{array} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ P \end{array} & \left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & -\frac{1}{4} & 0 & 1 \\ 0 & 1 & -\frac{1}{4} & \frac{5}{4} & 0 & 4 \\ \hline 0 & 0 & \frac{1}{4} & \frac{3}{4} & 1 & 6 \end{array}\right] \end{array}$$

Exercise

The initial tableau of a linear programing is given. Use the simplex method to solve it

$$\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & z \end{array} \\
\left[\begin{array}{cccccc|c} 1 & 2 & 4 & 1 & 0 & 0 & 8 \\ 5 & 8 & 1 & 0 & 1 & 0 & 10 \\ \hline -3 & -24 & 1 & 0 & 0 & 1 & 0 \end{array}\right]
\end{array}$$

Solution

$$\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & z \end{array} \\
\left[\begin{array}{cccccc|c} 1 & 2 & 4 & 1 & 0 & 0 & 8 \\ 5 & 8 & 1 & 0 & 1 & 0 & 10 \\ \hline -3 & -24 & 1 & 0 & 0 & 1 & 0 \end{array}\right]
\end{array}
\begin{array}{ll} \frac{8}{2}=4 & \\ \frac{10}{8} & \frac{1}{8}R_2 \\ & \end{array}$$

$$\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & z \end{array} \\
\left[\begin{array}{cccccc|c} 1 & 2 & 4 & 1 & 0 & 0 & 8 \\ \frac{5}{8} & 1 & \frac{1}{8} & 0 & \frac{1}{8} & 0 & \frac{5}{4} \\ \hline -3 & -24 & 1 & 0 & 0 & 1 & 0 \end{array}\right]
\end{array}
\begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 24R_2 \end{array}$$

$$\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & z \end{array} \\
\left[\begin{array}{cccccc|c} -\frac{1}{4} & 0 & \frac{15}{4} & 1 & -\frac{1}{4} & 0 & \frac{11}{2} \\ \frac{5}{8} & 1 & \frac{1}{8} & 0 & \frac{1}{8} & 0 & \frac{5}{4} \\ \hline 12 & 0 & 4 & 0 & 3 & 1 & 30 \end{array}\right]
\end{array}$$

$$\textit{Optimal Solution}: \boxed{\textit{Max } z = 30, \textit{ when } x_2 = \frac{5}{4} \textit{ and } x_1,\ x_3 = 0}$$

Exercise

Carrie is working to raise money for the homeless by sending information letters and making follow-up calls to local labor organizations and church groups. She discovers that each church group requires 2 hours of letter writing and 1 hour of follow-up, while for each labor union she needs 2 hours of letter writing and 3 hours of follow-up. Carrie can raise $100 from each church group and $200 from each union local, and she has a maximum of 16 hours of letter-writing time and a maximum of 12 hours of follow-up time available per month. Determine the most profitable mixture of groups she should contact and the most money she can raise in a month.

Solution

	Church Group x_1	***Labor Union*** x_2	***Max. Time***
Letter Writing	2	2	16
Follow-up	1	3	12
$$$ raised	$100	$200	

$$\begin{aligned}
&\textit{Maximize}: && P = 100x_1 + 200x_2 \\
&\textit{subject to}: && 2x_1 + 2x_2 \le 16 \\
& && x_1 + 3x_2 \le 12 \\
&\quad \textit{with} && x_1, x_2 \ge 0
\end{aligned}$$

$$\begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array}
\begin{array}{c}
\begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\
\left[\begin{array}{ccccc|c}
2 & 2 & 1 & 0 & 0 & 16 \\
1 & 3 & 0 & 1 & 0 & 12 \\ \hline
-100 & -200 & 0 & 0 & 1 & 0
\end{array}\right]
\end{array}$$

$$\begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array}
\begin{array}{c}
\begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\
\left[\begin{array}{ccccc|c}
1 & 0 & \frac{3}{4} & -\frac{1}{2} & 0 & 6 \\
0 & 1 & -\frac{1}{4} & \frac{1}{2} & 0 & 2 \\ \hline
0 & 0 & 25 & 50 & 1 & 1000
\end{array}\right]
\end{array}$$

The maximum amount of money raised is $1,000/moth when $x_1 = 6$ and $x_2 = 2$

Exercise

The Texas Poker Company assembles three different poker sets. Each Royal Flush poker set contains 1000 poker chips, 4 decks of cards, 10 dice, and 2 dealer buttons. Each Deluxe Diamond poker set contains 600 poker chips, 2 decks of cards, 5 dice, and one dealer button. The full House poker set contains 300 poker chips, 2 decks of cards, 5 dice, and one dealer button. The Texas Poker Company has 2,800,000 poker chips, 10,000 decks of cards, 25,000 dice, and 6000 dealer buttons in stock. They earn a profit of \$38 for each Royal Flush poker set, \$22 for each Deluxe Diamond poker set, and \$12 for each Full House poker set.

a. How many of each type of poker set should they assemble to maximize profit? What is the maximum profit?

b. Find the values of any nonzero slack variables and describe what they tell you about any unused components.

Solution

$$\begin{aligned}
&\textit{Maximize}: && P = 38x_1 + 22x_2 + 12x_3 \\
&\textit{subject to}: && 100x_1 + 600x_2 + 300x_3 \le 2{,}800{,}000 \\
& && 4x_1 + 2x_2 + 2x_3 \le 10{,}000 \\
& && 10x_1 + 5x_2 + 5x_3 \le 25{,}000 \\
& && 2x_1 + x_2 + x_3 \le 10{,}000 \\
&\textit{with} && x_1, x_2, x_3 \ge 0
\end{aligned}$$

$$\begin{array}{c}
\begin{array}{ccccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & s_4 & z & \end{array} \\
\left[\begin{array}{cccccccc|c}
100 & 600 & 300 & 1 & 0 & 0 & 0 & 0 & 2{,}800{,}000 \\
4 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 10{,}000 \\
10 & 5 & 5 & 0 & 0 & 1 & 0 & 0 & 25{,}000 \\
2 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 10{,}000 \\
\hline
-38 & -22 & -12 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\end{array}$$

$$\begin{array}{c}
\begin{array}{ccccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & s_4 & z & \end{array} \\
\left[\begin{array}{cccccccc|c}
0 & 1 & -2 & .01 & 0 & -1 & 0 & 0 & 3{,}000 \\
0 & 0 & 0 & 0 & 1 & -.4 & 0 & 0 & 0 \\
1 & 0 & 1.5 & -.005 & 0 & .6 & 0 & 0 & 1{,}000 \\
0 & 0 & 0 & 0 & 0 & -.2 & 1 & 0 & 1{,}000 \\
\hline
0 & 0 & 1 & .03 & 0 & .8 & 0 & 1 & 104{,}000
\end{array}\right]
\end{array}$$

The maximum profit is \$104,000 and it is obtained when 1000 Royal Flush poker sets, 3000 Deluxe Diamond poker sets, and no Full House poker are assembled.

Exercise

The Muro Manufacturing Company makes two kinds of plasma screen TV sets. It produces the Flexscan set that sells for \$350 profit and the Panoramic I that sells for \$500 profit. On the assembly line, the Flexscan requires 5 hours, and the Panoramic I takes 7 hours. The cabinet shop spends 1 hour on the cabinet for the Flexscan and 2 hours on the cabinet for the Panoramic I. Both sets require 4 hours for testing and packing. On a particular production run, the Muro Company has available 3600 work-hours on the assembly line, 900 work-hours in the cabinet shop, and 2600 work-hours in the testing and packing department.

a. How many sets of each type should it produce to make a maximum profit? What is the maximum profit?

b. Find the values of any nonzero slack variables and describe what they tell you about unused time.

Solution

a) Let x_1 : number of Flexscan

x_2 : number of Panoramic

$$Maximize: z = 350x_1 + 500x_2$$

$$Subject\ to \begin{cases} 5x_1 + 7x_2 \le 3600 \\ x_1 + 2x_2 \le 900 \\ 4x_1 + 4x_2 \le 2600 \end{cases}$$

$$with \qquad x_1, x_2 \ge 0$$

$$\begin{array}{c} \begin{array}{cccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & z & \end{array} \\ \left[\begin{array}{cccccc|c} 5 & 7 & 1 & 0 & 0 & 0 & 3600 \\ 1 & \boxed{2} & 0 & 1 & 0 & 0 & 900 \\ 4 & 4 & 0 & 0 & 1 & 0 & 2600 \\ \hline -350 & -500 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{r} \\ -7R_2+2R_1 \to R_1 \\ \\ -2R_2+R_3 \to R_3 \\ 250R_2+R_4 \to R_4 \end{array} \begin{array}{c} \begin{array}{cccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & z & \end{array} \\ \left[\begin{array}{cccccc|c} \boxed{3} & 0 & 2 & -7 & 0 & 0 & 900 \\ 1 & 2 & 0 & 1 & 0 & 0 & 900 \\ 2 & 0 & 0 & -2 & 1 & 0 & 800 \\ \hline -100 & 0 & 0 & 250 & 0 & 1 & 225{,}000 \end{array}\right] \end{array}$$

Pivot on the 3 in row 1, column 1.

$$\begin{array}{r} \\ \\ -R_1+3R_2 \to R_2 \\ -2R_1+3R_3 \to R_3 \\ 100R_1+3R_4 \to R_4 \end{array} \begin{array}{c} \begin{array}{cccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & z & \end{array} \\ \left[\begin{array}{cccccc|c} 3 & 0 & 2 & -7 & 0 & 0 & 900 \\ 0 & 6 & -2 & 10 & 0 & 0 & 1800 \\ 0 & 0 & -4 & 8 & 3 & 0 & 600 \\ \hline 0 & 0 & 200 & 50 & 0 & 3 & 765{,}000 \end{array}\right] \end{array}$$

$$\begin{array}{l} \\ \frac{1}{3}R_1 \to R_1 \\ \frac{1}{6}R_2 \to R_2 \\ \\ \frac{1}{3}R_4 \to R_4 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & s_3 & z \end{array} \\ \left[\begin{array}{cccccc|c} 1 & 0 & \frac{2}{3} & -\frac{7}{3} & 0 & 0 & 300 \\ 0 & 1 & -\frac{2}{3} & \frac{5}{3} & 0 & 0 & 300 \\ 0 & 0 & -4 & 8 & 3 & 0 & 600 \\ \hline 0 & 0 & \frac{200}{3} & \frac{50}{3} & 0 & 1 & 255{,}000 \end{array}\right] \end{array}$$

The optimal solution is \$255,000 when 300 Flexscan and 300 Panoramic I sets are produced.

b) $3s_3 = 600 \Rightarrow s_3 = 200$ leftover hours in the testing and packing department.

Exercise

A baker has 150 units of flour, 90 of sugar, and 150 of raisins. A loaf of raisin bread requires 1 unit of flour, 1 of sugar, and 2 of raisins, while a raisin cake needs 5, 2, and 1 units, respectively.

a) If raisin bread sells for \$1.75 a loaf and raisin cake for \$4.00 each, how many of each should baked so that gross income is maximized?

b) What is the maximum gross income?

c) Does it require all of the available units of flour, sugar, and raisins to produce the number the maximum profit? If not, how much of each ingredient is left over? Compare any leftover to the value of the relevant slack variable.

Solution

a) x_1 : Number of loaves of raisin bread

x_2 : Number of loaves of raisin cake

$$\textit{Maximize}: \ z = 1.75x_1 + 4x_2$$

$$\textit{Subject to} \quad \begin{cases} x_1 + 5x_2 \le 150 \\ x_1 + 2x_2 \le 90 \\ 2x_1 + x_2 \le 150 \end{cases}$$

$$\textit{with} \qquad x_1, x_2 \ge 0$$

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & s_3 & z \end{array} \\ \left[\begin{array}{cccccc|c} 1 & 5 & 1 & 0 & 0 & 0 & 150 \\ 1 & 2 & 0 & 1 & 0 & 0 & 90 \\ 2 & 1 & 0 & 0 & 1 & 0 & 150 \\ \hline -1.75 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & s_3 & z \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 1 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 20 \\ 1 & 0 & -\frac{2}{3} & \frac{5}{3} & 0 & 0 & 50 \\ 0 & 0 & 1 & -3 & 1 & 0 & 30 \\ \hline 0 & 0 & \frac{1}{6} & \frac{19}{12} & 0 & 1 & 167.5 \end{array}\right] \end{array}$$

The optimal solution occurs when $x_1 = 50 \quad and \quad x_2 = 20$.

That is, when 50 loaves of raisin bread and 20 raisin cakes are baked.

b) The maximum gross income is $167.50

c) When $x_1 = 50 \quad and \quad x_2 = 20$

The total amount for each ingredient:

Flour: $50 + 5(20) = 150$

Sugar: $50 + 2(20) = 90$

Raisins: $2(50) + 20 = 120$

Exercise

A farmer owns a 100 acre farm and plans to plant at most three crops. The seed for crops A, B, and C costs $24, $40, and $30 per acre, respectively. A maximum of $3,600 can be spent on seed. Crops A, B, and C require 1, 2, and 2 workdays per acre, respectively, and three are a maximum of 160 workdays available. If the farmer can make a profit of $140 per acre on crop A, $200 per acre on crop B, and $160 per acre on crop C, how many acres of each crop that should be planted to maximize the profit?

Solution

Maximize $\quad P = 140x_1 + 200x_2 + 160x_3$

$$\text{Subject to } \begin{cases} x_1 + x_2 + x_3 \le 100 \\ 24x_1 + 40x_2 + 30x_3 \le 3600 \\ x_1 + 2x_2 + 2x_3 \le 160 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$$\begin{array}{cl} \begin{array}{ccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & P \end{array} & \\ \left[\begin{array}{ccccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 100 \\ 24 & 40 & 30 & 0 & 1 & 0 & 0 & 3,600 \\ 1 & 2 & 2 & 0 & 0 & 1 & 0 & 160 \\ \hline -140 & \langle -200 \rangle & -160 & 0 & 0 & 0 & 1 & 0 \end{array}\right] & \begin{array}{l} \\ \\ \frac{1}{2}R_3 \\ \\ \end{array} \end{array}$$

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & P & \\ \end{array} \\ \left[\begin{array}{ccccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 100 \\ 24 & 40 & 30 & 0 & 1 & 0 & 0 & 3,600 \\ .5 & 1 & 1 & 0 & 0 & .5 & 0 & 80 \\ \hline -140 & -200 & -160 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} R_1 - R_3 \\ \\ \\ \\ \end{array}$$

$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 100 \\ -.5 & -1 & -1 & 0 & 0 & -.5 & 0 & -80 \\ \hline .5 & 0 & 0 & 1 & 0 & -.5 & 0 & 20 \end{array}$$

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & P & \\ \end{array} \\ \left[\begin{array}{ccccccc|c} .5 & 0 & 0 & 1 & 0 & -.5 & 0 & 20 \\ 24 & 40 & 30 & 0 & 1 & 0 & 0 & 3,600 \\ .5 & 1 & 1 & 0 & 0 & .5 & 0 & 80 \\ \hline -140 & -200 & -160 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ R_2 - 40R_3 \\ \\ \\ \end{array}$$

$$\begin{array}{cccccccc} 24 & 40 & 30 & 0 & 1 & 0 & 0 & 3600 \\ -20 & -40 & -40 & 0 & 0 & -20 & 0 & -3200 \\ \hline 4 & 0 & -10 & 0 & 1 & -20 & 0 & 400 \end{array}$$

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & P & \\ \end{array} \\ \left[\begin{array}{ccccccc|c} .5 & 0 & 0 & 1 & 0 & -.5 & 0 & 20 \\ 4 & 0 & -10 & 0 & 1 & -20 & 0 & 400 \\ .5 & 1 & 1 & 0 & 0 & .5 & 0 & 80 \\ \hline -140 & -200 & -160 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \\ \\ R_4 + 200R_3 \end{array}$$

$$\begin{array}{cccccccc} -140 & -200 & -160 & 0 & 0 & 0 & 1 & 0 \\ 100 & 200 & 200 & 0 & 0 & 100 & 0 & 1600 \\ \hline -40 & 0 & 40 & 0 & 0 & 100 & 1 & 1600 \end{array}$$

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & P & \\ \end{array} \\ \left[\begin{array}{ccccccc|c} .5 & 0 & 0 & 1 & 0 & -.5 & 0 & 20 \\ 4 & 0 & -10 & 0 & 1 & -20 & 0 & 400 \\ .5 & 1 & 1 & 0 & 0 & .5 & 0 & 80 \\ \hline -40 & 0 & 40 & 0 & 0 & 100 & 1 & 16000 \end{array}\right] \end{array} \begin{array}{l} 2R_1 \\ \\ \\ \\ \end{array}$$

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & P & \\ \end{array} \\ \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 2 & 0 & -1 & 0 & 40 \\ 4 & 0 & -10 & 0 & 1 & -20 & 0 & 400 \\ .5 & 1 & 1 & 0 & 0 & .5 & 0 & 80 \\ \hline -40 & 0 & 40 & 0 & 0 & 100 & 1 & 16000 \end{array}\right] \end{array} \begin{array}{l} \\ R_2 - 4R_1 \\ R_3 - .5R_1 \\ R_4 + 40R_1 \end{array}$$

$$\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & P & \\ \end{array} \\ \left[\begin{array}{ccccccc|c} \cancel{1} & \cancel{0} & \cancel{0} & \cancel{2} & \cancel{0} & \cancel{-1} & \cancel{0} & \cancel{40} \\ 0 & 0 & -10 & -8 & 1 & -16 & 0 & 240 \\ 0 & 1 & 1 & -1 & 0 & 1 & 0 & 60 \\ \hline 0 & 0 & 40 & 80 & 0 & 60 & 1 & 14400 \end{array}\right] \end{array} \begin{array}{l} x_1 = 40 \\ \\ x_2 = 60 \\ \\ \end{array}$$

$\therefore$ 40 acres of crop ***A***, 60 acres of crop ***B***, *no* crop ***C***

$$P = 140x_1 + 200x_2 + 160x_3$$
$$= 140(40) + 200(60) + 160(0)$$
$$= \$17,600$$

Exercise

A candy company makes three types of candy, solid, fruit, and cream filled, and packages these candies in three different assortments. A box of assortment I contains 4 solid, 4 fruit, and 12 cream and sells for \$9.40. A box of assortment II contains 12 solid, 4 fruit, and 4 cream and sells for \$7.60. A box of assortment III contains 8 solid, 8 fruit, and 8 cream and sells for \$11.00. The manufacturing costs per piece of candy are \$0.20 for solid, \$0.25 for fruit, and \$0.30 for cream. The company can manufacture up to 4800 solid, 4000 fruit, and 5600 cream candies weekly. How many boxes of each type should the company produce in order to maximize profit? What is their maximum profit?

Solution

This one is a bit more complicated simply because the profit per box is not given directly. To determine the profit per box, you must use the equation, Profit= Revenue – Cost.

	Assortment I {x_1}	Assortment II {x_2}	Assortment III {x_3}
# Solid Candies @ cost	4 @ 0.20=0.80	12 @ 0.20=2.40	8 @ 0.20=1.60
# Fruit Candies @cost	4 @ 0.25=1.00	4 @ 0.25=1.00	8 @ 0.25=2.00
# Cream Candies @cost	12 @ 0.30=3.60	4 @ 0.30=1.20	8 @ 0.30=2.40
Total Cost Per Box	\$5.40	\$4.60	\$6.00
Total Revenue Per Box	\$9.40	\$7.60	\$11.00
Total Profit Per Box {R-C}	9.40-5.40=\$4.00	7.60-4.60=\$3.00	11.00-6.00=\$5.00

$$Maximize: P = 4x_1 + 3x_2 + 5x_3$$

$$st: \begin{cases} 4x_1 + 12x_2 + 8x_3 \le 4800 & solid \\ 4x_1 + 4x_2 + 8x_3 \le 4000 & fruit \\ 12x_1 + 4x_2 + 8x_3 \le 5600 & cream \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

Initial Tableau:

$$\left[\begin{array}{ccccccc|c} 4 & 12 & 8 & 1 & 0 & 0 & 0 & 4800 \\ 4 & 4 & 8 & 0 & 1 & 0 & 0 & 4000 \\ 12 & 4 & 8 & 0 & 0 & 1 & 0 & 5600 \\ \hline -4 & -3 & -5 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Use the simplex method to solve:

$x_1 = 200 \quad x_2 = 100 \quad x_3 = 350 \quad P = 2850 \quad s_1, s_2, s_3 = 0$

Exercise

A small company manufactures three different electronic components for computers. Component A requires 2 hours of fabrication and 1 hour of assembly; component B requires 3 hours of fabrication and 1 hour of assembly; and component C requires 2 hours of fabrication and 2 hours of assembly. The company has up to 1,000 labor-hours of fabrication time and 800 labor-hours of assembly time available per week. The profit on each component, A, B, and C is $7, $8, and $10, respectively. How many components of each week in order to maximize its profit (assuming that all components that it manufactures can be sold)? What is the maximum profit?

Solution

Let x_1 : number of A components

x_2 : number of B components

x_3 : number of C components

$$\textit{Maximize}: \ P = 7x_1 + 8x_2 + 10x_3$$

$$\textit{subject to} \ \begin{cases} 2x_1 + 3x_2 + 2x_3 \le 1000 \\ x_1 + x_2 + 2x_3 \le 800 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{cccccc|c} 2 & 3 & 2 & 1 & 0 & 0 & 1000 \\ 1 & 1 & 2 & 0 & 1 & 0 & 800 \\ \hline -7 & -8 & -10 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{cccccc|c} 1 & 2 & 0 & 1 & -1 & 0 & 200 \\ 0 & -.5 & 1 & -.5 & 1 & 0 & 300 \\ \hline 0 & 1 & 0 & 2 & 3 & 1 & 4400 \end{array}\right]$$

The optimal solution: The maximum profit is $4400 when 200 ***A*** components and 0 ***B*** components and 300 ***C*** components are manufactured.

Exercise

An investor has at most \$100,000 to invest in government bonds, mutual funds, and money market funds. The average yields for government bonds, mutual funds, and money market funds are 8%, 13%,, and 15%, respectively. The investor's policy requires that the total amount invested in mutual and money market funds not exceed the amount invested in government bonds. How much should be invested in each type of investment in order to maximize the return? What is the maximum return?

Solution

Let x_1 : government bonds

x_2 : mutual funds

x_3 : money market funds

$$\textit{Maximize}: \ P = .08x_1 + .13x_2 + .150x_3$$

$$\textit{subject to} \ \begin{cases} x_1 + x_2 + x_3 \le 100,000 \\ x_2 + x_3 \le x_1 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 100,000 \\ -1 & 1 & 1 & 0 & 1 & 0 & 0 \\ \hline -.08 & -.13 & -.15 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ \end{array}$$
$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & .5 & -.5 & 0 & 50,000 \\ 0 & 1 & 1 & .5 & .5 & 0 & 50,000 \\ \hline 0 & .02 & 0 & .115 & .035 & 1 & 11,500 \end{array}\right]$$

The optimal solution: The maximum return is \$11,500 when $x_1 = \$50,000$ is invested in government bonds, $x_2 = \$0$ is invested in mutual bonds, $x_3 = \$50,000$ is invested in money market funds.

Exercise

A department store chain up to $20,000 to spend on television advertising for a sale. All ads will be placed with one television station, where 30-second as costs $1,000 on daytime TV and is viewed by 14,000 potential customers, $2000 on prime-time TV and is viewed by 24,000 potential customer, and $1,500 on late-night TV and is viewed by 18,000 potential customers. The television station will not accept a total of more than 15 ads in all three time periods. How many ads should be placed in each time period in order to maximize the number of potential customers who will see the ads? How many potential customers will see the ads?

Solution

Let x_1 : daytime ads

x_2 : prime-time ads

x_3 : late-night ads

$$\textit{Maximize}: \ P = 14,000x_1 + 24,000x_2 + 18,000x_3$$

$$\textit{subject to} \ \begin{cases} 1000x_1 + 2000x_2 + 1500x_3 \le 20,000 \\ x_1 + x_2 + x_3 \le 15 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ \hline 1000 & 2000 & 1500 & 1 & 0 & 0 & 20,000 \\ 1 & 1 & 1 & 0 & 1 & 0 & 15 \\ \hline -14,000 & -24,000 & -18,000 & 0 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ \hline 0 & 1 & .5 & .001 & -1 & 0 & 5 \\ 1 & 0 & .5 & 0 & 2 & 0 & 10 \\ \hline 0 & 0 & 1000 & 10 & 4000 & 1 & 260,000 \end{array}$$

Optimal Solution: maximum number of potential customers is 260,000 when $x_1 = 10$ daytime ads, $x_2 = 5$ prime-time ads, and $x_3 = 0$ late-night ads are placed.

Exercise

A political scientist has received a grant to find a research project involving voting trends. The budget of the grant includes \$3,200 for conducting door-to-door interviews the day before an election. Undergraduate students, graduate students, and faculty members will be hired to conduct the interviews. Each undergraduate student will conduct 18 interviews and be paid \$100. Each graduate student will conduct 25 interviews and be paid \$150. Each faculty members will conduct 30 interviews and be paid \$200. Due to limited transportation facilities, no more than 20 interviews can be hired. How many undergraduate students, graduate students, and faculty members should be hired in order to maximize the number of interviews that will be conducted? What is the maximum number of interviews?

Solution

Let x_1 : government bonds

x_2 : mutual funds

x_3 : money market funds

$$\textit{Maximize}: \ P = 18x_1 + 25x_2 + 30x_3$$

$$\textit{subject to} \ \begin{cases} x_1 + x_2 + x_3 \le 20 \\ 100x_1 + 150x_2 + 200x_3 \le 3200 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & P \end{array} \\ \left[\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 20 \\ 100 & 150 & 200 & 0 & 1 & 0 & 3200 \\ \hline -18 & -25 & -30 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & P \end{array} \\ \left[\begin{array}{cccccc|c} 2 & 1 & 0 & 4 & -1 & 0 & 16 \\ -1 & 0 & 1 & -3 & 1 & 0 & 4 \\ \hline 2 & 0 & 0 & 10 & 5 & 1 & 520 \end{array}\right] \end{array}$$

Optimal Solution: maximum number of interviews is 520 when $x_1 = 0$ undergraduates, $x_2 = 16$ graduate students, and $x_3 = 4$ faculty members.