2.6 Applications of Congruences

Hashing fctn $h(k)$

$h(k) = k \bmod m$

$h(064212848)$

$064212848 \bmod 111 = 14$

if location is exist $\Rightarrow +1 \rightarrow 15$

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pseudorandom: $\boxed{X_{n+1} = (a x_n + c) \bmod m}$

Ex

$m = 9$, $a = 7$, inc: $c = 4$, $x_0 = 3$

$\boxed{X_{n+1} = (7 X_n + 4) \bmod 9}$

$x_1 = (7x_0 + 4) \bmod 9 = 25 \bmod 9 = 7$

$x_2 = (7(7) + 4) \bmod 9 = 53 \bmod 9 = 8$

$x_3 = 60 \bmod 9 = 6$

$x_4 = 46 \bmod 9 = 1$

$x_5 = 11 \bmod 9 = 2$

$x_6 = 18 \bmod 9 = 0$

$x_7 = 4 \bmod 9 = 4$

$x_8 = 32 \bmod 9 = 5$

$x_9 = 39 \bmod 9 = 3$

3 7 8 6 1 2 0 4 5 3 7 8 6 - - -

y

12 - decimal

$$3x_1 + x_2 + 3x_3 + x_4 + 3x_5 + x_6 + 3x_7 + x_8 + 3x_9 + x_{10} + 3x_{11} + x_{12} \equiv 0 \pmod{10}$$

a) 7 9 3 5 7 3 3 4 3 1 0 4

$$3(7) + 9 + 3(3) + 5 + 3(7) + 3 + 3(3) + 4 + 3(3) + 1 + 3(0) + 4 \stackrel{?}{\equiv} 0 \bmod 10$$

$$94 \not\equiv 0 \bmod 10$$

2×3

b) 7 9 3 5 7 3 4 3 1 0 4

$$3(7) + 9 + 3(3) + 5 + 3(7) + 3 + 3(4) + 3 + 3(1) + 0 + 3(4) + x_{12} \equiv 0 \bmod 10$$

$$98 + x_{12} \equiv 0 \bmod 10$$

$$\begin{array}{r} 65 \\ 18 \\ \hline 84 \end{array}$$

$$98 + x_{12} = 100$$

$$x_{12} = 2$$

Exam 2

2.1 $1 \to 3$, $6 \to 7$, $8 \to 10$

2.3 1, show/ $2 \to 4$ 5, 8, 9 12

2) $(\quad)_2 = ?$

$(\quad)_8 \to (\quad)_2$

$(\quad)_{16} \to (\quad)_2$

prime nb

gcd

$\{a_n\}$ $a_n = 2(-3)^n + 5^n$

$a_1 = 2(-3) + 5$

$= \boxed{-1}$

$a_n = a_{n-1} + a_{n-3}$ $a_0 = 1$, $a_1 = 2$, $a_2 = 0$

$a_3 = a_2 + a_0$

$= 0 + 1$

$= \boxed{1}$

$a_4 = a_3 + a_1$

$= 1 + 2$

$= \boxed{3}$

17 divides 68

$68 = 17(4)$ yes

-111 divided 11

$-111 = -11(11) + 10$ $\quad q = -11 \quad r = 10$

$-17 \bmod 2 = 1$

$-17 = 2(-9) + 1$

$a \equiv 43 \pmod{23}$ $\quad -22 \le a \le 0$

$43 = 23(2) - 3$

$a = 43 - 23(2) = -3$

$(11\ 1011\ 1110)_2 = 0 \cdot 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \cdot 2^4 + 1 \cdot 2^5 + 0 \cdot 2^6 + 1 \cdot 2^7 + 1 \times 2^8 + 1 \times 2^9$

$= 2 + 4 + 8 + 16 + 32 + 128 + 256 + 512$

$= \boxed{958}$

$(572)_8 = (101\ 111\ 010)_2$

	1				
0	0	0	0	0	8
1	0	0	0	1	9
2	0	0	1	0	A
3	0	0	1	1	B
4	0	1	0	0	C
5	0	1	0	1	D
6	0	0	1	1	E
7		1	1	1	F

$(135AB)_{16} = (0001\ 0011\ 0101\ 1010\ 1011)_2$

0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1			
A				
B				
C				
D				
E				
F	1	1	1	1

A 1 | 0 0 | 0 0 0

1 1 2 1 1

$< 30 = 5 \times 3 \times 2 , 1$

gcd(1000, 625)

$1000 = 2^3 \cdot 5^3$ $\quad (10)^3 = (2 \cdot 5)^3$

$625 = 5^7$

$\gcd(1000, 625) = 5^3 = 125$