

3.1

Maxima & Minima
(Maximum) Min
MAX

extreme points

Crit point \Rightarrow Max. pt. or Min. pt.
more than 1 \Rightarrow $\begin{cases} RMAX & RMIN \\ \text{or} \\ LMAX & LMIN \end{cases}$

Absolute extreme

abs. Max, abs Min

[a, b]

Critical number. CN
 x, t

Critical point CP
 (x, y)

How to find CN by forcing 1st derivative to be
equal 0.
solve for the variable

Ex $f(x) = x^2$ $[-2, 1]$

$$f'(x) = 2x = 0 \Rightarrow \text{CN: } x = 0$$

x	$f(x)$	
-2	4	\rightarrow abs. Max: $(-2, 4)$
0	0	\rightarrow abs. Min: $(0, 0)$
1	1	

Ex $g(t) = 8t - t^4$ $[-2, 1]$

$$g'(t) = 8 - 4t^3 = 0$$

$$t^3 = 2 \Rightarrow \underline{t = \sqrt[3]{2} \text{ CN}}$$

t	g(t)
-2	-32
1	7

→ abs Min @ (-2, -32)
→ abs Max @ (1, 7)

Ex $f(x) = x^{2/3}$ $[-2, 3]$

$$f'(x) = \frac{2}{3} x^{-1/3}$$

$$= \frac{2}{3} \frac{1}{x^{1/3}} \neq 0$$

CN: x = 0

x	f(x)
-2	$\sqrt[3]{4}$
0	0
3	$\sqrt[3]{9}$

→ abs. Min (0, 0)
→ abs. Max (3, $\sqrt[3]{9}$)

$f(\theta) = \sin \theta$ $-\frac{\pi}{2} \leq \theta \leq \frac{5\pi}{6}$

$$f'(\theta) = \cos \theta = 0$$

CN: $\theta = \pm \frac{\pi}{2}$

θ	f(θ)
$-\frac{\pi}{2}$	-1
$\frac{\pi}{2}$	1
$\frac{5\pi}{6}$	$\frac{1}{2}$

→ abs. Min @ $(-\frac{\pi}{2}, -1)$
→ abs Max @ $(\frac{\pi}{2}, 1)$