Solution *Section* 3.1 – Sets

Exercise

Let $U=\{1,2,3,4,5,6,7,8,9\}$, $X=\{2,4,6,8\}$, $Y=\{2,3,4,5,6\}$, and $Z=\{1,2,3,8,9\}$

Solution

a) $X\cap Y=\{2,4,6\}$

b) $X\cup Y=\{2,3,4,5,6,8\}$

c) $Y'=\{1,7,8,9\}$

d) $X'\cap Z=\{1,3,9\}$

e) $Y\cap(X\cup Z)=\{2,3,4,6\}$

f) $X'\cap(Y'\cup Z)=\{1,3,7,9\}$

g) $(X\cap Y')\cup Z'=\{1,8\}$

Exercise

Given $A = \{0, 2, 4, 6\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{2, 6, 0, 4\}$, determine if the statement is true or false?

Solution

a) $A \subset B$ ***True***
b) $A \subset C$ ***True***
c) $A = C$ ***True***
d) $C \subset B$ ***True***
e) $B \not\subset A$ ***True***
f) $\varnothing \subset B$ ***True***

Exercise

Given $R = \{1, 2, 3, 4\}$, $S = \{1, 3, 5, 7\}$, $T = \{2, 4\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, find the following:

Solution

a) $R\cup S$ $\{1, 2, 3, 4, 5, 7\}$

b) $R\cap S$ $\{1, 3\}$

c) $S\cap T$ $\varnothing$

d) S' $\{2, 4, 6, 8, 9\}$

Exercise

Write true or false for each statement

a) $3 \in \{2,\ 5,\ 7,\ 9,\ 10\}$

b) $6 \in \{-2,\ 5,\ 6,\ 9\}$

c) $9 \notin \{2,\ 1,\ 5,\ 8\}$

d) $3 \notin \{7,\ 6,\ 5,\ 4,\ 10\}$

e) $\{2,\ 5,\ 8,\ 9\} = \{2,\ 5,\ 9,\ 8\}$

f) $\{3,\ 7,\ 12,\ 14\} = \{3,\ 7,\ 12,\ 14,\ 9\}$

Solution

a) ***False***, since the number 3 is not an element of the set

b) ***True***, since the number 6 is an element of the set

c) ***True***, since the number 9 is not an element of the set

d) ***True***, since the number 3 is not an element of the set

e) ***True***, since the set contain exactly the same elements

f) ***False***, since 9 is an element of the second set but not the first

Solution   *Section* 3.2 – Applications of Venn Diagrams

Exercise

Use the union rule to answer the following

a) If $n(A)=5;\quad n(B)=12,\quad \textit{and}\quad n(A\cap B)=4$ what is $n(A\cup B)$?

b) If $n(A)=15;\quad n(B)=30,\quad \textit{and}\quad n(A\cup B)=33$ what is $n(A\cap B)$?

c) $n(B)=9;\quad n(A\cap B)=5,\quad \textit{and}\quad n(A\cup B)=22$ what is $n(A)$?

Solution

a) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$=5+12-4$

$\underline{=13}$

b) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$33=15+30-n(A\cap B)$

$33=45-n(A\cap B)$

$n(A\cap B)=45-33\ \underline{=12}$

c) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$22=n(A)+9-5$

$22=n(A)+4$

$n(A)=22-4\ \underline{=18}$

Exercise

Draw a Venn diagram and use the given information to fill in the number of elements

a) $n(U)=41;\quad n(A)=16,\quad n(A\cap B)=12,\quad n(B')=20$

b) $n(A)=28;\quad n(B)=12,\quad n(A\cup B)=32,\quad n(A')=19$

c) $n(A)=11;\quad n(A\cap B)=6,\quad n(A\cup B)=24,\quad n(A'\cup B')=25$

d) $n(A)=28,\quad n(B)=34,\quad n(C)=25,\quad n(A\cap B)=14,\quad n(B\cap C)=15$
$n(A\cap C)=11;\quad n(A\cap B\cap C)=9,\quad n(U)=59$

e) $n(A)=54,\quad n(B')=63,\quad n(C)=44,\quad n(A\cap B)=22,\quad n(B\cap C)=16,$
$n(A\cap C)=15;\quad n(A\cap B\cap C)=4,\quad n(A\cup B)=85$

f) $n(A\cap C')=11,\quad n(B\cap C')=8,\quad n(C)=15,\quad n(A\cap B)=6,\quad n(B\cap C)=4$
$n(A\cap C)=7;\quad n(A\cap B\cap C)=4,\quad n(A'\cap B'\cap C')=5$

Solution

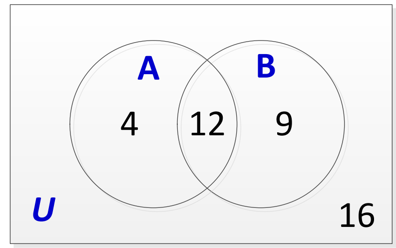
a) $n(U)=41;\quad n(A)=16,\quad n(A\cap B)=12,\quad n(B')=20$

Start with $n(A\cap B)=12$

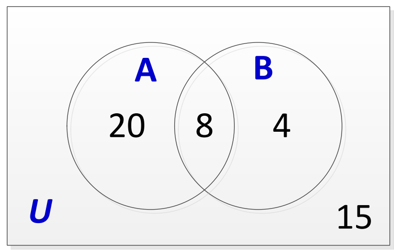
Since $n(A)=16$ and 12 in $A\cap B$, there must be $(16-12=4)$ 4 elements in *A* but not in $A\cap B$.

$n(B')=20$, so there are 20 not in *B*, 4 already in *A* which leave us with $20-4=16$.

$n(U)=41 \rightarrow 41-4-12-16=9$ in *B* but not in $A\cap B$



b) $n(A)=28;\quad n(B)=12,\quad n(A\cup B)=32,\quad n(A')=19$



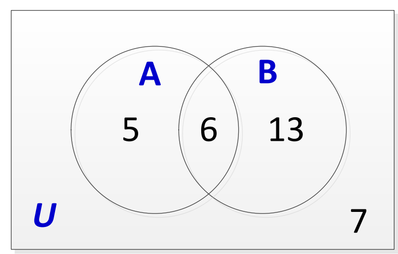
c) $n(A)=11;\quad n(A\cap B)=6,\quad n(A\cup B)=24,\quad n(A'\cup B')=25$

Start with $n(A\cap B)=6$

Since $n(A)=11$ and 6 in $A\cap B$, there must be $11-6=5$ elements in *A* but not in $A\cap B$.

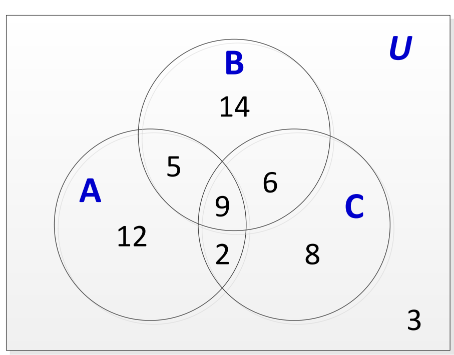
$n(A\cup B)=24$, we already have 11 so $24-11=13$ in *B* but not in $A\cap B$

$A'\cup B'=U-(A\cap B)\rightarrow 25-5-13=7$

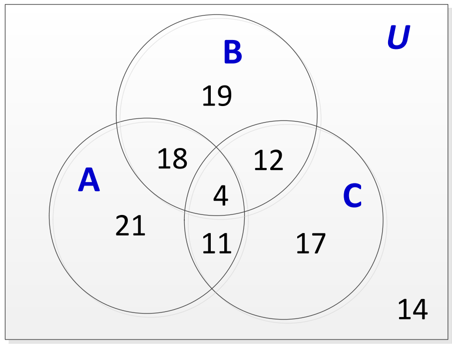


d) $n(A)=28,\quad n(B)=34,\quad n(C)=25,\quad n(A\cap B)=14,\quad n(B\cap C)=15$

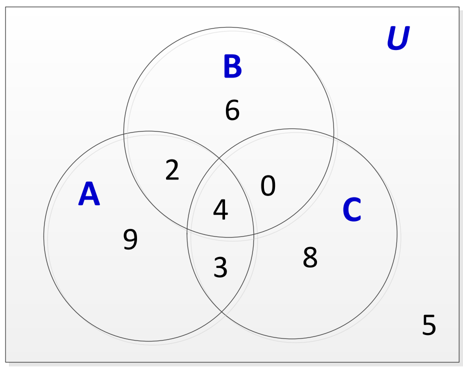
$n(A\cap C)=11;\quad n(A\cap B\cap C)=9,\quad n(U)=59$



e) $n(A)=54,\quad n(B')=63,\quad n(C)=44,\quad n(A\cap B)=22,\quad n(B\cap C)=16,$
$n(A\cap C)=15;\quad n(A\cap B\cap C)=4,\quad n(A\cup B)=85$



f) $n(A\cap C')=11,\quad n(B\cap C')=8,\quad n(C)=15,\quad n(A\cap B)=6,\quad n(B\cap C)=4$
$n(A\cap C)=7;\quad n(A\cap B\cap C)=4,\quad n(A'\cap B'\cap C')=5$



Exercise

Toward the middle of the harvesting season peaches for canning come in three types, early, late, and extra late, depending on the expected date of ripening. During a certain week, the following data were recorded at a fruit delivery station:

34 trucks went out carrying early peaches;
61 carried late peaches;
50 carried extra late;
25 carried early and late;
30 carried late and extra late;
8 carried early and extra late;
6 carried all three
9 carried only figs (no peaches at all).

a) How many trucks carried only variety peaches?
b) How many carried only extra late?
c) How many carried only one type of peach?
d) How many trucks (in all) went during the week?

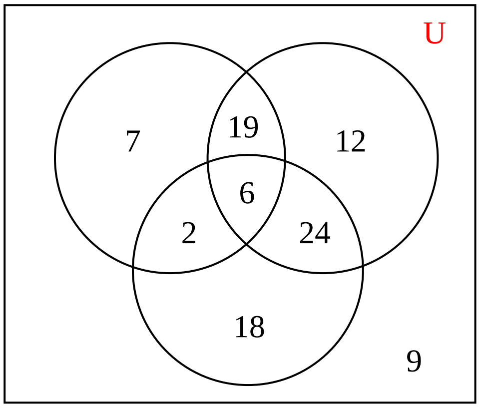
Solution

a) 12

b) 18

c) $7+12+18=37$

d) $7+2+6+19+12+24+18+9=97$



Exercise

In a survey of 100 randomly chosen students, a marketing questionnaire included the following:

- ✓ 75 own a TV
- ✓ 45 own a car
- ✓ 35 own a TV and a car

a) How many students owned a car but not a TV set?
b) How many students did not own both a car and a TV set?

Solution

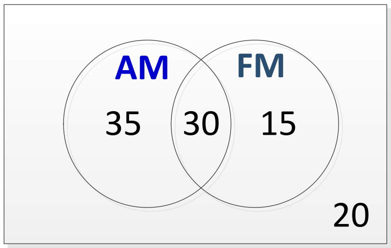
a) 10

b) 65

Exercise

A small town has two radio stations, an AM station and an FM station. A survey of 100 residents of the town produced the following results: In the last 30 days, 65 people have listened to the AM station, 45 have listened to the FM station, and 30 have listened to both stations.

Solution



Total 100, $AM = 65$, $FM = 45$, $AM \cap FM = 30$

Exercise

In a class of 35 students, 19 are married and 20 are blondes. Given that there are 7 students that are both married and blonde, answer the following questions.

a) How many are married, but not blonde?

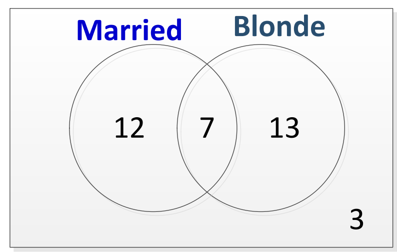
b) How many are blonde but not married?

c) How many are blonde or married?

d) How many are neither blonde nor married?

e) How many are not blonde?

Solution



a) 12

b) 13

c) 12 + 7 + 13 = 32

d) 35 – 32 = 3

e) 12 + 3 = 15

Exercise

In a survey of 500 businesses it was found that 250 had copiers and 300 had fax machines. It was also determined that 100 businesses had both copiers and fax machines.

a) How many had either a copier or a fax machine?

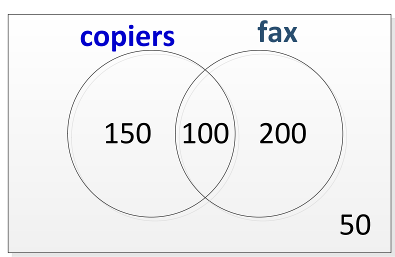
b) How many had neither a copier nor a fax machine?

c) How many had a copier, but no fax machine?

d) How many had a fax machine, but no copier?

e) How many had no fax machines?

Solution



a) 450

b) 50

c) 150

d) 200

e) 200

Exercise

Given: $n(U)=105, \quad n(A)=50, \quad n(B)=75, \quad n(A\cup B)=105$, find the following:

Solution

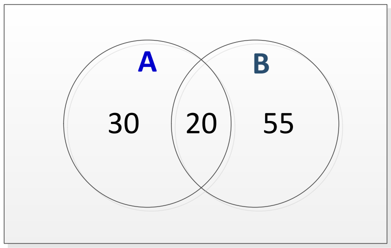
a) $n(A\cap B)=20$

b) $n(A'\cap B)=55$

c) $n(A'\cap B')=5$

d) $n(A\cup B')=55$

e) $n(B')=35$



Exercise

Fred interviewed 140 people in a shopping center to discover some of their cooking habits. He obtained the following results:

58 use microwave ovens
63 use electric ranges
58 use gas ranges
19 use microwave ovens and electric ranges
17 use microwave ovens and gas ranges
4 use both gas and electric ranges
1 uses all three
2 use none of the three

Should he be reassigned one more time? Why or why not?

Solution

Let M: use microwave ovens

E: use electric ranges

G: use gas ranges

$n(U)=140$

2 use none of the three $\Rightarrow n(M'\cap E'\cap G')=2$

1 uses all three $\Rightarrow n(M\cap E\cap G)=1$

4 use both gas and electric ranges $=n(E\cap G)$

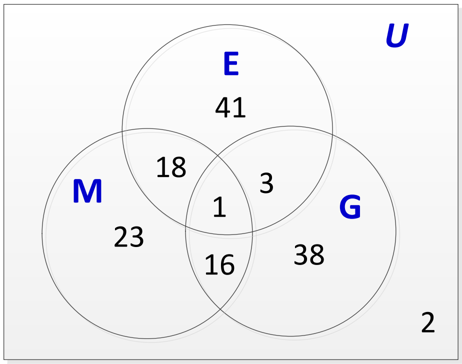
17 use microwave ovens and gas ranges $\Rightarrow n(M\cap G)=17$

19 use microwave ovens and electric ranges $\Rightarrow n(M\cap E)=19$

58 use gas ranges $\Rightarrow n(G)=58$

63 use electric ranges $\Rightarrow n(E)=63$

58 use microwave ovens $\Rightarrow n(M)=58$



Exercise

Toward the middle of the harvesting season, peaches for canning come in three types, early, late, and extra late, depending on the expected date of ripening. During a certain week, the following data were recorded at a fruit delivery station:

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50 carried extra late
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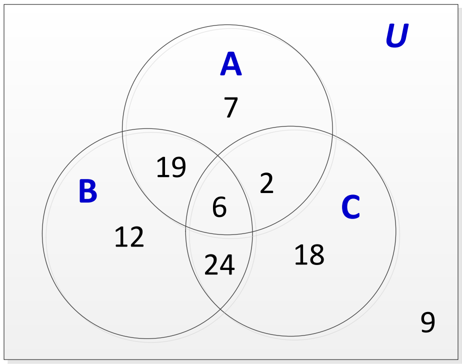
30 carried late and extra late
8 carried early and extra late
6 carried all three
9 carried only figs (no peaches at all)

a) How many trucks carried only late variety peaches?
b) How many carried only extra late?
c) How many carried only one type of peach?
d) How many trucks (in all) went out during the week?

Solution



Let A: use early peaches
B: use late peaches
C: use extra late peaches

9 carried only figs $\Rightarrow n(A' \cap B' \cap C') = 9$

6 carried all three $\Rightarrow n(A \cap B \cap C) = 6$

8 carried early and extra late $\Rightarrow n(A \cap C) = 8$

30 carried late and extra late $\Rightarrow n(B \cap C) = 30$

25 carried early and late $\Rightarrow n(A \cap B) = 25$

50 carried extra late $\Rightarrow n(C) = 50$

61 carried late peaches $\Rightarrow n(B) = 61$

34 trucks went out carrying early peaches $\Rightarrow n(A) = 34$

Exercise

Most mathematics professors love to invest their hard earned money. A recent survey of 150 math professors revealed that

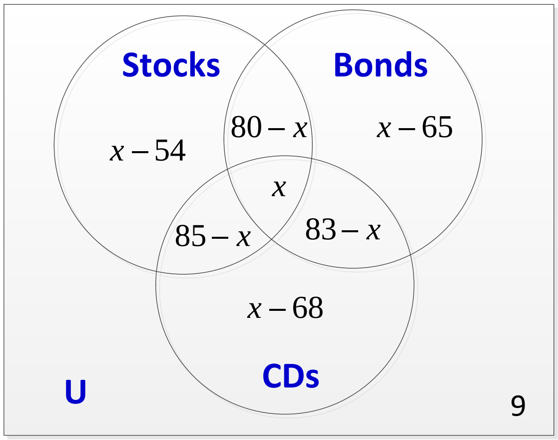
111 invested in stocks
98 invested in bonds
100 invested in certificates of deposit
80 invested in stocks and bonds
83 invested in bonds and certificates of deposit
85 invested in stocks and certificates of deposit
9 did not invest in any of three

How many mathematics professors invested in stocks and bonds and certificates of deposit?

Solution

Let A: Stocks
B: Bonds
C: CDs

$n(A \cap B \cap C) = x$



9 did not invest in any of three

$\Rightarrow n(A' \cap B' \cap C') = 9$

85 invested in stocks and CDs

$\Rightarrow n(A \cap C) = 85$

$85-x$ invested in only stocks and CDs

83 invested in bonds and CDs

$\Rightarrow n(B \cap C) = 83$

$83-x$ invested in only bonds and CDs

80 invested in stocks and bonds $\Rightarrow n(A \cap B) = 80$

$80-x$ invested in only stocks and bonds

100 invested in CDs $\Rightarrow n(C) = 100$

$100-\left[(85-x)+(83-x)+x\right] = 100-85+x-83+x-x = x-68$ invested in only CDs

98 invested in bonds $\Rightarrow n(B) = 98$

$98-\left[(80-x)+(83-x)+x\right] = 111-80+x-83+x-x = x-65$ invested in only bonds

111 invested in stocks $\Rightarrow n(A) = 111$

$111-\left[(80-x)+(85-x)+x\right] = 111-80+x-85+x-x = x-54$ invested in only stocks

$n(U) = 150$

$150 = x-54+80-x+x-65+85-x+83-x+x-68+x+9$

$$150 = 70 + x$$

$$\boxed{x = 80}$$

80 professors invested in stocks, bonds and CDs

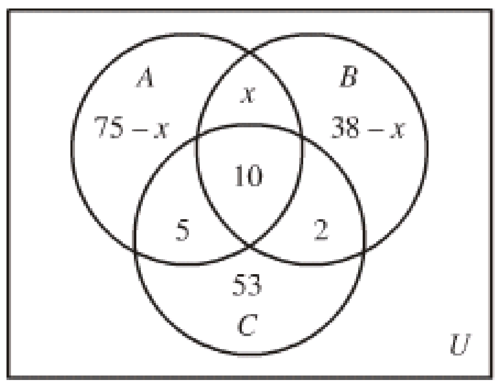
Exercise

Suppose that a group of 150 students have joined at least one of three chat rooms; one on auto-racing, one on bicycling, and one for college students. For simplicity, we will call these rooms *A*, *B*, and *C*. In addition,

90 students joined room *A*;
50 students joined room *B*;
70 students joined room *C*;
15 students joined room *A* and *C*;
12 students joined room *B* and *C*;
10 students joined all three rooms;

Determine how many students joined both chat rooms *A* and *B*.

Solution



$$(75 - x) + 5 + x + 10 + (38 - x) + 2 + 53 = 150$$

$$75 - x + 5 + x + 10 + 38 - x + 2 + 53 = 150$$

$$183 - x = 150$$

$$183 - 150 = x$$

$$\boxed{x = 33}$$

Solution   *Section* 3.3 – Counting; Multiplication Principle

Exercise

How many different types of homes are available if a builder offers a choice of 6 basic plans, 3 roof styles, and 2 exterior finishes?

Solution

$$6.3.2 = 36 \text{ different homes types}$$

Exercise

A menu offers a choice of 3 salads, 8 main dishes, and 7 desserts. How many different meals consisting of one salad, one main dish, and one dessert are possible?

Solution

$$3.8.7 = 168 \text{ different meals}$$

Exercise

A couple has narrowed down the choice of a name for their new baby to 4 first names and 5 middle names. How many different first- and middle-name arrangements are possible?

Solution

$$4.5 = 20 \text{ possible arrangements}$$

Exercise

An automobile manufacturer produces 8 models, each available in 7 different exterior colors, with 4 different upholstery fabrics and 5 interior colors. How many varieties of automobile are available?

Solution

$$8.7.4.5 = \underline{1120}$$

Exercise

A biologist is attempting to classify 52,000 species of insects by assigning 3 initials to each species. Is it possible to classify all the species in this way? If not, how many initials should be used?

Solution

$26^3 = 17,576$ *This would not be enough.*

$26^4 = 456,976$ *Which is more than enough*

Exercise

How many 4-letter code words are possible using the first 10 letters of the alphabet under:

a) No letter can be repeated

b) Letters can be repeated

c) Adjacent can't be alike

Solution

a) 10.9.8.7 = 5040

b) 10.10.10.10 = 10,000

c) 10.9.9.9 = 7290

Exercise

3 letters license plate without repeats:

Solution

26.25.24 = 15600 possible

Exercise

How many ways can 2 coins turn up heads, H, or tails, T – if the combined outcome (H, T) is to be distinguished from the outcome (T, H)?

Solution

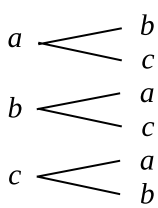
2x2 = 4 ***outcomes***

Exercise

How many 2-letter code words can be formed from the first 3 letters of the alphabet if no letter can be used more than once?

Solution

3x2 = 6 ***outcomes***



Exercise

A coin is tossed with possible outcomes heads, H, or tails, T. Then a single die is tossed with possible outcomes 1, 2, 3, 4, 5, or 6. How many combined outcomes are there?

Solution

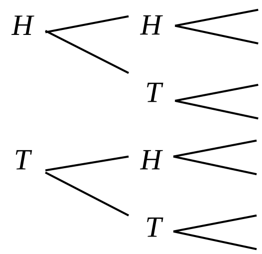
2x6 = 12 ***outcomes***

Exercise

In how many ways can 3 coins turn up heads, H, or tails, T – if combined outcomes such as (H,T,H), (H, H, T), and (T, H, H) are to be considered different?

Solution

$2.2.2 = 8$



Exercise

An entertainment guide recommends 6 restaurants and 3 plays that appeal to a couple.

a) If the couple goes to dinner or to a play, how many selections are possible?

b) If the couple goes to dinner and then to a play, how many combined selections are possible?

Solution

a) $3 + 6 = 9$

b) $6.3 = 18$

Exercise

There are 18 mathematics majors and 325 computer science majors at a college

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

b) In how many ways can one representative be picked who either a mathematics major or a computer science major?

Solution

a) $18 \cdot 325 \ \boxed{= 5850 \ \ ways}$

b) $18 + 325 \ \boxed{= 343 \ \ ways}$

Exercise

An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

Solution

Using the product rule: there are $27 \cdot 37 \ \boxed{= 999 \ \ offices}$

Exercise

A multiple-choice test contains 10 questions. There are four possible answers for each question

a) In how many ways can a student answer the questions on the test if the student answers every question?

b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

Solution

a) $4\cdot 4\cdot 4\cdots 4=4^{10}=\boxed{1,048,576\ ways}$

b) There are 5 ways to answer each question 0 give any if the 4 answers or give no answer at all

$5^{10}=\boxed{9,765,625\ ways}$

Exercise

A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of the shirts are made?

Solution

$12\cdot 2\cdot 3=\boxed{72}$ different types of shirt.

Exercise

How many different three-letter initials can people have?

Solution

$26\cdot 26\cdot 26=\boxed{17,576}$ different initials.

Exercise

How many different three-letter initials with none of the letters repeated can people have?

Solution

$26\cdot 25\cdot 24=\boxed{15,600\ \ ways}$

Exercise

How many different three-letter initials are there that begin with an A?

Solution

$1\cdot 26\cdot 26=\boxed{676\ \ ways}$

Exercise

How many strings are there of four lowercase letters that have the letter *x* in them?

Solution

Number of strings of length of 4 lowercase: 26^4

Number of strings of length of 4 lowercase other than ***x***: 25^4

$26^4 - 25^4 = \underline{66,351 \ \ strings}$

Exercise

How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

Solution

$10^3 \cdot 26^3 + 26^3 \cdot 10^3 \underline{= 35,152,000 \ \ license\ plates}$

Exercise

How many license plates can be made using either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters?

Solution

Letters		***Digits***			
L	L	D	D	D	D
26	26	10	10	10	10

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 6,760,000$

Digits		***Letters***			
D	D	L	L	L	L
10	10	26	26	26	26

$10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 45,697,600$

Therefore: $6,760,000 + 45,697,600 \ \underline{= 52,457,600 \ \ license\ plates}$

Exercise

How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

Solution

$26^3 \cdot 10^3 + 26^4 \cdot 10^2 \underline{= 63,273,600 \ \ license\ plates}$

Exercise

How many strings of eight English letter are there

a) That contains no vowels, if letters can be repeated?

b) That contains no vowels, if letters cannot be repeated?

c) That starts with a vowel, if letters can be repeated?

d) That starts with a vowel, if letters cannot be repeated?

e) That contains at least one vowel, if letters can be repeated?
f) That contains at least one vowel, if letters cannot be repeated?

Solution

a) There are 8 slots which can be filled with $26-5=21$ non-vowels.

By the product rule: $21^8 = \underline{37,822,859,361 \ \ strings}$

b) $21\cdot 20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15\cdot 14 = \underline{8,204,716,800 \ \ strings}$

c) $5\cdot 26^7 = \underline{40,159,050,880 \ \ strings}$

d) $5\cdot 25\cdot 24\cdot 23\cdot 22\cdot 21\cdot 20\cdot 19 = \underline{12,113,640,000 \ \ strings}$

e) By the product rule: $26^8 - 21^8 = \underline{171,004,205,215 \ \ strings}$

f) $8\cdot 5\cdot 21^7 = \underline{72,043,541,640 \ \ strings}$

	1	2	3	4	5	6	7	8
	NV	**NV**	**NV**	**NV**	**NV**	**NV**	**NV**	**NV**
a	21	21	21	21	21	21	21	21
b	21	20	19	18	17	16	15	14
	V	**L**	**L**	**L**	**L**	**L**	**L**	**L**
c	5	26	26	26	26	26	26	26
d	5	25	24	23	22	21	20	19

Exercise

In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) The bride must be in the picture?
b) Both the bride and groom must be in the picture?
c) Exactly one of the bride and the groom is in the picture?

Solution

a) The bride is in any of the 6 positions.
Then, it will leave us with 5 remaining positions.
This can be done in $9\cdot 8\cdot 7\cdot 6\cdot 5 = 15120$ ways.
Therefore $6\cdot 15120 = \underline{90,720 \ \ ways}$

1	2	3	4	5	6
B	***P***	***P***	***P***	***P***	***P***
1	9	8	7	6	5

b) The bride is in any of the 6 positions.
Then place the groom in any of the 5 remaining positions.
Then, it will leave us with 4 remaining positions in the picture.
This can be done in $8\cdot 7\cdot 6\cdot 5 = 1680$ ways.
Therefore $6\cdot 5\cdot 1680 = \underline{50,400 \ \ ways}$

1	2	3	4	5	6
B	***G***	***P***	***P***	***P***	***P***
1	1	8	7	6	5

c) For the just the bride to be in the picture: $90720 - 50400 = 40,320$ ways. There are 40,320 ways for just the groom to be in the picture. Therefore $40320 + 40320 = \underline{80,640 \ \ ways}$

Solution *Section* 3.4 – Permutations and Combinations

Exercise

Decide whether the situation involves ***permutations*** or ***combinations***

a) A batting order for 9 players for a baseball game
b) An arrangement of 8 people for a picture
c) A committee of 7 delegates chosen from a class of 30 students to bring a petition to the administration
d) A selection of a chairman and a secretary from a committee of 14 people
e) A sample of 5 items taken from 71 items on an assembly line
f) A blend of 3 spices taken from 7 spices on a spice rack
g) From the 7 male and 10 female sales representatives for an insurance company, team of 8 will be selected to attend a national conference on insurance fraud.
h) Marbles are being drawn without replacement from a bag containing 15 marbles.
i) The new university president named 3 new officers a vice-president of finance, a vice-president of academic affairs, and a vice-president of student affairs.
j) A student checked out 4 novels from the library to read over the holiday.
k) A father ordered an ice cream cone (chocolate, vanilla, or strawberry) for each of his 4 children.

Solution

a) Permutation

b) Permutation

c) Combination

d) Permutation

e) Combination

f) Combination

g) Combination

h) Combination

i) Permutation

j) Combination

k) Neither

Exercise

Wing has different books to arrange on a shelf: 4 blue, 3 green, and 2 red.

a) In how many ways can the books be arranged on a shelf?
b) If books of the same color are to be grouped together, how many arrangements are possible?
c) In how many distinguishable ways can the books be arranged if books of the same color are identical but need not be grouped together?

d) In how many ways can you select 3 books, one of each color, if the order in which the books are selected does not matter?

e) In how many ways can you select 3 books, one of each color, if the order in which the books are selected matters?

Solution

a) $P(9,9) = 362,880$ *ways*

b) $4!.3!.2!.3! = 1728$ *possibilities*

c) $\frac{9!}{4!3!2!} = 1260$

d) $4.3.2 = 24$

e) $24.3! = 144$ ***(24 from part-d)***

Exercise

A child has a set of differently shaped plastic objects. There are 3 pyramids, 4 cubes, and 7 spheres.

a) In how many ways can she arrange the objects in a row if each is a different color?

b) How many arrangements are possible if objects of the same shape must be grouped together and each object is a different color?

c) In how many distinguishable ways can the objects be arranged in a row if objects of the same shape are also the same color, but need not be grouped together?

d) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected does not matter and each object is a different color?

e) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected matters and each object is a different color?

Solution

a) $P(14,14) = 8.7178291 \times 10^{10}$

b) $3!4!7!3! = 4,354,560$ ***(3! number of ways to arrange the order of 3 groups)***

c) $\frac{14!}{3!4!7!} = 120,120$

d) $3.4.7 = 84$

e) $84.3! = 504$

Exercise

In a club with 16 members, how many ways can a slate of 3 officers consisting of president, vice-president, and secretary/treasurer be chosen?

Solution

$P(16,3) = \underline{3360}$

Exercise

Twelve drugs have been found to be effective in the treatment of a disease. It is believed that the sequence in which the drugs are administered is important in the effectiveness of the treatment. In how many different sequences can 5 of the 12 drugs be administered?

Solution

$$P(12,5) = \underline{95,040}$$

Exercise

In how many ways can 7 of 11 monkeys be arranged in a row for a genetics experiment?

Solution

$$P(11,7) = \underline{1,663,200}$$

Exercise

In an experiment on social interaction, 6 people will sit in 6 seats in a row. In how many ways can this be done?

Solution

$$P(6,6) = \underline{720}$$

Exercise

In an election with 3 candidates for one office and 6 candidates for another office, how many different ballots may be printed?

Solution

Office 1: $P(3,3)$

Office2: $P(6,6)$

Multiplication principle: $2.P(3,3)P(6,6)\ \underline{=8640}$

Exercise

A business school gives courses in typing, shorthand, transcription, business English, technical writing, and accounting. In how many ways can a student arrange a schedule if 3 courses are taken? assume that the order in which courses are schedules matters.

Solution

$$P(6,3)\ \underline{=120}$$

Exercise

If your college offers 400 courses, 25 of which are in mathematics, and your counselor arranges your schedule of 4 courses by random selection, how many schedules are possible that do not include a math course? Assume that the order in which courses are scheduled matters.

Solution

$$P(nonmath) = P(375,4) = \underline{1.946 \times 10^{10}}$$

Exercise

A baseball team has 19 players. How many 9-player batting orders are possible?

Solution

$$P(19,9) = \underline{3.352 \times 10^{10}}$$

Exercise

A chapter of union Local 715 has 35 members. In how many different ways can the chapter select a president, a vice-president, a treasurer, and a secretary?

Solution

$$P(35,4) = \underline{1,256,640}$$

Exercise

A concert to raise money for an economics prize is to consist of 5 works; 2 overtures, 2 sonatas, and a piano concerto.

a) In how many ways can the program be arranged?
b) In how many ways can the program be arranged if an overture must come first?

Solution

a) $P(5,5) = \underline{120}$

b) $P(2,1).P(4,4) = \underline{48}$

Exercise

A zydeco band from Louisiana will play 5 traditional and 3 original Cajun compositions at a concert. In how many ways can they arrange the program if

a) The begin with a traditional piece?
b) An original piece will be played last?

Solution

a) $P(5,1).P(7,7) = \underline{25,200}$

b) $P(7,7).P(3,1) = \underline{15,120}$

Exercise

Given the set $\{A, B, C, D\}$, how many permutations are there of this set of 4 object taken 2 at a time?

a) Using the multiplication principle

b) Using the Permutation

Solution

a) 4.3 = 12

b) $P_{4,2} = \frac{4!}{2!} = \underline{12}$

Exercise

Find the number of permutations of 30 objects taken 4 at a time.

Solution

$$P_{30,4} = \frac{30!}{(30-4)!} \underline{= 657,720}$$

Exercise

Five cards are marked with the numbers 1, 2, 3, 4, and 5, then shuffled, and 2 cards are drawn.

a) How many different 2-card combinations are possible?

b) How many 2-card hands contain a number less than 3?

Solution

a) $C_{5,2} = 10$

b) $\left\{\begin{array}{l} \{1,2\},\ \{1,3\},\ \{1,4\},\ \{1,5\},\ \{2,3\} \\ \{2,4\},\ \{2,5\},\ \{3,4\},\ \{3,5\},\ \{4,5\} \end{array}\right\}$

7 contain a card numbered less than 3.

Exercise

An economics club has 31 members.

a) If a committee of 4 is to be selected, in how many ways can the selection be made?

b) In how many ways can a committee of at least 1 and at most 3 be selected?

Solution

a) $C_{31,4} = 31,465$

b) $P(\text{at least } 1 \text{ and at most } 3 \text{ be selected}) = C_{31,1} + C_{31,2} + C_{31,3}$

$$= 31 + 465 + 4495$$

$$\underline{= 4991}$$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be chosen that have

a) All men?

b) All women?

c) 3 men and 2 women?

Solution

a) $C(9,5) = 126$

b) $C(11,5) = 462$

c) $C(9,3).C(11,2) = (84)(55) = 4620$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be selected that have

a) At least 4 women?

b) No more than 2 men?

Solution

a) $C(11,4)C(9,1) + C(11,5)C(9,0) = 3432$

b) $C(9,0)C(11,5) + C(9,1)C(11,4) + C(9,2)C(11,3) = 9372$

Exercise

Use a tree diagram for the following

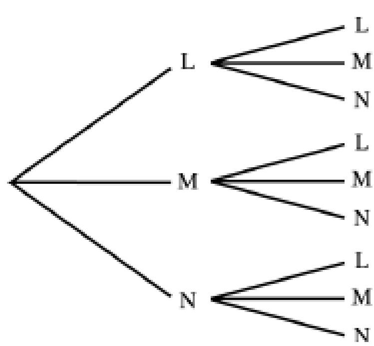
a) Find the number of ways 2 letters can be chosen from the set $\{L, M, N\}$ if order is important and repetition is allowed.

b) Reconsider part a if no repeats are allowed

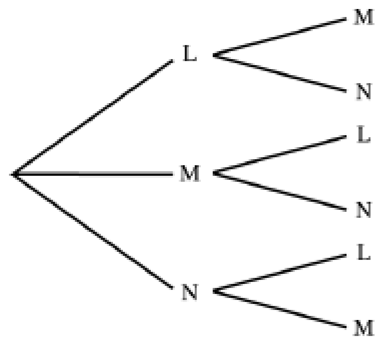
c) Find the number of combinations of 3 elements taken 2 at a time. Does this answer differ from part *a* or *b*?

Solution

a) There are 9 ways to choose 2 letters if repetition is allowed



b) There are 9 ways to choose 2 letters if repetition is allowed



c) The number of 3 elements taken 2 at a time is:

$$C_{3,2} = \underline{3}$$

Exercise

In a game of musical chairs, 12 children will sit in 11 chairs arranged in a row (one will be left out). In how many ways can this happen, if we count rearrangements of the children in the chairs as different outcomes?

Solution

$$P(12,11) = \underline{479,001,600}$$

Exercise

A group of 3 students is to be selected from a group of 14 students to take part in a class in cell biology.

a) In how many ways can this be done?

b) In how many ways can the group who will not take part be chosen?

Solution

a) $\binom{14}{3} = 364$

b) $\binom{14}{11} = 364$

Exercise

Marbles are being drawn without replacement from a bag containing 16 marbles.

a) How many samples of 2 marbles can be drawn?

b) How many samples of 2 marbles can be drawn?

c) If the bag contains 3 yellow, 4 white, and 9 blue marbles, how many samples of 2 marbles can be drawn in which both marbles are blue?

Solution

a) $C(16,2)=120$

b) $C(16,4)=1820$

c) $C(9,2)=36$

Exercise

There are 7 rotten apples in a crate of 26 apples

a) How many samples of 3 apples can be drawn from the crate?
b) How many samples of 3 could be drawn in which all 3 are rotten?
c) How many samples of 3 could be drawn in which there are two good apples and one rotten one?

Solution

a) $C_{26,3}=2600$

b) $C_{7,3}=35$

c) $C_{26,3}=2600$

Exercise

A bag contains 5 black, 1 red, and 3 yellow jelly beans; you take 3 at random. How many samples are possible in which the jelly beans are

a) All black?
b) All red?
c) All yellow?
d) 2 black and 1 red?
e) 2 black and 1 yellow?
f) 2 yellow and 1 black?
g) 2 red and 1 yellow?

Solution

a) $C_{5,3}=10$

b) No 3 red. $C_{1,3}=0$

c) $C_{3,3}=1$

d) $C_{5,2}C_{1,1}=10$

e) $C_{5,2}C_{3,1}=30$

f) $C_{3,2}C_{5,1}=15$

g) *There is only 1 red.*

Exercise

In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?

Solution

$$P_{9,5} = \underline{15,120}|$$

Exercise

From a pool of 8 secretaries, 3 are selected to be assigned to 3 managers, one per manager. In how many ways can they be selected and assigned?

Solution

$$P_{8,3} = \underline{336}|$$

Exercise

A salesperson has the names of 6 prospects.

a) In how many ways can she arrange her schedule if she calls on all 6?

b) In how many ways can she arrange her schedule if she can call on only 4 of the 6?

Solution

a) $P_{6,6} = \underline{720}|$

b) $P_{6,4} = 360$

Exercise

Five items are to be randomly selected from the first 50 items on an assembly line to determine the defect rate. How many different samples of 5 items can be chosen?

Solution

$$C_{50,5} = \underline{2,118,760}|$$

Exercise

A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.

a) How many delegations are possible?

b) If it is decided that a particular worker must be in the delegation, how many different delegations are possible?

c) If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?

Solution

a) $C_{9,3} \underline{= 84}$

b) $1.C_{8,2} \underline{= 28}$

c) $C_{4,1}C_{5,2} + C_{4,2}C_{5,1} + C_{4,3} \underline{= 74}$

Exercise

From a group of 16 smokers and 22 nonsmokers, a researcher wants to randomly select 8 smokers and 8 nonsmokers for a study. in how many ways can the study group be selected?

Solution

$$C_{16,8}C_{22,8} \underline{= 4{,}115{,}439{,}900}$$

Exercise

Hamburger Hut sells regular hamburgers as well as a larger burger. Either type can include cheese, relish, lettuce, tomato, mustard, or catsup.

a) How many different hamburgers can be ordered with exactly three extras?
b) How many different regular hamburgers can be ordered with exactly three extras?
c) How many different regular hamburgers can be ordered with at least five extras?

Solution

a) $C_{2,1}C_{6,3} \underline{= 40}$

b) $C_{6,3} \underline{= 20}$

c) $C_{6,5} + C_{6,6} \underline{= 7}$

Exercise

In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.

a) In how many ways can this be done?
b) In how many ways can this be done if exactly 2 wheat plants must be included?

Solution

a) $C_{11,4} \underline{= 330}$

b) $C_{6,2}C_{5,2} \underline{= 150}$

Exercise

A legislative committee consists of 5 Democrats and 4 Republicans. A delegation of 3 is to be selected to visit a small Pacific island republic.

a) How many different delegations are possible?
b) How many delegations would have all Democrats?
c) How many delegations would have 2 Democrats and 1 Republican?
d) How many delegations would have at least 1 Republican?

Solution

a) $C_{9,3} = \underline{84}$

b) $C_{5,3} = \underline{10}$

c) $C_{5,2}C_{4,1} = \underline{40}$

d) $C_{9,3} - C_{5,3} = 84 - 10 = \underline{74}$

Exercise

From 10 names on a ballot, 4 will be elected to a political party committee. in how many ways can the committee of 4 be formed if each person will have a different responsibility, and different assignments of responsibility are considered different committees?

Solution

$P_{10,4} = \underline{5040}$

Exercise

How many different 13-card bridge hands can be selected from an ordinary deck?

Solution

$C_{52,13} = \underline{635,013,559,600}$

Exercise

Five cards are chosen from an ordinary deck to form a hand in poker. In how many ways is it possible to get the following results?

a) 4 queens
b) No face card
c) Exactly 2 face cards
d) At least 2 face cards
e) 1 heart, 2 diamonds, and 2 clubs

Solution

a) $C_{4,4}C_{48,1} = \underline{48}$

b) $C_{40,5} = \underline{658,008}$

c) $C_{12,2}C_{40,3} = \underline{652,080}$

d) $C_{12,2}C_{40,3} + C_{12,3}C_{40,2} + C_{12,4}C_{40,1} + C_{12,5} = \underline{844,272}$

e) $C_{13,1}C_{13,2}C_{13,2} = \underline{79,092}$

Exercise

In poker, a flush consists of 5 cards with the same suit, such as 5 diamonds.

a) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by listing all the possibilities.

b) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by using combinations

Solution

a) $\{(5,6,7,8,9);(5,6,7,8,10);(5,7,8,9,10);(5,6,8,9,10);(5,7,8,9,10);(6,7,8,9,10)\}$

There are 6 possibilities for each suit and there are 4 suits: 4.6 = 24

b) $4C_{6,5} = \underline{24}$

Exercise

If a baseball coach has 5 good hitters and 4 poor hitters on the bench and chooses 3 players at random, in how many ways can be choose at least 2 good hitters?

Solution

$C_{5,2}C_{4,1} + C_{5,3}C_{4,0} = \underline{50}$

Exercise

The coach of a softball team has 6 good hitters and 8 poor hitters. He chooses 3 hitters at random.

a) In how many ways can he choose 2 good hitters and 1 poor hitter?

b) In how many ways can he choose 3 good hitters?

c) In how many ways can he choose at least 3 good hitters?

Solution

a) $C_{6,2}C_{8,1} = \underline{120}$

b) $C_{6,3} = \underline{20}$

c) $C_{6,2}C_{8,1} + C_{6,3} = \underline{140}$

Exercise

How many 5 card hands will have 3 aces and 2 kings?

Solution

$$\text{Number of hands } = C_{4,3} \cdot C_{4,2}$$
$$\underline{= 24|}$$

Exercise

How many 5 card hands will have 3 hearts and 2 spades?

Solution

$$\text{Number of hands } = C_{13,3} \cdot C_{13,2} \ \underline{= 22,308|}$$

Exercise

2 letters follow by 3 numbers; 2 letters out of 8 & 3 numbers out of 10

Solution

$$\text{Number } = P_{8,2} \cdot P_{10,3} \ \underline{= 40320|}$$

Exercise

Serial numbers for a product are to be made using 3 letters follow by 2 digits (0 – 9 no repeats). If the letters are to be taken from the first 8 letters of the alphabet with no repeats, how many serial numbers are possible?

Solution

$$\text{Possible } = P_{8,3} \cdot P_{10,2} \ \underline{= 30,240|}$$

Exercise

A company has 7 senior and 5 junior officers. An ad hoc legislative committee is to be formed.

a) How many 4-officer committees with 1 senior officer and 3 junior officers can be formed?

b) How many 4-officer committees with 4 junior officers can be formed?

c) How many 4-officer committees with at least 2 junior officers can be formed?

Solution

a) $C_{7,1} \cdot C_{5,3} \ \underline{= 70|}$

b) $C_{5,4} \ \underline{= 5|}$

c) $C_{7,2}.C_{5,2} + C_{7,1}.C_{5,3} + C_{7,0}.C_{5,4} \ \underline{= 285|}$

Exercise

From a committee of 12 people,

a) In how many ways can we choose a chairperson, a vice-chairperson, a secretary, and a treasurer, assuming that one person can't hold more than on position

b) In how many ways can we choose a subcommittee of 4 people?

Solution

a) $P_{12,4} = \underline{11,880\ ways}|$

b) $C_{12,4} = 495\ \ ways$

Exercise

Find the number of combinations of 30 objects taken 4 at a time.

Solution

$$C_{30,4} = \frac{30!}{4!(30-4)!} \underline{= 27,405}|$$

Exercise

How many different permutations are the of the set $\{a, b, c, d, e, f, g\}$?

Solution

$$P(7,\ 7) \underline{= 5040}|$$

Exercise

How many permutations of $\{a, b, c, d, e, f, g\}$ end with a?

Solution

To find the permutation to with a, then we may forget about the a, and leave us $\{b, c, d, e, f, g\}$

$$P(6,\ 6) \underline{= 720}|$$

Exercise

Find the number of 5-permutations of a set with nine elements

Solution

$$P(9,\ 5) \underline{= 15,120}|$$

Exercise

In how many different orders can five runners finish a race if no ties are allowed?

Solution

$$P(5,\ 5) \underline{=120|}$$

Exercise

A coin flipped eight times where each flip comes up either heads or tails. How many possible outcomes

a) Are there in total?

b) Contain exactly three heads?

c) Contain at least three heads?

d) Contain the same number of heads and tails?

Solution

a) Each flip can be either heads or tails: There are $2^8 \underline{= 256\ \ possible\ coutcomes|}$

b) $C(8,\ 3) \underline{=56\ \ outcomes|}$

c) At least three heads means: 3, 4, 5, 6, 7, 8 heads.

$C(8,\ 3)+C(8,\ 4)+C(8,\ 5)+C(8,\ 6)+C(8,\ 7)+C(8,\ 8) \underline{=219\ \ outcomes|}$

OR

$256-C(8,\ 0)-C(8,\ 1)-C(8,\ 2)=256-28-8-1 \underline{=219\ \ outcomes|}$

d) To have an equal number of heads and tails means 4 heads and 4 tails.

Therefore; $C(8,\ 4) \underline{=70\ \ outcomes|}$

Exercise

In how many ways can a set of two positive integers less than 100 be chosen?

Solution

$$C_{99,\ 2} \underline{=4851\ \ ways|}$$

Exercise

In how many ways can a set of five letters be selected from the English alphabet?

Solution

$$C_{26,\ 5} \underline{=65,780\ \ ways|}$$

Solution Section 3.5 – Probability

Exercise

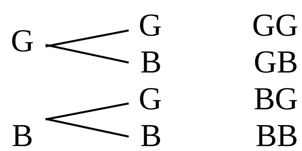
An experiment consists of recording the boy-girl composition of a two-child family.

a) What is an appropriate sample space if we are interested in the genders of the children in the order of their births? Draw a tree diagram.

b) What is an appropriate sample space if we are interested only in the *number* of the girls in a family?

Solution

a) What is an appropriate sample space if we are interested in the genders of the children in the order of their births? Draw a tree diagram.



$$S_1 = \{GG, GB, BG, BB\}$$

b) What is an appropriate sample space if we are interested only in the *number* of the girls in a family?

$$S_2 = \{0, 1, 2\}$$

Exercise

Given $S = \{1, 2, 3, \cdots, 17, 18\}$

a) The outcome is a number divisible by 12

b) The outcome is an even number greater than 15

c) Is divisible by 4

d) Is divisible by 5

Solution

a) $E = \{12\}$

b) $E = \{16, 18\}$

c) $E = \{4, 8, 12, 16\}$ $\quad$ E: Compound event

d) $F = \{5, 10, 15\}$

Exercise

Consider rolling 2 Dice.

a) What is the event that a sum of 5 turns up

b) What is the event that a sum that is a prime number greater than 7 turns up

Solution

a) Sum of 5: {(4, 1), (3, 2), (2, 3), (1, 4)}

b) {(6, 5), (5, 6)}

Exercise

A single fair die is rolled. Find the probability of each event

a) Getting a 2

b) Getting an odd number

c) Getting a number less than 5

d) Getting a number greater than 2

e) Getting a 3 or a 4

f) Getting any number except 3

Solution

a) $P = \frac{1}{6}$

b) $P(Odd) = \frac{3}{6} = \frac{1}{2}$

c) $P(<5) = \frac{4}{6} = \frac{2}{3}$

d) $P(>2) = \frac{4}{6} = \frac{2}{3}$

e) $P(3 \text{ or } 4) = \frac{2}{6} = \frac{1}{3}$

f) $P(no\ 3) = \frac{5}{6}$

Exercise

A card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing the following

a) A 9

b) A black card

c) A black 9

d) A heart

e) The 9 of hearts

f) A face card

g) A 2 or a queen

h) A black 7 or red 8
i) A red card or a 10
j) A spade or a king

Solution

a) $P(9)=\frac{4}{52}\ \boxed{=\frac{1}{13}}$

b) $P(black)=\frac{26}{52}\ \boxed{=\frac{1}{2}}$

c) $P(black-9)=\frac{2}{52}\ \boxed{=\frac{1}{26}}$

d) $P(heart)=\frac{13}{52}\ \boxed{=\frac{1}{4}}$

e) $P(9-heart)\ \boxed{=\frac{1}{52}}$

f) $P(\,face)=\frac{12}{52}\ \boxed{=\frac{3}{13}}$

g) $P(2\ or\ queen)=\frac{8}{52}\ \boxed{=\frac{2}{13}}$

h) $P(black-7\ or\ red-8)=\frac{4}{52}\ \boxed{=\frac{1}{13}}$

i) $P(red\ or\ 10)=\frac{28}{52}\ \boxed{=\frac{7}{13}}$

j) $P(spade\ or\ king)=\frac{16}{52}\ \boxed{=\frac{4}{13}}$

Exercise

The student sitting next to you in class concludes that the probability of the ceiling falling down on both of you before class ends is 1/2, because there are two possible outcomes - the ceiling will fall or not fall. What is wrong with this reasoning?

Solution

The outcomes are not equally likely.

Exercise

A jar contains 3 white, 4 orange, 5 yellow, and 8 black marbles. If a marble is drawn at random, find the probability that it is the following.

a) White
b) Orange
c) Yellow
d) Black
e) Not black

f) Orange or Yellow

Solution

a) $P(white) \boxed{= \frac{3}{20}}$

b) $P(orange) = \frac{4}{20} \boxed{= \frac{1}{5}}$

c) $P(yellow) = \frac{5}{20} \boxed{= \frac{1}{4}}$

d) $P(black) = \frac{8}{20} \boxed{= \frac{2}{5}}$

e) $P(no\ black) = \frac{12}{20} \boxed{= \frac{3}{5}}$ $\quad 1 - P(black)$

f) $P(orange\ \ or\ yellow) \boxed{= \frac{9}{20}}$

Exercise

Let consider rolling 2 dice. Find the probabilities of the following events

a) E = Sum of 5 turns up

b) F = a sum that is a prime number greater than 7 turns up

Solution

$1-1$	$1-2$	$1-3$	$1-4$	$1-5$	$1-6$
$2-1$	$2-2$	$2-3$	$2-4$	$2-5$	$2-6$
$3-1$	$3-2$	$3-3$	$3-4$	$3-5$	$3-6$
$4-1$	$4-2$	$4-3$	$4-4$	$4-5$	$4-6$
$5-1$	$5-2$	$5-3$	$5-4$	$5-5$	$5-6$
$6-1$	$6-2$	$6-3$	$6-4$	$6-5$	$6-6$

a) $P(E) = \frac{4}{36} \boxed{= \frac{1}{9}}$

b) $P(F) = \frac{2}{36} \boxed{= \frac{1}{18}}$

Exercise

The board of regents of a university is made up of 12 men and 16 women. If a committee of 6 chosen at random, what is the probability that it will contain 4 men and 2 women?

Solution

S: the set of 6 out of 28 $n(S) = C_{28,6} = 376,740$

4 men out of 12 men: $C_{12,4}$

2 women out of 16 men: $C_{16,2}$

$$P(E) = \frac{C_{12,4} . C_{16,2}}{C_{28,6}} \boxed{\approx 0.158}$$

Exercise

In drawing 7 cards from a 52-card deck without replacement, what is the probability of getting 7 hearts.

Solution

$$P(7\ hearts) = \frac{C_{13,7}}{C_{52,7}} \approx 0.00013$$

Exercise

A committee of 4 people is to be chosen from a group of 5 men and 6 women. What is the probability that the committee will consist of 2 men and 2 women?

E is the set of all possible ways to select 2 men and 2 women

S is the set of all possible ways to select 4 people from 11

Solution

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}_5C_2 \cdot {}_6C_2}{{}_{11}C_4} = 0.4545$$

Exercise

A department store receives a shipment of 27 new portable radios. There are 4 defective radios in the shipment. If 6 radios are selected for display, what is the probability that 2 of them are defective?

E is the set of all possible ways to have 2 defective and 4 not defective.

S is the set of all possible ways to select 6 radios from 27.

Solution

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}_4C_2 \cdot {}_{23}C_4}{{}_{27}C_6} = 0.1795$$

Exercise

Eight cards are drawn from a standard deck of cards. What is the probability that there are 4 face cards and 4 non-face cards?

E is the set of all possible ways to have 4 faces and 4 nonfaces.

S is the set of all possible ways to select 8 cards from 52.

Solution

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}_{12}C_4 \cdot {}_{40}C_4}{{}_{52}C_8} = 0.0601$$

Exercise

Five cards are drawn from a standard deck of cards. What is the probability that there are exactly 3 hearts?

Solution

$$P(E)=\frac{n(E)}{n(S)}=\frac{{}_{13}C_3 \cdot {}_{39}C_2}{{}_{52}C_5}\ \underline{=0.0815}$$

Exercise

A poll was conducted preceding an election to determine the relationship between voter persuasion concerning a controversial issue and the area of the city in which the voter lives. Five hundred registered voters were interviewed from three areas of the city. The data are shown below. Compute the probability of having no opinion on the issue or living in the inner city.

Area of city	***Favor***	***Oppose***	***No Opinion***
East	30	40	55
North	25	45	50
Inner	95	65	85

Solution

$$\Pr=\frac{Total\ Inner+No\ Opinion\ East+No\ Opinion\ North}{500}$$

$$=\frac{95+65+85+55+50}{500}$$

$$=\frac{350}{500}$$

$$\underline{\approx 0.7}$$

Exercise

There are 11 members on the board of directors for the Coca Cola Company.

a) If they must select a chairperson, first vice chairperson, second vice chairperson, and secretary, how many different slates of candidates are possible?

b) If they must form an ethics subcommittee of 4 members, how many different subcommittees are possible?

Solution

a) Since order makes a difference, there are 4 different offices ${}_{11}P_4=\frac{11!}{7!}\ \underline{=7920}$

b) Since the order in which the 4 are picked makes no differences ${}_{11}C_4=\frac{11!}{7!4!}\ \underline{=330}$

Exercise

When testing for current in a cable with five color-coded wires, the author used a meter to test two wires at a time. How many different tests are required for every possible pairing of two wires?

Solution

Since the order of the 2 wires being tested is irrelevant:

$$_5C_2 = \frac{5!}{3!\cdot 2!} \boxed{= 10 \ \ different\ tests}$$

Exercise

Identity theft often begins by someone discovering your 9-digit social security number. Answer each of the following. Express probabilities as fractions.

a) What is the probability of randomly generating 9 digits and getting your social security number?

b) In the past, many teachers posted grades along with the last 4 digits of your social security number, what is the probability that if they randomly generated the order digits, they would match yours? Is that something to worry about?

Solution

a) Let G = generating a given social security number in a single trial.

Total number of possible sequences $= 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10$

$= 1,000,000,000$

Since only one sequence is correct: $P(G) \boxed{= \frac{1}{1,000,000,000}}$

b) Let F = generating first 5 digits of a given social security number in a single trial.

Total number of possible sequences $= 10\cdot 10\cdot 10\cdot 10\cdot 10$

$= 100,000$

Since only one sequence is correct: $P(F) \boxed{= \frac{1}{100,000}}$

Since this probability is so small, need not worry about the given scenario

Exercise

You become suspicious when a genetics researcher randomly selects groups of 20 newborn babies and seems to consistently get 10 girls and 10 boys. The researchers claims that it is common to get 10 girls and 10 boys in such cases,

a) If 20 newborn babies are randomly selected, how many different gender sequences are possible?

b) How many different ways can 10 girls and 10 boys be arranged in sequence?

c) What is the probability of getting 10 girls and 10 boys when 10 babies are born?

Solution

a) There are 20 tasks to perform, and each task can be performed in either of 2 ways

Total number of possible sequences is $2\cdot 2\cdot 2\cdot \cdots \cdot 2 = 2^{20}$

$\boxed{= 1,048,576 \ \ possibilities}$

b) The number of possible sequences of n objects is when some are alike is

$$\frac{n!}{n_1!\cdot n_2!\cdots n_k!}: \frac{20!}{10!10!} \underline{=184,756 \ \ possibilities|}$$

c) $P(10G,\ 10B)=\frac{184,756}{1,048,576}\ \underline{=0.176|}$

Exercise

Two dice are rolled. Find the probabilities of the following events.

a) The first die is 3 or the sum is 8

b) The second die is 5 or the sum is 10.

Solution

a) $P(3\ or\ sum\ is\ 8) = P(3) + P(sum\ 8) - P(3\ and\ sum\ 8)$

$$= \frac{6}{36}+\frac{5}{36}-\frac{1}{36}$$

$$= \frac{10}{36}$$

$$\underline{= \frac{5}{18}|}$$

b) $P(5\ or\ sum\ 10) = P(5) + P(sum\ 10) - P(5\ and\ sum\ 10)$

$$= \frac{6}{36}+\frac{3}{36}-\frac{1}{36}$$

$$= \frac{8}{36}$$

$$\underline{= \frac{2}{9}|}$$

Exercise

One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) A 9 or 10

b) A red card or a 3

c) A 9 or a black 10

d) A heart or a black card

e) A face card or a diamond

Solution

a) $P(9\ or\ 10) = \frac{8}{52}\ \underline{= \frac{2}{13}|}$

b) $P(red\ or\ 3) = \frac{28}{52}\ \underline{= \frac{7}{13}|}$

c) $P(9 \text{ or } black\text{-}10) = \frac{6}{52} = \frac{3}{26}$

d) $P(heart \text{ or } black) = \frac{39}{52} = \frac{3}{4}$

e) $P(face \text{ or } diamond) = \frac{22}{52} = \frac{11}{26}$

Exercise

One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) Less than a 4 (count aces as ones)
b) A diamond or a 7
c) A black card or an ace
d) A heart or a jack
e) A red card or a face card

Solution

a) $P(<4) = P(ace, 2, 3) = \frac{12}{52} = \frac{3}{13}$

b) $P(diamond \text{ or } 7) = \frac{16}{52} = \frac{4}{13}$

c) $P(black \text{ or } ace) = \frac{28}{52} = \frac{7}{13}$

d) $P(heart \text{ or } jack) = \frac{16}{52} = \frac{4}{13}$

e) $P(red \text{ or } face) = \frac{32}{52} = \frac{8}{13}$

Exercise

Pam invites relatives to a party: her mother, 2 aunts, 3 uncles, 2 brothers, 1 male cousin, and 4 female cousins. If the chances of any one guest first equally likely, find the probabilities that the first guest to arrive is as follows.

a) A brother or an uncle
b) A brother or a cousin
c) A brother or her mother
d) An uncle or a cousin
e) A male or a cousin
f) A female or a cousin

Solution

a) $P(brother \text{ or } uncle) = \frac{5}{13}$

b) $P(brother\ or\ cousin) = \boxed{\frac{7}{13}}$

c) $P(brother\ or\ mother) = \boxed{\frac{3}{13}}$

d) $P(uncle\ or\ cousin) = \boxed{\frac{8}{13}}$

e) $P(male\ or\ cousin) = \boxed{\frac{10}{13}}$

f) $P(brother\ or\ cousin) = \boxed{\frac{8}{13}}$

Exercise

The numbers $\{1, 2, 3, 4, and\ 5\}$ are written on slips of paper, and 2 slips are drawn at random one at a time without replacement. Find the probabilities:

a) The sum of the numbers is 9.
b) The sum of the numbers is 5 or less.
c) The first number is 2 or the sum is 6
d) Both numbers are even.
e) One of the numbers is even or greater than 3.
f) The sum is 5 or the second number is 2.

Solution

a) $P(sum = 9) = \frac{2}{20} = \boxed{\frac{1}{10}}$

b) $P(sum \le 5) = \frac{8}{20} = \boxed{\frac{2}{5}}$

c) $P(2\ or\ sum\ 6) = \boxed{\frac{7}{20}}$

d) $P(even) = \frac{2}{20} = \boxed{\frac{1}{10}}$

e) $P(even\ or\ > 3) = \frac{18}{20} = \boxed{\frac{9}{10}}$

f) $P(sum 5\ or\ 2nd\ \#2) = \boxed{\frac{7}{20}}$

Exercise

Suppose $P(E)=0.26$, $P(F)=0.41$, and $P(E\cap F)=0.16$. Find the following

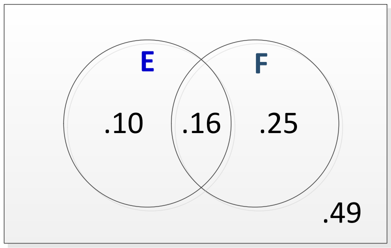
a) $P(E\cup F)$

b) $P(E'\cap F)$

c) $P(E\cap F')$

d) $P(E'\cup F')$

Solution



a) $P(E\cup F)=.1+.16+.25=\underline{.51}$

b) $P(E'\cap F)=\underline{.25}$

c) $P(E\cap F')=\underline{.10}$

d) $P(E'\cup F')=.74+.59-.49=\underline{.84}$

Exercise

Suppose $P(E)=0.42$, $P(F)=0.35$, and $P(E\cup F)=0.59$. Find the following

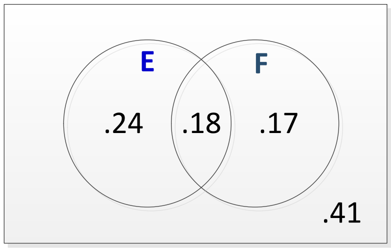
a) $P(E'\cap F')$

b) $P(E'\cup F')$

c) $P(E'\cup F)$

d) $P(E\cap F')$

Solution



$$\begin{aligned}P(E\cap F)&=P(E)+P(F)-P(E\cup F)\\&=.42+.35-.59\\&=\underline{.18}\end{aligned}$$

a) $P(E'\cap F')=\underline{.41}$

b) $P(E'\cup F')=1-.18=\underline{.82}$

c) $P(E'\cup F)=.17+.41+.18=\underline{.76}$

d) $P(E\cap F')=\underline{.24}$

Exercise

A single fair die is rolled. Find the odds in favor of getting the results

a) 3

b) 4, 5, or 6

c) 2, 3, 4, or 5

d) Some number less than 6

Solution

a) $P(E)=\frac{1}{6} \quad \to P(E')=\frac{5}{6}$

$$\Rightarrow odds=\frac{1}{5} \quad \to \boxed{1:5}$$

b) $P(E)=\frac{3}{6}=\frac{1}{2} \quad \to P(E')=\frac{1}{2}$

$$\Rightarrow odds=\frac{1/2}{1/2}=1 \quad \to \boxed{1:1}$$

c) $P(E)=\frac{4}{6}=\frac{2}{3} \quad \to P(E')=\frac{1}{3}$

$$\Rightarrow odds=\frac{2}{1} \quad \to \boxed{2:1}$$

d) $P(E)=\frac{5}{6} \quad \to P(E')=\frac{1}{6}$

$$\Rightarrow odds=\frac{5}{1} \quad \to \boxed{5:1}$$

Exercise

If in repeated rolls of two fair dice the odds against rolling a 6 before rolling a 7 are 6 to 5, what is the probability of rolling a 6 before rolling 7?

Solution

$$odds\ 6:5 \to \frac{6}{5}$$

$$\Rightarrow Against \to \quad =\frac{5}{6}$$

$$P(E)=\frac{5}{5+6}=\frac{5}{11}$$

Exercise

From survey involving 1,000 people in the certain city, it was found that 500 people had tried a certain brand of diet cola, 600 had tried a certain brand of regular cola, and 200 had tried both types of cola. If a resident of the city is selected at random, what is the empirical probability that

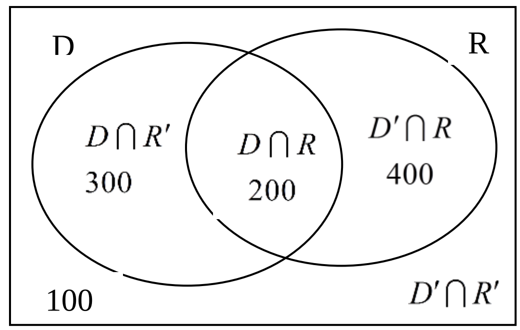
a) The resident has not tried either cola? What are the empirical odds for this event?

b) The resident has tried the diet or has not tried the regular cola? What are the empirical odds against this event?

Solution

a) $n(S) = 1000$ $\quad D \cap R = 200$

$D \cap R' = 300 \quad D' \cap R = 400$



$$P(\textit{neither } \mathrm{D} \textit{ or } R) = P(D' \cap R')$$

$$= \frac{100}{1000}$$

$$= .1$$

$$P(E') = 1 - .1 = 0.9$$

Odds for $\boldsymbol{E}$: $\dfrac{P(E)}{P(E')} = \dfrac{.1}{.9} = \dfrac{1}{9}$ *or* $\boxed{1:9}$

b) $P(E) = P(D \cup R')$

$$= P(D) + P(R') - P(D \cap R')$$

$$= \frac{500}{1000} + \frac{400}{1000} - \frac{300}{1000}$$

$$= .6$$

$$\Rightarrow P(E') = 1 - .6 = .4$$

Against Odds for P(E): $\dfrac{P(E)}{P(E')} = \dfrac{.4}{.6} = \dfrac{2}{3}$ *or* $\boxed{2:3}$

Exercise

The odds in favor of a particular horse winning a race are 4:5.

a) Find the probability of the horse winning.

b) Find the odds against the horse winning.

Solution

a) $P(E) = \dfrac{a}{a+b} = \dfrac{4}{4+5} = \dfrac{4}{9}$

b) The odds against the horse winning $\boxed{5:4}$

Exercise

Consider the sample space of equally likely events for the rolling of a single fair die.

a) What is the probability of rolling an odd number **and** a prime number?

b) What is the probability of rolling an odd number **or** a prime number?

Solution

a) $odd = \{1,\ 3,\ 5\} \quad prime = \{3,\ 5\}$

$$P(odd \cap prime) = \frac{n(A \cap B)}{n(S)} = \frac{2}{6} \ \underline{= \frac{1}{3}}$$

b) $P(odd \cup prime) = \frac{n(A \cup B)}{n(S)} = \frac{4}{6} \ \underline{= \frac{2}{3}}$

Exercise

Suppose that 2 fair Dice are rolled

a) What is the probability of that a sum of 2 or 3 turns up?

b) What is the probability of that both dice turn up the same or that a sum greater than 8 turns up?

Solution

a)
$$\begin{aligned} P(\Sigma=2 \text{ or } 3 \text{ turns up}) &= P(\Sigma=2) + P(\Sigma=3) \\ &= \frac{1}{36} + \frac{2}{36} \\ &= \frac{3}{36} \\ &= \frac{1}{12} \end{aligned}$$

b)
$$\begin{aligned} P(same \text{ or } \Sigma > 8) &= P(same \cup \Sigma > 8) \\ &= P(same) + P(\Sigma > 8) - P(same \cap \Sigma > 8) \\ &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\ &= \frac{7}{18} \end{aligned}$$

Exercise

A single card is drawn from an ordinary of 52 cards. Calculate the probabilities of and odds for each event

a) A face card or a club is drawn

b) A king or a heart is drawn

c) A black card or an ace is drawn

d) A heart or a number less than 7 (count an ace as 1) is drawn.

Solution

a) $\Pr(Face \text{ or } Club) = \Pr(F \cup C)$

$$= P(F) + P(C) - P(F \cap C)$$

$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52}$$

$$= \frac{11}{26}$$

$$P\left[(F \cup C)'\right] = 1 - \frac{11}{26} = \frac{15}{26}$$

Odds for $F \cup C = \dfrac{\frac{11}{26}}{\frac{15}{26}} = \dfrac{11}{15}$ $\boxed{11:15}$

b) $\Pr(\textit{King or Heart}) = \mathrm{P}(K \cup H)$

$$= P(K) + P(H) - P(K \cap H)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

$$P\left[(K \cup H)'\right] = 1 - \frac{4}{13} = \frac{9}{13}$$

Odds for $K \cup H = \dfrac{\frac{4}{13}}{\frac{9}{13}} = \dfrac{4}{9}$ $\boxed{4:9}$

c) $\Pr(\textit{Black card or Ace}) = \mathrm{P}(B \cup A)$

$$= P(B) + P(A) - P(B \cap A)$$

$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$$

$$= \frac{28}{52}$$

$$= \frac{7}{13}$$

$$P\left[(B \cup A)'\right] = 1 - \frac{7}{13} = \frac{6}{13}$$

Odds for $B \cup A = \dfrac{\frac{7}{13}}{\frac{6}{13}} = \dfrac{7}{6}$ $\boxed{7:6}$

d) $\Pr(\textit{Heart or } < 7) = \mathrm{P}(H \cup < 7)$

$$= P(B) + P(A) - P(B \cap A)$$

$$= \frac{13}{52} + \frac{6*4}{52} - \frac{6}{52}$$

$$= \frac{31}{52}$$

$$P\left[\left(H \cup \# < 7\right)'\right] = 1 - \frac{31}{52} = \frac{21}{52}$$

$$\text{Odds for } B \cup A = \frac{\frac{31}{52}}{\frac{21}{52}} = \frac{31}{21} \quad \boxed{31:21}$$

Exercise

What is the probability of getting at least 1 black card in a 7-card hand dealt from a standard 52-card deck?

Solution

There are 26 black cards.

Let A = "at least 1 black card in a 7-card hand dealt"

A' = "0 black cards in a 7-card hand dealt"

$$n(A') = C_{26,7}$$

$$n(S) = C_{52,7}$$

$$P(A) = 1 - P(A')$$

$$= 1 - \frac{n(A')}{n(S)}$$

$$= 1 - \frac{C_{26,7}}{C_{52,7}}$$

$$= 1 - .005$$

$$= .995$$

Exercise

What is the probability that a number selected at random from the first 600 positive integers is (exactly) divisible by 6 or 9?

Solution

Let A = ″Number divisible by 6″

B = ″Number divisible by 9″

A number divisible by 6 $\Rightarrow n(A) = \frac{600}{6} = 100$

A number divisible by 9 $\Rightarrow n(B) = \frac{600}{9} \approx 66$

A number divisible by 6 and by 9 $\rightarrow 18k \quad \Rightarrow n(A \cap B) = \frac{600}{18} \approx 33$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{100}{600} + \frac{66}{600} - \frac{33}{600}$$

$$= \frac{133}{600}$$

$$\approx 0.2217$$

Exercise

What is the probability that a number selected at random from the first 1,000 positive integers is (exactly) divisible by 6 or 8?

Solution

Let A = ″Number divisible by 6″

B = ″Number divisible by 8″

A number divisible by 6 $\Rightarrow n(A) = \frac{1,000}{6} = 166$

A number divisible by 8 $\Rightarrow n(B) = \frac{1,000}{8} \approx 125$

A number divisible by 6 and by 8 $\rightarrow 24k \quad \Rightarrow n(A \cap B) = \frac{1,000}{24} \approx 41$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{166}{1000} + \frac{125}{1000} - \frac{41}{1000}$$

$$= \frac{250}{1000}$$

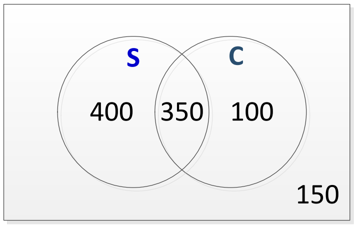
$$\approx 0.25$$

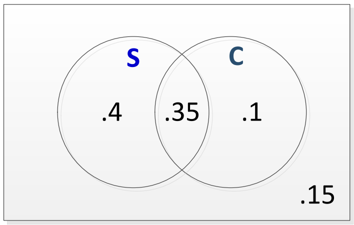
Exercise

From a survey involving 1,000 students at a large university, a market research company found that 750 students owned stereos, 450 owned cars, and 350 owned cars and stereos. If a student at the university is selected at random, what is the (empirical) probability that

a) The student owns either a car or a stereo?
b) The student owns neither a car nor a stereo?

Solution





Let S = "Number of stereos "
C = "Number of cars"

a) $P(S \cup C) = P(S) + P(C) - P(S \cap C)$

$$= \frac{750}{1000} + \frac{450}{1000} - \frac{350}{1000}$$

$$= \frac{850}{1000}$$

$$\approx 0.85$$

b) $P(S' \cap C') = .15$

Exercise

In order to test a new car, an automobile manufacturer wants to select 4 employees to test drive the car for 1 year. If 12 management and 8 union employees volunteer to be test drivers and the selection is made at random, what is the probability that at least 1 union employee is selected.

Solution

Let A = "at least 1 union employee is selected"
A' = "no union employee is selected"

$$\Rightarrow n(A') = C_{12,4}, \quad n(S) = C_{20,4}$$

$$P(A) = 1 - P(A')$$

$$= 1 - \frac{C_{12,4}}{C_{20,4}}$$

$$\approx 0.90$$

Exercise

A shipment of 60 inexpensive digital watches, including 9 that are defective, is sent to a department store. The receiving department selects 10 at random for testing and rejects the whole shipment if 1 or more in the sample are found defective. What is the probability that the shipment will be rejected?

Solution

The sample space: $S = C_{60,10}$

Let E = "Event that contains at least 1 defective watch".

E' = "Event that contains no defective watches".

$n(E') = C_{51,10}$

Probability that the shipment will be rejected:

$$\begin{aligned} P(E) &= 1 - P(E') \\ &= 1 - \frac{n(E')}{n(S)} \\ &= 1 - \frac{C_{51,10}}{C_{60,10}} \\ &= .83 \end{aligned}$$

Exercise

If you bet \$5 on the number 13 in roulette, your probability of winning is $\frac{1}{38}$ and the payoff odds are given by the casino as 35:1.

a) Find the actual odds against the outcome of 13.

b) How much net profit would you make if you win by betting on 13?

c) If the casino was not operating for profit, and the payoff odds were changed to match the actual odds against 13, how much would you win if the outcome were 13?

Solution

a) With odds: $P(13) = \frac{1}{38}$ and $P(not\ 13) = \frac{37}{38}$

Actual odds against 13 $\frac{P(not\ 13)}{P(13)} = \frac{37/38}{1/38} = \frac{37}{1}$ or 37:1

b) Because the payoffs odds against 13 are 35:1, we have:

35:1 = (*net profit*) : (*amount bet*)

So there is a \$35 profit for each \$1 bet. For \$5 bet, the net profit is $5 \times 35 = \$175$.
The winning bettor would collect \$175 plus the original \$5 bet. That is, the total amount collected would be \$180, for the net profit of \$175.

c) If the casino were not operating for profit, the payoff odds would be equal to the actual odds against the outcome of 13, or 37:1. So there is a net profit of $37 for each $1 bet.
For a $5 bet the net profit would be $185. (The casino makes its profit by paying only $175 instead of the $185 that would be paid with a roulette game that is fair instead of favoring the casino.)

Solution   *Section* 3.6 – Conditional Probability, Independent Events

Exercise

In building the space shuttle, NASA contracts for certain guidance components to be supplied by three different companies: 41% by company *A*, 25% by company *B*, and 34% by company *C*. It has been found that 1%, 1.75%, and 2% of the components from companies *A*, *B*, and *C*, respectively, are defective. If one of these guidance components is selected at random, what is the probability that it is defective?

Solution

D *= defective;* ***A*** *= company A;* ***B*** *= company B;* ***C*** *= company C*

$$\begin{aligned} P(D) &= P(A\cap D)\cup P(B\cap D)\cup P(C\cap D) \\ &= 0.41(.01)+0.25(.0175)+.34(.02) \\ &= \underline{.0153} \end{aligned}$$

Exercise

Suppose the probability of *A* is $P(A)=\frac{1}{4}$ and the probability of *B* is $P(B)=\frac{2}{3}$. What would the probability of *A* intersect *B* need to be for *A* and *B* to be independent events?

Solution

Since *A* and *B* to be independent events:

$$\begin{aligned} P(A\cap B) &= P(A)P(B) \\ &= \frac{1}{4}\frac{2}{3} \\ &= \underline{\frac{1}{6}} \end{aligned}$$

Exercise

In 2 throws of a fair die, what is the probability that you will get at least 5 on each throw? At least 5 on the first or second throw?

Solution

Let $A=$" At least 5 on the first throw". $\{5,6\}\to P(A)=\frac{2}{6}=\frac{1}{3}$

$B=$" At least 5 on the second throw". $\{5,6\}\to P(B)=\frac{2}{6}=\frac{1}{3}$

Since the events A and B are independent: $P(A\cap B)=P(A)\cdot P(B)$

$$\begin{aligned} P(A\cup B) &= P(A)+P(B)-P(A\cap B) \\ &= \frac{1}{3}+\frac{1}{3}-\frac{1}{3}\frac{1}{3} \\ &= \underline{\frac{5}{9}} \end{aligned}$$

Exercise

2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that the second ball was red, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

Solution

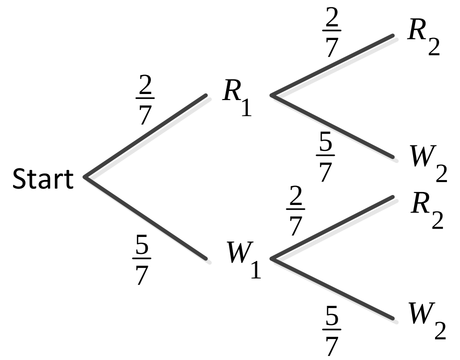
a) $P(R_2) = P(R_1 \cap R_2) + P(W_1 \cap R_2)$

$= P(R_1)P(R_2|R_1) + P(W_1)P(R_2|W_1)$

$= \frac{2}{7}\frac{2}{7} + \frac{5}{7}\frac{2}{7}$

$= \frac{14}{49}$

$= \frac{2}{7}$



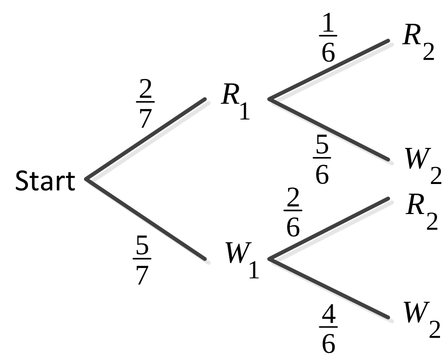
b) $P(R_2) = P(R_1 \cap R_2) + P(W_1 \cap R_2)$

$= P(R_1)P(R_2|R_1) + P(W_1)P(R_2|W_1)$

$= \frac{2}{7}\frac{1}{6} + \frac{5}{7}\frac{2}{6}$

$= \frac{12}{42}$

$= \frac{2}{7}$



Exercise

2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that at least 1 ball was red, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

Solution

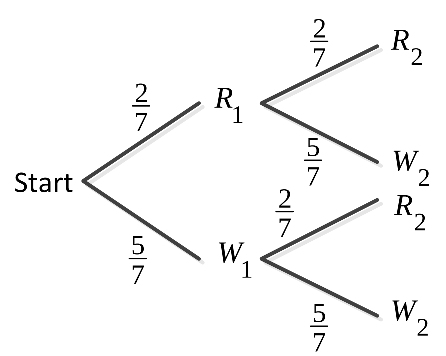
Let $E =$ " At least 1 ball was red".

a) With replacement:

$P(E) = P(R_1 \cap R_2) + P(R_1 \cap W_2) + P(W_1 \cap R_2)$

$= \frac{2}{7}\frac{2}{7} + \frac{2}{7}\frac{5}{7} + \frac{5}{7}\frac{2}{7}$

$= \frac{24}{49}$

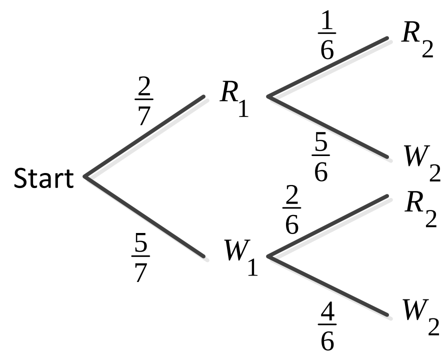


b) $P(E) = P\left(R_1 \cap R_2\right) + P\left(R_1 \cap W_2\right) + P\left(W_1 \cap R_2\right)$

$$= \frac{2}{7}\frac{1}{6} + \frac{2}{7}\frac{5}{6} + \frac{5}{7}\frac{2}{6}$$

$$= \frac{22}{42}$$

$$\boxed{= \frac{11}{21}}$$



Exercise

2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that both balls were the same color, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

Solution

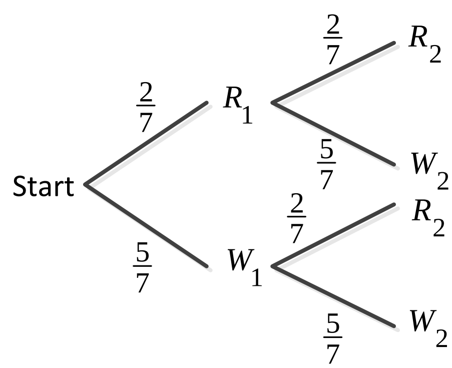
Let E = ″ both balls were the same color″.

a) With replacement:

$$P(E) = P\left(R_1 \cap R_2\right) + P\left(W_1 \cap W_2\right)$$

$$= \frac{2}{7}\frac{2}{7} + \frac{5}{7}\frac{5}{7}$$

$$\boxed{= \frac{29}{49}}$$



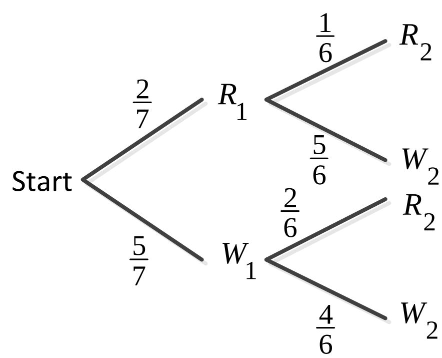
b) Without replacement:

$$P(E) = P\left(R_1 \cap R_2\right) + P\left(W_1 \cap W_2\right)$$

$$= \frac{2}{7}\frac{1}{6} + \frac{5}{7}\frac{4}{6}$$

$$= \frac{22}{42}$$

$$\boxed{= \frac{11}{21}}$$



Exercise

An automobile manufacturer produces 37% of its cars at plant A. If 5% of the cars manufactured at plant A have defective emission control devices, what is the probability that one of this manufacturer's cars was manufactured at plant A and has a defective emission control device?

Solution

Let A = "car is produced at plant A".
B = "car is defective".

$$P(A) = .37, \quad P(B|A) = .05$$

$$P(A \cap B) = P(A)P(B|A)$$

$$= (.37)(.05)$$

$$= .0185$$

Exercise

To transfer into a particular department, a company requires an employee to pass a screening test. A maximum of 3 attempts are allowed at 6-month intervals between trials. From past records it is found that 40% pass on the first trial; of those that fail the first trial and take the test a second time, 60% pass; and of those that fail on the second trial and take the test a third time, 20% pass. For an employee wishing to transfer:

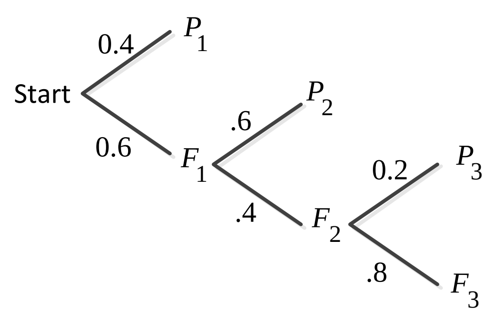
a) What is the probability of passing the test on the first or second try?
b) What is the probability of failing on the first 2 trials and passing on the third?
c) What is the probability of failing on all 3 attempts?

Solution

a) $P\left(\textit{passing } 1^{st} \textit{ or } 2^{nd} \textit{ try}\right) = P\left(P_1\right) + P\left(F_1 \cap P_2\right)$

$$= .4 + (.6)(.6)$$

$$= .76$$



b) $P\left(\textit{failing } 1^{st} \textit{ 2 trials or passing } 3^{nd}\right)$

$$= P\left(F_1 \cap F_2 \cap P_3\right)$$

$$= (.6)(.4)(.2)$$

$$= .048$$

c) $P\left(\textit{failing all trials}\right) = P\left(F_1 \cap F_2 \cap F_3\right)$

$$= (.6)(.4)(.8)$$

$$= .192$$

Exercise

A survey of the residents of a precinct in a large city revealed that 55% of the residents where members of the Democratic Party and that 60% of the Democratic Party members voted in the last election. What is the probability that a person selected at random from the residents of this precinct is a member of the Democratic Party and voted in the last election?

Solution

Let D = ″member of Democratic Party″.

V = ″voted in the last election″.

Then $P(D) = .55, \quad P(V|D) = .6$

$$\begin{aligned} P(D \cap V) &= P(D)P(V|D) \\ &= (.55)(.6) \\ &= 0.33 \end{aligned}$$

Exercise

In a two-child family, if we assume that the probabilities of a make child and a female child are each 0.5, are the events both children are the same sex and at most one male independent? Are they independent for a three-child family?

Solution

The sample space for 2 child is $\{MM, MF, FM, FF\}$

$P(same\ sex) = \frac{2}{4} = \frac{1}{2}$

$P(same\ sex | at\ most\ 1\ male) = \frac{1}{3}$

The events are not independent.

The sample space for 2 child is $\{MMM, MMF, MFM, FMM, MFF, FMF, FFM, FFF\}$

$P(same\ sex) = \frac{2}{8} = \frac{1}{4}$

$P(same\ sex | at\ most\ 1\ male) = \frac{1}{4}$

The events are dependent.

Exercise

Among users of ATMs, 92% use ATMs to withdraw cash, and 32% use them to check their account balance. Suppose that 96% use ATMs to either withdraw cash or check their account balance (or both). Given a woman who uses an ATM to check her account balance, what is the probability that she also uses an ATM to get cash?

Solution

Let ***W*** be the event withdraw cash from ATM

C be the vent "check account balance

$$P(C \cup W) = P(C) + P(W) - P(C \cap W)$$

$$0.96 = 0.32 + 0.92 - P(C \cap W)$$

$$0.96 = 1.24 - P(C \cap W)$$

$$P(C \cap W) = 1.24 - 0.96 \underline{= 0.28}$$

$$P(W|C) = \frac{P(C \cap W)}{P(C)} = \frac{0.28}{0.32} \underline{\approx 0.875}$$

The probability that she uses an ATM to get cash given that she checked her account balance is 0/875

Exercise

A car factory runs two assembly lines, ***A*** and ***B***. If 95% of line A's products pass inspection, while only 85% of line ***B***'s products pass inspection, and 60% of the factory's cars come off assembly line ***A*** (the rest off ***B***), find the probabilities that one of the factory's cars did not pass inspection and came off the following.

a) Assembly line ***A***

b) Assembly line ***B***

c) Find the probability that one of the factory's cars did not pass inspection.

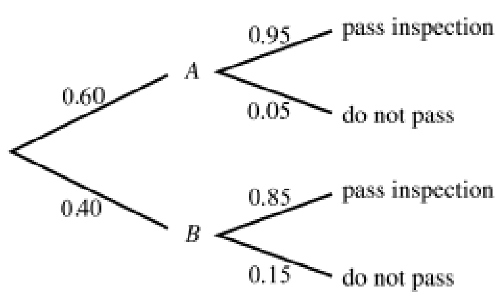
Solution

a) Given: $P(A) = 0.6;\quad P(\textit{pass inspection}|A) = 0.95;\quad P(\textit{not pass}|A) = 0.05$

$$P(A \cap \textit{not pass}) = P(A) \cdot P(\textit{not pass}|A)$$

$$= (0.6)(0.05)$$

$$\underline{= 0.03}$$



b) Since 40% of the production comes from B $P(B) = 0.4$

$$P(\textit{not pass} \cap B) = P(B) \cdot P(\textit{not pass}|B)$$

$$= (0.4)(0.15)$$

$$\underline{= 0.06}$$

c) $P(\textit{car did not pass inspection}) = (0.6)(0.05) + (0.4)(0.15)$

$$\underline{= 0.09}$$

Solution Section 3.7 – Probability Applications of Counting

Exercise

A basket contains 7 red apples and 4 yellow apples. A sample of 3 apples is drawn. Find the probabilities that the sample contains the following.

a) All red apples
b) All yellow apples
c) 2 yellow and 1 red apple
d) More red than yellow apples

Solution

a) $P(\text{All red apples}) = \dfrac{C_{7,3}}{C_{11,3}} = \dfrac{7}{33}$

b) $P(\text{All yellow apples}) = \dfrac{C_{4,3}}{C_{11,3}} = \dfrac{4}{165}$

c) $P(\text{1 red \& 2 yellow apples}) = \dfrac{C_{7,1}C_{4,2}}{C_{11,3}} = \dfrac{42}{165} = \dfrac{14}{55}$

d) $P(\text{red} > \text{yellow}) = \dfrac{C_{7,2}C_{4,1} + C_{7,3}C_{4,0}}{C_{11,3}} = \dfrac{119}{165}$

Exercise

Two cards are drawn at random from a ordinary deck of 52. How many 2-card hands are possible?

Solution

$C_{52,2} = 1326$ *possibilities*

Exercise

Find the probability that the 2-card hand contains the following.

a) 2 aces
b) At least 1 ace
c) All spades
d) 2 cards of the same suit
e) Only face cards
f) No face cards
g) No card higher than 8 (count ace as 1)

Solution

a) $P(2\ aces) = \dfrac{C_{4,2}}{C_{52,2}} = \dfrac{6}{1326} \approx 0.0045$

b) $P(>1\ ace) = \dfrac{C_{4,1}C_{48,1} + C_{4,2}C_{48,0}}{C_{52,2}} \approx 0.149$

c) $P(2\ spades) = \dfrac{C_{13,2}}{C_{52,2}} \approx 0.059$

d) $P(2\ same\ suit) = \dfrac{4C_{13,2}}{C_{52,2}} \approx 0.235$

e) $P(\ face\ cards) = \dfrac{C_{12,2}}{C_{52,2}} \approx 0.0498$

f) $P(No\ face\ cards) = \dfrac{C_{40,2}}{C_{52,2}} \approx 0.588$

g) $P(<8) = \dfrac{C_{32,2}}{C_{52,2}} \approx 0.374$

Exercise

A reader wrote to the "Ask Marilyn" column in a magazine. "You have six envelopes to pick from. Two-thirds (= 4) are empty. One-third (= 2) contain a \$100 bill. You're told to choose 2 envelopes at random. Which is more likely: **(1)** that you'll get at least one \$100 bill, or **(2)** that you'll get no \$100 bill at all?" Find the two probabilities.

Solution

$$P(at\ least\ one\ \$100\text{–}bill) = P(1\ \$100\text{–}bill) + P(2\ \$100\text{–}bill)$$

$$= \frac{C_{2,1}C_{4,1} + C_{2,2}C_{4,0}}{C_{6,2}}$$

$$= 0.6$$

$$P(no\ \$100\text{–}bill) = \frac{C_{2,0}C_{4,0}}{C_{6,2}} = 0.4$$

Exercise

After studying all night for a final exam, a bleary-eyed student randomly grabs 2 socks from a drawer containing 9 black, 6 brown, and 2 blue socks, all mixed together. What is the probability that she grabs a matched pair?

Solution

$$P(matched\ pair) = P(2\ black\ or\ 2\ brown\ or\ 2\ blue)$$

$$= P(2\ black) + P(2\ brown) + P(2\ blue)$$

$$= \frac{C_{9,2} + C_{6,2} + C_{2,2}}{C_{17,2}}$$

$$= 0.38$$

Exercise

At a conference of writers, special-edition books were selected to be given away in contests. There were 9 books written by Hughes, 5 books by Baldwin, and 7 books by Morrison. The judge of one contest selected 6 books at random for prizes. Find the probabilities that he selection consisted of the following.

a) 3 Hughes and 3 Morrison books
b) Exactly 4 Baldwin books
c) 2 Hughes, 3 Baldwin, and 1 Morrison book
d) At least 4 Hughes books
e) Exactly 4 books written by males (Morrison is female)
f) No more than 2 books written by Baldwin

Solution

a) $P(3H\ \&\ 3M) = \dfrac{C_{9,3}C_{7,3}}{C_{21,6}} \approx 0.0542$

b) $P(4B) = \dfrac{C_{5,4}C_{16,2}}{C_{21,6}} \approx 0.0111$

c) $P(2H, 3B, \&1M) = \dfrac{C_{9,2}C_{5,3}C_{7,1}}{C_{21,6}} \approx 0.0464$

d) $P(> 4H) = \dfrac{C_{9,4}C_{12,2} + C_{9,5}C_{12,1} + C_{9,6}C_{12,0}}{C_{21,6}} \approx 0.1827$

e) $P(4\ by\ males) = \dfrac{C_{14,4}C_{7,2}}{C_{21,6}} \approx 0.3874$

f) $P(< 2\ by\ B) = \dfrac{C_{5,2}C_{16,4} + C_{5,1}C_{16,5} + C_{5,0}C_{16,6}}{C_{21,6}} \approx 0.8854$

Exercise

A school in Bangkok requires that students take an entrance examination. After the examination, there is a drawing in which 5 students are randomly selected from each group of 40 for automatic acceptance into the school, regardless of their performance on the examination. The drawing consists of pacing 35 red and 5 green pieces of paper into a box. Each student picks a piece of paper from the box and then does not return the piece of paper to the box. The 5 lucky students who pick the green pieces are automatically accepted into the school.

a) What is the probability that the first person wins automatic acceptance?
b) What is the probability that the last person wins automatic acceptance?
c) If the students are chosen by the order of their seating does this give the student who goes first a better chance of winning than the second, third… person?

(*Hint*: Imagine that the 40 pieces of paper have been mixed up and laid in a row so that the first student picks the first piece of paper, the second student picks the second piece of paper, and so on.)

Solution

a) $P(\textit{first person}) = \frac{5}{40} = \frac{1}{8}$

b) $P(\textit{last person}) = \frac{5(39!)}{40!}$

$$= \frac{5}{40}$$

$$= \frac{1}{8}$$

c) No one can have the same chance.

Exercise

A controversy arose in 1992 over the Teen Talk Barbie doll, each of which was programmed with four saying randomly picked from a set of 270 sayings. The controversy was over the saying, "Math class is tough," which some felt gave a negative message toward girls doing well in math. In an interview with Science, a spokeswoman for Mattel, the makers of Barbie, said that "There is a less than 1% chance you're going to get a doll that says math class is tough". Is this figure correct? If not, give the correct figure.

Solution

$$P(\textit{Math class is tough}) = \frac{\binom{1}{1}\binom{269}{3}}{\binom{270}{4}} \approx .0148$$

No, it is not correct.

The correct figure is 1.48%

Exercise

Bingo has become popular in the U.S., and it is an efficient way for many organizations to raise money. The bingo card has 5 rows and 5 columns of numbers from 1 to 75, with the center given as a free cell. Balls showing one of the 75 numbers are picked at random from a container. If the drawn number appears on a player's card, then the player covers the number. In general, the winner is the person who first has a card with an entire row, column, or diagonal covered.

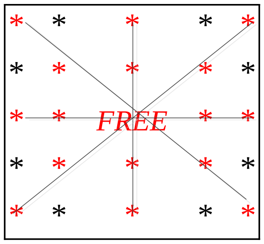
a) Find the probability that a person will win bingo after just four numbers are called.

b) An L occurs when the first column and the bottom row are both covered. Find the probability that an L will occur in the fewest number of calls.

c) An X-out occurs when both diagonals are covered. Find the probability than an X-out occurs in the fewest number of calls.

d) If bingo cards are constructed so that column one has 5 of the numbers from 1 to 15, column two has 5 of the numbers from 16 to 30, column three has 4 of the numbers from 31 to 45, column four has 5 of the numbers from 46 to 60, column five has 5 of the numbers from 61 to 75, how many different bingo cards could be constructed? (*Hint*: Order matters!)

Solution



a) There are only 4 ways to win in just 4 calls:

There are $C_{75,4}$ combinations of 4 numbers that can occur.

$$P(\text{win bingo}) = \frac{4}{C_{75,4}} \approx 3.291\times 10^{-6}$$

b) There is only 1 way to get an L. It can occur in as few as 9 calls.

There are $C_{75,9}$ combinations of 9 numbers.

$$P(L\ \text{occurs}) = \frac{1}{C_{75,9}} \approx 7.962\times 10^{-12}$$

c) There is only 1 way to get an X-out. It can occur in as few as 8 calls.

There are $C_{75,8}$ combinations of 8 numbers.

$$P(X-\text{out occurs}) = \frac{1}{C_{75,8}} \approx 5.927\times 10^{-11}$$

d) Four columns contain a permutation of 15 numbers taken 5 at a time. One column contains a permutation of 15 numbers taken 4 at a time.

$$\text{Number of differnt cards} = P^4_{15,5} \bullet P_{15,4} \approx 5.524\times 10^{26}$$

Solution Section 3.8 – Bayes' Theorem

Exercise

One urn has 4 red balls and 1 white ball; a second urn has 2 red balls and 3 white balls. A single card is randomly selected from a standard deck. If the card is less than 5 (aces count as 1), a ball is drawn out of the first urn; otherwise a ball is drawn out of the second urn. If the drawn ball is red, what is the probability that it came out of the second urn?

Solution

U1= selected from urn 1; U2 = selected from urn 2; R= red selected

$$P\left(U_2 \mid R\right)=\frac{P\left(U_2 \cap R\right)}{P(R)}$$

$$=\frac{P\left(U_2\right)P\left(R \mid U_2\right)}{P\left(U_2\right)P\left(R \mid U_2\right)+P\left(U_1\right)P\left(R \mid U_1\right)}$$

$$=\frac{\frac{9}{13}\cdot\frac{2}{5}}{\frac{9}{13}\cdot\frac{2}{5}+\frac{4}{13}\cdot\frac{4}{5}}$$

$$=\frac{\frac{18}{65}}{\frac{34}{65}}$$

$$=0.53$$

Exercise

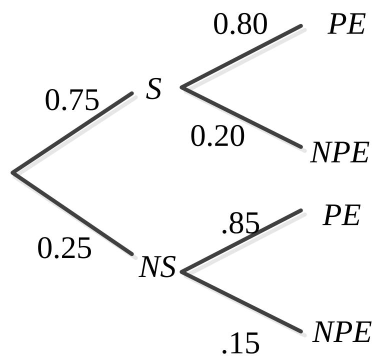
A small manufacturing company has rated 75% of its employees as satisfactory (S) and 25% as unsatisfactory (S'). Personnel records show that 80% of the satisfactory workers had previous work experience (E) in the job they are now doing, while 15% of the unsatisfactory workers had no work experience (E') in the job they are now doing. If a person who has had previous work experience is hired, what is the approximate empirical probability that this person will be an unsatisfactory employee?

Solution

$$P(S' \mid E)=\frac{P(S')P(E \mid S')}{P(S')P(E \mid S')+P(S)P(E \mid S)}$$

$$=\frac{(.25)\cdot(0.85)}{(0.25)(0.85)+(0.75)\cdot(0.80)}$$

$$=0.26$$



Exercise

A basketball team is to play two games in a tournament. The probability of winning the first game is .10. If the first game is won, the probability of winning the second game is .15. If the first game is lost, the probability of winning the second game is .25. What is the probability the first game was won if the second game is lost?

Solution

*W*1 *= win first game; L*1*= lose first game; L*2*=lose second game*

$$P(W1 \mid L2) = \frac{P(W1)P\left(L2 \mid W1\right)}{P(W1)P(L2 \mid W1) + P(L1)P(L2 \mid L1)}$$

$$= \frac{(.1)\cdot(0.85)}{(0.1)(0.85) + (0.9)\cdot(0.75)}$$

$$= 0.112$$

Exercise

To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

Solution

H = has Hansen's; H' = does not have Hansen's; T = test says has Hansen's

$$P(H \mid T) = \frac{P(H)P(T \mid H)}{P(H)P(T \mid H) + P(H')P(T \mid H')}$$

$$= \frac{(.05)\cdot(0.98)}{(0.05)(0.98) + (0.95)\cdot(0.03)}$$

$$= 0.632$$

Exercise

An urn contains 4 red and 5 white balls. Two balls are drawn in succession without replacement. If the second ball is white, what is the probability that the first ball was white?

Solution

$$P\left(W_1 \mid W_2\right) = \frac{P\left(W_1\right)P\left(W_2 \mid W_1\right)}{P\left(R_1\right)P\left(W_2 \mid R_1\right)+P\left(W_1\right)P\left(W_2 \mid W_1\right)}$$

$$= \frac{\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)}{\left(\frac{4}{9}\right)\left(\frac{5}{8}\right)+\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)}$$

$$= 0.5$$

Exercise

An urn contains 4 red and 5 white balls. Two balls are drawn in succession without replacement. If the second ball is red, what is the probability that the first ball was red?

Solution

$$P\left(R_1 \mid R_2\right)=\frac{P\left(R_1\right)P\left(R_2 \mid R_1\right)}{P\left(R_1\right)P\left(R_2 \mid R_1\right)+P\left(W_1\right)P\left(R_2 \mid W_1\right)}$$

$$=\frac{\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)}{\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)+\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)}$$

$$=\frac{3}{8}$$

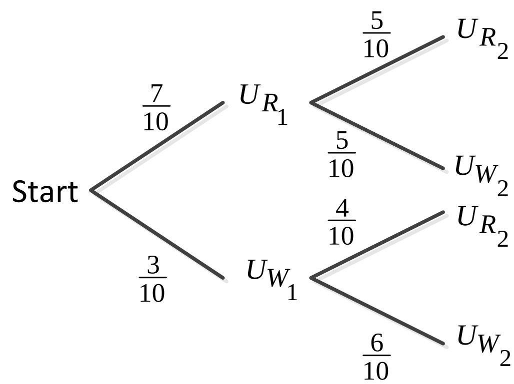
Exercise

Urn 1 contains 7 red and 3 white balls. Urn 2 contains 4 red and 5 white balls. A ball is drawn from urn 1 and placed in urn 2. Then a ball is drawn from urn2. If the ball drawn from urn 2 is red, what is the probability that the ball drawn from urn 1 was red?

Solution

$$P\left(R_1 \mid R_2\right)=\frac{\left(\frac{7}{10}\right)\left(\frac{5}{10}\right)}{\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)+\left(\frac{7}{10}\right)\left(\frac{5}{10}\right)}$$

$$\approx .745$$



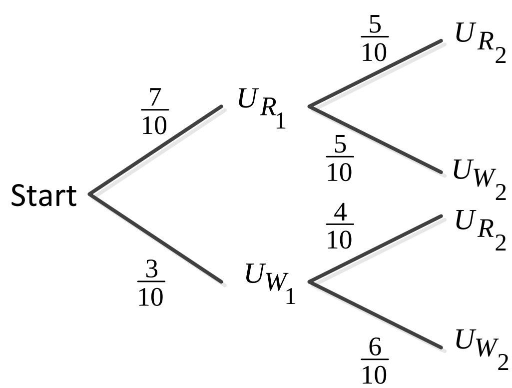
Exercise

Urn 1 contains 7 red and 3 white balls. Urn 2 contains 4 red and 5 white balls. A ball is drawn from urn 1 and placed in urn 2. Then a ball is drawn from urn2. If the ball drawn from urn 2 is white, what is the probability that the ball drawn from urn 1 was white?

Solution

$$P\left(W_1 \mid W_2\right)=\frac{\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)}{\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)+\left(\frac{7}{10}\right)\left(\frac{5}{10}\right)}$$

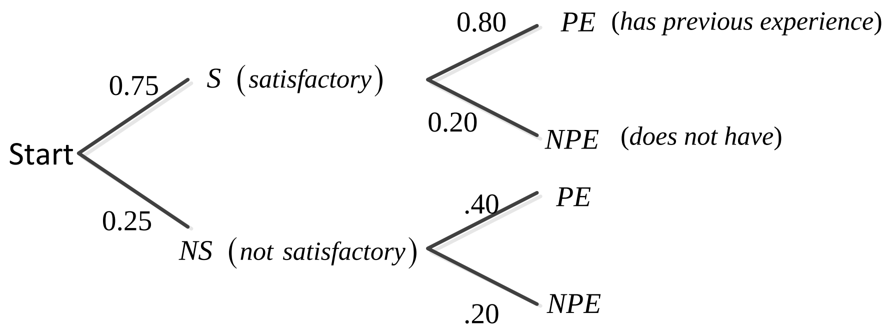
$$\approx .34$$



Exercise

A company has rated 75% of its employees as satisfactory and 25% as unsatisfactory. Personnel records indicate that 80% of the satisfactory workers had previous work experience, while only 40% of the unsatisfactory workers had any previous work experience. If a person with previous work experience is hired, what is the probability that this person will be a satisfactory employee? If a person with no previous work experience is hired, what is the probability that this person will be a satisfactory employee?

Solution



$$P(S \mid A) = \frac{(0.75)(.80)}{(0.75)(.80)+(.25)(.40)} \approx 0.86$$

$$P(S \mid A') = \frac{(0.75)(.20)}{(.75)(.20)+(.25)(.60)} \approx 0.50$$

Exercise

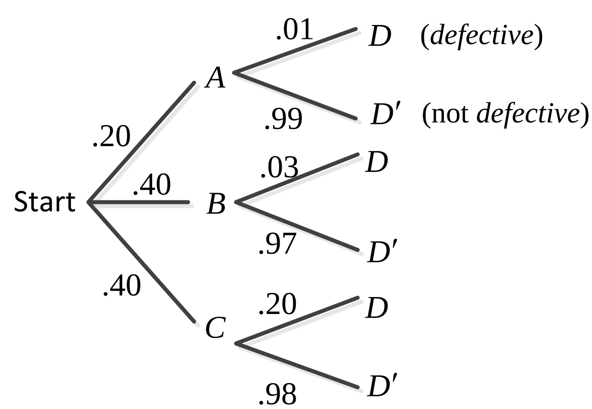
A manufacturer obtains clock-radios from three different subcontractors: 20% from A, 40% from B, and 40% from C. The defective rates for these subcontractors are 1%, 3%, and 2%, respectively. If a defective clock-radio is returned by a customer, what is the probability that it came from subcontractor A? From B? From C?

Solution

$$P(A \mid D) = \frac{(0.2)(.01)}{(0.2)(.01)+(0.4)(.03)+(0.4)(.02)} = 0.91$$

$$P(B \mid D) = \frac{(0.4)(.03)}{(0.2)(.01)+(0.4)(.03)+(0.4)(.02)} = 0.545$$

$$P(B \mid D) = \frac{(0.4)(.02)}{(0.2)(.01)+(0.4)(.03)+(0.4)(.02)} = 0.364$$



Exercise

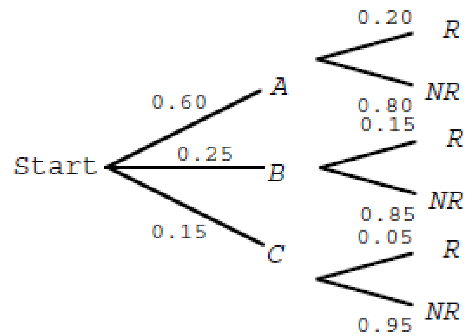
A computer store sells three types of microcomputer, brand A, brand B, brand C. Of the computers sell, 60% are brands A, 25% are brand B, 15% are brand C. They have found that 20% of the brand A computers, 15% of the brand B computers, and 5% of the brand C computers are returned for service during the warranty period. If a computer is returned for service during the warranty period, what is the probability that it is a brand A computer, A brand B computer? A brand C computer?

Solution

$$P(A \mid R) = \frac{(.6)(0.2)}{(.6)(0.2)+(.25)(0.15)+(.15)(0.05)} \underline{=0.73}$$

$$P(B \mid R) = \frac{(.25)(0.15)}{(.6)(0.2)+(.25)(0.15)+(.15)(0.05)} \underline{=0.23}$$

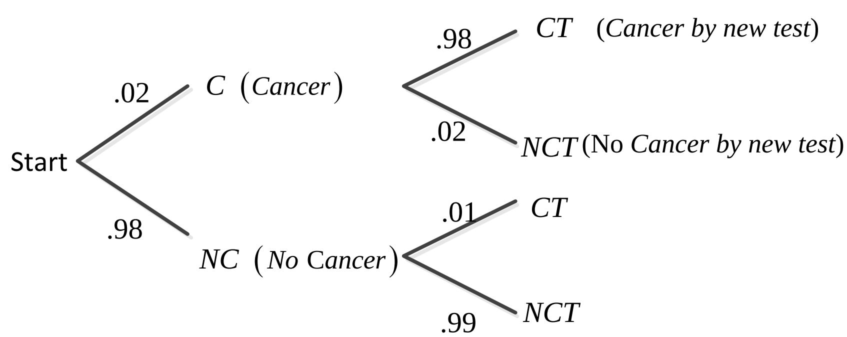
$$P(A \mid R) = \frac{(.15)(0.05)}{(.6)(0.2)+(.25)(0.15)+(.15)(0.05)} \underline{=0.05}$$



Exercise

A new, simple test has been developed to detect a particular type of cancer. The test must be evaluated before it is put into use. A medical researcher selects a random sample of 1,000 adults and finds (by other means) that 2% have this type of cancer. Each of the 1,000 adults is given test, and it is found that the test indicates cancer in 98% of those who have it and in 1% of those who do not. Based on these results, what is the probability of a randomly chosen person having cancer given that the test indicates cancer? Of a person having cancer given that the test does not indicate cancer?

Solution



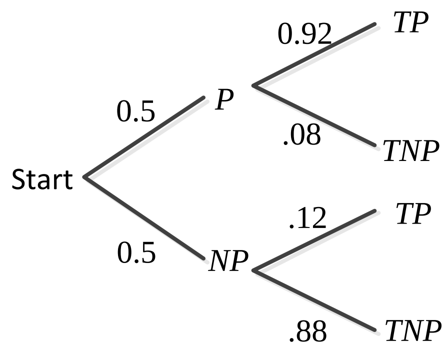
$$P(C \mid CT) = \frac{(0.02)(.98)}{(0.02)(.98)+(.98)(.01)} \underline{\approx 0.667}$$

$$P(C \mid NCT) = \frac{(0.02)(.02)}{(0.02)(.02)+(.98)(.99)} \underline{\approx 0.000412}$$

Exercise

In a random sample of 200 women who suspect that they are pregnant, 100 turn out to be pregnant. A new pregnancy test given to these women indicated pregnancy in 92 of the 100 pregnant women and in 12 of the 100 non-pregnant women. If a woman suspects she is pregnant and this test indicates that she is pregnant, what is the probability that she is pregnant? If the test indicates that she is not pregnant, what is the probability that she is not pregnant?

Solution



$$P(P \mid TP) = \frac{(0.5)(.92)}{(0.5)(.92)+(.5)(.12)} \approx 0.88$$

$$P(NP \mid NTP) = \frac{(0.5)(.88)}{(0.5)(.88)+(.5)(.08)} \approx 0.92$$

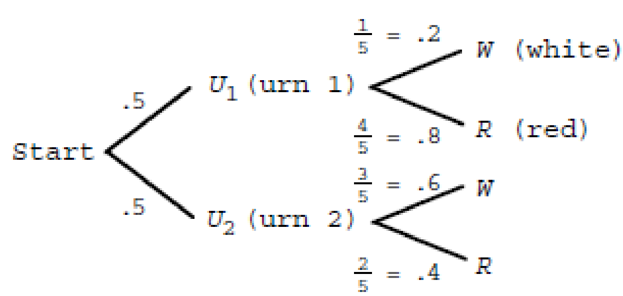
Exercise

One of two urns is chosen at random with one as likely to be chosen as the other. Then a ball is drawn from the chosen urn, Urn 1 contains 1 white and 4 red balls, and urn 2 has 3 white and 2 red balls.

a) If a white ball is drawn, what is the probability that it came from urn 1?
b) If a white ball is drawn, what is the probability that it came from urn 2?
c) If a red ball is drawn, what is the probability that it came from urn 2?
d) If a red ball is drawn, what is the probability that it came from urn 1?

Solution

a)
$$\begin{aligned}P(U_1 \mid W) &= \frac{P(U_1 \cap W)}{P(U_1 \cap W)+P(U_2 \cap W)} \\ &= \frac{P(U_1)P(W \mid U_1)}{P(U_1)P(W \mid U_1)+P(U_2)P(W \mid U_2)} \\ &= \frac{(.5)(.2)}{(.5)(.2)+(.5)(.6)} \\ &= .25\end{aligned}$$



b) $P(U_2 \mid W) = \dfrac{(.5)(.6)}{(.5)(.2)+(.5)(.6)} = .75$

c) $P(U_2 \mid R) = \dfrac{(.5)(.4)}{(.5)(.4)+(.5)(.8)} = .333$

d) $P(U_1 \mid R) = \dfrac{(.5)(.8)}{(.5)(.8)+(.5)(.4)} = 0.67$