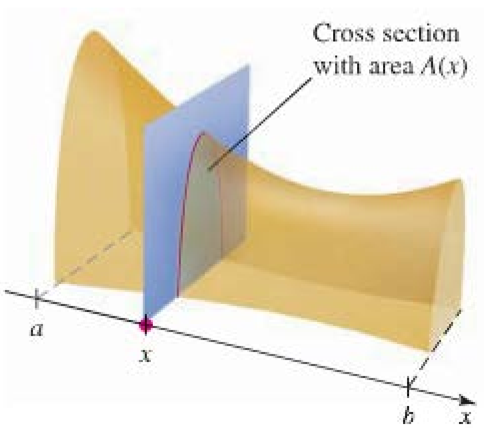
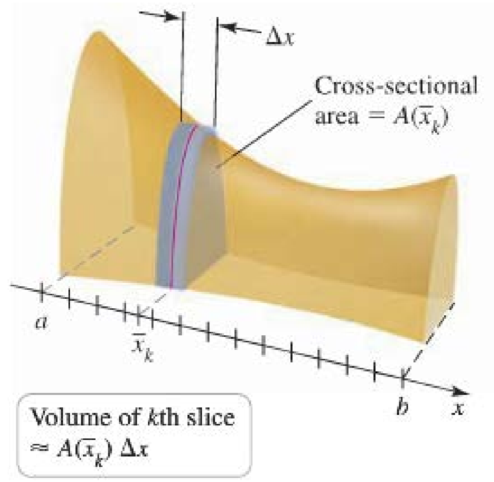
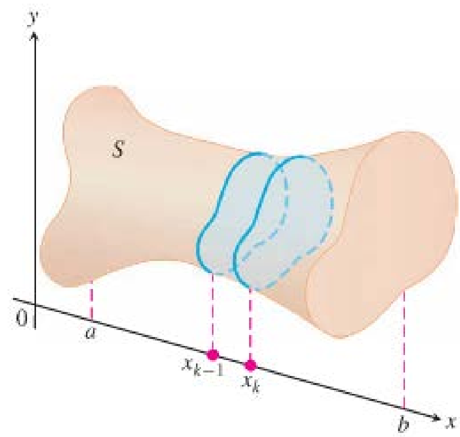
Section 1.3 – Volumes by Slicing

If we want to find a volume of a solid S and if the cylindrical solid has known base area A and height h, then the volume of the cylindrical solid is

$$Volume = area \times height = A \bullet h$$



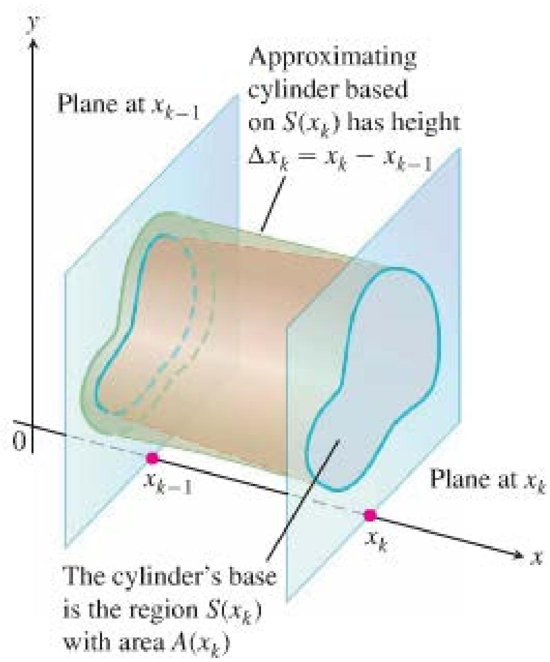




Definition

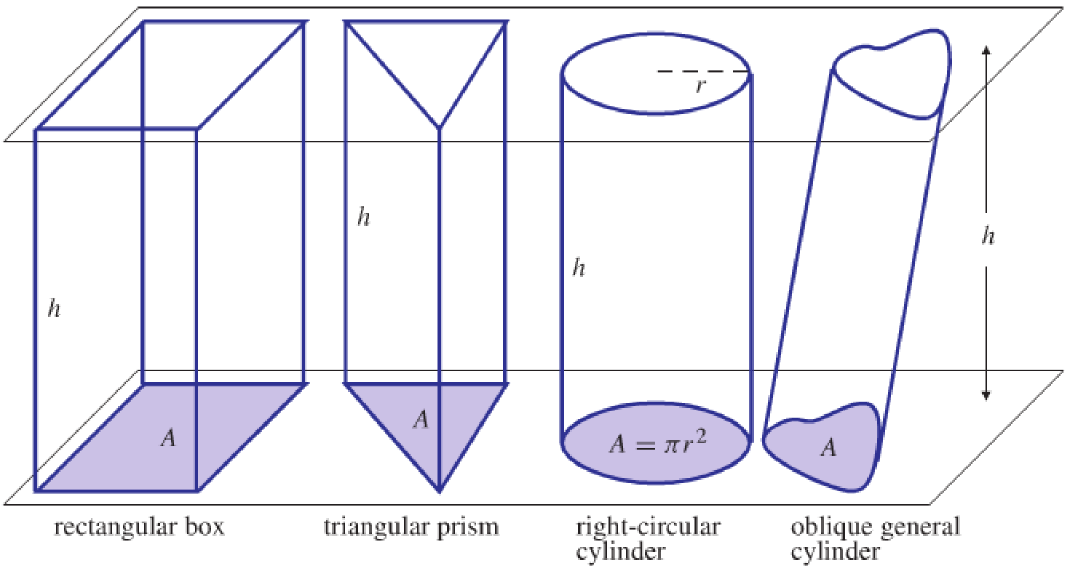
The volume of a solid of integrable cross-sectional area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b

$$V = \int_a^b A(x)\,dx$$



Calculating the volume of a solid

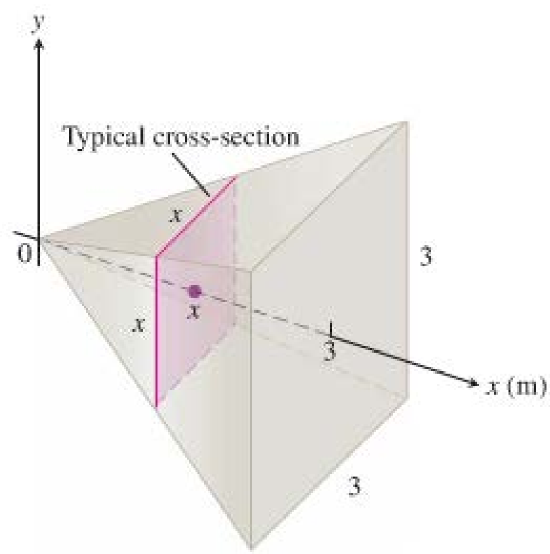
1. Sketch the solid and a typical cross-section
2. Find a formula for $A(x)$, the area of a typical cross-section
3. Find the limits of integration
4. Integrate $A(x)$ to find the volume



Example

A pyramid 3*m* high has a square base that is 3*m* on a side. The cross-section of the pyramid perpendicular to the altitude ***x*** *m* down from the vertex is a square ***x*** *m* on a side. Find the volume of the pyramid.

Solution



The area of the square is given by the formula:

$A(x) = x^2$

The volume: $V = \int_0^3 A(x)\,dx$

$= \int_0^3 x^2 dx$

$= \frac{1}{3}x^3 \Big]_0^3$

$= \frac{1}{3}\left(3^3 - 0\right)$

$= 9 \ \ m^3$

Example

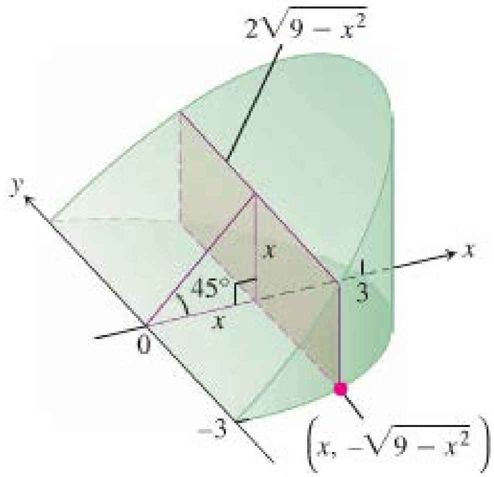
A curved wedge is cut from a circular cylinder of radius 3 by two planes. On plane is perpendicular to the axis of the cylinder. The second place crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.

Solution

The base of the cylinder is a circle $x^2 + y^2 = 9$.

Since the second plane cut the base at the cylinder at the center, therefore, the base of the wedge is semi-circle. $y = \pm\sqrt{9 - x^2} = radius$.

When we slice the wedge by a plane perpendicular to the axis of the cylinder, we obtained a cross-section at *x* which is a rectangle of height *x*.



The area of this cross-section is: $A(x) = height \times width$

$$= x\left(2\sqrt{9 - x^2}\right)$$

$$= 2x\sqrt{9 - x^2}$$

The rectangles run from $x = 0$ to $x = 3$, so

$$V = \int_0^3 A(x)dx$$

$$= \int_0^3 2x\sqrt{9 - x^2}dx \qquad or\ \ u = 9 - x^2 \quad \rightarrow \quad du = -2xdx$$

$$= -\int_0^3 \left(9 - x^2\right)^{1/2} d\left(9 - x^2\right)$$

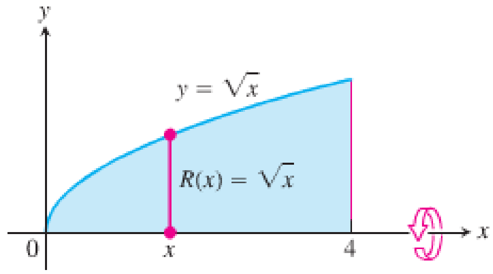
$$= -\frac{2}{3}\left[\left(9 - x^2\right)^{3/2}\right]_0^3$$

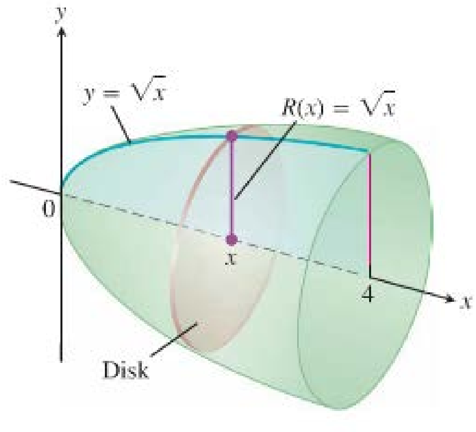
$$= -\frac{2}{3}\left[\left(9 - 3^2\right)^{3/2} - \left(9 - 0^2\right)^{3/2}\right]$$

$$= 18\ \ unit^3$$

Solids of Revolution: The Disk Method

The solid generated by rotating (or revolving) a plane region about an axis in its plane is called a ***solid of revolution***.





The cross-sectional area $A(x)$ is the area of a disk of radius $R(x)$, the distance of the planar region's boundary from the axis of revolution. The area then

$$A(x) = \pi(radius)^2 = \pi[R(x)]^2$$

And the volume

$$V = \int_a^b A(x)dx = \int_a^b \pi[R(x)]^2 dx$$

Example

The region between the curve $y = \sqrt{x}, \quad 0 \le x \le 4$, and the *x*-axis is revolved about the *x*-axis to generate a solid. Find its volume.

Solution

$$V = \int_a^b \pi[R(x)]^2 dx \quad = \pi \int_0^4 [\sqrt{x}]^2 dx$$

$$= \pi \int_0^4 x dx$$

$$= \pi \left[\frac{x^2}{2} \right]_0^4$$

$$= \frac{\pi}{2}\left[4^2 - 0\right]$$

$$\boxed{= 8\pi \ \ unit^3}$$

Example

The circle $x^2+y^2=a^2$ is rotated about x-axis to generate a sphere. Find its volume

Solution

$$A(x)=\pi y^2=\pi\left(a^2-x^2\right)$$

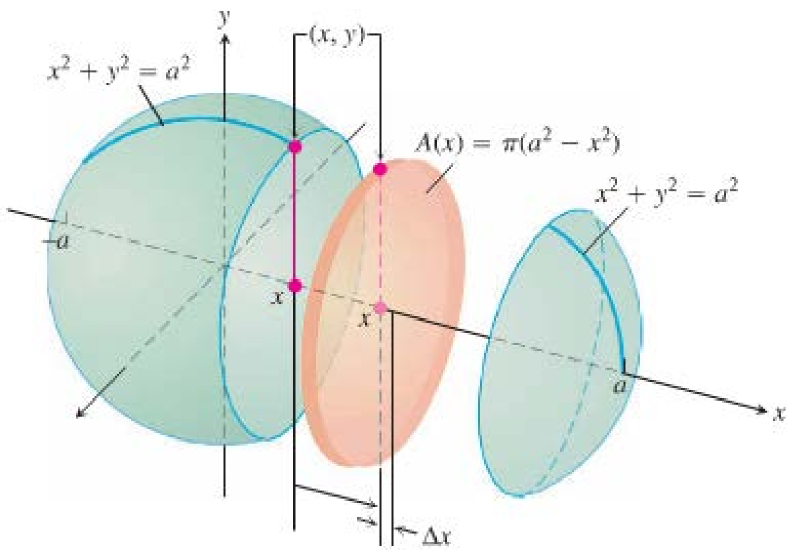
$$V=\int_{-a}^{a}\pi\left(a^2-x^2\right)dx$$

$$=2\pi\left[a^2x-\frac{1}{3}x^3\right]_0^a$$

$$=2\pi\left(a^2(a)-\frac{1}{3}a^3\right)$$

$$=2\pi\left(\frac{2}{3}a^3\right)$$

$$=\frac{4}{3}\pi a^3\ \ unit^3$$



Example

The region R is bounded by the graphs of $f(x)=\sqrt{x}$ and $g(x)=x^2$ between $x=0$ and $x=1$. What is the volume of the solid that results when R is revolved about the x-axis?

Solution

$$V=\pi\int_0^1\left(f(x)^2-g(x)^2\right)dx$$

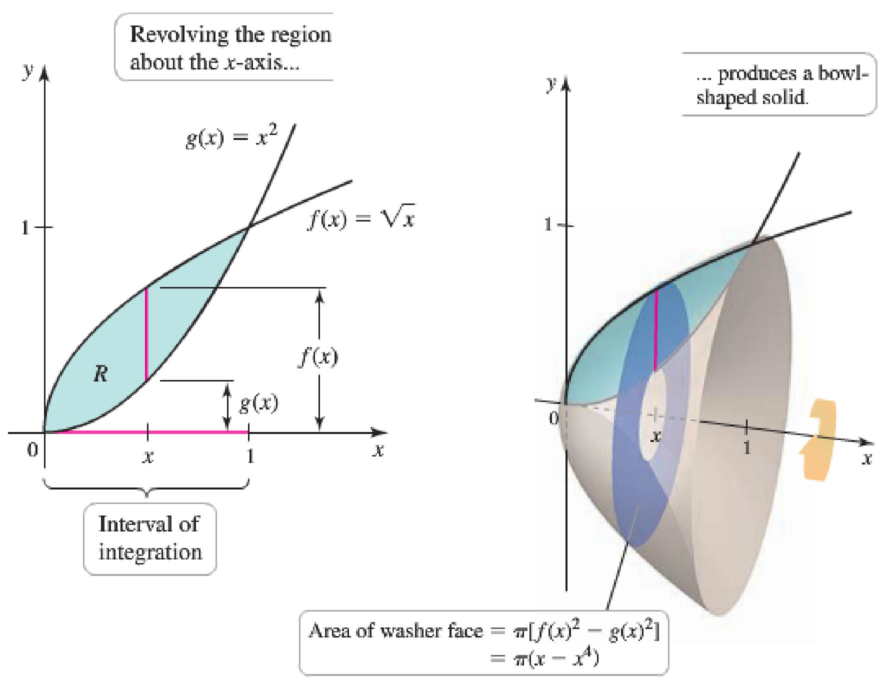
$$=\pi\int_0^1\left(\left(\sqrt{x}\right)^2-\left(x^2\right)^2\right)dx$$

$$=\pi\int_0^1\left(x-x^4\right)dx$$

$$=\pi\left(\frac{x^2}{2}-\frac{x^5}{5}\right)\Big|_0^1$$

$$=\pi\left(\frac{1}{2}-\frac{1}{5}\right)$$

$$=\frac{3\pi}{10}\ \ unit^3$$



Volume by Disks for Rotation about the *y*-axis

$$V=\int_c^d A(y)\,dy=\int_c^d \pi\left[R(y)\right]^2 dy$$

Example

Find the volume of the solid generated by revolving the region between the *y*-axis and the curve $x=\frac{2}{y},\quad 1\le y\le 4$ about the *y*-axis.

Solution

$$V=\int_1^4 \pi\left[R(y)\right]^2 dy$$

$$=\pi\int_1^4 \left(\frac{2}{y}\right)^2 dy$$

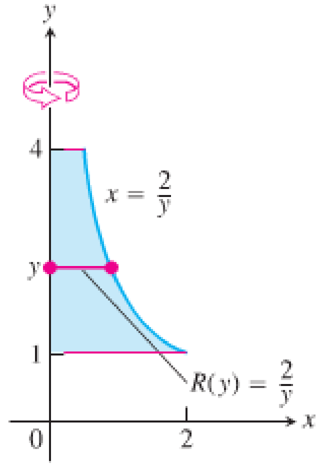
$$=\pi\int_1^4 \frac{4}{y^2}dy$$

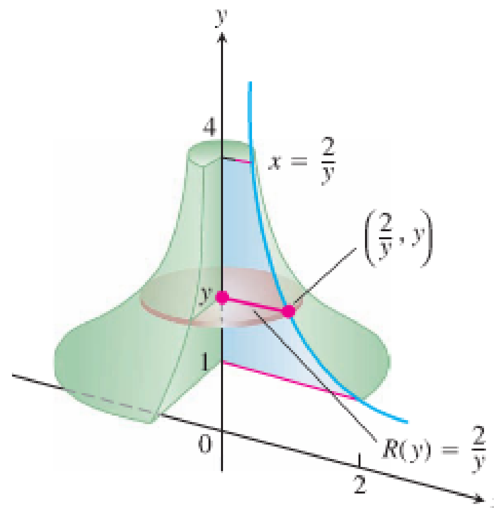
$$=4\pi\left[-\frac{1}{y}\right]_1^4$$

$$=4\pi\left[-\frac{1}{4}-\left(-\frac{1}{1}\right)\right]$$

$$=4\pi\left(\frac{3}{4}\right)$$

$$=3\pi\ \ unit^3$$

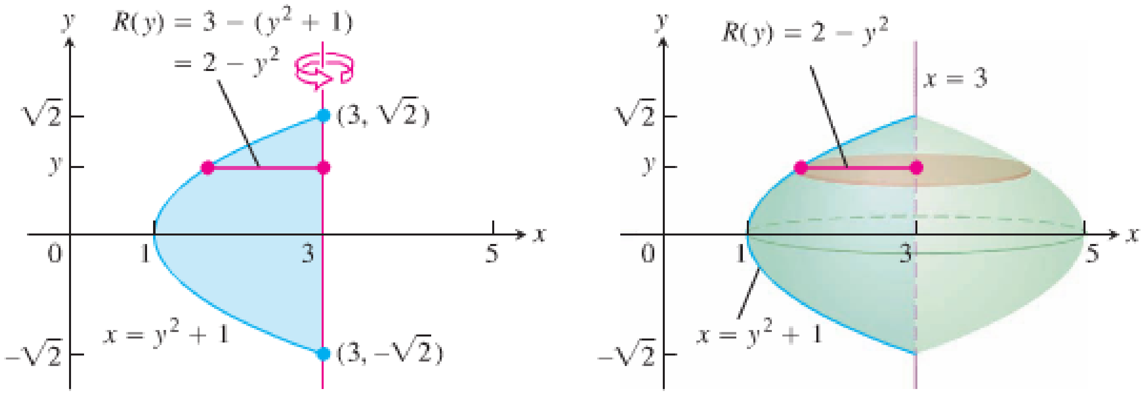




Example

Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

Solution



$$x = y^2 + 1 \quad \to \quad y^2 = x - 1$$

When $x = 3 \quad \Rightarrow \quad y^2 = 2 \to y = \pm\sqrt{2}$

$$V = \int_{-\sqrt{2}}^{\sqrt{2}} \pi \left[R(y)\right]^2 dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left[3 - \left(y^2 + 1\right)\right]^2 dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left[2 - y^2\right]^2 dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left(4 - 4y^2 + y^4\right) dy \qquad \textit{Even Function}$$

$$= 2\pi \left[4y - \frac{4}{3}y^3 + \frac{1}{5}y^5\right]_0^{\sqrt{2}}$$

$$= 2\pi \left[4\left(\sqrt{2}\right) - \frac{4}{3}\left(\sqrt{2}\right)^3 + \frac{1}{5}\left(\sqrt{2}\right)^5\right]$$

$$= 2\pi \left(4\sqrt{2} - \frac{8}{3}\sqrt{2} + \frac{4}{5}\sqrt{2}\right)$$

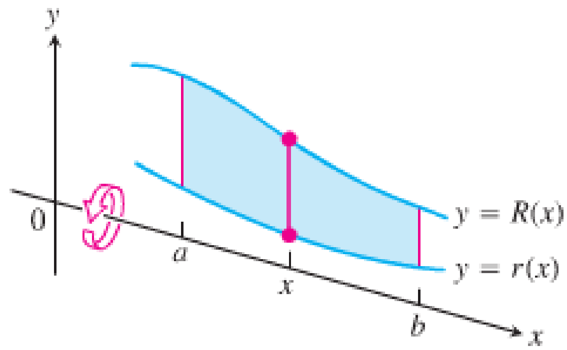
$$= \frac{64\pi\sqrt{2}}{15} \ \ unit^3$$

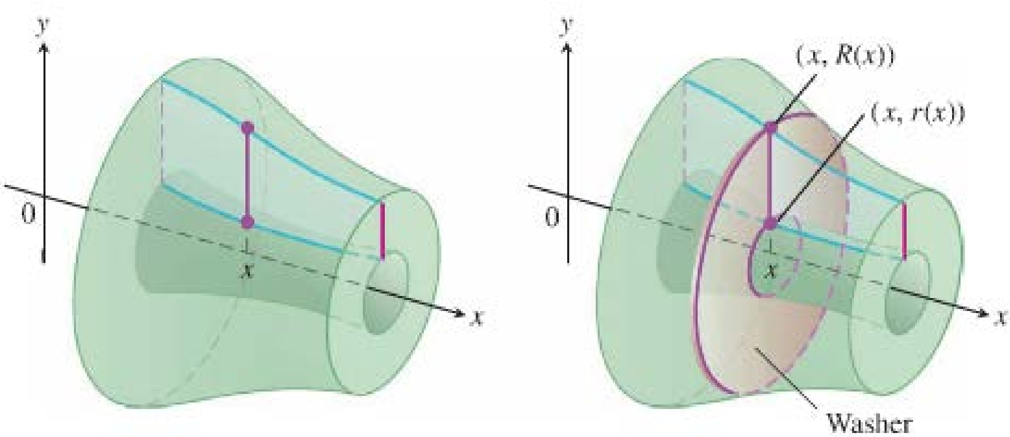
Solids of Revolution: The *Washer Method*

If the region we revolved to generate a solid does not border on or cross the axis of revolution, the solid has a hole in it. The cross-sections perpendicular to the axis of revolution are ***washers*** instead of disks. The dimensions of a typical washer are:

Outer radius: $R(x)$

Inner radius: $r(x)$





The washer's area is:

$$A(x) = \pi\left[R(x)\right]^2 - \pi\left[r(x)\right]^2 = \pi\left(\left[R(x)\right]^2 - \left[r(x)\right]^2\right)$$

The washer's volume:

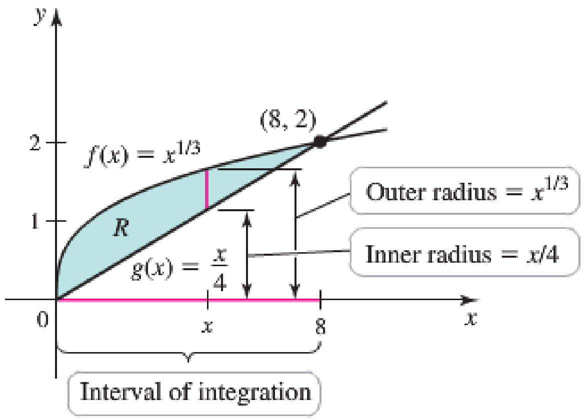
$$V = \int_a^b A(x)\,dx = \int_a^b \pi\left(\left[R(x)\right]^2 - \left[r(x)\right]^2\right)dx$$

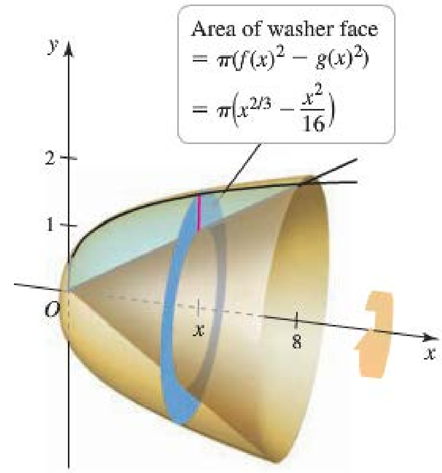
Example

The R be the region in the first quadrant bounded by the graphs of $x=y^3$ and $x=4y$. Which is the greater, the volume of the solid generated when R is revolved about the x-axis or the y-axis?

Solution

About the ***x-axis***





$$x=y^3=\left(\frac{x}{4}\right)^3 \quad\rightarrow\quad 4^3x-x^3=0$$

$$x\left(64-x^2\right)=0 \quad\Rightarrow\quad \underline{x=0,\ 8}|,\ \cancel{-8}\ (\notin QI)$$

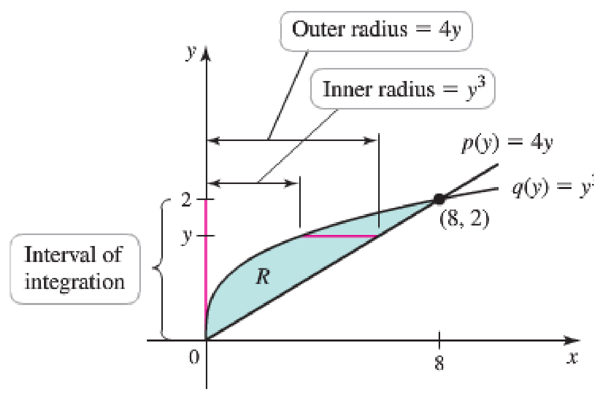
$$V=\pi\int_0^8\left(f(x)^2-g(x)^2\right)dx$$

$$=\pi\int_0^8\left(x^{2/3}-\frac{x^2}{16}\right)dx$$

$$=\pi\left(\frac{3}{5}x^{5/3}-\frac{1}{48}x^3\right)\Big|_0^8$$

$$=\pi\left(\frac{96}{5}-\frac{32}{3}\right)$$

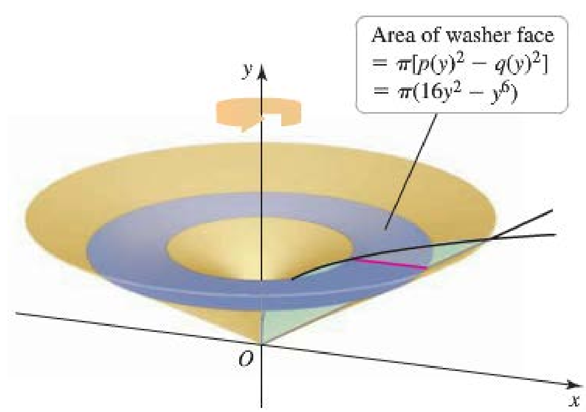
$$=\frac{128\pi}{15}\approx 26.81\ \ unit^3$$



About the ***y-axis***

$$x=y^3=4y \quad\rightarrow\quad y\left(y^2-4\right)=0$$

$$\underline{y=0,\ 2}|,\ \cancel{-2}\ (\notin QI)$$



$$V=\pi\int_0^2\left(p(y)^2-q(y)^2\right)dy$$

$$=\pi\int_0^2\left(16y^2-y^6\right)dy$$

$$= \pi \left(\frac{16}{3} y^3 - \frac{1}{7} y^7 \right) \Bigg|_0^2$$

$$= \pi \left(\frac{128}{3} - \frac{128}{7} \right)$$

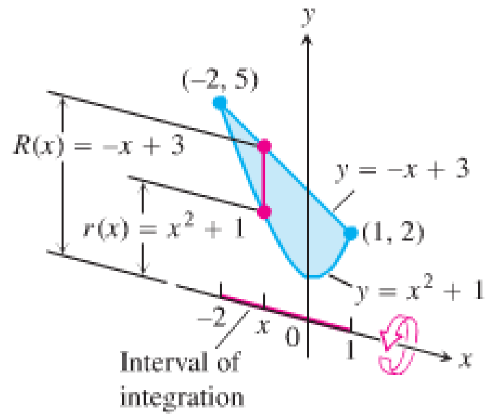
$$\boxed{= \frac{512\pi}{12} \approx 76.60 \ \ unit^3}$$

The region that is revolving about the ***y-axis*** produces a solid of greater volume.

Example

The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

Solution



$$y = x^2 + 1 = -x + 3$$

$$x^2 + x - 2 = 0 \qquad \textit{Solve for x}$$

$$\boxed{x = -2,\ 1}$$

$$V = \int_{-2}^{1} \pi \left((-x+3)^2 - \left(x^2+1\right)^2 \right) dx$$

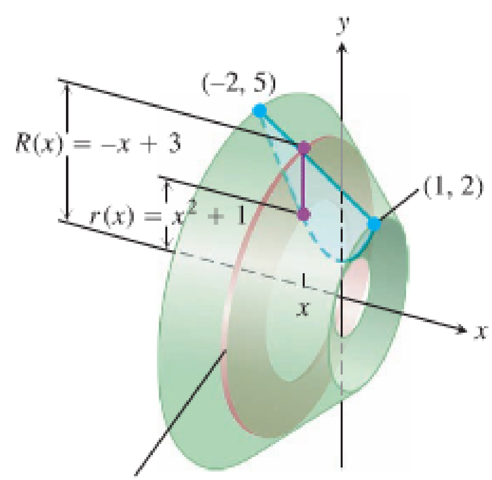
$$= \pi \int_{-2}^{1} \left(x^2 - 6x + 9 - x^4 - 2x^2 - 1 \right) dx$$

$$= \pi \int_{-2}^{1} \left(-x^4 - x^2 - 6x + 8 \right) dx$$

$$= \pi \left[-\frac{x^5}{5} - \frac{x^3}{3} - 3x^2 + 8x \right]_{-2}^{1}$$

$$= \pi \left[\left(-\frac{1}{5} - \frac{1}{3} - 3 + 8 \right) - \left(-\frac{(-2)^5}{5} - \frac{(-2)^3}{3} - 3(-2)^2 + 8(-2) \right) \right]$$

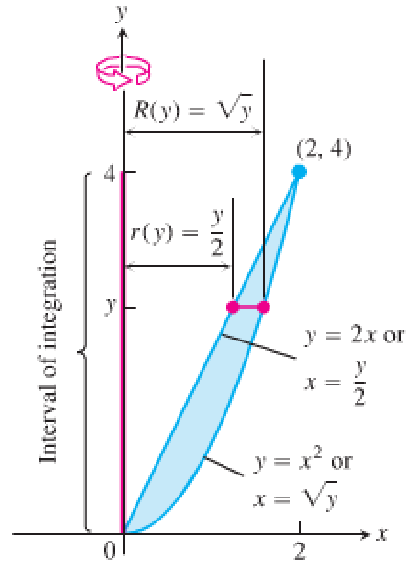
$$\boxed{= \frac{117\pi}{5}} \ \ unit^3$$



Washer cross-section
Outer radius: $R(x) = -x + 3$
Inner radius: $r(x) = x^2 + 1$

Example

The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.



Solution

$$y = x^2 \quad \to \quad x = \sqrt{y} = R(y)$$

$$y = 2x \quad \to \quad x = \frac{1}{2}y = r(y)$$

$$\sqrt{y} = \frac{1}{2}y \quad \to \quad 4y = y^2$$

$$y^2 - 4y = 0 \qquad \boxed{y = 0,\ 4}$$

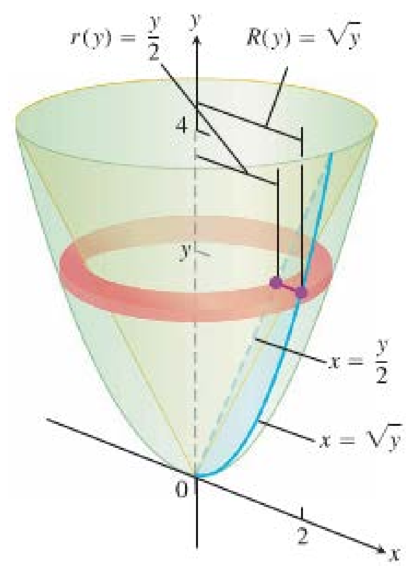
$$V = \int_0^4 \pi\left(\left(\sqrt{y}\right)^2 - \left(\frac{y}{2}\right)^2\right)dy$$

$$= \pi\int_0^4 \left(y - \frac{1}{4}y^2\right)dy$$

$$= \pi\left[\frac{y^2}{2} - \frac{1}{12}y^3\right]_0^4$$

$$= \pi\left(\frac{4^2}{2} - \frac{1}{12}(4)^3\right)$$

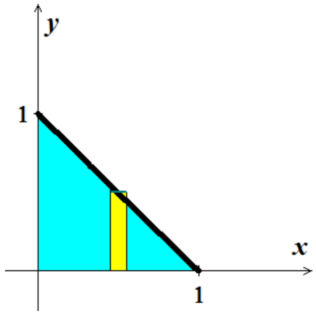
$$= \frac{8\pi}{3}\ \ unit^3$$



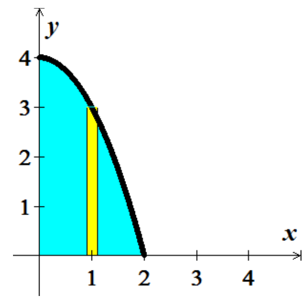
Exercises Section 1.3 – Volume by Slicing

Find the volume of the solid formed by revolving the region about the *x-axis*

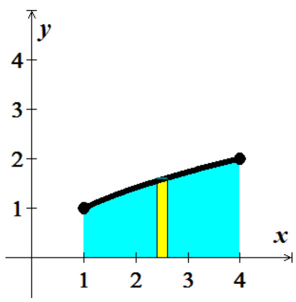
1. $y = -x + 1$



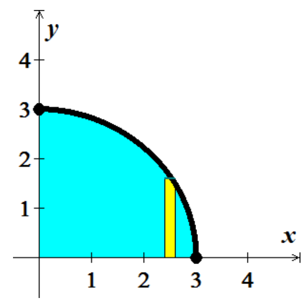
2. $y = 4 - x^2$



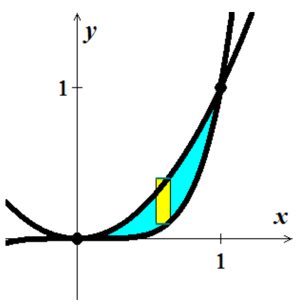
3. $y = \sqrt{x}$



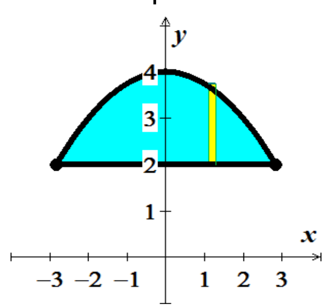
4. $y = \sqrt{9 - x^2}$



5. $y = x^2, \quad y = x^5$

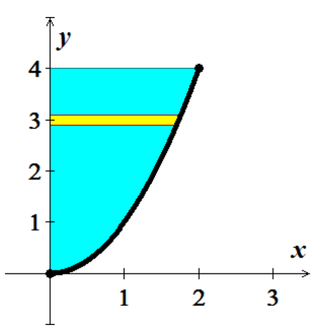


6. $y = 2, \quad y = 4 - \dfrac{x^2}{4}$

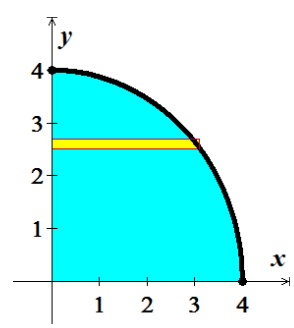


Find the volume of the solid formed by revolving the region about the *y-axis*

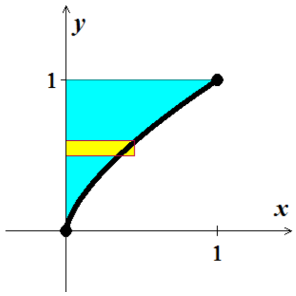
7. $y = x^2$



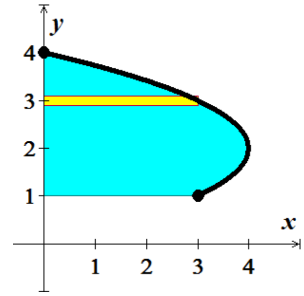
8. $y = \sqrt{16 - x^2}$



9. $y = x^{2/3}$



10. $x = -y^2 + 4y$



11. Find the volumes of the solids generated by revolving the region by the graphs of the equations about the given lines. $y = \sqrt{x}, \quad y = 0, \quad x = 3$

a) the *x-axis*   *b)* the *y-axis*   *c)* the line $x = 3$ *d)* the line $x = 6$

12. Find the volumes of the solids generated by revolving the region by the graphs of the equations about the given lines. $y = 2x^2, \quad y = 0, \quad x = 2$

a) the *x-axis*   *b)* the *y-axis*   *c)* the line $y = 8$ *d)* the line $x = 2$

13. Find the volumes of the solids generated by revolving the region by the graphs of the equations about the given lines. $y = x^2, \quad y = 4x - x^2$

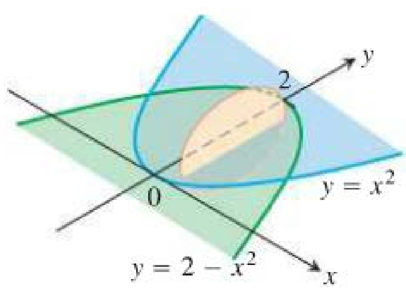
a) the *x-axis*   *b)* the line $y = 6$

14. Find the volumes of the solids generated by revolving the region by the graphs of the equations about the given lines. $y = -x^2 + 2x + 4, \quad y = 4 - x$

a) the *x-axis*   *b)* the line $y = 1$

15. The solid lies between planes perpendicular to the *x*-axis at *x* = 0 and *x* = 4. The cross-sections perpendicular to the axis on the interval $0 \le x \le 4$ are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$. Find the volume of the solid.

16. The solid lies between planes perpendicular to the *x*-axis at *x* = –1 and *x* = 1. The cross-sections perpendicular to the *x*-axis are circular whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$. Find the volume of the solid.



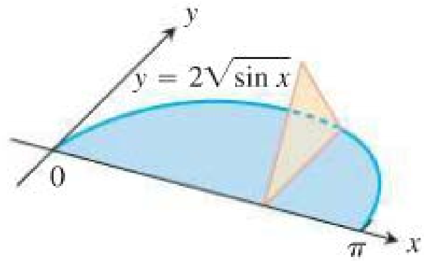
17. The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the axis between these planes are squares whose bases run from the semicircle $y = -\sqrt{1-x^2}$ to the semicircle $y = \sqrt{1-x^2}$. Find the volume of the solid.

18. The base of a solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross-sections perpendicular to the x-axis are

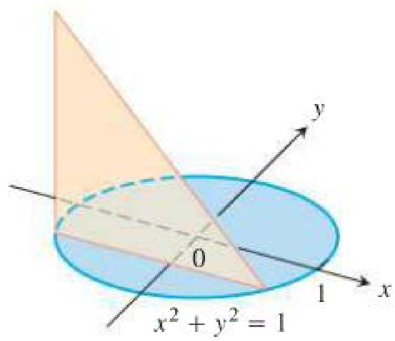
a) Equilateral triangles with bases running from the x-axis to the curve as shown

b) Squares with bases running from the x-axis to the curve.

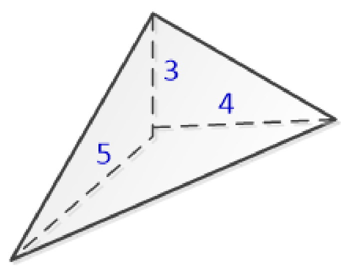
Find the volume of the solid.



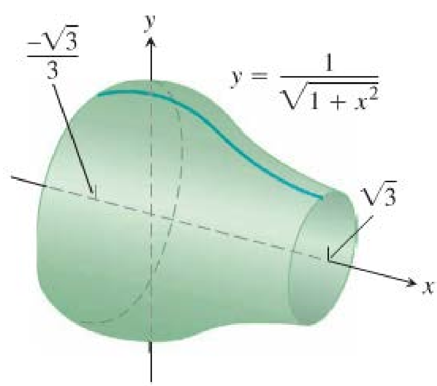
19. The base of the solid is the disk $x^2 + y^2 \le 1$. The cross-sections by planes perpendicular to the y-axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disk.



20. Find the volume of the given tetrahedron. (***Hint***: Consider slices perpendicular to one of the labeled edges)



21. Find the volume of the solid of revolution



22. Find the volume of the solid generated by revolving the region bounded by $y = x^2$ and the lines $y = 0, \quad x = 2$ about the x-axis.

23. Find the volume of the solid generated by revolving the region bounded by $y = x - x^2$ and the line $y = 0$ about the x-axis.

24. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{\cos x}$ and the lines $0 \le x \le \frac{\pi}{2}, \quad y = 0, \quad x = 0$ about the x-axis.

25. Find the volume of the solid generated by revolving the region bounded by $y = \sec x$ and the lines $y = 0, \quad x = -\frac{\pi}{4}, \quad x = \frac{\pi}{4}$ about the x-axis.

26. Find the volume of the solid generated by revolving the region bounded by $x = \sqrt{5}\, y^2$ and the lines $x = 0, \quad y = -1, \quad y = 1$ about the y-axis.

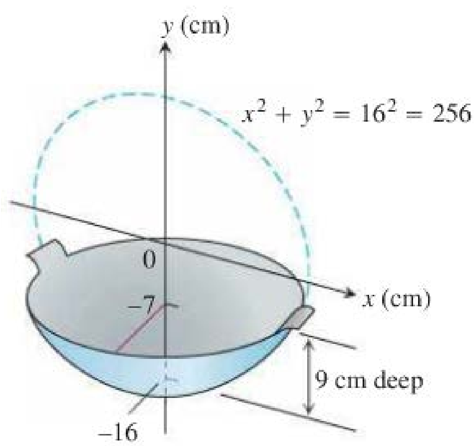
27. Find the volume of the solid generated by revolving the region bounded by $y = 2\sqrt{x}$ and the lines $y = 2, \quad x = 0$ about the x-axis.

28. Find the volume of the solid generated by revolving the region bounded by $y = \sec x, \quad y = \tan x$ and the lines $x = 0, \quad x = 1$ about the x-axis.

29. Find the volume of the solid generated by revolving the region bounded by $x = \sqrt{2 \sin 2y}$ and the lines $0 \le y \le \frac{\pi}{2}, \quad x = 0$ about the y-axis.

30. Find the volume of a solid ball having radius a.

31. You are designing a wok frying pan that will be shaped like a spherical bowl with handles. A bit of experimentation at home persuades you that you can get one that hold about 3 L if you make it 9 cm deep and give the sphere a radius of 16 cm. To be sure, you picture the wok as a solid of revolution, and calculate its volume with an integral. To the nearest cubic centimeter, what volume do you really get? $\left(1\,L = 1{,}000\ cm^3\right)$



32. A curved wedge is cut from a circular cylinder of radius 3 by two planes. On plane is perpendicular to the axis of the cylinder. The second place crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.

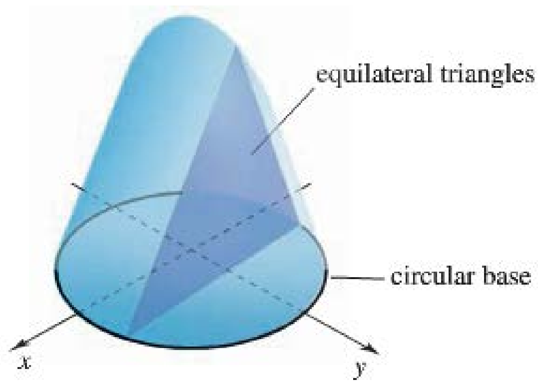
33. Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=1$, $x = 4$ about the line $y = 1$.

34. Let R be the region bounded by $y=\sin^{-1}x$, $x=0$, $y=\frac{\pi}{4}$. Use the disk method to find the volume of the solid generated when R is revolved about y-axis.

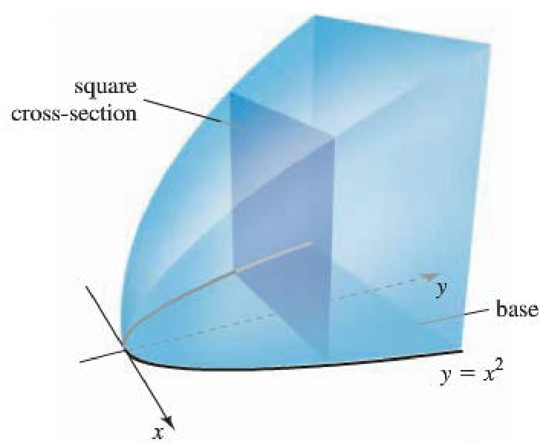
35. Find the volume of the solid of revolution bounded by $y=\frac{\ln x}{\sqrt{x}}$, $y=0$, *and* $x=2$ revolved about the x-axis. Sketch the region

36. Find the volume of the solid of revolution bounded by $y=e^{-x}$, $y=e^{x}$, $x=0$, $x=\ln 4$ revolved about the x-axis. Sketch the region

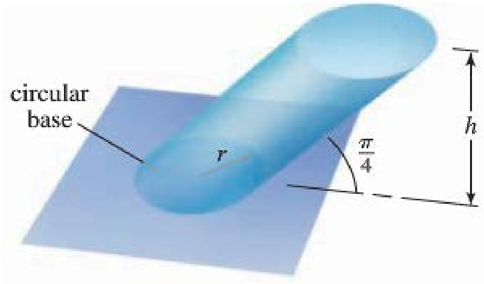
37. The solid with a circular base of radius 5 whose cross sections perpendicular to the base and parallel to the x-axis are equilateral triangles. Use the general slicing method to find the volume of the solid.



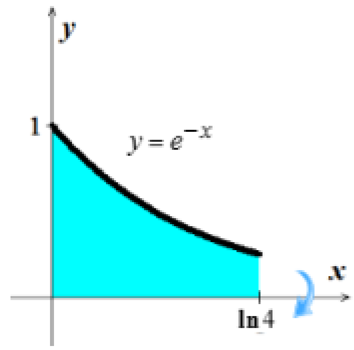
38. The solid whose base is the region bounded by $y=x^2$ and the line $y=1$ and whose cross sections perpendicular to the base and parallel to the x-axis squares. Use the general slicing method to find the volume of the solid.



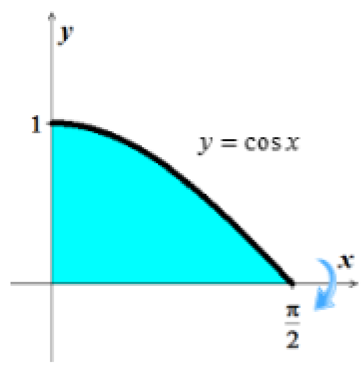
39. A circular cylinder of radius r and height h whose curved surface is at an angle of $\frac{\pi}{4}$ rad . Use the general slicing method to find the volume of the solid



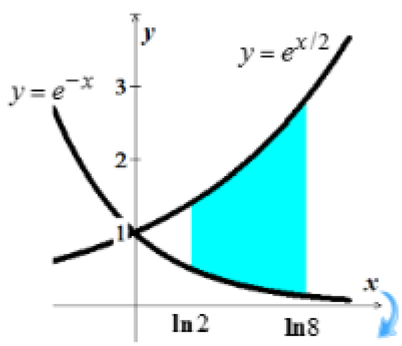
40. Let R be the region bounded by $y = e^{-x}$, $y = 0$, $x = 0$, $x = \ln 4$. Use the disk method to find the volume of the solid generated when R is revolved about x-axis.



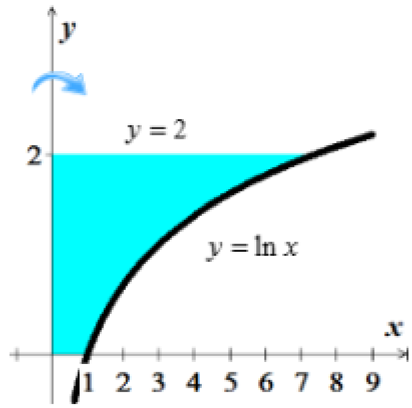
41. Let R be the region bounded by $y = \cos x$, $y = 0$, $x = 0$. Use the disk method to find the volume of the solid generated when R is revolved about x-axis,



42. Let R be the region bounded by $y = e^{x/2}$, $y = e^{-x}$, $x = \ln 2$, $x = \ln 8$. Use the disk method to find the volume of the solid generated when R is revolved about x-axis.



43. Let R be the region bounded by $y=0, \quad y=\ln x, \quad y=2, \quad x=0$. Use the disk method to find the volume of the solid generated when R is revolved about y-axis.



Find the volume of the solid generated by revolving the region boubbded by the graphs of the equations about the line $y=4$

44. $y=x, \quad y=3, \quad x=0$

45. $y=\frac{1}{2}x^3, \quad y=4, \quad x=0$

46. $y=\frac{3}{1+x}, \quad y=0, \quad x=0, \quad x=3$

47. $y=\sec x, \quad y=0, \quad 0\le x\le \frac{\pi}{3}$

Find the volume of the solid generated by revolving the region boubbded by the graphs of the equations about the line $x=5$

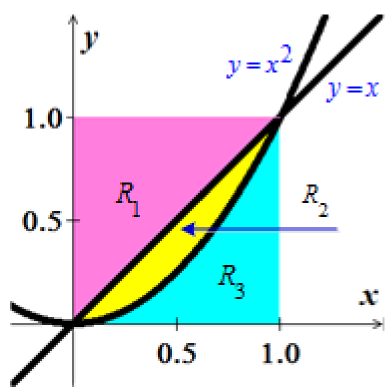
48. $y=x, \quad y=0, \quad y=4, \quad x=5$

49. $y=3-x, \quad y=0, \quad y=2, \quad x=0$

50. $x=y^2, \quad x=4$

51. $xy=3, \quad y=1, \quad y=4, \quad x=5$

52. Find the volume generated by rotating the given region $y=x^2$ *and* $y=x$ about the specified line.



a) R_1 *about* $x=0$

b) R_1 *about* $x=1$

c) R_2 *about* $y=0$

d) R_2 *about* $y=1$

e) R_3 *about* $x=0$

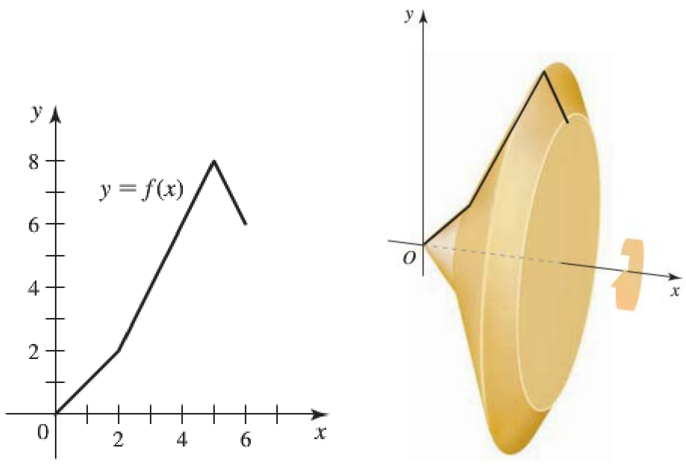
f) R_3 *about* $x=1$

g) R_2 *about* $x=0$

h) R_2 *about* $x=1$

53. Let $f(x) = \begin{cases} x & \text{if} \quad 0 \le x \le 2 \\ 2x - 2 & \text{if} \quad 2 < x \le 5 \\ -2x + 18 & \text{if} \quad 5 < x \le 6 \end{cases}$

Find the volume of the solid formed when the region bounded by the graph of *f*, the *x*-axis, and the line $x = 6$ is revolved about the *x*-axis

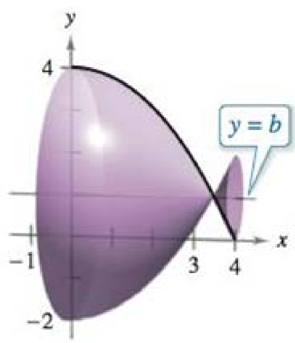


54. Consider the solid formed by revolving the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the *x-axis*

a) Find the value of *x* in the interval $[0,\ 4]$ that divides the solids into two parts of equal volume.

b) Find the values of *x* in the interval $[0,\ 4]$ that divide the solids into three parts of equal volume.

55. The arc of $y = 4 - \frac{1}{4}x^2$ on the interval $[0,\ 4]$ is revolved about the line $y = b$

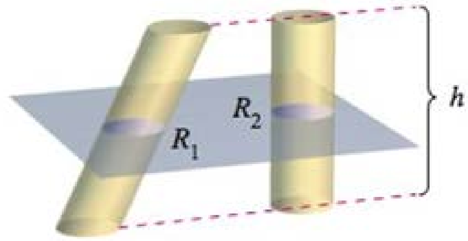


a) Find the volume of the resulting solid as a function of *b*.

b) Graph the function in part (*a*), and approximate the value of *b* that minimizes the volume of the solid.

c) Find the value of *b* that minimizes the volume of the solid, and compare the result with the answer in part (*b*).

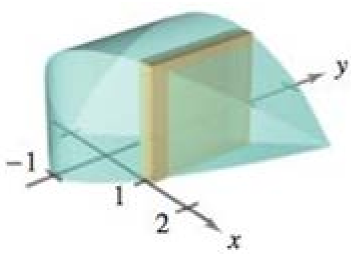
56. Prove that if two solids have equal altitudes and all plane sections parallel to their bases and at equal distances from their bases have equal areas, then the solids hace the same volume.



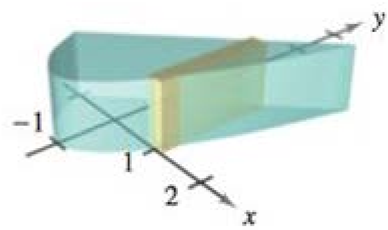
Area of R_1 = *Area of* R_2

57. Find the volumes of the solids whose bases are bounded by the graph of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the *x-axis*

a) Squares

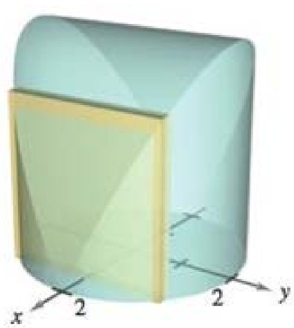


b) Rectangles of height 1

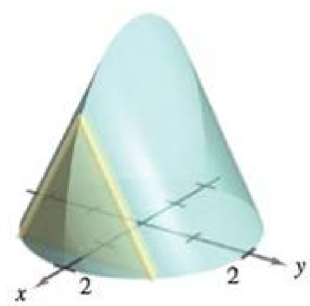


58. Find the volumes of the solids whose bases are bounded by the circle $x^2 + y^2 = 4$, with the indicated cross sections taken perpendicular to the *x-axis*

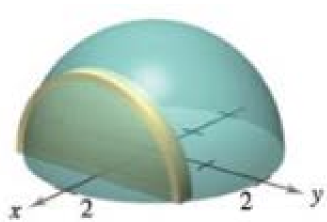
a) Squares



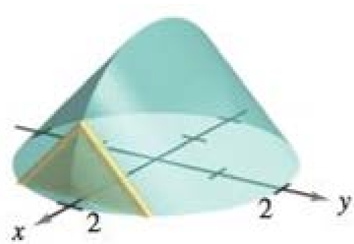
b) Equilateral triangles



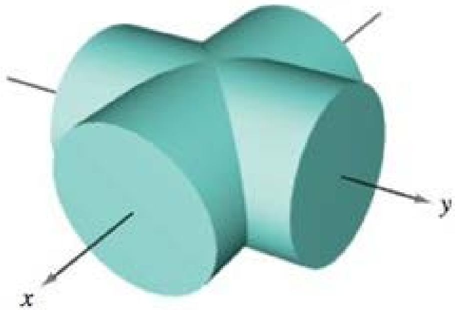
c) Semicircles



d) Iscosceles right triangles



59. Find the volume of the solid of intersection (the solid common to both) of the two right circular cylinders of radius r whose axes meet at right angles.



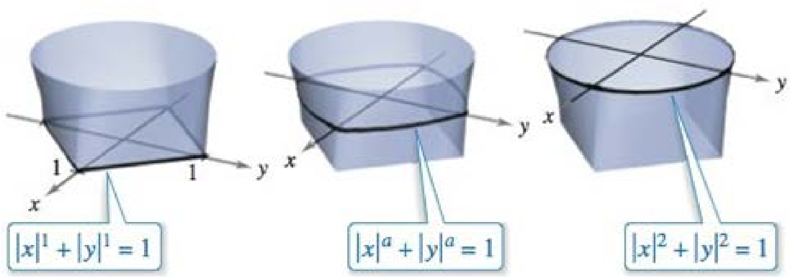
Two intersecting cylinders

Solid of intersection

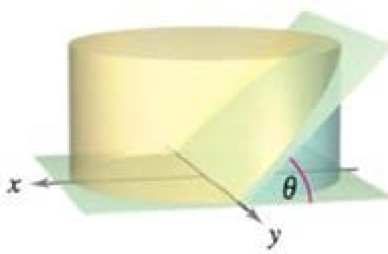
60. The solid shown in the figure has cross sections bounded by the graph $|x|^a + |y|^a = 1$ where $1 \le a \le 2$.

a) Describe the cross section when $a = 1$ and $a = 2$.

b) Describe a procedure for approximating the volume of the solid.



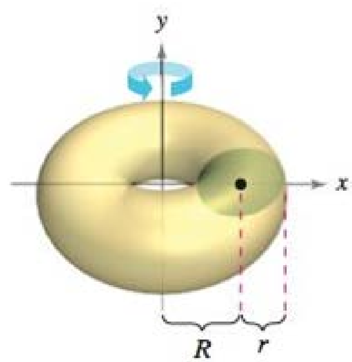
61. Two planes cut a right circular cylinder to form a wedge. One plane is perpendicular to the axis of the cylinder and the second makes an angle of θ degrees with the first.



a) Find the volume of the wedge if $\theta = 45°$.

b) Find the volume of the wedge for an arbitrary angle θ. Assuming that the cylinder has sufficient length, how does the volume of the wedge change as θ increases from 0° to 90°?

62. For the given torus (donut).



a) Show that the volume of the torus is given by the integral

$$8\pi R \int_0^r \sqrt{r^2 - y^2}\,dy \quad where \quad R > r > 0$$

b) Find the volume of the torus