2.3 – *Electric Potential*

Since electric force is conservative, let's start by reviewing some of the properties of conservative forces.

1. Work done by a conservative force is independent of the path followed. It depends only on the difference between the potential energies (energy due to location) at its initial and final locations. This implies that the work done on a closed path is zero because the initial and final locations are the same.

2. The work done by a conservative force is defined to be the negative of the change in the potential energy

$$W_c = -\Delta u$$

W_c : Work done by a conservative force

$\Delta u = u_f - u_i$ change in potential energy

3. If all the forces with non-zero contribution to the work done acting on an object are conservative, then mechanical energy is conserved

$$ME_i = ME_f$$

ME : Mechanical energy

$$ME = KE + PE$$

KE : Kinetic energy

PE : Potential energy

$$KE = \frac{1}{2}mv^2$$

$$PE = u$$

The conservative of mechanical energy may be written as

$$\frac{1}{2}mv_i^2 + u_i = \frac{1}{2}mv_f^2 + u_f$$

Or

$$\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = -\left(u_i - u_f\right) = -\Delta u$$

$$\Delta KE = -\Delta u$$

Work Done by Electrical Force

The work done by electrical force $\left(\vec{F}_e\right)$ is displacing a charge by an infinite small displacement $d\vec{r}$ is given by

$$dW_e = \vec{F}_e \cdot d\vec{r}$$

And the work done in displacing the charge from position $\vec{r}_i$ to position $\vec{r}_f$ is obtained by integration

$$W_e = \int dW_e = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_e \cdot d\vec{r}$$

But the electric force can be written as in terms of electric field

$$\vec{F}_e = q\vec{E} \qquad q\text{: charge} \qquad \vec{E}\text{: Electric field}$$

$$W_e = q\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\cdot d\vec{r}$$

If the electric field is a constant, then it can be taken out of the integral

$$W_e = q\vec{E}\int_{\vec{r}_1}^{\vec{r}_2} d\vec{r}$$

$$= q\vec{E}\left[r\right]_{\vec{r}_1}^{\vec{r}_2}$$

$$= q\vec{E}\left(\vec{r}_2 - \vec{r}_1\right)$$

$$\boxed{= q\vec{E}\bullet\Delta\vec{r}}$$

Or $$W_e = qEd\cos\theta$$

Where E is magnitude of the electric field
$d = \Delta r$ is the magnitude of the displacement
Θ is the angle between the electric filed and displacement

If $\vec{E}$ and $\Delta\vec{r}$ have the same direction, then $\theta = 0$ $\left(i.e \ \ \cos(0) = 1\right)$ and

$$\boxed{W_e = qEd} \qquad If \ \ \vec{E} \parallel \Delta\vec{r}$$

Unit of measurement for work is the Joule (J)

Example

A $6\mu C$ charge is displaced horizontally by a distance of 2mm in a region where there is an electric field of stream is $100N/C$ that makes an angle of 60° with the horizontal

Solution

Given: $q = 6\times10^{-6}C, \quad d = 2\times10^{-3}m, \quad E = 100N/C, \quad \theta = 60°$

$$W_e = qEd\cos\theta$$

$$= \left(6\times10^{-6}\right)(100)\left(2\times10^{-3}\right)\cos 60°$$

$$\boxed{= 2\times10^{-9}\ J}$$

Example

A $2\mu C$ charge is displaced from the location $\left(2\hat{i}-3\hat{j}\right)m$ to the location $\left(-4\hat{i}+9\hat{j}\right)m$ in a region where there is a constant electric field $\left(-10\hat{i}-4\hat{j}\right)N/m$. Calculate the work done by the electric force.

<u>Solution</u>

Given: $q=2\times10^{-6}C,\quad \vec{r}_i=\left(2\hat{i}-3\hat{j}\right)m,\quad \vec{r}_f=\left(-4\hat{i}+9\hat{j}\right)m,\quad E=\left(-10\hat{i}-4\hat{j}\right)N/C$

$$\begin{aligned}\Delta\vec{r}&=\vec{r}_f-\vec{r}_i\\&=\left(-4\hat{i}+9\hat{j}\right)-\left(2\hat{i}-3\hat{j}\right)\\&=\left[-6\hat{i}+12\hat{j}\right]m\end{aligned}$$

$$\begin{aligned}W_e&=q\vec{E}\bullet\Delta\vec{r}\\&=\left(2\times10^{-6}\right)\left[\left(-10\hat{i}-4\hat{j}\right)\bullet\left(-6\hat{i}+12\hat{j}\right)\right]\\&=\left(2\times10^{-6}\right)\left(60-48\right)\\&=24\times10^{-6}\ J\end{aligned}$$

Electrical Potential Energy

The change in the electrical potential energy of charge, q, as it is placed from an initial location $\vec{r}_i$ to a final location $\vec{r}_f$ is equal to the negative of the work done by the electrical force in displacing the charge from the initial location $\vec{r}_i$ to a final location $\vec{r}_f$

$$\Delta u=u_f-u_i-W_e=-q\int_{\vec{r}_1}^{\vec{r}_2}\vec{E}\cdot d\vec{r}$$

If the electric field is a constant, then $\vec{E}$ can be taken out of the integral and

$$\Delta u=-q\vec{E}\int_{\vec{r}_1}^{\vec{r}_2}d\vec{r}\qquad\qquad \int_{\vec{r}_1}^{\vec{r}_2}d\vec{r}=\vec{r}_2-\vec{r}_1=\Delta\vec{r}$$

$$\Delta u=-q\vec{E}\cdot\Delta\vec{r}$$

Or $\quad \Delta u=-qEd\cos\theta$

Where E is magnitude of the electric field

$d=\Delta r$ is the magnitude of the displacement

Θ is the angle formed between the filed and displacement

If the electric field and displacement are parallel, then $\theta = 0$ and

$$\boxed{\Delta u = -qEd}$$

If we are interested in the numerical value only then

$$\boxed{|\Delta u| = |q|\,Ed}$$

Potential Difference (ΔV)

Between two points is defined to be as the change in potential energy per a uit charge for a charge displaced between the two points.

$$\Delta V = \frac{\Delta u}{q}$$

ΔV : Potential difference

Δu : Change in potential energy

q : charge

Unit of measurement for potential difference is *J/C* which is defined to the Volt (*V*).

Since $\Delta u = -q\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\cdot d\vec{r}$

$$\Delta V = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\cdot d\vec{r}$$

Where $\Delta V = V(\vec{r}_2) - V(\vec{r}_1)$

If $\vec{E}$ is a constant, then

$$\Delta V = -\vec{E}\int_{\vec{r}_1}^{\vec{r}_2} d\vec{r} \ \underline{= -\vec{E}\Delta\vec{r}}$$

Or $\Delta V = -Ed\cos\theta$

Where *E* is magnitude of the electric field

$d = \Delta r$ is the magnitude of the displacement or straight line distance between the two points

Θ is the angle formed between the field and displacement

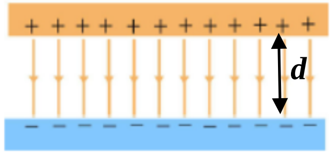
If $\vec{E} \parallel \Delta\vec{r}$

$$\Delta V = -Ed$$

If we are interested in the numerical value only then

$$\boxed{|\Delta V| = Ed}$$

Example



Two oppositely charged parallel plates are separated by a distance of 4*mm*. The electric field between the plates is uniform and ⊥ to both plates. Its strength is 20*N/C*.

a) Calculate the potential difference between the plates

b) Calculate the work done by the electric force as a $2\mu C$ charge is displaced from the positive plate to the negative plate

c) Calculate the change in potential energy for a $2\mu C$ charge as it is displaced from the positive plate to the negative plate

d) A $mass = 1.67\times 10^{-27}\,kg$ is released at rest from the positive plate. Calculate its speed by the time it reaches the negative plate

Solution

a) **Given**: $E = 20N/C, \quad d = 4\times 10^{-3}m$

$$|\Delta V| = Ed$$

$$= (20)\left(4\times 10^{-3}\right)$$

$$\boxed{= 8\times 10^{-2}\ V}$$

b) The field and the force have the same direction for a positive charge

$$W_e = qEd \qquad \left(q = 2\times 10^{-6}C\right)$$

$$= \left(2\times 10^{-6}\right)(20)\left(4\times 10^{-3}\right)$$

$$\boxed{= 1.6\times 10^{-7}\ J}$$

c) $\Delta u = -W_e \ \boxed{= -1.6\times 10^{-7}\ J}$

Or $\Delta u = q\Delta V == \left(2\times 10^{-6}\right)\left(8\times 10^{-2}\right)\ \boxed{= -1.6\times 10^{-7}\ J}$

ΔV is negative because it is being displaced from a higher to a lower potential

d) Since electric force is conservative, mechanical energy is conserved

$$\Delta u = +q_P\Delta V; \qquad \Delta V = -8\times 10^{-2}\ V$$

$$\Delta KE = -\Delta u$$

$$\frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2} = -\Delta u \qquad v_i = 0 \text{ (released at rest)}$$

$$\frac{1}{2}mv_f^2 = -q_p\Delta V$$

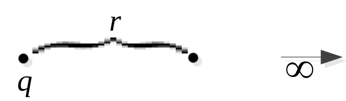
$$\frac{1}{2}\left(1.67\times 10^{-27}\right)v_f^2 = -\left(1.6\times 10^{-19}\right)\left(-8\times 10^{-2}\right)$$

$$v_f^2 = \frac{12.8\times 10^{-21}}{0.835\times 10^{-27}} \approx 15.3\times 10^6$$

$$\boxed{v_f \approx 1237\ m/s}$$

Potential Due to a Point Charge

Potential difference is independent of the choice of a reference point. But to specific the potential at a given point. A reference point is needed. The value of the potential at a certain point can be fixed arbitrary, then potential ρ at other points are specified with respect to the reference point. For potentials due to point charges, the reference point is taken to be at infinity, and the potential at infinity is set to zero. $\left(V \big|_{r \to \infty} = 0 \right)$. To obtain the potential at a point a distance r from the charge, first we obtain the potential difference between this point L the point at infinity.



$$\Delta V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

For a point charge

$$\vec{E} = \frac{kq}{r^2} \vec{e}_r, \quad d\vec{r} = dr\ \vec{e}_r, \quad where \quad \vec{e}_r = \frac{\vec{r}}{r}$$

$$\vec{E} \cdot d\vec{r} = \frac{kq}{r^2} dr \left(\vec{e}_r \cdot \vec{e}_r \right)$$

$$= \frac{kq}{r^2} dr$$

$$\Delta V = V(r) - V(\infty) = -\int_{\infty}^{r} \frac{kq}{r^2} dr$$

$$= -\left[-\frac{kq}{r} \right]_{\infty}^{r}$$

$$= \frac{kq}{r} - \frac{kq}{\infty}$$

$$= \frac{kq}{r}$$

$with \quad V(\infty) = 0$ (a reference point chosen arbitrary)

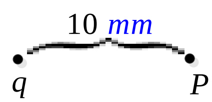
$\boxed{V(r) = \frac{kq}{r}}$ Potential due to a point charge q at a distance r from the charge.

Superposition Principle

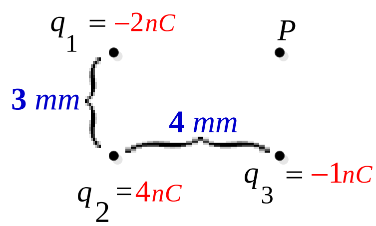
If there are a number of point charges in the vicinity of a point, the net potential at the point is obtained by adding the potentials due to the charges algebraically.

Example

a) Calculate the potential at point *P* due to the point charge *q*.



b) Point *P* is in the vicinity of 3 charges as shown. Calculate the net potential at point *P*.



Solution

a) Given: $q = -5\times10^{-6}C;\quad r = 10\times10^{-3}m = 10^{-2}m$

$$V = \frac{kq}{r} = \frac{\left(9\times10^{9}\right)\left(-5\times10^{-6}C\right)}{10^{-2}} \underline{= -4.5\times10^{6}V}$$

b) Given: $q_1 = -2\times10^{-9}C;\quad r_1 = 4\times10^{-3}m$

$$q_2 = 4\times10^{-9}C;\quad r_2 = \sqrt{3^2+4^2}\times10^{-3}m = 5\times10^{-3}m$$

$$q_3 = -1\times10^{-9}C;\quad r_1 = 3\times10^{-3}m$$

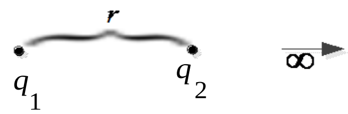
$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$= \left(9\times10^{9}\right)\left[\frac{-2\times10^{-9}}{4\times10^{-3}} + \frac{4\times10^{-9}}{5\times10^{-3}} + \frac{-1\times10^{-9}}{3\times10^{-3}}\right]$$

$$\underline{= -0.3\times10^{3}\ V}$$

Energy Stored by Point Charge

The energy stored by two point charges separated by a distance *r* is defined to the work needed by an external force to being one of the charges from infinity to a separation of distance *r* between the charges



The work done by the external force opposes the work done by the electric force

$$w_{ext} = -w_e \quad \text{but} \quad w_e = -\Delta u = -q_2 \Delta V = -q_2\left(\frac{kq_1}{r} - 0\right)$$

$$u = w_{ext} = -w_e = \Delta u = q_2\left(\frac{kq_1}{r}\right)$$

$$\boxed{u = \frac{kq_1 q_2}{r}}$$ Energy stored by two point charges separated by a distance *r*.

Energy stored by more than two point charges is equal to the external work needed to bring the charges from infinity so their respective charges with respect to one of the charges.

For example, if there are 3 charges, the energy required to bring q_2 at a distance of r_{12} from q_1 is

And the energy required to bring q_3 from ∞ to a distance of r_{23} from q_2 is given $\frac{kq_1 q_2}{r_{12}}$

The total energy stored by the 3 charges

$$\boxed{u = \frac{kq_1 q_2}{r_{12}} + \frac{kq_2 q_3}{r_{23}} + \frac{kq_1 q_3}{r_{13}}}$$

If there are *n* charges, by a similar process it can be shown that the energy stored by the n charges is given by

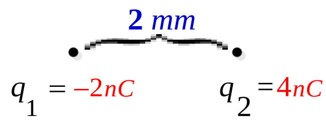
$$\boxed{u = \frac{1}{2}\sum_{i=1}^{n}\sum_{i=j}^{n}\frac{kq_i q_j}{r_{ij}}}$$

The $\frac{1}{2}$ is needed because of summation contribute two of the same term $-\frac{kq_i q_j}{r_{ij}}$ & $\frac{kq_i q_j}{r_{ji}}$

Example

Calculate the energy stored by the following system of point charges

a) ***Given***: $r_{12} = 2\times 10^{-3} m, \quad q_1 = -2\times 10^{-9} C, \quad q_3 = 4\times 10^{-9} C$



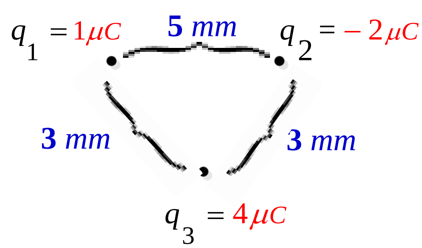
$$u = \frac{kq_1q_2}{r_{12}}$$

$$= \frac{\left(9\times 10^{9}\right)\left(-2\times 10^{-9}\right)\left(4\times 10^{-9}\right)}{2\times 10^{-3}}$$

$$\boxed{= -36\times 10^{-6}\ J}$$

b) ***Given***: $r_{12} = 5\times 10^{-3} m, \quad r_{13} = 3\times 10^{-3} m, \quad r_{23} = 3\times 10^{-3} m$

$q_1 = 10^{-6} C, \quad q_2 = -2\times 10^{-6} C, \quad q_3 = 4\times 10^{-6} C$



$$u = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_3}{r_{13}}\right)$$

$$= \left(9\times 10^{9}\right)\left[\frac{\left(10^{-6}\right)\left(-2\times 10^{-6}\right)}{5\times 10^{-3}} + \frac{\left(10^{-6}\right)\left(4\times 10^{-6}\right)}{3\times 10^{-3}} + \frac{\left(-2\times 10^{-6}\right)\left(4\times 10^{-6}\right)}{3\times 10^{-3}}\right]$$

$$= (9)\left(-\frac{2}{5} + \frac{4}{3} - \frac{8}{3}\right)$$

$$\boxed{= -\frac{134}{15}\ J}$$

Potential due to a Continuous Distribution of Charges

Is obtained by taking a small charge dq and obtaining the potential due to this charge which is

$$dV = k\frac{q}{r}$$

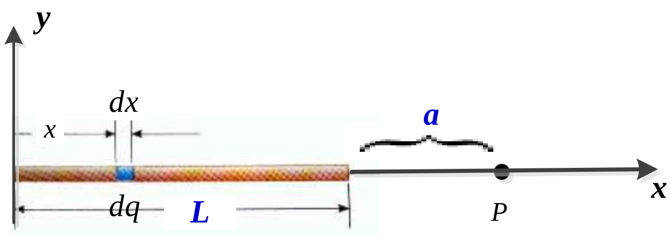
Then the net potential is obtained by interpreting over the total volume. To convert dq into a volume integral, dq can be expressed in terms of the volume element and the charge density ρ

$$\rho = \frac{dq}{dV} \quad \Rightarrow dq = \rho dV$$

$$V_{\rho} = k\int \frac{dq}{r}$$

Example

Obtain an expression for the potential at point P due to the uniformly charged rod of density λ as shown. The total charge is Q and the length of the rod is L.



<u>Solution</u>

The potential due to the charge dq is

$$dV = \frac{kdq}{a+(L-x)}$$

The distance between point P & dq is $a + L - x$

$$dq = \lambda dx$$

$$V = \int_{x=0}^{x=L} \frac{k\lambda dx}{a+L-x}$$

$Let \quad u = a + L - x \quad du = -dx$

$$= -k\lambda \int_0^L \frac{du}{u}$$

$$= -k\lambda \ln|a+L-x|\Big|_0^L$$

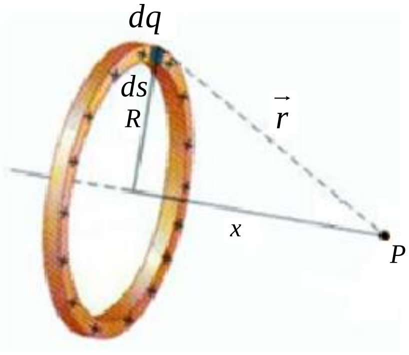
$$= -k\lambda\left(\ln|a| - \ln|a+L|\right)$$

$$= k\lambda\left(\ln|a+L| - \ln|a|\right)$$

$$= k\lambda \ \ln\left(\frac{a+L}{a}\right)$$

Example

Obtain an expression for the potential at point P due to Q uniformly charged ring of density λ as shown.



Solution

The potential due to the charge dq is

$$dV_P = \frac{kdq}{r} \qquad \text{But } r^2 = x^2 + R^2$$

The total potential is obtained by integrating over dS . S varies from zero to $2\pi R$ which is the circumference.

$$V_P = \int_{S=0}^{S=2\pi R} \frac{k\lambda dS}{\sqrt{x^2 + R^2}}$$

$$= \frac{k\lambda}{\sqrt{x^2 + R^2}} \int_0^{2\pi R} dS$$

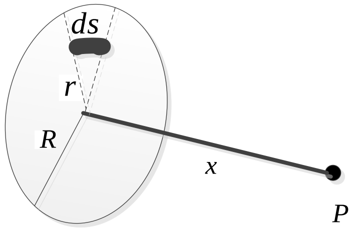
$$= \frac{k\lambda}{\sqrt{x^2 + R^2}} S \Big|_0^{2\pi R}$$

$$= \frac{2\pi R k\lambda}{\sqrt{x^2 + R^2}}$$

$$= \frac{kQ}{\sqrt{x^2 + R^2}} \quad \textit{because} \quad Q = 2\pi R\lambda \quad (\textit{Total charge})$$

Example

Find an expression for the net potential at point *P* due to a uniformly charged disc of charge density δ.



Solution

The potential due to a small charge element dq at point *P* is given by

$$dV_P = \frac{kdq}{\sqrt{x^2 + r^2}}$$

$$dq = \sigma dA$$

dA can be taken to be the product of the arc length pass element ds & radius path element dr

$$dA = ds\ dr$$

$$dV_P = \frac{k\sigma dsdr}{\sqrt{x^2 + r^2}}$$

$$V_P = \iint \frac{k\sigma dsdr}{\sqrt{x^2 + r^2}}$$

$$= k\sigma \int_0^R \frac{dr}{\sqrt{x^2 + r^2}} \int_0^{2\pi r} ds$$

$$= k\sigma \int_0^R \frac{dr}{\sqrt{x^2 + r^2}} (2\pi r)$$

$$= \pi k\sigma \int_0^R \frac{2rdr}{\sqrt{x^2 + r^2}}$$

$$= \pi k\sigma \int_0^R \frac{d\left(x^2 + r^2\right)}{\sqrt{x^2 + r^2}}$$

$$= 2\pi k\sigma \left[\sqrt{x^2 + r^2} \right]_0^R$$

$$\boxed{= 2\pi k\sigma \left(\sqrt{x^2 + R^2} - x \right)}$$

Obtaining Potential from Electric Field

$$V = -\int \vec{E} \cdot d\vec{r} \quad \Rightarrow \quad dV = -\vec{E} \cdot d\vec{r}$$

For simplicity, let's consider straight line displacements (say along x-axis) then

$$\vec{E} \cdot d\vec{r} = -Edx$$
$$dV = -Edx$$
$$\boxed{E = -\frac{dV}{dx}}$$

This implies that electric field can be obtained as the derivative of potential with respect to x.

Example

The potential due to a certain charge varies with position according to the equation

$$V = 3\cos(x) + 1$$

Obtain an expression for the electric field as a function of position

Solution

$$\begin{aligned} E(x) &= -\frac{dV}{dx} \\ &= -\frac{d}{dx}\left[3\cos(x) + 1\right] \\ &= 3\sin x \end{aligned}$$

Example

The electric field due to a uniformly charged solid sphere of radius R and total charge Q is given by

$$E = \begin{cases} \dfrac{kQr}{R^3} & for \quad r < R \\ \dfrac{kQ}{r^2} & for \quad r > R \end{cases}$$

Obtain an expression for the potential as a function of r by assuming the potential at infinity to be zero.

Solution

For $r > R$

$$V = -\int \vec{E} \cdot d\vec{r}$$

$$\vec{E} = \frac{kQ}{r^2}\hat{e}_r, \quad d\vec{r} = dr\ \hat{e}_r \quad \Rightarrow \quad \vec{E} \cdot d\vec{r} = Edr$$

$$\Delta V = V(r) - V(0) = \int_r^{\infty} \frac{kQ}{r^2} dr$$

$$= kQ\left[\frac{-1}{r}\right]_r^{\infty}$$

$$= kQ\left(0 + \frac{1}{r}\right)$$

$$\boxed{= \frac{kQ}{r}}$$

Since $V(\infty)$ is assumed to be zero.

$$V(r) = \frac{kQ}{r} \quad for \quad r > R$$

For $r < R$

$$\Delta V = V(r) - V(0) = -\int_r^R \vec{E} \cdot d\vec{r} = -\int_r^R E \cdot dr \qquad E = \frac{kQr}{R^3}$$

$$V(r) - V(0) = -\frac{kQ}{R^3}\int_r^R r \cdot dr$$

$$= -\frac{1}{2}\frac{kQ}{R^3}\left[r^2\right]_r^R$$

$$= -\frac{1}{2}\frac{kQ}{R^3}\left(R^2 - r^2\right)$$

But from the result for $r \geq R$; $V(R) = \frac{kQ}{R}$

$$\frac{kQ}{R} - V(r) = -\frac{1}{2}\frac{kQ}{R^3}R^2 + \frac{1}{2}\frac{kQ}{R^3}r^2$$

$$-V(r) = -\frac{kQ}{R} - \frac{1}{2}\frac{kQ}{R} + \frac{1}{2}\frac{kQ}{R^3}r^2$$

$$V(r) = \frac{3}{2}\frac{kQ}{R} - \frac{kQr^2}{2R^3}$$

$$\boxed{= \frac{1}{2}\frac{kQ}{R}\left(3 - \frac{r^2}{R^2}\right)}$$

Potential of a conductor in Electrostatic equilibrium

A conductor in electrostatic equilibrium is a conductor where the charges are not moving. This implies the electric field inside is zero (because if there was the charges would be moving). This also implies all the points in a conductor in electrostatic equilibrium are at the same potential. Because if points A and B are inside the conductor then

$$\Delta V = V(B) - V(A) = -\int_A^B \vec{E} \cdot d\vec{r} \qquad But \;\; \vec{E} = 0$$

$$V(B) - V(A) = 0$$

$$V(B) = V(A)$$

Also no work is required to transport a charge from one point to another point inside a conductor in electrostatic equilibrium; because

$$w_e = -\Delta u = -q\Delta V \qquad But \;\; \Delta V = 0$$

$$w_e = 0$$

As noted in the previous, excess charges of a conductor in electrostatic equilibrium must reside on the surface of the conduct. (If there were excess charges inside, from Gauss's law it will follow that there is a non-zero electric field inside).

The electric field just outside a spherical charged sphere is $\frac{kQ}{R^2}$ as shown previously. That is the electric filed decreases as $\frac{1}{R^2}$ at the surface. Therefore the smaller the radius of curvature the greater the electric field at the surface. This indicates that for surfaces of irregular shape the electric field just outside the conductor is greater for sharp surface then for dull surfaces.

Also since the electric field just outside a conductor in electrostatic equilibrium $(E = 4\pi k\sigma)$ is proportional to the charge density, it follows that the charge density is greater at sharp surfaces then it is at dull surfaces.

*E*quipotential surfaces are surfaces that contain points at the same potential only.

The work done is taking a charge from one point of an equipotential surface to another point of the equipotential surface is zero because $\Delta V = 0$ $\left(\Delta V = 0 \; \textit{because all the points are at the same potential}\right)$

$$i.e. \quad w_e = -q\Delta V = 0$$

Now let's consider a charge q displaced by a small displacement $d\vec{r}$ along an equipotential surface

$$dw_e = -qdV = 0 \quad because \quad dV = 0$$

But also

$$dw_e = q\vec{E} \cdot d\vec{r} = 0$$

$$\vec{E} \cdot d\vec{r} = 0$$

$$Edr\cos\theta = 0 \quad \Rightarrow \quad \cos\theta = 0 \;\rightarrow\; \theta = 90°$$

It follows that the electric field must be perpendicular to the equipotential lines at any point. For example of the equipotential lines are horizontal lines, the electric field lines must be vertical lines.

Example

Show that the electric field lines and equipotential lines for a positive point charge