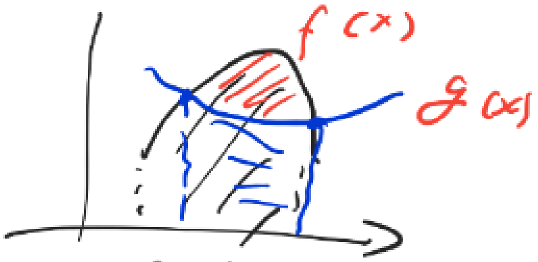
9/2 - Cal 2

1.2 Area between 2 curves.



To do so: for 2 fctns
intersection letting $y = f(x) = g(x)$
Solve for the variable

Ex Given: $y = 2 - x^2$ ($x = 0$) $y = -x$ ($x = 0$) (A?)

Find: A?

Soln

$$y = 2 - x^2 = -x$$

$$x^2 - x - 2 = 0$$

$$x = -1, 2$$

$a + b + c = 0$ $x = 1, c/a$

$a - b + c = 0$ $x = -1, -c/a$

$\sqrt{b^2 - 4ac}$

$\frac{1 \pm 3}{2}$

$$A = \int_{-1}^{2} (2 - x^2 + x)\,dx$$

$$= 2x - \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_{-1}^{2}$$

$$= 4 - \frac{8}{3} + 2 - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$$

$$= 8 - 3 - \frac{1}{2}$$

$$= \frac{9}{2} \text{ unit}^2$$

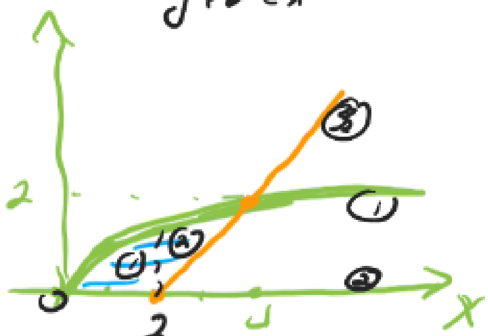
$5 - \frac{1}{2} = \frac{9}{2}$

1- X   A? $y = \sqrt{x}$ (1)   x-axis $y=0$ (2) $y = x-2$ (3)

$y+2=x$

Soln

(1) ∩ (3) $(4,2)$



$$(\sqrt{x})^2 = (x-2)^2$$

$$x = \cancel{1}, 4$$

$$A = \int_0^2 (y+2-y^2)\,dy$$

$$\boxed{\begin{array}{l} y=\sqrt{x} \\ x=y^2 \end{array}}$$

$$= \frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big|_0^2$$

$$= 2 + 4 - \frac{8}{3}$$

$$= \frac{10}{3} \text{ unit}^2$$

Ex #2 $y = 7-2x^2$ $y = x^2+4$ A?

$$y: 7-2x^2 = x^2+4 \qquad -(x^2+4)$$

$$-3x^2 = -3$$

$$x^2 = 1 \rightarrow x = \pm 1$$

$$A = \int_{-1}^{1} (7-2x^2 - x^2 - 4)\,dx$$

even fct

$$= \int_{-1}^{1} (3-3x^2)\,dx$$

$\int 3x^2$ → x^3 $\cancel{\frac{3}{3}}$

$$= 3x - x^3 \Big|_{-1}^{1} \quad \Big\} \; 2(3x - x^3 \Big|_0^1$$

$$= 3-1-(-3+1)$$

$$= 2+2$$

$$= 4 \text{ unit}^2$$

Hwk 1.2 name

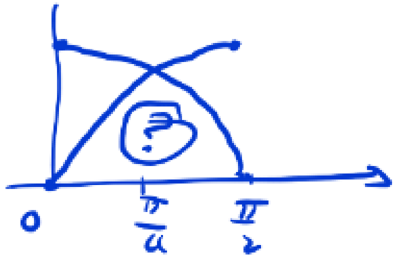
① Given

#1 $f(x) = \cos x \quad g(x) = \sin x \quad 0 \le x \le \frac{\pi}{2}$ A?

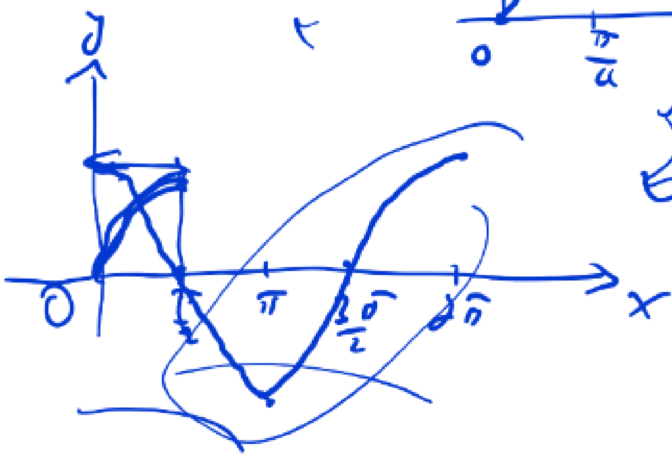
$\cos x = \sin x$

$x = ?$

$A = \int \qquad +$



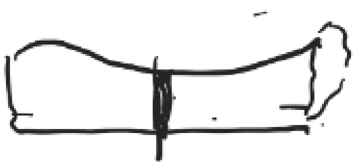
$y = \cos x$



1.3 Volume

Volume = Area × height

$V = A \cdot h$



$V = \int^{b} A(x)\, dx$

$\int_a$

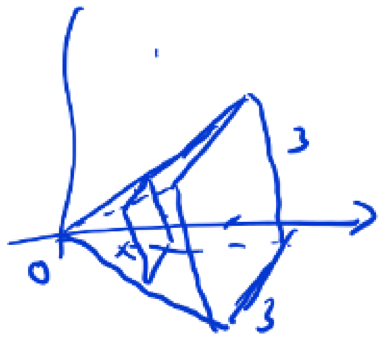
Ex 3 m sq: 3 m (side)

$A(x) = x^2$

$V = \int_0^3 x^2 dx$

$= \frac{1}{3}x^3 \Big|_0^3$

$= 9 \ m^3$

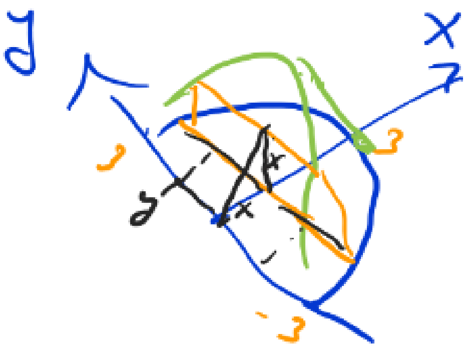


Ex $\theta = 45^\circ$

$x^2 + y^2 = 9$

$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$



$A = x(2y)$

$A(x) = 2x\sqrt{9 - x^2}$

$d(9 - x^2) = -2x\,dx$

$V = 2\int_0^3 x(9 - x^2)^{1/2} dx$

$= -\int_0^3 (9 - x^2)^{1/2} d(9 - x^2)$

$= -\frac{2}{3}(9 - x^2)^{3/2} \Big|_0^3$

$= -\frac{2}{3}[0 - 27]$

$9^{3/2}$

$(3^2)^{3/2}$

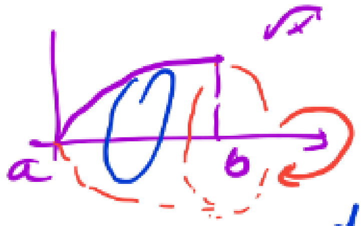
3^3

$= 18 \text{ unit}^3$

Disk Method

solid

revolution



$$V = \pi \int_a^b [R(x)]^2\, dx \quad = \pi \int_c^d R_{(y)}^2\, dy$$

Ex V? $y = \sqrt{x}$ $0 \le x \le 4$ rev x-axis

$$V = \pi \int_0^4 (\sqrt{x})^2\, dx$$

setup

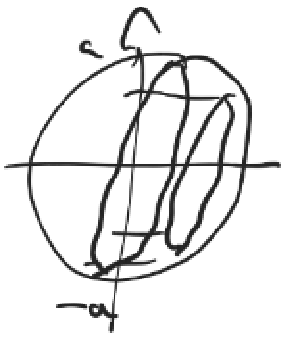
$$= \pi \int_0^4 x\, dx$$

$$= \frac{\pi}{2} x^2 \Big|_0^4$$

$$= 8\pi \text{ unit}^3$$

Ex Volume of a sphere w/ $r = a$

$$\boxed{\begin{aligned} A &= \pi r^2 \\ &= \pi a^2 \end{aligned}}$$



$$x^2 + y^2 = a^2$$

$$A(x) = \pi y^2$$

$$= \pi (a^2 - x^2)$$

$$V = \pi \int_{-a}^{a} (a^2 - x^2)\, dx \quad \to \quad \text{even fctn}$$

$$= 2\pi \left(a^2 x - \frac{1}{3} x^3\right) \Big|_0^a$$

$= 2\pi \left(a^{3} - \frac{1}{3} a^{3} \right)$

$= \frac{4\pi}{3} a^3 \text{ unit}^3$

Ex V? $x=0$, $\left(x = \frac{2}{y}\right)^2$ $1 \le y \le 4$
about y-axis

Soln

$$V = \pi \int_1^4 \frac{4}{y^2} \, dy \qquad \left(\frac{2}{y}\right)^2$$

$$= -4\pi \left. \frac{1}{y} \right|_1^4 \qquad \int \frac{dx}{x^2} = \frac{-1}{x} \quad \int x^{-2} dx$$

$$= -4\pi \left(\frac{1}{4} - 1 \right)$$

$$= 3\pi \text{ unit}^3$$

revolve about line $\begin{cases} x = a \\ y = b \end{cases}$

Ex V? $x = y^2 + 1$, $x = 3$ about $x=3$ dy

Soln $y^2 + 1 = 3$

$y^2 = 2 \Rightarrow y = \pm\sqrt{2}$

$$V = \pi \int_{-\sqrt{2}}^{\sqrt{2}} (3 - y^2 - 1)^2 \, dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (2 - y^2)^2 \, dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (4 - 4y^2 + y^4)\, dy$$

$$= 2\pi \left(4y - \frac{4}{3}y^3 + \frac{1}{5}y^5\right)\Big|_0^{\sqrt{2}}$$

$$= 2\pi\left(4\sqrt{2} - \frac{8}{3}\sqrt{2} + \frac{4}{5}\sqrt{2}\right)$$

$$= 2\pi\sqrt{2}\left(4 - \frac{8}{3} + \frac{4}{5}\right)$$

$$= 2\pi\sqrt{2}\left(\frac{60 - 40 + 12}{15}\right)$$

$$= \frac{64\pi\sqrt{2}}{15} \text{ unit}^3$$

Washer Method

$$V = \pi \int_a^b \left(R_{(x)}^2 - r_{(x)}^2\right) dx \leftarrow \text{rev } x\text{-axis}$$

$$= \pi \int_c^d \left([R(y)]^2 - [r(y)]^2\right) dy$$

Ex (Q I) $\begin{cases} x = y^3 & ① \\ x = 4y & ② \end{cases}$ rev: x-axis, y-axis

Soln

boundary?

$x = y^3 = 4y$ $\quad y = 0 \quad y^2 = 4$

$y^3 - 4y = 0$

$y(y^2 - 4) = 0 \quad y^2 = 4$

$y = 0, \pm 2)$ — $\quad (0, 0)\ y = x^{1/3}$

$y = 0, 2$ $\qquad$ $x = 0, 8$ $\qquad$ ② $\to y = \frac{x}{4}$

$$V = \pi \int_0^8 \left[x^{2/3} - \frac{x^2}{16} \right] dx$$

$$= \pi \left[\frac{3}{5} x^{5/3} - \frac{1}{48} x^3 \right|_0^8$$

$(2^{\cancel{3}})^{5/\cancel{3}}$

$\frac{8^2}{2}$

$$= \pi \left[\frac{3}{5} 2^5 - \frac{32}{3} \right)$$

$$= \pi \left(\frac{96}{5} - \frac{32}{3} \right)$$

$$= 32\pi \left(\frac{3}{5} - \frac{1}{3} \right)$$

$$= \boxed{\frac{128\pi}{15} \text{ unit}^3}$$

$$V = \pi \int_0^2 \left(16y^2 - y^6 \right) dy$$

$$= \pi \left(\frac{16}{3} y^3 - \frac{1}{7} y^7 \right|_0^2$$

$$= \pi \left(\frac{128}{3} - \frac{128}{7} \right) \qquad \frac{1}{3} - \frac{1}{7}$$

$$= 128\pi \left(\frac{4}{21} \right)$$

$$= \boxed{\frac{512\pi}{21} \text{ unit}^3}$$

$$\frac{\cancel{128}^{1}}{\cancel{15}_{5}} \; ? \; \frac{\cancel{512}^{4}}{\cancel{21}_{7}}$$

$$7 \; ? \; 20$$

x-axis $\qquad$ y-axis

Washer → shell 2 fetn →

αx ⇒ ✓ org

Pg 35 ✓ cut

tobe