

## ***Solution***      ***Section 3.2 - Exponential Functions***

### ***Exercise***

Evaluate to four decimal places using a calculator  $2^{3.4}$

### **Solution**

$$2^{3.4} = 10.5561$$

### ***Exercise***

Evaluate to four decimal places using a calculator  $5^{\sqrt{3}}$

### **Solution**

$$5^{\sqrt{3}} = 16.2425$$

### ***Exercise***

Evaluate to four decimal places using a calculator  $6^{-1.2}$

### **Solution**

$$6^{-1.2} = 0.1165$$

### ***Exercise***

Evaluate to four decimal places using a calculator:  $e^{-0.75}$

### **Solution**

$$e^{-0.75} = .4724$$

### ***Exercise***

Evaluate to four decimal places using a calculator:  $e^{2.3}$

### **Solution**

$$e^{2.3} = 9.9742$$

### ***Exercise***

Evaluate to four decimal places using a calculator:  $e^{-0.95}$

**Solution**

$$e^{-0.95} = 0.3867$$

***Exercise***

Evaluate to four decimal places using a calculator:  $\pi^{\sqrt{\pi}}$

**Solution**

$$\pi^{\sqrt{\pi}} = 7.6063$$

***Exercise***

Evaluate to four decimal places using a calculator:  $e^{\sqrt{2}}$

**Solution**

$$e^{\sqrt{2}} = 4.1133$$

***Exercise***

Sketch the graph:  $f(x) = 2^x + 3$

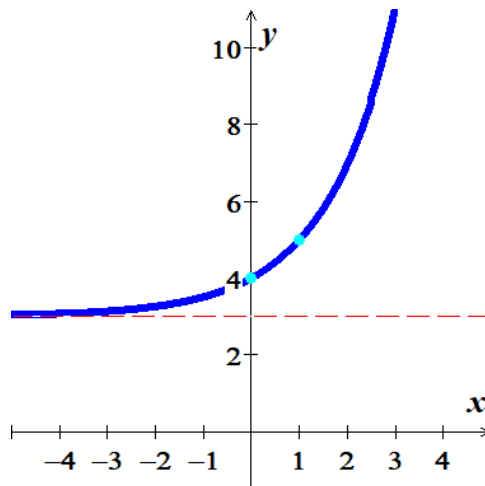
**Solution**

*Asymptote:*  $y = 3$

*Domain:*  $(-\infty, \infty)$

*Range:*  $(3, \infty)$

$x$	$f(x)$
-1	3.5
0	4
1	5
2	7



### Exercise

Sketch the graph:  $f(x) = 2^{3-x}$

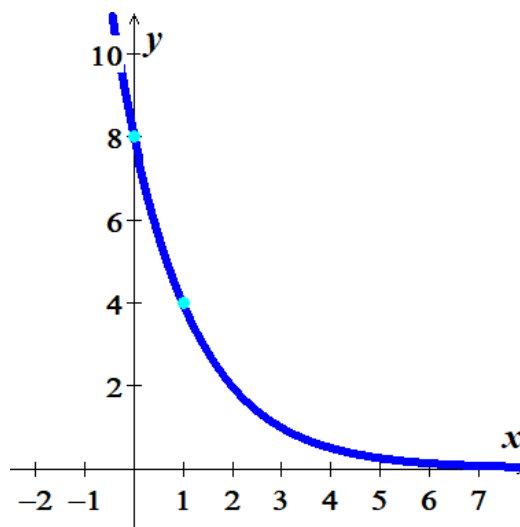
### Solution

**Asymptote:**  $y = 0$

**Domain:**  $(-\infty, \infty)$

**Range:**  $(0, \infty)$

$x$	$f(x)$
1	4
2	2
0	8



### Exercise

Sketch the graph:  $f(x) = \left(\frac{2}{5}\right)^{-x}$

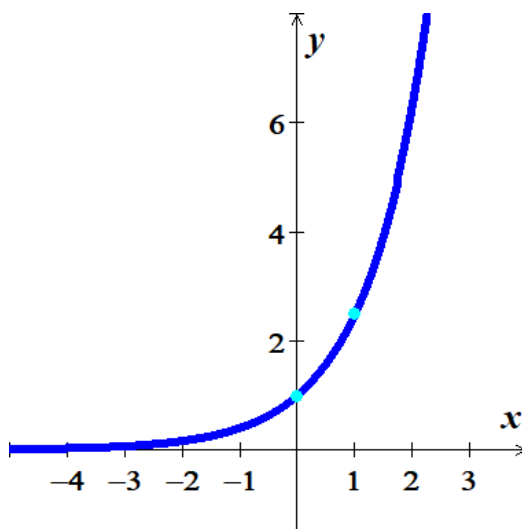
### Solution

**Asymptote:**  $y = 0$

**Domain:**  $(-\infty, \infty)$

**Range:**  $(0, \infty)$

$x$	$f(x)$
-1	0.4
0	1
1	2.5



### Exercise

Sketch the graph:  $f(x) = -\left(\frac{1}{2}\right)^x + 4$

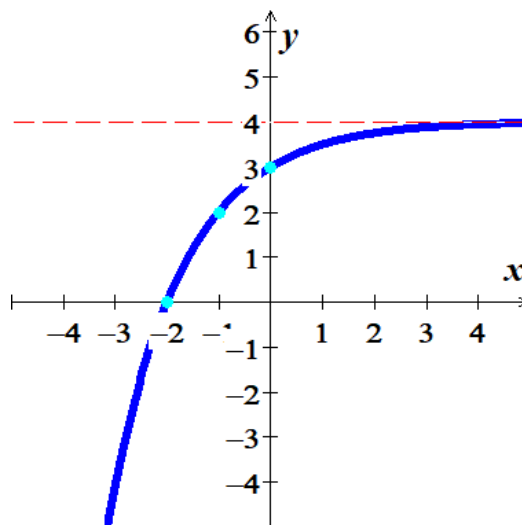
### Solution

**Asymptote:**  $y = 4$

**Domain:**  $(-\infty, \infty)$

**Range:**  $(-\infty, 4)$

$x$	$f(x)$
-2	0
-1	2
0	3



### Exercise

Sketch the graph of  $f(x) = 4^x$

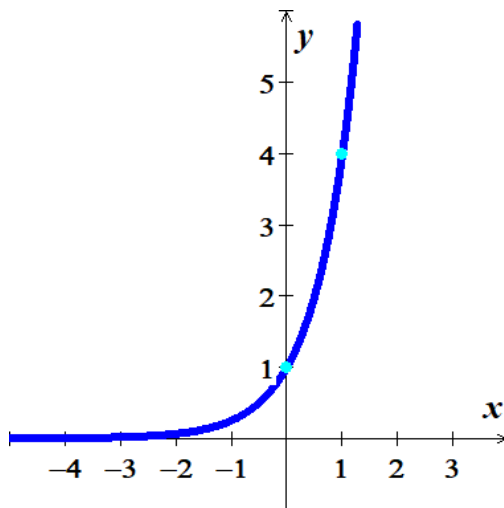
#### Solution

*Asymptote:*  $y = 0$

*Domain:*  $(-\infty, \infty)$

*Range:*  $(0, \infty)$

$x$	$f(x)$
0	1
1	4



### Exercise

Sketch the graph of  $f(x) = 2 - 4^x$

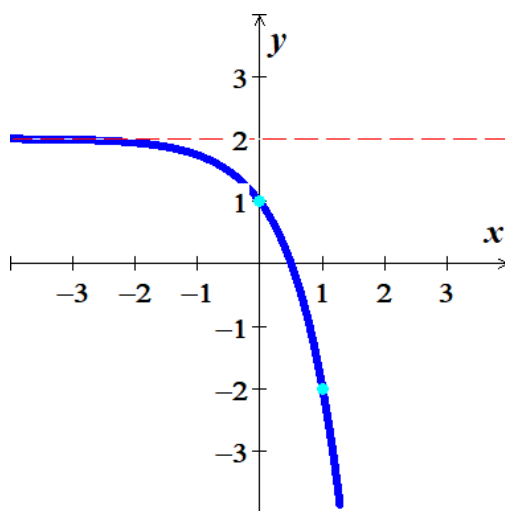
#### Solution

*Asymptote:*  $y = 2$

*Domain:*  $(-\infty, \infty)$

*Range:*  $(-\infty, 2)$

$x$	$f(x)$
0	1
1	-2



### Exercise

Sketch the graph of  $f(x) = -3 + 4^{x-1}$

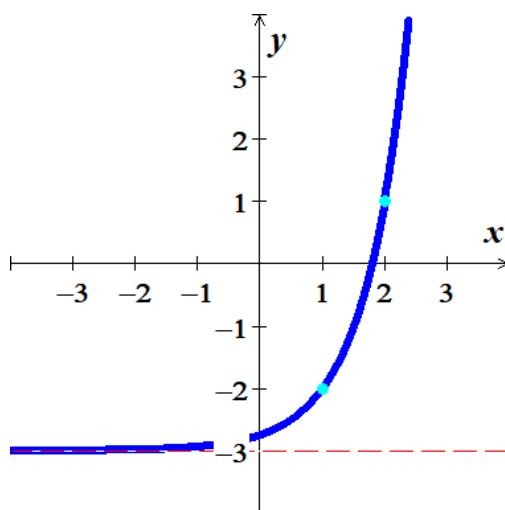
#### Solution

*Asymptote:*  $y = -3$

*Domain:*  $(-\infty, \infty)$

*Range:*  $(-3, \infty)$

$x$	$f(x)$
1	-2
2	1



### Exercise

Sketch the graph of  $f(x) = 1 + \left(\frac{1}{4}\right)^{x+1}$

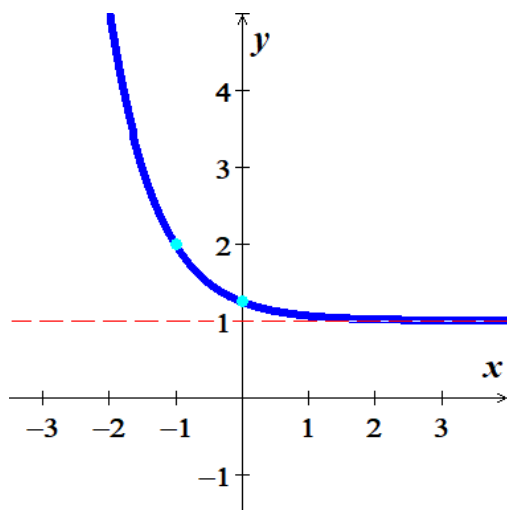
#### Solution

**Asymptote:**  $y = 1$

**Domain:**  $(-\infty, \infty)$

**Range:**  $(1, \infty)$

$x$	$f(x)$
-1	2
0	$\frac{5}{4}$



### Exercise

Sketch the graph of  $f(x) = e^{x-2}$

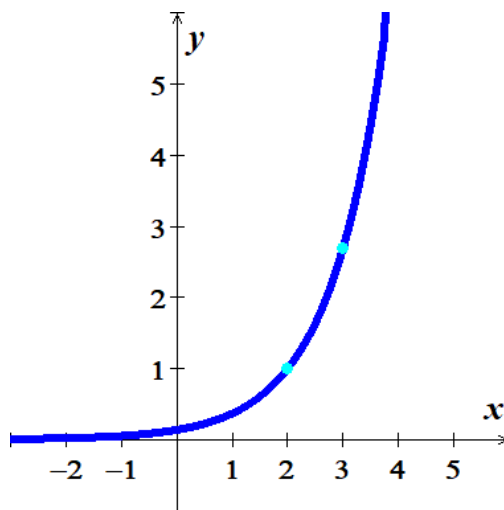
#### Solution

**Asymptote:**  $y = 0$

**Domain:**  $(-\infty, \infty)$

**Range:**  $(0, \infty)$

$x$	$f(x)$
2	1
3	2.7



### Exercise

Sketch the graph of  $f(x) = 3 - e^{x-2}$

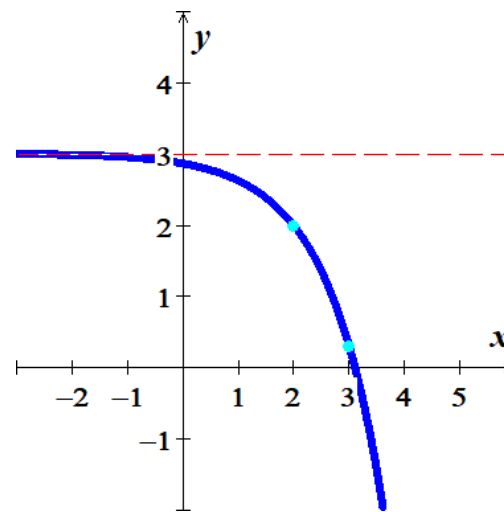
#### Solution

**Asymptote:**  $y = 3$

**Domain:**  $(-\infty, \infty)$

**Range:**  $(-\infty, 3)$

$x$	$f(x)$
2	2
3	1.7



### Exercise

Sketch the graph of  $f(x) = e^{x+4}$

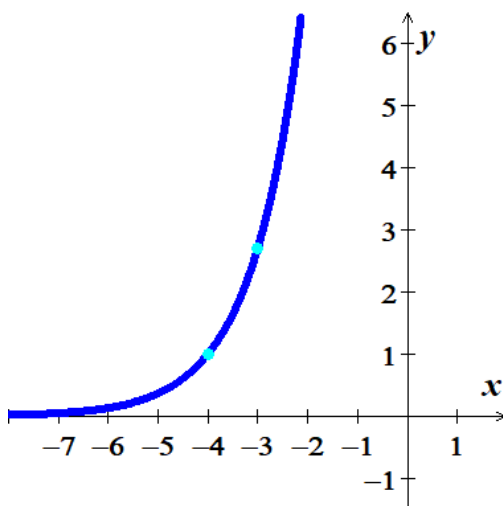
#### Solution

*Asymptote:*  $y = 0$

*Domain:*  $(-\infty, \infty)$

*Range:*  $(0, \infty)$

$x$	$f(x)$
-4	1
-3	2.7



### Exercise

Sketch the graph of  $f(x) = 2 + e^{x-1}$

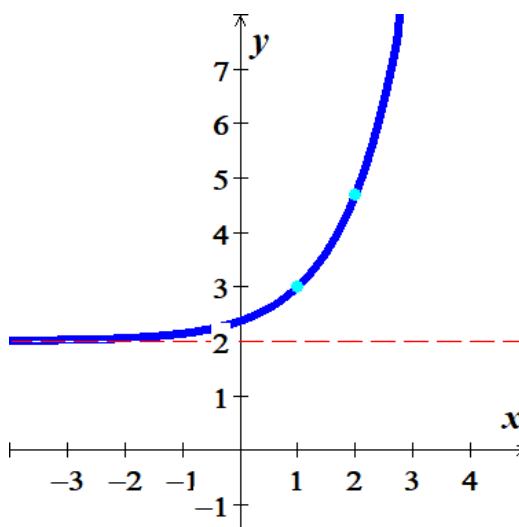
#### Solution

*Asymptote:*  $y = 2$

*Domain:*  $(-\infty, \infty)$

*Range:*  $(2, \infty)$

$x$	$f(x)$
1	3
2	4.7



### Exercise

The exponential function  $f(x) = 1066e^{0.042x}$  models the gray wolf population of the Western Great Lakes,  $f(x)$ , in *billions*,  $x$  years after 1978. Project the gray population in the recovery area in 2012.

#### Solution

$$x = 2012 - 1978 = 34$$

$$\begin{aligned} f(x = 34) &= 1066e^{0.042(34)} \\ &= 4445.6 \\ &\approx 4446 \text{ billions} \end{aligned}$$

### Exercise

The function  $f(x) = 6.4e^{0.0123x}$  describes world population,  $f(x)$ , in billions,  $x$  years after 2004 subject to a growth rate of 1.23% annually. Use the function to predict world population in 2050.

### Solution

$$x = 2050 - 2004 = 46$$

$$f(x = 46) = 6.4e^{0.0123(46)} \\ \approx 11.27 \text{ billion}$$

### Exercise

A cup of coffee is heated to  $160^\circ F$  and placed in a room that maintains a temperature of  $70^\circ F$ . The temperature  $T$  of the coffee, in degree Fahrenheit, after  $t$  minutes is given by

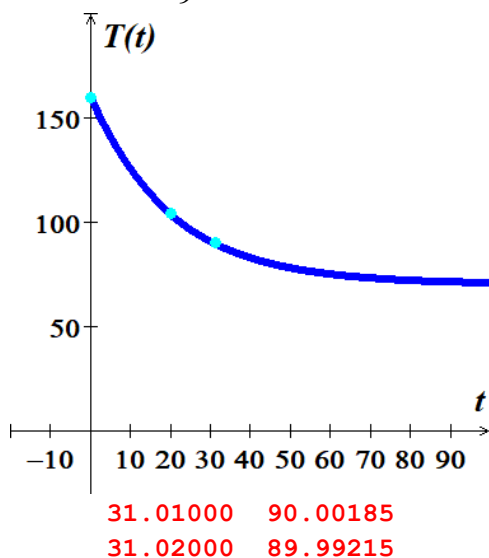
$$T(t) = 70 + 90e^{-0.0485t}$$

- a) Find the temperature of the coffee 20 minutes after it is placed in the room
- b) Determine when the temperature of the coffee will reach  $90^\circ F$

### Solution

$$a) \quad T(20) = 70 + 90e^{-0.0485(20)} \\ \approx 104^\circ F$$

$$b) \quad T(t) = 70 + 90e^{-0.0485t} = 90 \\ 90e^{-0.0485t} = 20 \\ e^{-0.0485t} = \frac{2}{9}$$



$\therefore$  The temperature of the coffee will reach  $90^\circ F$  in about **31.01** minutes.

### Exercise

A cup of coffee is heated to  $180^{\circ}F$  and placed in a room that maintains a temperature of  $65^{\circ}F$ . The temperature  $T$  of the coffee, in *degree Fahrenheit*, after  $t$  minutes is given by

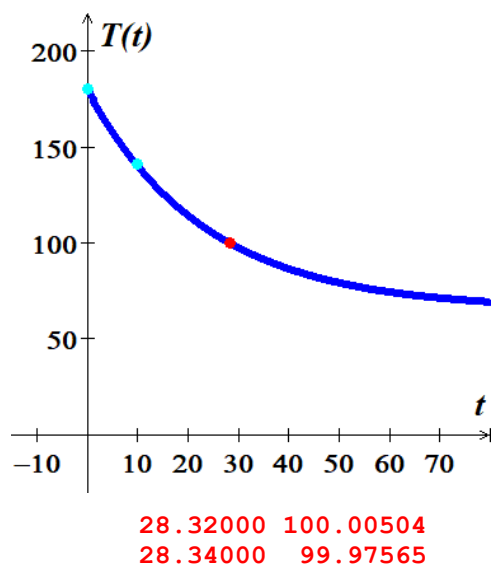
$$T(t) = 65 + 115e^{-0.042t}$$

- a) Find the temperature of the coffee 10 *minutes* after it is placed in the room
- b) Determine when the temperature of the coffee will reach  $100^{\circ}F$

### Solution

a)  $T(10) = 65 + 115e^{-0.042(10)}$   
 $\approx 141^{\circ}F$

b)  $T(t) = 65 + 115e^{-0.042t} = 100$   
 $115e^{-0.042t} = 35$   
 $e^{-0.042t} = \frac{7}{23}$



$\therefore$  The temperature of the coffee will reach  $100^{\circ}F$  in about **31.01** *minutes*.

### Exercise

The percent  $I(x)$  of the original intensity of light striking the surface of a lake that is available  $x$  *feet* below the surface of the lake is given by the equation

$$I(x) = 100e^{-.95x}$$

- a) What percentage of the light is available 2 *feet* below the surface of the lake?
- b) At what depth is the intensity of the light one-half the intensity at the surface?

### Solution

a)  $I(2) = 100e^{-.95(2)}$

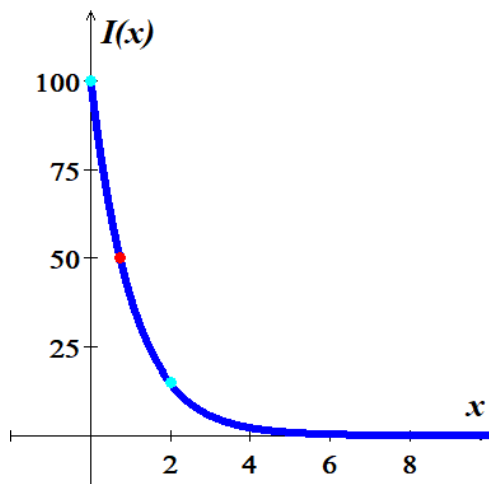


$$\approx 14.96$$

$\therefore$  The percentage of the light is available 2 feet below the surface of the lake is **15%**

$$b) I(x) = 100e^{-.95x} = \frac{1}{2}(100)$$

$$e^{-.95x} = \frac{1}{2}$$



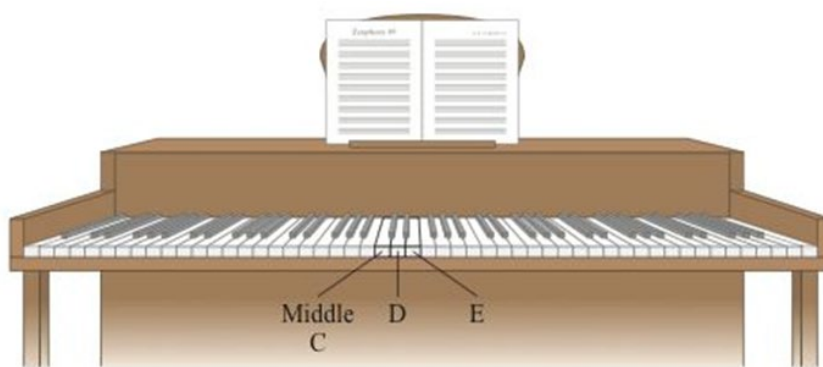
$$\begin{array}{ll} 0.72800 & 50.07742 \\ 0.73200 & 49.88749 \end{array}$$

$\therefore$  The depth is **0.73 feet** when the intensity of the light one-half the intensity at the surface

### Exercise

Starting on the left side of a standard 88-key piano, the frequency, in vibrations per second, of the  $n$ th note is given by

$$f(n) = (2.75) 2^{\frac{n-1}{12}}$$



- Determine the frequency of middle C, key number 40 on an 88-key piano.
- Is the difference in frequency between middle C (key number 40) and D (key number 42) the same as the difference in frequency between D (key number 42) and E (key number 44)?

### Solution

$$a) f(40) = (2.75) 2^{\frac{40-1}{12}}$$

$$\approx 26.16 \mid$$

the frequency of middle *C* is  $\approx 26$  vibrations per second.

$$b) f(42) = (2.75) 2^{(41/12)}$$

$$\approx 29.37 \mid$$

The difference between the frequency of middle *C* and *D* is:

$$29.37 - 26.16 \approx 3.21$$

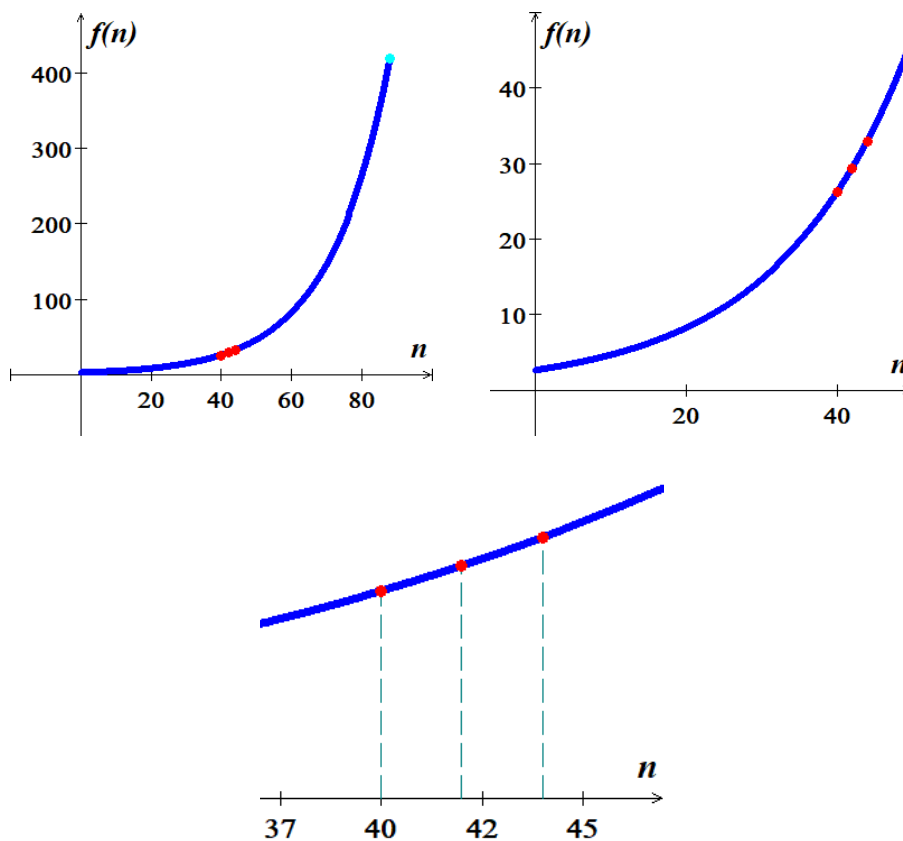
$$f(44) = (2.75) 2^{(43/12)}$$

$$\approx 32.96 \mid$$

The difference between the frequency of middle *D* and *E* is:

$$32.96 - 29.37 \approx 3.59$$

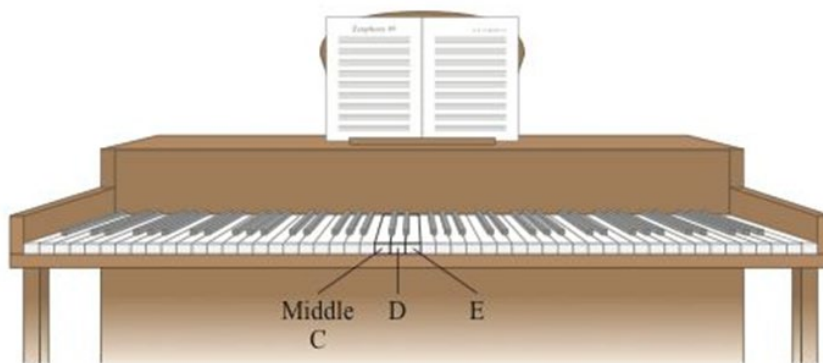
$\therefore$  the differences are **not** the same since the function is **not** linear function



## Exercise

Starting on the left side of a standard 88-key piano, the frequency, in *vibrations per second*, of the  $n$ th note is given by

$$f(n) = (27.5) 2^{\frac{n-1}{12}}$$



- c) Determine the frequency of middle C, key number 40 on an 88-key piano.
- d) Is the difference in frequency between middle C (key number 40) and D (key number 42) the same as the difference in frequency between D (key number 42) and E (key number 44)?

## Solution

$$\begin{aligned} \text{c) } f(40) &= (27.5) 2^{\frac{40-1}{12}} \\ &\approx 261.63 \end{aligned}$$

the frequency of middle C is  $\approx 262$  vibrations per second.

$$\begin{aligned} \text{d) } f(42) &= (27.5) 2^{(41/12)} \\ &\approx 293.66 \end{aligned}$$

The difference between the frequency of middle C and D is:  $293.66 - 261.66 \approx 32$

$$\begin{aligned} f(44) &= (27.5) 2^{(43/12)} \\ &\approx 329.63 \end{aligned}$$

The difference between the frequency of middle D and E is:  $329.63 - 293.66 \approx 36$

$\therefore$  The differences are **not** the same since the function is *not* linear function.

