

$$\begin{aligned}\lim_{x \rightarrow \pi} \frac{\cos x + 1}{(x - \pi)^2} &= \frac{0}{0} = \lim_{x \rightarrow \pi} \frac{-\sin x}{2(x - \pi)} = \frac{0}{0} \\ &= \lim_{x \rightarrow \pi} \frac{-\cos x}{2} \\ &= \underline{\underline{\frac{1}{2}}}\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin x - x}{7x^3} &= \frac{0}{0} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{21x^2} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{-\sin x}{42x} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{-\cos x}{42} \\ &= \underline{\underline{-\frac{1}{42}}}\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{\tan^{-1} x - \frac{\pi}{4}}{4x} &= \frac{0}{0} = \lim_{x \rightarrow \infty} \frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} \\ &= \lim_{x \rightarrow \infty} -\frac{x^2}{1+x^2} \\ &= \underline{\underline{-1}}\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 3} \frac{x-1-\sqrt{x^2-5}}{x-3} &= \frac{2-2}{0} = \frac{0}{0} \\ &= \lim_{x \rightarrow 3} \frac{1 - \frac{x}{\sqrt{x^2-5}}}{1} \\ &= 1 - \frac{3}{2} \\ &= \underline{\underline{-\frac{1}{2}}}\end{aligned}$$

$$\begin{aligned}\lim_{y \rightarrow 2} \frac{y^2 + y - 6}{\sqrt{8-y^2} - y} &= \frac{0}{0} = \lim_{y \rightarrow 2} \frac{2y+1}{\frac{-y}{\sqrt{8-y^2}} - 1} \\ &= \frac{5}{-1-1} = \underline{\underline{-\frac{5}{2}}}\end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{\sin^2 \pi x} &= \frac{0}{0} = \lim_{x \rightarrow 2} \frac{2x - 4}{2\pi \sin \pi x \cos \pi x} \\
 &= \lim_{x \rightarrow 2} \frac{2x - 4}{\pi \sin 2\pi x} = \frac{0}{0} \\
 &= \lim_{x \rightarrow 2} \frac{2}{2\pi^2 \cos 2\pi x} \\
 &= \frac{1}{\pi^2}
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow 2} \frac{(3x+2)^{1/3} - 2}{x-2} &= \frac{2-2}{0} = \frac{0}{0} \\
 &= \lim_{x \rightarrow 2} \frac{(3x-2)^{-2/3}}{1} \\
 &= 8^{-2/3} \\
 &= \frac{1}{4}
 \end{aligned}$$

$$\lim_{x \rightarrow \infty} \frac{3x^4 - x^2}{6x^4 + 12} = \frac{3}{6} = \frac{1}{2}$$

$$\lim_{x \rightarrow \infty} \frac{4x^3 - 2x^2 + 6}{\pi x^3 + 4} = \frac{4}{\pi}$$

$$\begin{aligned}
 \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\tan x}{3/(2x-\pi)} &= \frac{\infty}{\infty} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sec^2 x}{\frac{-6}{(2x-\pi)^2}} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}^-} \left(-\frac{1}{6}\right) \frac{(2x-\pi)^2}{\cos^2 x} = \frac{0}{0} \\
 &= -\frac{1}{6} \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{4(2x-\pi)}{-\sin 2x} = \frac{0}{0} \\
 &= \frac{2}{3} \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{2}{2\cos 2x} \\
 &= \frac{-2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow \infty} x \ln\left(1 + \frac{1}{x}\right) &= \lim_{x \rightarrow \infty} \frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} = \frac{0}{0} \\
 &= \lim_{x \rightarrow \infty} \frac{-\frac{1}{x^2}}{1 + \frac{1}{x}} \cdot \frac{1}{-\frac{1}{x^2}} \\
 &= \lim_{x \rightarrow \infty} \frac{x}{x+1} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow \frac{\pi}{2}^-} \left(\frac{\pi}{2} - x\right) \sec x &= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\frac{\pi}{2} - x}{\cos x} = \frac{0}{0} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{-1}{-\sin x} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow \infty} \frac{e^{1/x} - 1}{\sin \frac{1}{x}} &= \frac{1-1}{0} = \frac{0}{0} = \lim_{x \rightarrow \infty} \frac{-\frac{1}{x^2} e^{1/x}}{-\frac{1}{x^2} \cos \frac{1}{x}} \\
 &= \lim_{x \rightarrow \infty} \frac{e^{1/x}}{\cos \frac{1}{x}} = \frac{1}{1} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow 0^+} \sin x \sqrt{\frac{1-x}{x}} &= \lim_{x \rightarrow 0} \sin x \sqrt{\frac{x(1-x)}{x^2}} \\
 &= \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \sqrt{x(1-x)} \\
 &= 1 \cdot 0 \\
 &= 0
 \end{aligned}$$

$$\lim_{x \rightarrow 0} \left(\cot x - \frac{1}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{\cos x}{\sin x} - \frac{1}{x} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{x \cos x - \sin x}{x \sin x} \right) = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\cos x - x \sin x - \cos x}{\sin x + x \cos x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{-x \sin x}{\sin x + x \cos x} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x - x \cos x}{\cos x + \cos x - x \sin x}$$

$$= \frac{-0}{2}$$

$$= 0$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + 1}) = \infty - \infty = \lim_{x \rightarrow \infty} x \left(1 - \sqrt{1 + \frac{1}{x^2}} \right)$$

$$= \lim_{t \rightarrow 0} \frac{1 - \sqrt{1+t^2}}{t} = \frac{0}{0} \quad t = \frac{1}{x}$$

$$= \lim_{t \rightarrow 0} \frac{-t(1+t^2)^{-1/2}}{1}$$

$$= 0$$

$$\text{or } \lim_{x \rightarrow \infty} \left(1 - \frac{x}{\sqrt{x^2 + 1}} \right) = 1 - 1 = 0$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}^-} (\tan \theta - \sec \theta) = \infty - \infty = \lim_{\theta \rightarrow \frac{\pi}{2}^-} \left(\frac{\sin \theta}{\cos \theta} - \frac{1}{\cos \theta} \right)$$

$$= \lim_{\theta \rightarrow \frac{\pi}{2}^-} \left(\frac{\sin \theta - 1}{\cos \theta} \right) = \frac{0}{0}$$

$$= \lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{\cos \theta}{-\sin \theta} = \frac{0}{-1}$$

$$= 0$$

$$\begin{aligned}
 \lim_{x \rightarrow 0^+} \ln x^{2x} &= \lim_{x \rightarrow 0} 2x \ln x \\
 &= 2 \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x}} = \frac{\infty}{\infty} \\
 &= 2 \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} \\
 &= -2 \lim_{x \rightarrow 0} x \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow 0} \ln(1+4x)^{3/x} &= 3 \lim_{x \rightarrow 0} \frac{\ln(1+4x)}{x} \\
 &= 3 \lim_{x \rightarrow 0} \frac{\frac{4}{1+4x}}{1} \\
 &= 12
 \end{aligned}$$

$$\begin{aligned}
 \lim_{\theta \rightarrow \frac{\pi}{2}^-} \ln(\tan \theta)^{\cos \theta} &= \lim_{\theta \rightarrow \frac{\pi}{2}^-} \cos \theta \ln(\tan \theta) \\
 &= \lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{\ln(\tan \theta)}{\sec \theta} \\
 &= \lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{\sec^2 \theta / \tan \theta}{\sec \theta \tan \theta} \\
 &= \lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{\sec \theta}{\tan^2 \theta} \\
 &= \frac{0}{\infty} \\
 &= 0
 \end{aligned}$$

$$\lim_{x \rightarrow 0^+} (1+x)^{\cot x} = e^L$$

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \ln(1+x)^{\cot x} \\ &= \lim_{x \rightarrow 0} \cot x \ln(1+x) \\ &= \lim_{x \rightarrow 0} \frac{\ln(1+x)}{\tan x} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{1+x}}{\sec^2 x} = 1 \end{aligned}$$

$$\lim_{x \rightarrow 0} (1+x)^{\cot x} = e^1 = \underline{e}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\ln x} = e^L$$

$$\begin{aligned} L &= \lim_{x \rightarrow \infty} \ln \left(1 + \frac{1}{x}\right)^{\ln x} \\ &= \lim_{x \rightarrow \infty} (\ln x) \ln \left(1 + \frac{1}{x}\right) \\ &= \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{1}{x}\right)}{1/\ln x} \end{aligned}$$

$$= \lim_{x \rightarrow \infty} \frac{-\frac{1}{x^2}}{1 + \frac{1}{x}} \cdot \frac{1}{\frac{-1}{(\ln x)^2} \cdot \frac{1}{x}}$$

$$\frac{x \cdot 1}{x} \cdot \frac{1}{x}$$

$$\begin{aligned} &= \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x+1} \\ &= \lim_{x \rightarrow \infty} 2(\ln x) \frac{1}{x} = 2 \lim_{x \rightarrow \infty} \frac{1/x}{1} \\ &= 0 \end{aligned}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\ln x} = e^0 = \underline{1}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = e^L$$

$$\begin{aligned} L &= \lim_{x \rightarrow \infty} x \ln \left(1 + \frac{a}{x}\right) \\ &= \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{a}{x}\right)}{\frac{1}{x}} = \frac{0}{0} \\ &= \lim_{x \rightarrow \infty} \frac{-\frac{a}{x^2}}{1 + \frac{a}{x}} \cdot \frac{1}{-\frac{1}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{a}{1 + \frac{a}{x}} \\ &= a \end{aligned}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$

$$\lim_{x \rightarrow 0} (e^{5x} + x)^{1/x} = e^L$$

$$\begin{aligned} \lim_{x \rightarrow 0} \ln (e^{5x} + x)^{1/x} &= \lim_{x \rightarrow 0} \frac{\ln (e^{5x} + x)}{x} \\ &= \lim_{x \rightarrow 0} \frac{5e^{5x} + 1}{e^{5x} + x} \cdot \frac{1}{1} \\ &= \frac{6}{1} = 6 \end{aligned}$$

$$\lim_{x \rightarrow 0} (e^{5x} + x)^{1/x} = e^6$$

$$\lim_{x \rightarrow 0} (e^{ax} + x)^{1/x} = e^L$$

$$\begin{aligned} \lim_{x \rightarrow 0} \ln (e^{ax} + x)^{1/x} &= \lim_{x \rightarrow 0} \frac{\ln (e^{ax} + x)}{x} \\ &= \lim_{x \rightarrow 0} \frac{ae^{ax} + 1}{e^{ax} + x} \cdot \frac{1}{1} \\ &= a + 1 \end{aligned}$$

$$\lim_{x \rightarrow 0} (e^{ax} + x)^{1/x} = e^{a+1}$$

$$\lim_{x \rightarrow 0} (2^{ax} + x)^{1/x} = e^L$$

$$\begin{aligned} \lim_{x \rightarrow 0} \ln(2^{ax} + x)^{1/x} &= \lim_{x \rightarrow 0} \frac{\ln(2^{ax} + x)}{x} \\ &= \lim_{x \rightarrow 0} \frac{a 2^{ax} \ln 2 + 1}{2^{ax} + x} \cdot \frac{1}{1} \end{aligned}$$

$$= a \ln 2 + 1$$

$$\begin{aligned} \lim_{x \rightarrow 0} (2^{ax} + x)^{1/x} &= e^{1 + a \ln 2} \\ &= e e^{a \ln 2} \\ &= \underline{e \cdot 2^a} \end{aligned}$$

$$\lim_{x \rightarrow 0^+} (\tan x)^x = e^L$$

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \ln(\tan x)^x \\ &= \lim_{x \rightarrow 0} x \ln \tan x \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{\ln \tan x}{1/x}$$

$$= \lim_{x \rightarrow 0} \frac{\sec^2 x}{\tan x} \cdot \frac{1}{-1/x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-x^2}{\sin x \cos x}$$

$$= - \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \lim_{x \rightarrow 0} \frac{x}{\cos x}$$

$$= -1 \cdot 0$$

$$= 0$$

$$\lim_{x \rightarrow 0^+} (\tan x)^x = e^0 = \underline{1}$$