Review

10 $\left\{ \begin{array}{l} 5 \quad t\text{ - parametric} \\ 5 \quad \text{polar} \end{array} \right.$

$x = t^2 \qquad y = t^6 - 2t^4 \qquad -\infty \le t \le \infty$

$y = (t^2)^3 - 2(t^2)^2 \qquad x \geq 0$

$= x^3 - 2x^2 \qquad x \geq 0 \qquad y \in \mathbb{R}$?

$$\left\{ \begin{array}{l} x = 1 + \sin t \\ y = \cos t - 2 \end{array} \right. \qquad 0 \le t \le 2\pi$$

$\sin t = x - 1$

$\cos t = y + 2$

$\cos^2 t + \sin^2 t = 1$

$(y+2)^2 + (x-1)^2 = 1$

$\therefore$ circle center @ $(1, -2)$ + Radius 1

$$x = e^t \qquad y = \ln(t+1) \qquad t=0$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{(t+1)e^t}$$

$$\frac{dy}{dt} = \frac{1}{t+1}$$

$$\frac{dx}{dt} = e^t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$$

$$\left(\frac{1}{u}\right)' = \frac{-u'}{u^2}$$

$$\frac{dy'}{dt} = -\frac{1+t+1}{(t+1)^2 e^{2t}}\, e^t$$

$$\frac{d^2y}{dx^2} = -\frac{t+2}{(t+1)^2 e^t}\,\frac{1}{e^t}$$

$$= -\frac{t+2}{(t+1)^2 e^{2t}}\Bigg|_{t=0}$$

$$= \boxed{-2}$$

Area? $x = t - t^2$ $\quad y = 1 + e^{-t}$

y-axis $\Rightarrow x = 0$

$$A = \int_a^b x\,dy$$

$$t - t^2 = 0 \Rightarrow \boxed{t = 0, 1}$$

$$A = \int_0^1 (t - t^2)\, d(1 + e^{-t})$$

$$= \int_0^1 (t - t^2)(-e^{-t})\, dt$$

$$= \int_0^1 (t^2 - t)\, e^{-t}\, dt$$

		$\int e^{-t}$
+	$t^2 - t$	$-e^{-t}$
−	$2t - 1$	e^{-t}
+	2	$-e^{-t}$

$$= e^{-t}\left(-t^2 + t - 2t + 1 - 2\right)\Big|_0^1$$

$$= e^{-t}\left(-t^2 - t - 1\right)\Big|_0^1$$

$$= e^{-1}(-3) + 1$$

$$= \boxed{1 - \frac{3}{e} \text{ unit}^2}$$

Length

$$\begin{cases} x = a\cos^3 t \\ y = a\sin^3 t \end{cases}$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = -3a\cos^2 t \sin t$$

$$\frac{dy}{dt} = 3a\sin^2 t \cos t$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{9a^2\cos^4 t\sin^2 t + 9a^2\sin^4 t\cos^2 t}$$

$$= 3a\cos t\sin t\sqrt{\cos^2 t + \sin^2 t}$$

$$= 3a\cos t\sin t$$

$$= \frac{3}{2}a\sin 2t$$

$$L = \frac{3}{2}\int_0^{\pi/2} a\sin 2t\, dt$$

$$= -\frac{3a}{4}4\cos 2t \Big|_0^{\pi/2}$$

$$= -3a(-1-1)$$

$$= 6a \text{ unit}$$

Surface $x = \frac{2}{3} t^{3/2}$ $y = 2\sqrt{t}$

$0 \le t \le \sqrt{3}$ x-axis

$$S = 2\pi \int_a^b x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(t^{1/2}\right)^2 + \left(\frac{1}{\sqrt{t}}\right)^2}$$

$$= \sqrt{t + \frac{1}{t}}$$

$$S = 2\pi \frac{2}{3} \int_0^{\sqrt{3}} t^{3/2} \left(\frac{t^2+1}{t}\right)^{1/2} dt$$

$$= \frac{4\pi}{3} \int_0^{\sqrt{3}} t^{3/2} \frac{(t^2+1)^{1/2}}{t^{1/2}}\, dt$$

$$= \frac{4\pi}{3} \int_0^{\sqrt{3}} t \, (t^2+1)^{1/2}\, dt$$

$$= \frac{2\pi}{3} \int_0^{\sqrt{3}} (t^2+1)^{1/2}\, d(t^2+1)$$

$$= \frac{4\pi}{9} (t^2+1)^{3/2} \Big|_0^{\sqrt{3}}$$

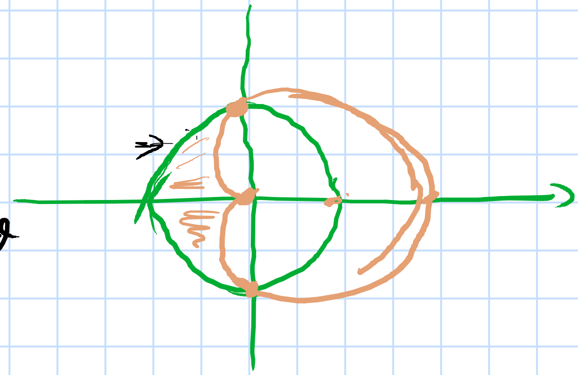
$$= \frac{4\pi}{9} \left(4^{3/2} - 1\right)$$

$$= \frac{28\pi}{9} \text{ unit}^2$$

Area? inside: $r=1$
out: $r = 1+\cos\theta$

$r = 1+\cos\theta = 1 \Rightarrow \cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$



$$A = 2\int_{\pi/2}^{\pi} \frac{1}{2}\left(1-(1+\cos\theta)^2\right)d\theta$$

$$= \int_{\pi/2}^{\pi} (-2\cos\theta - \cos^2\theta)\,d\theta$$

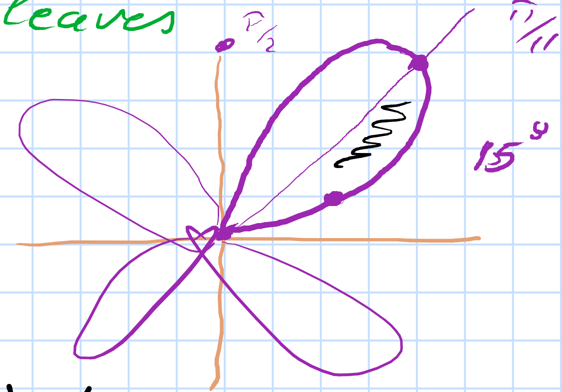
$$= \int_{\pi/2}^{\pi} \left(-2\cos\theta - \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= -2\sin\theta - \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \Big|_{\pi/2}^{\pi}$$

$$= -\frac{\pi}{2} + 2 + \frac{\pi}{4}$$

$$= 2 - \frac{\pi}{4} \text{ unit}^2$$

Area? $r = 3\sin \underbrace{2\theta}_{4\text{ leaves}}$ 1 leave



$$A = 2\cdot\frac{1}{2}\int_0^{\pi/4} 9\sin^2 2\theta\, d\theta$$

$$= \frac{9}{2}\int_0^{\pi/4}(1-\cos 4\theta)\, d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{1}{4}\sin 4\theta\right)\Big|_0^{\pi/4}$$

$$= \frac{9}{2}\left(\frac{\pi}{4}\right)$$

$$= \boxed{\frac{9\pi}{8}\ \text{unit}^2}$$

Area $y = 2 - \cos\theta$

$$A = \frac{1}{2}\int_0^{2\pi} (2-\cos\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (4 - 4\cos\theta + \cos^2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \left(\frac{9}{2} - 4\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{2}\left(\frac{9}{2}\theta - 4\sin\theta + \frac{1}{4}\sin 2\theta \right|_0^{2\pi}$$

$$= \frac{1}{2}(9\pi)$$

$$= \frac{9\pi}{2} \text{ unit}^2$$

Length: $r = a\sin^2\frac{\theta}{2}$ $\quad 0 \le \frac{\theta}{2} \le \frac{\pi}{2}$

$a > 0$

$$L = \int_\alpha^\beta \sqrt{r^2 + (r')^2}\, d\theta$$

$$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{a^2\sin^4\frac{\theta}{2} + \left(a\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^2}$$

$$= \sqrt{a^2\sin^4\frac{\theta}{2} + a^2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}$$

$$= a\left|\sin\frac{\theta}{2}\right|\sqrt{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}$$

$$\dot{=}\ a\sin\frac{\theta}{2}$$

$$L = a\int_0^\pi \sin\frac{\theta}{2}\, d\theta$$

$$= -2a\cos\frac{\theta}{2}\Big|_0^\pi$$

$$= -2a(0 - 1)$$

$$= 2a \text{ unit}$$

Surface, rev. polar $a > 0$

$$r = a(1+\cos\theta) \qquad 0 \le \theta \le \pi$$

$$S = 2\pi \int_{\alpha}^{\beta} f(\theta) \sin\theta \sqrt{r^2 + (r')^2}\, d\theta$$

$$\sqrt{r^2+(r')^2} = \sqrt{a^2(1+\cos\theta)^2 + a^2\sin^2\theta}$$

$$= a\sqrt{1+2\cos\theta+\underline{\cos^2\theta+\sin^2\theta}}$$

$$= a\sqrt{2+2\cos\theta}$$

$$S = 2\pi \int_0^{\pi} a^2(1+\cos\theta)\sin\theta\,(\sqrt{2})\,(1+\cos\theta)^{1/2}\,d\theta$$

$$= 2\pi a^2\sqrt{2}\int_0^{\pi} \sin\theta\,(1+\cos\theta)^{3/2}\,d\theta$$

$$= -2\pi a^2\sqrt{2}\int_0^{\pi} (1+\cos\theta)^{3/2}\,d(1+\cos\theta)$$

$$= -\frac{4\pi}{5}a^2\sqrt{2}\,(1+\cos\theta)^{5/2}\Big|_0^{\pi}$$

$$= -\frac{4\pi}{5}a^2\sqrt{2}\left(0-2^{5/2}\right) \qquad 4\sqrt{2}$$

$$= \boxed{\frac{32\pi a^2}{5} \text{ unit}^2}$$

Surface $\theta = \frac{\pi}{2}$

$r = 2\sin\theta \qquad 0 \le \theta \le \frac{\pi}{2}$

$$S = 2\pi \int_{\alpha}^{\beta} f(\theta)\cos\theta \sqrt{r^2 + (r')^2}\, d\theta$$

$$\sqrt{r^2 + (r')^2} = \sqrt{4\sin^2\theta + 4\cos^2\theta}$$
$$= \boxed{2}$$

$$S = 2\pi \int_{0}^{\pi/2} \underbrace{2\sin\theta\cos\theta}\,(2)\, d\theta$$
$$= 4\pi \int_{0}^{\pi/2} \sin 2\theta \, d\theta$$
$$= -2\pi \cos 2\theta \Big|_{0}^{\pi/2}$$
$$= -2\pi(-1 - 1)$$
$$= \boxed{4\pi \text{ unit}^2}$$