Solution   Section 8.6 – Polar Coordinates

Exercise

Convert to rectangular coordinates. $(4,\ 30°)$

Solution

$$x = r\cos\theta$$
$$= 4\cos 30°$$
$$= 4\left(\frac{\sqrt{3}}{2}\right)$$
$$= 2\sqrt{3}$$

$$y = r\sin\theta$$
$$= 4\sin 30°$$
$$= 4\left(\frac{1}{2}\right)$$
$$= 2$$

∴ The point $(2\sqrt{3},\ 2)$ in rectangular coordinates is equivalent to $(4,\ 30°)$ in polar coordinates.

Exercise

Convert to rectangular coordinates $\left(-\sqrt{2},\ \frac{3\pi}{4}\right)$.

Solution

$$x = -\sqrt{2}\cos\frac{3\pi}{4}$$
$$= -\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right)$$
$$= 1$$

$$y = -\sqrt{2}\sin\frac{3\pi}{4}$$
$$= -\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$
$$= -1$$

∴ The point $(1,\ -1)$ in rectangular coordinates is equivalent to $\left(-\sqrt{2},\ \frac{3\pi}{4}\right)$ in polar coordinates.

Exercise

Convert to rectangular coordinates $(3,\ 270°)$.

Solution

$$x = 3\cos 270°$$
$$= 3(0)$$
$$\underline{= 0}$$

$$y = 3\sin 270°$$
$$= 3(-1)$$
$$\underline{= -3}$$

$\therefore$ The point $(3,\ 270°)$ in polar coordinates is equivalent to $(0,\ -3)$ in rectangular coordinates.

Exercise

Convert to rectangular coordinates $\left(2,\ 60°\right)$

Solution

$$x = 2\cos 60°$$
$$= 2\left(\frac{1}{2}\right)$$
$$\underline{= 0}$$

$$y = 2\sin 60°$$
$$= 2\frac{\sqrt{3}}{2}$$
$$\underline{= \sqrt{3}}$$

$\therefore$ The point $\left(2,\ 60°\right)$ in polar coordinates is equivalent to $\left(1,\ \sqrt{3}\right)$ in rectangular coordinates.

Exercise

Convert to rectangular coordinates $\left(\sqrt{2},\ -225°\right)$

Solution

$$x = \sqrt{2}\cos\left(-225°\right)$$
$$= \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right)$$
$$\underline{= -1}$$

$$y = \sqrt{2}\sin\left(-225°\right)$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$\boxed{= 1}$$

$\therefore$ The point $\left(\sqrt{2},\ -225°\right)$ in polar coordinates is equivalent to $(-1,\ 1)$ in rectangular coordinates.

Exercise

Convert to rectangular coordinates $\left(4\sqrt{3},\ -\frac{\pi}{6}\right)$

Solution

$$x = 4\sqrt{3}\cos\left(-\frac{\pi}{6}\right)$$

$$= 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{= 6}$$

$$y = 4\sqrt{3}\sin\left(-\frac{\pi}{6}\right)$$

$$= 4\sqrt{3}\left(-\frac{1}{2}\right)$$

$$\boxed{= -2\sqrt{3}}$$

$\therefore$ The point $\left(4\sqrt{3},\ -\frac{\pi}{6}\right)$ in polar coordinates is equivalent to $\left(6,\ -2\sqrt{3}\right)$ in rectangular coordinates.

Exercise

Change the polar coordinates to rectangular coordinates $\left(-2,\ \frac{7\pi}{6}\right)$

Solution

$$x = -2\cos\left(\frac{7\pi}{6}\right)$$

$$= -2\left(-\frac{\sqrt{3}}{2}\right)$$

$$\boxed{= \sqrt{3}}$$

$$y = -2\sin\left(\frac{7\pi}{6}\right)$$

$$= -2\left(-\frac{1}{2}\right)$$

$$\boxed{= 1}$$

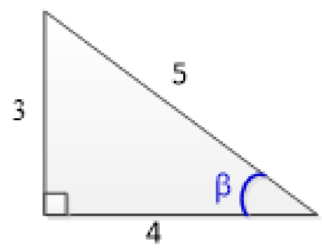
$\therefore$ The point $\left(-2,\ \frac{7\pi}{6}\right)$ in polar coordinates is equivalent to $\left(\sqrt{3},\ 1\right)$ in rectangular coordinates.

Exercise

Change the polar coordinates to rectangular coordinates $\left(6,\ \arctan\frac{3}{4}\right)$

Solution

$$\arctan\frac{3}{4} = \beta \quad \Rightarrow \quad \tan\beta = \frac{3}{4}$$



$$\begin{aligned} x &= 2\cos\beta \\ &= 2\left(\tfrac{4}{5}\right) \\ &= \frac{8}{5} \end{aligned}$$

$$\begin{aligned} y &= 2\sin\beta \\ &= 2\left(\tfrac{3}{5}\right) \\ &= \frac{6}{5} \end{aligned}$$

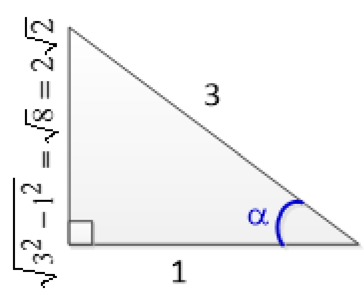
$\therefore$ The point $\left(6,\ \arctan\frac{3}{4}\right)$ in polar coordinates is equivalent to $\left(\frac{8}{5},\ \frac{6}{5}\right)$ in rectangular coordinates.

Exercise

Change the polar coordinates to rectangular coordinates $\left(10,\ \arccos\left(-\frac{1}{3}\right)\right)$

Solution

$$\arccos\left(-\tfrac{1}{3}\right) = \alpha \quad \Rightarrow \quad \cos\alpha = -\frac{1}{3} \quad (QII)$$



$$\begin{aligned} x &= 10\cos\alpha \\ &= 10\left(-\tfrac{1}{3}\right) \\ &= -\frac{10}{3} \end{aligned}$$

$$\begin{aligned} y &= 10\sin\alpha \\ &= 10\left(\frac{2\sqrt{2}}{3}\right) \\ &= \frac{20\sqrt{2}}{3} \end{aligned}$$

$\therefore$ The point $\left(10,\ \arccos\left(-\frac{1}{3}\right)\right)$ in polar coordinates is equivalent to $\left(-\frac{10}{3},\ \frac{20\sqrt{2}}{3}\right)$ in rectangular coordinates.

Exercise

Convert to polar coordinates $(3,\ 3)$.

Solution

$$r = \sqrt{3^2 + 3^2}$$
$$= \sqrt{18}$$
$$\boxed{= 3\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{3}{3}\right)$$
$$= \tan^{-1}(1)$$
$$\boxed{= 45^\circ}$$

$\therefore$ The point $(3,\ 3)$ in rectangular coordinates is equivalent to $\left(3\sqrt{2},\ 45^\circ\right)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-2,\ 0)$.

Solution

$$r = \pm\sqrt{4+0}$$
$$\boxed{= \pm\ 2}$$

$$\theta = \tan^{-1}\frac{0}{-2}$$
$$\boxed{= 0^\circ}$$

$\therefore$ The point $(-2,\ 0)$ in rectangular coordinates is equivalent to $\left(-2,\ 0^\circ\right)$ $\left(2,\ 180^\circ\right)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-1,\ \sqrt{3})$.

Solution

$$r = \pm\sqrt{1+3}$$
$$\boxed{= \pm\ 2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$$
$$\boxed{= 120^\circ}$$

$\therefore$ The point $\left(-1,\ \sqrt{3}\right)$ in rectangular coordinates is equivalent to $\left(2,\ 120^\circ\right)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-3,\ -3)$ $r \ge 0$ $0° \le \theta < 360°$

Solution

$$r = \sqrt{(-3)^2 + (-3)^2}$$

$$= 3\sqrt{2}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{3}{3}\right)$$

$$= \tan^{-1}(1)$$

$$= 45°$$

The angle is in quadrant III

Therefore, $\theta = 180° + 45°$

$$= 225°$$

$\therefore$ The point $(-3,\ 3)$ in rectangular coordinates is equivalent to $\left(3\sqrt{2},\ 225°\right)$ in polar coordinates.

Exercise

Convert to polar coordinates $\left(2,\ -2\sqrt{3}\right)$ $r \ge 0$ $0° \le \theta < 360°$

Solution

$$r = \sqrt{2^2 + \left(-2\sqrt{3}\right)^2}$$

$$= 4$$

$$\hat{\theta} = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)$$

$$= \tan^{-1}\left(\sqrt{3}\right)$$

$$= 60°$$

The angle is in quadrant *IV*

Therefore, $\theta = 360° - 60°$

$$= 300°$$

$\therefore$ The point $\left(2,\ -2\sqrt{3}\right)$ in rectangular coordinates is equivalent to $(4,\ 300°)$ in polar coordinates.

Exercise

Convert to polar coordinates $(-2,\ 0)$ $r \ge 0$ $0 \le \theta < 2\pi$

Solution

$$r = \sqrt{(-2)^2 + 0^2}$$

$$\underline{= 2}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{0}{2}\right)$$

$$\underline{= 0}$$

$$\underline{\theta = \pi}$$

$\therefore$ The point $(-2,\ 0)$ in rectangular coordinates is equivalent to $(2,\ \pi)$ in polar coordinates.

Exercise

Convert to polar coordinates $\left(-1,\ -\sqrt{3}\right)$ $r \ge 0$ $0 \le \theta < 2\pi$

Solution

$$r = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2}$$

$$\underline{= 2}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$\underline{= \frac{\pi}{3}}$$

The angle is in quadrant *III*

Therefore, $\theta = \pi + \frac{\pi}{3}$

$$\underline{= \frac{4\pi}{3}}$$

$\therefore$ The point $\left(-1,\ -\sqrt{3}\right)$ in rectangular coordinates is equivalent to $\left(2,\ \frac{4\pi}{3}\right)$ in polar coordinates.

Exercise

Change the rectangular coordinates to polar coordinates $\left(7,\ -7\sqrt{3}\right)$ $r > 0$ $0 \le \theta < 2\pi$

Solution

$$r = \sqrt{(7)^2 + \left(-7\sqrt{3}\right)^2}$$

$= \sqrt{196}$

$\boxed{= 14}$

$\hat{\theta} = \tan^{-1}\left(\frac{7\sqrt{3}}{7}\right)$

$\boxed{= \frac{\pi}{3}}$

The angle is in quadrant *IV*; therefore,

$\theta = 2\pi - \frac{\pi}{3}$

$\boxed{= \frac{5\pi}{3}}$

∴ The point $\left(7,\ -7\sqrt{3}\right)$ in rectangular coordinates is equivalent to $\left(14,\ \frac{5\pi}{3}\right)$ in polar coordinates.

Exercise

Change the rectangular coordinates to polar coordinates $\left(-2\sqrt{2},\ -2\sqrt{2}\right) \quad r > 0 \quad 0 \le \theta < 2\pi$

Solution

$r = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(-2\sqrt{2}\right)^2}$

$\boxed{= 4}$

$\hat{\theta} = \tan^{-1}\left(\frac{-2\sqrt{2}}{-2\sqrt{2}}\right)$

$= \tan^{-1}(1)$

$\boxed{= \frac{\pi}{4}}$

The angle is in quadrant *III*; therefore,

$\theta = \pi + \frac{\pi}{4}$

$\boxed{= \frac{5\pi}{4}}$

∴ The point $\left(7,\ -7\sqrt{3}\right)$ in rectangular coordinates is equivalent to $\left(4,\ \frac{5\pi}{4}\right)$ in polar coordinates.

Exercise

The point $(0,\ -3)$ in rectangular coordinates is equivalent to $(3,\ 270°)$ in polar coordinates.

Solution

$$r = \sqrt{0 + (-3)^2}$$
$$\underline{= 3}|$$
$$\hat{\theta} = \tan^{-1}\frac{0}{3}$$
$$\underline{= 90°}|$$

The polar point is $\underline{(3,\ 270°)}|$

Exercise

The point $(1,\ -1)$ in rectangular coordinates is equivalent to $\left(-\sqrt{2},\ \frac{3\pi}{4}\right)$ in polar coordinates.

Solution

$$r = \sqrt{(1)^2 + (-1)^2}$$
$$\underline{= \sqrt{2}}|$$
$$\hat{\theta} = \tan^{-1}\left(\frac{1}{1}\right)$$
$$\underline{= \frac{\pi}{4}}|$$
$$\theta \in QIV \quad \to \quad \underline{\theta = \frac{7\pi}{4}}|$$
$$\left(\sqrt{2},\ \frac{7\pi}{4}\right) \Leftrightarrow \left(-\sqrt{2},\ \frac{3\pi}{4}\right)$$

Exercise

A point lies at (4, 4) on a rectangular coordinate system. Give its address in polar coordinates $(r,\ \theta)$

Solution

$$r = \sqrt{4^2 + 4^2}$$
$$= \sqrt{32}$$
$$\underline{= 4\sqrt{2}}|$$
$$\theta = \tan^{-1}\left(\frac{4}{4}\right)$$

$$= \tan^{-1}(1)$$

$$\boxed{= 45^\circ}$$

$\therefore$ The point $(4,\ 4)$ in rectangular coordinates is equivalent to $\left(4\sqrt{2},\ 45^\circ\right)$ in polar coordinates.

Exercise

Write the equation in rectangular coordinates $r^2 = 4$

Solution

$$r^2 = 4$$

$$\boxed{x^2 + y^2 = 4}$$

Exercise

Write the equation in rectangular coordinates $r = 6\cos\theta$

Solution

$$r = 6\cos\theta$$

$$r = 6\frac{x}{r}$$

$$r^2 = 6x$$

$$\boxed{x^2 + y^2 = 6x}$$

Exercise

Write the equation in rectangular coordinates $r^2 = 4\cos 2\theta$

Solution

$$r^2 = 4\cos 2\theta$$

$$= 4\left(\cos^2\theta - \sin^2\theta\right) \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$= 4\left(\frac{x^2}{r^2} - \frac{y^2}{r^2}\right)$$

$$= 4\left(\frac{x^2 - y^2}{r^2}\right)$$

$$r^4 = 4\left(x^2 - y^2\right) \qquad r^2 = x^2 + y^2$$

$$\left(x^2+y^2\right)^4=4x^2-4y^2$$

Exercise

Write the equation in rectangular coordinates $r(\cos\theta - \sin\theta) = 2$

Solution

$$r(\cos\theta - \sin\theta) = 2 \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$r\left(\frac{x}{r} - \frac{y}{r}\right) = 2$$

$$r\left(\frac{x-y}{r}\right) = 2$$

$$\boxed{x - y = 2}$$

Exercise

Write the equation in rectangular coordinates $r^2 = 4\sin 2\theta$

Solution

$$r^2 = 4\sin 2\theta \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$= 4(2\sin\theta\cos\theta) \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$= 8\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$$

$$= 8\frac{xy}{r^2}$$

$$r^4 = 8xy \qquad r^2 = x^2 + y^2$$

$$\boxed{\left(x^2 + y^2\right)^2 = 8xy}$$

Exercise

Find an equation in x and y that has the same graph as polar equation. $r\sin\theta = -2$

Solution

$$r\sin\theta = -2 \qquad y = r\sin\theta$$

$$\boxed{y = -2}$$

Exercise

Find an equation in x and y that has the same graph as polar equation. $\theta = \frac{\pi}{4}$

Solution

$$\tan\theta = \tan\frac{\pi}{4}$$

$$\frac{y}{x} = 1$$

$$\boxed{y = x}$$

Exercise

Find an equation in x and y that has the same graph as polar $r^2\left(4\sin^2\theta - 9\cos^2\theta\right) = 36$

Solution

$$r^2\left(4\sin^2\theta - 9\cos^2\theta\right) = 36 \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$r^2\left(4\frac{y^2}{r^2} - 9\frac{x^2}{r^2}\right) = 36$$

$$r^2\left(\frac{4y^2 - 9x^2}{r^2}\right) = 36$$

$$\boxed{4y^2 - 9x^2 = 36}$$

Exercise

Find an equation in x and y that has the same graph as polar $r^2\left(\cos^2\theta + 4\sin^2\theta\right) = 16$

Solution

$$r^2\left(\cos^2\theta + 4\sin^2\theta\right) = 16 \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$r^2\left(\frac{x^2}{r^2} + 4\frac{y^2}{r^2}\right) = 16$$

$$r^2\left(\frac{x^2 + 4y^2}{r^2}\right) = 16$$

$$\boxed{x^2 + 4y^2 = 16}$$

Exercise

Find an equation in x and y that has the same graph as polar $r(\sin\theta - 2\cos\theta) = 6$

Solution

$$r(\sin\theta - 2\cos\theta) = 6 \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$r\left(\frac{y}{r} - 2\frac{x}{r}\right) = 6$$

$$r\left(\frac{y-2x}{r}\right) = 6$$

$$\underline{y - 2x = 6}$$

Exercise

Find an equation in x and y that has the same graph as polar $r\left(\sin\theta + r\cos^2\theta\right) = 1$

Solution

$$r\left(\sin\theta + r\cos^2\theta\right) = 1 \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$r\left(\frac{y}{r} + r\frac{x^2}{r^2}\right) = 1$$

$$r\left(\frac{y}{r} + \frac{x^2}{r}\right) = 1$$

$$r\left(\frac{y + x^2}{r}\right) = 1$$

$$\underline{y + x^2 = 1}$$

Exercise

Find an equation in x and y that has the same graph as polar $r = 8\sin\theta - 2\cos\theta$

Solution

$$r = 8\sin\theta - 2\cos\theta \qquad \cos\theta = \frac{x}{r} \quad \sin\theta = \frac{y}{r}$$

$$r = 8\frac{y}{r} - 2\frac{x}{r}$$

$$r^2 = 8y - 2x \qquad r^2 = x^2 + y^2$$

$$\underline{x^2 + y^2 = 8y - 2x}$$

Exercise

Find an equation in x and y that has the same graph as polar $r = \tan\theta$

Solution

$$r = \tan\theta$$

$$x^2 + y^2 = \frac{y^2}{x^2}$$

$$x^4 + x^2y^2 = y^2$$

$$\boxed{\sqrt{x^2 + y^2} = \frac{y}{x}}$$

Exercise

Find a polar equation that has the same graph as the equation in x and y. $y^2 = 6x$

Solution

$$y^2 = 6x \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$(r\sin\theta)^2 = 6(r\cos\theta)$$

$$r^2\sin^2\theta = 6r\cos\theta$$

$$\boxed{r = 6\frac{\cos\theta}{\sin^2\theta}}$$

Exercise

Find a polar equation that has the same graph as the equation in x and y. $xy = 8$

Solution

$$xy = 8 \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$(r\cos\theta)(r\sin\theta) = 8$$

$$\boxed{r^2 = \frac{8}{\cos\theta\sin\theta}}$$

Exercise

Find a polar equation that has the same graph as the equation in x and y. $(x+2)^2 + (y-3)^2 = 13$

Solution

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 13$$

$$x^2+4x+y^2-6y=13-9-4$$
$$x^2+4x+y^2-6y=0$$
$$x^2+y^2=6y-4x \qquad x=r\cos\theta \quad y=r\sin\theta$$
$$r^2=6r\sin\theta-4r\cos\theta$$
$$r^2=r\left(6\sin\theta-4\cos\theta\right) \qquad \textit{\textbf{Divide by r}}$$
$$\boxed{r=6\sin\theta-4\cos\theta}$$

Exercise

Find a polar equation that has the same graph as the equation in x and y. $y^2-x^2=4$

Solution

$$y^2-x^2=4 \qquad x=r\cos\theta \quad y=r\sin\theta$$
$$r^2\sin^2\theta-r^2\cos^2\theta=4$$
$$r^2\left(\sin^2\theta-\cos^2\theta\right)=4 \qquad \cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$
$$r^2\left(-\cos 2\theta\right)=4$$
$$\boxed{r^2=-\frac{4}{\cos 2\theta}}$$

Exercise

Write the equation in polar coordinates $x+y=5$

Solution

$$x+y=5$$
$$r\cos\theta+r\sin\theta=5 \qquad x=r\cos\theta \quad y=r\sin\theta$$
$$r\left(\cos\theta+\sin\theta\right)=5$$
$$\boxed{r=\frac{5}{\cos\theta+\sin\theta}}$$

Exercise

Write the equation in polar coordinates $x^2+y^2=9$

Solution

$$x^2+y^2=9 \qquad r^2=x^2+y^2$$
$$\boxed{r^2=9}$$

Exercise

Write the equation in polar coordinates $x^2 + y^2 = 4x$

Solution

$x^2 + y^2 = 4x \qquad\qquad r^2 = x^2 + y^2 \quad x = r\cos\theta$

$r^2 = 4r\cos\theta$

$\frac{r^2}{r} = \frac{4r\cos\theta}{r}$

$r = 4\cos\theta$

Exercise

Write the equation in polar coordinates $y = -x$

Solution

$y = -x \qquad\qquad x = r\cos\theta \quad y = r\sin\theta$

$r\sin\theta = -r\cos\theta$

$\sin\theta = -\cos\theta$

Exercise

Write the equation in polar coordinates $x + y = 4$

Solution

$x + y = 4$

$r\cos\theta + r\sin\theta = 4 \qquad\qquad x = r\cos\theta \quad y = r\sin\theta$

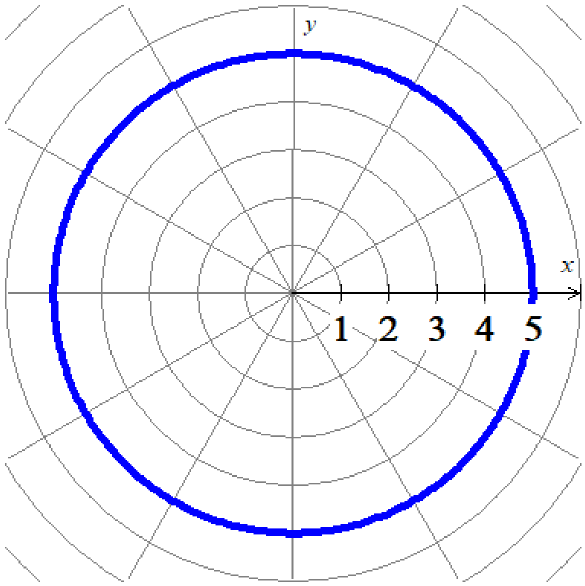
$r(\cos\theta + \sin\theta) = 4$

$r = \frac{4}{\cos\theta + \sin\theta}$

Exercise

Sketch the graph of the polar equation $r = 5$

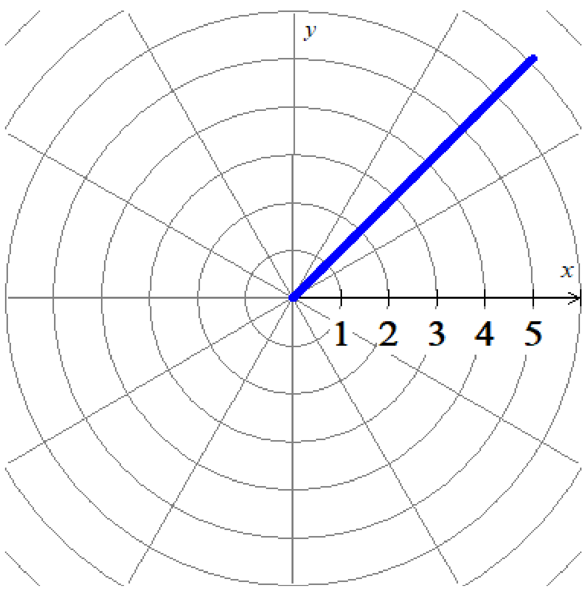
Solution



Exercise

Sketch the graph of the polar equation $\theta = \frac{\pi}{4}$

Solution

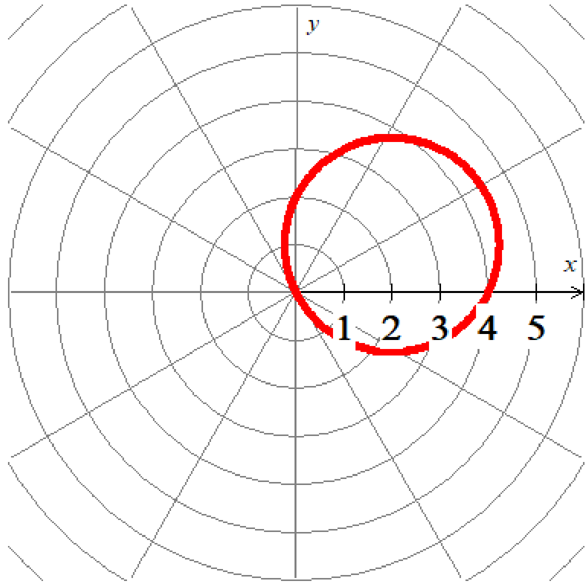


Exercise

Sketch graph $r = 4\cos\theta + 2\sin\theta$

Solution

θ	r
0	4
$\frac{\pi}{4}$	$3\sqrt{2}$
$\frac{\pi}{2}$	2
$\frac{3\pi}{4}$	$-\sqrt{2}$
π	-4
$\frac{3\pi}{2}$	-2

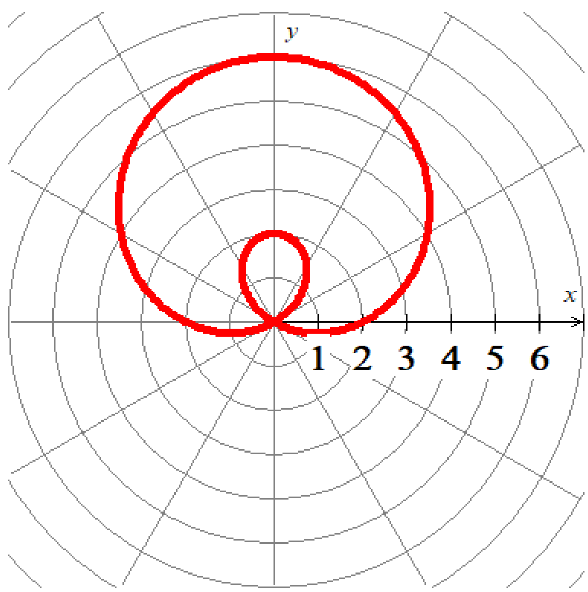


Exercise

Sketch the graph of the polar $r = 2 + 4\sin\theta$

Solution

θ	r
0	2
$\frac{\pi}{6}$	4
$\frac{\pi}{4}$	$2+2\sqrt{2}$
$\frac{\pi}{2}$	6
$\frac{5\pi}{6}$	4
π	2
$\frac{7\pi}{6}$	0
$\frac{3\pi}{2}$	-2
$\frac{11\pi}{6}$	0

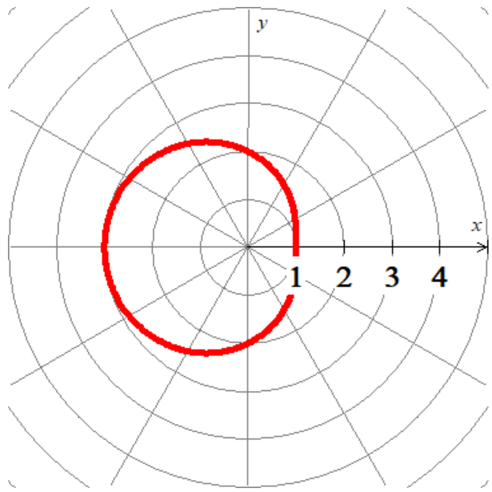


Exercise

Sketch the graph $r = 2 - \cos\theta$

Solution

θ	r
0	1
$\frac{\pi}{3}$	$\frac{3}{2}$
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	$\frac{5}{2}$
π	3
$\frac{4\pi}{3}$	$\frac{5}{2}$
$\frac{3\pi}{2}$	2
$\frac{5\pi}{3}$	$\frac{3}{2}$



Exercise

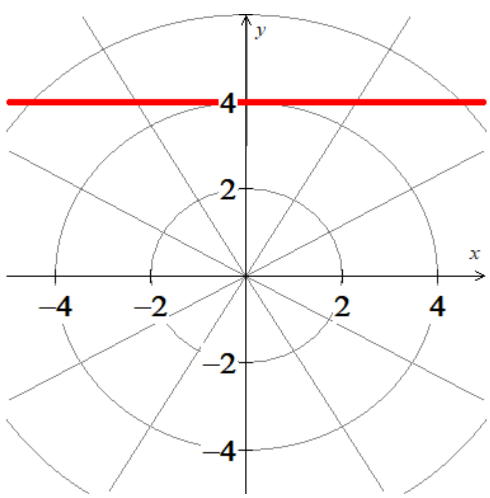
Sketch the graph $r = 4\csc\theta$

Solution

$$r = 4\csc\theta$$

$$= \frac{4}{\sin\theta}$$

$$r\sin\theta = \boxed{4 = y}$$



Exercise

Sketch the graph $r^2 = 4\cos 2\theta$

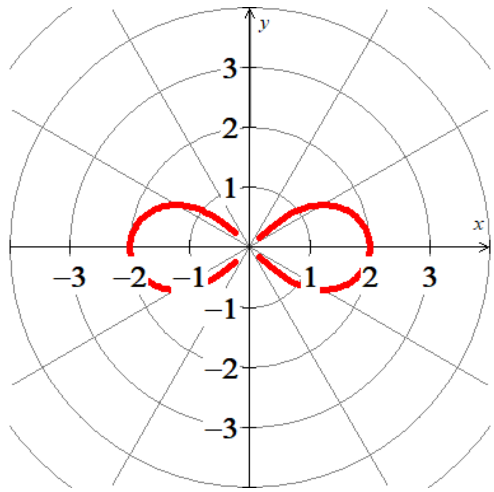
Solution

$r^2 = 4\cos 2\theta \ge 0$

$-\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$ & $\frac{3\pi}{4} \le \theta \le \frac{5\pi}{4}$

θ	r
0	2
$\frac{\pi}{6}$	$\sqrt{2}$
$\frac{\pi}{4}$	0
$\frac{3\pi}{4}$	0
π	2
$\frac{5\pi}{4}$	0
$\frac{7\pi}{4}$	0

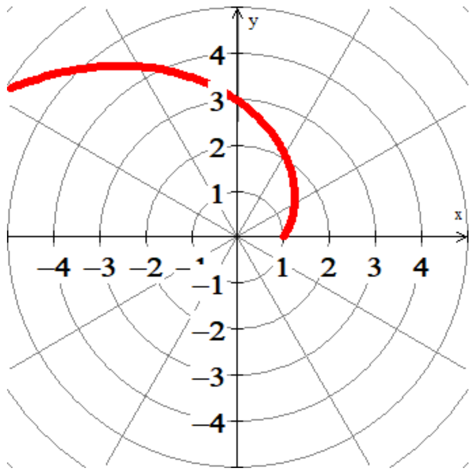


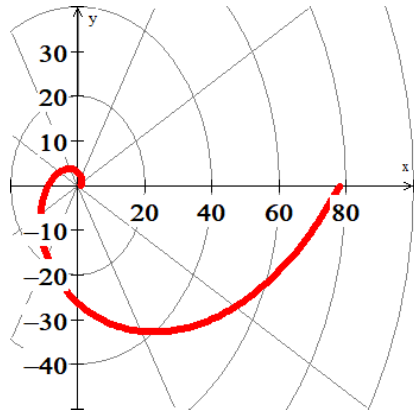
Exercise

Sketch the graph $r = 2^{\theta} \quad \theta \ge 0$

Solution

θ	r
0	1
$\frac{\pi}{2}$	$2^{\pi/2}$

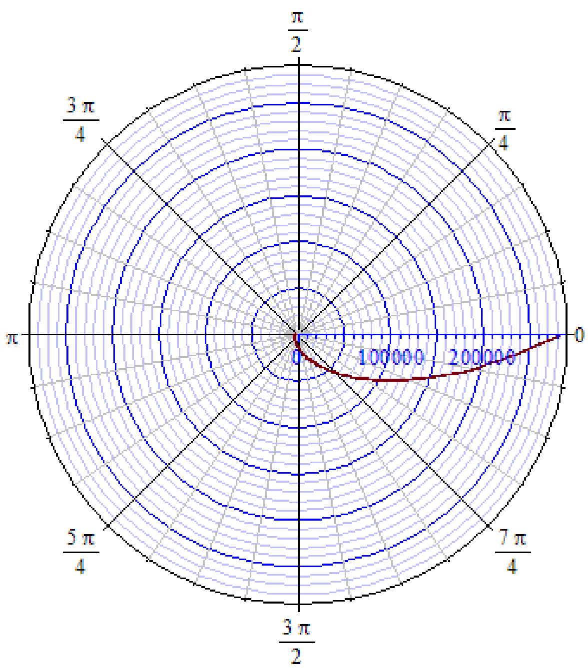


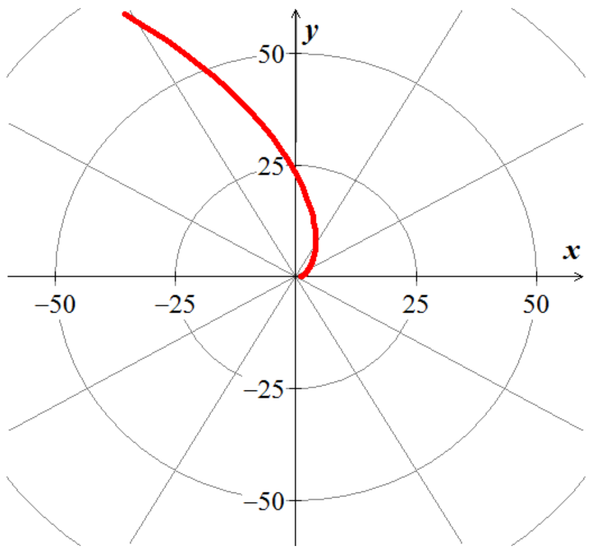


Exercise

Sketch the graph of the polar equation $\qquad r = e^{2\theta} \quad \theta \ge 0$

Solution

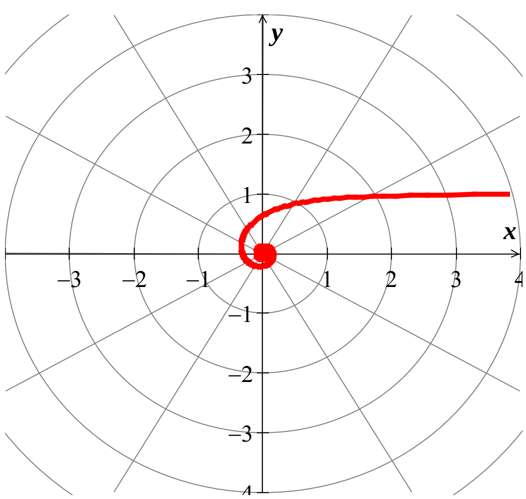


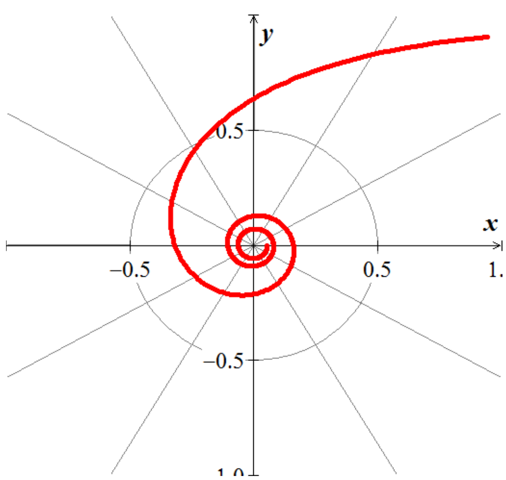


Exercise

Sketch the graph of the polar equation $\qquad r\theta = 1 \quad \theta > 0$

Solution





Exercise

Sketch the graph of the polar equation $\qquad r = 2 + 2\sec\theta$

Solution