Solution *Section* 2.1 – Functions and Graphs

Exercise

Determine whether each relation is a function and *find the domain and the range*.

a) {(1, 2), (3, 4), (5, 6), (5, 8)}

b) {(1, 2), (3, 4), (6, 5), (8, 5)}

c) {(9, -5), (9, 5), (2, 40)}

d) {(-2, 5), (5, 7), (0, 1), (4, -2)}

e) {(-5,3), (0, 3), (6, 3)}

Solution

a) {(1, 2), (3, 4), (5, 6), (5, 8)}

Not a function
Domain: {1, 3, 5}
Range: {2, 4, 6, 8}

b) {(1, 2), (3, 4), (6, 5), (8, 5)}

Function
Domain: {1, 3, 6, 8}
Range: {2, 4, 5}

c) {(9, -5), (9, 5), (2, 40)}

It is **not** a function
Domain = {2, 9}
Range = {-5, 5, 40}

d) {(-2, 5), (5, 7), (0, 1), (4, -2)}

It is a function
Domain = {-2, 0, 4, 5}
Range = {-2, 1, 5, 7}

e) {(-5,3), (0, 3), (6, 3)}

It is a function
Domain = {-5, 0, 6}
Range = {3}

Exercise

Find the domain and the range of the relation: {(5, 12.8), (10, 16.2), (15, 18.9), (20, 20.7), (25, 21.81)}

Solution

Domain: {5, 10, 15, 20, 25}
Range: {12.8, 16.2, 18.9, 20.7, 21.81}

Exercise

Let $f(x) = -3x + 4$, find $f(0)$

Solution

$$f(0) = -3(0) + 4$$
$$\underline{= 4|}$$

Exercise

Let $g(x) = -x^2 + 4x - 1$, find $g(-x)$

Solution

$$g(-x) = -(-x)^2 + 4(-x) - 1$$
$$\underline{= -x^2 - 4x - 1|}$$

Exercise

Let $f(x) = -3x + 4$, find $f(a+4)$

Solution

$$f(a+4) = -3(a+4) + 4$$
$$= -3a - 12 + 4$$
$$\underline{= -3a - 8|}$$

Exercise

Given: $f(x) = 2|x| + 3x$, find $f(2-h)$.

Solution

$$f(2-h) = 2|2-h| + 3(2-h)$$
$$\underline{= 2|2-h| + 6 - 3h|}$$

Exercise

Given: $g(x)=\frac{x-4}{x+3}$, find $g(x+h)$

Solution

$$g(x+h)=\boxed{\frac{x+h-4}{x+h+3}}$$

Exercise

Given: $g(x)=\frac{x}{\sqrt{1-x^2}}$, find $g(0)$ *and* $g(-1)$

Solution

a) $g(0)=\frac{0}{\sqrt{1-0^2}}=0$

b) $g(-1)=\frac{-1}{\sqrt{1-(-1)^2}}=\frac{-1}{0}$ *undefined*

Exercise

Given that $g(x)=2x^2+2x+3$. Find $g(p+3)$

Solution

$$
\begin{aligned}
g(p+3)&=2(p+3)^2+2(p+3)+3 \qquad (a+b)^2=a^2+2ab+b^2\\
&=2\left(p^2+2(p)(3)+3^2\right)+2p+6+3\\
&=2\left(p^2+6p+9\right)+2p+9\\
&=2p^2+12p+18+2p+9\\
&=\boxed{2p^2+14p+27}
\end{aligned}
$$

Exercise

If $f(x)=x^2-2x+7$, evaluate each of the following: $f(-5)$, $f(x+4)$, $f(-x)$

Solution

a. $f(-5)=?$

$f(x)=x^2-2x+7$

$$f(-5) = (-5)^2 - 2(-5) + 7$$
$$= 25 + 10 + 7$$
$$= 42$$

b. $f(x+4) = ?$

$$f(x+4) = (x+4)^2 - 2(x+4) + 7 \qquad (a+b)^2 = a^2 + 2ab + b^2$$
$$= x^2 + 2(4)x + 4^2 - 2x - 8 + 7$$
$$= x^2 + 8x + 16 - 2x - 1$$
$$\underline{= x^2 + 6x + 15}$$

c. $f(-x) = (-x)^2 - 2(-x) + 7$

$$\underline{= x^2 + 2x + 7}$$

Exercise

Find $g(0),\ g(-4),\ g(7),\ and\ g\left(\frac{3}{2}\right)\quad for\quad g(x) = \frac{x}{\sqrt{16-x^2}}$

Solution

$$g(0) = \frac{0}{\sqrt{16-0^2}} = \frac{0}{\sqrt{16}} = \underline{0}$$

$$g(-4) = \frac{-4}{\sqrt{16-(-4)^2}} = \frac{-4}{\sqrt{16-16}} = \frac{-4}{0} \quad \underline{doesn't\ \ exist}$$

$$g(7) = \frac{7}{\sqrt{16-7^2}} = \frac{7}{\sqrt{16-49}} = \frac{7}{\sqrt{-33}} = \underline{doesn't\ exist\ in\ real\ number}$$

$$g\left(\frac{3}{2}\right) = \frac{\frac{3}{2}}{\sqrt{16-\left(\frac{3}{2}\right)^2}}$$
$$= \frac{\frac{3}{2}}{\sqrt{16-\frac{9}{4}}}$$
$$= \frac{\frac{3}{2}}{\sqrt{\frac{4(16)-9}{4}}}$$

$$= \frac{\frac{3}{2}}{\sqrt{\frac{55}{4}}}$$

$$= \frac{\frac{3}{2}}{\frac{\sqrt{55}}{2}}$$

$$= \frac{3}{2} \cdot \frac{2}{\sqrt{55}}$$

$$= \frac{3}{\sqrt{55}}$$

$$\boxed{= \frac{3\sqrt{55}}{55}}$$

Exercise

$$f(x) = \begin{cases} 2 + x & \text{if } x < -4 \\ -x & \text{if } -4 \le x \le 2 \\ 3x & \text{if } x > 2 \end{cases}$$

Find: $f(-5),\ f(-1),\ f(0),\ and\ f(3)$

Solution

a) $f(-5) = 2 - 5 = -3$

b) $f(-1) = -(-1) = 1$

c) $f(0) = -0 = 0$

d) $f(3) = 3(3) = 9$

Exercise

$$f(x) = \begin{cases} -2x & \text{if } x < -3 \\ 3x - 1 & \text{if } -3 \le x \le 2 \\ -4x & \text{if } x > 2 \end{cases}$$

Find: $f(-5),\ f(-1),\ f(0),\ and\ f(3)$

Solution

a) $f(-5) = -2(-5) = 10$

b) $f(-1) = 3(-1) - 1 = -4$

c) $f(0) = 3(0) - 1 = -1$

d) $f(3) = -4(3) = -12$

Exercise

$$f(x)=\begin{cases} x^3+3 & \text{if} \quad -2\le x\le 0 \\ x+3 & \text{if} \quad 0<x<1 \\ 4+x-x^2 & \text{if} \quad 1\le x\le 3 \end{cases}$$ Find: $f(-5),\ f(-1),\ f(0),\ and\ f(3)$

Solution

a) $f(-5)=\ doesn't\ exist$

b) $f(-1)=(-1)^3+3=2$

c) $f(0)=(0)^3+3=3$

d) $f(3)=4+(3)-(3)^2=-2$

Exercise

$$h(x)=\begin{cases} \dfrac{x^2-9}{x-3} & \text{if} \quad x\ne 3 \\ 6 & \text{if} \quad x=3 \end{cases}$$ Find: $h(5),\ h(0),\ and\ h(3)$

Solution

a) $h(5)=\dfrac{5^2-9}{5-3}=8$

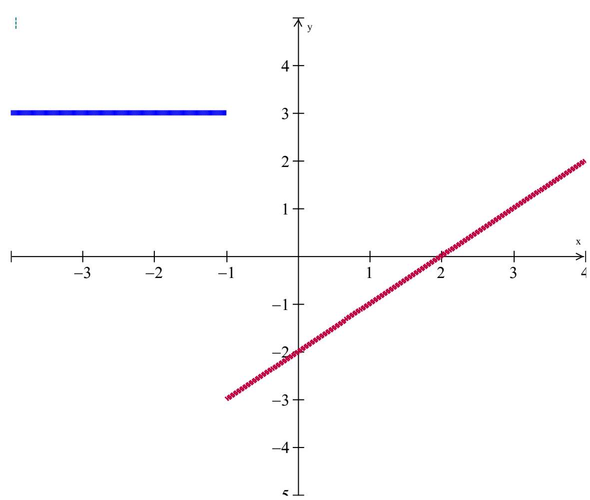
b) $h(0)=\dfrac{0^2-9}{0-3}=3$

c) $h(3)=6$

Exercise

Graph the piecewise function defined by $f(x)=\begin{cases} 3 & \text{if} \quad x\le -1 \\ x-2 & \text{if} \quad x>-1 \end{cases}$

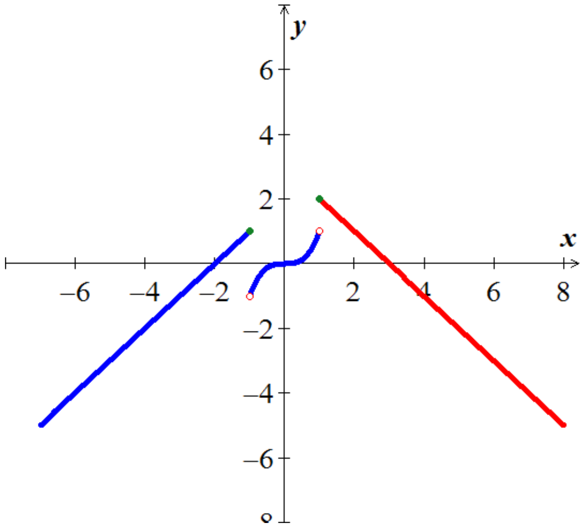
Solution



Exercise

Sketch the graph $f(x)=\begin{cases} x+2 & if \quad x\le -1 \\ x^3 & if \quad -1<x<1 \\ -x+3 & if \quad x\ge 1 \end{cases}$

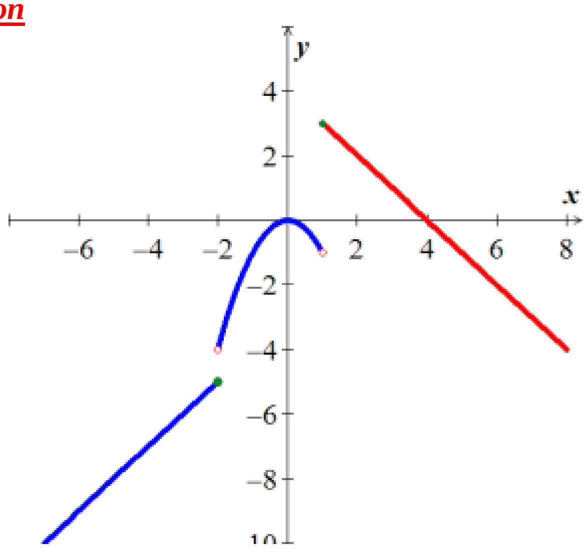
Solution



Exercise

Sketch the graph $f(x)=\begin{cases} x-3 & if \quad x\le -2 \\ -x^2 & if \quad -2<x<1 \\ -x+4 & if \quad x\ge 1 \end{cases}$

Solution



Exercise

The elevation H, in meters, above sea level at which the boiling point of water is in t degrees Celsius is given by the function

$$H(t) = 1000(100-t) + 580(100-t)^2$$

At what elevation is the boiling point 99.5°.

Solution

$$H(99.5) = 1000(100-99.5) + 580(100-99.5)^2$$

$$\underline{= 645\ m}$$

Exercise

A hot-air balloon rises straight up from the ground at a rate of 120 ft./min. The balloon is tracked from a rangefinder on the ground at point P, which is 400 ft. from the release point Q of the balloon. Let d = the distance from the balloon to the rangefinder and t – the time, in minutes, since the balloon was released. Express d as a function of t.

Solution

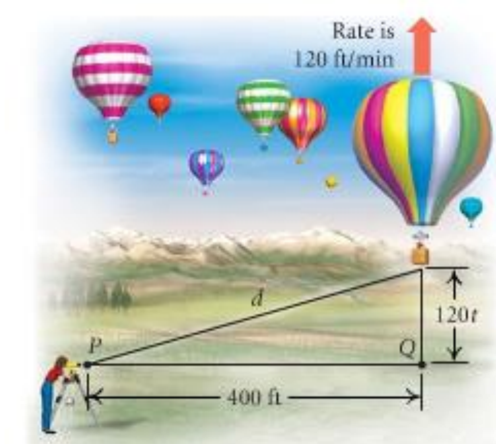
$$d^2 = (120t)^2 + 400^2$$

$$d(t) = \sqrt{(120t)^2 + 400^2}$$

$$d = \sqrt{14400t^2 + 160000}$$

$$d = \sqrt{1600(9t^2 + 100)}$$

$$d(t) = 40\sqrt{9t^2 + 100}$$



Exercise

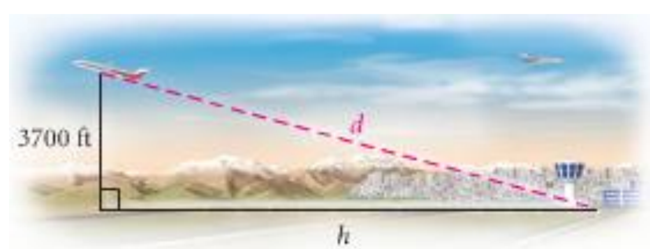
An airplane is flying at an altitude of 3700 *ft*. The slanted distance directly to the airport is d feet. Express the horizontal distance h as a function of d.

Solution

$$d^2 = (3700)^2 + h^2$$

$$d^2 - (3700)^2 = h^2$$

$$h(t) = \sqrt{d^2 - (3700)^2}$$



Exercise

A device used in golf to estimate the distance $\boldsymbol{d}$, in yards, to a hole measures the size $\boldsymbol{s}$, in inches, that the 7-ft pin appears to be in a viewfinder. Express the distance $\boldsymbol{d}$ as a function of $\boldsymbol{s}$.

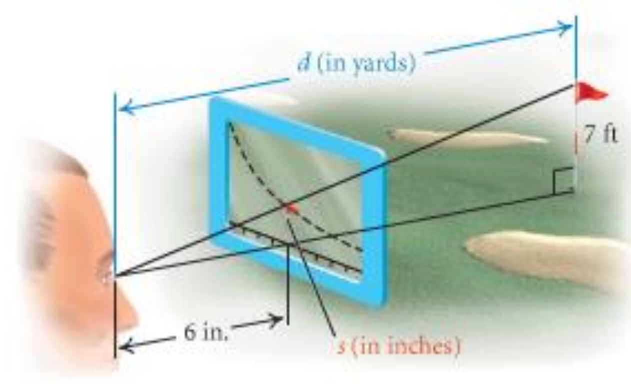
Solution

$$\frac{d}{6} = \frac{7}{s}\frac{ft}{in}$$

$$d = \frac{7}{s}\frac{ft}{in}6in$$

$$d = \frac{42}{s}ft\frac{1yd}{3ft}$$

$$\boxed{d(s) = \frac{14}{s}}$$



Exercise

A rancher has 360 yd. of fencing with which to enclose two adjacent rectangular corrals, one for sheep and one for cattle. A river forms one side of the corrals. Suppose the width of each corral is $\boldsymbol{x}$ yards.

a) Express the total area of the two corrals as a function of $\boldsymbol{x}$.

b) Find the domain of the function.

Solution

a) Express the total area of the two corrals as a function of x.

$$P = 3x + l = 360$$

$$l = 360 - 3x$$

$$A = xl = x(360 - 3x)$$

$$\rightarrow A(x) \boxed{= 360x - 3x^2}$$

b) Find the domain of the function.

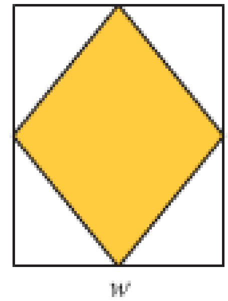
$$x(360 - 3x) = 0$$

$$x = 0 \qquad 360 - 3x = 0 \Rightarrow 3x = 360 \Rightarrow x = 120$$

Domain: $0 < x < 120$

Exercise

A *rhombus* is inscribed in a rectangle that is ***w*** meters wide with a perimeter of 40 m. Each vertex of the rhombus is a midpoint of a side of the rectangle. Express the area of the *rhombus* as a function of the rectangle's width.



Solution

The area of the rhombus = $\frac{1}{2}$ area of the rectangle, since each vertex of the rhombus is a midpoint of a side of the rectangle.

$Perimter:$

$2l + 2w = 40$ ***Divide both sides by 2***

$l + w = 20$

$l = 20 - w$

$Area\ of\ the\ rectangle = lw = (20 - w)w$

$Area\ of\ the\ rhombus = \frac{1}{2}(20w - w^2)$

$= -\frac{1}{2}w^2 + 10w$

Solution Section 2.2 – Transformations of Functions

Exercise

Write an equation for a function that has the shape of $f(x) = x^2$, but upside-down and shifted right 2 units and down 3 units.

Solution

$$f(x) = -(x-2)^2 - 3$$

Exercise

Describe how the graph of f(x) = $\sqrt{x-3}+2$ can be obtained from the graph of $y = \sqrt{x}$

Solution

Right 3 units and up 2 units

Exercise

Describe how the graph of $f(x) = -\frac{1}{2}(x-2)^2 + 1$ can be obtained from the graph of $f(x) = x^2$

Solution

Reflected across x-axis (Upside down), shrink by factor 2, and shifted right 2 units and up 1 unit

Exercise

Describe how the graph of $f(x) = -(x-8)^2$ can be obtained from the graph of $f(x) = x^2$

Solution

Reflected across x-axis and shifted right 8 units

Exercise

Describe how the graph of $y = \sqrt{x+6} - 5$ can be obtained from the graph of $y = \sqrt{x}$

Solution

Shifted left 6 units and down 5 units

Exercise

Describe how the graph of $y = \sqrt{-(x+2)} - 1$ can be obtained from the graph of $y = \sqrt{x}$

Solution

Reflected across the y-axis, and shifted left 2 units and down 1 unit

Exercise

Describe how the graph of $y = \left|\frac{1}{2}x\right| - 5$ can be obtained from the graph of $y = |x|$

Solution

Stretched horizontally by a factor of 2 and shifted down 5 units

Exercise

Explain how the graph $y = f\left(x-2\right)+3$ compares to the graph of $y = f(x)$

Solution

Shifted right 2 units and up 3 units.

Exercise

Explain how the graph $y = f\left(-x\right)-2$ compares to the graph of $y = f(x)$

Solution

Reflected across y-axis and shifted down 2 units.

Exercise

Explain how the graph $y = -\frac{1}{2} f\left(x\right)$ compares to the graph of $y = f(x)$

Solution

Reflected across x-axis and shrunk vertically by $\frac{1}{2}$ units.

Exercise

Explain how the graph $y = f\left(\frac{1}{2}x\right)-3$ compares to the graph of $y = f(x)$

Solution

Stretched horizontally by (1/2) units and down 3 units.

Exercise

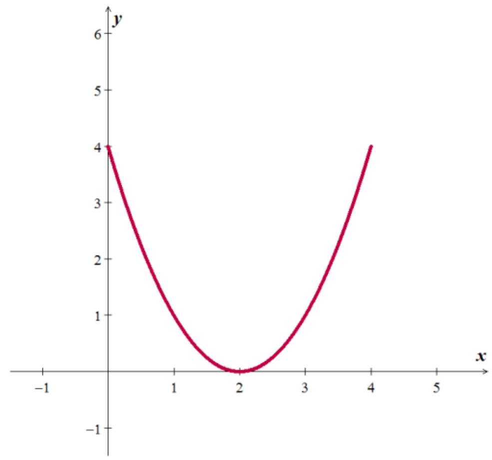
Explain how the graph $y=-2f\left[\frac{1}{2}(x-3)\right]+5$ compares to the graph of $y=f(x)$

Solution

Reflected across x-axis, stretched vertically by 2 units, stretched horizontally by $\frac{1}{2}$ units, shifted right 3 units, and up 5 units.

Exercise

The graph of a function $f(x)=x^2$ with domain [0, 4]:



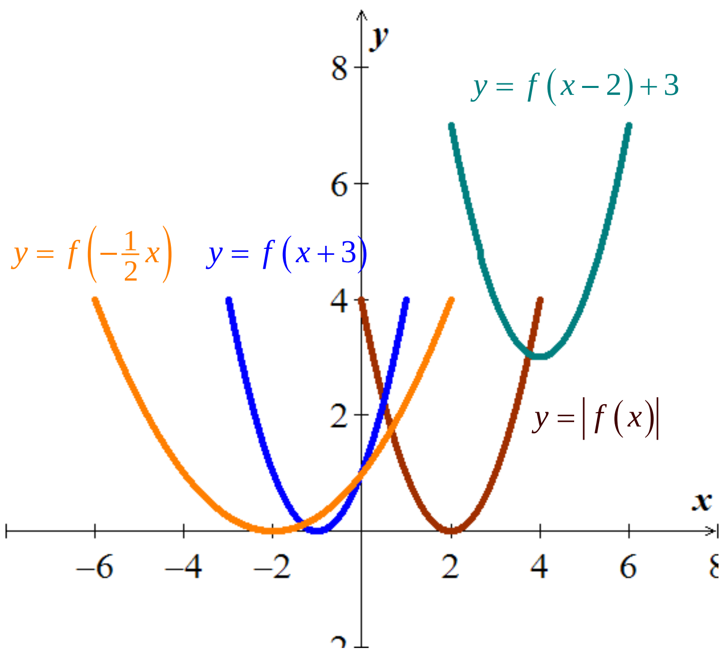
a) $y=f(x+3)$

b) $y=f(x-2)+3$

c) $y=f\left(-\frac{1}{2}x\right)$

d) $y=|f(x)|$

Solution



Solution Section 2.3 – Function Operations and Composition

Exercise

Find the domain: $f(x) = 7x + 4$

Solution

Domain: $(-\infty,\ \infty)$

Exercise

Find the domain: $f(x) = |3x - 2|$

Solution

Domain: $(-\infty,\ \infty)$

Exercise

Find the domain: $f(x) = x^2 - 2x - 15$

Solution

Domain: $(-\infty,\ \infty)$

Exercise

Find the domain: $g(x) = \dfrac{3}{x-4}$

Solution

Domain: $(-\infty, 4) \cup (4, \infty)$

Exercise

Find the domain $y = \dfrac{2}{x-3}$

Solution

$x - 3 \neq 0 \Rightarrow x \neq 3$

Domain: $\Rightarrow (-\infty, 3) \cup (3, \infty)$

Exercise

Find the domain $y = \dfrac{-7}{x-5}$

Solution

$x - 5 \neq 0 \quad \Rightarrow x \neq 5$

Domain: $\underline{(-\infty, 5) \cup (5, \infty)}|$

Exercise

Find the domain $f(x) = 4 - \dfrac{2}{x}$

Solution

$x \neq 0$

Domain: $\underline{(-\infty, 0) \cup (0, \infty)}|$

Exercise

Find the domain $f(x) = \dfrac{1}{x^4}$

Solution

$x \neq 0$

Domain: $\underline{(-\infty, 0) \cup (0, \infty)}|$

Exercise

Find the domain $f(x) = \dfrac{x+5}{2-x}$

Solution

$2 - x \neq 0 \Rightarrow x \neq 2$

Domain: $\underline{(-\infty, 2) \cup (2, \infty)}|$

Exercise

Find the domain $f(x) = \dfrac{8}{x+4}$

Solution

$x + 4 \neq 0 \Rightarrow x \neq -4$

Domain: $(-\infty, -4) \cup (-4, \infty)$

Exercise

Find the domain $f(x) = \dfrac{1}{x^2 - 4x - 5}$

Solution

$x^2 - 4x - 5 \neq 0$

$(x + 1)(x - 5) \neq 0$

$x \neq -1$ and $x \neq 5$

Domain: $\boxed{(-\infty, \ -1) \cup (-1, \ 5) \cup (5, \ \infty)}$

Exercise

Find the domain $g(x) = \dfrac{2}{x^2 + x - 12}$

Solution

$x^2 + x - 12 \neq 0$

$(x+4)(x-3) \neq 0$

$x \neq -4 \qquad x \neq 3$

Domain: $\boxed{(-\infty, -4) \cup (-4, 3) \cup (3, \infty)}$

Exercise

Find the domain $h(x) = \dfrac{5}{\frac{4}{x} - 1}$

Solution

$x \neq 0 \qquad \dfrac{4}{x} - 1 \neq 0$

$\dfrac{4 - x}{x} \neq 0$

$4 - x \neq 0$

$x \neq 4$

Domain: $\boxed{(-\infty, 0) \cup (0, 4) \cup (4, \infty)}$

Exercise

Find the domain $y = \sqrt{x}$

Solution

$x \geq 0$

Domain: $\boxed{[0, \ \infty)}$

Exercise

Find the domain $y = \sqrt{4x+1}$

Solution

$$4x+1 \ge 0 \Rightarrow x \ge -\frac{1}{4}$$

Domain: $\left[-\frac{1}{4}, \infty\right)$

Exercise

Find the domain $y = \sqrt{7-2x}$

Solution

$$7-2x \ge 0$$

$$\Rightarrow -2x \ge -7 \Rightarrow \boxed{x \le \frac{7}{2}}$$

Domain: $\left(-\infty, \frac{7}{2}\right]$

Exercise

Find the domain $f(x) = \sqrt{8-x}$

Solution

$$8-x \ge 0 \Rightarrow -x \ge -8$$

$$x \le 8$$

Domain: $(-\infty,\ 8]$

Exercise

Find the domain $f(x) = \dfrac{\sqrt{x+1}}{x}$

Solution

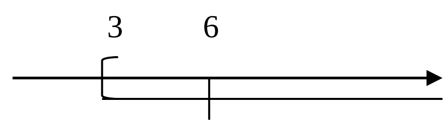
$x + 1 \ge 0 \qquad\qquad x \ne 0$

$x \ge -1$

Domain: $\boxed{[-1,\ 0) \cup (0,\ \infty)}$

Exercise

Find the domain $g(x) = \dfrac{\sqrt{x-3}}{x-6}$



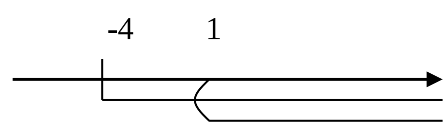
Solution

$$\rightarrow \begin{cases} x \ge 3 \\ x \ne 6 \end{cases}$$

Domain: $\boxed{[3,\ 6) \cup (6,\ \infty)}$

Exercise

Find the domain $f(x) = \dfrac{\sqrt{x+4}}{\sqrt{x-1}}$



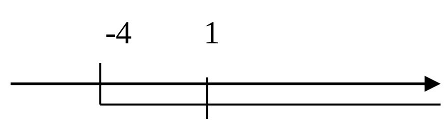
Solution

$$\rightarrow \begin{cases} x \ge -4 \\ x > 1 \end{cases}$$

Domain: $\boxed{(1,\ \infty)}$

Exercise

Find the domain $\sqrt{x+4} - \sqrt{x-1}$



Solution

$$\rightarrow \begin{cases} x \ge -4 \\ x \ge 1 \end{cases}$$

Domain: $\boxed{[1,\ \infty)}$

Exercise

Find the domain of $f(x)=\sqrt{2x+7}$

Solution

$$2x+7\ge 0 \Rightarrow 2x\ge -7 \to x\ge -\frac{7}{2}$$

Domain: $x\ge -\frac{7}{2}$

Exercise

Find the domain of $f(x)=\sqrt{8-3x}$

Solution

$$8-3x\ge 0 \Rightarrow -3x\ge 8 \to x\le -\frac{8}{3}$$

Domain: $x\ge -\frac{7}{2}$

Exercise

Find the domain of $f(x)=\sqrt{9-x^2}$

Solution

$$\begin{aligned} 9-x^2\ge 0 \Rightarrow -x^2 &\ge -9 \\ x^2 &\le 9 \\ -3\le x &\le 3 \end{aligned}$$

Domain: $-3\le x\le 3$ *or* $[-3,\ 3]$

Exercise

Find the domain of $f(x)=\sqrt{x^2-25}$

Solution

$$\begin{aligned} x^2-25\ge 0 &\Rightarrow x^2\ge 25 \\ &\Rightarrow x\le -5 \quad x\ge 5 \end{aligned}$$

Domain: $x\le -5,\ x\ge 5$ *or* $(-\infty,\ -5]\cup[5,\ \infty)$

Exercise

Find the domain of $f(x)=\dfrac{x+1}{x^3-4x}$

Solution

$$x^3-4x\neq 0 \Rightarrow x\left(x^2-4\right)\neq 0$$

$$x\neq 0 \quad x^2-4\neq 0$$

$$x\neq 0; \quad x\neq 2; \quad x\neq -2$$

Domain: $\quad (-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$

Exercise

Find the domain of $f(x)=\dfrac{4x}{6x^2+13x-5}$

Solution

$$6x^2+13x-5\neq 0 \Rightarrow \boxed{x\neq -\frac{5}{2},\ \frac{1}{3}}$$

Exercise

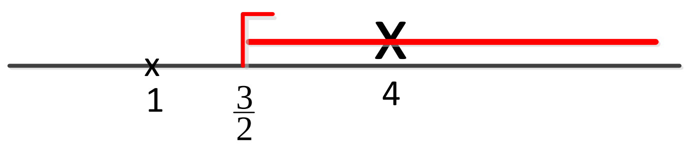
Find the domain of $f(x)=\dfrac{\sqrt{2x-3}}{x^2-5x+4}$

Solution

$$2x-3\geq 0 \qquad x^2-5x+4\neq 0$$

$$2x\geq 3 \qquad x\neq 1,\ 4$$

$$x\geq \frac{3}{2}$$



Domain: $\left[\frac{3}{2},\ 4\right)\cup(4,\ \infty)$

Exercise

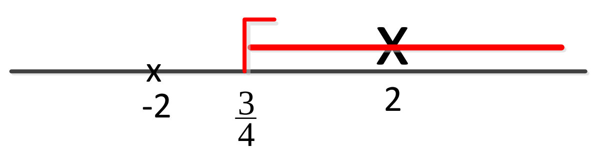
Find the domain of $f(x)=\dfrac{\sqrt{4x-3}}{x^2-4}$

Solution

$$4x-3\geq 0 \qquad x^2-4\neq 0$$

$$4x\geq 3 \qquad x\neq \pm 2$$

$$x\geq \frac{3}{4}$$



Domain: $\left[\frac{3}{4},\ 2\right)\cup(2,\ \infty)$

Exercise

Find the domain of $f(x) = \dfrac{x-4}{\sqrt{x-2}}$

Solution

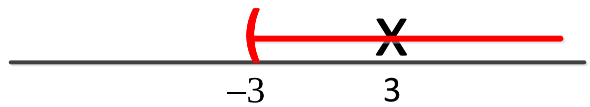
$$x-2 \neq 0 \Rightarrow x \neq 2$$

Domain: $\boxed{x \neq 2}$ $\quad (-\infty,\ 2) \cup (2,\ \infty)$

Exercise

Find the domain of $f(x) = \dfrac{1}{(x-3)\sqrt{x+3}}$

Solution



$$x-3 \neq 0 \qquad x+3 > 0$$

$$x \neq 3 \qquad x > -3$$

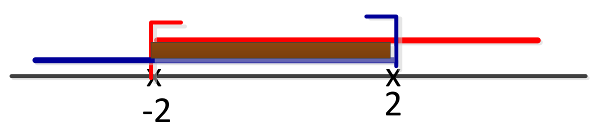
Domain: $\{x \mid x > -3 \text{ and } x \neq 3\}$

$$(-3,\ 3) \cup (3,\ \infty)$$

Exercise

Find the domain of $f(x) = \sqrt{x+2} + \sqrt{2-x}$

Solution



$$x+2 \geq 0 \qquad 2-x \geq 0$$

$$x \geq -2 \qquad -x \geq -2 \rightarrow x \leq 2$$

Domain: $\{x \mid -2 \leq x \leq 2\}$

Exercise

Find the domain of $f(x) = \sqrt{(x-2)(x-6)}$

Solution

$$x-2 \geq 0 \quad x-6 \geq 0$$

$$x \geq 2 \quad x \geq 6$$

Domain: $\{x \mid x \leq 2, x \geq 6\}$

	2	6
–	+	+
–	–	+
+	–	+

Exercise

Let $f(x)=\sqrt{4x-1}$ and $g(x)=\frac{1}{x}$. Find each of the following and give the domain

a) $(f+g)(x)$

b) $(f-g)(x)$

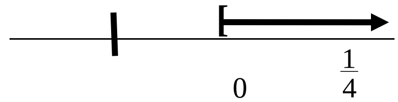
c) $(fg)(x)$

d) $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f+g)(x)$

$$(f+g)(x)=\sqrt{4x-1}+\frac{1}{x}$$



$4x-1\ge 0$ $\qquad$ $x\neq 0$

$x\ge \frac{1}{4}$

Domain: $\left[\frac{1}{4},\infty\right)$

b) $(f-g)(x)$

$$(f-g)(x)=\sqrt{4x-1}-\frac{1}{x}$$

$4x-1\ge 0$ $\qquad$ $x\neq 0$

$x\ge \frac{1}{4}$

Domain: $\left[\frac{1}{4},\infty\right)$

c) $(fg)(x)$

$$(fg)(x)=\sqrt{4x-1}\left(\frac{1}{x}\right)=\frac{\sqrt{4x-1}}{x}$$

$4x-1\ge 0$ $\qquad$ $x\neq 0$

$x\ge \frac{1}{4}$

Domain: $\left[\frac{1}{4},\infty\right)$

d) $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{4x-1}}{\frac{1}{x}}$$

Domain: $x\neq 0$

$$= x\sqrt{4x-1}$$

$$4x-1 \geq 0$$

$$x \geq \frac{1}{4}$$

Domain: $\left[\frac{1}{4}, \infty\right)$

Exercise

Given that $f(x) = x+1$ and $g(x) = \sqrt{x+3}$

a) Find $(f+g)(x)$

b) Find the domain of $(f+g)(x)$

c) Find: $(f+g)(6)$

Solution

a) $(f+g)(x) = f(x) + g(x)$

$$= x+1+\sqrt{x+3}$$

b) $x+3 \geq 0 \rightarrow x \geq -3$

Domain = $[-3, \ \infty)$

c) $(f+g)(6) = 6+1+\sqrt{6+3} = 10$

Exercise

Given that $f(x) = x^2 - 4$ and $g(x) = x+2$

a) Find $(f+g)(x)$ and its domain

b) Find $(f/g)(x)$ and its domain

Solution

a) $(f+g)(x) = x^2 - 4 + x + 2$

$$= x^2 + x - 2$$

Domain = $(-\infty, \ \infty)$

b) $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2-4}{x+2}$

$x \neq -2$

Domain: $(-\infty, -2) \cup (-2, \infty)$

Exercise

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find $(f + g)(1)$, $(f - g)(-3)$, $(fg)(5)$, and $\left(\frac{f}{g}\right)(0)$

Solution

a) $(f+g)(1) = f(1) + g(1)$

$$= 1^2 + 1 + 3(1) + 5$$

$$= 1 + 1 + 3 + 5$$

$$= 10$$

b) $(f-g)(-3) = f(-3) - g(-3)$

$$= (-3)^2 + 1 - (3(-3) + 5)$$

$$= 10$$

c) $(fg)(5) = f(5) \cdot g(5)$

$$= \left(5^2 + 1\right) \cdot (3(5) + 5)$$

$$= (26) \cdot (20)$$

$$= 520$$

d) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)}$

$$= \frac{0^2 + 1}{3(0) + 5}$$

$$= \frac{1}{5}$$

Exercise

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, *and* $(f/g)(x)$ and the domain of

$f(x) = \sqrt{3 - 2x}, \quad g(x) = \sqrt{x + 4}$

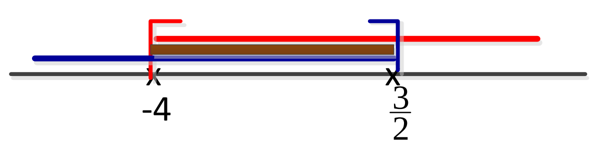
Solution

$$f(x) + g(x) = \sqrt{3 - 2x} + \sqrt{x + 4}$$

$3 - 2x \ge 0 \qquad x + 4 \ge 0$

$-2x \ge -3 \qquad x \ge -4$

$x \le \frac{3}{2}$



Domain: $\left\{ x \mid -4 \le x \le \frac{3}{2} \right\}$

$$f(x)-g(x)=\sqrt{3-2x}-\sqrt{x+4}$$

$3-2x\ge 0 \qquad x+4\ge 0$

$-2x\ge -3 \qquad x\ge -4$

$x\le \frac{3}{2}$

Domain: $\left\{x\,|\,-4\le x\le \frac{3}{2}\right\}$

$$(f\cdot g)(x)=\left(\sqrt{3-2x}\right)\left(\sqrt{x+4}\right)=\sqrt{(3-2x)(x+4)}=\sqrt{-2x^2-5x+12}$$

$3-2x\ge 0 \qquad x+4\ge 0$

$-2x\ge -3 \qquad x\ge -4$

$x\le \frac{3}{2}$

Domain: $\left\{x\,|\,-4\le x\le \frac{3}{2}\right\}$

$$(f/g)(x)=\frac{\sqrt{3-2x}}{\sqrt{x+4}}\frac{\sqrt{x+4}}{\sqrt{x+4}}=\frac{\sqrt{-2x^2-5x+12}}{x+4}$$

$3-2x\ge 0 \qquad x+4>0$

$-2x\ge -3 \qquad x>-4$

$x\le \frac{3}{2}$

Domain: $\left\{x\,|\,-4< x\le \frac{3}{2}\right\}$

$$\left(-4,\ \frac{3}{2}\right]$$

Exercise

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, *and* $(f/g)(x)$ and the domain of

$f(x)=\frac{2x}{x-4},\quad g(x)=\frac{x}{x+5}$

Solution

$$(f+g)(x)=\frac{2x}{x-4}+\frac{x}{x+5}$$

$$=\frac{2x(x+5)+x(x-4)}{(x-4)(x+5)}$$

$$=\frac{2x^2+10x+x^2-4x}{(x-4)(x+5)}$$

$$=\frac{3x^2+6x}{(x-4)(x+5)}$$

$x-4\ne 0 \qquad x+5\ne 0$

$x \neq 4 \qquad x \neq -5$

Domain: $\{x \mid x \neq -5, 4\}$ $\qquad (-\infty,\ -5) \cup (-5,\ 4) \cup (4,\ \infty)$

$$(f-g)(x) = \frac{2x}{x-4} - \frac{x}{x+5}$$

$$= \frac{2x(x+5) - x(x-4)}{(x-4)(x+5)}$$

$$= \frac{2x^2 + 10x - x^2 + 4x}{(x-4)(x+5)}$$

$$= \frac{x^2 + 14x}{(x-4)(x+5)}$$

$x \neq 4 \qquad x \neq -5$

Domain: $\{x \mid x \neq -5, 4\}$

$$(f \bullet g)(x) = \frac{2x}{x-4}\frac{x}{x+5} = \frac{2x^2}{(x-4)(x+5)}$$

$x \neq 4 \qquad x \neq -5$

Domain: $\{x \mid x \neq -5, 4\}$

$$(f / g)(x) = \frac{2x}{x-4} \div \frac{x}{x+5}$$

$$= \frac{2x}{x-4}\frac{x+5}{x} = 2\frac{x+5}{x-4}$$

$$= 2\frac{x+5}{x-4}$$

$x \neq 4 \qquad x \neq -5$

Domain: $\{x \mid x \neq -5, 4\}$

Exercise

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, *and* $(f / g)(x)$ of $f(x) = x - 5$ and $g(x) = x^2 - 1$

Solution

a. $(f+g)(x) = f(x) + g(x)$

$$= x - 5 + x^2 - 1$$

$$= x^2 + x - 6$$

b. $(f-g)(x)=f(x)-g(x)$

$$=x-5-(x^2-1)$$
$$=x-5-x^2+1$$
$$\underline{=-x^2+x-4}$$

c. $(fg)(x)=f(x)g(x)$

$$=(x-5)(x^2-1)$$
$$=x^3-x-5x^2+5$$
$$\underline{=x^3-5x^2-x+5}$$

d. $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$

$$\underline{=\frac{x-5}{x^2-1}}$$

Exercise

Given the function: $f(x)=2x^2$. Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$f(x+h)=2(x+h)^2$$
$$=2\left(x^2+2hx+h^2\right)$$
$$=2x^2+4hx+2h^2$$

$$\frac{f(x+h)-f(x)}{h}=\frac{2x^2+4hx+2h^2-2x^2}{h}$$
$$=\frac{4hx+2h^2}{h}$$
$$=\frac{4hx}{h}+\frac{2h^2}{h}$$
$$\underline{=4x+2h}$$

Exercise

For the function *f* given by $f(x)=9x+5$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$f(\mathbf{x}+\mathbf{h})=9(\mathbf{x}+\mathbf{h})+5=9x+9h+5$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\overbrace{9x+9h+5}^{f(x+h)}\ -\ \overbrace{(9x+5)}^{f(x)}}{h}$$

$$=\frac{9x+9h+5\ -\ 9x-5}{h}$$

$$=\frac{9h}{h}$$

$$\underline{=9}$$

Exercise

For the function *f* given by $f(x)=6x+2$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$\frac{f(x+h)-f(x)}{h}=\frac{6(x+h)+2-(6x+2)}{h}$$

$$=\frac{6x+6h+2-6x-2}{h}$$

$$=\frac{6h}{h}$$

$$\underline{=6}$$

Exercise

For the function *f* given by $f(x)=4x+11$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$\frac{f(x+h)-f(x)}{h}=\frac{4(x+h)+11-(4x+11)}{h}$$

$$=\frac{4x+4h+11-4x-11}{h}$$

$$=\frac{4h}{h}$$

$$\underline{=4}$$

Exercise

For the function f given by $f(x)=2x^2-x-3$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned} f(x+h) &= 2(x+h)^2-(x+h)-3 \\ &= 2\left(x^2+2hx+h^2\right)-x-h-3 \\ &= 2x^2+4hx+2h^2-x-h-3 \end{aligned}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{2x^2+2h^2+4hx-x-h-3-\left(2x^2-x-3\right)}{h} \\ &= \frac{2x^2+2h^2+4hx-x-h-3-2x^2+x+3}{h} \\ &= \frac{2h^2+4hx-h}{h} \\ &= \frac{2h^2}{h}+\frac{4hx}{h}-\frac{h}{h} \\ &= 2h+4x-1 \end{aligned}$$

Exercise

Given $f(x)=\sqrt{x}$ and $g(x)=x+3$, find $(f\circ g)(x)$, $(g\circ f)(x)$ and their domain.

Solution

$$\begin{aligned} (f\circ g)(x) &= f(g(x)) \\ &= f(x+3) \qquad \textbf{Domain}: (-\infty,\infty) \\ &= \sqrt{x+3} \end{aligned}$$

$$x+3\ge 0$$
$$x\ge -3$$

Domain: $[-3,\infty)$

$$\begin{aligned} (g\circ f)(x) &= g(f(x)) \\ &= g\left(\sqrt{x}\right) \qquad \textbf{Domain}: x\ge 0 \\ &= \sqrt{x}+3 \end{aligned}$$

$$x\ge 0$$

Domain: $[0,\infty)$

Exercise

Given that $f(x)=\sqrt{x}$ and $g(x)=2-3x$, find $(f\circ g)(x)$, $(g\circ f)(x)$ and their domain.

Solution

$$(f\circ g)(x)=f(g(x))$$

$$=f(2-3x) \qquad \textbf{\textit{Domain}}: (-\infty,\infty)$$

$$=\sqrt{2-3x}$$

$$2-3x\ge 0\to -3x\ge -2\Rightarrow \boxed{x\le \frac{2}{3}}$$

Domain: $\left(-\infty,\ \frac{2}{3}\right]$

$$g(f(x))=g\left(\sqrt{x}\right) \qquad \textbf{\textit{Domain}}: x\ge 0$$

$$=2-3\sqrt{x}$$

$$x\ge 0$$

Domain: $[0,\infty)$

Exercise

Given that $f(x)=\dfrac{1}{x-2}$ and $g(x)=\dfrac{x+2}{x}$, find $(f\circ g)(x)$, $(g\circ f)(x)$ and their domain.

Solution

$$f(g(x))=f\left(\tfrac{x+2}{x}\right) \qquad \textbf{\textit{Domain}}: \boxed{x\ne 0}$$

$$=\frac{1}{\frac{x+2}{x}-2}$$

$$=\frac{1}{\frac{x+2-2x}{x}}$$

$$=\frac{x}{2-x} \qquad \textbf{\textit{Domain}}: \boxed{x\ne 2}$$

Domain: $(-\infty,0)\cup(0,2)\cup(2,\infty)$

$$g(f(x))=g\left(\tfrac{1}{x-2}\right) \qquad \textbf{\textit{Domain}}: \boxed{x\ne 2}$$

$$=\frac{\frac{1}{x-2}+2}{\frac{1}{x-2}}$$

$$= \frac{\frac{1+2x-4}{x-2}}{\frac{1}{x-2}}$$

$$= 2x-3$$

Domain: $\boxed{\mathbb{R}}$

Domain: $(-\infty,\ 2)\cup(2,\ \infty)$

Exercise

Given that $f(x)=2x-5$ and $g(x)=x^2-3x+8$, find $(f\circ g)(x)$, $(g\circ f)(x)$ and their domain then find $(f\circ g)(7)$

Solution

$$f(g(x)) = f\left(x^2-3x+8\right)$$

Domain: $(-\infty,\infty)$

$$= 2(\text{------------})-5$$

$$= 2\left(2x^2-3x+8\right)-5$$

$$= 2x^2-6x+16-5$$

$$= 2x^2-6x+11$$

Domain: $(-\infty,\infty)$

Domain: $(-\infty,\infty)$

$$g(f(x)) = g(2x-5)$$

Domain: $(-\infty,\infty)$

$$= (---)^2-3(---)+8$$

$$= (2x-5)^2-3(2x-5)+8$$

$$= 4x^2-20x+25-6x+15+8$$

$$= 4x^2-26x+48$$

Domain: $(-\infty,\infty)$

Domain: $(-\infty,\infty)$

$$f(g(7)) = 2(7)^2-6(7)+11 \ \underline{= 67}$$

Exercise

Given that $f(x)=\sqrt{x}$ and $g(x)=x-1$, find

a) $(f\circ g)(x)=f(g(x))$

b) $(g\circ f)(x)=g(f(x))$

c) $(f\circ g)(2)=f(g(2))$

Solution

a) $(f\circ g)(x)=f(g(x))$

$$=f(x-1)$$

$$\boxed{=\sqrt{x-1}}$$

b) $(g\circ f)(x)=g(f(x))$

$$=g(\sqrt{x})$$

$$\boxed{=\sqrt{x}-1}$$

c) $(f\circ g)(2)=f(g(2))$ $\qquad =\sqrt{x-1}$

$$=\sqrt{2-1}$$

$$=1$$

Exercise

Given that $f(x)=\dfrac{x}{x+5}$ and $g(x)=\dfrac{6}{x}$, find

a) $(f\circ g)(x)=f(g(x))$

b) $(g\circ f)(x)=g(f(x))$

c) $(f\circ g)(2)=f(g(2))$

Solution

a) $(f\circ g)(x)=f(g(x))$

$$=f\left(\frac{6}{x}\right)$$

$$=\frac{\frac{6}{x}}{\frac{6}{x}+5}$$

$$=\frac{\frac{6}{x}}{\frac{6+5x}{x}}$$

$$\boxed{=\frac{6}{6+5x}}$$

b) $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{x}{x+5}\right)$$

$$= \frac{6}{\frac{x}{x+5}}$$

$$= \frac{6(x+5)}{x}$$

$$\boxed{= \frac{6x+30}{x}}$$

c) $(f \circ g)(2) = f(g(2))$

$$= \frac{6}{6+5(2)}$$

$$= \frac{6}{16}$$

$$\boxed{= \frac{3}{8}}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x) = 2x^2 + 3x - 4$, $g(x) = 2x - 1$

Solution

$$f(g(x)) = f(2x-1)$$

$$= 2(2x-1)^2 + 3(2x-1) - 4$$

$$= 2\left(4x^2 - 4x + 1\right) + 6x - 3 - 4$$

$$= 8x^2 - 8x + 2 + 6x - 7$$

$$= 8x^2 - 2x - 5$$

$$g(f(x)) = g\left(2x^2 + 3x - 4\right)$$

$$= 2\left(2x^2 + 3x - 4\right) - 1$$

$$= 4x^2 + 6x - 8 - 1$$

$$= 4x^2 + 6x - 9$$

$$f(g(-2)) = 8(-2)^2 - 2(-2) - 5 \ \boxed{= 31}$$

$$g(f(3)) = 4(3)^2 + 6(3) - 9 \ \boxed{= 45}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x) = x^3 + 2x^2$, $g(x) = 3x$

Solution

$$f(g(x)) = f(3x)$$
$$= (3x)^3 + 2(3x)^2$$
$$= 27x^3 + 18x^2$$

$$g(f(x)) = g\left(x^3 + 2x^2\right)$$
$$= 3\left(x^3 + 2x^2\right)$$
$$= 3x^3 + 6x^2$$

$$f(g(-2)) = 27(-2)^3 + 18(-2)^2 \ \underline{= 288}$$

$$g(f(3)) = 3(3)^3 + 6(3)^2 \ \underline{= 135}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x) = |x|$, $g(x) = -7$

Solution

$$f(g(x)) = f(-7)$$
$$= |-7|$$
$$= 7$$

$$g(f(x)) = g(|x|)$$
$$= -7$$

$$f(g(-2)) \ \underline{= 7}$$

$$g(f(3)) \ \underline{= -7}$$

Exercise

Let $f(x) = x^2 - 3x$ *and* $g(x) = \sqrt{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt{x+2}\right)$ $\qquad x+2 \ge 0 \Rightarrow x \ge -2$

$= \left(\sqrt{x+2}\right)^2 - 3\sqrt{x+2}$

$= x + 2 - 3\sqrt{x+2}$ $\qquad x+2 \ge 0 \Rightarrow x \ge -2$

Domain: $\{x \mid x \ge -2\}$

b) $g(f(x)) = g\left(x^2 - 3x\right)$ $\qquad \mathbb{R}$

$= \sqrt{x^2 - 3x + 2}$ $\qquad x^2 - 3x + 2 \ge 0 \Rightarrow (x = 1,\ 2) \leftrightarrow x \le 1,\ x \ge 2$

Domain: $\{x \mid x \le 1,\ x \ge 2\}$

Exercise

Let $f(x) = \sqrt{x-2}$ *and* $g(x) = \sqrt{x+5}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt{x+5}\right)$ $\qquad x+5 \ge 0 \Rightarrow x \ge -5$

$= \sqrt{\sqrt{x+5} - 2}$ $\qquad \sqrt{x+5} - 2 \ge 0 \Rightarrow \sqrt{x+5} \ge 2$

$x + 5 \ge 4$

$x \ge -1$

Domain: $\{x \mid x \ge -1\}$

b) $g(f(x)) = g\left(\sqrt{x-2}\right)$ $\qquad x - 2 \ge 0 \Rightarrow x \ge 2$

$= \sqrt{\sqrt{x-2} + 5}$ $\qquad \sqrt{x-2} + 5 \ge 0 \Rightarrow \sqrt{x-2} \ge -5$ *Always true when* $x \ge 2$

Domain: $\{x \mid x \ge 2\}$

Exercise

Let $f(x)=\frac{3x+5}{2}$ *and* $g(x)=\frac{2x-5}{3}$

a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

Solution

a) $f\left(g(x)\right)=f\left(\frac{2x-5}{3}\right)$ $\qquad\mathbb{R}$

$$=\frac{3\frac{2x-5}{3}+5}{2}$$

$$=\frac{2x-5+5}{2}$$

$$=\frac{2x}{2}$$

$$=x$$

Domain: $\mathbb{R}$

b) $g\left(f(x)\right)=g\left(\frac{3x+5}{2}\right)$ $\qquad\mathbb{R}$

$$=\frac{2\frac{3x+5}{2}-5}{3}$$

$$=\frac{3x+5-5}{3}$$

$$=\frac{3x}{3}$$

$$=x$$

Domain: $\mathbb{R}$

Exercise

Let $f(x)=\dfrac{x-1}{x-2}$ *and* $g(x)=\dfrac{x-3}{x-4}$

a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

Solution

a)

$$f(g(x))=f\left(\frac{x-3}{x-4}\right) \qquad x-4\neq 0 \Rightarrow x\neq 4$$

$$=\frac{\frac{x-3}{x-4}-1}{\frac{x-3}{x-4}-2}$$

$$=\frac{\frac{x-3-(x-4)}{x-4}}{\frac{x-3-2(x-4)}{x-4}}$$

$$=\frac{x-3+x+4}{x-3-2x+8}$$

$$=\frac{2x+1}{-x+5} \qquad -x+5\neq 0 \Rightarrow x\neq 5$$

Domain: $\{x \mid x\neq 4,\ 5\}$

b)

$$g(f(x))=g\left(\frac{x-1}{x-2}\right) \qquad x-2\neq 0 \Rightarrow x\neq 2$$

$$=\frac{\frac{x-1}{x-2}-3}{\frac{x-1}{x-2}-4}$$

$$=\frac{\frac{x-1-3(x-2)}{x-2}}{\frac{x-1-4(x-2)}{x-2}}$$

$$=\frac{x-1-3x+6}{x-1-4x+8}$$

$$=\frac{-2x+5}{-3x+7} \qquad -3x+7\neq 0 \Rightarrow -3x\neq -7 \to x\neq \frac{7}{3}$$

Domain: $\left\{x \mid x\neq 2,\ \frac{7}{3}\right\}$

Exercise

Let $f(x)=\dfrac{6}{x-3}$ *and* $g(x)=\dfrac{1}{x}$

a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

Solution

a) $(f\circ g)(x)$

$$f(g(x))=f\left(\frac{1}{x}\right)$$ ***Domain***: $x\neq 0$

$$=\frac{6}{\frac{1}{x}-3}$$

$$=\frac{6}{\frac{1-3x}{x}}$$

$$=\frac{6x}{1-3x}$$ ***Domain***: $1-3x\neq 0 \quad\Rightarrow\quad x\neq\frac{1}{3}$

Domain: $\boxed{(-\infty,0)\cup\left(0,\frac{1}{3}\right)\cup\left(\frac{1}{3},\infty\right)}$

b) $(g\circ f)(x)$

$$g(f(x))=g\left(\frac{6}{x-3}\right)$$ ***Domain***: $x\neq 3$

$$=\frac{1}{\frac{6}{x-3}}$$

$$=\frac{x-3}{6}$$ ***Domain***: $(-\infty,\infty)$

Domain: $\boxed{(-\infty,3)\cup(3,\infty)}$

Solution Section 2.4 – Quadratic Functions

Exercise

Give the vertex, axis, domain, and range. Then, graph the function $f(x) = x^2 + 6x + 5$

Solution

Vertex: $x = -\frac{b}{2a}$

$= -\frac{6}{2(1)}$

$= -3$

$y = f(-3) = (-3)^2 + 6(-3) + 5$

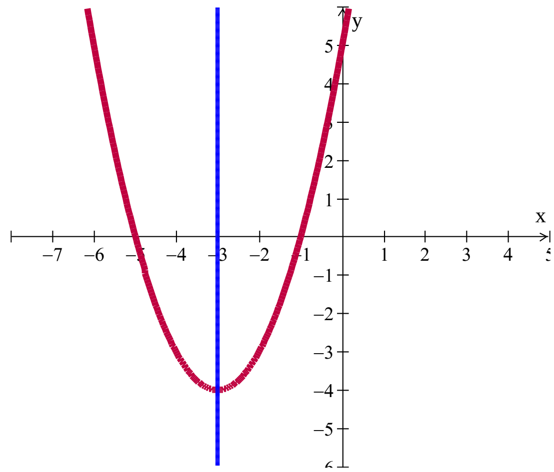
$= -4$

Vertex point: $(-3, -4)$

Axis of symmetry: $x = -3$

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$



Exercise

Give the vertex, axis, domain, and range. Then, graph the function $f(x) = -x^2 - 6x - 5$

Solution

Vertex: $x = -\frac{b}{2a}$

$= -\frac{-6}{2(-1)}$

$= -3$

$y = f(-3) = -(-3)^2 - 6(-3) - 5$

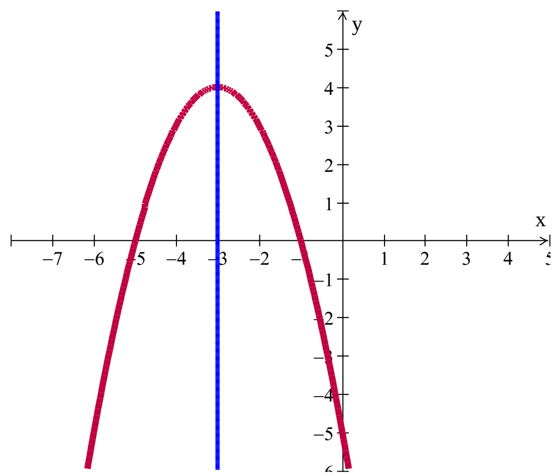
$= 4$

Vertex point: $(-3,\ 4)$

Axis of symmetry: $x = -3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$



Exercise

Graph the quadratic. Give the vertex, axis of symmetry, domain, and range:

$f(x)=x^2-4x+2$

Solution

Vertex point:

$$x=-\frac{b}{2a}=-\frac{-4}{2(1)}=2$$

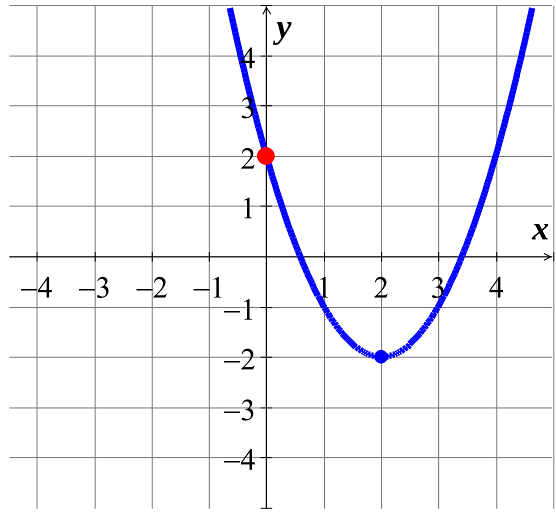
$$f(2)=2^2-4(2)+2=-2$$

The ***vertex point***: $(2,\ -2)$

Axis of symmetry is: $x=2$

Domain: $(-\infty,\ \infty)$

Range: $[-2,\ \infty)$ ***(Since function has a minimum)***



To graph: find another point:

$x=0 \quad \Rightarrow \quad y=f(0)=2$

Exercise

Graph the quadratic. Give the vertex, axis of symmetry, domain, and range:

$f(x)=-2x^2+16x-26$

Solution

Vertex point:

$$x=-\frac{b}{2a}=-\frac{16}{2(-2)}=4$$

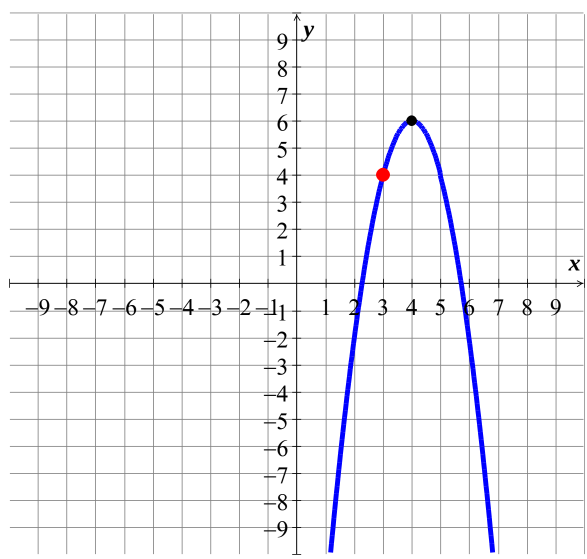
$$f(4)=-2\left(4^2\right)+16(4)-26=6$$

The ***vertex point***: $(4,\ 6)$

Axis of symmetry is: $x=4$

Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ 6]$ ***(Since function has a maximum)***



To graph: find another point:

$x=3 \quad \Rightarrow \quad y=f(3)=4$

Exercise

You have 600 *ft* of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

Solution

$$P = l + 2w$$

$$600 = l + 2w \qquad \to l = 600 - 2w$$

$$\begin{aligned} A &= lw \\ &= (600 - 2w)w \\ &= 600w - 2w^2 \\ &= -2w^2 + 600w \end{aligned}$$

Vertex: $w = -\frac{600}{2(-2)} = 150$

$$\to l = 600 - 2w = 300$$

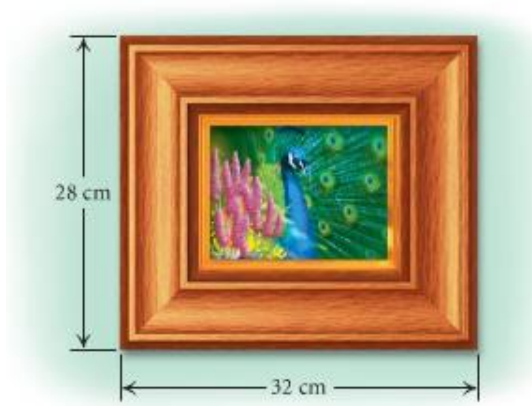
$$\begin{aligned} A &= lw = (300)(150) \\ &= 45000 \ \ ft^2 \end{aligned}$$

Exercise

A picture frame measures 28 cm by 32 cm and is of uniform width. What is the width of the frame if 192 cm^2 of the picture shows?

Solution

Area of the picture = $(32 - 2x)(28 - 2x) = 192$



$$896 - 64x - 56x + 4x^2 = 192$$

$$896 - 120x + 4x^2 - 192 = 0$$

$$4x^2 - 120x + 704 = 0$$

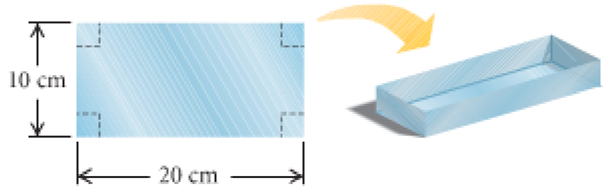
$$x^2 - 30x + 176 = 0$$

$$(x - 8)(x - 22) = 0$$

$$\begin{cases} x - 8 = 0 \to \boxed{x = 8} \\ x - 22 = 0 \to \boxed{x = 22} \end{cases}$$

Exercise

An open box is made from a 10-cm by 20-cm of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2. What is the length of the sides of the squares?



Solution

$$\text{Area of the base } = (20-2x)(10-2x) = 96$$

$$200 - 40x - 20x + 4x^2 = 96$$

$$4x^2 - 60x + 200 - 96 = 0$$

$$4x^2 - 60x + 104 = 0 \qquad \textit{Solve for x}$$

$$\boxed{x = 2,\ \cancel{13}}$$

The length of the sides of the squares is 3-cm

Exercise

A fourth-grade class decides to enclose a rectangular garden, using the side of the school as one side of the rectangle. What is the maximum area that the class can enclose with 32 ft. of fence? What should the dimensions of the garden be in order to yield this area?

Solution

$$\text{Perimeter: } P = l + 2w = 32$$

$$l = 32 - 2w$$

$$\text{Area: } A = lw$$

$$A = (32-2w)w$$

$$= 32w - 2w^2$$

$$= -2w^2 + 32w$$

$$\textbf{Vertex}: \ w = -\frac{32}{2(-2)} = 8$$

$$\rightarrow l = 32 - 2(8) = 16$$

$$A = lw = (16)(8)$$

$$= 128\ ft^2$$

Exercise

A rancher needs to enclose two adjacent rectangular corrals, one for cattle and one for sheep. If a river forms one side of the corrals and 240 yd of fencing is available, what is the largest total area that can be enclosed?

Solution

Perimeter: $P = l + 3w = 240$

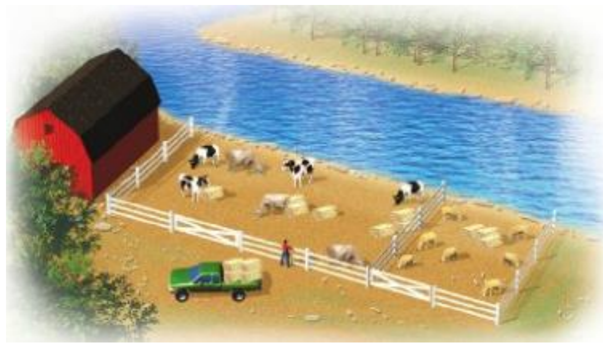
$$l = 240 - 3w$$

Area: $A = lw$

$$A = (240 - 3w)w$$

$$= 240w - 3w^2$$

$$= -3w^2 + 240w$$



Vertex: $\underline{\lfloor w = -\frac{240}{2(-3)} = 40 \rfloor}$

$\rightarrow \underline{\lfloor l = 240 - 3(40) = 120 \rfloor}$

$$A = lw = (120)(40)$$

$$\underline{= 4800\ yd^2 \rfloor}$$

Exercise

A Norman window is a rectangle with a semicircle on top. Sky Blue Windows is designing a Norman window that will require 24 *ft.* of trim on the outer edges. What dimensions will allow the maximum amount of light to enter a house?

Solution

Perimeter of the semi-circle $= \frac{1}{2}(2\pi x)$

Perimeter of the rectangle $= 2x + 2y$

Total perimeter: $\pi x + 2x + 2y = 24$

$$2y = 24 - \pi x - 2x$$

$$y = 12 - \frac{\pi}{2}x - x$$

A Norman window

Area $= \frac{1}{2}\left(\pi x^2\right) + (2x)y$

$$= \frac{\pi}{2}x^2 + 2x\left(12 - \frac{\pi}{2}x - x\right)$$

$$= \frac{\pi}{2}x^2 + 24x - \pi x^2 - 2x^2$$

$$= 24x - \left(\frac{\pi}{2} + 2\right)x^2$$

$$= -\left(\frac{\pi}{2}+2\right)x^2 + 24x$$

$$x = -\frac{b}{2a} = -\frac{24}{2\left(-\frac{\pi}{2}-2\right)} = -\frac{24}{-2\left(\frac{\pi+4}{2}\right)} = \boxed{\frac{24}{\pi+4}}$$

$$y = 12 - \frac{\pi}{2}\frac{24}{\pi+4} - \frac{24}{\pi+4}$$

$$= \frac{24\pi + 96 - 24\pi - 48}{2(\pi+4)}$$

$$\boxed{= \frac{24}{\pi+4}}$$

Exercise

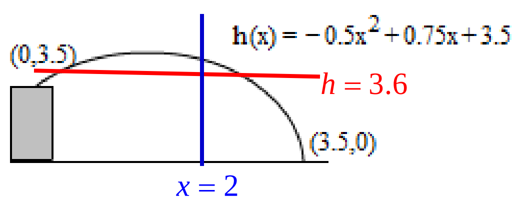
A frog leaps from a stump 3.5 ft. high and lands 3.5 ft. from the base of the stump.
It is determined that the height of the frog as a function of its distance, *x*, from the base of the stump is given by the function $h(x) = -0.5x^2 + 0.75x + 3.5$ where *h* is in feet.

a) How high is the frog when its horizontal distance from the base of the stump is 2 ft.?
b) At what two distances from the base of the stump after is jumped was the frog 3.6 ft. above the ground?
c) At what distance from the base did the frog reach its highest point?
d) What was the maximum height reached by the frog?

Solution

a) At $x = 2\, ft.$ *Find* $h(x=2)$

$$h(2) = -0.5\left(2^2\right) + 0.75(2) + 3.5 \boxed{= 3\ ft}$$



b) $h(x) = -0.5x^2 + 0.75x + 3.5 = 3.6$

$$-0.5x^2 + 0.75x + 3.5 - 3.6 = 0$$

$$-0.5x^2 + 0.75x - .1 = 0$$

Solve for *x*: $\boxed{x = 0.1,\ \ 1.4\ ft}$

c) The distance from the base for the frog to reach the highest point is

$$x = -\frac{b}{2a} = -\frac{.75}{2(-.5)} \boxed{= .75\ ft}$$

d) Maximum height:

$$h(x = .75) = -0.5(.75)^2 + 0.75(.75) + 3.5 \boxed{= 3.78\ ft}$$

Exercise

For the graph of the function $f(x) = x^2 + 6x + 3$

a. **Find the vertex point**

$$x = -\frac{6}{2(1)} = -3$$

$$y = f(-3) = (-3)^2 + 6(-3) + 3 = -6$$

Vertex point (–3, –6)

b. **Find the line of symmetry:** $x = -3$

c. **State whether there is a maximum or minimum value *and* find that value**

Minimum point, value (–3, –6)

d. **Find the zeros of *f*(*x*)**

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(3)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 12}}{2} = \frac{-6 \pm \sqrt{24}}{2} = \frac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$$

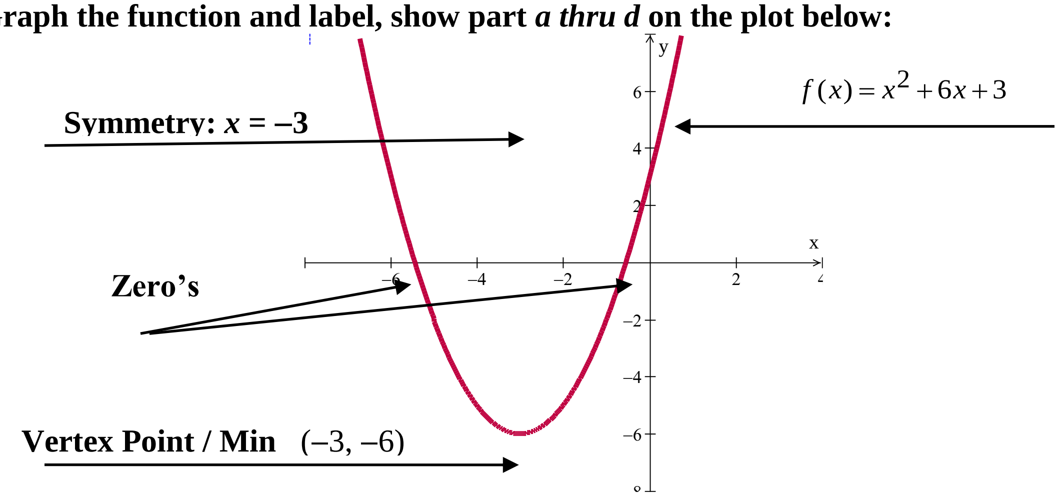
$$x = \begin{cases} -3 + \sqrt{6} = -0.5 \\ -3 - \sqrt{6} = -5.45 \end{cases}$$

e. **Find the *y*-intercept** $y = 3$

f. **Find the range and the domain of the function.**

Range: [–6, ∞) *Domain*: (–∞, ∞)

g. **Graph the function and label, show part *a thru d* on the plot below:**



h. **On what intervals is the function increasing? Decreasing?**

Decreasing: (–∞, –3)

Increasing: (–3, ∞)

Solution   *Section* 2.5 – Polynomial Functions

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^3 + 7x^2 - x + 9$

Solution

Leading term: $5x^3$ with 3rd degree (n is odd)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 11x^3 - 6x^2 + x + 3$

Solution

Leading term: $11x^3$ with 3rd degree (n is odd)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -11x^3 - 6x^2 + x + 3$

Solution

Leading term: $-11x^3$ with 3rd degree (n is odd)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty$ $\qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^4 + 7x^2 - x + 9$

Solution

Leading term: $5x^4$ with 4rd degree (n is even)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty$ $\qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 11x^4 - 6x^2 + x + 3$

Solution

Leading term: $11x^4$ with 4^{rd} degree (*n* is even)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty \qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -5x^4 + 7x^2 - x + 9$

Solution

Leading term: $-5x^4$ with 4^{rd} degree (*n* is even)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty \qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -11x^4 - 6x^2 + x + 3$

Solution

Leading term: $-11x^4$ with 4^{rd} degree (*n* is even)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty \qquad f(x)$ falls right

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - x - 1; \quad between \ 1 \ and \ 2$

Solution

$$f(1) = (1)^3 - (1) - 1 = -1$$

$$f(2) = (2)^3 - (2) - 1 = 5$$

Since $f(1)$ *and* $f(2)$ have opposite signs; therefore, the polynomial has a real zero between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - 4x^2 + 2;\quad between\ \ 0\ and\ 1$

Solution

$$f(0) = (0)^3 - 4(0)^2 + 2 \ \underline{=2}$$

$$f(1) = (1)^3 - 4(1)^2 + 2 \ \underline{=-1}$$

Since $f(0)\ and\ f(1)$ have opposite signs; therefore, the polynomial has a real zero between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 2x^4 - 4x^2 + 1;\quad between\ \ -1\ and\ 0$

Solution

$$f(-1) = 2(-1)^4 - 4(-1)^2 + 1 \ \underline{=-1}$$

$$f(0) = 2(0)^4 - 4(0)^2 + 1 \ \underline{=1}$$

Since $f(0)\ and\ f(-1)$ have opposite signs; therefore, the polynomial has a real zero between -1 and 0.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^4 + 6x^3 - 18x^2;\quad between\ \ 2\ and\ 3$

Solution

$$f(2) = (2)^4 + 6(2)^3 - 18(2)^2 \ \underline{=-8}$$

$$f(3) = (3)^4 + 6(3)^3 - 18(3)^2 \ \underline{=81}$$

Since $f(2)\ and\ f(3)$ have opposite signs; therefore, the polynomial has a real zero between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 + x^2 - 2x + 1;\quad between\ \ -3\ and\ -2$

Solution

$$f(-3)=(-3)^3+(-3)^2-2(-3)+1 \ \underline{=-11}$$

$$f(-2)=(-2)^3+(-2)^2-2(-2)+1 \ \underline{=1}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs; therefore, the polynomial has a real zero between –2 and –3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^5-x^3-1;\quad \textit{between}\ \ 1\ \textit{and}\ \ 2$

Solution

$$f(1)=(1)^5-(1)^3-1 \ \underline{=-1}$$

$$f(2)=(2)^5-(2)^3-1 \ \underline{=23}$$

Since $f(1)$ *and* $f(2)$ have opposite signs; therefore, the polynomial has a real zero between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-10x+9;\quad \textit{between}\ \ -3\ \textit{and}\ \ -2$

Solution

$$f(-3)=3(-3)^3-10(-3)+9 \ \underline{=-42}$$

$$f(-2)=3(-2)^3-10(-2)+9 \ \underline{=5}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs; therefore, the polynomial has a real zero between –3 and –2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-8x^2+x+2;\quad \textit{between}\ \ 2\ \textit{and}\ \ 3$

Solution

$$f(2)=3(2)^3-8(2)^2+(2)+2 \ \underline{=-4}$$

$$f(3)=3(3)^3-8(3)^2+(3)+2 \ \underline{=14}$$

Since $f(2)$ *and* $f(3)$ have opposite signs; therefore, the polynomial has a real zero between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-8x^2+x+2;\quad between\ \ 1\ and\ \ 2$

Solution

$$f(1)=3(1)^3-8(1)^2+(1)+2\ \underline{=-2}$$

$$f(2)=3(2)^3-8(2)^2+(2)+2\ \underline{=-4}$$

Since $f(1)\ and\ f(2)$ have same signs; therefore, cannot be determined.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^5+2x^4-6x^3+2x-3;\quad between\ \ 0\ and\ \ 1$

Solution

$$f(0)=(0)^5+2(0)^4-6(0)^3+2(0)-3\ \underline{=-3}$$

$$f(1)=(1)^5+2(1)^4-6(1)^3+2(1)-3\ \underline{=-4}$$

Since $f(0)\ and\ f(1)$ have same signs; therefore, cannot be determined.

Solution Section 2.6 – Properties of Division

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=2x^4-x^3+7x-12;\quad p(x)=x^2-3$

Solution

$$\begin{array}{r|l}
 & 2x^2-x+6 \\
\hline
x^2-3 & 2x^4-x^3+0x^2+7x-12 \\
 & \underline{2x^4 \qquad -6x^2} \\
 & \qquad -x^3+6x^2+7x \\
 & \qquad \underline{-x^3 \qquad\quad +3x} \\
 & \qquad\qquad 6x^2+4x-12 \\
 & \qquad\qquad \underline{6x^2 \qquad -18} \\
 & \qquad\qquad\qquad 4x+6
\end{array}$$

$Q(x)=2x^2-x+6;\quad R(x)=4x+6$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=3x^3+2x-4;\quad p(x)=2x^2+1$

Solution

$$\begin{array}{r|l}
 & \frac{3}{2}x \\
\hline
2x^2+1 & 3x^3+0x^2+2x-4 \\
 & \underline{3x^3 \qquad\quad +\frac{3}{2}x \qquad} \\
 & \qquad\qquad \frac{1}{2}x-4
\end{array}$$

$Q(x)=\frac{3}{2}x;\quad R(x)=\frac{1}{2}x-4$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=7x+2; \quad p(x)=2x^2-x-4$

Solution

$$Q(x)=0; \quad R(x)=7x+2$$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=9x+4; \quad p(x)=2x-5$

Solution

$$\begin{array}{r|l} & \frac{9}{2} \\ \hline 2x-5 & 9x+4 \\ & 9x-\frac{45}{2} \\ \hline & -\frac{37}{2} \end{array}$$

$$Q(x)=\frac{9}{2}; \quad R(x)=-\frac{37}{2}$$

Exercise

Use the remainder theorem to find $f(c)$: $f(x)=x^4-6x^2+4x-8; \quad c=-3$

Solution

$$f(-3)=(-3)^4-6(-3)^2+4(-3)-8 \ \underline{=7}$$

Exercise

Use the remainder theorem to find $f(c)$: $f(x)=x^4+3x^2-12; \quad c=-2$

Solution

$$f(-2)=(-2)^4+3(-2)^2-12 \ \underline{=16}$$

Exercise

Use the factor theorem to show that $x-c$ is a factor of $f(x)$: $f(x)=x^3+x^2-2x+12;\quad c=-3$

Solution

$$f(-3)=(-3)^3+(-3)^2-2(-3)+12 \ \underline{=0}$$

From the factor theorem; $x+3$ is a factor of $f(x)$.

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second: $2x^3-3x^2+4x-5;\quad x-2$

Solution

$$\begin{array}{r|rrrr} 2 & 2 & -3 & 4 & -5 \\ & & 4 & 2 & 12 \\ \hline & 2 & 1 & 6 & \boxed{7} \end{array}$$

$Q(x)=2x^2+x+6 \qquad R(x)=7$

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second: $5x^3-6x^2+15;\quad x-4$

Solution

$$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 15 \\ & & 20 & 56 & 224 \\ \hline & 5 & 14 & 56 & \boxed{239} \end{array}$$

$Q(x)=5x^2+14x+56 \qquad R(x)=239$

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second: $9x^3 - 6x^2 + 3x - 4; \quad x - \frac{1}{3}$

Solution

$$\begin{array}{r|rrrr} \frac{1}{3} & 9 & -6 & 3 & -4 \\ & & 3 & -1 & \frac{2}{3} \\ \hline & 9 & -3 & 2 & \boxed{-\frac{10}{3}} \end{array}$$

$Q(x) = 9x^2 - 3x + 2 \qquad R(x) = -\frac{10}{3}$

Exercise

Use the synthetic division to find $f(c)$: $f(x) = 2x^3 + 3x^2 - 4x + 4; \quad c = 3$

Solution

$$\begin{array}{r|rrrr} 3 & 2 & 3 & -4 & 4 \\ & & 9 & 36 & 93 \\ \hline & 3 & 12 & 32 & \boxed{97} \end{array}$$

$\boxed{f(3) = 97}$

Exercise

Use the synthetic division to find $f(c)$: $f(x) = 8x^5 - 3x^2 + 7; \quad c = \frac{1}{2}$

Solution

$$\begin{array}{r|rrrrrr} \frac{1}{2} & 8 & 0 & 0 & -3 & 0 & 7 \\ & & 4 & 2 & 1 & -1 & -\frac{1}{2} \\ \hline & 8 & 4 & 2 & -2 & -1 & \boxed{\frac{13}{2}} \end{array}$$

$\boxed{f\left(\frac{1}{2}\right) = \frac{13}{2}}$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=x^3-3x^2-8;\quad c=1+\sqrt{2}$

Solution

$$\begin{array}{r|rrrr} 1+\sqrt{2} & 3 & -3 & 0 & -8 \\ & & 3+3\sqrt{2} & 6+3\sqrt{2} & 12+9\sqrt{2} \\ \hline & 3 & 3\sqrt{2} & 6+3\sqrt{2} & \boxed{4+9\sqrt{2}} \end{array}$$

$$\boxed{f\left(1+\sqrt{2}\right)=4+9\sqrt{2}}$$

Exercise

Use the synthetic division to show that c is a zero of $f(x)$: $f(x)=3x^4+8x^3-2x^2-10x+4;\quad c=-2$

Solution

$$\begin{array}{r|rrrrr} -2 & 3 & 8 & -2 & -10 & 4 \\ & & -6 & -4 & 12 & -4 \\ \hline & 3 & 2 & -6 & 2 & \boxed{0} \end{array}$$

$$\boxed{f(-2)=0}$$

Exercise

Use the synthetic division to show that c is a zero of $f(x)$: $f(x)=27x^4-9x^3+3x^2+6x+1;\quad c=-\frac{1}{3}$

Solution

$$\begin{array}{r|rrrrr} -\frac{1}{3} & 27 & -9 & 3 & 6 & 1 \\ & & -9 & 6 & -3 & -1 \\ \hline & 27 & -18 & 9 & 3 & \boxed{0} \end{array}$$

$$\boxed{f\left(-\tfrac{1}{3}\right)=0}$$

Exercise

Find all values of k such that $f(x)$ is divisible by the given linear polynomial:

$f(x)=kx^3+x^2+k^2x+3k^2+11;\quad x+2$

Solution

$$\begin{array}{r|cccc} -2 & k & 1 & k^2 & 3k^2+11 \\ & & -2k & 4k-2 & -2k^2-8k+4 \\ \hline & k & 1-2k & k^2+4k-2 & k^2-8k+15 \end{array}$$

$k^2-8k+15=0 \Rightarrow \boxed{k=3,\ 5}$

Exercise

Find all solutions of the equation: $x^3-x^2-10x-8=0$

Solution

$possibilities\ for\ \frac{c}{d}: \dfrac{\pm1,\ \pm2,\ \pm4,\ \pm8}{\pm1}$

Using the calculator, the result will show that the solutions are: $x=-1,\ -2,\ 4$

Exercise

Find all solutions of the equation: $x^3+x^2-14x-24=0$

Solution

$possibilities\ for\ \frac{c}{d}: \dfrac{\pm1,\ \pm2,\ \pm3,\ \pm4,\ \pm6,\ \pm8,\ \pm12,\ \pm24}{\pm1}$

Using the calculator, the result will show that the solutions are: $\boxed{x=-2}$

$$\begin{array}{r|cccc} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

We have the factorization of: $(x+2)\left(x^2-x-12\right)=0$

$x^2-x-12=0 \Rightarrow \boxed{x=-3,\ 4}$

Exercise

Find all solutions of the equation: $2x^3 - 3x^2 - 17x + 30 = 0$

Solution

$$\textit{possibilities for } \frac{c}{d}: \frac{\pm 1,\ \pm 2,\ \pm 3,\ \pm 5,\ \pm 6,\ \pm 10,\ \pm 15,\ \pm 30}{\pm 1,\ \pm 2}$$

Using the calculator, the result will show that the solutions are: $\boxed{x = 2}$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & \boxed{0} \end{array}$$

We have the factorization of: $(x-2)\left(2x^2 + x - 15\right) = 0$

$$2x^2 + x - 15 = 0 \Rightarrow \boxed{x = -3,\ \frac{5}{2}}$$

Exercise

Find all solutions of the equation: $12x^3 + 8x^2 - 3x - 2 = 0$

Solution

$$\textit{possibilities for } \frac{c}{d}: \frac{\pm 1,\ \pm 2}{\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12}$$

Using the calculator, the result will show that the solutions are: $\boxed{x = \frac{1}{2}}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 12 & 8 & -3 & -2 \\ & & 6 & 7 & 2 \\ \hline & 12 & 14 & 4 & \boxed{0} \end{array}$$

We have the factorization of: $\left(x - \frac{1}{2}\right)\left(12x^2 + 14x + 4\right) = 0$

$$12x^2 + 14x + 4 = 0 \Rightarrow \boxed{x = -\frac{2}{3},\ -\frac{1}{2}}$$

Exercise

Find all solutions of the equation: $x^4 + 3x^3 - 30x^2 - 6x + 56 = 0$

Solution

$$\textit{possibilities for } \frac{c}{d}: \frac{\pm 1,\ \pm 2,\ \pm 4,\ \pm 7,\ \pm 8,\ \pm 14, \cdots, \pm 56}{\pm 1}$$

Using the calculator, the result will show that the solutions are: $\boxed{x=4}$

$$\begin{array}{r|rrrrr} 4 & 1 & 3 & -30 & -6 & 56 \\ & & 4 & 28 & -8 & -56 \\ \hline & 1 & 7 & -2 & -14 & \boxed{0} \end{array}$$

We have the factorization of: $(x-4)\left(x^3+7x^2-2x-14\right)=0$

For $x^3+7x^2-2x-14 \quad \Rightarrow \quad \frac{c}{d}=\frac{\pm1,\ \pm2,\ \pm7,\ \pm14}{\pm1}$

$\boxed{x=-7}$ is another solution.

$$\begin{array}{r|rrrr} -7 & 1 & 7 & -2 & -14 \\ & & -7 & 0 & 14 \\ \hline & 1 & 0 & -2 & \boxed{0} \end{array}$$

We have the factorization of: $(x+4)(x+7)\left(x^2-2\right)=0$

By applying quadratic formula to solve: $x^2-2=0 \ \Rightarrow \boxed{x=\pm\sqrt{2}}$

Exercise

Find all solutions of the equation: $3x^5-10x^4-6x^3+24x^2+11x-6=0$

Solution

$$\boxed{x=-1,\ -1,\ \frac{1}{3},\ 2,\ 3}$$

Exercise

Find all solutions of the equation: $6x^5+19x^4+x^3-6x^2=0$

Solution

$$x^2\left(6x^3+19x^2+x-6\right)=0$$

$$\boxed{x=0,\ 0,\ -\frac{2}{3},\ -3,\ \frac{1}{2}}$$

Solution Section 2.7 – Rational Functions

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \frac{3x}{1-x}$

Solution

$1 - x = 0 \Rightarrow x = 1$

$y = \frac{3x}{-x} = \frac{3}{-1} = -3$

VA	$x = 1$
HA	$y = -3$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \frac{x^2}{x^2 + 9}$

Solution

VA: n / a

HA: $y = 1$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \frac{x-2}{x^2 - 4x + 3}$

Solution

$x^2 - 4x + 3 = 0 \Rightarrow x = 1, \ 3$

$y = \frac{x}{x^2} \to 0$

VA: $x = 1, \ x = 3$

HA: $y = 0$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{3}{x-5}$

Solution

VA: $x = 5$

HA: $y = 0$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{x^3 - 1}{x^2 + 1}$

Solution

VA: *none*

HA: *none*

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{3x^2 - 27}{(x+3)(2x+1)}$

Solution

$$x + 3 = 0 \rightarrow x = -3$$

$$2x + 1 = 0 \rightarrow x = -\frac{1}{2}$$

$$y = \frac{3x^2}{(x)(2x)} = \frac{3x^2}{2x^2} = \frac{3}{2}$$

VA: $x = -3, \ -\frac{1}{2}$

HA: $y = \frac{3}{2}$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{x^3 + 3x^2 - 2}{x^2 - 4}$

Solution

VA: $x = \pm 2$

HA: n / a

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{x-3}{x^2-9}$

Solution

$$x^2 - 9 = 0 \to \boxed{x = \pm 3}$$

$$y = \frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3}$$

VA: $x = 3$

HA: $y = 0$

Hole: $x = 3 \to y = \frac{1}{6}$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{6}{\sqrt{x^2-4x}}$

Solution

$$x^2 - 4x = 0 \Rightarrow x(x-4) = 0 \to \boxed{x = 0, 4}$$

VA: $x = 0,\ x = 4$

HA: $y = 0$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{5x-1}{1-3x}$

Solution

VA: $x = \frac{1}{3}$

HA: $y = -\frac{5}{3}$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $f(x) = \dfrac{2x-11}{x^2+2x-8}$

Solution

VA: $x = 2,\ x = -4$

HA: $y = 0$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $f(x)=\dfrac{x^2-4x}{x^3-x}$

Solution

$$f(x)=\frac{x(x-4)}{x\left(x^2-1\right)}=\frac{x-4}{x^2-1}$$

VA: $x=-1,\ x=1$

HA: $y=0$

Hole: $x=0\rightarrow y=4$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $f(x)=\dfrac{x-2}{x^3-5x}$

Solution

VA: $x=0,\ x=\pm\sqrt{5}$

HA: $y=0$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{4x}{x^2+10x}$

Solution

$x^2+10x=0 \quad\rightarrow\quad x=0,\ -10$ ***Domain***: $(-\infty,\ -10)\cup(-10,\ 0)\cup(0,\ \infty)$

$$f(x)=\frac{4x}{x(x+10)}=\frac{4}{x+10}$$

VA: $x=-10$

HA: $y=0$

Hole: $x=0 \quad\rightarrow\quad y=\frac{4}{10}\Rightarrow$ *hole* $\left(0,\ \frac{2}{5}\right)$

Oblique asymptote: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{3-x}{(x-4)(x+6)}$

Solution

Domain: $(-\infty,\ -6)\cup(-6,\ 4)\cup(4,\ \infty)$

VA: $x=-6$ *and* $x=4$

HA: $y=0$

Hole: n/a

Oblique asymptote: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3}{2x^3-x^2-3x}$

Solution

$$2x^3-x^2-3x=x\left(2x^2-x-3\right)=0 \quad\rightarrow\quad x=0,\ -1,\ \frac{3}{2}$$

Domain: $(-\infty,\ -1)\cup(-1,\ 0)\cup\left(0,\ \frac{3}{2}\right)\cup\left(\frac{3}{2},\ \infty\right)$

$$f(x)=\frac{x^3}{2x^3-x^2-3x}=\frac{x^3}{x\left(2x^2-x-3\right)}=\frac{x^2}{2x^2-x-3}$$

VA: $x=-1$ *and* $x=\frac{3}{2}$

HA: $y=\frac{1}{2}$

Hole: $x=0 \quad\rightarrow\quad y=0\Rightarrow$ *hole* $(0,\ 0)$

Oblique asymptote: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{3x^2+5}{4x^2-3}$

Solution

$4x^2-3=0 \quad\rightarrow\quad x=\pm\frac{\sqrt{3}}{2}$ ***Domain***: $\left(-\infty,\ -\frac{\sqrt{3}}{2}\right)\cup\left(-\frac{\sqrt{3}}{2},\ \frac{\sqrt{3}}{2}\right)\cup\left(\frac{\sqrt{3}}{2},\ \infty\right)$

VA: $x=-\frac{\sqrt{3}}{2}$ *and* $x=\frac{\sqrt{3}}{2}$

HA: $y = \frac{3}{4}$

Hole: n/a

Oblique asymptote: n/a

Exercise

Determine all asymptotes of the function $f(x) = \frac{x+6}{x^3 + 2x^2}$

Solution

$x^3 + 2x^2 = x^2(x+2) = 0 \quad \rightarrow \quad x = 0, -2 \qquad$ ***Domain***: $(-\infty,\ -2) \cup (-2,\ 0) \cup (0,\ \infty)$

VA: $x = 0$ *and* $x = 2$

HA: $y = 0$

Hole: n/a

Oblique asymptote: n/a

Exercise

Determine all asymptotes of the function $f(x) = \frac{x^2 + 4x - 1}{x+3}$

Solution

$x + 3 = 0 \quad \rightarrow \quad x = -3 \qquad$ ***Domain***: $(-\infty,\ -3) \cup (-3,\ \infty)$

VA: $x = -3$

HA: n/a

Hole: n/a

$$\begin{array}{r|l} & x+1 \\ \hline x+3 & x^2 + 4x - 1 \\ & \underline{-x^2 - 3x} \\ & \quad x - 1 \\ & \quad \underline{-x - 3} \\ & \qquad -4 \end{array}$$

$$f(x) = \frac{x^2 + 4x - 1}{x+3} = x + 1 - \frac{4}{x+3}$$

Oblique asymptote: $y = x + 1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2-6x}{x-5}$

Solution

$x-5=0 \quad \rightarrow \quad x=5 \qquad\qquad$ ***Domain***: $(-\infty,\ 5)\cup(5,\ \infty)$

VA: $x=5$

HA: N/A

Hole: N/A

$$\begin{array}{r|l} & x-1 \\ x-5 & x^2-6x \\ & \underline{-x^2+5x} \\ & \quad -x \\ & \quad \underline{x-5} \\ & \quad -5 \end{array}$$

$$f(x)=\frac{x^2-6x}{x-5}=x-1-\frac{5}{x-5}$$

Oblique asymptote: $y=x-1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3-x^2+x-4}{x^2+2x-1}$

Solution

$x^2+2x-1=0 \quad \rightarrow \quad x=-1\pm\sqrt{2}$

Domain: $\left(-\infty,\ -1-\sqrt{2}\right)\cup\left(-1-\sqrt{2},\ -1+\sqrt{2}\right)\cup\left(-1+\sqrt{2},\ \infty\right)$

VA: $x=-1\pm\sqrt{2}$

HA: N/A

Hole: N/A

$$\begin{array}{r|l} & x-3 \\ x^2+2x-1 & x^3-x^2+x-4 \\ & \underline{-x^3-2x^2+x} \\ & \quad -3x^2+2x-4 \\ & \quad \underline{3x^2+6x-3} \\ & \qquad 8x-7 \end{array}$$

$$f(x)=\frac{x^3-x^2+x-4}{x^2+2x-1}=x-3+\frac{8x-7}{x^2+2x-1}$$

Oblique asymptote: $y=x-3$

Exercise

Determine all asymptotes of the function $f(x)=\frac{-3x}{x+2}$

Solution

$x+2=0\to x=-2$

$y=\frac{-3x}{x}\to -3$

VA	$x=-2$
HA	$y=-3$

Exercise

Determine all asymptotes of the function $f(x)=\frac{x+1}{x^2+2x-3}$

Solution

$x^2+2x-3=0\Rightarrow x=1,\ -3$

VA	$x=-3 \quad x=1$
HA	$y=0$

Exercise

Determine all asymptotes of the function $f(x)=\frac{2x^2-2x-4}{x^2+x-12}$

Solution

$x^2+x-12=0\Rightarrow x=-4,\ 3$

VA	$x=-4 \quad x=3$
HA	$y=2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{-2x^2+10x-12}{x^2+x}$

Solution

$$x^2+x=0 \Rightarrow x=0,\ -1$$

VA	$x=-1 \quad x=0$
HA	$y=-2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2-x-6}{x+1}$

Solution

$$\begin{array}{r|l} & x-2 \\ \hline x+1 & x^2-x-6 \\ & \underline{x^2+x} \\ & \quad -2x-6 \\ & \quad \underline{-2x-2} \\ & \qquad \boxed{-4} \end{array}$$

$$f(x)=\frac{x^2-x-6}{x+1}=x-2-\frac{4}{x+1}$$

The oblique asymptote is: $y=x-2$

The vertical asymptote is: $x=-1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3+1}{x-2}$

Solution

$$\begin{array}{r|l} & x^2+2x+4 \\ \hline x-2 & x^3-1 \\ & \underline{x^3-2x^2} \\ & \quad 2x^2 \\ & \quad \underline{2x^2-4x} \\ & \qquad 4x-1 \\ & \qquad \underline{4x-8} \\ & \qquad\quad 7 \end{array}$$

$$f(x)=x^2+2x+4+\frac{7}{x-2}$$

The oblique asymptote is:

$y=x^2+2x+4$

The vertical asymptote is: $x=2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{2x^2+x-6}{x^2+3x+2}$

Solution

$$f(x)=\frac{(2x-3)(x+2)}{(x+1)(x+2)}$$

$$=\frac{2x-3}{x+1}$$

VA	$x=-1$
HA	$y=2$
Hole	$x=-2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x-1}{1-x^2}$

Solution

$$f(x)=\frac{x-1}{(1-x)(1+x)}$$

$$f(x)=-\frac{1}{1+x}$$

VA	$x=-1$
HA	$y=0$
Hole	$x=1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2+x-2}{x+2}$

Solution

$$f(x)=\frac{(x+2)(x-1)}{x+2}$$

$$=x-1$$

VA	*na*
HA	*na*
Hole	$x=-2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3-2x^2-4x+8}{x-2}$

Solution

$$f(x)=\frac{\left(x^2-4\right)(x-2)}{x-2}=x^2-4$$

Hole	$x=2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{2x^2-3x-1}{x-2}$

Solution

$$\begin{array}{r|l} & 2x+1 \\ \hline x-2 & 2x^2-3x-1 \\ & \underline{-2x^2+4x} \\ & \quad x-1 \\ & \quad \underline{-x+2} \\ & \qquad 1 \end{array}$$

$$f(x)=\frac{2x^2-3x-1}{x-2}=(2x+1)+\frac{1}{x-2}$$

The ***oblique asymptote*** is the line $y=2x+1$

VA: $x=2$

HA: $y=1$

Hole: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{2x+3}{3x^2+7x-6}$

Solution

$$3x^2+7x-6=0 \quad\Rightarrow\quad x=-3,\ \frac{2}{3}$$

VA: $x=-3$ *and* $x=\dfrac{2}{3}$

HA: $y=0$

Hole: n/a

OA: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2-1}{x^2+x-6}$

Solution

$x^2+x-6=0 \quad \Rightarrow \quad x=-3,\ 2$

VA: $x=-3$ *and* $x=2$

HA: $y=1$

$$1=\frac{x^2-1}{x^2+x-6} \Rightarrow x^2+x-6=x^2-1$$

$$\boxed{x=5}$$

Hole: n/a

OA: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{-2x^2-x+15}{x^2-x-12}$

Solution

$x^2-x-12=0 \quad \Rightarrow \quad x=-3,\ 4$

Domain: $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

$$f(x)=\frac{(-2x+5)(x+3)}{(x-4)(x+3)}=\frac{-2x+5}{x-4}$$

VA: $x=4$

HA: $y=-2$

Hole: $x=-3 \quad \rightarrow \quad y=-\frac{11}{7}$

hole $\left(-3,\ -\frac{11}{7}\right)$

OA: n/a

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \text{vertical asymptote}: \ x=4 \\ \text{horizontal asymptote}: \ y=-1 \\ x-\text{intercept}: \ 3 \end{cases}$$

Solution

Vertical Asymptote: $f(x) = \dfrac{}{x-4}$

Horizontal Asymptote: $f(x) = \dfrac{-x+a}{x-4}$

x-intercept: $f(x=3) = \dfrac{-3+a}{3-4}$

$$0 = -3 + a$$

$$a = 3$$

$$f(x) = \frac{-x+3}{x-4}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \text{vertical asymptote}: \ x = -3, x = 1 \\ \text{horizontal asymptote}: \ y = 0 \\ x - \text{intercept}: \ -1, \ f(0) = -2 \\ \text{hole at} \ \ x = 2 \end{cases}$$

Solution

Vertical Asymptote: $f(x) = \dfrac{}{(x+3)(x-1)}$

Horizontal Asymptote: $f(x) = \dfrac{ax+b}{(x+3)(x-1)}$

x-intercept: $f(x=-1) = \dfrac{a(-1)+b}{(-1+3)(-1-1)} = \dfrac{-a+b}{-4} = 0$

$$-a + b = 0 \ \Rightarrow \ a = b$$

$$f(x=0) = \frac{a(0)+b}{(0+3)(0-1)} = \frac{b}{-3} = -2$$

$$\boxed{b = 6 = a}$$

$$f(x) = \frac{6x+6}{(x+3)(x-1)}$$

Hole at $x = 2$: $f(x) = \dfrac{6x+6}{(x+3)(x-1)} \dfrac{x-2}{x-2}$

$$= \frac{6(x+1)(x-2)}{(x^2+2x-3)(x-2)}$$

$$= \frac{6(x^2-x-2)}{x^3-7x+6}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} vertical\ asymptote:\ x=-4, x=5 \\ horizontal\ asymptote:\ y=\frac{3}{2} \\ x-intercept:\ -2 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\frac{}{(x+4)(x-5)}$

Horizontal Asymptote: $f(x)=\frac{3}{2}\frac{(x+a)(x+b)}{(x+4)(x-5)}$

x-intercept: $f(x=-2)=\frac{3}{2}\frac{(-2+a)(-2+b)}{}$

$$0=(-2+a)(-2+b)$$

$$\boxed{a=b=2}$$

$$f(x)=\frac{3}{2}\frac{(x-2)^2}{x^2-x-20}$$

$$=\frac{3x^2-12x+12}{2x^2-2x-40}$$

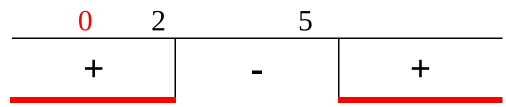
Solution — *Section* 2.8 – Polynomial and Rational Inequalities

Exercise

Solve: $x^2 - 7x + 10 > 0$

Solution

$$x^2 - 7x + 10 > 0$$
$$(x-5)(x-2) > 0$$
$$x = 2,\ 5$$



Solution: $x < 2$ *and* $x > 5$

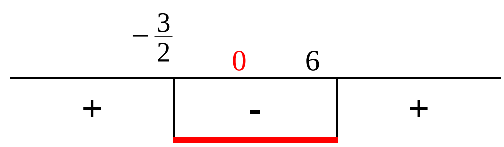
$$(-\infty, 2) \cup (5, \infty)$$

Exercise

Solve: $2x^2 - 9x \le 18$

Solution

$$2x^2 - 9x - 18 \le 0$$
$$(2x+3)(x-6) \le 0$$
$$2x + 3 = 0 \qquad x - 6 = 0$$
$$x = -\frac{3}{2} \qquad x = 6$$



$$\left[-\frac{3}{2},\ 6\right]$$

Solution: $\left[-\frac{3}{2},\ 6\right]$

Exercise

Solve the inequality: $x^2 - 5x + 4 > 0$

Solution

$$x^2 - 5x + 4 > 0$$
$$x = 1, 4$$

Solution: $x < 1$; $x > 4$

$$(-\infty, 1) \cup (4, \infty)$$

Exercise

Solve $x^2 + x - 2 > 0$

Solution

$$x^2 + x - 2 = 0 \quad \to x = -2, 1$$

0

-2 1

\+ - +

Solution: $(-\infty, -2) \cup (1, \infty)$

Exercise

Solve $x^2 - 4x + 12 < 0$

Solution

$$x^2 - 4x + 12 = 0$$

$$\to x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 48}}{2} = \frac{4 \pm \sqrt{-32}}{2} \quad \textit{Complex number}$$

No solution

Exercise

Solve: $x^3 - 3x^2 - 9x + 27 < 0$

Solution

$$x^3 - 3x^2 - 9x + 27 = 0$$

$$x^2(x-3) - 9(x-3) = 0$$

$$(x-3)(x^2 - 9) = 0 \to \begin{cases} x - 3 = 0 \to \underline{x = 3} \\ x^2 - 9 = 0 \to x^2 = 9 \to \underline{x = \pm 3} \end{cases}$$

$-\infty$ -3 3 ∞

\- + +

Solution: : $(-\infty, -3)$

Exercise

Solve $x^3 - x > 0$

Solution

$$x(x^2 - 1) = 0 \to \begin{cases} x = 0 \\ x^2 - 1 = 0 \to x^2 = 1 \to x = \pm 1 \end{cases}$$

-1 0 1 2

\- | + | - | +

$(-1, 0) \cup (1, \infty)$

Exercise

$x^3 + 3x^2 \le x + 3$

Solution

$$\Rightarrow x^3 + 3x^2 - x - 3 = 0$$

$$\Rightarrow x^2(x+3) - (x+3) = 0$$

$$\Rightarrow (x+3)(x^2 - 1) = 0$$

$$\begin{cases} x + 3 = 0 \rightarrow x = -3 \\ x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1 \end{cases}$$

-3	-1	0	1	
-	+	-		+

Solution: $(-\infty, -3] \cup [-1, 1]$

Exercise

Solve $x^3 + x^2 \ge 48x$

Solution

$$x^3 + x^2 - 48x = 0$$

$$x\left(x^2 + x - 48\right) = 0$$

$$x = 0 \qquad x^2 + x - 48 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(48)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{193}}{2}$$

$\frac{-1-\sqrt{193}}{2}$		0	**1**	$\frac{-1+\sqrt{193}}{2}$
$-$	$+$	$-$		$+$

Since the symbol is $\ge$ which means the positive sign:

The solution: $\left[\frac{-1-\sqrt{193}}{2},\ 0\right] \cup \left[\frac{-1+\sqrt{193}}{2},\ \infty\right)$

Exercise

Solve: $\dfrac{x}{x-3} > 0$

Solution

$$\frac{x}{x-3} = 0$$

	0		3	
+		–		+

$$x = 0, \quad x \neq 3$$

$$\Rightarrow \boxed{(-\infty,0)\cup(3,\infty)}$$

Exercise

Solve: $\dfrac{x-2}{x+2} \leq 2$

Solution

$$\frac{x-2}{x+2} \leq 2 \quad Cond.\ x \neq -2$$

$$\frac{x-2}{x+2} - 2 \leq 0$$

$$\frac{x-2}{x+2} - 2 = 0$$

	–6		–2	
–		+		–

$$x - 2 - 2(x+2) = 0$$

$$x - 2 - 2x - 4 = 0$$

$$-x - 6 = 0$$

$$x = -6 \qquad \boxed{(-\infty,-6]\cup(-2,\infty)}$$

Exercise

Solve $\dfrac{x+2}{3+2x} \leq 5$

Solution

$$\frac{x+2}{3+2x} - 5 \leq 0$$

$$\frac{x+2}{3+2x} - 5\frac{3+2x}{3+2x} \leq 0$$

$$\frac{x+2-5(3+2x)}{3+2x} \leq 0$$

$$\frac{x+2-15-10x}{3+2x} \leq 0$$

$$\frac{-9x-13}{3+2x} \leq 0$$

$$-9x - 13 = 0 \qquad 3 + 2x = 0$$
$$-9x = 13 \qquad 2x = -3$$
$$x = -\frac{13}{9} \qquad x = -\frac{3}{2}$$

$-\frac{3}{2}$	$-\frac{13}{9}$	0
-	+	-

Solution: $\left(-\infty,\ -\frac{3}{2}\right] \cup \left[-\frac{13}{9},\ \infty\right)$

Exercise

Solve: $\frac{x-3}{x+4} \ge \frac{x+2}{x-5}$

Solution

Conditions: $x + 4 \ne 0 \rightarrow x \ne -4$ *and* $x - 5 \ne 0 \rightarrow x \ne 5$

$$\frac{x-3}{x+4} - \frac{x+2}{x-5} = 0$$

$$\frac{0-3}{0+4} - \frac{0+2}{0-5} = \frac{-3}{4} - \frac{2}{-5} = \frac{-3}{4} + \frac{2}{5} = -$$

$$(x+4)(x-5)\left[\frac{x-3}{x+4} - \frac{x+2}{x-5}\right] = 0$$

$$(x-5)(x-3) - (x+4)(x+2) = 0$$

$$x^2 - 3x - 5x + 15 - (x^2 + 2x + 4x + 8) = 0$$

$$x^2 - 3x - 5x + 15 - x^2 - 2x - 4x - 8 = 0$$

$$-14x + 7 = 0$$

$$-14x = -7$$

$$x = \frac{-7}{-14} = \frac{1}{2}$$

-4	0	1/2	5
+	-	+	-

Solution $(-\infty, -4) \cup \left[\frac{1}{2}, 5\right)$

Exercise

Solve: $\frac{x-4}{x+3} - \frac{x+2}{x-1} \le 0$

Solution

Conditions: $x \ne -3$ and $x \ne 1$

$$\frac{x-4}{x+3} - \frac{x+2}{x-1} = 0$$

$$\frac{0-4}{0+3} - \frac{0+2}{0-1} = -\frac{4}{3} + 2 > 0$$

$$(x+3)(x-1)\left[\frac{x-4}{x+3} - \frac{x+2}{x-1}\right] = 0$$

$$(x-1)(x-4) - (x+3)(x+2) = 0$$

$$x^2-5x+4-(x^2+5x+6)=0$$

$$x^2-5x+4-x^2-5x-6=0$$

$$-10x-2=0 \to x=-\frac{1}{5}$$

	-3		$-1/5$		1	
$+$		$-$		$+$		$-$

0

Solution: $\left(-3,-\frac{1}{5}\right] \cup (1,\infty)$

Exercise

Solve: $\frac{2x-1}{x+3} \ge \frac{x+1}{3x+1}$

Solution

Conditions: $x \ne -3$ and $x \ne -\frac{1}{3}$

$$\frac{2x-1}{x+3}-\frac{x+1}{3x+1} \ge 0$$

$$(x+3)(3x+1)\frac{2x-1}{x+3}-(x+3)(3x+1)\frac{x+1}{3x+1}=0$$

$$(3x+1)(2x-1)-(x+3)(x+1)=0$$

$$6x^2-3x+2x-1-\left(x^2+x+3x+3\right)=0$$

$$6x^2-x-1-x^2-4x-3=0$$

$$5x^2-5x-4=0$$

$$x=\frac{5\pm\sqrt{105}}{10} \to -.5 \quad 1.5$$

	-3		$\frac{5-\sqrt{105}}{10}$		$-\frac{1}{3}$	**0**	$\frac{5+\sqrt{105}}{10}$	
+		-		+		-		+

$$(-\infty,-3) \cup \left[\frac{5-\sqrt{105}}{10},-\frac{1}{3}\right) \cup \left[\frac{5+\sqrt{105}}{10},\infty\right)$$

Exercise

Solve the inequality $\frac{x+6}{x-14} \ge 1$

Solution

Restriction: $x-14 \ne 0 \Rightarrow \boxed{x \ne 14}$

$$\frac{x+6}{x-14}-1=0$$

$$(x-14)\frac{x+6}{x-14}-1(x-14)=0(x-14)$$

$$x+6-x+14=0$$

$20=0$ $(Implossible)$ No Solution

$$\frac{0+6}{0-14}-1=\frac{6}{-14}-1=-\frac{3}{7}-1=-\frac{10}{7}$$

0	14	
—		+

Solution: $\boxed{(14,\ \infty)}$

Exercise

A car can be rented from Basic Rental for \$260 per week with no extra charge for mileage. Continental charges \$80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

Solution

x: number of miles driven

For Continental, cost: 80 + .25x

Basic Rental a better deal than Continental's

$260 < 80 + 0.25\,x$

$260 - 80 < 0.25\,x$

$180 < .25\,x$

$720 < x$

Solution: more than 720 miles per week.

Exercise

If a projectile is launched from ground level with an initial velocity of 96 *ft* per *sec*, its height in feet t seconds after launching is s feet, where

$$s=-16t^2+96t$$

When will the projectile be greater than 80 ft above the ground?

Solution

Projectile be greater than 80 ft above the ground

$$s>80$$

$$-16t^2+96t>80$$

$$-16t^2+96t-80>0$$

$$\frac{-16}{-16}t^2+\frac{96}{-16}t-\frac{80}{-16}<0$$

$$t^2 - 6t + 5 < 0$$

$$t^2 - 6t + 5 = 0$$

$$(t-1)(t-5) = 0$$

$$t = 1,\ 5$$

0	1		5	
	+	-	+	

Solution $(1,\ 5)$

Exercise

A projectile is fired straight up from ground level. After t seconds, its height above the ground is s *ft*, where

$$s = -16t^2 + 220t$$

For what time period is the projectile at least 624 ft above the ground?

Solution

Projectile at least 624 ft.

$$s \ge 624$$

$$-16t^2 + 220t \ge 624$$

$$-16t^2 + 220t - 624 \ge 0 \qquad \textit{Divide by } -4$$

$$4t^2 - 55t + 156 \le 0$$

$$t = \frac{-(-55) \pm \sqrt{(-55)^2 - 4(4)(156)}}{2(4)} = \frac{55 \pm 23}{16}$$

$$t = \frac{55+23}{16} \qquad t = \frac{55-23}{16}$$

$$= \frac{78}{16} \qquad = \frac{32}{16}$$

$$= \frac{39}{8} \qquad = 2$$

Solution: $\left[2,\ \frac{39}{8}\right]$