

## Solution

### Section 4.5 – Comparing Three or More Means

#### Exercise

Fill in the ANOVA table

<i>Source of Variation</i>	<i>Sum of Squares</i>	<i>Degrees of Freedom</i>	<i>Mean Squares</i>	<i>F-Test Statistic</i>
Treatment	565	5		
Error	3560	32		
Total				

#### Solution

$$MST = \frac{SST}{k-1} = \frac{565}{5} = 113$$

$$MSE = \frac{SSE}{n-k} = \frac{3560}{32} = 111.25$$

$$F_0 = \frac{MST}{MSE} = \frac{113}{111.25} = 1.016$$

<i>Source of Variation</i>	<i>Sum of Squares</i>	<i>Degrees of Freedom</i>	<i>Mean Squares</i>	<i>F-Test Statistic</i>
Treatment	565	5	113	1.016
Error	3560	32	111.25	
Total	4125	37		

#### Exercise

Fill in the ANOVA table

<i>Source of Variation</i>	<i>Sum of Squares</i>	<i>Degrees of Freedom</i>	<i>Mean Squares</i>	<i>F-Test Statistic</i>
Treatment	490	4		
Error	7267	21		
Total				

#### Solution

$$MST = \frac{SST}{k-1} = \frac{490}{4} = 122.5$$

$$MSE = \frac{SSE}{n-k} = \frac{7267}{21} = 346.048$$

$$F_0 = \frac{MST}{MSE} = \frac{122.5}{346.048} = 0.354$$

<i>Source of Variation</i>	<i>Sum of Squares</i>	<i>Degrees of Freedom</i>	<i>Mean Squares</i>	<i>F-Test Statistic</i>
Treatment	490	4	122.5	0.354
Error	7267	21	346.048	
Total	7757	25		

### Exercise

Determine the  $F$ -test statistic based on the given summary statistics

Population	Sample Size	Sample Mean	Sample Variance
1	10	42	35
2	10	41	40
3	10	22	23

Compute  $\bar{x}$ , the sample mean of the combined data set, by adding up all the observations and dividing by the number of the observations.

### Solution

$$\bar{x} = \frac{\sum n_i x_i}{\sum n_i} = \frac{10(42) + 10(41) + 10(22)}{10 + 10 + 10} = \underline{35}$$

$$MST = \frac{\sum n_i (x_i - \bar{x})^2}{k - 1} = \frac{10(42 - 35)^2 + 10(41 - 35)^2 + 10(22 - 35)^2}{3 - 1} = \underline{1270}$$

$$MSE = \frac{\sum (n_i - 1) s_i^2}{n - k} = \frac{(10 - 1)(35) + (10 - 1)(40) + (10 - 1)(23)}{30 - 3} = \underline{32.667}$$

$$F = \frac{MST}{MSE} = \frac{1270}{32.667} = \underline{38.88}$$

Therefore, the value of the  $F$ -test statistic is 38.8

### Exercise

An engineer wants to know if the mean strengths of three concrete mix designs differ significantly. He randomly selects 9 cylinders that measure 6 inches in diameter and 12 inches in heights in which mixture A is poured, 9 cylinders of mixture B, and 9 cylinders of mixture C. After 28 days, he measures the strength (in pounds per square inch) of the cylinders. The results are presented in the table below.

Mixture A	Mixture B	Mixture C
3,980	4,070	4,130
4,040	4,340	3,820
3,760	4,620	4,020
3,870	3,730	4,150
3,990	4,870	4,190
4,090	4,120	3,840
3,820	4,640	3,750
3,940	4,180	3,990
4,080	3,850	4,320

- Write the null and alternative hypotheses
- Explain why the one-way ANOVA cannot be used to test these hypotheses

### Solution

a)  $H_0 : \mu_A = \mu_B = \mu_C$

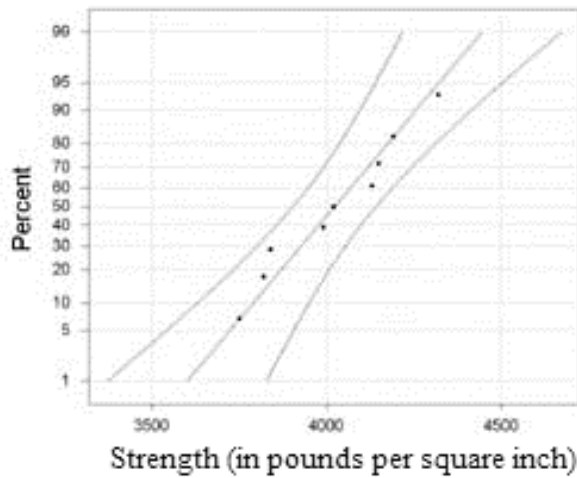
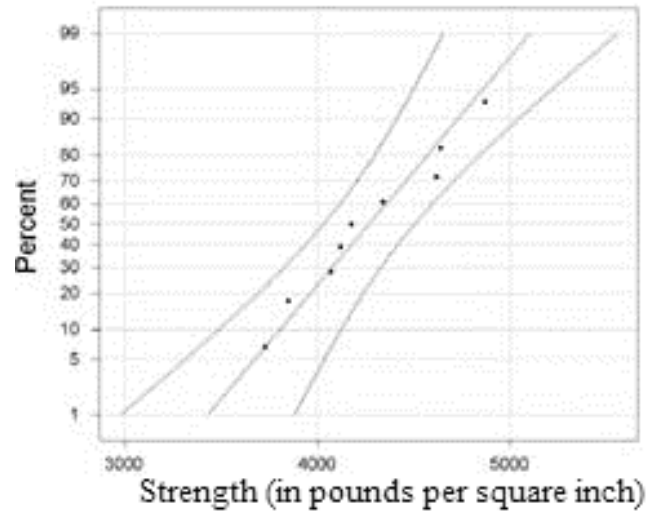
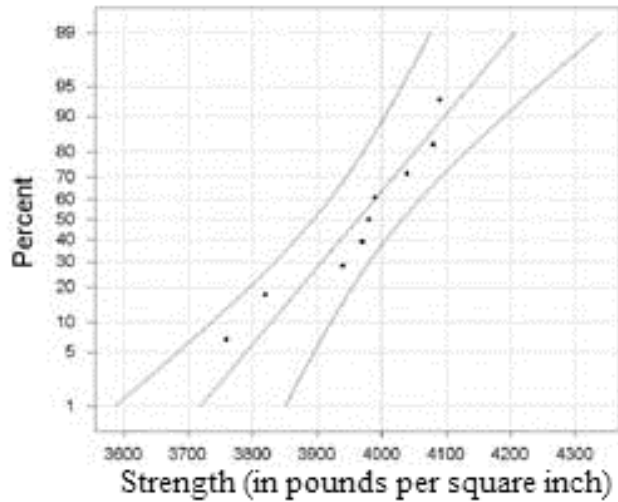
$H_1$  : at least one of the means is different

b) The standard deviation for mixture A is 115.41

The standard deviation for mixture B is 381.26

The standard deviation for mixture C is 191.71

The standard deviation for mixture B is more than 2 times larger than the standard deviation for mixture A.



### Exercise

At a community college, the mathematics department has been experimenting with four different delivery mechanisms for content in their Elementary Statistics courses. One method is the traditional lecture (method I), the second is a hybrid format in which half the class time is online and the other half is face-to-face (method II), the third is online (method III), and the fourth is an emporium model from which students obtain their lectures and do their work in a lab with an instructor available for assistance (method IV). To assess the effectiveness of the four methods, students in each approach are given a final exam with the results shown in the accompanying table. Do the data suggest that any method has a different mean score from the others?

Method I	76	81	85	68	88	73	80	65	60	92	83	51	71	63	71	65
Method II	88	52	77	73	64	38	57	63	83	65	78	64	87	92		
Method III	78	60	73	70	62	82	74	80	53	46	84	80	78			
Method IV	89	90	79	62	83	75	54	70	80	94	76	78	81			

- Write the null and alternative hypotheses
- State the requirements that must be satisfied to use one-way ANOVA procedure
- Assuming the requirements stated in part (b) are satisfied, use the following one-way ANOVA table to test the hypothesis of equal means at the  $\alpha = 0.05$  level of significance.
- Interpret the  $P$ -value.
- Verify that the residuals are normally distributed

### Solution

- $H_0 : \mu_I = \mu_{II} = \mu_{III} = \mu_{IV}$   
 $H_1$  : at least one of the means is different
- There must be  $k$  simple samples, one from each of  $k$  populations.  
The populations must be normally distributed.  
The populations must have the same variance
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Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Test Statistic	P-value
Treatment	3	481.24	160.41	1.03	0.387
Error	52	8089.54	155.74		
Total	55	8579.78			

Since the  $P$ -value is greater than  $\alpha$ , do not reject  $H_0$ , there is insufficient evidence to support  $H_1$

- Since the  $P$ -value is greater than or equal to  $\alpha$ , conclude that there is insufficient evidence that any one method is more or less effective than the others.
- $\mu_I = 73.250$ ;  $\mu_{II} = 70.071$ ;  $\mu_{III} = 70.769$ ;  $\mu_{IV} = 77.769$

