Lect 2.

2.1 $m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

$f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{x - x - h}{x(x+h)}\right)$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left(\frac{-\cancel{h}}{x(x+h)}\right)$$

$$= -\lim_{h \to 0} \frac{1}{x^2 + hx}$$

$$= \boxed{-\frac{1}{x^2}}$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$f(x)$ derivative: $f'(x)$, f' $\frac{d f(x)}{dx}$

$\frac{dy}{dx}$, y' $\underbrace{D_x(y)}$

$f(x) = \frac{x}{x-1}$

determinant 2x2

$$f'(x) = \lim_{h\to 0} \frac{1}{h}\left(\frac{x+h}{x+h-1} - \frac{x}{x-1}\right)$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{x^2 - x + hx - h - x^2 - hx + x}{(x-1)(x+h-1)}$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}} \frac{-\cancel{h}}{(x-1)(x+h-1)}$$

$$= \lim_{h\to 0} \frac{-1}{(x-1)(x+h-1)}$$

$$= \frac{-1}{(x-1)(x-1)}$$

$$= \frac{-1}{(x-1)^2}$$

1. def of limit. $\lim_{x \to x_0} f(x) = L$

$\forall \varepsilon > 0, \exists \delta > 0 \ni \forall x$

$|x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$f(x) = \sqrt{x-7}$ $\quad L = 4 \quad x_0 = 23 \quad \varepsilon = 1$

$-\delta < x - 23 < \delta$

$23 - \delta < x < 23 + \delta$

$-1 < \sqrt{x-7} - 4 < 1$

$3 < \sqrt{x-7} < 5$

$9 < x - 7 < 25$

$16 < x < 32$

$23 - \delta = 16 \rightarrow \delta = 7$

$23 + \delta = 32 \rightarrow \delta = 9$

$\therefore \delta = 7$

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4. $\lim_{x \to 0} \frac{\sin 5x}{3x} = \boxed{\frac{5}{3}}$

5. $\lim_{\theta \to 0} \frac{\theta \cot 4\theta}{\sin^2\theta \cot^2\theta} = \frac{0}{0}$

$\cos 0$

$= \lim_{\theta \to 0} \frac{\theta}{\sin\theta} \frac{1}{\sin\theta} \frac{\cos 4\theta}{\sin 4\theta} \frac{1}{\frac{\cos^2\theta}{\sin^2\theta}}$

$= \lim_{\theta \to 0} \frac{\theta}{\sin 4\theta} = \frac{0}{0}$

$= \lim_{\theta \to 0} \frac{\theta}{2\sin 2\theta \cos 2\theta}$

$= \frac{1}{2} \lim_{\theta \to 0} \frac{1}{\cos 2\theta} \lim_{2\theta \to 0} \frac{1}{2 \frac{\sin 2\theta}{2\theta}}$

$= \frac{1}{\;}$

6 - $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4} = \frac{a^2 - a^2}{a^4 - a^4} = \frac{0}{0}$

$= \lim_{x \to a} \frac{x^2 - a^2}{(x^2 - a^2)(x^2 + a^2)}$

$= \lim_{x \to a} \frac{1}{x^2 + a^2}$

$= \frac{1}{2a^2}$

11) $\lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = \frac{\frac{1}{2} - \frac{1}{2}}{0} = \frac{0}{0}$

$= \lim_{x \to 0} \frac{1}{x} \left(\frac{2 - 2 - x}{2(x+2)} \right)$

$= \lim_{x \to 0} \frac{-x}{2x(x+2)}$

$= -\frac{1}{2} \lim_{x \to 0} \frac{1}{x+2}$

$= -\frac{1}{4}$

#12 $\lim_{x \to -1} \frac{\sqrt{x^2+8} - 3}{x+1} = \frac{3-3}{0} = \frac{0}{0}$

$= \lim_{x \to -1} \frac{\sqrt{x^2+8} - 3}{x+1} \cdot \frac{\sqrt{x^2+8} + 3}{\sqrt{x^2+8} + 3}$

$= \lim_{x \to -1} \frac{x^2 + 8 - 9}{(x+1)(\sqrt{x^2+8} + 3)}$

$= \lim_{x \to -1} \frac{(x+1)(x-1)}{(x+1)(\sqrt{x^2+8} + 3)}$

$= \lim_{x \to -1} \frac{x-1}{\sqrt{x^2+8} + 3}$

$= \frac{-2}{6}$

$= -\frac{1}{3}$

15. $\lim_{x\to\infty} \frac{x+\cancel{\sin x}+2\sqrt{x}}{x+\sin x} = \lim_{x\to\infty} \frac{x}{x}$

$= \underline{1}$

#16 $\lim_{x\to-\infty} \left(\frac{x^2+x-1}{8x^2-3}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3}$

$= \frac{1}{2}$

$8^{1/3} = 2^{3 \cdot 1/3}$

#17 $\lim_{x\to-\infty} \frac{4-3x^3}{\sqrt{x^6+9}} = \lim_{x\to-\infty} \frac{-3x^3}{x^3}$

$= \underline{-3}$

$f = |x-1| + \sin x$

fctn is continuous everywhere

$f = \frac{x}{x^2-3x+2}$ $(x \neq 1, 2)$

fctn is continuous everywhere except when $x = 1, 2$.

#18 $\lim_{x\to-\infty} \frac{x^2-4x+8}{3x^3} = \underline{0}$

#19 $\lim_{x\to\infty} \frac{2x^2+3}{5x^2+7} = \underline{\frac{2}{5}}$

20. $\lim_{x\to\infty} \frac{x^4+x^3}{12x^3+128} = \underline{\infty}$

1.2
20/ $\lim_{x \to -2} (x^3 - 2x^2 + 4x + 8) = -8 - 8 - 8 + 8$

$= \underline{-16}$

$\frac{0}{0}$ } factor, conjugate $\frac{+}{+}$

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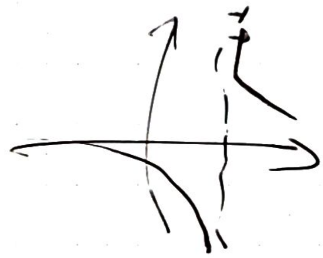
$\lim_{x \to +1} \frac{x-1}{\sqrt{x+3} + 2} = \frac{0}{4} = 0$

$\frac{\neq 0}{0} = \infty$

sign $\infty \to a^+$ or a^-

1.3 17

$\lim_{x \to 3^-} \frac{2}{(x-3)^3} = \frac{2}{0^-} = -\infty$



$\lim_{x \to 3^+} \frac{2}{(x-3)^3} = \frac{2}{0^+} = \infty$