Solution Section 2.1 – Sequences and Summations

Exercise

Find these terms of the sequence $\{a_n\}$, where $a_n = 2\cdot(-3)^n + 5^n$

a) a_0 b) a_1 c) a_4 d) a_5

Solution

a) $a_0 = 2\cdot(-3)^0 + 5^0$

$\underline{=3}$

b) $a_1 = 2\cdot(-3)^1 + 5^1$

$= -6 + 5$

$\underline{= -1}$

c) $a_4 = 2\cdot(-3)^4 + 5^4$

$= 162 + 625$

$\underline{= 787}$

d) $a_4 = 2\cdot(-3)^5 + 5^5$

$= -486 + 3125$

$\underline{= 2639}$

Exercise

What is the term a_8 of the sequence $\{a_n\}$, if a_n equals

a) 2^{n-1} b) 7 c) $1+(-1)^n$ d) $-(2)^n$

Solution

a) $a_8 = 2^{8-1}$

$\underline{= 128}$

b) $\underline{a_8 = 7}$

c) $a_8 = 1 + (-1)^8$

$\underline{= 2}$

d) $a_8 = -(2)^8$

$\underline{= -256}$

Exercise

What are the terms a_0, a_1, a_2, *and* a_3 of the sequence $\{a_n\}$, if a_n equals

$a)\ 2^n+1 \quad b)\ (n+1)^{n+1} \quad c)\ \frac{n}{2} \quad d)\ \frac{n}{2}+\frac{n}{2}$

$e)\ (-2)^n \quad f)\ 3 \quad g)\ 7+4^n \quad h)\ 2^n+(-2)^n$

Solution

a) $a_0=2^0+1\ \underline{=2}$

$a_1=2^1+1\ \underline{=3}$

$a_2=2^2+1\ \underline{=5}$

$a_3=2^3+1\ \underline{=9}$

b) $a_0=(0+1)^{0+1}\ \underline{=1}$

$a_1=(1+1)^{1+1}\ \underline{=4}$

$a_2=(2+1)^{2+1}\ \underline{=27}$

$a_3=(3+1)^{3+1}\ \underline{=256}$

c) $a_0=\frac{0}{2}\ \underline{=0}$

$\underline{a_1=\frac{1}{2}}$

$a_2=\frac{2}{2}\ \underline{=1}$

$\underline{a_3=\frac{3}{2}}$

d) $a_0=\frac{0}{2}+\frac{0}{2}\ \underline{=0}$

$a_1=\frac{1}{2}+\frac{1}{2}\ \underline{=1}$

$a_2=\frac{2}{2}+\frac{2}{2}\ \underline{=1}$

$a_3=\frac{3}{2}+\frac{3}{2}\ \underline{=3}$

e) $a_0=(-2)^0\ \underline{=1}$

$a_1=(-2)^1\ \underline{=-2}$

$a_2=(-2)^2\ \underline{=4}$

$a_3=(-2)^3\ \underline{=-8}$

f) $\underline{a_0 = 3}|$

$\underline{a_1 = 3}|$

$\underline{a_2 = 3}|$

$\underline{a_3 = 3}|$

g) $a_0 = 7 + 4^0 \ \underline{= 8}|$

$a_1 = 7 + 4^1 \ \underline{= 11}|$

$a_2 = 7 + 4^2 \ \underline{= 23}|$

$a_3 = 7 + 4^3 \ \underline{= 71}|$

h) $a_0 = 2^0 + (-2)^0 \ \underline{= 2}|$

$a_1 = 2^1 + (-2)^1 \ \underline{= 0}|$

$a_2 = 2^2 + (-2)^2 \ \underline{= 0}|$

$a_3 = 2^3 + (-2)^3 \ \underline{= 0}|$

Exercise

Find at least three different sequences beginning with the terms 1, 2, 4 whose terms are generated by a simple formula or rule.

Solution

1. $2^{n-1} \quad \rightarrow \quad 1,\ 2,\ 4,\ 8,\ 16,\ \ldots$

2. The second pattern, $2-1=1 \quad 4-2=2$, as we see the difference to the previous increasing by value of 1.

So, the next term $4+3=7 \quad 7+4=11$.

Therefore; the sequence is 1, 2, 4, 7, 11, 16, …

3. 1, 2, 4, 1, 2, 4, … Repeating the terms

Exercise

Find at least three different sequences beginning with the terms 3, 5, 7 whose terms are generated by a simple formula or rule.

Solution

One rule should be that each term is greater than the previous term by 2; the sequence would be 3, 5, 7, 9, 11, 13, . . .

Another rule could be that the n^{th} old prime.

The sequence would be 3, 5, 7, 11, 13, 17, …

The sequence: 3, 5, 7, 12, 23, 43, 75, 122, 187, 273 from an equation $\frac{1}{2}\left(x^3-6x^2+15x-4\right)$

Exercise

Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = 6a_{n-1}, \quad a_0 = 2$

b) $a_n = a_{n-1}^2, \quad a_1 = 2$

c) $a_n = a_{n-1} + 3a_{n-2}, \quad a_0 = 1,\ a_1 = 2$

d) $a_n = na_{n-1} + n^2 a_{n-2}, \quad a_0 = 1,\ a_1 = 1$

e) $a_n = a_{n-1} + a_{n-3}, \quad a_0 = 1,\ a_1 = 2,\ a_2 = 0$

Solution

a) $a_n = 6a_{n-1}, \quad a_0 = 2$

$a_1 = 6a_0 = 6(2) \ \underline{= 12}$

$a_2 = 6a_1 = 6(12) \ \underline{= 72}$

$a_3 = 6a_2 = 6(72) \ \underline{= 432}$

$a_4 = 6a_3 = 6(432) \ \underline{= 2592}$

b) $a_n = a_{n-1}^2, \quad a_1 = 2$

$a_2 = a_1^2 = 2^2 \ \underline{= 4}$

$a_3 = a_2^2 = 4^2 \ \underline{= 16}$

$a_4 = a_3^2 = 16^2 \ \underline{= 256}$

$a_5 = a_4^2 = 256^2 \ \underline{= 65536}$

c) $a_n = a_{n-1} + 3a_{n-2}, \quad a_0 = 1,\ a_1 = 2$

$a_2 = a_1 + 3a_0 = 2 + 3(1) \ \underline{= 5}$

$a_3 = a_2 + 3a_1 = 5 + 3(2) \ \underline{= 11}$

$a_4 = a_3 + 3a_2 = 11 + 3(5) \ \underline{= 26}$

$a_5 = a_4 + 3a_3 = 26 + 3(11) \ \underline{= 59}$

d) $a_n = na_{n-1} + n^2 a_{n-2}, \quad a_0 = 1,\ a_1 = 1$

$a_2 = 2a_1 + 2^2 a_0 = 2(1) + 4(1) \ \underline{= 6}$

$a_3 = 3a_2 + 3^2 a_1 = 3(6) + 9(1) \ \underline{= 27}$

$a_4 = 4a_3 + 4^2 a_2 = 4(27) + 16(6) \ \underline{= 204}$

$a_5 = 5a_4 + 5^2 a_3 = 5(204) + 25(27) \ \underline{= 1695}$

e) $a_n = a_{n-1} + a_{n-3}, \quad a_0 = 1,\ a_1 = 2,\ a_2 = 0$

$a_3 = a_2 + a_0 = 0 + 1 \ \underline{= 1}$

$a_4 = a_3 + a_1 = 1 + 2 \ \underline{= 3}$

$a_5 = a_4 + a_2 = 3 + 0 \ \underline{= 3}$

$a_6 = a_5 + a_3 = 3 + 3 \ \underline{= 6}$

Exercise

Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = -2a_{n-1}, \quad a_0 = -1$

b) $a_n = a_{n-1} - a_{n-2}, \quad a_0 = 2, \ a_1 = -1$

c) $a_n = 3a_{n-1}^2, \quad a_0 = 1$

d) $a_n = na_{n-1} + n^2 a_{n-2}, \quad a_0 = -1,\ a_1 = 0$

e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}, \quad a_0 = 1,\ a_1 = 2,\ a_2 = 2$

Solution

a) $a_0 = -1$

$a_1 = -2a_0 \ \underline{= 2}$

$a_2 = -2a_1 \ \underline{= -4}$

$a_3 = -2a_2 \ \underline{= 8}$

$a_4 = -2a_3 \ \underline{= -16}$

$a_5 = -2a_4 \ \underline{= 32}$

b) $a_0 = 2, \quad a_1 = -1$

$a_2 = a_1 - a_0 \ \underline{= -3}$

$a_3 = a_2 - a_1 \ \underline{= -2}$

$a_4 = a_3 - a_2 \ \underline{= 1}$

$a_5 = a_4 - a_3 \ \underline{= 3}$

c) $a_0 = 1$

$a_1 = 3a_0^2 \ \underline{= 3}$

$a_2 = 3a_1^2 = 27 \ \underline{= 3^3}$

$a_3 = 3a_2^2 = 2187 \ \underline{= 3^7}$

$a_4 = 3a_3^2 = 14348907 \ \underline{= 3^{15}}$

$a_5 = 3a_4^2 \ \underline{= 3^{31}}$

d) $a_0 = -1, \quad a_1 = 0$

$a_2 = 2a_1 + a_0^2 \ \underline{= 1}$

$a_3 = 3a_2 + a_1^2 \ \underline{= 3}$

$a_4 = 4a_3 + a_2^2 \ \underline{= 13}$

$a_5 = 5a_4 + a_3^2 \ \underline{= 74}$

e) $a_0 = 1, \quad a_1 = 1, \quad a_2 = 2$

$a_3 = a_2 - a_1 + a_0 \ \underline{= 2}$

$a_4 = a_3 - a_2 + a_1 \ \underline{= 1}$

$a_5 = a_4 - a_3 + a_2 \ \underline{= 1}$

Exercise

Let $a_n = 2^n + 5 \cdot 3^n \quad for \ \ n = 0, 1, 2, \ldots$

a) Find $a_0, a_1, a_2, a_3, and\ a_4$

b) Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$

c) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \ge 2$

Solution

a) $a_0 = 2^0 + 5 \cdot 3^0 = 1 + 5 \ \underline{= 6}$

$a_1 = 2^1 + 5 \cdot 3^1 = 2 + 15 \ \underline{= 17}$

$a_2 = 2^2 + 5\cdot 3^2 = 4 + 45 \underline{= 49}$

$a_3 = 2^3 + 5\cdot 3^3 = 8 + 5(27) \underline{= 143}$

$a_4 = 2^4 + 5\cdot 3^4 = 16 + 5(81) \underline{= 421}$

b)

$a_2 = 5a_1 - 6a_0$

$49 \overset{?}{=} 5(17) - 6(6)$

$49 = 49 \quad \checkmark$

Or

$$\begin{aligned}
5a_1 - 6a_0 &= 5\left(2^1 + 5\cdot 3^1\right) - 6\left(2^0 + 5\cdot 3^0\right) \\
&= 5\cdot 2 + 5\cdot 3 - 2\cdot 3 - 2\cdot 3\cdot 5 \\
&= (5\cdot 2 - 2\cdot 3) + 5\cdot 5\cdot 3 - 5\cdot 2\cdot 3 \\
&= 2(5-3) + 5\cdot 3(5-2) \\
&= 2\cdot 2 + 5\cdot 3\cdot 3 \\
&= 2^2 + 5\cdot 3^2 \quad \checkmark
\end{aligned}$$

$a_3 = 5a_2 - 6a_1$

$143 \overset{?}{=} 5(49) - 6(17)$

$143 = 143 \quad \checkmark$

Or

$$\begin{aligned}
5a_2 - 6a_1 &= 5\left(2^2 + 5\cdot 3^2\right) - 6\left(2^1 + 5\cdot 3^1\right) \\
&= 5\cdot 4 + 5\cdot 9 - 2\cdot 3\cdot 2 - 2\cdot 3\cdot 5\cdot 3 \\
&= (5\cdot 2 - 4\cdot 3) + 5\cdot 9 - 5\cdot 2\cdot 9 \\
&= 2(5-3) + 5\cdot 3(5-2) \\
&= 2\cdot 2 + 5\cdot 3\cdot 3 \\
&= 2^2 + 5\cdot 3^2 \quad \checkmark
\end{aligned}$$

$a_4 = 5a_3 - 6a_2$

$421 \overset{?}{=} 5(143) - 6(49)$

$421 = 421 \quad \checkmark$

Or

$$5a_3 - 6a_2 = 5\left(2^3 + 5\cdot 3^3\right) - 6\left(2^2 + 5\cdot 3^2\right)$$
$$= 5\cdot 2^3 + 5^2\cdot 3^3 - 3\cdot 2^3 - 2\cdot 3^3\cdot 5$$
$$= 5\cdot 2^3 - 3\cdot 2^3 + 5^2\cdot 3^3 - 2\cdot 3^3\cdot 5$$
$$= 2^3(5-3) + 5\cdot 3^3(5-2)$$
$$= 2^4 + 5\cdot 3^4 \quad \surd$$

Exercise

Is the sequence $\{a_n\}$ a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ if

a) $a_n = 0$?

b) $a_n = 1$?

c) $a_n = 2^n$?

d) $a_n = 4^n$?

e) $a_n = n4^n$?

f) $a_n = 2\cdot 4^n + 3n4^n$?

g) $a_n = (-4)^n$?

h) $a_n = n^2 4^n$?

Solution

a) Let $a_n = 8a_{n-1} - 16a_{n-2} = 0$ We get $0 = 0$ which is a true statement.

$\therefore\ a_n = 0$ is a solution of the recurrence relation.

b) Let $a_n = 8a_{n-1} - 16a_{n-2} = 1$

We get $1 = 8\cdot 1 - 16\cdot 1 = -8$ which is a false statement.

$\therefore\ a_n = 1$ is not a solution.

c) Let $a_n = 8a_{n-1} - 16a_{n-2} = 2^n$

We get $2^n = 8\cdot 2^{n-1} - 16\cdot 2^{n-2} = 2^{n-2}(8\cdot 2 - 16) = 0$

which is a false statement.

$\therefore\ a_n = 1$ is not a solution.

d) Let $a_n = 8a_{n-1} - 16a_{n-2} = 4^n$ We get

$$4^n = 8\cdot 4^{n-1} - 16\cdot 4^{n-2}$$

$= 4^{n-2}(8 \cdot 4 - 16)$

$= 4^{n-2} \cdot (16)$

$= 4^{n-2} \cdot 4^2$

$= 4^n$ which is a true statement

$\therefore\ a_n = 4^n$ is a solution of the recurrence relation.

e) Let $a_n = 8a_{n-1} - 16a_{n-2} = n4^n$ We get

$n4^n = 8 \cdot n4^{n-1} - 16 \cdot n4^{n-2}$

$= n4^{n-2}(8 \cdot 4 - 16)$

$= n4^{n-2} \cdot \left(4^2\right)$

$= n4^n$ which is a true statement

$\therefore\ a_n = n4^n$ is a solution of the recurrence relation.

f) Let $a_n = 8a_{n-1} - 16a_{n-2} = 2 \cdot 4^n + 3n4^n$ We get

$2 \cdot 4^n + 3n4^n = 8 \cdot \left(2 \cdot 4^{n-1} + 3(n-1)4^{n-1}\right) - 16 \cdot \left(2 \cdot 4^{n-2} + 3(n-2)4^{n-2}\right)$

$= 8 \cdot 4^{n-2}\left(2 \cdot 4 + 3 \cdot 4(n-1) - 2 \cdot (2 + 3(n-2))\right)$

$= 8 \cdot 4^{n-2}(8 + 12n - 12 - 4 - 6n + 12)$

$= 8 \cdot 4^{n-2}(4 + 6n)$

$= 4^2 4^{n-2}(2 + 3n)$

$= 2 \cdot 4^n + 3n \cdot 4^n$ which is a true statement

$\therefore\ a_n = 2 \cdot 4^n + 3n \cdot 4^n$ is a solution of the recurrence relation.

g) Let $a_n = 8a_{n-1} - 16a_{n-2} = (-4)^n$ We get

$(-4)^n = 8 \cdot (-4)^{n-1} - 16 \cdot (-4)^{n-2}$

$= (-4)^{n-2}(8 \cdot (-4) - 16)$

$= (-4)^{n-2}(-48)$

$= (-4)^{n-2}(-16 \cdot 3)$

$= (-4)^{n-2}(-4)^2 \cdot 3$

$= (-4)^n \cdot 3$ which is a false statement

$\therefore\ a_n = 4^n$ is not a solution.

h) Let $a_n = 8a_{n-1} - 16a_{n-2} = n^2 4^n$ We get

$$n^2 4^n = 8\cdot(n-1)^2 4^{n-1} - 16\cdot(n-2)^2 4^{n-2}$$

$$= 8\cdot 4^{n-2}\left(\left(n^2-2n+1\right)\cdot 4 - 2\cdot\left(n^2-4n+4\right)\right)$$

$$= 16\cdot 4^{n-2}\left(2n^2-4n+2-n^2+4n-4\right)$$

$$= 4^n\left(n^2-2\right)$$

$= n4^n$ which is a false statement

$\therefore\ a_n = n^2 4^n$ is a solution of the recurrence relation.

Exercise

Is the sequence $\{a_n\}$ a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ if

a) $a_n = -n + 2$

b) $a_n = 5(-1)^n - n + 2$

c) $a_n = 3(-1)^n + 2^n - n + 2$

d) $a_n = 7\cdot 2^n - n + 2$

Solution

a) $a_{n-1} + 2a_{n-2} + 2n - 9 = -(n-1) + 2 + 2\left[-(n-2)+2\right] + 2n - 9$

$$= -n + 1 + 2 - 2n + 4 + 4 + 2n - 9$$

$$= -n + 2$$

$$= a_n$$

b) $a_{n-1} + 2a_{n-2} + 2n - 9 = 5(-1)^{n-1} - (n-1) + 2 + 2\left[5(-1)^{n-2} - (n-2) + 2\right] + 2n - 9$

$$= 5(-1)^{n-1} - n + 3 + 2\left[5(-1)^{n-2} - n + 4\right] + 2n - 9$$

$$= 5(-1)^{n-1} - n + 3 + 10(-1)^{n-2} - 2n + 8 + 2n - 9$$

$$= 5(-1)^{n-1} + 10(-1)^{n-1}(-1)^{-1} - n + 2$$

$$= 5(-1)^{n-1} - 10(-1)^{n-1} - n + 2$$

$$= -5(-1)^{n-1} - n + 2$$

$$= (-1)^1 5(-1)^{n-1} - n + 2$$

$$= 5(-1)^n - n + 2$$
$$= a_n$$

c) $a_{n-1} + 2a_{n-2} + 2n - 9 = 3(-1)^{n-1} + 2^{n-1} - (n-1) + 2$
$$+2\left[3(-1)^{n-2} + 2^{n-2} - (n-2) + 2\right] + 2n - 9$$
$$= 3(-1)^{n-1} + 2^{n-1} + 6(-1)^{n-2} + 2^{n-1} - 2n + 8 + n - 6$$
$$= 3(-1)^{n-1} - 6(-1)^{n-1} + 2^n - n + 2$$
$$= -3(-1)^{n-1} + 2^n - n + 2$$
$$= 3(-1)^n + 2^n - n + 2$$
$$= a_n$$

d) $a_{n-1} + 2a_{n-2} + 2n - 9 = 7\cdot 2^{n-1} - (n-1) + 2 + 2\left[7\cdot 2^{n-2} - (n-2) + 2\right] + 2n - 9$
$$= 7\cdot 2^{n-1} + 7\cdot 2^{n-1} - 2n + 8 + n - 6$$
$$= 2\cdot 7\cdot 2^{n-1} - n + 2$$
$$= 7\cdot 2^n - n + 2$$
$$= a_n$$

Exercise

A person deposits $1,000.00 in an account that yields 9% interest compounded annually.

a) Set up a recurrence relation for the amount in the account at the end of *n* years.

b) Find an explicit formula for the amount in the account at the end of *n* years.

c) How much money will the account contain after 100 years?

Solution

a) The amount after $n-1$ years multiplied by 1.09 to give the amount after n years, since 9% of the value must be added to account for the interest. Therefore, we have $a_n = 1.09a_{n-1}$. The initial condition is $a_0 = 1000$.

b) Since multiplying by 1.09 for each year, the solution is $a_n = 1000(1.09)^n$.

c) $a_{100} = 1000(1.09)^{100}$
$$\approx \$5,529,041$$

Exercise

Suppose that the number of bacteria in a colony triples every hour.

a) Set up a recurrence relation for the number of bacteria after *n* hours have elapsed.

b) If 100 bacteria are used to begin new colony, how many bacteria will be in the colony in 10 hours?

Solution

a) Since the number of bacteria triples every hour, the recurrence relation should say that the number of bacteria after *n* hours is 3 times the number of bacteria after *n* – 1 hours.

Let a_n denote the number of bacteria after *n* hours, this statement translates into the recurrence relation $a_n = 3a_{n-1}$

b) The initial condition is $a_0 = 100$.

$$\begin{aligned} a_n &= 3 \cdot a_{n-1} \\ &= 3^2 \cdot a_{n-2} \\ &\vdots \quad \vdots \\ &= 3^n \cdot a_0 \end{aligned}$$

$n = 10$

$$\begin{aligned} a_{10} &= 100 \cdot 3^{10} \\ &= 5{,}904{,}900 \end{aligned}$$

Exercise

A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with *n* cars made in the *n*th month.

a) Set up a recurrence relation for the number of cars produced in the first *n* months by this factory.

b) How many cars are produced in the first year?

c) Find an explicit formula for the number of cars produced in the first *n* months by this factory

Solution

a) Let c_n be the number of cars produced in the first *n* months.

The initial condition is $c_0 = 0$.

Since *n* cars are made in the *n*th month then $c_n = c_{n-1} + n$, where c_{n-1} is the first $n-1$ months

b) The number of cars produced in the first year is c_{12}.

Plug in $n = 12$, we get

$$\begin{aligned} c_n &= n + c_{n-1} \\ &= n + (n-1) + c_{n-2} \end{aligned}$$

$$= n + (n-1) + (n-2) + c_{n-3}$$

$$\vdots \quad \vdots$$

$$= n + (n-1) + (n-2) + \cdots + 1 + c_0 \qquad\qquad n + (n-1) + (n-2) + \cdots + 1 = \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} + 0$$

$$\boxed{= \frac{n^2+n}{2}}$$

$$c_{12} = \frac{12^2+12}{2}$$

$$\boxed{= 78}$$

c) $\boxed{c_n = \frac{n^2+n}{2}}$

Exercise

For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

a) 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, …

b) 1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, …

c) 1, 0, 2, 0, 4, 0, 8, 0, 16, 0, …

d) 3, 6, 12, 24, 48, 96, 192, …

e) 15, 8, 1, –6, –13, –20, –27, …

f) 3, 5, 8, 12, 17, 23, 30, 38, 47, …

g) 2, 16, 54, 128, 250, 432, 686, …

h) 2, 3, 7, 25, 121, 721, 5041, 40321, …

i) 3, 6, 11, 18, 27, 38, 51, 66, 83, 102, …

j) 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, …

k) 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, …

Solution

a) We have one 1 and one 0, then two 1 and two 0, then three of each, and so on increasing the repetition by one each time. Since we have only one at the end, then we need three 1 and four 0 to continue the sequence.

b) A pattern is that the positive integers are increasing order, with odd number showing once and each even number repeated.
Thus, the next terms are 9, 10, 10.

c) The terms in the odd locations are the successive terms in the geometric sequence that starts with 1 and has ratio 2, and the terms in the even locations are all 0. The *n*th term is 0 if *n* even and is $2^{(n-1)/2}$ if *n* is odd.

Thus, the next three terms are 32, 0, 64.

d) The first term is 3 and each successive term is twice the predecessor. The *n*th term is

$3 \cdot 2^{n-1} \quad n > 0$.

Thus, the next three terms are 384, 768, 1536.

e) The first term is 15 and each successive term is 7 less than its predecessor. The *n*th term is $15 - 7(n-1) = 22 - 7n$.

Thus, the next three terms are –34, –41, –48.

f) The first term is 3 and each successive term by adding *n* to its predecessor.

$3,\ 3+2,\ 5+3,\ 8+4,\ 12+5 \quad nth \quad 3+2+3+4+5+\cdots+n = 2+1+2+3+4+5+\cdots+n$

$$= 2 + \frac{n^2+n}{2}$$

The *n*th term is $\frac{1}{2}\left(n^2+n+4\right)$.

Thus, the next three terms are 57, 68, 80.

g) Since all numbers are even, then if we divide by 2 the sequence becomes: 1, 8, 27, 64, 125, 216, 343, …. This sequence appears to be n^3, therefore the *n*th term is $2n^3$.

Thus, the next three terms are 1024, 1458, 2000.

h) The *n*th term appears to be $n! + 1$.

Thus, the next three terms are 362881, 3628801, 39916801.

i) The first term is 3 then by adding 3 to the predecessor, then 5, then 7, and so on.

$3,\ 3+3,\ 6+5,\ 11+7,\ \ldots \rightarrow\ 1+2,\ 4+2,\ 9+2,\ 16+2,\ \ldots$

Then the *n*th term is n^2+2.

Thus, the next three terms are 123, 146, 171.

j) This an arithmetic sequence whose difference is 4. Thus, the *n*th term is $7+4(n-1) = 4n+3$.

Thus, the next three terms are 47, 51, 55.

k) This is a binary expansion of *n*. Thus, the next three terms are 1100, 1101, 1110.