

Review ☺

$$\cos A = -\frac{15}{17} \quad A \in QII \quad \cos B = \frac{12}{13} \quad B \in QI$$

$$\sin A = \frac{8}{17}$$

$$\sin B = \frac{5}{13}$$

$$\begin{aligned} a) \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{8}{17} \cdot \frac{12}{13} + \left(-\frac{15}{17}\right) \left(\frac{5}{13}\right) \\ &= \frac{96 - 75}{221} \\ &= \frac{21}{221} \end{aligned}$$

$$\begin{aligned} b) \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \frac{-15}{17} \cdot \frac{12}{13} - \frac{8}{17} \cdot \frac{5}{13} \\ &= \frac{-180 - 40}{221} \\ &= \frac{-220}{221} \end{aligned}$$

$$c) \tan(A+B) = -\frac{21}{220}$$

$$\begin{aligned}
 d) \sin(A-B) &= \sin A \cos B - \cos A \sin B \\
 &= \frac{46 + 75}{221} \\
 &= \frac{121}{221}
 \end{aligned}$$

$$\begin{aligned}
 e) \cos(A-B) &= \cos A \cos B + \sin A \sin B \\
 &= \frac{-180 + 40}{221} \\
 &= -\frac{140}{221}
 \end{aligned}$$

$$f) \tan(A-B) = -\frac{121}{140}$$

$$\cos A = -\frac{4}{5} \quad A \in QII$$

$$\sin A = \frac{3}{5}$$

$$\frac{90^\circ}{2} < \frac{A}{2} < \frac{180^\circ}{2}$$

$$\frac{A}{2} \in QI$$

$$\begin{aligned} a) \sin 2A &= 2 \sin A \cos A \\ &= 2 \left(\frac{3}{5} \right) \left(-\frac{4}{5} \right) \\ &= -\frac{24}{25} \end{aligned}$$

$$\begin{aligned} b) \cos 2A &= \cos^2 A - \sin^2 A \\ &= \frac{16}{25} - \frac{9}{25} \\ &= \frac{7}{25} \end{aligned}$$

$$c) \tan 2A = -\frac{24}{7}$$

$$\begin{aligned} d) \sin \frac{A}{2} &= \sqrt{\frac{1}{2} (1 - \cos A)} \\ &= \sqrt{\frac{1}{2} \left(1 + \frac{4}{5} \right)} \\ &= \frac{3}{\sqrt{10}} \end{aligned}$$

$$\begin{aligned} e) \cos \frac{A}{2} &= \sqrt{\frac{1}{2} (1 + \cos A)} \\ &= \sqrt{\frac{1}{2} \left(1 - \frac{4}{5} \right)} \\ &= \frac{1}{\sqrt{10}} \end{aligned}$$

$$f) \tan \frac{A}{2} = 3$$

$$4 \sin^2 x + 4 \cos x - 5 = 0 \quad ([0, 2\pi])$$

$$4(1 - \cos^2 x) + 4 \cos x - 5 = 0$$

$$-4 \cos^2 x + 4 \cos x - 1 = 0$$

$$\cos x = \frac{-4 \pm 0}{-8} = \frac{1}{2}$$

$$x = \left[\frac{\pi}{3}, \frac{5\pi}{3} \right]$$

$$\tan^2 x \sin x = \sin x$$

$$\tan^2 x \sin x - \sin x = 0$$

$$\sin x (\tan^2 x - 1) = 0$$

$$\sin x = 0$$

$$\tan x = \pm 1$$

$$x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$2 \tan x \csc x + 2 \csc x + \tan x + 1 = 0$$

$$2 \csc x (\tan x + 1) + (\tan x + 1) = 0$$

$$(\tan x + 1) (2 \csc x + 1) = 0$$

$$\tan x = -1$$

$$\csc x = -\frac{1}{2}$$

$$\sin x = -2 \quad \#$$

$$x = \left[\frac{3\pi}{4}, \frac{7\pi}{4} \right]$$

$$1 \sin \theta - \sqrt{3} \cos \theta = 1$$

$$\frac{1}{2} \sin \theta - \frac{\sqrt{3}}{2} \cos \theta = \frac{1}{2}$$

$$\sin \theta \cos \frac{\pi}{3} - \cos \theta \sin \frac{\pi}{3} = \frac{1}{2}$$

$$\sin \left(\theta - \frac{\pi}{3} \right) = \left(\frac{1}{2} \right) ?$$

$$\theta - \frac{\pi}{3} = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6} + \frac{\pi}{3}$$

$$= \frac{\pi}{2}$$

$$\theta - \frac{\pi}{3} = \frac{5\pi}{6}$$

$$\theta = \frac{5\pi}{6} + \frac{\pi}{3}$$

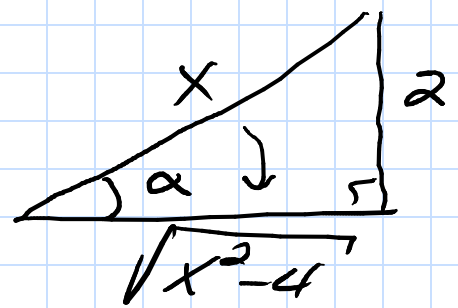
$$= \frac{7\pi}{6}$$

$$\sec \left(\tan^{-1} \frac{2}{\sqrt{x^2-4}} \right)$$

α

$$\tan \alpha = \frac{2}{\sqrt{x^2-4}}$$

$$\sec \alpha = \frac{x}{\sqrt{x^2-4}}$$

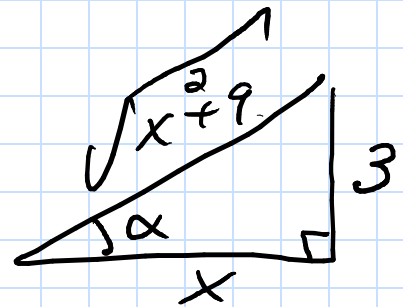


$$\sin \left(\cos^{-1} \frac{x}{\sqrt{x^2+9}} \right)$$

α

$$\cos \alpha = \frac{x}{\sqrt{x^2+9}}$$

$$\sin \alpha = \frac{3}{\sqrt{x^2+9}}$$



$$(3, 270^\circ)$$

$$(x, y)?$$

$$\begin{aligned}x &= r \cos \theta \\&= 3 \cos 270^\circ \\&= \underline{0}\end{aligned}$$

$$\begin{aligned}y &= r \sin \theta \\&= 3 \sin 270^\circ \\&= \underline{-3}\end{aligned}$$

$$(x, y) = (0, -3)$$

$$(-1, \sqrt{3})$$

$$(r, \theta)?$$

$$\begin{aligned}r &= \sqrt{x^2 + y^2} \\&= \sqrt{1 + 3} \\&= \underline{2}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1} \left| \frac{y}{x} \right| \quad \sqrt{3}/2 \\&= \tan^{-1} \frac{\sqrt{3}}{1} \\&= 60^\circ\end{aligned}$$

$$(r, \theta) = (2, 120^\circ) \quad \odot$$

$$r = 8 \sin \theta - 2 \cos \theta$$

$$r^2 = 8r \sin \theta - 2r \cos \theta$$

$$x^2 + y^2 = 8y - 2x$$

$$x^2 + 2x + (1)^2 + y^2 - 8y + (-4)^2 = 1 + 16$$

$$(x+1)^2 + (y-4)^2 = 17$$

$$\text{center } (-1, 4) \quad \text{radius } \sqrt{17}$$

$$x^2 + y^2 = 4x$$

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 4r \cos \theta \quad (r \neq 0)$$

$$r (\cos^2 \theta + \sin^2 \theta) = 4 \cos \theta$$

$$r = 4 \cos \theta$$

$$\sqrt{3} - i$$

$$x = \sqrt{3}$$

$$y = -1$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{3 + 1} \\ &= 2 \end{aligned}$$

$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) \quad \frac{\pi}{6}$$

$$\sqrt{3} - i = 2 \operatorname{cis} \frac{11\pi}{6}$$

$$3 \operatorname{cis} 210^\circ = 3(\cos 210^\circ + i \sin 210^\circ)$$

$$= -3 \frac{\sqrt{3}}{2} - i 3 \frac{1}{2}$$

$$= -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$$

$$\begin{aligned}
 \frac{15 \operatorname{cis}(210^\circ)}{3 \operatorname{cis}(150^\circ)} &= 5 \operatorname{cis}(210^\circ - 150^\circ) \\
 &= 5 \operatorname{cis} 60^\circ \checkmark \\
 &= 5 \cos 60^\circ + i 5 \sin 60^\circ \\
 &= \frac{5}{2} + \frac{5}{2} i
 \end{aligned}$$

$$\begin{aligned}
 (15 \operatorname{cis} 210^\circ) (5 \operatorname{cis} 150^\circ) &= 75 \operatorname{cis} 360^\circ \\
 &= 75 (\cos 360^\circ + i \sin 360^\circ) \\
 &= 75
 \end{aligned}$$