$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\} \qquad \begin{aligned} r &= \sqrt{x^2+y^2} \\ \theta &= \tan^{-1}\frac{y}{x} \end{aligned}$$

Ex. $(r,\theta) = \left(4, \frac{7\pi}{6}\right)$ $(x?, y?)$

$x = r\cos\theta$
$= 4\cos\frac{7\pi}{6}$
$= -4\left(\frac{\sqrt{3}}{2}\right)$
$= -2\sqrt{3}$

$y = r\sin\theta$
$= 4\sin\frac{7\pi}{6}$
$= 4\left(-\frac{1}{2}\right)$
$= -2$

$(x,y) = (-2\sqrt{3}, -2)$

Ex $(x,y) = (-1, \sqrt{3})$ $(r,\theta)?$

$r = \sqrt{x^2+y^2}$
$= \sqrt{1+3}$
$= 2$

$\theta = \tan^{-1}\frac{\sqrt{3}}{1}$
$= \frac{\pi}{3}$

$(r,\theta) = \left(2, \frac{2\pi}{3}\right)$

$\left(2, -\frac{4\pi}{3}\right)$

$\left(-2, -\frac{\pi}{3}\right)$ $\left(-2, \frac{5\pi}{3}\right)$

Ex

$$ax + by = c$$

$$ar\cos\theta + br\sin\theta = c$$

$$r(a\cos\theta + b\sin\theta) = c$$

$$r = \frac{c}{a\cos\theta + b\sin\theta}$$

Ex $x^2 - y^2 = 16$

$$r^2\cos^2\theta - r^2\sin^2\theta = 16$$

$$r^2(\cos^2\theta - \sin^2\theta) = 16$$

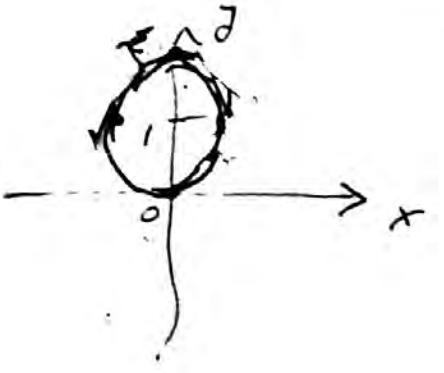
$$r^2\cos 2\theta = 16$$

$$r^2 = \frac{16}{\cos 2\theta}$$

$$= 16\sec 2\theta$$

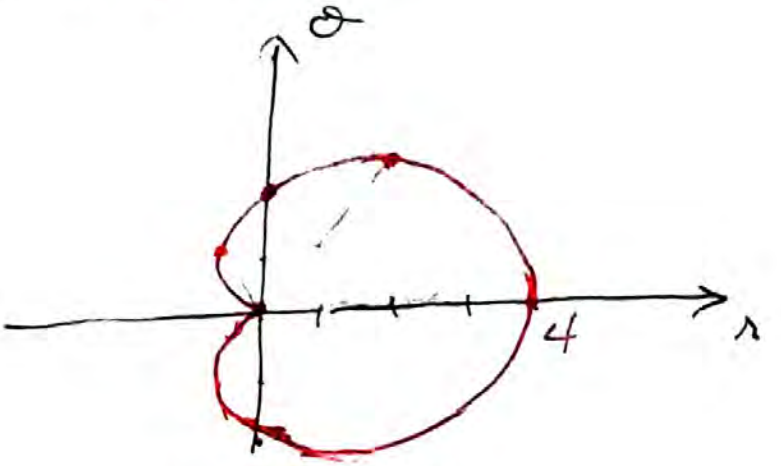
$$r = 2\sin\theta$$

θ	r
0	0
$\pi/6$	1
$\pi/2$	2
$5\pi/6$	1
π	0
$7\pi/6$	-1



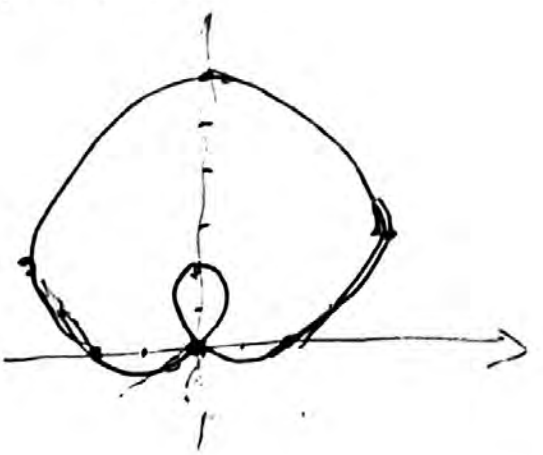
Graph $r = 2 + 2\cos\theta$

θ	r
0	4
$\pi/3$	3
$\pi/2$	2
$2\pi/3$	1
π	0



Cardioid

$r = 2 + 4\sin\theta$



8.6

6) $(4\sqrt{3}, -\frac{\pi}{6})$

$x = r\cos\theta$
$= 4\sqrt{3}\cos(-\frac{\pi}{6})$
$= 4\sqrt{3}(\frac{\sqrt{3}}{2})$
$= 6$

$y = r\sin\theta$
$= 4\sqrt{3}\sin(-\frac{\pi}{6})$
$= -4\sqrt{3}(\frac{1}{2})$
$= -2\sqrt{3}$

$(x,y) = (6, -2\sqrt{3})$

12) $(-1, \sqrt{3})$ (r, θ) ?

$r = \sqrt{x^2+y^2}$
$= \sqrt{1+3}$
$= 2$

$\hat{\theta} = \tan^{-1}\frac{\sqrt{3}}{1}$
$= \frac{\pi}{3}$
$\theta \in QII$

$(r,\theta) = (2, \frac{2\pi}{3})$

---

24) $r^2 = 4\cos 2\theta$
$= 4(\cos^2\theta - \sin^2\theta)$

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2+y^2$

$r^2 r^2 = 4(r^2\cos^2\theta - r^2\sin^2\theta)$
$(x^2+y^2)^2 = 4(x^2 - y^2)$
$x^4 + 2x^2y^2 + y^4 = 4x^2 - 4y^2$

41- $x^2+y^2=4x$

$r^2 = 4r\cos\theta \quad (r \neq 0)$

$r = 4\cos\theta$

38. $y^2 - x^2 = 4$

$r^2\sin^2\theta - r^2\cos^2\theta = 4$

$r^2(\underbrace{\sin^2\theta - \cos^2\theta}_{-\cos 2\theta}) = 4$

$r^2 = -\frac{4}{\cos 2\theta}$

8.7 Complex

$\sqrt{-1} = i$

$\sqrt{-1}\sqrt{-1} = i \cdot i$

$= i^2$

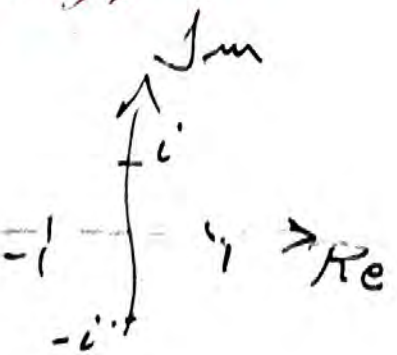
$= -1$

you can't do ~~$\sqrt{-1}\sqrt{-1}$~~ $= \sqrt{(-1)(-1)}$

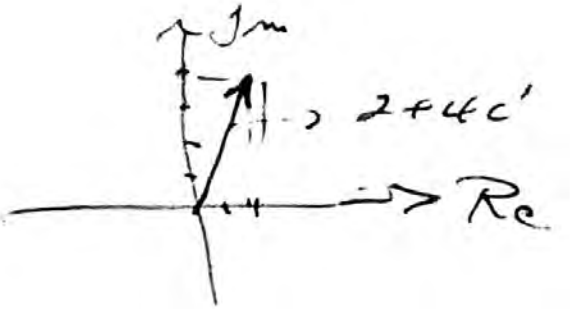
$z = a + ib$

Real part Re — Imaginary part Im

$a, b \in \mathbb{R}$



$z = 2 + 4i$



$z = x + iy$.

$r = \sqrt{x^2 + y^2}$ modulus

$\theta = \tan^{-1}\frac{y}{x}$ Argument

$z = x + iy$

$= r\cos\theta + i\, r\sin\theta$

$= r\,\mathrm{cis}\,\theta$ Trig Form

$z = -1 + i$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{1 + 1}$

$= \sqrt{2}$

$\hat{\theta} = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}$

$-1 + i = \sqrt{2}\,\mathrm{cis}\left(\frac{3\pi}{4}\right)$

$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4}$

$z = 2\,\text{cis}\,60^\circ$

$= 2\cos 60^\circ + i\,2\sin 60^\circ$

$= \underline{1 + i\sqrt{3}}$

$z = 7 \Rightarrow r = 7 \quad \theta = 0$

$= 7 + i\,0$

$5i \Rightarrow r = 5, \ \theta = \frac{\pi}{2}$

Product.

$a^2 + b^2 = (a + ib)(a - ib)$

$a + b = (1\vec{a} + i\sqrt{b})(1\vec{a} - i\sqrt{b})$

$z_1 z_2 = (r_1\,\text{cis}\,\theta_1)(r_2\,\text{cis}\,\theta_2)$

$= r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$

$(3\,\text{cis}\,45^\circ)(2\,\text{cis}\,135^\circ)$

$= 6\,\text{cis}(180^\circ)$ $\quad 45^\circ + 135^\circ =$

$= 6\cos 180^\circ + i\,6\sin 180^\circ$

$= \underline{-6}$

Quotient

$$\frac{r_1 \operatorname{cis}\theta_1}{r_2 \operatorname{cis}\theta_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$\frac{10 \operatorname{cis}(-60^\circ)}{5 \operatorname{cis}(100^\circ)} = \frac{10}{5} \operatorname{cis}(-60 - 100)$$

$$= 2 \operatorname{cis}(-210^\circ)$$

$$= 2\cos(-210^\circ) + i\, 2\sin(-210^\circ)$$

$$= 2\left(-\frac{\sqrt{3}}{2}\right) - i\, 2\left(-\frac{1}{2}\right)$$

$$= -\sqrt{3} + i$$

De Moivre's Theorem

$$(r \operatorname{cis}\theta)^n = r^n \operatorname{cis} n\theta$$

$$(1 + i\sqrt{3})^8$$

$$r = \sqrt{1+3} \qquad \theta = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{6}$$

$$= 2$$

$$(1 + i\sqrt{3})^8 = \left(2 \operatorname{cis}\frac{\pi}{6}\right)^8$$

$$= 2^8 \operatorname{cis}\frac{8\pi}{6} \qquad \frac{4\pi}{3}$$

$$= 256\cos\frac{4\pi}{3} + i\, 256\sin\frac{4\pi}{3}$$

$$= -\frac{256}{2} + i\,\frac{256\sqrt{3}}{2}$$

$$= -128 + i\,128\sqrt{3}$$

n) 4th root

$$(r\,\mathrm{cis}\,\theta)^{1/n} = \sqrt[n]{r}\,\mathrm{cis}\,\alpha$$

$$\alpha = \frac{\theta + 2\pi k}{n}$$

Info

4th root $-8 + 8i\sqrt{3}$

$$r = \sqrt{64 + 64(3)} \qquad 8(2)$$

$$= 16$$

$$\hat{\theta} = \tan^{-1}\frac{8\sqrt{3}}{8} = \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

$$\sqrt[4]{16} = 2$$

$$\alpha = \frac{\theta + 2\pi k}{n} =$$

$$= \frac{1}{4}\left(\frac{2\pi}{3} + 2\pi k\right) \qquad \left(\frac{1}{3} + k\right)$$

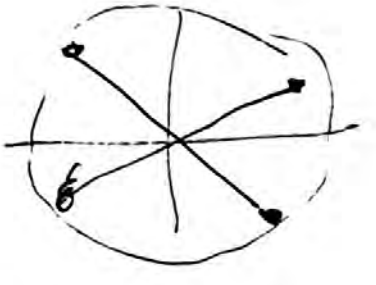
$$= \frac{\pi}{2}\left(\frac{1}{3} + k\right)$$

$k = 0 \rightarrow \alpha = \pi/6$

$k = 1 \rightarrow \alpha = 2\pi/3$

$k = 2 \rightarrow \alpha = 7\pi/6$

$k = 3 \rightarrow \alpha = 5\pi/3$



$$2\,\mathrm{cis}\frac{\pi}{6},\ 2\,\mathrm{cis}\frac{2\pi}{3},\ 2\,\mathrm{cis}\frac{7\pi}{6},\ 2\,\mathrm{cis}\frac{5\pi}{3}$$

8.6 (2) $(-\sqrt{2}, \frac{3\pi}{4})$

$x = r\cos\theta$
$= -\sqrt{2}\cos\frac{3\pi}{4}$
$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$
$= 1$

$y = r\sin\theta$
$= -\sqrt{2}\sin\frac{3\pi}{4}$
$= -\sqrt{2}\,\frac{1}{\sqrt{2}}$
$= -1$

$(x, y) = (1, -1)$

$(2, -2\sqrt{3})$ $\quad r \geq 0 \quad 0^\circ \leq \theta \leq 360^\circ.$

$r = \sqrt{x^2 + y^2}$
$= \sqrt{4 + 12}$
$= 4$

$\theta = \tan^{-1}\left|\frac{2\sqrt{3}}{2}\right|$
$= 60^\circ$
$\theta = 300^\circ.$

$(r, \theta) = (4, 300^\circ)$

31 $r(\sin\theta - 2\cos\theta) = 6$

$r\sin\theta - 2r\cos\theta = 6$

$y - 2x = 6$

32 $r = 8\sin\theta - 2\cos\theta$

$r^2 = 8r\sin\theta - 2r\cos\theta \qquad r \neq 0$

$x^2 + y^2 = 8y - 2x$

37 $(x+2)^2 + (y-3)^2 = 13$

$(r\cos\theta + 2)^2 + (r\sin\theta - 3)^2 = 13$

$r^2\cos^2\theta + 4r\cos\theta + 4 + r^2\sin^2\theta - 6r\sin\theta + 9 = 13$

$r^2(\underbrace{\cos^2\theta + \sin^2\theta}_{=1}) + r(4\cos\theta - 6\sin\theta) = 0$

$r^2 + r(4\cos\theta - 6\sin\theta) = 0 \quad (r \neq 0)$

$r + 4\cos\theta - 6\sin\theta = 0$

$r = 6\sin\theta - 4\cos\theta$

$x^2 + y^2 = r^2$

$\in \mathbb{R}$ and

May 6

5/6/20