Article

1) $\int x^5 e^{4x}$

	$\int e^{4x}$
x^5	$\frac{1}{4} e^{4x}$
$5x^4$	$\frac{1}{16} e^{4x}$
$\vdots$	$\vdots$

$$= e^{4x}\left(\frac{x^5}{4} - \frac{5}{16}x^4 + \frac{5}{16}x^3 - \frac{15}{64}x^2 + \frac{15}{128}x - \frac{15}{512}\right) + C$$

2) $\int \cos 2x\, e^{3x}\, dx$

	$\int \cos 2x$
$+\ e^{3x}$	$\frac{1}{2}\sin 2x$
$-\ 3e^{3x}$	$-\frac{1}{4}\cos 2x$
$+\ 9e^{3x}$	

$$= e^{3x}\left(\frac{1}{2}\sin 2x + \frac{3}{4}\cos 2x\right) - \frac{9}{4}\int e^{3x}\cos 2x\, dx$$

$$\int \cos 2x\, e^{3x} dx = \frac{4}{13} e^{3x}\left(\frac{1}{2}\sin 2x + \frac{3}{4}\cos 2x\right)$$

3)

	$\int \cos dx$
x^6	$+\frac{1}{4}\sin 4x$
$6x^5$	$-\frac{1}{24}\cos 4x$
$\vdots$	$\vdots$
720	

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4) $\int \cos(\ln x)\, dx$

$u = \cos(\ln x)$ $\quad v = \int dx$

$du = -\frac{1}{x}\sin(\ln x)$ $\quad = x$

$$\int \cos(\ln x)\, dx = x\cos(\ln x) + \int \sin(\ln x)\, dx$$

$u = \sin \ln x$ $\quad v = x$

$du = \frac{1}{x}\cos(\ln x)$

$$= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\, dx$$

$$\int \cos(\ln x)\, dx = \frac{1}{2}\left(x\sin(\ln x) - x\cos(\ln x)\right) + C$$

5/ $\int_0^{\pi} e^{\cos t} \sin 2t \, dt = 2\int_0^{\pi} e^{\cos t} \cos t \sin t \, dt$

$= -2\int_0^{\pi} \cos t \, e^{\cos t} \, d(\cos t)$

$= -2\int u e^u du$ $\quad u = \cos t$

$\int e^u$
$+$ u
$-$ 1
e^u

$= -2 e^{\cos t}(\cos t - 1)\Big|_0^{\pi}$

$= -2\left(\frac{1}{e}(-2) - e(0)\right)$

$= \frac{4}{e}$

2.2 Trig.

$$\int \sin^m x \cos^n x \, dx$$

Ex $\int \cos^2 x \sin^3 x \, dx$ $\qquad \cos^2 x + \sin^2 x = 1$

$$= \int \cos^2 x \sin^2 x \sin x \, dx \qquad d(\cos x) = -\sin x \, dx$$

$$= -\int \cos^2 x (1 - \cos^2 x) \, d(\cos x)$$

$$= \int (\cos^4 x - \cos^2 x) \, d(\cos x)$$

$$= \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$$

Ex $\int \cos^5 x \, dx = \int \cos^4 x \cos x \, dx \qquad (\cos^2 x)^2$

$d(\sin x) = \cos x \, dx$

$$= \int (1 - \sin^2 x)^2 \, d(\sin x)$$

$$= \int (1 - 2\sin^2 x + \sin^4 x) \, d(\sin x)$$

$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$$

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① odd → $(\,)^1 dx$

$$\cos x \, dx = d(\sin x)$$

$$\sin x \, dx = -d(\cos x)$$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x) = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

Ex. $\int \sin^2 x \cos^4 x \, dx$

$$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{8}\int (1-\cos 2x)(1+2\cos 2x+\cos^2 2x)\, dx$$

$$= \frac{1}{8}\int (1+\cos 2x-\cos^2 2x-\cos^3 2x)\, dx$$

$$= \frac{1}{8}\int \left(1+\cos 2x-\frac{1}{2}-\frac{1}{2}\cos 4x-\cos^3 2x\right) dx$$

($1 = 1/2 + 1/2$)

$$\int \cos^3 2x\, dx = \int \cos^2 2x \cos 2x \, dx$$

$$= \frac{1}{2}\int (1-\sin^2 2x)\, d(\sin 2x)$$

$$d(\sin 2x) = 2\cos 2x\, dx$$

$$= \frac{1}{2}\left(\sin 2x - \frac{1}{3}\sin^3 2x\right)$$

$$= \frac{1}{8}\left[\frac{1}{2}x+\frac{1}{2}\sin 2x-\frac{1}{8}\sin 4x-\frac{1}{2}\sin 2x+\frac{1}{6}\sin^3 2x\right]$$

$$= \frac{1}{8}\left(\frac{1}{2}x-\frac{1}{8}\sin 4x+\frac{1}{6}\sin^3 2x\right)+C$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

Q5 $\int_0^{\pi/4} \sqrt{1+\cos 4x}\, dx$

$= \int_0^{\pi/4} \sqrt{2\cos^2 2x}\, dx$

$= \sqrt{2} \int_0^{\pi/4} \cos 2x\, dx$

$= \frac{\sqrt{2}}{2} \sin 2x \Big|_0^{\pi/4}$

$= \frac{\sqrt{2}}{2}(1-0)$

$= \frac{\sqrt{2}}{2}$

Ex $\int \sin^3 x \cos^{-2} x\, dx = \int \sin^2 x \cos^{-2} x \sin x\, dx$

$= -\int (1-\cos^2 x)\cos^{-2} x\, d(\cos x)$

$= -\int (\cos^{-2} x - 1)\, d(\cos x)$

$= -(-\cos^{-1} x - \cos x)$

$= \frac{1}{\cos x} + \cos x + C$

$= \sec x + \cos x + C$

$d(\tan x) = \sec^2 x\,dx$

$d(\sec x) = \sec x \tan x\,dx$

$1 + \tan^2 x = \sec^2 x \quad \text{or} \quad \tan^2 \alpha = \sec^2 - 1$

$\cos^2 x + \sin^2 x = 1$

$\frac{\cos^2}{\cos^2} \quad \frac{\sin^2}{\cos^2} \quad \frac{1}{\cos^2}$

$\cos^2 \alpha + \sin^2 \alpha = 1$

Ex $\int \tan^4 x\,dx = \int \tan^2 x \tan^2 x\,dx$

$= \int \tan^2 x (\sec^2 x - 1)\,dx$

$= \int (\tan^2 x \sec^2 x - \tan^2 x)\,dx$

$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx$

$= \int \tan^2 x\,d(\tan x) - \int (\sec^2 x - 1)\,dx$

$= \frac{1}{3}\tan^3 x - (\tan x - x) + C$

$= \frac{1}{3}\tan^3 x - \tan x + x + C$

$\int \sec^3 x\,dx$

$u = \sec x \qquad v = \int \sec^2 x\,dx$

$du = \sec x \tan x\,dx \qquad = \tan x$

$\int \sec^3 x\,dx = \sec x \tan x - \int \sec x \tan^2 x\,dx$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1)\,dx$

$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$

$2\int \sec^3 x\,dx = \sec x \tan x + \int \sec x\,dx$

$= \sec x \tan x + \ln|\sec x + \tan x| + C_1$

$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$

$$\int \sec x\,dx = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x}\,dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$$

$$= \int \frac{d(\sec x + \tan x)}{\sec x + \tan x}$$

$$= \ln|\sec x + \tan x| + C$$

$$\sin mx \cos nx = \frac{1}{2}\left(\sin(m-n)x + \sin(m+n)x\right)$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) + \cos(a+b) = 2\cos a \cos b$$

$$\cos a \cos b = \frac{1}{2}\left(\cos(a-b) + \cos(a+b)\right)$$

Ex $\int \sin 3x \cos 5x\,dx = \frac{1}{2}\int (-\sin 2x + \sin 8x)\,dx$

$$= \frac{1}{2}\left(+\frac{1}{2}\cos 2x - \frac{1}{8}\cos 8x\right) + C$$

#143 $\int_0^{\pi/2} \cos^{10}\theta\,d\theta = \frac{\pi}{2}\cdot\frac{9}{10}\cdot\frac{7}{8}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}$

$$= \frac{63\pi}{2^9}$$

145 $\int_0^{\pi/2} \cos^9 x\,dx = \frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}$

$$= \frac{256}{315}$$

2.3 Trig. Substitutions

$\sqrt{a^2+x^2}$	$\sqrt{x^2-a^2}$
$x = a\tan\theta$	$x = a\sec\theta$
$\sqrt{a^2-x^2}$	
$x = a\sin\theta$	

Ex $\int \frac{dx}{\sqrt{4+x^2}}$ $\quad x = 2\tan\theta \quad \sqrt{4+x^2} = 2\sec\theta$

$dx = 2\sec^2\theta\, d\theta$

$$\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{2\sec^2\theta\, d\theta}{2\sec\theta}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$$

Q7 $\int \frac{x^2 dx}{\sqrt{9-x^2}}$ $\quad x = 3\sin\theta$, $\sqrt{9-x^2} = 3\cos\theta$

$dx = 3\cos\theta\, d\theta$

$$\int \frac{x^2 dx}{\sqrt{9-x^2}} = \int \frac{9\sin^2\theta \, 3\cos\theta \, d\theta}{3\cos\theta}$$

$\sin\theta = \frac{x}{3}$

$\theta = \sin^{-1}\frac{x}{3}$

$$= \frac{9}{2}\int (1-\cos 2\theta)\, d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{9}{2}\left(\theta - \sin\theta\cos\theta\right) + C$$

$$= \frac{9}{2}\left(\sin^{-1}\frac{x}{3} - \frac{x\sqrt{9-x^2}}{9}\right) + C$$

$$= \frac{9}{2}\sin^{-1}\frac{x}{3} - \frac{1}{2}x\sqrt{9-x^2} + C$$

$$\int \frac{dx}{\sqrt{25x^2-4}}$$

$5x = 2\sec\theta$

$dx = \frac{2}{5}\sec\theta\tan\theta\,d\theta$

$\sqrt{25x^2-4} = 2\tan\theta$

$$\int \frac{dx}{\sqrt{25x^2-4}} = \frac{2}{5}\int \frac{\sec\theta\tan\theta\,d\theta}{2\tan\theta}$$

$$= \frac{1}{5}\int \sec\theta\,d\theta$$

$$= \frac{1}{5}\ln|\sec\theta+\tan\theta| + C$$

$$= \frac{1}{5}\ln\left|\frac{5}{2}x + \frac{1}{2}\sqrt{25x^2-4}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{1-2x^2}}$$

$\sqrt{2}\,x = \sin\theta$

$dx = \frac{1}{\sqrt{2}}\cos\theta\,d\theta$

$\sqrt{1-2x^2} = \cos\theta$

$$\int \frac{dx}{\sqrt{1-2x^2}} = \frac{1}{\sqrt{2}}\int \frac{\cos\theta\,d\theta}{\cos\theta}$$

$$= \frac{1}{\sqrt{2}}\int d\theta$$

$$= \frac{1}{\sqrt{2}}\theta$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}(\sqrt{2}\,x)$$

#12 $\int \frac{dx}{x\sqrt{4x^2+9}}$

$2x = 3\tan\theta$

$dx = \frac{3}{2}\sec^2\theta\, d\theta$

$\sqrt{4x^2+9} = 3\sec\theta$

$$\int \frac{dx}{x\sqrt{4x^2+9}} = \frac{3}{2}\int \frac{\sec^2\theta\, d\theta}{\frac{3}{2}\tan\theta\, 3\sec\theta}$$

$$= \frac{1}{3}\int \frac{\sec\theta}{\tan\theta}\, d\theta$$

$$= \frac{1}{3}\int \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\, d\theta$$

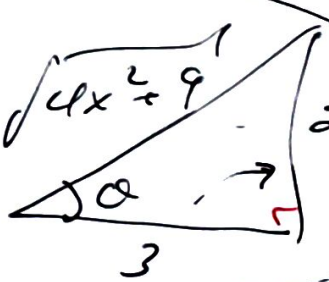
$$= \frac{1}{3}\int \csc\theta\, d\theta$$

$$= -\frac{1}{3}\ln|\csc\theta + \cot\theta| + C$$

$$= -\frac{1}{3}\ln\left|\frac{2x}{\sqrt{4x^2+9}} + \frac{3}{2x}\right| + C$$

$2x = 3\tan\theta$

$\tan\theta = \frac{2x}{3}$



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$$\int \frac{dx}{\sqrt{x^2-25}}$$

$x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-25} = 5\tan\theta$

$$\int \frac{dx}{\sqrt{x^2-25}} = \int \frac{5\sec\theta\tan\theta\, d\theta}{5\tan\theta}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{x}{5} + \frac{\sqrt{x^2-25}}{5}\right| + C$$