*Derivative*: Rational Function to Power '$n$' in the form $\left(\frac{ax^n+b}{cx^n+d}\right)^m$

$$\frac{d}{dx}\left(\frac{ax^n+b}{cx^n+d}\right)^m = mn(ad-bc)x^{n-1}\frac{\left(ax^n+b\right)^{m-1}}{\left(cx^n+d\right)^{m+1}}$$

***Proof***

$$u = ax^n+b \quad v = cx^n+d$$

$$u' = nax^{n-1} \quad v' = ncx^{n-1}$$

$$\frac{d}{dx}\left(\frac{ax^n+b}{cx^n+d}\right)^m = m\frac{nax^{n-1}\left(cx^n+d\right)-ncx^{n-1}\left(ax^n+b\right)}{\left(cx^n+d\right)^2}\left(\frac{ax^n+b}{cx^n+d}\right)^{m-1} \qquad \left(\frac{u}{v}\right)' = \frac{u'v-v'u}{v^2}$$

$$= \frac{m\left(nacx^{2n-1}+nadx^{n-1}-nacx^{2n-1}-nbcx^{n-1}\right)\left(ax^n+b\right)^{m-1}}{\left(cx^n+d\right)^2\left(cx^n+d\right)^{m-1}}$$

$$= \frac{m\left(nadx^{n-1}-nbcx^{n-1}\right)\left(ax^n+b\right)^{m-1}}{\left(cx^n+d\right)^{m+1}}$$

$$= \frac{mn(ad-bc)x^{n-1}\left(ax^n+b\right)^{m-1}}{\left(cx^n+d\right)^{m+1}}$$

### *Example*

Find $\dfrac{d}{dx}\left(\dfrac{x+2}{3x-2}\right)^4$

***Solution***

$$\frac{d}{dx}\left(\frac{x+2}{3x-2}\right)^4 = (1)(4)(-2-6)\frac{(x+2)^3}{(3x-2)^5}$$

$$= -\frac{32(x+2)^3}{(3x-2)^5}$$

$$\frac{d}{dx}\left(\frac{x+2}{3x-2}\right)^4 = 4\left(\frac{x+2}{3x-2}\right)^3 \frac{d}{dx}\left(\frac{x+2}{3x-2}\right)$$

$$= 4\frac{(x+2)^3}{(3x-2)^3}\frac{3x-2-3(x+2)}{(3x-2)^2}$$

$$= 4\frac{(x+2)^3}{(3x-2)^3}\frac{3x-2-3x-6}{(3x-2)^2}$$

$$= 4(-8)\frac{(x+2)^3}{(3x-2)^5}$$

$$= -\frac{32(x+2)^3}{(3x-2)^5}$$

### *Example*

Find $\dfrac{d}{dx}\left(\dfrac{5x^2-3}{2x^2-4}\right)^5$

***Solution***

$$\frac{d}{dx}\left(\frac{5x^2-3}{2x^2-4}\right)^5 = \frac{-140x\left(5x^2-3\right)^4}{\left(2x^2-4\right)^6}$$