

## ***Solution***      **Section 4.4 – Determinants**

### ***Exercise***

Evaluate  $\begin{vmatrix} -1 & 3 \\ -2 & 9 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} -1 & 3 \\ -2 & 9 \end{vmatrix} = -9 - (-6) \\ \underline{\underline{= -3}}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} 6 & -4 \\ 0 & -1 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} 6 & -4 \\ 0 & -1 \end{vmatrix} = -6 - (0) \\ \underline{\underline{= -6}}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} x & 4x \\ 2x & 8x \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} x & 4x \\ 2x & 8x \end{vmatrix} = x(8x) - 4x(2x) \\ = 8x^2 - 8x^2 \\ \underline{\underline{= 0}}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} x & 2x \\ 4 & 3 \end{vmatrix}$

### **Solution**

$$\begin{aligned}\begin{vmatrix} x & 2x \\ 4 & 3 \end{vmatrix} &= 3x - 2x(4) \\ &= 3x - 8x \\ &= \underline{-5x}\end{aligned}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} x^4 & 2 \\ x & -3 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} x^4 & 2 \\ x & -3 \end{vmatrix} = \underline{-3x^4 - 2x}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} -8 & -5 \\ b & a \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} -8 & -5 \\ b & a \end{vmatrix} = \underline{-8a + 5b}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} 5 & 7 \\ 2 & 3 \end{vmatrix}$

### **Solution**

$$\begin{aligned}\begin{vmatrix} 5 & 7 \\ 2 & 3 \end{vmatrix} &= 15 - 14 \\ &= \underline{1}\end{aligned}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} 1 & 4 \\ 5 & 5 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} 1 & 4 \\ 5 & 5 \end{vmatrix} = 5 - 20$$

$$\underline{\underline{= -16}}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} 5 & 3 \\ -2 & 3 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} 5 & 3 \\ -2 & 3 \end{vmatrix} = 15 + 6$$

$$\underline{\underline{= 21}}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} -4 & -1 \\ 5 & 6 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} -4 & -1 \\ 5 & 6 \end{vmatrix} = -24 + 5$$

$$\underline{\underline{= -19}}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} \sqrt{3} & -2 \\ -3 & \sqrt{3} \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} \sqrt{3} & -2 \\ -3 & \sqrt{3} \end{vmatrix} = 3 - 6$$

$$\underline{\underline{= -3}}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} \sqrt{7} & 6 \\ -3 & \sqrt{7} \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} \sqrt{7} & 6 \\ -3 & \sqrt{7} \end{vmatrix} = 7 + 18$$

$$\underline{= 25}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} \sqrt{5} & 3 \\ -2 & 2 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} \sqrt{5} & 3 \\ -2 & 2 \end{vmatrix} = \underline{2\sqrt{5} + 6}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{8} & -\frac{3}{4} \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{8} & -\frac{3}{4} \end{vmatrix} = -\frac{3}{8} - \frac{1}{16}$$

$$\underline{= -\frac{7}{16}}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} \frac{1}{5} & \frac{1}{6} \\ -6 & -5 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} \frac{1}{5} & \frac{1}{6} \\ -6 & -5 \end{vmatrix} = -1 + 1$$

$$\underline{= 0}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{2} & \frac{3}{4} \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{2} & \frac{3}{4} \end{vmatrix} = \frac{1}{2} + \frac{1}{6}$$
$$= \frac{2}{3}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} x & x^2 \\ 4 & x \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} x & x^2 \\ 4 & x \end{vmatrix} = x^2 - 4x^2$$
$$= -3x^2$$

### ***Exercise***

Evaluate  $\begin{vmatrix} x & x^2 \\ x & 9 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} x & x^2 \\ x & 9 \end{vmatrix} = 9x - x^3$$

### ***Exercise***

Evaluate  $\begin{vmatrix} x^2 & x \\ -3 & 2 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} x^2 & x \\ -3 & 2 \end{vmatrix} = 2x^2 + 3x$$

### ***Exercise***

Evaluate  $\begin{vmatrix} x+2 & 6 \\ x-2 & 4 \end{vmatrix}$

### **Solution**

$$\begin{aligned} \begin{vmatrix} x+2 & 6 \\ x-2 & 4 \end{vmatrix} &= 4(x+2) - 6(x-2) \\ &= 4x + 8 - 6x + 12 \\ &= \underline{-2x + 20} \end{aligned}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} x+1 & -6 \\ x+3 & -3 \end{vmatrix}$

### **Solution**

$$\begin{aligned} \begin{vmatrix} x+1 & -6 \\ x+3 & -3 \end{vmatrix} &= -3x - 3 + 6x + 18 \\ &= \underline{-2x + 20} \end{aligned}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} 3 & 0 & 0 \\ 2 & 1 & -5 \\ 2 & 5 & -1 \end{vmatrix}$

### **Solution**

$$\begin{aligned} \begin{vmatrix} 3 & 0 & 0 \\ 2 & 1 & -5 \\ 2 & 5 & -1 \end{vmatrix} &= \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} - \begin{vmatrix} 3 & 0 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 2 & 5 \end{vmatrix} \\ &= -3 + 0 + 0 - 0 + 75 - 0 \\ &= \underline{72} \end{aligned}$$

### ***Exercise***

Evaluate  $\begin{vmatrix} 4 & 0 & 0 \\ 3 & -1 & 4 \\ 2 & -3 & 6 \end{vmatrix}$

### **Solution**

$$\begin{vmatrix} 4 & 0 & 0 \\ 3 & -1 & 4 \\ 2 & -3 & 6 \end{vmatrix} \begin{matrix} 4 & 0 \\ 3 & -1 \\ 2 & -3 \end{matrix}$$

$$= -24 + 48$$

$$= \underline{24}$$

$$\text{or} = 4 \begin{vmatrix} -1 & 4 \\ -3 & 6 \end{vmatrix}$$

### Exercise

Evaluate  $\begin{vmatrix} 3 & 1 & 0 \\ -3 & -4 & 0 \\ -1 & 3 & 5 \end{vmatrix}$

### Solution

$$\begin{vmatrix} 3 & 1 & 0 \\ -3 & -4 & 0 \\ -1 & 3 & 5 \end{vmatrix} \begin{matrix} 3 & 1 \\ -3 & -4 \\ -1 & 3 \end{matrix}$$

$$= -60 + 15$$

$$= \underline{-45}$$

### Exercise

Evaluate  $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & -4 & 5 \end{vmatrix}$

### Solution

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & -4 & 5 \end{vmatrix} \begin{matrix} 1 & 1 \\ 2 & 2 \\ 3 & -4 \end{matrix}$$

$$= 10 + 6 - 8 - 6 + 8 - 10$$

$$= \underline{0}$$

### Exercise

Evaluate  $\begin{vmatrix} x & 0 & -1 \\ 2 & 1 & x^2 \\ -3 & x & 1 \end{vmatrix}$

### Solution

$$\begin{vmatrix} x & 0 & -1 \\ 2 & 1 & x^2 \\ -3 & x & 1 \end{vmatrix} \begin{matrix} x & 0 \\ 2 & 1 \\ -3 & x \end{matrix} \\
 = x - 2x - 3 - x^4 \\
 = \underline{-x^4 - x - 3}$$

### Exercise

Evaluate  $\begin{vmatrix} x & 1 & -1 \\ x^2 & x & x \\ 0 & x & 1 \end{vmatrix}$

### Solution

$$\begin{vmatrix} x & 1 & -1 \\ x^2 & x & x \\ 0 & x & 1 \end{vmatrix} \begin{matrix} x & 1 \\ x^2 & x \\ 0 & x \end{matrix} \\
 = x^2 - x^3 - x^3 - x^2 \\
 = \underline{-2x^3}$$

### Exercise

Evaluate  $\begin{vmatrix} 4 & -7 & 8 \\ 2 & 1 & 3 \\ -6 & 3 & 0 \end{vmatrix}$

### Solution

$$\begin{vmatrix} 4 & -7 & 8 \\ 2 & 1 & 3 \\ -6 & 3 & 0 \end{vmatrix} = 0 + 126 + 48 - (-48 + 36 + 0) \\
 = \underline{90}$$

### Exercise

Evaluate  $\begin{vmatrix} 2 & 1 & -1 \\ 4 & 7 & -2 \\ 2 & 4 & 0 \end{vmatrix}$



**Solution**

$$\begin{vmatrix} 2 & 1 & -1 \\ 4 & 7 & -2 \\ 2 & 4 & 0 \end{vmatrix} = 0 - 4 - 16 - (-14 - 16 + 0) \\ = 10$$

***Exercise***

Evaluate  $\begin{vmatrix} 3 & 1 & 2 \\ -2 & 3 & 1 \\ 3 & 4 & -6 \end{vmatrix}$

**Solution**

$$\begin{vmatrix} 3 & 1 & 2 \\ -2 & 3 & 1 \\ 3 & 4 & -6 \end{vmatrix} \begin{matrix} 3 & 1 \\ -2 & 3 \\ 3 & 4 \end{matrix} \\ = -54 + 3 - 16 - 18 - 12 - 12 \\ = -109$$

***Exercise***

Evaluate  $\begin{vmatrix} 2x & 1 & -1 \\ 0 & 4 & x \\ 3 & 0 & 2 \end{vmatrix}$

**Solution**

$$\begin{vmatrix} 2x & 1 & -1 \\ 0 & 4 & x \\ 3 & 0 & 2 \end{vmatrix} \begin{matrix} 2x & 1 \\ 0 & 4 \\ 3 & 0 \end{matrix} \\ = 16x + 3x + 12 \\ = 19x + 12$$

***Exercise***

Evaluate  $\begin{vmatrix} 0 & x & x \\ x & x^2 & 5 \\ x & 7 & -5 \end{vmatrix}$

**Solution**

$$\begin{vmatrix} 0 & x & x \\ x & x^2 & 5 \\ x & 7 & -5 \end{vmatrix} \begin{matrix} 0 & x \\ x & x^2 \\ x & 7 \end{matrix} \\
 = 5x^2 + 7x^2 - x^4 + 5x^2 \\
 = \underline{17x^2 - x^4}$$

### Exercise

Evaluate  $\begin{vmatrix} 2 & x & 1 \\ -3 & 1 & 0 \\ 2 & 1 & 4 \end{vmatrix}$

### Solution

$$\begin{vmatrix} 2 & x & 1 \\ -3 & 1 & 0 \\ 2 & 1 & 4 \end{vmatrix} \begin{matrix} 2 & x \\ -3 & 1 \\ 2 & 1 \end{matrix} \\
 = 8 - 3 - 2 + 12x \\
 = \underline{12x + 3}$$

### Exercise

Evaluate  $\begin{vmatrix} 1 & x & -2 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix}$

### Solution

$$\begin{vmatrix} 1 & x & -2 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix} \begin{matrix} 1 & x \\ 3 & 1 \\ 0 & -2 \end{matrix} \\
 = 2 + 12 + 2 - 6x \\
 = \underline{-6x + 16}$$

### Exercise

Solve for x.  $\begin{vmatrix} x & 3 \\ 2 & 1 \end{vmatrix} = 12$

### Solution

$$\begin{vmatrix} x & 3 \\ 2 & 1 \end{vmatrix} = x - 6 = 12$$

$$\therefore \text{Solution: } \underline{x = 18}$$

### Exercise

Solve for  $x$ .  $\begin{vmatrix} x & 1 \\ 2 & x \end{vmatrix} = -1$

### Solution

$$\begin{vmatrix} x & 1 \\ 2 & x \end{vmatrix} = x^2 - 2 = -1$$

$$x^2 = 1$$

$$\therefore \text{Solution: } \underline{x = \pm 1}$$

### Exercise

Solve for  $x$ .  $\begin{vmatrix} 3 & x \\ x & 4 \end{vmatrix} = -13$

### Solution

$$\begin{vmatrix} 3 & x \\ x & 4 \end{vmatrix} = 12 - x^2 = -13$$

$$x^2 = 25$$

$$\therefore \text{Solution: } \underline{x = \pm 5}$$

### Exercise

Solve for  $x$ .  $\begin{vmatrix} x & 2 \\ 3 & x \end{vmatrix} = x$

### Solution

$$\begin{vmatrix} x & 2 \\ 3 & x \end{vmatrix} = x^2 - 6 = x$$

$$x^2 - x - 6 = 0$$

$$\therefore \text{Solution: } \underline{x = -2, 3}$$

### ***Exercise***

Solve for  $x$ .  $\begin{vmatrix} 4 & 6 \\ -2 & x \end{vmatrix} = 32$

#### **Solution**

$$\begin{vmatrix} 4 & 6 \\ -2 & x \end{vmatrix} = 4x + 12 = 32$$

$$4x = 20$$

$$\therefore \text{Solution: } \underline{x = 5}$$

### ***Exercise***

Solve for  $x$ .  $\begin{vmatrix} x+2 & -3 \\ x+5 & -4 \end{vmatrix} = 3x - 5$

#### **Solution**

$$\begin{vmatrix} x+2 & -3 \\ x+5 & -4 \end{vmatrix} = -4x - 8 + 3x + 15 = 3x - 5$$

$$-4x = -12$$

$$\therefore \text{Solution: } \underline{x = 3}$$

### ***Exercise***

Solve for  $x$ .  $\begin{vmatrix} x+3 & -6 \\ x-2 & -4 \end{vmatrix} = 28$

#### **Solution**

$$\begin{vmatrix} x+3 & -6 \\ x-2 & -4 \end{vmatrix} = -4x - 12 + 6x - 12 = 28$$

$$2x = 52$$

$$\therefore \text{Solution: } \underline{x = 26}$$

### ***Exercise***

Solve for  $x$ .  $\begin{vmatrix} x & -3 \\ -1 & x \end{vmatrix} \geq 0$

#### **Solution**

$$\begin{vmatrix} x & -3 \\ -1 & x \end{vmatrix} = x^2 - 3 \geq 0$$

$$x^2 \geq 3$$

$$\therefore \text{Solution: } \underline{x \leq -\sqrt{3} \quad x \geq \sqrt{3}}$$

### Exercise

Solve for  $x$ . 
$$\begin{vmatrix} 2 & x & 1 \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -6$$

### Solution

$$\begin{vmatrix} 2 & x & 1 \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -8 - 3x + 4 - 6 + 8 + 2x = -6$$

$$-x = -4$$

$$\therefore \text{Solution: } \underline{x = 4}$$

### Exercise

Solve for  $x$ . 
$$\begin{vmatrix} 1 & x & -3 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix} = 8$$

### Solution

$$\begin{vmatrix} 1 & x & -3 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix} = 2 + 18 + 2 - 6x = 8$$

$$-6x = -14$$

$$\therefore \text{Solution: } \underline{x = \frac{7}{3}}$$

### Exercise

Solve for  $x$ . 
$$\begin{vmatrix} 2 & x & 1 \\ -3 & 1 & 0 \\ 2 & 1 & 4 \end{vmatrix} = 39$$

### Solution

$$\begin{vmatrix} 2 & x & 1 \\ -3 & 1 & 0 \\ 2 & 1 & 4 \end{vmatrix} = 8 - 3 - 2 + 12x = 39$$

$$12x = 36$$

$$\therefore \text{Solution: } \underline{x = 3}$$

### ***Exercise***

Solve for  $x$ .  $\begin{vmatrix} x & 0 & 0 \\ 7 & x & 1 \\ 7 & 2 & 1 \end{vmatrix} = -1$

### **Solution**

$$\begin{vmatrix} x & 0 & 0 \\ 7 & x & 1 \\ 7 & 2 & 1 \end{vmatrix} = x^2 - 2x = -1$$

$$x^2 - 2x + 1 = 0$$

$$\therefore \text{Solution: } \underline{x = 1}$$