$$\int_{-\infty}^{\infty} x e^{-x^2} dx = -\frac{1}{2}\int_{-\infty}^{\infty} e^{-x^2} d(-x^2)$$

$$= -\frac{1}{2} e^{-x^2}\Big|_{-\infty}^{\infty}$$

$$= -\frac{1}{2}\left(e^{-\infty} - e^{-\infty}\right)$$

$$= -\frac{1}{2}(1-1)$$

$$= \boxed{0}$$

$$\int_0^{\infty} \cos x \, dx = \sin x \Big|_0^{\infty}$$

$\sin\infty \nexists$

$$= \sin\infty - \sin 0$$

$= \text{doesn't exist} \Rightarrow \text{diverges}$

$$\int_2^{\infty} \frac{\cos(\pi/x)}{x^2} dx = -\frac{1}{\pi}\int_2^{\infty} \cos\left(\frac{\pi}{x}\right) d\left(\frac{\pi}{x}\right)$$

$d\left(\frac{\pi}{x}\right) = -\frac{\pi}{x^2} dx$

$$= -\frac{1}{\pi} \sin\frac{\pi}{x}\Big|_2^{\infty}$$

$\frac{\pi}{\infty} \to 0.$

$$= -\frac{1}{\pi}\left(\sin 0 - \sin\frac{\pi}{2}\right)$$

$$= \boxed{\frac{1}{\pi}}$$

$$\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+5} = \int_{-\infty}^{\infty} \frac{dx}{(x+1)^2+4}$$

$x+1 = 2\tan\theta$

$dx = 2\sec^2\theta \, d\theta$

$(x+1)^2+4 = 4\sec^2\theta$

$$= \int_{-\infty}^{\infty} \frac{2\sec^2\theta \, d\theta}{4\sec^2\theta}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} d\theta$$

$$= \frac{1}{2}\theta\Big|_{-\infty}^{\infty}$$

$$= \frac{1}{2}\tan^{-1}\frac{x+1}{2}\Big|_{-\infty}^{\infty}$$

$$= \frac{1}{2}\left(\tan\infty - \tan(-\infty)\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$= \boxed{\pi/2}$$

$$\int_{-\infty}^{a} \sqrt{e^x}\,dx = \int_{-\infty}^{a} e^{x/2}\,dx \qquad a \in \mathbb{R}.$$

$$= 2e^{x/2}\Big|_{-\infty}^{a}$$

$$= 2(e^{a/2} - 0)$$

$$= \boxed{2e^{a/2}}$$

$$\int_0^\infty \frac{e^u}{e^{2u}+1}\,du = \int_0^\infty \frac{d(e^u)}{(e^u)^2+1} \qquad \int \frac{dx}{x^2+1} = \tan^{-1}x$$

$$= \tan^{-1}e^u\Big|_0^\infty$$

$$= \tan^{-1}\infty - \tan^{-1}1$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \boxed{\frac{\pi}{4}}$$

$$\int_1^\infty \frac{dx}{x(x+1)} = \int_1^\infty \left(\frac{1}{x} - \frac{1}{x+1}\right)dx$$

$$= \ln|x| - \ln|x+1|\Big|_1^\infty$$

$$= \ln\frac{x}{x+1}\Big|_1^\infty \qquad \lim_{x\to\infty} \ln\frac{x}{x+1} = \ln 1 = 0$$

$$= 0 - \ln\frac{1}{2}$$

$$= \boxed{\ln 2}$$

$$\int_1^\infty \frac{dx}{x^2(x+1)} = \int_1^\infty \left(\frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}\right)dx \qquad \frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$1 = Ax^2 + Ax + Bx + B + Cx^2$$

$$\begin{cases} A + C = 0 \to C = 1 \\ A + B = 0 \to A = -1 \\ B = 1 \end{cases}$$

$$= -\ln x - \frac{1}{x} + \ln(x+1)\Big|_1^\infty$$

$$= \ln\frac{x+1}{x} - \frac{1}{x}\Big|_1^\infty$$

$$= \ln 1 - 0 - \ln 2 + 1 \qquad \lim_{x\to\infty} \ln\frac{x+1}{x} = \ln 1$$

$$= \boxed{1 - \ln 2}$$

$$\int_1^\infty \frac{3x^2+1}{x^3+x}\,dx = \int_1^\infty \frac{d(x^3+x)}{x^3+x}$$
$$= \ln(x^3+x)\Big|_1^\infty$$
$$= \infty - \ln 2$$
$$= \boxed{\infty} \text{ diverges.}$$

$$\int_1^\infty \frac{1}{x^2}\sin\frac{\pi}{x}\,dx = \frac{-1}{\pi}\int_1^\infty \sin\left(\frac{\pi}{x}\right)d\left(\frac{\pi}{x}\right) \qquad d\left(\frac{\pi}{x}\right) = -\frac{\pi}{x^2}dx$$
$$= \frac{1}{\pi^2}\cos\frac{\pi}{x}\Big|_1^\infty$$
$$= \frac{1}{\pi^2}(\cos 0 - \cos\pi)$$
$$= \boxed{\frac{2}{\pi^2}}$$

$$\int_2^\infty \frac{dx}{(x+2)^2} = \int_2^\infty \frac{d(x+2)}{(x+2)^2}$$
$$= -\frac{1}{x+2}\Big|_2^\infty$$
$$= -\left(0 - \frac{1}{4}\right)$$
$$= \boxed{\frac{1}{4}}$$

$$\int_1^\infty \frac{\tan^{-1}x}{x^2+1}\,dx = \int_1^\infty \tan^{-1}x \, d(\tan^{-1}x)$$
$$= \frac{1}{2}(\tan^{-1}x)^2\Big|_1^\infty$$
$$= \frac{1}{2}\left[\left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{4}\right)^2\right]$$
$$= \frac{1}{2}\left(\frac{\pi^2}{4} - \frac{\pi^2}{16}\right)$$
$$= \boxed{\frac{3\pi^2}{32}}$$

$$\int_0^8 \frac{dx}{\sqrt[3]{x}} = \int_0^8 x^{-1/3}\, dx$$

$$= \frac{3}{2}\, x^{2/3} \Big|_0^8$$

$$= \frac{3}{2}\,(4-0)$$

$$= \boxed{6}$$

$$\int_{-3}^{1} \frac{dx}{(2x+6)^{2/3}} = \frac{1}{2} \int_{-3}^{1} (2x+6)^{-2/3}\, d(2x+6)$$

$$= \frac{3}{2}\,(2x+6)^{1/3} \Big|_{-3}^{1}$$

$$= \frac{3}{2}\,[2-0]$$

$$= \boxed{3}$$

$$\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2\int_0^1 e^{\sqrt{x}}\, d(\sqrt{x}) \qquad d(\sqrt{x}) = \frac{1}{2\sqrt{x}}\, dx$$

$$= 2\, e^{\sqrt{x}} \Big|_0^1$$

$$= \boxed{2(e-1)}$$

$$\int_0^{\ln 3} \frac{e^x}{(e^x-1)^{2/3}}\, dx = \int_0^{\ln 3} (e^x-1)^{-2/3}\, d(e^x-1)$$

$$= 3\,(e^x-1)^{1/3} \Big|_0^{\ln 3}$$

$$= 3\,(2^{1/3} - 0)$$

$$= \boxed{3\sqrt[3]{2}}$$