Solution *Section* 3.2 - Exponential Functions

Exercise

Evaluate to four decimal places using a calculator $2^{3.4}$

Solution

$$2^{3.4} = 10.5561$$

Exercise

Evaluate to four decimal places using a calculator $5^{\sqrt{3}}$

Solution

$$5^{\sqrt{3}} = 16.2425$$

Exercise

Evaluate to four decimal places using a calculator $6^{-1.2}$

Solution

$$6^{-1.2} = 0.1165$$

Exercise

Evaluate to four decimal places using a calculator: $e^{-0.75}$

Solution

$$e^{-0.75} = .4724$$

Exercise

Evaluate to four decimal places using a calculator: $e^{2.3}$

Solution

$$e^{2.3} = 9.9742$$

Exercise

Evaluate to four decimal places using a calculator: $e^{-0.95}$

Solution

$e^{-0.95} = 0.3867$

Exercise

Evaluate to four decimal places using a calculator: $\pi^{\sqrt{\pi}}$

Solution

$\pi^{\sqrt{\pi}} = 7.6063$

Exercise

Evaluate to four decimal places using a calculator: $e^{\sqrt{2}}$

Solution

$e^{\sqrt{2}} = 4.1133$

Exercise

Sketch the graph: $f(x) = 2^x + 3$

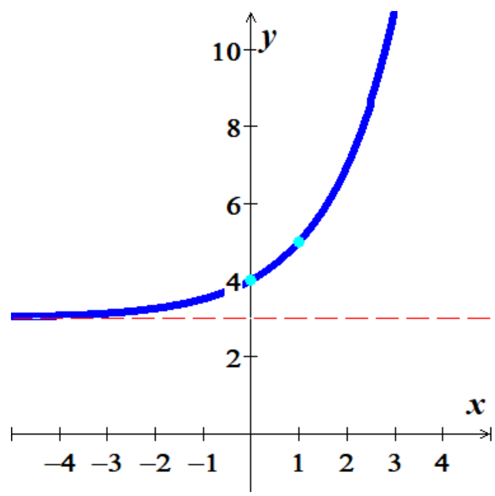
Solution

Asymptote: $y = 3$

Domain: $(-\infty, \infty)$

Range: $(3, \infty)$

x	$f(x)$
–1	3.5
0	4
1	5
2	7



Exercise

Sketch the graph: $f(x) = 2^{3-x}$

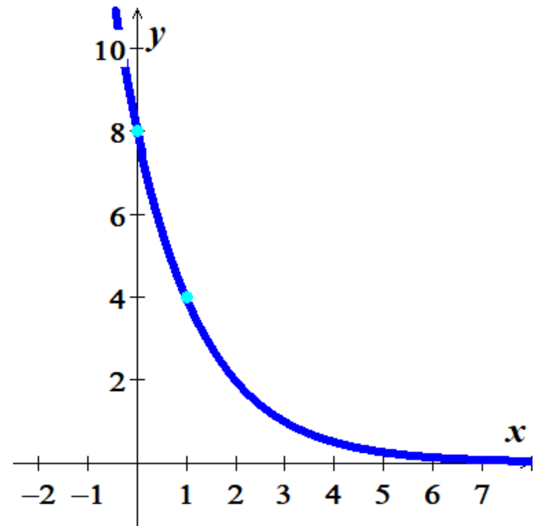
Solution

Asymptote: $y = 0$

Domain: $(-\infty,\ \infty)$

Range: $(0,\ \infty)$

x	$f(x)$
1	4
2	2
0	8



Exercise

Sketch the graph: $f(x) = \left(\frac{2}{5}\right)^{-x}$

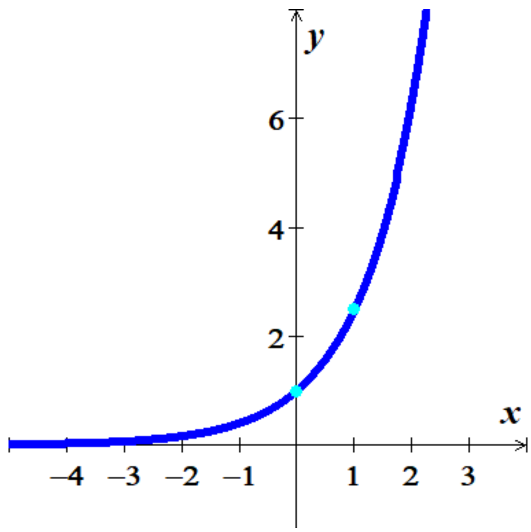
Solution

Asymptote: $y = 0$

Domain: $(-\infty,\ \infty)$

Range: $(0,\ \infty)$

x	$f(x)$
–1	0.4
0	1
1	2.5



Exercise

Sketch the graph: $f(x) = -\left(\frac{1}{2}\right)^{x} + 4$

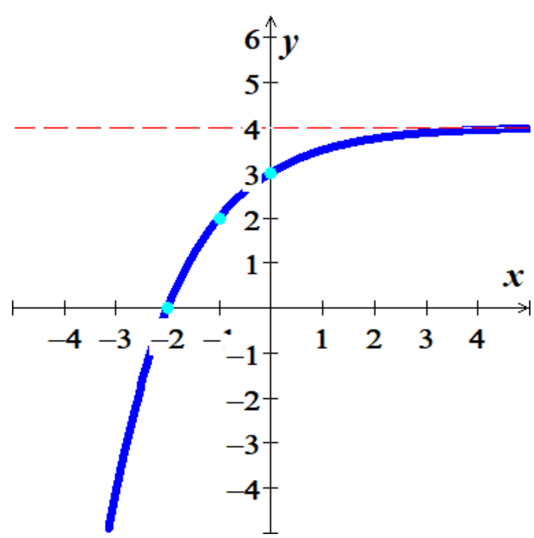
Solution

Asymptote: $y = 4$

Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ 4)$

x	$f(x)$
–2	0
–1	2
0	3



Exercise

Sketch the graph of $f(x) = 4^x$

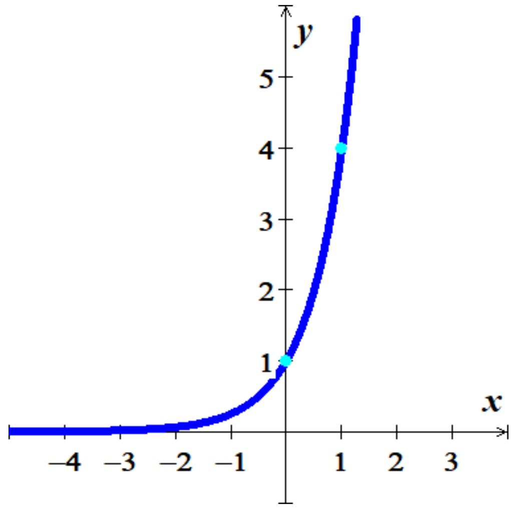
Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
0	1
1	4



Exercise

Sketch the graph of $f(x) = 2 - 4^x$

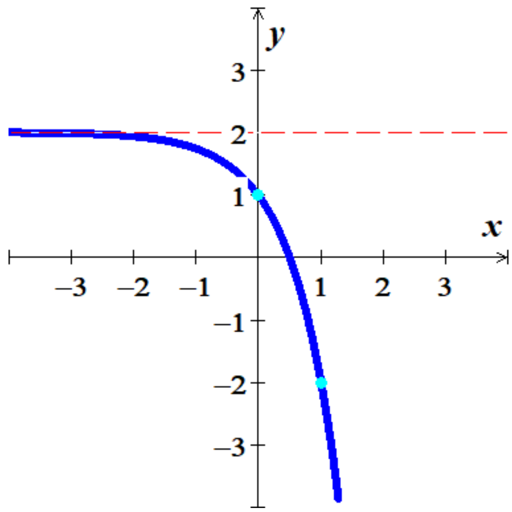
Solution

Asymptote: $y = 2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 2)$

x	$f(x)$
0	1
1	–2



Exercise

Sketch the graph of $f(x) = -3 + 4^{x-1}$

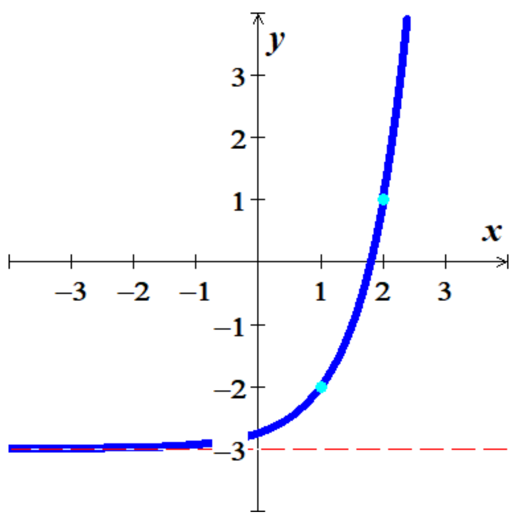
Solution

Asymptote: $y = -3$

Domain: $(-\infty, \infty)$

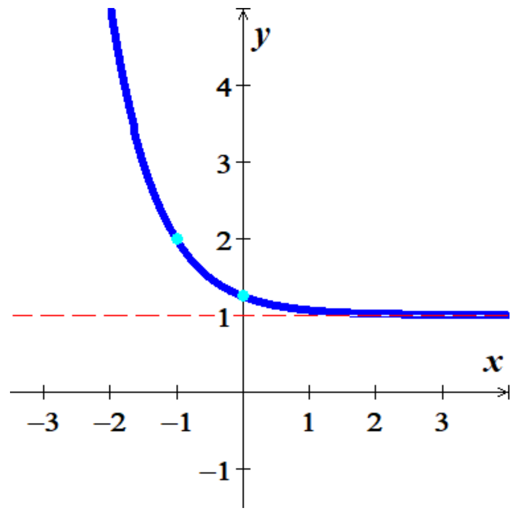
Range: $(-3, \infty)$

x	$f(x)$
1	–2
2	1



Exercise

Sketch the graph of $f(x) = 1 + \left(\frac{1}{4}\right)^{x+1}$



Solution

Asymptote: $y = 1$

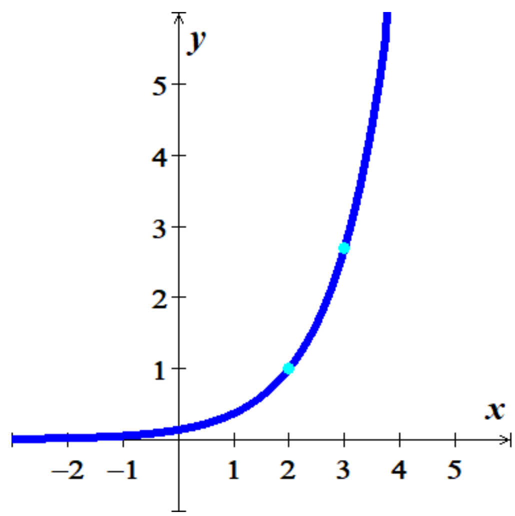
Domain: $(-\infty, \infty)$

Range: $(1, \infty)$

x	$f(x)$
–1	2
0	$\frac{5}{4}$

Exercise

Sketch the graph of $f(x) = e^{x-2}$



Solution

Asymptote: $y = 0$

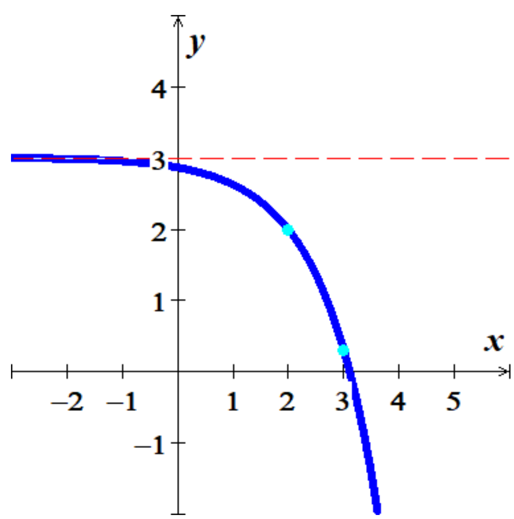
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
2	1
3	2.7

Exercise

Sketch the graph of $f(x) = 3 - e^{x-2}$



Solution

Asymptote: $y = 3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

x	$f(x)$
2	2
3	.3

Exercise

Sketch the graph of $f(x) = e^{x+4}$

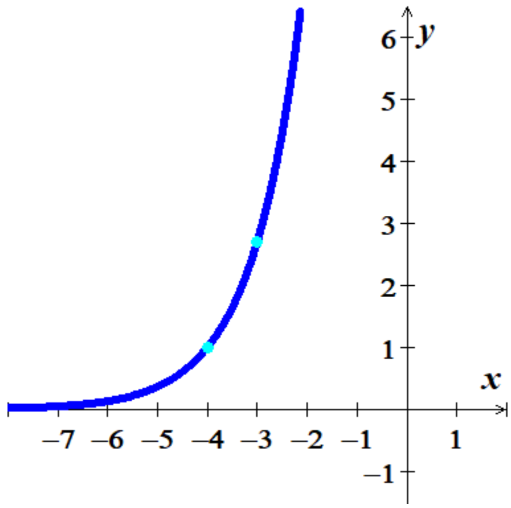
Solution

Asymptote: $y = 0$

Domain: $(-\infty,\ \infty)$

Range: $(0,\ \infty)$

x	$f(x)$
–4	1
–3	2.7



Exercise

Sketch the graph of $f(x) = 2 + e^{x-1}$

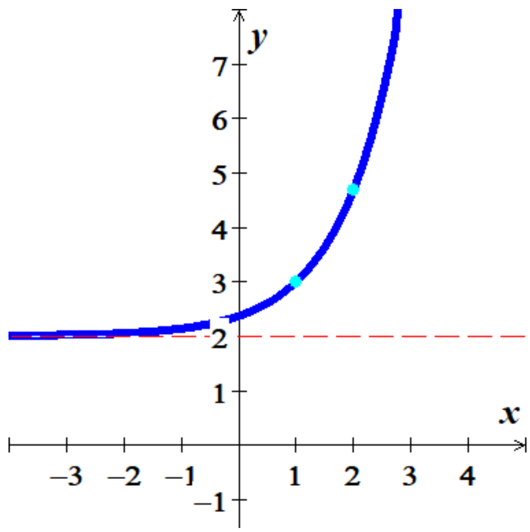
Solution

Asymptote: $y = 2$

Domain: $(-\infty,\ \infty)$

Range: $(2,\ \infty)$

x	$f(x)$
1	3
2	4.7



Exercise

The exponential function $f(x) = 1066e^{0.042x}$ models the gray wolf population of the Western Great Lakes, $f(x)$, in *billions*, x *years* after 1978. Project the gray population in the recovery area in 2012.

Solution

$x = 2012 - 1978 = 34$

$f(x = 34) = 1066e^{0.042(34)}$

$= 4445.6$

≈ 4446 *billions*

Exercise

The function $f(x) = 6.4e^{0.0123x}$ describes world population, $f(x)$, in billions, x *years* after 2004 subject to a growth rate of 1.23% annually. Use the function to predict world population in 2050.

Solution

$$x = 2050 - 2004 = 46$$

$$f(x=46) = 6.4e^{0.0123(46)}$$

$$\approx 11.27 \ \ billion$$

Exercise

A cup of coffee is heated to $160°F$ and placed in a room that maintains a temperature of $70°F$. The temperature T of the coffee, in *degree Fahrenheit*, after t minutes is given by

$$T(t) = 70 + 90e^{-0.0485t}$$

a) Find the temperature of the coffee 20 *minutes* after it is placed in the room

b) Determine when the temperature of the coffee will reach $90°F$

Solution

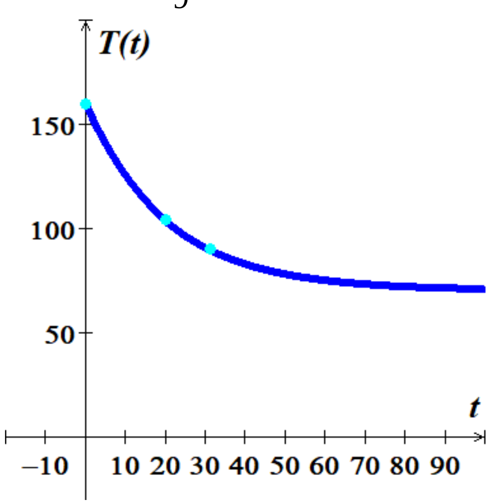
a) $T(20) = 70 + 90e^{-0.0485(20)}$

$$\approx 104°F$$

b) $T(t) = 70 + 90e^{-0.0485t} = 90$

$$90e^{-0.0485t} = 20$$

$$e^{-0.0485t} = \frac{2}{9}$$



31.01000	90.00185
31.02000	89.99215

∴ The temperature of the coffee will reach $90°F$ in about **31.01** *minutes*.

Exercise

A cup of coffee is heated to $180°F$ and placed in a room that maintains a temperature of $65°F$. The temperature *T* of the coffee, in *degree Fahrenheit,* after *t* minutes is given by

$$T(t) = 65 + 115e^{-0.042t}$$

a) Find the temperature of the coffee 10 *minutes* after it is placed in the room

b) Determine when the temperature of the coffee will reach $100°F$

Solution

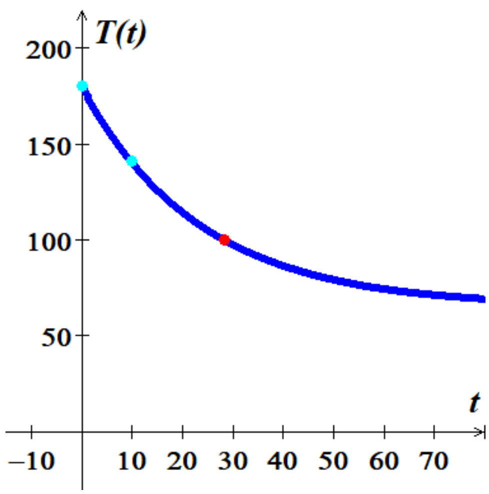
a) $T(10) = 65 + 115e^{-0.042(10)}$

$\approx 141°F$

b) $T(t) = 65 + 115e^{-0.042t} = 100$

$115e^{-0.042t} = 35$

$e^{-0.042t} = \frac{7}{23}$

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28.32000 100.00504
28.34000  99.97565
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$\therefore$ The temperature of the coffee will reach $100°F$ in about **31.01** *minutes*.

Exercise

The percent $I(x)$ of the original intensity of light striking the surface of a lake that is available *x feet* below the surface of the lake is given by the equation

$$I(x) = 100e^{-.95x}$$

a) What percentage of the light is available 2 *feet* below the surface of the lake?

b) At what depth is the intensity of the light one-half the intensity at the surface?

Solution

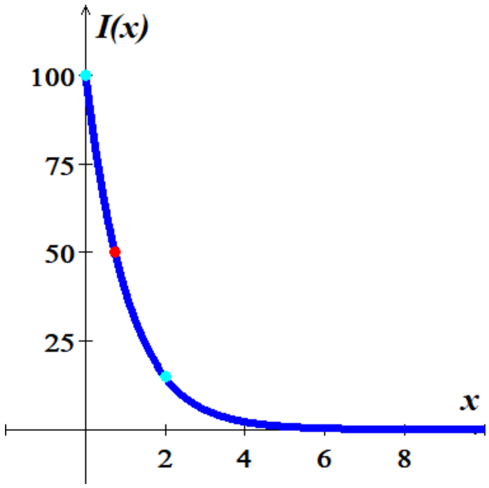
a) $I(2) = 100e^{-.95(2)}$

≈ 14.96

$\therefore$ The percentage of the light is available 2 *feet* below the surface of the lake is **15%**

b) $I(x) = 100e^{-.95x} = \frac{1}{2}(100)$

$e^{-.95x} = \frac{1}{2}$

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0.72800   50.07742
0.73200   49.88749
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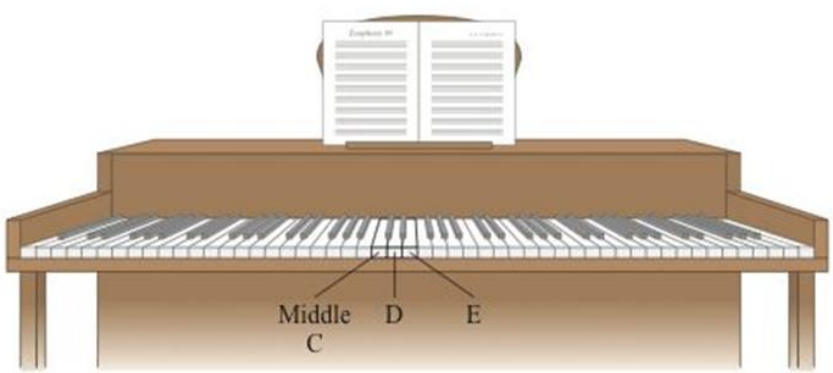
$\therefore$ The depth is **0.73** *feet* when the intensity of the light one-half the intensity at the surface

Exercise

Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

$$f(n) = (2.75)\ 2^{\frac{n-1}{12}}$$



a) Determine the frequency of middle *C*, key number 40 on an 88–*key* piano.

b) Is the difference in frequency between middle *C* (key number 40) and *D* (key number 42) the same as the difference in frequency between *D* (key number 42) and *E* (key number 44)?

Solution

a) $f(40) = (2.75)\ 2^{\frac{40-1}{12}}$

≈ 26.16

the frequency of middle C is ≈ 26 *vibrations* per *second*.

b) $f(42) = (2.75)\ 2^{(41/12)}$

≈ 29.37

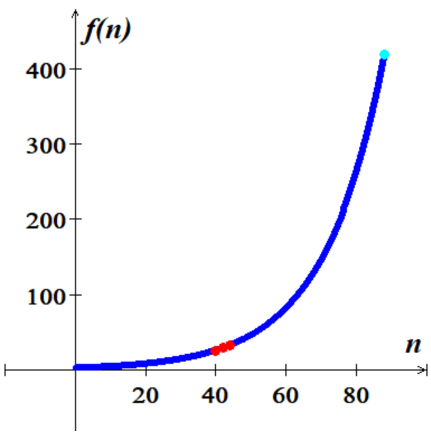
The difference between the frequency of middle C and D is:
$29.37 - 26.16 \approx 3.21$

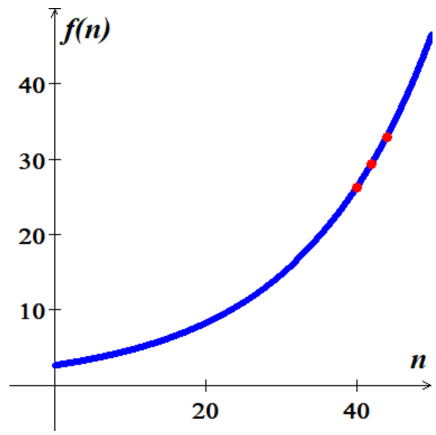
$f(44) = (2.75)\ 2^{(43/12)}$

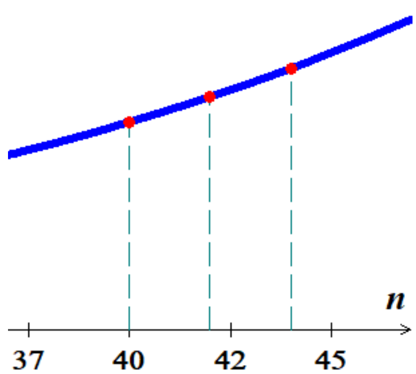
≈ 32.96

The difference between the frequency of middle D and E is:
$32.96 - 29.37 \approx 3.59$

$\therefore$ the differences are ***not*** the same since the function is *not* linear function



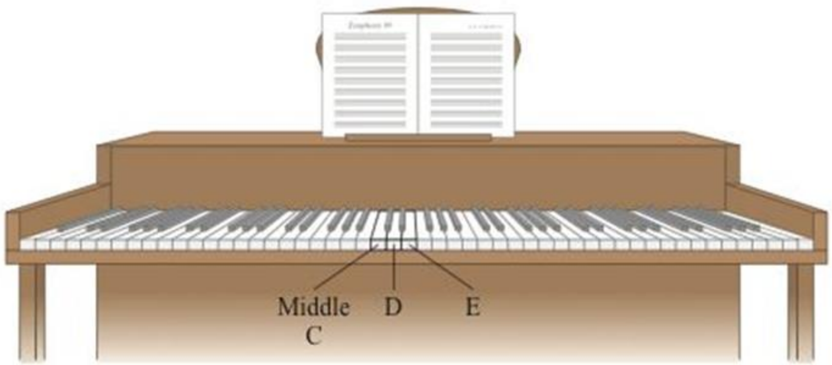




Exercise

Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

$$f(n) = (27.5)\ 2^{\frac{n-1}{12}}$$



c) Determine the frequency of middle *C*, key number 40 on an 88–*key* piano.

d) Is the difference in frequency between middle *C* (key number 40) and *D* (key number 42) the same as the difference in frequency between *D* (key number 42) and *E* (key number 44)?

Solution

c) $f(40) = (27.5)\ 2^{\frac{40-1}{12}}$

≈ 261.63

the frequency of middle *C* is ≈ 262 *vibrations* per *second*.

d) $f(42) = (27.5)\ 2^{(41/12)}$

≈ 293.66

The difference between the frequency of middle *C* and *D* is: $293.66 - 261.66 \approx 32$

$f(44) = (27.5)\ 2^{(43/12)}$

≈ 329.63

The difference between the frequency of middle *D* and *E* is: $329.63 - 293.66 \approx 36$

$\therefore$ The differences are ***not*** the same since the function is *not* linear function.