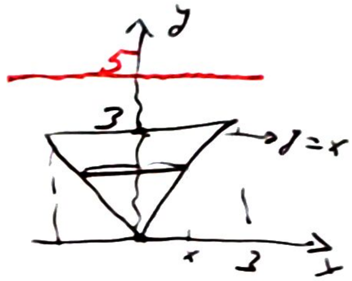
$$F = \rho g \int_0^a \underbrace{(a-y)}_{\text{depth}}\ \underbrace{w(y)}_{\text{width}}\, dy$$

Ex



$$\rho g = 62.4 \ lb/ft^3 = \frac{624}{10} = \frac{312}{5}$$

$$L(y) = 2x = 2y \qquad (x = y)$$

$$F = \frac{312}{5} \int_0^3 (5-y)(2y)\, dy$$

$$= \frac{624}{5} \int_0^3 (10y - 2y^2)\, dy$$

$$= \frac{624}{5} \left(5y^2 - \frac{2}{3} y^3 \right|_0^3$$

$$= \frac{624}{5} (45 - 18)$$

$$= \frac{(624)(27)}{5}$$

1.8 Ex

$$\int_0^4 \frac{x}{x^2+9}\, dx = \frac{1}{2} \int_0^4 \frac{d(x^2+9)}{x^2+9}$$

$$= \frac{1}{2} \ln(x^2+9) \Big|_0^4$$

$$= \frac{1}{2} (\ln 25 - \ln 9)$$

$$= \frac{1}{2} (\ln 5^2 - \ln 3^2)$$

$$= \frac{1}{2} (2\ln 5 - 2\ln 3)$$

$$= \ln \frac{5}{3}$$

$\ln x^n = n \ln x$

$\ln a - \ln b = \ln \frac{a}{b}$

$\ln a + \ln b = \ln ab.$

$\ln \frac{1}{x} = -\ln x$

$\ln e^x = x$

$\ln e = 1$

$\ln 1 = 0$

$$\int \frac{e^x}{1+e^x}\,dx = \int \frac{d(1+e^x)}{1+e^x} \qquad d(1+e^x) = e^x dx$$

$$= \ln(1+e^x) + C$$

#1 $y = \ln\left(\frac{\sqrt{\sin\theta\cos\theta}}{1+2\ln\theta}\right)$

$$= \ln(\sin\theta\cos\theta)^{1/2} - \ln(1+2\ln\theta)$$

$$= \frac{1}{2}\ln\left(\frac{1}{2}\sin 2\theta\right) - \ln(1+2\ln\theta)$$

$$y' = \frac{1}{2}\,\frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} - \frac{2\,\frac{1}{\theta}}{1+2\ln\theta}$$

$$= \cot 2\theta - \frac{2}{\theta(1+2\ln\theta)}$$

#2 $f(x) = e^{4\sqrt{x}+x^2}$ $\qquad (e^u)' = u'e^u$

$$f'(x) = \left(\frac{2}{\sqrt{x}} + 2x\right)e^{4\sqrt{x}+x^2} \qquad (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$(u^n)' = n\,u'u^{n-1}$$

$$\left(\frac{1}{u^n}\right)' = -\frac{n\,u'}{u^{n+1}}$$

#4 $f(x) = \frac{e^{\sqrt{x}}}{\ln(\sqrt{x}+1)}$ $\qquad \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

$$f'(x) = \frac{1}{(\ln(\sqrt{x}+1))^2}\left(\frac{1}{2\sqrt{x}}e^{\sqrt{x}}\ln(\sqrt{x}+1) - \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}+1}\,e^{\sqrt{x}}\right)$$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}\,(\ln(\sqrt{x}+1))^2}\left(\ln(\sqrt{x}+1) - \frac{1}{\sqrt{x}+1}\right)$$

$$= \frac{e^{\sqrt{x}}\left((\sqrt{x}+1)\ln(\sqrt{x}+1) - 1\right)}{2(x+\sqrt{x})(\ln(\sqrt{x}+1))^2}$$

$\sinh x = \frac{e^{x} - e^{-x}}{2}$ $\qquad$ $\cosh = \frac{e^{x} + e^{-x}}{2}$

$\frac{d}{dx}\left(\tanh \sqrt{1+t^{2}}\right) = \frac{1}{2} 2t\,(1+t^{2})^{-1/2}\, \operatorname{sech}^{2} \sqrt{1+t^{2}}$

$\boxed{(u^{n})' = n u' u^{n-1}}$ $\qquad = \frac{t}{\sqrt{1+t^{2}}}\, \operatorname{sech}^{2} \sqrt{1+t^{2}}$

$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$

$\int \coth 5x \, dx = \frac{1}{5}\int \frac{\cosh 5x}{\sinh 5x}\, d(5x)$ $\qquad d(5x) = 5dx$

$= \frac{1}{5} \int \frac{d(\cosh 5x)}{\sinh 5x}$

$= \frac{1}{5} \ln |\sinh 5x| + C$

$\int_{0}^{\ln 2} 4 e^{x} \sinh x \, dx = 4\int_{0}^{\ln 2} e^{x} \left(\frac{e^{x} - e^{-x}}{2}\right) dx$

$= 2 \int_{0}^{\ln 2} (e^{2x} - 1)\, dx$

$= 2 \left(\frac{1}{2} e^{2x} - x\right) \Big|_{0}^{\ln 2}$

$\boxed{e^{\ln x} = x}$ $\qquad = 2\left(\frac{1}{2} e^{2\ln 2} - \ln 2 - \left(\frac{1}{2}\right)\right)$

$= e^{\ln 2^{2}} - 2\ln 2 - 1$

$= 4 - 2\ln 2 - 1$

$= 3 - 2\ln 2$

$\cosh 1 = \frac{1}{2}\left(e + \frac{1}{e}\right)$

1 - Area

2 → Volume

2 → length $ax^n + bx^m$, $ae^{-nx} + be^{nx}$

1 - surface

1 mass $= \int \rho\, dx$

1 - $F = \rho g \int (a-y)\, L(y)\, dy$

1 - der. ln/e

2 - $\int$ hyp + ln + e

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4 83. $\int_0^{\ln 2} 2e^{-x} \cosh x\, dx = \int_0^{\ln 2} 2e^{-x} \frac{1}{2}(e^{x} + e^{-x})\, dx$

$$= \int_0^{\ln 2} (1 + e^{-2x})\, dx$$

$$= x - \frac{1}{2} e^{-2x} \Big|_0^{\ln 2}$$

$$= \ln 2 - \frac{1}{2} e^{-2\ln 2} + \frac{1}{2}$$

$$= \ln 2 - \frac{1}{2} \underbrace{e^{\ln 2^{-2}}}_{\frac{1}{4}} + \frac{1}{2}$$

$$= \ln 2 - \frac{1}{8} + \frac{1}{2}$$

$$= \ln 2 + \frac{3}{8}$$

pg 107 #44

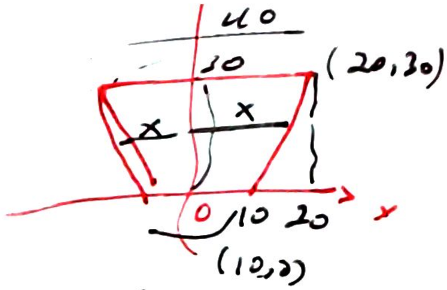
$y = m(x - x_0) + y_0$

$y = \frac{30-0}{20-10}(x-10)$

$= 3x - 30$

$x = \frac{y+30}{3}$

$L = 2x = \frac{2}{3}(y+30)$



$F = \rho g \int_0^a (a-y)\, L(y)\, dy$

$F = 10^3 (9.8) \int_0^{30} (30-y) \frac{2}{3} (30+y)\, dy$ $\quad 98 \; 10^{-1}$

$= \frac{98 \; 10^2 \times 2}{3} \int_0^{30} (900 - y^2)\, dy$

$= \frac{196}{3} 10^2 \left(900y - \frac{1}{3} y^3 \right|_0^{30}$ $\qquad 3(10^1)$

$= \frac{196}{3} 10^2 (27 \times 10^3 - 9 \; 10^3)$ $\qquad \frac{196}{6}$

$= \frac{196}{3} 10^5 (18)$

$= 1176 \times 10^5$

1.7 #5 $\rho(x) = x\sqrt{2-x^2}$ $\quad 0 \le x \le 1$

$m = \int_0^1 x (2-x^2)^{1/2} dx$

$= -\frac{1}{2} \int_0^1 (2-x^2)^{1/2} d(2-x^2)$ $\qquad d(2-x^2) = -2x\,dx$

$= -\frac{1}{3} (2-x^2)^{3/2} \Big|_0^1$

$= -\frac{1}{3} (1 - 2^{3/2})$

$= \frac{1}{3} (2^{3/2} - 1)$ unit.

18 (1.6)

$$y = \frac{1}{3}x^3 + \frac{1}{4x} \qquad \frac{1}{2} \le x \le 2$$

$a = \frac{1}{3}$ $m = 3$ $b = \frac{1}{4}$ $n = -1$

$$\begin{cases} m + n = 3 - 1 = 2 \checkmark \\ abmn = \frac{1}{3}\left(\frac{1}{4}\right)(3)(-1) = -\frac{1}{4} \checkmark \end{cases}$$

$$y' = x^2 - \frac{1}{4x^2}$$

$$S = 2\pi \int_{1/2}^{2} \left(\frac{1}{3}x^3 + \frac{1}{4x}\right)\left(x^2 + \frac{1}{4x^2}\right)dx$$

$$= 2\pi \int_{1/2}^{2} \left(\frac{1}{3}x^5 + \underbrace{\frac{1}{2}x + \frac{1}{4}x}_{3/4} + \frac{1}{16}x^{-3}\right)dx$$

$$= 2\pi \left(\frac{1}{18}x^6 + \frac{3}{8}x^2 - \frac{1}{32}x^{-2} \right|_{1/2}^{2} \qquad \frac{1}{32x^2}$$

$$= 2\pi\left(\frac{32}{9} + \frac{3}{2} - \frac{1}{128} - \left(\frac{1}{64 \times 18} + \frac{3}{8} - \frac{1}{8}\right)\right)$$

$$= 2\pi\left(\frac{32}{9} + \frac{191}{128} - \frac{1}{1,152} - \frac{1}{4}\right)$$

$$\overline{1,152}$$

5 #10

$$y = e^{2x} + \frac{1}{16} e^{-2x} \qquad 0 \le x \le \ln 3$$

$a = 1 \quad m = 2 \quad b = \frac{1}{16} \quad n = -2$

$m = -n$ ✓

$abmn = 2\left(\frac{1}{16}\right)(-2) = -\frac{1}{4}$ ✓

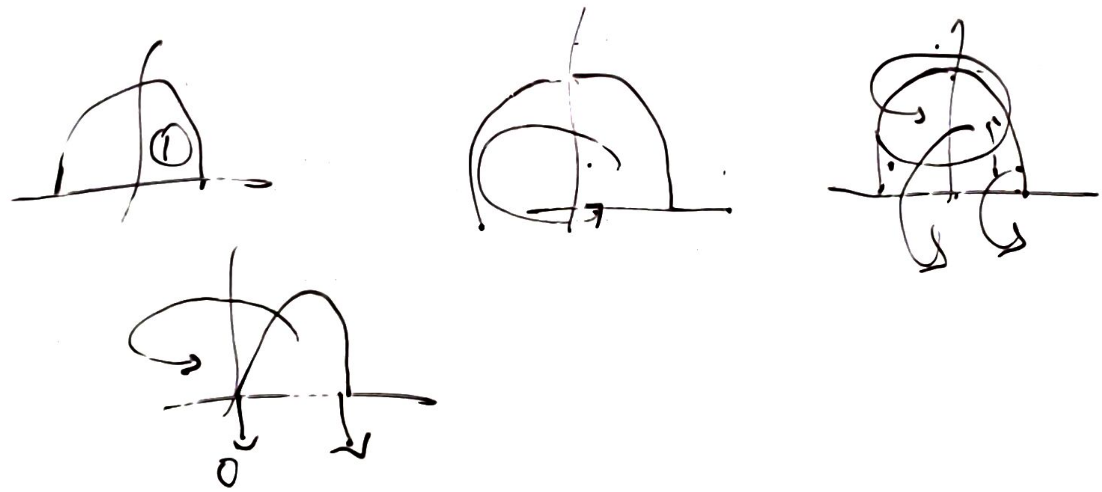
$$L = e^{2x} - \frac{1}{16} e^{-2x} \Big|_0^{\ln 3}$$

$$e^{2\ln 3} = e^{\ln 3^2}$$

$$= e^{\ln 9} - \frac{1}{16} e^{\ln 3^{-2}} - 1 + \frac{1}{16}$$

$$= 9 - \frac{1}{16}\frac{1}{9} - \frac{15}{16}$$

$$= \frac{1}{16}\left(9(16) - \frac{1}{9} - 15\right)$$



5) $y = \sqrt{x}$ $y = 2$ $x = 0$

c) $x = 4$ (y-axis)

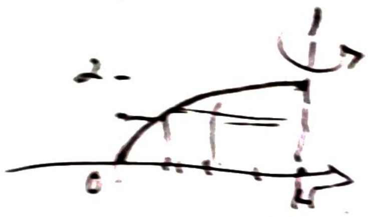
$$V = 2\pi \int_0^4 (4-x)(2-\sqrt{x})\,dx$$

$$= 2\pi \int_0^4 (8 - 4x^{1/2} - 2x + x^{3/2})\,dx$$

$$= 2\pi \left(8x - \frac{8}{3}x^{3/2} - x^2 + \frac{2}{5}x^{5/2}\right)\Big|_0^4 = 2^2$$

$y =$ (x axis)

25) $y = \sqrt{x}$ $y = 0$ $x = 0$ and $x = 4$



$V =$ dx