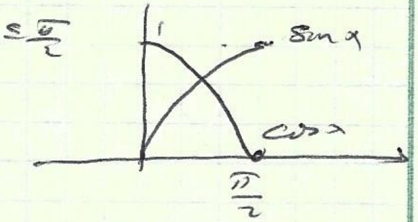
1/ $f(x) = \cos x \qquad g(x) = \sin x \qquad 0 \le x \le \frac{\pi}{2}$



A. $\cos x = \sin x \Rightarrow x = \frac{\pi}{4}$

$$\text{Area} = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$

$$= \sin x + \cos x \Big|_0^{\pi/4} + (-\cos x - \sin x)\Big|_{\pi/4}^{\pi/2}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 + \left(-1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$= 2\sqrt{2} - 2 \text{ unit}^2$$

2/ $y^2 = 2x + 6 \qquad y = x - 1$

$x = \frac{1}{2}y^2 - 3 = x = y + 1$

$\frac{1}{2}y^2 - y - 4 = 0 \Rightarrow y^2 - 2y - 8 = 0$

$y = -2, 4$

$$\text{Area} = \int_{-2}^{4} \left(y + 1 - \frac{1}{2}y^2 + 3\right) dy$$

$$= -\frac{1}{6}y^3 + \frac{1}{2}y^2 + 4y \Big|_{-2}^{4}$$

$$= -\frac{32}{3} + 8 + 16 - \frac{4}{3} - 2 + 8$$

$$= 18 \text{ unit}^2$$

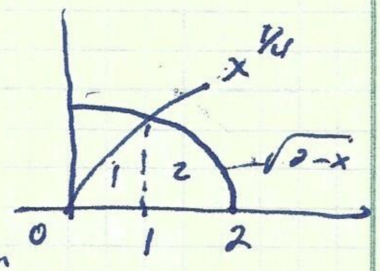
(or) $y^2 = 2x + 6 = (x-1)^2 \Rightarrow 2x + 6 = x^2 - 2x + 1$

$x^2 - 4x - 5 = 0 \Rightarrow x = -1, 5$

$$A = \int_{-1}^{5} (2x + 6 - x^2 + 2x - 1)\,dx$$

3/ $y^4 = x \qquad y = \sqrt{2-x} \; (x \le 2) \qquad y = 0$

$y^2 = 2 - x$



$y^4 = x = (2-x)^2$

$= 4 - 4x + x^2$

$x^2 - 5x + 4 = 0 \Rightarrow x = 1, 4$ → not in domain

$$\text{Area} = \int_0^1 x^{1/4}\,dx + \int_1^2 -(2-x)^{1/2}\,d(2-x)$$

$$= \frac{4}{5}x^{5/4}\Big|_0^1 - \frac{2}{3}(2-x)^{3/2}\Big|_1^2$$

$$= \frac{4}{5} - \frac{2}{3}(0-1)$$

$$= \frac{4}{5} + \frac{2}{3}$$

$$= \frac{22}{15} \text{ unit}^2.$$

or $\quad x = y^4 = -y^2 + 2$

$y^4 + y^2 - 2 = 0 \Rightarrow y^2 = 1, \cancel{-2}$

$y = 1$ only.

$$\text{Area} = \int_0^1 (2 - y^2 - y^4)\,dy$$

$$= 2y - \frac{1}{3}y^3 - \frac{1}{5}y^5 \Big|_0^1$$

$$= 2 - \frac{1}{3} - \frac{1}{5}$$

$$= \frac{22}{15} \text{ unit}^2$$