#19 $x = 1 - 3\sin 4\pi t \Rightarrow \sin 4\pi t = \frac{1-x}{3}$

$y = 2 + 3\cos 4\pi t \Rightarrow \cos 4\pi t = \frac{y-2}{3}$

$\sin^2 4\pi t + \cos^2 4\pi t = 1$

$\left(\frac{1-x}{3}\right)^2 + \left(\frac{y-2}{3}\right)^2 = 1$

$\frac{(1-x)^2}{9} + \frac{(y-2)^2}{9} = 1$

$(x-1)^2 + (y-2)^2 = 9$

circle w/ center @ $(1,2)$, radius of 3

#27 $\begin{cases} x = 2\sin t - 3 \Rightarrow \sin t = \frac{x+3}{2} \\ y = 5 + \cos 2t \end{cases}$

$= 5 + 2\cos^2 t - 1$

$\cos^2 t = \frac{y-4}{2}$

$\cos^2 t + \sin^2 t = 1$

$\frac{1}{2}y - 2 + \frac{(x+3)^2}{4} = 1$

$\frac{1}{2}y = 3 - \frac{1}{4}(x+3)^2$

$y = 6 - \frac{1}{2}(x+3)^2$ Parabola

#16 $x = e^{2t} = (e^t)^2$

$y = e^t + 1 \Rightarrow e^t = y - 1$

$x = (y-1)^2$

$= y^2 - 2y + 1$ Parabola.

4.2

\# 13 $\begin{cases} x = \sin 2\pi t \\ y = \cos 2\pi t \end{cases}$ @ $t = -\frac{1}{6}$ $\frac{d^2y}{dx^2}$

$$\frac{dx}{dt} = 2\pi \cos 2\pi t$$

$$\frac{dy}{dt} = -2\pi \sin 2\pi t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\pi \sin 2\pi t}{2\pi \cos 2\pi t}$$

$$= -\tan 2\pi t$$

$$\frac{d^2y}{dx^2} = \frac{-2\pi \sec^2 2\pi t}{2\pi \cos 2\pi t}$$

$\frac{dy'/dt}{dx/dt}$

$$= -\frac{1}{\cos^3 2\pi t}\Bigg|_{t=-\frac{1}{6}}$$

$\cos(-\alpha) = \cos\alpha$

$$= -\frac{1}{\left(\cos\frac{\pi}{3}\right)^3}$$

$$= -8$$

#20 $\begin{cases} x = \sqrt{t} \\ y = 3t - 1 \end{cases}$ $t = 1$

$\frac{dx}{dt} = \frac{1}{2\sqrt{t}}$ $\frac{dy}{dt} = 3$

$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= 3(2\sqrt{t}) \Big|_{t=1}$

$= 6$

@ $t=1$ $x = 1$, $y = 2$ $(1, 2)$

$y = 6(x-1) + 2$

$= 6x - 4$

$\frac{dy'}{dt} = \frac{d}{dt}(6\sqrt{t})$

$= \frac{3}{\sqrt{t}}$

$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$

$= \frac{3}{\sqrt{t}} \cdot 2\sqrt{t}$

$= 6$

2nd method

$x = \sqrt{t} \Rightarrow t = x^2 \Big|_{t=1}^{x=1}$

$y = 3t - 1$

$= 3x^2 - 1$

$m = y' = 6x = 6$

$y'' = 6$

#18

$$\begin{cases} x = t+e^t = 1 \\ y = 1-e^t = 0 \end{cases} \qquad t=0$$

$$\frac{dx}{dt} = 1+e^t \qquad \frac{dy}{dt} = -e^t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{-e^t}{1+e^t}\Big|_{t=0}$$

$$m = \frac{-1}{2}$$

$$y = -\frac{1}{2}(x-1)$$

$$\boxed{= -\frac{1}{2}x + \frac{1}{2}}$$

$$\frac{dy'}{dt} = \frac{d}{dt}\left(\frac{-e^t}{1+e^t}\right)$$

$$\frac{e^t+0}{e^t+1}$$

$$e^t(1+e^t)-e^te^t$$

$$= -\frac{e^t}{(1+e^t)^2}$$

$$\frac{dy^2}{dx^2} = \frac{dy'/dt}{dx/dt}$$

$$= -\frac{e^t}{(1+e^t)^2}\cdot\frac{1}{1+e^t}$$

$$= -\frac{e^t}{(1+e^t)^3}\Big|_{t=0}$$

$$\boxed{= -\frac{1}{8}}$$

#51 A?

$$\begin{cases} x = t - t^2 \\ y = 1 + e^{-t} \end{cases}$$

y-axis

$x = 0$

$x = y$

$t - t^2 = 1 + e^{-t}$

$x = 0 = t - t^2 \Rightarrow \boxed{t = 0, 1}$

$$A = \int_0^1 x \, dy \qquad \frac{dy}{dt} = \frac{d(1+e^{-t})}{dt} = -e^{-t}$$

$$= \int_0^1 (t - t^2)(-e^{-t})\,dt$$

		$\int e^{-t}$
$-$	$t - t^2$	$-e^{-t}$
$+$	$1 - 2t$	e^{-t}
$-$	-2	$-e^{-t}$

$$= (t - t^2 + 1 - 2t - 2)e^{-t} \Big|_0^1$$

$$= (1 - 1 + 1 - 2 - 2)e^{-1} - (1 - 2)$$

$$\boxed{= -\frac{3}{e} + 1} \text{ unit}^2$$

#60 L?

$$\begin{cases} x = t^3 \\ y = \frac{3}{2}t^2 \end{cases} \qquad 0 \le t \le \sqrt{3}$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(3t^2)^2 + (3t)^2}$$

$$= 3\sqrt{t^4 + t^2}$$

$$= 3t\sqrt{t^2+1}$$

$$L = \int_0^{\sqrt{3}} 3t(t^2+1)\,dt$$

$$= \frac{3}{2}\int_0^{\sqrt{3}} (t^2+1)^{1/2}\,d(t^2+1)$$

$$= (t^2+1)^{3/2}\Big|_0^{\sqrt{3}}$$

$$= 8 - 1$$

$$= 7 \text{ unit}$$

#62

$$\begin{cases} x = \ln(\sec t + \tan t) - \sin t \\ y = \cos t \end{cases} \qquad 0 \le t \le \frac{\pi}{3}$$

$$\frac{dx}{dt} = \frac{\sec t \tan t + \sec^2 t}{\sec t + \tan t} - \cos t$$

$$\cancel{= \frac{\sec t \tan t + \sec^2 t}{\sec}}$$

$$= \frac{\sec t(\tan t + \sec t)}{\sec t + \tan t} - \cos t$$

$$= \sec t - \cos t$$

$$\frac{dy}{dt} = -\sin t$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\sec^2 t - 2 + \underbrace{\cos^2 t + \sin^2 t}_{=1}}$$

$$= \sqrt{\sec^2 t - 1}$$

$$= \sqrt{\tan^2 t}$$

$$= \tan t$$

$$L = \int_0^{\pi/3} \tan t \, dt$$

$$= -\ln|\cos t| \Big|_0^{\pi/3}$$

$$= -\left(\ln\frac{1}{2} - \ln 1\right) \qquad \ln\frac{1}{x} = -\ln x$$

$$= -\ln\frac{1}{2}$$

$$= \boxed{\ln 2} \text{ unit}$$

#70 $\begin{cases} x = \frac{2}{3} t^{3/2} \\ y = 2\sqrt{t} \end{cases}$ $0 \le t \le \sqrt{3}$ x-axis

$\frac{dx}{dt} = t^{1/2}$ $\frac{dy}{dt} = \frac{1}{\sqrt{t}} = t^{-1/2}$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{t + \frac{1}{t}}$$

$$= \sqrt{\frac{t^2+1}{t}}$$

$$S = 2\pi \int_0^{\sqrt{3}} 2\sqrt{t} \frac{\sqrt{t^2+1}}{\sqrt{t}} dt .$$

$t = \tan\theta$
$dt = \sec^2\theta \, d\theta$
$\sqrt{1+t^2} = \sec\theta$

$$= 4\pi \int_0^{\sqrt{3}} \sqrt{t^2+1} \, dt$$

$$= 4\pi \int_0^{\sqrt{3}} \sec^3\theta \, d\theta$$

$$= 4\pi \frac{1}{2}\left(\sec\theta \tan\theta + \ln(\sec\theta + \tan\theta)\right)$$

$$= 2\pi \left(t\sqrt{t^2+1} + \ln(\sqrt{t^2+1} + t)\right)\Big|_0^{\sqrt{3}}$$

$$= 2\pi \left(2\sqrt{3} + \ln(2+\sqrt{3})\right) \text{ unit}^2$$

#76

$$\begin{cases} x = 5\cos\theta \\ y = 5\sin\theta \end{cases} \qquad y\text{-axis} \quad 0 \le \theta \le \frac{\pi}{2}$$

$$\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} = \sqrt{25\sin^2\theta + 25\cos^2\theta}$$

$25(\sin^2 t + \cos^2 t)$

$$= 5$$

$$S = 2\pi \int_0^{\pi/2} 5\cos\theta \,(5)\, d\theta$$

$$= 50\pi \sin\theta \Big|_0^{\pi/2}$$

$$\boxed{= 50\pi} \text{ unit}^2$$

4.4 ☺

#11 inside: $r = \sqrt{\cos\theta} \rightarrow$

$\cos\theta \ge 0 \rightarrow$ even fctn

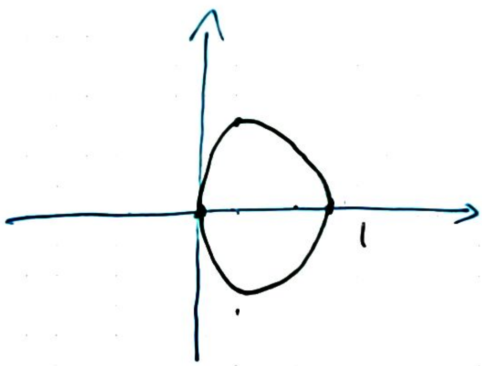
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$A = \frac{1}{2}\int_\alpha^\beta r^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} 2\cos\theta \, d\theta$$

$$= \sin\theta \Big|_0^{\pi/2}$$

$$\boxed{= 1 \text{ unit}^2}$$



#14 inside Limaçons $r = 2 + \cos\theta$: even fctn

$$A = \frac{1}{2}\,2\int_0^{\pi}(2+\cos\theta)^2\,d\theta$$

$$= \int_0^{\pi}\Big(4 + 2\cos\theta + \overbrace{\tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta}^{\cos^2\theta}\Big)\,d\theta$$

$$= \int_0^{\pi}\left(\frac{9}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= \frac{9}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\,\Big|_0^{\pi}$$

$$= \frac{9\pi}{2}\ \text{unit}^2$$

#18 shared circles: $r = 2\cos\theta$, $r = 2\sin\theta$

$$r = 2\cos\theta = 2\sin\theta$$

$$\theta = \frac{\pi}{4} \qquad \frac{5\pi}{4}$$

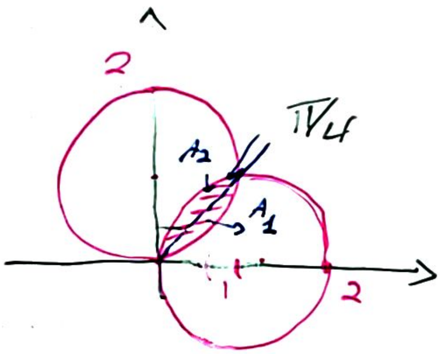
$$A = \frac{1}{2}\int_0^{\pi/4} 2\,(2\sin\theta)^2\,d\theta$$

$$= 4\int_0^{\pi/4}\frac{1-\cos 2\theta}{2}\,d\theta$$

$$= 2\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/4}$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \frac{\pi}{2} - 1$$



$$A_2 = \frac{1}{2}\int_{\pi/4}^{\pi/2} 2\,(2\cos\theta)^2$$