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1. Find the first four terms and the seventh term of the sequence:

$$a) \left\{ \frac{2^n}{(n+1)(n+2)(n+3)} \right\}$$

$$b) \{(-1)^{n+1} - (0.1)^n\}$$

2. Find the specified term of the arithmetic sequence that has two given terms:

$$a) a_{11}; a_1 = 2 + \sqrt{2}, a_2 = 3$$

$$b) a_{15}; a_3 = 7, a_{20} = 43$$

3. Express the sum in terms of summation notation:

$$a) 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$b) \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{99 \cdot 100}$$

$$c) 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7}$$

$$d) 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n}$$

$$e) 1 - \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^6}{6} + \cdots + (-1)^n \frac{x^{2n}}{2n}$$

4. Find the n th term, and the tenth term of the sequence:

$$a) \log 1000, \log 100, \log 10, \log 1, \dots$$

$$b) x - 8, x - 3, x + 2, x + 7, \dots$$

$$c) 1, -\frac{x}{3}, \frac{x^2}{9}, -\frac{x^3}{27}, \dots$$

$$d) 2, 2^{x+1}, 2^{2x+1}, 2^{3x+1}, \dots$$

5. Evaluate:

$$a) \sum_{k=2}^6 \frac{2k-8}{k-1}$$

$$b) \sum_{k=1}^4 (2^k - 10)$$

$$c) \sum_{k=1}^7 (3^{-k})$$

6. Find the sum of the infinite geometric series if it exists:

$$a) 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots$$

$$b) 1 - 0.1 + 0.01 - 0.001 + \dots$$

$$c) 250 - 100 + 40 - 16$$

7. Find the center and the radius of

a) $x^2 + y^2 + 6x + 8y + 9 = 0$

c) $x^2 + y^2 - 4x + 12y = -4$

b) $x^2 + y^2 + 8x - 6y + 16 = 0$

d) $4x^2 + 4y^2 + 4x - 16y - 19 = 0$

8. Find the vertex, focus, and directrix of the parabola.

a) $y^2 = -16x$

c) $(x + 2)^2 = \frac{1}{2}(y + 3)$

b) $x^2 = \frac{1}{9}y$

d) $(y - 2)^2 = -8(x + 1)$

9. Find an equation of a parabola that satisfies the given conditions

a) Vertex: $V(0, 1)$ focus: $F(0, 2)$

b) Focus: $F(1, 1)$ directrix: $x = -1$

c) Vertex: $V(1, 1)$ directrix: $y = 4$

10. Find the vertices, minors and foci of the ellipse, and then sketch the graph of

a) $\frac{x^2}{36} + \frac{y^2}{16} = 1$

b) $9x^2 + 6y^2 = 54$

c) $\frac{25y^2}{36} + \frac{64x^2}{9} = 1$

d) $9x^2 + 4y^2 + 18x - 8y - 23 = 0$

e) $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{16} = 1$

11. Find the vertices, the endpoints, the foci, and the equations of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes.

a) $49x^2 - 36y^2 = 1764$

d) $4x^2 + 16x - 9y^2 + 18y = 29$

b) $\frac{4x^2}{9} - \frac{25y^2}{16} = 1$

e) $16(x+5)^2 - (y-3)^2 = 1$

c) $25y^2 - 9x^2 = 1$

12. Find the partial fraction decomposition

a) $\frac{x+34}{x^2-4x-12}$

b) $\frac{4x^2-5x-15}{x^3-4x^2-5x}$

c) $\frac{2x^2+x}{(x-1)^2(x+1)^2}$

d) $\frac{x^2+19x+20}{x(x+2)(x-5)}$

e) $\frac{4x^3+3x^2+5x-2}{x^3(x+2)}$

f) $\frac{x^5-5x^4+7x^3-x^2-4x+12}{x^3-3x^2}$

g) $\frac{2x}{(x-1)^3}$

h) $\frac{x^2+3x-1}{(x+1)(x^2+2)}$

i) $\frac{2x}{(x-1)(x^2+1)^2}$

j) $\frac{4x+2}{(x+2)(2x-1)}$

k) $\frac{x}{x^2+4x-5}$

l) $\frac{2x^5+3x^4-3x^3-2x^2+x}{2x^2+4x+2}$

Solution

1. a) $\left\{\frac{1}{12}, \frac{1}{15}, \frac{1}{15}, \frac{8}{105}, \frac{8}{45}\right\}$ b) $\{0.9, -1.01, 0.999, -1.0001, 0.99999999\}$

2. a) $d = 1 - \sqrt{2}$ $a_{11} = 12 - 9\sqrt{2}$ b) $d = \frac{36}{17}$; $a_{15} = \frac{551}{17}$

3. a) $\sum_{n=1}^6 2^{3-n}$ b) $\sum_{n=1}^{99} \frac{1}{n(n+1)}$ c) $\sum_{n=1}^7 (-1)^{n-1} \frac{1}{n}$ d) $1 + \sum_{k=1}^n \frac{x^k}{k}$
e) $1 + \sum_{k=1}^n (-1)^k \frac{x^{2k}}{2k}$

4. a) $d = -1$; $a_n = -n + 4$; $a_{10} = -6$

b) $d = 5$; $a_n = x + 5n - 13$; $a_{10} = x + 37$

c) $r = -\frac{x}{3}$; $a_n = (-1)^{n-1} \left(\frac{x}{3}\right)^{n-1}$; $a_{10} = -\frac{x^9}{19683}$

d) $r = 2^x$; $a_n = 2^{(n-1)x+1}$; $a_{10} = 2^{9x+1}$

5. a) $-\frac{37}{10}$ b) -10 c) $\frac{1093}{2187}$

6. a) $S = 3$ b) $S = \frac{10}{11}$ c) $S = \frac{1250}{7}$

7. a) Center $(-3, -4)$; radius: 4 b) Center $(-4, 3)$; radius: 3

c) Center $(2, -6)$; radius: 6 d) Center $\left(-\frac{1}{2}, 2\right)$; radius: 3

8. a) $V(0,0)$ $F(-4,0)$ directrix: $x = 4$

b) $V(0,0)$ $F\left(0, \frac{1}{36}\right)$ directrix: $y = -\frac{1}{36}$

c) $V(-2, -3)$ $F\left(-2, -\frac{23}{8}\right)$ directrix: $y = -\frac{25}{8}$

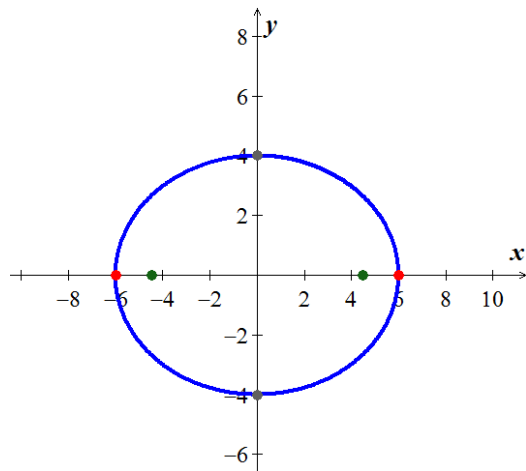
d) $V(-1, 2)$ $F(-3, 2)$ directrix: $x = 1$

9. a) $x^2 = 4(y-1)$

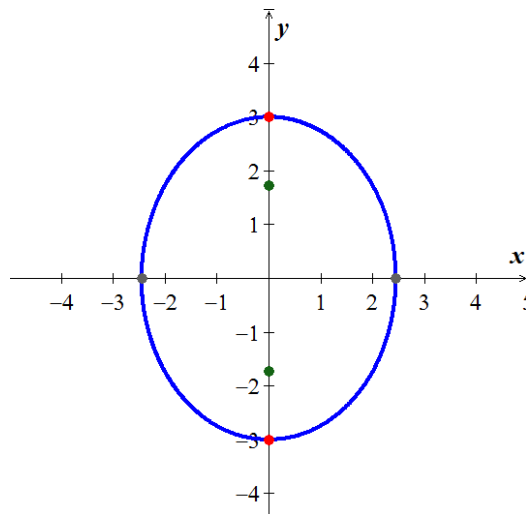
b) $(y-1)^2 = 4x$

c) $(x-1)^2 = -12(y-1)$

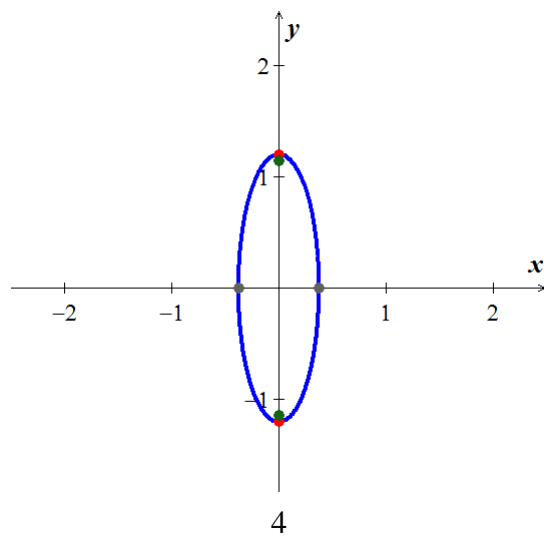
10. a) $V(\pm 6, 0)$ $M(0, \pm 4)$ $F(\pm 2\sqrt{5}, 0)$



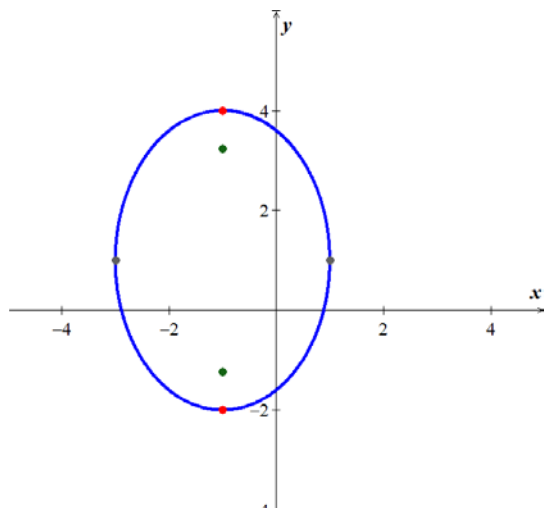
b) $V(0, \pm 3)$ $M(\pm\sqrt{6}, 0)$ $F(0, \pm\sqrt{3})$



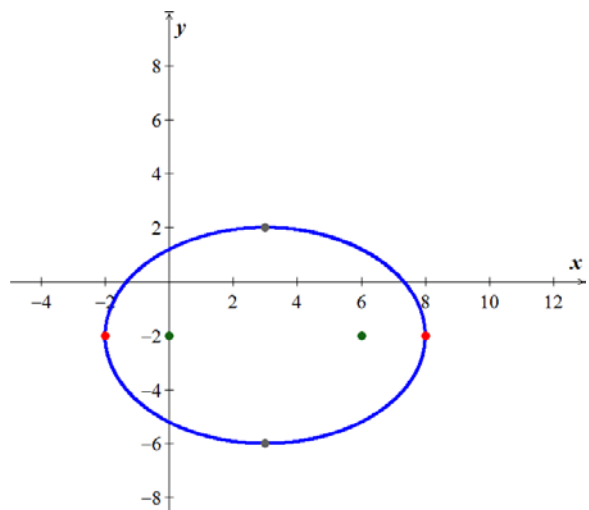
c) $V(0, \pm\frac{6}{5})$ $M(\pm\frac{3}{8}, 0)$ $F(0, \pm\frac{3\sqrt{231}}{40})$



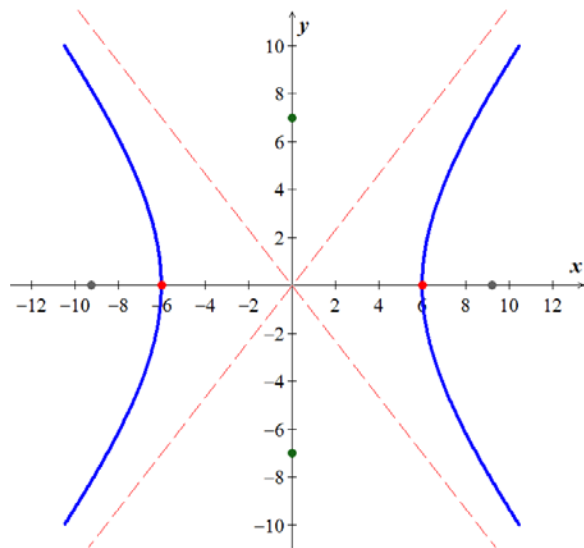
d) center $(-1, 1)$ $V(-1, -2), (-1, 4)$ $F(-1, 1 \pm \sqrt{5})$ $M(-3, 1), (1, 1)$



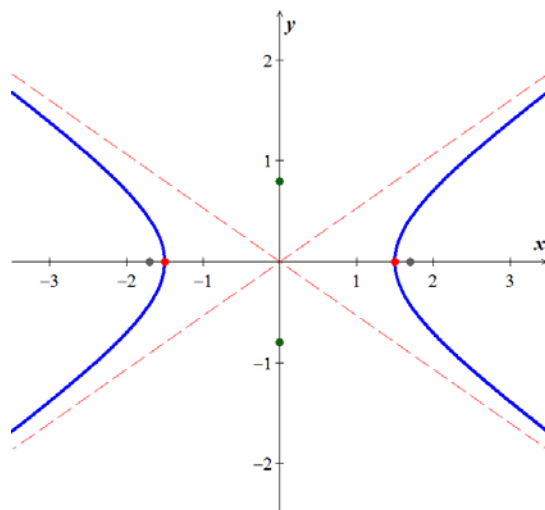
e) center $(3, -2)$ $V(-2, -2), (8, -2)$ $F(0, -2), (6, -2)$ $M(3, -6), (3, 2)$



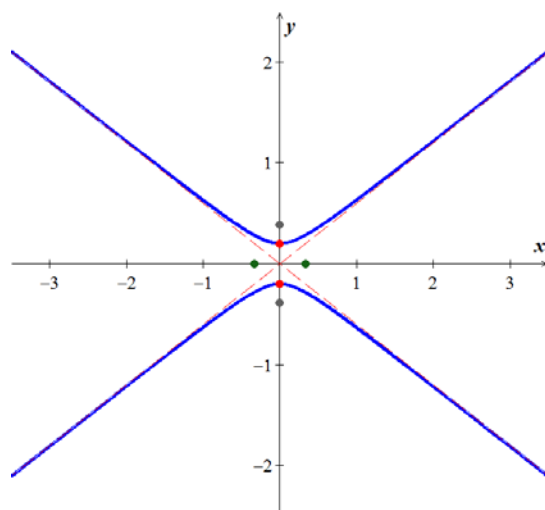
11. a) $V(\pm 6, 0)$ $W(0, \pm 7)$ $F(\pm\sqrt{85}, 0)$ $y = \pm\frac{7}{6}x$



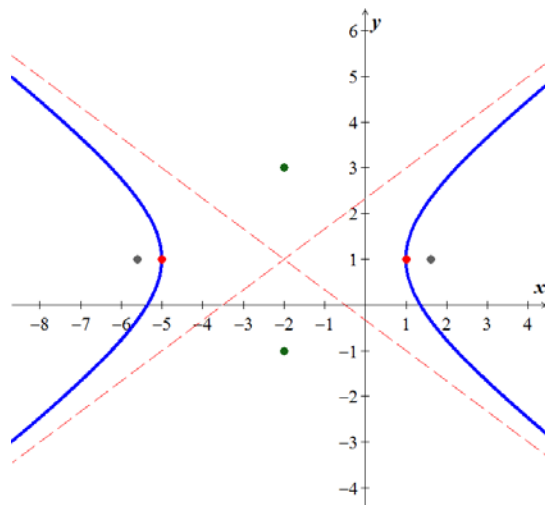
b) $V(\pm\frac{3}{2}, 0)$ $W(0, \pm\frac{4}{5})$ $F(\pm\frac{\sqrt{289}}{10}, 0)$ $y = \pm\frac{8}{15}x$



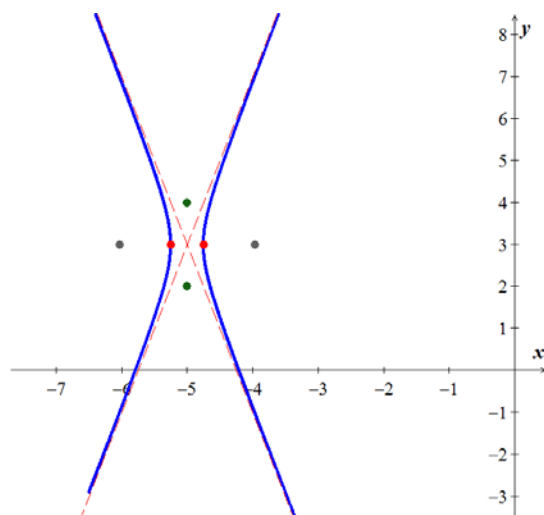
c) $V(0, \pm\frac{1}{5})$ $W(\pm\frac{1}{3}, 0)$ $F(0, \pm\frac{\sqrt{34}}{15})$ $y = \pm\frac{3}{5}x$



d) center $(-2, 1)$ $V(-5, 1), (1, 1)$ $W(-2, -1), (-2, 3)$ $F(-2 \pm \sqrt{13}, 1)$ $y = \pm \frac{2}{3}(x+2) + 1$



e) center $(-5, 3)$ $V(-\frac{21}{4}, 3), (-\frac{19}{4}, 3)$ $W(-5, 2), (-5, 4)$ $F(-5 \pm \frac{\sqrt{17}}{4}, 3)$ $y = \pm 4(x+5) + 3$



12. a) $\frac{5}{x-6} - \frac{4}{x+2}$ b) $\frac{3}{x} + \frac{2}{x-5} - \frac{1}{x+1}$ c) $\frac{1}{x-1} + \frac{\frac{3}{4}}{(x-1)^2} - \frac{1}{x+1} + \frac{\frac{1}{4}}{(x+1)^2}$
- d) $-\frac{2}{x} - \frac{1}{x+2} + \frac{4}{x-5}$ e) $\frac{3}{x^2} - \frac{1}{x^3} + \frac{4}{x+2}$ f) $x^2 - 2x + 1 - \frac{4}{x^2} + \frac{2}{x-3}$
- g) $\frac{2}{(x-1)^2} + \frac{2}{(x-1)^3}$ h) $\frac{-1}{x+1} + \frac{2x+1}{x^2+2}$ i) $\frac{1}{x-1} + \frac{-\frac{1}{2}x - \frac{1}{2}}{x^2+1} + \frac{-x+1}{(x^2+1)^2}$
- j) $\frac{\frac{6}{5}}{x+2} + \frac{\frac{8}{5}}{2x-1} = \frac{6}{5(x+2)} + \frac{8}{5(2x-1)}$
- k) $\frac{5}{6(x+5)} + \frac{1}{6(x-1)}$ l) $x^3 - x^2 + \frac{-1}{3(2x+1)} + \frac{2}{3(x+2)}$