Inverse Trig.

$y = \sin x \implies x = \sin^{-1} y$ (angle)

(or)

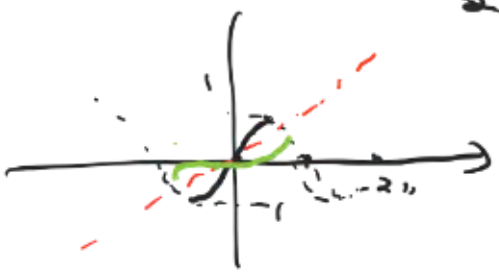
$x = \arcsin y$

$-1 \le y \le 1$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$-1 \le y \le 1$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$



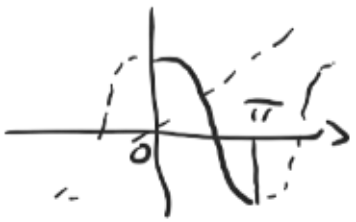
$\sin(\sin^{-1}\frac{1}{2}) = \frac{1}{2}$

$\sin(\sin^{-1}\frac{1}{2}) = \sin(\frac{\pi}{6})$

$= \frac{1}{2}$ ✓

$y = \cos^{-1} x$ iff $x = \cos y$

$-1 \le x \le 1$ $\quad 0 \le y \le \pi$



Acos x

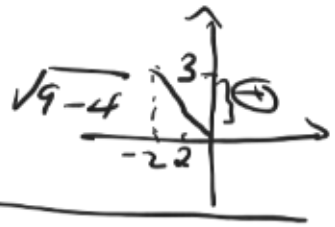
$\cos^{-1} x = \arccos x$

$\cos(\cos^{-1} x) = x$

$\sin\left(\underbrace{\arccos\left(-\frac{2}{3}\right)}_{\alpha}\right)$ $\quad \alpha = \arccos\left(-\frac{2}{3}\right)$

$\cos\alpha = -\frac{2}{3} \qquad \frac{\pi}{2} < \theta < \pi$

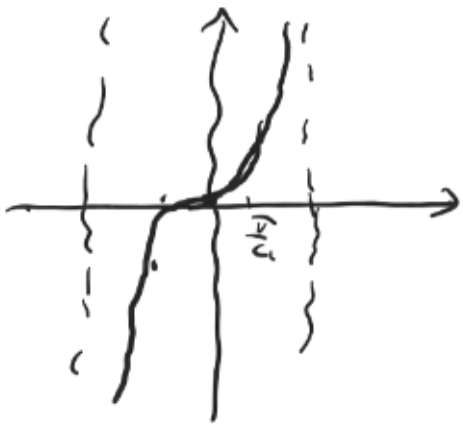
$\sin\alpha = \frac{\sqrt{5}}{3}$



Inverse Tangent

$x = \tan y \Rightarrow y = \tan^{-1} x$

$y = \arctan x$



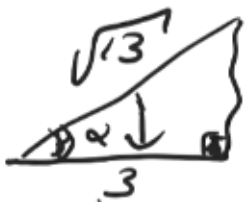
$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$\tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$

$\sec\left(\underbrace{\arctan\frac{2}{3}}_{\alpha}\right) \qquad \alpha = \arctan\frac{2}{3}$

$\tan\alpha = \frac{2}{3}$

$\sec\alpha = \frac{\sqrt{13}}{3}$



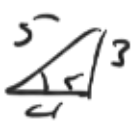
$r = \sqrt{x^2+y^2} \qquad \sqrt{9+4}$

Ex $\sin\left(\underbrace{\arctan\frac{1}{2}}_{\alpha} - \underbrace{\arccos\frac{4}{5}}_{\beta}\right)$

$\tan\alpha = \frac{1}{2}$

$\cos\beta = \frac{4}{5}$

$\sin\beta = \frac{3}{5}$



$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$= \frac{1}{\sqrt{5}}\cdot\frac{4}{5} - \frac{2}{\sqrt{5}}\cdot\frac{3}{5}$$

$$= \frac{4-6}{5\sqrt{5}}$$

$$= -\frac{2}{5\sqrt{5}} \quad (a) \quad -\frac{2\sqrt{5}}{25}$$

---

x $\quad \cos(\underbrace{\sin^{-1}x}_{\alpha}) = ?$

$\alpha = \sin^{-1}x$

$\sin\alpha = x \quad \left(\frac{x}{1}\right)$

$\cos\alpha = \sqrt{1-x^2}$

#34 $\operatorname{cot}\left(\underbrace{\sin^{-1}\frac{\sqrt{x^2-9}}{x}}_{\alpha}\right)$

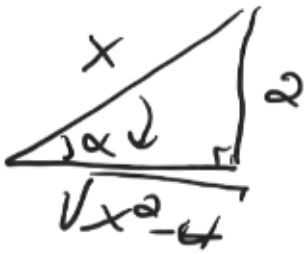
$\sin\alpha = \frac{\sqrt{x^2-9}}{x}$

$\cot\alpha = \frac{3}{x}$

#39 $\sec\left(\underbrace{\tan^{-1}\frac{2}{\sqrt{x^2-4}}}_{\alpha}\right)$

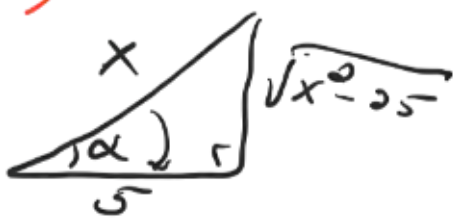
$\tan\alpha = \frac{2}{\sqrt{x^2-4}}$

$\sec\alpha = \frac{x}{\sqrt{x^2-4}}$



#40 $\sec\left(\sin^{-1}\frac{\sqrt{x^2-25}}{x}\right)$

$\sin\alpha = \frac{\sqrt{x^2-25}}{x}$



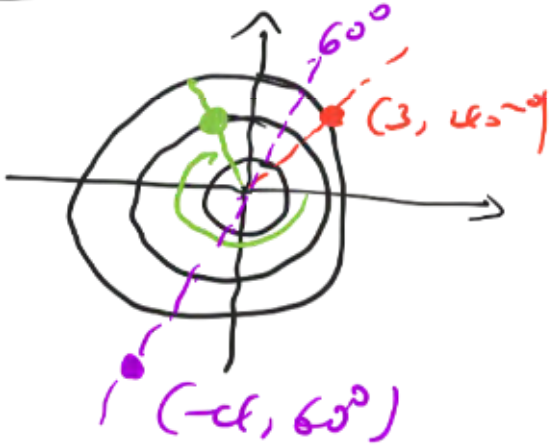
$\sec\alpha = \frac{x}{5}$

8.6 Polar Coordinates

(r, θ) → (x, y) Relation

rectangular

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\begin{cases} r = \sqrt{x^2+y^2} \\ \hat{\theta} = \tan^{-1}\frac{y}{x} \end{cases}$



(3, 45°)

(2, −240°)

(−4, 60°)

Ex $(r,\theta) = \left(4, \frac{7\pi}{6}\right)$ (x,y)?

$x = r\cos\theta$ $\quad y = r\sin\theta$

$= 4\cos\frac{7\pi}{6}$ $\quad = 4\sin\frac{7\pi}{6}$

$= 4\left(-\frac{\sqrt{3}}{2}\right)$ $\quad = 4\left(-\frac{1}{2}\right)$

$= 4\left(+\frac{\sqrt{3}}{2}\right)$ $= 4\left(-\frac{1}{2}\right)$

$= -2\sqrt{3}$ $= -2$

$(x, y) = (-2\sqrt{3}, -2)$

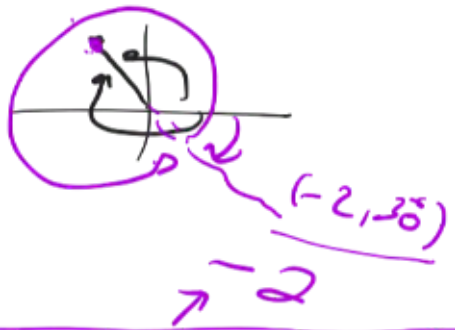
$(x, y) = (-1, \sqrt{3})$ (r, θ) ?

$r = \sqrt{x^2 + y^2}$ $\hat{\theta} = \tan \frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$= \sqrt{1+3}$ $\theta = \frac{2\pi}{3}$

$= 2$

$(r, \theta) = (2, \frac{2\pi}{3})$ $120°$



$x =$

$\vdots$

$=$

---

line $ax + by = c$

$a r\cos\theta + b r\sin\theta = c$

$r(a\cos\theta + b\sin\theta) = c$

$r(a\cos\theta + b\sin\theta) = c$

$$r = \frac{c}{a\cos\theta + b\sin\theta}$$

$x^2 - y^2 = 16$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 16$

$r^2\cos^2\theta - r^2\sin^2\theta = 16$

$r^2(\cos^2\theta - \sin^2\theta) = 16$

$$r^2 = \frac{16}{\cos 2\theta}$$

$r = a\sin\theta \quad (a \neq 0)$

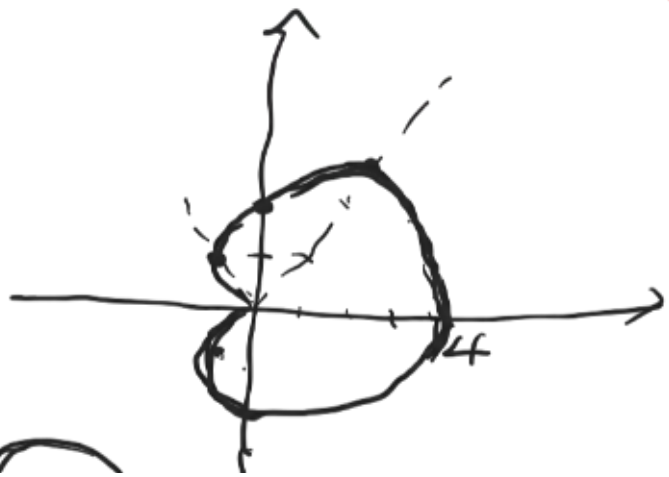
$r^2 = ar\sin\theta$

$x^2 + y^2 = ay$

Graph $0, 1, \frac{1}{2}$ ← $\cos 60° = \frac{1}{2}$, $\sin 30° = \frac{1}{2}$

$r = 2 + 2\cos\theta$ (cardioid)

θ	r
0	4
60°	3
90°	2
120°	1
$\frac{11}{11}$	0
	1



$a(1+\cos\theta)$
$a(1-\cos\theta)$ →
$a(1+\sin\theta)$
$a(1-\sin\theta)$

#1 $(4, 30°)$

$x = r\cos\theta$
$= 4\cos 30°$
$= 4\left(\frac{\sqrt{3}}{2}\right)$
$= 2\sqrt{3}$

$y = r\sin\theta$
$= 4\sin 30°$
$= 4\left(\frac{1}{2}\right)$
$= 2$

$(x, y) = (2\sqrt{3}, 2)$

#2 $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$

$x = r\cos\theta$
$= -\sqrt{2}\cos\frac{3\pi}{4}$
$= -\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right)$
$= 1$

$y = r\sin\theta$
$= -\sqrt{2}\sin\frac{3\pi}{4}$
$= -\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)$
$= -1$

$(x, y) = (1, -1)$

$(x, y) = (-1, \sqrt{3})$ $\quad \theta \in QII$

$r = \sqrt{x^2+y^2}$
$= \sqrt{1+3}$
$= 2$

$\theta' = \tan^{-1}\frac{\sqrt{3}}{1}$
$= \frac{\pi}{3}$
$\theta = 2\pi$

$= 2$ $\quad\quad$ $\sqrt{3}$

$(r, \theta) = (2, \frac{2\pi}{3})$

#(3) $(-3, -3) = (x, y)$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{9+9}$

$= 3\sqrt{2}$

$\theta = \tan^{-1}\frac{3}{3}$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

$\theta = \frac{5\pi}{4}$

$(r, \theta) = (3\sqrt{2}, \frac{5\pi}{4})$

---

23 $\quad r = 6\cos\theta$

$r^2 = 6r\cos\theta$

$x^2 + y^2 = 6x$

---

25 $\quad r^2(4\sin^2\theta - 9\cos^2\theta) = 36$

$4r^2\sin^2\theta - 9r^2\cos^2\theta = 36$

$4y^2 - 9x^2 = 36$

$\frac{y^2}{9} - \frac{x^2}{4} = 1$

---

34 $\quad r(\sin\theta + r\cos^2\theta) = 1$

$r\sin\theta + r^2\cos^2\theta = 1$

$y + x^2 = 1$

#50 $y^2 = 6x$

$(r\sin\theta)^2 = 6r\cos\theta$

$r^2\sin^2\theta = 6r\cos\theta \quad (r \neq 0)$

$r\sin\theta = 6\cos\theta$

$r = \frac{6\cos\theta}{\sin\theta}$

$= 6\cot\theta$

#37 $(x+2)^2 + (y-3)^2 = 13$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 13$

$x^2 + y^2 + 4x - 6y = 0$

$r^2 + 4r\cos\theta - 6r\sin\theta = 0$

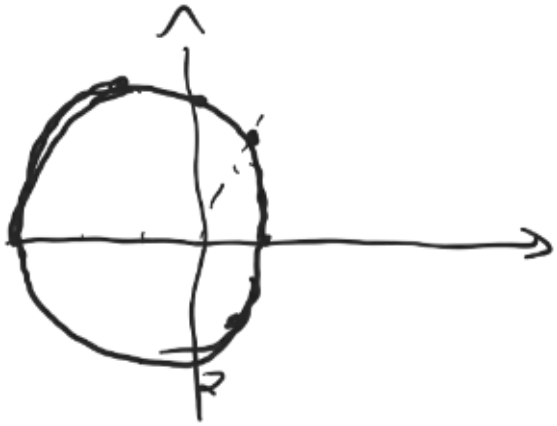
$(r \neq 0)$

$r + 4\cos\theta - 6\sin\theta = 0$

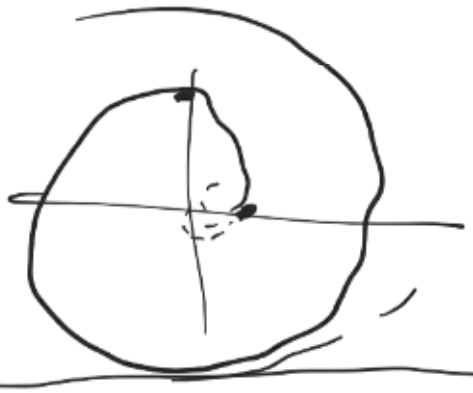
$r = 6\sin\theta - 4\cos\theta = f(\theta)$

#48 $r = 2 - \cos\theta$

θ	r
0	1
$\frac{\pi}{3}$	$3/2$
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	$5/2$
π	3
$\frac{4\pi}{3}$	$5/2$



$r = e^{2\theta}$



8.7 Trig Form.

$\mathbb{C}$ $\quad \sqrt{-1} = i$

$-1 = i^2$

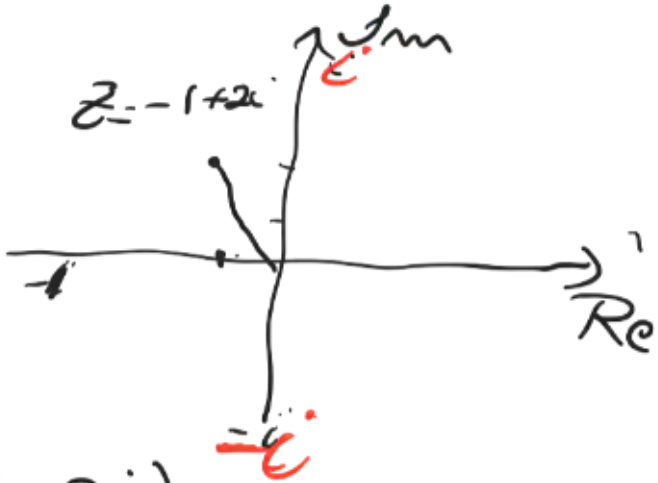
Real part $\quad -i = i^3 \quad i \cdot i^2$

$1 = i^4$

$z = \underbrace{a}_{Re} + \underbrace{ib}_{Im}$ imaginary part

$a, b \in \mathbb{R}$

$z = -1+2i$



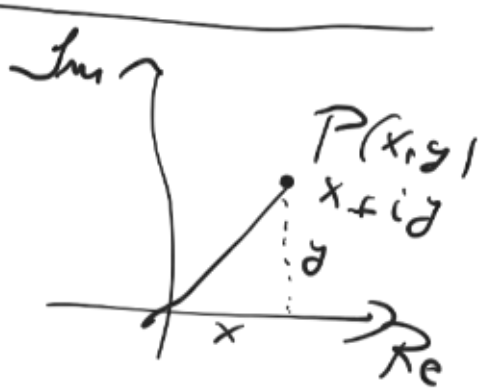
$(6-2i) + (-4-3i)$

$= 6-4 + i(-2-3)$

$= 2 - 5i$

$r = \sqrt{x^2 + y^2}$

modulus.



θ: argument

$\theta = \tan^{-1}\frac{y}{x}$

$z = x + yi$

$= r\cos\theta + i(r\sin\theta)$

$= r(\cos\theta + i\sin\theta)$

$= r\,\mathrm{cis}\,\theta$ } Trig Form

$z = -1 + i$ Trig Form

$r = \sqrt{(-1)^2 + 1^2}$

$= \sqrt{2}$

$\theta = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}$

$z = \sqrt{2}\,\mathrm{cis}\,\frac{3\pi}{4}$ $\left(\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right)$

$z = 6\,\mathrm{cis}\,60^\circ$

$= 6(\cos 60^\circ + i\sin 60^\circ)$

$= 6\cos 60 + 6i\sin 60$

$= 6\left(\frac{1}{2}\right) + i6\left(\frac{\sqrt{3}}{2}\right)$

$= 3 + 3i\sqrt{3}$

$|z|$? $z = 5i$ $z = 7$

$r = 5$ $r = 7$

$3 + 4i \rightarrow r = 5$

$\sqrt{9+16} = \sqrt{25}$

$\cos z \leftarrow z \in \mathbb{C}$

$(r_1 \operatorname{cis}\theta_1)(r_2 \operatorname{cis}\theta_2) = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$

Ex $3(\operatorname{cis}45^\circ)(2\operatorname{cis}135^\circ) = 6\operatorname{cis}(45^\circ + 135^\circ)$

$= 6\operatorname{cis}180^\circ$

$= 6\cos 180^\circ + i\,6\sin 180^\circ$

$= -6$

$a^2 + b^2 = (a+ib)(a-ib)$

$= a^2 - (ib)^2$

$a^2 - i^2 b^2$

$= a^2 - (-1)b^2$

$= a^2 + b^2$

$a^2 + b^2$

$= (a+b)^2$

$a+b = (\sqrt{a} - i\sqrt{b})(\sqrt{a} + i\sqrt{b})$

$a-b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$

$$\frac{r_1 \operatorname{cis}\theta_1}{r_2 \operatorname{cis}\theta_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$\frac{10 \operatorname{cis}(-60^\circ)}{5 \operatorname{cis}(150^\circ)} = \frac{10}{5} \operatorname{cis}(-60^\circ - 150^\circ)$$

$$= 2 \operatorname{cis}(-210^\circ)$$

$$= 2\cos(-210^\circ) + i\,2\sin(-210^\circ)$$

$$= 2\left(-\frac{\sqrt{3}}{2}\right) + i\,2\left(+\frac{1}{2}\right)$$

$$= -\sqrt{3} + i$$

210° III (−)

De Moivre's Theorem

$(r \operatorname{cis}\theta)^n = r^n \operatorname{cis} n\theta$

$(1 + i\sqrt{3})^8 \qquad \theta \in$ QI

$r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\frac{\sqrt{3}}{1} = 60^\circ$

$$(1+i\sqrt{3})^6 = (2\,\mathrm{cis}\,60^\circ)^6$$

$$= 2^6\,\mathrm{cis}\,(360^\circ)$$

$$= 256\,\mathrm{cis}\,(120^\circ)$$

$$= 256(\cos 120^\circ + i\sin 120^\circ)$$

$$= 256\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$= -128 + i\,128\sqrt{3}$$

---

n^{th} root $\quad (r\,\mathrm{cis}\,\theta)^{1/n} = \sqrt[n]{r}\,\mathrm{cis}\,\alpha$

$$\alpha = \frac{\theta}{n} + \frac{360^\circ k}{n}$$

α

$$\frac{\theta + 2\pi k}{n}$$