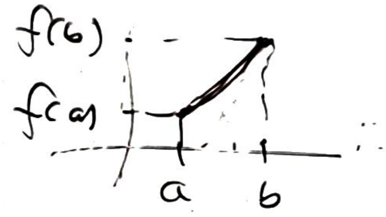
Lecture 2 Functions

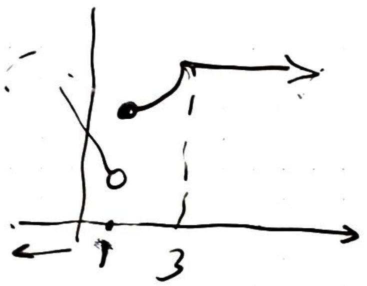
2.1 Increasing and decreasing (constant)
Inc dec. Function

Over x

if $a < b \Rightarrow f(a) < f(b)$
increasing

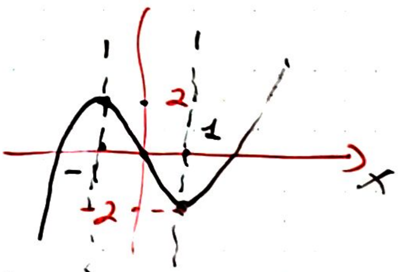


$a < b \Rightarrow f(a) > f(b)$ fctn decreasing



$(-\infty, 1)$ dec.
$[1, 3)$ inc
$(3, \infty)$ constant

$f(x) = x^3 - 3x$



Relative extreme
(RMAX, RMIN)
(LMAX, LMIN)

RMAX: $(-1, 2)$ RMIN: $(1, -2)$

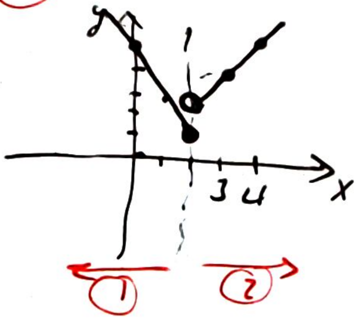
Inc: $(-\infty, -1) \cup (1, \infty)$ Dec: $(-1, 1)$

Piecewise fctn (no repeated x)

$$f(x) = \begin{cases} -2x+5 \ ① & \text{if } x \leq 2 \\ x+1 \ ② & \text{if } x > 2 \end{cases}$$

② $x \mid y$

draw table



satisfies cond.

$x = 0$?

$f(0) = -2(0)+5 = \underline{5}$

$f(2) = -4+5 = \underline{1}$

$f(3) = 3+1 = \underline{4}$

Ex

$$C(x) = \begin{cases} 20 & \text{if } 0 \leq t \leq 60 \\ 20+0.4(t-60) & t > 60 \end{cases}$$

a) $C(40) = \underline{20}$

b) $C(60) = \underline{20}$

c) $C(80) = 20 + .4(80-60)$

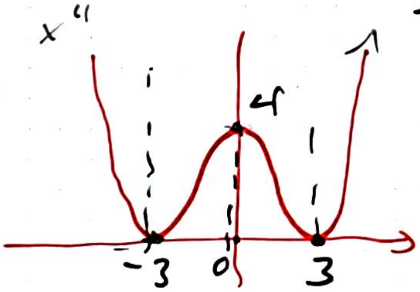
$= 20 + .4(20)$

$= 20 + 8$

$= \underline{28}$

#16

x^4



$x^{④}$

(4-1) RMAX : $(0, 4)$

RMIN : $(-3, 0)$ & $(3, 0)$

Inc : $(-3, 0)$ $(3, \infty)$

Dec : $(-\infty, -3)$ $(0, 3)$

Domain : $\mathbb{R}$

Range : $[0, \infty)$

2.2 1- Domain. (x's)

1 Rational fctn $\frac{\quad}{h(x)}$ Domain: $h(x) \neq 0$

ex $f(x) = \frac{1}{x-3}$ Domain: $x \neq 3$

① Interval: $(-\infty, 3) \cup (3, \infty)$

② $\{x \mid \underbrace{x \neq 3}_{\text{condi}}\}$

set $\{\}$ of all x such that

③ $\mathbb{R} - \{3\}$

index

2 Irrational: $\sqrt{g(x)}$ (index: even)

Domain: $g(x) \geq 0$

ex $g(x) = \sqrt{3-x} + 5$

Domain: $x \leq 3$

− point + Point

③ $\mathbb{R}$ domain $f(x) = x^3 + |x| + \frac{\pi}{5} + \sqrt{2}$

① x index. No ② $\sqrt{\quad}$ x No

Domain: $\mathbb{R}$

Comibe ① & ② $\frac{\quad}{\sqrt{h(x)}}$ $h(x) > 0$

ex $f(x) = \frac{x+1}{\sqrt{x-3}}$ Domain: $x > 3$

Ex $f(x) = x^2 + 3x - 17$ Domain: $\mathbb{R}$

Ex $g(x) = \frac{5x}{x^2-49}$ Domain: $x \neq \pm 7$

$x^2 - 49 = 0$

$x^2 = 49$

$x = \pm 7$

Ex $h(x) = \sqrt{9x-27}$

Domain: $x \geq 3$

$9x - 27 = 0$

$9x = 27$

Algebra $(+, -, *, \div)$

$f(x) + g(x) = (f+g)(x)$

$-$ $-$

$\cdot$ fg

$\frac{f(x)}{g(x)} = \frac{f}{g}(x)$

Ex $f(x) = x^2+1$ $g(x) = 3x+5$

Domain: $\mathbb{R}$ $\mathbb{R}$.

$(f+g)(1) = f(1) + g(1)$

$= 1+1 + 3+5$

$= 10$

$(f-g)(-3) = f(-3) - g(-3)$

$= 9+1 - (-9+5)$

$= 10 + 4$

$= 14$

$f(-3) = (-3)^2 + 1$

$g(-3) = 3(-3) + 5$

$fg(5) = (5^2+1)(15+5)$

$= 26(20)$

$= 520$

$$\frac{f(0)}{g(0)} = \frac{1}{5}$$

Ex $f(x) = 8x - 9$ $\qquad$ $g(x) = \sqrt{2x-1}$ $\qquad$ $2x = 1$

Domain: $\mathbb{R}$ $\qquad$ Domain: $x \geqslant \frac{1}{2}$

a) $(f+g)(x) = 8x - 9 + \sqrt{2x-1}$ $\qquad$ Domain: $x \geqslant \frac{1}{2}$

b) $(f-g)(x) = 8x - 9 - \sqrt{2x-1}$ $\qquad$ " : $x \geqslant 1/2$

c) $fg(x) = (8x-9)\sqrt{2x-1}$ $\qquad$ " : $x \geqslant 1/2$

d) $\frac{f}{g}(x) = \frac{8x-9}{\sqrt{2x-1}}$ $\qquad$ " : $x > \frac{1}{2}$

Ex $f(x) = \sqrt{x-3}$ $\qquad$ $g(x) = \sqrt{x+1}$

Domain: $x \geqslant 3$ $\qquad$ Domain: $x \geqslant -1$

a) $(f+g)(x) = \sqrt{x-3} + \sqrt{x+1}$

Domain: $x \geqslant 3$

$\frac{f}{g}(x) = \frac{\sqrt{x-3}}{\sqrt{x+1}}$ $\rightarrow$ $x \geqslant 3$, $x > -1$ $\qquad$ $x \geqslant 3$

$\frac{g}{f}(x) = \frac{\sqrt{x+1}}{x-3}$ $\qquad$ $x \geqslant -1$, $x > 3$ $\qquad$ $x > 3$

Ex $f(x) = 2x - 3$

$$\frac{f(x+h) - f(x)}{h}$$

difference quotient

$f(x+h) = 2(x+h) - 3$
$= 2x + 2h - 3$ copy

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{h}\left(2x + 2h - 3 - 2x + 3\right)$$
$$= \frac{1}{h}(2h)$$
$$= 2$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(x_1 + h) - f(x_1)}{h}$$

$h = x_2 - x_1$
$x_2 = x_1 + h$

Ex $f(x) = -2x^2 + x + 5$

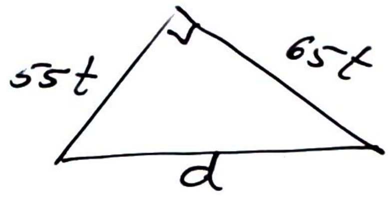
$f(x+h) = -2(x+h)^2 + (x+h) + 5$
$= -2(x^2 + 2hx + h^2) + x + h + 5$
$= -2x^2 - 4hx - 2h^2 + x + h + 5$

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{h}\left(-2x^2 - 4hx - 2h^2 + x + h + 5 + 2x^2 - x - 5\right)$$
$$= \frac{1}{h}\left(-4hx - 2h^2 + h\right)$$
$$= -4x - 2h + 1$$

$f(x) = ax + b$

$= a$

Ex $\quad 65 \frac{mi}{h} \cdot h = mi$



a) $d^2 = (65t)^2 + (55t)^2$

$= 65^2 t^2 + 55^2 t^2$

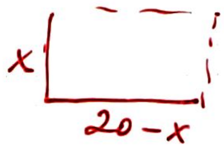
$= (65^2 + 55^2) t^2$

$d(t) = \sqrt{(65^2 + 55^2) t^2}$ $\qquad$ $t \}$ distance ≥ 0

$= t\sqrt{65^2 + 55^2}$ $\quad ; t \geq 0$

domain: $t \geq 0$

Ex $\quad$ 20 ft



a) $A(x) = x(20 - x)$

$= 20x - x^2$

b) Domain: $0 < x < 20$ $\qquad \neq 0, 20$

Homework due before (6/22) 9 AM

Exam 2 review: #1, 2, 3, 5, 8, 9

2.2 Domain

Domain

① $\frac{}{h(x)} \neq 0$

② $\sqrt{g}$ $g \geq 0$

③ $\mathbb{R}$

④ $\frac{1}{\sqrt{f}}$ $f > 0$

#2/ $f(x) = |3x-2|$, $\mathbb{R}$

8/ $f(x) = 4 - \frac{2}{x}$ $x \neq 0$

10/ $g(x) = \frac{3}{x-4}$ $x \neq 4$

13/ $f(x) = \frac{x+5}{2-x}$ $x \neq 2$

21/ $f(x) = \frac{1}{x-3} - \frac{8}{x-7}$ $x \neq 3, 7$

25/ $f(x) = \frac{x}{x^2+3x+2}$ $x \neq -1, -2$

30/ $y = \sqrt{x}$ $x \geq 0$ $\boxed{\sqrt{0} = 0}$

31/ $f(x) = \sqrt{8-3x}$ $x \leq \frac{8}{3}$ $-3x = -8$

45 $f(x) = \sqrt{x^2-5x+4}$ $x \leq 1$ $x \geq 4$

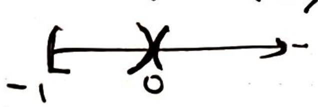
$x^2-5x+4 \geq 0$

55 $f(x) = \frac{\sqrt{x+1}}{x}$ $\to x \geq -1$

$x \neq 0$

$x \geq -1,\ x \neq 0$

$[-1, 0) \cup (0, \infty)$



59 $f(x) = \frac{x}{\sqrt{5-x}}$ $x < 5$

41) $f(x) = 9x+5$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left(\overbrace{9(x+h)+5}^{f(x+h)} - \overbrace{(9x+5)}^{f(x)}\right)$$

$$= \frac{1}{h}(9x+9h+5-9x-5)$$

$$= \frac{1}{h}(9h)$$

$$= 9$$

44) $f(x) = 2x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left(2(x+h)^2 - 2x^2\right)$$

$$= \frac{1}{h}\left(2(x^2+2hx+h^2)-2x^2\right)$$

$$= \frac{1}{h}(2x^2+4hx+2h^2-2x^2)$$

$$= \frac{4hx}{h} + \frac{2h^2}{h}$$

$$= 4x+2h$$