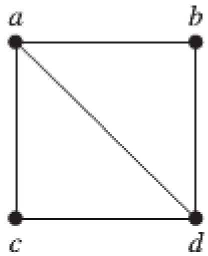
SOLUTION *Section* 4.7 – Representing Graphs and Graph Isomorphism

Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



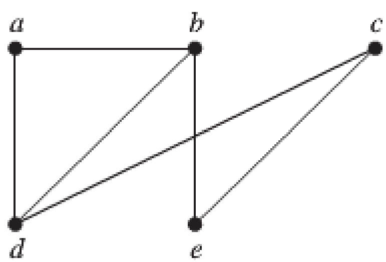
Solution

Vertex	*Adjacent Vertices*
a	*b*, *c*
b	*a*, *d*
c	*a*, *d*
d	*a*, *b*, *c*

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



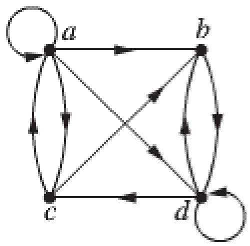
Solution

Vertex	*Adjacent Vertices*
a	*b*, *d*
b	*a*, *d*, *e*
c	*d*, *e*
d	*a*, *b*, *c*
e	*b*, *c*

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



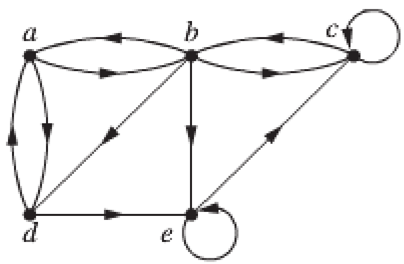
Solution

Initial Vertex	*Terminal Vertices*
a	*a, b , c, d*
b	*d*
c	*a, b*
d	*b, c, d*

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



Solution

Initial Vertex	*Terminal Vertices*
a	*b, d*
b	*a, c, d, e*
c	*b, c*
d	*a, e*
e	*c , e*

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

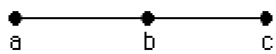
Exercise

Draw a graph with the given adjacency

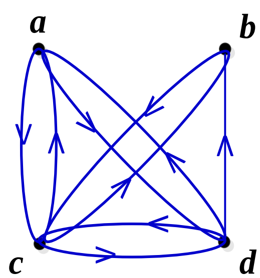
a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ *b*) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ *c*) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

Solution

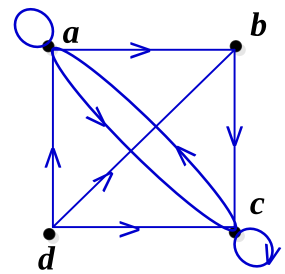
a)



b)



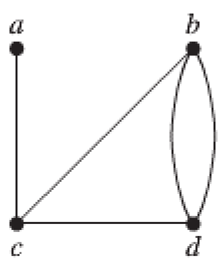
c)



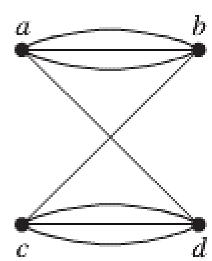
Exercise

Represent the given graph using adjacency matrix

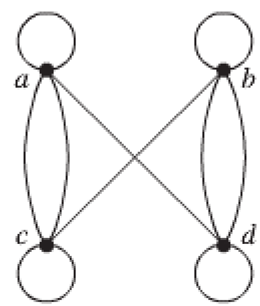
a)



b)



c)



Solution

a) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$ ***b***) $\begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$ ***c***) $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

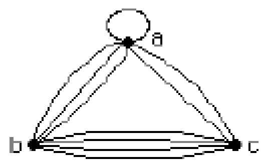
Exercise

Draw an undirected graph represented by the given adjacency

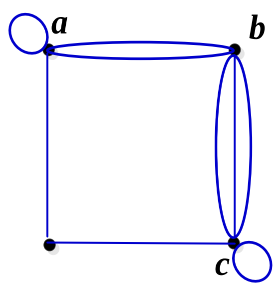
$a)\ \begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix}$ $\quad b)\ \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ $\quad c)\ \begin{bmatrix} 0 & 1 & 3 & 0 & 4 \\ 1 & 2 & 1 & 3 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$

Solution

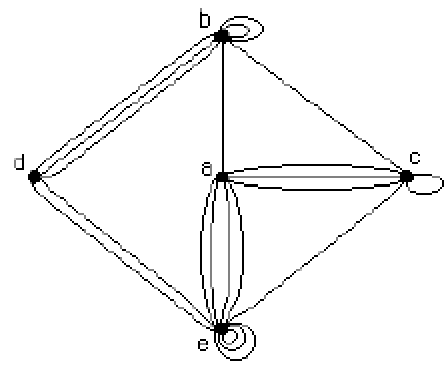
a)



b)



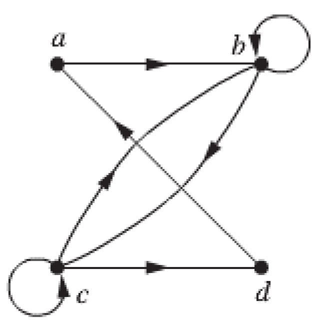
c)



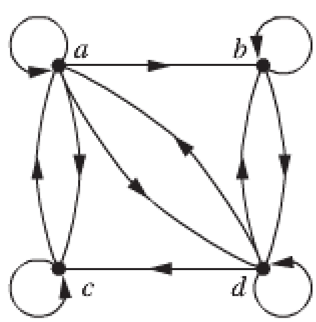
Exercise

Find the adjacency matrix of the given directed multigraph with respect to the vertices listed in alphabetic order.

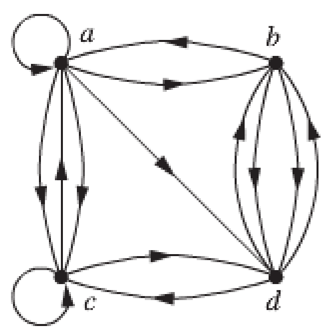
a)



b)



c)

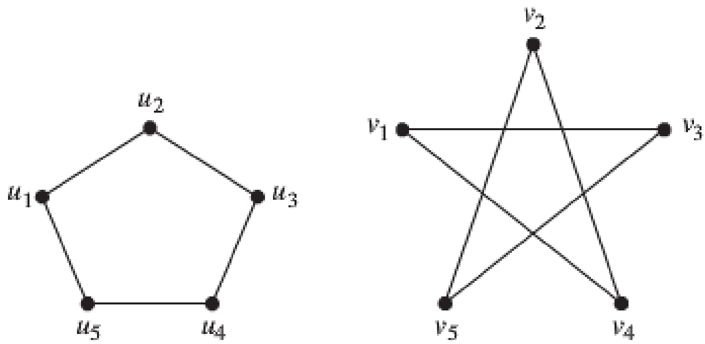


Solution

$\boldsymbol{a})\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ $\quad \boldsymbol{b})\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ $\quad \boldsymbol{c})\ \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

Exercise

Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

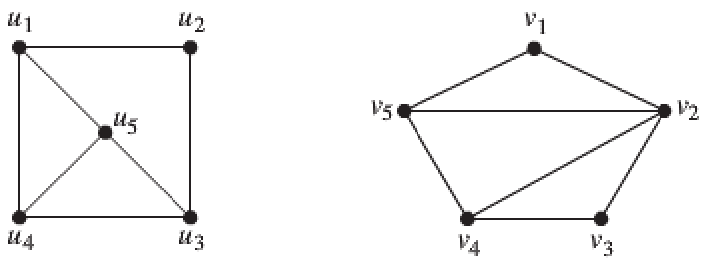


Solution

Both graphs have 5 vertices and 5 edges. However, each vertex in the second graph has of degree 2, whereas the first does not.

Exercise

Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

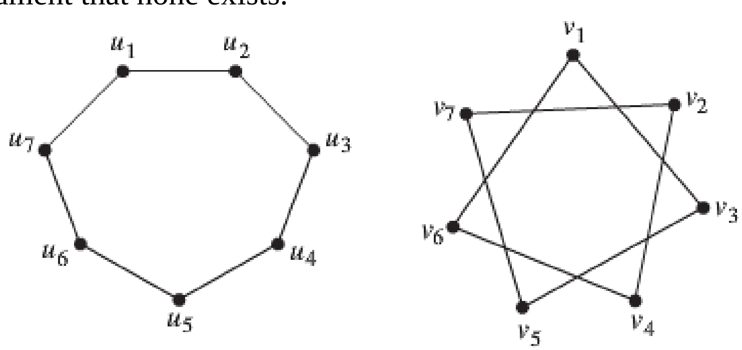


Solution

Both graphs have 5 vertices and 7 edges. However, the second graph has a vertex of degree 4, whereas the first does not.

Exercise

Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



Solution

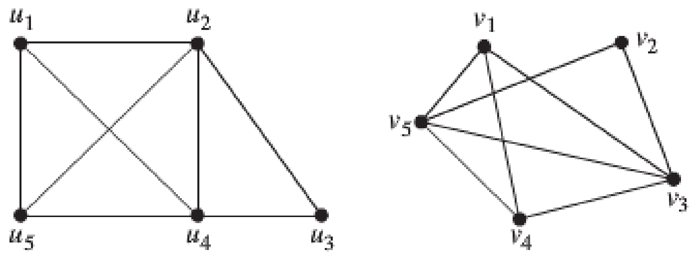
Both graphs have 7 vertices and 7 edges.

$f(u_1)=v_1$, $f(u_2)=v_3$, $f(u_3)=v_5$, $f(u_4)=v_7$, $f(u_5)=v_2$, $f(u_6)=v_4$, and $f(u_7)=v_6$

$\therefore$ The graphs are isomorphic.

Exercise

Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



Solution

Both graphs have 5 vertices and 8 edges.

$f(u_1)=v_1$, $f(u_2)=v_3$, $f(u_3)=v_2$, $f(u_4)=v_5$, and $f(u_5)=v_4$

$\therefore$ The graphs are isomorphic.