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Determine if the sequence converges or diverges. Find the limit of the convergent sequence

1. $a_n = 1 + \frac{(-1)^n}{n}$

5. $a_n = \left(1 + \frac{1}{n}\right)^n$

9. $a_n = n - \sqrt{n^2 - 1}$

2. $a_n = \frac{1 - (-1)^n}{\sqrt{n}}$

6. $a_n = \frac{(n+1)!}{n!}$

10. $a_n = \left(\frac{1}{n}\right)^{1/\ln n}$

3. $a_n = \sin n\pi$

7. $a_n = \frac{n^2 + 4}{\sqrt{4n^4 + 1}}$

11. $a_n = \tan^{-1} n$

4. $a_n = \frac{n + \ln n}{n}$

8. $a_n = \left(1 + \frac{3}{n}\right)^{2n}$

Find the sums of the series

12. $\sum_{n=3}^{\infty} \frac{1}{(2n-3)(2n-1)}$

15. $\sum_{n=0}^{\infty} e^{-n}$

17. $\sum_{n=0}^{\infty} \left(-\frac{1}{5}\right)^n$

13. $\sum_{n=2}^{\infty} \frac{-2}{n(n+1)}$

16. $\sum_{n=1}^{\infty} (-1)^n \frac{3}{4^n}$

18. $\sum_{n=1}^{\infty} \frac{2^n}{3^{n+2}}$

14. $\sum_{n=3}^{\infty} \frac{-8}{(4n-3)(4n+1)}$

Determine if the series converges or diverges. Give reasons for your answers

19. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

23. $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2 + 1)}{2n^2 + n - 1}$

27. $\sum_{n=1}^{\infty} \frac{2n^2 + 1}{\sqrt{n^3 + 2}}$

20. $\sum_{n=1}^{\infty} \frac{1}{2n^3}$

24. $\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

28. $\sum_{n=1}^{\infty} \left(\frac{n}{n+3}\right)^{2n}$

21. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

25. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2 - 1}}$

29. $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

22. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2 + 1}}$

26. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

30. $\sum_{n=1}^{\infty} \frac{3}{2 + e^n}$

$$31. \sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$$

$$33. \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n^2+4)}{2n^2+1}$$

$$35. \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

$$32. \sum_{n=1}^{\infty} \frac{\ln n^2}{n^2}$$

$$34. \sum_{n=1}^{\infty} \frac{n+1}{n!}$$

Find the radius of convergence of the series

$$36. \sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdots (3n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} x^n$$

$$37. \sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdots (2n+1)}{4 \cdot 9 \cdot 14 \cdots (5n-1)} (x-1)^n$$

Finding Taylor and Maclaurin Series generated by f at $x=0$

$$38. \frac{1}{1-2x}$$

$$39. \frac{1}{1+x^3}$$

$$40. \sin \frac{2x}{3}$$

$$41. e^{-x^2}$$

Finding Taylor and Maclaurin Series generated by f at $x=a$

$$42. f(x) = \sqrt{3+x^2}, \quad a = -1$$

$$43. f(x) = \frac{1}{1-x}, \quad a = 2$$

$$44. f(x) = \frac{1}{x+1}, \quad a = 3$$

Answers

1. *converges to 1*
2. *converges to 0*
3. *converges to 0*
4. *converges to 1*
5. *converges to $\frac{1}{e}$*
6. *diverges*
7. *converges to $\frac{1}{2}$*
8. *converges to e^6*
9. *converges to 0*
10. *converges to e^{-1}*
11. *converges to $\frac{\pi}{2}$*
12. $\frac{1}{6}$
13. -1
14. $-\frac{2}{9}$
15. $\frac{e}{e-1}$
16. $-\frac{3}{5}$
17. $\frac{5}{6}$
18. $\frac{2}{9}$
19. *diverges $p = \frac{1}{2}$*
20. *converges absolutely by Direct Comparison*
21. *converges to $\frac{1}{\ln 2}$*
22. *converges to 1*
23. *diverges $\rightarrow \frac{1}{2} \neq 0$*
24. *converges to 0*
25. *converges to 1*
26. *converges to 0*
27. *diverges $\rightarrow \infty$*
28. *diverges $\rightarrow \frac{1}{e^6} \neq 0$*
29. *converges to $\frac{2}{e}$*
30. *converges*
31. *diverges $\rightarrow 1 \neq 0$*
32. *converges*
33. *diverges $\rightarrow \frac{1}{2} \neq 0$*
34. *converges Comparison (p – series)*
35. *converges RT*
36. *converges conditionally*

$$37. \quad \text{radius} = \frac{2}{3}$$

$$38. \quad \text{radius} = \frac{5}{2}$$

$$39. \quad \sum_{n=0}^{\infty} 2^n x^n$$

$$40. \quad \sum_{n=0}^{\infty} (-1)^n x^{3n}$$

$$41. \quad \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{3^{2n+1} (2n+1)!}$$

$$42. \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$$

$$43. \quad 2 - \frac{x+1}{2 \cdot 1!} + \frac{3(x+1)^2}{2^3 \cdot 2!} + \frac{9(x+1)^3}{2^5 \cdot 3!} + \dots$$

$$44. \quad -1 + (x-2) - (x-2)^2 + (x-2)^3 - \dots$$

$$45. \quad \frac{1}{4} - \frac{1}{4^2}(x-3) + \frac{1}{4^3}(x-3)^2 - \frac{1}{4^4}(x-3)^3 + \dots$$