Solution *Section* 3.7 – Hypothesis Tests for a Population Mean

Exercise

Because the amounts of nicotine in king size cigarettes listed below

1.1	1.7	1.7	1.1	1.1	1.4	1.1	1.4	1	1.2	1.1	1.1	1.1
1.1	1.1	1.8	1.6	1.1	1.2	1.5	1.3	1.1	1.3	1.1	1.1	

We must satisfy the requirement that the population is normally distributed. How do we verify that a population is normally distributed?

Solution

We consider the normality requirement to be satisfied if there are no outliers and the histogram of the sample data is approximately bell-shaped. More formally, a normal quantile plot could be used to determine whether the sample data are approximately normally distributed.

Exercise

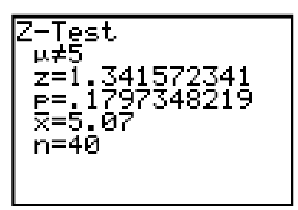
If you want to construct a confidence interval to be used for testing the claim that college students have a mean IQ score that is greater than 100, and you want the test conducted with a 0.01 significance level, what confidence level should be used for the confidence interval?

Solution

A one-tailed test at the 0.01 level of significance rejects the null hypothesis if the sample statistic falls into the extreme 1% of the sampling distribution in the appropriate tail. The corresponding (two-sided) confidence interval test that places 1% each tail would be a 98% confidence interval.

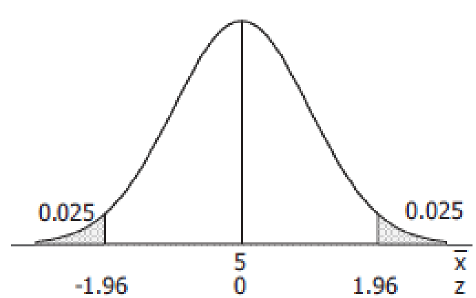
Exercise

A jewelry designer claims that women have wrist breadths with a mean equal to 5 cm. A simple random sample of the wrist breadths of 40 women has a mean of 5.07 cm. Assume that the population standard deviation is 0.33 cm. Use the accompanying TI display to test the designer's claim.



Identify the null hypothesis, alternative hypothesis, test statistic, *P*-value or critical value(s), conclusion about the null hypothesis, and final conclusion that address the original claim.

Solution



Original claim: $\mu = 5$ cm

$H_0: \mu = 5cm$

$H_1: \mu \neq 5cm$

Assume: $\alpha = 0.05$

Critical value: $z = \pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$

P-value = 0.1797 [*TI*]

$$z_{\overline{x}} = \frac{\overline{x} - \mu_{\overline{x}}}{\sigma_{\overline{x}}} = 1.34 \quad [TI]$$

Conclusion

Do not reject H_0; there is not sufficient evidence to reject the claim that $\mu = 5$. There is not sufficient evidence the claim that women have a mean wrist breadth equal to 5 cm.

Exercise

The U.S. Mint has a specification that pennies have a mean weight of 2.5 g. Assume that weights of pennies have a standard deviation of 0.0165 g and use the accompanying Minitab display to test the claim that the sample is from a population with a mean that is less than 2.5 g. These Minitab results were obtained using the 37 weights of post 1983 pennies.

```
Test of mu = 2.5 vs < 2.5. Assumed s.d. = 0.0165
                             95% Upper
 N     Mean      StDev      Bound        Z        P
37   2.49910   0.01648    2.50356    -0.33    0.370
```

Solution

Original claim: $\mu < 2.5$ g.

$H_0: \mu = 2.5\, g$

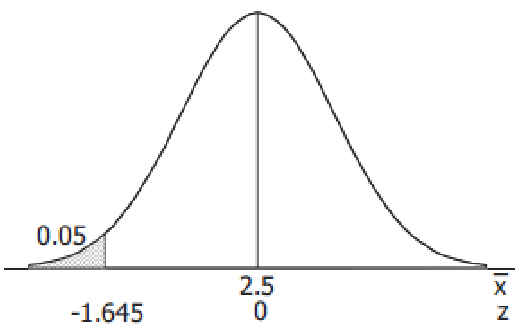
$H_1: \mu < 2.5\, g$

Assume: $\alpha = 0.05$

Critical value: $z = -z_{\alpha} = -z_{0.05} = -1.645$

$$z_{\overline{x}} = \frac{\overline{x} - \mu_{\overline{x}}}{\sigma_{\overline{x}}} = -0.33 \quad [Minitab]$$

P-value = 0.370 [***Minitab***]



Conclusion

Do not reject H_0; there is not sufficient evidence to reject the claim that $\mu < 2.5$. There is not sufficient evidence to support the claim the sample is from a population with a mean less than 2.5 g.

Exercise

In the manual "How long to have a Number One the Easy Way," by KLF Publications, it is stated that a song "must be no longer than 3 minutes and 30 seconds" (or 210 seconds). A simple random sample of 40 current hit songs results in a mean length of 252.5 sec. Assume that the standard deviation of song lengths is 54.5 sec. Use a 0.05 significance level to test the claim that the sample is from a population of songs with a mean greater than 210 sec. What do these results suggest about the advice given in the manual?

Solution

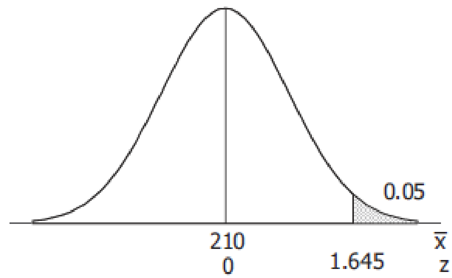
Original claim: $\mu > 210$ sec.

$H_0: \mu = 210$ sec

$H_1: \mu > 210$ sec

Given: $\alpha = 0.05$

Critical value: $z = z_{\alpha} = z_{0.05} = 1.645$



z score	Area
1.645	0.9500

$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{252.5 - 210}{\frac{54.5}{\sqrt{40}}} = 4.93$$

$$P - value = P(z > 4.93) = 1 - 0.9999 = 0.0001$$

Conclusion

Reject H_0; there is sufficient evidence to conclude that $\mu > 210$. There is sufficient evidence to support the claim the sample is from a population of songs with a mean greater than 210 sec. These results suggest that the advice given in the manual is not good advice.

Exercise

A simple random sample of 50 adults is obtained, and each person's red blood cell count (in cells per microliter) is measured. The sample mean is 5.23. The population standard deviation for red blood cell counts is 0.54. Use a 0.01 significance level to test the claim that the sample is from population with a mean less than 5.4, which is value often used for the upper limit of the range of normal values. What do the results suggest about the sample group?

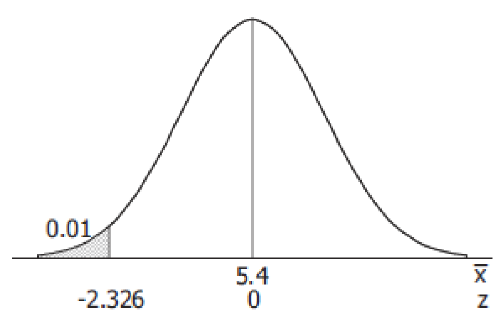
Solution

Original claim: $\mu < 5.4$ cells/microliter.

$H_0: \mu = 5.4 \ cells / microliter$

$H_1: \mu < 5.4 \ cells / microliter$

Given: $\alpha = 0.01$



z	0.00	0.01	0.02	0.03
−2.3	0.0107	0.0104	0.0102	0.0099

Critical value: $z = -z_{\alpha} = -z_{0.01} = -2.326$

$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{5.23 - 5.4}{\frac{0.54}{\sqrt{50}}} = -2.23$$

$(5.23 - 5.4) / (0.54 / \sqrt{50})$

$$P - value = P(z < -2.23) = 0.0129$$

Conclusion

Do not reject H_0; there is not sufficient evidence to reject the claim that $\mu < 5.4$. There is not sufficient evidence to support the claim the sample is from a population with a mean red blood cell count less than 504 cells/microliter. If 5.4 is the upper limit for the range of normal individuals, and if the mean of the sample group is not significantly below the upper limit for an individual and the mean of the sample group may have manually high red cell counts. The $\mu \pm 2\sigma$ guideline for normal values suggests that the population mean is approximately $5.4-2(0.54)=4.32$, and the sample mean of 5.23 is considerably higher than that.

Exercise

A simple random sample of 106 body temperature with a mean of 98.20 °F. Assume that σ is known to be 0.62 °F. Use a 0.05 significance level to test the claim that the mean body temperature of the population is equal to 98.6 °F, as is commonly believed. Is there sufficient evidence to conclude that the common belief is wrong?

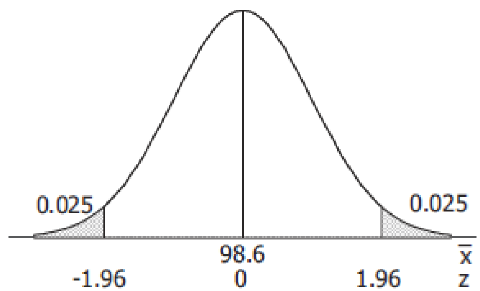
Solution

Original claim: μ = 98.6 °F.

$H_0:\ \mu = 98.6°F$

$H_1:\ \mu \neq 98.6°F$

Given: $\alpha = 0.05$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250

Critical value:

$$z = \pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$$

$$z_{\bar{x}} = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

$$= \frac{98.20-98.6}{\frac{0.62}{\sqrt{106}}}$$

$$(98.2-98.6)/\left(.62/\sqrt{106}\right)$$

$$\underline{=-6.64}$$

$$P-value = 2P(z<-6.64)$$

$$= 2(0.0001)$$

$$\underline{= 0.0002}$$

Conclusion

Reject H_0; there is sufficient evidence to reject the claim that $\mu = 98.6$ and conclude that $\mu \neq 98.6$ (in fact, that $\mu < 98.6$). There is sufficient evidence to reject the claim that the mean body temperature of the population is 98.6°F. Yes; it appears there is sufficient evidence to conclude that the common belief is wrong.

Exercise

When 40 people used the Weight Watchers diet for one year, their mean weight loss was 3.0 lb. Assume that the standard deviation of all such weight changes is $\sigma = 4.9$ lb. and use a 0.01 significance level to test the claim that the mean weight loss is greater than 0. Based on these results, does the diet appear to be effective? Does the diet appear to have a practical significance?

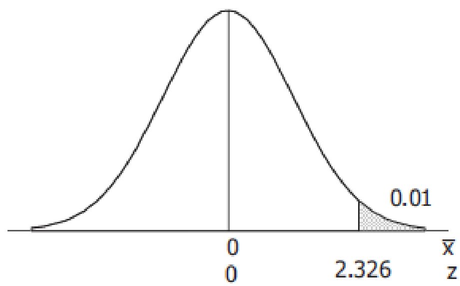
Solution

Original claim: $\mu > 0$ lb.

$H_0: \mu = 0\ lb$

$H_1: \mu > 0\ lb$

Given: $\alpha = 0.01$



Critical value:

$z = z_\alpha = z_{0.01} = 2.326$

$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{3.0 - 0}{\frac{4.9}{\sqrt{40}}} = 3.87$$

$30 / \left(4.9 / \sqrt{40}\right)$

$P - value = P(z > 3.87) = 1 - 0.9999 = 0.0001$

Conclusion

Reject H_0; there is sufficient evidence to reject the claim that $\mu > 0$. There is sufficient evidence to support the claim that the mean weight lost is greater than 0. The diet is effective in that the weight loss is statistically significant – but a mere 3.0 lbs. weight loss after following the regimen for an entire year suggests the diet may have no practical significance.

Exercise

The health of the bear population in Yellowstone National Park is monitored by periodic measurements taken from anesthetized bears. A sample of 54 bears has a mean weight of 182.9 lb. Assuming that σ is known to be 121.8 lb. use a 0.05 significance level to test the claim that the population mean of all such bear weights is greater than 150 lb.

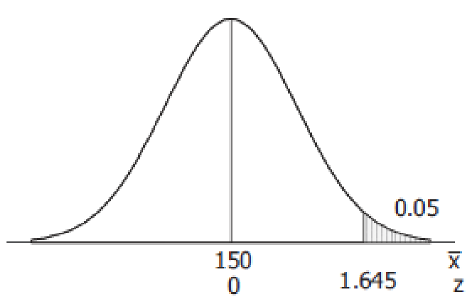
Solution

Original claim: $\mu > 150$ lbs.

$H_0: \mu = 150\ lbs$

$H_1: \mu > 150\ lbs$

Given: $\alpha = 0.05$



Critical value:

z score	Area
1.645	0.9500

$z = z_\alpha = z_{0.05} = 1.645$

$$z_{\overline{x}} = \frac{\overline{x}-\mu_{\overline{x}}}{\sigma_{\overline{x}}} = \frac{182.9-150}{\frac{121.8}{\sqrt{54}}} = 1.98$$

$(182.9-150)/(121.8/\sqrt{54})$

$$P-value = P(z>1.98) = 1-0.9761 = 0.0239$$

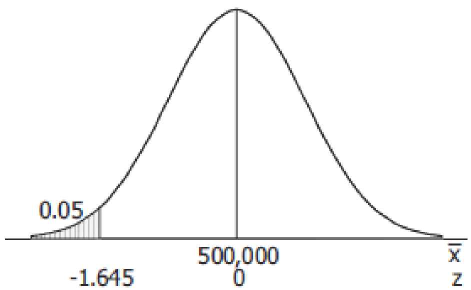
Conclusion

Reject H_0; there is sufficient evidence to conclude that $\mu > 150$. There is sufficient evidence to support the claim that the mean weight of all such bears is greater than 150 lbs.

Exercise

A simple random sample of 401 salaries of NCAA football coaches in the NCAA has a mean of \$415,953. The standard deviation of all salaries of NCAA football coaches is \$463,364. Use a 0.05 significance level to test the claim that the mean salary of a football coach in the NCAA is less than \$500,000.

Solution



Original claim: μ < \$500,000.

$H_0: \mu = \$500,000$

$H_1: \mu < \$500,000$

Given: $\alpha = 0.05$

Critical value: $z = -z_{\alpha} = -z_{0.05} = -1.645$

$$z_{\overline{x}} = \frac{\overline{x}-\mu_{\overline{x}}}{\sigma_{\overline{x}}} = \frac{415,953-500,000}{\frac{463,364}{\sqrt{40}}} = -1.15$$

$(415,953-500,000)/(463,364/\sqrt{40})$

$$P-value = P(z<-1.15) = 0.1251$$

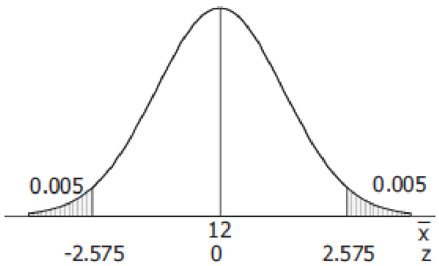
Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $\mu < 500,000$. There is not sufficient evidence to support the claim that the mean salary of an NCAA football coach is less than \$500,000.

Exercise

A simple random sample of 36 cans of regular Coke has a mean volume of 12.19 oz. Assume that the standard deviation of all cans of regular Coke is 0.11 oz. Use a 0.01 significance level to test the claim that cans if regular Coke have volumes with a mean of 12 oz., as stated on the label. If there is a difference, is it substantial?

Solution



Original claim: $\mu < 12$ oz.

$H_0: \mu = 12\ oz$

$H_1: \mu \neq 12\ oz$

Given: $\alpha = 0.01$

Critical value: $z = \pm z_{\alpha/2} = \pm z_{0.025} = \pm 2.575$

$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{12.19 - 12}{\frac{0.11}{\sqrt{36}}} = 10.36$$

$$(415,953 - 500,000) / \left(463,364 / \sqrt{40}\right)$$

$$P - value = 2 \cdot P(z > 10.36) = 2(1 - 0.999) = 0.0002$$

Conclusion

Reject H_0; there is sufficient evidence to reject the claim that $\mu = 12$ and conclude that $\mu \neq 12$ (in fact, that $\mu < 12$). There is sufficient evidence to reject the claim that cans of regular Coke have a mean volume of 12 oz. The difference is statistically significant, but the difference is of practical significance only in that it guarantees that virtually 100% of the product meets the volume stated on the label.

Exercise

A simple random sample of FICO credit rating scores is obtained, and the scores are listed below.

714 751 664 789 818 779 698 836 753 834 693 802

As the writing, the mean FICO score was reported to be 678. Assuming the standard deviation of all FICO scores is known to be 58.3, use a 0.05 significance level to test the claim that these sample FICO scores come from a population with a mean equal to 678.

Identify the null hypothesis, alternative hypothesis, test statistic, *P*-value or critical value(s), conclusion about the null hypothesis, and final conclusion that address the original claim.

Solution

$$n = 12; \quad \sum x = 9131 \quad \sum x^2 = 6,985.297 \quad \bar{x} = \frac{\sum x}{n} = \frac{9131}{12} = 760.9$$

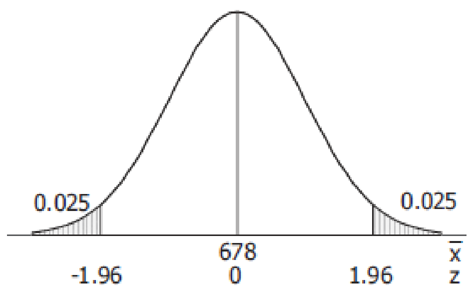
Original claim: $\mu <$ 678 FICO units.

$H_0: \mu = 678\ \ FICO\ units$

$H_1: \mu \neq 678\ \ FICO\ units$

Given: $\alpha = 0.05$

Critical value: $z = \pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$



$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{760.9 - 678}{\frac{58.3}{\sqrt{12}}} \underline{= 4.93|}$$

$$P-value = 2 \cdot P(z > 4.93) = 2(1 - 0.9999) \underline{= 0.0002|}$$

Conclusion

Reject H_0; there is sufficient evidence to reject the claim that $\mu = 678$ and conclude that $\mu \neq 678$ (in fact, that $\mu > 678$). There is sufficient evidence to reject the claim that these FICO scores come from a population with a mean equal to 678.

Exercise

Listed below are recorded speeds (in mi/h) of randomly selected cars traveling on a section of Highway 405 in Los Angeles.

68	68	72	73	65	74	73	72	68	65	65	73	66	71	68	74	66	71	65	73
59	75	70	56	66	75	68	75	62	72	60	73	61	75	58	74	60	73	58	75

That part of the highway has posted speed limit of 65 mi/h. Assume that the standard deviation od speeds is 5.7 mi/h and use a 0.01 significance level to test the claim that the sample is from a population with a mean that is greater than 65 mi/h.

Identify the null hypothesis, alternative hypothesis, test statistic, *P*-value or critical value(s), conclusion about the null hypothesis, and final conclusion that address the original claim.

Solution

$$n = 40; \quad \sum x = 2735 \quad \sum x^2 = 188,259 \quad \bar{x} = \frac{\sum x}{n} = \frac{2735}{40} = 68.375$$

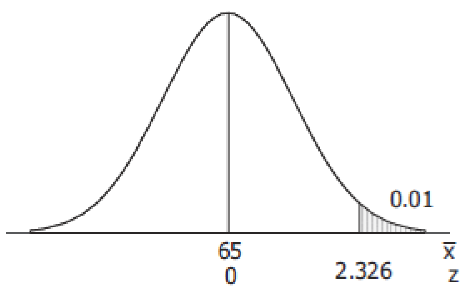
Original claim: $\mu > 65$ mph..

$H_0 : \mu = 65\ mph$

$H_1 : \mu > 65\ mph$

Given: $\alpha = 0.01$

Critical value: $z = z_{\alpha} = z_{0.01} = 2.326$



$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{68.375 - 65}{\frac{5.7}{\sqrt{40}}} \underline{= 3.74|}$$

$$P-value = P(z > 3.74) = 1 - 0.9999 \underline{= 0.0001|}$$

Conclusion

Reject H_0; there is sufficient evidence to conclude that $\mu > 65$. There is sufficient evidence to support the claim that the sample is from a population with a mean greater than 65 mph.

Exercise

Assume the resting metabolic rate (RMR) of healthy males in complete silence is 5710 kJ/day. Researchers measured the RMR of 45 healthy males who were listening to calm classical music and found their mean RMR to be 5708.07 with a standard deviation of 992.05.

At the α = 0.05 level of significance, is there evidence to conclude that the mean RMR of males listening to calm classical music is different than 5710 kJ/day?

Solution

We assume that the RMR of healthy males is 5710 kJ/day. This is a two-tailed test since we are interested in determining whether the RMR differs from 5710 kJ/day.

Since the sample size is large, we follow the steps for testing hypotheses about a population mean for large samples.

Step 1: $H_0 : \mu = 5710$ *vs.* $H_1 : \mu \neq 5710$

Step 2: The level of significance is α = 0.05.

Step 3: The sample mean is $\bar{x} = 5708.7$ and $s = 992.05$. The test statistic is

$$t_0 = \frac{5708.7 - 5710}{\frac{992.05}{\sqrt{45}}} = -0.013$$

Classical Approach

Step 4: Since this is the two-tailed test, we determine the critical value at the α = 0.05 level of significance with $df = 45 - 1 = 44$ to be $-t_{0.025} = -2.021$ *and* $t_{0.025} = 2.021$

Step 5: Since the test statistic $t_0 = -0.013$, is between the critical values, we fail to reject the null hypothesis.

P-Value Approach

Step 4: Since this is the two-tailed test, the *P*–value is the area under the *t*–distribution with $df = 44$ to the left of $-t_{0.025} = -2.021$ and to the right of $t_{0.025} = 2.021$.

That is, P–value $= P(t < -0.013) + P(t > 0.013)$

$$= 2P(t > 0.013)$$

$$= 0.50 < P - value$$

Step 5: Since the *P*-value is greater than the level of significance (0.05 < 0.5), we fail to reject the null hypothesis.

Step 6: There is insufficient evidence at the α = 0.05 level of significance to conclude that the mean RMR of males listening to calm classical music differs from 5710 kJ/day.

Exercise

People have died in boat accidents because an obsolete estimate of the mean weight of men was used. Using the weights of the simple random sample of men from Data Set 1 in Appendix B, we obtain these sample statistics: n = 40 and $\overline{x} = 172.55\ lb.$, and σ = 26.33 *lb*. Do not assume that the value of σ is known. Use these results to test the claim that men have a mean weight greater than 166.3 *lb*., which was the weight in the National Transportation and Safety Board's recommendation M-04-04. Use a 0.05 significance level, and the traditional method.

Solution

Requirements are satisfied: simple random sample, population standard deviation is not known, sample size is 40 ($n > 30$)

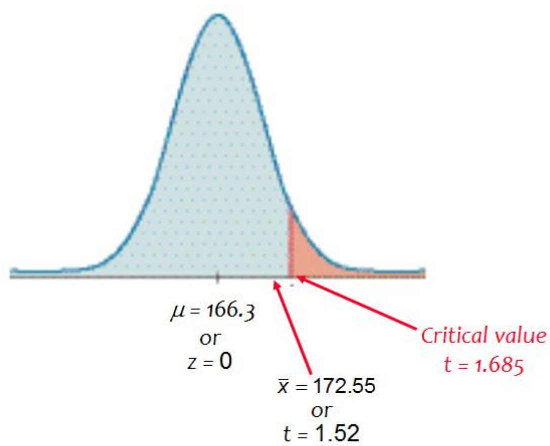
Step 1: Express claim as $\mu > 166.3\ lb.$

Step 2: Alternative to claim is $\mu \le 166.3\ lb.$

Step 3: $\mu > 166.3\ lb.$ does not contain equality, it is the alternative hypothesis:

$H_0:\ \mu = 166.3\ lb.$ null hypothesis

$H_1:\ \mu > 166.3\ lb.$ alternative hypothesis and original claim



Step 4: Significance level is $\alpha = 0.05$

Step 5: Claim is about the population mean, so the relevant statistic is the sample mean, 172.55 lb.

Step 6: Calculate t

$$t = \frac{\overline{x} - \mu_{\overline{x}}}{\frac{s}{\sqrt{n}}} = \frac{172.55 - 166.3}{\frac{26.33}{\sqrt{40}}} = 1.501$$

df = $n - 1$ = 39, area of 0.05, one-tail yields t = 1.685;

Step 7: t = 1.501 does not fall in the critical region bounded by t = 1.685, we fail to reject the null hypothesis.

✓ Because we fail to reject the null hypothesis, we conclude that there is not sufficient evidence to support a conclusion that the population mean is greater than 166.3 lb, as in the National Transportation and Safety Board's recommendation.

Exercise

Given a simple random sample of speeds of cars on Highway in CA, you want to test the claim that the sample that the sample values are from a population with a mean greater than the posted speed limit of 65 mi/hr. Is it necessary to determine whether the sample is from a normally distributed population? If so, what methods can be used to make that determination?

Solution

Yes, since $n \le 30$, the sample should be from a population that is approximately normally distributed. We consider the normality requirement to be satisfied for such data if there are no outliers and the histogram of the sample data is approximately bell-shaped. More formally, a normal quantile plot could be used to determine whether the sample data are approximately normally distributed.

Exercise

In statistics, what does ***df*** denote. If a simple random sample of 20 speeds of cars is to be used to test the claim that the sample values are from a population with a mean greater than the posted speed limit of 65 mi/h, what is the specific value of ***df***?

Solution

In statistics, ***df*** denotes the degrees of freedom. In general, the degrees of freedom give the number of pieces of information that are free to vary without changing the mathematical constraint of the problem. When using a *t* test with $n = 20$ sample values to test a claim about the mean of a population, $df = 19$

Exercise

Claim about IQ scores of statistics instructors: $\mu > 100$, sample data: $n = 15, \quad \bar{x} = 118, \quad s = 11$. The sample data appear to come from a normally distributed population with unknown μ and σ. Determine whether the hypotheses test involves a sampling distribution of means that is a normal distribution, student *t* distribution, or neither.

Solution

Use *t*. When σ is unknown and the *x*'s approximately normally distributed, use *t*.

Exercise

Claim about FICO credit scores of adults: $\mu = 678$, sample data: $n = 12$, $\bar{x} = 719$, $s = 92$. The sample data appear to come from a population with a distribution that is not normal, and σ Is unknown Determine whether the hypotheses test involves a sampling distribution of means that is a normal distribution, student *t* distribution, or neither.

Solution

Neither the *z* nor the *t* applies. When σ is unknown and the *x*'s are not normally distributed sample sizes $n \le 30$ cannot be used with these techniques.

Exercise

Claim about daily rainfall amounts in Boston: $\mu < 0.20\, in$, sample data: $n = 19$, $\bar{x} = 0.10\, in$, $s = 0.26\, in$. The sample data appear to come from a population with a distribution that is very far from normal, and σ is unknown Determine whether the hypotheses test involves a sampling distribution of means that is a normal distribution, student *t* distribution, or neither.

Solution

Neither the *z* nor the *t* applies. When σ is unknown and the *x*'s are not normally distributed sample sizes $n \le 30$ cannot be used with these techniques.

Exercise

Testing a claim about the mean weight of M&M's: Right-tailed test with *n* = 25 and test statistic *t* = 0.430. Find the *P*-value and find a range of values for the *P*-value.

Solution

$P - value = P\left(t_{24} > 0.430\right)$

Table for area in one tail: [0.430 < 1.318] $P - value > 0.10$

TI: ***tcdf***(0.430, 99, 24) = 0.3355

```
tcdf(0.430,99,24
                .3355
```

Exercise

Test a claim about the mean body temperature of healthy adults: left-tailed test with *n* = 11 and test statistic *t* = –3.158. Find the *P*-value and find a range of values for the *P*-value.

Solution

$P - value = P\left(t_{10} < -3.518\right)$

Table for area in one tail: [–3.518 < –3.169]

$P - value < 0.01$

TI: ***tcdf***(–99, –3.518, 10) = 0.0027

```
tcdf(-99,-3.518,
10
      .0027784746
```

Exercise

Two-tailed test with n = 15 and test statistic t = 1.495. Find the P-value and find a range of values for the P-value.

Solution

$$P-value = 2 \cdot P\left(t_{14} < 1.495\right)$$

```
2tcdf(1.495,99,1
4
            .1571
```

Table for area in two tails: [1.345 < 1.495 < 1.761]

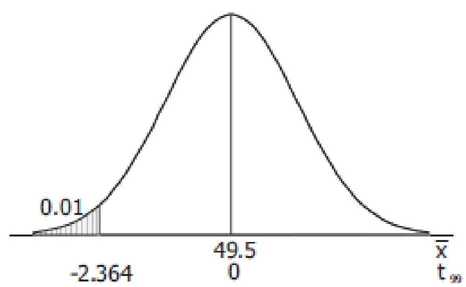
$$P-value < 0.20$$

TI: 2·***tcdf***(1.495, 99, 14) = 0.1571

Exercise

In an analysis investigating the usefulness of pennies, the cents portions of 100 randomly selected checks are recorded. The sample has mean of 23.8 cents and a standard deviation of 32.0 cents. If the amounts from 0 cents to 99 cents are all equally likely, the mean is expected to be 49.5 cents. Use a 0.01 significance level to test the claim that the sample is from a population with a mean less than 49.5 cents. What does the result suggest about the cents portions of the checks?

Solution



Original claim: μ = 49.5 cents.

$H_0: \ \mu = 49.5 \ cents$

$H_1: \ \mu < 49.5 \ cents$

Given: $\alpha = 0.01 \quad and \quad df = 99$

Critical value: $t = -t_{\alpha} = -t_{0.01} = -2.364$

$$t_{\bar{x}} = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{23.8 - 49.5}{\frac{32}{\sqrt{100}}} = -8.031$$

$(23.8 - 49.5) / \left(32 / \sqrt{100}\right)$

$$P-value = 2 \cdot P\left(t_{99} < -8.031\right) = 2 \cdot tcdf\,(-99, \ -8.031, \ 99) = 2.06\,E-12$$

```
tcdf(-99,-8.031,
99
            1.03E-12
```

Conclusion

Reject H_0; there is sufficient evidence to conclude that μ < 49.5. There is sufficient evidence to that the cents portion of all checks has a mean that is less than 49.5 cents. The results suggest that the cents portions of checks are not uniformly distributed from 0 to 99 cents.

Exercise

A simple random sample of 40 recorded speeds (in mi/h) is obtained from cars traveling on a section of Highway 405 in Los Angeles. The sample has a mean of 68.4 mi/h and a standard deviation of 5.7 mi/h.

Use a 0.05 significance level to test the claim that the mean speed of all cars is greater than the posted speed limit of 65 mi/h.

Solution

$$n = 40; \quad \sum x = 2735 \quad \sum x^2 = 188,259 \quad \bar{x} = \frac{\sum x}{n} = \frac{2735}{40} = 68.375$$

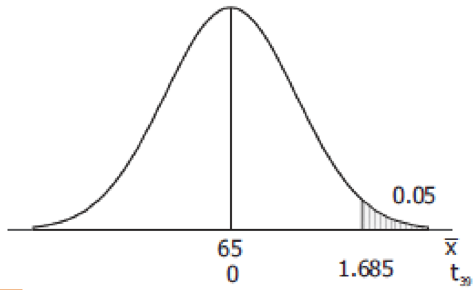
Original claim: $\mu > 65$ mph.

$H_0 : \mu = 65\ mph$

$H_1 : \mu > 65\ mph$

Given: $\alpha = 0.05$ *and* $df = 39$

Critical value: $t = t_{\alpha} = t_{0.05} = 1.685$



Degrees of Freedom	t Distribution: Critical t Values				
	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
39	2.708	2.426	2.023	1.685	1.304

$$t_{\bar{x}} = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{68.4 - 65}{\frac{5.7}{\sqrt{40}}} = \underline{3.773}$$

```
tcdf(3.773,99,39
   2.681717486E-4
```

$$P - value = P\left(t_{39} > 3.7763\right) = tcdf\left(3.773,\ 99,\ 39\right) \underline{= 0.0003}$$

Conclusion

Reject H_0; there is sufficient evidence to conclude that $\mu > 65$. There is sufficient evidence to support the claim that the mean speed of all such cars is greater than the posted speed of 65 mph.

Exercise

The heights are measured for the simple random sample of supermodels. They have mean height of 70.0 in. and a standard deviation of 1.5 in. Use a 0.01 significance level to test the claim that supermodels have heights with a mean that is greater than the mean heights of 63.6 in. for women in general population. Given that there are only nine heights represented, can we really conclude that supermodels are taller than the typical woman?

Solution

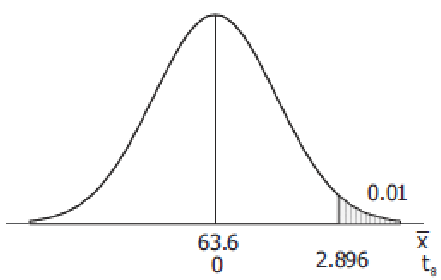
Original claim: $\mu > 63.6$ in.

$H_0 : \mu = 63.6\ in.$

$H_1 : \mu > 63.6\ in.$

Given: $\alpha = 0.01$ *and* $df = 9 - 1 = 8$

Critical value: $t = t_{\alpha} = t_{0.01} = 2.896$



Degrees of Freedom	t Distribution: Critical t Values				
	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
8	3.355	2.896	2.306	1.860	1.397

$$t_{\overline{x}} = \frac{\overline{x} - \mu}{s_{\overline{x}}} = \frac{70 - 63.6}{\frac{1.5}{\sqrt{9}}} = \underline{12.80}$$

$$P - value = P\left(t_8 > 12.8\right) = tcdf\,(12.8,\ 99,\ 8) = 6.5465E\text{ - }7 \approx \underline{0.0000007}$$

Conclusion

Reject H_0; there is sufficient evidence to conclude that $\mu > 63.6$. There is sufficient evidence to support the claim that supermodels have a mean height that is greater than the 63.6 in. of the general population of women. Yes; assuming that the heights of supermodels are approximately normally distributed around their mean, the test and the conclusion are valid.

Exercise

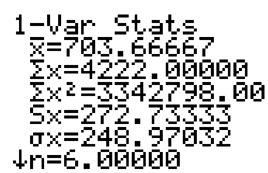
The National Highway Traffic Safety Administration conducted crash tests of child booster seats for cars. Listed below are results from those tests, with measurement given in hic (standard *head injury condition* units). The safety requirement is that the hic measurement should be less than 1000 hic. Use a 0.01 significance level to test the claim that the sample is from a population with a mean less than 1000 hic.

774 649 1210 546 431 612

Do the results suggest that all of the child booster seats meet the specified requirement?

Solution

$n = 6;\quad \sum x = 4222 \quad \sum x^2 = 3{,}342{,}798 \quad \overline{x} = 703.67 \quad s = 272.73$



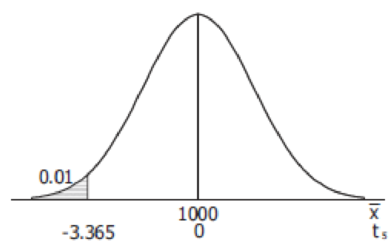
Original claim: $\mu < 1000$ hic.

$H_0 : \mu = 1000\ hic.$

$H_1 : \mu < 1000\ hic.$

Given: $\alpha = 0.01 \quad and \quad df = 6 - 1 = 5$

Critical value: $t = -t_\alpha = -t_{0.01} = -3.365$



Degrees of Freedom	*t* Distribution: Critical *t* Values				
			Area in One Tail		
	0.005	0.01	0.025	0.05	0.10
5	**4.032**	3.365	**2.571**	2.015	1.476

$$t_{\overline{x}} = \frac{\overline{x} - \mu}{s_{\overline{x}}} = \frac{703.67 - 1000}{\frac{272.73}{\sqrt{6}}} = \underline{-2.661}$$

$$P - value = P\left(t_5 < -2.661\right) = tcdf\,(-99, -2.661, 5) = \underline{0.0224}$$

Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $\mu < 1000$. There is not sufficient evidence to support the claim that the population mean is less than 1000 hic. No; since one of the sample values is 1210, there is proof that not all of the child booster seats meet the specified requirement.

Exercise

The trend of thinner Miss America winners has generated charges that the contest encourages unhealthy diet habits among young women. Listed below are body mass indexes (BMI) for recent Miss America winners. Use a 0.01 significance level to test the claim that recent Miss America winners are from a population with a mean BMI less than 20.16, which was the BMI for winners from the 1920s and 1930s.

19.5 20.3 19.6 20.2 17.8 17.9 19.1 18.8 17.6 16.8

Do recent winners appear to be significantly different from those in the 1920s and 1930s?

Solution

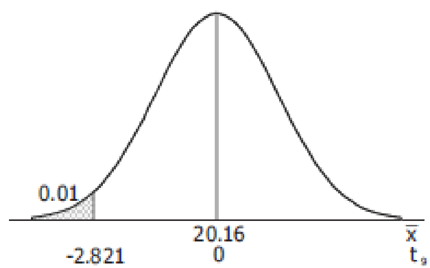
$n = 10;\quad \sum x = 187.6 \quad \overline{x} = 18.76 \quad s = 1.1862$

Original claim: $\mu < 20.16$.

$H_0: \mu = 20.16$

$H_1: \mu < 20.16$

Given: $\alpha = 0.01$ *and* $df = 9$



Critical value: $t = -t_{\alpha} = -t_{0.01} = -2.821$

Degrees of Freedom	*t* Distribution: Critical *t* Values				
	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
9	**3.250**	2.821	**2.262**	1.833	1.383

$$t_{\overline{x}} = \frac{\overline{x} - \mu}{s_{\overline{x}}} = \frac{18.76 - 20.16}{\frac{1.1862}{\sqrt{10}}} = -3.732$$

$$P - value = P\left(t_9 < -3.732\right) = tcdf\left(-99, -3.732, 9\right) = 0.0023$$

Conclusion

Reject H_0; there is sufficient evidence to conclude that $\mu < 20.16$. There is sufficient evidence to support the claim that the recent winners are from a population with BMI less than 20.16, which was the BMI for winners in the 1920's and 1930's.

Yes; recent winners appear to be significantly different from those in the earlier years.

Exercise

The list measured voltage amounts supplied directly to the author's home

123.8	123.9	123.9	123.3	123.4	123.3	123.3	123.6	123.5	123.5	123.5	123.7
123.6	123.7	123.9	124.0	124.2	123.9	123.8	123.8	124.0	123.9	123.6	123.5
123.4	123.4	123.4	123.4	123.3	123.3	123.5	123.6	123.8	123.9	123.9	123.8
123.9	123.7	123.8	123.8								

The Central Hudson power supply company states that it has a target power supply of 120 volts. Using those home voltage amounts, test the claim that the mean is 120 volts. Use a 0.01 significance level.

Solution

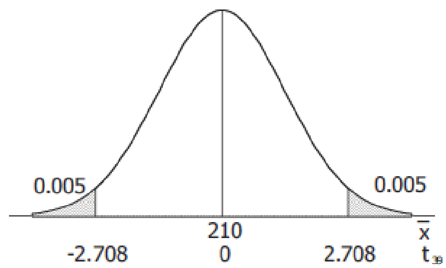
$n = 40; \quad \overline{x} = 123.6625 \quad s = 0.24039$

Original claim: μ = 120 volts.

$H_0 : \mu = 120\ volts$

$H_1 : \mu \neq 120\ volts$

Given: $\alpha = 0.01 \quad and \quad df = 39$



Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.005} = \pm 2.708$

$$t_{\overline{x}} = \frac{\overline{x} - \mu}{s_{\overline{x}}} = \frac{123.6625 - 120}{\frac{120}{\sqrt{40}}} \underline{= 96.359}$$

$P - value = 2 \cdot P\left(t_{39} > 96.359\right) = 2 \cdot tcdf\left(96.359, 999, 39\right) = 5.284E - 48 \underline{\approx 0}$

Conclusion

Reject H_0; there is sufficient evidence to conclude that μ = 120 and conclude that $\mu \neq 120$ (in fact the $\mu > 120$). There is sufficient evidence to reject the claim that the mean home voltage amount is 120 volts.

Exercise

When testing a claim about a population mean with a simple random sample selected from a normally distributed population with unknown σ, the student t distribution should be used for finding critical values and/or a P–value. If the standard normal distribution is incorrectly used instead, does that mistake make you more or less likely to reject the null hypothesis, or does it not make a difference? Explain.

Solution

Because the *z* distribution has less spread than a *t* distribution, z_α is less than t_α for any α. This makes the critical *z* value smaller (closer to 0) than the corresponding critical *t* value, which means that rejection is more likely with *z* than with *t*.

Exercise

The list measured human body temperature.

98.6	98.6	98.0	98.0	99.0	98.4	98.4	98.4	98.4	98.6	98.6	98.8	98.6	97.0	97.0	97.0
98.8	97.6	97.7	98.8	98.0	98.0	98.3	98.5	97.3	98.7	97.4	98.9	98.6	99.5	97.5	98.0
97.3	97.6	98.2	99.6	98.7	99.4	98.2	98.0	98.6	98.6	97.2	98.4	98.6	98.2	98.0	97.4
97.8	98.0	98.4	98.6	98.6	97.8	99.0	96.5	97.6	98.0	96.9	97.6	97.1	97.9	98.4	98.4
97.3	98.0	97.5	97.6	98.2	98.5	98.8	98.7	97.8	98.0	97.1	97.4	99.4	98.4	98.6	97.8
98.4	98.5	98.6	98.3	98.7	98.8	99.1	98.6	97.9	98.8	98.0	98.7	98.5	98.9	98.4	98.4
98.6	97.1	97.9	98.8	98.7	97.6	98.2	99.2	97.8	98.0						

Use the temperatures listed for 12 AM on day 2 to test the common belief that the mean body temperature is 98.6 °F. Does that common belief appear to be wrong?

Solution

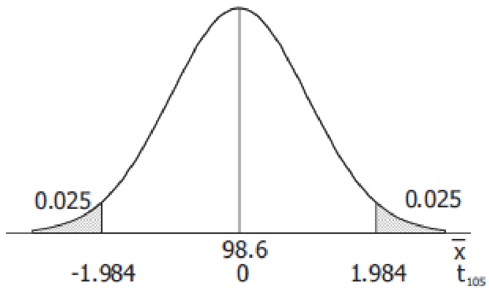
$n=106; \quad \bar{x}=98.2 \quad s=0.6229$

Original claim: $H_0: \ \mu = 98.6\ °F$ F.

$H_1: \ \mu \neq 98.6\ °F$

Given: $\alpha = 0.05$ (*assume*) *and* $df = 105$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 1.984$



$$t_{\bar{x}} = \frac{\bar{x}-\mu}{s_{\bar{x}}} = \frac{98.2-98.6}{\frac{0.6229}{\sqrt{106}}} = -6.611$$

$$P-value = 2\cdot P\left(t_{105} < -6.611\right) = 2\cdot tcdf\left(-99,-6.611,105\right) = 1.627E-9 \approx 0.0000002$$

Conclusion

Reject H_0; there is sufficient evidence to conclude that $\mu = 98.6$ and conclude that $\mu \neq 98.6$ (in fact the $\mu < 98.6$). There is sufficient evidence to reject the claim that the mean body temperature of the population is 98.6 °F. Yes; it appears there is sufficient evidence to conclude that the common belief is wrong.

Exercise

Determine whether the hypothesis test involves a sampling distribution of means that is a normal distribution, student *t* distribution, or neither

a) Claim $\mu = 981$. Sample data: $n=20, \quad \bar{x}=946, \quad s=27$. The sample data appear to come from a normally distributed population with $\sigma = 30$.

b) Claim $\mu = 105$. Sample data: $n=16, \quad \bar{x}=101, \quad s=15.1$. The sample data appear to come from a normally distributed population with unknown μ *and* σ.

Solution

a) Normal

b) Student *t*. (σ in unknown and the *x*'s approximately normally distributed, use *t*.