Lecture Two

Section 2.1 – Simple and Compound Interest

Simple Interest simply means money is not compounded only once between the beginning and the end of the note (or investment). With simple interest, the interest is calculated only at the end of the time period {rather than periodically as is done with compounding.}

$$I = \Pr t$$

$$A = P(1+rt) \quad \Rightarrow \quad P = \frac{A}{1+rt}$$

A: *Amount* is the balance of the account (including the interest).

P: *Principal* is the initial amount of principal that is borrowed or invested.

I: *Interest* is the fee that is applied to the note or investment.

r: *Rate* is the interest rate

t: *Time* is time in years.

Example

Find the total amount due on a loan of $500 at 12% simple interest at the end of 30 months.

Solution

$$A = P(1+rt)$$

$$= 500\left(1+.12\frac{30}{12}\right)$$

$$= \$650$$

500 * (1 + .12(30/12))

Example

To buy furniture for a new apartment, you borrowed $5,000 at rate of 8% simple interest for 11 months. How much should you pay?

Solution

$$I = \Pr t$$

$$= 5000(.08)\left(\frac{11}{12}\right)$$

5000 * .08 * 11 / 12

$$\approx \$366.67$$

Example

T-Bills are one of the instrument of the U.S Treasury Department uses to finance the public debit. If you buy a 180-day T-bill with a maturity value of $10,000 for $9,828.74, what annual simple interest rate will you earn?

Solution

$$t = \frac{180}{365} \approx .5$$

$$A = P(1+rt)$$

$$10000 = 9828.74(1+.5r)$$

$$\frac{10000}{9828.74} = 1+.5r$$

$$\frac{10000}{9828.74} - 1 = .5r$$

$$\frac{\frac{10000}{9828.74} - 1}{.5} = r$$

(10000 / 9828.74 – 1) / .5

$$r = .03485 \text{ or } 3.485\%$$

Example

Find the maturity value for a loan of $2500 to be repaid in 8 months with interest of 9.2%.

Solution

Given: $P = 2,500 \quad r = 0.092 \quad t = \frac{8}{12} = \frac{2}{3}$

$$A = P(1+rt)$$

$$= 2500\left(1+.092\left(\frac{2}{3}\right)\right)$$

$$\approx \$2,653.33$$

Compounded Interest

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

***A**:* Amount in the account (also called future value)

***P**:* *A*mount invested ($)

***r**:* *A*nnual simple interest rate

***t**:* *T*ime in years

m: Number of times a year the interest is compounded

- Daily: $m = 365$
- Monthly: $m = 12$
- Quarterly: $m = 4$
- Semi-annually: $m = 2$
- Annually: $m = 1$

Compounded Continuously

$$A = Pe^{rt}$$

Example

If $1,000 is invested at 6% compounded over an 8-year period.

Solution

a) Annually

$$A = 1000\left(1+\frac{.06}{1}\right)^{1(8)} = \$1,593.85$$

1000(1+.06/1)^(1*8)

b) Semiannually

$$A = 1000\left(1+\frac{.06}{2}\right)^{2(8)} = \$1,604.71$$

c) Quarterly

$$A = 1000\left(1+\frac{.06}{4}\right)^{4(8)} = \$1,610.32$$

d) Monthly

$$A = 1000\left(1+\frac{.06}{12}\right)^{12(8)} = \$1,614.14$$

Example

What amount will an account have after 1.5 years, if \$8,000 is invested at annual rate of 9%

a) Compounded ***Weekly***

$$A = 8000\left(1+\frac{.09}{52}\right)^{52(1.5)} \quad = \$9{,}155.23$$

8000(1+.09/52) ^ (52*1.5)

b) Compounded ***Continuously***

$$A = 8000e^{.09(1.5)} \quad = \$9{,}156.29$$

8000 e ^ (.09*1.5) (*e:* 2nd ln)

Example

How much should new parents invest now at 8% to have \$80,000 toward their child's college education in 17 years if compounded?

a) ***Semiannually***: $m = 2$

Given: $r = 8\% = 0.08$ **A = 80,000** $t = 17$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$80{,}000 = P\left(1+\frac{0.08}{2}\right)^{2(17)}$$

$$\frac{80{,}000}{\left(1+\frac{0.08}{2}\right)^{2(17)}} = P$$

$$P = \frac{80{,}000}{\left(1+\frac{0.08}{2}\right)^{2(17)}} \approx \$21{,}084.17$$

80000 / (1+ .08/2) ^ (2*17)

b) ***Continuously*** : $A = Pe^{rt}$

$$80{,}000 = Pe^{(.08)(17)}$$

$$P = \frac{80{,}000}{e^{(.08)(17)}} \approx \$20{,}532.86$$

80000 / *e* ^ (.08*17)

Annual Percentage Yield (APY)

Annual $\Rightarrow t=1 \Rightarrow A=P\left(1+\frac{r}{m}\right)^{mt}=P\left(1+\frac{r}{m}\right)^{m}$

Amount @ simple interest = Amount @ Compound interest

$$P(1+APY)=P\left(1+\frac{r}{m}\right)^{m} \qquad \textit{Divide P both side}$$

$$1+APY=\left(1+\frac{r}{m}\right)^{m}$$

$$APY=\left(1+\frac{r}{m}\right)^{m}-1$$

Continuously: $APY=r_e=e^{r}-1$

Effective Rate

The ***effective rate*** corresponding to a started rate of interest r compounded m times per year is

$$r_e=\left(1+\frac{r}{m}\right)^{m}-1$$

APY is also referred to as ***effective rate*** or true interest rate.

Example

How much should you invest now at 10% to have \$8,000 toward the purchase of a car in 5 years if compounded?

Solution

Given: r = 10% = 0.1 $\quad A = 8000 \quad$ t = 5

a) ***Quarterly***: $m = 4$

$$A=P\left(1+\frac{r}{m}\right)^{mt}$$

$$8000=P\left(1+\frac{0.1}{4}\right)^{4(5)}$$

$$P=\frac{8000}{\left(1+\frac{0.1}{4}\right)^{20}} \approx \$4882.17$$

8000 / (1 + 0.1 / 4) ∧ 20

b) ***Continuously*** : $A=Pe^{rt}$

$$8000=Pe^{(0.1)(5)}$$

$$P=\frac{8000}{e^{0.5}} \approx \$4852.25$$

8000 / (*e* ∧ (.1 * 5))

Example

A $10,000 investment in a particular growth mutual fund over a recent 10-year period would have grown to $126,000. What annual nominal rate would produce the same growth if

Solution

a) ***Annually***: $m = 1$

$$126000 = 10000\left(1+\frac{r}{1}\right)^{1(10)}$$

$$\frac{126000}{10000} = (1+r)^{10}$$

$$12.6 = (1+r)^{10}$$

$$(1+r)^{10} = 12.6$$

$$1+r = \sqrt[10]{12.6}$$

$$r = (12.6)^{1/10} - 1 \approx .28836 \quad or \quad \boxed{28.84\%}$$

(12.6) ∧ (1 / 10) − 1

b) ***Continuously*** : $A = Pe^{rt}$

$$126000 = 10000e^{10r}$$

$$12.6 = e^{10r}$$

$$\ln 12.6 = \ln e^{10r}$$

$$\ln 12.6 = 10r \ln e$$

$$\ln 12.6 = 10r$$

$$r = \frac{\ln 12.6}{10} \approx .25337 \quad or \quad \boxed{25.337\%}$$

ln(12.6) / 10

Example

A bank pays interest of 4.9% compounded monthly. Find the effective rate.

Solution

Given: $r = 0.049, \quad m = 12$

$$r_e = \left(1+\frac{r}{m}\right)^m - 1$$

$$= \left(1+\frac{.049}{12}\right)^{12} - 1$$

(1 + .049 / 12) ^ 12 − 1

$$\approx 0.0501156$$

Or $\boxed{r_e = 5.01\%}$

Example

You need to borrow money. Bank ***A*** charges 8% compounded semiannually. Another bank ***B*** charges 7.9% compounded monthly. At which bank will you pay the lesser amount of interest?

Solution

Bank ***A***: $r_e = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{.08}{2}\right)^2 - 1 = .0816 \quad \rightarrow \underline{r_e = 8.16\%}$

Bank ***B***: $r_e = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{.079}{12}\right)^{12} - 1 = .0819 \quad \rightarrow \underline{r_e = 8.19\%}$

Bank ***A*** has the lower effective rate, although it has a higher stated rate.

Exercises *Section* 2.1 – Simple and Compound Interest

1. If you want to earn an annual rate of 10% on your investments, how much should you pay for a note that will be worth $5,000 in 6 month?

2. How much should you deposit initially in an account paying 10% compounded semiannually in order to have $1,000,000 in 30 years? ***b***) Compounded monthly? ***c***) Compounded daily?

3. You have $7,000 toward the purchase of a $10,000 automobile. How long will it take the $7000 to grow to the $10,000 if it is invested at 9% compounded quarterly? (*Round up to the next highest quarter if not exact*.)

4. How long, to the nearest tenth of a year, will it take $1000 to grow to $3600 at 8% annual interest compounded quarterly?

5. Jennifer invested $4,000 in her savings account for 4 years. When she withdrew it, she had $4,350.52. Interest was compounded continuously. What was the interest rate on the account? *Round to the nearest tenth of a percent*.

6. An actuary for a pension fund need to have $14.6 million grows to $22 million in 6 years. What interest rate compounded annually does he need for this investment to growth as specified? *Round your answer to the nearest hundredth of a percent*.

7. Which is the better investment: 9% compounded monthly or 9.1% compounded quarterly?

8. Sun Kang borrowed $5200 from his friend to pay for remodeling work on his house. He repaid the loan 10 months later with simple interest at 7%. His friend then invested the proceeds in a 5-year CD paying 6.3% compounded quarterly. How much will his friend have at the end of the 5 years?

9. The consumption of electricity has increased historically at 6% per year. If it continues to increase at this rate indefinitely, find the number of years before the electric utilities will need to double their generating capacity. *Round up to the next highest year*.

10. In the New Testament, Jesus commends a widow who contributed 2 mites (roughly ¼ cent) to the temple treasury. Suppose the temple invested those mites at 4% compounded quarterly. How much would the money be worth 2000 years later?

11. If $1,000 is invested in an account that earns 9.75% compounded annually for 6 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

12. If $2,000 is invested in an account that earns 8.25% compounded annually for 5 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

13. If an investment company pays 6% compounded semiannually, how much you should deposit now to have $10,000
    a) 5 years from now?
    b) 10 years from now?

14. If an investment company pays 8% compounded quarterly, how much you should deposit now to have $6,000
    a) 3 years from now?
    b) 6 years from now?

15. What is the annual percentage yield (APY) for money invested at
    a) 4.5% compounded monthly?
    b) 5.8% compounded quarterly?

16. What is the annual percentage yield (APY) for money invested at
    a) 6.2% compounded semiannually?
    b) 7.1% compounded monthly?

17. A newborn child receives a $20,000 gift toward a college education from her grandparents. How much will the $20,000 be worth in 17 years if it is invested at 7% compounded quarterly?

18. A person with $14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy more expensive car. How much will be available for the purchase of a car at the end of 3 years?

19. You borrowed $7200 from a bank to buy a car. You repaid the bank after 9 months at an annual interest rate of 6.2%. Find the total amount you repaid. How much of this amount is interest?

20. An account for a corporation forgot to pay the firm's income tax of $321,812.85 on time. The government changed a penalty based on an annual interest rate of 13.4% for the 29 days the money was late. Find the total amount (tax and penalty) that was paid. (Use 365 days a year.)

21. A bond with a face value of $10,000 in 10 years can be purchased now for $5,988.02. What is the simple interest rate?

22. A stock that sold for $22 at the beginning of the year was selling for $24 at the end of the year. If the stock paid a dividend of $0.50 per share, what is the simple interest rate on an investment in this stock?

23. The Frank Russell Company is an investment fund that tracks the average performance of various groups of stocks. On average, a $10,000 investment in midcap growth funds over a recent 10-year period would have grown to $63,000. What annual nominal rate would produce the same growth if