Solution *Section* 1.1 – Propositional Logic

Exercise

Which of these sentences are propositions? What are truth values of those that are propositions?

a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida
c) $2+3=5$
d) $5+7=10$
e) $x+2=11$
f) Answer this question
g) Do not pass go
h) What time is it?
i) The moon is made of green cheese
j) $2^n \geq 100$

Solution

a) This is a true proposition

b) This is a false proposition, Tallahassee is the capital

c) This is a true proposition

d) This is a false proposition since $5+7=12 \neq 10$

e) This is not a proposition. (The truth value depends on the value assigned to x.)

f) This is not a proposition, it is a command

g) This is not a proposition; it is a command

h) This is not a proposition; it's a question

i) This is a proposition that is false

j) This is not a proposition; its truth value depends on the value of n.

Exercise

What is the negation if each of these propositions?

a) Mei has an MP3 player
b) There is no pollution in Texas
c) $2+1=3$
d) There are 13 items in a baker's dozen,
e) 121 is a perfect square

Solution

a) Mei does not have an MP3 player

b) There is pollution in Texas

c) $2+1 \neq 3$

d) There are not 13 items in a baker's dozen.

e) 121 is not a perfect square

Exercise

Suppose the Smartphone *A* has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone *B* has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; Smartphone *C* has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

a) Smartphone *B* has the most RAM of these three smartphones

b) Smartphone *C* has more ROM or higher resolution camera than Smartphone *B*.

c) Smartphone *B* has more RAM, more ROM, and a higher resolution camera than Smartphone *A*.

d) If Smartphone *B* has more RAM and more ROM than Smartphone *C*, then it also has a higher resolution camera.

e) Smartphone *A* has more RAM than Smartphone *B* if and only if Smartphone *B* has more RAM than Smartphone *A*.

Solution

a) This is a true proposition because 288 > 256 and 288 > 128

b) This is a true proposition, because the resolution is higher, C has 5 MP resolution compared to B's 4 MP.

c) This is not a true proposition. The resolution is not higher that *A*.

d) This is not a true proposition. Not necessary, the resolution in *B* is not higher that *C*.

e) This is not a true proposition

Exercise

Let p and q be the proposition

p: I bought a lottery ticket this week

q: I won the million dollars jackpot

$a)\ \neg p$ $b)\ p \vee q$ $c)\ p \rightarrow q$ $d)\ p \wedge q$

$e)\ p \leftrightarrow q$ $f)\ \neg p \rightarrow \neg q$ $g)\ \neg p \wedge \neg q$ $h)\ \neg p \vee (p \wedge q)$

Solution

a) I did not buy a lottery ticket this week.

b) I bought a lottery ticket this week or I won the million dollars jackpot.

c) I bought a lottery ticket this week then I won the million dollars jackpot.

d) I bought a lottery ticket this week and I won the million dollars jackpot.

e) I bought a lottery ticket this week if and only if I won the million dollars jackpot.

f) I did not buy a lottery ticket this week then I did not win the million dollars jackpot.

g) I did not buy a lottery ticket this week and I did not win the million dollars jackpot.

h) I did not buy a lottery ticket this week or either I bought a lottery ticket this week and I won the million dollars jackpot.

Exercise

Let p and q be the proposition

p: Swimming at the New Jersey shore is allowed

q: Sharks have been spotted new the shore

$a)\ \neg q$ $b)\ p \wedge q$ $c)\ \neg p \vee q$ $d)\ p \rightarrow \neg q$

$e)\ \neg q \rightarrow p$ $f)\ \neg p \rightarrow \neg q$ $g)\ p \leftrightarrow \neg q$ $h)\ \neg p \wedge (p \vee \neg q)$

Solution

a) Sharks have not been spotted new the shore.

b) Swimming at the New Jersey shore is allowed, and Sharks have been spotted new the shore.

c) Swimming at the New Jersey shore is not allowed, or Sharks have been spotted new the shore.

d) Swimming at the New Jersey shore is allowed then Sharks have not been spotted new the shore.

e) Sharks have not been spotted new the shore then Swimming at the New Jersey shore is allowed.

f) Swimming at the New Jersey shore is not allowed then Sharks have not been spotted new the shore.

g) Swimming at the New Jersey shore is allowed if and only if Sharks have not been spotted new the shore.

h) Swimming at the New Jersey shore is not allowed and either Swimming at the New Jersey shore is allowed, or Sharks have not been spotted new the shore

Exercise

Let $p,\ q$ and r be the proposition

p: You have the flu

q: You miss the final examination

r: You pass the course

Express each of these propositions as an English sentence

$a)\ p \rightarrow q$ $b)\ \neg q \leftrightarrow r$ $c)\ q \rightarrow \neg r$ $d)\ p \vee q \vee r$

$e)\ (p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ $f)\ (p \wedge q) \vee (\neg q \wedge r)$

Solution

a) You have the flu then you miss the final examination.

b) You don't miss the final examination if and only if you pass the course.

c) You miss the final examination then you don't pass the course.

d) You have the flu or you miss the final examination or you pass the course.

e) You have the flu then you don't pass the course or you miss the final examination then you don't pass the course.

f) You have the flu and you miss the final examination or both you don't miss the final examination and you pass the course.

Exercise

Determine whether each of these conditional statements is true or false.

a) If $1+1=2$, then $2+2=5$

b) If $1+1=3$, then $2+2=4$

c) If $1+1=3$, then $2+2=5$

d) If monkeys can fly, then $1+1=3$

e) If $1+1=3$, then unicorns exist

f) If $1+1=3$, then dogs can fly

g) If $1+1=2$, the dogs can fly

h) If $2+2=4$, then $1+2=3$

Solution

a) Since the hypothesis is true and the conclusion is false, this implication is false.

b) Since the hypothesis is false and the conclusion is true, this implication is true.

c) Since the hypothesis is false and the conclusion is false, this implication is true.

d) Since the hypothesis is false and the conclusion is false, this implication is true.

e) Since the hypothesis is false and the conclusion is false, this implication is true.

f) Since the hypothesis is false and the conclusion is false, this implication is true.

g) Since the hypothesis is true and the conclusion is false, this implication is false.

h) Since the hypothesis is true and the conclusion is true, this implication is true.

Exercise

Write each of these propositions in the form "p if and only if q" in English

a) If it is hot outside you buy an ice cream cone, and if you buy an ice cream cone it is hot outside.

b) For you to win the contest it is necessary and sufficient that you have only winning ticket.

c) You get promoted only if you have connections, and you have connections only if you get promoted.

d) If you watch television your mind will decay, and conversely.

e) The trains run late on exactly those days when I take it.

f) For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.

g) If you read the newspaper every day, you will be informed, and conversely.
h) It rains if it is a weekend day, and it is a weekend day if it rains.
i) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him

Solution

a) You buy an ice cream cone if and only if it is hot outside.

b) You win the contest if and only if you hold the only winning ticket.

c) You get promoted if and only if you have connection.

d) Your mind will decay if and only if you watch television.

e) The trains run late if and only if you watch television.

f) For you to get an *A* in this course if and only if you learn how to solve discrete mathematics problems.

Exercise

Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$
b) $p \vee \neg p$
c) $p \rightarrow \neg p$
d) $p \leftrightarrow \neg p$
e) $p \rightarrow \neg q$
f) $\neg p \leftrightarrow q$
g) $(p \vee \neg q) \rightarrow q$
h) $(p \vee q) \rightarrow (p \wedge q)$
i) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
j) $(p \rightarrow q) \rightarrow (q \rightarrow p)$
k) $p \oplus (p \vee q)$
l) $(p \wedge q) \rightarrow (p \vee q)$
m) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
n) $(p \rightarrow q) \vee (\neg p \rightarrow q)$
o) $(p \rightarrow q) \wedge (\neg p \rightarrow q)$
p) $(p \vee q) \vee r$
q) $(p \vee q) \wedge r$
r) $(p \wedge q) \vee r$
s) $(p \wedge q) \wedge r$
t) $(p \vee q) \wedge \neg r$

Solution

a)

p	$\neg p$	$p \wedge \neg p$
T	F	***F***
F	T	***F***

b)

p	$\neg p$	$p \vee \neg p$
T	F	***T***
F	T	***T***

c)

p	$\neg p$	$p \rightarrow \neg p$
T	F	***F***
F	T	***T***

d)

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	**F**
F	T	**F**

e)

p	q	$\neg q$	$p \rightarrow \neg q$
T	T	F	**F**
T	F	T	**T**
F	T	F	**T**
F	F	T	**T**

f)

p	q	$\neg q$	$\neg p \leftrightarrow q$
T	T	F	***F***
T	F	T	***T***
F	T	F	***T***
F	F	T	***F***

g)

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	F	T	***T***
T	F	T	T	***F***
F	T	F	F	***T***
F	F	T	T	***F***

h)

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	***T***
T	F	T	F	***F***
F	T	T	F	***F***
F	F	F	F	***T***

i)

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$	$(p \rightarrow q) \leftrightarrow (\neg q \wedge \neg p)$
T	T	**T**	F	F	**T**	***T***
T	F	**F**	T	F	**F**	***T***
F	T	**T**	F	T	**F**	***T***
F	F	**T**	T	T	**F**	***T***

j)

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	**T**	**T**	***T***
T	F	**F**	**T**	***T***
F	T	**T**	**F**	***F***
F	F	**T**	**T**	***T***

k)

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	**T**	***F***
T	F	**T**	***F***
F	T	**T**	***T***
F	F	**F**	***F***

l)

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	**T**	**T**	***T***
T	F	**F**	**T**	***T***
F	T	**F**	**T**	***T***
F	F	**F**	**F**	***T***

m)

p	q	$\neg p$	$q \to \neg p$	$p \leftrightarrow q$	$(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	**F**	**T**	***F***
T	F	F	**T**	**F**	***F***
F	T	T	**T**	**F**	***F***
F	F	T	**T**	**T**	***T***

n)

p	q	$\neg p$	$p \to q$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
T	T	F	**T**	**T**	***T***
T	F	F	**F**	**T**	***T***
F	T	T	**T**	**T**	***T***
F	F	T	**T**	**F**	***T***

o)

p	q	$\neg p$	$p \to q$	$\neg p \to q$	$(p \to q) \wedge (\neg p \to q)$
T	T	F	**T**	**T**	***T***
T	F	F	**F**	**T**	***F***
F	T	T	**T**	**T**	***T***
F	F	T	**T**	**F**	***F***

p)

p	q	r	$p \vee q$	$(p \vee q) \vee r$
T	T	T	**T**	***T***
T	T	F	**T**	***T***
T	F	T	**T**	***T***
T	F	F	**T**	***T***
F	T	T	**T**	***T***
F	T	F	**T**	***T***
F	F	T	**F**	***T***
F	F	F	**F**	***F***

q)

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	**T**	***T***
T	T	F	**T**	***F***
T	F	T	**T**	***T***
T	F	F	**T**	***F***
F	T	T	**T**	***T***
F	T	F	**T**	***F***
F	F	T	**F**	***F***
F	F	F	**F**	***F***

r)

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	**T**	***T***
T	T	F	**T**	***T***
T	F	T	**F**	***T***
T	F	F	**F**	***F***
F	T	T	**F**	***T***
F	T	F	**F**	***F***
F	F	T	**F**	***T***
F	F	F	**F**	***F***

s)

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	**T**	***T***
T	T	F	**T**	***F***
T	F	T	**F**	***F***
T	F	F	**F**	***F***
F	T	T	**F**	***F***
F	T	F	**F**	***F***
F	F	T	**F**	***F***
F	F	F	**F**	***F***

t)

p	q	r	$\neg r$	$p \vee q$	$(p \vee q) \wedge \neg r$
T	T	T	**F**	**T**	***F***
T	T	F	**T**	**T**	***T***
T	F	T	**F**	**T**	***F***
T	F	F	**T**	**T**	***T***
F	T	T	**F**	**T**	***F***
F	T	F	**T**	**T**	***T***
F	F	T	**F**	**F**	***F***
F	F	F	**T**	**F**	***F***