# Solution Section 2.7 – Binomial Probability Distribution

### Exercise

20 different Senators are randomly selected from the 100 Senators in the current Congress, and each was asked whether he or she is in favor of abolishing estate taxes. Does this procedure result in a binomial distribution, if it is not binomial, identify at least one requirement that is not satisfied?

**Solution**

NO; requirements (2) and (4) are not met.

Since $\frac{20}{100} = 0.20 > 0.05$ and the selections are done without replacement, the trials cannot be considered independent. The value of $p$ of obtaining a success changes from trial to trial as each selection without replacement changes the population from which the next selection is made.

### Exercise

15 different Governors are randomly selected from the 50 Governors in the currently office and the sex of each Governor is recorded. Does this procedure result in a binomial distribution, if it is not binomial, identify at least one requirement that is not satisfied?

**Solution**

NO; requirements (2) and (4) are not met.

Since $\frac{15}{50} = 0.30 > 0.05$ and the selections are done without replacement, the trials cannot be considered independent. The value of $p$ of obtaining a success changes from trial to trial as each selection without replacement changes the population from which the next selection is made.

### Exercise

200 statistics students are randomly selected and each asked if he or she owns a TI-84 Plus calculator. Does this procedure result in a binomial distribution, if it is not binomial, identify at least one requirement that is not satisfied?

**Solution**

Yes; all requirements are met. Since 200 statistics students are assumed to be less than 5% of the population of all statistics students, the selections can considered to be independent – even though ther are made without replacement.

### Exercise

Multiple choice questions on the SAT test have 5 possible answers (*a*, *b*, *c*, *d*, *e*), 1 of which is correct. Assume that you guess the answers to 3 such questions.

*a)* Use the multiplication rule to find the probability that the first 2 guesses are wrong and the third is correct. That is, find $P(WWC)$, where $C$ denotes a correct answer and $W$ denotes a wrong answer.

*b)* Beginning with $WWC$, make a complete list of the different possible arrangements of 2 wrong answers and 1 correct answer, then find the probability for each entry in the list.

*c)* Based on the proceeding results, what is the probability of getting exactly 1 correct answer when 3 guesses are made?

***Solution***

$$P(C: correct) = \frac{1}{5}, \quad P(W: wrong) = \frac{4}{5}$$

**a)** $P(WWC) = P(W) \cdot P(W) \cdot P(C)$

$$= \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{1}{5}$$

$$\underline{= 0.128}$$

**b)** There are: $WWC$, $WCW$, $CWW$ – 3 possible arrangements

$$P(WWC) = P(W) \cdot P(W) \cdot P(C) = \frac{16}{125}$$

$$P(WCW) = P(W) \cdot P(C) \cdot P(W) = \frac{4}{5} \cdot \frac{1}{5} \cdot \frac{4}{5} = \frac{16}{125}$$

$$P(WCW) = P(C) \cdot P(W) \cdot P(W) = \frac{1}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{16}{125}$$

**c)** $P(exactly\ one\ correct) = P(WWC\ or\ WCW\ or\ CWW)$

$$= P(WWC) + P(WCW) + P(CWW)$$

$$= \frac{16}{125} + \frac{16}{125} + \frac{16}{125}$$

$$= \frac{48}{125}$$

$$\underline{= 0.384}$$

## *Exercise*

A psychology test consists of multiple choice questions, each having 4 possible answers (*a, b, c, d*), 1 of which is correct. Assume that you guess the answers to 6 such questions.

*a)* Use the multiplication rule to find the probability that the first 2 guesses are wrong and the last 4 guesses are correct. That is, find $P(WWCCCC)$, where $C$ denotes a correct answer and $W$ denotes a wrong answer.

*b)* Beginning with $WWCCCC$, make a complete list of the different possible arrangements of 2 wrong answers and 4 correct answers, then find the probability for each entry in the list.

*c)* Based on the proceeding results, what is the probability of getting exactly 4 correct answers when 6 guesses are made?

***Solution***

$$P(C: correct) = \frac{1}{4}, \quad P(W: wrong) = \frac{3}{4}$$

***a)*** $P(WWCCCC) = P(W)\cdot P(W)\cdot P(C)\cdot P(C)\cdot P(C)\cdot P(C)$

$$= \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}$$

$$= \frac{9}{4096}$$

$$\underline{= 0.00220}$$

***b)*** There are ${}_6C_2 = 15$ possible arrangements.

*WWCCCC WCWCCC WCCWCC WCCCWC WCCCCW CWCCCW CCWCCW*
*CCCWCW CCCCWW CWWCCC CCWWCC CCCWWC CCCWWC CWCWCC CCWCWC*

$P(\textit{each arrangement}) = \dfrac{9}{4096}$ (part $a$)

***c)*** $P(\textit{exactly } 4 \textit{ correct}) = P(WWCCCC \textit{ or } WCWCCC \textit{ or } \cdots \textit{ or } CCWCWC)$

$$= P(WWCCCC) + P(WCWCCC) + \ldots + P(CCWCWC)$$

$$= \frac{9}{4096} + \frac{9}{4096} + \cdots + \frac{9}{4096}$$

$$= 15\cdot\frac{9}{4096}$$

$$\underline{= 0.0330}$$

## *Exercise*

Use the Binomial Probability Table to find the probability of $x$ success given the probability $p$ of success on a single trial

$a)$ $n=2,\ \ x=1,\ \ p=.30$ $b)$ $n=5,\ \ x=1,\ \ p=0.95$

$c)$ $n=15,\ \ x=11,\ \ p=0.99$ $d)$ $n=14,\ \ x=4,\ \ p=0.60$

$e)$ $n=10,\ \ x=2,\ \ p=0.05$ $f)$ $n=12,\ \ x=12,\ \ p=0.70$

***Solution***

***a)*** $n=2,\ \ x=1,\ \ p=.30$

| n | x | .01 | .05 | .10 | .20 | .30 | .40 | .50 | .60 | .70 | .80 | .90 | .95 | .99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | .980 | .902 | .810 | .640 | .490 | .360 | .250 | .160 | .090 | .040 | .010 | .002 | 0+ |
| | 1 | .020 | .095 | .180 | .320 | .420 | .480 | .500 | .480 | .420 | .320 | .180 | .095 | .020 |

$P(x=1)$ $\underline{= 0.420}$

***b)*** $n=5,\ \ x=1,\ \ p=0.95$

From the Table: $P(x=1)$ $\underline{= 0+}$

***c)*** $n=15,\ \ x=11,\ \ p=0.99$

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 0+ | 0+ | 0+ | 0+ | 0+ | 0+ | 0+ | 0+ | .005 | .035 | .206 | .463 | .860 |

From the Table: $P(x=11)$ $\underline{= 0+}$

$d)\quad n=14, \quad x=4, \quad p=0.60$

From the Table: $P(x=4)\ \underline{=0.014}$

$e)\quad n=10, \quad x=2, \quad p=0.05$

From the Table: $P(x=2)\ \underline{=0.075}$

$f)\quad n=12, \quad x=12, \quad p=0.70$

From the Table: $P(x=12)\ \underline{=0.014}$

## *Exercise*

Use the Binomial Probability Formula to find the probability of *x* success given the probability *p* of success on a single trial

$a)\quad n=12, \quad x=10, \quad p=\frac{3}{4}$ $\qquad b)\quad n=9, \quad x=2, \quad p=0.35$

$c)\quad n=20, \quad x=4, \quad p=0.15$ $\qquad d)\quad n=15, \quad x=13, \quad p=\frac{1}{3}$

***Solution***

$a)\quad n=12, \quad x=10, \quad p=\frac{3}{4} \rightarrow q=1-\frac{3}{4}=\frac{1}{4}$

$$P(x)=\frac{n!}{(n-x)!\ x!}\cdot p^x\cdot q^{n-x}$$

$$=\frac{12!}{(12-10)!\ 10!}\cdot\left(\frac{3}{4}\right)^{10}\left(\frac{1}{4}\right)^{12-10}$$

$$=\frac{12!}{2!\ 10!}\cdot\left(\frac{3}{4}\right)^{10}\left(\frac{1}{4}\right)^{2}$$

$\underline{=0.232}$

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12!(3/4)^10*(1/4
)^2/(2!10!
             .232
```

$b)\quad n=9, \quad x=2, \quad p=0.35 \rightarrow q=1-0.35=.65$

$$P(x)=\frac{n!}{(n-x)!\ x!}\cdot p^x\cdot q^{n-x}$$

$$=\frac{9!}{(9-2)!\ 2!}\cdot(.35)^2(.65)^{9-2}$$

$$=\frac{9!}{7!\ 2!}\cdot(.35)^2(.65)^7$$

$\underline{=0.216}$

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9!(.35²)*.65^7/(
7!2!
             .216
```

9! .35 ^ 2 * .65 ^ 7 / (7!* 2!)

$c)\quad n=20, \quad x=4, \quad p=0.15 \rightarrow q=1-0.15=.85$

$$P(x)=\frac{n!}{(n-x)!\ x!}\cdot p^x\cdot q^{n-x}$$

$$=\frac{20!}{16!\ 4!}\cdot(.15)^4(.85)^{16}$$

$\underline{=0.182}$

20! .15 ^ 4 * .85 ^ 4 / (16! 4!)

**d)** $n=15, \quad x=13, \quad p=\frac{1}{3} \to q=1-\frac{1}{3}=\frac{2}{3}$

$$P(x)=\frac{n!}{(n-x)!\,x!}\cdot p^{x}\cdot q^{n-x}$$

$$=\frac{15!}{2!\,13!}\cdot\left(\frac{1}{3}\right)^{13}\left(\frac{2}{3}\right)^{2}$$

15! (1/3)^13*(2/3)^2/(2! 13!)

$$\underline{=0.0000293}$$

## *Exercise*

In the US, 40% of the population have brown eyes. If 14 people are randomly selected, find the probability that at least 12 of them have brown eyes. Is it unusual to randomly select 14 people and find that at least 12 of them have brown eyes? Why or why not?

***Solution***

Let $x$ = number of people with brown eyes.

Binomial: $n=14; \quad p=0.4$

From the Binomial Probability Table:

$$P(x\ge 12)=P(x=12)+P(x=13)+P(x=14)$$

$$=0.001+0^{+}+0^{+}$$

$$\underline{=0.001}$$

Yes, since $0.001\le 0.05$, getting at least 12 persons with brown eyes would be unusual.

## *Exercise*

When blood donors were randomly selected, 45% of them had blood that is Group *O*. The display shows that the probabilities obtained by entering the values of $n$ = 5 and $p$ = 0.45.

*a)* Find the probability that at least 1 of the 5 donors has Group *O* blood. If at least 1 Group *O* donor is needed, is it reasonable to expect that at least 1 will be obtained?

*b)* Find the probability that at least 3 of the 5 donors have Group *O* blood. If at least 3 Group *O* donors are needed, is it very likely to expect that at least 3 will be obtained?

*c)* Find the probability that all donors have Group *O* blood. Is it unusual to get 5 Group *O* donors from 5 randomly selected donors? Why or Why not?

*d)* Find the probability that at most 2 of the 5 donors have Group *O* blood.

| ***x*** | ***P(x)*** |
|---|---|
| 0 | 0.050328 |
| 1 | 0.205889 |
| 2 | 0.336909 |
| 3 | 0.275653 |
| 4 | 0.112767 |
| 5 | 0.018453 |

***Solution***

***a)*** $P(x\ge 1)=1-P(x=0)$

$$=1-0.050328$$

$$\underline{\approx 0.95}$$

Yes, it is reasonable to expect that at least one group O donor will be obtained.

**b)** $P(x \ge 3) = P(x=3) + P(x=4) + P(x=5)$

$$= 0.275653 + 0.112767 + 0.018453$$
$$= 0.406873$$
$$\underline{\approx 0.407}$$

No; it is not *very likely* that at least 3 group of O donors will be obtained.
Since $0.407 > 0.05$ getting at least 3 such donors would not be an unusual event – but it would not be considered *very likely*.

**c)** $P(x=5) = 0.018453$ $\underline{\approx 0.018}$

Yes, since $0.018 \le 0.05$ getting all 5 donors from group O would be considered unusual event.

**d)** $P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$

$$= 0.050328 + 0.205889 + 0.336909$$
$$= 0.593126$$
$$\underline{\approx 0.593}$$

## *Exercise*

There is 1% delinquency rate for consumers with FICO credit rating scores above 800. If a bank provides large loans 12 people with FICL scores above 800, what is the probability that at least one of them becomes delinquent? Based on that probability, should the bank plan on dealing with a delinquency?

***Solution***

Let *x* = number of delinquencies
Binomial: $n = 12$; $p = 0.01$; from the Binomial Probability Table:

$P(x \ge 1) = 1 - P(x=0)$

$$= 1 - .886$$
$$\underline{= 0.114}$$

Yes, since $0.114 > 0.05$, it would be unusual for at least one of the people to become delinquent. The bank should make plans for dealing with a delinquency.

## Exercise

Ten peas are generated from parents having the green/yellow pair of genes, so there is a 0.75 probability that an individual pea will have a green pod. Find the probability that among the 10 offspring peas, at least 9 have green pods. Is it unusual to get at least 9 peas with green pods when 10 offspring peas are generated? Why or why not?

***Solution***

Let $x$ = number of delinquencies

Binomial: $n=10; \quad p=0.75$; using the binomial formula:

$$P(x)=\frac{n!}{(n-x)!\,x!}\cdot p^{x}\cdot q^{n-x}$$

$$\begin{aligned} P(x\geq 1) &= 1-P(x=0) \\ &= 1-\frac{10!}{10!\ 0!}\cdot(.75)^{0}\cdot(.25)^{10} \\ &= 0.999999046 \end{aligned}$$

The usual rounded rule (to 3 significant digits) is not satisfactory in this case since applying that rule would suggest $P(x\geq 1)=1.00$, which is a certainly.

In this case, 6 significant digits are necessary to differentiate the probability of this very likely event from the probability of an event that is a certainly.

## Exercise

You purchased a slot machine configured so that there is a $\frac{1}{2,000}$ probability of winning the jackpot on any individual trial. Although no one would seriously consider tricking the author, suppose that a guest claims that she played the slot machine 5 times and hit the jackpot twice

*a)* Find the probability of exactly 2 jackpots in 5 trials.

*b)* Find the probability of at least 2 jackpots in 5 trials.

*c)* Does the guest's claim of hitting 2 jackpots in 5 trials seem valid? Explain.

***Solution***

Let $x$ = number of jackpots hit.

Binomial: $n=5; \quad p=\frac{1}{2000}=0.0005$; using the binomial formula: $P(x)=\frac{n!}{(n-x)!\,x!}\cdot p^{x}\cdot q^{n-x}$

**a)** $P(x=2)=\frac{5!}{3!\ 2!}\cdot(.0005)^{2}\cdot(.9995)^{3}$

$\qquad = 0.00002496$

**b)**
$$\begin{aligned} P(x\geq 2) &= 1-P(x<2) \\ &= 1-\left[P(x=0)+P(x=1)\right] \\ &= 1-\left[\frac{5!}{5!\ 0!}\cdot(.0005)^{0}\cdot(.9995)^{5}+\frac{5!}{4!\ 1!}\cdot(.0005)^{1}\cdot(.9995)^{4}\right] \\ &= 1-[.997502499+.002495004] \end{aligned}$$

$\underline{= 0.000002497}$

***c)*** No; since $0.00002497 \le 0.05$, it would be unusual to hit 2 jackpots. If the machine is functioning as it is supposed to, either the guest is not telling the truth or an extremely rare event has occurred.

## *Exercise*

In a survey of 320 college graduates, 36% reported that they stayed on their first full-time job less than one year.

*a)* If 15 of those survey subjects are randomly selected without replacement for a follow-up survey, find the probability that 5 of them stayed on their first full-time job less than one year.

*b)* If part (a) is changed so that 20 different survey subjects are selected, explain why the binomial probability formula *cannot* be used.

***Solution***

Let $x$ = number who stayed less than one year.

Binomial: $n = 15$; $p = 0.36$; using the binomial formula: $P(x) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x}$

***a)*** $P(x=5) = \frac{15!}{10!\ 5!} \cdot (0.36)^5 \cdot (.64)^{10}$ $\underline{= 0.209}$

***b)*** The binomial distribution requires that the repeated selections be independent. Since these persons are selected from the original group of 320 without replacement, the repeated selections are not independent and the binomial distribution should not be used.

The sample size is $\frac{15}{320} = .046 \le .05$ of the population and the repeated samples may be treated as though they are independent.

If the sample size is increased to 20, the sample is $\frac{20}{320} = .0625 > .05$ of the population and the criteria for using independence to get an approximate probability is no longer met.

## *Exercise*

In a survey of 150 senior executives, 47% said that the most common job interview mistake is to have little or no knowledge of the company.

*a)* If 6 of those surveyed executives are randomly selected without replacement for a follow-up survey, find the probability that 3 of them said that the most common job interview mistake is to have little or no knowledge of the company.

*b)* If part (a) is changed so that 9 different surveyed executives are selected, explain why the binomial probability formula *cannot* be used.

***Solution***

Let $x$ = number who said the most common mistake is not to know the company

Binomial: $n = 6$; $p = 0.47$; using the binomial formula: $P(x) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x}$

***a)*** $P(x=3)=\frac{6!}{3!\ 3!}\cdot(0.47)^3\cdot(0.53)^3\ \underline{=0.309}|$

***b)*** The binomial distribution requires that the repeated selections be independent. Since these persons are selected from the original group of 150 without replacement, the repeated selections are not independent and the binomial distribution should not be used. In part (*a*), however, the sample size is $\frac{6}{150}=0.04\le 0.05$ of the population and the repeated samples may be treated as though they are independent. If the sample size is increased to 9, the sample is $\frac{9}{150}=0.06>0.05$ of the population and the criteria for using independence to get an approximate probability is no longer met.

## *Exercise*

In a Gallup poll of 1236 adults, it was found that 5% of those polled said that bad luck occurs after breaking a mirror. Based on these results, such randomly selected groups of 1236 adults will have a mean of 61.8 people with that belief, and a standard deviation of 7.7 people. What is the variance?

***Solution***

Exact formula: $\sigma^2=npq=1236(0.05)(0.95)\ \underline{=58.71\ \ people^2}|$

Using the rounded value: $\sigma^2=(\sigma)^2=(7.7)^2\ \underline{=59.29\ \ people^2}|$

## *Exercise*

Random guesses are made for 50 SAT multiple choice questions, so $n=50$ and $p=0.2$.

*a)* Find the mean μ and standard deviation σ.

*b)* Use the range rule of thumb to find the minimum usual number and the maximum usual number.

***Solution***

***a)*** *Mean*: $\mu=np=(50)(0.2)\ \underline{=10.0}|$

*Standard deviation*: $\sigma=\sqrt{npq}=\sqrt{50(0.2)(0.8)}\ \underline{=2.8}|$

***b)*** *Minimum usual value* $=\mu-2\sigma=10-2(2.828)\ \underline{=4.3}|$

*Maximum usual value* $=\mu+2\sigma=10+2(2.828)\ \underline{=15.7}|$

## *Exercise*

In an analysis of test result from the YSORT gender selection method, 152 babies are born and it is assumed that boys and girls are equally likely, so $n=152$ and $p=0.5$.

*a)* Find the mean μ and standard deviation σ.

*b)* Use the range rule of thumb to find the minimum usual number and the maximum usual number.

***Solution***

**a)** *Mean*: $\mu = np = (152)(0.5)\ \underline{= 76.0|}$

*Standard deviation*: $\sigma = \sqrt{npq} = \sqrt{152(0.5)(0.5)}\ \underline{= 6.164|}$

**b)** *Minimum usual value* $= \mu - 2\sigma = 76 - 2(6.164)\ \underline{= 63.7|}$

*Maximum usual value* $= \mu + 2\sigma = 76 + 2(6.164)\ \underline{= 88.3|}$

### Exercise

In a Gallup poll of 1236 adults, it showed that 145% believe that bad luck follows if your path is crossed by a black car, so $n = 1236$ and $p = 0.14$.

*a)* Find the mean μ and standard deviation σ.

*b)* Use the range rule of thumb to find the minimum usual number and the maximum usual number.

**Solution**

**a)** *Mean*: $\mu = np = (1236)(0.14)\ \underline{= 173.04|}$

*Standard deviation*: $\sigma = \sqrt{npq} = \sqrt{1236(0.14)(0.86)}\ \underline{= 12.199|}$

**b)** *Minimum usual value* $= \mu - 2\sigma = 173.04 - 2(12.199)\ \underline{= 148.6|}$

*Maximum usual value* $= \mu + 2\sigma = 173.04 + 2(12.199)\ \underline{= 197.4|}$

### Exercise

The midterm exam in a nursing course consists of 75 true/false questions. Assume that an unprepared student makes random guesses for each of the answers.

*a)* Find the mean and standard deviation for the number of correct answers for such students.

*b)* Would it be unusual for a student to pass this exam by guessing and getting at least 45 correct answers? Why or why not?

**Solution**

Since the questions are T/F, then $p = \frac{1}{2} = 0.5 = q$ and $n = 75$

**a)** *Mean*: $\mu = np = (75)(0.5)\ \underline{= 37.5|}$

*Standard deviation*: $\sigma = \sqrt{npq} = \sqrt{75(0.5)(0.5)}\ \underline{= 4.33|}$

**b)** *Minimum usual value* $= \mu - 2\sigma = 37.5 - 2(4.33)\ \underline{= 28.8|}$

*Maximum usual value* $= \mu + 2\sigma = 37.5 + 2(4.33)\ \underline{= 46.2|}$

No, Since 45 is within the above limits, it would not be unusual for a student to pass by getting at least 45 correct answers.

## Exercise

The final exam in a nursing course consists of 100 multiple-choice questions. Each question has 5 possible answers, and only 1 of them is correct. An unprepared student makes random guesses for all of the answers.

*a)* Find the mean and standard deviation for the number of correct answers for such students.

*b)* Would it be unusual for a student to pass this exam by guessing and getting at least 60 correct answers? Why or why not?

**Solution**

Since there are 5 questions, and only 1 of them is correct $p = \frac{1}{5} = 0.2; \quad q = 0.8$ and $n = 100$

**a)** *Mean*: $\mu = np = (100)(0.2) \; \underline{= 20.0}$

*Standard deviation*: $\sigma = \sqrt{npq} = \sqrt{100(0.2)(0.8)} \; \underline{= 4.0}$

**b)** *Minimum usual value* $= \mu - 2\sigma = 20 - 2(4) \; \underline{= 12.0}$

*Maximum usual value* $= \mu + 2\sigma = 20 + 2(4) \; \underline{= 28.0}$

Yes, Since 60 is not within the above limits, it would be unusual for a student to pass by getting at least 60 correct answers.

## Exercise

In a test of the XSORT method of gender selection, 574 babies are born to couples trying to have baby girls, and 525 of those babies are girls.

*a)* If the gender-selection method has no effect and boys and girls are equally likely, find the mean and standard deviation for the numbers of girls born in groups of 574.

*b)* Is the result of 525 girls unusual? Does it suggest that the gender-selection method appears to be effective?

**Solution**

Binomial problem: since 2 genders, $p = \frac{1}{2} = 0.5; \quad q = 0.5$ and $n = 574$

**a)** *Mean*: $\mu = np = (574)(0.5) \; \underline{= 287.0}$

*Standard deviation*: $\sigma = \sqrt{npq} = \sqrt{574(0.5)(0.5)} \; \underline{= 11.979}$

**b)** *Minimum usual value* $= \mu - 2\sigma = 287.0 - 2(11.979) \; \underline{= 263.0}$

*Maximum usual value* $= \mu + 2\sigma = 287.0 + 2(11.979) \; \underline{= 311.0}$

Yes, Since 525 is not within the above limits, it would be unusual for 574 births to include 525 girls. The results suggest that the gender selection method is effective.

### *Exercise*

In a test of the YSORT method of gender selection, 152 babies are born to couples trying to have baby boys, and 127 of those babies are boys.

*a)* If the gender-selection method has no effect and boys and girls are equally likely, find the mean and standard deviation for the numbers of boys born in groups of 152.

*b)* Is the result of 127 boys unusual? Does it suggest that the gender-selection method appears to be effective?

***Solution***

Binomial problem; and since 2 genders, $p = \frac{1}{2} = 0.5; \quad q = 0.5$ and $n = 152$

**a)** *Mean*: $\mu = np = (152)(0.5) \ \underline{= 76.0}$

*Standard deviation*: $\sigma = \sqrt{npq} = \sqrt{152(0.5)(0.5)} \ \underline{= 6.164}$

**b)** *Minimum usual value* $= \mu - 2\sigma = 76.0 - 2(6.164) \ \underline{= 63.7}$

*Maximum usual value* $= \mu + 2\sigma = 76.0 + 2(6.164) \ \underline{= 86.3}$

Yes, Since 127 is not within the above limits, it would be unusual for 152 births to include 127 boys. The results suggest that the gender selection method is effective.

### *Exercise*

A headline in USA Today states that "most stay at first job less than 2 years." That headline is based on a poll of 320 college graduates. Among those polled, 78% stayed at their full-time job less than 2 years.

*a)* Assuming that 50% is the true percentage of graduates who stay at their first job less than 2 years, find the mean and the standard deviation of the numbers of such graduates in randomly selected groups of 320 graduates.

*b)* Assuming that the 50% rate in part (*a*) is correct; find the range of usual values for the numbers of graduates among 320 who stay at their first job less than 2 years.

*c)* Find the actual number of surveyed who stayed at their first job less 2 years. Use the range of values from part (*b*) to determine whether that number is unusual. Does the result suggest that the headline is not justified?

*d)* This statement was given as part of the description of the survey methods used: "Alumni who opted-in to receive communications from Experience were invited to participate in the online poll, and 320 of them completed the survey." What does that statement suggest about the result?

***Solution***

Let $x$ = the number who stay at their job less than 2 years.

Binomial problem, $p = 0.5; \quad q = 0.5$ and $n = 320$

**a)** *Mean*: $\mu = np = (320)(0.5) \ \underline{= 160.0}$

*Standard deviation*: $\sigma = \sqrt{npq} = \sqrt{320(0.5)(0.5)} \ \underline{= 8.944}$

**b)** *Minimum unusual value* $= \mu - 2\sigma = 160.0 - 2(8.944) \ \underline{= 142.1}$

$Maximum\ unusual\ value\ = \mu + 2\sigma = 160.0 + 2(8.944)\ \underline{=177.9}$

**c)** $x = (.78)(320) \approx 250$

Since 250 is not within the above limits, it would be unusual for 320 graduates to include 250 persons who stayed at their job less than 2 years if the true proportion were 50%.
Since 250 is greater than above limits, the true proportion is most likely greater than 50%. The result suggests that the headline is justified.

**d)** The statement suggests that the 320 participants were a voluntary response sample, and so the results might not be representative of the target population.

## Exercise

In a study of 420,095 cell phone users in Denmark, it was found that 135 developed cancer of the brain or nervous system. If we assume that the use of cell phones has no effect on developing such cancer, then the probability of a person having such cancer is 0.000340.

*a)* Assuming that the cell phones have no effect on developing cancer, find the mean and the standard deviation of the numbers of people in groups of 420,095 that can be expected to have cancer of the brain or nervous system.

*b)* Based on the result from part (*a*), is it unusual to find that among 420,095 people, there are 135 cases of cancer of the brain or nervous system? Why or why not?

*c)* What do these results suggest about the publicized concern that cell phones are a health danger because they increase the risk of cancer of the brain or nervous system?

***Solution***

Binomial problem, $p = 0.00034$; $q = 0.99966$ and $n = 420{,}095$

**a)** *Mean*: $\mu = np = (420{,}095)(0.00034)\ \underline{=142.8}$

*Standard deviation*: $\sigma = \sqrt{npq} = \sqrt{(420{,}095)(0.00034)(0.99966)}\ \underline{=11.949}$

**b)** $Minimum\ unusual\ value\ = \mu - 2\sigma = 142.8 - 2(11.949)\ \underline{=118.9}$

$Maximum\ unusual\ value\ = \mu + 2\sigma = 142.8 + 2(11.949)\ \underline{=166.7}$

No, since 135 is within the above limits, it is not an unusual result.

**c)** These results do not provide evidence that cell phone use increases the risk of such cancers.

## Exercise

Mario's Pizza Parlor has just opened. Due to a lack of employee training, there is only a 0.8 probability that a pizza will be edible. An order for 5 pizzas has just been placed. What is the minimum number of pizzas that must be made in order to be at least 99% sure that there will be 5 that are edible?

***Solution***

Let $x$ = the number of edible pizzas.

Binomial problem, $p = 0.8$; $\rightarrow q = 0.2$ and $n = unknown$

Find: $P(x \geq 5) \geq 0.99$

Using *Binomial Probability Table*:

For $n=5$, $P(x \geq 5) = P(x=5)$

$$= 0.328$$

For $n=6$, $P(x \geq 5) = P(x=5) + P(x=6)$

$$= 0.393 + 0.262$$
$$= 0.655$$

For $n=7$, $P(x \geq 5) = P(x=5) + P(x=6) + P(x=7)$

$$= 0.275 + 0.367 + 0.210$$
$$= 0.852$$

For $n=8$, $P(x \geq 5) = P(x=5) + P(x=6) + P(x=7) + P(x=8)$

$$= 0.147 + 0.294 + 0.336 + 0.168$$
$$= 0.945$$

For $n=9$, $P(x \geq 5) = P(x=5) + P(x=6) + P(x=7) + P(x=8) + P(x=9)$

$$= 0.066 + 0.176 + 0.302 + 0.302 + 0.134$$
$$= 0.980$$

For $n=10$, $P(x \geq 5) = P(x=5) + P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$$= 0.026 + 0.088 + 0.201 + 0.302 + 0.268 + 0.107$$
$$= 0.992$$

The minimum number of pizza necessary to be at least 99% sure that there will be 5 edible pizzas available is $n$ = 10.

*This procedure may not be the most efficient, but it is easy to follow and promotes better understanding of the concepts involved.*