Distance Formula: $\left|P_1P_2\right| = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$

Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

Standard Equation for the Sphere $\left(x - x_0\right)^2 + \left(y - y_0\right)^2 + \left(z - z_0\right)^2 = a^2$

Magnitude: $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$

Direction / Unit vector: $\frac{\vec{v}}{|\vec{v}|}$

Angle between Vectors: $\theta = \cos^{-1}\left(\frac{u_1v_1 + u_2v_2 + u_3v_3}{|\vec{u}|\ |\vec{v}|}\right)$

Dot Product: $\vec{u} \bullet \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$

Vector Projection: $proj_{\vec{v}}\vec{u} = \left(|\vec{u}|\cos\theta\right)\frac{\vec{u}}{|\vec{v}|} = \left(\frac{\vec{u} \bullet \vec{v}}{|\vec{v}|^2}\right)\vec{v}$

The scalar component of $\vec{u}$ in the direction of $\vec{v}$ is the scalar $|\vec{u}|\cos\theta = \frac{\vec{u} \bullet \vec{v}}{|\vec{v}|} = \vec{u} \bullet \frac{\vec{v}}{|\vec{v}|}$

Work: $W = \vec{F} \bullet \vec{D} = \left|\vec{F}\right|\left|\vec{D}\right|\cos\theta$

Cross Product: $\vec{u} \times \vec{v} = \left(|\vec{u}||\vec{v}|\sin\theta\right)\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

Magnitude of torque vector: $|\vec{r}|\left|\vec{F}\right|\sin\theta$

Torque vector: $\left(|\vec{r}|\left|\vec{F}\right|\sin\theta\right)\vec{n}$

Triple scalar product: $\left(\vec{u} \times \vec{w}\right) \bullet \vec{w} = |\vec{u} \times \vec{v}||\vec{w}||\cos\theta|$

Volume: $V = \left(\vec{u} \times \vec{v}\right) \bullet \vec{w} = \left|\det\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix}\right|$

Vector equation for the line ***L***: $r(t) = r_0 + tv, \quad -\infty < t < \infty$

The distance from a Point to a line: $d = \frac{\left|\overrightarrow{PS} \times \vec{v}\right|}{|\vec{v}|}$

The distance from a Point to a Plane: $d = \left| \overrightarrow{PS} \bullet \frac{\vec{n}}{|\vec{n}|} \right|$

Angle between the planes: $\theta = \cos^{-1}\left(\frac{\vec{n}_1 \bullet \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} \right)$

Arc Length: $s(t) = \int_{t_0}^{t} \sqrt{\left[x'(\tau)\right]^2 + \left[y'(\tau)\right]^2 + \left[z'(\tau)\right]^2}\, d\tau = \int_{t_0}^{t} |\vec{v}(\tau)|\, d\tau$

Maximum height: $y_{\max} = \frac{(v_0 \sin\alpha)^2}{2g}$

Maximum time: $t = \frac{v_0 \sin\alpha}{g}$

Flight time: $t = \frac{2v_0 \sin\alpha}{g}$

Range: $R = \frac{v_0^2}{g} \sin 2\alpha$

Unit tangent vector: $\vec{T} = \frac{\vec{v}}{|\vec{v}|}$

Principal unit normal vector: $\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|} = \frac{1}{\kappa}\frac{d\vec{T}}{ds}$

Binormal vector: $\vec{B} = \vec{T} \times \vec{N}$

Curvature: $\kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3} = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$

Torsion: $\tau = -\frac{d\vec{B}}{ds} \bullet \vec{N} = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|\vec{v} \times \vec{a}|^2}$

Acceleration vector: $\vec{a} = a_T \vec{T} + a_N \vec{N}$

Tangential acceleration: $a_T = \frac{d}{dt}|\vec{v}|$

Normal acceleration: $a_N = \kappa |\vec{v}|^2 = \sqrt{|\vec{a}|^2 - a_T^2}$

$$\frac{\partial^2 f}{\partial y \partial x} \quad or \quad f_{xy} = \left(f_x\right)_y$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}, \quad \frac{dz}{dx} = -\frac{F_x}{F_z}, \quad \frac{dz}{dy} = -\frac{F_y}{F_z}$$

Gradient Vector: $\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$$\left(\frac{df}{ds}\right)_{\vec{u},P_0} = \left(\nabla f\right)_{P_0} \bullet \vec{u}$$

Directional Derivative: $D_{\boldsymbol{u}} f = \nabla f \bullet \vec{u} = |\nabla f||\vec{u}|\cos\theta = |\nabla f|\cos\theta$

Tangent Plane: $f_x\left(P_0\right)\left(x - x_0\right) + f_y\left(P_0\right)\left(y - y_0\right) + f_z\left(P_0\right)\left(z - z_0\right) = 0$

Normal Line: $x = x_0 + f_x\left(P_0\right)t, \quad y = y_0 + f_y\left(P_0\right)t, \quad z = z_0 + f_z\left(P_0\right)t$

Linearization: $L(x, y) = f\left(x_0, y_0\right) + f_x\left(x_0, y_0\right)\left(x - x_0\right) + f_y\left(x_0, y_0\right)\left(y - y_0\right)$

f has a local maximum at (a, b) if $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b).

f has a local minimum at (a, b) if $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b).

f has a saddle point at (a, b) if $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a, b).

The test is inconclusive at (a, b) if $f_{xx}f_{yy} - f_{xy}^2 = 0$ at (a, b).

Lagrange Multipliers: $\nabla f = \lambda \nabla g \quad and \quad g(x, y, z) = 0$

Volume: $$V=\int_a^b A(x)dx=\int_a^b\int_{g_1(x)}^{g_2(x)} f(x,y)dydx$$

$$V=\int_{x=a}^{x=b}\int_{y=g_1(x)}^{y=g_2(x)}\int_{z=f_1(x,y)}^{z=f_2(x,y)} F(x,y,z)dzdydx$$

Average values of f over $R=\dfrac{1}{area\ of\ R}\iint\limits_R fdA$

Average value of F over D $=\dfrac{1}{volume\ of\ D}\iiint\limits_D FdV$

$$A=\iint\limits_R r\ dr\ d\theta$$

Cartesian Integrals into Polar: $$\iint\limits_R f(x,y)dxdy=\iint\limits_G f(r\cos\theta,r\sin\theta)r\ drd\theta$$

$$r^2=x^2+y^2,\quad \tan\theta=\frac{y}{x}$$

$$r=\rho\sin\phi,\quad x=r\cos\theta=\rho\sin\phi\cos\theta,$$

$$z=\rho\cos\phi,\quad y=r\sin\theta=\rho\sin\phi\sin\theta,$$

$$\rho=\sqrt{x^2+y^2+z^2}=\sqrt{r^2+z^2}$$

$$dV=\rho^2\sin\phi\ d\rho\ d\phi\ d\theta$$

$$\iiint\limits_D f(\rho,\ \phi,\ \theta)dV=\int_{\theta=\alpha}^{\theta=\beta}\int_{\phi=\phi_{\min}}^{\phi=\phi_{\max}}\int_{\rho=g_1(\phi,\theta)}^{\rho=g_2(\phi,\theta)} f(\rho,\ \phi,\ \theta)\rho^2 sin\phi\ d\rho d\phi d\theta$$

$$\iiint\limits_R f(x,y)dxdy=\iiint\limits_R H(u,v,w)\left|J(u,v,w)\right|dudvdw$$

Jacobian: $$J(u,v,w)=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w}\\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w}\end{vmatrix}=\frac{\partial(x,y,z)}{\partial(u,v,w)}$$

$$W=\int_C \vec{T}\bullet\vec{T}\ ds=\int_a^b \vec{F}(\vec{r}(t))\cdot\frac{d\vec{r}}{dt}dt$$

$$\vec{F}=M(x,y)\ \hat{i}+N(x,y)\ \hat{j}$$

$$\text{Flux of } \vec{F} \text{ across } C = \int_C \vec{F} \bullet \vec{n} \, ds$$

$$\vec{F} \bullet \vec{n} = M(x,y)\frac{dy}{ds} - N(x,y)\frac{dx}{ds}$$

$$\left(\text{Flux of } \vec{F} = M\,\hat{i} + N\,\hat{j} \text{ across } C\right) = \oint_C M dy - N dx$$

Divergence $\quad div\vec{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y}$

$$\underbrace{\oint_C F \bullet N ds}_{\text{Outward flux}} = \oint_C M dy - N dx = \underbrace{\iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx dy}_{\text{Divergence integral}}$$

$$\underbrace{\oint_C F \bullet T ds = \oint_C M dx + N dy}_{\text{Counterclockwise circulation}} = \underbrace{\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx dy}_{\text{Curl integral}}$$

$$\boldsymbol{A} = \iint_R \left|\vec{r}_u \times \vec{r}_v\right| dA = \int_c^d \int_a^b \left|\vec{r}_u \times \vec{r}_v\right| du dv$$

$$\text{Surface area} = \iint_R \frac{|\nabla F|}{|\nabla F \cdot \boldsymbol{p}|} dA \qquad \boldsymbol{p} = \boldsymbol{i},\ \boldsymbol{j},\ \text{or}\ \boldsymbol{k}$$

$$A = \iint_R \sqrt{f_x^2 + f_y^2 + 1}\ dxdy$$