

$$f(x) = \frac{ax+b}{cx+d} \Rightarrow f^{-1}(x) = \frac{-dx+b}{cx-a}$$

Proof

$$y = \frac{ax+b}{cx+d}$$

$$x = \frac{ay+b}{cy+d}$$

$$cxy + dx = ay + b$$

$$cxy - ay = -dx + b$$

$$(cx - a)y = -dx + b$$

$$y = \frac{-dx+b}{cx-a}$$

$$\boxed{f^{-1}(x) = \frac{-dx+b}{cx-a}} \quad \checkmark$$

$$A = A_0 e^{kt} \Rightarrow kT = \ln \frac{A}{A_0}$$

Proof

$$A = A_0 e^{kt}$$

$$\frac{A}{A_0} = e^{kt}$$

$$\ln \frac{A}{A_0} = \ln e^{kt}$$

$$\boxed{\ln \frac{A}{A_0} = kt} \quad \checkmark$$

$$a^{mx+n} = b^{px+q} \Rightarrow x = \frac{q \ln b - n \ln a}{m \ln a - p \ln b}$$

Proof

$$\ln a^{mx+n} = \ln b^{px+q}$$

$$(mx+n)\ln a = (px+q)\ln b$$

$$mx \ln a + n \ln a = px \ln b + q \ln b$$

$$mx \ln a - px \ln b = q \ln b - n \ln a$$

$$x(m \ln a - p \ln b) = q \ln b - n \ln a$$

$$x = \frac{q \ln b - n \ln a}{m \ln a - p \ln b}$$
