

## Section R.4 – Circles

### The *Distance* Formula

The distance,  $d$ , between the points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the rectangular coordinate system is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

To complete the distance between two points. Find the square of the difference between the  $x$ -coordinate plus the square of the difference between the  $y$ -coordinates. The principal square root of this sum is the distance.

#### *Example*

Find the distance between  $P(-8, 4)$  and  $Q(3, -2)$

#### *Solution*

$$\begin{aligned} d &= \sqrt{(3 - (-8))^2 + (-2 - 4)^2} \\ &= \sqrt{(11)^2 + (-6)^2} \\ &= \sqrt{121 + 36} \\ &= \sqrt{157} \end{aligned}$$

#### *Example*

Find the distance between  $(-4, 9)$  and  $(1, -3)$

#### *Solution*

$$\begin{aligned} d &= \sqrt{(1 + 4)^2 + (-3 - 9)^2} \\ &= \sqrt{25 + 144} \\ &= 13 \end{aligned}$$

## **Midpoint Formula**

Consider a line segment whose endpoints are  $(x_1, y_1)$  and  $(x_2, y_2)$ . The coordinates of the segment's midpoint are

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

To find the midpoint, take the average of the two  $x$ -coordinates and the average of the  $y$ -coordinates

### **Example**

Find the midpoint of the line segment with endpoints  $(1, 2)$  and  $(7, -3)$

#### **Solution**

$$M = \left( \frac{1+7}{2}, \frac{2-3}{2} \right)$$

$$\rightarrow \left( \frac{8}{2}, \frac{-1}{2} \right)$$

$$\text{The midpoint: } \left( 4, -\frac{1}{2} \right)$$

### **Example**

Find the midpoint  $M$  of the segment with endpoints  $(8, -4)$  and  $(-6, 1)$

#### **Solution**

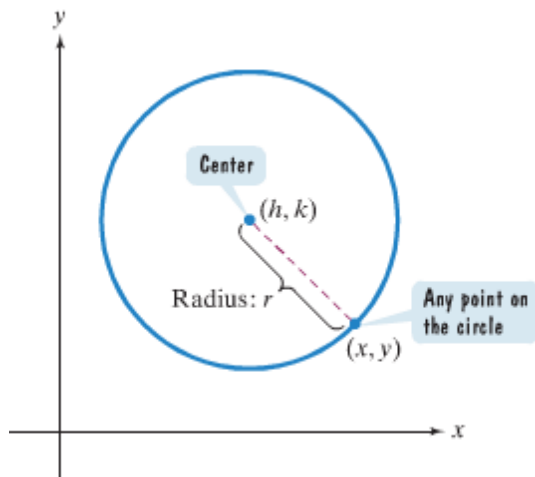
$$M \left( \frac{8 + (-6)}{2}, \frac{-4 + 1}{2} \right)$$

$$\left( 1, -\frac{3}{2} \right)$$

## The Standard Form of the Equation of a Circle

The standard form of the equation of a circle with center  $(h, k)$  and radius  $r$  is

$$(x - h)^2 + (y - k)^2 = r^2$$



A circle with center  $(0, 0)$  and radius  $r$  has equation:  $x^2 + y^2 = r^2$

### Example

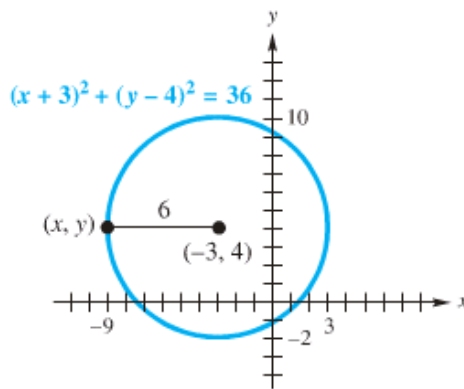
Find the center-radius form of the equation of each circle.

- a) Center at  $(-3, 4)$ , radius 6

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-3))^2 + (y - 4)^2 = 6^2$$

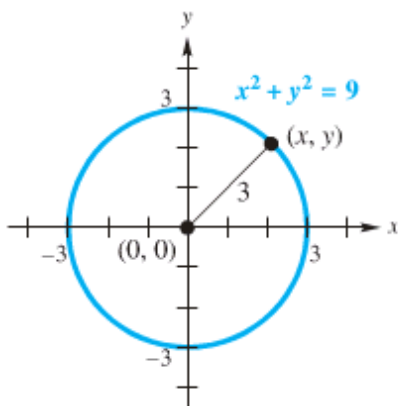
$$(x + 3)^2 + (y - 4)^2 = 36$$



- b) Center at  $(0, 0)$ , radius 3

$$(x - 0)^2 + (y - 0)^2 = 3^2$$

$$x^2 + y^2 = 9$$



### ***Example***

Find the equation of a circle with center  $(-1, 4)$  that passes through  $(3, 7)$ .

### **Solution**

$$\begin{aligned}r = d &= \sqrt{(-1-3)^2 + (4-7)^2} \\&= \sqrt{(-4)^2 + (-3)^2} \\&= \sqrt{16+9} \\&= \sqrt{25} \\&= 5\end{aligned}$$

$$\boxed{(x+1)^2 + (y-4)^2 = 25}$$

### ***Example***

Find an equation of the circle with endpoints **A**  $(1, -1)$  and **B**  $(2, -3)$

### **Solution**

$$\text{Center} = \text{Midpoint: } \left(\frac{1+2}{2}, \frac{-1-3}{2}\right) \rightarrow \left(\frac{3}{2}, -2\right)$$

$$\begin{aligned}r = d &= \sqrt{\left(\frac{3}{2}-1\right)^2 + (-2+1)^2} \\&= \sqrt{\left(\frac{1}{2}\right)^2 + (-1)^2} \\&= \sqrt{\frac{1}{4}+1} \\&= \sqrt{\frac{5}{4}}\end{aligned}$$

$$\boxed{\left(x-\frac{3}{2}\right)^2 + (y+2)^2 = \frac{5}{4}}$$

***Example***

Write in standard form:  $x^2 + y^2 + 4x - 4y - 1 = 0$

**Solution**

$$x^2 + 4x + y^2 - 4y = 1$$

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 + y^2 - 4y + \left(\frac{-4}{2}\right)^2 = 1 + \left(\frac{4}{2}\right)^2 + \left(\frac{-4}{2}\right)^2$$

$$x^2 + 4x + 2^2 + y^2 - 4y + 2^2 = 1 + 4 + 4$$

$$\underline{(x+2)^2 + (y-2)^2 = 9}$$

## Exercises      Section R.4 – Circles

(1 – 10) Find the distance between the two given points

1.  $P(8, 2)$  and  $Q(3, 5)$
2.  $P(-4, 3)$  and  $Q(2, -5)$
3.  $(-4, -1)$  and  $(2, -3)$
4.  $(2\sqrt{3}, \sqrt{6})$  and  $(-\sqrt{3}, 5\sqrt{6})$
5.  $(-1, -5)$  and  $(-1, 2)$
6.  $(-4, 9)$  and  $(1, -3)$
7.  $(-2, 2)$  and  $(3, -6)$
8.  $(\sqrt{5}, -\sqrt{3})$  and  $(-\sqrt{7}, \sqrt{7})$
9.  $(\sqrt{7}, -\sqrt{2})$  and  $(-\sqrt{3}, \sqrt{3})$
10.  $(\sqrt{7}, -\sqrt{2})$  and  $(-\sqrt{3}, \sqrt{7})$

(11 – 17) Find the midpoint of the line segment with endpoints

11.  $(1, 2)$  and  $(7, -3)$
12.  $P(8, 2)$  and  $Q(3, 5)$
13.  $P(-4, 3)$  and  $Q(2, -5)$
14.  $(4, -9)$  and  $(-12, -3)$
15.  $(7, -2)$  and  $(9, 5)$ .
16.  $(-2, -1)$  and  $(-8, 6)$
17.  $(7\sqrt{3}, -6)$  and  $(3\sqrt{3}, -2)$

(18 – 27) Write the standard form of the equation of the circle

18. center  $(-\sqrt{3}, -\sqrt{3})$ , radius  $\sqrt{3}$
19. center  $(0, 0)$  and radius 4.
20. center  $(5, -6)$  and radius 10.
21. center  $(2, -1)$  and  $r = 4$ .
22. center  $(-5, -3)$  and  $r = \sqrt{5}$ .
23. radius 5 and center  $(3, -7)$
24. center  $(6, -5)$  that passes through  $(1, 7)$ .
25. center  $(-2, -3)$  that passes through  $(-3, 2)$ .
26. center  $(-5, 2)$  passing through  $(-1, 5)$
27. diameter whose endpoints are  $(4, 4)$  and  $(-2, 3)$

(28 – 31) Find the center and the radius of

28.  $x^2 + y^2 + 6x + 2y + 6 = 0$
29.  $x^2 + y^2 + 8x + 4y + 16 = 0$
30.  $x^2 + y^2 - 10x - 6y - 30 = 0$
31.  $x^2 - 6x + y^2 + 10y + 25 = 0$