Math 2318 – Linear Algebra **_Exam 2_** **_Review_**

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1. Which of the following are linear combinations?

a) $(2,1,4)$ $(1,-1,3)$ $(3,2,5)$ $w=(5,9,5)$

b) $(1,-1,3)$ $(2,4,0)$ $w=(4,2,6)$

c) $(1,-1,3)$ $(2,4,0)$ $w=(1,5,6)$

d) $(2,1,4)$ $(1,-1,3)$ $(3,2,5)$ $w=(2,2,3)$

2. Show that the vector $\boldsymbol{w}$ is a subspace of $\boldsymbol{R}^3$?

a) All vectors of the form $\boldsymbol{w} = (a, 0, 0)$

b) $\boldsymbol{w} = (a, b, c)$, where $a + c + b = 0$, a, b, c are real numbers

c) $\boldsymbol{w} = (a, b, c)$, where $b = a + c$, a, b, c are real numbers

3. Determine whether the given vectors span $\boldsymbol{R}^3$

a) $v_1 = (1,1,1)$, $v_2 = (2,2,0)$, $v_3 = (3,0,0)$

b) $v_1 = (1,3,3)$, $v_2 = (1,3,4)$, $v_3 = (1,4,3)$, $v_4 = (6,2,1)$

4. Determine whether the vectors are linearly independent or linearly dependent

a) $(1, 1, -1), (2, -3, 1), (8, -7, 1)$

b) $(1, -2, -3), (2, 3, -1), (3, 2, 1)$

c) $(1, -2, 1), (1, 2, -1), (7, -4, 1)$

d) $(1, -3, 7), (2, 0, -6), (3, -1, -1), (2, 4, -5)$

e) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

5. Find the coordinate vector of $\boldsymbol{w}$ relative to the basis $S = \{u_1, u_2\}$ for $\boldsymbol{R}^2$

a) $u_1 = (1,-1)$, $u_2 = (1,1)$, $w = (1,0)$

b) $u_1 = (2,-4)$, $u_2 = (3,8)$, $w = (1,1)$

6. Find the coordinate vector of ***v*** relative to the basis $S = \{v_1, v_2, v_3\}$

a) $v = (2, -1, 1), \quad v_1 = (2, 1, 3), \quad v_2 = (1, 0, 1), \quad v_3 = (1, 1, 1)$

b) $v = (2, 1, 0), \quad v_1 = (1, 2, 1), \quad v_2 = (-1, 1, 2), \quad v_3 = (1, 2, 3)$

7. Given the matrix A and b:

a) Reduce A to row-reduced echelon form.
b) What is the dimension of A?
c) What is the rank of A?
d) What are the pivots?
e) What are the free variables?
f) Find the special (homogeneous) solutions.
g) What is the nullspace $N(A)$?
h) Find the particular solution to $Ax = b$
i) Give the complete solution.

i. $A = \begin{pmatrix} -1 & 2 & 5 & 0 \\ 2 & 1 & 0 & 0 \\ 6 & -1 & -8 & -1 \\ 0 & 2 & 4 & 3 \end{pmatrix} \qquad b = \begin{pmatrix} 5 \\ -15 \\ -47 \\ 16 \end{pmatrix}$

ii. $A = \begin{pmatrix} 1 & -2 & 1 & 2 \\ 2 & -4 & 2 & 4 \\ -1 & 2 & -1 & -2 \\ 3 & -6 & 3 & 6 \end{pmatrix} \qquad b = \begin{pmatrix} -1 \\ -2 \\ 1 \\ -3 \end{pmatrix}$

iii. $A = \begin{pmatrix} 1 & 2 & -3 & 1 \\ -2 & 1 & 2 & 1 \\ -1 & 3 & -1 & 2 \\ 4 & -7 & 0 & -5 \end{pmatrix} \qquad b = \begin{pmatrix} 4 \\ -1 \\ 3 \\ -5 \end{pmatrix}$

8. Find the standard matrix for the operator T defined by the formula

a) $T(x_1, x_2, x_3, x_4) = (7x_1 + 2x_2 - x_3 + x_4, \;\; x_2 + x_3, \;\; -x_1)$

b) $T(x_1, x_2, x_3) = (2x_1 + x_3, \;\; x_1 + x_2 - x_3, \;\; x_1 - x_2 + x_3)$

Solution

1. $a)$ $(5,9,5)=3(2,1,4)-4(1,-1,3)+1(3,2,5)$

$b)$ $(4,2,6)=2(1,-1,3)+1(2,4,0)$

$c)$ *not a linear combination*

$d)$ $(2,2,3)=\frac{1}{2}(2,1,4)-\frac{1}{2}(1,-1,3)+\frac{1}{2}(3,2,5)$

2. $a)$ *Yes*

$b)$ *Yes*

$c)$ *Yes*

3. $a)$ $\det=-6,$ *Yes*

$b)$ $\begin{pmatrix} 1 & 0 & 0 & 39 & 7b_1-b_2-b_3 \\ 0 & 1 & 0 & -17 & b_3-3b_1 \\ 0 & 0 & 1 & -16 & b_2-3b_1 \end{pmatrix}$, *Yes*

4. $a)$ *Lineraly dependent*

$b)$ *Lineraly independent*

$c)$ *Lineraly dependent*

$d)$ *Lineraly dependent*

$e)$ *Lineraly independent*

5. $a)$ $(w)_S=\left(\frac{5}{28},\ \frac{3}{14}\right)$

$b)$ $(w)_S=\left(\frac{1}{2},\ \frac{1}{2}\right)$

6. $a)$ $(v)_S=(-1,\ 4,\ 0)$

$b)$ $(v)_S=\left(\frac{1}{2},\ -1,\ \frac{1}{2}\right)$

7.

i) $A=\begin{pmatrix} -1 & 2 & 5 & 0 \\ 2 & 1 & 0 & 0 \\ 6 & -1 & -8 & -1 \\ 0 & 2 & 4 & 3 \end{pmatrix}$ $\quad b=\begin{pmatrix} 5 \\ -15 \\ -47 \\ 16 \end{pmatrix}$

a) $\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

b) $Dim = 1$

c) $Rank = 3$

d) $x_1,\ x_2,\ x_4$

e) x_3

f) $s_1 = (1, -2, 1, 0)$

g) $\boldsymbol{x}_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$

h) $\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -7 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow x_p = (-7, -1, 6, 0)$

i) $\boldsymbol{x} = \begin{bmatrix} -7 \\ -1 \\ 6 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -6 \\ -3 \\ 1 \\ 0 \end{bmatrix}$

ii) $A = \begin{pmatrix} 1 & -2 & 1 & 2 \\ 2 & -4 & 2 & 4 \\ -1 & 2 & -1 & -2 \\ 3 & -6 & 3 & 6 \end{pmatrix} \quad b = \begin{pmatrix} -1 \\ -2 \\ 1 \\ -3 \end{pmatrix}$

a) $\left[\begin{array}{cccc|c} 1 & -2 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

b) $Dim = 3$

c) $Rank = 1$

d) x_1

e) $x_2,\ x_3,\ x_4$

f) $s_1 = (1, 1, 0, 0) \quad s_2 = (-2, 0, 1, 0) \quad s_3 = (-3, 0, 0, 1)$

g) $\boldsymbol{x}_2\begin{bmatrix}1\\1\\0\\0\end{bmatrix}+\boldsymbol{x}_3\begin{bmatrix}-2\\0\\1\\0\end{bmatrix}+\boldsymbol{x}_4\begin{bmatrix}-3\\0\\0\\1\end{bmatrix}$

h) $x_p=(-1,0,0,0)$

i) $\boldsymbol{x}=\begin{bmatrix}-1\\0\\0\\0\end{bmatrix}+x_2\begin{bmatrix}1\\1\\0\\0\end{bmatrix}+x_3\begin{bmatrix}-2\\0\\1\\0\end{bmatrix}+x_4\begin{bmatrix}-3\\0\\0\\1\end{bmatrix}$

iii) $A=\begin{pmatrix}1&2&-3&1\\-2&1&2&1\\-1&3&-1&2\\4&-7&0&-5\end{pmatrix}$ $b=\begin{pmatrix}4\\-1\\3\\-5\end{pmatrix}$

a) $\left[\begin{array}{cccc|c}1&0&-\frac{7}{5}&-\frac{1}{5}&\frac{6}{5}\\0&1&-\frac{4}{5}&\frac{3}{5}&\frac{7}{5}\\0&0&0&0&0\\0&0&0&0&0\end{array}\right]$

b) $Dim=2$

c) $Rank=2$

d) $x_1,\quad x_2$

e) $x_3,\quad x_4$

f) $s_1=\left(\frac{7}{5},\frac{4}{5},1,0\right)\quad s_2=\left(\frac{1}{5},-\frac{3}{5},0,1\right)$

g) $\boldsymbol{x}_3\begin{bmatrix}\frac{7}{5}\\\frac{4}{5}\\1\\0\end{bmatrix}+\boldsymbol{x}_4\begin{bmatrix}\frac{1}{5}\\-\frac{3}{5}\\0\\1\end{bmatrix}$

h) $x_p=\left(\frac{6}{5},\frac{7}{5},0,0\right)$

i) $\boldsymbol{x}=\begin{bmatrix}\frac{6}{5}\\\frac{7}{5}\\0\\0\end{bmatrix}+x_3\begin{bmatrix}\frac{7}{5}\\\frac{4}{5}\\1\\0\end{bmatrix}+x_4\begin{bmatrix}\frac{1}{5}\\-\frac{3}{5}\\0\\1\end{bmatrix}$

8. *a*) $\begin{pmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$