Geometric Sequences.

$$a_{k+1} = a_k r$$

$a_1, a_2, \ldots, a_n, \ldots$

$r = \dfrac{a_{k+1}}{a_k}$ common Ratio.

$$\boxed{a_n = a_1 r^{n-1}}$$

$6, -12, 24, -48, \ldots, (-1)^{n-1} 6, \ldots$

$r = -\frac{12}{6} = -2$

Ex

1st term

$a_1 = 3 \quad r = -\frac{1}{2}$

$a_n = a_1 r^{n-1}$

$= 3\left(-\frac{1}{2}\right)^{n-1}$

$n = 1 \qquad a_1 = 3$

$a_2 = 3\left(-\frac{1}{2}\right)^1 = -\frac{3}{2}$

$a_3 = 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$

$$a_4 = 3\left(-\frac{1}{2}\right)^3 = -\frac{3}{8}$$

$$a_5 = 3\left(-\frac{1}{2}\right)^4 = \frac{3}{16}$$

Ex $a_3 = 5$, $a_6 = -40$ a_8??

$$r = \left(\frac{-40}{5}\right)^{1/(6-3)}$$

$$= (-8)^{1/3}$$

$$= -2$$

$$r = \left(\frac{y_2}{y_1}\right)^{1/(x_2 - x_1)}$$

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 (-2)^{\overset{3-1}{2}} = 5$$

$$4a_1 = 5$$

$$a_1 = \frac{5}{4}$$

$$a_8 = \frac{5}{4}(-2)^7$$

$$= -5(2^5)$$

$$= -160$$

$$S_n = a_1 \frac{1 - r^n}{1 - r} \qquad \boxed{r \neq 1}$$

ex $1, 0.3, 0.09, .0027, \ldots$

1st 5 terms $\quad a_1 = 1, \; r = .3$

$$S_5 = \frac{1-(.3)^5}{1-.3}$$

$$= \frac{1-\left(\frac{3}{10}\right)^5}{1-\frac{3}{10}}$$

$$= \frac{1-\frac{3^5}{10^5}}{\frac{7}{10}}$$

$$= \frac{10^5-3^5}{10^5} \cdot \frac{10}{7}$$

$$= \frac{10{,}000-243}{7 \times 10^4}$$

$a_1, a_2, \ldots, a_n, \ldots$

$$S = \frac{a}{1-r} \qquad |r| < 1$$

if $|r| \geq 1$; $S = \infty$

ex, $$\sum_{n=1}^{\infty} 3\left(-\frac{2}{3}\right)^{n-1} = \frac{3}{1+\frac{2}{3}} \qquad \left|-\frac{2}{3}\right| = \frac{2}{3} < 1$$

$$= \frac{3}{\frac{5}{3}}$$

$$= \frac{7}{5}$$

110 $\displaystyle\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1} = \frac{5}{1-\frac{1}{4}}$ $\qquad \left|\frac{1}{4}\right| < 1$

$$= \frac{5}{\frac{3}{4}}$$

$$= \frac{20}{3}$$

~~108~~ $\displaystyle\sum_{n=1}^{\infty} 3\left(\frac{3}{2}\right)^{n-1} = \infty$ $\qquad \frac{3}{2} \geq 1$

Ex $5.4\overline{27}$

$5.4272727 = 5.4 + .02727272727\ldots$

$= \frac{54}{10} + (.027 + .00027 + \cdots)$

$a_1 = .027 = 27 \times 10^{-3}$

$r = \frac{.00027}{.027} = .01$ $\qquad \boxed{\frac{27 \times 10^{-5+3}}{27 \times 10^{-3}}}$

$S = \frac{54}{10} + \frac{27 \times 10^{-3}}{1 - .01}$

$= \frac{54}{10} + \frac{27 \times 10^{-3}}{.99}$

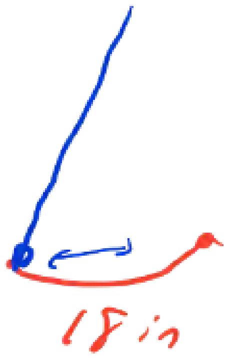
$= \frac{54}{10} + \frac{27}{99}\,\frac{10^{-3}}{10^{-2}}$ $\quad -3+2$

$= \frac{54}{10} + \frac{3}{110}$ $\qquad -2+3$

$$= \frac{594+3}{110}$$

$$\boxed{5.4\overline{27} = \frac{597}{110}}$$

Fr $l = .98 = r$



$$a_1 = 18$$

$$a_2 = 18(.98)$$

$$1: a_{10} = 18(.98)^9$$

$$\boxed{a_n = 18(.98)^{n-1}}$$

$$18(.98)^{n-1} = 12$$

$$(.98)^{n-1} = \frac{12}{18} = \frac{2}{3}$$

$$n-1 = \log_{.98}\frac{2}{3} = \frac{\ln(2/3)}{\ln .98}$$

$\ln(0<<1) = <0$

$\ln 1 = 0$

$\ln e = 1$

$$n \approx 21.07$$

22^{nd} swing

$$r = .98 < 1$$

$$T = \frac{18}{1-.98}$$

$$= \frac{18}{.02}$$

$$= \frac{18}{2\times10^{-2}}$$

$$= 900$$

5.7 Mathematical Induction

- Show that P_1 is true
- Assume P_k is true, prove P_{k+1} is also true

Ex

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

Soln

For $n=1 \Rightarrow$

$$1 = \frac{1(2)}{2}$$

$$1 = 1 \checkmark$$

P_1 is true

Assume P_k is true: $1+2+\dots+k = \frac{k(k+1)}{2}$

Is P_{k+1}: $1+\dots+k+(k+1) \overset{?}{=} \frac{(k+1)(k+2)}{2}$

$$1+\dots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= (k+1)\left(\frac{k}{2}+1\right)$$

$$= (k+1)\left(\frac{k+2}{2}\right) \checkmark$$

Hence, P_{k+1} is also true.

$\therefore$ By the mathematical induction, the given proof is completed

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Ex $1^2+3^2+\cdots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

For $n=1 \Rightarrow 1^2=\frac{1(2-1)(3)}{3}$

$1=1$ ✓ P_1 is true.

Assume P_k is true:

$1^2+\cdots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}$

is P_{k+1} is also true

$1^2+\cdots+(2k-1)^2+(2(k+1)-1)^2=\frac{1}{3}(k+1)(2(k+1)-1)(2(k+1)+1)$

$1^2+\cdots+(2k-1)^2+(2k+1)^2\overset{?}{=}\frac{1}{3}(k+1)(2k+1)(2k+3)$

$1^2+\cdots+(2k-1)^2+(2k+1)^2=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$

$=\frac{1}{3}(2k+1)\left[2k^2-k+3(2k+1)\right]$

$=\frac{1}{3}(2k+1)(2k^2+5k+3)$

$=\frac{1}{3}(2k+1)(k+1)(2k+3)$ ✓

P_{k+1} is also true

∴ By the Mathematical induction, the given proof is completed.

Ex 2 is a factor of n^2+5n

$n\in\mathbb{Z}^+$

$n=1 \Rightarrow 1^2+5=6$

$=2(3)$ ✓

P_1 is true.

$k^2+5k=2a$

Assume P_k is true: $k^2+5k=2p$

Is P_{k+1} is also true if $(k+1)^2+5(k+1)$ is 2 a factor

$$(k+1)^2+5(k+1)=k^2+2k+1+5k+5$$
$$=(k^2+5k)+2k+6$$
$$=2p+2k+6$$
$$=2(p+k+3)$$ ✓

P_{k+1} is also true.

∴ By the mathematical induction, the given proof is completed.

Exam 1

(2) - Partial Fraction (5.2)

(1) - Ellipse (5.3)

(1) - Hyperbola (5.4)

(2) - (5.5) $1^{st}d$ + a_{10}

(3) = $\sum_{1}^{10} 5 \quad \sum_{10}^{50} 5 \quad \sum_{1}^{4}$, $a_n=a_1+(n-1)d$

(1) (5.6) arithmetic $d=\frac{y_2-y_1}{x_2-x_1}$

$a_{x} : a_a \quad a_b$

(1) geometric : $r=\left(\frac{y_2}{y_1}\right)^{1/(x_2-x_1)}$

$\left\{ a_n=a_1 r^{n-1} \right.$

(2) $\sum$ $\left\{ \begin{array}{ll} S=\frac{a_1}{1-r} & |r|<1 \\ S=\infty & |r|\geq 1 \end{array} \right.$

(1) $a_1 + a_2 + \cdots + a_n = \Sigma$

(1) Prove

Hwk 5.6 due 9/17
5.7 (2) Thursday