Solution *Section* 1.1 – Functions

Exercise

Find the domain: $f(x) = 7x + 4$

Solution

Domain: $(-\infty, \infty)$

Exercise

Find the domain: $f(x) = |3x - 2|$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = 3x + \pi$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = \sqrt{7}x + \frac{1}{2}$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = -2x^2 + 3x - 5$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = x^3 - 2x^2 + x - 3$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain: $f(x) = x^2 - 2x - 15$

Solution

Domain: $\mathbb{R}$

Exercise

Find the domain $f(x) = 4 - \frac{2}{x}$

Solution

Domain: $x \neq 0$

Exercise

Find the domain $f(x) = \frac{1}{x^4}$

Solution

Domain: $x \neq 0$

Exercise

Find the domain: $g(x) = \frac{3}{x - 4}$

Solution

Domain: $x \neq 4$

Exercise

Find the domain $y = \frac{2}{x - 3}$

Solution

Domain: $x \neq 3$

Exercise

Find the domain $y = \dfrac{-7}{x-5}$

Solution

Domain: $\boxed{x \ne 5}$

Exercise

Find the domain $f(x) = \dfrac{x+5}{2-x}$

Solution

$2 - x \ne 0$

Domain: $\boxed{x \ne 2}$

Exercise

Find the domain $f(x) = \dfrac{8}{x+4}$

Solution

$x + 4 \ne 0$

Domain: $\boxed{x \ne -4}$

Exercise

Find the domain $f(x) = \dfrac{1}{x+4}$

Solution

Domain: $\boxed{x \ne -4}$

Exercise

Find the domain $f(x) = \dfrac{1}{x-4}$

Solution

Domain: $\boxed{x \ne 4}$

Exercise

Find the domain $f(x) = \frac{3x}{x+2}$

Solution

Domain: $\underline{x \neq -2}$

Exercise

Find the domain $f(x) = x - \frac{2}{x-3}$

Solution

Domain: $\underline{x \neq 3}$

Exercise

Find the domain $f(x) = x + \frac{3}{x-5}$

Solution

Domain: $\underline{x \neq 5}$

Exercise

Find the domain $f(x) = \frac{1}{2}x - \frac{8}{x+7}$

Solution

Domain: $\underline{x \neq -7}$

Exercise

Find the domain $f(x) = \frac{1}{x-3} - \frac{8}{x+7}$

Solution

Domain: $\underline{x \neq -7,\ 3}$

Exercise

Find the domain $f(x) = \frac{1}{x+4} - \frac{2x}{x-4}$

Solution

Domain: $\underline{x \neq \pm 4}$

Exercise

Fib+cnd the domain $f(x)=\dfrac{3x^2}{x+3}-\dfrac{4x}{x-2}$

Solution

Domain: $\underline{x \neq -3,\ 2}$

Exercise

Find the domain $f(x)=\dfrac{1}{x^2-2x+1}$

Solution

$x^2-2x+1 \neq 0 \qquad a+b+c=0 \;\rightarrow\; x=1,\ \dfrac{c}{a}$

Domain: $\underline{x \neq 1}$

Exercise

Find the domain $f(x)=\dfrac{x}{x^2+3x+2}$

Solution

$x^2+3x+2 \neq 0 \qquad a-b+c=0 \;\rightarrow\; x=-1,\ -\dfrac{c}{a}$

Domain: $\underline{x \neq -1,\ -2}$

Exercise

Find the domain $f(x)=\dfrac{x^2}{x^2-5x+4}$

Solution

$x^2-5x+4 \neq 0 \qquad a+b+c=0 \;\rightarrow\; x=1,\ \dfrac{c}{a}$

Domain: $\underline{x \neq -1,\ -2}$

Exercise

Find the domain $f(x)=\dfrac{1}{x^2-4x-5}$

Solution

$x^2-4x-5 \neq 0 \qquad a-b+c=0 \;\rightarrow\; x=-1,\ -\dfrac{c}{a}$

Domain: $\underline{x \neq -1,\ 5}$

Exercise

Find the domain $g(x)=\dfrac{2}{x^2+x-12}$

Solution

$$x^2+x-12\neq 0$$
$$(x+4)(x-3)\neq 0$$

Domain: $\boxed{x\neq -4,\ 3}$ $\boxed{(-\infty,-4)\cup(-4,3)\cup(3,\infty)}$

Exercise

Find the domain $h(x)=\dfrac{5}{\frac{4}{x}-1}$

Solution

$x\neq 0$ $\quad \dfrac{4}{x}-1\neq 0$

$$\frac{4-x}{x}\neq 0$$
$$4-x\neq 0$$
$$x\neq 4$$

Domain: $\boxed{x\neq 0,\ 4}$ $\boxed{(-\infty,0)\cup(0,4)\cup(4,\infty)}$

Exercise

Find the domain $y=\sqrt{x}$

Solution

$$x\geq 0$$

Domain: $\boxed{x\geq 0}$ $\boxed{[0,\ \infty)}$

Exercise

Find the domain $f(x)=\sqrt{8-3x}$

Solution

$$8-3x\geq 0$$
$$8\geq 3x$$

Domain: $\boxed{x\leq \frac{8}{3}}$ $\boxed{\left(-\infty,\ \frac{8}{3}\right]}$

Exercise

Find the domain $y=\sqrt{4x+1}$

Solution

$$4x+1\ge 0 \Rightarrow x\ge -\frac{1}{4}$$

Domain: $\underline{x\ge -\frac{1}{4}}$ $\left[-\frac{1}{4},\infty\right)$

Exercise

Find the domain $y=\sqrt{7-2x}$

Solution

$$7-2x\ge 0$$

$$-2x\ge -7$$

Domain: $\underline{x\le \frac{7}{2}}$ $\left(-\infty,\frac{7}{2}\right]$

Exercise

Find the domain $f(x)=\sqrt{8-x}$

Solution

$$8-x\ge 0$$

Domain: $\underline{x\le -8}$ $(-\infty,\ 8]$

Exercise

Find the domain $f(x)=\sqrt{3-2x}$

Solution

Domain: $\underline{x\le \frac{3}{2}}$

Exercise

Find the domain $f(x)=\sqrt{3+2x}$

Solution

Domain: $\underline{x\ge -\frac{3}{2}}$

Exercise

Find the domain $f(x)=\sqrt{5-x}$

Solution

Domain: $x \le 5$

Exercise

Find the domain $f(x)=\sqrt{x-5}$

Solution

Domain: $x \ge 5$

Exercise

Find the domain $f(x)=\sqrt{6-3x}$

Solution

Domain: $x \le 2$

Exercise

Find the domain $f(x)=\sqrt{3x-6}$

Solution

Domain: $x \ge 2$

Exercise

Find the domain $f(x)=\sqrt{2x+7}$

Solution

Domain: $x \ge -\frac{7}{2}$

Exercise

Find the domain $f(x)=\sqrt{x^2-16}$

Solution

$$x^2-16=0$$

$x^2 = 16$

$x = \pm 4$

Domain: $x \le -4 \quad x \ge 4$

Exercise

Find the domain $\qquad f(x) = \sqrt{16 - x^2}$

Solution

$x = \pm 4$

Domain: $-4 \le x \le 4$

Exercise

Find the domain $\qquad f(x) = \sqrt{9 - x^2}$

Solution

$x = \pm 3$

Domain: $-3 \le x \le 3$

Exercise

Find the domain $\qquad f(x) = \sqrt{x^2 - 25}$

Solution

$x = \pm 5$

Domain: $-5 \le x \le 5$

Exercise

Find the domain $\qquad f(x) = \sqrt{x^2 - 5x + 4}$

Solution

$x^2 - 5x + 4 \qquad\qquad a + b + c = 0 \quad \rightarrow \quad x = 1, \ \frac{c}{a}$

$x = 1, \ 4$

Domain: $x \le 1 \quad x \ge 4$

Exercise

Find the domain $f(x)=\sqrt{x^2+5x+4}$

Solution

x^2+5x+4 $\quad a-b+c=0 \ \rightarrow \ x=-1, \ -\frac{c}{a}$

$x=-1, \ -4$

Domain: $\underline{x\le -4 \quad x\ge -1}$

Exercise

Find the domain $f(x)=\sqrt{x^2+3x+2}$

Solution

x^2+3x+2 $\quad a-b+c=0 \ \rightarrow \ x=-1, \ -\frac{c}{a}$

$x=-1, \ -2$

Domain: $\underline{x\le -2 \quad x\ge -1}$

Exercise

Find the domain $f(x)=\sqrt{x^2-3x+2}$

Solution

x^2-3x+2 $\quad a+b+c=0 \ \rightarrow \ x=1, \ \frac{c}{a}$

$x=1, \ 2$

Domain: $\underline{x\le 1 \quad x\ge 2}$

Exercise

Find the domain $f(x)=\sqrt{x-4}+\sqrt{x+1}$

Solution

$x\ge 4 \quad x\ge -1$

Domain: $\underline{x\ge 4}$

Exercise

Find the domain $f(x) = \sqrt{3-x} + \sqrt{x-2}$

Solution

$x \le 3 \quad x \ge 2$

Domain: $\boxed{2 \le x \le 3}$

Exercise

Find the domain $f(x) = \sqrt{1-x} + \sqrt{4-x}$

Solution

$x \le 1 \quad x \le 4$

Domain: $\boxed{x \le 1}$

Exercise

Find the domain $f(x) = \sqrt{1-x} - \sqrt{x-3}$

Solution

$x \le 1 \quad x \ge 3$

Domain: $\boxed{\varnothing}$

Exercise

Find the domain $f(x) = \sqrt{x+4} - \sqrt{x-1}$

Solution

$x \ge -4 \quad x \ge 1$

Domain: $\boxed{x \ge 1}$

Exercise

Find the domain $f(x) = \dfrac{\sqrt{x+1}}{x}$

Solution

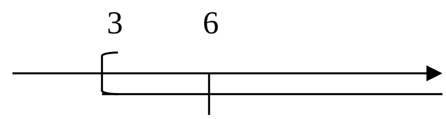
$x + 1 \ge 0 \qquad\qquad x \ne 0$

$x \ge -1$

Domain: $\boxed{x \ge -1 \quad x \ne 0}$ $\qquad \boxed{[-1,\ 0) \cup (0,\ \infty)}$

Exercise

Find the domain $g(x) = \dfrac{\sqrt{x-3}}{x-6}$



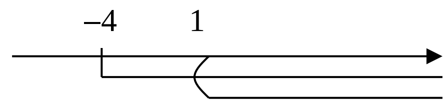
Solution

$$\rightarrow \begin{cases} x \geq 3 \\ x \neq 6 \end{cases}$$

Domain: $x \geq 3 \quad x \neq 6$ $\quad [3,\ 6) \cup (6,\ \infty)$

Exercise

Find the domain $f(x) = \dfrac{\sqrt{x+4}}{\sqrt{x-1}}$



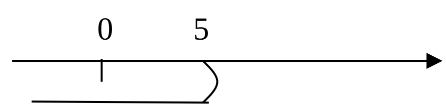
Solution

$$\rightarrow \begin{cases} x \geq -4 \\ x > 1 \end{cases}$$

Domain: $x > 1$ $\quad (1,\ \infty)$

Exercise

Find the domain $f(x) = \dfrac{\sqrt{5-x}}{x}$



Solution

$x \leq 5 \quad x \neq 0$

Domain: $x \leq 5 \quad x \neq 0$ $\quad (-\infty,\ 0) \cup (0,\ 5]$

Exercise

Find the domain $f(x) = \dfrac{x}{\sqrt{5-x}}$

Solution

Domain: $x < 5$ $\quad (-\infty,\ 5)$

Exercise

Find the domain $f(x)=\dfrac{1}{x\sqrt{5-x}}$

Solution

$x<5 \quad x\neq 0$

Domain: $\underline{x<5 \quad x\neq 0}$ $\quad \underline{(-\infty,\ 0)\cup(0,\ 5)}$

Exercise

Find the domain $f(x)=\dfrac{x+1}{x^3-4x}$

Solution

$x^3-4x\neq 0$

$x\left(x^2-4\right)\neq 0$

Domain: $\underline{x\neq 0,\ \pm 2}$ $\quad (-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$

Exercise

Find the domain $f(x)=\dfrac{\sqrt{x+5}}{x}$

Solution

$x\geq -5 \quad x\neq 0$

Domain: $\underline{x\geq -5 \quad x\neq 0}$

Exercise

Find the domain $f(x)=\dfrac{x}{\sqrt{x+5}}$

Solution

$x>-5$

Domain: $\underline{x>-5}$

Exercise

Find the domain $f(x)=\dfrac{1}{x\sqrt{x+5}}$

Solution

$x>-5 \quad x\neq 0$

Domain: $\underline{x>-5 \quad x\neq 0}$

Exercise

Find the domain $f(x)=\dfrac{x+3}{\sqrt{x-3}}$

Solution

Domain: $\underline{x>3}$

Exercise

Find the domain $f(x)=\dfrac{\sqrt{x+3}}{\sqrt{x-3}}$

Solution

$x\geq -3 \quad x>3$

Domain: $\underline{x>3}$

Exercise

Find the domain $f(x)=\dfrac{\sqrt{x-2}}{\sqrt{x+2}}$

Solution

$x\geq 2 \quad x>-2$

Domain: $\underline{x\geq 2}$

Exercise

Find the domain $f(x)=\dfrac{\sqrt{2-x}}{\sqrt{x+2}}$

Solution

$x\leq 2 \quad x>-2$

Domain: $\underline{-2<x\leq 2}$

Exercise

Find the domain $\quad f(x)=\dfrac{x-4}{\sqrt{x-2}}$

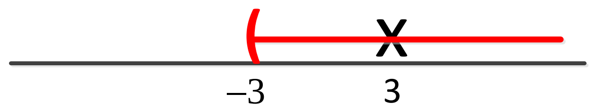
Solution

Domain: $\underline{x>2\,\rfloor}$

Exercise

Find the domain of $f(x)=\dfrac{1}{(x-3)\sqrt{x+3}}$

Solution



$x-3\neq 0 \qquad x+3>0$

$x\neq 3 \qquad x>-3$

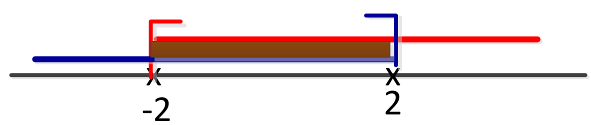
Domain: $\{x \mid x>-3 \ \ \textit{and} \ \ x\neq 3\}$

$(-3,\ 3)\cup(3,\ \infty)$

Exercise

Find the domain of $f(x)=\sqrt{x+2}+\sqrt{2-x}$

Solution



$x+2\geq 0 \qquad 2-x\geq 0$

$x\geq -2 \qquad -x\geq -2\rightarrow x\leq 2$

Domain: $\{x \mid -2\leq x\leq 2\}$

Exercise

Find the domain of $f(x)=\sqrt{(x-2)(x-6)}$

Solution

$x-2\geq 0 \quad x-6\geq 0$

$x\geq 2 \quad x\geq 6$

Domain: $\{x \mid x\leq 2,\ x\geq 6\}$

	2	6
–	+	+
–	–	+
+	–	+

Exercise

Find the domain of $f(x)=\sqrt{x+3}-\sqrt{4-x}$

Solution

$$x \ge -3 \quad x \le 4$$

Domain: $-3 \le x \le 4$

Exercise

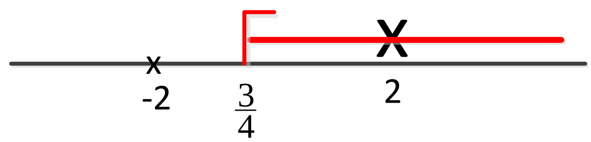
Find the domain of $f(x)=\dfrac{\sqrt{4x-3}}{x^2-4}$

Solution

$$4x-3 \ge 0 \qquad x^2-4 \ne 0$$

$$4x \ge 3 \qquad x \ne \pm 2$$

$$x \ge \frac{3}{4}$$



Domain: $\left[\frac{3}{4},\ 2\right) \cup (2,\ \infty)$

Exercise

Find the domain of $f(x)=\dfrac{4x}{6x^2+13x-5}$

Solution

$$6x^2+13x-5 \ne 0$$

$$x=\frac{-13 \pm \sqrt{169+120}}{12}$$

$$=\begin{cases} \dfrac{-13-17}{12}=-\dfrac{5}{2} \\[2mm] \dfrac{-13+17}{12}=\dfrac{1}{3} \end{cases}$$

Domain: $x \ne -\frac{5}{2},\ \frac{1}{3}$

Exercise

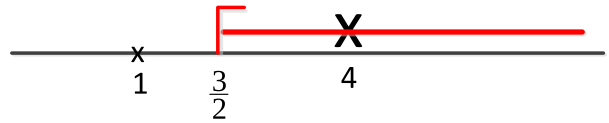
Find the domain of $f(x)=\dfrac{\sqrt{2x-3}}{x^2-5x+4}$

Solution

$$2x-3 \ge 0 \qquad x^2-5x+4 \ne 0$$

$2x \ge 3 \qquad x \ne 1,\ 4$

$x \ge \frac{3}{2}$



Domain: $x \ge \frac{3}{2},\ \ x \ne 4$ $\qquad \left[\frac{3}{2},\ 4\right) \cup (4,\ \infty)$

Exercise

Find the domain of $\quad f(x) = \dfrac{x^2}{\sqrt{x^2 - 5x + 4}}$

Solution

$x^2 - 5x + 4 \qquad\qquad a + b + c = 0 \ \rightarrow\ x = 1,\ \frac{c}{a}$

$x = 1,\ 4$

Domain: $x < 1 \quad x > 4$

Exercise

Find the domain of $\quad f(x) = \dfrac{x+2}{\sqrt{x^2 + 5x + 4}}$

Solution

$x^2 + 5x + 4 \qquad\qquad a - b + c = 0 \ \rightarrow\ x = -1,\ -\frac{c}{a}$

$x = -1,\ -4$

Domain: $x < -4 \quad x > -1$

Exercise

Find the domain of $\quad f(x) = \dfrac{\sqrt{x+2}}{\sqrt{x^2 + 3x + 2}}$

Solution

$x^2 + 3x + 2 \qquad\qquad a - b + c = 0 \ \rightarrow\ x = -1,\ -\frac{c}{a}$

$x < -2 \quad x > -1$

$\sqrt{x+2} \ \rightarrow\ x \ge -2$

Domain: $x > -1$

Exercise

Find the domain of $f(x)=\dfrac{\sqrt{2x+3}}{x^2-6x+5}$

Solution

$x^2-6x+5 \qquad\qquad a+b+c=0 \quad\rightarrow\quad x=1,\ \frac{c}{a}$

$x \neq 1,\ 5$

$\sqrt{2x+3} \quad\rightarrow\quad x \geq -\frac{3}{2}$

Domain: $\boxed{x \geq -\frac{3}{2} \quad x \neq 1,\ 5}$

Exercise

For the function *f* given by $f(x)=9x+5$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$f(x+h)=9(x+h)+5=9x+9h+5$

$$\frac{f(x+h)-f(x)}{h}=\frac{\overbrace{9x+9h+5}^{f(x+h)} \;-\; \overbrace{(9x+5)}^{f(x)}}{h}$$

$$=\frac{9x+9h+5 \;-\; 9x-5}{h}$$

$$=\frac{9h}{h}$$

$$\boxed{=9}$$

Exercise

For the function *f* given by $f(x)=6x+2$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\frac{f(x+h)-f(x)}{h}=\frac{6(x+h)+2-(6x+2)}{h}$$

$$=\frac{6x+6h+2-6x-2}{h}$$

$$=\frac{6h}{h}$$

$$\boxed{=6}$$

Exercise

For the function *f* given by $f(x)=4x+11$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{4(x+h)+11-(4x+11)}{h}\\ &= \frac{4x+4h+11-4x-11}{h}\\ &= \frac{4h}{h}\\ &= 4\end{aligned}$$

Exercise

For the function *f* given by $f(x)=3x-5$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)-5-3x+5}{h}\\ &= \frac{3x+3h-5-3x+5}{h}\\ &= \frac{3h}{h}\\ &= 3\end{aligned}$$

Exercise

For the function *f* given by $f(x)=-2x-3$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{-2(x+h)-3+2x+3}{h}\\ &= \frac{-2x-2h-3+2x+3}{h}\\ &= \frac{-2h}{h}\\ &= -2\end{aligned}$$

Exercise

For the function *f* given by $f(x) = -4x + 3$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-4(x+h) + 3 + 4x - 3}{h} \\ &= \frac{-4x - 4h + 3 + 4x - 3}{h} \\ &= \frac{-4h}{h} \\ &= -4\end{aligned}$$

Exercise

For the function *f* given by $f(x) = 3x - 6$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{3(x+h) - 6 - 3x + 6}{h} \\ &= \frac{3x + 3h - 6 - 3x + 6}{h} \\ &= \frac{3h}{h} \\ &= 3\end{aligned}$$

Exercise

For the function *f* given by $f(x) = -5x - 7$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-5(x+h) - 7 + 5x + 7}{h} \\ &= \frac{-5x - 5h - 7 + 5x + 7}{h} \\ &= \frac{-5h}{h} \\ &= -5\end{aligned}$$

Exercise

Given the function: $f(x)=2x^2$. Find and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned} f(x+h) &= 2(x+h)^2 \\ &= 2\left(x^2+2hx+h^2\right) \\ &= 2x^2+4hx+2h^2 \end{aligned}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{2x^2+4hx+2h^2-2x^2}{h} \\ &= \frac{4hx+2h^2}{h} \\ &= \frac{4hx}{h}+\frac{2h^2}{h} \\ &= 4x+2h \end{aligned}$$

Exercise

For the function *f* given by $f(x)=5x^2$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{5(x+h)^2-5x^2}{h} \\ &= \frac{5\left(x^2+2hx+h^2\right)-5x^2}{h} \\ &= \frac{5x^2+10hx+5h^2-5x^2}{h} \\ &= \frac{10hx+5h^2}{h} \\ &= 10x+5h \end{aligned}$$

Exercise

For the function *f* given by $f(x)=3x^2-4x$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2-4(x+h)-3x^2+4x}{h}$$

$$= \frac{3\left(x^2 + 2hx + h^2\right) - 4x - 4h - 3x^2 + 4x}{h}$$

$$= \frac{3x^2 + 6hx + 3h^2 - 4x - 4h - 3x^2 + 4x}{h}$$

$$= \frac{6hx + 3h^2 - 4h}{h}$$

$$\underline{= 6x + 3h - 4}$$

Exercise

For the function *f* given by $f(x) = 2x^2 - 3x$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$f(x+h) = 2(\text{—})^2 - 3(\text{—})$$

$$= 2(x+h)^2 - 3(x+h) \qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$= 2\left(x^2 + 2xh + h^2\right) - 3x - 3h$$

$$= 2x^2 + 4xh + 2h^2 - 3x - 3h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{2x^2 + 4xh + 2h^2 - 3x - 3h}^{f(x+h)} - \overbrace{(2x^2 - 3x)}^{f(x)}}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \frac{4xh + 2h^2 - 3h}{h}$$

$$= \frac{4xh}{h} + \frac{2h^2}{h} - \frac{3h}{h}$$

$$\underline{= 4x + 2h - 3}$$

Exercise

For the function f given by $f(x) = 2x^2 - x - 3$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Solution

$$f(x+h) = 2(x+h)^2 - (x+h) - 3$$

$$= 2\left(x^2 + 2hx + h^2\right) - x - h - 3$$

$$= 2x^2 + 4hx + 2h^2 - x - h - 3$$

$$\frac{f(x+h)-f(x)}{h}=\frac{2x^2+2h^2+4hx-x-h-3-\left(2x^2-x-3\right)}{h}$$

$$=\frac{2x^2+2h^2+4hx-x-h-3-2x^2+x+3}{h}$$

$$=\frac{2h^2+4hx-h}{h}$$

$$=\frac{2h^2}{h}+\frac{4hx}{h}-\frac{h}{h}$$

$$\underline{=2h+4x-1}$$

Exercise

For the given function $f(x)=2x^2-x-3$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)^2-(x+h)-3-2x^2+x+3}{h}$$

$$=\frac{2\left(x^2+2hx+h^2\right)-x-h-2x^2+x}{h}$$

$$=\frac{2x^2+4hx+2h^2-h-2x^2}{h}$$

$$=\frac{4hx+2h^2-h}{h}$$

$$\underline{=4x+2h-1}$$

Exercise

For the given function $f(x)=x^2-2x+5$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-2(x+h)+5-x^2+2x-5}{h}$$

$$=\frac{x^2+2hx+h^2-2x-2h-x^2+2x}{h}$$

$$=\frac{2hx+h^2-2h}{h}$$

$$\underline{=2x+h-2}$$

Exercise

For the given function $f(x)=3x^2-2x+5$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)^2-2(x+h)+5-3x^2+2x-5}{h}\\ &= \frac{3\left(x^2+2hx+h^2\right)-2x-2h-3x^2+2x}{h}\\ &= \frac{3x^2+6hx+3h^2-2h-3x^2}{h}\\ &= \frac{6hx+3h^2-2h}{h}\\ &= 6x+3h-2\end{aligned}$$

Exercise

For the given function $f(x)=-2x^2-3x+7$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{-2(x+h)^2-3(x+h)+7+2x^2+3x-7}{h}\\ &= \frac{-2\left(x^2+2hx+h^2\right)-3x-3h+2x^2+3x}{h}\\ &= \frac{-2x^2-4hx-2h^2-3h+2x^2}{h}\\ &= \frac{-4hx-2h^2-3h}{h}\\ &= -4x-2h-3\end{aligned}$$

Exercise

For the function *f* given by $f(x)=\sqrt{x-3}$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}$$

Exercise

Let $f(x)=4x-3$ and $g(x)=5x+7$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f+g)(x)=4x-3+5x+7$

$\qquad = 9x+4$

Domain: $\mathbb{R}$

b) $(f-g)(x)=4x-3-(5x+7)$

$\qquad = 4x-3-5x-7$

$\qquad = -x-10$

Domain: $\mathbb{R}$

c) $(fg)(x)=(4x-3)(5x+7)$

$\qquad = 20x^2+13x-21$

Domain: $\mathbb{R}$

d) $\left(\frac{f}{g}\right)(x)=\frac{4x-3}{5x+7}$

Domain: $x\neq-\frac{7}{5}$

Exercise

Let $f(x)=2x^2+3$ and $g(x)=3x-4$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f+g)(x)=2x^2+3+3x-4$

$\qquad = 2x^2+3x-1$

Domain: $\mathbb{R}$

b) $(f-g)(x)=2x^2+3-(3x-4)$

$\qquad = 2x^2+3-3x+4$

$$= 2x^2 - x + 7$$

Domain: $\mathbb{R}$

c) $(fg)(x) = \left(2x^2 + 3\right)(3x - 4)$

$$= 6x^2 + x - 12$$

Domain: $\mathbb{R}$

d) $\left(\frac{f}{g}\right)(x) = \frac{2x^2 + 3}{3x - 4}$

Domain: $x \neq -\frac{4}{3}$

Exercise

Let $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 + 3x - 2$. Find each of the following and give the domain

a) $(f + g)(x)$ *b)* $(f - g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f + g)(x) = x^2 - 2x - 3 + x^2 + 3x - 2$

$$= 2x^2 + x - 5$$

Domain: $\mathbb{R}$

b) $(f - g)(x) = x^2 - 2x - 3 - x^2 - 3x + 2$

$$= -5x - 1$$

Domain: $\mathbb{R}$

c) $(fg)(x) = \left(x^2 - 2x - 3\right)\left(x^2 + 3x - 2\right)$

$$= x^4 + 3x^3 - 2x^2 - 2x^3 - 6x^2 + 4x - 3x^2 - 9x + 6$$

$$= x^4 + x^3 - 11x^2 - 5x + 6$$

Domain: $\mathbb{R}$

d) $\left(\frac{f}{g}\right)(x) = \frac{x^2 - 2x - 3}{x^2 + 3x - 2}$

Domain: $x \neq \frac{-3 \pm \sqrt{17}}{2}$

Exercise

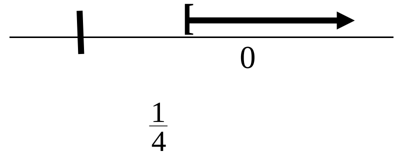
Let $f(x)=\sqrt{4x-1}$ and $g(x)=\frac{1}{x}$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\frac{f}{g}\right)(x)$

Solution

a) $(f+g)(x)$

$$(f+g)(x)=\sqrt{4x-1}+\frac{1}{x}$$



$4x-1\ge 0$ $\quad$ $x\ne 0$

$x\ge \frac{1}{4}$

Domain: $\left[\frac{1}{4},\infty\right)$

b) $(f-g)(x)$

$$(f-g)(x)=\sqrt{4x-1}-\frac{1}{x}$$

$4x-1\ge 0$ $\quad$ $x\ne 0$

$x\ge \frac{1}{4}$

Domain: $\left[\frac{1}{4},\infty\right)$

c) $(fg)(x)=\sqrt{4x-1}\left(\frac{1}{x}\right)$

$$=\frac{\sqrt{4x-1}}{x}$$

$4x-1\ge 0$ $\quad$ $x\ne 0$

$x\ge \frac{1}{4}$

Domain: $\left[\frac{1}{4},\infty\right)$

d) $\left(\frac{f}{g}\right)(x)=\dfrac{\sqrt{4x-1}}{\frac{1}{x}}$ $\qquad$ *Domain:* $x\ne 0$

$$=x\sqrt{4x-1}$$

$4x-1\ge 0$

$x\ge \frac{1}{4}$

Domain: $\left[\frac{1}{4},\ \infty\right)$

Exercise

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, *and* $(f/g)(x)$ and the domain of

$f(x)=\sqrt{3-2x}$, $g(x)=\sqrt{x+4}$

Solution

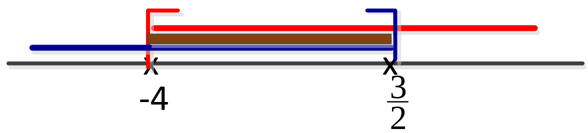
$$f(x)+g(x)=\sqrt{3-2x}+\sqrt{x+4}$$

$$3-2x\ge 0 \qquad x+4\ge 0$$

$$-2x\ge -3 \qquad x\ge -4$$

$$x\le \frac{3}{2}$$

Domain: $\left\{x \mid -4\le x\le \frac{3}{2}\right\}$



$$f(x)-g(x)=\sqrt{3-2x}-\sqrt{x+4}$$

$$3-2x\ge 0 \qquad x+4\ge 0$$

$$-2x\ge -3 \qquad x\ge -4$$

$$x\le \frac{3}{2}$$

Domain: $\left\{x \mid -4\le x\le \frac{3}{2}\right\}$

$$(f\cdot g)(x)=\left(\sqrt{3-2x}\right)\left(\sqrt{x+4}\right)=\sqrt{(3-2x)(x+4)}=\sqrt{-2x^2-5x+12}$$

$$3-2x\ge 0 \qquad x+4\ge 0$$

$$-2x\ge -3 \qquad x\ge -4$$

$$x\le \frac{3}{2}$$

Domain: $\left\{x \mid -4\le x\le \frac{3}{2}\right\}$

$$(f/g)(x)=\frac{\sqrt{3-2x}}{\sqrt{x+4}}\frac{\sqrt{x+4}}{\sqrt{x+4}}=\frac{\sqrt{-2x^2-5x+12}}{x+4}$$

$$3-2x\ge 0 \qquad x+4>0$$

$$-2x\ge -3 \qquad x>-4$$

$$x\le \frac{3}{2}$$

Domain: $\left\{x \mid -4< x\le \frac{3}{2}\right\}$

$$\boxed{\left(-4,\ \frac{3}{2}\right]}$$

Exercise

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, *and* $(f/g)(x)$ and the domain of

$$f(x)=\frac{2x}{x-4}, \quad g(x)=\frac{x}{x+5}$$

Solution

$$\begin{aligned}(f+g)(x) &= \frac{2x}{x-4}+\frac{x}{x+5} \\ &= \frac{2x(x+5)+x(x-4)}{(x-4)(x+5)} \\ &= \frac{2x^2+10x+x^2-4x}{(x-4)(x+5)} \\ &= \frac{3x^2+6x}{(x-4)(x+5)}\end{aligned}$$

$x-4\neq 0 \qquad x+5\neq 0$

$x\neq 4 \qquad x\neq -5$

Domain: $\{x \mid x\neq -5,4\}$ $\quad (-\infty,\ -5)\cup(-5,\ 4)\cup(4,\ \infty)$

$$\begin{aligned}(f-g)(x) &= \frac{2x}{x-4}-\frac{x}{x+5} \\ &= \frac{2x(x+5)-x(x-4)}{(x-4)(x+5)} \\ &= \frac{2x^2+10x-x^2+4x}{(x-4)(x+5)} \\ &= \frac{x^2+14x}{(x-4)(x+5)}\end{aligned}$$

$x\neq 4 \qquad x\neq -5$

Domain: $\{x \mid x\neq -5,4\}$

$$(f\bullet g)(x)=\frac{2x}{x-4}\,\frac{x}{x+5}=\frac{2x^2}{(x-4)(x+5)}$$

$x\neq 4 \qquad x\neq -5$

Domain: $\{x \mid x\neq -5,4\}$

$$(f/g)(x)=\frac{2x}{x-4}\div\frac{x}{x+5}==\frac{2x}{x-4}\,\frac{x+5}{x}=2\,\frac{x+5}{x-4}$$

$x\neq 4 \qquad x\neq -5$

Domain: $\{x \mid x\neq -5,4\}$

Exercise

Given that $f(x) = x+1$ and $g(x) = \sqrt{x+3}$

a) Find $(f+g)(x)$

b) Find the domain of $(f+g)(x)$

c) Find: $(f+g)(6)$

Solution

a) $(f+g)(x) = f(x) + g(x)$

$$= x + 1 + \sqrt{x+3}$$

b) $x+3 \ge 0 \to x \ge -3$

Domain = $[-3, \infty)$

c) $(f+g)(6) = 6+1+\sqrt{6+3} = 10$

Exercise

Given that $f(x) = x^2 - 4$ and $g(x) = x+2$

a) Find $(f+g)(x)$ and its domain

b) Find $(f/g)(x)$ and its domain

Solution

a) $(f+g)(x) = x^2 - 4 + x + 2$

$$= x^2 + x - 2$$

Domain = $\mathbb{R}$

b) $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2-4}{x+2}$

Domain: $x \ne 2$

Exercise

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find $(f+g)(1)$, $(f-g)(-3)$, $(fg)(5)$, and $\left(\frac{f}{g}\right)(0)$

Solution

a) $(f+g)(1) = f(1) + g(1)$

$$= 1^2 + 1 + 3(1) + 5$$

$$= 1+1+3+5$$

$$= 10$$

b) $(f-g)(-3) = f(-3) - g(-3)$

$$= (-3)^2 + 1 - (3(-3) + 5)$$

$$\underline{= 10}$$

c) $(fg)(5) = f(5) \cdot g(5)$

$$= \left(5^2 + 1\right) \cdot (3(5) + 5)$$

$$= (26) \cdot (20)$$

$$\underline{= 520}$$

d) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)}$

$$= \frac{0^2 + 1}{3(0) + 5}$$

$$\underline{= \frac{1}{5}}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x) = 2x^2 + 3x - 4$, $g(x) = 2x - 1$

Solution

$$f(g(x)) = f(2x - 1)$$

$$= 2(2x - 1)^2 + 3(2x - 1) - 4$$

$$= 2\left(4x^2 - 4x + 1\right) + 6x - 3 - 4$$

$$= 8x^2 - 8x + 2 + 6x - 7$$

$$\underline{= 8x^2 - 2x - 5}$$

$$g(f(x)) = g\left(2x^2 + 3x - 4\right)$$

$$= 2\left(2x^2 + 3x - 4\right) - 1$$

$$= 4x^2 + 6x - 8 - 1$$

$$\underline{= 4x^2 + 6x - 9}$$

$$f(g(-2)) = 8(-2)^2 - 2(-2) - 5$$

$$\underline{= 31}$$

$$g(f(3)) = 4(3)^2 + 6(3) - 9$$

$$\underline{= 45}$$

Exercise

Find $\left(f \circ g\right)(x)$, $\left(g \circ f\right)(x)$, $f\left(g(-2)\right)$ *and* $g\left(f(3)\right)$: $f(x) = x^3 + 2x^2$, $g(x) = 3x$

Solution

$$f\left(g(x)\right) = f\left(3x\right)$$
$$= (3x)^3 + 2(3x)^2$$
$$= 27x^3 + 18x^2$$

$$g\left(f(x)\right) = g\left(x^3 + 2x^2\right)$$
$$= 3\left(x^3 + 2x^2\right)$$
$$= 3x^3 + 6x^2$$

$$f\left(g(-2)\right) = 27(-2)^3 + 18(-2)^2$$
$$= -144$$

$$g\left(f(3)\right) = 3(3)^3 + 6(3)^2$$
$$= 135$$

Exercise

Find $\left(f \circ g\right)(x)$, $\left(g \circ f\right)(x)$, $f\left(g(-2)\right)$ *and* $g\left(f(3)\right)$: $f(x) = |x|$, $g(x) = -7$

Solution

$$f\left(g(x)\right) = f\left(-7\right)$$
$$= |-7|$$
$$= 7$$

$$g\left(f(x)\right) = g\left(|x|\right)$$
$$= -7$$

$$f\left(g(-2)\right) = 7$$

$$g\left(f(3)\right) = -7$$

Exercise

Given $f(x) = x - 3$ *and* $g(x) = x + 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(x+3)$ ***Domain***: $\mathbb{R}$

$= (x-3)+3$

$= x$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

b) $g(f(x)) = g(x-3)$ ***Domain***: $\mathbb{R}$

$= (x+3)-3$

$= x$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = \frac{2}{3}x$ *and* $g(x) = \frac{3}{2}x$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{3}{2}x\right)$ ***Domain***: $\mathbb{R}$

$= \frac{2}{3}\left(\frac{3}{2}x\right)$

$= x$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

b) $g(f(x)) = g\left(\frac{2}{3}x\right)$ ***Domain***: $\mathbb{R}$

$= \frac{3}{2}\left(\frac{2}{3}x\right)$

$= x$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = x - 1$ *and* $g(x) = 3x^2 - 2x - 1$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g(x)\right) = f\left(3x^2 - 2x - 1\right)$ ***Domain***: $\mathbb{R}$

$$= 3(x-1)^2 - 2(x-1) - 1$$

$$= 3\left(x^2 - 2x + 1\right) - 2x + 2 - 1$$

$$= 3x^2 - 6x + 3 - 2x + 1$$

$$= 3x^2 - 8x + 4$$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

b) $g\left(f(x)\right) = g(x-1)$ ***Domain***: $\mathbb{R}$

$$= 3x^2 - 2x - 1 - 1$$

$$= 3x^2 - 2x - 2$$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = 3x - 2$ *and* $g(x) = x^2 - 5$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g(x)\right) = f\left(x^2 - 5\right)$ ***Domain***: $\mathbb{R}$

$$= 3\left(x^2 - 5\right) - 2$$

$$= 3x^2 - 15 - 2$$

$$= 3x^2 - 17$$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

b) $g\left(f(x)\right) = g(3x-2)$ ***Domain***: $\mathbb{R}$

$$= (3x-2)^2 - 5$$

$= 9x^2 - 12x + 4 - 5$

$\boxed{= 9x^2 - 12x - 1}$ **Domain**: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

Exercise

Given $f(x) = x^2 - 2$ *and* $g(x) = 4x - 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(4x - 3)$ **Domain**: $\mathbb{R}$

$= (4x - 3)^2 - 2$

$= 16x^2 - 24x + 9 - 2$

$\boxed{= 16x^2 - 24x + 7}$ **Domain**: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

b) $g(f(x)) = g(x^2 - 2)$ **Domain**: $\mathbb{R}$

$= 4(x^2 - 2) - 3$

$= 4x^2 - 8 - 3$

$\boxed{= 4x^2 - 11}$ **Domain**: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

Exercise

Given $f(x) = 4x^2 - x + 10$ *and* $g(x) = 2x - 7$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(2x - 7)$ **Domain**: $\mathbb{R}$

$= 4(2x - 7)^2 - (2x - 7) + 10$

$= 4(4x^2 - 28x + 49) - 2x + 7 + 10$

$= 16x^2 - 112x + 196 - 2x + 17$

$$\boxed{= 16x^2 - 114x + 213}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

b) $g(f(x)) = g(4x^2 - x + 10)$ ***Domain***: $\mathbb{R}$

$$= 2(4x^2 - x + 10) - 7$$

$$= 8x^2 - 2x + 20 - 7$$

$$\boxed{= 8x^2 - 2x + 13}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

Exercise

Given $f(x) = \sqrt{x}$ *and* $g(x) = x + 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(x + 3)$ ***Domain***: $\mathbb{R}$

$$\boxed{= \sqrt{x + 3}}$$ ***Domain***: $x \ge -3$

Domain: $\boxed{x \ge -3}$

b) $g(f(x)) = g(\sqrt{x})$ ***Domain***: $x \ge 0$

$$\boxed{= \sqrt{x} + 3}$$ ***Domain***: $x \ge 0$

Domain: $\boxed{x \ge 0}$

Exercise

Given $f(x) = \sqrt{x}$ *and* $g(x) = 2 - 3x$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(2 - 3x)$ ***Domain***: $\mathbb{R}$

$$\boxed{= \sqrt{2 - 3x}}$$ ***Domain***: $x \le \frac{2}{3}$

Domain: $\boxed{x \le \frac{2}{3}}$

b) $g\left(f(x)\right) = g\left(\sqrt{x}\right)$ ***Domain***: $x \ge 0$

$\underline{= 2 - 3\sqrt{x}}$ ***Domain***: $x \ge 0$

Domain: $\underline{x \ge 0}$

Exercise

Given $f(x) = 3x + 2$ *and* $g(x) = \sqrt{x}$

a) Find $\left(f \circ g\right)(x)$ and the domain of $f \circ g$

b) Find $\left(g \circ f\right)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g(x)\right) = f\left(\sqrt{x}\right)$ ***Domain***: $x \ge 0$

$\underline{= 3\sqrt{x} + 2}$ ***Domain***: $x \ge 0$

Domain: $\underline{x \ge 0}$

b) $g\left(f(x)\right) = g\left(3x + 2\right)$ ***Domain***: $\mathbb{R}$

$\underline{= \sqrt{3x + 2}}$ ***Domain***: $x \ge -\frac{2}{3}$

Domain: $\underline{x \ge -\frac{2}{3}}$

Exercise

Given $f(x) = x^4$ *and* $g(x) = \sqrt[4]{x}$

a) Find $\left(f \circ g\right)(x)$ and the domain of $f \circ g$

b) Find $\left(g \circ f\right)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g(x)\right) = f\left(\sqrt[4]{x}\right)$ ***Domain***: $x \ge 0$

$= \left(\sqrt[4]{x}\right)^4$

$\underline{= x}$ ***Domain***: $\mathbb{R}$

Domain: $\underline{x \ge 0}$

b) $g\left(f(x)\right) = g\left(x^4\right)$ ***Domain***: $\mathbb{R}$

$= \sqrt[4]{x^4}$

$\underline{= x}$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = x^n$ *and* $g(x) = \sqrt[n]{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt[n]{x}\right)$ ***Domain***: $\begin{cases} \text{If } n \text{ is even} & x \ge 0 \\ \text{If } n \text{ is odd} & \mathbb{R} \end{cases}$

$= \left(\sqrt[n]{x}\right)^n$

$= x$ ***Domain***: $\mathbb{R}$

Domain: $\begin{cases} \text{If } n \text{ is even} & x \ge 0 \\ \text{If } n \text{ is odd} & \mathbb{R} \end{cases}$

b) $g(f(x)) = g\left(x^n\right)$ ***Domain***: $\mathbb{R}$

$= \sqrt[n]{x^n}$

$= x$ ***Domain***: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = x^2 - 3x$ *and* $g(x) = \sqrt{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt{x+2}\right)$ $x + 2 \ge 0 \Rightarrow x \ge -2$

$= \left(\sqrt{x+2}\right)^2 - 3\sqrt{x+2}$

$= x + 2 - 3\sqrt{x+2}$ $x + 2 \ge 0 \Rightarrow x \ge -2$

Domain: $\{x \mid x \ge -2\}$

b) $g(f(x)) = g\left(x^2 - 3x\right)$ $\mathbb{R}$

$$= \sqrt{x^2 - 3x + 2} \quad\Big|$$

$$x^2 - 3x + 2 \ge 0 \Rightarrow (x = 1,\ 2) \leftrightarrow x \le 1,\ x \ge 2$$

Domain: $\{x \mid x \le 1,\ x \ge 2\}$

Exercise

Given $f(x) = \sqrt{x-2}$ *and* $g(x) = \sqrt{x+5}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt{x+5}\right)$ $\qquad x + 5 \ge 0 \Rightarrow x \ge -5$

$$= \sqrt{\sqrt{x+5} - 2} \qquad \sqrt{x+5} - 2 \ge 0 \Rightarrow \sqrt{x+5} \ge 2$$

$$x + 5 \ge 4$$

$$x \ge -1$$

Domain: $\{x \mid x \ge -1\}$

b) $g(f(x)) = g\left(\sqrt{x-2}\right)$ $\qquad x - 2 \ge 0 \Rightarrow x \ge 2$

$$= \sqrt{\sqrt{x-2} + 5} \qquad \sqrt{x-2} + 5 \ge 0 \Rightarrow \sqrt{x-2} \ge -5 \quad \textit{Always true when } x \ge 2$$

Domain: $\{x \mid x \ge 2\}$

Exercise

Given $f(x) = x^2 + 2$ *and* $g(x) = \sqrt{3-x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt{3-x}\right)$ $\qquad$ ***Domain***: $x \le 3$

$$= \left(\sqrt{3-x}\right)^2 + 2$$

$$= 3 - x + 2$$

$$= 5 - x \quad\Big|$$

Domain: $\mathbb{R}$

Domain: $x \le 3$ $\Big|$

b) $g(f(x)) = g\left(x^2 + 2\right)$ $\qquad$ ***Domain***: $\mathbb{R}$

$$= \sqrt{3 - x^2 - 2}$$

$$= \sqrt{1 - x^2} \qquad \textbf{\textit{Domain}}: -1 \le x \le 1$$

Domain: $-1 \le x \le 1$

Exercise

Given $f(x) = x^5 - 2$ *and* $g(x) = \sqrt[5]{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt[5]{x+2}\right)$ ***Domain***: $\mathbb{R}$

$$= \left(\sqrt[5]{x+2}\right)^5 - 2$$

$$= x + 2 - 2$$

$$= x \qquad \textbf{\textit{Domain}}: \mathbb{R}$$

Domain: $\mathbb{R}$

b) $g(f(x)) = g\left(x^5 - 2\right)$ ***Domain***: $\mathbb{R}$

$$= \sqrt[5]{x^5 - 2 + 2}$$

$$= \sqrt[5]{x^5}$$

$$= x \qquad \textbf{\textit{Domain}}: \mathbb{R}$$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = 1 - x^2$ *and* $g(x) = \sqrt{x^2 - 25}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt{x^2 - 25}\right)$ ***Domain***: $x \le -5 \quad x \ge 5$

$$= 1 - \left(\sqrt{x^2 - 25}\right)^2$$

$$=1-\left(x^2-25\right)$$

$$=1-x^2+25$$

$$\boxed{=26-x^2}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{x \le -5 \quad x \ge 5}$

b) $g\left(f(x)\right)=g\left(1-x^2\right)$ ***Domain***: $\mathbb{R}$

$$=\sqrt{\left(1-x^2\right)^2-25}$$

$$=\sqrt{1-2x^2+x^4-25}$$

$$\boxed{=\sqrt{x^4-2x^2-24}}$$

$$x^2=\frac{2\pm\sqrt{4+96}}{2}$$

$$=\begin{cases}\frac{2-10}{2}=-4 \times \\ \frac{2+10}{2}=6\end{cases}$$

$$x^2=6 \quad \to \quad \boxed{x=\pm\sqrt{6}}$$ ***Domain***: $x \le -\sqrt{6} \quad x \ge \sqrt{6}$

Domain: $\boxed{x \le -\sqrt{6} \quad x \ge \sqrt{6}}$

Exercise

Given $f(x)=2x+3$ *and* $g(x)=\frac{x-3}{2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g(x)\right)=f\left(\frac{x-3}{2}\right)$ ***Domain***: $\mathbb{R}$

$$=2\left(\frac{x-3}{2}\right)+3$$

$$=x-3+3$$

$$\boxed{=x}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

b) $g\left(f(x)\right)=g(2x+3)$ ***Domain***: $\mathbb{R}$

$$=\frac{1}{2}(2x+3-3)$$

$\boxed{= x}$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

Exercise

Given $f(x) = 4x - 5$ *and* $g(x) = \dfrac{x+5}{4}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{x+5}{4}\right)$ ***Domain***: $\mathbb{R}$

$$= 4\left(\frac{x+5}{4}\right) - 5$$

$$= x + 5 - 5$$

$\boxed{= x}$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

b) $g(f(x)) = g(4x-5)$ ***Domain***: $\mathbb{R}$

$$= \frac{1}{4}(4x - 5 + 5)$$

$\boxed{= x}$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

Exercise

Given $f(x) = \dfrac{4}{1-5x}$ *and* $g(x) = \dfrac{1}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{1}{x}\right)$ ***Domain***: $x \neq 0$

$$= \frac{4}{1 - 5\frac{1}{x}}$$

$\boxed{= \dfrac{4x}{x-5}}$ ***Domain***: $x \neq 5$

Domain: $\boxed{x \neq 0,\ 5}$

b) $g(f(x)) = g\left(\dfrac{4}{1-5x}\right)$ ***Domain***: $x \neq \dfrac{1}{5}$

$$=\boxed{\dfrac{1-5x}{4}}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{x\neq\dfrac{1}{5}}$

Exercise

Given $f(x)=\dfrac{1}{x-2}$ *and* $g(x)=\dfrac{x+2}{x}$

a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

Solution

a) $f(g(x))=f\left(\dfrac{x+2}{x}\right)$ ***Domain***: $x\neq 0$

$$=\frac{1}{\frac{x+2}{x}-2}$$

$$=\frac{1}{\frac{x+2-2x}{x}}$$

$$=\boxed{\frac{x}{2-x}}$$ ***Domain***: $x\neq 2$

Domain: $\boxed{x\neq 0,\ 2}$ $(-\infty,\ 0)\cup(0,\ 2)\cup(2,\ \infty)$

b) $g(f(x))=g\left(\dfrac{1}{x-2}\right)$ ***Domain***: $x\neq 2$

$$=\frac{\frac{1}{x-2}+2}{\frac{1}{x-2}}$$

$$=\frac{\frac{1+2x-4}{x-2}}{\frac{1}{x-2}}$$

$$=\boxed{2x-3}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{x\neq 2}$

$$\boxed{(-\infty,\ 2)\cup(2,\ \infty)}$$

Exercise

Given $f(x) = \frac{3x+5}{2}$ *and* $g(x) = \frac{2x-5}{3}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{2x-5}{3}\right)$ ***Domain***: $\mathbb{R}$

$$= \frac{3\frac{2x-5}{3}+5}{2}$$

$$= \frac{2x-5+5}{2}$$

$$= \frac{2x}{2}$$

$$\underline{= x}$$

Domain: $\underline{\mathbb{R}}$

b) $g(f(x)) = g\left(\frac{3x+5}{2}\right)$ ***Domain***: $\mathbb{R}$

$$= \frac{2\frac{3x+5}{2}-5}{3}$$

$$= \frac{3x+5-5}{3}$$

$$= \frac{3x}{3}$$

$$\underline{= x}$$

Domain: $\underline{\mathbb{R}}$

Exercise

Given $f(x) = \frac{1}{1+x}$ *and* $g(x) = \frac{1-x}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{1-x}{x}\right)$ ***Domain***: $x \neq 0$

$$= \frac{1}{1+\frac{1-x}{x}}$$

$$= \frac{x}{x+1-x}$$

$\underline{= x}$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{x \neq 0}$

b) $g\left(f(x)\right) = g\left(\frac{1}{x+1}\right)$ ***Domain***: $x \neq -1$

$$= \frac{1 - \frac{1}{x+1}}{\frac{1}{x+1}}$$

$$= x + 1 - 1$$

$$\boxed{= x}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

Exercise

Given $f(x) = \frac{x-1}{x-2}$ *and* $g(x) = \frac{x-3}{x-4}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g(x)\right) = f\left(\frac{x-3}{x-4}\right)$ ***Domain***: $x \neq 4$

$$= \frac{\frac{x-3}{x-4} - 1}{\frac{x-3}{x-4} - 2}$$

$$= \frac{\frac{x-3-(x-4)}{x-4}}{\frac{x-3-2(x-4)}{x-4}}$$

$$= \frac{x-3+x+4}{x-3-2x+8}$$

$$\boxed{= \frac{2x+1}{-x+5}}$$ ***Domain***: $x \neq 5$

Domain: $\{x \mid x \neq 4,\ 5\}$

b) $g\left(f(x)\right) = g\left(\frac{x-1}{x-2}\right)$ ***Domain***: $x \neq 2$

$$= \frac{\frac{x-1}{x-2} - 3}{\frac{x-1}{x-2} - 4}$$

$$= \frac{x-1-3(x-2)}{x-1-4(x-2)}$$

$$= \frac{x-1-3x+6}{x-1-4x+8}$$

$$= \frac{-2x+5}{-3x+7}$$ **Domain**: $x \neq \frac{7}{3}$

Domain: $\left\{x \mid x \neq 2,\ \frac{7}{3}\right\}$

Exercise

Given $f(x) = \frac{6}{x-3}$ *and* $g(x) = \frac{1}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $(f \circ g)(x)$

$$f(g(x)) = f\left(\frac{1}{x}\right)$$ **Domain**: $x \neq 0$

$$= \frac{6}{\frac{1}{x}-3}$$

$$= \frac{6}{\frac{1-3x}{x}}$$

$$= \frac{6x}{1-3x}$$ **Domain**: $x \neq \frac{1}{3}$

Domain: $x \neq 0,\ \frac{1}{3}$ $\quad (-\infty, 0) \cup \left(0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$

b) $(g \circ f)(x)$

$$g(f(x)) = g\left(\frac{6}{x-3}\right)$$ **Domain**: $x \neq 3$

$$= \frac{1}{\frac{6}{x-3}}$$

$$= \frac{x-3}{6}$$ **Domain**: $(-\infty, \infty)$

Domain: $x \neq 3$ $\quad (-\infty, 3) \cup (3, \infty)$

Exercise

Given $f(x)=\frac{6}{x}$ *and* $g(x)=\frac{1}{2x+1}$

a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

Solution

a) $f\left(g(x)\right)=f\left(\frac{1}{2x+1}\right)$ ***Domain***: $x\neq -\frac{1}{2}$

$$=\frac{6}{\frac{1}{2x+1}}$$

$$\boxed{=12x+6}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{x\neq -\frac{1}{2}}$

b) $g\left(f(x)\right)=g\left(\frac{6}{x}\right)$ ***Domain***: $x\neq 0$

$$=\frac{1}{2\frac{6}{x}+1}$$

$$\boxed{=\frac{x}{12+x}}$$ ***Domain***: $x\neq -12$

Domain: $\boxed{x\neq -12,\ 0}$

Exercise

Given $f(x)=3x-7$ *and* $g(x)=\frac{x+7}{3}$

a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

Solution

a) $f\left(g(x)\right)=f\left(\frac{x+7}{3}\right)$ ***Domain***: $\mathbb{R}$

$$=3\frac{x+7}{3}-7$$

$$=x+7-7$$

$$\boxed{=x}$$ ***Domain***: $\mathbb{R}$

Domain: $\boxed{\mathbb{R}}$

b) $g\left(f(x)\right)=g(3x-7)$ ***Domain***: $\mathbb{R}$

$$=\frac{3x-7+7}{3}$$

$= x$ **Domain**: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = \dfrac{2x+3}{x-4}$ *and* $g(x) = \dfrac{4x+3}{x-2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{4x+3}{x-2}\right)$ **Domain**: $x \neq 2$

$$= \dfrac{2\dfrac{4x+3}{x-2}+3}{\dfrac{4x+3}{x-2}-4}$$

$$= \dfrac{8x+6+3x-6}{4x+3-4x+8}$$

$$= \dfrac{11x}{11}$$

$= x$ **Domain**: $\mathbb{R}$

Domain: $x \neq 2$

b) $g(f(x)) = g\left(\dfrac{2x+3}{x-4}\right)$ **Domain**: $x \neq 4$

$$= \dfrac{4\dfrac{2x+3}{x-4}+3}{\dfrac{2x+3}{x-4}-2}$$

$$= \dfrac{8x+12+3x-4}{2x+3-2x+8}$$

$$= \dfrac{11x}{11}$$

$= x$ **Domain**: $\mathbb{R}$

Domain: $x \neq 4$

Exercise

Given $f(x) = \dfrac{2x+3}{x+4}$ *and* $g(x) = \dfrac{-4x+3}{x-2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g(x)\right) = f\left(\dfrac{-4x+3}{x-2}\right)$ **Domain**: $x \neq 2$

$$= \dfrac{2\dfrac{-4x+3}{x-2}+3}{\dfrac{4x+3}{x-2}+4}$$

$$= \dfrac{-8x+6+3x-6}{4x+3+4x-8}$$

$$= \dfrac{-5x}{-5}$$

$= x$ **Domain**: $\mathbb{R}$

Domain: $x \neq 2$

b) $g\left(f(x)\right) = g\left(\dfrac{2x+3}{x+4}\right)$ **Domain**: $x \neq -4$

$$= \dfrac{-4\dfrac{2x+3}{x+4}+3}{\dfrac{2x+3}{x+4}-2}$$

$$= \dfrac{-8x-12+3x+12}{2x+3-2x-8}$$

$$= \dfrac{-5x}{-5}$$

$= x$ **Domain**: $\mathbb{R}$

Domain: $x \neq -4$

Exercise

Given $f(x) = x+1$ *and* $g(x) = x^3 - 5x^2 + 3x + 7$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g(x)\right) = f\left(x^3 - 5x^2 + 3x + 7\right)$ **Domain**: $\mathbb{R}$

$= x^3 - 5x^2 + 3x + 7 + 1$

$= x^3 - 5x^2 + 3x + 8$ **Domain**: $\mathbb{R}$

Domain: $\mathbb{R}$

b) $g\left(f(x)\right) = g(x+1)$ **Domain**: $\mathbb{R}$

$= (x+1)^3 - 5(x+1)^2 + 3(x+1) + 7$

$= x^3 + 3x^2 + 3x + 1 - 5\left(x^2 + 2x + 1\right) + 3x + 3 + 7$

$= x^3 + 3x^2 + 6x + 11 - 5x^2 - 10x - 5$

$= x^3 - 2x^2 - 4x + 6$ **Domain**: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = x - 1$ *and* $g(x) = x^3 + 2x^2 - 3x - 9$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(x^3 + 2x^2 - 3x - 9\right)$ **Domain**: $\mathbb{R}$

$= x^3 + 2x^2 - 3x - 9 - 1$

$= x^3 + 2x^2 - 3x - 10$ **Domain**: $\mathbb{R}$

Domain: $\mathbb{R}$

b) $g(f(x)) = g(x - 1)$ **Domain**: $\mathbb{R}$

$= (x-1)^3 + 2(x-1)^2 - (x-1) - 9$

$= x^3 - 3x^2 + 3x - 1 + 2\left(x^2 - 2x + 1\right) - 3x + 3 - 9$

$= x^3 - 3x^2 - 7 + 2x^2 - 4x + 2$

$= x^3 - x^2 - 4x - 5$ **Domain**: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = \sqrt{x}$ and $g(x) = x + 3$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain.

Solution

$(f \circ g)(x) = f(g(x))$

$= f(x + 3)$ **Domain**: $(-\infty, \infty)$

$= \sqrt{x+3}$

$x + 3 \ge 0 \quad \Rightarrow \quad x \ge -3$

Domain: $x \ge -3$

$(g \circ f)(x) = g(f(x))$

$= g\left(\sqrt{x}\right)$ ***Domain***: $x \ge 0$

$= \sqrt{x} + 3$

Domain: $\boxed{x \ge 0}$

Exercise

Given that $f(x) = \sqrt{x}$ and $g(x) = 2 - 3x$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain.

Solution

$(f \circ g)(x) = f(g(x))$

$= f(2 - 3x)$ ***Domain***: $(-\infty, \infty)$

$\boxed{= \sqrt{2 - 3x}}$

$2 - 3x \ge 0 \to -3x \ge -2 \Rightarrow \boxed{x \le \frac{2}{3}}$

Domain: $\left(-\infty, \ \frac{2}{3}\right]$

$g(f(x)) = g\left(\sqrt{x}\right)$ ***Domain***: $x \ge 0$

$= 2 - 3\sqrt{x}$

$x \ge 0$

Domain: $[0, \infty)$

Exercise

Given that $f(x) = \dfrac{1}{x - 2}$ and $g(x) = \dfrac{x + 2}{x}$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain.

Solution

$f(g(x)) = f\left(\frac{x+2}{x}\right)$ ***Domain***: $\boxed{x \ne 0}$

$$= \frac{1}{\frac{x + 2}{x} - 2}$$

$$= \frac{1}{\frac{x + 2 - 2x}{x}}$$

$= \dfrac{x}{2 - x}$ ***Domain***: $\boxed{x \ne 2}$

Domain: $\boxed{x \ne 0, \ 2}$

$$g\left(f(x)\right) = g\left(\frac{1}{x-2}\right) \qquad \textit{Domain}: \boxed{x \neq 2}$$

$$= \frac{\frac{1}{x-2}+2}{\frac{1}{x-2}} \quad (-\infty,0) \cup (0,2) \cup (2,\infty)$$

$$= \frac{\frac{1+2x-4}{x-2}}{\frac{1}{x-2}}$$

$$= 2x-3 \qquad \textit{Domain}: \boxed{\mathbb{R}}$$

Domain: $x \neq 2$

Exercise

Given that $f(x) = 2x-5$ and $g(x) = x^2 - 3x + 8$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain then find $(f \circ g)(7)$

Solution

$$f\left(g(x)\right) = f\left(x^2 - 3x + 8\right) \qquad \textit{Domain: } (-\infty, \infty)$$

$$= 2(\text{------------}) - 5$$

$$= 2\left(2x^2 - 3x + 8\right) - 5$$

$$= 2x^2 - 6x + 16 - 5$$

$$= 2x^2 - 6x + 11 \qquad \textit{Domain: } (-\infty, \infty)$$

Domain: $\mathbb{R}$

$$g\left(f(x)\right) = g(2x-5) \qquad \textit{Domain: } (-\infty, \infty)$$

$$= (---)^2 - 3(---) + 8$$

$$= (2x-5)^2 - 3(2x-5) + 8$$

$$= 4x^2 - 20x + 25 - 6x + 15 + 8$$

$$= 4x^2 - 26x + 48 \qquad \textit{Domain: } (-\infty, \infty)$$

Domain: $\mathbb{R}$

$$f\left(g(7)\right) = 2(7)^2 - 6(7) + 11$$

$$= 67$$

Exercise

Given that $f(x)=\sqrt{x}$ and $g(x)=x-1$, find

a) $(f\circ g)(x)=f(g(x))$ *b)* $(g\circ f)(x)=g(f(x))$ *c)* $(f\circ g)(2)=f(g(2))$

Solution

a) $(f\circ g)(x)=f(g(x))$

$$=f(x-1)$$

$$\boxed{=\sqrt{x-1}}$$

b) $(g\circ f)(x)=g(f(x))$

$$=g(\sqrt{x})$$

$$\boxed{=\sqrt{x}-1}$$

c) $(f\circ g)(2)=f(g(2))$ $=\sqrt{x-1}$

$$=\sqrt{2-1}$$

$$\boxed{=1}$$

Exercise

Given that $f(x)=\dfrac{x}{x+5}$ and $g(x)=\dfrac{6}{x}$, find

a) $(f\circ g)(x)=f(g(x))$ *b)* $(g\circ f)(x)=g(f(x))$ *c)* $(f\circ g)(2)=f(g(2))$

Solution

a) $(f\circ g)(x)=f(g(x))$

$$=f\left(\frac{6}{x}\right)$$

$$=\frac{\frac{6}{x}}{\frac{6}{x}+5}$$

$$=\frac{\frac{6}{x}}{\frac{6+5x}{x}}$$

$$\boxed{=\frac{6}{6+5x}}$$

b) $(g\circ f)(x)=g(f(x))$

$$=g\left(\frac{x}{x+5}\right)$$

$$=\frac{6}{\frac{x}{x+5}}$$

$$\boxed{= \frac{6(x+5)}{x}}$$

c) $(f \circ g)(2) = f(g(2))$

$$= \frac{6}{6+5(2)} \quad = \frac{6}{16}$$

$$\boxed{= \frac{3}{8}}$$

Exercise

Determine whether f is even, odd, or neither: $f(x) = 3x^4 + 2x^2 - 5$

Solution

$$\begin{aligned} f(-x) &= 3(-x)^4 + 2(-x)^2 - 5 \\ &= 3x^4 + 2x^2 - 5 \\ &= f(x) \end{aligned}$$

$\therefore$ The function is ***even***.

Exercise

Determine whether f is even, odd, or neither: $f(x) = 8x^3 - 3x^2$

Solution

$$\begin{aligned} f(-x) &= 8(-x)^3 - 3(-x)^2 \\ &= -8x^3 - 3x^2 \end{aligned}$$

$\therefore$ The function is ***neither***.

Exercise

Determine whether f is even, odd, or neither: $f(x) = \sqrt{x^2 + 4}$

Solution

$$\begin{aligned} f(-x) &= \sqrt{(-x)^2 + 4} \\ &= \sqrt{x^2 + 4} \\ &= f(x) \end{aligned}$$

$\therefore$ The function is ***even***.

Exercise

Determine whether f is even, odd, or neither: $f(x)=3x^2-5x+1$

Solution

$$f(-x)=3(-x)^2-5(-x)+1$$
$$=3x^2+5x+1$$

$\therefore$ The function is ***neither***.

Exercise

Determine whether f is even, odd, or neither: $f(x)=\sqrt[3]{x^3-x}$

Solution

$$f(-x)=\sqrt[3]{(-x)^3-(-x)}$$
$$=\sqrt[3]{-x^3+x}$$
$$=\sqrt[3]{-\left(x^3-x\right)}$$
$$=-\sqrt[3]{x^3-x}$$
$$=-f(x)$$

$\therefore$ The function is ***odd***.

Exercise

Determine whether f is even, odd, or neither: $f(x)=|x|-3$

Solution

$$f(-x)=|-x|-3$$
$$=|(-)x|-3$$
$$=|-1||x|-3$$
$$=|x|-3$$
$$=f(x)$$

$\therefore$ The function is ***even***.

Exercise

Determine whether f is even, odd, or neither: $f(x)=x^3-\frac{1}{x}$

Solution

$$f(-x) = (-x)^3 - \frac{1}{(-x)}$$
$$= -x^3 + \frac{1}{x}$$
$$= -\left(x^3 - \frac{1}{x}\right)$$
$$= -f(x)$$

$\therefore$ The function is ***odd***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = -x^3 + 2x$

Solution

$$f(-x) = -(-x)^3 + 2(-x)$$
$$= x^3 - 2x$$
$$= -f(x)$$

$\therefore$ The function is ***odd***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^5 - 2x^3$

Solution

$$f(-x) = (-x)^5 - 2(-x)^3$$
$$= -x^5 + 2x^3$$
$$= -f(x)$$

$\therefore$ The function is ***odd***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = .5x^4 - 2x^2 + 6$

Solution

$$f(-x) = .5(-x)^4 - 2(-x)^2 + 6$$
$$= .5x^4 - 2x^2 + 6$$
$$= f(x)$$

$\therefore$ The function is ***even***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = .75x^2 + |x| + 4$

Solution

$$f(-x) = .75(-x)^2 + |-x| + 4$$
$$= .75x^2 + |x| + 4$$
$$= f(x) \qquad \therefore \text{ The function is } \textbf{\textit{even}}.$$

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^3 - x + 9$

Solution

$$f(-x) = (-x)^3 - (-x) + 9$$
$$= -x^3 + x + 9$$

$\therefore$ The function is ***neither***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^4 - 5x + 8$

Solution

$$f(-x) = (-x)^4 - 5(-x) + 8$$
$$= x^4 + 5x + 8$$

$\therefore$ The function is ***neither***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^3 + x$

Solution

$$f(-x) = (-x)^3 + (-x)$$
$$= -x^3 - x$$
$$= -f(x)$$

$\therefore$ The function is ***odd***.

Exercise

Decide whether each function is even, odd, or neither $g(x) = x^2 - x$

Solution

$$g(-x) = (-x)^2 + (-x)$$
$$= x^2 - x$$

$\therefore$ The function is ***neither***.

Exercise

Decide whether each function is even, odd, or neither $h(x) = 2x^2 + x^4$

Solution

$$h(-x) = 2(-x)^2 + (-x)^4$$
$$= 2x^2 + x^4$$
$$= h(x)$$

$\therefore$ The function is ***even***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = 2x^2 + x^4 + 1$

Solution

$$f(-x) = 2(-x)^2 + (-x)^4 + 1$$
$$= 2x^2 + x^4 + 1$$
$$= f(x)$$

$\therefore$ The function is ***even***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = \frac{1}{5}x^6 - 3x^2$

Solution

$$f(-x) = \frac{1}{5}(-x)^6 - 3(-x)^2$$
$$= \frac{1}{5}x^6 - 3x^2$$
$$= f(x)$$

$\therefore$ The function is ***even***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x\sqrt{1 - x^2}$

Solution

$$f(-x) = -x\sqrt{1-(-x)^2}$$
$$= -x\sqrt{1-x^2}$$
$$= -f(x)$$

$\therefore$ The function is ***odd***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^2\sqrt{1-x^2}$

Solution

$$f(-x) = (-x)^2\sqrt{1-(-x)^2}$$
$$= x^2\sqrt{1-x^2}$$
$$= f(x)$$

$\therefore$ The function is ***even***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = 5x^7 - 6x^3 - 2x$

Solution

$$f(-x) = 5(-x)^7 - 6(-x)^3 - 2(-x)$$
$$= -5x^7 + 6x^3 + 2x$$
$$= -(5x^7 - 6x^3 - 2x)$$
$$= -f(x)$$

$\therefore$ The function is ***odd***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = 5x^6 - 3x^2 - 7$

Solution

$$f(-x) = 5(-x)^6 - 3(-x)^2 - 7$$
$$= 5x^6 - 3x^2 - 7$$
$$= f(x)$$

$\therefore$ The function is ***even***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^2 + 6$

Solution

$$\begin{aligned} f(-x) &= (-x)^2 + 6 \\ &= x^2 + 6 \\ &= f(x) \end{aligned}$$

$\therefore$ The function is ***even***.

Exercise

Decide whether each function is even, odd, or neither $f(x) = 7x^3 - x$

Solution

$$\begin{aligned} f(-x) &= 7(-x)^3 - (-x) \\ &= -7x^3 + x \\ &= -(7x^3 - x) \\ &= -f(x) \end{aligned}$$

$\therefore$ The function is ***odd***.

Exercise

Decide whether each function is even, odd, or neither $h(x) = x^5 + 1$

Solution

$$\begin{aligned} h(-x) &= (-x)^5 + 1 \\ &= -x^5 + 1 \quad \begin{cases} \neq x^5 + 1 \\ \neq -(x^5 + 1) \end{cases} \end{aligned}$$

$\therefore$ The function is ***neither***.

Exercise

$$f(x) = \begin{cases} 2 + x & \text{if } \ x < -4 \\ -x & \text{if } \ -4 \le x \le 2 \\ 3x & \text{if } \ x > 2 \end{cases}$$

Find: $f(-5),\ f(-1),\ f(0),\ and\ f(3)$

Solution

$$f(-5) = 2 - 5 = -3$$

$f(-1) = -(-1) = 1$

$f(0) = -0 = 0$

$f(3) = 3(3) = 9$

Exercise

$$f(x) = \begin{cases} -2x & \text{if} \quad x < -3 \\ 3x - 1 & \text{if} \quad -3 \le x \le 2 \\ -4x & \text{if} \quad x > 2 \end{cases} \qquad \text{Find: } f(-5),\ f(-1),\ f(0),\ and\ f(3)$$

Solution

$f(-5) = -2(-5) = 10$

$f(-1) = 3(-1) - 1 = -4$

$f(0) = 3(0) - 1 = -1$

$f(3) = -4(3) = -12$

Exercise

$$f(x) = \begin{cases} x^3 + 3 & \text{if} \quad -2 \le x \le 0 \\ x + 3 & \text{if} \quad 0 < x < 1 \\ 4 + x - x^2 & \text{if} \quad 1 \le x \le 3 \end{cases} \qquad \text{Find: } f(-5),\ f(-1),\ f(0),\ and\ f(3)$$

Solution

$f(-5) =$ *doesn't exist*

$f(-1) = (-1)^3 + 3 = 2$

$f(0) = (0)^3 + 3 = 3$

$f(3) = 4 + (3) - (3)^2 = -2$

Exercise

$$h(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if} \quad x \ne 3 \\ 6 & \text{if} \quad x = 3 \end{cases} \qquad \text{Find: } h(5),\ h(0),\ and\ h(3)$$

Solution

$h(5) = \dfrac{5^2 - 9}{5 - 3} = 8$

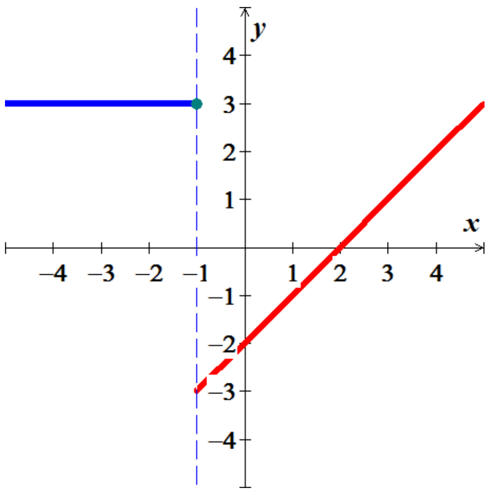
$$h(0) = \frac{0^2 - 9}{0 - 3} = 3$$

$$h(3) = 6$$

Exercise

Graph the piecewise function defined by $f(x) = \begin{cases} 3 & if \quad x \le -1 \\ x - 2 & if \quad x > -1 \end{cases}$

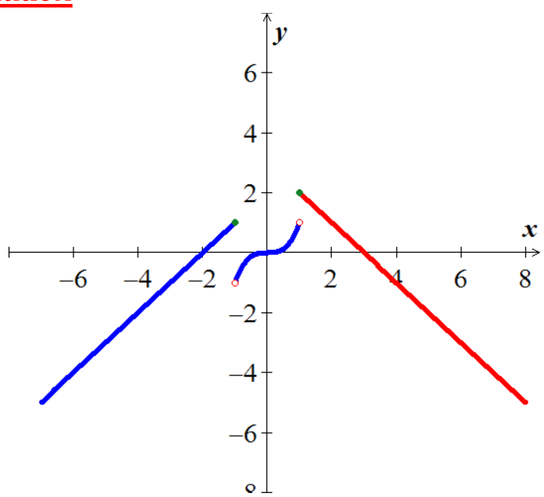
Solution



Exercise

Sketch the graph $f(x) = \begin{cases} x + 2 & if \quad x \le -1 \\ x^3 & if \quad -1 < x < 1 \\ -x + 3 & if \quad x \ge 1 \end{cases}$

Solution



Exercise

Sketch the graph $f(x)=\begin{cases} x-3 & if \quad x\le -2 \\ -x^2 & if \quad -2<x<1 \\ -x+4 & if \quad x\ge 1 \end{cases}$

Solution