

$$\int 4y^{-3} dy = -2y^{-2} + C$$

$$\int \sqrt{t} xy dt = \sqrt{t} xy + C$$

$$\int \frac{x^4 - 2\sqrt{x} + 2}{x^2} dx = \int (x^2 - 2x^{-3/2} + \frac{2}{x^2}) dx$$

$$= \frac{1}{3} x^3 + \frac{4}{\sqrt{x}} - \frac{2}{x} + C$$

$(\frac{1}{x})' = -\frac{1}{x^2}$

$$\int \sec 2x \tan 2x dx = \frac{1}{2} \int \sec 2x \tan 2x d(2x) \left\{ d(2x) = 2 dx \right.$$

$$= \frac{1}{2} \sec 2x + C$$

$$\int (2 \sin \theta - 5e^\theta) d\theta = -2 \cos \theta - 5e^\theta + C$$

$$\int (\frac{3}{x} + \sec^2 x) dx = 3 \ln|x| + \tan x + C$$

$$\int (a^2 - b^2) e^{(a-b)x} = \frac{a^2 - b^2}{a-b} e^{(a-b)x} + C$$

$$= \frac{(a-b)(a+b)}{a-b} e^{(a-b)x} + C$$

$$= (a+b) e^{(a-b)x} + C$$

$$\begin{aligned}\int_0^2 x(x-2) dx &= \int_0^2 (x^2 - 2x) dx \\ &= \left[ \frac{1}{3}x^3 - x^2 \right]_0^2 \\ &= \frac{8}{3} - 4 \\ &= \underline{\underline{-\frac{4}{3}}}\end{aligned}$$

$$\begin{aligned}\int_0^{\pi/3} 4 \sec u \tan u du &= 4 \sec u \Big|_0^{\pi/3} \\ &= 4(2 - 1) \\ &= \underline{\underline{4}}\end{aligned} \quad \left( \frac{f}{2} - \frac{g}{2} \right)$$

$$\begin{aligned}\int_{-\pi/3}^{-\pi/4} \left( 4 \sec^2 t + \frac{\pi}{t^2} \right) dt &= 4 \tan t - \frac{\pi}{t} \Big|_{-\pi/3}^{-\pi/4} \\ &= -4 + 4 - (-4\sqrt{3} + 3) \\ &= \underline{\underline{4\sqrt{3} - 3}}\end{aligned}$$

$$\begin{aligned}\int_{-2}^{-1} \left( 3e^{3x} + \frac{2}{x} \right) dx &= e^{3x} + 2 \ln|x| \Big|_{-2}^{-1} \\ &= e^{-3} + 2 \ln 1 - (e^{-6} + 2 \ln 2) \\ &= \underline{\underline{e^{-3} - e^{-6} - 2 \ln 2}}\end{aligned}$$

$$2x^2 - 4x + 2 = 0$$

$$0 \leq x \leq 2 \quad \checkmark?$$

$$\begin{aligned}I &= 2 \int_0^2 (x^2 - 2x + 1) dx \\ &= 2 \left( \frac{1}{3}x^3 - x^2 + x \right) \Big|_0^2 \\ &= 2 \left( \frac{8}{3} - 4 + 2 \right) \\ &= \underline{\underline{\frac{4}{3} \text{ unit}^3}}\end{aligned}$$

$$6) f(x) = x^2 + 4x + 3 = 0 \quad -3 \leq x \leq 0$$

$$x = -1, -3$$

$$A = - \int_{-3}^{-1} (x^2 + 4x + 3) dx + \int_{-1}^0 (x^2 + 4x + 3) dx$$

$$= - \left( \frac{1}{3} x^3 + 2x^2 + 3x \right) \Big|_{-3}^{-1} + \left( \frac{1}{3} x^3 + 2x^2 + 3x \right) \Big|_{-1}^0$$

$$= - \left( -\frac{1}{3} + 2 - 3 - (-9 + 18 - 9) \right) - \left( -\frac{1}{3} + 2 - 3 \right)$$

$$= \frac{4}{3} + \frac{4}{3}$$

$$= \frac{8}{3} \text{ unit}^2$$

A  $x$ -axis  $y = x^4 - 16 = 0$   
 $x^4 = 2^4$

$$x = \pm 2$$



$$A = - \int_{-2}^2 (x^4 - 16) dx$$

$$= -2 \left( \frac{1}{5} x^5 - 16x \right) \Big|_0^2$$

$$= -2 \left( \frac{32}{5} - 32 \right)$$

$$= -64 \left( \frac{1}{5} - 1 \right)$$

$$= \frac{256}{5} \text{ unit}^2$$

$$\int_{-a}^a \text{even}$$

$$= 2 \int_0^a \text{even}$$

if for odd  $\int_{-a}^a \text{odd} = 0$

$$\begin{aligned}
 \int_0^1 (2t+3)^3 dt &= \frac{1}{2} \int_0^1 (2t+3)^2 d(2t+3) & d(2t+3) &= 2dt \\
 &= \frac{1}{6} (2t+3)^3 \Big|_0^1 \\
 &= \frac{1}{6} (125 - 27) \\
 &= \frac{49}{3}
 \end{aligned}$$

$$\begin{aligned}
 \int_{-\pi}^{\pi} \frac{\cos z}{\sqrt{4+3\sin z}} dz & & d(4+3\sin z) &= 3 \cos z dz \\
 &= \frac{1}{3} \int_{-\pi}^{\pi} (4+3\sin z)^{-1/2} dz \\
 &= \frac{2}{3} (4+3\sin z)^{1/2} \Big|_{-\pi}^{\pi} \\
 &= \frac{2}{3} (2 - 2) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \int_0^1 \sqrt{t^5+2t} (5t^4+2) dt & & d(t^5+2t) &= (5t^4+2) dt \\
 &= \int_0^1 (t^5+2t)^{1/2} d(t^5+2t) \\
 &= \frac{2}{3} (t^5+2t)^{3/2} \Big|_0^1 \\
 &= \frac{2}{3} (3^{3/2} - 0) \\
 &= 2\sqrt{3}
 \end{aligned}$$

$$\int_0^{\pi/2} e^{\sin x} \cos x dx$$

$$d(\sin x) = \cos x dx$$

$$\begin{aligned} &= \int_0^{\pi/2} e^{\sin x} d(\sin x) \\ &= e^{\sin x} \Big|_0^{\pi/2} \\ &= e - 1 \end{aligned}$$

$$\int_0^{\pi/2} \tan \frac{x}{2} dx = 2 \int_0^{\pi/2} \tan \frac{x}{2} d\left(\frac{x}{2}\right)$$

$$d\left(\frac{x}{2}\right) = \frac{1}{2} dx$$

$$\begin{aligned} &= 2 \ln |\sec \frac{x}{2}| \Big|_0^{\pi/2} \\ &= 2 (\ln \sqrt{2} - \ln 1) \\ &= 2 \left(\frac{1}{2} \ln 2\right) \\ &= \ln 2 \end{aligned}$$

$$- \ln |\cos x|$$

$$\ln 2^{1/2} = \frac{1}{2} \ln 2$$

$$\begin{aligned} \int_1^{e^x} \frac{1}{t} dt &= \ln(t) \Big|_1^{e^x} \\ &= \ln e^x - \ln 1 \\ &= x \end{aligned}$$

$$\int_0^{\sqrt{\ln 10}} 2x e^{x^2} \cos(e^{x^2}) dx$$

$$d(e^{x^2}) = 2x e^{x^2} dx$$

$$\begin{aligned} &= \int_0^{\sqrt{\ln 10}} \cos(e^{x^2}) d(e^{x^2}) \\ &= \sin(e^{x^2}) \Big|_0^{\sqrt{\ln 10}} \\ &= \sin e^{\ln 10} - \sin e^0 \\ &= \sin 10 - 1 \end{aligned}$$

$$e^{\ln x} = x$$

$$\begin{aligned}
 \int_0^{\ln 2} \frac{e^x}{1+(e^x)^2} dx & \quad d(e^x) = e^x dx \\
 &= \int_0^{\ln 2} \frac{d(e^x)}{1+(e^x)^2} \\
 &= \tan^{-1} e^x \Big|_0^{\ln 2} \\
 &= \tan^{-1} 2 - \tan^{-1} 1 \\
 &= \tan^{-1} 2 - \frac{\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 \int_0^2 e^{4x+8} dx &= \frac{1}{4} \int_0^2 e^{4x+8} d(4x+8) \quad d(4x+8) = 4 dx \\
 &= \frac{1}{4} e^{4x+8} \Big|_0^2 \\
 &= \frac{1}{4} (e^{16} - e^8)
 \end{aligned}$$

$$\begin{aligned}
 \int_0^{\ln 4} \frac{e^x}{3+2e^x} dx &= \int_0^{\ln 4} \frac{d(3+2e^x)}{3+2e^x} \quad d(3+2e^x) = 2e^x dx \\
 &= \ln(3+2e^x) \Big|_0^{\ln 4} \\
 &= \ln 11 - \ln 5 = \ln \frac{11}{5}
 \end{aligned}$$



$$\int_{-1}^1 (x-1) (x^2-2x)^2 dx$$

$$d(x^2-2x) = (2x-2)dx \\ = 2(x-1)dx$$

$$= \frac{1}{2} \int_{-1}^1 (x^2-2x)^2 d(x^2-2x)$$

$$= \frac{1}{16} (x^2-2x)^8 \Big|_{-1}^1$$

$$= \frac{1}{16} (1-3^8)$$

$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^2 x} dx = \int_{\pi/4}^{\pi/2} \frac{d(\sin x)}{\sin^2 x}$$

$$d(\sin x) = \cos x dx$$

$$= -\frac{1}{\sin x} \Big|_{\pi/4}^{\pi/2}$$

$$\frac{1}{\sin} \quad \frac{1}{2}$$

$$= -(1 - \sqrt{2})$$

$$= \sqrt{2} - 1$$

$$\int_{-1}^2 x^2 e^{x^3+1} dx = \frac{1}{3} \int_{-1}^2 e^{x^3+1} d(x^3+1)$$

$$d(x^3+1) = 3x^2 dx$$

$$= \frac{1}{3} e^{x^3+1} \Big|_{-1}^2$$

$$= \frac{1}{3} (e^9 - 1)$$

$$\int_1^{e^2} \frac{\ln x}{x} dx = \int_1^{e^2} (\ln x) d(\ln x)$$

$$d(\ln x) = \frac{1}{x} dx$$

$$= \frac{1}{2} (\ln x)^2 \Big|_1^{e^2}$$

$$= \frac{1}{2} (4 - 0)$$

$$= 2$$

$$\begin{aligned}
 \int_0^{\pi/4} \frac{\sin \theta}{\cos^3 \theta} d\theta &= - \int_0^{\pi/4} \cos^{-3} \theta d(\cos \theta) \\
 &= \frac{1}{2} \cos^{-2} \theta \Big|_0^{\pi/4} \\
 &= \frac{1}{2} \frac{1}{\cos^2 \theta} \Big|_0^{\pi/4} \\
 &= \frac{1}{2} (2 - 1) \\
 &= \frac{1}{2}
 \end{aligned}$$

$$d(\cos \theta) = -\sin \theta d\theta$$

$$\int_0^{\pi/4} e^{\sin^2 x} \sin 2x dx$$

$$\begin{aligned}
 d(\sin^2 x) &= 2 \sin x \cos x dx \\
 &= \sin 2x dx
 \end{aligned}$$

$$= \int_0^{\pi/4} e^{\sin^2 x} d(\sin^2 x)$$

$$= e^{\sin^2 x} \Big|_0^{\pi/4}$$

$$= e^{1/2} - 1$$

or

$$\sqrt{e} - 1$$

$$\begin{aligned}
 \int_{-1}^2 x^2 e^{x^3+1} dx &= \frac{1}{3} \int_{-1}^2 e^{x^3+1} d(x^3+1) \\
 &= \frac{1}{3} e^{x^3+1} \Big|_{-1}^2 \\
 &= \frac{1}{3} (e^9 - e)
 \end{aligned}$$

$$d(x^3+1) = 3x^2 dx$$