

Solution

Section 2.3 – Composition Functions

Exercise

Given that $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 8$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain then find $(f \circ g)(7)$

Solution

$$\begin{aligned} f(g(x)) &= f(x^2 - 3x + 8) & \text{Domain: } (-\infty, \infty) \\ &= 2(\text{-----}) - 5 \\ &= 2(2x^2 - 3x + 8) - 5 \\ &= 2x^2 - 6x + 16 - 5 \\ &= 2x^2 - 6x + 11 & \text{Domain: } (-\infty, \infty) \end{aligned}$$

Domain: \mathbb{R}

$$\begin{aligned} g(f(x)) &= g(2x - 5) & \text{Domain: } (-\infty, \infty) \\ &= (\text{---})^2 - 3(\text{---}) + 8 \\ &= (2x - 5)^2 - 3(2x - 5) + 8 \\ &= 4x^2 - 20x + 25 - 6x + 15 + 8 \\ &= 4x^2 - 26x + 48 & \text{Domain: } (-\infty, \infty) \end{aligned}$$

Domain: \mathbb{R}

$$f(g(7)) = 2(7)^2 - 6(7) + 11 = \underline{67}$$

Exercise

Given that $f(x) = \sqrt{x}$ and $g(x) = x - 1$, find

- a) $(f \circ g)(x) = f(g(x))$
- b) $(g \circ f)(x) = g(f(x))$
- c) $(f \circ g)(2) = f(g(2))$

Solution

$$\begin{aligned} \text{a) } (f \circ g)(x) &= f(g(x)) \\ &= f(x - 1) \\ &= \underline{\sqrt{x - 1}} \end{aligned}$$

$$\begin{aligned}
 b) \quad (g \circ f)(x) &= g(f(x)) \\
 &= g(\sqrt{x}) \\
 &= \sqrt{x-1}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad (f \circ g)(2) &= f(g(2)) \\
 &= \sqrt{2-1} \\
 &= 1
 \end{aligned}$$

Exercise

Given that $f(x) = \frac{x}{x+5}$ and $g(x) = \frac{6}{x}$, find

$$a) \quad (f \circ g)(x) = f(g(x))$$

$$b) \quad (g \circ f)(x) = g(f(x))$$

$$c) \quad (f \circ g)(2) = f(g(2))$$

Solution

$$\begin{aligned}
 a) \quad (f \circ g)(x) &= f(g(x)) \\
 &= f\left(\frac{6}{x}\right) \\
 &= \frac{\frac{6}{x}}{\frac{6}{x}+5} \\
 &= \frac{\frac{6}{x}}{\frac{6+5x}{x}} \\
 &= \frac{6}{6+5x}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad (g \circ f)(x) &= g(f(x)) \\
 &= g\left(\frac{x}{x+5}\right) \\
 &= \frac{6}{\frac{x}{x+5}} \\
 &= \frac{6(x+5)}{x} \\
 &= \frac{6x+30}{x}
 \end{aligned}$$

$$c) \quad (f \circ g)(2) = f(g(2))$$

$$\begin{aligned}
 &= \frac{6}{6+5(2)} \\
 &= \frac{6}{16} \\
 &= \frac{3}{8} \quad |
 \end{aligned}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = 2x^2 + 3x - 4$, $g(x) = 2x - 1$

Solution

$$\begin{aligned}
 f(g(x)) &= f(2x - 1) \\
 &= 2(2x - 1)^2 + 3(2x - 1) - 4 \\
 &= 2(4x^2 - 4x + 1) + 6x - 3 - 4 \\
 &= 8x^2 - 8x + 2 + 6x - 7 \\
 &= 8x^2 - 2x - 5 \quad |
 \end{aligned}$$

$$\begin{aligned}
 g(f(x)) &= g(2x^2 + 3x - 4) \\
 &= 2(2x^2 + 3x - 4) - 1 \\
 &= 4x^2 + 6x - 8 - 1 \\
 &= 4x^2 + 6x - 9 \quad |
 \end{aligned}$$

$$\begin{aligned}
 f(g(-2)) &= 8(-2)^2 - 2(-2) - 5 \\
 &= 31 \quad |
 \end{aligned}$$

$$\begin{aligned}
 g(f(3)) &= 4(3)^2 + 6(3) - 9 \\
 &= 45 \quad |
 \end{aligned}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = x^3 + 2x^2$, $g(x) = 3x$

Solution

$$\begin{aligned}
 f(g(x)) &= f(3x) \\
 &= (3x)^3 + 2(3x)^2 \\
 &= 27x^3 + 18x^2 \quad |
 \end{aligned}$$

$$\begin{aligned}
 g(f(x)) &= g(x^3 + 2x^2) \\
 &= 3(x^3 + 2x^2) \\
 &= \underline{3x^3 + 6x^2}
 \end{aligned}$$

$$\begin{aligned}
 f(g(-2)) &= 27(-2)^3 + 18(-2)^2 \\
 &= \underline{288}
 \end{aligned}$$

$$\begin{aligned}
 g(f(3)) &= 3(3)^3 + 6(3)^2 \\
 &= \underline{135}
 \end{aligned}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = |x|$, $g(x) = -7$

Solution

$$\begin{aligned}
 f(g(x)) &= f(-7) \\
 &= |-7| \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 g(f(x)) &= g(|x|) \\
 &= -7
 \end{aligned}$$

$$f(g(-2)) = \underline{7}$$

$$g(f(3)) = \underline{-7}$$

Exercise

Given $f(x) = x - 3$ and $g(x) = x + 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned}
 \text{a) } f(g(x)) &= f(x + 3) & \text{Domain: } \mathbb{R} \\
 &= (x + 3) - 3 \\
 &= \underline{x} & \text{Domain: } \mathbb{R}
 \end{aligned}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

$$\begin{aligned}
 b) \quad g(f(x)) &= g(x-3) & \text{Domain: } \mathbb{R} \\
 &= (x+3)-3 \\
 &= x & \text{Domain: } \mathbb{R} \\
 \text{Domain: } &\mathbb{R}
 \end{aligned}$$

Exercise

Given $f(x) = \frac{2}{3}x$ and $g(x) = \frac{3}{2}x$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
 b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned}
 a) \quad f(g(x)) &= f\left(\frac{3}{2}x\right) & \text{Domain: } \mathbb{R} \\
 &= \frac{2}{3}\left(\frac{3}{2}x\right) \\
 &= x & \text{Domain: } \mathbb{R} \\
 \text{Domain: } &\mathbb{R}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad g(f(x)) &= g\left(\frac{2}{3}x\right) & \text{Domain: } \mathbb{R} \\
 &= \frac{3}{2}\left(\frac{2}{3}x\right) \\
 &= x & \text{Domain: } \mathbb{R} \\
 \text{Domain: } &\mathbb{R}
 \end{aligned}$$

Exercise

Given $f(x) = x-1$ and $g(x) = 3x^2 - 2x - 1$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
 b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned}
 a) \quad f(g(x)) &= f(3x^2 - 2x - 1) & \text{Domain: } \mathbb{R} \\
 &= 3(x-1)^2 - 2(x-1) - 1 \\
 &= 3(x^2 - 2x + 1) - 2x + 2 - 1 \\
 &= 3x^2 - 6x + 3 - 2x + 1
 \end{aligned}$$

$$= \underline{3x^2 - 8x + 4} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

$$b) \quad g(f(x)) = g(x-1) \quad \text{Domain: } \mathbb{R}$$

$$= 3x^2 - 2x - 1 - 1$$

$$= \underline{3x^2 - 2x - 2} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

Exercise

Given $f(x) = 3x - 2$ and $g(x) = x^2 - 5$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(x^2 - 5) \quad \text{Domain: } \mathbb{R}$$

$$= 3(x^2 - 5) - 2$$

$$= 3x^2 - 15 - 2$$

$$= \underline{3x^2 - 17} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

$$b) \quad g(f(x)) = g(3x - 2) \quad \text{Domain: } \mathbb{R}$$

$$= (3x - 2)^2 - 5$$

$$= 9x^2 - 12x + 4 - 5$$

$$= \underline{9x^2 - 12x - 1} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

Exercise

Given $f(x) = x^2 - 2$ and $g(x) = 4x - 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned}
 a) \quad f(g(x)) &= f(4x-3) & \text{Domain: } \mathbb{R} \\
 &= (4x-3)^2 - 2 \\
 &= 16x^2 - 24x + 9 - 2 \\
 &= \underline{16x^2 - 24x + 7} & \text{Domain: } \mathbb{R}
 \end{aligned}$$

Domain: \mathbb{R}

$$\begin{aligned}
 b) \quad g(f(x)) &= g(x^2 - 2) & \text{Domain: } \mathbb{R} \\
 &= 4(x^2 - 2) - 3 \\
 &= 4x^2 - 8 - 3 \\
 &= \underline{4x^2 - 11} & \text{Domain: } \mathbb{R}
 \end{aligned}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = 4x^2 - x + 10$ and $g(x) = 2x - 7$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned}
 a) \quad f(g(x)) &= f(2x-7) & \text{Domain: } \mathbb{R} \\
 &= 4(2x-7)^2 - (2x-7) + 10 \\
 &= 4(4x^2 - 28x + 49) - 2x + 7 + 10 \\
 &= 16x^2 - 112x + 196 - 2x + 17 \\
 &= \underline{16x^2 - 114x + 213} & \text{Domain: } \mathbb{R}
 \end{aligned}$$

Domain: \mathbb{R}

$$\begin{aligned}
 b) \quad g(f(x)) &= g(4x^2 - x + 10) & \text{Domain: } \mathbb{R} \\
 &= 2(4x^2 - x + 10) - 7 \\
 &= 8x^2 - 2x + 20 - 7 \\
 &= \underline{8x^2 - 2x + 13} & \text{Domain: } \mathbb{R}
 \end{aligned}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = \sqrt{x}$ and $g(x) = x + 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} a) \quad f(g(x)) &= f(x+3) \\ &= \sqrt{x+3} \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \geq -3$

Domain: $x \geq -3$

$$\begin{aligned} b) \quad g(f(x)) &= g(\sqrt{x}) \\ &= \sqrt{x} + 3 \end{aligned}$$

Domain: $x \geq 0$

Domain: $x \geq 0$

Domain: $x \geq 0$

Exercise

Given $f(x) = \sqrt{x}$ and $g(x) = 2 - 3x$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} a) \quad f(g(x)) &= f(2-3x) \\ &= \sqrt{2-3x} \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \leq \frac{2}{3}$

Domain: $x \leq \frac{2}{3}$

$$\begin{aligned} b) \quad g(f(x)) &= g(\sqrt{x}) \\ &= 2 - 3\sqrt{x} \end{aligned}$$

Domain: $x \geq 0$

Domain: $x \geq 0$

Domain: $x \geq 0$

Exercise

Given $f(x) = 3x + 2$ and $g(x) = \sqrt{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt{x}) \quad \text{Domain: } x \geq 0$$

$$= 3\sqrt{x} + 2 \quad \text{Domain: } x \geq 0$$

$$\text{Domain: } x \geq 0$$

$$b) \quad g(f(x)) = g(3x + 2) \quad \text{Domain: } \mathbb{R}$$

$$= \sqrt{3x + 2} \quad \text{Domain: } x \geq -\frac{2}{3}$$

$$\text{Domain: } x \geq -\frac{2}{3}$$

Exercise

Given $f(x) = x^4$ and $g(x) = \sqrt[4]{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt[4]{x}) \quad \text{Domain: } x \geq 0$$

$$= (\sqrt[4]{x})^4$$

$$= x \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } x \geq 0$$

$$b) \quad g(f(x)) = g(x^4) \quad \text{Domain: } \mathbb{R}$$

$$= \sqrt[4]{x^4}$$

$$= x \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \mathbb{R}$$

Exercise

Given $f(x) = x^n$ and $g(x) = \sqrt[n]{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt[n]{x})$$

$$= (\sqrt[n]{x})^n$$

$$= x$$

$$\text{Domain: } \begin{cases} \text{If } n \text{ is even} & x \geq 0 \\ \text{If } n \text{ is odd} & \mathbb{R} \end{cases}$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \begin{cases} \text{If } n \text{ is even} & x \geq 0 \\ \text{If } n \text{ is odd} & \mathbb{R} \end{cases}$$

$$b) \quad g(f(x)) = g(x^n)$$

$$= \sqrt[n]{x^n}$$

$$= x$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \mathbb{R}$$

Exercise

Given $f(x) = x^2 - 3x$ and $g(x) = \sqrt{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt{x+2})$$

$$x+2 \geq 0 \Rightarrow x \geq -2$$

$$= (\sqrt{x+2})^2 - 3\sqrt{x+2}$$

$$= x+2 - 3\sqrt{x+2}$$

$$x+2 \geq 0 \Rightarrow x \geq -2$$

$$\text{Domain: } \{x \mid x \geq -2\}$$

$$b) \quad g(f(x)) = g(x^2 - 3x)$$

$$\mathbb{R}$$

$$= \sqrt{x^2 - 3x + 2}$$

$$x^2 - 3x + 2 \geq 0 \Rightarrow (x-1)(x-2) \geq 0 \Leftrightarrow x \leq 1, x \geq 2$$

$$\text{Domain: } \{x \mid x \leq 1, x \geq 2\}$$

Exercise

Given $f(x) = \sqrt{x-2}$ and $g(x) = \sqrt{x+5}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
- b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} \text{a) } f(g(x)) &= f(\sqrt{x+5}) \\ &= \sqrt{\sqrt{x+5}-2} \end{aligned}$$

$$\begin{aligned} x+5 &\geq 0 \Rightarrow x \geq -5 \\ \sqrt{x+5}-2 &\geq 0 \Rightarrow \sqrt{x+5} \geq 2 \\ x+5 &\geq 4 \\ x &\geq -1 \end{aligned}$$

$$\text{Domain: } \{x \mid x \geq -1\}$$

$$\begin{aligned} \text{b) } g(f(x)) &= g(\sqrt{x-2}) \\ &= \sqrt{\sqrt{x-2}+5} \end{aligned}$$

$$\begin{aligned} x-2 &\geq 0 \Rightarrow x \geq 2 \\ \sqrt{x-2}+5 &\geq 0 \Rightarrow \sqrt{x-2} \geq -5 \quad \text{Always true when } x \geq 2 \end{aligned}$$

$$\text{Domain: } \{x \mid x \geq 2\}$$

Exercise

Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{3-x}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
- b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} \text{a) } f(g(x)) &= f(\sqrt{3-x}) \\ &= (\sqrt{3-x})^2 + 2 \\ &= 3-x+2 \\ &= 5-x \end{aligned}$$

$$\text{Domain: } x \leq 3$$

$$\text{Domain: } x \leq 3$$

$$\begin{aligned} \text{b) } g(f(x)) &= g(x^2+2) \\ &= \sqrt{3-x^2-2} \\ &= \sqrt{1-x^2} \end{aligned}$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } -1 \leq x \leq 1$$

$$\text{Domain: } -1 \leq x \leq 1$$

Exercise

Given $f(x) = x^5 - 2$ and $g(x) = \sqrt[5]{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt[5]{x+2}) \quad \text{Domain: } \mathbb{R}$$

$$= (\sqrt[5]{x+2})^5 - 2$$

$$= x + 2 - 2$$

$$= x$$

Domain: \mathbb{R}

Domain: \mathbb{R}

$$b) \quad g(f(x)) = g(x^5 - 2) \quad \text{Domain: } \mathbb{R}$$

$$= \sqrt[5]{x^5 - 2 + 2}$$

$$= \sqrt[5]{x^5}$$

$$= x$$

Domain: \mathbb{R}

Domain: \mathbb{R}

Exercise

Given $f(x) = 1 - x^2$ and $g(x) = \sqrt{x^2 - 25}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt{x^2 - 25}) \quad \text{Domain: } x \leq -5 \quad x \geq 5$$

$$= 1 - (\sqrt{x^2 - 25})^2$$

$$= 1 - (x^2 - 25)$$

$$= 1 - x^2 + 25$$

$$= 26 - x^2$$

Domain: \mathbb{R}

Domain: $x \leq -5 \quad x \geq 5$

$$b) \quad g(f(x)) = g(1-x^2) \quad \text{Domain: } \mathbb{R}$$

$$= \sqrt{(1-x^2)^2 - 25}$$

$$= \sqrt{1-2x^2+x^4-25}$$

$$= \sqrt{x^4-2x^2-24}$$

$$x^2 = \frac{2 \pm \sqrt{4+96}}{2}$$

$$= \begin{cases} \frac{2-10}{2} = -4 \text{ } \times \\ \frac{2+10}{2} = 6 \end{cases}$$

$$x^2 = 6 \rightarrow x = \pm\sqrt{6}$$

$$\text{Domain: } x \leq -\sqrt{6} \quad x \geq \sqrt{6}$$

$$\text{Domain: } \underline{x \leq -\sqrt{6} \quad x \geq \sqrt{6}}$$

Exercise

$$\text{Given } f(x) = 2x+3 \quad \text{and} \quad g(x) = \frac{x-3}{2}$$

$$a) \quad \text{Find } (f \circ g)(x) \text{ and the domain of } f \circ g$$

$$b) \quad \text{Find } (g \circ f)(x) \text{ and the domain of } g \circ f$$

Solution

$$a) \quad f(g(x)) = f\left(\frac{x-3}{2}\right) \quad \text{Domain: } \mathbb{R}$$

$$= 2\left(\frac{x-3}{2}\right) + 3$$

$$= x-3+3$$

$$= x$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

$$b) \quad g(f(x)) = g(2x+3) \quad \text{Domain: } \mathbb{R}$$

$$= \frac{1}{2}(2x+3-3)$$

$$= x$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

Exercise

Given $f(x) = 4x - 5$ and $g(x) = \frac{x+5}{4}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{x+5}{4}\right) \quad \text{Domain: } \mathbb{R}$$

$$= 4\left(\frac{x+5}{4}\right) - 5$$

$$= x + 5 - 5$$

$$= x$$

Domain: \mathbb{R}

Domain: \mathbb{R}

$$b) \quad g(f(x)) = g(4x - 5) \quad \text{Domain: } \mathbb{R}$$

$$= \frac{1}{4}(4x - 5 + 5)$$

$$= x$$

Domain: \mathbb{R}

Domain: \mathbb{R}

Exercise

Given $f(x) = \frac{4}{1-5x}$ and $g(x) = \frac{1}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{1}{x}\right) \quad \text{Domain: } x \neq 0$$

$$= \frac{4}{1 - 5\frac{1}{x}}$$

$$= \frac{4x}{x-5}$$

Domain: $x \neq 5$

Domain: $x \neq 0, 5$

$$b) \quad g(f(x)) = g\left(\frac{4}{1-5x}\right) \quad \text{Domain: } x \neq \frac{1}{5}$$

$$= \frac{1-5x}{4}$$

Domain: \mathbb{R}

Domain: $x \neq \frac{1}{5}$

Exercise

Given $f(x) = \frac{1}{x-2}$ and $g(x) = \frac{x+2}{x}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
- b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{x+2}{x}\right) \quad \text{Domain: } x \neq 0$$

$$= \frac{1}{\frac{x+2}{x} - 2}$$

$$= \frac{1}{\frac{x+2-2x}{x}}$$

$$= \frac{x}{2-x} \quad \text{Domain: } x \neq 2$$

$$\text{Domain: } \underline{x \neq 0, 2} \mid (-\infty, 0) \cup (0, 2) \cup (2, \infty)$$

$$b) \quad g(f(x)) = g\left(\frac{1}{x-2}\right) \quad \text{Domain: } x \neq 2$$

$$= \frac{\frac{1}{x-2} + 2}{\frac{1}{x-2}}$$

$$= \frac{1 + 2x - 4}{\frac{1}{x-2}}$$

$$= \underline{2x-3} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{x \neq 2} \mid (-\infty, 2) \cup (2, \infty)$$

Exercise

Given $f(x) = \frac{3x+5}{2}$ and $g(x) = \frac{2x-5}{3}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
- b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{2x-5}{3}\right) \quad \text{Domain: } \mathbb{R}$$

$$= \frac{3 \frac{2x-5}{3} + 5}{2}$$

$$\begin{aligned}
 &= \frac{2x-5+5}{2} \\
 &= \frac{2x}{2} \\
 &= x
 \end{aligned}$$

Domain: \mathbb{R}

b) $g(f(x)) = g\left(\frac{3x+5}{2}\right)$ **Domain:** \mathbb{R}

$$\begin{aligned}
 &= \frac{2 \cdot \frac{3x+5}{2} - 5}{3} \\
 &= \frac{3x+5-5}{3} \\
 &= \frac{3x}{3} \\
 &= x
 \end{aligned}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = \frac{1}{1+x}$ and $g(x) = \frac{1-x}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{1-x}{x}\right)$ **Domain:** $x \neq 0$

$$\begin{aligned}
 &= \frac{1}{1 + \frac{1-x}{x}} \\
 &= \frac{x}{x+1-x} \\
 &= x
 \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \neq 0$

b) $g(f(x)) = g\left(\frac{1}{x+1}\right)$ **Domain:** $x \neq -1$

$$\begin{aligned}
 &= \frac{1 - \frac{1}{x+1}}{\frac{1}{x+1}} \\
 &= x+1-1 \\
 &= x
 \end{aligned}$$

Domain: \mathbb{R}

Domain: \mathbb{R}

Exercise

Given $f(x) = \frac{x-1}{x-2}$ and $g(x) = \frac{x-3}{x-4}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{x-3}{x-4}\right)$ **Domain:** $x \neq 4$

$$\begin{aligned} &= \frac{\frac{x-3}{x-4} - 1}{\frac{x-3}{x-4} - 2} \\ &= \frac{\frac{x-3-(x-4)}{x-4}}{\frac{x-3-2(x-4)}{x-4}} \\ &= \frac{x-3+x+4}{x-3-2x+8} \\ &= \frac{2x+1}{-x+5} \end{aligned}$$

Domain: $x \neq 5$

Domain: $\{x \mid x \neq 4, 5\}$

b) $g(f(x)) = g\left(\frac{x-1}{x-2}\right)$ **Domain:** $x \neq 2$

$$\begin{aligned} &= \frac{\frac{x-1}{x-2} - 3}{\frac{x-1}{x-2} - 4} \\ &= \frac{\frac{x-1-3(x-2)}{x-2}}{\frac{x-1-4(x-2)}{x-2}} \\ &= \frac{x-1-3x+6}{x-1-4x+8} \\ &= \frac{-2x+5}{-3x+7} \end{aligned}$$

Domain: $x \neq \frac{7}{3}$

Domain: $\left\{x \mid x \neq 2, \frac{7}{3}\right\}$

Exercise

Given $f(x) = \frac{6}{x-3}$ and $g(x) = \frac{1}{x}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $(f \circ g)(x)$

$$f(g(x)) = f\left(\frac{1}{x}\right) \quad \text{Domain: } x \neq 0$$

$$= \frac{6}{\frac{1}{x} - 3}$$

$$= \frac{6}{\frac{1-3x}{x}}$$

$$= \frac{6x}{1-3x} \quad \text{Domain: } x \neq \frac{1}{3}$$

$$\text{Domain: } \underline{x \neq 0, \frac{1}{3}} \quad \underline{(-\infty, 0) \cup \left(0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)}$$

b) $(g \circ f)(x)$

$$g(f(x)) = g\left(\frac{6}{x-3}\right) \quad \text{Domain: } x \neq 3$$

$$= \frac{1}{\frac{6}{x-3}}$$

$$= \frac{x-3}{6} \quad \text{Domain: } (-\infty, \infty)$$

$$\text{Domain: } \underline{x \neq 3} \quad \underline{(-\infty, 3) \cup (3, \infty)}$$

Exercise

Given $f(x) = \frac{6}{x}$ and $g(x) = \frac{1}{2x+1}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{1}{2x+1}\right) \quad \text{Domain: } x \neq -\frac{1}{2}$

$$= \frac{6}{\frac{1}{2x+1}}$$

$$= \underline{12x+6} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{x \neq -\frac{1}{2}} \mid$$

$$b) \quad g(f(x)) = g\left(\frac{6}{x}\right) \quad \text{Domain: } x \neq 0$$

$$= \frac{1}{2\frac{6}{x} + 1}$$

$$= \underline{\frac{x}{12+x}} \mid$$

$$\text{Domain: } x \neq -12$$

$$\text{Domain: } \underline{x \neq -12, 0} \mid$$

Exercise

$$\text{Given } f(x) = 3x - 7 \quad \text{and} \quad g(x) = \frac{x+7}{3}$$

$$a) \quad \text{Find } (f \circ g)(x) \text{ and the domain of } f \circ g$$

$$b) \quad \text{Find } (g \circ f)(x) \text{ and the domain of } g \circ f$$

Solution

$$a) \quad f(g(x)) = f\left(\frac{x+7}{3}\right) \quad \text{Domain: } \mathbb{R}$$

$$= 3\frac{x+7}{3} - 7$$

$$= x + 7 - 7$$

$$= \underline{x} \mid$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}} \mid$$

$$b) \quad g(f(x)) = g(3x - 7) \quad \text{Domain: } \mathbb{R}$$

$$= \frac{3x - 7 + 7}{3}$$

$$= \underline{x} \mid$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}} \mid$$

Exercise

$$\text{Given } f(x) = \frac{2x+3}{x-4} \quad \text{and} \quad g(x) = \frac{4x+3}{x-2}$$

$$a) \quad \text{Find } (f \circ g)(x) \text{ and the domain of } f \circ g$$

$$b) \quad \text{Find } (g \circ f)(x) \text{ and the domain of } g \circ f$$

Solution

$$a) \quad f(g(x)) = f\left(\frac{4x+3}{x-2}\right) \quad \text{Domain: } x \neq 2$$

$$\begin{aligned}
&= \frac{2 \frac{4x+3}{x-2} + 3}{\frac{4x+3}{x-2} - 4} \\
&= \frac{8x+6+3x-6}{4x+3-4x+8} \\
&= \frac{11x}{11} \\
&= x
\end{aligned}$$

Domain: \mathbb{R}

Domain: $x \neq 2$

b) $g(f(x)) = g\left(\frac{2x+3}{x-4}\right)$ **Domain:** $x \neq 4$

$$\begin{aligned}
&= \frac{4 \frac{2x+3}{x-4} + 3}{\frac{2x+3}{x-4} - 2} \\
&= \frac{8x+12+3x-4}{2x+3-2x+8} \\
&= \frac{11x}{11} \\
&= x
\end{aligned}$$

Domain: \mathbb{R}

Domain: $x \neq 4$

Exercise

Given $f(x) = \frac{2x+3}{x+4}$ and $g(x) = \frac{-4x+3}{x-2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{-4x+3}{x-2}\right)$ **Domain:** $x \neq 2$

$$\begin{aligned}
&= \frac{2 \frac{-4x+3}{x-2} + 3}{\frac{4x+3}{x-2} + 4} \\
&= \frac{-8x+6+3x-6}{4x+3+4x-8} \\
&= \frac{-5x}{-5} \\
&= x
\end{aligned}$$

Domain: \mathbb{R}

Domain: $x \neq 2$

$$\begin{aligned}
 b) \quad g(f(x)) &= g\left(\frac{2x+3}{x+4}\right) & \text{Domain: } x \neq -4 \\
 &= -4\frac{2x+3}{x+4} + 3 \\
 &= \frac{\frac{2x+3}{x+4} - 2}{\frac{2x+3}{x+4} - 2} \\
 &= \frac{-8x-12+3x+12}{2x+3-2x-8} \\
 &= \frac{-5x}{-5} \\
 &= x
 \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \neq -4$

Exercise

Given $f(x) = x+1$ and $g(x) = x^3 - 5x^2 + 3x + 7$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned}
 a) \quad f(g(x)) &= f(x^3 - 5x^2 + 3x + 7) & \text{Domain: } \mathbb{R} \\
 &= x^3 - 5x^2 + 3x + 7 + 1 \\
 &= x^3 - 5x^2 + 3x + 8
 \end{aligned}$$

Domain: \mathbb{R}

Domain: \mathbb{R}

$$\begin{aligned}
 b) \quad g(f(x)) &= g(x+1) & \text{Domain: } \mathbb{R} \\
 &= (x+1)^3 - 5(x+1)^2 + 3(x+1) + 7 \\
 &= x^3 + 3x^2 + 3x + 1 - 5(x^2 + 2x + 1) + 3x + 3 + 7 \\
 &= x^3 + 3x^2 + 6x + 11 - 5x^2 - 10x - 5 \\
 &= x^3 - 2x^2 - 4x + 6
 \end{aligned}$$

Domain: \mathbb{R}

Domain: \mathbb{R}

Exercise

Given $f(x) = x - 1$ and $g(x) = x^3 + 2x^2 - 3x - 9$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(x^3 + 2x^2 - 3x - 9) \quad \text{Domain: } \mathbb{R}$$

$$= x^3 + 2x^2 - 3x - 9 - 1$$

$$= \underline{x^3 + 2x^2 - 3x - 10} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \mathbb{R}$$

$$b) \quad g(f(x)) = g(x - 1) \quad \text{Domain: } \mathbb{R}$$

$$= (x - 1)^3 + 2(x - 1)^2 - (x - 1) - 9$$

$$= x^3 - 3x^2 + 3x - 1 + 2(x^2 - 2x + 1) - 3x + 3 - 9$$

$$= x^3 - 3x^2 - 7 + 2x^2 - 4x + 2$$

$$= \underline{x^3 - x^2 - 4x - 5} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \mathbb{R}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(f \circ g)(4)$

Solution

$$(f \circ g)(4) = f(g(4))$$

$$= f(16 - 20)$$

$$= f(-4)$$

$$= -8 - 3$$

$$= \underline{-11}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(g \circ f)(4)$

Solution

$$(g \circ f)(4) = g(f(4))$$

$$\begin{aligned}
 &= g(8-3) \\
 &= g(5) \\
 &= 25-25 \\
 &= 0
 \end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(f \circ g)(-2)$

Solution

$$\begin{aligned}
 (f \circ g)(-2) &= f(g(-2)) \\
 &= f(4+10) \\
 &= f(14) \\
 &= 28-3 \\
 &= 25
 \end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(g \circ f)(-2)$

Solution

$$\begin{aligned}
 (g \circ f)(-2) &= g(f(-2)) \\
 &= g(-4-3) \\
 &= g(-7) \\
 &= 49+35 \\
 &= 84
 \end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(f \circ f)(-3)$

Solution

$$\begin{aligned}
 (f \circ f)(-3) &= f(f(-3)) \\
 &= f(-6-3) \\
 &= f(-9) \\
 &= -18-3 \\
 &= -21
 \end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(g \circ g)(7)$

Solution

$$\begin{aligned}(g \circ g)(7) &= g(g(7)) \\ &= g(49 - 35) \\ &= g(14) \\ &= 196 - 70 \\ &= \underline{126}\end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(f \circ g)(\sqrt{2})$

Solution

$$\begin{aligned}(f \circ g)(\sqrt{2}) &= f(g(\sqrt{2})) \\ &= f(2 - 5\sqrt{2}) \\ &= 2(2 - 5\sqrt{2}) - 3 \\ &= 4 - 10\sqrt{2} - 3 \\ &= \underline{1 - 10\sqrt{2}}\end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(g \circ f)(\sqrt{3})$

Solution

$$\begin{aligned}(g \circ f)(\sqrt{3}) &= g(f(\sqrt{3})) \\ &= g(2\sqrt{3} - 3) \\ &= (2\sqrt{3} - 3)^2 - 5(2\sqrt{3} - 3) \\ &= 12 - 12\sqrt{3} + 9 - 10\sqrt{3} + 15 \\ &= \underline{36 - 22\sqrt{3}}\end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(f \circ g)(2a)$

Solution

$$\begin{aligned}(f \circ g)(2a) &= f(g(2a)) \\&= f(4a^2 - 10a) \\&= 2(4a^2 - 10a) - 3 \\&= \underline{8a^2 - 20a - 3}\end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(g \circ f)(3b)$

Solution

$$\begin{aligned}(g \circ f)(3b) &= g(f(3b)) \\&= g(6b - 3) \\&= (6b - 3)^2 - 5(6b - 3) \\&= 36b^2 - 36b + 9 - 30b + 15 \\&= \underline{36b^2 - 66b + 24}\end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(f \circ g)(k + 1)$

Solution

$$\begin{aligned}(f \circ g)(k + 1) &= f(g(k + 1)) \\&= f((k + 1)^2 - 5k - 5) \\&= 2((k + 1)^2 - 5k - 5) - 3 \\&= 2(k^2 + 2k + 1) - 10k - 10 - 3 \\&= 2k^2 + 4k + 2 - 10k - 13 \\&= \underline{2k^2 - 6k - 11}\end{aligned}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$: $(g \circ f)(k - 1)$

Solution

$$\begin{aligned}(g \circ f)(k - 1) &= g(f(k - 1)) \\&= g(2k - 2 - 3) \\&= g(2k - 5) \\&= (2k - 5)^2 - 5(2k - 5) \\&= 4k^2 - 20k + 25 - 10k + 25 \\&= \underline{4k^2 - 30k + 50}\end{aligned}$$