# *Lecture Six* – Trigonometric

## *Section* 6.1 – Introduction

### Basic Terminology

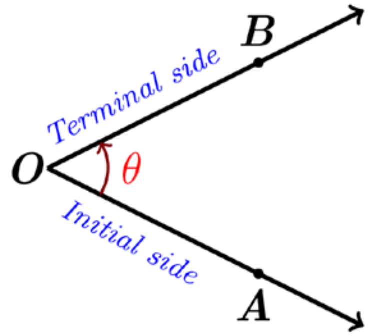
Two distinct points determine line *AB*.


***Line segment AB:*** portion of the line between *A* and *B*.

***Ray AB***: portion of the line *AB* starts at *A* and continues through *B*, and past *B*.


### *Angles* in General

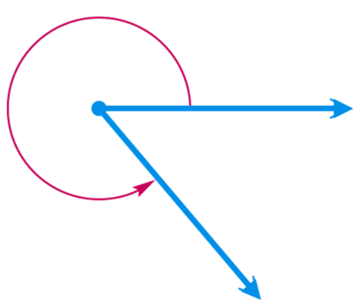
An angle is formed by 2 rays with the same end point.



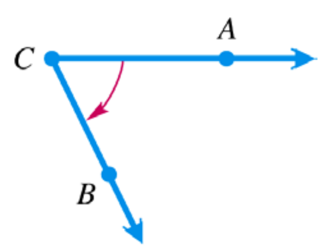

The two rays are the sides of the angle, angle $\theta$ = *AOB*
***O*** is the common endpoint and it is called ***vertex*** of the angle**.**

An angle is in a Counterclockwise (***CCW***) direction: positive angle.

An angle is in a Clockwise (***CW***) direction: negative angle.

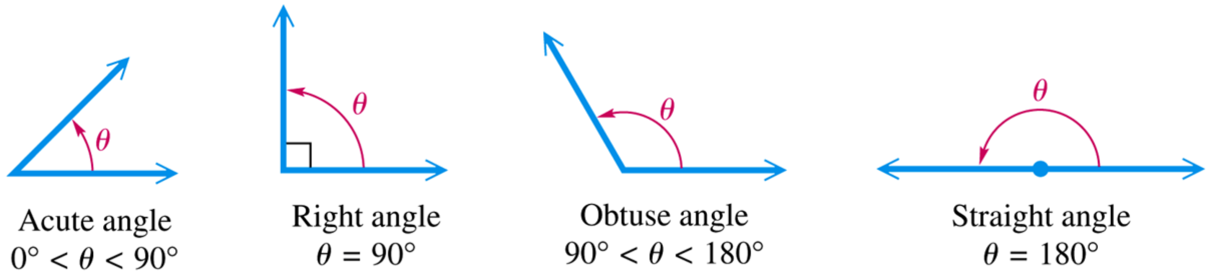


Positive angle




Negative angle

## Type of Angles: ***Degree***




***Complementary angles***: $\alpha + \beta = 90°$

***Supplementary angles***: $\alpha + \beta = 180°$

### *Example*

Give the complement and the supplement of each angle: 40° 110°

***Solution***

***a.*** 40° Complement: $90° - 40° = 50°$ Supplement: $180° - 40° = 140°$

***b.*** 110° Complement: $90° - 110° = -20°$ Supplement: $180° - 110° = 70°$

### ***Degrees, Minutes, Seconds***

$1°$: 1 *degree*
$1'$: 1 *minute*
$1''$: 1 *second*

**1 *full Rotation*** or ***Revolution* = 360°** $\qquad 1° = 60' = 3600'' \qquad 1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ}$

### *Example*

Change 27.25° to degrees and minutes

***Solution***

$$\begin{aligned} 27.25° &= 27° + .25° \\ &= 27° + .25(\mathbf{60'}) \\ &= 27° + 15' \\ &= 27°\ 15' \end{aligned}$$

***Example***

Add $48° \ 49'$ and $72° \ 26'$

***Solution***

$$\begin{array}{rr} & 48° \quad 49' \\ + & 72° \quad 26' \\ \hline & 120° \quad 75' \end{array}$$

$$\begin{aligned} 120° \ 75' &= 120° \ \ 60' + 15' \\ &= 121° \ \ 15' \end{aligned}$$

***Example***

Subtract $24° \ 14'$ and $90°$

***Solution***

$$\begin{array}{rr} & 90° \qquad \\ - & 24° \quad 14' \\ \hline \end{array} = \begin{array}{rr} & 89° \quad 60' \\ - & 24° \quad 14' \\ \hline & 65° \quad 46' \end{array}$$

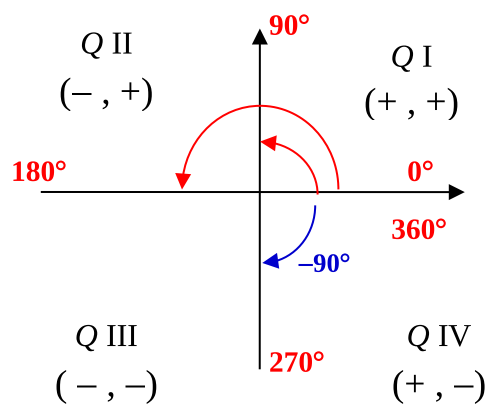
## Angles in Standard Position

An angle is said to be in standard position if its initial side is along the positive *x*-axis and its vertex is at the origin. If angle $\theta$ is in standard position and the terminal side of $\theta$ lies in quadrant I, then we say $\theta$ lies in QI

$$\theta \in QI$$

If the terminal side of an angle in standard position lies along one of the axes (*x*-axis or *y*-axis), such as angles with measures 90°, 180°, 270°, then that called a ***quadrantal*** *angle*.

Two angles in standard position with the same terminal side are called ***coterminal*** *angles*.




### *Example*

Find all angles that are coterminal with 120°.

***Solution***

$120° + 360°k$

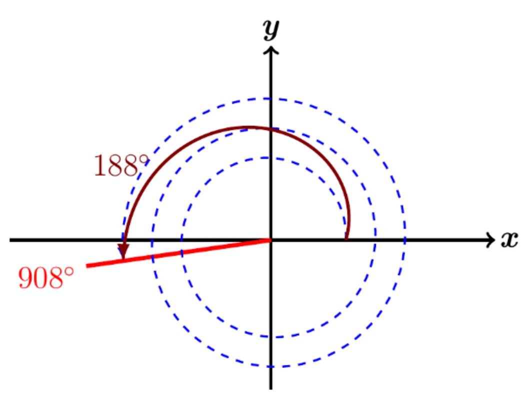
### *Example*

Find the angle of least possible positive measure coterminal with an angle of 908°.

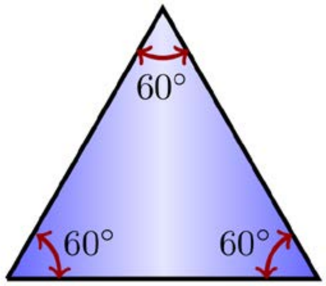
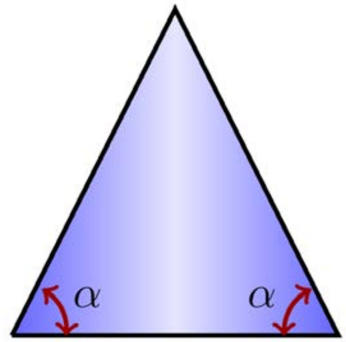
***Solution***

$908° - 2.360° = 188°$

An angle of 908° is coterminal with an angle of 188°



## *Triangles*

<table>
<tr>
<td>Equilateral – All angles always equal to 60°& all sides are equals<br><br></td>
<td>Isosceles: 2 sides and angles are equals<br><br></td>
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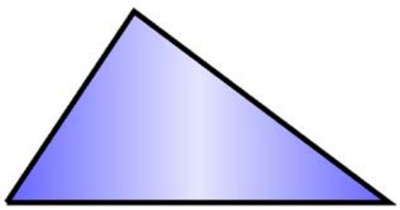
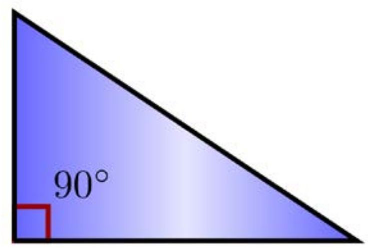
<td>Scalene: No equal sides or angles<br></td>
<td>Right: Has a right angle 90°.<br><br></td>
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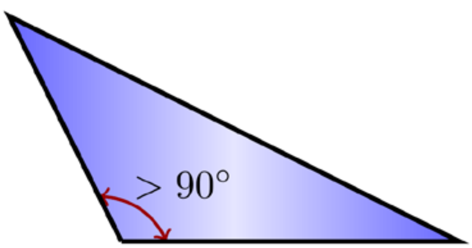
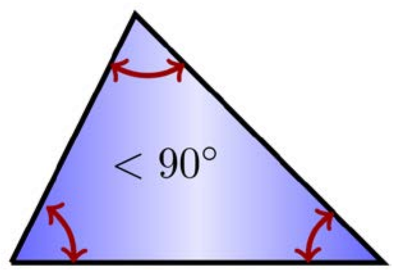
<td>Obtuse: Has an angle more than 90°.<br><br></td>
<td>Acute: All angles are less than 90°.<br><br></td>
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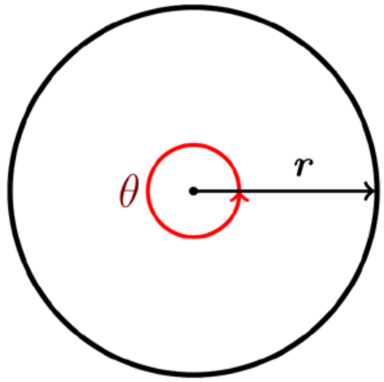
# *Radians*

## Degrees - Radians




*θ measures one full rotation* $\qquad \theta = 2\pi \qquad$ *The measure of θ in radians is* $2\pi$

$$\boxed{1 = 1\ rad}$$

$$1^\circ = 1\ degree$$

***If no unit of angle measure is specified, then the angle is to be measured in radians.***

$$Full\ Rotation:\ 360^\circ = 2\pi\ rad$$

$$180^\circ = \pi\ rad$$

### *Converting from Degrees to Radians*

$$\frac{180^\circ}{180} = \frac{\pi}{180}\ rad \qquad \Rightarrow 1^\circ = \frac{\pi}{180}\ rad$$

Multiply a degree measure by $\frac{\pi}{180}$ *rad* and simplify to convert to radians.

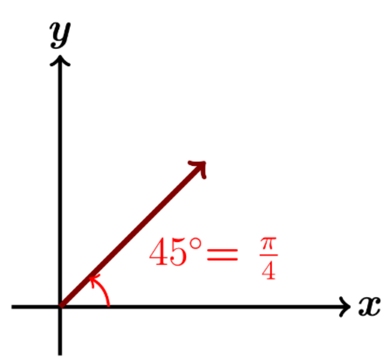
### *Example*

Convert 45° to radians

***Solution***

$$45^\circ = 45\left(\frac{\pi}{180}\right) rad$$

$$= \frac{\pi}{4}\ rad$$



### *Example*

Convert -450° to radians

**Solution**

$$-450^\circ = -450\left(\frac{\pi}{180}\right) rad$$

$$= -\frac{5\pi}{2}\ \ rad$$

### *Example*

Convert 249.8° to radians

**Solution**

$$249.8^\circ = \frac{2498}{10}\left(\frac{\pi}{180}\right) rad$$

$$= \frac{1,249\pi}{900}\ \ rad$$

$$\approx 4.360\ \ rad$$

### *Converting from Radians to Degrees*

Multiply a radian measure by $\frac{180^\circ}{\pi}$ radian and simplify to convert to degrees.

$$\frac{180^\circ}{\pi} = \frac{\pi}{\pi}\ \ rad$$

$$\boxed{\frac{180^\circ}{\pi} = 1\ \ rad}$$

### *Example*

Convert 1 to *degrees*

**Solution**

$$1\ rad = 1\left(\frac{180^\circ}{\pi}\right)$$

$$= 1\left(\frac{180^\circ}{3.14}\right)$$

$$= 57.3^\circ$$

### *Example*

Convert $\frac{4\pi}{3}$ to *degrees*

***Solution***

$$\frac{4\pi}{3} = \frac{4\pi}{3}\left(\frac{180^{\circ}}{\pi}\right)$$

$$= 240^{\circ}$$

### *Example*

Convert $-4.5$ to *degrees*

***Solution***

$$-4.5 = -4.5\left(\frac{180^{\circ}}{\pi}\right)$$

$$\approx -257.8^{\circ}$$

# *Exercises* *Section* 6.1– Introduction

**1.** Indicate the angle if it is an acute or obtuse. Then give the complement and the supplement of each angle.

*a*) $10°$ *b*) $52°$ *c*) $90°$ *d*) $120°$ *e*) $150°$

**2.** Change to decimal degrees.

*a)* $10°\ 45'$ *c)* $274°\ 18'\ 59''$ *e)* $98°\ 22'\ 45''$ *g)* $1°\ 2'\ 3''$

*b)* $34°\ 51'\ 35''$ *d)* $74°\ 8'\ 14''$ *f)* $9°\ 9'\ 9''$ *h)* $73°\ 40'\ 40''$

**3.** Convert to degrees, minutes, and seconds.

*a)* $89.9004°$ *c)* $122.6853°$ *e)* $44.01°$ *g)* $29.411°$

*b)* $34.817°$ *d)* $178.5994°$ *f)* $19.99°$ *h)* $18.255°$

**4.** Perform each calculation

*a)* $51°\ 29' + 32°\ 46'$ *b)* $90° - 73°12'$ *c)* $90° - 36°\ 18'\ 47''$ *d)* $75°\ 15' + 83°\ 32'$

**5.** Find the angle of least possible positive measure coterminal with an angle of

*a)* $-75°$ *b)* $-800°$ *c)* $270°$

**6.** Convert to radians

*a)* $256°\ 20'$ *b)* $-78.4°$ *c)* $330°$ *d)* $-60°$ *e)* $-225°$

**7.** Convert to degrees

*a)* $\frac{11\pi}{6}$ *c)* $\frac{\pi}{6}$ *e)* $\frac{\pi}{3}$ *g)* $-4\pi$

*b)* $-\frac{5\pi}{3}$ *d)* $2.4$ *f)* $-\frac{5\pi}{12}$ *h)* $\frac{7\pi}{13}$

**8.** A vertical rise of the Forest Double chair lift 1,170 *feet* and the length of the chair lift as 5,570 *feet*. To the nearest foot, find the horizontal distance covered by a person riding this lift.

**9.** A tire is rotating 600 times per *minute*. Through how many degrees does a point of the edge of the tire move in $\frac{1}{2}$ second?

**10.** A windmill makes 90 *revolutions* per *minute*. How many revolutions does it make per second?