

***Solution***      **Section 2.9 – Derivatives of Inverse Trigonometric Functions**

***Exercise***

Find the value of  $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

**Solution**

$$\begin{aligned}\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) &= \sin\left(\frac{\pi}{4}\right) \\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

***Exercise***

Find the value of  $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

**Solution**

$$\begin{aligned}\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) &= \cot\left(-\frac{\pi}{3}\right) \\ &= -\frac{1}{\sqrt{3}}\end{aligned}$$

***Exercise***

Find the limit:  $\lim_{x \rightarrow -1^+} \cos^{-1} x$

**Solution**

$$\begin{aligned}\lim_{x \rightarrow -1^+} \cos^{-1} x &= \cos^{-1}(-1) \\ &= \pi\end{aligned}$$

***Exercise***

Find the limit:  $\lim_{x \rightarrow -\infty} \tan^{-1} x$

**Solution**

$$\lim_{x \rightarrow -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

**Exercise**

Find the limit:  $\lim_{x \rightarrow \infty} \csc^{-1} x$

**Solution**

$$\begin{aligned} \lim_{x \rightarrow \infty} \csc^{-1} x &= \lim_{x \rightarrow \infty} \sin^{-1} \left( \frac{1}{x} \right) \\ &= \sin^{-1} \left( \frac{1}{\infty} \right) \\ &= \underline{0} \end{aligned}$$

**Exercise**

Find the derivative  $y = \cos^{-1} \left( \frac{1}{x} \right)$

**Solution**

$$\begin{aligned} y &= \cos^{-1} \left( \frac{1}{x} \right) \\ &= \sec^{-1}(x) \\ y' &= \frac{1}{|x| \cdot \sqrt{x^2 - 1}} \end{aligned}$$

**Exercise**

Find the derivative  $y = \sin^{-1} \sqrt{2}t$

**Solution**

$$\begin{aligned} y' &= \frac{\sqrt{2}}{\sqrt{1 - (\sqrt{2}t)^2}} \\ &= \frac{\sqrt{2}}{\sqrt{1 - 2t^2}} \end{aligned}$$

**Exercise**

Find the derivative  $y = \sec^{-1}(5s)$

**Solution**

$$y' = \frac{5s}{|5s| \sqrt{(5s)^2 - 1}}$$

$$= \frac{s}{|s|\sqrt{25s^2 - 1}} \Big|$$

### Exercise

Find the derivative  $y = \cot^{-1} \sqrt{t-1}$

### Solution

$$\begin{aligned} y' &= -\frac{\frac{1}{2}(t-1)^{-1/2}}{1 + \left[(t-1)^{1/2}\right]^2} \\ &= -\frac{1}{2(t-1)^{1/2}(1+t-1)} \\ &= -\frac{1}{2t\sqrt{t-1}} \Big| \end{aligned}$$

### Exercise

Find the derivative  $y = \ln(\tan^{-1} x)$

### Solution

$$\begin{aligned} y' &= \frac{\frac{1}{1+x^2}}{\tan^{-1} x} \\ &= \frac{1}{(1+x^2)\tan^{-1} x} \Big| \end{aligned}$$

### Exercise

Find the derivative  $y = \tan^{-1}(\ln x)$

### Solution

$$\begin{aligned} y' &= \frac{\frac{1}{x}}{1 + (\ln x)^2} \\ &= \frac{1}{x[1 + (\ln x)^2]} \Big| \end{aligned}$$

$$\left(\tan^{-1} u\right)' = \frac{u'}{1+u^2}$$

### Exercise

Find the derivative  $y = \csc^{-1}(e^t)$

### Solution

$$\begin{aligned} y' &= -\frac{e^t}{|e^t|\sqrt{(e^t)^2 - 1}} \\ &= -\frac{1}{\sqrt{e^{2t} - 1}} \end{aligned}$$

### Exercise

Find the derivative  $y = x\sqrt{1-x^2} + \cos^{-1}x$

### Solution

$$\begin{aligned} y' &= \sqrt{1-x^2} + x\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x) - \frac{1}{\sqrt{1-x^2}} \\ &= \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \\ &= \frac{1-x^2-x^2-1}{\sqrt{1-x^2}} \\ &= \frac{-2x^2}{\sqrt{1-x^2}} \end{aligned}$$

### Exercise

Find the derivative  $y = \ln(x^2 + 4) - x \tan^{-1}\left(\frac{x}{2}\right)$

### Solution

$$\begin{aligned} y' &= \frac{2x}{x^2 + 4} - \tan^{-1}\left(\frac{x}{2}\right) - x \frac{\frac{1}{2}}{1 + \left(\frac{x}{2}\right)^2} \\ &= \frac{2x}{x^2 + 4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{x}{2} \cdot \frac{1}{1 + \frac{x^2}{4}} \\ &= \frac{2x}{x^2 + 4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{x}{2} \cdot \frac{4}{4 + x^2} \\ &= \frac{2x}{x^2 + 4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{2x}{4 + x^2} \end{aligned}$$

$$\underline{= -\tan^{-1}\left(\frac{x}{2}\right) \Big|}$$

### Exercise

Find the derivative  $f(x) = \sin^{-1} \frac{1}{x}$

#### Solution

$$f'(x) = -\frac{1}{x^2} \frac{1}{\sqrt{1 - \left(\frac{1}{x}\right)^2}}$$

$$\underline{= \frac{-1}{|x|\sqrt{x^2 - 1}} \Big|}$$

### Exercise

Find the derivative  $\left. \frac{d}{dx} (x \sec^{-1} x) \right|_{x=\frac{2}{\sqrt{3}}}$

#### Solution

$$\left. \frac{d}{dx} (x \sec^{-1} x) = \sec^{-1} x + \frac{x}{x\sqrt{x^2 - 1}} \right|_{x=\frac{2}{\sqrt{3}}}$$

$$= \sec^{-1} \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{\frac{4}{3} - 1}}$$

$$\underline{= \frac{\pi}{6} + \sqrt{3} \Big|}$$

### Exercise

Find the derivative  $\left. \frac{d}{dx} (\tan^{-1} e^{-x}) \right|_{x=0}$

#### Solution

$$\left. \frac{d}{dx} (\tan^{-1} e^{-x}) = \frac{-e^{-x}}{1 + e^{-2x}} \right|_{x=0}$$

$$\underline{= -\frac{1}{2} \Big|}$$

### Exercise

Find the angle  $\alpha$

### Solution

$$65^\circ + (90^\circ - \beta) + (90^\circ - \alpha) = 180^\circ$$

$$65^\circ + 180^\circ - \beta - \alpha = 180^\circ$$

$$\beta + \alpha = 65^\circ$$

$$\underline{\alpha = 65^\circ - \beta}$$

$$\tan \beta = \frac{21}{50}$$

$$\beta = \tan^{-1}\left(\frac{21}{50}\right)$$

$$\underline{\approx 22.78^\circ}$$

$$\underline{\alpha \approx 65^\circ - 22.78^\circ}$$

$$\underline{\approx 42.22^\circ}$$

