Lecture Two

Section 2.1 – Definition of the Derivative

Derivative

The derivative of the function f at x is defined as

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

A function is differentiable @ x if its derivative exists at x.

The process of finding derivatives is called ***differentiation***.

$$f'(x), \quad f', \quad \frac{d}{dx}[f(x)], \quad \frac{d}{dx}f, \quad \frac{dy}{dx}, \quad y', \quad \dot{y}, \text{ and } D_x[y]$$

Differentiability ⇒ Continuity

If a function f is differentiable @ $x = c \Rightarrow f$ is continuous @ $x = c$

Example

Find the derivative of $f(x) = x^2$

<u>*Solution*</u>

$$\begin{aligned} f(x+h) &= (x+h)^2 \\ &= x^2 + 2hx + h^2 \end{aligned}$$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2hx + h^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{2hx + h^2}{h} \\ &= \lim_{h\to 0} (2x + h) \\ &= 2x \end{aligned}$$

Example

Find the derivative of $f(x) = 3x^2 - 2x$

Solution

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2 - 2(x+\Delta x) - (3x^2 - 2x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3(x^2 + \Delta x^2 + 2x\Delta x) - 2x - 2\Delta x - 3x^2 + 2x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2 + 3\Delta x^2 + 6x\Delta x - 2x - 2\Delta x - 3x^2 + 2x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3\Delta x^2 + 6x\Delta x - 2\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 3\Delta x + 6x - 2$$

$$= 6x - 2$$

Exercises Section 2.1 – Definition of the Derivative

1. Find the derivative of y with the respect to t for the function $y = \frac{4}{t}$

2. Find the equation of the tangent line to $f(x) = x^2 + 1$ that is parallel to $2x + y = 0$

3. Use the definition of limits to find the derivative: $f(x) = \dfrac{3}{\sqrt{x}}$

4. Use the definition of limits to find the derivative: $f(x) = \sqrt{x+2}$