Math 2318 – Linear Algebra **Exam 3** **Review**

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1. Let $\langle \boldsymbol{u}, \boldsymbol{v}\rangle$ be the Euclidean inner product on R^2, and let $\boldsymbol{u}=(3,\ -2)$, $\boldsymbol{v}=(4,\ 5)$, and $k=4$.

Verify the following for the weighted Euclidean inner product $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = 3u_1v_1 + 4u_2v_2$

a) $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = \langle \boldsymbol{v}, \boldsymbol{u}\rangle$ *b)* $\langle k\boldsymbol{u}, \boldsymbol{v}\rangle = k\langle \boldsymbol{u}, \boldsymbol{v}\rangle$

2. Which of the following form orthonormal sets?

a) $\left(\frac{1}{\sqrt{3}},\ -\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{6}},\ \frac{2}{\sqrt{6}},\ \frac{1}{\sqrt{6}}\right)$ in $\boldsymbol{R}^3$

b) $\left(\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right), \left(0,\frac{\sqrt{6}}{3},\frac{1}{\sqrt{6}},-\frac{1}{\sqrt{6}}\right), \left(\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{6},\frac{\sqrt{3}}{6},-\frac{\sqrt{3}}{6}\right), \left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

3. Use the Gram-Schmidt process to find an orthonormal basis for the subspaces of $\boldsymbol{R}^m$.

a) $x_1=(1,\ 1),\quad x_2=(1,\ 2)$

b) $x_1=(1,\ 2),\quad x_2=(1,\ 3)$

c) $x_1=(1,\ 2,\ 2),\quad x_2=(2,\ 1,\ 3)$

d) $v_1=(1,-1,-1,1),\quad v_2=(2,1,0,1),\quad v_3=(2,2,1,2)$

4. Find the ***QR***-decomposition of

a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ *b)* $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

5. Determine if the matrix is orthogonal. For those that is orthogonal find the inverse

a) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ *b)* $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

c) $\begin{pmatrix} \cos\theta\sin\theta & -\cos\theta & -\sin^2\theta \\ \cos^2\theta & \sin\theta & -\cos\theta\sin\theta \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$

Solution

1.

2. $a)$ *Orthonormal* $b)$ *Orthonormal*

3. $a)$ $v_1 = (1,1),\ v_1 = \left(-\frac{1}{2}, \frac{1}{2}\right);\quad q_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right),\ q_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$b)$ $q_1 = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right),\ q_2 = \left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

$c)$ $q_1 = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right),\quad q_2 = \left(\frac{8}{3\sqrt{26}}, -\frac{11}{3\sqrt{26}}, \frac{7}{3\sqrt{26}}\right)$

$d)$ $q_1 = \left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right),\quad q_2 = \left(\frac{3\sqrt{5}}{10}, \frac{3\sqrt{5}}{10}, \frac{\sqrt{5}}{10}, \frac{\sqrt{5}}{10}\right)\quad q_3 = \left(-\frac{\sqrt{6}}{6}, 0, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}\right)$

4. $a)$ $Q = \begin{pmatrix} \frac{1}{2} & -\frac{3}{\sqrt{12}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{6}} \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{6}} \end{pmatrix}\quad R = \begin{pmatrix} 2 & \frac{3}{2} & 1 \\ 0 & \frac{3}{\sqrt{12}} & \frac{2}{\sqrt{12}} \\ 0 & 0 & \frac{2}{\sqrt{6}} \end{pmatrix}$

$b)$ $Q = \begin{bmatrix} \frac{1}{2} & \frac{3\sqrt{5}}{10} & -\frac{\sqrt{6}}{6} \\ -\frac{1}{2} & \frac{3\sqrt{5}}{10} & 0 \\ -\frac{1}{2} & \frac{\sqrt{5}}{10} & \frac{\sqrt{6}}{6} \\ \frac{1}{2} & \frac{\sqrt{5}}{10} & \frac{\sqrt{6}}{3} \end{bmatrix}\quad R = \begin{pmatrix} 2 & 1 & \frac{1}{2} \\ 0 & \sqrt{5} & \frac{3\sqrt{5}}{2} \\ 0 & 0 & \frac{\sqrt{6}}{2} \end{pmatrix}$

5. $a)$ *Orthogonal* $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$b)$ *Orthogonal* $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$c)$ *Orthogonal* $\begin{pmatrix} \cos\theta\sin\theta & \cos^2\theta & \sin\theta \\ -\cos\theta & \sin\theta & 0 \\ -\sin^2\theta & -\cos\theta\sin\theta & \cos\theta \end{pmatrix}$