$$Cos(A+B) = cos A cos B - sin A sin B$$

$$Sin(A+B) = sin A cos B + cos A sin B$$

$$Cos\,75° = cos(45° + 30°)$$

$$= cos\,45° cos\,30° - sin\,45°sin\,30°$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

show $Cos(x+2\pi) = Cos\,x$

$$Cos(x+2\pi) = Cos\,x\,Cos\,2\pi - sin\,x\,\underset{=0}{\underline{sin\,2\pi}}$$

$$= Cos\,x \checkmark$$

$$Cos\,3x\,Cos\,2x - sin\,3x\,sin\,2x = Cos(3x+2x)$$

$$= Cos\,5x$$

$$Cos(90°-x) = sin\,x$$

$$Cos(90°-x) = \underset{=0}{cos\,90°}\,Cos\,x + \overset{=1}{sin\,90°}\,sin\,x$$

$$= sin\,x$$

Ex) $\cos 15° = ?$

$$\cos 15° = \cos(45° - 30°)$$
$$= \cos 45° \cos 30° + \sin 45° \sin 30°$$
$$= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2}$$
$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$\underline{Ex}$ $\sin A = \dfrac{3}{5}$, $A \in I$ / $\cos B = -\dfrac{5}{13}$ $B \in III$

$$\cos A = \dfrac{4}{5} \qquad\qquad \sin B = -\dfrac{12}{13}$$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \dfrac{3}{5}\left(\dfrac{-5}{13}\right) + \left(\dfrac{4}{5}\right)\left(-\dfrac{12}{13}\right)$$

$$= \dfrac{-15-48}{65}$$

$$= -\dfrac{63}{65}$$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \dfrac{4}{5}\left(\dfrac{-5}{13}\right) - \left(\dfrac{3}{5}\right)\left(-\dfrac{12}{13}\right)$$

$$= \dfrac{-20+36}{65}$$

$$= \dfrac{16}{65}$$

c) $\tan(A+B) = -\dfrac{63}{16}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= \dfrac{3}{5}\left(\dfrac{-5}{13}\right) - \left(\dfrac{4}{5}\right)\left(-\dfrac{12}{13}\right)$$

$$= \dfrac{-15+48}{65}$$

$$= \dfrac{33}{65}$$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$= \dfrac{-20-36}{65}$$

$$= \dfrac{-56}{65}$$

f) $\tan(A-B) = -\dfrac{33}{56}$

5.
$$\frac{\cos(x-y)}{\sin x \sin y} = \cot x \cot y + 1$$

$$\frac{\cos(x-y)}{\sin x \sin y} = \frac{\cos x \cos y + \sin x \sin y}{\sin x \sin y}$$

$$= \frac{\cos x \cos y}{\sin x \sin y} + \frac{\sin x \sin y}{\sin x \sin y}$$

$$= \cot x \cot y + 1 \checkmark$$

6. Prove: $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

$$\cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y}$$

$$= \frac{\dfrac{\cos x \cos y}{\sin x \sin y} - \dfrac{\sin x \sin y}{\sin x \sin y}}{\dfrac{\sin x \cos y}{\sin x \sin y} + \dfrac{\cos x \sin y}{\sin x \sin y}}$$

$$= \frac{\cot x \cot y - 1}{\cot y + \cot x} \checkmark$$

$$\sec(x-y) = \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \sin^2 y}$$

$$\sec(x-y) = \frac{1}{\cos(x-y)} \cdot \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{(\cos x \cos y + \sin x \sin y)(\cos x \cos y - \sin x \sin y)}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \sin^2 y} \checkmark$$

#9 $\cos A = \frac{15}{17}$ $A \in QI$ $\cos B = -\frac{12}{13}$ $B \in QII$

$\sin A = \frac{8}{17}$ $\sin B = \frac{5}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{8}{17} \cdot \frac{-12}{13} + \frac{15}{17} \cdot \frac{5}{13}$

$= \frac{-96 + 75}{221}$

$= -\frac{21}{221}$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{15}{17} \cdot \frac{-12}{13} - \frac{8}{17} \cdot \frac{5}{13}$

$= \frac{-180 - 40}{221}$

$= -\frac{220}{221}$

c) $\tan(A+B) = \frac{21}{220}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \frac{-96 - 75}{221}$

$= -\frac{171}{221}$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{-180 + 40}{221}$

$= -\frac{140}{221}$

f) $\tan(A-B) = \frac{171}{140}$

#12

$$\frac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B$$

$$\frac{\sin(A-B)}{\cos A \cos B} = \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B}$$

$$= \tan A - \tan B \checkmark$$

15

$$\frac{\cos(x+y)}{\cos(x-y)} = \frac{\cot y - \tan x}{\cot y + \tan x}$$

$$\frac{\cos(x+y)}{\cos(x-y)} = \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y + \sin x \sin y}$$

$$= \frac{\dfrac{\cos x \cos y}{\cos x \sin y} - \dfrac{\sin x \sin y}{\cos x \sin y}}{\dfrac{\cos x \cos y}{\cos x \sin y} + \dfrac{\sin x \sin y}{\cos x \sin y}}$$

$$= \frac{\cot y - \tan x}{\cot y + \tan x} \checkmark$$

#20

$$\csc(x-y) = \frac{\sin x \cos y + \cos x \sin y}{\sin^2 x - \sin^2 y}$$

$$\csc(x-y) = \frac{1}{\sin(x-y)} \cdot \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y + \cos x \sin y}$$

$$= \frac{\sin x \cos y + \cos x \sin y}{(\sin x \cos y - \cos x \sin y)(\sin x \cos y + \cos x \sin y)}$$

$$= \frac{\sin x \cos y + \cos x \sin y}{\sin^2 x \cos^2 y - \cos^2 x \sin^2 y}$$

$$= \frac{\sin x \cos y + \cos x \sin y}{\sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y}$$

$$= \frac{\sin x \cos y + \cos x \sin y}{\sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y}$$

$$= \frac{\sin x \cos y + \cos x \sin y}{\sin^2 x - \sin^2 y} \quad \checkmark$$

2.3

$$\boxed{\sin 2A = \sin(A + A)}$$
$$= \sin A \cos A + \cos A \sin A$$
$$= 2 \sin A \cos A$$

$$\cos 2A = \cos(A + A)$$
$$= \cos A \cos A - \sin A \sin A$$
$$= \cos^2 A - \sin^2 A$$

$$= \cos^2 A - (1 - \cos^2 A)$$
$$= 2\cos^2 A - 1$$

$$= 1 - \sin^2 A - \sin^2 A$$
$$= 1 - 2\sin^2 A$$

$$\cos 2A = 2\cos^2 A - 1$$
$$\cos 2A + 1 = 2\cos^2 A \implies \boxed{\cos^2 A = \frac{1 + \cos 2A}{2}}$$
$$\boxed{\sin^2 A = \frac{1 - \cos 2A}{2}}$$

$$(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$$

$$(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$$

$$= 1 + \sin 2\theta \checkmark$$

Prove

$$\sin 2x = \frac{2\cot x}{1 + \cot^2 x}$$

$$2\frac{\cot x}{1 + \cot^2 x} = 2\frac{\frac{\cos x}{\sin x}}{1 + \frac{\cos^2 x}{\sin^2 x}}$$

$$= 2\frac{\frac{\cos x}{\sin x}}{\frac{\{\sin^2 x + \cos^2 x\}}{\sin^2 x}} \rightarrow 1$$

$$= 2\frac{\cos x}{\sin x} \cdot \sin^2 x$$

$$= 2\cos x \sin x$$

$$= \sin 2x \checkmark .$$

$$\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$$

$$\cos(4x) = \cos 2(2x)$$

$$= 2\cos^2 2x - 1 \qquad (\cos 2x)^2$$

$$= 2(2\cos^2 x - 1)^2 - 1$$

$$= 2(4\cos^4 x - 4\cos^2 x + 1) - 1$$

$$= 8\cos^4 x - 8\cos^2 x + 1 \qquad 2-1$$

$$\tan 2A = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$= \frac{2\tan A}{1 - \tan^2 A}$$