

Inverse Functions

$$f(x) = \frac{ax+b}{cx+d} \Rightarrow f^{-1}(x) = \frac{-dx+b}{cx-a}$$

Proof

$$y = \frac{ax+b}{cx+d}$$

$$x = \frac{ay+b}{cy+d}$$

$$cxy + dx = ay + b$$

$$cxy - ay = -dx + b$$

$$(cx - a)y = -dx + b$$

$$y = \frac{-dx + b}{cx - a}$$

$$\boxed{f^{-1}(x) = \frac{-dx + b}{cx - a}} \quad \checkmark$$

Interchange ***a*** and ***d*** and change there signs.

Example

Find the inverse function of: $f(x) = \frac{1}{3x-2}$

Solution

$$f^{-1}(x) = \frac{2x+1}{3x}$$

$$f(x) = \frac{0x+1}{3x-2}$$

Example

Find the inverse function of: $f(x) = \frac{3x+2}{2x-5}$

Solution

$$f^{-1}(x) = \frac{5x+2}{2x-3}$$

$$f(x) = \frac{3x+2}{2x-5}$$

Example

Find the inverse function of: $f(x) = \frac{4x}{x+2}$

Solution

$$f^{-1}(x) = \frac{-2x}{x-4}$$

$$f(x) = \frac{4x}{x+2}$$