Ex Given: $y = x^2 + 1$ $y = -x + 3$

rev. x-axis $\to dx$

Find: Vol?

$\boxed{\begin{matrix} a+b+c=0 \\ x=1,\ c/a \end{matrix}}$

$\boxed{\begin{matrix} a-b+c=0 \\ x=-1 \\ -c/a \end{matrix}}$

Soln $y = x^2 + 1 = -x + 3$

$x^2 + x - 2 = 0 \Rightarrow \boxed{x = 1, -2}$

$$V = \pi \int_{-2}^{1} [(-x+3)^2 - (x^2+1)^2]\, dx$$

$$= \pi \int_{-2}^{1} (x^2 - 6x + 9 - x^4 - 2x^2 - 1)\, dx$$

$$= \pi \int_{-2}^{1} (-x^4 - x^2 - 6x + 8)\, dx$$

$$= \pi \left(-\frac{1}{5}x^5 - \frac{1}{3}x^3 - 3x^2 + 8x \right|_{-2}^{1}$$

$$= \pi \left[-\frac{1}{5} - \frac{1}{3} - 3 + 8 - \left(\frac{32}{5} + \frac{8}{3} - 12 - 16\right)\right]$$

$$= \pi \left(-\frac{33}{5} + 2 + 28\right)$$

$$= \boxed{\frac{117\pi}{5} \text{ unit}^3}$$

Ex Given: $y = x^2$, $y = 2x$ QI y-axis $\underline{}$ dy

Find: Vol?

Soln $y = x^2 \to x = \sqrt{y}$ (QI)

$y = 2x \to x = \frac{1}{2}y$

$y = x^2 = 2x \Rightarrow \begin{cases} x = 0 \to y = 0 \\ x = 2 \to y = 4 \end{cases}$

$$V = \pi \int_0^4 \left[(\sqrt{y})^2 - (\tfrac{1}{2}y)^2 \right] dy$$

$$= \pi \int_0^4 \left(y - \tfrac{1}{4}y^2 \right) dy$$

$$= \pi \left(\tfrac{1}{2}y^2 - \tfrac{1}{12}y^3 \right|_0^4$$

$$= \pi \left(8 - \tfrac{16}{3} \right)$$

$$= \frac{8\pi}{3} \text{ unit}^3$$

$\frac{4^3}{3 \cdot 4}$

Boundary: $0 \to 4$ dy

select $1 \to$

$x = (1)^2 - (\tfrac{1}{2})^2$

Ex Given: $f(x) = \sqrt{x}$ $g(x) = x^2$ $0 \le x \le 1$

Find: Vol: x-axis $\to dx$

soln

$$V = \pi \int_0^1 (x - x^4)\, dx$$

$\tfrac{1}{4}$

$\tfrac{1}{2} > \tfrac{1}{16}$

$(\sqrt{x})^2 - (x^2)^2$

$$= \pi \left(\tfrac{1}{2}x^2 - \tfrac{1}{5}x^5 \right|_0^1$$

$$= \pi \left(\tfrac{1}{2} - \tfrac{1}{5} \right)$$

$$= \frac{3\pi}{10} \text{ unit}^3$$

1.4 Shell Method vs Washer Method

$$V = 2\pi \int_a^b (\text{shell Rad})(\text{shell height})\, dx$$

$$= 2\pi \int^b x \; f(x) \; dx$$

$\to$ y-axis

$$= 2\pi \int_c^d y \; g(y) \, dy$$ $\rightarrow$ x-axis

Ex $f(x) = \sin x^2$ $\quad y=0, \; x = \sqrt{\frac{\pi}{2}}$

Vol? y-axis $\rightarrow dx$

Soln

$$V = 2\pi \int_0^{\sqrt{\pi/2}} x \sin x^2 \, dx \qquad \boxed{d(x^2) = 2x\,dx}$$

$$= \pi \int_0^{\sqrt{\pi/2}} \sin(x^2) \, d(x^2)$$

$$= -\pi \cos x^2 \Big|_0^{\sqrt{\pi/2}}$$

$$= -\pi (0 - 1)$$

$$= \pi \text{ unit}^3$$

Ex $y = \sqrt{x-2}$ $\quad y = 2$

a) x-axis $\rightarrow dy$ shell method.

$$y = \sqrt{x-2} \Rightarrow y^2 = x - 2$$
$$x = y^2 + 2$$

$$V = 2\pi \int_0^2 y\,(y^2 + 2) \, dy$$

$$= 2\pi \int_0^2 (y^3 + 2y) \, dy$$

$$= 2\pi \left(\frac{1}{4} y^4 + y^2 \Big|_0^2 \right.$$

$$= 2\pi (4 + 4)$$

$$= \boxed{16\pi \text{ unit}^3}$$

b) $y = -2$

$$V = 2\pi \int_0^2 (y+2)(y^2+2)\,dy$$

$$= 2\pi \int_0^2 (y^3+2y^2+2y+4)\,dy$$

$$= 2\pi \left(\frac{1}{4}y^4 + \frac{2}{3}y^3 + y^2 + 4y\right)\Big|_0^2$$

$$= 2\pi \left(4 + \frac{16}{3} + 4 + 8\right)$$

$$= 32\pi\left(1 + \frac{1}{3}\right)$$

$$= \boxed{\frac{128\pi}{3} \text{ unit}^3}$$

Ex $f(x) = 2x - x^2 \quad g(x) = x \quad [0, 1]$

$1 - \frac{1}{4} \qquad \frac{1}{2}$

$V?$ x-axis

Soln

$$2x - x^2 = x \Rightarrow x = 0$$
$$2 - x = 1 \Rightarrow x = 1$$

Washer Method (dx)

$$V = \pi \int_0^1 \left[(2x - x^2)^2 - x^2\right] dx$$

$$= \pi \int_0^1 (3x^2 - 4x^3 + x^4)\,dx$$

$$= \pi \left(x^3 - x^4 + \frac{1}{5}x^5\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{\ }\right)$$

$$= \frac{\pi}{5} \text{ unit}^3$$

shell Method $\rightarrow dy$ $\quad 0 < x \le 1$

$y = 2x - x^2$ $\qquad$ $y = x$ $\qquad$ $1 + \cancel{x} > 1$

$x^2 - 2x + y = 0$

$$x = \frac{2 \pm \sqrt{4 - 4y}}{2} = 1 \pm \sqrt{1-y}$$

$x = 0 \rightarrow y = 0$ $\qquad$ $\boxed{x = 1 - \sqrt{1-y}}$

$x = 1 \rightarrow y = 1$

$$V = 2\pi \int_0^1 y\left[y - (1 - \sqrt{1-y})\right] dy$$

$$= 2\pi \int_0^1 \left[y^2 - y + y(1-y)^{1/2}\right] dy$$

$$\int y(1-y)^{1/2} dy \qquad \begin{cases} u = 1-y \\ du = -dy \\ y = 1-u \end{cases}$$

$$= \int (1-u)\, u^{1/2} (-dy)$$

$$= -\int (u^{1/2} - u^{3/2})\, dy$$

$$= -\left(\frac{2}{3}(1-y)^{3/2} - \frac{2}{5}(1-y)^{5/2}\right)$$

$$= 2\pi \left(\frac{1}{3}y^3 - \frac{1}{2}y^2 - \frac{2}{3}(1-y)^{3/2} + \frac{2}{5}(1-y)^{5/2}\right)\Bigg|_0^1$$

$$= 2\pi\left[\frac{1}{3} - \frac{1}{2} - \left(-\frac{2}{3} + \frac{2}{5}\right)\right]$$

$$= 2\pi \left(1 - \frac{1}{2} - \frac{2}{5}\right)$$

$$= 2\pi \left(\frac{1}{2} - \frac{2}{5}\right)$$

$$\boxed{= \frac{\pi}{5} \text{ unit}^3}$$

$$\frac{5-4}{10}$$

$f(x) = 2x - x^2$ $\qquad g(x) = x$

$dx \rightarrow$ { x-axis → washer. / y-axis → shell.

$dy \rightarrow$ { x-axis → shell. / y-axis → washer.

1.5 Length (2 Exam)

$$L = \int_a^b \sqrt{1 + (f')^2}\, dx$$

Ex L? $\quad y = \frac{4\sqrt{2}}{3} x^{3/2} - 1 \quad 0 \le x \le 1$

$$\frac{dy}{dx} = 2\sqrt{2}\, x^{1/2}$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + 8x}$$

$$L = \int_0^1 (1 + 8x)^{1/2}\, dx$$

$d(1+8x) = 8dx$

$$= \frac{1}{8}\int_0^1 (1+8x)^{1/2}\, d(1+8x)$$

$$= \frac{1}{12}(1+8x)^{3/2}\Big|_0^1$$

$$= \frac{1}{12}(27-1)$$

$$= \frac{13\pi}{6} \text{ unit}$$

$$y = ax^m + bx^n$$

$$\begin{cases} ① & m+n=2 \\ ② & abmn = -\frac{1}{4} \end{cases}$$

$$L = \int_c^d \sqrt{1+(y')^2}\,dx$$

$$= ax^m - bx^n \Big|_c^d$$

$$y = ae^{mx} + be^{nx}$$

$$\begin{cases} ① & m=-n \\ ② & ambn = -\frac{1}{4} \end{cases}$$

$$L = ae^{mx} - be^{nx}\Big|_c^d$$

Ex $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \le x \le 4$

Soln

$$\begin{cases} m+1 = 3-1 = 2 \checkmark \\ a\,m\,b\,n = \frac{1}{12}(3)(1)(-1) = -\frac{1}{4} \checkmark \end{cases}$$

$$L = \frac{x^3}{12} - \frac{1}{x}\Big|_1^4$$

$$= \frac{16}{3} - \frac{1}{4} - \left(\frac{1}{12} - 1\right)$$

$$= \frac{61}{12} - \frac{1}{12} + 1$$

$$= 6 \text{ unit}$$

Ex #9 $y = 2e^{x} + \frac{1}{8}e^{-x} \quad 0 \le x \le \ln 2$

Soln

$$\begin{cases} m = -n \checkmark \\ a\,b\,m\,n = 2(1)\left(\frac{1}{8}\right)(-1) = -\frac{1}{4} \checkmark \end{cases}$$

$$L = 2e^{x} - \frac{1}{8}e^{-x}\Big|_0^{\ln 2}$$

$$\boxed{e^{\ln x} = x}$$

$$e^{-\ln x} = \frac{1}{x}$$

$$= 2e^{\ln 2} - \frac{1}{8}e^{-\ln 2} - \left(2 - \frac{1}{8}\right)$$

$$= 2(2) - \frac{1}{16} - 2 + \frac{1}{8}$$

$$= \frac{63}{16} - \frac{15}{8}$$

$$= \frac{33}{16} \text{ unit}$$

Ex $y = \ln\left(x + \sqrt{x^2+1}\right) \quad [1, \sqrt{2}]$

$$L = \int^{b} \sqrt{1+(y')^2}\,dx$$

$$L = \int_a \sqrt{1+(y')^2}\,dx$$

or $\displaystyle\int_c^d \sqrt{1+(x')^2}\,dy$

$$y = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$x + \sqrt{x^2-1} = e^y$$

$$\left(\sqrt{x^2-1}\right)^2 = (e^y - x)^2$$

$$x^2 - 1 = e^{2y} - 2xe^y + x^2$$

$$-1 = e^{2y} - 2xe^y$$

$$2xe^y = e^{2y} + 1$$

$$x = \frac{1}{2}\left(e^y + e^{-y}\right)$$

$$= \frac{1}{2}e^y + \frac{1}{2}e^{-y}$$

$$\begin{cases} m = -n \checkmark \\ a_m b_n = \frac{1}{2}(1)\left(+\frac{1}{2}\right)(-1) = -\frac{1}{4} \checkmark \end{cases}$$

$$L = \frac{1}{2}e^y - \frac{1}{2}e^{-y}\Big|_0^{\ln(\sqrt{2}+1)} \quad \begin{cases} x=1 \to y=0 \\ x=\sqrt{2} \to y=\ln(\sqrt{2}+1) \end{cases}$$

$$= \frac{1}{2}(\sqrt{2}+1) - \frac{1}{2}\left(\frac{1}{(\sqrt{2}+1)}\right) - \left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{1}{2(1+\sqrt{2})}$$

$$= \frac{\sqrt{2}+1}{2} - \frac{1}{2(\sqrt{2}+1)}$$

$$= \frac{1}{2}\left(\frac{2+2\sqrt{2}+1-1}{\sqrt{2}+1}\right)$$

$$= \frac{1}{2}(2)$$

$$= 1 \text{ unit}$$

Ex $\quad f(x) = \frac{1}{12}x^3 + \frac{1}{x} \qquad A = \left(1, \frac{13}{12}\right)$ initial

soln

$m+n = 3-1 = 2$ ✓

$a_m b_n = \frac{1}{12}(3)(1)(-1) = -\frac{1}{4}$ ✓

$$L = \frac{1}{12}x^3 - \frac{1}{x}\Big|_1^x$$

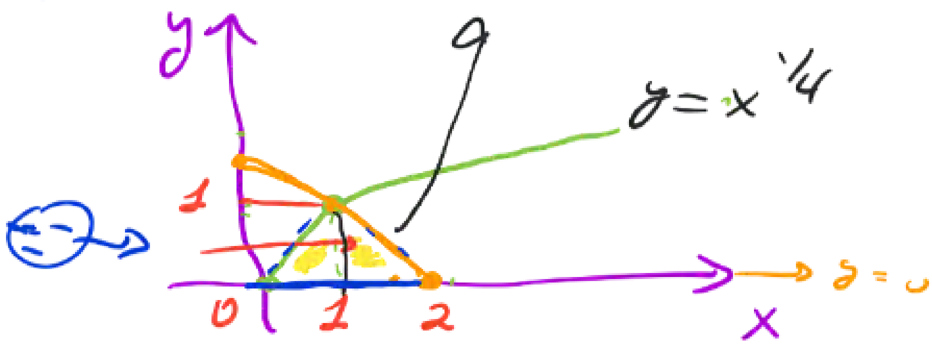
$$= \frac{1}{12}x^3 - \frac{1}{x} - \left(\frac{1}{12} - 1\right)$$

$$= \frac{1}{12}x^3 - \frac{1}{x} + \frac{11}{12} \text{ unit}$$

Hwk #3 $\quad y^4 = x \qquad y = \sqrt{2-x} \qquad y = 0$

$2 - x = y^2$

$x = 2 - y^2$



$A \quad \int^1 \quad {}^{1/4} \qquad \int^2$

$$A = \int_0 x \, dx + \int_1 \sqrt{2-x^2}\, dx$$

or

$$A = \int_0^1 (2-y^2-y^4)\, dy.$$

3 fctns Plot