Ex $y = x^2$ @ O

Soln $x = t \Rightarrow y = t^2$

$\vec{r} = t\hat{i} + t^2\hat{j}$

$\vec{v} = \hat{i} + 2t\hat{j}$

$|\vec{v}| = \sqrt{1+4t^2}\Big|_{t=0} = 1$

$$\left(\frac{1}{u^n}\right)' = -\frac{nu'}{u^{n+1}}$$

$$\left(\frac{u}{v}\right)' =$$

$$(u^n v^m)' = u^{n-1}v^{m-1}(nu'v + mu\ldots$$

$$\vec{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{1+4t^2}}\hat{i} + \frac{2t}{\sqrt{1+4t^2}}\hat{j}$$

$$\frac{d\vec{T}}{dt} = \frac{-4t}{(1+4t^2)^{3/2}}\hat{i} + 2\,\frac{1+4t^2-\frac{1}{2}(8t)t}{(1+4t^2)^{3/2}}\hat{j}$$

$$= \frac{4t}{(1+4t^2)^{3/2}}\hat{i} + \frac{2}{(1+4t^2)^{3/2}}\hat{j}$$

@ origin $\Rightarrow t = 0$

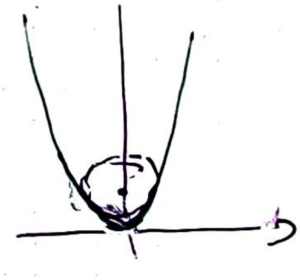
$$\left.\frac{dT}{dt}\right|_{t=0} = 2\hat{j}$$

$$\left|\frac{dT}{dt}\right| = 2$$

$$\kappa = \frac{|dT/dt|}{|\vec{v}|} = 2$$

radius of curvature $\rho = \frac{1}{\kappa} = \frac{1}{2}$

$\vec{T}\big|_{t=0} = \hat{i} \rightarrow \vec{N} = \hat{j}$



Ex $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$

$a, b > 0 \qquad a^2+b^2 \neq 0$

Soln

$\vec{v}(t) = -a\sin t\,\hat{i} + a\cos t\,\hat{j} + b\,\hat{k}$

$|\vec{v}(t)| = \sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2}$

$= \sqrt{a^2+b^2}$

$\vec{T} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{1}{\sqrt{a^2+b^2}}\left(-a\sin t\,\hat{i} + a\cos t\,\hat{j} + b\hat{k}\right)$

$\dfrac{d\vec{T}}{dt} = \dfrac{1}{\sqrt{a^2+b^2}}\left(-a\cos t\,\hat{i} - a\sin t\,\hat{j}\right)$

$= \boxed{\dfrac{-a}{\sqrt{a^2+b^2}}\left(\cos t\,\hat{i} + \sin t\,\hat{j}\right)}$

$\left|\dfrac{d\vec{T}}{dt}\right| = \dfrac{a}{\sqrt{a^2+b^2}}\sqrt{\cos^2 t + \sin^2 t}$

$= \dfrac{a}{\sqrt{a^2+b^2}}$

$\kappa = \dfrac{|d\vec{T}/dt|}{|\vec{v}|} = \dfrac{a}{\sqrt{a^2+b^2}} \cdot \dfrac{1}{\sqrt{a^2+b^2}}$

$= \dfrac{a}{a^2+b^2}$

$\rho = \dfrac{a^2+b^2}{a}$

$\vec{N} = \dfrac{d\vec{T}/dt}{|d\vec{T}/dt|} = -\cos t\,\hat{i} - \sin t\,\hat{j}$

Defn

$$\vec{a}(t) = a_T \vec{T} + a_N \vec{N}$$

$$a_T = \frac{d^2 s}{dt} = \frac{d|\vec{v}|}{dt}$$

$$a_N = \kappa |\vec{v}|^2$$

$$= \sqrt{|\vec{a}|^2 - a_T^2}$$

Ex w/o finding $\vec{T}$ & $\vec{N}$ $\quad \vec{a} = a_T \vec{T} + a_N \vec{N}$

$$\vec{r}(t) = (\cos t + t \sin t)\,\hat{i} + (\sin t - t\cos t)\,\hat{j}$$

$$(t > 0)$$

soln

$$\vec{v}(t) = (\underbrace{-\sin t + \sin t}\, + t\cos t)\,\hat{i} + (\cos t - \cos t + t\sin t)\,\hat{j}$$

$$= t\cos t\,\hat{i} + t\sin t\,\hat{j}$$

$$|\vec{v}(t)| = \sqrt{t^2\cos^2 t + t^2 \sin^2 t}$$

$$= t$$

$$\boxed{a_T = \frac{d}{dt}|\vec{v}(t)| = 1}$$

$$\vec{a}(t) = (\cos t - t\sin t)\,\hat{i} + (\sin t + t\cos t)\,\hat{j}$$

$$|\vec{a}|^2 = (\cos t - t\sin t)^2 + (\sin t + t\cos t)^2$$

$$= \cos^2 t - 2t\sin t\cos t + t^2\sin^2 t$$

$$+ \sin^2 t + 2t\sin t\cos t + t^2\cos^2 t$$

$$= 1 + t^2$$

$$\boxed{a_N = \sqrt{1 + t^2 - 1} = t}$$

$$\boxed{\vec{a} = \vec{T} + t\vec{N}}$$

1-8

15 $\vec{r}(t) = e^t\cos t\,\hat{i} + (e^t\sin t)\hat{j} + \sqrt{2}\,e^t\,\hat{k}$ $\quad t=0$

$\vec{v}(t) = e^t(\cos t - \sin t)\,\hat{i} + e^t(\sin t + \cos t)\,\hat{j} + \sqrt{2}\,e^t\,\hat{k}$

$$\boxed{\cancel{\begin{aligned}\vec{v}(0) &= \hat{i} + \hat{j} + \sqrt{2}\,\hat{k}\\ |\vec{v}(0)| &= \sqrt{1+1+2} = 2\end{aligned}}}$$

$|\vec{v}| = e^t\sqrt{(\cos t - \sin t)^2 + (\sin t + \cos t)^2 + 2}$

$= e^t\sqrt{\cos^2 t - 2\cos t\sin t + \sin^2 t + \sin^2 t + 2\cos t\sin t + \cos^2 t + 2}$

$= e^t\sqrt{4}$

$= 2e^t$

$\vec{a}_T = \frac{d}{dt}|\vec{v}|$

$= 2e^t \big|_{t=0}$

$= 2$

$\vec{a}(t) = e^t(\cos t - \sin t - \sin t - \cos t)\,\hat{i}$

$+ e^t(\sin t + \cos t + \cos t - \sin t)\,\hat{j}$

$+ \sqrt{2}\,e^t\,\hat{k}$

$= e^t\left(-2\sin t\,\hat{i} + 2\cos t\,\hat{j} + \sqrt{2}\,\hat{k}\right)$

$|\vec{a}|^2 = e^t\left(4\sin^2 t + 4\cos^2 t + 2\right)$

$= 6e^t\big|_{t=0}$

$= 6$

$\vec{a}_N = \sqrt{6-4} = \sqrt{2}$

$\vec{a} = 2\vec{T} + \sqrt{2}\,\vec{N}$

#16 $\vec{r}(t) = (2+3t+3t^2)\hat{i} + (4t+4t^2)\hat{j} - 6\cos t\,\hat{k}$

$t=0$

$\vec{v}(t) = (3+6t)\hat{i} + (4+8t)\hat{j} + 6\sin t\,\hat{k}$

$|\vec{v}| = \sqrt{(3+6t)^2 + (4+8t)^2 + 36\sin^2 t}$

$= \sqrt{9+36t+36t^2+16+64t+64t^2+36\sin^2 t}$

$= \sqrt{100t^2+100t+25+36\sin^2 t}$

$= \sqrt{(10t+5)^2+36\sin^2 t}$ $\quad (u)^{1/2}$

$\frac{1}{2}u'u^{-1/2}$

$\frac{d|\vec{v}|}{dt} = \frac{1}{2}\,\frac{20(10t+5)+72\sin t\cos t}{\sqrt{(10t+5)^2+36\sin^2 t}}\Bigg|_{t=0}$

$= \frac{1}{2}\,\frac{100}{5}$

$a_T = \boxed{10}$

$\vec{a}(t) = 6\hat{i} + 8\hat{j} + 6\cos t\,\hat{k}$

$|\vec{a}|^2 = 36+64+36\cos^2 t\,\Big|_{t=0}$

$= 136$

$a_N = \sqrt{136-100}$

$= 6$

$\boxed{\vec{a} = 10\vec{T} + 6\vec{N}}$

#35 $\vec{r}(t) = \cos t\,\hat{i} + 2\cos t\,\hat{j} + \sqrt{5}\sin t\,\hat{k}$

a) $\vec{v} = -\sin t\,\hat{i} - 2\sin t\,\hat{j} + \sqrt{5}\cos t\,\hat{k}$

$|\vec{v}| = \sqrt{\sin^2 t + 4\sin^2 t + 5\cos^2 t}$

$= \sqrt{5}$

$\boxed{\vec{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{5}}\left(-\sin t\,\hat{i} - 2\sin t\,\hat{j} + \sqrt{5}\cos t\,\hat{k}\right)}$

b) $\kappa = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$

$\frac{d\vec{T}}{dt} = \frac{1}{\sqrt{5}}\left(-\cos t\,\hat{i} - 2\cos t\,\hat{j} - \sqrt{5}\sin t\,\hat{k}\right)$

$\left|\frac{d\vec{T}}{dt}\right| = \frac{1}{\sqrt{5}}\sqrt{\cos^2 t + 4\cos^2 t + 5\sin^2 t}$

$= 1$

$\boxed{\kappa = \frac{1}{\sqrt{5}}}$

c) $\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|}$

$= \frac{1}{\sqrt{5}}\left(-\cos t\,\hat{i} - 2\cos t\,\hat{j} - \sqrt{5}\sin t\,\hat{k}\right)$

d) Prove $|\vec{N}| = 1$

$|\vec{N}| = \frac{1}{\sqrt{5}}\sqrt{\cos^2 t + 4\cos^2 t + 5\sin^2 t}$

$= \frac{1}{\sqrt{5}}\sqrt{5\cos^2 t + 5\sin^2 t}$

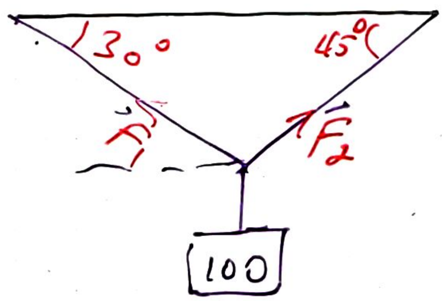
$= 1$ ✓.

$\vec{T}\cdot\vec{N} = \frac{1}{\sqrt{5}}\left(-\sin t\,\hat{i} - 2\sin t\,\hat{j} + \sqrt{5}\cos t\,\hat{k}\right)$

$\cdot \frac{1}{\sqrt{5}}\left(-\cos t\,\hat{i} - 2\cos t\,\hat{j} - \sqrt{5}\sin t\,\hat{k}\right)$

$$\vec{T} \cdot \vec{N} = \frac{1}{5}(\sin t \cos t + 4 \sin t \cos t - 5 \cos t \sin t)$$
$$= \frac{1}{5}(5 \sin t \cos t - 5 \cos t \sin t)$$
$$= 0 \checkmark.$$

1.1 # 28



$$\vec{F_1} = \langle -|\vec{F_1}|\cos 30^\circ,\ |\vec{F_1}|\sin 30^\circ \rangle$$

$$= \langle -\tfrac{\sqrt{3}}{2}|\vec{F_1}|,\ \tfrac{1}{2}|\vec{F_1}| \rangle$$

$$\vec{F_2} = \langle |\vec{F_2}|\cos 45^\circ,\ |\vec{F_2}|\sin 45^\circ \rangle$$

$$= \langle \tfrac{\sqrt{2}}{2}|\vec{F_2}|,\ \tfrac{\sqrt{2}}{2}|\vec{F_2}| \rangle$$

$$\vec{W} = \langle 0, -100 \rangle$$

$$\begin{cases} -\frac{\sqrt{3}}{2}|\vec{F_1}| + \frac{\sqrt{2}}{2}|\vec{F_2}| = 0 \\ \frac{1}{2}|\vec{F_1}| + \frac{\sqrt{2}}{2}|\vec{F_2}| = 100 \end{cases}$$

$$\begin{cases} \overset{+}{-}\sqrt{3}|\vec{F_1}| \overset{-}{+}\sqrt{2}|\vec{F_2}| = 0 \quad (1) \\ |\vec{F_1}| + \sqrt{2}|\vec{F_2}| = 200 \end{cases}$$

$$(\sqrt{3}+1)|\vec{F_1}| = 200$$

$$|\vec{F_1}| = \frac{200}{1+\sqrt{3}}$$

$$(1)\ |\vec{F_2}| = \frac{\sqrt{3}}{\sqrt{2}}|\vec{F_1}|$$

$$= \frac{\sqrt{6}}{2}\,\frac{200}{1+\sqrt{3}}$$

$$= \frac{100\sqrt{6}}{1+\sqrt{3}}$$

$$\vec{F_1} = \langle -\frac{100\sqrt{3}}{1+\sqrt{3}},\ \frac{100}{1+\sqrt{3}} \rangle$$

$$\vec{F_2} = \langle \frac{100\sqrt{3}}{1+\sqrt{3}},\ \frac{100\sqrt{3}}{1+\sqrt{3}} \rangle$$