functions odd & even. $f(x)$

if $f(x)$ is even $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

odd $\Rightarrow$ $\int_{-a}^{a} f(x)\,dx = 0.$

$$\int_{-1}^{1} (x^4 - x^2)\,dx = 2\left(\frac{1}{5}x^5 - \frac{1}{3}x^3\right)\Big|_0^1$$

$$= 2\left(\frac{1}{5} - \frac{1}{3}\right)$$

$$= -\frac{4}{15}$$

$$\int_{-2}^{2} (x^4 - 4x^2 + 6)\,dx = 2\int_0^2 (x^4 - 4x^2 + 6)\,dx$$

$$= 2\left(\frac{1}{5}x^5 - \frac{4}{3}x^3 + 6x\right)\Big|_0^2$$

$$= 2\left(\frac{32}{5} - \frac{32}{3} + 12\right)$$

$$= 2\left(-\frac{64}{15} + 12\right)$$

$$= \frac{232}{15}$$

$$\int_{-2}^{2} (3x^4 - 2x + 1)\,dx = 2\left(\frac{3}{5}x^5 + x\right|_0^2$$

$$= 2\left(\frac{96}{5} + 2\right)$$

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$$= \boxed{\frac{212}{5}}$$

#5 $$\int_{-200}^{200} 2x^5\,dx = 0$$

#6 $$\int_{-\pi/4}^{\pi/4} \cos x\,dx = 2\left(\sin x\right|_0^{\pi/4}$$

$$= \sqrt{2}$$

4.6 Substitution Rule

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

Ex $\int (x^3+x)^5 \underline{(3x^2+1)dx}$

$u = x^3 + x$

$du = \underbrace{(3x^2+1)dx}$

$$\int (x^3+x)^5 (3x^2+1)dx = \int u^5 \, du$$

$$= \frac{1}{6} u^6 + C$$

$$= \frac{1}{6}(x^3+x)^6 + C$$

$$\int (x^3+x)^5 (3x^2+1)dx = \int (x^3+x)^5 \, d(x^3+5) \qquad \frac{d(x^3+x)}{dx} = (3x^2+1)$$

(2) (1)

$$= \frac{1}{6}(x^3+x)^6 + C \quad (3)$$

Ex $\int \sqrt{2x+1}\, dx = \frac{1}{2}\int (2x+1)^{1/2} d(2x+1) \qquad \frac{1}{2} d(2x+1) = 2dx \frac{1}{2}$

$$= \frac{1}{3}(2x+1)^{3/2} + C$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx \qquad d(\cos x) = -\sin x\,dx$$

$$= -\int \frac{d(\cos x)}{\cos x} \qquad \int \frac{du}{u} = \ln|u|$$

$$= -\ln|\cos x| + C$$

or

$$= \ln|\cos x|^{-1} + C$$

$$= \boxed{\ln|\sec x| + C}$$

Ex
$$\int_0^2 \frac{2x}{x^2-5}\,dx = \int_0^2 \frac{d(x^2-5)}{x^2-5} \qquad d(x^2-5) = 2x\,dx$$

$$= \ln|x^2-5|\Big|_0^2$$

$$= \ln 1 - \ln 5$$

$$= \boxed{-\ln 5}$$

$$u = x^2-5$$
$$du = 2x\,dx$$

$$x = 2 \rightarrow u = -1$$
$$x = 0 \rightarrow u = -5$$

$$\int_{-5}^{-1} \frac{du}{u} = \ln|u|\Big|_{-5}^{-1} \qquad = \ln|u|\Big|_0^2$$

$$= -\ln 5 \qquad = \ln|x^2-5|\Big|_0^2$$

Ex $\int \sec^2(5t+1)\, 5\,dt =$

$d(5t+1) = 5\,dt$

$\int \sec^2(5t+1)\, d(5t+1) = \tan(5t+1) + C$

$\int e^u\, du = e^u + C$

Ex $\int_0^{\ln 2} e^{3x}\, dx = \frac{1}{3}\int_0^{\ln 2} e^{3x}\, d(3x)$ $\quad d(3x) = 3\,dx$

$= \frac{1}{3} e^{3x} \Big|_0^{\ln 2}$

$= \frac{1}{3}\left(e^{3\ln 2} - 1\right)$ $\quad e^{\ln u} = u$

$= \frac{1}{3}(8-1)$

$= \frac{7}{3}$

$$\int \cos(7\theta+3)\,d\theta = \frac{1}{7}\int \cos(7\theta+3)\,d(7\theta+3) \qquad d(7\theta+3) = 7\,d\theta$$

$$= \boxed{\frac{1}{7}\sin(7\theta+3) + C}$$

$$\int x^2 \sin(x^3)\,dx = \frac{1}{3}\int \sin(x^3)\,d(x^3) \qquad d(x^3) = 3x^2\,dx$$

$$= \boxed{-\frac{1}{3}\cos(x^3) + C}$$

$$\int x\sqrt{2x+1}\,dx \qquad \begin{aligned} u &= 2x+1 \rightarrow x = \frac{u-1}{2} \\ du &= 2\,dx \end{aligned}$$

$$\int x\sqrt{2x+1}\,dx = \int \frac{1}{2}(u-1)\,u^{1/2}\,\overbrace{\frac{1}{2}du}^{dx}$$

$$= \frac{1}{4}\int \left(u^{3/2} - u^{1/2}\right) du$$

$$= \frac{1}{4}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \boxed{\frac{1}{4}\left(\frac{2}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2}\right) + C}$$

$$\int \frac{2z\,dz}{\sqrt[3]{z^2+1}} = \int (z^2+1)^{-1/3}\, d(z^2+1) \qquad d(z^2+1) = 2z\,dz$$

$$= \frac{3}{2}(z^2+1)^{2/3} + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int 2^x\,dx = \frac{2^x}{\ln 2} + C$$

$$\int 2^{\sin x}\cos x\,dx = \int 2^{\sin x}\, d(\sin x) \qquad \left\{ d(\sin x) = \cos x\,dx \right.$$

$$= \frac{2^{\sin x}}{\ln 2} + C$$

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx = \int_{-1}^{1} (x^3+1)^{1/2}\, d(x^3+1) \qquad d(x^3+1) = 3x^2 dx$$

$$= \frac{2}{3}(x^3+1)^{3/2}\Big|_{-1}^{1}$$

$$= \frac{2}{3}\left(2^{3/2}\right)$$

$$= \frac{2^{5/2}}{3} = \frac{4}{3}\sqrt{2}$$

$$\int_{\pi/4}^{\pi/2} \cot\theta \csc^2\theta \, d\theta = -\int_{\pi/4}^{\pi/2} \cot\theta \, d(\cot\theta) \qquad d(\cot\theta) = -\csc^2\theta \, d\theta$$

$$= -\frac{1}{2}(\cot\theta)^2 \Big|_{\pi/4}^{\pi/2}$$

$$= -\frac{1}{2}\left[0 - 1\right]$$

$$= \frac{1}{2}$$

$$\cot\theta\csc\theta\csc\theta\,d\theta \qquad d(\csc\theta) = -\csc\theta\cot\theta\,d\theta$$

$$-\int_{\pi/4}^{\pi/2} \csc\theta \, d(\csc\theta)$$

Ex $\int_0^{\pi/6} \tan 2x \, dx = \frac{1}{2}\int_0^{\pi/6} \tan 2x \, d(2x) \qquad d(2x) = 2dx$

$$= \frac{1}{2} \ln|\sec 2x| \Big|_0^{\pi/6}$$

$$= -\frac{1}{2} \ln|\cos 2x| \Big|_0^{\pi/6}$$

$$= -\frac{1}{2}\left(\ln\frac{1}{2}\right) \qquad \ln\left(\frac{1}{x}\right) = -\ln x$$

$$= \frac{1}{2}\ln 2$$

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$
$$= \tfrac{1}{2} - \tfrac{1}{2}\cos 2x$$

$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$$
$$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$$

$$\int \cos^2 x\,dx = \frac{1}{2}\int (1+\cos 2x)\,dx$$
$$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$$

#6 $\int \frac{ds}{\sqrt{5s+4}} = \frac{1}{5}\int (5s+4)^{-1/2} d(5s+4)$ $\quad d(5s+4) = 5ds$

$= \frac{2}{5}(5s+4)^{1/2} + C$

#7 $\int \theta \sqrt[4]{1-\theta^2}\, d\theta = \frac{1}{2}\int (1-\theta^2)^{1/4} d(1-\theta^2)$ $\quad \{ d(1-\theta^2) = 2\theta d\theta$

$= \frac{2}{5}(1-\theta^2)^{5/4} + C$

#8 $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2} = 2\int \frac{d(1+\sqrt{x})}{(1+\sqrt{x})^2}$ $\quad d(1+\sqrt{x}) = \frac{1}{2\sqrt{x}}dx$

$\int \frac{du}{u^2} = -\frac{1}{u}$

$= -\frac{2}{1+\sqrt{x}} + C$

9/ $\int \tan^2 x \sec^2 x\, dx = \int \tan^2 x\, d(\tan x)$ $\quad d(\tan x) = \sec^2 x\, dx$

$= \frac{1}{3}\tan^3 x + C$

10/ $\int \sin^5 \frac{x}{3} \cos \frac{x}{3}\, dx =$ $\quad d(\sin \frac{x}{3}) = \frac{1}{3}\cos\frac{x}{3}dx$

$= 3\int \sin^5 \frac{x}{3}\, d(\sin \frac{x}{3})$

$= \frac{1}{2}\sin^6\left(\frac{x}{3}\right) + C$

11/ $\int \tan^7\frac{x}{2}\sec^2\frac{x}{2}\,dx$ $\quad d(\tan\frac{x}{2}) = \frac{1}{2}\sec^2\frac{x}{2}\,dx$

$= 2\int \tan^7\frac{x}{2}\,d(\tan\frac{x}{2})$

$= \frac{1}{4}\tan^8(\frac{x}{2}) + C$

12/ $\int r^4(7 - \frac{r^5}{10})^3 dr$ $\quad d(7 - \frac{r^5}{10}) = -\frac{1}{2}r^4 dr$

$= -2\int (7 - \frac{r^5}{10})^3 d(7 - \frac{r^5}{10})$

$= -\frac{1}{2}(7 - \frac{r^5}{10})^4 + C$

13/ $\int x^{1/2}\sin(x^{3/2}+1)\,dx$ $\quad d(x^{3/2}+1) = \frac{3}{2}x^{1/2}dx$

$= \frac{2}{3}\int \sin(x^{3/2}+1)\,d(x^{3/2}+1)$

$= -\frac{2}{3}\cos(x^{3/2}+1) + C$

14/ $\int \csc(\frac{v-\pi}{2})\cot(\frac{v-\pi}{2})\,dv$ $\quad d(\frac{v-\pi}{2}) = \frac{1}{2}dv$

$= 2\int \csc(\frac{v-\pi}{2})\cot(\frac{v-\pi}{2})\,d(\frac{v-\pi}{2})$

$= -2\csc(\frac{v-\pi}{2}) + C$