Solution *Section* 2.7 – Definition of Laplace Transform

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=3$

Solution

$$F(s)=\int_0^\infty 3e^{-st}dt$$

$$=\lim_{T\to\infty}\int_0^T 3e^{-st}dt$$

$$=\lim_{T\to\infty}\left(-\frac{3e^{-st}}{s}\right)_{t=0}^{T}$$

$$=\lim_{T\to\infty}\left(-\frac{3}{s}e^{-sT}+\frac{3}{s}\right) \qquad \lim_{T\to\infty}\left(e^{-sT}\right)=0$$

$$\boxed{=\frac{3}{s}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=t$

Solution

		$\int e^{-st}dt$
+	t	$-\frac{1}{s}e^{-st}$
–	1	$\frac{1}{s^2}e^{-st}$

$$F(s)=\int_0^\infty te^{-st}dt$$

$$=\lim_{T\to\infty}\left(\left(-\frac{t}{s}-\frac{1}{s^2}\right)e^{-st}\right)_{t=0}^{T}$$

$$=\lim_{T\to\infty}\left(\left(-\frac{T}{s}-\frac{1}{s^2}\right)e^{-sT}+\frac{1}{s^2}\right) \qquad \lim_{T\to\infty}\left(e^{-sT}\right)=0$$

$$\boxed{=\frac{1}{s^2}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=t^2$

Solution

$$F(s)=\int_0^\infty t^2 e^{-st}\,dt$$

$$=\left(-\frac{t^2}{s}-\frac{2t}{s^2}-\frac{2}{s^3}\right)e^{-st}\Big|_0^\infty$$

$$=\frac{2}{s^3}$$

		$\int e^{-st}dt$
+	t^2	$-\frac{1}{s}e^{-st}$
−	$2t$	$\frac{1}{s^2}e^{-st}$
+	2	$-\frac{1}{s^3}e^{-st}$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=e^{6t}$

Solution

$$F(s)=\int_0^\infty e^{6t}e^{-st}\,dt$$

$$=\int_0^\infty e^{-(s-6)t}\,dt$$

$$=-\frac{e^{-(s-6)t}}{s-6}\Big|_0^\infty$$

$$=\frac{1}{s-6} \qquad with: s>6$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=e^{-2t}$

Solution

$$F(s)=\int_0^\infty e^{-2t}e^{-st}\,dt$$

$$=\lim_{T\to\infty}\int_0^T e^{-(s+2)t}\,dt$$

$$=\lim_{T\to\infty}\left(\frac{-e^{-(s+2)t}}{s+2}\right)_{t=0}^{T}$$

$$=\lim_{T\to\infty}\left(-\frac{e^{-(s+2)T}}{s+2}+\frac{1}{s+2}\right) \qquad \lim_{T\to\infty}\left(e^{-(s+2)T}\right)=0$$

$$=\frac{1}{s+2} \qquad with: s>-2$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = te^{-3t}$

Solution

$$F(s) = \int_0^\infty te^{-3t}e^{-st}dt$$

$$= \int_0^\infty te^{-(s+3)t}dt$$

		$\int e^{-(s+3)t}dt$
+	t	$-\frac{1}{s+3}e^{-(s+3)t}$
−	1	$\frac{1}{(s+3)^2}e^{-(s+3)t}$

$$F(s) = \left(-\frac{1}{s+3}te^{-(s+3)t} - \frac{1}{(s+3)^2}e^{-(s+3)t}\right)_0^\infty$$

$$= \frac{1}{(s+3)^2} \quad \text{with } s > -3 \qquad e^{-\infty} = 0 \quad e^0 = 1$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = te^{3t}$

Solution

$$F(s) = \int_0^\infty te^{3t}e^{-st}dt$$

$$= \int_0^\infty te^{-(s-3)t}dt$$

		$\int e^{-(s-3)t}dt$
+	t	$-\frac{1}{s-3}e^{-(s-3)t}$
−	1	$\frac{1}{(s-3)^2}e^{-(s-3)t}$

$$F(s) = -\frac{1}{s-3}te^{-(s-3)t} - \frac{1}{(s-3)^2}e^{-(s-3)t}\Big|_0^\infty$$

$$= \frac{1}{(s-3)^2} \quad \text{with } s > 3 \qquad e^{-\infty} = 0 \quad e^0 = 1$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{2t}\cos 3t$

Solution

$$F(s) = \int_0^\infty \left(e^{2t}\cos 3t\right)e^{-st}dt$$

$$= \int_0^\infty e^{-(s-2)t}\cos 3t\ dt$$

		$\int \cos 3t\ dt$
+	$e^{-(s-2)t}$	$\frac{1}{3}\sin 3t$
−	$-(s-2)e^{-(s-2)t}$	$-\frac{1}{9}\cos 3t$
+	$(s-2)^2e^{-(s-2)t}$	$-\frac{1}{9}\int \cos 3t$

$$\int e^{-(s-2)t}\cos 3t\ dt = \tfrac{1}{3}e^{-(s-2)t}\sin 3t - \tfrac{1}{9}(s-2)e^{-(s-2)t}\cos 3t - \tfrac{1}{9}(s-2)^2\int e^{-(s-2)t}\cos 3t\ dt$$

$$\left(1+\tfrac{1}{9}(s-2)^2\right)\int e^{-(s-2)t}\cos 3t\ dt = \tfrac{1}{3}e^{-(s-2)t}\sin 3t - \tfrac{1}{9}(s-2)e^{-(s-2)t}\cos 3t$$

$$\left(9+(s-2)^2\right)\int e^{-(s-2)t}\cos 3t\ dt = 3e^{-(s-2)t}\sin 3t - (s-2)e^{-(s-2)t}\cos 3t$$

$$\int e^{-(s-2)t}\cos 3t\ dt = \frac{1}{9+(s-2)^2}\left[3e^{-(s-2)t}\sin 3t - (s-2)e^{-(s-2)t}\cos 3t\right]$$

$$F(s) = \left(\frac{3}{9+(s-2)^2}e^{-(s-2)t}\sin 3t - \frac{s-2}{9+(s-2)^2}e^{-(s-2)t}\cos 3t\right)_0^{\infty}$$

$$\boxed{= \frac{s-2}{9+(s-2)^2}} \qquad s > 2$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = \sin 3t$

Solution

$$F(s) = \int_0^{\infty} (\sin 3t)e^{-st}dt$$

$$\int \sin 3t\ e^{-st}dt = -\frac{1}{3}e^{-st}\cos 3t - \frac{s}{9}e^{-st}\sin 3t + \frac{s^2}{9}\int e^{-st}\sin 3t\ dt$$

		$\int \sin 3t\ dt$
+	e^{-st}	$-\frac{1}{3}\cos 3t$
−	$-se^{-st}$	$-\frac{1}{9}\sin 3t$
+	s^2e^{-st}	$-\frac{1}{9}\int \sin 3t$

$$\int \sin 3t\ e^{-st}dt + \frac{1}{9}s^2\int \sin 3t\ e^{-st}dt = -\frac{1}{3}e^{-st}\cos 3t - \frac{1}{9}se^{-st}\sin 3t$$

$$\left(9+s^2\right)\int \sin 3t\ e^{-st}dt = -(3\cos 3t - s\sin 3t)e^{-st}$$

$$\int \sin 3t\ e^{-st}dt = -\frac{3\cos 3t - s\sin 3t}{s^2+9}e^{-st}$$

$$F(s) = -\frac{3\cos 3t - s\sin 3t}{s^2+9}e^{-st}\Big|_0^{\infty} \qquad = -0 + \frac{3\cos 3(0) - s\sin 3(0)}{s^2+9}e^{-s(0)}$$

$$\boxed{= \frac{3}{s^2+9}} \qquad s > 0$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = \sin 2t$

Solution

$$F(s) = \int_0^\infty (\sin 2t) e^{-st} dt$$

$$\int \sin 2t \ e^{-st} dt = -\frac{1}{2} e^{-st} \cos 2t - \frac{s}{4} e^{-st} \sin 2t + \frac{s^2}{4} \int e^{-st} \sin 2t \ dt$$

$$\left(4 + s^2\right) \int \sin 2t \ e^{-st} dt = -\left(2\cos 2t - s\sin 2t\right) e^{-st}$$

$$\int \sin 2t \ e^{-st} dt = -\frac{2\cos 2t - s\sin 2t}{s^2 + 4} e^{-st}$$

		$\int \sin 2t \ dt$
+	e^{-st}	$-\frac{1}{2}\cos 2t$
−	$-se^{-st}$	$-\frac{1}{4}\sin 2t$
+	$s^2 e^{-st}$	

$$F(s) = -\frac{2\cos 2t - s\sin 2t}{s^2 + 4} e^{-st} \Big|_0^\infty$$

$$= -0 + \frac{2\cos 2(0) - s\sin 2(0)}{s^2 + 4} e^{-s(0)}$$

$$= \frac{2}{s^2 + 4}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = \cos 2t$

Solution

$$F(s) = \int_0^\infty (\cos 2t) e^{-st} dt$$

$$\int \cos 2t \ e^{-st} dt = \frac{1}{2} e^{-st} \sin 2t - \frac{s}{4} e^{-st} \cos 2t - \frac{s^2}{4} \int e^{-st} \cos 2t \ dt$$

$$\left(4 + s^2\right) \int \cos 2t \ e^{-st} dt = \left(2\sin 2t - s\cos 2t\right) e^{-st}$$

$$\int \cos 2t \ e^{-st} dt = \frac{2\sin 2t - s\cos 2t}{s^2 + 4} e^{-st}$$

		$\int \cos 2t \ dt$
+	e^{-st}	$\frac{1}{2}\sin 2t$
−	$-se^{-st}$	$-\frac{1}{4}\cos 2t$
+	$s^2 e^{-st}$	

$$F(s) = \frac{2\sin 2t - s\cos 2t}{s^2 + 4} e^{-st} \Big|_0^\infty$$

$$= \frac{s}{s^2 + 4}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = \cos bt$

Solution

$$F(s) = \int_0^\infty (\cos bt) e^{-st} dt$$

$$\int \cos bt\ e^{-st} dt = \frac{1}{b} e^{-st} \sin bt - \frac{s}{b^2} e^{-st} \cos bt - \frac{s^2}{b^2} \int e^{-st} \cos bt\ dt$$

$$\left(b^2 + s^2\right) \int \cos bt\ e^{-st} dt = (b \sin bt - s \cos bt) e^{-st}$$

$$\int \cos bt\ e^{-st} dt = \frac{b \sin bt - s \cos bt}{s^2 + b^2} e^{-st}$$

$$F(s) = \frac{b \sin bt - s \cos bt}{s^2 + b^2} e^{-st} \Big|_0^\infty$$

$$\boxed{= \frac{s}{s^2 + b^2}}$$

		$\int \cos bt\ dt$
+	e^{-st}	$\frac{1}{b} \sin bt$
−	$-se^{-st}$	$-\frac{1}{b^2} \cos bt$
+	$s^2 e^{-st}$	

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{t+7}$

Solution

$$F(s) = \int_0^\infty e^{t+7} e^{-st} dt$$

$$= \int_0^\infty e^7 e^{-(s-1)t} dt$$

$$= -\frac{e^7}{s-1} e^{-(s-1)t} \Big|_0^\infty \qquad e^{-\infty} = 0 \quad e^0 = 1$$

$$\boxed{= \frac{e^7}{s-1}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{-2t-5}$

Solution

$$F(s) = \int_0^\infty e^{-2t-5} e^{-st} dt$$

$$= e^{-5}\int_0^\infty e^{-(s+2)t}dt$$

$$= -\frac{1}{e^5}\cdot\frac{1}{s+2}\left(e^{-(s+2)t}\right)_0^\infty \qquad e^{-\infty}=0 \quad e^0=1$$

$$= \frac{1}{e^5}\cdot\frac{1}{s+2}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=te^{4t}$

Solution

$$F(s)=\int_0^\infty te^{4t}e^{-st}dt$$

$$=\int_0^\infty te^{-(s-4)t}dt$$

$$=\left(-\frac{t}{s-4}-\frac{1}{(s-4)^2}\right)e^{-(s-4)t}\Big|_0^\infty$$

$$=\frac{1}{(s-4)^2}$$

	$\int e^{-(s-4)t}dt$
t	$-\frac{1}{s-4}e^{-(s-4)t}$
1	$\frac{1}{(s-4)^2}e^{-(s-4)t}$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=t^2e^{-2t}$

Solution

$$F(s)=\int_0^\infty t^2e^{-2t}e^{-st}dt = \int_0^\infty t^2e^{-(s+2)t}dt$$

$$=\left(-\frac{t^2}{s+2}-\frac{2t}{(s+2)^2}-\frac{2}{(s+2)^3}\right)e^{-(s+2)t}\Big|_0^\infty$$

$$=\frac{2}{(s+2)^3}$$

	$\int e^{-(s+2)t}dt$
t^2	$-\frac{1}{s+2}e^{-(s+2)t}$
$2t$	$\frac{1}{(s+2)^2}e^{-(s+2)t}$
2	$-\frac{1}{(s+2)^3}e^{-(s+2)t}$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=e^{-t}\sin t$

Solution

$$F(s)=\int_0^{\infty} e^{-t}\sin t\ e^{-st}dt$$
$$=\int_0^{\infty} \sin t\ e^{-(s+1)t}dt$$

$$\int \sin t\ e^{-(s+1)t}dt=\left(-\cos t-(s+1)\sin t\right)e^{-(s+1)t}-(s+1)^2\int \sin t\ e^{-(s+1)t}dt$$
$$\left((s+1)^2+1\right)\int \sin t\ e^{-(s+1)t}dt=\left(-\cos t-(s+1)\sin t\right)e^{-(s+1)t}$$

$$\int_0^{\infty} \sin t\ e^{-(s+1)t}dt=-\frac{\cos t+(s+1)\sin t}{(s+1)^2+1}e^{-(s+1)t}$$

	$\int \sin t\ dt$
$e^{-(s+1)t}$	$-\cos t$
$-(s+1)e^{-(s+1)t}$	$-\sin t$
$(s+1)^2e^{-(s+1)t}$	$-\int \sin t\ dt$

$$F(s)=-\frac{\cos t+(s+1)\sin t}{(s+1)^2+1}e^{-(s+1)t}\Big|_0^{\infty}$$
$$=\frac{1}{(s+1)^2+1}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=e^{2t}\cos 3t$

Solution

$$F(s)=\int_0^{\infty} e^{2t}\cos 3t\ e^{-st}dt$$
$$=\int_0^{\infty} \cos 3t\ e^{-(s-2)t}dt$$

$$\int \cos 3t\ e^{-(s-2)t}dt=\left(\tfrac{1}{3}\sin 3t+\tfrac{1}{9}(s-2)\cos 3t\right)e^{-(s-2)t}-\tfrac{1}{9}(s-2)^2\int \cos 3t\ e^{-(s-2)t}dt$$
$$\left((s-2)^2+9\right)\int \sin t\ e^{-(s-2)t}dt=\left(3\sin 3t+(s-2)\cos 3t\right)e^{-(s-2)t}$$

$$\int_0^{\infty} \cos 3t\ e^{-(s-2)t}dt=\frac{3\sin 3t+(s-2)\cos 3t}{(s-2)^2+9}e^{-(s-2)t}$$

		$\int \cos 3t\ dt$
+	$e^{-(s-2)t}$	$\frac{1}{3}\sin 3t$
−	$-(s-2)e^{-(s-2)t}$	$-\frac{1}{9}\cos 3t$
+	$(s-2)^2e^{-(s-2)t}$	

$$F(s)=\frac{3\sin 3t+(s-2)\cos 3t}{(s-2)^2+9}e^{-(s-2)t}\Big|_0^{\infty}$$
$$=\frac{s-2}{(s-2)^2+9}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=e^{-t}\sin 2t$

Solution

$$F(s)=\int_0^{\infty} e^{-t}\sin 2t\ e^{-st}dt$$

$$=\int_0^{\infty} \sin 2t\ e^{-(s+1)t}dt$$

$$\int \sin 2t\ e^{-(s+1)t}dt=\left(-\tfrac{1}{2}\cos 2t-\tfrac{1}{4}(s+1)\sin 2t\right)e^{-(s+1)t}-\tfrac{1}{4}(s+1)^2\int \sin 2t\ e^{-(s+1)t}dt$$

$$\left((s+1)^2+4\right)\int \sin 2t\ e^{-(s+1)t}dt=-\left(2\cos 2t+(s+1)\sin 2t\right)e^{-(s+1)t}$$

$$\int_0^{\infty} \sin 2t\ e^{-(s+1)t}dt=-\frac{2\cos 2t+(s+1)\sin 2t}{(s+1)^2+4}e^{-(s+1)t}$$

	$\int \sin 2t\ dt$
$e^{-(s+1)t}$	$-\frac{1}{2}\cos t$
$-(s+1)e^{-(s+1)t}$	$-\frac{1}{4}\sin t$
$(s+1)^2e^{-(s+1)t}$	

$$F(s)=-\frac{2\cos 2t+(s+1)\sin 2t}{(s+1)^2+4}e^{-(s+1)t}\Big|_0^{\infty}$$

$$=\frac{2}{(s+1)^2+4}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=t\sin t$

Solution

$$F(s)=\int_0^{\infty} t\sin t\ e^{-st}dt$$

$$\int t\sin t\ e^{-st}dt=\left(-t\cos t+(1-st)\sin t\right)e^{-st}-s^2\int t\sin t\ e^{-st}dt+2s\int \sin t\ e^{-st}dt$$

$$\int \sin t\ e^{-st}dt=\left(-\cos t-s\sin t\right)e^{-st}-s^2\int \sin t\ e^{-st}dt$$

$$\left(s^2+1\right)\int \sin t\ e^{-st}dt=\left(-\cos t-s\sin t\right)e^{-st}$$

$$\int \sin t\ e^{-st}dt=-\frac{\cos t+s\sin t}{s^2+1}e^{-st}$$

	$\int \sin t\ dt$
te^{-st}	$-\cos t$
$(1-st)e^{-st}$	$-\sin t$
$\left(s^2t-2s\right)e^{-st}$	$-\int \sin t\ dt$

$$\left(s^2+1\right)\int t\sin t\ e^{-st}dt=\left(-t\cos t+(1-st)\sin t\right)e^{-st}-\frac{2s}{s^2+1}\left(\cos t+s\sin t\right)e^{-st}$$

$$\int t\sin t\, e^{-st}dt = \frac{1}{s^2+1}\left(-t\cos t + (1-st)\sin t\right)e^{-st} - \frac{2s}{\left(s^2+1\right)^2}\left(\cos t + s\sin t\right)e^{-st}$$

$$F(s) = \left(\frac{(1-st)\sin t - t\cos t}{s^2+1} - \frac{2s\left(\cos t + s\sin t\right)}{\left(s^2+1\right)^2}\right)e^{-st}\Bigg|_0^{\infty}$$

$$= \boxed{\frac{2s}{\left(s^2+1\right)^2}}$$

	$\int \sin t\, dt$
e^{-st}	$-\cos t$
$-se^{-st}$	$-\sin t$
s^2e^{-st}	$-\int \sin t\, dt$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = t\cos t$

Solution

$$F(s) = \int_0^{\infty} t\cos t\, e^{-st}dt$$

$$\int t\cos t\, e^{-st}dt = \left(t\sin t - (1-st)\cos t\right)e^{-st} - s^2\int t\cos t\, e^{-st}dt + 2s\int \cos t\, e^{-st}dt$$

$$\int \cos t\, e^{-st}dt = \left(\sin t + s\cos t\right)e^{-st} - s^2\int \cos t\, e^{-st}dt$$

$$\left(s^2+1\right)\int \cos t\, e^{-st}dt = \left(\sin t + s\cos t\right)e^{-st}$$

$$\int \cos t\, e^{-st}dt = \frac{\sin t + s\cos t}{s^2+1}e^{-st}$$

	$\int \cos t\, dt$
te^{-st}	$\sin t$
$(1-st)e^{-st}$	$-\cos t$
$\left(s^2t-2s\right)e^{-st}$	

$$\left(s^2+1\right)\int t\cos t\, e^{-st}dt = \left(t\sin t - (1-st)\cos t\right)e^{-st} + \frac{2s\left(\sin t + s\cos t\right)}{s^2+1}e^{-st}$$

$$\int t\cos t\, e^{-st}dt = \frac{1}{s^2+1}\left(t\sin t - (1-st)\cos t\right)e^{-st} + \frac{2s\left(\sin t + s\cos t\right)}{\left(s^2+1\right)^2}e^{-st}$$

$$F(s) = \left(\frac{t\sin t - (1-st)\cos t}{s^2+1} + \frac{2s\left(\sin t + s\cos t\right)}{\left(s^2+1\right)^2}\right)e^{-st}\Bigg|_0^{\infty}$$

$$= \frac{1}{s^2+1} + \frac{2s^2}{\left(s^2+1\right)^2}$$

	$\int \cos t\, dt$
e^{-st}	$\sin t$
$-se^{-st}$	$-\cos t$
s^2e^{-st}	

$$= \frac{-s^2 - 1 + 2s^2}{\left(s^2+1\right)^2}$$

$$\boxed{= \frac{s^2 - 1}{\left(s^2+1\right)^2}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = 2t^4$

Solution

$$F(s) = \int_0^{\infty} 2t^4 \, e^{-st} dt$$

$$= 2\left(-\frac{t^4}{s} - \frac{4t^3}{s^2} - \frac{12t^2}{s^3} - \frac{24t}{s^4} - \frac{24}{s^5}\right)e^{-st}\Big|_0^{\infty}$$

$$= 2\left(0 + \frac{24}{s^5}\right)$$

$$\boxed{= \frac{48}{s^5}}$$

	$\int e^{-st} dt$
t^4	$-\frac{1}{s}e^{-st}$
$4t^3$	$\frac{1}{s^2}e^{-st}$
$12t^2$	$-\frac{1}{s^3}e^{-st}$
$24t$	$\frac{1}{s^4}e^{-st}$
24	$-\frac{1}{s^5}e^{-st}$

Exercise

Use Definition of Laplace Transform to show the Laplace transform of $f(t) = \cos \omega t$ is $F(s) = \frac{s}{s^2 + \omega^2}$

Solution

$$F(s) = \int_0^{\infty} (\cos \omega t) e^{-st} dt$$

$$\int \cos \omega t \; e^{-st} dt = \frac{1}{\omega} e^{-st} \sin \omega t - \frac{s}{\omega^2} e^{-st} \cos \omega t + \frac{s^2}{\omega^2} \int e^{-st} \cos \omega t \; dt$$

$$\left(1 - \frac{s^2}{\omega^2}\right) \int e^{-st} \cos \omega t \; dt = \frac{1}{\omega} e^{-st} \sin \omega t - \frac{s}{\omega^2} e^{-st} \cos \omega t$$

$$\left(\frac{\omega^2 - s^2}{\omega^2}\right) \int e^{-st} \cos \omega t \; dt = \frac{1}{\omega}\left(\sin \omega t - \frac{s}{\omega} \cos \omega t\right) e^{-st}$$

$$\int e^{-st} \cos \omega t \; dt = \frac{\omega^2}{\omega^2 - s^2} \frac{1}{\omega^2} (\omega \sin \omega t - s \cos \omega t) e^{-st}$$

		$\int \cos \omega t \; dt$
+	e^{-st}	$\frac{1}{\omega} \sin \omega t$
−	$-se^{-st}$	$-\frac{1}{\omega^2} \cos \omega t$
+	$s^2 e^{-st}$	$-\frac{1}{\omega^2} \int \cos \omega t$

$$= \frac{e^{-st}}{\omega^2 - s^2}(\omega \sin \omega t - s \cos \omega t)$$

$$F(s) = \lim_{T\to\infty} \frac{e^{-st}}{\omega^2 - s^2}(\omega \sin \omega t - s \cos \omega t)\Bigg|_0^T$$

$$= \lim_{T\to\infty}\left[\frac{e^{-sT}}{\omega^2 - s^2}(\omega \sin \omega T - s \cos \omega T) - \frac{1}{\omega^2 - s^2}(\omega \sin 0 - s \cos 0)\right]$$

$$= 0 - \frac{1}{\omega^2 - s^2}(-s) \qquad \lim_{T\to\infty} e^{-sT} = \lim_{T\to\infty} \frac{1}{e^{-sT}} = 0$$

$$\boxed{= \frac{s}{s^2 + \omega^2}} \quad s > 0$$