

## Section 4.2 – Matrix operations and Their Applications

### Matrix Notation

The Matrix:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix} \text{ is called the coefficient matrix of the system}$$

The matrix is said to be of order  $m \times n$

$m$ : numbers of rows,

$n$ : number of columns

A matrix  $A$  with  $m$  rows and  $n$  columns can be written in a general form

$$A = \begin{bmatrix} a_{ij} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

The matrix  $A$  above has 3 rows and 3 columns; therefore, the order of the matrix  $A$  is  $(3 \times 3)$

When  $m = n$ , then matrix is said to be **square**.

The numbers in a matrix are called **entries**.

### Example

$$\text{Let } A = \begin{bmatrix} 5 & -2 \\ -3 & \pi \\ 1 & 6 \end{bmatrix}$$

a. What is the order of  $A$ ?

3 rows and 2 columns  $\Rightarrow A$  is  $3 \times 2$

b.  $a_{12} = -2$                        $a_{31} = 1$

## Equality of Matrices

### Definition of Equality of Matrices

Two matrices **A** and **B** are equal if and only if they have the same order (size)  $m \times n$  and if each pair corresponding elements is equal

$$a_{ij} = b_{ij} \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

### Example

Find the values of the variables for which each statement is true, if possible.

$$a) \begin{bmatrix} 2 & 1 \\ p & q \end{bmatrix} = \begin{bmatrix} x & y \\ -1 & 0 \end{bmatrix}$$

$$x = 2, y = 1, p = -1, q = 0$$

$$b) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

*can't be true*

$$c) \begin{bmatrix} w & x \\ 8 & -12 \end{bmatrix} = \begin{bmatrix} 9 & 17 \\ y & z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} w=9 & x=17 \\ 8=y & -12=z \end{bmatrix}$$

## Matrix Addition and Subtraction

Given two  $m \times n$  matrices  $A = [a_{ij}]$  and  $B = [b_{ij}]$  their sum is  $A + B = [a_{ij} + b_{ij}]$

And their difference is  $A - B = [a_{ij} - b_{ij}]$

The matrices have to be the *same order*

***Example***

Find  $\begin{bmatrix} -4 & 3 \\ 7 & -6 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & -4 \end{bmatrix}$

**Solution**

$$\begin{bmatrix} -4 & 3 \\ 7 & -6 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} -4+6 & 3+(-3) \\ 7+2 & -6+(-4) \end{bmatrix} \\ = \begin{bmatrix} 2 & 0 \\ 9 & -10 \end{bmatrix}$$

***Example***

Find  $\begin{bmatrix} 5 & 4 \\ -3 & 7 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 8 \\ 6 & 0 \\ -5 & 3 \end{bmatrix}$

**Solution**

$$\begin{bmatrix} 5 & 4 \\ -3 & 7 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 8 \\ 6 & 0 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 5-(-4) & 4-8 \\ -3-6 & 7-0 \\ 0-(-5) & 1-3 \end{bmatrix} \\ = \begin{bmatrix} 9 & -4 \\ -9 & 7 \\ 5 & -2 \end{bmatrix}$$

***Example***

Find  $\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix}$

**Solution**

$$\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 5-4 & -6+6 \\ 8+8 & 9-3 \end{bmatrix} \\ = \begin{bmatrix} 1 & 0 \\ 16 & 6 \end{bmatrix}$$

## ***Scalar Multiplication***

The scalar product of a number  $k$  and a matrix  $A$  is denoted by  $kA$ .

$$kA = k \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

### ***Example***

Find  $5 \begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}$

### **Solution**

$$\begin{aligned} 5 \begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix} &= \begin{bmatrix} 2(5) & -3(5) \\ 0(5) & 4(5) \end{bmatrix} \\ &= \begin{bmatrix} 10 & -15 \\ 0 & 20 \end{bmatrix} \end{aligned}$$

### ***Example***

Find  $\frac{3}{4} \begin{bmatrix} 20 & 36 \\ 12 & -16 \end{bmatrix}$

### **Solution**

$$\frac{3}{4} \begin{bmatrix} 20 & 36 \\ 12 & -16 \end{bmatrix} = \begin{bmatrix} 15 & 27 \\ 9 & -12 \end{bmatrix}$$

### ***Example***

Given:  $A = \begin{bmatrix} -4 & 1 \\ 3 & 0 \end{bmatrix}$        $B = \begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix}$

Find:

a)  $-6B$

b)  $3A + 2B$

### **Solution**

$$\begin{aligned} \text{a) } -6B &= -6 \begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix} \\ &= \begin{bmatrix} -1(-6) & -2(-6) \\ 8(-6) & 5(-6) \end{bmatrix} \end{aligned}$$

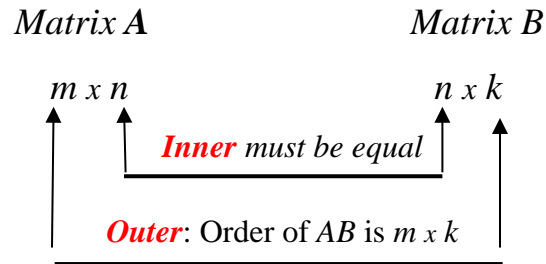
$$= \begin{bmatrix} 6 & 12 \\ -48 & -30 \end{bmatrix}$$

$$\begin{aligned} \textbf{b)} \quad 3A + 2B &= 3 \begin{bmatrix} -4 & 1 \\ 3 & 0 \end{bmatrix} + 2 \begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix} \\ &= \begin{bmatrix} -4(3) & 1(3) \\ 3(3) & 0(3) \end{bmatrix} + \begin{bmatrix} -1(2) & -2(2) \\ 8(2) & 5(2) \end{bmatrix} \\ &= \begin{bmatrix} -12 & 3 \\ 9 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ 16 & 10 \end{bmatrix} \\ &= \begin{bmatrix} -12-2 & 3-4 \\ 9+16 & 0+10 \end{bmatrix} \\ &= \begin{bmatrix} -14 & -1 \\ 25 & 10 \end{bmatrix} \end{aligned}$$

## Matrix Multiplication

### Product of Two Matrices

Let  $A$  be an  $m \times n$  matrix and let  $B$  be an  $n \times k$  matrix. To find the element in the  $i^{th}$  row and  $j^{th}$  column of the product matrix  $AB$ , multiply each element in the  $i^{th}$  row of  $A$  by the corresponding element in the  $j^{th}$  column of  $B$ , and then add these products. The product matrix  $AB$  is an  $m \times k$  matrix.



$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$2 \times 2 \quad 2 \times 2 \quad \rightarrow \quad 2 \times 2$

$$a_{11} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & - \\ - & - \end{bmatrix}$$

$$a_{12} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} - & af + bh \\ - & - \end{bmatrix}$$

$$a_{21} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} - & - \\ ce + dg & - \end{bmatrix}$$

$$a_{22} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} - & - \\ - & cf + dh \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

### ***Example***

Given:  $A = \begin{bmatrix} 1 & -3 \\ 7 & 2 \end{bmatrix}$      $B = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 4 & -1 \end{bmatrix}$

Find  $AB$  and  $BA$ .

### **Solution**

$$\begin{aligned} AB &= \begin{bmatrix} 1 & -3 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 4 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1(1) + (-3)3 & 1(0) + (-3)1 & 1(-1) + (-3)4 & 1(2) + (-3)(-1) \\ 7(1) + 2(3) & 7(0) + 2(1) & 7(-1) + 2(4) & 7(2) + 2(-1) \end{bmatrix} \\ &= \begin{bmatrix} -8 & -3 & -13 & 5 \\ 13 & 2 & 1 & 12 \end{bmatrix} \end{aligned}$$

$BA$  can be found since:  $B$ :  $2 \times 4$  and  $A$ :  $2 \times 2$

**Note:**  $AB \neq BA$

### ***Example***

Given:  $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$      $B = \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$

Find  $AB$ .

### **Solution**

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1(4) + 3(1) & 1(6) + 3(0) \\ 2(4) + 5(1) & 2(6) + 5(0) \end{bmatrix} \\ &= \begin{bmatrix} 7 & 6 \\ 13 & 12 \end{bmatrix} \end{aligned}$$

### Example

Given:  $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$   $B = \begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}$  Find  $AB$ .

### Solution

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3(1) + 1(3) - 1(-2) & 3(6) + 1(-5) - 1(4) \\ 2(1) + 0(3) + 3(-2) & 2(6) + 0(-5) + 3(4) \end{bmatrix} \\ &= \begin{bmatrix} 8 & 9 \\ -4 & 24 \end{bmatrix} \end{aligned}$$

### Example

Suppose  $A$  is a  $3 \times 2$  matrix, while  $B$  is a  $2 \times 4$  matrix.

- a) Can the product  $AB$  be calculated?
- b) If  $AB$  can be calculated, what size is it?
- c) Can  $BA$  be calculated?
- d) If  $BA$  can be calculated, what size is it?

### Solution

a)



b) The product  $AB$  size is  $3 \times 4$

c)



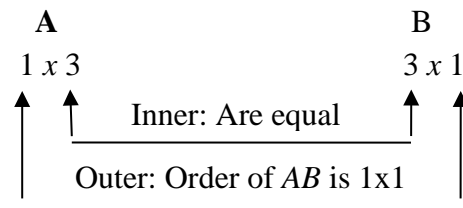
d) Can't be calculated



### Example

Given:  $A_{1 \times 3} = \begin{bmatrix} 2 & 0 & 4 \end{bmatrix}$      $B_{3 \times 1} = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$     Find  $AB$  and  $BA$ .

### Solution



$$AB = [2(1) + 0(3) + 4(7)] = [30]$$

$$BA : 3 \times 1 \text{ ---- } 1 \times 3$$

$$\begin{aligned} BA &= \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix} \begin{bmatrix} 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(0) & 1(4) \\ 3(2) & 3(0) & 3(4) \\ 7(2) & 7(0) & 7(4) \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 & 4 \\ 6 & 0 & 12 \\ 14 & 0 & 28 \end{bmatrix} \end{aligned}$$

### Example

Given:  $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$      $B = \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix}$     Find  $AB$  and  $BA$ .

### Solution

$$\begin{aligned} a) \quad AB &= \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix} \quad 2 \times 2 \text{ --- } 2 \times 4 \\ &= \begin{bmatrix} 1(2) + 3(0) & 1(3) + 3(5) & 1(-1) + 3(4) & 1(6) + 3(1) \\ 0(2) + 2(0) & 0(3) + 2(5) & 0(-1) + 2(4) & 0(6) + 2(1) \end{bmatrix} \\ &= \begin{bmatrix} 2 & 18 & 11 & 9 \\ 0 & 10 & 8 & 2 \end{bmatrix} \end{aligned}$$

$$b) \quad BA = \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} + \text{Undefined} \quad 2 \times 4 \text{ --- } 2 \times 2 \text{ (Inner order are not equal 2, 4)}$$

## ***Properties of Matrix***

### **Addition and Scalar Multiplication**

$$A + B = B + A \quad \text{Commutative Property of Addition}$$

$$A + (B + C) = (A + B) + C \quad \text{Associative Property of Addition}$$

$$(kl)A = k(lA) \quad \text{Associative Property of Scalar Multiplication}$$

$$k(A + B) = kA + kB \quad \text{Distributive Property}$$

$$(k + l)A = kA + lA \quad \text{Distributive Property}$$

$$A + 0 = 0 + A = A \quad \text{Additive Identity Property}$$

$$A + (-A) = (-A) + A = 0 \quad \text{Additive Inverse Property}$$

### **Multiplication**

$$A(BC) = (AB)C \quad \text{Associative Property of Multiplication}$$

$$A(B + C) = AB + AC \quad \text{Distributive Property}$$

$$(B + C)A = BA + CA \quad \text{Distributive Property}$$

## Exercises      Section 4.2 – Matrix operations and Their Applications

(1 – 7) Find values for the variables so that the matrices are equal.

1.  $\begin{bmatrix} w & x \\ 8 & -12 \end{bmatrix} = \begin{bmatrix} 9 & 17 \\ y & z \end{bmatrix}$

2.  $\begin{bmatrix} x & y+3 \\ 2z & 8 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 6 & 8 \end{bmatrix}$

3.  $\begin{bmatrix} 5 & x-4 & 9 \\ 2 & -3 & 8 \\ 6 & 0 & 5 \end{bmatrix} = \begin{bmatrix} y+3 & 2 & 9 \\ z+4 & -3 & 8 \\ 6 & 0 & w \end{bmatrix}$

4.  $\begin{bmatrix} a+2 & 3b & 4c \\ d & 7f & 8 \end{bmatrix} + \begin{bmatrix} -7 & 2b & 6 \\ -3d & -6 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 25 & 6 \\ -8 & 1 & 6 \end{bmatrix}$

5.  $\begin{bmatrix} a+11 & 12z+1 & 5m \\ 11k & 3 & 1 \end{bmatrix} + \begin{bmatrix} 9a & 9z & 4m \\ 12k & 5 & 3 \end{bmatrix} = \begin{bmatrix} 41 & -62 & 72 \\ 92 & 8 & 4 \end{bmatrix}$

6.  $\begin{bmatrix} x+2 & 3y+1 & 5z \\ 8w & 2 & 3 \end{bmatrix} + \begin{bmatrix} 3x & 2y & 5z \\ 2w & 5 & -5 \end{bmatrix} = \begin{bmatrix} 10 & -14 & 80 \\ 10 & 7 & -2 \end{bmatrix}$

7.  $\begin{bmatrix} 2x-3 & y-2 & 2z+1 \\ 5 & 2w & 7 \end{bmatrix} + \begin{bmatrix} 3x-3 & y+2 & z-1 \\ -5 & 5w+1 & 3 \end{bmatrix} = \begin{bmatrix} 20 & 8 & 9 \\ 0 & 8 & 10 \end{bmatrix}$

8. Given  $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix}$   $B = \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix}$  Find:  $A - B$ ,  $3A + 2B$

9. Given  $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$   $F = \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$  Find:  $3F + 2A$

(10 – 22) Evaluate

10.  $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 12 \end{bmatrix}$

14.  $\begin{bmatrix} -5 & 6 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 5 & -8 \end{bmatrix}$

11.  $\begin{bmatrix} 5 & 8 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 & 1 \\ 4 & 2 & 5 \end{bmatrix}$

15.  $\begin{bmatrix} 8 & 6 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -8 \end{bmatrix}$

12.  $\begin{bmatrix} -5 & 0 \\ 4 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix}$

16.  $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$

13.  $\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix}$

17.  $\begin{bmatrix} -3 & 4 & 2 \\ 5 & 0 & 4 \end{bmatrix} \begin{bmatrix} -6 & 4 \\ 2 & 3 \\ 3 & -2 \end{bmatrix}$

$$18. \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$$

$$19. \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}$$

$$22. \begin{bmatrix} x & 2x+1 & 4 \\ 5 & x-1 & 8 \\ -2 & 3x & 2x+1 \end{bmatrix} + \begin{bmatrix} 2x-1 & -2x-1 & 4x \\ -5 & 6 & x+1 \\ -5 & 2 & -2x \end{bmatrix}$$

$$20. \begin{bmatrix} -2 & -3 & -4 \\ 2 & -1 & 0 \\ 4 & -2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix}$$

$$21. \begin{bmatrix} \sqrt{2} & \sqrt{2} & -\sqrt{18} \\ \sqrt{3} & \sqrt{27} & 0 \end{bmatrix} \begin{bmatrix} 8 & -10 \\ 9 & 12 \\ 0 & 2 \end{bmatrix}$$

(23 – 33) Find  $AB$  and  $BA$ , if possible

$$23. A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 7 \\ 0 & 2 \end{bmatrix}$$

$$24. A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} \quad B = \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix}$$

$$25. A = \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -1 & -1 \\ 0 & 4 \end{pmatrix}$$

$$26. A = \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 1 \\ 2 & -3 \end{pmatrix}$$

$$27. A = \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix}$$

$$28. A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

$$29. A = \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$$

$$30. A = \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ -3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 0 & -2 \\ -2 & 6 & 2 \end{pmatrix}$$

$$31. A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$$

$$32. A = \begin{pmatrix} -1 & 2 & 0 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

$$33. A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & -2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} -3 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$34. \text{ Given } A = \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}, \text{ Find}$$

$$a) A + B$$

$$b) A - B$$

$$c) 3A$$

$$d) -2B$$

$$e) 2A + 3B$$

$$f) A^2$$

$$g) AB$$

$$h) BA$$

35. Given  $A = \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix}$   $B = \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix}$ , Find

a)  $A + B$

c)  $3A$

e)  $2A + 3B$

g)  $AB$

b)  $A - B$

d)  $-2B$

f)  $A^2$

h)  $BA$

36. Given  $A = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix}$   $B = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$ , Find

a)  $A + B$

c)  $3A$

e)  $2A + 3B$

g)  $AB$

b)  $A - B$

d)  $-2B$

f)  $A^2$

h)  $BA$

37. Given  $A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix}$   $B = \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix}$ , Find

a)  $A + B$

c)  $3A$

e)  $2A + 3B$

g)  $AB$

b)  $A - B$

d)  $-2B$

f)  $A^2$

h)  $BA$

38. Given  $A = \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$   $B = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$   $C = \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix}$   $D = \begin{pmatrix} -2 & 3 \\ 2 & -1 \\ 3 & 2 \end{pmatrix}$ , Find

a)  $4A - 2B$

d)  $2A - 3B$

g)  $A^2$

j)  $CA$

b)  $3A + C$

e)  $AB$

h)  $B^3$

k)  $CD$

c)  $3A + B$

f)  $BA$

i)  $AC$

l)  $DC$

39. Given  $A = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$   $B = \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$   $C = \begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix}$   $D = \begin{pmatrix} 2 & 4 & -2 \\ 0 & 3 & 5 \\ -3 & 1 & 1 \end{pmatrix}$ , Find

a)  $4A - 2B$

d)  $2A - 3B$

g)  $A^2$

j)  $CB$

b)  $3A + C$

e)  $AB$

h)  $B^3$

k)  $CD$

c)  $3A + B$

f)  $BA$

i)  $AC$

l)  $DC$

40. A contractor builds three kinds of houses, models  $A$ ,  $B$ , and  $C$ , with a choice of two styles, Spanish and contemporary. Matrix  $P$  shows the number of each kind of house planned for a new 100-home subdivision. The amounts for each of the exterior materials depend primarily on the style of the house. These amounts are shown in matrix  $Q$ . (concrete is in *cubic yards*, lumber in units of 1000 board *feet*, brick in 1000s, and shingles in units of 100  $ft^2$ .) Matrix  $R$  gives the cost in dollars for each kind of material.

- a) What is the total cost of these materials for each model?

- b) How much of each of four kinds of material must be ordered
- c) What is the total cost for exterior materials?

41. Mitchell Fabricators manufactures three styles of bicycle frames in its two plants. The following table shows the number of each style produced at each plant

	<i>Mountain Bike</i>	<i>Racing Bike</i>	<i>Touring Bike</i>
<i>North Plant</i>	150	120	100
<i>South Plant</i>	180	90	130

- a) Write a  $2 \times 3$  matrix  $A$  that represents the information in the table
  - b) The manufacturer increased production of each style by 20%. Find a Matrix  $M$  that represents the increased production figures.
  - c) Find the matrix  $A + M$  and tell what it represents
42. Sal's Shoes and Fred's Footwear both have outlets in California and Arizona. Sal's sells shoes for \$80, sandals for \$40, and boots for \$120. Fred's prices are \$60, \$30, and \$150 for shoes, sandals and boots, respectively. Half of all sales in California stores are shoes,  $1/4$  are *sandals*, and  $1/4$  are *boots*. In Arizona, the fractions are  $1/5$  *shoes*,  $1/5$  are *sandals*, and  $3/5$  are *boots*.
- a) Write a  $2 \times 3$  matrix called  $P$  representing prices for the two stores and three types of footwear.
  - b) Write a  $2 \times 3$  matrix called  $F$  representing fraction of each type of footwear sold in each state.
  - c) Only one of the two products  $PF$  and  $FP$  is meaningful. Determine which one it is, calculate the product, and describe what the entries represent.