Solution Section 2.1 - Radians & Degrees, Circular Functions

Exercise

Use a calculator to convert 256° 20′ to radians to the nearest hundredth of a radian.

Solution

$$\begin{aligned}256^\circ\ 20' &= 256^\circ + \frac{20^\circ}{60}\\ &= 256^\circ + \frac{2^\circ}{6}\\ &= \frac{1538^\circ}{6}\end{aligned}$$

$$\frac{1538^\circ}{6}\,\frac{\pi}{180^\circ} \underline{= 4.47\ rad}|$$

Exercise

Convert -78.4° to radians

Solution

$$\begin{aligned}-78.4^\circ &= -78.4\left(\frac{\pi}{180}\right) rad\\ &\approx -1.37\ rad\end{aligned}$$

Exercise

Convert $\frac{11\pi}{6}$ to degrees

Solution

$$\begin{aligned}\frac{11\pi}{6}\ \text{rad} &= \frac{11\pi}{6} \bullet \frac{180^\circ}{\pi}\\ &= \underline{330^\circ}|\end{aligned}$$

Exercise

Convert $-\frac{5\pi}{3}$ to degrees

Solution

$$\begin{aligned}-\frac{5\pi}{3}\ \text{rad} &= -\frac{5\pi}{3} \bullet \frac{180^\circ}{\pi}\\ &\underline{= -300^\circ}|\end{aligned}$$

Exercise

Convert $\frac{\pi}{6}$ to degrees

Solution

$$\frac{\pi}{6}(rad) = \frac{\pi}{6}\left(\frac{180}{\pi}\right)^{\circ}$$

$$\boxed{= 30^{\circ}}$$

Exercise

Use the calculator to convert 2.4 to degree measure to the nearest tenth of a degree.

Solution

$$2.4\ rad = 2.4 \bullet \frac{180^{\circ}}{\pi}$$

$$= \frac{432^{\circ}}{\pi}$$

$$\boxed{\approx 137.5^{\circ}}$$

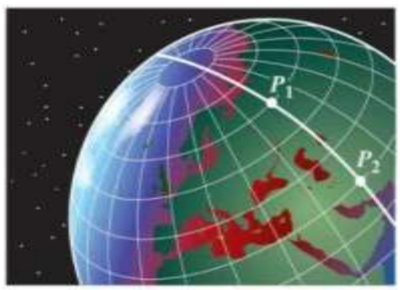
Exercise

In navigation, distance is not usually measured along a straight line, but along a great circle because the Earth is round. The formula to determine the great circle distance between two points $P_1\left(LT_1, LN_1\right)$ and $P_2\left(LT_2, LN_2\right)$ whose coordinates are given as latitudes and longitudes involves the expression

$$\sin\left(LT_1\right)\sin\left(LT_2\right)+\cos\left(LT_1\right)\cos\left(LT_2\right)\cos\left(LN_1-LN_2\right)$$

To use this formula, the latitudes and longitudes must be entered as angles in radians. However, most GPS units give these coordinates in degrees and minutes. To use this formula thus requires converting from degrees to radians.

Evaluate this expression for $P_1(N\ 32°\ 22.108', W\ 64°\ 41.178')$ and $P_2(N\ 13°\ 0.4809', W\ 59°\ 29.263')$ corresponding to Bermuda and Barbados, respectively.



Solution

$LT_1 = 32°\ 22.108'$ $=32° + \left(\frac{22.108}{60}\right)^{\circ}$ $=32.3685°$ $=32.3685\left(\frac{\pi}{180}\right)\ rad$ $\approx 0.565\ rad$	$LN_1 = 64°\ 41.178'$ $=64° + \frac{41.178}{60}^{\circ}$ $=64.6863°$ $=64.6863\left(\frac{\pi}{180}\right)\ rad$ $\approx 1.13\ rad$
$LT_2 = 13°\ 0.4809'$ $=13° + \frac{0.4809}{60}^{\circ}$ $=13.008°$ $=13.008\left(\frac{\pi}{180}\right)\ rad$ $\approx 0.228\ rad$	$LN_2 = 59°\ 29.263'$ $=59° + \frac{29.263}{60}^{\circ}$ $=59.4877°$ $=59.4877\left(\frac{\pi}{180}\right)\ rad$ $\approx 1.04\ rad$

$$\sin\left(LT_1\right)\sin\left(LT_2\right)+\cos\left(LT_1\right)\cos\left(LT_2\right)\cos\left(LN_1-LN_2\right)$$

$$=\sin(0.565)\sin(0.228)+\cos(0.565)\cos(0.228)\cos(1.13-1.04)$$

$$\underline{\approx 0.9404}$$

Exercise

If the angle θ is in standard position and the terminal side of θ intersects the unit circle at the point $\left(-\frac{1}{\sqrt{10}},-\frac{3}{\sqrt{10}}\right)$

Solution

$$\sin\theta = -\frac{3}{\sqrt{10}}$$

$$\cos\theta = -\frac{1}{\sqrt{10}}$$

$$\tan\theta = \frac{-\frac{3}{\sqrt{10}}}{-\frac{1}{\sqrt{10}}}$$

$$= 3$$

Exercise

Find the exact values of $\sin\frac{3\pi}{2},\ \cos\frac{3\pi}{2},\ \textit{and}\ \tan\frac{3\pi}{2}$

Solution

$$\frac{3\pi}{2} = \pi + \frac{\pi}{2}$$

$$\sin\frac{3\pi}{2} = -\sin\left(\frac{\pi}{2}\right)\ = -1$$

$$\cos\frac{3\pi}{2} = -\cos\left(\frac{\pi}{2}\right)\ = 0$$

$$\tan\frac{3\pi}{2} = \tan\left(\frac{\pi}{2}\right)\ = \textit{undefined}$$

Exercise

Use reference angles and degree/radian conversion to find exact value of $\cos\frac{2\pi}{3}$

Solution

$$\frac{2\pi}{3} = \pi - \frac{\pi}{3}$$

$$\cos\frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right)$$

$$= -\cos\frac{\pi}{3}$$

$$= -\frac{1}{2}$$

$$\frac{2\pi}{3}=\frac{2\pi}{3}\frac{180^\circ}{\pi}=120^\circ$$

$$\hat{\theta}=180^\circ-120^\circ=60^\circ$$

$$\cos 120^\circ=-\cos 60^\circ \underline{=-\frac{1}{2}}$$

Exercise

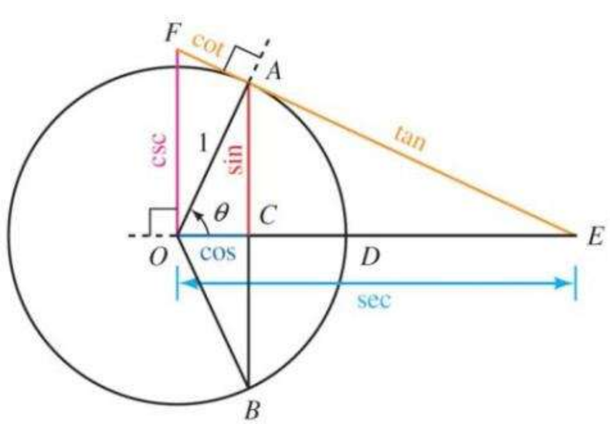
Evaluate $\sin\frac{13\pi}{6}$. Identify the function, the argument of the function, and the function value.

Solution

$$\sin\frac{13\pi}{6}=\frac{1}{2}$$

$\rightarrow$ The function is sine, the argument is $\frac{13\pi}{6}$, and the value is $\frac{1}{2}$

Exercise



Show why $OF=\csc\theta$

Solution

ΔOAF is similar to ΔACO

$$\Rightarrow \frac{OF}{OA}=\frac{AO}{AC}.$$

$$OA=1 \rightarrow AC=\sin\theta$$

$$\Rightarrow \frac{OF}{1}=\frac{1}{\sin\theta}$$

$$\Rightarrow OF=\frac{1}{\sin\theta}\ \underline{=\csc\theta}$$

Exercise

Evaluate $\sin\frac{9\pi}{4}$. Identify the function, the argument of the function, and the value of the function.

Solution

$$\begin{aligned}\frac{9\pi}{4} &= \frac{\pi}{4}+\frac{8\pi}{4}\\ &= \frac{\pi}{4}+2\pi\end{aligned}$$

$$\begin{aligned}\sin\frac{9\pi}{4} &= \sin\frac{\pi}{4}\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

$\rightarrow$ The function is sine, the argument is $\frac{9\pi}{4}$, and the value is $\frac{1}{\sqrt{2}}$

Exercise

The function is the sine function, $\frac{9\pi}{4}$ is the argument, and $\frac{1}{\sqrt{2}}$ is the value of the function

Solution

$$\sin\frac{9\pi}{4} = \frac{1}{\sqrt{2}}$$

Exercise

Evaluate $\cot 2.37$.

Solution

$$\begin{aligned}\cot 2.37 &= \frac{1}{\tan 2.37}\\ &\approx -1.0280\end{aligned}$$