Solution   *Section* 2.4 –Translation of Trigonometric Functions

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = 2\sin(x - \pi)$

Solution

Amplitude: $A = 2$

Period: $P = \frac{2\pi}{1} = 2\pi$

Phase Shift: $\phi = -\frac{-\pi}{1} = \pi$

VT: $y = 0$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = \frac{2}{3}\sin\left(x + \frac{\pi}{2}\right)$

Solution

Amplitude: $A = \frac{2}{3}$

Period: $P = \frac{2\pi}{1} = 2\pi$

Phase Shift: $\phi = -\frac{\frac{\pi}{2}}{1} = -\frac{\pi}{2}$

VT: $y = 0$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = 4\cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

Solution

Amplitude: $A = 4$

Period: $P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Phase Shift: $\phi = -\frac{\frac{\pi}{2}}{\frac{1}{2}} = -\pi$

VT: $y = 0$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = \frac{1}{2}\sin\left(\frac{1}{2}x + \pi\right)$

Solution

Amplitude: $A = \frac{1}{2}$

Period: $P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Phase Shift: $\phi = -\frac{\pi}{\frac{1}{2}} = -2\pi$

VT: $y = 0$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = 3\cos\frac{\pi}{2}\left(x - \frac{1}{2}\right)$

Solution

Amplitude: $A = 3$

Period: $P = \frac{2\pi}{1} = 2\pi$

Phase Shift: $\phi = -\frac{-\frac{1}{2}}{1} = \frac{1}{2}$

VT: $y = 0$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = -\cos\pi\left(x - \frac{1}{3}\right)$

Solution

Amplitude: $A = 1$

Period: $P = \frac{2\pi}{1} = 2\pi$

Phase Shift: $\phi = -\frac{-\frac{1}{3}}{1} = \frac{1}{3}$

VT: $y = 0$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = 2 - \sin\left(3x - \frac{\pi}{5}\right)$

Solution

Amplitude: $A = 1$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\phi = -\frac{-\frac{\pi}{5}}{3} = \frac{\pi}{15}$

VT: $y = 2$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = -\frac{2}{3}\sin\left(3x - \frac{\pi}{2}\right)$

Solution

Amplitude: $A = \frac{2}{3}$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\phi = -\frac{-\frac{\pi}{2}}{3} = \frac{\pi}{6}$

VT: $y = 0$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y = -1 + \frac{1}{2}\cos(2x - 3\pi)$

Solution

Amplitude: $A = \frac{1}{2}$

Period: $P = \frac{2\pi}{2} = \pi$

Phase Shift: $\phi = -\frac{-3\pi}{2} = \frac{3\pi}{2}$

VT: $y = -1$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y=2-\frac{1}{3}\cos\left(\pi x+\frac{3\pi}{2}\right)$

Solution

Amplitude: $A=\frac{1}{3}$

Period: $P=\frac{2\pi}{\pi}=2$

Phase Shift: $\phi=-\frac{\frac{3\pi}{2}}{\pi}=-\frac{3}{2}$

VT: $y=2$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y=\frac{5}{2}-3\cos\left(\pi x-\frac{\pi}{4}\right)$

Solution

Amplitude: $A=3$

Period: $P=\frac{2\pi}{\pi}=2$

Phase Shift: $\phi=-\frac{-\frac{\pi}{4}}{\pi}=\frac{1}{4}$

VT: $y=\frac{5}{2}$

Exercise

Find the amplitude, the period, any vertical translation, and any phase shift of $y=\frac{2}{3}-\frac{4}{3}\cos(3x-\pi)$

Solution

Amplitude: $A=\frac{4}{3}$

Period: $P=\frac{2\pi}{3}$

Phase Shift: $\phi=-\frac{-\pi}{3}=\frac{\pi}{3}$

VT: $y=\frac{2}{3}$

Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

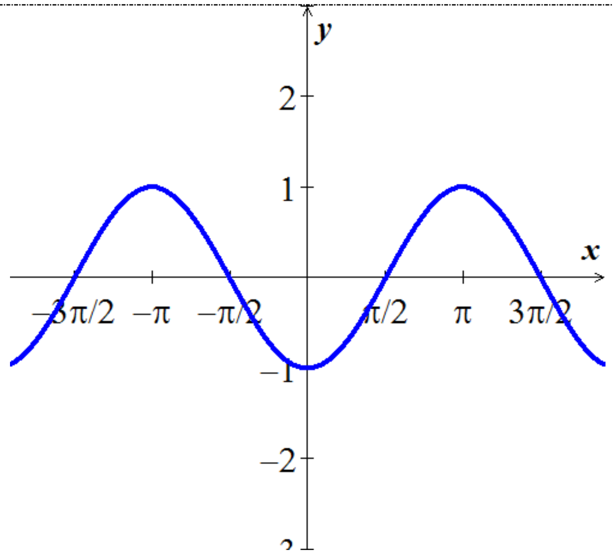
$y = \sin\left(x - \frac{\pi}{2}\right)$

Solution

Amplitude = 1

Period $= \frac{2\pi}{1} = 2\pi$

Phase shift $= \frac{\pi}{2}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

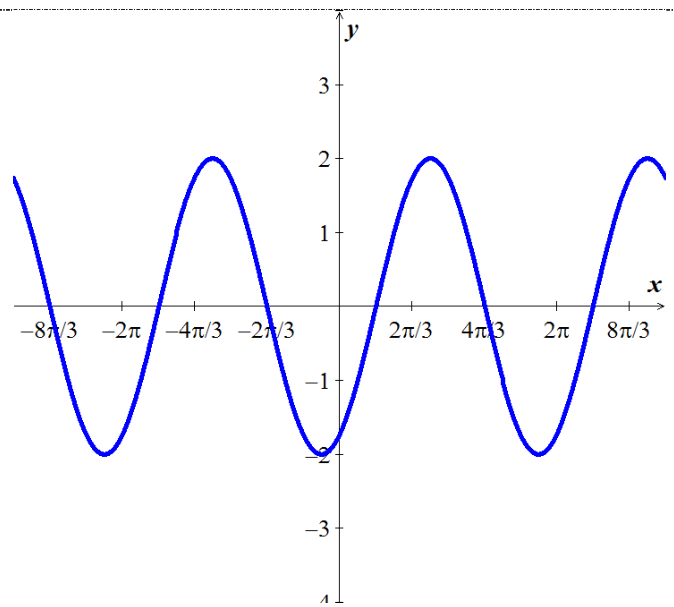
$y = 2\sin\left(x - \frac{\pi}{3}\right)$

Solution

Amplitude = 2

Period $= \frac{2\pi}{1} = 2\pi$

Phase shift $= \frac{\pi}{3}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

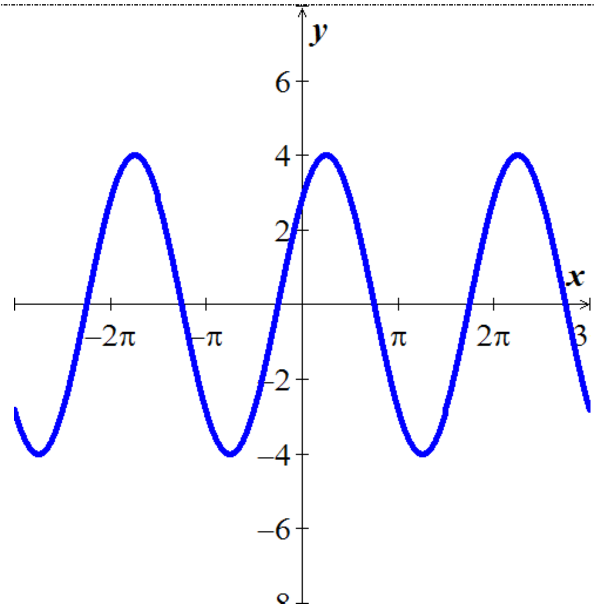
$y = 4\cos\left(x - \frac{\pi}{4}\right)$

Solution

Amplitude = 4

Period $= \frac{2\pi}{1} = 2\pi$

Phase shift $= \frac{\pi}{4}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

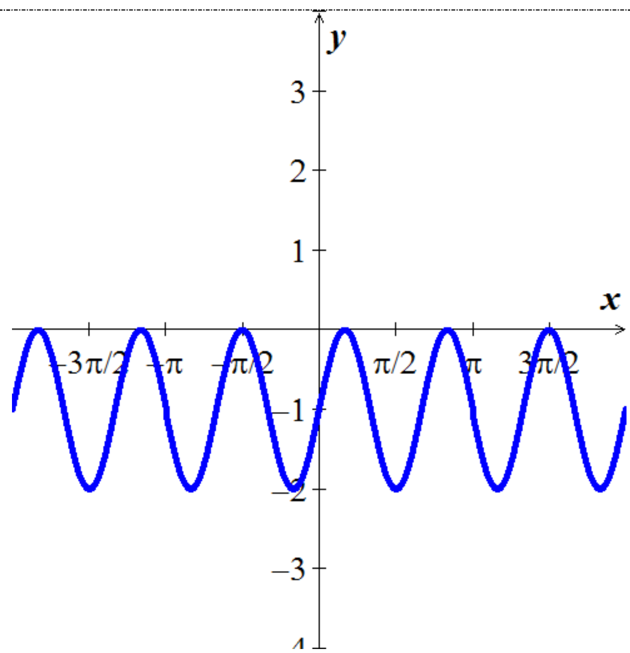
$y = -\sin(3x + \pi) - 1$

Solution

Amplitude = 1

Period $= \frac{2\pi}{3}$

Phase shift $= -\frac{\pi}{3}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

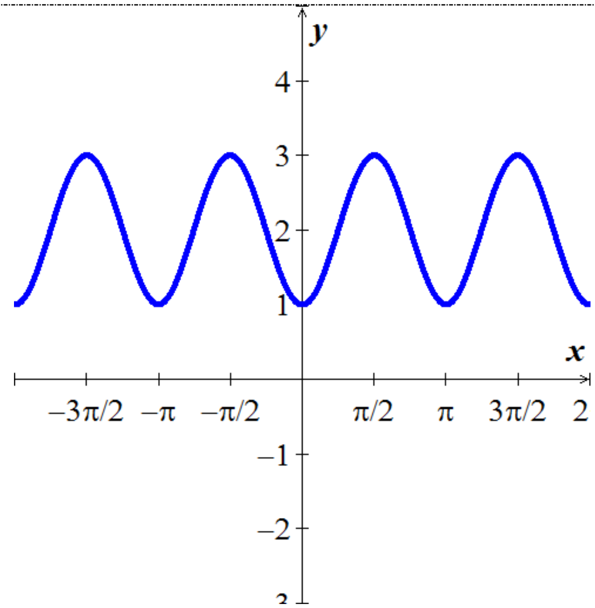
$y = \cos(2x - \pi) + 2$

Solution

Amplitude = 1

Period $= \dfrac{2\pi}{2} = \pi$

Phase shift $= \dfrac{\pi}{2}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

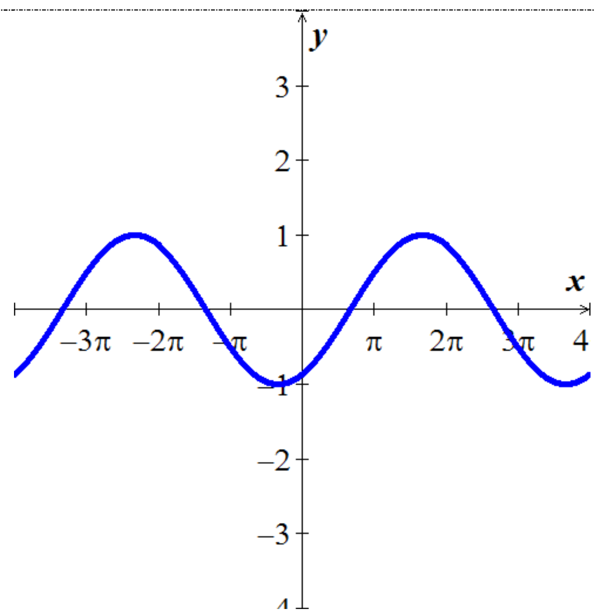
$y = \sin\left(\frac{1}{2}x - \frac{\pi}{3}\right)$

Solution

Amplitude = 1

Period $= \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

Phase shift $= \dfrac{2\pi}{3}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

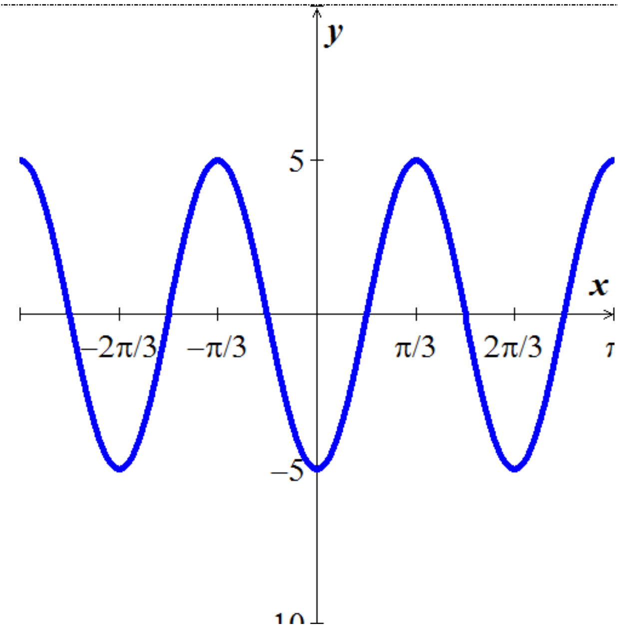
$y = 5\sin\left(3x - \frac{\pi}{2}\right)$

Solution

Amplitude = 3

Period $= \frac{2\pi}{3}$

Phase shift $= \frac{\pi}{6}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

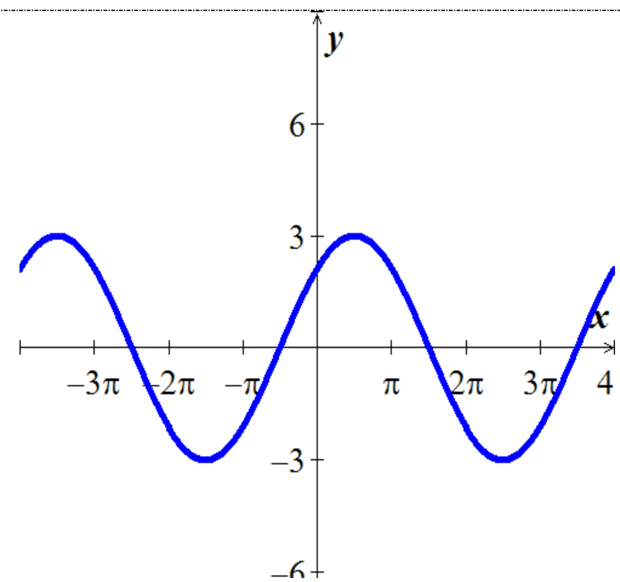
$y = 3\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

Solution

Amplitude = 3

Period $= \frac{2\pi}{1/2} = 4\pi$

Phase shift $= \frac{\pi}{2}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

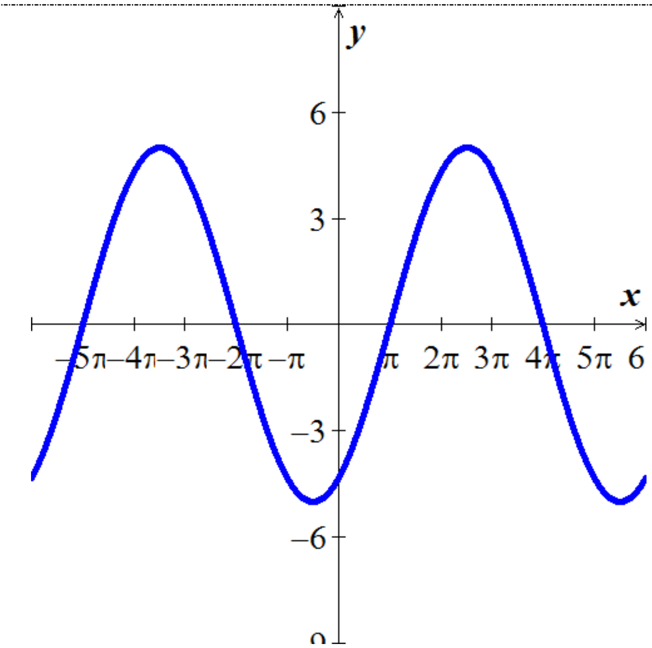
$y = -5\cos\left(\frac{1}{3}x + \frac{\pi}{6}\right)$

Solution

Amplitude = 5

$\text{Period } = \frac{2\pi}{1/3} = 6\pi$

$\text{Phase shift } = -\frac{\pi}{2}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

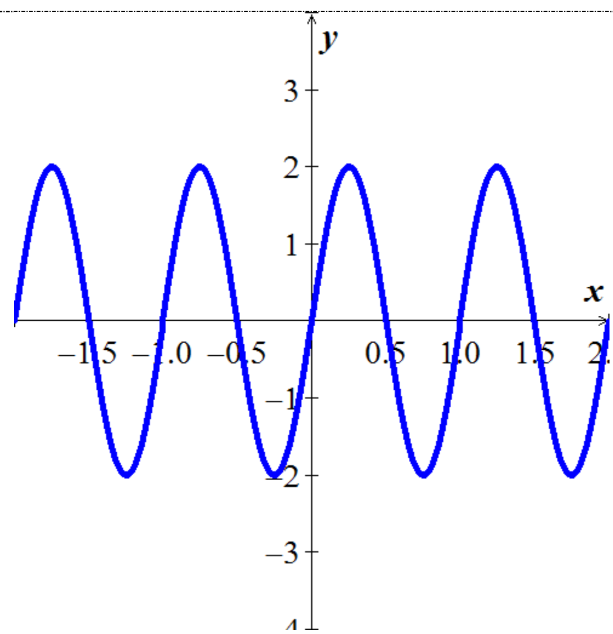
$y = -2\sin(2\pi x + \pi)$

Solution

Amplitude = 2

$\text{Period } = \frac{2\pi}{2\pi} = 1$

$\text{Phase shift } = -\frac{1}{2}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

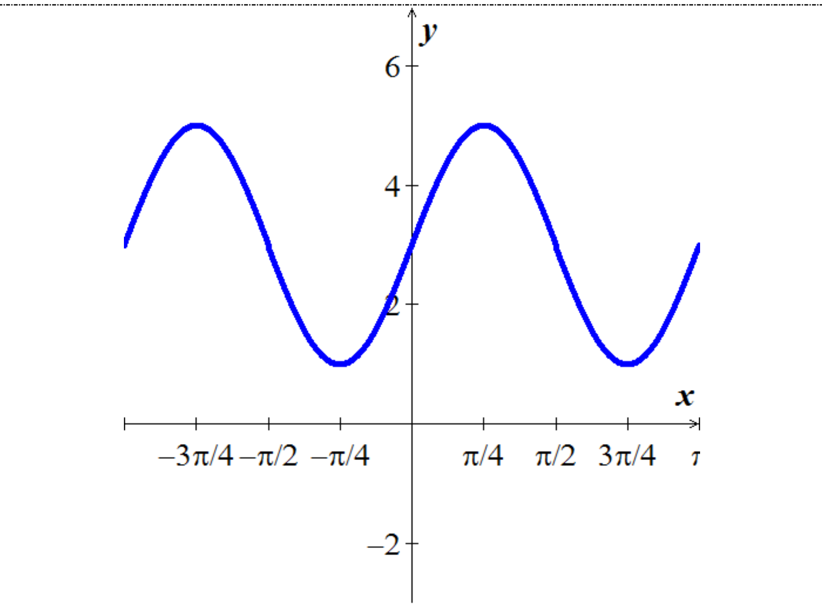
$y = -2\sin(2x - \pi) + 3$

Solution

Amplitude = 2

Period $= \frac{2\pi}{2} = \pi$

Phase shift $= \frac{\pi}{2}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

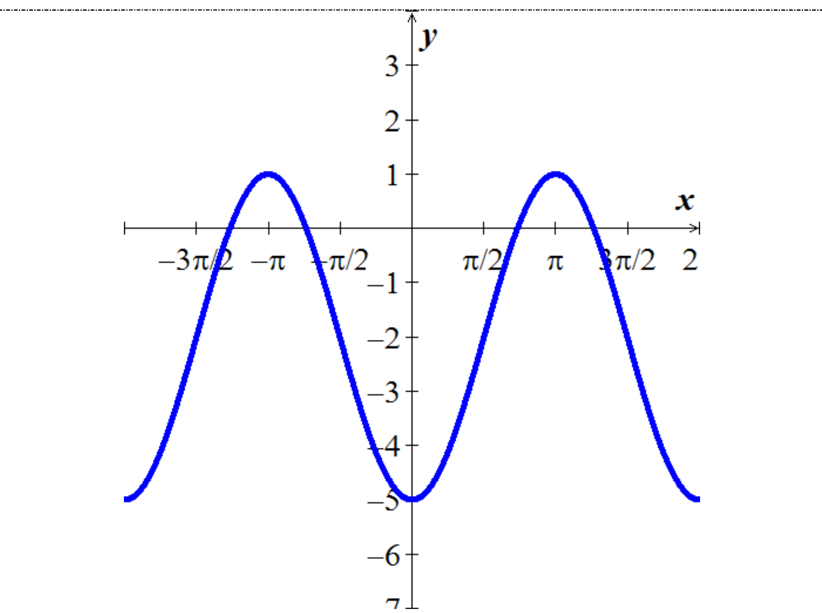
$y = 3\cos(x + 3\pi) - 2$

Solution

Amplitude = 3

Period $= \frac{2\pi}{1} = 2\pi$

Phase shift $= -3\pi$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

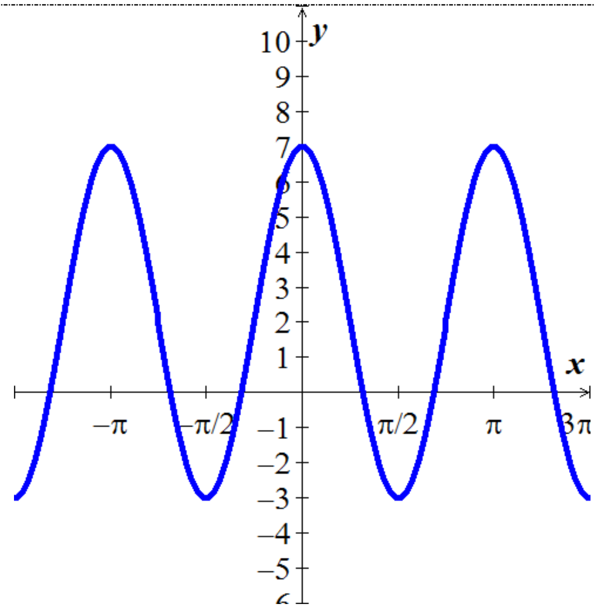
$y = 5\cos(2x + 2\pi) + 2$

Solution

Amplitude = 5

Period $= \dfrac{2\pi}{2} = \pi$

Phase shift $= -\pi$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

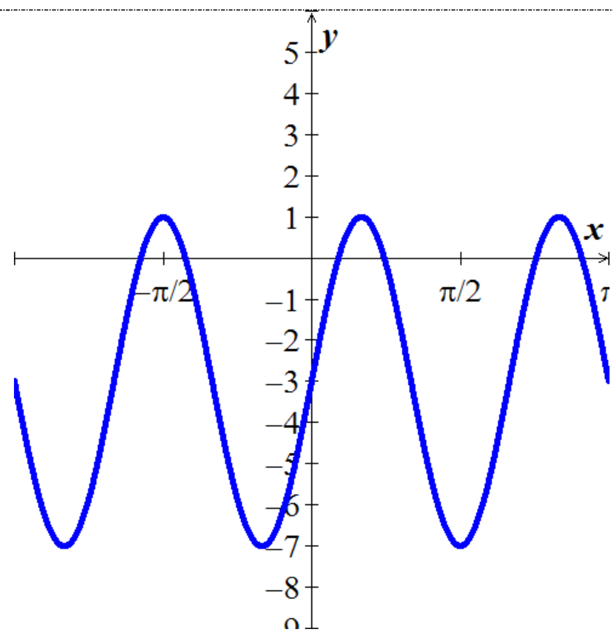
$y = -4\sin(3x - \pi) - 3$

Solution

Amplitude = 4

Period $= \dfrac{2\pi}{3}$

Phase shift $= \dfrac{\pi}{3}$



Exercise

Find the amplitude, the period, any vertical translation, and any phase shift. Then graph a one complete cycle of $y = \cos\frac{1}{2}x$

Solution

One cycle: $0 \le \arg ument \le 2\pi$

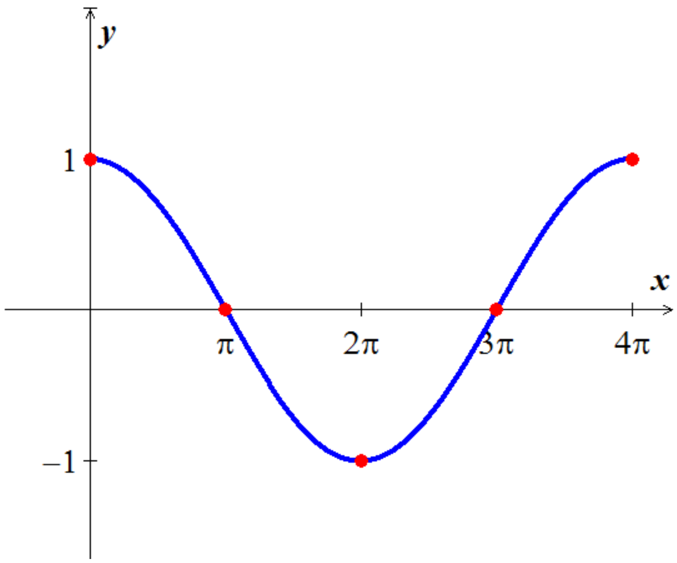
$$0 \le \frac{1}{2}x \le 2\pi \qquad \textit{Multiply by 2}$$

$$0 \le x \le 4\pi$$

Amplitude: $A = 1$

Period: $P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

x	x	$y = \cos\frac{1}{2}x$
0	0	1
$\frac{1}{4}P$	$\frac{1}{4}4\pi = \pi$	0
$\frac{1}{2}P$	$\frac{1}{2}4\pi = 2\pi$	-1
$\frac{3}{4}P$	$\frac{3}{4}4\pi = 3\pi$	0
P	4π	1



Exercise

Find the amplitude, the period, any vertical translation, and any phase shift. Then graph

$y = 2\sin(-\pi x) \quad \textit{for} \ -3 \le x \le 3$

Solution

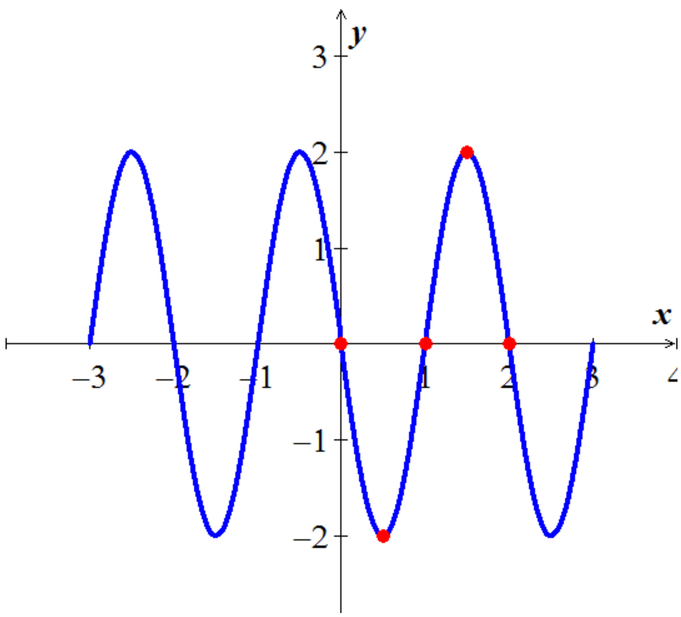
$y = 2\sin(-\pi x) \quad \textit{for} \ -3 \le x \le 3$

$$\begin{aligned} y &= 2\sin(-\pi x) \\ &= -2\sin(\pi x) \end{aligned}$$

Amplitude: $A = 2$

Period: $P = \frac{2\pi}{\pi} = 2$

x	$y = -2\sin(\pi x)$
0	0
$\frac{1}{2}$	-2
1	0
$\frac{3}{2}$	2
2	0



Exercise

Find the amplitude, the period, any vertical translation, and any phase shift. Then graph

$y = 4\cos\left(-\frac{2}{3}x\right)$ *for* $-\frac{15\pi}{4} \le x \le \frac{15\pi}{4}$

Solution

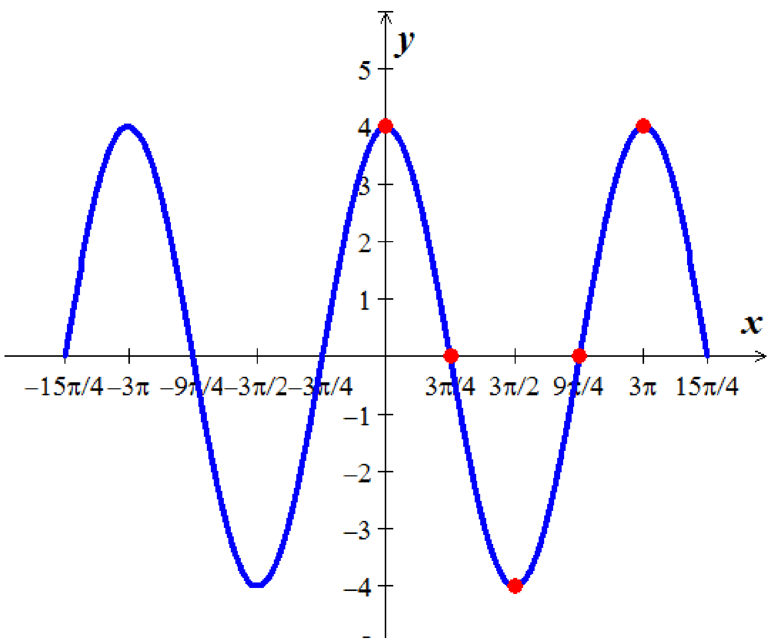
Amplitude: $A = 4$

Period: $P = \dfrac{2\pi}{\frac{2}{3}} = 3\pi$

$\frac{3\pi}{4} = section$

for $-\frac{15\pi}{4} \le x \le \frac{15\pi}{4}$

x	$y = 4\cos\left(-\frac{2}{3}x\right)$
0	4
$\frac{1}{4}3\pi = \frac{3\pi}{4}$	0
$\frac{1}{2}3\pi = \frac{3\pi}{2}$	-4
$\frac{3}{4}3\pi = \frac{9\pi}{4}$	0
3π	4



Exercise

Graph one complete cycle $y = \cos\left(x - \frac{\pi}{6}\right)$

Solution

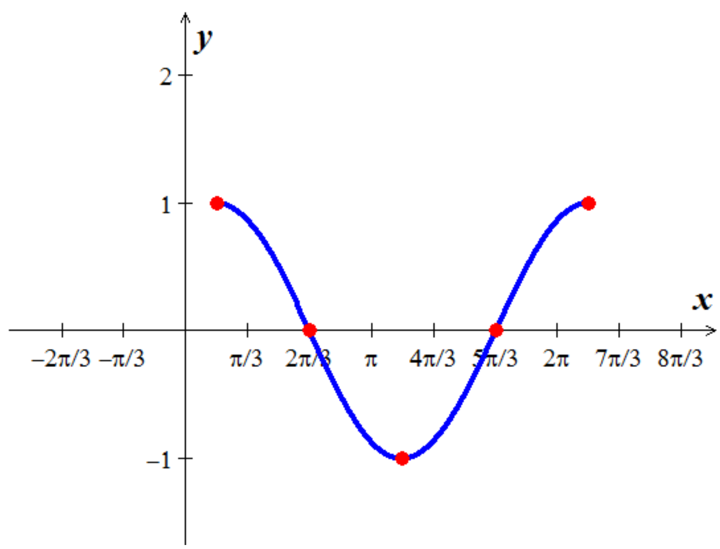
Amplitude: $A = 1$

Period: $P = \frac{2\pi}{1} = 2\pi$

Phase Shift = $\frac{\pi}{6}$

$x - \frac{\pi}{6} = 0 \rightarrow x = \frac{\pi}{6}$

x	x	$y = \cos\left(x - \frac{\pi}{6}\right)$
$\frac{\pi}{6} + 0$	$\frac{\pi}{6}$	1
$\frac{\pi}{6} + \frac{1}{2}\pi$	$\frac{2\pi}{3}$	0
$\frac{\pi}{6} + \pi$	$\frac{7\pi}{6}$	-1
$\frac{\pi}{6} + \frac{3}{2}\pi$	$\frac{5\pi}{3}$	0
$\frac{\pi}{6} + 2\pi$	$\frac{13\pi}{6}$	1



Exercise

Graph one complete cycle $y = \frac{2}{3} - \frac{4}{3}\cos(3x - \pi)$

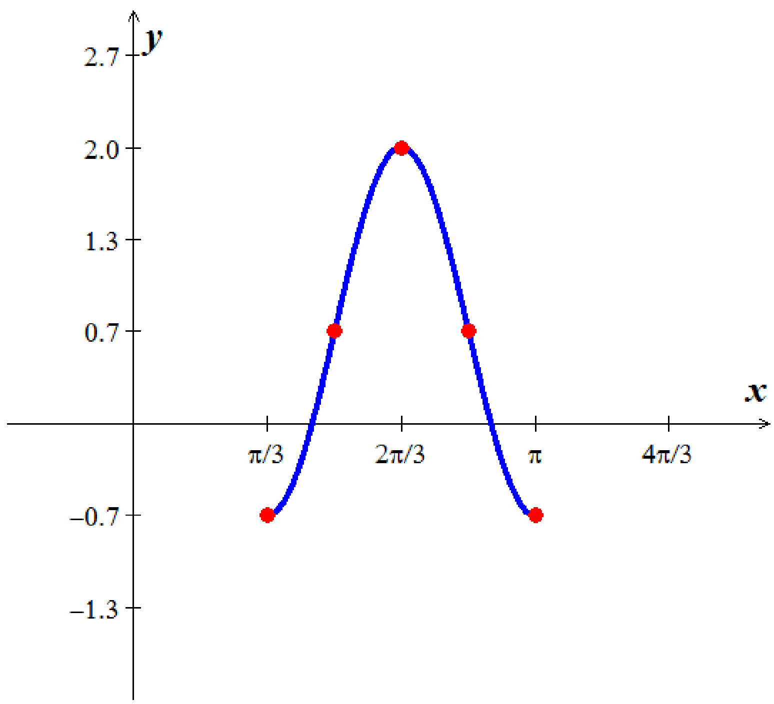
Solution

Amplitude: $A = \frac{4}{3}$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\phi = -\frac{-\pi}{3} = \frac{\pi}{3}$

x	$y = \frac{2}{3} - \frac{4}{3}\cos(3x - \pi)$
$\frac{\pi}{3}$	$-\frac{2}{3}$
$\frac{\pi}{2}$	$\frac{2}{3}$
$\frac{2\pi}{3}$	2
$\frac{5\pi}{6}$	$\frac{2}{3}$
π	$-\frac{2}{3}$



Exercise

Graph one complete cycle $y = -3 + \sin\left(\pi x + \frac{\pi}{2}\right)$

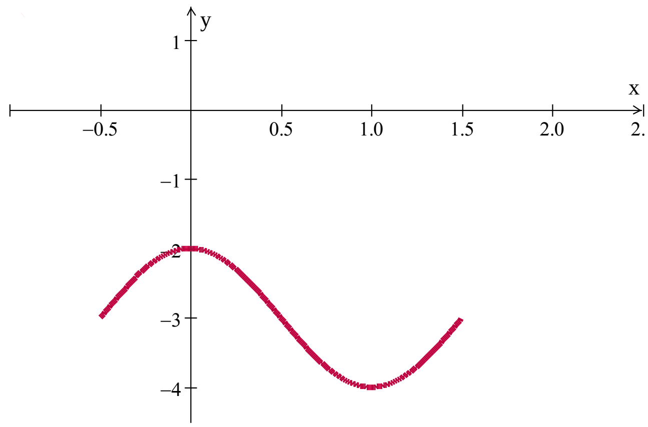
Solution

Amplitude: $A = 1$

Period: $P = \frac{2\pi}{\pi} = 2$

Phase Shift: $\phi = -\frac{\frac{\pi}{2}}{\pi} = -\frac{1}{2}$

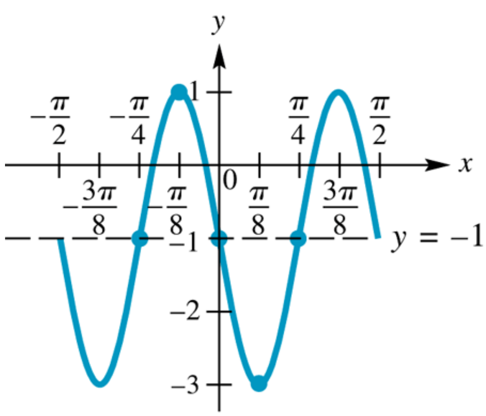
x	$y = -3 + \sin\left(\pi x + \frac{\pi}{2}\right)$
$-\frac{1}{2}$	-3
0	-2
$\frac{1}{2}$	-3
1	-4
$\frac{3}{2}$	-3



Exercise

Graph $y = -1 + 2\sin(4x + \pi)$ over two periods.

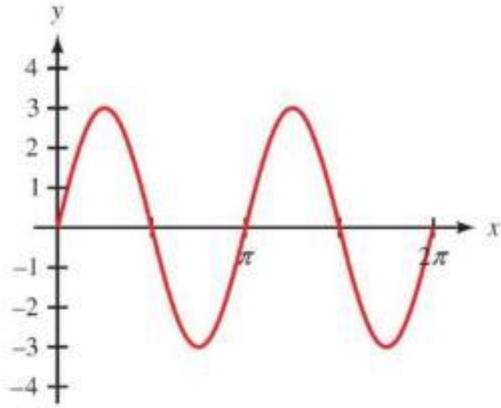
Solution



$y = -1 + 2\sin(4x + \pi)$

Exercise

Find an equation $y = k + A\sin(Bx + C)$ or $y = k + A\cos(Bx + C)$ to match the graph



Solution

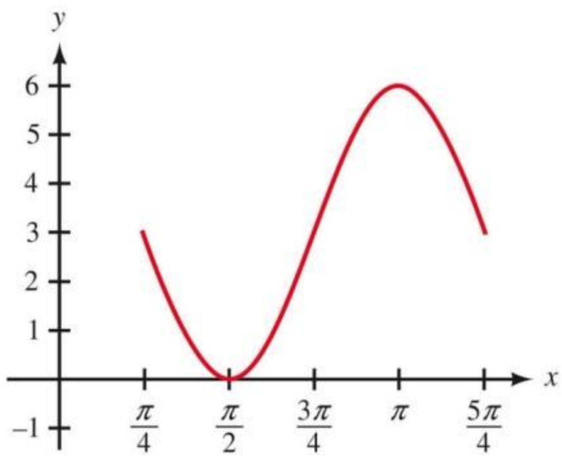
$$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$$

Amplitude = 3

$$y = 3\sin 2x \qquad 0 \le x \le 2\pi$$

Exercise

Find an equation $y = k + A\sin(Bx + C)$ or $y = k + A\cos(Bx + C)$ to match the graph



Solution

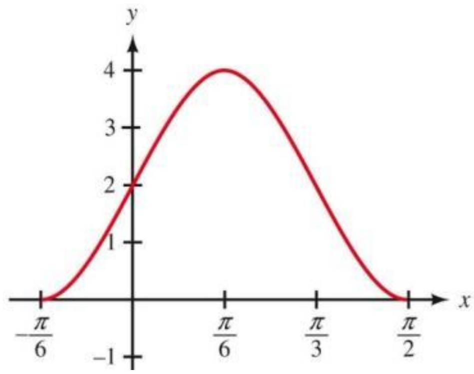
$$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$$

Amplitude = 3

$$y = 3\sin 2x \qquad \frac{\pi}{4} \le x \le \frac{5\pi}{4}$$

Exercise

Find an equation $y = k + A\sin(Bx + C)$ or $y = k + A\cos(Bx + C)$ to match the graph



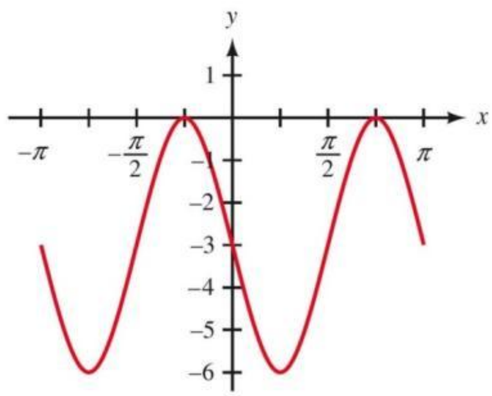
Solution

$P = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$	$B = \frac{2\pi}{P} = \frac{2\pi}{\frac{2\pi}{3}} = 3$
$\phi = -\frac{\pi}{6} = -\frac{C}{B} \Rightarrow C = \frac{\pi B}{6} = \frac{\pi}{2}$	***Amplitude*** = 2

$$y = 2 - 2\cos\left(3x + \frac{\pi}{2}\right) \quad -\frac{\pi}{6} \le x \le \frac{\pi}{2}$$

Exercise

Find an equation $y = k + A\sin(Bx + C)$ or $y = k + A\cos(Bx + C)$ to match the graph



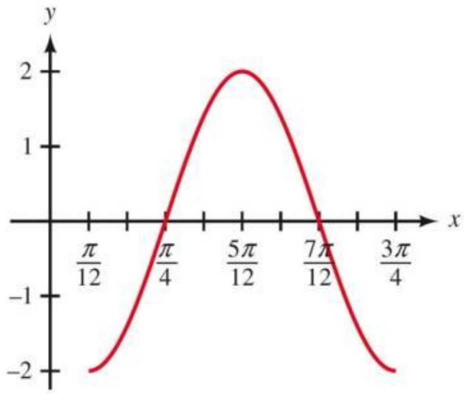
Solution

$P = \pi$	$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$
$\phi = 0$	***Amplitude*** = 3

$$y = -3 - 3\sin 2x \qquad -\pi \le x \le \pi$$

Exercise

Find an equation $y = k + A\sin(Bx + C)$ or $y = k + A\cos(Bx + C)$ to match the graph



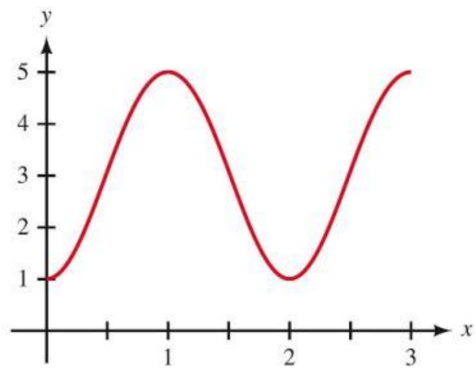
Solution

$P = \frac{3\pi}{4} - \frac{\pi}{12} = \frac{2\pi}{3}$	$B = \frac{2\pi}{P} = \frac{2\pi}{\frac{2\pi}{3}} = 3$
$\phi = \frac{\pi}{12} \Rightarrow C = -B\phi = -3\frac{\pi}{12} = -\frac{\pi}{4}$	***Amplitude*** $= 2$

$$y = -2\cos\left(3x - \frac{\pi}{4}\right) \qquad \frac{\pi}{12} \le x \le \frac{3\pi}{4}$$

Exercise

Find an equation $y = k + A\sin(Bx + C)$ or $y = k + A\cos(Bx + C)$ to match the graph



Solution

$P = 2$	$B = \frac{2\pi}{P} = \frac{2\pi}{2} = \pi$
$\phi = 0$	***Amplitude*** $= 2$

$$y = 3 - 2\cos(\pi x) \qquad 0 \le x \le 3$$

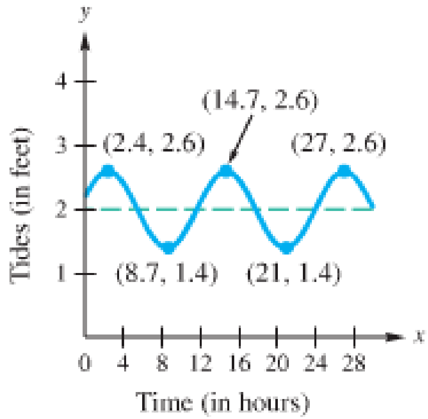
Exercise

The figure shows a function ***f*** that models the tides in feet at Clearwater Beach, *x* hours after midnight starting on Aug. 26,

a) Find the time between high tides.

b) What is the difference in water levels between high tide and low tide?

c) The tides can be modeled by $f(x)=0.6\cos[0.511x-2.4]+2$. Estimate the tides when $x = 10$.



Solution

a) Time between high tides = 14.7 – 2.4 = 12.3 hrs.

b) Difference in water levels between high tide and low tide = 2.6 – 1.4 = 1.2 *ft*.

c) $f(x=10)=0.6\cos[0.511(10)-2.4]_{rad}+2\approx 1.45$

Exercise

The maximum afternoon temperature in a given city might be modeled by $t=60-30\cos\frac{\pi x}{6}$

Where *t* represents the maximum afternoon temperature in month *x*, with *x* = 0 representing January, *x* = 1 representing February, and so on.. Find the maximum afternoon temperature to the nearest degree for each month.

a) Jan.

b) Apr.

c) May.

d) Jun.

e) Oct.

Solution

a) Jan. $t=60-30\cos\frac{\pi(0)}{6}=30°$

b) Apr. $t=60-30\cos\frac{\pi(4)}{6}=75°$

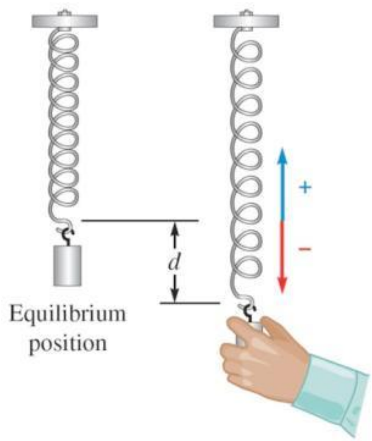
c) May. $t = 60 - 30\cos\frac{\pi(5)}{6} = 86°$

d) Jun. $t = 60 - 30\cos\frac{\pi(6)}{6} = 90°$

e) Oct. $t = 60 - 30\cos\frac{\pi(10)}{6} = 45°$

Exercise

A mass attached to a spring oscillates upward and downward. The length L of the spring after t seconds is given by the function $L = 15 - 3.5\cos(2\pi t)$, where L is measured in cm.



a) Sketch the graph of this function for $0 \le t \le 5$

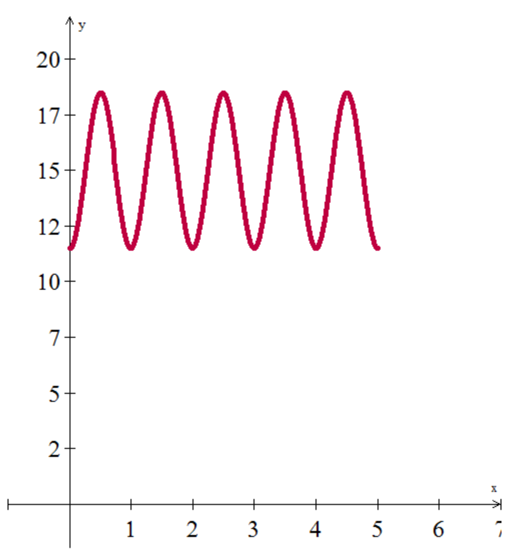
b) What is the length the spring when it is at equilibrium?

c) What is the length the spring when it is shortest?

d) What is the length the spring when it is longest?

Solution

a)



b) The length the spring when it is at equilibrium L = 15 cm

c) $\boxed{L = 15 - 3.5 = 11.5\ cm}$

d) $\boxed{L = 15 + 3.5 = 18.5\ cm}$

Exercise

The diameter of the Ferris wheel is 250 *ft*, the distance from the ground to the bottom of the wheel is 14 *ft*. We found the height of a rider on that Ferris wheel was given by the function:

$$H = 139 - 125\cos\left(\frac{\pi}{10}t\right)$$

Where t is the number of minutes from the beginning of a ride. Graph a complete cycle of this function.

Solution

Amplitude: $A = 125$

Period: $P = \frac{2\pi}{\frac{\pi}{10}} = 20$

Phase Shift: $\phi = 0$

VT: $H = 139$

t	$H = 139 - 125\cos\left(\frac{\pi}{10}t\right)$
0	139-125=14
5	139
10	139+125=264
15	139
20	14