Math 1325 - Calculus with Application **Exam 4** ***Review***

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Find the indefinite integral (general solution)

1. $\int \left(6x^5 + 5x^4 + 4x^3 + 2x - 6\right) dx$
2. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$
3. $\int \left(x^{-3} - 4x^2 + 2x - 5\right) dx$
4. $\int \frac{2x^3 + 1}{x^3}\, dx$
5. $\int 3x^2 \sqrt{x^3 + 1}\, dx$
6. $\int \left(1 + \sqrt{x}\right)^3 \left(\frac{1}{2\sqrt{x}}\right) dx$
7. $\int \frac{x^2}{\left(1 + x^3\right)^2} dx$
8. $\int u^3 \sqrt{u^4 + 2}\, du$
9. $\int \left(1 + \frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt$
10. $\int 3\left(x - 4\right) e^{x^2 - 8x} dx$
11. $\int e^{-0.25x} dx$
12. $\int \frac{5}{2x - 1} dx$
13. $\int \frac{e^{-x}}{1 - e^{-x}} dx$
14. $\int \frac{2}{1 + e^{-x}} dx$
15. $\int \frac{x^2}{5x^3 + 1} dx$
16. $\int \frac{2\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2} dx$

Find the particular solution $y = f(x)$ **at the given initial condition(s)**

17. $f'(x) = (2x - 3)(2x + 3);\ f(3) = 0$
18. $f''(x) = x^2;\ f'(0) = 6,\ f(0) = 0$
19. $f'(x) = \frac{x^2 - 5}{x^2}, x > 0;\ \ f(1) = 2$
20. $f''(x) = x^{-3/2};\ \ f'(1) = 2,\ f(9) = -4$
21. $f'(x) = \frac{e^{2x} + 2e^x - 1}{e^x};\ \ f(0) = 4$

Find the indefinite integral (use integration by parts):

22. $\int \ln x \, dx$

23. $\int xe^{2x} dx$

24. $\int x^3 e^x dx$

25. $\int \ln x^2 dx$

26. $\int x^2 e^{-3x} dx$

Evaluate the definite integral

27. $\int_2^5 (3x+4)\,dx$

28. $\int_3^6 \frac{2x}{x^2+3}\,dx$

29. $\int_3^6 \frac{x}{3\sqrt{x^2-8}}\,dx$

30. $\int_0^{\ln 5} e^{x/5} dx$

31. $\int_0^1 e^{5x} dx$

32. $\int_0^2 \left(2x^2+x+4\right) dx$

33. Find the area of the region bounded by $f(x) = -x^2 + 2x + 6$ *and* $g(x) = x^2 - 4x + 6$. The points of intersection are (0, 6) and (3, 3).

34. Find the area of the region bounded by $f(x) = -x^2 + 6x + 7$ *and* $g(x) = -2x + 19$. The points of intersection are (2, 15) and (6, 7).

35. If the monthly profit in thousands of dollars from the January 2000 until December 2007 can be approximated by the function $P(x) = -0.2x^2 + 10x + 1000,\ 0 \le x \le 96$, where x = 0 represents January 2000 and $x = 96$ represents December 2007. What is the average monthly profit for the years 2000 – 2007?

Solution

1. $x^6 + x^5 + x^4 + x^2 - 6x + C$
2. $\frac{2}{3}\sqrt{x^3} + \sqrt{x} + C$
3. $-\frac{1}{2}x^{-2} - \frac{4}{3}x^3 + x^2 - 5x + C$
4. $2x - \frac{1}{2x^2} + C$
5. $\frac{2}{3}\sqrt{\left(x^3+1\right)^3} + C$
6. $\frac{1}{4}\left(1+\sqrt{x}\right)^4 + C$
7. $-\frac{1}{3\left(1+x^3\right)} + C$
8. $\frac{1}{6}\sqrt{\left(u^4+2\right)^3} + C$
9. $-\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + C$
10. $\frac{3}{2}e^{x^2-8x} + C$
11. $-4e^{-0.25x} + C$
12. $\frac{5}{2}\ln\left(2x-1\right) + C$
13. $\ln\left(1-e^{-x}\right) + C$
14. $2\ln\left(e^x+1\right) + C$
15. $\frac{1}{15}\ln\left(5x^3+1\right) + C$
16. $-\frac{2}{e^x + e^{-x}} + C$
17. $f(x) = 43x^3 - 9x - 9$
18. $f(x) = \frac{1}{12}x^4 + 6x$
19. $f(x) = x + \frac{5}{x} - 4$
20. $f(x) = -4\sqrt{x} + 4x - 28$
21. $f(x) = e^x + 2x + e^{-x} + 2$
22. $x\ln x - x + C$
23. $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$
24. $e^x\left(x^3 - 3x^2 + 6x - 6\right) + C$
25. $2x(\ln x - 1) + C$
26. $-\frac{9x^2+6x+2}{27}e^{-3x} + C$
27. $\frac{87}{2}$
28. $\ln\frac{39}{12} \approx 1.179$
29. 1.43
30. 1.8986
31. $\frac{1}{5}\left(e^5 - 1\right)$
32. $\frac{46}{3}$
33. 9
34. 10.67
35. \$865.60

$$f'(x) = (2x-3)(2x+3);\ f(3) = 0$$

$$f(x) = \int f'(x)dx = \int (2x-3)(2x+3)dx$$

$$f(x) = \int (4x^2 - 9)dx = 4\frac{x^3}{3} - 9x + C$$

$$f(3) = 4\frac{3^3}{3} - 9(3) + C = 0$$

$$36 - 27 + C = 0 \to C = -9$$

$$f(x) = \frac{4}{3}x^3 - 9x - 9$$

$$f''(x) = x^2;\ f'(0) = 6,\ f(0) = 0$$

$$f'(x) = \int f''(x)dx = \int x^2 dx$$

$$f'(x) = \frac{x^3}{3} + C_1$$

$$f'(0) = \frac{0^3}{3} + C_1 = 6 \to C_1 = 6$$

$$f'(x) = \frac{x^3}{3} + 6$$

$$f(x) = \int f'(x) = \int \left(\frac{x^3}{3} + 6\right)dx$$

$$f(x) = \frac{x^4}{12} + 6x + C$$

$$f(0) = \frac{0^4}{12} + 60 + C = 0 \to C = 0$$

$$f(x) = \frac{1}{12}x^4 + 6x$$

$$f'(x) = \frac{x^2 - 5}{x^2},\ x > 0;\ \ f(1) = 2$$

$$f(x) = \int \frac{x^2 - 5}{x^2}dx = \int \left(\frac{x^2}{x^2} - \frac{5}{x^2}\right)dx = \int (1 - 5x^{-2})dx = x - 5\frac{x^{-1}}{-1} + C = x + \frac{5}{x} + C$$

$$f(x=1) = 1 + \frac{5}{1} + C = 2 \to 6 + C = 2 \to C = -4$$

$$f(x) = x + \frac{5}{x} - 4$$

$$f''(x) = x^{-3/2};\ \ f'(1) = 2,\ f(9) = -4$$

$$f'(x) = \int x^{-3/2}dx = \frac{x^{-1/2}}{-1/2} + C_1 = -\frac{2}{x^{1/2}} + C_1$$

$$f'(1) = -\frac{2}{1^{1/2}} + C_1 = 2 \to -2 + C_1 = 2 \to C_1 = 4$$

$$f'(x) = -\frac{2}{x^{1/2}} + 4$$

$$f(x) = \int \left(-2x^{-1/2} + 4\right)dx = -2\frac{x^{1/2}}{1/2} + 4x + C = -4x^{1/2} + 4x + C$$

$$f(9) = -4\sqrt{9} + 4(9) + C = -4$$

$$-12 + 36 + C = -4$$

$$24 + C = -4$$

$$C = -28$$

$$f(x) = -4\sqrt{x} + 4x - 28$$

$$f'(x) = \frac{e^{2x} + 2e^{x} - 1}{e^{x}}; \quad f(0) = 4$$

$$f(x) = \int\left(\frac{e^{2x}}{e^{x}} + \frac{2e^{x}}{e^{x}} - \frac{1}{e^{x}}\right)dx = \int\left(e^{x} + 2 - e^{-x}\right)dx = e^{x} + 2x + e^{-x} + C$$

$$f(0) = e^{0} + 2(0) + e^{-0} + C = 4$$

$$1 + 0 + 1 + C = 4 \Rightarrow C = 2$$

$$f(x) = e^{x} + 2x + e^{-x} + 2$$

Evaluate the integral $\int xe^{2x}dx$

Solution

Let: $u = x \Rightarrow du = dx$

$$dv = e^{2x}dx \Rightarrow v = \int dv = \int e^{2x}dx = \frac{1}{2}e^{2x}$$

$$\int udv = uv - \int vdu$$

$$\int xe^{2x}dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\frac{1}{2}e^{2x} + C$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

		$\int e^{2x}dx$
x	$(+)$	$\frac{1}{2}e^{2x}$
1	$(-)$	$\frac{1}{4}e^{2x}$

Evaluate the integral $\int \ln x^2 dx$

Solution

$$\int \ln x^2 dx = 2\int \ln x dx$$

$$u = \ln x \Rightarrow du = \frac{1}{x}dx$$

$$dv = dx \Rightarrow v = x$$

$$\int \ln x^2 dx = 2\left[x\ln x - \int x\frac{1}{x}dx \right]$$

$$= 2\left[x\ln x - \int dx \right]$$

$$= 2(x\ln x - x) + C$$

$$= 2x(\ln x - 1) + C$$

Evaluate the integral $\int x^2 e^{-3x} dx$

Solution

		$\int e^{-3x} dx$
x^2	$(+)$	$-\frac{1}{3}e^{-3x}$
$2x$	$(-)$	$\frac{1}{9}e^{-3x}$
2	$(+)$	$-\frac{1}{27}e^{-3x}$

$$= -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C = -\frac{9x^2+6x+2}{27}e^{-3x} + C$$