Review

arithmetic: $d = \frac{y_2 - y_1}{x_2 - x_1}$

$a_n = a_1 + (n-1)d$

Geometric: $r = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$

$a_n = a_1 \, r^{n-1}$

if $|r| < 1 \rightarrow S = \frac{a_1}{1-r}$

$|r| \geq 1 \rightarrow S = \infty$

$$\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1} = \frac{2}{1-\frac{2}{3}} \quad \checkmark \qquad \left|\frac{2}{3}\right| < 1 \ \checkmark$$

$$= 6$$

$$\sum_{n=1}^{\infty} 2\left(\frac{3}{2}\right)^{n-1} = \infty \qquad \left|\frac{3}{2}\right| \geq 1$$

$$\sum_{n=5}^{60} 3 = 3(60-5+1)$$
$$= \boxed{168}$$

$$\sum_{n=1}^{50} 4 = 4(50)$$
$$= \boxed{200}$$

$(2-1)+(4-1)$

$$\sum_{k=1}^{4} (2k-1) = 1+3+5+7$$
$$= \boxed{16}$$

$$\frac{x}{x^2-3x+4} = \frac{A}{x+1} + \frac{B}{x-4}$$

$$x = A(x-4) + B(x+1)$$

$x^1 \quad A + B = 1$

$x^0 \quad -4A + B = 0$

$5A = 1$

$$\boxed{A = \frac{1}{5}}$$

$$\boxed{B = \frac{4}{5}}$$

$$\boxed{\frac{x}{x^2-3x+4} = \frac{1/5}{x+1} + \frac{4/5}{x-4}}$$

$3 + 3^2 + \cdots + 3^n = \frac{3}{2}(3^n - 1)$ coeff 1^{st} n

For $n = 1 \Rightarrow 3 \stackrel{?}{=} \frac{3}{2}(3-1)$

$3 = 3$ ✓ P_1 is true.

P_k is true: $3 + \cdots + 3^k = \frac{3}{2}(3^k - 1)$

is P_{k+1}: $3 + \cdots + 3^k + 3^{k+1} \boxed{= \frac{3}{2}(3^{k+1} - 1)}$?

$$
\begin{aligned}
3 + \cdots + 3^k + 3^{k+1} &= \frac{3}{2}(3^k - 1) + 3^{k+1} \\
&= \frac{1}{2}(3^{k+1} - 3) + \frac{2}{2}3^{k+1} \\
&= \frac{1}{2}(3^{k+1} - 3 + 2 \cdot 3^{k+1}) \\
&= \frac{1}{2}(3 \cdot 3^{k+1} - 3) \\
&= \frac{3}{2}(3^{k+1} - 1) \checkmark
\end{aligned}
$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the given proof is completed.

~~Wrong~~

$3 + \cdots + 3^k + 3^{k+1} = \frac{3}{2}(3^k - 1) + 3^{k+1}$

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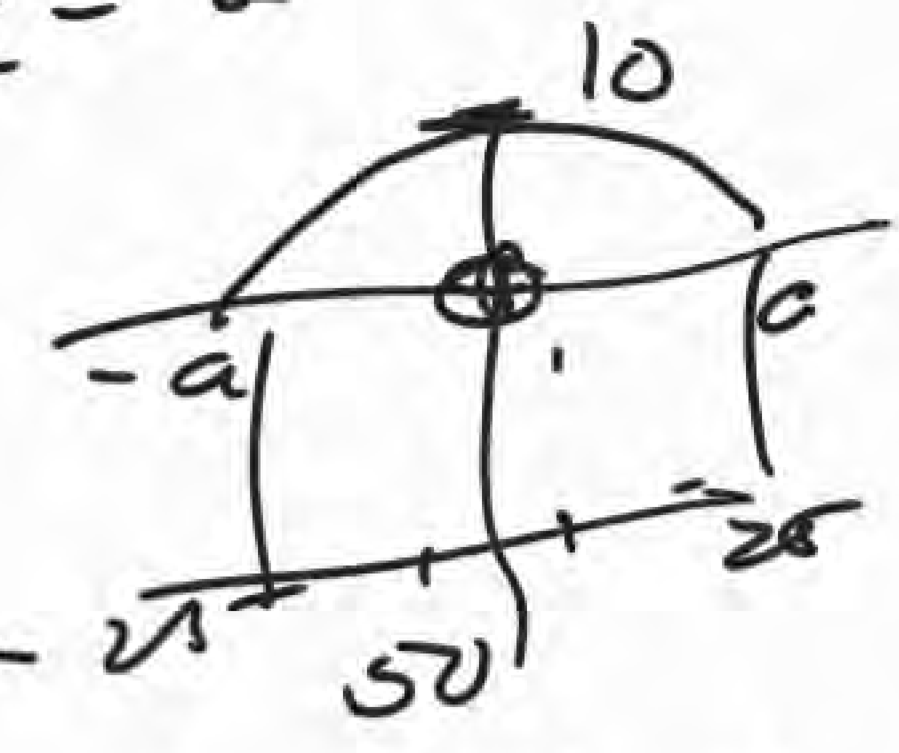
ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\boxed{a > b}$



$a = 25$

$b = 10$

$\dfrac{y^2}{b^2} = 1 - \left(\dfrac{x}{a}\right)^2 \longrightarrow$ simplify.

$\frac{a}{2} = x$

$y^2 = b^2 \dfrac{a_1^2 - x_1^2}{a_1^2}$ $\quad \left(\dfrac{x}{a}\right)^2 = \left(\dfrac{x_1}{a_1}\right)^2$

$(h)^2 ? \left(\dfrac{b}{a_1}\right)^2 (\#)$

$\begin{cases} w = 10 \to x = 5 \\ h = 9 \end{cases}$

$\dfrac{y^2}{10^2} = 1 - \left(\dfrac{\cancel{5}^{\,1}}{\cancel{25}_{\,5}}\right)^2$

$= 1 - \dfrac{1}{25}$

$= \dfrac{24}{5^2}$

$y^2 = \left(\dfrac{10}{5}\right)^2 (24)$

$81 < 96$ clear. —