

Solution **Section 5.5 – Arithmetic and Geometric Sequences**

Exercise

Show that the sequence $-6, -2, 2, \dots, 4n-10, \dots$ is arithmetic, and find the common difference.

Solution

We to show that $a_{k+1} - a_k$ equals to a constant.

$$\begin{aligned}a_{k+1} - a_k &= 4(k+1) - 10 - (4k - 10) \\&= 4k + 4 - 10 - 4k + 10 \\&= 4\end{aligned}$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $2, 6, 10, 14, \dots$

Solution

$$\begin{aligned}d &= 6 - 2 \\&= 4\end{aligned}$$

$$\begin{aligned}a_n &= 2 + (n-1)4 \\&= 2 + 4n - 4 \\&= 4n - 2\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 4(10) - 2 \\&= 38\end{aligned}$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $3, 2.7, 2.4, 2.1, \dots$

Solution

$$\begin{aligned}d &= 2.7 - 3 = -0.3 \\&= -0.3\end{aligned}$$

$$\begin{aligned}a_n &= 3 + (n-1)(-0.3) \\&= 3 - 0.3n + 0.3 \\&= 3.3 - 0.3n\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 3.3 - 0.3(10) \\&= 0\end{aligned}$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $-6, -4.5, -3, -1.5, \dots$

Solution

$$d = -4.5 - (-6)$$

$$= 1.5$$

$$a_n = -6 + (n-1)(1.5)$$

$$= -6 + 1.5n - 1.5$$

$$= 1.5n - 7.5$$

$$a_{10} = 1.5(10) - 7.5$$

$$= 7.5$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $\ln 3, \ln 9, \ln 27, \ln 81, \dots$

Solution

$$\ln 3, \ln 3^2, \ln 3^3, \ln 3^4, \dots$$

$$\ln 3, 2\ln 3, 3\ln 3, 4\ln 3, \dots$$

$$d = 2\ln 3 - \ln 3$$

$$= \ln 3$$

$$a_n = \ln 3 + (n-1)\ln 3$$

$$= \ln 3 + n\ln 3 - \ln 3$$

$$= n\ln 3$$

$$a_{10} = 10\ln 3$$

$$= \ln 3^{10}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = 2, \quad d = 3$

Solution

$$a_n = 2 + 3(n-1)$$

$$= 2 + 3n - 3$$

$$= 3n - 1$$

$$a_{10} = 29$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = 5$, $d = -3$

Solution

$$\begin{aligned}a_n &= 5 + (n-1)(-3) \\&= 5 - 3n + 3 \\&= \underline{8 - 3n}\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 8 - 30 \\&= \underline{-22}\end{aligned}$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = 1$, $d = -\frac{1}{2}$

Solution

$$\begin{aligned}a_n &= 1 + (n-1)\left(-\frac{1}{2}\right) \\&= 1 - \frac{1}{2}n + \frac{1}{2} \\&= \underline{\frac{3}{2} - \frac{1}{2}n}\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= \frac{3}{2} - \frac{1}{2}(10) \\&= \underline{-\frac{7}{2}}\end{aligned}$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = -2$, $d = 4$

Solution

$$\begin{aligned}a_n &= -2 + (n-1)(4) \\&= -2 + 4n - 4 \\&= \underline{4n - 6}\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_{10} &= 4(10) - 6 \\&= \underline{36}\end{aligned}$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = \sqrt{2}$, $d = \sqrt{2}$

Solution

$$\begin{aligned}a_n &= \sqrt{2} + (n-1)\sqrt{2} \\&= \sqrt{2} + \sqrt{2}n - \sqrt{2} \\&= \sqrt{2}n\end{aligned}$$

$$a_{10} = 10\sqrt{2}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = 0$, $d = \pi$

Solution

$$\begin{aligned}a_n &= 0 + (n-1)(\pi) \\&= \pi n - \pi\end{aligned}$$

$$\begin{aligned}a_{10} &= 10\pi - \pi \\&= 9\pi\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = 13$, $d = 4$

Solution

$$\begin{aligned}a_n &= 13 + (n-1)(4) \\&= 4n + 9\end{aligned}$$

$$\begin{aligned}a_{10} &= 4(10) + 9 \\&= 49\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = -40$, $d = 5$

Solution

$$\begin{aligned}a_n &= -40 + (n-1)(5) \\&= 5n - 45\end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$a_{10} = 4(10) - 45$$

$$\underline{= -5}$$

Exercise

Find the n th term, and the tenth term of the arithmetic sequence: $a_1 = -32$, $d = 4$

Solution

$$a_n = -32 + (n-1)(4)$$

$$a_n = a_1 + (n-1)d$$

$$\underline{= 4n - 36}$$

$$a_{10} = 4(10) - 36$$

$$\underline{= 4}$$

Exercise

Find the common difference for the arithmetic sequence with the specified terms: $a_4 = 14$, $a_{11} = 35$

Solution

$$a_n = a_1 + (n-1)d$$

$$a_{11} = a_1 + 10d \rightarrow 35 = a_1 + 10d$$

$$a_4 = a_1 + 3d \rightarrow \frac{14 = a_1 + 3d}{21 = 7d}$$

$$\underline{d = 3}$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_{12} ; $a_1 = 9.1$, $a_2 = 7.5$

Solution

$$d = a_2 - a_1$$

$$= 7.5 - 9.1$$

$$\underline{= -1.6}$$

$$a_n = a_1 + (n-1)d$$

$$a_{12} = 9.1 + (11)(-1.6)$$

$$\underline{= -8.5}$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_1 ; $a_8 = 47$, $a_9 = 53$

Solution

$$\begin{aligned}d &= a_9 - a_8 \\&= 53 - 47 \\&= 6\end{aligned}$$

$$a_8 = a_1 + (7)(6)$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned}a_1 &= 47 - 42 \\&= 5\end{aligned}$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_{10} ; $a_2 = 1$, $a_{18} = 49$

Solution

$$a_2 = a_1 + d$$

$$a_1 = a_2 - d$$

$$\begin{aligned}a_{18} &= a_1 + (17)d \\&= a_2 - d + 17d \\&= a_2 + 16d\end{aligned}$$

$$49 = 1 + 16d$$

$$16d = 48$$

$$d = \frac{48}{16} = 3$$

$$\begin{aligned}a_1 &= a_2 - d \\&= 1 - 3 \\&= -2\end{aligned}$$

$$\begin{aligned}a_{10} &= -2 + 9(3) \\&= 25\end{aligned}$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_{10} ; $a_8 = 8$, $a_{20} = 44$

Solution

$$\begin{aligned}
 d &= \frac{44-8}{20-8} \\
 &= \frac{36}{12} \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 a_8 &= a_1 + (8-1)(3) \\
 8 &= a_1 + 21 \\
 a_1 &= -13 \\
 a_{10} &= -13 + 9(3) \\
 &= 14
 \end{aligned}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_{12} ; $a_8 = 4$, $a_{18} = -96$

Solution

$$\begin{aligned}
 d &= \frac{-96-4}{18-8} \\
 &= \frac{-100}{10} \\
 &= -10
 \end{aligned}$$

$$\begin{aligned}
 a_8 &= a_1 + (8-1)(-10) \\
 4 &= a_1 - 70 \\
 a_1 &= 74 \\
 a_{12} &= 74 + (11)(-10) \\
 &= -36
 \end{aligned}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_8 ; $a_{15} = 0$, $a_{40} = -50$

Solution

$$\begin{aligned}
 d &= \frac{-50-0}{40-15} \\
 &= \frac{-50}{25} \\
 &= -2
 \end{aligned}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_{15} = a_1 + (15-1)(-2)$$

$$0 = a_1 - 28$$

$$\underline{a_1 = 28}$$

$$a_8 = 28 + (7)(-2)$$

$$\underline{= 14}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_{20} ; $a_9 = -5$, $a_{15} = 31$

Solution

$$d = \frac{31+5}{15-9}$$

$$= \frac{36}{6}$$

$$\underline{= 6}$$

$$a_9 = a_1 + (9-1)(6)$$

$$-5 = a_1 + 42$$

$$\underline{a_1 = -47}$$

$$a_{20} = -47 + (19)(6)$$

$$\underline{= 67}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_n ; $a_8 = 8$, $a_{20} = 44$

Solution

$$d = \frac{44-8}{20-8}$$

$$= \frac{36}{12}$$

$$\underline{= 3}$$

$$a_8 = a_1 + 3(8-1)$$

$$8 = a_1 + 21$$

$$\underline{a_1 = -13}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_n = a_1 + (n-1)d$$

$$a_n = -13 + 3(n-1)$$

$$\underline{= 3n - 16}$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_n ; $a_8 = 4$, $a_{18} = -96$

Solution

$$d = \frac{-96 - 4}{18 - 8}$$

$$\underline{= -10}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_8 = a_1 - 10(8-1)$$

$$a_n = a_1 + (n-1)d$$

$$4 = a_1 - 70$$

$$\underline{a_1 = 74}$$

$$a_n = 74 - 10(n-1)$$

$$\underline{= -10n + 84}$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_n ; $a_{14} = -1$, $a_{15} = 31$

Solution

$$d = \frac{31 - (-1)}{15 - 14}$$

$$\underline{= 32}$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_{14} = a_1 + 32(14-1)$$

$$a_n = a_1 + (n-1)d$$

$$-1 = a_1 + 416$$

$$\underline{a_1 = -417}$$

$$a_n = -417 + 32(n-1)$$

$$\underline{= 32n - 449}$$

Exercise

Find the specified term of the arithmetic sequence that has two given terms: a_n ; $a_9 = -5$, $a_{15} = 31$

Solution

$$d = \frac{31+5}{15-9}$$

$$= 6$$

$$a_9 = a_1 + 6(9-1)$$

$$-5 = a_1 + 48$$

$$a_1 = -53$$

$$a_n = -53 + 6(n-1)$$

$$= 6n - 59$$

$$d = \frac{a_y - a_x}{y - x}$$

$$a_n = a_1 + (n-1)d$$

Exercise

Find the sum S_n of the arithmetic sequence that satisfies the conditions: $a_1 = 40$, $d = -3$, $n = 30$

Solution

$$S_n = \frac{30}{2} [2(40) + (30-1)(-3)]$$

$$= -105$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

Exercise

Find the sum S_n of the arithmetic sequence that satisfies the conditions: $a_7 = \frac{7}{3}$, $d = -\frac{2}{3}$, $n = 15$

Solution

$$a_7 = a_1 + (6)\left(-\frac{2}{3}\right) = \frac{7}{3}$$

$$\frac{7}{3} = a_1 - 4$$

$$a_1 = \frac{7}{3} + 4$$

$$= \frac{19}{3}$$

$$S_n = \frac{15}{2} \left[2\left(\frac{19}{3}\right) + (15-1)\left(-\frac{2}{3}\right) \right]$$

$$= 25$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

Exercise

Find the number of integers between 32 and 390 that are divisible by 6, find their sum

Solution

$$\text{Number of terms: } n = \frac{390-36}{6} + 1 = 60$$

$$S_n = \frac{60}{2}(36 + 390) \qquad S_n = \frac{n}{2}(a_1 + a_n)$$
$$\underline{= 12780}$$

Exercise

Find the number of terms in the arithmetic sequence with the given conditions:

$$a_1 = -2, \quad d = \frac{1}{4}, \quad S = 21$$

Solution

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

$$21 = \frac{n}{2} [2(-2) + (n-1)\frac{1}{4}]$$

$$21 = -2n + \frac{1}{8}n(n-1)$$

$$(8)21 = -2n(8) + \frac{1}{8}n(n-1)(8)$$

$$168 = -16n + (n^2 - n)$$

$$0 = n^2 - 17n - 168$$

$$\boxed{n = 24} \qquad n = -7$$

Exercise

Express the sum in terms of summation notation and find the sum $2 + 11 + 20 + \dots + 16,058$.

Solution

Difference in terms:

$$d = 11 - 2 = 9$$

Number of terms:

$$n = \frac{16058-2}{9} + 1 = \underline{1785}$$

$$a_n = 2 + (n-1)(9)$$

$$= 2 + 9n - 9$$

$$\underline{= 9n - 7}$$

$$a_n = a_1 + (n-1)d$$

Hence the n th term is: $\sum_{n=1}^{1785} (9n - 7)$

$$S_{1785} = \frac{1789}{2}(2 + 16058) \\ = \underline{14,333,550}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Express the sum in terms of summation notation and find the sum $60 + 64 + 68 + 72 + \cdots + 120$.

Solution

Difference in terms:

$$d = 64 - 60 = 4$$

Number of terms:

$$n = \frac{120 - 60}{4} + 1 = \underline{16}$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$a_n = 60 + (n - 1)(4) \\ = \underline{4n - 54}$$

Hence the n th term is: $\sum_{n=1}^{16} (4n - 54)$

$$S = \frac{16}{2}(60 + 120) \\ = \underline{1440}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $1 + 3 + 5 + \cdots + (2n - 1)$

Solution

Difference in terms:

$$d = 3 - 1 = 2$$

Number of terms:

$$n = \frac{(2n - 1) - 1}{2} + 1 \\ = \frac{2n - 2}{2} + 1 \\ = n - 1 + 1 \\ = \underline{n}$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$\begin{aligned}
 S &= \frac{n}{2}(1 + (2n - 1)) \\
 &= \frac{n}{2}(2n) \\
 &= \underline{n^2}
 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $2 + 4 + 6 + \cdots + 2n$

Solution

Difference in terms: $d = 4 - 2 = 2$

Number of terms:

$$\begin{aligned}
 n &= \frac{2n - 2}{2} + 1 & n &= \frac{a_n - a_1}{d} + 1 \\
 &= n - 1 + 1 \\
 &= \underline{n}
 \end{aligned}$$

$$\begin{aligned}
 S &= \frac{n}{2}(2 + 2n) & S_n &= \frac{n}{2}(a_1 + a_n) \\
 &= n(n + 1) \\
 &= \underline{n^2 + n}
 \end{aligned}$$

Exercise

Find each arithmetic sum $2 + 5 + 8 + \cdots + 41$

Solution

Difference in terms:

$$d = 5 - 2 = 3$$

Number of terms:

$$\begin{aligned}
 n &= \frac{41 - 2}{3} + 1 & n &= \frac{a_n - a_1}{d} + 1 \\
 &= \underline{14}
 \end{aligned}$$

$$\begin{aligned}
 S &= \frac{14}{2}(2 + 41) & S_n &= \frac{n}{2}(a_1 + a_n) \\
 &= \underline{301}
 \end{aligned}$$

Exercise

Find each arithmetic sum $7 + 12 + 17 + \cdots + (2 + 5n)$

Solution

Difference in terms:

$$d = 12 - 7 = 5$$

Number of terms:

$$\begin{aligned} n &= \frac{2 + 5n - 7}{5} + 1 \\ &= \frac{5n - 5}{5} + 1 \\ &= \frac{5n}{5} - \frac{5}{5} + 1 \\ &= n \end{aligned}$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$\begin{aligned} S &= \frac{n}{2}(7 + 2 + 5n) \\ &= \frac{n}{2}(9 + 5n) \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $73 + 78 + 83 + 88 + \cdots + 558$

Solution

Difference in terms:

$$d = 78 - 73 = 5$$

Number of terms:

$$\begin{aligned} n &= \frac{558 - 73}{5} + 1 \\ &= 98 \end{aligned}$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$\begin{aligned} S &= \frac{98}{2}(73 + 558) \\ &= 30,919 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $7 + 1 - 5 - 11 - \cdots - 299$

Solution

Difference in terms:

$$d = 1 - 7 = -6$$

Number of terms:

$$n = \frac{-299 - 7}{-6} + 1$$

$$= 52$$

$$n = \frac{a_n - a_1}{d} + 1$$

$$S = \frac{52}{2}(7 - 299)$$

$$= -7592$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $-1 + 2 + 7 + \dots + (4n - 5)$

Solution

$$S = \frac{n}{2}(-1 + 4n - 5)$$

$$= n(2n - 3)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $5 + 9 + 13 + \dots + 49$

Solution

Difference in terms: $d = 9 - 5 = 4$

Number of terms: $n = \frac{49 - 5 + 4}{4} = 12$

$$S = \frac{12}{2}(5 + 49)$$

$$= 324$$

$$n = \frac{a_n - a_1 + d}{d}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $2 + 4 + 6 + \dots + 70$

Solution

Difference in terms: $d = 4 - 2 = 2$

Number of terms:

$$n = \frac{70 - 2 + 2}{2}$$

$$= 35$$

$$n = \frac{a_n - a_1 + d}{d}$$

$$S = \frac{35}{2}(70 + 2)$$

$$= 1,260$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $1 + 3 + 5 + \cdots + 59$

Solution

Difference in terms: $d = 3 - 1 = 2$

Number of terms:

$$\begin{aligned} n &= \frac{59 - 1 + 2}{2} \\ &= 30 \end{aligned}$$

$$n = \frac{a_n - a_1 + d}{d}$$

$$\begin{aligned} S &= \frac{30}{2}(59 + 1) \\ &= 900 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $4 + 4.5 + 5 + 5.5 + \cdots + 100$

Solution

Difference in terms: $d = 4.5 - 4 = 0.5$

Number of terms:

$$\begin{aligned} n &= \frac{100 - 4 + 0.5}{0.5} \\ &= 193 \end{aligned}$$

$$n = \frac{a_n - a_1 + d}{d}$$

$$\begin{aligned} S &= \frac{193}{2}(4 + 100) \\ &= 10,036 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find each arithmetic sum $8 + 8\frac{1}{4} + 8\frac{1}{2} + 8\frac{3}{4} + 9 + \cdots + 50$

Solution

Difference in terms: $d = 8\frac{1}{4} - 8 = \frac{1}{4}$

Number of terms:

$$\begin{aligned} n &= \frac{50 - 8 + 0.25}{0.25} \\ &= 169 \end{aligned}$$

$$n = \frac{a_n - a_1 + d}{d}$$

$$\begin{aligned} S &= \frac{169}{2}(8 + 50) \\ &= 4,901 \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Show that the given sequence is geometric, and find the common ratio

$$5, -\frac{5}{4}, \frac{5}{16}, \dots, 5\left(-\frac{1}{4}\right)^{n-1}, \dots$$

Solution

To be geometric, we must show that $\frac{a_{k+1}}{a_k} = r$ is equal to some constant, which is the common ratio.

The common ratio:

$$\begin{aligned} r &= \frac{a_2}{a_1} & r &= \frac{a_{k+1}}{a_k} \\ &= \frac{-\frac{5}{4}}{5} \\ &= -\frac{1}{4} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence 8, 4, 2, 1, ...

Solution

$$\text{Given: } a_1 = 8, \quad r = \frac{4}{8} = \frac{1}{2}$$

$$\begin{aligned} a_n &= a_1 r^{n-1} = 8\left(\frac{1}{2}\right)^{n-1} \\ &= 2^3 \left(2^{-1}\right)^{n-1} \\ &= 2^3 2^{-n+1} \\ &= 2^{4-n} \end{aligned}$$

$$\begin{aligned} a_5 &= 2^{4-5} \\ &= 2^{-1} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} a_8 &= 2^{4-8} \\ &= 2^{-4} \\ &= \frac{1}{16} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence

$$300, -30, 3, -0.3, \dots$$

Solution

$$\text{Given: } a_1 = 300, \quad r = \frac{-30}{300} = -0.1$$

$$\begin{aligned} a_n &= a_1 r^{n-1} = 300(-0.1)^{n-1} \\ &= 3(10^2)(-10^{-1})^{n-1} = 3(10)^2(-10)^{-n+1} = 3(-10)^{-n+3} \end{aligned}$$

$$\begin{aligned} a_5 &= 300(-0.1)^{5-1} \\ &= 300(-10^{-1})^4 \\ &= \underline{0.03} \end{aligned}$$

$$\begin{aligned} a_8 &= 3(-10)^{-8+3} \\ &= 3(-10)^{-5} \\ &= \underline{-0.00003} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $1, -\sqrt{3}, 3, -3\sqrt{3}, \dots$

Solution

$$\text{Given: } a_1 = 1, \quad r = \frac{a_2}{a_1} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

$$\begin{aligned} a_n &= 1(-\sqrt{3})^{n-1} & a_n &= a_1 r^{n-1} \\ &= a_1 r^{n-1} \end{aligned}$$

$$\begin{aligned} a_5 &= 1(-\sqrt{3})^{5-1} \\ &= \underline{9} \end{aligned}$$

$$\begin{aligned} a_8 &= 1(-\sqrt{3})^{8-1} \\ &= (-\sqrt{3})^7 \\ &= \underline{-27\sqrt{3}} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence 4, -6, 9, -13.5, ...

Solution

$$\text{Given: } a_1 = 4, \quad r = \frac{a_2}{a_1} = \frac{-6}{4} = -\frac{3}{2}$$

$$a_n = 4\left(-\frac{3}{2}\right)^{n-1} \qquad a_n = a_1 r^{n-1}$$

$$\begin{aligned} a_5 &= 4\left(-\frac{3}{2}\right)^{5-1} \\ &= 4\left(-\frac{3}{2}\right)^4 \\ &= 4\left(\frac{3^4}{2^4}\right) \\ &= \frac{81}{4} \end{aligned}$$

$$\begin{aligned} a_8 &= 4\left(-\frac{3}{2}\right)^7 \\ &= -4\left(\frac{3^7}{2^7}\right) \\ &= -\frac{2187}{32} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence 1, $-x^2$, x^4 , $-x^6$, ...

Solution

$$\text{Given: } a_1 = 1, \quad r = \frac{a_2}{a_1} = \frac{-x^2}{1} = -x^2$$

$$a_n = \left(-x^2\right)^{n-1} \qquad a_n = a_1 r^{n-1}$$

$$\begin{aligned} a_5 &= \left(-x^2\right)^4 \\ &= x^8 \end{aligned}$$

$$\begin{aligned} a_8 &= \left(-x^2\right)^7 \\ &= -x^{14} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence

$$10, 10^{2x-1}, 10^{4x-3}, 10^{6x-5}, \dots$$

Solution

Given: $a_1 = 10$

$$r = \frac{a_2}{a_1} = \frac{10^{2x-1}}{10} = 10^{2x-1-1} = 10^{2x-2}$$

$$a_n = 10 \left(10^{2x-2} \right)^{n-1} \qquad a_n = a_1 r^{n-1}$$

$$= 10 \left(10^{(2x-2)(n-1)} \right)$$

$$= 10 \left(10^{(2x-2)n-2x+2} \right)$$

$$= 10^{2nx-2n-2x+2+1}$$

$$= \underline{10^{2(n-1)x-2n+3}}$$

$$a_5 = 10^{2(5-1)x-2(5)+3}$$

$$= \underline{10^{8x-7}}$$

$$a_8 = 10^{2(8-1)x-2(8)+3}$$

$$= \underline{10^{14x-13}}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $a_1 = 2, \quad r = 3$

Solution

Given: $a_1 = 2, \quad r = 3$

$$a_n = \underline{2 \cdot 3^{n-1}}$$

$$a_n = a_1 r^{n-1}$$

$$a_5 = 2 \cdot 3^4$$

$$= \underline{162}$$

$$a_8 = 2 \cdot 3^7$$

$$= \underline{4374}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $a_1 = 1, \quad r = -\frac{1}{2}$

Solution

Given: $a_1 = 1, \quad r = -\frac{1}{2}$

$$\underline{a_n = \left(-\frac{1}{2}\right)^{n-1}} \qquad a_n = a_1 r^{n-1}$$

$$\begin{aligned} a_5 &= \left(-\frac{1}{2}\right)^4 \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} a_8 &= \left(-\frac{1}{2}\right)^7 \\ &= -\frac{1}{128} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $a_1 = -2, \quad r = 4$

Solution

Given: $a_1 = -2, \quad r = 4$

$$\underline{a_n = -2 \cdot (4)^{n-1}} \qquad a_n = a_1 r^{n-1}$$

$$\begin{aligned} a_5 &= \left(-\frac{1}{2}\right)^4 \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} a_8 &= \left(-\frac{1}{2}\right)^7 \\ &= -\frac{1}{128} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $a_1 = \sqrt{2}, \quad r = \sqrt{2}$

Solution

Given: $a_1 = \sqrt{2}, \quad r = \sqrt{2}$

$$a_n = \sqrt{2}(\sqrt{2})^{n-1}$$

$$= (\sqrt{2})^n$$

$$a_n = a_1 r^{n-1}$$

$$a_5 = (\sqrt{2})^5$$

$$= 4\sqrt{2}$$

$$a_8 = (\sqrt{2})^8$$

$$= 16$$

Exercise

Find the *n*th term, the *fifth* term, and the *eighth* term of the geometric sequence $a_1 = 0, \quad r = \pi$

Solution

Given: $a_1 = 0, \quad r = \pi$

$$a_n = 0(\pi)^{n-1}$$

$$= 0$$

$$a_n = a_1 r^{n-1}$$

$$a_5 = 0^5$$

$$= 0$$

$$a_8 = 0^8$$

$$= 0$$

Exercise

Find the *n*th term, the *fifth* term, and the *eighth* term of the geometric sequence $\{s_n\} = \{3^n\}$

Solution

$$a_n = 3^n$$

$$a_5 = 3^5$$

$$a_8 = 3^8$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $\{s_n\} = \{(-5)^n\}$

Solution

$$\underline{a_n = 3^n}$$

$$\begin{aligned} a_5 &= (-5)^5 \\ &= -5^5 \end{aligned}$$

$$\begin{aligned} a_8 &= (-5)^8 \\ &= 5^8 \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $\{s_n\} = \left\{-3\left(\frac{1}{2}\right)^n\right\}$

Solution

$$\underline{a_n = -3\left(\frac{1}{2}\right)^n}$$

$$\begin{aligned} a_5 &= -3\left(\frac{1}{2}\right)^5 \\ &= -\frac{3}{32} \end{aligned}$$

$$\begin{aligned} a_8 &= -3\left(\frac{1}{2}\right)^8 \\ &= -\frac{3}{256} \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $\{u_n\} = \left\{\frac{3^{n-1}}{2^n}\right\}$

Solution

$$\underline{a_n = \frac{3^{n-1}}{2^n}}$$

$$a_5 = \frac{3^4}{2^5}$$

$$\begin{aligned}
 &= \frac{81}{32} \\
 a_8 &= \frac{3^7}{2^8} \\
 &= \frac{3^7}{256}
 \end{aligned}$$

Exercise

Find the n th term, the *fifth* term, and the *eighth* term of the geometric sequence $\{u_n\} = \left\{ \frac{2^n}{3^{n-1}} \right\}$

Solution

$$\begin{aligned}
 a_n &= \frac{2^n}{3^{n-1}} \\
 a_5 &= \frac{2^5}{3^4} \\
 &= \frac{32}{81} \\
 a_8 &= \frac{2^8}{3^7} \\
 &= \frac{256}{3^7}
 \end{aligned}$$

Exercise

Find all possible values of r for a geometric sequence with the two given terms $a_4 = 3$, $a_6 = 9$

Solution

$$\begin{aligned}
 \frac{a_6}{a_4} &= \frac{a_1 r^5}{a_1 r^3} \\
 \frac{9}{3} &= r^2 \\
 r^2 &= 3 \\
 r &= \pm\sqrt{3}
 \end{aligned}$$

Exercise

Find the *sixth* term of the geometric sequence whose first two terms are 4 and 6

Solution

$$\text{Given: } a_1 = 4, \quad a_2 = 6$$

$$r = \frac{a_2}{a_1}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

$$a_6 = a_1 r^{n-1}$$

$$= 4 \left(\frac{3}{2} \right)^5$$

$$= \frac{243}{8}$$

Exercise

Given a geometric sequence with $a_4 = 4$, $a_7 = 12$, find r and a_{10}

Solution

$$r = \left(\frac{12}{4} \right)^{1/(7-4)}$$

$$= 3^{1/3}$$

$$= \sqrt[3]{3}$$

$$r = \left(\frac{a_y}{a_x} \right)^{1/(y-x)}$$

$$a_1 = \frac{a_4}{r^3}$$

$$= \frac{4}{3}$$

$$a_4 = a_1 r^{n-1}$$

$$a_{10} = \frac{4}{3} \left(\sqrt[3]{3} \right)^9$$

$$= 36$$

$$a_{10} = a_1 r^{n-1}$$

Exercise

Find the specified term of the geometric sequence a_6 ; $a_1 = 4$, $a_2 = 6$

Solution

$$r = \left(\frac{6}{4}\right)^{1/(2-1)}$$
$$= \frac{3}{2}$$

$$r = \left(\frac{a_y}{a_x}\right)^{1/(y-x)}$$

$$a_6 = 4\left(\frac{3}{2}\right)^5$$
$$= \frac{3^5}{8}$$

$$a_n = a_1 r^{n-1}$$

Exercise

Find the specified term of the geometric sequence a_7 ; $a_2 = 3$, $a_3 = -\sqrt{3}$

Solution

$$r = \left(\frac{-\sqrt{3}}{3}\right)^{1/(3-2)}$$
$$= -\frac{\sqrt{3}}{3}$$

$$r = \left(\frac{a_y}{a_x}\right)^{1/(y-x)}$$

$$a_2 = a_1 \left(-\frac{\sqrt{3}}{3}\right)^1$$

$$a_n = a_1 r^{n-1}$$

$$3 = -\frac{\sqrt{3}}{3} a_1$$

$$a_1 = -\frac{9}{\sqrt{3}}$$
$$= -3\sqrt{3}$$

$$a_7 = -3\sqrt{3} \left(-\frac{\sqrt{3}}{3}\right)^6$$
$$= -3\sqrt{3} \frac{3^3}{3^6}$$
$$= -\frac{\sqrt{3}}{9}$$

Exercise

Find the specified term of the geometric sequence a_6 ; $a_2 = 3$, $a_3 = -\sqrt{2}$

Solution

$$r = \left(\frac{-\sqrt{2}}{3} \right)^{1/(3-2)}$$
$$= -\frac{\sqrt{2}}{3}$$

$$r = \left(\frac{a_y}{a_x} \right)^{1/(y-x)}$$

$$a_2 = a_1 \left(-\frac{\sqrt{2}}{3} \right)^1$$

$$a_n = a_1 r^{n-1}$$

$$3 = -\frac{\sqrt{2}}{3} a_1$$

$$a_1 = -\frac{9}{\sqrt{2}}$$

$$a_6 = -\frac{9}{\sqrt{2}} \left(-\frac{\sqrt{2}}{3} \right)^5$$
$$= 9 \frac{\sqrt{2}^4}{3^5}$$
$$= \frac{4}{27}$$

Exercise

Find the specified term of the geometric sequence a_5 ; $a_1 = 4$, $a_2 = 7$

Solution

$$r = \frac{7}{4}$$

$$r = \left(\frac{a_y}{a_x} \right)^{1/(y-x)}$$

$$a_5 = 4 \left(\frac{7}{4} \right)^4$$
$$= \frac{7^4}{64}$$

$$a_n = a_1 r^{n-1}$$

Exercise

Find the specified term of the geometric sequence a_9 ; $a_2 = 3$, $a_5 = -81$

Solution

$$r = \left(\frac{-81}{3}\right)^{1/(5-2)}$$

$$= (-27)^{1/3}$$

$$= \underline{-3}$$

$$r = \left(\frac{a_y}{a_x}\right)^{1/(y-x)}$$

$$a_2 = a_1 (-3)^3$$

$$3 = -27a_1$$

$$a_1 = \underline{-\frac{1}{9}}$$

$$a_9 = -\frac{1}{9}(-3)^8$$

$$= \underline{-3^6}$$

$$a_n = a_1 r^{n-1}$$

Exercise

Find the specified term of the geometric sequence a_7 ; $a_1 = -4$, $a_3 = -1$

Solution

$$r = \left(\frac{-1}{-4}\right)^{1/(3-1)}$$

$$= \left(\frac{1}{4}\right)^{1/2}$$

$$= \underline{\frac{1}{2}}$$

$$r = \left(\frac{a_y}{a_x}\right)^{1/(y-x)}$$

$$a_7 = -4\left(\frac{1}{2}\right)^6$$

$$= \underline{-\frac{1}{16}}$$

$$a_n = a_1 r^{n-1}$$

Exercise

Find the specified term of the geometric sequence a_8 ; $a_2 = 3$, $a_4 = 6$

Solution

$$\begin{aligned} r &= \left(\frac{-81}{3} \right)^{1/(5-2)} & r &= \left(\frac{a_y}{a_x} \right)^{1/(y-x)} \\ &= (-27)^{1/3} \\ &= \underline{-3} \end{aligned}$$

$$\begin{aligned} a_2 &= a_1 (-3)^3 & a_n &= a_1 r^{n-1} \\ 3 &= -81a_1 \\ a_1 &= \underline{-\frac{1}{9}} \end{aligned}$$

$$\begin{aligned} a_8 &= -\frac{1}{9}(-3)^7 \\ &= \underline{3^5} \end{aligned}$$

Exercise

Express the sum in terms of summation notation: $4 + 11 + 18 + 25 + 32$. (Answers are not unique)

Solution

$$\begin{aligned} n &= 5 \\ d &= 11 - 4 = 7 \\ a_n &= 4 + (n-1)7 & a_n &= a_1 + (n-1)d \\ &= 4 + 7n - 7 \\ &= \underline{7n - 3} \end{aligned}$$

$$4 + 11 + 18 + 25 + 32 = \sum_{n=1}^5 (7n - 3)$$

Exercise

Express the sum in terms of summation notation: $4 + 11 + 18 + \dots + 466$. (Answers are not unique)

Solution

Difference in terms: $d = 11 - 4 = 7$

$$\text{Number of terms: } n = \frac{466-4}{7} + 1 = \underline{67}$$

$$\begin{aligned} a_n &= 4 + (n-1)7 \\ &= 4 + 7n - 7 \\ &= \underline{7n - 3} \end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$4 + 11 + 18 + \dots + 466 = \sum_{n=1}^{67} (7n - 3)$$

Exercise

Express the sum in terms of summation notation (*Answers are not unique*) $2 + 4 + 8 + 16 + 32 + 64 + 128$

Solution

$$\begin{aligned} 2 + 4 + 8 + 16 + 32 + 64 + 128 &= 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 \\ &= \sum_{n=1}^7 2^n \end{aligned}$$

Exercise

Express the sum in terms of summation notation (*Answers are not unique*) $2 - 4 + 8 - 16 + 32 - 64$

Solution

$$r = \frac{-4}{2} = -2$$

$$r = \frac{a_2}{a_1}$$

$$\begin{aligned} a_n &= 2(-2)^{n-1} \\ &= (-1)^{n-1} 2^n \end{aligned}$$

$$a_n = a_1 r^{n-1}$$

$$2 - 4 + 8 - 16 + 32 - 64 = \sum_{n=1}^6 (-1)^{n-1} 2^n$$

Exercise

Express the sum in terms of summation notation (*Answers are not unique*) $3 + 8 + 13 + 18 + 23$

Solution

$$\begin{aligned} d &= 8 - 3 \\ &= \underline{5} \end{aligned}$$

$$d = a_2 - a_1$$

$$a_n = 3 + 5(n-1)$$

$$a_n = a_1 + (n-1)d$$

$$= 5n - 2$$

$$3 + 8 + 13 + 18 + 23 = \sum_{n=1}^5 (5n - 2)$$

Exercise

Express the sum in terms of summation notation (*Answers are not unique*) $256 + 192 + 144 + 108 + \dots$

Solution

$$r = \frac{192}{256}$$

$$= \frac{3}{4}$$

$$a_n = 256 \left(\frac{3}{4} \right)^{n-1}$$

$$r = \frac{a_2}{a_1}$$

$$a_n = a_1 r^{n-1}$$

$$256 + 192 + 144 + 108 + \dots = \sum_{n=1}^{\infty} 256 \left(\frac{3}{4} \right)^{n-1}$$

Exercise

Express the sum in terms of summation notation (*Answers are not unique*): $\frac{5}{13} + \frac{10}{11} + \frac{15}{9} + \frac{20}{7}$

Solution

Number of terms: $n = 4$

Numerators : 5, 10, 15, 20 *common difference 5*

Denominators : 13, 11, 9, 7 *common difference -2*

Numerator:

$$a_n = 5 + (n-1)5$$

$$= 5 + 5n - 5$$

$$= 5n$$

$$a_n = a_1 + (n-1)d$$

Denominator:

$$a_n = 13 + (n-1)(-2)$$

$$= 13 - 2n + 2$$

$$= 15 - 2n$$

Hence the n th term is: $\frac{5}{13} + \frac{10}{11} + \frac{15}{9} + \frac{20}{7} = \sum_{n=1}^4 \frac{5n}{15-2n}$

Exercise

Express the sum in terms of summation notation (Answers are not unique.) $\frac{1}{4} - \frac{1}{12} + \frac{1}{36} - \frac{1}{108}$

Solution

$$\begin{aligned}\frac{1}{4} - \frac{1}{12} + \frac{1}{36} - \frac{1}{108} &= \frac{1}{4} - \frac{1}{4} \frac{1}{3^1} + \frac{1}{4} \frac{1}{3^2} - \frac{1}{4} \frac{1}{3^3} \\ &= \sum_{n=1}^4 (-1)^{n+1} \frac{1}{4} \left(\frac{1}{3}\right)^{n-1}\end{aligned}$$

Exercise

Express the sum in terms of summation notation (Answers are not unique.) $3 + \frac{3}{5} + \frac{3}{25} + \frac{3}{125} + \frac{3}{625}$

Solution

$$\begin{aligned}3 + \frac{3}{5} + \frac{3}{25} + \frac{3}{125} + \frac{3}{625} &= \frac{3}{5^0} + \frac{3}{5^1} + \frac{3}{5^2} + \frac{3}{5^3} + \frac{3}{5^4} \\ &= \sum_{n=0}^4 \frac{3}{5^n}\end{aligned}$$

Exercise

Express the sum in terms of summation notation (*Answers are not unique*): $\frac{3}{7} + \frac{6}{11} + \frac{9}{15} + \frac{12}{19} + \frac{15}{23} + \frac{18}{27}$

Solution

Numerators : 3, 6, 9, 12, 15, 18 *common difference 3*

Denominators : 7, 11, 15, 19, 23, 27 *common difference 4*

Numerator:

$$\begin{aligned}a_n &= 3 + 3(n-1) & a_n &= a_1 + (n-1)d \\ &= 3n\end{aligned}$$

Denominator:

$$\begin{aligned}a_n &= 7 + 4(n-1) \\ &= 4n + 3\end{aligned}$$

$$\frac{3}{7} + \frac{6}{11} + \frac{9}{15} + \frac{12}{19} + \frac{15}{23} + \frac{18}{27} = \sum_{n=1}^6 \frac{3n}{4n+3}$$

Exercise

Express the sum in terms of summation notation (*Answers are not unique.*) $\frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \dots, \quad |x| < 3$

Solution

$$\begin{aligned}\frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \dots &= \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \dots \\ &= \sum_{n=1}^{\infty} \left(\frac{x}{3}\right)^n\end{aligned}$$

Exercise

Express the sum in terms of summation notation (*Answers are not unique.*) $2x + 4x^2 + 8x^3 + \dots, \quad |x| < \frac{1}{2}$

Solution

$$\begin{aligned}2x + 4x^2 + 8x^3 + \dots &= 2x + (2x)^2 + (2x)^3 + \dots \\ &= \sum_{n=1}^{\infty} (2x)^n\end{aligned}$$

Exercise

Find the sum of the infinite geometric series if it exists: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

Solution

$$\begin{aligned}a_1 &= 1, \quad r = -\frac{1}{2} \\ S &= \frac{1}{1 + \frac{1}{2}} & S &= \frac{a_1}{1 - r} \\ &= \frac{1}{\frac{3}{2}} \\ &= \frac{2}{3}\end{aligned}$$

Exercise

Find the sum of the infinite geometric series if it exists: $1.5 + 0.015 + 0.00015 + \dots$

Solution

$$a_1 = 0.015$$

$$a_2 = .00015$$

$$r = \frac{.00015}{.015}$$

$$= .01$$

$$S = 1.5 + \frac{a_1}{1-r}$$

$$= 1.5 + \frac{.015}{1-.01}$$

$$= \frac{15}{10} + \frac{.015}{.99}$$

$$= \frac{15}{10} + \frac{15}{990}$$

$$= \frac{15}{10} + \frac{15}{990}$$

$$= \frac{1500}{990}$$

$$= \frac{50}{33}$$

Exercise

Find the sum of the infinite geometric series if it exists: $\sqrt{2} - 2 + \sqrt{8} - 4 + \dots$

Solution

$$r = \frac{-2}{\sqrt{2}}$$

$$= -\sqrt{2}$$

$$r = \frac{a_2}{a_1}$$

$|r| = \sqrt{2} > 1 \Rightarrow$ The sum *doesn't exist*.

Exercise

Find the sum of the infinite geometric series if it exists: $256 + 192 + 144 + 108 + \dots$

Solution

$$r = \frac{192}{256}$$

$$= \frac{3}{4}$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{256}{1 - \frac{3}{4}}$$

$$= 1024$$

$$S = \frac{a_1}{1-r}$$

Exercise

Find the sum of the infinite geometric series if it exists: $\frac{1}{4} + \frac{2}{4} + \frac{2^2}{4} + \frac{2^3}{4} + \dots + \frac{2^{n-1}}{4}$

Solution

$$r = \frac{\frac{2}{4}}{\frac{1}{4}}$$
$$= 2$$

$$r = \frac{a_2}{a_1}$$

$$S_n = \frac{1}{4} \left(\frac{1-2^n}{1-2} \right)$$
$$= -\frac{1}{4} (1-2^n)$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

Exercise

Find the sum of the infinite geometric series if it exists: $\frac{3}{9} + \frac{3^2}{9} + \frac{3^3}{9} + \dots + \frac{3^n}{9}$

Solution

$$r = \frac{\frac{3^2}{9}}{\frac{3}{9}}$$
$$= 3$$

$$r = \frac{a_2}{a_1}$$

$$S_n = \frac{3}{9} \left(\frac{1-3^n}{1-3} \right)$$
$$= -\frac{1}{6} (1-3^n)$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

Exercise

Find the sum of the infinite geometric series if it exists: $-1 - 2 - 4 - 8 - \dots - 2^{n-1}$

Solution

$$r = \frac{-2}{-1}$$
$$= 2$$

$$r = \frac{a_2}{a_1}$$

$$S_n = -1 \left(\frac{1-2^n}{1-2} \right)$$
$$= 1-2^n$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

Exercise

Find the sum of the infinite geometric series if it exists: $2 + \frac{6}{5} + \frac{18}{25} + \dots + 2\left(\frac{3}{5}\right)^{n-1}$

Solution

$$r = \frac{\frac{6}{5}}{2}$$

$$= \frac{3}{5} < 1$$

$$r = \frac{a_2}{a_1}$$

$$S_n = 2 \cdot \frac{1 - \left(\frac{3}{5}\right)^n}{1 - \frac{3}{5}}$$

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

$$= 2 \cdot \frac{1 - \left(\frac{3}{5}\right)^n}{\frac{2}{5}}$$

$$= 5 \left(1 - \left(\frac{3}{5}\right)^n \right)$$

Exercise

Find the sum of the infinite geometric series if it exists: $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

Solution

$$r = \frac{1}{3} < 1$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{1}{1 - \frac{1}{3}}$$

$$S = \frac{a_1}{1 - r}$$

$$= \frac{3}{2}$$

The series *converges*

Exercise

Find the sum of the infinite geometric series if it exists: $2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \dots$

Solution

$$r = \frac{\frac{4}{3}}{2}$$

$$r = \frac{a_2}{a_1}$$

$$= \frac{2}{3} < 1$$

$$S = \frac{2}{1 - \frac{2}{3}}$$

$$S = \frac{a_1}{1 - r}$$

$$= 6 \quad \text{The series *converges*}$$

Exercise

Find the sum of the infinite geometric series if it exists: $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \dots$

Solution

$$a_1 = 2$$

$$|r| = \left| -\frac{1}{4} \right| < 1$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{2}{1 + \frac{1}{4}}$$

$$S = \frac{a_1}{1 - r}$$

$$= \frac{8}{5} \quad \text{The series *converges*}$$

Exercise

Find the sum of the infinite geometric series if it exists: $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \dots$

Solution

$$a_1 = 1$$

$$|r| = \left| -\frac{3}{4} \right| < 1$$

$$r = \frac{a_2}{a_1}$$

$$S = \frac{1}{1 + \frac{3}{4}}$$

$$S = \frac{a_1}{1 - r}$$

$$= \frac{4}{7} \quad \text{The series *converges*}$$

Exercise

Find the sum of the infinite geometric series if it exists: $9 + 12 + 16 + \frac{64}{3} + \dots$

Solution

$$a_1 = 9$$

$$|r| = \left| \frac{4}{3} \right| > 1 \quad \text{The series *diverges*}$$

Exercise

Find the sum of the infinite geometric series if it exists: $8 + 12 + 18 + 27 + \dots$

Solution

$$a_1 = 8$$

$$\begin{aligned} r &= \frac{12}{8} & r &= \frac{a_2}{a_1} \\ &= \frac{3}{2} > 1 \end{aligned} \quad \text{The series *diverges*}$$

Exercise

Find the sum of the infinite geometric series if it exists: $6 + 2 + \frac{2}{3} + \frac{2}{9} + \dots$

Solution

$$a_1 = 6$$

$$|r| = \frac{2}{6} \quad r = \frac{a_2}{a_1}$$

$$= \frac{1}{3} < 1$$

$$S = \frac{6}{1 - \frac{1}{3}} \quad S = \frac{a_1}{1 - r}$$

$$= \frac{6}{\frac{2}{3}}$$

$$= 9 \quad \text{The series *converges*}$$

Exercise

Find the sum: $\sum_{k=1}^{20} (3k - 5)$

Solution

$$a_1 = 3(1) - 5 = \underline{-2}$$

$$a_{20} = 3(20) - 5 = \underline{55}$$

$$\begin{aligned} \sum_{k=1}^{20} (3k - 5) &= \frac{20}{2}(-2 + 55) \\ &= \underline{530} \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find the sum: $\sum_{k=1}^{18} \left(\frac{1}{2}k + 7\right)$

Solution

$$a_1 = \frac{1}{2}(1) + 7 = \underline{\frac{15}{2}}$$

$$a_{18} = \frac{1}{2}(18) + 7 = \underline{16}$$

$$\begin{aligned} \sum_{k=1}^{18} \left(\frac{1}{2}k + 7\right) &= \frac{18}{2}\left(\frac{15}{2} + 16\right) \\ &= \underline{\frac{423}{2}} \end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find the sum: $\sum_{k=1}^{80} (2k - 5)$

Solution

$$a_1 = 2(1) - 5 = \underline{-3}$$

$$a_{80} = 2(80) - 5 = \underline{155}$$

$$\begin{aligned}\sum_{k=1}^{80} (2k-5) &= \frac{80}{2}(-3+155) \\ &= 40(152) \\ &= \underline{6080}\end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find the sum: $\sum_{n=1}^{90} (3-2n)$

Solution

$$a_1 = 3 - 2(1) = \underline{1}$$

$$a_{90} = 3 - 2(90) = \underline{-177}$$

$$\begin{aligned}\sum_{n=1}^{90} (3-2n) &= \frac{90}{2}(1-177) \\ &= 45(-176) \\ &= \underline{-7920}\end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find the sum: $\sum_{n=1}^{100} \left(6 - \frac{1}{2}n\right)$

Solution

$$a_1 = 6 - \frac{1}{2}(1) = \underline{\frac{11}{2}}$$

$$a_{100} = 6 - \frac{1}{2}(100) = \underline{-44}$$

$$\begin{aligned}\sum_{n=1}^{100} \left(6 - \frac{1}{2}n\right) &= \frac{100}{2}\left(\frac{11}{2} - 44\right) \\ &= 50\left(-\frac{77}{2}\right) \\ &= \underline{-1925}\end{aligned}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise

Find the sum: $\sum_{n=1}^{80} \left(\frac{1}{3}n + \frac{1}{2} \right)$

Solution

$$a_1 = \frac{1}{3}(1) + \frac{1}{2} = \underline{\frac{5}{6}}$$

$$a_{80} = \frac{1}{3}(80) + \frac{1}{2} = \underline{\frac{163}{6}}$$

$$\begin{aligned} \sum_{n=1}^{80} \left(\frac{1}{3}n + \frac{1}{2} \right) &= \frac{80}{2} \left(\frac{5}{6} + \frac{163}{6} \right) \\ &= 40 \left(\frac{168}{6} \right) \\ &= \underline{1,120} \end{aligned}$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

Exercise

Find the sum: $\sum_{k=1}^{10} 3^k$

Solution

$$\begin{aligned} \sum_{k=1}^{10} 3^k &= 3 \frac{1-3^{10}}{1-3} \\ &= 3 \frac{-59048}{-2} \\ &= \underline{88,572} \end{aligned}$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

Exercise

Find the sum: $\sum_{k=1}^9 (-\sqrt{5})^k$

Solution

$$\begin{cases} a_1 = -\sqrt{5} \\ a_2 = (-\sqrt{5})^2 = 5 \end{cases}$$

$$r = \frac{5}{-\sqrt{5}}$$

$$r = \frac{a_2}{a_1}$$

$$= -\sqrt{5}$$

$$\begin{aligned} \sum_{k=1}^9 (-\sqrt{5})^k &= (-\sqrt{5}) \frac{1 - (-\sqrt{5})^9}{1 - (-\sqrt{5})} \\ &= \frac{(-\sqrt{5})(1 + 625\sqrt{5})}{1 + \sqrt{5}} \cdot \frac{1 - \sqrt{5}}{1 - \sqrt{5}} \\ &= \frac{3124\sqrt{5} - 3120}{-4} \\ &= \underline{780 - 781\sqrt{5}} \end{aligned}$$

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

Exercise

Find the sum: $\sum_{k=0}^9 \left(-\frac{1}{2}\right)^{k+1}$

Solution

$$\begin{aligned} \sum_{k=0}^9 \left(-\frac{1}{2}\right)^{k+1} &= \left(-\frac{1}{2}\right) \frac{1 - \left(-\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}} \\ &= -\frac{1}{2} \frac{1 - \frac{1}{2^{10}}}{\frac{3}{2}} \\ &= -\frac{\frac{1024-1}{1024}}{3} \\ &= -\frac{1023}{3072} \\ &= \underline{-\frac{341}{1024}} \end{aligned}$$

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

Exercise

Find the sum : $\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1}$

Solution

$$|r| = \frac{2}{3} < 1$$

$$\begin{aligned} \sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1} &= \frac{2}{1 - \frac{2}{3}} & S &= \frac{a_1}{1-r} \\ &= \frac{2}{\frac{1}{3}} \\ &= \underline{6}, \quad \text{the series } \textit{converges} \end{aligned}$$

Exercise

Find the sum: $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

Solution

$$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \sum_{n=1}^{\infty} \frac{2}{3} \left(\frac{2}{3}\right)^{n-1}$$

$$|r| = \frac{2}{3} < 1$$

$$\begin{aligned} \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n &= \frac{2}{3} \frac{1}{1 - \frac{2}{3}} & S &= \frac{a_1}{1-r} \\ &= \frac{2}{3}(3) \\ &= \underline{2} \quad \text{The series } \textit{converges} \end{aligned}$$

Exercise

Find the sum: $\sum_{n=1}^{\infty} 3\left(\frac{3}{2}\right)^n$

Solution

Since $|r| = \frac{3}{2} > 1$, the series *diverges*

Exercise

Find the sum: $\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1}$

Solution

$$|r| = \frac{1}{4} < 1$$

$$a_1 = 5$$

$$\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1} = \frac{5}{1 - \frac{1}{4}} \\ = \frac{20}{3}$$

$$S = \frac{a_1}{1 - r}$$

The series *converges*

Exercise

Find the sum: $\sum_{n=1}^{\infty} 8\left(\frac{1}{3}\right)^{n-1}$

Solution

$$|r| = \frac{1}{3} < 1$$

$$a_1 = 8$$

$$\sum_{n=1}^{\infty} 8\left(\frac{1}{3}\right)^{n-1} = \frac{8}{1 - \frac{1}{3}} \\ = 12$$

$$S = \frac{a_1}{1 - r}$$

The series *converges*

Exercise

Find the sum: $\sum_{k=1}^{\infty} \frac{1}{2} \cdot 3^{k-1}$

Solution

Since $|r| = 3 > 1$, the series *diverges*

Exercise

Find the sum: $\sum_{k=1}^{\infty} 6\left(-\frac{2}{3}\right)^{k-1}$

Solution

$$|r| = \frac{2}{3} < 1$$

$$a_1 = 6$$

$$\sum_{k=1}^{\infty} 6\left(-\frac{2}{3}\right)^{k-1} = \frac{6}{1 + \frac{2}{3}} = \frac{18}{5}$$

$$S = \frac{a_1}{1-r}$$

The series *converges*

Exercise

Find the sum: $\sum_{k=1}^{\infty} 4\left(-\frac{1}{2}\right)^{k-1}$

Solution

$$|r| = \frac{1}{2} < 1$$

$$a_1 = 4$$

$$\sum_{k=1}^{\infty} 4\left(-\frac{1}{2}\right)^{k-1} = \frac{4}{1 + \frac{1}{2}} = \frac{8}{3}$$

$$S = \frac{a_1}{1-r}$$

The series *converges*

Exercise

Find the sum: $\sum_{k=8}^{14} (3^{k-7} + 2j^2)$

Solution

$$a_n = 3^{n-7} \rightarrow \underline{a_1 = 3^{-6}}$$

$$r = 3$$

$$n = 14 - 8 + 1 = 7$$

$$\begin{aligned}
\sum_{k=8}^{14} (3^{k-7} + 2j^2) &= \sum_{k=8}^{14} 3^{k-7} + 2 \sum_{k=8}^{14} j^2 \\
&= 3^{-6} \cdot \frac{1-3^7}{1-3} + 2(7)j^2 \\
&= -\frac{1}{2} \left(\frac{1-3^7}{3^6} \right) + 14j^2 \\
&= -\frac{1}{2} \left(\frac{-2,186}{729} \right) + 14j^2 \\
&= \frac{1,093}{729} + 14j^2
\end{aligned}$$

Exercise

Find the sum of the first 120 terms of: 14, 16, 18, 20, ...

Solution

$$n = 120$$

$$a_1 = 14$$

$$d = 16 - 14 = 2$$

$$\begin{aligned}
S_{120} &= \frac{120}{2} [2(14) + 2(120-1)] \\
&= 60(48 + 238) \\
&= 17,160
\end{aligned}$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

Exercise

Find the sum of the first 46 terms of 2, -1, -4, -7, ...

Solution

$$n = 46$$

$$a_1 = 2$$

$$d = -1 - 2 = -3$$

$$\begin{aligned}
S_{46} &= \frac{46}{2} [2(2) - 3(46-1)] \\
&= 23(4 - 135) \\
&= -3,013
\end{aligned}$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

Exercise

Find the rational number represented by the repeating decimal $0.\overline{23}$

Solution

$$0.\overline{23} = 0.23 + 0.0023 + .000023 + \dots$$

$$a_1 = 0.23$$

$$r = \frac{.0023}{.23} = 0.01$$

$$S = \frac{0.23}{1 - 0.01}$$

$$= \frac{0.23}{0.99}$$

$$= \frac{23}{99}$$

$$S = \frac{a_1}{1-r}$$

Exercise

Find the rational number represented by the repeating decimal $0.0\overline{71}$

Solution

$$0.0\overline{71} = 0.071 + 0.00071 + .0000071 + \dots$$

$$a_1 = 0.071$$

$$r = \frac{.00071}{.071} = 0.01$$

$$S = \frac{0.071}{1 - 0.01}$$

$$= \frac{0.071}{0.990}$$

$$= \frac{71}{990}$$

$$S = \frac{a_1}{1-r}$$

Exercise

Find the rational number represented by the repeating decimal $2.4\overline{17}$

Solution

$$2.4\overline{17} = 2.4 + 0.017 + 0.00017 + .0000017 + \dots$$

$$a_1 = 0.017$$

$$r = \frac{.00017}{.017} = 0.01$$

$$\begin{aligned}
 S &= 2.4 + \frac{0.017}{1 - 0.01} \\
 &= \frac{24}{10} + \frac{0.017}{0.990} \\
 &= \frac{24}{10} + \frac{17}{990} \\
 &= \frac{240 + 17}{990} \\
 &= \frac{2,393}{990}
 \end{aligned}$$

$$S = \frac{a_1}{1-r}$$

Exercise

Find the rational number represented by the repeating decimal $10.\overline{5}$

Solution

$$10.\overline{5} = 10 + 0.5 + 0.05 + .005 + \dots$$

$$a_1 = 0.5$$

$$r = \frac{0.05}{0.5} = 0.1$$

$$\begin{aligned}
 S &= 10 + \frac{0.5}{1 - 0.1} \\
 &= 10 + \frac{0.5}{0.9} \\
 &= 10 + \frac{5}{9} \\
 &= \frac{95}{9}
 \end{aligned}$$

$$S = \frac{a_1}{1-r}$$

Exercise

Find the rational number represented by the repeating decimal $5.\overline{146}$

Solution

$$5.\overline{146} = 5 + 0.146 + 0.000146 + .000000146 + \dots$$

$$a_1 = 0.146$$

$$r = \frac{0.000146}{0.146} = 0.001$$

$$\begin{aligned}
 S &= 5 + \frac{0.146}{1 - 0.001} \\
 &= 5 + \frac{0.146}{0.999}
 \end{aligned}$$

$$S = \frac{a_1}{1-r}$$

$$= 5 + \frac{146}{999}$$

$$= \frac{5,141}{999}$$

Exercise

Find the rational number represented by the repeating decimal $3.\overline{2394}$

Solution

$$3.\overline{2394} = 3.2 + 0.0394 + 0.0000394 + \dots$$

$$a_1 = 0.0394$$

$$r = \frac{0.0000394}{0.0394} = 0.001$$

$$S = 3.2 + \frac{0.0394}{1 - 0.001}$$

$$S = \frac{a_1}{1 - r}$$

$$= \frac{32}{10} + \frac{0.0394}{0.9990}$$

$$= \frac{32}{10} + \frac{394}{9990}$$

$$= \frac{31968 + 394}{9990}$$

$$= \frac{32,362}{9,990}$$

$$= \frac{16,181}{4,995}$$

Exercise

Find the rational number represented by the repeating decimal $1.\overline{6124}$

Solution

$$1.\overline{6124} = 1 + 0.6124 + 0.00006124 + \dots$$

$$a_1 = 0.6124$$

$$r = \frac{0.00006124}{0.6124} = 0.0001$$

$$S = 1 + \frac{0.6124}{1 - 0.0001}$$

$$S = \frac{a_1}{1 - r}$$

$$= 1 + \frac{0.6124}{0.9999}$$

$$= 1 + \frac{6124}{9999}$$

$$= \frac{16,123}{9,999}$$

Exercise

Find x so that $x + 3$, $2x + 1$, and $5x + 2$ are consecutive terms of an arithmetic sequence.

Solution

$$d = 2x + 1 - (x + 3)$$

$$= x - 2$$

$$d = 5x + 2 - (2x + 1)$$

$$= 3x + 1$$

$$d = 3x + 1 = x - 2$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

Exercise

Find x so that $2x$, $3x + 2$, and $5x + 3$ are consecutive terms of an arithmetic sequence.

Solution

$$d = 3x + 2 - 2x$$

$$= x + 2$$

$$d = 5x + 3 - (3x + 2)$$

$$= 2x + 1$$

$$d = 2x + 1 = x + 2$$

$$x = 1$$

Exercise

Find x so that x , $x + 2$, and $x + 3$ are consecutive terms of a geometric sequence.

Solution

$$r = \frac{x+2}{x}$$

$$r = \frac{x+3}{x+2}$$

$$r = \frac{x+2}{x} = \frac{x+3}{x+2}$$

$$(x+2)^2 = x^2 + 3x$$

$$x^2 + 4x + 4 - x^2 - 3x = 0$$

$$x + 4 = 0$$

$$\underline{x = -4} \mid$$

Exercise

Find x so that $x-1$, x and $x+2$ are consecutive terms of a geometric sequence.

Solution

$$r = \frac{x}{x-1} = \frac{x+2}{x}$$

$$x^2 = x^2 + x - 2$$

$$x - 2 = 0$$

$$\underline{x = 2} \mid$$

Exercise

How many terms must be added in an arithmetic sequence whose first term is 11 and whose common difference is 3 to obtain a sum of 1092?

Solution

Given: $a_1 = 11$; $d = 3$; $S = 1092$

$$1092 = \frac{n}{2}(22 + 3(n-1))$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$n(3n+19) = 2184$$

$$3n^2 + 19n - 2184 = 0$$

$$n = \frac{-19 \pm \sqrt{361 + 26208}}{6}$$

$$= \frac{-19 \pm 163}{6}$$

$$\underline{n = 24} \mid \quad \& \quad \cancel{n = \frac{91}{3}}$$

Exercise

How many terms must be added in an arithmetic sequence whose first term is 78 and whose common difference is -4 to obtain a sum of 702?

Solution

Given: $a_1 = 78$; $d = -4$; $S = 702$

$$702 = \frac{n}{2}(2(78) - 4(n-1)) \quad S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$n(160 - 4n) = 1404$$

$$-4n^2 + 160n - 1404 = 0$$

$$\begin{aligned} n &= \frac{-160 \pm \sqrt{25,600 - 22464}}{-8} \\ &= \frac{160 \pm 56}{8} \end{aligned}$$

$$\underline{n = 13} \quad \& \quad \underline{n = 27}$$

Exercise

The first ten rows of seating in a certain section of a stadium have 30 seats, 32 seats, 34 seats, and so on. The eleventh through the twentieth rows each contain 50 seats. Find the total number of seats in the section.

Solution

Given: $a_1 = 30$; $d = 2$

$$S = S_{10} + 50(20 - 11 + 1) \quad S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

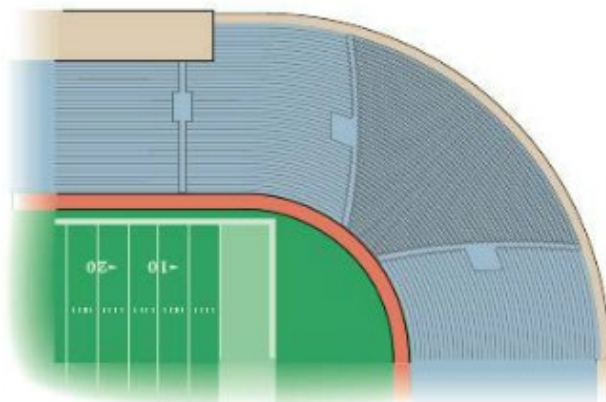
$$= \frac{10}{2}(2(30) + 2(9)) + 50(10)$$

$$= 5(78) + 500$$

$$\underline{= 890 \text{ seats}}$$

Exercise

The corner section of a football stadium has 15 seats in the first row and 40 rows in all. Each successive row contains two additional seats. How many seats are in this section?



Solution

Given: $a_1 = 15$; $d = 2$; $n = 40$

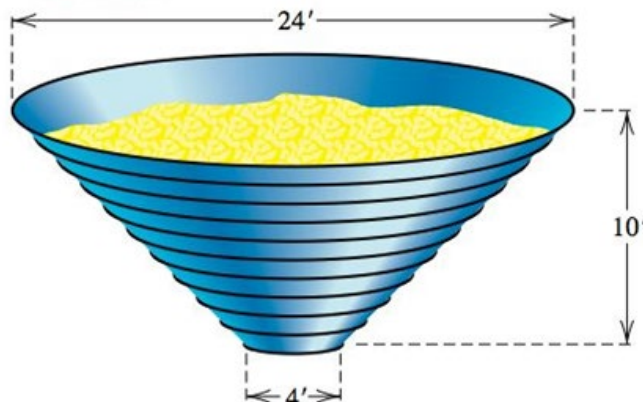
$$\begin{aligned} S_{40} &= \frac{40}{2}(30 + 2(40 - 1)) \\ &= 20(30 + 78) \\ &= 20(108) \\ &= \underline{2,160} \end{aligned}$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

The corner section has 2,160 seats.

Exercise

A grain bin is to be constructed in the shape of a frustum of a cone.



The bin is to be 10 feet tall with 11 metal rings positioned uniformly around it, from the 4-foot opening at the bottom to the 24-foot opening at the top. Find the total length of metal needed to make the rings.

Solution

The circumference of each ring is πD .

$$a_1 = 4\pi; \quad a_{11} = 24\pi$$

$$24 = 4 + (11 - 1)d$$

$$a_n = a_1 + (n - 1)d$$

$$10d = 20$$

$$\underline{d = 2}$$

$$S_{11} = \frac{11}{2}(4\pi + 24\pi)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$\underline{= 154\pi \text{ ft}}$$

Exercise

A bicycle rider coasts downhill, traveling 4 *feet* the first second. In each succeeding second, the rider travels 5 *feet* farther than in the preceding second. If the rider reaches the bottom of the hill in 11 *seconds*, find the total distance traveled.

Solution

Given: $a_1 = 4 \text{ ft}$ & $d = 5 \text{ ft}$

$$S_{11} = \frac{11}{2}(8 + 5(10))$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$\underline{= 319 \text{ ft}}$$

∴ the total distance traveled 319 *feet*.

Exercise

A contest will have five each prizes totaling \$5,000, and there will be a \$100 difference between successive prizes. Find the first prize.

Solution

Given: $n = 5$ $S_5 = 5000$ $d = -100$

$$5,000 = \frac{5}{2}[2a_1 + 4(-100)]$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$2,000 = 2a_1 - 400$$

$$\underline{a_1 = \$1,200}$$

Exercise

A Company is to distribute \$46,000 in bonuses to its top ten salespeople. The tenth salesperson on the list will receive \$1,000, and the difference in bonus money between successively ranked salesperson is to be constant. Find the bonus for each salesperson.

Solution

Given: $n = 10$ $S_{10} = 46,000$ $a_{10} = 1,000$

$$46,000 = \frac{10}{2}(a_1 + 1000) \qquad S_n = \frac{n}{2}(a_1 + a_n)$$

$$9,200 = a_1 + 1000$$

$$a_1 = 8,200$$

$$d = \frac{1,000 - 8,200}{9} \qquad a_n = a_1 + (n-1)d$$

$$= -800 \quad |$$

$$\text{\$8,200} \quad \text{\$7,400} \quad \text{\$6,600} \quad \text{\$5,800} \quad \text{\$5,000} \quad \text{\$4,200} \quad \text{\$3,400} \quad \text{\$2,600} \quad \text{\$1,800} \quad \text{\$1,000}$$

Exercise

Assuming air resistance is negligible, a small object that is dropped from a hot air balloon falls 16 *feet* during the first second, 48 *feet* during the second second, 80 *feet* during the third second, 112 *feet* during the fourth second, and so on. Find an expression for the distance the object falls in n seconds.

Solution

Given the sequence: 16, 48, 80, 112, ...

This is an arithmetic sequence with:

$$a_1 = 16 \quad \& \quad d = 48 - 16 = 32$$

$$S_n = \frac{n}{2}(32 + 32(n-1)) \qquad S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$= \frac{n}{2}(32n)$$

$$= 16n^2 \quad |$$

Exercise

A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two fewer bricks than the prior step.

- How many bricks are required for the top step?
- How many bricks are required to build the staircase?

Solution

a) **Given:** $n = 30$ $a_1 = 100$ $d = -2$

$$a_n = 100 - 2(n-1) \qquad a_n = a_1 + (n-1)d$$

$$= -2n + 102 \quad |$$

$$a_{30} = 102 - 60$$

$$= 42 \quad |$$

$$b) S_{30} = 15(100 + 42)$$

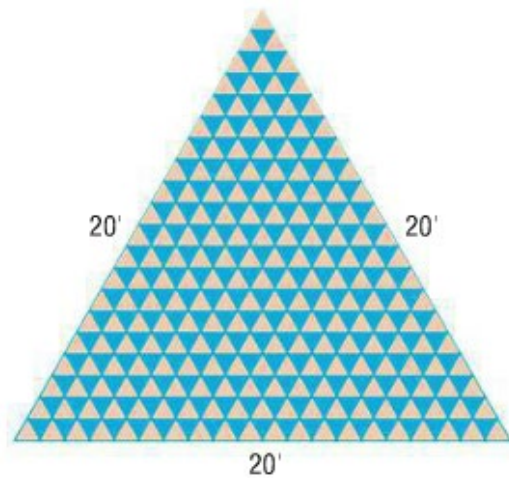
$$= 2,130$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

It required 2130 *bricks* to build the staircase.

Exercise

A mosaic is designed in the shape of an equilateral triangle, 20 *feet* on each side. Each tile in the mosaic is in the shape of an equilateral triangle, 12 *inches* to a side. The tiles are to alternate in color as shown below.



How many tiles of each color will be required?

Solution

Bottom row has 20 lighter colored tiles.

Top row has 1 lighter colored tile.

The number decreases by 1 as we move up the triangle.

∴ This is an arithmetic sequence with: $a_1 = 20$; $d = -1$; $n = 20$

$$S_{20} = \frac{20}{2}(40 + (-1)(20 - 1))$$

$$= 10(40 - 19)$$

$$= 10(21)$$

$$= 210$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

∴ There are 210 *lighter colored* tiles.

Bottom row has 19 darker colored tiles.

Top row has 1 darker colored tile.

∴ This is an arithmetic sequence with: $a_1 = 1$; $d = -1$; $n = 19$

$$S_{19} = \frac{19}{2}(2(19) + (-1)(19 - 1))$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$= \frac{19}{2}(38 - 18)$$

$$= \underline{190}$$

∴ There are 190 *darker colored* tiles.