11. Potential due to a point source EXTERIOR to a metal box — an example of a non-separable problem.

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We found the potential due to a point source inside of a metal box using separation of variables.

Can we do the corresponding exterior problem?
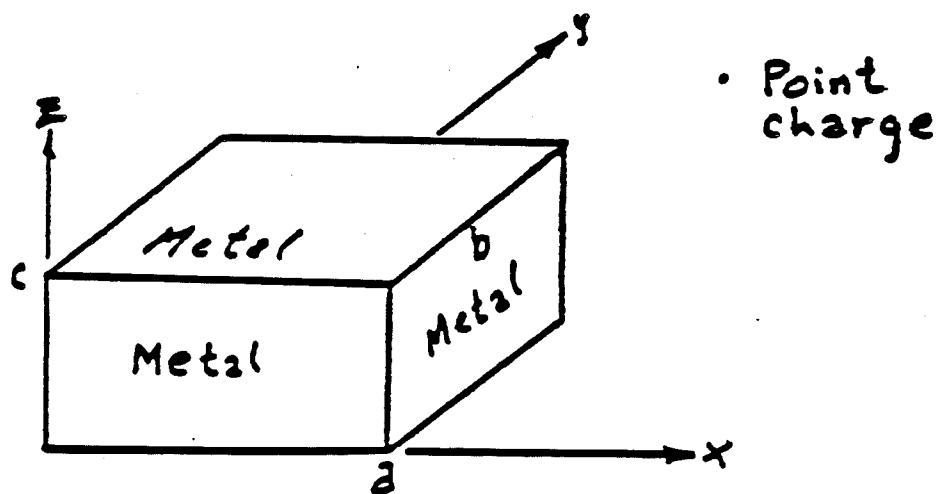
The boundary conditions are that $\varphi(x, y, z) = \varphi_0$ on the metal. (We can take $\varphi_0 = 0$ for convenience as before).

If we assume, $\varphi = X(x) Y(y) Z(z)$,
then the differential equation
separates over any region which ex-
cludes the source point as before.

Now let us attempt to satisfy the
boundary conditions.

$$\varphi(0, y, z) = 0 \quad \text{for } 0 \leq y \leq b, \ 0 \leq z \leq c \Rightarrow$$
$$X(0) Y(y) Z(z) = 0.$$

Now if we take $X(0) = 0$, as we did
in the interior problem, then

$$\varphi(0, y, z) = 0 \cdot Y(y) \cdot Z(z) = 0$$

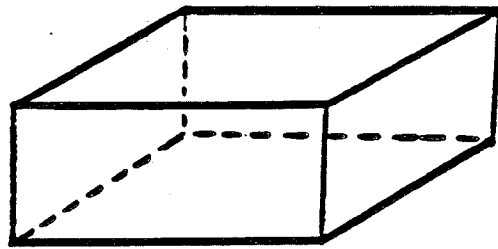everywhere!

Of course, this is not correct.

The remaining boudary conditions
can similarly be shown to be im-
possible to satisfy when a
separated solution is assumed.

Thus, though the differential equa-
tion is formally separable, the
boundary conditions are not!

Why are the boundary conditions
separable for the interior problem
and not for the exterior problem?

For the interior problem, the
potential at any point on the plane
x=0, for example, OUTSIDE of the
box has NO bearing on the solution
INSIDE THE BOX.

As far as the potential interior
of the box is concerned,

Is equivalent to
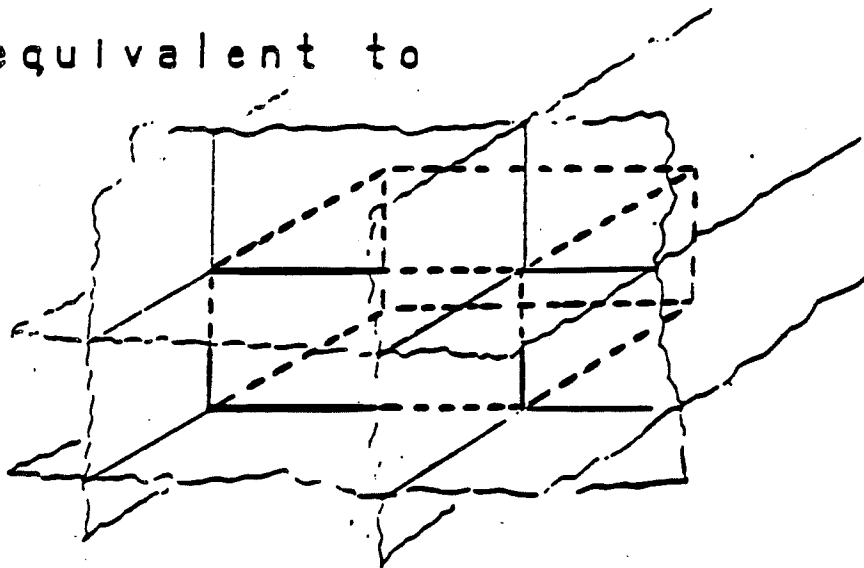
(Recall the uniqueness property of
this type of boundary value
problem: If the potential satis-
fies Poisson's equation everywhere
within a volume bounded by a closed
surface and is specified on that
closed surface, the solution is
unique).

Thus, the method of separation of variables does not work for every boundary value problem.

In general, the method of separation of variables will not work unless the boundaries at which the boundary conditions are specified can be described by ENTIRE CONSTANT COORDINATE SURFACES.

-OR- as in the case of the interior problem that we solved, the boundaries over which the boundary conditions are specified can be EXTENDED into complete constant coordinate surfaces without affecting the problem.

Some examples of separable problems are shown below.
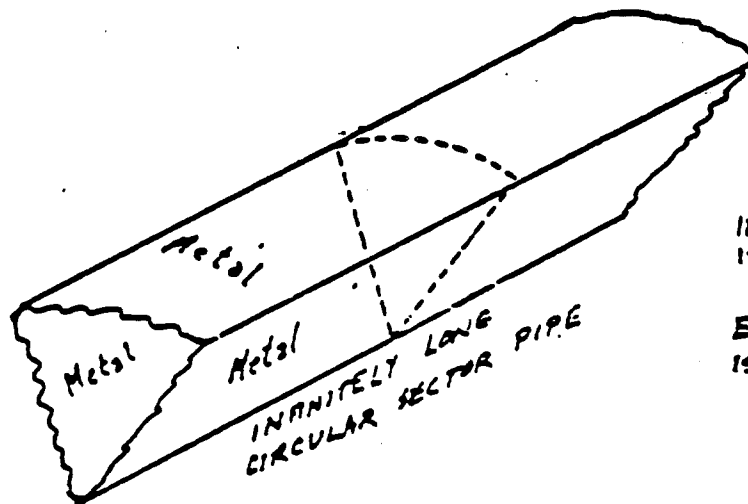
(1)
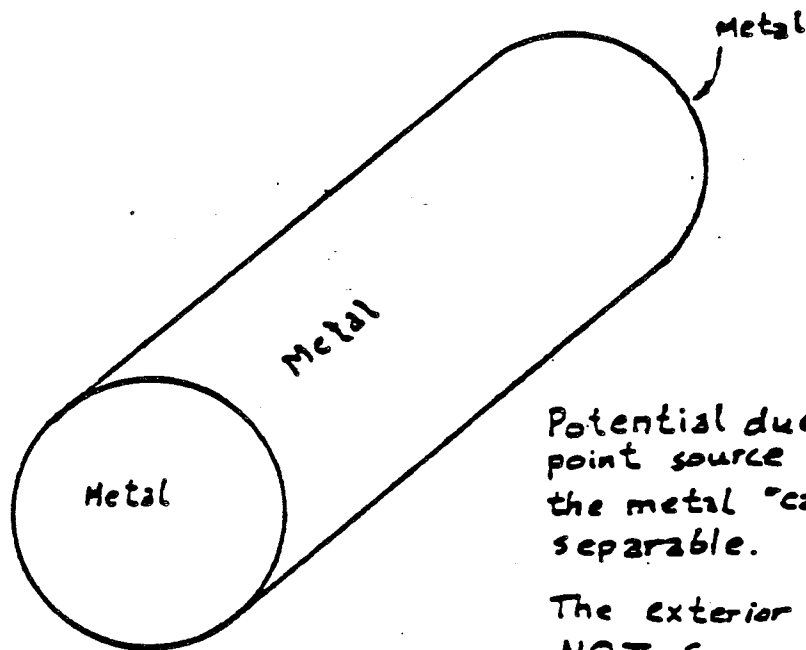
Metal

Metal

Metal

Potential due to
point source inside
the metal "can" is
separable.

The exterior problem is
NOT Separable

(2)   An infinitely long circular metal
Pipe.

Both interior and exterior problems
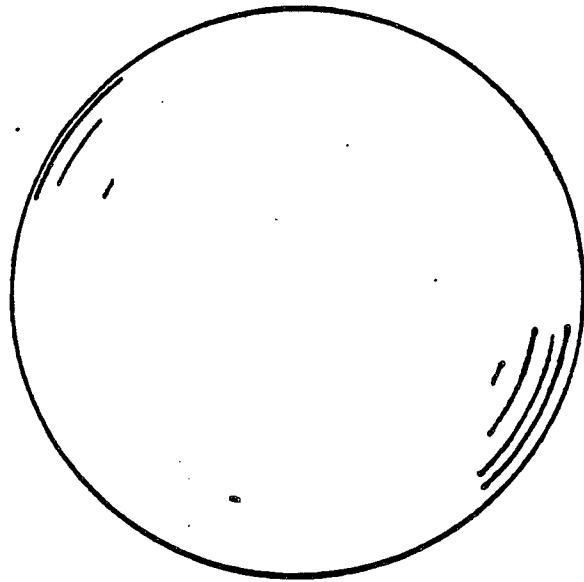are separable

(3)

Metal

Metal

Metal

Metal

INFINITELY LONG
CIRCULAR SECTOR PIPE

INTERIOR PROBLEM
IS SEPARABLE

EXTERIOR PROBLEM
IS NOT SEPARABLE.

(4)   Sphere: <u>both</u> interior and exterior problems are separable.

There are many other examples.