

Antiderivative

$$f(x) = x^3 \Rightarrow f'(x) = 3x^2$$

\xleftarrow{C}

$$f'(x) = (C)' = 0$$

$$f'(x) = f(x) \quad \text{differential}$$

$$\int \underbrace{f(x)}_{\text{integrand}} dx = \underbrace{f(x) + C}_{\text{Antiderivative}}$$

Integral
Sign

$$\int k dx = kx + C$$

$$\int k f(x) dx = k \int f(x) dx$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \underline{n \neq -1}$$

Ex

$$\begin{aligned} \int \frac{dx}{x^2} &= \int x^{-2} dx \\ &= \frac{x^{-2+1}}{-2+1} + C \\ &= -x^{-1} + C \\ &= -\frac{1}{x} + C \end{aligned}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

Ex $\int 3x^{1/3} dx = \int x^{1/3} dx$ $\frac{a}{b} \neq 1$
 $\frac{a+b}{b}$

$$= \frac{3}{4} x^{4/3} + C$$

$$= \frac{3}{4} x \sqrt[3]{x} + C$$

Ex $\int (4x^3 - 5x + 2) dx = x^4 - \frac{5}{2}x^2 + 2x + C$

Ex $\int (x^2 - 2x + 5) dx = \frac{1}{3}x^3 - x^2 + 5x + C$

Ex $\int \sin x dx = -\cos x + C$

Ex $\int \cos 3x dx = \frac{1}{3} \sin 3x + C$

$$\int \cos ax dx = \frac{1}{a} \sin ax + C$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C.$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$$

$$n = -1 \Rightarrow \int x^{-1} dx = \int \frac{dx}{x} \\ = \ln|x| + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$$

$$\underline{5x} \quad \int e^{-10x} dx = -\frac{1}{10} e^{-10x} + C$$

$$\underline{5x} \quad \int \frac{5}{x} dx = 5 \ln|x| + C$$

$$\underline{4x} \quad \int \frac{4}{\sqrt{9 - x^2}} dx = 4 \sin^{-1}\left(\frac{x}{3}\right) + C$$

$$\int \frac{dx}{16x^2+1} = \int \frac{dx}{16(x^2 + \frac{1}{16})}$$

$$= \frac{1}{16} 4 \tan^{-1} \frac{x}{\frac{1}{4}} + C$$

$$= \frac{1}{4} \tan^{-1}(4x) + C$$

1 $\int v^2 dv = \frac{1}{3} v^3 + C$

2 $\int x^{1/2} dx = \frac{2}{3} x^{3/2} + C$

3 $\int 4y^{-3} dy = -2y^{-2} + C$

4 $\int (x^3 - 4x + 2) dx = \frac{1}{4} x^4 - 2x^2 + 2x + C$

* $\int (ax+b) dy = (ax+b)y + C$

6 $\int (x^2-1)^2 dx = \int (x^4 - 2x^2 + 1) dx$
 $= \frac{1}{5} x^5 - \frac{2}{3} x^3 + x + C$

$$\begin{aligned}
 11 \int \frac{x^2+1}{\sqrt{x}} dx &= \int (x^{3/2} + x^{-1/2}) dx \\
 \left(\frac{x^2}{x^{1/2}} + \frac{1}{x^{1/2}} \right) &= \frac{2}{5} x^{5/2} + 2 x^{1/2} + C \\
 &= \frac{2}{5} x^2 \sqrt{x} + 2\sqrt{x} + C
 \end{aligned}$$

$$\begin{aligned}
 15 \int (\sqrt{x} + \frac{3}{x}) dx &= \int (x^{1/2} + x^{-1/3}) dx \\
 &= \frac{2}{3} x^{3/2} + \frac{3}{4} x^{4/3} + C
 \end{aligned}$$

$$18 \int (-2 \cos t) dt = -2 \sin t + C$$

$$19 \int 7 \sin \frac{\theta}{3} d\theta = -21 \cos \frac{\theta}{3} + C$$

$$\begin{aligned}
 21 \int (4 \sec x \tan x - 2 \sec^2 x) dx \\
 = 4 \sec x - 2 \tan x + C
 \end{aligned}$$

$$\begin{aligned}
 22 \int (2 \cos 2x - 3 \sin 3x) dx \\
 = \sin 2x + \cos 3x + C
 \end{aligned}$$

#23 $\int (1 + \tan^2 \theta) d\theta = \int \sec^2 \theta d\theta$
 $= \underline{\tan \theta + C}$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

ft 24 $\int \frac{\csc \theta}{\csc \theta - \sin \theta} d\theta = \int \frac{\frac{1}{\sin \theta}}{\frac{1}{\sin \theta} - \sin \theta} d\theta$
 $= \int \frac{\frac{1}{\sin \theta} \leftarrow}{\frac{1 - \sin^2 \theta}{\sin \theta} \leftarrow} d\theta$
 $= \int \frac{1}{\cos^2 \theta} d\theta$
 $= \int \sec^2 \theta d\theta$
 $= \underline{\tan \theta + C}$

$$\underline{\#25} \quad \int (2e^x - 3e^{-2x}) dx = \underline{2e^x + \frac{3}{2}e^{-2x} + C}$$

$$\underline{\#32} \quad \int \left(\frac{1}{x^2} - \frac{2}{x^{5/2}} \right) dx = -\frac{1}{x} - 2 \int x^{-5/2} dx$$

$$= \underline{-\frac{1}{x} + \frac{4}{3}x^{-3/2} + C}$$

$$\underline{\#36} \quad \int \sec 2x \tan 2x dx = \underline{\frac{1}{2} \sec 2x + C}$$

$$\underline{\#38} \quad \int \frac{12}{x} dx = \underline{12 \ln |x| + C}$$

$$\underline{\#41} \quad \int \frac{1 + \tan \theta}{\sec \theta} d\theta = \int \frac{1 + \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta}} d\theta$$

$$\left| \frac{\frac{\cos \theta + \sin \theta}{\cos \theta}}{\frac{1}{\cos \theta}} \right|$$

$$= \int (\cos \theta + \sin \theta) d\theta$$

$$= \underline{\sin \theta - \cos \theta + C}$$

$$\underline{\#42} \quad \int \left(\sqrt[4]{x^3} + \sqrt{x^5} \right) dx = \int (x^{3/4} + x^{5/2}) dx$$

$$= \frac{4}{7} x^{7/4} + \frac{2}{7} x^{7/2} + C$$

$$= \frac{4}{7} x \sqrt[4]{x^3} + \frac{2}{7} x^3 \sqrt{x} + C$$

$$\underline{46} \int (5x^{-4/3} + 3x^{-2/3} + 2x^{-1/3}) dx$$

$$= -15x^{-1/3} + 9x^{1/3} + 3x^{2/3} + C$$

$$\underline{54} \int \sec \theta (\sec \theta + \tan \theta) d\theta = \int (\sec^2 \theta + \sec \theta \tan \theta) d\theta$$

$$= \tan \theta + \sec \theta + C$$

$$\underline{56} \int (\cos^4 \theta - \sin^4 \theta) d\theta = \int (\cos^2 \theta - \sin^2 \theta) (\cos^2 \theta + \sin^2 \theta) d\theta$$

$\overset{=1}{d\theta}$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$= \int (\cos^2 \theta - \sin^2 \theta) d\theta$$

$$= \int \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta - \frac{1}{2} + \frac{1}{2} \cos 2\theta \right) d\theta$$

$$= \int \cos 2\theta d\theta$$

$$= \frac{1}{2} \sin 2\theta + C$$

$$\underline{59} \int (\cos 2x \cos 4x - \sin 2x \sin 4x) dx$$

$$= \int \cos (2x + 4x) dx$$

$$= \int \cos 6x dx$$

$$= \frac{1}{6} \sin 6x + C$$

$$6/2 \int \cos 2x \sin 2x dx = \frac{1}{2} \int \sin 4x dx$$

$$= -\frac{1}{8} \cos 4x + C$$

$$7/1 \int (4x - \frac{3}{x} - \csc^2 x) dx = 2x^2 - 3 \ln|x| + \cot x + C$$

$$7/2 \int (e^{4x} - \frac{3}{x} + 2 \cos x \cot x) dx = \frac{1}{4} e^{4x} - 3 \ln|x| - 2 \cos x + C$$

$$7/4 \int (a^2 - b^2) e^{(a-b)x} dx = \frac{a^2 - b^2}{a-b} e^{(a-b)x} + C$$

$$= (a+b) e^{(a-b)x} + C$$

$$V(r) = cr^2(r_0 - r)$$

$$= cr_0 r^2 - cr^3$$

$$V'(r) = 2cr_0 r - 3cr^2 = 0$$

$$2r_0 - 3r = 0$$

$$r = \frac{2}{3} r_0$$