$$df = f_x\Big|_{(x_0,y_0)} dx + f_y\Big|_{(x_0,y_0)} dy$$

ex Given: $r = 1$, $h = 5$ $dr = .03$ $dh = -.1$

ΔV?

$$V = \pi r^2 h$$

$$dV = 2\pi r h\, dr + \pi r^2 dh$$

$$= 2\pi (1)(5)\left(\frac{3}{100}\right) + \pi 1^2 \left(-\frac{1}{10}\right)$$

$$= \pi\left(\frac{3}{10} - \frac{1}{10}\right)$$

$$= \frac{\pi}{5} \text{ in}^3$$

$$\approx .63 \text{ in}^3$$

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ex $h = 25$ $r = 5$

Soln $V = \pi r^2 h$

$$dV = 2\pi r h\, dr + \pi r^2 dh$$

$$= 2\pi (5)(25)\, dr + \pi (5)^2 dh$$

$$= 250\pi\, dr + 25\pi\, dh$$

if $dr = 1 \to 250\pi$ unit
$1 \to$ (unit) in $h \to 25\pi$ unit
] change in Volume

$r = 25$ $h = 5$

$$dV = 250\pi\, dr + 625\pi\, dh.$$

1 in height $\to$ Change in volume 625π unit.

Ex $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$

Soln $f_x = y - 2x - 2 = 0$

$f_y = x - 2y - 2 = 0$

$$\begin{cases} -2x + y = +2 \\ x - 2y = 2 \end{cases}$$

$\boxed{x = \frac{-6}{3} = -2}$ $y = -2$

C.P. $(-2,-2)$

$f_{xx} = -2 < 0$ $f_{yy} = -2$ $f_{xy} = 1$

$f_{xx} f_{yy} - f_{xy}^2 = (-2)(-2) - 1$
$= 3 > 0$

fctn has a local Max @ $(-2,-2)$ w/

$f(-2,-2) = 4 - 4 - 4 + 4 + 4 + 4$
$= 8$

Ex $f(x,y) = 3y^2 - 2y^3 - 3x^2 + 6xy$

$f_x = -6x + 6y = 0$

$x = y$

$f_y = 6y - 6y^2 + 6x = 0$

① $y - y^2 + x = 0$

① $\rightarrow$ $y - y^2 + y = 0$

$-y^2 + 2y = 0 \Rightarrow \begin{cases} y = 0 = x \\ y = 2 = x \end{cases}$

CP: $(0,0)$ $(2,2)$

$f_{xx} = -6 < 0$ $\quad$ $f_{yy} = 6 - 12y$ $\quad$ $f_{xy} = 6$

@ $(0,0)$

$f_{xx} f_{yy} - f_{xy}^2 = (-6)(6) - 36$

$= -72 < 0$

The fctn has a saddle point @ $(0,0)$

@ $(2,2)$

$f_{xx} f_{yy} - f_{xy}^2 = (-6)(-18) - 36$

$= 72 > 0$

The fctn has a local max. @ $(2,2)$ w/

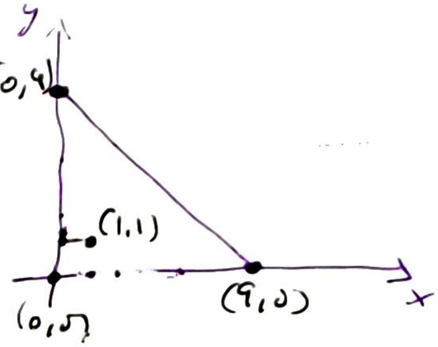
$f(2,2) = 12 - 16 - 12 + 24$

$= 8$

Ex abs. Max/Min

$$f(x,y) = 2 + 2x + 2y - x^2 - y^2$$

GI, $x = 0,\ y = 0,\ y = 9 - x$



$f_x = 2 - 2x = 0 \to x = 1$

$f_y = 2 - 2y = 0 \to y = 1$

$f(0,y) = 2 + 2y - y^2$

$f(x,0) = 2 + 2x - x^2$

$f(x, 9-x) = 2 + 2x + 18 - 2x - x^2 - 81 + 18x - x^2$

$= -2x^2 + 18x - 61$

$f'(x, 9-x) = -4x + 18 = 0 \to x = \frac{9}{2}$

$y = 9 - \frac{9}{2} = \frac{9}{2}$.

$f(0,0) = 2$

$f(9,0) = 2 + 18 - 81 = -61$

$f(0,9) = -61$

$f(1,1) = 2 + 2 + 2 - 1 - 1 = 4$

$f\left(\frac{9}{2}, \frac{9}{2}\right) = 2 + 9 + 9 - \frac{81}{4} - \frac{81}{4} = 20 - \frac{81}{2} = \frac{-41}{2}$

$f(0,1) = 3$

$f(1,0) = 3$

The fctn has a L.MIN @ (9,0) + (0,9) w/ value of −61

LMAX @ (1,1) w/ of 4

Ex girth : $P = 2y + 2z$

$x + \text{girth} = 108$

$x + 2y + 2z = 108$ (1)

Find dimensions. $(x, y, z?)$ Vol_{max}

$V = xyz$ (2)

(1) $\to x = 108 - 2y - 2z$

(2) $\to V = (108 - 2y - 2z)yz$

$= 108yz - 2y^2z - 2yz^2$

$V_y = 108z - 4yz - 2z^2$

$= 2z(54 - 2y - z) = 0$ (3)

$V_z = 108y - 2y^2 - 4yz$

$= 2y(54 - y - 2z) = 0$ (4)

(3) $\left| \; 2y + z = 54 \right. \longrightarrow y = 54 - 36 = \underline{18}$

(4) $\left| \; {-2}y + 2z = 54 \right.$ (multiplied by -2: -4, $-2z$...)

$-3z = -54 \Rightarrow \underline{z = 18}$

CP $(18, 18)$

(1) $x = 108 - 2(18) - 2(18)$

$= \underline{36}$

$x = 36, \; y = z = 18$

$V = 36(18)(18)$

$= \underline{11,664 \text{ in}^3}$

#1) $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$

$f_x = 2x + y + 3 = 0$ $\qquad$ $f_y = x + 2y - 3 = 0$

$$\begin{cases} 2x + y = -3 \\ x + 2y = 3 \end{cases} \qquad \boxed{x = \frac{-9}{3} = -3} \quad \boxed{y = -3 + 6 = 3}$$

C.P: $(-3, 3)$

$f_{xx} = 2 > 0$ $\qquad$ $f_{yy} = 2$ $\qquad$ $f_{xy} = 1$

$f_{xx} f_{yy} - f_{xy}^2 = 4 - 1 = 3 > 0$

The fctn has a LMIN @ $(-3,3)$ w/

$f(-3,3) = 9 - 9 + 9 - 9 - 9 + 4$
$= -5$

3/ $f(x,y) = x^3 + y^3 - 3xy + 15$

$f_x = 3x^2 - 3y = 0$ $\qquad$ $f_y = 3y^2 - 3x = 0$

$y = x^2$ $\qquad$ $y^2 = x$

$y = x^2$
$= y^4$

$y^4 - y = 0 \Rightarrow y(y^3 - 1) = 0$

$y = 0 \rightarrow x = 0$

$y = 1 \rightarrow x = 1$

CP : $(0,0)$ $\quad$ $(1,1)$

$f_{xx} = 6x$ $\qquad$ $f_{yy} = 6y$ $\qquad$ $f_{xy} = -3$

@ $(0,0)$ $\quad$ $f_{xx} = 0$, $f_{yy} = 0$

$f_{xx} f_{yy} - f_{xy}^2 = 0 - 9 = -9 < 0$

The fctn has a saddle point @ $(0,0)$

@ $(1,1)$ $\quad$ $f_{xx} = 6 > 0$ $\quad$ $f_{yy} = 6$

$f_{xx} f_{yy} - f_{xy}^2 = 36 - 9 = 27 > 0$

The fctn has a L MIN @ $(1,1)$ w/ value of

$f(1,1) = 1 + 1 - 3 + 15 = \boxed{14}$

#27 $f(x,y) = x^4 - 2x^2 + y^2 - 4y + 5$

$f_x = 4x^3 - 4x = 0$ $\quad\quad$ $f_y = 2y - 4 = 0$

$x(x^2-1) = 0$ $\quad\quad$ $y = 2$

$x = 0, \pm 1$

CP: $(0,2)$, $(\pm 1, 2)$

$f_{xx} = 12x^2 - 4$ $\quad$ $f_{yy} = 2$ $\quad$ $f_{xy} = 0$

@ $(0,2)$ $\quad$ $f_{xx} = -4 < 0$

$f_{xx} f_{yy} - f_{xy}^2 = -8 < 0$

The fctn has a saddle point @ $(0,2)$

@ $(-1,2)$ $\quad$ $f_{xx} = 8 > 0$

$f_{xx} f_{yy} - f_{xy}^2 = 16 > 0$

The fctn has a L Min. @ $(-1,2)$ w/ value

$f(-1,2) = 1 - 2 + 4 - 8 + 5 = 0$

@ $(1,2)$ $\quad$ $f_{xx} = 8 > 0$

$f_{xx} f_{yy} - f_{xy}^2 = 16 > 0$

The fctn has a L MIN @ $(1,2)$ w/ value

$f(1,2) = 1 - 2 + 4 - 4 + 5 = 4$

#30

$$f(x,y) = e^{x^2y^2 - 2xy^2 + y^2}$$

$$f_x = (2xy^2 - 2y^2)\, e^{x^2y^2 - 2xy^2 + y^2} = 0$$

$$\Rightarrow 2xy^2 - 2y^2 = 0 \quad (1)$$

$$f_{yy} = (2x^2y - 4xy + 2y)\, e^{x^2y^2 - 2xy^2 + y^2} = 0$$

$$\rightarrow 2x^2y - 4xy + 2y = 0 \quad (2)$$

(1) $y^2(x-1) = 0 \rightarrow y = 0,\ x = 1$

(2) $2y(x^2 - 2x + 1) = 0 \rightarrow y = 0,\ x = 1$

C.P : $(1,0)$

$$f_{xx} = \left(2y^2 + (2xy^2 - 2y^2)^2\right) e^{x^2y^2 - 2xy^2 + y^2} \Big|_{(1,0)}$$

$= 0$

$$f_{xy} = \left[(4xy - 4y) + (2xy^2 - 2y^2)^2\right) e^{x^2y^2 - 2xy^2 + y^2} \Big|_{(1,0)}$$

$= 0$

//

29. $f(x,y) = x^2 + 6x + y^2 + 8$

$f_x = 2x + 6 = 0$ $\quad\quad$ $f_y = 2y = 0$

$x = -3$ $\quad\quad$ $y = 0$

CP: $(-3, 0)$

$f_{xx} = 2 > 0$ $\quad$ $f_{yy} = 2$ $\quad$ $f_{xy} = 0$

$f_{xx} f_{yy} - f_{xy}^2 = 4 > 0$

The fctn has a LMIN @ $(-3, 0)$ w/ value of

$f(-3, 0) = 9 - 18 + 8$

$= \underline{-1}$