Lecture One - Trigonometric Functions

Section 1.1– Angles, Degrees, and Special Triangles

Basic Terminology

Two distinct points determine line *AB*.

Line segment AB: portion of the line between *A* and *B*.

Ray AB: portion of the line *AB* starts at *A* and continues through *B*, and past *B*.

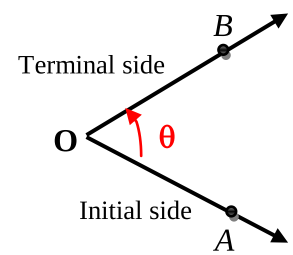
Angles in General

An angle is formed by 2 rays with the same end point.

The two rays are the sides of the angle.

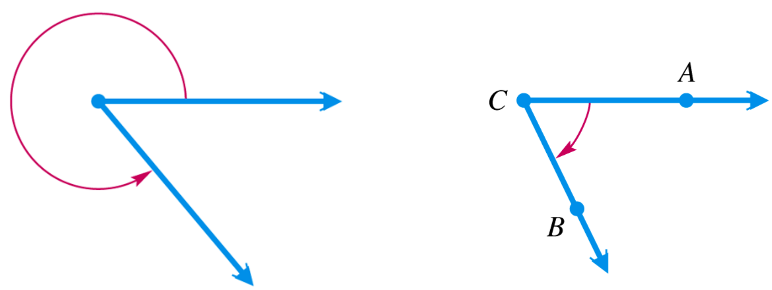
Angle $\theta = AOB$

O is the common endpoint and it is called ***vertex*** of the angle



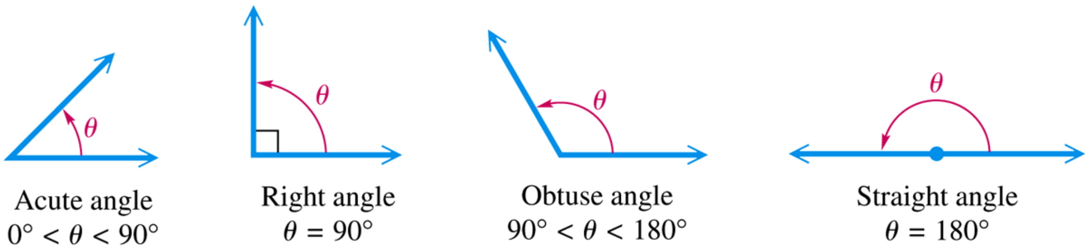
An angle is in a Counterclockwise (*CCW*) direction: positive angle

An angle is in a Clockwise (*CW*) direction: negative angle



Positive angle Negative angle

Type of Angles: ***Degree***



Complementary angles: $\alpha + \beta = 90°$

Supplementary angles: $\alpha + \beta = 180°$

Example

Give the complement and the supplement of each angle: 40° 110° θ

<u>*Solution*</u>

a. 40° Complement: 90° - 40° = 50° Supplement: 180° - 40° = 140°

b. 110° Complement: 90° - 110° = -20° Supplement: 180° - 110° = 70°

c. θ Complement: $90° - \theta$ Supplement: $180° - \theta$

Degrees, Minutes, Seconds

$1°$: 1 *degree* $1° = 60'$

$1'$: 1 *minute* $1' = 60''$

$1''$: 1 *second* $1 = 3600''$

1*full Rotation* or ***Revolution*= 360°**

$1° = 60' = 3600''$ $1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ}$

Example

Add $48°\ 49'$ and $72°\ 26'$

<u>*Solution*</u>

$$\begin{array}{rr} 48° & 49' \\ +\ 72° & 26' \\ \hline 120° & 75' \end{array}$$

$$\begin{aligned} 120°\ 75' &= 120°\ \ 60' + 15' \\ &= 121°\ \ 15' \end{aligned}$$

Example

Subtract $24° \, 14'$ and $90°$

<u>*Solution*</u>

$$\begin{array}{rr} & 90° \\ - & \underline{24° \quad 14'} \end{array} = \begin{array}{rr} & 89° \quad 60' \\ - & \underline{24° \quad 14'} \\ & 65° \quad 46' \end{array}$$

Example

Change 27.25° to degrees and minutes

<u>*Solution*</u>

$$\begin{aligned} 27.25° &= 27° + .25° \\ &= 27° + .25(\mathbf{60'}) \\ &= 27° + 15' \\ &= 27° \;\; 15' \end{aligned}$$

Angles in Standard Position

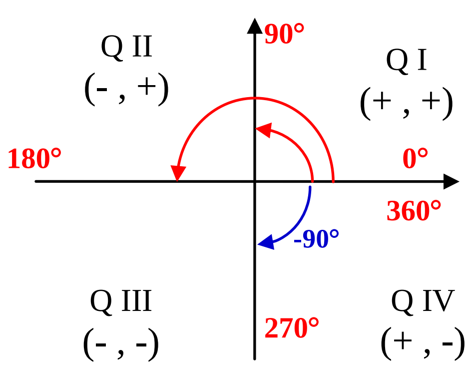
An angle is said to be in standard position if its initial side is along the positive *x*-axis and its vertex is at the origin.

If angle θ is in standard position and the terminal side of θ lies in quadrant I, then we say θ lies in QI

$$\theta \in QI$$

If the terminal side of an angle in standard position lies along one of the axes (*x*-axis or *y*-axis), such as angles with measures 90°, 180°, 270°, then that called a ***quadrantal*** *angle*.

Two angles in standard position with the same terminal side are called ***coterminal*** *angles*.



Example

Find all angles that are coterminal with 120°.

<u>Solution</u>:

$120° + 360°k$

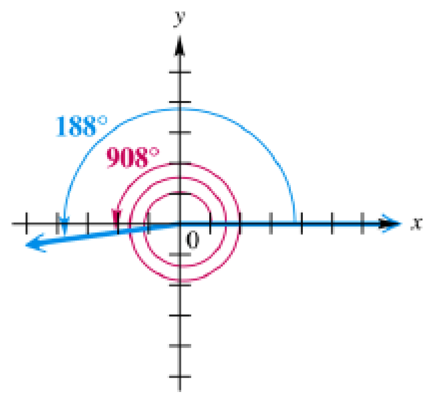
Example

Find the angle of least possible positive measure coterminal with an angle of 908°.

<u>Solution</u>

$908° - 2.360° = 188°$

An angle of 908° is coterminal with an angle of 188°



Example

CD players always spin at the same speed. Suppose a Constant Angular Velocity player makes 480 revolutions per minute. What degrees will a point on the edge of a CD spins for 2 seconds?

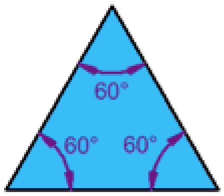
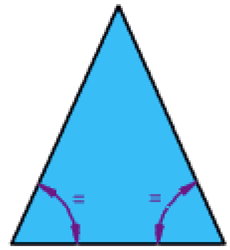
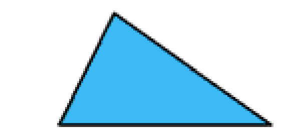
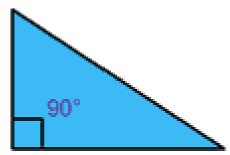
<u>Solution</u>

The player revolves 480 times in one minute $= \frac{480}{1'} = \frac{480}{60} = 8$ *times per* sec.

In 2 sec, the CD will spin: 2.8 = 16 times

Therefore; CD will revolve $16.360° = 5760°$

Triangles

<table>
<tr>
<td>Equilateral – All angles always equal to 60°& all sides are equals
</td>
<td>Isosceles: 2 sides and angles are equals
</td>
</tr>
<tr>
<td>Scalene: No equal sides or angles
</td>
<td>Right: Has a right angle 90°.
</td>
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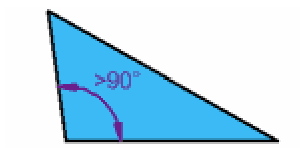
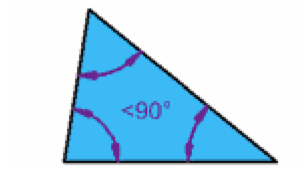
<tr>
<td>Obtuse: Has an angle more than 90°.
</td>
<td>Acute: All angles are less than 90°.
</td>
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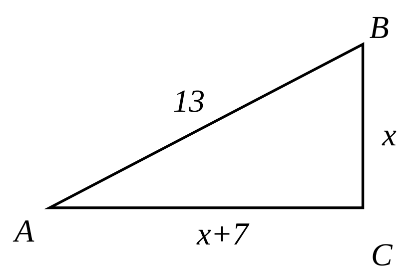
Pythagorean Theorem

$$C = 90^\circ \quad \Rightarrow c^2 = a^2 + b^2$$

Example

Solve for x in the right triangle



$$x^2 + (x+7)^2 = 13^2$$

$$x^2 + x^2 + 14x + 49 = 169$$

$$2x^2 + 14x + 49 - 169 = 0$$

$$2x^2 + 14x - 120 = 0$$

$$x^2 + 7x - 60 = 0$$

$$x = 5 \quad or \quad x = -12$$

Only $\boxed{x = 5}$ since we can't have -12 for a length

Exercises *Section* 1.1– Angles, Degrees, and Special Triangles

1. Indicate the angle if it is an acute or obtuse. Then give the complement and the supplement of each angle.

a) $10°$ *b*) $52°$ *c*) $90°$ *d*) $120°$ *e*) $150°$

2. Change $10°\ 45'$ to decimal degrees.

3. Convert $34°\ 51'\ 35''$ to decimal degrees.

4. Convert $274°\ 18'\ 59''$ to decimal degrees.

5. Change $74°\ 8'\ 14''$ to decimal degrees to the nearest thousandth.

6. Convert $89.9004°$ to degrees, minutes, and seconds.

7. Convert $34.817°$ to degrees, minutes, and seconds.

8. Convert $122.6853°$ to degrees, minutes, and seconds.

9. Convert $178.5994°$ to degrees, minutes, and seconds.

10. Perform each calculation

a) $51°\ 29' + 32°\ 46'$

b) $90° - 73°12'$

c) $90° - 36°\ 18'\ 47''$

d) $75°\ 15' + 83°\ 32'$

11. Find the angle of least possible positive measure coterminal with an angle of -75°.

12. Find the angle of least possible positive measure coterminal with an angle of -800°.

13. Find the angle of least possible positive measure coterminal with an angle of 270°.

14. A vertical rise of the Forest Double chair lift 1,170 feet and the length of the chair lift as 5,570 feet. To the nearest foot, find the horizontal distance covered by a person riding this lift.

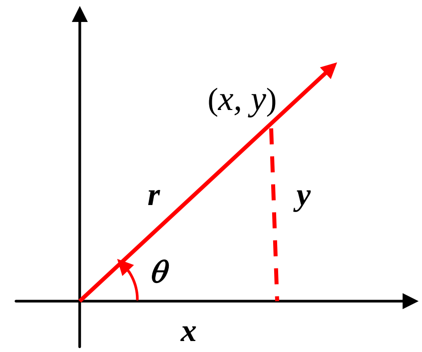
15. A tire is rotating 600 times per minute. Through how many degrees does a point of the edge of the tire move in $\frac{1}{2}$ second?

16. A windmill makes 90 revolutions per minute. How many revolutions does it make per second?

Section 1.2 – Trigonometric Functions

Let (x, y) be a point on the terminal side of an angle θ in standard position

The distance from the point to the origin is given by: $r = \sqrt{x^2 + y^2}$



Six Trigonometry Functions

$$\sin\theta = \frac{Opposite}{Hypotenuse} = \frac{opp}{hyp} = \frac{y}{r}$$

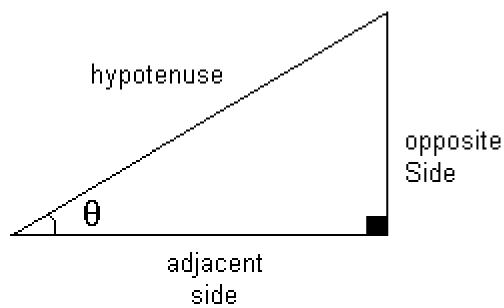
$$\cos\theta = \frac{Adjacent}{Hypotenuse} = \frac{adj}{hyp} = \frac{x}{r}$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

$$\cot\theta = \frac{adj}{opp} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} = \frac{x}{y}$$

$$\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta} = \frac{r}{x}$$

$$\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta} = \frac{r}{y}$$



Undefined Function Values

If the terminal side of a quadrantal angle lies along the ***y*-axis**, then the ***tangent*** and ***secant*** functions are undefined.

If the terminal side of a quadrantal angle lies along the ***x*-axis**, then the ***cotangent*** and ***cosecant*** functions are undefined.

Example

Find the six trigonometry functions of θ if θ is in the standard position and the point $(8, 15)$ is on the terminal side of θ.

<u>*Solution*</u>

$$\Rightarrow r = \sqrt{x^2 + y^2} = \sqrt{8^2 + 15^2} = 17$$

$$\sin\theta = \frac{y}{r} = \frac{15}{17} \qquad \cos\theta = \frac{x}{r} = \frac{8}{17} \qquad \tan\theta = \frac{y}{x} = \frac{15}{8}$$

$$\csc\theta = \frac{r}{y} = \frac{17}{15} \qquad \sec\theta = \frac{r}{x} = \frac{17}{8} \qquad \cot\theta = \frac{x}{y} = \frac{8}{15}$$

Example

Find the sine and cosine of 45° at the convenient point (1, 1)

<u>*Solution*</u>

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}} \qquad \cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}}$$

Example

Find the six trigonometry functions of 270°

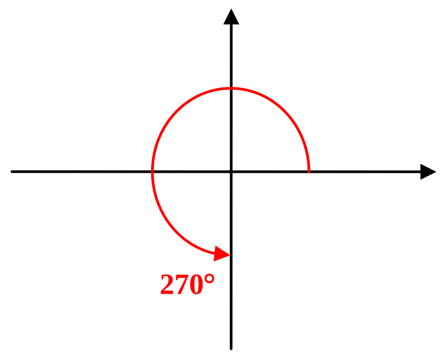
<u>*Solution*</u>

The convenient point (0, -1)

$$\Rightarrow r = \sqrt{0^2 + (-1)^2}$$

$$= \sqrt{1}$$

$$= 1$$



$$\sin 270^\circ = \frac{y}{r} = -1 \qquad \cot 270^\circ = \frac{x}{y} = \frac{0}{-1} = 0$$

$$\cos 270^\circ = \frac{x}{r} = \frac{0}{1} = 0 \qquad \sec 270^\circ = \frac{r}{x} = \frac{1}{0} = \infty$$

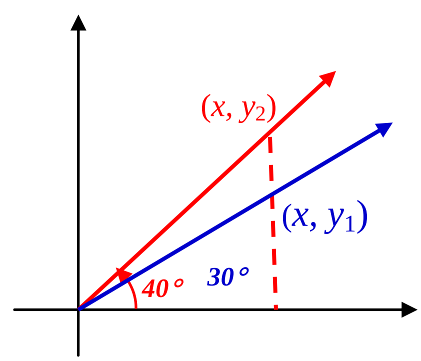
$$\tan 270^\circ = \frac{y}{x} = \frac{-1}{0} = \text{undefined} = -\infty \qquad \csc 270^\circ = \frac{r}{y} = \frac{1}{-1} = -1$$

Example

Which will be greater, tan 30° or tan 40°?

How large could $\tan\theta$ be?

<u>*Solution*</u>



$$\tan 30^\circ = \frac{y_1}{x}$$

$$\tan 40^\circ = \frac{y_2}{x}$$

$$Ratio: \frac{y_2}{x} > \frac{y_1}{x}$$

$\rightarrow$tan 40°> tan 30°

No limit as to how large $\tan\theta$ can be

Function	I	II	III	IV
$y = \sin x$	+	+	-	-
$y = \cos x$	+	-	-	+
$y = \tan x$	+	-	+	-
$y = \cot x$	+	-	+	-
$y = \csc x$	+	+	-	-
$y = \sec x$	+	-	-	+

Example

If $\cos\theta = \frac{\sqrt{3}}{2}$, and θ is QIV, find $\sin\theta$ and $\tan\theta$.

<u>*Solution*</u>

$$\cos\theta = \frac{\sqrt{3}}{2} = \frac{x}{r} \quad \to x = \sqrt{3}, \quad r = 2$$

$$r^2 = x^2 + y^2$$

$$\Rightarrow y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$y = \sqrt{2^2 - \left(\sqrt{3}\right)^2}$$

$$= \sqrt{4-3}$$

$$= 1 \qquad \text{Since } \theta \text{ is Q IV} \Rightarrow \boxed{y = -1}$$

$$\sin\theta = \frac{y}{r} = -\frac{1}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{\sqrt{3}} = -\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Ratio Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$x^2 + y^2 = r^2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1 \quad \Rightarrow \quad \boxed{\cos^2\theta + \sin^2\theta = 1}$$

Solving for $\cos\theta$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$$

Solving for $\sin\theta$

$$\sin^2\theta = 1 - \cos^2\theta \quad \Rightarrow \quad \boxed{\sin\theta = \pm\sqrt{1 - \cos^2\theta}}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{\cos\theta}{\cos\theta}\right)^2 + \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta}\right)^2$$

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

$\cos^2\theta + \sin^2\theta = 1$
$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$
$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$

Example

Prove $\sin\theta\cot\theta = \cos\theta$

<u>*Solution*</u>

$$\begin{aligned}\sin\theta\cot\theta &= \sin\theta\frac{\cos\theta}{\sin\theta}\\ &= \cos\theta\end{aligned}$$

Example

Write $\tan\theta$ in terms of $\sin\theta$.

<u>*Solution*</u>

$$\begin{aligned}\tan\theta &= \frac{\sin\theta}{\cos\theta}\\ &= \frac{\sin\theta}{\pm\sqrt{1-\sin^2\theta}}\\ &= \pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\end{aligned}$$

Example

If $\cos\theta = \frac{1}{2}$ and θ terminated in QIV, find the remaining trigonometric ratios for θ.

<u>*Solution*</u>

$$\begin{aligned}\sin\theta &= -\sqrt{1-\cos^2\theta}\\ &= -\sqrt{1-\left(\tfrac{1}{2}\right)^2}\\ &= -\sqrt{1-\frac{1}{4}}\\ &= -\sqrt{\frac{3}{4}}\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1/2} = 2$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

$$\cot\theta = -\frac{1}{\sqrt{3}}$$

Example

Find $\sin\theta$ and $\cos\theta$, given that $\tan\theta = \frac{4}{3}$ and θ is in QIII.

<u>*Solution*</u>

Using the identity $1+\tan^2\theta = \sec^2\theta$

$$\begin{aligned}\sec^2\theta &= 1+\left(\frac{4}{3}\right)^2\\ &= 1+\frac{16}{9}\\ &= \frac{25}{9}\end{aligned}$$

$$\sec\theta = -\sqrt{\frac{25}{9}} = -\frac{5}{3} \qquad \theta \in QIII \Rightarrow \cos\theta < 0 \to \sec\theta < 0$$

$$\cos\theta = -\frac{3}{5}$$

$$\begin{aligned}\sin^2\theta &= 1-\cos^2\theta\\ &= 1-\left(-\frac{3}{5}\right)^2\\ &= 1-\frac{9}{25}\\ &= \frac{16}{25}\end{aligned}$$

$$\sin\theta = -\frac{4}{5} \quad (\theta \in QIII)$$

Example

Show that the following statement is true by transforming the left side into the right side.

$$\cos\theta\tan\theta = \sin\theta$$

<u>*Solution*</u>

$$\begin{aligned}\cos\theta\tan\theta &= \cos\theta\frac{\sin\theta}{\cos\theta}\\ &= \sin\theta\end{aligned}$$

Example

Simplify the expression $\sqrt{x^2+9}$ as much as possible after substituting $3\tan\theta$ for x

<u>*Solution*</u>

$x = 3\tan\theta$

$$\begin{aligned}\sqrt{x^2+9} &= \sqrt{(3\tan\theta)^2+9}\\ &= \sqrt{9\tan^2\theta+9}\\ &= \sqrt{9\left(\tan^2\theta+1\right)}\\ &= 3\sqrt{\sec^2\theta}\\ &= 3\sec\theta\end{aligned}$$

Exercise *Section* 1.2 – Trigonometric Functions

1. Find the six trigonometry functions of θ if θ is in the standard position and the point $(-2, 3)$ is on the terminal side of θ.

2. Find the six trigonometry functions of θ if θ is in the standard position and the point $(-3, -4)$ is on the terminal side of θ.

3. Find the six trigonometry functions of θ in standard position with terminal side through the point $(-3,\ 0)$.

4. Find the six trigonometry functions of θ if θ is in the standard position and the point $(12, -5)$ is on the terminal side of θ.

5. Find the values of the six trigonometric functions for an angle of $90°$.

6. Indicate the two quadrants θ could terminate in if $\cos\theta = \frac{1}{2}$

7. Indicate the two quadrants θ could terminate in if $\csc\theta = -2.45$

8. Find the remaining trigonometric function of θ if $\sin\theta = \frac{12}{13}$ and θ terminates in QI.

9. Find the remaining trigonometric function of θ if $\cot\theta = -2$ and θ terminates in QII.

10. Find the remaining trigonometric function of θ if $\tan\theta = \frac{3}{4}$ and θ terminates in QIII.

11. Find the remaining trigonometric function of θ if $\cos\theta = \frac{24}{25}$ and θ terminates in QIV.

12. Find the remaining trigonometric functions of θ if $\cos\theta = \frac{\sqrt{3}}{2}$ and θ is terminates in QIV.

13. Find the remaining trigonometric functions of θ if $\tan\theta = -\frac{1}{2}$ and $\cos\theta > 0$.

14. If $\sin\theta = -\frac{5}{13}$, and θ is QIII, find $\cos\theta$ and $\tan\theta$.

15. If $\cos\theta = \frac{3}{5}$, and θ is QIV, find $\sin\theta$ and $\tan\theta$.

16. Use the reciprocal identities if $\cos\theta = \frac{\sqrt{3}}{2}$ find $\sec\theta$

17. Find $\cos\theta$, given that $\sec\theta = \frac{5}{3}$

18. Find $\sin\theta$, given that $\csc\theta = -\frac{\sqrt{12}}{2}$

19. Use a ratio identity to find $\tan\theta$ if $\sin\theta = \frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$

20. If $\cos\theta = -\frac{1}{2}$ and θ terminates in QII, find $\sin\theta$

21. If $\sin\theta = \frac{3}{5}$ and θ terminated in QII, find $\cos\theta$ and $\tan\theta$.

22. Find $\tan\theta$ if $\sin\theta = \frac{1}{3}$ and θ terminates in QI

23. Find the remaining trigonometric ratios of θ, if $\sec\theta = -3$ and $\theta \in QIII$

24. Using the calculator and rounding your answer to the nearest hundredth, find the remaining trigonometric ratios of θ if $\csc\theta = -2.45$ and $\theta \in QIII$

25. Write $\frac{\sec\theta}{\csc\theta}$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

26. Write $\cot\theta - \csc\theta$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

27. Write $\frac{\sin\theta}{\cos\theta} + \frac{1}{\sin\theta}$ in terms of $\sin\theta$ and/or $\cos\theta$, and then simplify if possible.

28. Write $\sin\theta\cot\theta + \cos\theta$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

29. Multiply $(1-\cos\theta)(1+\cos\theta)$

30. Multiply $(\sin\theta + 2)(\sin\theta - 5)$

31. Simplify the expression $\sqrt{25 - x^2}$ as much as possible after substituting $5\sin\theta$ for x.

32. Simplify the expression $\sqrt{4x^2 + 16}$ as much as possible after substituting $2\tan\theta$ for x

Section 1.3 – Evaluating Trigonometry Functions

$$\sin A = \frac{Opposite\ A}{Hypotenuse} = \frac{opp}{hyp} = \frac{a}{c} = \cos B$$

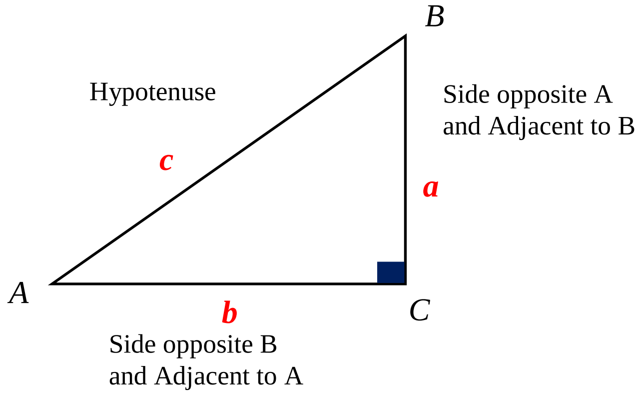
$$\cos A = \frac{Adjacent\ A}{Hypotenuse} = \frac{adj}{hyp} = \frac{b}{c} = \sin B$$

$$\tan A = \frac{opp\ A}{adj\ A} = \frac{a}{b} = \cot B$$

$$\cot A = \frac{adj\ A}{opp\ A} = \frac{b}{a} = \tan B$$

$$\sec A = \frac{hyp}{adj\ A} = \frac{c}{b} = \csc B$$

$$\csc A = \frac{hyp}{opp\ A} = \frac{c}{a} = \sec B$$



Example

Triangle ABC is a right triangle with C = 90°. If a = 6 and c = 10, find the six trigonometric functions of A.

$$\begin{aligned} b &= \sqrt{c^2 - a^2} \\ &= \sqrt{10^2 - 6^2} \\ &= 8 \end{aligned}$$

$$\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$$

$$\cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$$

$$\tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$$

$$\cot A = \frac{b}{a} = \frac{8}{6} = \frac{4}{3}$$

$$\sec A = \frac{c}{b} = \frac{10}{8} = \frac{5}{4}$$

$$\csc A = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$$

$$if\ \ A + B = 90° \Rightarrow \begin{cases} \sin A = \cos B \\ \sec A = \csc B \\ \tan A = \cot B \end{cases}$$

Cofunction Theorem

A trigonometric function of an angle is always equal to the cofunction of the complement of the angle.

Example

Fill in the blanks

a. $\sin(----) = \cos 30°$

Solution

$\sin 60° = \cos 30°$

b. $\tan y = \cot(----)$

Solution

$\tan y = \cot(90° - y)$

Example

Write each function in terms of its cofunction

a) $\cos 52°$

Solution

$\cos 52° = \sin(90° - 52°) = \sin 38°$

b) $\tan 71°$

Solution

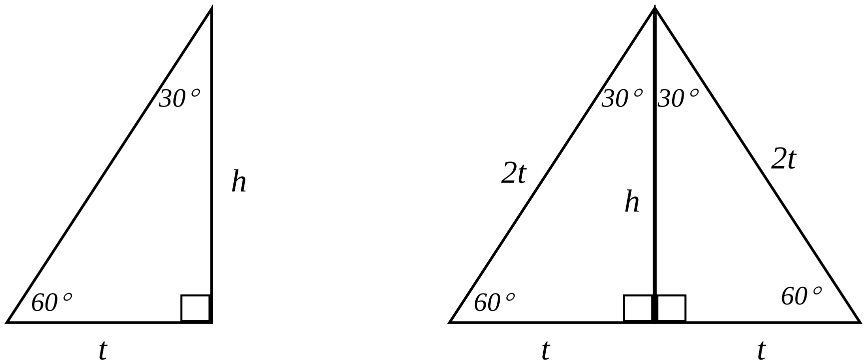
$\tan 71° = \cot(90° - 71°) = \cot 19°$

c) $\sec 24°$

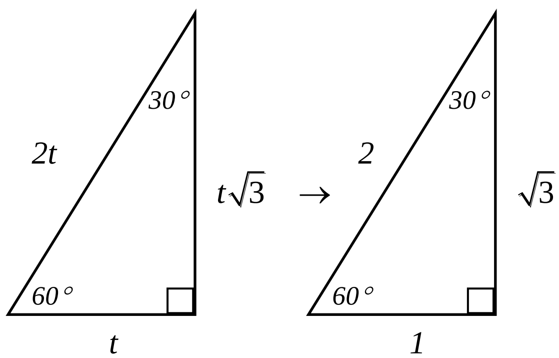
Solution

$\sec 24° = \csc(90° - 24°) = \csc 66°$

The 30° - 60° - 90° *Triangle*



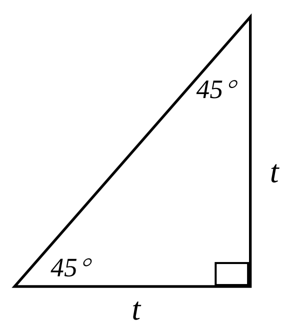
$$t^2 + h^2 = (2t)^2$$
$$t^2 + h^2 = 4t^2$$
$$h^2 = 4t^2 - t^2$$
$$h^2 = 3t^2$$
$$h = t\sqrt{3}$$

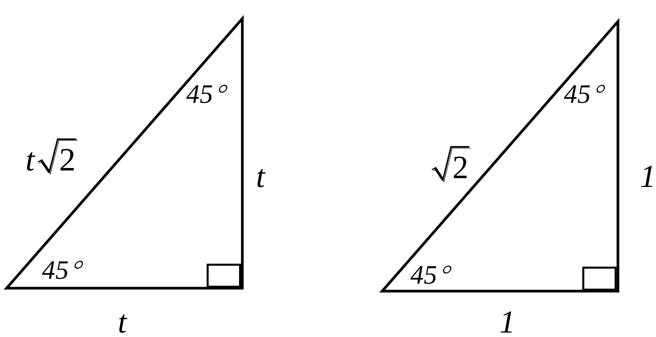


$$\Rightarrow \boxed{\sin 60° = \frac{\sqrt{3}}{2}}$$

The 45° - 45° - 90°*Triangle*

$$hypotenuse^2 = t^2 + t^2$$
$$hypotenuse = \sqrt{2t^2}$$
$$hypotenuse = t\sqrt{2}$$





$$\Rightarrow \boxed{\cos 45° = \frac{1}{\sqrt{2}}}$$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$45°$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Example

Show that the following are true

a. $\cos^2 30° + \sin^2 30° = 1$

$$\cos^2 30° + \sin^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$$

b. $\cos^2 45° + \sin^2 45° = 1$

$$\cos^2 45° + \sin^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

Example

Let $x = 30°$ and $y = 45°$ in each of the expressions that follow, and then simplify each expression as much as possible

a. $2\sin x$

$$2\sin 30° = 2\left(\frac{1}{2}\right) = 1$$

b. $\sin 2y$

$$\sin 2 \times 45° = \sin 90° = 1$$

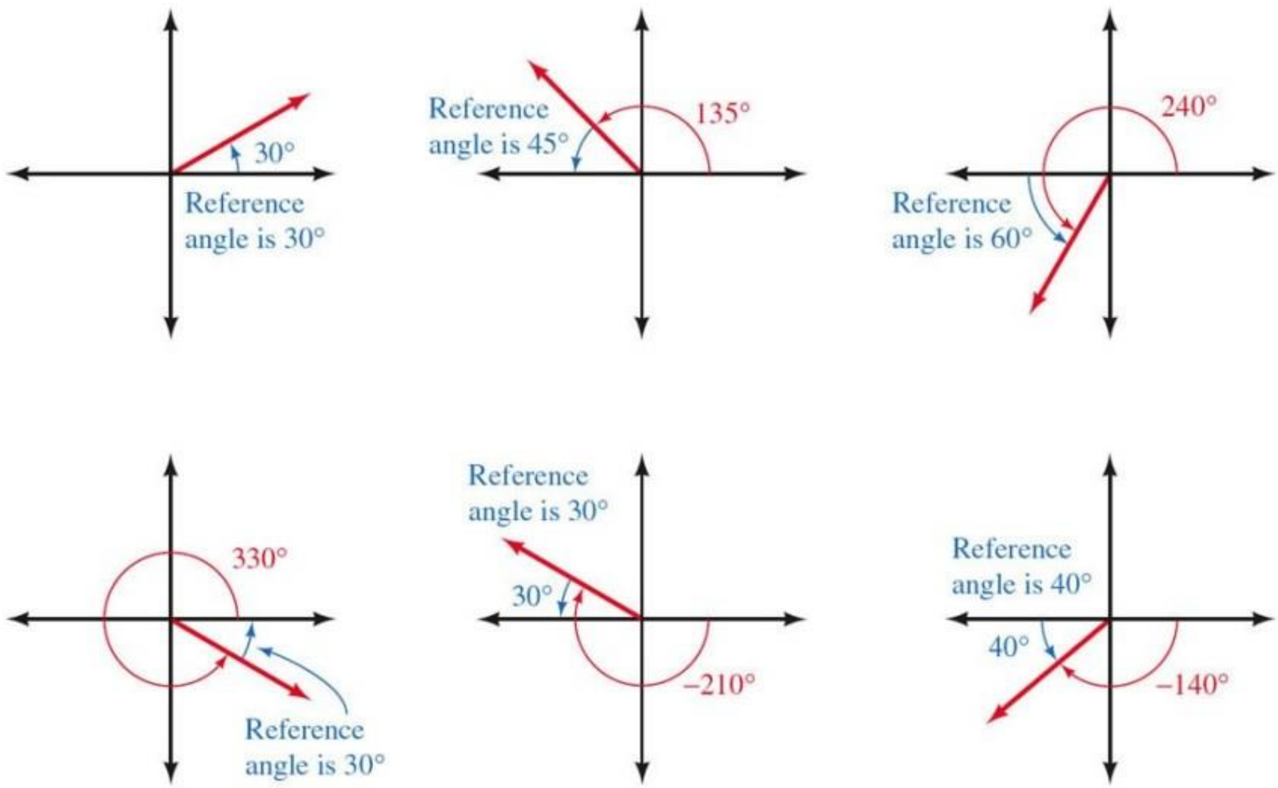
c. $4\sin(3x - 90°)$

$$4\sin(3(30°) - 90°) = 4\sin(0°) = 0$$

Reference Angle

Definition

The reference angle or related angle for any angle θ in standard position ifs the positive acute angle between the terminal side of θ and the x-axis, and it is denoted $\hat{\theta}$



$f\ \ \theta \in QI$ then $\hat{\theta} = \theta \quad \leftrightarrow \quad \theta = \hat{\theta}$

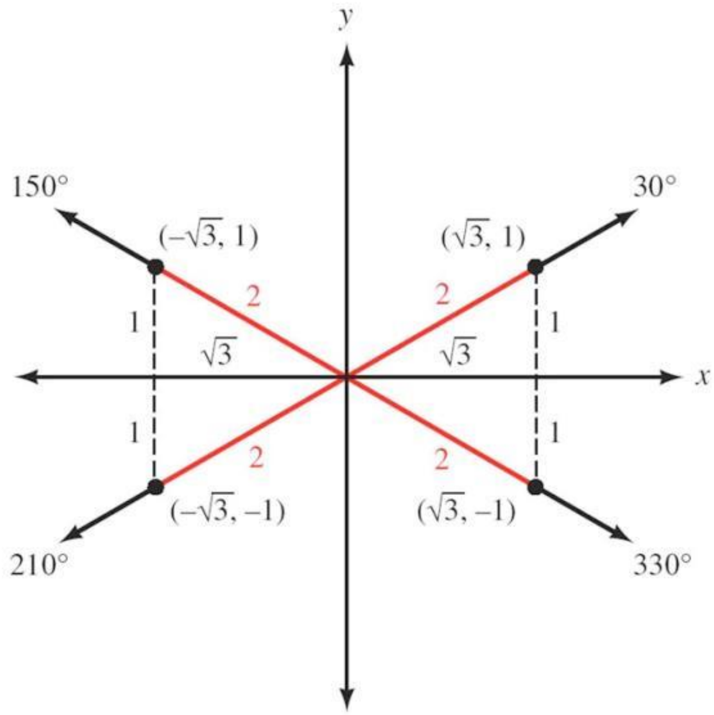
$f\ \ \theta \in QII$ then $\hat{\theta} = 180° - \theta \quad \leftrightarrow \quad \theta = 180° - \hat{\theta}$

$f\ \ \theta \in QIII$ then $\hat{\theta} = \theta - 180° \quad \leftrightarrow \quad \theta = \hat{\theta} + 180°$

$f\ \ \theta \in QIV$ then $\hat{\theta} = 360° - \theta \quad \leftrightarrow \quad \theta = 360° - \hat{\theta}$

Reference Angle Theorem

A trigonometric function of an angle and its reference angle are the same, except difference in sign.



$$\sin 150° = \sin 30° = \frac{1}{2}$$

$$\sin 210° = -\sin 30° = -\frac{1}{2}$$

Example

Find the exact value of $\sin 240°$

<u>*Solution*</u>

$$\hat{\theta} = 240° - 180° = 60° \qquad \rightarrow 240° \in QIII$$

$$\sin 240° = -\sin 60°$$

$$= -\frac{\sqrt{3}}{2}$$

Example

Find the exact value of $\tan 315°$

<u>*Solution*</u>

$$\hat{\theta} = 360° - 315° = 45° \qquad \rightarrow 315° \in QIV$$

$$\tan 315° = -\tan 45°$$

$$= -1$$

The trigonometry function of an angle and any coterminal to it are always equals.

$$\sin(\theta + 360°k) = \sin\theta$$

$$\cos(\theta + 360°k) = \cos\theta$$

Example

Find the exact value of $\cos 495°$

<u>*Solution*</u>

$495° - 360° = 135°$

$\rightarrow 135° \in QII$

$\hat{\theta} = 180° - 135° = 45°$

$\cos 495° = -\cos 45° = -\frac{1}{\sqrt{2}}$

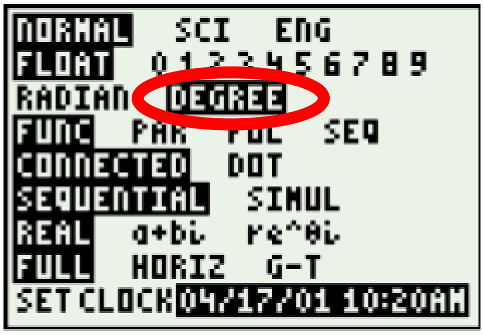
Approximation- Simply using calculator

$\sin 250° \approx -0.9397$

$\cos 250° \approx -0.3420$

$\tan 250° \approx 2.7475$

$\csc 250° = \frac{1}{\sin 250°} \approx -1.0642$



To find the angle by using the inverse trigonometry functions, always enter a ***positive*** *value.*

Example

Find θ if $\sin\theta = -0.5592$ and θ terminates in QIII with $0° \le \theta < 360°$.

<u>*Solution*</u>

$\hat{\theta} = \sin^{-1} 0.5592 \approx \underline{34°}$

$\theta \in$ QIII

$\Rightarrow \underline{\theta} = 180° + 34° = \underline{214°}$

Example

Find θ to the nearest degree if $\cot\theta = -1.6003$ and θ terminates in QII with $0° \le \theta < 360°$.

<u>*Solution*</u>

$$\cot\theta = -1.6003 = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{1}{-1.6003}$$

$$\hat{\theta} = \tan^{-1}\frac{1}{1.6003}$$

$$= 32°$$

$\theta \in$ QII

$$\Rightarrow \theta = 180° - 32° \,\underline{= 148°}$$

Angle θ in *degree*	*sin θ*	*cosθ*	*tan θ*	*cot θ*	*sec θ*	*cscθ*
$0°$	0	1	0	∞ (*undefined*)	1	∞
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
$90°$	1	0	$\pm\infty$	0	$\pm\infty$	1
$120°$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
$135°$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$150°$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
$180°$	0	-1	0	$\pm\infty$	-1	$\pm\infty$

Exercise *Section* 1.3 – Evaluating Trigonometry Functions

1. Simplify by using the table. $5\sin^2 30°$

2. Simplify by using the table $\sin^2 60° + \cos^2 60°$

3. Simplify by using the table $(\tan 45° + \tan 60°)^2$

4. Find the exact value of $\csc 300°$

5. Find θ if $\sin\theta = -\frac{1}{2}$ and θ terminates in QIII with $0° \le \theta \le 360°$.

6. Find θ to the nearest degree if $\sec\theta = 3.8637$ and θ terminates in QIV with $0° \le \theta < 360°$.

7. Find the exact value of $\cos 225°$

8. Find the exact value of $\tan 315°$

9. Find the exact value of $\cos 420°$

10. Find the exact value of $\cot 480°$

11. Use the calculator to find the value of $\csc 166.7°$

12. Use the calculator to find the value of $\sec 590.9°$

13. Use the calculator to find the value of $\tan 195°\,10'$

14. Use the calculator to find θ to the nearest degree if $\sin\theta = -0.3090$ with $\theta \in$ QIV with $0° \le \theta < 360°$

15. Use the calculator to find θ to the nearest degree if $\cos\theta = -0.7660$ with $\theta \in$ QIII with $0° \le \theta < 360°$

16. Use the calculator to find θ to the nearest degree if $\sec\theta = -3.4159$ with $\theta \in$ QII with $0° \le \theta < 360°$

17. Find θ to the nearest tenth of a degree if $\tan\theta = -0.8541$ and θ terminates in QIV with $0° \le \theta < 360°$

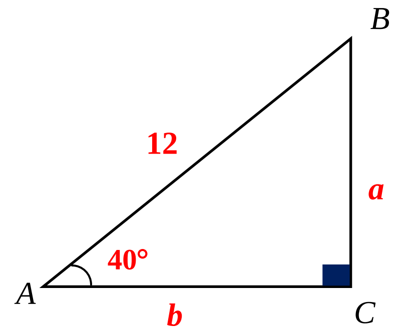
18. Use the calculator to find θ to the nearest degree if $\sin\theta = 0.49368329$ with $\theta \in$ QII with $0° \le \theta < 360°$

Section 1.4 – Solving Right Triangle Trigonometry

Example

In the right triangle *ABC*, $A = 40°$ and c = 12 cm. Find *a*, *b*, and *B*.

<u>*Solution*</u>



$$\sin 40° = \frac{a}{c} = \frac{a}{12}$$

$$a = 12\sin 40°$$
$$= 7.7cm$$

$$\cos 40° = \frac{b}{c}$$
$$= \frac{b}{12}$$

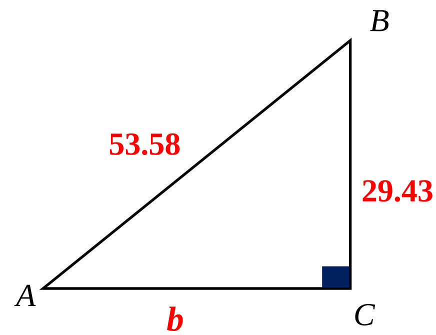
$$b = 12\cos 40°$$
$$= 9.2cm$$

$$B = 90° - A$$
$$= 90° - 40°$$
$$\approx 50°$$

Example

In the right triangle *ABC*, $a = 29.43$ and $c = 53.58$. Find the remaining side and angles.

<u>*Solution*</u>



$$c^2 = a^2 + b^2 \quad \Rightarrow \quad b^2 = c^2 - a^2$$

$$b = \sqrt{53.58^2 - 29.43^2} \approx 44.77$$

$$\sin A = \frac{a}{c}$$
$$= \frac{29.43}{53.58}$$

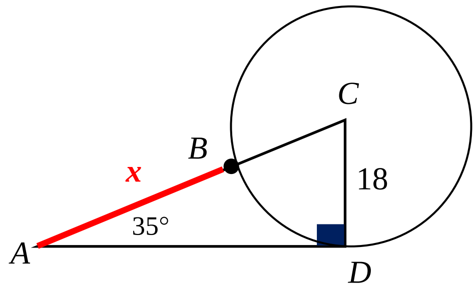
$$A = \sin^{-1}\left(\frac{29.43}{53.58}\right)$$
$$\approx 33.32°$$

$$B = 90° - A$$
$$= 90° - 33.32°$$
$$\approx 56.68°$$

Example

A circle has its center at C and a radius of 18 inches. If triangle ADC is a right triangle and $A = 35°$. Find x, the distance from A to B.

<u>*Solution*</u>



$$\sin 35° = \frac{18}{x+18}$$

$$(x+18)\sin 35° = 18$$

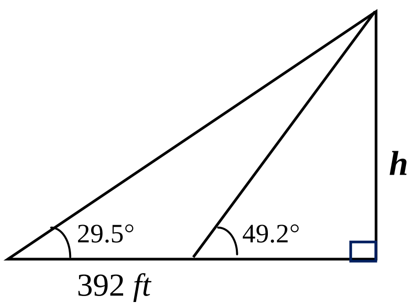
$$x + 18 = \frac{18}{\sin 35°}$$

$$x = \frac{18}{\sin 35°} - 18$$

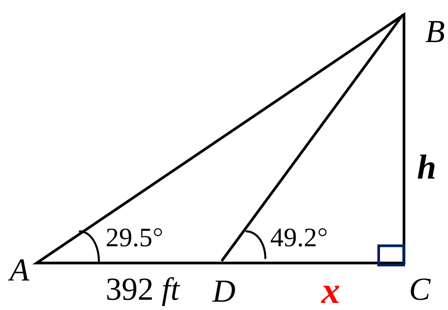
$$= 13\ in$$

Example

Find h as indicated in the figure.



<u>*Solution*</u>



Triangle DCB

$$\Rightarrow \tan 49.2° = \frac{h}{x}$$

$$h = x \tan 49.2°$$

Triangle ACB

$$\Rightarrow \tan 29.5° = \frac{h}{x+392}$$

$$h = (x+392)\tan 29.5°$$

$$h = x\tan 49.2° = (x+392)\tan 29.5°$$

$$x\tan 49.2° = x\tan 29.5° + 392\tan 29.5°$$

$$x\tan 49.2° - x\tan 29.5° = 392\tan 29.5°$$

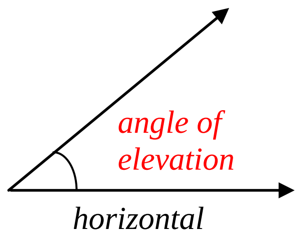
$$x\left(\tan 49.2° - \tan 29.5°\right) = 392\tan 29.5°$$

$$x = \frac{392\tan 29.5°}{\tan 49.2° - \tan 29.5°}$$

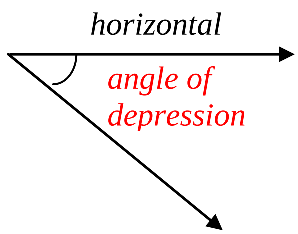
$$\boxed{h = \frac{392\tan 29.5°}{\tan 49.2° - \tan 29.5°}\tan 49.2° = 433.5\ ft}$$

Definitions

An angle measured from the horizontal up is called an ***angle of elevation***.



An angle measured from the horizontal down is called an ***angle of depression***.



Example

The two equal sides of an isosceles triangle are each 24 cm. If each of the two equal angles measures 52°, find the length of the base and the altitude.

Solution

$$\sin 52° = \frac{x}{24}$$

$$x = 24\sin 52°$$

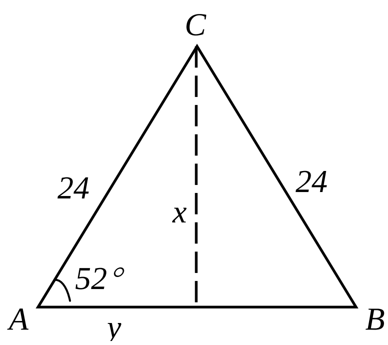
$$x = 19\ cm$$

$$\cos 52° = \frac{y}{24}$$

$$y = 24\cos 52°$$

$$y = 15\ cm$$

$$\Rightarrow \boxed{AB = 2y = 30\ cm}$$



Example

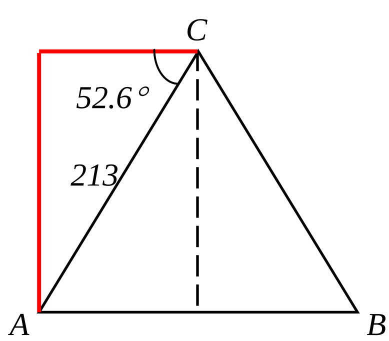
A man climbs 213 meters up the side of a pyramid. Find that the angle of depression to his starting point is 52.6°. How high off of the ground is he?

Solution

$$\sin 52.6° = \frac{h}{213}$$

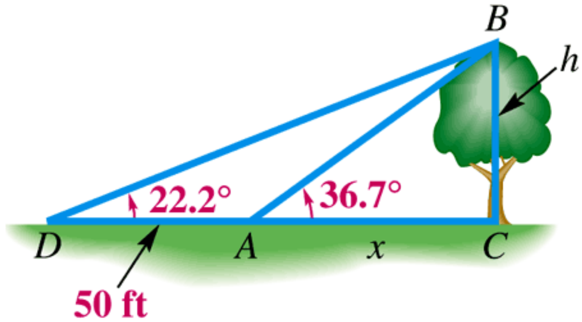
$$h = 213\sin 52.6°$$

$$h = 169\ m$$



Example

From a given point on the ground, the angle of elevation to the top of a tree is 36.7°. From a second point, 50 feet back, the angle of elevation to the top of the tree is 22.2°. Find the height of the tree to the nearest foot.



<u>*Solution*</u>

Triangle *DCB*

$$\Rightarrow \tan 22.2^\circ = \frac{h}{50+x}$$

$$h = (50+x)\tan 22.2^\circ$$

Triangle *ACB*

$$\Rightarrow \tan 36.7^\circ = \frac{h}{x}$$

$$h = x\tan 36.7^\circ$$

$$x\tan 36.7^\circ = (50+x)\tan 22.2^\circ$$

$$x\tan 36.7^\circ = 50\tan 22.2^\circ + x\tan 22.2^\circ$$

$$x\tan 36.7^\circ - x\tan 22.2^\circ = 50\tan 22.2^\circ$$

$$x(\tan 36.7^\circ - \tan 22.2^\circ) = 50\tan 22.2^\circ$$

$$x = \frac{50\tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}$$

$$h = x\tan 36.7^\circ$$

$$= \left(\frac{50\tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}\right)\tan 36.7^\circ$$

$$\approx 45\ ft$$

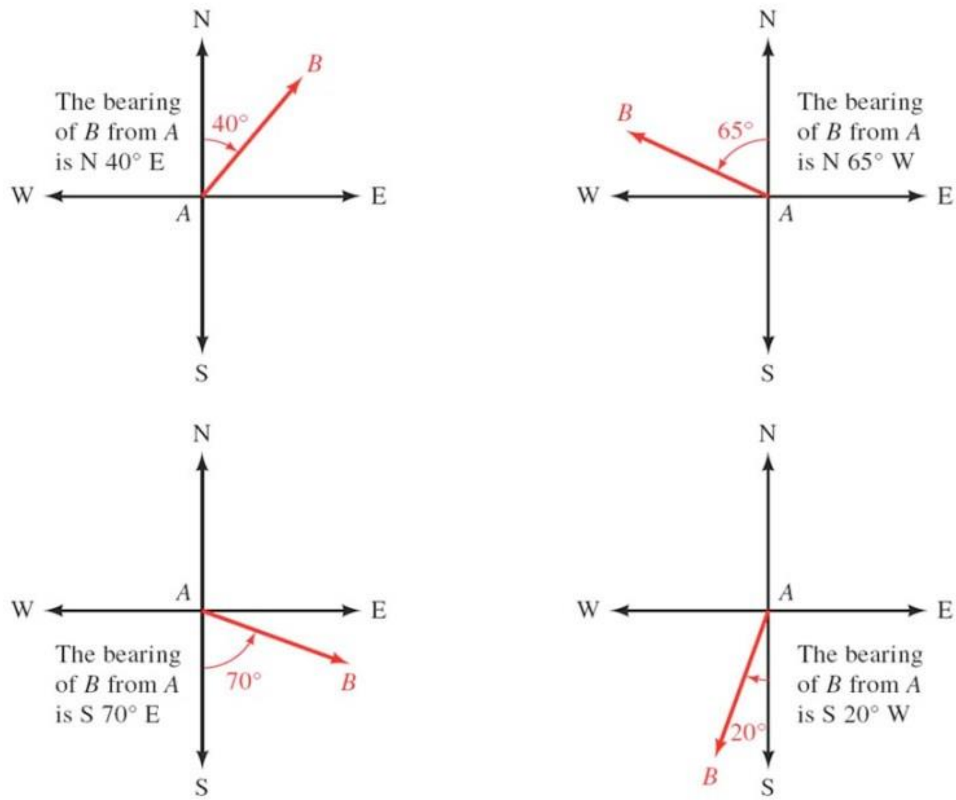
The tree is about 45 feet tall.

Bearing

Definition

The *bearing of a line* ℓ is the acute angle formed by the *north-south* line and the line ℓ.

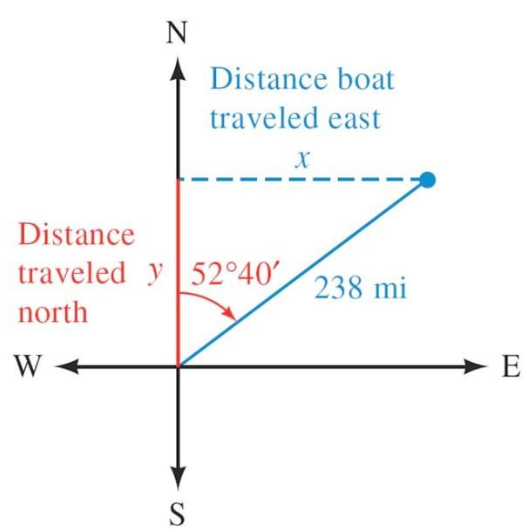
The notation used to designate the bearing of a line begins with N (for north) or S (for south), followed by the number of degrees in the angle, and ends with E (for east) or W (for west).



Example

A boat travels on a course of bearing N 52° 40′ E for distance of 238 miles. How many miles north and how many miles east have the boat traveled?

<u>*Solution*</u>



$$52°40' = 52° + 40'\frac{1°}{60'}$$

$$= 52.6667$$

$$\sin 52.6667 = \frac{x}{238}$$

$$x = 238 \sin 52.6667 \; \underline{= 189 \ mi|}$$

$$\cos 52.6667 = \frac{y}{238}$$

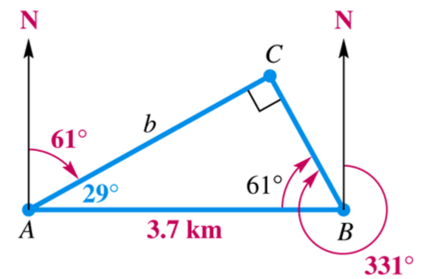
$$y = 238 \cos 52.6667$$

$$= 144 \ mi$$

Example

Radar stations A and B are on the east-west line, 3.7 km apart. Station A detects a place at C, on a bearing of 61°. Station B simultaneously detects the same plane, on a bearing of 331°. Find the distance from A to C.

<u>*Solution*</u>



$\lfloor \underline{A = 90° - 61° = 29°} \rfloor$

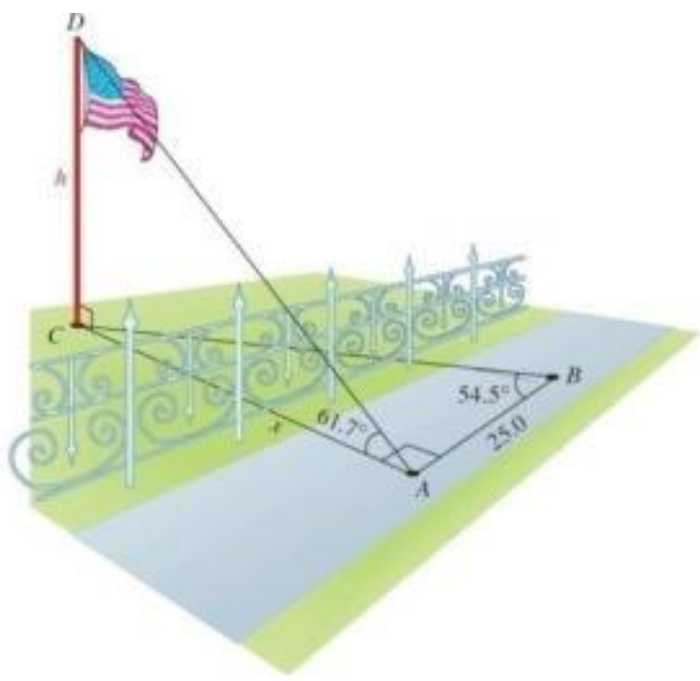
$\cos 29° = \dfrac{b}{3.7}$

$b = 3.7 \cos 29° \approx 3.2\ km$

Example

A diagram that shows how Diane estimates the height of a flagpole. She can't measure the distance between herself and the flagpole directly because there is a fence in the way. So she stands at point A facing the pole and finds the angle of elevation from point A to the top of the pole to be 61.7°. Then she turns 90° and walks 25.0 ft to point B, where she measures the angle between her path and a line from B to the base of the pole. She finds that angle is 54.5°. Use this information to find the height of the pole.

<u>*Solution*</u>



$\tan 54.5° = \dfrac{x}{25.0}$

$x = 25.0 \tan 54.5°$

$= 35.0487\ ft$

$\tan 61.7° = \dfrac{h}{35.0487}$

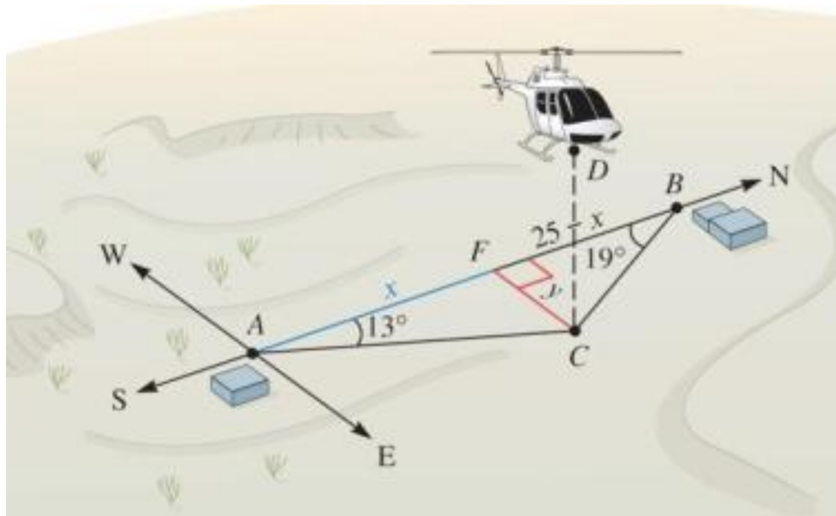
$h = 35.0487 \tan 61.7°$

$= 65.1\ ft$

Example

A helicopter is hovering over the desert when it develops mechanical problems and is forced to land. After landing, the pilot radios his position to a pair of radar station located 25 miles apart along a straight road running north and south. The bearing of the helicopter from one station is N 13° E, and from the other it is S 19° E. After doing a few trigonometric calculations, one of the stations instructs the pilot to walk due west for 3.5 miles to reach the road. Is this information correct?

<u>Solution</u>



In triangle AFC

$$\tan 13° = \frac{y}{x}$$

$$y = x \tan 13°$$

In triangle BFC

$$\tan 19° = \frac{y}{25 - x}$$

$$y = (25 - x) \tan 19°$$

$$y = y$$

$$(25 - x) \tan 19° = x \tan 13°$$

$$25 \tan 19° - x \tan 19° = x \tan 13°$$

$$25 \tan 19° = x \tan 13° + x \tan 19°$$

$$25 \tan 19° = x(\tan 13° + \tan 19°)$$

$$\frac{25 \tan 19°}{\tan 13° + \tan 19°} = x$$

$$x = 14.966$$

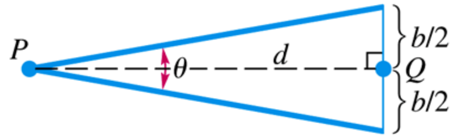
$$y = x \tan 13°$$

$$= 14.966 \tan 13°$$

$$= 3.5\ mi$$

Example

A method that surveyors use to determine a small distance d between two points P and Q is called the ***subtense bar method***. The subtense bar with length b is centered at Q and situated perpendicular to the line of sight between P and Q. Angle θ is measured, then the distance d can be determined.



a) Find d with $\theta = 1^\circ\ 23'\ 12''$ and $b = 2.000\ cm$

b) Angle θ usually cannot be measured more accurately than to the nearest $1''$. How much change would there be in the value of d if θ were measured $1''$ larger?

<u>*Solution*</u>

a) $\cot\frac{\theta}{2} = \frac{d}{b/2}$

$$d = \frac{b}{2}\cot\frac{\theta}{2}$$

$$\begin{aligned}\theta &= 1^\circ\ 23'\ 12'' \\ &= 1^\circ + \frac{23^\circ}{60} + \frac{12^\circ}{3600} \\ &\approx 1.38667^\circ\end{aligned}$$

$$\begin{aligned}d &= \frac{2}{2}\cot\frac{1.38667^\circ}{2} \\ &\approx 82.6341\ cm\end{aligned}$$

b) $\theta = 1^\circ\ 23'\ 12'' + 1'' = 1^\circ\ 23'\ 13''$

$$\approx 1.386944^\circ$$

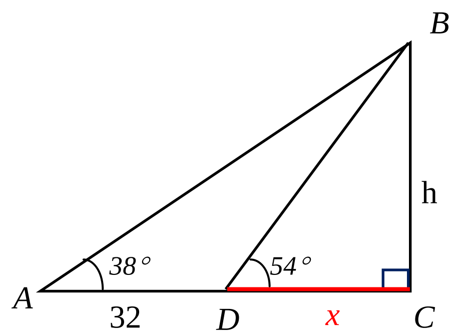
$$\begin{aligned}d &= \frac{2}{2}\cot\frac{1.386944^\circ}{2} \\ &\approx 82.617558\ cm\end{aligned}$$

The change is: $82.6341 - 82.6175 \approx 0.0166\ cm$

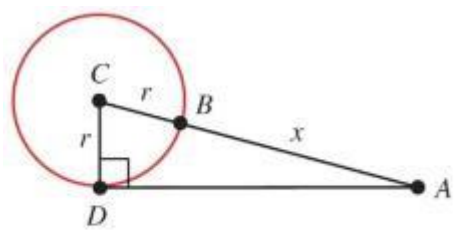
Exercises *Section* 1.4 – Solving Right Triangle Trigonometry

1. In the right triangle *ABC*, $a = 2.73$ and $b = 3.41$. Find the remaining side and angles.

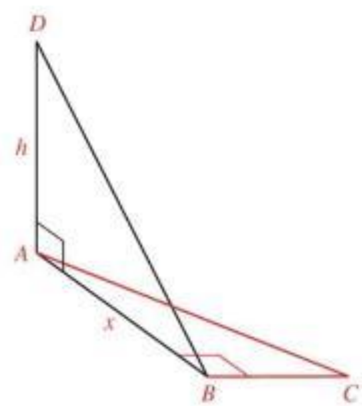
2. The distance from *A* to *D* is 32 feet. Use the information in figure to solve *x*, the distance between *D* and *C*.



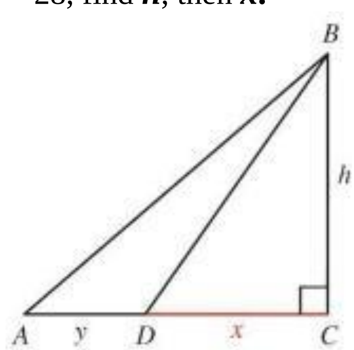
3. If C = 26° and $r = 19$, find *x*.



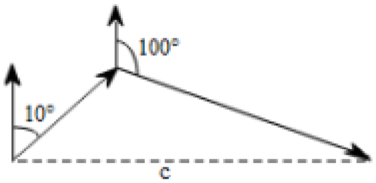
4. If ∠ABD = 53°, C = 48°, and BC = 42, find ***x*** and then find ***h***.



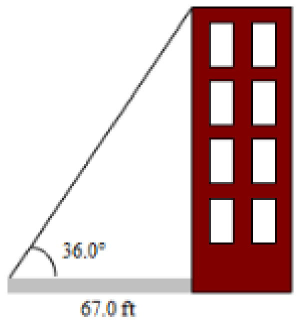
5. If A = 41°, ∠BDC = 58°, and AB = 28, find ***h***, then ***x***.



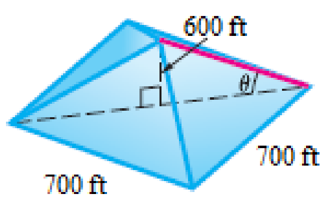
6. A plane flies 1.7 hours at 120 mph on a bearing of 10°. It then turns and flies 9.6 hours at the same speed on a bearing of 100°. How far is the plane from its starting point?



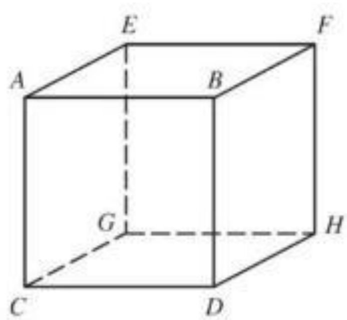
7. The shadow of a vertical tower is 67.0 *ft* long when the angle of elevation of the sun is 36.0°. Find the height of the tower.



8. The base of a pyramid is square with sides 700 ft long, and the height of the pyramid is 600 ft. Find the angle of elevation of the edge indicated in the figure to two significant digits. (Hint: The base of the triangle in the figure is half the diagonal of the square base of the pyramid.)

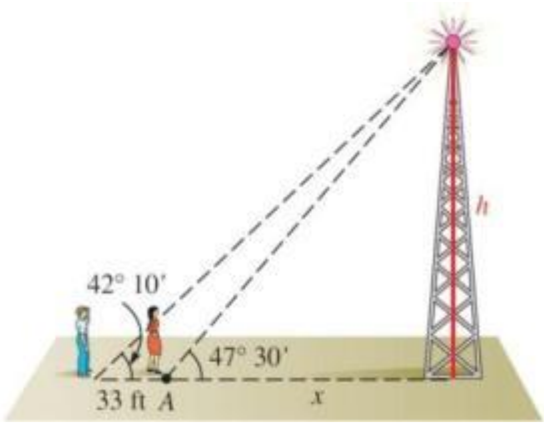


9. Suppose each edge of the cube is 3.00 inches long. Find the measure of the angle formed by diagonals DE and DG. *Round your answer to the nearest tenth of a degree*.

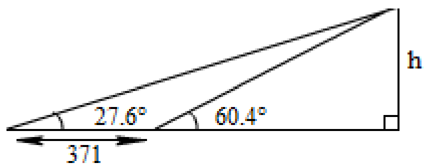


10. If a 73-foot flagpole casts a shadow 51 feet long, what is the angle of elevation of the sun (to the nearest tenth of a degree)?

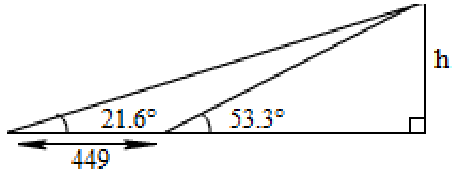
11. A person standing at point A notices that the angle of elevation to the top of the antenna is 47° 30′. A second person standing 33.0 feet farther from the antenna than the person at A finds the angle of elevation to the top of the antenna to be 42° 10′. How far is the person at A from the base of the antenna?



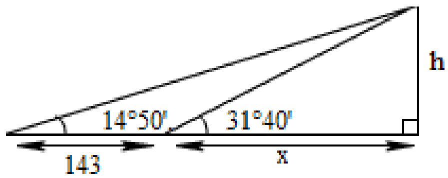
12. Find ***h*** as indicated in the figure.



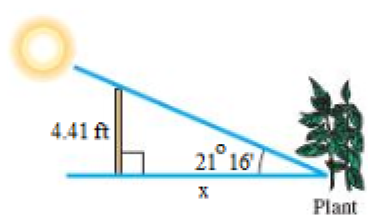
13. Find ***h*** as indicated in the figure.



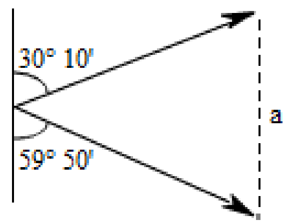
14. The angle of elevation from a point on the ground to the top of a pyramid is 31° 40′. The angle of elevation from a point 143 *ft* farther back to the top of the pyramid is 14° 50′. Find the height of the pyramid.



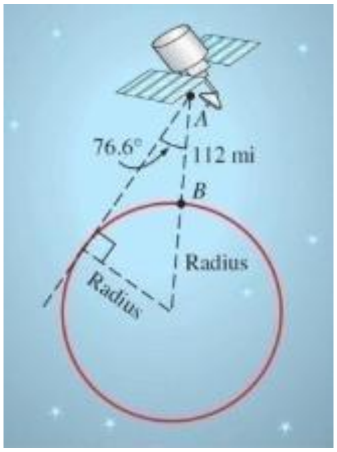
15. In one area, the lowest angle of elevation of the sun in winter is 21° 16′. Find the minimum distance, ***x***, that a plant needing full sun can be placed from a fence 4.41 *ft* high.



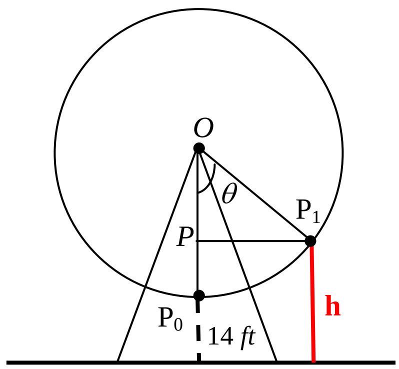
16. A ship leaves its port and sails on a bearing of N 30° 10′ E, at speed 29.4 mph. Another ship leaves the same port at the same time and sails on a bearing of S 59° 50′ E, at speed 17.1 mph. Find the distance between the two ships after 2 hrs.



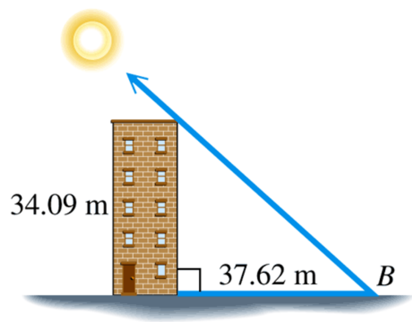
17. Suppose the figure below is exaggerated diagram of a plane flying above the earth. If the plane is 4.55 miles above the earth and the radius of the earth is 3,960 miles, how far is it from the plane to the horizon? What is the measure of angle A?



18. The Ferry wheel has a 250 feet diameter and 14 feet above the ground. If θ is the central angle formed as a rider moves from position P_0 to position P_1, find the rider's height above the ground h when θ is 45°.



19. The length of the shadow of a building 34.09 *m* tall is 37.62 *m*. Find the angle of the elevation of the sun.

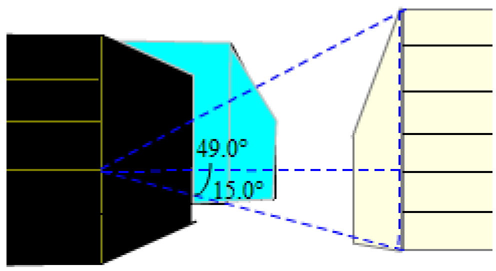


20. San Luis Obispo, California is 12 miles due north of Grover Beach. If Arroyo Grande is 4.6 miles due east of Grover Beach, what is the bearing of San Luis Obispo from Arroyo Grande?

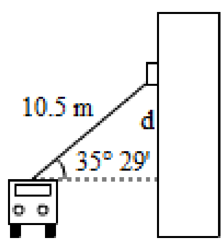
21. The bearing from A to C is S 52° E. The bearing from A to B is N 84° E. The bearing from B to C is S 38° W. A plane flying at 250 mph takes 2.4 hours to go from A to B. Find the distance from A to C.

22. A man wondering in the desert walks 2.3 miles in the direction S 31° W. He then turns 90° and walks 3.5 miles in the direction N 59° W. At that time, how far is he from his starting point, and what is his bearing from his starting point?

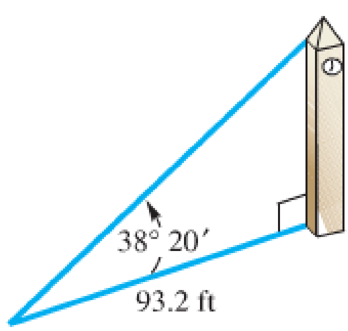
23. From a window 31.0 *ft.* above the street, the angle of elevation to the top of the building across the street is 49.0° and the angle of depression to the base of this building is 15.0°. Find the height of the building across the street.



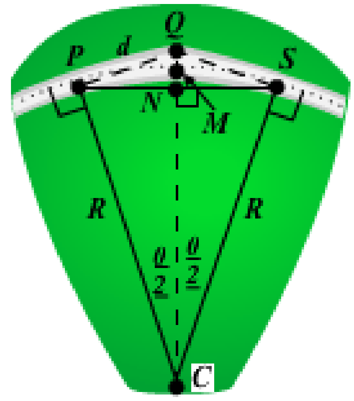
24. A 10.5-*m* fire truck ladder is leaning against a wall. Find the distance ***d*** the ladder goes up the wall (above the fire truck) if the ladder makes an angle of 35° 29′ with the horizontal.



25. The angle of elevation from a point 93.2 ft from the base of a tower to the top of the tower is 38° 20′. Find the height of the tower.



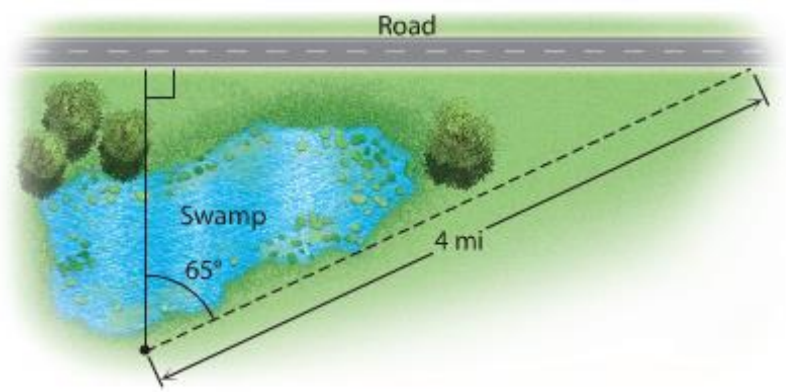
26. A basic curve connecting two straight sections of road is often circular. In the figure, the points ***P*** and ***S*** mark the beginning and end of the curve. Let ***Q*** be the point of intersection where the two straight sections of highway leading into the curve would meet if extended. The radius of the curve is ***R***, and the central angle denotes how many degrees the curve turns.



a) If ***R*** = 965 ft. and $\theta = 37°$, find the distance d between P and ***Q***.

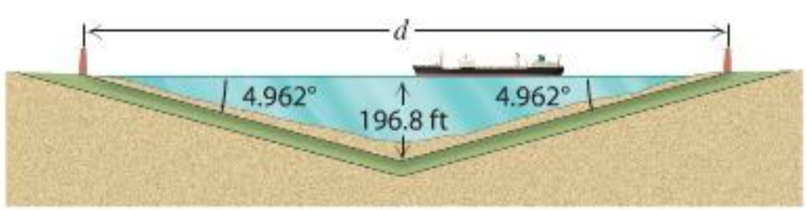
b) Find an expression in terms of ***R*** and θ for the distance between points ***M*** and ***N***.

27. Jane was hiking directly toward a long straight road when she encountered a swamp. She turned 65° to the right and hiked 4 mi in that direction to reach the road. How far was she form the road when she encountered the swamp?

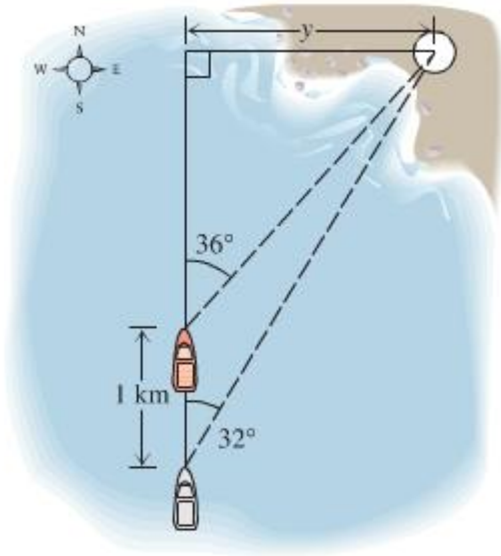


28. From a highway overpass, 14.3 m above the road, the angle of depression of an oncoming car is measured at 18.3°. How far is the car from a point on the highway directly below the observer?

29. A tunnel under a river is 196.8 ft. below the surface at its lowest point. If the angle of depression of the tunnel is 4.962° , then how far apart on the surface are the entrances to the tunnel? How long is the tunnel?

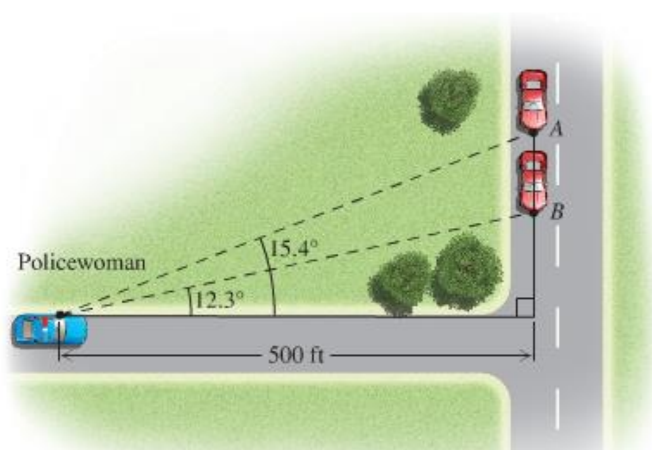


30. A boat sailing north sights a lighthouse to the east at an angle of 32° from the north. After the boat travels one more kilometer, the angle of the lighthouse from the north is 36°. If the boat continues to sail north, then how close will the boat come to the lighthouse?

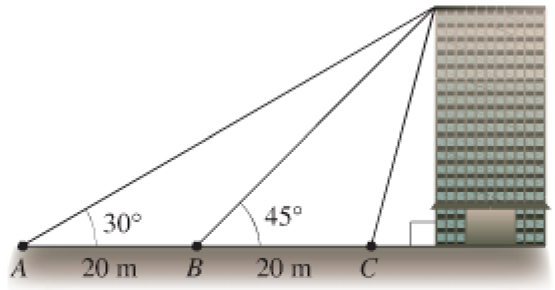


31. The angle of elevation of a pedestrian crosswalk over a busy highway is 8.34°, as shown in the drawing. If the distance between the ends of the crosswalk measured on the ground is 342 ft., then what is the height h of the crosswalk at the center?

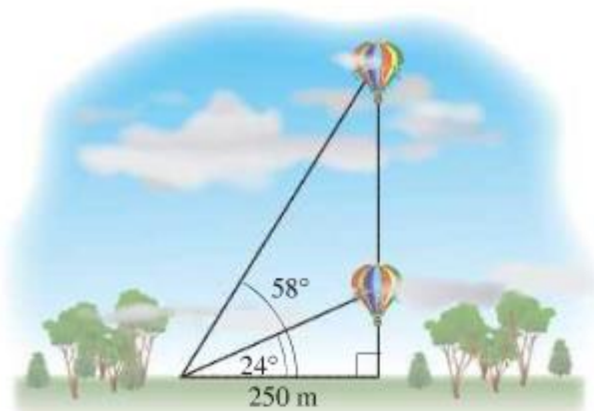
32. A policewoman has positioned herself 500 ft. from the intersection of two roads. She has carefully measured the angles of the lines of sight to points A and B. If a car passes from A to B is 1.75 sec and the speed limit is 55 mph, is the car speeding? (Hint: Find the distance from B to A and use R = D/T)



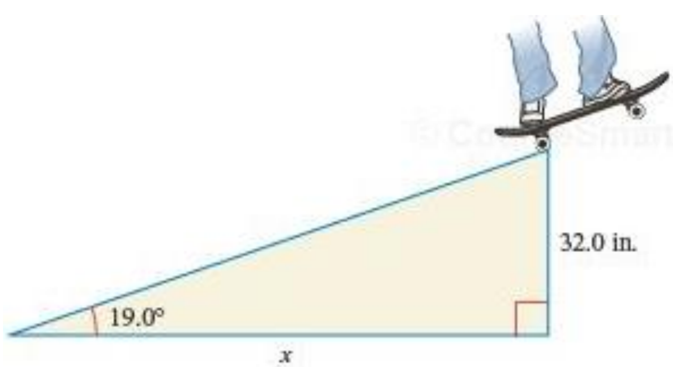
33. From point A the angle of elevation to the top of the building is 30°. From point B, 20 meters closer to the building, the angle of elevation is 45°. Find the angle of elevation of the building from point C, which is another 20 meters closer to the building.



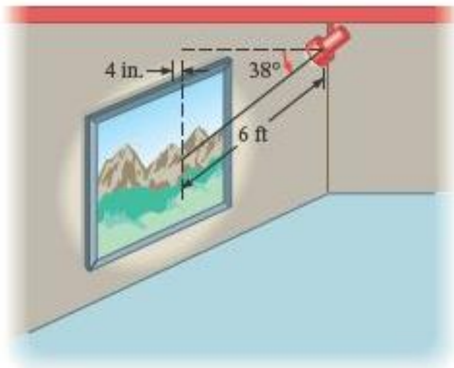
34. A hot air balloon is rising upward from the earth at a constant rate. An observer 250 m away spots the balloon at an angle of elevation of 24°. Two minutes later the angle of elevation of the balloon is 58°. At what rate is the balloon ascending?



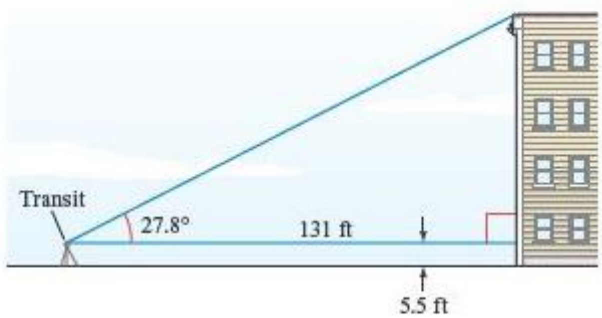
35. A skateboarder wishes to build a jump ramp that is inclined at a 19° angle and that has a maximum height of 32.0 inches. Find the horizontal width *x* of the ramp.



36. For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 ft from the piece of art and that the angle of depression of the light be 38°. How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 inches from the wall.



37. A surveyor determines that the angle of elevation from a transit to the top of a building is 27.8°. The transit is positioned 5.5 feet above ground level and 131 feet from the building. Find the height of the building to the nearest tenth of a foot.



38. From a point A on a line from the base of the Washington Monument, the angle of elevation to the top of the monument is 42.0°. From a point 100 ft away from A and on the same line, the angle to the top is 37.8°. Find the height, to the nearest foot, of the Monument.