$n \geq 1$ Evaluate: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)\ldots(k+n)}$

$n = 1 \quad \Rightarrow \quad \frac{k}{k+1}$ $\qquad \frac{k!}{n!\,(k+n)!} \; \frac{1}{n}$

$n = 2 \quad \Rightarrow \quad \frac{\frac{1}{2}k(k+3)}{2!(k+1)(k+2)} = \frac{k(k+3)}{2\;\;2!(k+1)(k+2)}$ $\qquad \frac{k!}{n!\,(k+n)!} \; \frac{1}{n} \; k(k+3)$

$n = 3 \quad \Rightarrow \quad \frac{\frac{1}{3}k(k^2+6k+11)}{3!(k+1)(k+2)(k+3)} = \frac{k(k^2+6k+11)}{3\;\;3!(k+1)(k+2)(k+3)}$ $\qquad \frac{k!}{n!\,(k+n)!} \; \frac{1}{n} \; k(k^2+6k+11)$

$n = 4 \quad \Rightarrow \quad \frac{\frac{1}{4}k(k+5)(k^2+5k+10)}{4!(k+1)(k+2)(k+3)(k+4)} = \frac{k(k+5)(k^2+5k+10)}{4\;\;4!(k+1)(k+2)(k+3)(k+4)}$ $\qquad \frac{k!}{n!\,(k+n)!} \; \frac{1}{n} \; k(k+5)(k^2+5k+10)$