1/ $$\int \sin^4 x\,dx = \frac{1}{4}\int (1-\cos 2x)^2\,dx$$

$$= \frac{1}{4}\int \Big(1 - 2\cos 2x + \overbrace{\tfrac{1}{2} + \tfrac{1}{2}\cos 4x}^{\cos^2 2x}\Big)\,dx$$

$$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x\right)dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x\right)$$

$$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

#2/ $$\int \cos^7 x\,dx = \int (\cos^2 x)^3 \cos x\,dx$$

$$= \int (1-\sin^2 x)^3\,d(\sin x)$$

$$= \int (1 - 3\sin^2 x + 3\sin^4 x - \sin^6 x)\,d(\sin x)$$

$$= \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$$

3/ $$\int_0^{\pi/2} \cos^{12} x\,dx = \frac{1}{2}\,\frac{3}{4}\,\frac{5}{6}\,\frac{7}{8}\,\frac{9}{10}\,\frac{11}{12}\,\frac{\pi}{2}$$

$$= \frac{231}{2^{11}}\,\pi$$

4/ $$\int_0^{\pi/2} \cos^{15} x\,dx = \frac{2}{3}\,\frac{4}{5}\,\frac{6}{7}\,\frac{8}{9}\,\frac{10}{11}\,\frac{12}{13}\,\frac{14}{15}$$

$$= \frac{2^{11}}{6{,}435}$$

5/ $\int \sqrt{9-4x^2}\,dx$

$2x = 3\sin\theta$ $\quad \sqrt{9-4x^2} = 3\cos\theta$

$dx = \frac{3}{2}\cos\theta\,d\theta$

$$\int \sqrt{9-4x^2}\,dx = \int 3\cos\theta\left(\tfrac{3}{2}\cos\theta\right)d\theta$$

$$= \frac{9}{2}\int \cos^2\theta\,d\theta$$

$$= \frac{9}{4}\int (1+\cos 2\theta)\,d\theta$$

$$= \frac{9}{4}\left(\theta + \frac{1}{2}\sin 2\theta\right)$$

$$= \frac{9}{4}\left(\theta + \sin\theta\cos\theta\right)$$

$$= \frac{9}{4}\sin^{-1}\frac{2x}{3} + \frac{9}{4}\,\frac{2x}{3}\,\frac{\sqrt{9-4x^2}}{3} + C$$

$$= \boxed{\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right) + \frac{1}{2}x\sqrt{9-4x^2} + C}$$

6/ $\int \frac{dx}{\sqrt{x^2-25}}$

$x = 5\sec\theta$ $\quad \sqrt{x^2-25} = 5\tan\theta$

$dx = 5\sec\theta\tan\theta\,d\theta$

$$\int \frac{dx}{\sqrt{x^2-25}} = \int \frac{5\sec\theta\tan\theta\,d\theta}{5\tan\theta}$$

$$= \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \boxed{\ln\left|\frac{x}{5} + \frac{\sqrt{x^2-25}}{5}\right| + C}$$

7/ $\int \frac{dx}{x^2\sqrt{x^2+36}}$

$x = 6\tan\theta$ $\qquad \sqrt{x^2+36} = 6\sec\theta$

$dx = 6\sec^2\theta\,d\theta$

$$\int \frac{dx}{x^2\sqrt{x^2+36}} = \int \frac{6\sec^2\theta\,d\theta}{36\tan^2\theta\,(6\sec\theta)}$$

$$= \frac{1}{36}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= \frac{1}{36}\int \frac{1}{\cos\theta}\,\frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$= \frac{1}{36}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$$= \frac{1}{36}\int \frac{d(\sin\theta)}{\sin^2\theta}$$

$$= -\frac{1}{36}\,\frac{1}{\sin\theta} \qquad \frac{\tan\theta}{\sec\theta} = \sin\theta$$

$$= -\frac{1}{36}\,\frac{\sec\theta}{\tan\theta}$$

$$= -\frac{1}{36}\,\frac{\sqrt{x^2+36}}{6}\cdot\frac{6}{x}$$

$$\boxed{= -\frac{1}{36}\,\frac{\sqrt{x^2+36}}{x} + C}$$