Adjacency matrix of a graph: Square matrix with $a_{ij} = 1$ when there is an edge from node ***i*** to node ***j***; otherwise $a_{ij} = 0$, $A = A^T$ for an undirected graph.

Affine Transformation: $T(v) = Av + v_0$ = linear transformation plus shift.

Back substitution: Upper triangular systems are solved in reverse order x_n *to* x_1.

Basis for V: Independent vectors $v_1, \ldots, v_n$ whose linear combinations give every ***v*** in V. A vector space has many bases.

Block matrix: A matrix can be partitioned into matrix blocks, by cuts between rows and/or between columns, ***Block multiplication*** of ***AB*** is allowed if the block shapes permit (the columns of ***A*** and rows of ***B*** must be matching blocks).

Cayley-Hamilton Theorem: $p(\lambda) = \det(A - \lambda I) = \text{zero matrix}$.

Change of basis matrix M: The old basis vectors v_j are combinations $\sum m_{ij} w_i$ of the new basis vectors. The coordinates of $c_1 v_1 + \ldots + c_n v_n = d_1 w_1 + \ldots + d_n w_n$ are related by $d = Mc$. (For n = 2 set $v_1 = m_{11} w_1 + m_{21} w_2$, $v_2 = m_{12} w_1 + m_{22} w_2$.)

Characteristic equation: $\det(A - \lambda I) = 0$. The n roots are the eigenvalues of ***A***.

Cholesky factorization: $A = CC^T = \left(L\sqrt{D}\right)\left(L\sqrt{D}\right)^T$ for positive eigenvalues of ***A***.

Circulant matric C: Constant diagonals wrap around as in cyclic shift S. Every Circulant is $c_0 I + c_1 S + \ldots + c_{n-1} S_{n-1}$. $C\boldsymbol{x}$ = ***convolution c*x***. Eigenvectors in F.

Cofactor C_{ij} ***:*** Remove row ***i*** and column ***j***; multiply the determinant by $(-1)^{i+j}$

Column picture of Ax = b: The vector ***b*** becomes a combination of the columns of ***A***. The system is solvable only when ***b*** is in the column space $C(\boldsymbol{A})$.

Column space $C(A)$: consists of all linear combinations of the columns. The combinations are all possible vectors Ax.

Commuting matrices $AB = BA$***:*** If diagonalizable, they share ***n*** eigenvectors.

Companion matrix: Put $c_1, \ldots, c_n$ in row n and put n – 1 1's along diagonal 1. Then

$$\det(A - \lambda I) = \pm\left(c_1 + c_2\lambda + c_3\lambda^2 + \ldots\right)$$

Complete solution: $x = x_p + x_n$ to $Ax = b$. $\left(\textit{Particular } x_p\right) + \left(x_n \textit{ in nullspace}\right)$

Complex conjugate: $\bar{z} = a - ib$ for any complex number $z = a + ib$. Then

$$z\bar{z} = |z|^2 \Rightarrow (a-ib)(a+ib) = a^2 + b^2$$

Covariance matrix $\sum$***:*** When random variables x_i have mean = average value = 0, their covariances $\sum_{ij}$ are the averages of $x_i x_j$. With means $\bar{x}_i$, the matrix $\sum$ = mean of $(x-\bar{x})(x-\bar{x})^T$ is positive (semi) definite; it is diagonal if the x_i are independent.

Cramer's Rule for Ax = b: B_j has ***b*** replacing column ***j*** of *A*, and $x_i = \dfrac{|B_j|}{|A|}$.

Cross product $\boldsymbol{u} \times \boldsymbol{v}$ ***in*** $\boldsymbol{R}^3$***:*** Vector perpendicular to ***u*** and ***v***, length $\|u\|\|v\||\sin\theta|$ = parallelogram area, computed as the "determinant" of $\left[\, \boldsymbol{i} \;\; \boldsymbol{j} \;\; \boldsymbol{k};\;\; u_1 \;\; u_2 \;\; u_3;\;\; v_1 \;\; v_2 \;\; v_3 \,\right]$

Diagonal matrix D: $d_{ij} = 0 \text{ if } i \neq j$. ***Block diagonal***: zero outside square blocks D_{ij}.

Dimension of a vector space: dim(***V***) = number of vectors in any basis for ***V***.

Dot Product: $x^T y = x_1 y_1 + \ldots + x_n y_n$. Complex dot product is $\bar{x}^T y$. Perpendicular vectors have zero dot product. $(AB)_{ij} =$ (row *i* of *A*)**.**(column *j* of *B*)

Echelon matrix U: The first nonzero entry (the pivot) in each row comes after the pivot in the previous row. All zero rows come last.

Eigenvalue λ and eigenvector x: $Ax = \lambda x \text{ with } x \neq 0$ so $\det(A - \lambda I) = 0$

Elimination: A sequence of row operations that reduces ***A*** to an upper triangular *U* or to the reduced form *R* = ***rref***(*A*). Then *A* = *LU* with multipliers ℓ_{ij} in *L*, or *PA* = *LU* with row exchanges in *P*, or *EA* = *R* with an invertible *E*.

Factorization: *A* = *LU*. If elimination takes *A* to *U* without row exchanges, then the lower triangular *L* with multipliers ℓ_{ij} $\left(\text{and } \ell_{ii} = 1\right)$ brings *U* back to *A*.

Fibonacci numbers: 0, 1, 1, 2, 3, 5, … satisfy $F_n = F_{n-1} + F_{n-2} = \dfrac{\lambda_1^n - \lambda_2^n}{\lambda_1 - \lambda_2}$. Growth rate $\lambda_1 = \frac{1+\sqrt{5}}{2}$ is the largest eigenvalue of the Fibonacci matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Four Fundamental subspaces of A $= C(A),\; N(A), C\left(A^T\right),\; N\left(A^T\right)$.

Free Columns of A: Columns without pivots; combinations of earlier columns.

Free variables x_i **:** Column i has no pivot in elimination. We can give the $n - r$ free variables any values, the $Ax = b$ determines the r pivot variables (if solvable!).

Full column rank $r = n$. Independent columns, $N(A) = \{0\}$, no free variables.

Full row rank $r = m$. Independent rows, at least one solution to $Ax = b$, column space is all of $\boldsymbol{R}^m$. Full rank means full column rank or full column rank or full row rank.

Fundamental Theorem: the nullspace $N(A)$ and row space $C\left(A^T\right)$ are orthogonal complements (perpendicular subspaces of $\boldsymbol{R}^n$ with dimensions r and $n - r$) from $Ax = \mathbf{0}$. Applied to A^T, the column space $\boldsymbol{C}(\boldsymbol{A})$ is the orthogonal complement of $N\left(A^T\right)$.

Independent vectors: $v_1, \ldots, v_n$. No combination $c_1v_1 + \ldots + c_nv_n = zero$ vector unless all $c_i = 0$. If the v's are the columns of A, the only solution to $Ax = \mathbf{0}$ is $\boldsymbol{x} = \mathbf{0}$.

Least squares solution $\hat{x}$: The vector $\hat{x}$ that minimizes the error $\|\boldsymbol{e}\|^2$ solves $A^T A\hat{x} = A^T b$. Then $e = b - A\hat{x}$ is orthogonal to all columns of $\boldsymbol{A}$.

Length $\|\boldsymbol{x}\|$: Square root of $x^T x$ (Pythagoras in n dimensions).

Linear combination $c\boldsymbol{v} + d\boldsymbol{w}$ *or* $\sum c_j v_j$. Vector addition and scalar multiplication.

Linear Transformation T: Each vector v in the input space transforms to $T(\boldsymbol{v})$ in the output space, and linearity requires $T(c\boldsymbol{v} + d\boldsymbol{w}) = cT(\boldsymbol{v}) + dT(\boldsymbol{w})$.

Linearly dependent $v_1, \ldots, v_n$. A combination other than all $c_i = 0$ gives $\sum c_i v_i = \mathbf{0}$

Linearly independent when the only solution to $Ax = 0$ is $x = 0$. ***No other combination*** Ax ***of the columns gives the zero vector***.

Nullspace of A consists of all solutins to $Ax = 0$. These solution vectors x are in $\boldsymbol{R}^n$. The Nullspace containing all solutions is denoted by $N(A)$ *or* $NS(A)$. $\left\{\vec{x} \in \boldsymbol{R}^n \mid Ax = 0\right\}$ is the nullspace of A, $NS(A)$ (Can also be called ***Kernel*** of A: $Ker(A)$)

Particular solution x_p Any solution to $Ax = b$; often x_p has free variables = 0.

Permutation matrix P. There are $\boldsymbol{n}$**!** orders of 1, …, n; the **n!** P's have the rows of I in those orders. PA puts the rows of A in the same order. P is a product of row exchanges P_{ij}; P is *even* or *odd* (det$P$ = 1 or -1) based on the number of exchanges.

Pivot columns of A: Columns that contain pivots after row reduction; not combinations of earlier columns. The pivot columns are a basis for the column space.

Pivot d: The diagonal entry (first nonzero) when a row is used in elimination.

Rank of a matrix A (***m*** by ***n***) is the number of ***nonzero rows*** in the row-reduced echelon form of A. (it is the number of pivot). $\boxed{rank(A) = r}$

R**educed **R**ow **E**chelon **F**orm* (rref***): is a matrix (R) with each pivot column has only one nonzero entry (the pivots which is always 1).

Row space $C\left(A^T\right)$ = all combinations of rows of A. Column vectors by convention.

Schwarz inequality $|v.w| = \|v\|.\|w\|$: Then $\left|v^T Aw\right|^2 \le \left(v^T Av\right)\left(w^T AW\right)$ $if\ A = C^T C$

***Singular matrix A*:** A square matrix that has no inverse: $\det(A) = 0$.

Spanning set $v_1,\ \ldots,\ v_m$ ***for V*:** Every vector in V is a combination of $v_1,\ \ldots,\ v_m$.

Subspace: of a vector space is a set of vectors (including 0) that satisfies two requirements: if v and w are vectors in the subspace and c is any scalar, then $v + w$ is in the subspace and cv is in the subspace

Symmetric matrix A: The transpose is $A^T = A$, and $a_{ij} = a_{ji}$. A^{-1} is also symmetric. All matrices of the form $R^T R$, LDL^T and $Q\Lambda Q^T$ are symmetric. Symmetric matrices have real eigenvalues in Λ and orthonormal eigenvectors in Q.

Trace of A: = sum of diagonal entries = sum of eigenvalues of ***A***. $Tr(AB) = Tr(BA)$.

Transpose matrix** A^T **: Entries $A^T_{ij} = A_{ji}$. A^T is n by m, $A^T A$ is square, symmetric, positive semi-definite. The transposes of AB and A^{-1} are $B^T A^T$ and $\left(A^T\right)^{-1}$