

$$\begin{aligned}
 \sum_{k=0}^{19} \frac{k-3}{4} &= \frac{1}{4} \left( \sum_{k=0}^{19} k - \sum_{k=0}^{19} 3 \right) \\
 &= \frac{1}{4} \left[ \frac{19(20)}{2} - 3(19-0+1) \right] \\
 &= \frac{1}{4} (190 - 60) \\
 &= \underline{\underline{\frac{65}{2}}}
 \end{aligned}$$

$$\begin{aligned}
 \sum_{k=2}^{50} (2,000 - 3k) &= \sum_{k=2}^{50} 210^3 - 3 \sum_{k=2}^{50} k \\
 1 &= 50 - 2 + 1 \\
 &= (50 - 2 + 1) 210^3 - 3 \left( \frac{50(51)}{2} - 1 \right) \\
 &= 98,000 - 3,822 \\
 &= \underline{\underline{94,178}}
 \end{aligned}$$

$$= \frac{49}{2} (1994 + 1850)$$

$$S_n = \frac{1}{2} (a_1 + a_n)$$

$$\cdot \overline{78} = .78787878$$

$$= .78 + .0078 + .000078 + \dots$$

$$a = .78$$

$$r = \frac{.0078}{.78} = .01$$

$$= \frac{1}{100}$$

$$= \frac{78 \times 10^{-4}}{78 \times 10^{-2}} = 10^{-2}$$

$$\cdot \overline{78} = \frac{.78}{1 - \frac{1}{100}}$$

$$= .78 \frac{1}{\frac{99}{100}}$$

$$100(.78)$$

$$= \frac{78}{99}$$

$$= \frac{26}{33}$$


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$$4 + 8 + 12 + \dots + 4n = 2n(n+1)$$

For  $n=1 \Rightarrow 4 \stackrel{?}{=} 2(1)(2)$   
 $4 = 4 \checkmark \quad P_1 \text{ is true}$

Assume  $P_k : 4 + 8 + \dots + 4k = 2k(k+1)$  is true.

Is  $P_{k+1} : 4 + \dots + 4k + 4(k+1) = 2(k+1)(k+2)$ ?

$$4 + \dots + 4k + 4(k+1) = 2k(k+1) + 4(k+1)$$

$$= 2(k+1)(k+2) \checkmark$$

$P_{k+1}$  is also true.

$\therefore$  By the mathematical induction, the proof is completed

$$1 + 5 + 9 + \dots + (4n-3) = n(2n-1)$$

For  $n=1 \Rightarrow 1 \stackrel{?}{=} 1(2-1)$   
 $1 = 1 \checkmark \quad P_1 \text{ is true}$

Let  $P_k : 1 + 5 + \dots + (4k-3) = k(2k-1)$  is true

Is  $P_{k+1} : 1 + \dots + (4k-3) + (4(k+1)-3) \stackrel{?}{=} (k+1)(2(k+1)-1)$   
 $1 + \dots + (4k-3) + (4k+1) \stackrel{?}{=} (k+1)(2k+1)$

$$1 + \dots + (4k-3) + (4k+1) = k(2k-1) + (4k+1)$$

$$= 2k^2 - k + 4k + 1$$

$$= 2k^2 + 3k + 1$$

$$= (k+1)(2k+1) \checkmark$$

$P_{k+1}$  is also true.

$\therefore$  By the mathematical induction, the proof is completed



$$\frac{x}{x^2 - 2x - 3} = \frac{A}{x+1} + \frac{B}{x-3} \quad (x+1)$$

$$x = A(x-3) + B(x+1)$$

$$x^1 \mid A + B = 1$$

$$x^0 \mid +3A + B = 0$$

$$\begin{aligned} 4A &= 1 \Rightarrow A = \frac{1}{4} \\ B &= -3A \\ &= -\frac{3}{4} \end{aligned}$$

$$\frac{x}{x^2 - 2x - 3} = \frac{1/4}{x+1} + \frac{-3/4}{x-3}$$

$$\frac{3x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$3x = A(x-1) + B(x+2)$$

$$x^1 \mid A + B = 3 \rightarrow A = 2$$

$$x^0 \mid -A + 2B = 0$$

$$B = 1$$

$$\frac{3x}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$$

$$\frac{x^n + \dots}{x^n + \dots}$$

← degree same or larger.

38 Given <sup>Truck</sup>  $h=14, w=10$

$$a = 25$$

$$b = 20$$

$$\frac{x^2}{25^2} + \frac{y^2}{20^2} = 1$$

$$\frac{y^2}{20^2} = 1 - \frac{x^2}{25^2} = \frac{25^2 - x^2}{25^2} \quad | \quad x=5$$

$$y^2 = \frac{20^2}{25^2} (25^2 - 25)$$

$$y = \frac{20}{25} \sqrt{25(25-1)}$$

$$\frac{4}{5} \cdot 5$$

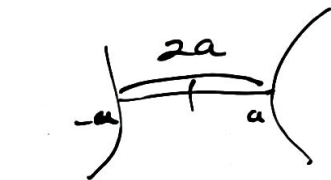
$$= 4\sqrt{24}$$

$$= 8\sqrt{6}$$

$$(14)^2 + (8\sqrt{6})^2$$

$$196 + 384$$

Truck will clear



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

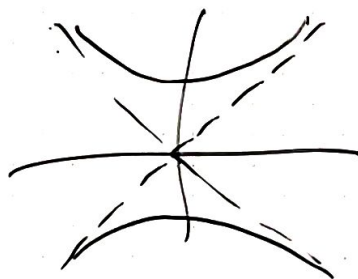
$$\frac{y^2}{a^2} = \frac{x^2}{b^2}$$

$$y = \pm \frac{a}{b} x$$

$$\frac{x^2}{a^2} = \frac{y^2}{b^2}$$

$$\frac{b^2 x^2}{a^2} = y^2$$

$$y = \pm \frac{b}{a} x$$



5.5  
~~11~~  $\{C_n\} = \left\{ \frac{(-1)^n}{(n+1)(n+2)} \right\}$

$$n=1 \rightarrow C_1 = \frac{-1}{2(3)} = -\frac{1}{6}$$

$$n=2 \rightarrow C_2 = \frac{1}{3(4)} = \frac{1}{12}$$

$$n=3 \rightarrow C_3 = \frac{(-1)^3}{(3+1)(3+2)} = -\frac{1}{20}$$

$$n=4 \rightarrow C_4 = \frac{(-1)^4}{5(6)} = \frac{1}{30}$$

$$n=8 \rightarrow C_8 = \frac{(-1)^8}{9(10)} = \frac{1}{90}$$

5.6 #22

Arithmetic

$$a_{20}; a_9 = -5, a_{15} = 31 \quad \left| \begin{array}{l} a_n = a_1 + (n-1)d \\ d = \frac{y_2 - y_1}{x_2 - x_1} \end{array} \right.$$

$$d = \frac{31 + 5}{15 - 9} = 6$$

$$d = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a_9 = a_1 + 8(6) = -5$$

$$\begin{aligned} a_1 &= -5 - 48 \\ &= -53 \end{aligned}$$

$$\begin{aligned} a_{20} &= -53 + 19(6) \\ &= 61 \end{aligned}$$



Geometric Seq.

66  $a_7 : a_2 = 3 \quad a_3 = -\sqrt{3}$

$$a_n = a_1 r^{n-1}$$

$$r = -\frac{\sqrt{3}}{3}$$

$$a_2 = a_1 \left(-\frac{\sqrt{3}}{3}\right) = 3$$

$$a_1 = -\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -3\sqrt{3}$$

$$a_7 = -3\sqrt{3} \left(-\frac{\sqrt{3}}{3}\right)^6$$

$$= -3\sqrt{3} \frac{3^3}{3^6}$$

$$= -\frac{\sqrt{3}}{9}$$

$$\rightarrow \frac{3^4}{3^6}$$

$$\rightarrow 3^2$$

$$\frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{x}$$

$$\frac{9\sqrt{3}}{3}$$

$$\frac{9}{\sqrt{3}} = 9 \frac{\sqrt{3}}{3}$$

$$\sqrt{8} = (2^{\frac{1}{2}})^6$$

$$r = \left(\frac{y_2}{y_1}\right)^{\left(\frac{1}{x_2 - x_1}\right)}$$

107  $\sum_{n=1}^{\infty} 2 \left(\frac{2}{3}\right)^{n-1} = \frac{2}{1 - \frac{2}{3}}$

$$r = \left|\frac{2}{3}\right| < 1$$

6

109  $\sum_{n=1}^{\infty} 3 \left(\frac{3}{2}\right)^n = \infty$

$$r = \frac{3}{2} \geq 1$$