

## ***Solution***      **Section 1.3 – Rational Functions**

### ***Exercise***

Determine all asymptotes of the function:  $y = \frac{3x}{1-x}$

### **Solution**

**VA:**  $x = 1$

**HA:**  $y = -3$

**Hole:**  $n/a$

**Oblique asymptote:**  $n/a$

### ***Exercise***

Determine all asymptotes of the function:  $y = \frac{x^2}{x^2 + 9}$

### **Solution**

**VA:**  $n/a$      $x^2 + 9 \neq 0$

**HA:**  $y = 1$

**Hole:**  $n/a$

**Oblique asymptote:**  $n/a$

### ***Exercise***

Determine all asymptotes of the function:  $y = \frac{x-2}{x^2 - 4x + 3}$

### **Solution**

$$x^2 - 4x + 3 = 0 \Rightarrow x = 1, 3$$

$$y = \frac{x}{x^2} \rightarrow 0$$

**VA:**  $x = 1, x = 3$

**HA:**  $y = 0$

**Hole:**  $n/a$

**Oblique asymptote:**  $n/a$

### ***Exercise***

Determine all asymptotes of the function:  $y = \frac{3}{x-5}$

### **Solution**

**VA:**  $x = 5$

**HA:**  $y = 0$

**Hole:**  $n/a$

**Oblique asymptote:**  $n/a$

### Exercise

Determine all asymptotes of the function:  $y = \frac{x^3 - 1}{x^2 + 1}$

#### Solution

VA: none

HA: none

Hole:  $n/a$

Oblique asymptote:  $y = x$

$$\begin{array}{r} x^2 + 1 \overline{) x^3 - 1} \\ \underline{-x^3 - x} \phantom{-1} \\ -x - 1 \end{array}$$
$$y = x - \frac{x+1}{x^2+1}$$

### Exercise

Determine all asymptotes of the function:  $y = \frac{x^3 + 3x^2 - 2}{x^2 - 4}$

#### Solution

VA:  $x = \pm 2$

HA:  $n/a$

Hole:  $n/a$

Oblique asymptote:  $y = x + 3$

$$\begin{array}{r} x + 3 \phantom{-2} \\ x^2 - 4 \overline{) x^3 + 3x^2 - 2} \\ \underline{-x^3 + 4x} \phantom{-2} \\ 3x^2 + 4x - 2 \\ \underline{-3x^2} \phantom{+12} \\ 4x + 10 \end{array}$$
$$y = x + 3 + \frac{4x+10}{x^2-4}$$

### Exercise

Determine all asymptotes of the function:  $y = \frac{3x^2 - 27}{(x+3)(2x+1)}$

#### Solution

$$y = \frac{3x^2 - 27}{(x+3)(2x+1)} = \frac{3(x^2 - 9)}{(x+3)(2x+1)} = \frac{3(x+3)(x-3)}{(x+3)(2x+1)} = \frac{3(x-3)}{(2x+1)}$$

VA:  $x = -3, -\frac{1}{2}$

HA:  $y = \frac{3}{2}$

Hole:  $n/a$

Oblique asymptote:  $n/a$

### Exercise

Determine all asymptotes of the function:  $y = \frac{x-3}{x^2-9}$

#### Solution

$$x^2 - 9 = 0 \rightarrow \boxed{x = \pm 3}$$

$$y = \frac{x-3}{(x-3)(x+3)} \\ = \frac{1}{x+3}$$

VA:  $x = 3$

HA:  $y = 0$

Hole:  $x = 3 \rightarrow y = \frac{1}{6}$

Oblique asymptote:  $n/a$

### Exercise

Determine all asymptotes of the function:  $y = \frac{6}{\sqrt{x^2-4x}}$

#### Solution

$$x^2 - 4x = 0$$

$$\Rightarrow x(x-4) = 0 \rightarrow \boxed{x = 0, 4}$$

VA:  $x = 0, x = 4$

HA:  $y = 0$

Hole:  $n/a$

Oblique asymptote:  $n/a$

### Exercise

Determine all asymptotes of the function:  $y = \frac{5x-1}{1-3x}$

#### Solution

VA:  $x = \frac{1}{3}$

HA:  $y = -\frac{5}{3}$

Hole:  $n/a$

Oblique asymptote:  $n/a$

### Exercise

Determine all asymptotes of the function:  $f(x) = \frac{2x-11}{x^2+2x-8}$

#### Solution

VA:  $x = 2, x = -4$

HA:  $y = 0$

Hole:  $n/a$

Oblique asymptote:  $n/a$

### Exercise

Determine all asymptotes of the function:  $f(x) = \frac{x^2 - 4x}{x^3 - x}$

#### Solution

$$\begin{aligned} f(x) &= \frac{x(x-4)}{x(x^2-1)} \\ &= \frac{x-4}{x^2-1} \end{aligned}$$

**VA:**  $x = -1, x = 1$       **HA:**  $y = 0$

**Hole:**  $x = 0 \rightarrow y = 4$       **Oblique asymptote:**  $n / a$

### Exercise

Determine all asymptotes of the function:  $f(x) = \frac{x-2}{x^3-5x}$

#### Solution

**VA:**  $x = 0, x = \pm\sqrt{5}$       **HA:**  $y = 0$

**Hole:**  $n / a$       **Oblique asymptote:**  $n / a$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{4x}{x^2 + 10x}$

#### Solution

$$x^2 + 10x = 0 \rightarrow x = 0, -10$$

**Domain:**  $(-\infty, -10) \cup (-10, 0) \cup (0, \infty)$

$$\begin{aligned} f(x) &= \frac{4x}{x(x+10)} \\ &= \frac{4}{x+10} \end{aligned}$$

**VA:**  $x = -10$       **HA:**  $y = 0$

**Hole:**  $x = 0 \rightarrow y = \frac{4}{10} \Rightarrow \text{hole} \left(0, \frac{2}{5}\right)$       **Oblique asymptote:**  $n / a$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{3-x}{(x-4)(x+6)}$

#### Solution

$$\text{VA: } x = -6 \text{ and } x = 4$$

$$\text{HA: } y = 0$$

$$\text{Hole: } n/a$$

$$\text{Oblique asymptote: } n/a$$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{x^3}{2x^3 - x^2 - 3x}$

#### Solution

$$2x^3 - x^2 - 3x = x(2x^2 - x - 3) = 0 \rightarrow x = 0, -1, \frac{3}{2}$$

$$\begin{aligned} f(x) &= \frac{x^3}{2x^3 - x^2 - 3x} \\ &= \frac{x^3}{x(2x^2 - x - 3)} \\ &= \frac{x^2}{2x^2 - x - 3} \end{aligned}$$

$$\text{VA: } x = -1 \text{ and } x = \frac{3}{2}$$

$$\text{HA: } y = \frac{1}{2}$$

$$\text{Hole: } x = 0 \rightarrow y = 0 \Rightarrow \text{hole } (0, 0)$$

$$\text{Oblique asymptote: } n/a$$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{3x^2 + 5}{4x^2 - 3}$

#### Solution

$$4x^2 - 3 = 0 \rightarrow x = \pm \frac{\sqrt{3}}{2}$$

$$\text{Domain: } \left(-\infty, -\frac{\sqrt{3}}{2}\right) \cup \left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right) \cup \left(\frac{\sqrt{3}}{2}, \infty\right)$$

$$\text{VA: } x = -\frac{\sqrt{3}}{2} \text{ and } x = \frac{\sqrt{3}}{2}$$

$$\text{HA: } y = \frac{3}{4}$$

$$\text{Hole: } n/a$$

$$\text{Oblique asymptote: } n/a$$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{x+6}{x^3 + 2x^2}$

#### Solution

$$x^3 + 2x^2 = x^2(x+2) = 0 \rightarrow x = 0, -2$$

**Domain:**  $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$

**VA:**  $x = 0$  and  $x = 2$

**HA:**  $y = 0$

**Hole:**  $n/a$

**Oblique asymptote:**  $n/a$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{x^2 + 4x - 1}{x + 3}$

#### Solution

**VA:**  $x = -3$

**HA:**  $n/a$

**Hole:**  $n/a$

**Oblique asymptote:**  $y = x + 1$

$$\begin{array}{r} x+1 \\ x+3 \overline{) x^2 + 4x - 1} \\ \underline{-x^2 - 3x} \phantom{-1} \\ x-1 \\ \underline{-x-3} \\ -4 \end{array}$$
$$f(x) = \frac{x^2 + 4x - 1}{x + 3} = x + 1 - \frac{4}{x + 3}$$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{x^2 - 6x}{x - 5}$

#### Solution

$$x - 5 = 0 \rightarrow x = 5$$

**Domain:**  $(-\infty, 5) \cup (5, \infty)$

$$\begin{aligned} f(x) &= \frac{x^2 - 6x}{x - 5} \\ &= x - 1 - \frac{5}{x - 5} \end{aligned}$$

**VA:**  $x = 5$

**HA:** N/A

**Hole:** N/A

**Oblique asymptote:**  $y = x - 1$

$$\begin{array}{r} x-1 \\ x-5 \overline{) x^2 - 6x} \\ \underline{-x^2 + 5x} \\ -x \\ \underline{x-5} \\ -5 \end{array}$$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{x^3 - x^2 + x - 4}{x^2 + 2x - 1}$

#### Solution

$$x^2 + 2x - 1 = 0 \rightarrow x = -1 \pm \sqrt{2}$$

$$\text{Domain: } (-\infty, -1 - \sqrt{2}) \cup (-1 - \sqrt{2}, -1 + \sqrt{2}) \cup (-1 + \sqrt{2}, \infty)$$

$$\begin{aligned} f(x) &= \frac{x^3 - x^2 + x - 4}{x^2 + 2x - 1} \\ &= x - 3 + \frac{8x - 7}{x^2 + 2x - 1} \end{aligned}$$

$$\text{VA: } x = -1 \pm \sqrt{2}$$

$$\text{HA: } n/a$$

$$\text{Hole: } n/a$$

$$\text{Oblique asymptote: } y = x - 3$$

$$\begin{aligned} x^2 + 2x - 1 &\overline{) \begin{array}{r} x^3 - x^2 + x - 4 \\ -x^3 - 2x^2 + x \\ \hline -3x^2 + 2x - 4 \\ 3x^2 + 6x - 3 \\ \hline 8x - 7 \end{array}} \end{aligned}$$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{4x}{x^2 + 10x}$

#### Solution

$$x^2 + 10x = 0 \rightarrow x = 0, -10 \quad \text{Domain: } (-\infty, -10) \cup (-10, 0) \cup (0, \infty)$$

$$f(x) = \frac{4x}{x(x+10)} = \frac{4}{x+10}$$

$$\text{VA: } x = -10$$

$$\text{HA: } y = 0$$

$$\text{Hole: } x = 0 \rightarrow y = \frac{4}{10} \Rightarrow \text{hole } \left(0, \frac{2}{5}\right)$$

$$\text{Oblique asymptote: } n/a$$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{3-x}{(x-4)(x+6)}$

#### Solution

$$\text{Domain: } (-\infty, -6) \cup (-6, 4) \cup (4, \infty)$$

$$\text{VA: } x = -6 \text{ and } x = 4$$

$$\text{HA: } y = 0$$

**Hole:**  $n/a$

**Oblique asymptote:**  $n/a$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{x^3}{2x^3 - x^2 - 3x}$

#### Solution

$$2x^3 - x^2 - 3x = x(2x^2 - x - 3) = 0 \rightarrow x = 0, -1, \frac{3}{2}$$

$$\text{Domain: } (-\infty, -1) \cup (-1, 0) \cup \left(0, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$$

$$f(x) = \frac{x^3}{2x^3 - x^2 - 3x} = \frac{x^3}{x(2x^2 - x - 3)} = \frac{x^2}{2x^2 - x - 3}$$

$$\text{VA: } x = -1 \text{ and } x = \frac{3}{2} \quad \text{HA: } y = \frac{1}{2}$$

$$\text{Hole: } x = 0 \rightarrow y = 0 \Rightarrow \text{hole } (0, 0)$$

**Oblique asymptote:**  $n/a$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{3x^2 + 5}{4x^2 - 3}$

#### Solution

$$4x^2 - 3 = 0 \rightarrow x = \pm \frac{\sqrt{3}}{2}$$

$$\text{Domain: } \left(-\infty, -\frac{\sqrt{3}}{2}\right) \cup \left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right) \cup \left(\frac{\sqrt{3}}{2}, \infty\right)$$

$$\text{VA: } x = -\frac{\sqrt{3}}{2} \text{ and } x = \frac{\sqrt{3}}{2}$$

$$\text{HA: } y = \frac{3}{4}$$

**Hole:**  $n/a$

**Oblique asymptote:**  $n/a$

### Exercise

Determine all asymptotes of the function  $f(x) = \frac{x+6}{x^3 + 2x^2}$

#### Solution

$$x^3 + 2x^2 = x^2(x+2) = 0 \rightarrow x = 0, -2 \quad \text{Domain: } (-\infty, -2) \cup (-2, 0) \cup (0, \infty)$$

$$\text{VA: } x = 0 \text{ and } x = -2$$

$$\text{HA: } y = 0$$

**Hole:**  $n/a$

**Oblique asymptote:**  $n/a$

### Exercise



Determine all asymptotes of the function  $f(x) = \frac{x^2 + 4x - 1}{x + 3}$

**Solution**

$$x + 3 = 0 \rightarrow x = -3$$

$$\text{Domain: } (-\infty, -3) \cup (-3, \infty)$$

$$x + 3 \overline{) x^2 + 4x - 1}$$

$$\underline{-x^2 - 3x}$$

$$x - 1$$

$$\underline{-x - 3}$$

$$-4$$

$$f(x) = \frac{x^2 + 4x - 1}{x + 3} = x + 1 - \frac{4}{x + 3}$$

$$\text{VA: } x = -3$$

$$\text{HA: } n/a$$

$$\text{Hole: } n/a$$

$$\text{Oblique asymptote: } y = x + 1$$

**Exercise**

Determine all asymptotes of the function  $f(x) = \frac{x^2 - 6x}{x - 5}$

**Solution**

$$x - 5 = 0 \rightarrow x = 5$$

$$\text{Domain: } (-\infty, 5) \cup (5, \infty)$$

$$x - 5 \overline{) x^2 - 6x}$$

$$\underline{-x^2 + 5x}$$

$$-x$$

$$\underline{x - 5}$$

$$-5$$

$$f(x) = \frac{x^2 - 6x}{x - 5} = x - 1 - \frac{5}{x - 5}$$

$$\text{VA: } x = 5$$

$$\text{HA: N/A}$$

$$\text{Hole: N/A}$$

$$\text{Oblique asymptote: } y = x - 1$$

**Exercise**

Determine all asymptotes of the function  $f(x) = \frac{x^3 - x^2 + x - 4}{x^2 + 2x - 1}$

**Solution**

$$x^2 + 2x - 1 = 0 \rightarrow x = -1 \pm \sqrt{2}$$

$$\text{Domain: } (-\infty, -1 - \sqrt{2}) \cup (-1 - \sqrt{2}, -1 + \sqrt{2}) \cup (-1 + \sqrt{2}, \infty)$$

$$\begin{array}{r}
 x^2 + 2x - 1 \overline{) x^3 - x^2 + x - 4} \\
 \underline{-x^3 - 2x^2 + x} \phantom{-4} \\
 -3x^2 + 2x - 4 \\
 \underline{3x^2 + 6x - 3} \\
 8x - 7
 \end{array}$$

$$\begin{aligned}
 f(x) &= \frac{x^3 - x^2 + x - 4}{x^2 + 2x - 1} \\
 &= x - 3 + \frac{8x - 7}{x^2 + 2x - 1}
 \end{aligned}$$

$$\text{VA: } x = -1 \pm \sqrt{2}$$

$$\text{HA: N/A}$$

$$\text{Hole: N/A}$$

$$\text{Oblique asymptote: } y = x - 3$$

### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{-3x}{x+2}$$

### Solution

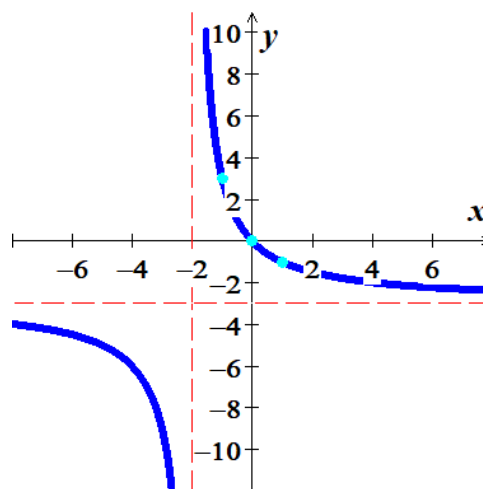
$$\text{VA: } x = -2$$

$$\text{HA: } y = -3$$

$$\text{Hole: } n/a$$

$$\text{OA: } n/a$$

$x$	$y$
0	0
1	-1
-1	3



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

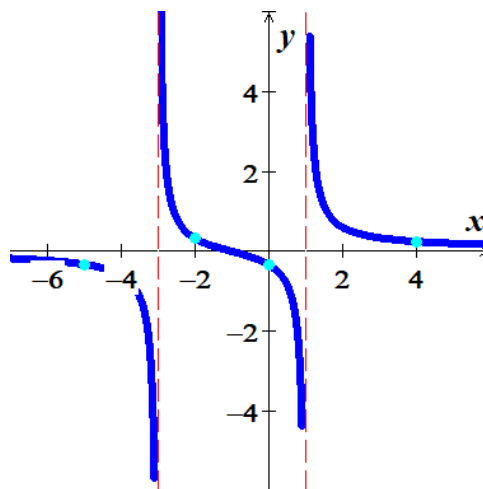
$$f(x) = \frac{x+1}{x^2 + 2x - 3}$$

### Solution

VA:  $x = 1, x = -3$       HA:  $y = 0$

Hole:  $n/a$       Oblique asymptote:  $n/a$

$x$	$y$
-5	-0.33
-2	0.33
0	-1/3
4	0.24



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

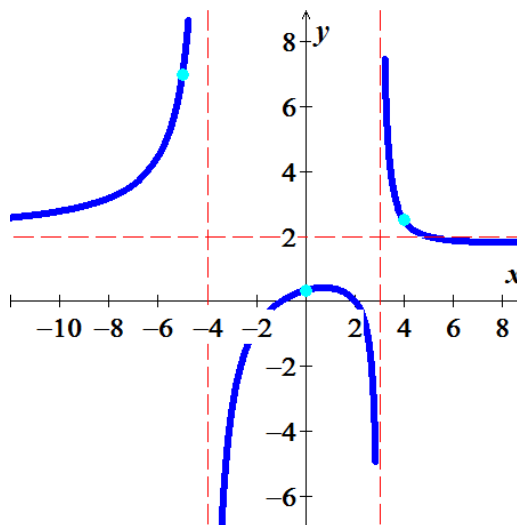
$$f(x) = \frac{2x^2 - 2x - 4}{x^2 + x - 12}$$

### Solution

VA:  $x = -4, 3$       HA:  $y = 2$

Hole:  $n/a$       OA:  $n/a$

$x$	$y$
-5	7
-2	-0.8
0	1/3
4	2.5
5	2



### Exercise

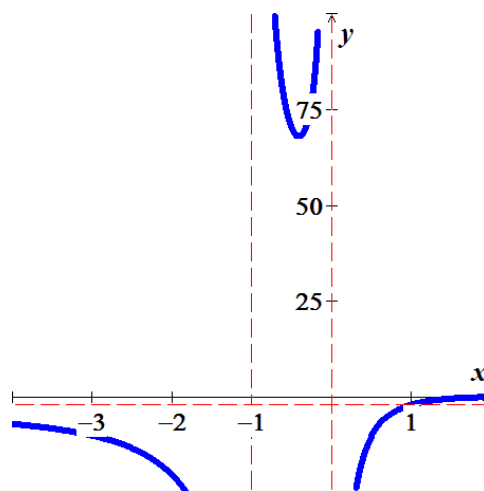
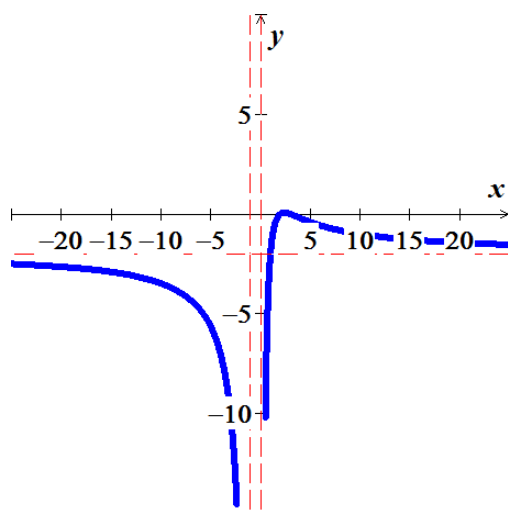
Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x) = \frac{-2x^2 + 10x - 12}{x^2 + x}$$

### Solution

**VA:**  $x = -1, 0$       **HA:**  $y = -2$

**Hole:**  $n/a$       **OA:**  $n/a$



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x) = \frac{x^2 - x - 6}{x + 1}$$

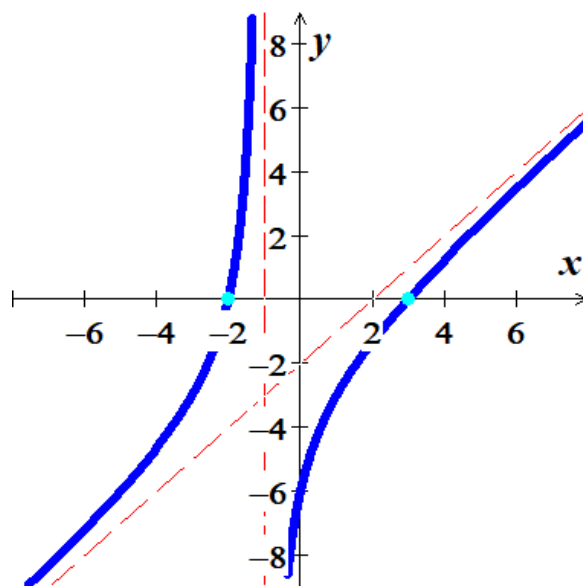
### Solution

$$\begin{array}{r} x-2 \\ x+1 \overline{) x^2 - x - 6} \\ \underline{x^2 + x} \phantom{-6} \\ -2x - 6 \\ \underline{-2x - 2} \\ -4 \end{array}$$

**VA:**  $x = -1$       **HA:**  $n/a$

**Hole:**  $n/a$       **OA:**  $y = x - 2$

$x$	$y$
2	0
-2	0
0	-6



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph

$$f(x) = \frac{x^3 + 1}{x - 2}$$

### Solution

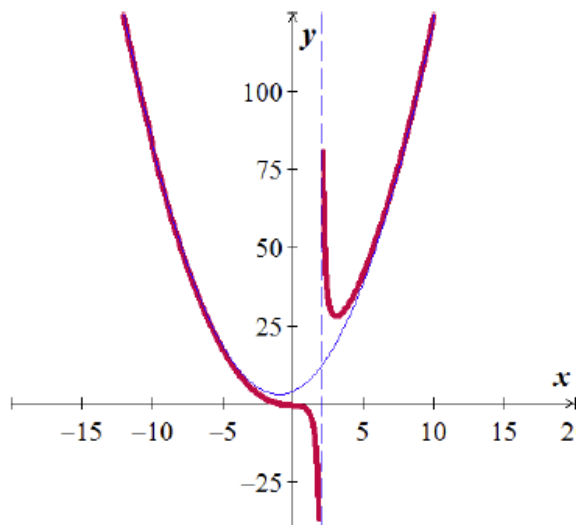
$$\begin{array}{r} x^2 + 2x + 4 \\ x-2 \overline{) x^3 - 1} \\ \underline{x^3 - 2x^2} \phantom{-1} \\ 2x^2 \phantom{-1} \\ \underline{2x^2 - 4x} \phantom{-1} \\ 4x - 1 \\ \underline{4x - 8} \\ 7 \end{array}$$

VA:  $x = 2$

HA:  $n/a$

Hole:  $n/a$

OA:  $y = x^2 + 2x + 4$



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph

$$f(x) = \frac{2x^2 + x - 6}{x^2 + 3x + 2}$$

### Solution

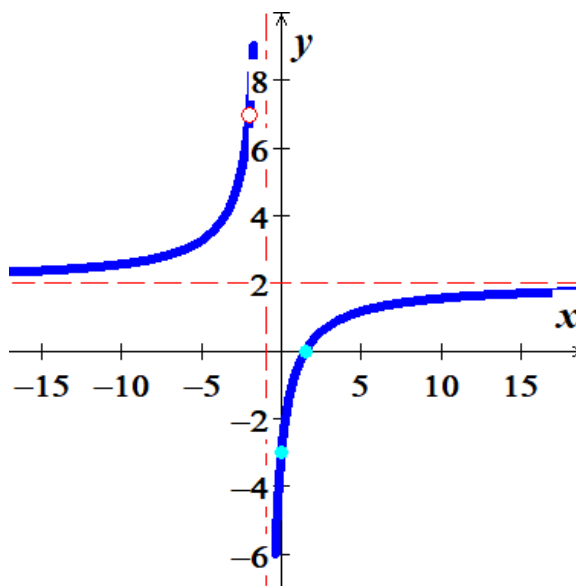
$$\begin{aligned} f(x) &= \frac{(2x-3)(x+2)}{(x+1)(x+2)} \\ &= \frac{2x-3}{x+1} \end{aligned}$$

VA:  $x = -1$

HA:  $y = 2$

Hole:  $(-2, 7)$

OA:  $n/a$



$x$	$y$
0	-3
$-\frac{3}{2}$	0

### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x) = \frac{x-1}{1-x^2}$$

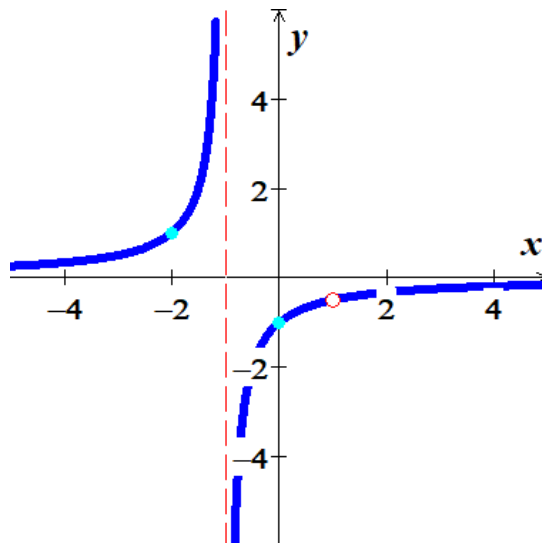
### Solution

$$\begin{aligned} f(x) &= \frac{x-1}{(x+1)(1-x)} \\ &= -\frac{1}{x+1} \end{aligned}$$

**VA:**  $x = -1$       **HA:**  $y = 0$

**Hole:**  $\left(1, -\frac{1}{2}\right)$       **OA:**  $n/a$

$x$	$y$
0	-1
-2	1



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph

$$f(x) = \frac{x^2 + x - 2}{x + 2}$$

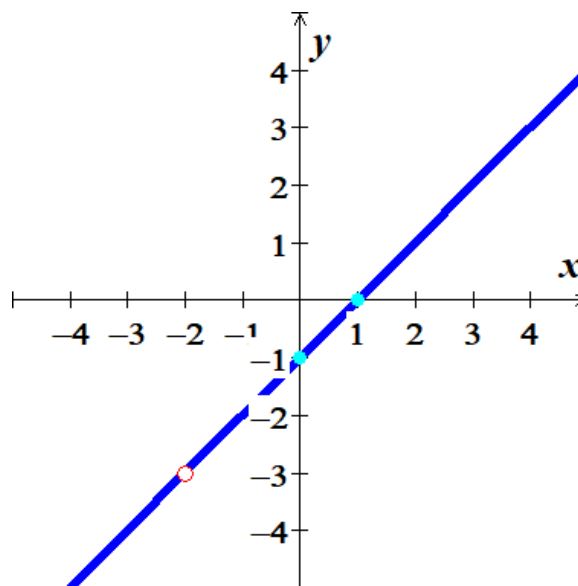
### Solution

$$\begin{aligned} f(x) &= \frac{(x+2)(x-1)}{x+2} \\ &= x-1 \end{aligned}$$

**VA:**  $n/a$       **HA:**  $n/a$

**Hole:**  $(-2, -3)$       **OA:**  $n/a$

$x$	$y$
0	-1
1	0



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph

$$f(x) = \frac{x^3 - 2x^2 - 4x + 8}{x - 2}$$

### Solution

$$\begin{aligned} f(x) &= \frac{(x^2 - 4)(x - 2)}{x - 2} \\ &= x^2 - 4 \end{aligned}$$

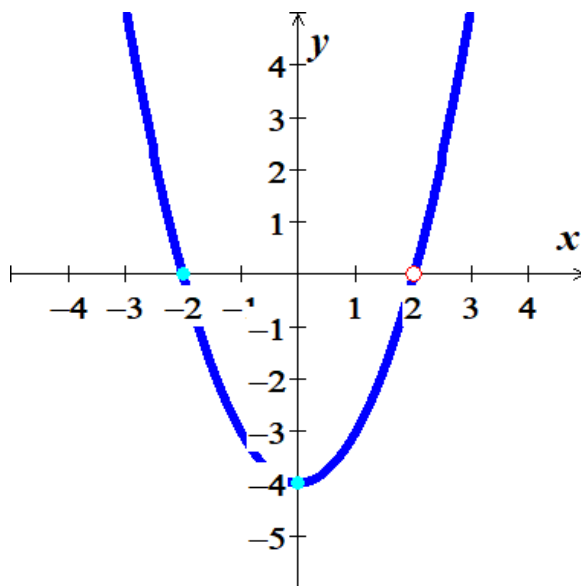
VA:  $n/a$

HA:  $n/a$

Hole:  $(2, 0)$

OA:  $n/a$

$x$	$y$
0	-4
-2	0



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph

$$f(x) = \frac{2x^2 - 3x - 1}{x - 2}$$

### Solution

$$\begin{array}{r} 2x+1 \\ x-2 \overline{) 2x^2 - 3x - 1} \\ \underline{-2x^2 + 4x} \phantom{-1} \\ x-1 \\ \underline{-x+2} \\ 1 \end{array}$$

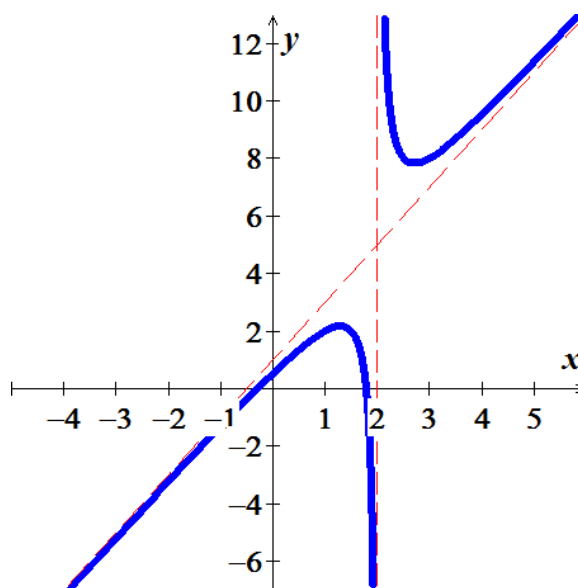
$$\begin{aligned} f(x) &= \frac{2x^2 - 3x - 1}{x - 2} \\ &= (2x + 1) + \frac{1}{x - 2} \end{aligned}$$

VA:  $x = 2$

HA:  $y = 1$

Hole:  $n/a$

OA:  $y = 2x + 1$



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote*) and sketch the graph

$$f(x) = \frac{2x+3}{3x^2+7x-6}$$

### Solution

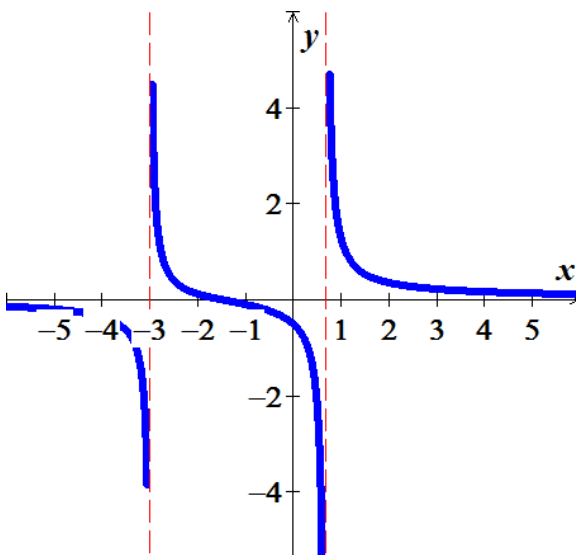
$$3x^2 + 7x - 6 = 0 \Rightarrow x = -3, \frac{2}{3}$$

$$\text{VA: } x = -3 \text{ and } x = \frac{2}{3}$$

$$\text{HA: } y = 0$$

$$\text{Hole: } n/a$$

$$\text{OA: } n/a$$



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote*) and sketch the graph

$$f(x) = \frac{x^2-1}{x^2+x-6}$$

### Solution

$$x^2 + x - 6 = 0 \Rightarrow x = -3, 2$$

$$\text{VA: } x = -3 \text{ and } x = 2 \quad \text{HA: } y = 1$$

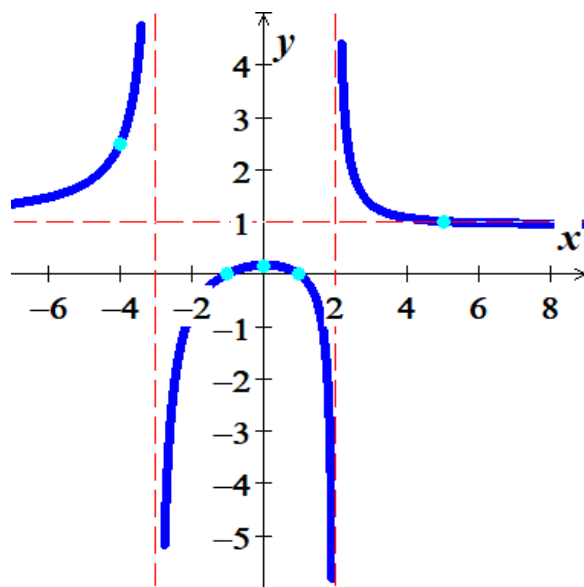
$$\text{Hole: } n/a \quad \text{OA: } n/a$$

$$1 = \frac{x^2-1}{x^2+x-6}$$

$$x^2 + x - 6 = x^2 - 1$$

$$x = 5$$

x	y
0	$\frac{1}{6}$
5	1
$\pm 1$	0
-4	$\frac{5}{2}$





## Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

$$f(x) = \frac{-2x^2 - x + 15}{x^2 - x - 12}$$

## Solution

$$x^2 - x - 12 = 0 \Rightarrow x = -3, 4$$

$$\text{Domain: } (-\infty, -3) \cup (-3, 4) \cup (4, \infty)$$

$$f(x) = \frac{(-2x+5)(x+3)}{(x-4)(x+3)}$$

$$= \frac{-2x+5}{x-4}$$

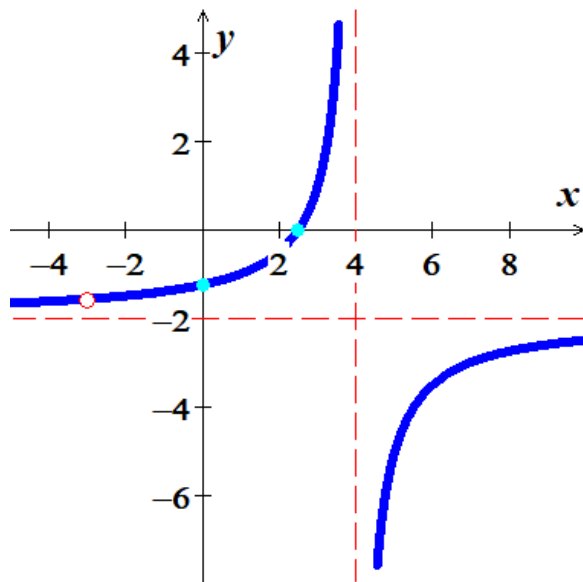
$$\text{VA: } x = 4$$

$$\text{HA: } y = -2$$

$$\text{Hole: } \left(-3, -\frac{11}{7}\right)$$

$$\text{OA: } n/a$$

$x$	$y$
0	$-\frac{5}{4}$
$\frac{5}{2}$	0



## Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

$$f(x) = \frac{1}{x-3}$$

## Solution

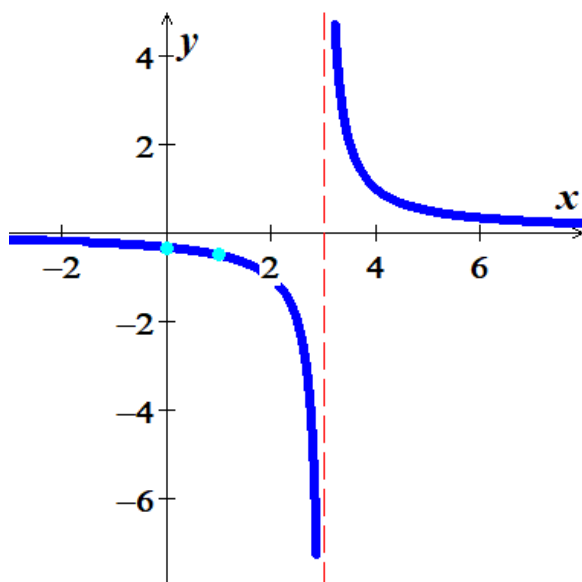
$$\text{VA: } x = 3$$

$$\text{HA: } y = 0$$

$$\text{Hole: } n/a$$

$$\text{OA: } n/a$$

$x$	$y$
0	$-\frac{1}{3}$
1	$-\frac{1}{2}$



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote*) and sketch the graph of

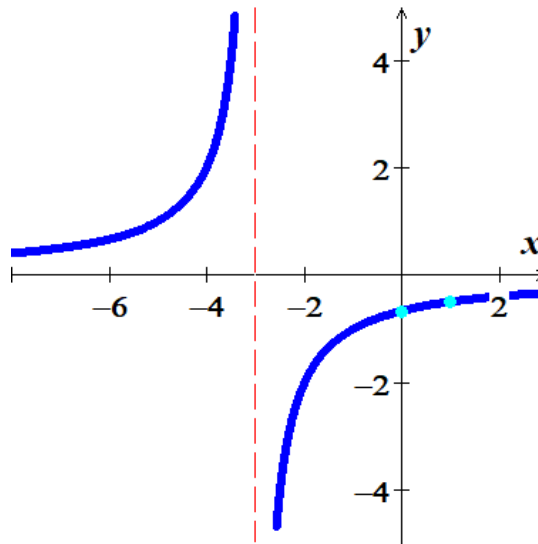
$$f(x) = \frac{-2}{x+3}$$

#### Solution

**VA:**  $x = -3$       **HA:**  $y = 0$

**Hole:**  $n/a$       **OA:**  $n/a$

$x$	$y$
0	$-\frac{2}{3}$
1	$-\frac{1}{2}$



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote*) and sketch the graph of

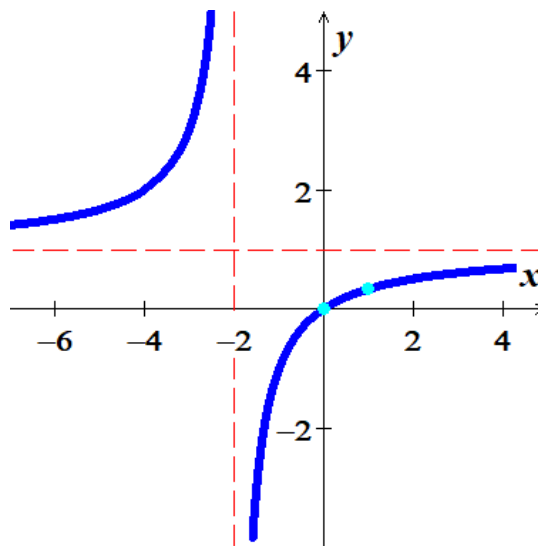
$$f(x) = \frac{x}{x+2}$$

#### Solution

**VA:**  $x = -2$       **HA:**  $y = 1$

**Hole:**  $n/a$       **OA:**  $n/a$

$x$	$y$
0	0
1	$\frac{1}{3}$



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

$$f(x) = \frac{x-5}{x+4}$$

### Solution

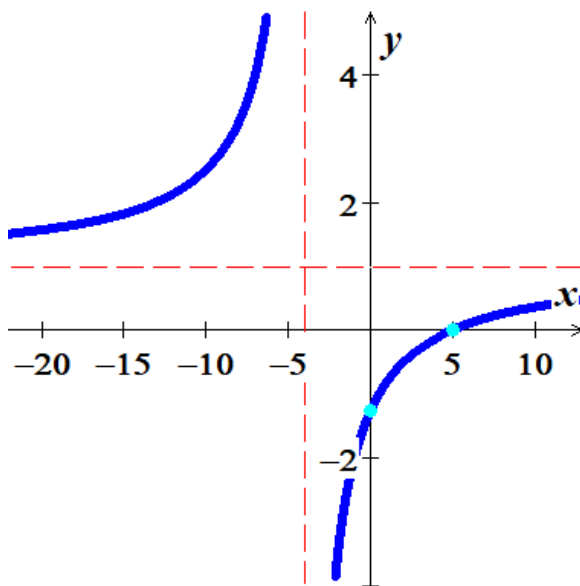
VA:  $x = -4$

HA:  $y = 1$

Hole:  $n/a$

OA:  $n/a$

$x$	$y$
0	$-\frac{5}{4}$
5	0



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

$$f(x) = \frac{2x^2 - 2}{x^2 - 9}$$

### Solution

$$x^2 = 9 \rightarrow x = \pm 3$$

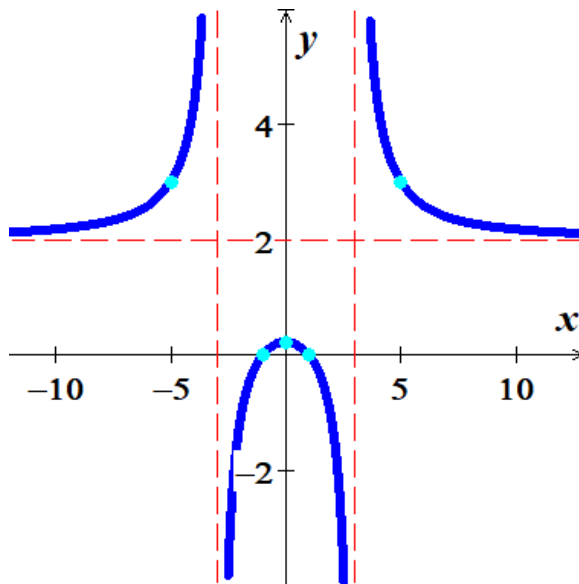
VA:  $x = \pm 3$

HA:  $y = 2$

Hole:  $n/a$

OA:  $n/a$

$x$	$y$
0	$\frac{2}{9}$
$\pm 1$	0
$\pm 5$	3



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

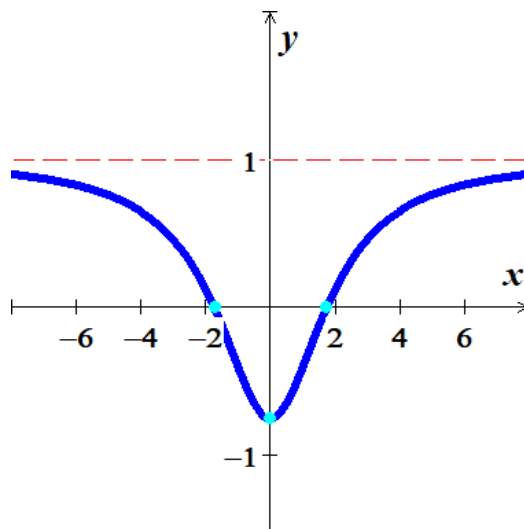
$$f(x) = \frac{x^2 - 3}{x^2 + 4}$$

### Solution

**VA:**  $n/a$       **HA:**  $y = 1$

**Hole:**  $n/a$       **OA:**  $n/a$

$x$	$y$
0	$-\frac{3}{4}$
$\pm\sqrt{3}$	0



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{x^2 + 4}{x^2 - 3}$$

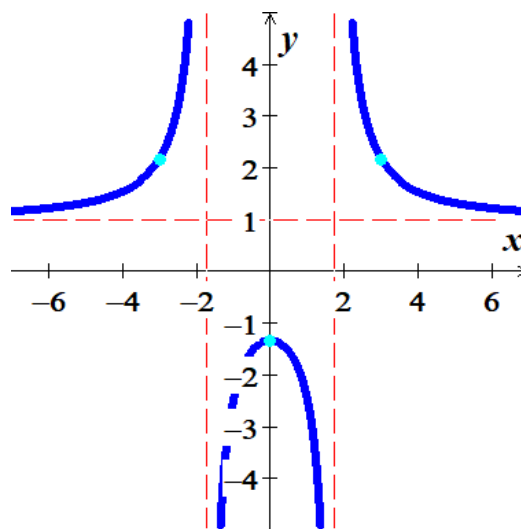
### Solution

$$x^2 - 3 = 0 \rightarrow x = \pm\sqrt{3}$$

**VA:**  $x = \pm\sqrt{3}$       **HA:**  $y = 1$

**Hole:**  $n/a$       **OA:**  $n/a$

$x$	$y$
0	$-\frac{4}{3}$
$\pm 3$	$\frac{13}{6}$



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

$$f(x) = \frac{x^2}{x^2 - 6x + 9}$$

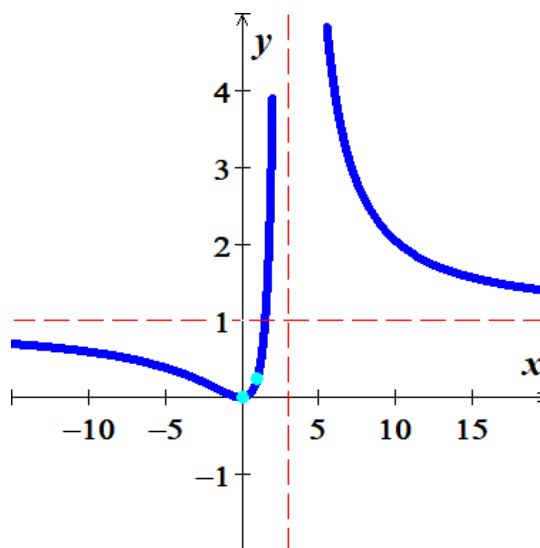
### Solution

$$x^2 - 6x + 9 = 0 \rightarrow x = 3$$

$$\text{VA: } x = 3 \quad \text{HA: } y = 1$$

$$\text{Hole: } n/a \quad \text{OA: } n/a$$

x	y
0	0
1	$\frac{1}{4}$



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

$$f(x) = \frac{x^2 + x + 4}{x^2 + 2x - 1}$$

### Solution

$$x^2 + 2x - 1 = 0$$

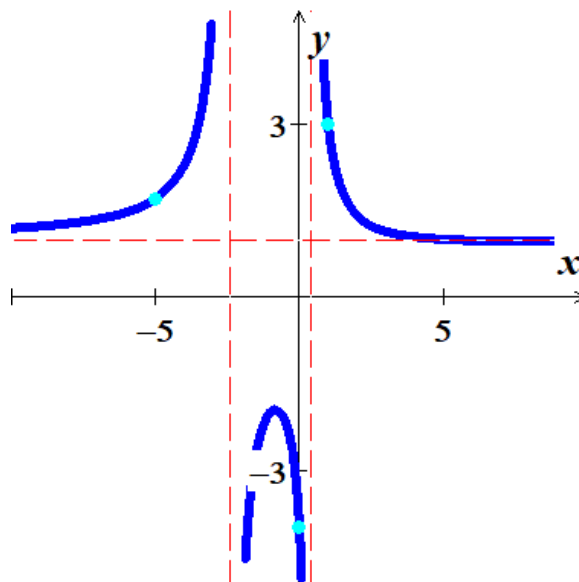
$$x = \frac{-2 \pm \sqrt{8}}{2}$$

$$= -1 \pm \sqrt{2}$$

$$\text{VA: } x = -1 \pm \sqrt{2} \quad \text{HA: } y = 1$$

$$\text{Hole: } n/a \quad \text{OA: } n/a$$

x	y
0	-4
1	3
-5	$\frac{12}{7}$



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{2x^2 + 14}{x^2 - 6x + 5}$$

### Solution

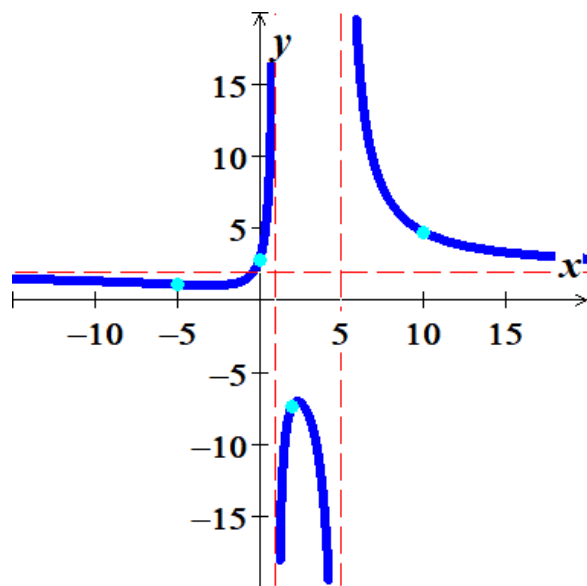
**VA:**  $x = 1, 5$

**HA:**  $y = 2$

**Hole:**  $n/a$

**OA:**  $n/a$

$x$	$y$
0	$\frac{14}{5}$
2	$-\frac{22}{3}$
-5	$\frac{16}{15}$
10	$\frac{214}{45}$



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{x^2 - 4x - 5}{2x + 5}$$

### Solution

$$\begin{array}{r} \frac{1}{2}x - \frac{13}{4} \\ 2x + 5 \overline{) x^2 - 4x - 5} \\ \underline{x^2 + \frac{5}{2}x} \phantom{-5} \\ -\frac{13}{2}x - 5 \end{array}$$

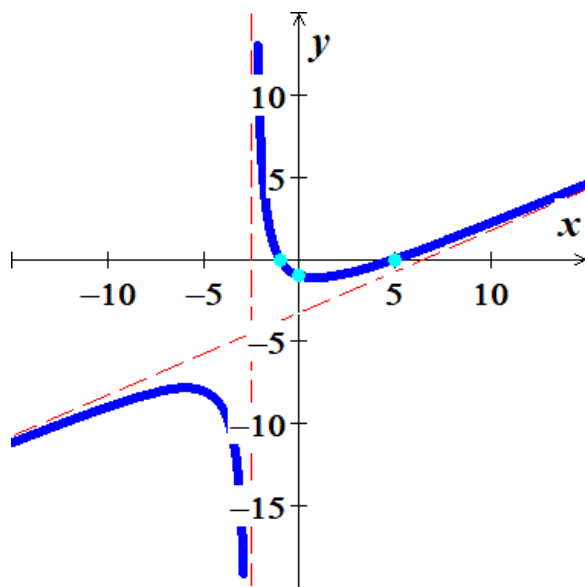
**VA:**  $x = -\frac{5}{2}$

**HA:**  $n/a$

**Hole:**  $n/a$

**OA:**  $y = \frac{1}{2}x - \frac{13}{2}$

$x$	$y$
0	-1
-1, 5	0



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{x-3}{x^2-3x+2}$$

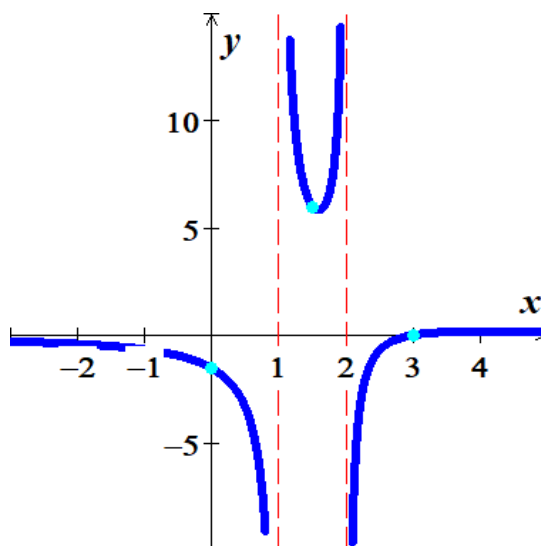
**Solution**

$$x^2 - 3x + 2 \rightarrow x = 1, 2$$

**VA:**  $x = 1, 2$       **HA:**  $y = 0$

**Hole:**  $n/a$       **OA:**  $n/a$

$x$	$y$
0	$-\frac{3}{2}$
3	0
$\frac{3}{2}$	6



***Exercise***

Determine all asymptotes (if any) (*Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{x^2+2}{x^2+3x+2}$$

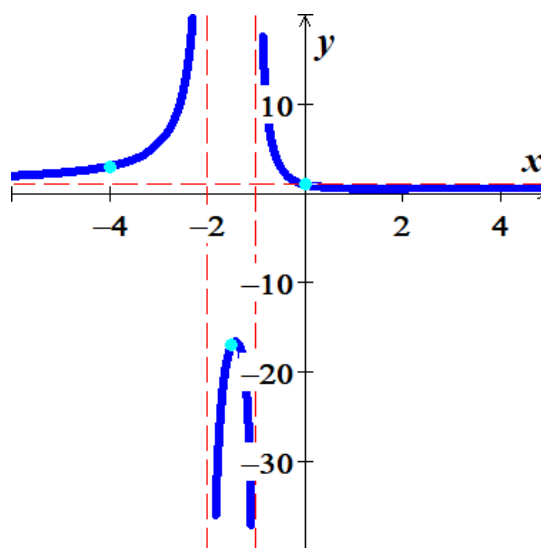
**Solution**

$$x^2 + 3x + 2 \rightarrow x = -1, -2$$

**VA:**  $x = -1, -2$       **HA:**  $y = 1$

**Hole:**  $n/a$       **OA:**  $n/a$

$x$	$y$
0	1
$-\frac{3}{2}$	-17
-4	3



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{x-2}{x^2-3x+2}$$

### Solution

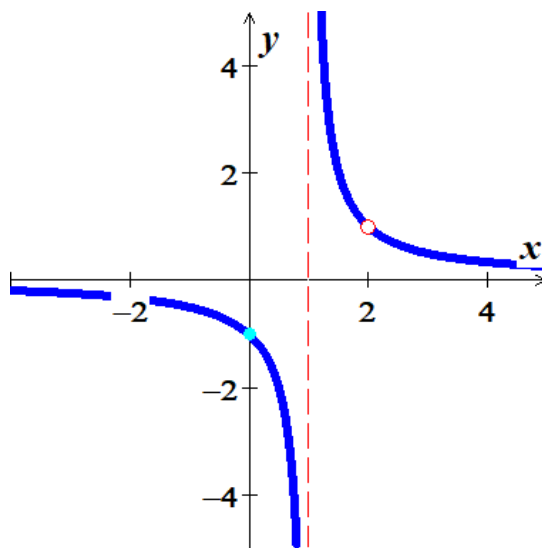
$$x^2 - 3x + 2 \rightarrow \underline{x=1, 2}$$

$$\begin{aligned} f(x) &= \frac{x-2}{(x-2)(x-1)} \\ &= \frac{1}{x-1} \end{aligned}$$

**VA:**  $x=1$       **HA:**  $y=0$

**Hole:**  $(2, 1)$       **OA:**  $n/a$

$x$	$y$
0	-1



### Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{x^2+x}{x+1}$$

### Solution

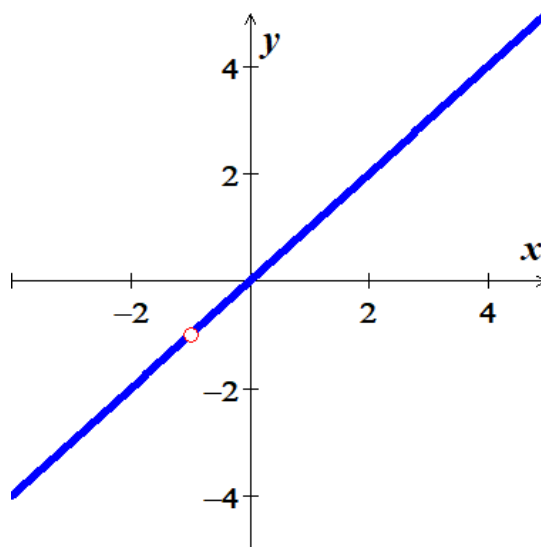
$$\begin{aligned} f(x) &= \frac{x(x+1)}{x+1} \\ &= \underline{x} \end{aligned}$$

**VA:**  $n/a$       **HA:**  $n/a$

**Hole:**  $(-1, -1)$       **OA:**  $n/a$

**Hole:**  $(-3, -\frac{11}{7})$       **OA:**  $n/a$

$x$	$y$
0	0





### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

$$f(x) = \frac{x^2 - 2x}{x - 2}$$

### Solution

$$f(x) = \frac{x(x-2)}{x-2}$$

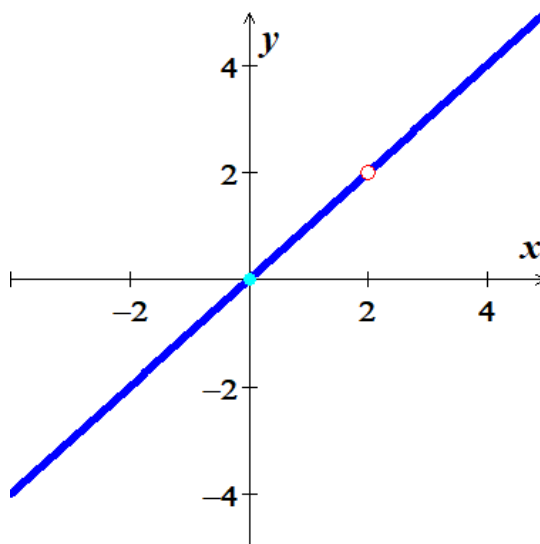
$$= \underline{x}$$

VA:  $n/a$       HA:  $n/a$

Hole:  $(2, 2)$       OA:  $n/a$

Hole:  $(-3, -\frac{11}{7})$       OA:  $n/a$

$x$	$y$
0	0



### Exercise

Determine all asymptotes (if any) (Vertical Asymptote, Horizontal Asymptote; Hole; Oblique Asymptote) and sketch the graph of

$$f(x) = \frac{x^2 - 3x}{x + 3}$$

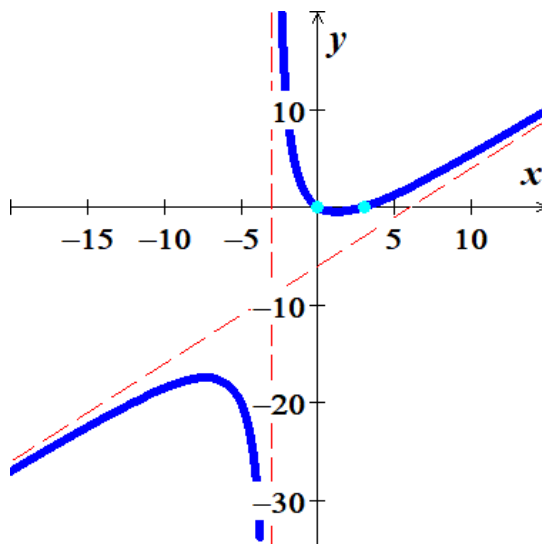
### Solution

$$x+3 \overline{) \begin{array}{r} x-6 \\ x^2-3x \\ \underline{x^2+3x} \\ -6x-5 \end{array}}$$

VA:  $x = -3$       HA:  $n/a$

Hole:  $n/a$       OA:  $y = x - 6$

$x$	$y$
0	0
3	0



## Exercise

Determine all asymptotes (if any) (*Vertical Asymptote*, *Horizontal Asymptote*; *Hole*; *Oblique Asymptote*) and sketch the graph of

$$f(x) = \frac{x^3 + 3x^2 - 4x + 6}{x + 2}$$

## Solution

$$\begin{array}{r} x^2 + x - 6 \\ x+2 \overline{) x^3 + 3x^2 - 4x + 6} \\ \underline{x^3 + 2x^2} \phantom{+ 6} \\ x^2 - 4x \phantom{+ 6} \\ \underline{x^2 + 2x} \phantom{+ 6} \\ -6x + 6 \end{array}$$

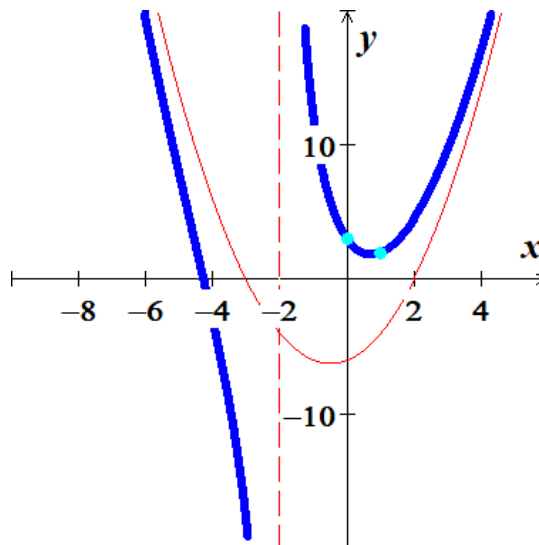
**VA:**  $x = -2$

**HA:**  $n/a$

**Hole:**  $n/a$

**OA:**  $y = x^2 + x - 6$

$x$	$y$
0	3
1	2



## Exercise

Find an equation of a rational function  $f$  that satisfies the given conditions

$$\begin{cases} \text{vertical asymptote : } x = 4 \\ \text{horizontal asymptote : } y = -1 \\ x\text{-intercept : } 3 \end{cases}$$

## Solution

**Vertical Asymptote:**  $f(x) = \frac{\quad}{x - 4}$

**Horizontal Asymptote:**  $f(x) = \frac{-x + a}{x - 4}$

**x-intercept:**  $f(x = 3) = \frac{-3 + a}{3 - 4} = 0 \Rightarrow \underline{a = 3}$

$$\underline{f(x) = \frac{-x + 3}{x - 4}}$$

### Exercise

Find an equation of a rational function  $f$  that satisfies the given conditions

$$\begin{cases} \text{vertical asymptote: } x = -4, x = 5 \\ \text{horizontal asymptote: } y = \frac{3}{2} \\ x\text{-intercept: } -2 \end{cases}$$

### Solution

**Vertical Asymptote:**  $f(x) = \frac{\quad}{(x+4)(x-5)}$

**Horizontal Asymptote:**  $f(x) = \frac{\textcolor{red}{3}(x+a)(x+b)}{\textcolor{red}{2}(x+4)(x-5)}$

**x-intercept:**  $f(x = \textcolor{red}{-2}) = \frac{3}{2} \frac{(\textcolor{red}{-2}+a)(\textcolor{red}{-2}+b)}{\quad}$

$$0 = (\textcolor{red}{-2}+a)(\textcolor{red}{-2}+b)$$

$$\boxed{a = b = 2}$$

$$\begin{aligned} f(x) &= \frac{3}{2} \frac{(x-2)^2}{x^2 - x - 20} \\ &= \frac{\textcolor{blue}{3x^2 - 12x + 12}}{\textcolor{blue}{2x^2 - 2x - 40}} \end{aligned}$$

### Exercise

Find an equation of a rational function  $f$  that satisfies the given conditions

$$\begin{cases} \text{vertical asymptote: } x = 5 \\ \text{horizontal asymptote: } y = -1 \\ x\text{-intercept: } 2 \end{cases}$$

### Solution

**Vertical Asymptote:**  $f(x) = \frac{\quad}{x-5}$

**x-intercept:**  $f(x) = \frac{x-2}{x-5}$

**Horizontal Asymptote:**  $f(x) = -\frac{x-2}{x-5}$

$$\underline{f(x) = -\frac{\textcolor{blue}{x-2}}{\textcolor{blue}{x-5}}}$$

### Exercise

Find an equation of a rational function  $f$  that satisfies the given conditions

$$\begin{cases} \text{vertical asymptote: } x = -2, x = 0 \\ \text{horizontal asymptote: } y = 0 \\ x\text{-intercept: } 2, \quad f(3) = 1 \end{cases}$$

### Solution

**Vertical Asymptote:**  $f(x) = \frac{\quad}{x(x+2)}$

**$x$ -intercept :**  $f(x) = \frac{x-2}{x(x+2)}$

**Horizontal Asymptote:**  $f(x) = \frac{a(x-2)}{x(x+2)}$

$$f(3) = 1 \rightarrow \frac{a(1)}{(3)(5)} = 1 \Rightarrow \underline{a = 15}$$

$$\underline{f(x) = \frac{15x-30}{x^2+2x}}$$

### Exercise

Find an equation of a rational function  $f$  that satisfies the given conditions

$$\begin{cases} \text{vertical asymptote: } x = -3, x = 1 \\ \text{horizontal asymptote: } y = 0 \\ x\text{-intercept: } -1, \quad f(0) = -2 \\ \text{hole: } x = 2 \end{cases}$$

### Solution

**Vertical Asymptote:**  $f(x) = \frac{\quad}{(x+3)(x-1)}$

**$x$ -intercept :**  $f(x) = \frac{(x+1)}{(x+3)(x-1)}$

**Horizontal Asymptote:**  $f(x) = \frac{a(x+1)}{(x+3)(x-1)}$

$$f(0) = -2 \rightarrow \frac{a}{-3} = -2 \Rightarrow \underline{a = 6}$$

**Hole at  $x = 2$ :**  $f(x) = \frac{6(x+1)(x-2)}{(x^2+2x-3)(x-2)}$

$$\underline{f(x) = \frac{6x^2-6x-12}{x^2+2x-3}}$$

### ***Exercise***

Find an equation of a rational function  $f$  that satisfies the given conditions

$$\begin{cases} \text{vertical asymptote: } x = -1, \ x = 3 \\ \text{horizontal asymptote: } y = 2 \\ x\text{-intercept: } -2, \ 1 \\ \text{hole: } x = 0 \end{cases}$$

### **Solution**

**Vertical Asymptote:**  $f(x) = \frac{\quad}{(x+1)(x-3)}$

**Horizontal Asymptote:**  $f(x) = \frac{2}{(x+1)(x-3)}$

**$x$ -intercept :**  $f(x) = \frac{2(x+2)(x-1)}{(x+1)(x-3)}$

**Hole at  $x = 0$ :**  $f(x) = \frac{2x(x+2)(x-1)}{x(x+1)(x-3)}$

$$\underline{f(x) = \frac{2x^3 + 2x^2 - 4x}{x^3 - 2x^2 - 3x}}$$