#1/ $\lim_{x \to 1^+} \frac{1}{x-1} = \frac{1}{0^+} = \infty$

2/ $\lim_{x \to 1^-} \frac{1}{x-1} = \frac{1}{0^-} = -\infty$

3/ $\lim_{x \to \infty} \left(x + \sqrt{x^2 - 4x + 1}\right) = \infty + \infty = \infty$

4/ $\lim_{x \to -\infty} \left(x + \sqrt{x^2 - 4x + 1}\right) = -\infty + \infty$

$= \lim_{x \to -\infty} \left(x + \sqrt{x^2 - 4x + 1}\right) \frac{x - \sqrt{x^2 - 4x + 1}}{x - \sqrt{x^2 - 4x + 1}}$

$= \lim_{x \to -\infty} \frac{x^2 - x^2 - 4x + 1}{x - \sqrt{x^2 - 4x + 1}}$

$= \lim_{x \to -\infty} \frac{-4x}{x - |x|}$ — (-) 4

$= \lim_{x \to -\infty} \frac{+4(x)}{-|x| - |x|}$

$= \lim_{x \to -\infty} \frac{4}{-2} \frac{|x|}{(x)}$

$= -2$

5/ $\lim_{x \to \infty} \frac{2x + 100}{x^2 + 3} = 0$

6/ $\lim_{x \to \infty} \frac{1 - x^2}{3x^2 - x - 1} = -\frac{1}{3}$

7/ $\lim_{x \to \infty} \frac{x^3 - 2}{x^2 + 5} = \infty$

8/ $\lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}} = 0$

9/ $\lim_{x\to\infty} \sqrt{x} = \infty$

10/ $\lim_{x\to\infty} \frac{\cos x}{x} = \frac{|1|\le 1}{\infty} = 0$

$-\frac{1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$ $\qquad \frac{1}{x} \to 0$

$\lim_{x\to\infty} -\frac{1}{x} = 0$

$0 \le \lim_{x\to\infty} \frac{\cos x}{x} \le 0$

$\lim_{x\to\infty} \frac{\cos x}{x} = 0$

11/ $\lim_{x\to 0} \sin \frac{1}{x^2} = \sin\infty$ ∄

$x \to 0 \Rightarrow \frac{1}{x^2} \to \infty$

$\lim_{\frac{1}{x^2}\to\infty} (\sin \frac{1}{x^2}) \frac{1/x^2}{1/x^2}$

12/ $\lim_{x\to\infty} \sin x = \sin\infty$ ∄

13/ $\lim_{x\to-\infty} e^{x^2} = e^{\infty} = \infty$

14/ $\lim_{x\to-\infty} e^{x^3} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

15/ $\lim_{x\to-\infty} \ln|x| = \ln\infty = \infty$

Ex $y = 2x - 1$ near $x_0 = 4$

y close to 7 $\quad$ x close 4

$$\lim_{x \to 4} (2x - 1) = 7$$

x? $\quad$ $y = 2x - 1$ differs from 7 < 2?

$$|y - 7| < 2$$

Soln

$$|y - 7| = |2x - 1 - 7|$$

$$= |2x - 8| \quad \text{(given)}$$

$$= 2|x - 4| < 2$$

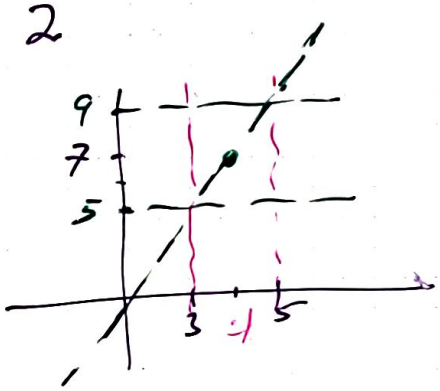
$$|x - 4| < 1$$

$$-1 < x - 4 < 1$$

$$+4 \qquad +4 \qquad +4$$

$$3 < x < 5$$

$$3 < 4 < 5$$



Defn $\lim_{x \to x_0} f(x) = L$

$\forall \varepsilon > 0, \exists \delta > 0 \ni \forall x$

$$|x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

Ex $\lim_{x \to 1} (5x - 3) = 2$

Given: $f(x) = 5x - 3$

$L = 2$

$x_0 = 1$

Soln $|x - x_0| < \delta$

$|x - 1| < \delta \Rightarrow -\delta < x - 1 < \delta$

$1 - \delta < x < 1 + \delta$

$|f(x) - L| < \varepsilon$

$|5x - 3 - 2| < \varepsilon$

$|5x - 5| < \varepsilon$

$5|x - 1| < \varepsilon$

$|x - 1| < \frac{\varepsilon}{5} = \delta$

$\lim_{x \to 1} (5x - 3) = 2$ $\quad \delta = \varepsilon/5$

Ex $\lim_{x \to 5} \sqrt{x-1} = 2$ $\varepsilon = 1$ $\delta > 0$?

Given: $L = 2$, $f(x) = \sqrt{x-1}$, $x_0 = 5$, $\varepsilon = 1$

Soln $|f(x) - L| < \varepsilon$ $|x-5| < \delta$?

$|\sqrt{x-1} - 2| < 1$

$-1 < \sqrt{x-1} - 2 < 1$

$1 < \sqrt{x-1} < 3$

$1 < x - 1 < 9$

$2 < x < 10$

$|x-5| < \delta$

$-\delta < x - 5 < \delta$

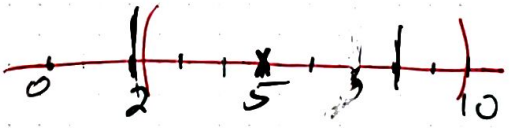
$5 - \delta < x < 5 + \delta$.

Compare $2 < x < 10$

$5 - \delta = 2 \Rightarrow \delta = 3$ ⇐

$5 + \delta = 10 \Rightarrow \delta = 5$

$\delta = 3$



Ex Prove $\lim_{x \to 2} f(x) = 4$

$$f(x) = \begin{cases} x^2 & , x \neq 2 \\ 1 & x = 2 \end{cases}$$

soln

$|x-2| < \delta \Rightarrow |f(x) - 4| < \varepsilon$

$|f(x) - 4| < \varepsilon$

$|x^2 - 4| < \varepsilon$

$-\varepsilon < x^2 - 4 < \varepsilon$

$4 - \varepsilon < x^2 < 4 + \varepsilon$

$\sqrt{4-\varepsilon} < |x| < \sqrt{4+\varepsilon}$

$x \in (\sqrt{4-\varepsilon}, \sqrt{4+\varepsilon})$

$|x-2| < \delta$

$-\delta < x - 2 < \delta$

$2 - \delta < x < \delta + 2$

$\begin{cases} \sqrt{4-\varepsilon} = 2 - \delta \\ \sqrt{4+\varepsilon} = 2 + \delta \end{cases}$

$\begin{cases} \delta = 2 - \sqrt{4-\varepsilon} \\ \delta = \sqrt{4+\varepsilon} - 2 \end{cases}$ $\quad \delta = 2 - \sqrt{4-\varepsilon}$

1.6 #4

$f(x) = x+1$, $L = 5$, $x_0 = 4$, $\varepsilon = 0.01$

soln $|x-4| < \delta$? $\Rightarrow$ $|f(x) - 5| < \varepsilon = 0.01$

$|x+1-5| < .01$

$-.01 < x-4 < .01$

$4 - .01 < x < 4.01$

$3.99 < x < 4.01$

$-\delta < x - 4 < \delta$

$4 - \delta < x < 4 + \delta$

$4 - \delta = 3.99 \rightarrow \delta = .01$

$4 + \delta = 4.01 \rightarrow \delta = -.01$

$\therefore \delta = .01$

$|x+1-5| < \frac{1}{100}$

$-\frac{1}{100} < x-4 < \frac{1}{100}$

$4 - \frac{1}{100} < x < \frac{1}{100} + 4$

$\frac{399}{100} < x < \frac{401}{100}$

$4 - \delta = \frac{399}{100}$

$4 + \delta = \frac{401}{100}$

$\delta = 4 - \frac{399}{100} = \frac{1}{100}$

$\delta = 4 - \frac{401}{100} = -\frac{1}{100}$

1.2

#53 $\lim_{x\to 0} 6x^2(\cot x)\csc 2x = 0 \cdot \frac{1}{0} \quad \frac{1}{0} = \frac{0}{0}$

$\cot x = \frac{\cos x}{\sin x}$ $\quad \csc 2x = \frac{1}{\sin 2x}$

$$= \lim_{x\to 0} 6x^2 \frac{\cos x}{\sin x} \frac{1}{\sin 2x}$$

$$= \lim_{x\to 0} \cos x \cdot \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}(x)} \cdot \frac{1}{\frac{\sin 2x}{2x}(2x)} (6x^2)$$

$\lim_{x\to 0} \cos x = 1$

$$= \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}} \frac{1}{\frac{\sin 2x}{2x}} \cdot \frac{6x^2}{2x2}$$

$$= 3$$

#56 $\lim_{\theta\to 0} \frac{\theta \cot 4\theta}{\sin^2\theta \cot^2\theta} = \frac{0}{0}$

$$= \lim_{\theta\to 0} \frac{1}{\frac{\sin\theta}{\theta}} \cdot \frac{1}{\sin\theta} \frac{\cos 4\theta}{\sin 4\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta}$$

$\frac{\sin\theta}{\theta} = 1$ $\quad \cos 4\theta \to 1$ $\quad \sin^2\theta \to 1$

$\sin 4\theta = 2\sin 2\theta \cos 2\theta$

$$= \lim_{\theta\to 0} \frac{\sin\theta}{2\sin 2\theta \cos 2\theta} \to 1$$

$$= \lim_{\theta\to 0} \frac{\sin\theta}{4\sin\theta\cos\theta\cos 2\theta}$$

$$= \frac{1}{4}$$