W ? $\vec{F} = \frac{\vec{r}}{|\vec{r}|^3}$

$\vec{r} = \langle t^2, 3t^2, -t^2 \rangle \qquad 1 \le t \le 2$

$W = \int \vec{F} \cdot d\vec{r}$

$\frac{d\vec{r}}{dt} = \langle 2t, 6t, -2t \rangle$

$\vec{F} = \frac{\langle t^2, 3t^2, -t^2 \rangle}{(t^4 + 9t^4 + t^4)^{3/2}}$

$W = \int_1^2 \frac{2t^3 + 18t^3 + 2t^3}{(11t^4)^{3/2}} \, dt$

$= \frac{1}{11^{3/2}} \int_1^2 \frac{22t^3}{t^6} \, dt$

$= \frac{2}{\sqrt{11}} \int_1^2 t^{-3} \, dt$

$= -\frac{1}{\sqrt{11}} t^{-2} \Big|_1^2$

$= -\frac{1}{\sqrt{11}} \left(\frac{1}{4} - 1 \right)$

$= \frac{3}{4\sqrt{11}}$

2/ $\int_C \vec{F}\cdot d\vec{r}$ $\quad f = \nabla(xyz)$

$$\vec{r}(t) = \left\langle \cos t, \sin t, \frac{t}{\pi} \right\rangle$$

$$0 \le t \le \pi$$

$$\vec{F} = \left\langle \frac{\partial}{\partial x}(xyz), \frac{\partial}{\partial y}(xyz), \frac{\partial}{\partial z}(x,y,z) \right\rangle$$

$$= \langle yz, xz, xy \rangle$$

$$= \left\langle \frac{t}{\pi}\sin t, \frac{t}{\pi}\cos t, \cos t \sin t \right\rangle$$

$$\frac{d\vec{r}}{dt} = \left\langle -\sin t, \cos t, \frac{1}{\pi} \right\rangle$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^{\pi} \left(-\frac{t}{\pi}\sin^2 t + \frac{t}{\pi}\cos^2 t + \frac{1}{\pi}\cos t \sin t \right) dt$$

$$= \frac{1}{\pi}\int_0^{\pi} \left(t\cos 2t + \frac{1}{2}\sin 2t \right) dt$$

		$\int \cos 2t$
$+$	t	$\frac{1}{2}\sin 2t$
$-$	1	$-\frac{1}{4}\cos 2t$

$$= \frac{1}{\pi}\left[\frac{1}{2}t\sin 2t + \underbrace{\frac{1}{4}\cos 2t - \frac{1}{4}\cos 2t}_{} \right]_0^{\pi}$$

$$= \underline{0}$$

$\vec{F} = \nabla \varphi$ $\qquad \phi = \frac{t}{\pi}\cos t \sin t.$

$\int_C \ldots = \varphi(b) - \phi(a)$

$$= \frac{\pi}{\pi}\cos\pi \sin\pi - \frac{0}{\pi}$$

$$= 0$$

3/ S? $z^2 = x^2 + y^2 = r^2$ $\boxed{2 \le z \le 4}$

$z = r = v$

$a = 0$

$\vec{r} = \langle v\cos u, v\sin u, v \rangle$ $| v = r$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -v\sin u & v\cos u & 0 \\ \cos u & \sin u & 1 \end{vmatrix}$$

$$= \langle v\cos u, v\sin u, -v(\cos^2 u + \sin^2 u) \rangle$$

$$= \langle v\cos u, v\sin u, -v \rangle$$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{v^2\cos^2 u + v^2\sin^2 u + v^2}$$

$$= \sqrt{2}\, v$$

$$S = \sqrt{2}\int_0^{2\pi} d\theta \int_2^4 v\, dv$$

$$= 2\pi\sqrt{2}\, \frac{1}{2}\left(v^2 \right|_2^4$$

$$= \pi\sqrt{2}\,(16 - 4)$$

$$= 12\pi\sqrt{2} \text{ unit}^2$$

-1/ $\iint_S (1+yz)\,ds$ $x, y, z \geq 0$

$x+y+z=2.$

$$z = 2-x-y$$

$$\sqrt{z_x^2+z_y^2+1} = \sqrt{1+1+1} = \sqrt{3}$$

$$\iint_S (1+yz)\,ds = \sqrt{3}\int_0^2\int_0^{2-x} (1+y(2-x-y))\,dy\,dx$$

$z=0 \Rightarrow 2-x-y=0 \rightarrow y = 2-x$

$0 = 2-x \Rightarrow x=2$

$$= \sqrt{3}\int_0^2\int_0^{2-x} (1+2y-xy-y^2)\,dy\,dx$$

$$= \sqrt{3}\int_0^2 \left(y+y^2-\frac{1}{2}xy^2-\frac{1}{3}y^3\right)\Big|_0^{2-x} dx$$

$$= \sqrt{3}\int_0^2 \Big(2-x+4-4x+x^2-\frac{1}{2}x(4-4x+x^2)$$
$$-\frac{1}{3}(8-12x+6x^2-x^3)\Big)\,dx$$

$$= \sqrt{3}\int_0^2 \left(\frac{10}{3}-3x+x^2-\frac{1}{6}x^3\right)dx$$

$$= \sqrt{3}\left(\frac{10}{3}x-\frac{3}{2}x^2+\frac{1}{3}x^3-\frac{1}{24}x^4\right)\Big|_0^2$$

$$= \sqrt{3}\left(\frac{20}{3}-6+\frac{8}{3}-\frac{2}{3}\right)$$

$$= \frac{8\sqrt{3}}{3}$$

4.4 #2

$$\vec{F} = \langle x^2 + 4y \,,\, x + y^2 \rangle \qquad \square \quad \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 1 \end{matrix}$$

$$\quad\;\; M \qquad\qquad N$$

$M = x^2 + 4y$ $\qquad$ $N = x + y^2$

$M_x = 2x$ $\qquad$ $N_x = 1$

$M_y = 4$ $\qquad$ $N_y = 2y$

$$\text{Flux} = \iint_R (M_x + N_y)\, dx\, dy$$

$$= \int_0^1 \int_0^1 (2x + 2y)\, dx\, dy$$

$$= \int_0^1 (x^2 + 2yx \Big|_0^1 \, dy$$

$$= \int_0^1 (1 + 2y)\, dy$$

$$= y + y^2 \Big|_0^1$$

$$= 2$$

$$\text{Cir} = \iint_R (N_x - M_y)\, dx\, dy$$

$$= \int_0^1 dy \int_0^1 (1 - 4)\, dx$$

$$= -3$$

$$\int_0^{\alpha} d\theta = \alpha$$

$\int\!\!\int 15$ $\vec{F} = \langle x, y, z\rangle$ $\displaystyle\iint_S (\nabla\times\vec{F})\cdot\hat{n}\,dS = \oint_C \vec{F}\cdot d\vec{r}$

$$\frac{x^2}{4}+\frac{y^2}{9}+z^2=1$$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= \langle 0, 0, 0\rangle$$

$$\boxed{\iint_S = 0.}$$

$$z=0 \Rightarrow \frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\vec{r}(t) = \langle 2\cos t, 3\sin t, 0\rangle$$

$$\vec{F} = \langle 2\cos t, 3\sin t, 0\rangle$$

$$\frac{d\vec{r}}{dt} = \langle -2\sin t, 3\cos t, 0\rangle$$

$$\oint_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi}(-4\cos t\sin t + 9\sin t\cos t)\,dt$$

$$= \frac{5}{2}\int_0^{2\pi}\sin 2t\,dt$$

$$= -\frac{5}{4}\cos 2t\Big|_0^{2\pi}$$

$$= \frac{-5}{4}(1-1)$$

$$\boxed{=0}$$

3 $\vec{F} = \frac{1}{3}\langle x^3, y^3, z^3 \rangle$

Sphere: $x^2+y^2+z^2 = 9$

$$\iint \vec{F}\cdot\vec{n}\,ds = \iiint_D \nabla\cdot\vec{F}\,dV.$$

$$\nabla\cdot\vec{F} = \frac{1}{3}\cdot\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\rangle\cdot\langle x^3, y^3, z^3\rangle$$

$$= \frac{1}{3}(3x^2+3y^2+3z^2)$$

$$= x^2+y^2+z^2$$

$$\iiint_D \nabla\cdot\vec{F}\,dV = \iiint (x^2+y^2+z^2)\,dV$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{3} \rho^2\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$$= \int_0^{2\pi}d\theta\int_0^{\pi}\sin\varphi\,d\varphi\int_0^{3}\rho^4\,d\rho$$

$$= 2\pi\left(-\cos\varphi\Big|_0^{\pi}\right)\left(\frac{1}{5}\rho^5\Big|_0^3\right)$$

$$= 4\pi\left(\frac{3^5}{5}\right)$$

$$= \frac{4\pi\cdot 3^5}{5}$$