

## Review

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, \\ x_1 + 5x_2, x_3)$$

$$= \begin{bmatrix} x_1 + 2x_2 + x_3 \\ x_1 + 5x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

standard matrix is:  $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$T$ -transf is Linear Domain  $A \ni T = f_n$

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$$

$$\text{let } \vec{u} = (u_1, u_2, u_3)$$

$$\vec{v} = (v_1, v_2, v_3)$$

$$\begin{aligned} T(\vec{u} + \vec{v}) &= T(u_1 + v_1, u_2 + v_2, u_3 + v_3) \\ &= (u_1 + v_1 - 5(u_2 + v_2) + 4(u_3 + v_3), \\ &\quad u_2 + v_2 - 6(u_3 + v_3)) \\ &= (u_1 + v_1 - 5u_2 - 5v_2 + 4u_3 + 4v_3, \\ &\quad u_2 + v_2 - 6u_3 - 6v_3) \\ &= ((u_1 - 5u_2 + 4u_3) + (v_1 - 5v_2 + 4v_3), \\ &\quad (u_2 - 6u_3) + (v_2 - 6v_3)) \\ &= (u_1 - 5u_2 + 4u_3, u_2 - 6u_3) + \\ &\quad (v_1 - 5v_2 + 4v_3, v_2 - 6v_3) \\ &= T(u_1, u_2, u_3) + T(v_1, v_2, v_3) \\ &= T(\vec{u}) + T(\vec{v}) \quad \checkmark \end{aligned}$$

$\vec{u}$ 's closed under addition

$$\begin{aligned}
T(\lambda \vec{u}) &= T(\lambda(u_1, u_2, u_3)) \\
&= T(\lambda u_1, \lambda u_2, \lambda u_3) \\
&= (\lambda u_1 - 5\lambda u_2 + 4\lambda u_3, \lambda u_2 - 6\lambda u_3) \\
&= \lambda(u_1 - 5u_2 + 4u_3, \underline{u_2 - 6u_3}) \\
&= \lambda T(u_1, u_2, u_3) \\
&= \lambda T(\vec{u})
\end{aligned}$$

It's closed under scalar multiplication

Since  $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$  and  $T(\lambda \vec{u}) = \lambda T(\vec{u})$ , then function  $T$  is a linear transformation

$$\hookrightarrow T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

Domain

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$$

$$= \begin{pmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{pmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{pmatrix} \quad R_3 - 3R_1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{pmatrix} = U$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= \lambda^2 - 4\lambda + 5$$

Characteristic eqn:  $\lambda^2 - 4\lambda + 5 = 0$

eigen values:  $\lambda_{1,2} = 2 \pm i$

~~$\frac{+4 \pm \sqrt{16-20}}{2}$~~   
 $\frac{4 \pm 2i}{2}$

For  $\lambda_1 = 2 - i \Rightarrow (A - \lambda_1 I) V_1 = 0$

$$\begin{pmatrix} i & -1 \\ 1 & i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$i^{\rightarrow} x = i^{\rightarrow} y \Leftarrow$$

eigenvectors

$$V_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$V_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$a \overrightarrow{x} = b \overrightarrow{y} \quad \begin{pmatrix} b \\ a \end{pmatrix}$$

$$A = \begin{pmatrix} 5 & 3 \\ -6 & -4 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 5-\lambda & 3 \\ -6 & -4-\lambda \end{vmatrix}$$

$$= \lambda^2 - \lambda - 2 = 0$$

Characteristic eqn:  $\lambda^2 - \lambda - 2 = 0$

Eigenvalues:  $\lambda_1 = -1, \lambda_2 = 2$

For  $\lambda_1 = -1 \Rightarrow (A - \lambda_1 I)v_1 = 0$

$$\begin{pmatrix} 6 & 3 \\ -6 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{aligned} 6x_1 &= -3y_1 \\ 2x_1 &= -y_1 \end{aligned}$$

$$v_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

For  $\lambda_2 = 2 \Rightarrow (A - \lambda_2 I)v_2 = 0$

$$\begin{pmatrix} 3 & 3 \\ -6 & -6 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad x_2 = -y_2$$

$$v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -6 - \lambda & 5 \\ -5 & 4 - \lambda \end{vmatrix} \\ = \lambda^2 + 2\lambda + 1 = 0$$

Eigen values:  $\lambda_{1,2} = -1$

$$\text{For } \lambda = -1 \Rightarrow (A - \lambda I)V_1 = 0$$

$$\begin{pmatrix} -5 & 5 \\ -5 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow x_1 = y_1$$

$$\underline{V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

$$AV_2 = V_1$$

$$\begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \\ -6x_2 + 5y_2 = 1 \quad (\text{line})$$

$$\underline{V_2 = \begin{pmatrix} 0 \\ 1/5 \end{pmatrix}} \quad \text{or} \quad \begin{pmatrix} 1/6 \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 2 & -5 & -6 \\ -2 & 3 & 4 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & 0 & 0 \\ 2 & -5-\lambda & -6 \\ -2 & 3 & 4-\lambda \end{vmatrix}$$

$$= (-1-\lambda)(-\lambda^2 + \lambda + 15)$$

$$= -(1+\lambda)(\lambda^2 + \lambda - 2) = 0 \leftarrow$$

$$\underline{\lambda_1 = -1, 1, -2}$$

$$\text{Characteristic eqn: } \underline{-\lambda^3 - 2\lambda^2 + \lambda + 2 = 0}$$