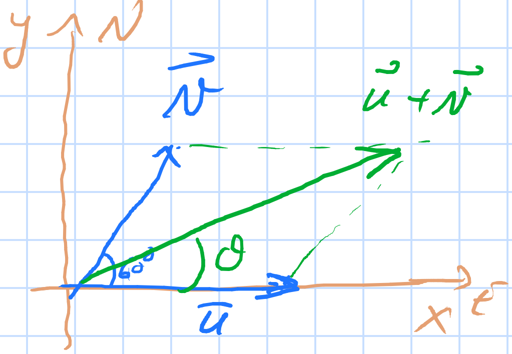
Ex Given: $|\vec{u}| = 500$

$|\vec{v}| = 70$

Find $|\vec{u}+\vec{v}|$ θ?



Soln

$$\vec{u} = \langle |\vec{u}|\cos 0°, |\vec{u}|\sin 0° \rangle$$
$$= \langle 500, 0 \rangle$$

$$\vec{v} = \langle |\vec{v}|\cos 60°, |\vec{v}|\sin 60° \rangle$$
$$= \langle 70\left(\tfrac{1}{2}\right), 70\left(\tfrac{\sqrt{3}}{2}\right) \rangle$$
$$= \langle 35, 35\sqrt{3} \rangle$$

$$\vec{u}+\vec{v} = \langle 535, 35\sqrt{3} \rangle$$

$$|\vec{u}+\vec{v}| = \sqrt{535^2 + 3(35^2)} \approx 538.4$$

$$\theta = \tan^{-1}\frac{35\sqrt{3}}{535} \approx 6.5°$$

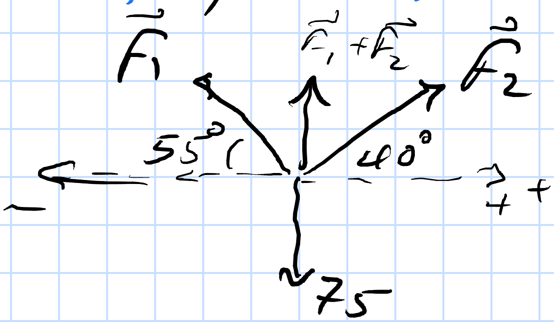
The new ground speed of the airplane is about ~~[illegible]~~ or 538.4 and in direction 6.5° N to east E 6.5° N

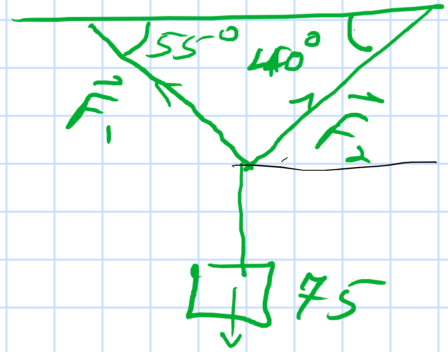
Ex Given: $|\vec{F}| = 20$ $\theta = 45^\circ$

$\vec{F} = \langle a, b \rangle$

$$\vec{F}_x = |\vec{F}| \cos 45^\circ$$
$$= 20\left(\frac{\sqrt{2}}{2}\right)$$
$$= \boxed{10\sqrt{2} \text{ lb}}$$

Ex Find $\vec{F}_1$, $\vec{F}_2$?





soln

$$\begin{cases} \vec{F}_1 = \langle -|\vec{F}_1|\cos 55^\circ, |\vec{F}_1|\sin 55^\circ \rangle \\ \vec{F}_2 = \langle |\vec{F}_2|\cos 40^\circ, |\vec{F}_2|\sin 40^\circ \rangle \end{cases}$$

$$\vec{F}_1 + \vec{F}_2 = \langle 0, 75 \rangle$$

$$\begin{cases} -|\vec{F}_1|\cos 55^\circ + |\vec{F}_2|\cos 40^\circ = 0 \\ |\vec{F}_1|\sin 55^\circ + |\vec{F}_2|\sin 40^\circ = 75 \end{cases}$$

$$\Delta = \begin{vmatrix} -\cos 55^\circ & \cos 40^\circ \\ \sin 55^\circ & \sin 40^\circ \end{vmatrix}$$

$$|\vec{F}_1| = \frac{-75\cos 40^\circ}{-\cos 55^\circ \sin 40^\circ - \cos 40^\circ \sin 55^\circ}$$

$$|\vec{F}_2| = \frac{75\cos 40^\circ \cos 55^\circ}{\cos 55^\circ \sin 40^\circ + \cos 40^\circ \sin 55^\circ}\left(\frac{1}{\cos 40^\circ}\right)$$

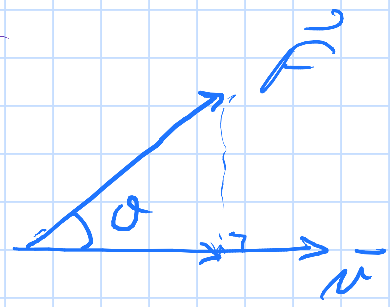
$|\vec{F}_1| \approx 57.67$ $\qquad$ $|\vec{F}_2| \approx 43.18\,N$

$$\vec{F}_1 \approx \langle -57.67\cos 55^\circ, 57.67\sin 55^\circ \rangle$$
$$= \langle -33.08, 47.24 \rangle$$

$$\vec{F}_2 \approx \langle 33.08, 27.76 \rangle$$

✓ 0 $\qquad$ 75 ✓

1.2 – Dot Products



length $= |\vec{F}| \cos\theta$

$\vec{u} = \langle u_1, u_2, u_3 \rangle$

$\vec{v} = \langle v_1, v_2, v_3 \rangle$

$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

angle:

$$\theta = \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

Cosine θ:

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

Ex

$$\langle 1, -2, 1 \rangle \cdot \langle -6, 2, -3 \rangle = -6 - 4 - 3 = \boxed{-13}$$

$$\left(\tfrac{1}{2}\hat{i} + 3\hat{j} + \hat{k}\right) \cdot \left(4\hat{i} - \hat{j} + 2\hat{k}\right) = 2 - 3 + 2 = \boxed{1}$$

Ex θ? $\vec{u} = \hat{i} - 2\hat{j} - 2\hat{k}$ $\quad \vec{v} = 6\hat{i} + 3\hat{j} + 2\hat{k}$

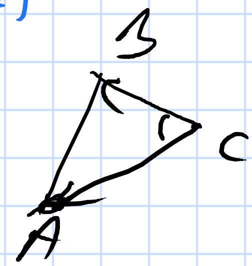
$$\theta = \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \cos^{-1} \frac{(\hat{i} - 2\hat{j} - 2\hat{k}) \cdot (6\hat{i} + 3\hat{j} + 2\hat{k})}{\sqrt{1+4+4}\ \sqrt{36+9+4}}$$

$$= \cos^{-1} \frac{6 - 6 - 4}{21}$$

$$= \boxed{\cos^{-1}\left(-\frac{4}{21}\right)}$$

Ex $C = \theta$? $A(0,0)$ $B(2,5)$ $C(5,2)$



$\vec{CA} = \langle -5, -2 \rangle$

$\vec{CB} = \langle -2, 3 \rangle$

$C = \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$

$$C = \theta = \cos^{-1} \frac{\langle -5, -2 \rangle \cdot \langle -2, 3 \rangle}{\sqrt{25+4}\ \sqrt{4+9}}$$

$$= \cos^{-1} \left(\frac{4}{\sqrt{29}\ \sqrt{13}} \right)$$

$\perp$ (orthogonal) $\perp$

$\vec{u} \cdot \vec{v} = 0$ when $\vec{u} \perp \vec{v}$

Ex $\vec{u} = \langle 3, -2 \rangle$ $\vec{v} = \langle 4, 6 \rangle$

$\vec{u} \cdot \vec{v} = 12 - 12$
$= 0$ ✓ $\vec{u} \perp \vec{v}$

$\vec{u} = 3\vec{i} - 2\vec{j} + \vec{k}$

$\vec{v} = 2\vec{j} + 4\vec{k}$

$\vec{u} \cdot \vec{v} = 0 - 4 + 4$
$= 0$ the 2 vectors are orthogonal

$$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u} \quad (1)$$

$$\vec{u}\cdot\vec{u} = |\vec{u}|^2$$

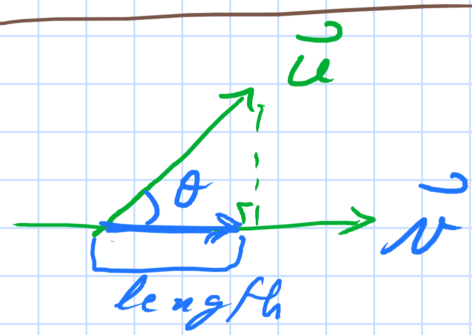
$$\vec{u}\cdot(\vec{v}\pm\vec{w}) = \vec{u}\cdot\vec{v} \pm \vec{u}\cdot\vec{w}$$

$$(\vec{v}+\vec{w})\cdot\vec{u} = \vec{v}\cdot\vec{u} + \vec{w}\cdot\vec{u}$$

$$\vec{u}\cdot(c\vec{v}) = c\,(\vec{u}\cdot\vec{v})$$

$$\vec{u}\cdot\vec{0} = 0 = \vec{0}\cdot\vec{v}$$

$$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v}$$



Ex
$$\vec{u} = 6\hat{i} + 3\hat{j} + 2\hat{k}$$
$$\vec{v} = \hat{i} - 2\hat{j} - 2\hat{k}$$

$\vec{u}\cdot\vec{v}$

$$\text{proj}_{\vec{v}}\vec{u} = \frac{6-6-4}{1+4+4}(\hat{i}-2\hat{j}-2\hat{k})$$

$|\vec{v}|^2$

$$= -\frac{4}{9}\hat{i} + \frac{8}{9}\hat{j} + \frac{8}{9}\hat{k}$$

scalar component of $\vec{u}$ ~~d~~irection $\vec{v}$

$$|\vec{u}|\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} \text{ or } \vec{u}\cdot\frac{\vec{v}}{|\vec{v}|}$$

$$|\vec{u}|\cos\theta = \frac{-4}{3}$$

Ex Given: $\vec{F} = 5\hat{i} + 2\hat{j}$ onto $\vec{v} = \hat{i} - 3\hat{j}$

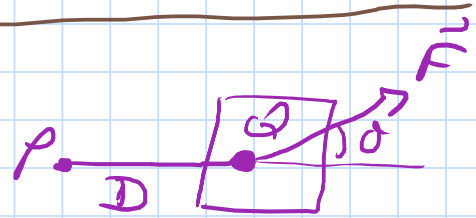
$$\text{proj}_{\vec{v}}\vec{F} = \frac{\vec{F}\cdot\vec{v}}{|\vec{v}|^2}\,\vec{v}$$

$$= \frac{5-6}{1+9}(\hat{i} - 3\hat{j})$$

$$= -\frac{1}{10}\hat{i} + \frac{3}{10}\hat{j}$$

$$|\vec{F}|\cos\theta = \frac{-1}{\sqrt{10}}$$

$$\text{Work} = \vec{F}\cdot\vec{D}$$



$$= |\vec{F}||D|\cos\theta$$

Ex $|\vec{F}| = 40\,N$ $\quad |\vec{D}| = 3$ $\quad \theta = 60^\circ$

$$W = (40)(3)\cos 60^\circ$$
$$= 60 \quad (N\cdot m)$$

(joules) $\left(\frac{b}{J}\right)$

1.3 - Cross-Product

$$\text{Volume} = |\vec{a} \cdot \underbrace{\vec{b} \times \vec{c}}_{\text{vector}}|$$

$$= \left| \underbrace{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}_{\text{determinant}} \right|$$

absolute Value

a_1
a_2
a_3

Def: $\vec{u} \times \vec{v} = (|\vec{u}|\,|\vec{v}| \sin\theta) \cdot \vec{n}$

$$\vec{u} \times \vec{v} = 0 \Rightarrow \vec{u} \parallel \vec{v}$$

$\Rightarrow \vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$ $\qquad \vec{u} \times \vec{v} = \vec{n}$

$\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$

$(\vec{v} + \vec{w}) \times \vec{u} = \vec{v} \times \vec{u} + \vec{w} \times \vec{u}$

$(r\vec{u}) \times \vec{v} = r(\vec{u} \times \vec{v})$

$(r\vec{u}) \times (s\vec{v}) = (rs)(\vec{u} \times \vec{v})$

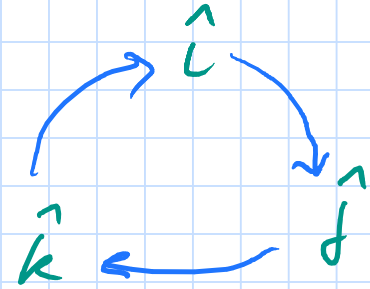
$\vec{u} \times \vec{0} = \vec{0}$

$\vec{u} \times \vec{u} = \vec{0}$

$\hat{i} \times \hat{i} = \vec{0}$

$\hat{j} \times \hat{j} = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$



$\hat{i} \times \hat{j} = \hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\vec{u} = \langle u_1, u_2, u_3 \rangle \qquad \vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\vec{u} \times \vec{v} = \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{matrix} \right| \begin{matrix} \\ u_1 & u_2 \\ v_1 & v_2 \end{matrix}$$

$$= (u_2 v_3 - u_3 v_2)\,\hat{i} + (u_3 v_1 - u_1 v_3)\,\hat{j} + (u_1 v_2 - v_1 u_2)\,\hat{k}$$

Ex $\vec{u} = 2\hat{j} + \hat{j} + \hat{k} \qquad \vec{v} = -4\hat{i} + 3\hat{j} + \hat{k}$

$$\vec{u} \times \vec{v} = \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{matrix} \right|$$

$$= -2\hat{i} - 6\hat{j} + 10\hat{k}$$

$$\vec{v} \times \vec{u} = 2\hat{i} + 6\hat{j} - 10\hat{k}$$

Ex $P(1,-1,0)\ Q(2,1,-1)\ R(-1,1,2)$

$\vec{n}$? $\vec{PQ} \times \vec{PR}$

$\vec{PQ} = \langle 1, 2, -1 \rangle$

$\vec{PR} = \langle -2, 2, 2 \rangle$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{vmatrix}$$

$$= 6\hat{i} + 6\vec{k}$$

b) Area of triangle PQR

$$\text{Area} = \frac{1}{2} |\vec{PQ} \times \vec{PR}|$$

$$= \frac{1}{2}\sqrt{36+36} \qquad 6\sqrt{2}$$

$$= 3\sqrt{2} \text{ unit}^2$$

c) unit vector $\perp$ PQR

$$\vec{n} = \frac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|}$$

$$= \frac{6}{6\sqrt{2}}\hat{i} + \frac{6}{6\sqrt{2}}\vec{k}$$

$$= \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\vec{k}$$

$$\text{Torque} = d \cdot |\vec{F}| \sin\theta$$

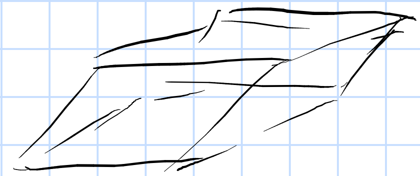
Ex $\quad d = 3 \quad \theta = 70^\circ \quad |\vec{F}| = 20\,\text{lb}$

$$\text{Torque} = 3(20)\sin 70^\circ$$

$$= \boxed{60 \sin 70^\circ \ \text{ft-lb}}$$

$$\approx 56.4$$

Parallelepiped



$$\vec{u} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{v} = -2\hat{i} + 3\hat{k}$$

$$\vec{w} = 7\hat{j} - 4\hat{k}$$

$$V = \left| \begin{vmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \\ 0 & 7 & -4 \end{vmatrix} \right|$$

$$= |-23|$$

$$= \boxed{23 \ \text{unit}^3}$$