MATH 1324 - Finite *Professor*: Fred Khoury

- 1. A shipment of 20 radios contains 7 defective radios. A sample of 5 radios is randomly selected.
 - a) Write the probability distribution table for the number of defective radios in the sample.
 - b) Find the expected number of defective radios.
- 2. A friend offer the following game: she wins \$1 from you if, on four rolls of a single die, a 6 turns up at least once; otherwise you win \$1 from her. What is the expected value of the game to you? To her?
- **3.** A carton of 20 watch batteries contains 2 dead ones. A random sample of 3 is selected from 20 and tested. Let *X* be the random variable associated with the number of dead batteries found in a sample.
 - a) Find the probability distribution of X.
 - b) Find the expected number of dead batteries in a sample.
- **4.** Find the mode(s), median, mean, and the standard deviation given the ungrouped data:
 - a) 1, 2, 3, 3, 3, 5, 6, 7, 7, 7, 9
 - b) 3, 5, 6, 4, 3, 6, 2, 2, 3, 5
 - c) 2, 3, 5, 5, 6, 4, 2, 3, 2, 7, 8
 - *d*) 2, 3, 6, 5, 6, 4, 2, 6, 2, 7, 8
- **5.** Find the mean and standard deviation for the grouped data in the following table:

a)

X	Frequency
12	2
17	6
22	12
27	7
32	2

b)

e)

Interval	Frequency
0.5 - 1.5	8
1.5 - 2.5	6
2.5 - 3.5	6
3.5 - 4.5	4
4.5 - 5.5	6

c)

Interval	Frequency
0.5 - 3.5	7
3.5 - 6.5	6
6.5 - 9.5	2

d)

Interval	Frequency
9.5 - 12.5	3
12.5 - 15.5	2
15.5 - 18.5	7
18.5 - 21.5	4

 Interval
 Frequency

 0.5 - 1.5
 2

 1.5 - 2.5
 5

 2.5 - 3.5
 6

 3.5 - 4.5
 4

 4.5 - 5.5
 1

- **6.** If a baseball player has a batting average of 0.287, what the probability that the player will get the following number of hits in the next 4 times at bat?
 - a) Exactly 3 hits
 - b) At least 3 hits

- 7. Each year a company selects a number of employees for a management training program given by a nearby university. On the average, 70% of those sent complete the program. Out of 7 people sent by the company, what is the probability that
 - a) Exactly 5 complete the program?
 - b) 5 or more complete the program?
- **8.** A basketball player has 75% chance of making any given free throw. If the player shoots five free throws in succession, find the following:
 - a) Probability the player makes exactly 3 free throws.
 - b) Probability the player makes at least 3 free throws.
 - c) The probability distribution
- 9. A survey was taken of employees ages 25 35 at a web-design company. The survey found that 85% of the employees had a bachelor's degree. If a random sample of 12 employees is taken.
 - a) What is the probability that 9 of them will have a bachelor's degree?
 - b) What is the probability that at least 9 of them will have a bachelor's degree?
- **10.** A survey of freshmen students at a large university found that 72% had gained at least 10 pounds since starting college. If a random sample of 6 freshmen were asked if they had gained at least 10 pounds since starting college, what is the probability that 3 had gained at least 10 pounds? What is the probability that 5 had gained at least 10 pounds?
- 11. In a group of registered voters it is found that 25% favor negative ads by the candidate that they support. In a random sample of 15 registered voters, what is the probability that 10 of them favor negative ads by the candidate they support? What is the probability that all 15 favor the negative ads? (You will have to use more than 4 decimal places to answer.)
- **12.** A multiple choice test is given with 5 choices only one is correct, for each of 5 questions. Answering each of the 5 questions by guessing constitutes a binomial experiment with an associated binomial distribution
 - a) Write the function defining the distribution.
 - b) Construct a table for the distribution.
 - c) Draw a histogram
 - d) Compute the mean and standard deviation
- 13. Suppose a fair die is rolled two times and a success on a single roll is considered to be rolling a number divisible by 2.
 - a) Write the function defining the distribution.
 - b) Construct a table for the distribution.
 - c) Draw a histogram
 - d) Compute the mean and standard deviation
- **14.** A cloth store claims that their card is used by 60% of the customer buying cloth in a particular town. A random sample of 25 customers is made. What is the probability that
 - a) From 10 to 18 customer in the sample use the card
 - b) More than 20 people

- 15. Bolts produced by a machine are acceptable provided that their length is within the range from 5.95 to 6.05 cm. Suppose that the lengths of the bolts produced are normally distributed with $\mu = 6$ cm and $\sigma = 0.02$. What is the probability that a bolt will be of an acceptable length?
- **16.** Finite tests are scaled so that the mean score is 400 and the standard deviation is 100. What percentage of the students taking the test should score 750 or more? Assume a normal distribution.
- 17. The average lifetime for a car battery of a certain brand is 170 weeks, with a standard deviation of 10 weeks. If the company guarantees the battery for 3 years, what percentage of the batteries sold would be expected to be returned before the end of the warranty period? Assume a normal distribution.
- **18.** The duration of routine operations in a certain hospital has approximately a normal distribution with a mean of 130 minutes and a standard deviation of 15 minutes. What percentage of operations last longer than 150 minutes?
- 19. The average healing time of a certain type of incision is 240 hours, with standard deviation of 20 hours. What percentage of the people having this incision would heal in 8 days or less? Assume a normal distribution.
- **20.** The average height of a hay crop is 38 inches, with a standard deviation of 1.5 inches. What percentage of the crop will be 40 inches or more? Assume a normal distribution
- 21. For certain types of fluorescent lights the number of hours a bulb will burn before requiring replacement has a mean of 3000 hours and a standard deviation of 250 hours. Suppose that 5000 such bulbs are installed in an office building. Estimate the number that will require replacement between 2250 and 3750 hours from the time of installation.
- 22. A union representative claims 60% of the union membership will vote in favor of a particular settlement. A random sample of 100 members is polled, and out of these, 47 favor the settlement. What is the approximate probability of 47 or fewer in a sample of 100 favoring the settlement when 60% of all the membership favors the settlement? Conclusion? Approximate a binomial distribution with a normal distribution.
- **23.** A certain brand of light bulb has a lifetime expectancy that is approximately normally distributed. The mean is 800 hours with standard deviation of 50 hours. What is the probability that a light bulb will work for
 - a) more than 950 hours?
 - b) more than 850 hours
 - c) less than 690 hours
 - d) less than 605 hours?
- **24.** The lifetime of a certain brand of computer is approximately normally distributed, with a mean of 5 years and standard deviation of 20 weeks. The computer carries a warranty of 4 years.
 - a) What percentage of the computers will fail before the warranty expires?
 - b) What percentage of the computers will still be operating after 6 years?

Solution

1.

X	0	1	2	3	4	5
P	0.083	0.323	0.387	0.176	0.029	0.0014

$$E(X) = 1.748$$

- 2. a) E(X: for him) = -0.036
- **b**) E(X: for her) = -0.036

3. a)

Γ	X	0	1	2
Ī	P	.716	.268	.016

- **b**) E(X) = .3
- **4.** *a*) Modes: 3, 7

Median: 5

Mean = 4.82

$$\sigma = 2.56$$

$$\sigma = \sqrt{\frac{\left(1 - 4.82\right)^2 + \left(2 - 4.82\right)^2 + 3\left(3 - 4.82\right)^2 + \left(5 - 4.82\right)^2 + \left(6 - 4.82\right)^2 + 3\left(7 - 4.82\right)^2 + \left(9 - 4.82\right)^2}{11 - 1}}$$

- **b**) Modes: 3
- Median: 3.5
- Mean = 3.9
- $\sigma = 1.523$

- *c*) Modes: 2
- Median: 4
- Mean = 4.2727
- $\sigma = 2.102$

- *d*) Modes: 2, 6 Median: 5
 - Mean = 4.6364
- $\sigma = 2.157$

- 5. *a*) Mean = 22.17
- $\sigma = 5.085$
- **b**) Mean = 2.8
- $\sigma = 1.495$
- c) Mean = 4
- $\sigma = 2.171$
- **d**) Mean = 16.25
- $\sigma = 3.19$
- *e*) Mean = 2.83
- $\sigma = 1.098$
- **6. a.** $P(X=3) = C_{4,3}(.287)^3(.713)^1 = 0.067$

b.
$$P(X = 3) + P(X = 4) = C_{4,3}(.287)^3(.713)^1 + C_{4,4}(.287)^4(.713)^0 = 0.074$$

- 7. a) P(5) = .318
 - **b**) P(x > 5) = .647
- a) $P(x=3) = C_{5,3}(0.75)^3(0.25)^2 \approx 0.2637$ 8.
 - **b**) 0.8965

$$P(x=2) = C$$
 $(0.75)^2 (0.25)^3 \approx 0.0879$

$$P(x=4) = C_{5,4} (0.75)^4 (0.25)^1 \approx 0.3955$$
 $P(x=5) = C_{5,5} (0.75)^5 (0.25)^0 \approx 0.2373$

c)
$$P(x=0) = C_{5,0} (0.75)^0 (0.25)^5 \approx 0.001$$
 $P(x=1) = C_{5,1} (0.75)^1 (0.25)^4 \approx 0.0146$

$$P(x=2) = C_{5,2} (0.75)^2 (0.25)^3 \approx 0.0879$$
 $P(x=3) = C_{5,3} (0.75)^3 (0.25)^2 \approx 0.2637$

$$P(x=5) = C_{5.5} (0.75)^5 (0.25)^0 \approx 0.2373$$

x	0	1	2	3	4	5
p	0.001	0.0146	0.0879	0.2637	03955	0.2373

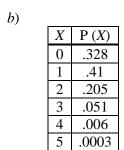
9. a)
$$P(X) = C_{12.9} (.85)^9 (.15)^3 = 0.172$$

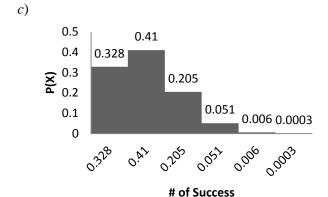
b)
$$P(X) = P(X=9) + P(X=10) + P(X=11) + P(X=12)$$

= $C_{12,9}(.85)^9(.15)^3 + C_{12,10}(.85)^{10}(.15)^2 + C_{12,11}(.85)^{11}(.15)^1 + C_{12,12}(.85)^{12}(.15)^0$
= .9078

- **10.** 0.1639; 0.3251
- **11.** .0007; .0000000009

12. *a*)
$$P(X) = C_{5,x}(.2)^x (.8)^{5-x}$$



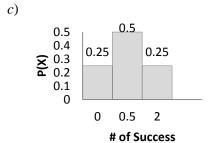


d)
$$\mu = 1$$
 $\sigma = \sqrt{npq} = \sqrt{5(.2)(.1)} = .894$

13. a)
$$\Rightarrow P(X = 2) = C_{2,x} p^x q^{2-x}$$
 $x \in \{0,1,2\}$

b)
$$p = \frac{3}{6} = .5 \rightarrow q = .5$$

$$\begin{array}{c|c}
X & P(X) \\
\hline
0 & .25 \\
\hline
1 & .5 \\
\hline
2 & .25 \\
\end{array}$$



d)
$$\mu = 1$$
 $\sigma = .707$

14. a)
$$\mu = 15$$
 $\sigma = 2.45$
$$z_1 = \frac{9.5 - 15}{2.45} = -2.24 \rightarrow A_1 = .0125 \quad z_2 = \frac{18.5 - 15}{2.45} = 1.43 \rightarrow A_2 = .9236 \quad A = .9111$$
 b) $z = 1.84 \rightarrow A = 1 - .9671 = 0.329 | \Rightarrow A = .0329$

15.
$$z_1 = -2.5 \rightarrow A_1 = .0062$$
 $z_2 = 2.5 \rightarrow A_2 = .9938$ $\Rightarrow A = .9876$

- **16.** $z = 3.5 \rightarrow A = 1 .9999 = 0.0001 \implies P = .01\%$
- **17.** $P(x \le 156) = 0.0808$ *or* 8.08%
- **18.** P = 0.0918 = 9.18%
- **19.** $P(x \le 192) = .0082 \text{ or } .82\%$
- **20.** $P(x \ge 40) = 1 0.9082 = 0.0918$ *or* 9.18%
- **21.** # of replacement = 4987
- 22. P(<47) = .0054; This a rare situation or the claim is false.
- **23.** *a*) .0013 *b*) .1587 *c*) .0139 *d*) 0
- **24.** *a*) .47% *b*) .47%