

Lecture Three – Probability

Section 3.1 – Sets

Set Properties & Notation

- $\{b \in B\}$ b is an element of B .
- \emptyset : The empty set
- $A \subset B$: Subset
- $A \not\subset B$: Not a Subset
- $\emptyset \subset A$
- $A = B$: Equal To
- $A \neq B$: Not Equal To
- $A \cup B$ Union
- $A \cap B$ Intersection
- $\{A'\}$ Complement

Set Notations

- U is the universal set. The universal set contains all of the elements.
- $A \cup B$ is read “ A union B ”. An element is in the set $A \cup B$ if the element lies in either set A or set B .
- $A \cap B$ is read “ A intersect B ”. An element is in the set $A \cap B$ if the element lies (simultaneously) in both set A and set B .
- A' or \overline{A} is read “ A complement” or “not A ”. An element is in the set A' if the element lies outside of set A .
- The capital letter notation A defines the set A , $n(A)$ is the notation used to give the number of elements in set A .
- Example: $A = \{2,4,6,8\}$, $n(A) = 4$
- \emptyset represents the empty set. {a set with no elements, like, the number of pregnant men in class}

Example

Write the elements $\{x \mid x \text{ is a natural number less than } 5\}$

Solution

$\{1, 2, 3, 4\}$

Subset

Set A is a subset of set B (written $A \subseteq B$) if every element of A is also an element of B . Set A is a proper subset (written $A \subset B$) if $A \subseteq B$ and $A \neq B$

Example

Decide whether the following statements are true or false

a) $\{3, 4, 5, 6\} = \{4, 6, 3, 5\}$

b) $\{5, 6, 9, 12\} \subseteq \{5, 6, 7, 8, 9, 10, 11\}$

Solution

a) $\{3, 4, 5, 6\} = \{4, 6, 3, 5\}$

True, since both sets contain exactly the same element.

b) $\{5, 6, 9, 12\} \subseteq \{5, 6, 7, 8, 9, 10, 11\}$

False, since first set contains 12 that doesn't belong to the second set.

Example

List all possible subsets for $\{7, 8\}$

Solution

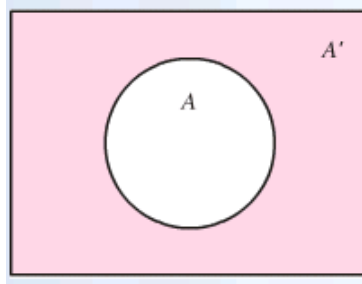
There are 4 subsets:

$$\emptyset, \{7\}, \{8\}, \{7, 8\}$$

Complement of a Set

Let A be any set, with U representing the universal set, then the complement of A .

$$A' = \{x \mid x \notin A \text{ and } x \in U\}$$



Example

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $A = \{1, 2, 3, 4, 5, 7\}$, and $B = \{2, 4, 5, 7, 9, 11\}$. Find each set.

- a) A'
- b) B'
- c) $\phi' = U$ and $U' = \phi$
- d) $(A')' = A$

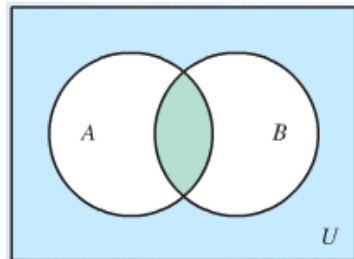
Solution

- a) $A' = \{6, 8, 9, 10, 11\}$
- b) $B' = \{1, 3, 6, 8, 10\}$
- c) $\phi' = U$ and $U' = \phi$
- d) $(A')' = A$

Intersection of Two Sets

The intersection of sets A and B , is:

$$A \cap B = \{e \in S \mid e \in A \text{ and } e \in B\}$$



Example

Let $A = \{3, 6, 9\}$, $B = \{2, 4, 6, 8\}$, and $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a) $A \cap B$

b) $A \cap B'$

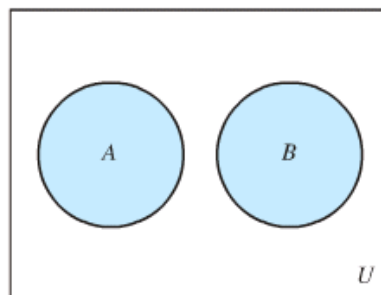
Solution

a) $A \cap B = \{6\}$

b) $A \cap B' = \{3, 6, 9\} \cap \{0, 1, 3, 5, 7, 9, 10\} = \{3, 9\}$
 $= \{3, 9\}$

Disjoint Sets

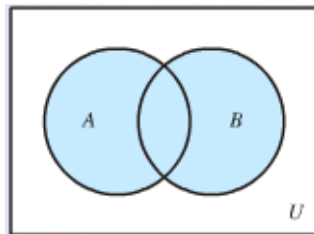
For any sets A and B , if A and B are disjoint sets, then $A \cap B = \phi$



Union of Two Sets

The union of sets A and B , is:

$$A \cup B = \{e \in S \mid e \in A \text{ or } e \in B\}$$



Example

Let $A = \{1, 3, 5, 7, 9, 11\}$, $B = \{3, 6, 9, 12\}$, $C = \{1, 2, 3, 4, 5\}$, and $U = \{0, 1, 2, \dots, 11, 12\}$ Find each set.

a) $A \cup B$

b) $(A \cup B) \cap C'$

Solution

a) $A \cup B = \{1, 3, 5, 6, 7, 9, 11, 12\}$

b) $(A \cup B) \cap C' = \{1, 3, 5, 6, 7, 9, 11, 12\} \cap \{0, 6, 7, 8, 9, 10, 11, 12\}$
 $= \{6, 7, 9, 11, 12\}$

Example

A department store classifies credit applicants by gender, marital status, and employment status. Let the universal set be the set of all applicants, M be the set of male applicants, S be the set of single applicants, and E be the set of employed applicants. Describe each set in words.

a) $M \cap E$

b) $M' \cup S'$

c) $M' \cap S'$

Solution

a) $M \cap E$

This set includes all applicants who are both male and employed; that is, employed male applicants

b) $M' \cup S'$

This set includes all applicants who are female (not male) or married, all married female applicants are in this set

c) $M' \cap S'$

This set includes all applicants who are female and married, this is the set of all married female applicants

Exercises Section 3.1 – Sets

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $X = \{2, 4, 6, 8\}$, $Y = \{2, 3, 4, 5, 6\}$, and $Z = \{1, 2, 3, 8, 9\}$
- a) $X \cap Y$ b) $X \cup Y$ c) Y' d) $X' \cap Z$
e) $Y \cap (X \cup Z)$ f) $X' \cap (Y' \cup Z)$ g) $(X \cap Y') \cup Z'$
2. Given $A = \{0, 2, 4, 6\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{2, 6, 0, 4\}$, determine if the statement is true or false?
- a) $A \subset B$ b) $A \subset C$ c) $A = B$ d) $C \subset B$
e) $B \not\subset A$ f) $\emptyset \subset B$
3. Given $R = \{1, 2, 3, 4\}$, $S = \{1, 3, 5, 7\}$, $T = \{2, 4\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, find the following:
- a) $R \cup S$ b) $R \cap S$ c) $S \cap T$ d) S'
4. Write true or false for each statement
- a) $3 \in \{2, 5, 7, 9, 10\}$ b) $6 \in \{-2, 5, 6, 9\}$
c) $9 \notin \{2, 1, 5, 8\}$ d) $3 \notin \{7, 6, 5, 4, 10\}$
e) $\{2, 5, 8, 9\} = \{2, 5, 9, 8\}$ f) $\{3, 7, 12, 14\} = \{3, 7, 12, 14, 9\}$