Solution *Section* 1.3 – Solving Linear Programming and Applications

Exercise

A manufacturing plant makes two types of inflatable boats, a two-person boat and a four-person boat. Each two-person boat requires 0.9 labor-hour in the cutting department and 0.8 labor-hour in the assembly department. Each four-person boat requires 1.8 labor-hours in the cutting department and 1.2 labor-hours in the assembly department. The maximum labor-hours available each month in the cutting and assembly departments are 864 and 672, respectively. The company makes a profit of $25 on each two-person boat and $40 on each four-person boat

a) Identify the decision variables
b) Summarize the relevant material in a table
c) Write the objective function P.
d) Write the problem constraints and the nonnegative constraints
e) Determine how many boats should be manufactured each month to maximize the profit. What is the maximum profit?

Solution

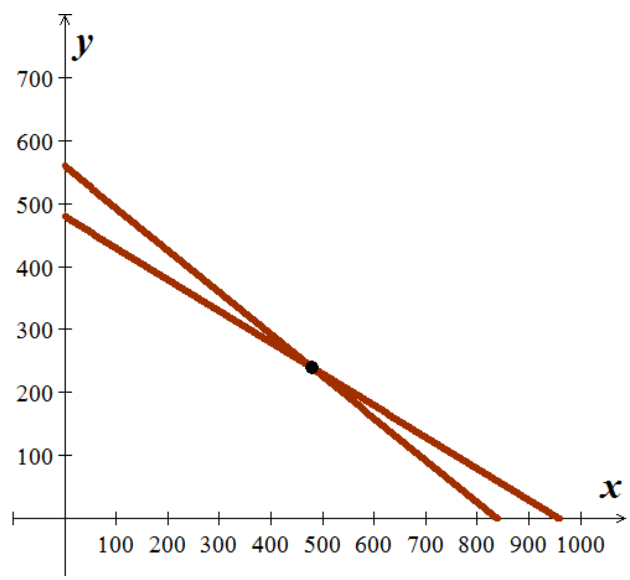
a) x = number of two- person boats produced each month
y = number of four- person boats produced each month

b)

	two-person	four-person		**Max**
Cutting	.9	1.8	$\le$	864
Assembly	.8	1.2	$\le$	672
Profit	25	40		

c) $P = 25x + 40y$

d) $$\begin{cases} .9x + 1.8y \le 864 \\ .8x + 1.2y \le 672 \\ x, y \ge 0 \end{cases}$$



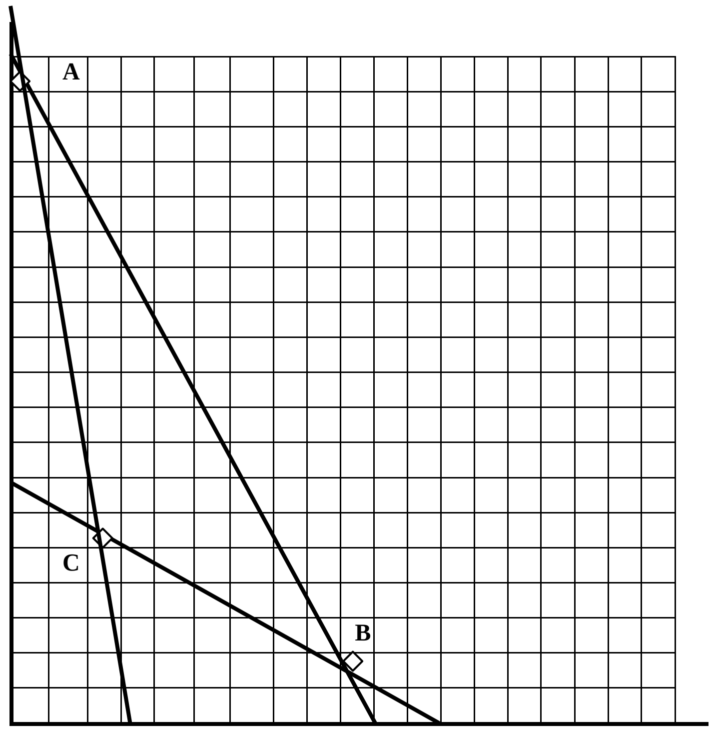
e) 480 two- person boats
240 four- person boats

$$P = 25(480) + 40(240)$$
$$= \$21,600 \ \ per\ month$$

Exercise

Maximize and minimize $z = 4x + 2y$ subject to the constraints

$$\begin{cases} 2x + y \le 20 & (1) \\ 10x + y \ge 36 & (2) \\ 2x + 5y \ge 36 & (3) \\ x, y \ge 0 \end{cases}$$



Solution

A $(1) \cap (2) \Rightarrow x = 2$ and $y = 16$

$\mathbf{z} = 4(2) + 2(16) = 40$

B $(1) \cap (3) \Rightarrow x = 8$ and $y = 4$

$\mathbf{z} = 4(8) + 2(4) = 40$

C $(3) \cap (2) \Rightarrow x = 3$ and $y = 6$

$\mathbf{z} = 4(3) + 2(6) = 24$

Minimize: $\mathbf{z} = 24$ @ (3, 6)

Maximize: $\mathbf{z} = 40$ @ (2, 16) & (8, 4)

Exercise

A chicken farmer can buy a special food mix A at 20¢ per pound and a special food mix B at 40¢ per pound. Each pound of mix A contains 3,000 units of nutrient N_1 and 1,000 units of nutrient N_2, and Each pound of mix B contains 4,000 units of nutrient N_1 and 4,000 units of nutrient N_2. If the minimum daily requirements for the chickens collectively are 36,000 units of nutrient N_1 and 20,000 units of nutrient N_2, how many pounds of each food mix should be used each day to minimize daily food costs while meeting (or exceeding) the minimum daily nutrient requirements? What is the minimum daily cost? Construct a mathematical model and solve using the geometric method.

Solution

Min $C = .2x + .4y$

$$\text{Subject: } \begin{array}{r} 3000x + 4000y \ge 36000 \\ 1000x + 4000y \ge 20000 \\ x, y \ge 0 \end{array}$$

$\begin{array}{r} 3x + 4y = 36 \\ x + 4y = 20 \end{array}$ Solve for x & y $\rightarrow (8, 3)$

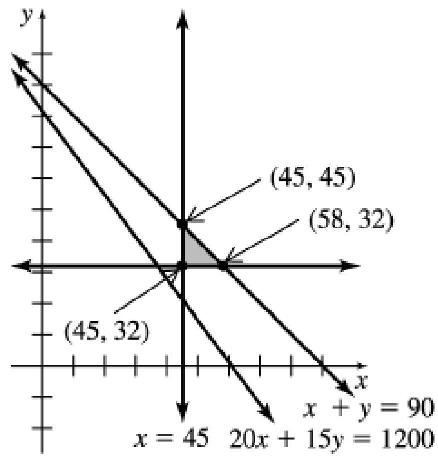
8 lb of mix A, 3 lb of mix B; min $C = .2(8) + .4(3) = \$2.80$ per day

Exercise

A company produces small engines for several manufacturers. The company receives orders from two assembly plants for their Top-flight engine. Plant I needs at least 45 engines, and plant II needs at least 32 engines. The company can send at most 90 engines to these two assembly plants. It costs \$30 per engine to ship to plant I and \$40 per engine to ship to plant II. Plant I gives the company \$20 in rebates toward its products for each engine they buy, while plant II gives similar \$15 rebates. The company estimates that they need at least \$1200 in rebates to cover products they plan to buy from the two plants. How many engines should be shipped to each plant to minimize shipping costs? What is the minimum cost?

Solution

$$\begin{aligned} &\textit{Minimize} && z = 30x + 40y \\ &\textit{Subject to} && x \ge 45 \\ &&& y \ge 32 \\ &&& x + y \le 90 \\ &&& 20x + 15y \ge 1200 \\ &&& x, y \ge 0 \end{aligned}$$



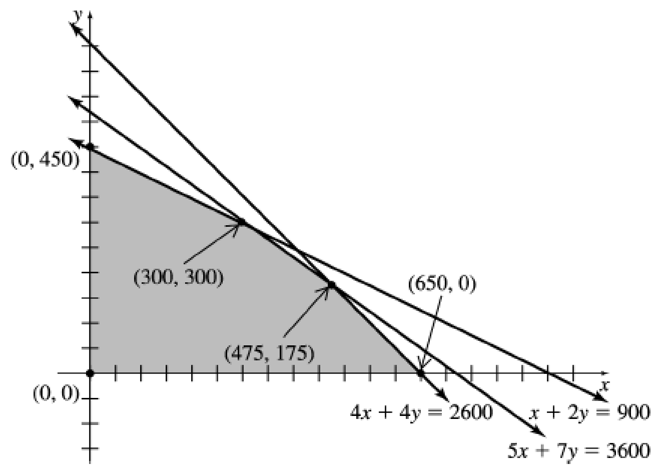
The minimum value is \$2,630, 45 engines to plan I, and 32 engines to plant II.

Exercise

The Muro Manufacturing Company makes two kinds of plasma screen TV sets. It produces the Flexscan set that sells for \$350 profit and the Panoramic I that sells for \$500 profit. On the assembly line, the Flexscan requires 5 hours, and the Panoramic I takes 7 hours. The cabinet shop spends 1 hour on the cabinet for the Flexscan and 2 hours on the cabinet for the Panoramic I. Both sets require 4 hours for testing and packing. On a particular production run, the Muro Company has available 3600 work-hours on the assembly line, 900 work-hours in the cabinet shop, and 2600 work-hours in the testing and packing department. How many sets of each type should it produce to make a maximum profit? What is the maximum profit?

Solution

$$\begin{aligned} &\textit{Maximize} && z = 350x + 500y \\ &\textit{subject to}: && 5x + 7y \le 3600 \\ &&& x + 2y \le 900 \\ &&& 4x + 4y \le 2600 \\ &\textit{with} && x, y \ge 0 \end{aligned}$$



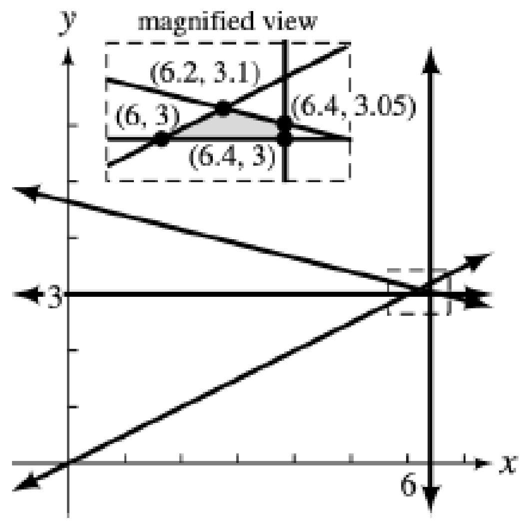
The maximum profit is \$255,000 when 300 Flexscan sets and 300 Panoramic/

Exercise

The manufacturing process requires that oil refineries must manufacture at least 2 gal of gasoline for every gallon of fuel oil. To meet the winter demand for fuel oil, at least 3 million gal a day must be produced. The demand for gasoline is no more than 6.4 million gal per day. It takes 0.25 hour to ship each million gal of gasoline and 1 hour to ship each million gal of fuel oil out of the warehouse. No more than 4.65 hours are available for shipping. If the refinery sells gasoline for \$2.50 per gal and fuel oil for \$2 per gal, how many of each should be produced to maximize revenue? Find the maximum revenue.

Solution

$$\begin{aligned} &\textit{Maximize} && z = 1.25x + 1.00y \\ &\textit{Subject to} && x \ge 2y \\ & && y \ge 3 \\ & && x \le 6.4 \\ & && .25x + y \ge 4.65 \\ & && x, y \ge 0 \end{aligned}$$



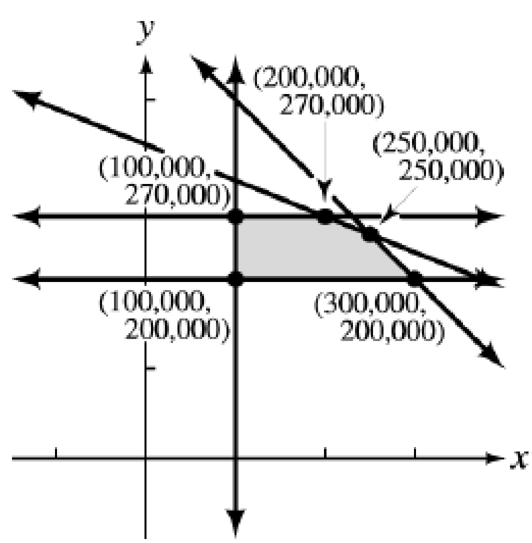
Produce 6.4 million gal
and 3.05 gal of fuel oil
for a maximum revenue of \$11.5 million.

Exercise

A small country can grow only two crops for export, coffee and cocoa. The country has 500,000 hectares of land available for the crops. Long-term contracts require that at least 100,000 hectares be devoted to coffee and at least 200,000 hectares to cocoa. Cocoa must be processed locally, and production bottlenecks limit cocoa to 270,000 hectares. Coffee requires two workers per hectare, with cocoa requiring five. No more than 1,750,000 people are available for working with these corps. Coffee produces a profit of \$220 per hectares and cocoa a profit of 4550 per hectare. How many hectares should the country devote to each crop in order to maximize profit? Find the maximum profit.

Solution

$$\begin{aligned} &\textit{Maximize} && z = 220x + 550y \\ &\textit{subject to}: && x + y \le 500,000 \\ & && x \ge 100,000 \\ & && 200,000 \le y \le 270,000 \\ & && 2x + 5y \le 1,750,000 \\ & && x, y \ge 0 \end{aligned}$$



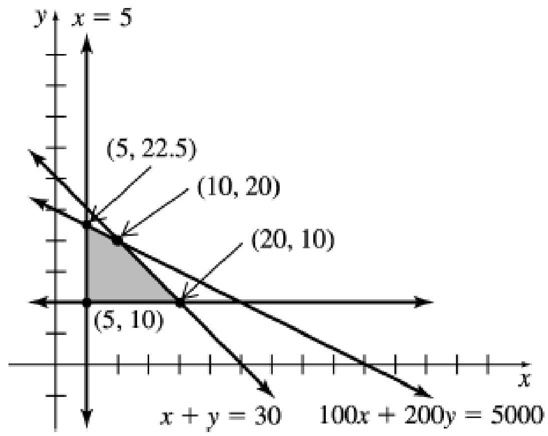
A maximum profit of \$192,500,000 is obtained
by growing 250,000 hectares of crop,
200,000 hectares of coffee, and
270,000 hectares of coffee.

Exercise

A pension fund manager decides to invest a total of at most \$39 million in U.S. treasury bonds paying 4% annual interest and in mutual funds paying 8% annual interest. He plans to invest at least \$5 million in bonds and at least \$10 million in mutual funds. Bonds have an initial fee of \$100 per million dollars, while the fee for mutual funds is \$200 per million. The fund manager is allowed to spend no more than \$5000 on fees. How much should be invested in each to maximize annual interest? What is the maximum annual interest?

Solution

$$\begin{aligned} &\textit{Maximize} && z = 0.04x + .08y \\ &\textit{subject to}: && x + y \le 30 \\ & && x \ge 5 \\ & && y \ge 10 \\ & && 100x + 200y \ge 5{,}000 \\ & && x, y \ge 0 \end{aligned}$$



$$\boxed{(5, 22.5) \,\&\, (10, 20)}$$

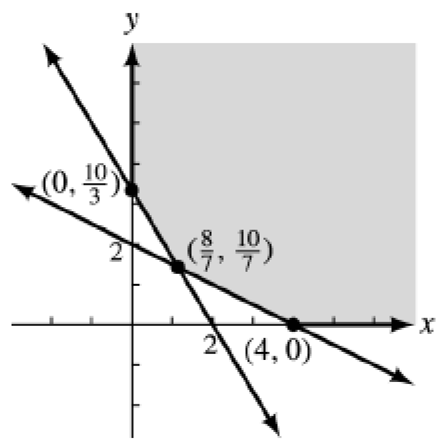
The Max. \$2 million can be achieved by investing \$5 million in Treasury bonds and 22.5 million in mutual funds.
Or \$10 million in Treasury bonds and 20 million in mutual funds.

Exercise

A certain predator requires at least 10 units of protein and 8 units of fat per day. One prey of species I provides 5 units of protein and 2 units of fat; one prey of species II provides 3 units of protein and 4 units of fat. Capturing and digesting each species-II prey requires 3 units of energy, and capturing and digesting each species-I prey requires 2 units of energy. How many of each prey would meet the predator's daily food requirements with the least expenditure of energy?

Solution

$$\begin{aligned} &\textit{Minimize} && z = 2x + 3y \\ &\textit{subject to}: && 5x + 3y \ge 10 \\ & && 2x + 4y \ge 8 \\ & && x, y \ge 0 \end{aligned}$$



$$\boxed{\left(\frac{8}{7}, \frac{10}{7}\right)}$$

Species I: $\frac{8}{7}$ units.

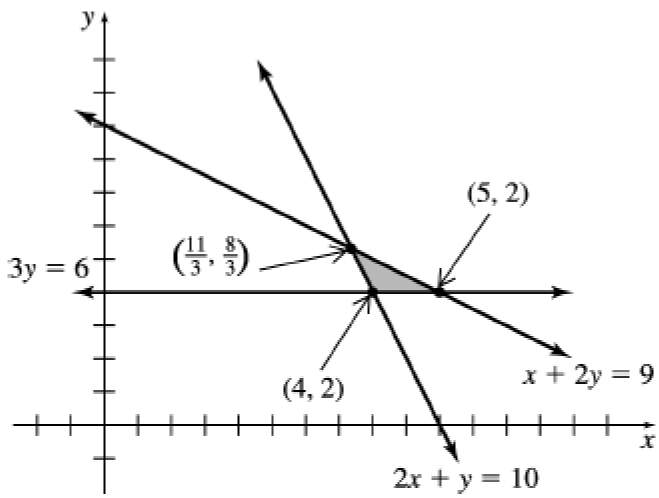
Species II: $\frac{10}{7}$ units.

Exercise

A dietician is planning a snack package of fruit and nuts. Each ounce of fruit will supply zero units of protein, 2 units of carbohydrates, and 1 unit of fat, and will contain 20 calories. Each ounce of nuts will supply 3 units of protein, 1 unit of carbohydrates, and 2 units of fat, and will contain 30 calories. Every package must provide at least 6 units of protein, at least 10 units of carbohydrates, and no more than 9 units of fat. Find the number of ounces of fruit and number of ounces of nuts that will meet the requirement with the least number of calories. What is the least number of calories?

Solution

$$\begin{aligned} &\textit{Minimize} \quad && z = 20x + 30y \\ &\textit{subject to}: && 3y \geq 6 \\ & && 2x + y \geq 10 \\ & && x + 2y \leq 9 \\ & && x,\ y \geq 0 \end{aligned}$$



$\boxed{(4,\ 2)}$

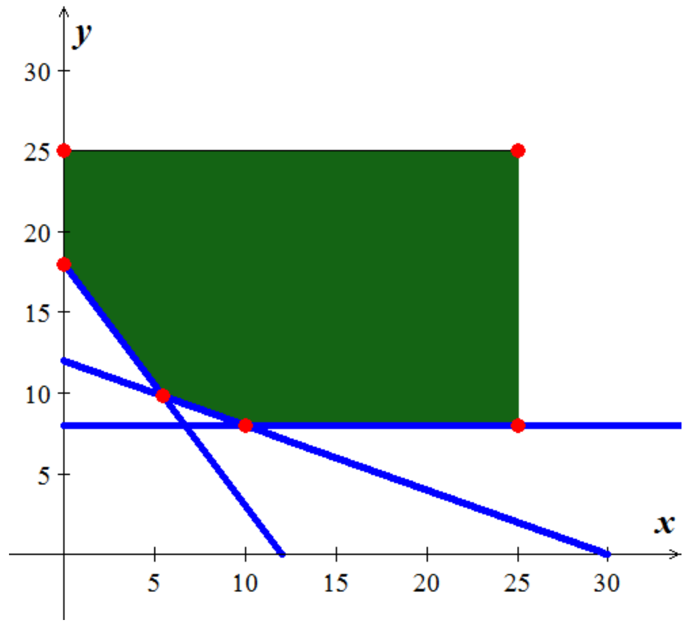
The dietician should use 4 oz. of fruit and 2 oz. of nuts for a minimum of $z = 20(4) + 30(2) = 140$ calories.

Exercise

An anthropology article presents a hypothetical situation that could be described by a linear programming model. Suppose a population gathers plants and animals for survival. They need at least 360 units of energy, 300 units of protein, and 8 hides during some time period. One unit of plants provides 30 units of energy, 10 units of protein, and no hides. One animal provides 20 units of energy, 25 units of protein, and 1 hide.

Solution

$$\begin{aligned} &\textit{Minimize} && z = 30x + 15y \\ &\textit{subject to}: && 30x + 20y \ge 360 \\ &&& 10x + 25y \ge 300 \\ &&& y \ge 8 \\ &&& 0 \le x \le 25 \\ &&& 0 \le y \le 25 \end{aligned}$$



Corner point	***Value*** $z = 30x + 15y$
$(0,18)$	**270 (Min)**
$(0,\ 25)$	375
$(25,\ 25)$	1125
$(25,\ 8)$	870
$\left(\frac{60}{11},\ \frac{108}{11}\right)$	310.91
$(10,\ 8)$	420

$\boxed{(0,\ 18)}$

The minimum is $z = 30(0) + 15(18) = 270$

Exercise

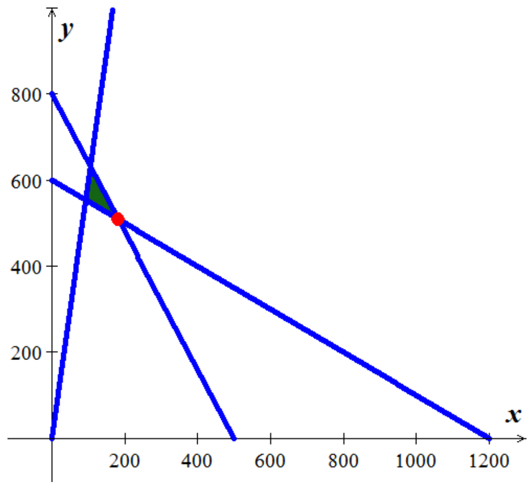
In a small town in South Carolina, zoning rules require that the window space (in square feet) in a house be at least one-sixth of the space used up by solid walls. The cost to build windows is \$10 per ft^2, while the cost to build solid walls is \$20 per ft^2. The total amount available for building walls and windows is no more than \$12,000. The estimated monthly cost to heat the house is \$0,32 for each square foot of windows and \$0.20 for each square foot of solid walls. Find the maximum total area (windows plus walls) if no more than \$160 per month is available to pay for heat.

Solution

$$\begin{aligned} &\textit{Maximize} && z = x + y \\ &\textit{subject to:} && x \ge \frac{1}{6}y \\ & && 10x + 20y \ge 12{,}000 \\ & && 0.32x + 0.2y \le 160 \\ & && x,\ y \ge 0 \end{aligned}$$

$\boxed{(181.82,\ 509.09)}$

The maximum total area is $181.82 + 509.09 = 690.91$



Exercise

A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The trick ski requires 9 labor-hours for fabricating and 1 labor-hour for finishing. The slalom ski requires 5 labor-hours for fabricating and 1 labor-hour for finishing. The maximum labor-hours available per day for fabricating and finishing are 135 and 20 respectively. If x is the number of trick skis and y is the number of slalom skis produced per day, write a system of linear inequalities that indicates appropriate restraints on x and y. Find the set of feasible solutions graphically for the number of each type of ski that can be produced.

Solution

$$\begin{cases} 9x + 5y \le 135 \\ x + y \le 20 \end{cases}$$

$\boxed{(8.75,\ 11.25)}$

The number of trick skis 8.75, and slalom is 11.25