Solution *Section* 2.5 – Graphing Polynomial Functions

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=5x^3+7x^2-x+9$

Solution

Leading term: $5x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=11x^3-6x^2+x+3$

Solution

Leading term: $11x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\quad$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-11x^3-6x^2+x+3$

Solution

Leading term: $-11x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=2x^3+3x^2-23x-42$

Solution

Leading term: $2x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=5x^4+7x^2-x+9$

Solution

Leading term: $5x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=11x^4-6x^2+x+3$

Solution

Leading term: $11x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-5x^4+7x^2-x+9$

Solution

Leading term: $-5x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-11x^4-6x^2+x+3$

Solution

Leading term: $-11x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=5x^5-16x^2-20x+64$

Solution

Leading term: $5x^5$ with 5^{th} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-5x^5-16x^2-20x+64$

Solution

Leading term: $-5x^5$ with 5^{th} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-3x^6-16x^3+64$

Solution

Leading term: $-3x^6$ with 6^{th} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=3x^6-16x^3+4$

Solution

Leading term: $3x^6$ with 6^{th} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises right

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - x - 1;\quad between\ \ 1\ and\ 2$

Solution

$$f(1) = (1)^3 - (1) - 1$$

$$\underline{= -1}\,|$$

$$f(2) = (2)^3 - (2) - 1$$

$$\underline{= 5}\,|$$

Since $f(1)$ *and* $f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - 4x^2 + 2;\quad between\ \ 0\ and\ 1$

Solution

$$f(0) = (0)^3 - 4(0)^2 + 2$$

$$\underline{= 2}\,|$$

$$f(1) = (1)^3 - 4(1)^2 + 2$$

$$\underline{= -1}\,|$$

Since $f(0)$ *and* $f(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 2x^4 - 4x^2 + 1;\quad between\ \ -1\ and\ 0$

Solution

$$f(-1) = 2(-1)^4 - 4(-1)^2 + 1$$

$$\underline{= -1}\,|$$

$$f(0) = 2(0)^4 - 4(0)^2 + 1$$

$$\underline{= 1}\,|$$

Since $f(0)$ *and* $f(-1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between −1 and 0.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^4 + 6x^3 - 18x^2; \quad between \ \ 2 \ and \ 3$

Solution

$$f(2) = (2)^4 + 6(2)^3 - 18(2)^2$$
$$\underline{= -8 \,|}$$

$$f(3) = (3)^4 + 6(3)^3 - 18(3)^2$$
$$\underline{= 81 \,|}$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 + x^2 - 2x + 1; \quad between \ \ -3 \ and \ -2$

Solution

$$f(-3) = (-3)^3 + (-3)^2 - 2(-3) + 1$$
$$\underline{= -11 \,|}$$

$$f(-2) = (-2)^3 + (-2)^2 - 2(-2) + 1$$
$$\underline{= 1 \,|}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between −2 and −3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^5 - x^3 - 1; \quad between \ \ 1 \ and \ 2$

Solution

$$f(1) = (1)^5 - (1)^3 - 1$$

$$\underline{= -1 \,\big|}$$

$$f(2) = (2)^5 - (2)^3 - 1$$

$$\underline{= 23 \,\big|}$$

Since $f(1)$ *and* $f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 10x + 9;$ *between* -3 *and* -2

Solution

$$f(-3) = 3(-3)^3 - 10(-3) + 9$$

$$\underline{= -42 \,\big|}$$

$$f(-2) = 3(-2)^3 - 10(-2) + 9$$

$$\underline{= 5 \,\big|}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between –3 and –2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 8x^2 + x + 2;$ *between* 2 *and* 3

Solution

$$f(2) = 3(2)^3 - 8(2)^2 + (2) + 2$$

$$\underline{= -4 \,\big|}$$

$$f(3) = 3(3)^3 - 8(3)^2 + (3) + 2$$

$$\underline{= 14 \,\big|}$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 8x^2 + x + 2;$ *between* 1 *and* 2

Solution

$$f(1) = 3(1)^3 - 8(1)^2 + (1) + 2$$

$$\underline{= -2}\,\rfloor$$

$$f(2) = 3(2)^3 - 8(2)^2 + (2) + 2$$

$$\underline{= -4}\,\rfloor$$

Since $f(1)$ *and* $f(2)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^5 + 2x^4 - 6x^3 + 2x - 3;$ *between* 0 *and* 1

Solution

$$f(0) = (0)^5 + 2(0)^4 - 6(0)^3 + 2(0) - 3$$

$$\underline{= -3}\,\rfloor$$

$$f(1) = (1)^5 + 2(1)^4 - 6(1)^3 + 2(1) - 3$$

$$\underline{= -4}\,\rfloor$$

Since $f(0)$ *and* $f(1)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 2x^3 + 3x^2 - 23x - 42,$ $a = 3,$ $b = 4$

Solution

$$P(3) = 54 + 27 - 69 - 42$$

$$\underline{= -30}\,\rfloor$$

$$P(4) = 128 + 48 - 92 - 42$$

$$\underline{= 90}\,\rfloor$$

Since $P(3)$ *and* $P(4)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and 4.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 4x^3 - x^2 - 6x + 1, \quad a = 0, \quad b = 1$

Solution

$P(0)\ \underline{= 1}\rvert$

$P(1) = 4 - 1 - 6 + 1$

$\underline{= -2}\rvert$

Since $P(0)$ *and* $P(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 3x^3 + 7x^2 + 3x + 7, \quad a = -3, \quad b = -2$

Solution

$P(-3) = -81 + 63 - 9 + 7$

$\underline{= -20}\rvert$

$P(-2) = -24 + 28 - 6 + 7$

$\underline{= 5}\rvert$

Since $P(-3)$ *and* $P(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between −3 and −2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 2x^3 - 21x^2 - 2x + 25, \quad a = 1, \quad b = 2$

Solution

$P(1) = 2 - 21 - 2 + 25$

$\underline{= 4}\rvert$

$$P(2) = 16 - 84 - 4 + 25$$
$$\boxed{= -47}$$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 4x^4 + 7x^3 - 11x^2 + 7x - 15, \quad a = 1, \quad b = \frac{3}{2}$

Solution

$$P(1) = 4 + 7 - 11 + 7 - 15$$
$$\boxed{= -8}$$

$$\begin{aligned} P\left(\frac{3}{2}\right) &= 81 + \frac{189}{8} - \frac{99}{4} + \frac{21}{2} - 15 \\ &= 66 + \frac{189 - 198 + 84}{8} \\ &= 66 + \frac{75}{8} \\ &\boxed{= \frac{603}{8}} \end{aligned}$$

Since $P(1)$ *and* $P\left(\frac{3}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and $\frac{3}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 5x^3 - 16x^2 - 20x + 64, \quad a = 3, \quad b = \frac{7}{2}$

Solution

$$P(3) = 135 - 144 - 60 + 64$$
$$\boxed{= -5}$$

$$\begin{aligned} P\left(\frac{7}{2}\right) &= \frac{1715}{8} - 196 - 70 + 64 \\ &= \frac{1715}{8} - 202 \\ &\boxed{= \frac{99}{8}} \end{aligned}$$

Since $P(3)$ *and* $P\left(\frac{7}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and $\frac{7}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^4-x^2-x-4,\quad a=1,\quad b=2$

Solution

$P(1)=1-1-1-4$

$\underline{=-5}$

$P(2)=16-4-2-4$

$\underline{=6}$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^3-x-8,\quad a=2,\quad b=3$

Solution

$P(2)=8-2-8$

$\underline{=-2}$

$P(3)=27-3-8$

$\underline{=16}$

Since $P(2)$ *and* $P(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^3-x-8,\quad a=0,\quad b=1$

Solution

$\underline{P(0)=-8}$

$$\begin{aligned} P(1) &= 1 - 1 - 8 \\ &\boxed{= -8} \end{aligned}$$

Since $P(0)$ *and* $P(1)$ have same sign.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^3 - x - 8, \quad a = 2.1, \quad b = 2.2$

Solution

$$\begin{aligned} P(2.1) &= P\left(\frac{21}{10}\right) \\ &= \frac{9261}{1000} - \frac{21}{10} - 8 \\ &= \frac{9261 - 2100 - 8000}{1000} \\ &\boxed{= -\frac{839}{1,000}} \end{aligned}$$

$$\begin{aligned} P(2.2) &= P\left(\frac{2.2}{10}\right) \\ &= \frac{10,648}{1000} - \frac{22}{10} - 8 \\ &= \frac{10,648 - 2,200 - 8,000}{1000} \\ &= \frac{448}{1,000} \\ &\boxed{= 0.448} \end{aligned}$$

Since $P(2.1)$ *and* $P(2.2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2.1 and 2.2.

Exercise

Let $f(x) = x^4 - 4x^2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

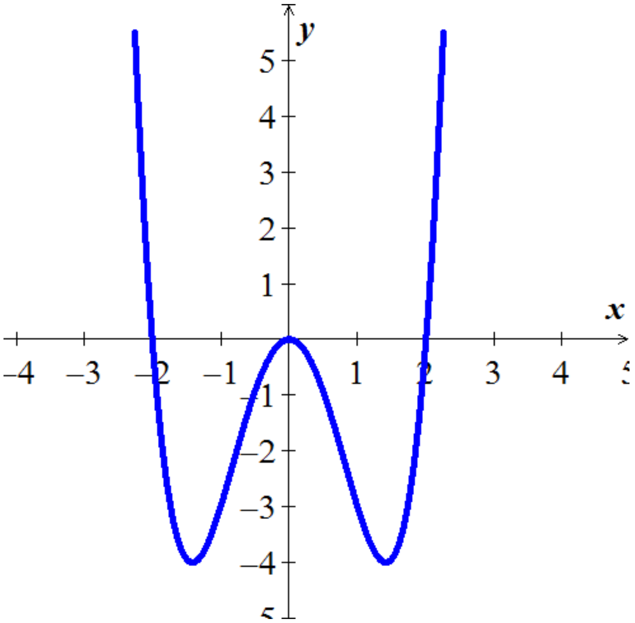
$$\begin{aligned} f(x) &= x^2\left(x^2 - 4\right) \\ &= x^2(x-2)(x+2) \end{aligned}$$

The zeros are: 0, 0, 2, –2.

$-\infty$ -2	$0,0$ 2	∞
+	–	+

$f(x) < 0$ $\quad \underline{(-2,\ 0) \cup (0,\ 2)}$

$f(x) > 0$ $\quad \underline{(-\infty,\ -2) \cup (2,\ \infty)}$



Exercise

Let $f(x) = x^4 + 3x^3 - 4x^2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

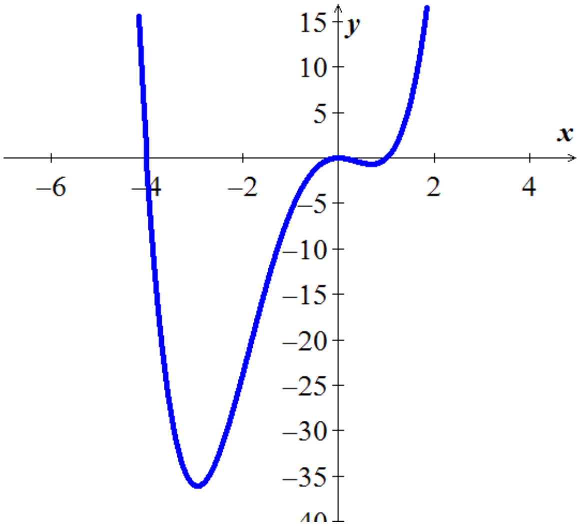
$$f(x) = x^2\left(x^2 + 3x - 4\right)$$

The zeros are: 0, 0, 1, –4.

$-\infty$ -4	$0,0$ 1	∞
+	–	+

$f(x) > 0$ $\quad \underline{(-\infty,\ -4) \cup (1,\ \infty)}$

$f(x) < 0$ $\quad \underline{(-4,\ 0) \cup (0,\ 1)}$



Exercise

Let $f(x)=x^3+2x^2-4x-8$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$\begin{aligned} f(x) &= x^2(x+2)-4(x+2) \\ &= (x+2)\left(x^2-4\right) \\ &= (x+2)(x+2)(x-2) = 0 \end{aligned}$$

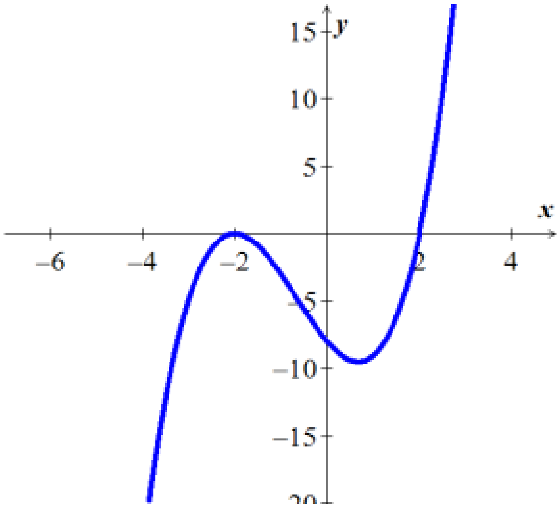
The zeros are: 2, −2, −2

$-\infty \qquad -2 \qquad 0 \qquad 2 \qquad \infty$

−	−	+

$f(x)>0 \quad (2,\ \infty)$

$f(x)<0 \quad (-\infty,\ -2)\cup(-2,\ 2)$



Exercise

Let $f(x)=x^3-3x^2-9x+27$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$\begin{aligned} f(x) &= x^2(x-3)-9(x-3) \\ &= (x-3)\left(x^2-9\right) \\ &= (x-3)(x-3)(x+3) \end{aligned}$$

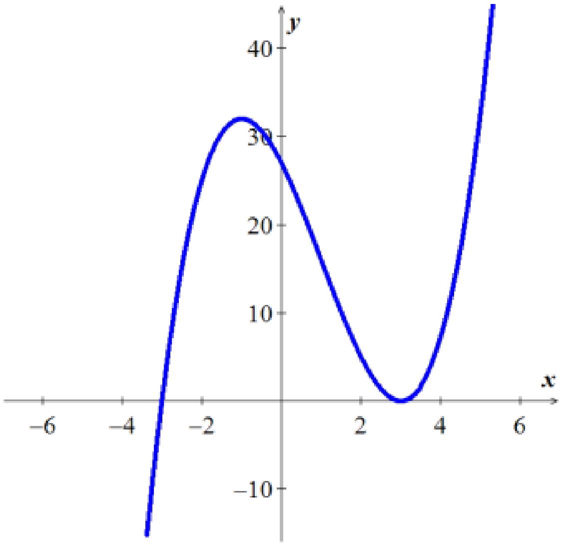
The zeros are: −3, 3 (multiplicity)

$-\infty \qquad -3 \qquad 0 \qquad 3 \qquad \infty$

−	+	+

$f(x)>0 \quad (-3,\ 3)\cup(3,\ \infty)$

$f(x)<0 \quad (-\infty,\ -3)$



Exercise

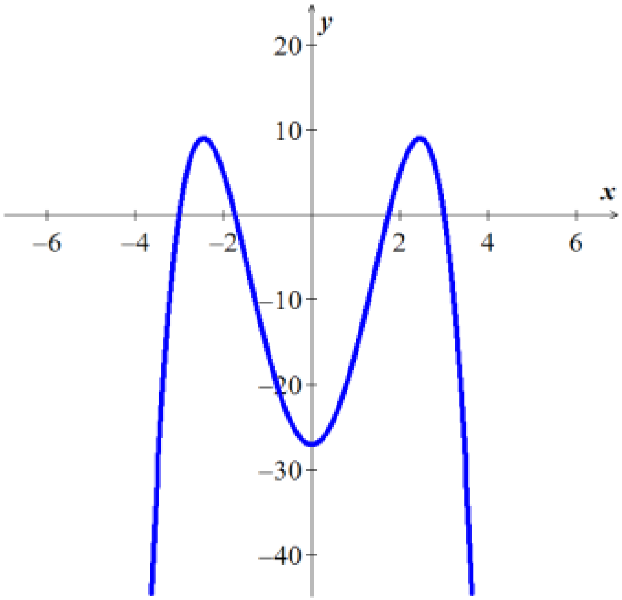
Let $f(x)=-x^4+12x^2-27$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$x^2=\frac{-12\pm\sqrt{36}}{-2}$$

$$=\begin{cases}\frac{-12-6}{-2}=9\\ \frac{-12+6}{-2}=3\end{cases}$$

$$\to\begin{cases}x^2=9\\ x^2=3\end{cases}\Rightarrow\quad\begin{matrix}x=\pm 3\\ x=\pm\sqrt{3}\end{matrix}$$



-3	$-\sqrt{3}$	$\sqrt{3}$	3	
−	+	−	+	−

$f(x)>0$ $\quad\underline{\left(-3,\ -\sqrt{3}\right)\cup\left(\sqrt{3},\ 3\right)}$

$f(x)<0$ $\quad\underline{\left(-\infty,\ -3\right)\cup\left(-\sqrt{3},\ \sqrt{3}\right)\cup\left(3,\ \infty\right)}$

Exercise

Let $f(x)=x^2(x+2)(x-1)^2(x-2)$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

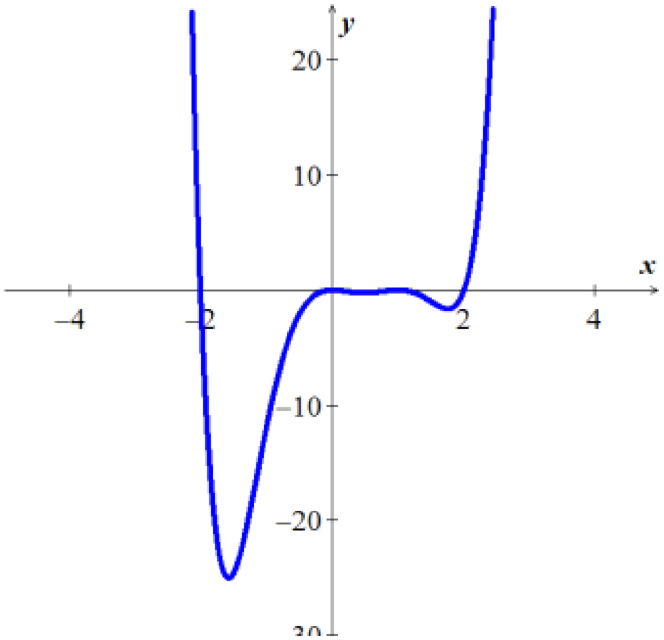
Solution

The zeros are: −2, 2, 0, 0, 1, 1

-2	**0,0 1,1**	2
+	−	+

$f(x)>0$ $\quad\underline{\left(-\infty,\ -2\right)\cup\left(2,\ \infty\right)}$

$f(x)<0$ $\quad\underline{\left(-2,\ 0\right)\cup\left(0,\ 1\right)\cup\left(1,\ 2\right)}$



Exercise

Let $f(x)=2x^3+11x^2-7x-6$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$\text{possibilities}:\pm\left\{\frac{6}{2}\right\}=\pm\left\{\frac{1,\ 2,\ 3,\ 6}{1,\ 2}\right\}$$

$$=\pm\left\{1,\ 2,\ 3,\ 5,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

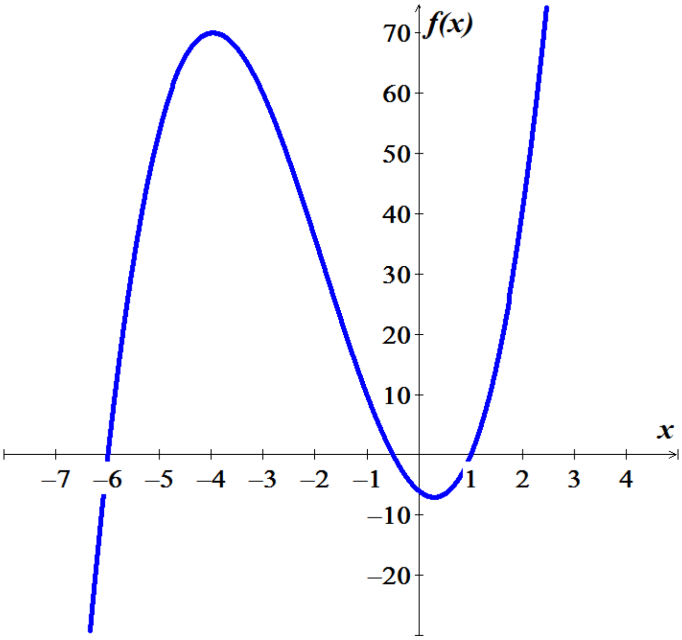
$$\begin{array}{r|rrrr} 1 & 2 & 11 & -7 & -6 \\ & & 2 & 13 & 6 \\ \hline & 2 & 13 & 6 & \boxed{0} \end{array} \to 2x^2+13x+6=0$$

The zeros are: $x=1,\ -\frac{1}{2},\ -6$

	-6		$-\frac{1}{2}$		1	
$-$		$+$		$-$		$+$

$f(x)>0$ $\left(-6,\ -\frac{1}{2}\right)\cup(1,\ \infty)$

$f(x)<0$ $(-\infty,\ -6)\cup\left(-\frac{1}{2},\ 1\right)$



Exercise

Let $f(x)=x^3+2x^2-5x-6$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$\text{possibilities}\ :\ \pm\left\{\frac{6}{1}\right\}$$

$$=\pm\{1,\ 2,\ 3,\ 6\}$$

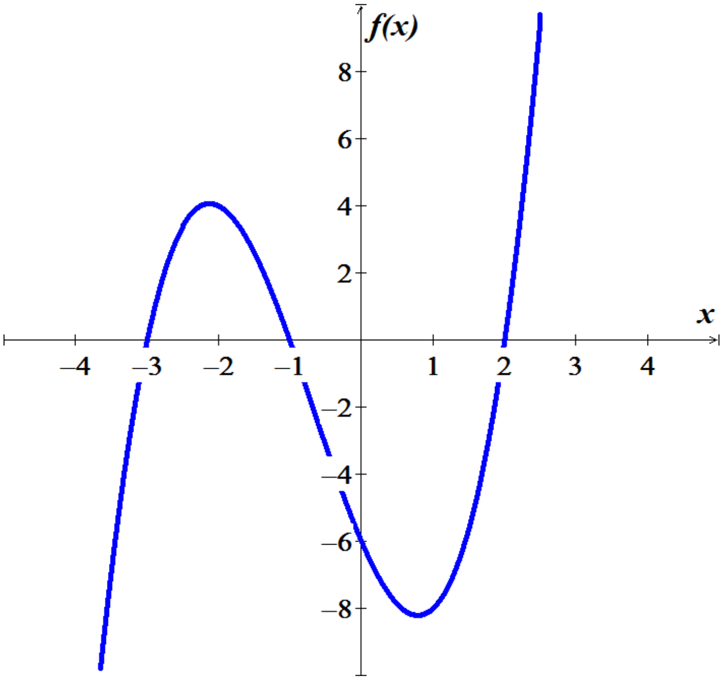
$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array} \to x^2+x-6=0$$

The zeros are: $x=-1,\ -3,\ 2$

	-3		-1		2	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{(-3,\ -1) \cup (2,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -3) \cup (-1,\ 2)}$



Exercise

Let $f(x) = x^3 + 8x^2 + 11x - 20$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\text{possibilities } : \pm\left\{\frac{20}{1}\right\} = \pm\{1,\ 2,\ 4,\ 5,\ 20,\ 20\}$$

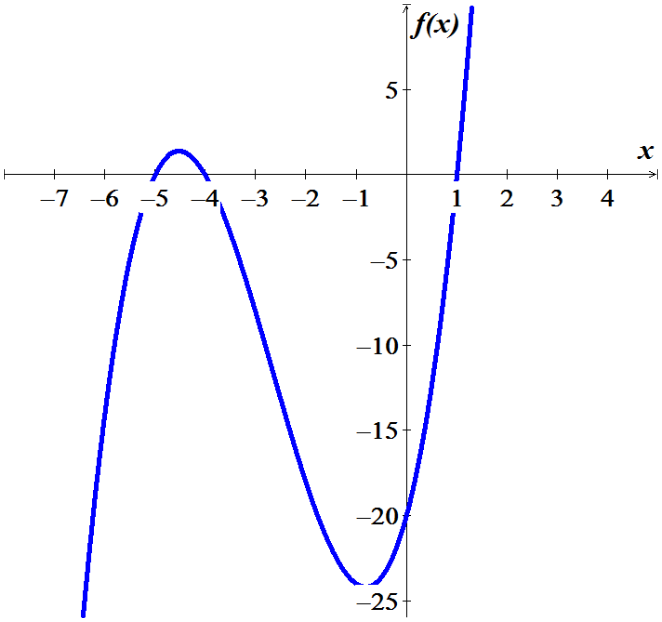
$$\begin{array}{r|rrrr} 1 & 1 & 8 & 11 & -20 \\ & & 1 & 9 & 20 \\ \hline & 1 & 9 & 20 & \boxed{0} \end{array} \quad \to\ x^2 + 9x + 20 = 0$$

The zeros are: $\boxed{x = -5,\ -4,\ 1}$

	-5		-4		1	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{(-5,\ -1) \cup (1,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -5) \cup (-4,\ 1)}$



Exercise

Let $f(x)=x^4+x^2-2$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

possibilities : $\pm\{1,\ 2\}$

$$\begin{array}{r|rrrrr}
1 & 1 & 0 & 1 & 0 & -2 \\
 & & 1 & 1 & 2 & 1 \\
\hline
-1 & 1 & 1 & 2 & 2 & 0 \\
 & & -1 & 0 & -2 & \\
\hline
 & 1 & 0 & 2 & 0 &
\end{array}
\quad \rightarrow x^3+x^2+2x+1=0 \ \rightarrow \pm\{1,\ 2\}$$

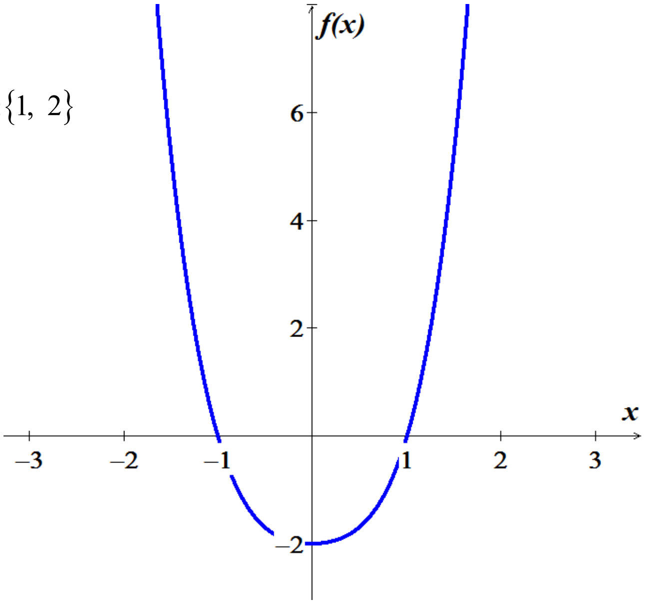
$$\rightarrow x^2+2=0 \quad \Rightarrow \quad \underline{x=\pm i\sqrt{2}|}$$

The zeros are: $\underline{x=\pm 1\ |}$

$$\begin{array}{c|c|c}
 & -1 \qquad 1 & \\
\hline
+ & - & +
\end{array}$$

$f(x)>0$ $\quad \underline{(-\infty,\ -1)\cup(1,\ \infty)\ |}$

$f(x)<0$ $\quad \underline{(-1,\ 1)\ |}$



Exercise

Let $f(x)=x^4-x^3-6x^2+4x+8$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

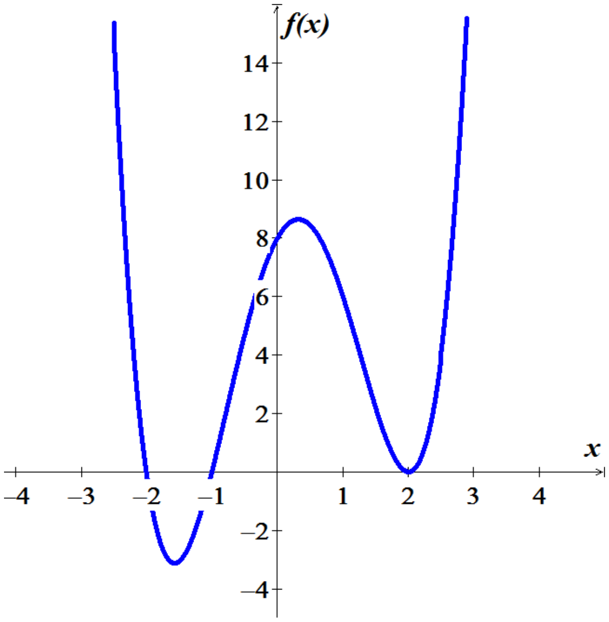
possibilities : $\pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrrr}
-1 & 1 & -1 & -6 & 4 & 8 \\
 & & -1 & 2 & 4 & -8 \\
\hline
-2 & 1 & -2 & -4 & 8 & 0 \\
 & & -2 & 8 & -8 & \\
\hline
 & 1 & -4 & 4 & 0 &
\end{array}
\quad \begin{array}{l} \rightarrow x^3-2x^2-4x+8=0 \\ \rightarrow\pm\{1,\ 2,\ 4,\ 8\} \end{array}$$

$$\rightarrow x^2-4x+4=0 \ \Rightarrow \ \underline{x=2,\ 2|}$$

The zeros are: $\underline{x=-2,\ -1,\ 2,\ 2\ |}$

$$\begin{array}{c|c|c|c}
 & -2 \qquad -1 & \qquad 2 & \\
\hline
+ & - & + & +
\end{array}$$



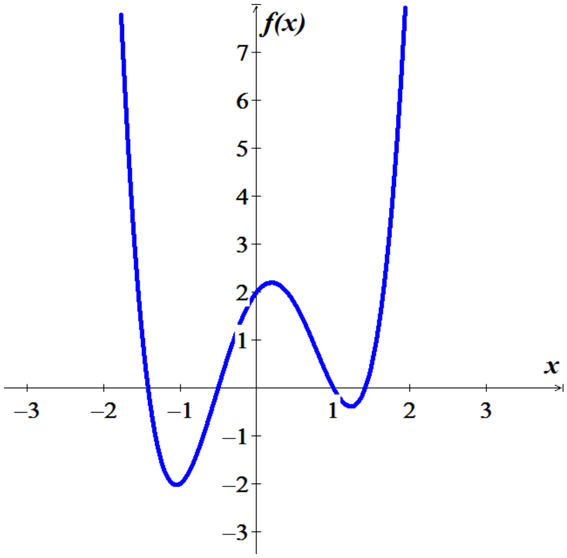
$f(x)>0 \quad \boxed{(-\infty,\ -1)\cup(1,\ \infty)}$

$f(x)<0 \quad \boxed{(-1,\ 1)}$

Exercise

Let $f(x)=2x^4-x^3-5x^2+2x+2$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution



$$possibilities:\ \pm\left\{\frac{2}{2}\right\}=\pm\left\{1,\ 2,\ \frac{1}{2}\right\}$$

$$\begin{array}{r|rrrrr} 1 & 2 & -1 & -5 & 2 & 2 \\ & & 2 & 1 & -4 & -2 \\ \hline -\frac{1}{2} & 2 & 1 & -4 & -2 & 0 \\ & & -1 & 0 & 2 & \\ \hline & 2 & 0 & -4 & 0 & \end{array}$$

$\to 2x^3+x^2-4x-2=0$

$\to\pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$$\to 2x^2-4=0 \Rightarrow \boxed{x=\pm\sqrt{2}}$$

The zeros are: $\boxed{x=-\frac{1}{2},\ 1,\ -\sqrt{2},\ \sqrt{2}}$

$$\begin{array}{ccccccccc} & -\sqrt{2} & & -\frac{1}{2} & & 1 & & \sqrt{2} & \\ + & | & - & | & + & | & - & | & + \end{array}$$

$f(x)>0 \quad \boxed{(-\infty,\ -\sqrt{2})\cup\left(-\frac{1}{2},\ 1\right)\cup(\sqrt{2},\ \infty)}$

$f(x)<0 \quad \boxed{\left(-\sqrt{2},\ -\frac{1}{2}\right)\cup(1,\ \sqrt{2})}$

Exercise

Let $f(x)=4x^5-8x^4-x+2$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$\begin{aligned} f(x)&=4x^4(x-2)-(x-2) \\ &=(x-2)\left(4x^4-1\right)=0 \end{aligned}$$

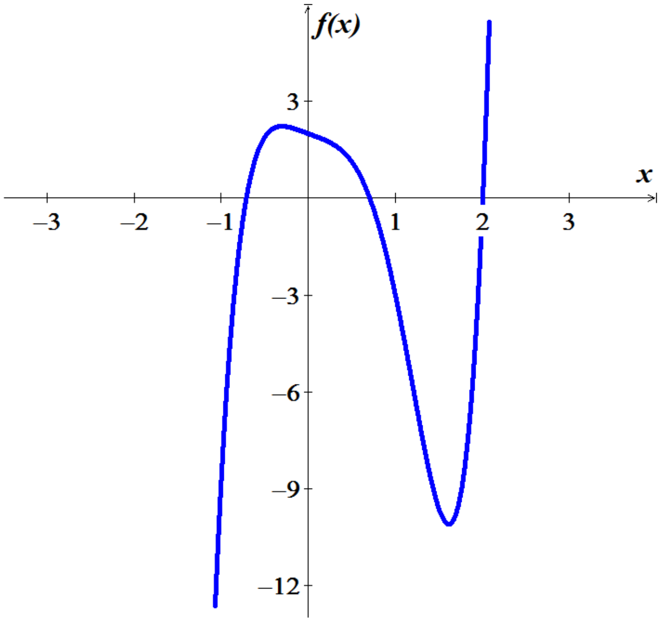
$$4x^4 - 1 = 0 \quad \Rightarrow \quad \begin{cases} x^2 = -\frac{1}{2} & \mathbb{C} \\ x^2 = \frac{1}{2} & x = \pm\frac{\sqrt{2}}{2} \end{cases}$$

The zeros are: $\underline{x = 2, \ -\frac{\sqrt{2}}{2}, \ \frac{\sqrt{2}}{2}}|$

	$-\frac{\sqrt{2}}{2}$		$\frac{\sqrt{2}}{2}$		2	
−		+		−		+

$f(x) > 0 \quad \underline{\left(-\frac{\sqrt{2}}{2}, \ \frac{\sqrt{2}}{2}\right) \cup (2, \ \infty)}|$

$f(x) < 0 \quad \underline{\left(-\infty, \ -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2}, \ 2\right)}|$



Exercise

Let $f(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$\textit{possibilities}: \ \pm\left\{\frac{36}{1}\right\} = \pm\{1, \ 2, \ 3, \ 4, \ 6, \ 9, \ 12, \ 18, \ 36\}$

1	1	−7	19	−37	60	−36
		1	−6	13	−24	36
3	1	−6	13	−24	36	0
		3	−9	12	−36	
3	1	−3	4	−12	0	
		3	0	12		
	1	0	4	0		

$x^4 - 6x^3 + 13x^2 - 24x + 36 = 0 \ \rightarrow \ \pm\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$x^3 - 3x^2 + 4x - 12 = 0 \ \rightarrow \ \pm\{1, \ 2, \ 3, \ 4, \ 6, \ 12\}$

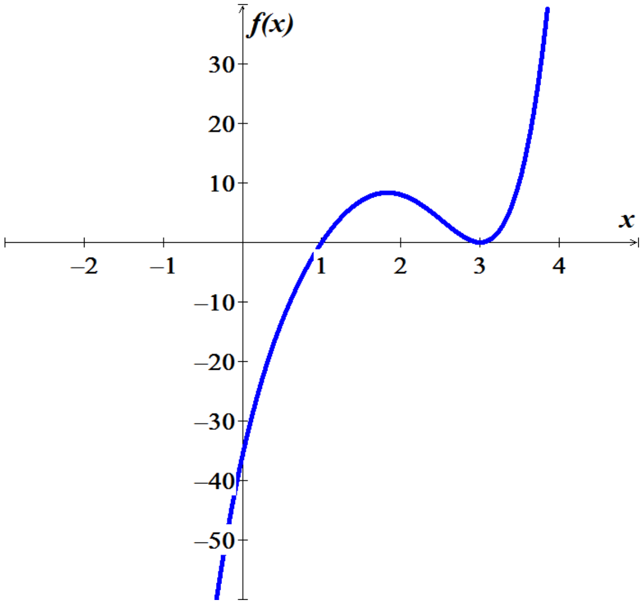
$x^2 + 4 = 0 \quad \Rightarrow \quad x = \pm 2i$

The zeros are: $\underline{x = 1, \ 3, \ 3}|$

	1		3	
−		+		+

$f(x) > 0 \quad \underline{(1, \ 3) \cup (3, \ \infty)}|$

$f(x) < 0 \quad \underline{(-\infty, \ 1)}|$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - x^2 - 10x - 8$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

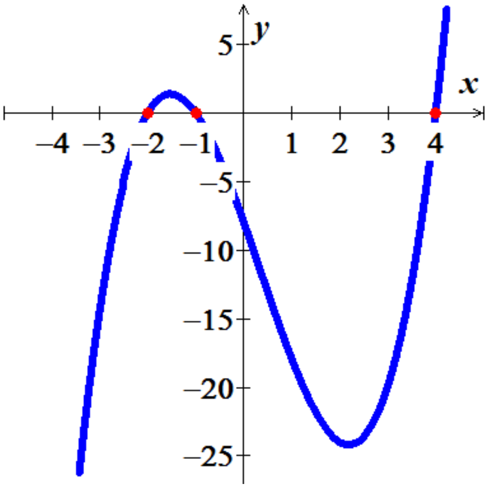
$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & 0 \end{array} \rightarrow \quad x^2 - 2x - 8 = 0$$

$x = -1,\ -2,\ 4$

$$\begin{array}{ccccccc} & -2 & & -1 & & 4 & \\ - & | & + & | & - & | & + \end{array}$$

$f(x) > 0 \quad (-2,\ -1) \cup (4,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -2) \cup (-1,\ 4)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + x^2 - 14x - 24$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

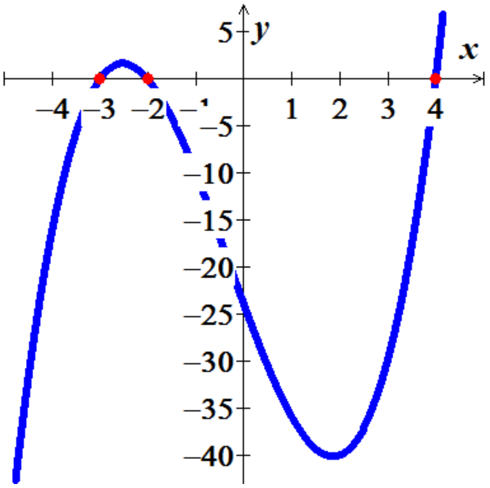
$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & 0 \end{array} \rightarrow \quad x^2 - x - 12 = 0$$

$x = -2,\ -3,\ 4$

$$\begin{array}{ccccccc} & -3 & & -2 & & 4 & \\ - & | & + & | & - & | & + \end{array}$$

$f(x) > 0 \quad (-3,\ -2) \cup (4,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -3) \cup (-2,\ 4)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=2x^3-3x^2-17x+30$$

Solution

$$\textit{possibilities for }\ \frac{c}{d}:\pm\left\{\frac{30}{2}\right\}=\pm\left\{1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30,\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{15}{2}\right\}$$

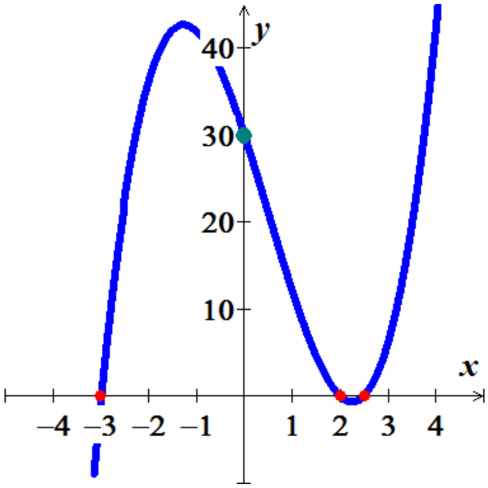
$$\begin{array}{r|rrrr} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & \boxed{0} \end{array} \rightarrow\ 2x^2+x-15=0$$

$$\underline{x=2,\ -3,\ \frac{5}{2}\Big|}$$

$$\begin{array}{c|c|c|c} & -3 & 2 & \frac{5}{2} & \\ \hline - & + & - & + \end{array}$$

$f(x)>0$ $\quad \underline{(-3,\ 2)\cup\left(\frac{5}{2},\ \infty\right)\Big|}$

$f(x)<0$ $\quad \underline{(-\infty,\ -3)\cup\left(2,\ \frac{5}{2}\right)\Big|}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=12x^3+8x^2-3x-2$$

Solution

$$\textit{possibilities for }\ \frac{c}{d}:\pm\left\{\frac{2}{12}\right\}=\pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ \frac{1}{6},\ \frac{1}{12}\right\}$$

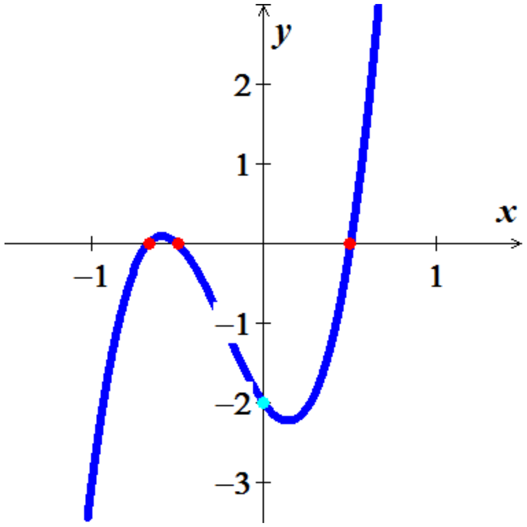
$$\begin{array}{r|rrrr} \frac{1}{2} & 12 & 8 & -3 & -2 \\ & & 6 & 7 & 2 \\ \hline & 12 & 14 & 4 & \boxed{0} \end{array} \rightarrow\ 12x^2+14x+4=0$$

$$\underline{x=\frac{1}{2},\ -\frac{1}{2},\ -\frac{2}{3}\Big|}$$

$$\begin{array}{c|c|c|c} & -\frac{2}{3} & -\frac{1}{2} & \frac{1}{2} & \\ \hline - & + & - & + \end{array}$$

$f(x)>0$ $\quad \underline{\left(-\frac{2}{3},\ -\frac{1}{2}\right)\cup\left(\frac{1}{2},\ \infty\right)\Big|}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ -\frac{2}{3}\right) \cup \left(-\frac{1}{2},\ \frac{1}{2}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + x^2 - 6x - 8$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

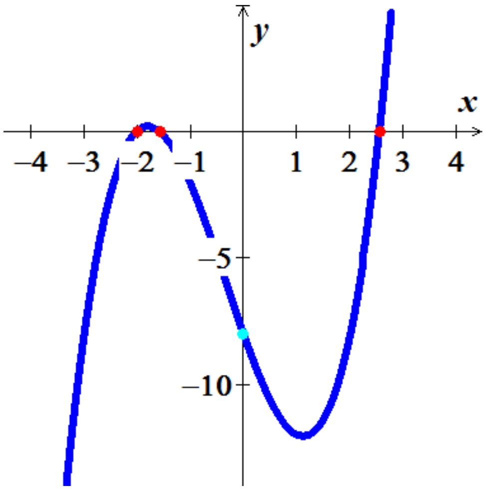
$$\begin{array}{r|rrrr} -2 & 1 & 1 & -6 & -8 \\ & & -2 & 2 & 8 \\ \hline & 1 & -1 & -4 & \boxed{0} \end{array} \rightarrow \quad x^2 - x - 4 = 0$$

$x = \dfrac{1 \pm \sqrt{1+16}}{2}$

$\boxed{x = -2,\ \dfrac{1 \pm \sqrt{17}}{2}}$

$-2 \qquad \frac{1-\sqrt{17}}{2} \qquad \frac{1+\sqrt{17}}{2}$

−	+	−	+



$f(x) > 0 \quad \boxed{\left(-2,\ \frac{1-\sqrt{17}}{2}\right) \cup \left(\frac{1+\sqrt{17}}{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -2) \cup \left(\frac{1-\sqrt{17}}{2},\ \frac{1+\sqrt{17}}{2}\right)}$

Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3-19x-30$$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{30}{1}\right\}=\pm\{1,\ 2,\ 3,\ 5,\ 6,\ 15,\ 30\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & \boxed{0} \end{array} \rightarrow\ x^2-2x-15$$

$$x=\frac{2\pm\sqrt{4+60}}{2}$$

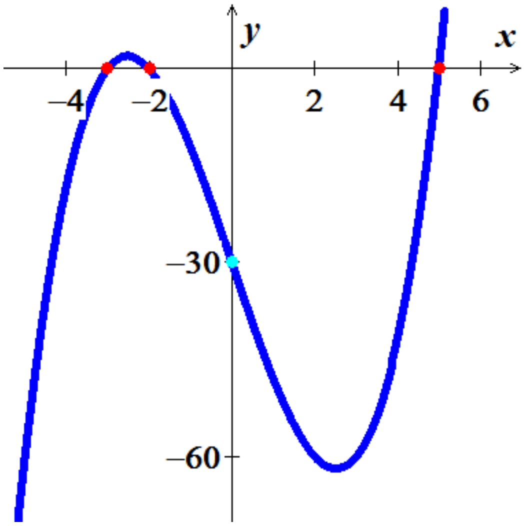
$$=\begin{cases} \frac{2-8}{2}=-3 \\ \frac{2+8}{2}=5 \end{cases}$$

$\underline{x=-2,\ -3,\ 5}$

	−3		−2		5	
−		+		−		+

$f(x)>0$ $\quad \underline{(-3,\ -2)\cup(5,\ \infty)}$

$f(x)<0$ $\quad \underline{(-\infty,\ -3)\cup(-2,\ 5)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 + x^2 - 25x + 12$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 12,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrr} 3 & 2 & 1 & -25 & 12 \\ & & 6 & 21 & -12 \\ \hline & 2 & 7 & -4 & \boxed{0} \end{array} \quad \to \quad 2x^2 + 7x - 4$$

$$x = \frac{-7 \pm \sqrt{49+32}}{4}$$

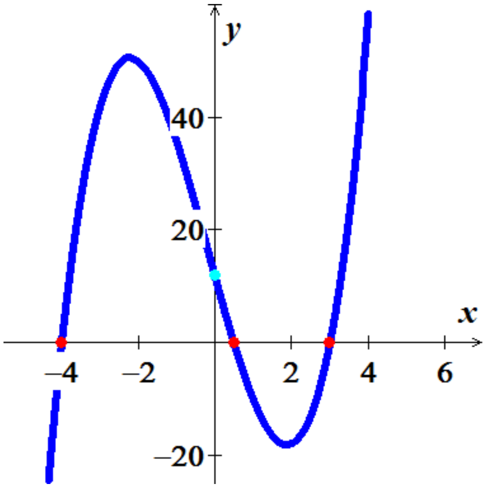
$$= \begin{cases} \frac{-7-9}{4} = -4 \\ \frac{-7+9}{4} = \frac{1}{2} \end{cases}$$

$$\underline{x = -4,\ \frac{1}{2},\ 3\ \big|}$$

	-4		$\frac{1}{2}$		3	
$-$		$+$		$-$		$+$

$f(x) > 0$ $\quad \left(-4,\ \frac{1}{2}\right) \cup (3,\ \infty)$

$f(x) < 0$ $\quad (-\infty,\ -1) \cup \left(\frac{1}{2},\ 3\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^3 + 11x^2 - 6x - 8$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ 8,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{8}{3}\right\}$

$$\begin{array}{c|cccc} 1 & 3 & 11 & -6 & -8 \\ & & 3 & 14 & 8 \\ \hline & 3 & 14 & 8 & \boxed{0} \end{array} \quad \rightarrow \quad 3x^2 + 14x + 8$$

$$x = \frac{-14 \pm \sqrt{196 - 96}}{6}$$

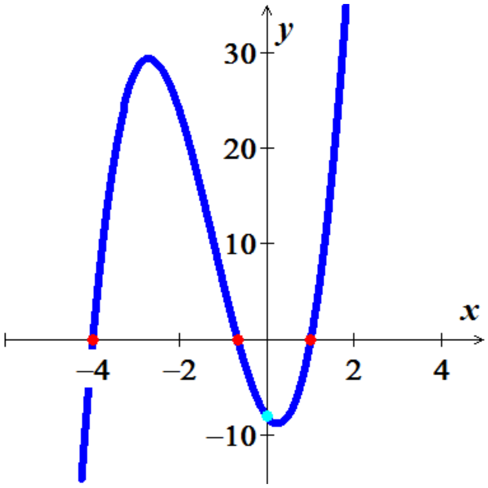
$$= \begin{cases} \frac{-14-10}{6} = -4 \\ \frac{-14+10}{6} = -\frac{2}{3} \end{cases}$$

$$\boxed{x = -4,\ -\frac{2}{3},\ 1}$$

	-4		$-\frac{2}{3}$		1	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{\left(-4,\ -\frac{2}{3}\right) \cup (1,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -4) \cup \left(-\frac{2}{3},\ 1\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 + 9x^2 - 2x - 9$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{9}{2}\right\} = \pm\left\{1,\ 3,\ 9,\ \frac{1}{2},\ \frac{3}{2},\ \frac{9}{2}\right\}$

$$\begin{array}{r|rrrr} 1 & 2 & 9 & -2 & -9 \\ & & 2 & 11 & 9 \\ \hline & 2 & 11 & 9 & \boxed{0} \end{array} \quad \rightarrow \quad 2x^2 + 11x + 9$$

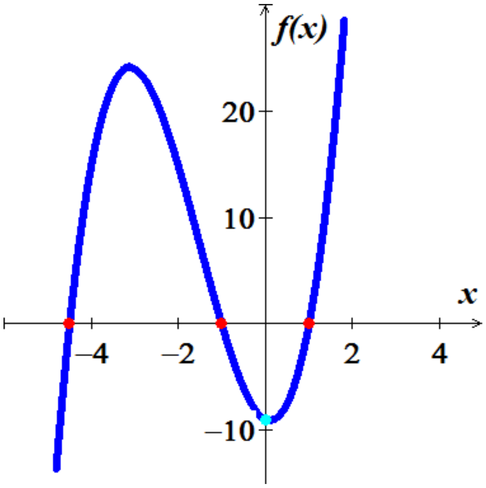
$x = -1,\ -\frac{9}{2}$ $\qquad\qquad$ $a - b + c = 0 \ \rightarrow \ x = -1,\ -\frac{c}{a}$

$x = -\frac{9}{2},\ -1,\ 1$

	$-\frac{9}{2}$		-1		1	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \left(-\frac{9}{2},\ -1\right) \cup (1,\ \infty)$

$f(x) < 0 \quad \left(-\infty,\ -\frac{9}{2}\right) \cup (-1,\ 1)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + 3x^2 - 6x - 8$$

Solution

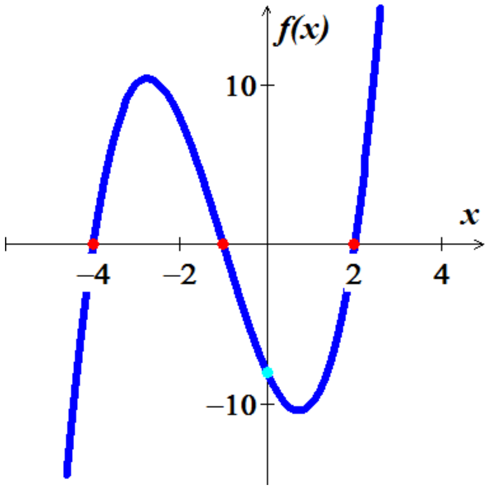
$$\textit{possibilities for } \frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ & & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & \boxed{0} \end{array} \rightarrow \ x^2 + 2x - 8 = 0$$

$$x = \frac{-2 \pm \sqrt{4+32}}{2}$$

$$= \begin{cases} \frac{-2-6}{2} = -4 \\ \frac{-2+6}{2} = 2 \end{cases}$$

$$\underline{x = -4,\ -1,\ 2}$$



$$\begin{array}{cccccccc} & -4 & & -1 & & 2 & \\ \hline - & | & + & | & - & | & + \end{array}$$

$f(x) > 0$ $\quad \underline{(-4,\ -1) \cup (2,\ \infty)}$

$f(x) < 0$ $\quad \underline{(-\infty,\ -4) \cup (-1,\ 2)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

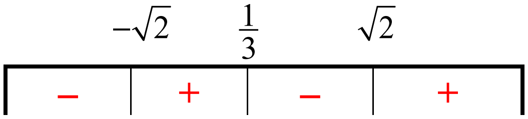
$$f(x) = 3x^3 - x^2 - 6x + 2$$

Solution

$$\textit{possibilities for } \frac{c}{d} : \pm\left\{\frac{2}{3}\right\} = \pm\left\{1,\ 2,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

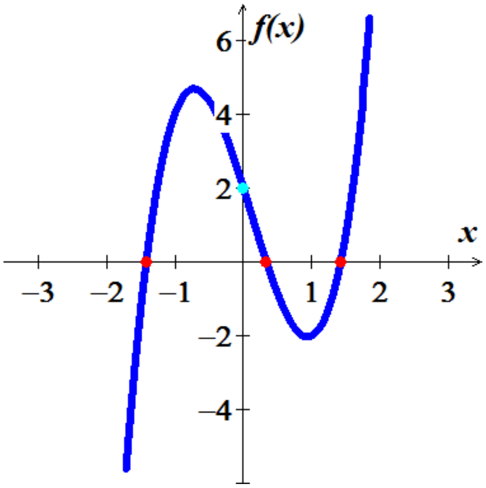
$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -6 & 2 \\ & & 1 & 0 & -2 \\ \hline & 3 & 0 & -6 & \boxed{0} \end{array} \rightarrow \ 3x^2 - 6 = 0$$

$$\underline{x = \frac{1}{3},\ \pm\sqrt{2}}$$



$f(x) > 0$ $\quad \left(-\sqrt{2},\ \frac{1}{3}\right) \cup \left(\sqrt{2},\ \infty\right)$

$f(x) < 0$ $\quad \left(-\infty,\ -\sqrt{2}\right) \cup \left(\frac{1}{3},\ \sqrt{2}\right)$



Exercise

Find all values of *x* such that $f(x) > 0$ and all *x* such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 8x^2 + 8x + 24$$

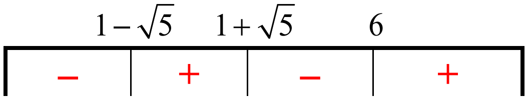
Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

$$\begin{array}{r|rrrr} 6 & 1 & -8 & 8 & 24 \\ & & 6 & -12 & -24 \\ \hline & 1 & -2 & -4 & \boxed{0} \end{array} \quad \to \quad x^2 - 2x - 4 = 0$$

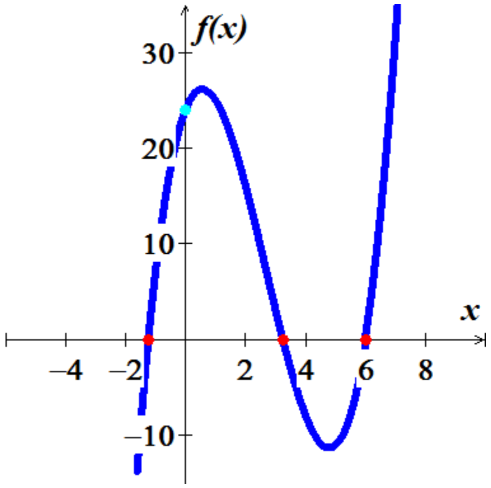
$$x = \frac{2 \pm \sqrt{4+16}}{2}$$

$x = 6,\ 1 \pm \sqrt{5}$



$f(x) > 0$ $\quad \left(1-\sqrt{5},\ 1+\sqrt{5}\right) \cup (6,\ \infty)$

$f(x) < 0 \quad \boxed{\left(-\infty,\ 1-\sqrt{5}\right) \cup \left(1+\sqrt{5},\ 6\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 7x^2 - 7x + 69$$

Solution

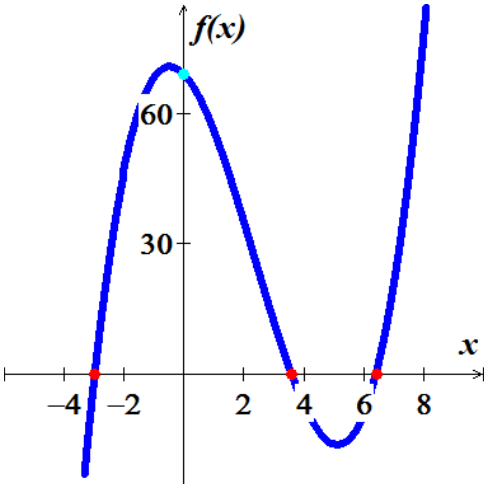
possibilities for $\frac{c}{d} : \pm\left\{\frac{69}{1}\right\} = \pm\{1,\ 3,\ 23,\ 69\}$

$$\begin{array}{r|rrrr} -3 & 1 & -7 & -7 & 69 \\ & & -3 & 30 & -69 \\ \hline & 1 & -10 & 23 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - 10x + 23 = 0$$

$$x = \frac{10 \pm \sqrt{100-92}}{2}$$

$$= \frac{10 \pm 2\sqrt{2}}{2}$$

$$\boxed{x = -3,\ 5 \pm \sqrt{2}}$$



	-3		$5-\sqrt{2}$		$5+\sqrt{2}$	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{\left(-3,\ 5-\sqrt{2}\right) \cup \left(5+\sqrt{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ -3\right) \cup \left(5-\sqrt{2},\ 5+\sqrt{2}\right)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 3x - 2$$

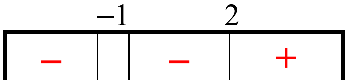
Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -3 & -2 \\ & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - x - 2 = 0$$

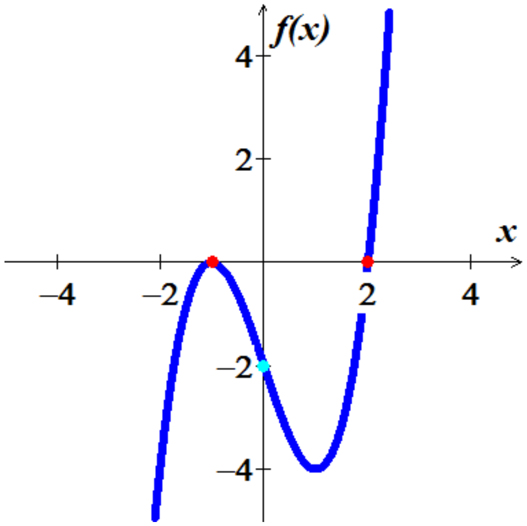
$x = -1,\ 2$ $\qquad\qquad$ $a - b + c = 0 \quad \rightarrow \quad x = -1,\ -\frac{c}{a}$

$\underline{x = -1,\ -1,\ 2}$



$f(x) > 0 \quad \underline{(2,\ \infty)}$

$f(x) < 0 \quad \underline{(-\infty,\ -1) \cup (-1,\ 2)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 2x + 1$$

Solution

possibilities for $\frac{c}{d} : \pm\{1\}$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -2 & 1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & -1 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 + x - 1 = 0$$

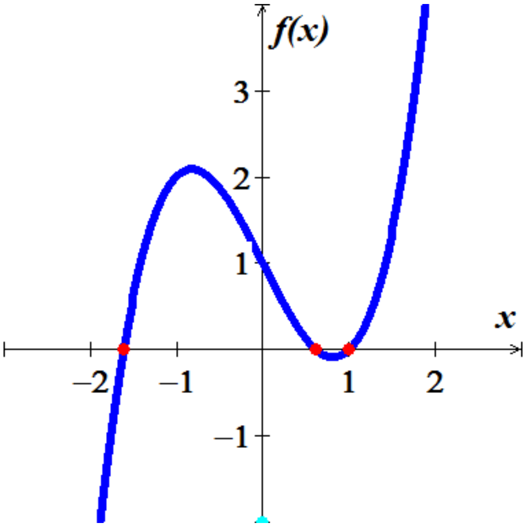
$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$\boxed{x = 1, \ \frac{-1 \pm \sqrt{5}}{2}}$$

	$\frac{-1-\sqrt{5}}{2}$		$\frac{-1+\sqrt{5}}{2}$		1	
$-$		$+$		$-$		$+$

$f(x) > 0$ $\quad \boxed{\left(\frac{-1-\sqrt{5}}{2}, \ \frac{-1+\sqrt{5}}{2}\right) \cup (1, \infty)}$

$f(x) < 0$ $\quad \boxed{\left(-\infty, \ \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \ 1\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 2x^2 - 11x + 12$$

Solution

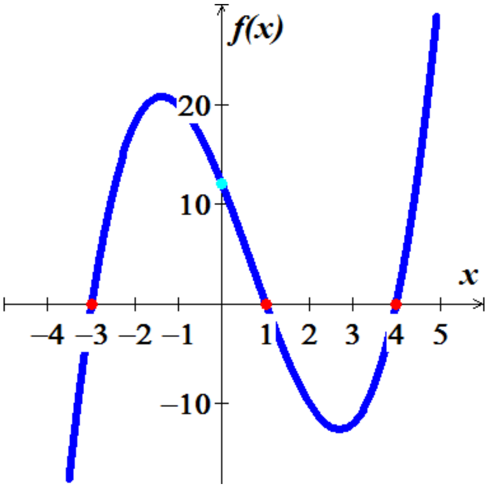
possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -11 & 12 \\ & & 1 & -1 & 12 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - x - 12 = 0$$

$$x = \frac{1 \pm \sqrt{1+48}}{2}$$

$$= \begin{cases} \frac{1-7}{2} = -3 \\ \frac{1+7}{2} = 4 \end{cases}$$

$\underline{x = -3,\ 1,\ 4}$



$-$	-3	$+$	1	$-$	4	$+$

$f(x) > 0 \quad \underline{(-3,\ 1) \cup (4,\ \infty)}$

$f(x) < 0 \quad \underline{(-\infty,\ -3) \cup (1,\ 4)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 2x^2 - 7x - 4$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{4}{1}\right\} = \pm\{1,\ 2,\ 4\}$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -7 & -4 \\ & & -1 & 3 & 4 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - 3x - 4 = 0$$

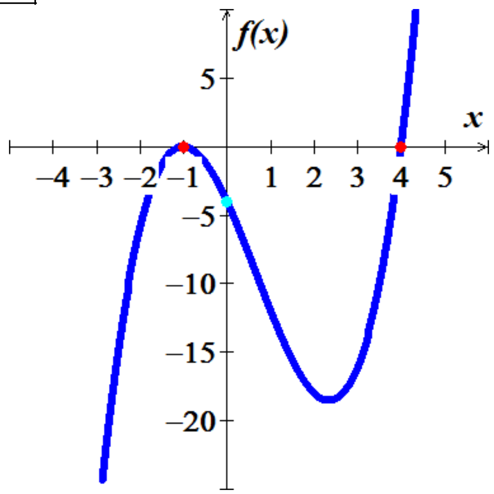
$\underline{x = -1,\ 4}$ $\qquad a - b + c = 0 \ \rightarrow \ x = -1,\ -\frac{c}{a}$

$\underline{x = -1,\ -1,\ 4}$

	-1		4	
$-$		$-$		$+$

$f(x) > 0 \quad \boxed{(4,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -1) \cup (-1,\ 4)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 10x - 12$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -10 & -12 \\ & & -2 & 4 & 12 \\ \hline & 1 & -2 & -6 & \boxed{0} \end{array} \quad \to \quad x^2 - 2x - 6 = 0$$

$$x = \frac{2 \pm \sqrt{4+24}}{2}$$

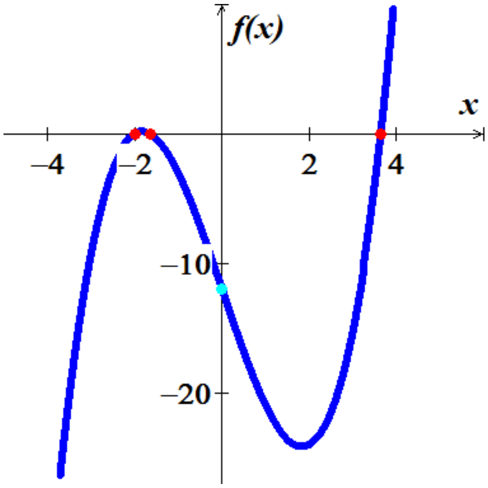
$$= \frac{2 \pm 2\sqrt{7}}{2}$$

$$\boxed{x = -2,\ 1 \pm \sqrt{7}}$$

	-2		$1-\sqrt{7}$		$1+\sqrt{7}$	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{\left(-2,\ 1-\sqrt{7}\right) \cup \left(1+\sqrt{7},\ \infty\right)}$

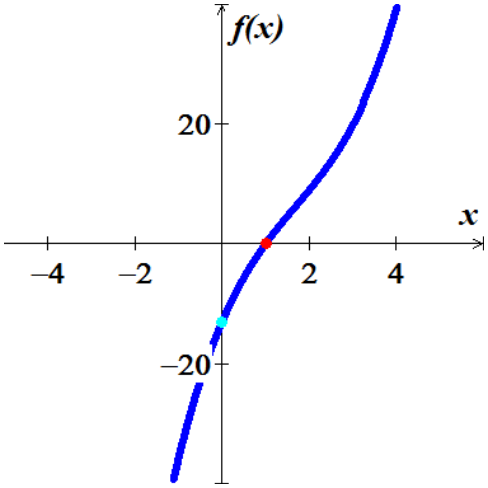
$f(x) < 0 \quad \underline{\left(-\infty,\ -2\right) \cup \left(1-\sqrt{7},\ 1+\sqrt{7}\right)\,|}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 5x^2 + 17x - 13$$



Solution

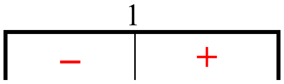
possibilities for $\frac{c}{d} : \pm\left\{\frac{13}{1}\right\} = \pm\{1,\ 13\}$

$$\begin{array}{c|cccc} 1 & 1 & -5 & 17 & -13 \\ & & 1 & -4 & 13 \\ \hline & 1 & -4 & 13 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - 4x + 13 = 0$$

$$x = \frac{4 \pm \sqrt{16-52}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$\underline{x = 1,\ 2 \pm 3i\,|}$



$f(x) > 0 \quad \underline{(1,\ \infty)\,|}$

$f(x) < 0 \quad \underline{(-\infty,\ 1)\,|}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^3 + 25x^2 - 24x + 5$$

Solution

$$\textit{possibilities for } \frac{c}{d} : \pm\left\{\frac{5}{6}\right\} = \pm\left\{1,\ 5,\ \frac{1}{2},\ \frac{5}{2},\ \frac{1}{3},\ \frac{5}{3},\ \frac{1}{6},\ \frac{5}{6}\right\}$$

$$\begin{array}{r|rrrr} -5 & 6 & 25 & -24 & 5 \\ & & -30 & 25 & -5 \\ \hline & 6 & -5 & 1 & \boxed{0} \end{array} \quad \rightarrow \quad 6x^2 - 5x + 1 = 0$$

$$x = \frac{5 \pm \sqrt{25-24}}{12}$$

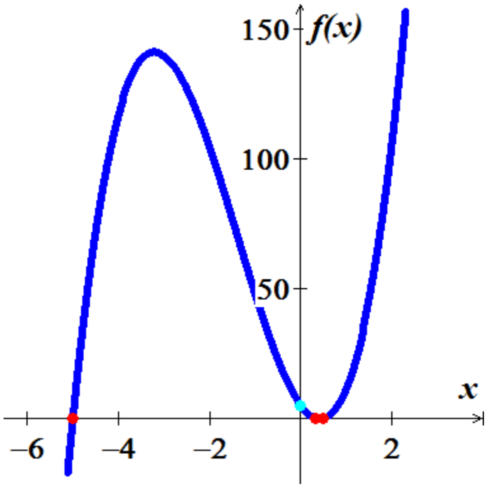
$$= \begin{cases} \dfrac{5-1}{12} = \dfrac{1}{3} \\ \dfrac{5+1}{12} = \dfrac{1}{2} \end{cases}$$

$$x = -5,\ \frac{1}{3},\ \frac{1}{2}$$

$$\begin{array}{c|c|c|c|c} & -5 & \frac{1}{3} & \frac{1}{2} & \\ - & & + & & - \quad\quad + \end{array}$$

$$f(x) > 0 \quad \left(-5,\ \frac{1}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)$$

$$f(x) < 0 \quad (-\infty,\ -5) \cup \left(\frac{1}{3},\ \frac{1}{2}\right)$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 8x^3 + 18x^2 + 45x + 27$$

Solution

$$\textit{possibilities}: \quad \pm\left\{\frac{27}{8}\right\} = \pm\left\{\frac{1,\ 3,\ 9,\ 27}{1,\ 2,\ 4,\ 8}\right\}$$

$$= \pm\left\{1,\ 3,\ 9,\ 27,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{3}{2},\ \frac{3}{4},\ \frac{3}{8},\ \frac{9}{2},\ \frac{9}{4},\ \frac{9}{8},\ \frac{27}{2},\ \frac{27}{4},\ \frac{27}{8}\right\}$$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 8 & 18 & 45 & 27 \\ & & -6 & -9 & -27 \\ \hline & 8 & 12 & 36 & \boxed{0} \end{array} \quad \rightarrow \quad 8x^2 + 12x + 36 = 0$$

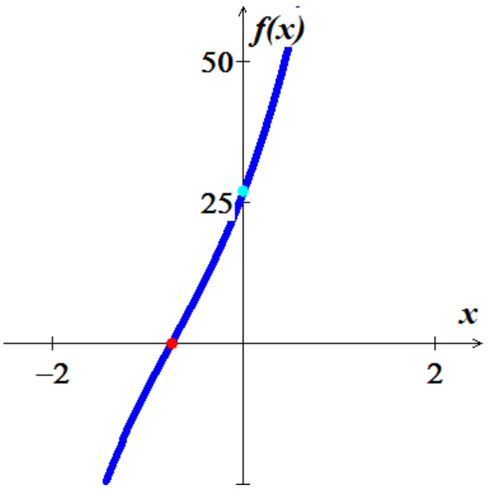
$$\boxed{x=-\frac{3}{4},\ \ -\frac{3}{4}\pm i\frac{3\sqrt{7}}{4}}$$

$-\frac{3}{4}$

−	+

$f(x)>0$ $\quad \boxed{\left(-\frac{3}{4},\ \infty\right)}$

$f(x)<0$ $\quad \boxed{\left(-\infty,\ -\frac{3}{4}\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=3x^3-x^2+11x-20$$

Solution

possibilities : $\pm\left\{\frac{20}{3}\right\}=\pm\left\{\frac{1,\ 2,\ 4,\ 5,\ 10,\ 20}{1,\ 3}\right\}$

$$=\pm\left\{1,\ 2,\ 4,\ 5,\ 10,\ 20,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{5}{3},\ \frac{10}{3},\ \frac{20}{3}\right\}$$

$$\begin{array}{c|cccc} \frac{4}{3} & 3 & -1 & 11 & -20 \\ & & 4 & 4 & 20 \\ \hline & 3 & 3 & 15 & \boxed{0} \end{array} \quad \rightarrow\ 3x^2+3x+15=0$$

$$x=\frac{-3\pm\sqrt{9-180}}{6}$$

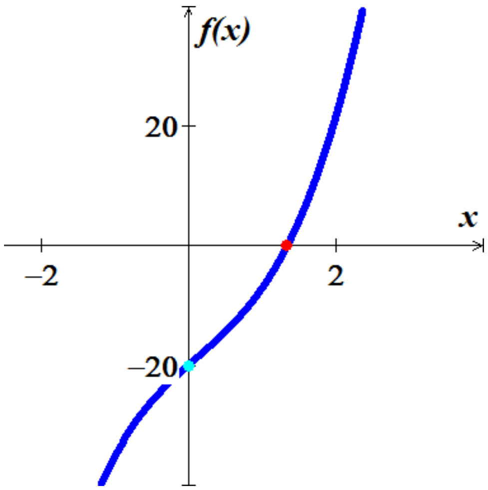
$$\boxed{x=\frac{4}{3},\ \ -\frac{1}{2}\pm i\frac{\sqrt{19}}{2}}$$

$\frac{4}{3}$

−	+

$f(x)>0$ $\quad \boxed{\left(\frac{4}{3},\ \infty\right)}$

$f(x)<0$ $\quad \boxed{\left(-\infty,\ \frac{4}{3}\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^4-x^3-9x^2+3x+18$$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{18}{1}\right\}=\pm\{1,\ 2,\ 3,\ 6,\ 9,\ 18\}$

$$\begin{array}{r|rrrrr}
-2 & 1 & -1 & -9 & 3 & 18 \\
 & & -2 & 6 & 6 & -18 \\
\hline
3 & 1 & -3 & -3 & 9 & 0 \\
 & & 3 & 0 & -9 & \\
\hline
 & 1 & 0 & -3 & 0 &
\end{array}$$

$\to x^3-3x^2-3x+9=0 \to \pm\left\{\frac{9}{1}\right\}=\pm\{1,\ 3,\ 9\}$

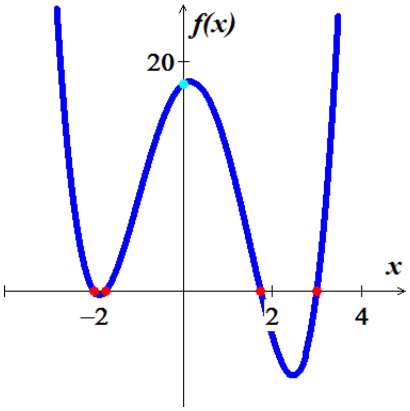
$\to x^2-3=0 \Rightarrow \underline{x=\pm\sqrt{3}}$

$\underline{x=-2,\ 3,\ \pm\sqrt{3}}$

$$\begin{array}{c|c|c|c|c}
 & -2 & -\sqrt{3} & \sqrt{3} & 3 \\
+ & - & + & - & +
\end{array}$$

$f(x)>0$ $\quad \underline{(-\infty,\ -2)\cup\left(-\sqrt{3},\ \sqrt{3}\right)\cup(3,\ \infty)}$

$f(x)<0$ $\quad \underline{\left(-2,\ -\sqrt{3}\right)\cup\left(\sqrt{3},\ 3\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=2x^4-9x^3+9x^2+x-3$$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{3}{2}\right\}=\pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & -9 & 9 & 1 & -3 \\
 & & 2 & -7 & 2 & 3 \\
\hline
1 & 2 & -7 & 2 & 3 & 0 \\
 & & 2 & -5 & -3 & \\
\hline
 & 2 & -5 & -3 & 0 &
\end{array}$$

$\to 2x^3-7x^2+2x+3=0 \to \pm\left\{\frac{3}{2}\right\}=\pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$\to 2x^2-5x-3=0$

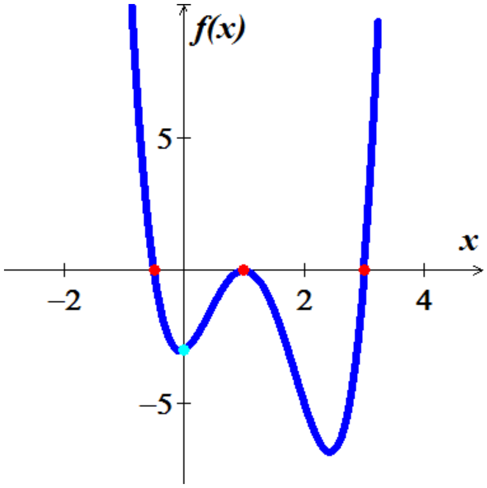
$$x=\frac{5\pm\sqrt{25+24}}{4}$$

$$=\begin{cases}\dfrac{5-7}{4}=-\dfrac{1}{2}\\[2mm] \dfrac{5+7}{4}=3\end{cases}$$

$x=1,\ 1,\ -\frac{1}{2},\ 3$

$-\frac{1}{2}$ 1 3

+	–		–	+



$f(x)>0$ $\left(-\infty,\ -\frac{1}{2}\right)\cup(3,\ \infty)$

$f(x)<0$ $\left(-\frac{1}{2},\ 1\right)\cup(1,\ 3)$

Exercise

Find all values of *x* such that $f(x)>0$ and all *x* such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=6x^4+5x^3-17x^2-6x$$

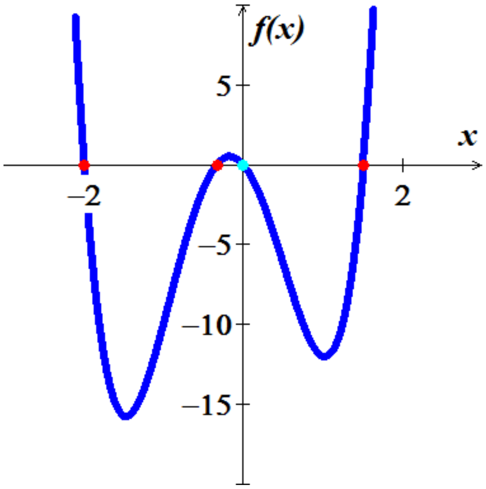
Solution

$x\left(6x^3+5x^2-17x-6\right)=0 \quad\to\quad x=0$

possibilities : $\pm\left\{\frac{6}{6}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrr} -2 & 6 & 5 & -17 & -6 \\ & & -12 & 14 & 6 \\ \hline & 6 & -7 & -3 & \boxed{0} \end{array} \quad\to\quad 6x^2-7x-3=0$$

$x=0,\ -2,\ -\frac{1}{3},\ \frac{3}{2}$



-2 $-\frac{1}{3}$ 0 $\frac{3}{2}$

+	–	+	–	+

$f(x)>0$ $(-\infty,\ -2)\cup\left(-\frac{1}{3},\ 0\right)\cup\left(\frac{3}{2},\ \infty\right)$

$f(x)<0$ $\left(-2,\ -\frac{1}{3}\right)\cup\left(0,\ \frac{3}{2}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 2x^2 - 16x - 15$$

Solution

$$possibilities: \ \pm\left\{\frac{15}{1}\right\} = \pm\{1,\ 3,\ 5,\ 15\}$$

$$\begin{array}{r|rrrrr}
-1 & 1 & 0 & -2 & -16 & -15 \\
 & & -1 & 1 & 1 & 15 \\ \hline
3 & 1 & -1 & -1 & -15 & 0 \\
 & & 3 & 6 & 15 & \\ \hline
 & 1 & 2 & 5 & 0 &
\end{array}$$

$$\to x^3 - x^2 - x - 15 = 0 \to \pm\left\{\frac{15}{1}\right\} = \pm\{1,\ 3,\ 5,\ 15\}$$

$$\to x^2 + 2x + 5 = 0$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

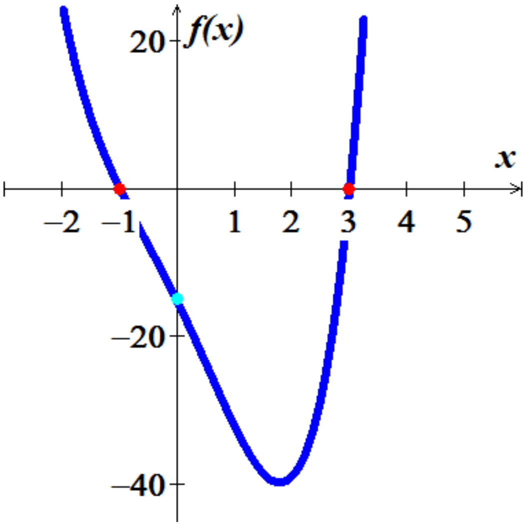
$$= -1 \pm 2i$$

$$\underline{x = -1,\ 3,\ -1 \pm 2i}$$

$$\begin{array}{c|c|c}
\multicolumn{1}{r}{-1} & \multicolumn{1}{r}{3} & \\
+ & - & +
\end{array}$$

$f(x) > 0$ $\quad \underline{(-\infty,\ -1) \cup (3,\ \infty)}$

$f(x) < 0$ $\quad \underline{(-1,\ 3)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 2x^3 - 5x^2 + 8x + 4$$

Solution

$$\text{possibilities}: \ \pm\left\{\frac{4}{1}\right\} = \pm\{1,\ 2,\ 4\}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -2 & -5 & 8 & 4 \\ & & 2 & 0 & -10 & -4 \\ \hline -2 & 1 & 0 & -5 & -2 & 0 \\ & & -2 & 4 & 2 & \\ \hline & 1 & -2 & -1 & 0 & \end{array}$$

$$\rightarrow x^3 - 5x - 2 = 0 \rightarrow \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$$

$$\rightarrow x^2 - 2x - 1 = 0$$

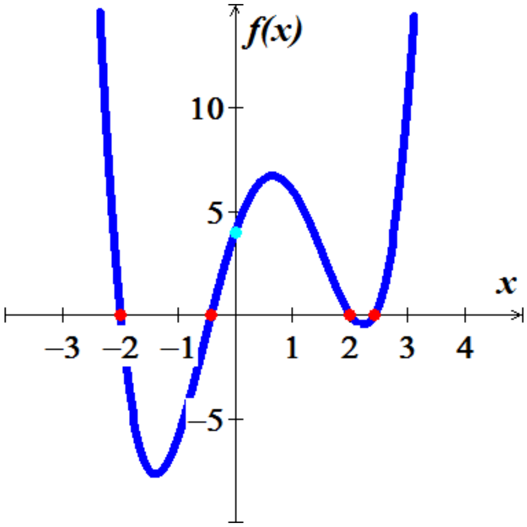
$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$\underline{x = -2,\ 2,\ 1 \pm \sqrt{2}}$$

$$\begin{array}{c|c|c|c|c} & -2 & 1-\sqrt{2} & 2 & 1+\sqrt{2} \\ + & - & + & - & + \end{array}$$

$f(x) > 0$ $\quad \underline{(-\infty,\ -2) \cup (1-\sqrt{2},\ 2) \cup (1+\sqrt{2},\ \infty)}$

$f(x) < 0$ $\quad \underline{(-2,\ 1-\sqrt{2}) \cup (2,\ 1+\sqrt{2})}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 17x^3 + 4x^2 + 35x - 24$$

Solution

$$possibilities: \ \pm\left\{\frac{24}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\begin{array}{r|rrrrr} 1 & 2 & -17 & 4 & 35 & -24 \\ & & 2 & -15 & -11 & 24 \\ \hline 1 & 2 & -15 & -11 & 24 & 0 \\ & & 2 & -13 & 24 & \\ \hline & 2 & -13 & -24 & 0 & \end{array}$$

$$\rightarrow\ 2x^3 - 15x^2 - 11x + 24 = 0\ \rightarrow\ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$$

$$\rightarrow\ 2x^2 - 13x - 24 = 0$$

$$x = \frac{13 \pm \sqrt{169 + 192}}{4}$$

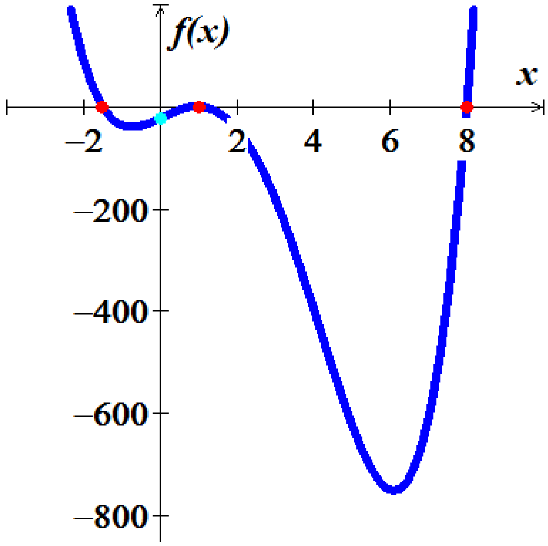
$$= \begin{cases} \frac{13 - 19}{4} = -\frac{3}{2} \\ \frac{13 + 19}{4} = 8 \end{cases}$$

$$\boxed{x = -\frac{3}{2},\ 1,\ 1,\ 8}$$

$$\begin{array}{ccccccc} & -\frac{3}{2} & & 1 & & 8 & \\ + & | & - & \| & - & | & + \end{array}$$

$$f(x) > 0 \quad \boxed{\left(-\infty,\ -\frac{3}{2}\right) \cup (8,\ \infty)}$$

$$f(x) < 0 \quad \boxed{\left(-\frac{3}{2},\ 1\right) \cup (1,\ 8)}$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 + x^3 - 3x^2 - 5x - 2$$

Solution

possibilities : $\pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$$\begin{array}{r|rrrrr} -1 & 1 & 1 & -3 & -5 & -2 \\ & & -1 & 0 & 3 & 2 \\ \hline -1 & 1 & 0 & -3 & -2 & 0 \\ & & -1 & 1 & 2 & \\ \hline & 1 & -1 & -2 & 0 & \end{array}$$

$\rightarrow x^3 - 3x - 2 = 0 \rightarrow \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$\rightarrow x^2 - x - 2 = 0$

$$x = \frac{1 \pm \sqrt{1+8}}{2}$$

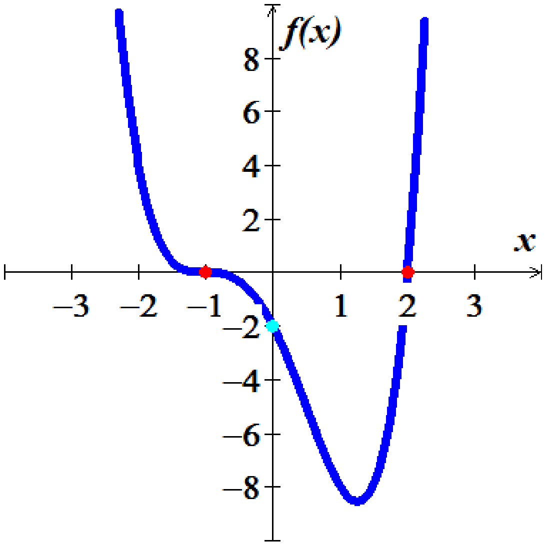
$$= \begin{cases} \frac{1-3}{2} = -1 \\ \frac{1+3}{2} = 2 \end{cases}$$

$x = -1,\ -1,\ -1,\ 2$

$$\begin{array}{|ccccc|} & -1 & & 2 & \\ + & | & - & | & + \end{array}$$

$f(x) > 0$ $(-\infty,\ -1) \cup (2,\ \infty)$

$f(x) < 0$ $(-2,\ 2)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 - 17x^3 - 11x^2 + 42x$$

Solution

$$x\left(6x^3 - 17x^2 - 11x + 42\right) = 0$$

$$x = 0 \quad 6x^3 - 17x^2 - 11x + 42 = 0$$

$$possibilities: \ \pm\left\{\frac{42}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ 7,\ 14,\ 21,\ 42,\ \frac{1}{2},\ \frac{3}{2},\ \frac{7}{2},\ \frac{21}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{7}{3},\ \frac{14}{3},\ \frac{1}{6},\ \frac{7}{6},\ \frac{21}{6}\right\}$$

$$\begin{array}{r|rrrr} 2 & 6 & -17 & -11 & 42 \\ & & 12 & -10 & -42 \\ \hline & 6 & -5 & -21 & \boxed{0} \end{array} \quad \to \ 6x^2 - 5x - 21 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 504}}{12}$$

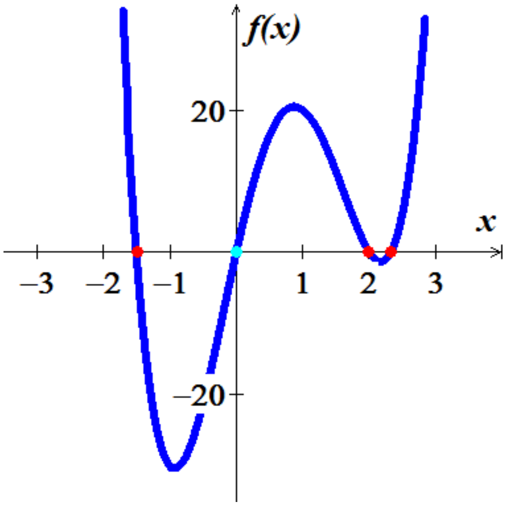
$$= \begin{cases} \dfrac{5 - 23}{12} = -\dfrac{3}{2} \\ \dfrac{5 + 23}{12} = \dfrac{7}{3} \end{cases}$$

$$\underline{x = -\frac{3}{2},\ 0,\ 2,\ \frac{7}{3}\ \Big|}$$

	$-\frac{3}{2}$		0		2		$\frac{7}{3}$	
+		−		+		−		+

$f(x) > 0 \quad \underline{\left(-\infty,\ -\frac{3}{2}\right) \cup (0,\ 2) \cup \left(\frac{7}{3},\ \infty\right)\ \Big|}$

$f(x) < 0 \quad \underline{\left(-\frac{3}{2},\ 0\right) \cup \left(2,\ \frac{7}{3}\right)\ \Big|}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f\left(x\right)$

$$f\left(x\right) = x^4 - 5x^2 - 2x$$

Solution

$$x\left(x^3 - 5x - 2\right) = 0$$

$$x = 0 \quad x^3 - 5x - 2 = 0$$

$$possibilities: \ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -5 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & \boxed{0} \end{array} \rightarrow x^2 - 2x - 1 = 0$$

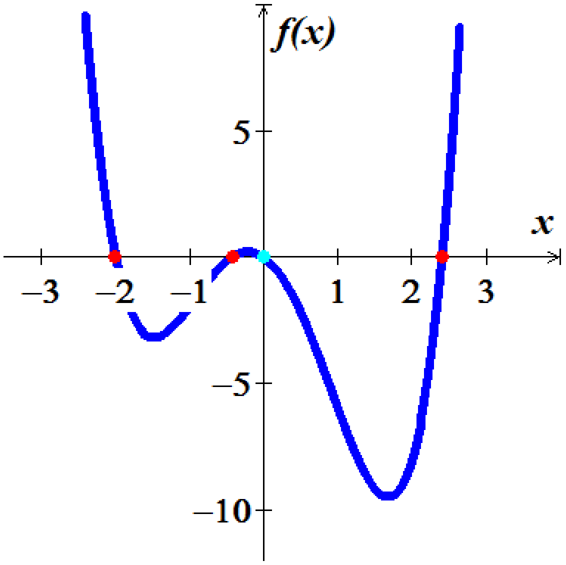
$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$\underline{x = -2,\ 0,\ 1 \pm \sqrt{2}}$$

$$\begin{array}{ccccccccc} & -2 & & 1-\sqrt{2} & & 2 & & 1+\sqrt{2} & \\ + & | & - & | & + & | & - & | & + \end{array}$$

$$f(x) > 0 \quad \underline{\left(-\infty,\ -2\right) \cup \left(1-\sqrt{2},\ 2\right) \cup \left(1+\sqrt{2},\ \infty\right)}$$

$$f(x) < 0 \quad \underline{\left(-2,\ 1-\sqrt{2}\right) \cup \left(2,\ 1+\sqrt{2}\right)}$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^4 - 4x^3 - 11x^2 + 16x - 4$$

Solution

$possibilities: \ \pm\left\{\frac{4}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3}\right\}$

$$\begin{array}{r|rrrrr} 1 & 3 & -4 & -11 & 16 & -4 \\ & & 3 & -1 & -12 & 4 \\ \hline 2 & 3 & -1 & -12 & 4 & 0 \\ & & 6 & 10 & -4 & \\ \hline & 3 & 5 & -2 & 0 & \end{array}$$

$\to 3x^3 - x^2 - 12x + 4 = 0 \to \pm\left\{\frac{4}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3}\right\}$

$\to 3x^2 + 5x - 2 = 0$

$$x = \frac{-5 \pm \sqrt{25+24}}{6}$$

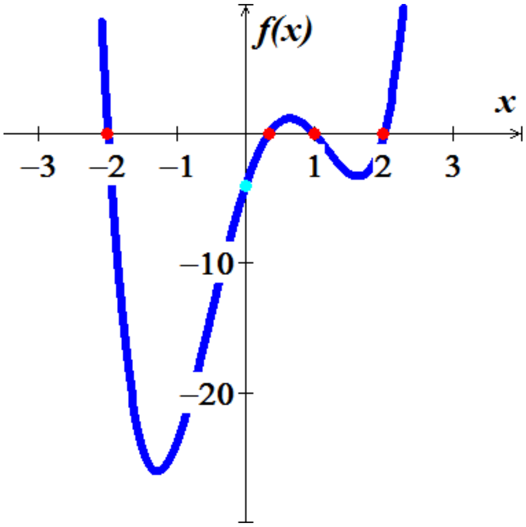
$$= \begin{cases} \frac{-5-7}{6} = -2 \\ \frac{-5+7}{6} = \frac{1}{3} \end{cases}$$

$$\boxed{x = -2,\ \frac{1}{3},\ 1,\ 2}$$

	-2		$\frac{1}{3}$		1		2	
+		−		+		−		+

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup \left(\frac{1}{3},\ 1\right) \cup (2,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-2,\ \frac{1}{3}\right) \cup (1,\ 2)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 + 23x^3 + 19x^2 - 8x - 4$$

Solution

$$\textit{possibilities}: \ \pm\left\{\frac{4}{6}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{6},\ \frac{2}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrrr}
-2 & 6 & 23 & 19 & -8 & -4 \\
 & & -12 & -22 & 6 & 4 \\
\hline
-2 & 6 & 11 & -3 & -2 & 0 \\
 & & -12 & 2 & 2 & \\
\hline
 & 6 & -1 & -1 & 0 &
\end{array}$$

$\rightarrow 6x^3 + 11x^2 - 3x - 2 = 0 \ \rightarrow \ \pm\left\{\frac{2}{6}\right\} = \pm\left\{1,\ 2,\ \frac{1}{6},\ \frac{1}{3}\right\}$

$\rightarrow 6x^2 - x - 1 = 0$

$$x = \frac{1 \pm \sqrt{25}}{12}$$

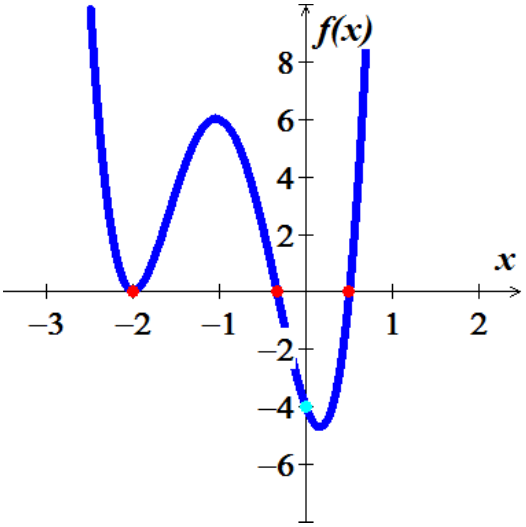
$$= \begin{cases} \frac{1-5}{12} = -\frac{1}{3} \\ \frac{1+5}{12} = \frac{1}{2} \end{cases}$$

$$\underline{x = -2,\ -2,\ -\frac{1}{3},\ \frac{1}{2}}\Big|$$

$$\begin{array}{c|c|c|c}
\multicolumn{1}{c}{} & -2 & -\frac{1}{3} & \frac{1}{2} \\
+ & + & - & +
\end{array}$$

$f(x) > 0 \quad (-\infty,\ -2) \cup \left(-2,\ -\frac{1}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)$

$f(x) < 0 \quad \left(-\frac{1}{3},\ \frac{1}{2}\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 4x^4 - 12x^3 + 3x^2 + 12x - 7$$

Solution

possibilities : $\pm\left\{\frac{7}{4}\right\} = \pm\left\{1,\ 7,\ \frac{1}{2},\ \frac{7}{2},\ \frac{1}{4},\ \frac{7}{4}\right\}$

$$\begin{array}{r|rrrrr} 1 & 4 & -12 & 3 & 12 & -7 \\ & & 4 & -8 & -5 & 7 \\ \hline -1 & 4 & -8 & -5 & 7 & 0 \\ & & -4 & 12 & -7 & \\ \hline & 4 & -12 & 7 & 0 & \end{array}$$

$\to 4x^3 - 8x^2 - 5x + 7 = 0 \ \to\ \pm\left\{\frac{7}{4}\right\} = \pm\left\{1,\ 7,\ \frac{1}{2},\ \frac{7}{2},\ \frac{1}{4},\ \frac{7}{4}\right\}$

$\to 4x^2 - 12x + 7 = 0$

$$x = \frac{12 \pm \sqrt{144 - 112}}{8}$$

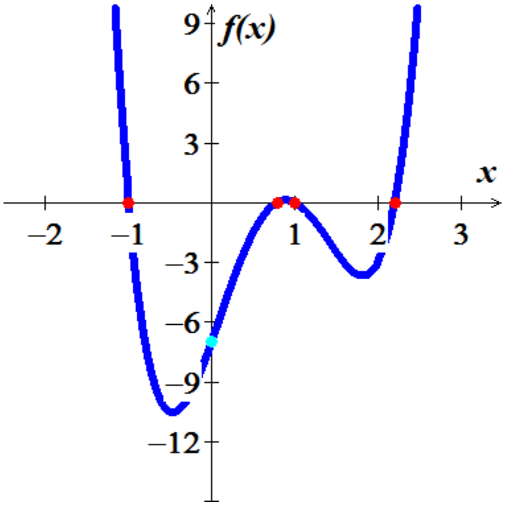
$$= \frac{12 \pm 4\sqrt{2}}{8}$$

$$\boxed{x = -1,\ 1,\ \frac{3 \pm \sqrt{2}}{2}}$$

$$\begin{array}{ccccccccc} & -1 & & \frac{3-\sqrt{2}}{2} & & 1 & & \frac{3+\sqrt{2}}{2} & \\ + & | & - & | & + & | & - & | & + \end{array}$$

$$f(x) > 0 \quad \boxed{(-\infty,\ -1) \cup \left(\frac{3-\sqrt{2}}{2},\ 1\right) \cup \left(\frac{3+\sqrt{2}}{2},\ \infty\right)}$$

$$f(x) < 0 \quad \boxed{\left(-1,\ \frac{3-\sqrt{2}}{2}\right) \cup \left(1,\ \frac{3+\sqrt{2}}{2}\right)}$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 9x^3 - 2x^2 + 27x - 12$$

Solution

possibilities : $\pm\left\{\frac{12}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 12,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrrr} 4 & 2 & -9 & -2 & 27 & -12 \\ & & 8 & -4 & -24 & 12 \\ \hline \frac{1}{2} & 2 & -1 & -6 & 3 & 0 \\ & & 1 & 0 & -3 & \\ \hline & 2 & 0 & -6 & 0 & \end{array}$$

$\rightarrow 2x^3 - x^2 - 6x + 3 = 0 \rightarrow \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

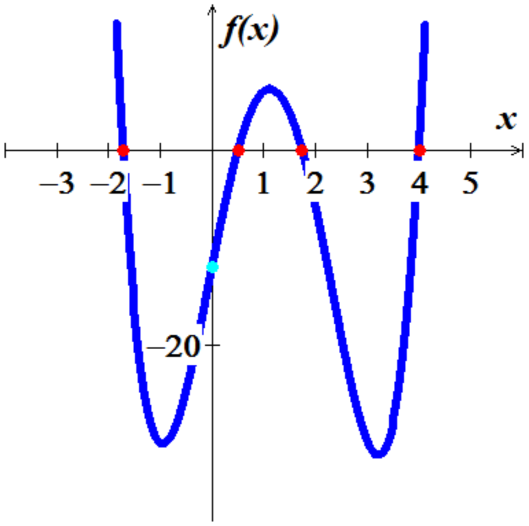
$\rightarrow 2x^2 - 6 = 0$

$x = \frac{1}{2},\ 4,\ \pm\sqrt{3}$

$$\begin{array}{ccccccccc} & -\sqrt{3} & & \frac{1}{2} & & \sqrt{3} & & 4 & \\ + & | & - & | & + & | & - & | & + \end{array}$$

$f(x) > 0$ $\quad \left(-\infty,\ -\sqrt{3}\right) \cup \left(\frac{1}{2},\ \sqrt{3}\right) \cup (4,\ \infty)$

$f(x) < 0$ $\quad \left(-\sqrt{3},\ \frac{1}{2}\right) \cup \left(\sqrt{3},\ 4\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 19x^3 + 51x^2 - 31x + 5$$

Solution

$$\textit{possibilities}: \ \pm\left\{\frac{5}{2}\right\} = \pm\left\{1,\ 5,\ \frac{1}{2},\ \frac{5}{2}\right\}$$

$$\begin{array}{r|rrrrr} 5 & 2 & -19 & 51 & -31 & 5 \\ & & 10 & -45 & 30 & -5 \\ \hline \frac{1}{2} & 2 & -9 & 6 & -1 & 0 \\ & & 1 & -4 & 1 & \\ \hline & 2 & -8 & 2 & 0 & \end{array}$$

$$\rightarrow 2x^3 - 9x^2 + 6x - 1 = 0 \rightarrow \pm\left\{\frac{1}{2}\right\}$$

$$\rightarrow 2x^2 - 8x + 2 = 0$$

$$x = \frac{8 \pm \sqrt{64-16}}{4}$$

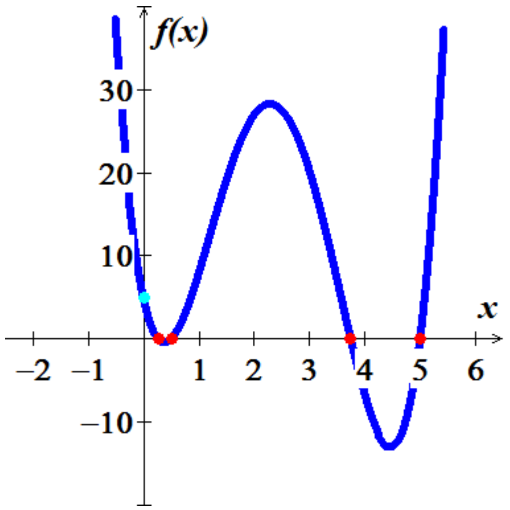
$$= \frac{8 \pm 4\sqrt{3}}{4}$$

$$\boxed{x = \frac{1}{2},\ 5,\ 2 \pm \sqrt{3}}$$

	$2-\sqrt{3}$	$\frac{1}{2}$	$2+\sqrt{3}$	5
+	−	+	−	+

$f(x) > 0$ $\boxed{\left(-\infty,\ 2-\sqrt{3}\right) \cup \left(\frac{1}{2},\ 2+\sqrt{3}\right) \cup (5,\ \infty)}$

$f(x) < 0$ $\boxed{\left(2-\sqrt{3},\ \frac{1}{2}\right) \cup \left(2+\sqrt{3},\ 5\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 4x^4 - 35x^3 + 71x^2 - 4x - 6$$

Solution

possibilities : $\pm\left\{\frac{6}{4}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{4},\ \frac{3}{4}\right\}$

$$\begin{array}{r|rrrrr} 3 & 4 & -35 & 71 & -4 & -6 \\ & & 12 & -69 & 6 & 6 \\ \hline -\frac{1}{4} & 4 & -23 & 2 & 2 & 0 \\ & & -1 & 6 & -2 & \\ \hline & 4 & -24 & 8 & 0 & \end{array}$$

$\rightarrow 4x^3 - 23x^2 + 2x + 2 = 0 \rightarrow \pm\left\{\frac{2}{4}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{4}\right\}$

$\rightarrow 4x^2 - 24x + 8 = 0$

$$x^2 - 6x + 2 = 0$$

$$x = \frac{6 \pm \sqrt{36-8}}{2}$$

$$= \frac{6 \pm 2\sqrt{7}}{4}$$

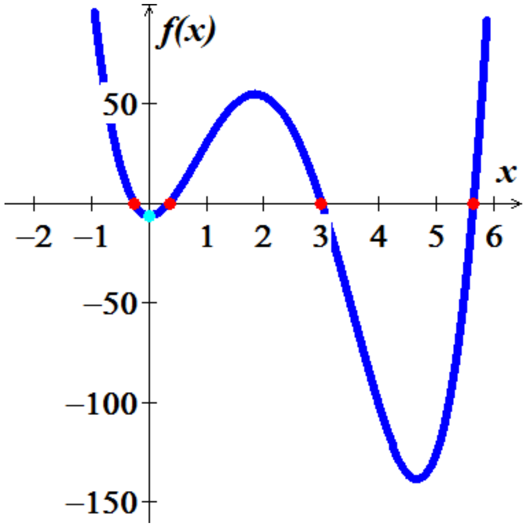
$$\boxed{x = -\frac{1}{4},\ 3,\ 3 \pm \sqrt{7}}$$

$-\frac{1}{4}$ $\quad 3-\sqrt{7}$ $\quad 3$ $\quad 3+\sqrt{7}$

+	−	+	−	+

$f(x) > 0$ $\quad \boxed{\left(-\infty,\ -\frac{1}{4}\right) \cup \left(3-\sqrt{7},\ 3\right) \cup \left(3+\sqrt{7},\ \infty\right)}$

$f(x) < 0$ $\quad \boxed{\left(-\frac{1}{4},\ 3-\sqrt{7}\right) \cup \left(3,\ 3+\sqrt{7}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$$

Solution

possibilities : $\pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & 3 & -4 & -3 & 2 \\
 & & 2 & 5 & 1 & -2 \\
\hline
-1 & 2 & 5 & 1 & -2 & 0 \\
 & & -2 & -3 & 2 & \\
\hline
 & 2 & 3 & -2 & 0 &
\end{array}$$

$\rightarrow 2x^3 - 23x^2 + 2x - 2 = 0 \rightarrow \pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$\rightarrow 2x^2 + 3x - 2 = 0$

$$x = \frac{-3 \pm \sqrt{9+16}}{4}$$

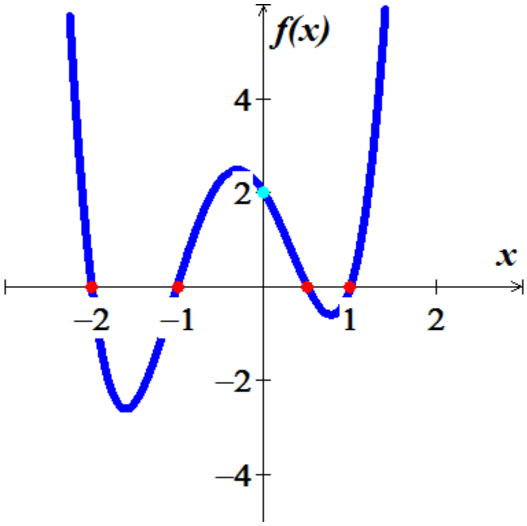
$$= \begin{cases} \frac{-3-5}{4} = -2 \\ \frac{-3+5}{4} = \frac{1}{2} \end{cases}$$

$$x = -2,\ -1,\ \frac{1}{2},\ 1$$

$$\begin{array}{ccccccccc}
 & -2 & & -1 & & \frac{1}{2} & & 1 & \\
+ & | & - & | & + & | & - & | & +
\end{array}$$

$f(x) > 0$ $\quad (-\infty,\ -2) \cup \left(-1,\ \frac{1}{2}\right) \cup (1,\ \infty)$

$f(x) < 0$ $\quad (-2,\ -1) \cup \left(\frac{1}{2},\ 1\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 + 3x^3 - 30x^2 - 6x + 56$$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{56}{1}\right\} = \pm\{1,\ 2,\ 4,\ 7,\ 8,\ 14,\ 28,\ 56\}$

$$\begin{array}{r|rrrrr} 4 & 1 & 3 & -3 & -6 & 56 \\ & & 4 & 28 & -8 & -56 \\ \hline -7 & 1 & 7 & -2 & -14 & 0 \\ & & -7 & 0 & 14 & \\ \hline & 1 & 0 & -2 & & \end{array}$$

$\rightarrow\ x^3 + 7x^2 - 2x - 14 = 0 \ \Rightarrow\ \frac{c}{d} = \pm\left\{\frac{14}{1}\right\} = \pm\{1,\ 2,\ 7,\ 14\}$

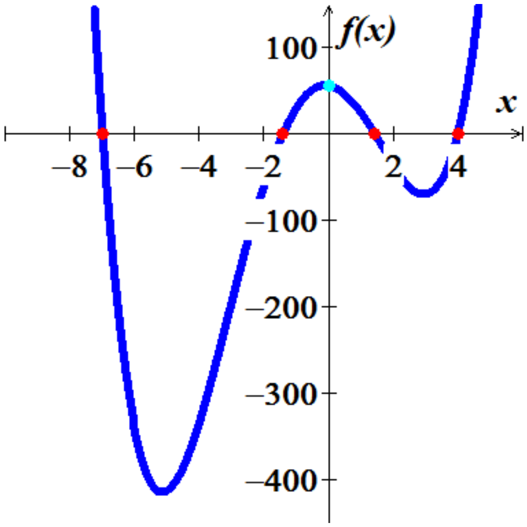
$\rightarrow\ x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}$

$x = 4,\ -7,\ \pm\sqrt{2}$

$$\begin{array}{ccccccccc} & -7 & & -\sqrt{2} & & \sqrt{2} & & 4 & \\ + & | & - & | & + & | & - & | & + \end{array}$$

$f(x) > 0 \quad (-\infty,\ -7) \cup (-\sqrt{2},\ \sqrt{2}) \cup (4,\ \infty)$

$f(x) < 0 \quad (-7,\ -\sqrt{2}) \cup (\sqrt{2},\ 4)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^5 - 10x^4 - 6x^3 + 24x^2 + 11x - 6$$

Solution

$$\text{possibilities for } \frac{c}{d} : \pm\left\{\frac{6}{3}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrrrr}
-1 & 3 & -10 & -6 & 24 & 11 & -6 \\
 & & -3 & 13 & -7 & -17 & 6 \\
\hline
-1 & 3 & -13 & 7 & 17 & -6 & 0 \\
 & & -3 & 16 & -23 & 6 & \\
\hline
2 & 3 & -16 & 23 & -6 & 0 & \\
 & & 6 & 20 & 6 & & \\
\hline
 & 3 & -10 & 3 & 0 & &
\end{array}$$

$$x^4 - 13x^3 + 7x^2 + 17x - 6 = 0 \to \pm\left\{\frac{6}{3}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^3 - 16x^2 + 26x - 6 = 0 \to \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^2 - 10x + 3 = 0$$

$$x = \frac{10 \pm \sqrt{100 - 36}}{6}$$

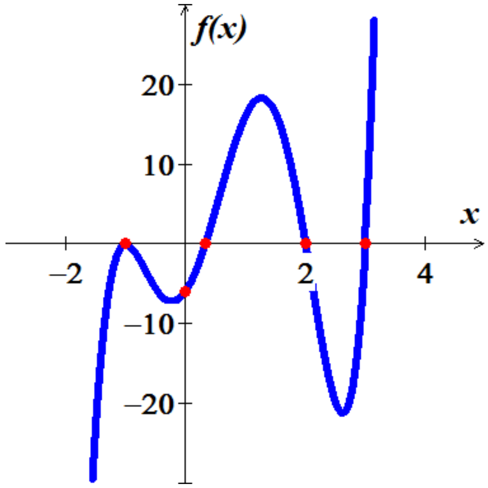
$$= \begin{cases} \dfrac{10 - 8}{6} = \dfrac{1}{3} \\ \dfrac{10 + 8}{4} = 3 \end{cases}$$

$$\boxed{x = -1,\ -1,\ \frac{1}{3},\ 2,\ 3}$$

$$\begin{array}{ccccccccc}
 & -1 & & \frac{1}{3} & & 2 & & 3 & \\
- & | & - & | & + & | & - & | & +
\end{array}$$

$f(x) > 0$ $\boxed{\left(\frac{1}{3},\ 2\right) \cup (3,\ \infty)}$

$f(x) < 0$ $\boxed{(-\infty,\ -1) \cup \left(-1,\ \frac{1}{3}\right) \cup (2,\ 3)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^5 + 19x^4 + x^3 - 6x^2$$

Solution

$$x^2\left(6x^3 + 19x^2 + x - 6\right) = 0 \quad \rightarrow \quad \underline{x = 0,\ 0|}$$

$$6x^3 + 19x^2 + x - 6 = 0$$

$$\textit{possibilities for } \tfrac{c}{d} : \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrr} -3 & 6 & 19 & 1 & -6 \\ & & -18 & -3 & 6 \\ \hline & 6 & 1 & -2 & \boxed{0} \end{array} \qquad 6x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1+48}}{12}$$

$$= \begin{cases} \dfrac{-1-7}{12} = -\dfrac{2}{3} \\ \dfrac{-1+7}{12} = \dfrac{1}{2} \end{cases}$$

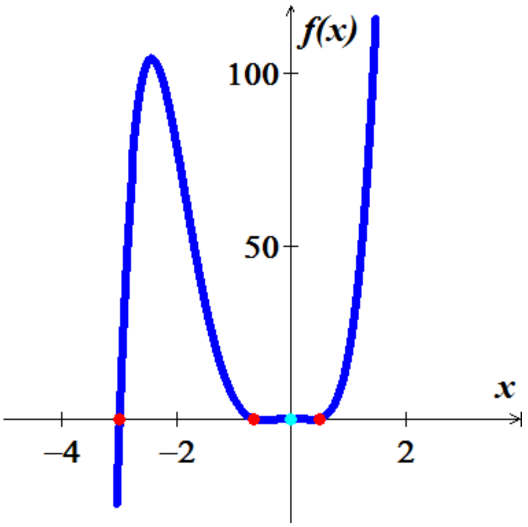
$$\underline{x = 0,\ 0,\ -\frac{2}{3},\ -3,\ \frac{1}{2}\ |}$$

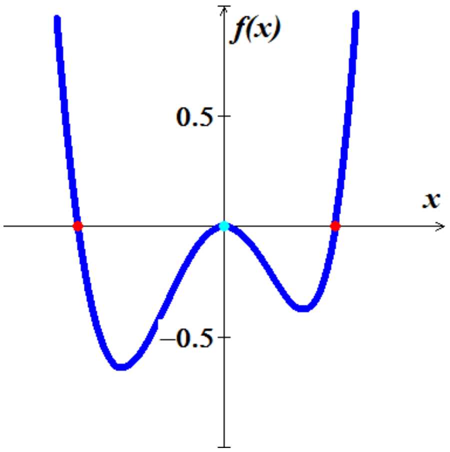
$$-3 \qquad -\frac{2}{3} \qquad 0 \qquad \frac{1}{2}$$

−	+	−	−	+

$f(x) > 0 \quad \underline{\left(-3,\ -\frac{2}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)\ |}$

$f(x) < 0 \quad \underline{(-\infty,\ -3) \cup \left(-\frac{2}{3},\ 0\right) \cup \left(0,\ \frac{1}{2}\right)\ |}$





Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Solution

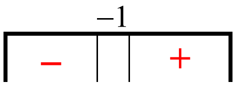
$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1 = (x+1)^5 = 0$$

possibilities for $\frac{c}{d} : \pm\{1\}$

$$\begin{array}{r|rrrrrrl}
-1 & 1 & 5 & 10 & 10 & 5 & 1 & \\
 & & -1 & -4 & -6 & -4 & -1 & \\
\hline
-1 & 1 & 4 & 6 & 4 & 1 & 0 & \to x^4 + 4x^3 + 6x^2 + 4x + 1 = 0 \to \pm\{1\} \\
 & & -1 & -3 & -3 & -1 & & \\
\hline
-1 & 1 & 3 & 3 & 1 & 0 & & \to x^3 + 3x^2 + 3x + 1 = 0 \to \pm\{1\} \\
 & & -1 & -2 & -1 & & & \\
\hline
 & 1 & 2 & 1 & 0 & & & \to x^2 + 2x + 1 = 0
\end{array}$$

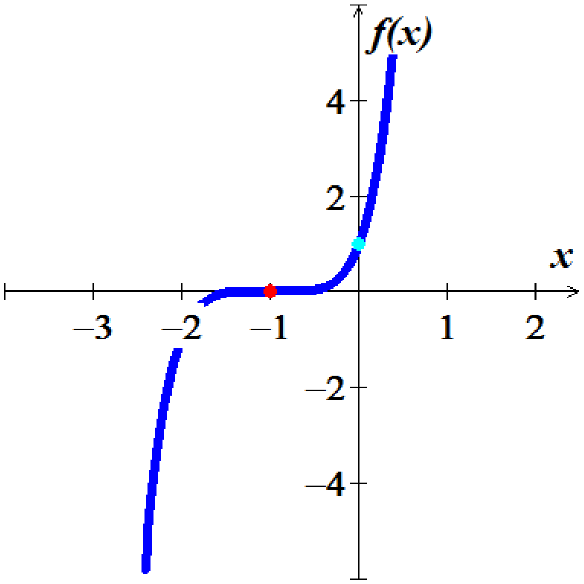
$$x^2 + 2x + 1 = (x+1)^2$$

$\underline{x = -1}$ (*multiplicity* of 5)



$f(x) > 0$ $\underline{(-1, \infty)}$

$f(x) < 0$ $\underline{(-\infty, -1)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^5 - x^4 - 7x^3 + 7x^2 + 12x - 12$$

Solution

$$\textit{possibilities for } \frac{c}{d} : \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$$

$$\begin{array}{r|rrrrrrl}
1 & 1 & -1 & -7 & 7 & 12 & -12 & \\
 & & 1 & 0 & -7 & 0 & 12 & \\
\hline
2 & 1 & 0 & -7 & 0 & 12 & 0 & \rightarrow\ x^4 - 7x^2 - 12 = 0 \rightarrow \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\} \\
 & & 2 & 4 & -6 & -12 & & \\
\hline
-2 & 1 & 2 & -3 & -6 & 0 & & \rightarrow\ x^3 + 2x^2 - 3x - 6 = 0 \rightarrow \pm\{1,\ 2,\ 3,\ 6\} \\
 & & -2 & 0 & 6 & & & \\
\hline
 & 1 & 0 & -3 & 0 & & & \rightarrow\ x^2 - 3 = 0
\end{array}$$

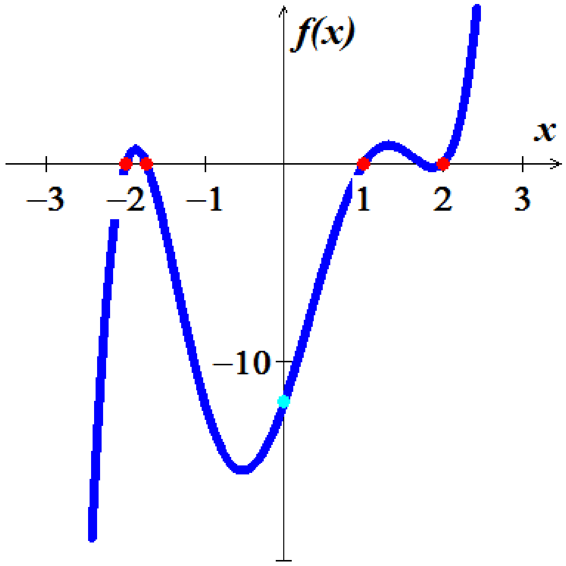
$x^2 = 3$

$\underline{x = -2,\ 1,\ 2,\ \pm\sqrt{3}\ |}$

$$\begin{array}{ccccccccccc}
 & -2 & & -\sqrt{3} & & 1 & & \sqrt{3} & & 2 & \\
- & | & + & | & - & | & + & | & - & | & +
\end{array}$$

$f(x) > 0$ $\quad (-2,\ -\sqrt{3}) \cup (1,\ \sqrt{3}) \cup (2,\ \infty)$

$f(x) < 0$ $\quad (-\infty,\ -2) \cup (-\sqrt{3},\ 1) \cup (\sqrt{3},\ 2)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^5-2x^3-8x$$

Solution

$$x\left(x^4-2x^2-8\right)=0$$

$$x=0$$

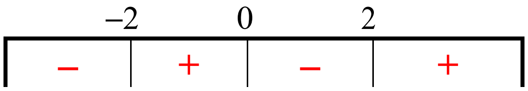
$$x^4-2x^2-8=0.$$

$$x^2=\frac{2\pm\sqrt{4+32}}{2}$$

$$=\begin{cases}\frac{2-6}{2}=-2\\ \frac{2+6}{2}=4\end{cases}$$

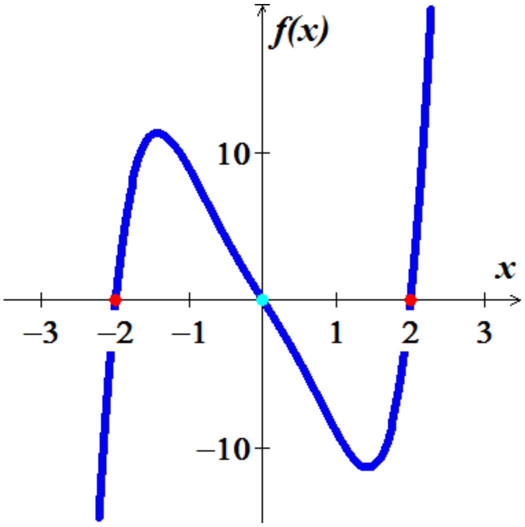
$$\begin{cases}x^2=-2 & \rightarrow & x=\pm i\sqrt{2}\\ x^2=4 & \rightarrow & x=\pm 2\end{cases}$$

$$x=0,\ \pm 2,\ \pm i\sqrt{2}$$



$f(x)>0$ $(-2,\ 0)\cup(2,\ \infty)$

$f(x)<0$ $(-\infty,\ -2)\cup(0,\ 2)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^6 - 10x^5 - 29x^4 + 34x^3 + 50x^2 - 24x - 24$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{3}\right\}$

$$= \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{8}{3}\right\}$$

$$\begin{array}{r|rrrrrrr}
1 & 3 & -10 & -29 & 34 & 50 & -24 & -24 \\
 & & 3 & -7 & -36 & -2 & 48 & 24 \\ \hline
-1 & 3 & -7 & -36 & -2 & 48 & 24 & 0 \\
 & & -3 & 10 & 26 & -24 & -24 & \\ \hline
-2 & 3 & -10 & -26 & 24 & 24 & 0 & \\
 & & -6 & 32 & -12 & -24 & & \\ \hline
-\frac{2}{3} & 3 & -16 & 6 & 12 & 0 & & \\
 & & -2 & 12 & -12 & & & \\ \hline
 & 3 & -18 & 18 & 0 & & &
\end{array}$$

$$\rightarrow\ 3x^5 - 7x^4 - 36x^3 - 2x^2 + 48x + 24 = 0$$

$$\rightarrow\ 3x^4 - 10x^3 - 26x^2 + 24x + 24 = 0$$

$$\rightarrow\ 3x^3 - 16x^2 + 12x - 12 = 0$$

$$\rightarrow\ 3x^2 - 18x + 18 = 0$$

$$x^2 - 6x + 6 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 24}}{2}$$

$$= \frac{6 \pm 2\sqrt{3}}{2}$$

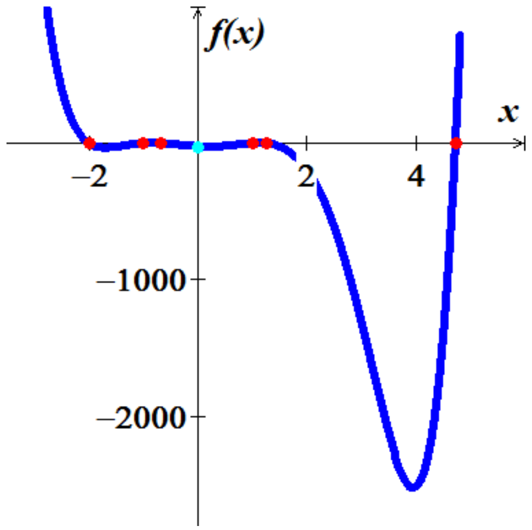
$$\boxed{x = -2,\ \ -1,\ 1,\ \ -\frac{2}{3},\ 3 \pm \sqrt{3}}$$

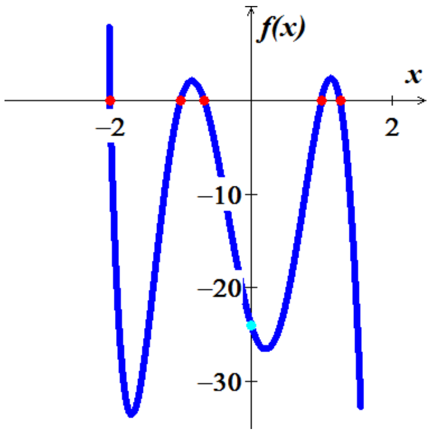
-2 $\quad -1$ $\quad -\frac{2}{3}$ $\quad 1$ $\quad 3-\sqrt{3}$ $\quad 3+\sqrt{3}$

+	–	+	–	+	–	+

$f(x) > 0$ $\quad \boxed{(-\infty,\ -2) \cup \left(-1,\ -\frac{2}{3}\right) \cup \left(1,\ 3-\sqrt{3}\right) \cup \left(3+\sqrt{3},\ \infty\right)}$

$f(x) < 0$ $\quad \boxed{(-2,\ -1) \cup \left(-\frac{2}{3},\ 1\right) \cup \left(3-\sqrt{3},\ 3+\sqrt{3}\right)}$


f(x)
x
−2
2
4
−1000
−2000


f(x)
x
−2
2
−10
−20
−30