Section 1.3 – Quadratic Functions

Basic Complex Number

$$i^2 = -1 \qquad \Rightarrow\ i = \sqrt{-1} \qquad \Rightarrow\ \sqrt{-1} = i$$

The number i is called the ***imaginary unit***.

Example

$\sqrt{-8} = 2i\ \sqrt{2}$

$$\begin{aligned}\sqrt{-7}\sqrt{-7} &= i\sqrt{7}\ i\sqrt{7}\\ &= i^2\left(\sqrt{7}\right)^2\\ &= -7\end{aligned}$$

Complex number is written in a form: $z = a + ib$

a is the real part

b is the imaginary part

Conjugate of a complex number $a + bi$ is $a - bi$

A ***quadratic equation*** in x is an equation that can be written in the general form:

$$ax^2 + bx + c = 0 \qquad \text{where } a,\ b, \text{ and c are real numbers,}$$

$$4x^2 - 3x + 2 = 0 \qquad a = 4 \quad b = -3 \quad c = 2$$

Solving Quadratic Equations by *Factoring*

The Zero-Product Principle

If $AB = 0$ then $A = 0$ or $B = 0$.

Example

Solve $6x^2 + 7x - 3 = 0$

Solution

$(3x - 1)(2x + 3) = 0$

$3x - 1 = 0 \qquad 2x + 3 = 0$

$\underline{x = \frac{1}{3}}\ \Big| \qquad \underline{x = -\frac{3}{2}}\ \Big|$

The Square Root Property

If u is an algebraic expression and d is a nonzero real number, then $u^2 = d$ has exactly two solutions:

$$\textit{If } u^2 = d, \textit{ then } u = \sqrt{d} \textit{ or } u = -\sqrt{d}$$

Equivalently,

$$\textit{If } u^2 = d \Rightarrow u = \pm\sqrt{d}.$$

Example

Solve $3x^2 - 21 = 0$

Solution

$3x^2 = 21$

$x^2 = 7$

$\boxed{x = \pm\sqrt{7}}$

Example

Solve $5x^2 + 45 = 0$

Solution

$5x^2 = -45$

$x^2 = -9$

$x = \pm\sqrt{-9}$

$\boxed{x = \pm 3i}$

Example

Solve $(x+5)^2 = 11$

Solution

$x + 5 = \pm\sqrt{11}$

$\boxed{x = -5 \pm \sqrt{11}}$

Completing the Square

If $x^2 + bx$ is a binomial, then by ***adding*** $\left(\frac{b}{2}\right)^2$ which is the square of half the coefficient of *x*. a perfect square trinomial will result. That is.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 \qquad\qquad x^2 + bx + \left(\frac{1}{2}b\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example

Solve: $x^2 + 4x - 1 = 0$

Solution

$$x^2 + 4x \quad = 1$$

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 1 + \left(\frac{4}{2}\right)^2$$

$$x^2 + 4x + (2)^2 = 1 + 4$$

$$(x+2)^2 = 5$$

$$x + 2 = \pm\sqrt{5}$$

$$x = -2 \pm \sqrt{5}$$

Quadratic Formula

(*Using Completing the Square*)

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{1}{2}\frac{b}{a}\right)^2 = -\frac{c}{a} + \left(\frac{1}{2}\frac{b}{a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$*** \quad ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{cases} b^2 - 4ac > 0 \rightarrow 2 \text{ Real numbers} \\ b^2 - 4ac < 0 \rightarrow 2 \text{ Complex numbers} \\ b^2 - 4ac = 0 \rightarrow \text{One solution (repeated)} \end{cases}$$

Example

Solve: $2x^2+2x-1=0$

Solution

$\Rightarrow a=2 \quad b=2 \quad c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}$$

$$=\frac{-2\pm\sqrt{4+8}}{4}$$

$$=-\frac{2}{4}\pm\frac{\sqrt{12}}{4}$$

$$=-\frac{1}{2}\pm\frac{2\sqrt{3}}{4}$$

$$\boxed{=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}}$$

or

$$=\frac{-2\pm\sqrt{12}}{4}$$

$$=\frac{-2\pm2\sqrt{3}}{4}$$

$$=\frac{2\left(-1\pm\sqrt{3}\right)}{4}$$

$$\boxed{=\frac{-1\pm\sqrt{3}}{2}}$$

Example

Solve $x^2-4x=-2$

Solution

$\Rightarrow a=1 \quad b=-4 \quad c=2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}$$

$$=\frac{4\pm\sqrt{16-8}}{2}$$

$$=\frac{4\pm\sqrt{8}}{2}$$

$$=\frac{4\pm2\sqrt{2}}{2}$$

$$=\frac{2\left(2\pm\sqrt{2}\right)}{2}$$

$$\boxed{=2\pm\sqrt{2}}$$

Example

Solve: $x^2 - 2x + 2 = 0$

Solution

$$\Rightarrow a = 1 \quad b = -2 \quad c = 2$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2}{2} \pm \frac{\sqrt{-4}}{2}$$

$$= 1 \pm \frac{2i}{2}$$

$$\underline{= 1 \pm i}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= \frac{2(1 \pm i)}{2}$$

$$\underline{= 1 \pm i}$$

$$ax^2 + bx + c = 0$$

If $\boldsymbol{a + b + c = 0} \quad \Rightarrow x = 1, \ \frac{c}{a}$

Example

$$2x^2 + x - 3 = 0$$

$$2 + 1 - 3 = 0 \qquad \Rightarrow \underline{x = 1, \ -\frac{3}{2}}$$

If $\boldsymbol{a - b + c = 0} \quad \Rightarrow x = -1, \ -\frac{c}{a}$

Example

$$2x^2 - x - 3 = 0$$

$$2 - (-1) - 3 = 0 \qquad \Rightarrow \underline{x = -1, \ \frac{3}{2}}$$

Exercises *Section* 1.3 – Quadratic Functions

(1 – 48) Solve

1. $x^2 = -25$

2. $x^2 = 49$

3. $9x^2 = 100$

4. $4x^2 + 25 = 0$

5. $5x^2 + 35 = 0$

6. $5x^2 - 45 = 0$

7. $(x-4)^2 = 12$

8. $(x+3)^2 = -16$

9. $(x-2)^2 = -20$

10. $(4x+1)^2 = 20$

11. $x^2 - 6x = -7$

12. $-6x^2 = 3x + 2$

13. $3x^2 + 2x = 7$

14. $3x^2 + 6 = 10x$

15. $5x^2 + 2 = x$

16. $5x^2 = 2x - 3$

17. $x^2 + 8x + 15 = 0$

18. $x^2 + 5x + 2 = 0$

19. $x^2 + x - 12 = 0$

20. $x^2 - 2x - 15 = 0$

21. $x^2 - 4x - 45 = 0$

22. $x^2 - 6x - 10 = 0$

23. $2x^2 + 3x - 4 = 0$

24. $x^2 - x + 8 = 0$

25. $2x^2 - 13x = 1$

26. $r^2 + 3r - 3 = 0$

27. $x^3 + 8 = 0$

28. $4x^2 - 12x + 9 = 0$

29. $9x^2 - 30x + 25 = 0$

30. $x^2 - 14x + 49 = 0$

31. $x^2 - 8x + 16 = 0$

32. $x^2 + 6x + 13 = 0$

33. $2x^2 - 2x + 13 = 0$

34. $x^2 + 2x + 29 = 0$

35. $4x^2 + 4x + 13 = 0$

36. $x^2 - 2x + 26 = 0$

37. $9x^2 - 4x + 20 = 0$

38. $x^2 + 6x + 21 = 0$

39. $9x^2 - 12x - 49 = 0$

40. $x(x-3) = 18$

41. $x(x-4) - 21 = 0$

42. $(x-1)(x+4) = 14$

43. $(x-3)(x+8) = -30$

44. $x(x+8) = 16(x-1)$

45. $x(x+9) = 4(2x+5)$

46. $(x+1)^2 = 2(x+3)$

47. $(x+1)^2 - 5(x+2) = 3x + 7$

48. $x(8x+1) = 3x^2 - 2x + 2$

(49 – 60) Solve using formula

49. $x^2 + 6x - 7 = 0$

50. $x^2 - 6x - 7 = 0$

51. $3x^2 + 4x - 7 = 0$

52. $3x^2 - 4x - 7 = 0$

53. $3x^2 - x - 2 = 0$

54. $3x^2 + x - 2 = 0$

55. $2x^2 + 3x - 5 = 0$

56. $2x^2 - 3x - 5 = 0$

57. $x^2 - 3x - 4 = 0$

58. $x^2 + 3x - 4 = 0$

59. $x^2 + 2x + 1 = 0$

60. $4x^2 - x - 5 = 0$

61. Solve for the specified variable $A = \frac{\pi d^2}{4}$, *for d*

62. Solve for the specified variable $rt^2 - st - k = 0$ $(r \neq 0)$, *for t*