

Section 2.3 – Composition Functions

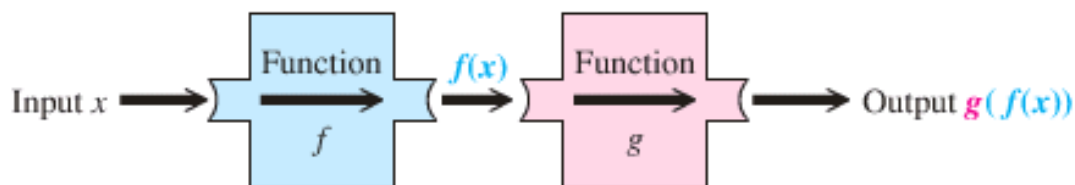
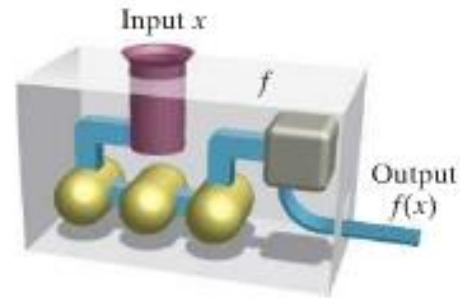
Composition of Functions

The composite function $g \circ f$, the composite of f and g , is defined as

$$(g \circ f)(x) = g(f(x))$$

Where x is in the domain of f

and $g(f(x))$ is in the domain of g



Example

Given that $f(x) = 5x + 6$ and $g(x) = 2x^2 - x - 1$, find $(f \circ g)(x)$ and $(g \circ f)(x)$

Solution

$$(f \circ g)(x) = f(g(x)) \qquad = f(2x^2 - x - 1) \qquad \text{Domain: All real numbers}$$

$$= 5(\text{-----}) + 6$$

$$= 5(2x^2 - x - 1) + 6$$

$$= 10x^2 - 5x - 5 + 6$$

$$= 10x^2 - 5x + 1$$

Domain: All real numbers

$$(g \circ f)(x) = g(f(x))$$

$$= g(5x + 6)$$

Domain: All real numbers

$$= 2(\quad)^2 - (\quad) - 1$$

$$= 2(5x + 6)^2 - (5x + 6) - 1$$

$$= 2(25x^2 + 60x + 36) - 5x - 6 - 1$$

$$= 50x^2 + 120x + 72 - 5x - 7$$

$$= 50x^2 + 115x + 65$$

Domain: All real numbers

Example

Let $f(x) = \sqrt{x}$ and $g(x) = 4x + 2$, find each of the following and its domain.

a) $(f \circ g)(x)$

b) $(g \circ f)(x)$

Solution

$$\begin{aligned} \text{a) } (f \circ g)(x) &= f(g(x)) \\ &= f(4x + 2) & (-\infty, \infty) \\ &= \sqrt{4x + 2} \end{aligned}$$

$$4x + 2 \geq 0$$

$$4x \geq -2$$

$$x \geq -\frac{2}{4}$$

$$\text{Domain: } \underline{x \geq -\frac{1}{2}} \quad \left[-\frac{1}{2}, \infty \right)$$

$$\begin{aligned} \text{b) } (g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x}) & x \geq 0 \\ &= 4\sqrt{x} + 2 & x \geq 0 \end{aligned}$$

$$\text{Domain: } \underline{x \geq 0} \quad [0, \infty)$$

Example

Let $f(x) = 2x - 1$ and $g(x) = \frac{4}{x-1}$ Find:

a) $(f \circ g)(2)$

b) $(g \circ f)(-3)$

Solution

$$\begin{aligned} \text{a) } (f \circ g)(2) &= f(g(2)) \\ &= f\left(\frac{4}{2-1}\right) \\ &= f(4) \\ &= 2(4) - 1 \\ &= 7 \end{aligned}$$

$$\begin{aligned} \text{b) } (g \circ f)(-3) &= g(f(-3)) \\ &= g(2(-3) - 1) \end{aligned}$$

$$\begin{aligned}
&= g(-7) \\
&= \frac{4}{-7-1} \\
&= \frac{4}{-8} \\
&= -\frac{1}{2}
\end{aligned}$$

Example

Given that $f(x) = \frac{4}{x+2}$ and $g(x) = \frac{1}{x}$, find

- a)** $(f \circ g)(x)$
- b)** Domain of $(f \circ g)(x)$

Solution

$$\begin{aligned}
\text{a) } (f \circ g)(x) &= f(g(x)) \\
&= f\left(\frac{1}{x}\right) && \text{Domain: } x \neq 0 \\
&= \frac{4}{\frac{1}{x}+2} \\
&= \frac{4}{\frac{1+2x}{x}} \\
&= 4 \div \frac{1+2x}{x} \\
&= 4 \frac{x}{1+2x} \\
&= \frac{4x}{1+2x} && \text{Domain: } x \neq -\frac{1}{2}
\end{aligned}$$

$$\text{b) Domain: } \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 0\right) \cup (0, \infty)$$

Exercises Section 2.3 – Composition Functions

1. Given that $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 8$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain then find $(f \circ g)(7)$
2. Given that $f(x) = \sqrt{x}$ and $g(x) = x - 1$, find
 - a) $(f \circ g)(x) = f(g(x))$
 - b) $(g \circ f)(x) = g(f(x))$
 - c) $(f \circ g)(2) = f(g(2))$
3. Given that $f(x) = \frac{x}{x+5}$ and $g(x) = \frac{6}{x}$, find
 - a) $(f \circ g)(x) = f(g(x))$
 - b) $(g \circ f)(x) = g(f(x))$
 - c) $(f \circ g)(2) = f(g(2))$
4. Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = 2x^2 + 3x - 4$, $g(x) = 2x - 1$
5. Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = x^3 + 2x^2$, $g(x) = 3x$
6. Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = |x|$, $g(x) = -7$
- (7 – 36) For the given function; find:
 - a) Find $(f \circ g)(x)$ and the **domain** of $f \circ g$
 - b) Find $(g \circ f)(x)$ and the **domain** of $g \circ f$
7. $f(x) = x - 3$ and $g(x) = x + 3$
8. $f(x) = \frac{2}{3}x$ and $g(x) = \frac{3}{2}x$
9. $f(x) = x - 1$ and $g(x) = 3x^2 - 2x - 1$
10. $f(x) = 3x - 2$ and $g(x) = x^2 - 5$
11. $f(x) = x^2 - 2$ and $g(x) = 4x - 3$
12. $f(x) = 4x^2 - x + 10$ and $g(x) = 2x - 7$
13. $f(x) = \sqrt{x}$ and $g(x) = x + 3$
14. $f(x) = \sqrt{x}$ and $g(x) = 2 - 3x$
15. $f(x) = 3x + 2$ and $g(x) = \sqrt{x}$
16. $f(x) = x^4$ and $g(x) = \sqrt[4]{x}$
17. $f(x) = x^n$ and $g(x) = \sqrt[n]{x}$
18. $f(x) = x^2 - 3x$ and $g(x) = \sqrt{x + 2}$
19. $f(x) = \sqrt{x - 2}$ and $g(x) = \sqrt{x + 5}$
20. $f(x) = x^2 + 2$ and $g(x) = \sqrt{3 - x}$
21. $f(x) = x^5 - 2$ and $g(x) = \sqrt[5]{x + 2}$
22. $f(x) = 1 - x^2$ and $g(x) = \sqrt{x^2 - 25}$

$$23. \quad f(x) = 2x + 3 \quad \text{and} \quad g(x) = \frac{x-3}{2}$$

$$24. \quad f(x) = 4x - 5 \quad \text{and} \quad g(x) = \frac{x+5}{4}$$

$$25. \quad f(x) = \frac{4}{1-5x} \quad \text{and} \quad g(x) = \frac{1}{x}$$

$$26. \quad f(x) = \frac{1}{x-2} \quad \text{and} \quad g(x) = \frac{x+2}{x}$$

$$27. \quad f(x) = \frac{1}{1+x} \quad \text{and} \quad g(x) = \frac{1-x}{x}$$

$$28. \quad f(x) = \frac{3x+5}{2} \quad \text{and} \quad g(x) = \frac{2x-5}{3}$$

$$29. \quad f(x) = \frac{x-1}{x-2} \quad \text{and} \quad g(x) = \frac{x-3}{x-4}$$

$$30. \quad f(x) = \frac{6}{x-3} \quad \text{and} \quad g(x) = \frac{1}{x}$$

$$31. \quad f(x) = \frac{6}{x} \quad \text{and} \quad g(x) = \frac{1}{2x+1}$$

$$32. \quad f(x) = 3x - 7 \quad \text{and} \quad g(x) = \frac{x+7}{3}$$

$$33. \quad f(x) = \frac{2x+3}{x-4} \quad \text{and} \quad g(x) = \frac{4x+3}{x-2}$$

$$34. \quad f(x) = \frac{2x+3}{x+4} \quad \text{and} \quad g(x) = \frac{-4x+3}{x-2}$$

$$35. \quad f(x) = x + 1 \quad \text{and} \quad g(x) = x^3 - 5x^2 + 3x + 7$$

$$36. \quad f(x) = x - 1 \quad \text{and} \quad g(x) = x^3 + 2x^2 - 3x - 9$$

(37 – 48) Evaluate each composite function, where $f(x) = 2x - 3$ and $g(x) = x^2 - 5x$

$$37. \quad (f \circ g)(4)$$

$$40. \quad (g \circ f)(-2)$$

$$43. \quad (f \circ g)(\sqrt{2})$$

$$46. \quad (g \circ f)(3b)$$

$$38. \quad (g \circ f)(4)$$

$$41. \quad (f \circ f)(-3)$$

$$44. \quad (g \circ f)(\sqrt{3})$$

$$47. \quad (f \circ g)(k+1)$$

$$39. \quad (f \circ g)(-2)$$

$$42. \quad (g \circ g)(7)$$

$$45. \quad (f \circ g)(2a)$$

$$48. \quad (g \circ f)(k-1)$$