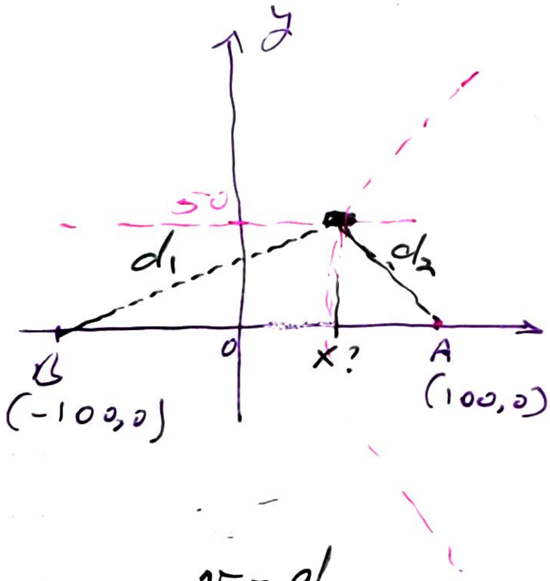
$v = 980 \ ft/\mu sec$

$t = 400 \ \mu sec$

$d_2 - d_1 = 980 \times 400$

$= 392 \times 10^3 = 2a$



$a = 196 \times 10^3 \ \frac{1 \ mi}{5280 ft}$

$\approx 37.\overline{12}$

$v = \frac{d}{t}$

$a^2 \approx 1378.$

$\} \ a^2 + b^2 = c^2$

$c = 100 \Rightarrow c^2 = 10^4$

$b^2 = c^2 - a^2$

$= 10^4 - 1378$

≈ 8622

$$\frac{x^2}{1378} - \frac{y^2}{8622} = 1$$

$$\frac{x^2}{1378} = 1 + \frac{50^2}{8622}$$

$$x = \sqrt{1378\left(1 + \frac{50^2}{8622}\right)}$$

≈ 42.16

$P(\cancel{4}42, 50)$

27/ $625y^2 - 400x^2 = 250{,}000$

$2a$?

$$\frac{y^2}{400} - \frac{x^2}{625} = 1$$

$a^2 = 400$ $\sqrt{\underbrace{00}_{1/2}}$

$a = 20$

distance $= 2a = 40$

5.5/ Sequence Infinite

$a_1, a_2, \ldots, a_n, \ldots$

Ex 1st 4 terms & 10th term $\left\{\frac{n}{n+1}\right\}$

$n = 1 \rightarrow a_1 = \frac{1}{2}$

$n = 2 \rightarrow a_2 = \frac{2}{3}$

$n = 3 \rightarrow a_3 = \frac{3}{3+1} = \frac{3}{4}$

$n = 4 \rightarrow a_4 = \frac{4}{5}$

$n = 10 \rightarrow a_{10} = \frac{10}{11}$

$\frac{1}{2}, \frac{2}{3}, \ldots, \frac{n}{n+1}, \ldots$

Ex 1st 4 term a_{10} $\{2+(.1)^n\}$

$a_1 = 2+.1 = 2.1$

$a_2 = 2+(.1)^2 = 2+.01 = \underline{2.01}$

$a_3 = 2+(.1)^3 = 2+.001 = \underline{2.001}$

$a_4 = 2+(.1)^4 = 2+.0001 = \underline{2.0001}$

$a_{10} = 2+(.1)^{10} = \underline{2.0000000001}$

Ex $\left\{(-1)^{n+1}\dfrac{n^2}{3n-1}\right\}$

$a_1 = (-1)^2\dfrac{1}{3-1} = \dfrac{1}{2}$

$a_2 = (-1)^3\dfrac{4}{5} = \underline{-\dfrac{4}{5}}$

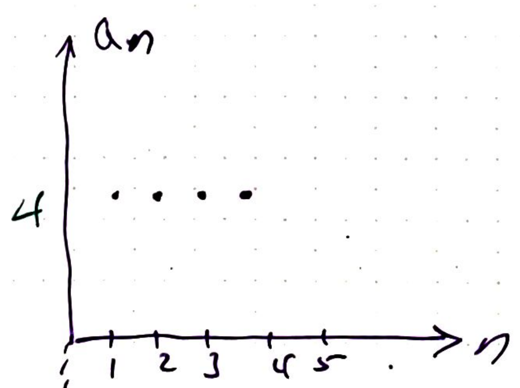
$a_3 = (-1)^4\dfrac{9}{8} = \underline{\dfrac{9}{8}}$

$a_4 = (-1)^5\dfrac{16}{11} = \underline{-\dfrac{16}{11}}$

$a_{10} = (-1)^{11}\dfrac{100}{29} = \underline{-\dfrac{100}{29}}$

Ex $\{4\}$

$a_1 = 4,\ a_2 = 4,\ a_3 = 4,\ a_4 = 4$



Ex 1^{st} 4 $a_1 = 3$ $a_{n+1} = (n+1)a_n$

$a_1 = 3$

$n=1$ $a_2 = 2a_1 = 2(3) = 6$

$n=2$ $a_3 = 3a_2 = 3(6) = 18$

$n=3$ $a_4 = 4a_3 = 4(18) = 72$

Summation Notation

$$a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$$

last value

a_n : formula

$$\sum_{n=1}^{5} (2n+3) = (2+3) + 7 + 9 + 11 + 13 = 45$$

$n=1$ $n=2$ 3 4 5

1st value

$k=1$

$$\sum_{k=1}^{4} k^2(k-3) = 1(-2) + 4(-1) + 9(0) + 16(1)$$
$$= -2 - 4 + 16$$
$$= 10$$

$$\sum_{k=1}^{n} c = \overbrace{c + \cdots + c}^{n \text{ times}} = nc$$

$$\sum_{k=m}^{n} c = c(n - m + 1)$$

$$\sum_{k=10}^{20} 5 = 5(20-10+1)$$
$$= 5(11)$$
$$= \boxed{55}$$

$$2^1 + 2^2 + 2^3 + \cdots + 2^{16} = \sum_{n=1}^{16} 2^n$$

$$2^1 + 2^2 + \cdots + 2^n + \cdots = \sum_{n=1}^{\infty} 2^n = \sum \cancel{2}^n$$

$$\sum a_n = \sum_{n=1}^{\infty} a_n$$

5.6

Arithmetic Sequence. $a_1, \ldots, a_n, \ldots$

$k \in \mathbb{Z}^+$

$$a_{k+1} = a_k + d$$ → common difference

<u>Ex</u> $1, 4, 7, 10, \ldots, 3n-2, \ldots$

$d = 4 - 1 = 3$ $\quad a_2 - a_1$

$a_{k+1} = a_k + d$

$$a_{k+1} - a_k = 3(k+1) - 2 - (3k-2)$$
$$= 3k + 3 - 2 - 3k + 2$$
$$= 3 \checkmark$$

Ex fourth term

Given: $a_4 = 5$, $a_9 = 20$ Find a_6?

$a_n = a_1 + (n-1)d$

$a_4 = a_1 + 3d = 5$

$a_9 = a_1 + 8d = 20$

$5d = 15$

$d = 3$

$\Rightarrow d = \frac{20-5}{9-4} = \frac{15}{5}$

$a_4 = a_1 + 3(3) = 5$

$a_1 = -4$

$a_6 = -4 + 5(3)$

$= 11$

#22 a_{20} : $a_9 = -5$ $a_{15} = 31$

$d = \frac{31+5}{15-9} = 6$ $\frac{36}{6}$

$a_9 = a_1 + 8(6) = -5$

$\Rightarrow a_1 = -53$

$a_{20} = -53 + 19(6)$

$= 61$

Theorem

$$S_n = \frac{1}{2} n (a_1 + a_n)$$
$$= \frac{n}{2} (2a_1 + (n-1)d)$$

$$S_n = a_1 + a_2 + \cdots + a_n$$
$$= a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + a_1 + (n-1)d$$
$$= \underbrace{a_1 + \cdots + a_1} + (d + 2d + \cdots + (n-1)d)$$
$$= n a_1 + d(1 + 2 + \cdots + (n-1))$$
$$= n a_1 + d \frac{(n-1)n}{2}$$
$$= \frac{n}{2} (2a_1 + (n-1)d)$$

Ex sum even $2 \to 100$

$$S_n = \frac{50}{2} (2 + 100)$$
$$= 2550$$

$$\frac{1}{4} + \frac{2}{9} + \frac{3}{14} + \frac{4}{19} + \frac{5}{24} + \frac{6}{29} = \sum_{n=1}^{6} \frac{n}{5n-1}$$

$n \to 1, 2, 3, 4, 5, 6 \to n$

den $4, 9, 14, 19, 24, 29$

$d = 5$

$$a_n = 4 + (n-1)5$$
$$= 5n - 5 + 4$$
$$= 5n - 1$$

Geometric Seq.

$$a_{k+1} = a_k r \rightarrow \text{common ratio}$$

$$r = \frac{a_{k+1}}{a_k}$$

$$a_n = a_1 r^{n-1} = \cdot$$

$a_1 = 3 \quad r = -\frac{1}{2}$ $\qquad a_n = 3\left(-\frac{1}{2}\right)^{n-1}$

$$a_1 = 3$$

$$a_2 = 3\left(-\frac{1}{2}\right) = -\frac{3}{2}$$

$$a_3 = 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$$

Ex $\quad a_3 = +5 \qquad a_6 = -40 \qquad a_8 = ?$

Geom.

$$a_3 = a_1 r^2 = 5$$

$$a_6 = a_1 r^5 = -40$$

$$\frac{a_1 r^5}{a_1 r^2} = \frac{-40}{5}$$

$$r^3 = -8 \qquad 2^3$$

$$r = -2$$

$$r = \left(-\frac{40}{5}\right)^{\frac{1}{6-3}} = (-8)^{1/3} = -2$$

$$a_1(-2)^2 = 5 \Rightarrow a_1 = \frac{5}{4}$$

$$a_8 = \frac{5}{4}(-2)^7 = -5(2^5)$$

$$= \underline{-160}$$