Pre-Cal 9/16

Geometric Sequence

$$a_{k+1} = a_k r$$

$$r = \frac{a_{k+1}}{a_k} \quad \text{common ratio}$$

$$\boxed{a_n = a_1 r^{n-1}}$$

Ex $6, -12, 24, \ldots, (-2)^{n-1}(6), \ldots$

$r = -\frac{12}{6} = -2$ $\quad n \in \mathbb{Z}^+$

Ex 1st term $a_1 = 3$ $\quad r = -\frac{1}{2}$ $\quad$ 1st 5.

$$a_n = 3\left(-\frac{1}{2}\right)^{n-1}$$

$a_1 = 3$

$a_2 = 3\left(-\frac{1}{2}\right)^1 = -\frac{3}{2}$

$a_3 = 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$

$a_4 = 3\left(-\frac{1}{2}\right)^3 = -\frac{3}{8}$

$a_5 = 3\left(-\frac{1}{2}\right)^4 = \frac{3}{16}$

Ex Geom. $a_3 = 5$ $\quad a_6 = -40$ $\quad a_8$?

$$r = \left(-\frac{40}{5}\right)^{\frac{1}{6-3}}$$

$$= (-8)^{1/3}$$

$$= -(2^3)^{1/3}$$

$$\underline{= -2}$$

$3-1$

$$r = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2-x_1}}$$

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 (-2)^2 = 5$$

$$4a_1 = 5$$

$$\underline{a_1 = \frac{5}{4}}$$

$$a_8 = \frac{5}{4}(-2)^7$$

$$= -5(2^5)$$

$$\underline{= -160}$$

Ex

Gem

$a_9 : a_2 = 3 \quad a_5 = -81$

$$r = (-27)^{1/3}$$

$$= -(3^3)^{1/3}$$

$$\underline{= -3}$$

$$\boxed{a_n = a_1 r^{n-1}}$$

$$a_2 = a_1(-3) = 3$$

$$\Rightarrow \boxed{a_1 = -1}$$

$$a_9 = -1\,(-3)^8$$
$$\boxed{= -3^8}$$

$$S_n = a_1 \frac{1-r^n}{1-r} \qquad \boxed{r \neq 1}$$

$$\begin{cases} S = \dfrac{a_1}{1-r} & |r| < 1 \\ S = \infty & |r| \geq 1 \end{cases}$$

Ex $\displaystyle\sum_{n=1}^{\infty} 3\left(-\frac{2}{3}\right)^{n-1} = \frac{3}{1+\frac{2}{3}}$ $\qquad \left|-\frac{2}{3}\right| = \frac{2}{3} < 1$

$$= \frac{2}{\frac{5}{3}}$$

$$\boxed{= \frac{6}{5}}$$

$$\boxed{\sum_{n=1}^{\infty} 3\left(\frac{3}{2}\right)^{n-1} = \infty} \qquad \frac{3}{2} \geq 1$$

Ex $5.4\overline{27}$ $\qquad = \dfrac{a}{b}$

$$5.4\overline{27} = 5.42727\ldots$$

$$= 5.4 + .02727\ldots$$

$$= \frac{54}{10} + .027 + .00027 + \ldots$$

$$a_1 = .027 = \frac{27}{1000} \to 10^3$$

$$= 27\times 10^{-3}$$

$$r = \frac{27\times 10^{-5}}{27\times 10^{-3}} \qquad -5+3$$

$$= 1\times 10^{-2}$$

$$5.4\overline{27} = \frac{54}{10} + \frac{27\times 10^{-3}}{1-.01}$$

$$= \frac{54}{10} + \frac{27\times 10^{-3}}{.99}$$

$$= \frac{54}{10} + \frac{\cancel{27}^{\,3}\times 10^{-3}}{\cancel{99}_{\,11}\times 10^{-2}}$$

$$= \frac{54}{10} + \frac{3}{110}$$

$$= \boxed{\frac{597}{110}}$$

5.7 Mathematical Induction

- P_1 is true
- Assume P_k is true, prove that P_{k+1} is also true.

Ex $n \in \mathbb{Z}^+$ $\quad 1+2+3+\ldots+n = n(n+1)$

$1+2+3+\cdots+n = \frac{\cdots}{2}$

Soln

For $n=1 \Rightarrow \quad 1 = \frac{1(2)}{2}$

$1 = 1$ ✓ P_1 is true

Assume P_k is true: $1+2+\cdots+k = \frac{k(k+1)}{2}$

is P_{k+1}: $1+\cdots+k+(k+1) \overset{?}{=} \frac{(k+1)(k+2)}{2}$

$$1+\cdots+k+(k+1) = \frac{1}{2}k(k+1) + (k+1)$$

$$= (k+1)\left(\frac{k}{2}+1\right)$$

$$= (k+1)\left(\frac{k+2}{2}\right) \checkmark$$

P_{k+1} is also true

$\therefore$ By the mathematical induction, the given proof is completed.

Ex $1^2+3^2+\cdots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

For $n=1 \Rightarrow \quad 1^2 = \frac{1(1)(3)}{3}$

$1 = 1$ ✓ P_1 is true

Assume P_k is true:

$$1^2+\cdots+(2k-1)^2 = \frac{1}{3}k(2k-1)(2k+1)$$

is P_{k+1}

$$1^2+\cdots+(2k-1)^2+(2(k+1)-1)^2 \overset{?}{=} \frac{1}{3}(k+1)(2(k+1)-1)(2(k+1)+1)$$

$$1^2+\cdots+(2k-1)^2+(2k+1)^2 \overset{?}{=} \frac{1}{3}(k+1)(2k+1)(2k+3)$$

$1^2 + \cdots + (2k-1)^2 + (2k+1)^2 = \frac{1}{3}(k+1)(2k+1)(2k+3)$

$1^2 + \cdots + (2k-1)^2 + (2k+1)^2 = \frac{1}{3}k(2k-1)(2k+1) + (2k+1)^2$

$= (2k+1)\left(\frac{1}{3}k(2k-1) + 2k+1\right)$

$= (2k+1)\left(\frac{2k^2-k+6k+3}{3}\right)$

$= \frac{1}{3}(2k+1)(2k^2+5k+3)$

$= \frac{1}{3}(2k+1)(k+1)(2k+3)$ ✓

P_{k+1} is also true

$\therefore$ By the mathematical induction, the given proof is completed

Ex 2 is factor of n^2+5n $n \in \mathbb{Z}^+$

For $n=1 \Rightarrow 1^2+5 = 6$
$= 2(3)$ ✓

P_1 is true

Assume P_k: 2 is factor of $k^2+5k = 2p$

is P_{k+1} : 2 is " $(k+1)^2+5(k+1)$?

$(k+1)^2+5(k+1) = k^2+2k+1+5k+5$

$= 2p + 2k + 6$

$= 2(p+k+3)$ ✓

hence, P_{k+1} is also true.

$\therefore$ By the mathematical induction, the proof is completed.

9/21 Review

9/23 6-1 / Rev.

9/25 →

---

(2) Partial Fraction 5.2

(1) ellipse Appl. 5.3

(1) hyperbola " 5.4

(2) 1st ... a_{10} 5.5

(5) $\Sigma = \begin{cases} \sum c \\ \sum L = \text{---} \\ \text{Geom} \begin{cases} S = \frac{a}{1-r} \\ S = \infty \end{cases} \end{cases}$

(1) Arithmetic 5.6 $d = \frac{y_2 - y_1}{x_2 - x_1}$

$a_n = a_1 + (n-1)d$

(1) Geom. $r = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$

$a_n = a_1 r^{n-1}$

(1) Proof.

Hwk due ON Monday

Proof each on 1 page