Lecture Two – Functions

Section 2.1 – Functions and Graphs

Increasing and *Decreasing* Functions

- A function *rises from left to right (x-coordinate)*, the function *f* is said to be ***increasing*** on an open interval ***I*** (*a, b*) (*x*-coordinate)

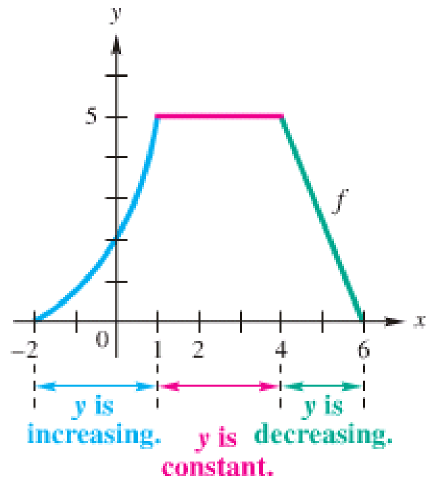
$$a < b \quad \Rightarrow \quad f(a) < f(b)$$

- A function *f* is said to be ***decreasing*** on an open interval ***I***

$$a < b \quad \Rightarrow \quad f(a) > f(b)$$

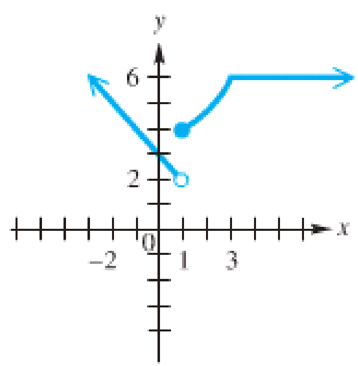
- A function *f* is said to be ***constant*** on an open interval ***I***

$$a < b \quad \Rightarrow \quad f(a) = f(b)$$



Example

Determine the intervals over which the function is increasing, decreasing, or constant



Increasing: [1, 3]

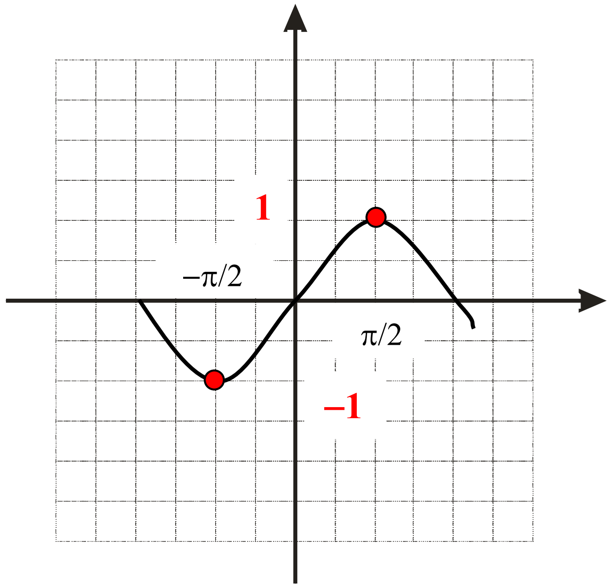
Decreasing: $(-\infty, 1)$

Constant: $[3, \infty)$

Relative *Maxima (um)* and *Minima (um)*

f(a) is a relative maximum if there exists an open interval I about *a* such that *f(a) > f(x),* for all *x* in I.

f(a) is a relative minimum if there exists an open interval I about *a* such that *f(a) < f(x),* for all *x* in I.

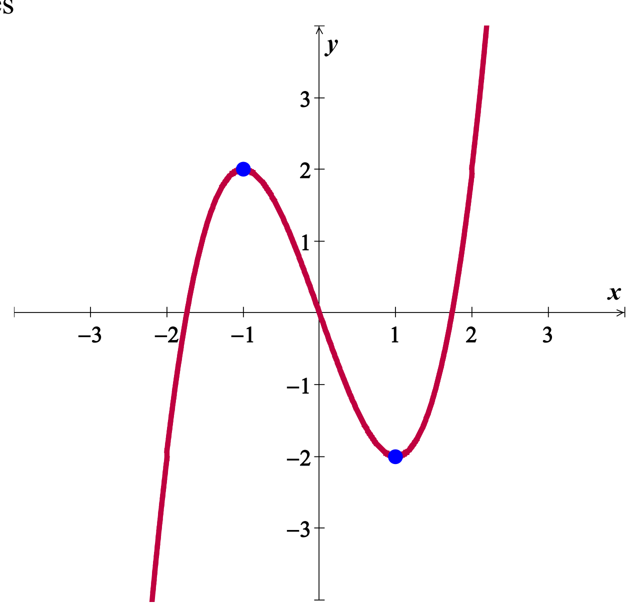


The relative minimum value of the function is −1 @ $x = -\pi/2$

The relative maximum value of the function is 1 @ $x = \pi/2$

Example

State the intervals on which the given function $f(x) = x^3 - 3x$ is increasing, decreasing, or constant, and determine the extreme values



Increasing $(-\infty,\ -1) \cup (1,\ \infty)$ ***RMIN*** $(1,\ -2)$

Decreasing $(-1,\ 1)$ ***RMAX*** $(-1,\ 2)$

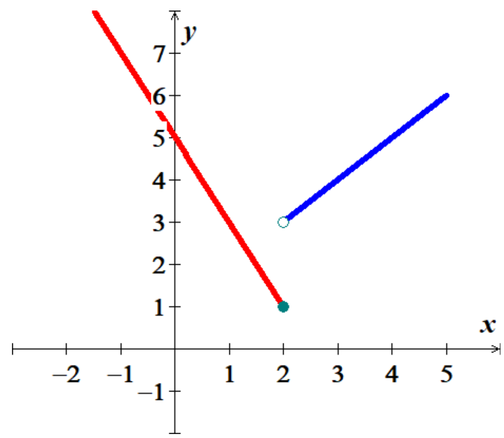
Piecewise-Defined Functions

Function are sometimes described by more than one expression, we call such functions ***piecewise-defined functions***.

Example

Graph function

$$f(x)=\begin{cases} -2x+5 & if \quad x\le 2 \\ x+1 & if \quad x>2 \end{cases}$$



Find: $f(2)=-2(2)+5=1$

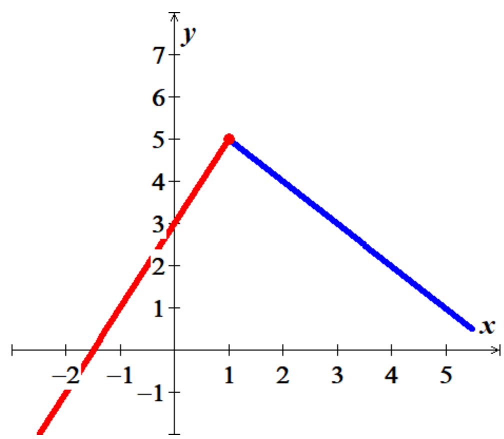
$f(0)=-2(0)+5=5$

$f(4)=4+1=5$

Example

Graph function

$$f(x)=\begin{cases} 2x+3 & if \quad x\le 1 \\ -x+6 & if \quad x>1 \end{cases}$$



Example

$$C(t)=\begin{cases} 20 & \text{if } 0\le t\le 60 \\ 20+0.40(t-60) & \text{if } t>60 \end{cases}$$

Find $C(40)$, $C(80)$, and $C(60)$

Solution

a) $C(40) = 20$

b) $C(80) = 20 + 0.40(80 – 60) = 28$

c) $C(60) = 20$

Exercise *Section* 2.1 – Functions and Graphs

1. $f(x)=\begin{cases} 2+x & if\ \ x<-4 \\ -x & if\ \ -4\le x\le 2 \\ 3x & if\ \ x>2 \end{cases}$ **Find**: $f(-5), f(-1), f(0),\ and\ f(3)$

2. $f(x)=\begin{cases} -2x & if\ \ x<-3 \\ 3x-1 & if\ \ -3\le x\le 2 \\ -4x & if\ \ x>2 \end{cases}$ **Find**: $f(-5), f(-1), f(0),\ and\ f(3)$

3. $f(x)=\begin{cases} x^3+3 & if\ \ -2\le x\le 0 \\ x+3 & if\ \ 0<x<1 \\ 4+x-x^2 & if\ \ 1\le x\le 3 \end{cases}$ **Find**: $f(-5), f(-1), f(0),\ and\ f(3)$

4. $h(x)=\begin{cases} \dfrac{x^2-9}{x-3} & if\ \ x\ne 3 \\ 6 & if\ \ x=3 \end{cases}$ **Find**: $h(5),\ h(0),\ and\ h(3)$

5. $f(x)=\begin{cases} 3x+5 & if\ \ x<0 \\ 4x+7 & if\ \ x\ge 0 \end{cases}$ **Find**

a) $f(0)$ *b)* $f(-2)$ *c)* $f(1)$ *d)* $f(3)+f(-3)$ *e)* Graph $f(x)$

6. $f(x)=\begin{cases} 6x-1 & if\ \ x<0 \\ 7x+3 & if\ \ x\ge 0 \end{cases}$ **Find**

a) $f(0)$ *b)* $f(-1)$ *c)* $f(4)$ *d)* $f(2)+f(-2)$ *e)* Graph $f(x)$

7. $f(x)=\begin{cases} 2x+1 & if\ \ x\le 1 \\ 3x-2 & if\ \ x>1 \end{cases}$ **Find**

a) $f(0)$ *b)* $f(2)$ *c)* $f(-2)$ *d)* $f(1)+f(-1)$ *e)* Graph $f(x)$

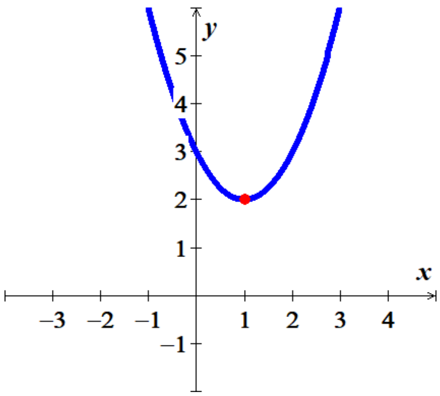
8. Graph the piecewise function defined by $f(x)=\begin{cases} 3 & if\ \ x\le -1 \\ x-2 & if\ \ x>-1 \end{cases}$

9. Sketch the graph $f(x)=\begin{cases} x+2 & if\ \ x\le -1 \\ x^3 & if\ \ -1<x<1 \\ -x+3 & if\ \ x\ge 1 \end{cases}$

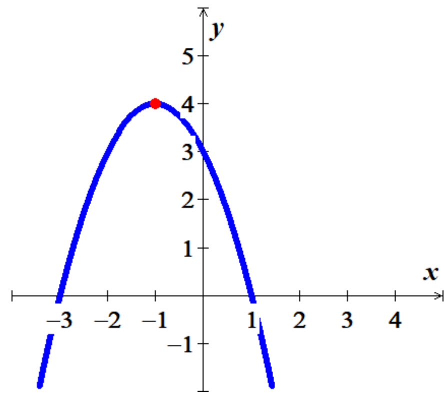
10. Sketch the graph $f(x)=\begin{cases} x-3 & \text{if} \quad x\le -2 \\ -x^2 & \text{if} \quad -2<x<1 \\ -x+4 & \text{if} \quad x\ge 1 \end{cases}$

(**37 – 42**) Determine any ***relative maximum*** or ***minimum*** of the function, determine the intervals on which the function ***increasing*** or ***decreasing***, and then find the ***domain*** and the ***range***.

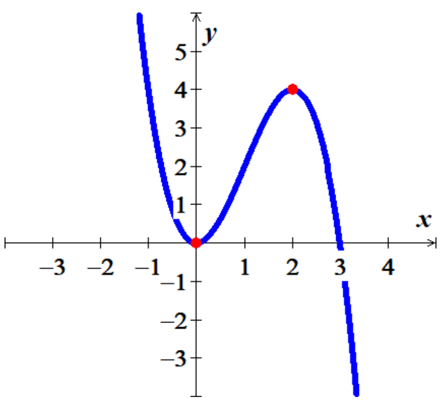
11. $f(x)=x^2-2x+3$



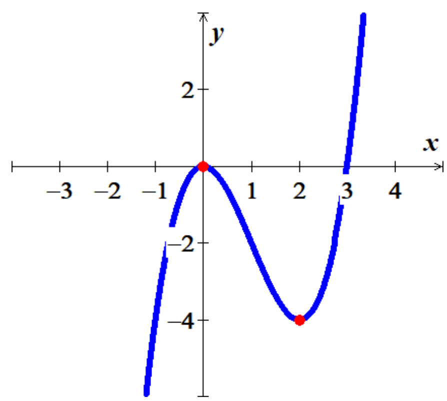
12. $f(x)=-x^2-2x+3$



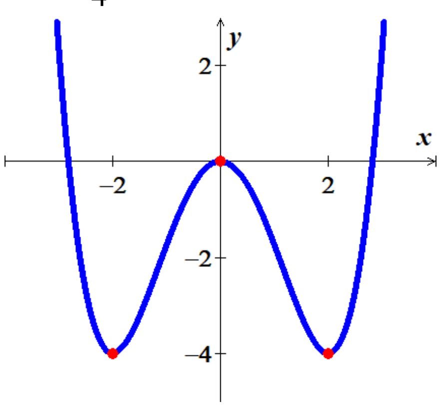
13. $f(x)=-x^3+3x^2$



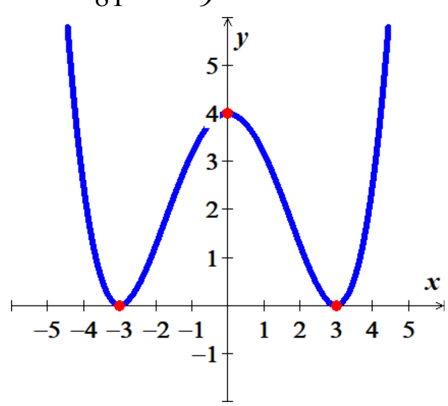
14. $f(x)=x^3-3x^2$



15. $f(x)=\frac{1}{4}x^4-2x^2$



16. $f(x)=\frac{4}{81}x^4-\frac{8}{9}x^2+4$

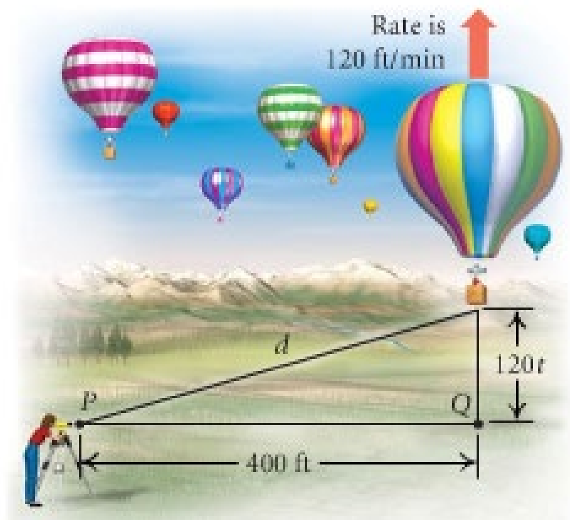


17. The elevation *H*, in *meters*, above sea level at which the boiling point of water is in ***t*** *degrees Celsius* is given by the function

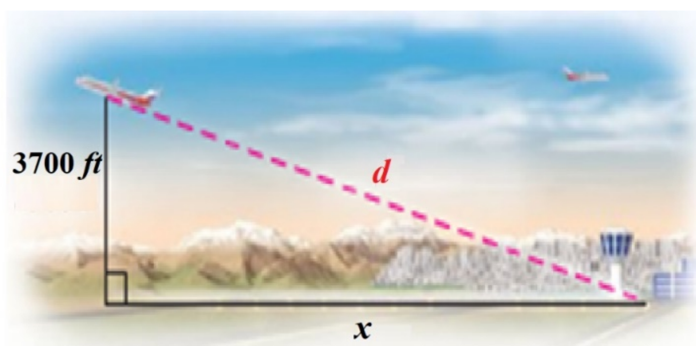
$$H(t) = 1000(100-t) + 580(100-t)^2$$

At what elevation is the boiling point 99.5°.

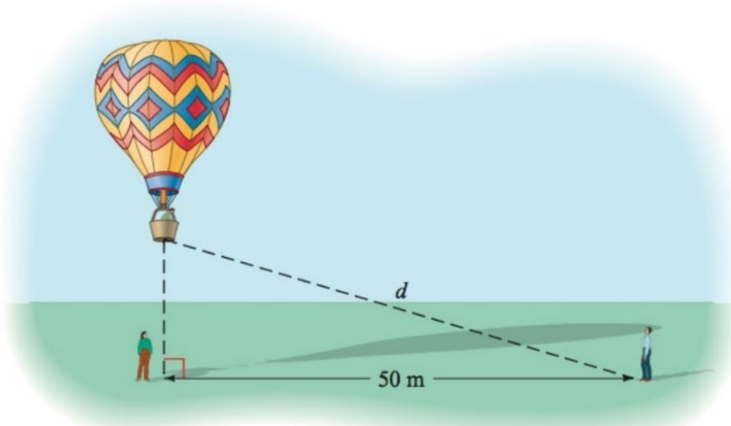
18. A hot-air balloon rises straight up from the ground at a rate of 120 *ft./min*. The balloon is tracked from a rangefinder on the ground at point *P*, which is 400 *feet*. from the release point *Q* of the balloon. Let d be the distance from the balloon to the rangefinder and t – the time, in *minutes*, since the balloon was released. Express d as a function of t.



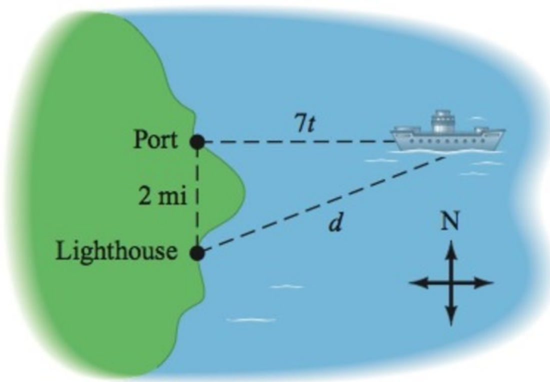
19. An airplane is flying at an altitude of 3700 *feet*. The slanted distance directly to the airport is d *feet*. Express the horizontal distance x as a function of d.



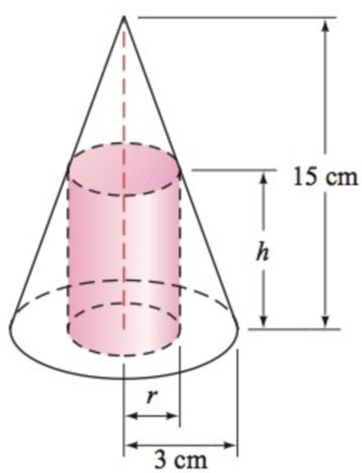
20. For the first minute of flight, a hot air balloon rises vertically at a rate of 3 *m/sec*. If t is the time in *seconds* that the balloon has been airborne, write the distance d between the balloon and a point on the ground 50 *meters* from the point to lift off as a function of t.



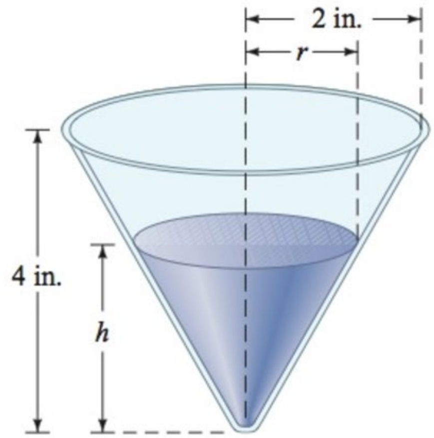
21. A light house is 2 *miles* south of a port. A ship leaves port and sails east at a rate of 7 *miles* per *hour*. Express the distance d between the ship and the lighthouse as a function of time, given that the ship has been sailing for t *hours*.



22. A cone has an altitude of 15 *cm* and a radius of 3 *cm*. A right circular cylinder of radius r and height h is inscribed in the cone. Use similar triangles to write h as a function of r.



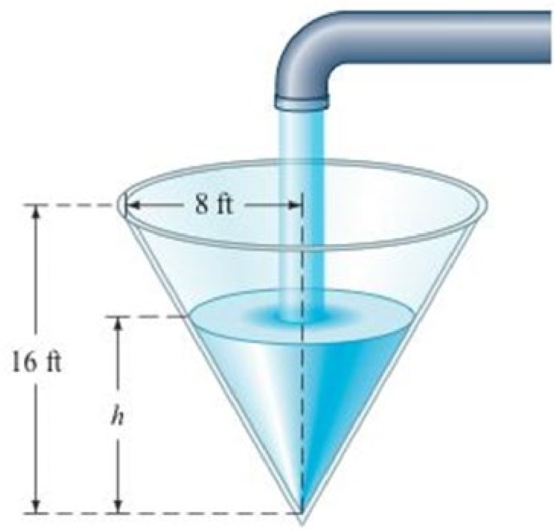
23. Water is flowing into a conical drinking cup with an altitude of 4 *inches* am a radius of 2 *inches*.



a) Write the radius r of the surface of the water as a function of its depth h.

b) Write the volume V of the water as a function of its depth h.

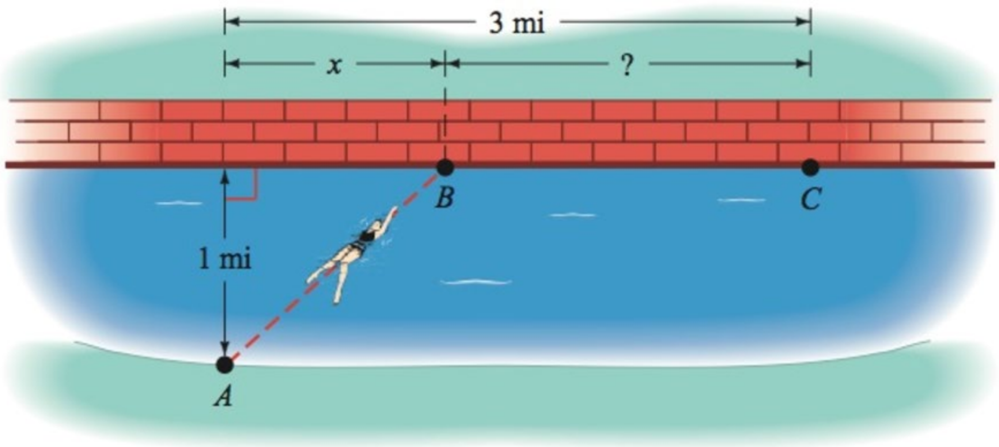
24. A water tank has the shape of a right circular cone with height 16 *feet* and radius 8 *feet*. Water is running into the tank so that the radius r (in *feet*) of the surface of the water is given by $r = 1.5t$, where t is the time (in *minutes*) that the water has been running.



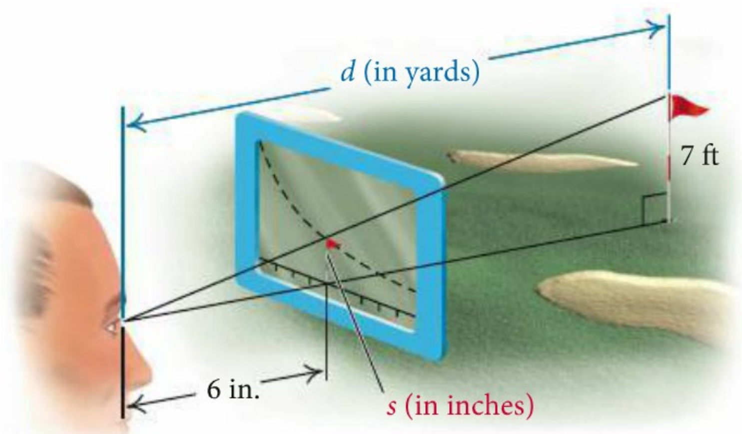
a) The area A of the surface of the water is $A = \pi r^2$. Find $A(t)$ and use it to determine the area of the surface of the water when $t = 2$ *minutes*.

b) The volume V of the water is given by $V = \frac{1}{3}\pi r^2 h$. Find $V(t)$ and use it to determine the volume of the water when $t = 3$ *minutes*

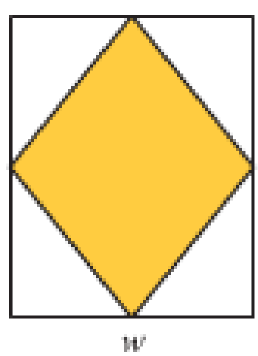
25. An athlete swims from point ***A*** to point ***B*** at a rate of 2 *miles* per *hour* and runs from point ***B*** to point ***C*** at a rate of 8 *miles* per *hour*. Use the dimensions in the figure to write the time t required to reach point ***C*** as a function of x.



26. A device used in golf to estimate the distance ***d***, in *yards*, to a hole measures the size ***s***, in *inches*, that the 7-*feet* pin appears to be in a viewfinder. Express the distance ***d*** as a function of ***s***.



27. A rhombus is inscribed in a rectangle that is ***w*** *meters* wide with a perimeter of 40 *m*. Each vertex of the rhombus is a midpoint of a side of the rectangle. Express the area of the rhombus as a function of the rectangle's width.

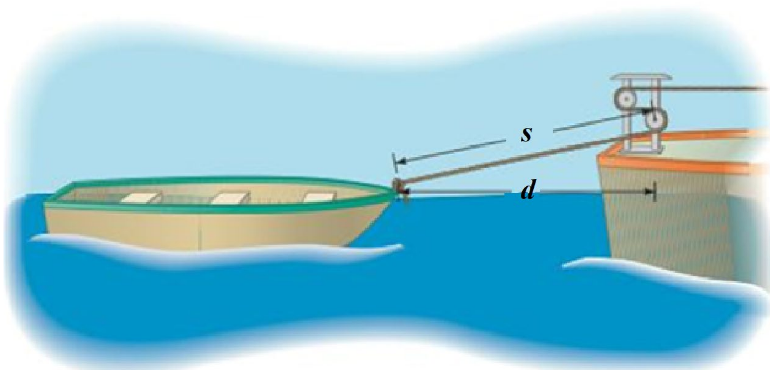


28. The surface area S of a right circular cylinder is given by the formula $S = 2\pi rh + 2\pi r^2$. if the height is twice the radius, find each of the following.

a) A function $S(r)$ for the surface area as a function of r.

b) A function $S(h)$ for the surface area as a function of h.

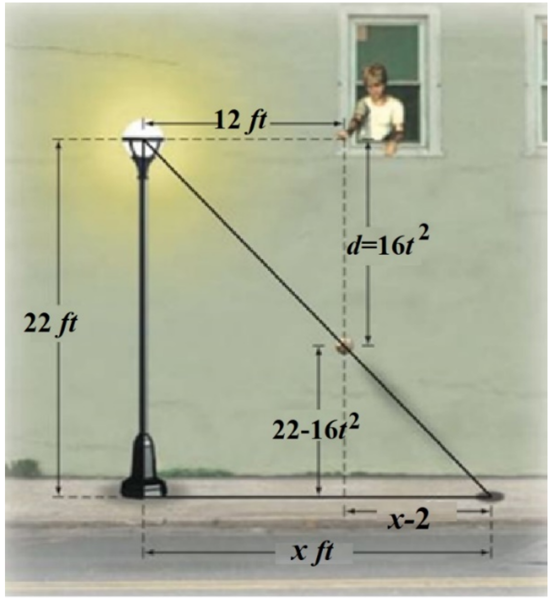
29. A boat is towed by a rope that runs through a pulley that is 4 *feet* above the point where the rope is tied to the boat. The length (in *feet*) of the rope from the boat to the pulley is given by $s = 48 - t$, where t is the time in *seconds* that the boat has been in tow. The horizontal distance from the pulley to the boat is d.



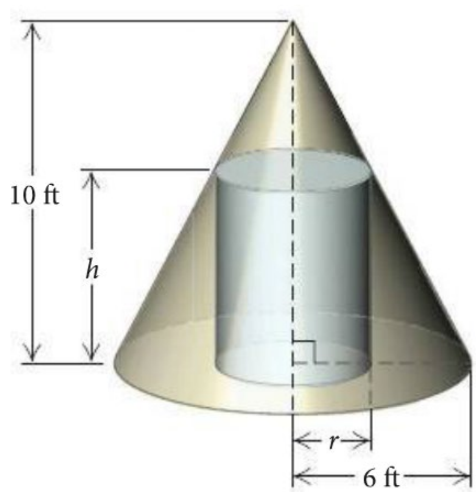
a) Find $d(t)$

b) Evaluate $s(35)$ and $d(35)$

30. The light from a lamppost casts a shadow from a ball that was dropped from a height of 22 *feet* above the ground. The distance d, in *feet*, the ball has dropped t *seconds* after it is released is given by $d(t) = 16t^2$. Find the distance x, in *feet*, of the shadow from the base of the lamppost as a function of time t.



31. *A right circular cylinder of height h and a radius r is inscribed in a right circular cone with a height of 10 *feet* and a base with radius 6 *feet*.



a) Express the height h of the cylinder as a function of r.
b) Express the volume V of the cylinder as a function of r.
c) Express the volume V of the cylinder as a function of h.