

8.d

#18  $2\cos^2 t - 9\cos t = 5 \quad [0, 2\pi)$

$$2\cos^2 t - 9\cos t - 5 = 0$$

$$\cos t = \frac{+9 \pm \sqrt{81 + 40}}{4}$$

$$= \begin{cases} \frac{9-11}{4} = -\frac{1}{2} \\ \frac{9+11}{4} = 5 > 1 \quad \cancel{\text{A}} \end{cases}$$

$$\cos t = -\frac{1}{2} \Rightarrow \underline{t = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

22  $2\cos x - 1 = \sec x$

$$= \frac{1}{\cos x}$$

$$\cos x \neq 0 \\ x \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2\cos^2 x - \cos x = 1$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$\cos x = 1 \quad \cos x = -\frac{1}{2}$$

$$\underline{x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}}$$

#23  $4\cos^2 x + 4\sin x - 5 = 0$

$$\cos^2 x + 5\sin^2 x = 1$$

$$4(1 - \sin^2 x) + 4\sin x - 5 = 0$$

$$-4\sin^2 x + 4\sin x - 1 = 0$$

$$\sin x = \frac{-4 \pm \sqrt{16 - 16}}{-8} = \frac{1}{2}$$

$$\underline{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

5.8

sine of an angle is between  $-1$  &  $1$

$$y = \sin x$$

$\swarrow$   $\searrow$   
 $-1 \leq y \leq 1$       angle  
 $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$$x = \arcsin y$$

$$= \sin^{-1} y$$

$$x = \sinh y$$

$\swarrow$   $\searrow$   
 $[-1, 1]$        $[-\frac{\pi}{2}, \frac{\pi}{2}]$

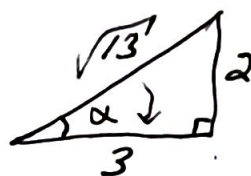
Ex exact  $\sec(\underbrace{\arctan \frac{2}{3}}_{\alpha})$

$$\alpha = \arctan \frac{2}{3}$$

$$\tan(\tan^{-1} \frac{2}{3}) = \frac{2}{3}$$

$$\tan \alpha = \frac{2}{3}$$

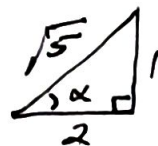
$$\sec \alpha = \frac{\sqrt{13}}{3}$$



Ex  $\sin(\underbrace{\arctan \frac{1}{2}}_{\alpha} - \underbrace{\arccos \frac{4}{5}}_{\beta})$

$$\alpha = \arctan \frac{1}{2} \Rightarrow \tan \alpha = \frac{1}{2}$$

$$\beta = \arccos \frac{4}{5} \rightarrow \cos \beta = \frac{4}{5} \rightarrow \sin \beta = \frac{3}{5}$$



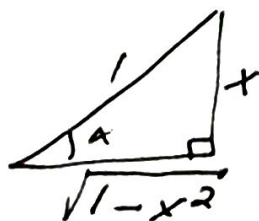
$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{4}{5} - \frac{2}{\sqrt{5}} \cdot \frac{3}{5}$$

$$= \frac{-2}{5\sqrt{5}} = -\frac{2\sqrt{5}}{25}$$

Ex  $\cos(\sin^{-1} x)$   
 $\sin^{-1} x = \alpha$   
 $\sin \alpha = x$

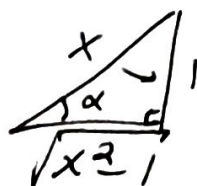
$$\cos(\sin^{-1} x) = \sqrt{1-x^2}$$



#31  $\csc(\sin^{-1} \frac{1}{x})$

$$\alpha = \sin^{-1} \frac{1}{x} \rightarrow \sin \alpha = \frac{1}{x}$$

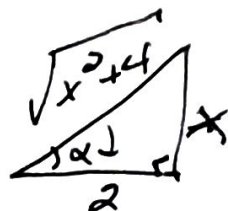
$$\csc(\sin^{-1} \frac{1}{x}) = x$$



33  $\sec(\sin^{-1} \frac{x}{\sqrt{x^2+4}})$   $x > 0$

$$\alpha = \sin^{-1} \frac{x}{\sqrt{x^2+4}} \rightarrow \sin \alpha = \frac{x}{\sqrt{x^2+4}}$$

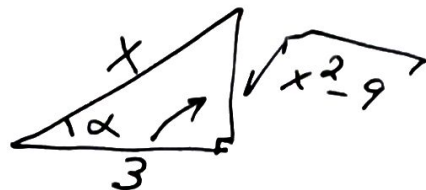
$$\sec \alpha = \frac{\sqrt{x^2+4}}{2}$$



34  $\cot(\sin^{-1} \frac{\sqrt{x^2-9}}{x})$

$$\sin \alpha = \frac{\sqrt{x^2-9}}{x}$$

$$\cot \alpha = \frac{3}{\sqrt{x^2-9}}$$

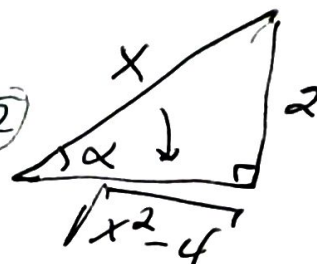


$$39/ \sec \left( \tan^{-1} \frac{2}{\sqrt{x^2-4}} \right)$$

$$\tan \alpha = \frac{2}{\sqrt{x^2-4}}$$

①

②

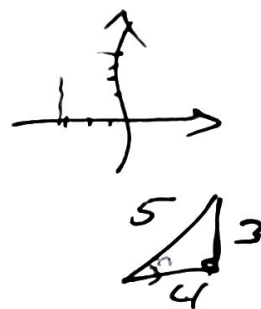


$$\sec \alpha = \frac{x}{\sqrt{x^2-4}}$$

$$13/ \cos \left( \underbrace{\arctan\left(-\frac{3}{4}\right)}_{\alpha} - \underbrace{\arcsin\left(\frac{4}{5}\right)}_{\beta} \right)$$

$$\tan \alpha = -\frac{3}{4}$$

$$\sin \beta = \frac{4}{5} \rightarrow \cos \beta = \frac{3}{5}$$



$$\cos(-\alpha - \beta) = \cos(-(\alpha + \beta))$$

$$= \cos(\alpha + \beta)$$

$$= \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \frac{4}{5} \cdot \frac{3}{5} - \frac{3}{5} \cdot \frac{4}{5}$$

$$= 0$$

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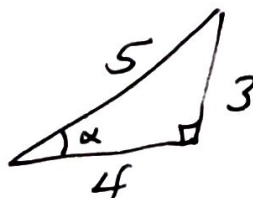

$$20/ \cos^{-1}(\cos \frac{7\pi}{6}) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{7\pi}{6}$$

26  $\tan(\sin^{-1} \frac{3}{5})$

$$\sin \alpha = \frac{3}{5}$$

$$\tan \alpha = \frac{3}{4}$$



53  $\xrightarrow{8.1}$

$$\frac{1 - 2\sin^2 x}{1 + 2\sin x \cos x} = \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$\frac{1 - 2\sin^2 x}{1 + 2\sin x \cos x} = \frac{\cos^2 x + \sin^2 x - 2\sin^2 x}{\cos^2 x + \sin^2 x + 2\sin x \cos x}$$

$$= \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2}$$

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2}$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}$$

64  $\cot^3 x = \cot x (\csc^2 x - 1)$   $\cot^2 x + 1 = \csc^2 x$

$$\cot x (\csc^2 x - 1) = \cot x (\cot^2 x)$$

$$= \cot^3 x$$

80  $10 \csc^2 x - 6 \cot^2 x = 4 \csc^2 x + 6$

$$\begin{aligned} 10 \csc^2 x - 6 \cot^2 x &= 10 \csc^2 x - 6 (\csc^2 x - 1) \\ &= 10 \csc^2 x - 6 \csc^2 x + 6 \\ &= 4 \csc^2 x + 6 \quad \checkmark \end{aligned}$$

8.2  $28, 15, 12$

#17  $\frac{\sin(x-y)}{\sin x \cos y} = 1 - \cot x \tan y$

$$\begin{aligned} \frac{\sin(x-y)}{\sin x \cos y} &= \frac{\sin x \cos y - \cancel{\cos x \sin y}}{\sin x \cos y} \\ &= 1 - \cot x \tan y \quad \checkmark \end{aligned}$$

20  $\frac{\sin(x+y)}{\cos(x-y)} = \frac{1 + \cot x \cot y}{\cot x + \tan y}$

$$\begin{aligned} \frac{\sin(x+y)}{\cos(x-y)} &= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y + \sin x \sin y} \\ &= \frac{\frac{\sin x \cos y}{\sin x \cos y} + \frac{\cos x \sin y}{\sin x \cos y}}{\frac{\cos x \cos y}{\sin x \cos y} + \frac{\sin x \sin y}{\sin x \cos y}} \\ &= \frac{1 + \cot x \tan y}{\cot x + \tan y} \quad \checkmark \end{aligned}$$



$$\underline{27} \quad \sec(x+y) = \frac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y} \leftarrow$$

$$\begin{aligned} \sec(x+y) &= \frac{1}{\cos(x+y)} \cdot \frac{\cos(x-y)}{\cos(x-y)} \\ &= \frac{\cos(x-y)}{(\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)} \\ &= \frac{\cos(x-y)}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y} \\ &= \frac{\cos(x-y)}{\cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y} \\ &= \frac{\cos(x-y)}{\cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y} \\ &= \frac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y} \checkmark \end{aligned}$$

$$\underline{\frac{8-3}{36}} \quad \sin 2\alpha \sin 2\beta = \sin^2(\alpha+\beta) - \sin^2(\alpha-\beta)$$

$$\sin^2(\alpha+\beta) - \sin^2(\alpha-\beta) = (\sin(\alpha+\beta) - \sin(\alpha-\beta)) (\sin(\alpha+\beta) + \sin(\alpha-\beta))$$

(1) (2)

$$\begin{aligned} \textcircled{1} &\rightarrow \sin \alpha \cos \beta + \cos \alpha \sin \beta - \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &\rightarrow 2 \cos \alpha \sin \beta \end{aligned}$$

$$\begin{aligned} \textcircled{2} &\sin \alpha \cos \beta + \cos \alpha \sin \beta + \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &\rightarrow 2 \sin \alpha \cos \beta \end{aligned}$$

$$\begin{aligned} \sin^2(\alpha+\beta) - \sin^2(\alpha-\beta) &= (2 \cos \alpha \sin \beta)(2 \sin \alpha \cos \beta) \\ &= (2 \cos \alpha \sin \alpha)(2 \sin \beta \cos \beta) \\ &= \sin 2\alpha \cos 2\beta \end{aligned}$$

$$\underline{46} \quad \frac{1 - \cos^2 \frac{x}{2}}{1 - \sin^2 \frac{x}{2}} = \frac{1 - \cos x}{1 + \cos x}$$

$$\begin{aligned} \frac{1 - \cos^2 \frac{x}{2}}{1 - \sin^2 \frac{x}{2}} &= \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}} \\ &= \tan^2 \frac{x}{2} \\ &= \tan \frac{x}{2} \tan \frac{x}{2} \\ &= \frac{1 - \cos x}{\sin x} \frac{\sin x}{1 + \cos x} \\ &= \frac{1 - \cos x}{1 + \cos x} \end{aligned}$$

$$\begin{aligned} \rightarrow & \frac{\frac{1}{2}(1 - \cos x)}{\frac{1}{2}(1 + \cos x)} \\ &= \frac{1 - \cos x}{1 + \cos x} \quad \checkmark \end{aligned}$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\frac{1 + \cos 2\theta}{2} = \cos^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$



$$32/ \frac{\cos 2x}{\sin^2 x} = 2 \cot^2 x - \csc^2 x$$

$$\frac{\cos 2x}{\sin^2 x} = \frac{2 \cos^2 x - 1}{\sin^2 x}$$

$$= 2 \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x}$$

$$= 2 \cot^2 x - \csc^2 x$$