Solution Section 2.5 – Graphing Polynomial Functions

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=5x^3+7x^2-x+9$

Solution

Leading term: $5x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \Rightarrow f(x)\to-\infty$ $\quad$ $f(x)$ falls left

$x\to\infty \Rightarrow f(x)\to\infty$ $\quad$ $f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=11x^3-6x^2+x+3$

Solution

Leading term: $11x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \Rightarrow f(x)\to-\infty$ $\quad$ $f(x)$ falls left

$x\to\infty \Rightarrow f(x)\to\infty$ $\quad$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-11x^3-6x^2+x+3$

Solution

Leading term: $-11x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \Rightarrow f(x)\to\infty$ $\quad$ $f(x)$ rises left

$x\to\infty \Rightarrow f(x)\to-\infty$ $\quad$ $f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=2x^3+3x^2-23x-42$

Solution

Leading term: $2x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \Rightarrow f(x)\to-\infty$ $\quad$ $f(x)$ falls left

$x\to\infty \Rightarrow f(x)\to\infty$ $\quad$ $f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^4 + 7x^2 - x + 9$

Solution

Leading term: $5x^4$ with 4^{rd} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 11x^4 - 6x^2 + x + 3$

Solution

Leading term: $11x^4$ with 4^{rd} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -5x^4 + 7x^2 - x + 9$

Solution

Leading term: $-5x^4$ with 4^{rd} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -11x^4 - 6x^2 + x + 3$

Solution

Leading term: $-11x^4$ with 4^{rd} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^5 - 16x^2 - 20x + 64$

Solution

Leading term: $5x^5$ with 5^{th} degree (n is ***odd***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -5x^5 - 16x^2 - 20x + 64$

Solution

Leading term: $-5x^5$ with 5^{th} degree (n is ***odd***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -3x^6 - 16x^3 + 64$

Solution

Leading term: $-3x^6$ with 6^{th} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 3x^6 - 16x^3 + 4$

Solution

Leading term: $3x^6$ with 6^{th} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises right

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^3-x-1;\quad between\ \ 1\ and\ \ 2$

Solution

$$f(1)=(1)^3-(1)-1$$

$$\underline{=-1}$$

$$f(2)=(2)^3-(2)-1$$

$$\underline{=5}$$

Since $f(1)\ and\ f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^3-4x^2+2;\quad between\ \ 0\ and\ 1$

Solution

$$f(0)=(0)^3-4(0)^2+2$$

$$\underline{=2}$$

$$f(1)=(1)^3-4(1)^2+2$$

$$\underline{=-1}$$

Since $f(0)\ and\ f(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=2x^4-4x^2+1;\quad between\ \ -1\ and\ \ 0$

Solution

$$f(-1)=2(-1)^4-4(-1)^2+1$$

$$\underline{=-1}$$

$$f(0)=2(0)^4-4(0)^2+1$$

$$\underline{=1}$$

Since $f(0)$ *and* $f(-1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between -1 and 0.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^4 + 6x^3 - 18x^2;$ *between* 2 *and* 3

Solution

$$f(2) = (2)^4 + 6(2)^3 - 18(2)^2$$
$$= -8$$

$$f(3) = (3)^4 + 6(3)^3 - 18(3)^2$$
$$= 81$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 + x^2 - 2x + 1;$ *between* -3 *and* -2

Solution

$$f(-3) = (-3)^3 + (-3)^2 - 2(-3) + 1$$
$$= -11$$

$$f(-2) = (-2)^3 + (-2)^2 - 2(-2) + 1$$
$$= 1$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between -2 and -3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^5 - x^3 - 1;$ *between* 1 *and* 2

Solution

$$f(1) = (1)^5 - (1)^3 - 1$$
$$\underline{= -1 \rfloor}$$

$$f(2) = (2)^5 - (2)^3 - 1$$
$$\underline{= 23 \rfloor}$$

Since $f(1)$ *and* $f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 10x + 9;$ *between* -3 *and* -2

Solution

$$f(-3) = 3(-3)^3 - 10(-3) + 9$$
$$\underline{= -42 \rfloor}$$

$$f(-2) = 3(-2)^3 - 10(-2) + 9$$
$$\underline{= 5 \rfloor}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between -3 and -2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 8x^2 + x + 2;$ *between* 2 *and* 3

Solution

$$f(2) = 3(2)^3 - 8(2)^2 + (2) + 2$$
$$\underline{= -4 \rfloor}$$

$$f(3) = 3(3)^3 - 8(3)^2 + (3) + 2$$
$$\underline{= 14 \rfloor}$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 8x^2 + x + 2; \quad between\ \ 1\ and\ \ 2$

Solution

$$f(1) = 3(1)^3 - 8(1)^2 + (1) + 2$$

$$= -2$$

$$f(2) = 3(2)^3 - 8(2)^2 + (2) + 2$$

$$= -4$$

Since $f(1)$ *and* $f(2)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^5 + 2x^4 - 6x^3 + 2x - 3; \quad between\ \ 0\ and\ \ 1$

Solution

$$f(0) = (0)^5 + 2(0)^4 - 6(0)^3 + 2(0) - 3$$

$$= -3$$

$$f(1) = (1)^5 + 2(1)^4 - 6(1)^3 + 2(1) - 3$$

$$= -4$$

Since $f(0)$ *and* $f(1)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 2x^3 + 3x^2 - 23x - 42, \quad a = 3, \quad b = 4$

Solution

$$P(3) = 54 + 27 - 69 - 42$$

$$= -30$$

$$P(4) = 128 + 48 - 92 - 42$$

$$= 90$$

Since $P(3)$ *and* $P(4)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and 4.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=4x^3-x^2-6x+1, \quad a=0, \quad b=1$

Solution

$P(0)=1$

$P(1)=4-1-6+1$

$=-2$

Since $P(0)$ *and* $P(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=3x^3+7x^2+3x+7, \quad a=-3, \quad b=-2$

Solution

$P(-3)=-81+63-9+7$

$=-20$

$P(-2)=-24+28-6+7$

$=5$

Since $P(-3)$ *and* $P(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between -3 and -2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=2x^3-21x^2-2x+25, \quad a=1, \quad b=2$

Solution

$P(1)=2-21-2+25$

$=4$

$P(2) = 16 - 84 - 4 + 25$

$\underline{= -47}$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 4x^4 + 7x^3 - 11x^2 + 7x - 15, \quad a = 1, \quad b = \frac{3}{2}$

Solution

$P(1) = 4 + 7 - 11 + 7 - 15$

$\underline{= -8}$

$$P\left(\frac{3}{2}\right) = 81 + \frac{189}{8} - \frac{99}{4} + \frac{21}{2} - 15$$

$$= 66 + \frac{189 - 198 + 84}{8}$$

$$= 66 + \frac{75}{8}$$

$$\underline{= \frac{603}{8}}$$

Since $P(1)$ *and* $P\left(\frac{3}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and $\frac{3}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 5x^3 - 16x^2 - 20x + 64, \quad a = 3, \quad b = \frac{7}{2}$

Solution

$P(3) = 135 - 144 - 60 + 64$

$\underline{= -5}$

$$P\left(\frac{7}{2}\right) = \frac{1715}{8} - 196 - 70 + 64$$

$$= \frac{1715}{8} - 202$$

$$\underline{= \frac{99}{8}}$$

Since $P(3)$ *and* $P\left(\frac{7}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and $\frac{7}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^4-x^2-x-4, \quad a=1, \quad b=2$

Solution

$$P(1)=1-1-1-4$$
$$\underline{=-5}$$
$$P(2)=16-4-2-4$$
$$\underline{=6}$$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^3-x-8, \quad a=2, \quad b=3$

Solution

$$P(2)=8-2-8$$
$$\underline{=-2}$$
$$P(3)=27-3-8$$
$$\underline{=16}$$

Since $P(2)$ *and* $P(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^3-x-8, \quad a=0, \quad b=1$

Solution

$$\underline{P(0)=-8}$$

$P(1) = 1 - 1 - 8$

$\quad = -8$

Since $P(0)$ *and* $P(1)$ have same sign.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^3 - x - 8, \quad a = 2.1, \quad b = 2.2$

Solution

$$P(2.1) = P\left(\frac{21}{10}\right)$$

$$= \frac{9261}{1000} - \frac{21}{10} - 8$$

$$= \frac{9261 - 2100 - 8000}{1000}$$

$$= -\frac{839}{1,000}$$

$$P(2.2) = P\left(\frac{2.2}{10}\right)$$

$$= \frac{10,648}{1000} - \frac{22}{10} - 8$$

$$= \frac{10,648 - 2,200 - 8,000}{1000}$$

$$= \frac{448}{1,000}$$

$$= 0.448$$

Since $P(2.1)$ *and* $P(2.2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2.1 and 2.2.

Exercise

Let $f(x) = x^4 - 4x^2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$f(x) = x^2\left(x^2 - 4\right)$$

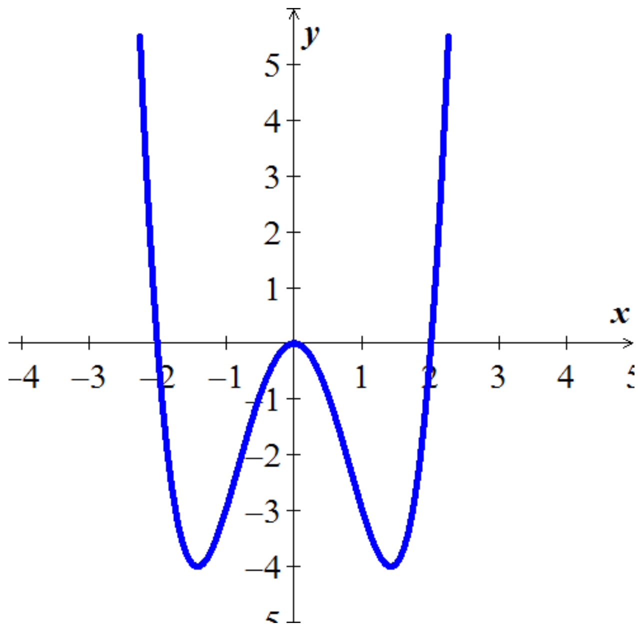
$$= x^2(x-2)(x+2)$$

The zeros are: 0, 0, 2, –2.

$-\infty$ -2	$0,0$ 2	∞
+	–	+

$f(x) < 0 \quad \underline{(-2,\ 0) \cup (0,\ 2)}$

$f(x) > 0 \quad \underline{(-\infty,\ -2) \cup (2,\ \infty)}$



Exercise

Let $f(x) = x^4 + 3x^3 - 4x^2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

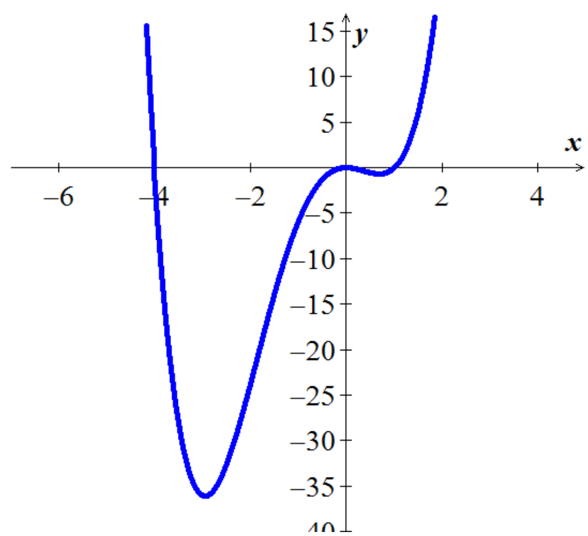
$$f(x) = x^2\left(x^2 + 3x - 4\right)$$

The zeros are: 0, 0, 1, –4.

$-\infty$ -4	$0,0$ 1	∞
+	–	+

$f(x) > 0 \quad \underline{(-\infty,\ -4) \cup (1,\ \infty)}$

$f(x) < 0 \quad \underline{(-4,\ 0) \cup (0,\ 1)}$



Exercise

Let $f(x) = x^3 + 2x^2 - 4x - 8$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

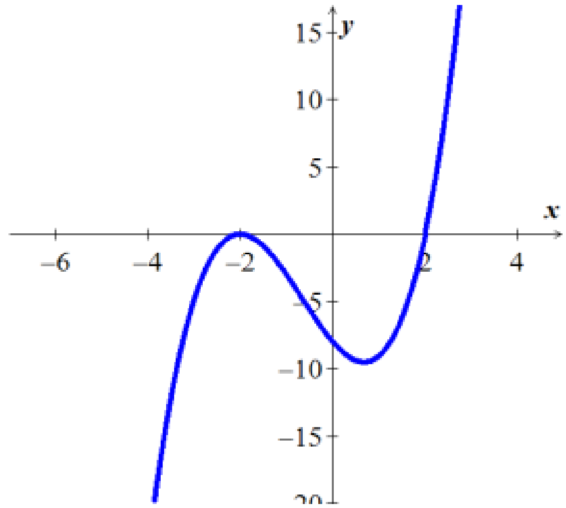
$$f(x) = x^2(x+2) - 4(x+2)$$
$$= (x+2)\left(x^2 - 4\right)$$
$$= (x+2)(x+2)(x-2) = 0$$

The zeros are: 2, –2, –2

$-\infty$	-2	0	2	∞
$-$		$-$		$+$

$f(x) > 0$ $\quad \boxed{(2,\ \infty)}$

$f(x) < 0$ $\quad \boxed{(-\infty,\ -2) \cup (-2,\ 2)}$



Exercise

Let $f(x) = x^3 - 3x^2 - 9x + 27$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

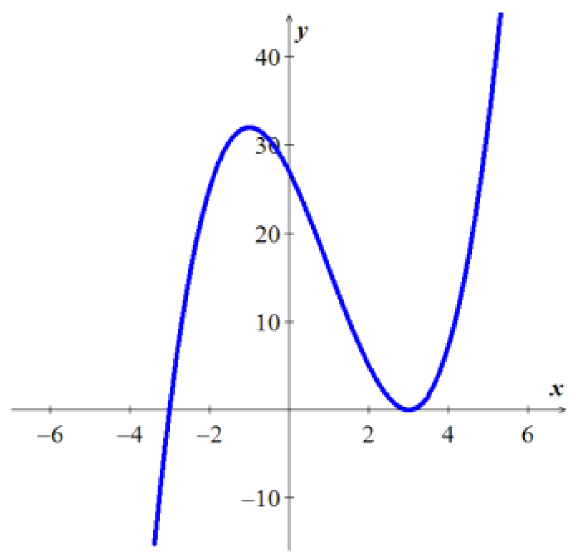
$$f(x) = x^2(x-3) - 9(x-3)$$
$$= (x-3)\left(x^2 - 9\right)$$
$$= (x-3)(x-3)(x+3)$$

The zeros are: –3, 3 (multiplicity)

$-\infty$	-3	0	3	∞
$-$		$+$		$+$

$f(x) > 0$ $\quad \boxed{(-3,\ 3) \cup (3,\ \infty)}$

$f(x) < 0$ $\quad \boxed{(-\infty,\ -3)}$



Exercise

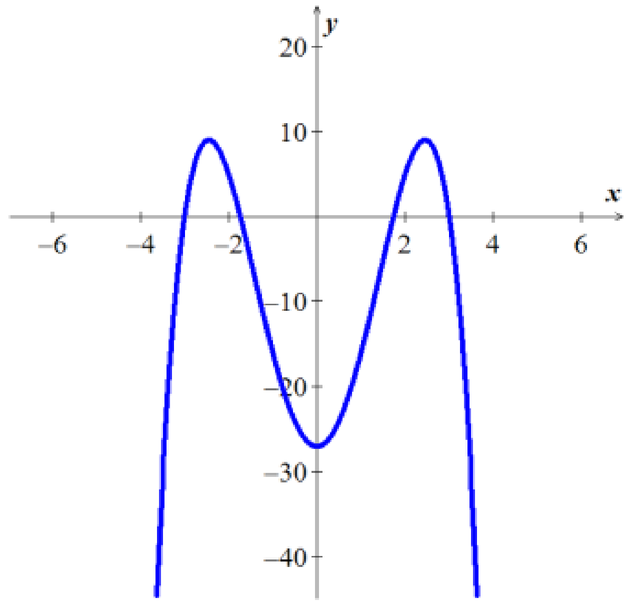
Let $f(x) = -x^4 + 12x^2 - 27$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$x^2 = \frac{-12 \pm \sqrt{36}}{-2}$$

$$= \begin{cases} \frac{-12-6}{-2} = 9 \\ \frac{-12+6}{-2} = 3 \end{cases}$$

$$\rightarrow \begin{cases} x^2 = 9 \\ x^2 = 3 \end{cases} \Rightarrow \begin{matrix} x = \pm 3 \\ x = \pm\sqrt{3} \end{matrix}$$



	-3	$-\sqrt{3}$	$\sqrt{3}$	3	
$-$	$+$		$-$	$+$	$-$

$f(x) > 0$ $\quad \left(-3,\ -\sqrt{3}\right) \cup \left(\sqrt{3},\ 3\right)$

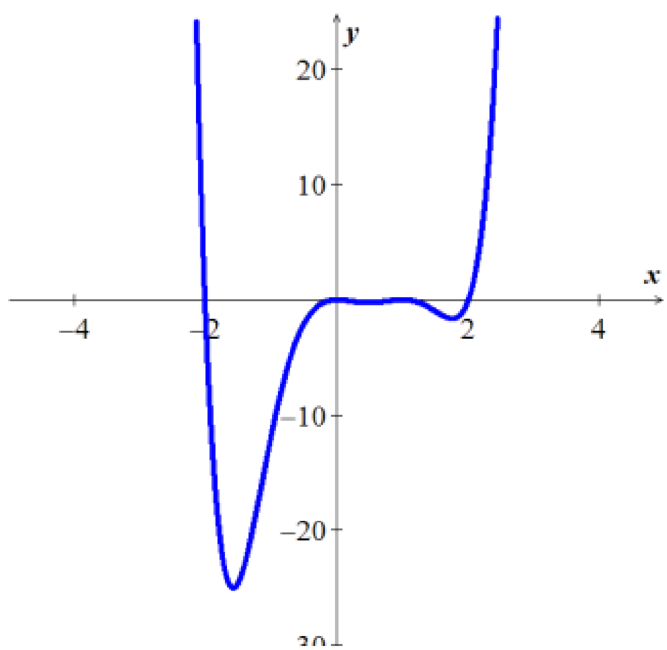
$f(x) < 0$ $\quad (-\infty,\ -3) \cup \left(-\sqrt{3},\ \sqrt{3}\right) \cup (3,\ \infty)$

Exercise

Let $f(x) = x^2(x+2)(x-1)^2(x-2)$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

The zeros are: $-2, 2, 0, 0, 1, 1$



	-2	**0,0** **1,1**	2	
$+$		$-$		$+$

$f(x) > 0$ $\quad (-\infty,\ -2) \cup (2,\ \infty)$

$f(x) < 0$ $\quad (-2,\ 0) \cup (0,\ 1) \cup (1,\ 2)$

Exercise

Let $f(x) = 2x^3 + 11x^2 - 7x - 6$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\textit{possibilities}: \pm\left\{\frac{6}{2}\right\} = \pm\left\{\frac{1,\ 2,\ 3,\ 6}{1,\ 2}\right\}$$

$$= \pm\left\{1,\ 2,\ 3,\ 5,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

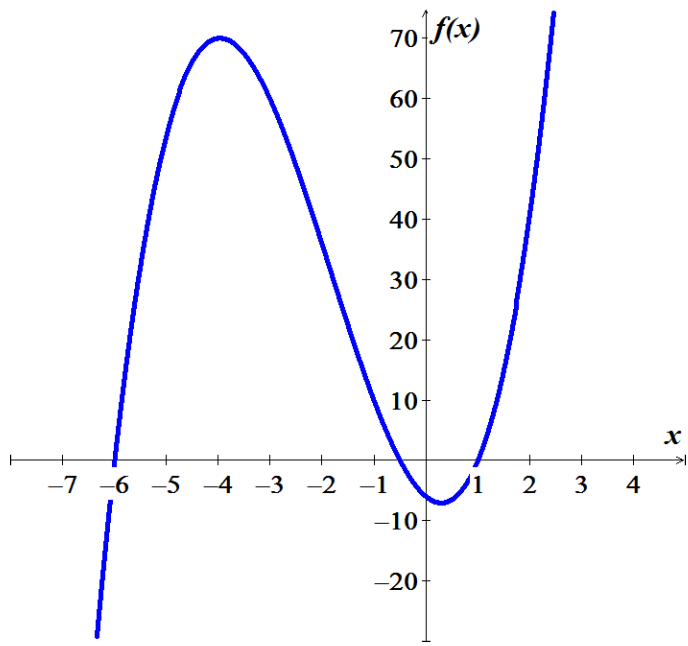
$$\begin{array}{r|rrrr} 1 & 2 & 11 & -7 & -6 \\ & & 2 & 13 & 6 \\ \hline & 2 & 13 & 6 & \boxed{0} \end{array} \rightarrow 2x^2 + 13x + 6 = 0$$

The zeros are: $\underline{x = 1,\ -\frac{1}{2},\ -6}$

	-6		$-\frac{1}{2}$		1	
$-$		$+$		$-$		$+$

$f(x) > 0$ $\quad \left(-6,\ -\frac{1}{2}\right) \cup (1,\ \infty)$

$f(x) < 0$ $\quad (-\infty,\ -6) \cup \left(-\frac{1}{2},\ 1\right)$



Exercise

Let $f(x) = x^3 + 2x^2 - 5x - 6$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\textit{possibilities}: \ \pm\left\{\frac{6}{1}\right\}$$

$$= \pm\{1,\ 2,\ 3,\ 6\}$$

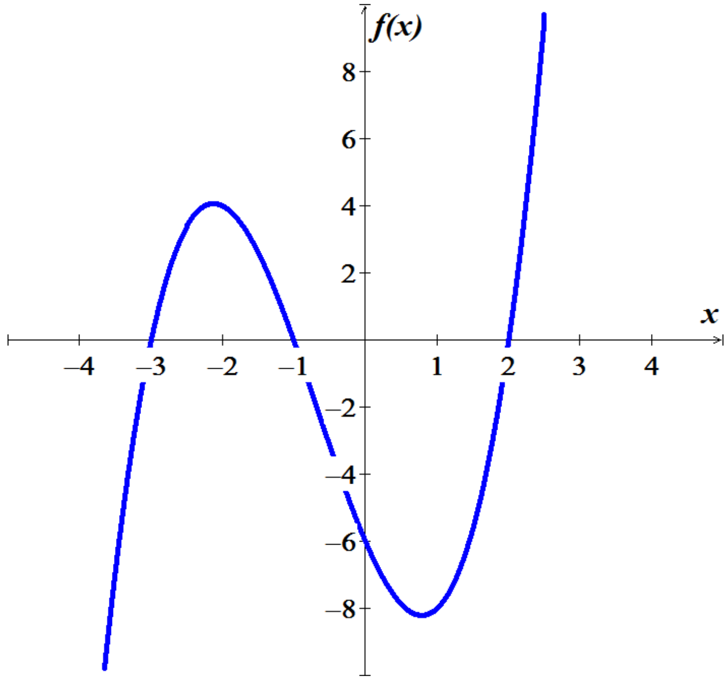
$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array} \rightarrow x^2 + x - 6 = 0$$

The zeros are: $\underline{x = -1,\ -3,\ 2}$

	-3		-1		2	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{(-3,\ -1) \cup (2,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -3) \cup (-1,\ 2)}$



Exercise

Let $f(x) = x^3 + 8x^2 + 11x - 20$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$possibilities\ : \pm\left\{\frac{20}{1}\right\} = \pm\{1,\ 2,\ 4,\ 5,\ 20,\ 20\}$

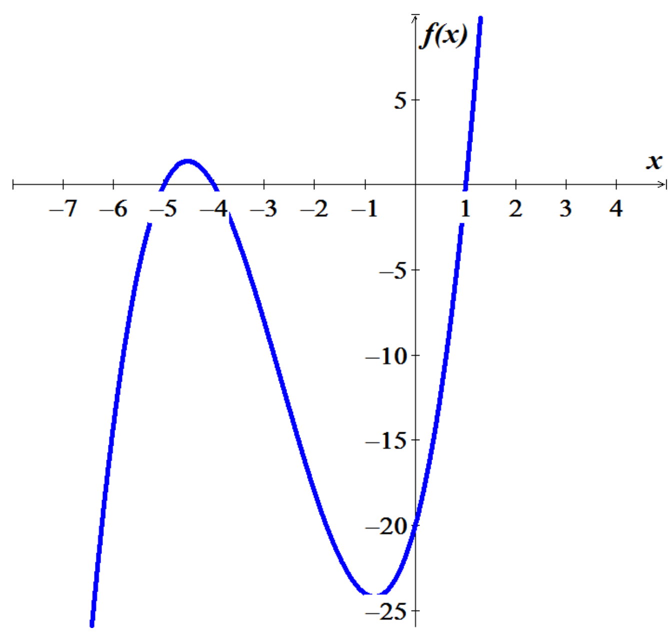
$$\begin{array}{r|rrrr} 1 & 1 & 8 & 11 & -20 \\ & & 1 & 9 & 20 \\ \hline & 1 & 9 & 20 & \boxed{0} \end{array} \quad \to\ x^2 + 9x + 20 = 0$$

The zeros are: $\boxed{x = -5,\ \ -4,\ 1}$

	−5		−4		1	
−		+		−		+

$f(x) > 0 \quad \boxed{(-5,\ -1) \cup (1,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -5) \cup (-4,\ 1)}$



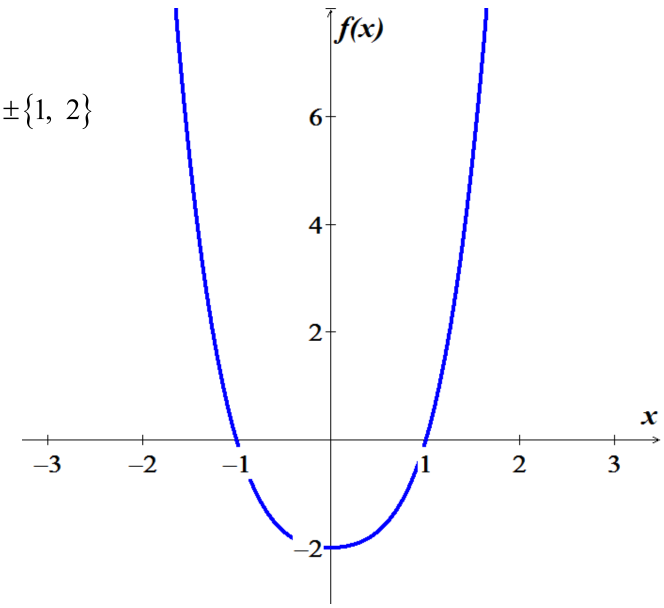
Exercise

Let $f(x) = x^4 + x^2 - 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

possibilities: $\pm\{1, 2\}$

$$\begin{array}{r|rrrrr}
1 & 1 & 0 & 1 & 0 & -2 \\
 & & 1 & 1 & 2 & 1 \\ \hline
-1 & 1 & 1 & 2 & 2 & 0 \\
 & & -1 & 0 & -2 & \\ \hline
 & 1 & 0 & 2 & 0 &
\end{array} \rightarrow x^3 + x^2 + 2x + 1 = 0 \rightarrow \pm\{1, 2\}$$



$\rightarrow x^2 + 2 = 0 \Rightarrow \underline{x = \pm i\sqrt{2}}$

The zeros are: $\underline{x = \pm 1}$

	-1		1	
$+$		$-$		$+$

$f(x) > 0 \quad \underline{(-\infty, -1) \cup (1, \infty)}$

$f(x) < 0 \quad \underline{(-1, 1)}$

Exercise

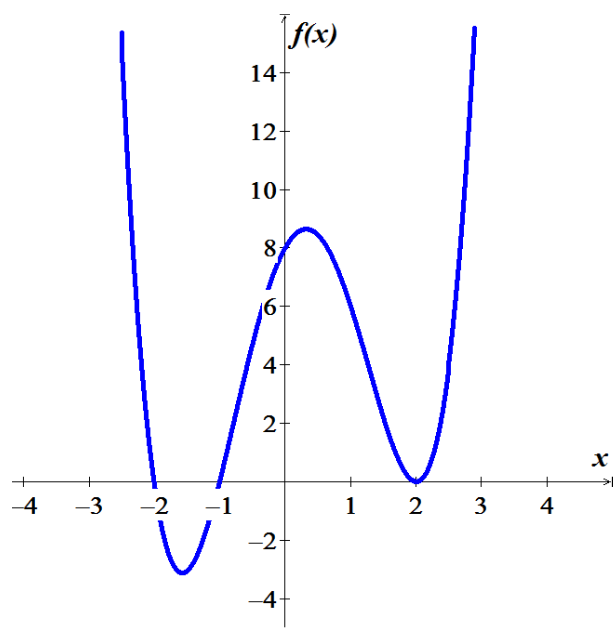
Let $f(x) = x^4 - x^3 - 6x^2 + 4x + 8$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

possibilities: $\pm\{1, 2, 4, 8\}$

$$\begin{array}{r|rrrrr}
-1 & 1 & -1 & -6 & 4 & 8 \\
 & & -1 & 2 & 4 & -8 \\ \hline
-2 & 1 & -2 & -4 & 8 & 0 \\
 & & -2 & 8 & -8 & \\ \hline
 & 1 & -4 & 4 & 0 &
\end{array} \rightarrow x^3 - 2x^2 - 4x + 8 = 0$$

$\rightarrow \pm\{1, 2, 4, 8\}$



$\rightarrow x^2 - 4x + 4 = 0 \Rightarrow \underline{x = 2, 2}$

The zeros are: $\underline{x = -2, -1, 2, 2}$

	-2		-1		2	
$+$		$-$		$+$		$+$

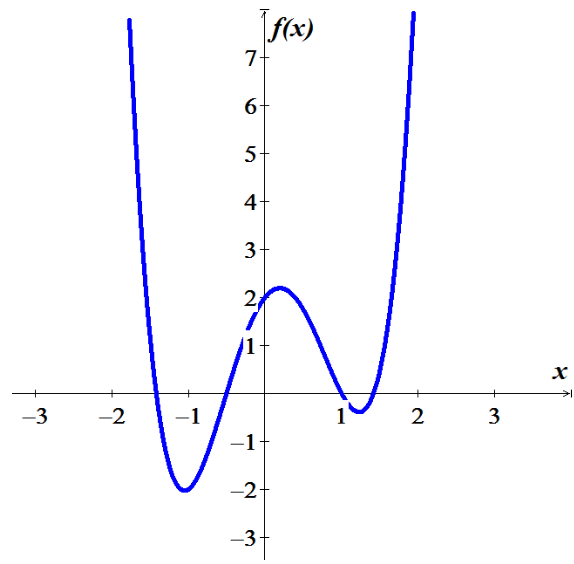
$f(x) > 0 \quad \boxed{(-\infty,\ -1) \cup (1,\ \infty)}$

$f(x) < 0 \quad \boxed{(-1,\ 1)}$

Exercise

Let $f(x) = 2x^4 - x^3 - 5x^2 + 2x + 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution



$$\textit{possibilities}: \ \pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$$

$$\begin{array}{c|ccccc}
1 & 2 & -1 & -5 & 2 & 2 \\
 & & 2 & 1 & -4 & -2 \\
\hline
-\frac{1}{2} & 2 & 1 & -4 & -2 & 0 \\
 & & -1 & 0 & 2 & \\
\hline
 & 2 & 0 & -4 & 0 &
\end{array}
\quad \begin{array}{l} \to 2x^3 + x^2 - 4x - 2 = 0 \\ \to \pm\left\{1,\ 2,\ \frac{1}{2}\right\} \end{array}$$

$$\to 2x^2 - 4 = 0 \Rightarrow \boxed{x = \pm\sqrt{2}}$$

The zeros are: $\boxed{x = -\frac{1}{2},\ 1,\ -\sqrt{2},\ \sqrt{2}}$

	$-\sqrt{2}$	$-\frac{1}{2}$	1	$\sqrt{2}$	
$+$		$-$	$+$	$-$	$+$

$f(x) > 0 \quad \boxed{(-\infty,\ -\sqrt{2}) \cup \left(-\frac{1}{2},\ 1\right) \cup (\sqrt{2},\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-\sqrt{2},\ -\frac{1}{2}\right) \cup (1,\ \sqrt{2})}$

Exercise

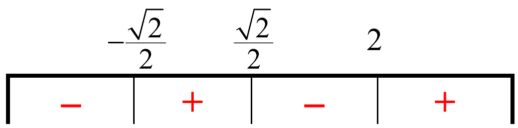
Let $f(x) = 4x^5 - 8x^4 - x + 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\begin{aligned} f(x) &= 4x^4(x-2) - (x-2) \\ &= (x-2)\left(4x^4 - 1\right) = 0 \end{aligned}$$

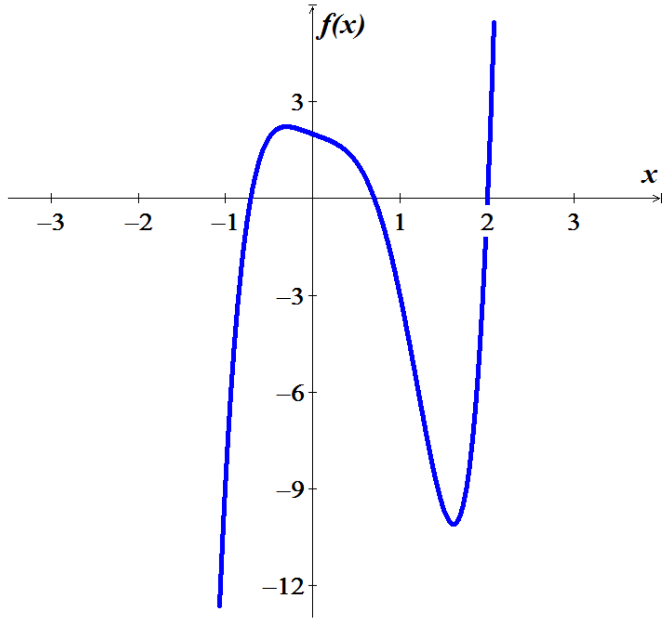
$$4x^4 - 1 = 0 \quad \Rightarrow \quad \begin{cases} x^2 = -\frac{1}{2} & \mathbb{C} \\ x^2 = \frac{1}{2} & x = \pm\frac{\sqrt{2}}{2} \end{cases}$$

The zeros are: $\boxed{x = 2,\ -\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}}$



$f(x) > 0 \quad \boxed{\left(-\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right) \cup (2,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2},\ 2\right)}$



Exercise

Let $f(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$possibilities: \ \pm\left\{\frac{36}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 9,\ 12,\ 18,\ 36\}$

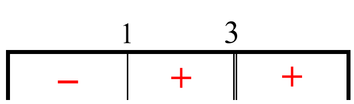
1	1	−7	19	−37	60	−36
		1	−6	13	−24	36
3	1	−6	13	−24	36	0
		3	−9	12	−36	
3	1	−3	4	−12	0	
		3	0	12		
	1	0	4	0		

$x^4 - 6x^3 + 13x^2 - 24x + 36 = 0 \ \rightarrow \ \pm\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$x^3 - 3x^2 + 4x - 12 = 0 \ \rightarrow \ \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

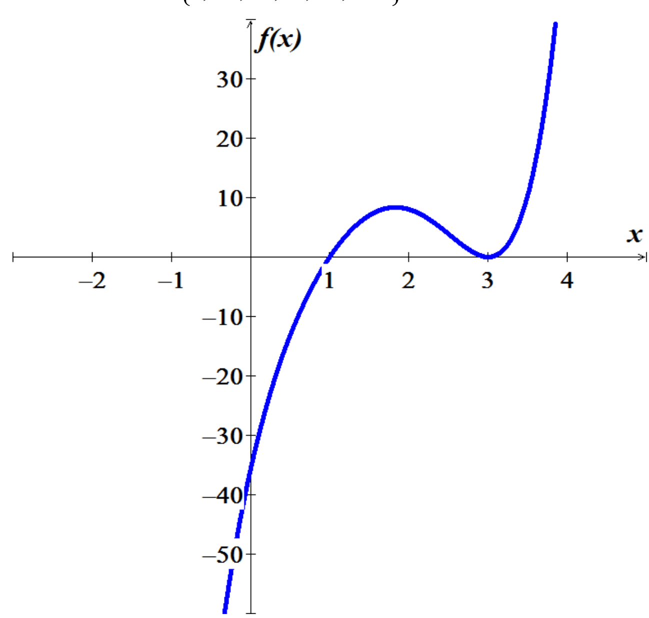
$x^2 + 4 = 0 \quad \Rightarrow \quad x = \pm 2i$

The zeros are: $\boxed{x = 1,\ 3,\ 3}$



$f(x) > 0 \quad \boxed{(1,\ 3) \cup (3,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ 1)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - x^2 - 10x - 8$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

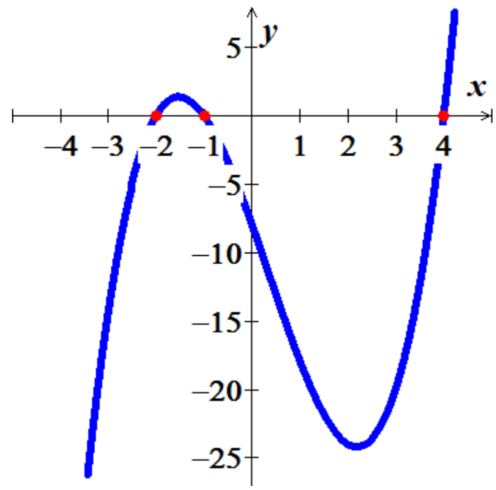
$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & 0 \end{array} \to \quad x^2 - 2x - 8 = 0$$

$\underline{x = -1,\ -2,\ 4}$

	-2		-1		4	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad (-2,\ -1) \cup (4,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -2) \cup (-1,\ 4)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + x^2 - 14x - 24$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

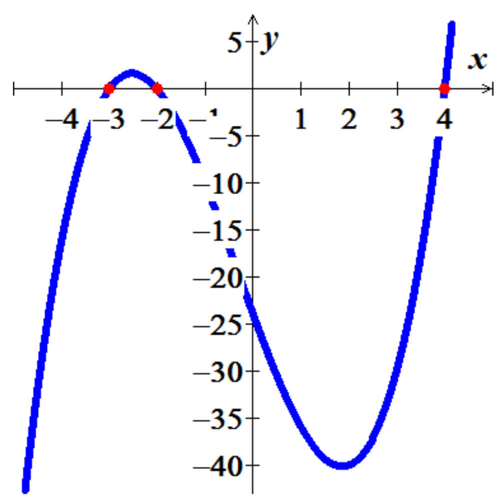
$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & 0 \end{array} \to \quad x^2 - x - 12 = 0$$

$\underline{x = -2,\ -3,\ 4}$

	-3		-2		4	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad (-3,\ -2) \cup (4,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -3) \cup (-2,\ 4)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 - 3x^2 - 17x + 30$$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{30}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30,\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{15}{2}\right\}$

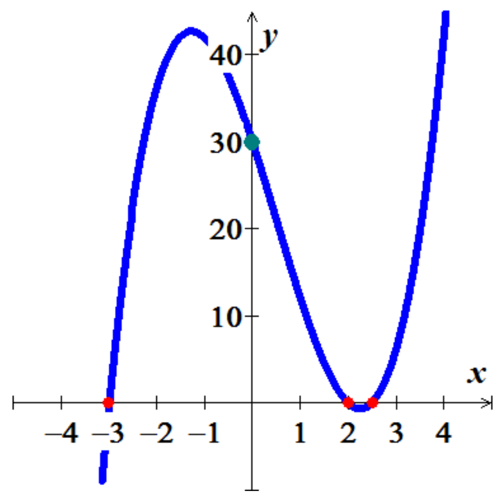
$$\begin{array}{r|rrrr} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & \boxed{0} \end{array} \rightarrow \quad 2x^2 + x - 15 = 0$$

$x = 2,\ -3,\ \frac{5}{2}$

	-3	2	$\frac{5}{2}$	
$-$		$+$	$-$	$+$

$f(x) > 0 \quad \left(-3,\ 2\right) \cup \left(\frac{5}{2},\ \infty\right)$

$f(x) < 0 \quad \left(-\infty,\ -3\right) \cup \left(2,\ \frac{5}{2}\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 12x^3 + 8x^2 - 3x - 2$$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{2}{12}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ \frac{1}{6},\ \frac{1}{12}\right\}$

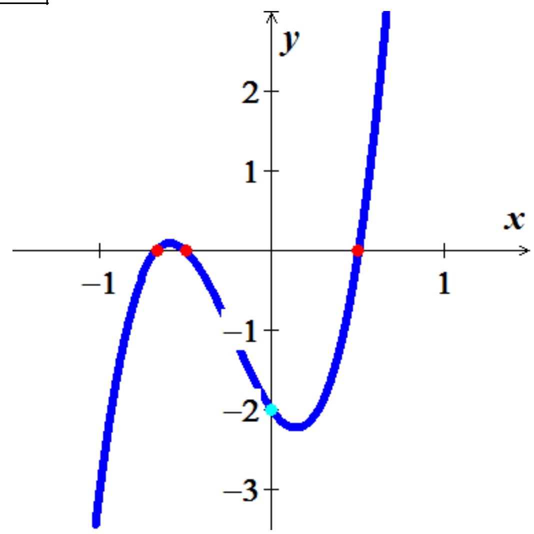
$$\begin{array}{r|rrrr} \frac{1}{2} & 12 & 8 & -3 & -2 \\ & & 6 & 7 & 2 \\ \hline & 12 & 14 & 4 & \boxed{0} \end{array} \rightarrow \quad 12x^2 + 14x + 4 = 0$$

$x = \frac{1}{2},\ -\frac{1}{2},\ -\frac{2}{3}$

	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	
$-$		$+$	$-$	$+$

$f(x) > 0 \quad \left(-\frac{2}{3},\ -\frac{1}{2}\right) \cup \left(\frac{1}{2},\ \infty\right)$

$f(x) < 0 \quad \boxed{\left(-\infty,\ -\frac{2}{3}\right) \cup \left(-\frac{1}{2},\ \frac{1}{2}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + x^2 - 6x - 8$$

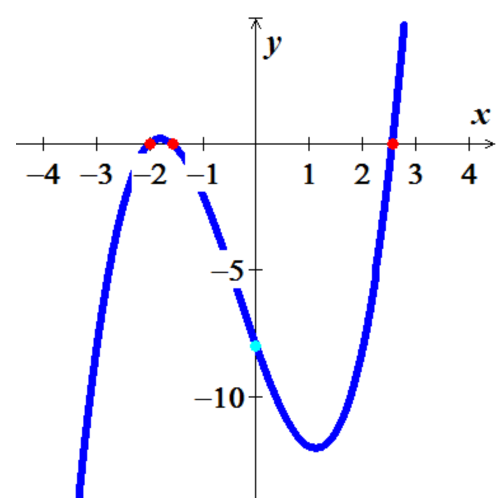
Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -6 & -8 \\ & & -2 & 2 & 8 \\ \hline & 1 & -1 & -4 & \boxed{0} \end{array} \to \quad x^2 - x - 4 = 0$$

$x = \dfrac{1 \pm \sqrt{1+16}}{2}$

$\boxed{x = -2,\ \dfrac{1 \pm \sqrt{17}}{2}}$



	-2		$\frac{1-\sqrt{17}}{2}$		$\frac{1+\sqrt{17}}{2}$	
−		+		−		+

$f(x) > 0 \quad \boxed{\left(-2,\ \frac{1-\sqrt{17}}{2}\right) \cup \left(\frac{1+\sqrt{17}}{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -2) \cup \left(\frac{1-\sqrt{17}}{2},\ \frac{1+\sqrt{17}}{2}\right)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 19x - 30$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{30}{1}\right\} = \pm\{1,\ 2,\ 3,\ 5,\ 6,\ 15,\ 30\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & \boxed{0} \end{array} \rightarrow\ x^2 - 2x - 15$$

$$x = \frac{2 \pm \sqrt{4+60}}{2}$$

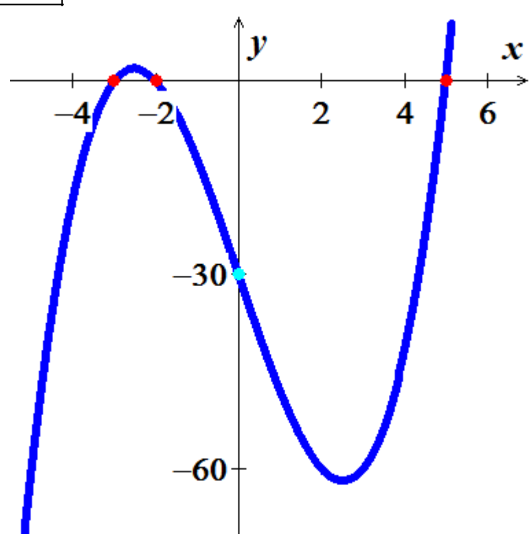
$$= \begin{cases} \frac{2-8}{2} = -3 \\ \frac{2+8}{2} = 5 \end{cases}$$

$x = -2,\ -3,\ 5$

	-3	-2	5	
$-$	$+$	$-$	$+$	

$f(x) > 0 \quad (-3,\ -2) \cup (5,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -3) \cup (-2,\ 5)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 + x^2 - 25x + 12$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 12,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrr} 3 & 2 & 1 & -25 & 12 \\ & & 6 & 21 & -12 \\ \hline & 2 & 7 & -4 & \boxed{0} \end{array} \quad \to \quad 2x^2 + 7x - 4$$

$$x = \frac{-7 \pm \sqrt{49 + 32}}{4}$$

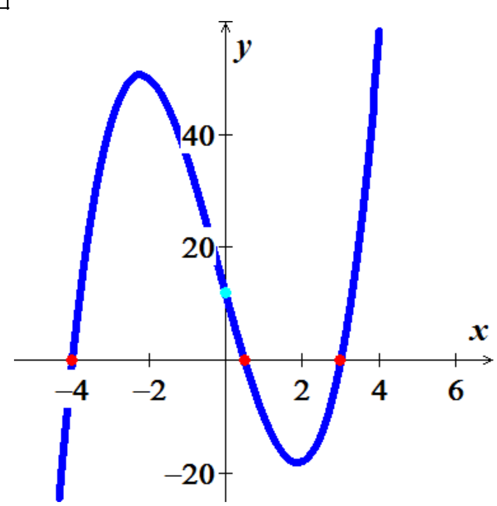
$$= \begin{cases} \frac{-7-9}{4} = -4 \\ \frac{-7+9}{4} = \frac{1}{2} \end{cases}$$

$$\boxed{x = -4,\ \frac{1}{2},\ 3}$$

	-4		$\frac{1}{2}$		3	
$-$		$+$		$-$		$+$

$f(x) > 0$ $\quad \boxed{\left(-4,\ \frac{1}{2}\right) \cup (3,\ \infty)}$

$f(x) < 0$ $\quad \boxed{(-\infty,\ -1) \cup \left(\frac{1}{2},\ 3\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^3 + 11x^2 - 6x - 8$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ 8,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{8}{3}\right\}$

$$\begin{array}{r|rrrr} 1 & 3 & 11 & -6 & -8 \\ & & 3 & 14 & 8 \\ \hline & 3 & 14 & 8 & \boxed{0} \end{array} \rightarrow 3x^2 + 14x + 8$$

$$x = \frac{-14 \pm \sqrt{196 - 96}}{6}$$

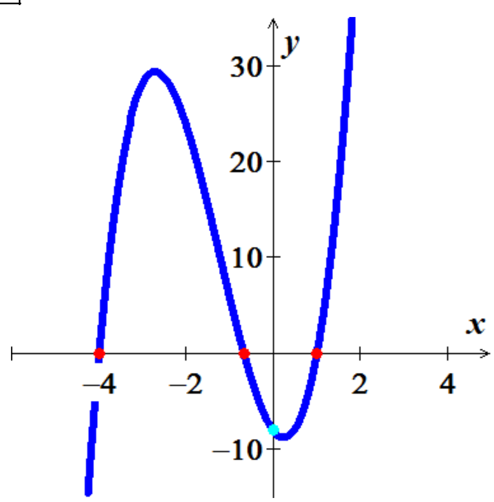
$$= \begin{cases} \frac{-14-10}{6} = -4 \\ \frac{-14+10}{6} = -\frac{2}{3} \end{cases}$$

$$\boxed{x = -4,\ -\frac{2}{3},\ 1}$$

$x<-4$	$-4<x<-\frac{2}{3}$	$-\frac{2}{3}<x<1$	$x>1$
−	+	−	+

$f(x) > 0$ $\quad \boxed{\left(-4,\ -\frac{2}{3}\right) \cup (1,\ \infty)}$

$f(x) < 0$ $\quad \boxed{(-\infty,\ -4) \cup \left(-\frac{2}{3},\ 1\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=2x^3+9x^2-2x-9$$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{9}{2}\right\}=\pm\left\{1,\ 3,\ 9,\ \frac{1}{2},\ \frac{3}{2},\ \frac{9}{2}\right\}$

$$\begin{array}{r|rrrr} 1 & 2 & 9 & -2 & -9 \\ & & 2 & 11 & 9 \\ \hline & 2 & 11 & 9 & \boxed{0} \end{array} \quad \to \quad 2x^2+11x+9$$

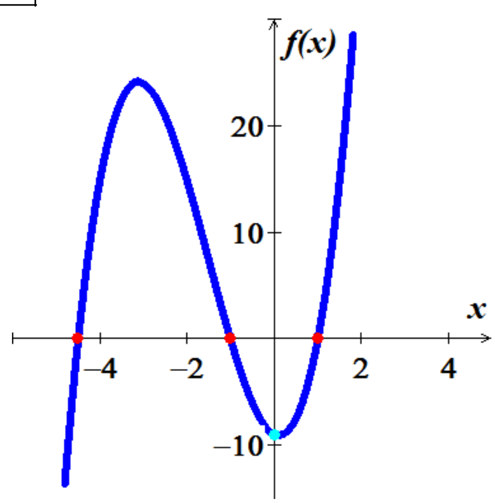
$x=-1,\ -\frac{9}{2}$ $\qquad a-b+c=0 \ \to \ x=-1,\ -\frac{c}{a}$

$x=-\frac{9}{2},\ -1,\ 1$

	$-\frac{9}{2}$	-1	1	
$-$		$+$	$-$	$+$

$f(x)>0 \quad \left(-\frac{9}{2},\ -1\right)\cup(1,\ \infty)$

$f(x)<0 \quad \left(-\infty,\ -\frac{9}{2}\right)\cup(-1,\ 1)$



Exercise

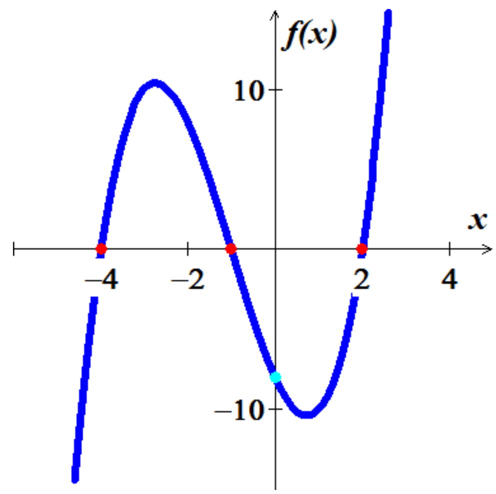
Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + 3x^2 - 6x - 8$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ & & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & \boxed{0} \end{array} \to \quad x^2 + 2x - 8 = 0$$



$$x = \frac{-2 \pm \sqrt{4+32}}{2}$$

$$= \begin{cases} \frac{-2-6}{2} = -4 \\ \frac{-2+6}{2} = 2 \end{cases}$$

$$\underline{x = -4,\ \ -1,\ \ 2\ |}$$

$$\begin{array}{c|c|c|c} & -4 & -1 & 2 \\ \hline - & + & - & + \end{array}$$

$-$	$+$	$-$	$+$

$f(x) > 0 \quad \underline{(-4,\ -1) \cup (2,\ \infty)\ |}$

$f(x) < 0 \quad \underline{(-\infty,\ -4) \cup (-1,\ 2)\ |}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^3 - x^2 - 6x + 2$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{3}\right\} = \pm\left\{1,\ 2,\ \frac{1}{3},\ \frac{2}{3}\right\}$

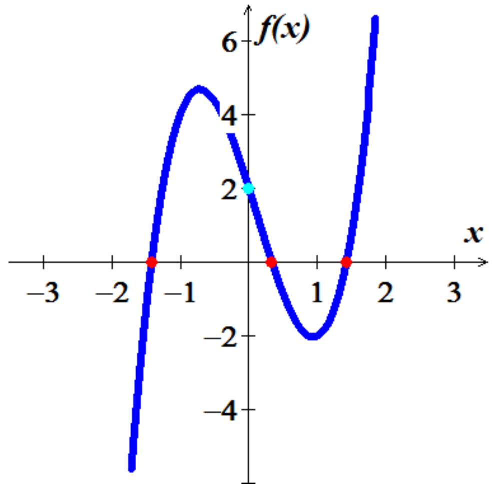
$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -6 & 2 \\ & & 1 & 0 & -2 \\ \hline & 3 & 0 & -6 & \boxed{0} \end{array} \quad \to \quad 3x^2 - 6 = 0$$

$$\underline{x = \frac{1}{3},\ \pm\sqrt{2}\ |}$$

$-\sqrt{2}$	$\frac{1}{3}$	$\sqrt{2}$	
$-$	$+$	$-$	$+$

$f(x) > 0 \quad \boxed{\left(-\sqrt{2},\ \frac{1}{3}\right) \cup \left(\sqrt{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ -\sqrt{2}\right) \cup \left(\frac{1}{3},\ \sqrt{2}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 8x^2 + 8x + 24$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

$$\begin{array}{r|rrrr} 6 & 1 & -8 & 8 & 24 \\ & & 6 & -12 & -24 \\ \hline & 1 & -2 & -4 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - 2x - 4 = 0$$

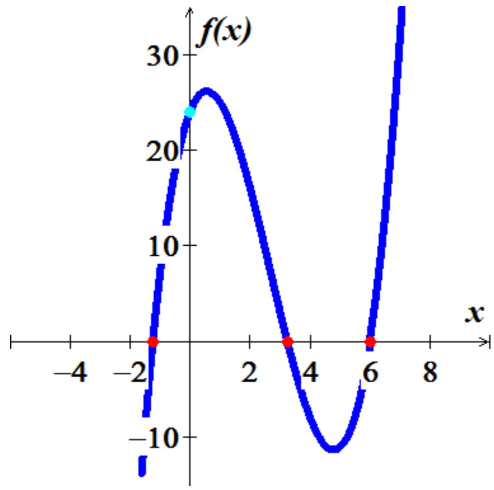
$$x = \frac{2 \pm \sqrt{4+16}}{2}$$

$\boxed{x = 6,\ 1 \pm \sqrt{5}}$

$1-\sqrt{5}$	$1+\sqrt{5}$	6	
$-$	$+$	$-$	$+$

$f(x) > 0 \quad \boxed{\left(1-\sqrt{5},\ 1+\sqrt{5}\right) \cup (6,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ 1-\sqrt{5}\right) \cup \left(1+\sqrt{5},\ 6\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 7x^2 - 7x + 69$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{69}{1}\right\} = \pm\{1,\ 3,\ 23,\ 69\}$

$$\begin{array}{r|rrrr} -3 & 1 & -7 & -7 & 69 \\ & & -3 & 30 & -69 \\ \hline & 1 & -10 & 23 & \boxed{0} \end{array} \quad \to \quad x^2 - 10x + 23 = 0$$

$$x = \frac{10 \pm \sqrt{100 - 92}}{2}$$

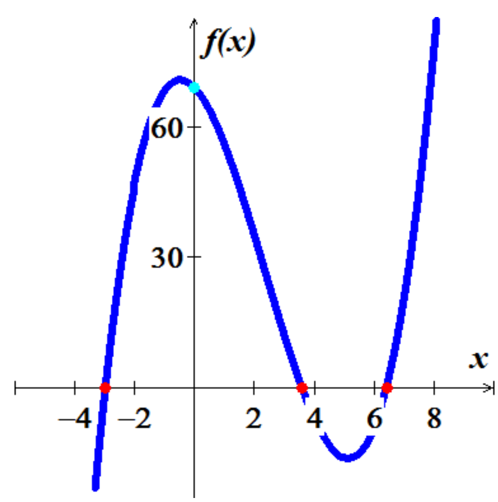
$$= \frac{10 \pm 2\sqrt{2}}{2}$$

$$\boxed{x = -3,\ 5 \pm \sqrt{2}}$$

	-3		$5-\sqrt{2}$		$5+\sqrt{2}$	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{\left(-3,\ 5-\sqrt{2}\right) \cup \left(5+\sqrt{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ -3\right) \cup \left(5-\sqrt{2},\ 5+\sqrt{2}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 3x - 2$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -3 & -2 \\ & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array} \quad \to \quad x^2 - x - 2 = 0$$

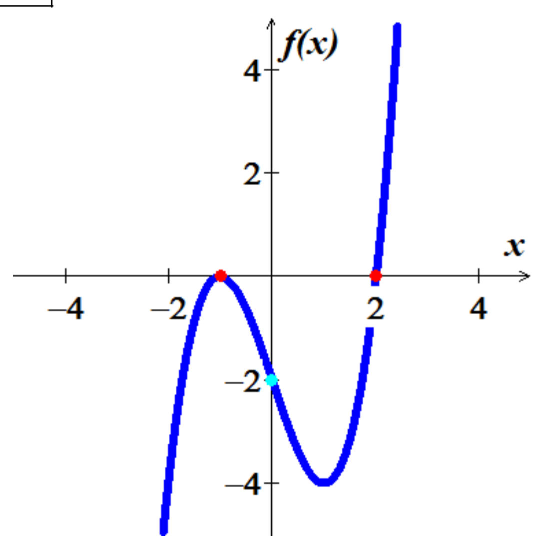
$x = -1,\ 2$ $\qquad a - b + c = 0 \ \to\ x = -1,\ -\frac{c}{a}$

$x = -1,\ -1,\ 2$

	-1		2	
$-$		$-$		$+$

$f(x) > 0 \quad (2,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -1) \cup (-1,\ 2)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3-2x+1$$

Solution

possibilities for $\frac{c}{d}:\pm\{1\}$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -2 & 1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & -1 & \boxed{0} \end{array} \quad \rightarrow \quad x^2+x-1=0$$

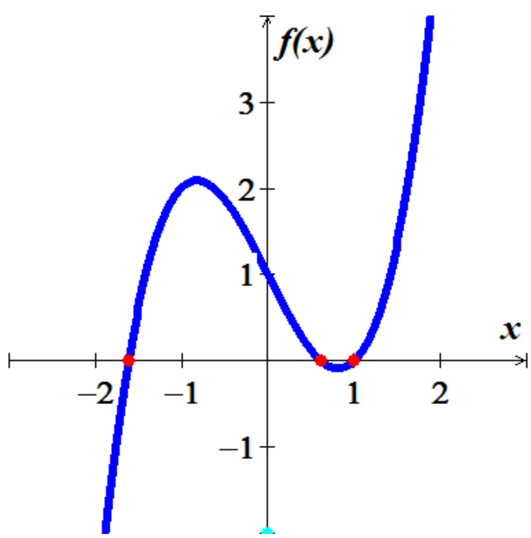
$$x=\frac{-1\pm\sqrt{5}}{2}$$

$$\boxed{x=1,\ \frac{-1\pm\sqrt{5}}{2}}$$

	$\frac{-1-\sqrt{5}}{2}$	$\frac{-1+\sqrt{5}}{2}$	1
$-$	$+$	$-$	$+$

$f(x)>0$ $\quad \boxed{\left(\frac{-1-\sqrt{5}}{2},\ \frac{-1+\sqrt{5}}{2}\right)\cup(1,\ \infty)}$

$f(x)<0$ $\quad \boxed{\left(-\infty,\ \frac{-1-\sqrt{5}}{2}\right)\cup\left(\frac{-1+\sqrt{5}}{2},\ 1\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3-2x^2-11x+12$$

Solution

$$\text{possibilities for } \tfrac{c}{d}: \pm\left\{\frac{12}{1}\right\}=\pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$$

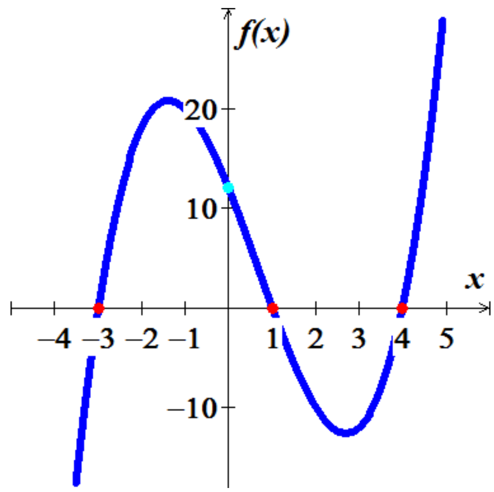
$$\begin{array}{r|rrrr} 1 & 1 & -2 & -11 & 12 \\ & & 1 & -1 & 12 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array} \quad \to \quad x^2-x-12=0$$

$$x=\frac{1\pm\sqrt{1+48}}{2}$$

$$=\begin{cases} \dfrac{1-7}{2}=-3 \\ \dfrac{1+7}{2}=4 \end{cases}$$

$$\underline{x=-3,\ 1,\ 4}$$

$-\infty$ to -3	-3 to 1	1 to 4	4 to ∞
$-$	$+$	$-$	$+$



$f(x)>0$ $\quad (-3,\ 1)\cup(4,\ \infty)$

$f(x)<0$ $\quad (-\infty,\ -3)\cup(1,\ 4)$

Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3-2x^2-7x-4$$

Solution

$$\text{possibilities for } \tfrac{c}{d}: \pm\left\{\frac{4}{1}\right\}=\pm\{1,\ 2,\ 4\}$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -7 & -4 \\ & & -1 & 3 & 4 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array} \quad \to \quad x^2-3x-4=0$$

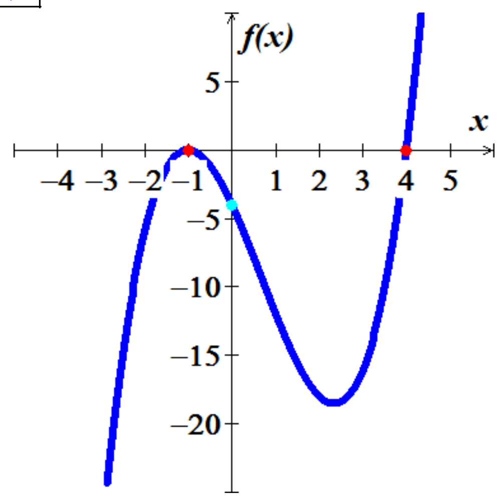
$\underline{x=-1,\ 4}$ $\qquad\qquad a-b+c=0 \ \to\ x=-1,\ -\frac{c}{a}$

$\underline{x=-1,\ -1,\ 4}$

	-1		4	
$-$		$-$		$+$

$f(x) > 0 \quad \boxed{(4,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -1) \cup (-1,\ 4)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 10x - 12$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -10 & -12 \\ & & -2 & 4 & 12 \\ \hline & 1 & -2 & -6 & \boxed{0} \end{array} \quad \to \quad x^2 - 2x - 6 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 24}}{2}$$

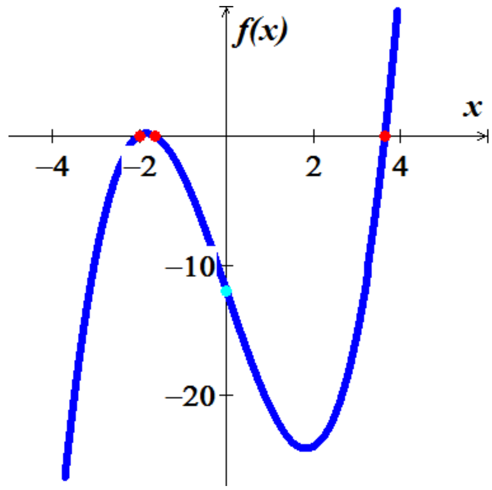
$$= \frac{2 \pm 2\sqrt{7}}{2}$$

$$\boxed{x = -2,\ 1 \pm \sqrt{7}}$$

	-2		$1-\sqrt{7}$		$1+\sqrt{7}$	
$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{\left(-2,\ 1-\sqrt{7}\right) \cup \left(1+\sqrt{7},\ \infty\right)}$

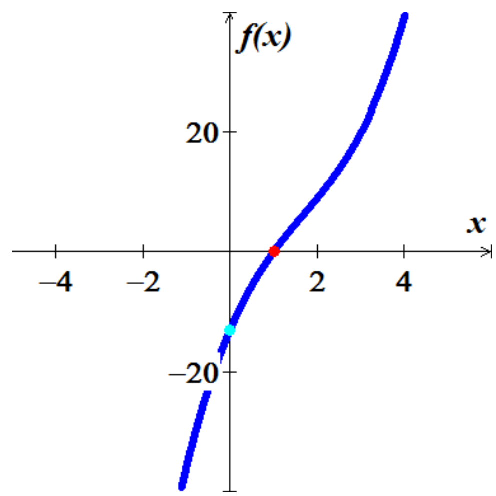
$f(x) < 0 \quad \boxed{\left(-\infty,\ -2\right) \cup \left(1-\sqrt{7},\ 1+\sqrt{7}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 5x^2 + 17x - 13$$



Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{13}{1}\right\} = \pm\{1,\ 13\}$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 17 & -13 \\ & & 1 & -4 & 13 \\ \hline & 1 & -4 & 13 & \boxed{0} \end{array} \quad \to \quad x^2 - 4x + 13 = 0$$

$$x = \frac{4 \pm \sqrt{16-52}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$\boxed{x = 1,\ 2 \pm 3i}$$

1	
$-$	$+$

$f(x) > 0 \quad \boxed{(1,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ 1)}$

Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=6x^3+25x^2-24x+5$$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{5}{6}\right\}=\pm\left\{1,\ 5,\ \frac{1}{2},\ \frac{5}{2},\ \frac{1}{3},\ \frac{5}{3},\ \frac{1}{6},\ \frac{5}{6}\right\}$

$$\begin{array}{r|rrrr} -5 & 6 & 25 & -24 & 5 \\ & & -30 & 25 & -5 \\ \hline & 6 & -5 & 1 & \boxed{0} \end{array} \quad \to \quad 6x^2-5x+1=0$$

$$x=\frac{5\pm\sqrt{25-24}}{12}$$

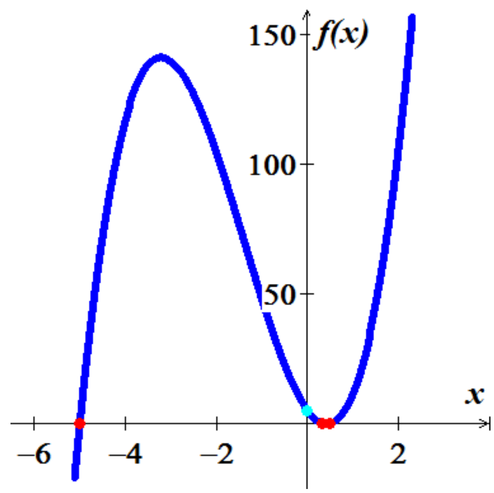
$$=\begin{cases} \frac{5-1}{12}=\frac{1}{3} \\ \frac{5+1}{12}=\frac{1}{2} \end{cases}$$

$$x=-5,\ \frac{1}{3},\ \frac{1}{2}$$

	-5		$\frac{1}{3}$		$\frac{1}{2}$	
$-$		$+$		$-$		$+$

$f(x)>0$ $\quad \left(-5,\ \frac{1}{3}\right)\cup\left(\frac{1}{2},\ \infty\right)$

$f(x)<0$ $\quad (-\infty,\ -5)\cup\left(\frac{1}{3},\ \frac{1}{2}\right)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=8x^3+18x^2+45x+27$$

Solution

possibilities : $\pm\left\{\frac{27}{8}\right\}=\pm\left\{\frac{1,\ 3,\ 9,\ 27}{1,\ 2,\ 4,\ 8}\right\}$

$$=\pm\left\{1,\ 3,\ 9,\ 27,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{3}{2},\ \frac{3}{4},\ \frac{3}{8},\ \frac{9}{2},\ \frac{9}{4},\ \frac{9}{8},\ \frac{27}{2},\ \frac{27}{4},\ \frac{27}{8}\right\}$$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 8 & 18 & 45 & 27 \\ & & -6 & -9 & -27 \\ \hline & 8 & 12 & 36 & \boxed{0} \end{array} \quad \to \quad 8x^2+12x+36=0$$

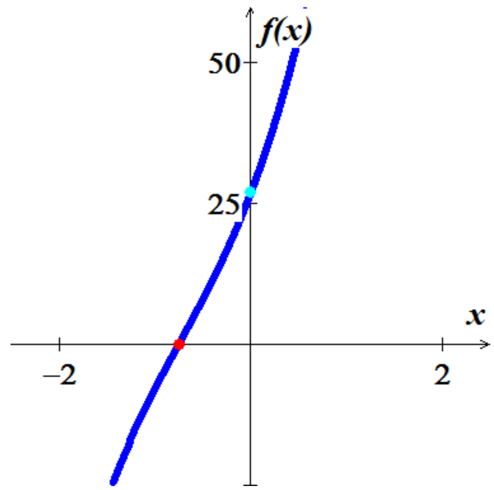
$$\boxed{x = -\frac{3}{4},\ \ -\frac{3}{4} \pm i\frac{3\sqrt{7}}{4}}$$

$$-\frac{3}{4}$$

–	+

$f(x) > 0$ $\boxed{\left(-\frac{3}{4},\ \infty\right)}$

$f(x) < 0$ $\boxed{\left(-\infty,\ -\frac{3}{4}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^3 - x^2 + 11x - 20$$

Solution

$$\text{possibilities}: \ \pm\left\{\frac{20}{3}\right\} = \pm\left\{\frac{1,\ 2,\ 4,\ 5,\ 10,\ 20}{1,\ 3}\right\}$$

$$= \pm\left\{1,\ 2,\ 4,\ 5,\ 10,\ 20,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{5}{3},\ \frac{10}{3},\ \frac{20}{3}\right\}$$

$$\begin{array}{c|cccc} \frac{4}{3} & 3 & -1 & 11 & -20 \\ & & 4 & 4 & 20 \\ \hline & 3 & 3 & 15 & \boxed{0} \end{array} \quad \to \ 3x^2 + 3x + 15 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 180}}{6}$$

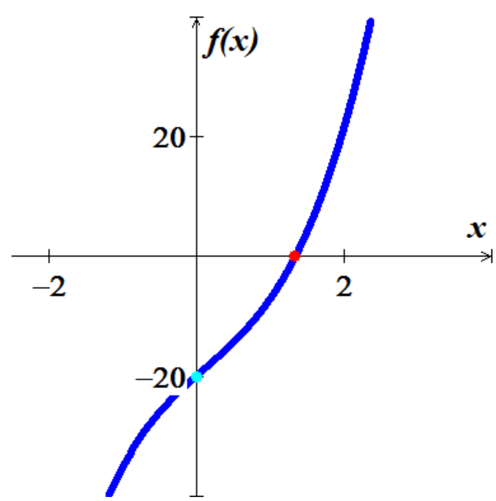
$$\boxed{x = \frac{4}{3},\ \ -\frac{1}{2} \pm i\frac{\sqrt{19}}{2}}$$

$$\frac{4}{3}$$

–	+

$f(x) > 0$ $\boxed{\left(\frac{4}{3},\ \infty\right)}$

$f(x) < 0$ $\boxed{\left(-\infty,\ \frac{4}{3}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - x^3 - 9x^2 + 3x + 18$$

Solution

$$\textit{possibilities for } \frac{c}{d} : \pm\left\{\frac{18}{1}\right\} = \pm\{1,\ 2,\ 3,\ 6,\ 9,\ 18\}$$

$$\begin{array}{r|rrrrr}
-2 & 1 & -1 & -9 & 3 & 18 \\
 & & -2 & 6 & 6 & -18 \\ \hline
3 & 1 & -3 & -3 & 9 & 0 \\
 & & 3 & 0 & -9 & \\ \hline
 & 1 & 0 & -3 & 0 &
\end{array}$$

$\rightarrow x^3 - 3x^2 - 3x + 9 = 0 \ \rightarrow\ \pm\left\{\frac{9}{1}\right\} = \pm\{1,\ 3,\ 9\}$

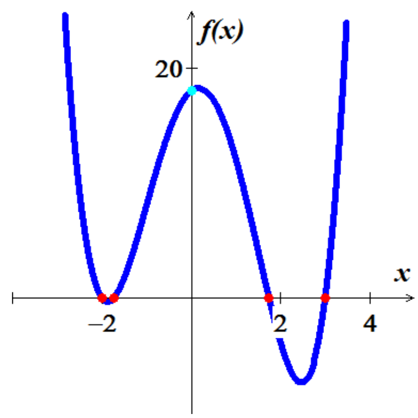
$\rightarrow x^2 - 3 = 0 \Rightarrow \boxed{x = \pm\sqrt{3}}$

$\boxed{x = -2,\ 3,\ \pm\sqrt{3}}$

	-2		$-\sqrt{3}$		$\sqrt{3}$		3	
$+$		$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup \left(-\sqrt{3},\ \sqrt{3}\right) \cup (3,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-2,\ -\sqrt{3}\right) \cup \left(\sqrt{3},\ 3\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 9x^3 + 9x^2 + x - 3$$

Solution

$$\textit{possibilities for } \frac{c}{d} : \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\begin{array}{r|rrrrr}
1 & 2 & -9 & 9 & 1 & -3 \\
 & & 2 & -7 & 2 & 3 \\ \hline
1 & 2 & -7 & 2 & 3 & 0 \\
 & & 2 & -5 & -3 & \\ \hline
 & 2 & -5 & -3 & 0 &
\end{array}$$

$\rightarrow 2x^3 - 7x^2 + 2x + 3 = 0 \ \rightarrow\ \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$\rightarrow 2x^2 - 5x - 3 = 0$

$$x = \frac{5 \pm \sqrt{25 + 24}}{4}$$

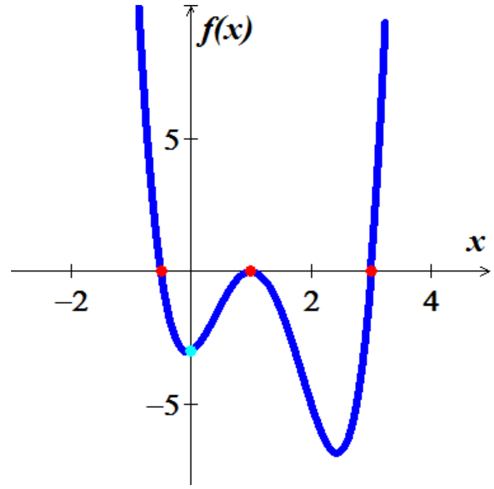
$$= \begin{cases} \dfrac{5-7}{4} = -\dfrac{1}{2} \\ \dfrac{5+7}{4} = 3 \end{cases}$$

$$x = 1,\ 1,\ -\frac{1}{2},\ 3$$

$-\frac{1}{2}$		1		3
$+$	$-$		$-$	$+$

$f(x) > 0 \quad \left(-\infty,\ -\frac{1}{2}\right) \cup (3,\ \infty)$

$f(x) < 0 \quad \left(-\frac{1}{2},\ 1\right) \cup (1,\ 3)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 + 5x^3 - 17x^2 - 6x$$

Solution

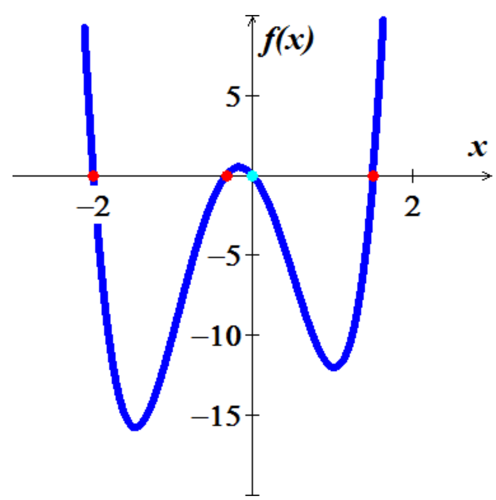
$$x\left(6x^3 + 5x^2 - 17x - 6\right) = 0 \quad \to \quad x = 0$$

possibilities : $\pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrr} -2 & 6 & 5 & -17 & -6 \\ & & -12 & 14 & 6 \\ \hline & 6 & -7 & -3 & \boxed{0} \end{array} \quad \to \quad 6x^2 - 7x - 3 = 0$$

$$x = 0,\ -2,\ -\frac{1}{3},\ \frac{3}{2}$$

-2	$-\frac{1}{3}$	0	$\frac{3}{2}$	
$+$	$-$	$+$	$-$	$+$



$f(x) > 0 \quad (-\infty,\ -2) \cup \left(-\frac{1}{3},\ 0\right) \cup \left(\frac{3}{2},\ \infty\right)$

$f(x) < 0 \quad \left(-2,\ -\frac{1}{3}\right) \cup \left(0,\ \frac{3}{2}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 2x^2 - 16x - 15$$

Solution

$$\textit{possibilities}: \ \pm\left\{\frac{15}{1}\right\} = \pm\{1,\ 3,\ 5,\ 15\}$$

$$\begin{array}{r|rrrrr}
-1 & 1 & 0 & -2 & -16 & -15 \\
 & & -1 & 1 & 1 & 15 \\
\hline
3 & 1 & -1 & -1 & -15 & 0 \\
 & & 3 & 6 & 15 & \\
\hline
 & 1 & 2 & 5 & 0 &
\end{array}$$

$\rightarrow x^3 - x^2 - x - 15 = 0 \rightarrow \pm\left\{\frac{15}{1}\right\} = \pm\{1,\ 3,\ 5,\ 15\}$

$\rightarrow x^2 + 2x + 5 = 0$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

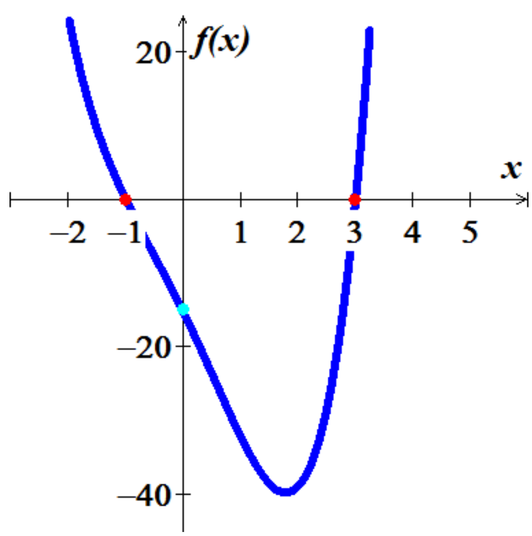
$$= -1 \pm 2i$$

$$\underline{x = -1,\ 3,\ -1 \pm 2i}$$

	-1		3	
$+$		$-$		$+$

$f(x) > 0 \quad \underline{(-\infty,\ -1) \cup (3,\ \infty)}$

$f(x) < 0 \quad \underline{(-1,\ 3)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 2x^3 - 5x^2 + 8x + 4$$

Solution

possibilities : $\pm\left\{\frac{4}{1}\right\} = \pm\{1,\ 2,\ 4\}$

$$\begin{array}{r|rrrrr}
2 & 1 & -2 & -5 & 8 & 4 \\
 & & 2 & 0 & -10 & -4 \\
\hline
-2 & 1 & 0 & -5 & -2 & 0 \\
 & & -2 & 4 & 2 & \\
\hline
 & 1 & -2 & -1 & 0 &
\end{array}$$

$\rightarrow x^3 - 5x - 2 = 0 \rightarrow \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$\rightarrow x^2 - 2x - 1 = 0$

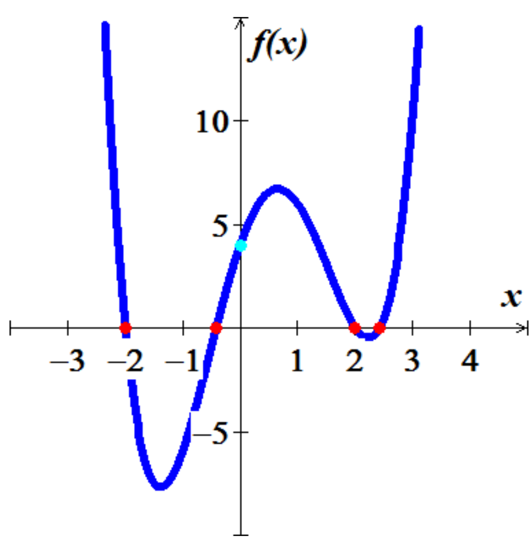
$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$\boxed{x = -2,\ 2,\ 1 \pm \sqrt{2}}$$

	-2		$1-\sqrt{2}$		2		$1+\sqrt{2}$	
$+$		$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup (1-\sqrt{2},\ 2) \cup (1+\sqrt{2},\ \infty)}$

$f(x) < 0 \quad \boxed{(-2,\ 1-\sqrt{2}) \cup (2,\ 1+\sqrt{2})}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 17x^3 + 4x^2 + 35x - 24$$

Solution

possibilities : $\pm\left\{\frac{24}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{c|ccccc}
1 & 2 & -17 & 4 & 35 & -24 \\
 & & 2 & -15 & -11 & 24 \\
\hline
1 & 2 & -15 & -11 & 24 & 0 \\
 & & 2 & -13 & 24 & \\
\hline
 & 2 & -13 & -24 & 0 &
\end{array}$$

$\to\ 2x^3 - 15x^2 - 11x + 24 = 0\ \to\ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$\to\ 2x^2 - 13x - 24 = 0$

$$x = \frac{13 \pm \sqrt{169 + 192}}{4}$$

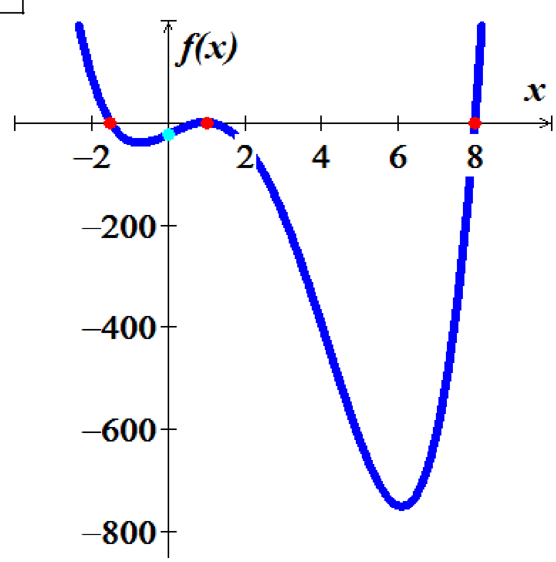
$$= \begin{cases} \frac{13 - 19}{4} = -\frac{3}{2} \\ \frac{13 + 19}{4} = 8 \end{cases}$$

$$\boxed{x = -\frac{3}{2},\ 1,\ 1,\ 8}$$

	$-\frac{3}{2}$		1		8	
$+$		$-$		$-$		$+$

$f(x) > 0$ $\quad \boxed{\left(-\infty,\ -\frac{3}{2}\right) \cup (8,\ \infty)}$

$f(x) < 0$ $\quad \boxed{\left(-\frac{3}{2},\ 1\right) \cup (1,\ 8)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 + x^3 - 3x^2 - 5x - 2$$

Solution

$possibilities: \ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$$\begin{array}{r|rrrrr}
-1 & 1 & 1 & -3 & -5 & -2 \\
 & & -1 & 0 & 3 & 2 \\
\hline
-1 & 1 & 0 & -3 & -2 & 0 \\
 & & -1 & 1 & 2 & \\
\hline
 & 1 & -1 & -2 & 0 &
\end{array}$$

$\to x^3 - 3x - 2 = 0 \to \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$\to x^2 - x - 2 = 0$

$$x = \frac{1 \pm \sqrt{1+8}}{2}$$

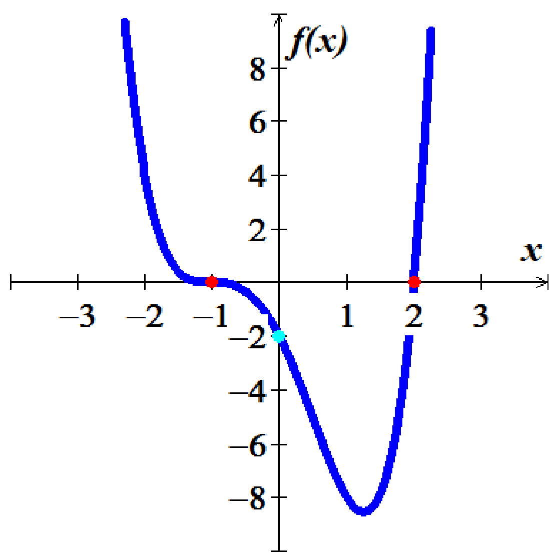
$$= \begin{cases} \frac{1-3}{2} = -1 \\ \frac{1+3}{2} = 2 \end{cases}$$

$\underline{x = -1,\ -1,\ -1,\ 2}$

	-1		2	
$+$		$-$		$+$

$f(x) > 0 \quad \boxed{(-\infty,\ -1) \cup (2,\ \infty)}$

$f(x) < 0 \quad \boxed{(-2,\ 2)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 - 17x^3 - 11x^2 + 42x$$

Solution

$$x\left(6x^3 - 17x^2 - 11x + 42\right) = 0$$

$$x = 0 \quad 6x^3 - 17x^2 - 11x + 42 = 0$$

possibilities : $\pm\left\{\frac{42}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ 7,\ 14,\ 21,\ 42,\ \frac{1}{2},\ \frac{3}{2},\ \frac{7}{2},\ \frac{21}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{7}{3},\ \frac{14}{3},\ \frac{1}{6},\ \frac{7}{6},\ \frac{21}{6}\right\}$

$$\begin{array}{r|rrrr} 2 & 6 & -17 & -11 & 42 \\ & & 12 & -10 & -42 \\ \hline & 6 & -5 & -21 & \boxed{0} \end{array} \to 6x^2 - 5x - 21 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 504}}{12}$$

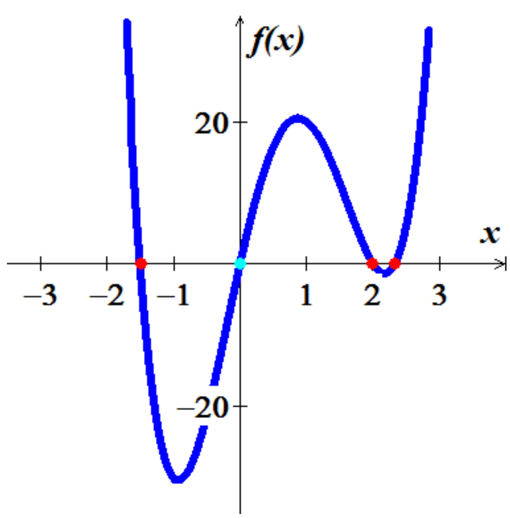
$$= \begin{cases} \frac{5-23}{12} = -\frac{3}{2} \\ \frac{5+23}{12} = \frac{7}{3} \end{cases}$$

$$x = -\frac{3}{2},\ 0,\ 2,\ \frac{7}{3}$$

$-\frac{3}{2}$	0	2	$\frac{7}{3}$	
+	−	+	−	+

$f(x) > 0 \quad \left(-\infty,\ -\frac{3}{2}\right) \cup (0,\ 2) \cup \left(\frac{7}{3},\ \infty\right)$

$f(x) < 0 \quad \left(-\frac{3}{2},\ 0\right) \cup \left(2,\ \frac{7}{3}\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 5x^2 - 2x$$

Solution

$$x\left(x^3 - 5x - 2\right) = 0$$

$$x = 0 \quad x^3 - 5x - 2 = 0$$

possibilities : $\pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -5 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & \boxed{0} \end{array} \rightarrow x^2 - 2x - 1 = 0$$

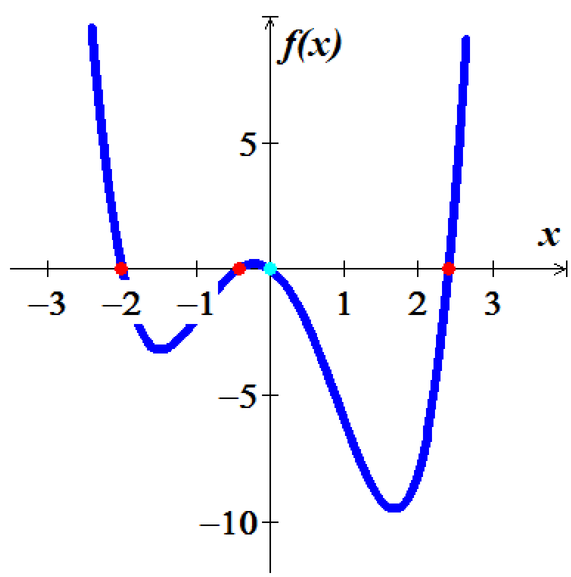
$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$\underline{x = -2,\ 0,\ 1 \pm \sqrt{2}}$$

	-2		$1-\sqrt{2}$		2		$1+\sqrt{2}$	
+		−		+		−		+

$f(x) > 0$ $\quad \underline{(-\infty,\ -2) \cup \left(1-\sqrt{2},\ 2\right) \cup \left(1+\sqrt{2},\ \infty\right)}$

$f(x) < 0$ $\quad \underline{\left(-2,\ 1-\sqrt{2}\right) \cup \left(2,\ 1+\sqrt{2}\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=3x^4-4x^3-11x^2+16x-4$$

Solution

possibilities : $\pm\left\{\frac{4}{3}\right\}=\pm\left\{1,\ 2,\ 4,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3}\right\}$

$$\begin{array}{r|rrrrr}
1 & 3 & -4 & -11 & 16 & -4 \\
 & & 3 & -1 & -12 & 4 \\
\hline
2 & 3 & -1 & -12 & 4 & 0 \\
 & & 6 & 10 & -4 & \\
\hline
 & 3 & 5 & -2 & 0 &
\end{array}$$

$\to 3x^3-x^2-12x+4=0 \to \pm\left\{\frac{4}{3}\right\}=\pm\left\{1,\ 2,\ 4,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3}\right\}$

$\to 3x^2+5x-2=0$

$$x=\frac{-5\pm\sqrt{25+24}}{6}$$

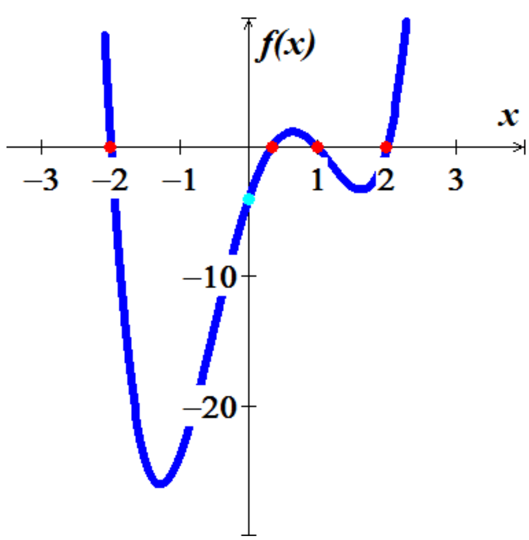
$$=\begin{cases}\frac{-5-7}{6}=-2\\ \frac{-5+7}{6}=\frac{1}{3}\end{cases}$$

$$\boxed{x=-2,\ \frac{1}{3},\ 1,\ 2}$$

$-\infty$ to -2	-2 to $\frac{1}{3}$	$\frac{1}{3}$ to 1	1 to 2	2 to ∞
+	−	+	−	+

$f(x)>0$ $\quad \boxed{(-\infty,\ -2)\cup\left(\frac{1}{3},\ 1\right)\cup(2,\ \infty)}$

$f(x)<0$ $\quad \boxed{\left(-2,\ \frac{1}{3}\right)\cup(1,\ 2)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 + 23x^3 + 19x^2 - 8x - 4$$

Solution

possibilities : $\pm\left\{\frac{4}{6}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{6},\ \frac{2}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrrr}
-2 & 6 & 23 & 19 & -8 & -4 \\
 & & -12 & -22 & 6 & 4 \\
\hline
-2 & 6 & 11 & -3 & -2 & 0 \\
 & & -12 & 2 & 2 & \\
\hline
 & 6 & -1 & -1 & 0 &
\end{array}$$

$\rightarrow 6x^3 + 11x^2 - 3x - 2 = 0 \rightarrow \pm\left\{\frac{2}{6}\right\} = \pm\left\{1,\ 2,\ \frac{1}{6},\ \frac{1}{3}\right\}$

$\rightarrow 6x^2 - x - 1 = 0$

$$x = \frac{1 \pm \sqrt{25}}{12}$$

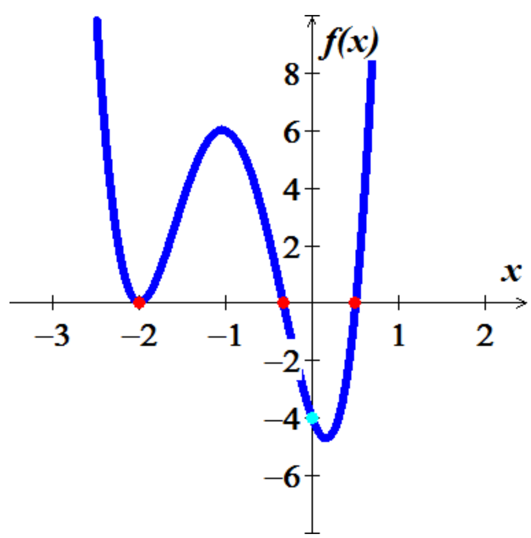
$$= \begin{cases} \frac{1-5}{12} = -\frac{1}{3} \\ \frac{1+5}{12} = \frac{1}{2} \end{cases}$$

$$\boxed{x = -2,\ -2,\ -\frac{1}{3},\ \frac{1}{2}}$$

	-2		$-\frac{1}{3}$		$\frac{1}{2}$	
+		+		−		+

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup \left(-2,\ -\frac{1}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-\frac{1}{3},\ \frac{1}{2}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 4x^4 - 12x^3 + 3x^2 + 12x - 7$$

Solution

possibilities : $\pm\left\{\frac{7}{4}\right\} = \pm\left\{1,\ 7,\ \frac{1}{2},\ \frac{7}{2},\ \frac{1}{4},\ \frac{7}{4}\right\}$

$$\begin{array}{r|rrrrr}
1 & 4 & -12 & 3 & 12 & -7 \\
 & & 4 & -8 & -5 & 7 \\
\hline
-1 & 4 & -8 & -5 & 7 & 0 \\
 & & -4 & 12 & -7 & \\
\hline
 & 4 & -12 & 7 & 0 &
\end{array}$$

$\to 4x^3 - 8x^2 - 5x + 7 = 0 \to \pm\left\{\frac{7}{4}\right\} = \pm\left\{1,\ 7,\ \frac{1}{2},\ \frac{7}{2},\ \frac{1}{4},\ \frac{7}{4}\right\}$

$\to 4x^2 - 12x + 7 = 0$

$$x = \frac{12 \pm \sqrt{144 - 112}}{8}$$

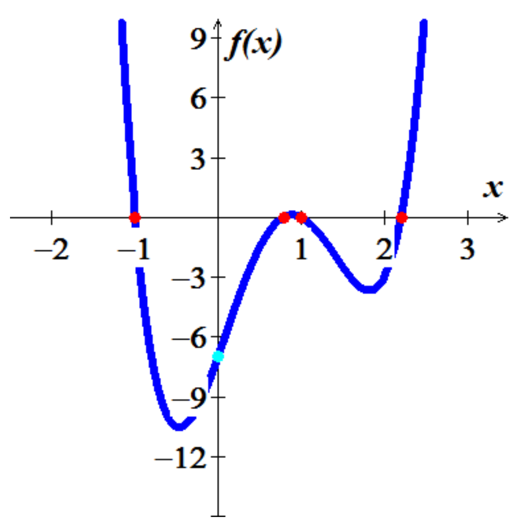
$$= \frac{12 \pm 4\sqrt{2}}{8}$$

$$x = -1,\ 1,\ \frac{3 \pm \sqrt{2}}{2}$$

	-1	$\frac{3-\sqrt{2}}{2}$	1	$\frac{3+\sqrt{2}}{2}$	
+		−	+	−	+

$f(x) > 0 \quad (-\infty,\ -1) \cup \left(\frac{3-\sqrt{2}}{2},\ 1\right) \cup \left(\frac{3+\sqrt{2}}{2},\ \infty\right)$

$f(x) < 0 \quad \left(-1,\ \frac{3-\sqrt{2}}{2}\right) \cup \left(1,\ \frac{3+\sqrt{2}}{2}\right)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=2x^4-9x^3-2x^2+27x-12$$

Solution

$$possibilities: \ \pm\left\{\frac{12}{2}\right\}=\pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 12,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\begin{array}{c|ccccc}
4 & 2 & -9 & -2 & 27 & -12 \\
 & & 8 & -4 & -24 & 12 \\
\hline
\frac{1}{2} & 2 & -1 & -6 & 3 & 0 \\
 & & 1 & 0 & -3 & \\
\hline
 & 2 & 0 & -6 & 0 &
\end{array}$$

$\rightarrow 2x^3-x^2-6x+3=0 \rightarrow \pm\left\{\frac{3}{2}\right\}=\pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

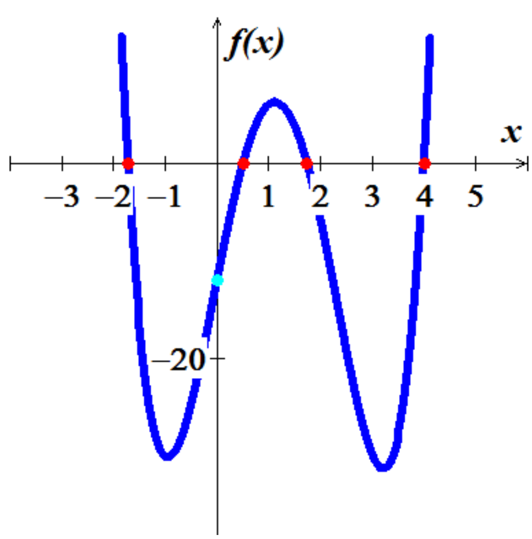
$\rightarrow 2x^2-6=0$

$$\boxed{x=\frac{1}{2},\ 4,\ \pm\sqrt{3}}$$

	$-\sqrt{3}$	$\frac{1}{2}$	$\sqrt{3}$	4	
+	–	+	–	+	

$f(x)>0 \quad \boxed{\left(-\infty,\ -\sqrt{3}\right)\cup\left(\frac{1}{2},\ \sqrt{3}\right)\cup(4,\ \infty)}$

$f(x)<0 \quad \boxed{\left(-\sqrt{3},\ \frac{1}{2}\right)\cup\left(\sqrt{3},\ 4\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 19x^3 + 51x^2 - 31x + 5$$

Solution

possibilities : $\pm\left\{\frac{5}{2}\right\} = \pm\left\{1,\ 5,\ \frac{1}{2},\ \frac{5}{2}\right\}$

$$\begin{array}{r|rrrrr}
5 & 2 & -19 & 51 & -31 & 5 \\
 & & 10 & -45 & 30 & -5 \\
\hline
\frac{1}{2} & 2 & -9 & 6 & -1 & 0 \\
 & & 1 & -4 & 1 & \\
\hline
 & 2 & -8 & 2 & 0 &
\end{array}$$

$\rightarrow 2x^3 - 9x^2 + 6x - 1 = 0 \ \rightarrow \ \pm\left\{\frac{1}{2}\right\}$

$\rightarrow 2x^2 - 8x + 2 = 0$

$$x = \frac{8 \pm \sqrt{64 - 16}}{4}$$

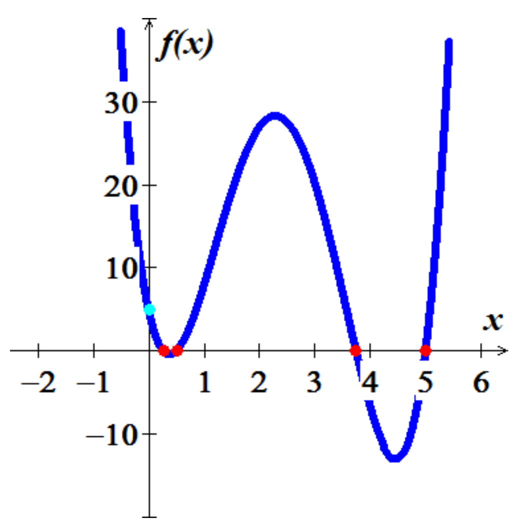
$$= \frac{8 \pm 4\sqrt{3}}{4}$$

$$\boxed{x = \frac{1}{2},\ 5,\ 2 \pm \sqrt{3}}$$

	$2-\sqrt{3}$		$\frac{1}{2}$		$2+\sqrt{3}$		5	
$+$		$-$		$+$		$-$		$+$

$f(x) > 0$ $\quad \boxed{\left(-\infty,\ 2-\sqrt{3}\right) \cup \left(\frac{1}{2},\ 2+\sqrt{3}\right) \cup (5,\ \infty)}$

$f(x) < 0$ $\quad \boxed{\left(2-\sqrt{3},\ \frac{1}{2}\right) \cup \left(2+\sqrt{3},\ 5\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 4x^4 - 35x^3 + 71x^2 - 4x - 6$$

Solution

possibilities : $\pm\left\{\frac{6}{4}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{4},\ \frac{3}{4}\right\}$

$$\begin{array}{r|rrrrr}
3 & 4 & -35 & 71 & -4 & -6 \\
 & & 12 & -69 & 6 & 6 \\ \hline
-\frac{1}{4} & 4 & -23 & 2 & 2 & 0 \\
 & & -1 & 6 & -2 & \\ \hline
 & 4 & -24 & 8 & 0 &
\end{array}$$

$\rightarrow 4x^3 - 23x^2 + 2x + 2 = 0 \ \rightarrow\ \pm\left\{\frac{2}{4}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{4}\right\}$

$\rightarrow 4x^2 - 24x + 8 = 0$

$$x^2 - 6x + 2 = 0$$

$$x = \frac{6 \pm \sqrt{36-8}}{2}$$

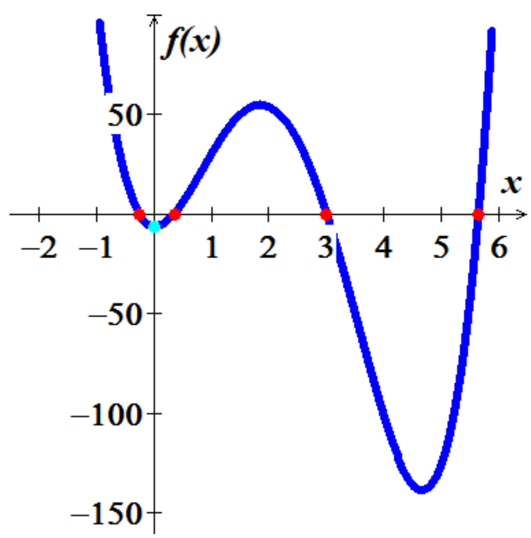
$$= \frac{6 \pm 2\sqrt{7}}{4}$$

$$\boxed{x = -\frac{1}{4},\ 3,\ 3 \pm \sqrt{7}}$$

$-\frac{1}{4}$	$3-\sqrt{7}$	3	$3+\sqrt{7}$	
+	−	+	−	+

$f(x) > 0$ $\quad \boxed{\left(-\infty,\ -\frac{1}{4}\right) \cup \left(3-\sqrt{7},\ 3\right) \cup \left(3+\sqrt{7},\ \infty\right)}$

$f(x) < 0$ $\quad \boxed{\left(-\frac{1}{4},\ 3-\sqrt{7}\right) \cup \left(3,\ 3+\sqrt{7}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$$

Solution

possibilities : $\pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & 3 & -4 & -3 & 2 \\
 & & 2 & 5 & 1 & -2 \\
\hline
-1 & 2 & 5 & 1 & -2 & 0 \\
 & & -2 & -3 & 2 & \\
\hline
 & 2 & 3 & -2 & 0 &
\end{array}$$

$\rightarrow 2x^3 - 23x^2 + 2x - 2 = 0 \ \rightarrow\ \pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$\rightarrow 2x^2 + 3x - 2 = 0$

$$x = \frac{-3 \pm \sqrt{9+16}}{4}$$

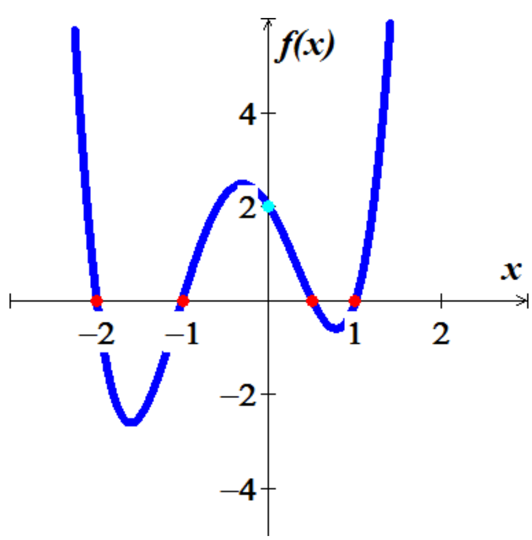
$$= \begin{cases} \frac{-3-5}{4} = -2 \\ \frac{-3+5}{4} = \frac{1}{2} \end{cases}$$

$$\boxed{x = -2,\ -1,\ \frac{1}{2},\ 1}$$

	-2		-1		$\frac{1}{2}$		1	
+		−		+		−		+

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup \left(-1,\ \frac{1}{2}\right) \cup (1,\ \infty)}$

$f(x) < 0 \quad \boxed{(-2,\ -1) \cup \left(\frac{1}{2},\ 1\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 + 3x^3 - 30x^2 - 6x + 56$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{56}{1}\right\} = \pm\{1,\ 2,\ 4,\ 7,\ 8,\ 14,\ 28,\ 56\}$

$$\begin{array}{r|rrrrr}
4 & 1 & 3 & -3 & -6 & 56 \\
 & & 4 & 28 & -8 & -56 \\
\hline
-7 & 1 & 7 & -2 & -14 & 0 \\
 & & -7 & 0 & 14 & \\
\hline
 & 1 & 0 & -2 & &
\end{array}$$

$\to x^3 + 7x^2 - 2x - 14 = 0 \Rightarrow \frac{c}{d} = \pm\left\{\frac{14}{1}\right\} = \pm\{1,\ 2,\ 7,\ 14\}$

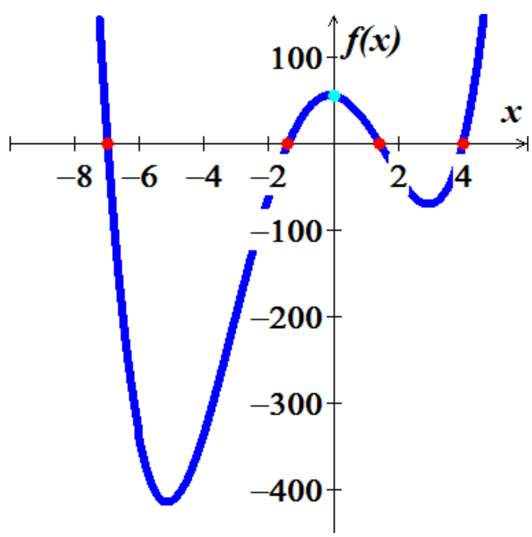
$\to x^2 - 2 = 0 \Rightarrow \underline{x = \pm\sqrt{2}}$

$\underline{x = 4,\ -7,\ \pm\sqrt{2}}$

	-7		$-\sqrt{2}$		$\sqrt{2}$		4	
$+$		$-$		$+$		$-$		$+$

$f(x) > 0 \quad \underline{(-\infty,\ -7) \cup \left(-\sqrt{2},\ \sqrt{2}\right) \cup (4,\ \infty)}$

$f(x) < 0 \quad \underline{\left(-7,\ -\sqrt{2}\right) \cup \left(\sqrt{2},\ 4\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=3x^5-10x^4-6x^3+24x^2+11x-6$$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{6}{3}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrrrr}
-1 & 3 & -10 & -6 & 24 & 11 & -6 \\
 & & -3 & 13 & -7 & -17 & 6 \\
\hline
-1 & 3 & -13 & 7 & 17 & -6 & 0 \\
 & & -3 & 16 & -23 & 6 & \\
\hline
2 & 3 & -16 & 23 & -6 & 0 & \\
 & & 6 & 20 & 6 & & \\
\hline
 & 3 & -10 & 3 & 0 & &
\end{array}$$

$x^4-13x^3+7x^2+17x-6=0\to\pm\left\{\frac{6}{3}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$

$3x^3-16x^2+26x-6=0\to\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$

$3x^2-10x+3=0$

$$x=\frac{10\pm\sqrt{100-36}}{6}$$

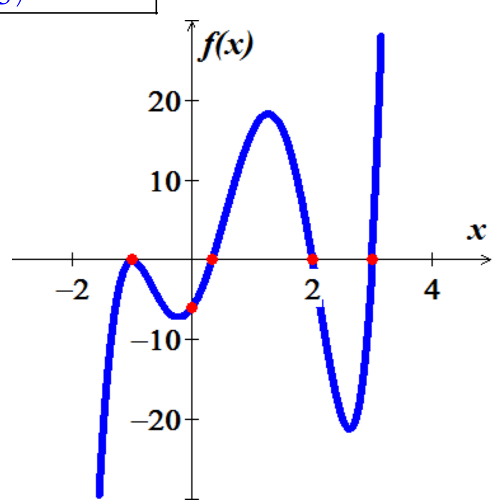
$$=\begin{cases}\dfrac{10-8}{6}=\dfrac{1}{3}\\[2mm] \dfrac{10+8}{4}=3\end{cases}$$

$$\boxed{x=-1,\ -1,\ \frac{1}{3},\ 2,\ 3}$$

	-1	$\frac{1}{3}$	2	3	
$-$	$-$	$+$	$-$	$+$	

$f(x)>0$ $\boxed{\left(\frac{1}{3},\ 2\right)\cup(3,\ \infty)}$

$f(x)<0$ $\boxed{(-\infty,\ -1)\cup\left(-1,\ \frac{1}{3}\right)\cup(2,\ 3)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^5 + 19x^4 + x^3 - 6x^2$$

Solution

$$x^2\left(6x^3 + 19x^2 + x - 6\right) = 0 \quad \rightarrow \quad \underline{x = 0,\ 0}$$

$$6x^3 + 19x^2 + x - 6 = 0$$

possibilities for $\frac{c}{d} : \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrr} -3 & 6 & 19 & 1 & -6 \\ & & -18 & -3 & 6 \\ \hline & 6 & 1 & -2 & \boxed{0} \end{array} \qquad 6x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{12}$$

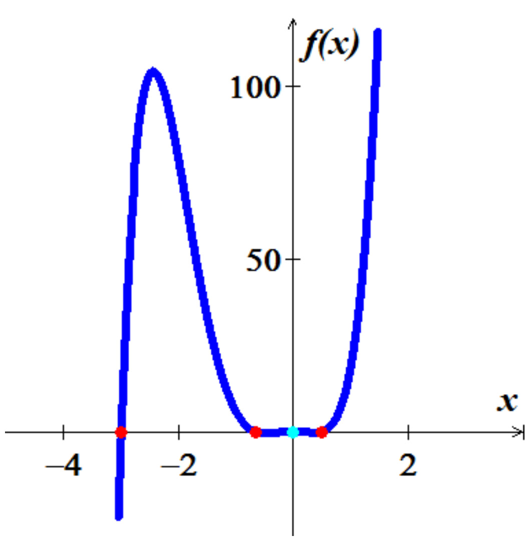
$$= \begin{cases} \dfrac{-1-7}{12} = -\dfrac{2}{3} \\ \dfrac{-1+7}{12} = \dfrac{1}{2} \end{cases}$$

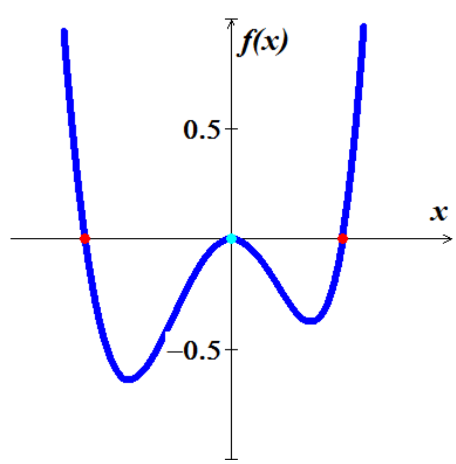
$$\underline{x = 0,\ 0,\ -\tfrac{2}{3},\ -3,\ \tfrac{1}{2}}$$

	-3		$-\frac{2}{3}$		0		$\frac{1}{2}$	
$-$		$+$		$-$		$-$		$+$

$f(x) > 0 \quad \underline{\left(-3,\ -\tfrac{2}{3}\right) \cup \left(\tfrac{1}{2},\ \infty\right)}$

$f(x) < 0 \quad \underline{\left(-\infty,\ -3\right) \cup \left(-\tfrac{2}{3},\ 0\right) \cup \left(0,\ \tfrac{1}{2}\right)}$





Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^5+5x^4+10x^3+10x^2+5x+1$$

Solution

$$x^5+5x^4+10x^3+10x^2+5x+1=(x+1)^5=0$$

possibilities for $\frac{c}{d}:\pm\{1\}$

$$\begin{array}{r|rrrrrr}
-1 & 1 & 5 & 10 & 10 & 5 & 1 \\
 & & -1 & -4 & -6 & -4 & -1 \\
\hline
-1 & 1 & 4 & 6 & 4 & 1 & 0 \\
 & & -1 & -3 & -3 & -1 & \\
\hline
-1 & 1 & 3 & 3 & 1 & 0 & \\
 & & -1 & -2 & -1 & & \\
\hline
 & 1 & 2 & 1 & 0 & &
\end{array}$$

$\to x^4+4x^3+6x^2+4x+1=0\to\pm\{1\}$

$\to x^3+3x^2+3x+1=0\to\pm\{1\}$

$\to x^2+2x+1=0$

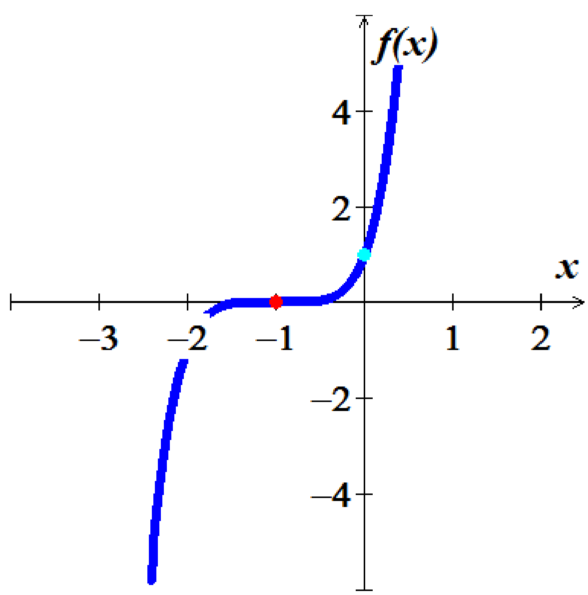
$$x^2+2x+1=(x+1)^2$$

$\underline{x=-1}$ $(\textit{multiplicity}\text{ of }5)$

	-1	
$-$		$+$

$f(x)>0$ $\quad (-1,\ \infty)$

$f(x)<0$ $\quad (-\infty,\ -1)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$f(x) = x^5 - x^4 - 7x^3 + 7x^2 + 12x - 12$

Solution

possibilities for $\frac{c}{d}: \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$
\begin{array}{r|rrrrrr}
1 & 1 & -1 & -7 & 7 & 12 & -12 \\
 & & 1 & 0 & -7 & 0 & 12 \\
\hline
2 & 1 & 0 & -7 & 0 & 12 & 0 \\
 & & 2 & 4 & -6 & -12 & \\
\hline
-2 & 1 & 2 & -3 & -6 & 0 & \\
 & & -2 & 0 & 6 & & \\
\hline
 & 1 & 0 & -3 & 0 & &
\end{array}
$$

$\to\ x^4 - 7x^2 - 12 = 0 \to \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$\to\ x^3 + 2x^2 - 3x - 6 = 0 \to \pm\{1,\ 2,\ 3,\ 6\}$

$\to\ x^2 - 3 = 0$

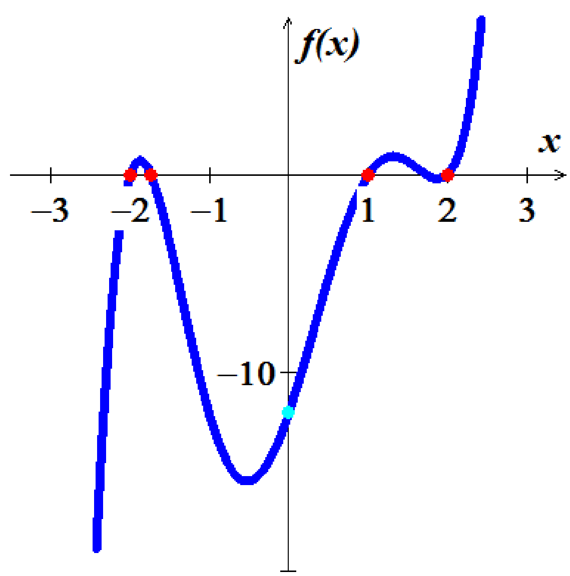
$x^2 = 3$

$\underline{x = -2,\ 1,\ 2,\ \pm\sqrt{3}}$

	-2		$-\sqrt{3}$		1		$\sqrt{3}$		2	
$-$		$+$		$-$		$+$		$-$		$+$

$f(x) > 0$ $\quad \underline{\left(-2,\ -\sqrt{3}\right) \cup \left(1,\ \sqrt{3}\right) \cup (2,\ \infty)}$

$f(x) < 0$ $\quad \underline{(-\infty,\ -2) \cup \left(-\sqrt{3},\ 1\right) \cup \left(\sqrt{3},\ 2\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^5-2x^3-8x$$

Solution

$x\left(x^4-2x^2-8\right)=0$

$\underline{x=0}$

$x^4-2x^2-8=0.$

$x^2=\dfrac{2\pm\sqrt{4+32}}{2}$

$=\begin{cases}\dfrac{2-6}{2}=-2\\ \dfrac{2+6}{2}=4\end{cases}$

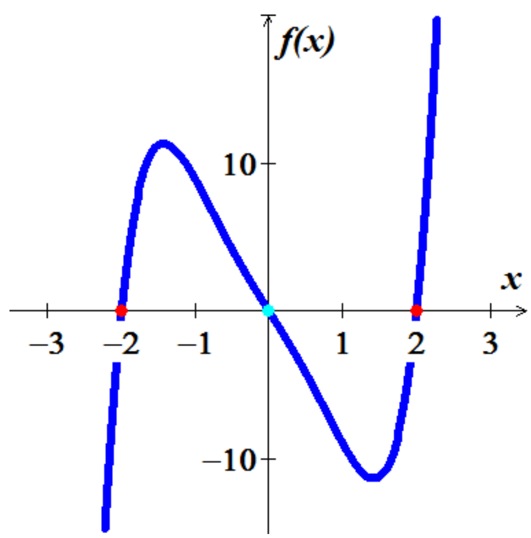
$\begin{cases}x^2=-2 & \rightarrow & x=\pm i\sqrt{2}\\ x^2=4 & \rightarrow & x=\pm 2\end{cases}$

$\underline{x=0,\ \pm 2,\ \pm i\sqrt{2}}$

	-2	0	2	
$-$	$+$	$-$	$+$	

$f(x)>0$ $\quad\underline{(-2,\ 0)\cup(2,\ \infty)}$

$f(x)<0$ $\quad\underline{(-\infty,\ -2)\cup(0,\ 2)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^6 - 10x^5 - 29x^4 + 34x^3 + 50x^2 - 24x - 24$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{3}\right\}$

$$= \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{8}{3}\right\}$$

1	3	-10	-29	34	50	-24	-24	
		3	-7	-36	-2	48	24	
-1	3	-7	-36	-2	48	24	0	$\to\ 3x^5 - 7x^4 - 36x^3 - 2x^2 + 48x + 24 = 0$
		-3	10	26	-24	-24		
-2	3	-10	-26	24	24	0		
		-6	32	-12	-24			$\to\ 3x^4 - 10x^3 - 26x^2 + 24x + 24 = 0$
$-\frac{2}{3}$	3	-16	6	12	0			$\to\ 3x^3 - 16x^2 + 12x - 12 = 0$
		-2	12	-12				
	3	-18	18	0				$\to\ 3x^2 - 18x + 18 = 0$

$x^2 - 6x + 6 = 0$

$$x = \frac{6 \pm \sqrt{36 - 24}}{2}$$

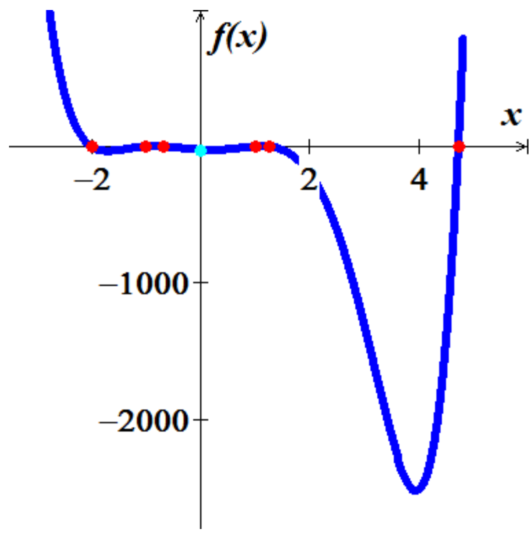
$$= \frac{6 \pm 2\sqrt{3}}{2}$$

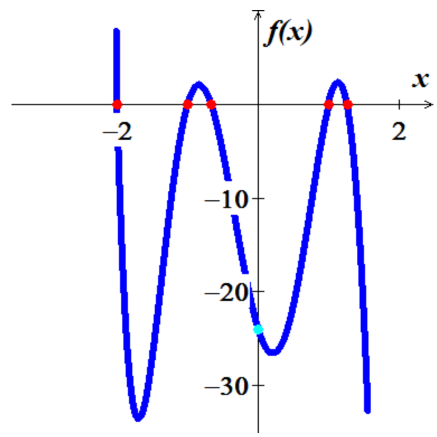
$$\boxed{x = -2,\ -1,\ 1,\ -\frac{2}{3},\ 3 \pm \sqrt{3}}$$

	-2		-1		$-\frac{2}{3}$		1		$3-\sqrt{3}$		$3+\sqrt{3}$	
+		−		+		−		+		−		+

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup \left(-1,\ -\frac{2}{3}\right) \cup \left(1,\ 3-\sqrt{3}\right) \cup \left(3+\sqrt{3},\ \infty\right)}$

$f(x) < 0 \quad \boxed{(-2,\ -1) \cup \left(-\frac{2}{3},\ 1\right) \cup \left(3-\sqrt{3},\ 3+\sqrt{3}\right)}$


f(x)
x
−2
2
4
−1000
−2000


f(x)
x
−2
2
−10
−20
−30