Solution Section 4.5 – Normal Distribution

Exercise

A manufacturing process produces light bulbs with life expectancies that are normally distributed with a mean of 500 hours and a standard deviation of 100 hours. What percentage of the light bulbs can be expected to last 500 to 750 hours?

Solution

$\mu = 500, \sigma = 100 \qquad \Rightarrow 500 \ \& \ 750$

$$z = \frac{x-\mu}{\sigma} = \frac{750-500}{100} = 2.5$$

$\Rightarrow$ **A = 49.38%**

Exercise

What is the probability of the light bulbs can be expected to last 400 to 500 hours?

Solution

$400 \to 500$

$$z = \frac{400-500}{100} = -1$$

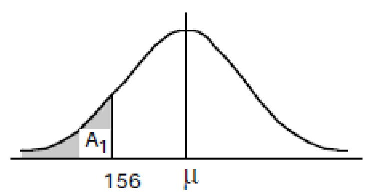
$\Rightarrow$ **A = .3413**

Exercise

The average lifetime for a car battery of a certain brand is 170 weeks, with a standard deviation of 10 weeks. If the company guarantees the battery for 3 years, what percentage of the batteries sold would be expected to be returned before the end of the warranty period? Assume a normal distribution

Solution

$\mu = 170, \sigma = 10; \quad x \le 3(52) = 156 \text{ weeks}$



$$z = \frac{156-170}{10} = -1.4$$

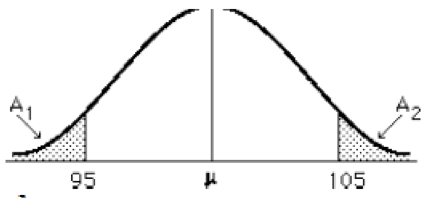
$$\begin{aligned} P(x \le 156) &= 0.5 - P(z = 1.4) \\ &= 0.5 - 0.4192 \\ &= 0.0808 \quad \text{or} \quad \boxed{8.08\%} \end{aligned}$$

Exercise

A manufacturing process produces a critical part of average length 100 millimeters, with a standard deviation of 2 millimeters. All parts deviating by more than 5 millimeters from the mean must be rejected. What percentage of the parts must be rejected, on the average? Assume a normal distribution.

Solution

$\mu = 100, \sigma = 2; \quad x = 105, x = 95$



$$z(x = 105) = \frac{105-100}{2} = 2.5 \quad \rightarrow A_1 = .4938$$

$$z(x = 95) = \frac{95-100}{2} = -2.5 \quad \rightarrow A_2 = .4938$$

$$\begin{aligned} Area &= 1 - \left(A_1 + A_2\right) \\ &= 1 - 2(.4938) \\ &= .0124 \end{aligned}$$

Percentage of the parts to be rejected: **1.24%**

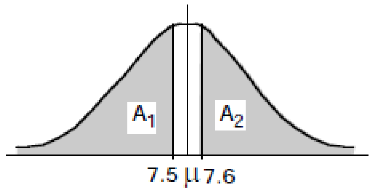
Exercise

An automated manufacturing process produces a component with an average width of 7.55 cm, with a standard deviation of 0.02 cm. All components deviating by more than 0.05 cm from the mean must be rejected. What percent of the parts must be rejected, on the average? Assume a normal distribution.

Solution

$\mu = 7.55, \ \sigma = 0.02;$

$x < \mu - 0.05, \ x > \mu + 0.05$



$P(x < \mu - 0.05) = P(x > \mu + 0.05)$

$$z(x = 7.6) = \frac{7.6-7.55}{0.02} = 2.5$$

$\rightarrow A_1 = .4938$

$$\begin{aligned} P(\text{parts rejected}) &= A_1 + A_2 \\ &= 2P(x > 7.6) \\ &= .5 - .4938 \\ &= .0062 \end{aligned}$$

$P = 2(.0062) = .0124$

Percentage of the parts to be rejected: **1.24%**

Exercise

A company claims that 60% of the households in a given community uses its product. A competitor surveys the community, using a random sample of 40 households, and finds only 15 households out of the 40 in the sample using the product. If the company's claim is correct, what is the probability of 15 or fewer households using the product in a sample of 40? Conclusion? Approximate a binomial distribution with a normal distribution.

Solution

$n = 40,\ p = 0.6, q = 0.4$

$\mu = np = 40(.6) = 24$

$\sigma = \sqrt{npq} = \sqrt{40(.6)(.4)} \approx 3.1$

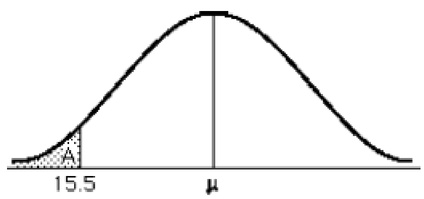
$z(x = 15.5) = \frac{15.5 - 24}{3.1} = -2.74$

$\to A = .4969$

$P(x \le 15) = .5 - A$

$= .5 - .4969$

$= 0.0031$



This is a rare event has occurred, e.g. the sample was not random, or the company's claim is false

Exercise

A union representative claims 60% of the union membership will vote in favor of a particular settlement. A random sample of 100 members is polled, and out of these, 47 favor the settlement. What is the approximate probability of 47 or fewer in a sample of 100 favoring the settlement when 60% of all the membership favor the settlement? Conclusion? Approximate a binomial distribution with a normal distribution.

Solution

$n = 100,\ p = 0.6,\ q = 0.4$

$\mu = np = 100(.6) = 60$

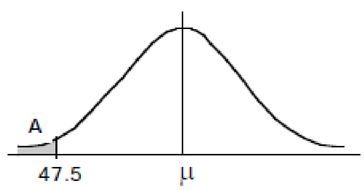
$\sigma = \sqrt{npq} = \sqrt{100(.6)(.4)} \approx 4.9$

$z(x = 47.5) = \frac{47.5 - 60}{4.9} = -2.55 \to A = .4946$

$P(x \le 47.5) = .5 - A$

$= .5 - .4946$

$= 0.0054$



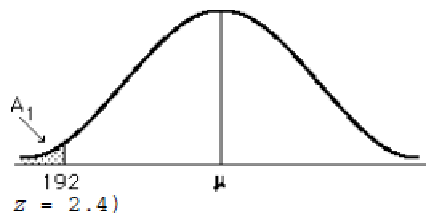
Conclusion: Either a rare event has happened or the claim of 60% is false

Exercise

The average healing time of a certain type of incision is 240 hours, with standard deviation of 20 hours. What percentage of the people having this incision would heal in 8 days or less? Assume a normal distribution.

Solution

$$\mu = 240, \sigma = 20; \quad x = 8\ days = 192\ hrs.$$



$$z(x=192) = \frac{192-240}{20} = -2.4$$

$$\rightarrow A = .4918$$

$$P(x \le 192) = .5 - .4918 \ \underline{= .0082}$$

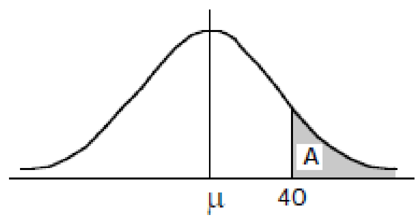
The percentage of the people having this incision would heal is 0.82%

Exercise

The average height of a hay crop is 38 inches, with a standard deviation of 1.5 inches. What percentage of the crop will be 40 inches or more? Assume a normal distribution

Solution

$$\mu = 38, \sigma = 1.5$$



$$z(x=40) = \frac{40-38}{1.5} = 1.33$$

$$\rightarrow A = .4082$$

$$P(x \ge 40) = .5 - .4082 = .0918 \ \ or \ \ \boxed{9.18\%}$$

Exercise

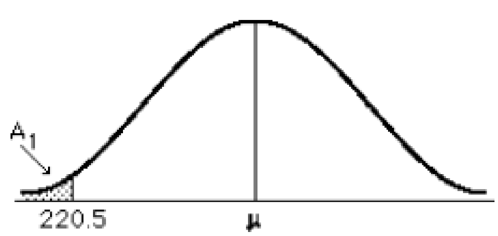
In a family with 2 children, the probability that both children are girls is approximately .25. In a random sample of 1,000 families with 2 children, what is the approximate probability that 220 or fewer will have 2 girls? Approximate a binomial distribution with a normal distribution.

Solution

$$n = 1000, \ p = 0.25, \ q = 0.75$$

$$\mu = np = 1000(.25) = 250$$

$$\sigma = \sqrt{npq} = \sqrt{1000(.25)(.75)} \approx 13.69$$



$$z(x=220.5) = \frac{220.5-250}{13.69} = -2.15 \ \rightarrow A = .4842$$

$$P(x \le 220.5) = .5 - .4842 \ \underline{= .0158}$$

Exercise

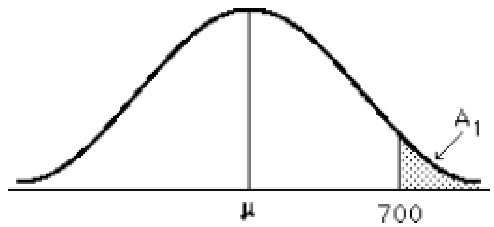
Aptitude Tests are scaled so that the mean score is 500 and the standard deviation is 100. What percentage of the students taking this test should score 700 or more? Assume a normal distribution

Solution

$\mu = 500, \sigma = 100$

$z(x = 500) = \dfrac{700-500}{100} = 2 \ \rightarrow A = .4722$

$P(x \geq 700) = .5 - .4722 = .0228 \ \ or \ \boxed{2.28\%}$



Exercise

Candidate Harkins claims a private poll shows that she will receive 52% of the vote for governor. Her opponent, Mankey, secures the services of another pollster, who finds that 470 out of a random sample of 1,000 registered voters favor Harkins. If Harkin's claim is correct, what is the probability that only 470 or fewer will favor her in a random sample of 1,000? Conclusion? Approximate a binomial distribution with a normal distribution.

Solution

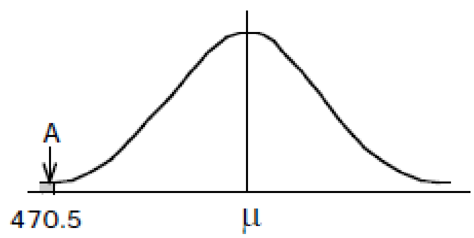
$n = 1000, \ p = 0.52, \ q = 0.48$

$\mu = np = 1000(.52) = 520$

$\sigma = \sqrt{npq} = \sqrt{1000(.52)(.48)} \approx 15.8$

$z(x = 470.5) = \dfrac{470.5-520}{15.8} = -3.13 \ \rightarrow A = .4991$

$P(x \leq 470.5) = .5 - .4991 = 0.0009$



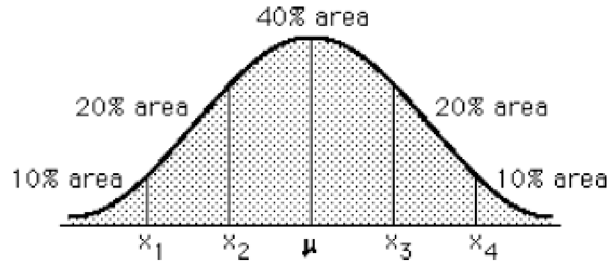
Conclusion: Either a rare event has happened or Harkin's claim is false

Exercise

An instructor grades on a curve by assuming the grades on a test are normally distributed. If the average grade is 70 and the standard deviation is 8, find the test scores for each grade interval if the instructor wishes to assign grades as follow: 10% A's, 20% B's, 40% C's, 20% D's, and 10% F's.

Solution

$\mu = 70, \sigma = 8$



Area between μ *and* $x_3 = 0.2 \to z = 0.52$

$$z\left(x = x_3\right) = \frac{x_3 - 70}{8} = 0.52$$

$8(.52) = x_3 - 70$

$\Rightarrow x_3 = 70 + 4.16 = 74.16$

$$z\left(x = x_2\right) = \frac{70 - x_2}{8} = 0.52$$

$4.16 = 70 - x_2$

$\Rightarrow x_2 = 70 - 4.16 = 65.84$

Area between μ *and* $x_4 = 0.4 \to z = 1.28$

$$\frac{x_4 - 70}{8} = 1.28$$

$8(1.28) = x_4 - 70$

$\Rightarrow x_4 = 70 + 10.24 = 80.24$

$$\frac{70 - x_1}{8} = 1.28$$

$10.24 = 70 - x_1$

$\Rightarrow x_1 = 70 - 10.2 = 59.8$

***A**: 80.2 or greater*

***B**: 74.2 – 80.2*

***C**: 65.8 – 74.2*

***D**: 59.8 – 65.8*

***F**: < 59.8*

Exercise

At the discount Market, the average weekly grocery bill is \$74.50, with a standard deviation of \$24.30. What are the largest and smallest amounts spent by the middle 50% of this market's customers?

Solution

Given: $\begin{cases} \mu = 74.5 \\ \sigma = 24.30 \end{cases}$

Middle 50% cutoffs at 25% below the mean $z = -0.67$

at 75% below the mean $z = 0.67$

$$\frac{x_1 - 74.5}{24.3} = -0.67 \qquad\qquad \frac{x_1 - 74.5}{24.3} = 0.67$$

$$x_1 - 74.5 = -16.281 \qquad\qquad x_1 - 74.5 = 16.281$$

$$x_1 \approx 58.22 \qquad\qquad x_2 \approx 90.78$$

The middle 50% of customers spend between \$58.22 and \$90.78

Exercise

A certain type of light bulb has an average life of 500 hours, with a standard deviation of 100 hours. The length of life of the bulb can be closely approximated by a normal curve. An amusement park buys and installs 10,000 such bulbs. Find the total number that can be expected to last for each period of time.

Solution

At least 500 hours

$$z = \frac{500-500}{100} = 0$$

$$P(x \ge 500) = 1 - P(z < 0) = 1 - 0.5 = 0.5$$

0.5 (10,000) = 5000 bulbs

Less than 500 hours

$$z = \frac{500-500}{100} = 0$$

$$P(z < 0) = 0.5$$

5000 bulbs

Between 680 and 780 hours

For $x = 680 \rightarrow z = \frac{680-500}{100} = 1.8$

For $x = 780 \rightarrow z = \frac{780-500}{100} = 2.8$

$$P(680 \le x \le 780) = P(1.8 \le z \le 2.8)$$

$$= P(z \le 2.8) - P(z \le 1.8)$$

$$= 0.9974 - 0.9641$$
$$= 0.0333$$

0.0333 (10,000) = 333 bulbs

Between 350 and 550 hours

For $x = 350 \rightarrow z = \frac{350-500}{100} = -1.5$

For $x = 550 \rightarrow z = \frac{550-500}{100} = 0.5$

$$P(350 \le x \le 550) = P(-1.5 \le z \le 0.5)$$
$$= P(z \le 0.5) - P(z \le -1.5)$$
$$= 0.6915 - 0.0668$$
$$= 0.6247$$

0.6247 (10,000) = 6247 bulbs

Less than 770 hours

For $x = 770 \rightarrow z = \frac{770-500}{100} = 2.7$

$$P(x < 770) = P(z < 2.7) = 0.9965$$

9965 bulbs

More than 440 hours

For $x = 440 \rightarrow z = \frac{440-500}{100} = -0.6$

$$P(x > 440) = P(z > -0.6)$$
$$= 1 - 0.2743$$
$$= 0.7257$$

7257 bulbs

Find the shortest and longest lengths of life for the middle 60% of the bulbs.

$P(z < z_1) = 0.2000$ (20%) $\Rightarrow z_1 = -0.84$

$P(z < z_2) = 0.8000$ (80%) $\Rightarrow z_2 = 0.84$

$\frac{x_1 - 500}{100} = -0.84$	$\frac{x_2 - 500}{100} = 0.84$
$x_1 - 500 = -84$	$x_2 - 500 = 84$
$x_1 = 416$	$x_2 = 584$

For the middle 60% of the bulbs: 416 hrs. and 584 hrs.

Exercise

A machine that fills quart milk cartons is set up to average 32.2 oz. per carton, with a standard deviation of 1.2 oz. What is the probability that a filled carton will contain less than 32 oz. of milk?

Solution

$$\text{Given: } \begin{cases} \mu = 32.2 \\ \sigma = 1.2 \end{cases}$$

$$x = 32 \rightarrow z = \frac{32-32.2}{1.2} = -0.17$$

$$P(x < 32) \underline{= 0.4325}$$

Exercise

A machine produces bolts with an average diameter of 0.25 in. and a standard deviation of 0.02 in. What is the probability that a bolt will be produced with a diameter greater than 0.3 in.?

Solution

$$\text{Given: } \begin{cases} \mu = 0.25 \\ \sigma = 0.02 \end{cases}$$

$$x = 0.3 \rightarrow z = \frac{0.3-0.25}{0.02} = 2.5$$

$$\begin{aligned} P(x > 0.3) &= 1 - P(x < 0.3) \\ &= 1 - 0.9938 \\ &\underline{= 0.0062} \end{aligned}$$