Ex $y'' + 2y' + 2y = \cos 2t$ $\begin{cases} y(0) = 0 \\ y'(0) = 1 \end{cases}$

$$\mathcal{L}\{y'' + 2y' + 2y\} = \mathcal{L}\{\cos 2t\}$$

$$\underbrace{s^2 Y(s) - s\,y(0) - y'(0)}_{y''} + 2\underbrace{[sY(s) - y(0)]}_{y'} + 2Y(s) = \frac{s}{s^2+4}$$

$$(s^2 + 2s + 2)Y(s) = \frac{s}{s^2+4} + 1$$

$$Y(s) = \frac{s^2 + s + 4}{(s^2+4)(s^2+2s+2)}$$

$$= \frac{As + B}{s^2+4} + \frac{Cs + D}{s^2+2s+2}$$

$$s^2 + s + 4 = (As + B)(s^2 + 2s + 2) + (Cs + D)(s^2 + 4)$$

s^3 $A + C = 0 \rightarrow C = -A$

s^2 $2A + B + D = 1$

s^1 $2A + 2B + 4C = 1$

s^0 $2B + 4D = 4 \rightarrow B = 2 - 2D$

$$\begin{cases} 2A + 2 - 2D + D = 1 \\ 2A + 4 - 4D - 4A = 1 \end{cases} \Rightarrow \begin{cases} 2A - D = -1 \\ -2A - 4D = -3 \end{cases}$$

$$-5D = -4$$

Ex cont $D = \frac{4}{5}$, $B = 2 - \frac{8}{5} = \frac{2}{5}$

$2A = \frac{4}{5} - 1 = -\frac{1}{5} \Rightarrow A = -\frac{1}{10}$

$C = \frac{1}{10}$

$$Y(s) = \frac{-\frac{1}{10}s + \frac{2}{5}}{s^2+4} + \frac{\frac{1}{10}s + \frac{4}{5}}{s^2+2s+2}$$

$$= \frac{-1}{10}\frac{s-4}{s^2+4} + \frac{1}{10}\frac{s+8}{s^2+2s+2}$$

$$= -\frac{1}{10}\frac{s}{s^2+4} + \frac{1}{5}\frac{2}{s^2+4}$$

$$+ \frac{1}{10}\left[\frac{s+1}{(s+1)^2+1} + \frac{7}{(s+1)^2+1}\right]$$

$$\mathcal{L}^{-1}\{Y(s)\} = -\frac{1}{10}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$+ \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} + \frac{7}{10}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$

$$y(t) = -\frac{1}{10}\cos 2t + \frac{1}{5}\sin 2t$$

$$+ \frac{1}{10}e^{-t}\cos t + \frac{7}{10}e^{-t}\sin t$$

5. $y^{(4)} - y = 0$ $\quad y(0) = 0,\ y'(0) = 1,\ y''(0) = 0,\ y'''(0) = 0$

$$\mathcal{L}\{y^{(4)} - y\} = 0$$

$$s^4 Y(s) - s^3 y(0) - s^2 y'(0) - s\, y''(0) - y'''(0) - Y(s) = 0$$

$$(s^4 - 1)\, Y(s) = s^2$$

$$Y(s) = \frac{s^2}{(s-1)(s+1)(s^2+1)}$$

$$= \frac{A}{s-1} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}$$

$$s^2 = A(s+1)(s^2+1) + B(s-1)(s^2+1) + (Cs+D)(s^2-1)$$

s^3: $A + B + C = 0$ ①

s^2: $A - B + D = 1$ ②

s^1: $A + B - C = 0$

s^0: $A - B - D = 0$

$A + B = 0$

$2A + 2B = 0$

$B = -\frac{1}{4}$

$4A = 1 \Rightarrow A = \frac{1}{4}$

① $\Rightarrow C = 0$

② $D = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$

$$\mathcal{L}^{-1}\{Y(s)\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$y(t) = \frac{1}{4}e^t - \frac{1}{4}e^{-t} + \frac{1}{2}\sin t$$

#1 $y' + y = te^t \qquad y(0) = -2$

$$\mathcal{L}\{y' + y\} = \mathcal{L}\{te^t\}$$

$$sY(s) - y(0) + Y(s) = \frac{1}{(s-1)^2}$$

$$(s+1)Y(s) = \frac{1}{(s-1)^2} - 2$$

$$Y(s) = \frac{1}{(s+1)(s-1)^2} - \frac{2}{s+1}$$

$$\frac{1}{(s+1)(s-1)^2} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$1 = A(s^2 - 2s + 1) + B(s^2 - 1) + C(s+1)$$

s^2: $A + B = 0 \rightarrow B = -A = -\frac{1}{4}$

s: $-2A + C = 0 \Rightarrow C = 2A = \frac{1}{2}$

so $A - B + C = 1$

$\hookrightarrow A + A + 2A = 1 \Rightarrow A = \frac{1}{4}$

$$\mathcal{L}^{-1}Y(s) = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{2}{s+1}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\}$$

$$y(t) = +\frac{1}{4}e^{-t} - 2e^{-t} - \frac{1}{4}e^t + \frac{1}{2}te^t$$

$$= -\frac{7}{4}e^{-t} - \frac{1}{4}e^t + \frac{1}{2}te^t$$

$t \to \frac{1}{s^2}$ $\quad t^n \to \frac{n!}{s^{n+1}}$

$e^{at} \to \frac{1}{s-a}$

$te^{at} \to \frac{1}{(s-a)^2}$

$\sin bt \to \frac{b}{s^2+b^2}$

$\cos bt \to \frac{s}{s^2+b^2}$

#17 $y'' + y = -2\cos 2t \quad | \quad y(0) = 1,\ y'(0) = -1$

$$\mathcal{L}\{y'' + y\} = -2\,\mathcal{L}\{\cos 2t\}$$

$$s^2 Y(s) - s\,y(0) - y'(0) + Y(s) = -2\,\frac{s}{s^2+4}$$

$$(s^2+1)\,Y(s) = \frac{-2s}{s^2+4} + s - 1$$

$$Y(s) = \underbrace{\frac{-2s}{(s^2+4)(s^2+1)}} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

$$\frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1} = \frac{-2s}{(\ )(\ )}$$

$$\begin{aligned} s^3 &: \quad A + C = 0 \\ s^2 &: \quad B + D = 0 \\ s^1 &: \quad A + 4C = -2 \\ s^0 &: \quad B + 4D = 0 \end{aligned} \Bigg\} \rightarrow -3C = +2$$

$$C = -\frac{2}{3}$$

$$A = +\frac{2}{3}$$

$$3D = 0 \Rightarrow D = 0 = B$$

$$\mathcal{L}^{-1}\{Y(s)\} = +\frac{2}{3}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - \frac{2}{3}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$y(t) = +\frac{2}{3}\cos 2t + \frac{1}{3}\cos t - \sin t$$