Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Circle: $(a=b)$ $x^2 + y^2 = a^2$

Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

major axis

5.5 Infinite Sequences

$a_1, a_2, \ldots, a_n, \ldots$

formula

Ex 1st 4 terms and 10th $\left\{\frac{n}{n+1}\right\}$

Soln $n=1 \Rightarrow \frac{1}{1+1} = \frac{1}{2}$

$n=2 \Rightarrow \frac{2}{2+1} = \frac{2}{3}$

$n=3 \Rightarrow \frac{3}{3+1} = \frac{3}{4}$

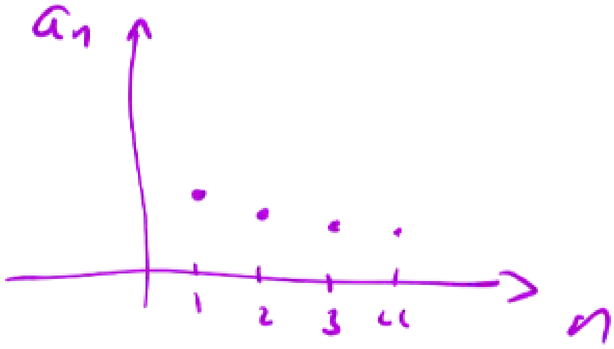
$n=4 \Rightarrow \frac{4}{4+1} = \frac{4}{5}$

$n=10 \Rightarrow \frac{10}{10+1} = \frac{10}{11}$

$\rightarrow$ a $\Rightarrow$

$a_n = \frac{n}{n+1}$ $f(x) = \frac{x}{x+1}$

$n=1 \quad a_n = \frac{2}{3} \qquad x=1 \rightarrow y = \frac{2}{3}$



Ex 1^{st} & 10^{th} $\{2 + (.1)^n\}$

$n=1 \rightarrow 2+\cancel{(.1)^1} = \underline{2+.1} = 2.1$

$n=2 \rightarrow 2+(.1)^2 = 2+.01 = 2.01$

$n=3 \rightarrow 2+(.1)^3 = 2+.001 = 2.001$

$n=4 \rightarrow 2+(.1)^4 = 2+.0001 = 2.0001$

$n=10 \rightarrow 2+(.1)^{10} = 2.0000000001$

Ex $\{c_n\} = \{(-1)^{n+1}\frac{n^2}{3n-1}\}$

$c_1 = (-1)^2 \frac{1}{3-1} = \frac{1}{2}$

$c_2 = (-1)^3 \frac{4}{6-1} = -\frac{4}{5}$

$c_3 = (-1)^4 \frac{9}{9-1} = \frac{9}{8}$

$c_4 = (-1)^5 \frac{4^2}{12-1} = -\frac{16}{11}$

$c_{10} = (-1)^{11} \frac{10^2}{30-1} = -\frac{100}{29}$

Ex $\{4\}$

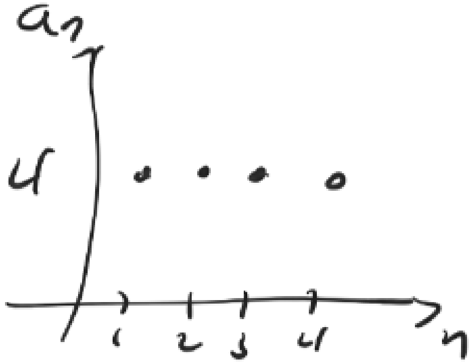
$n = 1 \rightarrow \{4\}$

$n = 2 \rightarrow \{4\}$

$n = 3 \rightarrow \{4\}$

$n = 4 \rightarrow \{4\}$

$n = 10 \rightarrow \{4\}$



Ex 1^{st} 4 terms $a_1 = 3$

$a_{n+1} = (n+1)a_n$

$\underline{a_1 = 3}$

$n=1 \quad a_2 = 2a_1 = 2(3) = \underline{6}$

$n=2 \quad a_3 = 3a_2 = 3(6) = \underline{18}$

$n=3 \quad a_4 = 4a_3 = 4(18) = \underline{72}$

#11 $\{c_n\} = \left\{ \frac{(-1)^n}{(n+1)(n+2)} \right\}$

1^{st} 4 c_5

$c_1 = \frac{(-1)^1}{2(3)} = \underline{-\frac{1}{6}}$

$c_2 = \frac{(-1)^2}{3(4)} = \underline{\frac{1}{12}}$

$c_3 = \frac{(-1)^3}{4(5)} = \underline{\frac{-1}{20}}$

$c_4 = \frac{(-1)^4}{5(6)} = \underline{\frac{1}{30}}$

$$c_8 = \frac{(-1)^{8}}{8(9)} = \frac{1}{72}$$

#21 $\quad a_1 = \sqrt{2} \qquad a_n = \sqrt{2 + a_{n-1}}$

$n=1 \quad a_1 = \sqrt{2}$

$n=2 \quad a_2 = \sqrt{2 + a_1} = \sqrt{2+\sqrt{2}}$

$n=3 \quad a_3 = \sqrt{2 + a_2}$

$\qquad = \sqrt{2 + \sqrt{2+\sqrt{2}}}$

$n=4 \quad a_4 = \sqrt{2 + a_3}$

$\qquad = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$

Summation Notation

$$\sum_{n=1}^{5} (2n+3)$$

5 ← last value

$(2n+3)$ } formula

$n=1$ ← first value

$\sum$ Sigma

Ex

$$\sum_{k=1}^{4} k^2(k-3) = \overbrace{1^2(1-3)}^{k=1} + \overbrace{2^2(2-3)}^{k=2}$$

$$+ \overbrace{3^2(3-3)}^{k=3} + \overbrace{4^2(4-3)}^{k=4}$$

$$= -2 - 4 + 16$$

$$= 10$$

$$\sum_{k=1}^{n} c = nc \qquad \sum_{k=m}^{n} c = (n-m+1)c$$

c : constant

$n - \underbrace{1 + 1}$

$$\sum_{k=10}^{20} 5 = 5(20-10+1)$$
$$= 55$$

$$\begin{array}{r} 72 \\ \hline 792 \end{array} \qquad \begin{array}{r} 75 \\ \hline 825 \end{array}$$

$$\sum_{k=1}^{n} (a_k + b_n) = \sum_{k=1}^{\infty} a_k + \sum_{k=1}^{n} b_n$$

$$\sum_{k=1}^{n} c\, a_k = c \sum_{k=1}^{n} a_k$$

$$2^1 + 2^2 + 2^3 + \cdots + 2^{16} = \sum_{n=1}^{16} 2^n$$

1 2 3 16

1, 3, 5, 17 ($n = 1 \to 9$)

#41

$$\sum_{k=253}^{571} \left(\frac{1}{13}\right) = \frac{1}{13}(571-253+1)$$

$$= \frac{319}{13}$$

#43

$$\sum_{k=1}^{40} k = \frac{1}{2}(40)(41) \quad n=40$$

$$= 820$$

#42

$$\sum_{k=1}^{50} 8 = 8(50)$$

$$= 400$$

5.6 Arithmetic Seq.

Defn $a_1, a_2, \ldots, a_n, \ldots$ is an arithmetic $\exists$ (exists) $d \in \mathbb{R}$ ⓚ

$$a_{k+1} = a_k + d$$

common difference

$d = a_{\ldots} - a_{\ldots}$

$d = a_{k+1} - a_k$

Ex $\quad 1, 4, 7, 10, \ldots, 3n-2, \ldots$

$\lfloor d = 4 - 1 = 3 \rfloor$

$d = a_{k+1} - a_k$

$= [3(k+1) - 2] - (3k - 2)$

$= 3k + 3 - 2 - 3k + 2$

$= 3$ ✓

$$\boxed{a_n = a_1 + (n-1)d}$$

Ex $\quad \overset{a_1}{20}, 16.5, 13 \qquad a_{15}?$

$d = 16.5 - 20$

$= -3.5$

$15 - 1$

$14 \cdot 2$
$\quad 7$

$a_{15} = 20 + (14)(-3.5)$

$= 20 - 49$

$= -29$

Ex $\quad a_4 = 5 \qquad a_9 = 20 \qquad a_6 ??$

$d = \dfrac{20 - 5}{9 - 4} = \boxed{\dfrac{15}{5}}$

slope formula

$= 3$

$a_n = a_1 + (n-1)d$

$(4-1) \, d$

$a_4 = a_1 + 3(3) = 5$

$a_1 = 5 - 9$
$= -4$

$a_6 = -4 + 5(3)$
$= 11$

Formula S_n

no $S_\infty = \infty$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$= \frac{n}{2}\left(a_1 + a_n\right)$$

$$\boxed{S_n = \frac{n}{2}(n+1)}$$

n : seq $1 \to n$

Ex S? Even $2 \to 100$ $n = ?$ $= 50$

$2 = a_1$, $100 = a_{50}$

$$S_{50} = \frac{50}{2}(2 + 100)$$

$$= \frac{50}{2}(102)$$

$$= 2550$$

$a_1 = 2$

$a_{50} = 100$

Ex $\frac{1}{4} + \frac{2}{9} + \frac{3}{14} + \frac{4}{19} + \frac{5}{24} + \frac{6}{29}$

$$= \sum_{n=1}^{6} \frac{n}{5n-1}$$

4, 9, 14, 19, 24, 29

$d = 9 - 4 = \underline{5}$

$a_n = 4 + (n-1)(5)$

$= 4 + 5n - 5$

$= \underline{5n - 1}$

#24 a_8 : $a_{15} = 0$ $a_{40} = -50$

$d = \frac{-50}{40-15} = -\frac{50}{25}$ $a_1 + (n-1)$

$= -2$

$a_{15} = a_1 + 14(-2) = 0$

$\underline{a_1 = 28}$

only 1

$a_8 = 28 + (7)(-2)$

$= \underline{14}$

Geometric Sequence

$\frac{a_{k+1}}{a_k} = r$ common Ratio

$+ \quad - \quad + \quad -$

$\lambda = -1 \quad \Rightarrow$