#1 $y = x^3$ $y = 8$ $x = 0$ y-axis

$$V = \pi \int_0^8 (y^{1/3})^2\, dy$$

#2 $y = x^2$ $y = x$ x-axis $x^2 = x \Rightarrow x = 0, 1$

$$V = \pi \int_0^1 (x^2 - x^4)\, dx$$

$$= \pi \left(\frac{1}{3}x^3 - \frac{1}{5}x^5 \right|_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{5}\right)$$

$$= \frac{2\pi}{15} \text{ unit}^3$$

#3 $y = x^2$, $y = x$ line $y = 2$

$x = 0, 1$

$$V = \pi \int_0^1 \left[(2-x)^2 - (2-x^2)^2\right] dx$$

#4 $y = \sqrt{x}$ $x = 1$ x-axis

$y^2 = x$

$$V = 2\pi \int_0^1 y\,(1 - y^2)\, dy$$

#5 $y = x - x^2$ $y = 0$ line $x = 2$ rot

$$V = 2\pi \int_0^1 (2-x)(x - x^2)\, dx$$

#6 $y = -x^2 + 6x - 8$, $y = 0$ y-axis

$x = 2, 4$

$$V = 2\pi \int_2^4 x\,(-x^2 + 6x - 8)\, dx$$

#7 $y = -x^2+6x-8 = 0$ $y = 0$ x - axis

$x = 2, 4$

$$V = \pi \int_2^4 (-x^2+6x-8)^2 \, dx$$

#8 $x = (y-3)^2 = x = 4$ line $y = 1$

$y - 3 = \pm 2 \Rightarrow y = 1, 5$

$$V = 2\pi \int_1^5 (y-1)\left(4-(y-3)^2\right) dy$$

1.5 length.

$$L = \int_a^b \sqrt{1+(f')^2}\, dx$$

Ex L? $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1 \qquad 0 \le x \le 1$

$$\sqrt{1+(y')^2} = \sqrt{1+\left(2\sqrt{2}\, x^{1/2}\right)^2}$$

$$= \sqrt{1+8x}$$

$$L = \frac{1}{8}\int_0^1 (1+8x)^{1/2}\, d(1+8x)$$

$$= \frac{1}{12}(1+8x)^{3/2}\Big|_0^1 \qquad 9^{3/2} = (3^2)^{3/2} = 3^3$$

$$= \frac{1}{12}(27-1)$$

$$= \frac{13}{6} \text{ unit}$$

Ex L? $f(x) = \frac{x^3}{12} + \frac{1}{x} \qquad 1 \le x \le 4$

$a = \frac{1}{12} \quad m = 3 \quad b = 1 \quad n = -1 \qquad ax^m + bx^n$

$$\begin{cases} m+n = 3-1 = 2 \; \checkmark \\ abmn = \frac{1}{12}(3)(1)(-1) = -\frac{1}{4} \; \checkmark \end{cases}$$

$$L = \frac{x^3}{12} - \frac{1}{x}\Big|_1^4$$

$$= \frac{16}{3} - \frac{1}{4} - \frac{1}{12} + 1$$

$$= \frac{64 - 3 - 1 + 12}{12}$$

$$= \frac{72}{12}$$

$$= 6 \text{ unit}$$

$ae^{mx}+be^{nx}$ $\quad \begin{cases} m=-n \\ abmn=-\frac{1}{4} \end{cases}$

Ex $\quad L?\quad f(x)=\ln\left(x+\sqrt{x^2-1}\right)\quad [1,\sqrt{2}]$

$$f'(x)=\frac{1+x(x^2-1)^{-1/2}}{x+\sqrt{x^2-1}} \qquad (u^n)'=nu^{n-1}$$

$$=\frac{\sqrt{x^2-1}+x}{\left(x+\sqrt{x^2-1}\right)\left(\sqrt{x^2-1}\right)}$$

$$y=\ln\left(x+\sqrt{x^2-1}\right)$$

$$x+\sqrt{x^2-1}=e^y$$

$$\left(\sqrt{x^2-1}\right)^2=(e^y-x)^2$$

$$x^2-1=e^{2y}-2xe^y+x^2$$

$$e^{2y}-2xe^y+1=0$$

$$2xe^y=1+e^{2y}$$

$$x=\frac{1}{2}\left(e^{-y}+e^y\right)=g(y)$$

$$g(y)=\frac{1}{2}\left(e^y+e^{-y}\right)=\frac{1}{2}e^y+\frac{1}{2}e^{-y}$$

$$x=1\to y=\ln(1+0)=0$$

$$x=\sqrt{2}\to y=\ln(\sqrt{2}+1)$$

$$a=b=\frac{1}{2}\qquad m=1,\ n=-1$$

$$\begin{cases} m=-n \\ abmn=\frac{1}{2}\cdot\frac{1}{2}(1)(-1)=-\frac{1}{4} \end{cases}$$

$$L=\frac{1}{2}\left(e^y-e^{-y}\right)\Big|_0^{\ln(\sqrt{2}+1)} \qquad e^{\ln ☺}=☺ \qquad e^{-\ln y}=e^{\ln\frac{1}{y}}$$

$$=\frac{1}{2}\left(\sqrt{2}+1-\frac{1}{\sqrt{2}+1}-(1-1)\right)$$

$$=\frac{1}{2}\,\frac{3+2\sqrt{2}-1}{\sqrt{2}+1}$$

$$=\frac{\sqrt{2}+1}{\,}=1 \text{ unit}$$

12. $f(y) = 2e^{\sqrt{2}y} + \frac{1}{16}e^{-\sqrt{2}y}$ $0 \le y \le \frac{\ln 2}{\sqrt{2}}$

$a = 2,\ b = \frac{1}{16},\ m = \sqrt{2}\quad n = -\sqrt{2}$

$$\begin{cases} m = -n \checkmark \\ abmn = 2\left(\frac{1}{16}\right)(\sqrt{2})(-\sqrt{2}) = -\frac{1}{4} \checkmark \end{cases}$$

$$L = 2e^{\sqrt{2}y} - \frac{1}{16}e^{-\sqrt{2}y}\Big|_0^{\frac{\ln 2}{\sqrt{2}}}$$

$e^{-\ln 2} = e^{\ln\frac{1}{2}}$

$$= 4 - \frac{1}{32} - \left(2 - \frac{1}{16}\right)$$

$\frac{1}{16} - \frac{1}{32}$

$$= 2 + \frac{1}{32}$$

$\frac{1}{16}\left(1 - \frac{1}{2}\right)$

$$= \frac{65}{32}\ \text{unit}$$

.6 Surface Area

$$S = 2\pi\int_a^b f(x)\sqrt{1+(f')^2}\,dx$$

Ex $y = 2\sqrt{x}$ $1 \le x \le 3$ x-axis

$$\sqrt{1+(y')^2} = \sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}$$

$$= \sqrt{1+\frac{1}{x}}$$

$$= \frac{\sqrt{x+1}}{\sqrt{x}}$$

$$S = 2\pi\int_1^3 2\sqrt{x}\,\frac{(x+1)^{1/2}}{\sqrt{x}}\,dx$$

$$= 4\pi\int_1^3 (x+1)^{1/2}\,d(x+1)$$

$$= \frac{8\pi}{3}(x+1)^{3/2}\Big|_1^3$$

$$= \frac{8\pi}{3}\left(8 - 2^{3/2}\right)$$

$$= \frac{16\pi}{3}\left(4 - \sqrt{2}\right)\ \text{unit}^2$$

$4\quad (2^2)^{3/2}$

$2^{(3/2)}\quad 2\sqrt{2}$

$\frac{x}{2}$

$f(x) = ax^m + bx^n \quad > \quad f'(x)$

$\begin{cases} m+n = 2 \checkmark \\ abmn = -\frac{1}{4} \checkmark \end{cases} \to \sqrt{1+(f')^2} = f'(x)$

#20 S ? $y = \frac{1}{6}x^3 + \frac{1}{2x} \quad 1 \le x \le 2$

$a = \frac{1}{6}, \; b = \frac{1}{2} \quad m = 3 \quad n = -1$

$\begin{cases} m+n = 3-1 = 2 \checkmark \\ abmn = \frac{1}{6}\frac{1}{2}(3)(-1) = -\frac{1}{4} \checkmark \end{cases}$

$f'(x) = \frac{1}{2}x^2 - \frac{1}{2x^2} \qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

$S = 2\pi \int_1^2 \left(\frac{1}{\cancel{6}_3}x^3 + \frac{1}{\cancel{2}x}\right)\left(\frac{1}{\cancel{2}}x^2 + \frac{1}{\cancel{2}x^2}\right)dx$

$= \pi \int_1^2 \left(\frac{1}{3}x^5 + \overbrace{\frac{1}{3}x + x}^{\frac{4}{3}x} + x^{-3}\right)dx$

$= \pi \left(\frac{1}{18}x^6 + \frac{2}{3}x^2 - \frac{1}{2}\frac{1}{x^2}\right|_1^2$

$= \pi\left(\frac{32}{9} + \underbrace{\frac{8}{3} - \frac{1}{8} - \frac{1}{18} - \frac{2}{3}}_{2} + \frac{1}{2}\right)$

$= \pi\left(2 + \frac{63}{18} + \frac{3}{8}\right)$

#15-4

$y = \frac{1}{3}x^{1/2} - x^{3/2} \qquad 0 \le x \le 1/3$

$\begin{cases} m+n = \frac{1}{2} + \frac{3}{2} = 2 \checkmark \\ abmn = \frac{1}{3}(-1)\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) = -1/4 \end{cases}$

$y' = \frac{1}{6}x^{-1/2} - \frac{3}{2}x^{1/2}$

$S = 2\pi \int_0^{1/3} \left(\frac{1}{3}x^{1/2} - x^{3/2}\right)\left(\frac{1}{6}x^{-1/2} + \frac{3}{2}x^{1/2}\right)dx$

$$S = 2\pi \int_0^{1/3} \left(\frac{1}{18} + \frac{3}{6}x - \frac{1}{6}x - \frac{3}{2}x^2\right) dx$$

$$= 2\pi \int_0^{1/3} \left(\frac{1}{18} + \frac{1}{3}x - \frac{3}{2}x^2\right) dx$$

$$= 2\pi \left(\frac{1}{6}x + \frac{1}{8}x^2 - \frac{1}{2}x^3 \Big|_0^{1/3}\right.$$

$$= 2\pi \left(\frac{1}{18} + \frac{1}{72} - \frac{1}{54}\right)$$

$$= \frac{\pi}{9}\left(1 + \frac{1}{4} - \frac{1}{3}\right)$$

$$= \frac{\pi}{9}\left(\frac{12+3-4}{12}\right)$$

$$= \frac{11\pi}{108} \text{ unit}^2$$

$$S = 2\pi \int_0^{1/3} \left(\frac{1}{18} + \frac{1}{3}x - \frac{3}{2}x^2\right) dx$$

$$= \frac{\pi}{9} \int_0^{1/3} (1 + 6x - 27x^2)\, dx$$

$$= \frac{\pi}{9}\left(x + 3x^2 - 9x^3 \Big|_0^{1/3}\right.$$

$$= \frac{\pi}{9}\left(\frac{1}{3} + \frac{1}{3} - \frac{1}{3}\right)$$

$$= \frac{\pi}{27} \text{ unit}^2$$

1.7

$$m = \text{density} \times \text{Volume}$$

$$m = \int_a^b \underbrace{\rho(x)}_{\text{density}}\, dx$$

Ex $0 \le x \le 2 \quad \rho(x) = 1 + x^2 \quad m?$

$$m = \int_0^2 (1+x^2)\,dx$$

$$= x + \frac{1}{3}x^3 \Big|_0^2$$

$$= 2 + \frac{8}{3}$$

$$= \boxed{\frac{14}{3} \text{ kg}}$$

$$W = \rho g \int_a^b A(y)\, D \cdot dy$$

$$= 57 \int_0^8 \frac{\pi}{4} y^2 (10 - y)\, dy$$

$$= \frac{57\pi}{4} \int_0^8 (10y^2 - y^3)\, dy$$

$$= \frac{57\pi}{4} \left(\frac{10}{3}y^3 - \frac{1}{4}y^4\right)\Big|_0^8$$

$$= \frac{57\pi}{4} \left(\frac{10}{3}8^3 - 2\,8^3\right)$$

$$= \frac{57\pi 8^3}{4}\left(\frac{10}{3} - 2\right)$$

$$= \boxed{19\pi 8^3 \text{ ft-lb}}$$