$4\sin^2 x \tan x - \tan x = 0 \qquad [0, 2\pi)$

$\tan x\,(4\sin^2 x - 1) = 0$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

ok

$\tan x = 0 \qquad \sin x = \pm\frac{1}{2}$

$x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\csc^4 2u - 4 = 0 \qquad [0, 2\pi)$

$(\csc^2 2u - 2)(\underbrace{\csc^2 2u + 2}_{\neq 0}) = 0$

$\csc 2u = \pm\sqrt{2}$

$\sin 2u = \pm\frac{1}{\sqrt{2}}$

$2u = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$u = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

$$\underbrace{5\sin\theta\tan\theta - 10\tan\theta} + \underbrace{3\sin\theta - 6} = 0$$

$$5\tan\theta\,(\underline{\sin\theta - 2}) + 3\,(\underline{\sin\theta - 2}) = 0$$

$$(\sin\theta - 2)\,(5\tan\theta + 3) = 0$$

$\sin\theta = 2$ # >1 $\qquad \tan\theta = -\frac{3}{5}$ $\qquad$ Q II, Q IV

$$\hat{\theta} = \underline{\tan^{-1}\left(\tfrac{3}{5}\right)}$$

$$\theta = \pi - \hat{\theta}$$
$$\theta = 2\pi - \hat{\theta}$$

#9 $\cos(\ln x) = 0$

$$\left\{\begin{array}{l} \ln x = \pi/2 \\ \ln x = \frac{3\pi}{2} \end{array}\right\} \to x = e^{\pi/2},\ e^{3\pi/2}$$

10/ $2\sin^2 x = 1 - \sin x$ $\qquad [0, 2\pi)$

$$2\sin^2 x + \sin x - 1 = 0$$

$\sin x = -1$ $\qquad \sin x = \frac{1}{2}$

$$\boxed{x = \frac{3\pi}{2},\ \frac{\pi}{6},\ \frac{5\pi}{6}}$$

11/ $\tan^2 x \sin x = \sin x$

$\tan^2 x \sin x - \sin x = 0$

$\sin x(\tan^2 x - 1) = 0$ $\quad \tan^2 x = 1$

$\sin x = 0 \qquad\qquad \tan x = \pm 1$

$$x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

12/ $1 - \sin x = \sqrt{3} \cos x$

( )^2 = ( )^2

at end

Test

$$\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{1}{2}$$

$$\cos \frac{\pi}{6} \cos x + \sin \frac{\pi}{6} \sin x = \frac{1}{2}$$

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2} \qquad \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x - \frac{\pi}{6} = \frac{\pi}{3} \qquad\qquad x - \frac{\pi}{6} = \frac{5\pi}{3}$$

$$x = \frac{\pi}{3} + \frac{\pi}{6} \qquad\qquad x = \frac{5\pi}{3} + \frac{\pi}{6}$$

$$= \frac{\pi}{2} \qquad\qquad = \frac{11\pi}{6}$$

15/ $2\sin^3 x + \sin^2 x - 2\sin x - 1 = 0$

$\sin^2 x\,(2\sin x + 1) - (2\sin x + 1) = 0$

$(2\sin x + 1)(\sin^2 x - 1) = 0$

$\sin x = -\frac{1}{2}$ $\qquad$ $\sin x = \pm 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$

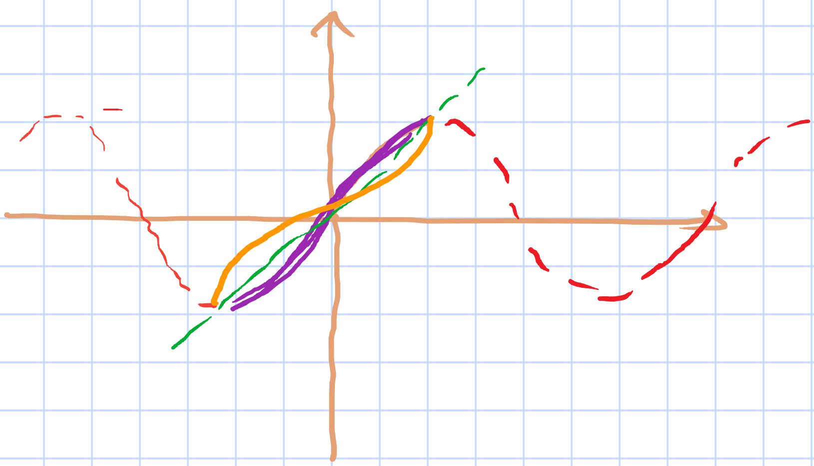
5.5 Inverse Trig fctns

$x = \sin y$ $\qquad$ $y = \sin^{-1} x$

$= \arcsin x$

$-1 \le x \le 1$

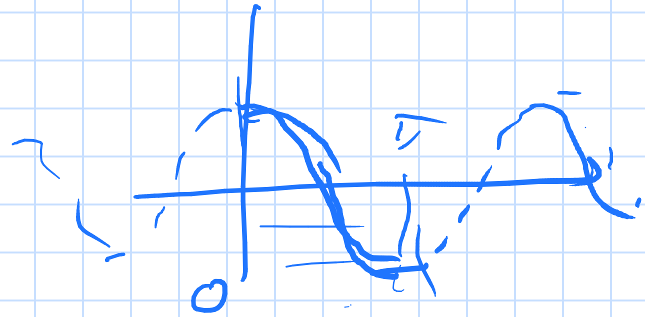
$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



$$\sin(\sin^{-1}x) = x$$

$$\arcsin(\sin y) = y$$

Cosine



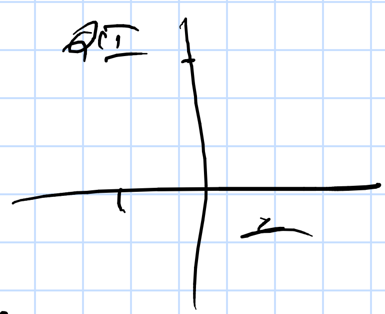
$$y = \cos^{-1}x$$
$$= \arccos x$$

$$-1 \le x \le 1 \qquad 0 \le y \le \pi$$

$$y = \tan^{-1}x \text{ or } \arctan x$$

$$x \in \mathbb{R} \qquad -\frac{\pi}{2} < y \le \frac{\pi}{2}$$

$$\sin(\underbrace{\arccos(-\tfrac{2}{3})}_{\alpha})$$



$$\alpha = \arccos(-\tfrac{2}{3})$$

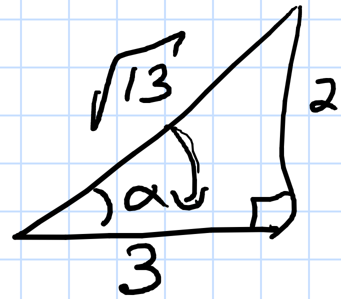
$$\cos\alpha = -\frac{2}{3}$$

$9 - 4$

$$\boxed{\sin\alpha = \frac{\sqrt{5}}{3}}$$

Ex $\sec\left(\arctan \frac{2}{3}\right)$

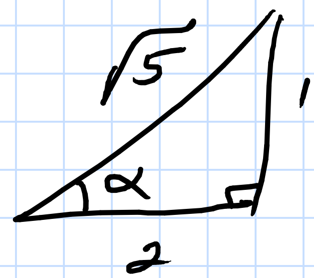
$\alpha = \arctan \frac{2}{3} \Rightarrow \tan\alpha = \frac{2}{3}$



$$\boxed{\sec\alpha = \frac{\sqrt{13}}{3}}$$

Ex $\sin\left(\underbrace{\arctan \tfrac{1}{2}}_{\alpha} - \underbrace{\arccos \tfrac{4}{5}}_{\beta}\right)$

$\alpha = \arctan \frac{1}{2} \rightarrow \tan\alpha = \frac{1}{2}$



$\beta = \arccos \frac{4}{5} \Rightarrow \cos\beta = \frac{4}{5}$

$\sin\beta = 3/5$

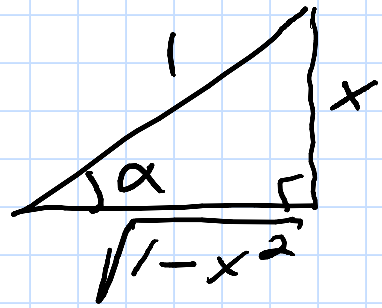
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$= \frac{1}{\sqrt{5}}\,\frac{4}{5} - \frac{2}{\sqrt{5}}\,\frac{3}{5}$$

$$= \boxed{\frac{-2}{5\sqrt{5}}}$$

Ex $\cos(\sin^{-1} x)$

$$\left.\begin{aligned} \alpha &= \sin^{-1} x \\ \sin\alpha &= \frac{x}{1} \end{aligned}\right\} \xrightarrow{1}$$



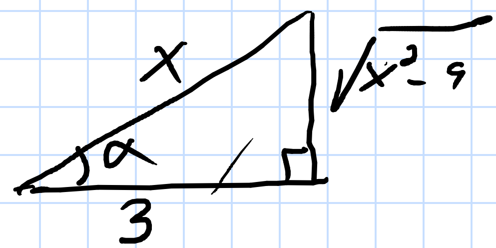
2

$$\boxed{\cos\alpha = \sqrt{1-x^2}}$$

34/ $\cot\left(\sin^{-1} \frac{\sqrt{x^2-9}}{x}\right)$ $x > 0$

$$\alpha = \sin^{-1} \frac{\sqrt{x^2-9}}{x}$$

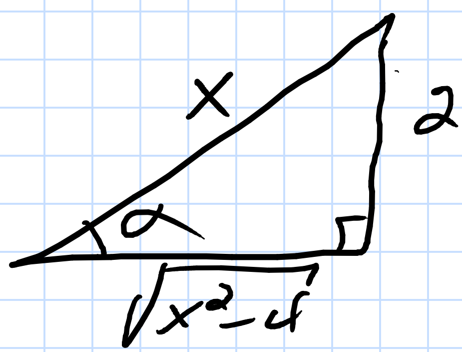
$$\sin\alpha = \frac{\sqrt{x^2-9}}{x}$$



$$\boxed{\cot\alpha = \frac{3}{\sqrt{x^2-9}}}$$

39/ $\sec\left(\tan^{-1} \frac{2}{\sqrt{x^2-4}}\right)$

$$\alpha = \tan^{-1} \frac{2}{\sqrt{x^2-4}}$$



$$\tan\alpha = \frac{2}{\sqrt{x^2-4}}$$

$$\boxed{\sec\alpha = \frac{x}{\sqrt{x^2-4}}}$$

16/ $\cos\left(2\sin^{-1}\frac{15}{17}\right) = \cos 2\alpha$

$\alpha = \sin^{-1}\frac{15}{17}$

$\sin\alpha = \frac{15}{17} \rightarrow \cos\alpha = \frac{8}{17}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

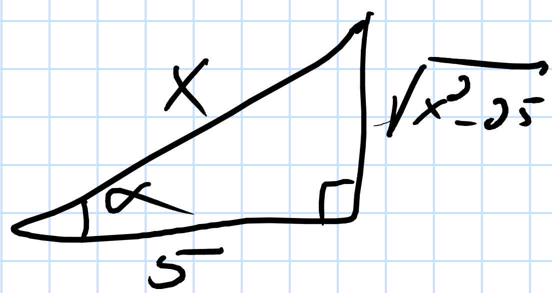
$= \frac{225}{289} - \frac{64}{289}$

$= \boxed{\frac{161}{289}}$

40/ $\sec\left(\sin^{-1}\frac{\sqrt{x^2-25}}{x}\right)$

$\alpha = \sin^{-1}\frac{\sqrt{x^2-25}}{x}$

$\sin\alpha = \frac{\sqrt{x^2-25}}{x}$

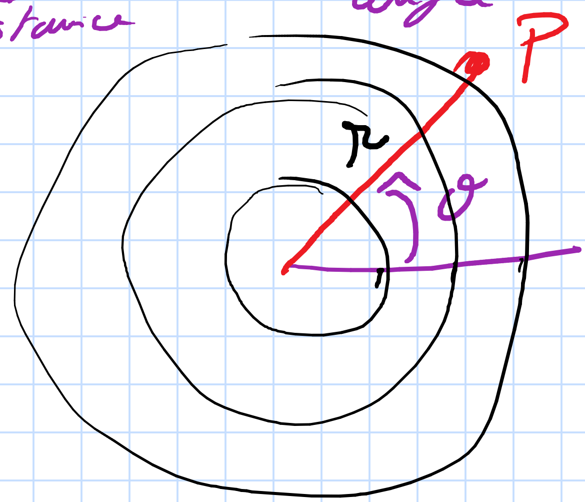


$\boxed{\sec\alpha = \frac{x}{5}}$

8.6 Polar Coordinates.

$$P(r, \theta)$$

distance   angle



$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \begin{cases} r^2 = x^2 + y^2 \\ \theta = \tan^{-1}\dfrac{y}{x} \end{cases}$$

Ex $(r, \theta) = \left(4, \frac{7\pi}{6}\right)$

$$\begin{aligned} x &= r\cos\theta \\ &= 4\cos\tfrac{7\pi}{6} \\ &= -4\left(\tfrac{\sqrt{3}}{2}\right) \\ &= -2\sqrt{3} \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= 4\sin\tfrac{7\pi}{6} \\ &= 4\left(-\tfrac{1}{2}\right) \\ &= -2 \end{aligned}$$

$$(x, y) = (-2\sqrt{3}, -2)$$

Ex $(x, y) = (-1, \sqrt{3})$

$$r = \sqrt{x^2 + y^2} = \sqrt{1+3} = 2$$

$$\hat{\theta} = \tan^{-1} \frac{\sqrt{3}}{1} = \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

$$(r, \theta) = \left(2, \frac{2\pi}{3}\right)$$

$$\left(-2, \frac{5\pi}{3}\right)$$

Ex $ax + by = c$

$$a\, r\cos\theta + b\, r\sin\theta = c$$

$$r(a\cos\theta + b\sin\theta) = c$$

$$r = \frac{c}{a\cos\theta + b\sin\theta}$$

Ex $x^2 - y^2 = 16$

$$(r\cos\theta)^2 - (r\sin\theta)^2 = 16$$

$$r^2\cos^2\theta - r^2\sin^2\theta = 16$$

$$r^2(\cos^2\theta - \sin^2\theta) = 16$$

$$r^2 = \frac{16}{\cos 2\theta}$$

30)

$r^2(\cos^2\theta + 4\sin^2\theta) = 16$

$r^2\underline{\cos^2\theta} + 4r^2\sin^2\theta = 16$

$x^2 + 4y^2 = 16$