Solution   *Section* **3.5 – Exponential and logarithmic Equations**

Exercise

Solve the equation: $2^x = 128$

Solution

$2^x = 2^7$

$\underline{x = 7}$

Exercise

Solve the equation: $3^x = 243$

Solution

$3^x = 3^5$

$\underline{x = 5}$

Exercise

Solve the equation: $5^x = 70$

Solution

$\underline{x = \log_5 70}$

Exercise

Solve the equation: $6^x = 50$

Solution

$\underline{x = \log_6 50}$

Exercise

Solve the equation: $5^x = 134$

Solution

$\underline{x = \log_5 134}$

Exercise

Solve the equation:　$7^x = 12$

Solution

$$x = \log_7 12$$

Exercise

Solve the equation:　$9^x = \dfrac{1}{\sqrt[3]{3}}$

Solution

$$\left(3^2\right)^x = \dfrac{1}{3^{1/3}}$$

$$3^{2x} = 3^{-1/3}$$

$$2x = -\dfrac{1}{3}$$

$$x = -\dfrac{1}{6}$$

Exercise

Solve the equation:　$49^x = \dfrac{1}{343}$

Solution

$$\left(7^2\right)^x = \dfrac{1}{7^3}$$

$$7^{2x} = 7^{-3}$$

$$2x = -3$$

$$x = -\dfrac{3}{2}$$

Exercise

Solve the equation:　$2^{5x+3} = \dfrac{1}{16}$

Solution

$$2^{5x+3} = 2^{-4}$$

$$5x + 3 = -4$$

$$5x = -7$$

$$x = -\dfrac{7}{5}$$

Exercise

Solve the equation: $\left(\dfrac{2}{5}\right)^x = \dfrac{8}{125}$

Solution

$$\left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^3$$

$\underline{x = 3}$

Exercise

Solve the equation: $2^{3x-7} = 32$

Solution

$$2^{3x-7} = 32$$
$$\phantom{2^{3x-7}} = 2^5$$
$$3x - 7 = 5 \qquad\qquad \text{\textit{add 7 on both sides}}$$
$$3x = 12 \qquad\qquad \text{\textit{Divide by 3}}$$
$\underline{x = 4}$

Exercise

Solve the equation: $4^{2x-1} = 64$

Solution

$$4^{2x-1} = 4^3$$
$$2x - 1 = 3$$
$$2x = 4$$
$\underline{x = 2}$

Exercise

Solve the equation: $3^{1-x} = \dfrac{1}{27}$

Solution

$$3^{1-x} = \frac{1}{3^3}$$
$$3^{1-x} = 3^{-3}$$
$$1 - x = -3$$
$\underline{x = 4}$

Exercise

Solve the equation: $2^{-x^2} = 5$

Solution

$\ln 2^{-x^2} = \ln 5$

$-x^2 \ln 2 = \ln 5$

$x^2 = -\dfrac{\ln 5}{\ln 2} \quad \Rightarrow \quad$ *No Solution*

Exercise

Solve the equation: $2^{-x} = 8$

Solution

$2^{-x} = 2^3$

$-x = 3$

$\underline{x = -3}$

Exercise

Solve the equation: $\left(\dfrac{1}{3}\right)^x = 81$

Solution

$\left(\dfrac{1}{3}\right)^x = 81$

$\left(3^{-1}\right)^x = 3^4$

$3^{-x} = 3^4$

$-x = 4$

$\underline{x = -4}$

Exercise

Solve the equation: $3^{-x} = 120$

Solution

$-x = \log_3 120 \qquad$ ***Convert to Log***

$x = -\log_3 120$

$\underline{= \log_3 \dfrac{1}{120}}$

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Exercise

Solve the equation: $27 = 3^{5x}\, 9^{x^2}$

Solution

$$3^3 = 3^{5x}\left(3^2\right)^{x^2}$$

$$= 3^{5x}\, 3^{2x^2}$$

$$= 3^{5x+2x^2}$$

$$2x^2 + 5x = 3$$

$$2x^2 + 5x - 3 = 0$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$x = \begin{cases} \dfrac{-5-7}{6} = \underline{-2} \\[2mm] \dfrac{-5+7}{6} = \underline{\dfrac{1}{3}} \end{cases}$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3)\ln 4 = -x\ln 3$$

$$x\ln 4 + 3\ln 4 = -x\ln 3$$

$$x\ln 4 + x\ln 3 = -3\ln 4$$

$$x(\ln 4 + \ln 3) = -3\ln 4$$

$$x = \frac{-3\ln 4}{(\ln 4 + \ln 3)}$$

Exercise

Solve the equation: $2^{x+4} = 8^{x-6}$

Solution

$$2^{x+4} = \left(2^3\right)^{x-6}$$

$$2^{x+4} = 2^{3x-18}$$

$$x + 4 = 3x - 18$$

$$2x = 22$$

$\underline{x = 11}$ |

Exercise

Solve the equation: $8^{x+2} = 4^{x-3}$

Solution

$$\left(2^3\right)^{x+2} = \left(2^2\right)^{x-3}$$

$2^{3(x+2)} = 2^{2(x-3)}$
$3(x+2) = 2(x-3)$
$3x+6 = 2x-6$
$3x-2x = -6-6$

$\underline{x = -12}$ |

Exercise

Solve the equation: $7^x = 12$

Solution

$\underline{x = \log_7 12}$ | ***Convert to Log***

Exercise

Solve the equation: $5^{x+4} = 4^{x+5}$

Solution

$\ln 5^{x+4} = \ln 4^{x+5}$
$(x+4)\ln 5 = (x+5)\ln 4$
$x\ln 5 + 4\ln 5 = x\ln 4 + 5\ln 4$
$(\ln 5 - \ln 4)x = 5\ln 4 - 4\ln 5$

$\underline{x = \dfrac{5\ln 4 - 4\ln 5}{\ln 5 - \ln 4}}$ |

Exercise

Solve the equation: $5^{x+2} = 4^{1-x}$

Solution

$$\ln 5^{x+2} = \ln 4^{1-x}$$

$$(x+2)\ln 5 = (1-x)\ln 4$$

$$x\ln 5 + 2\ln 5 = \ln 4 - x\ln 4$$

$$(\ln 5 + \ln 4)x = \ln 4 - 2\ln 5$$

$$\underline{x = \frac{\ln 4 - 2\ln 5}{\ln 5 + \ln 4}}$$

Exercise

Solve the equation: $3^{2x-1} = 0.4^{x+2}$

Solution

$$\ln 3^{2x-1} = \ln\left(0.4^{x+2}\right)$$

$$(2x-1)\ln 3 = (x+2)\ln\frac{4}{10}$$

$$2x\ln 3 - \ln 3 = x\ln\frac{2}{5} + 2\ln\frac{2}{5}$$

$$\left(2\ln 3 - \ln\frac{2}{5}\right)x = \ln 3 + 2\ln\frac{2}{5}$$

$$\underline{x = \frac{\ln 3 + 2\ln 0.4}{2\ln 3 - \ln 0.4}}$$

Exercise

Solve the equation: $4^{3x-5} = 16$

Solution

$$4^{3x-5} = 4^2$$

$$3x - 5 = 2$$

$$3x = 7$$

$$\underline{x = \frac{7}{3}}$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3)\ln 4 = -x\ln 3$$

$$x\ln 4 + 3\ln 4 = -x\ln 3$$

$$(\ln 4 + \ln 3)\, x = -3\ln 4$$

$$x = -\dfrac{3\ln 4}{\ln 4 + \ln 3}$$

Exercise

Solve the equation: $7^{2x+1} = 3^{x+2}$

Solution

$$\ln 7^{2x+1} = \ln 3^{x+2}$$

$$(2x+1)\ln 7 = (x+2)\ln 3$$

$$2x\ln 7 + \ln 7 = x\ln 3 + 2\ln 3$$

$$2x\ln 7 - x\ln 3 = 2\ln 3 - \ln 7$$

$$x(2\ln 7 - \ln 3) = 2\ln 3 - \ln 7$$

$$x = \dfrac{2\ln 3 - \ln 7}{2\ln 7 - \ln 3}$$

Exercise

Solve the equation: $3^{x-1} = 7^{2x+5}$

Solution

$$\ln 3^{x-1} = \ln 7^{2x+5}$$

$$(x-1)\ln 3 = (2x+5)\ln 7$$

$$x\ln 3 - \ln 3 = 2x\ln 7 + 5\ln 7$$

$$x\ln 3 - 2x\ln 7 = \ln 3 + 5\ln 7$$

$$x(\ln 3 - 2\ln 7) = \ln 3 + 5\ln 7$$

$$x = \dfrac{\ln 3 + 5\ln 7}{\ln 3 - 2\ln 7}$$

Exercise

Solve the equation: $4^{x-2} = 2^{3x+3}$

Solution

$$\left(2^2\right)^{x-2} = 2^{3x+3}$$

$$2^{2x-4} = 2^{3x+3}$$

$$2x - 4 = 3x + 3$$

$$2x - 3x = 4 + 3$$
$$-x = 7$$
$$\underline{x = -7}$$

Exercise

Solve the equation: $3^{5x-8} = 9^{x+2}$

Solution

$$3^{5x-8} = \left(3^2\right)^{x+2}$$

$$3^{5x-8} = 3^{2x+4}$$

$$5x - 8 = 2x + 4$$

$$5x - 2x = 8 + 4$$

$$3x = 12$$

$$\underline{x = 4}$$

Exercise

Solve the equation: $3^{x+4} = 2^{1-3x}$

Solution

$\ln 3^{x+4} = \ln 2^{1-3x}$	*'ln' both sides*
$(x+4)\ln 3 = (1-3x)\ln 2$	*Power Rule*
$x\ln 3 + 4\ln 3 = \ln 2 - 3x\ln 2$	*Distribute*
$x\ln 3 + 3x\ln 2 = \ln 2 - 4\ln 3$	
$x(\ln 3 + 3\ln 2) = \ln 2 - 4\ln 3$	
$\underline{x = \dfrac{\ln 2 - 4\ln 3}{\ln 3 + 3\ln 2}}$	

Exercise

Solve the equation: $3^{2-3x} = 4^{2x+1}$

Solution

$\ln 3^{2-3x} = \ln 4^{2x+1}$	*'ln' both sides*
$(2-3x)\ln 3 = (2x+1)\ln 4$	*Power Rule*
$2\ln 3 - 3x\ln 3 = 2x\ln 4 + \ln 4$	

$$-3x \ln 3 - 2x \ln 4 = \ln 4 - 2 \ln 3$$

$$-x \left(3 \ln 3 + 2 \ln 4\right) = \ln 4 - 2 \ln 3$$

$$x = -\frac{\ln 4 - 2 \ln 3}{3 \ln 3 + 2 \ln 4}$$

$$= -\frac{\ln 4 - \ln 3^2}{\ln 3^3 + \ln 4^2}$$

$$= -\frac{\ln 9 - \ln 4}{\ln 27 + \ln 16}$$

$$= \frac{\ln \frac{9}{4}}{\ln 432}$$

$$= \log_{432} \frac{9}{4}$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3) \ln 4 = -x \ln 3$$

$$x \ln 4 + 3 \ln 4 = -x \ln 3$$

$$x \ln 4 + x \ln 3 = -3 \ln 4$$

$$x(\ln 4 + \ln 3) = -3 \ln 4$$

$$x = \frac{-3 \ln 4}{(\ln 4 + \ln 3)}$$

Exercise

Solve the equation: $7^{x+6} = 7^{3x-4}$

$$x + 6 = 3x - 4$$
$$4 + 6 = 3x - x$$
$$10 = 2x$$
$$x = 5$$

Exercise

Solve the equation: $2^{-100x} = (0.5)^{x-4}$

Solution

$$2^{-100x} = \left(\frac{1}{2}\right)^{x-4}$$

$$2^{-100x} = \left(2^{-1}\right)^{x-4}$$

$$2^{-100x} = 2^{-x+4}$$

$$-100x = -x + 4$$

$$-100x + x = 4$$

$$-99x = 4$$

$$\underline{x = -\frac{4}{99}}$$

Exercise

Solve the equation: $4^x \left(\frac{1}{2}\right)^{3-2x} = 8.\left(2^x\right)^2$

Solution

$$\left(2^2\right)^x \left(2^{-1}\right)^{3-2x} = 2^3.2^{2x}$$

$$2^{2x} 2^{2x-3} = 2^{3+2x}$$

$$2^{2x+2x-3} = 2^{3+2x}$$

$$2^{4x-3} = 2^{3+2x}$$

$$4x - 3 = 3 + 2x$$

$$4x - 2x = 3 + 3$$

$$2x = 6$$

$$\underline{x = 3}$$

Exercise

$$5^x + 125\left(5^{-x}\right) = 30$$

Solution

$$5^x 5^x + 125\left(5^{-x}\right)5^x = 30\left(5^x\right)$$

$$5^{2x} + 125 = 30\left(5^x\right)$$

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$$5^{2x} - 30\left(5^x\right) + 125 = 0 \qquad \textit{\textcolor{green}{Solve for }} 5^x$$

$$5^x = 5 \qquad\qquad 5^x = 25 = 5^2$$
$$x = 1 \qquad\qquad\quad x = 2$$

$$\underline{x = 1,\ 2}$$

Exercise

$$4^x - 3\left(4^{-x}\right) = 8$$

Solution

$$4^x 4^x - 3\left(4^{-x}\right)4^x = 8\left(4^x\right)$$

$$4^{2x} - 3 = 8\left(4^x\right)$$

$$4^{2x} - 8\left(4^x\right) - 3 = 0 \qquad \textit{\textcolor{green}{Solve for }} 4^x$$

$$4^x = 4 + \sqrt{19} \qquad\qquad 4^x = 4 - \sqrt{19} < 0$$
$$x \ln 4 = \ln\left(4 + \sqrt{19}\right)$$

$$\underline{x = \dfrac{\ln\left(4 + \sqrt{19}\right)}{\ln 4}}$$

Exercise

Solve the equation: $\quad 5^{3x-6} = 125$

Solution

$$5^{3x-6} = 5^3$$
$$3x - 6 = 3$$
$$3x = 9$$
$$\underline{x = 3}$$

Exercise

Solve the equation: $\quad e^x = 15$

Solution

$$\underline{x = \ln 5} \qquad\qquad \textit{\textcolor{green}{Convert to Log}}$$

Exercise

Solve the equation: $e^{x+1} = 20$

$x + 1 = \ln 20$ ***Convert to Log***

$\underline{x = -1 + \ln 20}$

Exercise

Solve the equation: $9e^x = 107$

$e^x = \dfrac{107}{9}$

$\ln e^x = \ln\left(\dfrac{107}{9}\right)$

$x \ln e = \ln\left(\dfrac{107}{9}\right)$

$\underline{x = \ln\left(\dfrac{107}{9}\right)}$

Exercise

Solve the equation: $e^{x \ln 3} = 27$

$x \ln 3 = \ln 27$ ***Convert to Log***

$x \ln 3 = \ln 3^3$

$x = \dfrac{3 \ln 3}{\ln 3}$

$\underline{= 3}$

Exercise

Solve the equation: $e^{x^2} = e^{7x-12}$

$e^{x^2} = e^{7x-12}$

$x^2 = 7x - 12$

$x^2 - 7x + 12 = 0$

$\underline{x = 3, \ 4}$

Exercise

Solve the equation: $f(x) = xe^x + e^x$

Solution

$$xe^x + e^x = 0$$

$$e^x(x+1) = 0$$

$$e^x \neq 0 \qquad x+1 = 0$$

$\underline{x = -1}$ (*Only solution*)

Exercise

Solve the equation $f(x) = x^3\left(4e^{4x}\right) + 3x^2e^{4x}$

Solution

$$x^3\left(4e^{4x}\right) + 3x^2e^{4x} = 0$$

$$x^2e^{4x}(4x+3) = 0$$

$$x^2 = 0 \qquad 4x+3 = 0$$

$$x = 0, \ 0 \qquad x = -\frac{3}{4}$$

The solutions are: $\underline{x = 0, \ 0, \ -\dfrac{3}{4}}$

Exercise

Solve the equation: $e^{2x} - 2e^x - 3 = 0$

Solution

$$\left(e^x\right)^2 - 2e^x - 3 = 0$$

$$\begin{cases} e^{\,x} = -1 \ \times \ \rightarrow \ \textit{Impossible} \\ e^{\,x} = 3 \quad \rightarrow \ \underline{x = \ln 3} \end{cases}$$

Exercise

Solve the equation: $e^{0.08t} = 2500$

Solution

$$\ln\left(e^{0.08t}\right) = \ln 2500$$

$$0.08t = \ln(50)^2$$

$$t = \frac{200\ln 50}{8}$$

$$\underline{= 25\ln 50}\ |$$

Exercise

Solve the equation: $\quad e^{x^2} = 200$

Solution

$$\ln e^{x^2} = \ln 200 \qquad\qquad \textit{Natural Log both sides}$$

$$x^2 = \ln 200 \qquad\qquad \ln e = 1$$

$$\underline{x = \pm\sqrt{\ln 200}}\ |$$

Exercise

Solve the equation: $\quad e^{2x+1} \cdot e^{-4x} = 3e$

Solution

$$e^{2x+1-4x} = 3e$$

$$e^{-2x+1} = 3e$$

$$e^{-2x}e = 3e \qquad\qquad \textit{Divide by e}$$

$$e^{-2x} = 3$$

$$\ln e^{-2x} = \ln 3$$

$$-2x = \ln 3$$

$$\underline{x = -\frac{1}{2}\ln 3}\ |$$

Exercise

Solve the equation: $\quad e^{2x} - 8e^x + 7 = 0$

Solution

$$\left(e^x\right)^2 - 8e^x + 7 = 0 \qquad\qquad a+b+c = 0 \;\rightarrow\; x = 1,\ \frac{c}{a}$$

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$$\begin{cases} e^x = 1 & \rightarrow & \underline{x = 0} \\ e^x = 7 & \rightarrow & \underline{x = \ln 7} \end{cases}$$

Exercise

Solve the equation without using the calculator: $e^{2x} + 2e^x - 15 = 0$

Solution

$$\left(e^x\right)^2 + 2e^x - 15 = 0 \qquad \qquad \textbf{\textit{Solve for }} \boldsymbol{e^x}$$

$$e^x = 3 \qquad \qquad \qquad e^x \not= -5 < 0$$

$$\underline{x = \ln 3}$$

Exercise

Solve the equation: $\quad e^x + e^{-x} - 6 = 0$

Solution

$$e^x e^x + e^x e^{-x} - e^x 6 = e^x 0$$

$$e^{2x} + 1 - 6e^x = 0$$

$$\left(e^x\right)^2 - 6e^x + 1 = 0$$

$$e^x = \frac{6 \pm \sqrt{36 - 4}}{2}$$

$$= \frac{6 \pm 4\sqrt{2}}{2}$$

$$e^x = 3 \pm 2\sqrt{2}$$

$$\underline{x = \ln\left(3 \pm 2\sqrt{2}\right)}$$

Exercise

Solve the equation: $\quad e^{1-3x} \cdot e^{5x} = 2e$

Solution

$$e^{1-3x+5x} = 2e$$

$$e^{1+2x} = 2e$$

$$e^1 e^{2x} = 2e \qquad \qquad \textit{Divide by e}$$

$$e^{2x} = 2 \qquad \qquad \textit{Natural Log both sides}$$

$$\ln e^{2x} = \ln 2$$
$$2x = \ln 2$$
$$\underline{x = \tfrac{1}{2}\ln 2}$$

Exercise

Solve the equation: $6\ln(2x) = 30$

Solution

$$\ln(2x) = \frac{30}{6}$$
$$\ln(2x) = 5$$
$$2x = e^5$$
$$\underline{x = \tfrac{1}{2}e^5}$$

Exercise

Solve the equation: $\log_5(x-7) = 2$

Solution

$$x - 7 = 5^2$$
$$x = 25 + 7$$
$$\underline{x = 32}$$

Exercise

Solve the equation: $\log_4(5+x) = 3$

Solution

$$5 + x = 4^3$$
$$x = 64 - 5$$
$$\underline{= 59} \qquad \textbf{\textit{Check}}: \log_4(5+59)$$

Exercise

Solve the equation: $\log(4x-18) = 1$

Solution

$$4x - 18 = 10$$
$$4x = 28$$
$$\underline{x = 7}$$

Exercise

Solve the equation: $\log\left(x^2 + 19\right) = 2$

Solution

$$x^2 + 19 = 10^2$$
$$x^2 = 81$$
$$\underline{x = \pm 9} \qquad \left(\pm 9\right)^2 + 19 > 0$$

Exercise

Solve the equation: $\ln\left(x^2 - 12\right) = \ln x$

Solution

$$\ln\left(x^2 - 12\right) = \ln x$$
$$x^2 - 12 = x$$
$$x^2 - x - 12 = 0$$
$$\underline{x = -3, \ 4}$$

Check: $x = -3 \quad \ln\left(9 - 12\right) = \ln\left(-3\right)$ ✗

$\qquad\qquad x = 4 \quad \ln\left(16 - 12\right) = \ln\left(4\right)$

∴ *Solution*: $\underline{x = 4}$

Exercise

Solve the equation: $\log\left(2x^2 + 3x\right) = \log\left(10x + 30\right)$

Solution

$$\log\left(2x^2 + 3x\right) = \log\left(10x + 30\right)$$
$$2x^2 + 3x = 10x + 30$$
$$2x^2 - 7x - 30 = 0$$

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$$x = \frac{7 \pm \sqrt{49 + 240}}{4}$$

$$= \begin{cases} \dfrac{7-17}{4} = -\dfrac{5}{2} \\ \dfrac{7+17}{4} = 6 \end{cases}$$

Check: $x = -\dfrac{5}{2}$ $\quad \log\left(\dfrac{25}{2} - \dfrac{15}{2}\right) = \log(-25 + 30)$

$\qquad\qquad x = 4 \quad \log(32 + 12) = \log(40 + 30)$

$\therefore$ *Solution*: $x = -\dfrac{5}{2},\ 4$

Exercise

Solve the equation: $\quad \log_5 (2x + 3) = \log_5 11 + \log_5 3$

Solution

$\log_5 (2x + 3) = \log_5 (11 \times 3)$

$2x + 3 = 33$

$2x = 30$

$\underline{x = 15}$ $\qquad\qquad$ *Check:* $\quad \log_5 (30 + 3)$

Exercise

Solve the equation: $\quad \log_3 x - \log_9 (x + 42) = 0$

Solution

$\dfrac{\log x}{\log 3} - \dfrac{\log(x + 42)}{\log 9} = 0$

$\dfrac{\log x}{\log 3} - \dfrac{\log(x + 42)}{\log 3^2} = 0$

$\dfrac{\log x}{\log 3} - \dfrac{1}{2}\dfrac{\log(x + 42)}{\log 3} = 0$

$\log x - \dfrac{1}{2}\log(x + 42) = 0$

$2\log x = \log(x + 42)$

$\log x^2 = \log(x + 42)$

$x^2 = x + 42$

$x^2 - x - 42 = 0$

$$x = -6, \ 7 \ |$$

Check: $\quad x = -6 \quad \log_3(-6) - \log_9(-6 + 42) \ \text{X}$

$$x = 7 \quad \log_3 7 - \log_9(7 + 42) = 0$$

$\therefore$ *Solution*: $\underline{x = 7} \ |$

Exercise

Solve the equation: $\quad \log_5 x + \log_5(4x - 1) = 1$

Solution

$$\log_5 x(4x - 1) = 1$$

$$4x^2 - x = 5$$

$$4x^2 - x - 5 = 0 \qquad\qquad a - b + c = 0 \quad \rightarrow \quad x = -1, \ -\frac{c}{a}$$

$$x = -\frac{5}{2}, \ 4 \ |$$

Check: $\quad x = -\frac{5}{2} \quad \log_5\left(-\frac{5}{2}\right) + \log_5(10 - 1) \ \text{X}$

$$x = 4 \quad \log_5(4) + \log_5(15)$$

$\therefore$ *Solution*: $\underline{x = 4} \ |$

Exercise

Solve the equation: $\quad \log x - \log(x + 3) = 1$

Solution

$$\log \frac{x}{x + 3} = 1$$

$$\frac{x}{x + 3} = 10$$

$$x = 10x + 30$$

$$9x = -30$$

$$x = -\frac{10}{3} \ |$$

Check: $\quad x = -\frac{10}{3} \quad \log\left(-\frac{10}{3}\right) - \log(x + 3) \ \text{X}$

$\therefore$ *No Solution*

Exercise

Solve the equation: $\log x + \log(x - 9) = 1$

Solution

$\log x(x - 9) = 1$

$x^2 - 9x = 10$

$x^2 - 9x - 10 = 0$ $a - b + c = 0 \rightarrow x = -1, \ -\dfrac{c}{a}$

$\underline{x = -1, \ 10 \ |}$

     Check: $x = -1$ $\log(-1) + \log(x - 9)$ ✗

                  $x = 10$ $\log(10) + \log(10 - 9)$

$\therefore$ *Solution*: $\underline{x = 10 \ |}$

Exercise

Solve the equation: $\log_2(x + 1) + \log_2(x - 1) = 3$

Solution

$\log_2(x + 1)(x - 1) = 3$

$x^2 - 1 = 2^3$

$x^2 = 9$

$\underline{x = \pm 3 \ |}$

     Check: $x = -3$ $\log_2(-2) + \log_2(x - 1)$ ✗

                  $x = 3$ $\log_2(4) + \log_2(2)$

$\therefore$ *Solution*: $\underline{x = 3 \ |}$

Exercise

Solve the equation: $\log_8(x + 1) - \log_8 x = 2$

Solution

$\log_8 \dfrac{x + 1}{x} = 2$

$\dfrac{x + 1}{x} = 8^2$

$x + 1 = 64x$

$63x = 1$

$$x = \frac{1}{63}$$

Check: $x = \frac{1}{63}$ $\log_8 \left(\frac{1}{63} + 1 \right) - \log_8 \frac{1}{63}$

$\therefore$ *Solution*: $x = \frac{1}{63}$

Exercise

Solve the equation: $\ln(x+8) + \ln(x-1) = 2\ln x$

Solution

$$\ln(x+8)(x-1) = \ln x^2$$

$$x^2 + 7x - 8 = x^2$$

$$7x - 8 = 0$$

$$x = \frac{8}{7}$$

Check: $x = \frac{8}{7}$ $\ln \left(\frac{8}{7} + 8 \right) + \ln \left(\frac{8}{7} - 1 \right) = 2\ln \frac{8}{7}$

$\therefore$ *Solution*: $x = \frac{8}{7}$

Exercise

Solve the equation: $\ln(4x+6) - \ln(x+5) = \ln x$

Solution

$$\ln \frac{4x+6}{x+5} = \ln x$$

$$\frac{4x+6}{x+5} = x$$

$$4x + 6 = x^2 + 5x$$

$$x^2 + x - 6 = 0$$

$$x = -3, \ 2$$

Check: $x = -3$ $\ln(-6) - \ln(x+5) = \ln x$ ✗

$\qquad\qquad\quad x = 2$ $\ln(14) - \ln(7) = \ln 2$

$\therefore$ *Solution*: $x = 2$

Exercise

Solve the equation: $\ln(5+4x) - \ln(x+3) = \ln 3$

Solution

$\ln \dfrac{5+4x}{x+3} = \ln 3$

$\dfrac{5+4x}{x+3} = 3$

$5 + 4x = 3x + 9$

$\underline{x = 4\,|}$

   Check: $x = 4$ $\ln(21) - \ln(7) = \ln 3$

$\therefore$ *Solution*: $\underline{x = 4\,|}$

Exercise

Solve the equation: $\ln \sqrt[4]{x} = \sqrt{\ln x}$

Solution

Domain: $\underline{x \geq 1\,|}$

$\ln x^{1/4} = \sqrt{\ln x}$

$\dfrac{1}{4} \ln x = \sqrt{\ln x}$

$\left(\dfrac{1}{4} \ln x\right)^2 = \left(\sqrt{\ln x}\right)^2$

$\dfrac{1}{6} \ln^2 x = \ln x$

$\ln^2 x = 6 \ln x$

$\ln^2 x - 6 \ln x = 0$

$(\ln x)(\ln x - 6) = 0$

$\begin{cases} \ln x = 0 \;\rightarrow\; \underline{x = 1\,|} \\ \ln x = 6 \;\rightarrow\; \underline{x = e^6\,|} \end{cases}$

$\therefore$ *Solution*: $\underline{x = 1,\; e^6\,|}$

Exercise

Solve the equation: $\sqrt{\ln x} = \ln \sqrt{x}$

Solution

Domain: $\underline{x \geq 1\,|}$

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$$\sqrt{\ln x} = \ln x^{1/2}$$

$$\sqrt{\ln x} = \frac{1}{2} \ln x$$

$$\left(\sqrt{\ln x}\right)^2 = \left(\frac{1}{2} \ln x\right)^2$$

$$\ln x = \frac{1}{4} \ln^2 x$$

$$4 \ln x = \ln^2 x$$

$$\ln^2 x - 4 \ln x = 0$$

$$\ln x (\ln x - 4) = 0$$

$$\begin{cases} \ln x = 0 \ \rightarrow \ \underline{x = 1} \\ \ln x = 4 \ \rightarrow \ \underline{x = e^4} \end{cases}$$

$\therefore$ *Solution*: $\underline{x = 1, \ e^4}$

Exercise

Solve the equation: $\log x^2 = (\log x)^2$

Solution

Domain: $\underline{x \geq 1}$

$$2 \log x = (\log x)^2$$

$$(\log x)^2 - 2 \log x = 0$$

$$\log x (\log x - 2) = 0$$

$$\begin{cases} \log x = 0 \ \rightarrow \ \underline{x = 1} \\ \log x = 2 \ \rightarrow \ \underline{x = 100} \end{cases}$$

$\therefore$ *Solution*: $\underline{x = 1, \ 100}$

Exercise

Solve the equation: $\log x^3 = (\log x)^2$

Solution

Domain: $\underline{x \geq 1}$

$$3 \log x = (\log x)^2$$

$$(\log x)^2 - 3 \log x = 0$$

$$\log x (\log x - 3) = 0$$

$$\begin{cases} \log x = 0 \;\rightarrow\; \underline{x = 1} \\ \log x = 3 \;\rightarrow\; \underline{x = 10^3} \end{cases}$$ *Convert to exponential*

$\therefore$ *Solution*: $\underline{x = 1,\; 10^3}$

Exercise

Solve the equation: $\log\left(\log x\right) = 1$

Solution

$\log x = 10$ *Convert to exponential*

$\therefore$ *Solution*: $\underline{x = 10^{10}}$

Exercise

Solve the equation: $\log\left(\log x\right) = 2$

Solution

$\log x = 10^2$ *Convert to exponential*

$\therefore$ *Solution*: $\underline{x = 10^{100}}$

Exercise

Solve the equation: $\ln\left(\ln x\right) = 2$

Solution

$\ln x = e^2$ *Convert to exponential*

$\therefore$ *Solution*: $\underline{x = e^{e^2}}$

Exercise

Solve the equation: $\ln\left(e^{x^2}\right) = 64$

Solution

$e^{x^2} = e^{64}$ *Convert to exponential*

$x^2 = 64$

$\therefore$ *Solution*: $\underline{x = \pm 8}$

Exercise

Solve the equation: $e^{\ln(x-1)} = 4$

Solution

$$x - 1 = 4$$

$\therefore$ *Solution*: $\underline{x = 5}$

Exercise

Solve the equation: $10^{\log(2x+5)} = 9$

Solution

$$2x + 5 = 9$$

$$2x = 4$$

$\therefore$ *Solution*: $\underline{x = 2}$

Exercise

Solve the equation: $\log \sqrt{x^3 - 9} = 2$

Solution

$$\sqrt{x^3 - 9} = 10^2$$

$$x^3 - 9 = 10^4$$

$$x^3 = 10,009$$

$\therefore$ *Solution*: $\underline{x = \sqrt[3]{10,009}}$

Exercise

Solve the equation: $\log \sqrt{x^3 - 17} = \dfrac{1}{2}$

Solution

$$\log\left(x^3 - 17\right)^{1/2} = \frac{1}{2}$$

$$\frac{1}{2}\log\left(x^3 - 17\right) = \frac{1}{2}$$

$$\log\left(x^3 - 17\right) = 1$$

$$x^3 - 17 = 10$$

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$x^3 = 27$

$\underline{x = 3\,|}$

    Check: $x = 3$ $\log \sqrt{27} - 17$

$\therefore$ *Solution*: $\underline{x = 3\,|}$

Exercise

Solve the equation: $\log_4 x = \log_4 (8 - x)$

Solution

$x = 8 - x$

$x + x = 8$

$2x = 8$

$\underline{x = 4|}$

    Check: $x = 4$ $\log_4 4 = \log_4 (8 - 4)$

$\therefore$ *Solution*: $\underline{x = 4\,|}$

Exercise

Solve the equation: $\log_7 (x - 5) = \log_7 (6x)$

Solution

$x - 5 = 6x$

$x - 6x = 5$

$-5x = 5$

$\underline{x = -1|}$

    Check: $x = -1$ $\log_7 (-6) = \log_7 (6x)$ ✕

$\therefore$ *No Solution*

Exercise

Solve the equation: $\ln x^2 = \ln (12 - x)$

Solution

$\ln x^2 = \ln (12 - x)$

$x^2 = 12 - x$

$x^2 + x - 12 = 0$

$x = -4, \ 3$

> **Check:** $x = -4$ $\ln(16) = \ln(16)$
>
> $x = 3$ $\ln(9) = \ln(12 - 3)$

$\therefore$ *Solution*: $x = -4, \ 3$

Exercise

Solve the equation $\log_2 (x + 7) + \log_2 x = 3$

Solution

$\log_2 x(x + 7) = 3$

$x(x + 7) = 2^3$ ***Convert to Exponential Form***

$x^2 + 7x = 8$

$x^2 + 7x - 8 = 0$

$x = 1, \ -8$

> **Check:** $x = -8$ $\log_2 (x + 7) + \log_2 (-8)$ $\times$
>
> $x = 1$ $\log_2 (1 + 7) + \log_2 1$

$\therefore$ *Solution*: $x = 1$

Exercise

Solve the equation $\ln x = 1 - \ln(x + 2)$

Solution

$\ln x + \ln(x + 2) = 1$

$\ln x(x + 2) = 1$

$x^2 + 2x = e$

$x^2 + 2x - e = 0$

$$x = \frac{-2 \pm \sqrt{4 + 4e}}{2}$$

$$= \frac{-2 \pm 2\sqrt{1 + e}}{2}$$

$$= \begin{cases} -1 - \sqrt{1 + e} < 0 \\ -1 + \sqrt{1 + e} > 0 \end{cases}$$

$\therefore$ *Solution*: $x = -1 + \sqrt{1 + e}$

Exercise

Solve the equation $\ln x = 1 + \ln(x+1)$

$\ln x - \ln(x+1) = 1$

$\ln \dfrac{x}{x+1} = 1$

$\dfrac{x}{x+1} = e^1$

$x = (x+1)e$

$x = ex + e$

$x - ex = e$

$x(1-e) = e$

$x = \dfrac{e}{1-e} < 0$

∴ *No solution*

Exercise

Solve the equation $\log_6(2x-3) = \log_6 12 - \log_6 3$

$\log_6(2x-3) = \log_6 \dfrac{12}{3}$

$\log_6(2x-3) = \log_6 4$

$2x - 3 = 4$

$2x = 7$

$\underline{x = \dfrac{7}{2}}$

Check: $x = \dfrac{7}{2}$ $\log_6(7-3) = \log_6 12 - \log_6 3$

∴ *Solution*: $\underline{x = \dfrac{7}{2}}$

Exercise

Solve the equation: $\log(3x+2) + \log(x-1) = 1$

Domain: $x > 1$

$\log(3x+2)(x-1) = 1$ *Convert to exponential form*

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$$3x^2 - x - 2 = 10$$

$$3x^2 - x - 12 = 0 \qquad \textit{Solve for x}$$

$$x = \frac{1 \pm \sqrt{1+144}}{6}$$

$$= \begin{cases} \dfrac{1-\sqrt{145}}{6} < 0 \\ \dfrac{1+\sqrt{145}}{6} > 1 \end{cases}$$

$$\therefore \textit{Solution}: \ x = \frac{1+\sqrt{145}}{6}$$

Exercise

Solve the equation: $\quad \log_5 (x+2) + \log_5 (x-2) = 1$

Solution

$$\log_5 (x+2)(x-2) = 1$$

$$(x+2)(x-2) = 5^1$$

$$x^2 - 4 = 5$$

$$x^2 = 5 + 4$$

$$x^2 = 9$$

$$\underline{x = \pm 3}$$

$\qquad$ ***Check***: $\quad x = -3 \quad \log_5 (-1) + \log_5 (x-2)$ ✗

$$\qquad\qquad\qquad x = 3 \quad \log_5 (3+2) + \log_5 (3-2)$$

$$\therefore \textit{Solution}: \ \underline{x = 3}$$

Exercise

Solve the equation: $\quad \log_2 x + \log_2 (x-4) = 2$

Solution

$\quad$ *Domain*: $\ x > 4$

$$\log_2 x(x-4) = 2$$

$$x^2 - 4x = 2^2$$

$$x^2 - 4x - 4 = 0$$

115

$$x = \frac{4 \pm \sqrt{32}}{2}$$

$$= \begin{cases} 2 - 2\sqrt{2} < 4 \text{✗} \\ 2 + 2\sqrt{2} > 4 \end{cases}$$

$\therefore$ *Solution*: $\underline{x = 2 + 2\sqrt{2}}$

Exercise

Solve the equation: $\log_3 x + \log_3 (x + 6) = 3$

<u>*Solution*</u>

Domain: $x > 0$

$\log_3 x(x + 6) = 3$

$x^2 + 6x = 3^3$

$x^2 + 6x - 27 = 0$

$x = \frac{-6 \pm \sqrt{36 + 108}}{2}$

$= \begin{cases} \frac{-6 - 12}{2} = -9 < 0 \text{✗} \\ \frac{-6 + 12}{2} = 3 > 0 \end{cases}$

$\therefore$ *Solution*: $\underline{x = 3}$

Exercise

Solve the equation: $\log_3 (x + 3) + \log_3 (x + 5) = 1$

<u>*Solution*</u>

Domain: $x > -3$

$\log_3 (x + 3)(x + 5) = 1$

$x^2 + 8x + 15 = 3$

$x^2 + 8x + 12 = 0$

$x = \frac{-8 \pm \sqrt{64 - 48}}{2}$

$= \begin{cases} \frac{-8 - 4}{2} = -6 < -3 \text{✗} \\ \frac{-8 + 4}{2} = -2 > -3 \end{cases}$

$\therefore$ *Solution*: $\underline{x = -2}$

Exercise

Solve the equation: $\ln x = \frac{1}{2}\ln\left(2x + \frac{5}{2}\right) + \frac{1}{2}\ln 2$

Solution

> *Domain*: $x > 0$
>
> $2\ln x = \ln\left(2x + \frac{5}{2}\right) + \ln 2$
>
> $\ln x^2 = \ln 2\left(2x + \frac{5}{2}\right)$
>
> $x^2 = 4x + 5$
>
> $x^2 - 4x - 5 = 0$ $\qquad\qquad a - b + c = 0 \;\rightarrow\; x = -1, \; -\dfrac{c}{a}$
>
> $\underline{x = -1, \; 5 \;|}$
>
> $\therefore$ *Solution*: $\underline{x = 5 \;|}$

Exercise

Solve the equation $\ln(-4 - x) + \ln 3 = \ln(2 - x)$

Solution

> *Domain*: $x < 5$
>
> $\ln 3(-4 - x) = \ln(2 - x)$
>
> $-12 - 3x = 2 - x$
>
> $-12 - 2 = 3x - x$
>
> $-14 = 2x$
>
> $\therefore$ *Solution*: $\underline{x = -7 \;|}$

Exercise

Solve the equation: $\log_4 x + \log_4 (x - 2) = \log_4 (15)$

Solution

> *Domain*: $x > 2$
>
> $\log_4 x(x - 2) = \log_4 (15)$
>
> $x^2 - 2x = 15$
>
> $x^2 - 2x - 15 = 0$
>
> $x = \dfrac{2 \pm \sqrt{4 + 60}}{2}$

$$= \begin{cases} \dfrac{2-8}{2} = -4 < 2 \text{✗} \\ \dfrac{2+8}{2} = 5 > 2 \end{cases}$$

$\therefore$ *Solution*: $x = 5$

Exercise

Solve the equation: $\ln(x-5) - \ln(x+4) = \ln(x-1) - \ln(x+2)$

Solution

Domain: $x > 5$

$\ln \dfrac{x-5}{x+4} = \ln \dfrac{x-1}{x+2}$

$\dfrac{x-5}{x+4} = \dfrac{x-1}{x+2}$

$(x-5)(x+2) = (x-1)(x+4)$

$x^2 + 2x - 5x - 10 = x^2 + 4x - x - 4$

$x^2 - 3x - 10 = x^2 + 3x - 4$

$x^2 - 3x - 10 - x^2 - 3x + 4 = 0$

$-6x - 6 = 0$

$x = -1$

$\therefore$ *No solution*

Exercise

Solve the equation: $\log\left(x^2+4\right) - \log\left(x+2\right) = 2 + \log\left(x-2\right)$

Solution

Domain: $x > -2$

$\log\left(x^2+4\right) - \log\left(x+2\right) - \log\left(x-2\right) = 2$

$\log\left(x^2+4\right) - \left[\log\left(x+2\right) + \log\left(x-2\right)\right] = 2$

$\log\left(x^2+4\right) - \log\left(x+2\right)\left(x-2\right) = 2$

$\log\left(\dfrac{x^2+4}{x^2-4}\right) = 2$

$\dfrac{x^2+4}{x^2-4} = 10^2$

$x^2 + 4 = 100x^2 - 400$

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$$400 + 4 = 100x^2 - x^2$$

$$99x^2 = 404$$

$$x^2 = \frac{404}{99}$$

$\therefore$ *Solution*: $\underline{x = \dfrac{2\sqrt{101}}{3\sqrt{11}}}$ is the only solution

Exercise

Solve the equation $\log_3 (x-2) = \log_3 27 - \log_3 (x-4) - 5^{\log_5 1}$

Solution

Domain: $x > 4$

$$\log_3 (x-2) + \log_3 (x-4) = \log_3 3^3 - 1$$

$$\log_3 (x-2)(x-4) = 3 - 1$$

$$\log_3 \left(x^2 - 6x + 8\right) = 2$$

$$x^2 - 6x + 8 = 3^2$$

$$x^2 - 6x + 8 = 9$$

$$x^2 - 6x - 1 = 0$$

$\rightarrow \quad \underline{x = 3 \pm \sqrt{10}}$

Check: $\quad x = 3 + \sqrt{10} > 4$

$$x = 3 - \sqrt{10} < 4 \text{ ✗}$$

$\therefore$ *Solution*: $\underline{x = 3 + \sqrt{10}}$

Exercise

Solve the equation $\log_2 (x+3) = \log_2 (x-3) + \log_3 9 + 4^{\log_4 3}$

Solution

Domain: $x > 3$

$$\log_2 (x+3) - \log_2 (x-3) = 2 + 3$$

$$\log_2 \frac{x+3}{x-3} = 5$$

$$\frac{x+3}{x-3} = 2^5$$

$$x + 3 = 32(x-3)$$

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$$x + 3 = 32x - 96$$
$$96 + 3 = 32x - x$$
$$31x = 99$$
$$x = \frac{99}{31} > 3$$

$$\therefore Solution:\ x = \frac{99}{31}$$

Exercise

Solve for *t* using logarithms with base *a*: $2a^{t/3} = 5$

Solution

$$a^{t/3} = \frac{5}{2}$$
$$\log a^{t/3} = \log \frac{5}{2}$$
$$\frac{t}{3} \log a = \log \frac{5}{2}$$
$$\frac{t}{3} = \frac{\log \frac{5}{2}}{\log a}$$
$$\frac{t}{3} = \log_a \frac{5}{2}$$
$$t = 3 \log_a \frac{5}{2}$$

Exercise

Solve for *t* using logarithms with base *a*: $K = H - Ca^t$

Solution

$$Ca^t = H - K$$
$$a^t = \frac{H - K}{C}$$
$$\log a^t = \log \frac{H - K}{C}$$
$$t \log a = \log \frac{H - K}{C}$$
$$t = \frac{\log \frac{H - K}{C}}{\log a}$$
$$= \log_a \frac{H - K}{C}$$