Section 4.2 – Exponential and Logarithmic Integrals

Using the Exponential Rule

Let u be a differentiable function of x

$$\int e^x dx = e^x + C \qquad \textit{Simple Exponential Rule}$$

$$\int e^u \frac{du}{dx} dx = \int e^u du$$

$$= e^u + C \qquad \textit{General Exponential Rule}$$

Example

Find each indefinite integral.

a. $\int 3e^x dx = 3\int e^x dx$

$$= 3e^x + C$$

b. $\int 5e^{5x} dx$

Let $u = 5x \to du = 5dx$

$$\int e^u du = e^u + C$$

$$= e^{5x} + C$$

c. $\int \left(e^x - x\right) dx$

$$\int \left(e^x - x\right) dx = \int e^x dx - \int x dx$$

$$= e^x - \frac{x^2}{2} + C$$

Example

Find indefinite integral $\int e^{2x+3}dx$

<u>*Solution*</u>

Let $u = 2x + 3 \rightarrow du = 2dx$

$$\int e^{2x+3}dx = \int e^{u}\,\frac{1}{2}\,du$$

$$= \frac{1}{2}\int e^{u}du$$

$$= \frac{1}{2}e^{u} + C$$

$$= \frac{1}{2}e^{2x+3} + C$$

Using the Log Rule

Integrals of Logarithmic Functions

Let u be a differentiable function of x.

$$\int \frac{1}{x}\,dx = \ln|x| + C$$ ***Simple Logarithmic Rule***

$$\int \frac{du/dx}{u}\,dx = \int \frac{1}{u}du = \ln|u| + C$$

$$= \ln|u| + C$$ ***General Logarithmic Rule***

Example

Find each indefinite integral.

a) $\int \frac{2}{x}\,dx = 2\int \frac{1}{x}\,dx$

$= 2\ln|x| + C$

b) $\int \frac{3x^2}{x^3}\,dx = 3\int \frac{1}{x}\,dx$

$= 3\ln|x| + C$

c) $\int \frac{2}{2x+1}\,dx$

Let $u = 2x + 1 \to du = 2dx$

$\int \frac{2}{2x+1}\,dx = \int \frac{2dx}{2x+1}$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|2x+1| + C$

Example

Find the indefinite integral. $\int \frac{1}{4x+1}dx$

<u>*Solution*</u>

Let $u = 4x+1 \rightarrow du = 4dx \rightarrow \frac{1}{4}du = dx$

$$\int \frac{1}{4x+1}dx = \int \frac{1}{u}\frac{1}{4}du$$

$$= \frac{1}{4}\int \frac{1}{u}du$$

$$= \frac{1}{4}\ln|u| + C$$

$$= \frac{1}{4}\ln|4x+1| + C$$

Exercise *Section* 4.2 – Exponential and Logarithmic Integrals

Find each indefinite integral.

1. $\int (2x+1)e^{x^2+x}dx$

2. $\int \frac{1}{6x-5}dx$

3. $\int \frac{x^2+2x+3}{x^3+3x^2+9x+1}dx$

4. $\int \frac{1}{x(\ln x)^2}dx$

5. $\int \frac{e^x}{1+e^x}dx$

6. $\int \frac{1}{x^3}e^{1/4x^2}dx$

7. $\int \frac{e^{1/\sqrt{x}}}{x^{3/2}}dx$

8. $\int \frac{-e^{3x}}{2-e^{3x}}dx$

9. $\int (6x+e^x)\sqrt{3x^2+e^x}\,dx$

10. $\int \frac{2(e^x-e^{-x})}{(e^x+e^{-x})^2}dx$

11. $\int \frac{x-3}{x+3}dx$

12. $\int \frac{5}{e^{-5x}+7}dx$

13. $\int \frac{4x^2-3x+2}{x^2}dx$

14. $\int \frac{2}{e^{-x}+1}\,dx$

15. $\int \frac{4x^2+2x+4}{x+1}\,dx$

16. $\int 4xe^{x^2}\,dx$

17. $\int \frac{3x}{x^2+4}\,dx$

18. $\int 12t^3e^{-t^4}\,dt$

19. $\int \frac{7e^{7x}}{3+e^{7x}}dx$

20. Under certain conditions, the number of diseased cells $N(t)$ at time t increases at a rate $N'(t) = Ae^{kt}$, where A is the rate of increase at time 0 (in cells per day) and k is a constant.

a) Suppose $A = 60$, and at 4 days, the cells are growing at a rate of 180 per day. Find a formula for the number of cells after t days, given that 200 cells are present at $t = 0$.

b) Use the answer from part (*a*) to find the number of cells present after 9 days.