Matrix Factorization

$$A = LU = \begin{pmatrix} \text{lower triangular } L \\ 1\text{'s on the diagonal} \end{pmatrix} \begin{pmatrix} \text{upper triangular } U \\ \text{pivots on the diagonal} \end{pmatrix}$$

$$A = LDU = \begin{pmatrix} \text{lower triangular } L \\ 1\text{'s on the diagonal} \end{pmatrix} \begin{pmatrix} \text{pivot matrix} \\ D \text{ is diagonal} \end{pmatrix} \begin{pmatrix} \text{upper triangular } U \\ 1\text{'s on the diagonal} \end{pmatrix}$$

$PA = LU$ (Permutation matrix P to avoid zeros in the pivot positions).

$EA = R$ (m by m invertible E) (any A) = rref(A)

$A = CC^T$ = (lower triangular matrix C) (transpose is upper triangular)

$A = QR$ = (orthonormal columns in Q) (upper triangular R)

$A = S\Lambda S^{-1}$ = (eigenvectors in S) (eigenvalues in Λ) (left eigenvectors in S^{-1}).

$A = \mathbf{Q}\Lambda\mathbf{Q}^{\mathbf{T}}$ = (orthogonal matrix Q) (real eigenvalue matrix Λ) $\left(Q^T \text{ is } Q^{-1}\right)$

$A = MJM^{-1}$ = (generalized eigenvectors in M) (**J**ordan blocks in J) $\left(\mathrm{M}^{-1}\right)$

$$A = U\Sigma V^{\mathbf{T}} = \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ singular value matrix} \\ \delta_1,\cdots,\delta_r \text{ on its diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix}$$

$$A^{+} = V\Sigma^{+} U^{\mathbf{T}} = \begin{pmatrix} \text{orthogonal} \\ n\times n \end{pmatrix} \begin{pmatrix} n\times m \text{ pseudoinverse of } \Sigma \\ 1/\delta_1,\cdots,1/\delta_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ m\times m \end{pmatrix}$$

$A = QH$ = (orthogonal matrix Q) (symmetric positive definite matrix H)

$A = U\Lambda U^{-1}$ = (unitary U) (eigenvalue matrix Λ) (U^{-1}which $U^H = \overline{U}^T$).

$A = UTU^{-1}$ = (unitary U) (triangular T with λ 's on diagonal) ($U^{-1} = U^H$).

$$F_n = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_{n/2} & \\ & F_{n/2} \end{bmatrix} \begin{bmatrix} \text{even} - \text{odd} \\ \text{permutation} \end{bmatrix}$$ = one step of the **FFT**.