Derivation *of a projectile with wind from a point*

Let the wind is given by:

$$wind = w_x\ \hat{i} + w_y\ \hat{j}$$

Let initial position is given by:

$$\vec{r}_0 = x_0\ \hat{i} + y_0\ \hat{j}$$

The velocity is given by:

$$\vec{v} = \left(v_0 \cos\alpha\right)\ \hat{i} + \left(v_0 \sin\alpha\right)\ \hat{j} + w_x\ \hat{i} + w_y\ \hat{j}$$

$$= \left(v_0 \cos\alpha + w_x\right)\ \hat{i} + \left(v_0 \sin\alpha + w_y\right)\ \hat{j}$$

$$\vec{r}(t) = -\frac{1}{2} g t^2\ \hat{j} + \vec{v}\, t + \vec{r}_0$$

$$= -\frac{1}{2} g t^2\ \hat{j} + \left(\left(v_0 \cos\alpha + w_x\right)\ \hat{i} + \left(v_0 \sin\alpha + w_y\right)\ \hat{j}\right) t + x_0\ \hat{i} + y_0\ \hat{j}$$

$$= \underbrace{\left(x_0 + \left(v_0 \cos\alpha + w_x\right) t\right)}_{x(t)}\ \hat{i} + \underbrace{\left(-\frac{1}{2} g t^2 + \left(v_0 \sin\alpha + w_y\right) t + y_0\right)}_{y(t)}\ \hat{j}$$

$$y(t) = -\frac{1}{2} g t^2 + \left(v_0 \sin\alpha + w_y\right) t + y_0$$

Maximum time when the object is at maximum height.

$$y' = -gt + v_0 \sin\alpha + w_y = 0$$

$$\boxed{t_{Max} = \frac{v_0 \sin\alpha + w_y}{g}}$$

$$y(t) = -\frac{1}{2} g \left(\frac{v_0 \sin\alpha + w_y}{g}\right)^2 + \left(v_0 \sin\alpha + w_y\right)\left(\frac{v_0 \sin\alpha + w_y}{g}\right) + y_0$$

$$= -\frac{1}{2} \frac{\left(v_0 \sin\alpha + w_y\right)^2}{g} + \frac{\left(v_0 \sin\alpha + w_y\right)^2}{g} + y_0$$

$$\boxed{= \frac{\left(v_0 \sin\alpha + w_y\right)^2}{2g} + y_0} \qquad \textit{Maximum Height}$$

$$y(t) = -\frac{1}{2} g t^2 + \left(v_0 \sin\alpha + w_y\right) t + y_0 = 0$$

$$t_{1,\,2} = \frac{-\left(v_0 \sin\alpha + w_y\right) \pm \sqrt{\left(v_0 \sin\alpha + w_y\right)^2 - 4\left(-\frac{1}{2}g\right)y_0}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{v_0 \sin\alpha + w_y \mp \sqrt{\left(v_0 \sin\alpha + w_y\right)^2 + 2gy_0}}{g}$$

Range:

$$Range = x_0 + \left(v_0 \cos\alpha + w_x\right)t$$

$$= x_0 + \left(v_0 \cos\alpha + w_x\right)\frac{v_0 \sin\alpha + w_y + \sqrt{\left(v_0 \sin\alpha + w_y\right)^2 + 2gy_0}}{g}$$

If $y_0 = 0$

$$Range = x_0 + \left(v_0 \cos\alpha + w_x\right)\frac{v_0 \sin\alpha + w_y + \sqrt{\left(v_0 \sin\alpha + w_y\right)^2}}{g}$$

$$= x_0 + \frac{2\left(v_0 \sin\alpha + w_y\right)\left(v_0 \cos\alpha + w_x\right)}{g}$$

$$= x_0 + \frac{2}{g}\left(v_0^2 \sin\alpha\cos\alpha + v_0 w_x \sin\alpha + v_0 w_y \cos\alpha + w_x w_y\right)$$

$$= x_0 + \frac{v_0^2 \sin 2\alpha}{g} + \frac{2v_0\left(w_x \sin\alpha + w_y \cos\alpha\right)}{g} + \frac{2w_x w_y}{g}$$