$$3y'' + xy' - 4y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

$$3y'' + xy' - 4y = 0$$

$$3\sum_{n=0}^{\infty}(n+2)(n+1) a_{n+2} x^n + x\sum_{n=1}^{\infty} n a_n x^{n-1} - 4\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty}\left(3(n+1)(n+2) a_{n+2} - 4a_n\right)x^n + \sum_{n=0}^{\infty} n a_n x^n = 0$$

$$\sum_{n=0}^{\infty}\left[3(n+1)(n+2) a_{n+2} + (n-4) a_n\right] x^n = 0$$

$$a_{n+2} = -\frac{n-4}{3(n+1)(n+2)} a_n$$

$n$      $a_0$      $a_1$

0   $a_2 = +\frac{-4}{6} a_0 = \frac{2}{3} a_0$    $a_3 = +\frac{1}{6} a_1 = \frac{1}{3!} a_1$

2   $a_4 = \frac{2}{36} a_2 = \frac{1}{27} a_0$    3   $a_5 = \frac{1}{4 \cdot 5 \cdot 3} a_3 = \frac{1}{5!3} a_1$

4   $a_6 = 0$    5   $a_7 = \frac{-1}{3 \cdot 6 \cdot 7} a_5 = \frac{-1}{7!3^2} a_1$

7   $a_9 = -\frac{3}{3(8)(9)} a_7 = \frac{3}{9!3^3} a_1$

$$y(t) = a_0\left(1 + \frac{2}{3}x^2 + \frac{1}{27}x^4\right) + a_1\left(x + \frac{1}{3!}x^3 + \frac{1}{5!3}x^5 + \cdots\right)$$

$$y'' + x^2 y = 0 \qquad y(0) = 1 \quad y'(0) = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n + x^2 \sum_{n=0} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n + \sum_{n=0}^{\infty} a_n x^{n+2}$$

$$2a_2 + 6a_3 x + \sum_{n=2}^{\infty} (n+1)(n+2) a_{n+2} x^n + \sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$$2a_2 + 6a_3 x + \sum_{n=2}^{\infty} [(n+1)(n+2) a_{n+2} + a_{n-2}] x^n = 0$$

$$a_{n+2} = -\frac{1}{(n+1)(n+2)} a_{n-2}$$

$$2a_2 + 6a_3 x = 0$$

$\underline{a_2 = a_3 = 0}$ Given $y(0) = 1 = a_0$ $y'(0) = 0 = a_1$

$a_0 = 1$ $\qquad$ $a_1 = a_2 = a_3 = 0$

$n=2$ $\quad a_4 = -\frac{1}{12}$ $\qquad$ $n=3$ $\quad a_5 = -a_1 = 0$

$4 \quad a_6 = \cancel{0}\, a_2 = 0$

$5 \quad a_7 = 0 \cdot a_3 = 0$

$n=6$ $\quad a_8 = -\frac{1}{7.8} a_4$

$= \frac{+1}{3.4.7.8}$

$$y(x) = 1 - \frac{1}{12} x^4 + \frac{1}{3.4.7.8} x^8 - \cdots$$

[illegible] $y(0) = 1$ $\quad y'(0) = 0$

$a_0 = 1 \qquad a_1 = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n + x \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[(n+1)(n+2) a_{n+2} + a_n\right] x^n + \sum_{n=0}^{\infty} n a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[(n+1)(n+2) a_{n+2} + (n+1) a_n\right] x^n = 0$$

$$a_{n+2} = -\frac{1}{n+2} a_n$$

$a_0 = 1 \qquad\qquad a_1 = 0$

$n=0 \quad a_2 = -\frac{1}{2} \qquad\qquad a_3 = a_1 = 0$

$n=2 \quad a_4 = -\frac{1}{4} a_2 = \frac{1}{8}$

$n=4 \quad a_6 = -\frac{1}{6} a_4 = -\frac{1}{2^4 3}$

$$y(x) = 1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 - \frac{1}{48} x^6 + \cdots$$

$(x^2 y'' + y' + y = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$x^2 \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} n(n-1) a_n x^n + \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[ (n+1) a_{n+1} + (\underbrace{n(n-1)}_{n^2-n} + 1) a_n \right] x^n = 0$$

$$a_{n+1} = -\frac{n^2-n+1}{n+1} \cdot a_n$$

$a_0$

$n=0$ $\quad a_1 = -a_0$

$n=1$ $\quad a_2 = -\frac{1}{2} a_1 = \frac{1}{2} a_0$

$n=2$ $\quad a_3 = -\frac{3}{3} a_2 = -\frac{1}{2} a_0$

$n=3$ $\quad a_4 = -\frac{7}{4} a_3 = \frac{7}{8} a_0$

$$y(x) = a_0 \left( 1 - x + \frac{1}{2} x^2 - \frac{1}{2} x^3 + \frac{7}{8} x^4 - \cdots \right)$$