Section 1.2 – Exponents

Integer Exponents

Definition of exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdots \cdot a}_{n-times}$$ a appears as a factor n times

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^m \cdot a^n = a^{m+n}$$

$$\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

$$\left(a^m\right)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$(ab)^m = a^m b^m$$

a) 6^0

$$6^0 = 1$$

b) $(-9)^0$

$$(-9)^0 = 1$$

c) 3^{-2}

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

d) $\left(\frac{3}{4}\right)^{-1}$

$$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$$

a) $7^4 \cdot 7^6$

$$7^4 \cdot 7^6 = 7^{4+6} = 7^{10}$$

b) $\dfrac{9^{14}}{9^6}$

$$\frac{9^{14}}{9^6} = 9^{14-6} = 9^8$$

c) $\dfrac{r^9}{r^{17}}$

$$\frac{r^9}{r^{17}} = \frac{1}{r^{17-9}} = \frac{1}{r^8}$$

d) $\left(2m^3\right)^4$

$$\left(2m^3\right)^4 = (2)^4 \left(m^3\right)^4$$
$$= 16m^{12}$$

e) $\left(\dfrac{x^2}{y^3}\right)^6$

$$\left(\frac{x^2}{y^3}\right)^6 = \frac{\left(x^2\right)^6}{\left(y^3\right)^6}$$
$$= \frac{x^{2.6}}{y^{3.6}}$$
$$= \frac{x^{12}}{y^{18}}$$

f) $\dfrac{a^{-3}b^5}{a^4 b^{-7}}$

$$\frac{a^{-3}b^5}{a^4 b^{-7}} = \frac{b^5 b^7}{a^3 a^4}$$
$$= \frac{b^{5+7}}{a^{4+3}}$$
$$= \frac{b^{12}}{a^7}$$

g) $p^{-1} + q^{-1}$

$$p^{-1} + q^{-1} = \frac{1}{p} + \frac{1}{q}$$

$$= \frac{1}{p}\frac{q}{q} + \frac{1}{q}\frac{p}{p}$$

$$= \frac{q+p}{pq}$$

h) $\dfrac{x^{-2} - y^{-2}}{x^{-1} - y^{-1}}$

$$\frac{x^{-2} - y^{-2}}{x^{-1} - y^{-1}} = \frac{\dfrac{1}{x^2} - \dfrac{1}{y^2}}{\dfrac{1}{x} - \dfrac{1}{y}}$$

$$= \frac{\dfrac{y^2 - x^2}{x^2 y^2}}{\dfrac{y - x}{xy}}$$

$$= \frac{y^2 - x^2}{x^2 y^2} \cdot \frac{xy}{y - x}$$

$$= \frac{(y-x)(y+x)}{(xy)^2} \cdot \frac{xy}{y-x}$$

$$= \frac{y + x}{xy}$$

Calculations with exponents

a) $121^{1/2} = 11$

b) $625^{1/4} = 5$

c) $(-32)^{1/5} = -2$

d) $(-49)^{1/2}$ *is not a real number*

Rational Exponents

$$a^{m/n} = \left(a^{1/n}\right)^m$$

Calculations with Exponents

a) $27^{2/3}$ 27^(2/3)

$$27^{2/3} = \left(27^{1/3}\right)^2$$

$$= \left(\left(3^3\right)^{1/3}\right)^2$$

$$= \left(3^{3 \cdot \frac{1}{3}}\right)^2$$

$$= (3)^2$$

$$= 9$$

b) $32^{2/5}$ 32^(2/5)

$$32^{2/5} = \left(\left(2^5\right)^{1/5}\right)^2$$

$$= 2^2$$

$$= 4$$

c) $64^{4/3}$ 64^(4/3)

$$64^{4/3} = \left(\left(4^3\right)^{1/3}\right)^4$$

$$= (4)^4$$

$$= 256$$

a) $\dfrac{y^{1/3}y^{5/3}}{y^3}$

$$\dfrac{y^{1/3}y^{5/3}}{y^3} = \dfrac{y^{\frac{1}{3}+\frac{5}{3}}}{y^3}$$

$$= \dfrac{y^{\frac{6}{3}}}{y^3}$$

$$= \dfrac{y^2}{y^3}$$

$$= \dfrac{1}{y^{3-2}}$$

$$= \dfrac{1}{y}$$

b) $m^{2/3}\left(m^{7/3}+7m^{1/3}\right)$

$$m^{2/3}\left(m^{7/3}+7m^{1/3}\right) = m^{2/3}m^{7/3}+7m^{2/3}m^{1/3}$$

$$= m^{\frac{2}{3}+\frac{7}{3}}+7m^{\frac{2}{3}+\frac{1}{3}}$$

$$= m^{\frac{9}{3}}+7m^{\frac{3}{3}}$$

$$= m^3+7m$$

c) $\left(\dfrac{m^7 n^{-2}}{m^{-5}n^2}\right)^{1/4}$

$$\left(\dfrac{m^7 n^{-2}}{m^{-5}n^2}\right)^{1/4} = \left(\dfrac{m^{7+5}}{n^{2+2}}\right)^{1/4}$$

$$= \left(\dfrac{m^{12}}{n^4}\right)^{1/4}$$

$$= \dfrac{\left(m^{12}\right)^{1/4}}{\left(n^4\right)^{1/4}}$$

$$= \dfrac{m^{12/4}}{n^{4/4}}$$

$$= \dfrac{m^3}{n}$$

Simplify

a) $4m^{1/2} + 3m^{3/2}$

$$4m^{1/2} + 3m^{3/2} = m^{1/2}\left(4m^{1/2-1/2} + 3m^{3/2-1/2}\right)$$
$$= m^{1/2}(4 + 3m)$$

b) $9x^{-2} - 6x^{-3}$

$$9x^{-2} - 6x^{-3} = 3x^{-3}(3x - 2)$$

c) $2\left(x^2 + 5\right)(3x-1)^{-1/2} + (3x-1)^{1/2}(2x)$

$$2\left(x^2+5\right)(3x-1)^{-1/2} + (3x-1)^{1/2}(2x) = 2(3x-1)^{-1/2}\left[x^2 + 5 + x(3x-1)\right]$$
$$= 2(3x-1)^{-1/2}\left[x^2 + 5 + 3x^2 - x\right]$$
$$= 2(3x-1)^{-1/2}\left(4x^2 - x + 5\right)$$

Radicals

$$a^{1/n} = \sqrt[n]{a}$$

a) $\sqrt[4]{16}$

$$\sqrt[4]{16} = 16^{1/4} = 2$$

b) $\sqrt[5]{-32} = -2$

c) $\sqrt[3]{1000}$

$$\sqrt[3]{1000} = 1000^{1/3} = 10$$

d) $\sqrt[6]{\dfrac{64}{729}}$

$$\sqrt[6]{\frac{64}{729}} = \frac{\sqrt[6]{64}}{\sqrt[6]{729}} = \frac{2}{3}$$

Properties

$$\left(\sqrt[n]{a}\right)^n = a$$

$$\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n \text{ is even} \\ a & \text{if } n \text{ is odd} \end{cases}$$

$$\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

Simplify

a) $\sqrt{1000}$

$$\sqrt{1000} = \sqrt{100(10)}$$

$$= \sqrt{100}\sqrt{10}$$
$$= 10\sqrt{10}$$

b) $\sqrt{128}$

$$\sqrt{128} = \sqrt{64(2)}$$
$$= 8\sqrt{2}$$

c) $\sqrt{2}\sqrt{18}$

$$\sqrt{2}\sqrt{18} = \sqrt{2(18)}$$
$$= \sqrt{36}$$
$$= 6$$

d) $\sqrt[3]{54}$

$$\sqrt[3]{54} = \sqrt[3]{27(2)}$$
$$= 3\sqrt[3]{2}$$

e) $\sqrt{288m^5}$

$$\sqrt{288m^5} = \sqrt{144(2)m^4 m}$$
$$= 12m^2\sqrt{2m}$$

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f) $2\sqrt{18} - 5\sqrt{32}$

$$2\sqrt{18} - 5\sqrt{32} = 2\sqrt{9(2)} - 5\sqrt{16(2)}$$
$$= 6\sqrt{2} - 20\sqrt{2}$$
$$= -14\sqrt{2}$$