

Solution **Section 4.2 - Exponential Functions**

Exercise

Find $2^{3.4}$

Solution

$$2^{3.4} = 10.5561$$

Exercise

Find $5^{\sqrt{3}}$

Solution

$$5^{\sqrt{3}} = 16.2425$$

Exercise

Find $6^{-1.2}$

Solution

$$6^{-1.2} = 0.1165$$

Exercise

Evaluate to four decimal places using a calculator: $e^{-0.75}$

Solution

$$e^{-0.75} = .4724$$

Exercise

Evaluate to four decimal places using a calculator: $e^{2.3}$

Solution

$$e^{2.3} = 9.9742$$

Exercise

Evaluate to four decimal places using a calculator: $e^{-0.95}$

Solution

$$e^{-0.95} = 0.3867$$

Exercise

Sketch the graph: $f(x) = 2^x + 3$

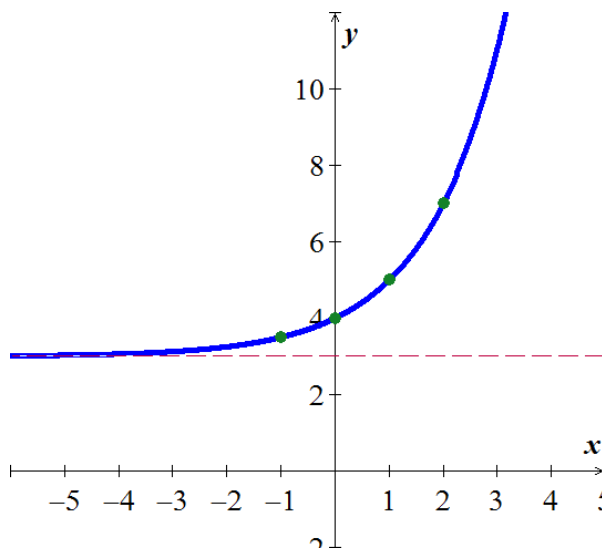
Solution

Asymptote: $y = 3$

Domain: $(-\infty, \infty)$

Range: $(3, \infty)$

x	$f(x)$
-1	3.5
0	4
1	5
2	7



Exercise

Sketch the graph: $f(x) = 2^{3-x}$

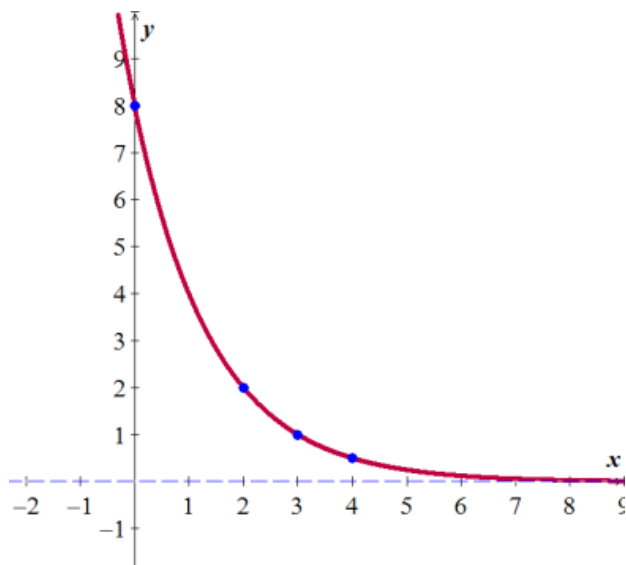
Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
1	4
2	2
3	1
4	.5



Exercise

Sketch the graph: $f(x) = \left(\frac{2}{5}\right)^{-x}$

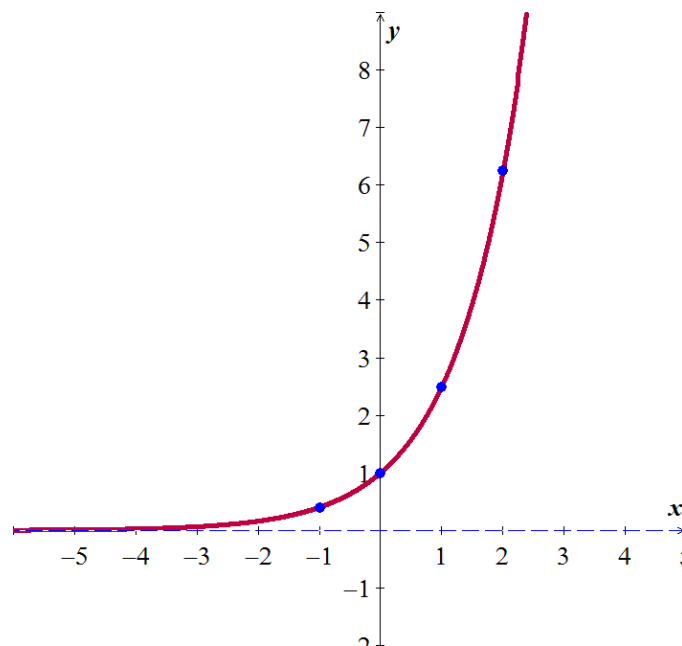
Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
-1	0.4
0	1
1	2.5
2	6.25



Exercise

Sketch the graph: $f(x) = -\left(\frac{1}{2}\right)^x + 4$

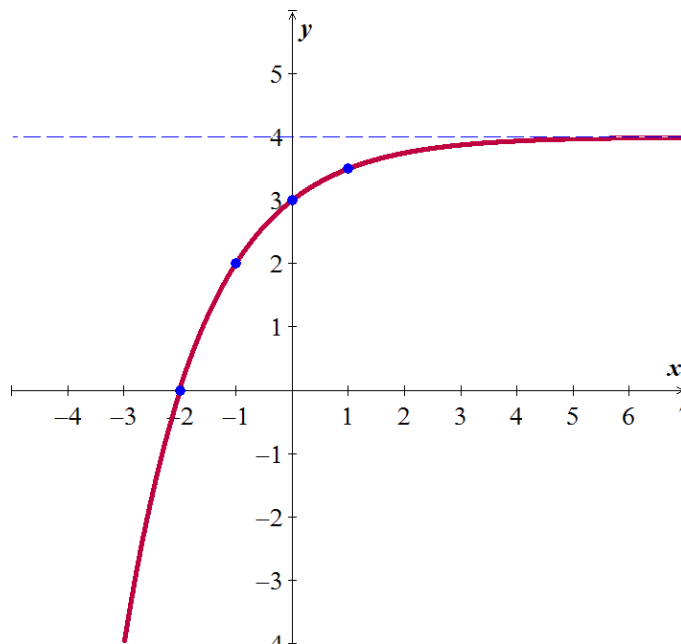
Solution

Asymptote: $y = 4$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4)$

x	$f(x)$
-2	0
-1	2
0	3
1	3.5



Exercise

Sketch the graph of $f(x) = e^{-x+4}$

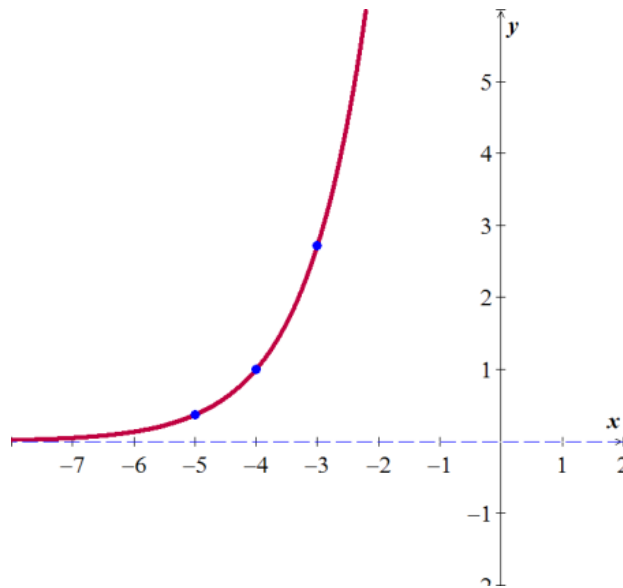
Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
-5	0.4
-4	1
-3	2.7



Exercise

The exponential function $f(x) = 1066e^{0.042x}$ models the gray wolf population of the Western Great Lakes, $f(x)$, in billions, x years after 1978. Project the gray population in the recovery area in 2012.

Solution

$$x = 2012 - 1978 = 34$$

$$\begin{aligned} f(x=34) &= 1066e^{0.042(34)} && 1066 e^{(.042 * 34)} \\ &= 4445.6 \\ &\approx 4446 \end{aligned}$$

Exercise

The function $f(x) = 6.4e^{0.0123x}$ describes world population, $f(x)$, in billions, x years after 2004 subject to a growth rate of 1.23% annually. Use the function to predict world population in 2050.

Solution

$$x = 2050 - 2004 = 46$$

$$\begin{aligned} f(x=46) &= 6.4e^{0.0123(46)} && 6.4 e^{(0.0123 * 46)} \\ &\approx 11.27 \text{ billion} \end{aligned}$$

Exercise

Find the accumulated value of an investment of \$10,000 for 5 years at an interest rate of 5.5% if the money is

Solution

Given: $P = 10000$

$$t = 5$$

$$r = 0.055$$

a. Semiannually: $n = 2$

$$\begin{aligned}\Rightarrow A &= 10000 \left(1 + \frac{0.055}{2}\right)^{2(5)} && 10000 (1 + .055 / 2) ^ (2 * 5) \\ &= \$13116.51\end{aligned}$$

b. Quarterly: $n = 4$

$$\begin{aligned}\Rightarrow A &= 10000 \left(1 + \frac{0.055}{4}\right)^{4(5)} && 10000 (1 + .055 / 4) ^ (4 * 5) \\ &= \$13140.67\end{aligned}$$

c. Monthly: $n = 12$

$$\begin{aligned}\Rightarrow A &= 10000 \left(1 + \frac{0.055}{12}\right)^{12(5)} && 10000 (1 + .055 / 12) ^ (12 * 5) \\ &= \$13157.04\end{aligned}$$

$$\begin{aligned}d. \quad A &= 10000e^{(0.055)(5)} = \$13165.31 && 10000 e ^ (0.055 * 5) \\ &= \$13165.31\end{aligned}$$

Exercise

Suppose \$1000 is deposited in an account paying 4% interest per year compounded quarterly.

a) Find the amount in the account after 10 years with no withdrawals.

b) How much interest is earned over the 10 years period?

Solution

Given:

$$P = 1000$$

$$r = .04$$

$$n = 4$$

a) $t = 10$

$$\begin{aligned}A &= P \left(1 + \frac{r}{n}\right)^{tn} \\ A &= 1000 \left(1 + \frac{.04}{4}\right)^{10(4)} && 1000(1 + .04 / 4) ^ (4 * 10)\end{aligned}$$

$$= \$1,488.86$$

b) The interest earned: $\$1488.86 - \$1000 = \$488.86$

Exercise

Becky must pay a lump sum of \$6000 in 5 yrs.

- a) What amount deposited today at 3.1% compounded annually will grow to \$6000 in 5 yrs.?
 b) If only \$5000 is available to deposit now, what annual interest rate is necessary for the money to increase to \$6000 in 5 yrs.?

Solution

$$a) \quad A = P \left(1 + \frac{r}{n} \right)^{tn}$$

$$6000 = P \left(1 + \frac{.031}{1} \right)^{5(1)}$$

$$6000 = P(1.031)^5$$

$$\frac{6000}{(1.031)^5} = P$$

$$6000 / 1.031^5$$

$$P \approx \$5,150.60$$

$$b) \quad A = P \left(1 + \frac{r}{n} \right)^{tn}$$

$$6000 = 5000 \left(1 + \frac{r}{1} \right)^{5(1)}$$

$$\frac{6000}{5000} = (1 + r)^5$$

$$\frac{6}{5} = (1 + r)^5$$

$$\left(\frac{6}{5} \right)^{1/5} = 1 + r$$

$$r = \left(\frac{6}{5} \right)^{1/5} - 1$$

$$(6/5)^{(1/5)} - 1$$

$$\approx .0371$$

The interest rate of 3.71% will produce enough to increase the \$5,000 to \$6,000 by the end of 5 yr.

Exercise

An investment of 1,000 increased to \$13,464 in 20 years. If the interest was compounded continuously, find the interest rate.

Solution

$$A = Pe^{rt}$$

$$13464 = 1000e^{20r}$$

$$13.464 = e^{20r}$$

$$\ln(13.464) = \ln e^{20r}$$

$$20r = \ln 13.464$$

$$r = \frac{\ln 13.464}{20} \approx 0.13$$

The interest rate is 13%.

Exercise

Find the present value of \$4,000 if the annual interest rate is 3.5% compounded quarterly for 6 years.

Solution

Given: $A = 4000.00$, $r = 0.035$, $t = 6$, $n = 4$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$4000 = P\left(1 + \frac{0.035}{4}\right)^{4(6)}$$

$$P = \frac{4000}{\left(1 + \frac{0.035}{4}\right)^{4(6)}}$$

$= \$3245.30$



4000/(1+.035/4)^{4*6}
3245.30

Exercise

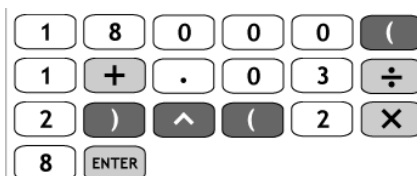
How much money will there be in an account at the end of 8 years if \$18,000 is deposited at 3% interest compounded semi-annually?

Solution

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 18000\left(1 + \frac{0.03}{2}\right)^{2(8)}$$

$= \$22,841.74$



18000(1+.03/2)^(2*8)
22841.73986

Exercise

The function defined by $P(x) = 908e^{-0.0001348x}$ approximates the atmospheric pressure (in millibars) at an altitude of x meters. Use P to predict the pressure:

- At 0 meters
- At 12,000 meters

Solution

- At 0 meters

$$P(x=0) = 908e^{-0.0001348(0)} = 908 \text{ millibars}$$

W 9 Q	CATALOG 0	V 8 P	2ND	e ^x LN S	ANS (-)
i .	CATALOG 0	CATALOG 0	CATALOG 0	L1 Y 1	L3 3
L4 4 T	V 8 P	[X R	CATALOG 0	ENTRY/SOLVE ENTER	

908e^{-0.0001348*0}
908.00

- At 12,000 meters

$$P(x=12,000) = 908e^{-0.0001348(12,000)} = 180 \text{ millibars}$$

CLEAR	W 9 Q	CATALOG 0	V 8 P	2ND	e ^x LN S
{ (K	ANS (-)	i .	CATALOG 0	CATALOG 0	CATALOG 0
L1 Y 1	L3 3	L4 4 T	V 8 P	[X R	L1 Y 1
L2 2 Z	CATALOG 0	CATALOG 0	CATALOG 0	ENTRY/SOLVE ENTER	

908e^(-0.0001348*12000)
180.12