Review

$$\frac{\cot^2\theta + 3\cot\theta - 4}{\cot\theta + 4} = \cot\theta - 1$$

$$\frac{\cot^2\theta + 3\cot\theta - 4}{\cot\theta + 4} = \frac{(\cot\theta + 4)(\cot\theta - 1)}{\cot\theta + 4}$$

$$= \cot\theta - 1 \quad \checkmark$$

$$\frac{\tan x - 1}{\tan x + 1} = \frac{1 - \cot x}{1 + \cot x}$$

$$\frac{\tan x - 1}{\tan x + 1} = \frac{\frac{1}{\cot x} - 1}{\frac{1}{\cot x} + 1}$$

$$= \frac{\frac{1 - \cot x}{\cot x}}{\frac{1 + \cot x}{\cot x}}$$

$$= \frac{1 - \cot x}{1 + \cot x} \quad \checkmark$$

$$(\sec x + \tan x)^2 = \frac{1+\sin x}{1-\sin x}$$

$$(\sec x + \tan x)^2 = \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)^2$$

$$= \left(\frac{1+\sin x}{\cos x}\right)^2$$

$$= \frac{(1+\sin x)^2}{\cos^2 x}$$

$$= \frac{(1+\sin x)^2}{1-\sin^2 x}$$

$$= \frac{(1+\sin x)^2}{(1+\sin x)(1-\sin x)}$$

$$= \frac{1+\sin x}{1-\sin x}$$ ✓

$$\frac{\csc x - 1}{\csc x + 1} = \frac{\cot^2 x}{\csc^2 x + 2\csc x + 1}$$

$$\frac{\csc x - 1}{\csc x + 1} = \frac{\csc x - 1}{\csc x + 1} \cdot \frac{\csc x + 1}{\csc x + 1}$$

$$= \frac{\csc^2 x - 1}{\csc^2 x + 2\csc x + 1}$$

$$= \frac{\cot^2 x}{\csc^2 x + 2\csc x + 1}$$ ✓

$$\csc^2 x - \cos x \sec x = \cot^2 x \qquad \cos x = \frac{1}{\sec x}$$

$$\csc^2 x - \cos x \sec x = \csc^2 x - 1$$

$$= \cot^2 x$$ ✓

$$\csc^2 x \sec^2 x = \sec^2 x + \csc^2 x$$

$$\csc^2 x \sec^2 x = \frac{1}{\sin^2 x} \cdot \frac{1}{\cos^2 x}$$

$$= \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$

$$= \csc^2 x + \sec^2 x$$ ✓

$$\frac{a}{\sin^2 x} + \frac{b}{\cos^2 x}$$

$$a\cos^2 x + b\sin^2 x = 1$$

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\cot y + \cot x}{\cot y - \cot x}$$

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\frac{\sin x\cos y}{\sin x\sin y} + \frac{\cos x\sin y}{\sin x\sin y}}{\frac{\sin x\cos y}{\sin x\sin y} - \frac{\cos x\sin y}{\sin x\sin y}}$$

$$= \frac{\cot y + \cot x}{\cot y - \cot x} \checkmark$$

$$\frac{\cos(x+y)}{\cos x\sin y} = \cot y - \tan x$$

$$\frac{\cos(x+y)}{\cos x\sin y} = \frac{\cos x\cos y - \sin x\sin y}{\cos x\sin y}$$

$$= \frac{\cos x\cos y}{\cos x\sin y} - \frac{\sin x\sin y}{\cos x\sin y}$$

$$= \cot y - \tan x \checkmark$$

$$\sec(x+y) = \frac{\cos y + \sin x \sin y}{\cos^2 x - \sin^2 y}$$

$$\sec(x+y) = \frac{1}{\cos(x+y)} \cdot \frac{\cos(x-y)}{\cos(x-y)}$$

$$= \frac{\cos(x-y)}{(\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)}$$

$$= \frac{\cos(x-y)}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}$$

$$= \frac{\cos(x-y)}{\cos^2 x (1-\sin^2 y) - (1-\cos^2 x)\sin^2 y}$$

$$= \frac{\cos(x-y)}{\cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y} \checkmark$$

$\cos 3x = \cos^3 x - 3\cos x \sin^2 x$

$\cos 3x = \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$= (\cos^2 x - \sin^2 x)\cos x - 2\sin^2 x \cos x$

$= \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x$

$= \cos^3 x - 3\sin^2 x \cos x$ ✓

$\frac{1}{4}\sin 4t = \cos^3 t \sin t - \sin^3 t \cos t$

$\frac{1}{4}\sin 4t = \frac{1}{4}\sin 2(2t)$

$= \frac{1}{4}(2\sin 2t \cos 2t)$

$= \frac{1}{2}(2\sin t \cos t)(\cos^2 t - \sin^2 t)$

$= \sin t \cos^3 t - \sin^3 t \cos t$ ✓

$$\frac{\cos 2x}{\cos^2 x} = \sec^2 x - 2\tan^2 x$$

$$\frac{\cos 2x}{\cos^2 x} = \frac{1 - 2\sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - 2\frac{\sin^2 x}{\cos^2 x}$$

$$= \sec^2 x - 2\tan^2 x \quad \checkmark$$

$$2\csc x \cos^2 \frac{x}{2} = \frac{\sin x}{1 - \cos x}$$

$$2\csc x \cos^2 \frac{x}{2} = 2\frac{1}{\sin x}\left(\frac{1}{2}\right)(1 + \cos x)$$

$$= \frac{1 + \cos x}{\sin x}\left(\frac{1 - \cos x}{1 - \cos x}\right)$$

$$= \frac{1 - \cos^2 x}{\sin x (1 - \cos x)}$$

$$= \frac{\sin^2 x}{\sin x (1 - \cos x)}$$

$$= \frac{\sin x}{1 - \cos x} \quad \checkmark$$

$$\sec^2\frac{x}{2} = \frac{2\sec x + 2}{\sec x + 2 + \cos x} \qquad 2(\sec x + 1)$$

$$\sec^2\frac{x}{2} = \frac{1}{\cos^2\frac{x}{2}}$$

$$= \frac{1}{\frac{1}{2}(1+\cos x)}$$

$$= \frac{2}{1+\cos x} \cdot \frac{1+\sec x}{1+\sec x}$$

$$= \frac{2 + 2\sec x}{1 + \cos x + \sec x + \underbrace{\cos x \sec x}_{=1}}$$

$$= \frac{2 + 2\sec x}{2 + \cos x + \sec x} \checkmark$$

$$\frac{1-\sin^2\frac{x}{2}}{1+\sin^2\frac{x}{2}} = \frac{1+\cos x}{3-\cos x}$$

$$\frac{1-\sin^2\frac{x}{2}}{1+\sin^2\frac{x}{2}} = \frac{1-\left(\frac{1}{2}-\frac{1}{2}\cos x\right)}{1+\left(\frac{1}{2}-\frac{1}{2}\cos x\right)}$$

$$= \frac{\frac{1}{2}+\frac{1}{2}\cos x}{\frac{3}{2}-\frac{1}{2}\cos x}$$

$$= \frac{1+\cos x}{3-\cos x} \quad \checkmark$$