Erick

#1) $\int x^5 e^{4x}$

$$\begin{array}{c|c} & \int e^{4x} \\ \hline x^5 & \frac{1}{4}e^{4x} \\ 5x^4 & \frac{1}{16}e^{4x} \\ \downarrow & \end{array}$$

$$= e^{4x}\left(\frac{x^5}{4} - \frac{5}{16}x^4 + \frac{5}{16}x^3 - \frac{15}{64}x^2 + \frac{15}{128}x - \frac{15}{512}\right) + C$$

2/ $\int \cos 2x \, e^{3x} dx$

$$\begin{array}{c|c} & \int \cos 2x \\ \hline + e^{3x} & \frac{1}{2}\sin 2x \\ -3e^{3x} & -\frac{1}{4}\cos 2x \\ +9e^{3x} & \end{array}$$

$$= e^{3x}\left(\frac{1}{2}\sin 2x + \frac{3}{4}\cos 2x\right) - \frac{9}{4}\int e^{3x}\cos 2x \, dx$$

$$\int \cos 2x \, e^{3x} dx = \frac{4}{13} e^{3x}\left(\frac{1}{2}\sin 2x + \frac{3}{4}\cos 2x\right)$$

3/

$$\begin{array}{c|c} & \int \cos 4x \\ \hline x^6 & +\frac{1}{4}\sin 4x \\ 6x^5 & -\frac{1}{24}\cos 4x \\ | & | \end{array}$$

720

-2

4/ $\int \cos(\ln x)\, dx$

$u = \cos(\ln x)$ $v = \int dx = x$

$du = -\frac{1}{x}\sin(\ln x)$

$$\int \cos(\ln x)\, dx = x\cos(\ln x) + \int \sin(\ln x)\, dx$$

$u = \sin \ln x$ $v = x$

$du = \frac{1}{x}\cos(\ln x)$

$$= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\, dx$$

$$\int \cos(\ln x)\, dx = \frac{1}{2}\left(x\sin(\ln x) - x\cos(\ln x)\right) + C$$

$$5/\quad \int_0^{\pi} e^{\cos t}\sin 2t\,dt = 2\int_0^{\pi} e^{\cos t}\cos t\,\underbrace{\sin t\,dt}$$

$$= -2\int_0^{\pi} \cos t\, e^{\cos t}\, d(\cos t)$$

$u = \cos t$

$$= -2\int u\,e^u\,du$$

$$= -2\, e^{\cos t}(\cos t - 1)\Big|_0^{\pi}$$

$$= -2\left(\frac{1}{e}(-2) - e(0)\right)$$

$$= \frac{4}{e}$$

7.2 Trig.

$$\int \sin^m x \cos^n x \, dx$$

Ex $\int \cos^2 x \sin^3 x \, dx$ $\qquad \cos^2 x + \sin^2 x = 1$

$= \int \cos^2 x \sin^2 x \underbrace{\sin x \, dx}$ $\qquad \frac{d\cos x}{dx} = -\sin x$

$= -\int \cos^2 x (1 - \cos^2 x) \, d(\cos x)$

$= \int (\cos^4 x - \cos^2 x) \, d(\cos x)$ $\qquad \cancel{\int (u^4 - u^2) du}$

$= \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

Ex $\int \cos^5 x \, dx = \int \cos^4 x \underline{\cos x \, dx}$ $\qquad (\cos^2 x)^2$ $\qquad d \sin x = \cos x \, dx$

$= \int (1 - \sin^2 x)^2 \, d(\sin x)$

$= \int (1 - 2\sin^2 x + \sin^4 x) \, d(\sin x)$

$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$

---

(1) odd $\to$ $(\ )^1 dx$

$\cos x \, dx = d(\sin x)$

$\sin x \, dx = -d(\cos x)$

$\sin^2 x = \frac{1 - \cos 2x}{2}$ $\qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\cos 2x = 2\cos^2 x - 1$ $\qquad = \frac{1}{2} + \frac{1}{2}\cos 2x$

$= 1 - 2\sin^2 x$

Ex. $\int \sin^2 x \cos^4 x \, dx$

$$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{8}\int (1-\cos 2x)(1+2\cos 2x+\cos^2 2x)\, dx$$

$$= \frac{1}{8}\int (1+\cos 2x-\cos^2 2x-\cos^3 2x)\, dx$$

$$= \frac{1}{8}\int (\underbrace{1+\cos 2x-\frac{1}{2}}_{1/2}-\frac{1}{2}\cos 4x-\cos^3 2x)\, dx$$

$$\int \cos^3 2x \, dx = \int \cos^2 2x \cos 2x \, dx$$

$$= \frac{1}{2}\int (1-\sin^2 2x)\, d(\sin 2x)$$

$$d(\sin 2x) = 2\cos 2x \, dx$$

$$= \frac{1}{2}\left(\sin 2x - \frac{1}{3}\sin^3 2x\right)$$

$$= \frac{1}{8}\left[\frac{1}{2}x + \frac{1}{2}\sin 2x - \frac{1}{8}\sin 4x - \frac{1}{2}\sin 2x + \frac{1}{6}\sin^3 2x\right]$$

$$= \frac{1}{8}\left(\frac{1}{2}x - \frac{1}{8}\sin 4x + \frac{1}{6}\sin^3 2x\right) + C$$

Ex $\displaystyle\int_0^{\pi/4} \sqrt{1+\cos 4x}\,dx$ $\qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \int_0^{\pi/4} \sqrt{2\cos^2 2x}\,dx$$

$$= \sqrt{2}\int_0^{\pi/4} \cos 2x\,dx$$

$$= \frac{\sqrt{2}}{2}\,\sin 2x\Big|_0^{\pi/4}$$

$$= \frac{\sqrt{2}}{2}(1-0)$$

$$= \frac{\sqrt{2}}{2}$$

Ex $\displaystyle\int \sin^3 x\cos^{-2} x\,dx = \int \sin^2 x\cos^{-2} x\,\underline{\sin x\,dx}$

$$= -\int (1-\cos^2 x)\cos^{-2} x\,d(\cos x)$$

$$= -\int (\cos^{-2} x - 1)\,d(\cos x)$$

$$= -(-\cos^{-1} x - \cos x)$$

$$= \frac{1}{\cos x} + \cos x + C$$

$$= \sec x + \cos x + C$$

$d(\tan x) = \sec^2 x\,dx$

$d(\sec x) = \sec x \tan x\,dx$

$1+\tan^2 x = \sec^2 x \Leftrightarrow \tan^2 x = \sec^2 - 1$

$\dfrac{\cos^2 x}{\cos^2} + \dfrac{\sin^2 x}{c^2} = \dfrac{1}{c^2}$

$c^2 + s^2 = 1$

Ex $\int \tan^4 x\,dx = \int \tan^2 x \tan^2 x\,dx$

$$= \int \tan^2 x(\sec^2 x - 1)\,dx$$

$$= \int (\tan^2 x \sec^2 x - \tan^2 x)\,dx$$

$$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx$$

$$= \int \tan^2 x\,d(\tan x) - \int (\sec^2 x - 1)\,dx$$

$$= \frac{1}{3}\tan^3 x - (\tan x - x) + C$$

$$= \frac{1}{3}\tan^3 x - \tan x + x + C$$

$\int \sec^3 x\,dx$

$u = \sec x \qquad v = \int \sec^2 x\,dx$

$du = \sec x \tan x\,dx \qquad = \tan x$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$$= \sec x \tan x - \int \sec x(\sec^2 x - 1)\,dx$$

$$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \boxed{\int \sec x\,dx}$$

$$= \sec x \tan x + \boxed{\ln|\sec x + \tan x|} + C_1$$

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

$$\int \sec x\,dx = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x}\,dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$$

$$= \int \frac{d(\sec x + \tan x)}{\sec x + \tan x}$$

$$= \ln|\sec x + \tan x| + C$$

$$\sin mx \cos nx = \frac{1}{2}\left(\sin(m-n)x + \sin(m+n)x\right)$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) + \cos(a+b) = 2\cos a \cos b$$

$$\cos a \cos b = \frac{1}{2}\left(\cos(a-b) + \cos(a+b)\right)$$

Ex $\int \sin 3x \cos 5x\,dx = \frac{1}{2}\int(-\sin 2x + \sin 8x)\,dx$

$$= \frac{1}{2}\left(+\frac{1}{2}\cos 2x - \frac{1}{8}\cos 8x\right) + C$$

#143 $\int_0^{\pi/2} \cos^{10}\theta\,d\theta = \frac{\pi}{2}\,\frac{9}{\cancel{10}_{2}}\,\frac{7}{8}\,\frac{\cancel{5}}{\cancel{6}_{2}}\,\frac{\cancel{3}}{4}\,\frac{1}{2}$

$$= \boxed{\frac{63\pi}{2^9}}$$

145 $\int_0^{\pi/2} \cos^9 x\,dx = \frac{8}{9}\,\frac{6}{7}\,\frac{4}{5}\,\frac{2}{\cancel{3}}$

$$= \boxed{\frac{2^7}{315}}$$

2.3 Trig. Substitutions

$\sqrt{a^2+x^2} = a\sec\theta$

$x = a\tan\theta$

$\sqrt{x^2-a^2} = a\tan\theta$

$x = a\sec\theta$

$\sqrt{a^2-x^2} = a\cos\theta$

$x = a\sin\theta$

Ex $\displaystyle\int \frac{dx}{\sqrt{4+x^2}}$ $\quad x = 2\tan\theta \quad \sqrt{4+x^2} = 2\sec\theta$

$dx = 2\sec^2\theta\, d\theta$

$$\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{2\sec^2\theta\, d\theta}{2\sec\theta}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$\boxed{= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C}$$

$:R \quad \int \frac{x^2 dx}{\sqrt{9-x^2}}$ $\quad x = 3\sin\theta \quad , \sqrt{9-x^2} = 3\cos\theta$

$dx = 3\cos\theta \, d\theta$

$$\int \frac{x^2 dx}{\sqrt{9-x^2}} = \int \frac{9\sin^2\theta \; 3\cos\theta \, d\theta}{3\cos\theta}$$

$\sin\theta = \frac{x}{3}$

$\theta = \sin^{-1}\frac{x}{3}$

$$= \frac{9}{2}\int (1-\cos 2\theta)\, d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{9}{2}\left(\theta - \sin\theta\cos\theta\right) + C$$

$$= \frac{9}{2}\left(\sin^{-1}\frac{x}{3} - \frac{x\sqrt{9-x^2}}{9}\right) + C$$

$$= \frac{9}{2}\sin^{-1}\frac{x}{3} - \frac{1}{2}x\sqrt{9-x^2} + C$$

$$\int \frac{dx}{\sqrt{25x^2-4}}$$

$$5x = 2\sec\theta$$
$$dx = \frac{2}{5}\sec\theta\tan\theta\, d\theta$$
$$\sqrt{25x^2-4} = 2\tan\theta$$

$$\int \frac{dx}{\sqrt{25x^2-4}} = \frac{2}{5}\int \frac{\sec\theta\tan\theta\, d\theta}{2\tan\theta}$$

$$= \frac{1}{5}\int \sec\theta\, d\theta$$

$$= \frac{1}{5}\ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \frac{1}{5}\ln\left|\frac{5}{2}x + \frac{1}{2}\sqrt{25x^2-4}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2-\underset{\to b^2}{x^2}}} = \frac{1}{B}\sin^{-1}\frac{x}{a}$$

test $$\int \frac{dx}{\sqrt{1-2x^2}}$$

$$\sqrt{2}\,x = \sin\theta$$
$$dx = \frac{1}{\sqrt{2}}\cos\theta\, d\theta$$
$$\sqrt{1-2x^2} = \cos\theta$$

$$\int \frac{dx}{\sqrt{1-2x^2}} = \frac{1}{\sqrt{2}}\int \frac{\cos\theta\, d\theta}{\cos\theta}$$

$$= \frac{1}{\sqrt{2}}\int d\theta$$

$$= \frac{1}{\sqrt{2}}\,\theta$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}(\sqrt{2}\,x)$$

#12 $\displaystyle\int \frac{dx}{x\sqrt{4x^2+9}}$

$$2x = 3\tan\theta$$
$$dx = \frac{3}{2}\sec^2\theta\, d\theta$$
$$\sqrt{4x^2+9} = 3\sec\theta$$

$$\int \frac{dx}{x\sqrt{4x^2+9}} = \frac{3}{2}\int \frac{\sec^2\theta\, d\theta}{\frac{3}{2}\tan\theta\; 3\sec\theta}$$

$$= \frac{1}{3}\int \frac{\sec\theta}{\tan\theta}\, d\theta$$

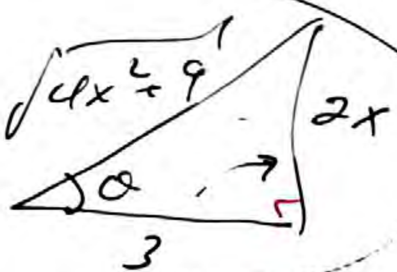
$$= \frac{1}{3}\int \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\, d\theta$$

$$= \frac{1}{3}\int \csc\theta\, d\theta$$

$$= -\frac{1}{3}\ln\left|\csc\theta + \cot\theta\right| + C$$

$$= -\frac{1}{3}\ln\left|\frac{2x}{\sqrt{4x^2+9}} + \frac{3}{2x}\right| + C$$

$$2x = 3\tan\theta$$
$$\tan\theta = \frac{2x}{3}$$



#35

$$\int \frac{dx}{\sqrt{x^2-25}}$$

$x = 5\sec\theta$
$dx = 5\sec\theta\tan\theta\,d\theta$
$\sqrt{x^2-25} = 5\tan\theta$

$$\int \frac{dx}{\sqrt{x^2-25}} = \int \frac{5\sec\theta\tan\theta\,d\theta}{5\tan\theta}$$

$$= \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{x}{5} + \frac{\sqrt{x^2-25}}{5}\right| + C$$