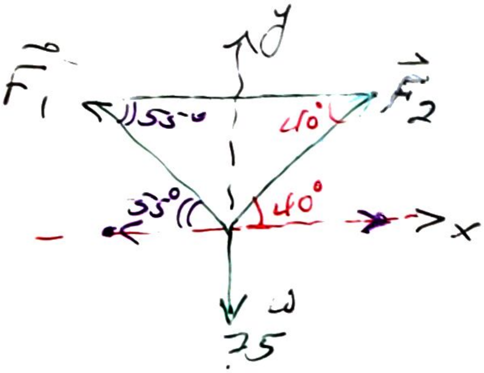
Ex

$\vec{F_1} = \langle -|\vec{F_1}|\cos 55^\circ, |\vec{F_1}|\sin 55^\circ \rangle$



$\vec{F_2} = \langle |\vec{F_2}|\cos 40^\circ, |\vec{F_2}|\sin 40^\circ \rangle$

$w = \langle 0, -75 \rangle$

$$\begin{cases} -|\vec{F_1}|\cos 55^\circ + |\vec{F_2}|\cos 40^\circ = 0 \\ |\vec{F_1}|\sin 55^\circ + |\vec{F_2}|\sin 40^\circ - 75 = 0 \end{cases}$$

$$\begin{cases} |\vec{F_2}|\cos 40^\circ = |\vec{F_1}|\cos 55^\circ & (1) \\ |\vec{F_1}|\sin 55^\circ + |\vec{F_2}|\sin 40^\circ = 75 & (2) \end{cases}$$

$(1) \rightarrow |\vec{F_2}| = |\vec{F_1}| \dfrac{\cos 55^\circ}{\cos 40^\circ}$ (3)

$(2) \rightarrow |\vec{F_1}| \left(\sin 55^\circ + \dfrac{\sin 40^\circ \cos 55^\circ}{\cos 40^\circ} \right) = 75$

$|\vec{F_1}| \left(\dfrac{\sin 55^\circ \cos 40^\circ + \sin 40^\circ \cos 55^\circ}{\cos 40^\circ} \right) = 75$

$$|\vec{F_1}| = \frac{75 \cos 40^\circ}{\underbrace{\sin 55^\circ \cos 40^\circ + \sin 40^\circ \cos 55^\circ}_{\sin(a+b)}}$$

$$= \frac{75 \cos 40^\circ}{\sin 95^\circ}$$

$$|\vec{F_2}| = 75 \frac{\cos 55^\circ}{\sin 95^\circ}$$

$$\vec{F_1} = \left\langle -\frac{75 \cos 40^\circ \cos 55^\circ}{\sin 95^\circ}, 75 \frac{\cos 40^\circ \sin 55^\circ}{\sin 95^\circ} \right\rangle$$

$$\vec{F_2} = \left\langle 75 \frac{\cos 55^\circ \cos 40^\circ}{\sin 95^\circ}, 75 \frac{\sin 40^\circ \cos 55^\circ}{\sin 95^\circ} \right\rangle$$

sect 1.2

$\vec{u} = \langle u_1, u_2, u_3 \rangle \quad \vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$\theta = \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$$

cosine: $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$

Ex

$$\langle 1, -2, -1 \rangle \cdot \langle -6, 2, -3 \rangle = -6 - 4 + 3 = -7$$

$$\left(\tfrac{1}{2}\hat{i} + 3\hat{j} + \hat{k}\right) \cdot \left(4\hat{i} - \hat{j} + 2\hat{k}\right) = 2 - 3 + 2 = 1$$

Ex θ? between $\vec{u} = \hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{v} = 6\hat{i} + 3\hat{j} + 2\hat{k}$

$$\vec{u} \cdot \vec{v} = 6 - 6 - 4 = -4$$

$$|\vec{u}| = \sqrt{1 + 4 + 4} = 3$$

$$|\vec{v}| = \sqrt{36 + 9 + 4} = 7$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

Ex $\quad A=(0,0) \quad B=(3,5) \quad C=(5,2)$

$\theta(C)$?

$\vec{CA} = \langle -5, -2 \rangle$

$\vec{CB} = \langle -2, 3 \rangle$

$\vec{CA} \cdot \vec{CB} = 10 - 6 = \underline{4}$

$|\vec{CA}| = \sqrt{25+4} = \sqrt{29}$

$|\vec{CB}| = \sqrt{4+9} = \sqrt{13}$

$$\theta = \cos^{-1} \frac{4}{\sqrt{29}\sqrt{13}}$$

Perpendicular ($\perp$) Orthogonal. vectors

$$\vec{u} \cdot \vec{v} = 0$$

Ex $\quad \vec{u} = 3\hat{i} - 2\hat{j} + \hat{k}$

$\vec{v} = 2\hat{j} + 4\hat{k}$

$\vec{u} \cdot \vec{v} = -4 + 4 = 0$

$$\text{Proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \vec{v}$$

$\vec{u} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ onto $\vec{v} = \hat{i} - 2\hat{j} - 2\hat{k}$

$$\text{Proj}_{\vec{v}} \vec{u} = \frac{6 - 6 - 4}{1+4+4} (\hat{i} - 2\hat{j} - 2\hat{k})$$

$$= -\frac{4}{9} (\hat{i} - 2\hat{j} - 2\hat{k})$$

$$= -\frac{4}{9}\hat{i} + \frac{8}{9}\hat{j} + \frac{8}{9}\hat{k}$$

Work $= \vec{F} \cdot \vec{D}$

$= |\vec{F}|\,|\vec{D}| \cos\theta$

Ex $|\vec{F}| = 40\,N$ $|\vec{D}| = 3\,m$ $\theta = 60°$

$W = (40)(3)\cos 60°$

$= 60\ J$ (joules)

1.3 Cross Product.

$\vec{u} = \langle u_1, u_2, u_3 \rangle$ $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \begin{matrix} \\ u_1 & u_2 \\ v_1 & v_2 \end{matrix}$$

$$= (u_2 v_3 - u_3 v_2)\,\hat{i} + (v_1 u_3 - u_1 v_3)\,\hat{j} + (u_1 v_2 - v_1 u_2)\,\hat{k}$$

Ex $\vec{u} = 2\hat{i} + \hat{j} + \hat{k}$ $\vec{v} = -4\hat{i} + 3\hat{j} + \hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix} \begin{matrix} \\ 2 & 1 \\ -4 & 3 \end{matrix}$$

$$= -2\hat{i} - 6\hat{j} + 10\hat{k}$$

$$\vec{v} \times \vec{u} = 2\hat{i} + 6\hat{j} - 10\hat{k}$$

Ex $P(1,-1,0)$ $Q(2,1,-1)$ $R(-1,1,2)$

a) Find a vector $\perp$ (PQR)

$$\vec{PQ} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{PR} = -2\hat{i} + 2\hat{j}\ 2\hat{k}$$

$$|\vec{PQ} \times \vec{PR}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{vmatrix} \begin{matrix} 1 & 2 \\ -2 & 2 \end{matrix}$$

$$= \boxed{6\hat{i} + 6\hat{k}}$$

b) Area of ΔPQR

$$\text{Area} = \frac{1}{2}|\vec{PQ} \times \vec{PR}|$$

$$= \frac{1}{2}\sqrt{36+36}$$

$$= \boxed{3\sqrt{2}\ \text{unit}^2}$$

c) unit vector $\perp$ PQR

$$\text{unit vector} = \frac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|}$$

$$\vec{n} = \frac{6\hat{i} + 6\hat{k}}{\sqrt{36+36}}$$

$$= \boxed{\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}}$$

Ex magnitude of Torque

$|\vec{F}| = 20 \text{ lb} \qquad |\vec{r}| = 3 \text{ ft} \qquad \theta = 70^\circ$

$$
\begin{aligned}
|\text{Torque}| &= |\vec{PQ} \times \vec{F}| \\
&= |\vec{F}| \cdot |\vec{r}| \sin\theta \\
&= 20\,(3)\,\sin 70^\circ \\
&= \boxed{60 \sin 70^\circ} \ \text{ft-lb}
\end{aligned}
$$

Ex Volume: $\vec{u} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{v} = -2\hat{i} + 3\hat{k}$

$\vec{w} = 7\hat{j} - 4\hat{k}$

absolute value

$$
\text{Volume} = \left| \underbrace{\begin{vmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \\ 0 & 7 & -4 \end{vmatrix}}_{\text{determinant}} \right| \quad \begin{matrix} 1 & 2 \\ -2 & 0 \\ 0 & 7 \end{matrix}
$$

$0 + 0 + 14 - 21$
-16

$$
\begin{aligned}
&= |-23| \\
&= \boxed{23 \text{ unit}^3}
\end{aligned}
$$