4.4 Polar Review

$r = 2\sin\theta \qquad \frac{\pi}{4} \le \theta \le \frac{\pi}{2} \qquad A?$

$$\text{Area} = \frac{1}{2}\int_{\pi/4}^{\pi/2} 4\sin^2\theta \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} (1 - \cos 2\theta) \, d\theta$$

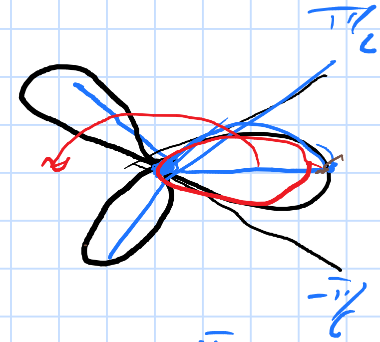
$$= \theta - \frac{1}{2}\sin 2\theta \Big|_{\pi/4}^{\pi/2}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} + \frac{1}{2}$$

$$= \frac{\pi}{4} + \frac{1}{2} \text{ unit}^2$$

$r = \cos 3\theta$ 1 leaf Area

$$\text{Area} = \frac{1}{3}\frac{1}{2}\int_0^{2\pi} \cos^2 3\theta \, d\theta$$



$\theta \quad \frac{\pi}{6}$

$$= \frac{1}{12}\int_0^{2\pi} (1+\cos 6\theta)\, d\theta$$

$$= \frac{1}{12}\left(\theta + \frac{1}{6}\sin 6\theta \right|_0^{2\pi}$$

$$= \frac{1}{12}(2\pi)$$

$\frac{1}{2}$

$$\cancel{= \frac{\pi}{6}}$$

$\int_0^{\pi}$ w/o graphing $\to \theta \to \pi$

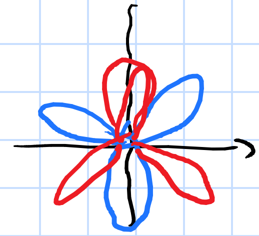
$$A = \frac{1}{2}\int_{-\pi/6}^{\pi/6} \cos^2 3\theta \, d\theta$$

$$= \frac{1}{4}\left(\theta + \frac{1}{6}\sin 6\theta \right|_{-\pi/6}^{\pi/6}$$

$$= \frac{1}{4}\left(\frac{\pi}{6} + \frac{\pi}{6}\right)$$

$$= \frac{\pi}{12} \text{ unit}^2$$ ✓

6-leaved Rose $r^2 = 2\sin 3\theta$ A ?

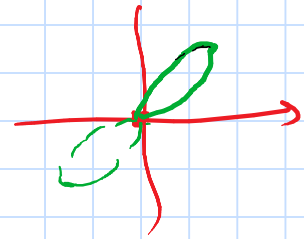


$$A = 6(2)\frac{1}{2}\int_0^{\pi/2} 2\sin 3\theta \, d\theta$$

$$= -\frac{12}{3}\cos 3\theta \Big|_0^{\pi/2}$$

$$= -4(0-1)$$

$$\boxed{= 4 \text{ unit}^2}$$



$r = \sqrt{\cos\theta} \geq 0$ I, IV

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

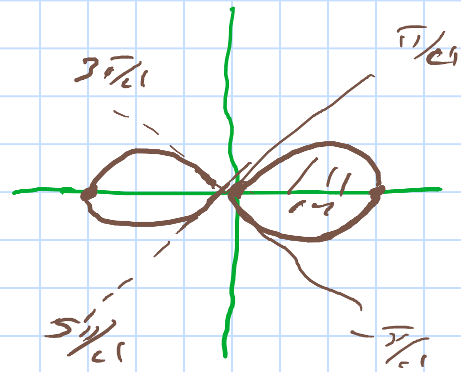
$$A = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= \frac{1}{2} \sin\theta \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}(1+1)$$

$$\boxed{= 1 \text{ unit}^2}$$

$r = \sqrt{\cos 2\theta}$ right lobe



$\cos 2\theta \geq 0$

$-\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$

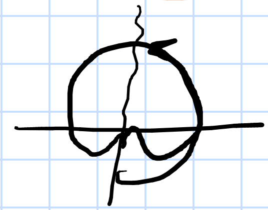
$-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

$$A = \frac{1}{2} 2 \int_0^{\pi/4} \cos 2\theta \, d\theta$$

$$= \frac{1}{2} \sin 2\theta \Big|_0^{\pi/4}$$

$$= \frac{1}{2} \text{ unit}^2$$

$r = 4 + 4 \sin \theta = 4(1 + \sin \theta)$



$$A = \frac{1}{2} \int_0^{2\pi} 16 \, (1 + \sin \theta)^2 \, d\theta$$

$$= 8 \int_0^{2\pi} \left(1 + 2 \sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta\right) d\theta$$

$$= 8 \int_0^{2\pi} \left(\frac{3}{2} + 2 \sin \theta - \frac{1}{2} \cos 2\theta\right) d\theta$$

$$= 8 \left(\frac{3}{2}\theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta\right) \Big|_0^{2\pi}$$

$$= 8(3\pi - 2 + 2)$$

$$= 24\pi \text{ unit}^2$$

$r_1 = 2\cos\theta \quad r_2 = 2\sin\theta \quad$ Area

$r = 2\cos\theta = 2\sin\theta$

$\theta = \frac{\pi}{4}$

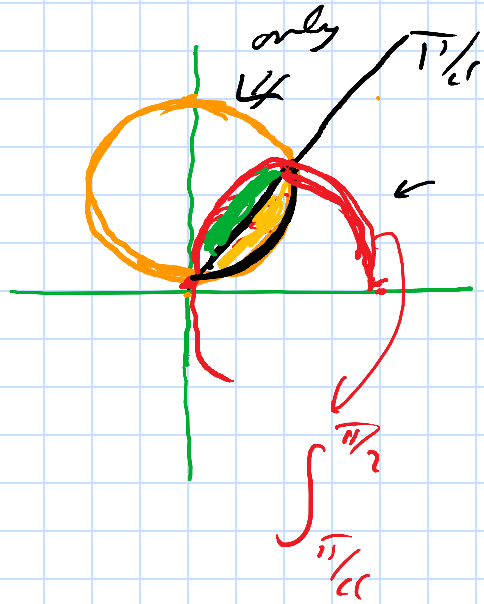
$$A = (2)\frac{1}{2}\int_0^{\pi/4} 4\sin^2\theta \, d\theta$$

$$= 2\int_0^{\pi/4} (1-\cos 2\theta)\, d\theta$$

$$= 2\left(\theta - \frac{1}{2}\sin 2\theta \right)\Big|_0^{\pi/4}$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \frac{\pi}{2} - 1 \quad \text{unit}^2$$



$r = 2\cos\theta$

$r^2 = 2r\cos\theta$

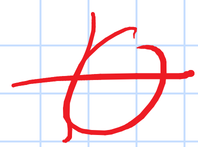
$x^2 + y^2 = 2x$

$x^2 - 2x + y^2 = 0$

$(x-1)^2 + y^2 = 1$

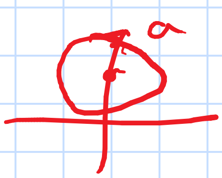
circle $(1,0)$ of $r = 1$

$r = a\cos\theta$



$r -$

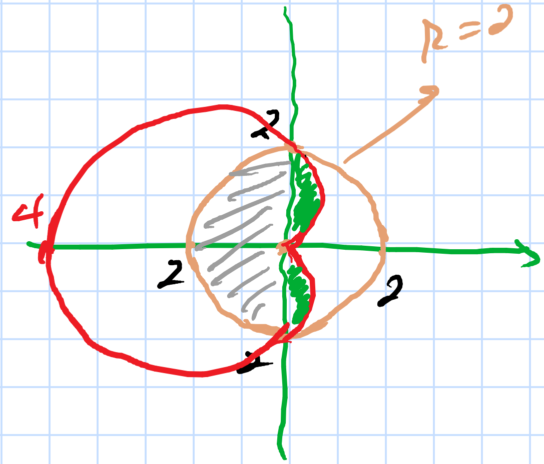
$r = a\sin\theta$



$r = 2 \qquad r = 2 - 2\cos\theta$

$2 = 2(1-\cos\theta)$

$\cos\theta = 0 \Rightarrow \theta = \pm\frac{\pi}{2}$



$A_1 = \frac{1}{2}\pi r^2$

$= 2\pi$

$$A_2 = 2\,\frac{1}{2}\int_0^{\pi/2} 4(1-\cos\theta)^2\,d\theta$$

$$= 4\int_0^{\pi/2}\left(1 - 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= 4\int_0^{\pi/2}\left(\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= 4\left(\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi/2}$$

$$= 4\left(\frac{3}{4}\pi - 2\right)$$

$$= 3\pi - 8$$

$$A = 2\pi + 3\pi - 8$$

$$= 5\pi - 8 \quad \text{unit}^2$$

in $r = 6$ line $r = 3\csc\theta$ above

$$r = 6 = 3\,\frac{1}{\sin\theta}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6},\ \frac{5\pi}{6}$$

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(36 - 9\csc^2\theta\right) d\theta$$

$$= \frac{9}{2}\left(4\theta + \cot\theta\right)\Big|_{\pi/6}^{5\pi/6}$$

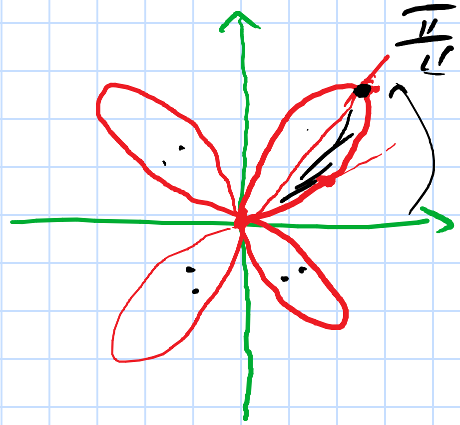
$\cot\theta = \frac{\sqrt{3}}{1}$

$$= \frac{9}{2}\left(\frac{10\pi}{3} - \sqrt{3} - \frac{2\pi}{3} - \sqrt{3}\right)$$

$$= \frac{9}{2}\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$$

$$= 12\pi - 9\sqrt{3}\ \text{unit}^2$$

Ell. $r = 3\sin 2\theta$



$$A = \frac{1}{2}(8)\int_0^{\pi/4} 9\sin^2 2\theta\, d\theta$$

$$= 18\int_0^{\pi/4} (1 - \cos 4\theta)\, d\theta$$

$$= 18\left(\theta - \frac{1}{4}\sin 4\theta \Big|_0^{\pi/4}\right.$$

$$= 18\left(\frac{\pi}{4}\right)$$

$$= \frac{9\pi}{2}\ \text{unit}^2$$

$r = a \qquad 0 \leq \theta \leq 2\pi$

$$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{a^2 + 0}$$

$$= a$$

$$L = \int_0^{2\pi} a\, d\theta$$

$$= 2\pi a \ \text{unit}$$

$$\boxed{c\int_0^{\alpha} d\theta = c\alpha}$$

$r = 4\sin\theta \qquad 0 \le \theta \le \pi$

$$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{16\sin^2\theta + 16\cos^2\theta}$$
$$= 4$$

$$L = \int_0^{\pi} 4\, d\theta$$

$$\boxed{= 4\pi \text{ unit}}$$

$r = 1 + \sin\theta \qquad 0 \le \theta \le 2\pi$

$$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{(1+\sin\theta)^2 + \cos^2\theta}$$
$$= \sqrt{1 + 2\sin\theta + \underline{\sin^2\theta + \cos^2\theta}}$$
$$= \sqrt{2 + 2\sin\theta} \qquad \sqrt{2}\sqrt{(1+\sin\theta)}$$

$$L = \sqrt{2}\int_0^{2\pi} \sqrt{1+\sin\theta}\, d\theta \qquad \frac{\sqrt{1-\sin\theta}}{\sqrt{1-\sin\theta}}$$

$$= \sqrt{2}\int_0^{2\pi} \frac{\sqrt{1-\sin^2\theta}}{(1-\sin\theta)^{1/2}}\, d\theta \qquad \sqrt{\cos^2\theta} = |\ |$$

$$= 2\sqrt{2}\int_{-\pi/2}^{\pi/2} \cos\theta\,(1-\sin\theta)^{-1/2}\, d\theta$$

$$= -2\sqrt{2}\int_{-\pi/2}^{\pi/2} (1-\sin\theta)^{-1/2}\, d(1-\sin\theta)$$

$$L = -4\sqrt{2}\,(1-\sin\theta)^{1/2}\Big|_{-\pi/2}^{\pi/2}$$

$$= -4\sqrt{2}\,(0-\sqrt{2})$$

$$= \boxed{8 \text{ unit}}$$

$r = e^{\theta} \qquad 0 \le \theta \le \pi$

$$\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2} = \sqrt{e^{2\theta}+e^{2\theta}}$$

$$= \sqrt{2}\,e^{\theta}$$

$$L = \sqrt{2}\int_0^{\pi} e^{\theta}\,d\theta$$

$$= \sqrt{2}\,e^{\theta}\Big|_0^{\pi}$$

$$= \boxed{\sqrt{2}\,(e^{\pi}-1) \text{ unit}}$$

$r = a\cos\theta \qquad 0 \le \theta \le \frac{\pi}{2} \qquad \boxed{\theta = \frac{\pi}{2}}$

$$\sqrt{r^2 + r'^2} = \sqrt{a^2\cos^2\theta + a^2\sin^2\theta}$$
$$= a$$

$$S = 2\pi \int_0^{\pi/2} r\cos\theta \sqrt{r^2 + r'^2}\, d\theta$$
$$= 2\pi \int_0^{\pi/2} a\cos^2\theta\,(a)\, d\theta$$
$$= \pi a^2 \int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta$$
$$= \pi a^2 \left(\theta + \tfrac{1}{2}\sin 2\theta \Big|_0^{\pi/2}\right.$$
$$= \pi a^2 \left(\frac{\pi}{2}\right)$$
$$= \frac{1}{2}(\pi a)^2 \text{ unit}^2$$

$r = a(1+\cos\theta) \quad 0 \le \theta \le \pi$ polar axis

$$\sqrt{r^2+(r')^2} = \sqrt{a^2(1+\cos\theta)^2 + a^2\sin^2\theta}$$

$$= a\sqrt{1+2\cos\theta+\cos^2\theta+\sin^2\theta}$$

$$= a\sqrt{2+2\cos\theta}$$

$$= a\sqrt{2}\sqrt{1+\cos\theta}$$

$$S = 2\pi\int_0^{\pi} a(1+\cos\theta)\sin\theta\left(a\sqrt{2}\sqrt{1+\cos\theta}\right)d\theta$$

$$= -2\pi a\sqrt{2}\int_0^{\pi}(1+\cos\theta)^{3/2}\,d(1+\cos\theta)$$

$$= -\frac{4}{5}a\pi\sqrt{2}\left(1+\cos\theta\right)^{5/2}\Big|_0^{\pi}$$

$$= -\frac{4}{5}a\pi\sqrt{2}\left(0-4\sqrt{2}\right)$$

$$= \frac{32}{5}\pi a \text{ unit}^2$$

$r = 1 + 4\cos\theta \qquad 0 \le \theta \le \frac{\pi}{2} \qquad$ Polar

$$\sqrt{r^2 + r'^2} = \sqrt{(1+4\cos\theta)^2 + 16\sin^2\theta}$$

$$= \sqrt{1 + 8\cos\theta + 16\cos^2\theta + 16\sin^2\theta}$$

$$= \sqrt{17 + 8\cos\theta}$$

$$S = 2\pi \int_0^{\pi/2} (1+4\cos\theta)\sin\theta \sqrt{17+8\cos\theta}\, d\theta$$

$$= 2\pi \int_0^{\pi/2} \sin\theta\,(17+8\cos\theta)^{1/2}\, d\theta \quad (1)$$

$$+ 8\pi \int_0^{\pi/2} \cos\theta\sin\theta\,(17+8\cos\theta)^{1/2}\, d\theta \quad (2)$$

$$(1) - \frac{\pi}{4}\int_0^{\pi/2} (17+8\cos\theta)^{1/2}\, d(17+8\cos\theta)$$

$$= -\frac{\pi}{6}(17+8\cos\theta)^{3/2}\Big|_0^{\pi/2}$$

$$= -\frac{\pi}{6}\left(17\sqrt{17} - 125\right)$$

$$= \frac{\pi}{6}\left(125 - 17\sqrt{17}\right)$$

$$② = 8\pi \int_0^{\pi/2} \cos\theta \sin\theta \,(17+8\cos\theta)^{1/2}\, d\theta$$

$$u = 17 + 8\cos\theta \leftarrow$$
$$du = -8\sin\theta\, d\theta$$

$$= -\frac{8\pi}{8} \int_0^{\pi/2} \left(\frac{u-17}{8}\right) u^{1/2}\, du$$

$$= -\frac{\pi}{8} \int_0^{\pi/2} \left(u^{3/2} - 17u^{1/2}\right) du$$

$$= -\frac{\pi}{8} \left(\frac{2}{5}(17+8\cos\theta)^{5/2} - \frac{34}{3}(17+8\cos\theta)^{3/2}\right)\Big|_0^{\pi/2}$$

$$= -\frac{\pi}{8}\left[\frac{2}{5}\, 17^{5/2} - \frac{34}{3}(17)^{3/2} - 2(5^4) + \frac{34}{3}(125)\right]$$

$$= -\frac{\pi}{8}\left(\frac{2}{5}\, 17^{5/2} - \frac{34}{3}\, 17^{3/2} - 1250 + \frac{1}{3}\, 4250\right)$$

$$S = \frac{\pi}{6}\left(125 - 17\sqrt{17}\right) -$$
$$\frac{\pi}{8}\left(\frac{2}{5}\, 17^{5/2} - \frac{34}{5}\, 17^{3/2} - 1250 + \frac{4250}{3}\right)$$

$$\boxed{\text{unit}^2}$$