$$2 - 2.7 + 2.7^2 + \cdots + 2(-7)^n = \frac{1-(-7)^{n+1}}{4}$$

$n \in \mathbb{Z}^+ + \{0\}$

For $n = 0 \Rightarrow 2 \stackrel{?}{=} \dfrac{1-(-7)^1}{4}$

$2 = 2$ ✓ P_0 is true

let P_k : $2 - 2.7 + \cdots + 2(-7)^k = \dfrac{1-(-7)^{k+1}}{4}$ true

Is P_{k+1} : $2 - 2.7 + \cdots + 2(-7)^k + 2(-7)^{k+1} \stackrel{?}{=} \dfrac{1-(-7)^{k+2}}{4}$

$$2 - 2.7 + \cdots + 2(-7)^k + 2(-7)^{k+1} = \frac{1-(-7)^{k+1}}{4} + 2(-7)^{k+1}$$

$$= \frac{1}{4}\left[1 - \underline{(-7)}^{k+1} + 8\underline{(-7)}^{k+1}\right]$$

$$= \frac{1}{4}\left(1 + 7(-7)^{k+1}\right)$$

$$= \frac{1}{4}\left(1 - (-7)(-7)^{k+1}\right)$$

$$= \frac{1}{4}\left(1 - (-7)^{k+2}\right) \checkmark$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the given proof is completed.

$3^n < n! \qquad n > 6$ $\qquad 3^5$

For $n = 7 \Rightarrow 3^7 \overset{?}{<} 7!$

$3^7 \overset{?}{<} 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot 2 \cdot 1$

$3^5 \overset{?}{<} 7 \cdot 5 \cdot 4 \cdot 4$

$243 < 480$ ✓

P_7 is true

let P_k is true: $3^k < k!$

Is P_{k+1}: $3^{k+1} < (k+1)!$

$$3^{k+1} = 3 \cdot 3^k$$
$$< 3\,(k!)$$
$$< (k+1)\,(k!)$$
$$= (k+1)!$$ ✓

$k > 6$
$1 + 6 < k + 1$
$7 < k + 1$
$3 < k + 1$

P_{k+1} is also true

By the mathematical induction, the given proof is completed

5 divides $n^5 - n$ $\quad n \in \mathbb{Z}^+$

For $n = 1 \Rightarrow \quad 1^5 - 1 = 0$

$= 0(5)$ ✓

0 is divisible by 5

P_1 is True

P_k is true : 5 divides $k^5 - k$

P_{k+1} is : 5 divides $(k+1)^5 - (k+1)$?

$$(k+1)^5 - (k+1) = \underline{k^5} + 5k^4 + 10k^3 + 10k^2 + 5k \cancel{+1} \underline{-k} \cancel{-1}$$

$$= k^5 - k + 5(k^4 + 2k^3 + 2k^2 + k)$$

5 divides $k^5 - k$: P_k

$5(\quad)$ is divisible by 5

5 divides $(k+1)^5 - (k+1)$

P_{k+1} is also true.

By the mathematical induction, the given proof is completed.

1 5 10 10 5 1

$2^5 \quad 32$

$n^5 \qquad -n$

$n^5 = \underbrace{xy\ldots n}$

$3^5 = 243$

$4^5 = 1024$

$n^5 - n = 5(x)$

$\boxed{n^5 - n = 0} \to$ last digit

3 divides $n^3 + 2n$

$27 + 2 \cdot 3$

$a_n = 2a_{n-1} \quad n \geq 1 \quad a_0 = 3$

$a_0 = 3$

$a_1 = 2a_0$
$= 2.(3)$

$a_2 = 2^2(3)$

$a_3 = 2a_2$
$= 3(2)^3$

$$\boxed{a_n = 3.2^n}$$

$\{a_n\} \quad n = 1, 2, \ldots$

a) $a_n = 6n$

$a_1 = 6$

$a_2 = 6(2) = 6+6$

$a_3 = 6(3) = \underbrace{6+6}+6$

$a_4 = 6(4) =$

$a_{n+1} = 6 + a_n$

$a_{n+1} \; ? \; a_n$

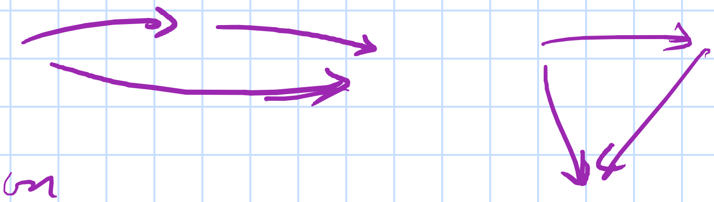
$a_1 = 6$

$a_{n+1} = 6 + a_n$

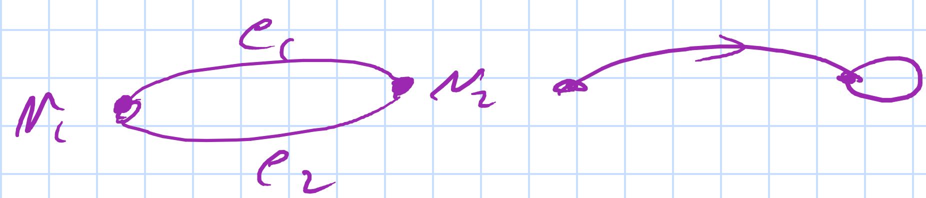
Relations R_i

Reflexive  loop

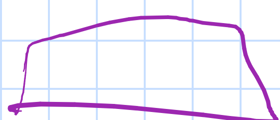
Symmetric 

Transitive 

closure relation



Lecture 4



4 letter 3 digit

repeated

ways = $26^4 \cdot 10^3$

no repeated: $26 \cdot 25 \cdot 24 \cdot 23 \cdot 10 \cdot 9 \cdot 8$