Lecture Two

Section 2.1 – Simple and Compound Interest

Simple Interest simply means money is not compounded only once between the beginning and the end of the note (or investment). With simple interest, the interest is calculated only at the end of the time period {*rather than periodically as is done with compounding*.}

$$I = \Pr t$$

$$A = P(1+rt) \quad \Rightarrow \; P = \frac{A}{1+rt}$$

A: ***Amount*** is the balance of the account (including the interest).

P: ***Principal*** is the initial amount of principal that is borrowed or invested.

I: ***Interest*** is the fee that is applied to the note or investment.

r: ***Rate*** is the interest rate

t: ***Time*** is time in years.

Example

Find the total amount due on a loan of $500 at 12% simple interest at the end of 30 months.

Solution

$$A = P(1+rt)$$

$$= 500\left(1+.12\,\frac{30}{12}\right)$$

$$\boxed{= \$650}$$

500 * (1 + .12(30/12))

Example

To buy furniture for a new apartment, you borrowed $5,000 at rate of 8% simple interest for 11 months. How much should you pay?

Solution

$$I = \Pr t$$

$$= 5000(.08)\left(\frac{11}{12}\right)$$

5000 * .08 * 11 / 12

$$\boxed{\approx \$366.67}$$

Example

T-Bills are one of the instrument of the U.S Treasury Department uses to finance the public debit. If you buy a 180-day T-bill with a maturity value of $10,000 for $9,828.74, what annual simple interest rate will you earn?

Solution

$$t = \frac{180}{365} \approx .5$$

$$A = P(1+rt)$$

$$10000 = 9828.74(1+.5r)$$

$$\frac{10000}{9828.74} = 1+.5r$$

$$\frac{10000}{9828.74} - 1 = .5r$$

$$\frac{\frac{10000}{9828.74} - 1}{.5} = r$$

(10000 / 9828.74 – 1) / .5

$$\boxed{r = .03485 \quad or \quad 3.485\%}$$

Example

Find the maturity value for a loan of $2500 to be repaid in 8 months with interest of 9.2%.

Solution

Given: $P = 2,500 \quad r = 0.092 \quad t = \frac{8}{12} = \frac{2}{3}$

$$\begin{aligned} A &= P(1+rt) \\ &= 2500\left(1+.092\left(\frac{2}{3}\right)\right) \\ &\boxed{\approx \$2,653.33} \end{aligned}$$

Compounded Interest

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

A*:* Amount in the account (also called future value)

P*:* *A*mount invested ($)

r*:* *A*nnual simple interest rate

t*:* *T*ime in years

m: Number of times a year the interest is compounded

- Daily: *m* = 365
- Monthly: *m* = 12
- Quarterly: *m* = 4
- Semi-annually: *m* = 2
- Annually: *m* = 1

Compounded Continuously

$$A = Pe^{rt}$$

Example

If $1,000 is invested at 6% compounded over an 8-year period.

<u>*Solution*</u>

a) Annually

$$A = 1000\left(1+\frac{.06}{1}\right)^{1(8)} \underline{= \$1,593.85}$$

1000(1+.06/1)^(1*8)

b) Semiannually

$$A = 1000\left(1+\frac{.06}{2}\right)^{2(8)} \underline{= \$1,604.71}$$

c) Quarterly

$$A = 1000\left(1+\frac{.06}{4}\right)^{4(8)} \underline{= \$1,610.32}$$

d) Monthly

$$A = 1000\left(1+\frac{.06}{12}\right)^{12(8)} \underline{= \$1,614.14}$$

Example

What amount will an account have after 1.5 years, if $8,000 is invested at annual rate of 9%

a) Compounded ***Weekly***

$$A = 8000\left(1+\frac{.09}{52}\right)^{52(1.5)} \underline{= \$9,155.23|}$$

8000(1+.09/52) ^ (52*1.5)

b) Compounded ***Continuously***

$$A = 8000e^{.09(1.5)} \underline{= \$9,156.29|}$$

8000 e ^ (.09*1.5) (e: 2nd ln)

Example

How much should new parents invest now at 8% to have $80,000 toward their child's college education in 17 years if compounded?

a) ***Semiannually***: $m = 2$

Given: $r = 8\% = 0.08$ **A = 80,000** $t = 17$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$80,000 = P\left(1+\frac{0.08}{2}\right)^{2(17)}$$

$$\frac{80,000}{\left(1+\frac{0.08}{2}\right)^{2(17)}} = P$$

$$\underline{|P} = \frac{80,000}{\left(1+\frac{0.08}{2}\right)^{2(17)}} \underline{\approx \$21,084.17|}$$

80000 / (1+ .08/2) ^ (2*17)

b) ***Continuously*** : $A = Pe^{rt}$

$$80,000 = Pe^{(.08)(17)}$$

$$\underline{|P} = \frac{80,000}{e^{(.08)(17)}} \underline{\approx \$20,532.86|}$$

80000 / e ^ (.08*17)

Annual Percentage Yield (**APY**)

Annual $\Rightarrow t=1 \Rightarrow A=P\left(1+\frac{r}{m}\right)^{mt}=P\left(1+\frac{r}{m}\right)^{m}$

Amount @ simple interest = Amount @ Compound interest

$$P(1+APY)=P\left(1+\frac{r}{m}\right)^{m}$$ ***Divide P both side***

$$1+APY=\left(1+\frac{r}{m}\right)^{m}$$

$$APY=\left(1+\frac{r}{m}\right)^{m}-1$$

Continuously: $APY=r_e=e^{r}-1$

Effective Rate

The ***effective rate*** corresponding to a started rate of interest r compounded m times per year is

$$r_e=\left(1+\frac{r}{m}\right)^{m}-1$$

APY is also referred to as ***effective rate*** or true interest rate.

Example

How much should you invest now at 10% to have $8,000 toward the purchase of a car in 5 years if compounded?

<u>*Solution*</u>

Given: r = 10% = 0.1 **A = 8000** $t = 5$

a) ***Quarterly***: $m = 4$

$$A=P\left(1+\frac{r}{m}\right)^{mt}$$

$$8000=P\left(1+\frac{0.1}{4}\right)^{4(5)}$$

$$P=\frac{8000}{\left(1+\frac{0.1}{4}\right)^{20}}\ \underline{\approx \$4882.17}$$

$8000/(1+0.1/4)\wedge 20$

b) ***Continuously*** : $A=Pe^{rt}$

$$8000=Pe^{(0.1)(5)}$$

$$P=\frac{8000}{e^{0.5}}\ \underline{\approx \$4852.25}$$

$8000/(e\wedge(.1*5))$

Example

A $10,000 investment in a particular growth mutual fund over a recent 10-year period would have grown to $126,000. What annual nominal rate would produce the same growth if

<u>Solution</u>

a) Annually: $m = 1$

$$126000 = 10000\left(1+\frac{r}{1}\right)^{1(10)}$$

$$\frac{126000}{10000} = (1+r)^{10}$$

$$12.6 = (1+r)^{10}$$

$$(1+r)^{10} = 12.6$$

$$1+r = \sqrt[10]{12.6}$$

$$r = (12.6)^{1/10} - 1 \approx .28836 \quad or \quad \underline{28.84\%}$$

$(12.6) \wedge (1/10) - 1$

b) Continuously : $A = Pe^{rt}$

$$126000 = 10000e^{10r}$$

$$12.6 = e^{10r}$$

$$\ln 12.6 = \ln e^{10r}$$

$$\ln 12.6 = 10r \ln e$$

$$\ln 12.6 = 10r$$

$$r = \frac{\ln 12.6}{10} \approx .25337 \quad or \quad \boxed{25.337\%}$$

$\ln(12.6) / 10$

Example

A bank pays interest of 4.9% compounded monthly. Find the effective rate.

<u>Solution</u>

Given: $r = 0.049, \quad m = 12$

$$r_e = \left(1+\frac{r}{m}\right)^m - 1$$

$$= \left(1+\frac{.049}{12}\right)^{12} - 1$$

$(1+.049/12)^\wedge 12 - 1$

$$\approx 0.0501156$$

Or $\boxed{r_e = 5.01\%}$

Example

You need to borrow money. Bank ***A*** charges 8% compounded semiannually. Another bank ***B*** charges 7.9% compounded monthly. At which bank will you pay the lesser amount of interest?

Solution

Bank ***A***: $r_e = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{.08}{2}\right)^2 - 1 = .0816 \quad \rightarrow \boxed{r_e = 8.16\%}$

Bank ***B***: $r_e = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{.079}{12}\right)^{12} - 1 = .0819 \quad \rightarrow \boxed{r_e = 8.19\%}$

Bank ***A*** has the lower effective rate, although it has a higher stated rate.

Exercises *Section* 2.1 – Simple and Compound Interest

1. If you want to earn an annual rate of 10% on your investments, how much should you pay for a note that will be worth $5,000 in 6 month?

2. How much should you deposit initially in an account paying 10% compounded semiannually in order to have $1,000,000 in 30 years? ***b***) Compounded monthly? ***c***) Compounded daily?

3. You have $7,000 toward the purchase of a $10,000 automobile. How long will it take the $7000 to grow to the $10,000 if it is invested at 9% compounded quarterly? (*Round up to the next highest quarter if not exact*.)

4. How long, to the nearest tenth of a year, will it take $1000 to grow to $3600 at 8% annual interest compounded quarterly?

5. Jennifer invested $4,000 in her savings account for 4 years. When she withdrew it, she had $4,350.52. Interest was compounded continuously. What was the interest rate on the account? *Round to the nearest tenth of a percent*.

6. An actuary for a pension fund need to have $14.6 million grows to $22 million in 6 years. What interest rate compounded annually does he need for this investment to growth as specified? *Round your answer to the nearest hundredth of a percent*.

7. Which is the better investment: 9% compounded monthly or 9.1% compounded quarterly?

8. Sun Kang borrowed $5200 from his friend to pay for remodeling work on his house. He repaid the loan 10 months later with simple interest at 7%. His friend then invested the proceeds in a 5-year CD paying 6.3% compounded quarterly. How much will his friend have at the end of the 5 years?

9. The consumption of electricity has increased historically at 6% per year. If it continues to increase at this rate indefinitely, find the number of years before the electric utilities will need to double their generating capacity. *Round up to the next highest year*.

10. In the New Testament, Jesus commends a widow who contributed 2 mites (roughly ¼ cent) to the temple treasury. Suppose the temple invested those mites at 4% compounded quarterly. How much would the money be worth 2000 years later?

11. If $1,000 is invested in an account that earns 9.75% compounded annually for 6 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

12. If $2,000 is invested in an account that earns 8.25% compounded annually for 5 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

13. If an investment company pays 6% compounded semiannually, how much you should deposit now to have $10,000
    a) 5 years from now?
    b) 10 years from now?

14. If an investment company pays 8% compounded quarterly, how much you should deposit now to have $6,000
    a) 3 years from now?
    b) 6 years from now?

15. What is the annual percentage yield (APY) for money invested at
    a) 4.5% compounded monthly?
    b) 5.8% compounded quarterly?

16. What is the annual percentage yield (APY) for money invested at
    a) 6.2% compounded semiannually?
    b) 7.1% compounded monthly?

17. A newborn child receives a $20,000 gift toward a college education from her grandparents. How much will the $20,000 be worth in 17 years if it is invested at 7% compounded quarterly?

18. A person with $14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy more expensive car. How much will be available for the purchase of a car at the end of 3 years?

19. You borrowed $7200 from a bank to buy a car. You repaid the bank after 9 months at an annual interest rate of 6.2%. Find the total amount you repaid. How much of this amount is interest?

20. An account for a corporation forgot to pay the firm’s income tax of $321,812.85 on time. The government changed a penalty based on an annual interest rate of 13.4% for the 29 days the money was late. Find the total amount (tax and penalty) that was paid. (Use 365 days a year.)

21. A bond with a face value of $10,000 in 10 years can be purchased now for $5,988.02. What is the simple interest rate?

22. A stock that sold for $22 at the beginning of the year was selling for $24 at the end of the year. If the stock paid a dividend of $0.50 per share, what is the simple interest rate on an investment in this stock?

23. The Frank Russell Company is an investment fund that tracks the average performance of various groups of stocks. On average, a $10,000 investment in midcap growth funds over a recent 10-year period would have grown to $63,000. What annual nominal rate would produce the same growth if

Section 2.2 – Future Value of an Annuity

Annuity is any sequence of equal periodic payments.

Deposit is equal payment each interval

There are two basic types of annuities.

An ***annuity due*** requires that the first payment be made at the beginning of the first period.

An ***ordinary annuity*** requires that the first payment is made at the end of the first period. We will only deal with **ordinary annuities**.

\$100 every 6 months, rate r = 0.06 compounded semiannually

$$A = P\left(1+\frac{.06}{2}\right)^{2t} = 100(1.03)^{2t}$$

Years			**1**		**2**		**3**	
# of Period	0	1	2	3	4	5	6	
								$=100$
								$=100(1.03)$
								$=100(1.03)^2$
								$=100(1.03)^{2\frac{3}{2}} = 100(1.03)^3$
								$=100(1.03)^4$
								$=100(1.03)^5$
								$\sum$

S = 100 + 100(1.03) + 100(1.03)^2 + 100(1.03)^3 + 100(1.03)^4 + 100(1.03)^5

= 100(1 + 1.03 + 1.03^2 + 1.03^3 + 1.03^4 + 1.03^5)

$$= 100\frac{1.03^6 - 1}{.03}$$

$$= \$646.84$$

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = a\frac{r^n - 1}{r - 1}$$

$$\therefore S = R + R(1+i) + R(1+i)^2 + R(1+i)^3 + \cdots + R(1+i)^{n-1}$$

$$= R\frac{(1+i)^n - 1}{1+i-1}$$

$$= R\frac{(1+i)^n - 1}{i}$$

Future Value of an Ordinary Annuity (*FV*)

$$FV = PMT\,\frac{(1+i)^n - 1}{i} \quad = PMT\, s_{\overline{n}|i}$$

PMT: Periodic payment

i: Rate per period $\quad i = \frac{r}{m}$

n: Number of payment

Example

What is the value of an annuity at the end of 10 years if $1,000 is deposited every 6 months into an account earning 8% compounded semiannually? How much of this value is interest?

Solution

Given: 8% compounded semiannually $\Rightarrow\ i = \frac{r}{m} = \frac{.08}{2} = .04$

a) ***Annuity***

$$FV = PMT\,\frac{(1+i)^n - 1}{i}$$

$$= 1000\,\frac{(1+0.04)^{20} - 1}{0.04}$$

$$\approx \$29{,}778.08$$

b) ***How much is the interest***?

Deposits = 20(1000) = 20,000.00

Interest = Value – Deposit

$$= 29{,}778.08 - 20{,}000$$

= $ 9,778.08

Sinking Funds

Sinking Fund is established for accumulating funds to meet future obligations or debts.

∴ How much I have to pay?

$$PMT = FV\frac{i}{(1+i)^n - 1} \quad ; \qquad i = \frac{r}{m}, \quad n = mt$$

Example

A bond issue is approved for building a marina in a city. The city is required to make regular payments every 3 months into a sinking fund paying 5.4% compounded quarterly. At the end of 10 years, the bond obligation will be retired with a cost of $5,000,000.

<u>*Solution*</u>

Given: Cost = $5,000,000.00 in 10 yrs

$\Rightarrow n = mt = 4(10) = 40$

$$i = \frac{r}{m} = \frac{.054}{4} = 0.0135$$

a) What should each payment be?

$$PMT = FV\frac{i}{(1+i)^n - 1}$$

$$= 5,000,000\frac{.0135}{(1+.0135)^{40} - 1}$$

$$\boxed{\approx \$95,094.67 \ \ per\ quarter}$$

5000000 (.0135 / ((1+.0135) ^ 40 - 1)

b) How much interest is earned during the 10th year?

1st 9 years = t = 4(9) = 36

$$FV = PMT\frac{(1+.0135)^{36} - 1}{.0135}$$

$$= 95094.67\frac{(1+.0135)^{36} - 1}{.0135}$$

$$\approx 4,370,992.44$$

5,000,000 – 4,370,992.44 = $629,007.56 *after 9 years*

3 *months* ⇒ (4) PMT

⇒ (4)(95094.67) = $380,378.68

Interest = 629,007.56 - 380,378.68 = $248,628.88

Example

Experts say the baby boom generation can't count on a company pension or Social Security to provide a comfortable retirement, as their parents did. It is recommended that they start to save early and regularly. Sarah, a baby boomer, has decided to deposit $200 each month for 20 years in an account that pays interest of 7.2% compounded monthly.

a) How much will be in the account at the end of 20 years?

b) Sarah believes she needs to accumulate $130,000 in the 20-year period to have enough for retirement. What interest rate would provide that amount?

<u>*Solution*</u>

a) **Given**: $PMT = 200 \quad m = 12 \quad r = 0.072$

$$i = \frac{r}{m} = \frac{.072}{12}$$

$$n = mt = 12(20) = 240$$

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 200\left[\frac{\left(1+\frac{.072}{12}\right)^{240} - 1}{\frac{.072}{12}}\right]$$

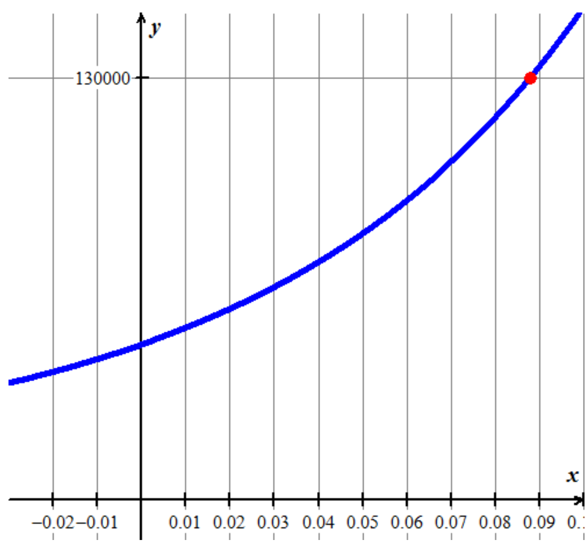
$$\underline{\approx \$106,752.47|}$$

b) $$130000 = 200\left[\frac{\left(1+\frac{r}{12}\right)^{240} - 1}{\frac{r}{12}}\right]$$

$$130000 = 200\frac{12}{r}\left[\left(\frac{12+r}{12}\right)^{240} - 1\right]$$

$$\frac{130000}{2400} = \frac{1}{r}\left[\left(\frac{12+r}{12}\right)^{240} - 1\right]$$

$$\frac{325}{6} = \frac{1}{r}\left[\left(\frac{12+r}{12}\right)^{240} - 1\right]$$



Using a calculator or program; the annual interest rate is 8.79%.

Exercises *Section* 2.2 – Future Value of an Annuity

1. Recently, Guaranty Income Life offered an annuity that pays 6.65% compounded monthly. If $500 is deposited into this annuity every month, how much is in the account after 10 years? How much of this is interest?

2. Recently, USG Annuity Life offered an annuity that pays 4.25% compounded monthly. If $1,000 is deposited into this annuity every month, how much is in the account after 15 years? How much of this is interest?

3. In order to accumulate enough money for a down payment on a house, a couple deposits $300 per month into an account paying 6% compounded monthly. If payments are made at the end of each period, how much money will be in the account in 5 years?

4. A self-employed person has a Keogh retirement plan. (This type of plan is free of taxes until money is withdrawn.) If deposits of $7,500 are made each year into an account paying 8% compounded annually, how much will be in the account after 20 years?

5. Sun America recently offered an annuity that pays 6.35% compounded monthly. What equal monthly deposit should be made into this annuity in order to have $200,000 in 15 years?

6. Recently, The Hartford offered an annuity that pays 5.5% compounded monthly. What equal monthly deposit should be made into this annuity in order to have $100,000 in 10 years?

7. Compu-bank, an online banking service, offered a money market account with an APY of 4.86%.
   a) If interest is compounded monthly, what is the equivalent annual nominal rate?
   b) If you wish to have $10,000 in the account after 4 years, what equal deposit should you make each month?

8. American Express's online banking division offered a money market account with an APY of 5.65%.
   a) If interest is compounded monthly, what is the equivalent annual nominal rate?
   b) If you wish to have $1,000,000 in the account after 8 years, what equal deposit should you make each month?

9. Find the future value of an annuity due if payments of $500 are made at the beginning of each quarter for 7 years, in an account paying 6% compounded quarterly.

10. A 45 year-old man puts $2500 in a retirement account at the end of each quarter until he reaches the age of 60, then makes no further deposits. If the account pays 6% interest compounded quarterly, how much will be in the account when the man retires at age 65?

11. A father opened a savings account for his daughter on the day she was born, depositing \$1000. Each year on her birthday he deposits another \$1000, making the last deposit on her 21st birthday. If the account pays 5.25% interest compounded annually, how much is in the account at the end of the day on his daughter's 21st birthday? How much interest has been earned?

12. You deposits \$10,000 at the beginning of each year for 12 years in an account paying 5% compounded annually. Then you put the total amount on deposit in another account paying 6% compounded semi-annually for another 9 years. Find the final amount on deposit after the entire 21-year period.

13. You need \$10,000 in 8 years.
    a) What amount should be deposit at the end of each quarter at 8% compounded quarterly so that he will have his \$10,000?
    b) Find your quarterly deposit if the money is deposited at 6% compounded quarterly.

14. You want to have a \$20,000 down payment when you buy a car in 6 years. How much money must you deposit at the end of each quarter in an account paying 3.2% compounded quarterly so that you will have the down payment you desire?

15. You sell a land and then you will be paid a lump sum of \$60,000 in 7 years. Until then, the buyer pays 8% simple interest quarterly.
    a) Find the amount of each quarterly interest payment on the \$60,000
    b) The buyer sets up a sinking fund so that enough money will be present to pay off the \$60,000. The buyer will make semiannual payments into the sinking fund; the account pays 6% compounded semiannually. Find the amount of each payment into the fund.

Section 2.3 – Present Value of an Annuity; Amortization

Principal Initial Value

PV is the present value or present sum of the payments.

PMT is the periodic payments.

Given

r = 6% semiannually, in order to withdraw \$1,000.00 every 6 months for next 3 years.

$i = \frac{r}{m} = \frac{.06}{2} = 0.03$

A = 1000 = PMT (periodic payment)

$$A = P(1+i)^n \Rightarrow P = \frac{A}{(1+i)^n} = A(1+i)^{-n} = 1000(1+.03)^{-n}$$

	Years			**1**		**2**		**3**
	Period	0	1	2	3	4	5	6
$=1000(1.03)^{-1}$								
$=1000(1.03)^{-2}$								
$=1000(1.03)^{-3}$								
$=1000(1.03)^{-4}$								
$=1000(1.03)^{-5}$								
$=1000(1.03)^{-6}$								

$$P = 1000(1.03)^{-1} + 1000(1.03)^{-2} + \cdots + 1000(1.03)^{-6}$$

$$P = R\frac{1-(1+i)^{-n}}{i}$$

Present Value (***PV***) of an ordinary annuity:

$$PV = PMT\frac{1-(1+i)^{-n}}{i}$$

i: Rate per period

n: Number of periods

Notes: Payments are made at the end of each period.

Example

A car costs \$12,000. After a down payment of \$2,000, the balance will be paid off in 36 equal monthly payments with interest of 6% per year on the unpaid balance, Find the amount of each payment.

Solution

Given: $P = 12,000 - 2,000 = 10,000$

$n = 36$

$i = \frac{.06}{12} = .005$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i} \approx \$13,577.71$$

$$10,000 = PMT\,\frac{1-(1.005)^{-66}}{.005}$$

$$PMT = \frac{10,000(.005)}{1-(1.005)^{-36}} \approx \$304.22 \qquad 10000(.005)/(1-(1.005)\wedge((-)36))$$

Example

An annuity that earned 6.5%. A person plans to make equal annual deposits into this account for 25 years in order to them make 20 equal annual withdrawals of \$25,000 reducing the balance in the amount to zero. How much must be deposited annually to accumulate sufficient funds to provide for these payments? How much total interest is earned during this entire 45-years process?

Solution

$r = 0.065$ annually

$$1 \xrightarrow[\text{Increasing}]{PMT} \underset{FV = PV}{25\,yrs} \xrightarrow[\text{decreasing}]{25\ k} 20\,yrs\,(=45)$$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i} = 25000\,\frac{1-(1.065)^{-20}}{.065} \qquad 25000\left(1-1.065\wedge((-)20)\right)/.065$$

$$\approx \$275,462.68$$

$$\boxed{FV = PV} \qquad FV = PMT\,\frac{(1+i)^n - 1}{i} \Rightarrow PMT = FV\,\frac{i}{(1+i)^n - 1}$$

$$\Rightarrow PMT = FV\,\frac{i}{(1+i)^n-1} = 275462.68\,\frac{.065}{(1.065)^{25}-1}$$

$$= \$4,677.76$$

$$\text{Total interest} = \underbrace{20(25000)}_{\textit{Withdraw}} - \underbrace{25(4677.76)}_{\textit{Deposit}} = \$383056.$$

Amortization

Amortization debt means the debt retired in given length (= payment),

Borrow money from a bank to buy and agree to payment period (36 months)

Example

Borrow \$5000 payment in 36 months, compounded monthly @ r = 12%. How much payment?

<u>*Solution*</u>

$i=\frac{.12}{12}=.01 \qquad n=36$

$$PV=PMT\frac{1-(1+i)^{-n}}{i}$$

$$\Rightarrow PMT=5000\frac{.01}{1-(1.01)^{-36}}$$

5000*.01 / (1-1.01^ (-)36))

$$\underline{=\$166.07 \ \ per\ \ month}|$$

Example

If you sell your car to someone for \$2,400 and agree to finance it at 1% per month on the unpaid balance, how much should you receive each month to amortize the loan in 24 months? How much interest will you receive?

<u>*Solution*</u>

$$PV=PMT\frac{1-(1+i)^{-n}}{i}$$

$$PMT=2400\frac{.01}{1-(1+.01)^{-24}}$$

2400*.01/(1- (1 + .01) ^ (-)24))

$$\underline{=\$112.98 \ \ per\ \ month}|$$

Total interest = amount of all payment – initial loan

= 24(112.98) – 2400

= \$311.52

Amortization Schedules

Pay off earlier last payment (lump sum) = Amortization schedules

Example

If you borrow $500 that you agree to repay in six equal monthly payments at 1% interest per month on the unpaid balance, how much of each monthly payment is used for interest and how much is used to reduce the unpaid balance

Solution

$$PMT = 500\frac{.01}{1-(1.01)^{-6}} = \$86.27 \text{ per month}$$

500(.01 / (1 – 1.01) ^ (–)6

@ The end of the 1st month interest due = 500(.01) = $5.00

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0				$500.00
1	$86.27	5.00	86.27 – 5 = 81.27	500-81.27 = $418.73
2	$86.27	418.73(.01) = 4.19	86.27 – 4.19 = 82.08	418.73-82.08 = $336.65
3	$86.27	336.65(.01) = 3.37	86.27 – 3.37 = 82.90	336.65-82.90 = $253.75
4	$86.27	2.54	86.27 – 2.54 = 83.73	$170.02
5	$86.27	1.7	86.27 – 1.7 = 84.57	$85.45
6	$86.27	.85	86.27 – .85 = 85.54	$0.0

Example

Construct an amortization schedule for a $1,000 debt that is to be amortized in six equal monthly payment at 1.25% interest rate per month on the unpaid balance.

Solution

$$PMT = 1000\frac{.0125}{1-(1.0125)^{-6}} = \$174.03 \text{ per month}$$

1000(.0125 / (1 – 1.0125) ^ (–)6

1st month interest due = 1000(.0125) = $12.50

#	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0				$1000.00
1	$174.03	$12.50	$ 161.53	$838.47
2	$174.03	10.48	163.55	$ 674.92
3	$174.03	8.44	165.59	$ 509.33
4	$174.03	6.37	167.66	$ 341.67
5	$174.03	4.27	169.76	$ 171.91
6	$174.03	2.15	171.91	$0.0
	$1044.21	**$44.21**	**Total = $1000**	

Equity

Equity = Current net market value – Unpaid balance

Example

A family purchase a home 10 years ago for $80,000.00. The home was financed by paying 20% down for 30-year mortgage at 9%, on the unpaid balance. The net market of the house is now $120,000.00 and the family wishes to sell the house. How much equity after making 120 monthly payments?

<u>*Solution*</u>

$Equity = Current\ Net - Unpaid\ Balance$

$$0 \longrightarrow 10 \xrightarrow{\quad Unpaid\ balance\ (20\ yrs) \quad} 30$$

Down Payment = 20 % $\Rightarrow$ Left 80% = .8(80000) = 64,000.00

$n = 12(30) = 360$

$i = \frac{.09}{12} = .0075$

Monthly Payment?

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 64,000 \frac{.0075}{1-(1.0075)^{-360}}$$

$64,000(.0075 / (1 - 1.0075) \wedge (12 * 30))$

$$\approx \boxed{\$514.96\ \ per\ month}$$

Unpaid balance – 10 years (now) $\Rightarrow$ 30 -10 = 20 years

$$PV = PMT \frac{1-(1+i)^{-n}}{i}$$

$$= 514.96 \frac{1-(1.0075)^{-240}}{.0075}$$

$514.96((1 - 1.0075) \wedge 240) / .0075$

$$\approx \boxed{\$57,235.00}$$

Equity = current – unpaid balance

= 120,000 – 57,235

= $ 62,765.

Exercises *Section* **2.3 – Present Value of an Annuity Amortization**

1. How much should you deposit in an account paying 8% compounded quarterly in order to receive quarterly payments of $1,000 for the next 4 years?

2. You have negotiated a price of $25,200 for a new truck. Now you must choose between 0% financing for 48 months or a $3,000 rebate. If you choose the rebate, you can obtain a loan for the balance at 4.5% compounded monthly for 48 months . Which option should you choose?

3. Suppose you have selected a new car to purchase for $19,500. If the car can be financed over a period of 4 years at an annual rate of 6.9% compounded monthly, how much will your monthly payments be? Construct an amortization table for the first 3 months.

4. Suppose your parents decide to give you $10,000 to be put in a college trust fund that will be paid in equally quarterly installments over a 5 year period. If you deposit the money into an account paying 1.5% per quarter, how much are the quarterly payments (Assume the account will have a zero balance at the end of period.)

5. You finally found your dream home. It sells for $120,000 and can be purchased by paying 10% down and financing the balance at an annual rate of 9.6% compounded monthly.
   a) How much are your payments if you pay monthly for 30 years?
   b) Determine how much would be paid in interest .
   c) Determine the payoff after 100 payments have been made.
   d) Change the rate to 8.4% and the time to 15 years and calculate the payment.
   e) Determine how much would be paid in interest and compare with the previous interest.

6. Sharon has found the perfect car for her family (anew mini-van) at a price of $24,500. She will receive a $3500 credit toward the purchase by trading in her old Gremlin, and will finance the balance at an annual rate of 4.8% compounded monthly.
   a) How much are her payments if she pays monthly for 5 years?
   b) How long would it take for her to pay off the car paying an extra $100 per mo., beginning with the first month?

7. Marie has determined that she will need $5000 per month in retirement over a 30-year period. She has forecasted that her money will earn 7.2% compounded monthly. Marie will spend 25-years working toward this goal investing monthly at an annual rate of 7.2%. How much should Marie's monthly payments be during her working years in order to satisfy her retirement needs?

8. American General offers a 10-year ordinary annuity with a guaranteed rate of 6.65% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $5,000 annually over the 10-year period?

9. American General offers a 7-year ordinary annuity with a guaranteed rate of 6.35% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $10,000 annually over the 7-year period?

10. You want to purchase an automobile for $27,300. The dealer offers you 0% financing for 60 months or a $5,000 rebate. You can obtain 6.3% financial for 60 months at the local bank. Which option should you choose? Explain.

11. You want to purchase an automobile for $28,500. The dealer offers you 0% financing for 60 months or a $6,000 rebate. You can obtain 6.2% financial for 60 months at the local bank. Which option should you choose? Explain.

12. Construct the amortization schedule for a $5,000 debt that is to be amortized in eight equal quarterly payments at 2.8% interest per quarter on the unpaid balance.

13. Construct the amortization schedule for a $10,000 debt that is to be amortized in six equal quarterly payments at 2.6% interest per quarter on the unpaid balance.

14. A loan of $37,948 with interest at 6.5% compounded annually, to be paid with equal annual payments over 10 years

15. A loan of $4,836 with interest at 7.25% compounded semi-annually, to be repaid in 5 years in equal semi-annual payments.