Solution   Section 2.5 – Polynomial Functions

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=5x^3+7x^2-x+9$

Solution

Leading term: $5x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=11x^3-6x^2+x+3$

Solution

Leading term: $11x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\quad$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-11x^3-6x^2+x+3$

Solution

Leading term: $-11x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=2x^3+3x^2-23x-42$

Solution

Leading term: $2x^3$ with 3^{rd} degree (n is ***odd***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^4 + 7x^2 - x + 9$

Solution

Leading term: $5x^4$ with 4^{rd} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 11x^4 - 6x^2 + x + 3$

Solution

Leading term: $11x^4$ with 4^{rd} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -5x^4 + 7x^2 - x + 9$

Solution

Leading term: $-5x^4$ with 4^{rd} degree (n is ***even***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty \qquad\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -11x^4 - 6x^2 + x + 3$

Solution

Leading term: $-11x^4$ with 4^{rd} degree (n is ***even***)

$x \to -\infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls left

$x \to \infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^5 - 16x^2 - 20x + 64$

Solution

Leading term: $5x^5$ with 5^{th} degree (n is ***odd***)

$x \to -\infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls left

$x \to \infty \Rightarrow f(x) \to \infty$ $f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -5x^5 - 16x^2 - 20x + 64$

Solution

Leading term: $-5x^5$ with 5^{th} degree (n is ***odd***)

$x \to -\infty \Rightarrow f(x) \to \infty$ $f(x)$ rises left

$x \to \infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -3x^6 - 16x^3 + 64$

Solution

Leading term: $-3x^6$ with 6^{th} degree (n is ***even***)

$x \to -\infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls left

$x \to \infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 3x^6 - 16x^3 + 4$

Solution

Leading term: $3x^6$ with 6^{th} degree (n is ***even***)

$x \to -\infty \;\Rightarrow\; f(x) \to \infty$ $\qquad f(x)$ rises left

$x \to \infty \;\Rightarrow\; f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $\quad f(x) = x^3 - x - 1; \quad between \;\; 1 \; and \;\; 2$

Solution

$$f(1) = (1)^3 - (1) - 1$$

$$\underline{= -1}$$

$$f(2) = (2)^3 - (2) - 1$$

$$\underline{= 5}$$

Since $f(1)$ *and* $f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $\quad f(x) = x^3 - 4x^2 + 2; \quad between \;\; 0 \; and \;\; 1$

Solution

$$f(0) = (0)^3 - 4(0)^2 + 2$$

$$\underline{= 2}$$

$$f(1) = (1)^3 - 4(1)^2 + 2$$

$$\underline{= -1}$$

Since $f(0)$ *and* $f(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 2x^4 - 4x^2 + 1;$ *between* -1 *and* 0

Solution

$$f(-1) = 2(-1)^4 - 4(-1)^2 + 1$$
$$\underline{= -1}$$

$$f(0) = 2(0)^4 - 4(0)^2 + 1$$
$$\underline{= 1}$$

Since $f(0)$ *and* $f(-1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between –1 and 0.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^4 + 6x^3 - 18x^2;$ *between* 2 *and* 3

Solution

$$f(2) = (2)^4 + 6(2)^3 - 18(2)^2$$
$$\underline{= -8}$$

$$f(3) = (3)^4 + 6(3)^3 - 18(3)^2$$
$$\underline{= 81}$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 + x^2 - 2x + 1;$ *between* -3 *and* -2

Solution

$$f(-3) = (-3)^3 + (-3)^2 - 2(-3) + 1$$
$$\underline{= -11}$$

$$f(-2) = (-2)^3 + (-2)^2 - 2(-2) + 1$$

$= 1$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between –2 and –3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^5 - x^3 - 1;$ *between* 1 *and* 2

Solution

$f(1) = (1)^5 - (1)^3 - 1$

$= -1$

$f(2) = (2)^5 - (2)^3 - 1$

$= 23$

Since $f(1)$ *and* $f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 10x + 9;$ *between* -3 *and* -2

Solution

$f(-3) = 3(-3)^3 - 10(-3) + 9$

$= -42$

$f(-2) = 3(-2)^3 - 10(-2) + 9$

$= 5$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between –3 and –2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 8x^2 + x + 2;$ *between* 2 *and* 3

Solution

$f(2) = 3(2)^3 - 8(2)^2 + (2) + 2$

$\underline{= -4}$

$f(3) = 3(3)^3 - 8(3)^2 + (3) + 2$

$\underline{= 14}$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 8x^2 + x + 2;$ *between* 1 *and* 2

Solution

$f(1) = 3(1)^3 - 8(1)^2 + (1) + 2$

$\underline{= -2}$

$f(2) = 3(2)^3 - 8(2)^2 + (2) + 2$

$\underline{= -4}$

Since $f(1)$ *and* $f(2)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^5 + 2x^4 - 6x^3 + 2x - 3;$ *between* 0 *and* 1

Solution

$f(0) = (0)^5 + 2(0)^4 - 6(0)^3 + 2(0) - 3$

$\underline{= -3}$

$f(1) = (1)^5 + 2(1)^4 - 6(1)^3 + 2(1) - 3$

$\underline{= -4}$

Since $f(0)$ *and* $f(1)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 2x^3 + 3x^2 - 23x - 42, \quad a = 3, \quad b = 4$

Solution

$P(3) = 54 + 27 - 69 - 42$

$\quad = -30$

$P(4) = 128 + 48 - 92 - 42$

$\quad = 90$

Since $P(3)$ *and* $P(4)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and 4.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 4x^3 - x^2 - 6x + 1, \quad a = 0, \quad b = 1$

Solution

$P(0) = 1$

$P(1) = 4 - 1 - 6 + 1$

$\quad = -2$

Since $P(0)$ *and* $P(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 3x^3 + 7x^2 + 3x + 7, \quad a = -3, \quad b = -2$

Solution

$P(-3) = -81 + 63 - 9 + 7$

$\quad = -20$

$P(-2) = -24 + 28 - 6 + 7$

$\quad = 5$

Since $P(-3)$ *and* $P(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between –3 and –2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=2x^3-21x^2-2x+25,\quad a=1,\quad b=2$

Solution

$$P(1)=2-21-2+25$$
$$\underline{=4}$$
$$P(2)=16-84-4+25$$
$$\underline{=-47}$$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=4x^4+7x^3-11x^2+7x-15,\quad a=1,\quad b=\frac{3}{2}$

Solution

$$P(1)=4+7-11+7-15$$
$$\underline{=-8}$$
$$P\left(\frac{3}{2}\right)=81+\frac{189}{8}-\frac{99}{4}+\frac{21}{2}-15$$
$$=66+\frac{189-198+84}{8}$$
$$=66+\frac{75}{8}$$
$$\underline{=\frac{603}{8}}$$

Since $P(1)$ *and* $P\left(\frac{3}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and $\frac{3}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=5x^3-16x^2-20x+64,\quad a=3,\quad b=\frac{7}{2}$

Solution

$P(3) = 135 - 144 - 60 + 64$

$\quad = -5$

$P\left(\frac{7}{2}\right) = \frac{1715}{8} - 196 - 70 + 64$

$\quad = \frac{1715}{8} - 202$

$\quad = \frac{99}{8}$

Since $P(3)$ *and* $P\left(\frac{7}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and $\frac{7}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^4 - x^2 - x - 4, \quad a = 1, \quad b = 2$

Solution

$P(1) = 1 - 1 - 1 - 4$

$\quad = -5$

$P(2) = 16 - 4 - 2 - 4$

$\quad = 6$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^3 - x - 8, \quad a = 2, \quad b = 3$

Solution

$P(2) = 8 - 2 - 8$

$\quad = -2$

$P(3) = 27 - 3 - 8$

$\quad = 16$

Since $P(2)$ *and* $P(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^3-x-8, \quad a=0, \quad b=1$

Solution

$$\boxed{P(0)=-8}$$

$$P(1)=1-1-8$$

$$\boxed{=-8}$$

Since $P(0)$ *and* $P(1)$ have same sign.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^3-x-8, \quad a=2.1, \quad b=2.2$

Solution

$$P(2.1)=P\left(\frac{21}{10}\right)$$

$$=\frac{9261}{1000}-\frac{21}{10}-8$$

$$=\frac{9261-2100-8000}{1000}$$

$$\boxed{=-\frac{839}{1,000}}$$

$$P(2.2)=P\left(\frac{2.2}{10}\right)$$

$$=\frac{10,648}{1000}-\frac{22}{10}-8$$

$$=\frac{10,648-2,200-8,000}{1000}$$

$$=\frac{448}{1,000}$$

$$\boxed{=0.448}$$

Since $P(2.1)$ *and* $P(2.2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2.1 and 2.2.