Math 2318 – Linear Algebra **_Exam 1_** **_Review_**

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1. Solve the system by Gaussian elimination

$$\textbf{\textit{a}})\ \begin{cases} 2x_1 - 4x_2 + 3x_3 - 4x_4 - 11x_5 = 28 \\ x_1 + 2x_2 - x_3 + 2x_4 + 5x_5 = -13 \\ \quad -3x_3 + x_4 + 6x_5 = -10 \\ 3x_1 - 6x_2 + 10x_3 - 8x_4 - 28x_5 = 61 \end{cases}$$

$$\textbf{\textit{b}})\ \begin{cases} x_1 + x_3 + x_4 - 2x_5 = 1 \\ 2x_1 + x_2 + 3x_3 - x_4 + x_5 = 0 \\ 3x_1 - x_2 + 4x_3 + x_4 + x_5 = 1 \end{cases}$$

2. Given the matrices

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix},\quad B = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix},\quad C = \begin{bmatrix} 2 & 1 \\ 4 & 0 \\ 8 & -1 \\ 3 & 2 \end{bmatrix},\quad D = \begin{bmatrix} 2 & 1 & 3 & 6 \\ 2 & 0 & 0 & 4 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & 1 & 2 \end{bmatrix}$$

a) $A - 3B$ ***b***) $3A + 4B$ ***c***) $D + C$ ***d***) AB ***e***) BA ***f***) CD ***g***) DC ***h***) CA ***i***) AC ***j***) CB

3. Find the inverse of the following matrices if they exist.

a) $A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$ ***b***) $B = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$ ***c***) $C = \begin{bmatrix} 2 & -4 \\ a & b \end{bmatrix}$

4. Evaluate the determinant

a) $\begin{vmatrix} 3 & 1 & 2 \\ -2 & 3 & 1 \\ 3 & 4 & -6 \end{vmatrix}$ ***b***) $\begin{vmatrix} x & 1 & -1 \\ x^2 & x & x \\ 0 & x & 1 \end{vmatrix}$ ***c***) $\begin{vmatrix} 1 & x & x \\ 2 & x^2 & 2x \\ x & 0 & -1 \end{vmatrix}$ ***d***) $\begin{vmatrix} a & c \\ -2 & -4 \end{vmatrix}$ ***e***) $\begin{vmatrix} 1 & k & k^2 \\ 1 & k & k^2 \\ 1 & k & k^2 \end{vmatrix}$

5. Find A^2, A^{-2}, *and* A^{-k} by inspection

a) $A = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ *b*) $A = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

6. Express $\left((AB)^{-1}\right)^T$ in terms of $\left(A^{-1}\right)^T$ and $\left(B^{-1}\right)^T$

7. Solve the system of equations using Cramer's Rule:

a.
$$\begin{aligned} x-y+2z&=0\\ x-2y+3z&=-1\\ 2x-2y+z&=-3 \end{aligned}$$

b.
$$\begin{aligned} x-y+z&=-4\\ 5x+y-2z&=12\\ 2x-3y+4z&=-15 \end{aligned}$$

***Prove*:**

a) $(ABC)^{-1}=C^{-1}B^{-1}A^{-1}$ where A, B, and C are invertible

b) $\left(A^T\right)^{-1}=\left(A^{-1}\right)^T$ where A is invertible

c) If A is invertible and $AB=AC$, prove that $B=C$

d) Prove if $A^TA=A$, then A is symmetric and $A=A^2$

e) $\det(A+B)\neq\det(A)+\det(B)$

f) $\det(AB)=\det(A)\det(B)$

g) $\det(kA)=k^n\det\left(A^T\right)$ A is $n\times n$

Solution

1. *a)* $\left(3+2x_2-2x_5,\ x_2,\ 2+x_5,\ -4-3x_5,\ x_5\right)$

a) $\left(3-\frac{7}{2}x_4+8x_5,\ \frac{1}{2}x_4+x_5,\ -2+\frac{5}{2}x_4-6x_5,\ x_4,\ x_5\right)$

2. ***a)*** $\begin{bmatrix}-1 & -3\\ -2 & -8\end{bmatrix}$ ***b)*** $\begin{bmatrix}10 & 17\\ 7 & 28\end{bmatrix}$ ***c)*** *can't be determined* ***d)*** $\begin{bmatrix}5 & 16\\ 5 & 18\end{bmatrix}$

e) $\begin{bmatrix}4 & 11\\ 6 & 19\end{bmatrix}$ ***f)*** *can't be determined* ***g)*** $\begin{bmatrix}48 & 11\\ 16 & 10\\ 3 & -2\\ 28 & 4\end{bmatrix}$ ***h)*** $\begin{bmatrix}5 & 10\\ 8 & 12\\ 15 & 20\\ 8 & 17\end{bmatrix}$

i) *can't be determined* ***j)*** $\begin{bmatrix}3 & 8\\ 4 & 8\\ 7 & 12\\ 5 & 14\end{bmatrix}$

3. *a)* $A^{-1}=\begin{bmatrix}1 & -2\\ 2 & -3\end{bmatrix}$ *b)* B^{-1} Does not exist *c)* $C^{-1}=\begin{bmatrix}\frac{b}{2b+4a} & \frac{2}{b+2a}\\ -\frac{a}{2b+4a} & \frac{1}{b+2a}\end{bmatrix}$

4. ***a)*** -109 ***b)*** $-2x^3$ ***c)*** $-x^4+2x^3-x^2+2x$ ***d)*** $-4a+2c$ ***e)*** 0

5. *a)* $A^2=\begin{bmatrix}\frac{1}{16} & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & \frac{1}{4}\end{bmatrix}$ $A^{-2}=\begin{bmatrix}16 & 0 & 0\\ 0 & \frac{1}{9} & 0\\ 0 & 0 & 4\end{bmatrix}$ $A^{-k}=\begin{bmatrix}4^k & 0 & 0\\ 0 & 3^{-k} & 0\\ 0 & 0 & 2^k\end{bmatrix}$

b) $A^2=\begin{bmatrix}9 & 0 & 0 & 0\\ 0 & 36 & 0 & 0\\ 0 & 0 & 9 & 0\\ 0 & 0 & 0 & 4\end{bmatrix}$ $A^{-2}=\begin{bmatrix}\frac{1}{9} & 0 & 0 & 0\\ 0 & \frac{1}{36} & 0 & 0\\ 0 & 0 & \frac{1}{9} & 0\\ 0 & 0 & 0 & \frac{1}{4}\end{bmatrix}$ $A^{-k}=\begin{bmatrix}(-3)^{-k} & 0 & 0 & 0\\ 0 & (6)^{-k} & 0 & 0\\ 0 & 0 & (-3)^{-k} & 0\\ 0 & 0 & 0 & (-2)^{-k}\end{bmatrix}$

6. $\left((AB)^{-1}\right)^T=\left(A^{-1}\right)^T\left(B^{-1}\right)^T$

7. *a.* $D=3\quad D_x=0\quad D_y=6\quad D_z=3\quad (0,\ 2,\ 1)$

b. $D=5\quad D_x=5\quad D_y=15\quad D_z=-10\quad (1,\ 3,\ -2)$