Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \tan\left(x - \frac{\pi}{4}\right)$

Solution

Amplitude: n / a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

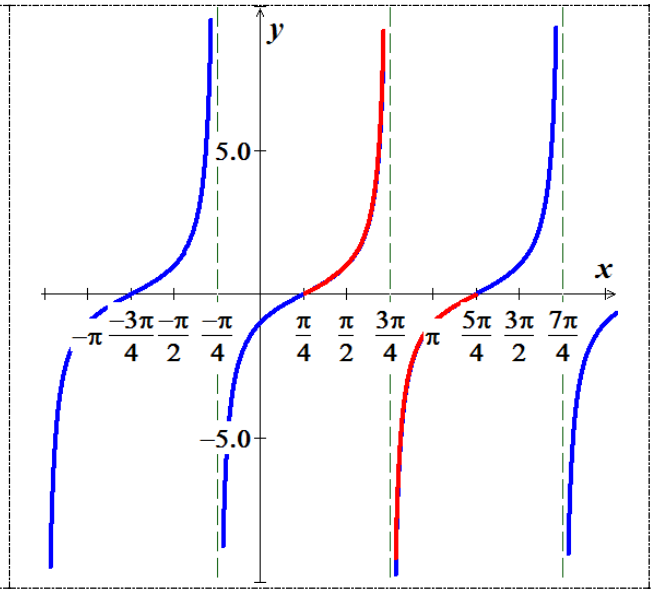
VT: $y = 0$

Asymptotes:

$x = -\frac{\pi}{4} + n\pi$

$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$

	x	y
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	1
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	∞
$\frac{3\pi}{4} + \frac{\pi}{4}$	π	–1
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\tan\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n / a

Period: $P = \frac{\pi}{2}$

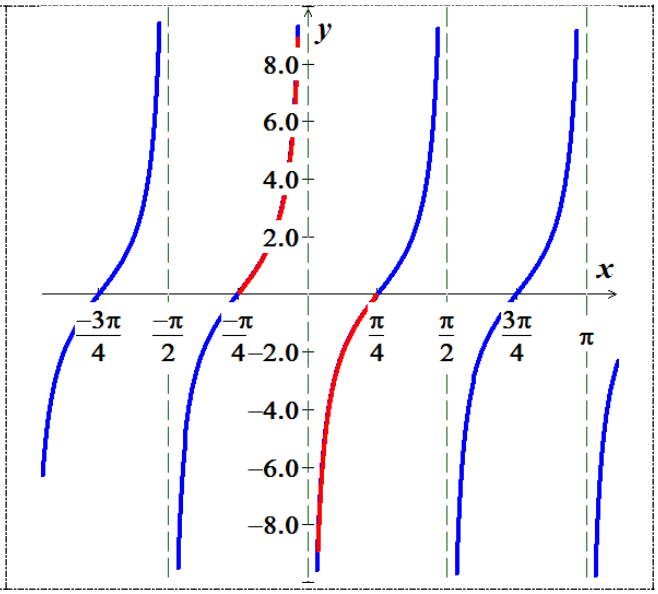
Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

Asymptotes: $x = n\frac{\pi}{2}$

$x = \ldots, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \ldots$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	0
$\frac{\pi}{8} - \frac{\pi}{4}$	$-\frac{\pi}{8}$	2
$\frac{\pi}{4} - \frac{\pi}{4}$	0	∞
$\frac{3\pi}{8} - \frac{\pi}{4}$	$\frac{\pi}{8}$	–2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{4}\tan\left(\frac{1}{2}x + \frac{\pi}{3}\right)$

Solution

Amplitude: n / a

Period: $P = 2\pi$

Phase Shift: $\varphi = -\frac{2\pi}{3}$

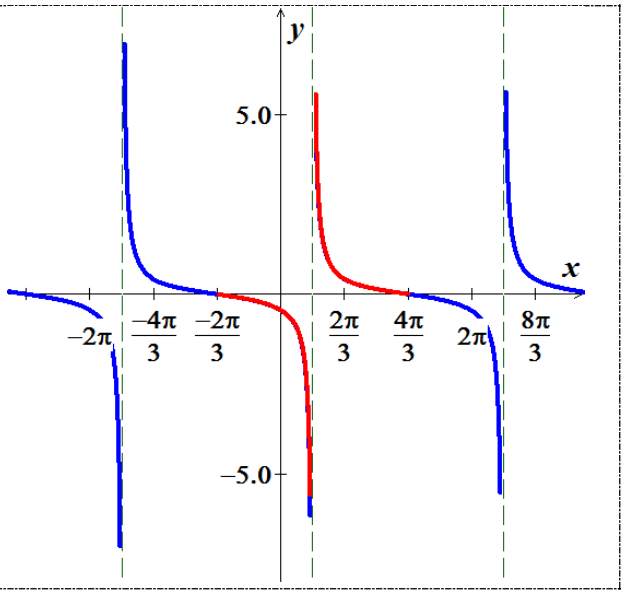
VT: $y = 0$

Asymptotes:

$x = -\frac{5\pi}{3} + 2n\pi$

$x = \ldots, -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3}, \ldots$

	x	y
$0 - \frac{2\pi}{3}$	$-\frac{2\pi}{3}$	0
$\frac{\pi}{2} - \frac{2\pi}{3}$	$-\frac{\pi}{6}$	$-\frac{1}{4}$
$\pi - \frac{2\pi}{3}$	$\frac{\pi}{3}$	∞
$\frac{3\pi}{2} - \frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{1}{4}$
$2\pi - \frac{2\pi}{3}$	$\frac{4\pi}{3}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \cot\left(x + \frac{\pi}{4}\right)$

Solution

Amplitude: n / a

Period: $P = \pi$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

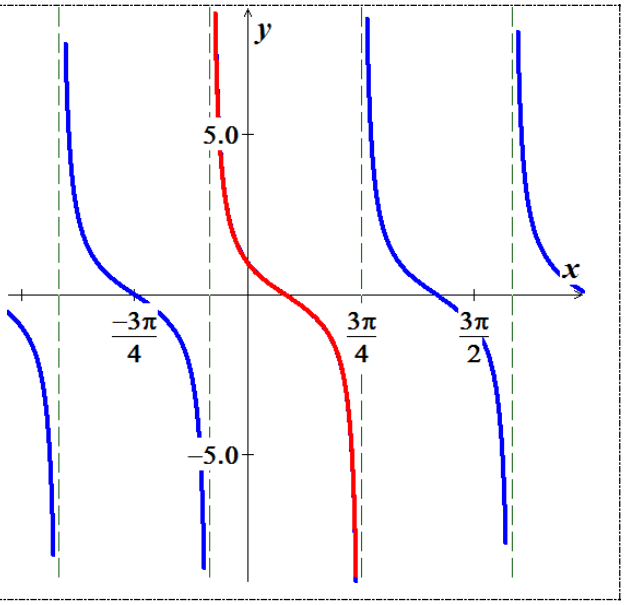
Asymptotes:

$x = 2n\pi + \pi - \frac{\pi}{4}$

$x = 2n\pi + \frac{3\pi}{4}$

$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	∞
$\frac{\pi}{4} - \frac{\pi}{4}$	0	1
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{3\pi}{4} - \frac{\pi}{4}$	$\frac{\pi}{2}$	−1
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	∞



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\cot\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n / a

Period: $P = \frac{\pi}{2}$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

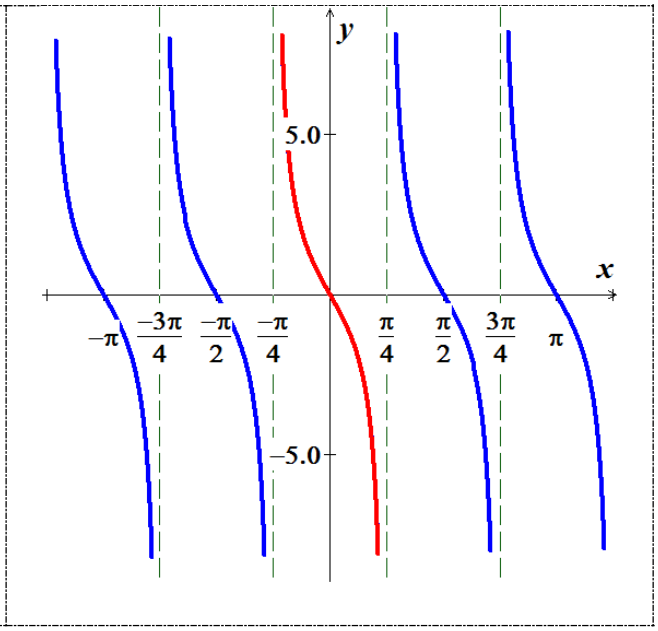
Asymptotes:

$2x + \frac{\pi}{2} = (2n+1)\pi$

$2x = 2n\pi + \frac{\pi}{2}$

$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	∞
$\frac{\pi}{8} - \frac{\pi}{4}$	$-\frac{\pi}{8}$	2
$\frac{\pi}{4} - \frac{\pi}{4}$	0	0
$\frac{3\pi}{8} - \frac{\pi}{4}$	$\frac{\pi}{8}$	-2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	∞



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{2}\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

Solution

Amplitude: n / a

Period: $P = 2\pi$

Phase Shift: $\varphi = -\frac{\pi}{2}$

VT: $y = 0$

Asymptotes:

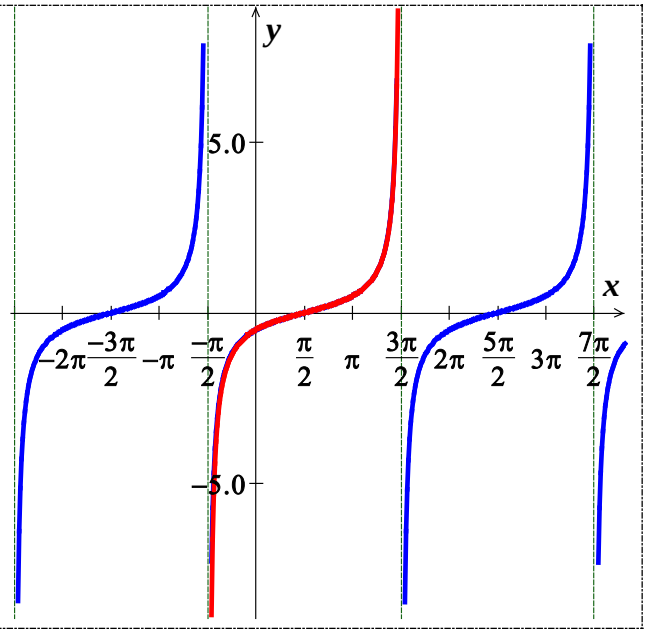
$\frac{1}{2}x + \frac{\pi}{4} = 2n\pi + \pi$

$\frac{1}{2}x = 2n\pi + \frac{3\pi}{4}$

$x = 4n\pi + \frac{3\pi}{2}$

$x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{11\pi}{2}, \ldots$

	x	y
$0 - \frac{\pi}{2}$	$-\frac{\pi}{2}$	∞
$\frac{\pi}{2} - \frac{\pi}{2}$	0	$-\frac{1}{2}$
$\pi - \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{3\pi}{2} - \frac{\pi}{2}$	π	$\frac{1}{2}$
$2\pi - \frac{\pi}{2}$	$\frac{3\pi}{2}$	∞



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \sec\left(x - \frac{\pi}{2}\right)$

Solution

Amplitude: n / a

Period: $P = 2\pi$

Phase Shift: $\varphi = \frac{\pi}{2}$

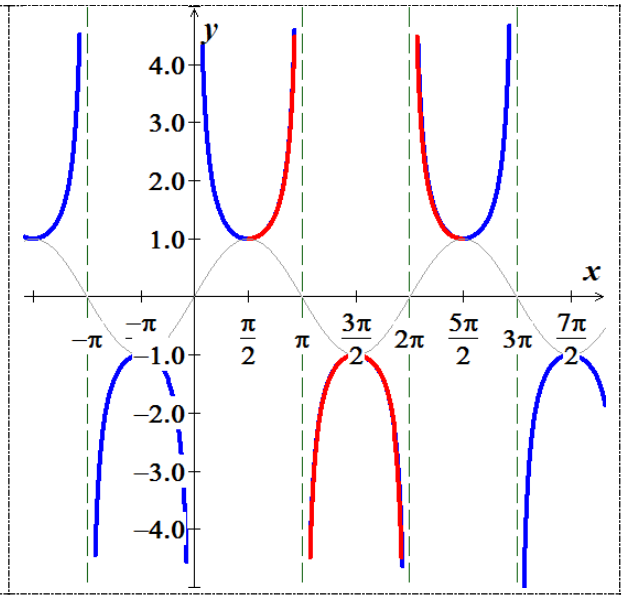
VT: $y = 0$

Asymptotes:

$x - \frac{\pi}{2} = n\pi + \frac{\pi}{2}$

$x = n\pi$

	x	$\cos\left(x - \frac{\pi}{2}\right)$
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	1
$\frac{\pi}{2} + \frac{\pi}{2}$	π	0
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	–1
$\frac{3\pi}{2} + \frac{\pi}{2}$	2π	0
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	1



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\sec\left(2x - \frac{\pi}{2}\right)$

Solution

Amplitude: n / a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

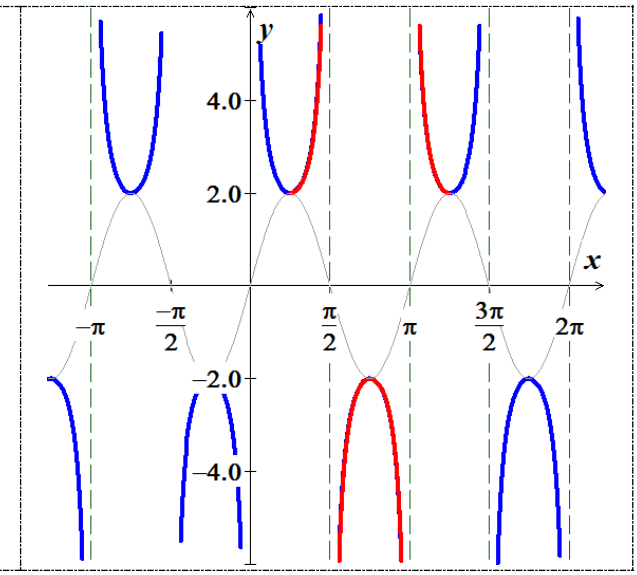
VT: $y = 0$

Asymptotes:

$2x - \frac{\pi}{2} = n\pi + \frac{\pi}{2}$

$x = \left(\frac{n+1}{2}\right)\pi$

	x	$2\cos\left(2x - \frac{\pi}{2}\right)$
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	2
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	–2
$\frac{3\pi}{4} + \frac{\pi}{4}$	2π	0
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	2



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -3\sec\left(\frac{1}{3}x + \frac{\pi}{3}\right)$

Solution

Amplitude: n/a

Period: $P = 6\pi$

Phase Shift: $\varphi = -\pi$

VT: $y = 0$

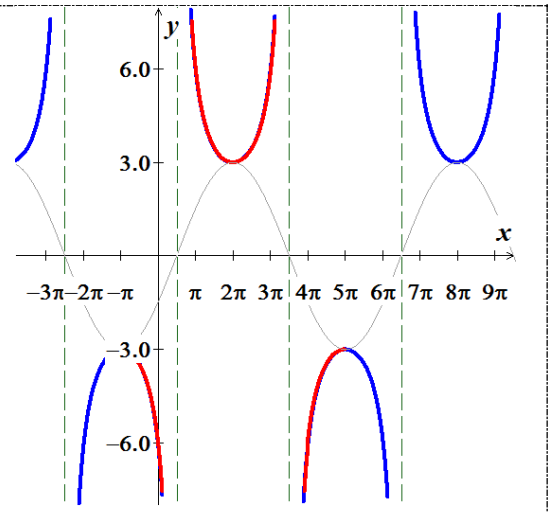
Asymptotes:

$\frac{1}{3}x + \frac{\pi}{3} = n\pi + \frac{\pi}{2}$

$x = 3n\pi + \frac{\pi}{2}$

$x = \ldots, -\frac{5\pi}{2}, \frac{\pi}{2}, \frac{7\pi}{2}, \ldots$

	x	$-3\cos\left(\frac{1}{3}x+\frac{\pi}{3}\right)$
$0-\pi$	$-\pi$	-3
$\frac{3\pi}{2}-\pi$	$\frac{\pi}{2}$	0
$3\pi-\pi$	2π	3
$\frac{9\pi}{2}-\pi$	$\frac{7\pi}{2}$	0
$6\pi-\pi$	5π	-3



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \csc\left(x - \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = \frac{\pi}{2}$

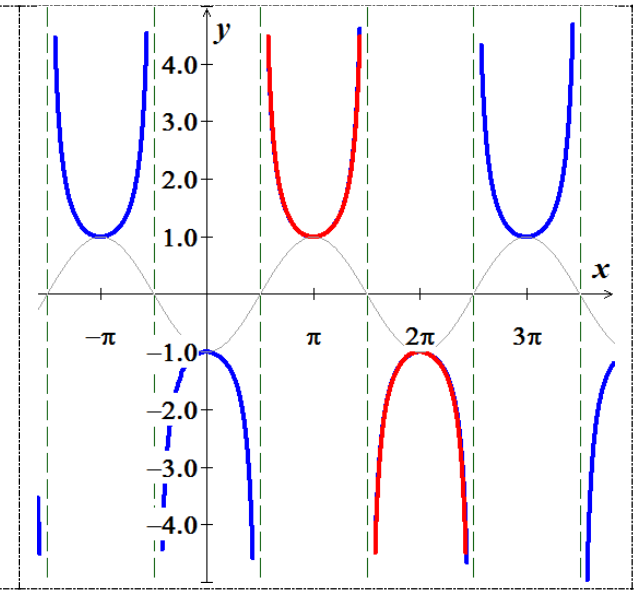
VT: $y = 0$

Asymptotes:

$x - \frac{\pi}{2} = n\pi$

$x = n\pi + \frac{\pi}{2}$

	x	$\sin\left(x-\frac{\pi}{3}\right)$
$0+\frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}+\frac{\pi}{2}$	π	1
$\pi+\frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$\frac{3\pi}{2}+\frac{\pi}{2}$	2π	-1
$2\pi+\frac{\pi}{2}$	$\frac{5\pi}{2}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\csc\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n / a

Period: $P = \pi$

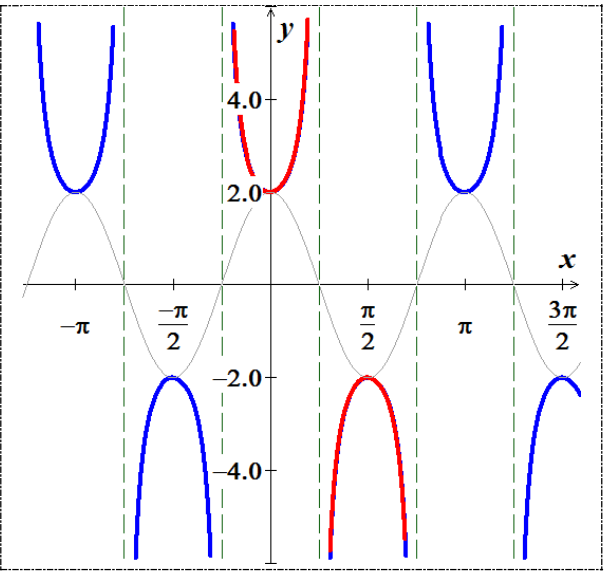
Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

Asymptotes:

$x = -\frac{\pi}{4} + n\frac{\pi}{2}$

	x	$2\sin\left(2x+\frac{\pi}{2}\right)$
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	0
$\frac{\pi}{4} - \frac{\pi}{4}$	0	2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{3\pi}{4} - \frac{\pi}{4}$	$\frac{\pi}{2}$	−2
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 4\csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

Solution

Amplitude: n / a

Period: $P = 4\pi$

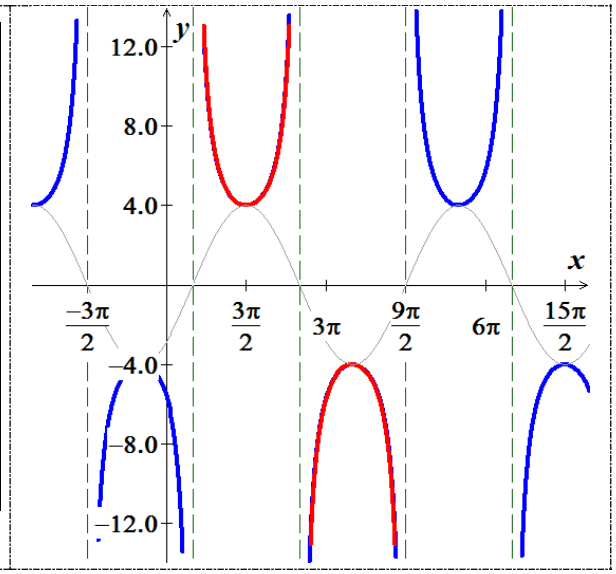
Phase Shift: $\varphi = \frac{\pi}{2}$

VT: $y = 0$

Asymptotes:

$x = \frac{\pi}{2} + 2\pi n$

	x	$4\sin\left(\frac{1}{2}x-\frac{\pi}{4}\right)$
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	4
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	0
$3\pi + \frac{\pi}{2}$	$\frac{7\pi}{2}$	−4
$4\pi + \frac{\pi}{2}$	$\frac{9\pi}{2}$	0



Exercise

Graph over a 2-period interval $y = 1 - 2\cot 2\left(x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \frac{\pi}{2}$

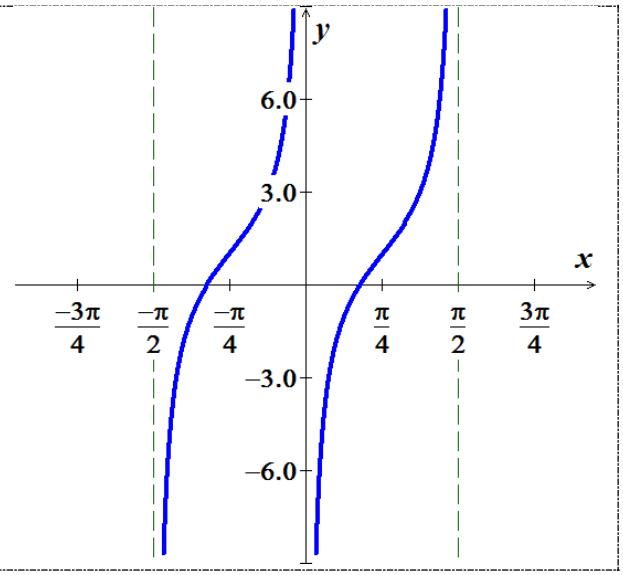
Phase Shift: $\varphi = -\frac{\pi}{2}$

VT: $y = 1$

Asymptotes:

$-\frac{\pi}{2} + n\pi$

	x	y
$0 - \frac{\pi}{2}$	$-\frac{\pi}{2}$	∞
$\frac{\pi}{8} - \frac{\pi}{2}$	$-\frac{3\pi}{8}$	-1
$\frac{\pi}{4} - \frac{\pi}{2}$	$-\frac{\pi}{4}$	1
$\frac{3\pi}{8} - \frac{\pi}{2}$	$-\frac{\pi}{8}$	3
$\frac{\pi}{2} - \frac{\pi}{2}$	0	∞



Exercise

Graph over a 2-period interval $y = \frac{2}{3}\tan\left(\frac{3}{4}x - \pi\right) - 2$

Solution

Amplitude: n/a

Period: $P = \frac{4\pi}{3}$

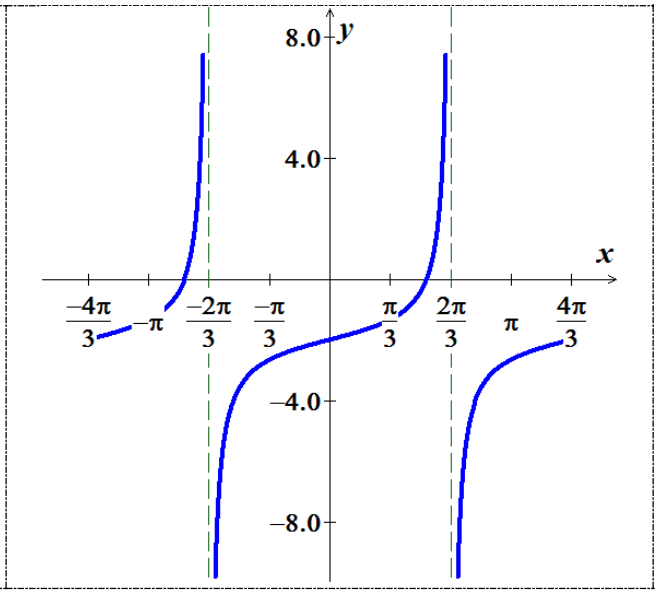
Phase Shift: $\varphi = -\frac{4\pi}{3}$

VT: $y = -2$

Asymptotes:

$x = \frac{3\pi}{4} + n\pi$

	x	y
$0 - \frac{4\pi}{3}$	$-\frac{4\pi}{3}$	-2
$\frac{\pi}{3} - \frac{4\pi}{3}$	$-\pi$	$-\frac{4}{3}$
$\frac{2\pi}{3} - \frac{4\pi}{3}$	$-\frac{2\pi}{3}$	∞
$\pi - \frac{4\pi}{3}$	$-\frac{\pi}{3}$	$-\frac{8}{3}$
$\frac{4\pi}{3} - \frac{4\pi}{3}$	0	-2



Exercise

Graph over a one-period interval $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

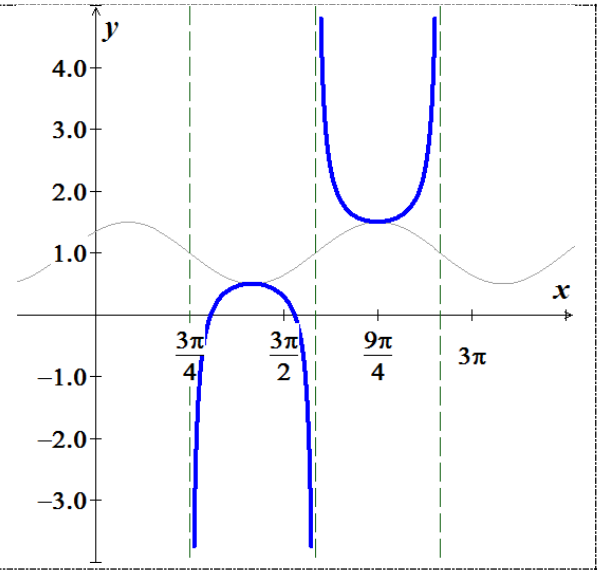
Phase Shift: $\varphi = \frac{3\pi}{4}$

VT: $y = 1$

Asymptotes:

$x = \frac{3\pi}{4} + 2\pi n$

	x	$4\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$
$0 + \frac{3\pi}{4}$	$\frac{3\pi}{4}$	1
$\frac{\pi}{2} + \frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{1}{2}$
$\pi + \frac{3\pi}{4}$	$\frac{7\pi}{4}$	1
$\frac{3\pi}{2} + \frac{3\pi}{4}$	$\frac{9\pi}{4}$	$-\frac{1}{2}$
$2\pi + \frac{3\pi}{4}$	$\frac{11\pi}{4}$	1



Exercise

Graph over a one-period interval $y = 2 + \frac{1}{4}\sec\left(\frac{1}{2}x - \pi\right)$

Solution

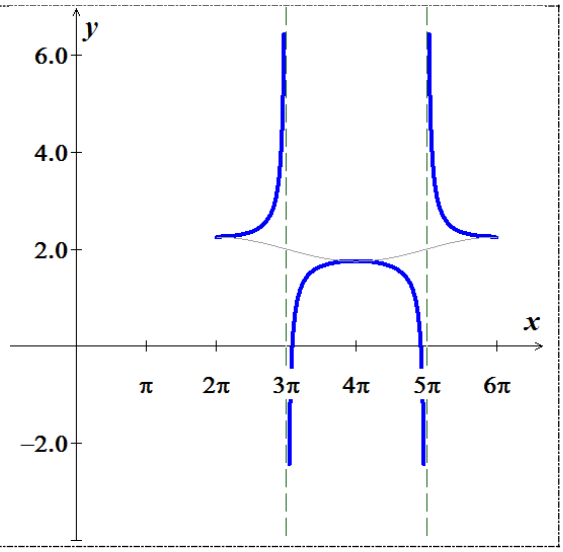
Amplitude: n/a

Period: $P = 4\pi$

Phase Shift: $\varphi = 2\pi$

VT: $y = 2$

	x	$2 + \frac{1}{4}\cos\left(\frac{1}{2}x - \pi\right)$
$0 + 2\pi$	2π	$\frac{9}{4}$
$\pi + 2\pi$	3π	2
$2\pi + 2\pi$	4π	$\frac{7}{4}$
$3\pi + 2\pi$	5π	2
$4\pi + 2\pi$	6π	$\frac{9}{4}$



Exercise

Graph one complete cycle $y = 3 + 2\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)$

Solution

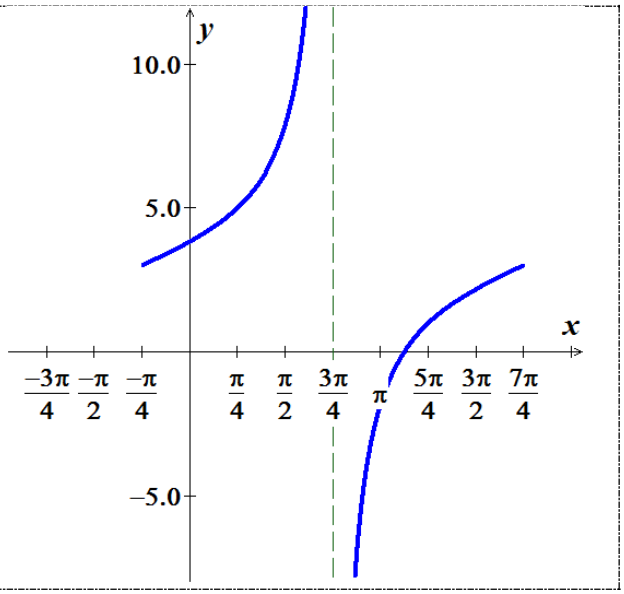
Amplitude: n / a

Period: $P = 2\pi$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 3$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	3
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	5
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	∞
$\frac{3\pi}{2} - \frac{\pi}{4}$	$\frac{5\pi}{4}$	1
$2\pi - \frac{\pi}{4}$	$\frac{7\pi}{4}$	3



Exercise

Graph two complete cycles $y = -2 - \cot\left(x - \frac{\pi}{4}\right)$

Solution

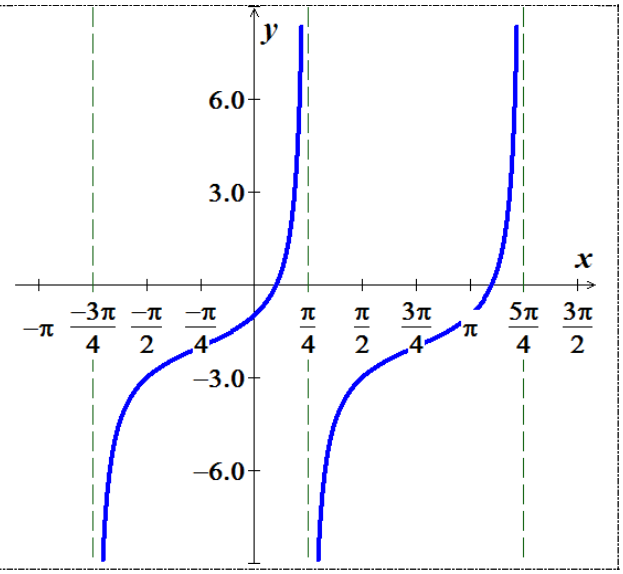
Amplitude: n / a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

VT: $y = -2$

	x	y
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	∞
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	−3
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	−2
$\frac{3\pi}{4} + \frac{\pi}{4}$	π	−1
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	∞



Exercise

Graph $y = \frac{1}{3}\sec 2x$ *for* $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$

Solution

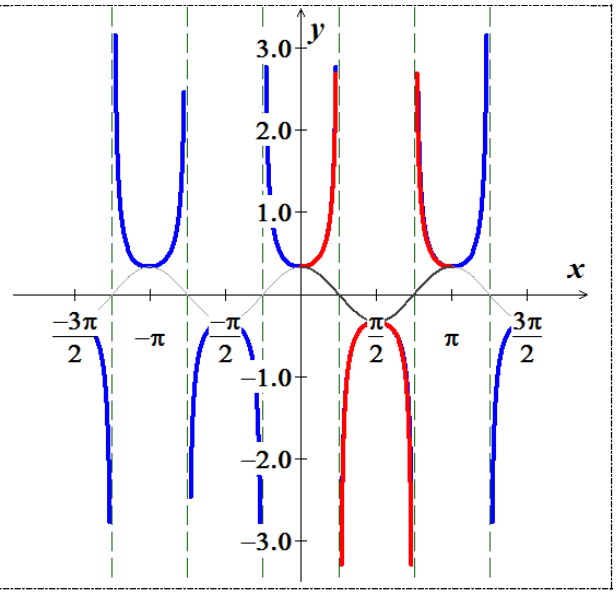
Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = 0$

VT: $y = 0$

x	$\frac{1}{3}\cos 2x$
0	$\frac{1}{3}$
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	$-\frac{1}{3}$
$\frac{3\pi}{4}$	0
π	$\frac{1}{3}$



Exercise

Graph one complete cycle $y = -1 - 3\csc\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$

Solution

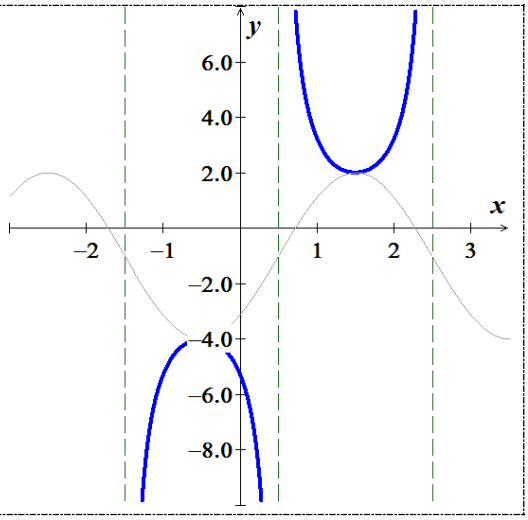
Amplitude: n/a

Period: $P = 4$

Phase Shift: $\varphi = -\frac{3}{2}$

VT: $y = -1$

	x	$-1-3\sin\left(\frac{\pi x}{2}+\frac{3\pi}{4}\right)$
$0-\frac{3}{2}$	$-\frac{3}{2}$	-1
$1-\frac{3}{2}$	$-\frac{1}{2}$	-4
$2-\frac{3}{2}$	$\frac{1}{2}$	-1
$3-\frac{3}{2}$	$\frac{3}{2}$	2
$4-\frac{3}{2}$	$\frac{5}{2}$	-1



Exercise

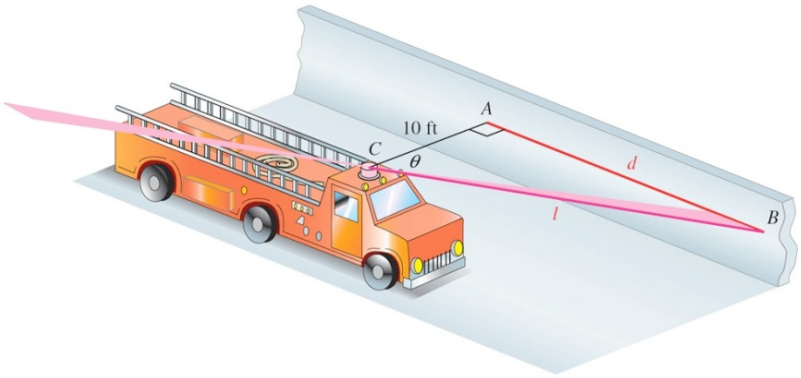
A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top is 10 *feet* from the wall and rotates through one complete revolution every 2 *seconds*. Graph the function that gives the length d in terms of time t from $t = 0$ to $t = 2$.

Solution

$$\omega = \frac{\theta}{t} = \frac{2\pi}{2} = \pi \ rad / sec$$

$$\tan\theta = \frac{d}{10} \quad \rightarrow \quad d = 10\tan\theta$$

$$d(t) = 10\tan\pi t$$

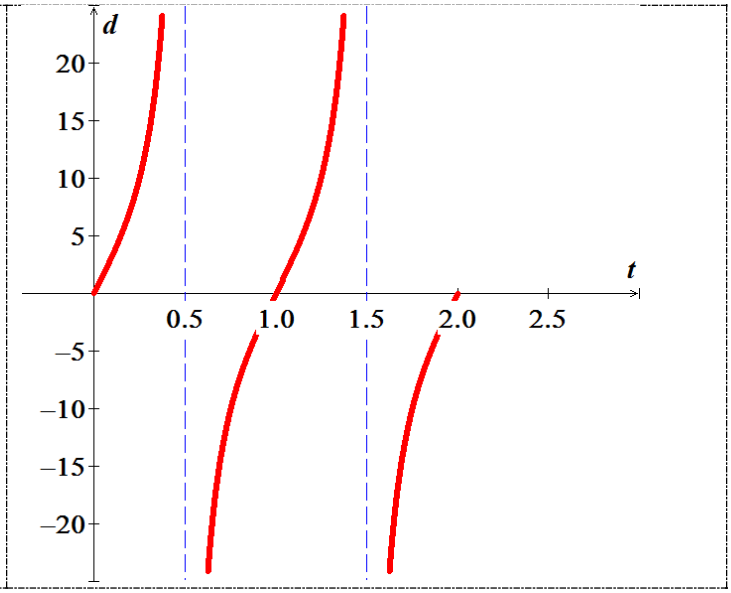


Period $= \frac{\pi}{\pi} = 1$

One cycle: $0 \le \pi t \le \pi$

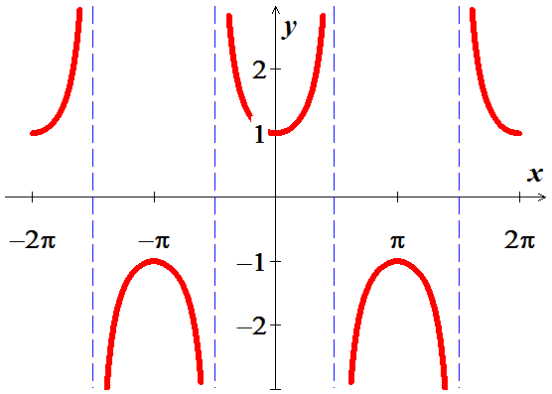
$0 \le t \le 1$

t	$d = 10\tan\pi t$
0	0
$\frac{1}{4}$	10
$\frac{1}{2}$	∞
$\frac{3}{4}$	-10
1	0



Exercise

Find an equation to match the graph



Solution

$P = 2\pi$

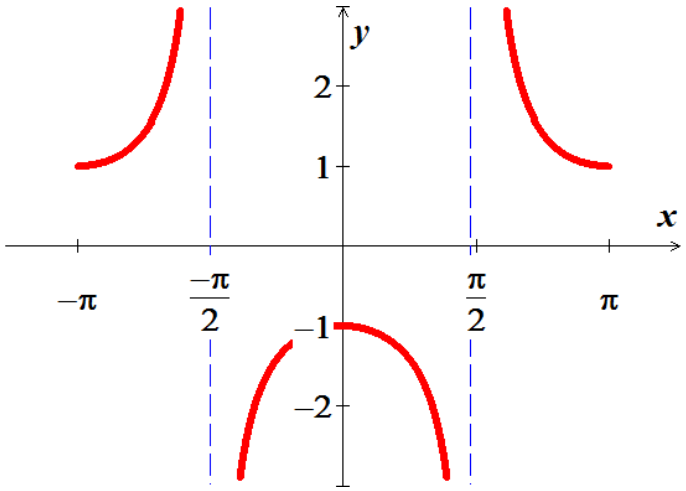
$\phi = 0$

$A = \frac{1+1}{2} = 1$

$y = \sec x \quad -2\pi \le x \le 2\pi$

Exercise

Find an equation to match the graph



Solution

$B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$

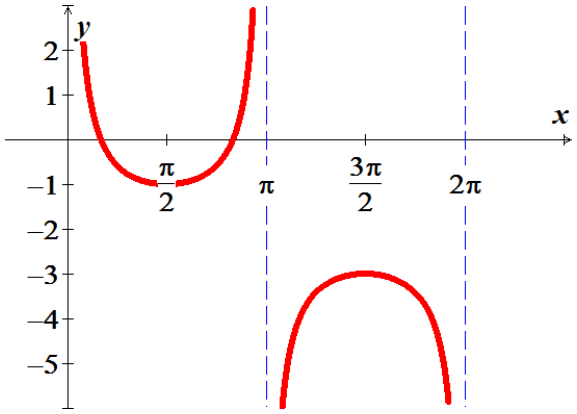
$\phi = 0 \rightarrow C = 0$

$A = \frac{1+1}{2} = 1$

$y = -\sec(x) \quad -\pi \le x \le \pi$

Exercise

Find an equation to match the graph



Solution

$$B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$$

$$\phi = 0 \rightarrow C = 0$$

$$A = \frac{-3-1}{2} = -2$$

$$y = -2 + \csc(x) \quad -2\pi \le x \le 2\pi$$

Exercise

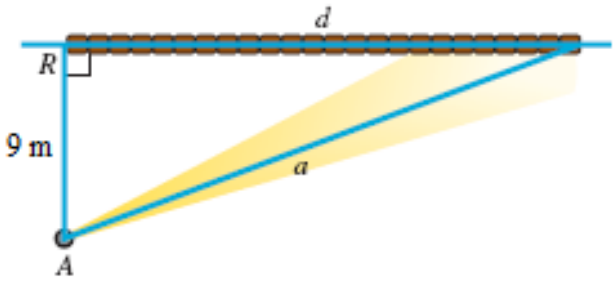
A rotating beacon is located at point A next to a long wall. The beacon is 9 m from the wall. The distance $\boldsymbol{a}$ is given by $a = 9\left|\sec 2\pi t\right|$, where t is time measured in seconds since the beacon started rotating. (When $t = 0$, the beacon is aimed at point R.) Find $\boldsymbol{a}$ for $t = 0.45$

Solution

$$a = 9\left|\sec\left(2\pi\left(0.45\right)\right)\right|$$

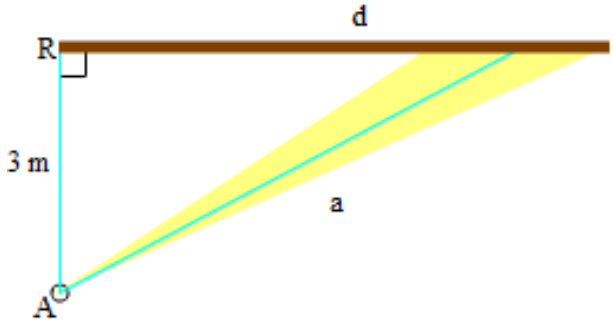
$$= \frac{9}{\left|\cos\left(2\pi\left(0.45\right)\right)\right|}$$

$$\underline{\approx 9.5\ m}$$



Exercise

A rotating beacon is located 3 *m* south of point *R* on an east-west wall. *d*, the length of the light display along the wall from *R*, is given by $d = 3\tan 2\pi t$, where *t* is time measured in seconds since the beacon started rotating. (When *t* = 0, the beacon is aimed at point *R*. When the beacon is aimed to the right of *R*, the value of *d* is positive; *d* is negative if the beacon is aimed to the left of *R*.) Find a for *t* = 0.8



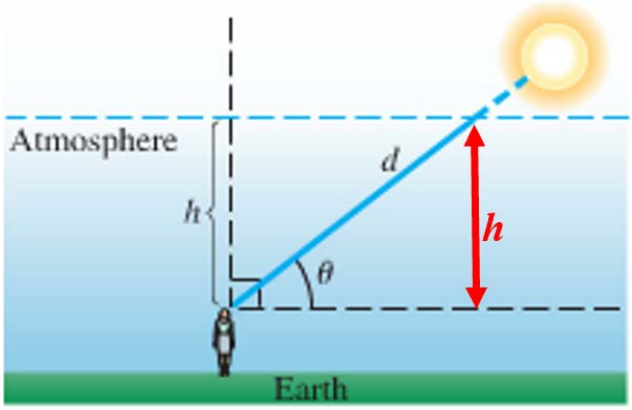
Solution

$$d = 3\tan\left(2\pi(0.8)\right)$$

$$\approx -9.23 \ m$$

Exercise

The shortest path for the sun's rays through Earth's atmosphere occurs when the sun is directly overhead. Disregarding the curvature of Earth, as the sun moves lower on the horizon, the distance that sunlight passes through the atmosphere increases by a factor of $\csc\theta$, where θ is the angle of elevation of the sun. This increased distance reduces both the intensity of the sun and the amount of ultraviolet light that reached Earth's surface.



a) Verify that $d = h\csc\theta$

b) Determine θ when $d = 2h$

c) The atmosphere filters out the ultraviolet light that causes skin to burn, Compare the difference between sunbathing when $\theta = \frac{\pi}{2}$ and when $\theta = \frac{\pi}{3}$. Which measure gives less ultraviolet light?

Solution

a) $\sin\theta = \frac{h}{d}$

$= \frac{1}{\csc\theta}$

$\underline{d = h\csc\theta}$ (***cross-multiplication***)

b) $\sin\theta = \frac{h}{d}$

$= \frac{h}{2h}$

$= \underline{\frac{1}{2}}$

$\theta = \sin^{-1}\frac{1}{2}$

$= \underline{\frac{\pi}{6}}$

c) $\begin{cases} \csc\frac{\pi}{2} = 1 \\ \csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3} \approx 1.15 \end{cases}$

When the distance to the sun is lager $\left(\theta = \frac{\pi}{3}\right)$, there is less ultraviolet light reaching the earth's surface. In this case, sunlight passes through 15% more atmosphere.

Exercise

Let a person whose eyes are h_1 feet from the ground stand ***d*** feet from an object h_1 feet tall, where $h_2 > h_1$ feet. Let θ be the angle of elevation to the top of the object.

a) Show that $d = \left(h_2 - h_1\right)\cot\theta$

b) Let $h_2 = 55$ and $h_1 = 5$. Graph ***d*** for the interval $0 < \theta \le \frac{\pi}{2}$

Solution

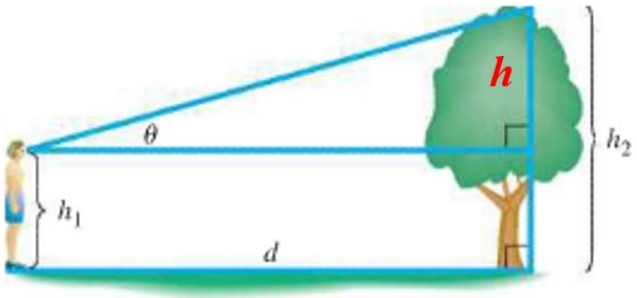
a) $h = h_2 - h_1$

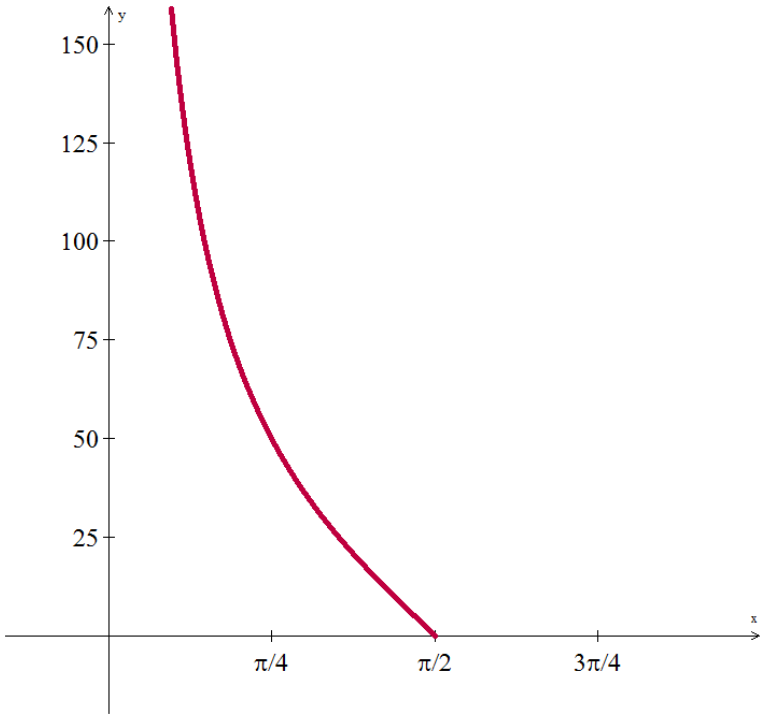
$\cot\theta = \frac{d}{h}$

$d = \left(h_2 - h_1\right)\cot\theta$

b) $d = (55 - 5)\cot\theta$

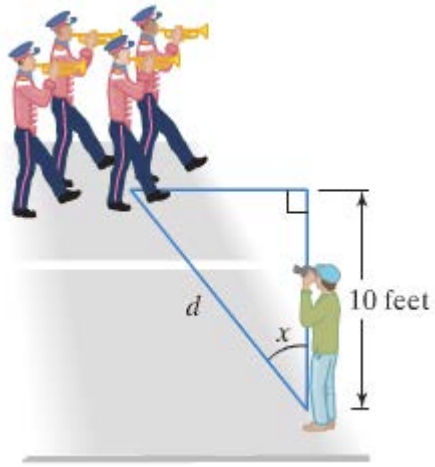
$d = 50\cot\theta \quad 0 < \theta \le \frac{\pi}{2}$





Exercise

Your friend is marching with a band and has asked you to film him. You have set yourself up 10 *feet* from the street where your friend will be passing from left to right. If d represents your distance, in feet, from your friend and x is the radian measure of the angle.



a) Express d in terms of a trigonometric function of x.

b) Graph the function for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Solution

a) $\cos x = \frac{10}{d}$

$d = 10 \sec x$

b)

$y = 10\cos x$

Amplitude: $|A| = 10$

Period: $P = 2\pi$

Phase Shift: $\phi = 0$

VT: $y = 0$

x	y
0	10
$\frac{\pi}{2}$	0
π	-10
$\frac{3\pi}{2}$	0
2π	10