1.   Use the binomial theorem to expand and simplify

   a) $\left(4x^2 - y\right)^6$ *b)* $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)^5$ *c)* $\left(\sqrt{2}x + \dfrac{1}{y}\right)^4$ *d)* $\left(7x + 2y\right)^4$

2.   Find the domain of:

   a) $f(x) = \dfrac{\sqrt{2x+3}}{x^2 - 6x + 5}$ *b)* $f(x) = \dfrac{1}{(x+4)\sqrt{x-2}}$

3.   For the function f given by $f(x) = -x^2 + x + 5$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

4.   Sketch the graph $f(x) = \begin{cases} x - 3 & if & x \le -2 \\ -x^2 & if & -2 < x < 1 \\ -x + 4 & if & x \ge 1 \end{cases}$

5.   Let $f(x) = \sqrt{3x+2}$ *and* $g(x) = \dfrac{1}{x^2}$

   a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

   b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

6.   Let $f(x) = \sqrt{25 - x^2}$ *and* $g(x) = \sqrt{x - 3}$

   a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

   b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

7.   Let $f(x) = \dfrac{x-1}{x-2}$ *and* $g(x) = \dfrac{x-3}{x-4}$

   a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

   b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

8.   Let $f(x) = \dfrac{1}{15}\left(x^5 - 20x^3 + 64x\right)$. Find all values of x such that $f(x) > 0$ and all x such that

   $f(x) < 0$, and then sketch the graph of f.

9.   Find the quotient and remainder if $f(x) = 3x^5 - 4x^3 + x + 5$ is divided by $p(x) = x^3 - 2x + 7$

10. Find the zeros of $f(x) = \left(x^2 - 2x + 1\right)^2 \left(x^2 + 2x - 3\right)$, and state the multiplicity of each zero.

11. Find all solutions of the equation: $x^4 + 9x^3 + 31x^2 + 49x + 30 = 0$

12. Find the vertical asymptotes, horizontal asymptotes, oblique asymptotes, intercepts, and holes (if any) of:

a) $y = \dfrac{x - 2}{x^2 - 4x + 3}$

b) $y = \dfrac{(x + 2)(x - 1)}{x^2 - 3x - 10}$

c) $f(x) = \dfrac{x^3 - 2x^2 - 4x + 8}{x - 2}$

13. Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x = -3 \\ \textit{horizontal asymptote}: \ y = \frac{3}{2} \\ x - \textit{intercept}: \ 5 \\ \textit{hole at} \ x = 2 \end{cases}$$

14. Let $f(x) = 2x^3 - 5$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists.

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

d) Sketch $f(x)$ and $f^{-1}(x)$

15. Let $f(x) = \dfrac{2x - 7}{9x + 1}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists.

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

d) Sketch $f(x)$ and $f^{-1}(x)$

16. Let $f(x) = 3^{-x^2}$, determine the asymptote, domain, range, increasing and decreasing, and sketch $f(x)$

17. Let $f(x) = \log_2 (x + 3)$, determine the asymptote, domain, range, increasing and decreasing, and sketch $f(x)$

18. Solve the equations:

a) $2^{5x+3} = 3^{2x+1}$

b) $8^{2x}\left(\frac{1}{4}\right)^{x-2} = 4^{-x}\left(\frac{1}{2}\right)^{2-x}$

c) $3^{2x-1} = \frac{1}{3}$

d) $2\ln(x+3) - \ln(x+1) = 3\ln 2$

e) $\log_4(x+1) = 2 + \log_4(3x-2)$

f) $x^2\left(-2xe^{-x^2}\right) + 2xe^{-x^2} = 0$

g) $e^x + 2 = 8e^{-x}$

h) $\log\sqrt{x} = \sqrt{\log x}$

i) $\log\left(x^2\right) = \left(\log x\right)^2$

j) $\log_2(x+3) = \log_2(x-3) + \log_3 9 + 4^{\log_4 3}$

19. Solve the equation for x in terms of y.

a) $y = \dfrac{1}{10^x + 10^{-x}}$

b) $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

20. Express the following in terms of sums and differences of logarithms $\log_a \sqrt[4]{\dfrac{m^8 n^{12}}{a^3 b^5}}$

Solution

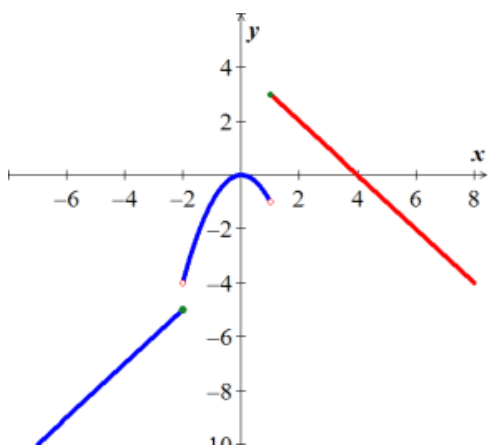**1.** a) $4096x^{12} - 6144x^{10}y + 3840x^8y^2 - 1280x^6y^3 + 240x^4y^4 - 24x^2y^5 + y^6$

b) $x^{5/2} - 5x^{3/2} + 10x^{1/2} - 10x^{-1/2} + 5x^{-3/2} - x^{-5/2}$

c) $4x^4 + \dfrac{8\sqrt{2}x^3}{y} + \dfrac{12x^2}{y^2} + \dfrac{4\sqrt{2}x}{y^3} + \dfrac{1}{y^4}$

d) $2401x^4 + 2744x^3y + 1176x^2y^2 + 224xy^3 + 16y^4$

2. a) $\left[-\dfrac{3}{2}, 1\right) \cup (1, 5) \cup (5, \infty)$ b) $\{x > 2\}$

3.

4. a) $f(g(x)) = \dfrac{\sqrt{2x^2 + 3}}{x}$ $Domain : \{x \neq 0\}$

b) $g(f(x)) = \dfrac{1}{3x + 2}$ $Domain : \left\{x \neq -\dfrac{2}{3}\right\}$

5. a) $f(g(x)) = \sqrt{28 - x}$ $Domain : \{3 \leq x \leq 28\}$

b) $f(g(x)) = \sqrt{\sqrt{25 - x^2} - 3}$ $Domain : \{-4 \leq x \leq 4\}$

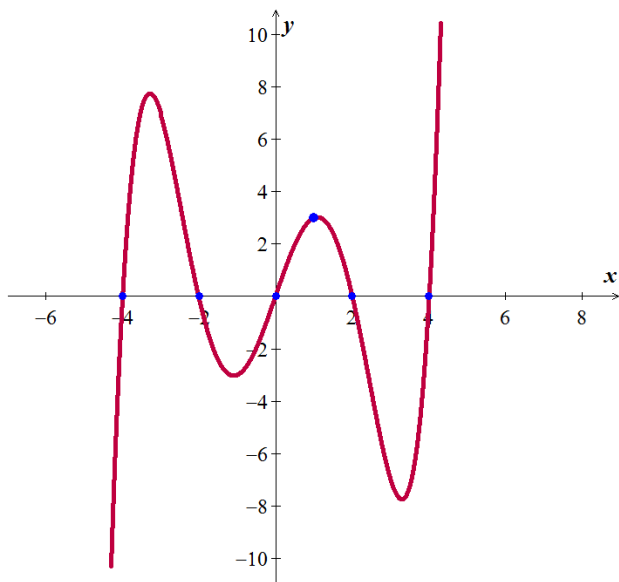**6.** a) $f(g(x)) = \dfrac{1}{-x + 5}$ $Domain : \{x \neq 4, \ 5\}$

b) $f(g(x)) = \dfrac{-2x + 5}{-3x + 7}$ $Domain : \left\{x \neq 2, \ \dfrac{7}{3}\right\}$

7. $f(x) = \frac{1}{15} x \left(x^4 - 20x^2 + 64 \right) \rightarrow$ *Zeros* : $0, \ \pm 2, \ \pm 4$

	-4		-2		0		2		4	
$-$		$+$		$-$		$+$		$-$		$+$

$f(x) > 0 \quad -4 < x < -2; \ \ 0 < x < 2; \ \ x > 4$

$f(x) < 0 \quad x < -4; \ \ -2 < x < 0; \ \ 2 < x < 4$

8. $Q(x) = 3x^2 + 2 \qquad R(x) = -21x^2 + 5x - 9$

9. $x = 1$ *(multiplicity of 5)* ; $x = -3$ *(multiplicity of 1)*
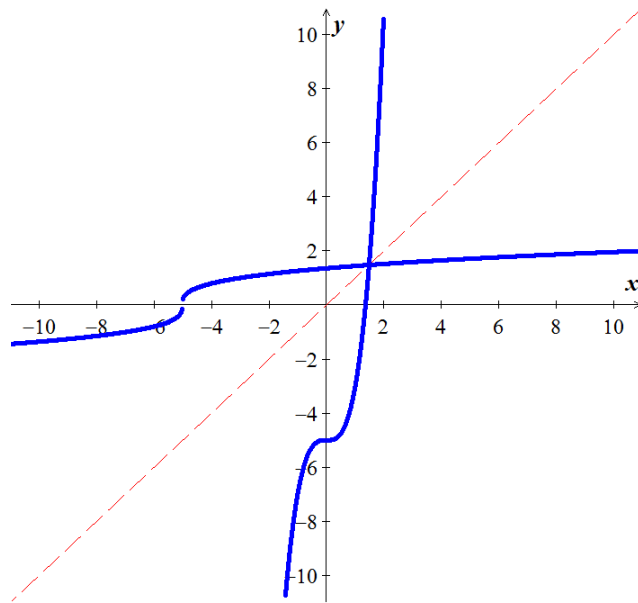
10. $-3, \ \ -2, \ \ -2 \pm i$

11. *a)* $VA : x = 1, \ x = 3,; \quad HA : y = 0; \quad x - \text{int} : 2; \quad y - \text{int} : -\frac{2}{3} \quad Hole : n/a \quad OA : n/a$

     b) $VA : x = 5; \quad HA : y = \frac{4}{3}; \quad x - \text{int} : 1; \quad y - \text{int} : \frac{4}{15}; \quad hole : \left(-2, \ \frac{4}{7} \right) \quad OA : n/a$

     c) $VA : n/a; \quad HA : n/a \quad x - \text{int} : -2; \quad y - \text{int} : -4; \quad hole : (2, \ 0) \ OA : y = x^2 - 4$

12. $f(x) = \dfrac{3x^2 - 21x + 30}{2x^2 + 2x - 12}$
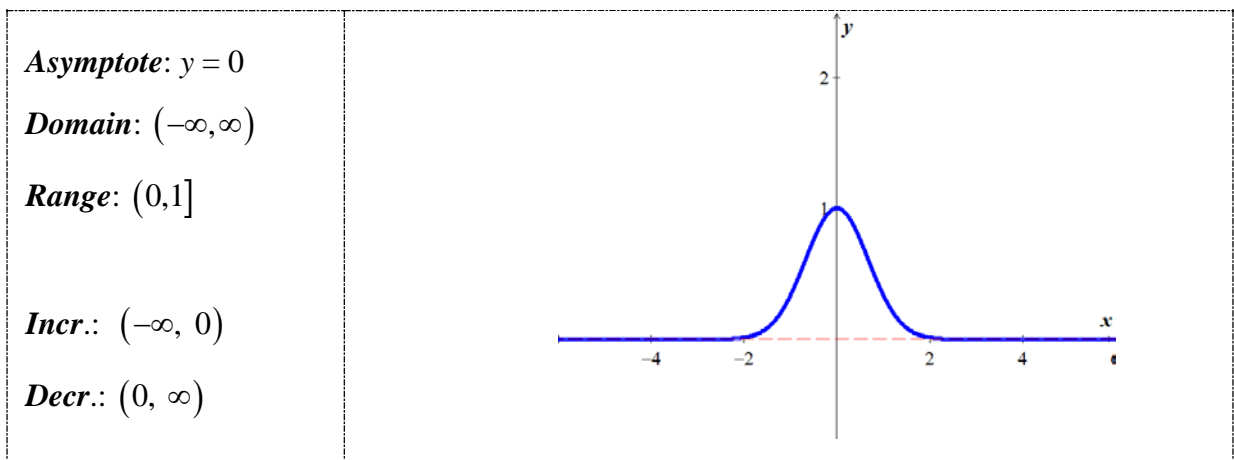
5

13. *a)* Yes   *b)* $f^{-1}(x) = \sqrt[3]{\dfrac{x+5}{2}}$ *c)* Domain & Range of $f(x)$ and $f^{-1}(x)$: $\mathbb{R}$
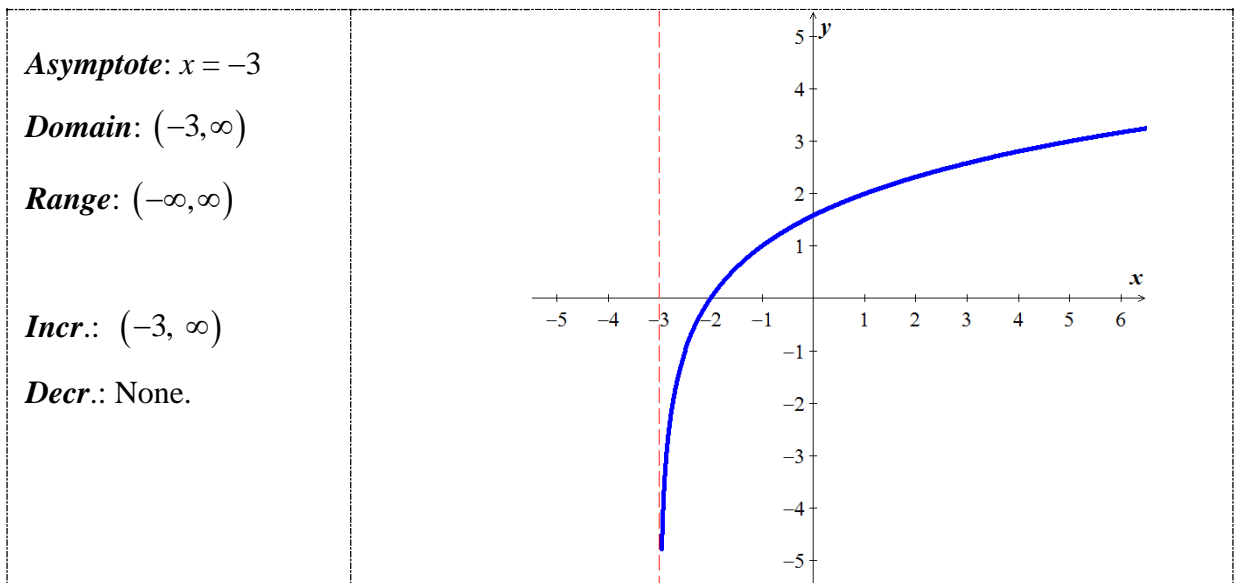

14. *a)* Yes   *b)* $f^{-1}(x) = \dfrac{-x-7}{9x-2}$ *c)* Domain of $f(x)$ = Range of $f^{-1}(x)$: $\mathbb{R} - \left\{-\dfrac{1}{9}\right\}$

Range of $f(x)$ = Domain of $f^{-1}(x)$: $\mathbb{R} - \left\{\dfrac{2}{9}\right\}$

15.

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, 1]$

Incr.: $(-\infty, 0)$

Decr.: $(0, \infty)$

16.

Asymptote: $x = -3$

Domain: $(-3, \infty)$

Range: $(-\infty, \infty)$

Incr.: $(-3, \infty)$

Decr.: None.
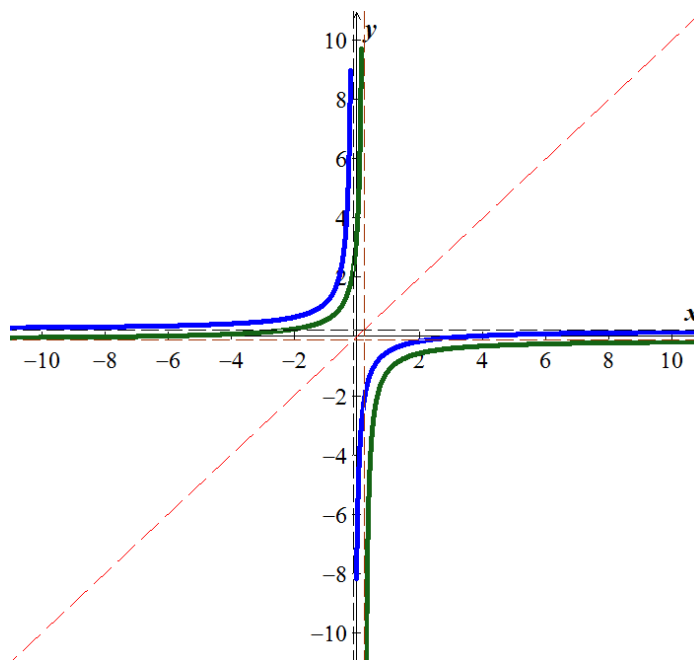
17. *a)* $x = \dfrac{\ln 3 - 3\ln 2}{5\ln 2 - 2\ln 3} = \dfrac{\ln\left(\frac{3}{8}\right)}{\ln\left(\frac{32}{9}\right)} \approx -0.7732$

b) $x = -\dfrac{6}{5}$

c) $x = 0$

d) $x = 1$

e) $x = \dfrac{33}{47}$

f) $x = 0, \ \pm 1$

g) $x = \ln 2$

h) $x = 1, \ 10{,}000$

i) $x = 1, \ 100$

j) $x = \dfrac{99}{31}$

18. a) $x = \log\left(\dfrac{1 \pm \sqrt{1 - 4y^2}}{2y}\right)$ 　　　　　b) $x = \dfrac{1}{2}\ln\left(\dfrac{1 + y}{1 - y}\right)$

19. $2\log_a m + 3\log_a n - \dfrac{3}{4} - \dfrac{5}{4}\log_a b$