Solution Section 4.1 – Random Variable & probability

Exercise

Suppose a random sample of 2 light bulbs is selected from a group of 8 bulbs that contain 3 defective bulbs.

a) What is the expected value of the number of defective bulbs in the sample?

b) Probability Distribution Table

c) What is your expected return?

Solution

a) Let *X* represents the number of defective bulbs that occur on a given trial. X = {0, 1, 2}

$$P(x=0)=\frac{{}_3C_0\cdot{}_5C_2}{{}_8C_2}=0.3571$$

$$P(x=1)=\frac{{}_3C_1\cdot{}_5C_1}{{}_8C_2}=0.5357$$

$$P(x=2)=\frac{{}_3C_2\cdot{}_5C_0}{{}_8C_2}=0.1071$$

b) Probability Distribution Table

x_i	0	1	2
P_i	.3571	.5357	.1071

c) $E[X]=0(.3571)+1(.5357)+2(.1071)\ \underline{=0.75}$

Exercise

Suppose 1000 raffle tickets are sold at a price of $10 each. Two first place tickets will be drawn, 5 second place tickets will be drawn and 10 third place tickets will be drawn. The first place prize is a $200 VCR, the second place prize is a $100 printer, and the third place prize is a $50 gift certificate.

Solution

Let *X* be the net gain to the ticketholder. Find the expected value of *X*.

X = {–10, 40. 90, 190} (you paid $10 to play)

$$P(x=-10)=\frac{983}{1000}=0.983 \qquad P(x=40)=\frac{10}{1000}=0.01$$

$$P(x=90)=\frac{5}{1000}=0.005 \qquad P(x=190)=\frac{2}{1000}=0.002$$

Payoff Table

x_i	-10	40	90	190
P_i	.983	.01	.005	.002

$$E[X] = -10(.983) + 40(.01) + 90(.005) + 190(.002)$$

$$= -\$8.60$$

Exercise

Find the expected value of each random variable.

a)

x	2	3	4	5
$P(x)$	0.1	0.4	0.3	0.2

b)

x	4	6	8	10
$P(x)$	0.4	0.4	0.05	0.15

c)

x	9	12	15	18	21
$P(x)$	0.14	0.22	0.38	0.19	0.07

d)

x	30	32	36	38	44
$P(x)$	0.31	0.29	0.26	0.09	0.05

Solution

a) $E(x) = 2(.1) + 3(0.4) + 4(0.3) + 5(0.2)$

$= 3.6$

b) $E(x) = 3(0.4) + 5(0.4) + 8(0.05) + 10(0.15)$

$= 5.9$

c) $E(x) = 9(0.14) + 12(0.22) + 15(0.38) + 18(0.19) + 21(0.07)$

$= 14.49$

d) $E(x) = 30(0.31) + 32(0.29) + 36(0.26) + 38(0.09) + 44(0.05)$

$= 33.56$

Exercise

A delegation of 3 selected from a city council made up of 5 liberals and 6 conservatives.

a) What is the expected number of liberals in the delegation?

b) What is the expected number of conservatives in the delegation?

Solution

a) The probability of the delegation:

$$P(0)=\frac{C_{5,0}C_{6,3}}{C_{11,3}}=0.1212 \qquad P(1)=\frac{C_{5,1}C_{6,2}}{C_{11,3}}=0.4545$$

$$P(2)=\frac{C_{5,2}C_{6,1}}{C_{11,3}}=0.3636 \qquad P(3)=\frac{C_{5,3}C_{6,0}}{C_{11,3}}=0.0606$$

x	0	1	2	3
$P(x)$	0.1212	0.4545	0.3636	0.0606

$$E(x)=0(0.1212)+1(0.4545)+2(0.3636)+3(0.0606)$$

$$\approx 1.3636 \ \textit{liberals}$$

b) $$P(0)=\frac{C_{5,3}C_{6,0}}{C_{11,3}}=0.0606 \qquad P(1)=\frac{C_{5,2}C_{6,1}}{C_{11,3}}=0.3636$$

$$P(2)=\frac{C_{5,1}C_{6,2}}{C_{11,3}}=0.4545 \qquad P(3)=\frac{C_{5,0}C_{6,3}}{C_{11,3}}=0.1212$$

x	0	1	2	3
$P(x)$	0.0606	0.3636	0.4545	0.1212

$$E(x)=0(0.0606)+1(0.3636)+2(0.4545)+3(0.1212)$$

$$\approx 1.6364 \ \textit{conservatives}$$

Exercise

From a group of 3 women and 5 men, a delegation of 2 is selected. Find the expected number of women in the delegation.

Solution

$$P(0)=\frac{C_{3,0}C_{5,2}}{C_{8,2}}=0.357 \qquad P(1)=\frac{C_{3,1}C_{5,1}}{C_{8,2}}=0.5357 \qquad P(2)=\frac{C_{3,2}C_{5,0}}{C_{8,2}}=0.107$$

x	0	1	2
$P(x)$	0.357	0.536	0.107

$$E(x)=0(0.357)+1(0.536)+2(0.107)$$

$$\approx 0.75$$

Exercise

In a club with 20 senior and 10 junior members, what is the expected number of junior members on a 4-member committee?

Solution

$$P(0) = \frac{C_{10,0}C_{20,4}}{C_{30,4}} = 0.1768 \qquad P(1) = \frac{C_{10,1}C_{20,3}}{C_{30,4}} = 0.416 \qquad P(2) = \frac{C_{10,2}C_{20,2}}{C_{30,4}} = 0.312$$

$$P(3) = \frac{C_{10,3}C_{20,1}}{C_{30,4}} = 0.0547 \qquad P(4) = \frac{C_{10,4}C_{20,0}}{C_{30,4}} = 0.0077$$

x	0	1	2	3	4
$P(x)$	0.1768	0.416	0.312	0.0547	.00766

$$E(x) = 0(0.1768) + 1(0.416) + 2(0.312) + 3(0.0547) + 4(0.00766)$$

$$\approx 1.333$$

Exercise

If 2 cards are drawn at one time from a deck of 52 cards, what is the expected number of diamonds?

Solution

$$P(0) = \frac{C_{13,0}C_{39,2}}{C_{52,2}} = 0.5588 \qquad P(1) = \frac{C_{13,1}C_{39,1}}{C_{52,2}} = 0.3823 \qquad P(2) = \frac{C_{13,2}C_{39,0}}{C_{52,2}} = 0.0588$$

x	0	1	2
$P(x)$	0.5588	0.3823	0.0588

$$E(x) = 0(0.5588) + 1(0.3823) + 2(0.0588)$$

$$\approx 0.5$$

Exercise

A local used-car dealer gets complaints about his car as shown in the table below. Find the expected number of complaints per day

Number of Complaints per day	0	1	2	3	4	5	6
Probability	0.02	0.06	0.16	0.25	0.32	0.13	0.06

Solution

$$E(x) = 0(0.02) + 1(0.06) + 2(0.16) + 3(0.25) + 4(0.32) + 5(0.13) + 6(0.06)$$

$$\approx 3.42 \ \textit{complaints per day}$$

Exercise

An insurance company has written 100 policies for \$100,000, 500 policies for \$50,000, and 1000 policies for \$10,000 for people of age 20. If experience shows that the probability that a person will die at age 20 is 0.0012, how much can the company expect to pay out during the ear policies were written?

Solution

For a single \$100,000 policy, we have:

	Pay	*Don't pay*
Outcome	\$100,000	\$100,000
Probability	0.0012	0.9998

$$E(x) = 100,000(0.0012) + 100,000(0.9998)$$

$$\approx 120$$

For all 100 such policies, the company can expect to pay out

$$100(120) = \$12,000$$

For a single \$50,000 policy, we have:

	Pay	*Don't pay*
Outcome	\$50,000	\$50,000
Probability	0.0012	0.9998

$$E(x) = 50,000(0.0012) + 50,000(0.9998)$$

$$\approx 60$$

For all 500 such policies, the company can expect to pay out

$$500(60) = \$30,000$$

For a single \$10,000 policy, we have:

	Pay	*Don't pay*
Outcome	\$10,000	\$10,000
Probability	0.0012	0.9998

$$E(x) = 10,000(0.0012) + 10,000(0.9998)$$

$$\approx 12$$

For all 1,000 such policies, the company can expect to pay out

$$1,000(12) = \$12,000$$

The total amount the company can expect to out is:

$$\$12,000 + \$30,000 + \$12,000 = \$54,000$$

Exercise

An insurance policy on an electrical device pays a benefit of \$4,000 if the device fails during the first year. The amount of the benefit decreases by \$1,000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure that year is 0.4. What the expected benefit under the policy? (Choose the appropriate)

a. \$2,234 *b.* \$2,400 *c.* \$2,500 *d.* \$2,667 *e.* \$2,694

Solution

$P(3{,}000) = 0.6 * 0.4 = \underline{0.24}$ $P(2{,}000) = 0.6^2 * 0.4 = \underline{0.144}$

$P(1{,}000) = 0.6^3 * 0.4 = \underline{0.0864}$ $P(0) = 1 - (0.4 + 0.24 + 0.144 + 0.0864) = \underline{0.1296}$

x	4,000	3,000	2,000	1,000	0
$P(x)$	0.4	0.24	0.144	0.0864	0.1296

$$E(x) = 4{,}000(0.4) + 3{,}000(0.24) + 2{,}000(0.144) + 1{,}000(0.0864) + 0(0.1296)$$

$$= \underline{2694.4}$$

The answer is ***e***.

Exercise

A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs \$50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay \$100 (ticket cost + \$50 penalty) to the tourist. What is the expected revenue of the tour operator? (Choose the appropriate)

a. \$935 *b.* \$950 *c.* \$967 *d.* \$976 *e.* \$985

Solution

Total earning: $21 * 50 = \$1{,}050$ (if 1 or more tourists do not show up).

The tour operator earns: $1{,}050 - 100 = \$950$

The probability that all tourist show up: $(1 - 0.02)^{21} \approx 0.6543$

The expected revenue is

$$1050(1 - 0.6453) + 650(0.6453) = \underline{984.57}$$

The answer is ***e***.