3.3 – Magnetic Fields

Magnetic Force is a force that exists between current carrying objects unlike electric force whose source is excess charges. The source of magnetic force is excess current.

Magnetic Field is a field used to represent magnetic force. The SI unit of magnetic field is the Tesla abbreviated *T*.

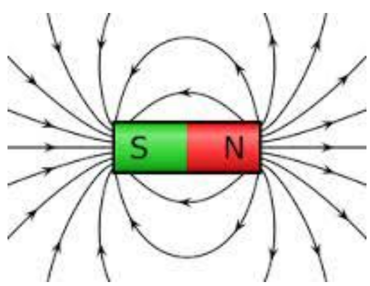
Magnetic Field Lines are lines used to represent magnetic field graphically. The lines are drawn in such a way that

1. The number of lines per a unit perpendicular area (density of lines) is proportional to the magnitude of the field and
2. The line tangent to the curve at a given point has the same line of action as magnetic field.

To distinguish between the two possible directions of the tangent line an arrow is placed on the curves. Unlike electric field lines (which originate in a positive charge and sink in a negative charge), magnetic field lines form complete loops.

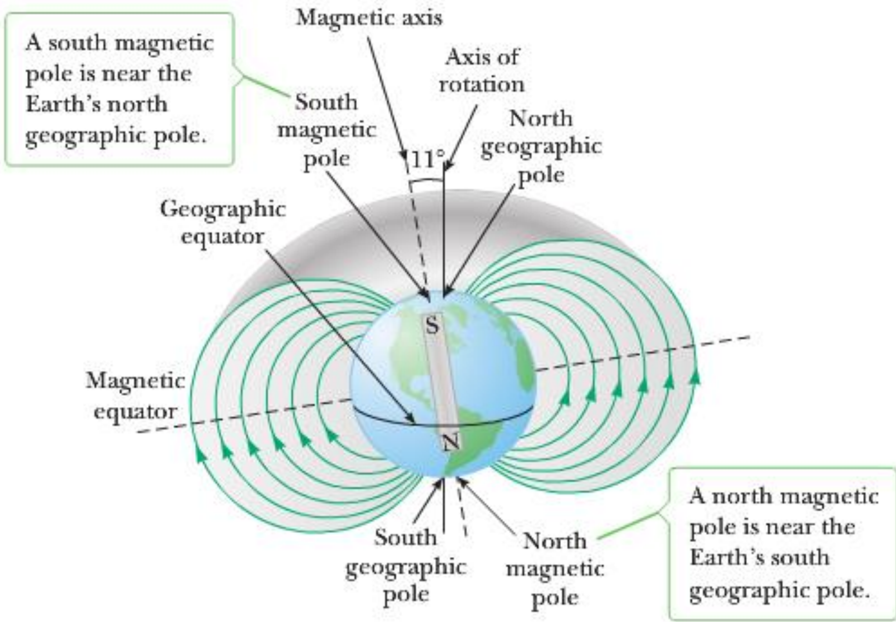
Permanent Magnets: The origin of the magnetic field due to permanent magnetic is the motion of electrons around the nucleus (which constitutes current). For most elements the currents due to the electrons of an atom cancel each and thus most elements do not have magnetic properties. But some elements such as nickel, iron and cobalt, the currents due to the electrons do not cancel each other as a result the atoms of these elements have net magnetic properties.

Normally, a sample of a magnetic material such as iron do not have magnetic properties even though their atoms do. The reason is that the atomic magnets are randomly distributed and they cancel each other. But if a magnetic material is placed in an external magnetic field, the atomic magnets are aligned in the direction of the field and the magnetic material acquires a net magnetic property becoming a magnet.



Permanent magnets of certain shapes (such as a rectangle) have two locations where the magnetic field is the strongest. These locations are called the poles of the magnet. These poles are identified as the North (***N***) and South (***S***) Pole of the magnate. If a magnet is free to rotate across a pivot, one of its ends will point towards the north pole of earth. (This is because earth has its own magnet). The pole that points towards the North Pole of earth is called the north pole of the north pole of the magnet and the other poleis called the south pole of the earth. Experiment shows that similar poles repel and opposite poles attract. Since the north pole of a magnet points towards the north pole of the earth its follows that the south pole of earth's magnet is located on the geographic north pole of earth.

Magnetic field lines come out of the North Pole and come in on the South Pole.

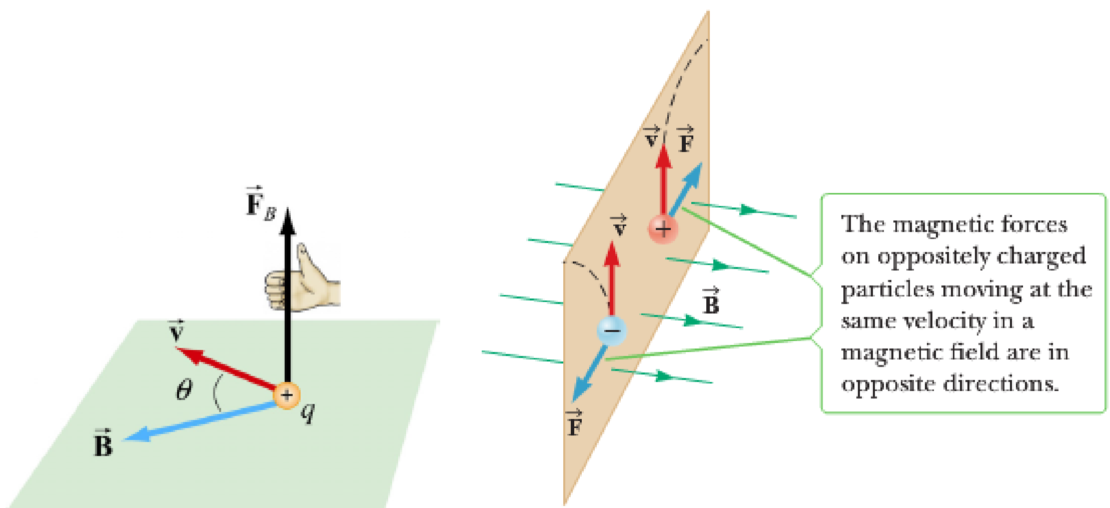


Magnetic Force on a Charge Moving in a Magnetic Field: is equal to the product of its charge and the cross product between its velocity and the magnetic field

$$\boxed{\vec{F}_B = q\vec{V} \times \vec{B}}$$

q: charge

$\vec{F}_B$ magnetic force on a charge q moving in a magnetic field $\vec{B}$ with a velocity $\vec{V}$.



If the angle between $\vec{V}$ and $\vec{B}$ is θ, then the magnitude of $\vec{V} \times \vec{B}$ is $VB\sin\theta$. Therefore the magnitude of the magnitude of the magnetic force is F_B is given by

$$\boxed{F_B = |q|VB\sin\theta}$$

F_B : Magnitude of magnetic force

V: Magnitude of velocity

B: Magnitude of magnetic field

Direction

If $\vec{V}$ and $\vec{B}$ are expressed in the $\hat{\imath} - \hat{\jmath}$ notation then evaluating $\vec{F}_B = q\vec{V} \times \vec{B}$ will give both the magnitude and direction of the magnetic force. Another alternative is to calculate the magnitude from $F_B = |q| VB \sin\theta$ and obtaining the direction from the screw or right hand rule. The direction of the magnetic force is always perpendicular to the plane determined by the velocity and magnetic field vectors. To distinguish between the two possible directions (perpendicularly out of the plane (0)) and perpendicularly into the plane (*x*)), the screw rule or the right hand rule can be used.

The Screw Rule

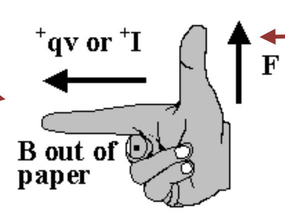
First connect the velocity vector and the magnetic field vector tail to tail as shown. Then place the screw perpendicularly at their tails, and rotate the screw from the velocity vector towards the field vector. Then, if the charge is positive the direction of movement of the screw gives direction of the magnetic force and if the charge is negative the direction of the magnetic force is opposite to the direction of movement of the screw.

Remember: a screw goes in if turned clockwise and goes out if turned counter clockwise.

The right hand rule

First arrange the thumb, index finger and middle finger of the right hand, in such a way that they are perpendicular to each other. Align the index finger in the direction of the velocity and the middle finger in the direction of the field; then if the charge is positive, direction of thumb gives direction of magnetic force and if the charge is negative the direction of the magnetic force is opposite to that of the thumb.

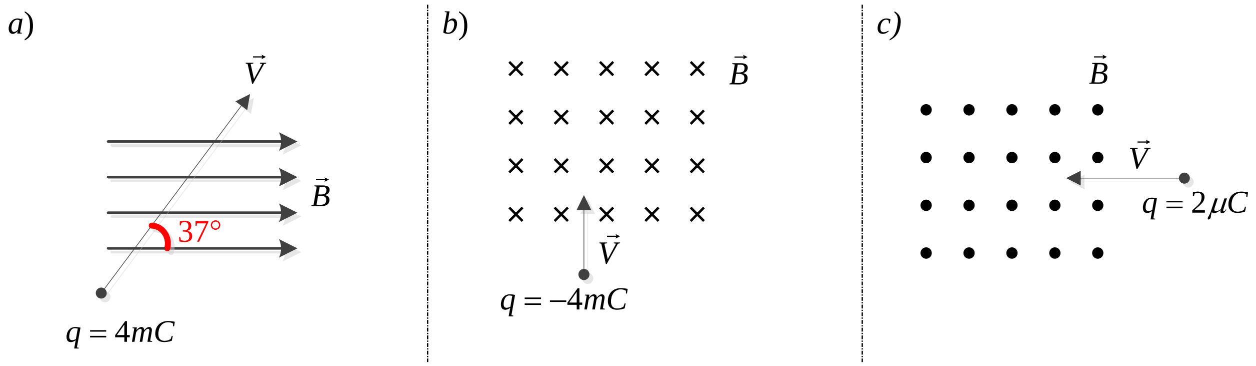
Point your fingers of your right hand in the direction of $\vec{v}$ and then curl them toward the direction of $\vec{B}$.



The magnetic force $\vec{F}$ points in the direction of your right thumb.

Example

If each of the following determine the magnetic force acting on the charge. In each case assume the speed to be 200 *m/s* and one strength of the field to be 2*T*. (Remember a dot $(\bullet)$ means perpendicularly outward or $\hat{k}$ and a cross $(\times)$ means perpendicularly into or $-\hat{k}$)



Solution

a) ***Given***: $V = 200\ m/s, \quad B = 2T, \quad q = 4\times10^{-3}C$

$$\vec{V} = 200\cos(37°)\hat{i} + 200\sin(37°)\hat{j}$$

$$\vec{B} = 2T\ \hat{i}$$

$$\vec{F}_B = q\vec{V}\times\vec{B}$$

$$= \left(4\times10^{-3}\right)\left[\left(200\cos(37°)\hat{i} + 200\sin(37°)\hat{j}\right)\times\left(2\ \hat{i}\right)\right]$$

$$= \left(4\times10^{-3}\right)\left(-240\ \hat{k}\right)$$

$$\boxed{= 0.96\left(-\hat{k}\right)\ N}$$

That is the force has a magnitude of 0.96*N* and its direction is ⊥'y into the plane of the paper $\left(-\hat{k}\right)$

Alternatively, the magnitude can be calculated from

$$F_B = |q|VB\sin\theta = \left(4\times10^{-3}\right)(200)(2)\sin 37° \ \boxed{= 0.96\ N}$$

Ant the direction from the screw rule or the tight hand rule to be ⊥'y into the paper (x)

b) $\vec{V} = 200\hat{j}$

$\vec{B} = 2T\left(-\hat{k}\right)$ (⊥'y into the paper)

$$\vec{F}_B = q\vec{V}\times\vec{B}$$

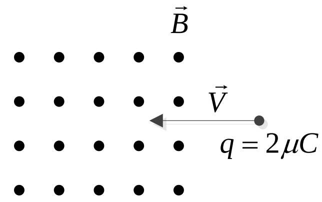
$$= \left(-4\times10^{-3}\right)\left[\left(200\hat{j}\right)\times\left(-2\ \hat{k}\right)\right]$$

$$=\left(-4\times10^{-3}\right)\left(-400\ \hat{i}\right)$$

$$\boxed{=1.6\left(\hat{i}\right)\ N} \qquad (East)$$

c) $\vec{V}=200\left(-\hat{i}\right)$

$\vec{B}=2\left(\hat{k}\right)$ (⊥'y out)



$$\vec{F}_B=q\vec{V}\times\vec{B}$$

$$=\left(2\times10^{-6}\right)\left[\left(200\left(-\hat{i}\right)\right)\times\left(2\ \hat{k}\right)\right]$$

$$=\left(2\times10^{-6}\right)\left(400\ \hat{j}\right)$$

$$\boxed{=0.08\left(\hat{j}\right)\ N} \qquad (North)$$

Magnetic Force on a current carrying wire placed on a magnetic field

Take a small element of the wire $d\vec{\ell}$ that contains charge dq. Suppose the charge dq is moving with a velocity $\vec{V}$, then the magnetic force acting on this element is

$$d\vec{F}_B = dq\left(\vec{V}\times\vec{B}\right) \qquad But\ \vec{V} = \frac{d\vec{\ell}}{dt}$$

$$d\vec{F}_B = dq\left(\frac{d\vec{\ell}}{dt}\times\vec{B}\right) = \frac{dq}{dt}\left(d\vec{\ell}\times\vec{B}\right) \qquad But\ \frac{dq}{dt} = I$$

$$\therefore\ d\vec{F}_B = I\left(d\vec{\ell}\times\vec{B}\right)$$

The total force on the wire is obtained by interpreting over the whole wire

$$\vec{F}_B = \int d\vec{F}_B = \int I\left(d\vec{\ell}\times\vec{B}\right)$$

$$= I\left(\int d\vec{\ell}\right)\times\vec{B} \qquad But \int d\vec{\ell} = \Delta\vec{\ell}$$

$\Delta\vec{\ell}$ is the vector connecting the ends of the wire & its direction is the direction of the current (or $\vec{r}$)
(Remember conventionally the direction of current is the direction of movement of positive charges)

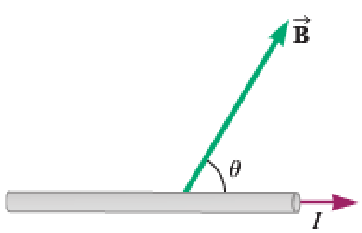
$$\boxed{\vec{F}_B = I\ \Delta\vec{\ell}\times\vec{B}}$$

$\vec{F}_B \rightarrow$ Magnetic force on a wire carrying a current & I placed in a magnetic field $\vec{B}$ where $\Delta\vec{\ell}$ is the straight line joining the two ends of the wire in the direction of the current.
($\Delta\vec{\ell} \rightarrow$ is the length of the straight line).

For a closed loop $\Delta\vec{\ell} = 0$ & it follows that the net force acting on a current carrying loop is zero.

If $\Delta\vec{\ell}$ & $\vec{B}$ are expressed in the $\hat{i}$-$\hat{j}$ notation then the magnitude & direction can be obtained directly from $\vec{F}_B = I\ \Delta\vec{\ell}\times\vec{B}$.

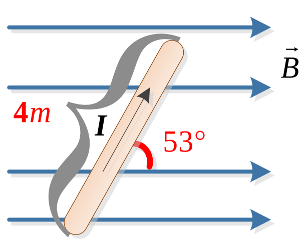
Alternatively the magnitude can be obtained from $F_B = I\ \Delta\ell\ B\sin\theta$ where θ is the angle between $\Delta\vec{\ell}$ & $\vec{B}$ & the direction can be obtained from the screw rule or the right hand rule. In using the screw rule, the screw is rotated from $\Delta\vec{\ell}$ towards $\vec{B}$ and in using the right hand rule $\Delta\vec{\ell}$ is represented by the index finger and the field is represented by the middle finger.



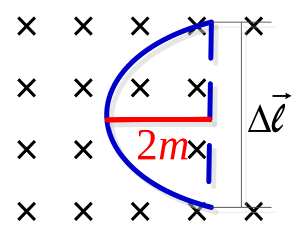
Example

If each of the following calculate the magnetic force acting on the wire shown placed in a magnetic field. Assume, in each case, the current is 2*A* & that the strength of the field is 5*T*.

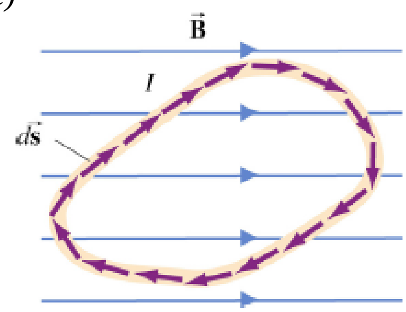
a)



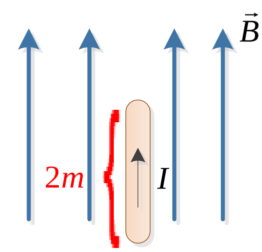
b)



c)



d)



<u>Solution</u>

a) ***Given***: $\Delta\ell = 4\ m,\quad B = 5\ T,\quad I = 2A$

$$\Delta\vec{\ell} = 4\cos(53°)\hat{i} + 4\sin(53°)\hat{j} = 2.4\hat{i} + 3.2\hat{j}$$

$$\vec{B} = 5\cos 0\ \hat{i} + 5\sin 0\ \hat{j} = 5\hat{i}$$

$$\vec{F}_B = I\ \Delta\vec{\ell} \times \vec{B}$$

$$= (2)\left[(2.4\hat{i} + 3.2\hat{j}) \times (5\ \hat{i})\right] \qquad \hat{i} \times \hat{i} = 0,\quad \hat{j} \times \hat{i} = -\hat{k}$$

$$= 2\left[16(-\hat{k})\right]$$

$$= 32(-\hat{k})\ N \qquad (\times \text{ or } \perp\text{'y into the paper})$$

Alternatively, the magnitude can be obtained from

$$F_B = I\ \Delta\ell\ B\sin\theta = 2(4)(5)\sin 53° = 32N$$

And then the direction can be obtained from the screw rule or the right hand rule.

b) Half circle with a radius of 2 *m*.

$\Delta\vec{\ell}$ is the vector joining the ends points (shown by dotted lined). Its length is twice the radius of the circle 4m. Its direction is down ward (south) as the current

Given: $\Delta l = 4\ m,\quad B = 5\ T,\quad I = 2A$

$$\Delta\vec{\ell} = 4\cos(-90°)\hat{i} + 4\sin(-90°)\hat{j} = \boxed{-4\hat{j}}$$

$$\vec{B} = 5(-\hat{k})$$

$$\vec{F}_B = I\ \Delta\vec{\ell} \times \vec{B}$$

$$= (2)\left[(-4\hat{j}) \times (5\ (-\hat{k}))\right] \qquad \hat{j} \times \hat{k} = \hat{i}$$

$$= 40(\hat{j} \times \hat{k})$$

$$\boxed{= 40(\hat{i})\ N} \qquad \text{(East)}$$

c) $\Delta\vec{\ell} = 0$, since it is a closed loop

$\Rightarrow\ \vec{F}_B = 0$

d) ***Given***: $\Delta\ell = 2\ m, \quad B = 5\ T, \quad I = 2A, \quad \theta_B = 90°$

$$\Delta\vec{\ell} = 2\hat{j}$$

$$\vec{B} = 5\hat{j}$$

$$\vec{F}_B = I\ \Delta\vec{\ell} \times \vec{B}$$

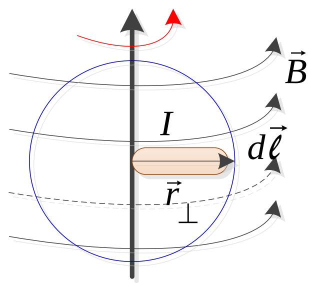
$$= (2)\left[2\hat{j} \times 5\hat{j}\right] \qquad \hat{j} \times \hat{j} = 0$$

$$\boxed{= 0}$$

Magnetic Torque on a current carrying loop placed in a magnetic field

Consider a current carrying loop placed in a magnetic field as shown

Consider a small element $d\vec{\ell}$ of the wire as shown.



The magnetic force acting on this element is given by

$$d\vec{F}_B = I\, d\vec{\ell} \times \vec{B}$$

The magnetic torque acting on this element about the axis of rotation shown is

$$d\vec{\tau} = \vec{r}_\perp \times d\vec{F}_B$$

Where $\vec{r}_\perp$ is a radial position vector of $d\vec{\ell}$ with respect to the axis of rotation

$$d\vec{\tau} = \vec{r}_\perp \times \left(I\, d\vec{\ell} \times \vec{B}\right)$$

The total torque on the loop is obtained by integrating $d\vec{r}$ over the entire loop

$$\vec{\tau} = \int d\vec{\tau} = \int \left[\vec{r}_\perp \times \left(I\, d\vec{\ell} \times \vec{B}\right)\right]$$

Assuming I & $\vec{B}$ are constants, they can be taken out of the integration

$$\vec{\tau} = I \int \left[\vec{r}_\perp \times d\vec{\ell}\right] \times \vec{B}$$

$\vec{r}_\perp \times d\vec{\ell}$ is equal to the area of the parallelogram determined by $\vec{r}_\perp$ & $d\vec{\ell}$ (It is the shaded parallelogram in the diagram)

$$\vec{r}_\perp \times d\vec{\ell} = d\vec{A}$$

$$\vec{\tau} = I\left\{\int d\vec{A}\right\} \times \vec{B} \qquad \& \qquad \int d\vec{A} = \vec{A}$$

Where $\vec{A}$ is the area of the loop

$$\boxed{\vec{\tau} = I\left(\vec{A} \times \vec{B}\right)}$$

Magnetic torque $(\vec{\tau})$ acting on a current carrying loop is equal to the product between the area of the loop and the magnetic field (provided the current & the field are constants).
Remember: Area is a vector quantity whose direction is perpendicular to the plane of the loop. Thumb gives the direction of area when the hand fingers are wrapped around the loop in a counterclockwise direction.

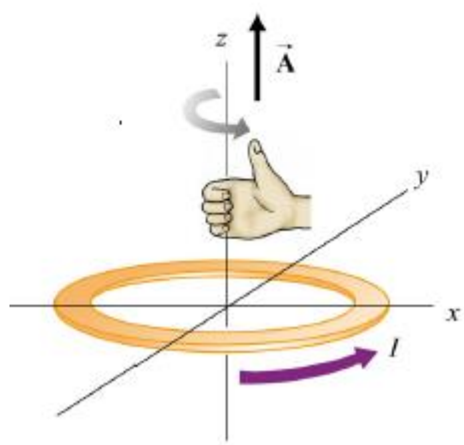
If the angle formed between $\vec{A}$ & $\vec{B}$ is θ, then the magnitude of the torque is given by

$$\boxed{\tau = IAB\sin\theta}$$

The maximum value of the torque occurs when $\theta = 90°$; that is when $\vec{A}$ is perpendicular to $\vec{B}$ or when the field is parallel to the plane of the loop

$$\boxed{\tau = IAB\sin 90° = IAB}$$

The direction of torque and the direction of rotation. That is either clockwise or counterclockwise is determined by the right hand rule when thumb is aligned in the direction of the torque, the direction of fingers represents direction of rotation. For example when thumb point up, fingers will be wrapped in a counterclockwise direction.



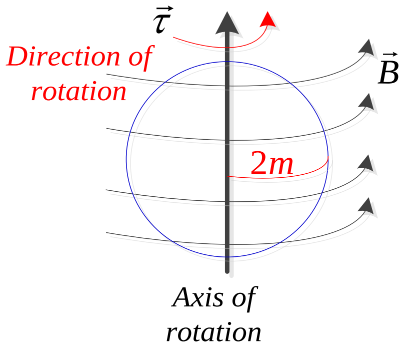
If $\vec{A}$ & $\vec{B}$ are expressed in the $\hat{i} - \hat{j}$ rotation, then the magnitude & direction of the torque can be obtained directly from $\vec{\tau} = I\left(\vec{A} \times \vec{B}\right)$. Alternatively the magnitude can be obtained from $\tau = IAB\sin\theta$ and the direction can be obtained either from the screw rule or the right hand rule.

In using the screw rule, the screw should be turned from $\vec{A}$ towards $\vec{B}$. The direction of movement of the screw will be the direction of torque if the current is counterclockwise and opposite to the direction of torque if the current is clockwise.

In using the right hand rule, $\vec{A}$ is represented by the index finger and $\vec{B}$ is represented by the middle finger. Then the direction of torque will be in the direction of thumb if the current is counterclockwise and the opposite to the direction of the thumb if current is clockwise.

Example

A circular loop of radius 2*m* is placed in a region where there is magnetic field of strength 10*T* as shown.



The loop is carrying a current of 4*A* in a counter clockwise direction.

a) Determine the magnetic torque acting on the loop.

b) From part (*a*), ids the loop rotating clockwise or counterclockwise.

<u>Solution</u>

a) Given: $I = 4A,\quad B = 10T,\quad \theta_B = 0°\ (east),\quad r = 2m$

$$A = \pi r^2 = \pi 2^2 = 4\pi$$

$$\vec{B} = B\cos(0°)\hat{i} + B\sin(0°)\hat{j}\ \underline{= 10\hat{i}|}$$

Direction of area is perpendicularly out or $\hat{k}$ from the tight hand rule

$$\vec{A} = 4\pi\hat{k}$$

$$\vec{\tau} = I\left(\vec{A} \times \vec{B}\right)$$

$$= \{4\}\left(4\pi\hat{k} \times 10\hat{i}\right)$$

$$= 160\pi\left(\hat{k} \times \hat{i}\right) \qquad \hat{k} \times \hat{i} = \hat{j}$$

$$\underline{= 160\pi\,\hat{j}\ \ Nm|}$$

That is direction of torque is north.

Alternatively, first the magnitude can be calculated from

$$\tau = IAB\sin\theta = 4(4\pi)(10)\sin 90° \underline{= 160\pi\,\hat{j}\ \ Nm|}$$

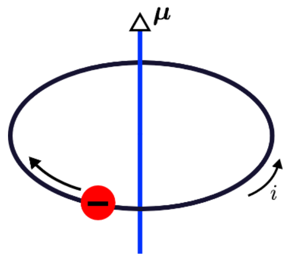
With $\vec{B} \perp \vec{A} \ \Rightarrow\ \theta = 90°$

And then the direction can be determined from the right hand rule or screw rule be north.

b) With thumb in the direction of torque (north) fingers are wrapped in a counterclockwise direction. Thus, the loop will be rotating in a counterclockwise direction,

Magnetic Moment

Magnetic moment of a current carrying loop is defined to be the product of the current in the loop and the area of the loop



$\boxed{\vec{\mu} = I\vec{A}}$ Where $\vec{\mu}$: is magnetic moment

If the current is in a counterclockwise direction then $\vec{\mu}$ has the same direction as the area; and if the current is clockwise the direction if $\vec{\mu}$ is opposite to that of $\vec{A}$. In other word, the direction of $\vec{\mu}$ is related with the direction of the current by the right hand rule. When fingers are wrapped in the direction of the current, thumb will give the direction of the magnetic moment. The unit of measurement of magnetic moment is Am^2.

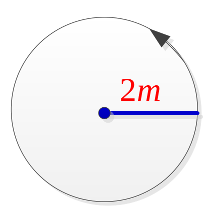
The torque acting on a current carrying loop placed in a magnetic field can now be written in terms of magnetic moment

$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$ Since $\vec{\tau} = I\left(\vec{A} \times \vec{B}\right)$ and $\vec{\mu} = I\vec{A}$

In using this formula, it is important to note that the angle used in $\tau = IAB\sin\theta$ is the angle between $\vec{\mu}$ and $\vec{B}$ & not between $\vec{A}$ and $\vec{B}$.

Example

The loop shown carries a current of 0.5 *A* in a counterclockwise direction, Determine its magnetic moment.



Solution

$$I = 0.5A$$

$$A = \pi r^2 = \pi(2)^2 = 4\pi$$

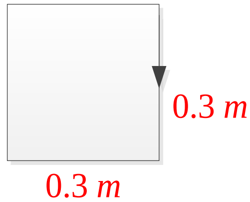
$$\vec{\mu} = I\vec{A}$$

$$= (0.5A)\left(4\pi m^2\right)$$

$$= 2\pi \ Am^2$$ (*Perpendicularly* out)

Example

The loop shown carries a current of 2 *A* in a clockwise direction, Determine its magnetic moment.



Solution

$I = -2A$ (negative because it is clockwise)

$A = (0.3m)^2 = \underline{0.09\hat{k}\ m^2}$

$\vec{\mu} = I\vec{A}$

$= (-2A)\left(0.09\ m^2\right)$

$\underline{= -0.18\ \hat{k}\ \ Am^2}$ (*Perpendicularly* out)

Magnetic Potential Energy

Consider a current carrying loop placed in a magnetic field such a way that the angle between its magnetic moment is different from zero or 180°. There will be a net torque active on it according to the equation $\tau = IAB\sin\theta$. If the loop is let go from this orientation, it will rotate because of the torque acting on it. In other words, this orientation is a source for rotational kinetic energy. This indicates that there is a magnetic potential energy associated with current carrying loops placed in a magnetic field.

A loop placed in a magnetic field rotates because there is a tangential magnetic force on the loop. The work done by this tangential force as the loop is displaced (rotated) by a small displacement $d\vec{r}$, then the work done is $dW = \vec{F}_t \bullet d\vec{r}$. Therefore the change in potential energy associated with this is

$$du = -dW = -\vec{F}_t \bullet d\vec{r}$$

Or

$$u = -\int \vec{F}_t \bullet d\vec{r}$$

Now consider a loop placed in a magnetic field in such a way that its magnetic moment is parallel to the magnetic field. At this position the torque acting on the loop is zero (because $\theta = 0$). Magnetic torque is a restoring force. If this loop is rotated counterclockwise (clockwise) the magnetic torque will be clockwise (counterclockwise). In other words, the tangential magnetic force is opposite to the displacement is rotational, $|d\vec{r}| = dS$ where dS represents arc length.

$$dW = -F_t r_\perp d\theta \qquad but \quad F_t r_\perp = \tau$$

$$dW = -\tau d\theta$$

$$du = -dW = \tau d\theta$$

$$u = \int \tau d\theta \qquad but \quad \tau = \mu B\sin\theta$$

$$u = \int \mu B\sin\theta d\theta$$

$$= -\mu B\cos\theta + C$$

It is easier to choose the reference point to be the orientation where $\cos\theta = 0$.

Therefore, let $u\Big|_{\theta=\frac{\pi}{2}} = 0$

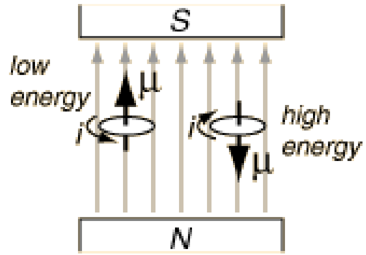
$$u\Big|_{\theta=\frac{\pi}{2}} = 0 = -\mu B\cos\theta + C \quad \Rightarrow \quad C = 0$$

$$u = -\mu B\cos\theta$$

This also can be written as a dot product

$$\boxed{u = -\vec{\mu}\bullet\vec{B}}$$

The minimum potential energy occurs when $\vec{\mu}$ & $\vec{B}$ are parallel ($\theta = 0$) $u\big|_{\theta=0} = -\mu B$



The maximum potential energy occurs when $\vec{\mu}$ & $\vec{B}$ are opposite ($\theta = 180°$) $u\big|_{\theta=180°} = \mu B$

These are orientations where the torque is zero. The former is a stable equilibrium orientation and the later is unstable equilibrium orientation. The reference point $\left(\theta = \frac{\pi}{2}\right)$ is the orientation where the maximum torque occurs.

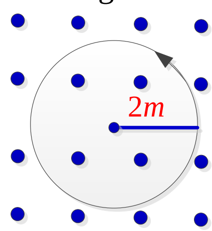
Even though this formula is obtained for orientations, it is generally valid. For example suppose the magnetic field changes along the x-axis. Then there will be a linear force along the x-axis because

$$F_x = -\frac{\partial u}{\partial x} = \vec{\mu} \bullet \frac{\partial \vec{B}}{\partial x}$$

This explains why iron is attracted (or repelled) by a magnet. The force is the result of the fact that the magnetic field gets stronger & stronger as the magnet is approached.

Example

The loop shown carries a current of $3A$ & the strength of the field is $0.4T$.



Calculate the magnetic potential energy stored by the loop.

Solution

Given: $I = -3A$ (*clockwise*), $B = 0.4\hat{k}$, $r = 2m$

$A = 4\pi\hat{k}$

$\vec{\mu} = IA = -12\pi\hat{k}$

$u = -\vec{\mu} \bullet \vec{B}$

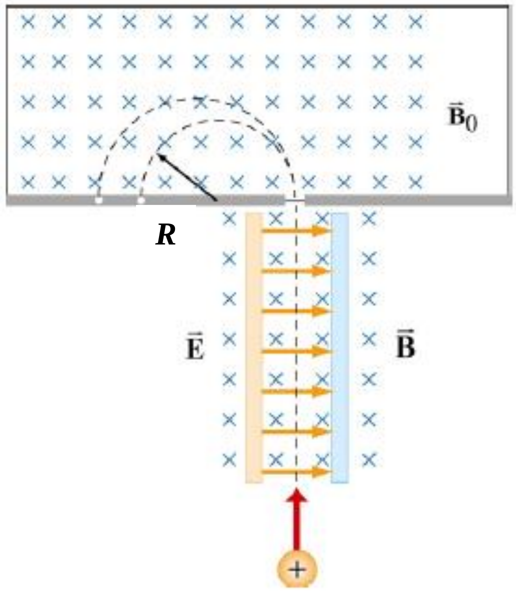
$= -\left(-12\pi\hat{k}\right) \bullet \left(0.4\hat{k}\right)$

$\underline{= 4.8\pi\ J}$

Mass Spectrometer

Mass Spectrometer is a device used to separate a mixture of charges according to their masses (assuming they all have the same charge).

Consider a particle of mass *m* and charge *q* propelled a magnetic field *B* perpendicularly as shown



As the charge enters the field it will be acted upon by a magnetic force $F_B = |q|vB\sin\theta$ but $\theta = 90°$ & $F_B = |q|vB$. Since this force is perpendicularly to the trajectory (velocity) of the particle, its effect is only to change direction & not magnitude. Such force is called a centripetal force and the resulting trajectory is circular. Let the radius of this trajectory be *R*. Centripetal force is related with the speed of the object according to the equation $F_C = \frac{mv^2}{R}$

$$F_B = F_C = \frac{mv^2}{R} = |q|vB$$

$$\boxed{R = \frac{mv}{|q|B}}$$

$m \rightarrow$ mass
$v \rightarrow$ velocity
$q \rightarrow$ charge
$B \rightarrow$ field

This equation shows that the radius of the trajectory depends on the mas of the particle. This if a mixture of charges of different masses is propelled to the field, the particles will have different trajectories according to their masses resulting in the separation of the particles according to their masses.

Example

An electron $\left(mass = 9.1\times10^{-31}kg,\ \ charge = -1.6\times10^{-19}C\right)$ is propelled perpendicularly into & for magnetic field of strength $2mT$ with a speed of $10^{6}\ m/s$

a) Calculate the radius of its trajectory

b) How long will it take to make one on plate revolution

<u>Solution</u>

a) Given: $m = 9.1\times10^{-31}kg,\quad |q| = -1.6\times10^{-19}C,\quad v = 10^{6}m/s,\quad B = 2\times10^{-3}T$

$$R = \frac{mv}{|q|B}$$

$$= \frac{\left(9.1\times10^{-31}\right)\left(10^{6}\right)}{\left(1.6\times10^{-19}\right)\left(2\times10^{-3}\right)}$$

$$= 2.84\times10^{-4}m$$

$$\approx 0.03\ \ cm$$

b) Let T = Time taken for one revolution

$$T = \frac{circumference}{speed} = \frac{2\pi R}{v}$$

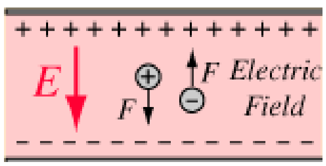
$$= \frac{2\pi\left(2.84\times10^{-4}\right)}{10^{6}}$$

$$= 1.78\times10^{-9}\ \ s$$

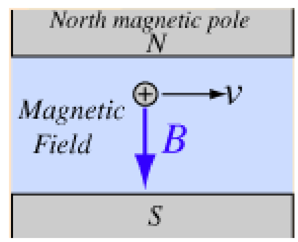
Electromagnetic force on a charge

A charge in a region where there are both electric and magnetic field, is subjected to both electrical and magnetic force. The electrical force is $q\vec{E}$ where $\vec{E}$ is electric field; and the magnetic force is $q\vec{v}\times\vec{B}$ where $\vec{v}$ is the velocity of the charge and $\vec{B}$ is the field. The net electromagnetic force is the sum of these two forces

$$\boxed{\vec{F}_{em} = q\vec{E} + q\vec{v}\times\vec{B}}$$



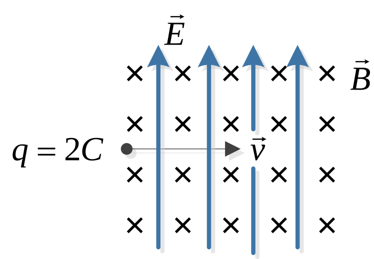
Electric force qE



Magnetic force of magnitude $qvB\sin\theta$ $\perp$ to both v and B, away from you

Example

A 2C charge is moving in a region where there is electric field directed towards north & a magnetic field that penetrates the paper perpendicularly inward. The magnitude of the electric field is 200 *N/C* and the magnitude of the magnetic field is 5*T*.



Calculate the electromagnetic force acting on the charge, at a time when it is moving towards east with a speed of 100 *m/s*.

<u>Solution</u>

Given: $\vec{E} = 200\,\hat{j}$ N/C, $\quad q = 3$ C, $\quad v = 100\hat{i}$ m / s, $\quad B = 5\left(-\hat{k}\right)$ T

$$\vec{F}_{em} = q\vec{E} + q\vec{v}\times\vec{B}$$

$$= 3\left(200\,\hat{j}\right) + 3\left(100\hat{i}\right)\times\left(5\left(-\hat{k}\right)\right)$$

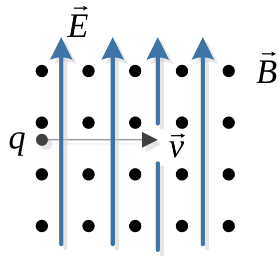
$$= 600\,\hat{j} - 1500\left(\hat{i}\times\hat{k}\right) \qquad but \quad \hat{i}\times\hat{k} = -\hat{j}$$

$$= 600\,\hat{j} + 1500\,\hat{j}$$

$$= 2100\,\hat{j}\ \ N$$

Example

A charge is moving undeflected with constant speed towards east in the fields shown. The strength of the electric field is 2000 *N/C* and the strength of the magnetic field is 0.005*T*.



Calculate the speed of the charge.

Solution

Given: $\vec{E} = 2000\,\hat{j}\ N/C,\quad \vec{v} = v\hat{i}\ m/s,\quad B = 0.005\left(\hat{k}\right)\ T$

If the charge is moving in a straight line with a constant speed, then the net force acting on it must be zero

$$\vec{F}_{em} = q\vec{E} + q\vec{v}\times\vec{B} = 0$$

$$\vec{E} + \vec{v}\times\vec{B} = 0$$

$$2000\,\hat{j} + \left(v\hat{i}\times 0.005\hat{k}\right) = 0$$

$$2000\,\hat{j} + 0.005v\left(\hat{i}\times\hat{k}\right) = 0 \qquad but \quad \hat{i}\times\hat{k} = -\hat{j}$$

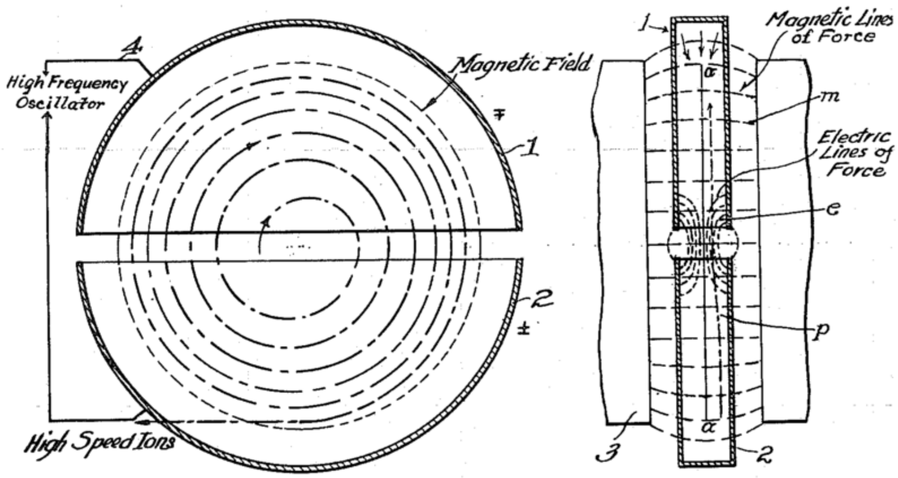
$$2000\,\hat{j} - 0.005v\hat{j} = 0$$

$$2000 - 0.005v = 0$$

$$v = \frac{2000}{0.005} \underline{= 4\times 10^{7}\ m/s|}$$

Cyclotron

A cyclotron is device used to accelerate charges to a high energy. It essentially consists of two half cylinders connected to an alternating potential difference paced in a magnetic field parallel to one axis of the cylinders. A charge is propelled perpendicular to the field. The charge will move in a circular path because of the magnetic force. Every time it goes from one of the half cylinders to the other it will be accelerated because of the potential difference between the half cylinders $(u = q\Delta v)$.



As the velocity increases because of the potential difference, the radius of revolution increases. Eventually, the radius of revolution is increased to the outer radius of the cyclotron. Let the outer radius of the cyclotron be R, then

$$\frac{mv^2}{R} = qvB$$

$$\boxed{v = \frac{qRB}{m}}$$

Thus, the energy of the charge by the time it leaves the cyclotron is

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qRB}{m}\right)^2 = \frac{1}{2m}q^2R^2B^2$$

$$\boxed{E = \frac{1}{2m}q^2R^2B^2}$$ Energy of a charge by the time it leaves a cyclotron

A common unit of energy for atomic particles is the electron Volt abbreviation as ***eV***.

An electron-volt is defined to be the energy needed to accelerate an electron through a potential difference of one volt.

$$Electron\ volt = q\Delta V = \left(1.6\times10^{-19}C\right)\left(1V\right) = 1.6\times10^{-19}J$$

$$\boxed{1\ eV = 1.6\times10^{-19}\ J}$$

Example

The strength of the field in a cyclotron is 10*T*. The outer radius the cyclotron is 0.8 *m*. Calculate the energy of a proton by the time it comes out of a cyclotron in electron volts.

Solution

Given: $B = 10T, \quad R = 0.8m, \quad q = 1.6\times 10^{-19}, \quad m_p = 1.67\times 10^{-27}\, kg$

$$E = \frac{1}{2m}q^2R^2B^2$$

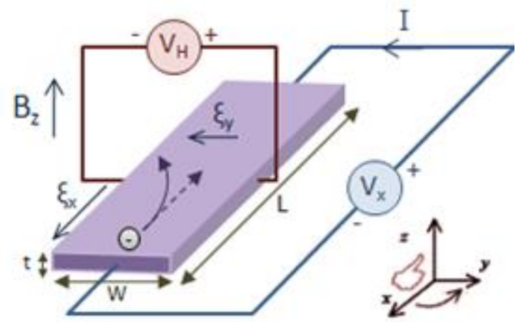
$$= \frac{1}{2\left(1.67\times 10^{-27}\right)}\left(1.6\times 10^{-19}\right)^2(0.8)^2(10)^2$$

$$= 4.9\times 10^{-10}\, J\, \frac{1\ eV}{1.6\times 10^{-19}\ J}$$

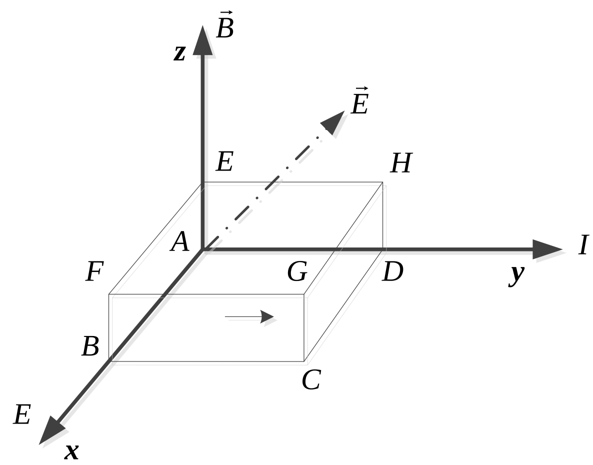
$$= 3.1\times 10^{9}\ eV$$

The Hall Effect

Hall effect refers to the fact that when a current carrying current is placed in a magnetic field perpendicular to the current, electric field (potential difference) is developed in a direction perpendicular to both the current and the magnetic field.



Consider the rectangular conductor shown which carries current in the positive *y* direction $\left(\vec{v}_d = v_d\,\hat{j}\right)$ and placed in a region where there is magnetic field whose direction is along the positive *z*-axis $\left(\vec{B} = B\hat{k}\right)$



The charges carrying the current will be subjected to a magnetic force according to the equation

$$F_B = q\vec{v}_d \times \vec{B}$$

Where $\vec{v}_d$ is the draft velocity of the charges

But $\vec{v}_d = v_d\,\hat{j}$ & $\vec{B} = B\hat{k}$. Therefore

$$F_B = q\left(v_d\,\hat{j} \times B\hat{k}\right) = qv_d B\left(\hat{j} \times \hat{k}\right) = qv_d B\hat{i}$$

Therefore the charges (assumed to be positive conventionally will be pushed towards the surface *BCGFB*. As a result the face *BCGFB* will be positively charges and the opposite dace *ADHEA* will be negatively charges because of the deficiency of positive charged result there will be ab electric field directed from the positively charges face *BCGFB* do the negatively charged face *ADHEA*. That is $\vec{E} = -E\hat{i}$.

Therefore the current carrier charges are subjected to both electric force & magnetic force. The electric force $\left(\vec{E} = -E\hat{i}\right)$ opposed the magnetic force $\left(F_B = qv_d B\hat{i}\right)$. The build up of the charges on the faces *BCGFB & ADHEA* can continue only until the magnetic force is balanced by the electric force. The balancing electric field is called the Hall electric field (denoted by E_H) When $E = E_H$; the sum of the electric force & magnetic force should be zero.

$$F_{EB} = \vec{F}_E + \vec{F}_B = 0$$

$$= -qE_H \hat{i} + qv_d B\hat{i} = 0$$

$$\Rightarrow \quad \boxed{E_H = v_d B}$$

The potential difference between the faces *BCGFB & ADHEA* is equal to the product of the electric field E_H & perpendicular distance between these faces $\left(\overline{AB}\right)$. Let $\overline{AB} = d$

$$\boxed{\Delta V_H = E_H d}$$ Where ΔV_H is the Hall potential difference

$$\boxed{\Delta V_H = v_d Bd}$$

The Hall Effect is often used to measure an unknown magnetic field

$$B = \frac{\Delta V_H}{v_d d}$$

v_d is a drift velocity related with the current

$$I = nqv_d A \quad or \quad v_d = \frac{I}{nqA}$$

n is concentration of charges & A is the cross-sectional area of the face perpendicular to the current

$$A = \overline{AB} \cdot \overline{AE} \qquad Let \quad \overline{AE} = t$$

$$\Rightarrow \; A = td$$

$$v_d = \frac{I}{nqtd}$$

$$B = \frac{\Delta V_H}{v_d d} = \frac{nqt\Delta V_H}{I}$$

Example

Consider the rectangular block shown in the above diagram. Suppose the block made up of copper & is carrying current of 2 A in the direction shown (the y-axis). And suppose there is a magnetic field along the positive z-axis as shown whose magnitude is unknown. The dimensions of the rectangular block are $\overline{AE} = 0.02\ m,\ \overline{AB} = 0.04\ m\ \ \&\ \ \overline{AD} = 0.06\ m$.

Calculate the magnitude of the magnetic field, if the Hall potential difference between the faces *ABCDA* & *ADHEA* is measured to be 5 nVolts.

(Atomic mass of Cu is 63.5 and its density is 8920 kg / m^3)

Solution

Given: $\Delta V_H = 5\times 10^{-9} V,\quad d = \overline{AB} = 0.04 m,\quad I = 2A$

n (concentration of charges) can be obtained as the ratio between Avogadro number & the volume of one gram molecular weight of copper (Cu has only one volume electron per atom)

$$n = \frac{6.02\times 10^{23}}{\dfrac{63.5\times 10^{-3} kg}{8920\ kg / m^3}} = 8.46\times 10^{28}$$

$$v_d = \frac{I}{nqA} = \frac{I}{nq\left(\overline{AB}\cdot\overline{AE}\right)}$$

$$= \frac{2}{\left(8.46\times 10^{28}\right)\left(1.6\times 10^{-19}\right)\left[(0.04)(0.02)\right]}$$

$$= 1.85\times 10^{-7}\ m / s$$

$$B = \frac{\Delta V_H}{v_d d}$$

$$= \frac{5\times 10^{-9}}{\left(1.85\times 10^{-7}\right)(0.04)}$$

$$= 0.68\ T$$