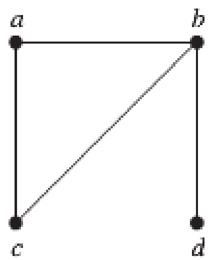
SOLUTION *Section* 4.6 – Graphs: Definitions and Basic Properties

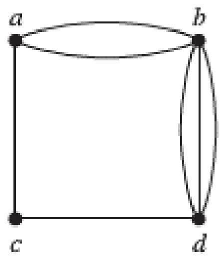
Exercise

Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops.

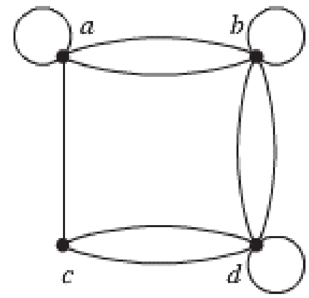
a)



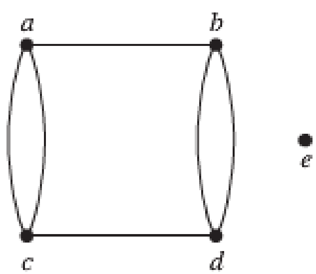
b)



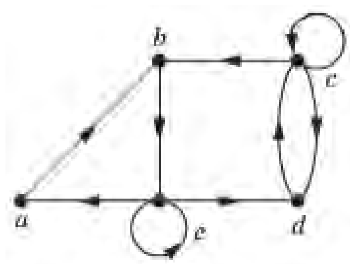
c)



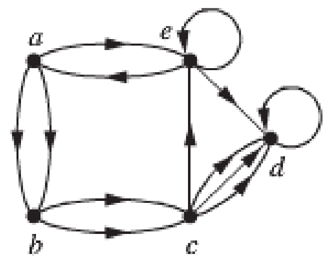
d)



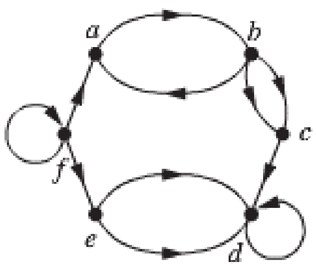
e)



f)



g)



Solution

a) This is a simple graph; the edges are undirected, and there are no parallel edges or loops.

b) This is a multigraph; the edges are undirected, and there are no loops, but there are parallel edges.

c) This is a pseudograph; the edges are undirected, and there are no parallel edges or loops.

d) This is a multigraph; the edges are undirected, and there are no loops, but there are parallel edges.

e) This is a directed graph; the edges are directed, and there are no parallel edges.

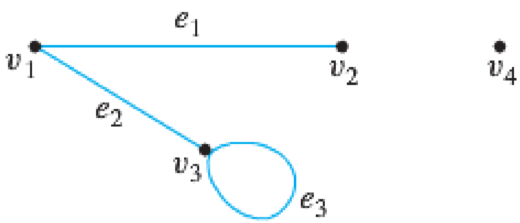
f) This is a directed multigraph; the edges are directed, and there are parallel edges.

g) This is a directed multigraph; the edges are directed, and there is a set of parallel edges.

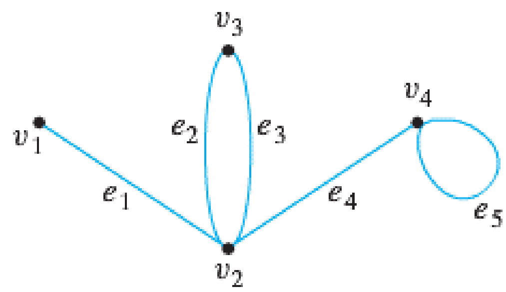
Exercise

Define each graph formally by specifying its vertex set, its edge set, and a table giving the edge-endpoint function

a)



b)



Solution

a) Vertex set $\{v_1, v_2, v_3, v_4\}$

Edge set $\{e_1, e_2, e_3\}$

Edge-endpoint function:

Edge	***Endpoints***
e_1	$\{v_1, v_2\}$
e_2	$\{v_1, v_3\}$
e_3	$\{v_3\}$

b) Vertex set $\{v_1, v_2, v_3, v_4\}$

Edge set $\{e_1, e_2, e_3, e_4, e_5\}$

Edge-endpoint function:

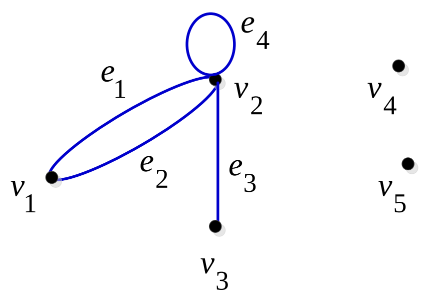
Edge	***Endpoints***
e_1	$\{v_1, v_2\}$
e_2	$\{v_2, v_3\}$
e_3	$\{v_2, v_3\}$
e_4	$\{v_2, v_4\}$
e_5	$\{v_4\}$

Exercise

Graph G has vertex set $\{v_1, v_2, v_3, v_4, v_5\}$ and edge set $\{e_1, e_2, e_3, e_4\}$, with edge-endpoint function as follow

Edge	***Endpoints***
e_1	$\{v_1, v_2\}$
e_2	$\{v_1, v_2\}$
e_3	$\{v_2, v_3\}$
e_4	$\{v_2\}$

Solution

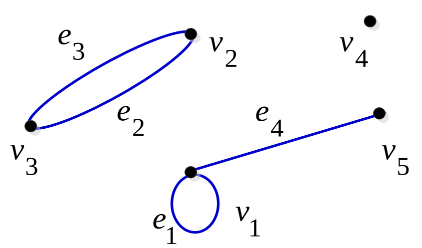


Exercise

Graph H has vertex set $\{v_1, v_2, v_3, v_4, v_5\}$ and edge set $\{e_1, e_2, e_3, e_4\}$, with edge-endpoint function as follow

Edge	***Endpoints***
e_1	$\{v_1\}$
e_2	$\{v_2, v_3\}$
e_3	$\{v_2, v_3\}$
e_4	$\{v_1, v_5\}$

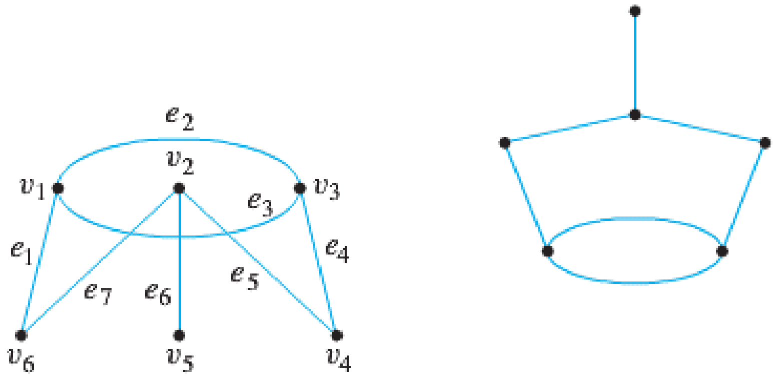
Solution



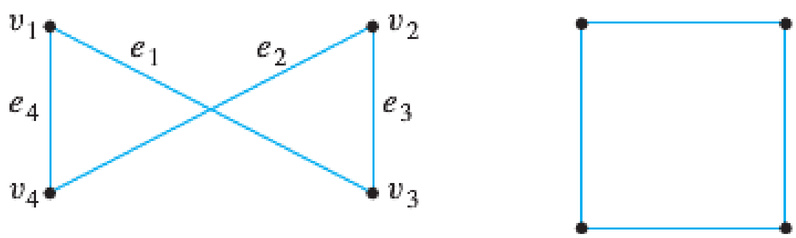
Exercise

Show that the 2 drawings represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to those of the left-hand drawing.

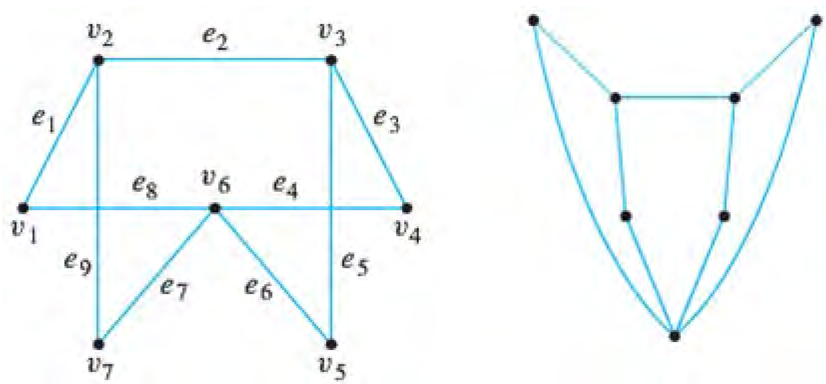
a)



b)

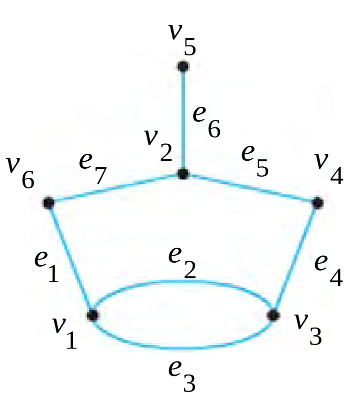


c)

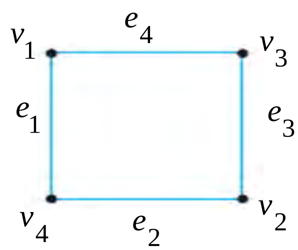


Solution

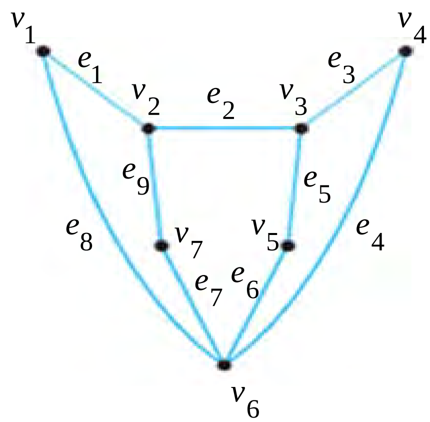
a) If you just hold the vertex v_5 turn it around to up position and stretch vertically little



b) Hold the edge e_4 and twisted as the vertices switch position.



c)



Exercise

For each of the graphs

i. Find all edges that are incident on v_1

ii. Find all vertices that are adjacent to v_3

iii. Find all edges that are adjacent to e_1

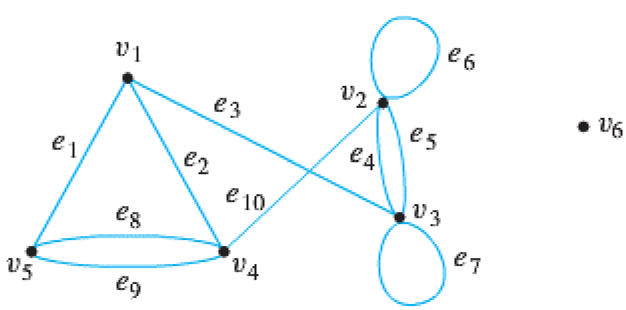
iv. Find all loops

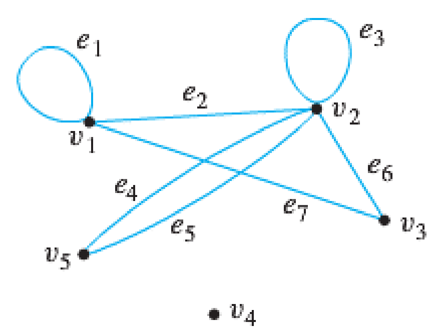
v. Find all parallel edges

vi. Find all isolated vertices

vii. Find the degree of v_3

viii. Find the total degree of the graph





Solution

a) e_1, e_2, and e_3 are incident on v_1

v_1, v_2 and v_3 are adjacent to v_3

e_2, e_3, e_8, and e_9 are adjacent to e_1

e_6 and e_7 are loops.

e_4 and e_5 are parallel; e_8 and e_9 are parallel

v_6 is an isolated vertex.

Degree of $v_3 = 5$

Total degree = 20

b) e_1, e_2, and e_7 are incident on v_1

v_1, v_2 and v_3 are adjacent to v_3

e_2 and e_7 are adjacent to e_1

e_1 and e_3 are loops.

e_4 and e_5 are parallel

Isolated vertex: none.

Degree of $v_3 = 2$

Total degree = 14

Exercise

Let *G* be a simple graph. Show that the relation *R* on the set of vertices of *G* such that uRv if and only if there is an edge associated to $\{u, v\}$ is a symmetric, irreflexive relation on *G*.

Solution

In a simple graph, edges are undirected.

If uRv, then there is edge associated with $\{u, v\}$. But $\{u, v\} = \{v, u\}$, so this edge is associated with $\{v, u\}$ and, therefore. So, *R* is symmetric.

A simple graph does not allow loops; that is if there is an edge associated with $\{u, v\}$, then $u \neq v$.

Thus uRu never holds, and so by definition *R* is irreflexive.

Exercise

Let G be an undirected graph with a loop at every vertex. Show that the relation R on the set of vertices of G such that uRv if and only if there is an edge associated to $\{u,\ v\}$ is a symmetric, reflexive relation on G.

Solution

If uRv, then there is edge associated with $\{u,\ v\}$, and since the graph is undirected, this is also edge joining vertices $\{v,\ u\}$ and therefore. So, R is symmetric.

The relation is reflexive because the loops guarantees that uRu for each vertex u.

Exercise

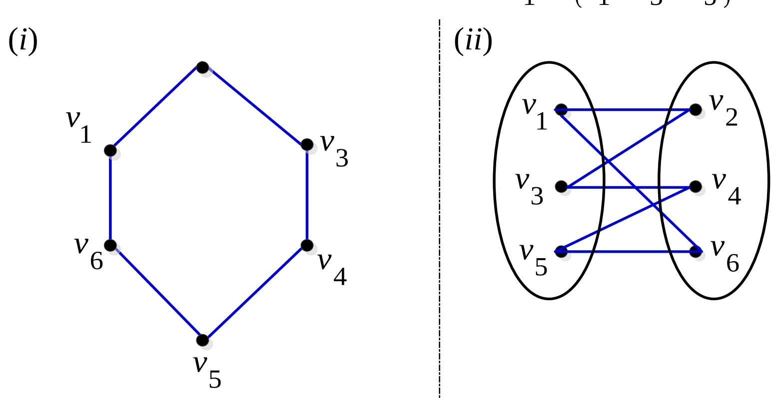
Explain how graphs can be used to model electronic mail messages in a network. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed? Describe a graph that models the electronic mail sent in a network in a particular week.

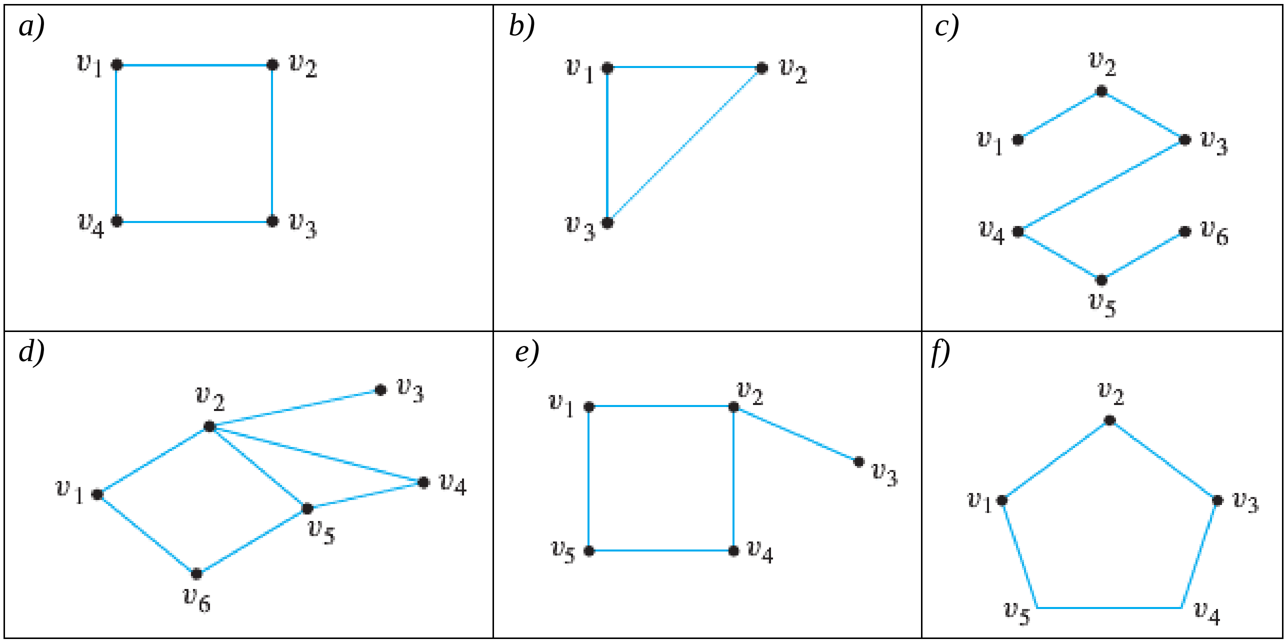
Solution

We can have a vertex for each mailbox or e-mail address in the network, with a directed edge between two vertices if a message is sent from the tail of the edge to the head.
We use directed edge for each message sent during the week.

Exercise

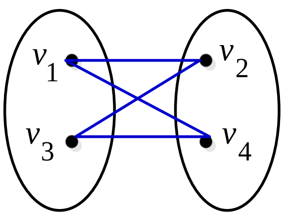
A bipartite graph G is a simple graph whose vertex set can be portioned into two disjoint nonempty subsets V_1 and V_2 such that vertices in V_1 may be connected to vertices in V_2, but no vertices in V_1 are connected to other vertices in V_1 and no vertices in V_2 are connected to other vertices in V_2. For example, the graph G illustrated in (i) can be redrawn as shown in (ii). From the drawing in (ii), you can see that G is bipartite with mutually disjoint vertex set $V_1 = \{v_1,\ v_3,\ v_5\}$ and $V_1 = \{v_2,\ v_4,\ v_6\}$





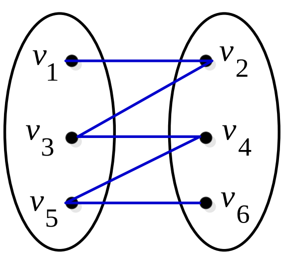
Solution

a)



b) $\{v_1, v_2, v_3\}$ form a triangle, we can't create a bipartite graph *G*.

c)



d) $\{v_2, v_4, v_5\}$ form a triangle, therefore we can't create a bipartite graph *G*.

e)