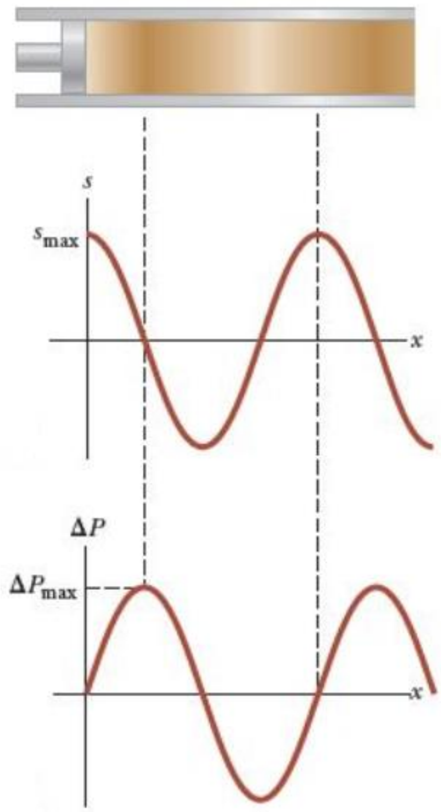
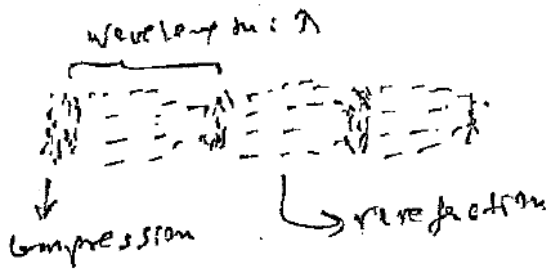
1.2 – Sound Waves

Sound waves are produced by vibrating objects. A vibrating object produces compression and reflections in the molecules in its vicinity creating pressure that propagates through the medium. In sound waves, the physical quantity that varies as a function of position and time is the pressure exerted on the molecules of the medium. The molecules of the medium carrying the pressure move back and forth about their equilibrium positions in the direction of propagation of the pressure. The distance between two compression (*maximum pressure location*) at a given instant of time is called the wavelength of the wave.





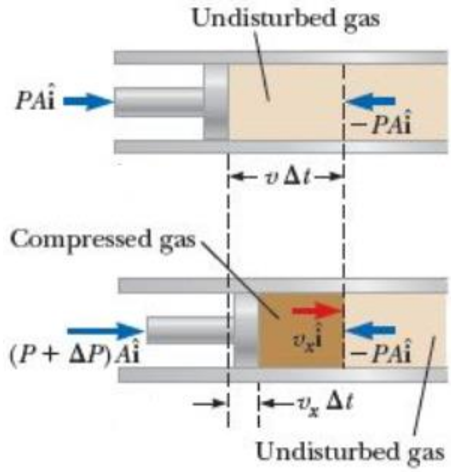
The time taken for the molecules to make one complete oscillation is called the period (T). Therefor the speed of sound (v) may be given as

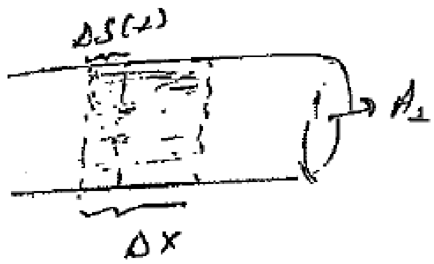
$$v = \frac{\pi}{T} - \lambda f$$

Sound waves are longitudinal wave, because the molecules move back and forth in the direction of propagation of sound.

Relationship between Displacement of the Molecules and the Pressure

The source of pressure is the difference between the displacements of neighboring of molecules (*if all the molecules displaced by the same out, it would be linear motion without additional pressure*). Let the displacement of the molecules be denoted by $s(x)$.





Consider of sample of molecules in a volume of $A_\perp \Delta x$ as shown. A change in the displacement between neighboring molecules will result in a compression change of volume of $A_\perp \Delta s(x)$. The Bulk modules (B) of the medium is defined as

$$B = -\frac{F/A}{\Delta V/V}$$

$\frac{F}{A}$ is the excess pressure ΔP corresponding to the compression. The initial volume before compression is $V = A_\perp \Delta x$ and the change in volume due to the difference in displacement between neighboring molecules is $\Delta V = As(x)$

$$\therefore B = -\frac{\Delta P}{\Delta V/V} \Rightarrow \Delta P = -B\frac{\Delta V}{V} = -B\frac{A_\perp \Delta s}{A_\perp \Delta x}$$

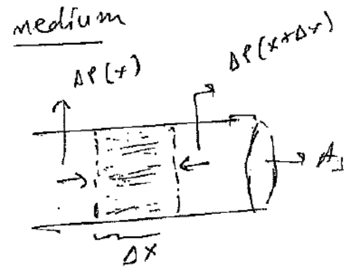
Taking the limit as Δx approaches zero gives.

$$\boxed{\Delta P = -B\frac{\partial s(x)}{\partial x}}$$

It is a partial derivative because the process considered is a constant time process.

Speed of Sound in a Medium

Again let's apply Newton's 2nd law to a small mass element Δm in a position as shown. If the identity of the medium is P, then $\Delta m = PA_{\perp}\Delta x$ from Newton's 2nd law



$$F_{net} = \Delta m \cdot a \qquad a = \frac{\partial^2 s}{\partial t^2}$$

$$F_{net} = \Delta m \frac{\partial^2 s}{\partial t^2} \Delta m = \rho A_{\perp} \Delta x$$

$$F_{net} = \rho A_{\perp} \Delta \frac{\partial^2 s}{\partial t^2}$$

The force on this mass element is caused by the pressure of both sides.

$$F_{net} = -\{\Delta P(x + \Delta x) - \Delta P(x)\}A_{\perp}$$

Where $\Delta P(x + \Delta x)$ and $\Delta P(x)$ are pressure on the left and right sides of the mass element respectively.

The negative sign is needed because the direction of the force due to $\Delta P(x + \Delta x)$ is to the left.

$$\Rightarrow -\{\Delta P(x + \Delta x) - \Delta P(x)\}A_{\perp} = \rho A_{\perp} \Delta x \frac{\partial^2 s}{\partial t^2}$$

$$\Rightarrow \left\{\frac{\Delta P(x + \Delta x) - \Delta P(x)}{\Delta x}\right\} = \rho \frac{\partial^2 s}{\partial t^2}$$

As Δx goes to zero, the left side becomes $-\frac{\partial \Delta \rho}{\partial x}$

$$\therefore -\frac{\partial}{\partial x} \Delta \rho = \rho \frac{\partial^2 s}{\partial t^2}$$

But as shown earlier $\Delta P = B \frac{\partial s}{\partial x}$

$$\Rightarrow \frac{\partial}{\partial x}\left(-B \frac{\partial s}{\partial x}\right) = \rho \frac{\partial^2 s}{\partial t^2}$$

$$\Rightarrow B \frac{\partial^2 s}{\partial x^2} = \rho \frac{\partial^2 s}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 s}{\partial x^2} = \frac{1}{B/\rho} \frac{\partial^2 s}{\partial t^2}$$

But this equation is the wave equation comparing it with the general wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$, it follows that $v^2 = \frac{B}{P}$ or

$$V = \sqrt{\frac{B}{\rho}}$$

V: speed of sound
ρ: density of the medium
B: Bulk modulus of the medium

Example

Density of water is 1,000 *kg/m*^{3}. Its bulk modulus is $0.21\times10^{10}\,Pa$. Calculate the speed of sound in water.

Solution

Given: $\rho = 1{,}000\ kg/m^3$; $B = 0.21\times10^{10}\,Pa$

$$V = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{0.21\times10^{10}}{1{,}000}} = 1.45\times10^{3}\ m/s$$

Harmonic Sound Wave

Since the displacement of the molecules (s), satisfies the wave equation (*as shown earlier*), the harmonic wave solution may be written as

$$s(x,t) = S_{max}\cos(kx - \omega t)$$

Where S_{max} is the maximum displacement of the molecules (*amplitude*) since

$$\Delta\rho = -B\frac{\partial s}{\partial x}$$

$$\Delta\rho = -B\frac{\partial}{\partial x}(s_{max}\cos(kx - \omega t))$$

$$\Delta\rho = BS_{max}R\sin(kx - \omega t)$$

Therefore the maximum value of the pressure (*amplitude*) is given by

$$\Delta\mathrm{P}_{net} = BS_{max}K$$

But $\quad k = \frac{\omega}{v}$ and $v = \sqrt{\frac{B}{\rho}}$ or $B = \rho v^2$

$$\therefore \Delta\mathrm{P}_{net} = (\rho v^2)S_{max}\frac{\omega}{v}$$

$$\Longrightarrow \Delta\mathrm{P}_{max} = \rho\omega v S_{max}$$

The amplitude of the pressure is proportionally to the density of the medium (ρ), frequency of the medium (ω), speed of sound and the amplitude of the displacement of the molecules (S_{max})

Example

Sound is travelling in a certain medium of density 2000 kg/m^3. If the displacement of the molecules vary as function of position and time according to the equation $s(x,y) = 10^{-6}\cos(0.4x - 160t)$

a) Calculate the maximum value of the pressure

b) Give a formula for the pressure as a function of position and time.

Solution

a) Given: ρ = 2,000 kg/m$^{3;}$ K= 0.4 1/m; ω = 1,600 1/s S_{max}= 10^{-6} m

$$\Delta P_{max} = \rho\omega v S_{max}$$

$$v = \frac{\omega}{K} = \frac{160}{0.4} = 4{,}000\frac{m}{s}$$

$$\therefore P_{max} = (2{,}000)(160)(4{,}000)(10^{-6}) = 1{,}280Pa$$

b) If $S(x,t) = S_{max}\cos(kx - \omega t)$ then

$\Delta P(x,t) = \Delta P_{max}\sin(kx - \omega t)$ therefore

$\Delta P(x,t) = 1{,}280P_{max}\sin(0.4x - 160t)$

The Dependence of speed of Sound in Air on Temperature

The speed of sound in air is proportional to the square root of the temperature in degree Kelvin. That is $V \propto \sqrt{T}$. Therefore if the speed at temperature T_1 and V_1 and that temperature T_2 is V_2, since

$$\frac{V}{\sqrt{T}} = constant$$

$$\frac{V_1}{\sqrt{T_1}} = \frac{V_2}{\sqrt{T_2}}$$

The speed of sound in air at 0°C is 331m/s. Therefore with v_2=331m/s and $T_2 = (0 + 273)°K = 273°$

$$\frac{V_1}{\sqrt{T_1}} = \frac{331}{\sqrt{273}}$$

Dropping the subscript which is not needed anymore

$$v = 331\sqrt{\frac{T}{273}}\frac{m}{s}$$

V: speed of sound in air
T: temperature

The temperature would probably be given in °C most of the time. In that case it should be converted to °K by adding 273 (°K = °C + 273) before using the formula.

Example

Calculate the wavelength of sound waves in air produced by a 500Hz turning fork when the temperature is 20˚C.

Solution

F=500Hz; T = 20˚C = 20 + 273 = 293˚K

$$v = 331\sqrt{\frac{T}{273}} = 331\sqrt{\frac{293}{273}} = 343\frac{m}{s}$$

$$\therefore \lambda = \frac{v}{f} = \frac{343}{500} = 0.686m$$

Intensity of Sound Waves (*Harmonic Wave*)

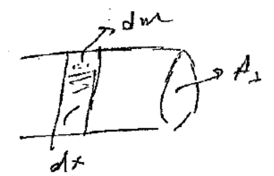
Intensity is defined to be amount of energy that crosses a unit perpendicular area per a unit time. Average intensity may be obtained as the energy that crosses a unit perpendicular area in one cycle. That is the intensity (I) may be given as

$$I = \frac{E_\lambda}{A_\perp T}$$

Where E_λ is amount of energy contained in one wavelength (*one cycle*), T is period or time taken for one cycle and $A_\perp$ the area of the cross-section through which the wave is travelling.

At a given location each molecule is oscillating like a harmonic oscillator, and its total energy is the mechanical energy of a harmonic oscillator. For a small mass element dm, the energy dE is given as

$$dE = \frac{dm}{2}\omega^2 S^2_{\max}$$



If then density of the meduim is ρ then $dm = \rho A_\perp dx$

$$dE = \frac{1}{2}\omega^2 S^2_{max} \rho A_\perp dx$$

Then the amount of energy that crosses a cross-sectional area in one cycle E_λ may be obtained by integrating this from x to $x + \lambda$

$$\therefore E_\lambda = \int_x^{x+\lambda} \frac{1}{2}\omega^2 S^2_{max}\rho A_\perp dx$$

$$= \frac{1}{2}\omega^2 S^2_{max}\rho A_\perp (x + \lambda - x)$$

$$\boxed{E_\lambda = \frac{1}{2}\omega^2 S^2_{max}\rho A_\perp \lambda}$$

And the intensity is given as

$$I = \frac{E_\lambda}{A_\perp T} = \frac{\frac{1}{2}\omega^2 S^2_{max}\rho A_\perp \lambda}{A_\perp T} = \frac{1}{2}\omega^2 S^2_{max}\rho A_\perp \left(\frac{\lambda}{T}\right)$$

But $\frac{\lambda}{T} = v$ (*speed of the wave*)

$$\boxed{I = \frac{1}{2}\rho\omega^2 S^2_{max} V}$$

I: intensity of a harmonic wave
ρ: density of medium
ω: angular frequency of the wave
S_{max}: amplitude of the displacement of the molecules
v: speed of the wave

The intensity also may be expressed in terms of amplitude of amplitude of pressure (ΔP_{max})

$$\Delta P^2_{net} = \rho \omega v S_{max}$$

$$\Delta P^2_{max} = \rho^2 \omega^2 v^2 S^2_{max}$$

$$\boxed{I = \frac{\Delta P^2_{max}}{2\rho v}}$$

Spherical waves

A very common type of sound wave is spherical wave. When sound is initiated at a certain point, it travels in all directions giving rise to a spherical wave. With transmission power (*rate of transfer of energy*) P given as $P = \frac{E}{T}$

$$I_{sp} = \frac{P}{A_\perp}$$

For a spherical wave $A_\perp$ is the surface area of a sphere of radius r, where r is the distance from the source.

$$\therefore A_\perp = 4\pi r^2$$

$$I_{sp} = \frac{\rho}{4\pi r^2}$$

I_{sp}: intensity due to a spherical wave
P: power of source
r: distance from source

The intensity of a spherical wave is inversely proportional to the square of the distance from the source.

Example

For a harmonic sound wave travelling through water, the water molecules are oscillating back and forth with a maximum displacement of $2\times10^{-7}\,m$. If conservative compression (*maximum pressure points*) are separated by a distance of 2m $\left(\rho_{h2o} = 1{,}000\frac{kg}{m^3},\ B_{h2o} = 1.2\times10^9\,Pa\right)$

a) Calculate the amount of energy that crosses a unit perpendicular cross-sectional area per a unit time.
b) Calculate the pressure in the compressions
c) Calculate the amount of energy that crosses a unit perpendicular cross-sectional area in one cycle.

Solution

a) S_{max}= 2E-7m λ= 2m I= ? P_{h2o}= 1,000 kg/m^3 B_{h2o}= 2.1E9 Pa

$$I = \frac{1}{2}\rho\omega^2 S_{max}^2 v$$

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1E9}{10E2}} = 1.45E3\ \frac{m}{s}$$

$$\omega = kv = \left(\frac{2\pi}{\lambda}\right)v = \left(\frac{2\pi}{\lambda}\right)(1.45E3) = 4.55E3\ \frac{rad}{s}$$

$$\therefore I = \frac{1}{2}\rho\omega^2 S_{max}^2 v = \frac{1}{2}(1{,}000)\left(4.55\times10^3\right)^2\left(2\times10^{-7}\right)_{max}^2\left(1.45\times10^3\right)$$

$$= 0.13\ \frac{watt}{m^2}$$

b) $I = \dfrac{\Delta P_{max}^2}{2\rho v}$

$$\Delta P_{max} = \sqrt{2\rho v I}$$

$$= \sqrt{2(1{,}000)\left(1.45\times 10^3\right)(.013)}$$

$$\boxed{= 1.94\times 10^2\ P}$$

c) $I = \dfrac{E_\lambda}{A_\perp T} = \left(\dfrac{E_\lambda}{A_\perp}\right)\dfrac{1}{T}$ $\qquad$ (*where T is the period*)

$$\frac{E_\lambda}{A_\perp} = IT$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.55\times 10^3} = 1.38\times 10^{-3}\,s$$

$$\frac{E_\lambda}{A_\perp} = IT = (.013)\left(1.33\times 10^{-3}\right) \boxed{= 1.794\times 10^{-5}\ \frac{J}{m^2}}$$

Example

A loudspeaker of power 100 watts is producing spherical waves. Calculate the intensity of the sound waves at a distance of 10m from the speaker.

Solution

Given: P = 100w; r = 10m

$$I = \frac{P}{4\pi r^2} = \frac{100}{4\pi 10^2} = \frac{1}{4\pi}\,\frac{watt}{m^2}$$

Example

The intensity due to a certain loudspeaker that produces spherical waves is found to 104 w/m^2 at a distance 4m from the speaker. Calculate the intensity at a distance of 8m.

Solution

Given: $r_1 = 4m;\quad I_1 = 10^4\,w/m^2;\quad r_2 = 8m$

Since it is the same speaker $P_1 = P_2 = P$

$$P_1 = 4\pi I_1 r_1^2 = 4\pi I_2 r_2^2 \quad\Rightarrow\quad I_1 r_1^2 = I_2 r_2^2$$

$$I_2 = \frac{I_1 r_1^2}{r_2^2} = \frac{10^4(4)^2}{8^2} \boxed{= 2{,}500\ \frac{w}{m^2}}$$

Relationships Between Properties of Sound and Human Sensation

The loudness of sound is related with the intensity of the sound. That is the greater the intensity the greater the sound. But they are not related linearly but logarithmically. Loudness (β) in decibels is related with the intensity of sound as

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

β: loudness in decibels
I: intensity of sound
I_0: 10^{-12} w/m^2 (*lowest intensity that can be detected by the human ear*)

Where I_0 is the minimum intensity of sound that can be detected by the human ear. Its value is 10^{-12} w/m^2. By the way, the maximum intensity that can be tolerated by the human ear is 1 w/m^2.
The pitch of sound is related with the frequency of sound. The greater the frequency the greater the pitch.

Brief Review of Logarithms

A logarithmic function is an inverse of an expanded function of

$$y = a^x \; then \; x = \log_a y$$

$\log_a^y$ is read as logarithm of y to the base a. The default base is 10 and is called common logarithm. That is if the base is not included it is taken to be 10. The following are some rules of logarithms.

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a \frac{y}{x} = \log_a x - \log_a y$$

$$\log_a x^b = b\log_a x$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

Logarithms to the base e (*natural numbers*) are customarily denoted as in i.e.

$$\log_e x = lnx$$

That is if $y = e^x$ then x = lny

(*remember e*=2.71828 *and* $\frac{de^x}{dt} = e^x$)

Example

Consider spherical waves produced by a 50 watt speaker.

a) Calculate the loudness in decibels at a distance of 10m from the speaker.
b) At what distance from the speaker would the loudness of the sound be 80db?

<u>Solution</u>

a) p= 50w r= ? β= ?

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

$$I_0 = 10^{-12}\frac{w}{m^2}$$

$$I = \frac{p}{4\pi r^2} = \frac{50}{4\pi 10^2} \approx 4E-2\ \frac{w}{m^2}$$

$$\beta = 10\log\left(\frac{4E-2}{1E-12}\right) = 106db$$

b) β= 80db   r= ?   p= 50w

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

$$80 = 10\log\left(\frac{I}{I_0}\right)$$

$$\log\left(\frac{I}{I_0}\right) = 8$$

$$\frac{I}{I_0} = 10^8$$

(*remember the default base is 10*)

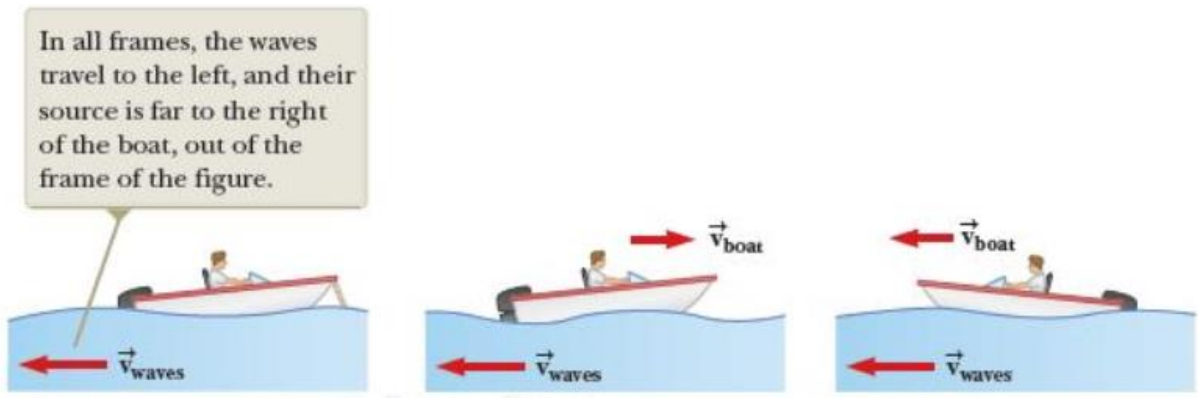
$$I = (10^8)(10^{-12}) = 10^{-4}\frac{v}{m^2}$$

$$I = \frac{p}{4\pi r^2}$$

$$r = \sqrt{\frac{p}{4\pi I}} = \sqrt{\frac{50}{4\pi(10)^{-4}}} = 199m$$

Doppler's Effect

Doppler's effect refers to the change in frequency (*pitch*) of the sound heard by an observer due to the speed of the observer or the source with respect to the medium carrying the sound wave which is air in this case. (*speed with respect to air basically means speed with respect to the ground*). It is a common experience that the pitch of a sound heard by an observer due to the horn of a car moving towards the observer increases (*and decreases when the car moves away from the observer.*)

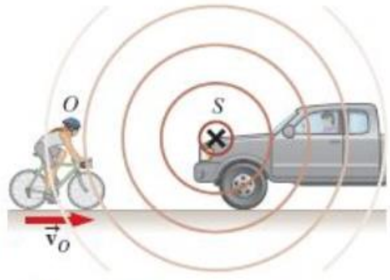


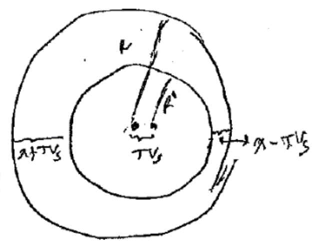
Let f be the frequency of the sound produced by the source, f' be the modified frequency heard by the observer and let v, v_o and v_s be the speeds of sound, observer and source respectively with respect to the medium carrying the sound wave (*air*). And further that $v_o(V_s)$ is positive when the observer (*source*) is moving in the direction of the sound heard by the observer and negative when the observer (*source*) is moving opposite to the direction of the sound heard by the observer.
There are 2 sources for Doppler's effect: the motion of observer and the motion of source. Let's first deal with them separately and then we will combine both effects.
If the source is stationary and observer is moving the speed with which sound is approaching (*going away from*) changes. Actually the new speed will become $v-v_o$. Remember according to any sign convention, v_o is negative if the observer is approaching the source and positive if going away from the source. The wavelength of the sound wave (*distance between consecutive peaks*) remains the same because it does not depend on the motion of the observer. Therefore it follows that $f=v/\lambda$ and $f'=(v-v_o)/\lambda$. Substituting for λ in the expression for f_1' we get

$$f' = f\left(\frac{v - v_0}{v}\right)$$

(*source stationary v_s= 0 and observer moving*)
f'= modified frequency
f= original frequency
v= speed of sound
v_0= speed of observer

If source is moving and observer is stationary the speed with which sound waves arrive the observer doesn't change (*the speed of sound depends on the medium and not on the of the source*) but the distance between consecutive peaks (*wavelength*) changes because the center from which the spherical sound waves spread is moving with the source. Since the center of the spherical wave moves a distance of v_s= v_s/f is one period (T), the wavelength will change from λ to λ-v_s/f. (*Remember according to our sign can reaction, v_s will be positive when source approaches observer and negative when source goes away from observer*).





Therefore f= v/λ and f̀= v/(λ-v_s/f). Substituting for λ in the expression for f̀, it follows that

$$f' = f\left(\frac{v}{v - v_s}\right)$$

(*source moving and observer stationary v_0=0*)

If both observer and source are moving, both effects can be combined as follows. Let f`` be the modified frequency due to observer moving from and source stationary. Then f``= f((v-v_0)/v). Now suppose observer is kept stationary and source moving. Then f̀= f``(v/(v-$v_s$)). Substituting for f``, the general expression combing both effects is obtained

$$f^{`} = f\left(\frac{v - v_0}{v - v_s}\right)$$

- $\grave{f}$ = modified frequency
- f_0 = original frequency
- v = speed of sound with respect to medium
- v_0 = speed of observed with respect to medium
- v_s = speed of source with respect to medium

(*Remember the above two formulas are special cases of this formula corresponding to v_s= 0 and v_0= 0 respectively*.)

Example

A stationary car is blowing its horn with a frequency of 1,000 Hz. Calculate the frequency of the sound heard by a cyclist (*when temperature is 20°C*)

a) When the cyclist is travelling towards the car with a speed of 10 m/s
b) When the cyclist is travelling away from the car with a speed of 10 m/s

Solution

a) *f*= 1,000 Hz v_s= 0

v_O= −10m/s (*negative because he is moving opposite to the direction of the sound received*)
T = 20° = 293°K

$$v = 331\sqrt{\frac{T}{293}} = \sqrt{\frac{293}{273}} = 343\frac{m}{s}$$

$$f' = f\left(\frac{v - v_0}{v - s}\right) = 1{,}000\left(\frac{343 - (10)}{343 - 0}\right) = 1{,}000\left(\frac{353}{343}\right) = 1{,}029Hz$$

b) f = 1,000 m/s; v_s= 0

v_o= 10 m/s (*positive because the direction of the sound received*)

$$f' = f\left(\frac{v - v_o}{v - v_s}\right) = 1{,}000\left(\frac{343 - 10}{343 - 0}\right) = 971\frac{m}{s}$$

Example

Calculate the frequency of the sound heard by a stationary observer (*at a temperature of 25°C)* due to a 200 Hz sound produced by a car.

a) When the car is travelling towards the observer with a speed of 40 m/s

b) When the car is travelling away from the observer with a speed of 40 m/s

Solution

a) f = 500 Hz v_O = 0 t = 25°C = 298°K

v_S= 4= m/s (*positive because the car is travelling in the direction of the sound received by the observer*)

$$v = \sqrt{\frac{T}{273}} = 331\sqrt{\frac{298}{273}} = 346\frac{m}{s}$$

$$f' = f\left(\frac{v - v_o}{v - v_s}\right) = 500\left(\frac{346 - 0}{346 - 40}\right) = 565\frac{m}{s}$$

b) f = 500 Hz v_O = 0 v = 346 m/s

v_S = –40 m/s (*negative because the car is moving opposite to the direction of the sound received by the observer*)

$$f' = f\left(\frac{v - v_o}{v - v_s}\right) = 500\left(\frac{346 - 0}{346 - (-40)}\right) = 448Hz$$

Example

A cyclist and a car are travelling in the same direction. The car is blowing its horn at a frequency of 800Hz. The cyclist is travelling at a speed of 10 m/s. The car is travelling at a speed of 40 m/s/ (*Assume* temperature is 20°*C*). Calculate the frequency of the sound heard by the cyclist?

a) When the car is behind the cyclist.

b) When the car overtakes him and is travelling in front of him.

Solution

a) f = 800 Hz

v_O = 10 m/s (*positive because he is moving in the direction of the sound received*)

v_S = 40 m/s (*positive because the car is moving in the direction of the sound received*)

T = 20°C =293°K

$$v = 331\sqrt{\frac{T}{273}} = 331\sqrt{\frac{293}{273}} = 343\frac{m}{s}$$

$$f' = f\left(\frac{v - v_o}{v - v_s}\right) = 800\left(\frac{343 - 10}{343 - 40}\right) = 879Hz$$

b) *f* = 800 Hz

v_O = –10 m/s (*negative because he is moving opposite to the direction of sound receives*)

v_S = –40m/s (*negative because the car is moving opposite to the direction of sound received*)

$$f' = f\left(\frac{v - v_o}{v - v_s}\right) = 800\left(\frac{343 - (-10)}{343 - (-40)}\right) = 737Hz$$