Solving Exponential Function with different bases

$$a^{mx+n} = b^{px+q} \quad \Rightarrow \quad x = \frac{q\ln b - n\ln a}{m\ln a - p\ln b} \qquad \text{coefficient} \frac{no\ x's}{x's}$$

Numerator: multiply q with $\ln b$ ***minus*** multiply n with $\ln a$
Denominator: multiply m with $\ln a$ ***minus*** multiply p with $\ln b$

Proof

$$\ln a^{mx+n} = \ln b^{px+q}$$

$$(mx+n)\ln a = (px+q)\ln b$$

$$mx\ln a + n\ln a = px\ln b + q\ln b$$

$$mx\ln a - px\ln b = q\ln b - n\ln a$$

$$x(m\ln a - p\ln b) = q\ln b - n\ln a$$

$$\underline{x = \frac{q\ln b - n\ln a}{m\ln a - p\ln b}}\Bigg|$$

$$mx\ln a + n\ln a = px\ln b + q\ln b$$

Example

Solve: $3^{2x-1} = 7^{x+1}$

Solution

$$\underline{x = \frac{\ln 3 + \ln 7}{2\ln 3 - \ln 7}}\Bigg|$$

Example

Solve: $4^{x+3} = 3^{-x}$

Solution

$$\underline{x = \frac{-3\ln 4}{\ln 4 + \ln 3}}\Bigg|$$

$$\ln 4^{\boxed{1}x+3} \quad -\ln 3^{\boxed{-1}x} \quad -\ln$$

Example

Solve: $4^{-x} = 3^{x+3}$

Solution

$$\underline{x = \frac{3\ln 3}{\ln 3 - \ln 4}}\Bigg|$$

$$\ln 4^{\boxed{-1}x} \quad -\ln 3^{\boxed{1}x+3} \quad \ln$$