

Section 2.5 – Counting Techniques

Fundamental Counting Rule

For a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of $m \cdot n$ ways.

Example

It's wise not to disclose social security numbers, because they are often used by criminals attempting identity theft. Assume that a criminal is found using your social security number and claims that all of the digits were randomly generated. What is the probability of getting your social security number when randomly generated 9 digits? Is the criminal's claim that your number was randomly generated likely to be true?

Solution

Each of the 9 digits has 10 possible outcomes: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

$$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 1,000,000,000$$

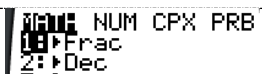
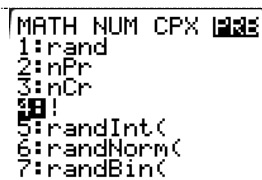


Notation

The **factorial symbol** (!) denotes the product of decreasing positive whole numbers.

For example,

$$0! = 1$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

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Factorial Rule

A collection of n different items can be arranged in order $n!$ different ways. (This factorial rule reflects the fact that the first item may be selected in n different ways, the second item may be selected in $n - 1$ ways, and so on.)

Example

During the summer, you are planning to visit these 6 national parks: Glacier, Yellowstone, Yosemite, Arches, Zion, and Grand Canyon. You would like to plan the most efficient route and you decide to list all of the possible routes. How many different routes are possible?

Solution

There 6 different parks can be arranged in order $6!$ different ways.

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \underline{720 \text{ ways}}$$

Permutations Rule (when items are all different)

Requirements:

1. There are n **different** items available. (This rule does not apply if some of the items are identical to others.)
2. We select r of the n items (without replacement).
3. We consider rearrangements of the same items to be different sequences. (The permutation of ABC is different from CBA and is counted separately.)

If the preceding requirements are satisfied, the number of permutations (or sequences) of r items selected from n available items (without replacement) is

$${}_nP_r = \frac{n!}{(n-r)!}$$

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Example

In horse racing, a bet on an exacta in a race is won by correctly selecting the horses that finish first and second, and you must select those 2 horses in the correct order. The 132nd running of the Kentucky Derby has a field of 20 horses. If a bettor randomly selects 2 of those horses for an exacta bet, what is the probability of winning?

Solution

We have $n = 20$ horses and we must select $r = 2$ of them without replacement.

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Permutations Rule (when some items are identical to others)

Requirements:

1. There are n items available, and some items are identical to others.
2. We select all of the n items (without replacement).
3. We consider rearrangements of distinct items to be different sequences.

If the preceding requirements are satisfied, and if there are n_1 alike, n_2 alike, \dots , n_k alike, the number of permutations (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}$$

Example

In a preliminary test of the MicroSort gender selection method developed by the Genetics and IVF Institute, 14 couples tried to have baby girls. Analysis of the effectiveness of the MicroSort method is based on a probability value, which in turn is based on numbers of permutations. Let's consider this simple problem: How many ways can 11 girls and 3 boys be arranged in sequence? That is, find the number of permutations of 11 girls and 3 boys.

Solution

$$n = 14; \quad n_1 = 11; \quad n_2 = 3$$

$$\begin{aligned} \frac{n!}{n_1! \cdot n_2!} &= \frac{14!}{11! \cdot 3!} \\ &= \frac{12 \cdot 13 \cdot 14}{1 \cdot 2 \cdot 3} \\ &= 364 \end{aligned}$$

There are 364 different ways to arrange 11 girls and 3 boys.

Example

In how many ways can the letters in the word *Mississippi* be arranged?

Solution

$$\frac{11!}{1!4!4!2!} = 34,650 \text{ ways}$$

<i>m</i>	<i>i</i>	<i>s</i>	<i>p</i>
1	4	4	2

Combinations Rule

Requirements:

1. There are n different items available.
2. We select r of the n items (without replacement).
3. We consider rearrangements of the same items to be the same. (The combination of ABC is the same as CBA.)

If the preceding requirements are satisfied, the number of combinations of r items selected from n different items is

$${}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

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	1: Frac	1: rand
	2: Dec	2: nPr
	3: 3	3: nCr
	4: 3J(4: 1
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	6: fMin(6: randNorm(
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Permutations versus Combinations

When different orderings of the same items are to be counted separately, we have a permutation problem, but when different orderings are not to be counted separately, we have a combination problem.

Permutation: *order matter.*

Combination: *Order doesn't matter.*

Examples

For each problem, tell whether permutations or combinations should be used to solve the problem.

- a) How many 4-digit code numbers are possible if no digits are repeated?

Permutation

- b) A sample of 3 light bulbs is randomly selected from batch of 15. How many different samples are possible?

Combination

- c) In a baseball conference with 8 teams, how many games must be played so that each team plays every other team exactly once?

Combination

- d) In how many ways can 4 patients be assigned to 6 different hospital rooms so that each patient has a private room?

Permutation

Example

A clinical test on humans of a new drug is normally done in 3 phases. Phase I is conducted with a relatively small number of healthy volunteers. Let's assume that we want to treat 8 healthy humans with a new drug, and we have 10 suitable volunteers available.

- a) If the subjects are selected and treated in sequence, so that the trial is discontinued if anyone displays adverse effects, how many different sequential arrangements are possible if 8 people are selected from the 10 that are available?
- b) If 8 subjects are selected from the 10 that are available, and the 8 selected subjects are all treated at the same time, how many different treatment groups are possible?

Solution

- a) Because order does count, we want the number of permutations of $r = 8$ people selected from the $n = 10$.

$${}_{10}P_8 = 1,814,400$$

- b) Because order does **not** count, we want the number of combinations of $r = 8$ people selected from the $n = 10$.

$${}_{10}C_8 = 45$$

Example

The Florida Lotto game is typical of state lotteries. You must select 6 different numbers between 1 and 53. You win the jackpot if the same 6 numbers are drawn in any order. Find the probability of winning the jackpot.

Solution

Because order does **not** count, we want the number of combinations of $r = 6$ people selected from the $n = 53$

$${}_{53}C_6 = 22,957,480$$

With 1 winning combination and 22,957,480 different possible combinations, the probability of winning the jackpot is

$$P = \frac{1}{22,957,480}$$

Exercises Section 2.5 – Counting Techniques

1. Decide whether the situation involves *permutations* or *combinations*
 - a) A batting order for 9 players for a baseball game
 - b) An arrangement of 8 people for a picture
 - c) A committee of 7 delegates chosen from a class of 30 students to bring a petition to the administration
 - d) A selection of a chairman and a secretary from a committee of 14 people
 - e) A sample of 5 items taken from 71 items on an assembly line
 - f) A blend of 3 spices taken from 7 spices on a spice rack
 - g) From the 7 male and 10 female sales representatives for an insurance company, team of 8 will be selected to attend a national conference on insurance fraud.
 - h) Marbles are being drawn without replacement from a bag containing 15 marbles.
 - i) The new university president named 3 new officers a vice-president of finance, a vice-president of academic affairs, and a vice-president of student affairs.
 - j) A student checked out 4 novels from the library to read over the holiday.
 - k) A father ordered an ice cream cone (chocolate, vanilla, or strawberry) for each of his 4 children.
2. Find the number of different ways that five test questions can be arranged in order by evaluating $5!$
3. In the game of blackjack played with one deck, a player is initially dealt 2 cards. Find the number of different two-card initial hands by evaluating ${}_{52}C_2$
4. A political strategist must visit state capitols, but she has time to visit only 3 of them. Find the number of different possible routes by evaluating ${}_{50}P_3$
5. Select the six winning numbers from 1, 2, ..., 54. Find the probability of winning lottery by buying one ticket. $\left(\text{of winning this lottery } \frac{1}{575,757} \right)$
6. Select the five winning numbers from 1, 2, ..., 36. Find the probability of winning lottery by buying one ticket. $\left(\text{of winning this lottery } \frac{1}{575,757} \right)$
7. In a club with 9 male and 11 female members, how many 5-member committees can be chosen that have
 - a) All men?
 - b) All women?
 - c) 3 men and 2 women?
8. In a club with 9 male and 11 female members, how many 5-member committees can be selected that have
 - a) At least 4 women?
 - b) No more than 2 men?
9. In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?

10. From a pool of 8 secretaries, 3 are selected to be assigned to 3 managers, one per manager. In how many ways can they be selected and assigned?
11. A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.
 - a) How many delegations are possible?
 - b) If it is decided that a particular worker must be in the delegation, how many different delegations are possible?
 - c) If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?
12. Hamburger Hut sells regular hamburgers as well as a larger burger. Either type can include cheese, relish, lettuce, tomato, mustard, or catsup.
 - a) How many different hamburgers can be ordered with exactly three extras?
 - b) How many different regular hamburgers can be ordered with exactly three extras?
 - c) How many different regular hamburgers can be ordered with at least five extras?
13. In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.
 - a) In how many ways can this be done?
 - b) In how many ways can this be done if exactly 2 wheat plants must be included?
14. A legislative committee consists of 5 Democrats and 4 Republicans. A delegation of 3 is to be selected to visit a small Pacific island republic.
 - a) How many different delegations are possible?
 - b) How many delegations would have all Democrats?
 - c) How many delegations would have 2 Democrats and 1 Republican?
 - d) How many delegations would have at least 1 Republican?
15. Five cards are chosen from an ordinary deck to form a hand in poker. In how many ways is it possible to get the following results?

a) 4 queens	c) Exactly 2 face cards	e) 1 heart, 2 diamonds, and 2 clubs
b) No face card	d) At least 2 face cards	
16. The student sitting next to you in class concludes that the probability of the ceiling falling down on both of you before class ends is $1/2$, because there are two possible outcomes - the ceiling will fall or not fall. What is wrong with this reasoning?
17. Identity theft often begins by someone discovering your 9-digit social security number. Answer each of the following. Express probabilities as fractions.
 - a) What is the probability of randomly generating 9 digits and getting your social security number?
 - b) In the past, many teachers posted grades along with the last 4 digits of your social security number, what is the probability that if they randomly generated the order digits, they would match yours? Is that something to worry about?

18. Credit card numbers typically have 16 digits, but not all of them are random. Answer the following and express probabilities as fractions.
- What is the probability of randomly generating 16 digits and getting your MasterCard number?
 - Receipts often show the last 4 digits of a credit card number. If those last 4 digits are known, what is the probability of randomly generating the order digits of your MasterCard number?
 - Discover cards begin with the digits 6011. If you also know the last 4 digits, what is the probability of randomly generating the other digits and getting all of them correct? Is this something to worry about?
19. When testing for current in a cable with five color-coded wires, the author used a meter to test two wires at a time. How many different tests are required for every possible pairing of two wires?
20. The starting 4 players for the Boston Celtics basketball team have agreed to make charity appearances tomorrow night. If you must send three players to the United Way event and the other 2 to a Heart Fund event, how many different ways can you make the assignments?
21. In phase I of a clinical trial with gene therapy used for treating HIV, 5 subjects were treated (based on data from Medical News Today). If 20 people were eligible for the Phase I treatment and a simple random of 5 is selected, how many different simple random samples are possible? What is the probability of each simple random sample?
22. Many newspapers carry “Jumble” a puzzle in which the reader must unscramble letters to form words. The letters BUJOM were included in newspapers. How many ways can the letters if BUJOM be arranged? Identify the correct unscrambling, then determine the probability of getting that result by randomly selecting one arrangement of the given letters.
23. There are 11 members on the board of directors for the Coca Cola Company.
- If they must select a chairperson, first vice chairperson, second vice chairperson, and secretary, how many different slates of candidates are possible?
 - If they must form an ethics subcommittee of 4 members, how many different subcommittees are possible?
24. The author owns a safe in which he stores his book. The safe combination consists of 4 numbers between 0 and 99. If another author breaks in and tries to steal this book, what is the probability that he or she will get the correct combination on the first attempt? Assume that the numbers are randomly selected. Given the number of possibilities, does it seem feasible to try opening the safe by making random guesses for the combination?
25. In a preliminary test of the MicroSort gender selection method, 14 babies were born and 13 of them were girls
- Find the number of different possible sequences of genders that are possible when 14 babies are born.
 - How many ways can 13 girls and 1 boy be arranged in a sequence?
 - If 14 babies are randomly selected, what is the probability that they consist of 13 girls and 1 boy?

- d) Does the gender-selection method appear to yield a result that is significantly different from a result that might be expected by random chance?
- 26.** You become suspicious when a genetics researcher randomly selects groups of 20 newborn babies and seems to consistently get 10 girls and 10 boys. The researcher's claim is that it is common to get 10 girls and 10 boys in such cases,
- If 20 newborn babies are randomly selected, how many different gender sequences are possible.
 - How many different ways can 10 girls and 10 boys be arranged in sequence?
 - What is the probability of getting 10 girls and 10 boys when 10 babies are born?
 - Based on the preceding results, do you agree with the researcher's explanation that it is common to get 10 girls and 10 boys when 20 babies are randomly selected?
- 27.** The Powerball lottery is run in 29 states. Winning the jackpot requires that you select the correct five numbers between 1 and 55 and, in a separate drawing, you must also select the correct single number between 1 and 42. Find the probability of winning the jackpot.
- 28.** The Mega Millions lottery is run in 12 states. Winning the jackpot requires that you select the correct 5 numbers between 1 and 56 and, in a separate drawing, you must also select the correct single number between 1 and 46. Find the probability of winning the jackpot.
- 29.** A state lottery involves the random selection of six different numbers between 1 and 31. If you select one six number combination, what is the probability that it will be the winning combination?
- 30.** How many ways can 6 people be chosen and arranged in a straight line if there are 8 people to choose from?
- 31.** 12 wrestlers compete in a competition. If each wrestler wrestles one match with each other wrestler, what are the total numbers of matches?
- 32.** Wing has different books to arrange on a shelf: 4 blue, 3 green, and 2 red.
- In how many ways can the books be arranged on a shelf?
 - If books of the same color are to be grouped together, how many arrangements are possible?
 - In how many distinguishable ways can the books be arranged if books of the same color are identical but need not be grouped together?
 - In how many ways can you select 3 books, one of each color, if the order in which the books are selected does not matter?
 - In how many ways can you select 3 books, one of each color, if the order in which the books are selected matters?
- 33.** A child has a set of differently shaped plastic objects. There are 3 pyramids, 4 cubes, and 7 spheres.
- In how many ways can she arrange the objects in a row if each is a different color?
 - How many arrangements are possible if objects of the same shape must be grouped together and each object is a different color?

- c) In how many distinguishable ways can the objects be arranged in a row if objects of the same shape are also the same color, but need not be grouped together?
 - d) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected does not matter and each object is a different color?
 - e) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected matters and each object is a different color?
34. Twelve drugs have been found to be effective in the treatment of a disease. It is believed that the sequence in which the drugs are administered is important in the effectiveness of the treatment. In how many different sequences can 5 of the 12 drugs be administered?
35. In a club with 16 members, how many ways can a slate of 3 officers consisting of president, vice-president, and secretary/treasurer be chosen?
36. In how many ways can 7 of 11 monkeys be arranged in a row for a genetics experiment?
37. In an experiment on social interaction, 6 people will sit in 6 seats in a row. In how many ways can this be done?
38. In an election with 3 candidates for one office and 6 candidates for another office, how many different ballots may be printed?
39. A business school gives courses in typing, shorthand, transcription, business English, technical writing, and accounting. In how many ways can a student arrange a schedule if 3 courses are taken? Assume that the order in which courses are scheduled matters.
40. If your college offers 400 courses, 25 of which are in mathematics, and your counselor arranges your schedule of 4 courses by random selection, how many schedules are possible that do not include a math course? Assume that the order in which courses are scheduled matters.
41. A baseball team has 19 players. How many 9-player batting orders are possible?
42. A chapter of union Local 715 has 35 members. In how many different ways can the chapter select a president, a vice-president, a treasurer, and a secretary?
43. A concert to raise money for an economics prize is to consist of 5 works; 2 overtures, 2 sonatas, and a piano concerto.
- a) In how many ways can the program be arranged?
 - b) In how many ways can the program be arranged if an overture must come first?
44. A zydeco band from Louisiana will play 5 traditional and 3 original Cajun compositions at a concert. In how many ways can they arrange the program if
- a) it begins with a traditional piece?
 - b) An original piece will be played last?
45. Given the set $\{A, B, C, D\}$, how many permutations are there of this set of 4 object taken 2 at a time?
- a) Using the multiplication principle
 - b) Using the Permutation

46. Find the number of permutations of 30 objects taken 4 at a time.
47. Five cards are marked with the numbers 1, 2, 3, 4, and 5, then shuffled, and 2 cards are drawn.
- How many different 2-card combinations are possible?
 - How many 2-card hands contain a number less than 3?
48. An economics club has 31 members.
- If a committee of 4 is to be selected, in how many ways can the selection be made?
 - In how many ways can a committee of at least 1 and at most 3 be selected?
49. Use a tree diagram for the following
- Find the number of ways 2 letters can be chosen from the set $\{L, M, N\}$ if order is important and repetition is allowed.
 - Reconsider part a if no repeats are allowed
 - Find the number of combinations of 3 elements taken 2 at a time. Does this answer differ from part a or b?

For each problem, decide whether permutations or combinations should be used to solve the problem.

50. In a club with 9 male and 11 female members, how many 5-member committees can be chosen that have
- All men?
 - All women?
 - 3 men and 2 women?
51. In a club with 9 male and 11 female members, how many 5-member committees can be selected that have
- At least 4 women?
 - No more than 2 men?
52. In a game of musical chairs, 12 children will sit in 11 chairs arranged in a row (one will be left out). In how many ways can this happen, if we count rearrangements of the children in the chairs as different outcomes?
53. A group of 3 students is to be selected from a group of 14 students to take part in a class in cell biology.
- In how many ways can this be done?
 - In how many ways can the group who will not take part be chosen?
54. Marbles are being drawn without replacement from a bag containing 16 marbles.
- How many samples of 2 marbles can be drawn?
 - How many samples of 2 marbles can be drawn?
 - If the bag contains 3 yellow, 4 white, and 9 blue marbles, how many samples of 2 marbles can be drawn in which both marbles are blue?
55. There are 7 rotten apples in a crate of 26 apples
- How many samples of 3 apples can be drawn from the crate?
 - How many samples of 3 could be drawn in which all 3 are rotten?
 - How many samples of 3 could be drawn in which there are two good apples and one rotten one?

- 56.** A bag contains 5 black, 1 red, and 3 yellow jelly beans; you take 3 at random. How many samples are possible in which the jelly beans are
- | | | |
|-----------------------|---------------------------------|---------------------------------|
| <i>a)</i> All black? | <i>d)</i> 2 black and 1 red? | <i>f)</i> 2 yellow and 1 black? |
| <i>b)</i> All red? | <i>e)</i> 2 black and 1 yellow? | <i>g)</i> 2 red and 1 yellow? |
| <i>c)</i> All yellow? | | |
- 57.** In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?
- 58.** From a pool of 8 secretaries, 3 are selected to be assigned to 3 managers, one per manager. In how many ways can they be selected and assigned?
- 59.** A salesperson has the names of 6 prospects.
- In how many ways can she arrange her schedule if she calls on all 6?
 - In how many ways can she arrange her schedule if she can call on only 4 of the 6?
- 60.** A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.
- How many delegations are possible?
 - If it is decided that a particular worker must be in the delegation, how many different delegations are possible?
 - If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?
- 61.** Hamburger Hut sells regular hamburgers as well as a larger burger. Either type can include cheese, relish, lettuce, tomato, mustard, or catsup.
- How many different hamburgers can be ordered with exactly three extras?
 - How many different regular hamburgers can be ordered with exactly three extras?
 - How many different regular hamburgers can be ordered with at least five extras?
- 62.** Five items are to be randomly selected from the first 50 items on an assembly line to determine the defect rate. How many different samples of 5 items can be chosen?
- 63.** From a group of 16 smokers and 22 nonsmokers, a researcher wants to randomly select 8 smokers and 8 nonsmokers for a study. In how many ways can the study group be selected?
- 64.** In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.
- In how many ways can this be done?
 - In how many ways can this be done if exactly 2 wheat plants must be included?
- 65.** A legislative committee consists of 5 Democrats and 4 Republicans. A delegation of 3 is to be selected to visit a small Pacific island republic.
- How many different delegations are possible?
 - How many delegations would have all Democrats?
 - How many delegations would have 2 Democrats and 1 Republican?
 - How many delegations would have at least 1 Republican?

66. From 10 names on a ballot, 4 will be elected to a political party committee. In how many ways can the committee of 4 be formed if each person will have a different responsibility, and different assignments of responsibility are considered different committees?
67. How many different 13-card bridge hands can be selected from an ordinary deck?
68. Five cards are chosen from an ordinary deck to form a hand in poker. In how many ways is it possible to get the following results?
- f) 4 queens
 - g) No face card
 - h) Exactly 2 face cards
 - i) At least 2 face cards
 - j) 1 heart, 2 diamonds, and 2 clubs
69. In poker, a flush consists of 5 cards with the same suit, such as 5 diamonds.
- a) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by listing all the possibilities.
 - b) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by using combinations
70. If a baseball coach has 5 good hitters and 4 poor hitters on the bench and chooses 3 players at random, in how many ways can he choose at least 2 good hitters?
71. The coach of a softball team has 6 good hitters and 8 poor hitters. He chooses 3 hitters at random.
- a) In how many ways can he choose 2 good hitters and 1 poor hitter?
 - b) In how many ways can he choose 3 good hitters?
 - c) In how many ways can he choose at least 3 good hitters?
72. How many 5 card hands will have 3 aces and 2 kings?
73. How many 5 card hands will have 3 hearts and 2 spades?
74. 2 letters follow by 3 numbers; 2 letters out of 8 & 3 numbers out of 10
75. Serial numbers for a product are to be made using 3 letters followed by 2 digits (0 – 9 no repeats). If the letters are to be taken from the first 8 letters of the alphabet with no repeats, how many serial numbers are possible?
76. A company has 7 senior and 5 junior officers. An ad hoc legislative committee is to be formed.
- a) How many 4-officer committees with 1 senior officer and 3 junior officers can be formed?
 - b) How many 4-officer committees with 4 junior officers can be formed?
 - c) How many 4-officer committees with at least 2 junior officers can be formed?
77. From a committee of 12 people,
- a) In how many ways can we choose a chairperson, a vice-chairperson, a secretary, and a treasurer, assuming that one person can't hold more than one position
 - b) In how many ways can we choose a subcommittee of 4 people?
78. Find the number of combinations of 30 objects taken 4 at a time.

79. How many different permutations are there of the set $\{a, b, c, d, e, f, g\}$?
80. How many permutations of $\{a, b, c, d, e, f, g\}$ end with a ?
81. Find the number of 5-permutations of a set with nine elements
82. In how many different orders can five runners finish a race if no ties are allowed?
83. A coin flipped eight times where each flip comes up either heads or tails. How many possible outcomes
- a) Are there in total?
 - b) Contain exactly three heads?
 - c) Contain at least three heads?
 - d) Contain the same number of heads and tails?
84. In how many ways can a set of two positive integers less than 100 be chosen?
85. In how many ways can a set of five letters be selected from the English alphabet?