

Section 2.2 – Function Operations

The *Domain* of a Function

1. **Rational** function: $\frac{f(x)}{h(x)} \Rightarrow \text{Domain: } \boxed{h(x) \neq 0}$

Example: $f(x) = \frac{1}{x-3}$

Domain: $x \neq 3 \mid \{x \mid x \neq 3\}$

Or $(-\infty, 3) \cup (3, \infty)$ *Interval Notation*

Or $\mathbb{R} - \{3\}$

2. **Irrational** function: $\sqrt{g(x)} \Rightarrow \text{Domain: } \boxed{g(x) \geq 0}$

Example: $g(x) = \sqrt{3-x} + 5$

$$3 - x \geq 0$$

$$-x \geq -3$$

Domain: $x < 3 \mid (-\infty, 3]$

3. **Otherwise:** Domain all real numbers $(-\infty, \infty)$

Example: $f(x) = x^3 + |x|$

Domain: All real numbers $\mathbb{R} \mid (-\infty, \infty)$

(1) & (2) \rightarrow Find the domain: $f(x) = \frac{x+1}{\sqrt{x-3}}$

$$x > 3$$

Domain: $(3, \infty)$

Example

Find the domain

a) $f(x) = x^2 + 3x - 17$

Domain: \mathbb{R}

b) $g(x) = \frac{5x}{x^2 - 49}$

$$x^2 \neq 49$$

$$\underline{x \neq \pm 7}$$

Domain: $\begin{cases} \{x \mid x \neq \pm 7\} \\ (-\infty, -7) \cup (-7, 7) \cup (7, \infty) \end{cases}$ **or**

c) $h(x) = \sqrt{9x - 27}$

$$9x - 27 \geq 0$$

$$9x \geq 27$$

Domain: $\underline{x \geq 3}$ $[3, \infty)$

The *Algebra* of Functions

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Example

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find each of the following $(f + g)(1)$, $(f - g)(-3)$, $(fg)(5)$, and $\left(\frac{f}{g}\right)(0)$

Solution

$$\begin{aligned}(f + g)(1) &= f(1) + g(1) \\ &= 1^2 + 1 + 3(1) + 5 \\ &= 1 + 1 + 3 + 5 \\ &= 10\end{aligned}$$

$$\begin{aligned}(f - g)(-3) &= f(-3) - g(-3) \\ &= (-3)^2 + 1 - (3(-3) + 5) \\ &= 14\end{aligned}$$

$$\begin{aligned}(fg)(5) &= f(5) \cdot g(5) \\ &= (5^2 + 1) \cdot (3(5) + 5) \\ &= (26) \cdot (20) \\ &= 520\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(0) &= \frac{f(0)}{g(0)} \\ &= \frac{0^2 + 1}{3(0) + 5} \\ &= \frac{1}{5}\end{aligned}$$

Example

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x - 1}$. Find each of the following and give the domain

$$(f + g)(x), \quad (f - g)(x), \quad (fg)(x), \quad \left(\frac{f}{g}\right)(x)$$

Solution

Domain of f : $(-\infty, \infty)$

Domain of g : $\left[\frac{1}{2}, \infty\right)$

$$\sqrt{2x - 1} \geq 0 \rightarrow 2x \geq 1 \Rightarrow x \geq \frac{1}{2}$$

a) $(f + g)(x) = 8x - 9 + \sqrt{2x - 1}$

Domain: $x \geq \frac{1}{2} \mid \left[\frac{1}{2}, \infty\right)$

b) $(f - g)(x) = 8x - 9 - \sqrt{2x - 1}$

Domain: $x \geq \frac{1}{2} \mid \left[\frac{1}{2}, \infty\right)$

c) $(fg)(x) = (8x - 9)\sqrt{2x - 1}$

Domain: $x \geq \frac{1}{2} \mid \left[\frac{1}{2}, \infty\right)$

d) $\left(\frac{f}{g}\right)(x) = \frac{8x - 9}{\sqrt{2x - 1}}$

Domain: $x > \frac{1}{2} \mid \left(\frac{1}{2}, \infty\right)$

Example

Let $f(x) = \sqrt{x - 3}$ and $g(x) = \sqrt{x + 1}$

Find $(f + g)(x)$ and its domain, $\left(\frac{f}{g}\right)(x)$ and its domain

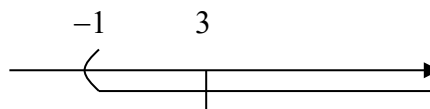
Solution

Domain $f(x)$: $x \geq 3$ and **Domain** $g(x)$: $x \geq -1$

a) $(f + g)(x) = \sqrt{x - 3} + \sqrt{x + 1}$

b) $x \geq 3$ and $x \geq -1 \Rightarrow$ **Domain:** $x \geq 3$

c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x - 3}}{\sqrt{x + 1}}$



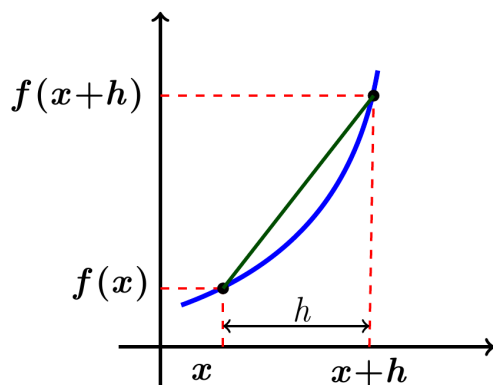
$$\rightarrow \begin{cases} x-3 \geq 0 \Rightarrow \underline{x \geq 3} \\ x+1 > 0 \Rightarrow \underline{x > -1} \end{cases}$$

Domain: $x \geq 3$ $[3, \infty)$

Difference Quotients

$$\frac{f(x+h) - f(x)}{(x+h) - x}$$

The difference quotient is given by: $\frac{f(x+h) - f(x)}{h}$



Example

For the function f given by $f(x) = 2x - 3$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned} f(x+h) &= 2(\text{---}) - 3 \\ &= 2(x+h) - 3 \\ &= 2x + 2h - 3 \end{aligned}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\underline{f(x+h)} - \underline{f(x)}}{h} \\ &= \frac{2x + 2h - 3 - (2x - 3)}{h} \\ &= \frac{2x + 2h - 3 - 2x + 3}{h} \\ &= \frac{2h}{h} \\ &= \underline{2} \end{aligned}$$

Example

For the function f given by $f(x) = -2x^2 + x + 5$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$f(\mathbf{x+h}) = -2(\mathbf{x+h})^2 + (\mathbf{x+h}) + 5$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

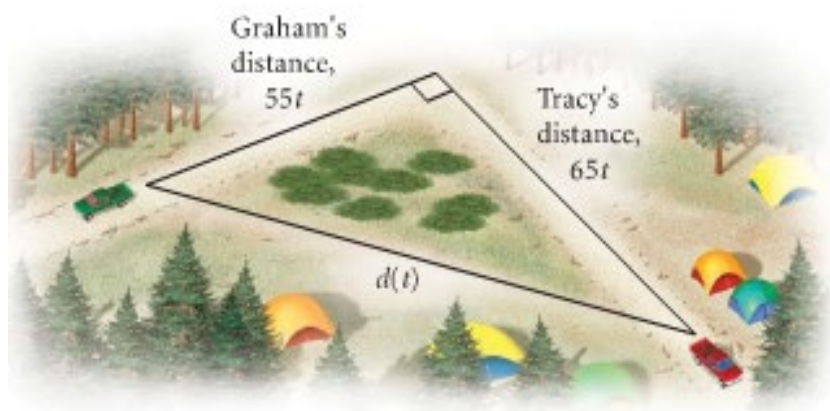
$$f(x+h) = -2(x^2 + 2hx + h^2) + x + h + 5$$

$$f(x+h) = -2x^2 - 4hx - 2h^2 + x + h + 5$$

$$\begin{aligned}\frac{f(\mathbf{x+h}) - \mathbf{f(x)}}{h} &= \frac{-2\mathbf{x^2} - 4\mathbf{hx} - 2\mathbf{h^2} + \mathbf{x+h+5} - (-2\mathbf{x^2} + \mathbf{x+5})}{h} \\&= \frac{-2x^2 - 4hx - 2h^2 + x + h + 5 + 2x^2 - x - 5}{h} \\&= \frac{-4hx - 2h^2 + h}{h} \\&= \frac{-4hx}{h} - \frac{2h^2}{h} + \frac{h}{h} \\&= \underline{-4x - 2h + 1}\end{aligned}$$

Example

Tracy and Graham drive away from a camp-ground at right angles to each other. Tracy's speed is 65 mph and Graham's is 55 *mph*.



- Express the distance between the cars as a function of time.
- Find the domain of the function.

Solution

a) $Distance = velocity * time$

Use Pythagorean Theorem:

$$d^2(t) = (65t)^2 + (55t)^2$$

$$d^2 = 4225t^2 + 3025t^2$$

$$= 7250t^2$$

$$d(t) = \sqrt{7250t^2}$$

$$= \sqrt{7250} \sqrt{t^2}$$

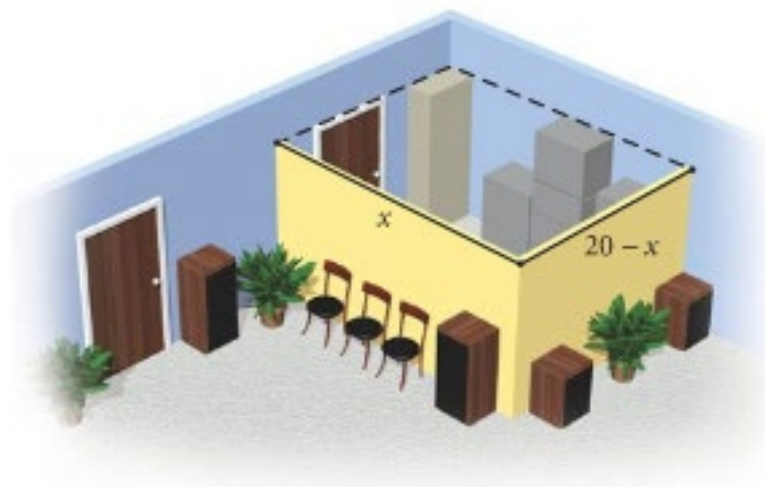
$$\approx 85.15|t|$$

$$= \underline{85.15 t}$$

b) Domain: $t \geq 0$

Example: (storage area)

The sound Shop has 20 *feet*. of dividers with which to set off a rectangular area for the storage of overstock. If a corner of the store is used for the storage area, the partition need only form two sides of a rectangle.



- Express the floor area of the storage space as a function of the length of the partition.
- Find the domain of the function.

Solution

Let x = the length

$$\text{width} + \text{length} = 20$$

$$\text{width} = 20 - \text{length}$$

a) Area = length * width

$$= x(20 - x)$$

$$= \underline{20x - x^2}$$

b) Domain: x value varies from 0 to 20 $\Rightarrow (0, 20)$

Exercises Section 2.2 – Function Operations

(1 – 80) Find the Domain

1. $f(x) = 7x + 4$
2. $f(x) = |3x - 2|$
3. $f(x) = 3x + \pi$
4. $f(x) = \sqrt{7}x + \frac{1}{2}$
5. $f(x) = -2x^2 + 3x - 5$
6. $f(x) = x^3 - 2x^2 + x - 3$
7. $f(x) = x^2 - 2x - 15$
8. $f(x) = 4 - \frac{2}{x}$
9. $f(x) = \frac{1}{x^4}$
10. $g(x) = \frac{3}{x-4}$
11. $y = \frac{2}{x-3}$
12. $y = \frac{-7}{x-5}$
13. $f(x) = \frac{x+5}{2-x}$
14. $f(x) = \frac{8}{x+4}$
15. $f(x) = \frac{1}{x+4}$
16. $f(x) = \frac{1}{x-4}$
17. $f(x) = \frac{3x}{x+2}$
18. $f(x) = x - \frac{2}{x-3}$
19. $f(x) = x + \frac{3}{x-5}$
20. $f(x) = \frac{1}{2}x - \frac{8}{x+7}$
21. $f(x) = \frac{1}{x-3} - \frac{8}{x+7}$
22. $f(x) = \frac{1}{x+4} - \frac{2x}{x-4}$
23. $f(x) = \frac{3x^2}{x+3} - \frac{4x}{x-2}$
24. $f(x) = \frac{1}{x^2 - 2x + 1}$
25. $f(x) = \frac{x}{x^2 + 3x + 2}$
26. $f(x) = \frac{x^2}{x^2 - 5x + 4}$
27. $f(x) = \frac{1}{x^2 - 4x - 5}$
28. $g(x) = \frac{2}{x^2 + x - 12}$
29. $h(x) = \frac{5}{\frac{4}{x} - 1}$
30. $y = \sqrt{x}$
31. $f(x) = \sqrt{8 - 3x}$
32. $y = \sqrt{4x + 1}$
33. $y = \sqrt{7 - 2x}$
34. $f(x) = \sqrt{8 - x}$
35. $f(x) = \sqrt{3 - 2x}$
36. $f(x) = \sqrt{3 + 2x}$
37. $f(x) = \sqrt{5 - x}$
38. $f(x) = \sqrt{x - 5}$
39. $f(x) = \sqrt{6 - 3x}$
40. $f(x) = \sqrt{3x - 6}$
41. $f(x) = \sqrt{2x + 7}$
42. $f(x) = \sqrt{x^2 - 16}$
43. $f(x) = \sqrt{16 - x^2}$
44. $f(x) = \sqrt{9 - x^2}$
45. $f(x) = \sqrt{x^2 - 25}$
46. $f(x) = \sqrt{x^2 - 5x + 4}$
47. $f(x) = \sqrt{x^2 + 5x + 4}$
48. $f(x) = \sqrt{x^2 + 3x + 2}$
49. $f(x) = \sqrt{x^2 - 3x + 2}$
50. $f(x) = \sqrt{x-4} + \sqrt{x+1}$
51. $f(x) = \sqrt{3-x} + \sqrt{x-2}$
52. $f(x) = \sqrt{1-x} + \sqrt{4-x}$
53. $f(x) = \sqrt{1-x} - \sqrt{x-3}$
54. $f(x) = \sqrt{x+4} - \sqrt{x-1}$
55. $f(x) = \frac{\sqrt{x+1}}{x}$
56. $g(x) = \frac{\sqrt{x-3}}{x-6}$
57. $f(x) = \frac{\sqrt{x+4}}{\sqrt{x-1}}$
58. $f(x) = \frac{\sqrt{5-x}}{x}$
59. $f(x) = \frac{x}{\sqrt{5-x}}$

$$60. f(x) = \frac{1}{x\sqrt{5-x}}$$

$$67. f(x) = \frac{\sqrt{x-2}}{\sqrt{x+2}}$$

$$75. f(x) = \frac{4x}{6x^2 + 13x - 5}$$

$$61. f(x) = \frac{x+1}{x^3 - 4x}$$

$$68. f(x) = \frac{\sqrt{2-x}}{\sqrt{x+2}}$$

$$76. f(x) = \frac{\sqrt{2x-3}}{x^2 - 5x + 4}$$

$$62. f(x) = \frac{\sqrt{x+5}}{x}$$

$$69. f(x) = \frac{x-4}{\sqrt{x-2}}$$

$$77. f(x) = \frac{x^2}{\sqrt{x^2 - 5x + 4}}$$

$$63. f(x) = \frac{x}{\sqrt{x+5}}$$

$$70. f(x) = \frac{1}{(x-3)\sqrt{x+3}}$$

$$78. f(x) = \frac{x+2}{\sqrt{x^2 + 5x + 4}}$$

$$64. f(x) = \frac{1}{x\sqrt{x+5}}$$

$$71. f(x) = \sqrt{x+2} + \sqrt{2-x}$$

$$79. f(x) = \frac{\sqrt{x+2}}{\sqrt{x^2 + 3x + 2}}$$

$$65. f(x) = \frac{x+3}{\sqrt{x-3}}$$

$$72. f(x) = \sqrt{(x-2)(x-6)}$$

$$80. f(x) = \frac{\sqrt{2x+3}}{x^2 - 6x + 5}$$

$$66. f(x) = \frac{\sqrt{x+3}}{\sqrt{x-3}}$$

$$74. f(x) = \frac{\sqrt{4x-3}}{x^2 - 4}$$

81. Let $f(x) = 4x - 3$ and $g(x) = 5x + 7$. Find each of the following and give the domain

$$a) (f+g)(x) \quad b) (f-g)(x) \quad c) (fg)(x) \quad d) \left(\frac{f}{g}\right)(x)$$

82. Let $f(x) = 2x^2 + 3$ and $g(x) = 3x - 4$. Find each of the following and give the domain

$$a) (f+g)(x) \quad b) (f-g)(x) \quad c) (fg)(x) \quad d) \left(\frac{f}{g}\right)(x)$$

83. Let $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 + 3x - 2$. Find each of the following and give the domain

$$a) (f+g)(x) \quad b) (f-g)(x) \quad c) (fg)(x) \quad d) \left(\frac{f}{g}\right)(x)$$

84. Let $f(x) = \sqrt{4x-1}$ and $g(x) = \frac{1}{x}$. Find each of the following and give the domain

$$a) (f+g)(x) \quad b) (f-g)(x) \quad c) (fg)(x) \quad d) \left(\frac{f}{g}\right)(x)$$

85. Given that $f(x) = x+1$ and $g(x) = \sqrt{x+3}$

$$a) \text{ Find } (f+g)(x)$$

$$b) \text{ Find the domain of } (f+g)(x)$$

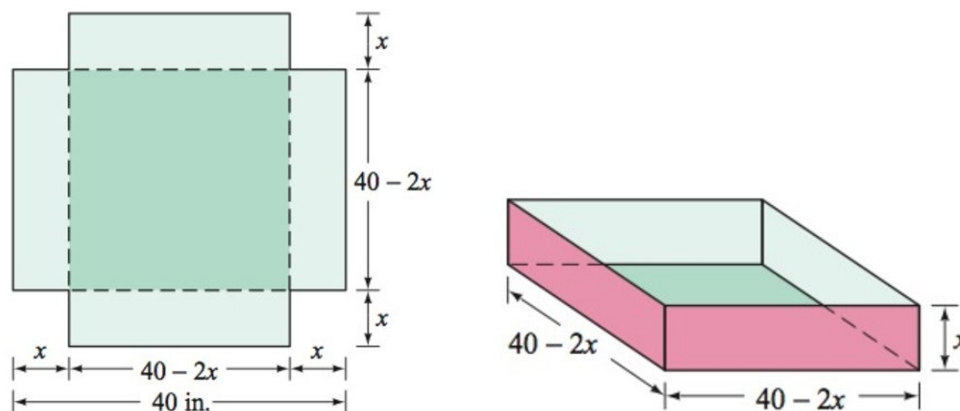
$$c) \text{ Find: } (f+g)(6)$$

86. Given that $f(x) = x^2 - 4$ and $g(x) = x + 2$
- Find $(f + g)(x)$ and its domain
 - Find $(f / g)(x)$ and its domain
87. Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find $(f + g)(1)$, $(f - g)(-3)$, $(fg)(5)$, and $\left(\frac{f}{g}\right)(0)$
88. Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $(f / g)(x)$ and the domain of
 $f(x) = \sqrt{3 - 2x}$, $g(x) = \sqrt{x + 4}$
89. Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $(f / g)(x)$ and the domain of
 $f(x) = \frac{2x}{x - 4}$, $g(x) = \frac{x}{x + 5}$
90. Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $(f / g)(x)$ of $f(x) = x - 5$ and $g(x) = x^2 - 1$

(88 – 103) Find and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$ for the given function

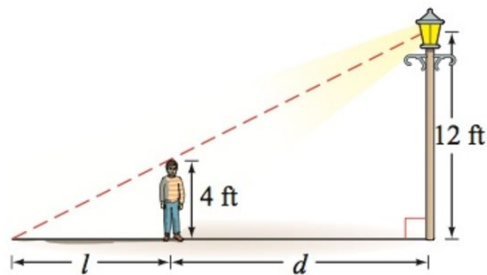
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|----------------------|-------------------------|------------------------------|
| 91. $f(x) = 9x + 5$ | 97. $f(x) = 3x - 6$ | 102. $f(x) = 2x^2 - 3x$ |
| 92. $f(x) = 6x + 2$ | 98. $f(x) = -5x - 7$ | 103. $f(x) = 2x^2 - x - 3$ |
| 93. $f(x) = 4x + 11$ | 99. $f(x) = 2x^2$ | 104. $f(x) = x^2 - 2x + 5$ |
| 94. $f(x) = 3x - 5$ | 100. $f(x) = 5x^2$ | 105. $f(x) = 3x^2 - 2x + 5$ |
| 95. $f(x) = -2x - 3$ | 101. $f(x) = 3x^2 - 4x$ | 106. $f(x) = -2x^2 - 3x + 7$ |
| 96. $f(x) = -4x + 3$ | | |

107. An open box is to be made from a square piece of cardboard that measures 40 inches on each side, to construct the box, squares that measure x inches on each side are cut from each corner of the cardboard.

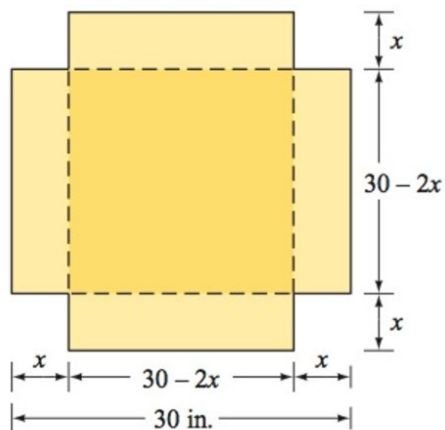


- Express the volume V of the box as a function of x .
- Determine the domain of V .

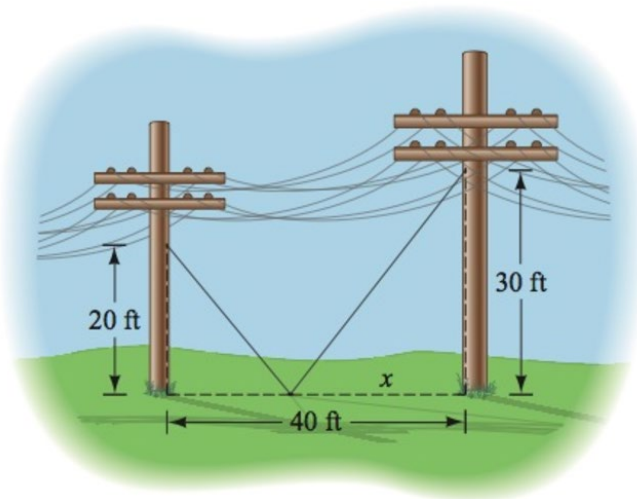
108. A child 4 feet tall is standing near a street lamp that is 12 feet high. The light from the lamp casts a shadow.



- Find the length l of the shadow as a function of the distance d of the child from the lamppost.
 - What is the domain of the function?
 - What is the length of the shadow when the child is 8 feet from the base of the lamppost?
109. An open box is to be made from a square piece of cardboard with the dimensions 30 inches by 30 inches by cutting out squares of area x^2 from each corner.



- Express the volume V of the box as a function of x .
 - Determine the domain of V .
110. Two guy wires are attached to utility poles that are 40 feet apart.



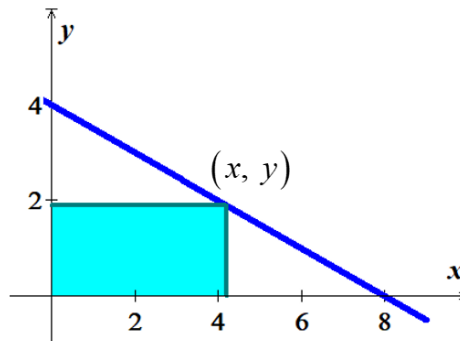
- a) Find the total length of the two guy wires as a function of x .
- b) What is the domain of this function?

- 111.** A rancher has 360 *yards*. of fencing with which to enclose two adjacent rectangular corrals, one for sheep and one for cattle. A river forms one side of the corrals. Suppose the width of each corral is x *yards*.



- a) Express the total area of the two corrals as a function of x .
- b) Find the domain of the function.

- 112.** A rectangle is bounded by the x - and y -axis of $y = -\frac{1}{2}x + 4$



- a) Find the area of the rectangle as a function of x .
- b) What is the domain of this function.