

Review Exam 4

Exam & Thursday 8/13

10:15 } lockdown & computer
Team → phone

$$\sin A = \frac{12}{13} \quad A \in QII,$$

$$\cos B = -\frac{15}{17} \quad B \in QIII$$

$$\sin A = \frac{12}{13} \quad QII \quad \cos B = -\frac{15}{17} \quad QIII$$

$$\cos A = -\frac{5}{13} \quad \sin B = -\frac{8}{17}$$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\begin{aligned} &= \left(\frac{12}{13}\right) \left(-\frac{15}{17}\right) + \left(-\frac{5}{13}\right) \left(-\frac{8}{17}\right) \\ &= -\frac{180}{221} + \frac{40}{221} \\ &= -\frac{140}{221} \end{aligned}$$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\begin{aligned} &= \left(-\frac{5}{13}\right) \left(-\frac{15}{17}\right) - \left(\frac{12}{13}\right) \left(-\frac{8}{17}\right) \\ &= \frac{75 + 96}{221} \\ &= \frac{171}{221} \end{aligned}$$

$$c) \tan(A+B) = -\frac{140}{171}$$

$$\begin{aligned} d) \sin(A-B) &= \sin A \cos B - \cos A \sin B \\ &= \frac{-180}{221} - \frac{40}{221} \\ &= -\frac{220}{221} \end{aligned}$$

$$\begin{aligned} e) \cos(A-B) &= \cos A \cos B + \sin A \sin B \\ &= \frac{75-96}{221} \\ &= -\frac{21}{221} \end{aligned}$$

$$f) \tan(A-B) = \frac{220}{21}$$

$$\cos \theta = -\frac{3}{5} \quad \theta \in 0 \cup 90^\circ \leq \theta < 180^\circ$$

$$\sin \theta = \frac{4}{5}$$

$$\begin{aligned} &45^\circ < \theta < 90^\circ \\ &\theta/2 \in 0 \cup \end{aligned}$$

$$\begin{aligned} a) \sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left(\frac{4}{5}\right) \left(-\frac{3}{5}\right) \\ &= -\frac{24}{25} \end{aligned}$$

$$\begin{aligned} b) \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \frac{9}{25} - \frac{16}{25} \\ &= -\frac{7}{5} \end{aligned}$$

$$c) \tan 2\theta = \frac{24}{7}$$

$$\theta/2 \in \mathbb{QI}$$

$$\begin{aligned} d) \sin \frac{\theta}{2} &= \sqrt{\frac{1}{2} (1 - \cos \theta)} \\ &= \sqrt{\frac{1}{2} (1 + \frac{3}{5})} \\ &= \sqrt{\frac{1}{2} (\frac{8}{5})} \\ &= \frac{2}{\sqrt{5}} \end{aligned}$$

$$\begin{aligned} e) \cos \frac{\theta}{2} &= \sqrt{\frac{1}{2} (1 + \cos \theta)} \\ &= \sqrt{\frac{1}{2} (1 - \frac{3}{5})} = \sqrt{\frac{1}{2} (\frac{2}{5})} \\ &= \frac{1}{\sqrt{5}} \end{aligned}$$

$$f) \tan \frac{\theta}{2} = 2$$

$$\left(\frac{2}{1} \right)$$

Solve \rightarrow 3 out of 5 $[0, 2\pi)$

$$\text{Solve: } 4\cos^2 x + 4\sin x - 5 = 0$$

$$\begin{aligned} 4(1 - \sin^2 x) + 4\sin x - 5 &= 0 \\ 4 - 4\sin^2 x + 4\sin x - 5 &= 0 \\ -4\sin^2 x + 4\sin x - 1 &= 0 \end{aligned}$$

$$\sin x = \frac{-d \pm \sqrt{16 - 4(4)}}{-8}$$

$$= \frac{-d}{-8} \} \#$$

$$= \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$2 \tan x \cos x + 2 \cos x + \tan x + 1 = 0$$

$$2 \cos x (\tan x + 1) + (\tan x + 1) = 0$$

$$(\tan x + 1)(2 \cos x + 1) = 0$$

$$\tan x = -1$$

$$\cos x = -\frac{1}{2} \#$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\sin x = -2$$

$$2 \cos \alpha + \tan \alpha = \sec \alpha \quad [0, 2\pi)$$

$$\cos \alpha \left(2 \cos \alpha + \frac{\sin \alpha}{\cos \alpha} \right) = \frac{1}{\cos \alpha}$$

$$\cos \alpha \neq 0$$

$$\alpha \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2 \cos^2 \alpha + \sin \alpha = 1$$

$$2(1 - \sin^2 \alpha) + \sin \alpha = 1$$

$$2 - 2 \sin^2 \alpha + \sin \alpha - 1 = 0$$

$$-2 \sin^2 \alpha + \sin \alpha + 1 = 0$$

$$\sin \alpha = 1$$

$$\sin \alpha = -\frac{1}{2}$$

$$\alpha = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\sec\left(\arcsin \frac{x}{\sqrt{x^2+9}}\right)$$

$$\sin \alpha = \frac{x}{\sqrt{x^2+9}}$$

$$\sec \alpha = \frac{\sqrt{x^2+9}}{3}$$



$$\sin\left(\tan^{-1} \frac{x}{\sqrt{16-x^2}}\right)$$

$$\tan \alpha = \frac{x}{\sqrt{16-x^2}}$$

$$\sin \alpha = \frac{x}{\sqrt{16-x^2}}$$



$$\left(2, \frac{\pi}{6}\right)$$

$$\begin{aligned} x &= r \cos \theta \\ &= 2 \cos \frac{\pi}{6} \\ &= 2 \left(\frac{\sqrt{3}}{2}\right) \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} y &= r \sin \theta \\ &= 2 \sin \frac{\pi}{6} \\ &= 2 \left(\frac{1}{2}\right) \\ &= 1 \end{aligned}$$

$$(x, y) = (\sqrt{3}, 1)$$

$$(2, 2)$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} \frac{2}{2} \\ &= \frac{\pi}{4} \end{aligned}$$

$$(r, \theta) = (2\sqrt{2}, \frac{\pi}{4})$$

$$r(\sin \theta + \cos \theta) = 4$$

$$r \sin \theta + r \cos \theta = 4$$

$$y + x = 4$$

$$x - y = 2$$

$$r \cos \theta - r \sin \theta = 2$$

$$r(\cos \theta - \sin \theta) = 2$$

$$r = \frac{2}{\cos \theta - \sin \theta}$$

$$\textcircled{3} \left. \begin{matrix} -1 \rightarrow \\ -2 \\ -3 \end{matrix} \right\} \text{ 2 out of 3}$$

$$1 + \sec^2 x \sin^2 x = \sec^2 x$$

$$\begin{aligned}
 1 + \sec^2 x \sin^2 x &= 1 + \frac{1}{\cos^2 x} \sin^2 x \\
 &= 1 + \tan^2 x \\
 &= \sec^2 x \quad \checkmark
 \end{aligned}$$

$$\tan^2 x = \sec^2 x - \sin^2 x - \cos^2 x$$

$$\begin{aligned}
 \sec^2 x - \sin^2 x - \cos^2 x &= \sec^2 x - (\sin^2 x + \cos^2 x) \\
 &= \sec^2 x - 1 \\
 &= \tan^2 x \quad \checkmark
 \end{aligned}$$

$$1 + \tan^2 x = \sec^2 x \quad \checkmark$$

$$\sec^4 x - \tan^4 x = \sec^2 x + \tan^2 x$$

$$\begin{aligned}
 \sec^4 x - \tan^4 x &= (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x) \\
 &= (\sec^2 x + \tan^2 x)(1) \\
 &= \sec^2 x + \tan^2 x \quad \checkmark
 \end{aligned}$$

$$\frac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B$$

$$\begin{aligned}
 \frac{\sin(A-B)}{\cos A \cos B} &= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} \\
 &= \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B} \\
 &= \frac{\sin A}{\cos A} - \frac{\sin B}{\cos B} \\
 &= \tan A - \tan B
 \end{aligned}$$

$$= \tan A - \tan B \quad \checkmark$$

$$\frac{\cos(x+y)}{\cos(x-y)} = \frac{\cot y - \tan x}{\cot y + \tan x}$$

$$\begin{aligned} \frac{\cos(x+y)}{\cos(x-y)} &= \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y + \sin x \sin y} \\ &= \frac{\frac{\cos x \cos y}{\cos x \sin y} - \frac{\sin x \sin y}{\cos x \sin y}}{\frac{\cos x \cos y}{\cos x \sin y} + \frac{\sin x \sin y}{\cos x \sin y}} \\ &= \frac{\cot y - \tan x}{\cot y + \tan x} \quad \checkmark \end{aligned}$$

$$\sec(x+y) = \frac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y}$$

$$\begin{aligned} \sec(x+y) &= \frac{1}{\cos(x+y)} \cdot \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y + \sin x \sin y} \\ &= \frac{\cos(x-y)}{(\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)} \\ &= \frac{\cos(x-y)}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y} \\ &= \frac{\cos(x-y)}{\cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y} \end{aligned}$$

$$= \frac{\cos(x-y)}{\cos^2 x - \cos x \sin^2 y - \sin^2 y + \cos x \sin^2 y}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y} \quad \checkmark$$

$$\cos 4x = \cos^4 x - 6 \sin^2 x \cos^2 x + \sin^4 x$$

$$\cos 4x = \cos 2(2x)$$

$$= \cos^2 2x - \sin^2 2x$$

$$= (\cos^2 x - \sin^2 x)^2 - (2 \sin x \cos x)^2$$

$$= \cos^4 x - 2 \sin^2 x \cos^2 x + \sin^4 x - 4 \sin^2 x \cos^2 x$$

$$= \cos^4 x - 6 \sin^2 x \cos^2 x + \sin^4 x \quad \checkmark$$

$$\frac{\cos 2x}{\cos^2 x} = \sec^2 x - 2 \tan^2 x$$

$$\frac{\cos 2x}{\cos^2 x} = \frac{1 - 2 \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - 2 \frac{\sin^2 x}{\cos^2 x}$$

$$= \sec^2 x - 2 \tan^2 x \quad \checkmark$$

$$2 \sin^2 \frac{x}{2} = \frac{\sin^2 x}{1 + \cos x}$$

$$2 \sin^2 \frac{x}{2} = 2 (1 - \cos x)$$

$$\begin{aligned}
 &= (1 - \cos x) \frac{1 + \cos x}{1 + \cos x} \\
 &= \frac{1 - \cos^2 x}{1 + \cos x} \\
 &= \frac{\sin^2 x}{1 + \cos x} \quad \checkmark
 \end{aligned}$$

$$\sec^2\left(\frac{\alpha}{2}\right) = \frac{2\sec\alpha + 2}{\sec\alpha + 2 + \cos\alpha}$$

$$\sec^2\left(\frac{\alpha}{2}\right) = \frac{1}{\cos^2\frac{\alpha}{2}}$$

$$\begin{aligned}
 &= \frac{1}{\frac{1}{2}(1 + \cos\alpha)} \\
 &= \frac{2}{1 + \cos\alpha} \cdot \frac{\sec\alpha + 1}{\sec\alpha + 1}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{2\sec\alpha + 2}{\sec\alpha + \underbrace{\sec\alpha\cos\alpha}_{=1} + \cos\alpha + 1}
 \end{aligned}$$

$$= \frac{2\sec\alpha + 2}{\sec\alpha + 2 + \cos\alpha}$$

$$\frac{\sin 4t}{4} = \cos^3 t \sin t - \sin^3 t \cos t$$

$$1 \sin 4t = 1 \sin (2t + 2t)$$

$$4 \sin^2 t = \frac{1}{4} \sin(2t + 2t)$$

$$= \frac{1}{4} (\sin 2t \cos 2t + \sin 2t \cos 2t)$$

$$= \frac{1}{2} \sin 2t \cos 2t$$

$$= \frac{1}{2} (2 \sin t \cos t)(\cos^2 t - \sin^2 t)$$

$$= \sin t \cos^3 t - \sin^3 t \cos t \quad \checkmark$$

$$\frac{\sin(x+y)}{\cos(x-y)} = \frac{1 + \cot x \tan y}{\cot x + \tan y}$$

$$\begin{aligned} \frac{\sin(x+y)}{\cos(x-y)} &= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y + \sin x \sin y} \\ &= \frac{\frac{\sin x \cos y}{\sin x \cos y} + \frac{\cos x \sin y}{\sin x \cos y}}{\frac{\cos x \cos y}{\sin x \cos y} + \frac{\sin x \sin y}{\sin x \cos y}} \\ &= \frac{1 + \cot x \tan y}{\cot x + \tan y} \quad \checkmark \end{aligned}$$