Solution Section 1.2 – Polynomial Functions & Graphs

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=2x^4-x^3+7x-12; \quad p(x)=x^2-3$

Solution

$$\begin{array}{r}
2x^2 - x + 6 \\
x^2-3 \overline{\smash{)}2x^4 - x^3 + 0x^2 + 7x - 12} \\
\underline{2x^4 \qquad -6x^2 \qquad\qquad} \\
-x^3 + 6x^2 + 7x \qquad \\
\underline{-x^3 \qquad +3x} \qquad \\
6x^2 + 4x - 12 \\
\underline{6x^2 \qquad -18} \\
4x + 6
\end{array}$$

$$\left.\underline{Q(x)=2x^2-x+6; \quad R(x)=4x+6}\right|$$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=3x^3+2x-4; \quad p(x)=2x^2+1$

Solution

$$\begin{array}{r}
\frac{3}{2}x \qquad\quad \\
2x^2+1 \overline{\smash{)}3x^3 + 0x^2 + 2x - 4} \\
\underline{3x^3 \qquad\quad + \frac{3}{2}x \qquad} \\
\frac{1}{2}x - 4 \quad
\end{array}$$

$$\left.\underline{Q(x)=\frac{3}{2}x; \quad R(x)=\frac{1}{2}x-4}\right|$$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=7x+2; \quad p(x)=2x^2-x-4$

Solution

$$P(x)=\frac{7x+2}{2x^2-x-4}$$

$$\left.\underline{Q(x)=0; \quad R(x)=7x+2}\right|$$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=9x+4; \quad p(x)=2x-5$

Solution

$$\begin{array}{r}
\frac{9}{2} \\
2x-5\overline{)9x+4} \\
\underline{9x-\frac{45}{2}} \\
-\frac{37}{2}
\end{array}$$

$$\boxed{Q(x)=\frac{9}{2}; \quad R(x)=-\frac{37}{2}}$$

Exercise

Use the remainder theorem to find $f(c)$: $f(x)=x^4-6x^2+4x-8; \quad c=-3$

Solution

$$f(-3)=(-3)^4-6(-3)^2+4(-3)-8$$

$$\boxed{=7}$$

Exercise

Use the remainder theorem to find $f(c)$: $f(x)=x^4+3x^2-12; \quad c=-2$

Solution

$$f(-2)=(-2)^4+3(-2)^2-12$$

$$\boxed{=16}$$

Exercise

Use the factor theorem to show that $x-c$ is a factor of $f(x)$: $f(x)=x^3+x^2-2x+12; \quad c=-3$

Solution

$$f(-3)=(-3)^3+(-3)^2-2(-3)+12$$

$$\boxed{=0}$$

From the factor theorem; $x+3$ is a factor of $f(x)$.

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second:

$2x^3 - 3x^2 + 4x - 5; \quad x - 2$

Solution

$$\begin{array}{r|rrrr} 2 & 2 & -3 & 4 & -5 \\ & & 4 & 2 & 12 \\ \hline & 2 & 1 & 6 & \boxed{7} \end{array}$$

$$Q(x) = 2x^2 + x + 6 \qquad R(x) = 7$$

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second:

$5x^3 - 6x^2 + 15; \quad x - 4$

Solution

$$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 15 \\ & & 20 & 56 & 224 \\ \hline & 5 & 14 & 56 & \boxed{239} \end{array}$$

$$Q(x) = 5x^2 + 14x + 56 \qquad R(x) = 239$$

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second:

$9x^3 - 6x^2 + 3x - 4; \quad x - \frac{1}{3}$

Solution

$$\begin{array}{r|rrrr} \frac{1}{3} & 9 & -6 & 3 & -4 \\ & & 3 & -1 & \frac{2}{3} \\ \hline & 9 & -3 & 2 & \boxed{-\frac{10}{3}} \end{array}$$

$$Q(x) = 9x^2 - 3x + 2 \qquad R(x) = -\frac{10}{3}$$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=2x^3+3x^2-4x+4; \quad c=3$

Solution

$$\begin{array}{r|rrrr}
3 & 2 & 3 & -4 & 4 \\
 & & 6 & 27 & 69 \\
\hline
 & 2 & 9 & 23 & \boxed{73}
\end{array}$$

$f(3)=73$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=8x^5-3x^2+7; \quad c=\frac{1}{2}$

Solution

$$\begin{array}{r|rrrrrr}
\frac{1}{2} & 8 & 0 & 0 & -3 & 0 & 7 \\
 & & 4 & 2 & 1 & -1 & -\frac{1}{2} \\
\hline
 & 8 & 4 & 2 & -2 & -1 & \boxed{\frac{13}{2}}
\end{array}$$

$f\left(\frac{1}{2}\right)=\frac{13}{2}$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=x^3-3x^2-8; \quad c=1+\sqrt{2}$

Solution

$$\begin{array}{r|cccc}
1+\sqrt{2} & 3 & -3 & 0 & -8 \\
 & & 3+3\sqrt{2} & 6+3\sqrt{2} & 12+9\sqrt{2} \\
\hline
 & 3 & 3\sqrt{2} & 6+3\sqrt{2} & \boxed{4+9\sqrt{2}}
\end{array}$$

$f\left(1+\sqrt{2}\right)=4+9\sqrt{2}$

Exercise

Use the synthetic division to show that c is a zero of $f(x)$: $f(x)=3x^4+8x^3-2x^2-10x+4; \quad c=-2$

Solution

$$\begin{array}{r|rrrrr} -2 & 3 & 8 & -2 & -10 & 4 \\ & & -6 & -4 & 12 & -4 \\ \hline & 3 & 2 & -6 & 2 & \boxed{0} \end{array}$$

$$\boxed{f(-2)=0}$$

Exercise

Use the synthetic division to show that c is a zero of $f(x)$: $f(x)=27x^4-9x^3+3x^2+6x+1;\quad c=-\frac{1}{3}$

Solution

$$\begin{array}{r|rrrrr} -\frac{1}{3} & 27 & -9 & 3 & 6 & 1 \\ & & -9 & 6 & -3 & -1 \\ \hline & 27 & -18 & 9 & 3 & \boxed{0} \end{array}$$

$$\boxed{f\left(-\frac{1}{3}\right)=0}$$

Exercise

Find all values of k such that $f(x)$ is divisible by the given linear polynomial:

$f(x)=kx^3+x^2+k^2x+3k^2+11;\quad x+2$

Solution

$$\begin{array}{r|llll} -2 & k & 1 & k^2 & 3k^2+11 \\ & & -2k & 4k-2 & -2k^2-8k+4 \\ \hline & k & 1-2k & k^2+4k-2 & k^2-8k+15 \end{array}$$

$$k^2-8k+15=0 \Rightarrow \boxed{k=3,\ 5}$$

Exercise

Find all values of k such that $f(x)$ is divisible by the given linear polynomial:

$f(x)=x^3+k^3x^2++2kx-2k^4;\quad x-1.6$

Solution

$$\begin{array}{r|llll} 1.6 & 1 & k^3 & 2k & -2k^4 \\ & & 1.6 & 1.6k^3+2.56 & 2.56k^3+3.2k+4.096 \\ \hline & 1 & k^3+1.6 & 1.6k^3+2k+2.56 & -2k^4+2.56k^3+3.2k+4.096 \end{array}$$

$$-2k^4 + 2.56k^3 + 3.2k + 4.096 = 0$$

Using the calculator, the result will show that the solutions are: $x = -0.75,\ \ 1.96$ $0.032 \pm 1.18i$

Exercise

Find all values of k such that $f(x)$ is divisible by the given linear polynomial:

$f(x) = k^2x^3 - 4kx + 3; \quad x - 1$

Solution

$$\begin{array}{r|llll} 1 & k^2 & 0 & -4k & 3 \\ & & k^2 & k^2 - 4k & k^2 - 4k \\ \hline & k^2 & k^2 & k^2 - 4k & k^2 - 4k + 3 \end{array}$$

$k^2 - 4k + 3 = 0 \quad \Rightarrow \quad k = 1,\ 3$

Exercise

Find all solutions of the equation: $x^3 - x^2 - 10x - 8 = 0$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & 0 \end{array} \rightarrow \quad x^2 - 2x - 8 = 0$$

The solutions are: $x = -1,\ -2,\ 4$

Exercise

Find all solutions of the equation: $x^3 + x^2 - 14x - 24 = 0$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & 0 \end{array} \rightarrow \quad x^2 - x - 12 = 0$$

The solutions are: $x = -2,\ -3,\ 4$

Exercise

Find all solutions of the equation: $2x^3 - 3x^2 - 17x + 30 = 0$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{30}{2}\right\}$

$$= \pm\left\{1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30,\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{15}{2}\right\}$$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & \boxed{0} \end{array} \rightarrow \quad 2x^2 + x - 15 = 0$$

The solutions are: $\underline{x = 2,\ \ -3,\ \ \frac{5}{2}}|$

Exercise

Find all solutions of the equation: $12x^3 + 8x^2 - 3x - 2 = 0$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{2}{12}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ \frac{1}{6},\ \frac{1}{12}\right\}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 12 & 8 & -3 & -2 \\ & & 6 & 7 & 2 \\ \hline & 12 & 14 & 4 & \boxed{0} \end{array} \rightarrow \quad 12x^2 + 14x + 4 = 0$$

$$6x^2 + 7x + 2 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 48}}{12}$$

$$= \begin{cases} \frac{-7-1}{12} = -\frac{2}{3} \\ \frac{-7+1}{12} = -\frac{1}{2} \end{cases}$$

The solutions are: $\underline{x = \frac{1}{2},\ \ -\frac{1}{2},\ \ -\frac{2}{3}}|$

Exercise

Find all solutions of the equation: $x^4 + 3x^3 - 30x^2 - 6x + 56 = 0$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{56}{1}\right\} = \pm\{1,\ 2,\ 4,\ 7,\ 8,\ 14,\ 28,\ 56\}$

$$\begin{array}{r|rrrrr}
4 & 1 & 3 & -3 & -6 & 56 \\
 & & 4 & 28 & -8 & -56 \\ \hline
-7 & 1 & 7 & -2 & -14 & 0 \\
 & & -7 & 0 & 14 & \\ \hline
 & 1 & 0 & -2 & &
\end{array}$$

$$\to \quad x^3+7x^2-2x-14=0 \quad \Rightarrow \quad \frac{c}{d}=\pm\left\{\frac{14}{1}\right\}=\pm\{1,\ 2,\ 7,\ 14\}$$

$$\to \quad x^2-2=0 \;\Rightarrow\; \underline{x=\pm\sqrt{2}}\,|$$

The solutions are: $\underline{x=4,\ \ -7,\ \ \pm\sqrt{2}}\,|$

Exercise

Find all solutions of the equation: $3x^5-10x^4-6x^3+24x^2+11x-6=0$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{6}{3}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrrrr}
-1 & 3 & -10 & -6 & 24 & 11 & -6 \\
 & & -3 & 13 & -7 & -17 & 6 \\ \hline
-1 & 3 & -13 & 7 & 17 & -6 & 0 \\
 & & -3 & 16 & -23 & 6 & \\ \hline
2 & 3 & -16 & 23 & -6 & 0 & \\
 & & 6 & 20 & 6 & & \\ \hline
 & 3 & -10 & 3 & 0 & &
\end{array}$$

$$x^4-13x^3+7x^2+17x-6=0\to\pm\left\{\frac{6}{3}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^3-16x^2+26x-6=0\to\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^2-10x+3=0$$

$$3x^2-10x+3=0$$

$$x=\frac{10\pm\sqrt{100-36}}{6}$$

$$=\begin{cases}\dfrac{10-8}{6}=\dfrac{1}{3}\\[2mm] \dfrac{10+8}{6}=3\end{cases}$$

The solutions are: $\underline{x=-1,\ \ -1,\ \ \frac{1}{3},\ 2,\ 3}\,|$

Exercise

Find all solutions of the equation: $6x^5+19x^4+x^3-6x^2=0$

Solution

$$x^2\left(6x^3+19x^2+x-6\right)=0 \quad\to\quad \boxed{x=0,\ 0}$$

$$6x^3+19x^2+x-6=0$$

$$possibilities\ for\ \frac{c}{d}: \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrr}
-3 & 6 & 19 & 1 & -6 \\
 & & -18 & -3 & 6 \\
\hline
 & 6 & 1 & -2 & \boxed{0}
\end{array}
\quad 6x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1+48}}{12}$$

$$= \begin{cases} \frac{-1-7}{12} = -\frac{2}{3} \\ \frac{-1+7}{12} = \frac{1}{2} \end{cases}$$

The solutions are: $x = 0,\ 0,\ -\frac{2}{3},\ -3,\ \frac{1}{2}$

Exercise

Find all solutions of the equation: $x^4 - x^3 - 9x^2 + 3x + 18 = 0$

Solution

$$possibilities\ for\ \frac{c}{d}: \pm\left\{\frac{18}{1}\right\} = \pm\{1,\ 2,\ 3,\ 6,\ 9,\ 18\}$$

$$\begin{array}{r|rrrrr}
-2 & 1 & -1 & -9 & 3 & 18 \\
 & & -2 & 6 & 6 & -18 \\
\hline
3 & 1 & -3 & -3 & 9 & 0 \\
 & & 3 & 0 & -9 & \\
\hline
 & 1 & 0 & -3 & 0 &
\end{array}
\begin{array}{l}
\to x^3 - 3x^2 - 3x + 9 = 0 \to \pm\left\{\frac{9}{1}\right\} = \pm\{1,\ 3,\ 9\} \\
\\
\to x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}
\end{array}$$

The solutions are: $x = -2,\ 3,\ \pm\sqrt{3}$

Exercise

Find all solutions of the equation: $2x^4 - 9x^3 + 9x^2 + x - 3 = 0$

Solution

$$possibilities\ for\ \frac{c}{d}: \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\begin{array}{r|rrrrr}
1 & 2 & -9 & 9 & 1 & -3 \\
 & & 2 & -7 & 2 & 3 \\
\hline
1 & 2 & -7 & 2 & 3 & 0 \\
 & & 2 & -5 & -3 & \\
\hline
 & 2 & -5 & -3 & 0 &
\end{array}
\quad \to \; 2x^3 - 7x^2 + 2x + 3 = 0 \;\to\; \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\to \; 2x^2 - 5x - 3 = 0$$

$$x = \frac{5 \pm \sqrt{25+24}}{4}$$

$$= \begin{cases} \dfrac{5-7}{4} = -\dfrac{1}{2} \\ \dfrac{5+7}{4} = 3 \end{cases}$$

The solutions are: $\boxed{x = 1,\ 1,\ -\frac{1}{2},\ 3}$

Exercise

Find all solutions of the equation: $8x^3 + 18x^2 + 45x + 27 = 0$

Solution

$$possibilities: \quad \pm\left\{\frac{27}{8}\right\} = \pm\left\{\frac{1,\ 3,\ 9,\ 27}{1,\ 2,\ 4,\ 8}\right\}$$

$$= \pm\left\{1,\ 3,\ 9,\ 27,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{3}{2},\ \frac{3}{4},\ \frac{3}{8},\ \frac{9}{2},\ \frac{9}{4},\ \frac{9}{8},\ \frac{27}{2},\ \frac{27}{4},\ \frac{27}{8}\right\}$$

$$\begin{array}{r|rrrr}
-\frac{3}{4} & 8 & 18 & 45 & 27 \\
 & & -6 & -9 & -27 \\
\hline
 & 8 & 12 & 36 & \boxed{0}
\end{array}
\quad \to \; 8x^2 + 12x + 36 = 0$$

$$2x^2 + 3x + 9 = 0$$

$$x = \frac{-3 \pm \sqrt{9-72}}{4}$$

$$= \frac{-3 \pm \sqrt{-63}}{4}$$

$$= \frac{-3 \pm 3i\sqrt{7}}{4}$$

The solutions are: $\boxed{x = -\frac{3}{4},\ -\frac{3}{4} \pm i\,\frac{3\sqrt{7}}{4}}$

Exercise

Find all solutions of the equation: $3x^3 - x^2 + 11x - 20 = 0$

Solution

$$possibilities: \ \pm\left\{\frac{20}{3}\right\} = \pm\left\{\frac{1,\ 2,\ 4,\ 5,\ 10,\ 20}{1,\ 3}\right\}$$

$$= \pm\left\{1,\ 2,\ 4,\ 5,\ 10,\ 20,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{5}{3},\ \frac{10}{3},\ \frac{20}{3}\right\}$$

A result will show that one solution is: $\boxed{x = \frac{4}{3}}$

$$\begin{array}{r|rrrr} \frac{4}{3} & 3 & -1 & 11 & -20 \\ & & 4 & 4 & 20 \\ \hline & 3 & 3 & 15 & \boxed{0} \end{array} \quad \rightarrow \ 3x^2 + 3x + 15 = 0$$

$$x^2 + x + 5 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 20}}{2}$$

The solutions are: $\boxed{x = \frac{4}{3},\ \ -\frac{1}{2} \pm i\frac{\sqrt{19}}{2}}$

Exercise

Find all solutions of the equation: $6x^4 + 5x^3 - 17x^2 - 6x = 0$

Solution

$$x\left(6x^3 + 5x^2 - 17x - 6\right) = 0 \ \rightarrow \ \boxed{x = 0}$$

$$possibilities: \ \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrr} -2 & 6 & 5 & -17 & -6 \\ & & -12 & 14 & 6 \\ \hline & 6 & -7 & -3 & \boxed{0} \end{array} \quad \rightarrow \ 6x^2 - 7x - 3 = 0$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$= \begin{cases} \frac{7 - 11}{12} = -\frac{1}{3} \\ \frac{7 + 11}{12} = \frac{3}{2} \end{cases}$$

The solutions are: $\boxed{x = 0,\ \ -2,\ \ -\frac{1}{3},\ \frac{3}{2}}$

Exercise

If $f(x)=3x^3-kx^2+x-5k$, find a number k such that the graph of f contains the point $(-1, 4)$.

Solution

$$f(-1)=3(-1)^3-k(-1)^2+(-1)-5k$$

$$4=-3-k-1-5k$$

$$4=-4-6k \qquad \textit{Add 4 on both side}$$

$$8=-6k$$

$$k=-\frac{8}{6}$$

$$\boxed{=-\frac{4}{3}}$$

Exercise

If $f(x)=kx^3+x^2-kx+2$, find a number k such that the graph of f contains the point $(2, 12)$.

Solution

$$f(2)=k(2)^3+(2)^2-k(2)+2$$

$$12=8k+4-2k+2$$

$$12=6k+6$$

$$6k=6$$

$$\boxed{k=1}$$

Exercise

If one zero of $f(x)=x^3-2x^2-16x+16k$ is 2, find two other zeros.

Solution

$$f(x)=x^2(x-2)-16(x-k) \qquad k=2$$

$$=(x-2)\left(x^2-16\right)$$

$$=(x-2)(x-4)(x+4)$$

The other zeros are: $4, -4$

Exercise

If one zero of $f(x)=x^3-3x^2-kx+12$ is -2, find two other zeros.

Solution

$$f(x) = x^2(x-3) - k\left(x - \frac{12}{k}\right) \qquad \frac{12}{k} \textit{ has to be equal to } 3. \Rightarrow k = 4$$

$$f(x) = x^2(x-3) - 4(x-3)$$

$$= (x-3)\left(x^2 - 4\right)$$

$$= (x-3)(x-2)(x+2)$$

The zeros of $f(x)$ are: 3, –2, 2

Exercise

Find a polynomial $f(x)$ of degree 3 that has the zeros –1, 2, 3; and satisfies the given condition: $f(-2) = 80$

Solution

$$f(x) = k(x+1)(x-2)(x-3)$$

$$= k\left(x^2 - x - 2\right)(x-3)$$

$$= k\left(x^3 - 3x^2 - x^2 + 3x - 2x + 6\right)$$

$$= k\left(x^3 - 4x^2 + x + 6\right)$$

$$f(-2) = k\left((-2)^3 - 4(-2)^2 + (-2) + 6\right)$$

$$80 = k(-20)$$

$$k = \frac{80}{-20} = -4$$

$$f(x) = -4\left(x^3 - 4x^2 + x + 6\right)$$

$$f(x) = -4x^3 + 16x^2 - 4x - 24$$

Exercise

Find a polynomial $f(x)$ of degree 3 that has the zeros $-2i$, $2i$, 3; and satisfies the given condition: $f(1) = 20$

Solution

$$f(x) = k(x+2i)(x-2i)(x-3)$$

$$= k\left(x^2 + 4\right)(x-3)$$

$$= k\left(x^3 - 3x^2 + 4x - 12\right)$$

$$f(1) = k\left((1)^3 - 3(1)^2 + 4(1) - 12\right)$$

$$20 = k(-10)$$

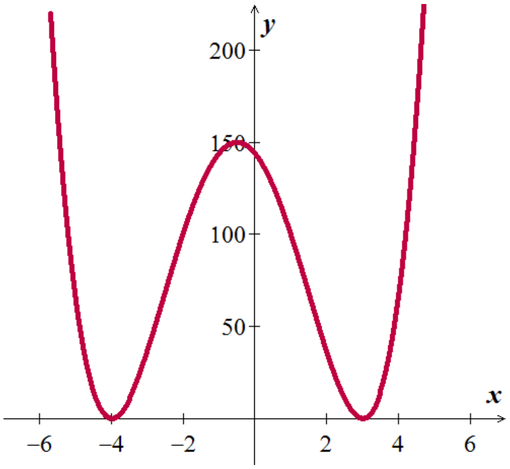
$k=-2$

$f(x)=-2\left(x^3-3x^2+4x-12\right)$

$\underline{f(x)=-2x^3+6x^2-8x+24}|$

Exercise

Find a polynomial $f(x)$ of degree 4 with leading coefficient 1 such that both -4 and 3 are zeros of multiplicity 2, and sketch the graph of f.



Solution

$$f(x)=a\left(x-x_1\right)\left(x-x_2\right)\left(x-x_3\right)\left(x-x_4\right)$$

$a=1 \quad x_1=x_2=-4 \quad x_3=x_4=3$

$$\begin{aligned}f(x)&=(x+4)(x+4)(x-3)(x-3)\\&=\left(x^2+8x+16\right)\left(x^2-6x+9\right)\\&=x^4-6x^3+9x^2+8x^3-48x^2+72x+16x^2-96x+144\\&=x^4+2x^3-23x^2-24x+144\end{aligned}$$

Exercise

Find the zeros of $f(x)=x^2(3x+2)(2x-5)^3$, and state the multiplicity of each zero.

Solution

$$f(x)=x^2(3x+2)(2x-5)^3=0$$

The zeros are: $x=0$ $(multiplicity\ of\ \ 2)$

$x=-\frac{2}{3}$

$x=\frac{5}{2}$ $(multiplicity\ of\ \ 3)$

Exercise

Find the zeros of $f(x)=4x^5+12x^4+9x^3$, and state the multiplicity of each zero.

Solution

$$\begin{aligned}f(x)&=x^3\left(4x^2+12x+9\right)=0\\&=x^3(2x+3)^2=0\end{aligned}$$

The zeros are: $x = 0$ $(\textit{multiplicity of } 3)$

$x = -\frac{3}{2}$ $(\textit{multiplicity of } 2)$

Exercise

Find the zeros of $f(x) = \left(x^2 + x - 12\right)^3 \left(x^2 - 9\right)^2$, and state the multiplicity of each zero.

Solution

$$f(x) = \left(x^2 + x - 12\right)^3 \left(x^2 - 9\right)^2 = 0$$

$x^2 + x - 12 = 0$ $\qquad$ $x^2 - 9 = 0$

$x = -4,\ 3$ $\qquad$ $x = \pm 3$

The zeros are: $x = -4$ $(\textit{multiplicity of } 3)$

$x = -3$ $(\textit{multiplicity of } 2)$

$x = 3$ $(\textit{multiplicity of } 5)$

Exercise

Find the zeros of $f(x) = \left(6x^2 + 7x - 5\right)^4 \left(4x^2 - 1\right)^2$, and state the multiplicity of each zero.

Solution

$$f(x) = \left(6x^2 + 7x - 5\right)^4 \left(4x^2 - 1\right)^2 = 0$$

$6x^2 + 7x - 5 = 0$ $\qquad$ $4x^2 - 1 = 0 \rightarrow x^2 = \frac{1}{4}$

$x = -\frac{5}{3},\ \frac{1}{2}$ $\qquad$ $x = \pm\frac{1}{2}$

The zeros are: $x = -\frac{5}{3}$ $(\textit{multiplicity of } 4)$

$x = -\frac{1}{2}$ $(\textit{multiplicity of } 2)$

$x = \frac{1}{2}$ $(\textit{multiplicity of } 6)$

Exercise

Find the zeros of $f(x) = x^4 + 7x^2 - 144$, and state the multiplicity of each zero.

Solution

$$\begin{aligned} f(x) &= x^4 + 7x^2 - 144 \\ &= \left(x^2 - 9\right)\left(x^2 + 16\right) = 0 \end{aligned}$$

$$x^2 - 9 = 0 \qquad\qquad x^2 + 16 = 0$$

$$x = \pm 3 \qquad\qquad x^2 = -16 \ \ (\mathbb{C})$$

The zeros are: $\underline{x = \pm 3}\,|$

Exercise

Find the zeros of $f(x) = x^4 + 21x^2 - 100$, and state the multiplicity of each zero.

Solution

$$\begin{aligned} f(x) &= x^4 + 21x^2 - 100 \\ &= \left(x^2 - 4\right)\left(x^2 + 25\right) = 0 \end{aligned}$$

$$x^2 - 4 = 0 \qquad\qquad x^2 + 25 = 0$$

$$x = \pm 2 \qquad\qquad x^2 = -25 \ \ (\mathbb{C})$$

The zeros are: $\underline{x = \pm 2}\,|$

Exercise

Let $f(x) = x^4 - 4x^2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

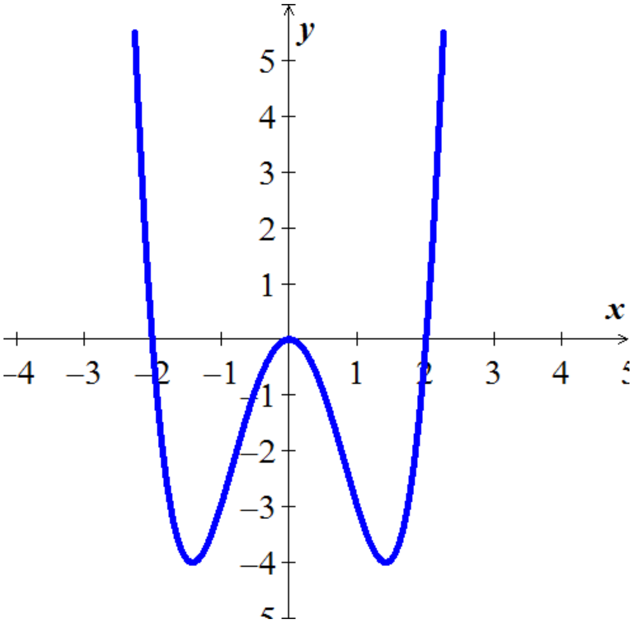
$$\begin{aligned} f(x) &= x^2\left(x^2 - 4\right) \\ &= x^2(x-2)(x+2) \end{aligned}$$

The zeros are: 0, 0, 2, –2.

<table>
<tr><td colspan="3">$-\infty$ -2 **0,0** 2 ∞</td></tr>
<tr><td>+</td><td>–</td><td>+</td></tr>
</table>

$f(x) < 0 \quad \underline{(-2,\ 0) \cup (0,\ 2)}\,|$

$f(x) > 0 \quad \underline{(-\infty,\ -2) \cup (2,\ \infty)}\,|$



Exercise

Let $f(x) = x^4 + 3x^3 - 4x^2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$f(x) = x^2\left(x^2 + 3x - 4\right)$$

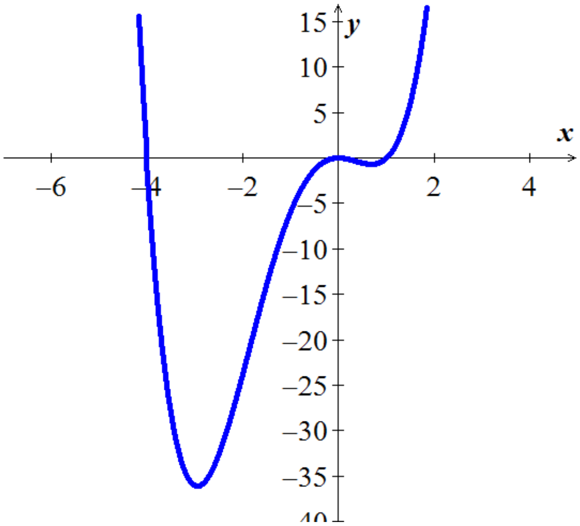
The zeros are: 0, 0, 1, −4.

$-\infty \qquad -4 \qquad 0,0 \qquad 1 \qquad \infty$

+	–	+

$f(x) > 0 \quad (-\infty,\ -4) \cup (1,\ \infty)$

$f(x) < 0 \quad (-4,\ 0) \cup (0,\ 1)$



Exercise

Let $f(x) = x^3 + 2x^2 - 4x - 8$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\begin{aligned} f(x) &= x^2(x+2) - 4(x+2) \\ &= (x+2)\left(x^2 - 4\right) \\ &= (x+2)(x+2)(x-2) = 0 \end{aligned}$$

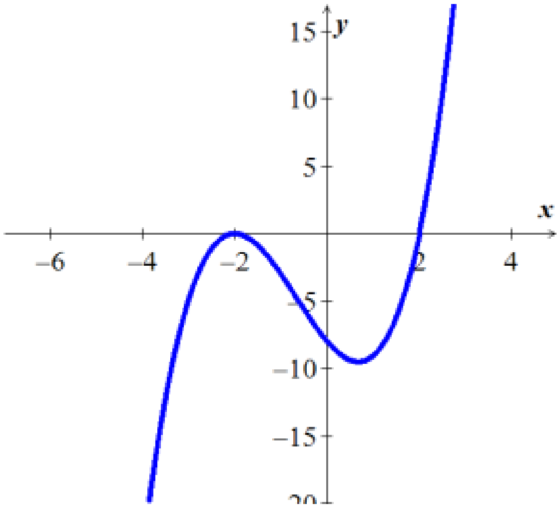
The zeros are: 2, −2, −2

$-\infty \qquad -2 \qquad 0 \qquad 2 \qquad \infty$

–	–	+

$f(x) > 0 \quad (2,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -2) \cup (-2,\ 2)$



Exercise

Let $f(x)=x^3-3x^2-9x+27$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

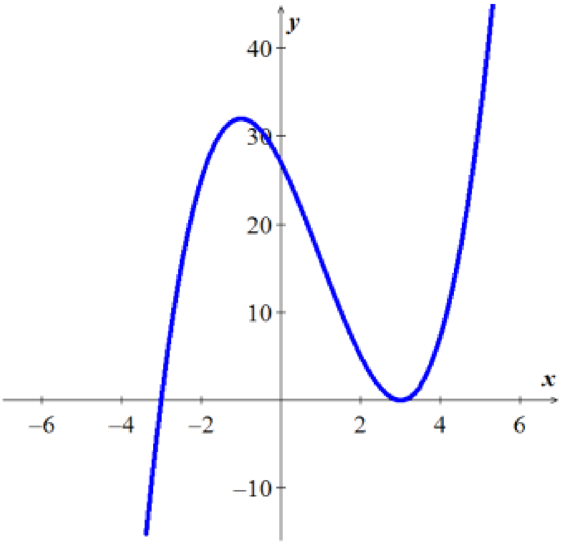
Solution

$$\begin{aligned} f(x) &= x^2(x-3)-9(x-3) \\ &= (x-3)\left(x^2-9\right) \\ &= (x-3)(x-3)(x+3) \end{aligned}$$

The zeros are: −3, 3 (multiplicity)

$-\infty \qquad -3 \qquad 0 \qquad 3 \qquad \infty$

−	+	+



$f(x)>0 \quad (-3,\ 3)\cup(3,\ \infty)$

$f(x)<0 \quad (-\infty,\ -3)$

Exercise

Let $f(x)=-x^4+12x^2-27$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

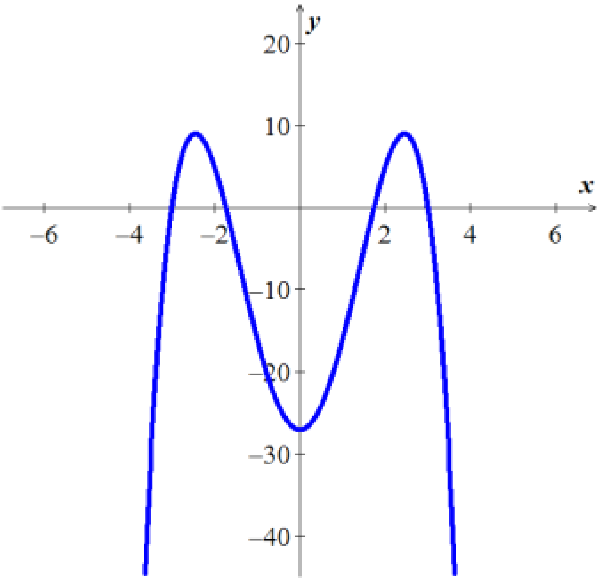
Solution

$$x^2=\frac{-12\pm\sqrt{36}}{-2}$$

$$=\begin{cases} \dfrac{-12-6}{-2}=9 \\ \dfrac{-12+6}{-2}=3 \end{cases}$$

$$\to\begin{cases} x^2=9 \\ x^2=3 \end{cases} \Rightarrow \begin{matrix} x=\pm3 \\ x=\pm\sqrt{3} \end{matrix}$$

	-3	$-\sqrt{3}$	$\sqrt{3}$	3	
−	+	−	+	−	

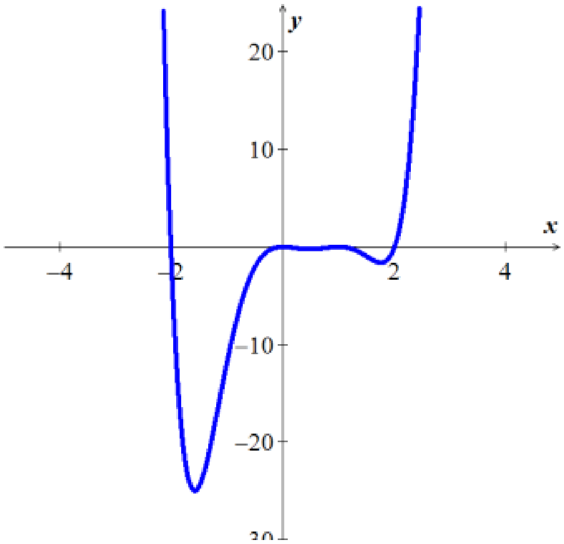


$f(x)>0 \quad \left(-3,\ -\sqrt{3}\right)\cup\left(\sqrt{3},\ 3\right)$

$f(x)<0 \quad (-\infty,\ -3)\cup\left(-\sqrt{3},\ \sqrt{3}\right)\cup(3,\ \infty)$

Exercise

Let $f(x)=x^2(x+2)(x-1)^2(x-2)$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.



Solution

The zeros are: −2, 2, 0, 0, 1, 1

−2 **0,0** **1,1** 2

+	−	+

$f(x)>0 \quad \boxed{(-\infty,\ -2)\cup(2,\ \infty)}$

$f(x)<0 \quad \boxed{(-2,\ 0)\cup(0,\ 1)\cup(1,\ 2)}$

Exercise

Let $f(x)=2x^3+11x^2-7x-6$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

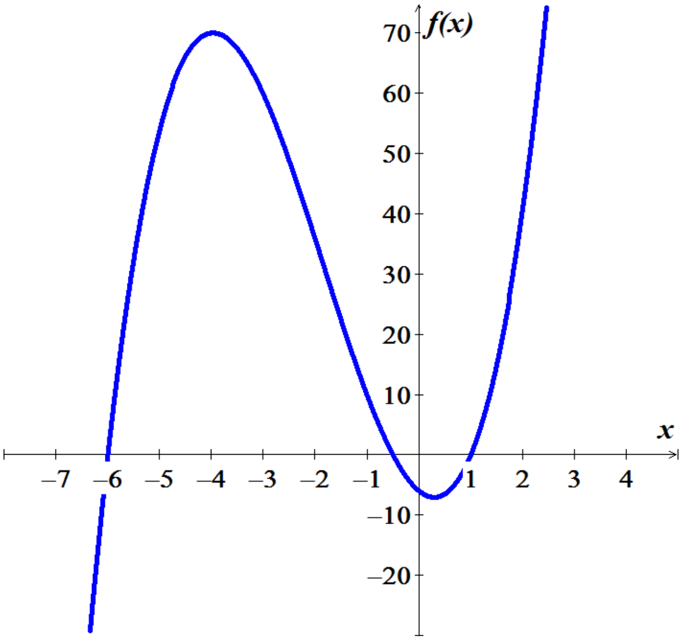
Solution

$$possibilities: \pm\left\{\frac{6}{2}\right\}=\pm\left\{\frac{1,\ 2,\ 3,\ 6}{1,\ 2}\right\}$$

$$=\pm\left\{1,\ 2,\ 3,\ 5,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\begin{array}{r|rrrr} 1 & 2 & 11 & -7 & -6 \\ & & 2 & 13 & 6 \\ \hline & 2 & 13 & 6 & \boxed{0} \end{array} \rightarrow 2x^2+13x+6=0$$

The zeros are: $\boxed{x=1,\ -\frac{1}{2},\ -6}$



−6		$-\frac{1}{2}$		1
−		+	−	+

$f(x)>0 \quad \boxed{\left(-6,\ -\frac{1}{2}\right)\cup(1,\ \infty)}$

$f(x)<0 \quad \boxed{(-\infty,\ -6)\cup\left(-\frac{1}{2},\ 1\right)}$

Exercise

Let $f(x)=x^3+2x^2-5x-6$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$\textit{possibilities}\ :\pm\left\{\frac{6}{1}\right\}=\pm\{1,\ 2,\ 3,\ 6\}$$

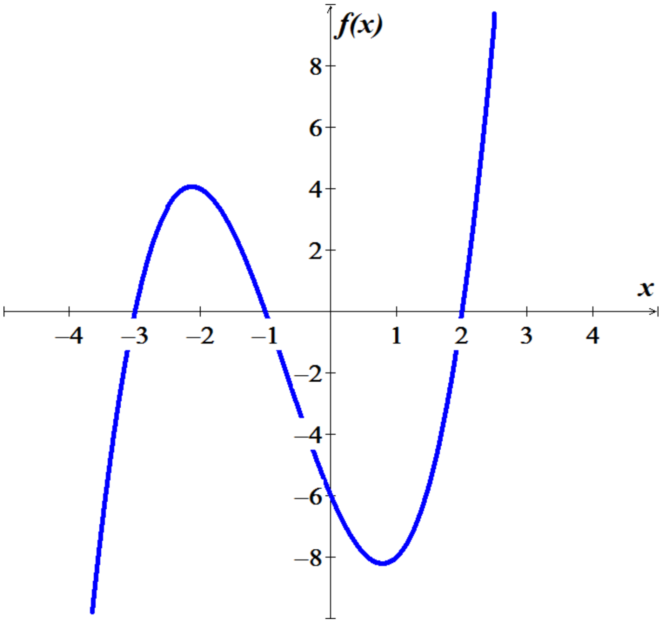
$$\begin{array}{r|rrrr}
-1 & 1 & 2 & -5 & -6 \\
 & & -1 & -1 & 6 \\
\hline
 & 1 & 1 & -6 & \boxed{0}
\end{array}\ \rightarrow\ x^2+x-6=0$$

The zeros are: $x=-1,\ -3,\ 2$

$$\begin{array}{c|c|c|c}
\multicolumn{1}{r}{-3} & \multicolumn{1}{r}{-1} & \multicolumn{1}{r}{2} & \\
- & + & - & +
\end{array}$$

$f(x)>0$ $\quad(-3,\ -1)\cup(2,\ \infty)$

$f(x)<0$ $\quad(-\infty,\ -3)\cup(-1,\ 2)$



Exercise

Let $f(x)=x^3+8x^2+11x-20$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$\textit{possibilities}\ :\pm\left\{\frac{20}{1}\right\}=\pm\{1,\ 2,\ 4,\ 5,\ 20,\ 20\}$$

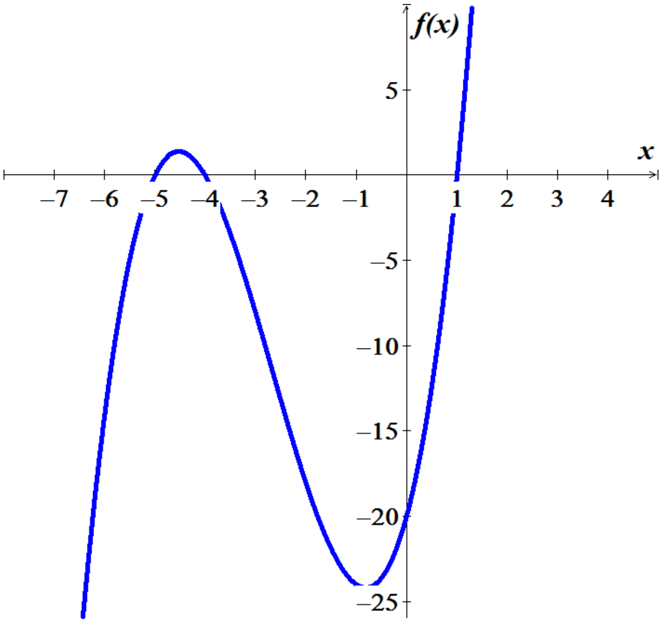
$$\begin{array}{r|rrrr}
1 & 1 & 8 & 11 & -20 \\
 & & 1 & 9 & 20 \\
\hline
 & 1 & 9 & 20 & \boxed{0}
\end{array}\ \rightarrow\ x^2+9x+20=0$$

The zeros are: $x=-5,\ -4,\ 1$

$$\begin{array}{c|c|c|c}
\multicolumn{1}{r}{-5} & \multicolumn{1}{r}{-4} & \multicolumn{1}{r}{1} & \\
- & + & - & +
\end{array}$$

$f(x)>0$ $\quad(-5,\ -1)\cup(1,\ \infty)$

$f(x)<0$ $\quad(-\infty,\ -5)\cup(-4,\ 1)$



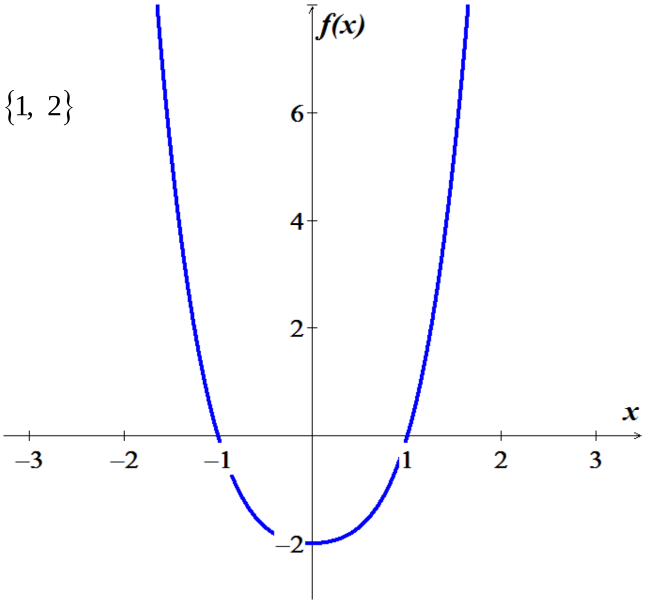
Exercise

Let $f(x) = x^4 + x^2 - 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

possibilities: $\pm\{1,\ 2\}$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & 1 & 0 & -2 \\ & & 1 & 1 & 2 & 1 \\ \hline -1 & 1 & 1 & 2 & 2 & 0 \\ & & -1 & 0 & -2 & \\ \hline & 1 & 0 & 2 & 0 & \end{array} \quad \to x^3 + x^2 + 2x + 1 = 0 \ \to \pm\{1,\ 2\}$$



$\to x^2 + 2 = 0 \quad \Rightarrow \quad \boxed{x = \pm i\sqrt{2}}$

The zeros are: $\boxed{x = \pm 1}$

$$\begin{array}{ccccc} & -1 & & 1 & \\ + & | & - & | & + \end{array}$$

$f(x) > 0 \quad \boxed{(-\infty,\ -1) \cup (1,\ \infty)}$

$f(x) < 0 \quad \boxed{(-1,\ 1)}$

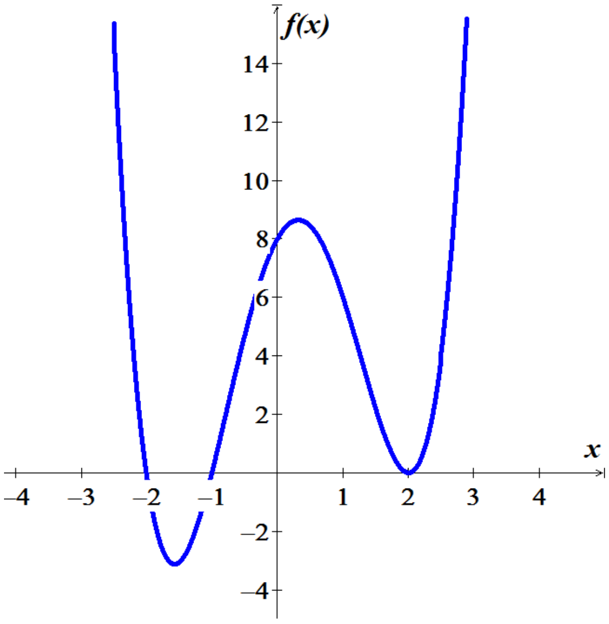
Exercise

Let $f(x) = x^4 - x^3 - 6x^2 + 4x + 8$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

possibilities: $\pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -6 & 4 & 8 \\ & & -1 & 2 & 4 & -8 \\ \hline -2 & 1 & -2 & -4 & 8 & 0 \\ & & -2 & 8 & -8 & \\ \hline & 1 & -4 & 4 & 0 & \end{array} \quad \to x^3 - 2x^2 - 4x + 8 = 0 \ \to \pm\{1,\ 2,\ 4,\ 8\}$$



$\to x^2 - 4x + 4 = 0 \quad \Rightarrow \quad \boxed{x = 2,\ 2}$

The zeros are: $\boxed{x = -2,\ -1,\ 2,\ 2}$

$$\begin{array}{ccccccc} & -2 & & -1 & & 2 & \\ + & | & - & | & + & \| & + \end{array}$$

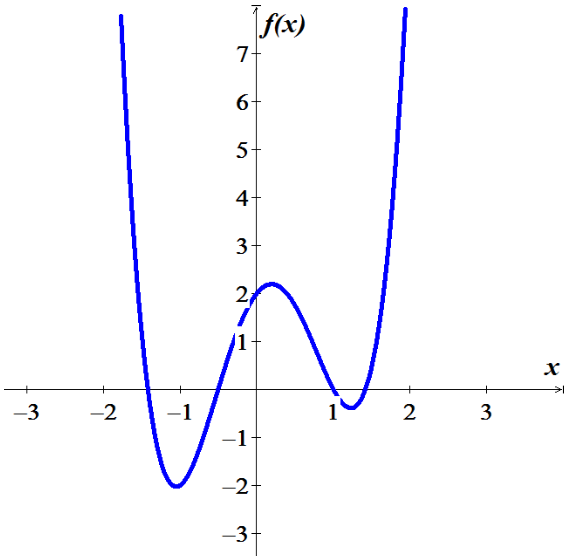
$f(x) > 0 \quad \underline{(-\infty,\ -1) \cup (1,\ \infty)}|$

$f(x) < 0 \quad \underline{(-1,\ 1)}|$

Exercise

Let $f(x) = 2x^4 - x^3 - 5x^2 + 2x + 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution



$possibilities: \ \pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & -1 & -5 & 2 & 2 \\
 & & 2 & 1 & -4 & -2 \\
\hline
-\frac{1}{2} & 2 & 1 & -4 & -2 & 0 \\
 & & -1 & 0 & 2 & \\
\hline
 & 2 & 0 & -4 & 0 &
\end{array}$$

$\to 2x^3+x^2-4x-2=0$

$\to \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$\to 2x^2 - 4 = 0 \Rightarrow \underline{x = \pm\sqrt{2}}|$

The zeros are: $\underline{x = -\frac{1}{2},\ 1,\ -\sqrt{2},\ \sqrt{2}}|$

$$\begin{array}{c|c|c|c|c|c|c|c|c}
 & -\sqrt{2} & & -\frac{1}{2} & & 1 & & \sqrt{2} & \\
+ & & - & & + & & - & & +
\end{array}$$

$f(x) > 0 \quad \underline{\left(-\infty,\ -\sqrt{2}\right) \cup \left(-\frac{1}{2},\ 1\right) \cup \left(\sqrt{2},\ \infty\right)}|$

$f(x) < 0 \quad \underline{\left(-\sqrt{2},\ -\frac{1}{2}\right) \cup \left(1,\ \sqrt{2}\right)}|$

Exercise

Let $f(x) = 4x^5 - 8x^4 - x + 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\begin{aligned} f(x) &= 4x^4(x-2) - (x-2) \\ &= (x-2)\left(4x^4 - 1\right) = 0 \end{aligned}$$

$$4x^4 - 1 = 0 \quad \Rightarrow \quad \begin{cases} x^2 = -\frac{1}{2} & \mathbb{C} \\ x^2 = \frac{1}{2} & x = \pm\frac{\sqrt{2}}{2} \end{cases}$$

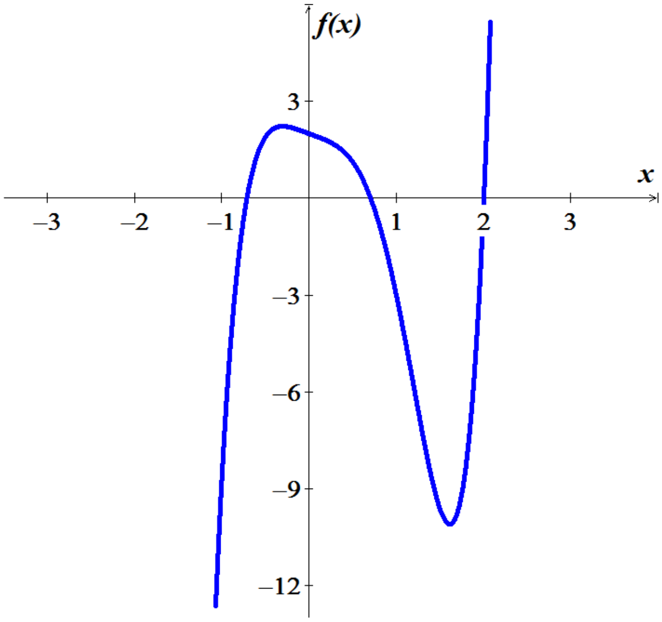
The zeros are: $\underline{x = 2,\ -\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}}$

$-\frac{\sqrt{2}}{2}$ $\quad\frac{\sqrt{2}}{2}\quad$ 2

− + − +

$f(x) > 0 \quad \underline{\left(-\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right) \cup (2,\ \infty)}$

$f(x) < 0 \quad \underline{\left(-\infty,\ -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2},\ 2\right)}$



Exercise

Let $f(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

possibilities: $\pm\left\{\frac{36}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 9,\ 12,\ 18,\ 36\}$

1	1	–7	19	–37	60	–36
		1	–6	13	–24	36
3	1	–6	13	–24	36	0
		3	–9	12	–36	
3	1	–3	4	–12	0	
		3	0	12		
	1	0	4	0		

$x^4 - 6x^3 + 13x^2 - 24x + 36 = 0 \ \rightarrow\ \pm\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$x^3 - 3x^2 + 4x - 12 = 0 \ \rightarrow\ \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$x^2 + 4 = 0 \quad \Rightarrow \quad x = \pm 2i$

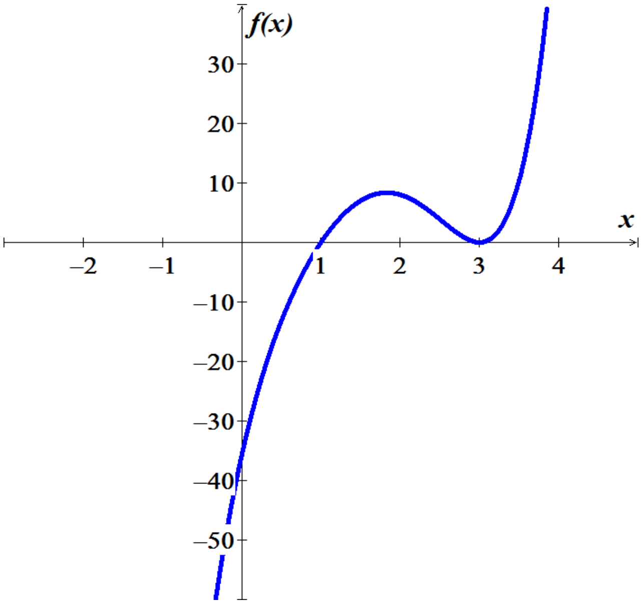
The zeros are: $\underline{x = 1,\ 3,\ 3}$

$1 \quad 3$

− + +

$f(x) > 0 \quad \underline{(1,\ 3) \cup (3,\ \infty)}$

$f(x) < 0 \quad \underline{(-\infty,\ 1)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - x^2 - 10x - 8$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & 0 \end{array} \rightarrow \quad x^2 - 2x - 8 = 0$$

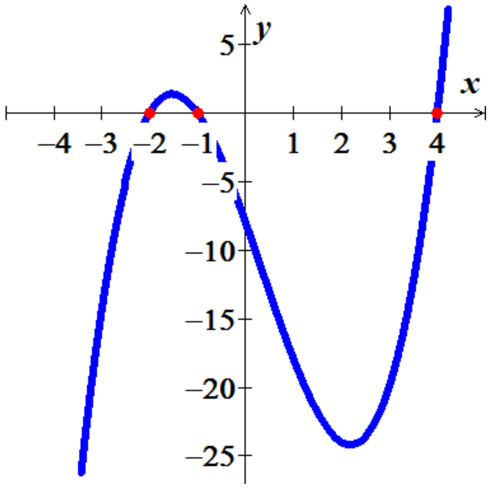
$\boxed{x = -1,\ -2,\ 4}$

−2 −1 4

−	+	−	+

$f(x) > 0 \quad \boxed{(-2,\ -1) \cup (4,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -2) \cup (-1,\ 4)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + x^2 - 14x - 24$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & 0 \end{array} \rightarrow \quad x^2 - x - 12 = 0$$

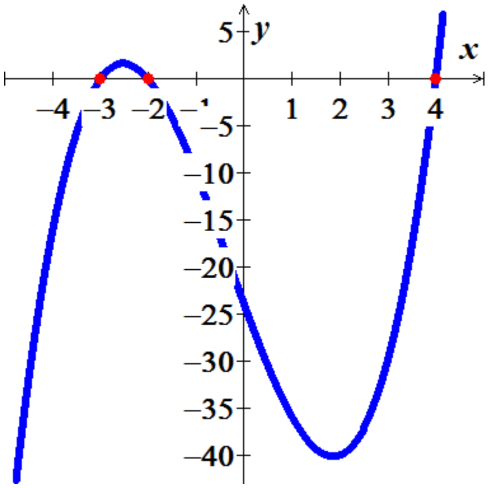
$\boxed{x = -2,\ -3,\ 4}$

−3 −2 4

−	+	−	+

$f(x) > 0 \quad \boxed{(-3,\ -2) \cup (4,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -3) \cup (-2,\ 4)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 - 3x^2 - 17x + 30$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{30}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30,\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{15}{2}\right\}$

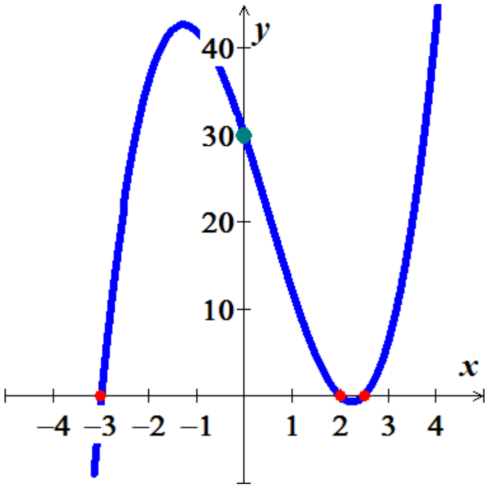
$$\begin{array}{r|rrrr} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & \boxed{0} \end{array} \rightarrow \quad 2x^2 + x - 15 = 0$$

$x = 2,\ -3,\ \frac{5}{2}$

$$\begin{array}{c|c|c|c} & -3 \quad\quad 2 & \quad\quad \frac{5}{2} & \\ \hline - & + & - & + \end{array}$$

$f(x) > 0 \quad (-3,\ 2) \cup \left(\frac{5}{2},\ \infty\right)$

$f(x) < 0 \quad (-\infty,\ -3) \cup \left(2,\ \frac{5}{2}\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 12x^3 + 8x^2 - 3x - 2$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{12}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ \frac{1}{6},\ \frac{1}{12}\right\}$

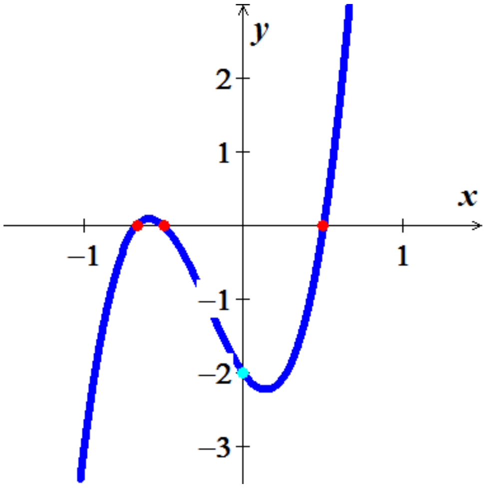
$$\begin{array}{r|rrrr} \frac{1}{2} & 12 & 8 & -3 & -2 \\ & & 6 & 7 & 2 \\ \hline & 12 & 14 & 4 & \boxed{0} \end{array} \rightarrow \quad 12x^2 + 14x + 4 = 0$$

$x = \frac{1}{2},\ -\frac{1}{2},\ -\frac{2}{3}$

$$\begin{array}{c|c|c|c} & -\frac{2}{3} \quad\quad -\frac{1}{2} & \quad\quad \frac{1}{2} & \\ \hline - & + & - & + \end{array}$$

$f(x) > 0 \quad \left(-\frac{2}{3},\ -\frac{1}{2}\right) \cup \left(\frac{1}{2},\ \infty\right)$

$f(x) < 0 \quad \left(-\infty,\ -\frac{2}{3}\right) \cup \left(-\frac{1}{2},\ \frac{1}{2}\right)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3+x^2-6x-8$$

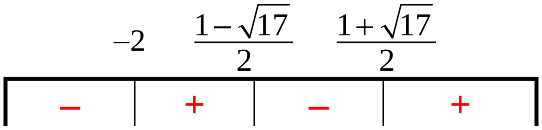
Solution

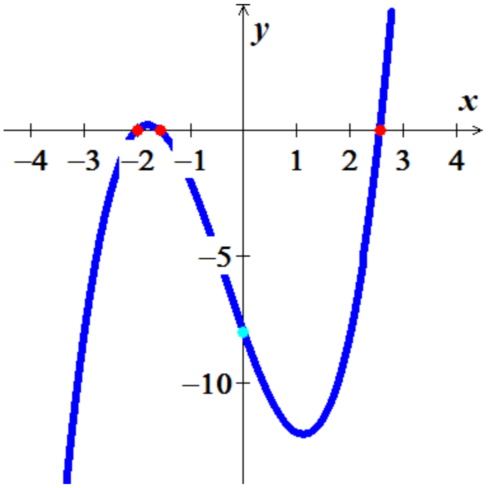
possibilities for $\frac{c}{d}:\pm\left\{\frac{8}{1}\right\}=\pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -6 & -8 \\ & & -2 & 2 & 8 \\ \hline & 1 & -1 & -4 & \boxed{0} \end{array} \to \quad x^2-x-4=0$$

$$x=\frac{1\pm\sqrt{1+16}}{2}$$

$$x=-2,\ \frac{1\pm\sqrt{17}}{2}$$





$f(x)>0 \quad \left(-2,\ \frac{1-\sqrt{17}}{2}\right)\cup\left(\frac{1+\sqrt{17}}{2},\ \infty\right)$

$f(x)<0 \quad (-\infty,\ -2)\cup\left(\frac{1-\sqrt{17}}{2},\ \frac{1+\sqrt{17}}{2}\right)$

Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3-19x-30$$

Solution

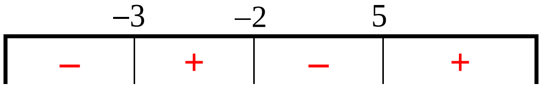
possibilities for $\frac{c}{d}:\pm\left\{\frac{30}{1}\right\}=\pm\{1,\ 2,\ 3,\ 5,\ 6,\ 15,\ 30\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & \boxed{0} \end{array} \to \quad x^2-2x-15$$

$$x=\frac{2\pm\sqrt{4+60}}{2}$$

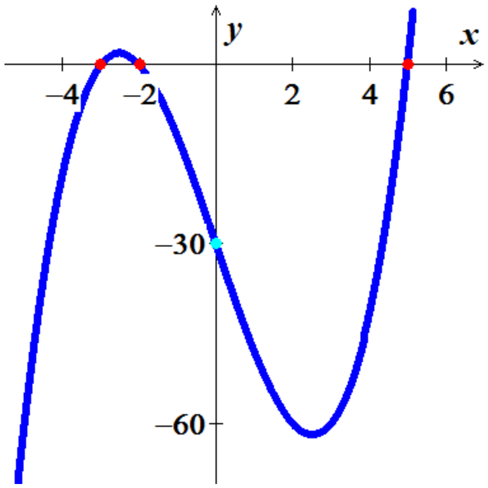
$$= \begin{cases} \frac{2-8}{2} = -3 \\ \frac{2+8}{2} = 5 \end{cases}$$

$\underline{x = -2, \ -3, \ 5}$



$f(x) > 0 \quad \underline{(-3, \ -2) \cup (5, \ \infty)}$

$f(x) < 0 \quad \underline{(-\infty, \ -3) \cup (-2, \ 5)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 + x^2 - 25x + 12$$

Solution

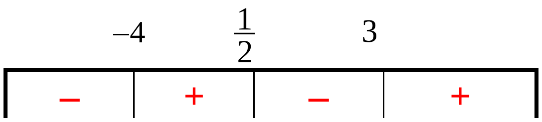
possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{2}\right\} = \pm\left\{1, \ 2, \ 3, \ 4, \ 6, \ 12, \ \frac{1}{2}, \ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrr} 3 & 2 & 1 & -25 & 12 \\ & & 6 & 21 & -12 \\ \hline & 2 & 7 & -4 & \boxed{0} \end{array} \quad \to \quad 2x^2 + 7x - 4$$

$$x = \frac{-7 \pm \sqrt{49+32}}{4}$$

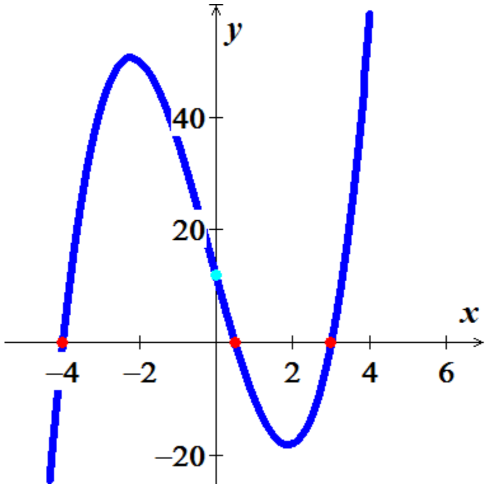
$$= \begin{cases} \frac{-7-9}{4} = -4 \\ \frac{-7+9}{4} = \frac{1}{2} \end{cases}$$

$\underline{x = -4, \ \frac{1}{2}, \ 3}$



$f(x) > 0 \quad \underline{\left(-4, \ \frac{1}{2}\right) \cup (3, \ \infty)}$

$f(x) < 0 \quad \underline{(-\infty, \ -1) \cup \left(\frac{1}{2}, \ 3\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^3 + 11x^2 - 6x - 8$$

Solution

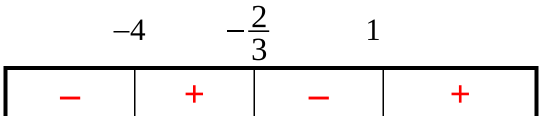
possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ 8,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{8}{3}\right\}$

$$\begin{array}{r|rrrr} 1 & 3 & 11 & -6 & -8 \\ & & 3 & 14 & 8 \\ \hline & 3 & 14 & 8 & \boxed{0} \end{array} \quad \rightarrow \quad 3x^2 + 14x + 8$$

$$x = \frac{-14 \pm \sqrt{196 - 96}}{6}$$

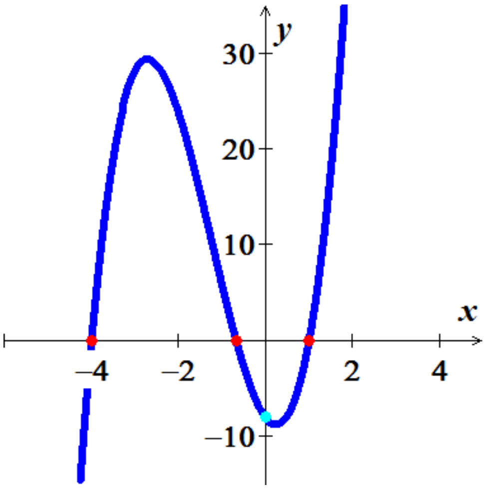
$$= \begin{cases} \frac{-14-10}{6} = -4 \\ \frac{-14+10}{6} = -\frac{2}{3} \end{cases}$$

$$x = -4,\ \ -\frac{2}{3},\ 1$$



$f(x) > 0$ $\left(-4,\ -\frac{2}{3}\right) \cup (1,\ \infty)$

$f(x) < 0$ $(-\infty,\ -4) \cup \left(-\frac{2}{3},\ 1\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 + 9x^2 - 2x - 9$$

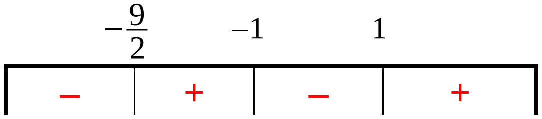
Solution

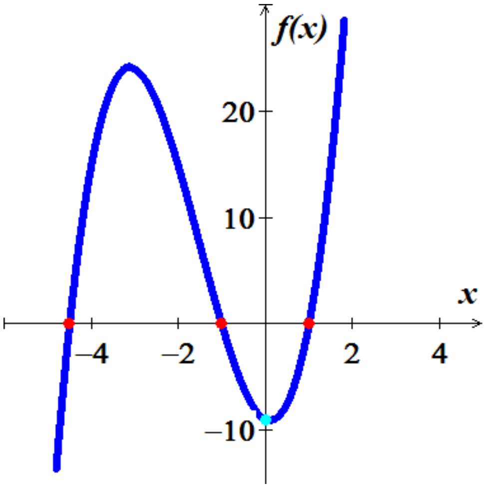
possibilities for $\frac{c}{d} : \pm\left\{\frac{9}{2}\right\} = \pm\left\{1,\ 3,\ 9,\ \frac{1}{2},\ \frac{3}{2},\ \frac{9}{2}\right\}$

$$\begin{array}{r|rrrr} 1 & 2 & 9 & -2 & -9 \\ & & 2 & 11 & 9 \\ \hline & 2 & 11 & 9 & \boxed{0} \end{array} \quad \rightarrow \quad 2x^2 + 11x + 9$$

$x = -1, \ -\frac{9}{2}$ $\qquad$ $a - b + c = 0 \ \rightarrow \ x = -1, \ -\frac{c}{a}$

$x = -\frac{9}{2}, \ -1, \ 1$





$f(x) > 0 \quad \left(-\frac{9}{2}, \ -1\right) \cup (1, \ \infty)$

$f(x) < 0 \quad \left(-\infty, \ -\frac{9}{2}\right) \cup (-1, \ 1)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + 3x^2 - 6x - 8$$

Solution

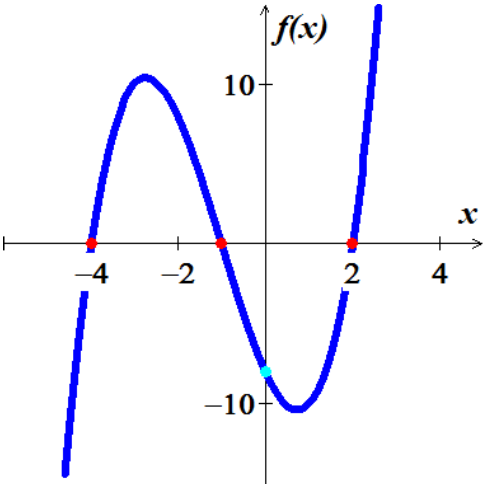
possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1, \ 2, \ 4, \ 8\}$

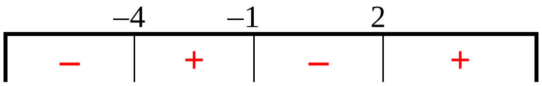
$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ & & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & \boxed{0} \end{array} \rightarrow \quad x^2 + 2x - 8 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 32}}{2}$$

$$= \begin{cases} \frac{-2-6}{2} = -4 \\ \frac{-2+6}{2} = 2 \end{cases}$$

$x = -4, \ -1, \ 2$





$f(x) > 0 \quad (-4, \ -1) \cup (2, \ \infty)$

$f(x) < 0 \quad (-\infty, \ -4) \cup (-1, \ 2)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

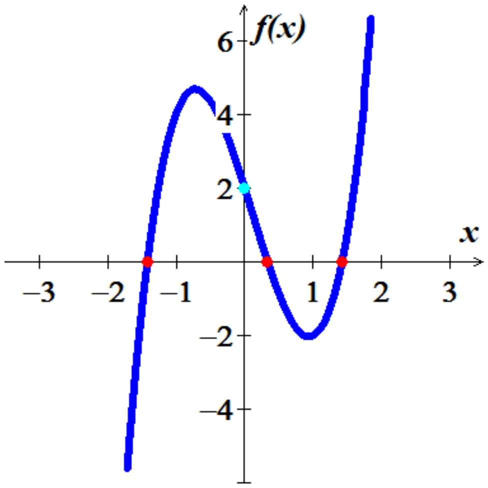
$$f(x) = 3x^3 - x^2 - 6x + 2$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{3}\right\} = \pm\left\{1,\ 2,\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -6 & 2 \\ & & 1 & 0 & -2 \\ \hline & 3 & 0 & -6 & \boxed{0} \end{array} \quad \rightarrow \quad 3x^2 - 6 = 0$$

$$x = \frac{1}{3},\ \pm\sqrt{2}$$



$-\sqrt{2}$ $\frac{1}{3}$ $\sqrt{2}$

$-$	$+$	$-$	$+$

$f(x) > 0$ $\quad \left(-\sqrt{2},\ \frac{1}{3}\right) \cup \left(\sqrt{2},\ \infty\right)$

$f(x) < 0$ $\quad \left(-\infty,\ -\sqrt{2}\right) \cup \left(\frac{1}{3},\ \sqrt{2}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 8x^2 + 8x + 24$$

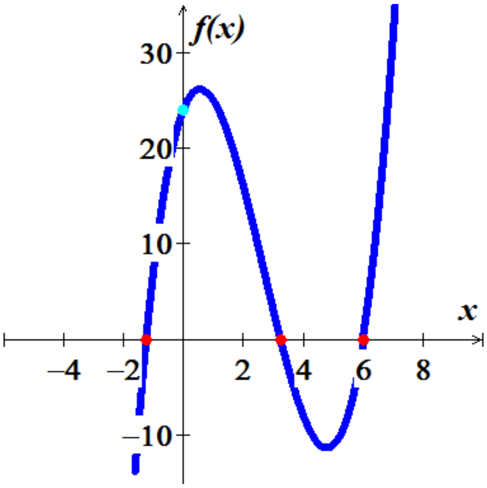
Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

$$\begin{array}{r|rrrr} 6 & 1 & -8 & 8 & 24 \\ & & 6 & -12 & -24 \\ \hline & 1 & -2 & -4 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - 2x - 4 = 0$$

$$x = \frac{2 \pm \sqrt{4+16}}{2}$$

$$x = 6,\ 1 \pm \sqrt{5}$$



$1-\sqrt{5}$ $1+\sqrt{5}$ 6

$-$	$+$	$-$	$+$

$f(x) > 0 \quad \boxed{\left(1-\sqrt{5},\ 1+\sqrt{5}\right) \cup (6,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ 1-\sqrt{5}\right) \cup \left(1+\sqrt{5},\ 6\right)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 7x^2 - 7x + 69$$

Solution

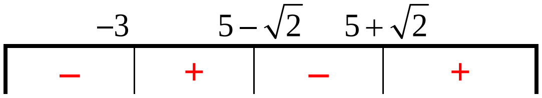
possibilities for $\frac{c}{d} : \pm\left\{\frac{69}{1}\right\} = \pm\{1,\ 3,\ 23,\ 69\}$

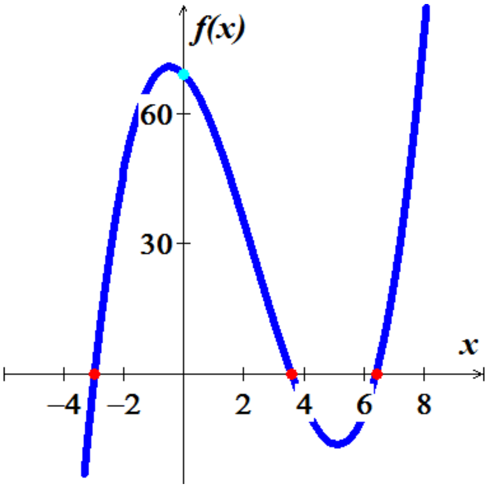
$$\begin{array}{r|rrrr} -3 & 1 & -7 & -7 & 69 \\ & & -3 & 30 & -69 \\ \hline & 1 & -10 & 23 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - 10x + 23 = 0$$

$$x = \frac{10 \pm \sqrt{100-92}}{2}$$

$$= \frac{10 \pm 2\sqrt{2}}{2}$$

$$\boxed{x = -3,\ 5 \pm \sqrt{2}}$$





$f(x) > 0 \quad \boxed{\left(-3,\ 5-\sqrt{2}\right) \cup \left(5+\sqrt{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ -3\right) \cup \left(5-\sqrt{2},\ 5+\sqrt{2}\right)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 3x - 2$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -3 & -2 \\ & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array} \quad \to \quad x^2 - x - 2 = 0$$

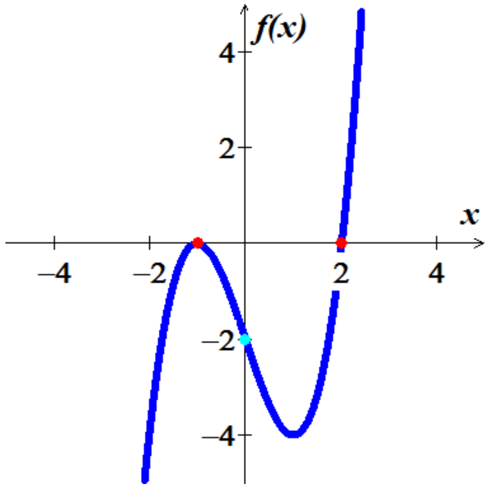
$x = -1,\ 2$ $\qquad\qquad$ $a - b + c = 0 \quad \to \quad x = -1,\ -\frac{c}{a}$

$x = -1,\ -1,\ 2$

$$\begin{array}{ccccc} & -1 & & 2 & \\ - & \| & - & | & + \end{array}$$

$f(x) > 0 \quad (2,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -1) \cup (-1,\ 2)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 2x + 1$$

Solution

possibilities for $\frac{c}{d} : \pm\{1\}$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -2 & 1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & -1 & \boxed{0} \end{array} \quad \to \quad x^2 + x - 1 = 0$$

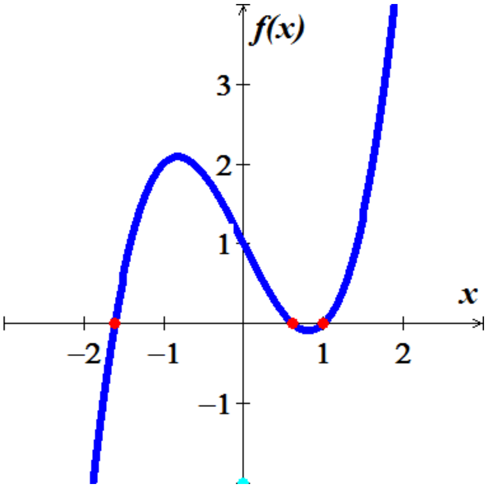
$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = 1,\ \frac{-1 \pm \sqrt{5}}{2}$$

$$\begin{array}{ccccccc} & \frac{-1-\sqrt{5}}{2} & & \frac{-1+\sqrt{5}}{2} & & 1 & \\ - & | & + & | & - & | & + \end{array}$$

$$f(x) > 0 \quad \left(\frac{-1-\sqrt{5}}{2},\ \frac{-1+\sqrt{5}}{2}\right) \cup (1,\ \infty)$$

$$f(x) < 0 \quad \left(-\infty,\ \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2},\ 1\right)$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f\left(x\right)$

$$f\left(x\right) = x^3 - 2x^2 - 11x + 12$$

Solution

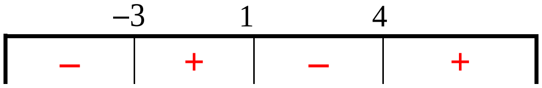
possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -11 & 12 \\ & & 1 & -1 & 12 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array} \quad \to \quad x^2 - x - 12 = 0$$

$$x = \frac{1 \pm \sqrt{1+48}}{2}$$

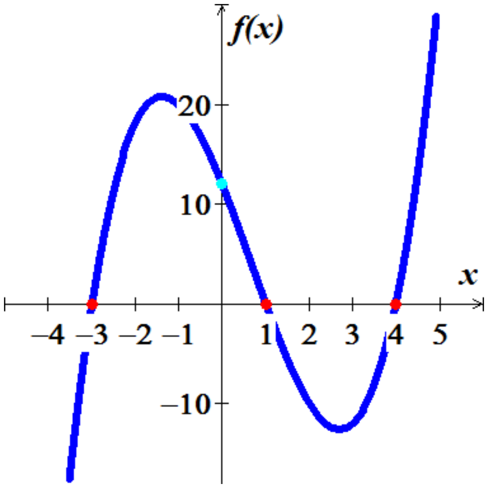
$$= \begin{cases} \frac{1-7}{2} = -3 \\ \frac{1+7}{2} = 4 \end{cases}$$

$x = -3,\ 1,\ 4$



$f(x) > 0 \quad (-3,\ 1) \cup (4,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -3) \cup (1,\ 4)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f\left(x\right)$

$$f\left(x\right) = x^3 - 2x^2 - 7x - 4$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{4}{1}\right\} = \pm\{1,\ 2,\ 4\}$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -7 & -4 \\ & & -1 & 3 & 4 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array} \quad \to \quad x^2 - 3x - 4 = 0$$

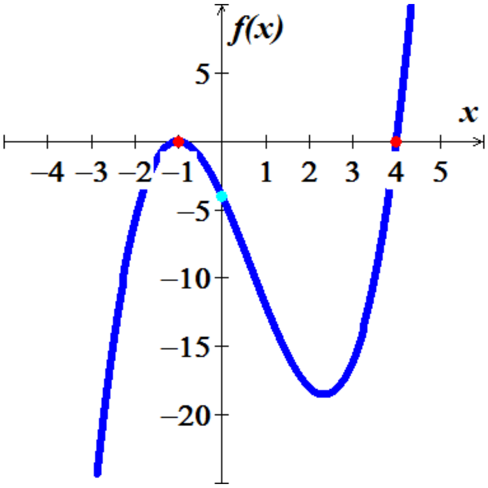
$x = -1,\ 4$ $\qquad a - b + c = 0 \quad \to \quad x = -1,\ -\frac{c}{a}$

$x = -1,\ -1,\ 4$

	-1		4	
$-$		$-$		$+$

$f(x) > 0 \quad (4,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -1) \cup (-1,\ 4)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 10x - 12$$

Solution

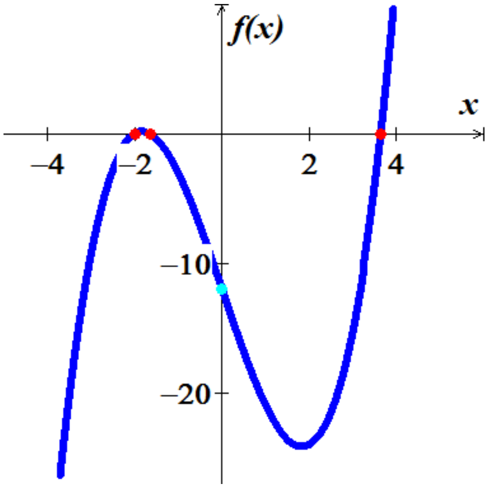
possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -10 & -12 \\ & & -2 & 4 & 12 \\ \hline & 1 & -2 & -6 & \boxed{0} \end{array} \quad \to \quad x^2 - 2x - 6 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 24}}{2}$$

$$= \frac{2 \pm 2\sqrt{7}}{2}$$

$$x = -2,\ 1 \pm \sqrt{7}$$



	-2		$1-\sqrt{7}$		$1+\sqrt{7}$	
$-$		$+$		$-$		$+$

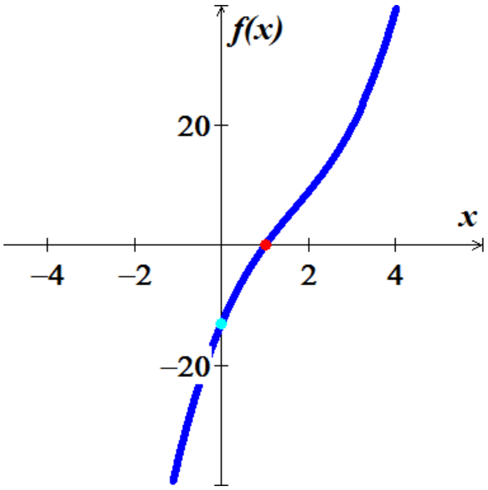
$f(x) > 0 \quad \left(-2,\ 1-\sqrt{7}\right) \cup \left(1+\sqrt{7},\ \infty\right)$

$f(x) < 0 \quad (-\infty,\ -2) \cup \left(1-\sqrt{7},\ 1+\sqrt{7}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 5x^2 + 17x - 13$$



Solution

$$\textit{possibilities for } \frac{c}{d} : \pm\left\{\frac{13}{1}\right\} = \pm\{1,\ 13\}$$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 17 & -13 \\ & & 1 & -4 & 13 \\ \hline & 1 & -4 & 13 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - 4x + 13 = 0$$

$$\begin{aligned} x &= \frac{4 \pm \sqrt{16 - 52}}{2} \\ &= \frac{4 \pm 6i}{2} \end{aligned}$$

$$\underline{x = 1,\ 2 \pm 3i\,}|$$

$$\begin{array}{c|c} \multicolumn{2}{c}{1} \\ \hline - & + \end{array}$$

$f(x) > 0 \quad (1,\ \infty)$

$f(x) < 0 \quad (-\infty,\ 1)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^3 + 25x^2 - 24x + 5$$

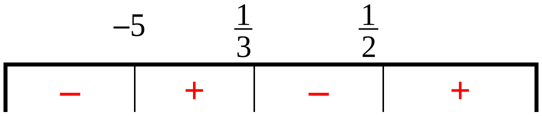
Solution

$$\textit{possibilities for } \frac{c}{d} : \pm\left\{\frac{5}{6}\right\} = \pm\left\{1,\ 5,\ \frac{1}{2},\ \frac{5}{2},\ \frac{1}{3},\ \frac{5}{3},\ \frac{1}{6},\ \frac{5}{6}\right\}$$

$$\begin{array}{r|rrrr} -5 & 6 & 25 & -24 & 5 \\ & & -30 & 25 & -5 \\ \hline & 6 & -5 & 1 & \boxed{0} \end{array} \quad \rightarrow \quad 6x^2 - 5x + 1 = 0$$

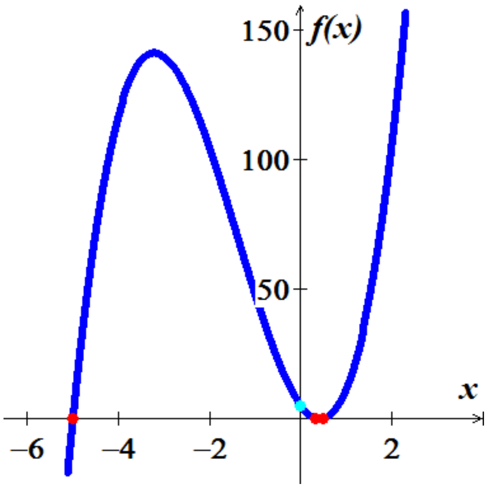
$$\begin{aligned} x &= \frac{5 \pm \sqrt{25 - 24}}{12} \\ &= \begin{cases} \dfrac{5-1}{12} = \dfrac{1}{3} \\ \dfrac{5+1}{12} = \dfrac{1}{2} \end{cases} \end{aligned}$$

$x = -5,\ \frac{1}{3},\ \frac{1}{2}$



$f(x) > 0 \quad \left(-5,\ \frac{1}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)$

$f(x) < 0 \quad (-\infty,\ -5) \cup \left(\frac{1}{3},\ \frac{1}{2}\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 8x^3 + 18x^2 + 45x + 27$$

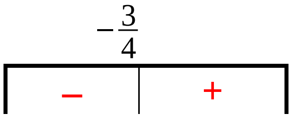
Solution

possibilities : $\pm\left\{\frac{27}{8}\right\} = \pm\left\{\frac{1,\ 3,\ 9,\ 27}{1,\ 2,\ 4,\ 8}\right\}$

$$= \pm\left\{1,\ 3,\ 9,\ 27,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{3}{2},\ \frac{3}{4},\ \frac{3}{8},\ \frac{9}{2},\ \frac{9}{4},\ \frac{9}{8},\ \frac{27}{2},\ \frac{27}{4},\ \frac{27}{8}\right\}$$

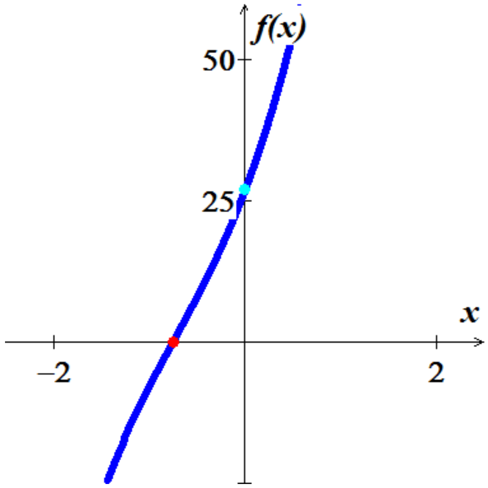
$$\begin{array}{r|rrrr} -\frac{3}{4} & 8 & 18 & 45 & 27 \\ & & -6 & -9 & -27 \\ \hline & 8 & 12 & 36 & \boxed{0} \end{array} \quad \rightarrow\ 8x^2 + 12x + 36 = 0$$

$x = -\frac{3}{4},\ -\frac{3}{4} \pm i\frac{3\sqrt{7}}{4}$



$f(x) > 0 \quad \left(-\frac{3}{4},\ \infty\right)$

$f(x) < 0 \quad \left(-\infty,\ -\frac{3}{4}\right)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=3x^3-x^2+11x-20$$

Solution

$$possibilities:\ \pm\left\{\frac{20}{3}\right\}=\pm\left\{\frac{1,\ 2,\ 4,\ 5,\ 10,\ 20}{1,\ 3}\right\}$$

$$=\pm\left\{1,\ 2,\ 4,\ 5,\ 10,\ 20,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{5}{3},\ \frac{10}{3},\ \frac{20}{3}\right\}$$

$$\begin{array}{c|cccc} \frac{4}{3} & 3 & -1 & 11 & -20 \\ & & 4 & 4 & 20 \\ \hline & 3 & 3 & 15 & \boxed{0} \end{array} \quad \to\ 3x^2+3x+15=0$$

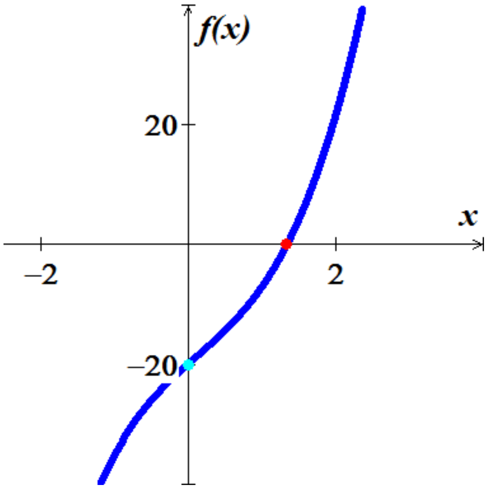
$$x=\frac{-3\pm\sqrt{9-180}}{6}$$

$$x=\frac{4}{3},\ -\frac{1}{2}\pm i\frac{\sqrt{19}}{2}$$

$$\begin{array}{c|c} \multicolumn{2}{r}{\frac{4}{3}} \\ \hline - & + \end{array}$$

$f(x)>0$ $\left(\frac{4}{3},\ \infty\right)$

$f(x)<0$ $\left(-\infty,\ \frac{4}{3}\right)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^4-x^3-9x^2+3x+18$$

Solution

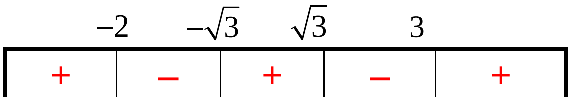
$$possibilities\ for\ \frac{c}{d}:\pm\left\{\frac{18}{1}\right\}=\pm\{1,\ 2,\ 3,\ 6,\ 9,\ 18\}$$

$$\begin{array}{c|ccccc} -2 & 1 & -1 & -9 & 3 & 18 \\ & & -2 & 6 & 6 & -18 \\ \hline 3 & 1 & -3 & -3 & 9 & 0 \\ & & 3 & 0 & -9 & \\ \hline & 1 & 0 & -3 & 0 & \end{array}$$

$\to x^3-3x^2-3x+9=0 \to \pm\left\{\frac{9}{1}\right\}=\pm\{1,\ 3,\ 9\}$

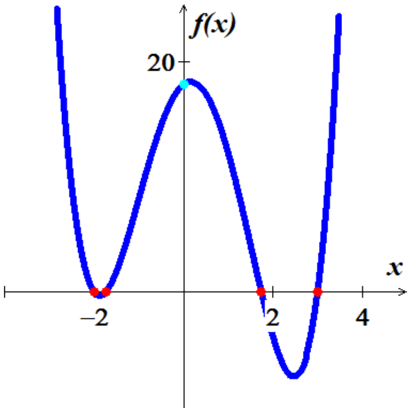
$\to x^2-3=0 \Rightarrow x=\pm\sqrt{3}$

$$x=-2,\ 3,\ \pm\sqrt{3}$$



$f(x) > 0 \quad \boxed{\left(-\infty,\ -2\right)\cup\left(-\sqrt{3},\ \sqrt{3}\right)\cup\left(3,\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-2,\ -\sqrt{3}\right)\cup\left(\sqrt{3},\ 3\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 9x^3 + 9x^2 + x - 3$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & -9 & 9 & 1 & -3 \\
 & & 2 & -7 & 2 & 3 \\
\hline
1 & 2 & -7 & 2 & 3 & 0 \\
 & & 2 & -5 & -3 & \\
\hline
 & 2 & -5 & -3 & 0 &
\end{array}$$

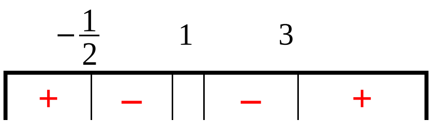
$\rightarrow\ 2x^3 - 7x^2 + 2x + 3 = 0 \ \rightarrow\ \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$\rightarrow\ 2x^2 - 5x - 3 = 0$

$$x = \frac{5 \pm \sqrt{25+24}}{4}$$

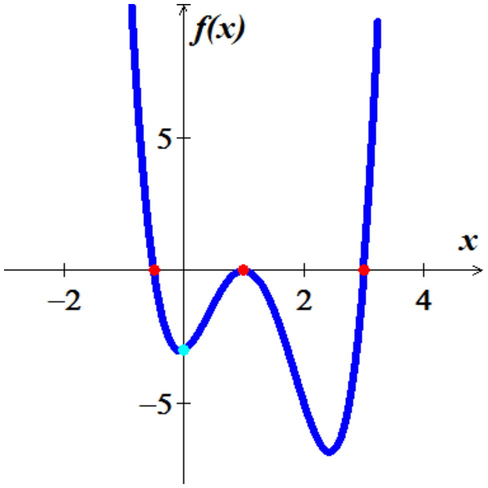
$$= \begin{cases} \frac{5-7}{4} = -\frac{1}{2} \\ \frac{5+7}{4} = 3 \end{cases}$$

$$\boxed{x = 1,\ 1,\ -\frac{1}{2},\ 3}$$



$f(x) > 0 \quad \boxed{\left(-\infty,\ -\frac{1}{2}\right)\cup\left(3,\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-\frac{1}{2},\ 1\right)\cup\left(1,\ 3\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 + 5x^3 - 17x^2 - 6x$$

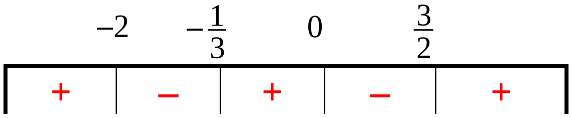
Solution

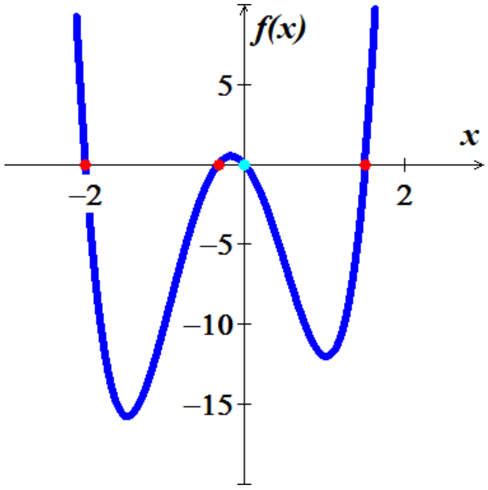
$$x\left(6x^3 + 5x^2 - 17x - 6\right) = 0 \quad \rightarrow \quad \underline{x = 0|}$$

$$possibilities: \ \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrr} -2 & 6 & 5 & -17 & -6 \\ & & -12 & 14 & 6 \\ \hline & 6 & -7 & -3 & \boxed{0} \end{array} \quad \rightarrow \ 6x^2 - 7x - 3 = 0$$

$$\underline{x = 0,\ -2,\ -\frac{1}{3},\ \frac{3}{2}|}$$





$f(x) > 0$ $\quad \underline{(-\infty,\ -2) \cup \left(-\frac{1}{3},\ 0\right) \cup \left(\frac{3}{2},\ \infty\right)|}$

$f(x) < 0$ $\quad \underline{\left(-2,\ -\frac{1}{3}\right) \cup \left(0,\ \frac{3}{2}\right)|}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 2x^2 - 16x - 15$$

Solution

$$possibilities: \ \pm\left\{\frac{15}{1}\right\} = \pm\{1,\ 3,\ 5,\ 15\}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -2 & -16 & -15 \\ & & -1 & 1 & 1 & 15 \\ \hline 3 & 1 & -1 & -1 & -15 & 0 \\ & & 3 & 6 & 15 & \\ \hline & 1 & 2 & 5 & 0 & \end{array} \quad \rightarrow \ x^3 - x^2 - x - 15 = 0 \ \rightarrow \ \pm\left\{\frac{15}{1}\right\} = \pm\{1,\ 3,\ 5,\ 15\}$$

$$\rightarrow \ x^2 + 2x + 5 = 0$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

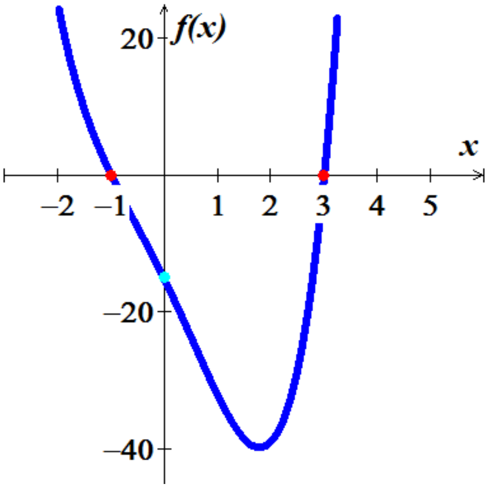
$$= -1 \pm 2i$$

$\boxed{x = -1,\ 3,\ \ -1 \pm 2i}$

	-1		3	
$+$		$-$		$+$

$f(x) > 0 \quad \boxed{(-\infty,\ -1) \cup (3,\ \infty)}$

$f(x) < 0 \quad \boxed{(-1,\ 3)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 2x^3 - 5x^2 + 8x + 4$$

Solution

possibilities : $\pm\left\{\frac{4}{1}\right\} = \pm\{1,\ 2,\ 4\}$

$$\begin{array}{r|rrrrr}
2 & 1 & -2 & -5 & 8 & 4 \\
 & & 2 & 0 & -10 & -4 \\
\hline
-2 & 1 & 0 & -5 & -2 & 0 \\
 & & -2 & 4 & 2 & \\
\hline
 & 1 & -2 & -1 & 0 &
\end{array}$$

$\rightarrow x^3 - 5x - 2 = 0 \rightarrow \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$\rightarrow x^2 - 2x - 1 = 0$

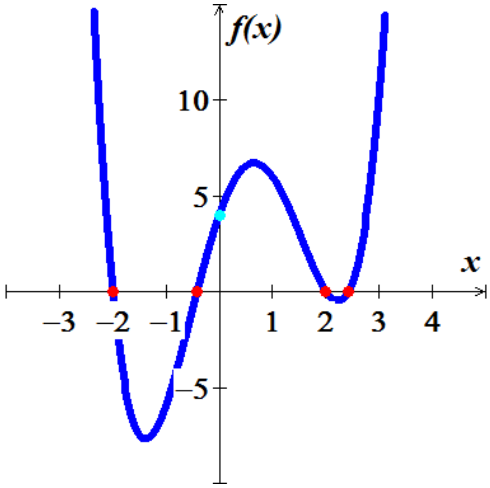
$$x = \frac{2 \pm \sqrt{8}}{2}$$

$\boxed{x = -2,\ 2,\ 1 \pm \sqrt{2}}$

	-2		$1-\sqrt{2}$		2		$1+\sqrt{2}$	
$+$		$-$		$+$		$-$		$+$

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup \left(1-\sqrt{2},\ 2\right) \cup \left(1+\sqrt{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-2,\ 1-\sqrt{2}\right) \cup \left(2,\ 1+\sqrt{2}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 17x^3 + 4x^2 + 35x - 24$$

Solution

possibilities : $\pm\left\{\frac{24}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{c|rrrrr}
1 & 2 & -17 & 4 & 35 & -24 \\
 & & 2 & -15 & -11 & 24 \\
\hline
1 & 2 & -15 & -11 & 24 & 0 \\
 & & 2 & -13 & 24 & \\
\hline
 & 2 & -13 & -24 & 0 &
\end{array}$$

$\rightarrow 2x^3 - 15x^2 - 11x + 24 = 0 \rightarrow \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$\rightarrow 2x^2 - 13x - 24 = 0$

$$x = \frac{13 \pm \sqrt{169 + 192}}{4}$$

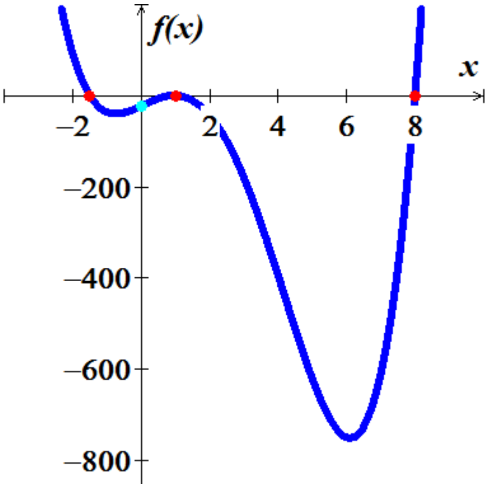
$$= \begin{cases} \frac{13-19}{4} = -\frac{3}{2} \\ \frac{13+19}{4} = 8 \end{cases}$$

$$\boxed{x = -\frac{3}{2},\ 1,\ 1,\ 8}$$

	$-\frac{3}{2}$		1		8	
+		−		−		+

$f(x) > 0 \quad \boxed{\left(-\infty,\ -\frac{3}{2}\right) \cup (8,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-\frac{3}{2},\ 1\right) \cup (1,\ 8)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 + x^3 - 3x^2 - 5x - 2$$

Solution

$$possibilities: \ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$$

$$\begin{array}{r|rrrrr}
-1 & 1 & 1 & -3 & -5 & -2 \\
 & & -1 & 0 & 3 & 2 \\
\hline
\overline{-1} & 1 & 0 & -3 & -2 & 0 \\
 & & -1 & 1 & 2 & \\
\hline
 & 1 & -1 & -2 & 0 &
\end{array}
\quad \to \ x^3 - 3x - 2 = 0 \ \to \ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$$

$$\to \ x^2 - x - 2 = 0$$

$$x = \frac{1 \pm \sqrt{1+8}}{2}$$

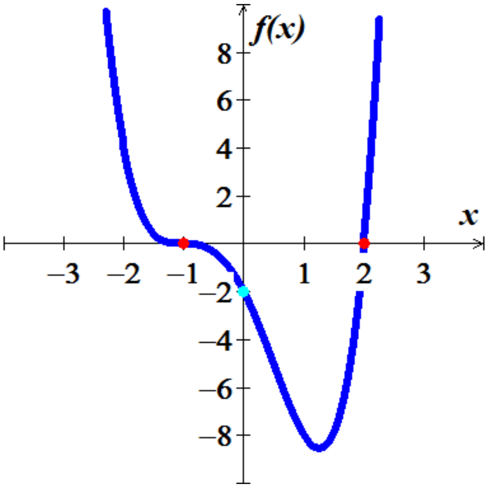
$$= \begin{cases} \frac{1-3}{2} = -1 \\ \frac{1+3}{2} = 2 \end{cases}$$

$$\underline{x = -1,\ \ -1,\ \ -1,\ 2 \ |}$$

$$\begin{array}{|c|c|c|}
 & -1 \quad\quad & 2 \\
\hline
+ & - & +
\end{array}$$

$$f(x) > 0 \quad \underline{(-\infty,\ -1) \cup (2,\ \infty) \ |}$$

$$f(x) < 0 \quad \underline{(-2,\ 2) \ |}$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 - 17x^3 - 11x^2 + 42x$$

Solution

$$x\left(6x^3 - 17x^2 - 11x + 42\right) = 0$$

$$x = 0 \quad 6x^3 - 17x^2 - 11x + 42 = 0$$

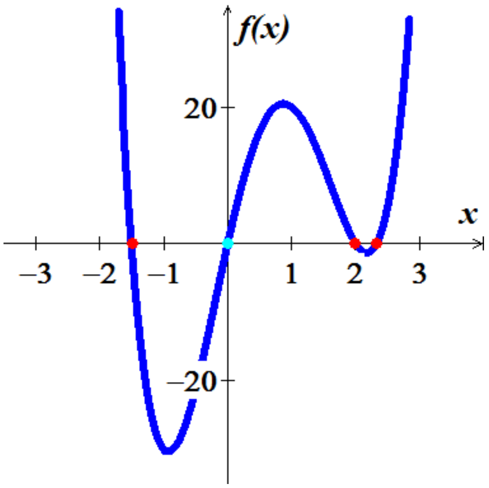
$$possibilities: \ \pm\left\{\frac{42}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ 7,\ 14,\ 21,\ 42,\ \frac{1}{2},\ \frac{3}{2},\ \frac{7}{2},\ \frac{21}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{7}{3},\ \frac{14}{3},\ \frac{1}{6},\ \frac{7}{6},\ \frac{21}{6}\right\}$$

$$\begin{array}{r|rrrr}
2 & 6 & -17 & -11 & 42 \\
 & & 12 & -10 & -42 \\
\hline
 & 6 & -5 & -21 & \boxed{0}
\end{array}
\quad \to \ 6x^2 - 5x - 21 = 0$$

$$x=\frac{5\pm\sqrt{25+504}}{12}$$

$$=\begin{cases}\dfrac{5-23}{12}=-\dfrac{3}{2}\\ \dfrac{5+23}{12}=\dfrac{7}{3}\end{cases}$$

$$x=-\frac{3}{2},\ 0,\ 2,\ \frac{7}{3}$$



	$-\frac{3}{2}$		0		2		$\frac{7}{3}$	
+		–		+		–		+

$f(x)>0$ $\quad \left(-\infty,\ -\frac{3}{2}\right)\cup(0,\ 2)\cup\left(\frac{7}{3},\ \infty\right)$

$f(x)<0$ $\quad \left(-\frac{3}{2},\ 0\right)\cup\left(2,\ \frac{7}{3}\right)$

Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^4-5x^2-2x$$

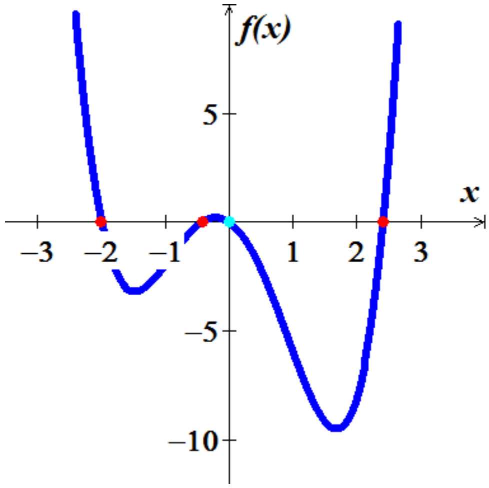
Solution

$$x\left(x^3-5x-2\right)=0$$

$$x=0 \quad x^3-5x-2=0$$

possibilities : $\pm\left\{\frac{2}{1}\right\}=\pm\{1,\ 2\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -5 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & \boxed{0} \end{array} \rightarrow x^2-2x-1=0$$



$$x=\frac{2\pm\sqrt{8}}{2}$$

$$x=-2,\ 0,\ 1\pm\sqrt{2}$$

	-2		$1-\sqrt{2}$		2		$1+\sqrt{2}$	
+		–		+		–		+

$f(x)>0$ $\quad \left(-\infty,\ -2\right)\cup\left(1-\sqrt{2},\ 2\right)\cup\left(1+\sqrt{2},\ \infty\right)$

$f(x)<0$ $\quad \left(-2,\ 1-\sqrt{2}\right)\cup\left(2,\ 1+\sqrt{2}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^4 - 4x^3 - 11x^2 + 16x - 4$$

Solution

$$\textit{possibilities}: \ \pm\left\{\frac{4}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3}\right\}$$

$$\begin{array}{r|rrrrr}
1 & 3 & -4 & -11 & 16 & -4 \\
 & & 3 & -1 & -12 & 4 \\
\hline
2 & 3 & -1 & -12 & 4 & 0 \\
 & & 6 & 10 & -4 & \\
\hline
 & 3 & 5 & -2 & 0 &
\end{array}$$

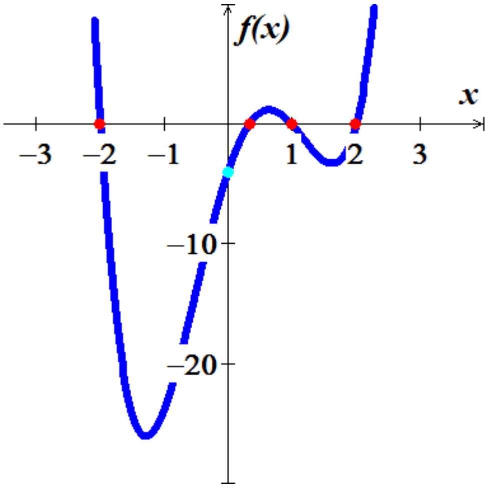
$$\to 3x^3 - x^2 - 12x + 4 = 0 \ \to \ \pm\left\{\frac{4}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3}\right\}$$

$$\to 3x^2 + 5x - 2 = 0$$

$$x = \frac{-5 \pm \sqrt{25+24}}{6}$$

$$= \begin{cases} \dfrac{-5-7}{6} = -2 \\ \dfrac{-5+7}{6} = \dfrac{1}{3} \end{cases}$$

$$x = -2,\ \frac{1}{3},\ 1,\ 2$$



$$\begin{array}{ccccccccc}
 & -2 & & \frac{1}{3} & & 1 & & 2 & \\
\hline
+ & | & - & | & + & | & - & | & +
\end{array}$$

$$f(x) > 0 \quad (-\infty,\ -2) \cup \left(\frac{1}{3},\ 1\right) \cup (2,\ \infty)$$

$$f(x) < 0 \quad \left(-2,\ \frac{1}{3}\right) \cup (1,\ 2)$$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 + 23x^3 + 19x^2 - 8x - 4$$

Solution

$$\textit{possibilities}: \ \pm\left\{\frac{4}{6}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{6},\ \frac{2}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrrr} -2 & 6 & 23 & 19 & -8 & -4 \\ & & -12 & -22 & 6 & 4 \\ \hline -2 & 6 & 11 & -3 & -2 & 0 \\ & & -12 & 2 & 2 & \\ \hline & 6 & -1 & -1 & 0 & \end{array}$$

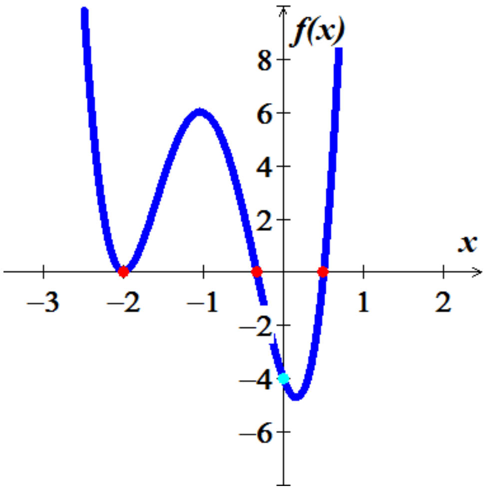
$$\rightarrow 6x^3 + 11x^2 - 3x - 2 = 0 \rightarrow \pm\left\{\frac{2}{6}\right\} = \pm\left\{1,\ 2,\ \frac{1}{6},\ \frac{1}{3}\right\}$$

$$\rightarrow 6x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{25}}{12}$$

$$= \begin{cases} \frac{1-5}{12} = -\frac{1}{3} \\ \frac{1+5}{12} = \frac{1}{2} \end{cases}$$

$$\boxed{x = -2,\ -2,\ -\frac{1}{3},\ \frac{1}{2}}$$



$-2 \qquad -\frac{1}{3} \qquad \frac{1}{2}$

+		+	–	+

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup \left(-2,\ -\frac{1}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-\frac{1}{3},\ \frac{1}{2}\right)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 4x^4 - 12x^3 + 3x^2 + 12x - 7$$

Solution

possibilities : $\pm\left\{\frac{7}{4}\right\} = \pm\left\{1,\ 7,\ \frac{1}{2},\ \frac{7}{2},\ \frac{1}{4},\ \frac{7}{4}\right\}$

$$\begin{array}{r|rrrrr} 1 & 4 & -12 & 3 & 12 & -7 \\ & & 4 & -8 & -5 & 7 \\ \hline -1 & 4 & -8 & -5 & 7 & 0 \\ & & -4 & 12 & -7 & \\ \hline & 4 & -12 & 7 & 0 & \end{array}$$

$$\rightarrow 4x^3 - 8x^2 - 5x + 7 = 0 \rightarrow \pm\left\{\frac{7}{4}\right\} = \pm\left\{1,\ 7,\ \frac{1}{2},\ \frac{7}{2},\ \frac{1}{4},\ \frac{7}{4}\right\}$$

$$\rightarrow 4x^2 - 12x + 7 = 0$$

$$x = \frac{12 \pm \sqrt{144 - 112}}{8}$$

$$= \frac{12 \pm 4\sqrt{2}}{8}$$

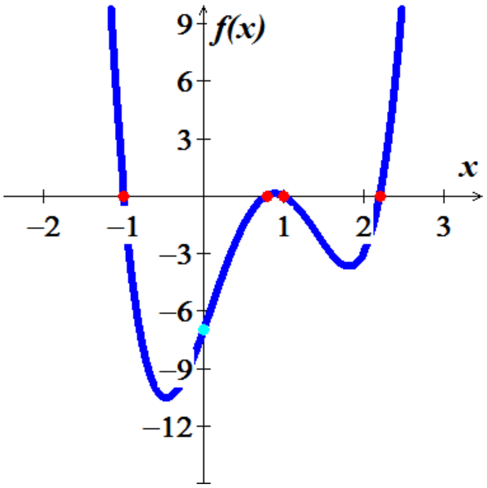
$$\boxed{x=-1,\ 1,\ \frac{3\pm\sqrt{2}}{2}}$$

$$-1 \qquad \frac{3-\sqrt{2}}{2} \qquad 1 \qquad \frac{3+\sqrt{2}}{2}$$

$$+ \quad | \quad - \quad | \quad + \quad | \quad - \quad | \quad +$$

$$f(x)>0 \quad \boxed{(-\infty,\ -1)\cup\left(\frac{3-\sqrt{2}}{2},\ 1\right)\cup\left(\frac{3+\sqrt{2}}{2},\ \infty\right)}$$

$$f(x)<0 \quad \boxed{\left(-1,\ \frac{3-\sqrt{2}}{2}\right)\cup\left(1,\ \frac{3+\sqrt{2}}{2}\right)}$$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=2x^4-9x^3-2x^2+27x-12$$

Solution

$$\textit{possibilities}:\ \pm\left\{\frac{12}{2}\right\}=\pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 12,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\begin{array}{r|rrrrr}
4 & 2 & -9 & -2 & 27 & -12 \\
 & & 8 & -4 & -24 & 12 \\
\hline
\frac{1}{2} & 2 & -1 & -6 & 3 & 0 \\
 & & 1 & 0 & -3 & \\
\hline
 & 2 & 0 & -6 & 0 &
\end{array}$$

$$\rightarrow 2x^3-x^2-6x+3=0 \ \rightarrow\ \pm\left\{\frac{3}{2}\right\}=\pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\rightarrow 2x^2-6=0$$

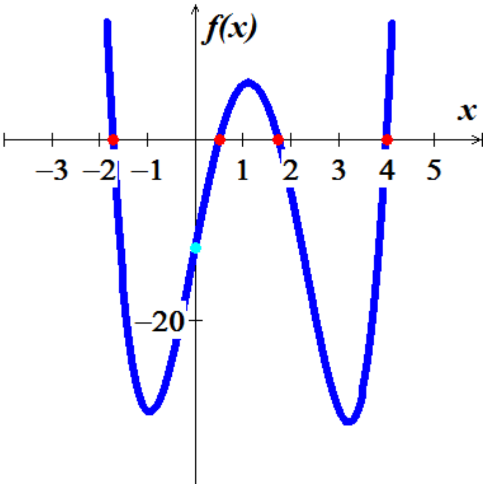
$$\boxed{x=\frac{1}{2},\ 4,\ \pm\sqrt{3}}$$

$$-\sqrt{3} \qquad \frac{1}{2} \qquad \sqrt{3} \qquad 4$$

$$+ \quad | \quad - \quad | \quad + \quad | \quad - \quad | \quad +$$

$$f(x)>0 \quad \boxed{\left(-\infty,\ -\sqrt{3}\right)\cup\left(\frac{1}{2},\ \sqrt{3}\right)\cup(4,\ \infty)}$$

$$f(x)<0 \quad \boxed{\left(-\sqrt{3},\ \frac{1}{2}\right)\cup\left(\sqrt{3},\ 4\right)}$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 19x^3 + 51x^2 - 31x + 5$$

Solution

$$\text{possibilities}: \ \pm\left\{\frac{5}{2}\right\} = \pm\left\{1,\ 5,\ \frac{1}{2},\ \frac{5}{2}\right\}$$

$$\begin{array}{r|rrrrr}
5 & 2 & -19 & 51 & -31 & 5 \\
 & & 10 & -45 & 30 & -5 \\
\hline
\frac{1}{2} & 2 & -9 & 6 & -1 & 0 \\
 & & 1 & -4 & 1 & \\
\hline
 & 2 & -8 & 2 & 0 &
\end{array}$$

$$\rightarrow 2x^3 - 9x^2 + 6x - 1 = 0 \ \rightarrow \ \pm\left\{\frac{1}{2}\right\}$$

$$\rightarrow 2x^2 - 8x + 2 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 16}}{4}$$

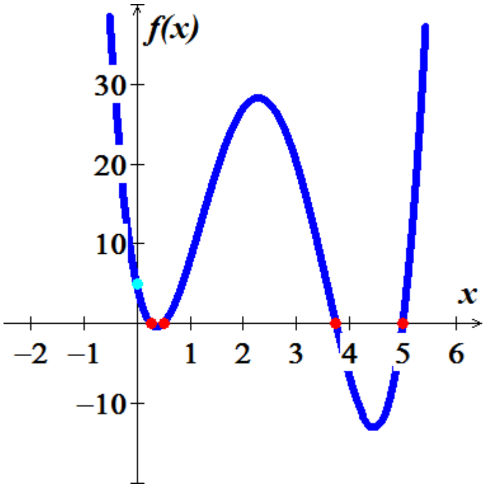
$$= \frac{8 \pm 4\sqrt{3}}{4}$$

$$\boxed{x = \frac{1}{2},\ 5,\ 2 \pm \sqrt{3}}$$

$$\begin{array}{ccccccccc}
 & 2-\sqrt{3} & & \frac{1}{2} & & 2+\sqrt{3} & & 5 & \\
\hline
+ & | & - & | & + & | & - & | & +
\end{array}$$

$$f(x) > 0 \quad \boxed{\left(-\infty,\ 2-\sqrt{3}\right) \cup \left(\frac{1}{2},\ 2+\sqrt{3}\right) \cup (5,\ \infty)}$$

$$f(x) < 0 \quad \boxed{\left(2-\sqrt{3},\ \frac{1}{2}\right) \cup \left(2+\sqrt{3},\ 5\right)}$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 4x^4 - 35x^3 + 71x^2 - 4x - 6$$

Solution

$$\text{possibilities}: \ \pm\left\{\frac{6}{4}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{4},\ \frac{3}{4}\right\}$$

$$\begin{array}{r|rrrrr}
3 & 4 & -35 & 71 & -4 & -6 \\
 & & 12 & -69 & 6 & 6 \\ \hline
-\frac{1}{4} & 4 & -23 & 2 & 2 & 0 \\
 & & -1 & 6 & -2 & \\ \hline
 & 4 & -24 & 8 & 0 &
\end{array}$$

$\to 4x^3 - 23x^2 + 2x + 2 = 0 \to \pm\left\{\frac{2}{4}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{4}\right\}$

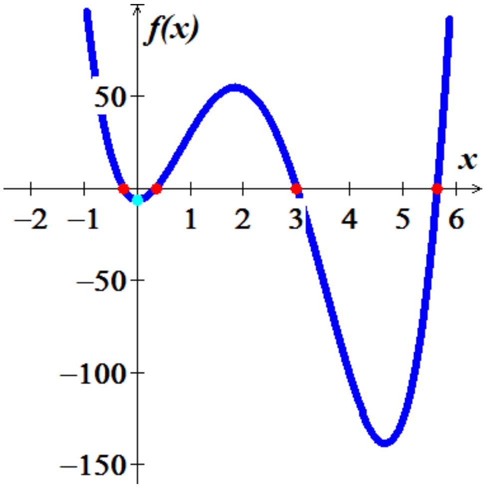
$\to 4x^2 - 24x + 8 = 0$

$$x^2 - 6x + 2 = 0$$

$$x = \frac{6 \pm \sqrt{36-8}}{2}$$

$$= \frac{6 \pm 2\sqrt{7}}{4}$$

$$\boxed{x = -\frac{1}{4},\ 3,\ 3 \pm \sqrt{7}}$$



	$-\frac{1}{4}$		$3-\sqrt{7}$		3		$3+\sqrt{7}$	
+		−		+		−		+

$f(x) > 0$ $\boxed{\left(-\infty,\ -\frac{1}{4}\right) \cup \left(3-\sqrt{7},\ 3\right) \cup \left(3+\sqrt{7},\ \infty\right)}$

$f(x) < 0$ $\boxed{\left(-\frac{1}{4},\ 3-\sqrt{7}\right) \cup \left(3,\ 3+\sqrt{7}\right)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$$

Solution

possibilities : $\pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & 3 & -4 & -3 & 2 \\
 & & 2 & 5 & 1 & -2 \\ \hline
-1 & 2 & 5 & 1 & -2 & 0 \\
 & & -2 & -3 & 2 & \\ \hline
 & 2 & 3 & -2 & 0 &
\end{array}$$

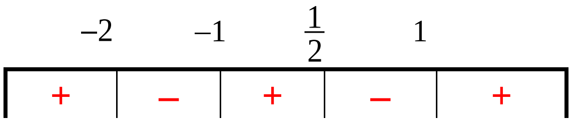
$\to 2x^3 - 23x^2 + 2x - 2 = 0 \to \pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$\to 2x^2 + 3x - 2 = 0$

$$x = \frac{-3 \pm \sqrt{9+16}}{4}$$

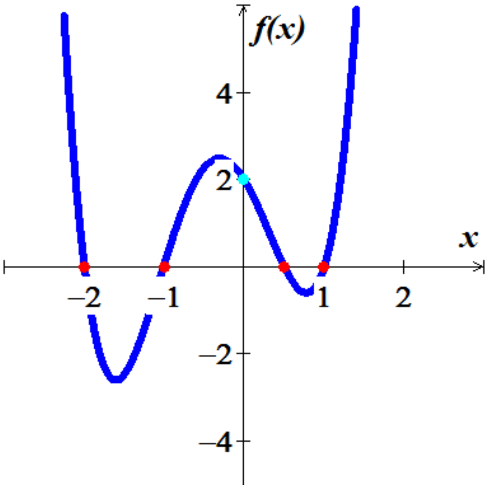
$$= \begin{cases} \frac{-3-5}{4} = -2 \\ \frac{-3+5}{4} = \frac{1}{2} \end{cases}$$

$x = -2,\ \ -1,\ \frac{1}{2},\ 1$



$f(x) > 0 \quad \left(-\infty,\ -2\right) \cup \left(-1,\ \frac{1}{2}\right) \cup \left(1,\ \infty\right)$

$f(x) < 0 \quad \left(-2,\ -1\right) \cup \left(\frac{1}{2},\ 1\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 + 3x^3 - 30x^2 - 6x + 56$$

Solution

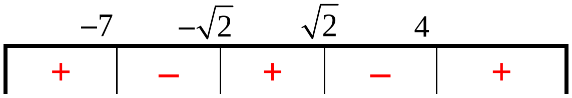
possibilities for $\frac{c}{d} : \pm\left\{\frac{56}{1}\right\} = \pm\{1,\ 2,\ 4,\ 7,\ 8,\ 14,\ 28,\ 56\}$

$$\begin{array}{r|rrrrr} 4 & 1 & 3 & -3 & -6 & 56 \\ & & 4 & 28 & -8 & -56 \\ \hline -7 & 1 & 7 & -2 & -14 & 0 \\ & & -7 & 0 & 14 & \\ \hline & 1 & 0 & -2 & & \end{array}$$

$\rightarrow \quad x^3 + 7x^2 - 2x - 14 = 0 \ \Rightarrow \ \frac{c}{d} = \pm\left\{\frac{14}{1}\right\} = \pm\{1,\ 2,\ 7,\ 14\}$

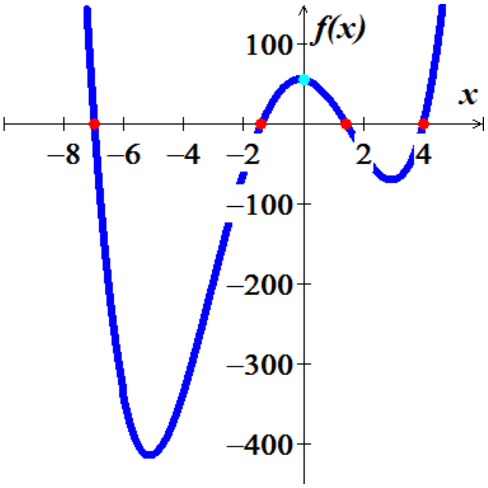
$\rightarrow \quad x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}$

$x = 4,\ \ -7,\ \ \pm\sqrt{2}$



$f(x) > 0 \quad \left(-\infty,\ -7\right) \cup \left(-\sqrt{2},\ \sqrt{2}\right) \cup \left(4,\ \infty\right)$

$f(x) < 0 \quad \left(-7,\ -\sqrt{2}\right) \cup \left(\sqrt{2},\ 4\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^5 - 10x^4 - 6x^3 + 24x^2 + 11x - 6$$

Solution

$$\textit{possibilities for } \frac{c}{d} : \pm\left\{\frac{6}{3}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrrrr}
-1 & 3 & -10 & -6 & 24 & 11 & -6 \\
 & & -3 & 13 & -7 & -17 & 6 \\ \hline
-1 & 3 & -13 & 7 & 17 & -6 & 0 \\
 & & -3 & 16 & -23 & 6 & \\ \hline
2 & 3 & -16 & 23 & -6 & 0 & \\
 & & 6 & 20 & 6 & & \\ \hline
 & 3 & -10 & 3 & 0 & &
\end{array}$$

$$x^4 - 13x^3 + 7x^2 + 17x - 6 = 0 \rightarrow \pm\left\{\frac{6}{3}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^3 - 16x^2 + 26x - 6 = 0 \rightarrow \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^2 - 10x + 3 = 0$$

$$x = \frac{10 \pm \sqrt{100 - 36}}{6}$$

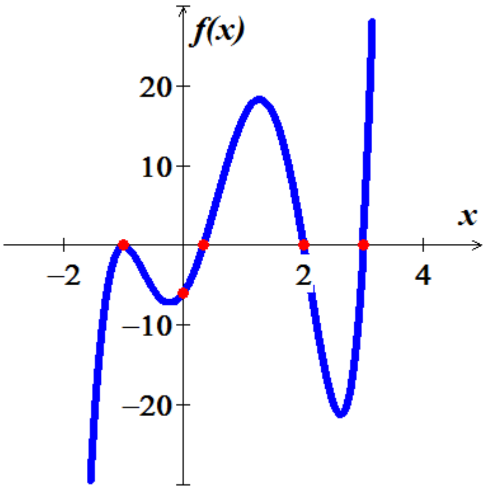
$$= \begin{cases} \frac{10-8}{6} = \frac{1}{3} \\ \frac{10+8}{4} = 3 \end{cases}$$

$$\boxed{x = -1,\ -1,\ \frac{1}{3},\ 2,\ 3}$$

$$\begin{array}{ccccccccc}
 & -1 & & \frac{1}{3} & & 2 & & 3 & \\
- & | & - & | & + & | & - & | & +
\end{array}$$

$f(x) > 0 \quad \boxed{\left(\frac{1}{3},\ 2\right) \cup (3,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -1) \cup \left(-1,\ \frac{1}{3}\right) \cup (2,\ 3)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^5 + 19x^4 + x^3 - 6x^2$$

Solution

$$x^2\left(6x^3 + 19x^2 + x - 6\right) = 0 \quad \rightarrow \quad \underline{x = 0,\ 0|}$$

$$6x^3 + 19x^2 + x - 6 = 0$$

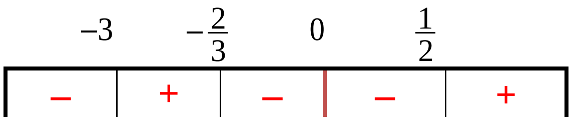
possibilities for $\frac{c}{d} : \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrr} -3 & 6 & 19 & 1 & -6 \\ & & -18 & -3 & 6 \\ \hline & 6 & 1 & -2 & \boxed{0} \end{array} \qquad 6x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1+48}}{12}$$

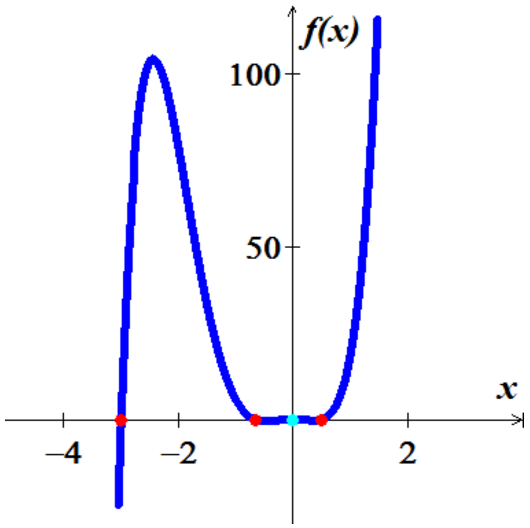
$$= \begin{cases} \frac{-1-7}{12} = -\frac{2}{3} \\ \frac{-1+7}{12} = \frac{1}{2} \end{cases}$$

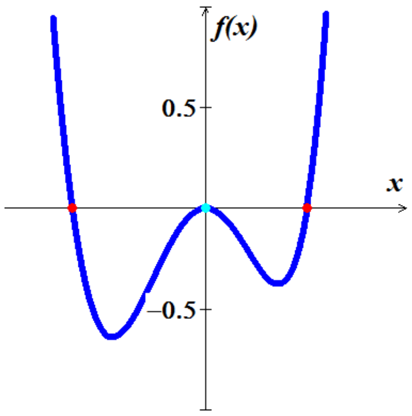
$$\underline{x = 0,\ 0,\ -\frac{2}{3},\ -3,\ \frac{1}{2}\,|}$$



$f(x) > 0 \quad \underline{\left(-3,\ -\frac{2}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)\,|}$

$f(x) < 0 \quad \underline{\left(-\infty,\ -3\right) \cup \left(-\frac{2}{3},\ 0\right) \cup \left(0,\ \frac{1}{2}\right)\,|}$





Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Solution

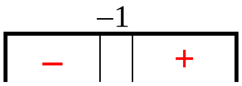
$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1 = (x+1)^5 = 0$$

possibilities for $\frac{c}{d} : \pm\{1\}$

$$\begin{array}{r|rrrrrrl}
-1 & 1 & 5 & 10 & 10 & 5 & 1 & \\
 & & -1 & -4 & -6 & -4 & -1 & \\
\hline
-1 & 1 & 4 & 6 & 4 & 1 & 0 & \rightarrow x^4 + 4x^3 + 6x^2 + 4x + 1 = 0 \rightarrow \pm\{1\} \\
 & & -1 & -3 & -3 & -1 & & \\
\hline
-1 & 1 & 3 & 3 & 1 & 0 & & \rightarrow x^3 + 3x^2 + 3x + 1 = 0 \rightarrow \pm\{1\} \\
 & & -1 & -2 & -1 & & & \\
\hline
 & 1 & 2 & 1 & 0 & & & \rightarrow x^2 + 2x + 1 = 0
\end{array}$$

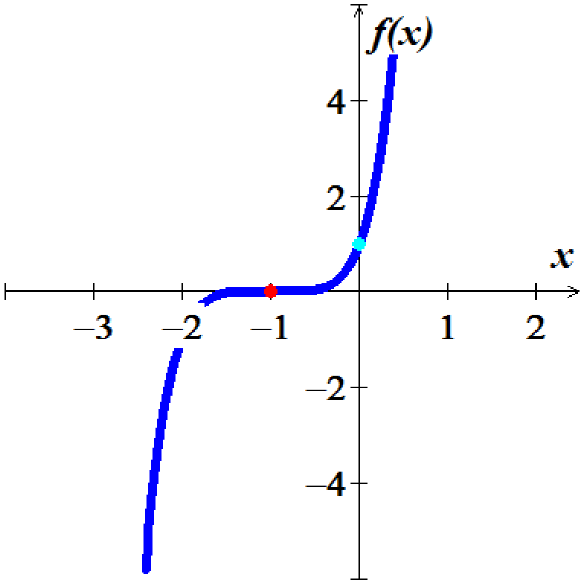
$$x^2 + 2x + 1 = (x+1)^2$$

$\underline{x = -1}$ $(multiplicity \text{ of } 5)$



$f(x) > 0$ $\underline{(-1, \infty)}$

$f(x) < 0$ $\underline{(-\infty, -1)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^5 - x^4 - 7x^3 + 7x^2 + 12x - 12$$

Solution

possibilities for $\frac{c}{d} : \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$\begin{array}{c|cccccc}
1 & 1 & -1 & -7 & 7 & 12 & -12 \\
 & & 1 & 0 & -7 & 0 & 12 \\
\hline
2 & 1 & 0 & -7 & 0 & 12 & 0 \\
 & & 2 & 4 & -6 & -12 & \\
\hline
-2 & 1 & 2 & -3 & -6 & 0 & \\
 & & -2 & 0 & 6 & & \\
\hline
 & 1 & 0 & -3 & 0 & &
\end{array}$$

$\rightarrow x^4 - 7x^2 - 12 = 0 \rightarrow \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$\rightarrow x^3 + 2x^2 - 3x - 6 = 0 \rightarrow \pm\{1,\ 2,\ 3,\ 6\}$

$\rightarrow x^2 - 3 = 0$

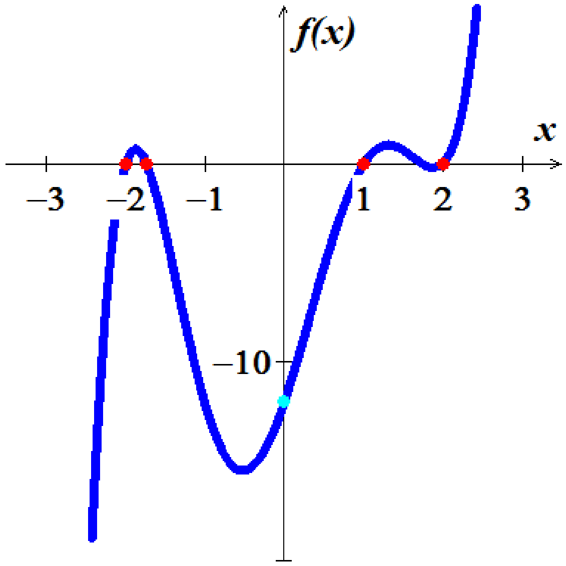
$x^2 = 3$

$\underline{x = -2,\ 1,\ 2,\ \pm\sqrt{3}\ |}$

$$\begin{array}{ccccccccccc}
 & -2 & & -\sqrt{3} & & 1 & & \sqrt{3} & & 2 & \\
- & | & + & | & - & | & + & | & - & | & +
\end{array}$$

$f(x) > 0 \quad \left(-2,\ -\sqrt{3}\right) \cup \left(1,\ \sqrt{3}\right) \cup (2,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -2) \cup \left(-\sqrt{3},\ 1\right) \cup \left(\sqrt{3},\ 2\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^5 - 2x^3 - 8x$$

Solution

$$x\left(x^4 - 2x^2 - 8\right) = 0$$

$$\boxed{x = 0}$$

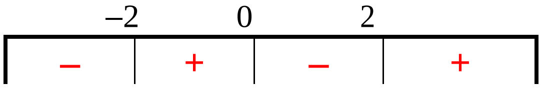
$$x^4 - 2x^2 - 8 = 0.$$

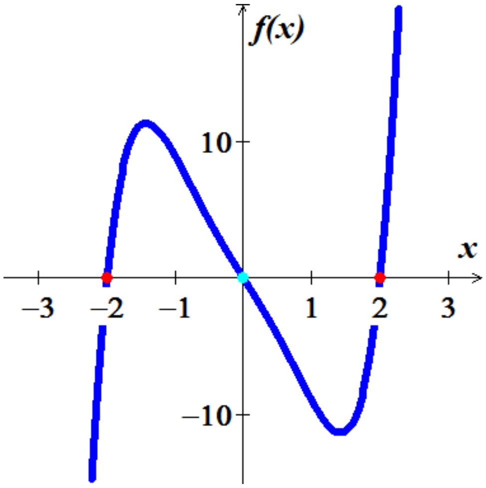
$$x^2 = \frac{2 \pm \sqrt{4+32}}{2}$$

$$= \begin{cases} \frac{2-6}{2} = -2 \\ \frac{2+6}{2} = 4 \end{cases}$$

$$\begin{cases} x^2 = -2 & \rightarrow & x = \pm i\sqrt{2} \\ x^2 = 4 & \rightarrow & x = \pm 2 \end{cases}$$

$$\boxed{x = 0, \ \pm 2, \ \pm i\sqrt{2}}$$





$f(x) > 0$ $\boxed{(-2, \ 0) \cup (2, \ \infty)}$

$f(x) < 0$ $\boxed{(-\infty, \ -2) \cup (0, \ 2)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^6 - 10x^5 - 29x^4 + 34x^3 + 50x^2 - 24x - 24$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{3}\right\} = \pm\left\{1, \ 2, \ 3, \ 4, \ 6, \ 8, \ 12, \ 24, \ \frac{1}{3}, \ \frac{2}{3}, \ \frac{4}{3}, \ \frac{8}{3}\right\}$

1	3	−10	−29	34	50	−24	−24
		3	−7	−36	−2	48	24
−1	3	−7	−36	−2	48	24	0
		−3	10	26	−24	−24	
−2	3	−10	−26	24	24	0	
		−6	32	−12	−24		
$-\frac{2}{3}$	3	−16	6	12	0		
		−2	12	−12			
	3	−18	18	0			

$\rightarrow\ 3x^5-7x^4-36x^3-2x^2+48x+24=0$

$\rightarrow\ 3x^4-10x^3-26x^2+24x+24=0$

$\rightarrow\ 3x^3-16x^2+12x-12=0$

$\rightarrow\ 3x^2-18x+18=0$

$x^2-6x+6=0$

$$x=\frac{6\pm\sqrt{36-24}}{2}$$

$$=\frac{6\pm2\sqrt{3}}{2}$$

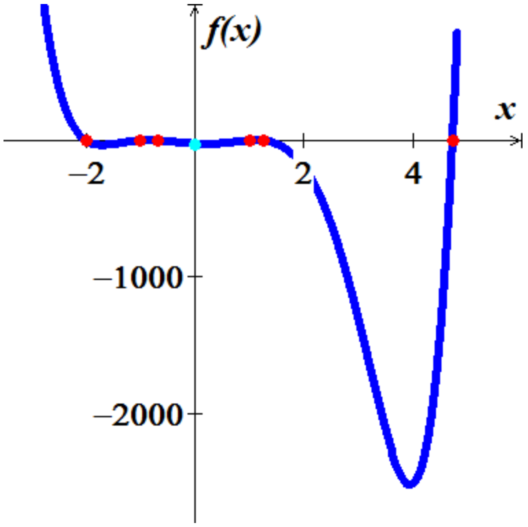
$$\boxed{x=-2,\ -1,\ 1,\ -\frac{2}{3},\ 3\pm\sqrt{3}}$$

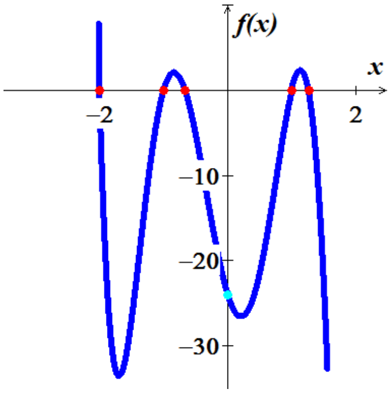
$-2 \qquad -1 \qquad -\frac{2}{3} \qquad 1 \qquad 3-\sqrt{3} \qquad 3+\sqrt{3}$

+	−	+	−	+	−	+

$f(x)>0\quad \boxed{(-\infty,\ -2)\cup\left(-1,\ -\frac{2}{3}\right)\cup\left(1,\ 3-\sqrt{3}\right)\cup\left(3+\sqrt{3},\ \infty\right)}$

$f(x)<0\quad \boxed{(-2,\ -1)\cup\left(-\frac{2}{3},\ 1\right)\cup\left(3-\sqrt{3},\ 3+\sqrt{3}\right)}$





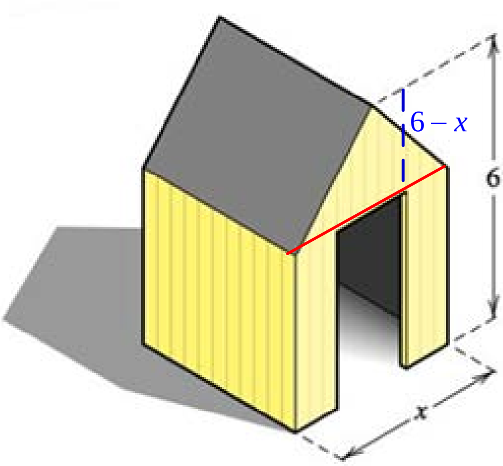
Exercise

A storage shelter is to be constructed in the shape of a cube with a triangular prism forming the roof. The length x of a side of the cube is yet to be determined.

a) If the total height of the structure is 6 *feet*, show that its volume V is given by $V = x^3 + \frac{1}{2}x^2(6-x)$

b) Determine x so that the volume is $80\ ft^3$

Solution



a) $V = V_{cube} + V_{triangle}$

$$= x^3 + \frac{1}{2}x(x)(6-x)$$

$$= \frac{1}{2}x^2(2x+6-x)$$

$$\underline{= \frac{1}{2}x^2(x+6)}$$

b) $V = \frac{1}{2}x^2(x+6) = 80$

$$x^3 + 6x^2 - 160 = 0$$

possibilities : $\pm\left\{\frac{160}{1}\right\} = \pm\{1,\ 2,\ 4,\ 5,\ 8,\ 10,\ 16,\ 20,\ 32,\ 40,\ 80,\ 160\}$

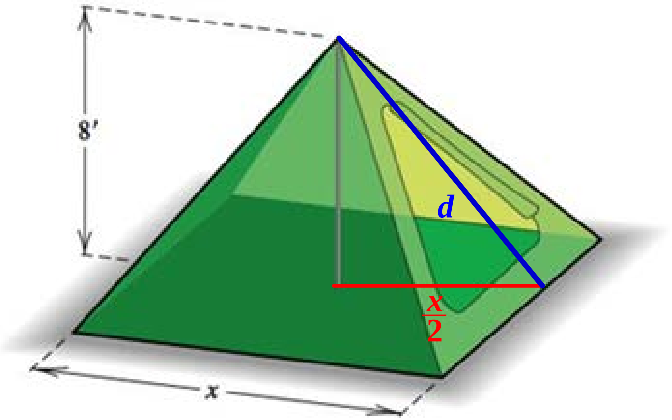
$$\begin{array}{r|rrrr} 4 & 1 & 6 & 0 & -160 \\ & & 4 & 40 & 160 \\ \hline & 1 & 10 & 40 & \boxed{0} \end{array} \quad \rightarrow\ x^2 + 10x + 40 = 0 \Rightarrow \underline{x = -5 \pm i\sqrt{15}}$$

The solution is: $\underline{x = 4}$

Exercise

A canvas camping tent is to be constructed in the shape of a pyramid with a square base. An 8–foot pole will form the center support. Find the length x of a side of the base so that the total amount of canvas needed for the sides and bottom is $384\ ft^2$.

Solution



$$d = \sqrt{64 + \frac{x^2}{4}} = \frac{1}{2}\sqrt{x^2 + 256}$$

$$A_{bottom} = x^2$$

$$A_{1-side} = \frac{1}{2}xd$$

$$= \frac{1}{4}x\sqrt{x^2 + 256}$$

$$A_{total} = A_{bottom} + 4A_{1-side}$$

$$= x^2 + x\sqrt{x^2+256} = 384$$

$$x\sqrt{x^2+256} = 384 - x^2$$

$$\left(x\sqrt{x^2+256}\right)^2 = \left(384 - x^2\right)^2$$

$$x^2\left(x^2+256\right) = 147,456 - 768x^2 + x^4$$

$$-1,024x^2 + 147,456 = 0$$

$$x = \pm\sqrt{\frac{147,456}{1,024}}$$

$$\underline{= 12 \ \ ft|}$$

Exercise

Glasses can be stacked to form a triangular pyramid. The total number of glasses in one of these pyramids is given by

$$T(k) = \frac{1}{6}\left(k^3 + 3k^2 + 2k\right)$$

Where k is the number of levels in the pyramid. If 220 glasses are used to form a triangle pyramid, how many levels are in the pyramid?

Solution

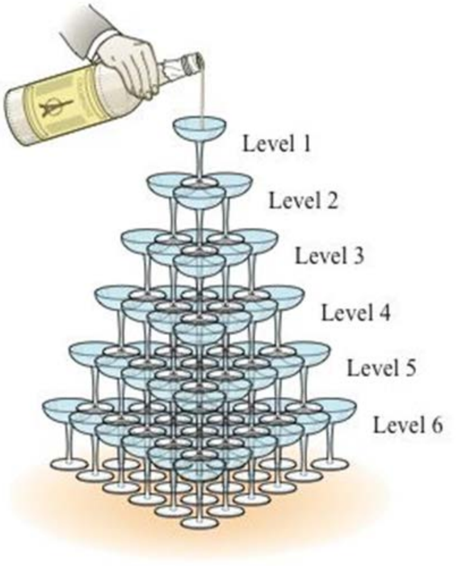
$$\frac{1}{6}\left(k^3 + 3k^2 + 2k\right) = 220$$

$$k^3 + 3k^2 + 2k - 1,320 = 0$$

$$\begin{array}{r|rrrr} 10 & 1 & 3 & 2 & -1320 \\ & & 10 & 130 & 1320 \\ \hline & 1 & 13 & 132 & 0 \end{array} \quad \rightarrow \quad k^2 + 13k + 132 = 0$$

$$k = \frac{-13 \pm \sqrt{-359}}{2} \quad \mathbb{C}$$

The are **10** levels in the pyramid.

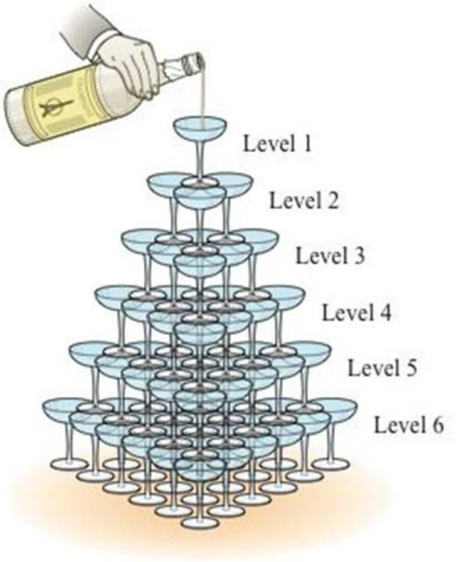


Exercise

Glasses can be stacked to form a triangular pyramid. The total number of glasses in one of these pyramids is given by

$$T(k) = \frac{1}{6}\left(2k^3 + 3k^2 + k\right)$$

Where k is the number of levels in the pyramid. If 140 glasses are used to form a triangle pyramid, how many levels are in the pyramid?



Solution

$$\frac{1}{6}\left(2k^3+3k^2+k\right)=150$$

$$2k^3+3k^2+k-840=0$$

$$\begin{array}{r|rrrr} 7 & 2 & 3 & 1 & -840 \\ & & 14 & 119 & 840 \\ \hline & 2 & 17 & 120 & 0 \end{array} \quad \rightarrow \quad 2k^2+17k+120=0$$

$$k=\frac{-17\pm\sqrt{-671}}{4} \quad \mathbb{C}$$

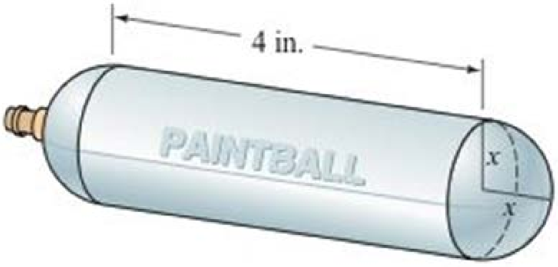
The are 7 levels in the pyramid.

Exercise

A carbon dioxide cartridge for a paintball rifle has the shape of a right circular cylinder with a hemisphere at each end. The cylinder is 4 *inches* long, and the volume of the cartridge is $2\pi \ \ in^3$.
The common interior radius of the cylinder and the hemispheres is denoted by x. Estimate the length of the radius x.

Solution

Volume of the Cartridge = 2 × (Volume of Hemisphere) + Volume of Cylinder



$$Volume\ of\ \ Sphere=\frac{4}{3}\pi x^3$$

$$Volume\ of\ \ Cylinder=4\pi x^2$$

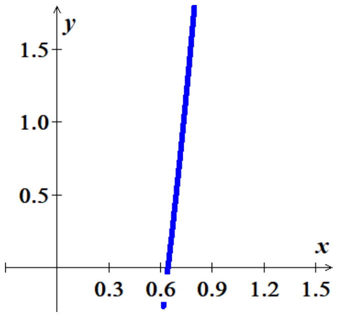
$$Volume\ of\ \ Cartridge=\frac{4}{3}\pi x^3+4\pi x^2$$

$$2\pi=\frac{4}{3}\pi x^3+4\pi x^2$$

$$2x^3+6x^2=3$$

$$2x^3+6x^2-3=0$$

Using Graph:



$\underline{x\approx 0.64\ \ in.}|$

Exercise

A propane tank has the shape of a circular cylinder with a hemisphere at each end. The cylinder is 6 *feet* long and the volume of the tank is $9\pi \ \ ft^3$. Find the length of the radius *x*.

Solution

Volume of the Cartridge = 2 × (Volume of Hemisphere) + Volume of Cylinder

$$Volume\ of\ \ Sphere = \frac{4}{3}\pi x^3$$

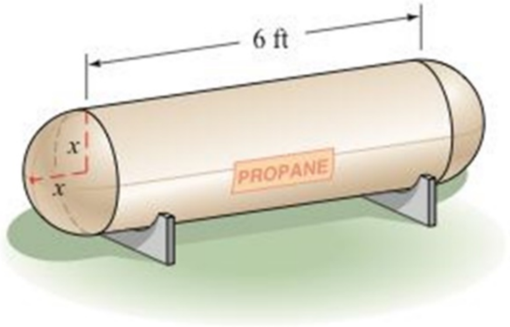
$$Volume\ of\ \ Cylinder = 6\pi x^2$$

$$Volume\ of\ \ Cartridge = \frac{4}{3}\pi x^3 + 6\pi x^2$$

$$9\pi = \frac{4}{3}\pi x^3 + 6\pi x^2$$

$$27 = 4x^3 + 18x^2$$

$$4x^3 + 18x^2 - 27 = 0$$



$$possibilities\ for\ \ \frac{c}{d} : \pm\left\{\frac{27}{4}\right\} = \pm\left\{1,\ 3,\ 9,\ 27,\ \frac{1}{2},\ \frac{3}{2},\ \frac{9}{2},\ \frac{27}{2},\ \frac{1}{4},\ \frac{3}{4},\ \frac{9}{4},\ \frac{27}{4}\right\}$$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 4 & 18 & 0 & -27 \\ & & -6 & -18 & 27 \\ \hline & 4 & 12 & -18 & 0 \end{array} \quad \rightarrow \quad 4x^2 + 12x - 18 = 0$$

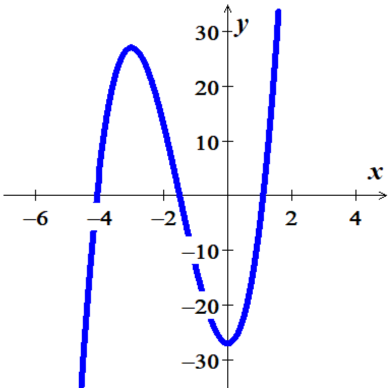
$$2x^2 + 6x - 9 = 0$$

$$x = \frac{-6 \pm \sqrt{36 + 72}}{4}$$

$$= \frac{-6 \pm 6\sqrt{3}}{4}$$

$$= \frac{-3 \pm 3\sqrt{3}}{2}$$

$$\boxed{x = -\frac{3}{2},\ \frac{-3 - 3\sqrt{3}}{2},\ \frac{-3 + 3\sqrt{3}}{2}}$$



∴ the length of the radius *x* is $\frac{-3 + 3\sqrt{3}}{2} \approx 1.1\ \ foot$

Exercise

A cube measures *n* inches on each edge. If a slice 2 *inches* thick is cut from one face of the cube, the resulting solid has a volume of $567\ \ in^3$. Find *n*.

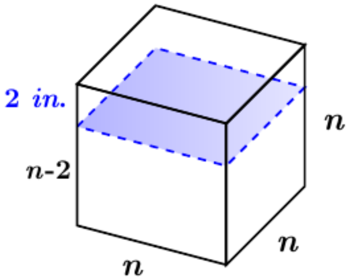
Solution

$$Volume = n^2(n-2)$$

$n^3 - 2n^2 = 567$

$n^3 - 2n^2 - 567 = 0$

$possibilities\ for\ \frac{c}{d} := \pm\{1,\ 3,\ 7,\ 9,\ 21,\ 27,\ 63,\ 81,\ 189,\ 567\}$



$$\begin{array}{r|rrrr} 9 & 1 & -2 & 0 & -567 \\ & & 9 & 63 & 567 \\ \hline & 1 & 7 & 63 & 0 \end{array} \quad \rightarrow \quad n^2 + 7n + 63 = 0$$

$$n = \frac{-7 \pm \sqrt{49 - 252}}{2}$$

$$= \frac{-7 \pm i\sqrt{203}}{2} \quad \times$$

$\therefore \quad \underline{n = 9}\rvert$

Exercise

A cube measures *n* inches on each edge. If a slice 1 *inch* thick is cut from one face of the cube and then a slice 3 inches thick is cut from another face of the cube, the resulting solid has a volume of 1560 in^3. Find the dimensions of the original cube.

Solution

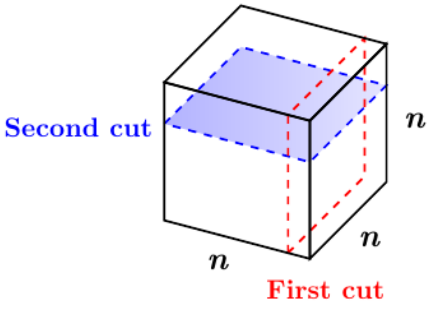
$Volume = n(n-1)(n-3)$

$n^3 - 4n^2 + 3n = 1560$

$n^3 - 4n^2 + 3n - 1560 = 0$

$$possibilities\ for\ \frac{c}{d} := \pm\left\{\begin{array}{l} 1,\ 2,\ 4,\ 5,\ 6,\ 8,\ 10,\ 12,\ 13,\ 15,\ 20,\ 24,\ 26,\ 30,\ 39, \\ 40,\ 52,\ 60,\ 65,\ 78,\ 104,\ 120,\ 130,\ 156,\ 195,\ 260,\ 312,\ 390,\ 780,\ 1560 \end{array}\right\}$$

$$\begin{array}{r|rrrr} 13 & 1 & -4 & 3 & -1560 \\ & & 13 & 117 & 1560 \\ \hline & 1 & 9 & 120 & 0 \end{array} \quad \rightarrow \quad n^2 + 9n + 120 = 0$$



$$n = \frac{-9 \pm \sqrt{81 - 480}}{2}$$

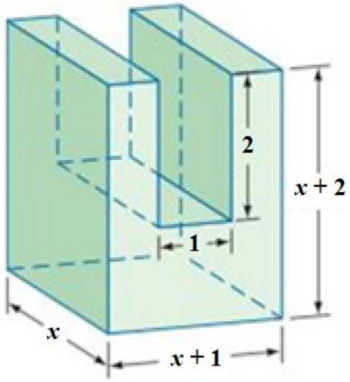
$$= \frac{-9 \pm i\sqrt{399}}{2} \quad \times$$

$\therefore \quad \underline{n = 13}\rvert$

Exercise

For what value of x will the volume of the following solid be $112\ in^3$

Solution



$$\text{Volume of the bottom portion } = x^2(x+1)$$

$$\text{Volume of one side portion } = 2x\left(\frac{1}{2}x\right)$$
$$= x^2$$

$$\text{Total Volume } = x^2(x+1) + 2x^2$$

$$112 = x^3 + 3x^2$$
$$x^3 + 3x^2 - 112 = 0$$

possibilities for $\frac{c}{d} := \pm\{1,\ 2,\ 4,\ 8,\ 14,\ 28,\ 56,\ 112\}$

$$\begin{array}{c|cccc} 4 & 1 & 3 & 0 & -112 \\ & & 4 & 28 & 112 \\ \hline & 1 & 7 & 28 & 0 \end{array} \quad \rightarrow \quad x^2 + 7x + 28 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 112}}{2}$$
$$= \frac{-7 \pm 3i\sqrt{7}}{2} \quad \times$$

$$\therefore \quad \underline{x = 4}|$$

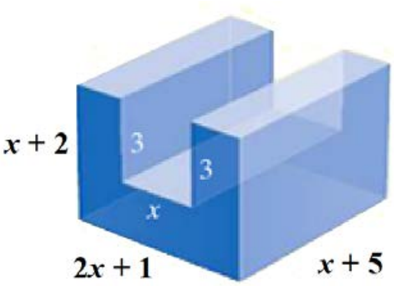
Exercise

For what value of x will the volume of the following solid be $208\ in^3$

Solution

$$\text{Volume of the bottom portion } = (2x+1)(x+5)(x+2-3)$$
$$= \left(2x^2 + 11x + 5\right)(x-1)$$
$$= 2x^3 + 11x^2 + 5x - 2x^2 - 11x - 5$$
$$= 2x^3 + 9x^2 - 6x - 5$$

$$\text{Volume of one side portion } = (3)\tfrac{1}{2}(2x+1-x)(x+5)$$
$$= \frac{3}{2}(x+1)(x+5)$$
$$= \frac{3}{2}\left(x^2 + 6x + 5\right)$$



$$\text{Total Volume } = 2x^3+9x^2-6x-5+2\left(\frac{3}{2}\right)\left(x^2+6x+5\right)$$

$$208 = 2x^3+9x^2-6x-5+3x^2+18x+15$$

$$2x^3+12x^2+12x-198=0$$

$$x^3+6x^2+6x-99=0$$

possibilities for $\frac{c}{d} := \pm\{1,\ 3,\ 9,\ 11,\ 33,\ 99\}$

$$\begin{array}{r|rrrr} 3 & 1 & 6 & 6 & -99 \\ & & 3 & 27 & 99 \\ \hline & 1 & 9 & 33 & 0 \end{array} \quad \rightarrow \quad x^2+9x+33=0$$

$$x = \frac{-9\pm\sqrt{81-132}}{2}$$

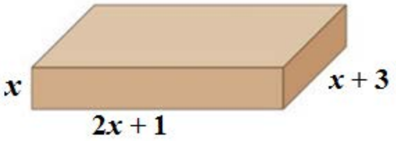
$$= \frac{-9\pm i\sqrt{51}}{2} \quad ✗$$

$$\therefore \quad \underline{x=3}$$

Exercise

The length of rectangular box is 1 *inch* more than twice the height of the box, and the width is 3 *inches* more than the height. If the volume of the box is 126 in^3, find the dimensions of the box.

Solution



$$Volume = x(2x+1)(x+3)$$

$$2x^3+7x^2+3x=126$$

$$2x^3+7x^2+3x-126=0$$

possibilities for $\frac{c}{d} := \pm\left\{\frac{126}{2}\right\}$

$$= \pm\left\{1,\ 2,\ 3,\ 6,\ 9,\ 14,\ 21,\ 42,\ 63,\ 126,\ \frac{1}{2},\ \frac{3}{2},\ \frac{9}{2},\ \frac{21}{2},\ \frac{63}{2}\right\}$$

$$\begin{array}{r|rrrr} 3 & 2 & 7 & 3 & -126 \\ & & 6 & 39 & 126 \\ \hline & 2 & 13 & 42 & 0 \end{array} \quad \rightarrow \quad 2x^2+13x+42=0$$

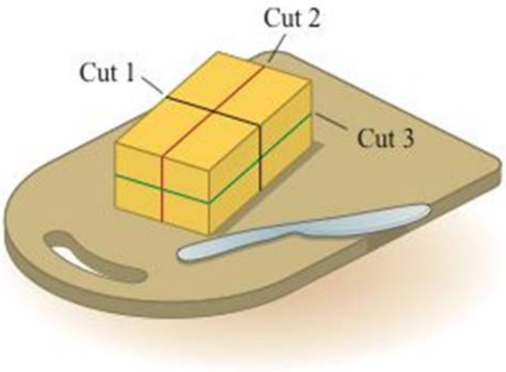
$$x = \frac{-13\pm\sqrt{169-336}}{4}$$

$$= \frac{-13\pm i\sqrt{167}}{4} \quad ✗$$

$$\therefore \quad \underline{x=3}$$

Exercise

One straight cut through a thick piece of cheese produces two pieces. Two straight cuts can produce a maximum of four pieces. Two straight cuts can produce a maximum of four pieces. Three straight cuts can produce a maximum of eight pieces.



You might be inclined to think that every additional cut doubles number of pieces. However, for four straight cuts, you get a maximum of 15 pieces. The maximum number of pieces P that can be produced by n straight cuts is given by

$$P(n)=\frac{n^3+5n+6}{6}$$

a) Determine number of pieces that can be produces by five straight cuts.

b) What is the fewest number of straight cuts that are needed to produce 64 pieces?

Solution

a) $P(5)=\dfrac{5^3+25+6}{6}$

$\qquad \underline{=26}\,\big|$

b) $\dfrac{n^3+5n+6}{6}=64$

$n^3+5n+6=384$

$n^3+5n-378=0$

possibilities for $\frac{c}{d} := \pm\{378\}$

$=\pm\{1,\ 2,\ 3,\ 6,\ 7,\ 9,\ 14,\ 18,\ 21,\ 27,\ 42,\ 54,\ 63,\ 126,\ 189,\ 378\}$

$$\begin{array}{r|rrrr} 7 & 1 & 0 & 5 & -378 \\ & & 7 & 49 & 378 \\ \hline & 1 & 7 & 54 & 0 \end{array} \quad \to \quad n^2+7n+54=0$$

$n=\dfrac{-7\pm\sqrt{49-216}}{2}$

$\quad =\dfrac{-7\pm i\sqrt{167}}{2} \qquad \text{X}$

$\therefore\ \underline{n=7}\,\big|$

Exercise

The number of ways one can select three cards from a group of *n* cards (the order of the selection matters), where $n \ge 3$, is given by $P(n) = n^3 - 3n^2 + 2n$. For a certain card trick, a magician has determined that there are exactly 504 *ways* to choose three cards from a given group. How many cards are in the group?

Solution

$$P(n) = n^3 - 3n^2 + 2n = 504$$

$$n^3 - 3n^2 + 2n - 504 = 0$$

possibilities for $\frac{c}{d} := \pm\{504\}$

$$= \pm\left\{\begin{array}{l} 1,\ 2,\ 3,\ 4,\ 6,\ 7,\ 8,\ 9,\ 12,\ 14,\ 18,\ 21, \\ \ \ 24,\ 28,\ 36,\ 42,\ 56,\ 63,\ 72,\ 84,\ 126,\ 168,\ 252,\ 504 \end{array}\right\}$$

$$\begin{array}{r|rrrr} 9 & 1 & -3 & 2 & -504 \\ & & 9 & 54 & 504 \\ \hline & 1 & 6 & 56 & 0 \end{array} \quad \rightarrow \quad n^2 + 6n + 56 = 0$$

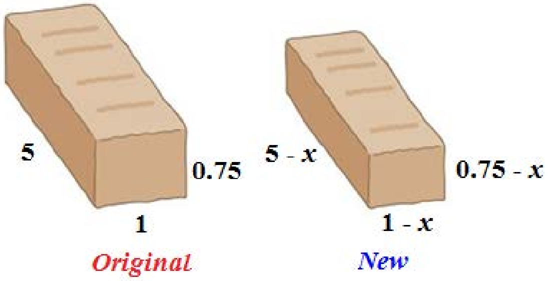
$$n = \frac{-6 \pm \sqrt{36 - 224}}{2}$$

$$= -3 \pm i\sqrt{47} \quad \times$$

$$\therefore \quad \underline{n = 9}\,\big|$$

Exercise

A nutrition bar in the shape of a rectangular solid measure 0.75 *in*. by 1 *in*. by 5 *inches*.



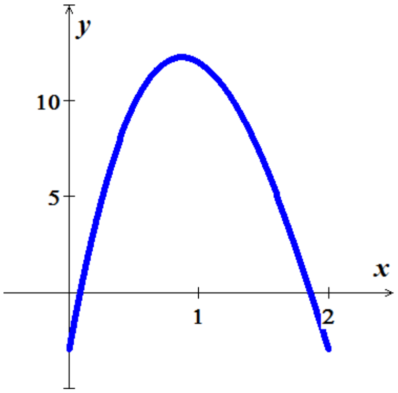
To reduce costs, the manufacturer has decided to decrease each of the dimensions of the nutrition bar by *x inches*, what value of *x* will produce a new bar with a volume that is $0.75\ \ in^3$ less than the present bar's volume.

Solution

$$V_{original} = (5)(1)\left(\frac{3}{4}\right)$$

$$= \frac{15}{4}$$

$$V_{new} = (5-x)(1-x)\left(\frac{3}{4}-x\right) \qquad \left(x < \frac{3}{4}\right)$$



$$\left(5-6x+x^2\right)\left(\frac{3-4x}{4}\right) = \frac{15}{4} - \frac{3}{4}$$

$$15-20x-18x+24x^2+3x^2-4x^3 = 4(3)$$

$$4x^3-27x^2+38x-3=0$$

From graph table:

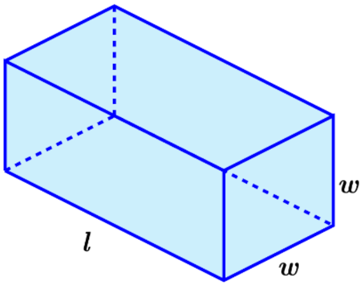
```
0.08200  -0.06334
0.08400   0.00386
```

$$\underline{x \approx 0.083 \ \ in.}$$

Exercise

A rectangular box is square on two ends and has length plus girth of 81 *inches*. (Girth: distance *around* the box). Determine the possible lengths $l \ (l > w)$ of the box if its volume is $4900 \ \ in^3$.

Solution



$$81 = l + 4w$$

$$l = 81 - 4w$$

$$V = lw^2$$

$$= (81-4w)w^2$$

$$-4w^3+81w^2 = 4900$$

$$4w^3-81w^2+4900=0$$

$$possibilities \ for \ \frac{c}{d} := \pm\left\{\frac{4900}{4}\right\} = \pm\left\{\begin{matrix}1, \ 2, \ 4, \ 7, \ 10, \ 14, \ 20, \ 28, \ 49, \\ 100, \ 175, \ 245, \ 350, \ 490, \ 700, \ 1225, \ 2450, \ 4900, \cdots\end{matrix}\right\}$$

$$\begin{array}{c|cccc} 14 & 4 & -81 & 0 & 4900 \\ & & 56 & -350 & -4900 \\ \hline & 4 & -25 & -350 & 0 \end{array} \rightarrow \ 4w^2-25w-350=0$$

$$w = \frac{25 \pm \sqrt{625+5600}}{8}$$

$$= \frac{25 \pm 5\sqrt{249}}{8}$$

$$= \begin{cases} \dfrac{25-5\sqrt{249}}{8} < 0 \\ \dfrac{25+5\sqrt{249}}{8} \approx 13 \end{cases}$$

$$l = 81 - 4(14) = 25$$

$l = 81 - 4(13) = 29$

$\therefore$ the possible lengths l are around **25** *in*. ***or*** **29** *in*.