Review

12/9 Exam 4

No class 12/7

$\cos A = \frac{15}{17}$ $A \in Q1$ $\cos B = -\frac{12}{13}$ $B \in Q2$

$\sin A = \frac{8}{17}$ $\sin B = \frac{5}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \left(\frac{8}{17}\right)\left(-\frac{12}{13}\right) + \left(\frac{15}{17}\right)\left(\frac{5}{13}\right)$

$= \frac{-96+75}{221}$

$= -\frac{21}{221}$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{15}{17}\left(\frac{-12}{13}\right) - \left(\frac{8}{17}\right)\frac{5}{13}$

$= \frac{-180-40}{221}$

$= -\frac{220}{221}$

c) $\tan(A+B) = \frac{21}{220}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= \frac{8}{17}\left(\frac{-12}{13}\right) - \frac{15}{17}\,\frac{5}{13}$$

$$= \frac{-96-75}{221}$$

$$= -\frac{171}{221}$$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$= \frac{15}{17}\left(\frac{-12}{13}\right) + \frac{8}{17}\,\frac{5}{13}$$

$$= \frac{-180+40}{221}$$

$$= -\frac{140}{221}$$

f) $\tan(A-B) = \dfrac{171}{140}$

#2/ $\sin A = \frac{3}{5}$ A in QII $\quad 90^\circ \le A \le 180^\circ$

$45^\circ \le \frac{A}{2} \le 90^\circ$

$\cos A = -\frac{4}{5}$

a) $\sin 2A = 2 \sin A \cos A$

$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= -\frac{24}{25}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

c) $\tan 2A = -\frac{24}{7}$

d) $\sin \frac{A}{2} = \sqrt{\frac{1}{2}(1 - \cos A)}$

$= \sqrt{\frac{1}{2}\left(1 + \frac{4}{5}\right)}$

$= \frac{3}{\sqrt{10}}$

e) $\cos \frac{A}{2} = \sqrt{\frac{1}{2}(1 + \cos A)}$

$= \sqrt{\frac{1}{2}\left(1 - \frac{4}{5}\right)}$

$= \frac{1}{\sqrt{10}}$

f) $\tan \frac{A}{2} = 3$

$1 - \sin x = \sqrt{3}\cos x \qquad [0, 2\pi)$

$\sqrt{3}\cos x + \sin x = 1$

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{1}{2}$

$\cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x = \frac{1}{2}$

$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

$x - \frac{\pi}{6} = \frac{\pi}{3}$

$x = \frac{\pi}{3} + \frac{\pi}{6}$

$= \frac{\pi}{2}$

$x - \frac{\pi}{6} = \frac{5\pi}{3}$

$x = \frac{5\pi}{3} + \frac{\pi}{6}$

$= \frac{11\pi}{6}$

$2\tan x \csc x + 2\csc x + \tan x + 1 = 0 \qquad [0, 2\pi)$

$2\csc x(\tan x + 1) + (\tan x + 1) = 0$

$(\tan x + 1)(2\csc x + 1) = 0$

$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\csc x = -\frac{1}{2}$

$\sin x = -2$ #

$2\sin^2 x - \cos x - 1 = 0 \qquad [0, 2\pi)$

$2(1-\cos^2 x) - \cos x - 1 = 0$

$2 - 2\cos^2 x - \cos x - 1 = 0$

$-2\cos^2 x - \cos x + 1 = 0$

$\cos x = -1 \qquad \cos x = \frac{1}{2}$

$x = \pi \quad , \frac{\pi}{3}, \frac{5\pi}{3}$

$\sin x \tan x = \sin x \qquad [0, 2\pi)$

$\sin x \tan x - \sin x = 0$

$\sin x (\tan x - 1) = 0$

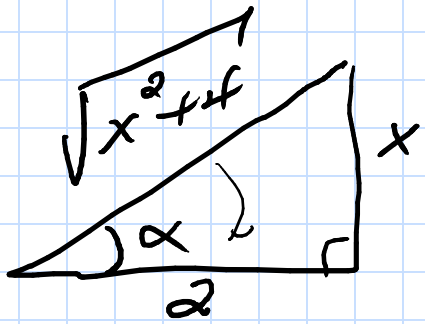
$\sin x = 0 \qquad \tan x = 1$

$x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$

$$\sec\left(\sin^{-1}\frac{x}{\sqrt{x^2+4}}\right)$$

α

$$\sin\alpha = \frac{x}{\sqrt{x^2+4}}$$

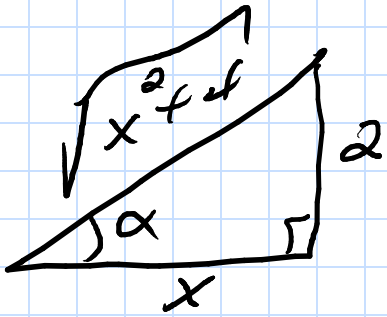


$$\sec\alpha = \frac{\sqrt{x^2+4}}{2}$$

$$\sin\left(\cos^{-1}\frac{x}{\sqrt{x^2+4}}\right)$$

α

$$\cos\alpha = \frac{x}{\sqrt{x^2+4}}$$



$$\sin\alpha = \frac{2}{\sqrt{x^2+4}}$$

$(4, 30°)$ (x, y)?

$x = r\cos\theta$

$= 4\cos 30°$

$= 4\left(\frac{\sqrt{3}}{2}\right)$

$= 2\sqrt{3}$

$y = r\sin\theta$

$= 4\sin 30°$

$= 4\left(\frac{1}{2}\right)$

$= 2$

$(x, y) = (2\sqrt{3}, 2)$

→ (4) all no need.

$(3, -3)$ (r, θ)?

$r = \sqrt{x^2 + y^2}$

$= \sqrt{9 + 9}$

$= 3\sqrt{2}$

$\theta = \tan^{-1}\left|\frac{3}{3}\right|$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

$(r, \theta) = \left(3\sqrt{2}, \frac{7\pi}{4}\right)$

$$r(\sin\theta - 2\cos\theta) = 6$$

$$r\sin\theta - 2r\cos\theta = 6$$

$$\boxed{y - 2x = 6}$$

$$y^2 = x$$

$$(r\sin\theta)^2 = r\cos\theta$$

$$r^2\sin^2\theta = r\cos\theta \qquad (r \neq 0)$$

$$r\sin^2\theta = \cos\theta$$

$$\boxed{r = \frac{\cos\theta}{\sin^2\theta}}$$

$-\sqrt{3} + i$

$x = -\sqrt{3}, \quad y = 1$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{3 + 1}$

$= \sqrt{4}$

$= 2$

$\theta = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$

$= \frac{\pi}{6}$

$-\sqrt{3} + i = 2 \operatorname{cis} \frac{5\pi}{6}$

$\sqrt{2} \operatorname{cis} \frac{7\pi}{4} = \sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

$= \sqrt{2}\left(\frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2}\right)$

$= 1 - i$

$\frac{20 \operatorname{cis}(75^\circ)}{4 \operatorname{cis}(40^\circ)} = 5 \operatorname{cis}(75^\circ - 40^\circ)$

$= 5 \operatorname{cis}(35^\circ)$

$(20 \operatorname{cis} 75^\circ)(4 \operatorname{cis} 40^\circ) = 80 \operatorname{cis}(75^\circ + 40^\circ)$

$= 80 \operatorname{cis}(115^\circ)$