# Geometric Sequences.

$$a_{k+1} = a_k r$$

$a_1, a_2, \ldots, a_n, \ldots$

$r = \frac{a_{k+1}}{a_k}$ common Ratio.

$$\boxed{a_n = a_1 r^{n-1}}$$

$6, -12, 24, -48, \ldots, (-1)^{n-1} 6, \ldots$

$r = -\frac{12}{6} = -2$

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Ex

1st term

$a_1 = 3 \quad r = -\frac{1}{2}$

$a_n = a_1 r^{n-1}$

$= 3\left(-\frac{1}{2}\right)^{n-1}$

$n = 1$

$a_1 = 3$

$a_2 = 3\left(-\frac{1}{2}\right)^1 = -\frac{3}{2}$

$a_3 = 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4},$

$$a_4 = 3\left(-\frac{1}{2}\right)^3 = \boxed{-\frac{3}{8}}$$

$$a_5 = 3\left(-\frac{1}{2}\right)^4 = \boxed{\frac{3}{16}}$$

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**Ex** $a_3 = 5$, $a_6 = -40$ $\quad a_8$??

$$r = \left(\frac{-40}{5}\right)^{\frac{1}{6-3}} \qquad r = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$$

$$= (-8)^{1/3} \qquad a_n = a_1 r^{n-1}$$

$$\boxed{= -2}$$

$$a_3 = a_1 (-2)^{\overset{3-1}{2}} = 5$$

$$4a_1 = 5$$

$$\boxed{a_1 = \frac{5}{4}}$$

$$a_8 = \frac{5}{4}(-2)^7$$

$$= -5(2^5)$$

$$= -160$$

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$$S_n = a_1 \frac{1 - r^n}{1 - r} \qquad \boxed{r \neq 1}$$

Ex $1, 0.3, 0.09, .0027, \ldots$

1st 5 terms $\quad \underline{a_1 = 1}, \underline{r = .3}$

$$S_5 = \frac{1-(.3)^5}{1-.3}$$

$$= \frac{1-\left(\frac{3}{10}\right)^5}{1-\frac{3}{10}}$$

$$= \frac{1-\frac{3^5}{10^5}}{\frac{7}{10}}$$

$$= \frac{10^5-3^5}{10^5} \cdot \frac{10}{7}$$

$$= \frac{10{,}000-243}{7 \times 10^4}$$

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$a_1, a_2, \ldots, a_n, \ldots$

$$S = \frac{a}{1-r} \qquad |r| < 1$$

if $|r| \geq 1$ ; $\boxed{S = \infty}$

Ex, $\displaystyle\sum_{n=1}^{\infty} 3\left(-\frac{2}{3}\right)^{n-1} = \frac{3}{1+\frac{2}{3}} \qquad \left|-\frac{2}{3}\right| = \frac{2}{3} < 1$

$$= \frac{3}{\frac{5}{3}}$$

$$= \frac{7}{5}$$

110 $\displaystyle\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1} = \frac{5}{1-\frac{1}{4}}$ $\quad \left|\frac{1}{4}\right| < 1$

$$= \frac{5}{\frac{3}{4}}$$

$$= \frac{20}{3}$$

~~109~~ $\displaystyle\sum_{n=1}^{\infty} 3\left(\frac{3}{2}\right)^{n-1} = \infty$ $\quad \frac{3}{2} \geq 1$

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Ex $\quad 5.4\overline{27}$

$5.4272727 = 5.4 + .02727272727\ldots$

$$= \frac{54}{10} + (.027 + .00027 + \cdots)$$

$$a_1 = .027 = 27 \times 10^{-3}$$

$$r = \frac{.00027}{.027} = .01 \qquad \frac{27 \times 10^{-5+3}}{27 \times 10^{-3+3}}$$

$$S = \frac{54}{10} + \frac{27 \times 10^{-3}}{1 - .01}$$

$$= \frac{54}{10} + \frac{27 \times 10^{-3}}{.99}$$

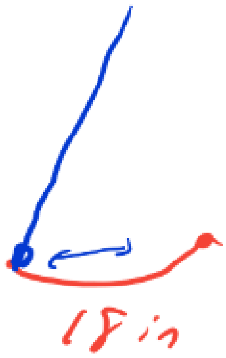
$$= \frac{54}{10} + \frac{27}{99}\,\frac{10^{-3}}{10^{-2}} \quad +3$$

$$= \frac{54}{10} + \frac{3}{110} \qquad -2+3$$

$$= \frac{594+3}{110}$$

$$\boxed{5.4\overline{27} = \frac{597}{110}}$$

Fr $\quad l = .98 = r$




$a_1 = 18$

$a_2 = 18(.98)$

$a_{10} = 18(.98)^9$

$$\boxed{a_n = 18(.98)^{n-1}}$$

$$18(.98)^{n-1} = 12$$

$$(.98)^{n-1} = \frac{12}{18} = \frac{2}{3}$$

$$n-1 = \log_{.98} \frac{2}{3} = \frac{\ln(2/3)}{\ln .98}$$

$\ln(0<<1) = <0$

$\ln 1 = 0$

$\ln e = 1$

$n \approx 21.07$

$22^{nd}$ swing

$r = .98 < 1$

$$T = \frac{18}{1-.98}$$

$.9$

$$= \frac{18}{.02}$$

$$= \frac{18}{2\times 10^{-2}}$$

$$= 900$$

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## 5.7 Mathematical Induction

Show that $P_1$ is true

Assume $P_k$ is true, Prove $P_{k+1}$ is also true

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**Ex** $1+2+3+ \cdots + n = \frac{n(n+1)}{2}$

**Soln**

For $n = 1 \Rightarrow$ $1 = \frac{1(2)}{2}$

$1 = 1$ ✓

$P_1$ is true

Assume $P_k$ is true: $1+2+\cdots+k = \frac{k(k+1)}{2}$

Is $P_{k+1}$: $1+\cdots+k+(k+1) \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$

$$1+\cdots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= (k+1)\left(\frac{k}{2}+1\right)$$

$$= (k+1)\left(\frac{k+2}{2}\right) \checkmark$$

Hence, $P_{k+1}$ is also true.

$\therefore$ By the mathematical induction, the given proof is completed

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Ex $1^2+3^2+\cdots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

For $n=1 \Rightarrow 1^2=\frac{1(2-1)(3)}{3}$

$1=1$ ✓ $P_1$ is true.

Assume $P_k$ is true:

$$1^2+\cdots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}$$

is $P_{k+1}$ is also true

$$1^2+\cdots+(2k-1)^2+(2(k+1)-1)^2=\frac{1}{3}(k+1)(2(k+1)-1)(2(k+1)+1)$$

$$1^2+\cdots+(2k-1)^2+(2k+1)^2\overset{?}{=}\frac{1}{3}(k+1)(2k+1)(2k+3)$$

$$1^2+\cdots+(2k-1)^2+(2k+1)^2=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$$

$$=\frac{1}{3}(2k+1)[2k^2-k+3(2k+1)]$$

$$=\frac{1}{3}(2k+1)(2k^2+5k+3)$$

$$=\frac{1}{3}(2k+1)(k+1)(2k+3) \checkmark$$

$P_{k+1}$ is also true

$\therefore$ By the Mathematical induction, the given proof is completed.

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Ex 2 is a factor of $n^2+5n$

$n \in \mathbb{Z}^+$

$n=1 \Rightarrow 1^2+5=6$

$=2(3)$ ✓

$P_1$ is true.

Assume $P_k$ is true: $k^2 + 5k = 2p$

Is $P_{k+1}$ is also true? $(k+1)^2 + 5(k+1)$ is 2 a factor

$$(k+1)^2 + 5(k+1) = k^2 + 2k + 1 + 5k + 5$$
$$= (k^2 + 5k) + 2k + 6$$
$$= 2p + 2k + 6$$
$$= 2(p + k + 3) \checkmark$$

$P_{k+1}$ is also true.

∴ By the mathematical induction, the given proof is completed.

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## Exam 1

2 - Partial Fraction (5.2)

1 - ellipse (5.3)

1 - Hyperbola (5.4)

2 - (5.5) 1st d + $a_{10}$

3 = $\sum_{1}^{10} 5 \quad \sum_{10}^{50} 5 \quad \sum_{1}^{4}$ $\quad a_n = a_1 + (n-1)d$

1 (5.6) arithmetic $d = \frac{y_2 - y_1}{x_2 - x_1}$

$a_{\cancel{x}} : a_a \quad a_6$

1 geometric : $r = \left(\frac{y_2}{y_1}\right)^{1/x_2 - x_1}$

$\{ a_n = a_1 r^{n-1}$

2 $\Sigma$ $\begin{cases} S = \frac{a_1}{1-r} & |r| < 1 \\ S = \infty & |r| \geq 1 \end{cases}$

① $a_1 + a_2 + \cdots + a_n = \Sigma$

① Prove

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Hwk 5.6 due 9/17
5.7 (2) Thursday