

$$16/ \quad A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$$

$$aA + bB = a \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} \\ = \begin{pmatrix} a & 2a+b \\ 3a-b & 4a+2b \end{pmatrix}$$

$$17/ \quad A = \begin{pmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{pmatrix}$$

$$a) \quad 3X + 2A = B$$

$$3 \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \\ x_5 & x_6 \end{pmatrix} + 2 \begin{pmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 3x_1 - 8 & 3x_2 \\ 3x_3 + 2 & 3x_4 - 10 \\ 3x_5 - 6 & 3x_6 + 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{pmatrix}$$

$$3x_1 - 8 = 1 \Rightarrow \underline{x_1 = 3}$$

$$3x_2 = 2 \rightarrow x_2 = \frac{2}{3}$$

$$3x_3 + 2 = -2 \rightarrow x_3 = -\frac{4}{3}$$

$$3x_4 - 10 = 1 \rightarrow x_4 = \frac{11}{3}$$

$$3x_5 - 6 = 4 \rightarrow x_5 = \frac{10}{3}$$

$$3x_6 + 4 = 4 \rightarrow x_6 = 0$$

$$X = \begin{pmatrix} 3 & \frac{2}{3} \\ -\frac{4}{3} & \frac{11}{3} \\ \frac{10}{3} & 0 \end{pmatrix}$$

14/ a) $\text{Tr}(A+B) = \text{Tr}(A) + \text{Tr}(B)$

$$\text{Tr}(A) = a_{11} + a_{22} + \dots + a_{nn}$$

$$\text{Tr}(B) = b_{11} + b_{22} + \dots + b_{nn}$$

$$A+B = [a_{ij} + b_{ij}]$$

$$\begin{aligned} \text{Tr}(A+B) &= (a_{11} + b_{11}) + (a_{22} + b_{22}) + \dots + (a_{nn} + b_{nn}) \\ &= (a_{11} + a_{22} + \dots + a_{nn}) + (b_{11} + b_{22} + \dots + b_{nn}) \\ &= \text{Tr}(A) + \text{Tr}(B) \checkmark \end{aligned}$$

b) $\text{Tr}(cA) = c \text{Tr}(A)$

$$\begin{aligned} c \text{Tr}(A) &= c(a_{11} + a_{22} + \dots + a_{nn}) \\ &= ca_{11} + ca_{22} + \dots + ca_{nn} \\ &= \text{Tr}(cA) \checkmark \end{aligned}$$

15/ $A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \quad B = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}$

$$\begin{aligned} AB &= \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \\ &= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\cos \alpha \sin \beta - \sin \alpha \cos \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & -\sin \alpha \sin \beta + \cos \alpha \cos \beta \end{pmatrix} \\ &= \begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \\ &= \begin{pmatrix} \cos \beta \cos \alpha - \sin \beta \sin \alpha & -\sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \cos \alpha \sin \beta + \sin \alpha \cos \beta & -\sin \alpha \sin \beta + \cos \alpha \cos \beta \end{pmatrix} \\ &= \begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} \end{aligned}$$

$$\Rightarrow AB = BA$$

$$\overline{13}, A = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \quad B = \begin{pmatrix} -5 & 0 \\ 0 & 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} -5 & 0 \\ 0 & 4 \end{pmatrix} \\ = \begin{pmatrix} -10 & 0 \\ 0 & -12 \end{pmatrix}$$

$$BA = \begin{pmatrix} -5 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \\ = \begin{pmatrix} -10 & 0 \\ 0 & -12 \end{pmatrix}$$

$$AB = BA$$

$$\text{11/} \quad \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} a_{11} + 2a_{21} = 1 \\ a_{12} + 2a_{22} = 0 \\ 3a_{11} + 5a_{21} = 0 \\ 3a_{12} + 5a_{22} = 1 \end{cases}$$

$$\begin{aligned} -3 \begin{cases} a_{11} + 2a_{21} = 1 \\ 3a_{11} + 5a_{21} = 0 \end{cases} \\ \hline -3a_{11} - 6a_{21} = -3 \\ 3a_{11} + 5a_{21} = 0 \\ \hline a_{21} = 3 \\ a_{11} = -5 \end{aligned}$$

$$A = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$$

$$\begin{aligned} -3 \begin{cases} a_{12} + 2a_{22} = 0 \\ 3a_{12} + 5a_{22} = 1 \end{cases} \end{aligned}$$

$$\Rightarrow a_{22} = -1, a_{12} = 2$$

$$\text{12/} \quad \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 19 & 2 \end{pmatrix}$$

$$\begin{aligned} -3 \begin{cases} a + 2c = 6 \\ 3a + 4c = 19 \end{cases} \\ \hline -2c = 1 \\ c = -\frac{1}{2} \\ a = 7 \end{aligned}$$

$$\begin{aligned} -3 \begin{cases} b + 2d = 3 \\ 3b + 4d = 2 \end{cases} \\ \hline -2d = -7 \\ d = \frac{7}{2} \\ b = -4 \end{aligned}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 7 & -4 \\ -\frac{1}{2} & \frac{7}{2} \end{pmatrix}$$

9. cont $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -3 & 1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

$$x_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} -1 \\ -3 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$$

$$\begin{cases} x_1 - x_2 + 2x_3 = -1 \\ 3x_1 - 3x_2 + x_3 = 7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 3 & -3 & 1 & 7 \end{array} \right] R_2 - 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 0 & 0 & -5 & 10 \end{array} \right] \xrightarrow{\textcircled{1}} \rightarrow x_3 = -2$$

$$\textcircled{1} \quad x_1 - x_2 = -1 - 2(-2) = 3$$

$$\text{let } x_2 = 0 \Rightarrow x_1 = 3$$

$$3 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 0 \begin{pmatrix} -1 \\ -3 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$$

$$\text{check!} \quad \begin{matrix} 3+0-4 = -1 \checkmark \\ 9-2 = 7 \checkmark \end{matrix}$$

$$\text{if we let } x_1 = 0 \Rightarrow x_2 = 3$$

$$0 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 3 \begin{pmatrix} -1 \\ -3 \end{pmatrix} - 2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$$

$$-3-4 \neq -1 \quad \#$$

$$\begin{cases} x_1 = 3, x_2 = 0, x_3 = -2 \end{cases}$$

(not unique) $\begin{cases} x_1 = 2, x_2 = -1, x_3 = -2 \checkmark \end{cases}$

6/ $A: 2 \times 4$ $E: 4 \times 3$

$E - 2A$ not possible, $E + A$ are not the same size

7/ $D: 4 \times 2$ $C: 4 \times 2$

$2D + C: 4 \times 2$

8/ $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & -2 & 2 \end{pmatrix}$ $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$Ax = 0$

$Ax: 2 \times 3 - 3 \times 1$

$\begin{pmatrix} 2 & -1 & -1 \\ 1 & -2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{cases} 2x_1 - x_2 - x_3 = 0 \\ x_1 - 2x_2 + 2x_3 = 0 \end{cases}$

let $x_3 = t$

$\Rightarrow \begin{cases} 2x_1 - x_2 = t \\ x_1 - 2x_2 = -2t \end{cases}$

$2x_1 - x_2 = t$

$-2x_1 + 4x_2 = 4t$

$3x_2 = 5t \Rightarrow x_2 = \frac{5}{3}t$

$x_1 = -2t + \frac{10}{3}t$

$= \frac{4}{3}t$

$\therefore x = \left(\frac{4}{3}t, \frac{5}{3}t, t \right)$

9/ $\begin{cases} -x_1 + x_2 = 4 \\ -2x_1 + x_2 = 0 \end{cases}$

$\begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$Ax = b$

$$3/ A = \begin{pmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 8 & -1 \\ 4 & -3 \end{pmatrix}$$

$2 \times 3 \qquad \qquad \qquad 2 \times 2$

a) $A + B$ not possible (A & B are not same size)

b) $A - B$ " " " " same "

$$c) 2A = 2 \begin{pmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{pmatrix} \\ = \begin{pmatrix} 12 & 0 & 6 \\ -2 & -8 & 0 \end{pmatrix}$$

d) $2A - B$ not possible

e) $B + \frac{1}{2}A$ " "

$$4/ A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -1 & 8 \end{pmatrix}$$

$$a) AB = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 8 \end{pmatrix} = \begin{pmatrix} 0 & 15 \\ 6 & 12 \end{pmatrix}$$

$$b) BA = \begin{pmatrix} 2 & -1 \\ -1 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 31 & 14 \end{pmatrix}$$

$$5/ A = \begin{pmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{pmatrix}$$

$3 \times 2 \qquad \qquad \qquad 3 \times 3$

a) $AB =$ not possible

$$b) BA = \begin{pmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{pmatrix}$$

$3 \times 3 \quad 3 \times 2$

$$= \begin{pmatrix} 3 & -4 \\ 10 & 16 \\ 26 & 46 \end{pmatrix}$$

$$\begin{bmatrix} 16 & 4 & 5 & 4 \\ -3 & 13 & 15 & 6 \\ 0 & 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 4 & 2x+1 & 4 \\ -3 & 13 & 15 & 3x \\ 0 & 2 & 3y-5 & 0 \end{bmatrix}$$

$$2x+1=5 \rightarrow x=2$$

$$3x=6$$

$$3y-5=4 \Rightarrow y=3$$

$$2/ \quad A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -3 & -2 \\ 4 & 2 \end{pmatrix}$$

$$a) \quad A+B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} -3 & -2 \\ 4 & 2 \end{pmatrix} \\ = \begin{pmatrix} -2 & 0 \\ 6 & 3 \end{pmatrix}$$

$$b) \quad A-B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} -3 & -2 \\ 4 & 2 \end{pmatrix} \\ = \begin{pmatrix} 4 & 4 \\ -2 & -1 \end{pmatrix}$$

$$c) \quad 2A = 2 \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \\ = \begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}$$

$$d) \quad 2A-B = \begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix} - \begin{pmatrix} -3 & -2 \\ 4 & 2 \end{pmatrix} \\ = \begin{pmatrix} 5 & 6 \\ 0 & 0 \end{pmatrix}$$

$$e) \quad B + \frac{1}{2}A = \begin{pmatrix} -3 & -2 \\ 4 & 2 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \\ = \begin{pmatrix} -3 & -2 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 1 \\ 1 & \frac{1}{2} \end{pmatrix} \\ = \begin{pmatrix} -\frac{5}{2} & -1 \\ 5 & \frac{5}{2} \end{pmatrix}$$