Section 8.4 – Solving Trigonometry Equations

Example

Find the solutions of the equation $\sin\theta = \frac{1}{2}$ if

a) θ is in the interval $[0,\ 2\pi)$

b) θ is any real number

Solution

a) $\theta = \sin^{-1}\frac{1}{2} \underline{= \frac{\pi}{6}}\,\big|$

$\theta = \pi - \frac{\pi}{6} \underline{= \frac{5\pi}{6}}\,\big|$

b) Since the sine function has period 2π.

$\theta = \frac{\pi}{6} + 2\pi n \quad and \quad \theta = \frac{5\pi}{6} + 2\pi n$

Example

Solve the equation $\sin x\ \tan x = \sin x$

Solution

$\sin x\ \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0$	$\tan x - 1 = 0$
	$\tan x = 1$
$\hat{x} = \sin^{-1} 0 = 0$	$\hat{x} = \tan^{-1} 1 = \frac{\pi}{4}$
$x = 0,\ \pm\pi,\ \pm 2\pi, \ldots$	$x = \pm\frac{\pi}{4},\ \pm\frac{5\pi}{4}, \ldots$
$x = \pi n$	$x = \frac{\pi}{4} + \pi n$

The solutions are: $x = \pi n \quad and \quad x = \frac{\pi}{4} + \pi n$ for every integer *n*.

Example

Solve the equation $2\sin^2 t - \cos t - 1 = 0$, and express the solutions both in radians and degrees.

Solution

$$2\sin^2 t - \cos t - 1 = 0$$

$$2\left(1-\cos^2 t\right) - \cos t - 1 = 0 \qquad \sin^2 t + \cos^2 t = 1$$

$$2 - 2\cos^2 t - \cos t - 1 = 0$$

$$-2\cos^2 t - \cos t + 1 = 0 \qquad \textit{Multiply by } -1$$

$$2\cos^2 t + \cos t - 1 = 0 \qquad \textit{Factor or use quadratic formula}$$

$$(2\cos t - 1)(\cos t + 1) = 0$$

$$2\cos t - 1 = 0 \qquad \cos t + 1 = 0$$

$$2\cos t = 1 \qquad \cos t = -1$$

$$\cos t = \frac{1}{2}$$

$$t = \frac{\pi}{3} \quad or \quad t = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \qquad t = \pi$$

$$t = \frac{\pi}{3} + 2\pi n, \quad t = \frac{5\pi}{3} + 2\pi n, \qquad t = \pi + 2\pi n$$

$$t = 60^\circ + 360^\circ n, \quad 300^\circ + 360^\circ n, \quad and \quad 180^\circ + 360^\circ n$$

Example

Solve the equation $4\sin^2 x \tan x - \tan x = 0$ in the interval $[0,\ 2\pi)$.

Solution

$$4\sin^2 x \tan x - \tan x = 0$$

$$\tan x\left(4\sin^2 x - 1\right) = 0 \qquad \textit{Factor out } \tan x$$

$$\tan x = 0 \qquad 4\sin^2 x - 1 = 0$$

$$\sin^2 x = \frac{1}{4}$$

$$\tan x = 0 \qquad \sin x = \frac{1}{2} \qquad \sin x = -\frac{1}{2}$$

$$x = 0,\ \pi \qquad x = \frac{\pi}{6},\ \frac{5\pi}{6} \qquad x = \frac{7\pi}{6},\ \frac{11\pi}{6}$$

Example

Find the solutions of $\csc^4 2u - 4 = 0$

Solution

$$\left(\csc^2 2u - 2\right)\left(\csc^2 2u + 2\right) = 0$$

$$\csc^2 2u - 2 = 0 \qquad \csc^2 2u + 2 = 0$$

$$\csc^2 2u = 2 \qquad \csc^2 2u = -2 \times$$

$$\csc 2u = \pm\sqrt{2}$$

$$\sin 2u = \pm\frac{1}{\sqrt{2}}$$

$$\sin 2u = \frac{\sqrt{2}}{2} \quad \Rightarrow 2u = \frac{\pi}{4} + 2\pi n \quad \to u = \frac{\pi}{8} + \pi n$$

$$\Rightarrow 2u = \frac{3\pi}{4} + 2\pi n \quad \to u = \frac{3\pi}{8} + \pi n$$

$$\sin 2u = -\frac{\sqrt{2}}{2} \quad \Rightarrow 2u = \frac{5\pi}{4} + 2\pi n \quad \to u = \frac{5\pi}{8} + \pi n$$

$$\Rightarrow 2u = \frac{7\pi}{4} + 2\pi n \quad \to u = \frac{7\pi}{8} + \pi n$$

Example

Approximate to the nearest degree, the solutions of the following equation in the interval $[0°,\ 360°)$:

$$5\sin\theta\tan\theta - 10\tan\theta + 3\sin\theta - 6 = 0$$

Solution

$$\tan\theta(5\sin\theta - 10) + (3\sin\theta - 6) = 0$$

$$5\tan\theta(\sin\theta - 2) + 3(\sin\theta - 2) = 0$$

$$(\sin\theta - 2)(5\tan\theta + 3) = 0$$

$$\sin\theta - 2 = 0 \qquad 5\tan\theta + 3 = 0$$

$$\sin\theta = 2 > 1 \qquad \tan\theta = -\frac{3}{5} \qquad \boldsymbol{\theta \in \textbf{QII, QIV}}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{3}{5}\right) = 31°$$

$$\begin{cases} \theta = 180° - 31° & = 149° \\ \theta = 360° - 31° & = 329° \end{cases}$$

Exercises *Section* 8.4 – Trigonometric Equations

(1 – 9) Find all solutions of the equation

1. $\sin x = \frac{\sqrt{2}}{2}$

2. $\cos x = -\frac{\pi}{3}$

3. $2\cos\theta - \sqrt{3} = 0$

4. $\sqrt{3}\tan\frac{1}{3}x = 1$

5. $\cos\left(4x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

6. $(\cos\theta - 1)(\sin\theta + 1) = 0$

7. $\cot^2 x - 3 = 0$

8. $\cos x + 1 = 2\sin^2 x$

9. $\cos(\ln x) = 0$

(10 – 24) Find the solutions of the equation that are in the interval $[0,\ 2\pi)$

10. $2\sin^2 x = 1 - \sin x$

11. $\tan^2 x \sin x = \sin x$

12. $1 - \sin x = \sqrt{3}\cos x$

13. $\sin x + \cos x \cot x = \csc x$

14. $2\sin^3 x + \sin^2 x - 2\sin x - 1 = 0$

15. $2\tan x \csc x + 2\csc x + \tan x + 1 = 0$

16. $5\cos t + \sqrt{12} = \cos t$

17. $2\sin^2 x - \cos x - 1 = 0$

18. $2\cos^2 t - 9\cos t = 5$

19. $\tan^2 x + \tan x - 2 = 0$

20. $\tan x + \sqrt{3} = \sec x$

21. $2\sin^2\theta + 2\sin\theta - 1 = 0$

22. $2\cos x - 1 = \sec x$

23. $4\cos^2 x + 4\sin x - 5 = 0$

24. $\sin\theta - \cos\theta = 1$

(25 – 35) Find the solutions of the equation that are in the interval *if* $0° \le \theta < 360°$

25. $2\cos\theta + \sqrt{3} = 0$

26. $\tan\theta - 2\cos\theta\tan\theta = 0$

27. $2\sin^2\theta - 2\sin\theta - 1 = 0$

28. $4\cos\theta - 3\sec\theta = 0$

29. $\sin\theta - \sqrt{3}\cos\theta = 1$

30. $7\sin^2\theta - 9\cos 2\theta = 0$

31. $\sin\theta\tan\theta = \sin\theta$

32. $2\sin\theta - 3 = 0$

33. $3\sin\theta - 2 = 7\sin\theta - 1$

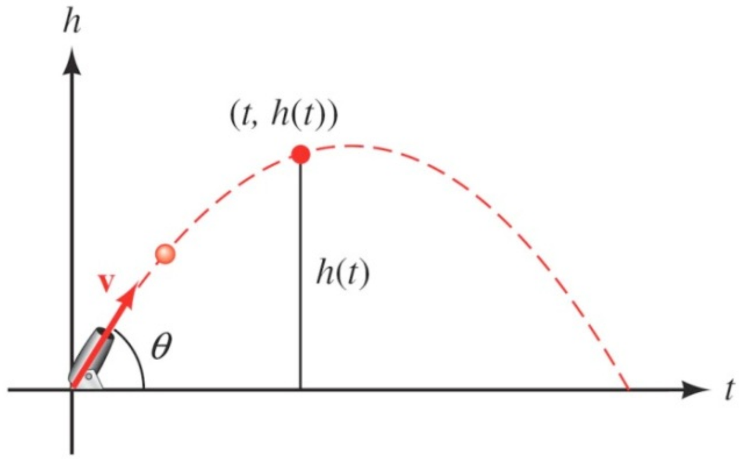
34. $\cos 2\theta + 3\sin\theta - 2 = 0$

35. $\sin 2\theta + \sqrt{2}\cos\theta = 0$

36. Solve $\cos\left(A - \frac{\pi}{9}\right) = -\frac{1}{2}$

37. Solve $\cos(A - 25°) = -\frac{1}{\sqrt{2}}$

38. If a projectile (such as a bullet) is fired into the air with an initial velocity $\boldsymbol{v}$ at an angle of elevation θ, then the height h of the projectile at time t is given by: $h(t) = -16t^2 + vt\sin\theta$



a) Give the equation for the height, if $\boldsymbol{v}$ is 600 *ft./sec* and $\theta = 45°$.

b) Use the equation in part (*a*) to find the height of the object after $\sqrt{3}$ seconds.

c) Find the angle of elevation of θ of a rifle barrel, if a bullet fired at 1,500 *ft./sec* takes 3 seconds to reach a height of 750 *feet*. Give your answer in the nearest of a degree.