1/27

#36 $\displaystyle \lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1} = \frac{\overbrace{3-3}^{\text{can avoid}}}{0} = \frac{0}{0}$

$$= \lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1} \cdot \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}$$

$$= \lim_{x\to -1} \frac{\overbrace{x^2+8-9}^{x^2-1}}{(x+1)\left(\sqrt{x^2+8}+3\right)}$$

$$= \lim_{x\to -1} \frac{(x+1)(x-1)}{(x+1)\left(\sqrt{x^2+8}+3\right)}$$

$$= \lim_{x\to -1} \frac{x-1}{\sqrt{x^2+8}+3}$$

$$= \frac{-2}{6}$$

$$= \boxed{-\frac{1}{3}}$$

Ex $\displaystyle \lim_{x\to 0} \frac{\cos x-1}{x} = 0$ show

$$\lim_{x\to 0} \frac{\cos x-1}{x} = \frac{0}{0} \qquad \cos x - 1 = -2\sin^2\frac{x}{2}$$

$$= -2 \lim_{x\to 0} \frac{\sin^2\frac{x}{2}}{x} \qquad \theta = \frac{x}{2}$$

$$= - \lim_{\frac{x}{2}\to 0} \underbrace{\frac{\sin x/2}{\frac{x}{2}}}_{=1} \sin\frac{x}{2}$$

$$= -(1)(0)$$

$$= \boxed{0} \checkmark$$

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\cos x - 1}{x} \cdot \frac{\cos x + 1}{\cos x + 1}$$

$$= \lim_{x\to 0} \frac{\cos^2 x - 1}{x(\cos x + 1)} \longrightarrow -\sin^2 x$$

$$= -\lim_{x\to 0} \frac{\sin x}{x} \frac{\sin x}{\cos x + 1}$$

$$= -(1)\left(\frac{0}{2}\right)$$

$$= 0$$

Ex $\lim_{x\to 0} \frac{\sin 2x}{5x} = \frac{2}{5}$ show.

$$\lim_{x\to 0} \frac{\sin 2x}{5x} = \frac{0}{0} \qquad (2)x \to 0$$

$$= \lim_{2x\to 0} \frac{\sin 2x}{2x} \frac{2}{5} \qquad \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$= \frac{2}{5}$$

$$\lim_{x\to 0} \frac{\sin ax}{bx} = \frac{a}{b}$$

$$\lim_{x\to 0} \frac{\tan x \sec 2x}{3x} = \frac{1}{3}$$

$$\lim_{x\to 0} \frac{\tan x \sec 2x}{3x} = \frac{0}{0} \qquad \tan x \quad \sec 2x$$

$$= \frac{1}{3} \lim_{x\to 0} \frac{1}{x} \frac{\sin x}{\cos x} \frac{1}{\cos 2x}$$

$$= \frac{1}{3} \underbrace{\lim_{x} \frac{\sin x}{x}}_{1} \lim_{x\to 0} \frac{1}{\cos x} \lim_{x\to 0} \frac{1}{\cos 2x}$$

$$= \frac{1}{3}(1)(1)(1)$$

$$= \frac{1}{3} \checkmark$$

#1 $\lim_{x \to 3} (-1) = -1$

#3 $\lim_{x \to 1000} 8\pi^2 = 8\pi^2$

#4 $\lim_{x \to 1} \sqrt{5x+6} = \sqrt{11}$

#11 $\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} = \frac{0}{0}$

$$= \lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

$$= \lim_{x \to 9} \frac{(x-9)(\sqrt{x}+3)}{x-9}$$

$$= \lim_{x \to 9} (\sqrt{x}+3)$$

$$= 6$$

$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} = \frac{0}{0}$

$$= \lim_{x \to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(\sqrt{x}-3)}$$

$$= \lim_{x \to 9} (\sqrt{x}+3)$$

$$= 6$$

#38 $\lim_{x \to 0} (2\sin x - 1) = -1$

#41 $\lim_{x \to 0} \frac{1+x+\sin x}{3\cos x} = \frac{1}{3}$

#42 $\lim_{x \to -\pi} (\sqrt{x+4})(\cos(x+\pi)) = \sqrt{-\pi+4}\, \overbrace{\cos 0}^{=1}$

$$= \sqrt{4-\pi}$$

#49 $\lim_{x\to 0} \frac{x}{\sin 3x} = \frac{0}{0}$

$= \lim_{3x\to 0} \frac{1}{3\frac{\sin 3x}{3x}}$

$= \frac{1}{3} \frac{1}{\lim_{3x\to 0} \frac{\sin 3x}{3x}}$

$= \frac{1}{3}$

#51 $\lim_{x\to 0} \frac{\sin 3x}{4x} = \frac{3}{4}$

86 $\lim_{x\to \pi} \frac{\cos^2 x + 3\cos x + 2}{\cos x + 1} = \frac{1-3+2}{-1+1} = \frac{0}{0}$

$= \lim_{x\to\pi} \frac{(\cos x+1)(\cos x+2)}{\cos x+1}$

$= \lim_{x\to\pi} (\cos x + 2)$

$= -1 + 2$

$= 1$

90 $\lim_{x\to 0} \frac{e^{2x}-1}{e^x-1} = \frac{1-1}{1-1} = \frac{0}{0}$

$= \lim_{x\to 0} \frac{(e^x-1)(e^x+1)}{e^x-1}$

$= \lim_{x\to 0} (e^x+1)$

$= 2$

110/ $\lim_{x\to 0} e^{x^3} = 1 \quad = e^0$

111/ $\lim_{x\to 1} e^{x^2} = e$

112/ $\lim_{x\to 1} e^{x^3-1} = e^0 = \boxed{1}$

113/ $\lim_{x\to -1} e^{x^3-1} = e^{-2} = \boxed{\frac{1}{e^2}}$

116/ $\lim_{x\to e} \ln x = \ln e$
$= 1$

117/ $\lim_{x\to 0^+} \ln x = -\infty$

118/ $\lim_{x\to 1} \ln x = \ln 1$
$= \boxed{0}$

$\lim_{x\to 0} \ln e^{x^3} = \ln e^0$
$= \ln 1$
$= 0$

$= \lim_{x\to 0} x^3 \cancel{\ln e}$
$= 0$

$e^{\infty} = \infty$

$e^{-\infty} = 0 = \frac{1}{e^{\infty}} = \frac{1}{\infty}$

$$\lim_{x \to \infty} f(x) = L$$

$\infty \,/\, -\infty \,/\, \pm\infty$

Limit of $f(x)$ as x approaches infinity equal L

$\forall \varepsilon > 0,\ \exists N \ni \forall x \quad x > M$

$$|f(x) - L| < \varepsilon$$

$$\boxed{\lim_{x \to \infty} f(x) = L \iff f(x) \longrightarrow L}$$

$$f(x) \xrightarrow[x \to a]{} L$$

$$\lim_{x \to \pm\infty} k = k \qquad \lim_{x \to \pm\infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2} = \lim_{x \to \infty} \frac{\frac{5x^2}{x^2} + \frac{8x}{x^2} - \frac{3}{x^2}}{\frac{3x^2}{x^2} + \frac{2}{x^2}}$$

$$= \lim_{x \to \infty} \frac{5 + \frac{8}{x} - \frac{3}{x^2}}{3 + \frac{2}{x^2}}$$

$$= \frac{5 + 0 + 0}{3 + 0}$$

$$= \frac{5}{3}$$

$\frac{\neq 0}{0}$ $\quad x \to a$

Vertical Asymptote : Domain $\frac{1}{x-a}$ $(x \neq a)$

$$\frac{1}{x-a} \xrightarrow{x \to a} \frac{1}{0} = \pm\infty$$

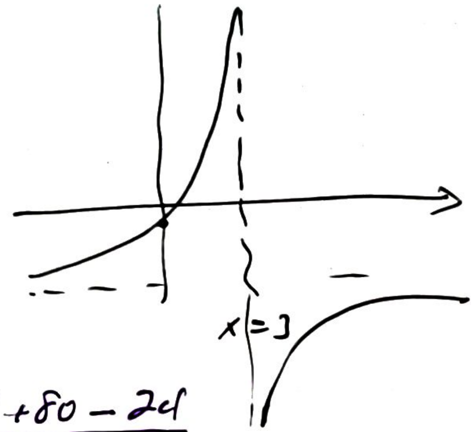
$$\lim_{x\to 3^+} \frac{2-5x}{x-3} = \frac{-13}{0} \longrightarrow$$

$\boxed{\text{sign } \frac{+}{-} \quad 3^+ \quad \frac{-}{+0}}$

$$= -\infty$$

$$\lim_{x\to 3^-} \frac{2-5x}{x-3} = \frac{-13}{0^-}$$

$$= \infty$$



$$\lim_{x\to 4^+} \frac{-x^3+5x^2-6x}{-x^3-4x^2} = \frac{-64+80-24}{-64-64}$$

$$= \frac{-8}{-128}$$

$$= \frac{2}{32} = \frac{1}{16}$$

$$\lim_{x\to -4^+} \frac{-x^3+5x^2-6x}{-x^3-4x^2} = \frac{64+80+24}{64-64}$$

$X = -3$

$$= \frac{168}{0}$$

$x \neq 0$

$$= -\infty$$

$$\lim_{\substack{x\to -4^+ \\ x \neq 0}} \frac{-x^2+5x-6}{-x^2-4x} = \lim \frac{-(x-2)(x+3)}{-x\,\cancel{(x+4)}}$$

$-$

$+$

$$\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 - 1} = \frac{0}{0}$$

$$= \lim_{x \to 1} \frac{(x-1)(x-3)}{(x-1)(x+1)}$$

$$= \lim_{x \to 1} \frac{x-3}{x+1}$$

$$= -1$$

$$\lim_{x \to -1} \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \to -1} \frac{x-3}{x+1}$$

$$\frac{8}{6}$$

$$= \frac{-4}{0}$$

$$= \mp\infty$$

$$\lim_{x \to -1^-} \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \to -1^-} \frac{x-3}{x+1}$$

$$= \infty$$

$$\lim_{x \to -1^+} \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \to -1^+} \frac{x-3}{x+1}$$

$$= -\infty$$