

Solution

Section 1.1 – Functions

Exercise

Find the domain: $f(x) = 7x + 4$

Solution

Domain: \mathbb{R}

Exercise

Find the domain: $f(x) = |3x - 2|$

Solution

Domain: \mathbb{R}

Exercise

Find the domain: $f(x) = 3x + \pi$

Solution

Domain: \mathbb{R}

Exercise

Find the domain: $f(x) = \sqrt{7}x + \frac{1}{2}$

Solution

Domain: \mathbb{R}

Exercise

Find the domain: $f(x) = -2x^2 + 3x - 5$

Solution

Domain: \mathbb{R}

Exercise

Find the domain: $f(x) = x^3 - 2x^2 + x - 3$

Solution

Domain: \mathbb{R}

Exercise

Find the domain: $f(x) = x^2 - 2x - 15$

Solution

Domain: \mathbb{R}

Exercise

Find the domain $f(x) = 4 - \frac{2}{x}$

Solution

Domain: $x \neq 0$

Exercise

Find the domain $f(x) = \frac{1}{x^4}$

Solution

Domain: $x \neq 0$

Exercise

Find the domain: $g(x) = \frac{3}{x-4}$

Solution

Domain: $x \neq 4$

Exercise

Find the domain $y = \frac{2}{x-3}$

Solution

Domain: $x \neq 3$

Exercise

Find the domain $y = \frac{-7}{x-5}$

Solution

Domain: $x \neq 5$

Exercise

Find the domain $f(x) = \frac{x+5}{2-x}$

Solution

$$2 - x \neq 0$$

Domain: $x \neq 2$

Exercise

Find the domain $f(x) = \frac{8}{x+4}$

Solution

$$x + 4 \neq 0$$

Domain: $x \neq -4$

Exercise

Find the domain $f(x) = \frac{1}{x+4}$

Solution

Domain: $x \neq -4$

Exercise

Find the domain $f(x) = \frac{1}{x-4}$

Solution

Domain: $x \neq 4$

Exercise

Find the domain $f(x) = \frac{3x}{x+2}$

Solution

Domain: $x \neq -2$

Exercise

Find the domain $f(x) = x - \frac{2}{x-3}$

Solution

Domain: $x \neq 3$

Exercise

Find the domain $f(x) = x + \frac{3}{x-5}$

Solution

Domain: $x \neq 5$

Exercise

Find the domain $f(x) = \frac{1}{2}x - \frac{8}{x+7}$

Solution

Domain: $x \neq -7$

Exercise

Find the domain $f(x) = \frac{1}{x-3} - \frac{8}{x+7}$

Solution

Domain: $x \neq -7, 3$

Exercise

Find the domain $f(x) = \frac{1}{x+4} - \frac{2x}{x-4}$

Solution

Domain: $x \neq \pm 4$

Exercise

Find the domain $f(x) = \frac{3x^2}{x+3} - \frac{4x}{x-2}$

Solution

Domain: $x \neq -3, 2$

Exercise

Find the domain $f(x) = \frac{1}{x^2 - 2x + 1}$

Solution

$$x^2 - 2x + 1 \neq 0 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

Domain: $x \neq 1$

Exercise

Find the domain $f(x) = \frac{x}{x^2 + 3x + 2}$

Solution

$$x^2 + 3x + 2 \neq 0 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

Domain: $x \neq -1, -2$

Exercise

Find the domain $f(x) = \frac{x^2}{x^2 - 5x + 4}$

Solution

$$x^2 - 5x + 4 \neq 0 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

Domain: $x \neq 1, 4$

Exercise

Find the domain $f(x) = \frac{1}{x^2 - 4x - 5}$

Solution

$$x^2 - 4x - 5 \neq 0 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

Domain: $x \neq -1, 5$

Exercise

Find the domain $g(x) = \frac{2}{x^2 + x - 12}$

Solution

$$x^2 + x - 12 \neq 0$$
$$(x + 4)(x - 3) \neq 0$$

Domain: $\underline{x \neq -4, 3} \mid \underline{(-\infty, -4) \cup (-4, 3) \cup (3, \infty)}$

Exercise

Find the domain $h(x) = \frac{5}{\frac{4}{x} - 1}$

Solution

$$x \neq 0 \quad \frac{4}{x} - 1 \neq 0$$
$$\frac{4 - x}{x} \neq 0$$
$$4 - x \neq 0$$
$$x \neq 4$$

Domain: $\underline{x \neq 0, 4} \mid \underline{(-\infty, 0) \cup (0, 4) \cup (4, \infty)}$

Exercise

Find the domain $y = \sqrt{x}$

Solution

$$x \geq 0$$

Domain: $\underline{x \geq 0} \mid \underline{[0, \infty)}$

Exercise

Find the domain $f(x) = \sqrt{8 - 3x}$

Solution

$$8 - 3x \geq 0$$

$$8 \geq 3x$$

Domain: $\underline{x \leq \frac{8}{3}} \mid \underline{\left(-\infty, \frac{8}{3}\right]}$

Exercise

Find the domain $y = \sqrt{4x+1}$

Solution

$$4x+1 \geq 0 \Rightarrow x \geq -\frac{1}{4}$$

$$\text{Domain: } \underline{x \geq -\frac{1}{4}} \quad \left[-\frac{1}{4}, \infty \right)$$

Exercise

Find the domain $y = \sqrt{7-2x}$

Solution

$$7-2x \geq 0$$

$$-2x \geq -7$$

$$\text{Domain: } \underline{x \leq \frac{7}{2}} \quad \left(-\infty, \frac{7}{2} \right]$$

Exercise

Find the domain $f(x) = \sqrt{8-x}$

Solution

$$8-x \geq 0$$

$$\text{Domain: } \underline{x \leq 8} \quad (-\infty, 8]$$

Exercise

Find the domain $f(x) = \sqrt{3-2x}$

Solution

$$\text{Domain: } \underline{x \leq \frac{3}{2}} \quad \left(-\infty, \frac{3}{2} \right]$$

Exercise

Find the domain $f(x) = \sqrt{3+2x}$

Solution

$$\text{Domain: } \underline{x \geq -\frac{3}{2}} \quad \left[-\frac{3}{2}, \infty \right)$$

Exercise

Find the domain $f(x) = \sqrt{5-x}$

Solution

Domain: $x \leq 5$ |

Exercise

Find the domain $f(x) = \sqrt{x-5}$

Solution

Domain: $x \geq 5$ |

Exercise

Find the domain $f(x) = \sqrt{6-3x}$

Solution

Domain: $x \leq 2$ |

Exercise

Find the domain $f(x) = \sqrt{3x-6}$

Solution

Domain: $x \geq 2$ |

Exercise

Find the domain $f(x) = \sqrt{2x+7}$

Solution

Domain: $x \geq -\frac{7}{2}$ |

Exercise

Find the domain $f(x) = \sqrt{x^2-16}$

Solution

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

$$\text{Domain: } \underline{x \leq -4 \quad x \geq 4}$$

Exercise

Find the domain $f(x) = \sqrt{16 - x^2}$

Solution

$$x = \pm 4$$

$$\text{Domain: } \underline{-4 \leq x \leq 4}$$

Exercise

Find the domain $f(x) = \sqrt{9 - x^2}$

Solution

$$x = \pm 3$$

$$\text{Domain: } \underline{-3 \leq x \leq 3}$$

Exercise

Find the domain $f(x) = \sqrt{x^2 - 25}$

Solution

$$x = \pm 5$$

$$\text{Domain: } \underline{x \leq -5 \quad x \geq 5}$$

Exercise

Find the domain $f(x) = \sqrt{x^2 - 5x + 4}$

Solution

$$x^2 - 5x + 4 = 0 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

$$x = 1, 4$$

$$\text{Domain: } \underline{x \leq 1 \quad x \geq 4}$$

Exercise

Find the domain $f(x) = \sqrt{x^2 + 5x + 4}$

Solution

$$x^2 + 5x + 4 \qquad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$
$$x = -1, -4$$

$$\text{Domain: } \underline{x \leq -4 \quad x \geq -1}$$

Exercise

Find the domain $f(x) = \sqrt{x^2 + 3x + 2}$

Solution

$$x^2 + 3x + 2 \qquad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$
$$x = -1, -2$$

$$\text{Domain: } \underline{x \leq -2 \quad x \geq -1}$$

Exercise

Find the domain $f(x) = \sqrt{x^2 - 3x + 2}$

Solution

$$x^2 - 3x + 2 \qquad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$
$$x = 1, 2$$

$$\text{Domain: } \underline{x \leq 1 \quad x \geq 2}$$

Exercise

Find the domain $f(x) = \sqrt{x-4} + \sqrt{x+1}$

Solution

$$x \geq 4 \quad x \geq -1$$

$$\text{Domain: } \underline{x \geq 4}$$

Exercise

Find the domain $f(x) = \sqrt{3-x} + \sqrt{x-2}$

Solution

$$x \leq 3 \quad x \geq 2$$

Domain: $\underline{2 \leq x \leq 3}$

Exercise

Find the domain $f(x) = \sqrt{1-x} + \sqrt{4-x}$

Solution

$$x \leq 1 \quad x \leq 4$$

Domain: $\underline{x \leq 1}$

Exercise

Find the domain $f(x) = \sqrt{1-x} - \sqrt{x-3}$

Solution

$$x \leq 1 \quad x \geq 3$$

Domain: $\underline{\emptyset}$

Exercise

Find the domain $f(x) = \sqrt{x+4} - \sqrt{x-1}$

Solution

$$x \geq -4 \quad x \geq 1$$

Domain: $\underline{x \geq 1}$

Exercise

Find the domain $f(x) = \frac{\sqrt{x+1}}{x}$

Solution

$$x+1 \geq 0$$

$$x \neq 0$$

$$x \geq -1$$

Domain: $\underline{x \geq -1 \quad x \neq 0} \quad \underline{[-1, 0) \cup (0, \infty)}$

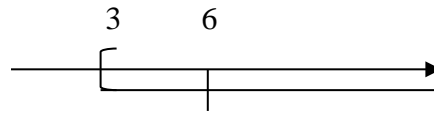
Exercise

Find the domain $g(x) = \frac{\sqrt{x-3}}{x-6}$

Solution

$$\rightarrow \begin{cases} x \geq 3 \\ x \neq 6 \end{cases}$$

Domain: $\underline{x \geq 3 \quad x \neq 6} \quad \underline{[3, 6) \cup (6, \infty)}$



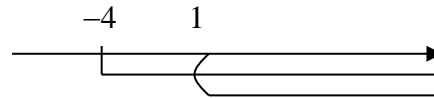
Exercise

Find the domain $f(x) = \frac{\sqrt{x+4}}{\sqrt{x-1}}$

Solution

$$\rightarrow \begin{cases} x \geq -4 \\ x > 1 \end{cases}$$

Domain: $\underline{x > 1} \quad \underline{(1, \infty)}$



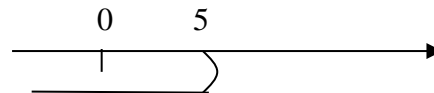
Exercise

Find the domain $f(x) = \frac{\sqrt{5-x}}{x}$

Solution

$$x \leq 5 \quad x \neq 0$$

Domain: $\underline{x \leq 5 \quad x \neq 0} \quad \underline{(-\infty, 0) \cup (0, 5]}$



Exercise

Find the domain $f(x) = \frac{x}{\sqrt{5-x}}$

Solution

Domain: $\underline{x < 5} \quad \underline{(-\infty, 5)}$

Exercise

Find the domain $f(x) = \frac{1}{x\sqrt{5-x}}$

Solution

$$x < 5 \quad x \neq 0$$

$$\text{Domain: } \underline{x < 5 \quad x \neq 0}$$

Exercise

Find the domain $f(x) = \frac{x+1}{x^3-4x}$

Solution

$$x^3 - 4x \neq 0$$

$$x(x^2 - 4) \neq 0$$

$$\text{Domain: } \underline{x \neq 0, \pm 2}$$

Exercise

Find the domain $f(x) = \frac{\sqrt{x+5}}{x}$

Solution

$$x \geq -5 \quad x \neq 0$$

$$\text{Domain: } \underline{x \geq -5 \quad x \neq 0}$$

Exercise

Find the domain $f(x) = \frac{x}{\sqrt{x+5}}$

Solution

$$x > -5$$

$$\text{Domain: } \underline{x > -5}$$

Exercise

Find the domain $f(x) = \frac{1}{x\sqrt{x+5}}$

Solution

$$x > -5 \quad x \neq 0$$

$$\text{Domain: } \underline{x > -5 \quad x \neq 0}$$

Exercise

Find the domain $f(x) = \frac{x+3}{\sqrt{x-3}}$

Solution

$$\text{Domain: } \underline{x > 3}$$

Exercise

Find the domain $f(x) = \frac{\sqrt{x+3}}{\sqrt{x-3}}$

Solution

$$x \geq -3 \quad x > 3$$

$$\text{Domain: } \underline{x > 3}$$

Exercise

Find the domain $f(x) = \frac{\sqrt{x-2}}{\sqrt{x+2}}$

Solution

$$x \geq 2 \quad x > -2$$

$$\text{Domain: } \underline{x \geq 2}$$

Exercise

Find the domain $f(x) = \frac{\sqrt{2-x}}{\sqrt{x+2}}$

Solution

$$x \leq 2 \quad x > -2$$

$$\text{Domain: } \underline{-2 < x \leq 2}$$

Exercise

Find the domain $f(x) = \frac{x-4}{\sqrt{x-2}}$

Solution

Domain: $x > 2$

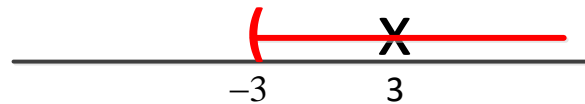
Exercise

Find the domain of $f(x) = \frac{1}{(x-3)\sqrt{x+3}}$

Solution

$$\begin{aligned} x-3 &\neq 0 & x+3 &> 0 \\ x &\neq 3 & x &> -3 \end{aligned}$$

Domain: $\{x \mid x > -3 \text{ and } x \neq 3\}$
 $(-3, 3) \cup (3, \infty)$



Exercise

Find the domain of $f(x) = \sqrt{x+2} + \sqrt{2-x}$

Solution

$$\begin{aligned} x+2 &\geq 0 & 2-x &\geq 0 \\ x &\geq -2 & -x &\geq -2 \rightarrow x \leq 2 \end{aligned}$$

Domain: $\{x \mid -2 \leq x \leq 2\}$



Exercise

Find the domain of $f(x) = \sqrt{(x-2)(x-6)}$

Solution

$$\begin{aligned} x-2 &\geq 0 & x-6 &\geq 0 \\ x &\geq 2 & x &\geq 6 \end{aligned}$$

Domain: $\{x \mid x \leq 2, x \geq 6\}$

	2	6	
-	+	+	
-	-	+	
+	-	+	

Exercise

Find the domain of $f(x) = \sqrt{x+3} - \sqrt{4-x}$

Solution

$$x \geq -3 \quad x \leq 4$$

$$\text{Domain: } \underline{-3 \leq x \leq 4}$$

Exercise

Find the domain of $f(x) = \frac{\sqrt{4x-3}}{x^2-4}$

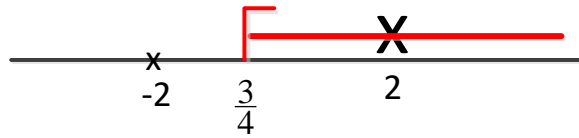
Solution

$$4x-3 \geq 0 \quad x^2-4 \neq 0$$

$$4x \geq 3 \quad x \neq \pm 2$$

$$x \geq \frac{3}{4}$$

$$\text{Domain: } \left[\frac{3}{4}, 2 \right) \cup (2, \infty)$$



Exercise

Find the domain of $f(x) = \frac{4x}{6x^2+13x-5}$

Solution

$$6x^2+13x-5 \neq 0$$

$$x = \frac{-13 \pm \sqrt{169+120}}{12}$$

$$= \begin{cases} \frac{-13-17}{12} = -\frac{5}{2} \\ \frac{-13+17}{12} = \frac{1}{3} \end{cases}$$

$$\text{Domain: } \underline{x \neq -\frac{5}{2}, \frac{1}{3}}$$

Exercise

Find the domain of $f(x) = \frac{\sqrt{2x-3}}{x^2-5x+4}$

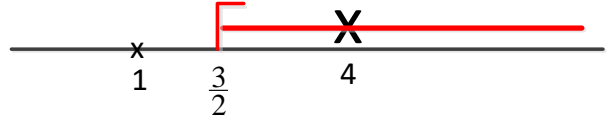
Solution

$$2x-3 \geq 0 \quad x^2-5x+4 \neq 0$$

$$2x \geq 3 \quad x \neq 1, 4$$

$$x \geq \frac{3}{2}$$

$$\text{Domain: } \underline{x \geq \frac{3}{2}, x \neq 4} \quad \left[\frac{3}{2}, 4 \right) \cup (4, \infty)$$



Exercise

Find the domain of $f(x) = \frac{x^2}{\sqrt{x^2 - 5x + 4}}$

Solution

$$x^2 - 5x + 4 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

$$x = 1, 4$$

$$\text{Domain: } \underline{x < 1 \quad x > 4}$$

Exercise

Find the domain of $f(x) = \frac{x+2}{\sqrt{x^2 + 5x + 4}}$

Solution

$$x^2 + 5x + 4 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x = -1, -4$$

$$\text{Domain: } \underline{x < -4 \quad x > -1}$$

Exercise

Find the domain of $f(x) = \frac{\sqrt{x+2}}{\sqrt{x^2 + 3x + 2}}$

Solution

$$x^2 + 3x + 2 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x < -2 \quad x > -1$$

$$\sqrt{x+2} \rightarrow x \geq -2$$

$$\text{Domain: } \underline{x > -1}$$

Exercise

Find the domain of $f(x) = \frac{\sqrt{2x+3}}{x^2 - 6x + 5}$

Solution

$$x^2 - 6x + 5 \qquad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

$$x \neq 1, 5$$

$$\sqrt{2x+3} \rightarrow x \geq -\frac{3}{2}$$

$$\text{Domain: } \underline{x \geq -\frac{3}{2} \quad x \neq 1, 5}$$

Exercise

For the function f given by $f(x) = 9x + 5$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$f(x+h) = 9(x+h) + 5 = 9x + 9h + 5$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\overbrace{9x+9h+5}^{f(x+h)} - \overbrace{(9x+5)}^{f(x)}}{h} \\ &= \frac{9x+9h+5 - 9x-5}{h} \\ &= \frac{9h}{h} \\ &= \underline{9} \end{aligned}$$

Exercise

For the function f given by $f(x) = 6x + 2$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{6(x+h) + 2 - (6x+2)}{h} \\ &= \frac{6x+6h+2-6x-2}{h} \\ &= \frac{6h}{h} \\ &= \underline{6} \end{aligned}$$

Exercise

For the function f given by $f(x) = 4x + 11$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{4(x+h) + 11 - (4x + 11)}{h} \\ &= \frac{4x + 4h + 11 - 4x - 11}{h} \\ &= \frac{4h}{h} \\ &= 4\end{aligned}$$

Exercise

For the function f given by $f(x) = 3x - 5$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{3(x+h) - 5 - 3x + 5}{h} \\ &= \frac{3x + 3h - 5 - 3x + 5}{h} \\ &= \frac{3h}{h} \\ &= 3\end{aligned}$$

Exercise

For the function f given by $f(x) = -2x - 3$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-2(x+h) - 3 + 2x + 3}{h} \\ &= \frac{-2x - 2h - 3 + 2x + 3}{h} \\ &= \frac{-2h}{h} \\ &= -2\end{aligned}$$

Exercise

For the function f given by $f(x) = -4x + 3$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-4(x+h) + 3 + 4x - 3}{h} \\ &= \frac{-4x - 4h + 3 + 4x - 3}{h} \\ &= \frac{-4h}{h} \\ &= -4\end{aligned}$$

Exercise

For the function f given by $f(x) = 3x - 6$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{3(x+h) - 6 - 3x + 6}{h} \\ &= \frac{3x + 3h - 6 - 3x + 6}{h} \\ &= \frac{3h}{h} \\ &= 3\end{aligned}$$

Exercise

For the function f given by $f(x) = -5x - 7$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-5(x+h) - 7 + 5x + 7}{h} \\ &= \frac{-5x - 5h - 7 + 5x + 7}{h} \\ &= \frac{-5h}{h} \\ &= -5\end{aligned}$$

Exercise

Given the function: $f(x) = 2x^2$. Find and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}f(x+h) &= 2(\textcolor{red}{x} + \textcolor{red}{h})^2 \\&= 2(x^2 + 2hx + h^2) \\&= 2x^2 + 4hx + 2h^2 \\ \frac{f(x+h) - f(x)}{h} &= \frac{\textcolor{red}{2}x^2 + 4hx + \textcolor{red}{2}h^2 - 2x^2}{h} \\&= \frac{4hx + 2h^2}{h} \\&= \frac{4hx}{h} + \frac{2h^2}{h} \\&= \textcolor{blue}{4x + 2h} \quad | \end{aligned}$$

Exercise

For the function f given by $f(x) = 5x^2$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{5(x+h)^2 - 5x^2}{h} \\&= \frac{5(x^2 + 2hx + h^2) - 5x^2}{h} \\&= \frac{5x^2 + 10hx + 5h^2 - 5x^2}{h} \\&= \frac{10hx + 5h^2}{h} \\&= \textcolor{blue}{10x + 5h} \quad | \end{aligned}$$

Exercise

For the function f given by $f(x) = 3x^2 - 4x$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 4(x+h) - 3x^2 + 4x}{h}$$

$$\begin{aligned}
&= \frac{3(x^2 + 2hx + h^2) - 4x - 4h - 3x^2 + 4x}{h} \\
&= \frac{3x^2 + 6hx + 3h^2 - 4x - 4h - 3x^2 + 4x}{h} \\
&= \frac{6hx + 3h^2 - 4h}{h} \\
&= \underline{6x + 3h - 4}
\end{aligned}$$

Exercise

For the function f given by $f(x) = 2x^2 - 3x$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}
f(x+h) &= 2(\text{---})^2 - 3(\text{---}) \\
&= 2(x+h)^2 - 3(x+h) & (a+b)^2 &= a^2 + 2ab + b^2 \\
&= 2(x^2 + 2xh + h^2) - 3x - 3h \\
&= 2x^2 + 4xh + 2h^2 - 3x - 3h \\
\frac{f(x+h) - f(x)}{h} &= \frac{\overbrace{2x^2 + 4xh + 2h^2 - 3x - 3h}^{f(x+h)} - \overbrace{(2x^2 - 3x)}^{f(x)}}{h} \\
&= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h} \\
&= \frac{4xh + 2h^2 - 3h}{h} \\
&= \frac{4xh}{h} + \frac{2h^2}{h} - \frac{3h}{h} \\
&= \underline{4x + 2h - 3}
\end{aligned}$$

Exercise

For the function f given by $f(x) = 2x^2 - x - 3$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}
f(x+h) &= 2(x+h)^2 - (x+h) - 3 \\
&= 2(x^2 + 2hx + h^2) - x - h - 3 \\
&= 2x^2 + 4hx + 2h^2 - x - h - 3
\end{aligned}$$

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{2x^2 + 2h^2 + 4hx - x - h - 3 - (2x^2 - x - 3)}{h} \\
&= \frac{2x^2 + 2h^2 + 4hx - x - h - 3 - 2x^2 + x + 3}{h} \\
&= \frac{2h^2 + 4hx - h}{h} \\
&= \frac{2h^2}{h} + \frac{4hx}{h} - \frac{h}{h} \\
&= \underline{2h + 4x - 1}
\end{aligned}$$

Exercise

For the given function $f(x) = 2x^2 - x - 3$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{2(x+h)^2 - (x+h) - 3 - 2x^2 + x + 3}{h} \\
&= \frac{2(x^2 + 2hx + h^2) - x - h - 3 - 2x^2 + x + 3}{h} \\
&= \frac{2x^2 + 4hx + 2h^2 - h - 2x^2}{h} \\
&= \frac{4hx + 2h^2 - h}{h} \\
&= \underline{4x + 2h - 1}
\end{aligned}$$

Exercise

For the given function $f(x) = x^2 - 2x + 5$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^2 - 2(x+h) + 5 - x^2 + 2x - 5}{h} \\
&= \frac{x^2 + 2hx + h^2 - 2x - 2h - x^2 + 2x}{h} \\
&= \frac{2hx + h^2 - 2h}{h} \\
&= \underline{2x + h - 2}
\end{aligned}$$

Exercise

For the given function $f(x) = 3x^2 - 2x + 5$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{3(x+h)^2 - 2(x+h) + 5 - 3x^2 + 2x - 5}{h} \\&= \frac{3(x^2 + 2hx + h^2) - 2x - 2h - 3x^2 + 2x}{h} \\&= \frac{3x^2 + 6hx + 3h^2 - 2h - 3x^2}{h} \\&= \frac{6hx + 3h^2 - 2h}{h} \\&= \underline{6x + 3h - 2} \quad | \end{aligned}$$

Exercise

For the given function $f(x) = -2x^2 - 3x + 7$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-2(x+h)^2 - 3(x+h) + 7 + 2x^2 + 3x - 7}{h} \\&= \frac{-2(x^2 + 2hx + h^2) - 3x - 3h + 2x^2 + 3x}{h} \\&= \frac{-2x^2 - 4hx - 2h^2 - 3h + 2x^2}{h} \\&= \frac{-4hx - 2h^2 - 3h}{h} \\&= \underline{-4x - 2h - 3} \quad | \end{aligned}$$

Exercise

For the function f given by $f(x) = \sqrt{x-3}$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$

Solution

$$\frac{f(x+h) - f(x)}{h} = \underline{\underline{\frac{\sqrt{x+h-3} - \sqrt{x-3}}{h}}}$$

Exercise

Let $f(x) = 4x - 3$ and $g(x) = 5x + 7$. Find each of the following and give the domain

$$a) (f + g)(x) \quad b) (f - g)(x) \quad c) (fg)(x) \quad d) \left(\frac{f}{g}\right)(x)$$

Solution

$$a) (f + g)(x) = 4x - 3 + 5x + 7 \\ = 9x + 4$$

Domain: \mathbb{R}

$$b) (f - g)(x) = 4x - 3 - (5x + 7) \\ = 4x - 3 - 5x - 7 \\ = -x - 10$$

Domain: \mathbb{R}

$$c) (fg)(x) = (4x - 3)(5x + 7) \\ = 20x^2 + 13x - 21$$

Domain: \mathbb{R}

$$d) \left(\frac{f}{g}\right)(x) = \frac{4x - 3}{5x + 7}$$

Domain: $x \neq -\frac{7}{5}$

Exercise

Let $f(x) = 2x^2 + 3$ and $g(x) = 3x - 4$. Find each of the following and give the domain

$$a) (f + g)(x) \quad b) (f - g)(x) \quad c) (fg)(x) \quad d) \left(\frac{f}{g}\right)(x)$$

Solution

$$a) (f + g)(x) = 2x^2 + 3 + 3x - 4 \\ = 2x^2 + 3x - 1$$

Domain: \mathbb{R}

$$b) (f - g)(x) = 2x^2 + 3 - (3x - 4) \\ = 2x^2 + 3 - 3x + 4$$

$$= 2x^2 - x + 7 \mid$$

Domain: $\mathbb{R} \mid$

$$c) (fg)(x) = (2x^2 + 3)(3x - 4)$$

$$= 6x^2 + x - 12 \mid$$

Domain: $\mathbb{R} \mid$

$$d) \left(\frac{f}{g}\right)(x) = \frac{2x^2 + 3}{3x - 4} \mid$$

$$\text{Domain: } x \neq -\frac{4}{3} \mid$$

Exercise

Let $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 + 3x - 2$. Find each of the following and give the domain

$$a) (f + g)(x)$$

$$b) (f - g)(x)$$

$$c) (fg)(x)$$

$$d) \left(\frac{f}{g}\right)(x)$$

Solution

$$a) (f + g)(x) = x^2 - 2x - 3 + x^2 + 3x - 2$$

$$= 2x^2 + x - 5 \mid$$

Domain: $\mathbb{R} \mid$

$$b) (f - g)(x) = x^2 - 2x - 3 - x^2 - 3x + 2$$

$$= -5x - 1 \mid$$

Domain: $\mathbb{R} \mid$

$$c) (fg)(x) = (x^2 - 2x - 3)(x^2 + 3x - 2)$$

$$= x^4 + 3x^3 - 2x^2 - 2x^3 - 6x^2 + 4x - 3x^2 - 9x + 6$$

$$= x^4 + x^3 - 11x^2 - 5x + 6 \mid$$

Domain: $\mathbb{R} \mid$

$$d) \left(\frac{f}{g}\right)(x) = \frac{x^2 - 2x - 3}{x^2 + 3x - 2} \mid$$

$$\text{Domain: } x \neq \frac{-3 \pm \sqrt{17}}{2} \mid$$

Exercise

Let $f(x) = \sqrt{4x-1}$ and $g(x) = \frac{1}{x}$. Find each of the following and give the domain

a) $(f+g)(x)$

b) $(f-g)(x)$

c) $(fg)(x)$

d) $\left(\frac{f}{g}\right)(x)$

Solution

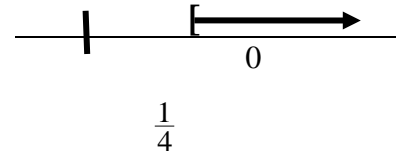
a) $(f+g)(x)$

$$(f+g)(x) = \sqrt{4x-1} + \frac{1}{x}$$

$$4x-1 \geq 0 \quad x \neq 0$$

$$x \geq \frac{1}{4}$$

Domain: $\left[\frac{1}{4}, \infty\right)$



b) $(f-g)(x)$

$$(f-g)(x) = \sqrt{4x-1} - \frac{1}{x}$$

$$4x-1 \geq 0 \quad x \neq 0$$

$$x \geq \frac{1}{4}$$

Domain: $\left[\frac{1}{4}, \infty\right)$

c) $(fg)(x) = \sqrt{4x-1} \left(\frac{1}{x}\right)$

$$= \frac{\sqrt{4x-1}}{x}$$

$$4x-1 \geq 0 \quad x \neq 0$$

$$x \geq \frac{1}{4}$$

Domain: $\left[\frac{1}{4}, \infty\right)$

d) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{4x-1}}{\frac{1}{x}}$

Domain: $x \neq 0$

$$= x\sqrt{4x-1}$$

$$4x-1 \geq 0$$

$$x \geq \frac{1}{4}$$

Domain: $\left[\frac{1}{4}, \infty\right)$

Exercise

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $(f / g)(x)$ and the domain of

$$f(x) = \sqrt{3-2x}, \quad g(x) = \sqrt{x+4}$$

Solution

$$f(x) + g(x) = \sqrt{3-2x} + \sqrt{x+4}$$

$$3-2x \geq 0 \quad x+4 \geq 0$$

$$-2x \geq -3 \quad x \geq -4$$

$$x \leq \frac{3}{2}$$

$$\text{Domain: } \left\{x \mid -4 \leq x \leq \frac{3}{2}\right\}$$

$$f(x) - g(x) = \sqrt{3-2x} - \sqrt{x+4}$$

$$3-2x \geq 0 \quad x+4 \geq 0$$

$$-2x \geq -3 \quad x \geq -4$$

$$x \leq \frac{3}{2}$$

$$\text{Domain: } \left\{x \mid -4 \leq x \leq \frac{3}{2}\right\}$$



$$(f \cdot g)(x) = (\sqrt{3-2x})(\sqrt{x+4}) = \sqrt{(3-2x)(x+4)} = \sqrt{-2x^2 - 5x + 12}$$

$$3-2x \geq 0 \quad x+4 \geq 0$$

$$-2x \geq -3 \quad x \geq -4$$

$$x \leq \frac{3}{2}$$

$$\text{Domain: } \left\{x \mid -4 \leq x \leq \frac{3}{2}\right\}$$

$$(f / g)(x) = \frac{\sqrt{3-2x}}{\sqrt{x+4}} \frac{\sqrt{x+4}}{\sqrt{x+4}} = \frac{\sqrt{-2x^2 - 5x + 12}}{x+4}$$

$$3-2x \geq 0 \quad x+4 > 0$$

$$-2x \geq -3 \quad x > -4$$

$$x \leq \frac{3}{2}$$

$$\text{Domain: } \left\{x \mid -4 < x \leq \frac{3}{2}\right\}$$

$$\underline{\left(-4, \frac{3}{2}\right]}$$

Exercise

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $(f / g)(x)$ and the domain of

$$f(x) = \frac{2x}{x-4}, \quad g(x) = \frac{x}{x+5}$$

Solution

$$\begin{aligned}(f + g)(x) &= \frac{2x}{x-4} + \frac{x}{x+5} \\&= \frac{2x(x+5) + x(x-4)}{(x-4)(x+5)} \\&= \frac{2x^2 + 10x + x^2 - 4x}{(x-4)(x+5)} \\&= \frac{3x^2 + 6x}{(x-4)(x+5)}\end{aligned}$$

$$x - 4 \neq 0 \quad x + 5 \neq 0$$

$$x \neq 4 \quad x \neq -5$$

$$\text{Domain: } \{x \mid x \neq -5, 4\} \quad (-\infty, -5) \cup (-5, 4) \cup (4, \infty)$$

$$\begin{aligned}(f - g)(x) &= \frac{2x}{x-4} - \frac{x}{x+5} \\&= \frac{2x(x+5) - x(x-4)}{(x-4)(x+5)} \\&= \frac{2x^2 + 10x - x^2 + 4x}{(x-4)(x+5)} \\&= \frac{x^2 + 14x}{(x-4)(x+5)}\end{aligned}$$

$$x \neq 4 \quad x \neq -5$$

$$\text{Domain: } \{x \mid x \neq -5, 4\}$$

$$(f \cdot g)(x) = \frac{2x}{x-4} \cdot \frac{x}{x+5} = \frac{2x^2}{(x-4)(x+5)}$$

$$x \neq 4 \quad x \neq -5$$

$$\text{Domain: } \{x \mid x \neq -5, 4\}$$

$$(f / g)(x) = \frac{2x}{x-4} \div \frac{x}{x+5} = \frac{2x}{x-4} \cdot \frac{x+5}{x} = 2 \frac{x+5}{x-4}$$

$$x \neq 4 \quad x \neq -5$$

$$\text{Domain: } \{x \mid x \neq -5, 4\}$$

Exercise

Given that $f(x) = x + 1$ and $g(x) = \sqrt{x + 3}$

- a) Find $(f + g)(x)$
- b) Find the domain of $(f + g)(x)$
- c) Find: $(f + g)(6)$

Solution

$$\begin{aligned} \text{a) } (f + g)(x) &= f(x) + g(x) \\ &= x + 1 + \sqrt{x + 3} \end{aligned}$$

$$\text{b) } x + 3 \geq 0 \rightarrow x \geq -3$$

$$\text{Domain} = [-3, \infty)$$

$$\text{c) } (f + g)(6) = 6 + 1 + \sqrt{6 + 3} = 10$$

Exercise

Given that $f(x) = x^2 - 4$ and $g(x) = x + 2$

- a) Find $(f + g)(x)$ and its domain
- b) Find $(f / g)(x)$ and its domain

Solution

$$\begin{aligned} \text{a) } (f + g)(x) &= x^2 - 4 + x + 2 \\ &= x^2 + x - 2 \end{aligned}$$

$$\text{Domain} = \mathbb{R}$$

$$\text{b) } \frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 4}{x + 2}$$

$$\text{Domain: } \underline{x \neq -2}$$

Exercise

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find $(f + g)(1)$, $(f - g)(-3)$, $(fg)(5)$, and $\left(\frac{f}{g}\right)(0)$

Solution

$$\begin{aligned} \text{a) } (f + g)(1) &= f(1) + g(1) \\ &= 1^2 + 1 + 3(1) + 5 \\ &= 1 + 1 + 3 + 5 \\ &= 10 \end{aligned}$$

$$\begin{aligned}
 b) \quad (f - g)(-3) &= f(-3) - g(-3) \\
 &= (-3)^2 + 1 - (3(-3) + 5) \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 c) \quad (fg)(5) &= f(5) \cdot g(5) \\
 &= (5^2 + 1) \cdot (3(5) + 5) \\
 &= (26) \cdot (20) \\
 &= 520
 \end{aligned}$$

$$\begin{aligned}
 d) \quad \left(\frac{f}{g}\right)(0) &= \frac{f(0)}{g(0)} \\
 &= \frac{0^2 + 1}{3(0) + 5} \\
 &= \frac{1}{5}
 \end{aligned}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = 2x^2 + 3x - 4$, $g(x) = 2x - 1$

Solution

$$\begin{aligned}
 f(g(x)) &= f(2x - 1) \\
 &= 2(2x - 1)^2 + 3(2x - 1) - 4 \\
 &= 2(4x^2 - 4x + 1) + 6x - 3 - 4 \\
 &= 8x^2 - 8x + 2 + 6x - 7 \\
 &= 8x^2 - 2x - 5
 \end{aligned}$$

$$\begin{aligned}
 g(f(x)) &= g(2x^2 + 3x - 4) \\
 &= 2(2x^2 + 3x - 4) - 1 \\
 &= 4x^2 + 6x - 8 - 1 \\
 &= 4x^2 + 6x - 9
 \end{aligned}$$

$$\begin{aligned}
 f(g(-2)) &= 8(-2)^2 - 2(-2) - 5 \\
 &= 31
 \end{aligned}$$

$$\begin{aligned}
 g(f(3)) &= 4(3)^2 + 6(3) - 9 \\
 &= 45
 \end{aligned}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = x^3 + 2x^2$, $g(x) = 3x$

Solution

$$\begin{aligned} f(g(x)) &= f(3x) \\ &= (3x)^3 + 2(3x)^2 \\ &= 27x^3 + 18x^2 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(x^3 + 2x^2) \\ &= 3(x^3 + 2x^2) \\ &= 3x^3 + 6x^2 \end{aligned}$$

$$\begin{aligned} f(g(-2)) &= 27(-2)^3 + 18(-2)^2 \\ &= -144 \end{aligned}$$

$$\begin{aligned} g(f(3)) &= 3(3)^3 + 6(3)^2 \\ &= 135 \end{aligned}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$: $f(x) = |x|$, $g(x) = -7$

Solution

$$\begin{aligned} f(g(x)) &= f(-7) \\ &= |-7| \\ &= 7 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(|x|) \\ &= -7 \end{aligned}$$

$$f(g(-2)) = 7$$

$$g(f(3)) = -7$$

Exercise

Given $f(x) = x - 3$ and $g(x) = x + 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} \text{a) } f(g(x)) &= f(x+3) & \text{Domain: } \mathbb{R} \\ &= (x+3)-3 \\ &= x \end{aligned}$$

Domain: \mathbb{R}

$$\begin{aligned} \text{b) } g(f(x)) &= g(x-3) & \text{Domain: } \mathbb{R} \\ &= (x-3)+3 \\ &= x \end{aligned}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = \frac{2}{3}x$ and $g(x) = \frac{3}{2}x$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} \text{a) } f(g(x)) &= f\left(\frac{3}{2}x\right) & \text{Domain: } \mathbb{R} \\ &= \frac{2}{3}\left(\frac{3}{2}x\right) \\ &= x \end{aligned}$$

Domain: \mathbb{R}

$$\begin{aligned} \text{b) } g(f(x)) &= g\left(\frac{2}{3}x\right) & \text{Domain: } \mathbb{R} \\ &= \frac{3}{2}\left(\frac{2}{3}x\right) \\ &= x \end{aligned}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = x - 1$ and $g(x) = 3x^2 - 2x - 1$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} \text{a) } f(g(x)) &= f(3x^2 - 2x - 1) & \text{Domain: } \mathbb{R} \\ &= 3(x-1)^2 - 2(x-1) - 1 \\ &= 3(x^2 - 2x + 1) - 2x + 2 - 1 \\ &= 3x^2 - 6x + 3 - 2x + 1 \\ &= \underline{3x^2 - 8x + 4} \end{aligned}$$

Domain: \mathbb{R}

$$\begin{aligned} \text{b) } g(f(x)) &= g(x-1) & \text{Domain: } \mathbb{R} \\ &= 3x^2 - 2x - 1 - 1 \\ &= \underline{3x^2 - 2x - 2} \end{aligned}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = 3x - 2$ and $g(x) = x^2 - 5$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} \text{a) } f(g(x)) &= f(x^2 - 5) & \text{Domain: } \mathbb{R} \\ &= 3(x^2 - 5) - 2 \\ &= 3x^2 - 15 - 2 \\ &= \underline{3x^2 - 17} \end{aligned}$$

Domain: \mathbb{R}

$$\begin{aligned} \text{b) } g(f(x)) &= g(3x - 2) & \text{Domain: } \mathbb{R} \\ &= (3x - 2)^2 - 5 \end{aligned}$$

$$= 9x^2 - 12x + 4 - 5$$

$$= \underline{9x^2 - 12x - 1} \quad \text{Domain: } \mathbb{R}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = x^2 - 2$ and $g(x) = 4x - 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(4x - 3) \quad \text{Domain: } \mathbb{R}$$

$$= (4x - 3)^2 - 2$$

$$= 16x^2 - 24x + 9 - 2$$

$$= \underline{16x^2 - 24x + 7} \quad \text{Domain: } \mathbb{R}$$

Domain: \mathbb{R}

$$b) \quad g(f(x)) = g(x^2 - 2) \quad \text{Domain: } \mathbb{R}$$

$$= 4(x^2 - 2) - 3$$

$$= 4x^2 - 8 - 3$$

$$= \underline{4x^2 - 11} \quad \text{Domain: } \mathbb{R}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = 4x^2 - x + 10$ and $g(x) = 2x - 7$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(2x - 7) \quad \text{Domain: } \mathbb{R}$$

$$= 4(2x - 7)^2 - (2x - 7) + 10$$

$$= 4(4x^2 - 28x + 49) - 2x + 7 + 10$$

$$= 16x^2 - 112x + 196 - 2x + 17$$

$$\underline{= 16x^2 - 114x + 213} \mid$$

Domain: \mathbb{R}

Domain: $\underline{\mathbb{R}} \mid$

$$\begin{aligned} b) \quad g(f(x)) &= g(4x^2 - x + 10) \\ &= 2(4x^2 - x + 10) - 7 \\ &= 8x^2 - 2x + 20 - 7 \\ &= \underline{8x^2 - 2x + 13} \mid \end{aligned}$$

Domain: \mathbb{R}

Domain: \mathbb{R}

Domain: $\underline{\mathbb{R}} \mid$

Exercise

Given $f(x) = \sqrt{x}$ and $g(x) = x + 3$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
- b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} a) \quad f(g(x)) &= f(x + 3) \\ &= \underline{\sqrt{x + 3}} \mid \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \geq -3$

Domain: $\underline{x \geq -3} \mid$

$$\begin{aligned} b) \quad g(f(x)) &= g(\sqrt{x}) \\ &= \underline{\sqrt{x} + 3} \mid \end{aligned}$$

Domain: $x \geq 0$

Domain: $x \geq 0$

Domain: $\underline{x \geq 0} \mid$

Exercise

Given $f(x) = \sqrt{x}$ and $g(x) = 2 - 3x$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
- b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} a) \quad f(g(x)) &= f(2 - 3x) \\ &= \underline{\sqrt{2 - 3x}} \mid \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \leq \frac{2}{3}$

Domain: $\underline{x \leq \frac{2}{3}} \mid$

$$\begin{aligned}
 b) \quad g(f(x)) &= g(\sqrt{x}) \\
 &= 2 - 3\sqrt{x} \\
 \text{Domain: } &\underline{x \geq 0}
 \end{aligned}$$

$$\text{Domain: } x \geq 0$$

$$\text{Domain: } x \geq 0$$

Exercise

Given $f(x) = 3x + 2$ and $g(x) = \sqrt{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned}
 a) \quad f(g(x)) &= f(\sqrt{x}) \\
 &= 3\sqrt{x} + 2
 \end{aligned}$$

$$\text{Domain: } x \geq 0$$

$$\text{Domain: } x \geq 0$$

$$\text{Domain: } \underline{x \geq 0}$$

$$\begin{aligned}
 b) \quad g(f(x)) &= g(3x + 2) \\
 &= \sqrt{3x + 2}
 \end{aligned}$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } x \geq -\frac{2}{3}$$

$$\text{Domain: } \underline{x \geq -\frac{2}{3}}$$

Exercise

Given $f(x) = x^4$ and $g(x) = \sqrt[4]{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned}
 a) \quad f(g(x)) &= f(\sqrt[4]{x}) \\
 &= (\sqrt[4]{x})^4 \\
 &= x
 \end{aligned}$$

$$\text{Domain: } x \geq 0$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{x \geq 0}$$

$$\begin{aligned}
 b) \quad g(f(x)) &= g(x^4) \\
 &= \sqrt[4]{x^4} \\
 &= x
 \end{aligned}$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \mathbb{R}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = x^n$ and $g(x) = \sqrt[n]{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt[n]{x})$$

$$\text{Domain: } \begin{cases} \text{If } n \text{ is even} & x \geq 0 \\ \text{If } n \text{ is odd} & \mathbb{R} \end{cases}$$

$$= \left(\sqrt[n]{x}\right)^n$$

$$= x$$

$$\text{Domain: } \mathbb{R}$$

$$\text{Domain: } \begin{cases} \text{If } n \text{ is even} & x \geq 0 \\ \text{If } n \text{ is odd} & \mathbb{R} \end{cases}$$

$$b) \quad g(f(x)) = g(x^n)$$

$$\text{Domain: } \mathbb{R}$$

$$= \sqrt[n]{x^n}$$

$$= x$$

$$\text{Domain: } \mathbb{R}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = x^2 - 3x$ and $g(x) = \sqrt{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt{x+2})$$

$$x+2 \geq 0 \Rightarrow x \geq -2$$

$$= \left(\sqrt{x+2}\right)^2 - 3\sqrt{x+2}$$

$$= x+2 - 3\sqrt{x+2}$$

$$x+2 \geq 0 \Rightarrow x \geq -2$$

$$\text{Domain: } \{x \mid x \geq -2\}$$

$$b) \quad g(f(x)) = g(x^2 - 3x)$$

$$\mathbb{R}$$

$$= \sqrt{x^2 - 3x + 2} \mid$$

Domain: $\{x \mid x \leq 1, x \geq 2\}$

$$x^2 - 3x + 2 \geq 0 \Rightarrow (x = 1, 2) \leftrightarrow x \leq 1, x \geq 2$$

Exercise

Given $f(x) = \sqrt{x-2}$ and $g(x) = \sqrt{x+5}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt{x+5})$$

$$x+5 \geq 0 \Rightarrow x \geq -5$$

$$= \sqrt{\sqrt{x+5} - 2}$$

$$\sqrt{x+5} - 2 \geq 0 \Rightarrow \sqrt{x+5} \geq 2$$

$$x+5 \geq 4$$

$$x \geq -1$$

Domain: $\{x \mid x \geq -1\}$

$$b) \quad g(f(x)) = g(\sqrt{x-2})$$

$$x-2 \geq 0 \Rightarrow x \geq 2$$

$$= \sqrt{\sqrt{x-2} + 5}$$

$$\sqrt{x-2} + 5 \geq 0 \Rightarrow \sqrt{x-2} \geq -5 \quad \text{Always true when } x \geq 2$$

Domain: $\{x \mid x \geq 2\}$

Exercise

Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{3-x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(\sqrt{3-x})$$

Domain: $x \leq 3$

$$= (\sqrt{3-x})^2 + 2$$

$$= 3 - x + 2$$

$$= 5 - x \mid$$

Domain: \mathbb{R}

Domain: $x \leq 3 \mid$

$$b) \quad g(f(x)) = g(x^2 + 2)$$

Domain: \mathbb{R}

$$= \sqrt{3 - x^2} - 2$$

$$= \sqrt{1 - x^2}$$

Domain: $-1 \leq x \leq 1$

Domain: $\underline{-1 \leq x \leq 1}$

Exercise

Given $f(x) = x^5 - 2$ and $g(x) = \sqrt[5]{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(\sqrt[5]{x+2})$ **Domain:** \mathbb{R}

$$= (\sqrt[5]{x+2})^5 - 2$$

$$= x + 2 - 2$$

$$= x$$

Domain: \mathbb{R}

Domain: $\underline{\mathbb{R}}$

b) $g(f(x)) = g(x^5 - 2)$ **Domain:** \mathbb{R}

$$= \sqrt[5]{x^5 - 2 + 2}$$

$$= \sqrt[5]{x^5}$$

$$= x$$

Domain: \mathbb{R}

Domain: $\underline{\mathbb{R}}$

Exercise

Given $f(x) = 1 - x^2$ and $g(x) = \sqrt{x^2 - 25}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(\sqrt{x^2 - 25})$ **Domain:** $x \leq -5 \quad x \geq 5$

$$= 1 - (\sqrt{x^2 - 25})^2$$

$$\begin{aligned}
 &= 1 - (x^2 - 25) \\
 &= 1 - x^2 + 25 \\
 &= 26 - x^2
 \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \leq -5 \quad x \geq 5$

b) $g(f(x)) = g(1 - x^2)$

Domain: \mathbb{R}

$$\begin{aligned}
 &= \sqrt{(1 - x^2)^2 - 25} \\
 &= \sqrt{1 - 2x^2 + x^4 - 25} \\
 &= \sqrt{x^4 - 2x^2 - 24}
 \end{aligned}$$

$$\begin{aligned}
 x^2 &= \frac{2 \pm \sqrt{4 + 96}}{2} \\
 &= \begin{cases} \frac{2 - 10}{2} = -4 \quad \text{X} \\ \frac{2 + 10}{2} = 6 \end{cases}
 \end{aligned}$$

$$x^2 = 6 \rightarrow x = \pm\sqrt{6}$$

Domain: $x \leq -\sqrt{6} \quad x \geq \sqrt{6}$

Domain: $x \leq -\sqrt{6} \quad x \geq \sqrt{6}$

Exercise

Given $f(x) = 2x + 3$ and $g(x) = \frac{x-3}{2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{x-3}{2}\right)$

Domain: \mathbb{R}

$$\begin{aligned}
 &= 2\left(\frac{x-3}{2}\right) + 3 \\
 &= x - 3 + 3 \\
 &= x
 \end{aligned}$$

Domain: \mathbb{R}

Domain: \mathbb{R}

b) $g(f(x)) = g(2x + 3)$

Domain: \mathbb{R}

$$= \frac{1}{2}(2x + 3 - 3)$$

$$= x \mid$$

Domain: \mathbb{R}

Domain: $\mathbb{R} \mid$

Exercise

Given $f(x) = 4x - 5$ and $g(x) = \frac{x+5}{4}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{x+5}{4}\right) \quad \text{Domain: } \mathbb{R}$$

$$= 4\left(\frac{x+5}{4}\right) - 5$$

$$= x + 5 - 5$$

$$= x \mid$$

Domain: \mathbb{R}

Domain: $\mathbb{R} \mid$

$$b) \quad g(f(x)) = g(4x - 5) \quad \text{Domain: } \mathbb{R}$$

$$= \frac{1}{4}(4x - 5 + 5)$$

$$= x \mid$$

Domain: \mathbb{R}

Domain: $\mathbb{R} \mid$

Exercise

Given $f(x) = \frac{4}{1-5x}$ and $g(x) = \frac{1}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{1}{x}\right) \quad \text{Domain: } x \neq 0$$

$$= \frac{4}{1-5\frac{1}{x}}$$

$$= \frac{4x}{x-5} \mid$$

Domain: $x \neq 5$

Domain: $x \neq 0, 5 \mid$

$$b) \quad g(f(x)) = g\left(\frac{4}{1-5x}\right) \quad \text{Domain: } x \neq \frac{1}{5}$$

$$\left| \frac{1-5x}{4} \right|$$

Domain: \mathbb{R}

Domain: $\left| x \neq \frac{1}{5} \right|$

Exercise

Given $f(x) = \frac{1}{x-2}$ and $g(x) = \frac{x+2}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{x+2}{x}\right)$

Domain: $x \neq 0$

$$= \frac{1}{\frac{x+2}{x} - 2}$$

$$= \frac{1}{\frac{x+2-2x}{x}}$$

$$= \frac{x}{2-x} \left| \right.$$

Domain: $x \neq 2$

Domain: $\left| x \neq 0, 2 \right| \quad (-\infty, 0) \cup (0, 2) \cup (2, \infty)$

b) $g(f(x)) = g\left(\frac{1}{x-2}\right)$

Domain: $x \neq 2$

$$= \frac{\frac{1}{x-2} + 2}{\frac{1}{x-2}}$$

$$= \frac{\frac{1+2x-4}{x-2}}{\frac{1}{x-2}}$$

$$= 2x-3 \left| \right.$$

Domain: \mathbb{R}

Domain: $\left| x \neq 2 \right|$

$$\left| (-\infty, 2) \cup (2, \infty) \right|$$

Exercise

Given $f(x) = \frac{3x+5}{2}$ and $g(x) = \frac{2x-5}{3}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
- b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} \text{a) } f(g(x)) &= f\left(\frac{2x-5}{3}\right) & \text{Domain: } \mathbb{R} \\ &= \frac{3\frac{2x-5}{3} + 5}{2} \\ &= \frac{2x-5+5}{2} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

Domain: \mathbb{R}

$$\begin{aligned} \text{b) } g(f(x)) &= g\left(\frac{3x+5}{2}\right) & \text{Domain: } \mathbb{R} \\ &= \frac{2\frac{3x+5}{2} - 5}{3} \\ &= \frac{3x+5-5}{3} \\ &= \frac{3x}{3} \\ &= x \end{aligned}$$

Domain: \mathbb{R}

Exercise

Given $f(x) = \frac{1}{1+x}$ and $g(x) = \frac{1-x}{x}$

- a) Find $(f \circ g)(x)$ and the domain of $f \circ g$
- b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$\begin{aligned} \text{a) } f(g(x)) &= f\left(\frac{1-x}{x}\right) & \text{Domain: } x \neq 0 \\ &= \frac{1}{1 + \frac{1-x}{x}} \\ &= \frac{x}{x+1-x} \\ &= x \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \neq 0$ |

$$b) \quad g(f(x)) = g\left(\frac{1}{x+1}\right)$$

$$= \frac{1 - \frac{1}{x+1}}{\frac{1}{x+1}}$$

$$= x + 1 - 1$$

$$= x \quad |$$

Domain: $x \neq -1$

Domain: \mathbb{R}

Domain: \mathbb{R} |

Exercise

Given $f(x) = \frac{x-1}{x-2}$ and $g(x) = \frac{x-3}{x-4}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{x-3}{x-4}\right)$$

Domain: $x \neq 4$

$$= \frac{\frac{x-3}{x-4} - 1}{\frac{x-3}{x-4} - 2}$$

$$= \frac{\frac{x-3-(x-4)}{x-4}}{\frac{x-3-2(x-4)}{x-4}}$$

$$= \frac{x-3+x+4}{x-3-2x+8}$$

$$= \frac{2x+1}{-x+5} \quad |$$

Domain: $x \neq 5$

Domain: $\{x \mid x \neq 4, 5\}$

$$b) \quad g(f(x)) = g\left(\frac{x-1}{x-2}\right)$$

Domain: $x \neq 2$

$$= \frac{\frac{x-1}{x-2} - 3}{\frac{x-1}{x-2} - 4}$$

$$= \frac{x-1-3(x-2)}{x-1-4(x-2)}$$

$$= \frac{x-1-3x+6}{x-1-4x+8}$$

$$= \frac{-2x+5}{-3x+7}$$

$$\text{Domain: } x \neq \frac{7}{3}$$

$$\text{Domain: } \left\{ x \mid x \neq 2, \frac{7}{3} \right\}$$

Exercise

Given $f(x) = \frac{6}{x-3}$ and $g(x) = \frac{1}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $(f \circ g)(x)$

$$f(g(x)) = f\left(\frac{1}{x}\right)$$

$$\text{Domain: } x \neq 0$$

$$= \frac{6}{\frac{1}{x} - 3}$$

$$= \frac{6}{\frac{1-3x}{x}}$$

$$= \frac{6x}{1-3x}$$

$$\text{Domain: } x \neq \frac{1}{3}$$

$$\text{Domain: } x \neq 0, \frac{1}{3}$$

$$(-\infty, 0) \cup \left(0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$$

b) $(g \circ f)(x)$

$$g(f(x)) = g\left(\frac{6}{x-3}\right)$$

$$\text{Domain: } x \neq 3$$

$$= \frac{1}{\frac{6}{x-3}}$$

$$= \frac{x-3}{6}$$

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Domain: } x \neq 3$$

$$(-\infty, 3) \cup (3, \infty)$$

Exercise

Given $f(x) = \frac{6}{x}$ and $g(x) = \frac{1}{2x+1}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{1}{2x+1}\right) \quad \text{Domain: } x \neq -\frac{1}{2}$$

$$= \frac{6}{\frac{1}{2x+1}}$$

$$= 12x + 6$$

Domain: \mathbb{R}

$$\text{Domain: } x \neq -\frac{1}{2}$$

$$b) \quad g(f(x)) = g\left(\frac{6}{x}\right) \quad \text{Domain: } x \neq 0$$

$$= \frac{1}{2\frac{6}{x} + 1}$$

$$= \frac{x}{12 + x}$$

Domain: $x \neq -12$

$$\text{Domain: } x \neq -12, 0$$

Exercise

Given $f(x) = 3x - 7$ and $g(x) = \frac{x+7}{3}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{x+7}{3}\right) \quad \text{Domain: } \mathbb{R}$$

$$= 3\frac{x+7}{3} - 7$$

$$= x + 7 - 7$$

$$= x$$

Domain: \mathbb{R}

$$\text{Domain: } \mathbb{R}$$

$$b) \quad g(f(x)) = g(3x - 7) \quad \text{Domain: } \mathbb{R}$$

$$= \frac{3x - 7 + 7}{3}$$

$$= x \mid$$

Domain: \mathbb{R}

Domain: $\mathbb{R} \mid$

Exercise

Given $f(x) = \frac{2x+3}{x-4}$ and $g(x) = \frac{4x+3}{x-2}$

- Find $(f \circ g)(x)$ and the domain of $f \circ g$
- Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\frac{4x+3}{x-2}\right)$ **Domain:** $x \neq 2$

$$\begin{aligned} &= \frac{2 \frac{4x+3}{x-2} + 3}{\frac{4x+3}{x-2} - 4} \\ &= \frac{8x+6+3x-6}{4x+3-4x+8} \\ &= \frac{11x}{11} \end{aligned}$$

$$= x \mid$$

Domain: \mathbb{R}

Domain: $x \neq 2 \mid$

b) $g(f(x)) = g\left(\frac{2x+3}{x-4}\right)$ **Domain:** $x \neq 4$

$$\begin{aligned} &= \frac{4 \frac{2x+3}{x-4} + 3}{\frac{2x+3}{x-4} - 2} \\ &= \frac{8x+12+3x-4}{2x+3-2x+8} \\ &= \frac{11x}{11} \end{aligned}$$

$$= x \mid$$

Domain: \mathbb{R}

Domain: $x \neq 4 \mid$

Exercise

Given $f(x) = \frac{2x+3}{x+4}$ and $g(x) = \frac{-4x+3}{x-2}$

- Find $(f \circ g)(x)$ and the domain of $f \circ g$
- Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f\left(\frac{-4x+3}{x-2}\right) \quad \text{Domain: } x \neq 2$$

$$\begin{aligned} &= \frac{2\frac{-4x+3}{x-2} + 3}{\frac{4x+3}{x-2} + 4} \\ &= \frac{-8x+6+3x-6}{4x+3+4x-8} \\ &= \frac{-5x}{-5} \\ &= x \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \neq 2$

$$b) \quad g(f(x)) = g\left(\frac{2x+3}{x+4}\right) \quad \text{Domain: } x \neq -4$$

$$\begin{aligned} &= \frac{-4\frac{2x+3}{x+4} + 3}{\frac{2x+3}{x+4} - 2} \\ &= \frac{-8x-12+3x+12}{2x+3-2x-8} \\ &= \frac{-5x}{-5} \\ &= x \end{aligned}$$

Domain: \mathbb{R}

Domain: $x \neq -4$

Exercise

Given $f(x) = x+1$ and $g(x) = x^3 - 5x^2 + 3x + 7$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(x^3 - 5x^2 + 3x + 7) \quad \text{Domain: } \mathbb{R}$$

$$\begin{aligned} &= x^3 - 5x^2 + 3x + 7 + 1 \\ &= x^3 - 5x^2 + 3x + 8 \end{aligned} \quad \text{Domain: } \mathbb{R}$$

Domain: \mathbb{R}

$$b) \quad g(f(x)) = g(x+1) \quad \text{Domain: } \mathbb{R}$$

$$\begin{aligned} &= (x+1)^3 - 5(x+1)^2 + 3(x+1) + 7 \\ &= x^3 + 3x^2 + 3x + 1 - 5(x^2 + 2x + 1) + 3x + 3 + 7 \end{aligned}$$

$$= x^3 + 3x^2 + 6x + 11 - 5x^2 - 10x - 5$$

$$= \underline{x^3 - 2x^2 - 4x + 6} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

Exercise

Given $f(x) = x - 1$ and $g(x) = x^3 + 2x^2 - 3x - 9$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$$a) \quad f(g(x)) = f(x^3 + 2x^2 - 3x - 9) \quad \text{Domain: } \mathbb{R}$$

$$= x^3 + 2x^2 - 3x - 9 - 1$$

$$= \underline{x^3 + 2x^2 - 3x - 10} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

$$b) \quad g(f(x)) = g(x - 1) \quad \text{Domain: } \mathbb{R}$$

$$= (x - 1)^3 + 2(x - 1)^2 - (x - 1) - 9$$

$$= x^3 - 3x^2 + 3x - 1 + 2(x^2 - 2x + 1) - 3x + 3 - 9$$

$$= x^3 - 3x^2 - 7 + 2x^2 - 4x + 2$$

$$= \underline{x^3 - x^2 - 4x - 5} \quad \text{Domain: } \mathbb{R}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

Exercise

Given $f(x) = \sqrt{x}$ and $g(x) = x + 3$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain.

Solution

$$(f \circ g)(x) = f(g(x))$$

$$= f(x + 3)$$

$$\text{Domain: } (-\infty, \infty)$$

$$= \sqrt{x + 3}$$

$$x + 3 \geq 0 \Rightarrow x \geq -3$$

$$\text{Domain: } \underline{x \geq -3}$$

$$(g \circ f)(x) = g(f(x))$$

$$= g(\sqrt{x})$$

$$\text{Domain: } x \geq 0$$

$$= \sqrt{x} + 3$$

$$\text{Domain: } \underline{x \geq 0}$$

Exercise

Given that $f(x) = \sqrt{x}$ and $g(x) = 2 - 3x$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain.

Solution

$$(f \circ g)(x) = f(g(x))$$

$$= f(2 - 3x)$$

$$\text{Domain: } (-\infty, \infty)$$

$$= \sqrt{2 - 3x}$$

$$2 - 3x \geq 0 \rightarrow -3x \geq -2 \Rightarrow \boxed{x \leq \frac{2}{3}}$$

$$\text{Domain: } \left(-\infty, \frac{2}{3}\right]$$

$$g(f(x)) = g(\sqrt{x})$$

$$\text{Domain: } x \geq 0$$

$$= 2 - 3\sqrt{x}$$

$$x \geq 0$$

$$\text{Domain: } [0, \infty)$$

Exercise

Given that $f(x) = \frac{1}{x-2}$ and $g(x) = \frac{x+2}{x}$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain.

Solution

$$f(g(x)) = f\left(\frac{x+2}{x}\right)$$

$$\text{Domain: } \boxed{x \neq 0}$$

$$= \frac{1}{\frac{x+2}{x} - 2}$$

$$= \frac{1}{\frac{x+2-2x}{x}}$$

$$= \frac{x}{2-x}$$

$$\text{Domain: } \boxed{x \neq 2}$$

$$\text{Domain: } \underline{x \neq 0, 2}$$

$$g(f(x)) = g\left(\frac{1}{x-2}\right)$$

$$\text{Domain: } \boxed{x \neq 2}$$

$$= \frac{\frac{1}{x-2} + 2}{\frac{1}{x-2}} \quad \underline{(-\infty, 0) \cup (0, 2) \cup (2, \infty)}$$

$$= \frac{1 + 2x - 4}{x - 2} \cdot \frac{1}{1}$$

$$= 2x - 3$$

$$\text{Domain: } \boxed{\mathbb{R}}$$

$$\text{Domain: } \boxed{x \neq 2}$$

Exercise

Given that $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 8$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain then find $(f \circ g)(7)$

Solution

$$f(g(x)) = f(x^2 - 3x + 8)$$

$$\text{Domain: } (-\infty, \infty)$$

$$= 2(\text{-----}) - 5$$

$$= 2(2x^2 - 3x + 8) - 5$$

$$= 2x^2 - 6x + 16 - 5$$

$$= 2x^2 - 6x + 11$$

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Domain: } \boxed{\mathbb{R}}$$

$$g(f(x)) = g(2x - 5)$$

$$\text{Domain: } (-\infty, \infty)$$

$$= (\text{---})^2 - 3(\text{---}) + 8$$

$$= (2x - 5)^2 - 3(2x - 5) + 8$$

$$= 4x^2 - 20x + 25 - 6x + 15 + 8$$

$$= 4x^2 - 26x + 48$$

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Domain: } \boxed{\mathbb{R}}$$

$$f(g(7)) = 2(7)^2 - 6(7) + 11$$

$$= \underline{67}$$

Exercise

Given that $f(x) = \sqrt{x}$ and $g(x) = x - 1$, find

$$a) \quad (f \circ g)(x) = f(g(x))$$

$$b) \quad (g \circ f)(x) = g(f(x))$$

$$c) \quad (f \circ g)(2) = f(g(2))$$

Solution

$$a) \quad (f \circ g)(x) = f(g(x))$$

$$= f(x - 1)$$

$$= \sqrt{x - 1}$$

$$b) \quad (g \circ f)(x) = g(f(x))$$

$$= g(\sqrt{x})$$

$$= \sqrt{x} - 1$$

$$c) \quad (f \circ g)(2) = f(g(2))$$

$$= \sqrt{2 - 1}$$

$$= \sqrt{1}$$

$$= 1$$

Exercise

Given that $f(x) = \frac{x}{x+5}$ and $g(x) = \frac{6}{x}$, find

$$a) \quad (f \circ g)(x) = f(g(x))$$

$$b) \quad (g \circ f)(x) = g(f(x))$$

$$c) \quad (f \circ g)(2) = f(g(2))$$

Solution

$$a) \quad (f \circ g)(x) = f(g(x))$$

$$= f\left(\frac{6}{x}\right)$$

$$= \frac{\frac{6}{x}}{\frac{6}{x} + 5}$$

$$= \frac{\frac{6}{x}}{\frac{6 + 5x}{x}}$$

$$= \frac{6}{6 + 5x}$$

$$b) \quad (g \circ f)(x) = g(f(x))$$

$$= g\left(\frac{x}{x+5}\right)$$

$$= \frac{6}{\frac{x}{x+5}}$$

$$= \frac{6(x+5)}{x}$$

$$\begin{aligned} \text{c) } (f \circ g)(2) &= f(g(2)) \\ &= \frac{6}{6+5(2)} = \frac{6}{16} \\ &= \frac{3}{8} \end{aligned}$$

Exercise

Determine whether f is even, odd, or neither: $f(x) = 3x^4 + 2x^2 - 5$

Solution

$$\begin{aligned} f(-x) &= 3(-x)^4 + 2(-x)^2 - 5 \\ &= 3x^4 + 2x^2 - 5 \\ &= f(x) \end{aligned}$$

\therefore The function is **even**.

Exercise

Determine whether f is even, odd, or neither: $f(x) = 8x^3 - 3x^2$

Solution

$$\begin{aligned} f(-x) &= 8(-x)^3 - 3(-x)^2 \\ &= -8x^3 - 3x^2 \end{aligned}$$

\therefore The function is **neither**.

Exercise

Determine whether f is even, odd, or neither: $f(x) = \sqrt{x^2 + 4}$

Solution

$$\begin{aligned} f(-x) &= \sqrt{(-x)^2 + 4} \\ &= \sqrt{x^2 + 4} \\ &= f(x) \end{aligned}$$

\therefore The function is **even**.

Exercise

Determine whether f is even, odd, or neither: $f(x) = 3x^2 - 5x + 1$

Solution

$$\begin{aligned}f(-x) &= 3(-x)^2 - 5(-x) + 1 \\&= 3x^2 + 5x + 1\end{aligned}$$

\therefore The function is **neither**.

Exercise

Determine whether f is even, odd, or neither: $f(x) = \sqrt[3]{x^3 - x}$

Solution

$$\begin{aligned}f(-x) &= \sqrt[3]{(-x)^3 - (-x)} \\&= \sqrt[3]{-x^3 + x} \\&= \sqrt[3]{-(x^3 - x)} \\&= -\sqrt[3]{x^3 - x} \\&= -f(x)\end{aligned}$$

\therefore The function is **odd**.

Exercise

Determine whether f is even, odd, or neither: $f(x) = |x| - 3$

Solution

$$\begin{aligned}f(-x) &= |-x| - 3 \\&= |(-)x| - 3 \\&= |-1||x| - 3 \\&= |x| - 3 \\&= f(x)\end{aligned}$$

\therefore The function is **even**.

Exercise

Determine whether f is even, odd, or neither: $f(x) = x^3 - \frac{1}{x}$

Solution

$$\begin{aligned}
 f(-x) &= (-x)^3 - \frac{1}{(-x)} \\
 &= -x^3 + \frac{1}{x} \\
 &= -\left(x^3 - \frac{1}{x}\right) \\
 &= -f(x)
 \end{aligned}$$

\therefore The function is *odd*.

Exercise

Decide whether each function is even, odd, or neither $f(x) = -x^3 + 2x$

Solution

$$\begin{aligned}
 f(-x) &= -(-x)^3 + 2(-x) \\
 &= x^3 - 2x \\
 &= -f(x)
 \end{aligned}$$

\therefore The function is *odd*.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^5 - 2x^3$

Solution

$$\begin{aligned}
 f(-x) &= (-x)^5 - 2(-x)^3 \\
 &= -x^5 + 2x^3 \\
 &= -f(x)
 \end{aligned}$$

\therefore The function is *odd*.

Exercise

Decide whether each function is even, odd, or neither $f(x) = .5x^4 - 2x^2 + 6$

Solution

$$\begin{aligned}
 f(-x) &= .5(-x)^4 - 2(-x)^2 + 6 \\
 &= .5x^4 - 2x^2 + 6 \\
 &= f(x)
 \end{aligned}$$

\therefore The function is *even*.

Exercise

Decide whether each function is even, odd, or neither $f(x) = .75x^2 + |x| + 4$

Solution

$$\begin{aligned} f(-x) &= .75(-x)^2 + |-x| + 4 \\ &= .75x^2 + |x| + 4 \\ &= f(x) \end{aligned} \quad \therefore \text{The function is *even*.}$$

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^3 - x + 9$

Solution

$$\begin{aligned} f(-x) &= (-x)^3 - (-x) + 9 \\ &= -x^3 + x + 9 \end{aligned} \quad \therefore \text{The function is *neither*.}$$

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^4 - 5x + 8$

Solution

$$\begin{aligned} f(-x) &= (-x)^4 - 5(-x) + 8 \\ &= x^4 + 5x + 8 \end{aligned} \quad \therefore \text{The function is *neither*.}$$

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^3 + x$

Solution

$$\begin{aligned} f(-x) &= (-x)^3 + (-x) \\ &= -x^3 - x \\ &= -f(x) \end{aligned}$$

\therefore The function is *odd*.

Exercise

Decide whether each function is even, odd, or neither $g(x) = x^2 - x$

Solution

$$\begin{aligned} g(-x) &= (-x)^2 + (-x) \\ &= x^2 - x \end{aligned}$$

\therefore The function is *neither*.

Exercise

Decide whether each function is even, odd, or neither $h(x) = 2x^2 + x^4$

Solution

$$\begin{aligned} h(-x) &= 2(-x)^2 + (-x)^4 \\ &= 2x^2 + x^4 \\ &= h(x) \end{aligned}$$

\therefore The function is *even*.

Exercise

Decide whether each function is even, odd, or neither $f(x) = 2x^2 + x^4 + 1$

Solution

$$\begin{aligned} f(-x) &= 2(-x)^2 + (-x)^4 + 1 \\ &= 2x^2 + x^4 + 1 \\ &= f(x) \end{aligned}$$

\therefore The function is *even*.

Exercise

Decide whether each function is even, odd, or neither $f(x) = \frac{1}{5}x^6 - 3x^2$

Solution

$$\begin{aligned} f(-x) &= \frac{1}{5}(-x)^6 - 3(-x)^2 \\ &= \frac{1}{5}x^6 - 3x^2 \\ &= f(x) \end{aligned}$$

\therefore The function is *even*.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x\sqrt{1-x^2}$

Solution

$$\begin{aligned}
 f(-x) &= -x\sqrt{1-(-x)^2} \\
 &= -x\sqrt{1-x^2} \\
 &= -f(x)
 \end{aligned}$$

∴ The function is **odd**.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^2\sqrt{1-x^2}$

Solution

$$\begin{aligned}
 f(-x) &= (-x)^2\sqrt{1-(-x)^2} \\
 &= x^2\sqrt{1-x^2} \\
 &= f(x)
 \end{aligned}$$

∴ The function is **even**.

Exercise

Decide whether each function is even, odd, or neither $f(x) = 5x^7 - 6x^3 - 2x$

Solution

$$\begin{aligned}
 f(-x) &= 5(-x)^7 - 6(-x)^3 - 2(-x) \\
 &= -5x^7 + 6x^3 + 2x \\
 &= -(5x^7 - 6x^3 - 2x) \\
 &= -f(x)
 \end{aligned}$$

∴ The function is **odd**.

Exercise

Decide whether each function is even, odd, or neither $f(x) = 5x^6 - 3x^2 - 7$

Solution

$$\begin{aligned}
 f(-x) &= 5(-x)^6 - 3(-x)^2 - 7 \\
 &= 5x^6 - 3x^2 - 7 \\
 &= f(x)
 \end{aligned}$$

∴ The function is **even**.

Exercise

Decide whether each function is even, odd, or neither $f(x) = x^2 + 6$

Solution

$$\begin{aligned}f(-x) &= (-x)^2 + 6 \\&= x^2 + 6 \\&= f(x)\end{aligned}$$

\therefore The function is **even**.

Exercise

Decide whether each function is even, odd, or neither $f(x) = 7x^3 - x$

Solution

$$\begin{aligned}f(-x) &= 7(-x)^3 - (-x) \\&= -7x^3 + x \\&= -(7x^3 - x) \\&= -f(x)\end{aligned}$$

\therefore The function is **odd**.

Exercise

Decide whether each function is even, odd, or neither $h(x) = x^5 + 1$

Solution

$$\begin{aligned}h(-x) &= (-x)^5 + 1 \\&= -x^5 + 1 \quad \begin{cases} \neq x^5 + 1 \\ \neq -(x^5 + 1) \end{cases}\end{aligned}$$

\therefore The function is **neither**.

Exercise

$$f(x) = \begin{cases} 2+x & \text{if } x < -4 \\ -x & \text{if } -4 \leq x \leq 2 \\ 3x & \text{if } x > 2 \end{cases} \quad \text{Find: } f(-5), f(-1), f(0), \text{ and } f(3)$$

Solution

$$f(-5) = 2 - 5 = -3$$

$$f(-1) = -(-1) = 1$$

$$f(0) = -0 = 0$$

$$f(3) = 3(3) = 9$$

Exercise

$$f(x) = \begin{cases} -2x & \text{if } x < -3 \\ 3x - 1 & \text{if } -3 \leq x \leq 2 \\ -4x & \text{if } x > 2 \end{cases} \quad \text{Find: } f(-5), f(-1), f(0), \text{ and } f(3)$$

Solution

$$f(-5) = -2(-5) = 10$$

$$f(-1) = 3(-1) - 1 = -4$$

$$f(0) = 3(0) - 1 = -1$$

$$f(3) = -4(3) = -12$$

Exercise

$$f(x) = \begin{cases} x^3 + 3 & \text{if } -2 \leq x \leq 0 \\ x + 3 & \text{if } 0 < x < 1 \\ 4 + x - x^2 & \text{if } 1 \leq x \leq 3 \end{cases} \quad \text{Find: } f(-5), f(-1), f(0), \text{ and } f(3)$$

Solution

$$f(-5) = \text{doesn't exist}$$

$$f(-1) = (-1)^3 + 3 = 2$$

$$f(0) = (0)^3 + 3 = 3$$

$$f(3) = 4 + (3) - (3)^2 = -2$$

Exercise

$$h(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases} \quad \text{Find: } h(5), h(0), \text{ and } h(3)$$

Solution

$$h(5) = \frac{5^2 - 9}{5 - 3} = 8$$

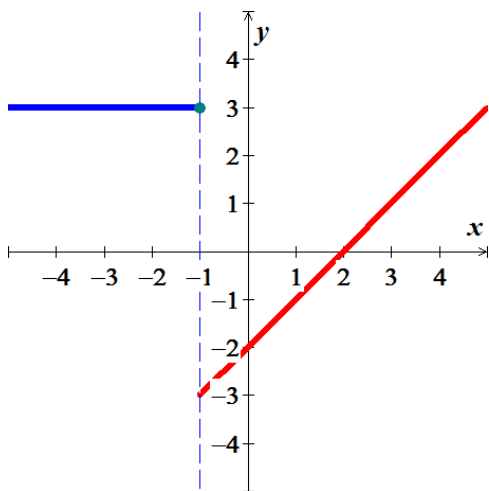
$$h(0) = \frac{0^2 - 9}{0 - 3} = 3$$

$$h(3) = 6$$

Exercise

Graph the piecewise function defined by $f(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ x - 2 & \text{if } x > -1 \end{cases}$

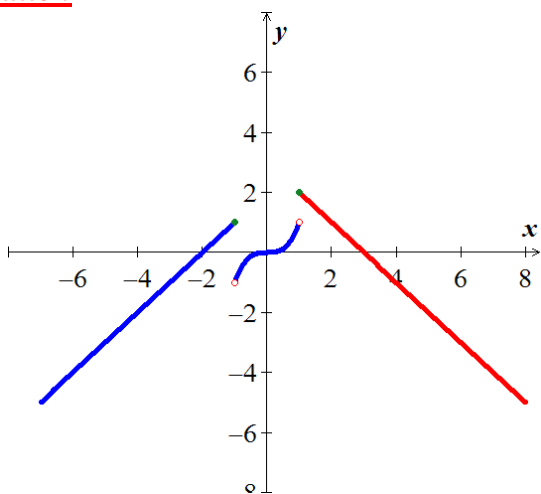
Solution



Exercise

Sketch the graph $f(x) = \begin{cases} x + 2 & \text{if } x \leq -1 \\ x^3 & \text{if } -1 < x < 1 \\ -x + 3 & \text{if } x \geq 1 \end{cases}$

Solution



Exercise

Sketch the graph $f(x) = \begin{cases} x-3 & \text{if } x \leq -2 \\ -x^2 & \text{if } -2 < x < 1 \\ -x+4 & \text{if } x \geq 1 \end{cases}$

Solution

