$$2\pi$$
 (radians) =  $360^{\circ}$  = 1 revolution

$$\theta = \frac{s}{s}$$
 (radians)

$$v = \frac{S}{t} = r\omega = r\frac{\theta}{t}$$

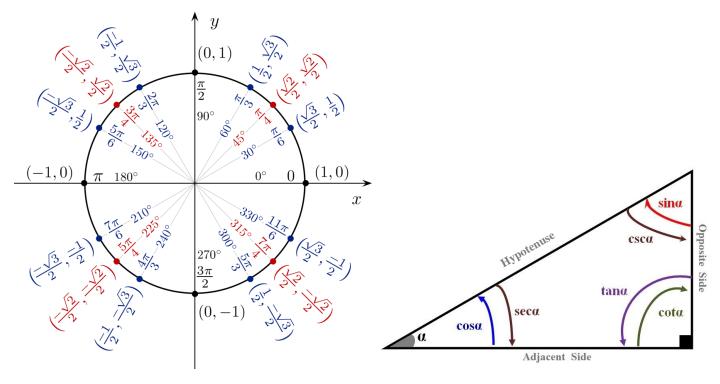
$$\theta = \frac{s}{r}$$
 (radians)  $v = \frac{s}{t} = r\omega = r\frac{\theta}{t}$   $\omega = \frac{\theta}{t} = \frac{v}{r} = \frac{s}{rt} = \frac{v\theta}{s}$ 

$$3600 \ rev \ / \ minute = \frac{3600 \ rev}{1 \ min} \frac{2\pi \ (radians)}{1 \ rev} \frac{1 \ min}{60 \ sec} = \frac{120\pi \ (radians)}{1 \ sec}$$

$$r = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$\sin \theta = \frac{Opposite}{Hypotenuse} = \frac{opp}{hyp}$	$\cos \theta = \frac{Adjacent}{Hypotenuse} = \frac{adj}{hyp}$	$\tan \theta = \frac{opposite}{adjacent} = \frac{opp}{adj} = \frac{\sin \theta}{\cos \theta}$
$\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$	$\sec \theta = \frac{hyp}{adj} = \frac{1}{\cos \theta}$	$\cot \theta = \frac{adj}{opp} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$



$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$1 + \tan^2 \alpha = \sec^2 \alpha$$

$$1 + \cot^2 \alpha = \csc^2 \alpha$$

$$\cos(-\alpha) = \cos\alpha$$

$$\sin(-\alpha) = -\sin\alpha$$

$$tan(-\alpha) = -tan\alpha$$

$$cos(90^{\circ} - \alpha) = sin\alpha$$

$$\sin(90^{\circ} - \alpha) = \cos\alpha$$

$$tan(90^{\circ} - \alpha) = cot\alpha$$

$$cos(\alpha - \beta) = cos\alpha \cos\beta + sin\alpha \sin\beta$$
$$cos(\alpha + \beta) = cos\alpha \cos\beta - sin\alpha \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$$
$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha$$
$$\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$
$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Half-Angle: 
$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$
  $\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$   $\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$   $a\sin x + b\cos x = k\sin\left(x+\alpha\right)$  where  $k = \sqrt{a^2+b^2}$ ,  $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$ , and  $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$ 

#### Double-Angle

$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ $= 1 - 2\sin^2 \alpha$ $= 2\cos^2 \alpha - 1$	$\sin 2\alpha = 2\sin \alpha \cos \alpha$	$\tan 2\alpha = \frac{2\tan \alpha}{1 - \tan^2 \alpha}$
$\cos^2\alpha = \frac{1 + \cos 2\alpha}{2}$	$\sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$	$\tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$

#### Product-to-Sum:

$\sin \alpha \cos \beta = \frac{1}{2} \left[ \sin (\alpha + \beta) + \sin (\alpha - \beta) \right]$	$\cos \alpha \sin \beta = \frac{1}{2} \left[ \sin (\alpha + \beta) - \sin (\alpha - \beta) \right]$
$\cos \alpha \cos \beta = \frac{1}{2} \left[ \cos (\alpha + \beta) + \cos (\alpha - \beta) \right]$	$\sin \alpha \sin \beta = \frac{1}{2} \left[ \cos (\alpha - \beta) - \cos (\alpha + \beta) \right]$

## Sum-to-Product:

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$	$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$	$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

**Law of Sines:**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$   $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 

## Law of Cosines:

$a^2 = b^2 + c^2 - 2bc \cos A$	$b^2 = a^2 + c^2 - 2ac\cos B$	$c^2 = a^2 + b^2 - 2ab\cos C$
$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$	$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$	$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$

**Area**:  $A = \frac{1}{2}r^2\theta$  (sector)

$$K = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$
  $K = \sqrt{s(s-a)(s-b)(s-c)}$   $s = \frac{1}{2}(a+b+c)$ 

#### Vectors:

Magnitude: 
$$|V| = \sqrt{a^2 + b^2}$$
 Angle:  $\cos \theta = \frac{U \cdot V}{|U||V|}$ 

Dot Product:  $U \cdot V = (ai + bj) \cdot (ci + dj) = ac + bd$ 

 $z = r(\cos\theta + i \sin\theta) = r \operatorname{cis}\theta \qquad \qquad r = \sqrt{x^2 + y^2} \qquad \cos\theta = \frac{x}{r}, \quad \sin\theta = \frac{y}{r}, \quad and \quad \tan\theta = \frac{y}{x}$   $\left(r_1 \operatorname{cis}\theta_1\right) \left(r_2 \operatorname{cis}\theta_2\right) = r_1 r_2 \operatorname{cis}\left(\theta_1 + \theta_2\right) \qquad \frac{r_1 \operatorname{cis}\theta_1}{r_2 \operatorname{cis}\theta_2} = \frac{r_1}{r_2} \operatorname{cis}\left(\theta_1 - \theta_2\right)$ 

**De Moivre's Theorem:**  $\left[rcis\theta\right]^n = r^n \left(cisn\theta\right)$   $\left[rcis\theta\right]^{1/n} = \sqrt[n]{r}cis\alpha$   $\alpha = \frac{\theta}{n} + \frac{360^{\circ}k}{n}$ 

The graphs of  $y = A\sin(Bx + C) + D$  and  $y = A\cos(Bx + C) + D$ , where B > 0, will have the following characteristics:

Period =  $\frac{2\pi}{|B|}$  Phase Shift =  $\varphi = -\frac{C}{B}$  One cycle:  $0 \le argument \le 2\pi$ Amplitude = |A|

Vertical Shift: y = D

# To graph "Sine or Cosine"

- 1- Find the Amplitude
- 2- Find the Period
- 3- Construct a table

х	$y = A\cos(Bx + C) + D$	$y = A\sin(Bx + C) + D$
φ	D+A	D
$\varphi + \frac{P}{4}$	D	D+A
$\varphi + \frac{P}{2}$	D-A	D
$\varphi + \frac{3P}{4}$	D	D-A
$\varphi + P$	D+A	D

- 4- Graph One Cycle
- 5- Extend the graph, if necessary

The graphs of  $y = A \tan(Bx + C) + D$  and  $y = A \cot(Bx + C) + D$ , where B > 0, will have the following characteristics:

Period  $=\frac{\pi}{|B|}$  Phase Shift  $=-\frac{C}{B}$  One cycle:  $0 \le argument \le \pi$ No Amplitude

Vertical Shift: y = D

х	$y = A \tan(Bx + C) + D$	$y = A\cot(Bx + C) + D$
$\varphi$	D	$\infty$
$\varphi + \frac{P}{4}$	D+A	D+A
$\varphi + \frac{P}{2}$	$\infty$	D
$\varphi + \frac{3P}{4}$	D-A	D-A
$\varphi + P$	D	∞