let wind $= \omega_x \hat{i} + \omega_y \hat{j}$

$$v = (v_0 \cos\alpha)\hat{i} + (v_0 \sin\alpha)\hat{j} + \omega_x \hat{i} + \omega_y \hat{j}$$
$$= (v_0 \cos\alpha + \omega_x)\hat{i} + (v_0 \sin\alpha + \omega_y)\hat{j}$$

$$r_0 = x_0 \hat{i} + y_0 \hat{j}$$

$$r(t) = -\frac{1}{2} g t^2 \hat{j} + v t + r_0$$
$$= -\frac{1}{2} g t^2 \hat{j} + (v_0 \cos\alpha + \omega_x) t \hat{i} + (v_0 \sin\alpha + \omega_y) t \hat{j} + x_0 \hat{i} + y_0 \hat{j}$$
$$= [\underbrace{x_0 + (v_0 \cos\alpha + \omega_x) t}_{x(t)}]\hat{i} + [\underbrace{-\frac{1}{2} g t^2 + (v_0 \sin\alpha + \omega_y) t + y_0}_{y(t)}]\hat{j}$$

$$y' = -gt + v_0 \sin\alpha + \omega_y = 0 \Rightarrow \boxed{t_{Max} = \frac{v_0 \sin\alpha + \omega_y}{g}}$$

$$y_{Max} = -\frac{1}{2} g \left(\frac{v_0 \sin\alpha + \omega_y}{g}\right)^2 + (v_0 \sin\alpha + \omega_y)\left(\frac{v_0 \sin\alpha + \omega_y}{g}\right) + y_0$$
$$= -\frac{1}{2} \frac{(v_0 \sin\alpha + \omega_y)^2}{g} + \frac{(v_0 \sin\alpha + \omega_y)^2}{g} + y_0$$

$$\boxed{y_{Max} = \frac{(v_0 \sin\alpha + \omega_y)^2}{2g} + y_0}$$

$$-\frac{1}{2} g t^2 + (v_0 \sin\alpha + \omega_y) t + y_0 = 0$$

$$t_{1,2} = \frac{-(v_0 \sin\alpha + \omega_y) \pm \sqrt{(v_0 \sin\alpha + \omega_y)^2 - 4\left(-\frac{1}{2}g\right) y_0}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{v_0 \sin\alpha + \omega_y \mp \sqrt{(v_0 \sin\alpha + \omega_y)^2 + 2 g y_0}}{g}$$

if $y_0 = 0 \Rightarrow$ $t = \dfrac{v_0 \sin\alpha + \omega_y + \sqrt{(v_0 \sin\alpha + \omega_y)^2}}{g}$

$$= \boxed{\frac{2(v_0 \sin\alpha + \omega_y)}{g}}$$

Flight Time

$$\boxed{\text{Range} = x_0 + (v_0 \cos\alpha + \omega_x)\,\frac{2(v_0 \sin\alpha + \omega_y)}{g}}$$

$$= x_0 + \frac{2 v_0^2 \sin\alpha\cos\alpha}{g} + \frac{2 v_0 \omega_y \cos\alpha + 2 v_0 \omega_x \sin\alpha + 2\omega_x \omega_y}{g}$$

$$\boxed{= x_0 + \frac{v_0^2 \sin 2\alpha}{g} + \frac{2 v_0(\omega_y \cos\alpha + \omega_x \sin\alpha)}{g} + \frac{2\omega_x \omega_y}{g}}$$

If $y_0 \neq 0 \Rightarrow$ $\boxed{t = \dfrac{v_0 \sin\alpha + \omega_y + \sqrt{(v_0 \sin\alpha + \omega_y)^2 + 2 g y_0}}{g}}$

$$\boxed{\text{Range} = x_0 + (v_0 \cos\alpha + \omega_x)\,\frac{v_0 \sin\alpha + \omega_y + \sqrt{(v_0 \sin\alpha + \omega_y)^2 + 2 g y_0}}{g}}$$

$$r(t) = \underbrace{\left(x_0 + (v_0 \cos\alpha)\, t\right)}_{x(t)} \hat{\imath} + \underbrace{\left(y_0 + (v_0 \sin\alpha) t - \frac{1}{2} g t^2\right)}_{y(t)} \hat{\jmath}$$

$$y' = -gt + v_0 \sin\alpha = 0$$

$$\Rightarrow \boxed{t_{Max} = \frac{v_0 \sin\alpha}{g}}$$

Maximum time when the object at Maximum height

$$y = -\frac{1}{2} g \left(\frac{v_0 \sin\alpha}{g}\right)^2 + (v_0 \sin\alpha) \frac{v_0 \sin\alpha}{g} + y_0$$

$$= -\frac{1}{2} \frac{(v_0 \sin\alpha)^2}{g} + \frac{(v_0 \sin\alpha)^2}{g} + y_0$$

$$\boxed{Y_{Max} = \frac{(v_0 \sin\alpha)^2}{2g} + y_0}$$

Maximum height.

$$y(t) = -\frac{1}{2} g t^2 + (v_0 \sin\alpha) t + y_0 = 0.$$

$$\Rightarrow t_{1,2} = \frac{-v_0 \sin\alpha \pm \sqrt{(v_0 \sin\alpha)^2 - 4\left(-\frac{1}{2} g\right)(y_0)}}{2\left(-\frac{1}{2} g\right)}$$

$$= \frac{-v_0 \sin\alpha \pm \sqrt{(v_0 \sin\alpha)^2 + 2 g y_0}}{-g}$$

$$= \frac{v_0 \sin\alpha \mp \sqrt{(v_0 \sin\alpha)^2 + 2 g y_0}}{g}$$

if $y_0 = 0 \Rightarrow t = \dfrac{v_0 \sin\alpha + \sqrt{(v_0 \sin\alpha)^2}}{g} = \dfrac{v_0 \sin\alpha + v_0 \sin\alpha}{g}$

$$= \boxed{\frac{2 v_0 \sin\alpha}{g}}$$

flight time

$$\text{Range} = x_0 + v_0 \cos\alpha \left(\frac{2 v_0 \sin\alpha}{g}\right)$$

$$= \boxed{x_0 + \frac{(v_0)^2 \sin 2\alpha}{g}}$$

$\sin 2\alpha = 2 \cos\alpha \sin\alpha.$

if $y_0 \neq 0 \Rightarrow t = \dfrac{v_0 \sin\alpha + \sqrt{(v_0 \sin\alpha)^2 + 2 g y_0}}{g}$