$\int x^5 e^{4x}\,dx$

	$\int e^{4x}\,dx$
$+\ x^5$	$\frac{1}{4} e^{4x}$
$-\ 5x^4$	$\frac{1}{2^4} e^{4x}$
$+\ 20x^3$	$\frac{1}{2^6} e^{4x}$
$-\ 60x^2$	$\frac{1}{2^8} e^{4x}$
$+\ 120x$	$\frac{1}{2^{10}} e^{4x}$
$-\ 120$	$\frac{1}{2^{12}} e^{4x}$

$$\int x^5 e^{4x}\,dx = e^{4x}\left(\frac{1}{4}x^5 - \frac{5}{2^4}x^4 + \frac{5}{2^4}x^3 - \frac{15}{2^6}x^2 + \frac{15}{2^7}x - \frac{15}{2^9}\right) + C$$

$\int \cos 2x\, e^{3x}\,dx$

	$\int \cos 2x$
$+\ e^{3x}$	$\frac{1}{2}\sin 2x$
$-\ 3e^{3x}$	$-\frac{1}{4}\cos 2x$
$+\ 9e^{3x}$	integration

$$\int \cos 2x\, e^{3x}\,dx = e^{3x}\left(\frac{1}{2}\sin 2x + \frac{3}{4}\cos 2x\right) - \frac{9}{4}\int \cos 2x\, e^{3x}\,dx$$

$$\left(1+\frac{9}{4}\right)\int \cos 2x\, e^{3x}\,dx = \frac{e^{3x}}{4}\left(2\sin 2x + 3\cos 2x\right)$$

$$\frac{13}{4}\int \cos 2x\, e^{2x}\,dx = \frac{1}{4}e^{3x}\left(2\sin 2x + 3\cos 2x\right)$$

$$\int \cos 2x\, e^{3x}\,dx = \frac{1}{13}e^{3x}\left(2\sin 2x + 3\cos 2x\right) + C$$

$\int x^6 \cos 4x\, dx$

		$\int \cos 4x\, dx$
$+$	x^6	$\frac{1}{4}\sin 4x$
$-$	$6x^5$	$-\frac{1}{2^4}\cos 4x$
$+$	$30x^4$	$-\frac{1}{2^6}\sin 4x$
$-$	$120x^3$	$\frac{1}{2^8}\cos 4x$
$+$	$360x^2$	$\frac{1}{2^{10}}\sin 4x$
$-$	$720x$	$-\frac{1}{2^{12}}\cos 4x$
$+$	720	$-\frac{1}{2^{14}}\sin 4x$

$$\int x^6 \cos 4x\, dx = \left(\frac{1}{4}x^6 - \frac{15}{2^5}x^4 + \frac{45}{2^7}x^2 - \frac{45}{2^{10}}\right)\sin 4x + \left(\frac{3}{2^3}x^5 - \frac{15}{2^5}x^3 + \frac{15}{2^8}\right)\cos 4x + C$$

$$\int \begin{cases} x^n e^{mx} \\ x^n \cos mx \\ e^{nx} \cos mx \\ \quad\quad\;\; \sin \end{cases}$$

$$\int \sin^4 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1-2\cos 2x + \cos^2 2x)\,dx$$

$$= \frac{1}{4}\int \left(\underbrace{1 - 2\cos 2x + \frac{1}{2}}{} + \frac{1}{2}\cos 4x\right) dx$$

$$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x\right) dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x\right)$$

$$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

$$\int \cos^7 x\,dx = \int \cos^6 x \cos x\,dx \quad (\cos^2 x)^3$$

$$= \int (1-\sin^2 x)^3 \, d(\sin x)$$

$$= \int (1 - 3\sin^2 x + 3\sin^4 x - \sin^6 x)\, d(\sin x)$$

$$= \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$$

$$\int_0^{\pi/2} \cos^{10} x\,dx = \frac{\pi}{2}\cdot\frac{9}{10}\cdot\frac{7}{8}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}$$

$$= \frac{63\pi}{2^9}$$

$$\int_0^{\pi/2} \cos^9 x\,dx = \frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}$$

$$= \frac{2^7}{315}$$

$\int \sqrt{9-4x^2}\,dx$

$2x = 3\sin\theta$ $\quad \sqrt{9-4x^2} = 3\cos\theta$

$dx = \frac{3}{2}\cos\theta\,d\theta$

$$\int \sqrt{9-4x^2}\,dx = \int 3\cos\theta\,\frac{3}{2}\cos\theta\,d\theta$$
$$= \frac{9}{2}\int \cos^2\theta\,d\theta$$
$$= \frac{9}{4}\int (1+\cos 2\theta)\,d\theta$$
$$= \frac{9}{4}\left(\theta + \frac{1}{2}\sin 2\theta\right)$$
$$= \frac{9}{4}\left(\theta + \sin\theta\cos\theta\right)$$
$$= \frac{9}{4}\left(\sin^{-1}\frac{2x}{3} + \frac{2x}{3}\,\frac{\sqrt{9-4x^2}}{3}\right)$$
$$= \frac{9}{4}\sin^{-1}\frac{2x}{3} + \frac{1}{2}x\sqrt{9-4x^2} + C$$

$\int \frac{dx}{\sqrt{x^2-25}}$

$x = 5\sec\theta$ $\quad \sqrt{x^2-25} = 5\tan\theta$

$dx = 5\sec\theta\tan\theta\,d\theta$

$$\int \frac{dx}{\sqrt{x^2-25}} = \int \frac{5\sec\theta\tan\theta\,d\theta}{5\tan\theta}$$
$$= \int \sec\theta\,d\theta$$
$$= \ln|\sec\theta + \tan\theta|$$
$$= \ln\left|\frac{x}{5} + \frac{\sqrt{x^2-25}}{5}\right| + C$$

$$\int \frac{dx}{x^2\sqrt{x^2+36}} \qquad \begin{array}{l} x = 6\tan\theta \\ dx = 6\sec^2\theta\, d\theta \end{array} \qquad \sqrt{x^2+36} = 6\sec\theta$$

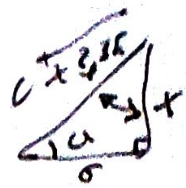
$$\int \frac{dx}{x^2\sqrt{x^2+36}} = \int \frac{6\sec^2\theta\, d\theta}{36\tan^2\theta\,(6\sec\theta)}$$

$$= \frac{1}{36}\int \frac{\sec\theta\, d\theta}{\tan^2\theta}$$

$$= \frac{1}{36}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$= \frac{1}{36}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$$= \frac{1}{36}\int \frac{d(\sin\theta)}{\sin^2\theta}$$



$$= -\frac{1}{36\sin\theta}$$

$$= -\frac{1}{36}\,\frac{\sqrt{x^2+36}}{x} + C$$

sin

cos

$$\int \frac{dx}{x^2-5x+6}$$

$$\frac{1}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$1 = Ax - 3A + Bx - 2B$$

$$x \quad A + B = 0$$

$$x^0 \quad -3A - 2B = 1$$

$$B = 1 \rightarrow A = -1$$

$$\begin{array}{ll} 0A + B & 0 \quad 1 \\ -3A - 2B & 1 \quad -2 \\ B = 1 & \\ A = -1 & \end{array}$$

$$\int \frac{dx}{x^2-5x+6} = -\int \frac{dx}{x-2} + \int \frac{d(x-3)}{x-3}$$

$$= -\ln|x-2| + \ln|x-3| + C$$

$$\int \frac{8x+5}{2x^2+3x+1}\,dx$$

$$\frac{8x+5}{2x^2+3x+1} = \frac{A}{x+1} + \frac{B}{2x+1}$$

$$8x+5 = 2Ax + A + Bx + B \qquad B(x+1)$$

$$\begin{array}{lll} x^1 & 2A + B = 8 & \rightarrow B = 8-6 \\ x^0 & \underline{-A \mp B = 5} & \quad\; = 2 \\ & A = 3 & \end{array}$$

$$\int \frac{8x+5}{2x^2+3x+1}\,dx = 3\int \frac{dx}{x+1} + \int \frac{d(2x+1)}{2x+1}$$

$$= 3\ln|x+1| + \ln|2x+1| + C$$

$$\int_0^\infty \frac{dx}{x^2+1} = \tan^{-1}x \Big|_0^\infty \qquad \rightarrow \tan \frac{\sin}{\cos} = \infty$$

$$= \tan^{-1}\infty - \tan^{-1}0 \qquad \infty = \frac{1}{0_\epsilon}$$

$$= \frac{\pi}{2}$$

$$\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+4)^{3/2}} = \frac{1}{2}\int_{-\infty}^{\infty} (x^2+4)^{-3/2}\, d(x^2+4)$$

$$= -(x^2+4)^{-1/2}\Big|_{-\infty}^{\infty}$$

$$= -\frac{1}{\sqrt{x^2+4}}\Big|_{-\infty}^{\infty}$$

$$= 0$$

$$\int_{-\infty}^{0} x e^{-4x}\,dx$$

$$\begin{array}{c|c} & \int e^{-4x} \\ \hline +x & -\frac{1}{4}e^{-4x} \\ -1 & \frac{1}{16}e^{-4x} \end{array}$$

$$\int_{-\infty}^{0} x e^{-4x}\,dx = \left(-\frac{1}{4}x - \frac{1}{16}\right)e^{-4x}\Big|_{-\infty}^{0}$$

$$= -\frac{1}{16} - \infty$$

$$= -\infty$$

$$y' = (1+y^2)e^x = \frac{dy}{dx}$$

$$\int \frac{dy}{1+y^2} = \int e^x\,dx$$

$$\tan^{-1} y = e^x + C$$

$$y = \tan(e^x + C)$$

$$\tan\theta \frac{dr}{d\theta} + r = \sin^2\theta$$

$$\frac{dr}{d\theta} + \frac{1}{\tan\theta} r = \sin^2\theta \cdot \frac{\cos\theta}{\sin\theta}$$

$$= \sin\theta\cos\theta$$

$$e^{\int \frac{\cos\theta}{\sin\theta}d\theta} = e^{\int \frac{d(\sin\theta)}{\sin\theta}}$$

$$= e^{\ln(\sin\theta)}$$

$$= \sin\theta$$

$$\int \sin^2\theta\cos\theta\,d\theta = \int \sin^2\theta\, d(\sin\theta)$$

$$= \frac{1}{3}\sin^3\theta$$

$$r(\theta) = \frac{1}{\sin\theta}\left(\frac{1}{3}\sin^3\theta + C\right)$$

$$= \frac{1}{3}\sin^2\theta + \frac{C}{\sin\theta}$$

$$\frac{dy}{dt} = \frac{t+2}{y} \qquad y(0)=2$$

$$\int y\,dy = \int (t+2)\,dt$$

$$\frac{1}{2}y^2 = \frac{1}{2}t^2 + 2t + C$$

$$y(0)=2$$

$$\underline{2 = C}$$

$$\frac{1}{2}y^2 = \frac{1}{2}t^2 + 2t + 2$$

$$\underline{y^2 = t^2 + 4t + 4}$$

$$y' + 4xy = x^3 e^{x^2} \qquad y(0) = -1$$

$$e^{\int 4x\,dx} = \underline{e^{2x^2}}$$

$$\int x^3 e^{x^2} e^{2x^2}\,dx = \int x^3 e^{3x^2}\,dx$$

$$= \frac{1}{6}\int x^2 e^{3x^2}\,d(3x^2)$$

$$= \frac{1}{18}\int (3x^2)\, e^{3x^2}\,d(3x^2)$$

u	$\int e^u$
$+u$	e^u
-1	e^u

$u = 3x^2$

$$= \frac{1}{18}\int u e^u\,du$$

$$= \frac{1}{18}(u-1)e^u$$

$$= \underline{\frac{1}{18}(3x^2-1)e^{3x^2}}$$

$$y(x) = \frac{1}{e^{2x^2}}\left(\frac{1}{18}(3x^2-1)e^{3x^2} + C\right)$$

$$= \frac{1}{18}(3x^2-1)e^{x^2} + \frac{C}{e^{2x^2}} \qquad \frac{1}{18} - 1$$

$$-1 = -\frac{1}{18} + C \rightarrow C = -\frac{17}{18}$$

$$\boxed{y(x) = \frac{1}{18}(3x^2-1)e^{x^2} - \frac{17}{18e^{2x^2}}}$$