Lecture One - Trigonometric Functions

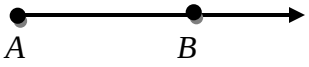
Section 1.1– Angles, Degrees, and Special Triangles

Basic Terminology

Two distinct points determine line *AB*.

Line segment AB: portion of the line between *A* and *B*.

Ray AB: portion of the line *AB* starts at *A* and continues through *B*, and past *B*.



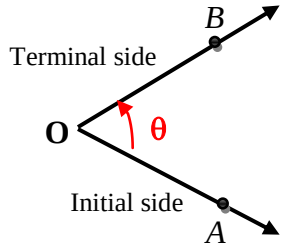
Angles in General

An angle is formed by 2 rays with the same end point.

The two rays are the sides of the angle.

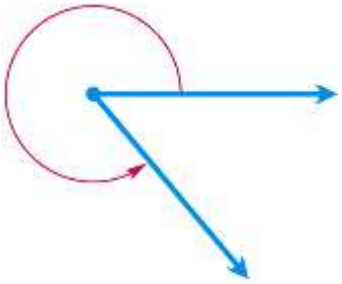
Angle $\theta = AOB$

O is the common endpoint and it is called ***vertex*** of the angle

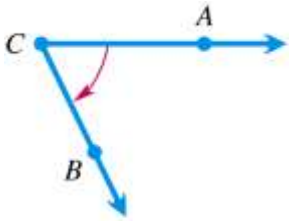


An angle is in a Counterclockwise (*CCW*) direction: positive angle

An angle is in a Clockwise (*CW*) direction: negative angle

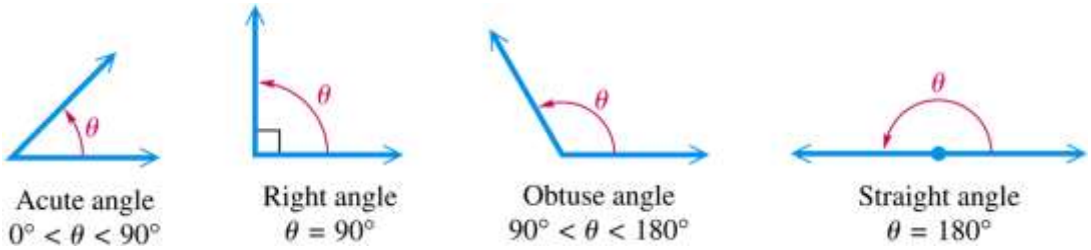


Positive angle



Negative angle

Type of Angles: *Degree*



Complementary angles: $\alpha + \beta = 90°$

Supplementary angles: $\alpha + \beta = 180°$

Example

Give the complement and the supplement of each angle: 40° 110° θ

Solution

a.	40°	Complement: 90° - 40° = 50°	Supplement:180° - 40° = 140°
b.	110°	Complement: 90° - 110° = -20°	Supplement:180° - 110° = 70°
c.	θ	Complement: 90° - θ	Supplement:180° - θ

Degrees, Minutes, Seconds

1°: 1 *degree* $1° = 60'$

1′ : 1 *minute* $1' = 60''$

1″ : 1 *second* $1 = 3600''$

1*full Rotation* or ***Revolution*= 360°**

$1° = 60' = 3600''$ $1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)°$

Example

Add 48° 49′ and 72° 26′

Solution

$$\begin{array}{rr} 48° & 49' \\ +\ 72° & 26' \\ \hline 120° & 75' \end{array}$$

$120°\ 75' = 120°\ 60' + 15'$

$= 121°\ 15'$

Example

Subtract $24° 14'$ and $90°$

Solution

$$\begin{array}{rr} 90° & \\ -24° & 14' \\ \hline \end{array} = \begin{array}{rr} 89° & 60' \\ -24° & 14' \\ \hline 65° & 46' \end{array}$$

Example

Change 27.25° to degrees and minutes

Solution

$$27.25° = 27° + .25°$$
$$= 27° + .25(\mathbf{60'})$$
$$= 27° + 15'$$
$$= 27° \ 15'$$

Angles in Standard Position

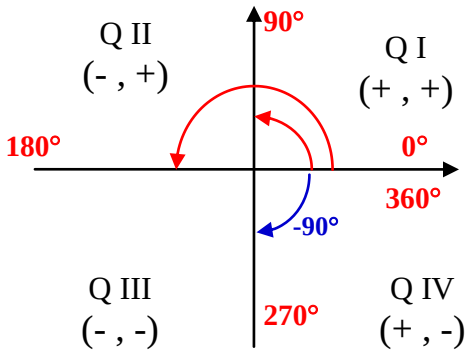
An angle is said to be in standard position if its initial side is along the positive *x*-axis and its vertex is at the origin.

If angle θ is in standard position and the terminal side of θ lies in quadrant I, then we say θ lies in QI

$$\theta \in QI$$

If the terminal side of an angle in standard position lies along one of the axes (*x*-axis or *y*-axis), such as angles with measures 90°, 180°, 270°, then that called a ***quadrantal****angle*.

Two angles in standard position with the same terminal side are called ***coterminal*** *angles*.



Example

Find all angles that are coterminal with 120°.

Solution:

120° + 360°k

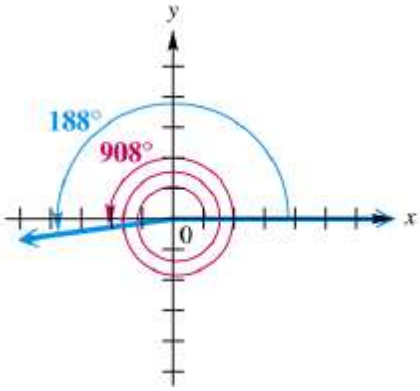
Example

Find the angle of least possible positive measure coterminal with an angle of 908°.

Solution

$908° - 2.360° = 188°$

An angle of 908° is coterminal with an angle of 188°



Example

CD players always spin at the same speed. Suppose a Constant Angular Velocity player makes 480 revolutions per minute. What degrees will a point on the edge of a CD spins for 2 seconds?

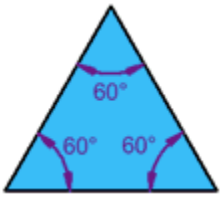
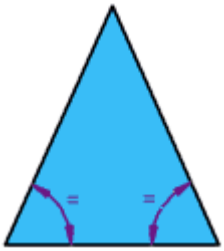
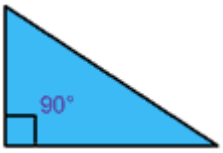
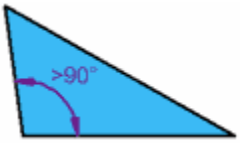
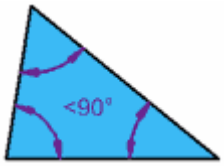
Solution

The player revolves 480 times in one minute $= \frac{480}{1'} = \frac{480}{60} = 8$ *times per* sec.

In 2 sec, the CD will spin: 2.8 = 16 times

Therefore; CD will revolve $16.360° = 5760°$

Triangles

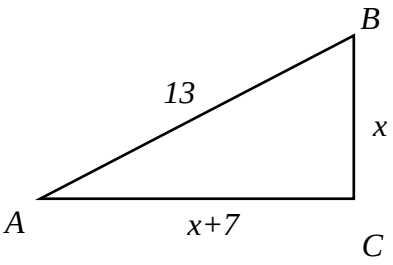
Equilateral – All angles always equal to 60°& all sides are equals 	***Isosceles***: 2 sides and angles are equals 
Scalene: No equal sides or angles 	***Right***: Has a right angle 90°. 
Obtuse: Has an angle more than 90°. 	***Acute***: All angles are less than 90°. 

Pythagorean Theorem

$$C = 90^\circ \Rightarrow c^2 = a^2 + b^2$$

Example

Solve for x in the right triangle



$$x^2 + (x+7)^2 = 13^2$$

$$x^2 + x^2 + 14x + 49 = 169$$

$$2x^2 + 14x + 49 - 169 = 0$$

$$2x^2 + 14x - 120 = 0$$

$$x^2 + 7x - 60 = 0$$

$$x = 5 \quad or \quad x = -12$$

Only $\boxed{x = 5}$ since we can't have -12 for a length

Exercises *Section* 1.1– Angles, Degrees, and Special Triangles

1. Indicate the angle if it is an acute or obtuse. Then give the complement and the supplement of each angle.

a) 10° *b*) 52° *c*) 90° *d*) 120° *e*) 150°

2. Change $10°\ 45'$ to decimal degrees.

3. Convert $34°\ 51'\ 35''$ to decimal degrees.

4. Convert $274°\ 18'\ 59''$ to decimal degrees.

5. Change $74°\ 8'\ 14''$ to decimal degrees to the nearest thousandth.

6. Convert $89.9004°$ to degrees, minutes, and seconds.

7. Convert $34.817°$ to degrees, minutes, and seconds.

8. Convert $122.6853°$ to degrees, minutes, and seconds.

9. Convert $178.5994°$ to degrees, minutes, and seconds.

10. Perform each calculation

a) $51°\ 29' + 32°\ 46'$
b) $90° - 73°12'$
c) $90° - 36°\ 18'\ 47''$
d) $75°\ 15' + 83°\ 32'$

11. Find the angle of least possible positive measure coterminal with an angle of -75°.

12. Find the angle of least possible positive measure coterminal with an angle of -800°.

13. Find the angle of least possible positive measure coterminal with an angle of 270°.

14. A vertical rise of the Forest Double chair lift 1,170 feet and the length of the chair lift as 5,570 feet. To the nearest foot, find the horizontal distance covered by a person riding this lift.

15. A tire is rotating 600 times per minute. Through how many degrees does a point of the edge of the tire move in $\frac{1}{2}$ second?

16. A windmill makes 90 revolutions per minute. How many revolutions does it make per second?