2.5 Graphing Polynomial fctns.

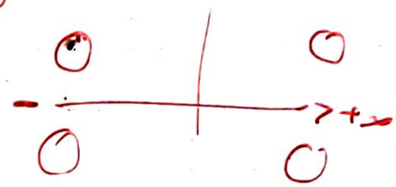
$$f(x) = \underbrace{a_n x^n} + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

Leading Term: $\underbrace{a_n}_{\text{leading Coefficient}} x^n \leftarrow$ degree

$f(x) = a_0 = y$

$f(x) = ax + b = a_1x + a_0$

$a_n x^n$ sign $(-)$ or $(+)$



$a_n > 0$ $x \to \pm\infty \Rightarrow f(x) \to$

$+(-)^n$

n is even $\left\{ \begin{array}{l} x \to +\infty \Rightarrow f(x) \to +\infty \\ x \to -\infty \Rightarrow f(x) \to +\infty \end{array} \right]$

n is odd $\left\{ \begin{array}{l} x \to -\infty \Rightarrow f(x) \to -\infty \\ x \to \infty \Rightarrow f(x) \to +\infty \end{array} \right]$

$a_n < 0$ $-(\pm)^n$ n even

$(+)$

$\left\{ \begin{array}{l} x \to +\infty \Rightarrow f(x) \to -\infty \\ x \to -\infty \quad f(x) \to -\infty \end{array} \right.$

$-(+)^n$ n is odd

$--$

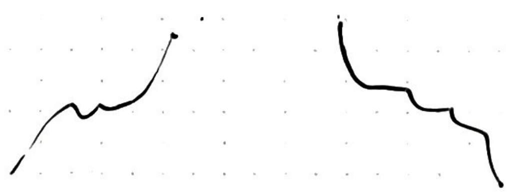
$x \to -\infty \Rightarrow f(x) \to +\infty$

$x \to \infty \Rightarrow f(x) \to -\infty$

Even odd

(+) U

(−) ∩

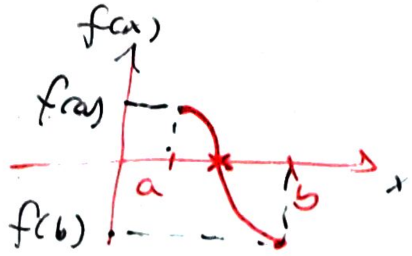


Intermediate Value Theorem

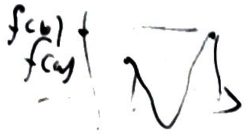
$f(a)$ & $f(b)$

opposite sign

at least 1 real zero



$f(a)$ & $f(b)$ are the same sign, can not be determined



Ex $f(x) = x^3 + x^2 - 6x \qquad a = -4,\ b = -2$

$f(-4) = -64 + 16 + 24 = -24$

$f(-2) = -8 + 4 + 12 = 8$

$f(x)$ has at least 1 real zero between -4 & -2

$a = -1 \qquad b = 3$

$f(-1) = -1 + 1 + 6 = 6$

$f(3) = 27 + 9 - 18 = 18$

can't be determined

sketch

$$f(x) = x^3 + x^2 - 4x - 4$$

Find all values of $x \ni f(x) > 0, \quad f(x) < 0$

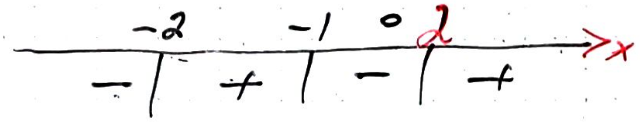
$$x^3 + x^2 - 4x - 4 = 0$$
$$x^2(x+1) - 4(x+1) = 0$$
$$(x+1)(x^2-4) = 0$$
$$x = -1, \pm 2$$

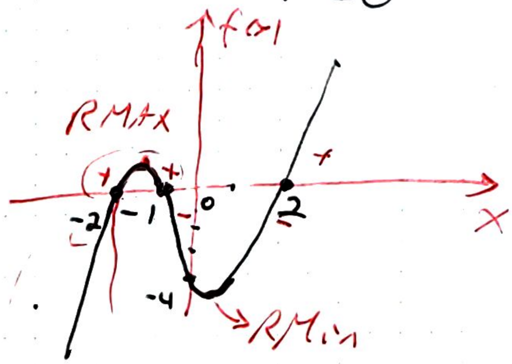
$\left\{\frac{\pm 4}{1}\right\} = \pm\{1, 2, 4\}$

$\times 3$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & \downarrow & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

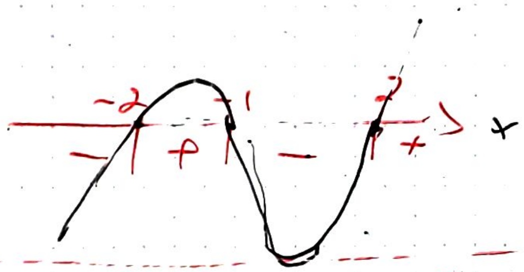
$$x^2 - 4 = 0$$





$f(x) < 0 \Rightarrow (-\infty, -2) \cup (-1, 2)$

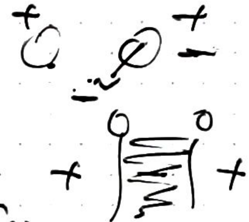
$\quad > 0 \Rightarrow (-2, -1) \cup (2, \infty)$

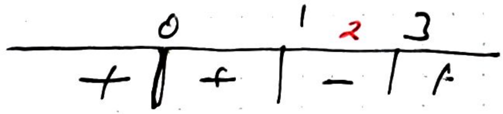


$$f(x) = x^4 - 4x^3 + 3x^2$$

$$x^2(x^2 - 4x + 3) = 0$$

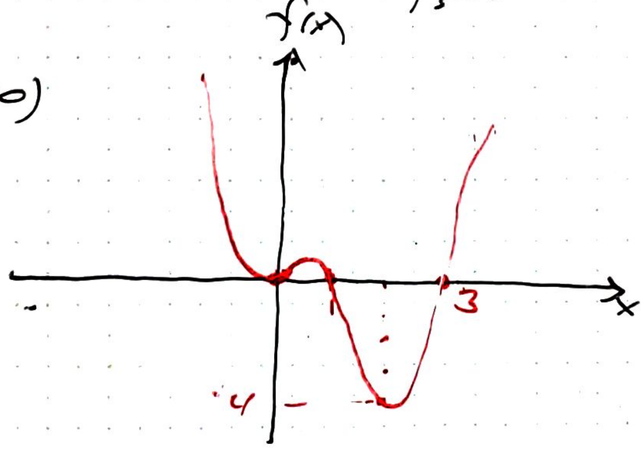
$$\underline{x = 0, 0, 1, 3}$$





$f(x) > 0 \quad (-\infty, 0) \cup (0, 1) \cup (3, \infty)$

$f(x) < 0 \quad (1, 3)$



$\cup$

33 - 91   2.5

#55) $f(x) = x^3 + 3x^2 - 6x - 8$

possibilities: $\{\frac{8}{1}\} = \pm\{1, 2, 4, 8\}$

$$
\begin{array}{r|rrrr}
 & \times 3 & & & \\
+2 & 1 & 3 & -6 & -8 \\
 & & 2 & 10 & 8 \\
\hline
 & 1 & 5 & 4 & \boxed{0}
\end{array}
$$

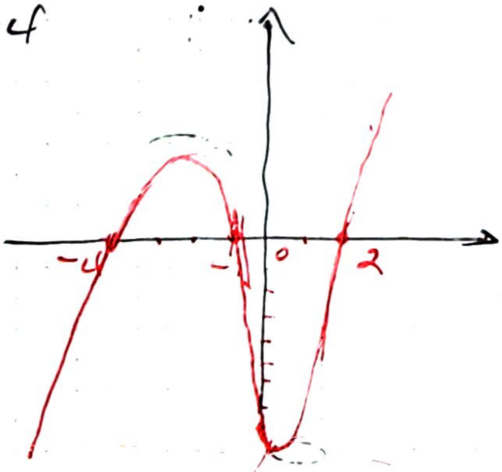
$x^2 + 5x + 4 = 0 \Rightarrow x = -1, -4$

$\boxed{x = -1, -4, 2}$

$$
\begin{array}{ccccccc}
 & -4 & & -1 \;\; 0 & & 2 & \\
\hline
- & | & + & | & - & | & +
\end{array}
$$

$f(x) > 0 \Rightarrow (-4, -1)\ (2, \infty)$

$f(x) < 0 \Rightarrow (-\infty, -4)\ (-1, 2)$



#46 $f(x) = 2x^4 - x^3 - 5x^2 + 2x + 2$

P: $\{\frac{2}{2}\} = \pm\{\frac{1, 2}{1, 2}\} = \pm\{1, \frac{1}{2}, 2\}$

$$
\begin{array}{r|rrrrr}
1 & 2 & -1 & -5 & 2 & 2 \\
 & & 2 & 1 & -4 & -2 \\
\hline
-\frac{1}{2} & 2 & 1 & -4 & -2 & \boxed{0} \\
 & 4 & -1 & 0 & 2 & \\
\hline
 & 2 & 0 & -4 & \boxed{0} &
\end{array}
$$

$2x^3 + x^2 - 4x - 2 = 0$

$\{\frac{2}{2}\} = \pm\{1, 2, \frac{1}{2}\}$

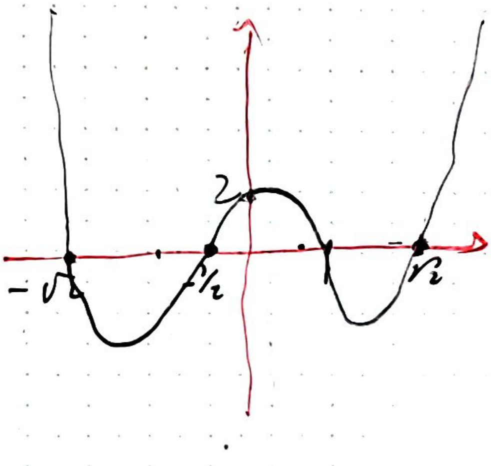
$2x^2 - 4 = 0 \Rightarrow \boxed{x = \pm\sqrt{2}}$

$x = \pm\sqrt{2}, 1, -\frac{1}{2}$

$$
\begin{array}{ccccccccc}
 & -\sqrt{2} & & -\frac{1}{2} \;\; 0 & & 1 & & \sqrt{2} & \\
\hline
+ & | & - & | & + & | & - & | & +
\end{array}
$$

$f(x) > 0 \Rightarrow (-\infty, -\sqrt{2})(-\frac{1}{2}, 1)\ (\sqrt{2}, \infty)$

$f(x) < 0 \Rightarrow (-\sqrt{2}, -\frac{1}{2})\ (1, \sqrt{2})$



#39. $f(x) = 2x^3 + 11x^2 - 7x - 6$

$P: \left\{\frac{6}{2}\right\} = \left\{\frac{1, 2, 3, 6}{1, 2}\right\} = \pm\left\{1, 2, 3, 6, \frac{1}{2}, \frac{3}{2}\right\}$

$$\begin{array}{r|rrrr} 1 & 2 & 11 & -7 & -6 \\ & & 2 & 13 & 6 \\ \hline & 2 & 13 & 6 & \boxed{0} \end{array}$$

$2x^2 + 13x + 6 = 0$

$x = \frac{-13 \pm \sqrt{169 - 48}}{4}$

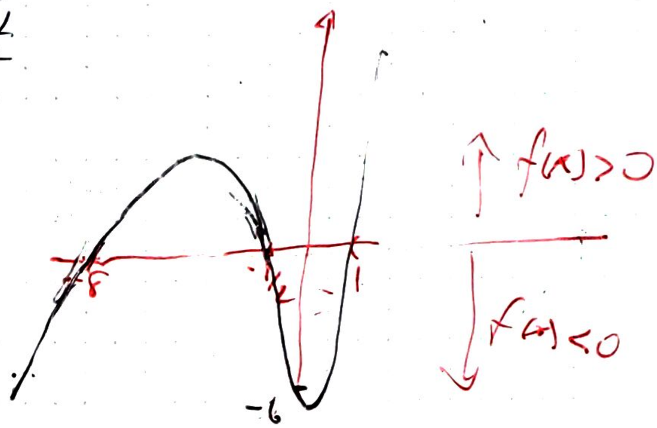
$x = \frac{-13 - 11}{4}$ $= -8$ $x = \frac{-13 + 11}{4}$ $= -\frac{1}{2}$

$\boxed{x = -8, -\frac{1}{2}, 1}$

-8	$-\frac{1}{2}$ 0	1	
$-$	$+$	$-$	$+$

$f(x) < 0 \Rightarrow (-\infty, -8) \; (-\frac{1}{2}, 1)$

$f(x) > 0 \Rightarrow (-8, -\frac{1}{2}) \; (1, \infty)$



2.6 Rational fctns

$$f(x) = \frac{g(x)}{h(x)}$$

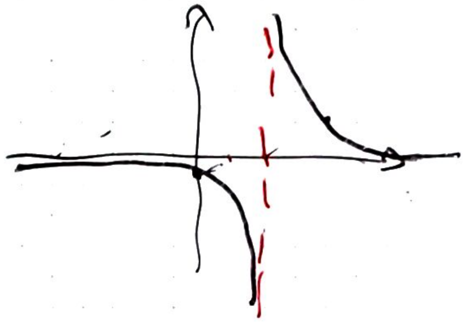
Asymptotes.

Vertical Asymptote (VA) ☺ domain

$$h(x) = 0$$

Ex $f(x) = \frac{1}{x-2}$ Domain $x \neq 2$

VA: $x = 2$



Horizontal Asymptote: (HA)

$$f(x) = \frac{a_n x^n + \cdots}{b_m x^m + \cdots}$$

1- $n < m \Rightarrow$ HA: $y = 0$

Ex $f(x) = \frac{2x+1}{4x^2+5} \Rightarrow \frac{2x}{4x^2} \quad \frac{1}{x}$

HA: $y = 0$

2- $n = m \Rightarrow$ HA: $y = \frac{a_n}{b_m}$

ex: $f(x) = \frac{2x^2+1}{4x^2+5} \Rightarrow$ HA: $y = \frac{1}{2}$

3- $n > m \Rightarrow$ HA: n/a.

↳ oblique Asymptote (OA)

$$y = \frac{2x^3+1}{4x^2+5}$$

OA: $y = \frac{1}{2}x$

$\frac{1}{2}x$

$4x^2+5 \overline{)\, 2x^3+1}$

$-2x^3 \mp \frac{5}{2}x$

$-\frac{5}{2}x + 1$

{VA, hole} {HA, OA}

hole $\Rightarrow \frac{0}{0}$

Ex $g(x) = \dfrac{3x^2+x-4}{2x^2-7x+5} \rightarrow x \neq 1, 5/2$

$= \dfrac{(3x+4)(x-1)}{(2x-5)(x-1)}$

$= \dfrac{3x+4}{2x-5}$

VA : $x = \cancel{1}$ $5/2$

hole: $x = 1 \Rightarrow y = -\frac{7}{3}$ $(1, -\frac{7}{3})$

VA, hole, HA, OA.

Ex $y = \dfrac{3x-1}{x^2-x-6}$

VA: $x = -2, 3$

hole: n/a

HA: $y = 0$

OA: n/a

Ex $f(x) = \dfrac{5x^2+1}{3x^2-4}$

$3x^2 - 4 = 0$

$3x^2 = 4 \Rightarrow x^2 = \frac{4}{3}$

VA: $x = \pm \frac{2}{\sqrt{3}}$

hole: n/a

HA: $y = 5/3$

OA: n/a

Ex $f(x) = \dfrac{2x^4-3x^2+5}{x^2+1}$

VA: n/a

hole: n/a

HA: n/a

OA: $y = 2x^2 - 5$

$$\begin{array}{r} 2x^2 - 5 \\ x^2+1 \overline{\smash{)}\, 2x^4 - 3x^2 + 5} \\ -2x^4 \mp 2x^2 \\ \hline -5x^2 + 5 \end{array}$$

#27 $f(x) = \frac{x+1}{x^2+2x-3}$

VA: $x=1, 3$ HA: $y=0$

hole: n/a O.A: n/a

#30 $f(x) = \frac{x^2+x-2}{x+2} = \frac{(x+2)(x-1)}{x+2} = x-1$

VA: n/a HA: n/a

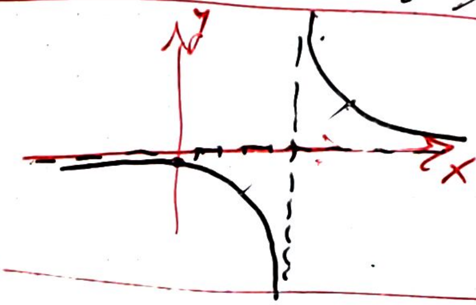
Hole: $x=-2, y=-3$ OA: n/a

$(-2,-3)$

#4 $y = \frac{3}{x-5}$

VA: $x=5$ HA: $y=0$

hole: n/a OA: n/a

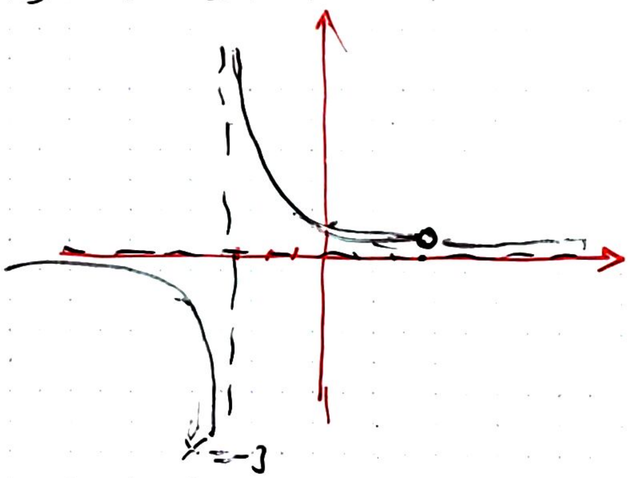


#8 $y = \frac{x-3}{x^2-9} = \frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3}$

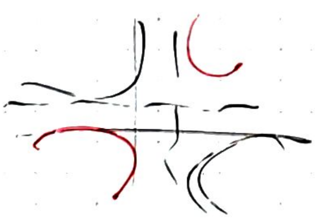
VA: $x = -3$ HA: $y=0$

hole: $x=3$ OA: n/a

$(3, \frac{1}{6})$



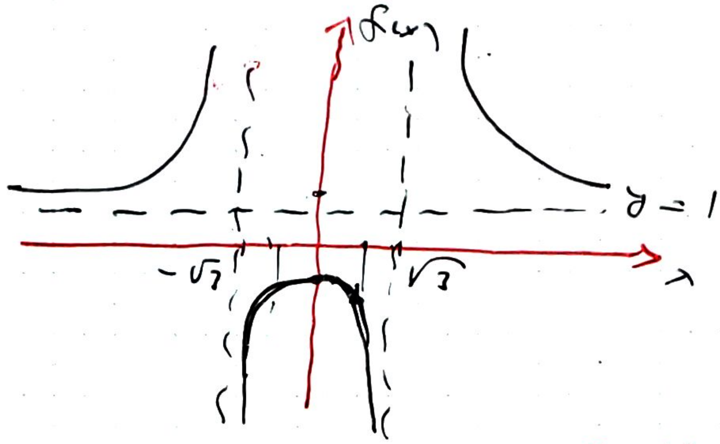
$f(x) = \frac{ax+b}{cx+d}$



42 $f(x)=\frac{x^2+4}{x^2-3}$ ≥ 0 $\frac{5}{-2}$

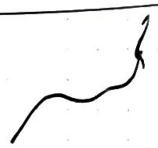
VA: $x=\pm\sqrt{3}$ HA: $y=1$

hole: n/a OA: n/a



$-\sqrt{3}<x<\sqrt{3}$

see 2.5

x^3  $-x^3$ 

61 $f(x)=x^3-2x^2-11x+12$

P: $\left\{\frac{12}{1}\right\}=\pm\{1,2,3,4,6,12\}$

$$\begin{array}{c|cccc} & x^3 & x^2 & x^1 & x^0 \\ 1 & 1 & -2 & -11 & 12 \\ & \downarrow & 1 & -1 & -12 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

$x^2-x-12=0$

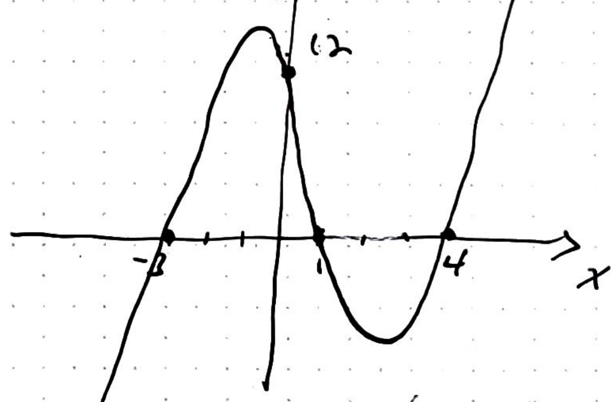
$x=-3,\ 4$

$\boxed{x=1,-3,4}$

$$\begin{array}{cccc} -3 & & 1 & & 4 \\ \hline - & + & - & + \end{array}$$

$f(x)<0 \Rightarrow (-\infty,-3)\ (1,4)$

$f(x)>0 \Rightarrow (-3,1)\ (4,\infty)$



#16 Review (Exam 2)

a) $y = \dfrac{x-2}{x^2-4x+3}$

VA: $x = 1, 3$ HA: $y = 0$

hole: n/a OA: n/a

b) $y = \dfrac{(x+2)(x-1)}{x^2-3x-10} = \dfrac{(x+2)(x-1)}{(x+2)(x-5)} = \dfrac{x-1}{x-5}$ $\underline{-3}$

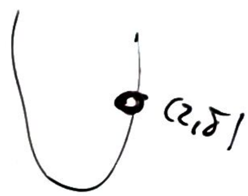
VA: $x = 5$ HA: $y = 1$

hole: $(-2, \frac{3}{7})$ OA: n/a.

c) $y = \dfrac{x^3-2x^2-4x+8}{x-2} = x^2+4$

VA: n/a HA: n/a

hole: $(2, 8)$ OA: n/a



$$\begin{array}{r|rrrr} 2 & 1 & -2 & -4 & 8 \\ & & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

d) $y = \dfrac{-x+1}{-2x^2+5x-3} = \dfrac{-x+1}{(-x+1)(2x-3)} = \dfrac{1}{2x-3}$

VA: $x = \cancel{1}, \frac{3}{2}$ HA: $y = 0$

hole: $(1, -1)$ OA: n/a