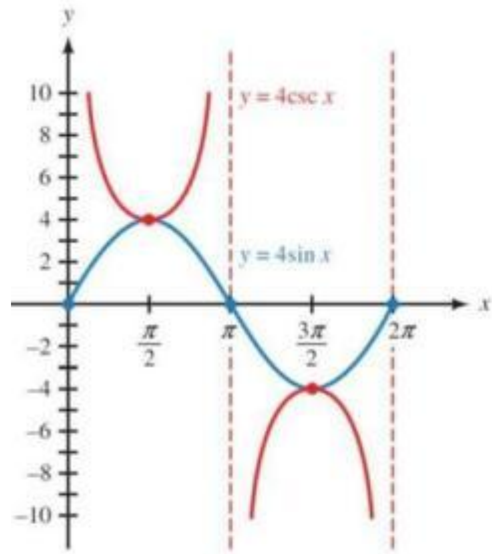
Exercise

Graph one complete cycle $y = 4\csc x$

Solution

Period $= \frac{2\pi}{1} = 2\pi$

First, graph $y = 4\sin x$



Exercise

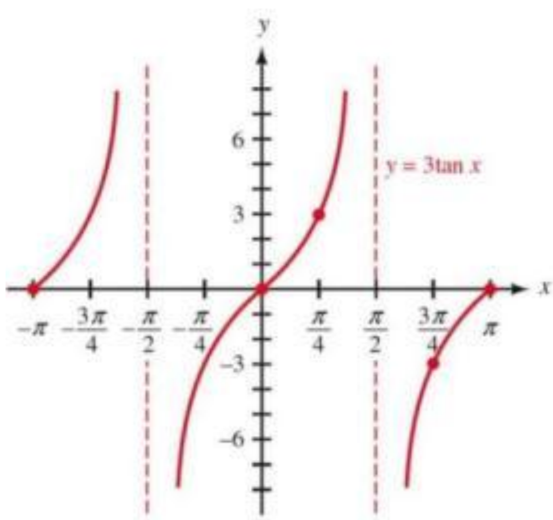
Graph $y = 3\tan x$ *for* $-\pi \le x \le \pi$

Solution

Period = π

One cycle: $0 \le x \le \pi$

x	$y = 3\tan x$
0	0
$\frac{\pi}{4}$	3
$\frac{\pi}{2}$	∞
$\frac{3\pi}{4}$	-3
π	0



Exercise

Graph one complete cycle $y = \frac{1}{2}\cot(-2x)$

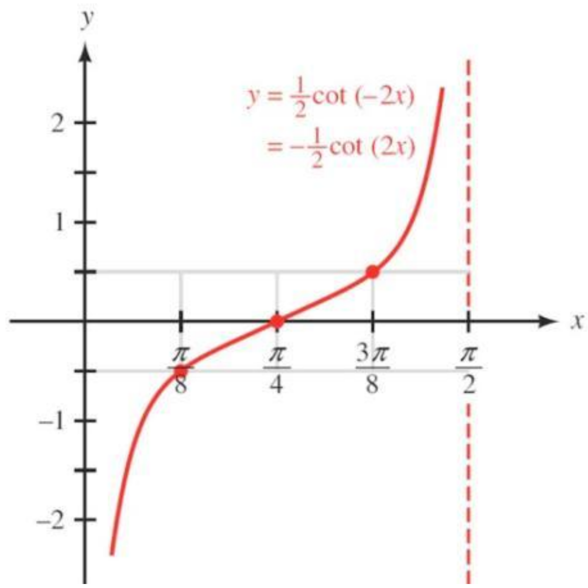
Solution

Period = $\frac{\pi}{2}$

One cycle: $0 \le 2x \le \pi$

$0 \le x \le \frac{\pi}{2}$

x	$y = \frac{1}{2}\cot(-2x)$
0	$-\infty$
$\frac{\pi}{8}$	-0.5
$\frac{\pi}{4}$	0
$\frac{3\pi}{8}$	0.5
$\frac{\pi}{2}$	∞



Exercise

Graph over a 2-period interval $y = 1 - 2\cot 2\left(x + \frac{\pi}{2}\right)$

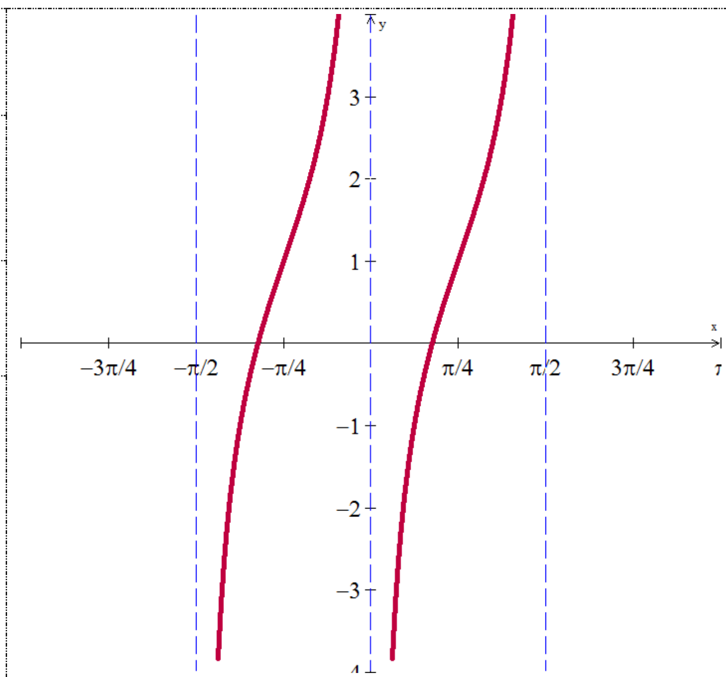
Solution

$P = \frac{\pi}{2}$

$\phi = -\frac{\frac{\pi}{2}}{1} = -\frac{\pi}{2}$

Vertical translation = 1

VA = $-\frac{\pi}{2} + n\pi$



Exercise

Graph over a 2-period interval $y = \frac{2}{3}\tan\left(\frac{3}{4}x - \pi\right) - 2$

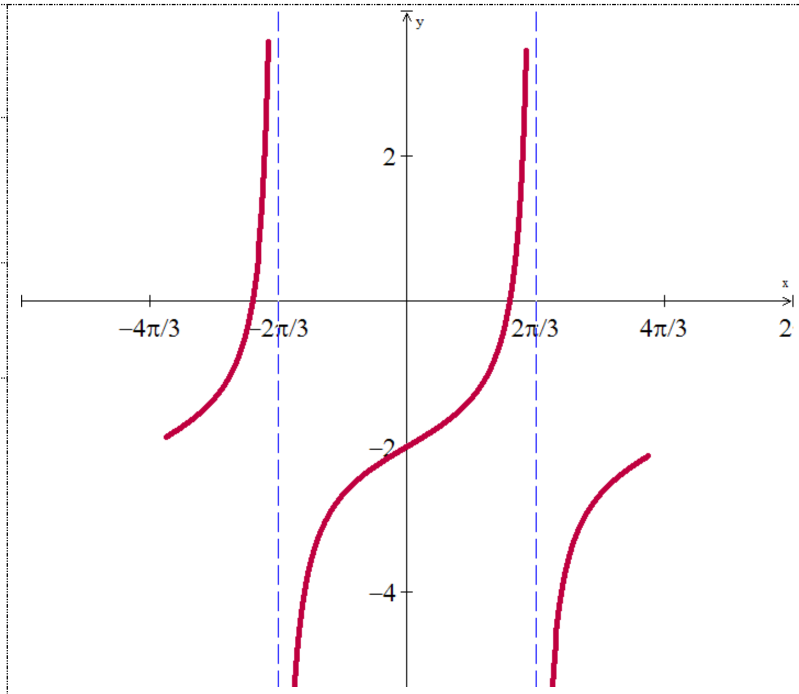
Solution

$$P = \frac{\pi}{\frac{3}{4}} = \frac{3}{4}\pi$$

$$\phi = -\frac{\pi}{\frac{3}{4}} = -\frac{4\pi}{3}$$

Vertical translation = -2

VA = $\frac{3\pi}{4} + n\pi$



Exercise

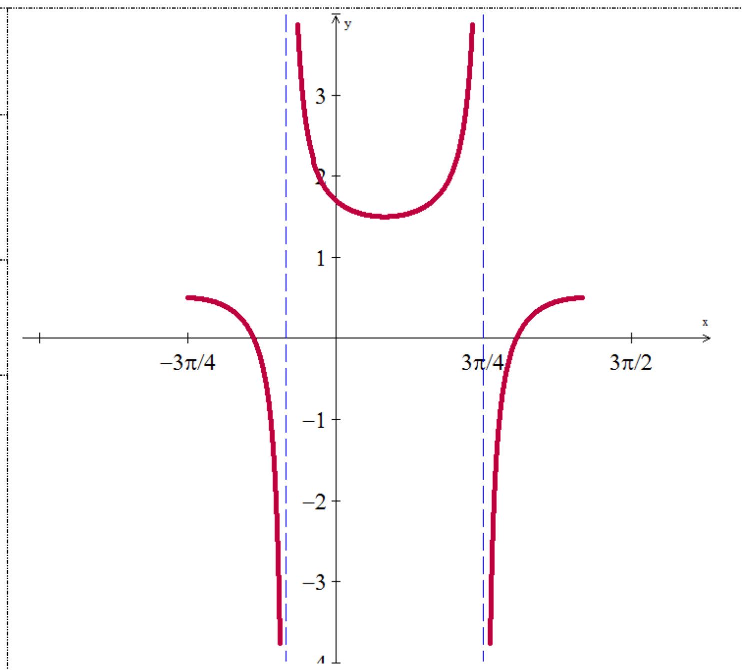
Graph over a one-period interval $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$

Solution

$$P = 2\pi$$

$$\phi = \frac{3\pi}{4}$$

Vertical translation = 1



Exercise

Graph over a one-period interval $y = 2 + \frac{1}{4}\sec\left(\frac{1}{2}x - \pi\right)$

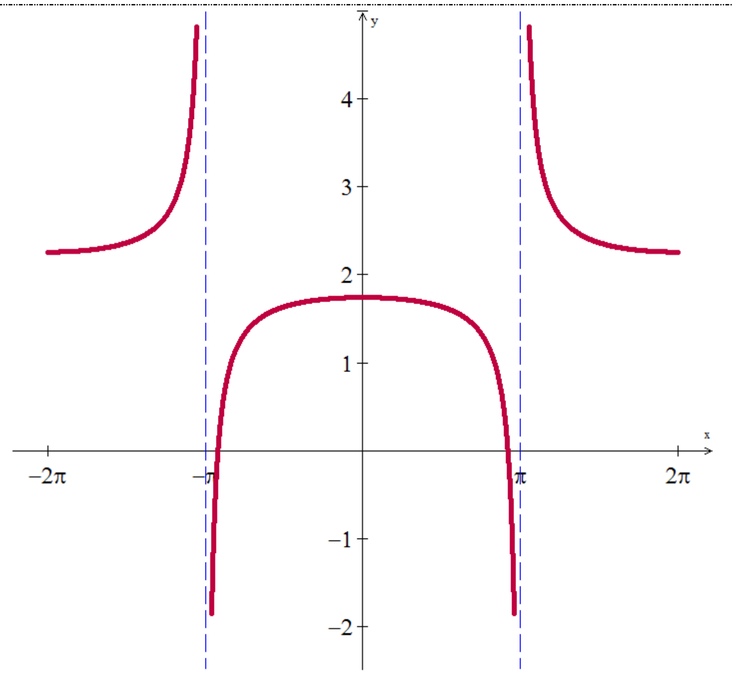
Solution

$P = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

$\phi = \dfrac{\pi}{\frac{1}{2}} = 2\pi$

Vertical translation = 1

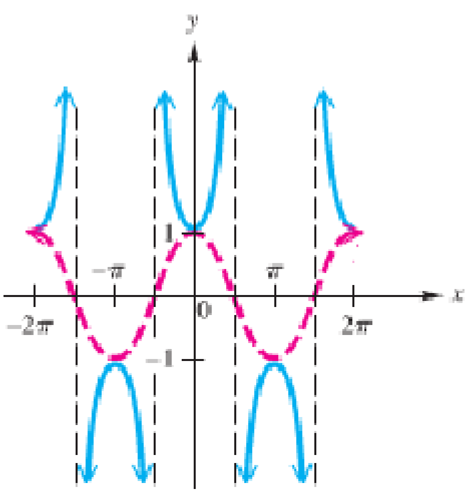
VA = $\dfrac{3\pi}{4} + 2n\pi$



Exercise

Find an equation to match the graph

Solution

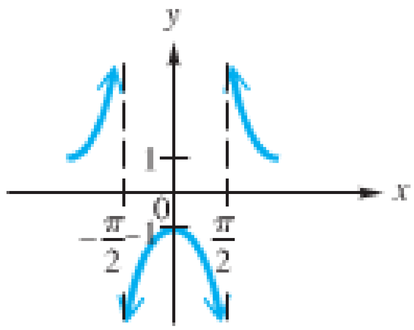


$P = 2\pi$

$\phi = 0$

$A = \dfrac{1+1}{2} = 1$

$y = \sec x \quad -2\pi \le x \le 2\pi$

<table>
<tr><td rowspan="4"></td><td>$B=\frac{2\pi}{P}=\frac{2\pi}{2\pi}=1$</td></tr>
<tr><td>$\phi=0\to C=0$</td></tr>
<tr><td>$A=\frac{1+1}{2}=1$</td></tr>
<tr><td>$y=-\sec(x)\quad -2\pi\le x\le 2\pi$</td></tr>
</table>

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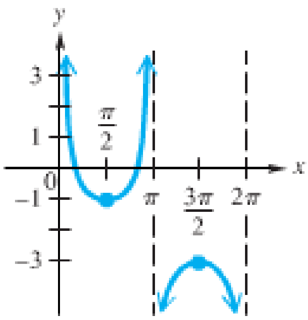
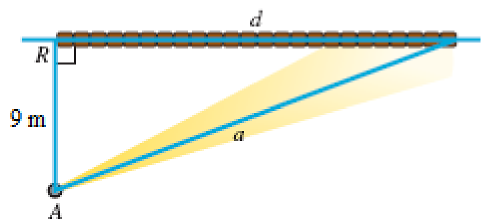
<tr><td rowspan="4"></td><td>$B=\frac{2\pi}{P}=\frac{2\pi}{2\pi}=1$</td></tr>
<tr><td>$\phi=0\to C=0$</td></tr>
<tr><td>$A=\frac{-3-1}{2}=-2$</td></tr>
<tr><td>$y=-2+\csc(x)\quad -2\pi\le x\le 2\pi$</td></tr>
</table>

Exercise

A rotating beacon is located at point A next to a long wall. The beacon is 9 m from the wall. The distance ***a*** is given by $a=9\left|\sec 2\pi t\right|$, where t is time measured in seconds since the beacon started rotating. (When t = 0, the beacon is aimed at point R.) Find ***a*** for t = 0.45



Solution

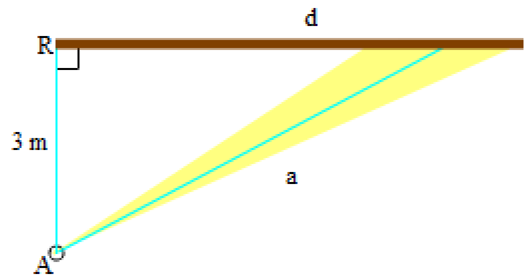
$$a=9\left|\sec\left(2\pi(0.45)\right)\right|$$

$$=\frac{9}{\left|\cos\left(2\pi(0.45)\right)\right|}$$

$$\approx 9.5\ m$$

Exercise

A rotating beacon is located 3 m south of point R on an east-west wall. d, the length of the light display along the wall from R, is given by $d = 3\tan 2\pi t$, where t is time measured in seconds since the beacon started rotating. (When $t = 0$, the beacon is aimed at point R. When the beacon is aimed to the right of R, the value of d is positive; d is negative if the beacon is aimed to the left of R.) Find a for $t = 0.8$



Solution

$$d = 3\tan\left(2\pi(0.8)\right)$$

$$\approx -9.23\ m$$

Exercise

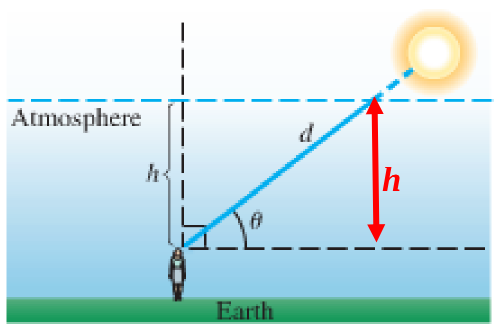
The shortest path for the sun's rays through Earth's atmosphere occurs when the sun is directly overhead. Disregarding the curvature of Earth, as the sun moves lower on the horizon, the distance that sunlight passes through the atmosphere increases by a factor of $\csc\theta$, where θ is the angle of elevation of the sun. This increased distance reduces both the intensity of the sun and the amount of ultraviolet light that reached Earth's surface.

a) Verify that $d = h\csc\theta$

b) Determine θ when $d = 2h$

c) The atmosphere filters out the ultraviolet light that causes skin to burn, Compare the difference between sunbathing when $\theta = \frac{\pi}{2}$ and when $\theta = \frac{\pi}{3}$. Which measure gives less ultraviolet light?

Solution



a) $\sin\theta = \frac{h}{d} = \frac{1}{\csc\theta}$

$d = h\csc\theta$ (***cross-multiplication***)

b) $\sin\theta = \frac{h}{d} = \frac{h}{2h} = \frac{1}{2}$

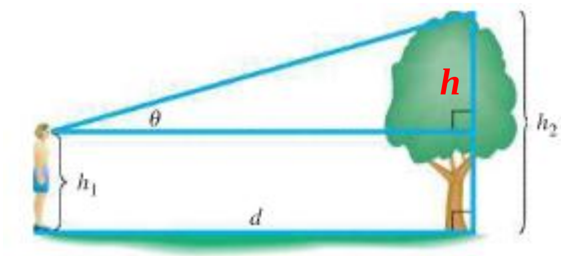
$$\theta = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

c) $$\begin{cases} \csc\frac{\pi}{2} = 1 \\ \csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3} \approx 1.15 \end{cases}$$

When the distance to the sun is lager $\left(\theta = \frac{\pi}{3}\right)$, there is less ultraviolet light reaching the earth's surface. In this case, sunlight passes through 15% more atmosphere.

Exercise

Let a person whose eyes are h_1 feet from the ground stand ***d*** feet from an object h_1 feet tall, where $h_2 > h_1$ feet. Let θ be the angle of elevation to the top of the object.



a) Show that $d = (h_2 - h_1)\cot\theta$

b) Let $h_2 = 55$ and $h_1 = 5$. Graph ***d*** for the interval $0 < \theta \le \frac{\pi}{2}$

Solution

a) $h = h_2 - h_1$

$\cot\theta = \dfrac{d}{h}$

$d = (h_2 - h_1)\cot\theta$

b) $d = (55 - 5)\cot\theta$

$d = 50\cot\theta \quad 0 < \theta \le \frac{\pi}{2}$