

Solution **Section 1.1 – The Binomial Theorem**

Exercise

Find the fifth term in the expansion $(x^3 + \sqrt{y})^{13}$

Solution

$$\begin{aligned}\binom{13}{4}(x^3)^9(\sqrt{y})^4 &= \frac{13!}{4!(13-4)!}x^{27}y^2 \\ &= \underline{715x^{27}y^2} \quad | \end{aligned}$$

Exercise

Find the term involving q^{10} in the binomial expansion $(\frac{1}{3}p + q^2)^{12}$

Solution

Given: $a = \frac{1}{3}p, \quad b = q^2, \quad n = 12$

$$q^{10} = (q^2)^5 = b^5$$

$$\begin{aligned}\binom{n}{k}a^{n-k}b^k &= \binom{12}{5}\left(\frac{1}{3}p\right)^{12-5}\left(q^2\right)^5 \\ &= \frac{12!}{5!(12-5)!}\left(\frac{1}{3}p\right)^7q^{10} \\ &= \underline{\frac{88}{243}p^7q^{10}} \quad | \end{aligned}$$

Exercise

Use the binomial theorem to expand and simplify: $(4x - y)^3$

Solution

$$\begin{aligned}(4x - y)^3 &= \binom{3}{0}(4x)^3(-y)^0 + \binom{3}{1}(4x)^2(-y)^1 + \binom{3}{2}(4x)^1(-y)^2 + \binom{3}{3}(4x)^0(-y)^3 \\ &= 64x^3 + 3(16x^2)(-y) + 3(4x)y^2 - y^3 \\ &= \underline{64x^3 - 48x^2y + 12xy^2 - y^3} \quad | \end{aligned}$$

Exercise

Use the binomial theorem to expand and simplify: $(x + y)^6$

Solution

$$(x + y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

Exercise

Use the binomial theorem to expand and simplify: $(a - b)^6$

Solution

$$(a - b)^6 = a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

Exercise

Use the binomial theorem to expand and simplify: $(x - y)^7$

Solution

$$(x - y)^7 = x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$$

Exercise

Use the binomial theorem to expand and simplify: $(a + b)^8$

Solution

$$(a + b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$$

Exercise

Use the binomial theorem to expand and simplify: $(3t - 5x)^4$

Solution

$$\begin{aligned}(3t - 5x)^4 &= (3t)^4 + 4(3t)^3(-5x)^1 + 6(3t)^2(-5x)^2 + 4(3t)^1(-5x)^3 + (-5x)^4 \\ &= \underline{81t^4 - 540t^3x + 1350t^2x^2 - 1500tx^3 + 625x^4}\end{aligned}$$

Exercise

Use the binomial theorem to expand and simplify: $\left(\frac{1}{3}x + y^2\right)^5$

Solution

$$\begin{aligned}\left(\frac{1}{3}x + y^2\right)^5 &= \left(\frac{1}{3}x\right)^5 + 5\left(\frac{1}{3}x\right)^4 y^2 + 10\left(\frac{1}{3}x\right)^3 (y^2)^2 + 10\left(\frac{1}{3}x\right)^2 (y^2)^3 + 5\frac{1}{3}x(y^2)^4 + (y^2)^5 \\ &= \frac{1}{243}x^5 + \frac{5}{81}x^4 y^2 + \frac{10}{27}x^3 y^4 + \frac{10}{9}x^2 y^6 + \frac{5}{3}xy^8 + y^{10} \quad | \end{aligned}$$

Exercise

Use the binomial theorem to expand and simplify: $\left(\frac{1}{x^2} + 3x\right)^6$

Solution

$$\begin{aligned}\left(\frac{1}{x^2} + 3x\right)^6 &= \left(x^{-2} + 3x\right)^6 \\ &= \left(x^{-2}\right)^6 + 6\left(x^{-2}\right)^5 3x + 15\left(x^{-2}\right)^4 (3x)^2 + 20\left(x^{-2}\right)^3 (3x)^3 \\ &\quad + 15\left(x^{-2}\right)^2 (3x)^4 + 15x^{-2} (3x)^5 + (3x)^6 \\ &= x^{-12} + 18x^{-9} + 135x^{-6} + 540x^{-3} + 1215 + 1458x^3 + 729x^6 \quad | \end{aligned}$$

Exercise

Use the binomial theorem to expand and simplify: $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^5$

Solution

$$\begin{aligned}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^5 &= \left(x^{1/2} + x^{-1/2}\right)^5 \\ &= \left(x^{1/2}\right)^5 + 5\left(x^{1/2}\right)^4 x^{-1/2} + 10\left(x^{1/2}\right)^3 \left(x^{-1/2}\right)^2 + 10\left(x^{1/2}\right)^2 \left(x^{-1/2}\right)^3 \\ &\quad + 5x^{1/2} \left(x^{-1/2}\right)^4 + \left(x^{-1/2}\right)^5 \\ &= x^{5/2} + 5x^2 x^{-1/2} + 10x^{3/2} x^{-1} + 10xx^{-3/2} + 5x^{1/2} x^{-2} + x^{-5/2} \\ &= x^{5/2} + 5x^{3/2} + 10x^{1/2} + 10x^{-1/2} + 5x^{-3/2} + x^{-5/2} \quad | \end{aligned}$$

ExerciseExpand and simplify: $(2y - 3)^4$ **Solution**

$$\begin{aligned}
 (2y - 3)^4 &= (2y)^4 + 4(2y)^3(-3) + 6(2y)^2(-3)^2 + 4(2y)(-3)^3 + (-3)^4 \\
 &= 16y^4 - 96y^3 + 216y^2 - 216y + 81
 \end{aligned}$$

ExerciseExpand and simplify: $(x + 2)^5$ **Solution**

$$\begin{aligned}
 (x + 2)^5 &= x^5 + 5x^4(2) + 10x^3(2)^2 + 10x^2(2)^3 + 5x(2)^4 + (2)^5 \\
 &= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32
 \end{aligned}$$

ExerciseExpand and simplify: $(x^2 - y^2)^6$ **Solution**

$$\begin{aligned}
 (x^2 - y^2)^6 &= (x^2)^6 + 6(x^2)^5(-y^2) + 15(x^2)^4(-y^2)^2 + 20(x^2)^3(-y^2)^3 \\
 &\quad + 15(x^2)^2(-y^2)^4 + 15(x^2)(-y^2)^5 + (-y^2)^6 \\
 &= x^{12} - 6x^{10}y^2 + 15x^8y^4 - 20x^6y^6 + 15x^4y^8 - 15x^2y^{10} + y^{12}
 \end{aligned}$$

ExerciseExpand and simplify: $(ax - by)^4$ **Solution**

$$\begin{aligned}
 (ax - by)^4 &= (ax)^4 + 4(ax)^3(-by) + 6(ax)^2(-by)^2 + 4(ax)(-by)^3 + (-by)^4 \\
 &= a^4x^4 - 4a^3x^3by + 6a^2x^2b^2y^2 - 4axb^3y^3 + b^4y^4
 \end{aligned}$$

ExerciseExpand and simplify: $(ax + by)^5$ **Solution**

$$\begin{aligned}(ax + by)^5 &= (ax)^5 + 5(ax)^4(by) + 10(ax)^3(by)^2 + 10(ax)^2(by)^3 + 5(ax)(by)^4 + (by)^5 \\ &= \underline{a^5x^5 + 5a^4x^4by + 10a^3x^3b^2y^2 + 10a^2x^2b^3y^3 + 5axb^4y^4 + b^5y^5}\end{aligned}$$

ExerciseExpand and simplify: $(\sqrt{x} - \sqrt{3})^4$ **Solution**

$$\begin{aligned}(\sqrt{x} - \sqrt{3})^4 &= (\sqrt{x})^4 + 4(\sqrt{x})^3(-\sqrt{3}) + 6(\sqrt{x})^2(-\sqrt{3})^2 + 4(\sqrt{x})(-\sqrt{3})^3 + (-\sqrt{3})^4 \\ &= \underline{x^2 - 4x\sqrt{3x} + 18x^2 - 13\sqrt{3x} + 9}\end{aligned}$$

ExerciseExpand and simplify: $(\sqrt{x} - \sqrt{2})^6$ **Solution**

$$\begin{aligned}(\sqrt{x} - \sqrt{2})^6 &= (\sqrt{x})^6 + 6(\sqrt{x})^5(-\sqrt{2}) + 15(\sqrt{x})^4(-\sqrt{2})^2 + 20(\sqrt{x})^3(-\sqrt{2})^3 \\ &\quad + 15(\sqrt{x})^2(-\sqrt{2})^4 + 15(\sqrt{x})(-\sqrt{2})^5 + (-\sqrt{2})^6 \\ &= \underline{x^3 - 6x^2\sqrt{2x} + 30x^2 - 40x\sqrt{2x} + 60x - 60\sqrt{2x} + 8}\end{aligned}$$

ExerciseExpand and simplify: $(2x - 1)^{12}$ **Solution**

$$\begin{aligned}(2x - 1)^{12} &= (2x)^{12} + 12(2x)^{11}(-1) + 66(2x)^{10}(-1)^2 + 240(2x)^9(-1)^3 + 535(2x)^8(-1)^4 \\ &\quad + 812(2x)^7(-1)^5 + 924(2x)^6(-1)^6 + 812(2x)^5(-1)^7 + 535(2x)^4(-1)^8 \\ &\quad + 240(2x)^3(-1)^9 + 66(2x)^2(-1)^{10} + 12(2x)(-1)^{11} + (-1)^{12} \\ &= \underline{4096x^{12} - 24576x^{11} + 67584x^{10} - 122880x^9 + 136960x^8 - 103936x^7} \\ &\quad + 59136x^6 - 25984x^5 + 8560x^4 - 1920x^3 + 264x^2 - 24x + 1\end{aligned}$$

Exercise

Expand and simplify: $\left(x - \frac{1}{x^2}\right)^9$

Solution

$$\begin{aligned}\left(x - \frac{1}{x^2}\right)^9 &= x^9 + 9x^8\left(-\frac{1}{x^2}\right) + 36x^7\left(-\frac{1}{x^2}\right)^2 + 84x^6\left(-\frac{1}{x^2}\right)^3 + 126x^5\left(-\frac{1}{x^2}\right)^4 + 126x^4\left(-\frac{1}{x^2}\right)^5 \\ &\quad + 84x^3\left(-\frac{1}{x^2}\right)^6 + 36x^2\left(-\frac{1}{x^2}\right)^7 + 9x\left(-\frac{1}{x^2}\right)^8 + \left(-\frac{1}{x^2}\right)^9 \\ &= \underline{x^9 - 9x^6 + 36x^3 - 84 + 126x^{-3} - 126x^{-6} + 84x^{-9} - 36x^{-12} + 9x^{-15} - x^{-18}}\end{aligned}$$

Exercise

Expand and simplify: $\left(\frac{2}{x} - 3y\right)^5$

Solution

$$\begin{aligned}\left(\frac{2}{x} - 3y\right)^5 &= \left(\frac{2}{x}\right)^5 + 5\left(\frac{2}{x}\right)^4(-3y) + 10\left(\frac{2}{x}\right)^3(-3y)^2 + 10\left(\frac{2}{x}\right)^2(-3y)^3 + 5\left(\frac{2}{x}\right)(-3y)^4 + (-3y)^5 \\ &= \underline{\frac{32}{x^5} - 240\frac{y}{x^4} + 720\frac{y^2}{x^3} - 1,080\frac{y^3}{x^2} + 810\frac{y^4}{x} - 243y^5}\end{aligned}$$

Exercise

Expand and simplify: $\left(3\sqrt{x} + \sqrt[4]{x}\right)^4$

Solution

$$\begin{aligned}\left(3\sqrt{x} + \sqrt[4]{x}\right)^4 &= \left(3\sqrt{x}\right)^4 + 4\left(3\sqrt{x}\right)^3\left(\sqrt[4]{x}\right) + 6\left(3\sqrt{x}\right)^2\left(\sqrt[4]{x}\right)^2 + 4\left(3\sqrt{x}\right)\left(\sqrt[4]{x}\right)^3 + \left(\sqrt[4]{x}\right)^4 \\ &= 81x^2 + 108x^{3/2}x^{1/4} + 54x\sqrt{x} + 12x^{1/2}x^{3/4} + x \\ &= 81x^2 + 108x^{7/4} + 54x\sqrt{x} + 12x^{5/4} + x \\ &= \underline{81x^2 + 108x\sqrt[4]{x^3} + 54x\sqrt{x} + 12x\sqrt[4]{x} + x}\end{aligned}$$

Exercise

Expand and simplify: $(x+1)^5$

Solution

$$(x+1)^5 = \underline{x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1}$$

Exercise

Expand and simplify: $(x-1)^5$

Solution

$$(x-1)^5 = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$$

Exercise

Expand and simplify: $(x-2)^6$

Solution

$$(x-2)^6 = x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$$

Exercise

Expand and simplify: $\left(\frac{1}{x^3} - 2x\right)^5$

Solution

$$\begin{aligned} \left(\frac{1}{x^3} - 2x\right)^5 &= \frac{1}{x^{15}} - 10\frac{x}{x^{12}} + 10\frac{4x^2}{x^9} - 10\frac{8x^3}{x^6} + 5\frac{16x^4}{x^3} - 32x^5 \\ &= \frac{1}{x^{15}} - \frac{10}{x^{11}} + \frac{40}{x^7} - \frac{80}{x^3} + 80x - 32x^5 \end{aligned}$$

Exercise

Expand and simplify: $\left(\frac{1}{x} - 2x\right)^6$

Solution

$$\begin{aligned} \left(\frac{1}{x} - 2x\right)^6 &= \frac{1}{x^6} - 6\frac{1}{x^5}(2x) + 15\frac{1}{x^4}(2x)^2 - 20\frac{1}{x^3}(2x)^3 + 15\frac{1}{x^2}(2x)^4 - 6\frac{1}{x}(2x)^5 + (2x)^6 \\ &= \frac{1}{x^6} - \frac{12}{x^4} + \frac{60}{x^2} - 160 + 240x^2 - 192x^4 + 64x^6 \end{aligned}$$

Exercise

Expand and simplify: $(x^2 - 2y)^5$

Solution

$$\underline{(x^2 - 2y)^5 = x^{10} - 10x^8y + 40x^6y^2 - 80x^4y^3 + 80x^2y^4 - 32y^5}$$

Exercise

Expand and simplify: $\left(\frac{2}{x} + 3\sqrt{x}\right)^4$

Solution

$$\begin{aligned}\left(\frac{2}{x} + 3\sqrt{x}\right)^4 &= \frac{16}{x^4} + \frac{32}{x^3}(3\sqrt{x}) + \frac{24}{x^2}(9x) + \frac{8}{x}(27x\sqrt{x}) + 81x^2 \\ &= \frac{16}{x^4} + \frac{96\sqrt{x}}{x^3} + \frac{216}{x} + 216\sqrt{x} + 81x^2\end{aligned}$$

Exercise

Expand and simplify: $(2x + 5y)^7$

Solution

$$\begin{aligned}(2x + 5y)^7 &= 128x^7 + 7(64x^6)(5y) + 21(32x^5)(25y^2) + 35(16x^4)(125y^3) \\ &\quad + 35(8x^3)(625y^4) + 21(4x^2)(3,125y^5) + 7(2x)(5^6y^6) + (5y)^7 \\ &= 128x^7 + 320x^6y + 16,800x^5y^2 + 70,000x^4y^3 + 175,000x^3y^4 + 262,500x^2y^5 \\ &\quad + 218,750xy^6 + 78,125y^7\end{aligned}$$

Exercise

Expand and simplify: $(2x - 3)^{11}$

Solution

$$\begin{aligned}(2x - 3)^{11} &= (2x)^{11} - 33(2x)^{10} + 495(2x)^9 - 4,995(2x)^8 + 350(3)^4(2x)^7 - 462(3)^5(2x)^6 \\ &\quad + 462(3)^6(2x)^5 - 350(3)^7(2x)^4 + 185(3)^8(2x)^3 - 55(3)^9(2x)^2 + 22(3)^{10}x - 3^{11} \\ &= 2,048x^{11} - 33,792x^{10} + 253,440x^9 - 1,278,720x^8 + 3,628,800x^7 - 7,185,024x^6 \\ &\quad + 462(3)^6 2^5x^5 - 5,600(3)^7x^4 + 1,480(3)^8x^3 - 220(3)^9x^2 + 22(3)^{10}x - 3^{11}\end{aligned}$$

Exercise

Expand and simplify: $(2x - 3y)^6$

Solution

$$(2x - 3y)^6 = 64x^6 - 576x^5y + 2,160x^4y^2 - 4,320x^3y^3 + 4,860x^2y^4 - 2,196xy^5 + 729y^6$$

Exercise

Expand and simplify: $(2x + 3y)^5$

Solution

$$(2x + 3y)^5 = 32x^5 + 240x^4y + 720x^3y^2 + 1,080x^2y^3 + 810xy^4 + 243y^5$$

Exercise

Expand and simplify: $(3x - 2y)^4$

Solution

$$(3x - 2y)^4 = 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$$

Exercise

Expand and simplify: $(x^2 + y^3)^3$

Solution

$$(x^2 + y^3)^3 = x^6 + 3x^4y^3 + 3x^2y^6 + y^9$$

Exercise

Expand and simplify: $(x^2 - y^2)^3$

Solution

$$(x^2 - y^2)^3 = x^6 - 3x^4y^2 + 3x^2y^4 - y^6$$

Exercise

Expand and simplify: $(2+i)^6$

Solution

$$\begin{aligned}(2+i)^6 &= 64 + 6(32)i + 15(16)i^2 + 20(8)i^3 + 15(4)i^4 + 12i^5 + i^6 \\ &= 64 + 192i - 240 - 160i + 60 + 12i - 1 \\ &= \underline{-117 + 44i} \quad | \end{aligned}$$

Exercise

Expand and simplify: $(2-i)^6$

Solution

$$\begin{aligned}(2-i)^6 &= 64 - 6(32)i + 15(16)i^2 - 20(8)i^3 + 15(4)i^4 - 12i^5 + i^6 \\ &= 64 - 192i - 240 + 160i + 60 - 12i - 1 \\ &= \underline{-117 - 44i} \quad | \end{aligned}$$

Exercise

Expand and simplify: $(\sqrt{2}+i)^5$

Solution

$$\begin{aligned}(\sqrt{2}+i)^5 &= 2\sqrt{2} + 20i + 20\sqrt{2}i^2 + 20i^3 + \sqrt{2}i^4 + i^5 \\ &= 2\sqrt{2} + 20i - 20\sqrt{2} - 20i + \sqrt{2} + i \\ &= \underline{-17\sqrt{2} + i} \quad | \end{aligned}$$

Exercise

Expand and simplify: $(3-i)^4$

Solution

$$\begin{aligned}(3-i)^4 &= 84 - 108i + 54i^2 - 12i^3 + i^4 \\ &= 84 - 108i - 54 + 12i + 1 \\ &= \underline{31 - 96i} \quad | \end{aligned}$$

Solution

Section 1.2 – Functions

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(1, 2), (3, 4), (5, 6), (5, 8)\}$$

Solution

Not a function

Domain: $\{1, 3, 5\}$

Range: $\{2, 4, 6, 8\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(1, 2), (3, 4), (6, 5), (8, 5)\}$$

Solution

It is a Function

Domain: $\{1, 3, 6, 8\}$

Range: $\{2, 4, 5\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(9, -5), (9, 5), (2, 4)\}$$

Solution

It is ***not*** a function

Domain = $\{2, 9\}$

Range = $\{-5, 5, 4\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(-2, 5), (5, 7), (0, 1), (4, -2)\}$$

Solution

It is a function

Domain = $\{-2, 0, 4, 5\}$

Range = $\{-2, 1, 5, 7\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(-5, 3), (0, 3), (6, 3)\}$$

Solution

It is a function

$$\text{Domain} = \{-5, 0, 6\}$$

$$\text{Range} = \{3\}$$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(1, 2), (3, 4), (6, 5), (8, 5), (1, 5)\}$$

Solution

It is **not** a function

$$\text{Domain} = \{1, 3, 6, 8\}$$

$$\text{Range} = \{2, 4, 5\}$$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(-1, 3), (3, 4), (6, 5), (8, 5), (1, 5)\}$$

Solution

It is a function

$$\text{Domain} = \{-1, 1, 3, 6, 8\}$$

$$\text{Range} = \{3, 4, 5\}$$

Exercise

Find the domain and the range of the relation:

$$\{(5, 12.8), (10, 16.2), (15, 18.9), (20, 20.7), (25, 21.81)\}$$

Solution

$$\text{Domain: } \{5, 10, 15, 20, 25\}$$

$$\text{Range: } \{12.8, 16.2, 18.9, 20.7, 21.81\}$$

Exercise

Let $f(x) = -3x + 4$, find $f(0)$

Solution

$$\begin{aligned} f(0) &= -3(0) + 4 \\ &= 4 \end{aligned}$$

Exercise

Let $g(x) = -x^2 + 4x - 1$, find $g(-x)$

Solution

$$\begin{aligned} g(-x) &= -(-x)^2 + 4(-x) - 1 \\ &= -x^2 - 4x - 1 \end{aligned}$$

Exercise

Let $f(x) = -3x + 4$, find $f(a + 4)$

Solution

$$\begin{aligned} f(a + 4) &= -3(a + 4) + 4 \\ &= -3a - 12 + 4 \\ &= -3a - 8 \end{aligned}$$

Exercise

Given: $f(x) = 2|x| + 3x$, find $f(2 - h)$.

Solution

$$\begin{aligned} f(2 - h) &= 2|2 - h| + 3(2 - h) \\ &= 2|2 - h| + 6 - 3h \end{aligned}$$

Exercise

Given: $g(x) = \frac{x-4}{x+3}$, find $g(x + h)$

Solution

$$g(x + h) = \frac{x + h - 4}{x + h + 3}$$

Exercise

Given: $g(x) = \frac{x}{\sqrt{1-x^2}}$, find $g(0)$ and $g(-1)$

Solution

$$g(0) = \frac{0}{\sqrt{1-0^2}} \\ = 0$$

$$g(-1) = \frac{-1}{\sqrt{1-(-1)^2}} \\ = \frac{-1}{0} \text{ undefined}$$

Exercise

Given that $g(x) = 2x^2 + 2x + 3$. Find $g(p+3)$

Solution

$$\begin{aligned} g(p+3) &= 2(p+3)^2 + 2(p+3) + 3 \\ &= 2(p^2 + 2(p)(3) + 3^2) + 2p + 6 + 3 \\ &= 2(p^2 + 6p + 9) + 2p + 9 \\ &= 2p^2 + 12p + 18 + 2p + 9 \\ &= 2p^2 + 14p + 27 \end{aligned}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

Exercise

If $f(x) = x^2 - 2x + 7$, evaluate each of the following: $f(-5)$, $f(x+4)$, $f(-x)$

Solution

$$\begin{aligned} f(-5) &= (-5)^2 - 2(-5) + 7 \\ &= 25 + 10 + 7 \\ &= 42 \end{aligned}$$

$$\begin{aligned} f(x+4) &= (x+4)^2 - 2(x+4) + 7 \\ &= x^2 + 2(4)x + 4^2 - 2x - 8 + 7 \\ &= x^2 + 8x + 16 - 2x - 1 \\ &= x^2 + 6x + 15 \\ &= x^2 + 2x + 7 \end{aligned}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

Exercise

Find $g(0)$, $g(-4)$, $g(7)$, and $g\left(\frac{3}{2}\right)$ for $g(x) = \frac{x}{\sqrt{16-x^2}}$

Solution

$$\begin{aligned}g(0) &= \frac{0}{\sqrt{16-0^2}} \\&= \frac{0}{\sqrt{16}} \\&= 0\end{aligned}$$

$$\begin{aligned}g(-4) &= \frac{-4}{\sqrt{16-(-4)^2}} \\&= \frac{-4}{\sqrt{16-16}} \\&= \frac{-4}{0} \quad \text{undefined}\end{aligned}$$

$$\begin{aligned}g(7) &= \frac{7}{\sqrt{16-7^2}} \\&= \frac{7}{\sqrt{16-49}} \\&= \frac{7}{\sqrt{-33}} \quad \text{doesn't exist in real number}\end{aligned}$$

$$\begin{aligned}g\left(\frac{3}{2}\right) &= \frac{\frac{3}{2}}{\sqrt{16-\left(\frac{3}{2}\right)^2}} \\&= \frac{\frac{3}{2}}{\sqrt{16-\frac{9}{4}}} \\&= \frac{\frac{3}{2}}{\sqrt{\frac{4(16)-9}{4}}} \\&= \frac{\frac{3}{2}}{\frac{\sqrt{55}}{2}} \\&= \frac{3}{\sqrt{55}} \\&= \frac{3\sqrt{55}}{55}\end{aligned}$$

Exercise

$$f(x) = 3x - 4$$

$$a) f(0)$$

$$b) f\left(\frac{5}{3}\right)$$

$$c) f(-2a)$$

$$d) f(x+h)$$

Solution

$$a) f(0) = \underline{-4}$$

$$\begin{aligned} b) f\left(\frac{5}{3}\right) &= 3\frac{5}{3} - 4 \\ &= 5 - 4 \\ &= \underline{1} \end{aligned}$$

$$\begin{aligned} c) f(-2a) &= 3(-2a) - 4 \\ &= \underline{-6a - 4} \end{aligned}$$

$$\begin{aligned} d) f(x+h) &= 3(x+h) - 4 \\ &= \underline{3x + 3h - 4} \end{aligned}$$

Exercise

$$f(x) = 3x^2 + 3x - 1$$

$$a) f(0)$$

$$b) f(x+h)$$

$$c) f(2)$$

$$d) f(h)$$

Solution

$$a) f(0) = \underline{-1}$$

$$\begin{aligned} b) f(x+h) &= 3(x+h)^2 + 3(x+h) - 1 \\ &= 3(x^2 + 2hx + h^2) + 3x + 3h - 1 \\ &= \underline{3x^2 + 6hx + 3h^2 + 3x + 3h - 1} \end{aligned}$$

$$\begin{aligned} c) f(2) &= 12 + 6 - 1 \\ &= \underline{17} \end{aligned}$$

$$d) f(h) = \underline{3h^2 + 3h - 1}$$

Exercise

$$f(x) = 2x^2 - 4$$

$$a) f(0) \qquad b) f(x+h) \qquad c) f(2) \qquad d) f(2) - f(-3)$$

Solution

$$a) f(0) = \underline{-4}$$

$$\begin{aligned} b) f(x+h) &= 2(x+h)^2 - 4 \\ &= 2(x^2 + 2hx + h^2) - 4 \\ &= \underline{2x^2 + 4hx + 2h^2 - 4} \end{aligned}$$

$$\begin{aligned} c) f(2) &= 8 - 4 \\ &= \underline{4} \end{aligned}$$

$$\begin{aligned} d) f(2) - f(-3) &= 8 - 4 - (18 - 4) \\ &= 4 - 14 \\ &= \underline{-10} \end{aligned}$$

Exercise

$$f(x) = 3x^2 + 4x - 2$$

$$a) f(0) \qquad b) f(x+h) \qquad c) f(3) \qquad d) f(-5)$$

Solution

$$a) f(0) = \underline{-2}$$

$$\begin{aligned} b) f(x+h) &= 3(x+h)^2 + 4(x+h) - 2 \\ &= 3(x^2 + 2hx + h^2) + 4x + 4h - 2 \\ &= \underline{3x^2 + 6hx + 3h^2 + 4x + 4h - 2} \end{aligned}$$

$$\begin{aligned} c) f(3) &= 27 + 12 - 2 \\ &= \underline{37} \end{aligned}$$

$$\begin{aligned} d) f(-5) &= 75 - 20 - 2 \\ &= \underline{53} \end{aligned}$$

Exercise

$$f(x) = -x^3 - x^2 - x + 10$$

$$a) f(0)$$

$$b) f(-1)$$

$$c) f(2)$$

$$d) f(1) - f(-2)$$

Solution

$$a) f(0) = \underline{10}$$

$$b) f(-1) = 1 - 1 + 1 + 10 \\ = \underline{11}$$

$$c) f(2) = -8 - 4 - 2 + 10 \\ = \underline{-4}$$

$$d) f(1) - f(-2) = -1 - 1 - 1 + 10 - (8 - 4 + 2 + 10) \\ = 7 - 16 \\ = \underline{-9}$$

Exercise

For $\frac{1}{10}x^{10} - \frac{1}{2}x^6 + \frac{2}{3}x^3 - 10x$, determine

$$a) f(2) - f(-2)$$

$$b) f(1) - f(-1)$$

$$c) f(2) - f(0)$$

Solution

$$a) f(2) - f(-2) = \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2}{3}2^3 - 20 - \left(\frac{2^{10}}{10} - \frac{2^6}{2} - \frac{2}{3}2^3 + 20 \right) \\ = \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2^4}{3} - 20 - \frac{2^{10}}{10} + \frac{2^6}{2} + \frac{2^4}{3} - 20 \\ = \frac{2^5}{3} - 40 \\ = \frac{32}{3} - 40 \\ = \underline{-\frac{88}{3}}$$

$$b) f(1) - f(-1) = \frac{1}{10} - \frac{1}{2} + \frac{2}{3} - 10 - \left(\frac{1}{10} - \frac{1}{2} - \frac{2}{3} + 10 \right) \\ = \frac{1}{10} - \frac{1}{2} + \frac{2}{3} - 10 - \frac{1}{10} + \frac{1}{2} + \frac{2}{3} - 10 \\ = \frac{4}{3} - 20 \\ = \underline{-\frac{56}{3}}$$

$$\begin{aligned}
 c) \quad f(2) - f(0) &= \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2}{3}2^3 - 20 - (0) \\
 &= \frac{2^9}{5} - 2^5 + \frac{2^4}{3} - 5(2^2) \\
 &= 2^2 \left(\frac{128}{5} - 8 + \frac{4}{3} - 5 \right) \\
 &= 4 \left(\frac{384 + 20 - 195}{15} \right) \\
 &= 4 \left(\frac{209}{15} \right) \\
 &= \underline{\underline{\frac{836}{15}}}
 \end{aligned}$$

Exercise

For $f(x) = 3x^4 + x^2 - 4$, determine

$$a) \quad f(2) - f(-2)$$

$$b) \quad f(1) - f(-1)$$

$$c) \quad f(2) - f(0)$$

Solution

$$\begin{aligned}
 a) \quad f(2) - f(-2) &= 3(16) + 4 - 4 - (3(16) + 4 - 4) \\
 &= 48 + 4 - 4 - 48 - 4 + 4 \\
 &= \underline{\underline{0}}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f(1) - f(-1) &= 3 + 1 - 4 - (3 + 1 - 4) \\
 &= \underline{\underline{0}}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad f(2) - f(0) &= 3(16) + 4 - 4 - (0) \\
 &= \underline{\underline{48}}
 \end{aligned}$$

Exercise

For $f(x) = -\frac{2}{3}x^3 + 4x$, determine

$$a) \quad f(2) - f(-2)$$

$$b) \quad f(1) - f(-1)$$

$$c) \quad f(2) - f(0)$$

Solution

$$\begin{aligned}
 a) \quad f(2) - f(-2) &= -\frac{2}{3}(2^3) + 8 - \left(-\frac{2}{3}(-2)^3 - 8 \right) \\
 &= -\frac{16}{3} + 8 - \frac{16}{3} + 8 \\
 &= 2 \left(-\frac{16}{3} + 8 \right)
 \end{aligned}$$

$$\begin{aligned}
 &= 16\left(-\frac{1}{3} + 1\right) \\
 &= 16\left(\frac{2}{3}\right) \\
 &= \frac{32}{3} \quad |
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f(1) - f(-1) &= -\frac{2}{3} + 4 - \left(\frac{2}{3} - 4\right) \\
 &= 2\left(-\frac{2}{3} + 4\right) \\
 &= \frac{20}{3} \quad |
 \end{aligned}$$

$$\begin{aligned}
 c) \quad f(2) - f(0) &= -\frac{16}{3} + 8 - (0) \\
 &= \frac{8}{3} \quad |
 \end{aligned}$$

Exercise

For $f(x) = \frac{2x-3}{x-4}$, determine

$$a) \quad f(0)$$

$$b) \quad f(3)$$

$$c) \quad f(x+h)$$

$$d) \quad f(-4)$$

Solution

$$a) \quad f(0) = \frac{3}{4} \quad |$$

$$\begin{aligned}
 b) \quad f(3) &= \frac{6-3}{3-4} \\
 &= -3 \quad |
 \end{aligned}$$

$$\begin{aligned}
 c) \quad f(x+h) &= \frac{2(x+h)-3}{x+h-4} \\
 &= \frac{2x+2h-3}{x+h-4} \quad |
 \end{aligned}$$

$$\begin{aligned}
 d) \quad f(-4) &= \frac{-8-3}{-4-4} \\
 &= \frac{11}{8} \quad |
 \end{aligned}$$

Exercise

For $f(x) = \frac{3x-1}{x-5}$, determine

a) $f(0)$

b) $f(3)$

c) $f(x+h)$

d) $f(-5)$

Solution

$$a) \quad f(0) = \frac{1}{5}$$

$$b) \quad f(3) = \frac{9-1}{3-5} \\ = -4$$

$$c) \quad f(x+h) = \frac{3(x+h)-1}{x+h-5} \\ = \frac{3x+3h-1}{x+h-5}$$

$$d) \quad f(-5) = \frac{-12-1}{-4-5} \\ = \frac{13}{9}$$

Solution

Section 1.3 – Quadratic Functions

Exercise

Solve: $x^2 = -25$

Solution

$$x = \pm\sqrt{-25}$$
$$\underline{= \pm 5i}$$

Exercise

Solve: $x^2 = 49$

Solution

$$\underline{x = \pm 7}$$

Exercise

Solve: $9x^2 = 100$

Solution

$$x^2 = \frac{100}{9}$$
$$x = \pm\sqrt{\frac{100}{9}}$$
$$\underline{= \pm \frac{10}{3}}$$

Exercise

Solve: $4x^2 + 25 = 0$

Solution

$$4x^2 = -25$$
$$x^2 = -\frac{25}{4}$$
$$x = \pm\sqrt{-\frac{25}{4}}$$
$$\underline{= \pm \frac{5}{2}i}$$

Exercise

Solve: $5x^2 + 35 = 0$

Solution

$$5x^2 = -35$$

$$x^2 = -7$$

$$\underline{x = \pm i\sqrt{7} \mid}$$

Exercise

Solve: $5x^2 - 45 = 0$

Solution

$$5x^2 = 45$$

$$x = \frac{45}{5}$$

$$x^2 = 9$$

$$\underline{x = \pm 3 \mid}$$

Exercise

Solve: $(x - 4)^2 = 12$

Solution

$$x - 4 = \pm\sqrt{12}$$

$$x = 4 \pm \sqrt{12} \quad \sqrt{12} = \sqrt{4(3)} = 2\sqrt{3}$$

$$\underline{x = 4 \pm 2\sqrt{3} \mid}$$

Exercise

Solve: $(x + 3)^2 = -16$

Solution

$$x + 3 = \pm\sqrt{-16}$$

$$\underline{x = -3 \pm 4i \mid}$$

Exercise

Solve: $(x-2)^2 = -20$

Solution

$$x - 2 = \pm\sqrt{-20}$$

$$x = \underline{2 \pm 4i\sqrt{5}} \quad |$$

Exercise

Solve: $(4x+1)^2 = 20$

Solution

$$4x + 1 = \pm\sqrt{20}$$

$$4x = -1 \pm 2\sqrt{5}$$

$$x = \underline{\frac{-1 \pm 2\sqrt{5}}{4}} \quad |$$

Exercise

Solve $x^2 - 6x = -7$

Solution

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{8}}{2}$$

$$= \frac{6 \pm 2\sqrt{2}}{2}$$

$$= \frac{2(3 \pm \sqrt{2})}{2}$$

$$= \underline{3 \pm \sqrt{2}} \quad |$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve $-6x^2 = 3x + 2$

Solution

$$6x^2 + 3x + 2 = 0$$

$$\begin{aligned}
 x &= \frac{-3 \pm \sqrt{3^2 - 4(6)(2)}}{2(6)} \\
 &= \frac{-3 \pm \sqrt{-39}}{12} \\
 &= \frac{-3}{12} \pm i \frac{\sqrt{39}}{12} \\
 &= \underline{-\frac{1}{4} \pm i \frac{\sqrt{39}}{12}}
 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $3x^2 + 2x = 7$

Solution

$$3x^2 + 2x - 7 = 0 \quad \Rightarrow \quad \mathbf{a = 3, \quad b = 2, \quad c = -7}$$

$$x = \frac{-2 \pm \sqrt{4 - 4(3)(-7)}}{2(3)}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{88}}{6}$$

$$= \frac{-2 \pm \sqrt{4(22)}}{6}$$

$$= \frac{-2 \pm 2\sqrt{22}}{6}$$

$$= \frac{2(-1 \pm \sqrt{22})}{6}$$

$$= \underline{-\frac{1 \pm \sqrt{22}}{3}}$$

$$x = -\frac{1}{3} \pm \frac{\sqrt{22}}{3}$$

Exercise

$3x^2 + 6 = 10x$

Solution

$$3x^2 - 10x + 6 = 0$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
&= \frac{10 \pm \sqrt{100 - 72}}{6} \\
&= \frac{10}{6} \pm \frac{\sqrt{28}}{6} \\
&= \frac{5}{3} \pm \frac{2\sqrt{7}}{6} \\
&= \frac{5}{3} \pm \frac{\sqrt{7}}{3}
\end{aligned}$$

Exercise

Solve: $5x^2 + 2 = x$

Solution

$$5x^2 - x + 2 = 0$$

$$\begin{aligned}
x &= \frac{1 \pm \sqrt{1 - 40}}{10} \\
&= \frac{1 \pm \sqrt{-39}}{10} \\
&= \frac{1 \pm i\sqrt{39}}{10} \\
&= \frac{1}{10} \pm i \frac{\sqrt{39}}{10}
\end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $5x^2 = 2x - 3$

Solution

$$5x^2 - 2x + 3 = 0$$

$$\begin{aligned}
x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(3)}}{2(5)} \\
&= \frac{2 \pm \sqrt{4 - 60}}{10} \\
&= \frac{2 \pm \sqrt{-56}}{10} \\
&= \frac{2 \pm i\sqrt{4(14)}}{10}
\end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
&= \frac{2 \pm i2\sqrt{14}}{10} \\
&= \frac{2}{10} \pm i \frac{2\sqrt{14}}{10} \\
&= \frac{1}{5} \pm i \frac{\sqrt{14}}{5}
\end{aligned}$$

Exercise

Solve: $x^2 + 8x + 15 = 0$

Solution

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{64 - 60}}{2}$$

$$= \frac{-8 \pm \sqrt{4}}{2}$$

$$= \frac{-8 \pm 2}{2}$$

$$= \left\{ \begin{array}{l} \frac{-8+2}{2} = \frac{-6}{2} = -3 \\ \frac{-8-2}{2} = \frac{-10}{2} = -5 \end{array} \right\}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 + 5x + 2 = 0$

Solution

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$= \frac{-5 \pm \sqrt{17}}{2}$$

$$= \frac{-5}{2} \pm \frac{\sqrt{17}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 + x - 12 = 0$

Solution

$$\begin{aligned}x &= \frac{-1 \pm \sqrt{1 + 48}}{2} \\&= \frac{-1 \pm 7}{2} \\&= \begin{cases} \frac{-1 - 7}{2} = -4 \\ \frac{-1 + 7}{2} = 3 \end{cases}\end{aligned}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 - 2x - 15 = 0$

Solution

$$\begin{aligned}x &= \frac{2 \pm \sqrt{4 + 60}}{2} \\&= \frac{2 \pm 8}{2} \\&= \begin{cases} \frac{2 + 8}{2} = 5 \\ \frac{2 - 8}{2} = -3 \end{cases}\end{aligned}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 - 4x - 45 = 0$

Solution

$$\begin{aligned}x &= \frac{4 \pm \sqrt{16 + 180}}{2} \\&= \frac{4 \pm \sqrt{196}}{2} \\&= \frac{4 \pm 14}{2} \\&= \begin{cases} \frac{4 + 14}{2} = 9 \\ \frac{4 - 14}{2} = -5 \end{cases}\end{aligned}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 - 6x - 10 = 0$

Solution

$$x^2 - 6x = 10$$

$$x^2 - 6x + \left(\frac{-6}{2}\right)^2 = 10 + \left(\frac{-6}{2}\right)^2$$

$$x^2 - 2(3)x + (3)^2 = 10 + 9$$

$$(x - 3)^2 = 19$$

$$x - 3 = \pm\sqrt{19}$$

$$\underline{x = 3 \pm \sqrt{19}}$$

Exercise

Solve: $2x^2 + 3x - 4 = 0$

Solution

$$x^2 + \frac{3}{2}x = 2$$

$$x^2 + \frac{3}{2}x + \left(\frac{1 \cdot 3}{2 \cdot 2}\right)^2 = 2 + \left(\frac{1 \cdot 3}{2 \cdot 2}\right)^2$$

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = 2 + \frac{9}{16}$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{41}{16}$$

$$x + \frac{3}{4} = \pm\sqrt{\frac{41}{16}}$$

$$\underline{x = -\frac{3}{4} \pm \frac{\sqrt{41}}{4}}$$

Exercise

Solve $x^2 - x + 8 = 0$

Solution

$$x = \frac{1 \pm \sqrt{1 - 32}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{-31}}{2}$$

$$= \frac{1 \pm i\sqrt{31}}{2}$$

Exercise

Solve $2x^2 - 13x = 1$

Solution

$$2x^2 - 13x - 1 = 0$$

$$x = \frac{13 \pm \sqrt{169 + 8}}{4}$$

$$= \frac{13 \pm \sqrt{169 + 8}}{4}$$

$$= \frac{13 \pm \sqrt{177}}{4}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve $r^2 + 3r - 3 = 0$

Solution

$$r = \frac{-3 \pm \sqrt{3^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9 + 12}}{2}$$

$$= \frac{-3 \pm \sqrt{21}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^3 + 8 = 0$

Solution

$$(x + 2)(x^2 - 2x + 4) = 0$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$x + 2 = 0$$

$$x^2 - 2x + 4 = 0$$

$$x = -2$$

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} \\ &= \frac{2 \pm \sqrt{-12}}{2} \\ &= \frac{2 \pm 2i\sqrt{3}}{2} \\ &= \frac{2(1 \pm i\sqrt{3})}{2} \\ &= 1 \pm i\sqrt{3} \end{aligned}$$

The solution set is $\{-2, 1 \pm i\sqrt{3}\}$

Exercise

Solve: $4x^2 - 12x + 9 = 0$

Solution

$$\begin{aligned} x &= \frac{12 \pm \sqrt{144 - 144}}{8} \\ &= \frac{12}{8} \\ &= \frac{3}{2} \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $9x^2 - 30x + 25 = 0$

Solution

$$\begin{aligned} x &= \frac{30 \pm \sqrt{900 - 900}}{18} \\ &= \frac{30}{18} \\ &= \frac{5}{3} \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 - 14x + 49 = 0$

Solution

$$\begin{aligned}
 x &= \frac{14 \pm \sqrt{196 - 196}}{2} \\
 &= \frac{14}{2} \\
 &= \underline{7}
 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 - 8x + 16 = 0$

Solution

$$\begin{aligned}
 x &= \frac{8 \pm \sqrt{64 - 64}}{2} \\
 &= \frac{8}{2} \\
 &= \underline{4}
 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 + 6x + 13 = 0$

Solution

$$\begin{aligned}
 x &= \frac{-6 \pm \sqrt{36 - 52}}{2} \\
 &= \frac{-6 \pm \sqrt{-16}}{2} \\
 &= \frac{-6 \pm 4i}{2} \\
 &= \underline{-3 \pm 2i}
 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $2x^2 - 2x + 13 = 0$

Solution

$$\begin{aligned}
 x &= \frac{2 \pm \sqrt{4 - 104}}{4} \\
 &= \frac{2 \pm \sqrt{-100}}{4}
 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm 10i}{4}$$

$$= \frac{1}{2} \pm \frac{5}{2}i$$

Exercise

Solve: $x^2 + 2x + 29 = 0$

Solution

$$x = \frac{-2 \pm \sqrt{4 - 116}}{2}$$

$$= \frac{-2 \pm \sqrt{-112}}{2}$$

$$= \frac{-2 \pm 4i\sqrt{7}}{2}$$

$$= -1 \pm 2i\sqrt{7}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $4x^2 + 4x + 13 = 0$

Solution

$$x = \frac{-4 \pm \sqrt{16 - 16(13)}}{8}$$

$$= \frac{-4 \pm 4\sqrt{-12}}{8}$$

$$= \frac{-4 \pm 8i\sqrt{3}}{8}$$

$$= -\frac{1}{2} \pm i\sqrt{3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 - 2x + 26 = 0$

Solution

$$x = \frac{2 \pm \sqrt{4 - 4(26)}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm 2\sqrt{-25}}{2}$$

$$= \underline{1 \pm 5i}$$

Exercise

Solve: $9x^2 - 4x + 20 = 0$

Solution

$$x = \frac{4 \pm \sqrt{16 - 16(45)}}{18}$$

$$= \frac{4 \pm 4\sqrt{-44}}{18}$$

$$= \frac{-4 \pm 8i\sqrt{11}}{8}$$

$$= \underline{-\frac{1}{2} \pm i\sqrt{11}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 + 6x + 21 = 0$

Solution

$$x = \frac{-6 \pm \sqrt{36 - 84}}{2}$$

$$= \frac{-6 \pm \sqrt{-48}}{2}$$

$$= \frac{-6 \pm 4i\sqrt{3}}{2}$$

$$= \underline{-3 \pm 2i\sqrt{3}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $9x^2 - 12x - 49 = 0$

Solution

$$x = \frac{12 \pm \sqrt{2^4 3^2 - 2^2 3^2 7^2}}{18}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
&= \frac{12 \pm 6\sqrt{4-49}}{18} \\
&= \frac{12 \pm 6\sqrt{-45}}{18} \\
&= \frac{12 \pm 18i\sqrt{5}}{18} \\
&= \underline{\underline{\frac{2}{3} \pm i\sqrt{5}}}
\end{aligned}$$

Exercise

Solve: $x(x-3) = 18$

Solution

$$x^2 - 3x - 18 = 0$$

$$x = \frac{3 \pm \sqrt{9+72}}{2}$$

$$= \frac{3 \pm 9}{2}$$

$$= \left\{ \begin{array}{l} \frac{3+9}{2} = \underline{6} \\ \frac{3-9}{2} = \underline{-3} \end{array} \right.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x(x-4) - 21 = 0$

Solution

$$x^2 - 4x - 21 = 0$$

$$x = \frac{4 \pm \sqrt{16+84}}{2}$$

$$= \frac{4 \pm 10}{2}$$

$$= \left\{ \begin{array}{l} \frac{4+10}{2} = \underline{7} \\ \frac{4-10}{2} = \underline{-3} \end{array} \right.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $(x-1)(x+4) = 14$

Solution

$$x^2 + 3x - 18 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 72}}{2}$$

$$= \frac{-3 \pm 9}{2}$$

$$= \left\{ \begin{array}{l} \frac{-3+9}{2} = 3 \\ \frac{-3-9}{2} = -6 \end{array} \right|$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $(x-3)(x+8) = -30$

Solution

$$x^2 + 5x + 6 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

$$= \frac{-5 \pm 1}{2}$$

$$= \left\{ \begin{array}{l} \frac{-5+1}{2} = -2 \\ \frac{-5-1}{2} = -3 \end{array} \right|$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x(x+8) = 16(x-1)$

Solution

$$x^2 + 8x = 16x - 16$$

$$x^2 - 8x + 16 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 64}}{2}$$

$$= \frac{8}{2}$$

$$= 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x(x+9) = 4(2x+5)$

Solution

$$x^2 + 9x = 8x + 20$$

$$x^2 + x - 20 = 0$$

$$x = \frac{-1 \pm \sqrt{1+80}}{2}$$

$$= \frac{-1 \pm 9}{2}$$

$$= \left\{ \begin{array}{l} \frac{-1+9}{2} = 4 \\ \frac{-1-9}{2} = -5 \end{array} \right.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $(x+1)^2 = 2(x+3)$

Solution

$$x^2 + 2x + 1 = 2x + 6$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

Exercise

Solve: $(x+1)^2 - 5(x+2) = 3x+7$

Solution

$$x^2 + 2x + 1 - 5x - 10 = 3x + 7$$

$$x^2 - 6x - 16 = 0$$

$$x = \frac{6 \pm \sqrt{36+64}}{2}$$

$$= \frac{6 \pm 10}{2}$$

$$= \left\{ \begin{array}{l} \frac{6+10}{2} = 8 \\ \frac{6-10}{2} = -2 \end{array} \right.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x(8x+1) = 3x^2 - 2x + 2$

Solution

$$8x^2 + x = 3x^2 - 2x + 2$$

$$5x^2 + 3x - 2 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{10}$$

$$= \frac{-3 \pm 7}{2}$$

$$= \left\{ \begin{array}{l} \frac{-3+7}{10} = \frac{2}{5} \\ \frac{-3-7}{10} = -1 \end{array} \right\}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Solve: $x^2 + 6x - 7 = 0$

Solution

$$1 + 6 - 7 = 0 \quad a + b + c = 0$$

$$\underline{x = 1, -7} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

Exercise

Solve: $x^2 - 6x - 7 = 0$

Solution

$$1 - (-6) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, 7} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Exercise

Solve: $3x^2 + 4x - 7 = 0$

Solution

$$3 + 4 - 7 = 0 \quad a + b + c = 0$$

$$\underline{x = 1, -\frac{7}{3}} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

Exercise

Solve: $3x^2 - 4x - 7 = 0$

Solution

$$3 - (-4) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, \frac{7}{3}} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Exercise

Solve: $3x^2 - x - 2 = 0$

Solution

$$3 - 1 - 2 = 0 \quad a + b + c = 0$$

$$\underline{x = 1, -\frac{2}{3}} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

Exercise

Solve: $3x^2 + x - 2 = 0$

Solution

$$3 - 1 - 2 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, \frac{2}{3}} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Exercise

Solve: $2x^2 + 3x - 5 = 0$

Solution

$$2 + 3 - 5 = 0 \quad a + b + c = 0$$

$$\underline{x = 1, -\frac{5}{2}} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

Exercise

Solve: $2x^2 - 3x - 5 = 0$

Solution

$$2 - (-3) - 5 = 0 \quad a + b + c = 0$$

$$\underline{x = -1, \frac{5}{2}} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Exercise

$$\text{Solve: } x^2 - 3x - 4 = 0$$

Solution

$$1 - (-3) - 4 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, 4} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Exercise

$$\text{Solve: } x^2 + 3x - 4 = 0$$

Solution

$$1 + 3 - 4 = 0 \quad a + b + c = 0$$

$$\underline{x = 1, -4} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

Exercise

$$\text{Solve: } x^2 + 2x + 1 = 0$$

Solution

$$1 - 2 + 1 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, -1} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Exercise

$$\text{Solve: } 4x^2 - x - 5 = 0$$

Solution

$$4 - (-1) - 5 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, \frac{5}{4}} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Exercise

Solve for the specified variable $A = \frac{\pi d^2}{4}$, for d

Solution

$$\frac{4}{\pi}A = \frac{4}{\pi} \frac{\pi d^2}{4}$$

$$4A = 4 \frac{\pi d^2}{4}$$

$$\frac{4A}{\pi} = d^2$$

$$4A = \pi d^2$$

$$d^2 = \frac{4A}{\pi}$$

$$\frac{4A}{\pi} = \frac{\pi}{\pi} d^2$$

$$d = \pm \sqrt{\frac{4A}{\pi}}$$

$$= \pm 2 \frac{\sqrt{A}}{\sqrt{\pi}}$$

$$= \pm 2 \frac{\sqrt{A}}{\sqrt{\pi}} \frac{\sqrt{\pi}}{\sqrt{\pi}}$$

$$= \pm \frac{2\sqrt{\pi A}}{\pi}$$

Exercise

Solve for the specified variable $rt^2 - st - k = 0$ ($r \neq 0$), for t

Solution

$$t = \frac{-(-s) \pm \sqrt{(-s)^2 - 4(r)(-k)}}{2(r)}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{s \pm \sqrt{s^2 + 4rk}}{2r}$$

Solution **Section 1.4 – Quadratic Functions**

Exercise

For the function $f(x) = x^2 + 6x + 3$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value and find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{6}{2(1)} = -3$

$y = f(-3) = (-3)^2 + 6(-3) + 3 = -6$ **Vertex point** $(-3, -6)$

b) Line of symmetry: $x = -3$

c) Minimum point, value $(-3, -6)$

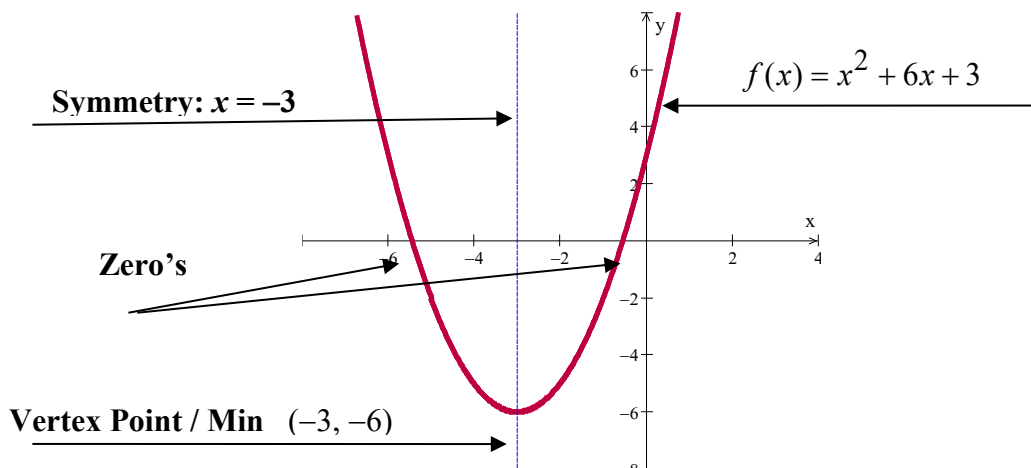
d) $x = \frac{-6 \pm \sqrt{36 - 12}}{2} = \frac{-6 \pm \sqrt{24}}{2} = \frac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$

$$x = \begin{cases} -3 + \sqrt{6} = -0.5 \\ -3 - \sqrt{6} = -5.45 \end{cases}$$

e) y -intercept $y = 3$

f) Range: $[-6, \infty)$ Domain: $(-\infty, \infty)$

g)



h) Decreasing: $(-\infty, -3)$ Increasing: $(-3, \infty)$

Exercise

For the function $f(x) = x^2 + 6x + 5$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{6}{2}$ $x = -\frac{b}{2a}$
 $\quad \quad \quad = -3$

$y = f(-3) = (-3)^2 + 6(-3) + 5$
 $\quad \quad \quad = -4$

Vertex point: $(-3, -4)$

b) Axis of symmetry: $x = -3$

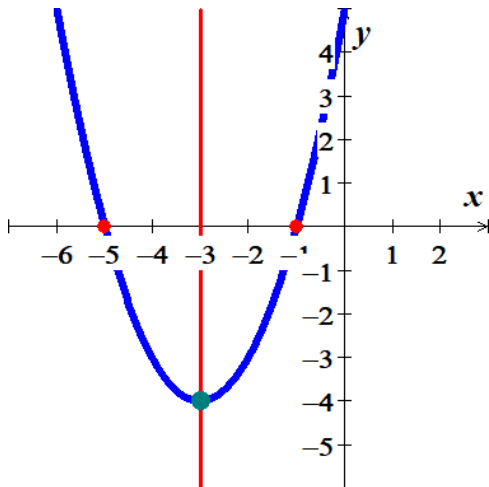
c) Minimum point @ $(-3, -4)$

d) $x^2 + 6x + 5 = 0$
 $\quad \quad \quad x = -5, -1$

e) $x = 0 \rightarrow y = 5$

f) Domain: \mathbb{R} Range: $[-4, \infty)$

g)



h) Increasing: $(-3, \infty)$ Decreasing: $(-\infty, -3)$

Exercise

For the function $f(x) = -x^2 - 6x - 5$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{-6}{-2} = -3$ $x = -\frac{b}{2a}$

$y = f(-3) = -9 + 18 - 5 = 4$

Vertex point: $(-3, 4)$

b) Axis of symmetry: $x = -3$

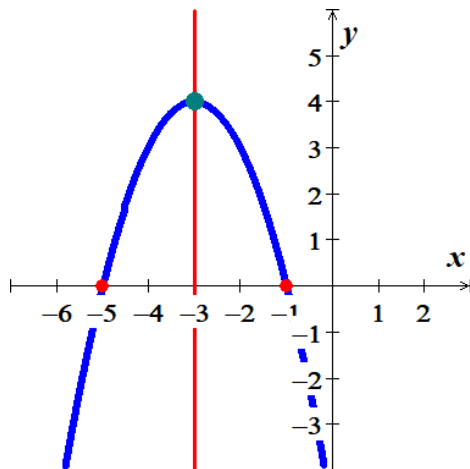
c) Maximum point @ $(-3, 4)$

d) $-(x^2 + 6x + 5) = 0$
 $x = -5, -1$

e) $x = 0 \rightarrow y = -5$

f) Domain: \mathbb{R} Range: $(-\infty, 4]$

g)



h) Increasing: $(-\infty, -3)$ Decreasing: $(-3, \infty)$

Exercise

For the function $f(x) = x^2 - 4x + 2$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{-4}{2}$ $x = -\frac{b}{2a}$

$= 2$

$f(2) = 4 - 8 + 2$

$= -2$

Vertex point: $(2, -2)$

b) Axis of symmetry: $x = 2$

c) Minimum point @ $(2, -2)$

d) $x^2 - 4x + 2 = 0$

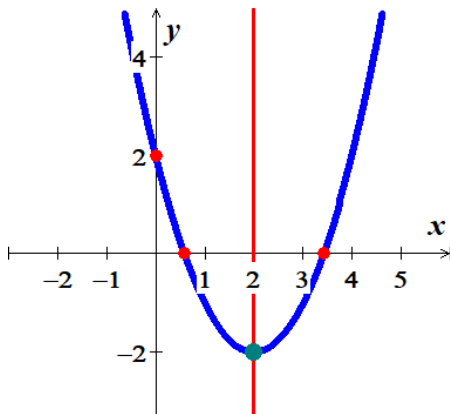
$x = \frac{4 \pm \sqrt{8}}{2}$

$x = 2 \pm \sqrt{2}$

e) $x = 0 \rightarrow y = 2$

f) Domain: \mathbb{R} Range: $[-2, \infty)$

g)



h) Increasing: $(2, \infty)$ Decreasing: $(-\infty, 2)$

Exercise

For the function $f(x) = -2x^2 + 16x - 26$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y-intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{16}{-4}$ $x = -\frac{b}{2a}$
 $\quad \quad \quad = 4$

$f(4) = -32 + 64 - 26$
 $\quad \quad \quad = 6$

Vertex point: $(4, 6)$

b) Axis of symmetry: $x = 4$

c) Maximum point @ $(4, 6)$

d) $-2x^2 + 16x - 26 = 0$

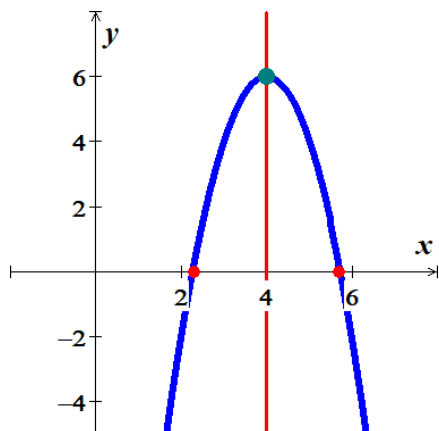
$x = \frac{-16 \pm \sqrt{128}}{-4}$

$x = 4 \pm 2\sqrt{2}$

e) $x = 0 \rightarrow y = -26$

f) Domain: \mathbb{R} Range: $(-\infty, 6]$

g)



h) Increasing: $(-\infty, 4)$ Decreasing: $(4, \infty)$

Exercise

For the function $f(x) = x^2 + 4x + 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{4}{2}$ $x = -\frac{b}{2a}$
 $\quad \quad \quad = -2$

$f(-2) = 4 - 8 + 1$
 $\quad \quad \quad = -3$

Vertex point: $(-2, -3)$

b) Axis of symmetry: $x = -2$

c) Minimum point @ $(-2, -3)$

d) $x^2 + 4x + 1 = 0$

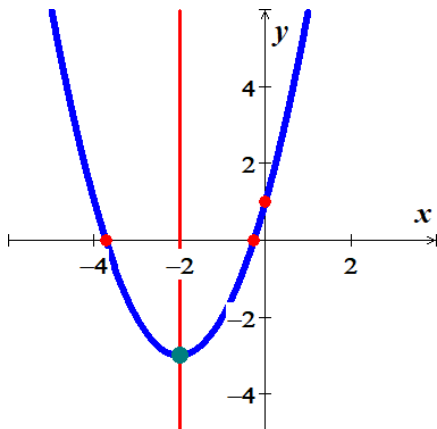
$x = \frac{-4 \pm \sqrt{12}}{2}$

$x = -2 \pm \sqrt{3}$

e) $x = 0 \rightarrow y = 1$

f) Domain: \mathbb{R} Range: $[-3, \infty)$

g)



h) Increasing: $(-2, \infty)$ Decreasing: $(-\infty, -2)$

Exercise

For the function $f(x) = x^2 - 8x + 5$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y-intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{-8}{2}$ $x = -\frac{b}{2a}$

$= 4$

$f(4) = 16 - 32 + 5$

$= -11$

Vertex point: $(4, -11)$

b) Axis of symmetry: $x = 4$

c) Minimum point @ $(4, -11)$

d) $x^2 - 8x + 5 = 0$

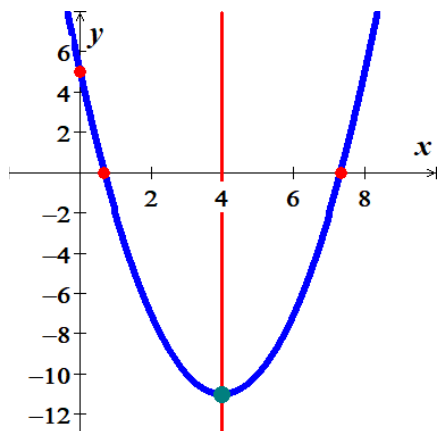
$x = \frac{8 \pm \sqrt{44}}{2}$

$x = 4 \pm \sqrt{11}$

e) $x = 0 \rightarrow y = 5$

f) Domain: \mathbb{R} Range: $[-11, \infty)$

g)



h) Increasing: $(4, \infty)$ Decreasing: $(-\infty, 4)$

Exercise

For the function $f(x) = x^2 + 6x - 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{6}{2}$ $x = -\frac{b}{2a}$
 $= -3$

$f(-3) = 9 - 18 - 1$
 $= -10$

Vertex point: $(-3, -10)$

b) Axis of symmetry: $x = -3$

c) Minimum point @ $(-3, -10)$

d) $x^2 + 6x - 1 = 0$

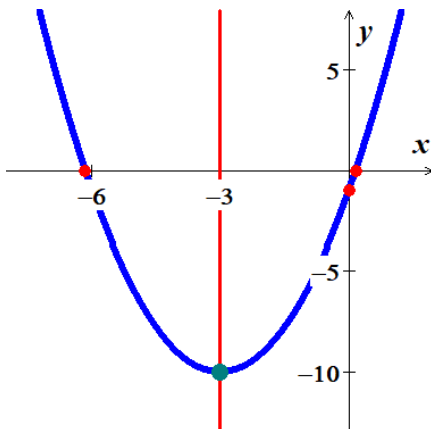
$x = \frac{-6 \pm \sqrt{40}}{2}$

$x = -3 \pm \sqrt{10}$

e) $x = 0 \rightarrow y = -1$

f) Domain: \mathbb{R} Range: $[-10, \infty)$

g)



h) Increasing: $(-3, \infty)$ Decreasing: $(-\infty, -3)$

Exercise

For the function $f(x) = x^2 + 6x + 3$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y-intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{6}{2}$ $x = -\frac{b}{2a}$
 $= -3$

$f(-3) = 9 - 18 + 3$
 $= -6$

Vertex point: $(-3, -6)$

b) Axis of symmetry: $x = -3$

c) Minimum point @ $(-3, -6)$

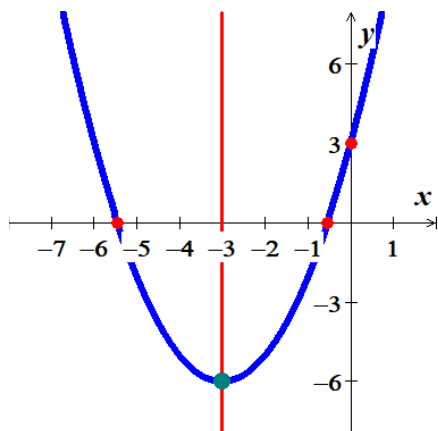
d) $x^2 + 6x + 3 = 0$
 $x = \frac{-6 \pm \sqrt{24}}{2}$

$x = -3 \pm \sqrt{6}$

e) $x = 0 \rightarrow y = 3$

f) Domain: \mathbb{R} Range: $[-6, \infty)$

g)



h) Increasing: $(-3, \infty)$ Decreasing: $(-\infty, -3)$

Exercise

For the function $f(x) = x^2 - 10x + 3$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{-10}{2}$ $x = -\frac{b}{2a}$

$= 5$

$f(5) = 25 - 50 + 3$

$= -22$

Vertex point: $(5, -22)$

b) Axis of symmetry: $x = 5$

c) Minimum point @ $(5, -22)$

d) $x^2 - 10x + 3 = 0$

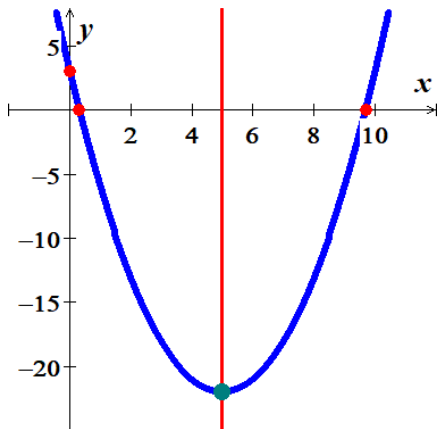
$x = \frac{10 \pm \sqrt{88}}{2}$

$x = 5 \pm \sqrt{22}$

e) $x = 0 \rightarrow y = 3$

f) Domain: \mathbb{R} Range: $[-22, \infty)$

g)



h) Increasing: $(5, \infty)$ Decreasing: $(-\infty, 5)$

Exercise

For the function $f(x) = x^2 - 3x + 4$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y-intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = \frac{3}{2}$ $x = -\frac{b}{2a}$

$$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} + 4$$
$$= \frac{7}{4}$$

Vertex point: $\left(\frac{3}{2}, \frac{7}{4}\right)$

b) Axis of symmetry: $x = \frac{3}{2}$

c) Minimum point @ $\left(\frac{3}{2}, \frac{7}{4}\right)$

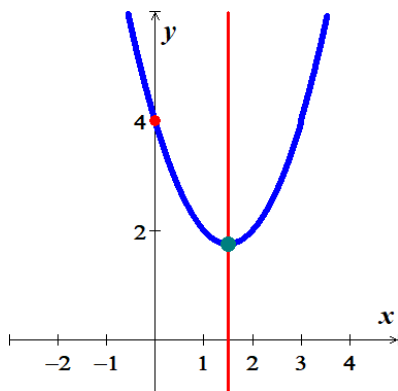
d) $x^2 - 3x + 4 = 0$

$$x = \frac{3 \pm \sqrt{-7}}{2} \quad \mathbb{C}$$

e) $x = 0 \rightarrow y = 4$

f) Domain: \mathbb{R} Range: $\left[\frac{7}{4}, \infty\right)$

g)



h) Increasing: $\left(\frac{3}{2}, \infty\right)$ Decreasing: $\left(-\infty, \frac{3}{2}\right)$

Exercise

For the function $f(x) = x^2 - 3x - 4$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = \frac{3}{2}$ $x = -\frac{b}{2a}$

$$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} - 4$$
$$= -\frac{25}{4}$$

Vertex point: $\left(\frac{3}{2}, -\frac{25}{4}\right)$

b) Axis of symmetry: $x = \frac{3}{2}$

c) Minimum point @ $\left(\frac{3}{2}, -\frac{25}{4}\right)$

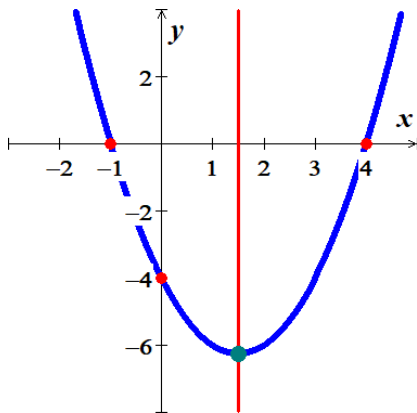
d) $x^2 - 3x - 4 = 0$

$$x = -1, 4$$

e) $x = 0 \rightarrow y = -4$

f) Domain: \mathbb{R} Range: $\left[-\frac{25}{4}, \infty\right)$

g)



h) Increasing: $\left(\frac{3}{2}, \infty\right)$ Decreasing: $\left(-\infty, \frac{3}{2}\right)$

Exercise

For the function $f(x) = x^2 - 4x - 5$

- Find the vertex point
- Find the line of symmetry
- State whether there is a *maximum* or *minimum* value *and* find that value
- Find the zeros of $f(x)$
- Find the y-intercept
- Find the *range* and the *domain* of the function.
- Graph the function
- On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = 2$ | $x = -\frac{b}{2a}$

$$f(2) = 4 - 8 - 5$$

$$= -9$$

Vertex point: $(2, -9)$ |

b) Axis of symmetry: $x = 2$ |

c) Minimum point @ $(2, -9)$ |

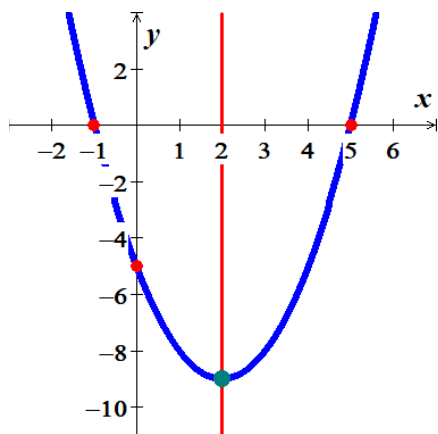
d) $x^2 - 4x - 5 = 0$

$$x = -1, 5$$

e) $x = 0 \rightarrow y = -5$ |

f) Domain: \mathbb{R} | Range: $[-9, \infty)$ |

g)



h) Increasing: $(2, \infty)$ | Decreasing: $(-\infty, 2)$ |

Exercise

For the function $f(x) = 2x^2 - 3x + 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = \frac{3}{4}$ $x = -\frac{b}{2a}$

$$f\left(\frac{3}{4}\right) = \frac{9}{8} - \frac{9}{4} + 1$$
$$= -\frac{1}{8}$$

Vertex point: $\left(\frac{3}{4}, -\frac{1}{8}\right)$

b) Axis of symmetry: $x = \frac{3}{4}$

c) Minimum point @ $\left(\frac{3}{4}, -\frac{1}{8}\right)$

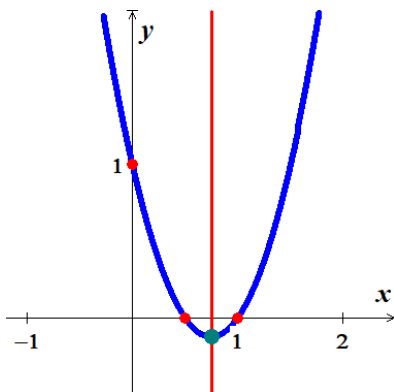
d) $2x^2 - 3x + 1 = 0$

$x = 1, \frac{1}{2}$

e) $x = 0 \rightarrow y = 1$

f) Domain: \mathbb{R} Range: $\left[-\frac{1}{8}, \infty\right)$

g)



h) Increasing: $\left(\frac{3}{4}, \infty\right)$ Decreasing: $\left(-\infty, \frac{3}{4}\right)$

Exercise

For the function $f(x) = -x^2 - 3x + 4$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y-intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{3}{2}$ $x = -\frac{b}{2a}$

$$f\left(-\frac{3}{2}\right) = -\frac{9}{4} + \frac{9}{2} + 4$$
$$= \frac{7}{2}$$

Vertex point: $\left(-\frac{3}{2}, \frac{7}{2}\right)$

b) Axis of symmetry: $x = -\frac{3}{2}$

c) Maximum point @ $\left(-\frac{3}{2}, \frac{7}{2}\right)$

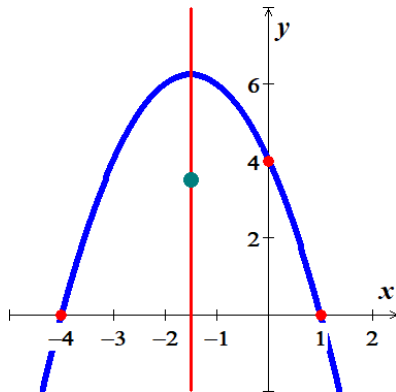
d) $-x^2 - 3x + 4 = 0$

$$x = 1, -4$$

e) $x = 0 \rightarrow y = 4$

f) Domain: \mathbb{R} Range: $\left(-\infty, \frac{7}{2}\right]$

g)



h) Increasing: $\left(-\infty, -\frac{3}{2}\right)$ Decreasing: $\left(-\frac{3}{2}, \infty\right)$

Exercise

For the function $f(x) = -2x^2 + 3x - 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = \frac{3}{4}$ $x = -\frac{b}{2a}$

$$f\left(\frac{3}{4}\right) = -\frac{9}{8} + \frac{9}{4} - 1$$
$$= \frac{1}{8}$$

Vertex point: $\left(\frac{3}{4}, \frac{1}{8}\right)$

b) Axis of symmetry: $x = \frac{3}{4}$

c) Maximum point @ $\left(\frac{3}{4}, \frac{1}{8}\right)$

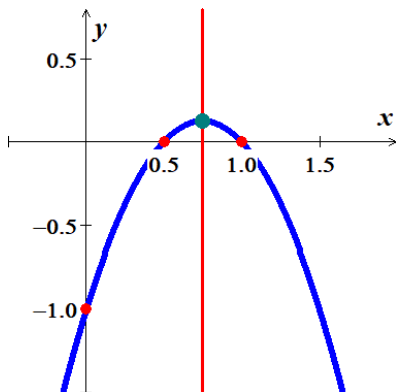
d) $-2x^2 + 3x - 1 = 0$

$$x = 1, \frac{1}{2}$$

e) $x = 0 \rightarrow y = -1$

f) Domain: \mathbb{R} Range: $\left(-\infty, \frac{1}{8}\right]$

g)



h) Increasing: $\left(-\infty, \frac{3}{4}\right)$ Decreasing: $\left(\frac{3}{4}, \infty\right)$

Exercise

For the function $f(x) = -2x^2 - 3x - 1$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y-intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{3}{4}$ $x = -\frac{b}{2a}$

$$f\left(-\frac{3}{4}\right) = -\frac{9}{8} + \frac{9}{4} - 1$$
$$= \frac{1}{8}$$

Vertex point: $\left(-\frac{3}{4}, \frac{1}{8}\right)$

b) Axis of symmetry: $x = -\frac{3}{4}$

c) Maximum point @ $\left(-\frac{3}{4}, \frac{1}{8}\right)$

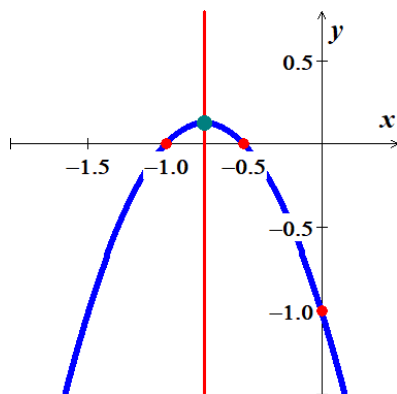
d) $-2x^2 - 3x - 1 = 0$

$$x = -1, -\frac{1}{2}$$

e) $x = 0 \rightarrow y = -1$

f) Domain: \mathbb{R} Range: $\left(-\infty, \frac{1}{8}\right]$

g)



h) Increasing: $\left(-\infty, -\frac{3}{4}\right)$ Decreasing: $\left(-\frac{3}{4}, \infty\right)$

Exercise

For the function $f(x) = -x^2 - 4x + 5$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -2$ $x = -\frac{b}{2a}$

$$f(-2) = -4 + 8 + 5 \\ = 9$$

Vertex point: $(-2, 9)$

b) Axis of symmetry: $x = -2$

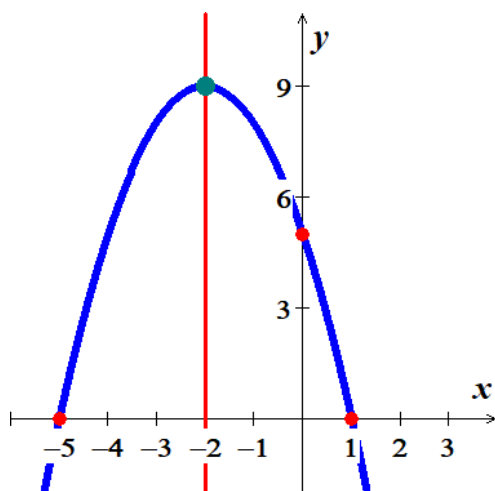
c) Maximum point @ $(-2, 9)$

d) $-x^2 - 4x + 5 = 0$
 $x = 1, -5$

e) $x = 0 \rightarrow y = 5$

f) Domain: \mathbb{R} Range: $(-\infty, 9]$

g)



h) Increasing: $(-\infty, -2)$ Decreasing: $(-2, \infty)$

Exercise

For the function $f(x) = -x^2 + 4x + 2$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y-intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = 2$ | $x = -\frac{b}{2a}$

$$f(2) = -4 + 8 + 2$$

$$= 6$$

Vertex point: $(2, 6)$ |

b) Axis of symmetry: $x = 2$ |

c) Maximum point @ $(2, 6)$ |

d) $-x^2 + 4x + 2 = 0$

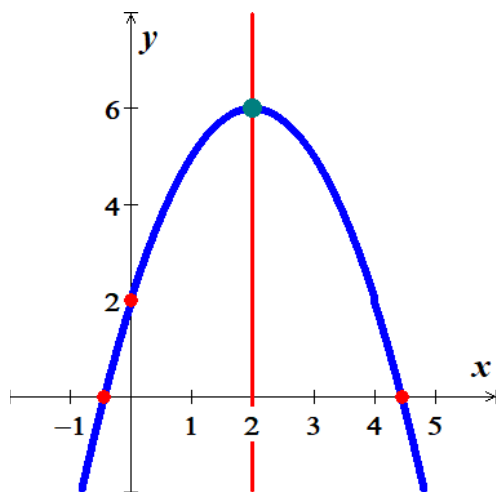
$$x = \frac{-4 \pm \sqrt{16 + 8}}{-2}$$

$$x = 2 \pm \sqrt{6}$$

e) $x = 0 \rightarrow y = 2$ |

f) Domain: \mathbb{R} | Range: $(-\infty, 6]$ |

g)



h) Increasing: $(-\infty, 2)$ | Decreasing: $(2, \infty)$ |

Exercise

For the function $f(x) = -3x^2 + 3x + 7$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y -intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = \frac{1}{2}$ $x = -\frac{b}{2a}$

$$f\left(\frac{1}{2}\right) = -\frac{3}{4} + \frac{3}{2} + 7 = \frac{31}{4}$$

Vertex point: $\left(\frac{1}{2}, \frac{31}{4}\right)$

b) Axis of symmetry: $x = \frac{1}{2}$

c) Maximum point @ $\left(\frac{1}{2}, \frac{31}{4}\right)$

d) $-3x^2 + 3x + 7 = 0$

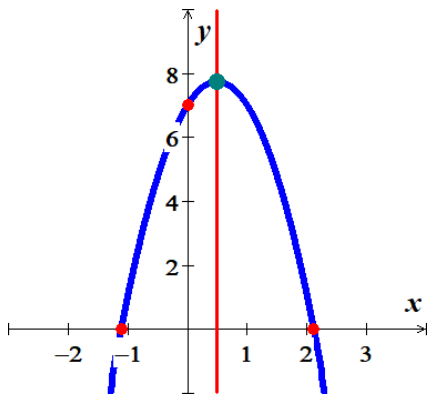
$$x = \frac{-3 \pm \sqrt{93}}{-6}$$

$$x = \frac{3 \pm \sqrt{93}}{6}$$

e) $x = 0 \rightarrow y = 7$

f) Domain: \mathbb{R} Range: $\left(-\infty, \frac{31}{4}\right]$

g)



h) Increasing: $\left(-\infty, \frac{1}{2}\right)$ Decreasing: $\left(\frac{1}{2}, \infty\right)$

Exercise

For the function $f(x) = -x^2 + 2x - 2$

- a) Find the vertex point
- b) Find the line of symmetry
- c) State whether there is a *maximum* or *minimum* value *and* find that value
- d) Find the zeros of $f(x)$
- e) Find the y-intercept
- f) Find the *range* and the *domain* of the function.
- g) Graph the function
- h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = 1$ $x = -\frac{b}{2a}$

$$f(1) = -1 + 2 - 2$$

$$= -1$$

Vertex point: $(1, -1)$

b) Axis of symmetry: $x = 1$

c) Maximum point @ $(1, -1)$

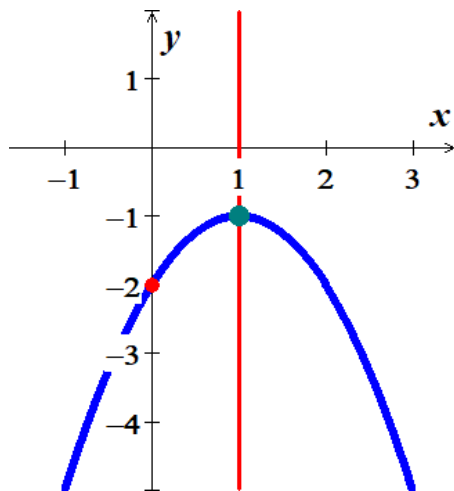
d) $-x^2 + 2x - 2 = 0$

$$x = \frac{-2 \pm \sqrt{-4}}{-2} \quad \mathbb{C}$$

e) $x = 0 \rightarrow y = -2$

f) Domain: \mathbb{R} Range: $(-\infty, -1]$

g)



h) Increasing: $(-\infty, 1)$ Decreasing: $(1, \infty)$

Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $20x = y^2$

Solution

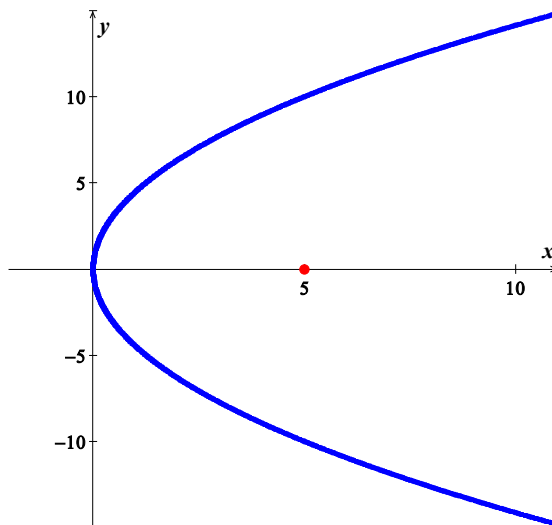
$$20x = y^2 \quad 4px = y^2$$

$$4p = 20 \Rightarrow \boxed{p = 5}$$

Vertex: $(0, 0)$

Focus $(5, 0)$

Directrix: $x = -5$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $2y^2 = -3x$

Solution

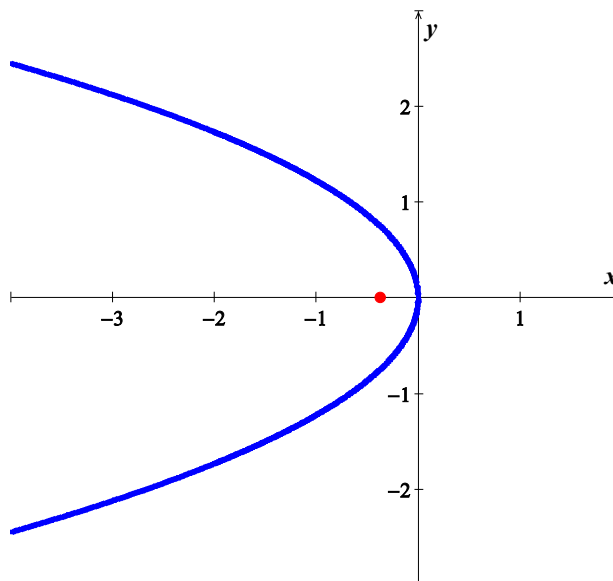
$$y^2 = -\frac{3}{2}x = 4px$$

$$4p = -\frac{3}{2} \Rightarrow \boxed{p = -\frac{3}{8}}$$

Vertex: $(0, 0)$

Focus: $\left(-\frac{3}{8}, 0\right)$

Directrix: $x = \frac{3}{8}$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $(x+2)^2 = -8(y-1)$

Solution

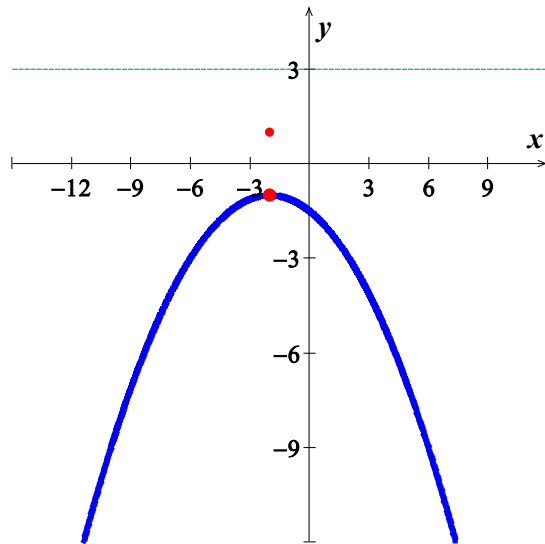
$$(x+2)^2 = 4p(y-1)$$

$$4p = -8 \Rightarrow \boxed{p = -2}$$

$$\text{Vertex: } (-2, 1)$$

$$\text{Focus: } (-2, 1-2) = (-2, -1)$$

$$\text{Directrix: } y = 1+2 \\ \boxed{= 3}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $(x-3)^2 = \frac{1}{2}(y+1)$

Solution

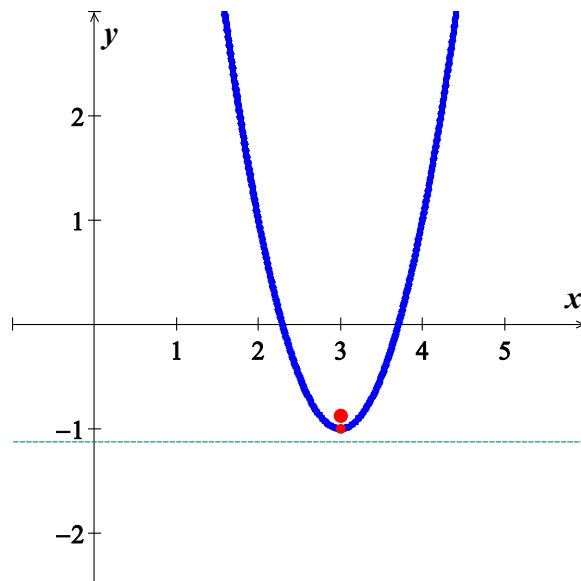
$$(x-3)^2 = 4p(y+1)$$

$$4p = \frac{1}{2} \Rightarrow \boxed{p = \frac{1}{8}}$$

$$\text{Vertex: } (3, -1)$$

$$\text{Focus: } \left(3, -1 + \frac{1}{8}\right) = \left(3, -\frac{7}{8}\right)$$

$$\text{Directrix: } y = -1 - \frac{1}{8} \\ \boxed{= -\frac{9}{8}}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $(y+1)^2 = -12(x+2)$

Solution

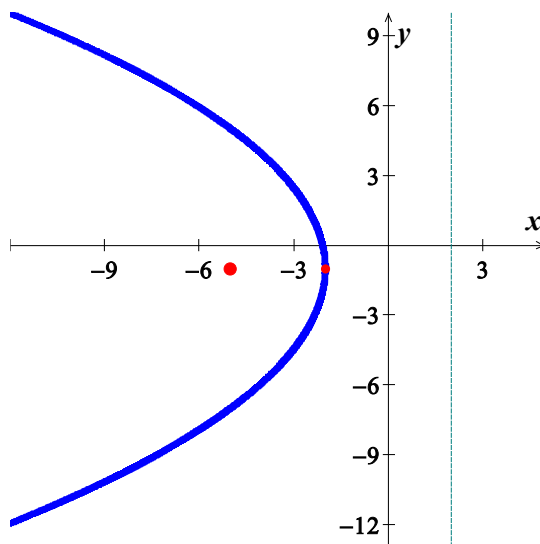
$$(y+1)^2 = 4p(x+2)$$

$$4p = -12 \Rightarrow \boxed{p = -3}$$

$$\text{Vertex: } (-2, -1)$$

$$\text{Focus: } (-2-3, -1) = \underline{\underline{(-5, -1)}}$$

$$\text{Directrix: } x = -1+3 \\ = \underline{\underline{2}}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $y = x^2 - 4x + 2$

Solution

$$y = ax^2 + bx + c \Rightarrow a = 1$$

$$p = \frac{1}{4a} = \frac{1}{4}$$

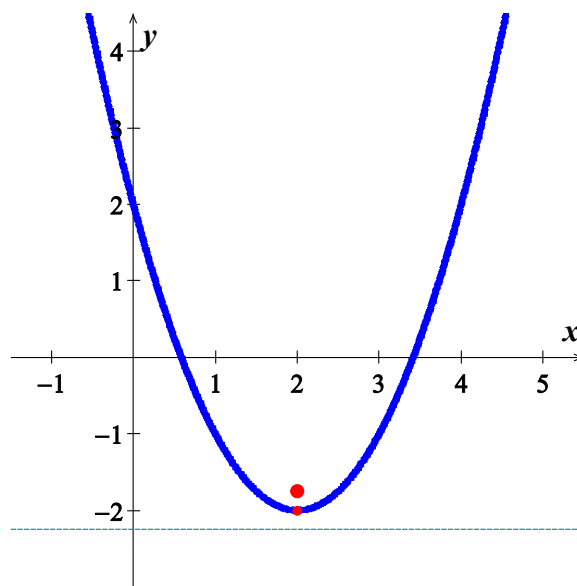
$$\boxed{p = \frac{1}{4}}$$

$$\text{Vertex: } \begin{cases} h = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2 \\ k = 2^2 - 4(2) + 2 = -2 \end{cases}$$

$$\underline{\underline{V = (2, -2)}}$$

$$\text{Focus: } \left(2, -2 + \frac{1}{4}\right) = \left(2, -\frac{7}{4}\right)$$

$$\text{Directrix: } y = -2 - \frac{1}{4} \\ = \underline{\underline{-\frac{9}{4}}}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $y^2 + 14y + 4x + 45 = 0$

Solution

$$y^2 + 14y = -4x - 45$$

$$y^2 + 14y + (7)^2 = -4x - 45 + (7)^2$$

$$(y + 7)^2 = -4x + 4$$

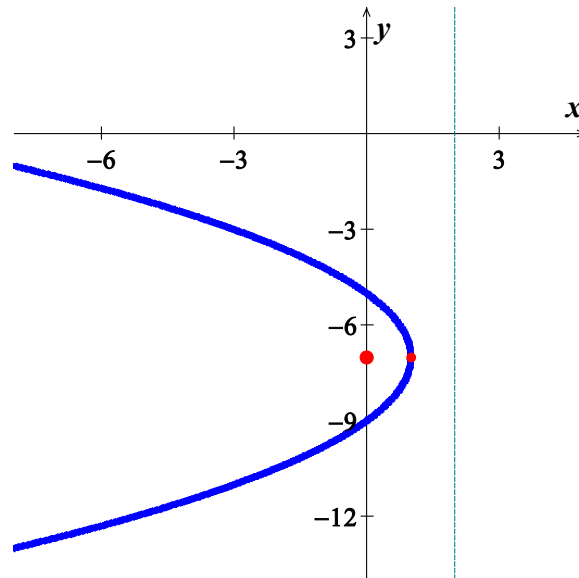
$$(y + 7)^2 = -4(x - 1)$$

$$4p = -4 \Rightarrow \boxed{p = -1}$$

$$\text{Vertex: } (1, -7)$$

$$\text{Focus: } (1 - 1, -7) = \underline{(0, -7)}$$

$$\text{Directrix: } x = 1 + 1 \\ = \underline{2}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $x^2 + 20y = 10$

Solution

$$x^2 = -20y + 10$$

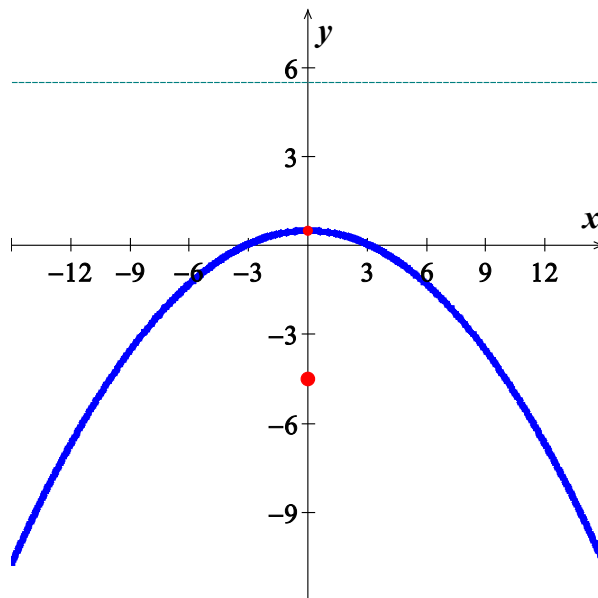
$$x^2 = -20\left(y - \frac{1}{2}\right)$$

$$4p = -20 \Rightarrow \boxed{p = -5}$$

$$\text{Vertex: } \underline{\left(0, \frac{1}{2}\right)}$$

$$\text{Focus: } \left(0, \frac{1}{2} - 5\right) = \underline{\left(0, -\frac{9}{2}\right)}$$

$$\text{Directrix: } y = \frac{1}{2} + 5 \\ = \underline{\frac{11}{2}}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $x^2 = 16y$

Solution

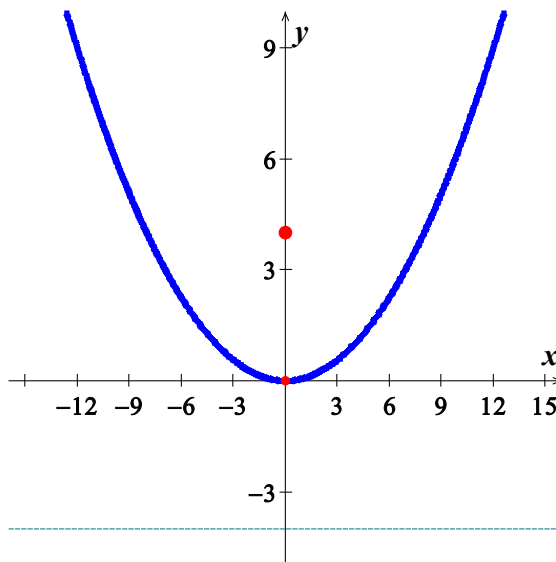
$$x^2 = 16y = 4py$$

$$4p = 16 \Rightarrow \boxed{p = 4}$$

$$\text{Vertex: } \boxed{(0, 0)}$$

$$\text{Focus: } \boxed{(0, 4)}$$

$$\text{Directrix: } \boxed{y = -4}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $x^2 = -\frac{1}{2}y$

Solution

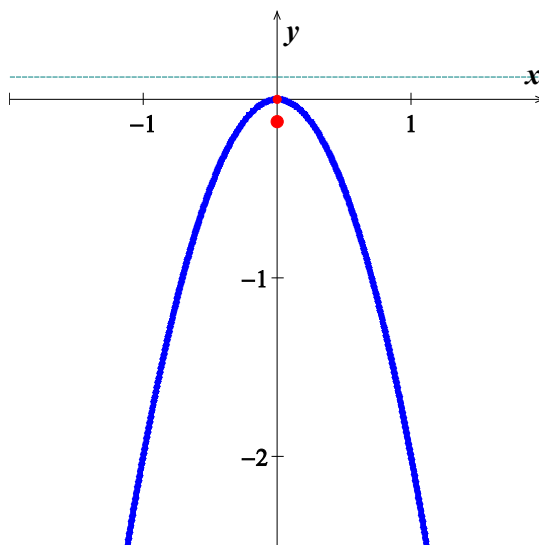
$$x^2 = -\frac{1}{2}y = 4py$$

$$4p = -\frac{1}{2} \Rightarrow \boxed{p = -\frac{1}{8}}$$

$$\text{Vertex: } \boxed{(0, 0)}$$

$$\text{Focus: } \boxed{(0, -\frac{1}{8})}$$

$$\text{Directrix: } \boxed{y = \frac{1}{8}}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $(y+1)^2 = -4(x-2)$

Solution

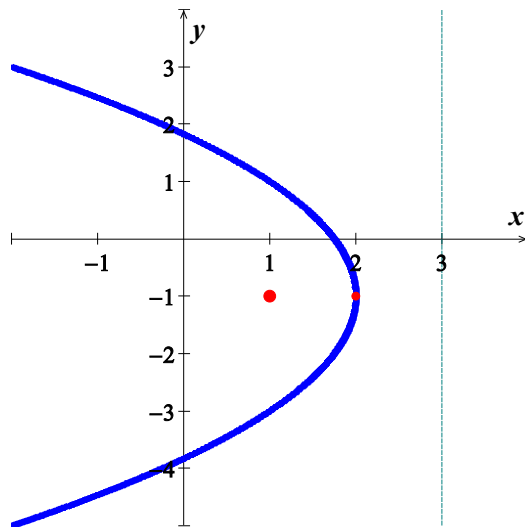
$$(y+1)^2 = 4p(x-2)$$

$$4p = -4 \Rightarrow \boxed{p = -1}$$

$$\text{Vertex: } \boxed{(2, -1)}$$

$$\text{Focus: } (2-1, -1) = \boxed{(1, -1)}$$

$$\text{Directrix: } x = 2+1 \\ = \boxed{3}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $x^2 + 6x - 4y + 1 = 0$

Solution

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 4y - 1 + (3)^2$$

$$(x+3)^2 = 4y + 8$$

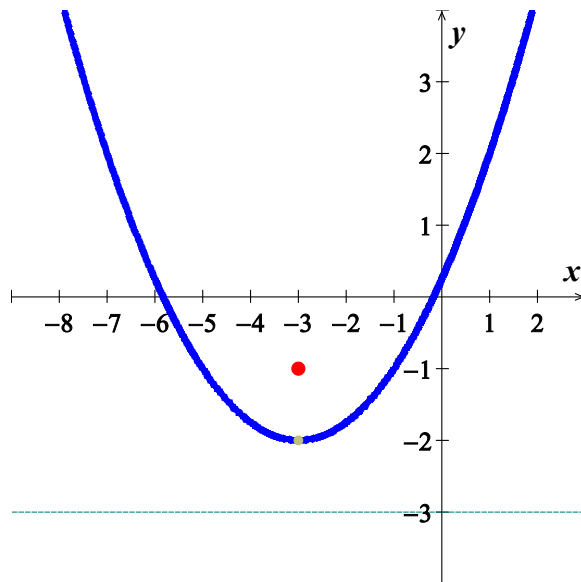
$$(x+3)^2 = 4(y+2)$$

$$4p = 4 \Rightarrow \boxed{p = 1}$$

$$\text{Vertex: } \boxed{(-3, -2)}$$

$$\text{Focus: } (-3, -2+1) = \boxed{(-3, -1)}$$

$$\text{Directrix: } y = -2-1 \\ = \boxed{-3}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $y^2 + 2y - x = 0$

Solution

$$y^2 + 2y = x$$

$$y^2 + 2y + \left(\frac{2}{2}\right)^2 = x + (1)^2$$

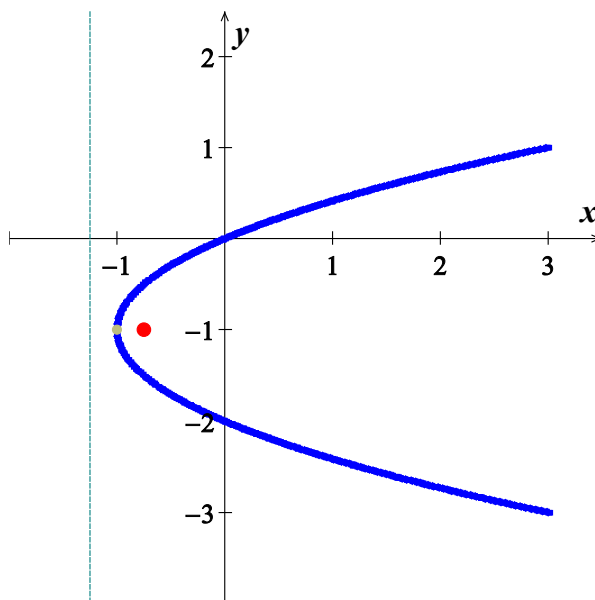
$$(y+1)^2 = (x+1)$$

$$4p = 1 \Rightarrow \boxed{p = \frac{1}{4}}$$

$$\text{Vertex: } \underline{V = (-1, -1)}$$

$$\text{Focus: } F = \left(-1 + \frac{1}{4}, -1\right) \\ = \left(-\frac{3}{4}, -1\right)$$

$$\text{Directrix: } x = -1 - \frac{1}{4} \\ = -\frac{5}{4}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $y^2 - 4y + 4x + 4 = 0$

Solution

$$y^2 - 4y = -4x - 4$$

$$y^2 - 4y + \left(\frac{-4}{2}\right)^2 = -4x - 4 + (-2)^2$$

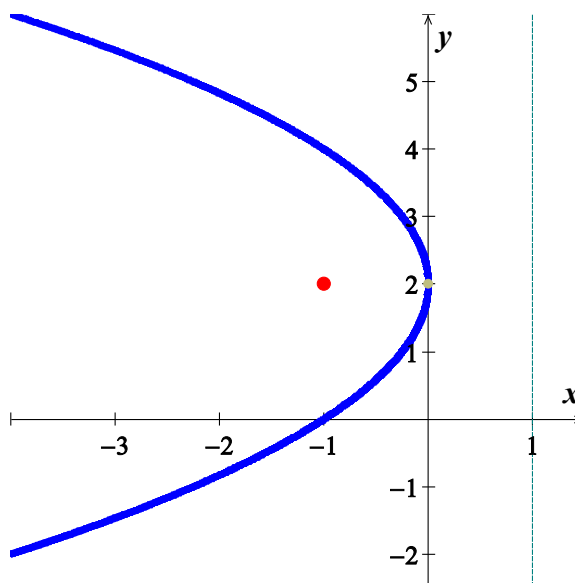
$$(y-2)^2 = -4x$$

$$4p = -4 \Rightarrow \boxed{p = -1}$$

$$\text{Vertex: } \underline{V = (0, 2)}$$

$$\text{Focus: } \underline{F = (-1, 2)}$$

$$\text{Directrix: } \underline{x = 1}$$



Exercise

Find the **vertex**, **focus**, and **directrix** of the parabola. Sketch its graph. $x^2 - 4x - 4y = 4$

Solution

$$x^2 - 4x = 4y + 4$$

$$x^2 - 4x + \left(\frac{-4}{2}\right)^2 = 4y + 4 + (-2)^2$$

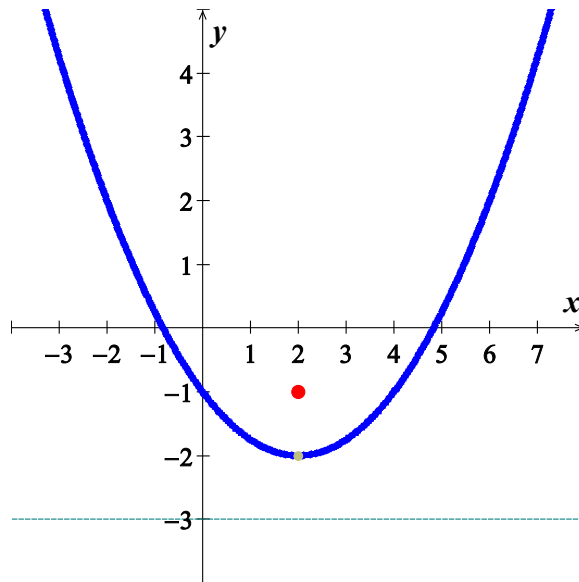
$$(x - 2)^2 = 4(y + 2)$$

$$4p = 4 \Rightarrow \boxed{p = 1}$$

$$\text{Vertex: } V = (2, -2) \mid$$

$$\text{Focus: } F = (2, -2 + 1) \\ = (2, -1) \mid$$

$$\text{Directrix: } y = -2 - 1 \\ = -3 \mid$$



Exercise

Find an equation of the parabola that satisfies the given conditions **Focus** : $F(2, 0)$ **directrix** : $x = -2$

Solution

$$x = -2 = -p \rightarrow p = 2$$

$$y^2 = 4px$$

$$\underline{y^2 = 8x} \mid$$

Exercise

Find an equation of the parabola that satisfies the given conditions **Focus** : $F(0, -4)$ **directrix** : $y = 4$

Solution

$$y = 4 = -p \rightarrow p = -4$$

$$x^2 = 4py$$

$$\underline{x^2 = -16y} \mid$$

Exercise

Find an equation of the parabola that satisfies the given conditions *Focus* : $F(-3, -2)$ *directrix* : $y = 1$

Solution

$$y = 1 = k - p \rightarrow k - p = 1$$

$$\left\{ \begin{array}{l} \boxed{h = -3} \\ k + p = -2 \end{array} \right. \rightarrow \left\{ \begin{array}{l} k + p = -2 \\ k - p = 1 \end{array} \right.$$

$$\Rightarrow 2k = -1 \rightarrow \underline{k = -\frac{1}{2}}$$

$$k - p = 1 \rightarrow p = k - 1$$

$$p = -\frac{1}{2} - 1$$

$$\underline{= -\frac{3}{2}}$$

$$\text{Vertex: } \underline{V = \left(-3, -\frac{1}{2}\right)}$$

$$(x + 3)^2 = 4\left(-\frac{3}{2}\right)\left(y + \frac{1}{2}\right)$$

$$\underline{(x + 3)^2 = -6\left(y + \frac{1}{2}\right)}$$

Exercise

Find an equation of the parabola that satisfies the given conditions *Vertex* : $V(3, -5)$ *directrix* : $x = 2$

Solution

$$\text{Vertex: } V(3, -5) \quad \left\{ \begin{array}{l} h = 3 \\ k = -5 \end{array} \right.$$

$$\text{directrix: } x = 2 = h - p$$

$$p = h - 2$$

$$= 3 - 2$$

$$\underline{= 1}$$

$$(y - k)^2 = 4p(x - h)$$

$$\underline{(y + 5)^2 = 4(x - 3)}$$

Exercise

Find an equation of the parabola that satisfies the given conditions *Vertex* : $V(-2, 3)$ *directrix* : $y = 5$

Solution

$$\text{Vertex : } V(-2, 3) \quad \begin{cases} h = -2 \\ k = 3 \end{cases}$$

$$\text{directrix : } y = 5 = k - p$$

$$p = k - 5$$

$$= 3 - 5$$

$$= -2 \quad |$$

$$(x - h)^2 = 4p(y - k)$$

$$\underline{(x + 2)^2 = -8(y - 3) \quad |}$$

Exercise

Find an equation of the parabola that satisfies the given conditions *Vertex* : $V(-1, 0)$ *focus* : $F(-4, 0)$

Solution

$$\text{Vertex : } V(-1, 0) \quad \begin{cases} h = -1 \\ k = 0 \end{cases}$$

$$\text{focus : } F(-4, 0) \quad \begin{cases} h + p = -4 \\ k = 0 \end{cases}$$

$$p = -4 - h$$

$$= -4 + 1$$

$$= -3 \quad |$$

$$(y - k)^2 = 4p(x - h)$$

$$\underline{y^2 = -12(x + 1) \quad |}$$

Exercise

Find an equation of the parabola that satisfies the given conditions *Vertex* : $V(1, -2)$ *focus* : $F(1, 0)$

Solution

$$\text{Vertex : } V(1, -2) \quad \begin{cases} h = 1 \\ k = -2 \end{cases}$$

$$\text{focus} : F(1, 0) \quad \begin{cases} h=1 \\ k+p=0 \Rightarrow \underline{p=-k=2} \end{cases}$$

$$(x-h)^2 = 4p(y-k)$$

$$\underline{(x-1)^2 = 8(y+2)}$$

Exercise

Find an equation of the parabola that satisfies the given conditions $\text{Vertex} : V(0, 1)$ $\text{focus} : F(0, 2)$

Solution

$$\text{Vertex} : V(0, 1) \quad \begin{cases} h=0 \\ k=1 \end{cases}$$

$$\text{focus} : F(0, 2) \quad \begin{cases} h=0 \\ k+p=2 \Rightarrow \underline{p=2-1=1} \end{cases}$$

$$(x-h)^2 = 4p(y-k)$$

$$\underline{x^2 = 4(y-1)}$$

Exercise

Find an equation of the parabola that satisfies the given conditions $\text{Vertex} : V(3, 2)$ $\text{focus} : F(-1, 2)$

Solution

$$\text{Vertex} : V(3, 2) \quad \begin{cases} h=3 \\ k=2 \end{cases}$$

$$\text{focus} : F(-1, 2) \quad \begin{cases} h+p=-1 \Rightarrow \underline{p=-1-3=-4} \\ k=2 \end{cases}$$

$$(y-k)^2 = 4p(x-h)$$

$$\underline{(y-2)^2 = -16(x-3)}$$

Solution ***Section 1.5 – Applications and Models***

Exercise

A rectangular park is 6 *miles* long and 2 *miles* wide. How long is a pedestrian route that runs diagonally across the park?

Solution

$$d^2 = 6^2 + 2^2$$

$$d^2 = 40$$

$$d = \sqrt{40}$$

$$\approx 6.32 \text{ miles}$$

Exercise

What is the width of a 25-*inch* television set whose height is 15 *inches*?

Solution

$$w^2 + 15^2 = 25^2$$

$$w^2 = 25^2 - 15^2$$

$$w = \sqrt{625 - 225}$$

$$= 20 \text{ in}$$

Exercise

The length of a rectangular sign is 3 *feet* longer than the width. If the sign's area is 54 square *feet*, find its length and width.

Solution

$$\ell = w + 3$$

$$\text{Area} = \ell w = 54$$

$$(w + 3)w = 54$$

$$w^2 + 3w - 54 = 0$$

$$\begin{aligned} w &= \frac{-3 \pm \sqrt{9 + 216}}{2} \\ &= \frac{-3 \pm 15}{2} \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \begin{cases} \frac{-3-15}{2} = -\cancel{9} \\ \frac{-3+15}{2} = \underline{6} \end{cases}$$

$$\begin{aligned} \ell &= 6 + 3 \\ &= \underline{9} \end{aligned}$$

\therefore the length of sign is **6 feet** and width is **3 feet**.

Exercise

A rectangular parking lot has a length that is 3 *yards* greater than the width. The area of the parking lot is 180 square *yards*, find the length and the width.

Solution

$$\ell = w + 3$$

$$\text{Area} = \ell w = 180$$

$$(w + 3)w = 180$$

$$w^2 + 3w - 180 = 0$$

$$w = \frac{-3 \pm \sqrt{9 + 720}}{2}$$

$$= \frac{-3 \pm 27}{2}$$

$$= \begin{cases} \frac{-3-27}{2} = -\cancel{15} \\ \frac{-3+27}{2} = \underline{12} \end{cases}$$

$$\begin{aligned} \ell &= 12 + 3 \\ &= \underline{15} \end{aligned}$$

\therefore the length of sign is **15 feet** and width is **12 feet**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise

Each side of a square is lengthened by 3 *inches*. The area of this new, larger square is 64 square *inches*. Find the length of a side of the original square.

Solution

The new length of each side of a square is $= x + 3$

$$A = (x + 3)^2 = 64$$

$$x + 3 = \pm 8$$

$$x = -3 \pm 8$$

$$= \begin{cases} -3 - 8 = -11 \\ -3 + 8 = 5 \end{cases}$$

∴ the length of the original square side is **5 inches**.

Exercise

Each side of a square is lengthened by 2 inches. The area of this new, larger square is 36 square inches. Find the length of a side of the original square.

Solution

The new length of each side of a square is $= x + 2$

$$A = (x + 2)^2 = 36$$

$$x + 2 = \pm 6$$

$$x = -2 \pm 6$$

$$= \begin{cases} -2 - 6 = -8 \\ -2 + 6 = 4 \end{cases}$$

∴ the length of the original square side is **4 inches**.

Exercise

One number is 5 greater than another. The product of the numbers is 36. Find the numbers.

Solution

$$n = m + 5$$

$$P = mn = 36$$

$$m(m + 5) = 36$$

$$m^2 + 5m - 36 = 0$$

$$m = \frac{-5 \pm \sqrt{25 + 144}}{2}$$

$$= \frac{-5 \pm 13}{2}$$

$$= \begin{cases} \frac{-5 - 13}{2} = -9 \\ \frac{-5 + 13}{2} = 4 \end{cases}$$

$$n = -9 + 5 = -4$$

$$n = 4 + 5 = 9$$

∴ The numbers are **4 & 9 or -4 & -9**

Exercise

One number is 6 less than another. The product of the numbers is 72. Find the numbers.

Solution

$$n = m - 6$$

$$P = mn = 72$$

$$m(m - 6) = 72$$

$$m^2 - 6m - 72 = 0$$

$$m = \frac{6 \pm \sqrt{36 + 288}}{2}$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm 18}{2}$$

$$= \left\{ \begin{array}{l} \frac{6-18}{2} = -6 \\ \frac{6+18}{2} = 12 \end{array} \right\}$$

$$n = -6 - 6 = -12 \quad n = 12 - 6 = 6$$

\therefore The numbers are 6 & 12 or -6 & -12

Exercise

A vacant rectangular lot is being turned into a community vegetable garden measuring 15 meters by 12 meters. A path of uniform width is to surround the garden. If the area of the garden and path combined is 378 square meters, find the width of the path.

Solution

$$\text{Area} = (15 + 2x)(12 + 2x)$$

$$378 = (15 + 2x)(12 + 2x)$$

$$378 = 180 + 30x + 24x + 4x^2$$

$$0 = 180 + 54x + 4x^2 - 378$$

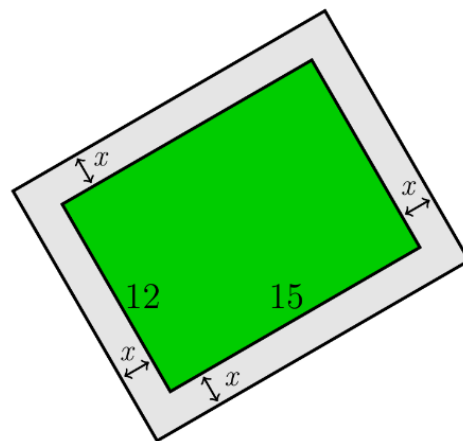
$$0 = 4x^2 + 54x - 198$$

$$4x^2 + 54x - 198 = 0$$

$$x = \frac{-(54) \pm \sqrt{(54)^2 - 4(4)(-198)}}{2(4)}$$

$$= \frac{-54 \pm \sqrt{6084}}{8}$$

$$= \frac{-54 \pm 78}{8}$$



$$= \begin{cases} \frac{-54+78}{8} = \underline{3} \\ \frac{-54-78}{8} = -16.5 \end{cases}$$

\therefore the width of the path is **3 meters**.

Exercise

A pool measuring 10 m by 20 m is surrounded by a path of uniform width. If the area of the pool and the path combined is 600 m^2 , what is the width of the path?

Solution

$$A = lw$$

$$600 = (20 + 2x)(10 + 2x)$$

$$600 = 200 + 40x + 20x + 4x^2$$

$$0 = -600 + 200 + 60x + 4x^2$$

$$0 = -400 + 60x + 4x^2$$

$$0 = -100 + 15x + x^2$$

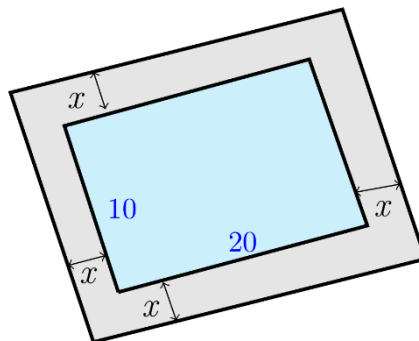
$$x^2 + 15x - 100 = 0$$

$$x = \frac{-15 \pm \sqrt{15^2 + 400}}{2(1)}$$

$$= \frac{-15 \pm \sqrt{625}}{2}$$

$$= \begin{cases} \frac{-15-25}{2} = -20 \\ \frac{-15+25}{2} = 5 \end{cases}$$

\therefore The width of the path is **5 m**



Exercise

You put in flower bed measuring 10 feet by 12 feet. You plan to surround the bed with uniform border of low-growing plants.

- Write a polynomial that describes the area of the uniform border that surrounds your flowers.
- The low growing plants surrounding the flower bed require 1 square foot each when mature. If you have 168 of these plants, how wide a strip around the flower bed should you prepare for the border?

Solution

$$a) \text{ Area} = 4x^2 + 2(12x) + 2(10x)$$

$$= 4x^2 + 44x$$

$$b) \quad A = 4x^2 + 44x = 168 \times 1$$

$$4x^2 + 44x - 168 = 0$$

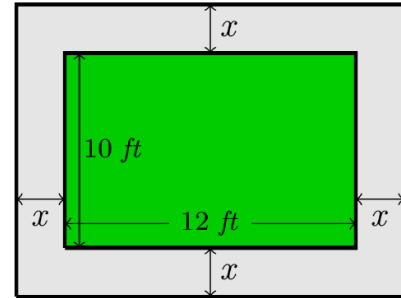
$$x^2 + 11x - 42 = 0$$

$$x = \frac{-11 \pm \sqrt{121 + 168}}{2}$$

$$= \frac{-11 \pm 17}{2}$$

$$= \begin{cases} \frac{-11-17}{2} = -\cancel{14} \\ \frac{-11+17}{2} = \underline{3} \end{cases}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



\therefore The width of the path is **3** feet.

Exercise

A rectangular garden measures 80 feet by 60 feet. A large path of uniform width is to be added along both shorter sides and one longer side of the garden. The landscape designer doing the work wants to double the garden's area with the addition of this path. How wide should the path be?

Solution

$$\text{Total Area} = 2 \times (\text{area of the garden})$$

$$(80 + 2x)(60 + x) = 2(60)(80)$$

$$4800 + 200x + 2x^2 = 9600$$

$$2x^2 + 200x - 4800 = 0$$

$$x^2 + 100x - 2400 = 0$$

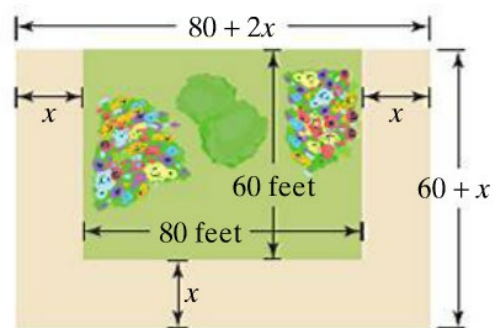
$$x = \frac{-100 \pm \sqrt{10,000 + 9,600}}{2}$$

$$= \frac{-100 \pm 10\sqrt{196}}{2}$$

$$= \frac{-100 \pm 140}{2}$$

$$= \begin{cases} \frac{-100-140}{2} = -\cancel{120} \\ \frac{-100+140}{2} = \underline{20} \end{cases}$$

\therefore the path should be **20** feet.



Exercise

The length of a rectangular poster is 1 *foot* more than the width, and a diagonal of the poster is 5 *feet*. Find the length and the width.

Solution

$$\text{Given: } \ell = w + 1 \quad d = 5$$

$$\ell^2 + w^2 = d^2$$

$$(w + 1)^2 + w^2 = 25$$

$$w^2 + 2w + 1 + w^2 = 25$$

$$2w^2 + 2w - 24 = 0$$

$$w^2 + w - 12 = 0$$

$$w = 3, \quad \cancel{4} \quad |$$

$$\ell = 3 + 1 = \underline{4} \quad |$$

\therefore The length is **4 feet** and the width is **3 feet**.

Exercise

One leg of a right triangle is 7 *cm* less than the length of the other leg. The length of the hypotenuse is 13 *cm*. find the lengths of the legs.

Solution

$$\text{Given: } x = y - 7 \quad d = 13$$

$$x^2 + y^2 = d^2$$

$$(y - 7)^2 + y^2 = 169$$

$$y^2 - 14y + 49 + y^2 - 169 = 0$$

$$2y^2 - 14y - 120 = 0$$

$$y^2 - 7y - 60 = 0$$

$$y = \frac{7 \pm \sqrt{49 + 240}}{2}$$

$$= \frac{7 \pm \sqrt{289}}{2}$$

$$= \frac{7 \pm 17}{2}$$

$$= \left\{ \begin{array}{l} \frac{7-17}{2} = -\cancel{5} \\ \frac{7+17}{2} = \underline{12} \end{array} \right. |$$

$$y = 12$$

$$x = 12 - 7 = 5$$

∴ The length of the leg: **5** & **12** cm.

Exercise

A tent with wires attached to help stabilize it, as shown below. The length of each wire is 8 feet greater than the distance from the ground to where it is attached to the tent.

The distance from the base of the tent to where the wire is anchored exceeds this height by 7 feet, Find the length of each wire used to stabilize the tent.

Solution

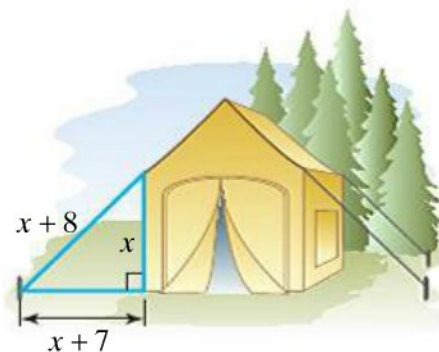
$$x^2 + (x + 7)^2 = (x + 8)^2$$

$$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$$

$$x^2 - 2x - 15 = 0$$

$$x = 5, \text{ } \cancel{3}$$

∴ The length of each wire: **5** feet, **12** feet, and **13** feet.



Exercise

A boat is being pulled into a dock with a rope attached to the boat at water level. Where the boat is 12 feet from the dock, the length of the rope from the boat to the dock is 3 feet longer than twice the height of the dock above the water. Find the height of the dock.

Solution

$$(2h + 3)^2 = h^2 + 12^2$$

$$4h^2 + 12h + 9 = h^2 + 144$$

$$4h^2 + 12h + 9 - h^2 - 144 = 0$$

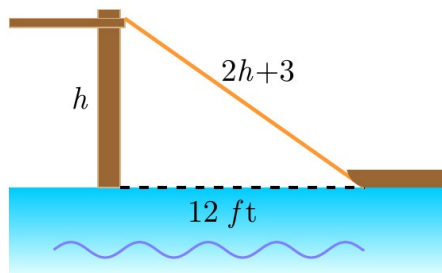
$$3h^2 + 12h - 135 = 0$$

$$h^2 + 4h - 45 = 0$$

$$(h + 9)(h - 5) = 0$$

$$h = -9, \text{ } 5$$

Height = 5 feet.



Exercise

A piece of wire measuring 20 feet is attached to a telephone pole as a guy wire. The distance along the ground from the bottom of the pole to the end of the wire is 4 feet greater than the height where the wire is attached to the pole. How far up the pole does the guy wire reach?

Solution

$$(x+4)^2 + x^2 = 20^2$$

$$x^2 + 8x + 16 + x^2 = 400$$

$$2x^2 + 8x - 384 = 0$$

$$x^2 + 4x - 192 = 0$$

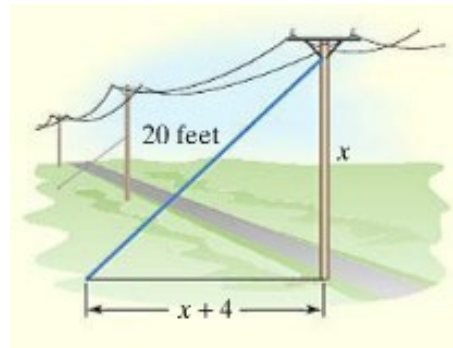
$$x = \frac{-4 \pm \sqrt{16 + 768}}{2}$$

$$= \frac{-4 \pm \sqrt{784}}{2}$$

$$= \frac{-4 \pm 28}{2}$$

$$= \begin{cases} \frac{-4 - 28}{2} = -16 \\ \frac{-4 + 28}{2} = 12 \end{cases}$$

∴ the guy wire reaches the pole at 12 feet high.



Exercise

Logan and Cassidy leave a campsite, Logan biking due north and Cassidy biking due east. Logan bikes 7 km/h slower than Cassidy. After 4 hr, they are 68 km apart. Find the speed of each bicyclist.

Solution

$$4r^2 + [4(r-7)]^2 = 68^2$$

$$16r^2 + 16(r^2 - 14r + 49) = 4624$$

$$16r^2 + 16r^2 - 224r + 784 = 4624$$

$$32r^2 - 224r + 784 - 4624 = 0$$

$$32r^2 - 224r - 3840 = 0$$

$$r^2 - 7r - 120 = 0$$

$$\rightarrow r = -8, 15$$

$$\Rightarrow \text{Cassidy's} = 15 \text{ km/h}$$

$$\Rightarrow \text{Logan's} = 8 \text{ km/h}$$



Exercise

Two trains leave a station at the same time. One train travels due west, and the other travels due south. The train traveling west travels 20 km/hr faster than the train traveling south. After 2 hr. , the trains are 200 km apart. Find the speed of each train.

Solution

Given: $w = s + 20$ & $t = 2$

$$[2(s + 20)]^2 + (2s)^2 = 200^2$$

$$4(s^2 + 40s + 400) + 4s^2 = 40,000$$

$$s^2 + 40s + 400 + s^2 = 10,000$$

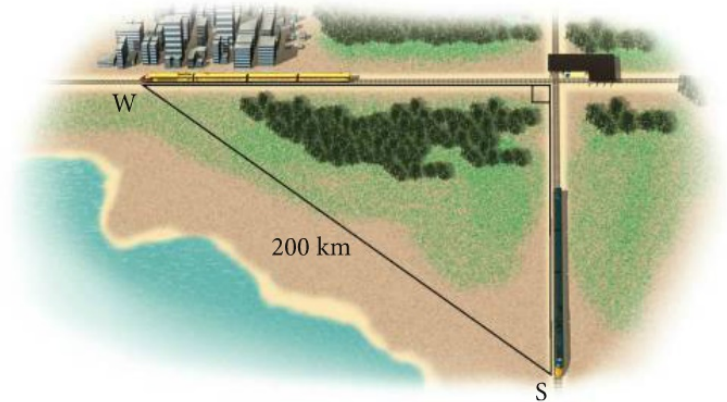
$$2s^2 + 40s + 9,600 = 0$$

$$s^2 + 20s + 4,800 = 0$$

$$\rightarrow s = \cancel{80}, 60$$

\therefore Speed of south train: 60 km/hr

Speed of west train: $60 + 20 = 80 \text{ km/hr}$



Exercise

Towers are 1482 feet tall. How long would it take an object dropped from the top to reach the ground?

Given $s = t^2$

Solution

$$1482 = 16t^2$$

$$\frac{1482}{16} = t^2$$

$$t = \sqrt{\frac{1482}{16}}$$

$$= \frac{\sqrt{1482}}{4}$$

$$\approx 9.624 \text{ sec}$$

Exercise

The formula $P = 0.01A^2 + .05A + 107$ models a woman's normal Point systolic blood pressure, P , an age A . Use this formula to find the age, to the nearest year, of a woman whose normal systolic blood pressure is 115 mm Hg.

Solution

$$0.01A^2 + 0.05A + 107 = 115 \Rightarrow 0.01A^2 + 0.05A - 8 = 0$$

$$\begin{aligned} A &= \frac{-0.05 \pm \sqrt{.05^2 - 4(.01)(-8)}}{2(.01)} \\ &= \frac{-0.05 \pm \sqrt{.0025 + .32}}{.02} \\ &= \frac{-0.05 \pm .567}{.02} \\ &= \begin{cases} \frac{-0.05 - .567}{.02} = \text{~~-31~~ (Not a Solution)} \\ \frac{-0.05 + .567}{.02} = 25.89 \approx 26 \end{cases} \end{aligned}$$

Exercise

A rectangular piece of metal is 10 *in.* longer than it is wide. Squares with sides 2 *in.* long are cut from the four corners, and the flaps folded upward to form an open box. If the volume of the box is 832 *in*³, what were the original dimensions of the piece of metal?

Solution

$$l = w + 10$$

$$\text{Bottom width: } w - 4$$

$$\text{Bottom length: } l - 4 = w + 10 - 4 = w + 6$$

$$V = lwh = (w + 6)(w - 4)2$$

$$= 2(w^2 - 4w + 6w - 24)$$

$$= 2w^2 + 4w - 48$$

$$2w^2 + 4w - 48 = 832$$

$$2w^2 + 4w - 880 = 0$$

$$w^2 + 2w - 440 = 0$$

$$(w + 22)(w - 20) = 0$$

$$w + 22 = 0 \quad w - 20 = 0$$

$$w = -22 \quad w = 20$$

Width of the metal is 20 *in* by the length (20+10) 30 *in*.

Exercise

An astronaut on the moon throws a baseball upward. The astronaut is 6 ft., 6 in., tall, and the initial velocity of the ball is 30 ft/sec . The height s of the ball in feet is given by the equation

$$s = -2.7t^2 + 30t + 6.5$$

Where t is the number of seconds after the ball was thrown.

- a) After how many seconds is the ball 12 feet above the moon's surface?
- b) How many seconds will it take for the ball to return to the surface?

Solution

- a) After how many seconds is the ball 12 feet above the moon's surface?

$$12 = -2.7t^2 + 30t + 6.5$$

$$0 = -2.7t^2 + 30t + 6.5 - 12$$

$$0 = -2.7t^2 + 30t - 5.5$$

$$t = \frac{-30 \pm \sqrt{(30)^2 - 4(-2.7)(-5.5)}}{2(-2.7)}$$

$$\approx \frac{-30 \pm 29}{-5.4}$$

$$t \approx \frac{-30-29}{-5.4}$$

$$t \approx 10.9 \text{ sec}$$

$$t \approx \frac{-30+29}{-5.4}$$

$$t \approx 0.12 \text{ sec}$$

- b) How many seconds will it take for the ball to return to the surface?

$$0 = -2.7t^2 + 30t + 6.5$$

$$t = \frac{-30 \pm \sqrt{(30)^2 - 4(-2.7)(6.5)}}{2(-2.7)} \approx \frac{-30 \pm 31.15}{-5.4}$$

$$t \approx \frac{-30-31.15}{-5.4}$$

$$t \approx 11.32$$

$$t \approx \frac{-30+31.15}{-5.4}$$

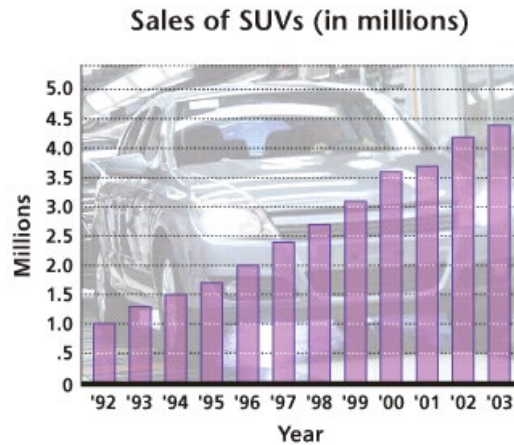
$$t \approx -0.212$$

It will take 11.32 sec.

Exercise

The bar graph shows of SUVs (sport utility vehicles in the US, in *millions*. The quadratic equation

$S = .00579x^2 + .2579x + .9703$ models sales of SUVs from 1992 to 2003, where S represents sales in *millions*, and $x = 0$ represents 1992, $x = 1$ represents 1993 and so on.



- a) Use the model to determine sales in 2002 and 2003. Compare the results to the actual figures of 4.2 million and 4.4 million from the graph.
- b) According to the model, in what year do sales reach 3.5 million? Is the result accurate?

Solution

- a) For 2002 $\Rightarrow x = 10$

$$S = .00579(10)^2 + .2579(10) + .9703$$
$$\approx 4.1 \text{ million}$$

For 2003 $\Rightarrow x = 11$

$$S = .00579(11)^2 + .2579(11) + .9703$$
$$\approx 4.5 \text{ million}$$

- b) $3.5 = .00579x^2 + .2579x + .9703$

$$0 = .00579x^2 + .2579x + .9703 - 3.5$$

$$0 = .00579x^2 + .2579x - 2.5297$$

$$x = \frac{-.2579 \pm \sqrt{(.2579)^2 - 4(.00579)(-2.5297)}}{2(.00579)}$$

$$= \frac{-.2579 \pm \sqrt{.1251}}{.01158}$$

$$x = \frac{-.2579 - .3537}{.01158}$$

$$x \approx -52.8$$

$$x = \frac{-.2579 + .3537}{.01158}$$

$$x \approx 8.3$$

According to the model, the number reached 3.5 *million* in the year 2000. The model closely matches the graph, so it is accurate

Exercise

Cynthia wants to buy a rug for a room that is 20 feet wide and 27 feet long. She wants to leave a uniform strip of floor around the rug. She can afford to buy 170 square feet of carpeting. What dimension should the rug have?

Solution

The area of the rug is:

$$(27 - 2x)(20 - 2x) = 170$$

$$540 - 54x - 40x + 4x^2 = 170$$

$$540 - 94x + 4x^2 - 170 = 0$$

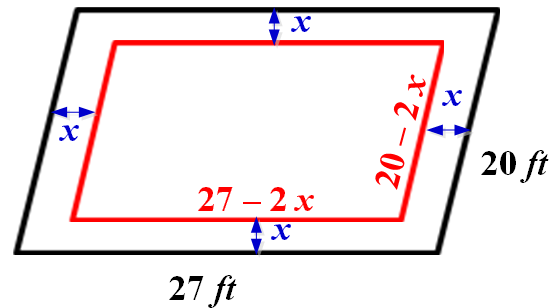
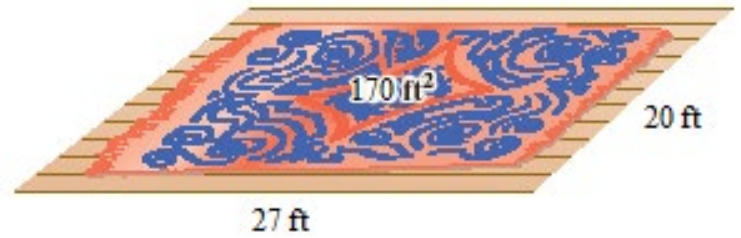
$$4x^2 - 94x + 370 = 0 \quad \text{Solve for } x.$$

$$\cancel{x = 18.5} \quad \text{or} \quad x = 5$$

$$20 - 2x = 20 - 2(5) = 10$$

$$\text{and } 27 - 2x = 27 - 2(5) = 17$$

Therefore, the dimensions are: 10, 20 feet.



Exercise

Erik finds a piece of property in the shape of a right triangle. He finds that the longer leg is 20 m longer than twice the length of the shorter leg. The hypotenuse is 10 m longer than the length of the longer leg. Find the lengths of the sides of the triangular lot.

Solution

l : longer leg

s : shorter leg

Longer leg is 20 m longer than twice the length of the shorter leg

$$l = 2s + 20$$

The hypotenuse is 10 m longer than the length of the longer leg

$$h = l + 10$$

$$= 2s + 20 + 10$$

$$= 2s + 30$$

$$l^2 + s^2 = h^2$$

$$(2s + 20)^2 + s^2 = (2s + 30)^2$$

$$4s^2 + 80s + 400 + s^2 = 4s^2 + 120s + 900$$

$$4s^2 + 80s + 400 + s^2 - 4s^2 - 120s - 900 = 0$$

$$s^2 - 40s - 500 = 0$$

$$(s + 10)(s - 50) = 0$$

$$s + 10 = 0$$

$$s = -10$$

$$s - 50 = 0$$

$$s = 50$$

The shorter length is 50 m.

The longer length is $l = 2s + 20 = 2(50) + 20 = 120$

$$h = l + 10 = 120 + 10 = 130 \text{ m}$$

Exercise

An open box is made from a 10-cm by 20-cm piece of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2 . What is the length of the sides of the squares?

Solution

$$\begin{aligned} \text{Area of the base} &= (10 - 2x)(20 - 2x) \\ &= 200 - 20x - 40x + 4x^2 \\ &= 4x^2 - 60x + 200 \end{aligned}$$

$$4x^2 - 60x + 200 = 96$$

$$4x^2 - 60x + 104 = 0$$

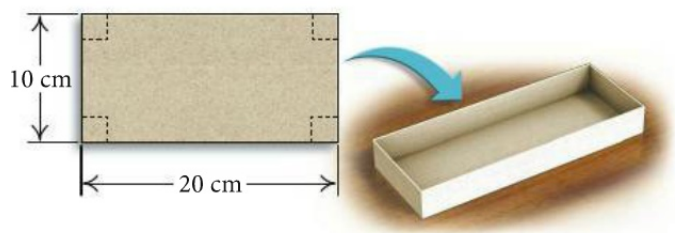
$$x^2 - 15x + 26 = 0$$

$$(x - 13)(x - 2) = 0$$

$$\begin{cases} x - 13 = 0 \rightarrow x = 13 \\ x - 2 = 0 \rightarrow x = 2 \end{cases}$$

$$\Rightarrow x = 2 \text{ (only)}$$

Therefore, the length of the sides are 2 cm.



Exercise

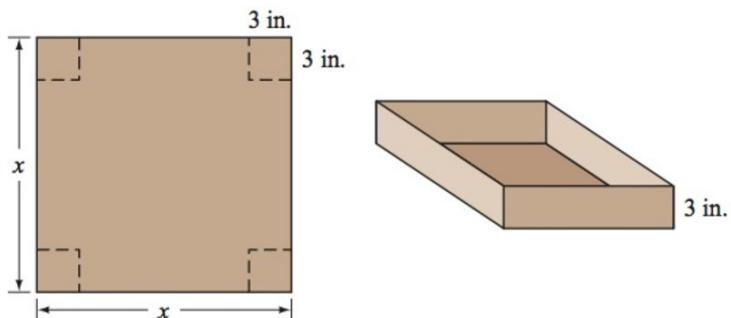
A square piece of cardboard is formed into a box by cutting out 3-inch squares from each of the corners and folding up the sides. If the volume of the box needs to be 126.75 cubic inches, what size square piece of cardboard is needed?

Solution

$$V = 3(x - 6)^2 = 126.75$$

$$(x - 6)^2 = 42.25$$

$$x - 6 = \sqrt{\frac{4225}{100}}$$



$$\begin{aligned}
 x &= 6 + \frac{65}{10} \\
 &= 6 + \frac{13}{2} \\
 &= \frac{25}{2} \\
 &= \underline{12.5 \text{ in.}}
 \end{aligned}$$

Exercise

You want to use 132 *feet* of chain-link fencing to enclose a rectangular region and subdivide the region into two smaller rectangular regions. If the total enclosed area is 576 *square feet*, find the dimensions of the enclosed region.

Solution

$$P = 2l + 3w = 132$$

$$l = \frac{1}{2}(132 - 3w)$$

$$A = lw = 576$$

$$\frac{w}{2}(132 - 3w) = 576$$

$$132w - 3w^2 = 1,152$$

$$3w^2 - 132w + 1,152 = 0$$

$$w^2 - 44w + 384 = 0$$

$$w = \frac{44 \pm \sqrt{1936 - 1536}}{2}$$

$$= \frac{44 \pm \sqrt{400}}{2}$$

$$= \frac{44 \pm 20}{2}$$

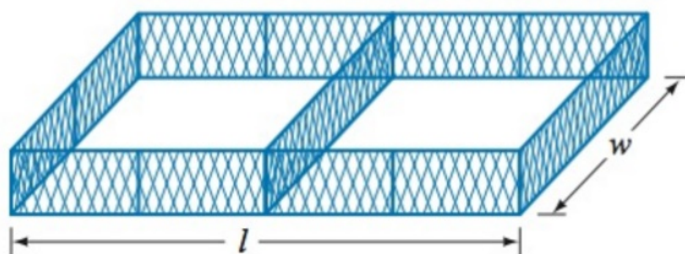
$$= \begin{cases} \frac{44 - 20}{2} = 12 \\ \frac{44 + 20}{2} = 32 \end{cases}$$

$$w = 12 \rightarrow l = \frac{1}{2}(132 - 36) = \underline{48}$$

$$w = 32 \rightarrow l = \frac{1}{2}(132 - 96) = \underline{18}$$

∴ the dimensions: Length **48 feet**, width **12 feet**.

Or Length **18 feet**, width **32 feet**.



Exercise

How far is it from home plate to second base on a baseball diamond?

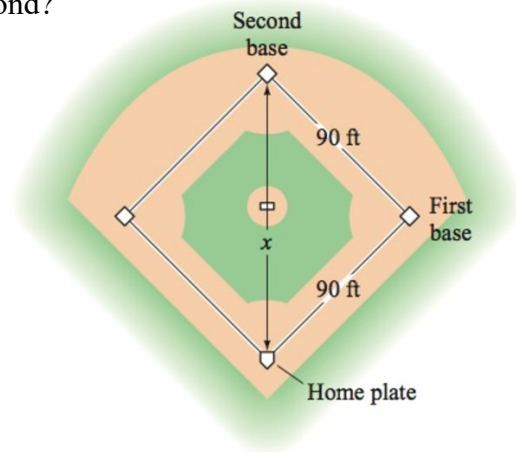
Solution

$$\begin{aligned}x^2 &= 90^2 + 90^2 \\&= 2(90^2)\end{aligned}$$

$$x = 90\sqrt{2}$$

∴ The distance between home plate and second base is

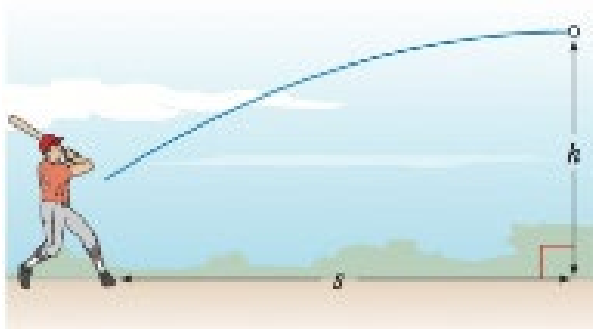
$$90\sqrt{2} \text{ feet}$$



Exercise

Two equations can be used to track the position of a baseball t seconds after it is hit.

For instance, suppose $h = -16t^2 + 50t + 4.5$ gives the height, in *feet*, of a baseball t seconds after it is hit and $s = 103.9t$ gives the horizontal distance, in *feet*, of the ball from home plate t seconds after it is hit.



Use these equations to determine whether this particular baseball will clear a 10-*foot* fence positioned 360 *feet* from home plate.

Solution

$$s = 103.9t = 360$$

$$\begin{aligned}t &= \frac{360}{103.9} \\&= \frac{3600}{1039} \text{ sec} \quad \approx 3.46 \text{ sec}\end{aligned}$$

$$\begin{aligned}h(3.46) &= -16(3.46)^2 + 50(3.46) + 4.5 \\&\approx -14.05\end{aligned}$$

Since the height is negative, then the ball hit the ground before the fence.

∴ The baseball will **not** clear the 10-*foot* fence.

Exercise

A ball is thrown downward with an initial velocity of 5 feet per second from the Golden Gate Bridge, which is 220 feet above the water. How long will it take for the ball to hit the water?

Solution

$$s(t) = -16t^2 - 5t + 220$$

$$s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

$$-16t^2 - 5t + 220 = 0$$

$$t = \frac{5 \pm \sqrt{25 + 4(16)(220)}}{-32}$$

$$= \frac{5 \pm \sqrt{25 + 14,080}}{-32}$$

$$= \frac{-5 + \sqrt{14,105}}{32}$$

$$\therefore \text{It will take for the ball to hit the water } \frac{-5 + \sqrt{14,105}}{32} \approx 3.56 \text{ sec}$$

Exercise

A television screen measures 60 inches diagonally, and its aspect ratio is 16 to 9. This means that the ratio of the width of the screen to the height of the screen is 16 to 9.

Find the width and height of the screen.

Solution

$$(16x)^2 + (9x)^2 = 60^2$$

$$256x^2 + 81x^2 = 3600$$

$$337x^2 = 3600$$

$$x^2 = \frac{3600}{337}$$

$$x = \sqrt{\frac{3600}{337}}$$

$$= \frac{60}{\sqrt{337}} \text{ in.} \approx 3.268 \text{ in.}$$



$$\therefore \text{The width of TV is } 16 \times \frac{60}{\sqrt{337}} = \frac{960}{\sqrt{337}} \text{ in.} \approx 52 \text{ in.}$$

$$\text{The height of TV is } 9 \times \frac{60}{\sqrt{337}} = \frac{540}{\sqrt{337}} \text{ in.} \approx 29.4 \text{ in.}$$

Exercise

A company makes rectangular solid candy bars that measures 5 inches by 2 inches by 0.5 inch. Due to difficult financial times, the company has decided to keep the price of the candy bar fixed and reduce the volume of the bar by 20%. What should the dimensions be for the new candy bar if the company keeps the height at 0.5 inch and makes length of the candy bar 3 inches longer than the width?

Solution

The original volume is given:

$$\begin{aligned} V &= 5 \times 2 \times \frac{1}{2} \\ &= 5 \text{ in}^3 \end{aligned}$$

Reduction the volume of the bar by 20% which leave 80% of the new candy.

$$\begin{aligned} V_{new} &= (.8)(5) \\ &= 4 \text{ in}^3 \end{aligned}$$

$$V = lwh$$

$$4 = (w+3)(w)\left(\frac{1}{2}\right)$$

$$w^2 + 3w = 8$$

$$w^2 + 3w - 8 = 0$$

$$w = \frac{-3 \pm \sqrt{9+32}}{2}$$

$$= \frac{-3 \pm \sqrt{41}}{2}$$

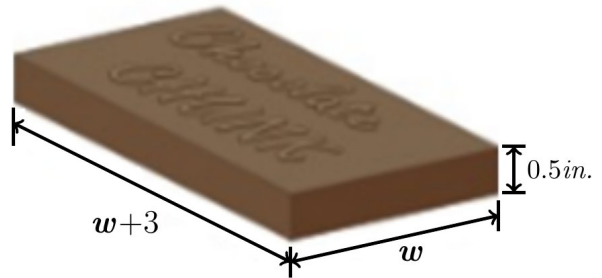
$$w = \frac{-3 + \sqrt{41}}{2}$$

$$\cancel{w = \frac{-3 - \sqrt{41}}{2}} < 0$$

$$\therefore \text{The new width of the chocolate bar is } = \frac{-3 + \sqrt{41}}{2} \text{ in.}$$

$$\approx 1.7 \text{ in.}$$

$$\begin{aligned} \text{The new length of the chocolate bar is } & \frac{-3 + \sqrt{41}}{2} + 3 = \frac{3 + \sqrt{41}}{2} \text{ in.} \\ & \approx 4.7 \text{ in.} \end{aligned}$$



Exercise

A company makes rectangular solid candy bars that measures 5 inches by 2 inches by 0.5 inch. Due to difficult financial times, the company has decided to keep the price of the candy bar fixed and reduce the volume of the bar by 20%. What should the dimensions be for the new candy bar if the company keeps the height at 0.5 inch and makes length of the candy bar 2.5 times as long as its width?

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Reduction the volume of the bar by 20% which leave 80% of the new candy.

$$\begin{aligned} V_{\text{new}} &= (.8)(5) \\ &= 4 \text{ in}^3 \end{aligned}$$

$$V = lwh$$

$$4 = \left(\frac{3}{2}w\right)(w)\left(\frac{1}{2}\right)$$

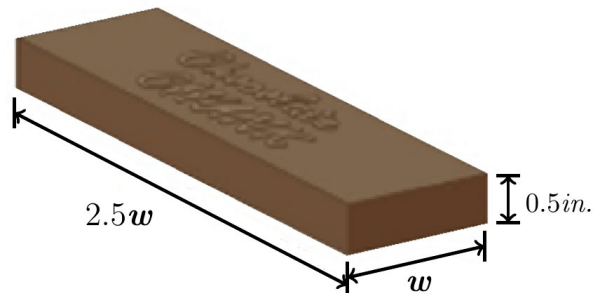
$$3w^2 = 16$$

$$w^2 = \frac{16}{3}$$

$$w = \frac{4}{\sqrt{3}}$$

$$\therefore \text{The new width of the chocolate bar is } \frac{4\sqrt{3}}{3} \text{ in.}$$

$$\text{The new length of the chocolate bar is } 3\frac{4\sqrt{3}}{3} = 4\sqrt{3} \text{ in.}$$



Exercise

A picture frame measures 28 cm by 32 cm and is of uniform width. What is the width of the frame if 192 cm² of the picture shows?

Solution

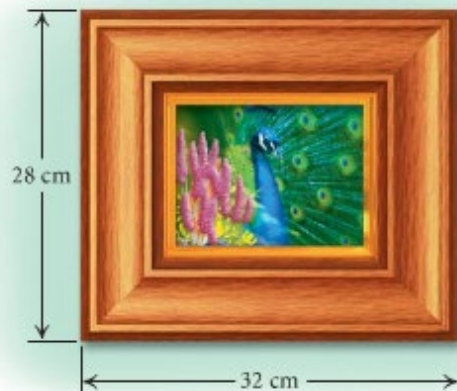
$$\text{Area of the picture} = (32 - 2x)(28 - 2x) = 192$$

$$896 - 64x - 56x + 4x^2 = 192$$

$$896 - 120x + 4x^2 - 192 = 0$$

$$4x^2 - 120x + 704 = 0$$

$$x^2 - 30x + 176 = 0$$

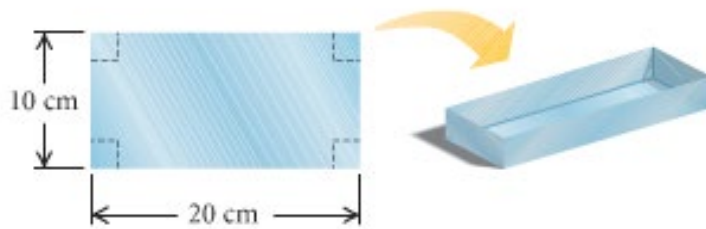


$$\begin{cases} x - 8 = 0 \rightarrow \underline{x = 8} \\ x - 22 = 0 \rightarrow \underline{x = 22} \end{cases}$$

\therefore The width of the frame is **8 cm**.

Exercise

An open box is made from a 10-cm by 20-cm of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2 . What is the length of the sides of the squares?



Solution

$$\text{Area of the base} = (20 - 2x)(10 - 2x) = 96$$

$$200 - 40x - 20x + 4x^2 = 96$$

$$4x^2 - 60x + 200 - 96 = 0$$

$$4x^2 - 60x + 104 = 0 \quad \text{Solve for } x$$

$$\boxed{x = 2, \cancel{x = 8}}$$

The length of the sides of the squares is 3-cm

Exercise

You have 600 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river.

- Find the length and width of the plot that will maximize the area.
- What is the largest area that can be enclosed?

Solution

$$a) \quad P = l + 2w$$

$$600 = l + 2w \rightarrow l = 600 - 2w$$

$$A = (600 - 2w)w \quad A = lw$$

$$= 600w - 2w^2$$

$$= -2w^2 + 600w$$

$$w = -\frac{600}{2(-2)} \quad x_{\text{vertex}} = -\frac{b}{2a}$$

$$\underline{= 150 \text{ feet}} \mid$$

$$l = 600 - 2w$$

$$\underline{= 300 \text{ feet}} \mid$$

$$b) \quad A = lw = (300)(150)$$

$$\underline{= 45000 \text{ ft}^2} \mid$$

Exercise

You have 60 yards of fencing to enclosed a rectangular region.

- Find the dimensions of the rectangle that maximize the enclosed area.
- What is the maximum area?

Solution

$$a) \quad P = 2(\ell + w)$$

$$60 = 2(\ell + w)$$

$$\ell + w = 30$$

$$\underline{\ell = 30 - w} \mid$$

$$A = (30 - w)w$$

$$= -w^2 + 30w$$

$$w = \frac{30}{2}$$

$$x_{\text{vertex}} = -\frac{b}{2a}$$

$$\underline{= 15 \text{ yards}} \mid$$

$$\ell = 30 - 15$$

$$\underline{= 15 \text{ yards}} \mid$$

The dimensions of the rectangle 15×15

$$b) \quad \text{Area} = 15 \times 15$$

$$\underline{= 225 \text{ yard}^2} \mid$$

Exercise

You have 80 yards of fencing to enclosed a rectangular region.

- Find the dimensions of the rectangle that maximize the enclosed area.
- What is the maximum area?

Solution

$$a) \quad P = 2(\ell + w)$$

$$80 = 2(\ell + w)$$

$$\ell + w = 40$$

$$\ell = 40 - w$$

$$A = (40 - w)w$$

$$= -w^2 + 40w$$

$$w = \frac{40}{2}$$

$$x_{\text{vertex}} = -\frac{b}{2a}$$

$$= 20 \text{ yards}$$

$$\ell = 40 - 20$$

$$= 20 \text{ yards}$$

The dimensions of the rectangle 20×20

$$b) \text{ Area} = 20 \times 20$$

$$= 400 \text{ yard}^2$$

Exercise

The sum of the length ℓ and the width w of a rectangle tangular area is 240 *meters*.

- Write w as a function of ℓ .
- Write the area A as a function of ℓ .
- Find the dimensions that produce the greatest area.

Solution

$$a) P = 2(\ell + w)$$

$$240 = 2(\ell + w)$$

$$\ell + w = 120$$

$$w = 120 - \ell$$

$$b) A = \ell(120 - \ell)$$

$$= -\ell^2 + 120\ell$$

$$c) \ell = \frac{120}{2}$$

$$x_{\text{vertex}} = -\frac{b}{2a}$$

$$= 60 \text{ m}$$

$$w = 120 - 60$$

$$= 60 \text{ m}$$

The dimensions of the rectangle 60×60

Exercise

You use 600 *feet* of chainlink fencing to enclose a rectangular region and to subdivide the region into two smaller rectangular regions by placing a fence parallel to one of the sides.

- Write w as a function of l .
- Write the area A as a function of l .
- Find the dimensions that produce the greatest area.

Solution

$$a) \quad P = 2\ell + 3w$$

$$600 = 2\ell + 3w$$

$$w = \frac{1}{3}(600 - 2\ell)$$

$$b) \quad A = \ell \frac{1}{3}(600 - 2\ell)$$

$$= -\frac{2}{3}\ell^2 + 200\ell$$

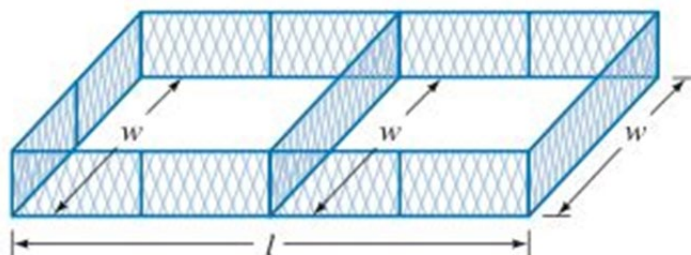
$$c) \quad \ell = 200 \frac{3}{4}$$

$$= 150 \text{ ft}$$

$$w = \frac{1}{3}(600 - 300)$$

$$= 100 \text{ ft}$$

$$x_{\text{vertex}} = -\frac{b}{2a}$$



Exercise

You use 1,200 *feet* of chainlink fencing to enclose a rectangular region and to subdivide the region into three smaller rectangular regions by placing a fence parallel to one of the sides.

- Write w as a function of l .
- Write the area A as a function of l .
- Find the dimensions that produce the greatest area.

Solution

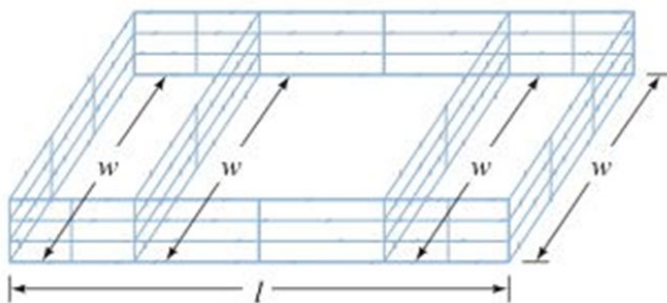
$$a) \quad P = 2\ell + 4w$$

$$1,200 = 2\ell + 4w$$

$$w = 300 - \frac{1}{2}\ell$$

$$b) \quad A = \ell \left(300 - \frac{1}{2}\ell \right)$$

$$= -\frac{1}{2}\ell^2 + 300\ell$$



$$c) \quad \underline{\ell = 300 \text{ ft}} \quad x_{\text{vertex}} = -\frac{b}{2a}$$

$$w = 300 - 150$$

$$\underline{= 150 \text{ ft}}$$

Exercise

A landscaper has enough stone to enclose a rectangular pond next to existing garden wall of the house with 24 feet of stone wall. If the garden wall forms one side of the rectangle.

- What is the maximum area that the landscaper can enclose?
- What dimensions of the pond will yield this area?

Solution

$$a) \quad P = \ell + 2w$$

$$24 = \ell + 2w$$

$$\underline{\ell = 24 - 2w}$$

$$A = (24 - 2w)w$$

$$= -2w^2 + 24w$$

$$w = \frac{24}{4}$$

$$\underline{= 6 \text{ ft}}$$

$$\ell = 24 - 12$$

$$\underline{= 12 \text{ ft}}$$

$$\text{Area} = 12 \times 6$$

$$\underline{= 72 \text{ ft}^2}$$

- The dimensions of the rectangle 6×12 feet



Exercise

A berry farmer needs to separate and enclose two adjacent rectangular fields, one for strawberries and one for blueberries. If a lake forms one side of the fields and 1,000 feet of fencing is available, what is the largest total area that can be enclosed?

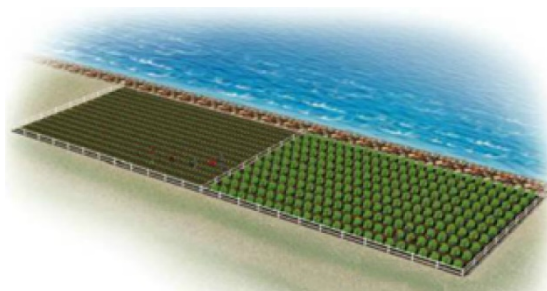
Solution

$$P = \ell + 3w$$

$$1,000 = \ell + 3w$$

$$\underline{\ell = 1,000 - 3w}$$

$$A = (1,000 - 3w)w$$



$$= -3w^2 + 1,000w$$

$$w = \frac{1,000}{6} \qquad x_{\text{vertex}} = -\frac{b}{2a}$$

$$= \frac{500}{3} \text{ ft}$$

$$\ell = 1,000 - 500$$

$$= 500 \text{ ft}$$

$$\text{Area} = 500 \times \frac{500}{3}$$

$$= \frac{250,000}{3} \text{ ft}^2$$

Exercise

A fourth-grade class decides to enclose a rectangular garden, using the side of the school as one side of the rectangle. What is the maximum area that the class can enclose with 32 feet of fence? What should the dimensions of the garden be in order to yield this area?

Solution

$$\text{Perimeter: } P = l + 2w = 32$$

$$l = 32 - 2w$$

$$\text{Area: } A = lw$$

$$A = (32 - 2w)w$$

$$= 32w - 2w^2$$

$$= -2w^2 + 32w$$

$$w = -\frac{32}{2(-2)} \qquad x_{\text{vertex}} = -\frac{b}{2a}$$

$$= 8$$

$$l = 32 - 2(8)$$

$$= 16$$

$$A = lw = (16)(8)$$

$$= 128 \text{ ft}^2$$



Exercise

A rancher needs to enclose two adjacent rectangular corrals, one for cattle and one for sheep. If a river forms one side of the corrals and 240 yards of fencing is available, what is the largest total area that can be enclosed?

Solution

Perimeter: $P = l + 3w = 240$

$$l = 240 - 3w$$

Area: $A = lw$

$$A = (240 - 3w)w$$

$$= 240w - 3w^2$$

$$= -3w^2 + 240w$$

$$w = -\frac{240}{2(-3)} \quad x_{\text{vertex}} = -\frac{b}{2a}$$

$$= 40$$

$$l = 240 - 3(40)$$

$$= 120$$

$$A = lw = (120)(40)$$

$$= 4800 \text{ yd}^2$$



Exercise

A Norman window is a rectangle with a semicircle on top. Sky Blue Windows is designing a Norman window that will require 24 feet of trim on the outer edges. What dimensions will allow the maximum amount of light to enter a house?

Solution

Perimeter of the semi-circle $= \frac{1}{2}(2\pi x)$

Perimeter of the rectangle $= 2x + 2y$

Total perimeter: $\pi x + 2x + 2y = 24$

$$2y = 24 - \pi x - 2x$$

$$y = 12 - \frac{\pi}{2}x - x$$

$$\text{Area} = \frac{1}{2}(\pi x^2) + (2x)y$$

$$= \frac{\pi}{2}x^2 + 2x\left(12 - \frac{\pi}{2}x - x\right)$$



A Norman window

$$= \frac{\pi}{2}x^2 + 24x - \pi x^2 - 2x^2$$

$$= 24x - \left(\frac{\pi}{2} + 2\right)x^2$$

$$= -\left(\frac{\pi}{2} + 2\right)x^2 + 24x$$

$$x = -\frac{24}{2\left(-\frac{\pi}{2} - 2\right)}$$

$$x_{\text{vertex}} = -\frac{b}{2a}$$

$$= -\frac{24}{-2\left(\frac{\pi + 4}{2}\right)}$$

$$= \frac{24}{\pi + 4}$$

$$y = 12 - \frac{\pi}{2} \frac{24}{\pi + 4} - \frac{24}{\pi + 4}$$

$$= \frac{24\pi + 96 - 24\pi - 48}{2(\pi + 4)}$$

$$= \frac{24}{\pi + 4}$$

Exercise

A Norman window has the shape of a rectangle surmounted by a semicircle. The exterior perimeter of the window is 48 feet.

Find the height h and the radius r that will allow the maximum amount of light to enter the window?

Solution

$$\begin{aligned} \text{Perimeter of the semi-circle} &= \frac{1}{2}(2\pi r) \\ &= \pi r \end{aligned}$$

$$\text{Perimeter of the rectangle} = 2r + 2h$$

Total perimeter:

$$\pi r + 2r + 2h = 48$$

$$2h = 48 - \pi r - 2r$$

$$h = 24 - \frac{1}{2}\pi r - r$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}\pi r^2 + (2r)h \\ &= \frac{1}{2}\pi r^2 + 2r\left(24 - \frac{1}{2}\pi r - r\right) \\ &= \frac{1}{2}\pi r^2 + 48r - \pi r^2 - 2r^2 \end{aligned}$$



$$= -\left(\frac{1}{2}\pi + 2\right)r^2 + 48r$$

$$r = -\frac{48}{2\left(-\frac{\pi}{2} - 2\right)} \quad x_{\text{vertex}} = -\frac{b}{2a}$$

$$= \frac{48}{\pi + 4}$$

$$\begin{aligned} h &= 24 - \left(\frac{\pi}{2} + 1\right)r \\ &= 24 - \left(\frac{\pi + 2}{2}\right)\frac{48}{\pi + 4} \\ &= 24 - 24\frac{\pi + 2}{\pi + 4} \\ &= 24\left(1 - \frac{\pi + 2}{\pi + 4}\right) \\ &= \frac{48}{\pi + 4} \end{aligned}$$

Exercise

A Norman window has the shape of a rectangle surmounted by a semicircle. It requires 36 feet of trim on the outer edges. What dimensions will allow the maximum amount of light to enter a house?

Solution

$$\begin{aligned} \text{Perimeter of the semi-circle} &= \frac{1}{2}(2\pi r) \\ &= \pi r \end{aligned}$$

$$\text{Perimeter of the rectangle} = 2r + 2h$$

Total perimeter:

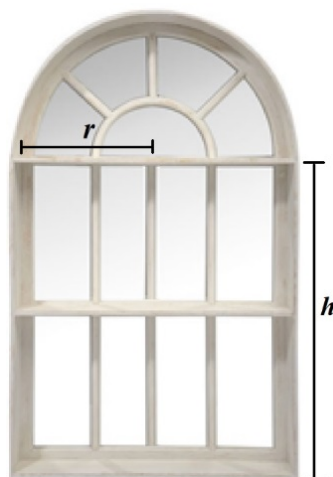
$$\pi r + 2r + 2h = 36$$

$$2h = 36 - \pi r - 2r$$

$$h = 18 - \frac{1}{2}\pi r - r$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}\pi r^2 + (2r)h \\ &= \frac{1}{2}\pi r^2 + 2r\left(18 - \frac{1}{2}\pi r - r\right) \\ &= \frac{1}{2}\pi r^2 + 36r - \pi r^2 - 2r^2 \\ &= -\left(\frac{1}{2}\pi + 2\right)r^2 + 36r \end{aligned}$$

$$r = -\frac{36}{2\left(-\frac{\pi}{2} - 2\right)} \quad r_{\text{vertex}} = -\frac{b}{2a}$$



$$= \frac{36}{\pi + 4} \quad |$$

$$\begin{aligned} h &= 18 - \left(\frac{\pi}{2} + 1 \right) r \\ &= 18 - \left(\frac{\pi + 2}{2} \right) \frac{36}{\pi + 4} \\ &= 18 - 18 \frac{\pi + 2}{\pi + 4} \\ &= 18 \left(1 - \frac{\pi + 2}{\pi + 4} \right) \\ &= \frac{36}{\pi + 4} \quad | \end{aligned}$$

Exercise

The temperature $T(t)$, in degrees Fahrenheit, during the day can be modeled by the equation

$$T(t) = -0.7t^2 + 9.4t + 59.3, \text{ where } t \text{ is the number of hours after 6:00 AM.}$$

- At what time the temperature a maximum?
- What is the maximum temperature?

Solution

$$\begin{aligned} a) \quad t &= -\frac{9.4}{2(-0.7)} \\ &= \frac{94}{14} \\ &= \frac{47}{7} \text{ hrs} \\ &= \left(6 + \frac{5}{7} \right) \text{ hrs} \\ &= 6 \text{ hrs} \quad \frac{5}{7} \text{ hr} \frac{60 \text{ min}}{\text{hr}} \\ &= 6 \text{ hrs} \quad \frac{300}{7} \text{ min} \\ &= 6 \text{ hrs} \quad 42 \text{ min} \quad \frac{6}{7} \text{ min} \\ &= 6 \text{ hrs} \quad 42 \text{ min} \quad \frac{6}{7} \text{ min} \frac{60 \text{ sec}}{\text{min}} \\ &= 6 \text{ hrs} \quad 42 \text{ min} \quad \frac{360}{7} \text{ sec} \\ &\approx 6 \text{ hrs} \quad 42 \text{ min} \quad 51 \text{ sec} \quad | \end{aligned}$$

The maximum temperature is around 12:43 PM |

$$b) \quad T\left(\frac{47}{7}\right) = -\frac{7}{10} \left(\frac{2209}{49} \right) + \frac{94}{10} \left(\frac{47}{7} \right) + \frac{593}{10}$$

$$\begin{aligned}
&= -\frac{2209}{70} + \frac{2209}{35} + \frac{593}{10} \\
&= \frac{2209}{70} + \frac{593}{10} \\
&= \frac{6360}{70} \\
&= \frac{636}{7} \text{ } ^\circ F \\
&\approx 90.86 \text{ } ^\circ F
\end{aligned}$$

Exercise

When a softball player swings a bat, the amount of energy $E(t)$, in *joules*, that is transferred to the bat can be approximated by the function

$$E(t) = -279.67t^2 + 82.86t$$

Where $0 \leq t \leq 0.3$ and t is measured in *seconds*. According to this model, what is the maximum energy of the bat?

Solution

$$\begin{aligned}
t &= -\frac{82.86}{2(-279.67)} & t_{\text{vertex}} &= -\frac{b}{2a} \\
&= \frac{8286}{2(27967)} \\
&= \frac{4243}{27967} \\
&\approx 0.15 \text{ sec}
\end{aligned}$$

The maximum energy is

$$\begin{aligned}
E(0.15) &= -279.67(0.15)^2 + 82.86(0.15) \\
&\approx 6.136 \text{ joules}
\end{aligned}$$

Exercise

Some softball fields are built in a parabolic mound shape so that water will drain off the field. A model for the shape of a certain field is given by

$$h(x) = -0.0002348x^2 + 0.0375x$$

Where $h(x)$ is the height, in *feet*, of the field at a distance of x *feet* from one sideline. Find the maximum height of the field.

Solution

$$x = -\frac{0.0375}{2(-0.0002348)} \quad x_{\text{vertex}} = -\frac{b}{2a}$$

$$\approx 79.86 \text{ ft}$$

The maximum height of the field is

$$h(79.86) = -0.0002348(79.86)^2 + 0.0375(79.86)$$

$$\approx 4.5 \text{ feet}$$

Exercise

The fuel efficiency for a certain midsize car is given by

$$E(v) = -0.018v^2 + 1.476v + 3.4$$

Where $E(v)$ is the fuel efficiency in *miles per gallon* for a car traveling v in *miles per hour*.

- What speed will yield the maximum fuel efficiency?
- What is the maximum fuel efficiency for this car?

Solution

$$a) \quad v = -\frac{1.476}{2(-0.018)} \quad v_{\text{vertex}} = -\frac{b}{2a}$$

$$= 41 \text{ mi/hr}$$

$$b) \quad E(41) = -0.018(41)^2 + 1.476(41) + 3.4$$

$$\approx 33.658 \text{ mi/gal}$$

Exercise

If the initial velocity of a projectile is 128 *feet per second*, then the height h , in *feet*, is a function of time t , in *seconds*, given by the equation

$$h(t) = -16t^2 + 128t$$

- Find the time t when the projectile achieves its maximum height.
- Find the maximum height of the projectile.
- Find the time t when the projectile hits the ground.

Solution

$$a) \quad t = -\frac{128}{-32} \quad t_{\text{vertex}} = -\frac{b}{2a}$$

$$= 4 \text{ sec}$$

$$b) \quad h(4) = -16(4) + 128(4)$$

$$= 256 \text{ ft} \mid$$

$$c) \quad h(t) = -16t^2 + 128t = 0$$

$$-16t(t - 8) = 0$$

$$t = 0 \quad t = 8$$

The projectile hits the ground in $t = 8 \text{ sec} \mid$

Exercise

If the initial velocity of a projectile is 64 feet per second and an initial height of 80 feet, then the height h , in feet, is a function of time t , in seconds, given by the equation

$$h(t) = -16t^2 + 64t + 80$$

a) Find the time t when the projectile achieves its maximum height.

b) Find the maximum height of the projectile.

c) Find the time t when the projectile hits the ground.

Solution

$$a) \quad t = -\frac{64}{-32} \\ = 2 \text{ sec} \mid$$

$$t_{\text{vertex}} = -\frac{b}{2a}$$

$$b) \quad h(2) = -16(4) + 64(2) + 80 \\ = 144 \text{ ft} \mid$$

$$c) \quad h(t) = -16t^2 + 64t + 80 = 0$$

$$t = \frac{-64 \pm \sqrt{4,096 + 5,120}}{-32} \\ = \frac{64 \pm \sqrt{9,216}}{32} \\ = \frac{64 \pm 96}{32} \\ = \begin{cases} \frac{64 - 96}{32} = - \\ \frac{64 + 96}{32} = 5 \end{cases}$$

The projectile hits the ground in $t = 5 \text{ sec} \mid$

Exercise

If the initial velocity of a projectile is 100 *feet per second* and an initial height of 20 *feet*, then the height h , in *feet*, is a function of time t , in *seconds*, given by the equation

$$h(t) = -16t^2 + 100t + 20$$

- a) Find the time t when the projectile achieves its maximum height.
- b) Find the maximum height of the projectile.
- c) Find the time t when the projectile hits the ground.

Solution

$$\begin{aligned} a) \quad t &= -\frac{100}{-32} & t_{\text{vertex}} &= -\frac{b}{2a} \\ &= \frac{25}{8} \text{ sec} \\ &= 3.125 \text{ sec} \end{aligned}$$

$$\begin{aligned} b) \quad h(3.125) &= -16(3.125)^2 + 100(3.125) + 20 \\ &= 176.25 \text{ ft} \end{aligned}$$

$$\begin{aligned} c) \quad h(t) &= -16t^2 + 100t + 20 = 0 \\ t &= \frac{-100 \pm \sqrt{10,000 + 1,280}}{-32} \\ &= \frac{64 \pm \sqrt{11,280}}{32} \\ &= \begin{cases} \frac{64 - 106.2}{32} = - \\ \frac{64 + 106.2}{32} = 5.3 \end{cases} \end{aligned}$$

The projectile hits the ground in $t = 5.3 \text{ sec}$

Exercise

A frog leaps from a stump 3.5-foot-high and lands 3.5 *feet* from the base of the stump.

It is determined that the height of the frog as a function of its distance, x , from the base of the stump is given by the function $h(x) = -0.5x^2 + 0.75x + 3.5$ where h is in feet.

- a) How high is the frog when its horizontal distance from the base of the stump is 2 *feet*?
- b) At what two distances from the base of the stump after is jumped was the frog 3.6 *feet* above the ground?
- c) At what distance from the base did the frog reach its highest point?
- d) What was the maximum height reached by the frog?

Solution

a) At $x = 2 \text{ ft}$. Find $h(x = 2)$

$$h(2) = -0.5(2^2) + 0.75(2) + 3.5$$

$$= 3 \text{ ft}$$

b) $h(x) = -0.5x^2 + 0.75x + 3.5 = 3.6$

$$-0.5x^2 + 0.75x + 3.5 - 3.6 = 0$$

$$-0.5x^2 + 0.75x - .1 = 0$$

Solve for x : $x = 0.1, 1.4 \text{ ft}$

c) The distance from the base for the frog to reach the highest point is

$$x = -\frac{b}{2a} = -\frac{.75}{2(-.5)} = .75 \text{ ft}$$

d) Maximum height:

$$h(x = .75) = -0.5(.75)^2 + 0.75(.75) + 3.5 = 3.78 \text{ ft}$$

Exercise

The height of an arch is given by

$$h(x) = -\frac{3}{64}x^2 + 27, \quad -24 \leq x \leq 24$$

Where $|x|$ is the horizontal distance in *feet* from the center of the arch to the ground

a) What is the maximum height of the arch?

b) What is the height of the arch 10 *feet* to the right of center?

c) How far from the center is the arch 8 *feet* tall?

Solution

a) $x = 0 \text{ ft}$ $x_{\text{vertex}} = -\frac{b}{2a}$

$$h(0) = 27 \text{ ft}$$

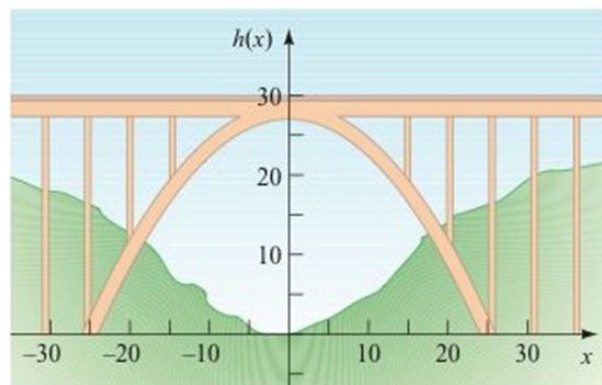
b) $h(10) = -\frac{3}{64}(100) + 27$

$$= -\frac{75}{16} + 27$$

$$= \frac{357}{16}$$

$$= 22.3125 \text{ ft}$$

c) $h(x) = -\frac{3}{64}x^2 + 27 = 8$



$$-\frac{3}{64}x^2 = -19$$

$$x^2 = \frac{1,216}{3}$$

$$x = \pm \sqrt{\frac{1,216}{3}}$$

$$= \pm 8 \sqrt{\frac{19}{3}}$$

$$\approx \pm 20.13 \text{ ft}$$

Exercise

A weightless environment can be created in an airplane by flying in a series of parabolic paths. This is one method that NASA uses to train astronauts for the experience of weightlessness. Suppose the height h , in feet, of NASA's airplane is modeled by

$$h(t) = -6.6t^2 + 430t + 28,000$$

Where t is the time, in seconds, after the plane enters its parabolic path.

Find the maximum height of the plane.

Solution

$$t = \frac{430}{13.2}$$

$$= \frac{4300}{132}$$

$$= \frac{1,075}{33}$$

$$\approx 32.58 \text{ sec}$$

$$t_{\text{vertex}} = -\frac{b}{2a}$$

$$h(32.58) = -6.6(32.58)^2 + 430(32.58) + 28,000$$

$$\approx 35,000 \text{ ft}$$

Exercise

You drop a screwdriver from the top of an elevator shaft. Exactly 5 seconds later, you hear the sound of the screwdriver hitting the bottom of the shaft. The speed of sound is 1,100 ft/sec. How tall is the elevator shaft?

Solution

$$t_1 + t_2 = 5$$

$$s(t) = 16t^2$$

$$t^2 = \frac{s}{16}$$

$$t_1 = \frac{\sqrt{s}}{4}$$

$$s = 1,100 t_2$$

$$t_2 = \frac{s}{1,100}$$

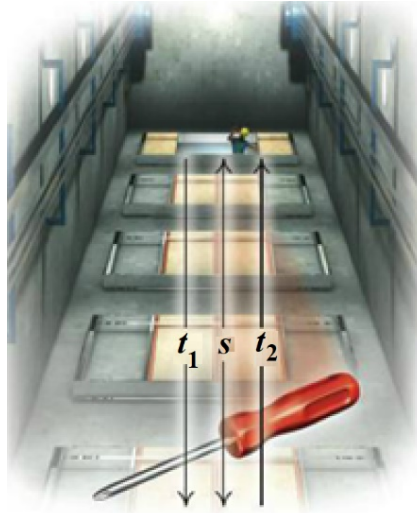
$$t_1 + t_2 = 5$$

$$\frac{\sqrt{s}}{4} + \frac{s}{1,100} = 5$$

$$s + 275\sqrt{s} - 5,500 = 0$$

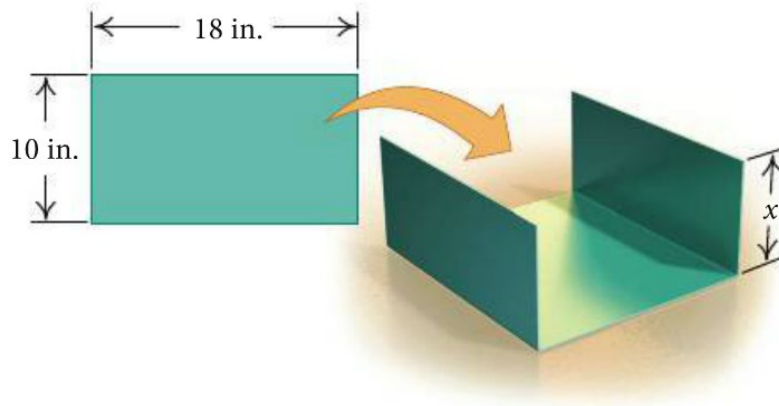
$$\sqrt{s} = \begin{cases} \frac{-275 - 312.5}{2} = - \\ \frac{-275 + 312.5}{2} = 18.725 \end{cases}$$

$$s = 350.6 \text{ feet}$$



Exercise

A company plans to produce a one-compartment vertical file by bending the long side of a 10-in. by 18-in. sheet of metal along two lines to form a U-shape. How tall should the file be in order to maximize the volume that it can hold?



Solution

Height = x

If the length is 18 in.

Width of the base = $10 - 2x$

$$\text{Volume} = 18x(10 - 2x)$$

$$= -36x^2 + 180x$$

$$x = \frac{180}{72}$$

$$x_{\text{vertex}} = -\frac{b}{2a}$$

$$= \frac{5}{2} \text{ in.} \mid$$

$$= 2.5 \text{ in.} \mid$$

$$\text{Max. Area} = 18 \frac{5}{2} (10 - 5)$$

$$= 225 \text{ in}^3 \mid$$

If the length is 10 in.

Width of the base = $18 - 2x$

$$\text{Volume} = 10x(18 - 2x)$$

$$= -20x^2 + 180x$$

$$x = \frac{180}{40} \qquad x_{\text{vertex}} = -\frac{b}{2a}$$

$$= \frac{9}{2} \text{ in.} \mid$$

$$= 4.5 \text{ in.} \mid$$

$$\text{Max. Area} = 10 \frac{9}{2} (18 - 9)$$

$$= 405 \text{ in}^3 \mid$$

To maximize the volume, the length should be 10 in. and bent on 18 in. side with 4.5 in. height to give a volume of 405 in³

Exercise

The sum of the base and the height of a triangle is 20 cm. Find the dimensions for which the area is a maximum.

Solution

$$b + h = 20$$

$$b = 20 - h$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(20 - h)h$$

$$= -\frac{1}{2}h^2 + 10h$$

$$h = 10 \text{ cm} \mid \qquad h_{\text{vertex}} = -\frac{b}{2a}$$

$$b = 20 - 10$$

$$= 10 \text{ cm} \mid$$

The triangle dimensions for the maximum area is 10 × 10 cm

Exercise

The sum of the base and the height of a parallelogram is 14 *in.* Find the dimensions for which the area is a maximum.

Solution

$$b + h = 14$$

$$b = 14 - h$$

$$\text{Area} = bh$$

$$= (14 - h)h$$

$$= -h^2 + 14h$$

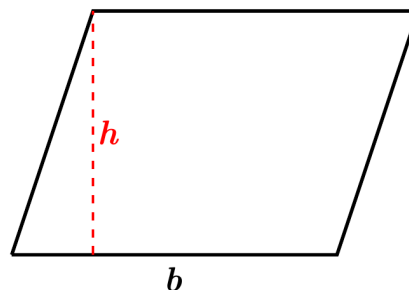
$$\underline{h = 7 \text{ in.}}$$

$$h_{\text{vertex}} = -\frac{b}{2a}$$

$$b = 14 - 7$$

$$\underline{= 7 \text{ in.}}$$

The parallelogram dimensions for the maximum area is $7 \times 7 \text{ cm}$



Exercise

An arch in the shape of a parabola has the dimensions shown in the figure. How wide is the arch 9 *feet* up?

Solution

$$\text{Vertex: } V(0, 12)$$

$$(x - h)^2 = 4p(y - k)$$

$$(x - 0)^2 = 4p(y - 12)$$

$$x^2 = 4p(y - 12)$$

The parabola passes through the point (6, 0)

$$6^2 = 4p(0 - 12)$$

$$-48p = 36$$

$$p = -\frac{36}{48} = -\frac{3}{4}$$

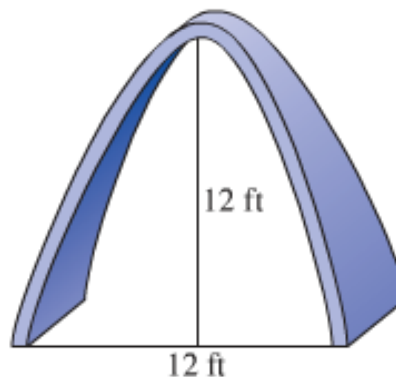
The equation is: $x^2 = -3(y - 12)$

The arch is 9 feet up that is the *y*-coordinate,

$$x^2 = -3(9 - 12) = 9$$

$$\underline{x = 3}$$

The width is $2(3) = \underline{6 \text{ feet}}$



Exercise

The cable in the center portion of a bridge is supported as shown in the figure to form a parabola. The center support is 10 *feet* high, the tallest supports are 210 *feet* high, and the distance between the two tallest supports is 400 *feet*. Find the height of the remaining supports if the supports are evenly spaced.

Solution

$$\text{Vertex: } V(0, 10)$$

$$(x - h)^2 = 4p(y - k)$$

$$(x - 0)^2 = 4p(y - 10) \Rightarrow x^2 = 4p(y - 10)$$

The parabola passes through the point (200, 210)

$$200^2 = 4p(210 - 10)$$

$$800p = 200^2$$

$$p = \frac{40000}{800}$$

$$= 50 \text{ |}$$

$$\text{The equation is: } x^2 = 200(y - 10)$$

The x -coordinate of one of the supports is 100.

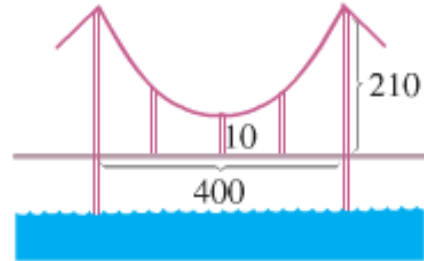
$$100^2 = 200(y - 10)$$

$$y - 10 = \frac{10000}{200} = 50$$

$$y = 50 + 10$$

$$= 60 \text{ feet |}$$

\therefore The height is 60 *feet*



Exercise

A headlight is being constructed in the shape of a paraboloid with depth 4 *inches* and diameter 5 *inches*. Determine the distance d that the bulb should be form the vertex in order to have the beam of light shine straight ahead.

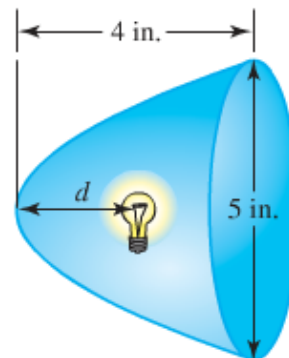
Solution

Let the vertex be at the origin $V(0, 0)$

$$\text{The equation is: } y^2 = 4px$$

Which it passes through the point $V(4, 2.5)$

$$(2.5)^2 = 4p(4)$$

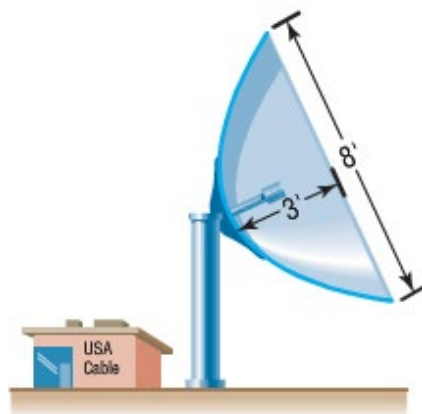


$$p = \frac{(2.5)^2}{16} = \frac{25}{64}$$

The bulb should be $\frac{25}{64} \approx 0.39$ inch from the vertex

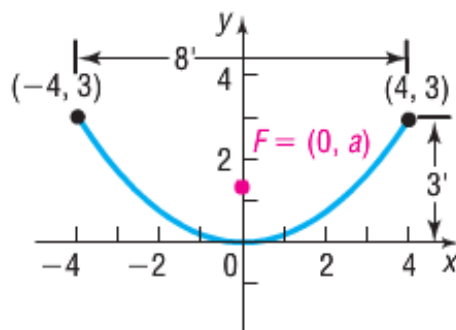
Exercise

A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point, where the receiver is located. If the dish is 8 feet across at its opening and 3 feet deep at its center, at what position should the receiver be placed? That is, where is the focus?



Solution

From the figure, we can draw the parabola used to form the dish on a rectangular coordinate system so that the vertex of the parabola is at the origin and its focus on the positive y -axis.



The equation from of the parabola is: $x^2 = 4py$

Since $(4, 3)$ is a point on the graph

$$4^2 = 4p(3)$$

$$p = \frac{16}{12} = \frac{4}{3}$$

Therefore, the receiver should be located $\frac{4}{3}$ ft from the base of the dish, along its axis of symmetry.

Exercise

A cable TV receiving dish is in the shape of a paraboloid of revolution. Find the location of the receiver, which is placed at the focus, if the dish is 6 feet across at its opening and 2 feet deep.

Solution

Given: Parabola is 6 feet across and 2 feet deep.

Let the vertex of the parabola is at $(0, 0)$ and it opens up, then the equation of the parabola has the form $x^2 = 4ay$

Therefore, the point $(3, 2)$ and $(-3, 2)$ are on the parabola.

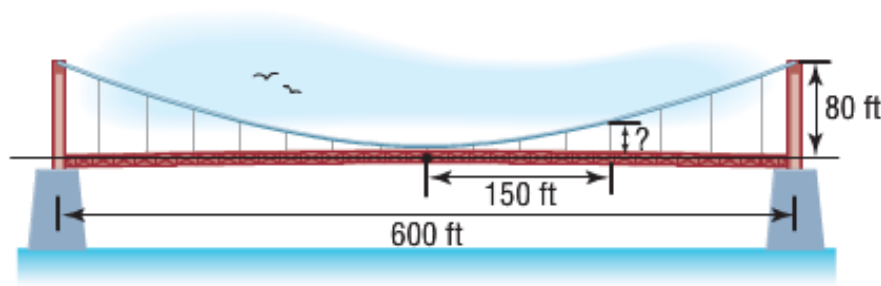
$$3^2 = 4a(2) \rightarrow a = \frac{9}{8} = 1.125$$

Where a is the distance from the vertex to the focus.

Thus, the receiver (located at the focus) is 1.125 feet or 13.5 inches from the base of the dish, along the axis of the parabola.

Exercise

The cables of a suspension bridge are in the shape of a parabola, as shown below. The towers supporting the cable are 600 feet apart and 80 feet high. If the cables touch the road surface midway between the towers, what is the height of the cable from the road at a point 150 feet from the center of the bridge?



Solution

Let the vertex of the parabola is at $(0, 0)$ and it opens up, then the equation of the parabola has the form $x^2 = cy$

The point $(300, 80)$ is a point on the parabola.

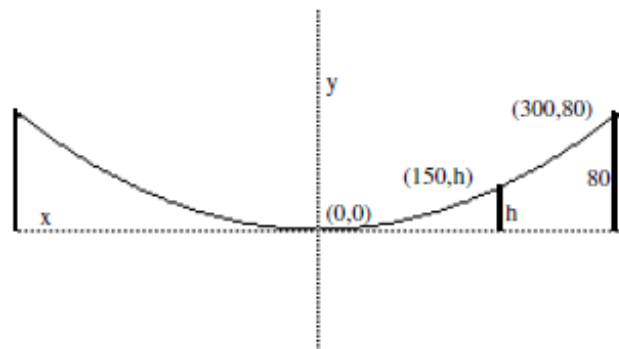
$$300^2 = c(80) \rightarrow c = \frac{300^2}{80} = 1125$$

$$x^2 = 1125y$$

The height of the cable 150 feet from the center is:

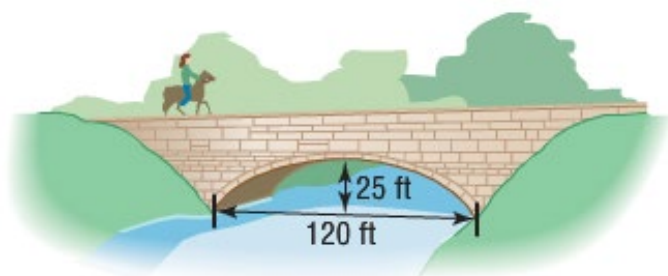
$$150^2 = 1125h \rightarrow h = \frac{150^2}{1125} = 20$$

The height of the cable 150 feet from the center is 20 feet.



Exercise

A bridge is built in the shape of a parabolic arch. The bridge has a span of 120 *feet* and a maximum height of 25 *feet*. Choose a suitable rectangular coordinate system and find the height of the arch at distances of 10, 30, and 50 *feet* from the center.



Solution

Let the vertex of the parabola is at $(0, 0)$ and it opens down, then the equation of the parabola has the form $x^2 = cy$

The point $(60, -25)$ is a point on the parabola.

$$60^2 = c(-25) \rightarrow c = \frac{60^2}{-25} = -144$$

$$x^2 = -144y$$

The height of the arch at

Distance 10:

$$10^2 = -144y$$

$$y = \frac{100}{-144} \approx -0.69$$

The height of the bridge 10 feet from the center is about $25 - 0.69 = 24.31$ ft

Distance 30:

$$30^2 = -144y$$

$$y = \frac{900}{-144} \approx -6.25$$

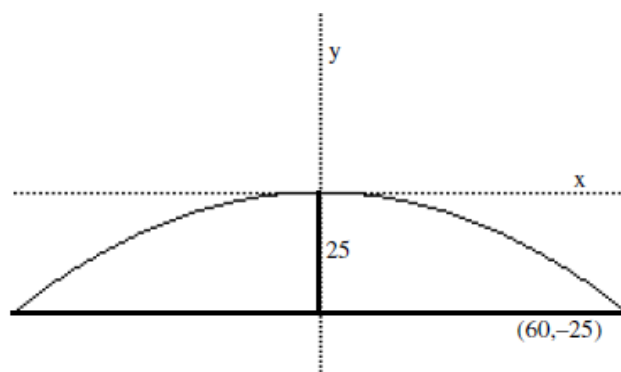
The height of the bridge 30 feet from the center is about $25 - 6.25 = 18.75$ ft

Distance 50:

$$50^2 = -144y$$

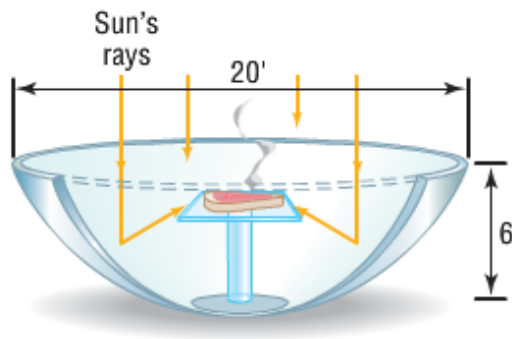
$$y = \frac{2500}{-144} \approx -17.36$$

The height of the bridge 50 feet from the center is about $25 - 17.36 = 7.64$ ft



Exercise

A mirror is shaped like a paraboloid of revolution and will be used to concentrate the rays of the sun at its focus, creating a heat source. If the mirror is 20 *feet* across at its opening and is 6 *feet* deep, where will the heat source be concentrated?



Solution

Let the vertex of the parabola is at $(0, 0)$ and it opens up, then the equation of the parabola has the form $x^2 = 4ay$

Since the parabola is 20 *feet* across and 6 feet deep.

The points $(10, 6)$ and $(-10, 6)$ are on the parabola.

$$10^2 = 4a(6) \rightarrow a = \frac{100}{24} \approx 4.17 \text{ ft}$$

The heat will be concentrated about 4.17 *feet* from the base, along the axis of symmetry.

Exercise

A reflecting telescope contains a mirror shaped a paraboloid of revolution. If the mirror is 4 *inches* across at its opening and is 3 *inches* deep, where will the collected light be concentrated?

Solution

Let the vertex of the parabola is at $(0, 0)$ and it opens up.

Then the equation of the parabola has the form $x^2 = 4ay$

Since the parabola is 4 *inches* across and 3 *inches* deep.

The points $(2, 3)$ and $(-2, 3)$ are on the parabola.

$$2^2 = 4a(3) \rightarrow a = \frac{4}{12} \approx \frac{1}{3} \text{ in}$$

The collected light will be concentrated $\frac{1}{3}$ inch from the base of the mirror along the axis of symmetry.

Exercise

Show that the graph of an equation of the form $Ax^2 + Dx + Ey + F = 0$ $A \neq 0$

- a) Is a parabola if $E \neq 0$
- b) Is a vertical line if $E = 0$ and $D^2 - 4AF = 0$
- c) Is two vertical lines if $E = 0$ and $D^2 - 4AF > 0$
- d) Contains no points if $E = 0$ and $D^2 - 4AF < 0$

Solution

a) If $E \neq 0 \rightarrow Ax^2 + Dx + Ey + F = 0$

The x-vertex: $x = -\frac{b}{2a} = -\frac{D}{2A}$

$$A\left(-\frac{D}{2A}\right)^2 + D\left(-\frac{D}{2A}\right) + Ey + F = 0$$

$$\frac{D^2}{4A} - \frac{D^2}{2A} + Ey + F = 0$$

$$Ey = \frac{D^2}{4A} - F$$

$$y = \frac{D^2 - 4AF}{4AE}$$

This is the equation of a parabola whose vertex is: $\left(-\frac{D}{2A}, \frac{D^2 - 4AF}{4AE}\right)$ and whose axis of symmetry is parallel to the y-axis.

b) If $E = 0 \rightarrow Ax^2 + Dx + F = 0$

$$x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$$

$$= -\frac{D}{2A} \quad \text{Since } D^2 - 4AF = 0$$

This is a single vertical line.

c) If $E = 0 \rightarrow Ax^2 + Dx + F = 0$

$$x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$$

If $D^2 - 4AF > 0$, then

$$x = \frac{-D - \sqrt{D^2 - 4AF}}{2A} \quad \text{and} \quad x = \frac{-D + \sqrt{D^2 - 4AF}}{2A} \quad \text{are two vertical lines.}$$

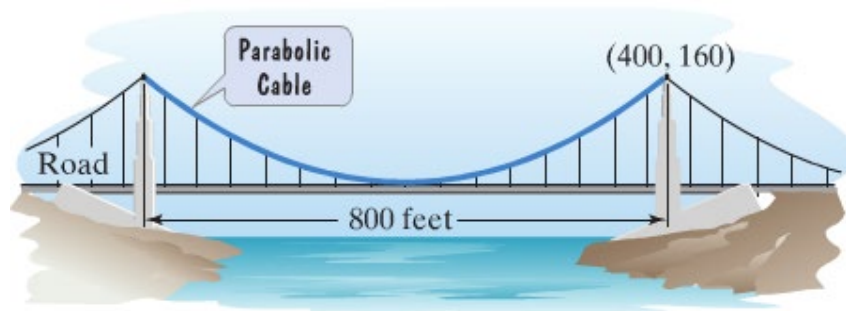
d) If $E = 0 \rightarrow Ax^2 + Dx + F = 0$

$$x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$$

If $D^2 - 4AF < 0$, then there is no real solution. The graph contains no points.

Exercise

The towers of a suspension bridge are 800 *feet* apart and rise 160 *feet* above the road. The cable between the towers has the shape of a parabola and the cable just touches the sides of the road midway between the towers. What is the height of the cable 100 *feet* from a tower?



Solution

Given the point: (400, 160)

$$(400)^2 = 4p(160) \quad x^2 = 4py$$

$$p = \frac{400^2}{640} = 250$$

$$x^2 = 1,000y$$

$$x = 400 - 100 = 300$$

$$(300)^2 = 1,000y \quad x^2 = 4py$$

$$y = \frac{300^2}{1,000} = 90$$

The height is 90 *feet*.

Exercise

The cables of a suspension bridge are in the shape of a parabola. The towers supporting the cable are 400 *feet* apart and 100 *feet* high. If the cables are at a height of 10 *feet* midway between the towers, what is the height of the cable at a point 50 *feet* from the center of the bridge?

Solution

Vertex point: (0, 10) and the parabola is open up

A point on parabola: $(200, 100)$

$$200^2 = c(100 - 10) \quad (x - h)^2 = c(y - k)$$

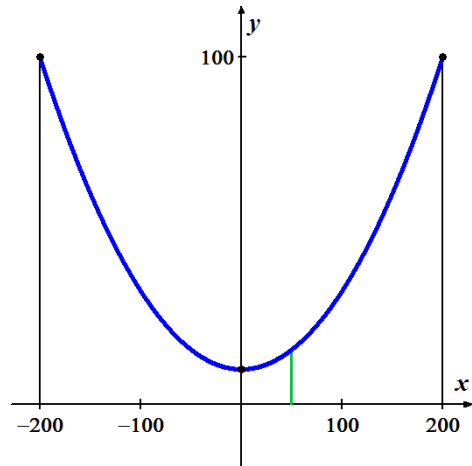
$$c = \frac{40,000}{90} = \frac{4000}{9}$$

$$\underline{x^2 = \frac{4000}{9}(y - 10) \quad |}$$

The height of the cable 50 feet from the center $-(50, h)$

$$y = \frac{9}{4000}x^2 + 10$$

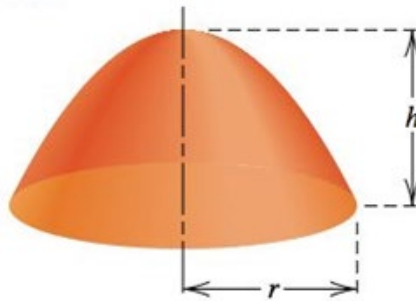
$$h = \frac{9}{4000}(50)^2 + 10 \approx \underline{15.625 \text{ ft} \quad |}$$



The height of the cable 50 feet from the center is about 15.625 feet.

Exercise

The focal length of the (finite) paraboloid is the distance p between its vertex and focus



- Express p in terms of r and h .
- A reflector is to be constructed with a focal length of 10 feet and a depth of 5 feet. Find the radius of the reflector.

Solution

- The point (r, h) is on the parabola.

$$r^2 = 4p(h) \quad x^2 = 4py$$

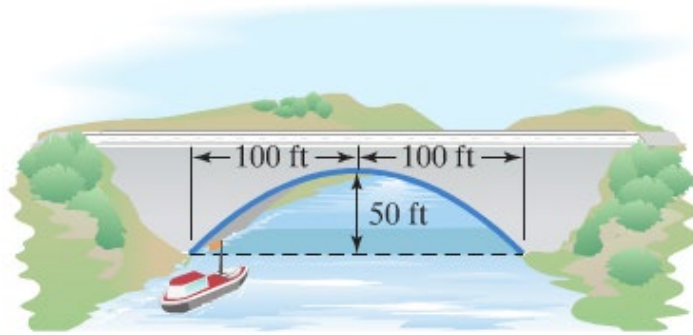
$$\underline{p = \frac{r^2}{4h} \quad |}$$

- Given: $p = 10$; $h = 5$

$$r = \sqrt{4(10)(5)} \\ \underline{= 10\sqrt{2} \quad |}$$

Exercise

The parabolic arch is 50 *feet* above the water at the center and 200 *feet* wide at the base. Will a boat that is 30 *feet* tall clear the arch 30 *feet* from the center?



Solution

$$\left(\frac{200}{2}\right)^2 = 4p(-50) \quad x^2 = 4py$$

$$p = \frac{200^2}{-200}$$
$$= -200$$

$$x^2 = -200y$$

Given the boat tall: $x = 30$

$$(30)^2 = -200y \quad x^2 = 4py$$

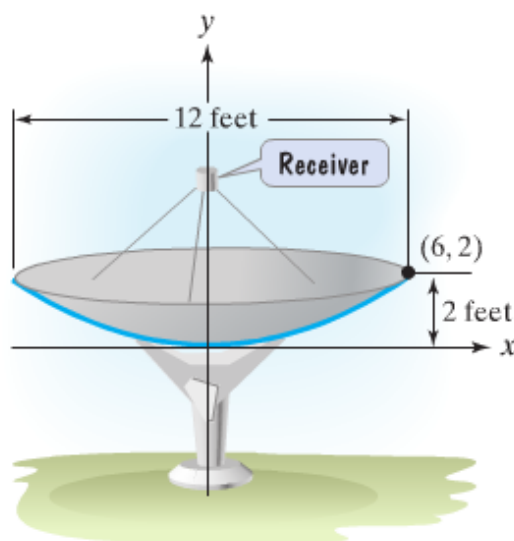
$$y = \frac{900}{-200}$$
$$= -4.5$$

$$\text{Height of bridge} = 50 - 4.5 = 45.5 \text{ ft}$$

Yes, the boat will clear the arch.

Exercise

A satellite dish, as shown below, is in the shape of a parabolic surface. Signals coming from a satellite strike the surface of the dish and are reflected to the focus, where the receiver is located. The satellite dish shown has a diameter of 12 feet and a depth of 2 feet. How far from the base of the dish should the receiver be placed?



Solution

$$6^2 = 4p(2) \quad x^2 = 4py$$

$$p = \frac{36}{8}$$

$$= 4.5$$

The receiver should be located 4.5 feet from the base of the dish.

Exercise

A searchlight is shaped like a paraboloid of revolution. If the light source is located 2 feet from the base along the axis of symmetry and the opening is 5 feet across, how deep should the searchlight be?

Solution

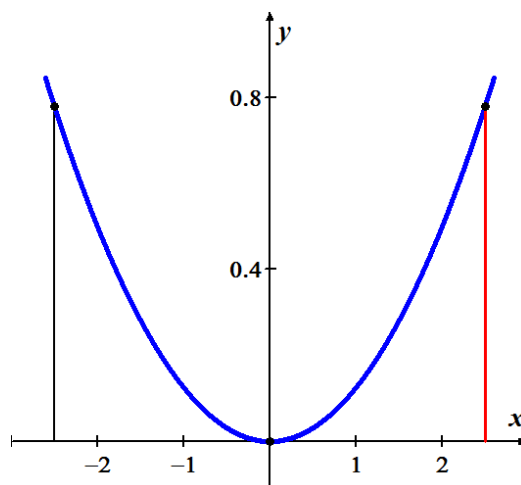
Vertex point: (0, 0) and the parabola is open up.

Given: $p = 2$

$$x^2 = 8y \quad x^2 = 4py$$

The opening is 5 feet across – (2.5, y)

$$y = \frac{x^2}{8}$$
$$= \frac{2.5^2}{8}$$



$$\underline{= 0.78125 \text{ ft}}|$$

The depth of the searchlight should be 0.78125 feet.

Exercise

A searchlight is shaped like a paraboloid of revolution. If the light source is located 2 feet from the base along the axis of symmetry and the depth of the searchlight is 4 feet across, how deep should the opening be?

Solution

Vertex point: $(0, 0)$ and the parabola is open up.

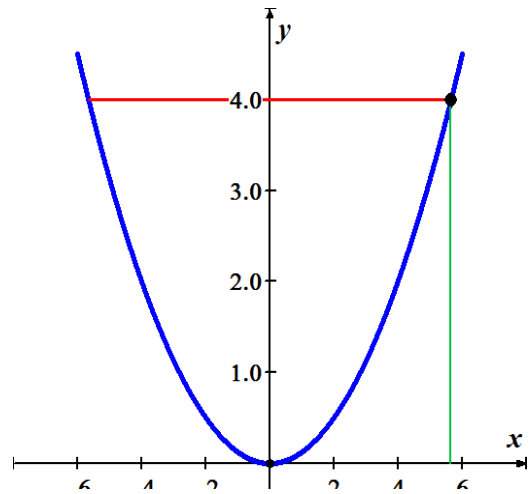
Given: $p = 2$

$$x^2 = 8y \qquad x^2 = 4py$$

The depth is 4 feet $\rightarrow (x, 4)$

$$\begin{aligned} x^2 &= 8(4) \\ &= 32 \end{aligned}$$

$$\underline{x = \pm 4\sqrt{2} \text{ ft}}|$$



The width of the opening of the searchlight should be $2(4\sqrt{2}) = 11.31$ feet.

Exercise

A searchlight is shaped like a paraboloid, with the light source at the focus. If the reflector is 3 feet across at the opening and 1 foot deep, where is the focus?

Solution

Vertex point: $(0, 0)$ and the parabola is open up.

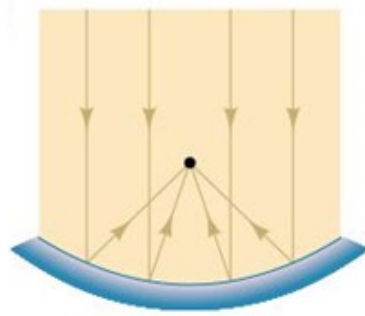
$$2x = 3 \rightarrow x = \frac{3}{2}$$

$$1 = \frac{1}{4p} \left(\frac{3}{2} \right)^2 \qquad y = \frac{1}{4p} x^2$$

$$\underline{p = \frac{9}{16} \text{ ft}}|$$

Exercise

A mirror for a reflecting telescope has the shape of a (finite) paraboloid of diameter 8 *inches* and depth 1 *inch*. How far from the center the mirror will the incoming light collect?



Solution

Vertex point: $(0, 0)$ and passing through $P\left(\frac{8}{2}, 1\right) = (4, 1)$

$$1 = \frac{1}{4p}(4)^2 \qquad y = \frac{1}{4p}x^2$$

$$p = \frac{16}{4} = 4$$

The light will collect 4 *inches* from the center of the mirror.

Solution

Section 1.6 – Other Types of Equations

Exercise

Solve $3x^3 + 2x^2 = 12x + 8$

Solution

$$3x^3 + 2x^2 - (12x + 8) = 0$$

$$x^2(3x + 2) - 4(3x + 2) = 0$$

$$(3x + 2)(x^2 - 4) = 0$$

$$3x + 2 = 0$$

$$x^2 - 4 = 0$$

$$3x = -2$$

$$x^2 = 4$$

$$\underline{x = -\frac{2}{3} \mid}$$

$$\underline{x = \pm 2 \mid}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{2}{3}, \pm 2 \mid}$$

Exercise

Solve: $x^3 + x^2 - 4x - 4 = 0$

Solution

$$x^2(x + 1) - 4(x + 1) = 0$$

$$(x + 1)(x^2 - 4) = 0$$

$$x + 1 = 0$$

$$\underline{x = -1 \mid}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\underline{x = \pm 2 \mid}$$

$$\therefore \text{Solutions: } \underline{x = -1, \pm 2 \mid}$$

Exercise

Solve: $x^3 + x^2 + 4x + 4 = 0$

Solution

$$x^2(x + 1) + 4(x + 1) = 0$$

$$(x+1)(x^2+4)=0$$

$$x+1=0$$

$$\underline{x=-1}$$

$$x^2+4=0$$

$$x^2=-4$$

$$\underline{x=\pm 2i}$$

$$\therefore \text{Solutions: } \underline{x=-1, \pm 2i}$$

Exercise

$$\text{Solve: } x^3 + 4x^2 - 25x - 100 = 0$$

Solution

$$x^2(x+4)-25(x+4)=0$$

$$(x+4)(x^2-25)=0$$

$$x+4=0$$

$$\underline{x=-4}$$

$$x^2-25=0$$

$$x^2=25$$

$$\underline{x=\pm 5}$$

$$\therefore \text{Solutions: } \underline{x=-4, \pm 5}$$

Exercise

$$\text{Solve: } x^3 - 2x^2 - x + 2 = 0$$

Solution

$$x^2(x-2)-(x-2)=0$$

$$(x-2)(x^2-1)=0$$

$$x-2=0$$

$$\underline{x=2}$$

$$x^2-1=0$$

$$x^2=1$$

$$\underline{x=\pm 1}$$

$$\therefore \text{Solutions: } \underline{x=2, \pm 1}$$

Exercise

Solve: $x^3 - x^2 - 25x + 25 = 0$

Solution

$$x^2(x-1) - 25(x-1) = 0$$

$$(x-1)(x^2 - 25) = 0$$

$$x-1=0$$

$$\underline{x=1}$$

$$x^2 - 25 = 0$$

$$x^2 = 25$$

$$\underline{x = \pm 5}$$

$$\therefore \text{Solutions: } \underline{x = 1, \pm 5}$$

Exercise

Solve: $x^3 - x^2 = 16x - 16$

Solution

$$x^3 - x^2 - 16x + 16 = 0$$

$$x^2(x-1) - 16(x-1) = 0$$

$$(x-1)(x^2 - 16) = 0$$

$$x-1=0$$

$$\underline{x=1}$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$\underline{x = \pm 4}$$

$$\therefore \text{Solutions: } \underline{x = 1, \pm 4}$$

Exercise

Solve: $x^3 + x^2 + 25x + 25 = 0$

Solution

$$x^2(x+1) + 25(x+1) = 0$$

$$(x+1)(x^2 + 25) = 0$$

$$x+1=0$$

$$\underline{x = -1}$$

$$x^2 + 25 = 0$$

$$x^2 = -25$$

$$\underline{x = \pm 5i}$$

∴ Solutions: $x = -1, \pm 5i$

Exercise

Solve: $x^3 + 2x^2 = 16x + 32$

Solution

$$x^3 + 2x^2 - 16x - 32 = 0$$

$$x^2(x + 2) - 16(x + 2) = 0$$

$$(x + 2)(x^2 - 16) = 0$$

$$x + 2 = 0$$

$$\underline{x = -2}$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$\underline{x = \pm 4}$$

∴ Solutions: $x = -2, \pm 4$

Exercise

Solve: $2x^3 + 3x^2 - 6x - 9 = 0$

Solution

$$x^2(2x + 3) - 3(2x + 3) = 0$$

$$(2x + 3)(x^2 - 3) = 0$$

$$2x + 3 = 0$$

$$\underline{x = -\frac{3}{2}}$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$\underline{x = \pm\sqrt{3}}$$

∴ Solutions: $x = -\frac{3}{2}, \pm\sqrt{3}$

Exercise

Solve: $2x^3 + x^2 - 8x - 4 = 0$

Solution

$$x^2(2x + 1) - 4(2x + 1) = 0$$

$$(2x + 1)(x^2 - 4) = 0$$

$$2x + 1 = 0$$

$$x = -\frac{1}{2}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\therefore \text{Solutions: } x = -\frac{1}{2}, \pm 2$$

Exercise

$$\text{Solve: } 2x^3 + 16x^2 + 30x = 0$$

Solution

$$2x(x^2 + 8x + 15) = 0$$

$$2x(x + 3)(x + 5) = 0$$

$$\therefore \text{Solutions: } x = 0, -3, -5$$

Exercise

$$\text{Solve: } 3x^3 - 9x^2 - 30x = 0$$

Solution

$$3x(x^2 - 3x - 10) = 0$$

$$3x(x + 2)(x - 5) = 0$$

$$\therefore \text{Solutions: } x = 0, -2, 5$$

Exercise

$$\text{Solve } x^4 + 3x^2 = 10$$

Solution

$$x^4 + 3x^2 - 10 = 0$$

$$(x^2 + 5)(x^2 - 2) = 0$$

$$x^2 + 5 = 0$$

$$x^2 = -5$$

$$x = \pm i\sqrt{5}$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

$$\therefore \text{Solutions: } x = \pm i\sqrt{5}, \pm \sqrt{2}$$

Exercise

Solve: $5x^4 = 40x$

Solution

$$5x^4 - 40x = 0$$

$$5x(x^3 - 8) = 0$$

$$5x(x^3 - 2^3) = 0$$

$$5x(x-2)(x^2 + 2x + 2^2) = 0$$

$$5x(x-2)(x^2 + 2x + 4) = 0$$

$$\underline{x = 0}$$

$$\underline{x = 2}$$

$$x^2 + 2x + 4 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$= \underline{-1 \pm i\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = 0, 2, -1 \pm i\sqrt{3}}$$

Exercise

Solve $9x^4 - 9x^2 + 2 = 0$

Solution

$$x^2 = \frac{9 \pm \sqrt{81 - 72}}{18}$$

$$= \frac{9 \pm \sqrt{9}}{18}$$

$$= \frac{9 \pm 3}{18}$$

$$= \left\{ \begin{array}{l} \frac{9-3}{18} = \frac{6}{18} = \frac{1}{3} \\ \frac{9+3}{18} = \frac{12}{18} = \frac{2}{3} \end{array} \right.$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{cases} x^2 = \frac{1}{3} \Rightarrow x = \pm \frac{1}{\sqrt{3}} \\ x^2 = \frac{2}{3} \Rightarrow x = \pm \frac{\sqrt{2}}{\sqrt{3}} \end{cases}$$

$$\begin{cases} x = \pm \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3} \\ x = \pm \frac{\sqrt{2}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{6}}{3} \end{cases}$$

$$\therefore \text{Solutions: } x = \pm \frac{\sqrt{3}}{3}, \pm \frac{\sqrt{6}}{3}$$

Exercise

Solve: $x^4 + 720 = 89x^2$

Solution

$$\begin{aligned} x^4 - 89x^2 + 720 &= 0 \\ x^2 &= \frac{89 \pm \sqrt{7,921 - 2,880}}{2} \\ &= \frac{89 \pm \sqrt{5041}}{2} \\ &= \frac{89 \pm 71}{2} \\ &= \begin{cases} \frac{89 - 71}{2} = 9 \\ \frac{89 + 71}{2} = 80 \end{cases} \\ x^2 = 9 &\Rightarrow x = \pm 3 \\ x^2 = 80 &\Rightarrow x = \pm \sqrt{80} \\ &= \pm \sqrt{(16)(5)} \\ &= \pm 4\sqrt{5} \end{aligned}$$

$$\therefore \text{Solutions: } x = \pm 3, \pm 4\sqrt{5}$$

Exercise

Solve $12x^4 - 11x^2 + 2 = 0$

Solution

$$x^2 = \frac{11 \pm \sqrt{121 - 96}}{24}$$

$$= \frac{11 \pm \sqrt{25}}{24}$$

$$= \frac{11 \pm 5}{24}$$

$$= \begin{cases} \frac{11-5}{24} = \frac{1}{4} \\ \frac{11+5}{24} = \frac{2}{3} \end{cases}$$

$$x^2 = \frac{1}{4}$$

$$x = \pm \frac{1}{2}$$

$$x^2 = \frac{2}{3}$$

$$x = \pm \sqrt{\frac{2}{3}}$$

$$x = \pm \frac{\sqrt{6}}{3}$$

$$\therefore \text{Solutions: } \left\{ \pm \frac{\sqrt{6}}{3}, \pm \frac{1}{2} \right\}$$

Exercise

Solve $2x^4 - 7x^2 + 5 = 0$

Solution

$$(2x^2 - 5)(x^2 - 1) = 0$$

$$2x^2 - 5 = 0$$

$$x^2 = \frac{5}{2}$$

$$x = \pm \frac{\sqrt{5}}{\sqrt{2}}$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$\therefore \text{Solutions: } \left\{ \pm \frac{\sqrt{10}}{2}, \pm 1 \right\}$$

Exercise

Solve $x^4 - 5x^2 + 4 = 0$

Solution

$$1 - 5 + 4 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, 4} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$\begin{array}{c|c} x^2 = 1 & x^2 = 4 \\ \hline \underline{x = \pm 1} & \underline{x = \pm 2} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm 2}$$

Exercise

Solve $x^4 + 3x^2 = 10$

Solution

$$x^4 + 3x^2 - 10 = 0$$

$$(x^2 + 5)(x^2 - 2) = 0$$

$$\begin{array}{c|c} x^2 + 5 = 0 & x^2 - 2 = 0 \\ \hline x^2 = -5 & x^2 = 2 \\ \hline x = \pm\sqrt{-5} & \underline{x = \pm\sqrt{2}} \\ \hline = \pm i\sqrt{5} & \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm i\sqrt{5}, \pm\sqrt{2}}$$

Exercise

Solve: $3x^4 - 48x^2 = 0$

Solution

$$3x^2(x^2 - 16) = 0$$

$$\begin{array}{c|c} x^2 = 0 & x^2 - 16 = 0 \\ \hline \underline{x = 0, 0} & x^2 = 16 \\ \hline & \underline{x = \pm 4} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, \pm 4}$$

Exercise

Solve: $5x^4 - 20x^2 = 0$

Solution

$$5x^2(x^2 - 4) = 0$$

$$\begin{array}{l} x^2 = 0 \\ \underline{x = 0, 0} \end{array}$$

$$\begin{array}{l} x^2 - 4 = 0 \\ x^2 = 4 \\ \underline{x = \pm 2} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, \pm 2}$$

Exercise

Solve: $x^4 - 4x^3 - 4x^2 = 0$

Solution

$$x^2(x^2 - 4x - 4) = 0$$

$$\begin{array}{l} x^2 = 0 \\ \underline{x = 0, 0} \end{array}$$

$$\begin{array}{l} x^2 - 4x - 4 = 0 \\ x = \frac{4 \pm \sqrt{16 + 16}}{2} \\ = \frac{4 \pm 4\sqrt{2}}{2} \\ \underline{x = 2 \pm 2\sqrt{2}} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 2 \pm 2\sqrt{2}}$$

Exercise

Solve: $x^4 - 6x^3 + 9x^2 = 0$

Solution

$$x^2(x^2 - 6x + 9) = 0$$

$$\begin{array}{l} x^2 = 0 \\ \underline{x = 0, 0} \end{array}$$

$$\begin{array}{l} x^2 - 6x + 9 = 0 \\ (x - 3)^2 = 0 \\ \underline{x = 3, 3} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 3, 3}$$

Exercise

Solve: $x^4 - 4x^3 + 3x^2 = 0$

Solution

$$x^2(x^2 - 4x + 3) = 0$$

$$x^2 = 0$$

$$\underline{x = 0, 0}$$

$$x^2 - 4x + 3 = 0$$

$$\underline{x = 1, 3}$$

$$\therefore \text{Solutions: } \underline{x = 0, 0, 1, 3}$$

Exercise

Solve: $x^4 - 4x^2 + 3 = 0$

Solution

$$1 - 4 + 3 = 0$$

$$a + b + c = 0$$

$$\underline{x^2 = 1, 3}$$

$$x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$x^2 = 3$$

$$\underline{x = \pm\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm\sqrt{3}}$$

Exercise

Solve: $x^4 + 4x^2 + 3 = 0$

Solution

$$1 - 4 + 3 = 0$$

$$a - b + c = 0$$

$$\underline{x^2 = -1, -3}$$

$$x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = -1$$

$$\underline{x = \pm i}$$

$$x^2 = -3$$

$$\underline{x = \pm i\sqrt{3}}$$

$$\therefore \text{Solutions: } \underline{x = \pm i, \pm i\sqrt{3}}$$

Exercise

Solve: $x^4 + 6x^2 - 7 = 0$

Solution

$$1 + 6 - 7 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, -7} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$\begin{array}{l} x^2 = 1 \\ x = \pm 1 \end{array}$$

$$\begin{array}{l} x^2 = -7 \\ x = \pm i\sqrt{7} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm 1, \pm i\sqrt{7}}$$

Exercise

Solve: $x^4 - 6x^2 - 7 = 0$

Solution

$$1 - (-6) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x^2 = -1, 7} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$\begin{array}{l} x^2 = -1 \\ x = \pm i \end{array}$$

$$\begin{array}{l} x^2 = 7 \\ x = \pm \sqrt{7} \end{array}$$

$$\therefore \text{Solutions: } \underline{x = \pm i, \pm \sqrt{7}}$$

Exercise

Solve: $3x^4 + 4x^2 - 7 = 0$

Solution

$$3 + 4 - 7 = 0 \quad a + b + c = 0$$

$$\underline{x^2 = 1, -\frac{7}{3}} \quad x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$\begin{array}{l} x^2 = 1 \\ x = \pm 1 \end{array}$$

$$\begin{array}{l} x^2 = -\frac{7}{3} \\ x = \pm i\sqrt{\frac{7}{3}} \frac{\sqrt{3}}{\sqrt{3}} \\ \underline{= \pm i\frac{\sqrt{21}}{3}} \end{array}$$

$$\therefore \text{Solutions: } x = \pm 1, \pm i \frac{\sqrt{21}}{3}$$

Exercise

Solve: $3x^4 - 4x^2 - 7 = 0$

Solution

$$3 - (-4) - 7 = 0 \quad a - b + c = 0$$

$$x^2 = -1, \frac{7}{3}$$

$$x^2 = -1$$

$$x = \pm i$$

$$x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = \frac{7}{3}$$

$$x = \pm \sqrt{\frac{7}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \pm \frac{\sqrt{21}}{3}$$

$$\therefore \text{Solutions: } x = \pm i, \pm \frac{\sqrt{21}}{3}$$

Exercise

Solve: $3x^4 - x^2 - 2 = 0$

Solution

$$3 - 1 - 2 = 0 \quad a + b + c = 0$$

$$x^2 = 1, -\frac{2}{3}$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x_1 = 1, \quad x_2 = \frac{c}{a}$$

$$x^2 = -\frac{2}{3}$$

$$x = \pm i \sqrt{\frac{2}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \pm i \frac{\sqrt{6}}{3}$$

$$\therefore \text{Solutions: } x = \pm 1, \pm i \frac{\sqrt{6}}{3}$$

Exercise

Solve: $3x^4 + x^2 - 2 = 0$

Solution

$$3 - 1 - 2 = 0$$

$$a - b + c = 0$$

$$x^2 = -1, \frac{2}{3}$$

$$x^2 = -1$$

$$x = \pm i$$

$$x_1 = -1, \quad x_2 = -\frac{c}{a}$$

$$x^2 = \frac{2}{3}$$

$$x = \pm \sqrt{\frac{2}{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \pm \frac{\sqrt{6}}{3}$$

$$\therefore \text{Solutions: } x = \pm i, \pm \frac{\sqrt{6}}{3}$$

Exercise

$$\text{Solve } x - 3\sqrt{x} - 4 = 0$$

Solution

$$(\sqrt{x} - 4)(\sqrt{x} + 1) = 0$$

$$\sqrt{x} - 4 = 0$$

$$\sqrt{x} + 1 = 0$$

$$\sqrt{x} = 4$$

$$\sqrt{x} = -1 \quad \text{Impossible}$$

$$x = 16$$

$$\therefore \text{Solution: } x = 16$$

Exercise

$$\text{Solve } (5x^2 - 6)^{1/4} = x$$

Solution

$$\left[(5x^2 - 6)^{1/4} \right]^4 = x^4$$

$$5x^2 - 6 = x^4$$

$$x^4 - 5x^2 + 6 = 0$$

$$(x^2 - 3)(x^2 - 2) = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$\therefore \text{Solutions: } \underline{x = \pm\sqrt{3}, \pm\sqrt{2}} \mid$$

Exercise

$$\text{Solve } \left(x^2 + 24x\right)^{1/4} = 3$$

Solution

$$\left[\left(x^2 + 24x\right)^{1/4}\right]^4 = 3^4$$

$$x^2 + 24x = 81$$

$$x^2 + 24x - 81 = 0$$

$$(x + 27)(x - 3) = 0$$

$$x + 27 = 0 \quad x - 3 = 0$$

$$x = -27 \quad x = 3$$

$$\therefore \text{Solutions: } \underline{x = -27, 3} \mid$$

Exercise

$$\text{Solve: } x^{5/2} = 32$$

Solution

$$\begin{aligned} x &= 32^{2/5} && \text{Reciprocal} \\ &= \left(2^5\right)^{2/5} \\ &= \underline{4} \mid \end{aligned}$$

Exercise

$$\text{Solve: } \sqrt[3]{2x+11} = 3$$

Solution

$$2x + 11 = 3^3$$

$$2x = 27 - 11$$

$$x = \frac{16}{2}$$

$$= \underline{8} \mid$$

Exercise

Solve: $\sqrt[3]{6x-3} = 3$

Solution

$$6x - 3 = 3^3$$

$$6x = 27 + 3$$

$$x = \frac{30}{6}$$

$$= 5$$

Exercise

Solve: $\sqrt[3]{2x-6} = 4$

Solution

$$2x - 6 = 4^3$$

$$2x = 64 + 6$$

$$x = \frac{70}{2}$$

$$= 35$$

Exercise

Solve: $\sqrt[3]{4x-3} - 5 = 0$

Solution

$$\sqrt[3]{4x-3} = 5$$

$$4x - 3 = 5^3$$

$$4x = 125 + 3$$

$$x = \frac{128}{4}$$

$$= 32$$

Exercise

Solve: $(3x-1)^{1/3} + 4 = 0$

Solution

$$(3x-1)^{1/3} = -4$$

$$3x - 1 = (-4)^3$$

$$3x = 1 - 64$$

$$x = -\frac{63}{3}$$

$$\underline{\underline{= -21}}$$

Exercise

Solve: $(2x + 3)^{1/3} + 4 = 6$

Solution

$$(2x + 3)^{1/3} = 2$$

$$2x + 3 = 2^3$$

$$2x = 8 - 3$$

$$\underline{\underline{x = \frac{5}{2}}}$$

Exercise

Solve: $(3x - 6)^{1/3} + 5 = 8$

Solution

$$(3x - 6)^{1/3} = 3$$

$$3x - 6 = 3^3$$

$$3x = 27 + 6$$

$$x = \frac{33}{3}$$

$$\underline{\underline{= 11}}$$

Exercise

Solve: $(3x + 1)^{1/4} + 7 = 9$

Solution

$$(3x + 1)^{1/4} = 2$$

$$3x + 1 = 2^4$$

$$3x = 16 - 1$$

$$x = \frac{15}{3}$$

$$= 5$$

∴ Solution set is: $\{5\}$

Exercise

Solve: $(2x + 3)^{1/4} + 7 = 10$

Solution

$$(2x + 3)^{1/4} = 3$$

$$2x + 3 = 3^4$$

$$2x = 81 - 3$$

$$x = \frac{78}{2}$$

$$= 39$$

∴ Solution set is: $\{39\}$

Exercise

Solve: $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$

Solution

$$\left(\sqrt[3]{4x^2 - 4x + 1}\right)^3 = \left(\sqrt[3]{x}\right)^3$$

$$4x^2 - 4x + 1 = x$$

$$4x^2 - 5x + 1 = 0$$

$$(4x - 1)(x - 1) = 0$$

$$4x - 1 = 0$$

$$x - 1 = 0$$

$$x = \frac{1}{4}$$

$$x = 1$$

Check

$$x = \frac{1}{4}$$

$$x = 1$$

$$\sqrt[3]{4\left(\frac{1}{4}\right)^2 - 4\frac{1}{4} + 1} - \sqrt[3]{\frac{1}{4}} = 0$$

$$\sqrt[3]{4(1)^2 - 4(1) + 1} - \sqrt[3]{1} = 0$$

$$\sqrt[3]{\frac{1}{4}} - \sqrt[3]{\frac{1}{4}} = 0$$

$$\sqrt[3]{4 - 4 + 1} - \sqrt[3]{1} = 0$$

$$0 = 0 \quad (\text{true})$$

$$0 = 0 \quad (\text{true})$$

$$\therefore \text{Solution set is: } \underline{\left\{-1, \frac{1}{4}\right\}}$$

Exercise

$$\text{Solve: } \sqrt{2x+3} = 5$$

Solution

$$2x+3=5^2 \qquad \sqrt[n]{u}=a \rightarrow u=a^n$$

$$2x=25-3$$

$$x=\frac{22}{2}$$

$$\underline{=11}$$

$$\text{Check: } \sqrt{2(\textcolor{red}{11})+3} \stackrel{?}{=} 5$$

$$\sqrt{25}=5 \quad \checkmark$$

$$\therefore \text{Solution set is: } \underline{\{11\}}$$

Exercise

$$\text{Solve: } \sqrt{x-3}+6=5$$

Solution

$$\sqrt{x-3}=-1 \quad \times$$

\therefore **No** solution.

Exercise

$$\text{Solve: } \sqrt{3x-2}=4$$

Solution

$$3x-2=4^2 \qquad \sqrt[n]{u}=a \rightarrow u=a^n$$

$$3x=16+2$$

$$x=\frac{18}{3}$$

$$\underline{=6}$$

$$\text{Check: } \sqrt{3(\textcolor{red}{6})-2} \stackrel{?}{=} 4$$

$$\sqrt{16}=4 \quad \checkmark$$

∴ Solution set is: $\{6\}$

Exercise

Solve: $\sqrt{5x-4} = 9$

Solution

$$5x - 4 = 9^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$5x = 81 + 4$$

$$x = \frac{85}{5}$$

$$= 17$$

Check: $\sqrt{5(17) - 4} = 9$

$$\sqrt{81} = 9 \quad \checkmark$$

∴ Solution set is: $\{17\}$

Exercise

Solve: $\sqrt{5x-1} = 8$

Solution

$$5x - 1 = 8^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$5x = 64 + 1$$

$$x = \frac{65}{5}$$

$$= 13$$

Check: $\sqrt{5(13) - 1} = 8$

$$\sqrt{64} = 8 \quad \checkmark$$

∴ Solution set is: $\{13\}$

Exercise

Solve: $\sqrt{3x-2} - 5 = 0$

Solution

$$\sqrt{3x-2} = 5$$

$$3x - 2 = 5^2$$

$$\sqrt[n]{u} = a \rightarrow u = a^n$$

$$3x = 25 + 2$$

$$x = \frac{27}{3}$$

$$= 9$$

Check: $\sqrt{3(9)-2}-5=0$

$$5-5=0 \quad \checkmark$$

\therefore Solution set is: $\{9\}$

Exercise

Solve: $\sqrt{2x+5}+11=6$

Solution

$$\sqrt{2x+5} = -5 \quad \times$$

\therefore **No** solution.

Exercise

Solve: $\sqrt{3x+7}+10=4$

Solution

$$\sqrt{3x+7} = -6 \quad \times$$

\therefore **No** solution.

Exercise

Solve: $x = \sqrt{7x+8}$

Solution

$$x^2 = 7x+8$$

$$x^2 - 7x - 8 = 0$$

$$1 - (-7) - 8 = 0 \quad a - b + c = 0$$

$$x = -1, 8 \quad \left| \quad x_1 = -1, \quad x_2 = -\frac{c}{a} \right.$$

Check:

$x = -1$ $-1 \neq \sqrt{7(-1)+8}$	$\left \right.$	$x = 8$ $8 = \sqrt{7(8)+8}$ $8 = \sqrt{64} \quad \checkmark$
--------------------------------------	------------------	--

\therefore Solution set is: $\{8\}$

Exercise

Solve: $x = \sqrt{6x+7}$

Solution

$$x^2 = 6x + 7$$

$$x^2 - 6x - 7 = 0$$

$$1 - (-6) - 7 = 0 \quad a - b + c = 0$$

$$\underline{x = -1, 7} \quad x_1 = -1, \quad x_2 = -\frac{c}{a}$$

Check:

$$\left. \begin{array}{l} x = -1 \\ -1 \neq \sqrt{} \end{array} \right| \begin{array}{l} x = 7 \\ ? \\ 7 = \sqrt{6(7)+7} \\ 7 = \sqrt{49} \quad \checkmark \end{array}$$

\therefore Solution is: $\underline{x = 7}$

Exercise

Solve: $\sqrt{5x+1} = x+1$

Solution

$$5x+1 = (x+1)^2$$

$$5x+1 = x^2 + 2x + 1$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$\underline{x = 0, 3}$$

Check:

$$\left. \begin{array}{l} x = 0 \\ \sqrt{1} = 1 \quad \checkmark \end{array} \right| \begin{array}{l} x = 3 \\ ? \\ \sqrt{5(3)+1} = 3+1 \\ \sqrt{16} = 4 \quad \checkmark \end{array}$$

\therefore Solutions are: $\underline{x = 0, 3}$

Exercise

Solve: $x = \sqrt{2x-2} + 1$

Solution

$$x-1=\sqrt{2x-2}$$

$$(x-1)^2=2x-2$$

$$x^2-2x+1=2x-2$$

$$x^2-4x+3=0$$

$$1-4+3=0 \quad a+b+c=0$$

$$\underline{x=1, 3} \quad x_1=1, \quad x_2=\frac{c}{a}$$

Check:

$$\begin{array}{l|l} x=1 & x=3 \\ 1=1 \quad \checkmark & \begin{array}{l} ? \\ 3=\sqrt{4}+1 \\ 3=3 \quad \checkmark \end{array} \end{array}$$

\therefore Solutions are: $\underline{x=1, 3}$

Exercise

Solve: $x-2\sqrt{x-3}=3$

Solution

$$x-3=2\sqrt{x-3}$$

$$(x-3)^2=(2\sqrt{x-3})^2$$

$$x^2-6x+9=4(x-3)$$

$$x^2-6x+9=4x-12$$

$$x^2-10x+21=0$$

$$\underline{x=3, 7}$$

Check:

$$\begin{array}{l|l} x=3 & x=7 \\ 3-2(0)=3 \quad ? & \begin{array}{l} 7-2\sqrt{4}=3 \quad ? \\ 7-4=3 \quad \checkmark \end{array} \\ 3=3 \quad \checkmark & \end{array}$$

\therefore Solution set is: $\underline{\{3, 7\}}$

Exercise

Solve: $x+\sqrt{26-11x}=4$

Solution

$$\sqrt{26-11x} = 4-x$$

$$26-11x = (4-x)^2$$

$$26-11x = 16-8x+x^2$$

$$x^2 + 3x - 10 = 0$$

$$(x-2)(x+5) = 0$$

$$\underline{x = 2, -5}$$

Check:

$x = 2$	$x = -5$
$2 + \sqrt{26-22} \stackrel{?}{=} 4$	$-5 + \sqrt{26+55} \stackrel{?}{=} 4$
$2 + 2 = 4 \quad \checkmark$	$-5 + 9 = 4 \quad \checkmark$

\therefore Solutions are: $\underline{x = 2, -5}$

Exercise

Solve $x - \sqrt{2x+3} = 0$

Solution

$$x = \sqrt{2x+3}$$

$$(x)^2 = (\sqrt{2x+3})^2$$

$$x^2 = 2x+3$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$x+1=0$	$x-3=0$
$x=-1$	$x=3$

Check

$x = -1$

$x = 3$

$$(-1) - \sqrt{2(-1)+3} = 0$$

$$(3) - \sqrt{2(3)+3} = 0$$

$$-1 - \sqrt{1} = 0$$

$$3 - \sqrt{9} = 0$$

*False**True*

\therefore Solution set is: $\underline{\{3\}}$

Exercise

Solve: $\sqrt{x+3} + 3 = x$

Solution

$$\sqrt{x+3} = x-3$$

$$(\sqrt{x+3})^2 = (x-3)^2$$

$$x+3 = x^2 - 6x + 9$$

$$x^2 - 7x + 6 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{25}}{2}$$

$$= \frac{7 \pm 5}{2} = \begin{cases} \frac{7+5}{2} = \frac{12}{2} = 6 \\ \frac{7-5}{2} = \frac{2}{2} = 1 \end{cases} \Rightarrow x = 1, 6$$

Check:

$$x = 1 \Rightarrow \sqrt{1+3} + 3 = 1 \Rightarrow 5 = 1 \text{ (Not a solution)}$$

$$x = 6 \Rightarrow \sqrt{6+3} + 3 = 6 \Rightarrow 6 = 6 \rightarrow x = 6 \text{ is the only solution}$$

Exercise

Solve $x - \sqrt{x+11} = 1$

Solution

$$-\sqrt{x+11} = 1 - x$$

Square both side

$$(-\sqrt{x+11})^2 = (1-x)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$x+11 = 1 - 2x + x^2$$

$$0 = x^2 - 2x + 1 - x - 11$$

$$0 = x^2 - 3x - 10$$

$$x^2 - 3x - 10 = 0$$

Solve for x

$$x = 5, -2$$

Check:

$$x = 5 \Rightarrow 5 - \sqrt{5+11} = 1 \Rightarrow 5 - \sqrt{16} = 1 \Rightarrow 5 - 4 = 1 \Rightarrow 1 = 1$$

$$x = -2 \Rightarrow -2 - \sqrt{-2+11} = 1 \Rightarrow -2 - \sqrt{9} = 1 \Rightarrow -2 - 3 = 1 \Rightarrow -5 = 1 \text{ (False)}$$

\therefore Solution set is: $\{5\}$

Exercise

Solve: $\sqrt{x-7} = 7 - \sqrt{x}$

Solution

$$(\sqrt{x-7})^2 = (7 - \sqrt{x})^2$$

$$x - 7 = 49 - 14\sqrt{x} + x$$

$$14\sqrt{x} = 56$$

$$\sqrt{x} = \frac{56}{14}$$

$$= 4$$

$$\underline{x = 16}$$

Check:

$$x = 16$$

$$\sqrt{16-7} \stackrel{?}{=} 7 - 4$$

$$3 = 3 \quad \checkmark$$

$$\therefore \text{Solution set is: } \underline{\{16\}}$$

Exercise

Solve: $\sqrt{x-8} = \sqrt{x} - 2$

Solution

$$(\sqrt{x-8})^2 = (\sqrt{x} - 2)^2$$

$$x - 8 = x - 4\sqrt{x} + 4$$

$$4\sqrt{x} = 12$$

$$\sqrt{x} = 3$$

$$\underline{x = 9}$$

Check:

$$x = 9$$

$$\sqrt{9-8} \stackrel{?}{=} \sqrt{9} - 2$$

$$1 = 3 - 2 \quad \checkmark$$

$$\therefore \text{Solution set is: } \underline{\{9\}}$$

Exercise

Solve: $\sqrt{2x-5} = \sqrt{x+4}$

Solution

$$(\sqrt{2x-5})^2 = (\sqrt{x+4})^2$$

$$2x - 5 = x + 4$$

$$\underline{x = 9}$$

Check:

$$x = 9$$

$$\sqrt{18-5} \stackrel{?}{=} \sqrt{9+4}$$

$$\sqrt{13} = \sqrt{13} \quad \checkmark$$

\therefore Solution set is: $\underline{\{9\}}$

Exercise

Solve: $\sqrt{6x+2} = \sqrt{5x+3}$

Solution

$$(\sqrt{6x+2})^2 = (\sqrt{5x+3})^2$$

$$6x + 2 = 5x + 3$$

$$\underline{x = 1}$$

Check:

$$x = 1$$

$$\sqrt{6+2} \stackrel{?}{=} \sqrt{5+3}$$

$$\sqrt{8} = \sqrt{8} \quad \checkmark$$

\therefore Solution set is: $\underline{\{1\}}$

Exercise

Solve: $\sqrt{3x+1} - \sqrt{x+4} = 1$

Solution

$$\sqrt{3x+1} = 1 + \sqrt{x+4}$$

$$(\sqrt{3x+1})^2 = (1 + \sqrt{x+4})^2$$

$$3x + 1 = 1 + 2\sqrt{x+4} + x + 4$$

$$2x - 4 = 2\sqrt{x+4}$$

$$x - 2 = \sqrt{x+4}$$

$$(x-2)^2 = x+4$$

$$x^2 - 4x + 4 = x + 4$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$\underline{x = 0, 5}$$

Check:

$$x = 0$$

$$1 - \sqrt{4} \stackrel{?}{=} 1$$

$$-1 \neq 1 \quad \times$$

$$x = 5$$

$$\sqrt{15+1} - \sqrt{5+4} \stackrel{?}{=} 1$$

$$4 - 3 = 1 \quad \checkmark$$

\therefore Solution is: $\underline{x = 5}$

Exercise

Solve: $\sqrt{x+2} + \sqrt{x-1} = 3$

Solution

$$\sqrt{x+2} = 3 - \sqrt{x-1}$$

$$x+2 = (3 - \sqrt{x-1})^2$$

$$x+2 = 9 - 6\sqrt{x-1} + x-1$$

$$6\sqrt{x-1} = 6$$

$$\sqrt{x-1} = 1$$

$$x-1 = 1^2$$

$$\underline{x = 2}$$

Check:

$$x = 2$$

$$\sqrt{4} + 1 \stackrel{?}{=} 3$$

$$2 + 1 = 3 \quad \checkmark$$

\therefore Solution is: $\underline{x = 2}$

Exercise

Solve: $\sqrt{x-4} + \sqrt{x+4} = 4$

Solution

$$\sqrt{x-4} = 4 - \sqrt{x+4}$$

$$x-4 = (4 - \sqrt{x+4})^2$$

$$x-4 = 16 - 8\sqrt{x+4} + x + 4$$

$$8\sqrt{x+4} = 24$$

$$\sqrt{x+4} = 3$$

$$x+4 = 9$$

$$x = 5 \quad |$$

Check:

$$x = 5$$

$$\sqrt{1} + \sqrt{9} = 4$$

$$1 + 3 = 4 \quad \checkmark$$

$$\therefore \text{Solution is: } x = 5 \quad |$$

Exercise

Solve: $\sqrt{2x-3} - \sqrt{x-2} = 1$

Solution

$$\sqrt{2x-3} = 1 + \sqrt{x-2}$$

$$2x-3 = (1 + \sqrt{x-2})^2$$

$$2x-3 = 1 + 2\sqrt{x-2} + x-2$$

$$x-2 = 2\sqrt{x-2}$$

$$(x-2)^2 = (2\sqrt{x-2})^2$$

$$x^2 - 4x + 4 = 4(x-2)$$

$$x^2 - 4x + 4 = 4x - 8$$

$$x^2 - 8x + 12 = 0$$

$$x = 2, 6 \quad |$$

Check:

$$x = 2 \quad | \quad x = 6$$

$$\begin{array}{l|l} \sqrt{4-3} - \sqrt{2-2} \stackrel{?}{=} 1 & \sqrt{12-3} - \sqrt{6-2} \stackrel{?}{=} 1 \\ 1 = 1 \quad \checkmark & 3 - 2 = 1 \quad \checkmark \end{array}$$

∴ Solution is: $x = 2, 6$

Exercise

Solve: $\sqrt{x+2} + \sqrt{3x+7} = 1$

Solution

$$\sqrt{x+2} = 1 - \sqrt{3x+7}$$

$$x+2 = (1 - \sqrt{3x+7})^2$$

$$x+2 = 1 - 2\sqrt{3x+7} + 3x+7$$

$$2\sqrt{3x+7} = 2x+6$$

$$\sqrt{3x+7} = x+3$$

$$3x+7 = (x+3)^2$$

$$3x+7 = x^2 + 6x+9$$

$$x^2 + 3x + 2 = 0$$

$$x = -1, -2$$

Check:

$$\begin{array}{l|l} x = -1 & x = -2 \\ \sqrt{-1+2} \stackrel{?}{=} 1 - \sqrt{-3+7} & \sqrt{-2+2} \stackrel{?}{=} 1 - \sqrt{-6+7} \\ 1 \neq 1 - 2 \quad \times & 0 = 1 - 1 \quad \checkmark \end{array}$$

∴ Solution is: $x = -2$

Exercise

Solve: $2\sqrt{4x+1} - 9 = x - 5$

Solution

$$2\sqrt{4x+1} = x+4$$

$$(2\sqrt{4x+1})^2 = (x+4)^2$$

$$4(4x+1) = x^2 + 8x + 16$$

$$16x + 4 = x^2 + 8x + 16$$

$$x^2 - 8x + 12 = 0$$

$$\underline{x = 2, 6}$$

Check:

$$\begin{array}{l|l} x = 2 & x = 6 \\ 2\sqrt{8+1} - 9 \stackrel{?}{=} 2 - 5 & 2\sqrt{24+1} - 9 \stackrel{?}{=} 6 - 5 \\ 6 - 9 = -3 \quad \checkmark & 10 - 9 = 1 \quad \checkmark \end{array}$$

$$\therefore \text{Solution is: } \underline{x = 2, 6}$$

Exercise

Solve: $x\sqrt{x-3} + 4 = x + 1$

Solution

$$x\sqrt{x-3} = x - 3$$

$$(x\sqrt{x-3})^2 = (x-3)^2$$

$$x^2(x-3) - (x-3)^2 = 0$$

$$(x-3)(x^2 - x + 3) = 0$$

$$x^2 - x + 3 = 0 \rightarrow x = \frac{1 \pm \sqrt{1-12}}{2} \in \mathbb{C}$$

$$\underline{x = 3} \quad \text{Only result.}$$

Check:

$$x = 3$$

$$3\sqrt{3-3} + 4 \stackrel{?}{=} 3 + 1$$

$$4 = 4 \quad \checkmark$$

$$\therefore \text{Solution is: } \underline{x = 3}$$

Exercise

Solve: $\sqrt{2x-3} + \sqrt{x-2} = 1$

Solution

$$\sqrt{2x-3} = 1 - \sqrt{x-2}$$

$$(\sqrt{2x-3})^2 = (1 - \sqrt{x-2})^2$$

$$2x - 3 = 1 - 2\sqrt{x-2} + (\sqrt{x-2})^2$$

$$2x - 3 - 1 = -2\sqrt{x-2} + x - 2$$

$$2x - 4 - x + 2 = -2\sqrt{x-2}$$

$$x - 2 = -2\sqrt{x-2}$$

$$(x-2)^2 = (-2\sqrt{x-2})^2$$

$$x^2 - 4x + 4 = 4(x-2)$$

$$x^2 - 4x + 4 = 4x - 8$$

$$x^2 - 4x + 4 - 4x + 8 = 0$$

$$x^2 - 8x + 12 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$$

$$x = 2, 6$$

Check

$$x = 2 \Rightarrow \sqrt{2(2)-3} + \sqrt{2-2} = 1 \Rightarrow 1 + 0 = 1$$

$$x = 6 \Rightarrow \sqrt{2(6)-3} + \sqrt{6-2} = 1 \Rightarrow 3 + 2 = 1 \Rightarrow 5 \neq 1$$

\therefore Solution is: $x = 2$

Exercise

Solve: $\sqrt{x+5} - \sqrt{x-3} = 2$

Solution

$$\sqrt{x+5} = 2 + \sqrt{x-3}$$

$$(\sqrt{x+5})^2 = (2 + \sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + (\sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + x-3$$

$$x-x+5-4-3 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$$1 = \sqrt{x-3}$$

$$1 = x-3$$

$$\Rightarrow x = 4$$

Check: $\sqrt{4+5} - \sqrt{4-3} = 2$

$$3 - 1 = 2 \text{ (True statement)}$$

∴ Solution is: $x = 4$

Exercise

Solve: $\sqrt{2x+3} = 1 + \sqrt{x+1}$

Solution

$$(\sqrt{2x+3})^2 = (1 + \sqrt{x+1})^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$2x+3 = 1 + 2\sqrt{x+1} + x+1$$

$$2x+3 = 2\sqrt{x+1} + x+2$$

$$x+1 = 2\sqrt{x+1}$$

$$(x+1)^2 = (2\sqrt{x+1})^2$$

$$x^2 + 2x + 1 = 4(x+1)$$

$$x^2 + 2x + 1 = 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x-3 = 0 \quad x+1 = 0$$

$$x = 3 \quad x = -1$$

Check

$$x = 3$$

$$\sqrt{2(3)+3} = 1 + \sqrt{(3)+1}$$

$$\sqrt{9} = 1 + \sqrt{4}$$

$$3 = 3 \quad (\text{true})$$

$$x = -1$$

$$\sqrt{2(-1)+3} = 1 + \sqrt{(-1)+1}$$

$$\sqrt{1} = 1 + \sqrt{0}$$

$$1 = 1 \quad (\text{true})$$

$$\therefore \text{Solution: } \underline{x = -1, 3}$$

Exercise

Solve: $\sqrt{x+5} - \sqrt{x-3} = 2$

Solution

$$\sqrt{x+5} = 2 + \sqrt{x-3}$$

$$(\sqrt{x+5})^2 = (2 + \sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + (\sqrt{x-3})^2$$

$$x+5 = 4 + 4\sqrt{x-3} + x-3$$

$$x-x+5-4-3 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$$1 = \sqrt{x-3}$$

$$1 = x - 3$$

$$\Rightarrow x = 4$$

Check: $\sqrt{4+5} - \sqrt{4-3} = 2$

$$3 - 1 = 2 \text{ (True statement)}$$

\therefore Solution is: $x = 4$

Exercise

Solve: $|x| = -9$

Solution

$|x| = -9$ Not True

\therefore **No** Solution

Exercise

Solve: $|x| = 9$

Solution

\therefore Solutions: $x = \pm 9$

Exercise

Solve: $|x-2| = 7$

Solution

$$x - 2 = 7 \qquad x - 2 = -7$$

$$x = 9 \qquad x = -5$$

\therefore Solutions: $x = -5, 9$

Exercise

Solve: $|x-2| = 0$

Solution

$$x - 2 = 0$$

\therefore Solution: $x = 2$

Exercise

Solve: $|2x - 3| = 6$

Solution

$$2x - 3 = 6 \quad 2x - 3 = -6$$

$$2x = 9 \quad 2x = -3$$

$$\underline{x = \frac{9}{2} \quad x = -\frac{3}{2}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{3}{2}, \frac{9}{2}}$$

Exercise

Solve: $|2x - 1| = 11$

Solution

$$2x - 1 = 11 \quad 2x - 1 = -11$$

$$2x = 12 \quad 2x = -10$$

$$\underline{x = 6 \quad x = -5}$$

$$\therefore \text{Solutions: } \underline{x = -5, 6}$$

Exercise

Solve $7|5x| + 2 = 16$

Solution

$$7|5x| = 16 - 2$$

$$7|5x| = 14$$

$$|5x| = \frac{14}{7}$$

$$|5x| = 2$$

$$5x = 2 \quad 5x = -2$$

$$x = \frac{2}{5} \quad x = -\frac{2}{5}$$

$$\therefore \text{Solution: } \underline{x = \pm \frac{2}{5}}$$

Exercise

Solve $4\left|1 - \frac{3}{4}x\right| + 7 = 10$

Solution

$$4\left|1 - \frac{3}{4}x\right| = 10 - 7$$

$$|4 - 3x| = 10 - 7 \quad \text{Distribute 4}$$

$$4\left|1 - \frac{3}{4}x\right| = 3$$

$$|4 - 3x| = 3$$

$$\left|1 - \frac{3}{4}x\right| = \frac{3}{4}$$

$$1 - \frac{3}{4}x = \frac{3}{4}$$

$$1 - \frac{3}{4}x = -\frac{3}{4}$$

$$-\frac{3}{4}x = \frac{3}{4} - 1$$

$$-\frac{3}{4}x = -\frac{3}{4} - 1$$

$$-\frac{3}{4}x = -\frac{1}{4}$$

$$-\frac{3}{4}x = -\frac{7}{4}$$

$$x = -\frac{1}{4}\left(-\frac{4}{3}\right)$$

$$x = -\frac{7}{4}\left(-\frac{4}{3}\right)$$

$$x = \frac{1}{3}$$

$$x = \frac{7}{3}$$

$$4 - 3x = -3$$

$$4 - 3x = 3$$

$$-3x = -7$$

$$-3x = -1$$

$$x = \frac{7}{3}$$

$$x = \frac{1}{3}$$

$$\therefore \text{Solutions: } x = \frac{1}{3}, \frac{7}{3}$$

Exercise

Solve $|x + 7| + 6 = 2$

Solution

$$|x + 7| = 2 - 6$$

$$|x + 7| = -4$$

\Rightarrow No solution or \emptyset , since the absolute value can't be equal to a negative.

Exercise

Solve equation: $|5 - 3x| = 12$

Solution

$$5 - 3x = 12$$

$$5 - 3x = -12$$

$$5 - 3x - 5 = 12 - 5$$

$$5 - 3x - 5 = -12 - 5$$

$$-3x = 7$$

$$-3x = -17$$

$$x = -\frac{7}{3}$$

$$x = \frac{17}{3}$$

$$\therefore \text{Solutions: } x = \frac{17}{3}, -\frac{7}{3} \mid$$

Exercise

Solve equation: $|4x + 2| = 5$

Solution

$$4x + 2 = -5 \qquad 4x + 2 = 5$$

$$4x = -7 \qquad 4x = 3$$

$$x = -\frac{7}{4} \qquad x = \frac{3}{4}$$

$$\therefore \text{Solutions: } x = -\frac{7}{4}, \frac{3}{4} \mid$$

Exercise

Solve: $3|x + 5| = 12$

Solution

$$|x + 5| = 4$$

$$x + 5 = 4 \qquad x + 5 = -4$$

$$\underline{x = -1} \mid \qquad \underline{x = -9} \mid$$

$$\therefore \text{Solutions: } x = -9, -1 \mid$$

Exercise

Solve: $2|x - 6| = 8$

Solution

$$x - 6 = 4$$

$$x - 6 = 4 \qquad x - 6 = -4$$

$$\underline{x = 10} \mid \qquad \underline{x = 2} \mid$$

$$\therefore \text{Solutions: } x = 2, 10 \mid$$

Exercise

Solve: $3|2x - 1| = 21$

Solution

$$|2x - 1| = 7$$

$$\begin{array}{ll}
 2x - 1 = 7 & 2x - 1 = -7 \\
 2x = 8 & 2x = -6 \\
 \underline{x = 4} & \underline{x = -3} \\
 \therefore \text{Solutions: } \underline{x = -3, 4}
 \end{array}$$

Exercise

Solve: $2|3x - 2| = 14$

Solution

$$\begin{array}{ll}
 |3x - 2| = 7 & \\
 3x - 2 = 7 & 3x - 2 = -7 \\
 3x = 9 & 3x = -5 \\
 \underline{x = 3} & \underline{x = -\frac{5}{3}} \\
 \therefore \text{Solutions: } \underline{x = -\frac{5}{3}, 3}
 \end{array}$$

Exercise

Solve: $|3x - 1| + 2 = 16$

Solution

$$\begin{array}{ll}
 |3x - 1| = 14 & \\
 3x - 1 = 14 & 3x - 1 = -14 \\
 3x = 15 & 3x = -13 \\
 \underline{x = 5} & \underline{x = -\frac{13}{3}} \\
 \therefore \text{Solutions: } \underline{x = -\frac{13}{3}, 5}
 \end{array}$$

Exercise

Solve: $|6x - 2| + 4 = 32$

Solution

$$\begin{array}{ll}
 |6x - 2| = 28 & \\
 6x - 2 = 28 & 6x - 2 = -28 \\
 6x = 30 & 6x = -26
 \end{array}$$

$$\begin{array}{l} \underline{x = 5} \quad \underline{x = -\frac{13}{3}} \\ \therefore \text{Solutions: } \underline{x = -\frac{13}{3}, 5} \end{array}$$

Exercise

Solve: $7|5x| + 2 = 16$

Solution

$$7|5x| = 14$$

$$|5x| = 2$$

$$5x = 2 \quad 5x = -2$$

$$\underline{x = \frac{2}{5}} \quad \underline{x = -\frac{2}{5}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{2}{5}, \frac{2}{5}}$$

Exercise

Solve: $|4x + 1| + 10 = 4$

Solution

$$|4x + 1| = -6$$

\therefore **No** Solution

Exercise

Solve: $|4x + 1| + 4 = 10$

Solution

$$|4x + 1| = 6$$

$$4x + 1 = 6 \quad 4x + 1 = -6$$

$$4x = 5 \quad 4x = -7$$

$$\underline{x = \frac{5}{4}} \quad \underline{x = -\frac{7}{4}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{7}{4}, \frac{5}{4}}$$

Exercise

Solve: $|3x - 2| + 8 = 1$

Solution

$$|3x - 2| = -7$$

\therefore **No** Solution

Exercise

Solve: $|3x - 2| + 1 = 8$

Solution

$$|3x - 2| = 7$$

$$4x + 1 = 6 \quad 4x + 1 = -6$$

$$4x = 5 \quad 4x = -7$$

$$\underline{x = \frac{5}{4} \quad \quad x = -\frac{7}{4}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{7}{4}, \frac{5}{4}}$$

Exercise

Solve equation: $\left| \frac{6x+1}{x-1} \right| = 3$

Solution

$$\frac{6x+1}{x-1} = -3$$

$$(x-1) \frac{6x+1}{x-1} = -3(x-1)$$

$$6x+1 = -3x+3$$

$$6x+1+3x = -3x+3+3x$$

$$9x+1 = 3$$

$$9x = 2$$

$$x = \frac{2}{9}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{4}{3}, \frac{2}{9}}$$

$$\frac{6x+1}{x-1} = 3$$

$$6x+1 = 3(x-1)$$

$$6x+1 = 3x-3$$

$$6x+1-3x = 3x-3-3x$$

$$3x+1 = -3$$

$$3x = -4$$

$$x = -\frac{4}{3}$$

Exercise

Solve equation: $|x + 1| = |1 - 3x|$

Solution

$$x + 1 = -(1 - 3x) \qquad x + 1 = 1 - 3x$$

$$x + 1 = -1 + 3x \qquad x + 3x = 1 - 1$$

$$x - 3x = -1 - 1 \qquad 4x = 0$$

$$-2x = -2 \qquad x = 0$$

$$x = 1$$

∴ Solutions: $\underline{x = 0, 1}$

Exercise

Solve: $|3x - 1| = |x + 5|$

Solution

$$3x - 1 = x + 5 \qquad 3x - 1 = -(x + 5)$$

$$2x = 6 \qquad 3x - 1 = -x - 5$$

$$\underline{x = 3} \qquad 4x = -4$$

$$\underline{x = -1}$$

∴ Solutions: $\underline{x = -1, 3}$

Exercise

Solve: $|5x - 8| = |3x + 2|$

Solution

$$5x - 8 = 3x + 2 \qquad 5x - 8 = -(3x + 2)$$

$$2x = 10 \qquad 5x - 8 = -3x - 2$$

$$\underline{x = 5} \qquad 8x = 6$$

$$\underline{x = \frac{3}{4}}$$

∴ Solutions: $\underline{x = \frac{3}{4}, 5}$

Exercise

Solve: $|4x - 9| = |2x + 1|$

Solution

$$4x - 9 = 2x + 1 \quad 4x - 9 = -(2x + 1)$$

$$2x = 10$$

$$5x - 8 = -3x - 2$$

$$\underline{x = 5}$$

$$8x = 6$$

$$\underline{x = \frac{3}{4}}$$

$$\therefore \text{Solutions: } \underline{x = \frac{3}{4}, 5}$$

Exercise

Solve: $|2x - 4| = |x - 1|$

Solution

$$2x - 4 = x - 1$$

$$2x - 4 = -x + 1$$

$$\underline{x = 3}$$

$$3x = -5$$

$$\underline{x = -\frac{5}{3}}$$

$$\therefore \text{Solutions: } \underline{x = -\frac{5}{3}, 3}$$

Exercise

Solve: $|3x - 4| = |3x + 4|$

Solution

$$3x - 4 = 3x + 4$$

$$3x - 4 = -3x + 4$$

$$-4 \neq 4$$

$$6x = 8$$

$$\underline{x = \frac{4}{3}}$$

$$\therefore \text{Solution: } \underline{x = \frac{4}{3}}$$

Exercise

Solve: $|3x - 5| = |3x + 5|$

Solution

$$3x - 5 = 3x + 5 \quad 3x - 5 = -3x + 5$$

$$-5 \neq 5 \quad 6x = 10$$

$$\underline{x = \frac{5}{3}}$$

$$\therefore \text{Solution: } \underline{x = \frac{5}{3}}$$

Exercise

Solve: $|x - 3| = |5 - x|$

Solution

$$x - 3 = 5 - x \quad x - 3 = -5 + x$$

$$2x = 8 \quad -3 \neq -5$$

$$\underline{x = 4}$$

$$\therefore \text{Solutions: } \underline{x = 4}$$

Exercise

Solve: $|x - 3| = |6 - x|$

Solution

$$x - 3 = 6 - x \quad x - 3 = -6 + x$$

$$2x = 9 \quad -3 \neq -6$$

$$\underline{x = \frac{9}{2}}$$

$$\therefore \text{Solutions: } \underline{x = \frac{9}{2}}$$

Exercise

Solve: $\left| \frac{2}{3}x - 2 \right| = \left| \frac{1}{3}x + 3 \right|$

Solution

$$\frac{2}{3}x - 2 = \frac{1}{3}x + 3 \quad \frac{2}{3}x - 2 = -\frac{1}{3}x - 3$$

$$\frac{2}{3}x - \frac{1}{3}x = 3 + 2 \quad \frac{2}{3}x + \frac{1}{3}x = -3 + 2$$

$$\frac{1}{3}x = 5 \quad \underline{x = -1}$$

$$\underline{x = 15}$$

∴ Solutions: $\underline{x = -1, 15}$

Exercise

Solve: $\left| \frac{1}{2}x - 2 \right| = \left| x - \frac{1}{2} \right|$

Solution

$$\frac{1}{2}x - 2 = x - \frac{1}{2} \quad \frac{1}{2}x - 2 = -x + \frac{1}{2}$$

$$\frac{1}{2}x - x = 2 - \frac{1}{2} \quad \frac{1}{2}x + x = 2 + \frac{1}{2}$$

$$\frac{1}{2}x = \frac{3}{2} \quad \frac{3}{2}x = \frac{5}{2}$$

$$\underline{x = 3} \quad \underline{x = \frac{5}{3}}$$

∴ Solutions: $\underline{x = 3, \frac{5}{3}}$

Solution ***Section 1.7 – Inequalities***

Exercise

Find: $(-3, 0) \cap [-1, 2]$

Solution

$$(-3, 0) \cap [-1, 2] = \underline{[-1, 0]}$$

Exercise

Find: $(-3, 0) \cup [-1, 2]$

Solution

$$(-3, 0) \cup [-1, 2] = \underline{(-3, 2]}$$

Exercise

Find: $(-4, 0) \cap [-2, 1]$

Solution

$$(-4, 0) \cap [-2, 1] = \underline{[-2, 0]}$$

Exercise

Find: $(-4, 0) \cup [-2, 1]$

Solution

$$(-4, 0) \cup [-2, 1] = \underline{(-4, 1]}$$

Exercise

Find: $(-\infty, 5) \cap [1, 8)$

Solution

$$(-\infty, 5) \cap [1, 8) = \underline{[1, 5)}$$

Exercise

Find: $(-\infty, 5) \cup [1, 8)$

Solution

$$(-\infty, 5) \cup [1, 8) = \underline{(-\infty, 8)}$$

Exercise

Solve $-3x + 5 > -7$

Solution

$$-3x > -7 - 5$$

$$-3x > -12$$

$$\frac{-3}{-3}x < \frac{-12}{-3}$$

$$\therefore \text{Solution: } \underline{x < 4} \quad (-\infty, 4)$$

Exercise

Solve $2 - 3x \leq 5$

Solution

$$-3x \leq 3 \quad \text{Divide by } -3 \text{ both sides}$$

$$\frac{-3}{-3}x \geq \frac{3}{-3}$$

$$\therefore \text{Solution: } \underline{x \geq -1} \quad \text{or } [-1, \infty)$$

Exercise

Solve $4 - 3x \leq 7 + 2x$

Solution

$$4 - 3x - 4 \leq 7 + 2x - 4$$

$$-3x \leq 3 + 2x$$

$$-3x - 2x \leq 3 + 2x - 2x$$

$$-5x \leq 3$$

$$\therefore \text{Solution: } \underline{x \geq -\frac{3}{5}} \quad \text{or } \left[-\frac{3}{5}, \infty\right)$$

Exercise

Solve the inequality equation $5x + 11 < 26$

Solution

$$5x < 15$$

$$\therefore \text{Solution: } \underline{x < 3} \mid \text{ or } (-\infty, 3)$$

Exercise

Solve the inequality equation $3x - 8 \geq 13$

Solution

$$3x \geq 21$$

$$\therefore \text{Solution: } \underline{x \geq 7} \mid \text{ or } [7, \infty)$$

Exercise

Solve the inequality equation $-9x \geq 36$

Solution

$$-\frac{9x}{9} \geq \frac{36}{9}$$

$$-x \geq 4$$

$$\therefore \text{Solution: } \underline{x \leq -4} \mid \text{ or } (-\infty, -4]$$

Exercise

Solve the inequality equation $-4x \leq 64$

Solution

$$-\frac{4x}{4} \leq \frac{64}{4}$$

$$-x \leq 16$$

$$\therefore \text{Solution: } \underline{x \geq -16} \mid \text{ or } [-16, \infty)$$

Exercise

Solve the inequality equation $8x - 11 \leq 3x - 13$

Solution

$$5x \leq -2$$

$$\therefore \text{Solution: } \underline{x \leq -\frac{2}{5}} \quad \text{or} \quad \left(-\infty, -\frac{2}{5}\right]$$

Exercise

Solve the inequality equation $18x + 45 \leq 12x - 8$

Solution

$$6x \leq -53$$

$$\therefore \text{Solution: } \underline{x \leq -\frac{53}{6}} \quad \text{or} \quad \left(-\infty, -\frac{53}{6}\right]$$

Exercise

Solve the inequality equation $4(x+1) + 2 \geq 3x + 6$

Solution

$$4x + 4 + 2 \geq 3x + 6$$

$$\therefore \text{Solution: } \underline{x \geq 0} \quad \text{or} \quad [0, \infty)$$

Exercise

Solve the inequality equation $8x + 3 > 3(2x + 1) + x + 5$

Solution

$$8x + 3 > 6x + 3 + x + 5$$

$$8x + 3 > 7x + 8$$

$$\therefore \text{Solution: } \underline{x > 5} \quad \text{or} \quad (5, \infty)$$

Exercise

Solve the inequality equation $2x - 11 < -3(x + 2)$

Solution

$$2x - 11 < -3x - 6$$

$$5x < 5$$

$$\therefore \text{Solution: } \underline{x < 1} \quad \text{or} \quad (-\infty, 1)$$

Exercise

Solve the inequality equation $-4(x + 2) > 3x + 20$

Solution

$$-4x - 8 > 3x + 20$$

$$-7x > 28$$

$$-x > 4$$

$$\therefore \text{Solution: } \underline{x < -4} \mid \text{ or } (-\infty, -4)$$

Exercise

Solve the inequality equation $1 - (x + 3) \geq 4 - 2x$

Solution

$$1 - x - 3 \geq 4 - 2x$$

$$-x - 2 \geq 4 - 2x$$

$$x \geq 6$$

$$\therefore \text{Solution: } \underline{x \geq 6} \mid \text{ or } [6, \infty)$$

Exercise

Solve the inequality equation $5(3 - x) \leq 3x - 1$

Solution

$$15 - 5x \leq 3x - 1$$

$$-8x \leq -16$$

$$-x \leq -2$$

$$\therefore \text{Solution: } \underline{x \geq 2} \mid \text{ or } [2, \infty)$$

Exercise

Solve the inequality equation $\frac{x}{4} - \frac{1}{2} \leq \frac{x}{2} + 1$

Solution

$$4 \times \frac{x}{4} - \frac{1}{2} \leq \frac{x}{2} + 1$$

$$x - 2 \leq 2x + 4$$

$$-x \leq 6$$

$$\therefore \text{Solution: } \underline{x \geq -6} \mid \text{ or } [-6, \infty)$$

Exercise

Solve the inequality equation $\frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$

Solution

$$10 \times \frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$$

$$3x + 10 \geq 2 - x$$

$$4x \geq -8$$

$$\therefore \text{Solution: } \underline{x \geq -2} \mid \text{ or } [-2, \infty)$$

Exercise

Solve $6x - (2x + 3) \geq 4x - 5$

Solution

$$6x - 2x - 3 \geq 4x - 5$$

$$4x - 3 \geq 4x - 5$$

$$4x - 4x \geq 3 - 5$$

$$0 \geq -2 \quad (\text{true})$$

$$\therefore \text{Solution: } \mathbb{R} \quad (-\infty, \infty)$$

Exercise

Solve $\frac{2x-5}{-8} \leq 1-x$

Solution

$$(-8) \frac{2x-5}{-8} \geq (-8)(1-x)$$

$$2x - 5 \geq -8 + 8x$$

$$2x - 8x \geq -8 + 5$$

$$-6x \geq -3$$

$$\frac{-6}{-6} x \leq \frac{-3}{-6}$$

$$\therefore \text{Solution: } \underline{x \leq \frac{1}{2}} \mid \left(-\infty, \frac{1}{2}\right]$$

Exercise

Solve the inequality equation $1 - \frac{x}{2} > 4$

Solution

$$2 \times 1 - \frac{x}{2} > 4$$

$$2 - x > 8$$

$$-x > 6$$

$$\therefore \text{Solution: } \underline{x < -6} \mid \text{ or } (-\infty, -6)$$

Exercise

Solve the inequality equation $7 - \frac{4}{5}x < \frac{3}{5}$

Solution

$$5 \times 7 - \frac{4}{5}x < \frac{3}{5}$$

$$35 - 4x < 3$$

$$-4x > -32$$

$$\therefore \text{Solution: } \underline{x < 8} \mid \text{ or } (-\infty, 8)$$

Exercise

Solve the inequality equation $\frac{x-4}{6} \geq \frac{x-2}{9} + \frac{5}{18}$

Solution

$$18 \times \frac{x-4}{6} \geq \frac{x-2}{9} + \frac{5}{18}$$

$$3x - 12 \geq 2x - 4 + 5$$

$$3x - 12 \geq 2x + 1$$

$$\therefore \text{Solution: } \underline{x \geq -13} \mid \text{ or } [-13, \infty)$$

Exercise

Solve the inequality equation $\frac{4x-3}{6} + 2 \geq \frac{2x-1}{12}$

Solution

$$12 \times \frac{4x-3}{6} + 2 \geq \frac{2x-1}{12}$$

$$8x - 6 + 24 \geq 2x - 1$$

$$8x + 18 \geq 2x - 1$$

$$6x \geq -19$$

$$\therefore \text{Solution: } \underline{x \geq -\frac{19}{6}} \mid \text{ or } \left[-\frac{19}{6}, \infty\right)$$

Exercise

Solve the inequality equation $4(3x - 2) - 3x < 3(1 + 3x) - 7$

Solution

$$12x - 8 - 3x < 3 + 9x - 7$$

$$9x - 8 < 9x - 4$$

$$-8 < -4 \quad \text{True}$$

$$\therefore \text{Solution: } \underline{\mathbb{R}}$$

Exercise

Solve the inequality equation $3(x - 8) - 2(10 - x) < 5(x - 1)$

Solution

$$3x - 24 - 20 + 2x < 5x - 5$$

$$5x - 44 < 5x - 5$$

$$-44 < -5 \quad \text{True}$$

$$\therefore \text{Solution: } \underline{\mathbb{R}}$$

Exercise

Solve the inequality equation $8(x + 1) \leq 7(x + 5) + x$

Solution

$$8x + 8 \leq 7x + 35 + x$$

$$8x + 8 \leq 8x + 35$$

$$8 \leq 35$$

$$\therefore \text{Solution: } \underline{\mathbb{R}}$$

Exercise

Solve the inequality equation $4(x - 1) \geq 3(x - 2) + x$

Solution

$$4x - 4 \geq 3x - 6 + x$$

$$4x - 4 \geq 4x - 6$$

$$-4 \geq -6 \quad \text{True}$$

$$\therefore \text{Solution: } \underline{\mathbb{R}}$$

Exercise

Solve the inequality equation $7(x + 4) - 13 > 12 + 13(3 + x)$

Solution

$$7x + 28 - 13 > 12 + 39 + 13x$$

$$7x + 15 > 51 + 13x$$

$$-6x > 36$$

$$\therefore \text{Solution: } \underline{x < -9} \quad \text{or } (-\infty, -9)$$

Exercise

Solve the inequality equation $-2[7x - (2x - 3)] < -2(x + 1)$

Solution

$$-2(7x - 2x + 3) < -2x - 2$$

$$-2(5x + 3) < -2x - 2$$

$$-10x - 6 < -2x - 2$$

$$-8x < 4$$

$$\therefore \text{Solution: } \underline{x > \frac{1}{2}} \quad \text{or } \left(\frac{1}{2}, \infty\right)$$

Exercise

Solve the inequality equation $6 - \frac{2}{3}(3x - 12) \leq \frac{2}{5}(10x + 50)$

Solution

$$15 \times \quad 6 - \frac{2}{3}(3x - 12) \leq \frac{2}{5}(10x + 50)$$

$$90 - 10(3x - 12) \leq 6(10x + 50)$$

$$90 - 30x + 120 \leq 60x + 300$$

$$210 - 30x \leq 60x + 300$$

$$-90x \leq 90$$

$$\therefore \text{Solution: } \underline{x \geq -1} \quad \text{or } [-1, \infty)$$

Exercise

Solve the inequality equation $\frac{2}{7}(7 - 21x) - 4 < 10 - \frac{3}{11}(11x - 11)$

Solution

$$77 \times \frac{2}{7}(7 - 21x) - 4 < 10 - \frac{3}{11}(11x - 11)$$

$$22(7 - 21x) - 308 < 770 - 21(11x - 11)$$

$$154 - 462x - 308 < 770 - 231x + 231$$

$$-462x - 154 < -231x + 1,001$$

$$-231x < 1,155$$

$$-x < 5$$

$$\therefore \text{Solution: } \underline{x > 5} \quad \text{or} \quad (5, \infty)$$

Exercise

Solve the inequality equation $3[3(x + 5) + 8x + 7] + 5[3(x - 6) - 2(3x - 5)] < 2(4x + 3)$

Solution

$$3(3x + 15 + 8x + 7) + 5(3x - 18 - 6x + 10) < 8x + 6$$

$$3(11x + 22) + 5(-3x - 8) < 8x + 6$$

$$33x + 66 - 15x - 40 < 8x + 6$$

$$18x + 26 < 8x + 6$$

$$10x < -20$$

$$\therefore \text{Solution: } \underline{x < -2} \quad \text{or} \quad (-\infty, -2)$$

Exercise

Solve the inequality equation $5[3(2 - 3x) - 2(5 - x)] - 6[5(x - 2) - 2(4x - 3)] < 3x + 19$

Solution

$$5(6 - 9x - 10 + 2x) - 6(5x - 10 - 8x + 6) < 3x + 19$$

$$5(-7x - 4) - 6(-3x - 4) < 3x + 19$$

$$-35x - 20 + 18x + 24 < 3x + 19$$

$$-17x + 4 < 3x + 19$$

$$-20x < 15$$

$$\therefore \text{Solution: } \underline{x > -\frac{3}{4}} \quad \text{or} \quad \left(-\frac{3}{4}, \infty\right)$$

Exercise

Solve the inequality equation $0 \leq 3x - 1 \leq 10$

Solution

$$0 + 1 \leq 3x - 1 + 1 \leq 10 + 1$$

$$1 \leq 3x \leq 11$$

$$\therefore \text{Solution: } \underline{\frac{1}{3} \leq x \leq \frac{11}{3}} \quad |$$

Exercise

Solve the inequality equation $0 \leq 1 - 3x \leq 10$

Solution

$$-1 \leq -3x \leq 9$$

$$-\frac{1}{3} \leq -x \leq 3$$

$$\therefore \text{Solution: } \underline{-3 \leq x \leq \frac{1}{3}} \quad |$$

Exercise

Solve the inequality equation $0 \leq 2x + 6 \leq 54$

Solution

$$-6 \leq 2x \leq 48$$

$$\therefore \text{Solution: } \underline{-3 \leq x \leq 24} \quad |$$

Exercise

Solve the inequality equation $-3 \leq \frac{2}{3}x - 5 \leq -1$

Solution

$$-3 \leq \frac{2}{3}x - 5 \leq -1$$

$$-3 + 5 \leq \frac{2}{3}x - 5 + 5 \leq -1 + 5$$

$$2 \leq \frac{2}{3}x \leq 4$$

$$2 \frac{3}{2} \leq \frac{3}{2} \frac{2}{3}x \leq \frac{3}{2} 4$$

$$\therefore \text{Solution: } \underline{3 \leq x \leq 6} \quad |$$

Exercise

Solve $-6 \leq 6x + 3 \leq 21$

Solution

$$-6 - 3 \leq 6x + 3 - 3 \leq 21 - 3$$

$$-9 \leq 6x \leq 18$$

$$-\frac{9}{6} \leq \frac{6}{6}x \leq \frac{18}{6}$$

$$\therefore \text{Solution: } \underline{-\frac{3}{2} \leq x \leq 3} \quad \left[-\frac{3}{2}, 3 \right]$$

Exercise

Solve the inequality equation: $1 \leq 2x + 3 < 11$

Solution

$$1 - 3 \leq 2x + 3 - 3 < 11 - 3$$

$$-2 \leq 2x < 8$$

$$-\frac{2}{2} \leq \frac{2}{2}x < \frac{8}{2}$$

$$\therefore \text{Solution: } \underline{-1 \leq x < 4}$$

Exercise

Solve the inequality equation $|x| < 2$

Solution

$$\therefore \text{Solution: } \underline{-2 < x < 2}$$

Exercise

Solve the inequality equation $|x| \geq 2$

Solution

$$\therefore \text{Solution: } \underline{x \leq -2 \quad x \geq 2}$$

Exercise

Solve the inequality equation $|x - 2| < 1$

Solution

$$-1 < x - 2 < 1$$

$$\therefore \text{Solution: } \underline{1 < x < 3}$$

Exercise

Solve the inequality equation $|x - 1| < 4$

Solution

$$-4 < x - 1 < 4$$

$$\therefore \text{Solution: } \underline{-3 < x < 5}$$

Exercise

Solve the inequality equation $|x + 2| \geq 1$

Solution

$$x + 2 \leq -1 \qquad x + 2 \geq 1$$

$$x \leq -3 \qquad x \geq -1$$

$$\therefore \text{Solution: } \underline{x \leq -3 \quad x \geq -1}$$

Exercise

Solve the inequality equation $|x + 1| \geq 4$

Solution

$$x + 1 \leq -4 \qquad x + 1 \geq 4$$

$$x \leq -5 \qquad x \geq 3$$

$$\therefore \text{Solution: } \underline{x \leq -5 \quad x \geq 3}$$

Exercise

Solve the inequality equation $|3x + 5| < 17$

Solution

$$-17 < 3x + 5 < 17$$

$$-22 < 3x < 12$$

$$\therefore \text{Solution: } \underline{-11 < x < 4}$$

Exercise

Solve the inequality equation $|5x - 2| < 13$

Solution

$$-13 < 5x - 2 < 13$$

$$-11 < 5x < 15$$

$$\therefore \text{Solution: } \underline{-\frac{11}{5} < x < 3}$$

Exercise

Solve the inequality equation $|5x - 2| \geq 13$

Solution

$$5x - 2 \leq -13 \quad 5x - 2 \geq 13$$

$$5x \leq -11 \quad 5x \geq 15$$

$$x \leq -\frac{11}{5} \quad x \geq 3$$

$$\therefore \text{Solution: } \underline{x \leq -\frac{11}{5} \quad x \geq 3}$$

Exercise

Solve the inequality equation $|2(x - 1) + 4| \leq 8$

Solution

$$-8 \leq 2x - 2 + 4 \leq 8$$

$$-8 \leq 2x + 2 \leq 8$$

$$-10 \leq 2x \leq 6$$

$$\therefore \text{Solution: } \underline{-5 \leq x \leq 3}$$

Exercise

Solve the inequality equation $|3(x - 1) + 2| \leq 20$

Solution

$$-20 \leq 3x - 3 + 2 \leq 20$$

$$-20 \leq 3x - 1 \leq 20$$

$$-19 \leq 3x \leq 21$$

$$\therefore \text{Solution: } \underline{-\frac{19}{3} \leq x \leq 7}$$

Exercise

Solve the inequality equation $\left| \frac{2x+6}{3} \right| > 2$

Solution

$$|2x+6| > 6$$

$$2x+6 < -6 \qquad 2x+6 > 6$$

$$2x < -12 \qquad 2x > 0$$

$$x < -6 \qquad x > 0$$

$$\therefore \text{Solution: } \underline{x < -6 \quad x > 0}$$

Exercise

Solve the inequality equation $\left| \frac{3x-3}{4} \right| < 6$

Solution

$$\frac{3}{4}|x-1| < 6$$

$$|x-1| < 8$$

$$-8 < x-1 < 8$$

$$\therefore \text{Solution: } \underline{-7 < x < 9}$$

Exercise

Solve the inequality equation $\left| \frac{2x+2}{4} \right| \geq 2$

Solution

$$\frac{1}{2}|x+1| \geq 2$$

$$|x+1| \geq 4$$

$$x+1 \leq -4 \qquad x \geq 3$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x \geq 3}$$

Exercise

Solve the inequality equation $\left| \frac{3x-3}{9} \right| \leq 1$

Solution

$$\frac{1}{3}|x-1| \leq 1$$

$$|x-1| \leq 3$$

$$-3 \leq x-1 \leq 3$$

$$\therefore \text{Solution: } \underline{-2 \leq x \leq 4}$$

Exercise

Solve the inequality equation $\left|3 - \frac{2x}{3}\right| > 5$

Solution

$$\frac{1}{3}|9 - 2x| > 5$$

$$|9 - 2x| > 15$$

$$9 - 2x < -15 \quad 9 - 2x > 15$$

$$-2x < -24 \quad -2x > 6$$

$$x > 12 \quad x < -3$$

$$\therefore \text{Solution: } \underline{x < -3 \quad x > 12}$$

Exercise

Solve the inequality equation $\left|3 - \frac{3x}{4}\right| < 9$

Solution

$$\frac{3}{4}|4 - x| < 9$$

$$|4 - x| < 12$$

$$-12 < 4 - x < 12$$

$$-16 < -x < 8$$

$$16 > x > -8$$

$$\therefore \text{Solution: } \underline{-8 < x < 16}$$

Exercise

Solve the inequality equation $|x - 2| < -1$

Solution

Impossible, since Absolute value can't be negative.

\therefore No **Solution**

Exercise

Solve the inequality equation $|x + 2| < -3$

Solution

Impossible, since Absolute value can't be negative.

\therefore No ***Solution***

Exercise

Solve the inequality equation $|x + 6| > -10$

Solution

\therefore ***Solution:*** \mathbb{R}

Exercise

Solve the inequality equation $|x + 2| > -8$

Solution

\therefore ***Solution:*** \mathbb{R}

Exercise

Solve the inequality equation $|x + 2| + 9 \leq 16$

Solution

$$|x + 2| \leq 5$$

$$-5 \leq x + 2 \leq 5$$

\therefore ***Solution:*** $-7 \leq x \leq 3$

Exercise

Solve the inequality equation $|x - 2| + 4 \geq 5$

Solution

$$|x - 2| \geq 1$$

$$x - 2 \leq -1 \quad x - 2 \geq 1$$

$$x \leq 1 \quad x \geq 3$$

\therefore ***Solution:*** $x \leq 1 \quad x \geq 3$

Exercise

Solve the inequality equation $2|2x - 3| + 10 > 12$

Solution

$$2|2x - 3| > 2$$

$$|2x - 3| > 1$$

$$2x - 3 < -1 \quad 2x - 3 > 1$$

$$2x < 2 \quad 2x > 4$$

$$x < 1 \quad x > 2$$

$$\therefore \text{Solution: } \underline{x < 1 \quad x > 2}$$

Exercise

Solve the inequality equation $3|2x - 1| + 2 < 8$

Solution

$$3|2x - 1| < 6$$

$$|2x - 1| < 2$$

$$-2 < 2x - 1 < 2$$

$$-1 < 2x < 3$$

$$\therefore \text{Solution: } \underline{-\frac{1}{2} < x < \frac{3}{2}}$$

Exercise

Solve the inequality equation $-4|1 - x| < -16$

Solution

$$|1 - x| > 4$$

$$1 - x < -4 \quad 1 - x > 4$$

$$-x < -5 \quad -x > 3$$

$$x > 5 \quad x < -3$$

$$\therefore \text{Solution: } \underline{x < -3 \quad x > 5}$$

Exercise

Solve the inequality equation $-2|5 - x| < -6$

Solution

$$|5 - x| > 3$$

$$5 - x < -3$$

$$5 - x > 3$$

$$-x < -8$$

$$-x > -2$$

$$x > 8$$

$$x < 2$$

$$\therefore \text{Solution: } \underline{x < 2 \quad x > 8}$$

Exercise

Solve the inequality equation $3 \leq |2x - 1|$

Solution

$$|2x - 1| \geq 3$$

$$2x - 1 \leq -3$$

$$2x - 1 \geq 3$$

$$2x \leq -2$$

$$2x \geq 4$$

$$x \leq -1$$

$$x \geq 2$$

$$\therefore \text{Solution: } \underline{x \leq -1 \quad x \geq 2}$$

Exercise

Solve the inequality equation $9 \leq |4x + 7|$

Solution

$$|4x + 7| \geq 9$$

$$4x + 7 \leq -9$$

$$4x + 7 \geq 9$$

$$4x \leq -16$$

$$4x \geq 2$$

$$x \leq -4$$

$$x \geq \frac{1}{2}$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x \geq \frac{1}{2}}$$

Exercise

Solve the inequality equation: $12 < \left| -2x + \frac{6}{7} \right| + \frac{3}{7}$

Solution

$$(7)12 < \left| -(7)2x + (7)\frac{6}{7} \right| + (7)\frac{3}{7}$$

Multiply by 7 both sides

$$84 < \left| -14x + 6 \right| + 3$$

$$81 < \left| -14x + 6 \right|$$

$$|-14x + 6| > 81$$

$$-14x + 6 < -81 \quad -14x + 6 > 81$$

$$-14x < -81 - 6 \quad -14x > 81 - 6$$

$$-14x < -87 \quad -14x > 75$$

$$x > \frac{87}{14} \quad x < -\frac{75}{14}$$

$$\therefore \text{Solution: } \underline{x < -\frac{75}{14} \quad x > \frac{87}{14}} \quad \left(-\infty, -\frac{75}{14}\right) \cup \left(\frac{87}{14}, \infty\right)$$

Exercise

Solve the inequality equation: $4 + \left|3 - \frac{x}{3}\right| \geq 9$

Solution

$$\left|3 - \frac{x}{3}\right| \geq 9 - 4$$

$$\left|3 - \frac{x}{3}\right| \geq 5$$

$$\left|(3)3 - (3)\frac{x}{3}\right| \geq (3)5$$

$$|9 - x| \geq 15$$

$$9 - x \leq -15 \quad 9 - x \geq 15$$

$$-x \leq -24 \quad -x \geq 6$$

$$x \geq 24 \quad x \leq -6$$

$$\therefore \text{Solution: } \underline{x \leq -6 \quad x \geq 24} \quad (-\infty, -6] \cup [24, \infty)$$

Exercise

Solve the inequality equation: $|x - 2| < 5$

Solution

$$-5 < x - 2 < 5$$

$$\therefore \text{Solution: } \underline{-3 < x < 7}$$

Exercise

Solve the inequality equation: $|2x + 1| < 7$

Solution

$$-7 < 2x + 1 < 7$$

$$-7 - 1 < 2x + 1 - 1 < 7 - 1$$

$$-8 < 2x < 6$$

$$-\frac{8}{2} < \frac{2}{2}x < \frac{6}{2}$$

$$\therefore \text{Solution: } \underline{-4 < x < 3}$$

Exercise

Solve the inequality equation: $|5x + 2| - 2 < 3$

Solution

$$|5x + 2| < 5$$

$$-5 < 5x + 2 < 5$$

$$-7 < 5x < 3$$

$$\therefore \text{Solution: } \underline{-\frac{7}{5} < x < \frac{3}{5}} \quad \left(-\frac{7}{5}, \frac{3}{5}\right)$$

Exercise

Solve the inequality equation: $|2 - 7x| - 1 > 4$

Solution

$$|2 - 7x| > 5$$

$$2 - 7x < -5$$

$$2 - 7x > 5$$

$$-7x < -7$$

$$-7x > 3$$

$$x > 1$$

$$x < -\frac{3}{7}$$

$$\therefore \text{Solution: } \underline{x < -\frac{3}{7} \quad x > \frac{1}{14}} \quad \left(-\infty, -\frac{3}{7}\right) \cup \left(\frac{1}{14}, \infty\right)$$

Exercise

Solve the inequality equation: $|3x - 4| < 2$

Solution

$$-2 < 3x - 4 < 2$$

$$-2 + 4 < 3x - 4 + 4 < 2 + 4$$

$$2 < 3x < 6$$

$$\therefore \text{Solution: } \underline{\frac{2}{3} < x < 2} \quad \underline{\left(\frac{2}{3}, 2\right)}$$

Exercise

Solve the inequality equation: $|2x + 5| \geq 3$

Solution

$$2x + 5 \leq -3 \qquad 2x + 5 \geq 3$$

$$2x \leq -8 \qquad 2x \geq -2$$

$$x \leq -4 \qquad x \geq -1$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x \geq -1} \quad (-\infty, -4] \cup [-1, \infty)$$

Exercise

Solve $|12 - 9x| \geq -12$

Solution

\therefore **Solution** set: $(-\infty, \infty)$ because the absolute value always greater than any negative number.

Exercise

Solve $|6 - 3x| < -11$

Solution

\therefore **No solution**, because the absolute value cannot be less than any negative number

Exercise

Solve the inequality equation $|7 + 2x| < 0$

Solution

\therefore **No solution**, because the absolute value cannot be any negative number

Exercise

Solve: $x^2 - 7x + 10 > 0$

Solution

$$x^2 - 7x + 10 > 0$$

$$(x - 5)(x - 2) > 0$$

$$x = 2, 5$$

$$\therefore \text{Solution: } \underline{x < 2 \quad x > 5} \quad (-\infty, 2) \cup (5, \infty)$$

0	2		5
+		-	+

Exercise

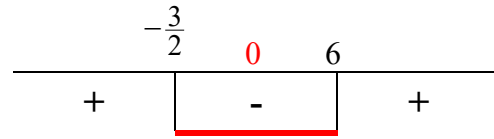
Solve: $2x^2 - 9x \leq 18$

Solution

$$2x^2 - 9x - 18 \leq 0$$

$$(2x + 3)(x - 6) \leq 0$$

$$\therefore \text{Solution: } \underline{-\frac{3}{2} \leq x \leq 6} \quad \left[-\frac{3}{2}, 6\right]$$



Exercise

Solve the inequality: $x^2 - 5x + 4 > 0$

Solution

$$x^2 - 5x + 4 > 0$$

$$x = 1, 4$$

$$\therefore \text{Solution: } \underline{x < 1 \quad x > 4} \quad \underline{(-\infty, 1) \cup (4, \infty)}$$

Exercise

Solve $x^2 + x - 2 > 0$

Solution

$$x^2 + x - 2 = 0 \rightarrow x = -2, 1$$

$$\therefore \text{Solution: } \underline{x < -2 \quad x > 1} \quad \underline{(-\infty, -2) \cup (1, \infty)}$$

Exercise

Solve $x^2 - 4x + 12 < 0$

Solution

$$x^2 - 4x + 12 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 48}}{2}$$

$$= \frac{4 \pm \sqrt{-32}}{2} \quad \text{Complex number}$$

\therefore No **Solution**

Exercise

Solve the inequality equation $x^2 + 7x > 0$

Solution

$$x^2 + 7x = 0$$

$$x(x + 7) = 0$$

$$\underline{x = 0, -7 \mid}$$

$$\therefore \text{Solution: } \underline{x < -7 \quad x > 0 \mid}$$

Exercise

Solve the inequality equation $x^2 - 49 < 0$

Solution

$$x^2 - 49 = 0$$

$$\underline{x = \pm 7 \mid}$$

$$\therefore \text{Solution: } \underline{-7 < x < 7 \mid}$$

Exercise

Solve the inequality equation $x^2 - 5x \geq 0$

Solution

$$x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$$\underline{x = 0, 5 \mid}$$

$$\therefore \text{Solution: } \underline{x \leq 0 \quad x \geq 5 \mid}$$

Exercise

Solve the inequality equation $x^2 - 16 \leq 0$

Solution

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$\underline{x = \pm 4 \mid}$$

$$\therefore \text{Solution: } \underline{-4 \leq x \leq 4 \mid}$$

Exercise

Solve the inequality equation $x^2 + 7x + 10 < 0$

Solution

$$x^2 + 7x + 10 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{2}$$

$$= \frac{-7 \pm 3}{2}$$

$$= \begin{cases} \frac{-7-3}{2} = -5 \\ \frac{-7+3}{2} = -2 \end{cases}$$

$$\therefore \text{Solution: } \underline{-5 < x < -2}$$

Exercise

Solve the inequality equation $x^2 - 3x \geq 28$

Solution

$$x^2 - 3x - 28 = 0$$

$$x = \frac{3 \pm \sqrt{121}}{2}$$

$$= \begin{cases} \frac{3-11}{2} = -4 \\ \frac{3+11}{2} = 7 \end{cases}$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x \geq 7}$$

Exercise

Solve the inequality equation $x^2 + 5x + 6 < 0$

Solution

$$x^2 + 5x + 6 = 0$$

$$x = \frac{-5 \pm 1}{2}$$

$$= \begin{cases} \frac{-5-1}{2} = -3 \\ \frac{-5+1}{2} = -2 \end{cases}$$

$$\therefore \text{Solution: } \underline{-3 < x < -2}$$

Exercise

Solve the inequality equation $x^2 < -x + 30$

Solution

$$x^2 + x - 30 = 0$$

$$x = \frac{-1 \pm 11}{2}$$

$$= \begin{cases} \frac{-1-11}{2} = -6 \\ \frac{-1+11}{2} = 5 \end{cases}$$

$$\therefore \text{Solution: } \underline{-6 < x < 5}$$

Exercise

Solve: $x^3 - 3x^2 - 9x + 27 < 0$

Solution

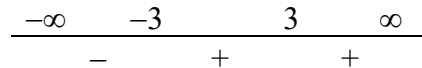
$$x^3 - 3x^2 - 9x + 27 = 0$$

$$x^2(x-3) - 9(x-3) = 0$$

$$(x-3)(x^2-9) = 0$$

$$\rightarrow \begin{cases} x-3=0 \rightarrow \underline{x=3} \\ x^2-9=0 \rightarrow x^2=9 \rightarrow \underline{x=\pm 3} \end{cases}$$

$$\therefore \text{Solution: } \underline{x < -3} \quad \underline{(-\infty, -3)}$$



Exercise

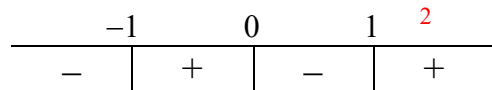
Solve $x^3 - x > 0$

Solution

$$x(x^2-1) = 0$$

$$\rightarrow \begin{cases} x=0 \\ x^2-1=0 \rightarrow x^2=1 \rightarrow x=\pm 1 \end{cases}$$

$$\therefore \text{Solution: } \underline{-1 < x < 0 \quad x > 1} \quad \underline{(-1, 0) \cup (1, \infty)}$$



Exercise

$$x^3 + 3x^2 \leq x + 3$$

Solution

$$x^3 + 3x^2 - x - 3 = 0$$

$$x^2(x+3) - (x+3) = 0$$

$$(x+3)(x^2 - 1) = 0$$

$$\begin{cases} x+3=0 \rightarrow x=-3 \\ x^2-1=0 \rightarrow x^2=1 \rightarrow x=\pm 1 \end{cases}$$

	-3		-1	0	1	
-		+	-		+	

$$\therefore \text{Solution: } \underline{-1 < x < 0 \quad x > 1} \quad \underline{(-\infty, -3] \cup [-1, 1]}$$

Exercise

$$\text{Solve } x^3 + x^2 \geq 48x$$

Solution

$$x^3 + x^2 - 48x = 0$$

$$x(x^2 + x - 48) = 0$$

$$x = 0 \quad x^2 + x - 48 = 0$$

$$x = \frac{-1 \pm \sqrt{1+192}}{2}$$

	$\frac{-1-\sqrt{193}}{2}$		0	1	$\frac{-1+\sqrt{193}}{2}$	
-	+	-	+			

$$\therefore \text{Solution: } \underline{\frac{-1-\sqrt{193}}{2} < x < 0 \quad x > \frac{-1+\sqrt{193}}{2}} \quad \underline{\left[\frac{-1-\sqrt{193}}{2}, 0 \right] \cup \left[\frac{-1+\sqrt{193}}{2}, \infty \right)}$$

Exercise

$$\text{Solve the inequality equation } x^3 - x^2 - 16x + 16 < 0$$

Solution

$$x^2(x-1) - 16(x-1) = 0$$

$$(x-1)(x^2 - 16) = 0$$

$$x = 1, \pm 4$$

	-4	0	1	4	
-	+	-	+		

$$\therefore \text{Solution: } \underline{x < -4 \quad 1 < x < 4}$$

Exercise

Solve the inequality equation $x^3 + x^2 - 9x - 9 > 0$

Solution

$$x^2(x+1) - 9(x+1) = 0$$

$$(x+1)(x^2 - 9) = 0$$

$$x = -1, \pm 3$$

$$\therefore \text{Solution: } \underline{-3 < x < -1 \quad x > 3}$$

	-3		-1	0		3
-		+		-		+

Exercise

Solve the inequality equation $x^3 + 3x^2 - 4x - 12 \geq 0$

Solution

$$x^2(x+3) - 4(x+3) = 0$$

$$(x+3)(x^2 - 4) = 0$$

$$x = -3, \pm 2$$

$$\therefore \text{Solution: } \underline{-3 < x < -2 \quad x > 2}$$

	-3		-2	0		2
-		+		-		+

Exercise

Solve the inequality equation $x^4 - 20x^2 + 64 \leq 0$

Solution

$$x^4 - 20x^2 + 64 = 0$$

$$x^2 = \frac{20 \pm \sqrt{400 - 256}}{2}$$

$$= \begin{cases} \frac{20-12}{2} = 4 \\ \frac{20+12}{2} = 16 \end{cases}$$

$$\begin{cases} x^2 = 4 \rightarrow x = \pm 2 \\ x^2 = 16 \rightarrow x = \pm 4 \end{cases}$$

$$\therefore \text{Solution: } \underline{-4 \leq x \leq -2 \quad 2 \leq x \leq 4}$$

	-4		-2	0		2		4
+		-		+		-		+

Exercise

Solve the inequality equation $x^4 - 10x^2 + 9 \geq 0$

Solution

$$x^4 - 10x^2 + 9 = 0$$

$$\begin{cases} x^2 = 1 \rightarrow x = \pm 1 \\ x^2 = 9 \rightarrow x = \pm 3 \end{cases}$$

$$\therefore \text{Solution: } \underline{x \leq -3 \quad -1 \leq x \leq 1 \quad x \geq 3}$$

-3	-1	0	1	3
+	-	+	-	+

Exercise

Solve the inequality equation $\frac{x+4}{x-1} < 0$

Solution

Restriction: $x \neq 1$

$$\frac{x+4}{x-1} = 0$$

$$\underline{x = -4}$$

$$\therefore \text{Solution: } \underline{-4 < x < 1}$$

	-4	1	
+	-		+

Exercise

Solve the inequality equation $\frac{x-2}{x+3} > 0$

Solution

Restriction: $x \neq -3$

$$\frac{x-2}{x+3} = 0$$

$$\underline{x = 2}$$

$$\therefore \text{Solution: } \underline{x < -3 \quad x > 2}$$

-3	0	2
+	-	+

Exercise

Solve the inequality equation $\frac{x-5}{x+8} \geq 3$

Solution

Restriction: $x \neq -8$

$$\frac{x-5}{x+8} - 3 = 0$$

$$x - 5 - 3x - 24 = 0$$

$$-2x = 29$$

$$x = -\frac{29}{2}$$

$$\therefore \text{Solution: } \underline{x \leq -\frac{29}{2} \quad x > -8}$$

	$-\frac{29}{2}$	0	-8
+		-	+

Exercise

Solve the inequality equation $\frac{x-4}{x+6} \leq 1$

Solution

Restriction: $x \neq -6$

$$\frac{x-4}{x+6} - 1 = 0$$

$$x - 4 - x - 6 = 0$$

$$-10 = 0 \quad \times$$

$$\therefore \text{Solution: } \underline{x > -6}$$

	-6	0
+		-

Exercise

Solve the inequality equation $\frac{x}{2x+7} \geq 4$

Solution

Restriction: $x \neq -\frac{7}{2}$

$$\frac{x}{2x+7} - 4 = 0$$

$$x - 8x - 28 = 0$$

$$7x = -28$$

$$x = -4$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x > -\frac{7}{2}}$$

	-4	0	$-\frac{7}{2}$
+		-	+

Exercise

Solve the inequality equation $\frac{x}{3x-5} \leq -5$

Solution

Restriction: $x \neq \frac{5}{3}$

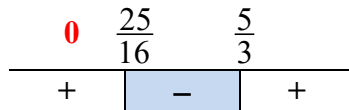
$$\frac{x}{3x-5} + 5 = 0$$

$$x + 15x - 25 = 0$$

$$16x = 25$$

$$x = \frac{25}{16}$$

$$\therefore \text{Solution: } \frac{25}{16} \leq x < \frac{5}{3}$$



Exercise

Solve the inequality equation $\frac{x+2}{x-5} \leq 2$

Solution

Restriction: $x \neq 5$

$$\frac{x+2}{x-5} - 2 = 0$$

$$x + 2 - 2x + 10 = 0$$

$$x = 12$$

$$\therefore \text{Solution: } 5 \leq x < 12$$



Exercise

Solve the inequality equation $\frac{3x+1}{x-2} \geq 4$

Solution

Restriction: $x \neq 2$

$$\frac{3x+1}{x-2} - 4 = 0$$

$$3x + 1 - 4x + 8 = 0$$

$$x = 9$$

$$\therefore \text{Solution: } 2 \leq x < 9$$



Exercise

Solve the inequality equation $\frac{x}{x-3} > 0$

Solution

Restriction: $x \neq 3$

$$\frac{x}{x-3} = 0$$

$$x = 0$$

$$\therefore \text{Solution: } \underline{x < 0 \quad x > 3} \quad | \quad \underline{(-\infty, 0) \cup (3, \infty)} \quad |$$

	0		3
+		-	+

Exercise

Solve the inequality equation $\frac{x-3}{x+2} \geq 0$

Solution

Restriction: $x \neq -2$

$$\frac{x-3}{x+2} = 0$$

$$x = 3$$

$$\therefore \text{Solution: } \underline{x < -2 \quad x \geq 3} \quad |$$

	-2	0	3
+		-	+

Exercise

Solve the inequality equation $\frac{x-2}{x+2} \leq 2$

Solution

Restriction: $x \neq -2$

$$\frac{x-2}{x+2} - 2 = 0$$

$$x - 2 - 2x - 4 = 0$$

$$-x - 6 = 0$$

$$x = -6$$

$$\therefore \text{Solution: } \underline{x \leq -6 \quad x > -2} \quad | \quad \underline{(-\infty, -6] \cup (-2, \infty)} \quad |$$

	-6		-2
-		+	-

Exercise

Solve the inequality equation $\frac{x+2}{x-2} \geq 2$

Solution

Restriction: $x \neq 2$

$$\frac{x+2}{x-2} - 2 = 0$$

$$x + 2 - 2x + 4 = 0$$

$$\underline{x = 6}$$

$$\therefore \text{Solution: } \underline{2 < x \leq 6}$$



Exercise

Solve the inequality equation $\frac{x+2}{3+2x} \leq 5$

Solution

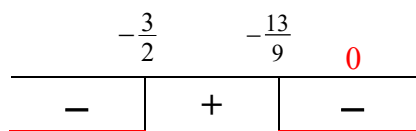
$$\text{Restriction: } x \neq -\frac{3}{2}$$

$$\frac{x+2}{3+2x} - 5 = 0$$

$$x+2-15-10x=0$$

$$-9x-13=0$$

$$\underline{x = -\frac{13}{9}}$$



$$\therefore \text{Solution: } \underline{x < -\frac{3}{2} \quad x > -\frac{13}{9}} \quad \left(-\infty, -\frac{3}{2} \right) \cup \left[-\frac{13}{9}, \infty \right)$$

Exercise

Solve the inequality $\frac{x+6}{x-14} \geq 1$

Solution

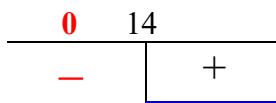
$$\text{Restriction: } x-14 \neq 0 \Rightarrow \boxed{x \neq 14}$$

$$\frac{x+6}{x-14} - 1 = 0$$

$$x+6-x+14=0$$

$$20=0 \quad (\text{Impossible}) \quad \text{No Solution}$$

$$\therefore \text{Solution: } \underline{x > 14} \quad \underline{(14, \infty)}$$



Exercise

Solve: $\frac{x-3}{x+4} \geq \frac{x+2}{x-5}$

Solution

$$\text{Conditions: } x+4 \neq 0 \rightarrow x \neq -4 \quad \text{and} \quad x-5 \neq 0 \rightarrow x \neq 5$$

$$\frac{x-3}{x+4} - \frac{x+2}{x-5} = 0$$

$$\frac{0-3}{0+4} - \frac{0+2}{0-5} = \frac{-3}{4} - \frac{2}{-5} = \frac{-3}{4} + \frac{2}{5} = -$$

$$(x+4)(x-5)\left[\frac{x-3}{x+4}-\frac{x+2}{x-5}\right]=0$$

$$(x-5)(x-3)-(x+4)(x+2)=0$$

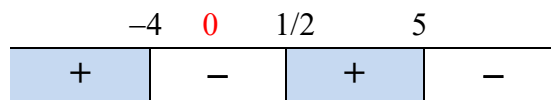
$$x^2-3x-5x+15-(x^2+2x+4x+8)=0$$

$$x^2-3x-5x+15-x^2-2x-4x-8=0$$

$$-14x+7=0$$

$$-14x=-7$$

$$x=\frac{-7}{-14}=\frac{1}{2}$$



$$\therefore \text{Solution: } \underline{x < -4 \quad \frac{1}{2} \leq x < 5} \quad \underline{(-\infty, -4) \cup \left[\frac{1}{2}, 5\right)}$$

Exercise

Solve: $\frac{x-4}{x+3}-\frac{x+2}{x-1} \leq 0$

Solution

Conditions: $x \neq -3$ and $x \neq 1$

$$\frac{x-4}{x+3}-\frac{x+2}{x-1}=0$$

$$\frac{0-4}{0+3}-\frac{0+2}{0-1}=-\frac{4}{3}+2 > 0$$

$$(x+3)(x-1)\left[\frac{x-4}{x+3}-\frac{x+2}{x-1}\right]=0$$

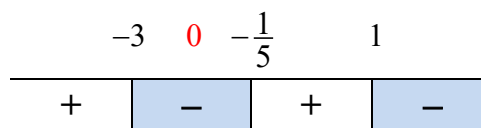
$$(x-1)(x-4)-(x+3)(x+2)=0$$

$$x^2-5x+4-(x^2+5x+6)=0$$

$$x^2-5x+4-x^2-5x-6=0$$

$$-10x-2=0$$

$$\underline{x = -\frac{1}{5}}$$



$$\therefore \text{Solution: } \underline{-3 < x \leq -\frac{1}{5} \quad x > 1} \quad \underline{\left(-3, -\frac{1}{5}\right] \cup (1, \infty)}$$

Exercise

Solve: $\frac{2x-1}{x+3} \geq \frac{x+1}{3x+1}$

Solution

Conditions: $x \neq -3$ and $x \neq -\frac{1}{3}$

$$\frac{2x-1}{x+3} - \frac{x+1}{3x+1} \geq 0$$

$$(x+3)(3x+1) \frac{2x-1}{x+3} - (x+3)(3x+1) \frac{x+1}{3x+1} = 0$$

$$(3x+1)(2x-1) - (x+3)(x+1) = 0$$

$$6x^2 - 3x + 2x - 1 - (x^2 + x + 3x + 3) = 0$$

$$6x^2 - x - 1 - x^2 - 4x - 3 = 0$$

$$5x^2 - 5x - 4 = 0$$

$$x = \frac{5 \pm \sqrt{105}}{10} \rightarrow -0.5 \quad 1.5$$

	-3	$\frac{5-\sqrt{105}}{10}$	$-\frac{1}{3}$	0	$\frac{5+\sqrt{105}}{10}$	
+		-	+	-	+	

$$\therefore \text{Solution: } \underline{x < -3 \quad \frac{5-\sqrt{105}}{10} \leq x < -\frac{1}{3} \quad x \geq \frac{5+\sqrt{105}}{10}}$$

$$(-\infty, -3) \cup \left[\frac{5-\sqrt{105}}{10}, -\frac{1}{3} \right) \cup \left[\frac{5+\sqrt{105}}{10}, \infty \right)$$

Exercise

Solve the inequality equation $\frac{(x+1)(x-4)}{x-2} < 0$

Solution

Restriction: $x \neq 2$

$$\frac{(x+1)(x-4)}{x-2} = 0$$

$$x = -1, 4$$

	-1	0	2	4	
-		+	-	+	

$$\therefore \text{Solution: } \underline{x < -1 \quad 2 < x < 4}$$

Exercise

Solve the inequality equation $\frac{x(x-4)}{x+5} > 0$

Solution

Restriction: $x \neq -5$

$$\frac{x(x-4)}{x+5} = 0$$

$$x = 0, 4$$

	-5	0	1	4	
-		+	-	+	

$$\therefore \text{Solution: } \underline{-5 < x < 0 \quad x > 4}$$

Exercise

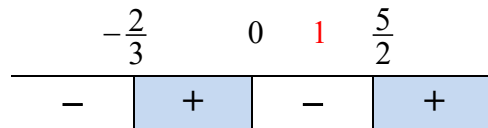
Solve the inequality equation $\frac{6x^2 - 11x - 10}{x} > 0$

Solution

Restriction: $x \neq 0$

$$6x^2 - 11x - 10 = 0$$

$$x = \frac{11 \pm \sqrt{121 + 240}}{12}$$
$$= \left\{ \begin{array}{l} \frac{11-19}{12} = -\frac{2}{3} \\ \frac{11+19}{12} = \frac{5}{2} \end{array} \right|$$



$$\therefore \text{Solution: } \underline{-\frac{2}{3} < x < 0 \quad x > \frac{5}{2}}$$

Exercise

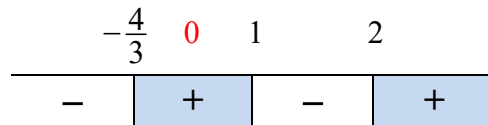
Solve the inequality equation $\frac{3x^2 - 2x - 8}{x-1} \geq 0$

Solution

Restriction: $x \neq 1$

$$3x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 96}}{6}$$
$$= \left\{ \begin{array}{l} \frac{2-10}{6} = -\frac{4}{3} \\ \frac{2+10}{6} = 2 \end{array} \right|$$



$$\therefore \text{Solution: } \underline{-\frac{4}{3} \leq x < 1 \quad x \geq 2}$$

Exercise

Solve the inequality equation $\frac{x^2 - 6x + 9}{x-5} \leq 0$

Solution

Restriction: $x \neq 5$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$\underline{x = 3 \mid}$$

$$\therefore \text{Solution: } \underline{x < 5 \mid}$$

	0	3	5	
	-	-	+	

Exercise

Solve the inequality equation $\frac{x^2 + 10x + 25}{x + 1} \leq 0$

Solution

Restriction: $x \neq -1$

$$x^2 + 10x + 25 = 0$$

$$(x + 5)^2 = 0$$

$$\underline{x = -5 \mid}$$

$$\therefore \text{Solution: } \underline{x < -1 \mid}$$

	-5	-1	0	
	-	-	+	

Solution

Section 1.8 – More Applications and Models

Exercise

Two vertical poles of lengths 4 feet and 10 feet stand 15 feet apart. A cable reaches from the top of one pole to some point on the ground between the poles and then to the top of the other pole. Where should this point be located to use 24 feet of cable?

Solution

$$l_1^2 = x^2 + 4^2 \qquad l_1 = \sqrt{x^2 + 16}$$

$$l_2^2 = (15 - x)^2 + 10^2 \qquad l_2 = \sqrt{(15 - x)^2 + 100}$$

$$l_1 + l_2 = 24$$

$$\sqrt{x^2 + 16} + \sqrt{(15 - x)^2 + 100} = 24$$

$$\sqrt{(15 - x)^2 + 100} = 24 - \sqrt{x^2 + 16}$$

$$\left(\sqrt{(15 - x)^2 + 100} \right)^2 = \left(24 - \sqrt{x^2 + 16} \right)^2$$

$$x^2 - 30x + 225 + 100 = 576 - 48\sqrt{x^2 + 16} + x^2 + 16$$

$$x^2 - 30x + 325 - x^2 - 576 - 16 = -48\sqrt{x^2 + 16}$$

$$-30x - 267 = -48\sqrt{x^2 + 16}$$

$$30x + 267 = 48\sqrt{x^2 + 16}$$

$$(30x + 267)^2 = 48^2(x^2 + 16)$$

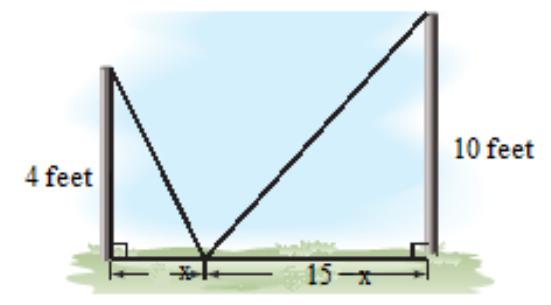
$$900x^2 + 16020x + 71289 = 2304(x^2 + 16)$$

$$900x^2 + 16020x + 71289 = 2304x^2 + 36864$$

$$900x^2 + 16020x + 71289 - 2304x^2 - 36864 = 0$$

$$-1404x^2 + 16020x + 34425 = 0$$

$$x \approx 13.259$$



Exercise

Towns **A** and **B** are located 6 miles and 3 miles, respectively, from a major expressway. The point on the expressway closet to town **A** is 12 miles from the point on the expressway closet to town **B**. Two new roads are to be built from **A** to the expressway and then to **B**.

- Express the combined lengths of the new road in terms of x .
- If the combined lengths of the new roads is 15 miles, what distance does x represent?

Solution



$$a) \quad d_1^2 = x^2 + 6^2 \rightarrow d_1 = \sqrt{x^2 + 36}$$

$$d_2^2 = (12 - x)^2 + 3^2 \rightarrow d_2 = \sqrt{(12 - x)^2 + 9}$$

$$d_1 + d_2 = \sqrt{x^2 + 36} + \sqrt{(12 - x)^2 + 9}$$

$$b) \quad \sqrt{x^2 + 36} + \sqrt{(12 - x)^2 + 9} = 15$$

$$\sqrt{x^2 + 36} = 15 - \sqrt{144 - 24x + x^2 + 9}$$

$$\left(\sqrt{x^2 + 36}\right)^2 = \left(15 - \sqrt{x^2 - 24x + 153}\right)^2$$

$$x^2 + 36 = 225 - 30\sqrt{x^2 - 24x + 153} + x^2 - 24x + 153$$

$$30\sqrt{x^2 - 24x + 153} = -24x + 342$$

$$\left(30\sqrt{x^2 - 24x + 153}\right)^2 = (-24x + 342)^2$$

$$900(x^2 - 24x + 153) = 576x^2 - 16416x + 116964$$

$$900x^2 - 21600x + 137700 = 576x^2 - 16416x + 116964$$

$$324x^2 - 5184x + 20736 = 0 \quad \text{Solve for } x:$$

$$x = 8$$

Exercise

A solid silver sphere has a diameter of 8 *millimeters*, and a second silver has a diameter of 12 *millimeters*. The spheres are melted down and recast to form a single cube. What is the length s of each edge of the cube?

Solution

$$\begin{aligned} V &= \frac{4\pi}{3} \left(\frac{8}{2}\right)^3 + \frac{4\pi}{3} \left(\frac{12}{2}\right)^3 & V &= \frac{4\pi}{3} r^3 \\ &= \frac{4\pi}{3} (64 + 216) \\ &= \frac{4\pi}{3} (280) \\ &= \frac{1,120\pi}{3} \end{aligned}$$

$$V_{\text{cube}} = s^3 = \frac{1,120\pi}{3}$$

$$s = \sqrt[3]{\frac{1,120\pi}{3}} \text{ mm} \approx 10.5 \text{ mm}$$

Exercise

The period T of the pendulum is the time it takes the pendulum to complete one swing from left to right and back. For a pendulum near the surface of Earth

$$T = 2\pi \sqrt{\frac{L}{32}}$$

Where T is measured in *seconds* and L is the length of the pendulum in *feet*. Find the length of a pendulum that has a period of 4 *seconds*.

Solution

$$T = 2\pi \sqrt{\frac{L}{32}} = 4$$

$$\sqrt{\frac{L}{32}} = \frac{2}{\pi}$$

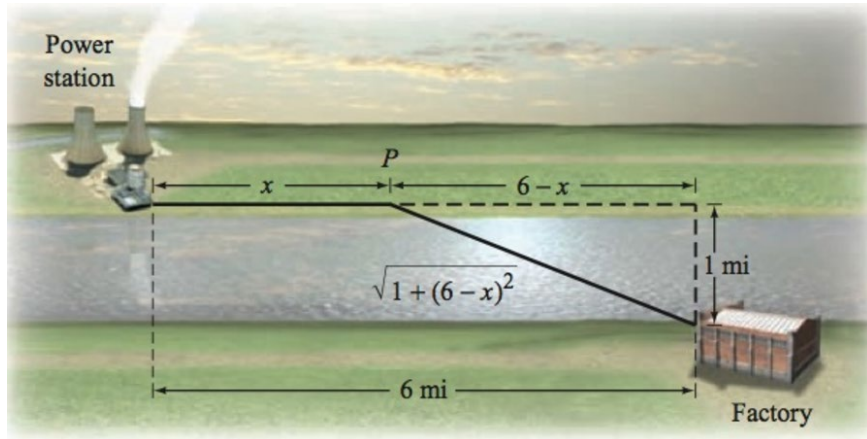
$$\frac{L}{32} = \frac{4}{\pi^2}$$

$$L = \frac{128}{\pi^2} \text{ feet}$$

$$\approx 13 \text{ feet}$$

Exercise

A power station is on one side of a river that is 1 *mile* wide, and a factory is 6 *miles* down-stream on the other side of the river, the cost is \$0.125 *million* per *mile* to run power lines over land and \$0.2 *million* per *mile* to run power lines under water. How far over the land should the power line be run if the total cost of the project is to be \$1 *million*?



Solution

Let x be the distance the power lines overland.

$\sqrt{1 + (6 - x)^2}$ the distance the power lines underwater.

The total cost is given:

$$0.125x + 0.2\sqrt{1 + (6 - x)^2} = 1$$

$$0.2\sqrt{1 + (6 - x)^2} = 1 - 0.125x$$

$$200\sqrt{1 + (6 - x)^2} = 1000 - 125x$$

$$8\sqrt{1 + 36 - 12x + x^2} = 40 - 5x$$

$$\left(8\sqrt{1 + 36 - 12x + x^2}\right)^2 = (40 - 5x)^2$$

$$64(37 - 12x + x^2) = 1600 - 400x + 25x^2$$

$$2,368 - 768x + 64x^2 = 1600 - 400x + 25x^2$$

$$39x^2 - 368x + 768 = 0$$

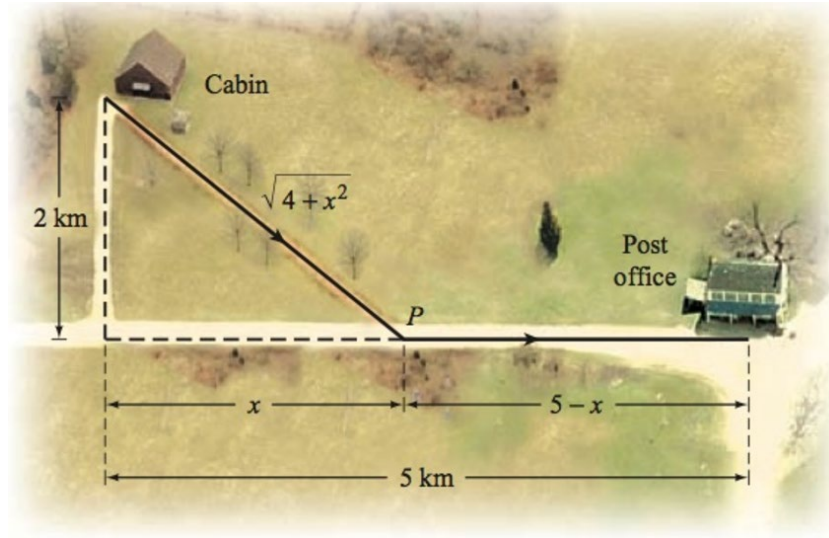
$$x = \frac{368 \pm \sqrt{15,616}}{78}$$

$$\approx \begin{cases} \frac{368 - 125}{78} \approx 3.11 \\ \frac{368 + 125}{78} \approx 6.32 > 6 \end{cases}$$

\therefore Distance of the power lines overland is **3.11** km.

Exercise

A cabin is located in a meadow at the end of a straight driveway 2 km long. A post office is located 5 km from the driveway along a straight road. A woman walks 2 km/hr through the meadow to point P and then 5 km/hr along the road to the post office. If it takes the woman 2.25 hours to reach the post office, what is the distance x of point P from the end of the driveway?



Solution

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

$$\text{Time to walk from cabin to } P = \frac{\sqrt{4+x^2}}{2}$$

$$\text{Time to walk from } P \text{ to Post Office} = \frac{5-x}{5}$$

$$\frac{\sqrt{4+x^2}}{2} + \frac{5-x}{5} = 2.25$$

$$5\sqrt{4+x^2} + 10 - 2x = 22.5$$

$$5\sqrt{4+x^2} = 2x + 12.5$$

$$\left(5\sqrt{4+x^2}\right)^2 = (2x + 12.5)^2$$

$$25(4+x^2) = 4x^2 + 50x + 156.25$$

$$100 + 25x^2 = 4x^2 + 50x + 156.25$$

$$21x^2 - 50x - 56.25 = 0$$

$$x = \frac{50 \pm \sqrt{7,225}}{42}$$

$$= \frac{50 \pm 85}{42}$$

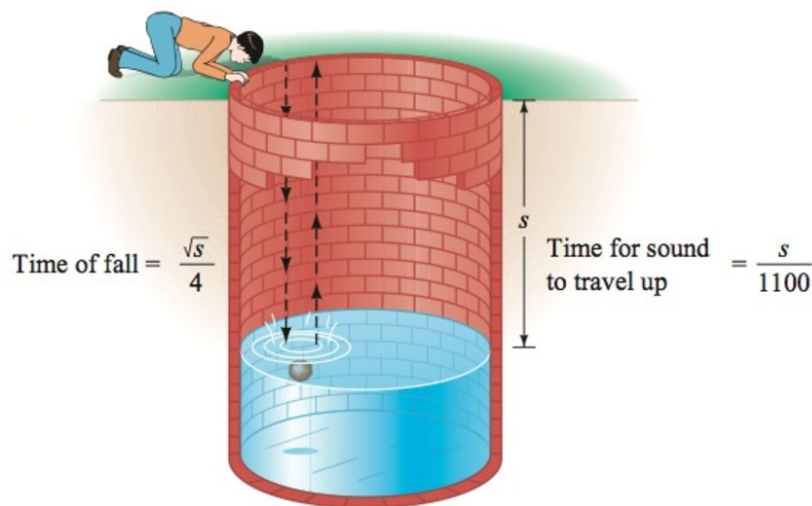
$$= \begin{cases} \frac{50+85}{42} = \frac{45}{14} \\ \frac{50-85}{42} = \frac{\cancel{35}}{\cancel{42}} < 0 \end{cases}$$

\therefore Distance of point **P** from the end of the driveway is $\frac{45}{14} \approx 3.21$ km.

Exercise

The depth s from the opening of a well to the water below can be determined by measuring the total time between the instant you drop a stone and the moment you hear it hit the water. The time, in *seconds*, it takes the stone to hit the water is given by $\frac{\sqrt{s}}{4}$, where s is measured in *feet*. The time, also in seconds, required for the sound of the impact to travel up to your ears is given by $\frac{s}{1,100}$. Thus, the total time T , in *seconds*, between the instant you drop the stone and the moment you hear its impact is

$$T = \frac{\sqrt{s}}{4} + \frac{s}{1,100}$$



- One of the world's deepest water wells is 7,320 *feet* deep. Find the time between the instant you drop a stone and the time you hear it hit the water if the surface of the water is 7,100 *feet* below the opening of the well.
- Find the depth from the opening of a well to the water level if the time between the instant you drop a stone and the moment you hear its impact is 3 *seconds*.

Solution

- Given:** $s = 7,100$

$$T = \frac{\sqrt{7,100}}{4} + \frac{7,100}{1,100}$$

$$= \frac{5\sqrt{71}}{2} + \frac{71}{11} \text{ sec} \Big|$$

$$\approx 27.52 \text{ sec} \Big|$$

b) Given: $T = 3$

$$T = \frac{\sqrt{s}}{4} + \frac{s}{1,100} = 3$$

$$\frac{\sqrt{s}}{4} = 3 - \frac{s}{1,100}$$

$$\frac{\sqrt{s}}{4} = \frac{3,300 - s}{1,100}$$

$$\sqrt{s} = \frac{3,300 - s}{275}$$

$$s = \left(\frac{3,300 - s}{275} \right)^2$$

$$275^2 s = 1,089 \times 10^4 - 6,600s + s^2$$

$$s^2 - 82,225s + 1,089 \times 10^4 = 0$$

$$s = \frac{82,225 \pm 1375\sqrt{3553}}{2}$$

$$= \begin{cases} \frac{82,225 + 1375\sqrt{3553}}{2} \approx 82,092.34 & \text{too large} \\ \frac{82,225 - 1375\sqrt{3553}}{2} \approx 132.66 \end{cases}$$

\therefore The depth from the opening of a well to the water level is about **133 feet**.

Exercise

On a ship, the distance d that you can see to the horizon is given by $d = \sqrt{1.5h}$, where h is the height of your eye measured in *feet* above the sea level and d is measured in *miles*. How high is the eye level of a navigator who can see 14 *miles* to the horizon?

Solution

Given: $d = 14$

$$d = \sqrt{1.5h} = 14$$

$$\frac{3}{2}h = (14)^2$$

$$h = \frac{2(196)}{3}$$

$$= \frac{392}{3} \text{ feet} \Big| \quad \approx 131 \text{ feet} \Big|$$

Exercise

A car can be rented from Basic Rental for \$260 per week with no extra charge for mileage. Continental charges \$80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

Solution

x : number of miles driven

For Continental, cost: $80 + .25x$

Basic Rental a better deal than Continental's

$$260 < 80 + 0.25x$$

$$260 - 80 < 0.25x$$

$$180 < .25x$$

$$720 < x$$

Solution: more than 720 miles per week.

Exercise

If a projectile is launched from ground level with an initial velocity of 96 ft per sec, its height in feet t seconds after launching is s feet, where

$$s = -16t^2 + 96t$$

When will the projectile be greater than 80 ft above the ground?

Solution

Projectile be greater than 80 ft above the ground

$$s > 80$$

$$-16t^2 + 96t > 80$$

$$-16t^2 + 96t - 80 > 0$$

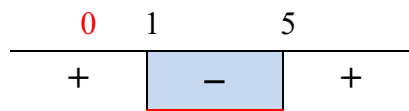
$$\frac{-16}{-16}t^2 + \frac{96}{-16}t - \frac{80}{-16} < 0$$

$$t^2 - 6t + 5 < 0$$

$$t^2 - 6t + 5 = 0$$

$$(t - 1)(t - 5) = 0$$

$$t = 1, 5$$



Solution (1, 5)

Exercise

A projectile is fired straight up from ground level. After t seconds, its height above the ground is s ft, where

$$s = -16t^2 + 220t$$

For what time period is the projectile at least 624 ft above the ground?

Solution

Projectile at least 624 ft.

$$s \geq 624$$

$$-16t^2 + 220t \geq 624$$

$$-16t^2 + 220t - 624 \geq 0 \quad \text{Divide by -4}$$

$$4t^2 - 55t + 156 \leq 0$$

$$t = \frac{-(-55) \pm \sqrt{(-55)^2 - 4(4)(156)}}{2(4)} = \frac{55 \pm 23}{16}$$

$$\begin{aligned} t &= \frac{55+23}{16} & t &= \frac{55-23}{16} \\ &= \frac{78}{16} & &= \frac{32}{16} \\ &= \frac{39}{8} & &= 2 \end{aligned}$$

$$\text{Solution: } \left[2, \frac{39}{8} \right]$$

Exercise

Your test scores of 70 and 81 in your math class. To receive a C grade, you must obtain an average greater than or equal to 72 but less than 82. What range of test scores on the one remaining test will enable you to get a C for the course.

Solution

$$72 \leq \frac{70+81+x}{3} < 82$$

$$216 \leq 151 + x < 246$$

$$65 \leq x < 95$$

\therefore The range of test scores on the one remaining test will enable you to get a C for the course is

$$65 \leq x < 95$$

Exercise

A truck can be rented from Basic Rental for \$50 a day plus \$0.20 per *mile*. Continental charges \$20 per day plus \$0.50 per *mile* to rent the same truck. How many miles must be driven in a day to make the rental cost for Basic Rental a better deal than Constiental's?

Solution

$$\text{Basic Rental: } BR = 50 + 0.2x$$

$$\text{Continental: } C = 20 + 0.5x$$

$$BR > C$$

$$50 + 0.2x > 20 + 0.5x$$

$$30 > 0.3x$$

$$x < 100 \mid$$

\therefore **100 miles** must be driven in a day to make the rental cost for Basic Rental a better deal than Constiental's.

Exercise

You are choosing between two telephone plans. Plan **A** has a monthly fee of \$15 with a charge of \$0.08 per *minute* for all calls. Plan **B** has a monthly fee of \$3 with a charge of \$0.12 per *minute* for all calls. How many calling minutes in a month make plan **A** the better deal?

Solution

$$\text{Plan A: } 15 + 0.08x$$

$$\text{Plan B: } 3 + .12x$$

$$A < B$$

$$15 + 0.08x < 3 + 0.12x$$

$$12 < 0.04x$$

$$x > 300 \mid$$

\therefore Plan **A** is a better deal when more than 300 minutes.

Exercise

A City commission has proposed two tax bills. The first bill requires that a homeowner pay \$1,800 plus 3% of the assesses home value in taxes. The second bill requires taxes of \$200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal for the homeowner?

Solution

$$\text{First bill: } B_1 = 1,800 + 0.03x$$

$$\text{Second bill: } B_2 = 200 + 0.08x$$

$$B_1 < B_2$$

$$1,800 + 0.03x < 200 + 0.08x$$

$$1,600 < 0.05x$$

$$0.05x > 1,600$$

$$x > \frac{1,600}{0.05}$$

$$= 32,000$$

∴ The first bill is a better deal for the homeowner when greater than **\$32,000**

Exercise

A local bank charges \$8 per month plus \$0.05 per check. The credit union charges \$2 per month \$0.08 per check. How many checks should be written each month to make the credit union a better deal?

Solution

Local bank: $C_1 = 8 + .05x$

Credit union: $C_2 = 2 + .08x$

$$C_1 > C_2$$

$$8 + .05x > 2 + .08x$$

$$6 > .03x$$

$$.03x < 6$$

$$x < \frac{6}{0.03}$$

$$= 200$$

∴ The credit union make less than **200** checks for a better deal.

Exercise

A company manufactures and sells blank audiocassette tapes. The weekly fixed cost is \$10,000 and it costs \$0.40 to produce each tape. The selling price is \$2.00 per tape. How many tapes must be produced and sold each week for the company to have a profit?

Solution

Cost: $C = 10,000 + .4x$

Revenue: $R = 2x$

$$C < R$$

$$10,000 + .4x < 2x$$

$$10,000 < 1.6x$$

$$1.6x > 10,000$$

$$x > \frac{10,000}{1.6}$$

$$= 6,250 \mid$$

∴ For the company to have a profit, they must sell more than **6,250** tapes.

Exercise

A company manufactures and sells stationery. The weekly fixed cost is \$3,000 and it costs \$3.00 to produce each package of stationery. The selling price is \$5.50 per package. How many packages of stationery must be produced and sold each week for the company to have a profit?

Solution

Cost: $C = 3,000 + 3x$

Revenue: $R = 5.5x$

$$C < R$$

$$3,000 + 3x < 5.5x$$

$$3,000 < 2.5x$$

$$2.5x > 3,000$$

$$x > \frac{3,000}{2.5}$$

$$= 1,200 \mid$$

∴ For the company to have a profit when it produces more than **1,200** packages each week.

Exercise

An elevator at a construction site has a maximum capacity of 3,000 *pounds*. If the elevator operator weighs 200 *pounds* and each cement bag weighs 70 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

Solution

The weight inside the elevator: $200 + 70x$

$$200 + 70x \leq 3,000$$

$$70x \leq 2,800$$

$$x \leq \frac{2,800}{70}$$

$$= 40 \mid$$

∴ **50** bags of cement or less.

Exercise

An elevator at a construction site has a maximum capacity of 2,500 *pounds*. If the elevator operator weighs 160 *pounds* and each cement bag weighs 60 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

Solution

The weight inside the elevator: $160 + 60x$

$$160 + 60x \leq 2,500$$

$$60x \leq 2,340$$

$$x \leq \frac{2,340}{60}$$

$$= 39 \mid$$

\therefore 39 bags of cement or less.

Exercise

You can rent a car for the day from Company *A* for \$29.00 plus \$0.12 a *mile*. Company *B* charges \$22.00 plus \$0.21 a *mile*. Find the number of miles *m* per day for which it is cheaper to rent from Company *A*.

Solution

Plan *A*: $29 + 0.12x$

Plan *B*: $22 + 0.21x$

$$A < B$$

$$29 + 0.12x < 22 + 0.21x$$

$$7 < 0.09x$$

$$0.09x > 7$$

$$x > \frac{7}{.09}$$

$$= \frac{700}{9} \mid \approx 78$$

\therefore Plan *A* is a better deal when more than 78 days.

Exercise

UPS will only ship packages for which the length is less than or equal to 108 *inches* and the length plus the girth is less than or equal to 130 *inches*. The length of a package is defined as the length of the longest side. The girth is defined as twice the width plus twice the height of the package. If a box has a length of 34 *inches* and a width of 22 *inches*, determine the possible range of heights *h* for this package if you wish to ship it by UPS.

Solution

Given: $\ell \leq 108$

$$\ell + 2w + 2h \leq 130$$

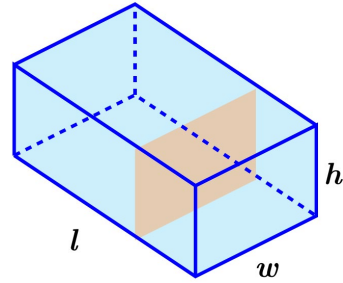
$$34 + 2(22) + 2h \leq 130$$

$$2h \leq 130 - 34 - 44$$

$$2h \leq 52$$

$$h \leq 26$$

∴ The possible range of heights h for this package $0 < h \leq 26$



Exercise

The sum of three consecutive odd integers is between 63 and 81. Find all possible sets of integers that satisfy these conditions.

Solution

Let the first odd number is given by: $2n + 1$

$$63 < (2n + 1) + (2n + 3) + (2n + 5) < 81$$

$$63 < 6n + 9 < 81$$

$$54 < 6n < 72$$

$$9 < n < 12$$

For $n = 10 \rightarrow 21, 23, 25$

For $n = 11 \rightarrow 23, 25, 27$

Exercise

Forensic specialists can estimate the height of a deceased person from the lengths of the person's bones. For instance, an inequality that relates the height h , in cm , of an adult female and the length f , in cm , of her femur is $|h - (2.47f + 54.10)| \leq 3.72$. Use the inequalities to estimate the possible range of heights for an adult female whose measures 32.24 cm .

Solution

Given: $f = 32.24$

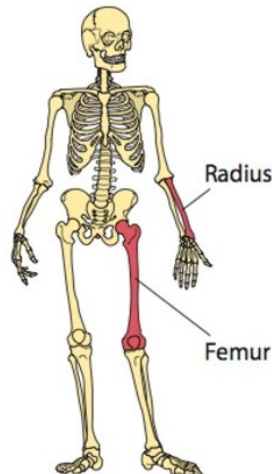
$$|h - 2.47(32.24) + 54.10| \leq 3.72$$

$$-3.72 \leq h - 79.6328 - 54.10 \leq 3.72$$

$$-3.72 \leq h - 133.7328 \leq 3.72$$

$$133.7328 - 3.72 \leq h \leq 3.72 + 133.7328$$

$$130.01 \leq h \leq 137.45 \text{ cm}$$



Exercise

An inequality that is used to calculate the height h of an adult male from the length r of his radius is

$$|h - (3.32r + 85.43)| \leq 4.57$$

Where h and r are both in cm . Use this inequality to estimate the possible range of heights for an adult male whose radius measures 26.36 cm .

Solution

Given: $r = 26.36$

$$|h - (3.32(26.36) + 85.43)| \leq 4.57$$

$$-4.57 \leq h - 87.5152 - 85.43 \leq 4.57$$

$$-4.57 \leq h - 172.9452 \leq 4.57$$

$$172.9452 - 4.57 \leq h \leq 172.9452 + 4.57$$

$$\underline{168.4 \leq h \leq 177.5 \text{ cm}}$$