$f(x) = \dfrac{(x+1)^2}{1+x^2}$ $\qquad \mathbb{R}$

$$\begin{matrix} 1 & 2 & 1 \\ 1 & 0 & 1 \end{matrix}$$

CN

$$f'(x) = \frac{-2x^2+2}{(1+x^2)^2} = 0 \qquad -2(x^2-1)(1+x^2)^{-2}$$

$u' \quad v''$

$x^2 = 1 \Rightarrow$ $\boxed{\text{CN}: x = \pm 1}$

$$f''(x) = -2\,\frac{2x(1+x^2) - 4x(x^2-1)}{(1+x^2)^3}$$

$$\begin{array}{c|c|c|c} & -1 & 0 & 1 \\ \hline - & & + & - \end{array}$$

$$= -2\,\frac{+6x - 2x^3}{(1+x^2)^3}$$

$$= 4\,\frac{x(x^2-3)}{(1+x^2)^3} = 0$$

Pt of Inf: $\boxed{x = 0,\ x = \pm\sqrt{3}}$

$$\begin{array}{c|c|c|c} & -\sqrt{3} & 0 & \sqrt{3} \\ \hline - & + & - & + \end{array}$$

	$f(x)$
-1	0
1	2

Incr $(-1, 1)$

Dec. $(-\infty, -1)\ (1, \infty)$

LMIN $(-1, 0)$

LMAX $(1, 2)$

Concave up $(-\sqrt{3}, 0)\ (\sqrt{3}, \infty)$

Concave down $(-\infty, -\sqrt{3})\ (0, \sqrt{3})$

Sec 3.3 Optimization

Minimize
Maximize $\rightarrow$ ☺ $f'(x)$ C.P.

Solve

1- Read.

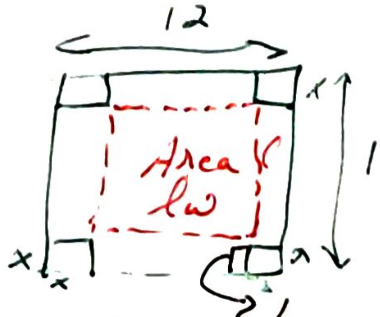
2- Draw (if)

3- read it, Given applied variables (introduction)

4- Write an eqn. (s)

5- Test C.P.

Ex



x? C.N.

Hold: Vol Max

$h = x \qquad l = w = 12 - 2x$

$$V = lwh$$
$$= (12-2x)^2 x \qquad (4x^2 - 48x + 144)x$$
$$= 4x^3 - 48x^2 + 144x$$

$$V' = 12x^2 - 96x + 144 = 0$$
$$\Rightarrow x^2 - 8x + 12 = 0$$
$$\underline{x = 2, 6} \Big\} \text{ C.N.}$$

$$x = 2 \Rightarrow V = (12-4)^2(2) = 128 \text{ in}^3$$

The Max. vol. is 128 in^3 w/ cutout square is equal to 2 in.

Ex $V = 1\,l. = 10^3\,cm^3$

dim: r, h

Least Material. Surf $= S_{min}$

soln $V = \pi r^2 h = 10^3$ (1)

$$S = \underbrace{2\pi r^2}_{\text{circle Top + Bott.}} + 2\pi r h \quad (2)$$

(1) $\rightarrow$ $h = \frac{10^3}{\pi}\frac{1}{r^2}$ (3)

$$S = 2\pi r^2 + 2\pi r \frac{10^3}{\pi r^2}$$

$$S(r) = 2\pi r^2 + \frac{2\cdot 10^3}{r}$$

$$S' = 4\pi r - \frac{\cancel{2}\times 10^3}{r^2} = 0$$

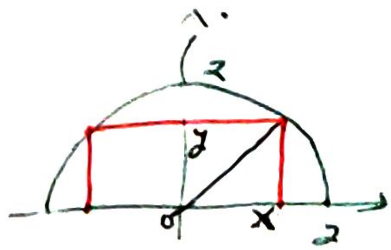
$$2\pi r = \frac{1000}{r^2}$$

$$r^3 = \frac{500}{\pi} \Rightarrow \boxed{r = \sqrt[3]{\frac{500}{\pi}}} \quad \frac{10}{\sqrt[3]{2\pi}}$$

(3) $\rightarrow$ $h = \frac{10^3}{\pi}\frac{1}{\left(\frac{500}{\pi}\right)^{2/3}}$

$$= \boxed{\frac{10^3}{(500)^{2/3}}\frac{1}{\sqrt[3]{\pi}}}$$

Ex $\quad A \; ? \; Max$



$$x^2 + y^2 = 4$$

$$y = \sqrt{4 - x^2}$$

$$A = 2xy.$$

$$A(x) = 2x\sqrt{4 - x^2}$$

$$= 2\underbrace{x}_{u}\underbrace{(4 - x^2)}_{v}^{1/2}$$

$$A'(x) = 2\,\frac{4 - x^2 - x^2}{\sqrt{4 - x^2}}$$

$$= 4\,\frac{2 - x^2}{\sqrt{4 - x^2}} = 0 \Rightarrow x^2 = 2$$

C.N: $x = \pm\sqrt{2}$

$$\underline{x = \sqrt{2} \Rightarrow y = \sqrt{2}}$$

$$A(\sqrt{2}) = 2\sqrt{2}\,\sqrt{2}$$

$$\underline{= 4 \text{ unit}^2}$$

Economics.

P = Profit
R = Revenue
C = Cost.

$\left.\begin{matrix} P = \text{Profit} \\ R = \text{Revenue} \\ C = \text{Cost.} \end{matrix}\right\} P = R - C$

$\frac{dP}{dx}$ = Marginal Profit

$\frac{dR}{dx}$ = " Revenue

$\frac{dC}{dx}$ = " Cost.

Point of inflection / Break-even point.
diminishing point

Ex $\quad r = 9x \qquad C(x) = x^3 - 6x^2 + 15x$

$x_{\text{mil.}}$

P_{Max} ?

$P(x) = R(x) - C(x)$

$= 9x - x^3 + 6x^2 - 15x$

$= -x^3 + 6x^2 - 6x$

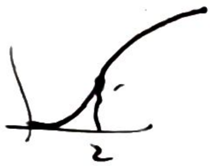
Break-even: $R'(x) = C'(x)$

$P'(x) = -3x^2 + 12x - 6$

$P''(x) = -6x + 12 = 0 \Rightarrow \underline{x = 2}$

0	1	2	
	+		−

concave up $(0, 2)$ concave down $(2, \infty)$



$R'(x) = C'(x)$

$9 = 3x^2 - 12x + 15$

$3x^2 - 12x + 6 = 0$

$x^2 - 4x + 2 = 0 \quad \Big| \; x = 2 \pm \sqrt{2}$

$x = 2 - \sqrt{2} \rightarrow$ Max lost

$x = 2 + \sqrt{2} \rightarrow$ Max Profit

3.3

#1

$x, y \geq 0$ $\quad \underbrace{2x + y = 30}_{①} \ni \underbrace{xy^2}_{②}$ Max?

① $\Rightarrow y = 30 - 2x$ ③

② $\Rightarrow \quad M = x\,(30 - 2x)^2$

$= 4x^3 - 120x^2 + 900x$

$M' = 12x^2 - 240x + 900 = 0$

$x^2 - 20x + 75 = 0$

$$\begin{cases} x = 5 \xrightarrow{③} y = 20 \\ x = 15 \xrightarrow{3} y = 0 \; \# \end{cases}$$

$\boxed{x = 5, \; y = 20}$

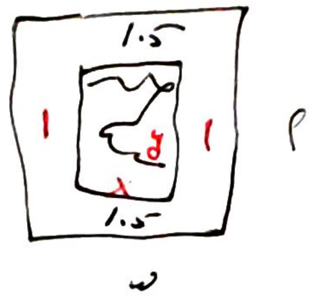
$$\left. \begin{aligned} M &= 5(20)^2 \\ &= 2{,}000 \end{aligned} \right\}$$

#2

$A_i = 54 \text{ in}^2$

dim?

$A_i = xy = 54$ (1) $\rightarrow y = \frac{54}{x}$



$A = (x+2)(y+3)$

$= xy + 3x + 2y + 6$

$= 54 + 3x + \frac{108}{x} + 6$

$A(x) = 3x + \frac{108}{x} + 60$

$A' = 3 - \frac{108}{x^2} = 0 \Rightarrow 3 = \frac{108}{x^2}$

$x^2 = 36$

$x = 6 \Rightarrow y = \frac{54}{6} = 9$

$\begin{cases} x+2 = 6+2 = 8 \\ y+3 = 12 \end{cases}$

dimension of paper: 8×12

#12 $S(x) = -x^3 + 6x^2 + 288x + 4{,}000$

$4 \le x \le 20$

$S' = -3x^2 + 12x + 288 = 0$

$-x^2 + 6x + 96 = 0$

$x = -8$ #

$\boxed{x = 12^\circ C}$

#24 girth (dist. around) = $P = 2(x+x)$

$l + G = 108$

$L + 4x = 108 \Rightarrow \boxed{L = 108 - 4x}$

a) $V = x^2 L$

$= x^2(108 - 4x)$

$= 108x^2 - 4x^3$

$V' = 216x - 12x^2 = 0$ $\quad x = 0$ #

$x = \frac{216}{12} = \boxed{18}$

$L = 108 - \underset{72}{4(18)} = \boxed{36 \text{ in}}$

$V = \boxed{18^2(36) \text{ in}^3}$

b) $L + \overset{\text{area}}{2\pi r} = 108 \rightarrow L = 108 - 2\pi r$

$V = \pi r^2 (108 - 2\pi r)$

$= 108\pi r^2 - 2\pi^2 r^3$

$V' = 216\pi r - 6\pi^2 r^2$

$= 6\pi r(36 - \pi r) = 0$ $\quad r = 0$ #, $\quad r = \frac{36}{\pi}$

$L = 108 - 72 = 36$