$\longrightarrow$ approach implies

$=$ equal

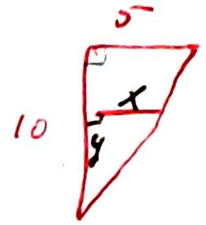
$\approx$

## 2.10 Related Rates

$$V = \frac{4\pi}{3} r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Ex. $\frac{dV}{dt} = 9 \frac{ft^3}{min}$ $\quad \frac{dy}{dt}$ ? $\;|_{y=6}$




$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi x^2 y$$

$$= \frac{\pi}{3}\frac{1}{4} y^3$$

$$\frac{y}{10} = \frac{x}{5}$$

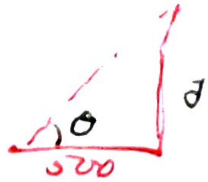
$$x = \frac{1}{2} y$$

$$\frac{dV}{dt} = \frac{\pi}{4} y^2 \frac{dy}{dt}$$

$$\frac{9 \times 4}{\pi (36)} = \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{1}{\pi} \; ft/min$$

$\theta = \frac{\pi}{4}$ $\quad \frac{d\theta}{dt} = 0.14 = \frac{14}{100} = \frac{7}{50}$




$\frac{dy}{dt}$ ? | $y$

$\tan\theta = \frac{y}{500}$

$y = 500 \tan\theta$

$\frac{dy}{dt} = 500 \sec^2\theta \frac{d\theta}{dt}$

$= 500\left(\sec^2 \frac{\pi}{4}\right) \frac{7}{50}$

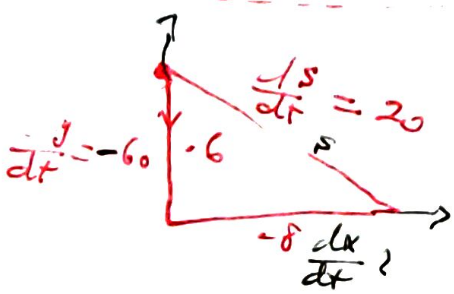
$= 70 (\sqrt{2})^2$

$= 140$ ft/min

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Ex Given: $y = \frac{6}{10} = \frac{3}{5} = .6$

$x = \frac{8}{10} = \frac{4}{5} = .8$




$\frac{dx}{dt}$ ?

$s^2 = x^2 + y^2$

$s = \sqrt{(.6)^2 + (.8)^2} = 1$

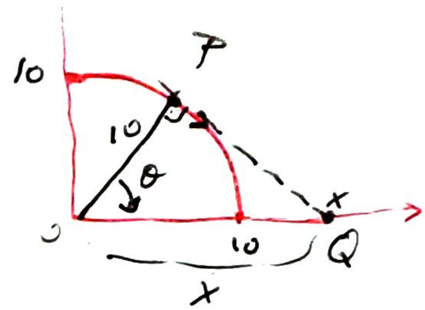
$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{4}{5}\frac{dx}{dt} = 20 - \frac{3}{5}(-60)$

$\frac{dx}{dt} = 56\left(\frac{5}{4}\right)$

$= 70$ mph

Ex $\quad t = 0 \to 30 \Rightarrow \frac{\pi}{2}$



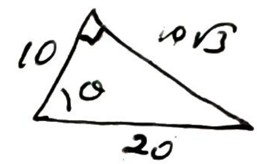

Find $\left.\frac{dx}{dt}\right|_{x=20}$ ?

$$\frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2}{1/2} = -\pi \text{ rad/min}$$

$\Delta OPQ$ : $\cos\theta = \frac{10}{x}$

$$x = \frac{10}{\cos\theta}$$

$$x = 10\sec\theta$$




$$\frac{dx}{dt} = 10\sec\theta\tan\theta\,\frac{d\theta}{dt}$$

$\sec\theta = 2$ $\qquad \tan\theta = \sqrt{\sec^2\theta - 1}$
$\qquad\qquad\qquad\qquad = \sqrt{3}$

$$\frac{dx}{dt} = 10\,(2)(\sqrt{3})(-\pi)$$

$$= -20\pi\sqrt{3} \text{ ft/min}$$

[illegible]

$\frac{dx}{dt}$ ?  $\frac{d\theta}{dt} = \frac{2}{3}$ deg/sec


$\tan\theta = \frac{12{,}000}{x}$

$x = \frac{12{,}000}{5280}\cot\theta$

$\frac{dx}{dt} = -\frac{1200}{528}\csc^2\theta\,\frac{d\theta}{dt}$

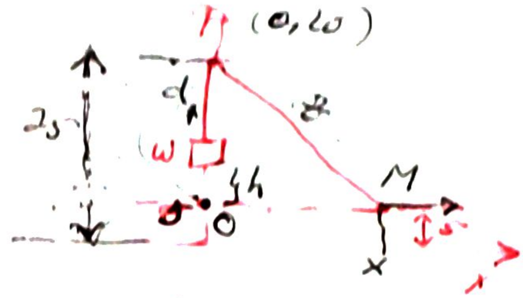
$= -\frac{300}{134}\cdot\frac{1}{\sin^2 10}\cdot\frac{2}{3}\frac{\text{deg}}{\text{sec}}\cdot\frac{3600\text{ sec}}{1\text{ hr}}$ ($\frac{100}{67}$, 4)

$= \frac{-14.1\times 10^4}{67}$ mph.

Ex. rope: $45 = d_1 + z$ (1

$\frac{dx}{dt} = 6$

$\frac{dh}{dt}\, ? \Big|_{x=21}$




$z^2 = 20^2 + x^2$  (1  $45 = 20 - h + z$, $z = 25 + h$)

$(25+h)^2 - 400 = x^2$

$2(25+h)\frac{dh}{dt} = 2x\frac{dx}{dt}$

$\frac{dh}{dt} = \frac{x}{25+h}\frac{dx}{dt}$

$(25+h)^2 = 21^2 + 400 = 21^2 + 20^2$

$25 + h = 29$

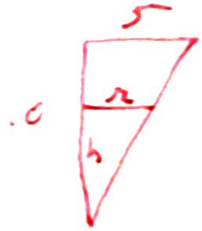
$= \frac{21}{29}(6)$

$= \frac{126}{29}$ ft/sec.

# 17

$$\frac{dV}{dt} = 2 \ \frac{ft^3}{min}$$

$$\frac{dh}{dt} - ? \Big|_{h=4}$$




$$\frac{r}{5} = \frac{h}{10}$$

$$r = \frac{1}{2} h$$

$$V = \frac{\pi}{3} r^2 h$$

$$= \frac{\pi}{3} \left(\frac{h}{2}\right)^2 h$$

$$= \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$2 = \frac{\pi}{4} 16 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2}{4\pi} = \frac{1}{2\pi} \ ft/min$$

HWK 2.6

$$\left(\frac{u}{v}\right)' \quad u'v - \quad (\sec u)' = u' \sec u \tan u$$

$$(u^n)' = n u^{n-1} u'$$

20 $$y = \sec\left(\frac{x^2+1}{x^4+2}\right)^3 \quad u$$

$$y' = 3\left(\frac{x^2+1}{x^4+2}\right)^2 \frac{2x(x^4+2) - 4x^3(x^2+1)}{(x^4+2)^2} \sec\left(\frac{x^2+1}{x^4+2}\right)^3 \tan\left(\frac{x^2+1}{x^4+2}\right)^3$$

$$= 3 \frac{(x^2+1)^2(-2x^5+4x-4x^3)}{(x^4+2)^4} \sec(\ )^3 \tan(\ )^3$$

#1 $f(x) = (x^2-2)^2$ $(u^n)' = u'(n)$

$f'(x) = 4x(x^2-2)$

#2 $f(x) = (2\sqrt{x}-1)(4x+1)^{-1}$ $(u^n v^m)' = u^{n-1}v^{m-1}(nu'v + muv')$

$$f'(x) = \frac{1}{(4x+1)^2}\left(\frac{1}{\sqrt{x}}(4x+1) - 4(2\sqrt{x}-1)\right)$$

$$= \frac{4x+1-8x+4\sqrt{x}}{\sqrt{x}(4x+1)^2}$$

$$= \frac{1-4x+4\sqrt{x}}{\sqrt{x}(4x+1)^2}$$

$(\sin x)' = \cos x$ $(\cos u)' = -u'\sin u$

$(\sin u)' = u'\cos u$

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#3 $f(x) = x^2\sqrt{1-x^2}$

$$f'(x) = \frac{x}{\sqrt{1-x^2}}\left(2-2x^2 + \frac{1}{2}(-2x)x^2\right)$$

$$= \frac{x(2-2x^2-x^3)}{\sqrt{1-x^2}}$$

#4 $f(t) = \sin^3 4t$ $(\sin u)^n = n(\sin u)'(\sin u)^{n-1}$

$f'(t) = 12\sin^2 4t\cos 4t$ $= n(u'\cos u)(\sin u)^{n-1}$

#5 $y = 3(4-9x)^4$

$y' = -108(4-9x)^3$

27/4

#6 $y = \sqrt[3]{6x^2+1} = (6x^2+1)^{1/3}$

$y' = \frac{1}{3}(12x)(6x^2+1)^{-2/3}$

$= \dfrac{4x}{\sqrt[3]{(6x^2+1)^2}}$

#7 $y = \left(\dfrac{1}{x-3}\right)^2$

$\dfrac{1}{(x-3)^2}$ $\left(\dfrac{1}{u^n}\right)' = -\dfrac{nu'}{u^{n+1}}$

$y' = -\dfrac{2}{(x-3)^3}$

#8 $y = \dfrac{1}{\sqrt{x+2}} = \dfrac{1}{(x+2)^{1/2}}$

$y' = -\frac{1}{2}\dfrac{1}{(x+2)^{3/2}}$ $= \cancel{\dfrac{-\frac{1}{2}}{\quad}}$

$(u^n v^m)'$

#9 $f(x) = x^3(x-4)^5$

$f'(x) = x^2(x-4)^4(3x-12+5x)$

$= x^2(x-4)^4(8x-12)$