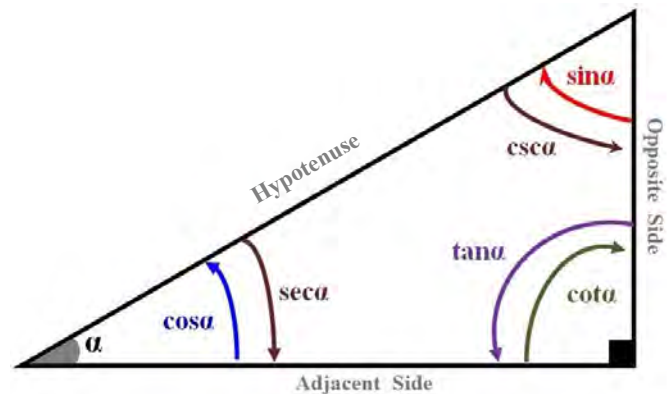
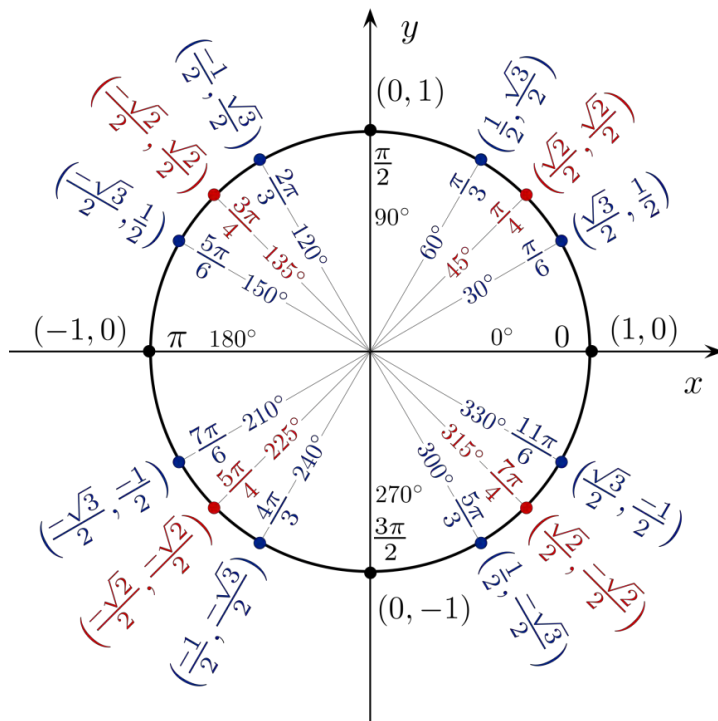


$$2\pi \text{ (radians)} \equiv 360^\circ \equiv 1 \text{ revolution} \quad \theta = \frac{s}{r} \text{ (radians)} \quad v = \frac{s}{t} = r\omega = r\frac{\theta}{t} \quad \omega = \frac{\theta}{t} = \frac{v}{r} = \frac{s}{rt} = \frac{v\theta}{s}$$

$$3600 \text{ rev / minute} = \frac{3600 \text{ rev}}{1 \text{ min}} \frac{2\pi \text{ (radians)}}{1 \text{ rev}} \frac{1 \text{ min}}{60 \text{ sec}} = \frac{120\pi \text{ (radians)}}{1 \text{ sec}}$$

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (x-h)^2 + (y-k)^2 = r^2$$

$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$	$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$	$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{opp}}{\text{adj}} = \frac{\sin \theta}{\cos \theta}$
$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin \theta}$	$\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos \theta}$	$\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$



$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$1 + \tan^2 \alpha = \sec^2 \alpha$$

$$1 + \cot^2 \alpha = \csc^2 \alpha$$

$$\cos(-\alpha) = \cos \alpha$$

$$\sin(-\alpha) = -\sin \alpha$$

$$\tan(-\alpha) = -\tan \alpha$$

$$\cos(90^\circ - \alpha) = \sin \alpha$$

$$\sin(90^\circ - \alpha) = \cos \alpha$$

$$\tan(90^\circ - \alpha) = \cot \alpha$$

$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$	$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$	$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$
$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$	$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$	$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

**Half-Angle:**  $\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$   $\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$   $\tan\left(\frac{\alpha}{2}\right) = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$

$a \sin x + b \cos x = k \sin(x + \alpha)$  where  $k = \sqrt{a^2 + b^2}$ ,  $\sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$ , and  $\cos \alpha = \frac{a}{\sqrt{a^2 + b^2}}$

## Double-Angle

$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ $= 1 - 2\sin^2 \alpha$ $= 2\cos^2 \alpha - 1$	$\sin 2\alpha = 2\sin \alpha \cos \alpha$	$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$
$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$	$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$	$\tan^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$

## Product-to-Sum:

$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$	$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$
$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$	$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

## Sum-to-Product:

$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$	$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$	$\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$

**Law of Sines:**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$        $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

## Law of Cosines:

$a^2 = b^2 + c^2 - 2bc \cos A$	$A = \cos^{-1} \left( \frac{b^2 + c^2 - a^2}{2bc} \right)$
$b^2 = a^2 + c^2 - 2ac \cos B$	$B = \cos^{-1} \left( \frac{a^2 + c^2 - b^2}{2ac} \right)$
$c^2 = a^2 + b^2 - 2ab \cos C$	$C = \cos^{-1} \left( \frac{a^2 + b^2 - c^2}{2ab} \right)$

## Vectors:

<b>Magnitude:</b> $ V  = \sqrt{a^2 + b^2}$	<b>Angle:</b> $\cos \theta = \frac{U \bullet V}{ U  V }$
<b>Dot Product:</b> $U \bullet V = (ai + bj) \bullet (ci + dj) = ac + bd$	

$$z = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta \quad r = \sqrt{x^2 + y^2} \quad \cos \theta = \frac{x}{r}, \quad \sin \theta = \frac{y}{r}, \quad \text{and} \quad \tan \theta = \frac{y}{x}$$

$$(r_1 \operatorname{cis} \theta_1)(r_2 \operatorname{cis} \theta_2) = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) \quad \frac{r_1 \operatorname{cis} \theta_1}{r_2 \operatorname{cis} \theta_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

**De Moivre's Theorem:**  $[r \operatorname{cis} \theta]^n = r^n (\operatorname{cis} n\theta)$        $[r \operatorname{cis} \theta]^{1/n} = \sqrt[n]{r} \operatorname{cis} \alpha$        $\alpha = \frac{\theta}{n} + \frac{360^\circ k}{n}$

The graphs of  $y = A\sin(Bx + C) + D$  and  $y = A\cos(Bx + C) + D$ , where  $B > 0$ , will have the following characteristics:

$$\text{Amplitude} = |A| \quad \text{Period} = \frac{2\pi}{|B|} \quad \text{Phase Shift} = \varphi = -\frac{C}{B} \quad \text{One cycle: } 0 \leq \text{argument} \leq 2\pi$$

$$\text{Vertical Shift: } y = D$$

### ***To graph “Sine or Cosine”***

- 1- Find the Amplitude
- 2- Find the Period
- 3- Construct a table

$x$	$y = A\cos(Bx + C) + D$	$y = A\sin(Bx + C) + D$
$\varphi$	$D + A$	$D$
$\varphi + \frac{P}{4}$	$D$	$D + A$
$\varphi + \frac{P}{2}$	$D - A$	$D$
$\varphi + \frac{3P}{4}$	$D$	$D - A$
$\varphi + P$	$D + A$	$D$

- 4- Graph *One Cycle*
- 5- Extend the graph, if necessary

The graphs of  $y = A\tan(Bx + C) + D$  and  $y = A\cot(Bx + C) + D$ , where  $B > 0$ , will have the following characteristics:

$$\text{No Amplitude} \quad \text{Period} = \frac{\pi}{|B|} \quad \text{Phase Shift} = -\frac{C}{B} \quad \text{One cycle: } 0 \leq \text{argument} \leq \pi$$

$$\text{Vertical Shift: } y = D$$

$x$	$y = A\tan(Bx + C) + D$	$y = A\cot(Bx + C) + D$
$\varphi$	$D$	$\infty$
$\varphi + \frac{P}{4}$	$D + A$	$D + A$
$\varphi + \frac{P}{2}$	$\infty$	$D$
$\varphi + \frac{3P}{4}$	$D - A$	$D - A$
$\varphi + P$	$D$	$\infty$