Differential Equations

		Solution
Bernoulli’s Equation	$y' + P(x)y = Q(x)y^n$	$y^{(1-n)}e^{(1-n)\int Pdx} = (1-n)\int Qe^{(1-n)\int Pdx}dx + C$ *If* $n=1 \Rightarrow \ln y = \int (Q-P)dx + c$
Bessel's Equation	$x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + \left(\lambda^2x^2 - n^2\right)y = 0$	$y = c_1J_n(\lambda x) + c_2Y_n(x)$
Euler - Cauchy Equation	$ax^2\ddot{y} + bx\dot{y} + cy = S(x)$ $a\lambda^2 + (b-a)\lambda + c = 0$	1. $\left(\lambda_1 \neq \lambda_2\right) \in \nabla \Rightarrow y = c_1x^{\lambda_1} + c_2x^{\lambda_2}$ 2. $\left(\lambda_1 = \lambda_2\right) \in \nabla \Rightarrow y = c_1x^{\lambda_1} + c_2x^{\lambda_2}\ln x$ 3. $\lambda_{1,2} = \alpha \pm i\beta \in \mathbb{R}$ $\Rightarrow y = x^{\alpha}\left(c_1\cos(\alpha\ln x) + c_2\sin(\beta\ln x)\right)$
Exact Equation	$M(x,y)dx + N(x,y)dy = 0$ $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$	$\int M\partial x + \int\left(N - \dfrac{\partial}{\partial y}\int Mdx\right)dy = c$
Homogeneous	$\dfrac{dy}{dx} = F\left(\dfrac{y}{x}\right)$	$\ln x = \int \dfrac{dv}{F(v)-v} + c$
	$y.F(xy)dx + x.G(xy)dy = 0$	$\ln x = \int \dfrac{G(v)dv}{v[G(v)-F(v)]} + c$ (where $v = xy$)
Legendre’s Eq.	$\left(1-x^2\right)y'' - 2xy' + n(n+1)y = 0$	$y = c_1P_n(x) + c_2Q_n(x)$
Linear First Order Equation	$\dfrac{dy}{dx} + P(x)y = Q(x)$	$ye^{\int Pdx} = \int Qe^{\int Pdx} + c$
Linear , Homogeneous Second Order Equation	$\dfrac{d^2y}{dx^2} + a\dfrac{dy}{dx} + by = 0$	1. $\lambda_{1,2} \in \nabla \Rightarrow y = c_1e^{\lambda_1x} + c_2e^{\lambda_2x}$ 2. $\lambda_{1,2} = \alpha \pm i\beta \in \mathbb{R}$ $\Rightarrow y = e^{Px}\left(c_1\cos\alpha x + c_2\sin\beta x\right)$

Linear , nonhomogeneous Second Order Equation	$\frac{d^2y}{dx^2}+a\frac{dy}{dx}+by=R(x)$	$y=c_1e^{m_1x}+c_2e^{m_2x}$ $+\frac{e^{m_1x}}{m_1-m_2}\int e^{-m_1x}R(x)dx$ $+\frac{e^{m_2x}}{m_2-m_1}\int e^{-m_2x}R(x)dx$
Separation of Variables	$f_1(x)g_1(y)dx+f_2(x)g_2(y)dy=0$	$\int\frac{f_1(x)}{f_2(x)}dx+\int\frac{g_1(y)}{g_2(y)}dy=c$