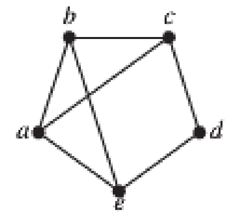
SOLUTION Section 4.9 – Euler and Hamilton Paths

Exercise

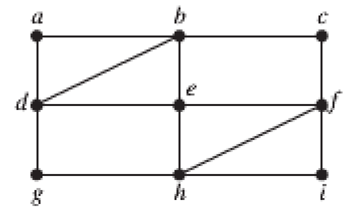
Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



Solution

The vertices *a, b, c, e* have degree 3, therefore the graph has no Euler circuit.
It is not Euler path since there is more than 2 vertices with an odd degree.

Exercise



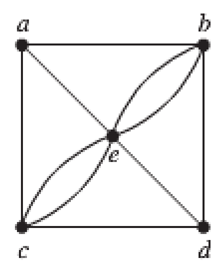
Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

Solution

All the vertex degrees are even, so there is an Euler circuit.
Circuit form: *a, b, c, f, i, h, g, d, e, h, f, e, b, d, a*

Exercise

Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

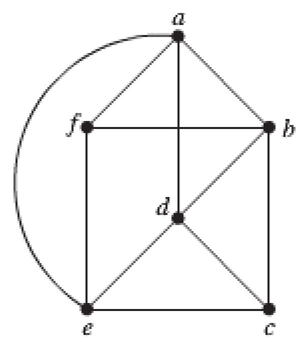


Solution

The vertices *a, d* have degree 3, therefore the graph has no Euler circuit.
It has an Euler path *a, e, c, e, b, e, d, b, a, c, d*. (it has exactly 2 vertices of odd degree)

Exercise

Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

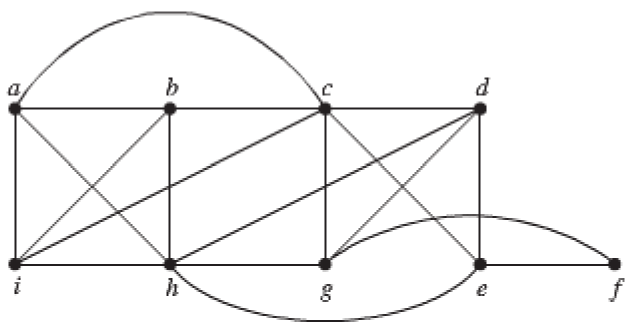


Solution

The vertices *c, f* have degrees 3, therefore the graph has no Euler circuit.
There is an Euler path between the two vertices of odd degree.
One such path is: *f, a, b, c, d, e, f, b, d, a, e, c*.

Exercise

Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

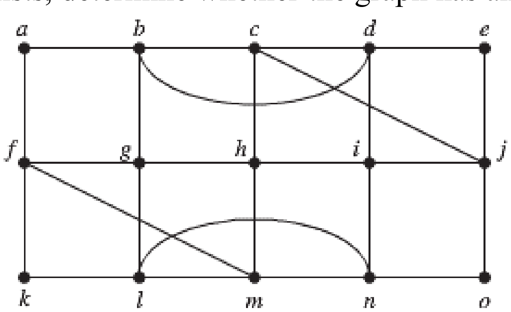


Solution

All the vertex degree are even, so there is an Euler circuit.
Form: *a, i, h, g, d, e, f, g, c, e, h, d, c, a, b, I, c, b, h, a*

Exercise

Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

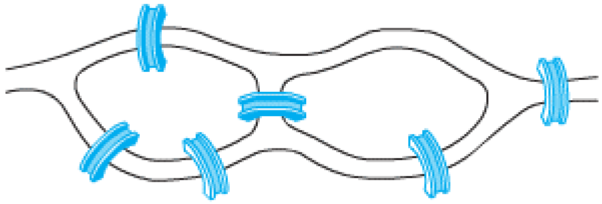


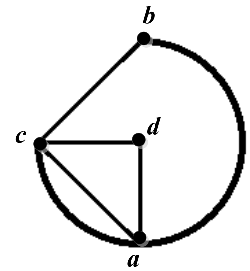
Solution

All the vertex degree are even, so there is an Euler circuit.
Circuit: *a, b, c, d, e, j, c, h, i, d, b, g, h, m, n, o, j, i, n, l, m, f, g, l, k, f, a*

Exercise

Can someone cross all the bridges shown in this map exactly once and return to the starting point?



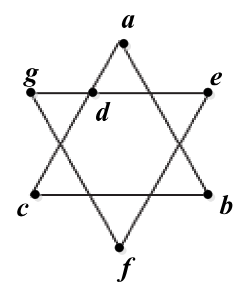


Solution

Vertices *a* and *b* are the banks of the river, and vertices *c* and *d* are the islands.
Each vertex has even degree, so the graph has an Euler circuit, such as: *a, c, b, a, d, c, a*.
Therefore, a walk of the type described is possible.

Exercise

Determine whether the picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture

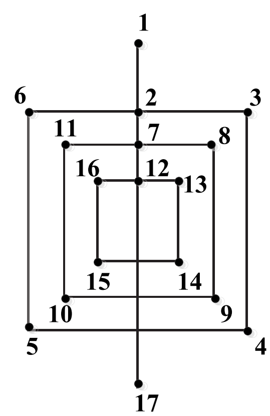


Solution

Yes, the path: *a, b, c, d, e, f, g, d, a*.

Exercise

Determine whether the picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture

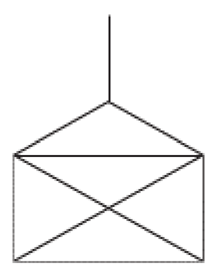


Solution

1, 2, 3, 4, 5, 6, 2, 7, 8, 9, 10, 11, 7, 12, 13, 14, 15, 16, 12, 17

Exercise

Determine whether the picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture

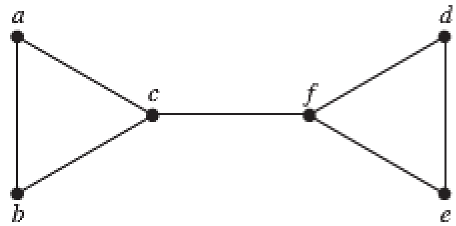


Solution

No

Exercise

Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.



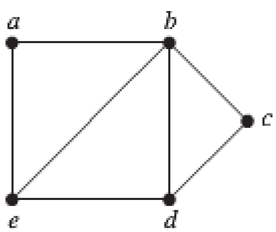
Solution

The graph is not a Hamilton circuit because of the cut edge $\{c, f\}$.

Every simple circuit must be confined to one of the 2 components obtained by deleting this edge.

Exercise

Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

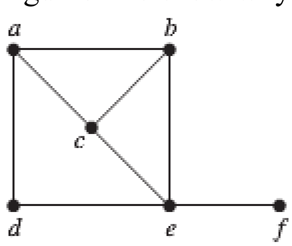


Solution

Hamilton circuit: a, b, c, d, e, a.

Exercise

Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

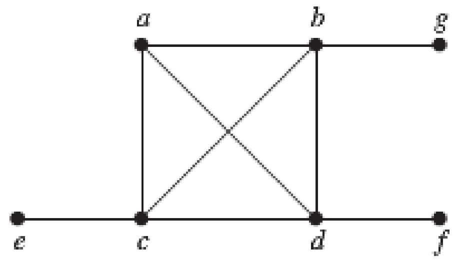


Solution

The graph is not a Hamilton circuit because of the cut edge $\{e, f\}$.

Exercise

Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

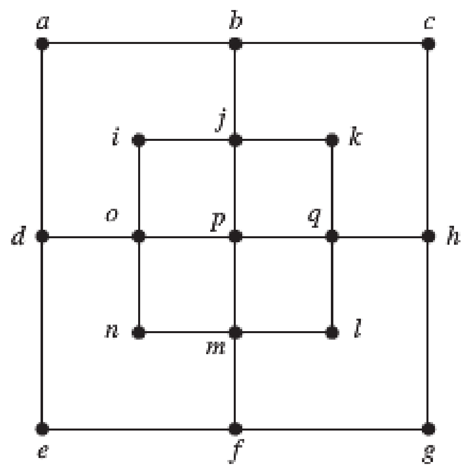


Solution

No Hamilton circuit exists, because once a purported circuit has reached *e* it would be nowhere to go.

Exercise

Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.



Solution

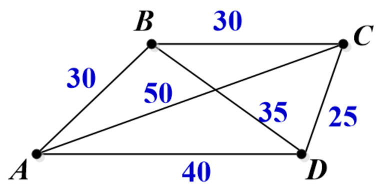
This graph has no Hamilton circuit.

If it did, then certainly the circuit would have to contain edges $\{d, a\}$ and $\{a, b\}$, since these are the only edges incident to vertex *a*. By the same reasoning, the circuit would have to contain the other six edges around the outside of the figure. These 8 edges already complete a circuit, and this circuit omits the 9 vertices on the inside.

Therefore, there is no Hamilton circuit.

Exercise

Imagine that the drawing below is a map showing 4 cities and the distances in kilometers between them. Suppose that a salesman must travel to each city exactly once, starting and ending in city *A*. Which route from city to city will minimize the total distance that must be traveled?



Solution

Route	*Total Distance (Km)*
ABCDA	30 + 30 + 25 + 40 = 125
ABDCA	30 + 35 + 25 + 50 = 140
ACBDA	50 + 30 + 35 + 40 = 155
ACDBA	50 + 25 + 35 + 30 = 140
ADBCA	40 + 35 + 30 + 50 = 155
ADCBA	40 + 25 + 30 + 30 = 125

Thus either route *ABCDA* or *ADCBA* fives the minimum total distance of 125 km.