Solution   Section 4.5 – Comparing Three or More Means

Exercise

Fill in the ANOVA table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-Test Statistic
Treatment	565	5		
Error	3560	32		
Total				

Solution

$$MST = \frac{SST}{k-1} = \frac{565}{5} = 113$$

$$MSE = \frac{SSE}{n-k} = \frac{3560}{32} = 111.25$$

$$F_0 = \frac{MST}{MSE} = \frac{113}{111.25} = 1.016$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-Test Statistic
Treatment	565	5	**113**	**1.016**
Error	3560	32	**111.25**	
Total	**4125**	**37**		

Exercise

Fill in the ANOVA table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-Test Statistic
Treatment	490	4		
Error	7267	21		
Total				

Solution

$$MST = \frac{SST}{k-1} = \frac{490}{4} = 122.5$$

$$MSE = \frac{SSE}{n-k} = \frac{7267}{21} = 346.048$$

$$F_0 = \frac{MST}{MSE} = \frac{122.5}{346.048} = 0.354$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-Test Statistic
Treatment	490	4	**122.5**	**0.354**
Error	7267	21	**346.048**	
Total	**7757**	**25**		

Exercise

Determine the ***F***-test statistic based on the given summary statistics

Population	***Sample Size***	***Sample Mean***	***Sample Variance***
1	10	42	35
2	10	41	40
3	10	22	23

Compute $\overline{x}$, the sample mean of the combined data set, by adding up all the observations and dividing by the number pf the observations.

Solution

$$\overline{x} = \frac{\sum n_i x_i}{\sum n_i} = \frac{10(42)+10(41)+10(22)}{10+10+10} = 35$$

$$MST = \frac{\sum n_i \left(x_i - \overline{x}\right)^2}{k-1} = \frac{10(42-35)^2+10(41-35)^2+10(22-35)^2}{3-1} = 1270$$

$$MSE = \frac{\sum \left(n_i - 1\right)s_i^2}{n-k} = \frac{(10-1)(35)+(10-1)(40)+(10-1)(23)}{30-3} = 32.667$$

$$F = \frac{MST}{MSE} = \frac{1270}{32.667} = 38.88$$

Therefore, the value of the ***F***-test statistic is 38.8

Exercise

An engineer wants to know if the mean strengths of three concrete mix designs differ significantly. He randomly selects 9 cylinders that measure 6 inches in diameter and 12 inches in heights in which mixture *A* is poured, 9 cylinders of mixture *B*, and 9 cylinders of mixture *C*. After 28 days, he measures the strength (in pounds per square inch) of the cylinders. The results are presented in the table below.

Mixture A	***Mixture B***	***Mixture C***
3,980	4,070	4,130
4,040	4,340	3,820
3,760	4,620	4,020
3,870	3,730	4,150
3,990	4,870	4,190
4,090	4,120	3,840
3,820	4,640	3,750
3,940	4,180	3,990
4,080	3,850	4,320

a) Write the null and alternative hypotheses

b) Explain why the one-way ANOVA cannot be used to test these hypotheses

Solution

a) $H_0: \mu_A = \mu_B = \mu_C$

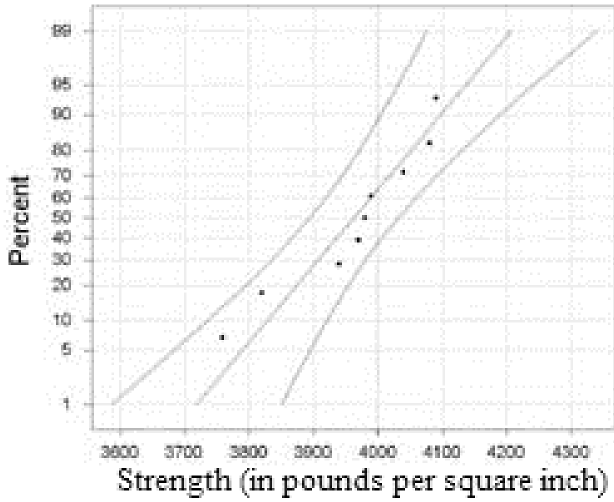
H_1 : at least one of the means is different

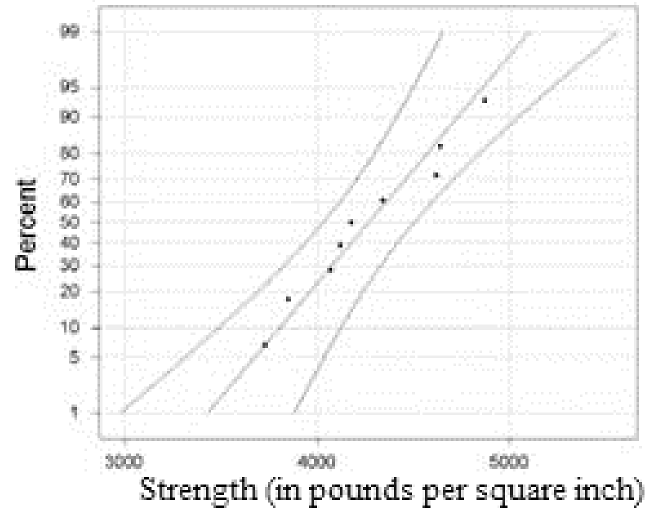
b) The standard deviation for mixture *A* is 115.41

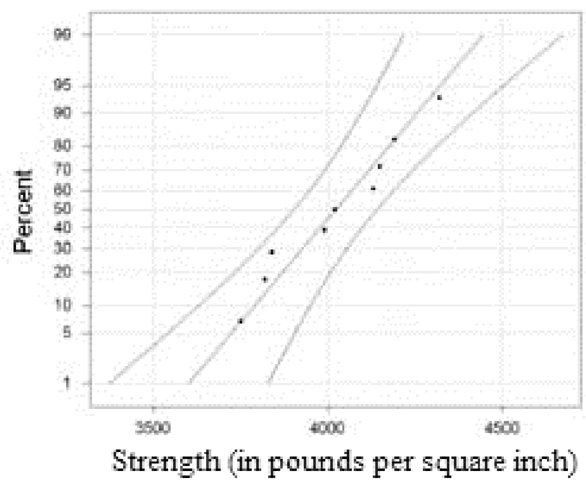
The standard deviation for mixture *B* is 381.26

The standard deviation for mixture *C* is 191.71

The standard deviation for mixture *B* is more than 2 times larger than the standard deviation for mixture *A*.







Exercise

At a community college, the mathematics department has been experimenting with four different delivery mechanisms for content in their Elementary Statistics courses. One method is the traditional lecture (method I), the second is a hybrid format in which half the class time is online and the other half is face-to-face (method II), the third is online (method III), and the fourth is an emporium model from which students obtain their lectures and do their work in a lab with an instructor available for assistance (method IV). To assess the effectiveness of the four methods, students in each approach are given a final exam with the results shown in the accompanying table. Do the data suggest that any method has a different mean score from the others?

Method I	76	81	85	68	88	73	80	65	60	92	83	51	71	63	71	65
Method II	88	52	77	73	64	38	57	63	83	65	78	64	87	92		
Method III	78	60	73	70	62	82	74	80	53	46	84	80	78			
Method IV	89	90	79	62	83	75	54	70	80	94	76	78	81			

a) Write the null and alternative hypotheses

b) State the requirements that must be satisfied to use one-way ANOVA procedure

c) Assuming the requirements stated in part (b) are satisfied, use the following one-way ANOVA table to test the hypothesis of equal means at the $\alpha = 0.05$ level of significance.

d) Interpret the *P*-value.

e) Verify that the residuals are normally distributed

Solution

a) $H_0 : \mu_I = \mu_{II} = \mu_{III} = \mu_{IV}$

H_1 : at least one of the means is different

b) There must be *k* simple samples, one from each of *k* populations.

The populations must be normally distributed.

The populations must have the same variance

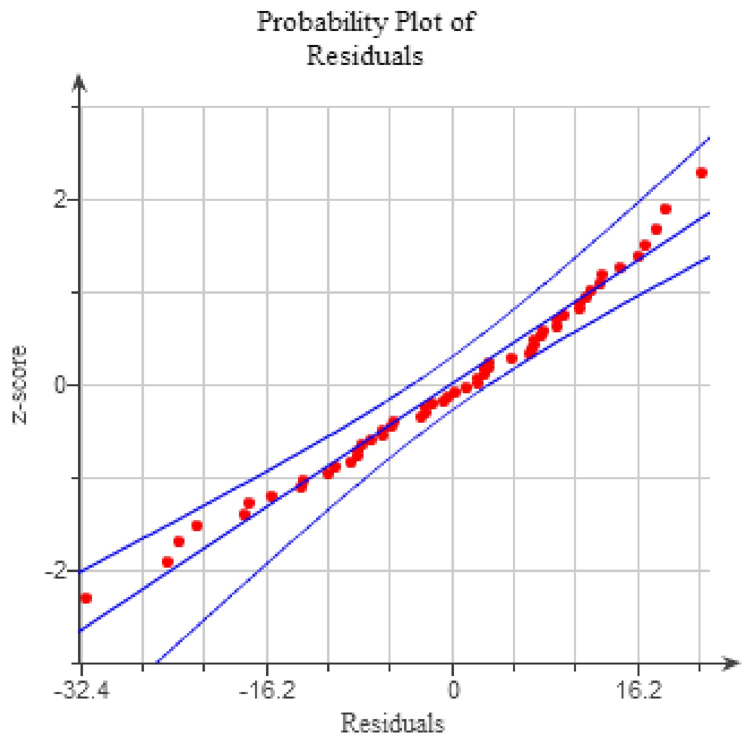
c)

Source of Variation	***Degrees of Freedom***	***Sum of Squares***	***Mean Squares***	***F-Test Statistic***	***P- value***
Treatment	3	481.24	**160.41**	**1.03**	**0.387**
Error	52	8089.54	**155.74**		
Total	**55**	**8579.78**			

Since the *P*-value is greater than α, do not reject H_0, there is insufficient evidence to support H_1

d) Since the *P*-value is greater than or equal to α, conclude that there is insufficient evidence that any one method is more or less effective than the others.

e) $\mu_I = 73.250$; $\mu_{II} = 70.071$; $\mu_{III} = 70.769$; $\mu_{IV} = 77.769$


Probability Plot of
Residuals
z-score
2
0
-2
-32.4
-16.2
0
16.2
Residuals