# **Solution** Section 1.2 – Quadratic Fundtions

# Exercise

Solve: 
$$x^2 = -25$$

# **Solution**

$$x = \pm \sqrt{-25}$$

$$=\pm 5i$$

#### Exercise

Solve:

$$x^2 = 49$$

# **Solution**

$$x = \pm 7$$

# Exercise

Solve: 
$$9x^2 = 100$$

#### **Solution**

$$x^2 = \frac{100}{9}$$

$$x = \pm \sqrt{\frac{100}{9}}$$

$$=\pm\frac{10}{3}$$

# Exercise

Solve: 
$$4x^2 + 25 = 0$$

$$4x^2 = -25$$

$$x^2 = -\frac{25}{4}$$

$$x = \pm \sqrt{-\frac{25}{4}}$$

$$=\pm\frac{5}{2}i$$

Solve:

$$5x^2 + 35 = 0$$

# **Solution**

$$5x^2 = -35$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

## Exercise

Solve: 
$$5x^2 - 45 = 0$$

# **Solution**

$$5x^2 = 45$$

$$x = \frac{45}{5}$$

$$x^2 = 9$$

$$x = \pm 3$$

# Exercise

Solve: 
$$(x-4)^2 = 12$$

# **Solution**

$$x - 4 = \pm \sqrt{12}$$

$$x = 4 \pm \sqrt{12}$$

$$x = 4 \pm \sqrt{12} \qquad \qquad \sqrt{12} = \sqrt{4(3)} = 2\sqrt{3}$$

$$x = 4 \pm 2\sqrt{3}$$

# Exercise

Solve:

$$\left(x+3\right)^2 = -16$$

$$x + 3 = \pm \sqrt{-16}$$

$$x = -3 \pm 4i$$

Solve:

$$\left(x-2\right)^2 = -20$$

#### **Solution**

 $x - 2 = \pm \sqrt{-20}$ 

 $x = 2 \pm 4i\sqrt{5}$ 

#### Exercise

Solve: 
$$(4x+1)^2 = 20$$

## **Solution**

 $4x + 1 = \pm \sqrt{20}$ 

 $4x = -1 \pm 2\sqrt{5}$ 

$$x = \frac{-1 \pm 2\sqrt{5}}{4}$$

# Exercise

Solve 
$$x^2 - 6x = -7$$

# **Solution**

 $x = \frac{-(-6)\pm\sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$ 

$$=\frac{6\pm\sqrt{8}}{2}$$

 $=\frac{6\pm 2\sqrt{2}}{2}$ 

$$=\frac{2(3\pm\sqrt{2})}{2}$$

 $=3\pm\sqrt{2}$ 

# Exercise

Solve 
$$-6x^2 = 3x + 2$$

# **Solution**

$$6x^2 + 3x + 2 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(6)(2)}}{2(6)}$$

$$= \frac{-3 \pm \sqrt{-39}}{12}$$

$$= \frac{-3}{12} \pm i \frac{\sqrt{39}}{12}$$

$$= -\frac{1}{4} \pm i \frac{\sqrt{39}}{12}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve:  $3x^2 + 2x = 7$ 

#### **Solution**

$$3x^{2} + 2x - 7 = 0 \Rightarrow a = 3, b = 2, c = -7$$

$$x = \frac{-2 \pm \sqrt{4 - 4(3)(-7)}}{2(3)} \qquad x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{88}}{6}$$

$$= \frac{-2 \pm \sqrt{4(22)}}{6}$$

$$= \frac{-2 \pm 2\sqrt{22}}{6}$$

$$= \frac{2(-1 \pm \sqrt{22})}{6}$$

$$= \frac{-1 \pm \sqrt{22}}{3}$$

$$x = -\frac{1}{3} \pm \frac{\sqrt{22}}{3}$$

#### Exercise

$$3x^2 + 6 = 10x$$

$$3x^{2} - 10x + 6 = 0$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^{2} - 4(3)(6)}}{2(3)}$$

$$x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$= \frac{10 \pm \sqrt{100 - 72}}{6}$$

$$= \frac{10}{6} \pm \frac{\sqrt{28}}{6}$$

$$= \frac{5}{3} \pm \frac{2\sqrt{7}}{6}$$

$$= \frac{5}{3} \pm \frac{\sqrt{7}}{3}$$

Solve:  $5x^2 + 2 = x$ 

#### **Solution**

$$5x^{2} - x + 2 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 40}}{10}$$

$$= \frac{1 \pm \sqrt{-39}}{10}$$

$$= \frac{1 \pm i\sqrt{39}}{10}$$

$$= \frac{1}{10} \pm i\frac{\sqrt{39}}{10}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Exercise

Solve:  $5x^2 = 2x - 3$ 

$$5x^{2} - 2x + 3 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^{2} - 4(5)(3)}}{2(5)}$$

$$= \frac{2 \pm \sqrt{4 - 60}}{10}$$

$$= \frac{2 \pm \sqrt{-56}}{10}$$

$$= \frac{2 \pm i\sqrt{4(14)}}{10}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm i2\sqrt{14}}{10}$$
$$= \frac{2}{10} \pm i\frac{2\sqrt{14}}{10}$$
$$= \frac{1}{5} \pm i\frac{\sqrt{14}}{5}$$

Solve:  $x^2 + 8x + 15 = 0$ 

# **Solution**

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{64 - 60}}{2}$$

$$= \frac{-8 \pm \sqrt{4}}{2}$$

$$= \frac{-8 \pm 2}{2}$$

$$= \begin{cases} \frac{-8 + 2}{2} = \frac{-6}{2} = -3 \\ \frac{-8 - 2}{2} = \frac{-10}{2} = -5 \end{cases}$$

# Exercise

Solve:  $x^2 + 5x + 2 = 0$ 

# **Solution**

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{-5 \pm \sqrt{25 - 8}}{2}$$
$$= \frac{-5 \pm \sqrt{17}}{2}$$
$$= \frac{-5}{2} \pm \frac{\sqrt{17}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve:

$$x^2 + x - 12 = 0$$

#### **Solution**

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2}$$

$$-1 + 7$$

$$= \frac{-1 \pm 7}{2}$$

$$= \begin{cases} \frac{-1 - 7}{2} = -4 \\ \frac{-1 + 7}{2} = 3 \end{cases}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Exercise

Solve: 
$$x^2 - 2x - 15 = 0$$

#### **Solution**

$$x = \frac{2 \pm \sqrt{4 + 60}}{2}$$

$$=\frac{2\pm8}{2}$$

$$= \begin{cases} \frac{2+8}{2} = 5 \\ \frac{2-8}{2} = -3 \end{cases}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# **Exercise**

Solve: 
$$x^2 - 4x - 45 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 180}}{2}$$

$$=\frac{4\pm\sqrt{196}}{2}$$

$$=\frac{4\pm14}{2}$$

$$= \begin{cases} \frac{4+14}{2} = 9 \\ \frac{4-14}{2} = -5 \end{cases}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve: 
$$x^2 - 6x - 10 = 0$$

#### **Solution**

$$x^2 - 6x = 10$$

$$x^{2} - 6x + \left(\frac{-6}{2}\right)^{2} = 10 + \left(\frac{-6}{2}\right)^{2}$$

$$x^2 - 2(3)x + (3)^2 = 10 + 9$$

$$(x-3)^2 = 19$$

$$x - 3 = \pm \sqrt{19}$$

$$x = 3 \pm \sqrt{19}$$

# Exercise

Solve: 
$$2x^2 + 3x - 4 = 0$$

#### Solution

$$x^2 + \frac{3}{2}x = 2$$

$$x^{2} + \frac{3}{2}x + \left(\frac{1}{2}\frac{3}{2}\right)^{2} = 2 + \left(\frac{1}{2}\frac{3}{2}\right)^{2}$$

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = 2 + \frac{9}{16}$$

$$\left(x+\frac{3}{4}\right)^2 = \frac{41}{16}$$

$$x + \frac{3}{4} = \pm \sqrt{\frac{41}{16}}$$

$$x = -\frac{3}{4} \pm \frac{\sqrt{41}}{4}$$

# Exercise

Solve 
$$x^2 - x + 8 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 32}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{-31}}{2}$$
$$= \frac{1 \pm i\sqrt{31}}{2}$$

Solve  $2x^2 - 13x = 1$ 

# **Solution**

$$2x^2 - 13x - 1 = 0$$

$$x = \frac{13 \pm \sqrt{169 + 8}}{4}$$
$$= \frac{13 \pm \sqrt{169 + 8}}{4}$$
$$= \frac{13 \pm \sqrt{177}}{4}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Exercise

Solve  $r^2 + 3r - 3 = 0$ 

# **Solution**

$$r = \frac{-3 \pm \sqrt{3^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{9 + 12}}{2}$$
$$= \frac{-3 \pm \sqrt{21}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Exercise

Solve:  $x^3 + 8 = 0$ 

#### **Solution**

$$(x+2)(x^2-2x+4) = 0$$

$$x+2=0 x^2-2x+4=0$$

 $a^{3} + b^{3} = (a+b)(a^{2} - ab + b^{2})$ 

$$x = -2$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm 2i\sqrt{3}}{2}$$

$$= \frac{2\left(1 \pm i\sqrt{3}\right)}{2}$$

$$= 1 \pm i\sqrt{3}$$

The solution set is  $\{-2, 1 \pm i\sqrt{3}\}$ 

# Exercise

Solve: 
$$4x^2 - 12x + 9 = 0$$

#### **Solution**

$$x = \frac{12 \pm \sqrt{144 - 144}}{8}$$

$$= \frac{12}{8}$$

$$= \frac{3}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Exercise

Solve: 
$$9x^2 - 30x + 25 = 0$$

# Solution

$$x = \frac{30 \pm \sqrt{900 - 900}}{18}$$

$$= \frac{30}{18}$$

$$= \frac{5}{3}$$

#### Exercise

Solve: 
$$x^2 - 14x + 49 = 0$$

$$x = \frac{14 \pm \sqrt{196 - 196}}{2}$$
$$= \frac{14}{2}$$
$$= 7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve: 
$$x^2 - 8x + 16 = 0$$

#### **Solution**

$$x = \frac{8 \pm \sqrt{64 - 64}}{2}$$
$$= \frac{8}{2}$$
$$= 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# **Exercise**

Solve: 
$$x^2 + 6x + 13 = 0$$

#### **Solution**

$$x = \frac{-6 \pm \sqrt{36 - 52}}{2}$$
$$= \frac{-6 \pm \sqrt{-16}}{2}$$
$$= \frac{-6 \pm 4i}{2}$$
$$= -3 \pm 2i$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Exercise

Solve: 
$$2x^2 - 2x + 13 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 104}}{4}$$
$$= \frac{2 \pm \sqrt{-100}}{4}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm 10i}{4}$$
$$= \frac{1}{2} \pm \frac{5}{2}i$$

Solve:  $x^2 + 2x + 29 = 0$ 

# **Solution**

$$x = \frac{-2 \pm \sqrt{4 - 116}}{2}$$
$$= \frac{-2 \pm \sqrt{-112}}{2}$$
$$= \frac{-2 \pm 4i\sqrt{7}}{2}$$
$$= -1 \pm 2i\sqrt{7}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### **Exercise**

Solve:  $4x^2 + 4x + 13 = 0$ 

#### **Solution**

$$x = \frac{-4 \pm \sqrt{16 - 16(13)}}{8}$$

$$= \frac{-4 \pm 4\sqrt{-12}}{8}$$

$$= \frac{-4 \pm 8i\sqrt{3}}{8}$$

$$= -\frac{1}{2} \pm i\sqrt{3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Exercise

Solve:  $x^2 - 2x + 26 = 0$ 

$$x = \frac{2 \pm \sqrt{4 - 4(26)}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$=\frac{2\pm2\sqrt{-25}}{2}$$
$$=1\pm5i$$

Solve:

$$9x^2 - 4x + 20 = 0$$

#### **Solution**

$$x = \frac{4 \pm \sqrt{16 - 16(45)}}{18}$$

$$= \frac{4 \pm 4\sqrt{-44}}{18}$$

$$= \frac{-4 \pm 8i\sqrt{11}}{8}$$

$$= -\frac{1}{2} \pm i\sqrt{11}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Exercise

Solve:

$$x^2 + 6x + 21 = 0$$

#### **Solution**

$$x = \frac{-6 \pm \sqrt{36 - 84}}{2}$$
$$= \frac{-6 \pm \sqrt{-48}}{2}$$
$$= \frac{-6 \pm 4i\sqrt{3}}{2}$$
$$= -3 \pm 2i\sqrt{3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Exercise

Solve:

$$9x^2 - 12x - 49 = 0$$

$$x = \frac{12 \pm \sqrt{2^4 3^2 - 2^2 3^2 7^2}}{18}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{12 \pm 6\sqrt{4 - 49}}{18}$$

$$= \frac{12 \pm 6\sqrt{-45}}{18}$$

$$= \frac{12 \pm 18i\sqrt{5}}{18}$$

$$= \frac{2}{3} \pm i\sqrt{5}$$

Solve: x(x-3) = 18

# **Solution**

$$x^{2} - 3x - 18 = 0$$

$$x = \frac{3 \pm \sqrt{9 + 72}}{2}$$

$$= \frac{3 \pm 9}{2}$$

$$= \begin{cases} \frac{3 + 9}{2} = 6 \\ \frac{3 - 9}{2} = -3 \end{cases}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Exercise

Solve: x(x-4)-21=0

$$x^{2} - 4x - 21 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 84}}{2}$$

$$= \frac{4 \pm 10}{2}$$

$$= \begin{cases} \frac{4 + 10}{2} = 7 \\ \frac{4 - 10}{2} = -3 \end{cases}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve: (x-1)(x+4) = 14

#### **Solution**

$$x^{2} + 3x - 18 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 72}}{2}$$

$$= \frac{-3 \pm 9}{2}$$

$$= \begin{cases} \frac{-3 + 9}{2} = 3 \\ \frac{-3 - 9}{2} = -6 \end{cases}$$

#### Exercise

Solve: (x-3)(x+8) = -30

#### **Solution**

$$x^{2} + 5x + 6 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{-5 \pm 1}{2}$$

$$= \begin{cases} \frac{-5 + 1}{2} = -2 \\ \frac{-5 - 1}{2} = -3 \end{cases}$$

#### Exercise

Solve: x(x+8) = 16(x-1)

$$x^{2} + 8x = 16x - 16$$

$$x^{2} - 8x + 16 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 64}}{2}$$

$$= \frac{8}{2}$$

$$= 4 \mid$$

Solve: 
$$x(x+9) = 4(2x+5)$$

#### **Solution**

$$x^{2} + 9x = 8x + 20$$
$$x^{2} + x - 20 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 80}}{2}$$
$$= \frac{-1 \pm 9}{2}$$

$$= \begin{cases} \frac{-1+9}{2} = 4 \\ \frac{-1-9}{2} = -5 \end{cases}$$

#### Exercise

Solve: 
$$(x+1)^2 = 2(x+3)$$

#### Solution

$$x^2 + 2x + 1 = 2x + 6$$

$$x^2 = 5$$

$$x = \pm \sqrt{5}$$

# Exercise

Solve: 
$$(x+1)^2 - 5(x+2) = 3x + 7$$

# **Solution**

$$x^2 + 2x + 1 - 5x - 10 = 3x + 7$$

$$x^2 - 6x - 16 = 0$$

$$x = \frac{6 \pm \sqrt{36 + 64}}{2}$$

$$=\frac{6\pm10}{2}$$

$$= \begin{cases} \frac{6+10}{2} = 8 \\ \frac{6-10}{2} = -2 \end{cases}$$

 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 

$$x(8x+1) = 3x^2 - 2x + 2$$

#### **Solution**

$$8x^2 + x = 3x^2 - 2x + 2$$

$$5x^2 + 3x - 2 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{10}$$

$$=\frac{-3\pm7}{2}$$

$$= \begin{cases} \frac{-3+7}{10} = \frac{2}{5} \\ \frac{-3-7}{10} = -1 \end{cases}$$

# Exercise

Solve: 
$$x^2 + 6x - 7 = 0$$

#### **Solution**

$$1+6-7=0$$
  $a+b+c=0$ 

$$a+b+c=0$$

$$x = 1, -7$$

$$x = 1, -7$$
  $x_1 = 1, x_2 = \frac{c}{a}$ 

# Exercise

Solve: 
$$x^2 - 6x - 7 = 0$$

# **Solution**

$$1-(-6)-7=0$$
  $a-b+c=0$ 

$$a - b + c = 0$$

$$x = -1, 7$$

$$x_1 = -1, 7$$
  $x_2 = -\frac{c}{a}$ 

# Exercise

Solve: 
$$3x^2 + 4x - 7 = 0$$

# **Solution**

$$3 + 4 - 7 = 0 \qquad a + b + c = 0$$

$$a+b+c=0$$

$$x = 1, -\frac{7}{3}$$

$$x = 1, -\frac{7}{3}$$
  $x_1 = 1, x_2 = \frac{c}{a}$ 

Solve: 
$$3x^2 - 4x - 7 = 0$$

#### Solution

$$3-(-4)-7=0$$
  $a-b+c=0$ 

$$x_1 = -1, \frac{7}{3}$$
  $x_1 = -1, x_2 = -\frac{c}{a}$ 

#### Exercise

Solve: 
$$3x^2 - x - 2 = 0$$

#### **Solution**

$$3-1-2=0$$
  $a+b+c=0$ 

$$x = 1, -\frac{2}{3}$$
  $x_1 = 1, x_2 = \frac{c}{a}$ 

#### Exercise

Solve: 
$$3x^2 + x - 2 = 0$$

# **Solution**

$$3-1-2=0$$
  $a-b+c=0$ 

$$x = -1, \frac{2}{3}$$
  $x_1 = -1, x_2 = -\frac{c}{a}$ 

# Exercise

Solve: 
$$2x^2 + 3x - 5 = 0$$

# **Solution**

$$2+3-5=0$$
  $a+b+c=0$ 

$$x = 1, -\frac{5}{2}$$
  $x_1 = 1, x_2 = \frac{c}{a}$ 

# Exercise

Solve: 
$$2x^2 - 3x - 5 = 0$$

$$2-(-3)-5=0$$
  $a+b+c=0$ 

$$x = -1, \frac{5}{2}$$
  $x_1 = -1, x_2 = -\frac{c}{a}$ 

Solve: 
$$x^2 - 3x - 4 = 0$$

#### **Solution**

$$1-(-3)-4=0$$
  $a-b+c=0$ 

$$x_1 = -1, 4$$
  $x_2 = -\frac{c}{a}$ 

#### Exercise

Solve: 
$$x^2 + 3x - 4 = 0$$

#### **Solution**

$$1+3-4=0$$
  $a+b+c=0$ 

$$x = 1, -4$$
  $x_1 = 1, x_2 = \frac{c}{a}$ 

#### Exercise

Solve: 
$$x^2 + 2x + 1 = 0$$

#### **Solution**

$$1-2+1=0$$
  $a-b+c=0$ 

$$x_1 = -1, -1$$
  $x_2 = -\frac{c}{a}$ 

#### Exercise

Solve: 
$$4x^2 - x - 5 = 0$$

$$4-(-1)-5=0$$
  $a-b+c=0$ 

$$x = -1, \frac{5}{4}$$
  $x_1 = -1, x_2 = -\frac{c}{a}$ 

Solve for the specified variable  $A = \frac{\pi d^2}{4}$ , for d

#### **Solution**

$$\frac{4}{\pi}A = \frac{4}{\pi}\frac{\pi d^2}{4}$$

$$\frac{4A}{\pi} = d^2$$

$$d^2 = \frac{4A}{\pi}$$

$$d = \pm \sqrt{\frac{4A}{\pi}}$$

$$d = \pm 2\frac{\sqrt{A}}{\sqrt{\pi}}$$

$$d = \pm 2\frac{\sqrt{A}}{\sqrt{\pi}} = \pm 2\frac{\sqrt{A}}{\sqrt{\pi}} = \pm 2\frac{\sqrt{A}}{\pi}$$

#### Exercise

Solve for the specified variable  $rt^2 - st - k = 0$   $(r \neq 0)$ , for t

lution
$$t = \frac{-(-s) \pm \sqrt{(-s)^2 - 4(r)(-k)}}{2(r)}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{s \pm \sqrt{s^2 + 4rk}}{2r}$$