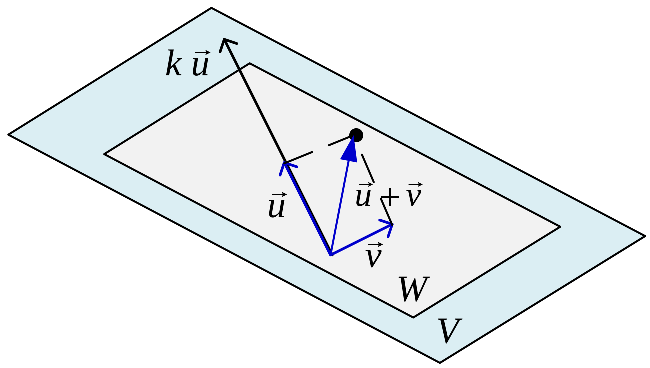
Section 2.5 – Subspaces, Span and Null Spaces

Subspaces

Definition

A subset W of a vector space V is called a ***subspace*** of V if W itself a vector space under the addition and scalar multiplication defined in V.



Theorem

If W is a set of one or more vectors in a vector space V, then W is a subspace of V iff the following conditions holds

1. If $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors in W, then $\boldsymbol{u} + \boldsymbol{v}$ is in W.
2. If k is any scalar and $\boldsymbol{v}$ is any vector in W, the $k\boldsymbol{v}$ is in the subspace in W.

- The most fundamental ideas in linear algebra are that the plane is a subspace of the full vector space $\boldsymbol{R}^n$.

- Every subspace contains the zero vector. The plane vector in $\boldsymbol{R}^3$ has to go through (0, 0, 0). From rule (**2**), if we choose $k = 0$ and the rule requires $0v$ to be in the subspace.

The ***axioms*** that are ***not*** inherited by W are

Axiom 1 – Closure of W under addition

Axiom 4 – Existence of a zero vector in W

Axiom 5 – Existence of a negative in W for every vector in W

Axiom 6 – Closure of W under scalar multiplication

Example

Keep only the vectors (x, y) whose components are positive or zero (first quadrant ***"quarter-plane"***).

The vector $(2, 3)$ is included but $(-2, -3)$ is not. So, rule (**2**) is violated when we try $k=-1$. ***The quarter-plane is not a subspace***.

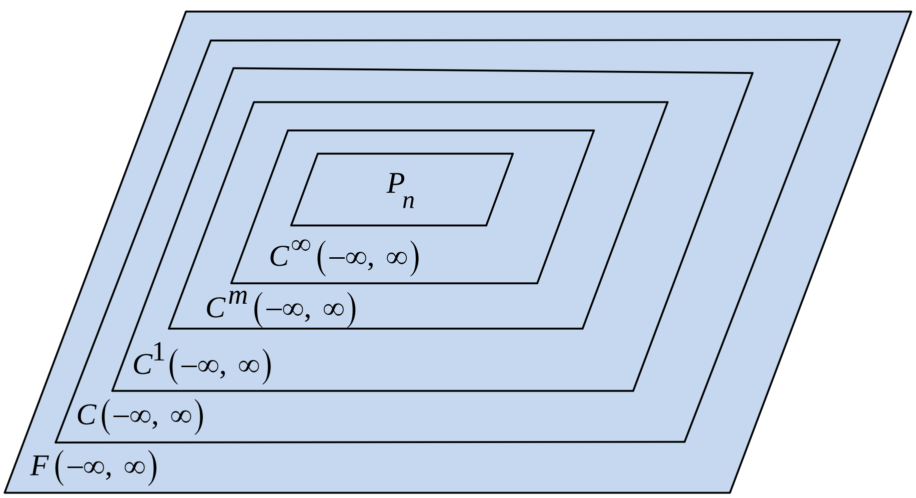
Example

Include also the vectors whose components are both negative. Now we have two quarter-planes. Rule (***ii***) satisfies when we multiply by any c. But rule (***i***) fails. The sum of $v=(2, 3)$ and $w=(-3, -2)$ is $(-1, 1)$ which is outside the quarter-plane. ***Two quarter-planes don't make a subspace***.

Example

The **Subspace** $C(-\infty, \infty)$

There is a theorem in calculus which states that a sum of continuous functions is continuous and than a constant times a continuous function is continuous. In vector word, the set of continuous functions on $(-\infty, \infty)$ is a subspace of $F(-\infty, \infty)$. We dente this subspace by $C(-\infty, \infty)$



Theorem

If $W_1, W_2, \ldots, W_n$ are subspaces of a vector space V, then intersection of these subspaces is also a subspace of V.

- ***A subspace containing*** $\vec{v}$ ***and*** $\vec{w}$ ***must contain all linear combination*** $c\vec{v}+d\vec{w}$.

Example

Inside the vector space M of all 2 by 2 matrices, given two subspaces:

U all upper triangular matrices $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$

D all diagonal matrices $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$

Solution

If we add 2 matrices in **U**: $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} + \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2d \end{bmatrix}$ is in **U**.

If we add 2 matrices in **D**: $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} 2a & 0 \\ 0 & 2d \end{bmatrix}$ is in **D**.

In this case **D** is also a subspace of **U**!. The zero matrix is in these subspaces, when a, b, and d all equal zero.

Span

Definition

The subspace of a vector space V that is formed from all possible linear combinations of the vectors in a nonempty set S is called the ***span of S***, and we say that the vectors in S *span* that subspace. If $S = \{w_1, w_2, \ldots, w_r\}$, then we denoted the span of S by

$$span\{w_1, w_2, \ldots, w_r\} \quad or \quad span(S)$$

Theorem

Let $\vec{v}_1, \ldots, \vec{v}_n$ be vectors in vector space V and S be their span. Then,

a) S is a subspace of V.

Proof: $\forall\ \vec{u}, \vec{v} \in S$, $\vec{u} = a_1\vec{v}_1 + \ldots + a_n\vec{v}_n$ and $\vec{v} = b_1\vec{v}_1 + \ldots + b_n\vec{v}_n$

$$\vec{u} + \vec{v} = (a_1 + b_1)\vec{v}_1 + \ldots + (a_n + b_n)\vec{v}_n \quad \in \boldsymbol{S}$$

$$k\vec{u} = ka_1\vec{v}_1 + \ldots + ka_n\vec{v}_n \quad \in \boldsymbol{S}$$

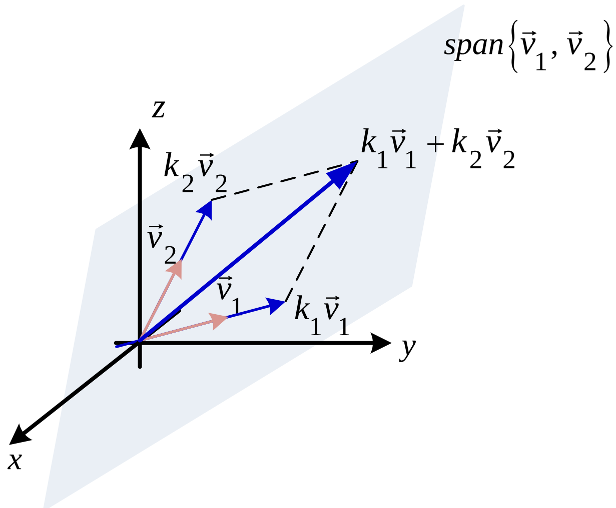
b) S is the smallest subspace of V that contains $\vec{v}_1, \ldots, \vec{v}_k$. i.e. any other subspace $\vec{w}$ containing $\vec{v}_1, \ldots, \vec{v}_n$ also contains S.

Proof: *let* $\vec{u} \in S$, $\vec{u} = a_1\vec{v}_1 + \ldots + a_n\vec{v}_n$

But $a_1\vec{v}_1,\ldots,a_n\vec{v}_n \in \vec{w}$ $\therefore$ $\vec{w}$ *closed under scalar multiplication.*

$a_1\vec{v}_1,\ldots,a_n\vec{v}_n \in \vec{w}$ $\therefore$ $\vec{w}$ *closed under addition.*

$\therefore$ $\underline{\vec{u} \in \vec{w}}$



Example

a) $\vec{v}_1 = \begin{pmatrix}1\\0\end{pmatrix}$ *and* $\vec{v}_2 = \begin{pmatrix}0\\1\end{pmatrix}$ span the full two-dimensional space $\boldsymbol{R}^2$.

b) $\vec{v}_1 = \begin{pmatrix}1\\0\end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix}0\\1\end{pmatrix}$, *and* $\vec{v}_3 = \begin{pmatrix}4\\7\end{pmatrix}$ span the full space $\boldsymbol{R}^2$.

c) $\vec{w}_1 = \begin{pmatrix}1\\1\end{pmatrix}$ *and* $\vec{w}_2 = \begin{pmatrix}-1\\-1\end{pmatrix}$ only span a line in $\boldsymbol{R}^2$.

Definition

The ***row space*** of a matrix is the subspace of $\boldsymbol{R}^n$ spanned by the rows.

Example

Determine whether $\vec{v}_1 = (1,\ 1,\ 2)$, $\vec{v}_2 = (1,\ 0,\ 1)$, *and* $\vec{v}_3 = (2,\ 1,\ 3)$ span the vector space $\boldsymbol{R}^3$

Solution

Let $b = (b_1,\ b_2,\ b_3)$ be the arbitrary vector in $\boldsymbol{R}^3$ can be expressed as a linear combination

$$\vec{b} = k_1\vec{v}_1 + k_2\vec{v}_2 + k_3\vec{v}_3$$

$$\left(b_1, b_2, b_3\right) = k_1(1, 1, 2) + k_2(1, 0, 1) + k_3(2, 1, 3)$$
$$\left(b_1, b_2, b_3\right) = \left(k_1 + k_2 + 2k_3, \ k_1 + k_3, \ 2k_1 + k_2 + 3k_3\right)$$
$$\rightarrow \begin{cases} k_1 + k_2 + 2k_3 = b_1 \\ k_1 \qquad + k_3 = b_2 \\ 2k_1 + k_2 + 3k_3 = b_3 \end{cases}$$
$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 0$$

Since the determinant is zero, the $\vec{v}_1, \vec{v}_2, \textit{ and } \vec{v}_3$ ***do not*** span space $\boldsymbol{R}^3$

Solution Spaces of *Homogeneous* (*Null Space*) Systems

Theorem

The solution set of a homogeneous linear system $A\boldsymbol{x} = \mathbf{0}$ in n unknowns is a subspace of $\boldsymbol{R}^n$

Proof

Let W be the solution set for the system. The set W is not empty because it contains at least the trivial solution $\boldsymbol{x} = \mathbf{0}$.

To show that W is a subspace of $\boldsymbol{R}^n$, we must show that it is closed under addition and scalar multiplication.

Let $\vec{x}_1$ and $\vec{x}_2$ be vectors in W and these vectors are solution of $A\boldsymbol{x} = \mathbf{0}$.

$$A\vec{x}_1 = 0 \quad \textit{and} \quad A\vec{x}_2 = 0$$

Therefore, $A\left(\vec{x}_1 + \vec{x}_2\right) = A\vec{x}_1 + A\vec{x}_2 = 0 + 0 = 0$

So, W is closed under addition.

$$A\left(k\vec{x}_1\right) = kA\vec{x}_1 = k0 = 0$$

So, W is closed under scalar multiplication.

Null Spaces

Definition

The nullspace of A consists of all solutins to $A\vec{x}=0$. These solution vectors x are in $\boldsymbol{R}^n$. The Nullspace containing all solutions is denoted by $N(A)$ *or* $NS(A)$.

$\left\{\vec{x}\in \boldsymbol{R}^n \mid Ax=0\right\}$ is the nullspace of A, $NS(A)$

(Can also be called ***Kernel*** of A: $Ker(A)$)

Theorem

Suppose $NS(A)$ is a subspace of $\boldsymbol{R}^n$ for $A_{m\times n}$

- ✓ Let $\vec{x}$ and $\vec{y}$ are in the nullspace $\left(\vec{x},\, \vec{y}\in NS(A)\right)$ then $A\left(\vec{x}+\vec{y}\right)=A\vec{x}+A\vec{y}=0+0=0$
- ✓ Let $\vec{x}\in NS(A)$ then $c\vec{x}\in NS(A)$ $\therefore A\left(c\vec{x}\right)=cA\vec{x}=c0=0$

Since we can add and multiply without leaving the Nullspace, it is a subspace.

Example

The equation $x+2y+3z=0$ comes from the 1 by 3 matrix $A=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$. This equation produces a plane through the origin. The plane is a subspace of $\boldsymbol{R}^3$. *It is the Nullspace* of A.

Solution

The solution to $x+2y+3z=6$ also form a plane, but not a subspace.

Example

Find the null space of

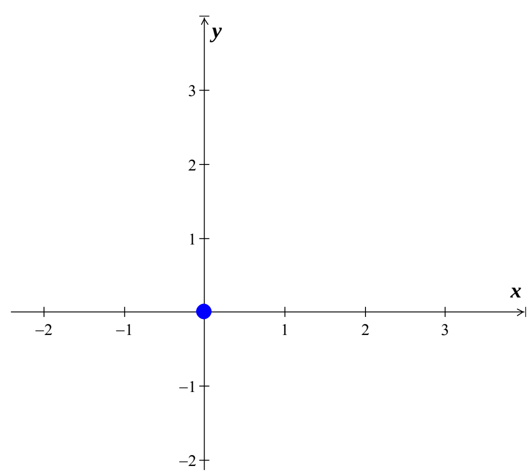
a) $A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ *b)* $B=\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$

Solution

a) $\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}\to \begin{matrix}x_1+x_2=0\\ 3x_2=0\end{matrix}$

$\Rightarrow x_1=x_2=0$

So $NS(A)=\{\mathbf{0}\}$

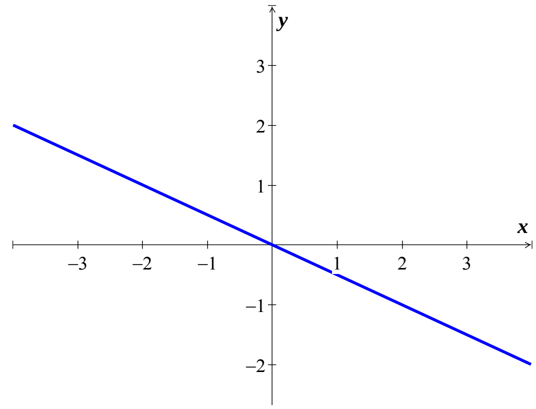


b) $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 6 & 0 \end{bmatrix} \xrightarrow[R_2 - 3R_1]{} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow x_1 = -2x_2$

If we let $x_2 = s$, then

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is in $NS(B)$ if and only if $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = s\begin{bmatrix} -2 \\ 1 \end{bmatrix}$



Example

Describe the nullspace of $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

Solution

Apply the elimination to the linear equations $Ax = 0$:

$$\begin{bmatrix} x_1 + 2x_2 = 0 \\ 3x_1 + 6x_2 = 0 \end{bmatrix} \xrightarrow[R_2 - 3R_1]{} \begin{bmatrix} x_1 + 2x_2 = 0 \\ 0 = 0 \end{bmatrix}$$

There is only one equation $\left(x_1 + 2x_2 = 0\right)$, this line is the Nullspace $N(A)$.

Example

Consider the linear system $\begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solution

$z = t, \quad y = s, \quad x = 2s - 3t$

$\Rightarrow x - 2y + 3z = 0$

This is the equation of a plane through the origin that has $\boldsymbol{n} = (1, -2, 3)$ as a normal.

Example

Consider the linear system $\begin{bmatrix} 1 & -2 & 3 \\ -3 & 7 & -8 \\ 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solution

$x = 0, \quad y = 0, \quad z = 0$

The solution space is $\{\mathbf{0}\}$

Exercises   *Section* 2.5 – Subspaces, Span and Null Spaces

1. Suppose S and T are two subspaces of a vector space **V**.

a) The sum $S+T$ contains all sums $\vec{s}+\vec{t}$ of a vector $\vec{s}$ in S and a vector $\vec{t}$ in T. Show that $S+T$ satisfies the requirements (addition and scalar multiplication) for a vector space.

b) If S and T are lines in $\boldsymbol{R}^m$, what is the difference between $S+T$ and $S\cup T$? That union contains all vectors from S and T or both. Explain this statement: The span of $S\cup T$ is $S+T$.

2. Determine which of the following are subspaces of $\boldsymbol{R}^3$?

a) All vectors of the form $(a, 0, 0)$

b) All vectors of the form $(a, 1, 1)$

c) All vectors of the form (a, b, c), where $b = a + c$

d) All vectors of the form (a, b, c), where $b = a + c + 1$

e) All vectors of the form $(a, b, 0)$

3. Determine which of the following are subspaces of $\boldsymbol{R}^\infty$?

a) All sequences $\boldsymbol{v}$ in $\boldsymbol{R}^\infty$ of the form $\boldsymbol{v} = (v, 0, v, 0, \ldots)$

b) All sequences $\boldsymbol{v}$ in $\boldsymbol{R}^\infty$ of the form $\boldsymbol{v} = (v, 1, v, 1, \ldots)$

c) All sequences $\boldsymbol{v}$ in $\boldsymbol{R}^\infty$ of the form $\boldsymbol{v} = (v, 2\mathrm{v}, 4v, 8v, 16v, \ldots)$

4. Which of the following are linear combinations of $\boldsymbol{u} = (0, -2, 2)$ and $\boldsymbol{v} = (1, 3, -1)$?

a) $(2, 2, 2)$ *b)* $(3, 1, 5)$ *c)* $(0, 4, 5)$ *d)* $(0, 0, 0)$

5. Which of the following are linear combinations of $\boldsymbol{u} = (2, 1, 4)$, $\boldsymbol{v} = (1, -1, 3)$ and $\boldsymbol{w} = (3, 2, 5)$?

a) $(-9, -7, -15)$ *b)* $(6, 11, 6)$ *c)* $(0, 0, 0)$

6. Determine whether the given vectors span $\boldsymbol{R}^3$

a) $\vec{v}_1 = (2,\ 2,\ 2),\quad \vec{v}_2 = (0,\ 0,\ 3),\quad \vec{v}_3 = (0,\ 1,\ 1)$

b) $\vec{v}_1 = (2,\ -1,\ 3),\quad \vec{v}_2 = (4,\ 1,\ 2),\quad \vec{v}_3 = (8,\ -1,\ 8)$

c) $\vec{v}_1 = (3,\ 1,\ 4),\quad \vec{v}_2 = (2,\ -3,\ 5),\quad \vec{v}_3 = (5,\ -2,\ 9),\quad \vec{v}_4 = (1,\ 4,\ -1)$

7. Which of the following are linear combinations of $A = \begin{pmatrix} 4 & 0 \\ -2 & -2 \end{pmatrix},\quad B = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix},\quad C = \begin{pmatrix} 0 & 2 \\ 1 & 4 \end{pmatrix}$

a) $\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$ *b)* $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ *c)* $\begin{bmatrix} 6 & 0 \\ 3 & 8 \end{bmatrix}$

8. Suppose that $\vec{v}_1 = (2,\ 1,\ 0,\ 3)$, $\vec{v}_2 = (3,\ -1,\ 5,\ 2)$, $\vec{v}_3 = (-1,\ 0,\ 2,\ 1)$. Which of the following vectors are in span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

a) (2, 3, –7, 3) *b)* (0, 0, 0, 0) *c)* (1, 1, 1, 1) *d)* (–4, 6, –13, 4)

9. Let $\boldsymbol{f} = \cos^2 x$ and $\boldsymbol{g} = \sin^2 x$. Which of the following lie in the space spanned by $\boldsymbol{f}$ and $\boldsymbol{g}$

a) $\cos 2x$ *b)* $3 + x^2$ *c)* $\sin x$ *d)* 0

10. $V = \mathbb{R}^3$, $S = \{(0,\ s,\ t) \mid s,\ t\ \text{are real numbers}\}$ where *V* is a vector space and *S* is subset of *V*

a) Is *S* closed under addition?
b) Is *S* closed under scalar multiplication?
c) Is *S* a subspace of *V*?

11. $V = \mathbb{R}^3$, $S = \{(x,\ y,\ z) \mid x,\ y\ ,\ z\ \geq 0\}$ where *V* is a vector space and *S* is subset of *V*

a) Is *S* closed under addition?
b) Is *S* closed under scalar multiplication?
c) Is *S* a subspace of *V*?

12. $V = \mathbb{R}^3$, $S = \{(x,\ y,\ z) \mid z = x + y + 1\}$ where *V* is a vector space and *S* is subset of *V*

a) Is *S* closed under addition?
b) Is *S* closed under scalar multiplication?
c) Is *S* a subspace of *V*?

13. Let $S = \{(a_1,\ a_2,\ a_3) \in \mathbb{R}^3 :\ a_1 = 3a_2\ \text{and}\ a_3 = -a_2\}$, Determine:

a) Is *S* closed under addition?
b) Is *S* closed under scalar multiplication?
c) Is *S* a subspace of $\boldsymbol{R}^3$?

14. Let $S = \{(a_1,\ a_2,\ a_3) \in \mathbb{R}^3 :\ a_1 = a_3 + 2\}$, Determine:

a) Is *S* closed under addition?
b) Is *S* closed under scalar multiplication?
c) Is *S* a subspace of $\boldsymbol{R}^3$?

15. Let $S = \{(a_1,\ a_2,\ a_3) \in \mathbb{R}^3 :\ 2a_1 - 7a_2 + a_3 = 0\}$, Determine:

a) Is *S* closed under addition?
b) Is *S* closed under scalar multiplication?
c) Is *S* a subspace of $\boldsymbol{R}^3$?

16. Let $S=\left\{\left(a_1,\ a_2,\ a_3\right)\in\mathbb{R}^3:\ a_1-4a_2-a_3=0\right\}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of $\boldsymbol{R}^3$?

17. Let $S=\left\{\left(a_1,\ a_2,\ a_3\right)\in\mathbb{R}^3:\ a_1+2a_2-3a_3=0\right\}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of $\boldsymbol{R}^3$?

18. Let $S=\left\{\left(a_1,\ a_2,\ a_3\right)\in\mathbb{R}^3:\ a_1+2a_2-3a_3=1\right\}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of $\boldsymbol{R}^3$?

19. Let $S=\left\{\left(a_1,\ a_2,\ a_3\right)\in\mathbb{R}^3:\ 5a_1^2-3a_2^2+6a_3^2=0\right\}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of $\boldsymbol{R}^3$?

20. Let $S=\left\{\left(a_1,\ a_2,\ a_3\right)\in\mathbb{R}^3:\ a_3=a_1+a_2\right\}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of $\boldsymbol{R}^3$?

21. Let $S=\left\{\left(a_1,\ a_2,\ a_3\right)\in\mathbb{R}^3:\ a_1+a_2+a_3=0\right\}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of $\boldsymbol{R}^3$?

22. $S=\left\{\left(x_1,\ x_2,\ 1\right):\ x_1 \text{ and } x_2 \text{ are real numbers}\right\}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of $\boldsymbol{R}^3$?

23. $S=\left\{\left(x_1, x_2, x_3\right)\in\mathbb{R}^3 : x_2 = x_1 + 2x_3\right\}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of $\boldsymbol{R}^3$?

24. $S=\left\{\begin{pmatrix} a & 1 \\ c & d\end{pmatrix}\in M_{2\times 2} \mid a,\ b,\ c\in\mathbb{R}\right\}$ and $V = M_{2,2}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of V ?

25. $S=\left\{\begin{pmatrix} a & 0 \\ c & d\end{pmatrix}\in M_{2\times 2} \mid a,\ b,\ c\in\mathbb{R}\right\}$ and $V = M_{2,2}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of V ?

26. Let $S=\left\{\begin{pmatrix} a & 0 \\ 0 & d\end{pmatrix}\in M_{2\times 2} \mid a,\ d\in\mathbb{R}\ \ \&\ \ ad\ge 0\right\}$ and $V = M_{2,2}$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of V ?

27. $V = M_{33}$, $S=\left\{A \mid A \text{ is invertible}\right\}$ where V is a vector space and S is subset of V

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of V?

28. Let $S=\left\{p(t)=a+2at+3at^3 \ \mid a\in\mathbb{R}\ \ \&\ \ p(t)\in P_2\right\}$ and $V = P_2$, Determine:

a) Is S closed under addition?
b) Is S closed under scalar multiplication?
c) Is S a subspace of V ?

29. Given: $A=\begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4\end{bmatrix}$

a) Find $NS(A)$

b) For which n is $NS(A)$ a subspace of $\mathbb{R}^n$

c) Sketch $NS(A)$ in $\mathbb{R}^2$ or $\mathbb{R}^3$

30. Determine which of the following are subspaces of M_{22}

a) All 2×2 matrices with integer entries

b) All matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a+b+c+d=0$

31. Let $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \ ad - bc = 1 \right\}$. Is V a vector space?

32. Let $V = \{(x, 0, y) : \ x \,\&\, y \text{ are arbitrary } \mathbb{R}\}$. Define addition and scalar multiplication as follows:

$$\begin{cases} (x_1,\ 0,\ y_1) + (x_2,\ 0,\ y_2) = (x_1 + x_2,\ y_1 + y_2) \\ \quad c(x,\ 0,\ y) = (cx,\ cy) \end{cases}$$

Is V a vector space?

33. Construct a matrix whose column space contains $(1,\ 1,\ 0)$ and $(0,\ 1,\ 1)$ and whose nullspace contains $(1,\ 0,\ 1)$ and $(0,\ 0,\ 1)$

34. How is the nullspace $N(C)$ related to the spaces $N(A)$ and $N(B)$, is $C = \begin{bmatrix} A \\ B \end{bmatrix}$?

35. True or False (check addition or give a counterexample)

a) If V is a vector space and W is a subset of V that is a vector space, then W is s subspace of V.

b) The empty set is a subspace of every vector space.

c) If V is a vector space other than the zero vector space, then V contains a subspace W such that $W \neq V$.

d) The intersection of any two subsets of V is a subspace of V.

e) Let W be the *xy*-plane in $\boldsymbol{R}^3$; that is, $W = \{(a_1,\ a_2,\ 0) : \ a_1,\ a_2 \in \boldsymbol{R}\}$. Then $W = \boldsymbol{R}^2$