1/ $\{(2,-4), (2,1)\}$

a) orthogonal?

$(2,-4)\cdot(2,1) = 4-4=0$

it's orthogonal

b) orthonormal? $\sqrt{4+16} \neq 1$

it's not orthonormal

c) Since it's orthogonal $\Rightarrow$ the set is a basis for R^2

$\left(\begin{vmatrix} 2 & 2 \\ -4 & 1 \end{vmatrix} = 10 \neq 0\right)$.

5/ $\{(4,-1,1), (-1,0,4), (-4,-17,-1)\}$

a) $(4,-1,1)\cdot(-1,0,4) = -4+4=0$

$(4,-1,1)\cdot(-4,-17,-1) = -16+17-1=0$

$(-1,0,4)\cdot(-4,-17,-1) = 4-4=0$

The set is orthogonal

b) $\sqrt{16+1+1} = \sqrt{18} \neq 0$

The set is not orthonormal

c) Since it's orthogonal $\Rightarrow$ the set is a basis in R^3

det $\neq 0$.

11) $\{(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}), (0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0), (-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2})\}$

a) $(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}) \cdot (0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0) = 0$

$(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}) \cdot (-\frac{1}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{1}{2}) = -\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} = 0$

$(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0) \cdot (-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}) = \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4} = 0$

The set is orthogonal

b) $\sqrt{\frac{2}{4} + \frac{2}{4}} = 1$ –

$\sqrt{\frac{2}{4} + \frac{2}{4}} = 1$

$\sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = 1$

The set is an orthonormal.

c) set is in R^4 & we only have 3 vectors (4x3)

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{-1}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{1}{2} \end{bmatrix} \xrightarrow{\times 2} \begin{bmatrix} \sqrt{2} & 0 & 1 \\ 0 & \sqrt{2} & 1 \\ 0 & \sqrt{2} & -1 \\ \sqrt{2} & 0 & 1 \end{bmatrix} \begin{matrix} \\ \\ R_3 - R_2 \\ R_4 - R_1 \end{matrix}$$

$$\begin{bmatrix} \sqrt{2} & 0 & 1 \\ 0 & \sqrt{2} & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

15) $\{(\sqrt{3}, \sqrt{3}, \sqrt{3}), (-\sqrt{2}, 0, \sqrt{2})\}$

a) $(\sqrt{3}, \sqrt{3}, \sqrt{3}) \cdot (-\sqrt{2}, 0, \sqrt{2}) = -\sqrt{6} + \sqrt{6} = 0$

set is orthogonal

b) $\frac{(\sqrt{3}, \sqrt{3}, \sqrt{3})}{\|\sqrt{3}, \sqrt{3}, \sqrt{3}\|} = \frac{(\sqrt{3}, \sqrt{3}, \sqrt{3})}{\sqrt{3+3+3}} = \frac{1}{3}(\sqrt{3}, \sqrt{3}, \sqrt{3})$

$\vec{u}_1 = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

$\|(-\sqrt{2}, 0, \sqrt{2})\| = \sqrt{2+2} = 2$

$\vec{u}_2 = \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$

25) $B = \{(3,4), (1,0)\}$

$\vec{v}_1 = (3,4) \quad \vec{v}_2 = (1,0)$

$\vec{w}_1 = \vec{v}_1 = (3,4)$

$\vec{u}_1 = \frac{\vec{w}_1}{\|w_1\|} = \left(\frac{3}{5}, \frac{4}{5}\right)$

$\vec{w}_2 = \vec{v}_2 - \frac{\vec{v}_2 \cdot \vec{w}_1}{\|\vec{w}_1\|^2}\vec{w}_1$

$= (1,0) - \frac{(1.0)\cdot(3,4)}{5}(3,4)$

$= (1,0) - \frac{3}{25}(3,4)$

$= (1,0) - \left(\frac{9}{25}, \frac{12}{25}\right)$

$= \left(\frac{16}{25}, -\frac{12}{25}\right)$

$\|\vec{w}_2\| = \sqrt{\frac{256+144}{25^2}}$

$= \frac{20}{25}$

$\vec{u}_2 = \frac{\vec{w}_2}{\|\vec{w}_2\|} = \frac{25}{20}\left(\frac{16}{25}, -\frac{12}{25}\right)$

$= \left(\frac{4}{5}, -\frac{3}{5}\right)$

29/ $B = \{(2,1,-2), (1,2,2), (2,-2,1)\}$ with $\vec{v}_1 = (2,1,-2)$, $\vec{v}_2 = (1,2,2)$, $\vec{v}_3 = (2,-2,1)$

$\vec{w}_1 = \vec{v}_1 = (2,1,-2)$ $\qquad \boxed{\vec{u}_1 = \frac{(2,1,-2)}{\sqrt{4+1+4}} = \left(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}\right)}$

$$\vec{w}_2 = \vec{v}_2 - \frac{\vec{v}_2 \cdot \vec{w}_1}{\|\vec{w}_1\|^2}\vec{w}_1$$

$$= (1,2,2) - \frac{(1,2,2)\cdot(2,1,-2)}{9}(2,1,-2)$$

$$= (1,2,2) - 0$$

$$= \boxed{(1,2,2)}$$

$$\boxed{\vec{u}_2 = \frac{(1,2,2)}{3} = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)}$$

$$\vec{w}_3 = \vec{v}_3 - \frac{\langle \vec{v}_3, \vec{w}_1 \rangle}{\|\vec{w}_1\|^2}\vec{w}_1 - \frac{\langle \vec{v}_3, \vec{w}_2 \rangle}{\|\vec{w}_2\|^2}\vec{w}_2$$

$$= (2,-2,1) - \frac{(2,-2,1)\cdot(2,1,-2)}{3}(2,1,-2)$$

$$- \frac{(2,-2,1)\cdot(1,2,2)}{3}(1,2,2)$$

$$= (2,-2,1) - 0 - 0$$

$$= (2,-2,1)$$

$$\vec{u}_3 = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$$

$$\left\{\left(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}\right), \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)\right\}$$

39/ $B = \{(1,2,-1,0),\ (2,2,0,1),\ (1,1,-1,0)\}$
$\quad\quad\quad v_1 \quad\quad v_2 \quad\quad v_3$

$\vec{w}_1 = \vec{v}_1 = (1,2,-1,0)$

$\|\vec{w}_1\| = \sqrt{1+4+1} = \sqrt{6}$

$$\boxed{\vec{u}_1 = \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, 0\right)}$$

$$\vec{w}_2 = \vec{v}_2 - \frac{\langle \vec{v}_2, \vec{w}_1 \rangle}{\|\vec{w}_1\|^2}\vec{w}_1$$

$$= (2,2,0,1) - \frac{(2,2,0,1)\cdot(1,2,-1,0)}{6}(1,2,-1,0)$$

$$= (2,2,0,1) - (1,2,-1,0)$$

$$= (1,0,1,1)$$

$\|\vec{w}_2\| = \sqrt{1+1+1} = \sqrt{3}$

$$\boxed{\vec{u}_2 = \left(\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$$

$$\vec{w}_3 = \vec{v}_3 - \frac{\langle \vec{v}_3, \vec{w}_1 \rangle}{\|\vec{w}_1\|^2}\vec{w}_1 - \frac{\langle \vec{v}_3, \vec{w}_2 \rangle}{\|\vec{w}_2\|^2}\vec{w}_2$$

$$= (1,1,-1,0) - \frac{(1,1,-1,0)\cdot(1,2,-1,0)}{6}(1,2,-1,0)$$
$$- \frac{(1,1,-1,0)\cdot(1,0,1,1)}{3}(1,0,1,1)$$

$$= (1,1,-1,0) - \frac{2}{3}(1,2,-1,0) - 0$$

$$= \left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0\right)$$

$$\|\vec{w}_3\| = \sqrt{\frac{1}{9} + \frac{1}{9}\ \frac{1}{9}} = \frac{\sqrt{3}}{3}$$

$$\vec{u}_3 = \frac{3}{\sqrt{3}}\left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0\right)$$

$$\boxed{= \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, 0\right)}$$