8. Potential Energy and Conservation of Mechanical Energy (Chapter 8 Lecture 1)

8.1 Conservative Force

Conservative forces are forces for which the work done is independent of the path followed. It depends only on the potential energies of the objects at the initial and final locations. Potential energy is energy that an object possesses just because of its location. Work done by a conservative force is equal to the negative of the change in the potential energy (u) of the object.

$$\boxed{w_c = -\Delta u = -(u_f - u_i)}$$

$w_c \rightarrow$ work done by a conservative force

$u_i(u_f) \rightarrow$ initial(final) potential energy

Work done on a closed path is zero, because the initial and final location are the same and $u_i = u_f \Rightarrow \Delta u = 0$

8.2 Relationship between a conservative force and the potential energy associated with it

The work done by a conservative force in displacing an object by an infinitely small displacement, dx, may be given as

$$dw_c = F_c dx = -du$$

$$\boxed{F_c = -\frac{du}{dx}}$$

$F_c \rightarrow$ conservative force

$u \rightarrow$ potential energy

A conservative force may be obtained as the negative derivative the potential energy of the object with respect to the position of the object.

Alternatively

$$F_c dx = -du \implies \int du = -\int F\, dx$$

$$\implies \boxed{u = -\int F\, dx + C}$$

Where C is an arbitrary constant

Or $\int_{u_i}^{u_f} du = -\int_{x_i}^{x_f} F\, dx$

$$u_f - u_i = \boxed{\Delta u = -\int_{x_i}^{x_f} F\, dx}$$

The constant in $u(x) = -\int F\, dx + C$ is determined by assigning an arbitrary potential energy to a certain point in space.

Example: The potential energy of a certain object varies with position according to the equation $u(x) = -x^2 + x + 5$. Find an expression for the conservative force acting on it as a function of position(x). Also fine the value of the force at x=2.

$$F_c = -\frac{du}{dx} \text{ and } u(x) = -x^2 + x + 5$$

$$\therefore F_c = -\frac{d}{dx}[-x^2 + x + 5] = -[-2x + 1]$$

$$F_c(x) = 2x - 1 \quad ; \quad F_c(x = 2) = 2(2) - 1 = \underline{3}$$

Example: The conservative force acting on an object varies with position (x) according to the equation $F(x) = -x^3$. Assuming the potential energy at the origin(x=0) is zero, find an expression for the dependence of its potential energy on position (x). Also obtain its potential energy at x=2.

$F(x) = -x^3 \quad u(0) = 0$

$u(x) = ??$

$$u(x) = -\int F\, dx + C = -\int (-x^3) dx + C$$

$$u(x) = \frac{x^4}{4} + C$$
$$u(0) = \frac{0^4}{4} + C = 0 \implies C = 0$$
$$\therefore \quad u(x) = \frac{x^4}{4}$$
$$u(x=2) = \frac{2^4}{4} = 4$$

Example: The conservative force acting on an object varies with position according to the equation $F(x) = -\frac{2}{x^3} + \frac{1}{x^2}$. Find an expression for its potential energy as a function of x. Assume the potential energy at infinity to be zero.

$u(x) = ??$
$F(x) = -\frac{2}{x^3} + \frac{1}{x^2}$
$u(x=\infty) = 0$

$$u(x) = -\int F\,dx + C$$
$$= -\int \left(-\frac{2}{x^3} + \frac{1}{x^2}\right) dx + C$$
$$= -\left(-\frac{2x^{-2}}{-2} + \frac{x^{-1}}{-1}\right) + C$$
$$u(x) = \frac{1}{x^2} - \frac{1}{x} + C$$

$$u|_{x=0} = 0 \quad \& \quad u|_{x=\infty} = \frac{1}{\infty^2} - \frac{1}{\infty} + C = 0$$
$$C = 0$$
$$\boxed{u(x) = \frac{1}{x^2} - \frac{1}{x}}$$

Example: The conservative force acting on an object varies with position according to the equation $F(x) = -x^2 - 5$. Find the change in its potential energy and the work done by the conservative force as it is displaced from x=2 to x=4

$F(x) = -x^2 - 5$
$\Delta u|_{x=2}^{x=4} = ??$
$w_c = ??$

$$\Delta u = -\int_{x_i}^{x_f} F\,dx = -\int_{x=2}^{x=4} (-x^2 - 5)dx$$
$$= -\left[-\frac{x^3}{3} - 5x\right]\Big|_{x=2}^{x=4}$$
$$= -\left[\left(-\frac{4^3}{3} - 5(4)\right) - \left(-\frac{2^3}{3} - 5(2)\right)\right]$$
$$= -\left[\frac{124}{3} + \frac{38}{3}\right] = -54$$
$$\therefore \quad \Delta u = -54\,J$$
$$w_c = -\Delta u = -(-54J) = 54J$$

8.3 Gravitational Potential Energy

Gravitational force acting on an object of mass m (i.e. weight) is given by
$$F_g = -m|g|\hat{j}$$
Gravitational potential energy (u_g) is given by

$$u_g = -\int F_g \cdot d\vec{r} + C$$

$$d\vec{r} = dx\hat{\imath} + dy\hat{\jmath}$$
$$\therefore \; F_g \cdot d\vec{r} = (-m|g|\hat{\jmath}) \cdot (dx\hat{\imath} + dy\hat{\jmath})$$
$$= -m|g|dy$$

$$u_g = -\int (-m|g|dy) + C$$
$$= m|g|y + C$$

Assuming gravitational potential energy at y = 0 is zero

$$u_g|_{y=0} = m|g|(0) + C = 0$$
$$C = 0$$
$$\therefore \; \boxed{u_g = m|g|y}$$

u_g → gravitational potential energy of an object of mass m located at a point whose y-coordinate is y

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Work done by gravitational force

$$w_g = -\Delta u_g = -\Delta(m|g|y)$$
$$= -m|g|(\Delta y) = -m|g|(y_f - y_i)$$
$$\therefore \; \boxed{w_g = -m|g|\Delta y = -m|g|(y_f - y_i)}$$

w_g → work done by gravity in displacing an object of mass, displaced from a location whose y-coordinate is y_i to a location whose y-coordinate is y_f

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Example: Calculate the gravitational potential energy of an object of mass 10kg located at a point whose position vector is

a) $\vec{r} = (2\hat{\imath} + 6\hat{\jmath})m$

$x = 2 \quad y = 6 \quad m = 10kg$

$u_g = ??$ $\qquad u_g = m|g|y$

$= (10)(9.8)(6)J$

$\underline{= 588\,J}$

b) $\vec{r} = 10\hat{\jmath}$

$x = 0 \quad y = 10$

$u_g = m|g|y = (10)(9.8)(10)J = 980J$

c) $\vec{r} = -5\hat{\imath}$

$x = -5\hat{\imath} \quad y = 0$

$u_g = m|g|y = (10)(9.8)(0)J = 0J$

Example: Calculate the work done by gravity on a 2kg object when it is displaced from the location $-2\hat{\imath} + 4\hat{\jmath})\ m$ to the location $(8\hat{\imath} + 2\hat{\jmath})m$

$\vec{r_i} = -2\hat{\imath} + 4\hat{\jmath} \Rightarrow y_i = 4m$

$\vec{r_f} = 8\hat{\imath} + 2\hat{\jmath} \Rightarrow y_f = 2m$

$\therefore w_g = -m|g|(y_f - y_i) = -2(9.8)(2 - 4)$

$= 39.2\,J$

Example: Obtain an expression for gravitational force starting from the expression from gravitational potential energy.

$u_g = m|g|y$

$F_g = ??$

$$F_g = -\frac{du_g}{dy} = -\frac{d}{dy}(m|g|y) = -m|g|$$
$$F_g = -m|g| \;\Rightarrow\; \vec{F_g} = -m|g|\hat{\jmath}$$

8.4 Elastic Potential Energy (u_e)

From Hook's Law, the force due to a spring depends on its displacement (extension or compression) according to the equation

$$F_{sp} = -kx$$

$$\therefore \quad u_c = -\int F_{sp}dx + C$$

$$= -\int (-kx)dx + C$$

$$u_e = \frac{1}{2}kx^2 + C$$

Assuming the elastic potential energy at relaxed position (x=0) is zero

$$u_e|_{x=0} = \frac{1}{2}k0^2 + C = 0$$

$$C = 0$$

$$\therefore \quad \boxed{u_e = \frac{1}{2}kx^2}$$

$u_e \rightarrow$ elastic potential energy of a spring of Hook's constant k extended or compressed by x

Work done by the force due to a spring

$$w_{sp} = -\Delta u_e = -\left(u_{ef} - u_{ei}\right)$$

$$= -\left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right)$$

$\boxed{w_{sp} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)}$	$w_{sp} \rightarrow$ work done by a spring as its displacement changed from x_i to x_f

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Example: Calculate the elastic potential energy stored by a spring of Hook's Constant 200 N/m when compressed by 2 cm

$k = 200\frac{N}{m}$

$x = -2\ cm = -.02m$

u_e

$$u_e = \frac{1}{2}kx^2$$

$$= \frac{1}{2}(200)(0.02)^2$$

$$u_e = 4 \times 10^{-2}$$

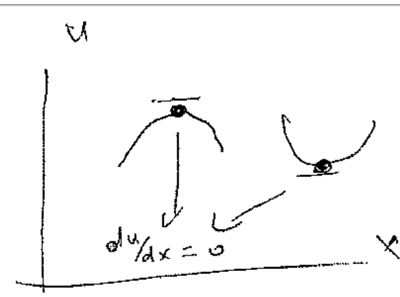
Example: Calculate the work done by the force due to a spring of Hook's constant 100 N/m when the spring is extended from x=1 cm to x=6 cm

$k = 100\frac{N}{m}$ $x_i = 1cm = 0.01m$ $x_f = 6cm = 0.06m$ $w_{sp} = ??$	$w_{sp} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$ $= -\frac{1}{2}(100)(.06^2 - .01^2)$ $w_{sp} = -.175\ J$

8.5 Conditions of Equilibrium

An object is said to be in equilibrium(i.e. either at rest or moving in a straight line with a constant speed) if the force acting on it is zero.

If a particle is being acted upon by a conservative force

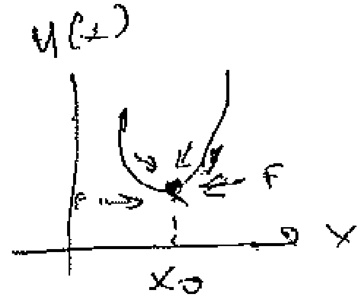


Since $F = -\frac{du}{dx}$

$$F = 0 \ \Rightarrow \frac{du}{dx} = 0$$

A particle will be in equilibrium if the derivative of the potential energy with respect to position is zero. In other words, the particle will be in equilibrium at the turning points of its potential energy.

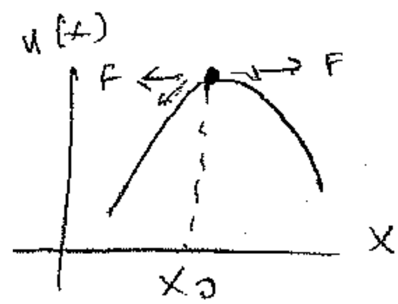
There are two kinds of equilibrium: stable and unstable equilibrium.



<u>Stable Equilibrium:</u> is equilibrium where the particle tends to return to its equilibrium position when displaced by a small displacement. This happens when the potential energy opens upward(as a function of x) at the turning point.

Mathematically this happens when the second derivative of the potential energy with respect to position is positive. Thus a particle will be in stable equilibrium at $x = x_0$ if

$$\boxed{\frac{du}{dx}|_{x=x_o} = 0 \quad \& \quad \frac{d^2u}{dx^2}|_{x=x_0} > 0}$$ Conditions of stable equilibrium



<u>Unstable Equilibrium:</u> is equilibrium where the particle tends to go away from its equilibrium position when displaced by a small displacement. This happens when the graph of potential energy as a function opens downward at the turning point.

Mathematically this happens when the second derivative at the equilibrium position (turning point) is negative.

Therefore a particle will be in unstable equilibrium at $x = x_o$ if

$$\boxed{\frac{du}{dx}|_{x=x_o} = 0 \quad \& \quad \frac{d^2u}{dx^2}|_{x=x_0} < 0}$$ Conditions of unstable equilibrium

An example of a stable equilibrium is a ball in a valley while an example of unstable equilibrium is a ball at the top of a hill.

<u>Example:</u> The potential energy of a certain particle varies with position according to the equation $u(x) = x^2 - 1$

a) Find the point(s) of equilibrium

Equilibrium point(s) → value for x for which $\frac{du}{dx} = 0$

$$\frac{du}{dx} = \frac{d(x^2 - 2)}{dx} = 2x = 0$$
$$\Rightarrow \ x = 0$$

b) Determine if the equilibrium point is stable or unstable

$$\frac{d^2u}{dx^2}|_{x=x_0} = \frac{d^2(x^2 - 1)}{dx^2}|_{x=x_0} = \frac{d(2x)}{dx}|_{x=x_o} = 2 > 0$$

Since $\frac{d^2u}{dx^2}|_{x=x_0} > 0$, the equilibrium is a <u>stable</u> equilibrium

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<u>Example:</u> The potential energy of a certain particle varies with position according to the equation $u(x) = \frac{x^3}{3} - 4x$

a) Determine the points of equilibrium

Equilibrium point(s)→ value of x for which $\frac{du}{dx} = 0$

$$\frac{du}{dx} = \frac{d\left(\frac{x^3}{3} - 4x\right)}{dx} = x^2 - 4 = 0$$
$$\Rightarrow x^2 - 4$$
$$x = \pm 2$$

Points of equilibrium are $x = -2$ & $x = 2$

b) Determine if the equilibrium points are stable or unstable

$$\frac{d^2u}{dx^2}\Big|_{x=x_0} = \frac{d^2\left(\frac{x^3}{3} - 4x\right)}{dx^2}\Big|_{x=x_0} = \frac{d(x^2-4)}{dx}\Big|_{x=x_o} = 2x$$

<u>At $x = 2$</u>

$$\frac{d^2u}{dx^2}\Big|_{x=2} = 2x|_{x=2} = 2(2) = 4 > 0$$

∴ the equilibrium at x = 2 is <u>stable</u> because $\frac{d^2u}{dx^2}\Big|_{x=2} > 0$

<u>At $x = 2$</u>

$$\frac{d^2u}{dx^2}\Big|_{x=-2} = 2x|_{x=-2} = 2(-2) = -4 < 0$$

∴ the equilibrium at x = -2 is <u>unstable</u> because $\frac{d^2u}{dx^2}\Big|_{x=-2} < 0$

<u>8.6 Central Forces</u>

Central forces are forces whose magnitude depend on the distance between the source (origin) and the point, and whose direction is along the direction of the position vector of the point. Central forces can generally be written as

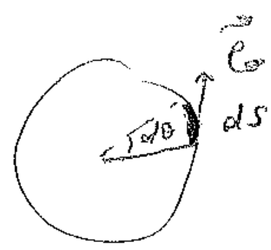
$$\boxed{\vec{F} = f(r)\vec{e}_r = f(r)\frac{\vec{r}}{r}}$$

Where $f(r)$ an arbitrary functions of r.

<u>Potential energy associated with 2 central forces</u>

$$u = -\int \vec{F} \cdot d\vec{r} = -\int f(r)\vec{e}_r \cdot d\vec{r} + C$$

The path element of $d\vec{r}$ can be expressed in terms of polar coordinates as follows, when θ is constant only the length of the position vector can change & the component of $d\vec{r}$ when θ is constant, can be written as $dr\vec{e}_r$



If r is kept constant, $\vec{r}$ can only rotate in a circle of radius r. If the arc length along the circular path is ds, then the component of $d\vec{r}$ along $\vec{e}_\theta$ can be written as $ds\vec{e}_\theta$. Thus $d\vec{r}$ can be written in terms of polar coordinates as

$$d\vec{r} = dr\vec{e}_r + rd\theta\vec{e}_\theta$$
$$u = -\int f(r)\,\vec{e}_r[dr\vec{e}_r + rd\theta\vec{e}_\theta] + C$$

Since $\vec{e}_r \cdot \vec{e}_r = 1$ & $\vec{e}_\theta \cdot \vec{e}_r = 0$

$$\boxed{u = -\int f(r)\,dr + C}$$

And if $u(r)$ is the potential energy associated with a central force, the central force can be obtained as

$$\boxed{\vec{F} = -\frac{du}{dr}\vec{e}_r}$$

<u>Note:</u> The direction of force that changes r only, must be along $\vec{e}_r$

Example: A central force is given as $\vec{F}(r) = -\frac{1}{r^2}\vec{e}_r$. Assuming the potential energy at infinity to be zero, calculate the potential energy to this force on a particle located at the point (3,4)m.

Solution

$$u(r) = -\int f(r)dr + C \qquad f(r) = -\frac{1}{r^2}$$

$$= -\int -\frac{1}{r^2}dr + C = \int \frac{1}{r^2}dr + C = -\frac{1}{r} + C$$

$$u(\infty) = 0 \quad \Rightarrow \quad u(\infty) = -\frac{1}{\infty} + C = 0 \quad \Rightarrow \quad C = 0$$

$$u(r) = -\frac{1}{r}$$

At the point (3,4)m $r = \sqrt{3^2 + 4^2} = 5$

$$u(r = 5) = -\frac{1}{5}$$

Example: The potential energy of a particle varies on the distance from the origin according to the equation $(r) = -\frac{1}{r}$. Calculate the force exerted on a particle located at the point (3,4)m

Solution

$$u(r) = -\frac{1}{r}$$

$$\vec{F}(r) = -\frac{du}{dr}\vec{e}_r = -\frac{d}{dr}\left(-\frac{1}{r}\right)\vec{e}_r = -\frac{1}{r^2}\vec{e}_r$$

$$\vec{F}(r) = -\frac{1}{r^2}\vec{e}_r = -\frac{1}{r^2}\left(\frac{\vec{r}}{r}\right) = -\frac{1}{r^3}\vec{r} = -\frac{1}{r^3}(x\hat{\imath} + y\hat{\jmath})$$

$$(x, y) = (3,4)m$$

$$r = \sqrt{3^2 + 4^2} = 5m$$

$$\therefore \quad \vec{F}(5) = -\frac{1}{5^3}(3\hat{\imath} + 4\hat{\jmath})N$$

$$\vec{F}(5) = (-\frac{3}{125}\hat{\imath} - \frac{4}{125}\hat{\jmath})N$$

8.6 Conservation of Mechanical Energy (Lecture 2)

Mechanical Energy of an object is defined to be the sum of its potential energy and kinetic energy.

$$ME = u + KE \text{ but } KE = \frac{1}{2}mv^2$$

$$\boxed{ME = u + \frac{1}{2}mv^2}$$

$ME \rightarrow$ mechanical energy
$u \rightarrow$ potential energy
$m \rightarrow$ mass
$v \rightarrow$ speed

If the force is gravitational force, then $u = m|g|y$ and

$$\boxed{ME_g = \frac{1}{2}mv^2 + m|g|y}$$

If the force is force due to a spring, $u = \frac{1}{2}kx^2$ and

$$\boxed{ME_e = \frac{1}{2}mv^2 + \frac{1}{2}kx^2}$$

Principle of conservation of Mechanical Energy: states that if all the forces with non-zero contribution to the work done acting on an object are conservative, then the mechanical energy of the object is conserved.

Proof

$$w_{net} = w_c + w_{nc}$$

Where $w_c(w_{nc})$ is work done by conservative(non-conservative) force

$$\text{If } w_{nc} = 0 \text{ then } w_{net} = w_c$$

$$\text{But } w_{net} = \Delta KE \quad \& \quad w_c = -\Delta PE$$

$$\therefore \quad \Delta KE = -\Delta PE$$

$$\Delta KE + \Delta PE = 0$$

$$\Delta(KE + PE) = 0$$

$$\Delta ME = 0$$

$$ME_f - ME_i = 0 \quad \Longrightarrow \quad ME_f = ME_i$$

$$ME_f = ME_i \quad \Longrightarrow \quad \boxed{\frac{1}{2}mv_f^2 + u_f = \frac{1}{2}mv_i^2 + u_i}$$

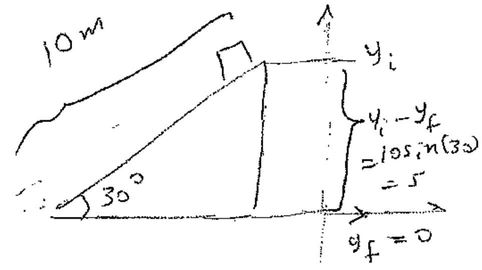
If the conservative force is gravity, u=$m|g|y$ and

$$\boxed{\frac{1}{2}mv_f^2 + m|g|y_f = \frac{1}{2}mv_i^2 + m|g|y_i}$$

If the conservative force is force due to a spring, $u = \frac{1}{2}kx^2$ and

$$\boxed{\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2}$$

Example: An object of mass 7 kg is sliding down a frictionless 10m 30° inclined plane. Calculate the speed of the object when it reaches the ground.



The net force acting on the object is the component of its weight parallel to the plane, which is conservative. Thus the principle of conservation of Mechanical Energy applies.

$$\frac{1}{2}mv_i^2 + m|g|y_i = \frac{1}{2}mv_f^2 + m|g|y_f$$

$v_i = 0$ (sliding from rest)
$y_i = 10 \sin 30 = 5m$
$y_f = 0$ (ground is the reference)
$v_f = ??$

Dividing the equation by m

$$\Rightarrow \frac{1}{2}v_f^2 + |g|y_f = \frac{1}{2}v_i^2 + |g|y_i$$

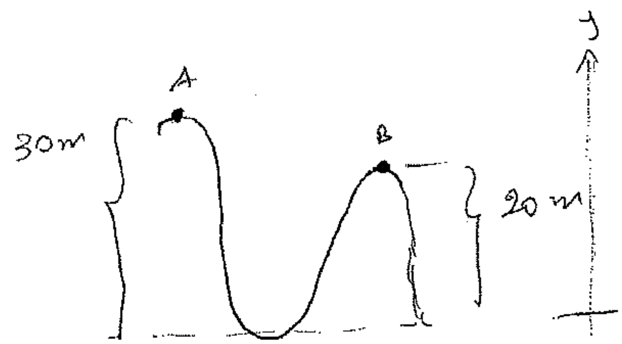
Since y_f & $v_i = 0$

$$\frac{1}{2}v_f^2 = |g|y_i$$

$$v_f^2 = 2|g|y_i = 2(10)(5) = 100$$

$$v_f = \sqrt{100} = \underline{10 \text{ m/s}}$$

Example: Consider the roller coaster shown. An object starts at point A with a speed of 5 m/s. If the roller coaster is frictionless, calculate its speed at point B.



The net force moving the object is gravity which is conservative. Therefore, its mechanical energy is conserved.

$$\frac{1}{2}mv_A^2 + m|g|y_A = \frac{1}{2}mv_B^2 + m|g|y_B$$

$y_A = 30m$
$v_A = 5$ m/s
$y_B = 20\ m$
$v_B = ??$

Dividing by m

$$\Rightarrow \frac{1}{2}v_A^2 + |g|y_A = \frac{1}{2}v_B^2 + |g|y_B$$

$$\frac{1}{2}(5)^2 + (10)(30) = \frac{1}{2}v_B^2 + (10)(20)$$

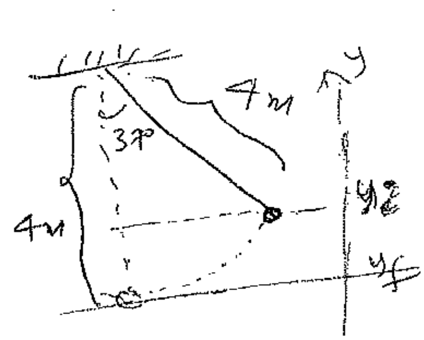
$$12.5 + 300 = \frac{1}{2}v_B^2 + 200$$

$$\frac{1}{2}v_B^2 = 112.5$$

$$v_B^2 = 225$$

$$v_B = \sqrt{225} = 15 \text{ m/s}$$

Example: A pendulum of length 4 m is let go from an angle of 37°. Calculate its speed at its lowest point



The force with non-zero contribution to the work done is gravity (the tension in the string does not contribute to the work done because it is perpendicular to the trajectory).

$\Rightarrow ME_f = ME_i$

With the reference line at the lowest point

$y_f = 0$
$y_i = 4m - 4m \cos 37° = 0.8\ m$
$v_i = 0\ (released)$
$v_f = ??$

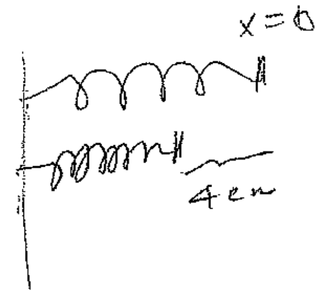
$$\frac{1}{2}mv_i^2 + m|g|y_i = \frac{1}{2}mv_f^2 + m|g|y_f$$

Dividing by m and since $y_f = 0$ & $v_i = 0$

$$\Rightarrow \frac{1}{2}v_f^2 = |g|y_i$$

$$v_f^2 = 2|g|y_i = 2(10)(0.8)$$
$$v_f = \sqrt{16} = \underline{4 \text{ m/s}}$$

Example: An object of mass 4 kg is attached to a spring of Hook's constant 200 N/m. The spring is then compressed by 4 cm and then let go. Calculate the speed of the object by the time it leaves the spring.



$x_i = 4cm$
$= 0.04\, m$
$x_f = 0$
$v_i = 0$
$k = 200$ N/m
$m = 4\, kg$
$v_f = ??$

$$\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2$$

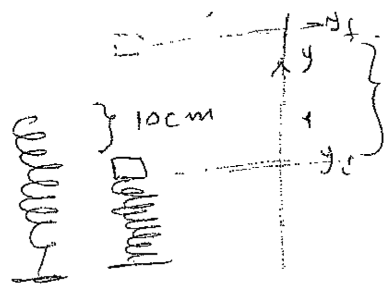
Dividing by $\frac{1}{2}$ and since $x_f = 0$ & $v_i = 0$

$$mv_f^2 = kx_i^2$$
$$v_f^2 = \frac{kx_i^2}{4} \Rightarrow v_f^2 = \frac{200(.04)^2}{4}$$
$$v_f = \sqrt{.08} \approx \underline{.283 \text{ m/s}}$$

Example: A 2 kg object is placed on a vertical spring of Hook's constant 600 N/m. The spring is then compressed further so that its compression from its relaxed position is 10 cm and then it is let go. How high will the object rise?

The only forces involved are gravity and the force due to the spring which are conservative. Therefore, mechanical energy is conserved.

With the reference (y=0) line set at the initial location.



$m = 2\, kg$
$k = 600$ N/m
$y_i = 0$
$v_i = 0$ (released)
$v_f = 0$ (maximum height)
$x_i = 0.1\, m$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 + m|g|y_i$$
$$= \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 + m|g|y_f$$

Since y_i, v_i, x_f & $v_f = 0$

$$\frac{1}{2}kx_i^2 = m|g|y_f$$
$$y_f = \frac{1}{2}\frac{600(0.1)^2}{2(10)}$$
$$= \underline{0.15 \text{ m}}$$

8.7 Work done by non-conservative forces

Examples of non-conservative forces are friction and air resistance. The net work done on an object is the sum of the work done by all conservative forces (w_c) and non-conservative forces(w_{nc}).

$$w_{net} = w_{nc} + w_c$$
$$w_{nc} = w_{net} - w_c$$

But $w_{net} = \Delta KE$ & $w_c = -\Delta PE$

$$w_{nc} = \Delta KE - (-\Delta PE) = \Delta KE + \Delta PE$$
$$= \Delta(KE + PE) = \Delta ME$$
$$\boxed{w_{nc} = \Delta ME = ME_f - ME_i}$$

Work done by non-conservative forces is equal to the change of mechanical energy. If the conservative force is gravitational force, then

$$w_{nc} = \left(\frac{1}{2}mv_f^2 + m|g|y_f\right) - \left(\frac{1}{2}mv_i^2 + m|g|y_i\right)$$

If the conservative force is force due to a spring

$$w_{nc} = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2\right) - \left(\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2\right)$$

Example: A 2 kg object is sliding down a 10 m 30° inclined plane. If it is found that its speed at the bottom is 5 m/s,

a) Calculate the work done by friction

$v_i = 0$ (released)
m= 2kg
$v_f = 5$ m/s
$y_i - y_f = 10m \sin 30° = 5m$
$y_f - y_i = -5m$

$$w_{nc} = w_f = \left(\frac{1}{2}mv_f^2 + m|g|y_f\right) - \left(\frac{1}{2}mv_i^2 + m|g|y_i\right)$$
$$= \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + m|g|(y_f - y_i)$$
$$w_f = \left(\frac{1}{2}(2)(5)^2 - 0\right) + 2(10)(-5)$$
$$= 25 - 100 = -75\,J$$

b) Calculate the friction

$$w_f = fd\cos\theta$$

$$-75 = f(10)\cos 180°$$
$$-75 = -10f$$
$$\underline{f = 7.5\,N}$$