#1 $f(x) = \cos x$ $g(x) = \sin x$ $0 \le x \le \frac{\pi}{2}$

$\cos x = \sin x \Rightarrow x = \frac{\pi}{4}$

$$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$

$$= \sin x + \cos x \Big|_0^{\pi/4} + (-\cos x - \sin x \Big|_{\pi/4}^{\pi/2}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 + \left(-1 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right)$$

$$= \sqrt{2} - 2 + \sqrt{2}$$

$$= 2\sqrt{2} - 2 \text{ unit}^2$$

#2 $y^2 = 2x + 6$ $y = x - 1$

$x = \frac{1}{2}y^2 - 3$ $x = y + 1$

$\frac{1}{2}y^2 - 3 = y + 1$

$\frac{1}{2}y^2 - y - 4 = 0$

$y^2 - 2y - 8 = 0 \Rightarrow y = -2, 4$

$-\left(\frac{1}{2}y^2 - 3\right)$

$$\text{Area} = \int_{-2}^{4} \left(y + 1 - \frac{1}{2}y^2 + 3\right) dy$$

$$= \frac{1}{2}y^2 - \frac{1}{6}y^3 + 4y \Big|_{-2}^{4}$$

$$= 8 - \frac{32}{3} + 16 - \left(2 + \frac{4}{3} - 8\right)$$

$$= 30 - 12$$

$$= 18 \text{ unit}^2$$

#3 $y^4 = x$ $\quad y = \sqrt{2-x}$ $\quad y = 0$

$y^2 = 2 - x$

$x = 2 - y^2$

$$A = \int_0^1 (2 - y^2 - y^4)\,dy$$

$$= 2y - \frac{1}{3}y^3 - \frac{1}{5}y^5 \Big|_0^1$$

$$= 2 - \frac{1}{3} - \frac{1}{5}$$

$$= \frac{30 - 5 - 3}{15}$$

$$= \frac{22}{15} \text{ unit}^2$$

Sec. 1.3 Volume

Ex $r = 3$ $\quad$ sq. : 3 m $\quad$ $0 \le x \le 3$ $\quad$ Vol?

$y = x^2$

$$V = \int_0^3 x^2\,dx$$

$$= \frac{1}{3}x^3 \Big|_0^3$$

$$= 9 \text{ m}^3$$

Ex $r = 3 \rightarrow x^2 + y^2 = 9$ $\theta = 45^\circ$

$0 \le x \le 3$

$y = \sqrt{9 - x^2}$ $(\pm)$

$A = x \cdot (2y)$

$V = 2\int_0^3 x\,(9-x^2)^{1/2}\,dx$ $\quad d(9-x^2) = -2x\,dx$

$= -\int_0^3 (9-x^2)^{1/2}\,d(9-x^2)$

$= -\frac{2}{3}(9-x^2)^{3/2}\Big|_0^3$ $\quad 9^{3/2}\ (3^2)^{3/2}$

$= -\frac{2}{3}(0 - 27)$

$= 18$ unit^3

Disk Method: $V = \pi \int_a^b R^2(x)\,dx$

Ex $y = \sqrt{x}$ $\quad 0 \le x \le 4$ $\quad$ rev. x axis

$V = \pi\int_0^4 x\,dx$ $\quad (\sqrt{x})^2$

$= \frac{\pi}{2} x^2\Big|_0^4$

$= 8\pi$ unit^3

$V_{sphere} = \frac{4}{3}\pi a^3$

$x^2 + y^2 = a^2$

$A = \pi y^2$

$= \pi (a^2 - x^2)$

$V = \pi \int_{-a}^{a} (a^2 - x^2)\, dx$

$= 2\pi \left(a^2 x - \frac{1}{3}x^3 \Big|_0^a \right)$

$= 2\pi \left(a^3 - \frac{1}{3}a^3 \right)$

$= \frac{4\pi}{3} a^3$ ✓

fctn: even

$\int_{-a}^{a} = 2\int_{0}^{a}$

odd: $\int_{-a}^{a} = 0$ (A/V)

Ex rev. y-axis $x = \frac{2}{y}$ $1 \le y \le 4$

$V = \pi \int_1^4 \frac{4}{y^2}\, dy$ ↝ $\left(\frac{2}{y}\right)^2$

$d\left(\frac{1}{0}\right) = -\frac{1}{y^2}$

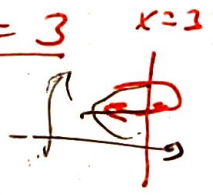
y^{-2}

$= -4\pi \frac{1}{y} \Big|_1^4$

$= -4\pi \left(\frac{1}{4} - 1 \right)$

$= 3\pi$ unit^3

Ex $x = y^2 + 1$ $x = 3$ rev. $x = 3$



$$V = \pi \int_a^b R^2 \, dy$$

$$y^2 = x - 1 \Big|_{x=3}$$
$$= 2$$
$$y = \pm\sqrt{2}$$

$$V = \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left(3 - (y^2+1)\right)^2 dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (2 - y^2)^2 dy$$

$$= 2\pi \int_0^{\sqrt{2}} (4 - 4y^2 + y^4) dy$$

$$= 2\pi \left(4y - \frac{4}{3}y^3 + \frac{1}{5}y^5\right)\Big|_0^{\sqrt{2}}$$

$(\sqrt{2})^5$

$$= 2\pi \left(4\sqrt{2} - \frac{8}{3}\sqrt{2} + \frac{4}{5}\sqrt{2}\right)$$

$$= 2\pi \left(4 - \frac{8}{3} + \frac{4}{5}\right)\sqrt{2}$$

$$= 8\pi \left(\frac{15 - 10 + 3}{15}\right)\sqrt{2}$$

$$= \frac{64\pi}{15}\sqrt{2} \text{ unit}^3$$

Washer: $V = \pi \int_a^b \left(R(x)^2 - r^2(x)\right) dx$

Ex

Q I $\begin{cases} x = y^3 \\ x = 4y \end{cases}$ a) x-axis b) y-axis

$x = y^3 = 4y \rightarrow y = 0$

$y^2 = 4 \quad (y = \pm 2)$

$y = 0, 2$

Q I $\rightarrow y = 2$

$y = x^{1/3}, \; y = \frac{x}{4}$

$\begin{cases} y = 0 \rightarrow x = 0 \\ y = 2 \rightarrow x = 8 \end{cases}$

$$V = \pi \int_0^8 \left((x^{1/3})^2 - \left(\frac{x}{4}\right)^2\right) dx$$

$$= \pi \int_0^8 \left(x^{2/3} - \frac{1}{16} x^2\right) dx$$

$$= \pi \left(\frac{3}{5} x^{5/3} - \frac{1}{48} x^3 \Big|_0^8 \right) \qquad 8 \rightarrow 2^3$$

$$= \pi \left(\frac{96}{5} - \frac{32}{3}\right) \qquad \frac{16 \times 3}{2^4} \qquad \frac{3}{2} - \frac{1}{3}$$

$$= 32\pi \left(\frac{9 - 5}{15}\right)$$

$$= \frac{128\pi}{15} \text{ unit}^3$$

$$V = \pi \int_0^2 (16y^2 - y^6)\,dy$$

$$= \pi \left(\frac{16}{3}y^3 - \frac{1}{7}y^7 \right)\Big|_0^2$$

$$= \pi \left(\frac{128}{3} - \frac{128}{7} \right)$$

$$= 128\pi \left(\frac{7-3}{21} \right)$$

$$= \frac{512}{21}\pi \text{ unit}^3$$

$\frac{512}{21} \times \frac{128}{15}$

Ex $y = x^2 + 1$ $y = -x + 3$ rev. x-axis

$$x^2 + 1 = -x + 3$$

$$x^2 + x - 2 = 0 \Rightarrow x = 1, -2$$

$$V = \pi \int_{-2}^{1} ((3-x)^2 - (x^2+1)^2)\,dx$$

$$= \pi \int_{-2}^{1} (9 - 6x + x^2 - x^4 - 2x^2 - 1)\,dx$$

$$= \pi \int_{-2}^{1} (-x^4 - x^2 - 6x + 8)\,dx$$

$$= \pi \left(-\frac{1}{5}x^5 - \frac{1}{3}x^3 - 3x^2 + 8x \right)\Big|_{-2}^{1}$$

$$= \pi \left(-\frac{1}{5} - \frac{1}{3} - 3 + 8 - \left(\frac{32}{5} + \frac{8}{3} - 12 - 16 \right) \right)$$

$$= \pi \left(-\frac{33}{5} - 3 + 5 + 28 \right)$$

$30 - \frac{33}{5}$

$$= \frac{117\pi}{5} \text{ unit}^3$$

$y = x^2$ $y = 2x$ QI y-axis $x = \sqrt{y}$ $x = \frac{1}{2}y$

$x^2 = 2x \Rightarrow x = 0, 2$ | $y = 0, 4$

$$V = \pi \int_0^4 \left(y - \frac{1}{4}y^2 \right) dy$$

$$= \pi \left(\frac{1}{2}y^2 - \frac{1}{12}y^3 \right)\Big|_0^4$$

$$= \pi \left(8 - \frac{16}{3} \right)$$

$$= \frac{8\pi}{3} \text{ unit}^3$$

Shell Method: $V = 2\pi \int (\text{radius})(\text{height})\, dx$ ↑ y-axis

Ex $f(x) = \sin x^2$, $y = 0$, $x = \sqrt{\frac{\pi}{2}}$ y-axis

$$V = 2\pi \int_0^{\sqrt{\pi/2}} x(\sin x^2)\, dx \qquad d(x^2) = 2x\,dx$$

$$= \pi \int_0^{\sqrt{\pi/2}} \sin(x^2)\, d(x^2)$$

$$= -\pi \cos x^2 \Big|_0^{\sqrt{\pi/2}}$$

$$= -\pi(0 - 1)$$

$$= \pi \text{ unit}^3$$

Ex $y = \sqrt{x-2}$ $y = 2$ $x - 2 = y^2$ $x = y^2 + 2$ Q2.

$V = 2\pi \int y f(y)\, dy$

$\int x g(x)\, dx$

a) Rev. x-axis

$$V = 2\pi \int_0^2 y(y^2+2)\, dy$$

$$= 2\pi \int_0^2 (y^3 + 2y)\, dy$$

$$= 2\pi \left(\frac{1}{4}y^4 + y^2 \right)\Big|_0^2$$

$$= 2\pi (4 + 4)$$

$$= 16\pi \text{ unit}^3$$

b) Rev. $y = -2$ ← $y + 2 \geq 0$

$$V = 2\pi \int_0^2 (y+2)(y^2+2)\, dy$$

$$= 2\pi \int_0^2 (y^3 + 2y + 2y^2 + 4)\, dy$$

$$= 2\pi \left(\frac{1}{4}y^4 + y^2 + \frac{2}{3}y^3 + 4y \right)\Big|_0^2$$

$$= 2\pi \left(4 + 4 + \frac{16}{3} + 8 \right)$$

$16\left(1 + \frac{1}{3}\right)$

$$= \frac{128\pi}{3} \text{ unit}^3$$

Ex $y = 2x - x^2$ $y = x$ $[0, 1]$

$2x - x^2 = x \Rightarrow x - x^2 = 0 \Rightarrow x = 0, 1$

Rev. x-axis

Washer. $V = \pi \int_a^b (R^2 - r^2)\,dx$

$V = \pi \int_0^1 ((2x - x^2)^2 - x^2)\,dx$

$= \pi \int_0^1 (4x^2 - 4x^3 + x^4 - x^2)\,dx$

$= \pi \int_0^1 (3x^2 - 4x^3 + x^4)\,dx$

$= \pi \left(x^3 - x^4 + \frac{1}{5}x^5 \right)\Big|_0^1$

$= \frac{\pi}{5}$ unit^3

shell method

$V = 2\pi \int y\,(R^2 - r^2)\,dy$

$R(y)$

$f(x) = x^n$ $g(x)$ | x-axis Washer dx | y-axis shell dx

$f(y)$ $g(y)$ | x-axis shell dy | y-axis Washer dy