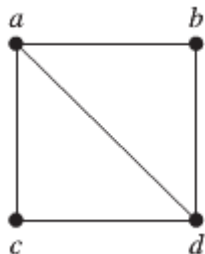


## ***SOLUTION*** Section 4.7 – Representing Graphs and Graph Isomorphism

### ***Exercise***

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



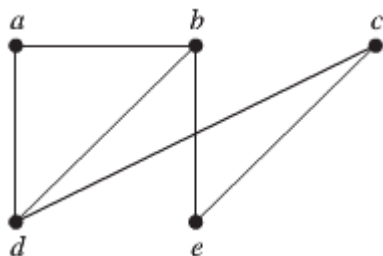
### ***Solution***

<i>Vertex</i>	<i>Adjacent Vertices</i>
<i>a</i>	<i>b, c</i>
<i>b</i>	<i>a, d</i>
<i>c</i>	<i>a, d</i>
<i>d</i>	<i>a, b, c</i>

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

### ***Exercise***

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



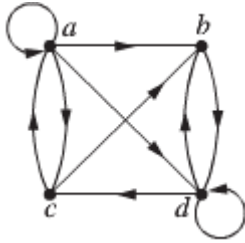
### ***Solution***

<i>Vertex</i>	<i>Adjacent Vertices</i>
<i>a</i>	<i>b, d</i>
<i>b</i>	<i>a, d, e</i>
<i>c</i>	<i>d, e</i>
<i>d</i>	<i>a, b, c</i>
<i>e</i>	<i>b, c</i>

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

### Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



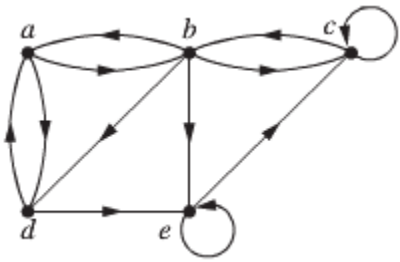
### Solution

<i>Initial Vertex</i>	<i>Terminal Vertices</i>
<i>a</i>	<i>a, b, c, d</i>
<i>b</i>	<i>d</i>
<i>c</i>	<i>a, b</i>
<i>d</i>	<i>b, c, d</i>

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

### Exercise

Use the adjacency list to represent the given graph, then represent with an adjacency matrix



### Solution

<i>Initial Vertex</i>	<i>Terminal Vertices</i>
<i>a</i>	<i>b, d</i>
<i>b</i>	<i>a, c, d, e</i>
<i>c</i>	<i>b, c</i>
<i>d</i>	<i>a, e</i>
<i>e</i>	<i>c, e</i>

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

### Exercise

Draw a graph with the given adjacency

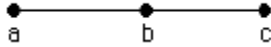
a) 
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

b) 
$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

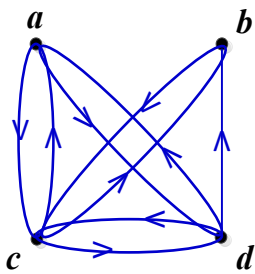
c) 
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

### Solution

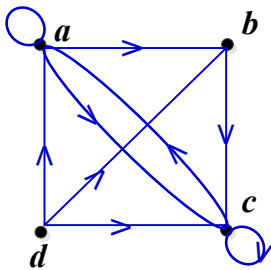
a)



b)



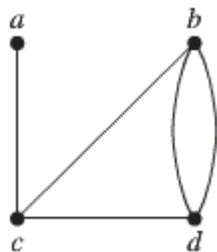
c)



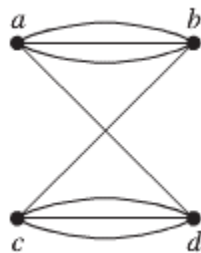
### Exercise

Represent the given graph using adjacency matrix

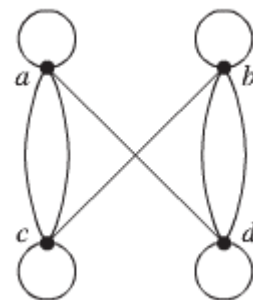
a)



b)



c)



### Solution

$$a) \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

$$b) \begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

$$c) \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

### ***Exercise***

Draw an undirected graph represented by the given adjacency

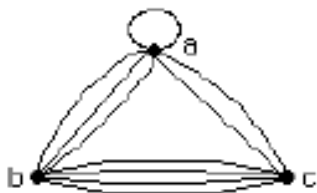
$$a) \begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix}$$

$$b) \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

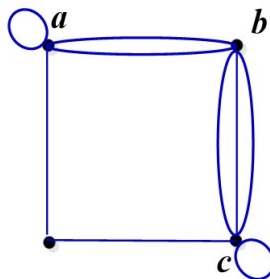
$$c) \begin{bmatrix} 0 & 1 & 3 & 0 & 4 \\ 1 & 2 & 1 & 3 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

### **Solution**

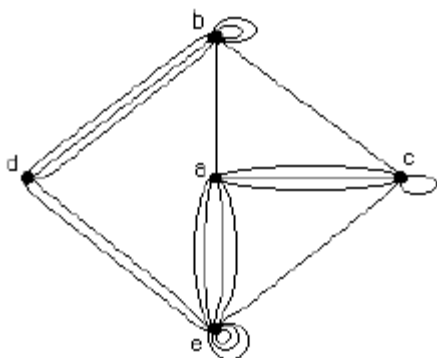
*a)*



*b)*



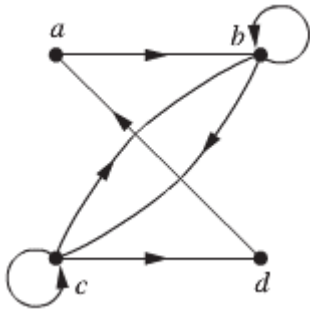
*c)*



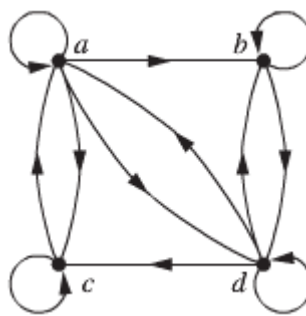
### Exercise

Find the adjacency matrix of the given directed multigraph with respect to the vertices listed in alphabetic order.

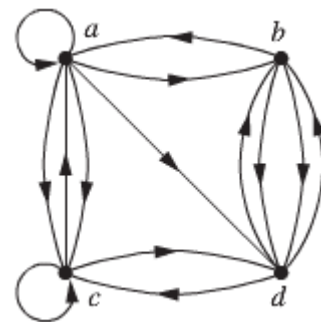
a)



b)



c)



### Solution

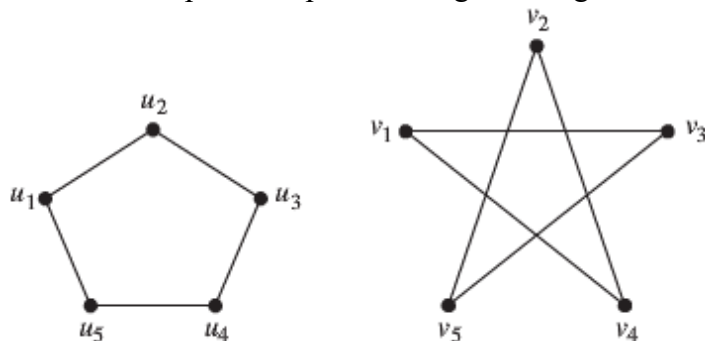
$$a) \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$b) \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$c) \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

### Exercise

Determine whether the given pair of graphs is isomorphic. Exhibit an isorphism or provide a rigorous argument that none exists.



### Solution

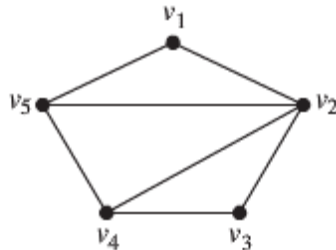
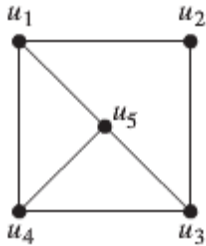
Both graphs have 5 vertices and 5 edges.

However, each vertex in the second graph has of degree 2, whereas the first does not.

### Exercise

Determine whether the given pair of graphs is isomorphic.

Exhibit an isomorphism or provide a rigorous argument that none exists.



### Solution

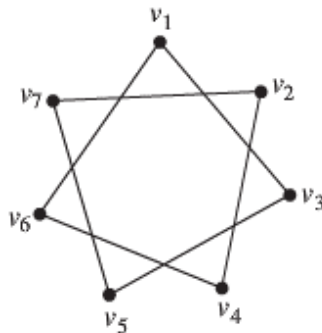
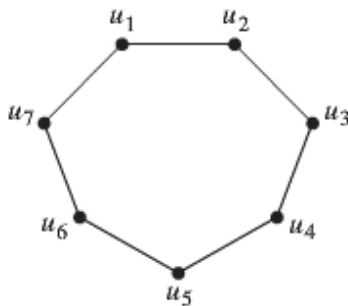
Both graphs have 5 vertices and 7 edges.

However, the second graph has a vertex of degree 4, whereas the first does not.

### Exercise

Determine whether the given pair of graphs is isomorphic.

Exhibit an isomorphism or provide a rigorous argument that none exists.



### Solution

Both graphs have 7 vertices and 7 edges.

$$f(u_1) = v_1$$

$$f(u_2) = v_3$$

$$f(u_3) = v_5$$

$$f(u_4) = v_7$$

$$f(u_5) = v_2$$

$$f(u_6) = v_4$$

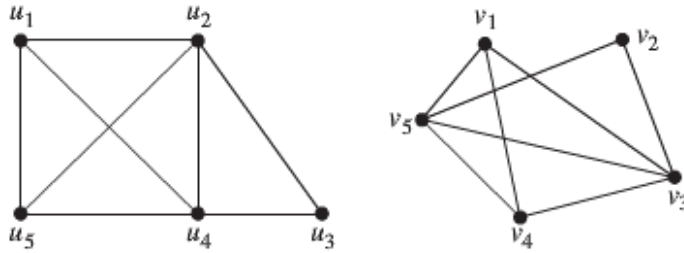
and  $f(u_7) = v_6$

$\therefore$  The graphs are isomorphic.

### ***Exercise***

Determine whether the given pair of graphs is isomorphic.

Exhibit an isomorphism or provide a rigorous argument that none exists.



### **Solution**

Both graphs have 5 vertices and 8 edges.

$$f(u_1) = v_1$$

$$f(u_2) = v_3$$

$$f(u_3) = v_2$$

$$f(u_4) = v_5$$

$$\text{and } f(u_5) = v_4$$

$\therefore$  The graphs are isomorphic.