Math 1325 - Calculus with Application

Exam 2
Review

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1. Find the derivative.

a) $f(t)=\sqrt{t-4}$

b) $f(x)=\dfrac{1}{x+2}$

c) $g(x)=3x^2-5x+7$

d) $g(t)=\left(4x^2+3x\right)^2$

e) $y=\sqrt{x}(x+2)^2$

f) $y=\dfrac{2}{\sqrt[3]{x^2}}$

g) $g(t)=\dfrac{t^2-1}{t+4}$

h) $y=\left(x^5-3x\right)\left(\dfrac{1}{x^2}\right)$

i) $Q(w)=\dfrac{w+1}{\sqrt{2w+3}}$

j) $f(x)=\sqrt{x^2-3x+5}$

k) $R(x)=\dfrac{x^3-2x^2+3}{\sqrt{x-2}}$

l) $y=\left(\dfrac{x+3}{x-4}\right)(x+5)$

m) $f(x)=8x^{-2}-3x^3+11x$

n) $f(x)=3x^4-3x^3+6x^2-x+5$

o) $f(x)=(x+3)\left(1-\dfrac{2}{x-3}\right)$

p) $f(x)=\left(5x^3+4\right)\left(3x^7-5\right)$

q) $y=\dfrac{2x-7}{3x-2}$

2. Find the derivative:

a) $y=4e^{x^2}$

b) $y=\sqrt[3]{2e^{3x}}$

c) $y=x^2e^x$

d) $y=e^{x^2+1}\sqrt{5x+2}$

e) $y=\dfrac{10}{1-2x}$

f) $y=\ln 3x^2$

g) $y=\ln\dfrac{x(x-1)}{x-2}$

h) $y=\dfrac{x^2}{\ln x}$

i) $y=\ln\left(e^{2x}\sqrt{e^{2x}-1}\right)$

j) $y=\ln\left[\dfrac{x^2(x+1)^3}{(x+3)^{1/2}}\right]$

k) $y=\dfrac{\ln x}{e^{2x}}$

l) $y=\dfrac{6e^x}{2e^x+1}$

m) $y=\ln x^4-5e^x+2x^3$

3. A brick becomes dislodged from the top of the Empire State Building at a height of 1250 feet. The position function is $h(t)=-16t^2+1250$ where $h(t)$ the bricks distance above the sidewalk in feet after *t* seconds. Find the velocity functions. What is the velocity of the brick after 5 seconds?

4. Suppose the quantity demanded weekly of the Super Titan radial tires is related to its unit price by the equation $p+x^2=196$ where *p* is measured in dollars and *x* is measured in units of a thousand. How fast is the quantity demanded changing when *x* = 4, *p* = 180, and the price/tire is increasing at the rate of $2/week?

5. Carlos is blowing air into a soap bubble at the rate of 8 cm^3/*sec*. Assume that the bubble is spherical $\left(V = \frac{4}{3}\pi r^3 \right)$. How fast is the radius changing at the instant of time when the radius is 10 cm?

6. The position function for an amusement ride moving on a horizontal track is $x = -0.01t^4 + 0.3t^3 + 0.4t^2 + 12t$ where x is in feet and t is in seconds. What is the velocity at 20 seconds?

7. Lynbrook West, an apartment complex, has 100 two-bedroom units. The monthly profit (in dollars) realized from renting x apartments is

$$P(x) = -10x^2 + 1760x - 50{,}000$$

Compute the marginal profit when $x = 50$.

8. The population of Americans age 55 and older as a percent of the total population is approximated by the function

$$f(t) = 10.72(0.9t + 10)^{0.3} \quad (0 \le t \le 20)$$

where t is measured in years, with t = 0 corresponding to the year 2000. At what rate will the percent of Americans age 55 and older be changing in 2010?

Solutions:

1.

a) $\dfrac{1}{2\sqrt{t-4}}$

b) $f' = -\dfrac{1}{(x+2)^2}$

c) $g'(x) = 3x-5$

d) $g'(t) = 64x^3 + 72x^2 + 18x$

e) $\dfrac{dy}{dx} = \dfrac{5x^2+12x+4}{2\sqrt{x}}$

f) $y' = \dfrac{-4}{3\sqrt[3]{x^5}}$

g) $g'(t) = \dfrac{t^2+8t+1}{(t+4)^2}$

h) $y' = 3x^2 + \dfrac{3}{x^2}$

i) $Q'(w) = \dfrac{w+2}{(2w+3)^{3/2}}$

j) $f'(x) = \dfrac{2x-3}{2\sqrt{x^2-3x+5}}$

k) $R'(x) = \dfrac{5x^3-18x^2+16x-3}{2(x-2)^{3/2}}$

l) $y' = \dfrac{x^2-8x-47}{(x-4)^2}$

m) $f'(x) = 16x^{-3} - 9x^2 + 11$

n) $f'(x) = 12x^3 - 9x^2 + 12x - 1$

o) $f'(x) = \dfrac{x^2-6x+21}{(x-3)^2}$

p) $f(x) = 150x^9 + 84x^6 - 75x^2$

q) $y' = \dfrac{17}{(3x-2)^2}$

2.

a) $y' = 8xe^{x^2}$

b) $y' = \sqrt[3]{2}\, e^x$

c) $y' = x(x+2)e^x$

d) $\dfrac{e^{x^2+1}\left(20x^2+8x+5\right)}{2\sqrt{5x+2}}$

e) $y' = \dfrac{20}{(1-2x)^2}$

f) $y' = \dfrac{2}{x}$

g) $y' = \dfrac{1}{x} + \dfrac{1}{x-1} - \dfrac{1}{x-2}$

h) $y' = \dfrac{2x\ln x - x}{(\ln x)^2}$

i) $y' = 2 + \dfrac{e^{2x}}{e^{2x}-1}$

j) $y' = \dfrac{2}{x} + \dfrac{3}{x+1} - \dfrac{1}{2(x+3)}$

k) $y' = \dfrac{1-2x\ln x}{xe^{2x}}$

l) $y = \dfrac{6e^x}{\left(2e^x+1\right)^2}$

m) $y' = \dfrac{4}{x} - 5e^x + 6x^2$

3. $v(t) = -32t$ $\qquad v(5) = -160\ ft/\sec$

4. $\frac{dx}{dt} = -250\ units\,/\,week$

5. $\frac{1}{50\pi} \approx .0064\ cm/\sec$

6. $v(t) = -0.04t^3 + 0.9t^2 + 0.8t + 12$
$v(20) = 68\ \ ft/\sec$

7. $P'(x) = -20x + 1760$
$P'(50) = \$760$

8. $f'(t) = 2.8944(0.9t + 10)^{-.7}$
$f'(10) = .3685$

Find the Derivatives of $y=\sqrt[3]{2e^{3x}}$

Solution

$$y=\left(2e^{3x}\right)^{1/3} \qquad \textit{Exponential Form}$$

$$=(2)^{1/3}\left(e^{3x}\right)^{1/3}$$

$$=\sqrt[3]{2}\left(e^{3x\frac{1}{3}}\right)$$

$$=\sqrt[3]{2}e^{x}$$

$$\boxed{y'=\sqrt[3]{2}\ e^{x}}$$

Find the Derivatives of $y=e^{x^2+1}\sqrt{5x+2}$

Solution

$$f=e^{x^2+1} \qquad U=x^2+1\Rightarrow U'=2x \qquad f'=2xe^{x^2+1}$$

$$g=\sqrt{5x+2}=(5x+2)^{1/2} \qquad U=5x+2\Rightarrow U'=5 \qquad g'=\frac{1}{2}5(5x+2)^{-1/2}$$

$$y'=2xe^{x^2+1}\sqrt{5x+2}+\frac{5}{2}e^{x^2+1}(5x+2)^{-1/2}$$

$$=\left[2xe^{x^2+1}\sqrt{5x+2}+\frac{5}{2}e^{x^2+1}(5x+2)^{-1/2}\right]\frac{2(5x+2)^{1/2}}{2(5x+2)^{1/2}}$$

$$=\left[2xe^{x^2+1}\sqrt{5x+2}(2)(5x+2)^{1/2}+\frac{5}{2}e^{x^2+1}(5x+2)^{-1/2}(2)(5x+2)^{1/2}\right]\frac{1}{2(5x+2)^{1/2}}$$

$$=\left[4xe^{x^2+1}(5x+2)+5e^{x^2+1}\right]\frac{1}{2\sqrt{5x+2}}$$

$$=\frac{e^{x^2+1}\left[4x(5x+2)+5\right]}{2\sqrt{5x+2}}$$

$$=\frac{e^{x^2+1}\left[20x^2+8x+5\right]}{2\sqrt{5x+2}}$$

Find the Derivatives of $y=\dfrac{10}{1-2x}$

Solution

$$f=10 \qquad f'=0$$
$$g=1-2x \qquad g'=-2$$

$$y'=\frac{0(1-2x)-(-2)(10)}{(1-2x)^2}$$ ***Quotient Rule***

$$y'=\frac{20}{(1-2x)^2}$$

Find the Derivatives of $y=\ln\dfrac{x(x-1)}{x-2}$

Solution

$$y=\ln\left[x(x-1)\right]-\ln(x-2)$$ ***Quotient Rule***

$$y=\ln x+\ln(x-1)-\ln(x-2)$$ ***Product Rule***

$$(\ln x)'=\frac{1}{x} \quad (\ln(x-1))'=\frac{1}{x-1} \quad (\ln(x-2))'=\frac{1}{x-2}$$

$$y'=\frac{1}{x}+\frac{1}{x-1}-\frac{1}{x-2}$$

Find the Derivatives of $y=\ln\left(e^{2x}\sqrt{e^{2x}-1}\right)$

Solution

$$y=\ln\left(e^{2x}\right)+\ln\left(e^{2x}-1\right)^{1/2}$$

$$=2x\ln(e)+\tfrac{1}{2}\ln\left(e^{2x}-1\right)$$ $\ln(e)=1$

$$=2x+\tfrac{1}{2}\ln\left(e^{2x}-1\right)$$

$$\left(\ln\left(e^{2x}-1\right)\right)'=\frac{\left(e^{2x}-1\right)'}{e^{2x}-1}$$

$$y'=2+\frac{1}{2}\frac{2e^{2x}}{e^{2x}-1}$$

$$=2+\frac{e^{2x}}{e^{2x}-1}$$

Find the Derivatives of $y = \ln\left[\dfrac{x^2(x+1)^3}{(x+3)^{1/2}}\right]$

Solution

$$y = \ln\left[x^2(x+1)^3\right] - \ln(x+3)^{1/2} \qquad \textbf{\textit{Quotient Rule}}$$

$$= \ln x^2 + \ln(x+1)^3 - \ln(x+3)^{1/2} \qquad \textbf{\textit{Product Rule}}$$

$$= 2\ln x + 3\ln(x+1) - \tfrac{1}{2}\ln(x+3) \qquad \textbf{\textit{Power Rule}}$$

$$y' = \frac{2}{x} + \frac{3}{x+1} - \frac{1}{2(x+3)}$$

Find the Derivatives of $y = \dfrac{\ln x}{e^{2x}}$

Solution

$$f = \ln x \qquad f' = \frac{1}{x}$$

$$g = e^{2x} \qquad g' = 2e^{2x}$$

$$y' = \frac{e^{2x}\left(\frac{1}{x}\right) - \ln x(2e^{2x})}{e^{4x}}$$

$$= \frac{e^{2x}\left(\frac{1}{x}\right) - \ln x(2e^{2x})}{e^{4x}}\,\frac{x}{x}$$

$$= \frac{e^{2x} - 2x\ln x(e^{2x})}{e^{4x}}$$

$$= \frac{e^{2x}\left(1 - 2x\ln x\right)}{e^{4x}}$$