Solution Section 1.4 – Inverse, Exponential & Logarithmic Functions

Exercise

Determine whether the function is one-to-one: $f(x)=3x-7$

Solution

$f(a)=f(b)$

$3a-7=3b-7$

$3a=3b-7+7$

$3a=3b$ *Divide both sides by 3*

$a=b$

$\therefore$ The function is one-to-one

Exercise

Determine whether the function is one-to-one: $f(x)=x^2-9$

Solution

$1\neq -1$ $1^2-9\neq(-1)^2-9$ $-8=-8\rightarrow$ Contradict the definition	$f(a)=f(b)$ $a^2-9=b^2-9$ $a^2=b^2$ $a=\pm b$

$\therefore$ The function is ***not*** one-to-one

Exercise

Determine whether the function is one-to-one: $f(x)=\sqrt{x}$

Solution

$f(a)=f(b)$

$\sqrt{a}=\sqrt{b}$

$\left(\sqrt{a}\right)^2=\left(\sqrt{b}\right)^2$ *Square both sides*

$a=b$

$\therefore$ The function is one-to-one

Exercise

Determine whether the function is one-to-one: $f(x)=\sqrt[3]{x}$

Solution

$f(a)=f(b)$

$\sqrt[3]{a}=\sqrt[3]{b}$

$\left(\sqrt[3]{a}\right)^3=\left(\sqrt[3]{b}\right)^3$ ***cube both sides***

$a=b$

$\therefore$ The function is one-to-one

Exercise

Determine whether the function is one-to-one: $f(x)=|x|$

Solution

$1\neq -1$

$|1|\neq|-1|$

$1\neq 1\ (\mathit{false})$

$\therefore$ The function is ***not*** one-to-one

Exercise

Determine whether the function is one-to-one $f(x)=\dfrac{2}{x+3}$

Solution

$f(a)=f(b)$

$\dfrac{2}{a+3}=\dfrac{2}{b+3}$

$2(b+3)=2(a+3)$

$b+3=a+3$

$a=b$

$\therefore f$ is one-to-one

Exercise

Determine whether the function is one-to-one $f(x)=(x-2)^3$

Solution

$f(a)=f(b)$

$(a-2)^3 = (b-2)^3$

$\left[(a-2)^3\right]^{1/3} = \left[(b-2)^3\right]^{1/3}$

$a-2 = b-2$ *Add 2 on both sides*

$a = b$

$\therefore$ Function is one-to-one

Exercise

Determine whether the function is one-to-one $y = x^2 + 2$

Solution

$f(a) = f(b)$

$a^2 + 2 = b^2 + 2$ *Subtract 2*

$a^2 = b^2$

$a = \pm\sqrt{b^2}$

$\therefore$ Function is ***not*** a one-to-one

The inverse function doesn't exist.

Exercise

Determine whether the function is one-to-one $f(x) = \dfrac{x+1}{x-3}$

Solution

$f(a) = f(b)$

$\dfrac{a+1}{a-3} = \dfrac{b+1}{b-3}$ ***Cross multiplication***

$(a+1)(b-3) = (b+1)(a-3)$

$ab - 3a + b - 3 = ab - 3b + a - 3$

$-4a = -4b$ ***Divide by -4***

$a = b$

$\therefore$ Function is *one*-to-*one*

Exercise

Given the function $f(x)=(x+8)^3$

a) Find $f^{-1}(x)$

b) Graph f and f^{-1} in the same rectangular coordinate system

c) Find the domain and the range of f and f^{-1}

Solution

a) $y=(x+8)^3$ **Replace f(*x*) with *y***

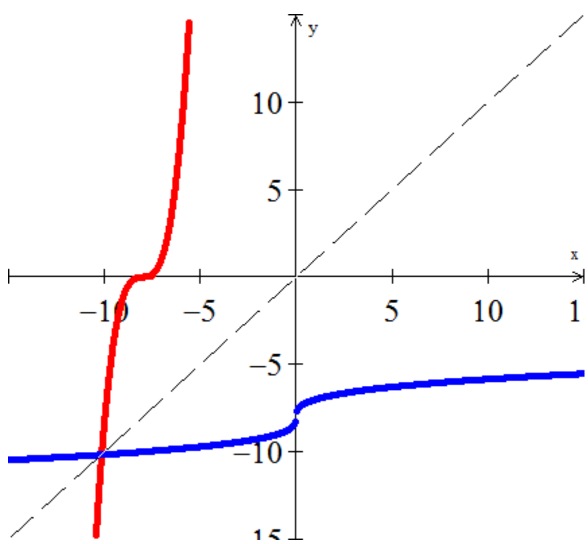
$x=(y+8)^3$ **Interchange *x* and *y***

$(x)^{1/3}=\left((y+8)^3\right)^{1/3}$

$x^{1/3}=y+8$ ***Subtract 8 from both sides.***

$\underline{f^{-1}(x)=x^{1/3}-8}$

b)



c) Domain of f = Range of f^{-1}: $(-\infty,\infty)$

Range of f = Domain of f^{-1}: $(-\infty,\infty)$

Exercise

For the given function $f(x)=\dfrac{2x}{x-1}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$$\frac{2a}{a-1}=\frac{2b}{b-1}$$

$$2ab-2a=2ab-2b$$

$$-2a=-2b$$

$$\boxed{a=b} \quad \checkmark$$

$\therefore$ $f(x)$ is one-to-one function.

b) $y=\dfrac{2x}{x-1}$

$$x=\frac{2y}{y-1}$$

$$xy-x=2y$$

$$(x-2)y=x$$

$$y=\boxed{\frac{x}{x-2}=f^{-1}(x)}$$

c) Domain of $f^{-1}(x)$ = Range of $f(x)$: $\boxed{\mathbb{R}-\{1\}}$

Range of $f^{-1}(x)$ = Domain of $f(x)$: $\boxed{\mathbb{R}-\{2\}}$

Exercise

For the given function $f(x)=\dfrac{x}{x-2}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$$\frac{a}{a-2}=\frac{b}{b-2}$$

$ab-2a=ab-2b$
$-2a=-2b$
$\boxed{a=b}$ ✓

$\therefore\ f(x)$ is one-to-one function.

b) $y=\dfrac{x}{x-2}$

$x=\dfrac{y}{y-2}$

$xy-2x=y$

$(x-1)y=2x$

$\boxed{f^{-1}(x)=\dfrac{2x}{x-1}}$

c) Domain of $f^{-1}(x)$ = Range of $f(x)$: $\boxed{\mathbb{R}-\{2\}}$

Range of $f^{-1}(x)$ = Domain of $f(x)$: $\boxed{\mathbb{R}-\{1\}}$

Exercise

For the given function $f(x)=\dfrac{x+1}{x-1}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$\dfrac{a+1}{a-1}=\dfrac{b+1}{b-1}$
$ab-a+b-1=ab-b+a-1$
$-2a=-2b$
$\boxed{a=b}$ ✓

$\therefore\ f(x)$ is one-to-one function.

b) $y=\dfrac{x+1}{x-1}$

$x=\dfrac{y+1}{y-1}$

$xy-x=y+1$

$(x-1)y=x+1$

$\boxed{f^{-1}(x) = \frac{x+1}{x-1}}$

c) Domain of $f^{-1}(x)$ = Range of $f(x)$: $\boxed{\mathbb{R} - \{1\}}$

Range of $f^{-1}(x)$ = Domain of $f(x)$: $\boxed{\mathbb{R} - \{1\}}$

Exercise $f(x) = \frac{2x+1}{x+3}$

For the given function

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a) = f(b)$

$$\frac{2a+1}{a+3} = \frac{2b+1}{b+3}$$

$$2ab + 6a + b + 3 = 2ab + 6b + a + 3$$

$$5a = 5b$$

$$\boxed{a = b} \quad \checkmark$$

$\therefore$ $f(x)$ is one-to-one function.

b) $y = \frac{2x+1}{x+3}$

$$x = \frac{2y+1}{y+3}$$

$$xy + 3x = 2y + 1$$

$$(x-2)y = -3x + 1$$

$$\boxed{f^{-1}(x) = \frac{-3x+1}{x-2}}$$

c) Domain of $f^{-1}(x)$ = Range of $f(x)$: $\boxed{\mathbb{R} - \{-3\}}$

Range of $f^{-1}(x)$ = Domain of $f(x)$: $\boxed{\mathbb{R} - \{2\}}$

Exercise

For the given function $f(x)=\dfrac{3x-1}{x-2}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$$\frac{3a-1}{a-2}=\frac{3b-1}{b-2}$$
$$3ab-6a-b+2=3ab-6b-a+2$$
$$-5a=-5b$$
$$\underline{a=b} \quad \checkmark$$

$\therefore$ $f(x)$ is one-to-one function.

b) $y=\dfrac{3x-1}{x-2}$

$$x=\frac{3y-1}{y-2}$$
$$xy-2x=3y-1$$
$$(x-3)y=2x-1$$
$$\underline{f^{-1}(x)=\frac{2x-1}{x-3}}$$

c) Domain of $f^{-1}(x)$ = Range of $f(x)$: $\underline{\mathbb{R}-\{2\}}$

Range of $f^{-1}(x)$ = Domain of $f(x)$: $\underline{\mathbb{R}-\{3\}}$

Exercise

For the given function $f(x)=\dfrac{3x-2}{x+4}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$$\frac{3a-2}{a+4}=\frac{3b-2}{b+4}$$
$$3ab+12a-2b-8=3ab+12b-2a-8$$

$14a = 14b$
$\boxed{a = b}$ ✓

$\therefore\ f(x)$ is one-to-one function.

b) $y = \dfrac{3x-2}{x+4}$

$x = \dfrac{3y-2}{y+4}$

$xy + 4x = 3y - 2$

$(x-3)y = -4x - 2$

$\boxed{f^{-1}(x) = \dfrac{-4x-2}{x-3}}$

c) Domain of $f^{-1}(x)$ = Range of $f(x)$: $\boxed{\mathbb{R} - \{-4\}}$

Range of $f^{-1}(x)$ = Domain of $f(x)$: $\boxed{\mathbb{R} - \{3\}}$

Exercise

For the given function $f(x) = \dfrac{-3x-2}{x+4}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a) = f(b)$

$\dfrac{-3a-2}{a+4} = \dfrac{-3b-2}{b+4}$

$-3ab - 12a - 2b - 8 = -3ab - 12b - 2a - 8$

$-10a = -10b$

$\boxed{a = b}$ ✓

$\therefore\ f(x)$ is one-to-one function.

b) $y = \dfrac{-3x-2}{x+4}$

$x = \dfrac{-3y-2}{y+4}$

$xy + 4x = -3y - 2$

$(x+3)y = -4x - 2$

$\boxed{f^{-1}(x) = \dfrac{-4x-2}{x+3}}$

c) Domain of $f^{-1}(x)$ = Range of $f(x)$: $\boxed{\mathbb{R}-\{-4\}}$

Range of $f^{-1}(x)$ = Domain of $f(x)$: $\boxed{\mathbb{R}-\{-3\}}$

Exercise

For the given function $f(x)=\sqrt{x-1} \quad x\ge 1$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$\sqrt{a-1}=\sqrt{b-1}$

$\left(\sqrt{a-1}\right)^2=\left(\sqrt{b-1}\right)^2$

$a-1=b-1$

$\boxed{a=b}$ ✓

$\therefore$ $f(x)$ is one-to-one function.

b) $y=\sqrt{x-1}$

$x=\sqrt{y-1}$

$x^2=y-1$

$y=x^2+1$

$\boxed{f^{-1}(x)=x^2+1 \quad x\ge 0}$

c) Domain of $f(x)$ = Range of $f^{-1}(x)$: $\boxed{[1,\ \infty)}$

Range of $f(x)$ = Domain of $f^{-1}(x)$: $\boxed{[0,\ \infty)}$

Exercise

For the given function $f(x)=\sqrt{2-x} \quad x\le 2$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a) = f(b)$

$\sqrt{2-a} = \sqrt{2-b}$

$\left(\sqrt{2-a}\right)^2 = \left(\sqrt{2-b}\right)^2$

$2-a = 2-b$

$\boxed{a = b}$ ✓

$\therefore$ $f(x)$ is one-to-one function.

b) $y = \sqrt{2-x}$

$x = \sqrt{2-y}$

$x^2 = 2-y$

$y = 2-x^2$

$\boxed{f^{-1}(x) = 2-x^2 \quad x \ge 0}$

c) Domain of $f(x)$ = Range of $f^{-1}(x)$: $\boxed{(-\infty,\ 2]}$

Range of $f(x)$ = Domain of $f^{-1}(x)$: $\boxed{[0,\ \infty)}$

Exercise

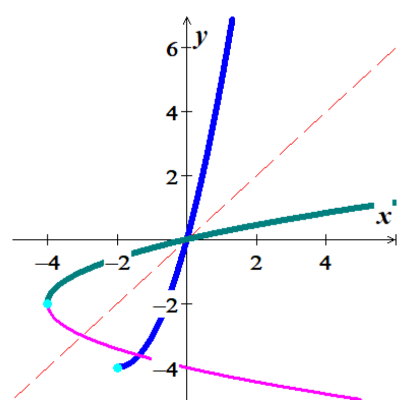
For the given function $f(x) = x^2 + 4x \quad x \ge -2$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution



$x_{vertex} = -\frac{4}{2}$

$\boxed{= -2}$

$f(-2) = 4 - 8$

$\boxed{= -4}$

$Vertex = (-2,\ -4)$

a) Since, $f(x)$ is a restricted function with $x \ge -2$.

$x = -2$ is the line symmetry, therefore; $f(x)$ is one-to-one function.

b) $y = x^2 + 4x$

$x = y^2 + 4y$

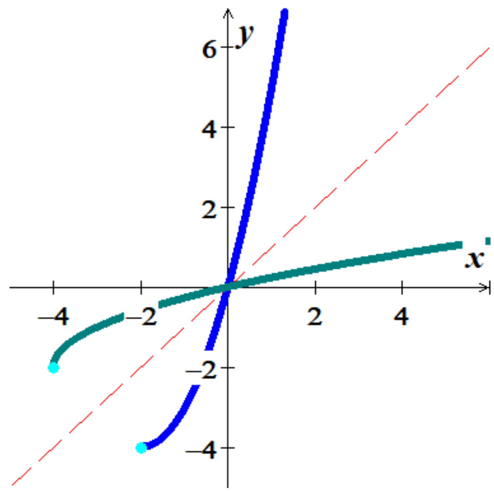
$y^2 + 4y - x = 0$

$y = \dfrac{-4 \pm \sqrt{16+4x}}{2}$

$= \dfrac{-4 \pm 2\sqrt{4+x}}{2}$

$= -2 + \sqrt{x+4}$

$\boxed{f^{-1}(x) = -2 + \sqrt{x+4} \quad x \ge 0}$



c) Domain of $f(x)$ = Range of $f^{-1}(x)$: $\boxed{[-2, \infty)}$

Range of $f(x)$ = Domain of $f^{-1}(x)$: $\boxed{[-4, \infty)}$

Exercise

For the given function $f(x) = 3x + 5$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a) = f(b)$

$3a + 5 = 3b + 5$

$3a = 3b$

$a = b$

$\therefore$ $f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y = 3x + 5$

$x = 3y + 5$ ***Interchange x and y***

$x - 5 = 3y$ ***Solve for y***

$\frac{x-5}{3} = y$

$\boxed{f^{-1}(x) = \dfrac{x-5}{3}}$

c) Domain of f^{-1} = Range of f: $\boxed{\mathbb{R}}$

Range of f^{-1} = Domain of f: $\boxed{\mathbb{R}}$

Exercise

For the given function $f(x)=\dfrac{1}{3x-2}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$$\frac{1}{3a-2}=\frac{1}{3b-2}$$

$$3b-2=3a-2$$

$$3b=3a$$

$$a=b$$

$\therefore f(x)$ is **1–1** **&** $f^{-1}(x)$ exists

b) $y=\dfrac{1}{3x-2}$

$x=\dfrac{1}{3y-2}$ *Interchange x and y*

$x(3y-2)=1$ *Solve for y*

$$3xy-2x=1$$

$$3xy=1+2x$$

$$f^{-1}(x)=\frac{1+2x}{3x}$$

c) Domain of f^{-1} = Range of f: $\mathbb{R}-\left\{\frac{2}{3}\right\}$

Range of f^{-1} = Domain of f: $\mathbb{R}-\{0\}$

Exercise

For the given function $f(x)=\dfrac{3x+2}{2x-5}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$$\frac{3a+2}{2a-5}=\frac{3b+2}{2b-5}$$

$$6ab-15a+4b-10=6ab-15b+4a-10$$

$19a = 19b$

$a = b$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y = \dfrac{3x+2}{2x-5}$

$x = \dfrac{3y+2}{2y-5}$ ***Interchange x and y***

$2xy - 5x = 2y + 2$ ***Solve for y***

$(2x-3)\,y = 5x + 2$

$\boxed{f^{-1}(x) = \dfrac{5x+2}{2x-3}}$

c) Domain of f^{-1} = Range of f: $\boxed{\mathbb{R} - \left\{\dfrac{5}{2}\right\}}$

Range of f^{-1} = Domain of f: $\boxed{\mathbb{R} - \left\{\dfrac{3}{2}\right\}}$

Exercise

For the given function $f(x) = \dfrac{4x}{x-2}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a) = f(b)$

$\dfrac{4a}{a-2} = \dfrac{4b}{b-2}$

$4ab - 8a = 4ab - 8b$

$-8a = -8b$

$a = b$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y = \dfrac{4x}{x-2}$

$x = \dfrac{4y}{y-2}$

$xy - 2x = 4y$

$(x-4)\,y = 2x$

$\boxed{f^{-1}(x) = \dfrac{2x}{x-4}}$

c) Domain of f^{-1} = Range of f : $\boxed{\mathbb{R}-\{2\}}$

Range of f^{-1} = Domain of f : $\boxed{\mathbb{R}-\{4\}}$

Exercise

For the given function $f(x)=2-3x^2; \quad x\le 0$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$2-3a^2=2-3b^2$

$-3a^2=-3b^2$

$a^2=b^2$

$a=b$ since $x\le 0$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y=2-3x^2$

$x=2-3y^2$

$3y^2=2-x$

$y^2=\dfrac{2-x}{3}$

$\boxed{f^{-1}(x)=-\sqrt{\dfrac{2-x}{3}}}$ ***Since x < 0***

c) Domain of f^{-1} = Range of f : $\boxed{\mathbb{R}}$

Range of f^{-1} = Domain of f : $\boxed{\mathbb{R}}$

Exercise

For the given function $f(x)=2x^3-5$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$2a^3 - 5 = 2b^3 - 5$

$a^3 = b^3$

$a = b$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y = 2x^3 - 5$

$y + 5 = 2x^3$

$\dfrac{y+5}{2} = x^3$

$\underline{f^{-1}(x) = \sqrt[3]{\dfrac{x+5}{2}}}\,\big|$

c) Domain of f^{-1} = Range of f : $\underline{\mathbb{R}}|$

Range of f^{-1} = Domain of f : $\underline{\mathbb{R}}|$

Exercise

For the given function $f(x) = \sqrt{3-x}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a) = f(b)$

$\left(\sqrt{3-a}\right)^2 = \left(\sqrt{3-b}\right)^2$

$3 - a = 3 - b$

$a = b$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y = \sqrt{3-x}$ $\qquad\qquad$ $\boldsymbol{y \ge 0}$

$y = \sqrt{3-x}$

$y^2 = 3 - x$

$x = 3 - y^2$ $\qquad\qquad$ $\boldsymbol{x \ge 0}$

$\underline{f^{-1}(x) = 3 - x^2}\,|$

c) Domain of f^{-1} = Range of f : $\underline{(-\infty,\ 3]}|$

Range of f^{-1} = Domain of f : $\underline{[0,\ \infty)}|$

Exercise

For the given function $f(x)=\sqrt[3]{x}+1$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$\sqrt[3]{a}+1=\sqrt[3]{b}+1$

$\left(\sqrt[3]{a}\right)^3=\left(\sqrt[3]{b}\right)^3$

$a=b$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y=\sqrt[3]{x}+1$

$y=\sqrt[3]{x}+1$

$y-1=\sqrt[3]{x}$

$(y-1)^3=x$

$\boxed{f^{-1}(x)=(x-1)^3}$

c) Domain of f^{-1} = Range of f: $\mathbb{R}$

Range of f^{-1} = Domain of f: $\mathbb{R}$

Exercise

For the given function $f(x)=\left(x^3+1\right)^5$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$\left(a^3+1\right)^5=\left(b^3+1\right)^5$

$a^3+1=b^3+1$

$a^3=b^3$

$a=b$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y = \left(x^3 + 1\right)^5$

$y = \left(x^3 + 1\right)^5$

$\sqrt[5]{y} = x^3 + 1$

$\sqrt[5]{y} - 1 = x^3$

$x = \sqrt[3]{\sqrt[5]{y} - 1}$

$\boxed{f^{-1}(x) = \sqrt[3]{\sqrt[5]{x} - 1}}$

c) Domain of f^{-1} = Range of f: $\mathbb{R}$

Range of f^{-1} = Domain of f: $\mathbb{R}$

Exercise

For the given function $f(x) = x^2 - 6x; \quad x \geq 3$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a) = f(b)$

$a^2 - 6a = b^2 - 6b$

$a^2 - b^2 = 6a - 6b$

$(a-b)(a+b) = 6(a-b)$

$a = b$

$\therefore$ $f(x)$ is **1–1** **&** $f^{-1}(x)$ exists

b) $y = x^2 - 6x$

$x^2 - 6x - y = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-y)}}{2(1)}$$

$$= \frac{6 \pm 4\sqrt{9+y}}{2}$$

$$= 3 \pm \sqrt{9+y}$$

Since $x \geq 3 \Rightarrow$ we can select $x = 3 + \sqrt{y+9}$

$\therefore$ $\boxed{f^{-1}(x) = 3 + \sqrt{x+9}}$

c) Domain of f^{-1} = Range of f: $\mathbb{R}:\ \geq 3$

Range of f^{-1} = Domain of f: ≥ -9

Exercise

For the given function $f(x)=(x-2)^3$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$(a-2)^3=(b-2)^3$

$a-2=b-2$

$a=b$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y=(x-2)^3$

$x=(y-2)^3$

$x^{1/3}=\left[(y-2)^3\right]^{1/3}$

$x^{1/3}=y-2$

$\sqrt[3]{x}+2=y$

$\therefore\ f^{-1}(x)=\sqrt[3]{x}+2$

c) Domain of f^{-1} = Range of f: $\mathbb{R}$

Range of f^{-1} = Domain of f: $\mathbb{R}$

Exercise

For the given function $f(x)=\dfrac{x+1}{x-3}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$$\frac{a+1}{a-3}=\frac{b+1}{b-3}$$
$$ab-3a+b-3=ab-3b+a-3$$
$$-4a=-4b$$
$$a=b$$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y=\dfrac{x+1}{x-3}$

$$x=\frac{y+1}{y-3}$$
$$x(y-3)=y+1$$
$$xy-3x=y+1$$
$$xy-y=3x+1$$
$$y(x-1)=3x+1$$
$$y=\boxed{\frac{3x+1}{x-1}=f^{-1}(x)}$$

c) Domain of f^{-1} = Range of f: $\boxed{\mathbb{R}-\{3\}}$

Range of f^{-1} = Domain of f: $\boxed{\mathbb{R}-\{1\}}$

Exercise

For the given function $f(x)=\dfrac{2x+1}{x-3}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

Solution

a) $f(a)=f(b)$

$$\frac{2a+1}{a-3}=\frac{2b+1}{b-3}$$
$$2ab-6a+b-3=2ab-6b+a-3$$
$$-7a=-7b$$
$$a=b$$

$\therefore\ f(x)$ is **1–1 &** $f^{-1}(x)$ exists

b) $y=\dfrac{2x+1}{x-3}$

$$x=\frac{2y+1}{y-3}$$
$$xy-3x=2y+1$$

$$y(x-2)=3x+1$$

$$\boxed{f^{-1}(x)=\frac{3x+1}{x-2}}$$

c) Domain of f^{-1} = Range of f: $\boxed{\mathbb{R}-\{3\}}$

Range of f^{-1} = Domain of f: $\boxed{\mathbb{R}-\{2\}}$

Exercise

Simplify the expression $\dfrac{\left(e^x+e^{-x}\right)\left(e^x+e^{-x}\right)-\left(e^x-e^{-x}\right)\left(e^x-e^{-x}\right)}{\left(e^x+e^{-x}\right)^2}$

Solution

$$\frac{\left(e^x+e^{-x}\right)\left(e^x+e^{-x}\right)-\left(e^x-e^{-x}\right)\left(e^x-e^{-x}\right)}{\left(e^x+e^{-x}\right)^2}=\frac{\left(e^x+e^{-x}\right)^2-\left(e^x-e^{-x}\right)^2}{\left(e^x+e^{-x}\right)^2}$$

$$=\frac{\left[\left(e^x+e^{-x}\right)-\left(e^x-e^{-x}\right)\right]\left[\left(e^x+e^{-x}\right)+\left(e^x-e^{-x}\right)\right]}{\left(e^x+e^{-x}\right)^2}$$

$$=\frac{\left(e^x+e^{-x}-e^x+e^{-x}\right)\left(e^x+e^{-x}+e^x-e^{-x}\right)}{\left(e^x+e^{-x}\right)^2}$$

$$=\frac{\left(2e^{-x}\right)\left(2e^x\right)}{\left(e^x+e^{-x}\right)^2} \qquad e^{-x}e^x=e^0=1$$

$$\boxed{=\frac{4}{\left(e^x+e^{-x}\right)^2}}$$

Exercise

Simplify the expression $\dfrac{\left(e^x-e^{-x}\right)^2-\left(e^x+e^{-x}\right)^2}{\left(e^x+e^{-x}\right)^2}$

Solution

$$\frac{\left(e^x-e^{-x}\right)^2-\left(e^x+e^{-x}\right)^2}{\left(e^x+e^{-x}\right)^2}=\frac{\left[\left(e^x-e^{-x}\right)-\left(e^x+e^{-x}\right)\right]\left[\left(e^x-e^{-x}\right)+\left(e^x+e^{-x}\right)\right]}{\left(e^x+e^{-x}\right)^2}$$

$$= \frac{\left(e^{x} - e^{-x} - e^{x} - e^{-x}\right)\left(e^{x} - e^{-x} + e^{x} + e^{-x}\right)}{\left(e^{x} + e^{-x}\right)^{2}}$$

$$= \frac{\left(-2e^{-x}\right)\left(2e^{x}\right)}{\left(e^{x} + e^{-x}\right)^{2}}$$

$$= \frac{-4}{\left(e^{x} + e^{-x}\right)^{2}}$$

Exercise

Write the equation in its equivalent logarithmic form $2^{6} = 64$

Solution

$6 = \log_2 64$

Exercise

Write the equation in its equivalent logarithmic form $5^{4} = 625$

Solution

$4 = \log_5 625$

Exercise

Write the equation in its equivalent logarithmic form $5^{-3} = \frac{1}{125}$

Solution

$-3 = \log_5 \frac{1}{125}$

Exercise

Write the equation in its equivalent logarithmic form $\sqrt[3]{64} = 4$

Solution

$64^{1/3} = 4$

$\log_{64} = \frac{1}{3}$

Exercise

Write the equation in its equivalent logarithmic form $b^3 = 343$

Solution

$$\boxed{\log_b 343 = 3}$$

Exercise

Write the equation in its equivalent logarithmic form $8^y = 300$

Solution

$$\boxed{\log_8 300 = y}$$

Exercise

Write the equation in its equivalent logarithmic form: $\sqrt[n]{x} = y$

Solution

$$(x)^{1/n} = y$$

$$\boxed{\log_x (y) = \frac{1}{n}}$$

Exercise

Write the equation in its equivalent logarithmic form: $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$

Solution

$$\boxed{\log_{\frac{2}{3}} \left(\frac{27}{8}\right) = -3}$$

Exercise

Write the equation in its equivalent logarithmic form: $\left(\frac{1}{2}\right)^{-5} = 32$

Solution

$$\boxed{\log_{\frac{1}{2}} (32) = -5}$$

Exercise

Write the equation in its equivalent logarithmic form: $e^{x-2} = 2y$

Solution

$\boxed{x-2=\ln|2y|}$

Exercise

Write the equation in its equivalent logarithmic form: $e=3x$

Solution

$\boxed{1=\ln|3x|}$

Exercise

Write the equation in its equivalent logarithmic form: $\sqrt[3]{e^{2x}}=y$

Solution

$e^{2x/3}=y$

$\boxed{\frac{2x}{3}=\ln|y|}$

Exercise

Write the equation in its equivalent exponential form $\log_5 125=y$

Solution

$\boxed{5^y=125}$

Exercise

Write the equation in its equivalent exponential form $\log_4 16=x$

Solution

$\boxed{16=4^x}$

Exercise

Write the equation in its equivalent exponential form $\log_5 \frac{1}{5}=x$

Solution

$\boxed{\frac{1}{5}=5^x}$

Exercise

Write the equation in its equivalent exponential form $\log_2 \frac{1}{8}=x$

Solution

$\frac{1}{8} = 2^x$

Exercise

Write the equation in its equivalent exponential form $\log_6 \sqrt{6} = x$

Solution

$\sqrt{6} = 6^x$

Exercise

Write the equation in its equivalent exponential form $\log_3 \frac{1}{\sqrt{3}} = x$

Solution

$3^{-1/2} = 3^x$

Exercise

Write the equation in its equivalent exponential form: $6 = \log_2 64$

Solution

$6 = \log_2 64 \iff 2^6 = 64$

Exercise

Write the equation in its equivalent exponential form: $2 = \log_9 x$

Solution

$2 = \log_9 x \iff x = 2^9$

Exercise

Write the equation in its equivalent exponential form: $\log_{\sqrt{3}} 81 = 8$

Solution

$\log_{\sqrt{3}} 81 = 8 \iff 81 = \left(\sqrt{3}\right)^8$

Exercise

Write the equation in its equivalent exponential form: $\log_4 \frac{1}{64} = -3$

<u>Solution</u>

$$\log_4 \frac{1}{64} = -3 \quad \Leftrightarrow \quad \boxed{\frac{1}{64} = x^{-3}}$$

Exercise

Write the equation in its equivalent exponential form: $\log_4 26 = y$

<u>Solution</u>

$$\log_4 26 = y \quad \Leftrightarrow \quad \boxed{26 = 4^y}$$

Exercise

Write the equation in its equivalent exponential form: $\ln M = c$

<u>Solution</u>

$$\ln M = c \quad \Leftrightarrow \quad \boxed{M = e^c}$$

Exercise

Evaluate the expression without using a calculator: $\log_4 16$

<u>Solution</u>

$$\log_4 16 = \log_4 4^2 \qquad\qquad \log_b b^x = x$$

$$\boxed{= 2}$$

Exercise

Evaluate the expression without using a calculator: $\log_2 \frac{1}{8}$

<u>Solution</u>

$$\log_2 \frac{1}{8} = \log_2 \frac{1}{2^3}$$

$$= \log_2 2^{-3} \qquad\qquad \log_b b^x = x$$

$$\boxed{= -3}$$

Exercise

Evaluate the expression without using a calculator: $\log_6 \sqrt{6}$

Solution

$$\log_6 \sqrt{6} = \log_6 6^{1/2}$$

$$\boxed{= \frac{1}{2}}$$

Exercise

Evaluate the expression without using a calculator: $\log_3 \frac{1}{\sqrt{3}}$

Solution

$$\log_3 \frac{1}{\sqrt{3}} = \log_3 \frac{1}{3^{1/2}}$$

$$= \log_3 3^{-1/2} \qquad \log_b b^x = x$$

$$\boxed{= -\frac{1}{2}}$$

Exercise

Evaluate the expression without using a calculator: $\log_3 \sqrt[7]{3}$

Solution

$$\log_3 3^{1/7} = x \qquad \textit{Converts to exponential}$$

$$3^{1/7} = 3^x$$

$$x = \frac{1}{7}$$

$$\boxed{\log_3 \sqrt[7]{3} = \frac{1}{7}}$$

Exercise

Evaluate the expression without using a calculator: $\log_3 \sqrt{9}$

Solution

$$\log_3 \sqrt{9} = \log_3 3 \qquad \log_b b^x = x$$

$$\boxed{= 1}$$

Exercise

Evaluate the expression without using a calculator: $\log_{\frac{1}{2}} \sqrt{\frac{1}{2}}$

Solution

$$\log_{\frac{1}{2}} \sqrt{\frac{1}{2}} = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{\frac{1}{2}} \qquad \log_b b^x = x$$

$$= \frac{1}{2}$$

Exercise

Simplify $\log_5 1$

Solution

$$\log_5 1 = 0$$

Exercise

Simplify $\log_7 7^2$

Solution

$$\log_7 7^2 = 2$$

Exercise

Simplify $3^{\log_3 8}$

Solution

$$3^{\log_3 8} = 8$$

Exercise

Simplify $10^{\log 3}$

Solution

$\underline{10^{\log 3} = 3}$

Exercise

Simplify $e^{2+\ln 3}$

Solution

$$e^{2+\ln 3} = e^2 e^{\ln 3}$$
$$= 3e^2$$

Exercise

Simplify $\ln e^{-3}$

Solution

$\ln e^{-3} = -3$

Exercise

Simplify $\ln e^{x-5}$

Solution

$\ln e^{x-5} = x-5$

Exercise

Simplify $\log_b b^n$

Solution

$\log_b b^n = n$

Exercise

Simplify $\ln e^{x^2+3x}$

Solution

$$\ln e^{x^2+3x} = x^2 + 3x$$

Exercise

Find the domain of $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Solution

$$e^x + e^{-x} > 0$$

Domain: $\mathbb{R}$

Exercise

Find the domain of $f(x) = \dfrac{e^{|x|}}{1+e^x}$

Solution

$$1 + e^x > 0$$

Domain: $\mathbb{R}$

Exercise

Find the domain of $f(x) = \sqrt{1-e^x}$

Solution

$$1 - e^x \ge 0$$

$$e^x \le 1$$

$$x \le \ln 1$$

Domain: $x \le 0$

Exercise

Find the domain of $f(x) = \sqrt{e^x - e^{-x}}$

Solution

$e^{x} - e^{-x} \ge 0$

$e^{x} \ge e^{-x}$

$e^{2x} \ge 1$

$2x \ge \ln 1$

Domain: $\boxed{x \ge 0}$

Exercise

Find the domain of $f(x) = \log_5 (x+4)$

Solution

Domain: $\boxed{x > -4}$

Exercise

Find the domain of $f(x) = \log_5 (x+6)$

Solution

Domain: $\boxed{x > -6}$

Exercise

Find the domain of $f(x) = \log(2-x)$

Solution

Domain: $\boxed{x < 2}$

Exercise

Find the domain of $f(x) = \log(7-x)$

Solution

Domain: $\boxed{x < 7}$

Exercise

Find the domain of $f(x) = \ln(x-2)^2$

Solution

Domain: $\boxed{\mathbb{R} - \{2\}}$ $\boxed{(-\infty,\ 2) \cup (2,\ \infty)}$

Exercise

Find the domain of $f(x) = \ln(x-7)^2$

Solution

Domain: $\mathbb{R} - \{7\}$

$(-\infty,\ 7) \cup (7,\ \infty)$

Exercise

Find the domain of $f(x) = \log\left(x^2 - 4x - 12\right)$

Solution

$$x^2 - 4x - 12 > 0$$

$$x = \frac{4 \pm \sqrt{16 + 48}}{2}$$

$$= \begin{cases} \frac{4-8}{2} = -2 \\ \frac{4+8}{2} = 6 \end{cases}$$

Domain: $x < -2 \quad x > 6$

$(-\infty,\ -2) \cup (6,\ \infty)$

Exercise

Find the domain of $f(x) = \log\left(\frac{x-2}{x+5}\right)$

Solution

$$\begin{cases} x \ne 2 \\ x \ne -5 \end{cases}$$

	-5	0	2
$+$		$-$	$+$

Domain: $x < -5 \quad x > 2$

$(-\infty,\ -5) \cup (2,\ \infty)$

Exercise

Find the domain of $f(x) = \log\left(\frac{3-x}{x-2}\right)$

Solution

$$\begin{cases} x \neq 3 \\ x \neq 2 \end{cases}$$

0	2	3
–	+	–

Domain: $\boxed{2 < x < 3}$

$\boxed{(2,\ 3)}$

Exercise

Find the domain of $f(x) = \ln\left(x^2 - 9\right)$

Solution

$x^2 - 9 > 0$

Domain: $\boxed{x < -3 \quad x > 3}$

Exercise

Find the domain of $f(x) = \ln\left(\dfrac{x^2}{x-4}\right)$

Solution

$\dfrac{x^2}{x-4} > 0$

$x^2 \ \rightarrow \ \mathbb{R}$

$x > 4$

Domain: $\boxed{x > 4}$

Exercise

Find the domain of $f(x) = \log_3\left(x^3 - x\right)$

Solution

0,0	1	2
–	–	+

$x^3 - x > 0$

$\boxed{x = 0,\ 0,\ 1}$

Domain: $\boxed{x > 1}$

Exercise

Find the domain of $f(x) = \log\sqrt{2x-5}$

Solution

$$2x-5>0$$

Domain: $\boxed{x>\frac{5}{2}}$

Exercise

Find the domain of $\quad f(x)=3\ln(5x-6)$

Solution

$$5x-6>0$$

Domain: $\boxed{x>\frac{6}{5}}$

Exercise

Find the domain of $\quad f(x)=\log\left(\frac{x}{x-2}\right)$

Solution

$$\frac{x}{x-2}>0$$

$$\boxed{x=0,\ 2}$$

Domain: $\boxed{x<0 \quad x>2}$

Exercise

Find the domain of $\quad f(x)=\log\left(4-x^2\right)$

Solution

$$4-x^2>0$$

$$4-x^2=0 \quad \rightarrow \quad x=\pm 2$$

Domain: $\boxed{-2<x<2}$

Exercise

Find the domain of $\quad f(x)=\ln\left(x^2+4\right)$

Solution

x^2+4 always positive.

Domain: $\boxed{\mathbb{R}}$

Exercise

Find the domain of $f(x) = \ln|4x - 8|$

Solution

$$4x - 8 = 0 \quad \to \quad x = 2$$

Domain: $\boxed{\mathbb{R} - \{2\}}$

Exercise

Find the domain of $f(x) = \ln|5 - x|$

Solution

$$5 - x = 0 \quad \to \quad x = 5$$

Domain: $\boxed{\mathbb{R} - \{5\}}$

Exercise

Find the domain of $f(x) = \ln(x - 4)^2$

Solution

$$x - 4 = 0 \quad \to \quad x = 4$$

Domain: $\boxed{\mathbb{R} - \{4\}}$

Exercise

Find the domain of $f(x) = \ln\left(x^2 - 4\right)$

Solution

$$x^2 - 4 > 0$$

$$x^2 - 4 = 0 \quad \to \quad x = \pm 2$$

Domain: $\boxed{x < -2 \quad x > 2}$

Exercise

Find the domain of $f(x) = \ln\left(x^2 - 4x + 3\right)$

Solution

$$x^2 - 4x + 3 = 0 \quad \to \quad \boxed{x = 1,\ 3}$$

$$x^2 - 4x + 3 > 0$$

Domain: $x < 1 \quad x > 3$

Exercise

Find the domain of $\quad f(x) = \ln\left(2x^2 - 5x + 3\right)$

Solution

$$2x^2 - 5x + 3 = 0 \quad \to \quad x = 1, \ \frac{3}{2}$$

$$2x^2 - 5x + 3 > 0$$

Domain: $x < 1 \quad x > \frac{3}{2}$

Exercise

Find the domain of $\quad f(x) = \log\left(x^2 + 4x + 3\right)$

Solution

$$x^2 + 4x + 3 = 0 \quad \to \quad x = -1, \ -3$$

$$x^2 + 4x + 3 > 0$$

Domain: $x < -3 \quad x > -1$

Exercise

Find the domain of $\quad f(x) = \ln\left(x^4 - x^2\right)$

Solution

$$x^4 - x^2 = 0$$

$$x^2\left(x^2 - 1\right) = 0$$

$$x = 0, \ 0, \ \pm 1$$

$$x^4 - x^2 > 0$$

Domain: $x < -1 \quad x > 1$

-1	$0,0$	1	2
$+$	$-$	$-$	$+$

Exercise

Sketch the graph: $f(x)=2^{x}+3$

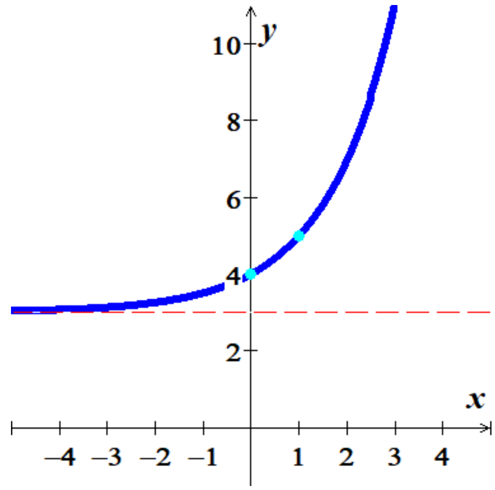
Solution

Asymptote: $y=3$

Domain: $(-\infty,\ \infty)$

Range: $(3,\ \infty)$

x	$f(x)$
–1	3.5
0	4
1	5
2	7



Exercise

Sketch the graph: $f(x)=2^{3-x}$

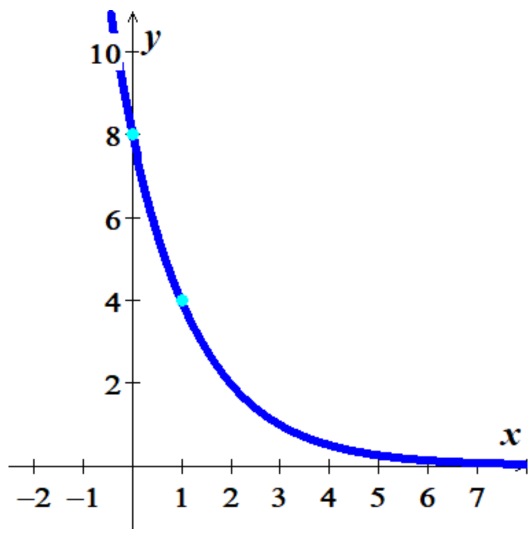
Solution

Asymptote: $y=0$

Domain: $(-\infty,\ \infty)$

Range: $(0,\ \infty)$

x	$f(x)$
1	4
2	2
0	8



Exercise

Sketch the graph: $f(x)=\left(\frac{2}{5}\right)^{-x}$

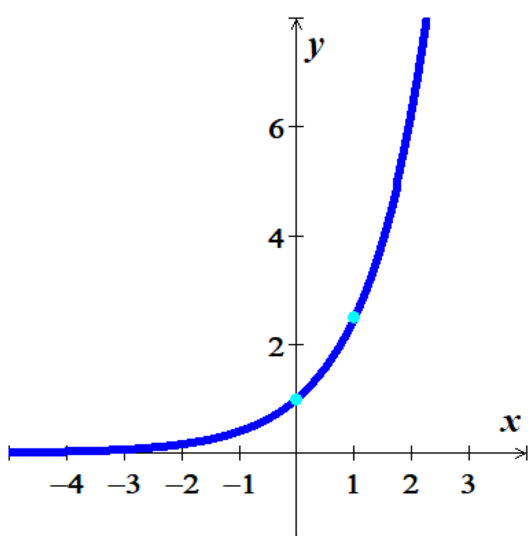
Solution

Asymptote: $y=0$

Domain: $(-\infty,\ \infty)$

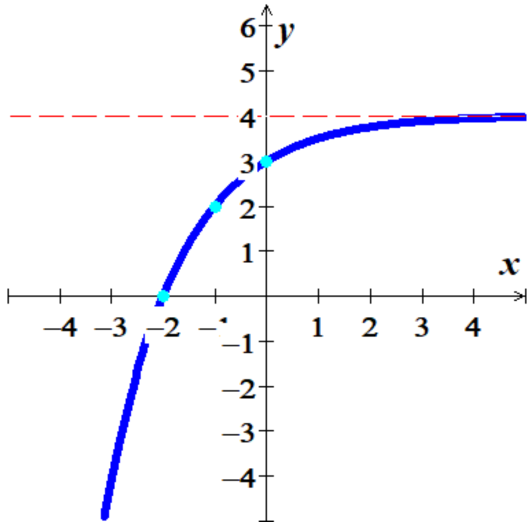
Range: $(0,\ \infty)$

x	$f(x)$
–1	0.4
0	1
1	2.5



Exercise

Sketch the graph: $f(x) = -\left(\frac{1}{2}\right)^x + 4$



Solution

Asymptote: $y = 4$

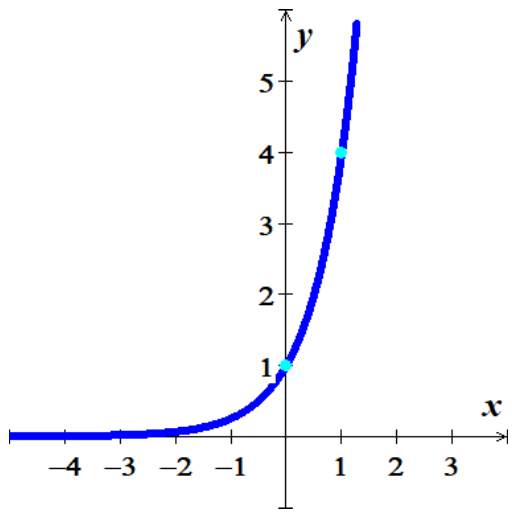
Domain: $(-\infty, \infty)$

Range: $(-\infty, 4)$

x	$f(x)$
–2	0
–1	2
0	3

Exercise

Sketch the graph of $f(x) = 4^x$



Solution

Asymptote: $y = 0$

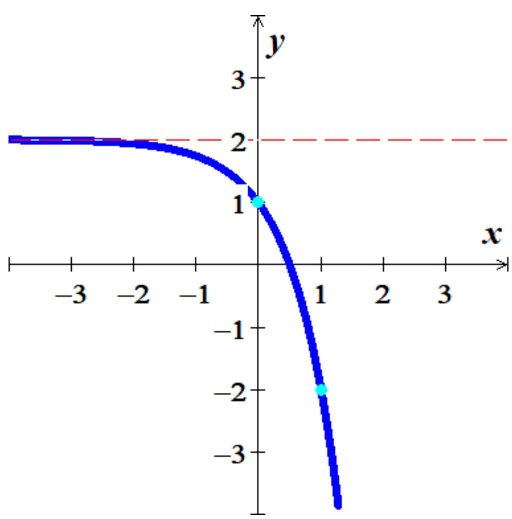
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
0	1
1	4

Exercise

Sketch the graph of $f(x) = 2 - 4^x$



Solution

Asymptote: $y = 2$

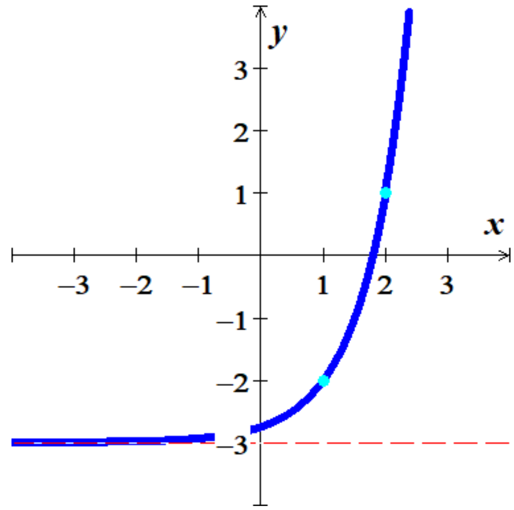
Domain: $(-\infty, \infty)$

Range: $(-\infty, 2)$

x	$f(x)$
0	1
1	–2

Exercise

Sketch the graph of $f(x) = -3 + 4^{x-1}$



Solution

Asymptote: $y = -3$

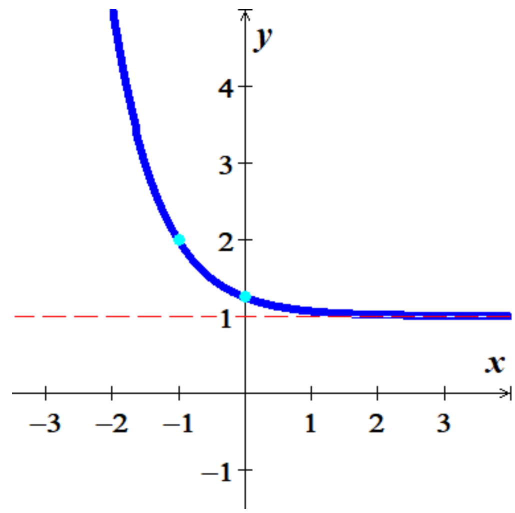
Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

x	$f(x)$
1	–2
2	1

Exercise

Sketch the graph of $f(x) = 1 + \left(\frac{1}{4}\right)^{x+1}$



Solution

Asymptote: $y = 1$

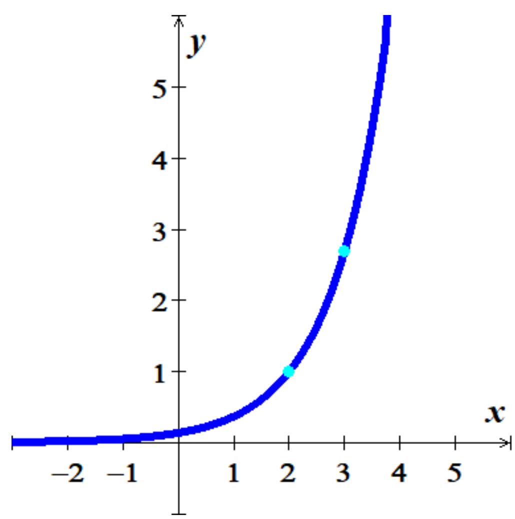
Domain: $(-\infty, \infty)$

Range: $(1, \infty)$

x	$f(x)$
–1	2
0	$\frac{5}{4}$

Exercise

Sketch the graph of $f(x) = e^{x-2}$



Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
2	1
3	2.7

Exercise

Sketch the graph of $f(x) = 3 - e^{x-2}$

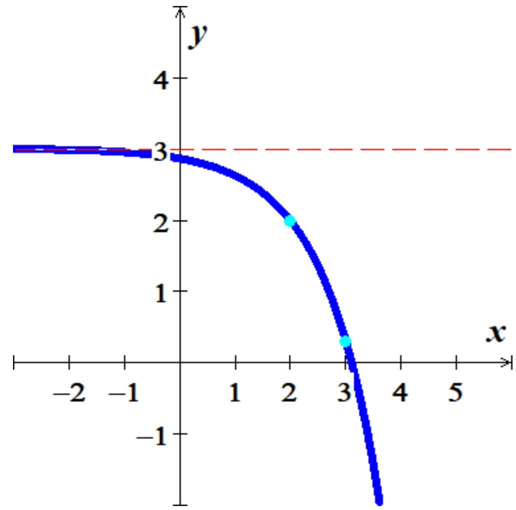
Solution

Asymptote: $y = 3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

x	$f(x)$
2	2
3	.3



Exercise

Sketch the graph of $f(x) = e^{x+4}$

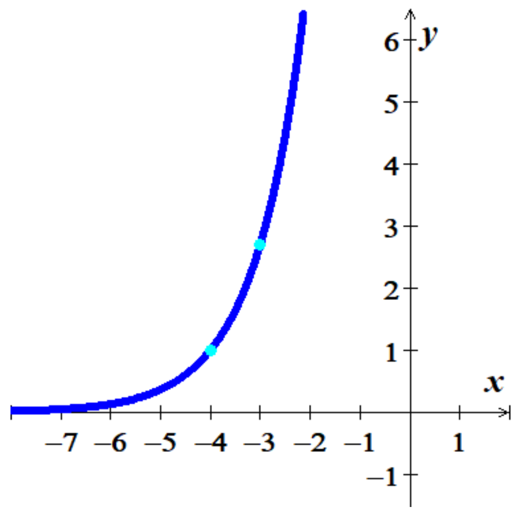
Solution

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x	$f(x)$
–4	1
–3	2.7



Exercise

Sketch the graph of $f(x) = 2 + e^{x-1}$

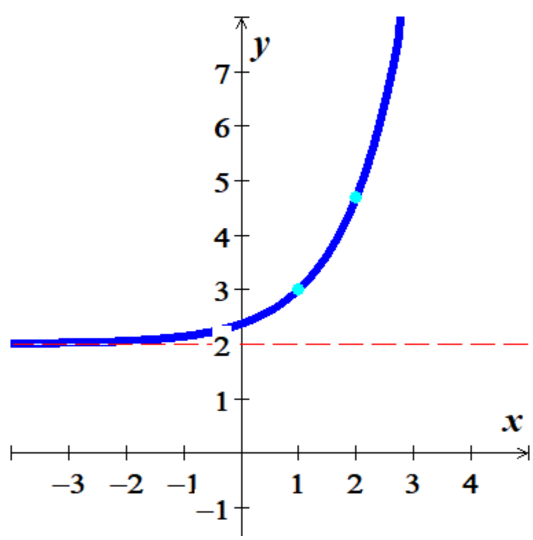
Solution

Asymptote: $y = 2$

Domain: $(-\infty, \infty)$

Range: $(2, \infty)$

x	$f(x)$
1	3
2	4.7



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \log_4 (x-2)$

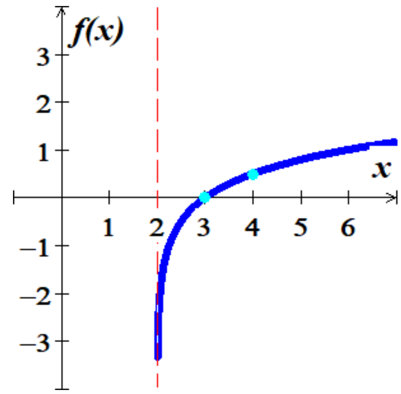
Solution

Asymptote: $x = 2$

Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

x	$f(x)$
2	
3	0
4	.5



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \log_4 |x|$

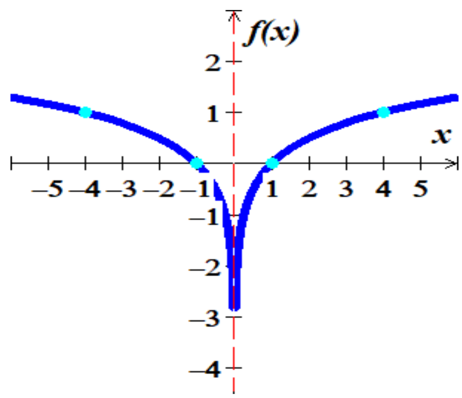
Solution

Asymptote: $x = 0$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, \infty)$

x	$f(x)$
0	
± 1	0
± 4	1



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \left(\log_4 x\right) - 2$

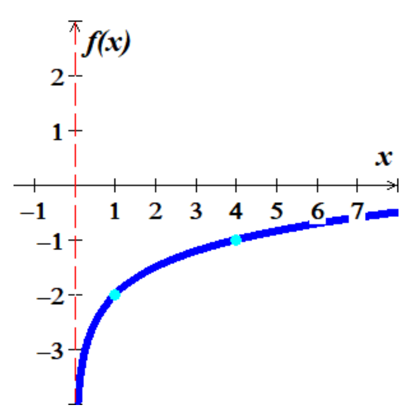
Solution

Asymptote: $x = 0$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x	$f(x)$
0	
1	0
4	–1



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \log(3-x)$

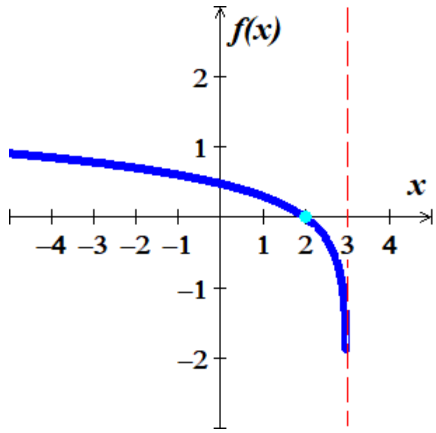
Solution

Asymptote: $x = 3$

Domain: $(-\infty,\ 3)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
3	
2	0



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = 2 - \log(x+2)$

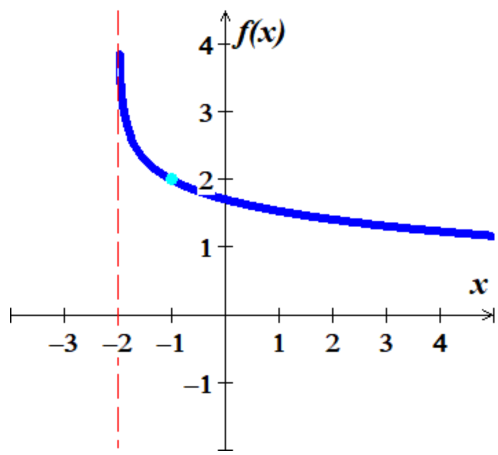
Solution

Asymptote: $x = -2$

Domain: $(-2,\ \infty)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
-2	
-1	2



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \ln(x-2)$

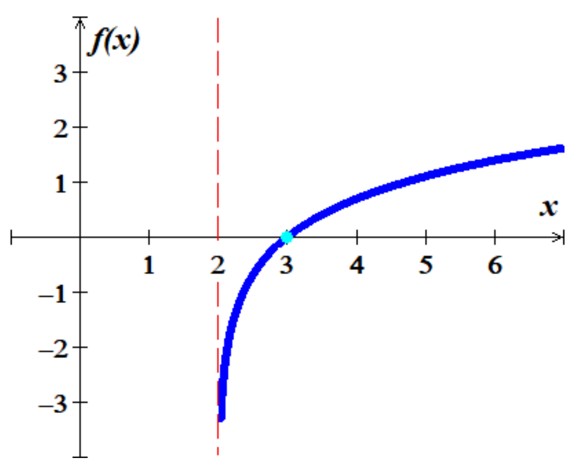
Solution

Asymptote: $x = 2$

Domain: $(2,\ \infty)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
2	
3	0



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x)=\ln(3-x)$

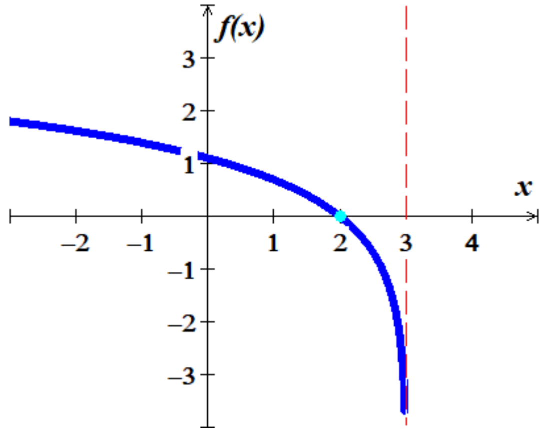
Solution

Asymptote: $x=3$

Domain: $(-\infty,\ 3)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
3	
2	0



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x)=2+\ln(x+1)$

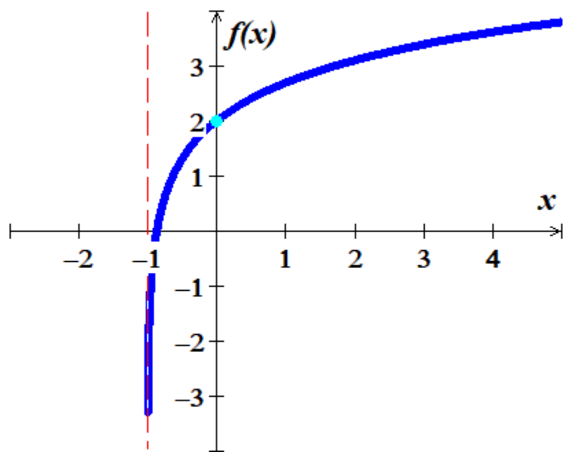
Solution

Asymptote: $x=-1$

Domain: $(-1,\ \infty)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
–1	
0	2



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x)=1-\ln(x-2)$

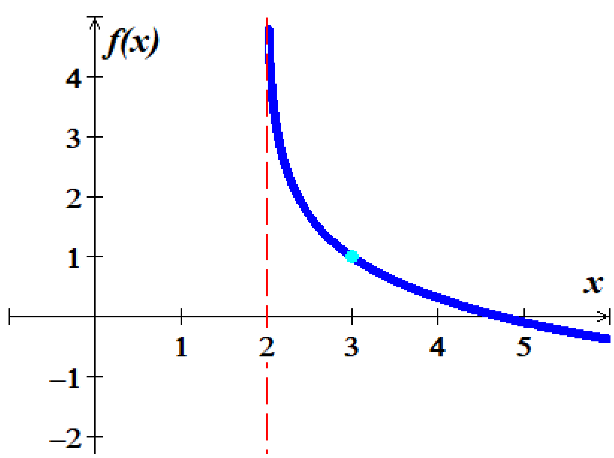
Solution

Asymptote: $x=2$

Domain: $(2,\ \infty)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
2	
3	1



Exercise

On a study by psychologists Bornstein and Bornstein, it was found that the average walking speed w, in feet per second, of a person living in a city of population P, in ***thousands***, is given by the function

$$w(P) = 0.37\ln P + 0.05$$

a) The population is 124,848. Find the average walking speed of people living in Hartford.
b) The population is 1,236,249. Find the average walking speed of people living in San Antonio.

Solution

124,848 = 124.848 ***thousand***

a) $w(124.848) = 0.37\ln(124.848) + 0.05$

≈ 1.8 *ft/sec*

b) $w(1,236.249) = 0.37\ln(1,236.249) + 0.05$

≈ 2.7 *ft/sec*

Exercise

The loudness of sounds is measured in a unit called a decibel. To measure with this unit, we first assign an intensity of I_0 to a very faint sound, called the threshold sound. If a particular sound has intensity I, then the decibel rating of this louder sound is

$$d = 10\log\frac{I}{I_0}$$

Find the exact decibel rating of a sound with intensity $10,000 I_0$

Solution

$$\begin{aligned} d &= 10\log\frac{10000 I_0}{I_0} \\ &= 10\log 10000 \\ &= 40 \ \ db \end{aligned}$$

Exercise

Students in an accounting class took a final exam and then took equivalent forms of the exam at monthly intervals thereafter. The average score *S(t)*, as a percent, after t months was found to be given by the function

$$S(t) = 78 - 15\ \log(t+1); \quad t \ge 0$$

a) What was the average score when the students initially took the test, $t = 0$?

b) What was the average score after 4 *months*? 24 *months*?

Solution

a) $S(0) = 78 - 15\ \log(1)$

$\approx 78\%$

b) After 4 *months*

$S(4) = 78 - 15\ \log(5)$

$\approx 67.5\%$

After 24 *months*

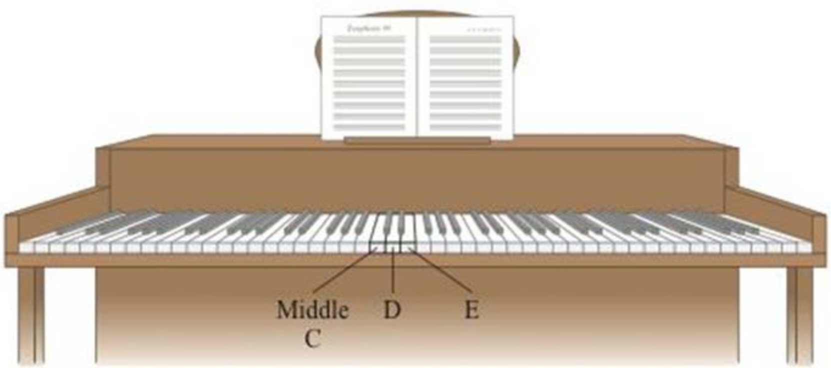
$S(24) = 78 - 15\ \log(25)$

$\approx 57\%$

Exercise

Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

$$f(n) = (27.5)\ 2^{\frac{n-1}{12}}$$



a) Determine the frequency of middle *C*, key number 40 on an 88–*key* piano.

b) Is the difference in frequency between middle *C* (key number 40) and *D* (key number 42) the same as the difference in frequency between *D* (key number 42) and *E* (key number 44)?

Solution

a) $f(40) = (27.5)\ 2^{\frac{40-1}{12}}$

≈ 261.63

the frequency of middle *C* is ≈ 262 *vibrations* per *second*.

b) $f(42) = (27.5)\ 2^{(41/12)}$

≈ 293.66

The difference between the frequency of middle *C* and *D* is: $293.66 - 261.66 \approx 32$

$f(44) = (27.5)\ 2^{(43/12)}$

≈ 329.63

The difference between the frequency of middle D and E is: $329.63 - 293.66 \approx 36$

$\therefore$ The differences are ***not*** the same since the function is *not* linear function.