4 ADDITIONAL DERIVATIVE TOPICS

EXERCISE 4-1

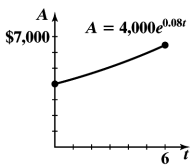
2. $A = \$5{,}000e^{0.08t}$

When $t = 1$, $A = \$5{,}000e^{(0.08)1} = \$5{,}000e^{0.08} = \$5{,}416.44$.

When $t = 4$, $A = \$5{,}000e^{(0.08)4} = \$5{,}000e^{0.32} = \$6{,}885.64$.

When $t = 10$, $A = \$5{,}000e^{(0.08)10} = \$5{,}000e^{0.8} = \$11{,}127.70$.

4.



6. $2 = e^{0.03t}$

Take the natural log of both sides of this equation

$$\begin{aligned} \ln(e^{0.03t}) &= \ln 2 \\ 0.03t \ln e &= \ln 2 \\ 0.03t &= \ln 2 \quad (\ln e = 1) \\ t &= \frac{\ln 2}{0.03} \approx 23.10 \end{aligned}$$

8. $3 = e^{0.25t}$

$$\begin{aligned} \ln(e^{0.25t}) &= \ln 3 \\ 0.25t &= \ln 3 \\ t &= \frac{\ln 3}{0.25} \approx 4.39 \end{aligned}$$

10. $3 = e^{10r}$

$$\begin{aligned} \ln(e^{10r}) &= \ln 3 \\ 10r &= \ln 3 \\ r &= \frac{\ln 3}{10} \approx 0.11 \end{aligned}$$

12.

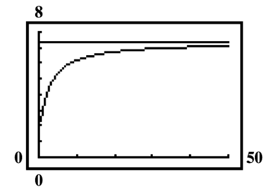
s	$(1 + s)^{1/s}$
0.01	2.70481
-0.01	2.73200
0.001	2.71692
-0.001	2.71964
0.0001	2.71815
-0.0001	2.71842
0.00001	2.71827
-0.00001	2.71830
⇓	⇓
0	$e = 2.7182818\ldots$

14.

s	0.1	0.01	0.001	0.0001
$\left(1+\frac{1}{s}\right)^{s}$	1.270982	1.047232	1.006933	1.000921

$$\lim_{s\to 0^+}\left(1+\frac{1}{s}\right)^{s} = 1$$

16. The graphs of $y_1 = \left(1+\frac{2}{n}\right)^n$,
$y_2 = 7.3890560999 \approx e^2$ for $1 \le n \le 50$ are given at the right.



18. (A) $A = Pe^{rt} = \$10{,}000e^{0.0364(3)}$
$= \$10{,}000e^{0.1092}$
$= \$11{,}153.85$

(B) $11{,}000 = 10{,}000e^{0.0364t}$
$e^{0.0364t} = 1.1$
$0.0364t = \ln 1.1$
$t = \frac{\ln 1.1}{0.0364} \approx 2.62$ years

20. $A = Pe^{rt}$
$\$50{,}000 = Pe^{0.064(5)} = Pe^{0.32}$
Therefore,
$P = \frac{\$50{,}000}{e^{0.32}} = \$50{,}000e^{-0.32} \approx \$36{,}307.45$

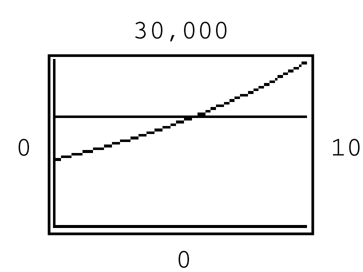
22. $195{,}000 = 99{,}000e^{15r}$
$e^{15r} \approx 1.97$
$r = \frac{\ln(1.97)}{15} \approx 0.0452$ or 4.52%

24. $P = 10{,}000e^{-0.08t} = 5{,}000$
$e^{-0.08t} = 0.5$
$-0.08t = \ln(0.5)$
$t = -\frac{\ln(0.5)}{0.08} \approx 8.66$ years

26. $2P = Pe^{0.05t}$
$e^{0.05t} = 2$
$\ln(e^{0.05t}) = \ln 2$
$0.05t = \ln 2$
$t = \frac{\ln 2}{0.05} \approx 13.86$ years

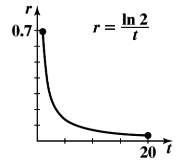
28. $2P = Pe^{r(10)}$
$\ln(e^{10r}) = \ln 2$
$10r = \ln 2$
$r = \frac{\ln 2}{10} \approx 0.0693$ or 6.93%

30. The total investment in the two accounts is given by
$A = 5{,}000e^{0.088t} + 7{,}000(1 + 0.096)^t$
On a graphing utility, locate the intersection point of
$y_1 = 5{,}000e^{0.088x} + 7{,}000(1 + 0.096)^x$ and $y_2 = 20{,}000$.
The result is: $x = t \approx 5.7$ years.



32. (A)
$$A = Pe^{rt};$$
$$2P = Pe^{rt}$$
$$2 = e^{rt};$$
$$rt = \ln 2;$$
$$r = \frac{\ln 2}{t}$$

(B)



Although t could be any positive number, the restrictions on t are reasonable in the sense that the doubling times for most investments would be expected to be between 1 and 20 years.

(C) $t = 2;\ r = \frac{\ln 2}{2} \approx 0.347$ or 34.7%

$t = 4;\ r = \frac{\ln 2}{4} \approx 0.173$ or 17.3%

$t = 6;\ r = \frac{\ln 2}{6} \approx 0.116$ or 11.6%

$t = 8;\ r = \frac{\ln 2}{8} \approx 0.087$ or 8.7%

$t = 10;\ r = \frac{\ln 2}{10} \approx 0.069$ or 6.9%

$t = 12;\ r = \frac{\ln 2}{12} \approx 0.058$ or 5.8%

34.
$$Q = Q_0 e^{-0.0001238t}$$
$$\frac{1}{2}Q_0 = Q_0 e^{-0.0001238t}$$
$$e^{-0.0001238t} = \frac{1}{2}$$
$$\ln(e^{-0.0001238t}) = \ln\left(\frac{1}{2}\right) = \ln 1 - \ln 2$$
$$-0.0001238t = -\ln 2 \quad (\ln 1 = 0)$$
$$t = \frac{\ln 2}{0.0001238} \approx 5{,}599 \text{ years}$$

36.
$$Q = Q_0 e^{rt} \quad (r < 0)$$
$$\frac{1}{2}Q_0 = Q_0 e^{r(90)}$$
$$e^{90r} = \frac{1}{2}$$
$$\ln(e^{90r}) = \ln\left(\frac{1}{2}\right) = \ln 1 - \ln 2$$
$$90r = -\ln 2 \quad (\ln 1 = 0)$$
$$r = -\frac{\ln 2}{90} \approx -0.0077$$

Thus, the continuous compound rate of decay of the strontium isotope is approximately -0.0077.

38. $P = P_0e^{rt}$

$2P_0 = P_0e^{0.00975t}$ or $e^{0.00975t} = 2$

Thus, $\ln(e^{0.00975t}) = \ln 2$

and $0.00975t = \ln 2$

Therefore, $t = \dfrac{\ln 2}{0.00975} \approx 71.09$ years

40. $2P_0 = P_0e^{r(200)}$

$e^{200r} = 2$

$\ln(e^{200r}) = \ln 2$

$200r = \ln 2$

$r = \dfrac{\ln 2}{200} \approx 0.0035$

or 0.35%

EXERCISE 4-2

2. $f(x) = -7e^x - 2x + 5$

$f'(x) = -7e^x - 2$

4. $f(x) = 6 \ln x - x^3 + 2$

$f'(x) = 6\left(\dfrac{1}{x}\right) - 3x^2 = \dfrac{6}{x} - 3x^2$

6. $f(x) = 9e^x + 2x^2$

$f'(x) = 9e^x + 4x$

8. $f(x) = \ln x + 2e^x - 3x^2$

$f'(x) = \dfrac{1}{x} + 2e^x - 6x$

10. $f(x) = \ln x^8 = 8 \ln x$

$f'(x) = 8\left(\dfrac{1}{x}\right) = \dfrac{8}{x}$

12. $f(x) = 4 + \ln x^9 = 4 + 9 \ln x$

$f'(x) = 9\left(\dfrac{1}{x}\right) = \dfrac{9}{x}$

14. $f(x) = \ln x^{10} + 2 \ln x = 10 \ln x + 2 \ln x = 12 \ln x$

$f'(x) = 12\left(\dfrac{1}{x}\right) = \dfrac{12}{x}$

16. $f(x) = 2 \ln x$

$f'(x) = 2\left(\dfrac{1}{x}\right) = \dfrac{2}{x}$

For $x = 1$, the slope of the tangent line is $m = f'(1) = \dfrac{2}{1} = 2$, and $f(1) = 2 \ln 1 = 2(0) = 0$. So, the equation of the tangent line at $x = 1$ is: $y - 0 = 2(x - 1)$ or $y = 2x - 2$.

18. $f(x) = e^x + 1$

$f'(x) = e^x$

For $x = 0$, $m = f'(0) = e^0 = 1$ and $f(0) = 2$, so the equation of the tangent line at $x = 0$ is: $y - 2 = 1(x - 0)$ or $y = x + 2$.

20. $f(x) = 1 + \ln x^4 = 1 + 4 \ln x$

$f'(x) = 4\left(\dfrac{1}{x}\right) = \dfrac{4}{x}$

For $x = e$, $m = f'(e) = \dfrac{4}{e}$ and $f(e) = 1 + 4 \ln e = 5$, so the equation of the tangent line at $x = e$ is:

$y - 5 = \dfrac{4}{e}(x - e) = \left(\dfrac{4}{e}\right)x - 4$ or $y = (4e^{-1})x + 1$

22. $f(x) = 5e^x$

$f'(x) = 5e^x$

For $x = 1$, $m = f'(1) = 5e$ and $f(1) = 5e$, so the equation of the tangent line at $x = 1$ is:

$y - 5e = 5e(x - 1)$ or $y = (5e)x$

24. $f(x) = e^x$

$f'(x) = e^x$

For $x = 1$, $f'(1) = e^1 = e$ and $f(1) = e$, so the equation of the tangent line at $x = 1$ is:

$y - e = e(x - 1)$ or $y = ex$. This line passes through the origin.

There is no other tangent line that will pass through the origin, since for any other value of x, the y-intercept of the tangent line will not be 0.

26. $f(x) = \ln x$

$f'(x) = \dfrac{1}{x}$

For $x = e$, $f'(e) = \dfrac{1}{e}$ and $f(e) = \ln e = 1$, so the equation of the tangent line at $x = e$ is:

$y - 1 = \dfrac{1}{e}(x - e)$ or $y = \dfrac{x}{e}$. This line passes through the origin.

There is no other tangent line that will pass through the origin, since for any other value of x, the y-intercept of the tangent line will not be 0.

28. $f(x) = 2 + 3 \ln \dfrac{1}{x}$

$= 2 + 3 \ln x^{-1}$

$= 2 + 3(-1) \ln x$

$= 2 - 3 \ln x$

$f'(x) = -3\left(\dfrac{1}{x}\right) = -\dfrac{3}{x}$

30. $f(x) = x + 5 \ln 6x$

$= x + 5(\ln 6 + \ln x)$

$= x + 5 \ln 6 + 5 \ln x$

$f'(x) = 1 + 5\left(\dfrac{1}{x}\right) = 1 + \dfrac{5}{x} = \dfrac{x+5}{x}$

32. $y = 3 \log_5 x$

$\dfrac{dy}{dx} = 3\left(\dfrac{1}{\ln 5} \cdot \dfrac{1}{x}\right) = \dfrac{3}{x \ln 5}$

34. $y = 4^x$

$\dfrac{dy}{dx} = 4^x \ln 4 = (2^2)^x \ln 2^2 = (2^{2x})2 \ln 2$

$= 2^{2x+1} \ln 2$

36. $y = \log x + 4x^2 + 1$

$\dfrac{dy}{dx} = \dfrac{1}{\ln 10} \cdot \dfrac{1}{x} + 8x = \dfrac{1 + 8x^2 \ln 10}{x \ln 10}$

38. $y = x^5 - 5^x$

$\dfrac{dy}{dx} = 5x^4 - 5^x \ln 5$

40. $y = -\log_2 x + 10 \ln x$

$\dfrac{dy}{dx} = -\dfrac{1}{\ln 2} \cdot \dfrac{1}{x} + 10\left(\dfrac{1}{x}\right) = \left(10 - \dfrac{1}{\ln 2}\right)\dfrac{1}{x}$

42. $y = e^3 - 3^x$

$\dfrac{dy}{dx} = -3^x \ln 3$

44. On a graphing utility, graph $y_1 = e^x$ and $y_2 = x^5$. Rounded off to two decimal places, the points of intersection are: (1.30, 3.65), (12.71, 332,105.11).

46. On a graphing utility, graph $y_1 = (\ln x)^3$ and $y_2 = \sqrt{x}$. Rounded off to two decimal places, the point of intersection is: (3.41, 1.85).

48. On a graphing utility, graph $y_1 = \ln x$ and $y_2 = x^{1/4}$. Rounded off to two decimal places, the point of intersection is: (4.18, 1.43).

50. $f(x) = e^{cx}$

Step 1. $f(x+h) = e^{c(x+h)} = e^{cx} \cdot e^{ch}$

Step 2. $f(x+h) - f(x) = e^{c(x+h)} - e^{cx}$

$$= e^{cx} \cdot e^{ch} - e^{cx}$$
$$= e^{cx}(e^{ch} - 1)$$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{e^{cx}(e^{ch} - 1)}{h} = e^{cx}\left(\dfrac{e^{ch} - 1}{h}\right)$

Step 4. $f'(x) = \lim\limits_{h \to 0} e^{cx}\left(\dfrac{e^{ch} - 1}{h}\right)$

$$= e^{cx} \lim_{h \to 0} \frac{e^{ch} - 1}{h}$$
$$= e^{cx}(c) \quad \text{(by problem 49)}$$
$$= ce^{cx}$$

52. $R(t) = 20{,}000(0.86)^t$

$R'(t) = 20{,}000(0.86)^t \ln(0.86)$

$R'(1) = 20{,}000(0.86) \ln(0.86) \approx -2{,}594$

The rate is \$2,594 after 1 year.

$R'(2) = 20{,}000(0.86)^2 \ln(0.86) \approx -2{,}231$

The rate is \$2,231 after 2 years.

$R'(3) = 20{,}000(0.86)^3 \ln(0.86) \approx -1{,}919$

The rate is \$1,919 after 3 years.

54. $A(t) = 1000 \cdot 2^{4t} = 1000 \cdot 16^t$

$A'(t) = 1000 \cdot 16^t \ln 16$

$A'(1) = 1000 \cdot 16 \ln 16 \approx 44{,}361$

$A'(5) = 1000 \cdot (16)^5 \ln 16 \approx 2{,}907{,}269{,}992$

56. $P(x) = 17.5(1 + \ln x)$, $10 \le x \le 100$

$P'(x) = 17.5\left(\dfrac{1}{x}\right) = \dfrac{17.5}{x}$

Given $P'(x) = 0.3$, we will solve the following equation for x:

$0.3 = \dfrac{17.5}{x}$ or $x = \dfrac{17.5}{0.3} \approx 58$ lb

58. $N(t) = 10 + 6 \ln t, t \geq 1$

$N'(t) = \frac{6}{t}$

$N'(10) = \frac{6}{10} = 0.6$

$N'(100) = \frac{6}{100} = 0.06$

After 10 hours of instruction and practice, the rate of learning is 0.6 words/minute per hour of instruction and practice.

After 100 hours of instruction and practice, the rate of learning is 0.06 words/minute per hour of instruction and practice.

EXERCISE 4-3

2. $f(x) = 5x^2(x^3 + 2)$

$f'(x) = (5x^2)'(x^3 + 2) + 5x^2(x^3 + 2)'$ (using product rule)

$= 10x(x^3 + 2) + 5x^2(3x^2)$

$= 10x^4 + 20x + 15x^4 = 25x^4 + 20x$

4. $f(x) = (3x + 2)(4x - 5)$

$f'(x) = (3x + 2)'(4x - 5) + (3x + 2)(4x - 5)'$ (using product rule)

$= 3(4x - 5) + (3x + 2)(4)$

$= 12x - 15 + 12x + 8 = 24x - 7$

6. $f(x) = \frac{3x}{2x+1}$

$f'(x) = \frac{(3x)'(2x+1) - (2x+1)'(3x)}{(2x+1)^2}$ (using quotient rule)

$= \frac{3(2x+1) - (2)(3x)}{(2x+1)^2} = \frac{6x+3-6x}{(2x+1)^2} = \frac{3}{(2x+1)^2}$

8. $f(x) = \frac{3x-4}{2x+3}$

$f'(x) = \frac{(3x-4)'(2x+3) - (2x+3)'(3x-4)}{(2x+3)^2}$ (using quotient rule)

$= \frac{3(2x+3) - 2(3x-4)}{(2x+3)^2} = \frac{6x+9-6x+8}{(2x+3)^2} = \frac{17}{(2x+3)^2}$

10. $f(x) = x^2e^x$

Use product formula to find $f'(x)$:

$f'(x) = x^2(e^x)' + (x^2)'e^x$

$= x^2e^x + 2xe^x = x(x + 2)e^x$

12. $f(x) = 5x \ln x$

Using product formula:

$f'(x) = 5[x(\ln x)' + (x)' \ln x]$

$= 5\left[x\left(\frac{1}{x}\right) + (1)\ln x\right]$

$= 5(1 + \ln x)$

14. $f(x) = (3x + 5)(x^2 - 3)$

$f'(x) = (3x + 5)'(x^2 - 3) + (3x + 5)(x^2 - 3)'$ (using product rule)

$= 3(x^2 - 3) + (3x + 5)(2x)$ $= 3x^2 - 9 + 6x^2 + 10x$

$= 9x^2 + 10x - 9$

16. $f(x) = (0.5x - 4)(0.2x + 1)$

$f'(x) = (0.5x - 4)'(0.2x + 1) + (0.5x - 4)(0.2x + 1)'$ (using product rule)

$= 0.5(0.2x + 1) + (0.5x - 4)(0.2)$

$= 0.10x + 0.5 + 0.10x - 0.8 = 0.20x - 0.30$

18. $f(x) = \dfrac{3x+5}{x^2-3}$

$f'(x) = \dfrac{(3x+5)'(x^2-3)-(x^2-3)'(3x+5)}{(x^2-3)^2}$ (using quotient rule)

$= \dfrac{3(x^2-1)-2x(3x+5)}{(x^2-3)^2} = \dfrac{3x^2-9-6x^2-10x}{(x^2-3)^2}$

$= \dfrac{-3x^2-10x-9}{(x^2-3)^2}$

20. $f(x) = (x^2 - 4)(x^2 + 5)$

$f'(x) = (x^2 - 4)'(x^2 + 5) + (x^2 - 4)(x^2 + 5)'$ (using product rule)

$= 2x(x^2 + 5) + (x^2 - 4)(2x)$

$= 2x^3 + 10x + 2x^3 - 8x = 4x^3 + 2x$

22. $f(x) = \dfrac{x^2-4}{x^2+5}$

$f'(x) = \dfrac{(x^2-4)'(x^2+5)-(x^2+5)'(x^2-4)}{(x^2+5)^2}$ (using quotient rule)

$= \dfrac{2x(x^2+5)-(2x)(x^2-4)}{(x^2+5)^2} = \dfrac{2x^3+10x-2x^3+8x}{(x^2+5)^2}$

$= \dfrac{18x}{(x^2+5)^2}$

24. $f(x) = \dfrac{1-e^x}{1+e^x}$

Use quotient formula to find $f'(x)$:

$$f'(x) = \frac{(1-e^x)'(1+e^x)-(1+e^x)'(1-e^x)}{(1+e^x)^2}$$

$$= \frac{-e^x(1+e^x)-e^x(1-e^x)}{(1+e^x)^2}$$

$$= \frac{-e^x-e^{2x}-e^x+e^{2x}}{(1+e^x)^2} = \frac{-2e^x}{(1+e^x)^2}$$

26. $f(x) = \dfrac{2x}{1+\ln x}$

Use quotient formula:

$$f'(x) = \frac{(2x)'(1+\ln x)-(1+\ln x)'(2x)}{(1+\ln x)^2}$$

$$= \frac{2(1+\ln x)-\left(\frac{1}{x}\right)(2x)}{(1+\ln x)^2} = \frac{2+2\ln x-2}{(1+\ln x)^2}$$

$$= \frac{2\ln x}{(1+\ln x)^2}$$

28. $h(x) = x^2 f(x)$

$h'(x) = 2xf(x) + x^2 f'(x)$ (Product Rule)

30. $h(x) = \dfrac{f(x)}{x}$

$h'(x) = \dfrac{xf'(x) - f(x)}{x^2}$ (Quotient Rule)

32. $h(x) = \dfrac{f(x)}{x^3}$

$h'(x) = \dfrac{x^3 f'(x) - 3x^2 f(x)}{x^6}$ (Quotient Rule)

34. $h(x) = \dfrac{x^2}{f(x)}$

$h'(x) = \dfrac{2xf(x) - x^2 f'(x)}{(f(x))^2}$ (Quotient Rule)

36. $h(x) = \dfrac{e^x}{f(x)}$

Use quotient formula:

$$h'(x) = \frac{(e^x)' f(x) - f'(x)(e^x)}{(f(x))^2} = \frac{e^x f(x) - e^x f'(x)}{(f(x))^2} = \frac{e^x(f(x) - f'(x))}{(f(x))^2}$$

38. $h(x) = \dfrac{f(x)}{\ln x}$

Use quotient formula:

$$h'(x) = \frac{f'(x)\ln x - (\ln x)' f(x)}{(\ln x)^2} = \frac{f'(x)\ln x - \left(\frac{1}{x}\right) f(x)}{(\ln x)^2} = \frac{f'(x)x\ln x - f(x)}{x(\ln x)^2}$$

40. $y = (x^3 + 2x^2)(3x - 1)$

$y' = (x^3 + 2x^2)'(3x - 1) + (x^3 + 2x^2)(3x - 1)'$

$= (3x^2 + 4x)(3x - 1) + (x^3 + 2x^2)(3)$

$= 9x^3 + 12x^2 - 3x^2 - 4x + 3x^3 + 6x^2$

$= 12x^3 + 15x^2 - 4x$

42. $\dfrac{d}{dt}[(3 - 0.4t^3)(0.5t^2 - 2t)]$

$= \left[\dfrac{d}{dt}(3 - 0.4t^3)\right](0.5t^2 - 2t) + (3 - 0.4t^3)\left[\dfrac{d}{dt}(0.5t^2 - 2t)\right]$

$= -1.2t^2(0.5t^2 - 2t) + (3 - 0.4t^3)(t - 2)$

$= -0.6t^4 + 2.4t^3 + 3t - 6 - 0.4t^4 + 0.8t^3$

$= -t^4 + 3.2t^3 + 3t - 6$

44. $f(x) = \dfrac{3x^2}{2x - 1}$

$$f'(x) = \frac{(3x^2)'(2x-1) - (2x-1)'(3x^2)}{(2x-1)^2}$$

$$= \frac{6x(2x-1) - 2(3x^2)}{(2x-1)^2} = \frac{12x^2 - 6x - 6x^2}{(2x-1)^2} = \frac{6x^2 - 6x}{(2x-1)^2}$$

46. $y = \dfrac{w^4 - w^3}{3w - 1}$

$$\frac{dy}{dw} = \frac{\left[\frac{d}{dw}(w^4 - w^3)\right](3w-1) - \left[\frac{d}{dw}(3w-1)\right](w^4 - w^3)}{(3w-1)^2}$$

$$= \frac{(4w^3 - 3w^2)(3w-1) - (3)(w^4 - w^3)}{(3w-1)^2}$$

$$= \frac{12w^4 - 4w^3 - 9w^3 + 3w^2 - 3w^4 + 3w^3}{(3w-1)^2}$$

$$= \frac{9w^4 - 10w^3 + 3w^2}{(3w-1)^2}$$

48. $y = (1 + e^t)\ln t$

Use product formula:

$$\frac{dy}{dt} = (1 + e^t)(\ln t)' + (1 + e^t)' \ln t$$

$$= (1 + e^t)\left(\frac{1}{t}\right) + e^t \ln t$$

$$= \frac{1+e^t}{t} + e^t \ln t = \frac{1+e^t+(t\ln t)e^t}{t}$$

50. $f(x) = (7 - 3x)(1 + 2x)$

First find $f'(x)$:

$f'(x) = (7 - 3x)'(1 + 2x) + (7 - 3x)(1 + 2x)'$

$= -3(1 + 2x) + (7 - 3x)(2)$

$= -3 - 6x + 14 - 6x = -12x + 11$

An equation for the tangent line at $x = 2$ is:

$y - y_1 = m(x - x_1)$

where $x_1 = 2$, $y_1 = f(2) = 5$, and $m = f'(x_1) = f'(2) = -13$.

Thus, we have:

$y - 5 = -13(x - 2)$ or $y = -13x + 31$

52. $f(x) = \dfrac{2x-5}{2x-3}$

First find $f'(x)$:

$$f'(x) = \frac{(2x-5)'(2x-3) - (2x-3)'(2x-5)}{(2x-3)^2}$$

$$= \frac{2(2x-3) - 2(2x-5)}{(2x-3)^2} = \frac{4x - 6 - 4x + 10}{(2x-3)^2} = \frac{4}{(2x-3)^2}$$

An equation for the tangent line at $x = 2$ is:

$y - y_1 = m(x - x_1)$

where $x_1 = 2$, $y_1 = f(2) = -1$, and $m = f'(x_1) = f'(2) = 4$.

Thus, we have:

$y + 1 = 4(x - 2)$ or $y = 4x - 9$

54. $f(x) = (x - 2)\ln x$

$f'(x) = (x - 2)(\ln x)' + (x - 2)' \ln x$

$$= (x-2)\left(\frac{1}{x}\right) + (1)\ln x = \frac{x-2}{x} + \ln x$$

For $x = 2$, $m = f'(2) = \dfrac{2-2}{2} + \ln 2 = \ln 2$ is the slope of the tangent line at $x = 2$. Since $f(2) = (2 - 2)\ln 2 = 0$, the equation of the tangent line at $x = 2$ is:

$y - 0 = (\ln 2)(x - 2)$ or $y = (\ln 2)x - 2\ln 2$

56. $f(x) = (2x - 3)(x^2 - 6)$

$f'(x) = (2x - 3)'(x^2 - 6) + (2x - 3)(x^2 - 6)'$

$= 2(x^2 - 6) + (2x - 3)(2x)$

$= 2x^2 - 12 + 4x^2 - 6x = 6x^2 - 6x - 12$

To find the value(s) of x where $f'(x) = 0$, set

$f'(x) = 6x^2 - 6x - 12 = 0$

or $x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

Thus, $x = -1$, $x = 2$.

58. $f(x) = \dfrac{x}{x^2+9}$

$$f'(x) = \frac{(x)'(x^2+9)-(x^2+9)'(x)}{(x^2+9)^2}$$

$$= \frac{x^2+9-(2x)(x)}{(x^2+9)^2} = \frac{x^2+9-2x^2}{(x^2+9)^2} = \frac{9-x^2}{(x^2+9)^2}$$

To find the value(s) of x where $f'(x) = 0$, set

$f'(x) = \dfrac{9-x^2}{(x^2+9)^2} = 0$ or $9 - x^2 = 0$,

$(3 - x)(3 + x) = 0$.

Thus, $x = -3$, $x = 3$.

60. $f(x) = x^4(x^3 - 1)$

First, we use the product rule:

$f'(x) = (x^4)'(x^3 - 1) + x^4(x^3 - 1)'$

$= 4x^3(x^3 - 1) + x^4(3x^2)$

$= 4x^6 - 4x^3 + 3x^6 = 7x^6 - 4x^3$

Next, simplifying $f(x)$, we have $f(x) = x^7 - x^4$.

Thus, $f'(x) = 7x^6 - 4x^3$.

62. $f(x) = \dfrac{x^4+4}{x^4}$

First, we use the quotient rule:

$$f'(x) = \frac{(x^4+4)'(x^4)-(x^4)'(x^4+4)}{(x^4)^2} = \frac{4x^3(x^4)-(4x^3)(x^4+4)}{x^8}$$

$$= \frac{4x^7-4x^7-16x^3}{x^8} = \frac{-16x^3}{x^8} = -\frac{16}{x^5}$$

Next, simplifying $f(x)$, we have $f(x) = 1 + \frac{4}{x^4} = 1 + 4x^{-4}$.

Thus, $f'(x) = 4(-4x^{-5}) = -16x^{-5} = -\frac{16}{x^5}$.

64. $g(w) = (w - 5)\log_3 w$

Use product formula:

$g'(w) = (w - 5)(\log_3 w)' + (w - 5)' \log_3 w$

$$= (w-5)\left(\frac{1}{\ln 3}\cdot\frac{1}{w}\right) + (1)\log_3 w = \frac{w-5}{w\ln 3} + \log_3 w$$

$$= \frac{w-5+w(\ln 3)(\log_3 w)}{w\ln 3} = \frac{w-5+w\ln w}{w\ln 3}$$

66. $y = \frac{x^3-3x+4}{2x^2+3x-2}$

$$y' = \frac{(x^3-3x+4)'(2x^2+3x-2)-(2x^2+3x-2)'(x^3-3x+4)}{(2x^2+3x-2)^2}$$

$$= \frac{(3x^2-3)(2x^2+3x-2)-(4x+3)(x^3-3x+4)}{(2x^2+3x-2)^2}$$

$$= \frac{6x^4+9x^3-6x^2-6x^2-9x+6-4x^4+12x^2-16x-3x^3+9x-12}{(2x^2+3x-2)^2}$$

$$= \frac{2x^4+6x^3-16x-6}{(2x^2+3x-2)^2}$$

68. $\frac{d}{dx}[(4x^{1/2} - 1)(3x^{1/3} + 2)]$

$$= \left[\frac{d}{dx}(4x^{1/2}-1)\right](3x^{1/3} + 2) + (4x^{1/2} - 1)\left[\frac{d}{dx}(3x^{1/3}+2)\right]$$

$$= (2x^{-1/2})(3x^{1/3} + 2) + (4x^{1/2} - 1)(x^{-2/3})$$

$$= 6x^{-1/6} + 4x^{-1/2} + 4x^{-1/6} - x^{-2/3}$$

$$= 10x^{-1/6} + 4x^{-1/2} - x^{-2/3}$$

$$= \frac{10}{x^{1/6}} + \frac{4}{x^{1/2}} - \frac{1}{x^{2/3}}$$

$$= \frac{10x+4x^{2/3}-x^{1/2}}{x^{7/6}}$$

70. $y = \frac{10^x}{1+x^4}$

Use quotient formula:

$$\frac{dy}{dx} = \frac{(10^x)'(1+x^4)-(1+x^4)'(10^x)}{(1+x^4)^2}$$

$$= \frac{10^x(\ln 10)(1+x^4)-(4x^3)10^x}{(1+x^4)^2}$$

$$= \frac{10^x[(1+x^4)\ln 10-4x^3]}{(1+x^4)^2}$$

72. $y = \dfrac{2\sqrt{x}}{x^2-3x+1} = \dfrac{2x^{1/2}}{x^2-3x+1}$

$$y' = \frac{(2x^{1/2})'(x^2-3x+1)-(x^2-3x+1)'(2x^{1/2})}{(x^2-3x+1)^2}$$

$$= \frac{x^{-1/2}(x^2-3x+1)-2(2x-3)x^{1/2}}{(x^2-3x+1)^2} = \frac{(x^2-3x+1)-2x(2x-3)}{(x^2-3x+1)^2 x^{1/2}}$$

$$= \frac{x^2-3x+1-4x^2+6x}{(x^2-3x+1)^2 x^{1/2}} = \frac{-3x^2+3x+1}{(x^2-3x+1)^2 x^{1/2}}$$

74. $h(t) = \dfrac{-0.05t^2}{2t+1}$

$$h'(t) = \frac{(-0.1t)(2t+1)-(2)(-0.05t^2)}{(2t+1)^2} \quad \text{(Quotient Rule)}$$

$$= \frac{-0.2t^2-0.1t+0.1t^2}{(2t+1)^2} = \frac{-0.1t^2-0.1t}{(2t+1)^2}$$

76. Use product formula:

$$\frac{d}{dt}[10^t \log t] = 10^t \frac{d}{dt}[\log t] + (\log t)\frac{d}{dt}[10^t]$$

$$= 10^t\left(\frac{1}{\ln 10}\cdot\frac{1}{t}\right) + (\log t)(10^t \ln 10)$$

$$= \frac{10^t + 10^t t(\log t)(\ln 10)^2}{t\ln 10}$$

$$= \frac{10^t(1+(\ln 10)^2 t\log t)}{t\ln 10}$$

$$= \frac{10^t(1+(\ln 10)t(\ln 10 \log t))}{t\ln 10}$$

$$= \frac{10^t(1+t\ln t(\ln 10))}{t\ln 10}$$

78. $y = \dfrac{x^2-3x+1}{\sqrt[4]{x}} = \dfrac{x^2-3x+1}{x^{1/4}}$

$$\frac{dy}{dx} = \frac{\left[\dfrac{d}{dx}(x^2-3x+1)\right]x^{1/4} - \left[\dfrac{d}{dx}(x^{1/4})\right](x^2-3x+1)}{x^{1/2}}$$

$$= \frac{(2x-3)x^{1/4} - \left(\dfrac{1}{4}x^{-3/4}\right)(x^2-3x+1)}{x^{1/2}}$$

$$= \frac{(2x-3)x - \frac{1}{4}(x^2-3x+1)}{(x^{1/2})(x^{3/4})}$$

$$= \frac{4(2x-3)x-(x^2-3x+1)}{4x^{5/4}} = \frac{8x^2-12x-x^2+3x-1}{4x^{5/4}}$$

$$= \frac{7x^2-9x-1}{4x^{5/4}}$$

80. $y = \dfrac{2x-1}{(x^3+2)(x^2-3)}$

$$y' = \frac{(2x-1)'(x^3+2)(x^2-3)-[(x^3+2)(x^2-3)]'(2x-1)}{[(x^3+2)(x^2-3)]^2}$$

$$= \frac{2(x^3+2)(x^2-3)-[(x^3+2)'(x^2-3)+(x^3+2)(x^2-3)'](2x-1)}{[(x^3+2)(x^2-3)]^2}$$

$$= \frac{2(x^5-3x^3+2x^2-6)-[(3x^2)(x^2-3)+(x^3+2)(2x)](2x-1)}{[(x^3+2)(x^2-3)]^2}$$

$$= \frac{2x^5-6x^3+4x^2-12-[3x^4-9x^2+2x^4+4x](2x-1)}{(x^3+2)^2(x^2-3)^2}$$

$$= \frac{2x^5-6x^3+4x^2-12-6x^5+3x^4+18x^3-9x^2-4x^5+2x^4-8x^2+4x}{(x^3+2)^2(x^2-3)^2}$$

$$= \frac{-8x^5+5x^4+12x^3-13x^2+4x-12}{(x^3+2)^2(x^2-3)^2}$$

82. $y = \dfrac{u^2e^u}{1+\ln u}$

Use quotient formula:

$$\frac{dy}{du} = \frac{(u^2e^u)'(1+\ln u)-(1+\ln u)'(u^2e^u)}{(1+\ln u)^2}$$

$$= \frac{(u^2(e^u)'+(u^2)'e^u)(1+\ln u)-\left(\frac{1}{u}\right)(u^2e^u)}{(1+\ln u)^2} \quad \text{(using product formula)}$$

$$= \frac{(u^2e^u+2ue^u)(1+\ln u)-ue^u}{(1+\ln u)^2} \quad \text{(factor } ue^u\text{)}$$

$$= \frac{ue^u[(u+2)(1+\ln u)-1]}{(1+\ln u)^2}$$

$$= \frac{ue^u[u+2+(u+2)\ln u-1]}{(1+\ln u)^2}$$

$$= \frac{ue^u[(u+2)\ln u+u+1]}{(1+\ln u)^2}$$

84. $N(t) = \dfrac{180t}{t+4}$

(A) $N'(t) = \dfrac{180(t+4)-180t}{(t+4)^2} = \dfrac{180t+720-180t}{(t+4)^2} = \dfrac{720}{(t+4)^2}$

(B) $N(16) = \dfrac{180(16)}{16+20} = 144$; $N'(16) = \dfrac{720}{(16+4)^2} = 1.8$;

after 16 months, the total number of subscribers is 144,000 and is increasing at a rate of 1,800 subscribers per month.

(C) The total subscribers after 17 months will be approximately 145,800.

86. $x = \dfrac{100p}{0.1p+1}$, $10 \le p \le 70$

(A) $\dfrac{dx}{dp} = \dfrac{100(0.1p+1)-0.1(100p)}{(0.1p+1)^2} = \dfrac{10p+100-10p}{(0.1p+1)^2} = \dfrac{100}{(0.1p+1)^2}$

(B) $x(40) = \dfrac{100(40)}{0.1(40)+1} = \dfrac{4{,}000}{5} = 800$;

$\left.\dfrac{dx}{dp}\right|_{40} = \dfrac{100}{(0.1(40)+1)^2} = \dfrac{100}{25} = 4$

At a price level of \$40, the supply is 800 DVD players and is increasing at the rate of 4 players per dollar.

(C) At a price of \$41, the demand will be approximately 804 DVD players.

88. $T(x) = x^2\left(1-\dfrac{x}{9}\right)$, $0 \le x \le 7$

(A) $T'(x) = 2x\left(1-\dfrac{x}{9}\right) + x^2\left(-\dfrac{1}{9}\right) = 2x - \dfrac{2}{9}x^2 - \dfrac{1}{9}x^2 = 2x - \dfrac{1}{3}x^2$;

(B) $T'(1) = 2(1) - \dfrac{1}{3}(1)^2 = 2 - \dfrac{1}{3} = \left(\dfrac{5}{3}\right)$ per mg of drug;

$T'(3) = 2(3) - \dfrac{1}{3}(3)^2 = 6 - 3 = 3$ per mg of drug;

$T'(6) = 2(6) - \dfrac{1}{3}(6)^2 = 12 - 12 = 0$ per mg of drug.

EXERCISE 4-4

2. $f(u) = u^4$, $g(x) = 1 - 4x^3$

$f[g(x)] = (1 - 4x^3)^4$

4. $f(u) = e^u$, $g(x) = 3x^3$

$f[g(x)] = e^{3x^3}$

6. Let $u = g(x) = 2x^3 + x + 3$ and $f(u) = u^5$. Then $y = f(u) = u^5$.

8. Let $u = g(x) = x^4 + 2x^2 + 5$ and $f(u) = e^u$. Then $y = f(u) = e^u$.

10. (-2); $\frac{d}{dx}(5 - 2x)^6 = 6(5 - 2x)^5(-2) = -12(6 - 2x)^5$

12. $6x$; $\frac{d}{dx}(3x^2 + 7)^5 = 5(3x^2 + 7)^4(6x) = 30x(3x^2 + 7)^4$

14. 4; $\frac{d}{dx}e^{4x-2} = e^{4x-2}\frac{d}{dx}(4x - 2) = e^{4x-2}(4) = 4e^{4x-2}$

16. $1 - 3x^2$; $\frac{d}{dx}\ln(x - x^3) = \frac{1}{x - x^3}\frac{d}{dx}(x - x^3) = \frac{1}{x - x^3}(1 - 3x^2) = \frac{1 - 3x^2}{x - x^3}$

18. $f(x) = (x - 6)^3$
$f'(x) = 3(x - 6)^2(x - 6)' = 3(x - 6)^2(1) = 3(x - 6)^2$

20. $f(x) = (3x - 7)^5$
$f'(x) = 5(3x - 7)^4(3x - 7)' = 5(3x - 7)^4(3) = 15(3x - 7)^4$

22. $f(x) = (9 - 5x)^2$
$f'(x) = 2(9 - 5x)(9 - 5x)' = 2(9 - 5x)(-5) = -10(9 - 5x)$

24. $f(x) = (6 - 0.5x)^4$
$f'(x) = 4(6 - 0.5x)^3(6 - 0.5x)' = 4(6 - 0.5x)^3(-0.5)$
$= -2(6 - 0.5x)^3$

26. $f(x) = (5x^2 - 3)^6$
$f'(x) = 6(5x^2 - 3)^5(5x^2 - 3)' = 6(5x^2 - 3)^5(10x)$
$= 60x(5x^2 - 3)^5$

28. $f(x) = 10 - 4e^x$
$f'(x) = 0 - 4e^x(x)'$
$= -4e^x(1) = -4e^x$

30. $f(x) = 6e^{-2x}$
$f'(x) = 6e^{-2x}(-2x)'$
$= 6e^{-2x}(-2) = -12e^{-2x}$

32. $f(x) = e^{x^2+3x+1}$
$f'(x) = e^{x^2+3x+1}(x^2 + 3x + 1)'$
$= e^{x^2+3x+1}(2x + 3) = (2x + 3)e^{x^2+3x+1}$

34. $f(x) = (4x + 3)^{1/2}$

$f'(x) = \frac{1}{2}(4x + 3)^{-1/2}(4x + 3)' = \frac{1}{2}(4x + 3)^{-1/2}(4)$

$= 2(4x + 3)^{-1/2} = \frac{2}{(4x+3)^{1/2}}$

36. $f(x) = (x^5 + 2)^{-3}$

$f'(x) = (-3)(x^5 + 2)^{-4}(x^5 + 2)' = (-3)(x^5 + 2)^{-4}(5x^4)$

$= -15x^4(x^5 + 2)^{-4}$

$= -\frac{15x^4}{(x^5+2)^4}$

38. $f(x) = 8\ln x$

$f'(x) = 8\left(\frac{1}{x}\right)(x)' = 8\left(\frac{1}{x}\right)(1) = \frac{8}{x}$

40. $f(x) = 2\ln(x^2 - 3x + 4)$

$f'(x) = 2\left[\frac{1}{x^2-3x+4}(x^2-3x+4)'\right]$

$= 2\left(\frac{2x-3}{x^2-3x+4}\right) = \frac{2(2x-3)}{x^2-3x+4}$

42. $f(x) = (x - 2\ln x)^4$

$f'(x) = 4(x - 2\ln x)^3(x - 2\ln x)'$

$= 4(x - 2\ln x)^3\left(1-\frac{2}{x}\right)$

$= \frac{4(x-2)(x-2\ln x)^3}{x}$

44. $f(x) = (3x - 1)^4$

$f'(x) = 4(3x - 1)^3(3) = 12(3x - 1)^3$

Tangent line at $x = 1$: $y - y_1 = m(x - x_1)$ where $x_1 = 1$,

$y_1 = f(1) = (3(1) - 1)^4 = 16$, $m = f'(1) = 12(3(1) - 1)^3 = 96$.

Thus, $y - 16 = 96(x - 1)$ or $y = 96x - 80$.

The tangent line is horizontal at the value(s) of x such that $f'(x) = 0$:

$12(3x - 1)^3 = 0$

$3x - 1 = 0$

$x = \frac{1}{3}$

46. $f(x) = (2x + 8)^{1/2}$

$f'(x) = \frac{1}{2}(2x + 8)^{-1/2}(2) = (2x + 8)^{-1/2} = \frac{1}{(2x+8)^{1/2}}$

Tangent line at $x = 4$: $y - y_1 = m(x - x_1)$ where $x_1 = 4$,

$y_1 = f(4) = [2(4) + 8]^{1/2} = 4$, $m = f'(4) = \frac{1}{[2(4)+8]^{1/2}} = \frac{1}{4}$.

Thus, $y - 4 = \frac{1}{4}(x - 4)$ or $y = \frac{1}{4}x + 3$.

The tangent line is horizontal at the value(s) of x such that

$f'(x) = 0$.

$f'(x) = \frac{1}{(2x+8)^{1/2}} \neq 0$, so there is none.

48. $f(x) = \ln(1 - x^2 + 2x^4)$

$f'(x) = \frac{(1-x^2+2x^4)'}{1-x^2+2x^4} = \frac{-2x+8x^3}{1-x^2+2x^4} = \frac{2x(4x^2-1)}{2x^4-x^2+1}$

Tangent line at $x = 1$: $y - y_1 = m(x - x_1)$ where $x_1 = 1$,

$y_1 = f(1) = \ln(1 - (1)^2 + 2(1)^4) = \ln(2)$, $m = f'(1) = \frac{2(1)\left(4(1)^2-1\right)}{2(1)^4-(1)^2+1} = \frac{6}{3} = 3$.

Thus, $y - \ln(2) = 3(x - 1)$ or $y = 3(x - 1) + \ln(2)$.

The tangent line is horizontal at the value(s) of x such that

$f'(x) = 0$.

$f'(x) = \frac{2x(4x^2-1)}{2x^4-x^2+1} = 0.$

$\frac{2x(2x-1)(2x+1)}{2x^4-x^2+1} = 0$

$x = 0, \pm\frac{1}{2}$

50. $y = 2(x^3 + 6)^5$

$y' = 2(5)(x^3 + 6)^4(3x^2) = 30x^2(x^3 + 6)^4$

52. $\frac{d}{dt}[3(t^3 + t^2)^{-2}] = 3(-2)(t^3 + t^2)^{-3}(3t^2 + 2t)$

$= \frac{-6(3t^2+2t)}{(t^3+t^2)^3}$

54. $g(w) = \sqrt[3]{3w-7} = (3w - 7)^{1/3}$

$\frac{dg}{dw} = \frac{1}{3}(3w - 7)^{-2/3}(3) = (3w - 7)^{-2/3} = \frac{1}{\sqrt[3]{(3w-7)^2}}$

56. $h(x) = \dfrac{e^{2x}}{x^2+9}$

Use quotient formula:

$$h'(x) = \frac{(e^{2x})'(x^2+9)-(x^2+9)'(e^{2x})}{(x^2+9)^2}$$

$$= \frac{2e^{2x}(x^2+9)-2xe^{2x}}{(x^2+9)^2} = \frac{2e^{2x}[(x^2+9)-x]}{(x^2+9)^2} = \frac{2e^{2x}(x^2-x+9)}{(x^2+9)^2}$$

58. Use product formula:

$$\frac{d}{dx}[x^4 \ln(1+x^4)] = (x^4)\frac{d}{dx}[\ln(1+x^4)] + \left[\frac{d}{dx}(x^4)\right]\ln(1+x^4)$$

$$= (x^4)\frac{4x^3}{1+x^4} + (4x^3)\ln(1+x^4)$$

$$= 4x^3\left[\frac{x^4}{1+x^4}+\ln(1+x^4)\right]$$

$$= \frac{4x^3}{1+x^4}[x^4 + (1+x^4)\ln(1+x^4)]$$

60. $G(t) = (1-e^{2t})^2$

$G'(t) = 2(1-e^{2t})(1-e^{2t})'$

$= 2(1-e^{2t})(-2e^{2t})$

$= -4e^{2t}(1-e^{2t}) = 4e^{2t}(e^{2t}-1)$

62. $y = [\ln(x^2+3)]^{3/2}$

$y' = \dfrac{3}{2}[\ln(x^2+3)]^{1/2}[\ln(x^2+3)]'$

$= \dfrac{3}{2}[\ln(x^2+3)]^{1/2} \cdot \dfrac{(x^2+3)'}{x^2+3}$

$= \dfrac{3}{2}[\ln(x^2+3)]^{1/2} \cdot \dfrac{2x}{x^2+3}$

$= \dfrac{3x[\ln(x^2+3)]^{1/2}}{x^2+3}$

64. $$\frac{d}{dw}\left[\frac{1}{(w^2-2)^6}\right] = \frac{d}{dw}[(w^2-2)^{-6}]$$

$$= (-6)\cdot(w^2-2)^{-7}(2w)$$

$$= -12w(w^2-2)^{-7} = \frac{-12w}{(w^2-2)^7}$$

66. $f(x) = x^2(1 - x)^4$

$f'(x) = (x^2)'(1 - x)^4 + x^2[(1 - x)^4]'$

$= 2x(1 - x)^4 + x^2[4(1 - x)^3(-1)]$

$= 2x(1 - x)^4 - 4x^2(1 - x)^3 = 2x(1 - x)^3[(1 - x) - 2x]$

$= 2x(1 - x)^3(1 - 3x)$

$= 2x(1 - 3x)(1 - x)^3$

An equation for the tangent line to the graph of f at $x = 2$ is:
$y - y_1 = m(x - x_1)$ where $x_1 = 2$,

$y_1 = f(2) = (2)^2(1 - 2)^4 = 4$, $m = f'(2) = 2(2)[1 - 3(2)](1 - 2)^3 = 20$.

Thus, $y - 4 = 20(x - 2)$ or $y = 20x - 36$

68. $f(x) = \dfrac{x^4}{(3x-8)^2}$

$$f'(x) = \frac{(x^4)'(3x-8)^2 - [(3x-8)^2]'(x^4)}{(3x-8)^4}$$

$$= \frac{4x^3(3x-8)^2 - [2(3x-8)(3)]x^4}{(3x-8)^4}$$

$$= \frac{4x^3(3x-8)^2 - 6x^4(3x-8)}{(3x-8)^4}$$

$$= \frac{2x^3(3x-8)[2(3x-8)-3x]}{(3x-8)^4} = \frac{2x^3[6x-16-3x]}{(3x-8)^3}$$

$$= \frac{2x^3(3x-16)}{(3x-8)^3}$$

An equation for the tangent line to the graph of f at $x = 4$ is:
$y - y_1 = m(x - x_1)$ where $x_1 = 4$,

$y_1 = f(4) = \dfrac{(4)^4}{[3(4)-8]^2} = \dfrac{4^4}{4^2} = 4^2 = 16$,

$m = f'(4) = \dfrac{2(4)^3[3(4)-16]}{[3(4)-8]^3} = \dfrac{-8(4)^3}{(4)^3} = -8$.

Thus, $y - 16 = -8(x - 4)$ or $y = -8x + 48$.

70. $f(x) = e^{\sqrt{x}}$

$f'(x) = e^{\sqrt{x}}(\sqrt{x})' = e^{\sqrt{x}}(x^{1/2})' = e^{\sqrt{x}}\left(\dfrac{1}{2}x^{-1/2}\right)$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

For $x = 1$, $m = f'(1) = \dfrac{e}{2}$ is the slope of the tangent line at $x = 1$. Since $f(1) = e$, the equation of the tangent line at $x = 1$ is:

$$y - e = \frac{e}{2}(x - 1) \text{ or } y - e = \left(\frac{e}{2}\right)x - \frac{e}{2}$$

or $y = \left(\frac{e}{2}\right)x + \frac{e}{2} = \frac{e}{2}(x + 1)$

72. $f(x) = x^3(x - 7)^4$

$$\begin{aligned} f'(x) &= (x^3)'(x - 7)^4 + x^3[(x - 7)^4]' \\ &= 3x^2(x - 7)^4 + x^3[4(x - 7)^3(1)] \\ &= 3x^2(x - 7)^4 + 4x^3(x - 7)^3 \\ &= x^2(x - 7)^3[3(x - 7) + 4x] \\ &= x^2(x - 7)^3[3x - 21 + 4x] = x^2(x - 7)^3(7x - 21) \\ &= 7x^2(x - 3)(x - 7)^3 \end{aligned}$$

The tangent line to the graph of f is horizontal at the value(s) of x such that $f'(x) = 0$. Thus, we set $7x^2(x - 3)(x - 7)^3 = 0$ and $x = 0$, $x = 3$, $x = 7$.

74. $f(x) = \dfrac{x-1}{(x-3)^3}$

$$\begin{aligned} f'(x) &= \frac{(x-1)'(x-3)^3 - [(x-3)^3]'(x-1)}{(x-3)^6} \\ &= \frac{(x-3)^3 - [3(x-3)^2(1)](x-1)}{(x-3)^6} \\ &= \frac{(x-3)^3 - 3(x-3)^2(x-1)}{(x-3)^6} = \frac{(x-3)^2[(x-3) - 3(x-1)]}{(x-3)^6} \\ &= \frac{[x - 3 - 3x + 3]}{(x-3)^4} = \frac{-2x}{(x-3)^4} \end{aligned}$$

The tangent line to the graph of f is horizontal at the value(s) of x such that $f'(x) = 0$. Thus, we set $-2x = 0$ and $x = 0$.

76. $f(x) = \sqrt{x^2 + 4x + 5} = (x^2 + 4x + 5)^{1/2}$

$$\begin{aligned} f'(x) &= \frac{1}{2}(x^2 + 4x + 5)^{-1/2}(2x + 4) \\ &= \frac{x+2}{(x^2 + 4x + 5)^{1/2}} \end{aligned}$$

The tangent line to the graph of f is horizontal at the value(s) of x such that $f'(x) = 0$. Thus, we set

$$\frac{x+2}{(x^2 + 4x + 5)^{1/2}} = 0$$

$$x + 2 = 0$$

and $x = -2$

78. $f'(x) = (1)\ln(x + 1) + (x + 1)\cdot\frac{1}{x+1} - 1 = \ln(x + 1)$

$g'(x) = \frac{1}{3}(x + 1)^{-2/3}$

which are not the same function. All four functions appear in the view window $0 \le x \le 5$, $0 \le y \le 3$.

80.
$$\begin{aligned}\frac{d}{dx}[2x^2(x^3 - 3)^4] &= \left[\frac{d}{dx}(2x^2)\right](x^3 - 3)^4 + 2x^2\left[\frac{d}{dx}(x^3 - 3)^4\right]\\ &= 4x(x^3 - 3)^4 + 2x^2[4(x^3 - 3)^3(3x^2)]\\ &= 4x(x^3 - 3)^4 + 24x^4(x^3 - 3)^3\\ &= 4x(x^3 - 3)^3[(x^3 - 3) + 6x^3]\\ &= 4x(x^3 - 3)^3(7x^3 - 3)\end{aligned}$$

82.
$$\begin{aligned}\frac{d}{dx}\left[\frac{3x^2}{(x^2+5)^3}\right] &= \frac{\left[\frac{d}{dx}(3x^2)\right](x^2+5)^3 - \left[\frac{d}{dx}(x^2+5)^3\right](3x^2)}{(x^2+5)^6}\\ &= \frac{6x(x^2+5)^3 - [3(x^2+5)^2(2x)](3x^2)}{(x^2+5)^6}\\ &= \frac{6x(x^2+5)^3 - 18x^3(x^2+5)^2}{(x^2+5)^6}\\ &= \frac{6x(x^2+5)^2[(x^2+5) - 3x^2]}{(x^2+5)^6}\\ &= \frac{6x(5-2x^2)}{(x^2+5)^4} = \frac{30x - 12x^3}{(x^2+5)^4}\end{aligned}$$

84. $\frac{d}{dx}\log(x^3 - 1) = \frac{1}{\ln 10}\cdot\frac{1}{x^3-1}(3x^2) = \frac{1}{\ln 10}\cdot\frac{3x^2}{x^3-1}$

86. $\frac{d}{dx}8^{1-2x^2} = 8^{1-2x^2}(-4x)(\ln 8) = -4x\,8^{1-2x^2}(\ln 8)$

88.
$$\begin{aligned}\frac{d}{dx}\log_5(5^{x^2-1}) &= \frac{1}{\ln 5}\cdot\frac{1}{5^{x^2-1}}(5^{x^2-1}(2x)(\ln 5))\\ &= 2x\end{aligned}$$

or

$$\begin{aligned}\frac{d}{dx}\log_5(5^{x^2-1}) &= \frac{d}{dx}[(x^2 - 1)\log_5 5]\\ &= \frac{d}{dx}(x^2 - 1) = 2x\end{aligned}$$

90. $\frac{d}{dx}10^{\ln x} = 10^{\ln x}\left(\frac{1}{x}\right)(\ln 10) = \frac{\ln 10}{x}10^{\ln x}$

92. $C(x) = 6 + \sqrt{4x+4} = 6 + (4x + 4)^{1/2}, 0 \le x \le 30$

(A) $C'(x) = \frac{1}{2}(4x + 4)^{-1/2}(4) = 2(4x + 4)^{-1/2} = \frac{2}{(4x+4)^{1/2}}$

(B) $C'(15) = \frac{2}{[4(15)+4]^{1/2}} = \frac{2}{8} = \frac{1}{4} = 0.25$ or \$25.

At a production level of 15 cameras, total costs are increasing at the rate of \$25 per camera; also, the cost of producing the 16th camera is approximately \$25.

$C'(24) = \frac{2}{[4(24)+4]^{1/2}} = \frac{2}{10} = 0.2$ or \$20.

At a production level of 24 cameras, total costs are increasing at the rate of \$20 per camera; also, the cost of producing the 25th camera is approximately \$20.

94. $x = 1{,}000 - 60\sqrt{p+25} = 1{,}000 - 60(p + 25)^{1/2}, 20 \le p \le 100.$

(A) $\frac{dx}{dp} = -60\left(\frac{1}{2}\right)(p + 25)^{-1/2} = \frac{-30}{(p+25)^{1/2}}$

(B) $x(75) = 1{,}000 - 60(75 + 25)^{1/2} = 1{,}000 - 600 = 400$

$\left.\frac{dx}{dp}\right|_{75} = \frac{-30}{(75+25)^{1/2}} = \frac{-30}{10} = -3$

At a price of \$75, the demand is 400 bicycle helmets and is decreasing at the rate of 3 helmets per dollar.

96. $C(t) = 250(1 - e^{-t}), t \ge 0$

(A) $C'(t) = 250e^{-t}$

$C'(1) = 250e^{-1} \approx 92$

$C'(4) = 250e^{-4} \approx 4.6$

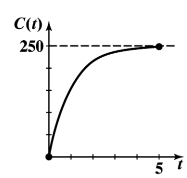
Thus, at the end of 1 minute concentration is increasing at the rate of 92 micrograms/milliliter per minute; At the end of 4 minutes the concentration is increasing at the rate of 4.6 micrograms/milliliter per minute.

(B) $C'(t) = 250e^{-t} > 0$ on (0, 5)

Thus, C is increasing on (0, 5); there are no local extrema.

$C''(t) = -250e^{-t} < 0$ on $(0, \infty)$.

Thus, the graph is concave downward on (0, 5).



t	$C(t)$
0	0
1	158.03
4	245.42
5	248.32

98. $T(t) = 30e^{-0.58t} + 38,\ t \geq 0$

$T'(t) = 30e^{-0.58t}(-0.58) = -17.4e^{-0.58t}$

$T'(1) = -17.4e^{-0.58(1)} \approx -9.74°$F per hour.

$T'(4) = -17.4e^{-0.58(4)} \approx -1.71°$F per hour.

EXERCISE 4-5

2. $-2x + 6y - 4 = 0$

(A) Implicit differentiation:

$$\frac{d}{dx}(-2x) + \frac{d}{dx}(6y) + \frac{d}{dx}(-4) = \frac{d}{dx}(0)$$
$$-2 + 6y' - 0 = 0$$
$$y' = \frac{2}{6} = \frac{1}{3}$$

(B) $6y = 2x + 4$

$$y = \frac{1}{3}x + \frac{2}{3}$$
$$y' = \frac{1}{3}$$

4. $2x^3 + 5y - 2 = 0$

(A) Implicit differentiation:

$$\frac{d}{dx}(2x^3) + \frac{d}{dx}(5y) + \frac{d}{dx}(-2) = \frac{d}{dx}(0)$$
$$6x^2 + 5y' + 0 = 0$$
$$5y' = -6x^2$$
$$y' = -\frac{6}{5}x^2$$

(B) $5y = -2x^3 + 2$

$$y = -\frac{2}{5}x^3 + \frac{2}{5}$$
$$y' = -\frac{6}{5}x^2$$

6. $5x^3 - y - 1 = 0$

$$\frac{d}{dx}(5x^3) + \frac{d}{dx}(-y) + \frac{d}{dx}(-1) = \frac{d}{dx}(0)$$
$$15x^2 - y' + 0 = 0$$
$$y' = 15x^2$$
$$y'\Big|_{(1,4)} = 15(1)^2 = 15$$

8. $y^2 + x^3 + 4 = 0$

$$\frac{d}{dx}(y^2) + \frac{d}{dx}(x^3) + \frac{d}{dx}(4) = \frac{d}{dx}(0)$$
$$2yy' + 3x^2 + 0 = 0$$
$$2yy' = -3x^2$$
$$y' = -\frac{3x^2}{2y}$$
$$y'\Big|_{(-2,2)} = -\frac{3(-2)^2}{2(2)} = -\frac{12}{4} = -3$$

10. $y^2 - y - 4x = 0$

$$\frac{d}{dx}(y^2) - \frac{d}{dx}(y) - \frac{d}{dx}(4x) = \frac{d}{dx}(0)$$

$$2yy' - y' - 4 = 0$$
$$y'(2y - 1) = 4$$
$$y' = \frac{4}{2y-1}$$
$$y'\Big|_{(0,1)} = \frac{4}{2(1)-1} = 4$$

12. $3xy - 2x - 2 = 0$

$$\frac{d}{dx}(3xy) - \frac{d}{dx}(2x) - \frac{d}{dx}(2) = \frac{d}{dx}(0)$$
$$3y + 3xy' - 2 - 0 = 0$$
$$3xy' = 2 - 3y$$
$$y' = \frac{2-3y}{3x}$$
$$y' \text{ at } (2, 1) = \frac{2-3(1)}{3(2)} = -\frac{1}{6}$$

14. $2y + xy - 1 = 0$

$$\frac{d}{dx}(2y) + \frac{d}{dx}(xy) - \frac{d}{dx}(1) = \frac{d}{dx}(0)$$
$$2y' + y + xy' - 0 = 0$$
$$y'(x + 2) = -y$$
$$y' = -\frac{y}{x+2}$$
$$y' \text{ at } (-1, 1) = -\frac{1}{-1+2} = -1$$

16. $2x^3y - x^3 + 5 = 0$

$$\frac{d}{dx}(2x^3y) - \frac{d}{dx}(x^3) + \frac{d}{dx}(5) = \frac{d}{dx}(0)$$
$$6x^2y + 2x^3y' - 3x^2 + 0 = 0$$
$$2x^3y' = 3x^2 - 6x^2y$$
$$y' = \frac{3x^2 - 6x^2y}{2x^3} = \frac{3(1-2y)}{2x}$$
$$y' \text{ at } (-1, 3) = \frac{3(1-2(3))}{2(-1)} = \frac{-15}{-2} = \frac{15}{2}$$

18. $x^2 - y = 4e^y$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(y) = \frac{d}{dx}(4e^y)$$
$$2x - y' = 4e^y y'$$
$$y'(1 + 4e^y) = 2x$$
$$y' = \frac{2x}{1+4e^y}$$
$$y' \text{ at } (2, 0) = \frac{2(2)}{1+4e^0} = \frac{4}{5}$$

20. $\ln y = 2y^2 - x$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(2y^2) - \frac{d}{dx}(x)$$
$$\frac{1}{y}\cdot y' = 4yy' - 1$$
$$y' = 4y^2y' - y$$
$$y'(4y^2 - 1) = y$$
$$y' = \frac{y}{4y^2-1}$$
$$y' \text{ at } (2, 1) = \frac{1}{4(1)^2-1} = \frac{1}{3}$$

22. $xe^y - y = x^2 - 2$

$$\frac{d}{dx}(xe^y) - \frac{d}{dx}(y) = \frac{d}{dx}(x^2) - \frac{d}{dx}(2)$$

$$e^y + xe^y \cdot y' - y' = 2x - 0$$

$$y'(xe^y - 1) = 2x - e^y$$

$$y' = \frac{2x - e^y}{xe^y - 1}$$

$$y' \text{ at } (2, 0) = \frac{2(2) - e^0}{2e^0 - 1} = \frac{4-1}{2-1} = 3$$

24. $x^3 - tx^2 - 4 = 0$

$$\frac{d}{dt}(x^3) - \frac{d}{dt}(tx^2) - \frac{d}{dt}(4) = \frac{d}{dt}(0)$$

$$3x^2x' - x^2 - 2txx' - 0 = 0$$

$$x'(3x^2 - 2tx) = x^2$$

$$x' = \frac{x^2}{x(3x-2t)} = \frac{x}{3x-2t}$$

x' at $(-3, -2) = \dfrac{-2}{3(-2)-2(-3)} = \dfrac{-2}{0}$, so x' is not defined at $(-3, -2)$

26. $(x - 1)^2 + (y - 1)^2 = 1.$

Differentiating implicitly, we have:

$$\frac{d}{dx}(x - 1)^2 + \frac{d}{dx}(y - 1)^2 = \frac{d}{dx}(1)$$

$$2(x - 1) + 2(y - 1)y' = 0$$

$$y' = -\frac{x-1}{y-1}$$

To find the points on the graph where $x = 0.2$, we solve the given equation for y:

$$(y - 1)^2 = 1 - (x - 1)^2$$

$$y - 1 = \pm\sqrt{1-(x-1)^2}$$

$$y = 1 \pm \sqrt{1-(x-1)^2}$$

Now, when $x = 0.2$, $y = 1 + \sqrt{1-0.64} = 1 + \sqrt{0.36}$
$= 1 + 0.6$
$= 1.6$

and $y = 1 - \sqrt{0.36} = 1 - 0.6 = 0.4$. Thus, the points are $(0.2, 1.6)$ and $(0.2, 0.4)$. These values can be verified on the graph.

$$y'\Big|_{(0.2,1.6)} = -\frac{0.2-1}{1.6-1} = \frac{0.8}{0.6} = \frac{4}{3}$$

$$y'\Big|_{(0.2,0.4)} = -\frac{0.2-1}{0.4-1} = -\frac{0.8}{0.6} = -\frac{4}{3}$$

28. $3x + xy + 1 = 0$

When $x = -1$, $3(-1) + (-1)y + 1 = 0$, so $y = -2$. Thus, we want to find the equation of the tangent line at $(-1, -2)$.

First, find y'.

$$\frac{d}{dx}(3x) + \frac{d}{dx}(xy) + \frac{d}{dx}(1) = \frac{d}{dx}(0)$$

$$\begin{aligned} 3 + y + xy' + 0 &= 0 \\ xy' &= -(y + 3) \\ y' &= -\frac{y+3}{x} \\ y'\Big|_{(-1,-2)} &= -\frac{-2+3}{-1} = 1 \end{aligned}$$

Thus, the slope of the tangent line at $(-1, -2)$ is $m = 1$. The equation of the line through $(-1, -2)$ with slope $m = 1$ is:

$$\begin{aligned} y + 2 &= (x + 1) \\ y &= x - 1 \end{aligned}$$

30. $xy^2 - y - 2 = 0$

$$\begin{aligned} \text{When } x = 1, y^2 - y - 2 &= 0 \\ (y + 1)(y - 2) &= 0 \\ y &= -1 \text{ or } 2 \end{aligned}$$

Thus, we have to find the equations of the tangent lines at $(1, -1)$ and $(1, 2)$. First find y':

$$\frac{d}{dx}(xy^2) - \frac{d}{dx}(y) - \frac{d}{dx}(0) = \frac{d}{dx}(0)$$

$$\begin{aligned} y^2 + 2xyy' - y' - 0 &= 0 \\ y'(2xy - 1) &= -y^2 \\ y' &= \frac{y^2}{1-2xy} \\ y'\Big|_{(1,-1)} &= \frac{(-1)^2}{1-2(1)(-1)} = \frac{1}{3} \end{aligned}$$

The equation of the tangent line at $(1, -1)$ with $m = \frac{1}{3}$ is:

$$\begin{aligned} y + 1 &= \frac{1}{3}(x - 1) \\ y &= \frac{1}{3}x - \frac{4}{3} \\ y'\Big|_{(1,2)} &= \frac{(2)^2}{1-2(1)(2)} = -\frac{4}{3} \quad \text{[slope at (1, 2)]} \end{aligned}$$

Thus, the equation of the tangent line at $(1, 2)$ with $m = -\frac{4}{3}$ is:

$$\begin{aligned} y - 2 &= -\frac{4}{3}(x - 1) \\ y &= -\frac{4}{3}x + \frac{10}{3} \end{aligned}$$

32. Since y appears in two places as polynomial of degree one and as exponent we cannot express y as an explicit function of x. We need to use implicit differentiation to find the slope of the tangent line to the graph of the equation at the point (0, 1).

$$x^3 + y + xe^y = 1$$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y) + \frac{d}{dx}(xe^y) = \frac{d}{dx}(1)$$

$$3x^2 + y' + e^y + xe^y \cdot y' = 0$$

$$y'(xe^y + 1) = -(e^y + 3x^2)$$

$$y' = -\frac{e^y + 3x^2}{xe^y + 1}$$

$$y'\Big|_{(0,1)} = -\frac{e^1 + 3(0)^2}{0e^1 + 1} = -e$$

34. $(y - 3)^4 - x = y$

$$\frac{d}{dx}(y - 3)^4 - \frac{d}{dx}(x) = \frac{d}{dx}(y)$$

$$4(y - 3)^3 y' - 1 = y'$$

$$y'[4(y - 3)^3 - 1] = 1$$

$$y' = \frac{1}{4(y-3)^3 - 1}$$

$$y'\Big|_{(-3,4)} = \frac{1}{4(4-3)^3 - 1} = \frac{1}{3}$$

36. $(2x - y)^4 - y^3 = 8$

$$\frac{d}{dx}(2x - y)^4 - \frac{d}{dx}(y^3) = \frac{d}{dx}(8)$$

$$4(2x - y)^3(2 - y') - 3y^2 y' = 0$$

$$[4(2x - y)^3 + 3y^2]y' = 8(2x - y)^3$$

$$y' = \frac{8(2x-y)^3}{4(2x-y)^3 + 3y^2}$$

$$y'\Big|_{(-1,-2)} = \frac{8(2(-1)-(-2))^3}{4(2(-1)-(-2))^3 + 3(-2)^2}$$

$$= \frac{8(0)^3}{4(0)^3 + 12} = 0$$

38. $6\sqrt{y^3 + 1} - 2x^{3/2} - 2 = 0$

$$6(y^3 + 1)^{1/2} - 2x^{3/2} - 2 = 0$$

$$\frac{d}{dx}(6(y^3 + 1)^{1/2}) - \frac{d}{dx}(2x^{3/2}) - \frac{d}{dx}(2) = \frac{d}{dx}(0)$$

$$6\left(\frac{1}{2}\right)(y^3 + 1)^{-1/2}(3y^2y') - 3x^{1/2} - 0 = 0$$

$$9y^2(y^3 + 1)^{-1/2}y' = 3x^{1/2}$$

$$y' = \frac{x^{1/2}}{3y^2(y^3+1)^{-1/2}} = \frac{x^{1/2}(y^3+1)^{1/2}}{3y^2}$$

$$y'\Big|_{(4,2)} = \frac{4^{1/2}(2^3+1)^{1/2}}{3(2)^2} = \frac{(2)(3)}{12} = \frac{1}{2}$$

40. $e^{xy} - 2x = y + 1$

$$\frac{d}{dx}(e^{xy}) - \frac{d}{dx}(2x) = \frac{d}{dx}(y) + \frac{d}{dx}(1)$$

$$e^{xy}(y + xy') - 2 = y' + 0$$

$$ye^{xy} + xy'e^{xy} - 2 = y'$$

$$y'(xe^{xy} - 1) = 2 - ye^{xy}$$

$$y' = \frac{2 - ye^{xy}}{xe^{xy} - 1}; \quad y'\Big|_{(0,0)} = \frac{2 - 0e^0}{0e^0 - 1} = -2$$

42. First find the point(s) on the graph of the equation with $y = -1$:
Setting $y = -1$, we have

$$(-1)^3 - x(-1) - x^3 = 2$$

$$-1 + x - x^3 = 2$$

$$x^3 - x + 3 = 0$$

Graphing this equation on a graphing utility, we get $x \approx -1.67$.

Now, differentiate implicitly to find the slope of the tangent line at the point (-1.67, -1):

$$\frac{d}{dx}(y^3) - \frac{d}{dx}(xy) - \frac{d}{dx}(x^3) = \frac{d}{dx}(2)$$

$$3y^2y' - y - xy' - 3x^2 = 0$$

$$(3y^2 - x)y' = y + 3x^2$$

$$y' = \frac{y + 3x^2}{3y^2 - x}$$

$$y'\Big|_{(-1.67,-1)} = \frac{-1 + 3(-1.67)^2}{3(-1)^2 - (-1.67)} \approx 1.58$$

Tangent line: $y + 1 = 1.58(x + 1.67)$ or $y = 1.58x + 1.64$

44. $x = p^3 - 3p^2 + 200$

$$\frac{d}{dx}(x) = \frac{d}{dx}(p^3) - \frac{d}{dx}(3p^2) + \frac{d}{dx}(200)$$

$$1 = 3p^2\frac{dp}{dx} - 6p\frac{dp}{dx} + 0$$

$$1 = 3p(p - 2)\frac{dp}{dx}$$

Thus,

$$p' = \frac{dp}{dx} = \frac{1}{3p(p-2)}.$$

46. $x = \sqrt[3]{1{,}500 - p^3} = (1{,}500 - p^3)^{1/3}$

$$\frac{dx}{dx} = \frac{d}{dx}(1{,}500 - p^3)^{1/3}$$

$$1 = \frac{1}{3}(1{,}500 - p^3)^{-2/3}\left(-3p^2\frac{dp}{dx}\right)$$

$$3(1{,}500 - p^3)^{2/3} = -3p^2\frac{dp}{dx}$$

$$p' = \frac{dp}{dx} = -\frac{(-1{,}500 - p^3)^{2/3}}{p^2} \quad \text{or} \quad p' = -\frac{x^2}{p^2}$$

48. $(L + m)(V + n) = k$

$$\frac{d}{dL}((L + m)(V + n)) = \frac{d}{dL}(k)$$

$$V + n + (L + m)\frac{dV}{dL} = 0$$

$$\frac{dV}{dL} = -\frac{(V + n)}{(L + m)}$$

50. $F = G\dfrac{m_1m_2}{r^2}$

$$\frac{d}{dF}(F) = \frac{d}{dF}\left(G\frac{m_1m_2}{r^2}\right)$$

$$\frac{d}{dF}(F) = Gm_1m_2\frac{d}{dF}\left(r^{-2}\right)$$

$$1 = Gm_1m_2\left(-2r^{-3}\right)\frac{dr}{dF}$$

$$1 = -\frac{2Gm_1m_2}{r^3}\frac{dr}{dF}$$

$$\frac{dr}{dF} = -\frac{r^3}{2Gm_1m_2}$$

52. $F = G\dfrac{m_1 m_2}{r^2}$

$\dfrac{d}{dr}(F) = \dfrac{d}{dr}\left(G\dfrac{m_1 m_2}{r^2}\right)$

$\dfrac{d}{dr}(F) = Gm_1 m_2 \dfrac{d}{dr}\left(r^{-2}\right)$

$\dfrac{dF}{dr} = Gm_1 m_2\left(-2r^{-3}\right)$

$\dfrac{dF}{dr} = -\dfrac{2Gm_1 m_2}{r^3}$ This is the reciprocal of the answer to Problem 50

EXERCISE 4-6

2. $y = x^3 - 3$

Differentiating with respect to t:

$\dfrac{dy}{dt} = 3x^2\dfrac{dx}{dt}$

Given: $\dfrac{dx}{dt} = -2$ when $x = 2$. Thus, we have

$\dfrac{dy}{dt} = 3(2)^2(-2) = -24$

4. $x^2 + y^2 = 4$

Differentiating with respect to t:

$2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

Given: $\dfrac{dy}{dt} = 5$ when $x = 1.2$ and $y = -1.6$. Therefore

$2(1.2)\dfrac{dx}{dt} + 2(-1.6)(5) = 0$

$2.4\dfrac{dx}{dt} = 16$

$\dfrac{dx}{dt} = \dfrac{16}{2.4} = \dfrac{160}{24} = \dfrac{20}{3}$

6. $x^2 - 2xy - y^2 = 7$

Differentiating with respect to t:

$$2x\frac{dx}{dt} - 2\frac{dx}{dt}y - 2x\frac{dy}{dt} - 2y\frac{dy}{dt} = 0$$

Given: $\frac{dy}{dt} = -1$ when $x = 2$ and $y = -1$. Therefore

$$\begin{aligned}
2(2)\frac{dx}{dt} - 2\frac{dx}{dt}(-1) - 2(2)(-1) - 2(-1)(-1) &= 0 \\
4\frac{dx}{dt} + 2\frac{dx}{dt} + 4 - 2 &= 0 \\
6\frac{dx}{dt} &= -2 \\
\frac{dx}{dt} &= -\frac{1}{3}
\end{aligned}$$

8. $4x^2 + 9y^2 = 36$

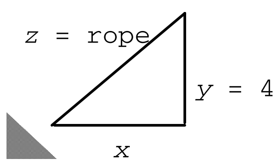
Differentiate with respect to t:

$$8x\frac{dx}{dt} + 18y\frac{dy}{dt} = 0$$

Given: $\frac{dy}{dt} = -2$ when $x = 3$ and $y = 0$. Therefore

$$\begin{aligned}
8(3)\frac{dx}{dt} + 18(0)(-2) &= 0 \\
24\frac{dx}{dt} &= 0 \\
\frac{dx}{dt} &= 0
\end{aligned}$$

The x coordinate does not change at that moment.

10. 

From the triangle,

$$\begin{aligned}
x^2 + y^2 &= z^2 \\
\text{or } x^2 + 16 &= z^2, \text{ since } y = 4.
\end{aligned}$$

Differentiate with respect to t:

$$\begin{aligned}
2x\frac{dx}{dt} &= 2z\frac{dz}{dt} \\
\text{or } x\frac{dx}{dt} &= z\frac{dz}{dt}
\end{aligned}$$

Given: $\frac{dx}{dt} = -3.05$ when $x = 10$ and $z = \sqrt{100+16} \approx 10.77$.

Therefore,

$$\begin{aligned}
10(-3.05) &= 10.77\frac{dz}{dt} \\
\frac{dz}{dt} &= -\frac{30.5}{10.77} \approx -2.83 \text{ feet per second.}
\end{aligned}$$

12. Circumference: $C = 2\pi R$

$\frac{dC}{dt} = 2\pi \frac{dR}{dt}$

Given: $\frac{dR}{dt} = 2$ ft/sec

$\frac{dC}{dt} = 2\pi(2) = 4\pi$

≈ 12.56 ft/sec

14. Surface area: $S = 4\pi R^2$

$\frac{dS}{dt} = 8\pi R \frac{dR}{dt}$

Given: $\frac{dR}{dt} = 3$ cm/min

$\frac{dS}{dt} = 8\pi R(3) = 24\pi R$

$\left.\frac{dS}{dt}\right|_{R = 10 \text{ cm}}$

$= 240\pi$

$\approx 753.6\,\text{cm}^2/\text{min}$

16. $VP = k$

Differentiating with respect to t:

$\frac{dV}{dt} P + V \frac{dP}{dt} = 0$

Given: $\frac{dV}{dt} = -5$ in^3/sec, $V = 1{,}000$ in^3, $P = 40$ pounds per square inch.

Thus, we have $(-5)(40) + 1{,}000 \frac{dP}{dt} = 0$

$\frac{dP}{dt} = \frac{200}{1{,}000} = 0.2$

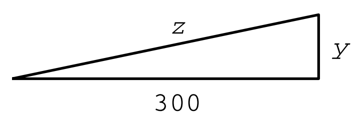
Pressure increases at 0.2 pound per square inch per second.

18. By the Pythagorean theorem,

$z^2 = (300)^2 + y^2$ (1)

Differentiating with respect to t:

$z\frac{dz}{dt} = y\frac{dy}{dt}$



Therefore, $\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt}$. Given: $\frac{dy}{dt} = 5$. Thus, $\frac{dz}{dt} = \frac{5y}{z}$.

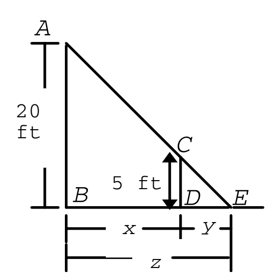
From (1), $z^2 = (300)^2 + y^2 = (300)^2 + (400)^2 = 250{,}000$ when $y = 400$.

Thus, $z = 500$ when $y = 400$, and $\left.\frac{dz}{dt}\right|_{(400,500)} = \frac{5(400)}{500} = 4$ m/sec.

20. y = length of shadow

x = distance of man from light

z = distance of tip of shadow from light



We want to compute $\frac{dy}{dt}$. Triangles ABE and CDE are similar triangles; thus, the ratios of corresponding sides are equal.

Therefore, $\frac{z}{20} = \frac{y}{5}$ or $\frac{x+y}{20} = \frac{y}{5}$ [Note: $z = x + y$]

or $x + y = 4y$ or $y = \frac{1}{3}x$

Differentiating with respect to t:

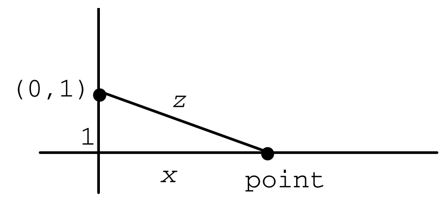
$$\frac{dy}{dt} = \frac{1}{3}\frac{dx}{dt}$$

Given: $\frac{dx}{dt} = 5$. Thus, $\frac{dy}{dt} = \frac{5}{3}$ ft/sec.

22. Observe that

$z^2 = x^2 + 1$

Differentiating with respect to t:



$$z\frac{dz}{dt} = x\frac{dx}{dt}$$

$$x = \frac{z\frac{dz}{dt}}{\frac{dx}{dt}}$$

Given: $\frac{dx}{dt} = 5$, $z = \sqrt{x^2+1}$.

Thus, $x = \frac{1}{5}\sqrt{x^2+1}\,\frac{dz}{dt}$ (1)

From (1), for $\frac{dz}{dt} = 2$, we have:

$x = \frac{2}{5}\sqrt{x^2+1}$ or $x^2 = \frac{4(x^2+1)}{25}$ or

$21x^2 = 4$, $x^2 = \frac{4}{21}$, $x \approx 0.4364$

From (1), for $\frac{dz}{dt} = 4$, we have

$x = \frac{4}{5}\sqrt{x^2+1}$ or $25x^2 = 16(x^2 + 1)$

$9x^2 = 16$, $x^2 = \frac{16}{9}$, $x = \frac{4}{3} = 1.3333$

From (1), for $\frac{dz}{dt} = 5$, we have $x = \sqrt{x^2+1}$ which is impossible.

Therefore, the distance from (0, 1) is never increasing at ≥ 5 units per second.

24. $x^3 + y^2 = 1$; $\frac{dy}{dt} = 2$, $\frac{dx}{dt} = 1$

Differentiating with respect to t:

$3x^2\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ [Note: this equation has a solution for x only when $y \leq 0$.]

or

$3x^2(1) + 2y(2) = 0$

or

$3x^2 + 4y = 0$

From $x^3 + y^2 = 1$, $y = -\sqrt{1-x^3}$ and hence

$$\begin{aligned} 3x^2 - 4\sqrt{1-x^3} &= 0 \\ 3x^2 &= 4\sqrt{1-x^3} \\ 9x^4 &= 16(1 - x^3) \\ 9x^4 + 16x^3 - 16 &= 0 \end{aligned}$$

Using a graphing utility, we find $x \approx 0.875$ and $x \approx -2$.

Therefore the points at which the x coordinate increasing at a rate of 1 unit per second are: (0.875, -0.574) and (-2, -3).

[Note: For $x = 0.875$, $y = -\sqrt{1-x^3} = -0.574$ and for $x = -2$, $y = -\sqrt{1-x^3} = -3$.]

26. $C = 72{,}000 + 60x$ (1)

$R = 200x - \dfrac{x^2}{30}$ (2)

$P = R - C$ (3)

(A) Differentiating (1) with respect to t:

$$\frac{dC}{dt} = 60\frac{dx}{dt}$$

Thus, $\dfrac{dC}{dt} = 60(500) \quad \left(\dfrac{dx}{dt} = 500\right)$

$= \$30{,}000$ per week.

Costs are increasing at \$30,000 per week at this production level.

(B) Differentiating (2) with respect to t:

$$\frac{dR}{dt} = 200\frac{dx}{dt} - \frac{1}{15}x\frac{dx}{dt} = \left(200 - \frac{x}{15}\right)\frac{dx}{dt}$$

Thus, $\dfrac{dR}{dt} = \left(200 - \dfrac{1{,}500}{15}\right)(500) \quad \left(x = 1{,}500, \dfrac{dx}{dt} = 500\right)$

$= \$50{,}000$ per week.

Revenue is increasing at \$50,000 per week at this production level.

(C) Differentiating (3) with respect to t:

$$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt}$$

Thus, from parts (A) and (B), we have:

$\frac{dP}{dt}$ = 50,000 - 30,000 = \$20,000

Profits are increasing at \$20,000 per week at this production level.

28. $S = 50{,}000 - 20{,}000e^{-0.0004x}$

Differentiating implicitly with respect to t, we have

$$\frac{dS}{dt} = -20{,}000e^{-0.0004x}(-0.0004)\frac{dx}{dt}$$
$$= 8e^{-0.0004x}\frac{dx}{dt}$$

Now, for $x = 2{,}000$ and $\frac{dx}{dt} = 300$, we have

$$\frac{dS}{dt} = 8e^{-0.0004(2000)}(300)$$
$$= 2{,}400e^{-0.8} \approx 1{,}078$$

Thus, sales are increasing at the rate of \$1,078 per week.

30. Price p and demand x are related by the equation

$$x^2 + 2xp + 25p^2 = 74{,}500 \qquad (1)$$

Differentiating implicitly with respect to t, we have

$$2x\frac{dx}{dt} + 2\frac{dx}{dt}p + 2x\frac{dp}{dt} + 50p\frac{dp}{dt} = 0 \qquad (2)$$

(A) From (2), $\frac{dx}{dt} = \frac{-(x+25p)\frac{dp}{dt}}{x+p}$

Setting $p = 30$ in (1), we get

$x^2 + 60x + 22{,}500 = 74{,}500$

or $x^2 + 60x - 52{,}000 = 0$

Thus, $x = -30 \pm \sqrt{(30)^2 + 52{,}000}$

$= -30 \pm 230 = 200, -260$

Since $x \geq 0$, $x = 200$

Now, for $x = 200$, $p = 30$ and $\frac{dp}{dt} = 2$, we have

$$\frac{dx}{dt} = \frac{-(200+25(30))(2)}{200+30} = -\frac{1{,}900}{230} \approx -8.26$$

The demand is decreasing at the rate of 8.26 units/month.

(B) From (2), $\dfrac{dp}{dt} = -\dfrac{(x+p)\dfrac{dx}{dt}}{x+25p}$

Setting $x = 150$ in (1), we get

$$(150)^2 + 2(150)p + 25p^2 = 74{,}500$$
$$22{,}500 + 300p + 25p^2 = 74{,}500$$
or $$p^2 + 12p - 2{,}080 = 0$$
and $$p = -6 \pm \sqrt{36+2080} = -6 \pm 46 = 40, -52$$

Since $p \geq 0$, $p = 40$.

Now, for $x = 150$, $p = 40$ and $\dfrac{dx}{dt} = -6$, we have

$$\frac{dp}{dt} = -\frac{(150+40)(-6)}{150+25(40)} \approx 0.99$$

Thus, the price is increasing at the rate of \$0.99 per month.

32. $T = 6\left(1+\dfrac{1}{\sqrt{x}}\right) = 6(1 + x^{-1/2})$

Differentiating with respect to t:

$$\frac{dT}{dt} = 6\left(-\frac{1}{2}\right)x^{-3/2}\left(\frac{dx}{dt}\right) = -3x^{-3/2}\frac{dx}{dt}$$

or

$$\frac{dT}{dt} = -\frac{3}{x^{3/2}} \cdot \frac{dx}{dt}$$

Given: $\dfrac{dx}{dt} = 6$, $x = 36$. Therefore,

$$\frac{dT}{dt} = -\frac{3}{(36)^{3/2}}(6) = -\frac{18}{216} = -\frac{1}{12}$$ of a minute/hour.

EXERCISE 4-7

2. $f(x) = 60x - 1.2x^2$

$f'(x) = 60 - 2.4x$

$$\frac{f'(x)}{f(x)} = \frac{60-2.4x}{60x-1.2x^2}.$$

4. $f(x) = 15 - 3e^{-0.5x}$

$f'(x) = 1.5e^{-0.5x}$

$$\frac{f'(x)}{f(x)} = \frac{1.5e^{-0.5x}}{15-3e^{-0.5x}}.$$

6. $f(x) = 25 - 2\ln x$

$f'(x) = \frac{-2}{x}$

$\frac{f'(x)}{f(x)} = \frac{\frac{-2}{x}}{25 - 2\ln x} = -\frac{2}{25x - 2x\ln x}$

8. $f(x) = 580$

$f'(x) = 0$

$\frac{f'(x)}{f(x)} = \frac{0}{580} = 0$

$\frac{f'(300)}{f(300)} = 0$

10. $f(x) = 500 - 6x$

$f'(x) = -6$

$\frac{f'(x)}{f(x)} = \frac{-6}{500 - 6x}$

$\frac{f'(40)}{f(40)} = \frac{-6}{500 - 6(40)} = -\frac{3}{130} \approx -0.023$

12. $f(x) = 500 - 6x$

$f'(x) = -6$

$\frac{f'(x)}{f(x)} = \frac{-6}{500 - 6x}$

$\frac{f'(75)}{f(75)} = \frac{-6}{500 - 6(75)} = -\frac{3}{25} = -0.12$

14. $f(x) = 9x - 5\ln x$

$f'(x) = 9 - \frac{5}{x}$

$\frac{f'(x)}{f(x)} = \frac{9 - \frac{5}{x}}{9x - 5\ln x} = \frac{9x - 5}{9x^2 - 5x\ln x}$

$\frac{f'(3)}{f(3)} = \frac{9(3) - 5}{9(3)^2 - 5(3)\ln(3)} \approx 0.341$

16. $f(x) = 9x - 5\ln x$

$f'(x) = 9 - \frac{5}{x}$

$\frac{f'(x)}{f(x)} = \frac{9 - \frac{5}{x}}{9x - 5\ln x} = \frac{9x - 5}{9x^2 - 5x\ln x}$

$\frac{f'(7)}{f(7)} = \frac{9(7) - 5}{9(7)^2 - 5(7)\ln(7)} \approx 0.156$

18. $f(x) = 75 + 110x$

$f'(x) = 110$

$\frac{f'(x)}{f(x)} = \frac{110}{75 + 110x}$

$100 \times \frac{f'(4)}{f(4)} = 100 \times \frac{110}{75 + 110(4)} \approx 21.4\%$

20. $f(x) = 75 + 110x$

$f'(x) = 110$

$\frac{f'(x)}{f(x)} = \frac{110}{75 + 110x}$

$100 \times \frac{f'(16)}{f(16)} = 100 \times \frac{110}{75 + 110(16)} \approx 6.00\%$

22. $f(x) = 3{,}000 - 8x^2$

$f'(x) = -16x$

$\frac{f'(x)}{f(x)} = \frac{-16x}{3{,}000 - 8x^2}$

$100 \times \frac{f'(12)}{f(12)} = 100 \times \frac{-16(12)}{3{,}000 - 8(12)^2} \approx -10.4\%$

24. $f(x) = 3{,}000 - 8x^2$

$f'(x) = -16x$

$\frac{f'(x)}{f(x)} = \frac{-16x}{3{,}000 - 8x^2}$

$100 \times \frac{f'(18)}{f(18)} = 100 \times \frac{-16(18)}{3000 - 8(18)^2} \approx -70.6\%$

26. $f(p) = 10{,}000 - 190p$

$f'(p) = -190$

$$E(p) = -\frac{pf'(p)}{f(p)} = \frac{190p}{10{,}000 - 190p}$$

28. $f(p) = 8{,}400 - 7p^2$

$f'(p) = -14p$

$$E(p) = -\frac{pf'(p)}{f(p)} = \frac{14p^2}{8{,}400 - 7p^2}$$

30. $f(p) = 160 - 35\ln p$

$$f'(p) = \frac{-35}{p}$$

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p\left(\frac{-35}{p}\right)}{160 - 35\ln p} = \frac{35}{160 - 35\ln p}$$

32. $x = f(p) = 1{,}875 - p^2$

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{-p(-2p)}{1{,}875 - p^2} = \frac{2p^2}{1{,}875 - p^2}$$

(A) $E(15) = \dfrac{2(15)^2}{1{,}875 - (15)^2} = 0.\overline{27} < 1$; INELASTIC

(B) $E(25) = \dfrac{2(25)^2}{1{,}875 - (25)^2} = 1$; UNIT ELASTICITY

(C) $E(40) = \dfrac{2(40)^2}{1{,}875 - (40)^2} = 11.64 > 1$; ELASTIC

34. $x = f(p) = 875 - p - 0.05p^2$

$$E(p) = \frac{-p(-1 - 0.10p)}{875 - p - 0.05p^2} = \frac{p + 0.10p^2}{875 - p - 0.05p^2}$$

(A) $E(50) = \dfrac{50 + 0.10(50)^2}{875 - 50 - 0.05(50)^2} = 0.43 < 1$; INELASTIC

(B) $E(70) = \dfrac{70 + 0.10(70)^2}{875 - 70 - 0.05(70)^2} = 1$; UNIT ELASTICITY

(C) $E(100) = \dfrac{100 + 0.10(100)^2}{875 - 100 - 0.05(100)^2} = 4$; ELASTIC

36. $p + 0.01x = 50$

(A) $0.01x = 50 - p$, $x = \dfrac{50}{0.01} - \dfrac{1}{0.01}p$

$= 5{,}000 - 100p$, $0 \le p \le 50$

(B) $E(p) = -\dfrac{p(-100)}{5{,}000-100p} = \dfrac{p}{50-p}$

(C) $E(10) = \dfrac{10}{50-10} = 0.25$; $(0.25)(5\%) = 1.25\%$ increase

(D) $E(45) = \dfrac{45}{50-45} = 9$; $9(5\%) = 45\%$ increase

(E) $E(25) = \dfrac{25}{50-25} = 1$; $1(5\%) = 5\%$ increase

38. $0.025x + p = 50$

(A) $0.025x = 50 - p$, $x = \dfrac{50}{0.025} - \dfrac{1}{0.025}p = 2{,}000 - 40p$, $0 \le p \le 50$

(B) $R(p) = px = p(2{,}000 - 40p) = 2{,}000p - 40p^2$

(C) $E(p) = \dfrac{-p(-40)}{2{,}000-40p} = \dfrac{40p}{2{,}000-40p} = \dfrac{p}{50-p}$

(D) $E(p) > 1$ if $\dfrac{p}{50-p} > 1$ or $p > 50 - p$ or $p > 25$.

Thus, Elastic on (25, 50) and Inelastic on (0, 25).

(E) Inelastic on (0, 25) implies revenue increase on (0, 25).
Elastic on (25, 50) implies revenue decrease on (25, 50).

(F) Since $p = \$10 < \$25$, a decrease in price results in decrease in revenue.

(G) Since $p = \$40 > \$25$, a decrease in price results in increase in revenue.

40. $x = f(p) = 480 - 8p$, $480 - 8p > 0$, so $0 < p < 60$.

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{-p(-8)}{480-8p} = \frac{p}{60-p}$$

$E(p) = \dfrac{p}{60-p} > 1$ implies that $p > 30$. Thus, Elastic on (30, 60) and Inelastic on (0, 30).

42. $x = f(p) = 2{,}400 - 6p^2,\ 2{,}400 - 6p^2 > 0$, so $0 < p < 20$.

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{-p(-12p)}{2{,}400-6p^2} = \frac{12p^2}{2{,}400-6p^2} = \frac{2p^2}{400-p^2}$$

$E(p) = \dfrac{2p^2}{400-p^2} > 1$ implies that $p > \dfrac{20}{\sqrt{3}}$. Thus, Elastic on $\left(\dfrac{20}{\sqrt{3}}, 20\right)$ and Inelastic on $\left(0, \dfrac{20}{\sqrt{3}}\right)$.

44. $x = f(p) = \sqrt{324-2p}$ $\quad 324 - 2p \ge 0$ or $0 \le p \le 162$

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{-p\left[\frac{1}{2}(-2)(324-2p)^{-1/2}\right]}{(324-2p)^{1/2}} = \frac{p}{324-2p}$$

$E(p) = \dfrac{p}{324-2p} > 1$ implies that $p > 108$. Thus, Elastic on (108, 162) and Inelastic on (0, 108).

46. $x = f(p) = \sqrt{3{,}600-2p^2}$, $3{,}600 - 2p^2 \ge 0$, $0 \le p \le 30\sqrt{2}$

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{-p\left[\frac{1}{2}(-4p)(3{,}600-2p^2)^{-1/2}\right]}{(3{,}600-2p^2)^{1/2}} = \frac{2p^2}{3{,}600-2p^2}$$

$E(p) = \dfrac{2p^2}{3{,}600-2p^2} > 1$ implies that $2p^2 > 3{,}600 - 2p^2$

or $4p^2 > 3{,}600$ or $p^2 > 900$ or $30 < p < 30\sqrt{2}$

Therefore, Elastic on $(30, 30\sqrt{2})$ and Inelastic on (0, 30).

48. $x = f(p) = 10(16 - p)$, $0 \le p \le 16$

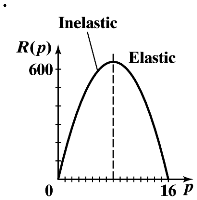
$R(p) = px = p[10(16 - p)] = 160p - 10p^2$.

$R'(p) = 160 - 20p$

Critical value in the interval (0, 16) is $p = 8$.

$R(p)$ is a parabola and its graph over [0, 16] is given below.

Note: $R'(p) > 0$ ($R(p)$ increasing) on (0, 8) corresponds to Inelastic and $R'(p) < 0$ ($R(p)$ decreasing) on (8, 16) corresponds to Elastic.



50. $x = f(p) = 10(p - 9)^2$, $0 \le p \le 9$

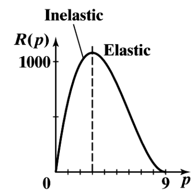
$R(p) = px = 10p(p - 9)^2$

$R'(p) = 10(p - 9)^2 + 20p(p - 9) = 10(p - 9)[p - 9 + 2p] = 10(p - 9)(3p - 9)$

Critical value in the interval (0, 9) is $p = 3$.

$R'(p) > 0$ for $3p - 9 < 0$ or $p < 3$ and $R'(p) < 0$ for $3p - 9 > 0$ or $p > 3$. (Note that $p - 9 \le 0$).

The graph of $R(p)$ is:



Note: $R'(p) > 0$ on (0, 3),
thus Inelastic.
$R'(p) < 0$ on (3, 9),
thus Elastic.

52. $x = f(p) = 30 - 5\sqrt{p} \geq 0$ implies $0 \leq p \leq 36$

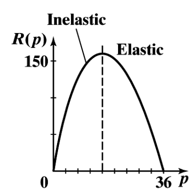
$(30 - 5\sqrt{p} \geq 0$ or $\sqrt{p} \leq 6$ or $p \leq 36)$

$R(p) = px = p(30 - 5\sqrt{p}) = 30p - 5p^{3/2}$

$R'(p) = 30 - \frac{15}{2}p^{1/2} > 0$ if $30 > \frac{15}{2}p^{1/2}$ or

$p^{1/2} < 4$ or $p < 16$. Thus $R'(p) > 0$ (or Inelastic) on (0, 16) and
$R'(p) < 0$ (Elastic) on (16, 36).

The graph of $R(p)$ is:



54. $p = g(x) = 30 - 0.05x$

$g'(x) = -0.05$

$$E(x) = -\frac{g(x)}{xg'(x)} = -\frac{30 - 0.05x}{-0.05x} = \frac{600}{x} - 1$$

For $x = 400$, $E(400) = \frac{600}{400} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$.

56. $p = g(x) = 20 - \sqrt{x}$

$$g'(x) = -\frac{1}{2\sqrt{x}}$$

$$E(x) = -\frac{20 - \sqrt{x}}{x\left(-\frac{1}{2\sqrt{x}}\right)} = \frac{20 - \sqrt{x}}{\frac{1}{2}\sqrt{x}} = \frac{2(20 - \sqrt{x})}{\sqrt{x}}$$

For $x = 100$, $E(100) = \frac{2(20 - 10)}{10} = 2$

58. $p = g(x) = 640 - 0.4x,\ 640 - 0.4x > 0$ so $0 < p < 1{,}600$.

$g'(x) = -0.4$

$$E(x) = -\frac{g(x)}{xg'(x)} = -\frac{640 - 0.4x}{x(-0.4)} = \frac{1{,}600 - x}{x}$$

$E(x) = \frac{1{,}600 - x}{x} > 1$ implies that $p < 800$. Thus, Elastic on (0, 800) and Inelastic on (800, 1,600).

60. $p = g(x) = 540 - 0.2x^2$, $540 - 0.2x^2 > 0$ so $0 < p < 30\sqrt{3}$.

$g'(x) = -0.4x$

$$E(x) = -\frac{g(x)}{xg'(x)} = -\frac{540 - 0.2x^2}{x(-0.4x)} = \frac{2{,}700 - x^2}{2x^2}$$

$E(x) = \dfrac{2{,}700 - x^2}{2x^2} > 1$ implies that $x < 30$. Thus, Elastic on (0, 30) and Inelastic on $\left(30, 30\sqrt{3}\right)$.

62. $x = f(p) = Ae^{-kp}$

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{-p\left[-kAe^{-kp}\right]}{Ae^{-kp}} = \frac{kApe^{-kp}}{Ae^{-kp}} = kp$$

64. (0.80)(45) = \$36 per day

66. $x = 3{,}000 - 400p$

$$E(p) = \frac{-p(-400)}{3{,}000 - 400p} = \frac{2p}{15 - 2p}$$

$$E(4) = \frac{2(4)}{15 - 2(4)} = \frac{8}{7} > 1$$

Thus, a 10% increase in the price will result in a decrease in revenue.

68. $x = 2{,}500 - 1{,}000p$

$$E(p) = \frac{-p(-1{,}000)}{2{,}500 - 1{,}000p} = \frac{2p}{5 - 2p}$$

$$E(1.29) = \frac{2(1.29)}{5 - 2(1.29)} = \frac{129}{121} > 1$$

Thus, a 10% decrease in the price will result in an increase in revenue.

70. From Problem 67, we have

$R(p) = p(2{,}500 - 1{,}000p) = 2{,}500p - 1{,}000p^2$

$R'(p) = 2{,}500 - 2{,}000p = 0$, $p = \dfrac{2{,}500}{2{,}000} = 1.25$

$R''(p) = -2{,}000 < 0$ for all p.

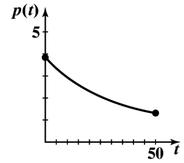
Thus, $p = \$1.25$ will maximize the revenue from selling fries.

72. $f(t) = 1.49t + 38.8$

$f'(t) = 1.49$

$$\frac{f'(t)}{f(t)} = \frac{1.49}{1.49t + 38.8}$$

$$p(t) = 100 \times \frac{f'(t)}{f(t)} = \frac{149}{1.49x + 38.8}$$



74. $a(t) = 18.2 - 5.2\ln t$

$$f'(t) = -\frac{5.2}{t}$$

$$\frac{f'(t)}{f(t)} = \frac{-\frac{5.2}{t}}{18.2 - 5.2\ln t} = \frac{-5.2}{18.2t - 5.2t\ln t}$$

For 2008, $t = 18$, and $\frac{f'(18)}{f(18)} = \frac{-520}{18.2(18) - 5.2(18)\ln(18)} \approx -0.09$.