SOLUTION Section 1.2 – Trigonometric Functions

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-2, 3)$ is on the terminal side of θ.

Solution

$$\Rightarrow r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

$$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} \qquad \tan\theta = \frac{y}{x} = -\frac{3}{2} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = -\frac{\sqrt{13}}{2}$$

$$\cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{13}} \qquad \cot\theta = \frac{x}{y} = -\frac{2}{3} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-3, -4)$ is on the terminal side of θ.

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-4)^2} = 5$$

$$\sin\theta = -\frac{4}{5} \qquad \tan\theta = \frac{-4}{-3} = \frac{4}{3} \qquad \csc\theta = -\frac{5}{4}$$

$$\cos\theta = -\frac{3}{5} \qquad \cot\theta = \frac{3}{4} \qquad \sec\theta = -\frac{5}{3}$$

Exercise

Find the six trigonometry functions of θ in standard position with terminal side through the point $(-3, 0)$.

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 0^2} = 3$$

$$\sin\theta = \frac{0}{3} = 0 \qquad \tan\theta = \frac{0}{-3} = 0 \qquad \csc\theta = \frac{1}{0} \to \infty$$

$$\cos\theta = \frac{-3}{3} = -1 \qquad \cot\theta = \frac{1}{0} = \infty \qquad \sec\theta = \frac{1}{-1} = -1$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(12, -5)$ is on the terminal side of θ.

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{12^2 + (-5)^2} = \underline{13|}$$

$$\sin\theta = -\frac{5}{13} \qquad \tan\theta = -\frac{5}{12} \qquad \csc\theta = -\frac{13}{5}$$

$$\cos\theta = \frac{12}{13} \qquad \cot\theta = -\frac{12}{5} \qquad \sec\theta = \frac{13}{12}$$

Exercise

Find the values of the six trigonometric functions for an angle of $90°$.

Solution

$$\sin 90° = 1 \qquad \tan 90° = \infty \qquad \csc 90° = 1$$

$$\cos 90° = 0 \qquad \cot 90° = 0 \qquad \sec 90° = \infty$$

Exercise

Indicate the two quadrants θ could terminate in if $\cos\theta = \frac{1}{2}$

Solution

$$\cos\theta = \frac{1}{2} \qquad \rightarrow \text{QI \& QIV}$$

Exercise

Indicate the two quadrants θ could terminate in if $\csc\theta = -2.45$

Solution

$$\csc\theta = -2.45 = \frac{1}{\sin\theta} \qquad \rightarrow \text{QIII \& QIV}$$

Exercise

Find the remaining trigonometric function of θ if $\sin\theta = \frac{12}{13}$ and θ terminates in QI

Solution

$$x = \sqrt{13^2 - 12^2} = 5$$

$$\sin\theta = \frac{12}{13} = \frac{y}{r} \qquad \tan\theta = \frac{y}{x} = \frac{12}{5} \qquad \csc\theta = \frac{13}{12}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{13} \qquad \cot\theta = \frac{x}{y} = \frac{5}{12} \qquad \sec\theta = \frac{13}{5}$$

Exercise

Find the remaining trigonometric function of θ if $\cot\theta = -2$ and θ terminates in QII.

Solution

$$\cot\theta = -2 = \frac{x}{y} \quad (\theta \in QII) \quad \Rightarrow \boxed{x = -2, \ \ y = 1}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (1)^2} = \underline{\sqrt{5}}|$$

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{5}}, \quad \cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{5}}$$

$$\tan\theta = \frac{y}{x} = -\frac{1}{2}, \quad \sec\theta = \frac{r}{x} = -\frac{\sqrt{5}}{2}, \quad \csc\theta = \frac{r}{y} = \sqrt{5}$$

Exercise

Find the remaining trigonometric function of θ if $\tan\theta = \frac{3}{4}$ and θ terminates in QIII.

Solution

$$\tan\theta = \frac{3}{4} = \frac{y}{x} \quad (\theta \in QIII) \quad \Rightarrow \boxed{x = -4, \ \ y = -3}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + (-3)^2} = \underline{5}|$$

$$\sin\theta = \frac{y}{r} = -\frac{3}{5}, \quad \cos\theta = \frac{x}{r} = -\frac{4}{5}$$

$$\cot\theta = \frac{x}{y} = \frac{4}{3}, \quad \sec\theta = \frac{r}{x} = -\frac{5}{4}, \quad \csc\theta = \frac{r}{y} = -\frac{5}{3}$$

Exercise

Find the remaining trigonometric function of θ if $\cos\theta = \frac{24}{25}$ and θ terminates in QIV.

Solution

$$\cos\theta = \frac{24}{25} = \frac{x}{r} \quad (\theta \in QIV) \quad \Rightarrow \boxed{x = 24}$$

$$y = -\sqrt{r^2 - x^2} = -\sqrt{(25)^2 - (24)^2} = \underline{-7|}$$

$$\sin\theta = \frac{y}{r} = -\frac{7}{25}$$

$$\tan\theta = \frac{y}{x} = -\frac{7}{24}, \quad \cot\theta = \frac{x}{y} = -\frac{24}{7}$$

$$\sec\theta = \frac{r}{x} = \frac{25}{24}, \quad \csc\theta = \frac{r}{y} = -\frac{25}{7}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos\theta = \frac{\sqrt{3}}{2}$ and θ is terminates in QIV.

Solution

$$\cos\theta = \frac{\sqrt{3}}{2} = \frac{x}{r} \quad \Rightarrow x = \sqrt{3}, \ r = 2$$

$$x^2 + y^2 = r^2$$

$$y^2 = r^2 - x^2$$

$$y = \pm\sqrt{r^2 - x^2}$$

Since θ is QIV $\Rightarrow y = -\sqrt{2^2 - \sqrt{3}^2}$

$$= -\sqrt{4-3}$$

$$\underline{= -1|}$$

$$\sin\theta = \frac{y}{r} = -\frac{1}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \qquad \cot\theta = -\sqrt{3}$$

$$\sec\theta = \frac{r}{x} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \qquad \csc\theta = \frac{r}{y} = \frac{2}{-1} = -2$$

Exercise

Find the remaining trigonometric functions of θ if $\tan\theta = -\frac{1}{2}$ and $\cos\theta > 0$.

Solution

$$\tan\theta = \frac{\sin\theta}{\cos\theta} < 0 \ \ \& \ \ \cos\theta > 0 \ \ \Rightarrow \sin\theta < 0 \Rightarrow \theta \text{ in QIV}$$

$$\tan\theta = -\frac{1}{2} = \frac{y}{x}$$

$$\Rightarrow y = 1, x = 2 \rightarrow r = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{5}} \qquad \cos\theta = \frac{x}{r} = \frac{2}{\sqrt{5}}$$

$$\cot\theta = -2$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{1} \qquad \csc\theta = \frac{r}{y} = \frac{2}{-1} = -2$$

Exercise

If $\sin\theta = -\frac{5}{13}$, and θ is QIII, find $\cos\theta$ and $\tan\theta$.

Solution

$$\sin\theta = -\frac{5}{13} = \frac{y}{r} \quad \rightarrow y = -5, \ \ r = 13$$

$$r^2 = x^2 + y^2$$

$$\Rightarrow x^2 = r^2 - y^2$$

$$\Rightarrow x = \sqrt{r^2 - y^2}$$

$$\Rightarrow x = \sqrt{13^2 - 5^2} = \pm 12 \quad \text{Since } \ \theta \text{ is Q III} \Rightarrow x = -12$$

$$\cos\theta = \frac{x}{r} = -\frac{12}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$$

Exercise

If $\cos\theta = \frac{3}{5}$, and θ is QIV, find $\sin\theta$ and $\tan\theta$.

Solution

$$\cos\theta = \frac{3}{5} = \frac{x}{r} \quad (\theta \in QIV) \quad \Rightarrow \boxed{x=3} \qquad y = \underline{-4|}$$

$$\sin\theta = -\frac{4}{5}, \quad \tan = -\frac{4}{3}$$

Exercise

Use the reciprocal identities if $\cos\theta = \frac{\sqrt{3}}{2}$ find $\sec\theta$

Solution

$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{2}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

Exercise

Find $\cos\theta$, given that $\sec\theta = \frac{5}{3}$

Solution

$$\cos\theta = \frac{1}{\sec\theta}$$

$$= \frac{1}{\frac{5}{3}}$$

$$= \frac{3}{5}$$

Exercise

Find $\sin\theta$, given that $\csc\theta = -\frac{\sqrt{12}}{2}$

Solution

$$\begin{aligned}\sin\theta &= \frac{1}{\csc\theta}\\ &= -\frac{2}{\sqrt{12}}\frac{\sqrt{12}}{\sqrt{12}}\\ &= -\frac{2\sqrt{12}}{12}\\ &= -\frac{\sqrt{12}}{6}\end{aligned}$$

Exercise

Use a ratio identity to find $\tan\theta$ if $\sin\theta = \frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$

Solution

$$\begin{aligned}\tan\theta &= \frac{\sin\theta}{\cos\theta}\\ &= \frac{\frac{3}{5}}{-\frac{4}{5}}\\ &= -\frac{3}{4}\end{aligned}$$

Exercise

If $\cos\theta = -\frac{1}{2}$ and θ terminates in QII, find $\sin\theta$

Solution

$$\begin{aligned}\sin\theta &= \sqrt{1-\cos^2\theta}\\ &= \sqrt{1-\frac{1}{4}}\\ &= \sqrt{\frac{3}{4}}\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

Exercise

If $\sin\theta = \frac{3}{5}$ and θ terminated in QII, find $\cos\theta$ and $\tan\theta$.

Solution

$$\cos\theta = -\sqrt{1-\sin^2\theta}$$

$$= -\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= -\sqrt{1-\frac{9}{25}}$$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{3/5}{-4/5}$$

$$= -\frac{3}{4}$$

Exercise

Find $\tan\theta$ if $\sin\theta = \frac{1}{3}$ and θ terminates in QI

Solution

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\frac{1}{9}}$$

$$= \sqrt{\frac{8}{9}}$$

$$= \frac{\sqrt{8}}{3}$$

$$= \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}$$

$$= \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}$$

Exercise

Find the remaining trigonometric ratios of θ, if $\sec\theta = -3$ and $\theta \in QIII$

Solution

$$\sec\theta = \frac{1}{\cos\theta} = -3 \qquad \Rightarrow \cos\theta = -\frac{1}{3}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

$$\cot\theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{3}{2\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

Exercise

Using the calculator and rounding your answer to the nearest hundredth, find the remaining trigonometric ratios of θ if $\csc\theta = -2.45$ and $\theta \in QIII$

Solution

$$\sin\theta = \frac{1}{\csc} = \frac{1}{-2.45} = -.41$$

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-.41^2} = -.91$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-.41}{-.91} = .45$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{.45} = 2.22$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-.91} = -1.1$$

Exercise

Write $\dfrac{\sec\theta}{\csc\theta}$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

Solution

$$\frac{sec\,\theta}{csc\,\theta} = \frac{\frac{1}{cos\,\theta}}{\frac{1}{sin\,\theta}}$$

$$= \frac{1}{cos\,\theta}\frac{sin\,\theta}{1}$$

$$= \frac{sin\,\theta}{cos\,\theta}$$

Exercise

Write $\cot\theta - \csc\theta$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

Solution

$$cot\,\theta - csc\,\theta = \frac{cos\,\theta}{sin\,\theta} - \frac{1}{sin\,\theta}$$

$$= \frac{cos\,\theta - 1}{sin\,\theta}$$

Exercise

Write $\dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\sin\theta}$ in terms of $\sin\theta$ and/or $\cos\theta$, and then simplify if possible.

Solution

$$\frac{\sin\theta}{\cos\theta} + \frac{1}{\sin\theta} = \frac{\sin^2\theta + \cos\theta}{\cos\theta\sin\theta}$$

Exercise

Write $\sin\theta\cot\theta + \cos\theta$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify if possible.

Solution

$$\sin\theta\cot\theta + \cos\theta = \sin\theta\frac{\cos\theta}{\sin\theta} + \cos\theta$$

$$= \cos\theta + \cos\theta$$

$$= 2\cos\theta$$

Exercise

Multiply $(1-\cos\theta)(1+\cos\theta)$

Solution

$$\begin{aligned}(1-\cos\theta)(1+\cos\theta) &= 1-\cos^2\theta \\ &= \sin^2\theta\end{aligned}$$

Exercise

Multiply $(\sin\theta+2)(\sin\theta-5)$

Solution

$$(\sin\theta+2)(\sin\theta-5)=\sin^2\theta-3\sin\theta-10$$

Exercise

Simplify the expression $\sqrt{25-x^2}$ as much as possible after substituting $5\sin\theta$ for x.

Solution

$$\begin{aligned}&\sqrt{25-x^2}=\sqrt{25-(5\sin\theta)^2} \\ &=\sqrt{25-25\sin^2\theta} \\ &=\sqrt{25(1-\sin^2\theta)} \\ &=\sqrt{25}\sqrt{\cos^2\theta} \\ &=5\cos\theta\end{aligned}$$

Exercise

Simplify the expression $\sqrt{4x^2+16}$ as much as possible after substituting $2\tan\theta$ for x

Solution

$$\begin{aligned}\sqrt{4x^2+16} &= \sqrt{4(2\tan\theta)^2+16} \\ &= \sqrt{16\tan^2\theta+16} \\ &= \sqrt{16(\tan^2\theta+1)} \\ &= 4\sqrt{\tan^2\theta+1} \\ &= 4\sqrt{\sec^2\theta} \\ &= 4\sec\theta\end{aligned}$$