Factorial

The Factorial of a non-negative integer n is denoted by $n!$. The factorial notation $n!$ was introduced by Christian Kramp in 1808.

Factorial on any number is the product of positive less than or equal to that number (n).

$$n! = \prod_{k=1}^{n} k$$

$$= \begin{cases} 1 & if \quad n = 0 \\ (n-1)! \times n & if \quad n > 0 \end{cases}$$

Example: $6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$

$0! = 1$

Double Factorial

The product of all odd integers up to some odd positive integer n is called the double factorial of n, denoted by $n!!$.

$$(2n-1)!! = \prod_{k=1}^{n} (2k-1)$$

$$n!! = \prod_{k=1}^{\frac{n+1}{2}} (2k-1)$$

$$= n(n-2)(n-4) \cdots 3 \cdot 1$$

Example: $9!! = 1 \times 3 \times 5 \times 7 \times 9 = 945$

For even positive integer n the double factorial is

$$(2n)!! = \prod_{k=1}^{n} (2k)$$

$$n!! = \prod_{k=1}^{\frac{n}{2}} (2k)$$

$$= n(n-2)(n-4) \cdots 4 \cdot 2$$

Example: $8!! = 1 \times 2 \times 4 \times 6 \times 8 = 384$

Triple Factorial

The product of all odd integers up to some odd positive integer n is called the triple factorial of n, denoted by $n!!!$.

$$n!!! = n(n-3)(n-6)\cdots$$

Example:

$$5!!! = 5\times 2 = 10$$
$$6!!! = 6\times 3\times 0! = 18$$
$$7!!! = 7\times 4\times 1 = 28$$
$$13!!! = 13\times 10\times 7\times 4\times 1 = 3{,}640$$

n	$n!!!$
1	1
2	2
3	3
4	4
5	10
6	18
7	28
8	80
9	162
10	280

Multifactorial

A common related notation is to use multiple exclamation points to denote a multifactorial, the product of integers in steps of two $(n!!)$, three $(n!!!)$

$$n! = n!!\times(n-1)!! \qquad n\geq 1$$
$$= n!!!\times(n-1)!!!\times(n-2)!!! \qquad n\geq 2$$
$$= \prod_{i=0}^{k-1}(n-i)!^{(k)}$$