Cal III

$$f(x,y) = x^2 + 3xy + y - 1$$

Find. $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ @ $(4,-5)$

Soln

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^2 + 3xy + \overbrace{y-1}^{\text{constant.}}\right)$$

$$= 2x + 3y \Big|_{(4,-5)}$$

$$= 8 - 15$$

$$= \boxed{-7}$$

ct: constant

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\overbrace{x^2}^{\text{constant}} + 3xy + y - \overbrace{1}^{ct}\right)$$

$$= 3x + 1 \Big|_{(4,-5)}$$

$$= \boxed{13}$$

$f(x,y) = y \sin xy$ $\qquad f_x$, f_y

$$f_x = y^2 \cos xy$$

$(xy)_x$

$(uv)' = u'v + v'u$

$$f_y = \boxed{\sin xy + xy\cos xy}$$

Ex $f(x,y) = \dfrac{2y}{y+\cos x}$

$f_x = \dfrac{+2y\sin x}{(y+\cos x)^2}$

$\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$f_x = \dfrac{2y\sin x}{(y+\cos x)^2}$

$\left(\dfrac{ay+b}{cy+d}\right)' = \dfrac{ady' - bcy'}{(cy+d)'}$

$\dfrac{0x+2y}{\cos x + y}$

$f_y = \dfrac{2\cos x}{(y+\cos x)^2}$

Ex $\dfrac{\partial z}{\partial x}$: $\underset{ct}{y}z - \ln z = x + \underset{ct}{y}$

Soln

$\dfrac{\partial}{\partial x}(yz) - \dfrac{\partial}{\partial x}(\ln z) = \dfrac{\partial}{\partial x}(x) + \dfrac{\partial}{\partial x}(y)$

$y\dfrac{\partial z}{\partial x} - \dfrac{1}{z}\dfrac{\partial z}{\partial x} = 1$

$\left(y - \dfrac{1}{z}\right)\dfrac{\partial z}{\partial x} = 1$

$\dfrac{yz-1}{z} = 1$

$\dfrac{\partial z}{\partial x} = \dfrac{z}{yz-1}$

Ex Plane: $x = 1$ $z = x^2 + y^2$ (paraboloid)

@ $(1, 2, 5)$

Soln

$$\frac{\partial z}{\partial y} = 2y \Big|_{(1,2,5)}$$

$$= \underline{4}$$

Ex $f(x, y, z) = x \sin(y + 3z)$

$$\frac{\partial f}{\partial x} = \sin(y + 3z)$$

$$\frac{\partial f}{\partial y} = x \cos(y + 3z)$$

$$\frac{\partial f}{\partial z} = 3x \cos(y + 3z)$$

Ex R_1, R_2, R_3 // equivalent R

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\boxed{\frac{\partial R}{\partial R_2}} \quad \frac{\partial}{\partial R_2}\left(\frac{1}{R}\right) = 0 + \frac{\partial}{\partial R_2}\left(\frac{1}{R_2}\right) + 0$$

$$-\frac{1}{R^2} \frac{\partial R}{\partial R_2} = -\frac{1}{R_2^2}$$

$$\frac{\partial R}{\partial R_2} = \frac{R^2}{R_2^2}$$

$$\frac{\partial^2 f}{\partial x^2} : f_{xx} \qquad \frac{\partial^2 f}{\partial y^2} : f_{yy}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} = f_{yx}$$

$$\frac{\partial^2 f}{\partial y \partial x} = f_{xy}$$

Ex $f(x,y) = x\cos y + y e^x$

$f_x = \cos y + y e^x$ $\qquad$ $f_y = -x\sin y + e^x$

$f_{xx} = y e^x$ $\qquad$ $f_{yy} = -x\cos y$

$\underline{f_{xy} = -\sin y + e^x}$ $\qquad$ $\underline{f_{yx} = -\sin y + e^x}$

$$f_{xy} = f_{yx}$$

Ex $\quad w = xy + \frac{e^y}{y^2+1} \qquad \frac{\partial^2 w}{\partial x \partial y}$

$$\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right) = \frac{\partial}{\partial x}\left(x + \frac{e^y \ldots}{(y^2+1)^2}\right)$$
$$= 1$$

$$w_{xy} = w_{yx} = w_y(y)$$
$$= 1$$

Ex $f(x,y,z) = 1 - 2xy^2z + x^2y$

f_{yxyz}

$f_y = 4xyz + x^2$

$f_{yx} = 4yz + 2x$

$f_{yxy} = 4z$

$f_{yxyz} = \underline{4}$

$w = f(\cancel{x(t)}, y(t))$

$(w')\ \dfrac{dw}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\cdot\dfrac{dy}{dt}$

$w' = f_x x' + f_y y' + f_z z'$

$w = xy \qquad x = \cos t \quad y = \sin t$

$$\begin{aligned}\frac{dw}{dt} &= w_x \frac{dx}{dt} + w_y \frac{dy}{dt}\\ &= y(-\sin t) + x\cos t\\ &= -\sin^2 t + \cos^2 t\\ &= \cos 2t.\end{aligned}$$

OR

$w = \cos t \sin t = \frac{1}{2}\sin 2t$

$\underline{w' = \cos 2t}$

$$w = xy + z \qquad \begin{cases} x = \cos t \\ y = \sin t \\ z = t \end{cases} \qquad w'? \frac{dw}{dt}$$

$$w = \cos t \sin t + t$$
$$= \frac{1}{2}\sin 2t + t$$

$$\frac{dw}{dt} = \boxed{\cos 2t + 1}$$

$$\frac{dw}{dt} = y(-\sin t) + x\cos t + 1$$
$$= -\sin^2 t + \cos^2 t + 1$$
$$= \boxed{\cos 2t + 1}$$

$$w = x + 2y + z^2 \qquad \begin{cases} x = \frac{r}{s} \\ y = r^2 + \ln s \\ z = 2r \end{cases}$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + w_y y_r + w_z z_r$$
$$= \frac{1}{s} + \cancel{2}4r + 2\overset{2r}{z}(2)$$
$$= \frac{1}{s} + 4r + 8r$$
$$= \boxed{\frac{1}{s} + 12r}$$

$$\frac{\partial w}{\partial s} = w_x x_s + w_y y_s + w_z z_s$$
$$= \boxed{-\frac{r}{s^2} + \frac{2}{s}}$$

$$w = \frac{r}{s} + 6r^2 + 2\ln s \cancel{+ 4r^2}$$
$$w_r = \frac{1}{s} + 12r \qquad w_s = -\frac{r}{s^2} + \frac{2}{s}$$

Ex $\quad w = x^2 + y^2 \qquad x = r - s, \quad y = r + s$

$$\boxed{w = (r-s)^2 + (r+s)^2}$$
$$= r^2 - 2rs + s^2 + r^2 + 2rs + s^2$$
$$= 2r^2 + 2s^2$$

$w_r = 4r \qquad w_s = 4s$

$$w_r = 2(r-s) + 2(r+s) \qquad (u^n)' = n u' u^{n-1}$$
$$= 4r$$

$F(x,y) = 0$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

$$\frac{dz}{dx} = -\frac{f_x}{f_z} \qquad \frac{dy}{dz} = -\frac{f_z}{f_y}$$

Ex $\quad x^3 + z^2 + ye^{xz} + z\cos y = 0$

$$\frac{dz}{dx} = -\left.\frac{3x^2 + yze^{xz}}{2z + xye^{xz} + \cos y}\right|_{(0,0,0)} \qquad -\frac{F_x}{F_z}$$

$$= \underline{0}$$

$$\frac{dz}{dy} = -\frac{e^{xz} - z\sin y}{2z + xye^{xz} + \cos y}$$

#31

$$y \ln(x^2+y^2+4) = 3$$

$$y \ln(x^2+y^2+4) - 3 = 0$$

$$\frac{dy}{dx} = - \frac{y \dfrac{2x}{x^2+y^2+4}}{\ln(x^2+y^2+4) + y \dfrac{2y}{x^2+y^2+4}} \qquad \frac{F_x}{F_y}$$

$$= \frac{-2xy}{(x^2+y^2+4)\ln(x^2+y^2+4) + 2y^2}$$

2.5

$$f(x,y) = f(g(t), h(t))$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$f' = f_x x' + f_y y'$$

$$(D_{\vec{u}} f)_{P_0} \qquad \left(\frac{df}{ds}\right)_{\vec{u}, P_0} = \lim_{s \to 0} \frac{f(x_0 + su_1, y_0 + su_2) - f(x_0, y_0)}{s}$$

Ex $f(x,y) = x^2 + xy$ @ $P_0(1,2)$

$$\vec{u} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$\left(\frac{df}{ds}\right)_{\vec{u}, P_0} = \lim_{s \to 0} \frac{f\left(1 + \frac{1}{\sqrt{2}}s, 2 + \frac{s}{\sqrt{2}}\right) - f(1,2)}{s}$$

$$= \lim_{s \to 0} \frac{1}{s}\left[\left(1+\frac{s}{\sqrt{2}}\right)^2 + \left(1+\frac{s}{\sqrt{2}}\right)\left(2+\frac{s}{\sqrt{2}}\right) - 3\right]$$

$$= \lim_{s \to 0} \frac{1}{s}\left(1 + \frac{1}{2}s^2 + \frac{2s}{\sqrt{2}} + 2 + \frac{3s}{\sqrt{2}} - 3\right)$$

$$= \lim_{s \to 0} \left(\frac{1}{2}s + \frac{s}{\sqrt{2}}\right)$$

$$= \frac{s}{\sqrt{2}}$$

Gradient of $f(x,y)$ $\quad$ $f(x,y,z)$

$$\nabla f = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$$

del