Solution   Section 2.6 – Discrete Random Variables

Exercise

Determine whether or not a probability distribution is given. If a probability is given, find its mean and standard deviation. If the probability is not given, identify the requirements that are not satisfied.

a)

x	$P(x)$
0	0.125
1	0.375
2	0.375
3	0.125

b)

x	$P(x)$
0	0.22
1	0.16
2	0.21
3	0.16

c)

x	$P(x)$
0	0.528
1	0.360
2	0.098
3	0.013
4	0.001
5	0^+

0^+ denotes a positive probability value that is very small.

d)

x	$P(x)$
0	0.02
1	0.15
2	0.29
3	0.26
4	0.16
5	0.12

Solution

a) Since $0 \le P(x) \le 1;\quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.125	0	0	0
1	0.375	0.375	1	0.375
2	0.375	0.750	4	1.500
3	0.125	0.375	9	1.125
	1.000	1.500		3.000

Mean: $\mu = \sum \left[x \cdot P(x) \right] \underline{= 1.50}$

Variance: $\sigma^2 = \sum \left[x^2 \cdot P(x) \right] - \mu^2 = 3.0 - (1.5)^2 \underline{= 0.75}$

Standard deviation: $\sigma = \sqrt{.75} \underline{= .866}$

b) Since $0 \le P(x) \le 1;$

$\sum P(x) = 0.22 + 0.16 + 0.21 + 0.16 = 0.75 \ne 1$

This given table is not a probability distribution.

c) Since $0 \le P(x) \le 1; \quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.528	0	0	0
1	0.360	0.360	1	0.360
2	0.098	0.196	4	0.392
3	0.013	0.039	9	0.117
4	0.001	.004	16	0.016
5	0+	0	25	0
	1.000	0.599		0.885

Mean: $\mu = \sum[x \cdot P(x)] \underline{= 0.599}$

Variance: $\sigma^2 = \sum\left[x^2 \cdot P(x)\right] - \mu^2 = 0.885 - (0.599)^2 \underline{= 0.526}$

Standard deviation: $\sigma = \sqrt{0.526} \underline{= 0.725}$

d) Since $0 \le P(x) \le 1; \quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.02	0	0	0
1	0.15	0.15	1	0.15
2	0.29	0.58	4	1.16
3	0.26	0.78	9	2.34
4	0.16	0.64	16	2.56
5	0.12	0.60	25	3.00
	1.000	2.75		9.21

Mean: $\mu = \sum[x \cdot P(x)] \underline{= 2.75}$

Variance: $\sigma^2 = \sum\left[x^2 \cdot P(x)\right] - \mu^2 = 9.21 - (2.75)^2 \underline{= 1.6475}$

Standard deviation: $\sigma = \sqrt{1.6475} \underline{= 1.284}$

Exercise

Based on past results found in the *Information Please Almanac*, there is a 0.1919 probability that a baseball World Series context will last 4 games, is a 0.2121 probability that it will last 5 games, a 0.2222 probability that it will last 6 games, a 0.3737 probability that us will last 7 games.

a) Does the given information describe a probability distribution?

b) Assuming that the given information describes a probability distribution, find the mean and standard deviation for the numbers of games in World Series contests.

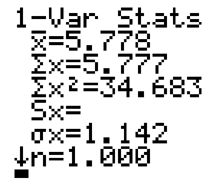
c) Is it unusual for a team to "sweep" by winning in four games? Why or why not?

Solution

a) Since $0 \le P(x) \le 1$; $\sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
4	0.1919	0.7676	16	3.0704
5	0.2121	1.0605	25	5.3025
6	0.2222	1.3332	36	7.9992
7	0.3737	2.6159	49	18.3113
	0.9999	5.7772		34.6834

Mean: $\mu = \sum[x \cdot P(x)] \underline{= 5.7772}$



Variance: $\sigma^2 = \sum\left[x^2 \cdot P(x)\right] - \mu^2 = 34.6834 - (5.7772)^2 \underline{= 1.3074}$

Standard deviation: $\sigma = \sqrt{1.3074} \underline{= 1.1434}$

b) $\mu = \underline{5.8 \;\; games}$ *and* $\sigma = \underline{1.1 \; games}$

c) No, since $P(x=4) = 0.1919 > 0.05$, winning in 4 games is not an unusual event.

Exercise

Based on information from MRI Network, some job applicants are required to have several interviews before a decision is made. The number of required interviews and the corresponding probabilities are: 1 (0.09); 2 (0.31); 3 (0.37); 4 (0.12); 5 (0.05); 6 (0.05).

a) Does the given information describe a probability distribution?
b) Assuming that a probability distribution is described, find its mean and standard deviation.
c) Use the range rule of thumb to identify the range of values for usual numbers of interviews.
d) Is it unusual to have a decision after just one interview? Explain?

Solution

a) Since $0 \le P(x) \le 1$; $\sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
1	0.09	0.09	1	0.09
2	0.31	0.62	4	1.24
3	0.37	1.11	9	3.33
4	0.12	0.48	16	1.92
5	0.05	0.25	25	1.25
6	0.05	0.30	36	1.80
	0.99	2.85		9.63

Mean: $\mu = \sum[x \cdot P(x)] \underline{= 2.85}$

Variance: $\sigma^2 = \sum\left[x^2 \cdot P(x)\right] - \mu^2 = 9.63 - (2.85)^2 \underline{= 1.5075}$

Standard deviation: $\sigma = \sqrt{1.5075} \underline{= 1.228}$

b) $\mu = \underline{2.9 \;\; interviews}$ *and* $\sigma = \underline{1.2 \; interviews}$

c) The range rule of thumb suggests that "usual" values are those within two standard deviations of the mean.

$$\textit{Minimum usual value } = \mu - 2\sigma = 2.9 - 2(1.2) \ \underline{= 0.5}$$

$$\textit{Maximum usual value } = \mu + 2\sigma = 2.9 + 2(1.2) \ \underline{= 5.3}$$

The range of values for usual numbers of interviews is from 0.5 to 5.3.

d) No, since $0.05 < 1 < 5.3$, it is not unusual to have a decision after just one interview.

Exercise

Based on information from Car dealer, when a car is randomly selected the number of bumper stickers and the corresponding probabilities are: 0 (0.824); 1 (0.083); 2 (0.039); 3 (0.014); 4 (0.012); 5 (0.008); 6 (0.008); 7 (0.004); 8 (0.004); 9 (0.004).

a) Does the given information describe a probability distribution?

b) Assuming that a probability distribution is described, find its mean and standard deviation.

c) Use the range rule of thumb to identify the range of values for usual numbers of bumper stickers.

d) Is it unusual for a car to have more than one bumper sticker? Explain?

Solution

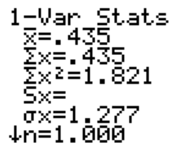
a) Since $0 \le P(x) \le 1; \quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.824	0	0	0
1	0.083	0.083	1	0.83
2	0.039	0.078	4	0.156
3	0.014	0.042	9	0.126
4	0.012	0.048	16	0.192
5	0.008	0.040	25	0.288
6	0.008	0.048	36	0.288
7	0.004	0.028	49	0.196
8	0.004	0.032	64	0.256
9	0.004	0.036	81	0.324
	1.000	0.435		1.821

Mean: $\mu = \sum\left[x \cdot P(x)\right] \ \underline{= 0.435}$

Variance: $\sigma^2 = \sum\left[x^2 \cdot P(x)\right] - \mu^2 = 1.821 - (0.435)^2 \ \underline{= 1.635}$

Standard deviation: $\sigma = \sqrt{1.635} \ \underline{= 1.279}$



b) $\mu = \underline{0.4 \ \textit{bumper stickers}}$ *and* $\sigma = \underline{1.3 \ \textit{bumper stickers}}$

c) The range rule of thumb suggests that "*usual*" values are those within two standard deviations of the mean.

$$\textit{Minimum usual value } = \mu - 2\sigma = 0.4 - 2(1.3) \ \underline{= -2.2}$$

$$\textit{Maximum usual value } = \mu + 2\sigma = 0.4 + 2(1.3) \ \underline{= 3.0}$$

The range of values for usual numbers of interviews is from 0 to 3.0.

d) No, since $0 < 1 < 3.0$, it is not unusual to have more than 1 bumper sticker.

Exercise

A Company hired 8 employees from a large pool of applicants with an equal numbers of males and females. If the hiring is done without regard to sex, the numbers of females hired and the corresponding probabilities are: 0 (0.004); 1 (0.0313); 2 (0.109); 3 (0.219); 4 (0.273); 5 (0.219); 6 (0.109); 7 (0.031); 8 (0.004).

a) Does the given information describe a probability distribution?

b) Assuming that a probability distribution is described, find its mean and standard deviation.

c) Use the range rule of thumb to identify the range of values for usual numbers of females hired in such groups of 8.

d) If the most recent group of 8 newly hired employees does not include any females, does there appear to be discrimination based on sex? Explain?

Solution

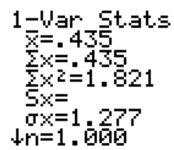
a) Since $0 \le P(x) \le 1; \quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.004	0	0	0
1	0.031	0.031	1	0.031
2	0.109	0.218	4	0.436
3	0.219	0.657	9	1.971
4	0.273	1.092	16	4.368
5	0.219	1.095	25	5.475
6	0.109	0.654	36	3.924
7	0.031	0.217	49	1.519
8	0.004	0.032	64	0.256
	0.999	3.996		17.980

Mean: $\mu = \sum \left[x \cdot P(x) \right] = 3.996$

Variance: $\sigma^2 = \sum \left[x^2 \cdot P(x) \right] - \mu^2 = 17.980 - (3.996)^2 = 2.012$

Standard deviation: $\sigma = \sqrt{2.02} = 1.418$



b) $\mu = 4$ *females* and $\sigma = 1.4$ *females*

c) The range rule of thumb suggests that "*usual*" values are those within two standard deviations of the mean.

$$\text{Minimum usual value } = \mu - 2\sigma = 4 - 2(1.4) = 1.2$$

$$\text{Maximum usual value } = \mu + 2\sigma = 4 + 2(1.4) = 6.8$$

The range of values for usual numbers of interviews is from 1.2 to 6.8.

d) Yes, since $0 < 1.2$, it would be unusual to hire no females if only factors were in operation.

Exercise

Let the random variable *x* represent the number of girls in a family of 4 children. Construct a table describing the probability distribution; then find the mean and the standard deviation. (Hint: List the different possible outcomes.) Is it unusual for a family of 3 children to consist of 3 girls?

Solution

The sample space is: $S = \{GGG,\ GGB,\ GBG,\ BGG,\ BBG,\ BGB,\ GBB,\ BBB\}$

Therefore, there are 8 equally likely possible outcomes.

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.125	0	0	0
1	0.375	0.375	1	0.375
2	0.375	0.750	4	1.500
3	0.125	0.375	9	1.125
	1.000	1.500		3.000

Mean: $\mu = \sum \left[x \cdot P(x) \right] \underline{= 1.50|}$

Variance: $\sigma^2 = \sum \left[x^2 \cdot P(x) \right] - \mu^2 = 3.0 - (1.5)^2 \underline{= 0.75|}$

Standard deviation: $\sigma = \sqrt{.75}\ \underline{= .866|}$

$\mu = \underline{1.5\ \ girls|}$ *and* $\sigma = \underline{0.9\ girls|}$

No, since $P(x=3) = 0.125 > 0.05$, it is not unusual for a family to have all girls.

Exercise

In 4 lottery game, you pay 50¢ to select a sequence of 4 digits, such 1332. If you select the same sequence of 4 digits that are drawn, you win and collect $2788.

a) How many different selections are possible?

b) What is the probability of winning?

c) If you win, what is your net profit?

d) Find the expected value.

Solution

a) Since each of the 4 positions could be filled with replacement by any of the 10 digits
$10 \cdot 10 \cdot 10 \cdot 10\ \ \underline{= 10{,}000\ \ possibilities|}$

b) Since only one possible selection: $P(winning) = \dfrac{1}{10{,}000}\ \underline{= 0.0001|}$

c) The net profit is the payoff minus the original bet. $2,788.00 – $0.50 = $2,787.50

d) The expected value is –22.1¢

Event	x	$P(x)$	$x \cdot P(x)$
Lose	–$0.50	0.9999	–$.49995
Gain	$2787.50	0.0001	$0.27875
Total			–$0.2212

Exercise

When playing roulette at casino, a gambler is trying to decide whether to bet \$5 on the number 13 or bet \$5 that the outcomes any one of these 5 possibilities: 0 or 00 or 1 or 2 or 3.the expected value of the \$5 bet for a single number is –26¢. For the \$5 bet that the outcome 0 or 00 or 1 or 2 or 3, there is a probability of $\frac{5}{38}$ of making a net profit of \$30 and a $\frac{33}{38}$ probability of losing \$5.

a) Find the expected value for the \$5 bet that the outcome is 0 or 00 or 1 or 2 or 3.

b) Which bet is better: A \$5 bet on the number 13 or a \$5 bet the outcome is 0 or 00 or 1 or 2 or 3? Why?

Solution

a)

Event	x	$P(x)$	$x \cdot P(x)$
Lose	–5	$\frac{33}{38}$	$-\frac{165}{38}$
Gain	30	$\frac{5}{38}$	$\frac{150}{38}$
Total		1	–0.3947

The expected value is –39.5¢

b) Since $-26 > -39.5$, wagering \$5 on the number 13 is the better bet.

Exercise

There is a 0.9986 probability that a randomly selected 30-year-old male lives through the year. As insurance company charges \$161 for insuring that the male will live through the year. If the male does not survive the year, the policy pays out \$100,000 as a death benefit.

a) From the perspective of the 30-year-old male, what are the values corresponding to the 2 events of surviving the year and not surviving?

b) If a 30-year-old male purchases the policy, what is his expected value?

c) Can the insurance company expect to make a profit from many such policies? Why?

Solution

a) From the 30-year-old male's perspective, the 2 possible outcome values are –\$161, if he lives $100,000 - 161 = \$99,839$ is he dies.

b)

Event	x	$P(x)$	$x \cdot P(x)$
Lose	–161	0.9986	–160.7446
Gain	99,839	0.0014	139.7746
Total		1.0000	–21

The expected value is –\$21.0

c) Yes; the insurance company can expect to make an average of \$21.00 per policy.

Exercise

An insurance company charges a 21-year-old male a premium of $500 for a one-year $100,000 life insurance policy. A 21-year-old male has a 0.9985 probability of living for a year.

a) From the perspective of a 21-year-old male (or estate), what are the values of the two different outcomes?

b) What is the expected value for a 21-year-old male who buys the insurance?

c) What would be the cost of the insurance if the company just breaks even (in the long run with many such policies), instead of making a profit?

d) Given that the expected value is negative (so the insurance company can make a profit), why

Solution

a) The value if he lives is $-\$500$

The value if he dies is $=100,000-500 \ \underline{=\$99,500}$

b) The expected value is:

$(.9985)(500)+(1-.9985)(99500) \ \underline{=-\$350}$

c) $(.9985)x-(1-.9985)(100,000-x)=0$

$.9985x-150+.0015x=0$

$\underline{x=\$150}$

d) Insuring the financial security of loved ones compensates for the negative expected value.