$\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\qquad$ $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \to \begin{vmatrix} a & \cdot & \cdot \\ b & \cdot & \cdot \\ c & & \cdot \end{vmatrix}$

Sec. 1.7 Cramer's Rule

$AX = B$

$|A| \neq 0$

if $|A| = 0$ $\begin{cases} |D_x| \text{ or } |D_y| = 0 & \text{unique infinite soln} \quad ! \ \infty \text{ soln} \\ \text{3var} \quad \text{2v} & \\ D_x \text{ or } D_y \neq 0 & \underline{\underline{\text{No}}} \text{ soln} \end{cases}$

$D_x = -(\quad)$ $\qquad -6$

$$\begin{cases} 2x + 2y = -4 \\ 2x - y = -5 \end{cases}$$

$x = -\frac{7}{3} \quad y = \frac{1}{3}$

$\therefore \left(-\frac{7}{3}, \frac{1}{3}\right)$

Ex

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ -2x_1 + x_2 = 0 \\ -4x_1 + x_3 = 0 \end{cases}$$

$D = 7$

$D_1 = 1$

$x = \frac{1}{7}$

Soln

$$D = \begin{vmatrix} 1 & 1 & 1 \\ -2 & 1 & 0 \\ -4 & 0 & 1 \end{vmatrix} = 7 \neq 0$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ -2 & 0 & 0 \\ -4 & 0 & 1 \end{vmatrix} = 2$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ -2 & 1 & 0 \\ -4 & 0 & 0 \end{vmatrix} = 4$$

$x = \frac{D_1}{D} = \frac{1}{7}, \quad y = \frac{2}{7}, \quad z = \frac{4}{7}$

$\therefore \left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right)$

Ex
$$\begin{cases} x_1 + 2x_3 = 6 \\ -3x_1 + 4x_2 + 6x_3 = 30 \\ -x_1 - 2x_2 + 3x_3 = 8 \end{cases}$$

$$D = \begin{vmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{vmatrix} = 44$$

$$D_1 = \begin{vmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{vmatrix} = -40$$

$$D_2 = \begin{vmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{vmatrix} = 72$$

$$D_3 = \begin{vmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{vmatrix} = 152$$

$$x = -\frac{10}{11}, \quad y = \frac{18}{11}, \quad z = \frac{38}{11}$$

$$\therefore \left(-\frac{10}{11}, \frac{18}{11}, \frac{38}{11}\right)$$

A^{-1} ? $\Rightarrow$ how

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\left[\begin{array}{cc} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \\ -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & \\ & \end{array}\right.$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \qquad |A| = 7$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 1 & -1 & -1 \\ 2 & 5 & -2 \\ 4 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} & -\frac{1}{7} \\ 2/7 & 5/7 & -2/7 \\ 4/7 & -4/7 & 3/7 \end{bmatrix}$$

adjoint Method

Equivalent:

A $n \times n$

a) A is invertible

b) $Ax = 0$ has only a trivial solution

c) RREF of I_n

d) as a product of elementary matrices

e) $Ax = b$ is consistent $n \times 1$ (b)

f) $\det(A) \neq 0$

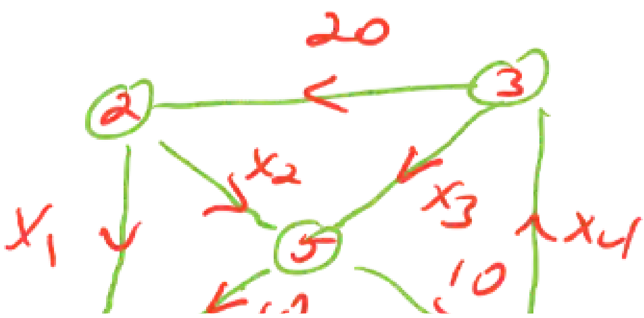
Ex) a) $=$ $=$ b)

b)

Sec 1.8 → Applications

Network.

Ex



1) $x_1 + 10 - x_5 = 0 \Rightarrow \quad x_1 - x_5 = -10$ (1)

2) $+x_1 + x_2 = 20$ (2)

3) $x_4 - x_3 = 20$ (3)

4) $-x_4 + x_5 = -10 \rightarrow$

5) $x_2 + x_3 = 20$ (5)

$$\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & -10 \\ 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 0 & -1 & 1 & -10 \\ 0 & 1 & 1 & 0 & 0 & 20 \end{array}\right] \quad R_2 - R_1$$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 0 & -1 & 1 & -10 \\ 0 & 1 & 1 & 0 & 0 & 20 \end{array}\right] \quad R_5 - R_2$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 0 & -1 & 1 & -10 \\ 0 & 0 & 1 & 0 & -1 & -10 \end{array}\right] \quad R_5 + R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 0 & -1 & 1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

$\to x_2 + x_5 = 30$

$\to -x_3 + x_4 = 20$

$-x_4 + x_5 = -10$

$x_4 = 10 + x_5$

$x_3 = 10 + x_5 - 20$
$= -10 + x_5$

$x_2 = 30 - x_5$

$x_1 = x_5 - 10$

$(x_5 - 10,\ 30 - x_5,\ -10 + x_5,\ 10 + x_5,\ x_5)$