$=$ equal

$\rightarrow$ imply / approach

$\approx$

$$x = b^y \iff y = \log_b x$$

$b \rightarrow$ base

ln : $\log_e$

log : $\log_{10}$

Ex $3 = \log_7 x \iff x = 7^3$

$\log_b 25 = 2 \iff 25 = b^2$

$\log_b b^2$

$2^5 = x \iff 5 = \log_2 x$

$27 = b^3 \iff 3 = \log_b 27$

$\log_b b = 1$ $\quad$ $b = b^1 = b$ ✓

$\log_b 1 = 0$ $\quad$ $1 = b^0 = 1$ ✓

$\log_b b^x = x \quad , \quad b^{\log_b x} = x$

Ex $\log_5 \frac{1}{125} = \log_5 \frac{1}{5^3}$

$= \log_5 5^{-3}$

$= -3$

$\log_3 \sqrt{3} = \log_3 3^{1/2} = \boxed{\frac{1}{2}}$

$\ln x = \log_e x$

$\ln e = 1$ $\qquad \ln e^x = x$

$\ln 1 = 0$

$\log_b x = x \rightarrow y = b^x$

$\log_b (>0) \rightarrow$ domain

$f(x) = \log_b (x)$

$\ln (0 < x < 1) = -$

$\log_b 1 = 0$

$\log > 1 = +$

$\log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$

$f(x) = \log_5$

Asymptote: inside $= 0$

Domain: inside > 0

Range: $\mathbb{R}$

$f(x) = \log_4 (x-5)$

Asymptote: $x = 5$

Domain: $x > 5$

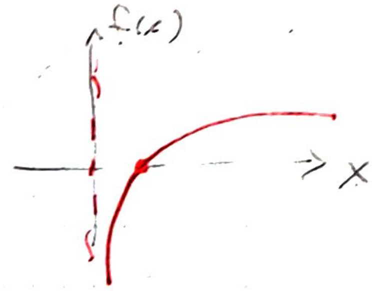
Range: $\mathbb{R}$

	Asymptote	Domain	Range
$f(x) = \ln(4-x)$	$x = 4$	$x < 4$	$\mathbb{R}$
$f(x) = \ln x^2$	$x = 0$	$\mathbb{R} - \{0\}$ $x \neq 0$	$\mathbb{R}$
# 42 $f(x) = \log_5(x+6)$	$x = -6$	$x > -6$	$\mathbb{R}$
43 $f(x) = \log(2-x)$	$x = 2$	$x < 2$	$\mathbb{R}$
$f(x) = e^{2x+1} + 3$	$y = 3$	$\mathbb{R}$	$y > 3$
$f(x) = 1 - e^{x-2}$	$y = 1$	$\mathbb{R}$	$y < 1$

Graph. $f(x) = \log x$

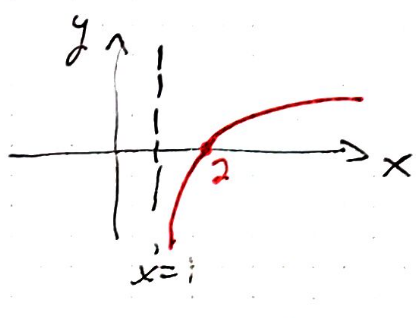
Asymptote: $x = 0$ Domain: $x > 0$ Range: $\mathbb{R}$

x	$f(x)$
$\cancel{0}$	
$1/2$	$-\log 2$
1	0
2	$\log 2$



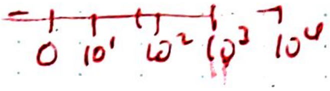
$f(x) = \log_2(x-1)$

Asymptote: $x = 1$ Domain: $x > 1$ Range: $\mathbb{R}$



5 0's $100,000$

$\log_{10} 10^5 = 5$



5.3. #1 $2^6 = 64 \iff 6 = \log_2 64$

#3 $5^{-3} = \frac{1}{125} \iff -3 = \log_5 \frac{1}{125}$

#11 $e^1 = 3x \iff 1 = \ln 3x = \log_e 3x$

#13 $\log_5 125 = y \iff 125 = 5^y$

exponential form b^x, a^b

20 $2 = \log_9 x \iff x = 9^2$

25 $\log_4 16 = \log_4 4^2 = 2$

29 $\log_3 \sqrt[7]{3} = \log_3 3^{1/7} = \frac{1}{7}$

Simplify

#32 $\log_5 1 = 0$

35 $\ln e^{x-5} = x-5$

40 $\ln e^{x^2+3x} = x^2+3x$

3.4 Properties.

Product Rules: $\log_b MN = \log_b M + \log_b N$

ex. $\log 100x = \log 100 + \log x$

$= \log_{10} 10^2 + \log x$

$= 2 + \log x$

Power Rule: $\log_b M^p = p \log_b M$

$\log 10^2 = 2 \log 10$

$= 2(1)$

$= 2$

Quotient Rule: $\log_b \frac{M}{N} = \log_b M - \log_b N$

$\log_b \frac{1}{x} = -\log_b x$

Ex $\ln \frac{e^5}{11} = \ln e^5 - \ln 11$

$= 5 - \ln 11$

$\log_6 (7 \times 9) = \log_6 7 + \log_6 9$

$\log_9 \left(\frac{15}{7}\right) = \log_9 15 - \log_9 7$

$\log_5 \sqrt{8} = \log_5 (2^3)^{1/2}$

$= \log_5 2^{3/2}$

$= \frac{3}{2} \log_5 2$

Ex $\log_b (x^4 \sqrt[3]{y}) = \log_b x^4 + \log_b y^{1/3}$

$= 4 \log_b x + \frac{1}{3} \log_b y$

Ex $\log_a \left(\frac{mnq}{p^2 r^4}\right) = \log_a (mnq) - \log_a (p^2 r^4)$

$= \log_a m + \log_a n + \log_a q - (\log_a p^2 + \log_a r^4)$

$= \log_a m + \log_a n + \log_a q - 2 \log_a p - 4 \log_a r$

Ex $\log_5 \frac{\sqrt{x}}{25 y^3} = \log_5 x^{1/2} - \log_5 (5^2 y^3)$

$= \frac{1}{2} \log_5 x - (\log_5 5^2 + \log_5 y^3)$

$= \frac{1}{2} \log_5 x - 2 - 3 \log_5 y$

Ex $\log_b \sqrt[5]{\frac{m^4 n^5}{x^2 a b^{10}}} = \log_b \left(\frac{m^4 n^5}{x^2 a b^{10}}\right)^{1/5}$

$= \frac{1}{5} \log_b \frac{m^4 n^5}{x^2 a b^{10}}$

$= \frac{1}{5} [\log_b m^4 n^5 - \log_b x^2 a b^{10}]$

$= \frac{1}{5} [\log_b m^4 + \log_b n^5 - (\log_b x^3 + \log_b a + \log_b b^{10})]$

$= \frac{1}{5} (4 \log_b m + 5 \log_b n - 3 \log_b x - \log_b a - 10)$

$= \frac{4}{5} \log_b m + \log_b n - \frac{3}{5} \log_b x - \frac{1}{5} \log_b a - 2$

Ex $\log(7x+6) - \log x = \log \dfrac{7x+6}{x}$

Ex $\underline{\log_3(x+2) + \log_3 x} - \log_3 2 = \log_3 x(x+2) - \log_3 2$

$= \log_3 \dfrac{x(x+2)}{2}$

Ex $2\ln x + \frac{1}{3}\ln(x+5) = \ln x^2 + \ln(x+5)^{1/3}$

$= \ln x^2 (x+5)^{1/3}$

$= \ln x^2 \sqrt[3]{x+5}$

Ex $2\log(x-3) - \log x = \log(x-3)^2 - \log x$

$= \log \dfrac{(x-3)^2}{x}$

#40 $5\log_a x - \frac{1}{2}\log_a(3x-4) - 3\log_a(5x+1)$

$= \log_a x^5 - \left(\log_a(3x-4)^{1/2} + \log_a(5x+1)^3\right)$

$= \log_a x^5 - \log_a\left((3x-4)^{1/2}(5x+1)^3\right)$

$= \log_a \dfrac{x^5}{\sqrt{3x-4}\,(5x+1)^3}$

#50 $\frac{2}{3}\left[\ln(x^2-9) - \ln(x+3)\right] + \ln(x+y)$

$= \frac{2}{3}\ln\dfrac{x^2-9}{x+3} + \ln(x+y)$

$= \ln\left(\dfrac{(x-3)\cancel{(x+3)}}{\cancel{x+3}}\right)^{2/3} + \ln(x+y)$

$= \ln(x-3)^{2/3} + \ln(x+y)$

$= \ln\left[(x-3)^{2/3}(x+y)\right]$

#44 $4\ln x + 7\ln y - 3\ln z = (\ln x^4 + \ln y^7) - \ln z^3$

$= \ln x^4 y^7 - \ln z^3$

$= \ln \dfrac{x^4 y^7}{z^3}$

#29 $\ln \sqrt{\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}} = \ln \left(\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}\right)^{1/2}$

$= \frac{1}{2} \ln \dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}$

$= \frac{1}{2} \left[\ln x(x+1)(x-2) - \ln (x^2+1)(2x+3) \right]$

$= \frac{1}{2} \left[\ln x + \ln(x+1) + \ln(x-2) - \left(\ln(x^2+1) + \ln(2x+3) \right) \right]$

$= \frac{1}{2} \left(\ln x + \ln(x+1) + \ln(x-2) - \ln(x^2+1) - \ln(2x+3) \right)$

#12 $\log_a \sqrt[4]{\dfrac{m^8 n^{12}}{a^3 b^5}} = \log_a \left(\dfrac{m^8 n^{12}}{a^3 b^5}\right)^{1/4}$

$= \frac{1}{4} \log_a \dfrac{m^8 n^{12}}{a^3 b^5}$

$= \frac{1}{4} \left(\log_a m^8 n^{12} - \log_a a^3 b^5 \right)$

$= \frac{1}{4} \left(\log_a m^8 + \log_a n^{12} - \left(\log_a a^3 + \log_a b^5 \right) \right)$

$= \frac{1}{4} \left(8 \log_a m + 12 \log_a n - 3 - 5 \log_a b \right)$

$= 2 \log_a m + 3 \log_a n - \frac{3}{4} - \frac{5}{4} \log_a b$

#36 $\ln xz - \ln x\sqrt{y} + 2\ln\frac{y}{z}$

$$= \ln\frac{xz}{x\sqrt{y}} + \ln\left(\frac{y}{z}\right)^2$$

$$= \ln\left(\frac{z}{\sqrt{y}}\cdot\frac{y^2}{z^2}\right)$$

$$\frac{y^2}{y^{1/2}} = y^{2-1/2}$$

$$= \ln\left(\frac{y^{3/2}}{z}\right)$$

Solve

$6^M = 6^N \Leftrightarrow M = N$

$6^M = a \Leftrightarrow \log_6 a = M$ 100%

$\ln b^x = x$, $\ln e^x = x$

ln. both sides or

$\log_6 M = a \Rightarrow M = 6^a$

$\log_6 M = \log_6 N \Rightarrow M = N$

$M > 0$

Ex $5^{3x-6} = 125 = 5^3$

$3x - 6 = 3$

$3x = 9$

$x = 3$

$8^{x+2} = 4^{x-3}$

$(2^3)^{x+2} = (2^2)^{x-3}$

$2^{3(x+2)} = 2^{2(x-3)}$

$3(x+2) = 2(x-3)$

$3x + 6 = 2x - 6$

$x = -12$