7.1 $\csc\theta = \dfrac{1}{\sin\theta}$ $\sin\theta = \dfrac{1}{\csc\theta}$ $\sin\theta\csc\theta = 1$

$\sec\theta = \dfrac{1}{\cos\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\sec\theta\cos\theta = 1$

$\tan\theta = \dfrac{1}{\cot\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta\tan\theta = 1$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Ex $\sec\theta\tan\theta = \dfrac{1}{\cos\theta}\dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos^2\theta}$ $(\cos\theta)^2$

Ex $\dfrac{1}{\sin\theta} + \dfrac{1}{\cos\theta} = \dfrac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}$

Ex/ $\tan x + \cot x$ in terms $\sin x$ + $\cos x$

$$\tan\alpha + \cot\alpha = \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}$$

$$= \frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha\sin\alpha} \rightarrow = 1$$

$$= \frac{1}{\cos\alpha\sin\alpha}$$

Prove: $\tan x + \cos x = \sin x(\sec x + \cot x)$

$$\sin x(\sec x + \cot x) = \sin x\sec x + \sin x\cot x$$

$$= \sin x\frac{1}{\cos x} + \sin x\frac{\cos x}{\sin x}$$

$$= \tan x + \cos x \ ✓$$

$$\tan x + \cos x = \frac{\sin x}{\cos x} + \cos x$$

$$= \sin x\frac{1}{\cos x} + \cos x\frac{\sin x}{\sin x}$$

$$= \sin x\left(\frac{1}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \sin x(\sec x + \cot x) \ ✓$$

Prove: $\cot\alpha + 1 = \csc\alpha\,(\cos\alpha + \sin\alpha)$

$$\csc\alpha\,(\cos\alpha + \sin\alpha) = \frac{1}{\sin\alpha}\cos\alpha + \frac{1}{\sin\alpha}\sin\alpha$$

$$= \cot\alpha + 1 \quad ✓$$

Prove: $\dfrac{\cos^4 t - \sin^4 t}{\cos^2 t} = 1 - \tan^2 t$.

$$\frac{\cos^4 t - \sin^4 t}{\cos^2 t} = \frac{(\cos^2 t - \sin^2 t)\overbrace{(\cos^2 t + \sin^2 t)}^{=1}}{\cos^2 t}$$

$$= \frac{\cos^2 t - \sin^2 t}{\cos^2 t}$$

$$= \frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t}$$

$$= 1 - \tan^2 t. \quad ✓$$

$1 + \cos\theta = \dfrac{\sin^2\theta}{1 - \cos\theta}$ $\qquad\qquad \cos^2\theta + \sin^2\theta = 1$

$$\frac{\sin^2\theta}{1-\cos\theta} = \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$

$$= 1 + \cos\theta \quad ✓$$

$*$ $(1+\cos\theta)(1-\cos\theta) \stackrel{?}{=} \sin^2\theta$

$$1 - \cos^2\theta \stackrel{?}{=} \sin^2\theta$$

$$\sin^2\theta = \sin^2\theta \quad ✓$$

$$+\cos\theta = (1+\cos\theta)\frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta} \quad ✓$$

Prove: $\tan^2\alpha\,(1+\cot^2\alpha) = \dfrac{1}{1-\sin^2\alpha}$

$$\tan^2\alpha\,(1+\cot^2\alpha) = \tan^2\alpha + \tan^2\alpha\cot^2\alpha$$

$$= \frac{\sin^2\alpha}{\cos^2\alpha} + 1$$

$$= \frac{\sin^2\alpha+\cos^2\alpha}{\cos^2\alpha}$$

$$= \frac{1}{1-\sin^2\alpha} \quad ✓$$

$$\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = 2\csc\alpha$$

$$\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha+(1+\cos\alpha)^2}{\sin\alpha\,(1+\cos\alpha)}$$

$$= \frac{\sin^2\alpha + 1 + 2\cos\alpha + \cos^2\alpha}{\sin\alpha\,(1+\cos\alpha)}$$

$$= \frac{2+2\cos\alpha}{\sin\alpha\,(1+\cos\alpha)}$$

$$= \frac{2\,(1+\cos\alpha)}{\sin\alpha\,(1+\cos\alpha)}$$

$$= 2\csc\alpha \quad ✓$$

Prove $\frac{1+\sin t}{\cos t} = \frac{\cos t}{1-\sin t}$

$$\frac{\cos t}{1-\sin t} = \frac{\cos t}{1-\sin t} \cdot \frac{1+\sin t}{1+\sin t}$$

$$= \frac{\cos t\,(1+\sin t)}{1-\sin^2 t}$$

$$= \frac{\cos t\,(1+\sin t)}{\cos^2 t}$$

$$= \frac{1+\sin t}{\cos t} \checkmark$$

$\cot^2\theta + \cos^2\theta \neq \cot^2\theta \cos^2\theta$

$\theta = \frac{\pi}{4}$ $\quad \cot^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} \overset{?}{\neq} \cot^2\frac{\pi}{4} \cos^2\frac{\pi}{4}$

$$1 + \left(\frac{1}{\sqrt{2}}\right)^2 \overset{?}{=} 1\left(\frac{1}{2}\right)$$

$$\frac{3}{2} \neq \frac{1}{2} \checkmark$$

#1 $\cos\theta\cot\theta + \sin\theta = \csc\theta$

$$\cos\theta\cot\theta + \sin\theta = \cos\theta\frac{\cos\theta}{\sin\theta} + \sin\theta$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta \checkmark$$

#? $\sec\theta\cot\theta - \sin\theta = \frac{\cos^2\theta}{\sin\theta}$

$$\sec\theta\cot\theta - \sin\theta = \frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta} - \sin\theta$$

$$= \frac{1}{\sin\theta} - \sin\theta$$

$$= \frac{1 - \sin^2\theta}{\sin\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta} \checkmark$$

#21 $\frac{\cot^2\theta + 3\cot\theta - 4}{\cot\theta + 4} = \cot\theta - 1$

$$\frac{\cot^2\theta + 3\cot\theta - 4}{\cot\theta + 4} = \frac{(\cot\theta + 4)(\cot\theta - 1)}{(\cot\theta + 4)}$$

$$= \cot\theta - 1 \checkmark$$

#42d $\sin x(\tan x\cos x - \cot x\cos x) = 1 - 2\cos^2 x$

$\sin x(\tan x\cos x - \cot x\cos x)$

$= \sin x \frac{\sin x}{\cos x}\cos x - \sin x \frac{\cos x}{\sin x}\cos x$

$= \sin^2 x - \cos^2 x$

$= 1 - \cos^2 x - \cos^2 x$

$= 1 - 2\cos^2 x$ ✓.

28/ $7\csc^2 x - 5\cot^2 x = 2\csc^2 x + 5$

$7\csc^2 x - 5\cot^2 x = 7\csc^2 x - 5(\csc^2 x - 1)$

$= 7\csc^2 x - 5\csc^2 x + 5$

$\csc^2 x - 1$

$= 2\csc^2 x + 5$ ✓.

35/ $\frac{\cot x + \csc x - 1}{\cot x - \csc x + 1} = \csc x + \cot x$ $\quad \frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$\frac{\cot x + \csc x - 1}{\cot x - \csc x + 1} = \frac{\frac{\cos x}{\sin x} + \frac{1}{\sin x} - 1}{\frac{\cos x}{\sin x} - \frac{1}{\sin x} + 1}$

$= \frac{\frac{\cos x + 1 - \sin x}{\sin x}}{\frac{\cos x - 1 + \sin x}{\sin x}}$

$= \frac{\cos x + 1 - \sin x}{\cos x + \sin x - 1} \cdot \frac{(\cos x + \sin x) + 1}{(\cos x + \sin x) + 1}$

$$= \frac{\cos^2 x + \cos x \sin x + \cos x + \cos x + \sin x + 1 - \sin x \cos x - \sin^2 x - \sin x}{\cos^2 x + 2\cos x \sin x + \sin^2 x - 1}$$

$$= \frac{\cos^2 x + 2\cos x + \cos^2 x}{2\cos x \sin x}$$

$$= \frac{2\cos^2 x + 2\cos x}{2\cos x \sin x}$$

$$= \frac{\cos x + 1}{\sin x}$$

$$= \frac{\cos x}{\sin x} + \frac{1}{\sin x}$$

$$= \cot x + \csc x \checkmark.$$

$$\cot x + \csc x - 1 \stackrel{?}{=} (\cot x - \csc x + 1)(\csc x + \cot x)$$

$$\stackrel{?}{=} \underline{\cot x \csc x} + \cot^2 x - \csc^2 x - \underline{\csc x \cot x} + \csc x + \cot x$$

$$= -1 + \csc x + \cot x \checkmark$$

$$1 + \cot^2 x = \csc^2 x$$

$$\cot^2 x - \csc^2 x = -1$$

39/ $$\frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 1 + \sin x \cos x$$

$$\frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = \frac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\sin x - \cos x}$$

$= 1$

$$= 1 + \sin x \cos x$$

71/ $$\frac{\csc x - 1}{\csc x + 1} = \frac{\cot^2 x}{\csc^2 x + 2\csc x + 1}$$

$$\frac{\csc x - 1}{\csc x + 1} = \frac{\csc x - 1}{\csc x + 1} \cdot \frac{\csc x + 1}{\csc x + 1}$$

$$= \frac{\csc^2 x - 1}{\csc^2 x + 2\csc x + 1}$$

$$= \frac{\cot^2 x}{\csc^2 x + 2\csc x + 1}$$

75/ $$\csc^2 x - \cos^2 x \csc^2 x = 1$$

$$\csc^2 x - \cos^2 x \csc^2 x = \csc^2 x (1 - \cos^2 x)$$

$$= \frac{1}{\sin^2 x} \cdot (\sin^2 x)$$

$$= 1$$

77/ $\csc^2 x - \cos x \sec x = \cot^2 x$

$$\csc^2 x - \cos x \sec x = \frac{1}{\sin^2 x} - \cos x \frac{1}{\cos x}$$

$$= \frac{1}{\sin^2 x} - 1$$

$$= \frac{1 - \sin^2 x}{\sin^2 x}$$

$$= \frac{\cos^2 x}{\sin^2 x}$$

$$= \cot^2 x \checkmark$$

78/ $(\sec x - \tan x)(\sec x + \tan x) = 1$

$$(\sec x - \tan x)(\sec x + \tan x) = \sec^2 x - \tan^2 x$$

$$= 1 \checkmark$$

$1 + \tan^2 = \sec^2$

$$= \sec^2 x - (-1 + \sec^2 x)$$

#48/ $\dfrac{3\csc^2 x - 5\csc x - 28}{\csc x - 4} = \dfrac{3}{\sin x} + 7$

$$\frac{3\csc^2 x - 5\csc x - 28}{\csc x - 4} = \frac{(\csc x - 4)(3\csc x + 7)}{\csc x - 4}$$

$$= 3\frac{1}{\sin x} + 7 \checkmark$$

PC/ $10\csc^2 x - 6\cot^2 x = 4\csc^2 x + 6$

$$10\csc^2 x - 6\cot^2 x = 10\csc^2 x - 6(\csc^2 x - 1)$$
$$= 10\csc^2 x - 6\csc^2 x + 6$$
$$= 4\csc^2 x + 6$$