# Solution    *Section* 1.2 – Dot Products

### *Exercise*

Find for $\boldsymbol{v} = 2\boldsymbol{i} - 4\boldsymbol{j} + \sqrt{5}\boldsymbol{k}, \quad \boldsymbol{u} = -2\boldsymbol{i} + 4\boldsymbol{j} - \sqrt{5}\boldsymbol{k}$

*a)* $\boldsymbol{v} \bullet \boldsymbol{u},\ |\boldsymbol{v}|,\ |\boldsymbol{u}|$

*b)* The cosine of the angle between $\boldsymbol{v}$ and $\boldsymbol{u}$

*c)* The scalar component of $\boldsymbol{u}$ in the direction of $\boldsymbol{v}$

*d)* The vector $proj_{\boldsymbol{v}}\boldsymbol{u}$

***Solution***

***a)***
$$\begin{aligned} \boldsymbol{v} \bullet \boldsymbol{u} &= \left(2\boldsymbol{i} - 4\boldsymbol{j} + \sqrt{5}\boldsymbol{k}\right) \cdot \left(-2\boldsymbol{i} + 4\boldsymbol{j} - \sqrt{5}\boldsymbol{k}\right) \\ &= -4 - 16 - 5 \\ &= -25 \end{aligned}$$

$$\begin{aligned} |\boldsymbol{v}| &= \sqrt{2^2 + (-4)^2 + \left(\sqrt{5}\right)^2} \\ &= \sqrt{4 + 16 + 5} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} |\boldsymbol{u}| &= \sqrt{(-2)^2 + 4^2 + \left(-\sqrt{5}\right)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

***b)*** $\cos\theta = \dfrac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|} = \dfrac{-25}{(5)(5)} = -1$

***c)*** $|\boldsymbol{u}|\cos\theta = (5)(-1) = -5$

***d)***
$$\begin{aligned} proj_{\boldsymbol{v}}\boldsymbol{u} &= \left(\frac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{v}|^2}\right)\boldsymbol{v} \\ &= \left(\frac{-25}{5^2}\right)\left(2\boldsymbol{i} - 4\boldsymbol{j} + \sqrt{5}\boldsymbol{k}\right) \\ &= -\left(2\boldsymbol{i} - 4\boldsymbol{j} + \sqrt{5}\boldsymbol{k}\right) \\ &= -2\boldsymbol{i} + 4\boldsymbol{j} - \sqrt{5}\boldsymbol{k} \end{aligned}$$

## *Exercise*

Find for $\vec{v} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}, \quad \vec{u} = 5\hat{i} + 12\hat{j}$

*a)* $\boldsymbol{v} \bullet \boldsymbol{u}, \ |\boldsymbol{v}|, \ |\boldsymbol{u}|$

*b)* The cosine of the angle between ***v*** and ***u***

*c)* The scalar component of ***u*** in the direction of ***v***

*d)* The vector $proj_{\boldsymbol{v}}\boldsymbol{u}$

***Solution***

**a)** $\vec{v} \bullet \vec{u} = \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}\right) \bullet \left(5\hat{i} + 12\hat{j}\right)$

$= 3$

$$|\vec{v}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{16}{25}}$$

$$= \sqrt{\frac{25}{25}}$$

$$= 1$$

$$|\vec{u}| = \sqrt{5^2 + 12^2}$$

$$= 13$$

**b)** $\cos\theta = \dfrac{\vec{u} \bullet \vec{v}}{|\vec{u}||\vec{v}|}$

$$= \frac{3}{(1)(13)}$$

$$= \frac{3}{13}$$

**c)** $|\vec{u}|\cos\theta = (13)\left(\frac{3}{13}\right) = 3$

**d)** $proj_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u} \bullet \vec{v}}{|\vec{v}|^2}\right)\vec{v}$

$$= \left(\frac{3}{1^2}\right)\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}\right)$$

$$= \frac{9}{5}\hat{i} + \frac{12}{5}\hat{k}$$

## *Exercise*

Find for $\boldsymbol{v} = 2\boldsymbol{i} + 10\boldsymbol{j} - 11\boldsymbol{k}, \quad \boldsymbol{u} = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$

*a)* $\boldsymbol{v} \cdot \boldsymbol{u},\ |\boldsymbol{v}|,\ |\boldsymbol{u}|$

*b)* The cosine of the angle between $\boldsymbol{v}$ and $\boldsymbol{u}$

*c)* The scalar component of $\boldsymbol{u}$ in the direction of $\boldsymbol{v}$

*d)* The vector $proj_{\boldsymbol{v}}\boldsymbol{u}$

***Solution***

***a)***
$$\begin{aligned}\boldsymbol{v} \cdot \boldsymbol{u} &= (2\boldsymbol{i} + 10\boldsymbol{j} - 11\boldsymbol{k}) \cdot (2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}) \\ &= 4 + 20 - 11 \\ &= 13\end{aligned}$$

$$\begin{aligned}|\boldsymbol{v}| &= \sqrt{2^2 + 10^2 + (-11)^2} \\ &= \sqrt{4 + 100 + 121} \\ &= \sqrt{225} \\ &= 15\end{aligned}$$

$$\begin{aligned}|\boldsymbol{u}| &= \sqrt{2^2 + 2^2 + 1^2} \\ &= 3\end{aligned}$$

***b)***
$$\begin{aligned}\cos\theta &= \frac{\boldsymbol{u} \cdot \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|} \\ &= \frac{13}{(3)(15)} \\ &= \frac{13}{45}\end{aligned}$$

***c)***
$$|\boldsymbol{u}|\cos\theta = (3)\left(\frac{13}{45}\right) = \frac{13}{15}$$

***d)***
$$\begin{aligned}proj_{\vec{v}}\vec{u} &= \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right)\vec{v} \\ &= \left(\frac{13}{15^2}\right)(2\boldsymbol{i} + 10\boldsymbol{j} - 11\boldsymbol{k}) \\ &= \frac{13}{225}(2\boldsymbol{i} + 10\boldsymbol{j} - 11\boldsymbol{k})\end{aligned}$$

## *Exercise*

Find for $\boldsymbol{v} = 5\boldsymbol{i} + \boldsymbol{j}, \quad \boldsymbol{u} = 2\boldsymbol{i} + \sqrt{17}\,\boldsymbol{j}$

*a)* $\boldsymbol{v} \bullet \boldsymbol{u},\ |\boldsymbol{v}|,\ |\boldsymbol{u}|$

*b)* The cosine of the angle between $\boldsymbol{v}$ and $\boldsymbol{u}$

*c)* The scalar component of $\boldsymbol{u}$ in the direction of $\boldsymbol{v}$

*d)* The vector $proj_{\boldsymbol{v}}\boldsymbol{u}$

***Solution***

***a)*** $\boldsymbol{v} \bullet \boldsymbol{u} = (5\boldsymbol{i} + \boldsymbol{j}) \cdot \left(2\boldsymbol{i} + \sqrt{17}\,\boldsymbol{j}\right) \ \boxed{= 10 + \sqrt{17}}$

$|\boldsymbol{v}| = \sqrt{25+1} \ \boxed{= \sqrt{26}}$

$|\boldsymbol{u}| = \sqrt{4+17} \ \boxed{= \sqrt{21}}$

***b)*** $\cos\theta = \dfrac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$

$$= \frac{10+\sqrt{17}}{\sqrt{21}\sqrt{26}}$$

$$\boxed{= \frac{10+\sqrt{17}}{\sqrt{546}}}$$

***c)*** $|\boldsymbol{u}|\cos\theta = \left(\sqrt{21}\right)\left(\dfrac{10+\sqrt{17}}{\sqrt{546}}\right)$

$$\boxed{= \frac{10+\sqrt{17}}{\sqrt{26}}}$$

***d)*** $proj_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u} \bullet \vec{v}}{|\vec{v}|^2}\right)\vec{v}$

$$\boxed{= \left(\frac{10+\sqrt{17}}{26}\right)(5\boldsymbol{i} + \boldsymbol{j})}$$

### *Exercise*

Find for $\boldsymbol{v}=\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}} \right\rangle, \quad \boldsymbol{u}=\left\langle \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{3}} \right\rangle$

*a)* $\boldsymbol{v}\bullet\boldsymbol{u},\ |\boldsymbol{v}|,\ |\boldsymbol{u}|$

*b)* The cosine of the angle between $\boldsymbol{v}$ and $\boldsymbol{u}$

*c)* The scalar component of $\boldsymbol{u}$ in the direction of $\boldsymbol{v}$

*d)* The vector $proj_{\boldsymbol{v}}\boldsymbol{u}$

***Solution***

***a)*** $\boldsymbol{v}\bullet\boldsymbol{u}=\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}} \right\rangle\cdot\left\langle \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{3}} \right\rangle=\frac{1}{2}-\frac{1}{3}\ \underline{=\frac{1}{6}}$

$$|\boldsymbol{v}|=\sqrt{\frac{1}{2}+\frac{1}{3}}=\frac{\sqrt{5}}{\sqrt{6}}\frac{\sqrt{6}}{\sqrt{6}}\ \underline{=\frac{\sqrt{30}}{6}}$$

$$|\boldsymbol{u}|=\sqrt{\frac{1}{2}+\frac{1}{3}}=\frac{\sqrt{5}}{\sqrt{6}}\frac{\sqrt{6}}{\sqrt{6}}\ \underline{=\frac{\sqrt{30}}{6}}$$

***b)*** 
$$\cos\theta=\frac{\boldsymbol{u}\bullet\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$$

$$=\frac{\frac{1}{6}}{\frac{\sqrt{30}}{6}\frac{\sqrt{30}}{6}}$$

$$=\frac{1}{6}\left(\frac{36}{30}\right)$$

$$\underline{=\frac{1}{5}}$$

***c)*** $|\boldsymbol{u}|\cos\theta=\left(\frac{\sqrt{30}}{6}\right)\left(\frac{1}{5}\right)=\frac{\sqrt{30}}{30}=\underline{\frac{1}{\sqrt{30}}}$

***d)*** 
$$proj_{\vec{v}}\vec{u}=\left(\frac{\vec{u}\bullet\vec{v}}{|\vec{v}|^2}\right)\vec{v}$$

$$=\frac{1}{6}\left(\frac{36}{30}\right)\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}} \right\rangle$$

$$\underline{=\frac{1}{5}\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}} \right\rangle}$$

### *Exercise*

Find the angles between the vectors $\vec{u} = 2\hat{i} + \hat{j}, \quad \vec{v} = \hat{i} + 2\hat{j} - \hat{k}$

***Solution***

$$\theta = \cos^{-1}\left(\frac{2+2+0}{\sqrt{4+1}\sqrt{1+4+1}}\right) \qquad \theta = \cos^{-1}\frac{\vec{u}\bullet\vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \cos^{-1}\left(\frac{4}{\sqrt{5}\sqrt{6}}\right)$$

$$= \cos^{-1}\left(\frac{4}{\sqrt{30}}\right)$$

$$\approx 0.84 \ \ rad$$

### *Exercise*

Find the angles between the vectors $\vec{u} = \sqrt{3}\hat{i} - 7\hat{j}, \quad \vec{v} = \sqrt{3}\hat{i} + \hat{j} + \hat{k}$

***Solution***

$$\theta = \cos^{-1}\left(\frac{3-7+0}{\sqrt{3+49}\sqrt{3+1+1}}\right) \qquad \theta = \cos^{-1}\frac{\vec{u}\bullet\vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \cos^{-1}\left(\frac{-4}{\sqrt{52}\sqrt{5}}\right)$$

$$= \cos^{-1}\left(-\frac{4}{\sqrt{260}}\right)$$

$$\approx 1.82 \ \ rad$$

### *Exercise*

Find the angles between the vectors $\vec{u} = \hat{i} + \sqrt{2}\hat{j} - \sqrt{2}\hat{k}, \quad \vec{v} = -\hat{i} + \hat{j} + \hat{k}$

***Solution***

$$\theta = \cos^{-1}\left(\frac{-1+\sqrt{2}-\sqrt{2}}{\sqrt{1+2+2}\sqrt{1+1+1}}\right) \qquad \theta = \cos^{-1}\frac{\vec{u}\bullet\vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \cos^{-1}\left(\frac{-1}{\sqrt{5}\sqrt{3}}\right)$$

$$= \cos^{-1}\left(-\frac{1}{\sqrt{15}}\right)$$

$$\approx 1.83 \ \ rad$$

## Exercise

Consider $\vec{u} = -3\hat{j} + 4\hat{k}$, $\vec{v} = -4\hat{i} + \hat{j} + 5\hat{k}$

*a)* Find the angle between $\vec{u}$ and $\vec{v}$.

*b)* Compute $proj_{\vec{v}}\vec{u}$ and $scal_{\vec{v}}\vec{u}$

*c)* Compute $proj_{\vec{u}}\vec{v}$ and $scal_{\vec{u}}\vec{v}$

***Solution***

**a)** $\theta = \cos^{-1}\dfrac{\left(-3\hat{j} + 4\hat{k}\right)\bullet\left(-4\hat{i} + \hat{j} + 5\hat{k}\right)}{\sqrt{9+16}\ \sqrt{16+1+25}}$ $\qquad\qquad \theta = \cos^{-1}\dfrac{\vec{u}\bullet\vec{v}}{|\vec{u}||\vec{v}|}$

$= \cos^{-1}\dfrac{-3+20}{\sqrt{25}\ \sqrt{42}}$

$\boxed{= \cos^{-1}\dfrac{17}{5\sqrt{42}}}$

$\boxed{\approx 1.02\ \ rad}$

**b)** $proj_{\vec{v}}\vec{u} = \dfrac{17}{42}\left(-4\hat{i} + \hat{j} + 5\hat{k}\right)$ $\qquad\qquad proj_{\vec{v}}\vec{u} = \dfrac{\vec{u}\bullet\vec{v}}{|\vec{v}|^2}\vec{v}$

$\boxed{= \dfrac{17}{42}\langle -4,\ \ 1,\ \ 5\rangle}$

$\boxed{scal_{\vec{v}}\vec{u} = \dfrac{17}{\sqrt{42}}}$ $\qquad\qquad scal_{\vec{v}}\vec{u} = \dfrac{\vec{u}\bullet\vec{v}}{|\vec{v}|}$

**c)** $proj_{\vec{u}}\vec{v} = \dfrac{17}{25}\left(-3\hat{j} + 4\hat{k}\right)$ $\qquad\qquad proj_{\vec{u}}\vec{v} = \dfrac{\vec{u}\bullet\vec{v}}{|\vec{u}|^2}\vec{u}$

$\boxed{= \dfrac{17}{25}\langle 0,\ \ -3,\ \ 4\rangle}$

$\boxed{scal_{\vec{u}}\vec{v} = \dfrac{17}{5}}$ $\qquad\qquad scal_{\vec{v}}\vec{u} = \dfrac{\vec{u}\bullet\vec{v}}{|\vec{v}|}$

## Exercise

Consider $\vec{u} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{v} = 3\hat{i} + 6\hat{j} + 6\hat{k}$

*a)* Find the angle between $\vec{u}$ and $\vec{v}$.

*b)* Compute $proj_{\vec{v}}\vec{u}$ and $scal_{\vec{v}}\vec{u}$

*c)* Compute $proj_{\vec{u}}\vec{v}$ and $scal_{\vec{u}}\vec{v}$

***Solution***

**a)** $\theta = \cos^{-1}\dfrac{\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)\bullet\left(3\hat{i} + 6\hat{j} + 6\hat{k}\right)}{\sqrt{1+4+4}\ \sqrt{9+36+36}}$ $\qquad\qquad \theta = \cos^{-1}\dfrac{\vec{u}\bullet\vec{v}}{|\vec{u}||\vec{v}|}$

$$= \cos^{-1}\frac{-3+12+12}{3\,(9)}$$

$$= \cos^{-1}\frac{21}{27}$$

$$\boxed{= \cos^{-1}\frac{7}{9}}$$

$$\boxed{\approx 0.68\ \ rad}$$

***b)*** $proj_{\vec{v}}\,\vec{u} = \frac{21}{81}\langle 3,\ 6,\ 6\rangle$ $\qquad proj_{\vec{v}}\,\vec{u} = \frac{\vec{u}\bullet\vec{v}}{|\vec{v}|^2}\vec{v}$

$$\boxed{= \frac{7}{9}\langle 1,\ \ 2,\ \ 2\rangle}$$

$scal_{\vec{v}}\,\vec{u} = \frac{21}{9}$ $\qquad scal_{\vec{v}}\,\vec{u} = \frac{\vec{u}\bullet\vec{v}}{|\vec{v}|}$

$$\boxed{= \frac{7}{3}}$$

***c)*** $proj_{\vec{u}}\,\vec{v} = \frac{21}{9}\langle -1,\ 2,\ 2\rangle$ $\qquad proj_{\vec{u}}\,\vec{v} = \frac{\vec{u}\bullet\vec{v}}{|\vec{u}|^2}\vec{u}$

$$\boxed{= \frac{7}{3}\langle -1,\ \ 2,\ \ 2\rangle}$$

$scal_{\vec{u}}\,\vec{v} = \frac{21}{3}$ $\qquad scal_{\vec{v}}\,\vec{u} = \frac{\vec{u}\bullet\vec{v}}{|\vec{v}|}$

$$\boxed{= 7}$$

## *Exercise*

The direction angles $\alpha$, $\beta$, and $\gamma$ of a vector $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ are defined as follows:

is the angle between $\vec{v}$ and the positive $x$-axis $(0 \le \alpha \le \pi)$

is the angle between $\vec{v}$ and the positive $y$-axis $(0 \le \beta \le \pi)$

is the angle between $\vec{v}$ and the positive $z$-axis $(0 \le \gamma \le \pi)$

*a)* Show that $\cos\alpha = \frac{a}{|\vec{v}|}$, $\quad \cos\beta = \frac{b}{|\vec{v}|}$, $\quad \cos\gamma = \frac{c}{|\vec{v}|}$, and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ . These cosines are called the direction cosines of $\vec{v}$ .

*b)* Show that if $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ is a unit vector, then $a$, $b$, and $c$ are the direction cosines of $\vec{v}$ .

***Solution***

***a)*** $\cos\alpha = \frac{\hat{i}\bullet\vec{v}}{|\hat{i}||\vec{v}|}$

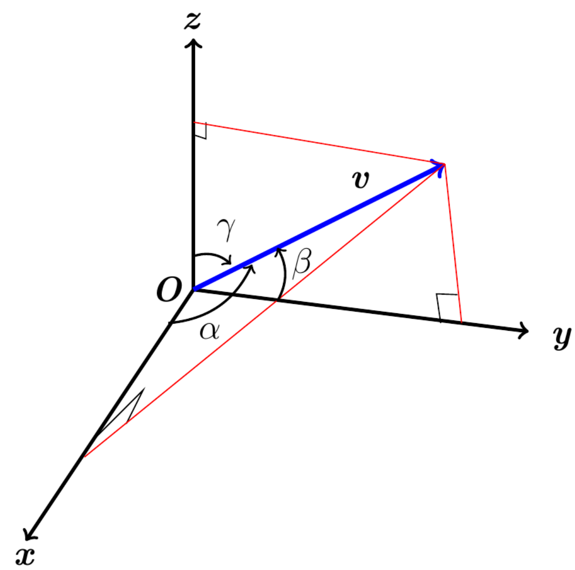
$$= \frac{\hat{i}\bullet\left(a\hat{i} + b\hat{j} + c\hat{k}\right)}{|\vec{v}|}$$

$$= \boxed{\frac{a}{|\vec{v}|}}$$

$$\cos\beta = \frac{\hat{j}\bullet\vec{v}}{|\hat{j}||\vec{v}|}$$

$$= \frac{\hat{j}\bullet\left(a\hat{i} + b\hat{j} + c\hat{k}\right)}{|\vec{v}|}$$

$$= \boxed{\frac{b}{|\vec{v}|}}$$




$$\cos\gamma = \frac{\hat{k}\bullet\vec{v}}{|\hat{k}||\vec{v}|}$$

$$= \frac{\hat{k}\bullet\left(a\hat{i} + b\hat{j} + c\hat{k}\right)}{|\vec{v}|}$$

$$= \boxed{\frac{c}{|\vec{v}|}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\frac{a}{|\vec{v}|}\right)^2 + \left(\frac{b}{|\vec{v}|}\right)^2 + \left(\frac{c}{|\vec{v}|}\right)^2$$

$$= \frac{a^2}{|\vec{v}|^2} + \frac{b^2}{|\vec{v}|^2} + \frac{c^2}{|\vec{v}|^2}$$

$$= \frac{a^2 + b^2 + c^2}{|\vec{v}|^2}$$

$$= \frac{a^2 + b^2 + c^2}{a^2 + b^2 + c^2}$$

$$\boxed{= 1}$$

***b)*** If $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ is a unit vector $\Rightarrow |\vec{v}| = 1$

$\cos\alpha = \frac{a}{|\vec{v}|} = a, \quad \cos\beta = \frac{b}{|\vec{v}|} = b, \quad \cos\gamma = \frac{c}{|\vec{v}|} = c$ are the direction cosines of $\vec{v}$.

## *Exercise*

A water main is to be constructed with 20% grade in the north direction and a 10% grade in the east direction. Determine the angle $\theta$ required in the water main for the turn from north to east.

***Solution***

20% grade in the north direction

$\Rightarrow zk = 20\%xi = .2xi \quad \rightarrow If \;\; x = 10 \quad z = 2$

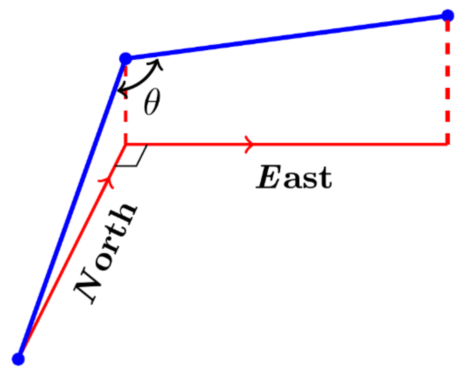
Let $\vec{u} = 10\hat{i} + 2\hat{k}$ be parallel to the pipe in the north direction.

$\vec{v} = 10\hat{j} + \hat{k}$ be parallel to the pipe in the east direction.

$$\theta = \cos^{-1}\frac{0+0+2}{\sqrt{100+4}\sqrt{100+1}} \qquad \theta = \cos^{-1}\frac{\vec{u}\bullet\vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \cos^{-1}\frac{2}{\sqrt{104}\sqrt{101}}$$

$$\approx 88.88^\circ$$




### Exercise

A gun with muzzle velocity of 1200 *ft/sec* is fired at an angle of 8° above the horizontal. Find the horizontal and vertical components of the velocity.

***Solution***

Horizontal component: $1200\cos 8^\circ \approx 1188\ ft/s$

Vertical component: $1200\sin 8^\circ \approx 167\ ft/s$

### Exercise

Suppose that a box is being towed up an inclined plane. Find the force $\boldsymbol{w}$ needed to make the component of the force parallel to the indicated plane equal to 2.5 *lb*.

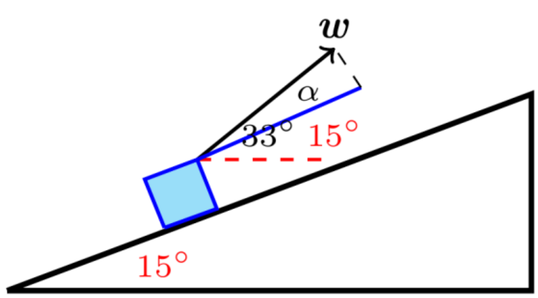
***Solution***

$$2.5 = |w|\cos\alpha$$

$$|\vec{w}| = \frac{2.5}{\cos(33^\circ - 15^\circ)} = \frac{2.5}{\cos 18^\circ}$$

$$\vec{w} = \frac{2.5}{\cos 18^\circ}\langle \cos 33^\circ,\ \sin 33^\circ \rangle$$

$$= \langle 2.205,\ 1.432 \rangle$$




### Exercise

Find the work done by a force $\boldsymbol{F} = 5\boldsymbol{i}$ (magnitude 5 *N*) in moving an object along the line from the origin to the point (1, 1) (distance in meters)

***Solution***

$$P(1,\ 1) \quad \Rightarrow \quad \overrightarrow{OP} = \boldsymbol{i} + \boldsymbol{j}$$

$$W = F \cdot \overrightarrow{OP}$$

$$= 5\boldsymbol{i} \cdot (\boldsymbol{i} + \boldsymbol{j})$$

$$= 5\ J$$

## Exercise

How much work does it take to slide a crate 20 *m* along a loading dock by pulling on it with a 200 *N* force at an angle of 30° from the horizontal?

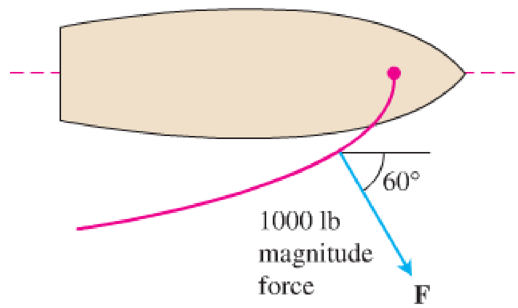
**Solution**

$$W = |F|\left|\overrightarrow{PQ}\right|\cos\theta$$
$$= (200)(20)\cos 30°$$
$$\boxed{= 3464.10\ J}$$

## Exercise

The wind passing over a boat's sail exerted a 1000-*lb* magnitude force *F*. How much work did the wind perform in moving the boat forward 1 *mi*? Answer in foot-pounds.

**Solution**




$$W = |F|\left|\overrightarrow{PQ}\right|\cos\theta$$
$$= (1000N)\left(1\ mi\,\frac{5280\ ft}{1\ mi}\right)\cos 60°$$
$$\boxed{= 2,640,000\ \ ft\cdot lb}$$

## Exercise

Use a dot product to find an equation of the line in the *xy*-plane passing through the point $(x_0,\ y_0)$ perpendicular to the vector $\langle a,\ b\rangle$.

**Solution**

$$\langle x-x_0,\ y-y_0\rangle \bullet \langle a,\ b\rangle = 0$$
$$\boxed{a(x-x_0)+b(y-y_0)=0}$$

## Exercise

A 180-*lb* man stands on a hillside that makes an angle of 30° with the horizontal, producing a force of $W = \langle 0,\ -180\rangle$ *lbs*.

*a)* Find the component of his weight in the downward direction perpendicular to the hillside and in the downward parallel to the hillside.

*b)* How much work is done when the man moves 10 *ft* up the hillside?

**Solution**

***a)*** $\left|F_{\perp}\right| = \left|F_y\right| = 180\cos 30°$

$$= 180\left(\frac{\sqrt{3}}{2}\right)$$

$$\underline{= 90\sqrt{3}\ \ lb}$$

$\left|F_{//}\right| = \left|F_x\right| = 180\sin 30°$

$$= 180\left(\frac{1}{2}\right)$$

$$\underline{= 90\ \ lb}$$

***b)*** $Work = d \cdot F_x$

$$= 10(90)$$

$$\underline{= 900\ \ ft\text{-}lbs}$$