Discrete 9/16

Set

Elements

$A = \{ \ , \ , \ , \ \}$

$\{ \ \ | \ \underline{\text{condition}} \ \}$

for all such that

$A = \{a_1, a_2\}$ $B = \{b_1, b_2, b_3\}$

$A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2), (a_3, b_3)\}$

1st 2nd

1.8 Set Operations

$A \cup B = \{x \mid x \in A \vee x \in B\}$

is element

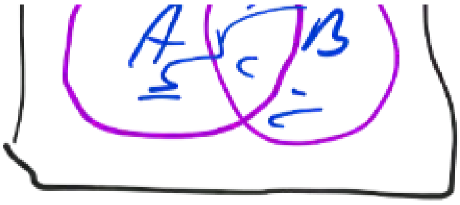
elt

Ex

$A = \{1, 3, 5, 7, 9, 11\}$

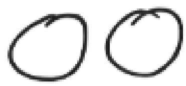
$B = \{3, 6, 9, 12\}$

$A \cup B = \{1, 3, 5, 7, 9, 11, 6, 12\}$

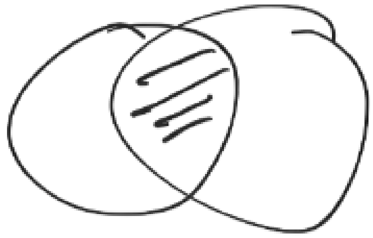


Intersection $\cap$

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$A \cap B = \{\phi\}$ disjoint. 

$A \cap B = \{3, 9\}$



Complement A'

$\bar{A} = \{x \mid x \notin A \wedge x \in U\}$

$A \cap \bar{A} = \{\phi\}$

$A \cup \bar{A} = U$

$\{1, 3, 5\} - \{1, 2, 3\} = \{5\}$

Ex $\cancel{A = \{x \mid x > 10, x \in \mathbb{Z}^+\}}$

$A = \{x \in \mathbb{Z}^+ \mid x > 10\}$

$\longrightarrow x \le 0$

$\bar{A} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$= \{x \in \mathbb{Z}^+ \mid x \le 10\}$

$A \cup A = A$

$A \cup \phi = A$

$A \cup U = U$

$A \cap A = A$

$A \cap \phi = \phi$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

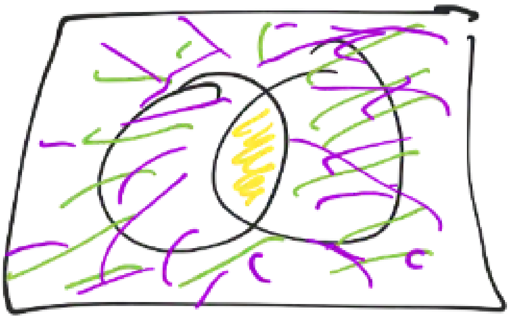
$\quad \vee \quad \wedge \qquad\qquad \vee \quad \wedge \quad \vee$

union $\bigcup$ $\qquad \bigcap$ int.

or $\vee$ $\qquad \wedge$ and

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

prove $\longrightarrow$



$x \in \overline{A \cap B}$

$x \notin A \cap B$

$x \notin ((x \in A) \wedge (x \in B))$

$\neg (x \in A) \vee \neg (x \in B)$

$x \notin A \quad \vee \quad x \notin B$

$x \in \bar{A} \quad \vee \quad x \in \bar{B}$

$$\bar{A} \cup \bar{B}$$

$$\left.\begin{array}{l} \overline{A \cap B} \subseteq \bar{A} \cup \bar{B} \\ \bar{A} \cup \bar{B} \subseteq \overline{A \cap B} \end{array}\right\} \overline{A \cap B} = \bar{A} \cup \bar{B}$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

$A = \{0, 2, 4, 6, 8\}$

$B = \{0, 1, 2, 3, 4\}$

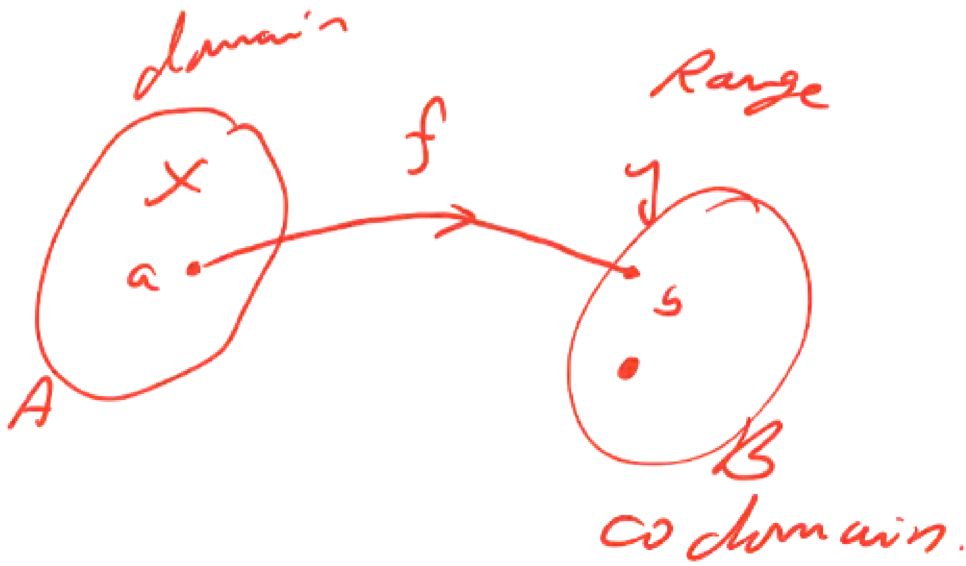
$C = \{0, 3, 6, 9\}$

$A \cup B \cup C = \{0, 1, 2, 3, 4, 6, 8, 9\}$

$A \cap B \cap C = \{0\}$

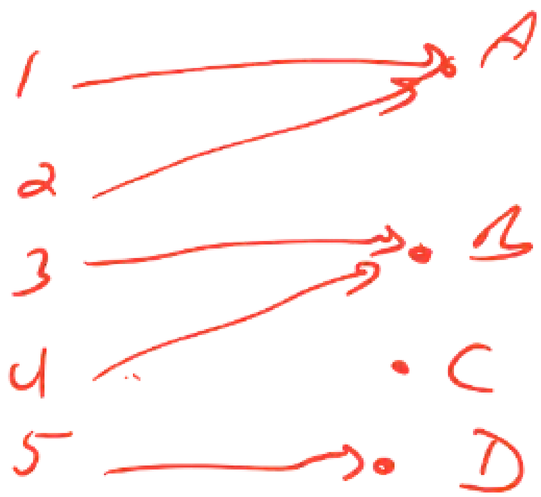
$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{k=1}^{n} A_k$$

1.9 Functions



range / codomain

$\{b\}$ $\quad$ B



Range $\{A, B, D\}$

Co-domain $\{A, B, C, D\}$

$$(f_1 + f_2)(x) = f_1(x) + f_2(x)$$

$$(f_1 f_2)(x) = f_1(x)\, f_2(x)$$

$f_1(x) = x^2 \qquad f_2 = x - x^2$

$(f_1 + f_2)(x) = x^2 + x - x^2$

$\qquad\qquad\qquad\qquad = x$

Defn $\quad f : A \to B \qquad S \subseteq A$

image of $S \subseteq B$

$$f(S) = \{ t \mid \exists s \in S \, (t = f(s)) \}$$

$$\{ y \mid \exists x \in S \, (y = f(x)) \}$$

One-to-One $\qquad x \underset{f^{-1}}{\overset{f}{\rightleftarrows}} y$

$1 - 1$

or $\begin{cases} f(a) = f(b) \Rightarrow a = b \\ \text{or} \\ a \neq b \Rightarrow f(a) \neq f(b) \end{cases}$

$f(x) = x^2 \qquad -1 \neq 1$

$\qquad\qquad f(-1) = f(1) \quad \#$

Not $1-1$ fctn

$$f(x) = \frac{ax+b}{cx+d} \Rightarrow f^{-1} = \frac{-dx+b}{cx-a}$$

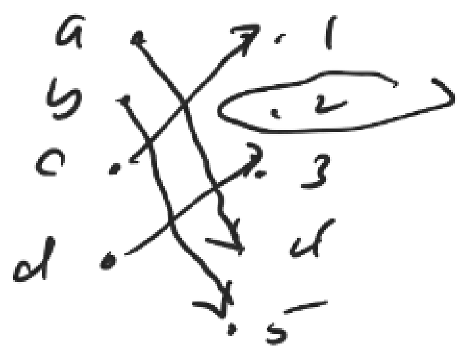
in function (1-1)

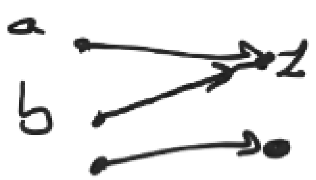
ex $\{a,b,c,d\} \to \{1,2,3,4,5\}$

$f(a)=4$, $f(b)=5$ $f(c)=1$

$f(d)=3$

1-1? ✓





not 1-1
onto

$f(x) = 2x+3$

$g(x) = 3x+2$

$(f\circ g)(x) = f(g(x))$

$= f(3x+2)$

$= 2(3x+2)+3$

$= 6x+7$

$\mathbb{R} \to \mathbb{R}$

$f(x) = \frac{1}{x}$? $x \neq 0 \Rightarrow$ not a fctn

$f(x) = \sqrt{x}$ not a fctn $x \geq 0$

$\mathbb{R}$

$f(x) = \pm\sqrt{x^2+1}$

$f(1) = \pm\sqrt{2}$ not a fctn

$-\sqrt{2}$ $\sqrt{2}$

$1 \to -\sqrt{2}$

$1 \to \sqrt{2}$

$\left.\begin{array}{l} \pm \\ x^2 \end{array}\right\}$ not

restriction $x > 0$

$y = \sqrt{x}$

$y = x^2$