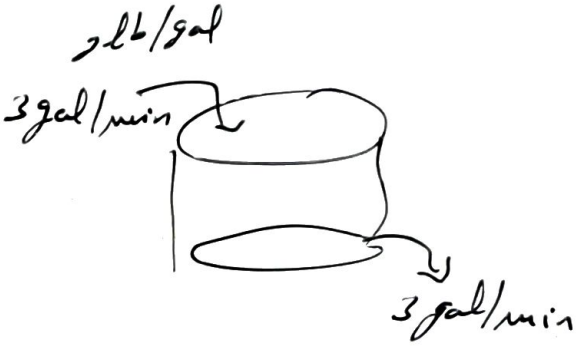
Rate of Change $= \frac{dx}{dt}$

$=$ Rate in $-$ Rate out.

Rate $=$ Vol $\frac{gal}{min}$ (Rate) $\times$ concentration $\frac{lb}{gal}$

Ex $x(t)$: lb



Volume $= 100 + (3-3)t$

$= 100$

concentration: $C(t) = \frac{x(t)}{V(t)}$

$= \frac{x(t)}{100}$

Rate in $= 2\frac{lb}{gal} \times 3\frac{gal}{min}$

$= 6$ lb/min

Rate out $= 3\frac{gal}{min} \times \frac{x(t)}{100} = \frac{3}{100}x(t)$

$\frac{dx}{dt} = R_{in} - R_{out}$

$= 6 - \frac{3}{100}x$

$x' + \frac{3}{100}x = 6$

$e^{\int \frac{3}{100}dt} = e^{3t/100}$

$\int 6e^{3t/100}dt = 200e^{\frac{3t}{100}}$

$x(t) = \frac{1}{e^{3t/100}}\left(200e^{\frac{3t}{100}} + C\right)$

$= 200 + \frac{C}{e^{3t/100}}$

$t = 0 \Rightarrow x(t) = 0$

$0 = 200 + \frac{C}{1} \Rightarrow C = -200$

$$x(t) = 200 - \frac{200}{e^{3t/100}} \qquad \frac{18}{10}$$

$$x(60) = 200\left(1 - e^{-1.8}\right)$$

$$\approx 167 \text{ lb}$$

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600 Tank w/ 300 $(x(0) = 0)$

In: 1.5 lb/gal $= \frac{3}{2}$, 3 gal/min

Out 1 gal/min

Soln $V(t) = 300 + (3-1)t$

$= 300 + 2t = 600$

$t = 150$ min Full.

$$c(t) = \frac{x}{300+2t}$$

$$R_{in} = \frac{3}{2} \times 3 = \frac{9}{2}$$

$$R_{out} = 1 \times \frac{x}{300+2t} = \frac{x}{300+2t}$$

$$x' = \frac{9}{2} - \frac{x}{300+2t}$$

$$x' + \frac{1}{300+2t}x = \frac{9}{2}$$

$$e^{\int \frac{dt}{300+2t}} = e^{\frac{1}{2}\int \frac{d(300+2t)}{300+2t}}$$

$$= e^{\frac{1}{2}\ln(300+2t)}$$

$$= e^{\ln(300+2t)^{1/2}}$$

$$= \sqrt{300+2t}$$

$$\frac{1}{2}\frac{9}{2}\int (300+2t)^{1/2}\, d(300+2t)$$

$$= \frac{3}{2}(300+2t)^{3/2}$$

$$x(t) = \frac{1}{\sqrt{300+2t}} \left(\frac{3}{2}(300+2t)^{3/2} + C \right)$$

$$= \frac{3}{2}(300+2t) + \frac{C}{\sqrt{300+2t}}$$

$$= 450 + 3t + \frac{C}{\sqrt{300+2t}}$$

$$0 = 450 + \frac{C}{\sqrt{300}}$$

$$C = -4500\sqrt{3}$$

$$x(t) = 450 + 3t - \frac{4500\sqrt{3}}{\sqrt{300+2t}}$$

1.6 Exact Differential Eqn.

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$M + N y' = 0$$

If exact, $M_y(x,y) = N_x(x,y)$

$$\psi_x = M(x,y) \qquad \psi_y = N(x,y)$$

$$\psi(x,y) = \int M(x,y)\,dx$$

Ex $2x + y^2 + 2xy\,y' = 0$

$M = 2x + y^2$ $\qquad N = 2xy$

$M_y = 2y$ $\qquad N_x = 2y$

$\Rightarrow M_y = N_x = 2y$

$$\psi = \int M\,dx$$

$$= \int (2x + y^2)\,dx$$

$$= x^2 + y^2 x + h(y)$$

$$\psi_y = N$$

$$2xy + h'(y) = 2xy$$

$h'(y) = 0 \rightarrow h(y) = \int 0\,dx = C$

$$\psi(x,y) = x^2 + y^2 x + C = 0$$

$$x^2 + xy^2 = C$$

Ex: $\underbrace{y\cos x + 2xe^y}_{M} + \underbrace{(\sin x + x^2e^y - 1)}_{N}y' = 0$

$M_y = \cos x + 2xe^y \qquad N_x = \cos x + 2xe^y$

$\therefore M_y = N_x$ ✓

$\psi = \int (y\cos x + 2xe^y)\,dx$

$= y\sin x + x^2e^y + h(y)$

$\psi_y = \sin x + x^2e^y + h'(y) = N$

$= \sin x + x^2e^y - 1$

$h'(y) = -1 \Rightarrow h(y) = -\int dy = -y$

$y\sin x + x^2e^y - y = C$

Ex $\underbrace{(3xy+y^2)}_{M} + \underbrace{(x^2+xy)}_{N}y' = 0$

$M_y = 3x + 2y \qquad N_x' = 2x + y$

$M_y \neq N_x'$

$$\frac{M_y - N_x'}{N} = \frac{3x+2y-2x-y}{x^2+xy}$$

$$= \frac{x+y}{x(x+y)}$$

$$= \underline{\frac{1}{x}}$$

No need

$$\frac{d\mu}{dx} = \frac{1}{x}\mu$$

$$\frac{d\mu}{\mu} = \frac{dx}{x}$$

$$\ln\mu = \ln x$$

$\mu = x$ ← Given

$x(3xy+y^2) + x(x^2+xy)y' = 0$

$(\underbrace{3x^2y + xy^2}_{M}) + \underbrace{(x^3+x^2y)}_{N}y' = 0$

$M_y = 3x^2 + 2xy \qquad N_x = 3x^2 + 2xy$

$M_y = N_x$

$\psi = \int (3x^2y + xy^2)dx$

$= x^3y + \frac{1}{2}x^2y^2 + h(y)$

$\psi_y = x^3 + x^2y + h'(y) = \underbrace{x^3+x^2y}_{N}$

$h'(y) = 0 \Rightarrow h(y) = C$

$$x^3 y + \frac{1}{2} x^2 y^2 = C$$