Lecture One

Section 1.1 – Functions

Relations

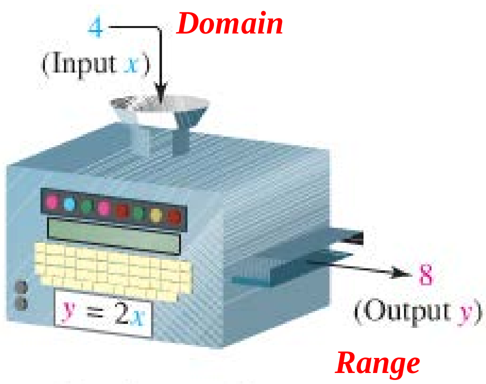
A ***relation*** is any set of ordered pairs. The set of all first components of ordered pairs is called the domain of the relation and the set of second components is called the range of the relation.

Definition of a Function

A ***function*** is a relation between two variables such that to matches each element of a first set (called ***domain***) to an element of a second set (called ***range***) in such way that no element in the first set is assigned to two different elements in the second set.

The ***domain*** of the function is the set of all values of the independent variable for which the function is defined.

The ***range*** of the function is the set of all values taken on by the dependent variable.



Function machine

Example

Determine whether each relation is a function and *find the domain and the range*.

a) $F = \{(1,\ 2),\ (-2,\ 4),\ (3,\ -1)\}$

Function: Yes

Domain: $\{-2,\ 1,\ 3\}$

Range: $\{-1,\ 2,\ 4\}$

b) $G = \{(1,\ 1),\ (1,\ 2),\ (1,\ 3),\ (2,\ 3)\}$

Function: No

Domain: {1, 2}

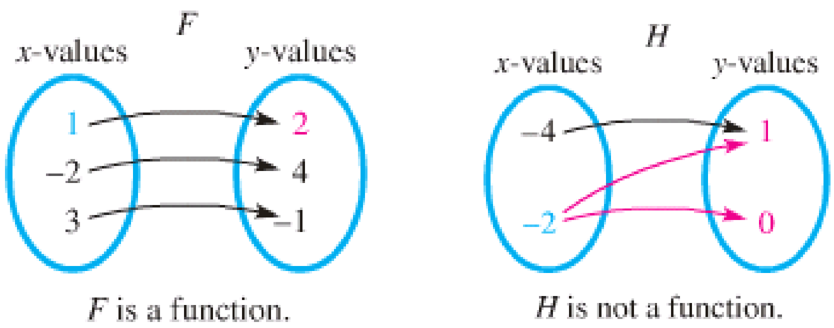
Range: {1, 2, 3}

c) $H = \{(-4,\ 1),\ (-2,\ 1),\ (-2,\ 0)\}$

Function: No

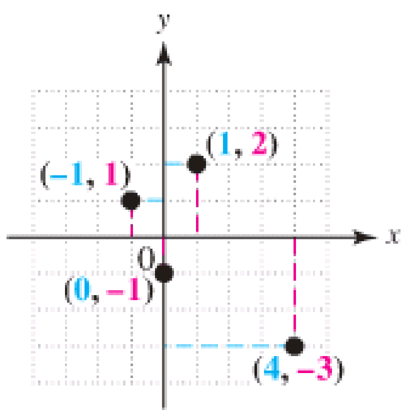
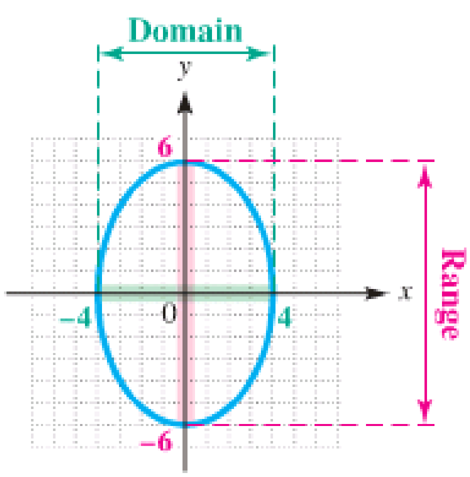
Domain: $\{-4,\ -2\}$

Range: {0, 1}



Example

Give the domain and range of each relation

	***Domain***: {-1, 0, 1, 4} ***Range***: {-3, -1, 1, 2}
	***Domain***: $[-4,\ 4]$ ***Range***: $[-6,\ 6]$

Functions as Equations $y = -0.016x^2 + 0.93x + 8.5$

x: independent

y: depend on x

Notation for Functions

***f*(*x*)** read " *f* of *x* " or " *f* at *x* " represents the value of the function at the number *x*.

Example

Let $f(x) = -x^2 + 5x - 3$

a) $f(2)$

$$f(x) = -x^2 + 5x - 3$$

$$f(\text{—}) = -(\text{—})^2 + 5(\text{—}) - 3$$

$$f(2) = -(2)^2 + 5(2) - 3$$

$$\underline{= 3}$$

b) $f(q)$

$$f(q) = -(q)^2 + 5(q) - 3$$

$$\underline{= -q^2 + 5q - 3}$$

Example

If $f(x) = x^2 - 2x + 7$, evaluate each of the following:

a) $f(-5)$

b) $f(x+4)$

Solution

a) $f(-5) = ?$

$$f(\text{—}) = (\text{—})^2 - 2(\text{—}) + 7$$

$$f(-5) = (-5)^2 - 2(-5) + 7$$

$$= 25 + 10 + 7$$

$$\underline{= 42}$$

b) $f(x+4)=?$

$$f(\_\_)=(\_\_)^2-2(\_\_)+7$$

$$f(x+4)=(x+4)^2-2(x+4)+7 \qquad (a+b)^2=a^2+2ab+b^2$$

$$=x^2+2(4)x+4^2-2x-8+7$$

$$=x^2+8x+16-2x-1$$

$$\underline{=x^2+6x+15}\,\big|$$

Example

Let $g(x)=2x+3$, find $g(a+1)$

Solution

$$g(x)=2x+3$$

$$g(a+1)=2(a+1)+3$$

$$=2a+2+3$$

$$\underline{=2a+5}\,\big|$$

Example

Given: $f(x)=2x^2-x+3$, find the following.

a) $f(0)$

b) $f(-7)$

c) $f(5a)$

Solution

a) $f(x=0)=2(0)^2-(0)+3$

$$\underline{=3}\,\big|$$

b) $f(-7)=2(-7)^2-(-7)+3$

$$\underline{=108}\,\big|$$

c) $f(5a)=2(5a)^2-(5a)+3$

$$\underline{=50a^2-5a+3}\,\big|$$

Exercises *Section* 1.1 – Functions

(1 – 7) Determine whether each relation is a function and *find the domain and the range*.

1. $\{(1,\ 2),\ (3,\ 4),\ (5,\ 6),\ (5,\ 8)\}$

2. $\{(1,\ 2),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5)\}$

3. $\{(9,\ -5),\ (9,\ 5),\ (2,\ 4)\}$

4. $\{(-2,\ 5),\ (5,\ 7),\ (0,\ 1),\ (4,\ -2)\}$

5. $\{(-5,\ 3),\ (0,\ 3),\ (6,\ 3)\}$

6. $\{(1,\ 2),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5),\ (1,\ 5)\}$

7. $\{(-1,\ 3),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5),\ (1,\ 5)\}$

8. Let $f(x)=-3x+4$, find $f(0)$, $f(-1)$, $f(h)$, and $f(a-1)$

9. Let $g(x)=-x^2+4x-1$, find $g(-x)$, $g(2)$, and $g(-2)$

10. Let $f(x)=-3x+4$, find $f(a+4)$

11. Given: $f(x)=2|x|+3x$, find $f(2-h)$.

12. Given: $g(x)=\dfrac{x-4}{x+3}$, find $g(x+h)$

13. Given: $g(x)=\dfrac{x}{\sqrt{1-x^2}}$, find $g(0)$ *and* $g(-1)$

14. Given that $g(x)=2x^2+2x+3$. Find $g(p+3)$

15. If $f(x)=x^2-2x+7$, evaluate each of the following: $f(-5)$, $f(x+4)$, $f(-x)$

16. Find $g(0)$, $g(-4)$, $g(7)$, *and* $g\left(\frac{3}{2}\right)$ *for* $g(x)=\dfrac{x}{\sqrt{16-x^2}}$

17. For $f(x)=3x-4$, determine

a) $f(0)$ *b)* $f\left(\frac{5}{3}\right)$ *c)* $f(-2a)$ *d)* $f(x+h)$

18. For $f(x)=3x^2+3x-1$, determine

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(2)$ *d)* $f(h)$

19. For $f(x)=2x^2-4$, determine

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(2)$ *d)* $f(2)-f(-3)$

20. For $f(x) = 3x^2 + 4x - 2$, determine

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(3)$ *d)* $f(-5)$

21. For $f(x) = -x^3 - x^2 - x + 10$, determine

a) $f(0)$ *b)* $f(-1)$ *c)* $f(2)$ *d)* $f(1) - f(-2)$

22. For $\frac{1}{10}x^{10} - \frac{1}{2}x^6 + \frac{2}{3}x^3 - 10x$, determine

a) $f(2) - f(-2)$ *b)* $f(1) - f(-1)$ *c)* $f(2) - f(0)$

23. For $f(x) = 3x^4 + x^2 - 4$, determine

a) $f(2) - f(-2)$ *b)* $f(1) - f(-1)$ *c)* $f(2) - f(0)$

24. For $f(x) = -\frac{2}{3}x^3 + 4x$, determine

a) $f(2) - f(-2)$ *b)* $f(1) - f(-1)$ *c)* $f(2) - f(0)$

25. For $f(x) = \frac{2x-3}{x-4}$, determine

a) $f(0)$ *b)* $f(3)$ *c)* $f(x+h)$ *d)* $f(-4)$

26. For $f(x) = \frac{3x-1}{x-5}$, determine

a) $f(0)$ *b)* $f(3)$ *c)* $f(x+h)$ *d)* $f(-5)$

Section 1.2 – Quadratic Functions

Basic Complex Number

$$i^2 = -1 \qquad \Rightarrow \; i = \sqrt{-1} \qquad \Rightarrow \; \sqrt{-1} = i$$

The number i is called the ***imaginary unit***.

Example

$\sqrt{-8} = 2i\,\sqrt{2}$

$$\begin{aligned} \sqrt{-7}\sqrt{-7} &= i\sqrt{7}\; i\sqrt{7} \\ &= i^2\left(\sqrt{7}\right)^2 \\ &= -7 \end{aligned}$$

Complex number is written in a form: $z = a + ib$

a is the real part

b is the imaginary part

Conjugate of a complex number $a + bi$ is $a - bi$

A ***quadratic equation*** in x is an equation that can be written in the general form:

$ax^2 + bx + c = 0$ where a, b, and c are real numbers,

$4x^2 - 3x + 2 = 0 \qquad a = 4 \quad b = -3 \quad c = 2$

Solving Quadratic Equations by *Factoring*

The Zero-Product Principle

If $AB = 0$ then $A = 0$ or $B = 0$.

Example

Solve $6x^2 + 7x - 3 = 0$

Solution

$(3x-1)(2x+3) = 0$

$3x - 1 = 0 \qquad 2x + 3 = 0$

$x = \frac{1}{3} \qquad x = -\frac{3}{2}$

The Square Root Property

If u is an algebraic expression and d is a nonzero real number, then $u^2 = d$ has exactly two solutions:

$$\text{If } u^2 = d, \text{ then } u = \sqrt{d} \text{ or } u = -\sqrt{d}$$

Equivalently,

$$\text{If } u^2 = d \Rightarrow u = \pm\sqrt{d}.$$

Example

Solve $3x^2 - 21 = 0$

Solution

$3x^2 = 21$

$x^2 = 7$

$\boxed{x = \pm\sqrt{7}}$

Example

Solve $5x^2 + 45 = 0$

Solution

$5x^2 = -45$

$x^2 = -9$

$x = \pm\sqrt{-9}$

$\boxed{x = \pm 3i}$

Example

Solve $(x + 5)^2 = 11$

Solution

$x + 5 = \pm\sqrt{11}$

$\boxed{x = -5 \pm \sqrt{11}}$

Completing the Square

If $x^2 + bx$ is a binomial, then by ***adding*** $\left(\frac{b}{2}\right)^2$ which is the square of half the coefficient of *x*. a perfect square trinomial will result. That is.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 \qquad\qquad x^2 + bx + \left(\frac{1}{2}b\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example

Solve: $x^2 + 4x - 1 = 0$

Solution

$$x^2 + 4x \quad = 1$$

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 1 + \left(\frac{4}{2}\right)^2$$

$$x^2 + 4x + (2)^2 = 1 + 4$$

$$(x+2)^2 = 5$$

$$x + 2 = \pm\sqrt{5}$$

$$\underline{x = -2 \pm \sqrt{5}\,}|$$

Quadratic Formula

(*Using Completing the Square*)

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{1}{2}\frac{b}{a}\right)^2 = -\frac{c}{a} + \left(\frac{1}{2}\frac{b}{a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$*** \quad ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{cases} b^2 - 4ac > 0 & \rightarrow \quad 2 \text{ Real numbers} \\ b^2 - 4ac < 0 & \rightarrow \quad 2 \text{ Complex numbers} \\ b^2 - 4ac = 0 & \rightarrow \quad \text{One solution (repeated)} \end{cases}$$

Example

Solve: $2x^2 + 2x - 1 = 0$

Solution

$\Rightarrow a = 2 \quad b = 2 \quad c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4+8}}{4}$$

$$= -\frac{2}{4} \pm \frac{\sqrt{12}}{4}$$

$$= -\frac{1}{2} \pm \frac{2\sqrt{3}}{4}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$$

or

$$= \frac{-2 \pm \sqrt{12}}{4}$$

$$= \frac{-2 \pm 2\sqrt{3}}{4}$$

$$= \frac{2\left(-1 \pm \sqrt{3}\right)}{4}$$

$$= \frac{-1 \pm \sqrt{3}}{2}$$

Example

Solve $x^2 - 4x = -2$

Solution

$\Rightarrow a = 1 \quad b = -4 \quad c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16-8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$$= \frac{2\left(2 \pm \sqrt{2}\right)}{2}$$

$$= 2 \pm \sqrt{2}$$

Example

Solve: $x^2 - 2x + 2 = 0$

Solution

$\Rightarrow a = 1 \quad b = -2 \quad c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2}{2} \pm \frac{\sqrt{-4}}{2}$$

$$= 1 \pm \frac{2i}{2}$$

$$\underline{= 1 \pm i}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= \frac{2(1 \pm i)}{2}$$

$$\underline{= 1 \pm i}$$

$$ax^2 + bx + c = 0$$

If $\mathbf{a + b + c = 0} \quad \Rightarrow x = 1, \ \frac{c}{a}$

Example

$$2x^2 + x - 3 = 0$$

$$2 + 1 - 3 = 0 \qquad \Rightarrow \underline{x = 1, \ -\frac{3}{2}}$$

If $\mathbf{a - b + c = 0} \quad \Rightarrow x = -1, \ -\frac{c}{a}$

Example

$$2x^2 - x - 3 = 0$$

$$2 - (-1) - 3 = 0 \qquad \Rightarrow \underline{x = -1, \ \frac{3}{2}}$$

Exercises *Section* 1.2 – Quadratic Functions

(**1 – 48**) Solve

1. $x^2 = -25$
2. $x^2 = 49$
3. $9x^2 = 100$
4. $4x^2 + 25 = 0$
5. $5x^2 + 35 = 0$
6. $5x^2 - 45 = 0$
7. $(x-4)^2 = 12$
8. $(x+3)^2 = -16$
9. $(x-2)^2 = -20$
10. $(4x+1)^2 = 20$
11. $x^2 - 6x = -7$
12. $-6x^2 = 3x + 2$
13. $3x^2 + 2x = 7$
14. $3x^2 + 6 = 10x$
15. $5x^2 + 2 = x$
16. $5x^2 = 2x - 3$
17. $x^2 + 8x + 15 = 0$
18. $x^2 + 5x + 2 = 0$
19. $x^2 + x - 12 = 0$
20. $x^2 - 2x - 15 = 0$
21. $x^2 - 4x - 45 = 0$
22. $x^2 - 6x - 10 = 0$
23. $2x^2 + 3x - 4 = 0$
24. $x^2 - x + 8 = 0$
25. $2x^2 - 13x = 1$
26. $r^2 + 3r - 3 = 0$
27. $x^3 + 8 = 0$
28. $4x^2 - 12x + 9 = 0$
29. $9x^2 - 30x + 25 = 0$
30. $x^2 - 14x + 49 = 0$
31. $x^2 - 8x + 16 = 0$
32. $x^2 + 6x + 13 = 0$
33. $2x^2 - 2x + 13 = 0$
34. $x^2 + 2x + 29 = 0$
35. $4x^2 + 4x + 13 = 0$
36. $x^2 - 2x + 26 = 0$
37. $9x^2 - 4x + 20 = 0$
38. $x^2 + 6x + 21 = 0$
39. $9x^2 - 12x - 49 = 0$
40. $x(x-3) = 18$
41. $x(x-4) - 21 = 0$
42. $(x-1)(x+4) = 14$
43. $(x-3)(x+8) = -30$
44. $x(x+8) = 16(x-1)$
45. $x(x+9) = 4(2x+5)$
46. $(x+1)^2 = 2(x+3)$
47. $(x+1)^2 - 5(x+2) = 3x + 7$
48. $x(8x+1) = 3x^2 - 2x + 2$

(**49 – 60**) Solve using formula

49. $x^2 + 6x - 7 = 0$
50. $x^2 - 6x - 7 = 0$
51. $3x^2 + 4x - 7 = 0$
52. $3x^2 - 4x - 7 = 0$
53. $3x^2 - x - 2 = 0$
54. $3x^2 + x - 2 = 0$
55. $2x^2 + 3x - 5 = 0$
56. $2x^2 - 3x - 5 = 0$
57. $x^2 - 3x - 4 = 0$
58. $x^2 + 3x - 4 = 0$
59. $x^2 + 2x + 1 = 0$
60. $4x^2 - x - 5 = 0$

61. Solve for the specified variable $A = \frac{\pi d^2}{4}$, *for* d

62. Solve for the specified variable $rt^2 - st - k = 0$ $(r \neq 0)$, *for* t

Section 1.3 – Quadratic Graphics

Quadratic Function

A function f is a ***quadratic function*** if $f(x)=ax^2+bx+c$

Formula	*Example*
Vertex of a Parabola	$f(x)=x^2-4x-2$
The ***vertex*** of the graph of $f(x)$ is	$x=-\frac{b}{2a}=-\frac{-4}{2(1)}=2$
V_x or $x_v=-\frac{b}{2a}$	$y=f\left(-\frac{b}{2a}\right)=f(2)$
V_y or $y_v=f\left(-\frac{b}{2a}\right)$	$=(2)^2-4(2)-2$
Vertex Point $\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$	$=-6$
	Vertex point: $(2,\ -6)$

Axis of Symmetry:

$$x=V_x=-\frac{b}{2a}$$

Axis of Symmetry: $x=2$

Minimum or *Maximum Point*

If $a>0\Rightarrow$ *f(x)* has a ***minimum*** point

If $a<0\Rightarrow$ *f(x)* has a ***maximum*** point

@ vertex point $(V_x,\ V_y)$

Minimum point @ $(2,\ -6)$

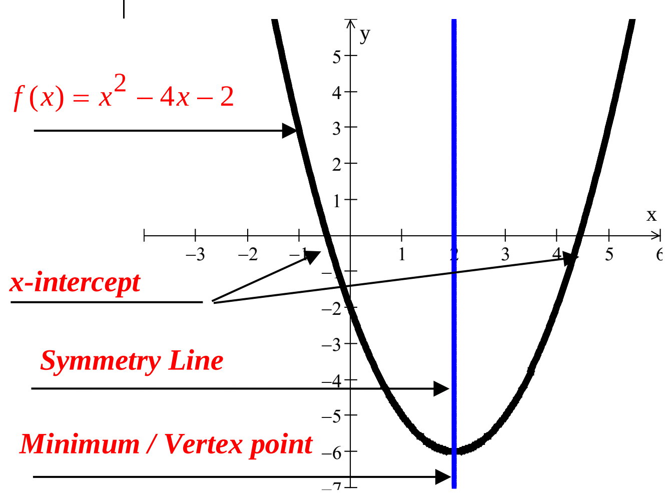
Range

If $a>0\Rightarrow[V_y,\ \infty)$

If $a<0\Rightarrow(-\infty,\ V_y]$

$[-6,\ \infty)$

Domain: $(-\infty,\ \infty)$



Example

For the graph of the function $f(x) = -x^2 - 2x + 8$

a. **Find the vertex point**

$$x = -\frac{-2}{2(-1)} = -1$$

$$y = f(-1) = -(-1)^2 - 2(-1) + 8 = 9$$

Vertex point (–1, 9)

b. **Find the line of symmetry:** $x = -1$

c. **State whether there is a maximum or minimum value *and* find that value**

Minimum point, value (–1, 9)

d. **Find the *x*-intercept**

$x = -4,\ 2$

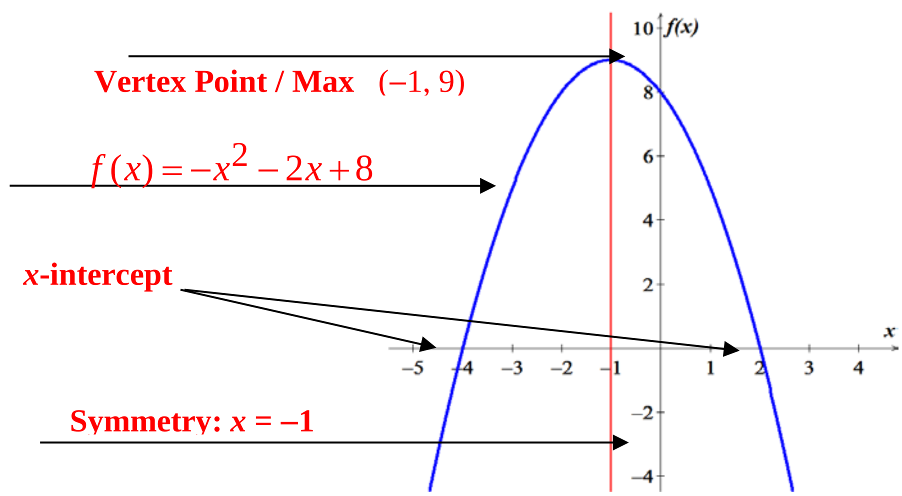
e. **Find the y-intercept**

$y = 8$

f. **Find the range and the domain of the function.**

Range: (–∞, 9] *Domain*: (–∞, ∞)

g. **Graph the function and label, show part *a thru d* on the plot below**



h. **On what intervals is the function increasing? Decreasing?**

Increasing: (–∞, –1) *Decreasing*: (–1, ∞)

Example

Find the axis and vertex of the parabola having equation $f(x) = 2x^2 + 4x + 5$

Solution

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{4}{2(2)} \\ &= -1 \end{aligned}$$

Axis of the parabola: $x = -1$

$$\begin{aligned} y &= f(-1) \\ &= 2(-1)^2 + 4(-1) + 5 \\ &= 3 \end{aligned}$$

Vertex point: $\underline{(-1,\ 3)}$

Exercises *Section* 1.3 – Quadratic Functions

(**1 – 21**) For the Given functions

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the *y*-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function and label, show part *a thru d*
h) On what intervals is the function *increasing*? *decreasing*?

1. $f(x)=x^2+6x+3$

2. $f(x)=x^2+6x+5$

3. $f(x)=-x^2-6x-5$

4. $f(x)=x^2-4x+2$

5. $f(x)=-2x^2+16x-26$

6. $f(x)=x^2+4x+1$

7. $f(x)=x^2-8x+5$

8. $f(x)=x^2+6x-1$

9. $f(x)=x^2+6x+3$

10. $f(x)=x^2-10x+3$

11. $f(x)=x^2-3x+4$

12. $f(x)=x^2-3x-4$

13. $f(x)=x^2-4x-5$

14. $f(x)=2x^2-3x+1$

15. $f(x)=-x^2-3x+4$

16. $f(x)=-2x^2+3x-1$

17. $f(x)=-2x^2-3x-1$

18. $f(x)=-x^2-4x+5$

19. $f(x)=-x^2+4x+2$

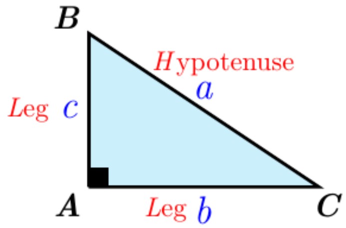
20. $f(x)=-3x^2+3x+7$

21. $f(x)=-x^2+2x-2$

Section 1.4 – Quadradic Applications and Models

Pythagorean Theorem

The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If the legs have lengths a and b, and the hypotenuse has length c, then:

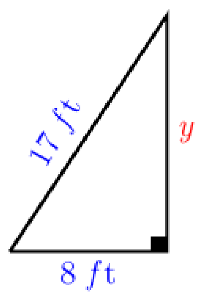


$$a^2 = b^2 + c^2$$

Example

A ladder that is 17 *feet* long is 8 *feet* from the base of a wall. How far up the wall does the ladder reach?

Solution



$$8^2 + y^2 = 17^2$$

$$y^2 = 17^2 - 8^2$$

$$y = \sqrt{17^2 - 8^2}$$

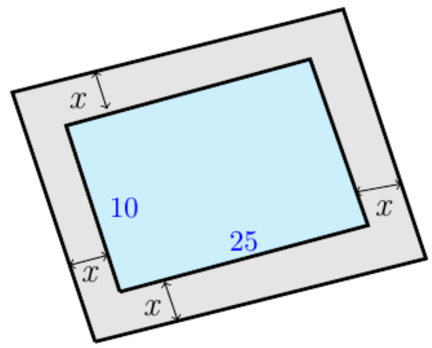
$$\underline{= 15\ ft}$$

∴ The ladder reach at 15 *feet* of the wall height.

Example

A pool measuring 10 *feet* by 25 *feet* is surrounded by a path of uniform width. If the area of the pool and the path combined is 496 $feet^2$, what is the width of the path?

Solution



$$A = lw$$

$$496 = (25+2x)(10+2x)$$

$$250 + 50x + 20x + 4x^2 = 496$$

$$4x^2 + 70x - 246 = 0$$

$$2x^2 + 35x - 123 = 0$$

$$x = \frac{-35 \pm \sqrt{35^2 + 4(2)(123)}}{2(2)}$$

$$= \frac{-35 \pm \sqrt{2,209}}{4}$$

$$= \begin{cases} \frac{-35-47}{4} = -\frac{82}{4} \\ \frac{-35+47}{4} = 3 \end{cases}$$

$\therefore$ The width of the path is **3** *feet*

Maximizing Area

Example

You have 120 *feet* of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

Solution

$$P = 2\ell + 2w$$

$$120 = 2\ell + 2w$$

$$60 = \ell + w$$

$$\underline{\ell = 60 - w}$$

$$A = \ell w$$

$$= (60-w)w$$

$$= 60w - w^2$$

$$= -w^2 + 60w$$

Vertex: $w = -\frac{60}{2(-1)} = 30$

$\ell = 60 - w$

$\quad \underline{= 30}|$

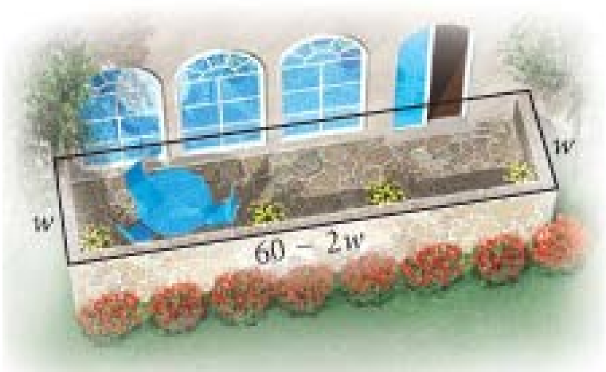
$A = (30)(30) \qquad\qquad\qquad A = \ell w$

$\quad \underline{= 900\ ft^2}|$

Example

A stone mason has enough stones to enclose a rectangular patio with 60 *feet* of stone wall. If the house forms one side of the rectangle, what is the maximum area that the mason can enclose? What should the dimensions of the patio be in order to yield this area?

Solution



$P = l + 2w = 60$

$\underline{l = 60 - 2w}|$

$A = lw$

$\quad = (60 - 2w)w$

$\quad = 60w - 2w^2$

$\quad = -2w^2 + 60w$

$w = -\dfrac{b}{2a}$

$\quad = -\dfrac{60}{2(-2)}$

$\quad \underline{= 15\ ft}|$

$l = 60 - 2w$

$\quad = 60 - 2(15)$

$\quad \underline{= 30\ ft}|$

$Area = (15)(30)$

$\qquad \underline{= 450\ \ ft^2}|$

Height of a Projected Object *(Position Function)*

An object that is falling or vertically projected into the air has its height above the ground, $s(t)$, in feet, given by

$$s(t) = -16t^2 + v_0 t + s_0$$

v_0 is the original velocity (initial velocity) of the object, in *feet* per *second*

t is the time that the object is in motion, in *second*

s_0 is the original height (initial height) of the object, in *feet*

Example

If a projectile is shot vertically upward from the ground with an initial velocity of 100 $ft/\sec$, neglecting air resistance, its height s (in *feet*) above the ground *t* seconds after projection is given by

$$s = -16t^2 + 100t$$

a) After how many seconds will it be 50 *feet*. above the ground?

b) How long will it take for the projectile to return to the ground?

c) Determine the time at which the rocket reaches its maximum height?

d) Find the maximum height?

Solution

a) *After how many seconds will it be 50 feet above the ground?*

$$50 = -16t^2 + 100t$$

$$16t^2 - 100t + 50 = 0$$

$$8t^2 - 50t + 25 = 0$$

$$t = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(8)(25)}}{2(8)}$$

$$= \frac{50 \pm \sqrt{1700}}{16}$$

$$t = \frac{50 - 10\sqrt{17}}{16} = \frac{25 - 5\sqrt{17}}{8} \approx 0.55$$

$$t = \frac{50 + 10\sqrt{17}}{16} = \frac{25 + 5\sqrt{17}}{8} \approx 5.70$$

b) *How long will it take for the projectile to return to the ground?*

$$0 = -16t^2 + 100t$$

$$0 = -4t(4t - 25)$$

$-4t = 0 \qquad\qquad 4t - 25 = 0$

$\boxed{t = 0} \qquad\qquad \boxed{t = \frac{25}{4} = 6.25}$

c) *Determine the time at which the rocket reaches its maximum height*?

$t = -\frac{100}{2(-16)} \qquad\qquad t = -\frac{b}{2a}$

$\boxed{= \frac{25}{8}\ sec} \quad \boxed{= 3.125\ sec}$

d) *Find the maximum height*?

$$s\left(\frac{25}{8}\right) = -16\left(\frac{25}{8}\right)^2 + 100\left(\frac{25}{8}\right)$$

$$= -\frac{625}{4} + \frac{625}{2}$$

$$\boxed{= \frac{625}{4}\ feet}$$

Exercise *Section* 1.4 – Quadratic Applications

1. A rectangular park is 6 *miles* long and 2 *miles* wide. How long is a pedestrian route that runs diagonally across the park?

2. What is the width of a 25-*inch* television set whose height is 15 *inches*?

3. The length of a rectangular sign is 3 *feet* longer than the width. If the sign's area is 54 square *feet*, find its length and width.

4. A rectangular parking lot has a length that is 3 *yards* greater than the width. The area of the parking lot is 180 square *yards*, find the length and the width.

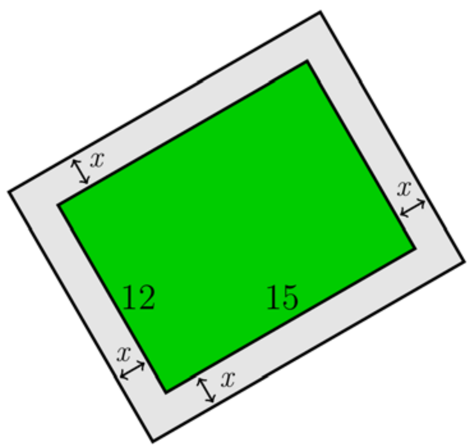
5. Each side of a square is lengthened by 3 *inches*. The area of this new, larger square is 64 square *inches*. Find the length of a side of the original square.

6. Each side of a square is lengthened by 2 *inches*. The area of this new, larger square is 36 square *inches*. Find the length of a side of the original square.

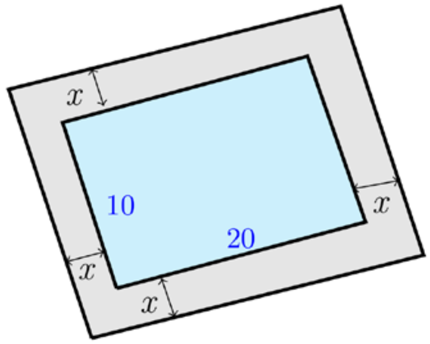
7. One number is 5 greater than another. The product of the numbers is 36. Find the numbers.

8. One number is 6 less than another. The product of the numbers is 72. Find the numbers.

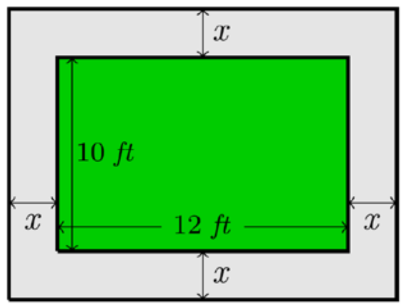
9. A vacant rectangular lot is being turned into a community vegetable garden measuring 15 *meters* by 12 *meters*. A path of uniform width is to surround the garden. If the area of the garden and path combined is 378 *square meters*, find the width of the path.



10. A pool measuring 10 *m* by 20 *m* is surrounded by a path of uniform width. If the area of the pool and the path combined is 600 m^2, what is the width of the path?



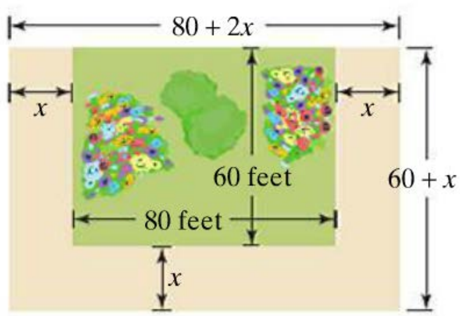
11. You put in flower bed measuring 10 *feet* by 12 *feet*. You plan to surround the bed with uniform border of low-growing plants.



a) Write a polynomial that describes the area of the uniform border that surrounds your flowers.

b) The low growing plants surrounding the flower bed require 1 square *foot* each when mature. If you have 168 of these plants, how wide a strip around the flower bed should you prepare for the border?

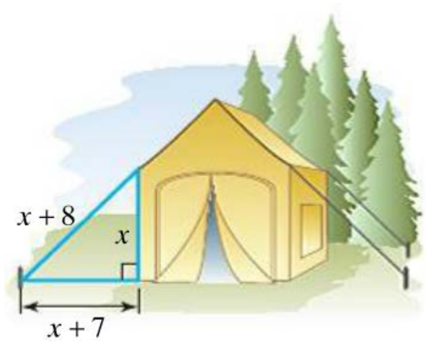
12. A rectangular garden measures 80 *feet* by 60 *feet*. A large path of uniform width is to be added along both shorter sides and one longer side of the garden. The landscape designer doing the work wants to double the garden's area with the addition of this path. How wide should the path be?



13. The length of a rectangular poster is 1 *foot* more than the width, and a diagonal of the poster is 5 *feet*. Find the length and the width.

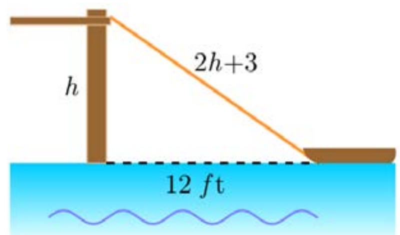
14. One leg of a right triangle is 7 *cm* less than the length of the other leg. The length of the hypotenuse is 13 *cm*. find the lengths of the legs.

15. A tent with wires attached to help stabilize it, as shown below. The length of each wire is 8 *feet* greater than the distance from the ground to where it is attached to the tent.

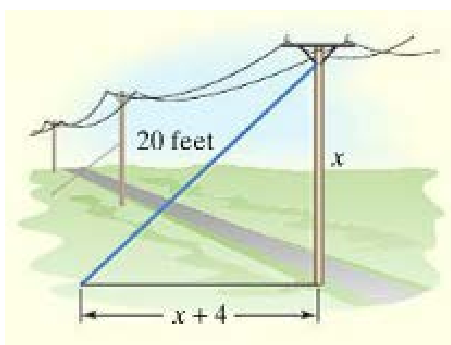


The distance from the base of the tent to where the wire is anchored exceeds this height by 7 *feet*, Find the length of each wire used to stabilize the tent.

16. A boat is being pulled into a dock with a rope attached to the boat at water level. Where the boat is 12 *feet*. from the dock, the length of the rope from the boat to the dock is 3 *feet*. longer than twice the height of the dock above the water. Find the height of the dock.

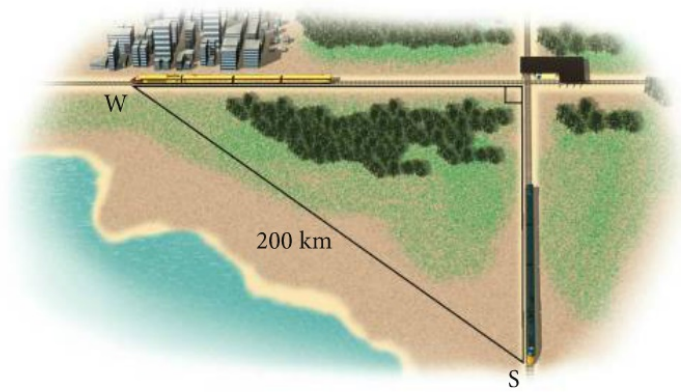


17. A piece of wire measuring 20 *feet* is attached to a telephone pole as a guy wire. The distance along the ground from the bottom of the pole to the end of the wire is 4 *feet* greater than the height where the wire is attached to the pole. How far up the pole does the guy wire reach?



18. Logan and Cassidy leave a campsite, Logan biking due north and Cassidy biking due east. Logan bikes 7 *km/h* slower than Cassidy. After 4 *hrs*, they are 68 *km* apart. Find the speed of each bicyclist.

19. Two trains leave a station at the same time. One train travels due west, and the other travels due south. The train traveling west travels 20 *km/hr* faster than the train traveling south. After 2 *hr*., the trains are 200 *km* apart. Find the speed of each train.



20. Towers are 1482 *feet.* tall. How long would it take an object dropped from the top to reach the ground? Given $s = 16t^2$

21. The formula $P = 0.01A^2 + .05A + 107$ models a woman's normal Point systolic blood pressure, P, an age A. Use this formula to find the age, to the nearest year, of a woman whose normal systolic blood pressure is 115 *mm* Hg.

22. A rectangular piece of metal is 10 *in*. longer than it is wide. Squares with sides 2 *in*. long are cut from the four corners, and the flaps folded upward to form an open box. If the volume of the box is 832 in^3, what were the original dimensions of the piece of metal?

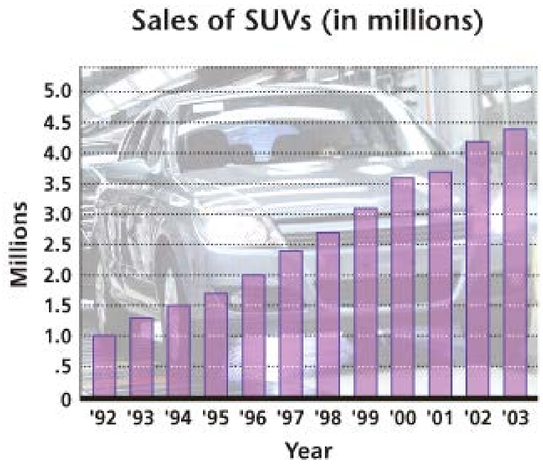
23. An astronaut on the moon throws a baseball upward. The astronaut is 6 *ft*., 6 *in*., tall, and the initial velocity of the ball is 30 *ft/sec* . The height s of the ball in feet is given by the equation

$$s = -2.7t^2 + 30t + 6.5$$

Where t is the number of seconds after the ball was thrown.

a) After how many seconds is the ball 12 *ft*. above the moon's surface?
b) How many seconds will it take for the ball to return to the surface?

24. The bar graph shows of SUVs (sport utility vehicles in the US, in *millions*. The quadratic equation $S = .00579x^2 + .2579x + .9703$ models sales of SUVs from 1992 to 2003, where S represents sales in *millions*, and x = 0 represents 1992, x = 1 represents 1993 and so on.

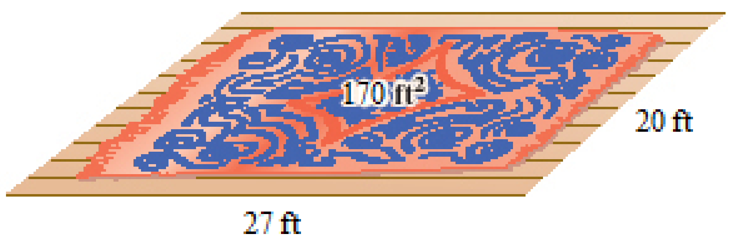


a) Use the model to determine sales in 2002 and 2003. Compare the results to the actual figures of 4.2 million and 4.4 *million* from the graph.

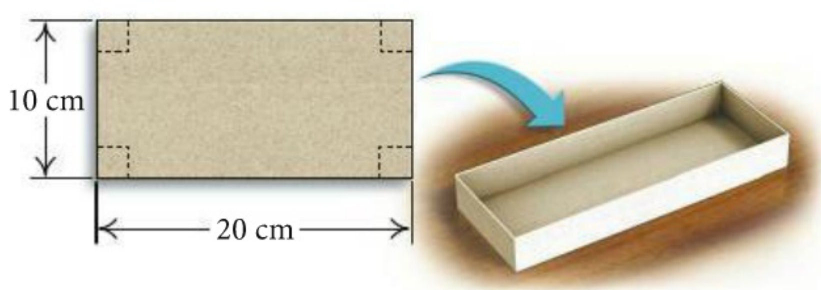
b) According to the model, in what year do sales reach 3.5 million? Is the result accurate?

25. Erik finds a piece of property in the shape of a right triangle. He finds that the longer leg is 20 *m* longer than twice the length of the shorter leg. The hypotenuse is 10 *m* longer than the length of the longer leg. Find the lengths of the sides of the triangular lot.

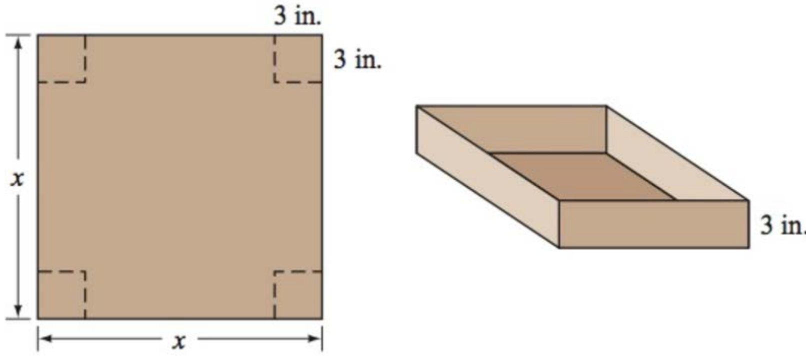
26. Cynthia wants to buy a rug for a room that is 20 *feet.* wide and 27 *feet*. long. She wants to leave a uniform strip of floor around the rug. She can afford to buy 170 square *feet* of carpeting. What dimension should the rug have?



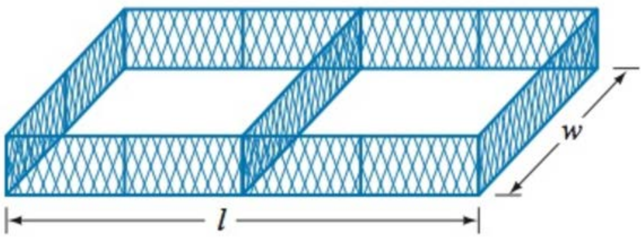
27. An open box is made from a 10-*cm* by 20-*cm* piece of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2. What is the length of the sides of the squares?



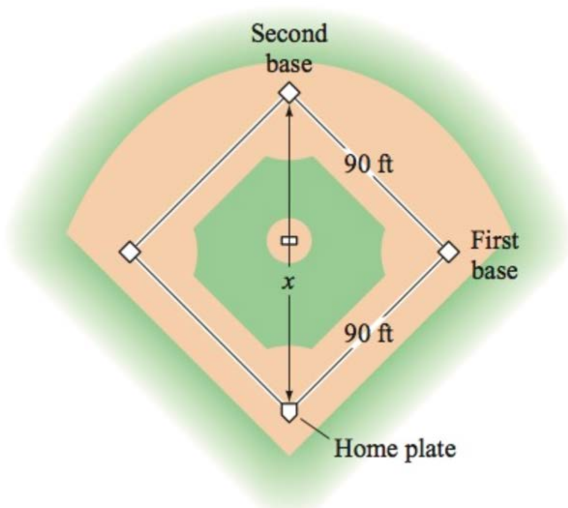
28. A square piece of cardboard is formed into a box by cutting out 3-*inch* squares from each of the corners and folding up the sides. If the volume of the box needs to be 126.75 cubic *inches*, what size square piece of cardboard is needed?



29. You want to use 132 *feet* of chain-link fencing to enclose a rectangular region and subdivide the region into two smaller rectangular regions. If the total enclosed area is 576 *square feet*, find the dimensions of the enclosed region.

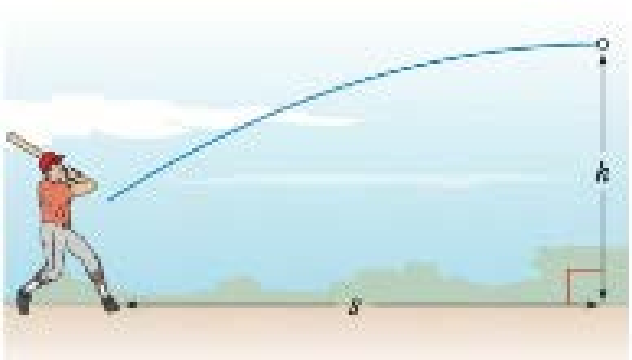


30. How far is it from home plate to second base on a baseball diamond?



31. Two equations can be used to track the position of a baseball t seconds after it is hit.

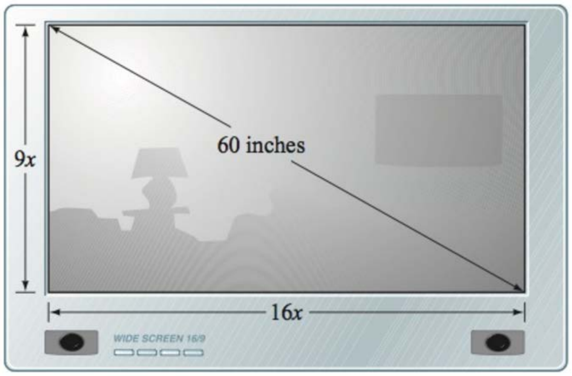
For instance, suppose $h = -16t^2 + 50t + 4.5$ gives the height, in *feet*, of a baseball t seconds after it is hit and $s = 103.9t$ gives the horizontal distance, in *feet*, of the ball from home plate t seconds after it is hit.



Use these equations to determine whether this particular baseball will clear a 10-*foot* fence positioned 360 *feet* from home plate.

32. A ball is thrown downward with an initial velocity of 5 *feet* per *second* from the Golden Gate Bridge, which is 220 *feet* above the water. How long will it take for the ball to hit the water?

33. A television screen measures 60 *inches* diagonally, and its aspect ratio is 16 to 9. This means that the ratio of the width of the screen to the height of the screen is 16 to 9.
Find the width and height of the screen.

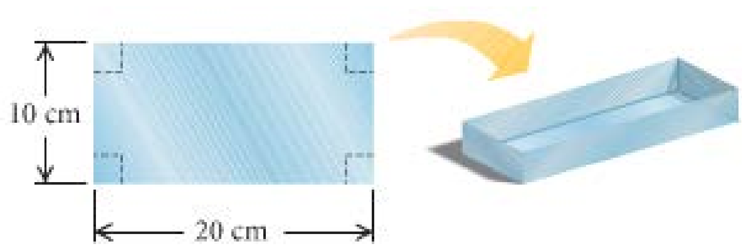


34. A company makes rectangular solid candy bars that measures 5 *inches* by 2 *inches* by 0.5 *inch*. Due to difficult financial times, the company has decided to keep the price of the candy bar fixed and reduce the volume of the bar by 20%. What should the dimensions be for the new candy bar if the company keeps the height at 0.5 *inch* and makes length of the candy bar 3 *inches* longer than the width?

35. A company makes rectangular solid candy bars that measures 5 *inches* by 2 *inches* by 0.5 *inch*. Due to difficult financial times, the company has decided to keep the price of the candy bar fixed and reduce the volume of the bar by 20%. What should the dimensions be for the new candy bar if the company keeps the height at 0.5 *inch* and makes length of the candy bar 2.5 times as long as its width?

36. A picture frame measures 28 *cm* by 32 *cm* and is of uniform width. What is the width of the frame if 192 cm^2 of the picture shows?

37. An open box is made from a 10-*cm* by 20-*cm* of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2. What is the length of the sides of the squares?



38. You have 600 *feet.* of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river.

a) Find the length and width of the plot that will maximize the area.

b) What is the largest area that can be enclosed?

39. You have 60 *yards* of fencing to enclosed a rectangular region.

a) Find the dimensions of the rectangle that maximize the enclosed area.

b) What is the maximum area?

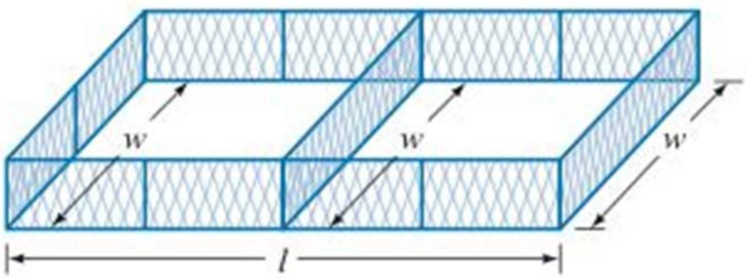
40. You have 80 *yards* of fencing to enclosed a rectangular region.

a) Find the dimensions of the rectangle that maximize the enclosed area.
b) What is the maximum area?

41. The sum of the length l and the width w of a rectangle tangular area is 240 *meters*.

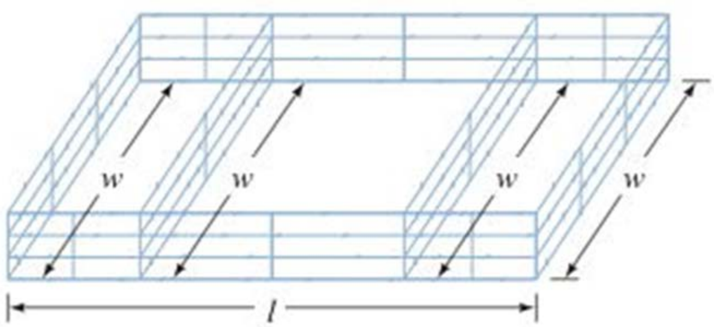
a) Write w as a function of l.
b) Write the area A as a function of l.
c) Find the dimensions that produce the greatest area.

42. You use 600 *feet* of chainlink fencing to enclose a rectangular region and to subdivide the region into two smallerrectangular regions by placing a fence parallel to one of the sides.



a) Write w as a function of l.
b) Write the area A as a function of l.
c) Find the dimensions that produce the greatest area.

43. You use 1,200 *feet* of chainlink fencing to enclose a rectangular region and to subdivide the region into three smallerrectangular regions by placing a fence parallel to one of the sides.



a) Write w as a function of l.
b) Write the area A as a function of l.
c) Find the dimensions that produce the greatest area.

44. A landscaper has enough stone to enclose a rectangular pond next to exiting garden wall of the house with 24 *feet* of stone wall. If the garden wall forms one side of the rectangle.

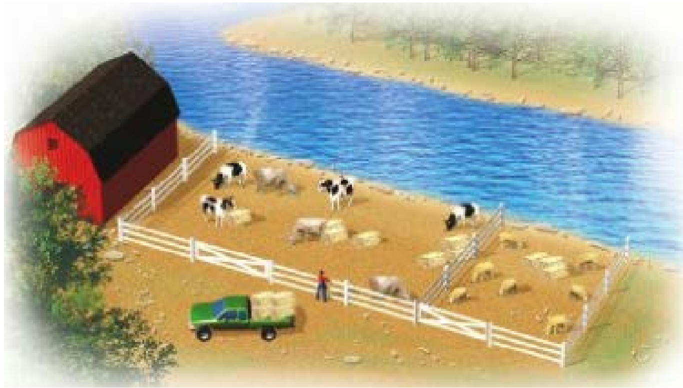
a) What is the maximum area that the landscaper can enclose?

b) What dimensions of the pond will yield this area?

45. A berry former needs to separate and enclose two adjacent rectangular fields, one for strawberries and one for blueberries. If a lake forms one side of the fields and 1,000 *feet* of fencing is available, what is the largest total area that can be enclosed?

46. A fourth-grade class decides to enclose a rectangular garden, using the side of the school as one side of the rectangle. What is the maximum area that the class can enclose with 32 *feet* of fence? What should the dimensions of the garden be in order to yield this area?

47. A rancher needs to enclose two adjacent rectangular corrals, one for cattle and one for sheep. If a river forms one side of the corrals and 240 *yard* of fencing is available, what is the largest total area that can be enclosed?



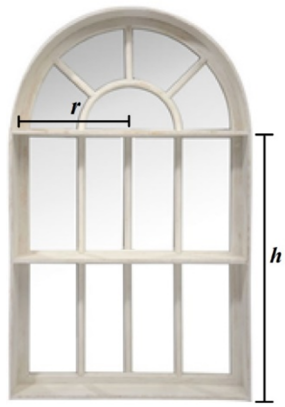
48. A Norman window is a rectangle with a semicircle on top. Sky Blue Windows is designing a Norman window that will require 24 *feet* of trim on the outer edges. What dimensions will allow the maximum amount of light to enter a house?

A Norman window

49. A Norman window has the shape of a rectangle surmounted by a semicircle. The exterior perimeter of the window is 48 *feet*.

Find the height h and the radius r that will allow the maximum amount of light to enter the window?

50. A Norman window has the shape of a rectangle surmounted by a semicircle. It requires 24 *feet* of trim on the outer edges.



What dimensions will allow the maximum amount of light to enter a house?

51. The temperature $T(t)$, in degrees Fahrenheit, during the day can be modeled by the equation $T(t) = -0.7t^2 + 9.4t + 59.3$, where t is the number of hours after 6:00 AM.

a) At what time the temperature a maximum?

b) What is the maximum temperature?

52. When a softball player swings a bat, the amount of energy $E(t)$, in *joules*, that is transferred to the bat can be approximated by the function

$$E(t) = -279.67t^2 + 82.86t$$

Where $0 \le t \le 0.3$ and t is measured in *seconds*. According to this model, what is the maximum energy of the bat?

53. Some softball fields are built in a parabolic mound shape so that water will drain off the field. A model for the shape of a certain field is given by

$$h(x) = -0.0002348x^2 + 0.0375x$$

Where $h(x)$ is the height, in *feet*, of the field at a distance of x *feet* from one sideline. Find the maximum height of the field.

54. The fuel efficiency for a certain midsize car is given by

$$E(v) = -0.018v^2 + 1.476v + 3.4$$

Where $E(v)$ is the fuel efficiency in *miles* per *gallon* for a car traveling v in *miles* per *hour*.

a) What speed will yield the maximum fuel efficiency?

b) What is the maximum fuel efficiency for this car?

55. If the initial velocity of a projectile is 128 *feet* per *second*, then the height h, in *feet*, is a function of time t, in *seconds*, given by the equation

$$h(t) = -16t^2 + 128t$$

a) Find the time t when the projectile achieves its maximum height.

b) Find the maximum height of the projectile.

c) Find the time t when the projectile hits the ground.

56. If the initial velocity of a projectile is 64 *feet* per *second* and an initial height of 80 *feet*, then the height h, in *feet*, is a function of time t, in *seconds*, given by the equation

$$h(t) = -16t^2 + 64t + 80$$

a) Find the time t when the projectile achieves its maximum height.

b) Find the maximum height of the projectile.

c) Find the time t when the projectile hits the ground.

57. If the initial velocity of a projectile is 100 *feet* per *second* and an initial height of 20 *feet*, then the height h, in *feet*, is a function of time t, in *seconds*, given by the equation

$$h(t) = -16t^2 + 100t + 20$$

a) Find the time t when the projectile achieves its maximum height.

b) Find the maximum height of the projectile.

c) Find the time t when the projectile hits the ground.

58. A frog leaps from a stump 3.5 *feet* high and lands 3.5 *feet* from the base of the stump.

It is determined that the height of the frog as a function of its distance, x, from the base of the stump is given by the function $h(x) = -0.5x^2 + 0.75x + 3.5$ where h is in *feet*.

a) How high is the frog when its horizontal distance from the base of the stump is 2 *feet*?

b) At what two distances from the base of the stump after is jumped was the frog 3.6 *feet* above the ground?

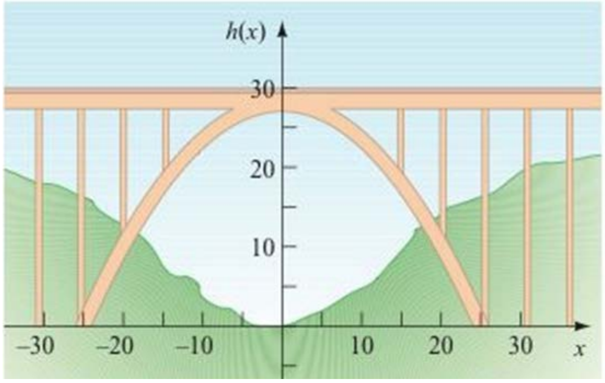
c) At what distance from the base did the frog reach its highest point?

d) What was the maximum height reached by the frog?

59. The height of an arch is given by

$$h(x) = -\frac{3}{64}x^2 + 27, \quad -24 \le x \le 24$$

Where $|x|$ is the horizontal distance in *feet* from the center of the arch to the ground.



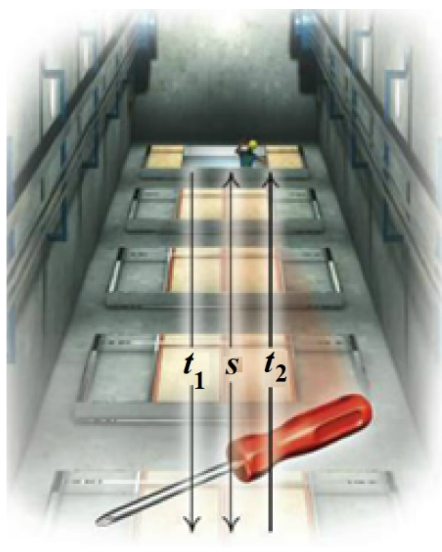
a) What is the maximum height of the arch?
b) What is the height of the arch 10 *feet* to the right of center?
c) How far from the center is the arch 8 *feet* tall?

60. A weightless environment can be created in an airplane by flying in a series of parabolic paths. This is one method that NASA uses to train astronauts for the experience of weightlessness. Suppose the height *h*, in *feet*, of NASA's airplane is modeled by

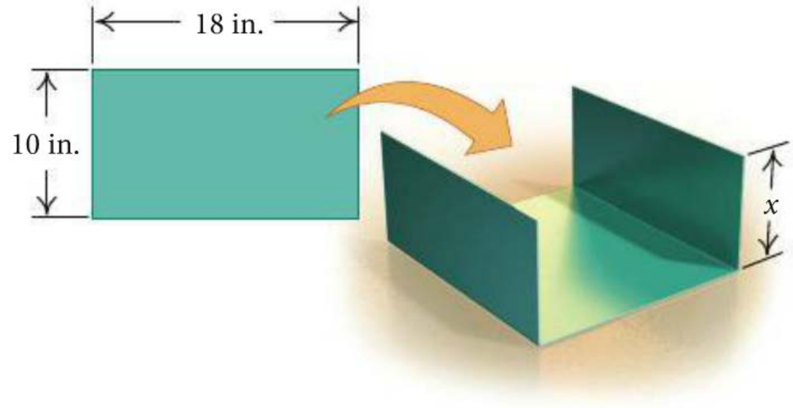
$$h(t) = -6.6t^2 + 430t + 28,000$$

Where t is the time, in *seconds*, after the plane enters its parabolic path.
Find the maximum height of the plane.

61. You drop a screwdriver from the top of an elevator shaft. Exactly 5 *seconds* later, you hear the sound of the screwdriver hitting the bottom of the shaft. The speed of sound is $1{,}100$ *ft/sec*. How tall is the elevator shaft?



62. A company plans to produce a one- compartment vertical file by bending the long side of a 10-*in*. by 18-*in*. sheet of metal along two lines to form a $\lfloor\_\rfloor$ − shape. How tall should the file be in order to maximize the volume that it can hold?



63. The sum of the base and the height of a triangle is 20 *cm*. Find the dimensions for which the area is a maximum.

64. The sum of the base and the height of a parallelogram is 14 *inches*. Find the dimensions for which the area is a maximum.

Section 1.5 – Other Types of Equations

The numbers of solutions to a polynomial with ***n*** degree, where ***n*** is Natural Number, are ***n*** solutions.

Solving a Polynomial Equation by factoring

Example

Solve: $4x^4 = 12x^2$

Solution

$$4x^4 - 12x^2 = 0$$

$$4x^2\left(x^2 - 3\right) = 0$$

$$4x^2 = 0 \qquad\qquad x^2 - 3 = 0$$

$$x^2 = 0 \qquad\qquad x^2 = 3$$

$$\rightarrow \underline{x = 0, 0} \qquad\qquad \underline{x = \pm\sqrt{3}}$$

Example

Solve: $2x^3 + 3x^2 = 8x + 12$

Solution

$$2x^3 + 3x^2 - 8x - 12 = 0$$

$$x^2(2x+3) - 4(2x+3) = 0$$

$$(2x+3)\left(x^2 - 4\right) = 0$$

$$2x + 3 = 0 \qquad\qquad x^2 - 4 = 0$$

$$2x = -3 \qquad\qquad x^2 = 4$$

$$\underline{x = -\frac{3}{2}} \qquad\qquad \underline{x} = \pm\sqrt{4} \ \ \underline{= \pm 2}$$

Equations that Are Quadratic in Form

$$ax^2+bx+c=0$$

$$a(x)^2+b(x)^1+c=0$$

$$a(u)^2+b(u)^1+c=0$$

$$a\left(x^n\right)^2+b\left(x^n\right)^1+c=0$$

$$au^2+bu+c=0$$

Example

Solve: $x^4-5x^2+6=0$

Solution

$$\left(x^2\right)^2-5\left(x^2\right)+6=0$$

$$(U)^2-5(U)+6=0$$

$$U^2-5U+6=0$$

Solve for U

$$\Rightarrow U=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(6)}}{2(1)}$$

$$=\frac{5\pm\sqrt{25-24}}{2}$$

$$=\frac{5\pm\sqrt{1}}{2}$$

$$\to\begin{cases}U=\frac{5-1}{2}=2\\ U=\frac{5+1}{2}=3\end{cases}$$

$$x^2=U \qquad \to\begin{cases}x^2=2\to \underline{x=\pm\sqrt{2}}\\ x^2=3\to \underline{x=\pm\sqrt{3}}\end{cases}$$

or $\left(x^2-2\right)\left(x^2-3\right)=0$

$x^2-2=0 \qquad x^2-3=0$

$x^2=2 \qquad x^2=3$

$x=\pm\sqrt{2} \qquad x=\pm\sqrt{3}$

Example

Solve: $(x+1)^{2/3}-(x+1)^{1/3}-2=0$

Solution

$u=(x+1)^{1/3}$

$\left((x+1)^{1/3}-2\right)\left((x+1)^{1/3}+1\right)=0$

$u^2-u-2=0$

$(u-2)(u+1)=0$

$u-2=0 \qquad u+1=0$

$u=2 \qquad u=-1$

$u=(x+1)^{1/3}=2 \qquad u=(x+1)^{1/3}=-1$

$x+1=2^3 \qquad x+1=(-1)^3$

$x+1=8 \qquad x+1=-1$

$\underline{x=7} \qquad \underline{x=-2}$

Example

Solve: $3x^{2/3}-11x^{1/3}-4=0$

Solution

$3\left(x^{1/3}\right)^2-11\left(x^{1/3}\right)-4=0$

$x^{1/3}=\dfrac{11\pm\sqrt{121+48}}{2(3)}$

$x^{1/3}=\dfrac{11-13}{6} \qquad x^{1/3}=\dfrac{11+13}{6}$

$=-\dfrac{1}{3} \qquad =4$

$x=\left(-\dfrac{1}{3}\right)^3 \qquad x=4^3$

$\underline{=-\dfrac{1}{27}} \qquad \underline{=64}$

Or factor

$\left(3x^{1/3}+1\right)\left(x^{1/3}-4\right)=0$

$3x^{1/3}+1=0 \qquad x^{1/3}-4=0$

Solving a *Radical* Equation

Power Property

If P and Q are algebraic expressions, then every solution of the equation $P = Q$ is also a solution of the equation $P^n = Q^n$; for any positive integer n.

Example

Solve $\quad x - \sqrt{15 - 2x} = 0$

Solution

$$x = \sqrt{15 - 2x}$$

$$x^2 = \left(\sqrt{15 - 2x}\right)^2$$

$$x^2 = 15 - 2x$$

$$x^2 + 2x - 15 = 0$$

$$(x - 3)(x + 5) = 0$$

$$x - 3 = 0 \qquad x + 5 = 0$$

$$x = 3 \qquad x = -5$$

Check

$x = 3$	$x = -5$
$3 - \sqrt{15 - 2(3)} = 0$	$-5 - \sqrt{15 - 2(-5)} = 0$
$3 - \sqrt{9} = 0$	$-5 - \sqrt{25} = 0$
$3 - 3 = 0 \quad (true)$	$-5 - 5 \neq 0 \quad (false)$

$\underline{|x = 3}$ is the only solution

Solving Radical Equations of the Form $x^{\frac{m}{n}} = k$

Assume that *m* and *n* are positive integers

If *m* is ***even***: $\quad x^{\frac{m}{n}} = k \quad \Rightarrow \left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = k^{\frac{n}{m}} \quad \Rightarrow x = \pm k^{\frac{n}{m}}$

If *m* is ***odd***: $\quad x^{\frac{m}{n}} = k \quad \Rightarrow \left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = k^{\frac{n}{m}} \quad \Rightarrow x = k^{\frac{n}{m}}$

Example

Solve: $\quad 5x^{3/2} - 25 = 0$

Solution

$$5x^{3/2} = 25$$

$$x^{3/2} = \frac{25}{5} = 5$$

$$x = 5^{\frac{2}{3}}$$

$$= \sqrt[3]{5^2}$$

$$\underline{= \sqrt[3]{25}}$$

Example

Solve: $\quad x^{2/3} - 8 = -4$

Solution

$$x^{2/3} = 4$$

$$x = \pm(4)^{3/2}$$

$$= \pm\left(2^2\right)^{3/2}$$

$$= \pm 2^3$$

$$\underline{= \pm 8}$$

Solving an Absolute Value Equation

If c is a positive real number and X represents any algebraic expression, then $|X| = c$ is equivalent to $X = c$ or $X = -c$

$$|X| = c \quad \rightarrow \quad X = c \text{ or } X = -c$$

Properties of Absolute Value

1. For b > 0, $|a| = b$ if and only if (*iff*) $a = b \ \text{ or } \ a = -b$
2. $|a| = |b| \ \text{iff} \ a = b \ \text{ or } \ a = -b$

For any positive number b:

3. $|a| < b \ \text{iff} \ -b < a < b$
4. $|a| < b \ \text{iff} \ a < -b \ \text{ or } \ a > b$

Example

Solve: $|2x - 1| = 5$

Solution

$2x - 1 = 5$ $\qquad\qquad$ $2x - 1 = -5$

$2\boldsymbol{x} = 6$ $\qquad\qquad$ $2\boldsymbol{x} = -4$

$\boldsymbol{x} = 3$ $\qquad\qquad$ $\boldsymbol{x} = -2$

Solutions: $x = -2, \ 3$

Example

Solve: $4|1 - 2x| - 20 = 0$

Solution

$4|1 - 2x| = 20$

$|1 - 2x| = 5$

$1 - 2x = 5$ $\qquad\qquad$ $1 - 2x = -5$

$-2x = 4$ $\qquad\qquad$ $-2x = -6$

$x = -2$ $\qquad\qquad$ $x = 3$

Solutions: $x = -2, \ 3$

Exercise *Section* 1.5 – Other Types of Equations

(**1 – 112**) Solve

1. $3x^3+2x^2=12x+8$

2. $x^3+x^2-4x-4=0$

3. $x^3+x^2+4x+4=0$

4. $x^3+4x^2-25x-100=0$

5. $x^3-2x^2-x+2=0$

6. $x^3-x^2-25x+25=0$

7. $x^3-x^2=16x-16$

8. $x^3+x^2+25x+25=0$

9. $x^3+2x^2=16x+32$

10. $2x^3+3x^2-6x-9=0$

11. $2x^3+x^2-8x-4=0$

12. $2x^3+16x^2+30x=0$

13. $3x^3-9x^2-30x=0$

14. $x^4+3x^2=10$

15. $5x^4=40x$

16. $9x^4-9x^2+2=0$

17. $x^4+720=89x^2$

18. $12x^4-11x^2+2=0$

19. $2x^4-7x^2+5=0$

20. $x^4-5x^2+4=0$

21. $x^4+3x^2=10$

22. $3x^4-48x^2=0$

23. $5x^4-20x^2=0$

24. $x^4-4x^3-4x^2=0$

25. $x^4-6x^3+9x^2=0$

26. $x^4-4x^3+3x^2=0$

27. $x^4-4x^2+3=0$

28. $x^4+4x^2+3=0$

29. $x^4+6x^2-7=0$

30. $x^4-6x^2-7=0$

31. $3x^4+4x^2-7=0$

32. $3x^4-4x^2-7=0$

33. $3x^4-x^2-2=0$

34. $3x^4+x^2-2=0$

---

35. $x-3\sqrt{x}-4=0$

36. $\left(5x^2-6\right)^{1/4}=x$

37. $\left(x^2+24x\right)^{1/4}=3$

38. $x^{5/2}=32$

39. $\sqrt[3]{2x+11}=3$

40. $\sqrt[3]{6x-3}=3$

41. $\sqrt[3]{2x-6}=4$

42. $\sqrt[3]{4x-3}-5=0$

43. $(3x-1)^{1/3}+4=0$

44. $(2x+3)^{1/3}+4=6$

45. $(3x-6)^{1/3}+5=8$

46. $(3x+1)^{1/4}+7=9$

47. $(2x+3)^{1/4}+7=10$

48. $\sqrt[3]{4x^2-4x+1}-\sqrt[3]{x}=0$

---

49. $\sqrt{2x+3}=5$

50. $\sqrt{x-3}+6=5$

51. $\sqrt{3x-2}=4$

52. $\sqrt{5x-4}=9$

53. $\sqrt{5x-1}=8$

54. $\sqrt{3x-2}-5=0$

55. $\sqrt{2x+5}+11=6$

56. $\sqrt{3x+7}+10=4$

57. $x=\sqrt{7x+8}$

58. $x=\sqrt{6x+7}$

59. $\sqrt{5x+1}=x+1$

60. $x=\sqrt{2x-2}+1$

61. $x-2\sqrt{x-3}=3$

62. $x+\sqrt{26-11x}=4$

63. $x-\sqrt{2x+3}=0$

64. $\sqrt{x+3}+3=x$

65. $x-\sqrt{x+11}=1$

66. $\sqrt{x-7}=7-\sqrt{x}$

67. $\sqrt{x-8}=\sqrt{x}-2$

68. $\sqrt{2x-5}=\sqrt{x+4}$

69. $\sqrt{6x+2}=\sqrt{5x+3}$

70. $\sqrt{3x+1}-\sqrt{x+4}=1$

71. $\sqrt{x+2}+\sqrt{x-1}=3$

72. $\sqrt{x-4}+\sqrt{x+4}=4$

73. $\sqrt{2x-3}-\sqrt{x-2}=1$

74. $\sqrt{x+2}+\sqrt{3x+7}=1$

75. $2\sqrt{4x+1}-9=x-5$

76. $\sqrt{2x-3}+\sqrt{x-2}=1$

77. $\sqrt{2x+3}=1+\sqrt{x+1}$

78. $\sqrt{x+5}-\sqrt{x-3}=2$

---

79. $|x| = -9$

80. $|x| = 9$

81. $|x - 2| = 7$

82. $|x - 2| = 0$

83. $|2x - 3| = 6$

84. $|2x - 1| = 11$

85. $7|5x| + 2 = 16$

86. $4\left|1 - \frac{3}{4}x\right| + 7 = 10$

87. $|x + 7| + 6 = 2$

88. $|5 - 3x| = 12$

89. $|4x + 2| = 5$

90. $3|x + 5| = 12$

91. $2|x - 6| = 8$

92. $3|2x - 1| = 21$

93. $2|3x - 2| = 14$

94. $|3x - 1| + 2 = 16$

95. $|6x - 2| + 4 = 32$

96. $7|5x| + 2 = 16$

97. $|4x + 1| + 10 = 4$

98. $|4x + 1| + 4 = 10$

99. $|3x - 2| + 8 = 1$

100. $|3x - 2| + 1 = 8$

101. $\left|\frac{6x + 1}{x - 1}\right| = 3$

102. $|x + 1| = |1 - 3x|$

103. $|3x - 1| = |x + 5|$

104. $|5x - 8| = |3x + 2|$

105. $|4x - 9| = |2x + 1|$

106. $|2x - 4| = |x - 1|$

107. $|3x - 4| = |3x + 4|$

108. $|3x - 5| = |3x + 5|$

109. $|x - 3| = |5 - x|$

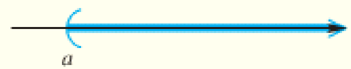
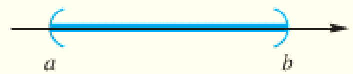
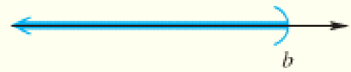
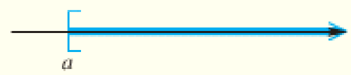
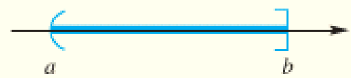
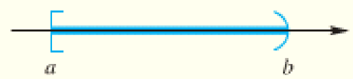
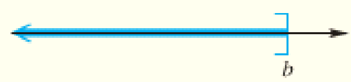
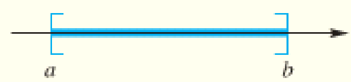
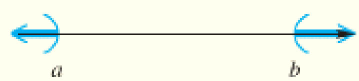
110. $|x - 3| = |6 - x|$

111. $\left|\frac{2}{3}x - 2\right| = \left|\frac{1}{3}x + 3\right|$

112. $\left|\frac{1}{2}x - 2\right| = \left|x - \frac{1}{2}\right|$

Section 1.6 – Inequalities

Notation

Type of Interval	Set	Interval Notation	Graph
Open interval	$\{x \mid x > a\}$	(a, ∞)	
	$\{x \mid a < x < b\}$	(a, b)	
	$\{x \mid x < b\}$	$(-\infty, b)$	
Other intervals	$\{x \mid x \geq a\}$	$[a, \infty)$	
	$\{x \mid a < x \leq b\}$	$(a, b]$	
	$\{x \mid a \leq x < b\}$	$[a, b)$	
	$\{x \mid x \leq b\}$	$(-\infty, b]$	
Closed interval	$\{x \mid a \leq x \leq b\}$	$[a, b]$	
Disjoint interval	$\{x \mid x < a \text{ or } x > b\}$	$(-\infty, a) \cup (b, \infty)$	
All real numbers	$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	

Properties of inequality

1. ***If a < b, then a + c < b + c***
2. ***If a < b and if c > 0, then ac < bc***
3. ***If a < b and if c < 0, then ac > bc***

Example

Solve $3x + 1 > 7x - 15$

Solution

$3x - 7x > -1 - 15$

$-4x > -16$ *Divide by – 4 both sides*

$x < 4$ *or* $(-\infty, 4)$ *or* $\{x \mid x < 4\}$

Example

Solve $\frac{x-4}{2} \ge \frac{x-2}{3} + \frac{5}{6}$ *LCD*: 2, 3, 6

Solution

$(6)\frac{x-4}{2} \ge (6)\frac{x-2}{3} + (6)\frac{5}{6}$

$3(x-4) \ge 2(x-2)+5$

$3x-12 \ge 2x-4+5$

$3x-12 \ge 2x+1$

$3x-2x \ge 12+1$

$\underline{x \ge 13}$

Example

a) $3(x+1) > 3x+2$

$3x+3 > 3x+2$

$3x-3x > -3+1$

$0 > -1$ (*True statement*)

Sol.: $\mathbb{R}$ ***or*** $\{x|$ *All Real numbers*$\}$ ***or*** $(-\infty, \infty)$

b) $x+1 \le x-1$

$x-x \le -1-1$

$0 \le -2$

Sol.: $\varnothing$

Example

Solve $-2 < 5+3x < 20$ Give the solution set in interval notation and graph it.

Solution

$-2-5 < 5+3x-5 < 20-5$

$-7 < 3x < 15$

$-\frac{7}{3} < \frac{3}{3}x < \frac{15}{3}$

$-\frac{7}{3} < x < 5$

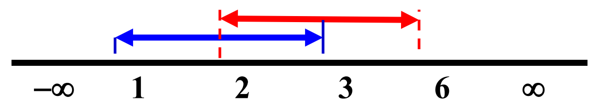
Solution: $\left(-\frac{7}{3},\ 5\right)$

Intersections of Interval $\bigcap$

To find the intersection, take the portion of the number line that the two graphs have in ***common***

Example

$[1, 3] \cap (2, 6) = (2, 3]$

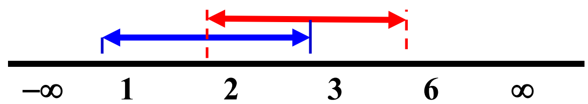


Unions of Interval $\bigcup$

To find the union, take the portion of the number line representing the total ***collection*** of numbers in the two graphs.

Example

$[1, 3] \cup (2, 6) = [1, 6)$



Solving an *Absolute Value* Inequality:

If X is an algebraic expression and c is a positive number,

1. The solutions of $|X| < c$ are the numbers that satisfy $-c < X < c$.
2. The solutions of $|X| > c$ are the numbers that satisfy $X < -c$ or $X > c$.

Example

Solve: $-3|5x-2| + 20 \geq -19$

Solution

$-3|5x-2| \geq -39$

$-|5x-2| \geq -13$

$|5x-2| \leq 13$

$-13 \leq 5x-2 \leq 13$

$-11 \leq 5x \leq 15$

$-\frac{11}{5} \leq x \leq 3$ or $\left[-\frac{11}{5},\ 3\right]$

Example

Solve: $18 < |6-3x|$

Solution

$|6-3x| > 18$

$6-3x < -18$	$6-3x > 18$
$-3x < -18-6$	$-3x > 18-6$
$-3x < -24$	$-3x > 12$
$\frac{-3}{-3}x > -\frac{24}{-3}$	$\frac{-3}{-3}x < \frac{12}{-3}$
$x > 8$	$x < -4$

Solution: $(-\infty,\ -4) \cup (8,\ \infty)$

Special Cases

Example

Solve the inequality $|2-5x| \ge -4$

Solution

$|2-5x| \ge -4$

It is always ***true***

$\therefore$ The solution set is: $\mathbb{R}$ All real numbers $(-\infty, \infty)$

Example

Solve the inequality $|4x-7| < -3$

Solution

$|4x-7| < -3$

Any absolute value can't be less than any negative number.

$\therefore$ No solution or $\varnothing$

Example

Solve the inequality $|5x+15| = 0$

Solution

$|5x+15| = 0$

$5x+15=0$

$5x=-15$

$\therefore$ Solution: $\underline{x=-3}$

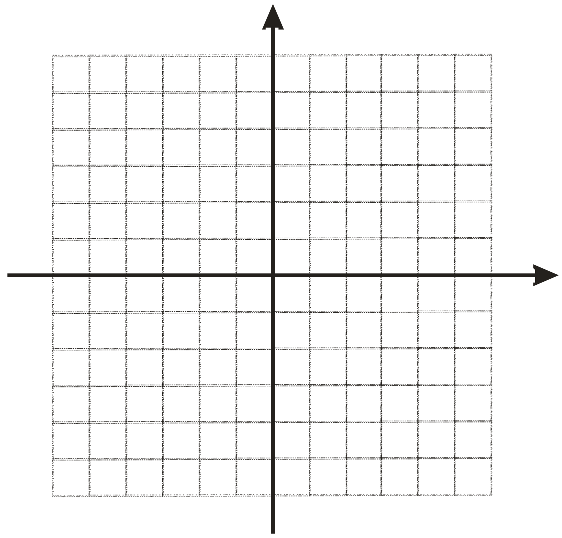
Definition of a Polynomial Inequality

A polynomial inequality is any inequality that can be put into one of the forms

$$f(x) < 0 \qquad f(x) > 0 \qquad f(x) \le 0 \qquad f(x) \ge 0$$

Where f is a polynomial function.

$f(x) = x^2 - 5x + 4 \quad (x = 1, 4)$



Procedure* for Solving Polynomial Inequalities** ***Example

1. Express the inequality in the form $f(x)$? 0	$x^2 - x < 12$ $x^2 - x - 12 < 0$
2. Solve $f(x) = 0$	$x^2 - x - 12 = 0$ $x = -3,\ 4$
3. Locate the boundary 4. Choose one test value	$-3 \quad 0 \quad 4$ $+ \quad - \quad +$
5. Write the solution set	$(-3,\ 4)$

✓ $ax^2 + bx + c \ge 0 \ \rightarrow\ if\ a > 0 \ \Rightarrow\ x \le x_1,\ x \ge x_2$

✓ $ax^2 + bx + c \le 0 \ \rightarrow\ if\ a > 0 \ \Rightarrow\ x_1 \le x \le x_2$

Example

Solve $2x^2 + 5x - 12 \ge 0$

Solution

$$2x^2 + 5x - 12 = 0$$
$$(2x-3)(x+4) = 0$$
$$x = -4,\ \frac{3}{2}$$

-4	0	$\frac{3}{2}$
$+$	$-$	$+$

$$\left(-\infty,\ -4\right] \cup \left[\frac{3}{2},\ \infty\right)$$

Solution: $x \le -4 \quad x \ge \frac{3}{2}$

Example

Solve: $x^3 + 3x^2 \le x + 3$

Solution

$$x^3 + 3x^2 - x - 3 = 0$$
$$x^2(x+3) - (x+3) = 0$$
$$(x+3)(x^2-1) = 0$$

$$x + 3 = 0 \qquad x^2 - 1 = 0$$
$$x = -3 \qquad x^2 = 1$$
$$x = -3 \qquad x = \pm 1$$

-3	-1	0	1
$-$	$+$	$-$	$+$

Solution: $x \le -3 \quad -1 \le x \le 1$ $\qquad \left(-\infty,\ -3\right] \cup \left[-1,\ 1\right]$

Rational Inequality

Example

Solve: $\dfrac{2x}{x+1} \ge 1$

Solution

$\dfrac{2x}{x+1} = 1 \quad \rightarrow \quad Cond.: x+1 \ne 0 \Rightarrow \boxed{x \ne -1}$

$(x+1)\dfrac{2x}{x+1} - 1(x+1) = 0$

$2x - x - 1 = 0$

$x - 1 = 0$

$x = 1$

-1	0	1
$+$	$-$	$+$

Solution: $\boxed{x \le -1 \quad x \ge 1}$ $\qquad \boxed{(-\infty,\ -1) \cup [1,\ \infty)}$

Example

Solve $\dfrac{5}{x+4} \ge 1$

Solution

$\dfrac{5}{x+4} - 1 = 0$ *Exception:* $x + 4 \ne 0 \ \Rightarrow x \ne -4$

$(x+4)\dfrac{5}{x+4} - 1(x+4) = 0$

$5 - x - 4 = 0$

$\boxed{x = 1}$

-4	0	1
$-$	$+$	$-$

Solution: $\boxed{-4 < x \le 1}$ $\quad \boxed{(-4,\ 1]}$

Example

Solve $\dfrac{2x-1}{3x+4} < 5$

Solution

$\dfrac{2x-1}{3x+4} - 5 = 0$ *Restriction:* $3x + 4 \ne 0 \ \Rightarrow \ \boxed{x \ne -\frac{4}{3}}$

$(3x+4)\dfrac{2x-1}{3x+4} - 5(3x+4) = 0$

$2x - 1 - 15x - 20 = 0$

$-13x - 21 = 0$

$\boxed{x = -\frac{21}{13}}$

$-\frac{21}{13}$	$-\frac{4}{3}$	0
$-$	$+$	$-$

Solution: $\boxed{x < -\frac{21}{13} \quad x > -\frac{4}{3}}$ $\qquad \boxed{\left(-\infty,\ -\frac{21}{13}\right) \cup \left(-\frac{4}{3},\ \infty\right)}$

Exercises *Section* 1.6 – Inequalities

(**1 – 6**) Find:

1. $(-3,0)\cap[-1,2]$

2. $(-3,0)\cup[-1,2]$

3. $(-4,0)\cap[-2,1]$

4. $(-4,0)\cup[-2,1]$

5. $(-\infty,5)\cap[1,8)$

6. $(-\infty,5)\cup[1,8)$

(**7 – 45**) Solve the inequality equation

7. $-3x+5>-7$

8. $2-3x\le 5$

9. $4-3x\le 7+2x$

10. $5x+11<26$

11. $3x-8\ge 13$

12. $-9x\ge 36$

13. $-4x\le 64$

14. $8x-11\le 3x-13$

15. $18x+45\le 12x-8$

16. $4(x+1)+2\ge 3x+6$

17. $8x+3>3(2x+1)+x+5$

18. $2x-11<-3(x+2)$

19. $-4(x+2)>3x+20$

20. $1-(x+3)\ge 4-2x$

21. $5(3-x)\le 3x-1$

22. $\frac{x}{4}-\frac{1}{2}\le\frac{x}{2}+1$

23. $\frac{3x}{10}+1\ge\frac{1}{5}-\frac{x}{10}$

24. $6x-(2x+3)\ge 4x-5$

25. $\frac{2x-5}{-8}\le 1-x$

26. $1-\frac{x}{2}>4$

27. $7-\frac{4}{5}x<\frac{3}{5}$

28. $\frac{x-4}{6}\ge\frac{x-2}{9}+\frac{5}{18}$

29. $\frac{4x-3}{6}+2\ge\frac{2x-1}{12}$

30. $4(3x-2)-3x<3(1+3x)-7$

31. $3(x-8)-2(10-x)<5(x-1)$

32. $8(x+1)\le 7(x+5)+x$

33. $4(x-1)\ge 3(x-2)+x$

34. $7(x+4)-13>12+13(3+x)$

35. $-2[7x-(2x-3)]<-2(x+1)$

36. $6-\frac{2}{3}(3x-12)\le\frac{2}{5}(10x+50)$

37. $\frac{2}{7}(7-21x)-4<10-\frac{3}{11}(11x-11)$

38. $3[3(x+5)+8x+7]+5[3(x-6)-2(3x-5)]<2(4x+3)$

39. $5[3(2-3x)-2(5-x)]-6[5(x-2)-2(4x-3)]<3x+19$

40. $0\le 3x-1\le 10$

41. $0\le 1-3x\le 10$

42. $0\le 2x+6\le 54$

43. $-3\le\frac{2}{3}x-5\le -1$

44. $-6\le 6x+3\le 21$

45. $1\le 2x+3\le 11$

(**46 – 85**) Solve the inequality equation

46. $|x| < 2$

47. $|x| \geq 2$

48. $|x-2| < 1$

49. $|x-1| < 4$

50. $|x+2| \geq 1$

51. $|x+1| \geq 4$

52. $|3x+5| < 17$

53. $|5x-2| < 13$

54. $|5x-2| \geq 13$

55. $|2(x-1)+4| \leq 8$

56. $|3(x-1)+2| \leq 20$

57. $\left|\frac{2x+6}{3}\right| > 2$

58. $\left|\frac{3x-3}{4}\right| < 6$

59. $\left|\frac{2x+2}{4}\right| \geq 2$

60. $\left|\frac{3x-3}{9}\right| \leq 1$

61. $\left|3-\frac{2x}{3}\right| > 5$

62. $\left|3-\frac{3x}{4}\right| < 9$

63. $|x-2| < -1$

64. $|x+2| < -3$

65. $|x+6| > -10$

66. $|x+2| > -8$

67. $|x+2|+9 \leq 16$

68. $|x-2|+4 \geq 5$

69. $2|2x-3|+10 > 12$

70. $3|2x-1|+2 < 8$

71. $-4|1-x| < -16$

72. $-2|5-x| < -6$

73. $3 \leq |2x-1|$

74. $9 \leq |4x+7|$

75. $12 < \left|-2x+\frac{6}{7}\right|+\frac{3}{7}$

76. $4+\left|3-\frac{x}{3}\right| \geq 9$

77. $|x-2| < 5$

78. $|2x+1| < 7$

79. $|5x+2|-2 < 3$

80. $|2-7x|-1 > 4$

81. $|3x-4| < 2$

82. $|2x+5| \geq 3$

83. $|12-9x| \geq -12$

84. $|6-3x| < -11$

85. $|7+2x| < 0$

(**86 – 107**) Solve the inequality equation

86. $x^2-7x+10 > 0$

87. $2x^2-9x \leq 18$

88. $x^2-5x+4 > 0$

89. $x^2+x-2 > 0$

90. $x^2-4x+12 < 0$

91. $x^2+7x > 0$

92. $x^2-49 < 0$

93. $x^2-5x \geq 0$

94. $x^2-16 \leq 0$

95. $x^2+7x+10 < 0$

96. $x^2-3x \geq 28$

97. $x^2+5x+6 < 0$

98. $x^2 < -x+30$

99. $x^3-3x^2-9x+27 < 0$

100. $x^3-x > 0$

101. $x^3+3x^2 \leq x+3$

102. $x^3+x^2 \geq 48x$

103. $x^3-x^2-16x+16 < 0$

104. $x^3+x^2-9x-9 > 0$

105. $x^3+3x^2-4x-12 \geq 0$

106. $x^4-20x^2+64 \leq 0$

107. $x^4-10x^2+9 \geq 0$

(**108 – 130**) Solve the inequality equation

108. $\frac{x+4}{x-1} < 0$

109. $\frac{x-2}{x+3} > 0$

110. $\frac{x-5}{x+8} \ge 3$

111. $\frac{x-4}{x+6} \le 1$

112. $\frac{x}{2x+7} \ge 4$

113. $\frac{x}{3x-5} \le -5$

114. $\frac{x+2}{x-5} \le 2$

115. $\frac{3x+1}{x-2} \ge 4$

116. $\frac{x}{x-3} > 0$

117. $\frac{x-3}{x+2} \ge 0$

118. $\frac{x-2}{x+2} \le 2$

119. $\frac{x+2}{x-2} \ge 2$

120. $\frac{x+2}{3+2x} \le 5$

121. $\frac{x+6}{x-14} \ge 1$

122. $\frac{x-3}{x+4} \ge \frac{x+2}{x-5}$

123. $\frac{x-4}{x+3} - \frac{x+2}{x-1} \le 0$

124. $\frac{2x-1}{x+3} \ge \frac{x+1}{3x+1}$

125. $\frac{(x+1)(x-4)}{x-2} < 0$

126. $\frac{x(x-4)}{x+5} > 0$

127. $\frac{6x^2-11x-10}{x} > 0$

128. $\frac{3x^2-2x-8}{x-1} \ge 0$

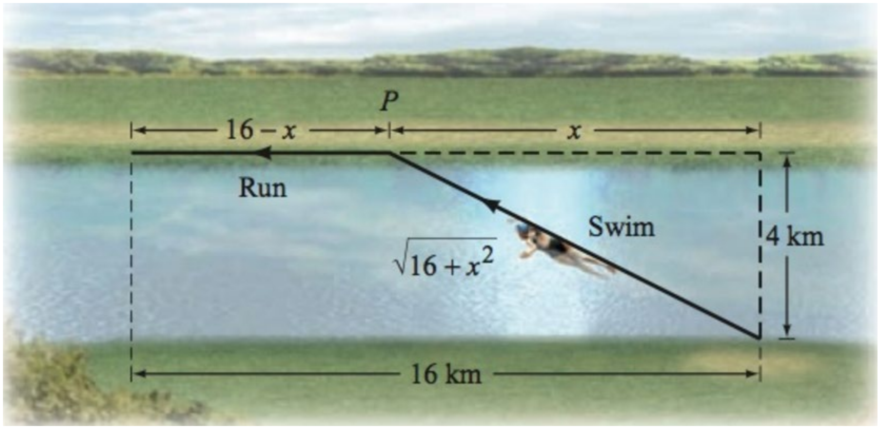
129. $\frac{x^2-6x+9}{x-5} \le 0$

130. $\frac{x^2+10x+25}{x+1} \le 0$

Section 1.7 – More Applications and Models

Example

To prepare for a triathlon, a person swims across a river to point ***P*** and then runs along a path.



The person swims at 7 km/hr and runs at 22 km/hr. For what distance x is the total time for swimming and running 2 *hours*?

Solution

$$time = \frac{distance}{rate}$$

$$\text{Time swimming } = \frac{\sqrt{16+x^2}}{7}$$

$$\text{Time runs } = \frac{16-x}{22}$$

$$\frac{\sqrt{16+x^2}}{7} + \frac{16-x}{22} = 2$$

$$22\sqrt{16+x^2} + 7(16-x) = 308$$

$$22\sqrt{16+x^2} + 112 - 7x = 308$$

$$22\sqrt{16+x^2} = 7x + 196$$

$$\left(22\sqrt{16+x^2}\right)^2 = (7x+196)^2$$

$$484\left(16+x^2\right) = 49x^2 + 2,744x + 38,416$$

$$7,744 + 484x^2 = 49x^2 + 2,744x + 38,416$$

$$435x^2 - 2,744x - 30,672 = 0$$

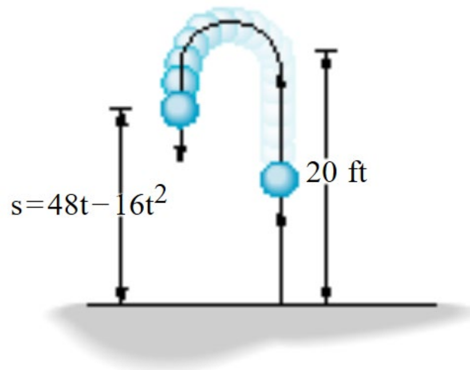
$$x \approx \frac{2,744 \pm 7,803.77}{870}$$

$$= \begin{cases} \dfrac{2,744 + 7,803.77}{870} = 12.1 \\ \dfrac{2,744 - 7,803.77}{870} = - < 0 \end{cases}$$

$\therefore$ The total distance is $12.1\ km$.

Example

A ball is thrown vertically upward with an initial velocity of 48 *feet* per *second*. The distance ***s*** (in *feet*) of the ball from the ground after ***t*** *seconds* is given by: $s(t) = 48t - 16t^2$.



a) At what time ***t*** will the ball strike the ground?

b) For what time ***t*** is the ball more than 20 *feet* above the ground?

Solution

a) $16t(3-t) = 0$

$t = 0,\ 3$

The ball will strike the ground when $t = 3$ *seconds*.

b) $48t - 16t^2 > 20$

$-4t^2 - 12t - 5 > 0$

$$t = \frac{12 \pm \sqrt{144 - 80}}{-8}$$

$$= \frac{12 \pm 8}{-8}$$

$$t = \frac{1}{2},\ \frac{5}{2}$$

$$\frac{1}{2} < t < \frac{5}{2}$$

The ball is more than 20 feet above the ground when $\frac{1}{2} < t < \frac{5}{2}$ *seconds*

Example

Suppose that the manufacturer of a gas clothes dryer has found that when the unit price is p dollars, the revenue R (in dollars) is

$$R(p) = -2p^2 + 4,000p$$

a) At what prices p is revenue zero?

b) For what range of prices will revenue exceed $500,000?

Solution

a) $-2p(p-2,000) = 0$

$$\boxed{p = 0, \quad 2,000}$$

The revenue is zero when $p = \$0 \ \ and \ \ \$2,000$

b) $-2p^2 + 4,000p > 500,000$

$$-p^2 + 2,000p - 250,000 > 0$$

$$p = \frac{-2,000 \pm \sqrt{4 \times 10^6 - 10^6}}{-2}$$

$$= \frac{-2,000 \pm 10^3\sqrt{3}}{-2}$$

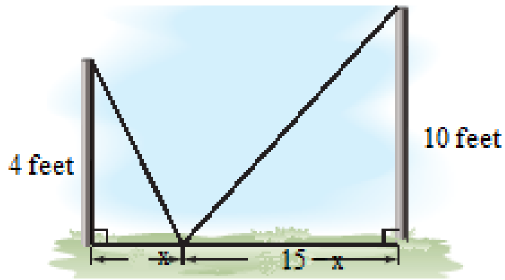
$$= 1,000 \mp 500\sqrt{3}$$

$\therefore$ The range of prices will revenue exceed 500,000 when $1,000 - 500\sqrt{3} < p < 1,000 + 500\sqrt{3}$

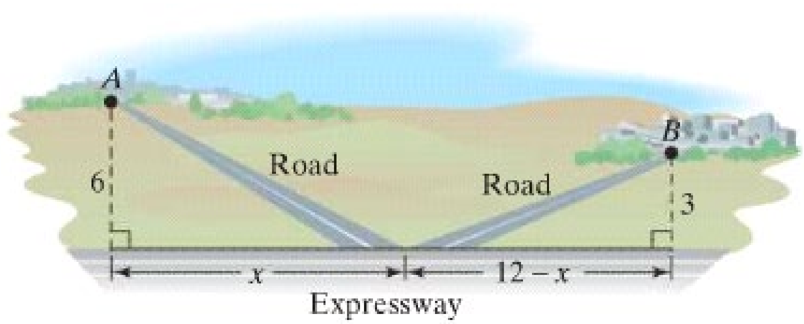
$\$133.97 < p < \$1,866.03$

Exercise Section 1.7 – More Applications and Models

1. Two vertical poles of lengths 4 *feet* and 10 *feet* stand 15 *feet* apart. A cable reaches from the top of one pole to some point on the ground between the poles and then to the top of the other pole. Where should this point be located to use 24 *feet* of cable?



2. Towns ***A*** and ***B*** are located 6 *miles* and 3 *miles*, respectively, from a major expressway. The point on the expressway closet to town ***A*** is 12 *miles* from the point on the expressway closet to town ***B***. Two new roads are to be built from ***A*** to the expressway and then to ***B***.



a) Express the combined lengths of the new road in terms of x.

b) If the combined lengths of the new roads is 15 *miles*, what distance does x represent?

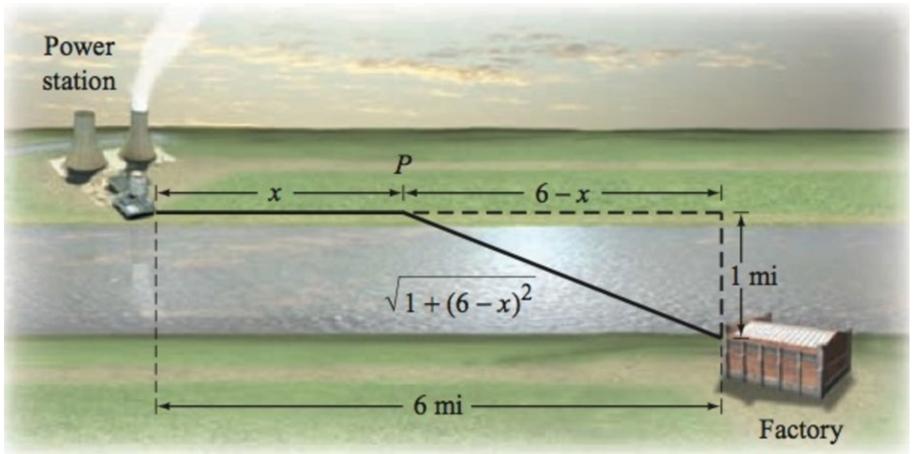
3. A solid silver sphere has a diameter of 8 *millimeters*, and a second silver has a diameter of 12 *millimeters*. The spheres are melted down and recast to form a single cube. What is the length s of each edge of the cube?

4. The period T of the pendulum is the time it takes the pendulum to complete one swing from left to right and back. For a pendulum near the surface of Earth

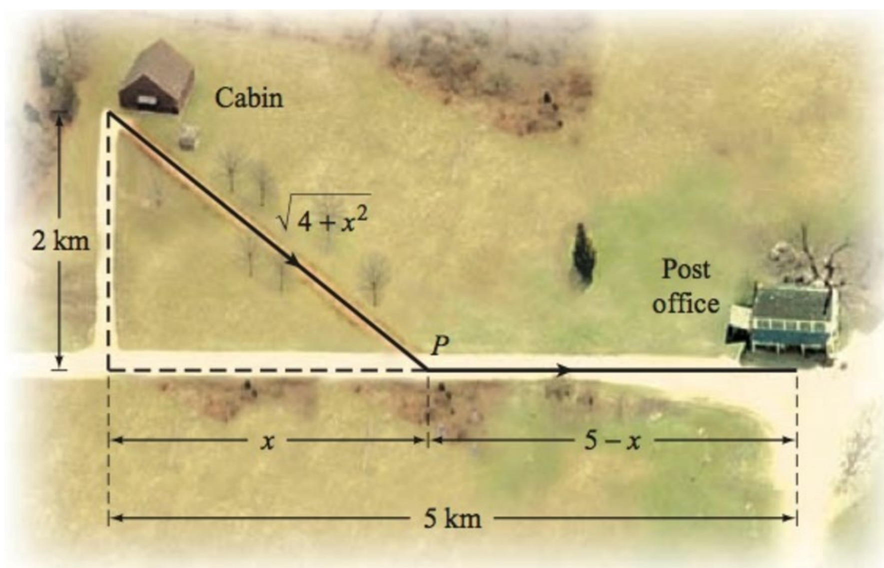
$$T = 2\pi\sqrt{\frac{L}{32}}$$

Where T is measured in *seconds* and L is the length of the pendulum in feet. Find the length of a pendulum that has a period of 4 *seconds*.

5. A power station is on one side of a river that is 1 *mile* wide, and a factory is 6 *miles* down-stream on the other side of the river, the cost is \$0.125 *million* per *mile* to run power lines over land and \$0.2 *million* per *mile* to run power lines under water. How far over the land should the power line be run if the total cost of the project is to be \$1 *million*?

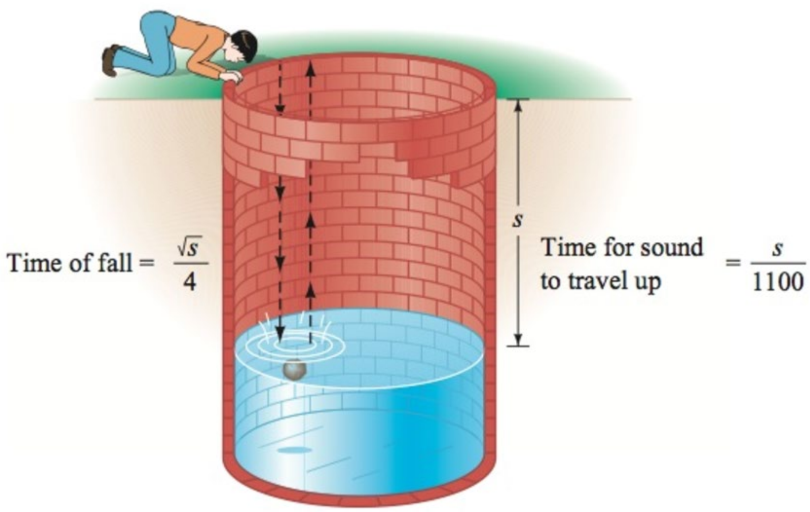


6. A cabin is located in a meadow at the end of a straight driveway 2 *km* long. A post office is located 5 *km* from the driveway along a straight road. A woman walks 2 *km/hr* through the meadow to point ***P*** and then 5 *km/hr* along the road to the post office. If it takes the woman 2.25 *hours* to reach the post office, what is the distance x of point ***P*** from the end of the driveway?



7. The depth s from the opening of a well to the water below can be determined by measuring the total time between the instant you drop a stone and the moment you heat it hit the water. The time, in *seconds*, it takes the stone to hit the water is given by $\frac{\sqrt{s}}{4}$, where s is measured in *feet*. The time, also in seconds, required for the sound of the impact to travel up to your ears is given by $\frac{s}{1{,}100}$. Thus, the total time T, in *seconds*, between the instant you drop the stone and the moment you hear its impact is

$$T = \frac{\sqrt{s}}{4} + \frac{s}{1{,}100}$$



a) One of the world's deepest water wells is 7,320 *feet* deep. Find the time between the instant you drop a stone and the time you hear it hit the water if the surface of the water if the surface of the water is 7,100 *feet* below the opening of the well.

b) Find the depth from the opening of a well to the water level if the time between the instant you drop a stone and the moment you heat its impact is 3 *seconds*.

8. On a ship, the distance d that you can see to the horizon is given by $d = \sqrt{1.5h}$, where h is the height of your eye measured in *feet* above the sea level and d is measured in *miles*. How high is the eye level of a navigator who can see 14 *miles* to the horizon?

9. A car can be rented from Basic Rental for \$260 per week with no extra charge for mileage. Continental charges \$80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

10. If a projectile is launched from ground level with an initial velocity of 96 *ft.* per *sec*, its height in feet ***t*** seconds after launching is ***s*** feet, where

$$s = -16t^2 + 96t$$

When will the projectile be greater than 80 *ft*. above the ground?

11. A projectile is fired straight up from ground level. After ***t*** seconds, its height above the ground is ***s*** *ft*., where

$$s = -16t^2 + 220t$$

For what time period is the projectile at least 624 *ft*. above the ground?

12. Your test scores of 70 and 81 in your math class. To receive a *C* grade, you must obtain an average greater than or equal to 72 but less than 82. What range of test scores on the one remaining test will enable you to get a *C* for the course.

13. A truck can be rented from Basic Rental for \$50 a day plus \$0.20 per *mile*. Continental charges \$20 per day plus \$0.50 per *mile* to rent the same truck. How many miles must be driven in a day to make the rental cost for Basic Rental a better deal than Constiental's?

14. You are choosing between two telephone plans. Plan ***A*** has a monthly fee of \$15 with a charge of \$0.08 per *minute* for all calls. Plan ***B*** has a monthly fee of \$3 with a charge of \$0.12 per *minute* for all calls. How many calling minutes in a month make plan ***A*** the better deal?

15. A City commission has proposed two tax bills. The first bill requires that a homeowner pay \$1,800 plus 3% of the assesses home value in taxes. The second bill requires taxes of \$200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal for the homeowner?

16. A local bank charges \$8 per month plus \$0.05 per check. The credit union charges \$2 per month \$0.08 per check. How many checks should be written each month to make the credit union a better deal?

17. A company manufactures and sells blank audiocassette tapes. The weekly fixed cost is \$10,000 and it costs \$0.40 to produce each tape. The selling price is \$2.00 per tape. How many tapes must be produced and sold each week for the company to have a profit?

18. A company manufactures and sells stationery. The weekly fixed cost is \$3,000 and it costs \$3.00 to produce each package of stationery. The selling price is \$5.50 per package. How many packages of stationery must be produced and sold each week for the company to have a profit?

19. An elevator at a construction site has a maximum capacity of 3,000 *pounds*. If the elevator operator weighs 200 *pounds* and each cement bag weighs 70 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

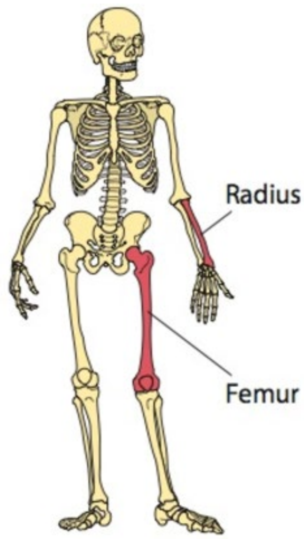
20. An elevator at a construction site has a maximum capacity of 2,500 *pounds*. If the elevator operator weighs 160 *pounds* and each cement bag weighs 60 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

21. You can rent a car for the day from Company ***A*** for \$29.00 plus \$0.12 a *mile*. Company ***B*** charges \$22.00 plus \$0.21 a *mile*. Find the number of miles m per day for which it is cheaper to rent from Company ***A***.

22. *UPS* will only ship packages for which the length is less than or equal to 108 *inches* and the length plus the girth is less than or equal to 130 *inches*. The length of a package is defined as the length of the longest side. The girth is defined as twice the width plus twice the height of the package. If a box has a length of 34 *inches* and a width of 22 *inches,* determine the possible range of heights h for this package if you wish to ship it by UPS.

23. The sum of three consecutive odd integers is between 63 and 81. Find all possible sets of integers that satisfy these conditions.

24. Forensic specialists can estimate the height of a deceased person from the lengths of the person's bones. For instance, an inequality that relates the height h, in *cm*, of an adult female and the length f, in *cm*, of her femur is $\left|h-(2.47f+54.10)\right|\le 3.72$. Use the inequalities to estimate the possible range of heights for an adult female whose measures 32.24 *cm*.



25. An inequality that is used to calculate the height h of an adult male from the length r of his radius is

$$\left|h-(3.32r+85.43)\right|\le 4.57$$

Where h and r are both in *cm*. Use this inequality to estimate the possible range of heights for an adult male whose radius measures 26.36 *cm*.