Jose's Method Integration by Part

Evaluate $\int e^{ax}\cos bx\,dx$

$$\int e^{ax}\cos bx\,dx=\frac{1}{b}e^{ax}\sin bx+\frac{a}{b^2}e^{ax}\cos bx-\frac{a^2}{b^2}\int e^{ax}\cos bx\,dx$$

$$\left(1+\frac{a^2}{b^2}\right)\int e^{ax}\cos bx\,dx=\frac{e^{ax}}{b^2}\left(b\sin bx+a\cos bx\right)$$

$$\int e^{ax}\cos bx\,dx=\boxed{\frac{e^{ax}}{a^2+b^2}\left(b\sin bx+a\cos bx\right)+C}$$

		$\int\cos bx\,dx$
+	e^{ax}	$\frac{1}{b}\sin bx$
-	ae^{ax}	$-\frac{1}{b^2}\cos bx$
+	a^2e^{ax}	$-\frac{1}{b^2}\int\cos bx\,dx$

Proof

Find $\int e^{ax}\cos bx\,dx$

Solution

Let: $u=e^{ax}$ $\quad dv=\cos bx dx$

$du=ae^{ax}dx \quad v=\int\cos bx dx=\frac{1}{b}\sin bx$

$$\int e^{ax}\cos bx\,dx=\frac{1}{b}e^{ax}\sin bx-\frac{a}{b}\int e^{ax}\sin bx dx \qquad \int udv = uv-\int vdu$$

Let: $u=e^{ax}$ $\quad dv=\sin bx dx$

$du=ae^{ax}dx \quad v=\int\sin bx dx=-\frac{1}{b}\cos bx$

$$\int e^{ax}\cos bx\,dx=\frac{1}{b}e^{ax}\sin bx-\frac{a}{b}\left[-\frac{1}{b}e^{ax}\cos bx+\frac{a}{b}\int e^{ax}\cos bx\,dx\right]$$

$$=\frac{1}{b}e^{ax}\sin bx+\frac{a}{b^2}e^{ax}\cos bx-\frac{a^2}{b^2}\int e^{ax}\cos bx\,dx$$

$$\int e^{ax}\cos bx\,dx+\frac{a^2}{b^2}\int e^{ax}\cos bx\,dx=\frac{1}{b}e^{ax}\sin bx+\frac{a}{b^2}e^{ax}\cos bx+C_1$$

$$\frac{a^2+b^2}{b^2}\int e^{ax}\cos bx\,dx=\frac{1}{b^2}e^{ax}\left(b\sin bx+a\cos bx\right)+C_1$$

$$\int e^{ax}\cos bx\,dx=\boxed{\frac{e^{ax}}{a^2+b^2}\left(b\sin bx+a\cos bx\right)+C}$$