08-10-2020

Inverse Trig.

angle

$y = \sin x \Longrightarrow x = \sin^{-1} y$

or

$x = \arcsin y$

$-1 \le y \le 1$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$-1 \le y \le 1$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

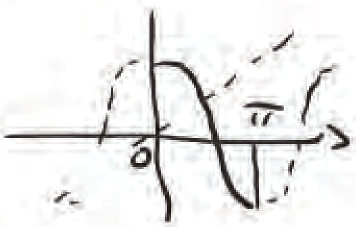
$\sin(\sin^{-1}\frac{1}{2}) = \frac{1}{2}$

$\sin(\sin^{-1}\frac{1}{2}) = \sin(\frac{\pi}{6})$

$= \frac{1}{2}$ ✓

$y = \cos^{-1} x$ iff $x = \cos y$

$-1 \le x \le 1 \qquad 0 \le y \le \pi$



ACos x

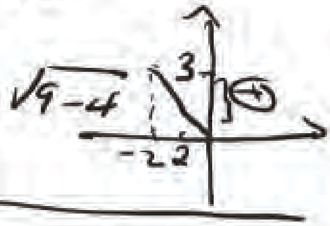
$\cos^{-1} x = \arccos x$

$\cos(\cos^{-1} x) = x$

$\sin\left(\underbrace{\arccos\left(-\frac{2}{3}\right)}_{\alpha}\right)$ $\qquad \alpha = \arccos\left(-\frac{2}{3}\right)$

$\cos\alpha = -\frac{2}{3} \qquad \frac{\pi}{2} < \theta < \pi$

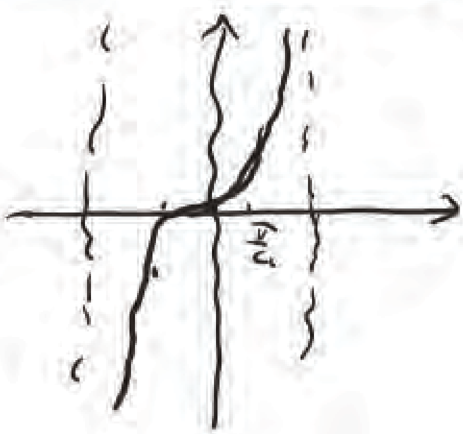
$\sin\alpha = \frac{\sqrt{5}}{3}$



Inverse Tangent

$x = \tan y \Rightarrow y = \tan^{-1} x$

$y = \arctan x$



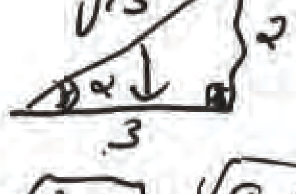
$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$\tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$

$\sec\left(\underbrace{\arctan\frac{2}{3}}_{\alpha}\right) \qquad \alpha = \arctan\frac{2}{3}$

$\tan\alpha = \frac{2}{3}$

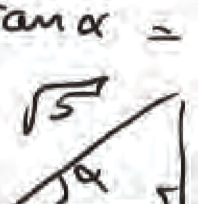
$\boxed{\sec\alpha = \frac{\sqrt{13}}{3}}$



$r = \sqrt{x^2+y^2} \qquad \sqrt{9+4}$

Ex $\sin\left(\underset{\alpha}{\arctan\frac{1}{2}} - \underset{\beta}{\arccos\frac{4}{5}}\right)$

$\tan\alpha = \frac{1}{2}$



$\cos\beta = \frac{4}{5}$

$\sin\beta = \frac{3}{5}$

2

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$= \frac{1}{\sqrt{5}}\,\frac{4}{5} - \frac{2}{\sqrt{5}}\cdot\frac{3}{5}$$

$$= \frac{4-6}{5\sqrt{5}}$$

$$= \underline{-\frac{2}{5\sqrt{5}}} \quad a \quad -\frac{2\sqrt{5}}{25}$$

Ex $\cos(\underbrace{\sin^{-1}x}_{\alpha}) = ?$

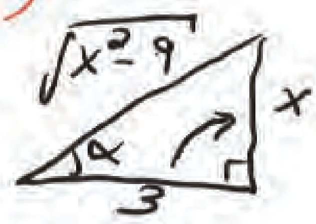
$$\alpha = \sin^{-1}x$$

$$\sin\alpha = x \quad \left(\frac{x}{1}\right)$$

$$\underline{\cos\alpha = \sqrt{1-x^2}}$$

#34 $\cot\left(\underbrace{\sin^{-1}\frac{\sqrt{x^2-9}}{x}}_{\alpha}\right)$

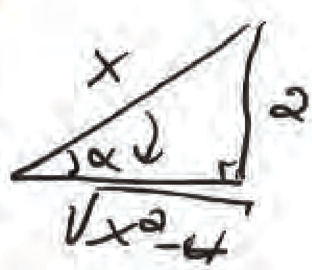
$$\sin\alpha = \frac{\sqrt{x^2-9}}{x}$$



$$\underline{\cot\alpha = \frac{3}{x}}$$

#39 $\sec\left(\underbrace{\tan^{-1}\frac{2}{\sqrt{x^2-4}}}_{\alpha}\right)$

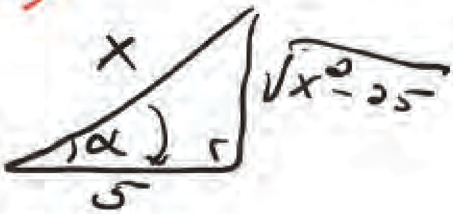
$$\tan\alpha = \frac{2}{\sqrt{x^2-4}}$$



$$\sec\alpha = \frac{x}{\sqrt{x^2-4}}$$

#22 $\sec\left(\sin^{-1}\frac{\sqrt{x^2-25}}{x}\right)$

$\sin\alpha = \frac{\sqrt{x^2-25}}{x}$



$\boxed{\sec\alpha = \frac{x}{5}}$

8.6 Polar Coordinates

$(r, \theta) \xrightarrow{\text{Relation}} (x, y)$ rectangular

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\quad\begin{cases} r = \sqrt{x^2+y^2} \\ \hat{\theta} = \tan^{-1}\frac{y}{x} \end{cases}$

$(3, 45°)$

$(2, -240°)$

$(-4, 60°)$

Ex $(r, \theta) = (4, \frac{7\pi}{6})$ $\quad (x, y)$?

$x = r\cos\theta$ $\qquad y = r\sin\theta$

$= 4\cos\frac{7\pi}{6}$ $\qquad = 4\sin\frac{7\pi}{6}$

$= 4\left(-\frac{\sqrt{3}}{2}\right) \qquad = 4\left(-\frac{1}{2}\right)$

$= -2\sqrt{3} \qquad = -2$

$(x, y) = (-2\sqrt{3}, -2)$

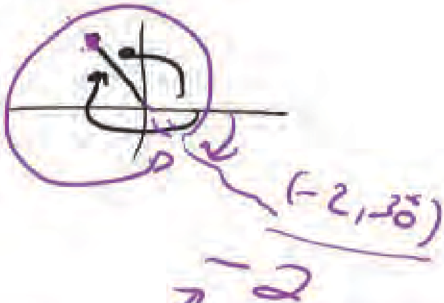
$(x, y) = (-1, \sqrt{3}) \qquad (r, \theta)?$

$r = \sqrt{x^2 + y^2} \qquad \hat{\theta} = \tan \frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$= \sqrt{1 + 3} \qquad \theta = \frac{2\pi}{3}$

$= 2$

$(r, \theta) = \left(2, \frac{2\pi}{3}\right) \qquad 120°$



$x =$

$\vdots$

$=$

(x, y)

---

line $ax + by = c$

$a\,r\cos\theta + b\,r\sin\theta = c$

$r(a\cos\theta + b\sin\theta) = c$

$r(a\cos\theta + b\sin\theta) = c$

$$\boxed{r = \frac{c}{a\cos\theta + b\sin\theta}}$$

$x^2 - y^2 = 16$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 16$

$r^2\cos^2\theta - r^2\sin^2\theta = 16$

$r^2(\underline{\cos^2 - \sin^2\theta}) = 16$

$$\boxed{r^2 = \frac{16}{\cos 2\theta}}$$

$r = a\sin\theta \qquad (a \neq 0)$

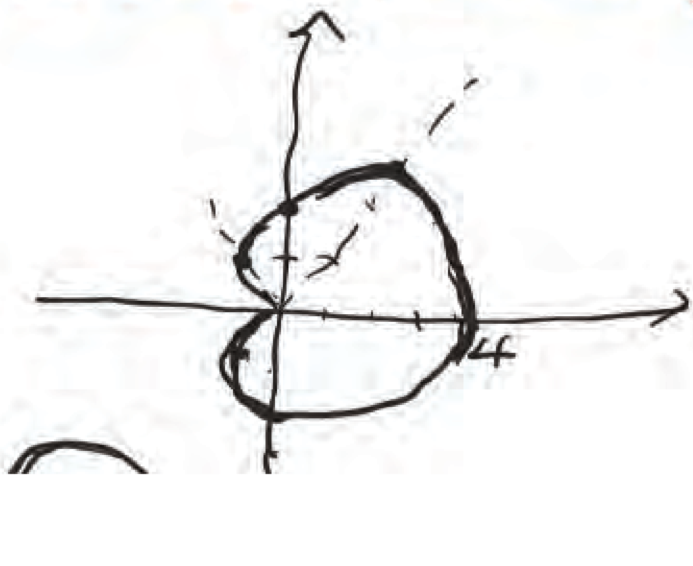
$\underline{r^2} = a\, r \sin\theta$

$$\boxed{x^2 + y^2 = a\, y}$$

Graph $\quad 0, 1, \frac{1}{2}$ $\quad \cos 60^\circ = \frac{1}{2}$, $\sin 30^\circ = \frac{1}{2}$

$r = 2 + 2\cos\theta$ (cardioid)

θ	r
0	4
60°	3
90°	2
120°	1
π	0
	1

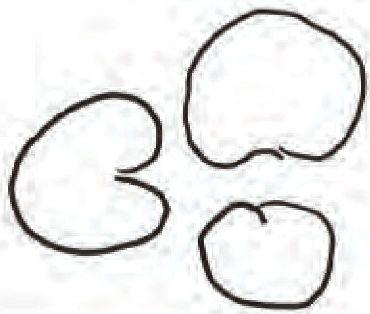


$a(1+\cos\theta)$

$a(1-\cos\theta) \longrightarrow$

$a(1+\sin\theta)$

$a(1-\sin\theta)$



#1 $(4, 30^\circ)$

$$x = r\cos\theta = 4\cos 30^\circ = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$$

$$y = r\sin\theta = 4\sin 30^\circ = 4\left(\frac{1}{2}\right) = 2$$

$$(x, y) = (2\sqrt{3}, 2)$$

#2 $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$

$$x = r\cos\theta = -\sqrt{2}\cos\frac{3\pi}{4} = -\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = 1$$

$$y = r\sin\theta = -\sqrt{2}\sin\frac{3\pi}{4} = -\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = -1$$

$$(x, y) = (1, -1)$$

$(x, y) = (-1, \sqrt{3})$ $\theta \in Q\text{II}$

$$r = \sqrt{x^2+y^2} = \sqrt{1+3} = 2$$

$$\theta' = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$$

$$\theta = 2\pi$$

$$(r, \theta) = \left(2, \frac{2\pi}{3}\right)$$

#13) $(-3, -3) = (x, y)$

$$r = \sqrt{x^2 + y^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$\hat{\theta} = \tan^{-1}\frac{3}{3} = \tan^{-1} 1 = \frac{\pi}{4}$$

$$\theta = \frac{5\pi}{4}$$

$$(r, \theta) = \left(3\sqrt{2}, \frac{5\pi}{4}\right)$$

#23 $r = 6\cos\theta$

$$r^2 = 6r\cos\theta$$

$$\boxed{x^2 + y^2 = 6x}$$

#29 $r^2(4\sin^2\theta - 9\cos^2\theta) = 36$

$$4r^2\sin^2\theta - 9r^2\cos^2\theta = 36$$

$$4y^2 - 9x^2 = 36$$

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

#31 $r(\sin\theta + r\cos^2\theta) = 1$

$$r\sin\theta + r^2\cos^2\theta = 1$$

$$y + x^2 = 1$$

#30 $y^2 = 6x$

$$(r\sin\theta)^2 = 6r\cos\theta$$

$$r^2\sin^2\theta = 6r\cos\theta \qquad (r \neq 0)$$

$$r\sin\theta = 6\cos\theta$$

$$r = \frac{6\cos\theta}{\sin\theta}$$

$$= \boxed{6\cot\theta}$$

#37 $(x+2)^2 + (y-3)^2 = 13$

$$x^2 + 4x + \underline{4} + y^2 - 6y + \underline{9} = 13$$

$$\underbrace{x^2 + y^2} + 4x - 6y = 0$$

$$r^2 + 4r\cos\theta - 6r\sin\theta = 0$$

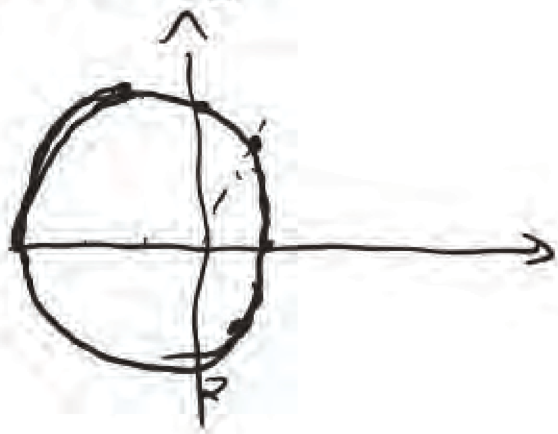
$$(r \neq 0)$$

$$r + 4\cos\theta - 6\sin\theta = 0$$

$$\boxed{r = 6\sin\theta - 4\cos\theta} = \underline{f(\theta)}$$

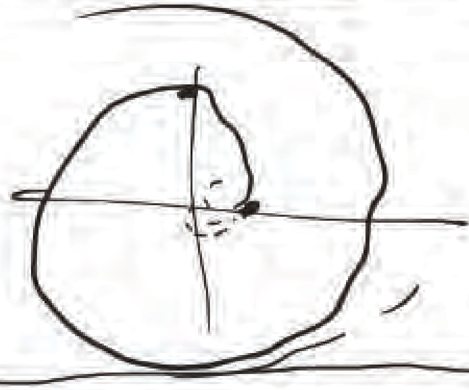
#48 $r = 2 - \cos\theta$

θ	r
0	1
$\pi/3$	$3/2$
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	$5/2$
π	3
$\frac{4\pi}{3}$	$5/2$



$\frac{3\pi}{2}$ | 2
$\frac{5\pi}{3}$ | $\frac{3}{2}$

$r = e^{2\theta}$



8.7 Trig Form

$\mathbb{C}$

$\sqrt{-1} = i$

$-1 = i^2$

Realpart $\quad -i = i^3 \quad i\, i^2$

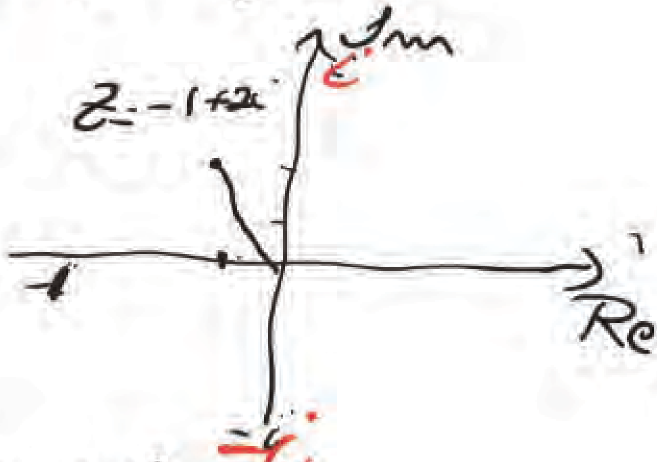
$1 = i^4$

$z = \overbrace{a}^{\text{Realpart}} + \underbrace{ib}_{Im}$ imaginary part

Re

$a, b \in \mathbb{R}$

$z = -1 + 2i$



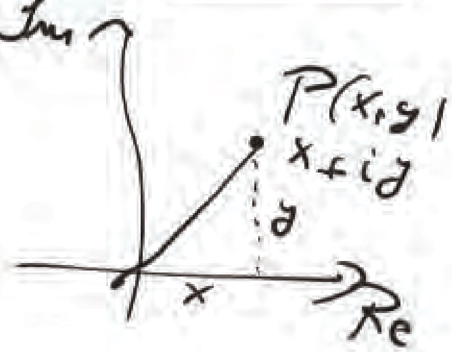
$(6 - 2i) + (-4 - 3i)$

$= 6 - 4 + i(-2 - 3)$

$= 2 - 5i$

$r = \sqrt{x^2 + y^2}$

modulus.



θ: argument

$\theta = \tan^{-1} \frac{y}{x}$

$z = x + yi$

$= r\cos\theta + i(r\sin\theta)$

$= r(\cos\theta + i\sin\theta)$

$= r \,\mathrm{cis}\, \theta$ } Trig Form

$z = -1 + i$ Trig Form

$r = \sqrt{(-1)^2 + 1^2}$

$= \sqrt{2}$

$\hat{\theta} = \tan^{-1} \frac{1}{1} = \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}$

$\boxed{z = \sqrt{2} \,\mathrm{cis}\, \frac{3\pi}{4}}$ $\left(\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right)$

$z = 6 \,\mathrm{cis}\, 60^\circ$

$= 6\cos 60^\circ + 6i\sin 60^\circ$

$= 6\cos 60 + 6i\sin 60$

$= 6\left(\frac{1}{2}\right) + i6\left(\frac{\sqrt{3}}{2}\right)$

$= 3 + 3i\sqrt{3}$

$|r|$? $z = 5i$ $z = 7$

$r = 5$ $r = 7$

$3 + 4i \to r = 5$

$\sqrt{9+16} = \sqrt{25}$

$\cos z \leftarrow z\ \mathbb{C}$

$(r_1 \operatorname{cis} \theta_1)(r_2 \operatorname{cis} \theta_2) = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$

Ex $3(\operatorname{cis} 45^\circ)(2 \operatorname{cis} 135^\circ) = 6 \operatorname{cis}(45^\circ + 135^\circ)$

$= 6 \operatorname{cis} 180^\circ$

$= 6\cos 180^\circ + i\, 6\sin 180^\circ$

$= -6$

$a^2 + b^2 = (a+ib)(a-ib)$

$= a^2 - (ib)^2$

$a^2 - i^2 b^2$

$= a^2 - (-1) b^2$

$= a^2 + b^2$

$a^2 + b^2 =$

$$= (a+b)^2$$

$$a+b = (\sqrt{a} - i\sqrt{b})(\sqrt{a} + i\sqrt{b})$$

$$a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$$

$$\frac{r_1 \operatorname{cis} \theta_1}{r_2 \operatorname{cis} \theta_2} = \frac{r_1}{r_2} \operatorname{cis} (\theta_1 - \theta_2)$$

$$\frac{10 \operatorname{cis} (-60^\circ)}{5 \operatorname{cis} (150^\circ)} = \frac{10}{5} \operatorname{cis} (-60^\circ - 150^\circ)$$

$$= 2 \operatorname{cis} (-210^\circ)$$

$$= 2\cos(-210^\circ) + i\,2\sin(-210^\circ)$$

$$= 2\left(-\frac{\sqrt{3}}{2}\right) + i\,2\left(+\frac{1}{2}\right)$$

$$= -\sqrt{3} + i$$

$\underset{(-)}{210^\circ}$ III

De Moivre's Theorem

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} n\theta$$

$$(1 + i\sqrt{3})^8 \qquad \theta \in Q\,I$$

$$r = \sqrt{1+3} = 2$$

$$\theta = \tan^{-1} \frac{\sqrt{3}}{1} = 60^\circ$$

$$(1+i\sqrt{3})^8 = (2\,\text{cis}\,60^\circ)^8$$

$$= 2^8\,\text{cis}\,(480^\circ)$$

$$= 256\,\text{cis}\,(120^\circ)$$

$$= 256\,(\cos 120^\circ + i\sin 120^\circ)$$

$$= 256\left(-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right)$$

$$= -128 + i\,128\sqrt{3}$$

n^{th} root $\quad (r\,\text{cis}\,\theta)^{1/n} = \sqrt[n]{r}\,\text{cis}\,\frac{\theta}{n}\,\alpha$

$$\alpha = \frac{\theta}{n} + \frac{360^\circ}{n}k$$

or

$$\frac{\theta + 2\pi k}{n}$$