1/ $\frac{x}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$

$$x = A(x-1) + B(x+3)$$

$$x^1 \quad A + B = 1$$

$$x^0 \quad -A + 3B = 0 \rightarrow \boxed{A = 3\left(\tfrac{1}{4}\right)}$$

$$4B = 1$$

$$\boxed{B = \tfrac{1}{4}}$$

$$\boxed{\frac{x}{x^2+2x-3} = \frac{3}{4}\,\frac{1}{x+3} + \frac{1/4}{x-1}}$$

$$\frac{1}{(2x+3)(4x-1)} = \frac{A}{2x+3} + \frac{B}{4x-1}$$

$$1 = A(4x-1) + B(2x+3)$$

$$x^1 \quad 4A + 2B = 0 \rightarrow (1)$$

$$x^0 \quad \overset{4}{-A} + \overset{12}{3}B = \overset{4}{1}$$

$$14B = 4 \Rightarrow \boxed{B = \tfrac{2}{7}}$$

$$(1) \quad 4A = -2\left(\tfrac{2}{7}\right)$$

$$\boxed{A = \frac{-4}{4(7)} = -1/7}$$

$$\boxed{\frac{1}{(2x+3)(4x-1)} = \frac{-1/7}{2x+3} + \frac{2/7}{4x-1}}$$

1st 4 terms? a_8? 8

$a_n = (-1)^{n+1} \frac{n}{n+1}$

$a_1 = (-1)^2 \frac{1}{2} = \frac{1}{2}$

$a_2 = (-1)^3 \frac{2}{3} = -\frac{2}{3}$

$a_3 = (-1)^4 \frac{3}{4} = \frac{3}{4}$

$a_4 = (-1)^5 \frac{4}{5} = \frac{-4}{5}$

$a_8 = (-1)^9 \frac{8}{9} = -\frac{8}{9}$

$2 + 5 + 8 + \cdots + 17 = \sum_{n=1}^{6} (3n-1)$

$d = 5 - 2 = 3$

$a_n = 2 + (n-1)(3)$

$= 2 + 3n - 3$

$= 3n - 1$

$\frac{17-2}{3} = 5+1$

Arithm. a_{12}: $a_8 = 4$ $a_{18} = -96$

$$d = \frac{-96-4}{18-8} = -\frac{100}{10} = -10$$

$a_n = a_1 + (n-1)d$

$$a_8 = a_1 + 7(-10)$$

$$4 = a_1 - 70$$

$$a_1 = 74$$

$$a_{12} = 74 + 11(-10)$$

$$= 74 - 110$$

$$= -36$$

Geom a_9 : $a_2 = 4$ $a_5 = 32$

$$r = \left(\frac{32}{4}\right)^{\frac{1}{5-2}}$$

$$= (8)^{1/3}$$

$$= 2$$

$(2^3)^{1/3}$

$a_n = a_1 r^{n-1}$

$$a_2 = a_1 \, 2^1 = 4$$

$$a_1 = 2$$

$$a_9 = 2(2)^8$$

$$= 2^9$$

$$\sum_{n=1}^{25} 5 = 5(25) = \boxed{125}$$

$$\sum_{k=15}^{45} 8 = 8(45-15+1) = 8(31) = \boxed{248}$$

$$\sum_{k=1}^{5} (2k+1) = \overbrace{\boxed{(2+1)+(4+1)+(6+1)+(8+1)+11}}^{\text{you can avoid this step}}$$

$$= 3+5+7+9+11$$

$$= \boxed{35}$$

$$\sum_{n=1}^{\infty} 5\left(\frac{3}{2}\right)^n = \infty \qquad \left|\frac{3}{2}\right| \geq 1$$

$$\sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right) = \frac{3}{1-\frac{2}{3}} = \frac{3}{\frac{1}{3}} = \boxed{9} \qquad \left|\frac{2}{3}\right| < 1 \qquad \frac{a}{1-r}$$

$4+8+12+\cdots+4n=2n(n+1)$

$n=1 \Rightarrow \quad 4 = 2(1+1)$

$\qquad\qquad 4 = 4$ ✓ P_1 is true.

Assume P_k is true: $4+\cdots+4k=2k(k+1)$

is P_{k+1}: $4+\cdots+4k+4(k+1)=2(k+1)(k+2)$?

$4+\cdots+4k+4(k+1)=2k(k+1)+4(k+1)$

$\qquad\qquad =2(k+1)(k+2)$ ✓

P_{k+1} is also true.

∴ By the mathematical model, the given proof is completed.

$h=15$ $\qquad w=10$

$\qquad\qquad x=5$

$$\frac{x^2}{25^2}+\frac{y^2}{20^2}=1$$

$$\frac{y^2}{20^2}=1-\left(\frac{\cancel{5}^{1}}{\cancel{25}_{5}}\right)^2$$

$$\left(\frac{15}{20}\right)^2 \; ? \; 1-\frac{1}{25}$$

$$\frac{24}{25}$$

$$\frac{225}{400} \; ? \; \frac{24}{25}$$

$$225 \; ? \; \frac{24\times \cancel{400}^{16}}{\cancel{25}}$$

$$225 < 384$$

will clear

$625y^2 - 400x^2 = 25 \times 10^4.$

$$\rightarrow \frac{y^2}{400} - \frac{x^2}{625} = 1$$

$a^2 = 400$

$a = 20$

the closest point : $2a = \underline{40}$ yards.

$1 + 5 + 9 + \cdots + (4n-3) = n(2n-1)$

For $n = 1 \Rightarrow \quad 1 = 1(2-1)$

$1 = 1$ ✓ P_1 is true

Assume P_k is true: $1 + \cdots + (4k-3) = k(2k-1)$

is P_{k+1}: $1 + \cdots + (4k-3) + (4(k+1)-3) = (k+1)(2(k+1)-1)$

$1 + \cdots + (4k-3) + (4k+1) \stackrel{?}{=} (k+1)(2k+1)$

$$\begin{aligned} 1 + \cdots + (4k-3) + (4k+1) &= k(2k-1) + (4k+1) \\ &= 2k^2 - k + 4k + 1 \\ &= 2k^2 + 3k + 1 \\ &= (k+1)(2k+1) \checkmark \end{aligned}$$

P_{k+1} is also true

$\therefore$ By the mathematical induction, the given proof is complete

$$1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$

For $n=1 \Rightarrow 1^2=\frac{1(2)(3)}{6}$

$1=1$ ✓ P_1 is true.

Assume: P_k is true: $1^2+\cdots+k^2=\frac{1}{6}k(k+1)(2k+1)$

is P_{k+1}: $1^2+\cdots+k^2+(k+1)^2=\frac{1}{6}(k+1)(k+2)(2k+3)$

$$\begin{aligned}1^2+\cdots+k^2+(k+1)^2&=\frac{1}{6}k(k+1)(2k+1)+(k+1)^2\frac{6}{6}\\&=\frac{1}{6}(k+1)\left[k(2k+1)+6(k+1)\right]\\&=\frac{1}{6}(k+1)(2k^2+7k+6)\\&=\frac{1}{6}(k+1)(k+2)(2k+3)\end{aligned}$$

P_{k+1} is also true

$\therefore$ By the mathematical induction, the given proof is completed.

$\boxed{\text{14 to 15 questions on Exam 1}}$

Proof always on separate (only) paper

$$\frac{3x-1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$3x-1 = A(x+2) + B(x-1)$$

$$x^1 \quad A + B = 3 \rightarrow B = 3 - \frac{2}{3}$$

$$x^0 \quad 2A - B = -1 \qquad = \frac{7}{3}$$

$$3A = 2$$

$$A = \frac{2}{3}$$

$$\frac{3x-1}{(x-1)(x+2)} = \frac{2/3}{x-1} + \frac{7/3}{x+2}$$

$$\frac{2x+1}{2x^2+x-3} = \frac{A}{2x+3} + \frac{B}{x-1}$$

$$2x+1 = A(x-1) + B(2x+3)$$

$$x^1 \quad A + 2B = 2 \rightarrow A = 2 - 2\left(\frac{3}{5}\right)$$

$$= 2 - \frac{6}{5}$$

$$x^0 \quad -A + 3B = 1 \qquad = \frac{4}{5}$$

$$5B = 3$$

$$B = \frac{3}{5}$$

$$\frac{2x+1}{2x^2+x-3} = \frac{4/5}{2x+3} + \frac{3/5}{x-1}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

simplify

$$\frac{y^2}{b^2} = 1 - \boxed{\frac{x^2}{a^2}} \quad \left(\frac{x}{a}\right)^2$$

$$= \frac{a_1^2 - x_1^2}{a_1^2}$$

$$y^2 = \frac{b^2}{a_1^2}(a_1^2 - x_1^2)$$

$$y^2 \stackrel{?}{} \left(\frac{b}{a_1}\right)^2 (a_1^2 - x_1^2)$$

simplify

$w = 10 \to x = 5$

$h = y = 9$

$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

$$\frac{y^2}{10^2} = 1 - \left(\frac{\cancel{5}^{1}}{\cancel{20}_{4}}\right)^2$$

$$= 1 - \frac{1}{16}$$

$$= \frac{15}{4^2}$$

$$9^2 \;?\; \left(\frac{10}{4}\right)^2 (15)$$

$$81 \;?\; \frac{25}{4}(15)$$

$$81 \times 4 \;?\; (15)(25)$$

$$324 < 375$$

clear ✓