1/ $W = \{(x_1, x_2, x_3, 0) : x_1, x_2, x_3 \in \mathbb{R}\}$

let $u = (x_1, x_2, x_3, 0)$ $v = (y_1, y_2, y_3, 0)$

1- $u + v = (x_1, x_2, x_3, 0) + (y_1, y_2, y_3, 0)$
$= (x_1+y_1, x_2+y_2, x_3+y_3, 0)$
$= (z_1, z_2, z_3, 0) \in W.$ $\left.\right\} \begin{array}{l} z_1 = x_1+y_1 \\ z_2 = x_2+y_2 \\ z_3 = x_3+y_3 \end{array}$

Set is closed under addition

2. let c scalar $\in \mathbb{R}$.

$cu = c(x_1, x_2, x_3, 0)$
$= (cx_1, cx_2, cx_3, 0)$
$= (z_1, z_2, z_3, 0) \in W$ $\left.\right\} \begin{array}{l} cx_1 = z_1 \\ cx_2 = z_2 \\ cx_3 = z_3 \end{array}$

set is closed under scalar multiplication

$\therefore$ set W is a subspace of $\mathbb{R}^4$

3/ $W : M_{2\times2} = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$

let $M_1 = \begin{pmatrix} 0 & a_1 \\ b_1 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & a_2 \\ b_2 & 0 \end{pmatrix}$

1- $M_1 + M_2 = \begin{pmatrix} 0 & a_1 \\ b_1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & a_2 \\ b_2 & 0 \end{pmatrix}$
$= \begin{pmatrix} 0 & a_1+a_2 \\ b_1+b_2 & 0 \end{pmatrix}$ $\left.\right\} \begin{array}{l} a = a_1+a_2 \\ b = b_1+b_2 \end{array}$
$= \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \in W$

set is closed under addition

2. let $c \in \mathbb{R}$

$cM_1 = c\begin{pmatrix} 0 & a_1 \\ b_1 & 0 \end{pmatrix}$
$= \begin{pmatrix} 0 & ca_1 \\ cb_1 & 0 \end{pmatrix}$ $\left.\right\} \begin{array}{l} ca_1 = a \\ cb_1 = b \end{array}$
$= \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \in W.$

set is closed under scalar multiplication

Since, the set is closed under addition & scalar multiplication therefore, set is a subspace

7/ W is the set of all vector $\mathbb{R}^3$ 3^{rd} coord $= -1$

let $u = (0,1,1)$ & $v = (1,0,-1)$

$u + v = (0,1,1) + (1,0,-1)$

$= (1,1,\underline{0}) \notin W$.

$0 \neq -1$.

$\therefore$ it's not closed under addition

9/ $W \in \mathbb{R}^2$; x, y are rational

let $(1, \frac{3}{2}) \in W$

let $c = \sqrt{2} \Rightarrow \sqrt{2}(1, \frac{3}{2}) = (1, \frac{3\sqrt{2}}{2})$

$\frac{3\sqrt{2}}{2} \notin Q$

it's not closed under scalar multiplication

#15/ $M_{3\times3} = \begin{bmatrix} 1 & a & b \\ c & 1 & d \\ e & f & 0 \end{bmatrix}$

let $M_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$M_1 + M_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{pmatrix} \Rightarrow 2 \neq 1$

it's not closed under addition

(or $c = 2 \Rightarrow$ "

#21/ $C(-\infty,\infty)$ set of all positive fctns: $f(x) > 0$

1. $f(x) > 0$, $g(x) > 0$

$f(x) + g(x) > 0$

closed under addition

2. let $c = -1$

$c f(x) = -f(x) < 0$.

it's not closed under scalar multiplication

$\therefore$ it's not a subspace

31/ $M_{n \times n}$ w/ integer entries $(a_{ij} \in \mathbb{Z})$

1. $M_1 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ $M_2 = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$; $a_{ij} + b_{ij} \in \mathbb{Z}$

$$M_1 + M_2 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

$M \; a_{ij}$

$$= \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}$$

$c_1 = a_{11} + b_{11} \in \mathbb{Z}$
$c_2 = a_{12} + b_{12} \in \mathbb{Z}$
$c_3 = a_{21} + b_{21} \in \mathbb{Z}$
$c_4 = a_{22} + b_{22} \in \mathbb{Z}$

$c_i \in \mathbb{Z}$

$\Rightarrow M_{n \times n}$ set is closed under addition.

2. let $c = \frac{2}{3}$, $M = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

$\Rightarrow cM = \frac{2}{3}\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} \frac{2}{3} & \\ & \end{pmatrix}$ $\frac{2}{3} \notin \mathbb{Z}$

the set is not closed under scalar multiplication

$\therefore$ The set is not a subspace.

39/ $W = \{(a, a-3b, b) : a, b \in \mathbb{R}\}$

let $u = (x_1, x_1 - 3y_1, y_1)$ $v = (x_2, x_2 - 3y_2, y_2)$

1. $u + v = (x_1, x_1 - 3y_1, y_1) + (x_2, x_2 - 3y_2, y_2)$

$= (x_1 + x_2, x_1 - 3y_1 + x_2 - 3y_2, y_1 + y_2)$

$= (x_1 + x_2, x_1 + x_2 - 3(y_1 + y_2), y_1 + y_2)$

$= (a, a-3b, b) \in W$ $\left\{ \begin{array}{l} x_1 + x_2 = a \\ y_1 + y_2 = b \end{array} \right.$

It's closed under addition

2. let $c \in \mathbb{R}$.

$cu = c(x_1, x_1 - 3y_1, y_1)$ let $\left\{ \begin{array}{l} cx_1 = a \\ cy_1 = b \end{array} \right.$

$= (cx_1, cx_1 - 3cy_1, cy_1)$

$= (a, a-3b, b) \in W$

It's closed under scalar multiplication

$\therefore$ ~~since~~ the set W is a subspace.

41/ $W = \{(x_1, x_2, x_1 x_2) : x_1, x_2 \in \mathbb{R}\}$

1. let $u = (u_1, u_2, u_1 u_2)$ $v = (v_1, v_2, v_1 v_2)$

$$u+v = (u_1, u_2, u_1 u_2) + (v_1, v_2, v_1 v_2)$$
$$= (u_1 + v_1, u_2 + v_2, u_1 u_2 + v_1 v_2)$$

let $\begin{cases} x_1 = u_1 + v_1 \\ x_2 = u_2 + v_2 \end{cases} \Rightarrow x_1 x_2 = (u_1 + v_1)(u_2 + v_2)$
$$= u_1 u_2 + u_1 v_2 + v_1 u_2 + v_1 v_2$$
$$\neq u_1 u_2 + v_1 v_2$$

$\therefore$ it's not closed under addition

2. let $c \in \mathbb{R}$

$$cu = c(u_1, u_2, u_1 u_2)$$
$$= (cu_1, cu_2, cu_1 u_2)$$

let $x_1 = c\, u_1$
$x_2 = c\, u_2$
$x_1 x_2 = c^2 u_1 u_2 \neq c u_1 u_2$

$\therefore$ it's not closed under scalar mult.

$\Rightarrow$ Since the set W is not closed neither under addition or scalar multiplication

$\therefore$ set W is not subspace.

1) $S = \{(2,-1,3), (5,0,4)\}$

a) $\vec{z} = (-1,-2,2)$

$$\left(\begin{array}{cc|c} 2 & 5 & -1 \\ -1 & 0 & -2 \\ 3 & 4 & 2 \end{array}\right) \begin{array}{l} \\ 2R_2+R_1 \\ 2R_3-3R_1 \end{array} \rightarrow \left(\begin{array}{cc|c} 2 & 5 & -1 \\ 0 & 5 & -5 \\ 0 & -7 & 7 \end{array}\right) \rightarrow C_2 = -1$$

$$2C_1 + 5C_2 = -1 \rightarrow \boxed{C_1 = 2}$$

$$\boxed{\vec{z} = 2(2,-1,3) - (5,0,4)}$$

b) $\vec{v} = \left(8, -\frac{1}{4}, \frac{27}{4}\right)$

$$\left(\begin{array}{cc|c} 2 & 5 & 8 \\ -1 & 0 & -1/4 \\ 3 & 4 & \frac{27}{4} \end{array}\right) \begin{array}{l} \\ 2R_2+R_1 \\ 2R_3-3R_1 \end{array} \rightarrow \left(\begin{array}{cc|c} 2 & 5 & 8 \\ 0 & 5 & 15/2 \\ 0 & -7 & -\frac{21}{2} \end{array}\right)$$

$$5C_2 = \frac{15}{2} \rightarrow \boxed{C_2 = \frac{3}{2}}$$

$$-7C_2 = -\frac{21}{2}$$

$$2C_1 = 8 - 5\left(\frac{3}{2}\right) \rightarrow \boxed{C_1 = \frac{1}{4}}$$

$$\boxed{\vec{v} = \frac{1}{4}(2,-1,3) + \frac{3}{2}(5,0,4)}$$

c) $\vec{w} = (1,-8,12)$

$$\left(\begin{array}{cc|c} 2 & 5 & 1 \\ -1 & 0 & -8 \\ 3 & 4 & 12 \end{array}\right) \begin{array}{l} \\ 2R_2+R_1 \\ 2R_3-3R_1 \end{array} \rightarrow \left(\begin{array}{cc|c} 2 & 5 & 1 \\ 0 & 5 & -15 \\ 0 & -7 & 21 \end{array}\right)$$

$(-3 \times -7 = 21 \checkmark)$

$$5C_2 = -15 \rightarrow \boxed{C_2 = -3}$$

$$2C_1 = 1 - 5(-3) \rightarrow \boxed{C_1 = 8}$$

$$\boxed{\vec{w} = 8(2,-1,3) - 3(5,0,4)}$$

d) $\vec{u} = (1,1,-1)$

$$\left(\begin{array}{cc|c} 2 & 5 & 1 \\ -1 & 0 & 1 \\ 3 & 4 & -1 \end{array}\right) \begin{array}{l} \\ 2R_2+R_1 \\ 2R_3-3R_1 \end{array} \rightarrow \left(\begin{array}{cc|c} 2 & 5 & 1 \\ 0 & 5 & 3 \\ 0 & -7 & -5 \end{array}\right) \quad \left.\begin{array}{l} C_2 = 3/5 \\ C_2 = 5/7 \end{array}\right] \#$$

No linear combination

3/ $S = \{(2,0,7), (2,4,5), (2,-12,13)\}$

a) $\vec{u} = (-1, 5, -6)$

$$\left(\begin{array}{ccc|c} 2 & 2 & 2 & -1 \\ 0 & 4 & -12 & 5 \\ 7 & 5 & 13 & -6 \end{array}\right) \begin{array}{l} \to (2) \\ \to (1) \\ \\ \end{array} \qquad \Delta = \begin{vmatrix} 2 & 2 & 2 \\ 0 & 4 & -12 \\ 7 & 5 & 13 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 2 & 2 & -1 \\ 0 & 4 & 5 \\ 7 & 5 & -6 \end{vmatrix} = 0$$

$c_3 = 0$

(1) $\to 4c_2 = 5 \to c_2 = \frac{5}{4}$

(2) $2c_1 = -1 - 2c_2 = -1 - \frac{5}{2}$

$c_1 = -\frac{7}{4}$

$\vec{u} = -\frac{7}{4}(2,0,7) + \frac{5}{4}(2,4,5) + 0(2,-12,13)$

b) $\vec{v} = (-3, 15, 18)$

$$\Delta_3 = \begin{vmatrix} 2 & 2 & -3 \\ 0 & 4 & 15 \\ 7 & 5 & 18 \end{vmatrix} = 288 \neq 0 \text{ (why)}$$

$\therefore$ No linear combination for the vector $\vec{v}$

c) $\vec{w} = \left(\frac{1}{3}, \frac{4}{3}, \frac{1}{2}\right)$

$$\Delta_3 = \begin{vmatrix} 2 & 2 & 1/3 \\ 0 & 4 & 4/3 \\ 7 & 5 & 1/2 \end{vmatrix} = 0 \qquad \left(\begin{array}{ccc|c} 2 & 2 & 2 & 1/3 \\ 0 & 4 & -12 & 4/3 \\ 7 & 5 & 13 & 1/2 \end{array}\right) \begin{array}{l} \to (2) \\ \to (1) \\ \\ \end{array}$$

$c_3 = 0$

(1) $\to 4c_2 = \frac{4}{3} \to c_2 = \frac{1}{3}$

(2) $\to 2c_1 = \frac{1}{3} - 2\left(\frac{1}{3}\right) = -\frac{1}{6}$

$\vec{w} = -\frac{1}{6}(2,0,7) + \frac{1}{3}(2,4,5) + 0(2,-12,13)$

d) $\vec{z} = (2, 20, -3)$

$$\Delta_3 = \begin{vmatrix} 2 & 2 & 2 \\ 0 & 4 & 20 \\ 7 & 5 & -3 \end{vmatrix} = 0 \to \left(\begin{array}{ccc|c} 2 & 2 & 2 & 2 \\ 0 & 4 & -12 & 20 \\ 7 & 5 & 13 & -3 \end{array}\right) \begin{array}{l} \to (2) \\ \to (1) \\ \\ \end{array}$$

$c_3 = 0$ (1) $\to 4c_2 = 20 \to c_2 = 5$

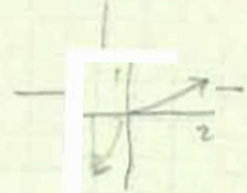
(2) $\to 2c_1 = 2 - 10 \to c_1 = -4$

$\vec{z} = -4(2,0,7) + 5(2,4,5) + 0(2,-12,13)$

9/ $S = \{(2,1), (-1,2)\}$

$\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 5 \neq 0$

S spans R^2



13/ $S = \{(-3,5)\}$

S doesn't span R^2 line

17/ $S = \{(1,3), (-2,-6), (4,12)\}$

$(-2,-6) = -2(1,3)$

$(4,12) = 4(1,3)$

S lies on a line

S doesn't span R^2

19/ $S = \{(4,7,3), (-1,2,6), (2,-3,5)\}$

$\begin{vmatrix} 4 & -1 & 2 \\ 7 & 2 & -3 \\ 3 & 6 & 5 \end{vmatrix} = 228 \neq 0$

S spans R^3