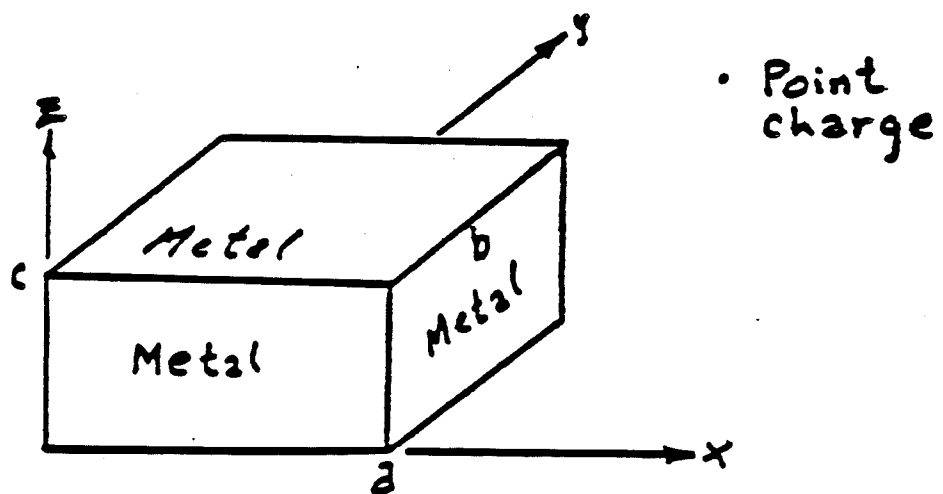


11. Potential due to a point source  
EXTERIOR to a metal box - an  
example of a non-separable  
problem.

+++++

We found the potential due to a  
point source inside of a metal box  
using separation of variables.

Can we do the corresponding ex-  
terior problem?



The boundary conditions are that  
 $\varphi(x, y, z) = \varphi_0$  on the metal. (We  
can take  $\varphi_0 = 0$  for convenience as  
before).

If we assume,  $\varphi = X(x)Y(y)Z(z)$ , then the differential equation separates over any region which excludes the source point as before.

Now let us attempt to satisfy the boundary conditions,

$$\varphi(0, y, z) = 0 \text{ for } 0 \leq y \leq b, 0 \leq z \leq c \Rightarrow \\ X(0)Y(y)Z(z) = 0.$$

Now if we take  $X(0) = 0$ , as we did in the interior problem, then

$$\varphi(0, y, z) = 0 \cdot Y(y) \cdot Z(z) = 0$$

everywhere!

Of course, this is not correct.

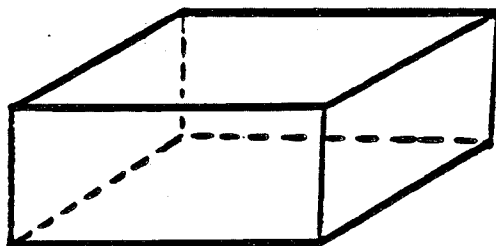
The remaining boundary conditions can similarly be shown to be impossible to satisfy when a separated solution is assumed.

Thus, though the differential equation is formally separable, the boundary conditions are not!

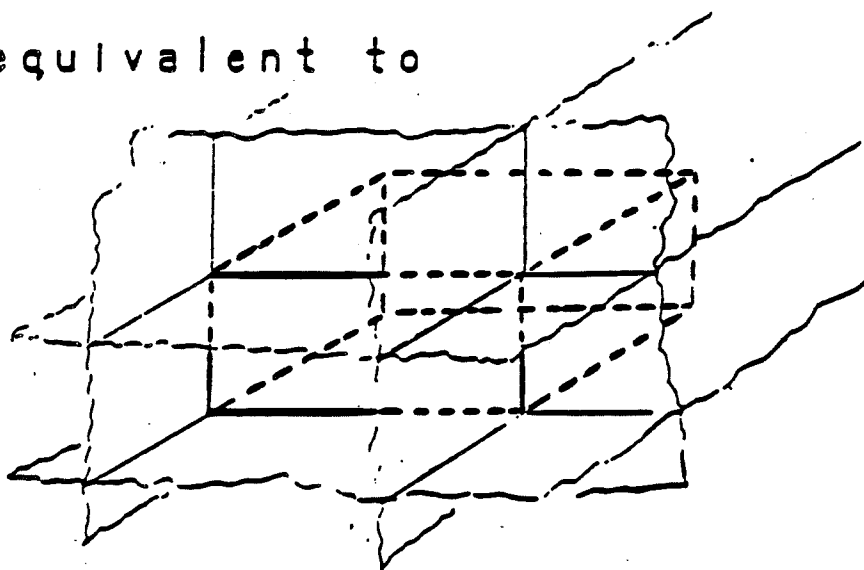
Why are the boundary conditions separable for the interior problem and not for the exterior problem?

For the interior problem, the potential at any point on the plane  $x=0$ , for example, OUTSIDE of the box has NO bearing on the solution INSIDE THE BOX.

As far as the potential interior  
of the box is concerned,



is equivalent to



(Recall the uniqueness property of  
this type of boundary value  
problem: If the potential satis-  
fies Poisson's equation everywhere  
within a volume bounded by a closed  
surface and is specified on that  
closed surface, the solution is  
unique).

Thus, the method of separation of variables does not work for every boundary value problem.

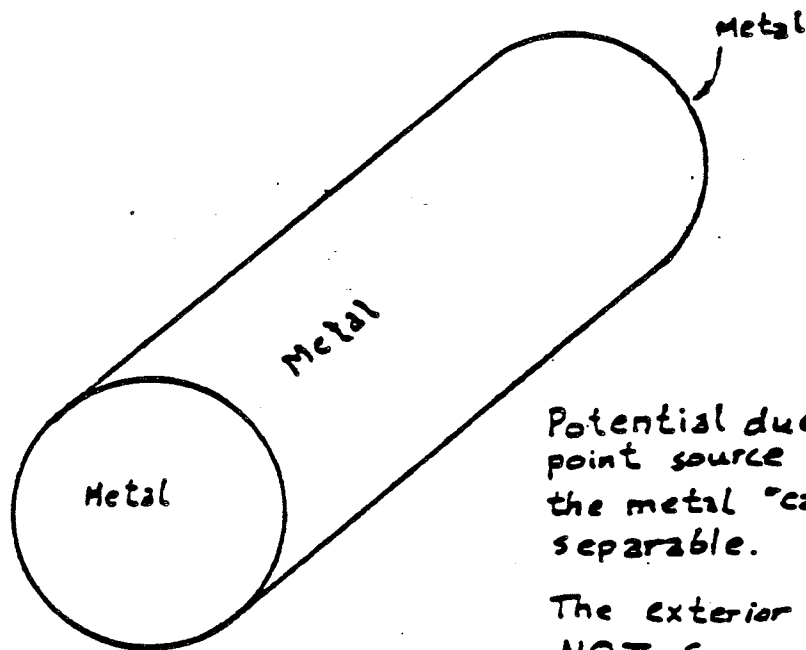
In general, the method of separation of variables will not work unless the boundaries at which the boundary conditions are specified can be described by ENTIRE CONSTANT COORDINATE SURFACES.

-OR- as in the case of the interior problem that we solved, the boundaries over which the boundary conditions are specified can be EXTENDED into complete constant coordinate surfaces without affecting the problem.

Some examples of separable problems are shown below.

(1)

11.6



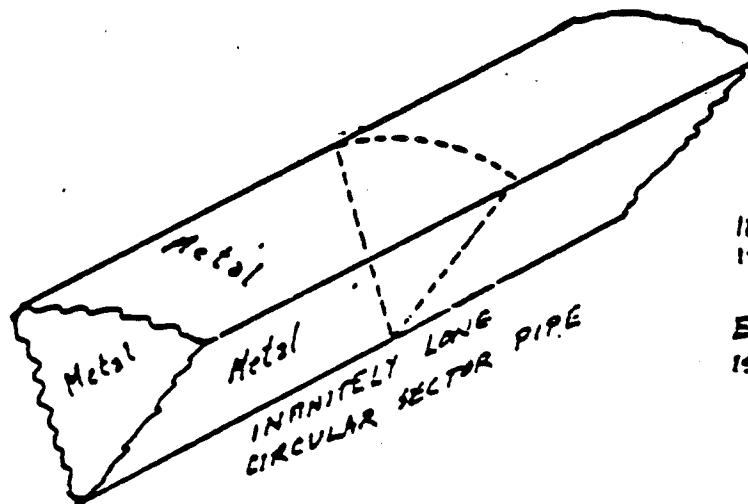
Potential due to point source inside the metal "can" is separable.

The exterior problem is NOT Separable

(2) An infinitely long circular metal pipe.

Both interior and exterior problems are separable

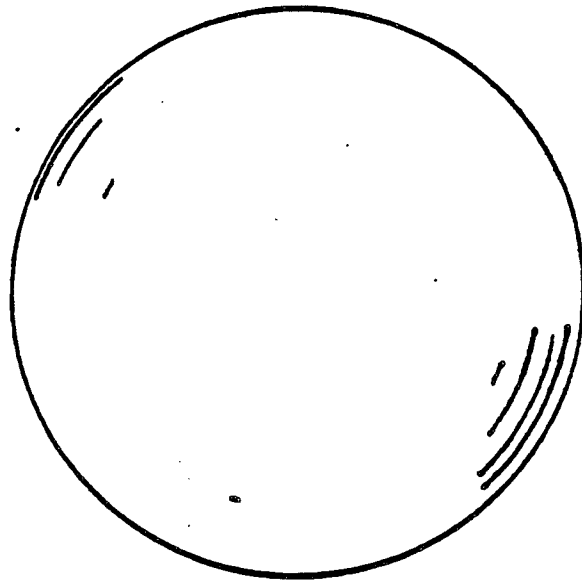
(3)



INTERIOR PROBLEM IS SEPARABLE

EXTERIOR PROBLEM IS NOT SEPARABLE.

14) Sphere: both interior and exterior problems are separable. 11.7



There are many other examples.