Solution   *Section* 7.1 – Graphing Sine & Cosine

Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 2\sin(x - \pi)$

Solution

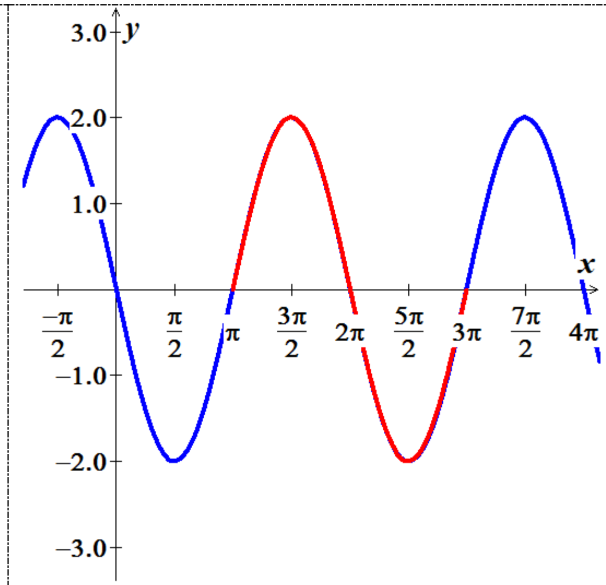
Amplitude: $|A| = 2$

Period: $P = 2\pi$

Phase Shift: $\phi = \pi$

VT: $y = 0$

	x	y
$0 + \pi$	π	0
$\frac{1}{2}\pi + \pi$	$\frac{3\pi}{2}$	2
$\frac{3}{2}\pi + \pi$	2π	0
$\frac{3}{4}\pi + \pi$	$\frac{5\pi}{2}$	-2
$2\pi + \pi$	3π	0



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = \frac{2}{3}\sin\left(x + \frac{\pi}{2}\right)$

Solution

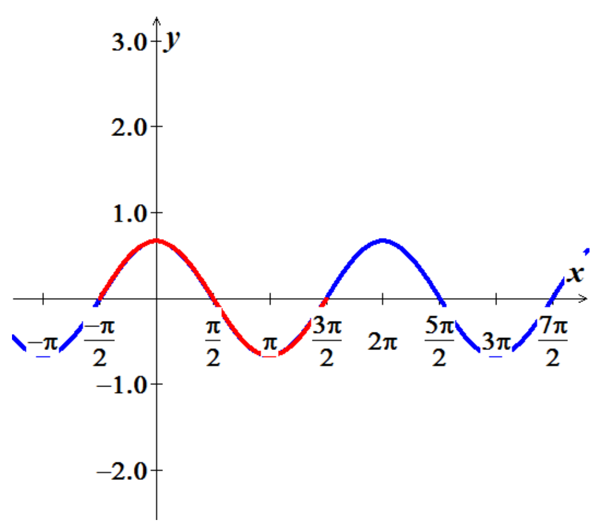
Amplitude: $|A| = \frac{2}{3}$

Period: $P = 2\pi$

Phase Shift: $\phi = -\frac{\pi}{2}$

VT: $y = 0$

	x	y
$0 - \frac{\pi}{2}$	$-\frac{\pi}{2}$	0
$\frac{1}{2}\pi - \frac{\pi}{2}$	0	$\frac{2}{3}$
$\frac{3}{2}\pi - \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{3}{4}\pi - \frac{\pi}{2}$	π	$-\frac{2}{3}$
$2\pi - \frac{\pi}{2}$	$\frac{3\pi}{2}$	0



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 4\cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

Solution

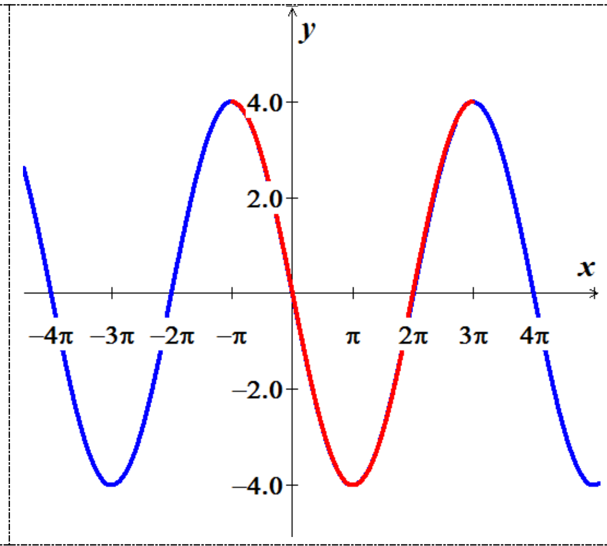
Amplitude: $|A| = 4$

Period: $P = 4\pi$

Phase Shift: $\phi = -\pi$

VT: $y = 0$

	x	y
$0-\pi$	$-\pi$	4
$\pi-\pi$	0	0
$2\pi-\pi$	π	-4
$3\pi-\pi$	2π	0
$4\pi-\pi$	3π	4



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = \frac{1}{2}\sin\left(\frac{1}{2}x + \pi\right)$

Solution

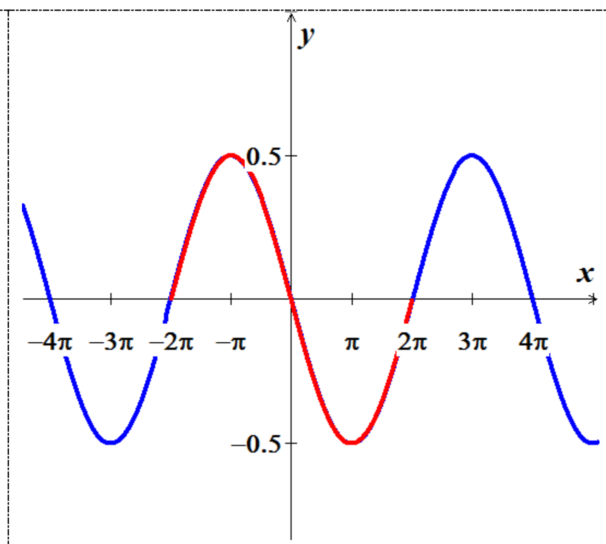
Amplitude: $|A| = \frac{1}{2}$

Period: $P = 4\pi$

Phase Shift: $\phi = -2\pi$

VT: $y = 0$

	x	y
$0-2\pi$	-2π	0
$\pi-2\pi$	$-\pi$	$\frac{1}{2}$
$2\pi-2\pi$	0	0
$3\pi-2\pi$	π	$-\frac{1}{2}$
$4\pi-2\pi$	2π	0



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 3\cos\left[\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right]$

Solution

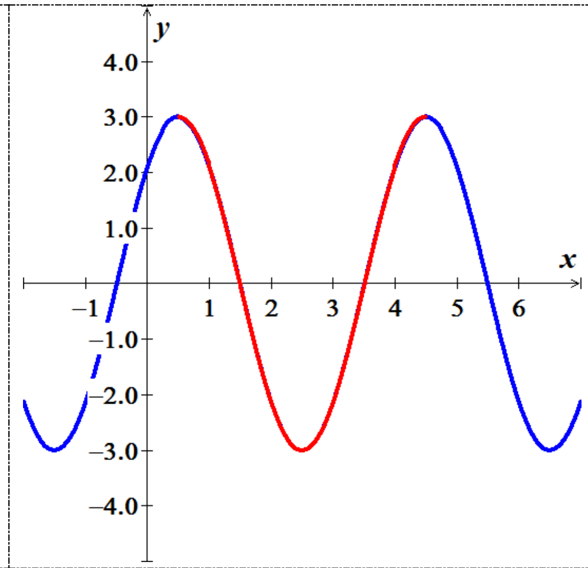
Amplitude: $|A| = 3$

Period: $P = \dfrac{2\pi}{\frac{\pi}{2}} = 4$

Phase Shift: $\varphi = \dfrac{1}{2}$

VT: $y = 0$

	x	***y***
$0 + \frac{1}{2}$	$\frac{1}{2}$	3
$1 + \frac{1}{2}$	$\frac{3}{2}$	0
$2 + \frac{1}{2}$	$\frac{5}{2}$	-3
$3 + \frac{1}{2}$	$\frac{7}{2}$	0
$4 + \frac{1}{2}$	$\frac{9}{2}$	3



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = -\cos \pi\left(x - \frac{1}{3}\right)$

Solution

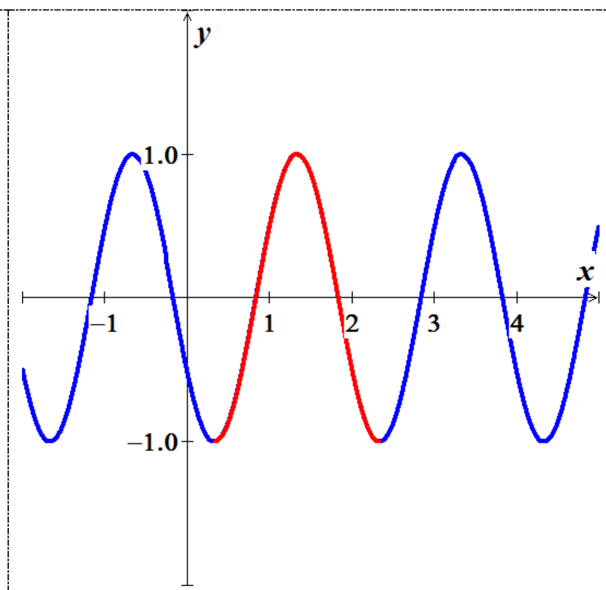
Amplitude: $|A| = 1$

Period: $P = 2$

Phase Shift: $\phi = \dfrac{1}{3}$

VT: $y = 0$

	x	***y***
$0 + \frac{1}{3}$	$\frac{1}{3}$	-1
$\frac{1}{2} + \frac{1}{3}$	$\frac{5}{6}$	0
$1 + \frac{1}{3}$	$\frac{4}{3}$	1
$\frac{3}{2} + \frac{1}{3}$	$\frac{11}{6}$	0
$2 + \frac{1}{3}$	$\frac{7}{3}$	-1



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 2 - \sin\left(3x - \frac{\pi}{5}\right)$

Solution

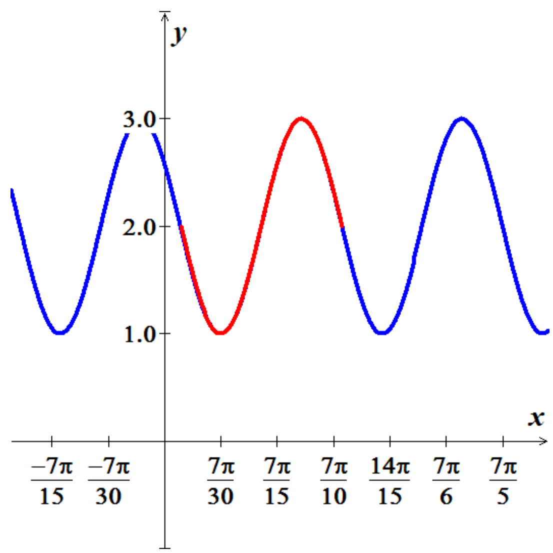
Amplitude: $|A| = 1$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\phi = \frac{\pi}{15}$

VT: $y = 2$

	x	y
$0 + \frac{\pi}{15}$	$\frac{\pi}{15}$	2
$\frac{\pi}{6} + \frac{\pi}{15}$	$\frac{7\pi}{30}$	1
$\frac{\pi}{3} + \frac{\pi}{15}$	$\frac{6\pi}{15}$	2
$\frac{\pi}{2} + \frac{\pi}{15}$	$\frac{17\pi}{30}$	3
$\frac{2\pi}{3} + \frac{\pi}{15}$	$\frac{11\pi}{15}$	2



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = -\frac{2}{3}\sin\left(3x - \frac{\pi}{2}\right)$

Solution

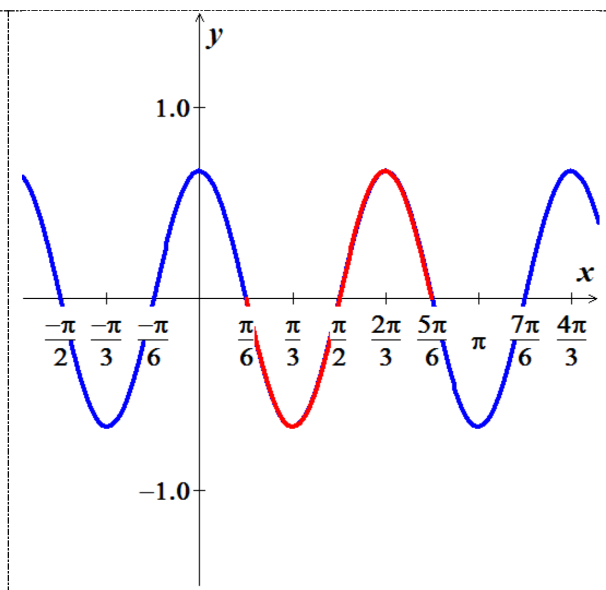
Amplitude: $|A| = \frac{2}{3}$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\phi = \frac{\pi}{6}$

VT: $y = 0$

	x	y
$0 + \frac{\pi}{6}$	$\frac{\pi}{6}$	0
$\frac{\pi}{6} + \frac{\pi}{6}$	$\frac{\pi}{3}$	$-\frac{2}{3}$
$\frac{\pi}{3} + \frac{\pi}{6}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2} + \frac{\pi}{6}$	$\frac{2\pi}{3}$	$\frac{2}{3}$
$\frac{2\pi}{3} + \frac{\pi}{6}$	$\frac{5\pi}{6}$	0



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y=-1+\frac{1}{2}\cos(2x-3\pi)$

Solution

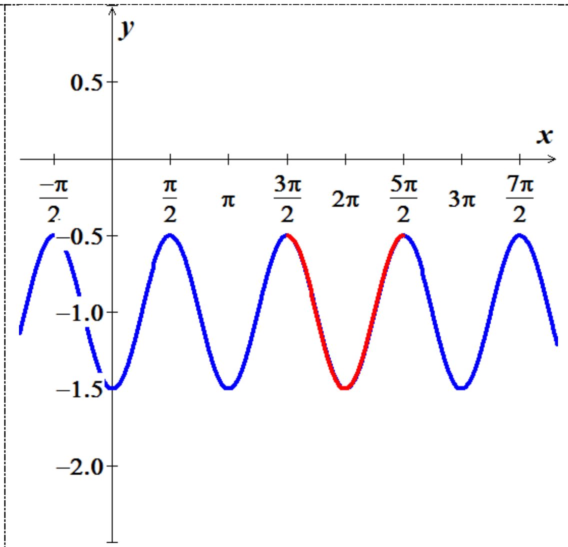
Amplitude: $|A|=\frac{1}{2}$

Period: $P=\pi$

Phase Shift: $\phi=\frac{3\pi}{2}$

VT: $y=-1$

	x	y
$0+\frac{3\pi}{2}$	$\frac{3\pi}{2}$	$-\frac{1}{2}$
$\frac{\pi}{4}+\frac{3\pi}{2}$	$\frac{7\pi}{4}$	-1
$\frac{\pi}{2}+\frac{3\pi}{2}$	2π	$-\frac{3}{2}$
$\frac{3\pi}{4}+\frac{3\pi}{2}$	$\frac{9\pi}{4}$	-1
$\pi+\frac{3\pi}{2}$	$\frac{5\pi}{2}$	$-\frac{1}{2}$



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y=2-\frac{1}{3}\cos\left(\pi x+\frac{3\pi}{2}\right)$

Solution

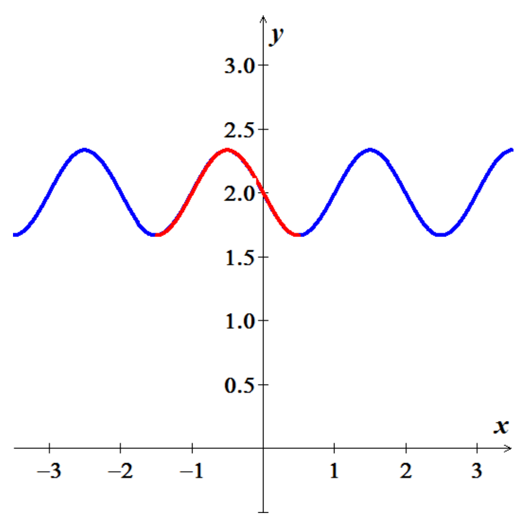
Amplitude: $|A|=\frac{1}{3}$

Period: $P=\frac{2\pi}{\pi}=2$

Phase Shift: $\phi=-\frac{\frac{3\pi}{2}}{\pi}=-\frac{3}{2}$

VT: $y=2$

	x	y
$0-\frac{3}{2}$	$-\frac{3}{2}$	$\frac{5}{3}$
$\frac{1}{2}-\frac{3}{2}$	-1	2
$1-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{7}{3}$
$\frac{3}{2}-\frac{3}{2}$	0	2
$2-\frac{3}{2}$	$\frac{1}{2}$	$\frac{5}{3}$



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = \frac{5}{2} - 3\cos\left(\pi x - \frac{\pi}{4}\right)$

Solution

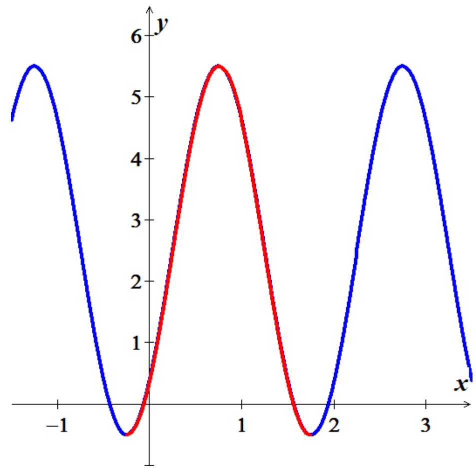
Amplitude: $|A| = 3$

Period: $P = \frac{2\pi}{\pi} = 2$

Phase Shift: $\phi = -\frac{-\frac{\pi}{4}}{\pi} = \frac{1}{4}$

VT: $y = \frac{5}{2}$

	x	y
$0 - \frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$
$\frac{1}{2} - \frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{2}$
$1 - \frac{1}{4}$	$\frac{3}{4}$	$\frac{11}{2}$
$\frac{3}{2} - \frac{1}{4}$	$\frac{5}{4}$	$\frac{5}{2}$
$2 - \frac{1}{4}$	$\frac{7}{4}$	$-\frac{1}{2}$



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = \frac{2}{3} - \frac{4}{3}\cos(3x - \pi)$

Solution

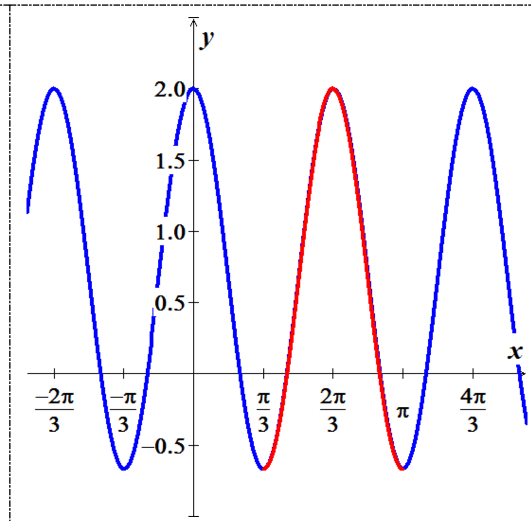
Amplitude: $|A| = \frac{4}{3}$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\phi = \frac{\pi}{3}$

VT: $y = \frac{2}{3}$

	x	y
$0 + \frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{2}{3} - \frac{4}{3} = -\frac{2}{3}$
$\frac{\pi}{6} + \frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}$
$\frac{\pi}{3} + \frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{2}{3} + \frac{4}{3} = 2$
$\frac{\pi}{2} + \frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{2}{3}$
$\frac{2\pi}{3} + \frac{\pi}{3}$	π	$-\frac{2}{3}$



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = \sin\left(x - \frac{\pi}{2}\right)$

Solution

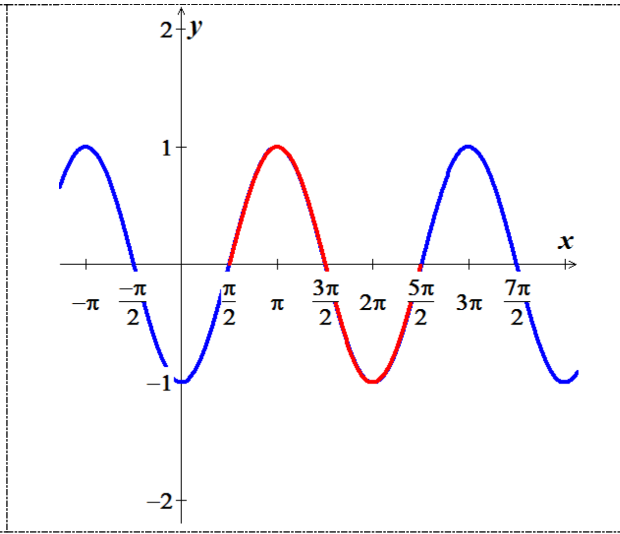
Amplitude: $|A| = 1$

Period: $P = 2\pi$

Phase Shift: $\phi = \frac{\pi}{2}$

VT: $y = 0$

	x	y
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2} + \frac{\pi}{2}$	π	1
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$\frac{3\pi}{2} + \frac{\pi}{2}$	2π	-1
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	0



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 2\sin\left(x - \frac{\pi}{3}\right)$

Solution

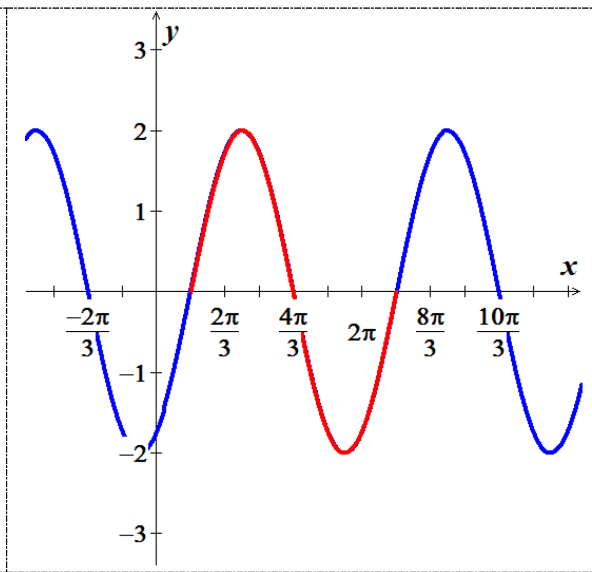
Amplitude: $|A| = 2$

Period: $P = 2\pi$

Phase Shift: $\phi = \frac{\pi}{3}$

VT: $y = 0$

	x	y
$0 + \frac{\pi}{3}$	$\frac{\pi}{3}$	0
$\frac{\pi}{2} + \frac{\pi}{3}$	$\frac{5\pi}{6}$	2
$\pi + \frac{\pi}{3}$	$\frac{4\pi}{3}$	0
$\frac{3\pi}{2} + \frac{\pi}{3}$	$\frac{11\pi}{6}$	-2
$2\pi + \frac{\pi}{3}$	$\frac{7\pi}{3}$	0



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 4\cos\left(x - \frac{\pi}{4}\right)$

Solution

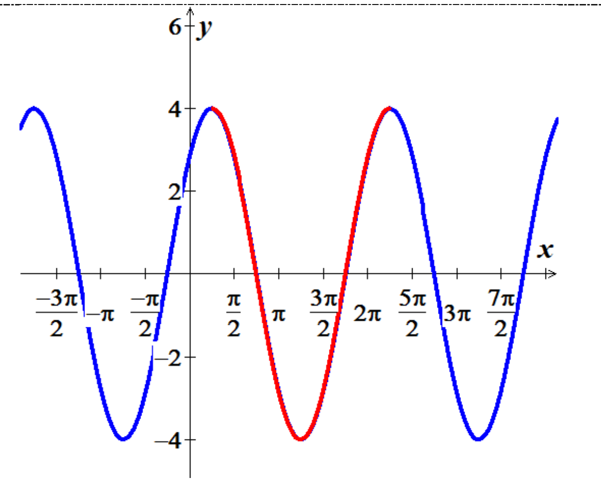
Amplitude: $|A| = 4$

Period: $P = 2\pi$

Phase Shift: $\phi = \frac{\pi}{4}$

VT: $y = 0$

	x	y
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	4
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	0
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	−4
$\frac{3\pi}{2} + \frac{\pi}{4}$	$\frac{7\pi}{4}$	0
$2\pi + \frac{\pi}{4}$	$\frac{9\pi}{4}$	4



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$y = -\sin(3x + \pi) - 1$

Solution

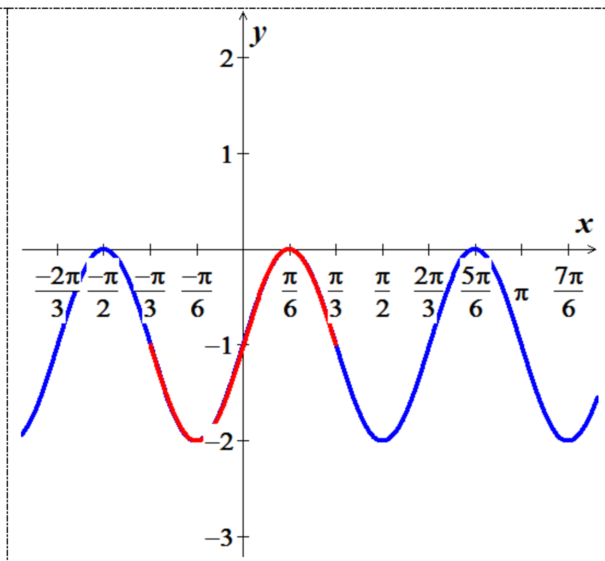
Amplitude: $|A| = 1$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\phi = -\frac{\pi}{3}$

VT: $y = -1$

	x	y
$0 - \frac{\pi}{3}$	$-\frac{\pi}{3}$	−1
$\frac{\pi}{6} - \frac{\pi}{3}$	$-\frac{\pi}{6}$	−2
$\frac{\pi}{3} - \frac{\pi}{3}$	0	−1
$\frac{\pi}{2} - \frac{\pi}{3}$	$\frac{\pi}{6}$	0
$\frac{2\pi}{3} - \frac{\pi}{3}$	$\frac{\pi}{3}$	−1



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = \cos(2x - \pi) + 2$

Solution

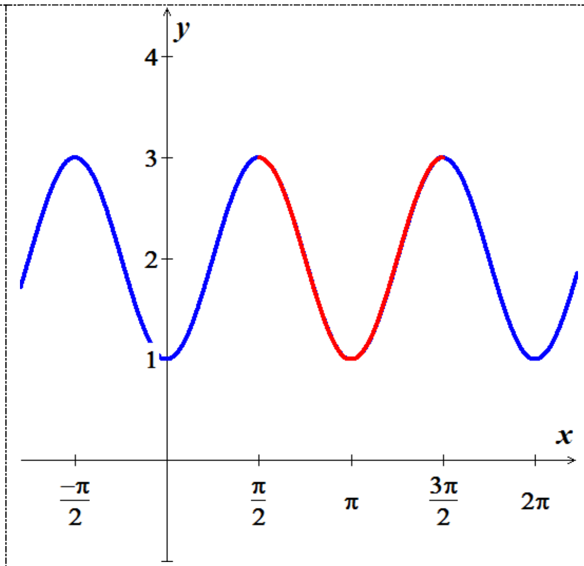
Amplitude: $|A| = 1$

Period: $P = \pi$

Phase Shift: $\phi = \frac{\pi}{2}$

VT: $y = 2$

	x	***y***
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	-1
$\frac{\pi}{4} + \frac{\pi}{2}$	$\frac{3\pi}{4}$	-2
$\frac{\pi}{2} + \frac{\pi}{2}$	π	-1
$\frac{3\pi}{4} + \frac{\pi}{2}$	$\frac{5\pi}{4}$	0
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	-1



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = \sin\left(\frac{1}{2}x - \frac{\pi}{3}\right)$

Solution

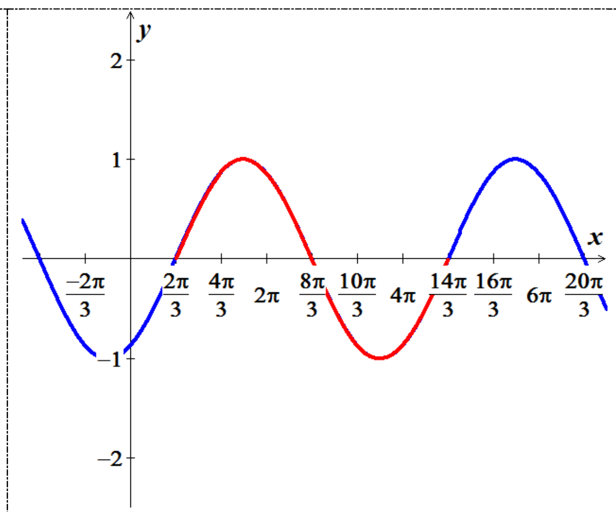
Amplitude: $|A| = 1$

Period: $P = 4\pi$

Phase Shift: $\phi = \frac{2\pi}{3}$

VT: $y = 0$

	x	***y***
$0 + \frac{2\pi}{3}$	$\frac{2\pi}{3}$	0
$\pi + \frac{2\pi}{3}$	$\frac{5\pi}{3}$	1
$2\pi + \frac{2\pi}{3}$	$\frac{8\pi}{3}$	0
$3\pi + \frac{2\pi}{3}$	$\frac{11\pi}{3}$	-1
$4\pi + \frac{2\pi}{3}$	$\frac{14\pi}{3}$	0



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 5\sin\left(3x - \frac{\pi}{2}\right)$

Solution

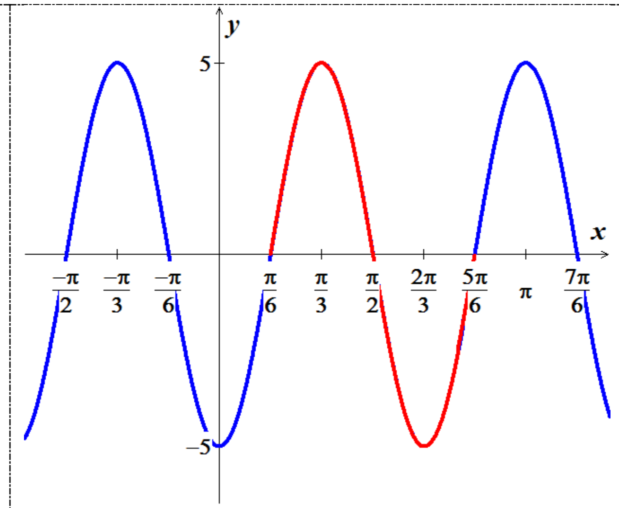
Amplitude: $|A| = 5$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\varphi = \frac{\pi}{6}$

VT: $y = 0$

	x	y
$0 + \frac{\pi}{6}$	$\frac{\pi}{6}$	0
$\frac{\pi}{6} + \frac{\pi}{6}$	$\frac{\pi}{3}$	5
$\frac{\pi}{3} + \frac{\pi}{6}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2} + \frac{\pi}{6}$	$\frac{2\pi}{3}$	-5
$\frac{2\pi}{3} + \frac{\pi}{6}$	$\frac{5\pi}{6}$	0



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 3\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

Solution

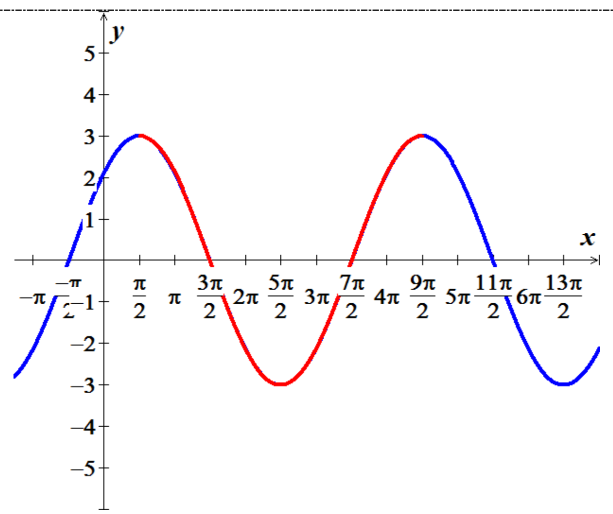
Amplitude: $|A| = 3$

Period: $P = 4\pi$

Phase Shift: $\varphi = \frac{\pi}{2}$

VT: $y = 0$

	x	y
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	3
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	-3
$3\pi + \frac{\pi}{2}$	$\frac{7\pi}{2}$	0
$4\pi + \frac{\pi}{2}$	$\frac{9\pi}{2}$	3



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$y = -5\cos\left(\frac{1}{3}x + \frac{\pi}{6}\right)$

Solution

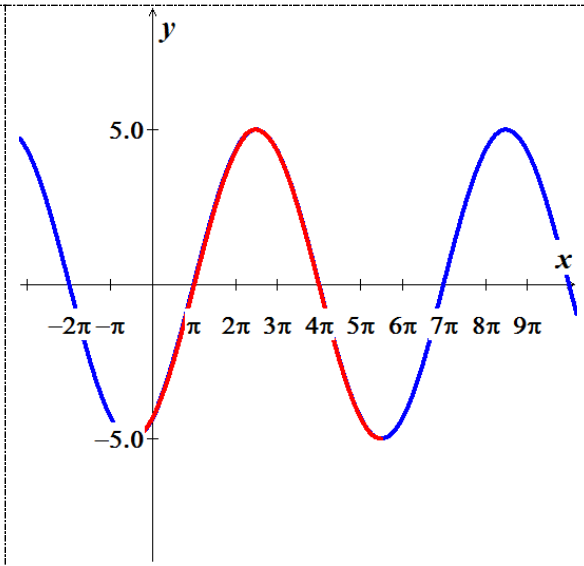
Amplitude: $|A| = 5$

Period: $P = \frac{2\pi}{1/3} = 6\pi$

Phase Shift: $\varphi = -\frac{\pi}{2}$

VT: $y = 0$

	x	y
$0 - \frac{\pi}{2}$	$-\frac{\pi}{2}$	-5
$\frac{3\pi}{2} - \frac{\pi}{2}$	π	0
$3\pi - \frac{\pi}{2}$	$\frac{5\pi}{2}$	5
$\frac{9\pi}{2} - \frac{\pi}{2}$	4π	0
$6\pi - \frac{\pi}{2}$	$\frac{11\pi}{2}$	-5



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$y = -2\sin(2\pi x + \pi)$

Solution

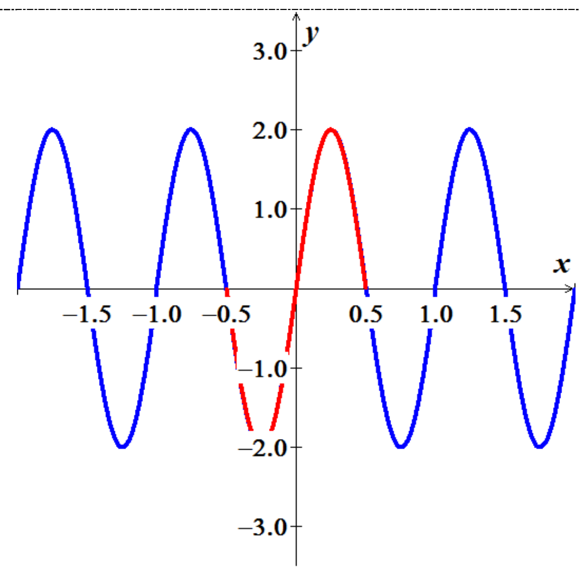
Amplitude: $|A| = 2$

Period: $P = \frac{2\pi}{2\pi} = 1$

Phase Shift: $\varphi = -\frac{1}{2}$

VT: $y = 0$

	x	y
$0 - \frac{1}{2}$	$-\frac{1}{2}$	0
$\frac{1}{4} - \frac{1}{2}$	$-\frac{1}{4}$	-2
$\frac{1}{2} - \frac{1}{2}$	0	0
$\frac{3}{4} - \frac{1}{2}$	$\frac{1}{4}$	2
$1 - \frac{1}{2}$	$\frac{1}{2}$	0



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$y = -2\sin(2x - \pi) + 3$

Solution

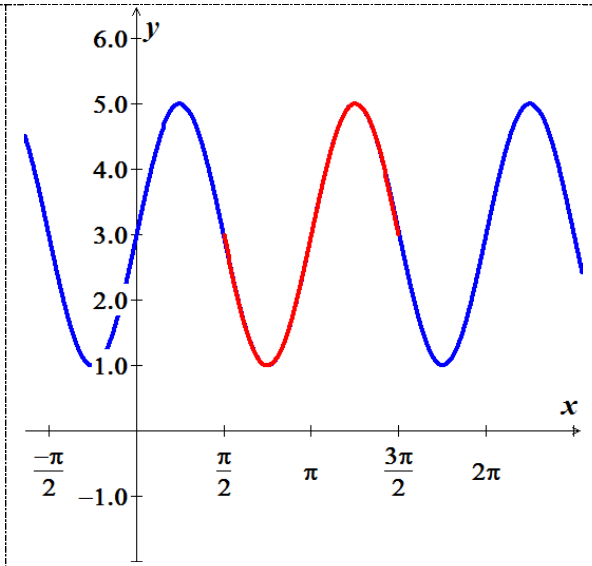
Amplitude: $|A| = 2$

Period: $P = \frac{2\pi}{2} = \pi$

Phase Shift: $\varphi = \frac{\pi}{2}$

VT: $y = 3$

	x	y
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	3
$\frac{\pi}{4} + \frac{\pi}{2}$	$\frac{3\pi}{4}$	1
$\frac{\pi}{2} + \frac{\pi}{2}$	π	3
$\frac{3\pi}{4} + \frac{\pi}{2}$	$\frac{5\pi}{4}$	5
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	3



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$y = 3\cos(x + 3\pi) - 2$

Solution

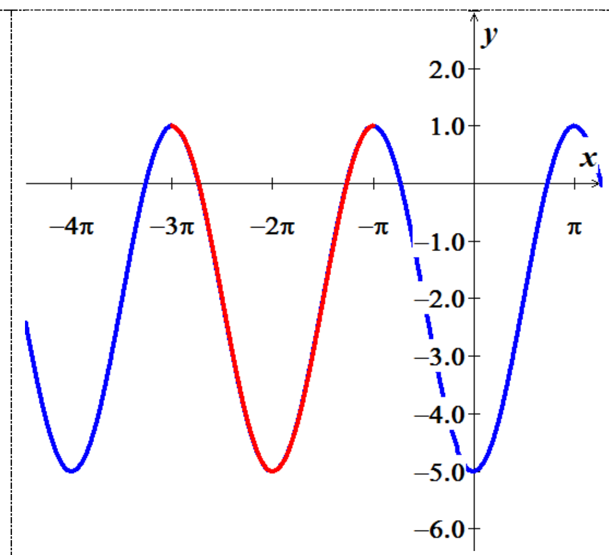
Amplitude: $|A| = 3$

Period: $P = 2\pi$

Phase Shift: $\varphi = -3\pi$

VT: $y = -2$

	x	y
$0 - 3\pi$	-3π	1
$\frac{\pi}{2} - 3\pi$	$-\frac{5\pi}{2}$	0
$\pi - 3\pi$	-2π	-5
$\frac{3\pi}{2} - 3\pi$	$-\frac{3\pi}{2}$	0
$2\pi - 3\pi$	$-\pi$	1



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$y = 5\cos(2x + 2\pi) + 2$

Solution

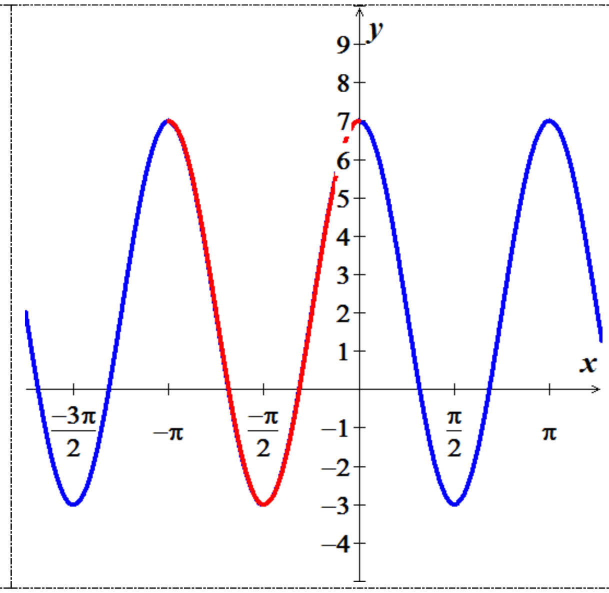
Amplitude: $|A| = 5$

Period: $P = \pi$

Phase Shift: $\varphi = -\pi$

VT: $y = 2$

	x	y
$0 - \pi$	$-\pi$	7
$\frac{\pi}{4} - \pi$	$-\frac{3\pi}{4}$	2
$\frac{\pi}{2} - \pi$	$-\frac{\pi}{2}$	-3
$\frac{3\pi}{4} - \pi$	$-\frac{\pi}{2}$	2
$\pi - \pi$	0	7



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$y = -4\sin(3x - \pi) - 3$

Solution

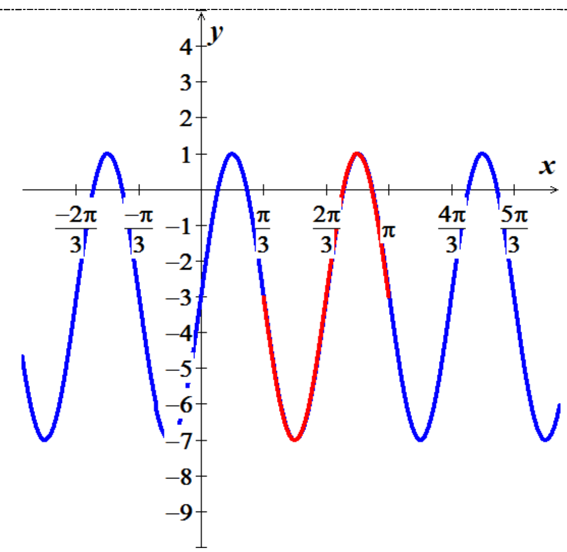
Amplitude: $|A| = 4$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\varphi = \frac{\pi}{3}$

VT: $y = -3$

	x	y
$0 + \frac{\pi}{3}$	$\frac{\pi}{3}$	-3
$\frac{\pi}{6} + \frac{\pi}{3}$	$\frac{\pi}{2}$	-7
$\frac{\pi}{3} + \frac{\pi}{3}$	$\frac{2\pi}{3}$	-3
$\frac{\pi}{2} + \frac{\pi}{3}$	$\frac{5\pi}{6}$	1
$\frac{2\pi}{3} + \frac{\pi}{3}$	π	-3



Exercise

Find the amplitude, the period, any vertical translation, and any phase shift. Then graph a one complete cycle of $y = \cos\frac{1}{2}x$

Solution

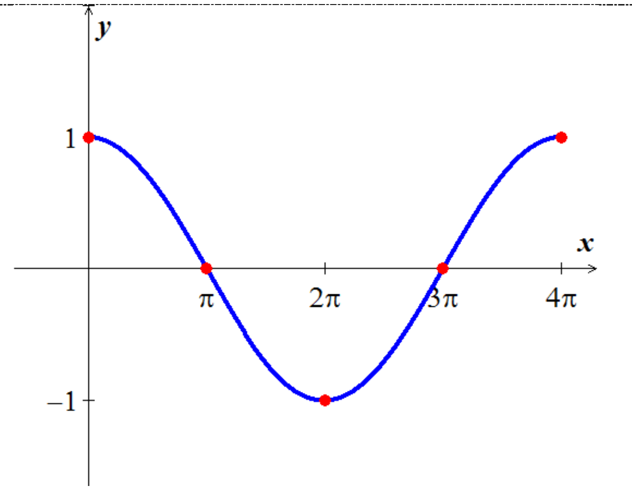
Amplitude: $|A| = 1$

Period: $P = 4\pi$

Phase Shift: $\varphi = 0$

VT: $y = 0$

x	y
0	1
π	0
2π	-1
3π	0
4π	1



Exercise

Find the amplitude, the period, any vertical translation, and any phase shift. Then graph
$y = 2\sin(-\pi\, x)$ *for* $-3 \le x \le 3$

Solution

$y = 2\sin(-\pi\, x) = -2\sin(\pi\, x)$

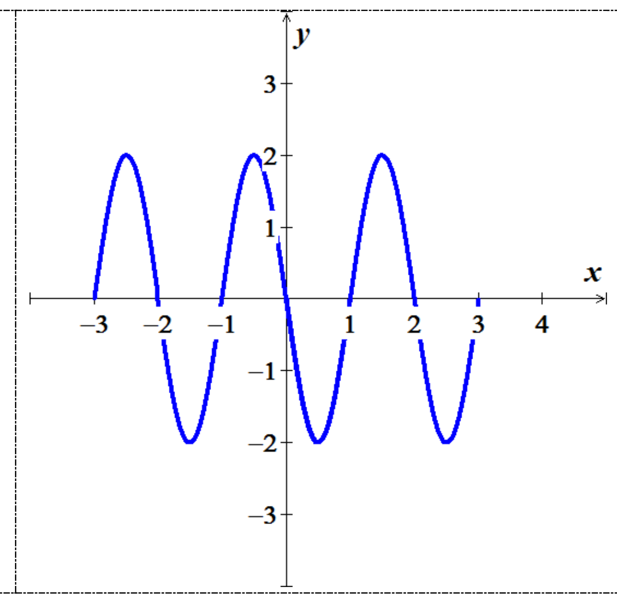
Amplitude: $|A| = 2$

Period: $P = 2$

Phase Shift: $\varphi = 0$

VT: $y = 0$

x	y
0	0
$\frac{1}{2}$	-2
1	0
$\frac{3}{2}$	2
2	



Exercise

Find the amplitude, the period, any vertical translation, and any phase shift. Then graph

$y = 4\cos\left(-\frac{2}{3}x\right) \quad for \quad -\frac{15\pi}{4} \le x \le \frac{15\pi}{4}$

Solution

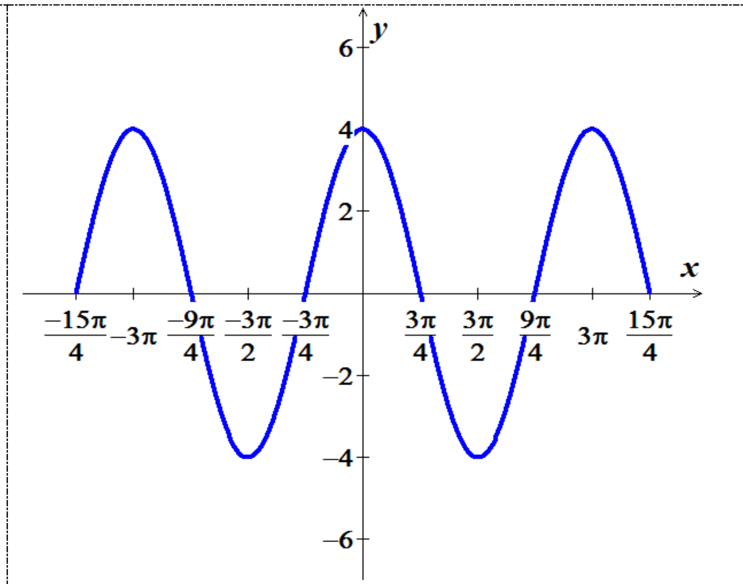
Amplitude: $|A| = 4$

Period: $P = 3\pi$

Phase Shift: $\varphi = 0$

VT: $y = 0$

x	y
0	4
$\frac{3\pi}{4}$	0
$\frac{3\pi}{2}$	-4
$\frac{9\pi}{4}$	0
3π	4



Exercise

Graph one complete cycle $y = \cos\left(x - \frac{\pi}{6}\right)$

Solution

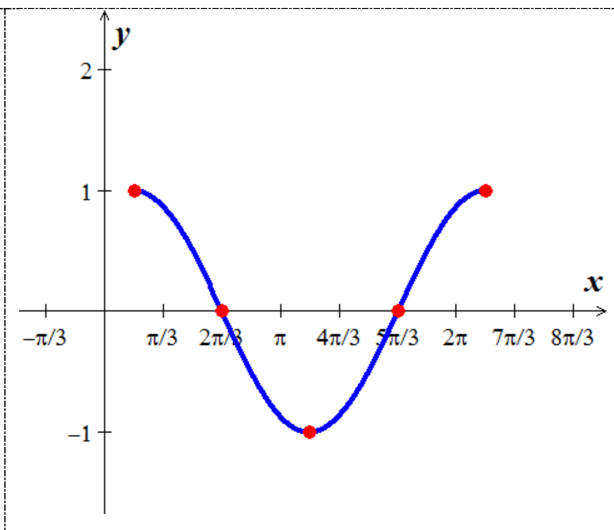
Amplitude: $|A| = 1$

Period: $P = 2\pi$

Phase Shift: $\varphi = \frac{\pi}{6}$

VT: $y = 0$

	x	y
$\frac{\pi}{6} + 0$	$\frac{\pi}{6}$	1
$\frac{\pi}{6} + \frac{1}{2}\pi$	$\frac{2\pi}{3}$	0
$\frac{\pi}{6} + \pi$	$\frac{7\pi}{6}$	-1
$\frac{\pi}{6} + \frac{3}{2}\pi$	$\frac{5\pi}{3}$	0
$\frac{\pi}{6} + 2\pi$	$\frac{13\pi}{6}$	1



Exercise

Graph one complete cycle $y = \frac{2}{3} - \frac{4}{3}\cos(3x - \pi)$

Solution

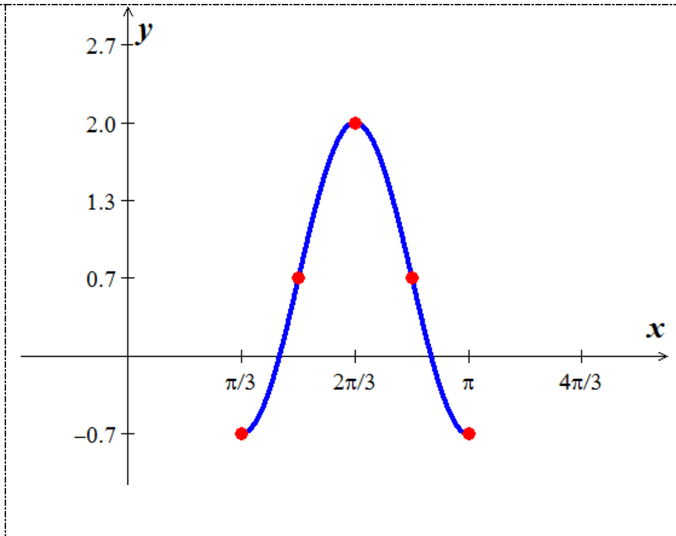
Amplitude: $|A| = \frac{4}{3}$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\varphi = \frac{\pi}{3}$

VT: $y = \frac{2}{3}$

	x	***y***
$0 + \frac{\pi}{3}$	$\frac{\pi}{3}$	$-\frac{2}{3}$
$\frac{\pi}{6} + \frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}$
$\frac{\pi}{3} + \frac{\pi}{3}$	$\frac{2\pi}{3}$	2
$\frac{\pi}{2} + \frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{2}{3}$
$\frac{2\pi}{3} + \frac{\pi}{3}$	π	$-\frac{2}{3}$



Exercise

Graph one complete cycle $y = -3 + \sin\left(\pi x + \frac{\pi}{2}\right)$

Solution

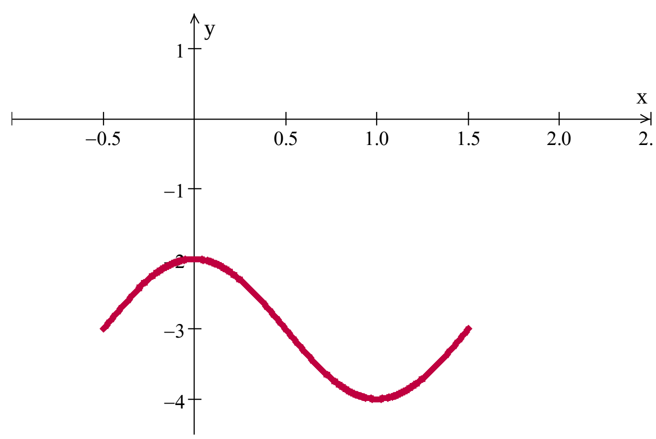
Amplitude: $|A| = 1$

Period: $P = 2$

Phase Shift: $\varphi = -\frac{1}{2}$

VT: $y = -3$

	x	***y***
$0 - \frac{1}{2}$	$-\frac{1}{2}$	-3
$\frac{1}{2} - \frac{1}{2}$	0	-2
$1 - \frac{1}{2}$	$\frac{1}{2}$	-3
$\frac{3}{2} - \frac{1}{2}$	1	-4
$2 - \frac{1}{2}$	$\frac{3}{2}$	-3



Exercise

Graph $y = -1 + 2\sin(4x + \pi)$ over two periods.

Solution

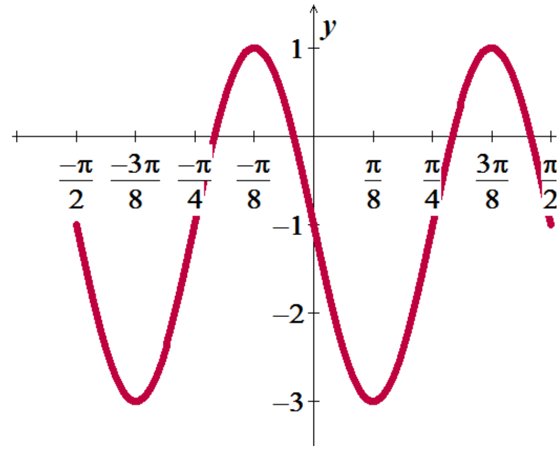
Amplitude: $|A| = 2$

Period: $P = \frac{\pi}{2}$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = -1$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	-1
$\frac{\pi}{8} - \frac{\pi}{4}$	$-\frac{\pi}{8}$	1
$\frac{\pi}{4} - \frac{\pi}{4}$	0	-1
$\frac{3\pi}{8} - \frac{\pi}{4}$	$\frac{\pi}{8}$	-3
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	-1



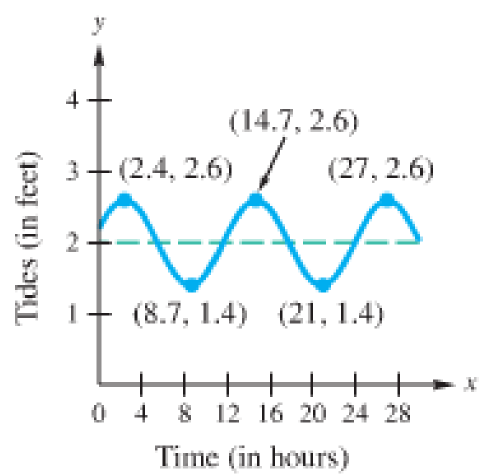
Exercise

The figure shows a function f that models the tides in feet at Clearwater Beach, x hours after midnight starting on Aug. 26,

a) Find the time between high tides.

b) What is the difference in water levels between high tide and low tide?

c) The tides can be modeled by $f(x) = 0.6\cos[0.511x - 2.4] + 2$. Estimate the tides when $x = 10$.



Solution

a) Time between high tides = 14.7 – 2.4

$= 12.3 \ \ hrs$

b) Difference in water levels between high tide and low tide = 2.6 – 1.4

$= 1.2 \ \ ft$

c) $f(x=10) = 0.6\cos\left[0.511(10) - 2.4\right]_{rad} + 2$

$\underline{\approx 1.45\ ft}$

Exercise

The maximum afternoon temperature in a given city might be modeled by $t = 60 - 30\cos\frac{\pi x}{6}$

Where t represents the maximum afternoon temperature in month x, with $x = 0$ representing January, $x = 1$ representing February, and so on.. Find the maximum afternoon temperature to the nearest degree for each month.

a) Jan. *b*) Apr. *c*) May. *d*) Jun. *e*) Oct.

Solution

a) Jan. $t = 60 - 30\cos\frac{\pi(0)}{6} = 30^\circ$

b) Apr. $t = 60 - 30\cos\frac{\pi(4)}{6} = 75^\circ$

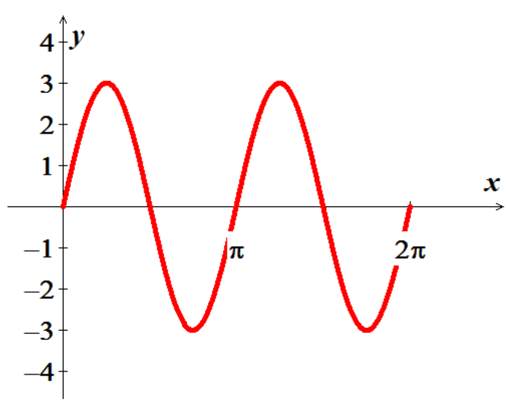
c) May. $t = 60 - 30\cos\frac{\pi(5)}{6} = 86^\circ$

d) Jun. $t = 60 - 30\cos\frac{\pi(6)}{6} = 90^\circ$

e) Oct. $t = 60 - 30\cos\frac{\pi(10)}{6} = 45^\circ$

Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



Solution

$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$

Amplitude $= 3$

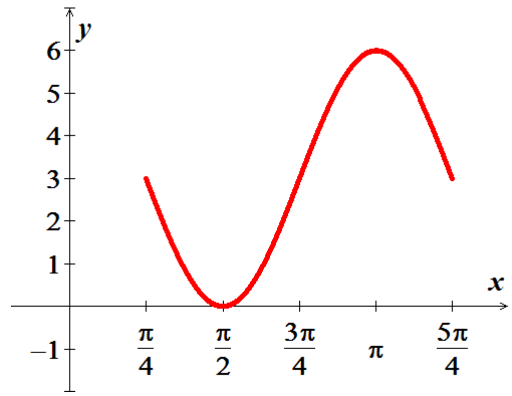
Phase shift: $\phi = 0$

$\underline{y = 3\sin 2x}$ $\quad 0 \le x \le 2\pi$

Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph

Solution



$$P = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$$

$$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$$

Amplitude = 3

$$A = -3$$

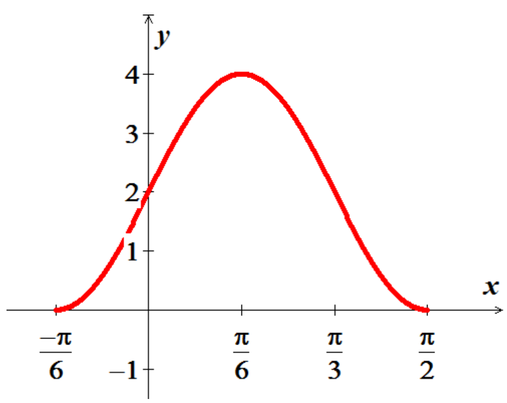
Phase shift: $\phi = \frac{\pi}{4} = -\frac{C}{B}$

$$C = -\frac{\pi}{2}$$

$y = -3\sin\left(2x - \frac{\pi}{2}\right)$ $\quad \frac{\pi}{4} \le x \le \frac{5\pi}{4}$ $\quad \frac{\pi}{4} \le x \le \frac{5\pi}{4}$

Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



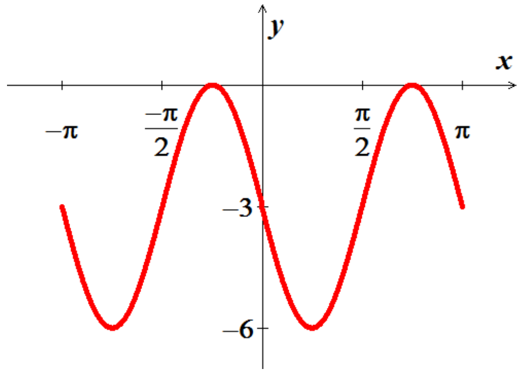
Solution

$P = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$	$B = \frac{2\pi}{P} = \frac{2\pi}{\frac{2\pi}{3}} = 3$	$D = 2$
$\phi = -\frac{\pi}{6} = -\frac{C}{B} \Rightarrow C = \frac{\pi B}{6} = \frac{\pi}{2}$	***Amplitude*** = 2 $\rightarrow \boxed{A = -2}$	

$y = 2 - 2\cos\left(3x + \frac{\pi}{2}\right)$ $\quad -\frac{\pi}{6} \le x \le \frac{\pi}{2}$

Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



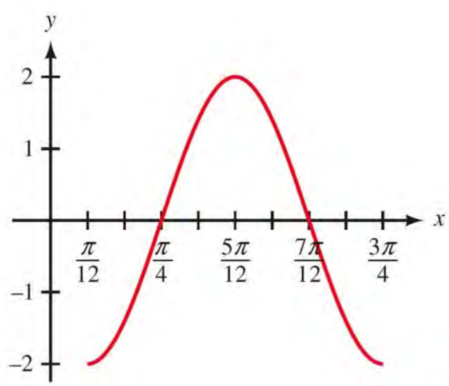
Solution

$P = \pi$	$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$	$D = -3$
$\phi = 0$	***Amplitude*** $= 3$ $\boxed{A = -3}$	

$\underline{y = -3 - 3\sin 2x}\,\big|$ $\quad -\pi \le x \le \pi$

Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



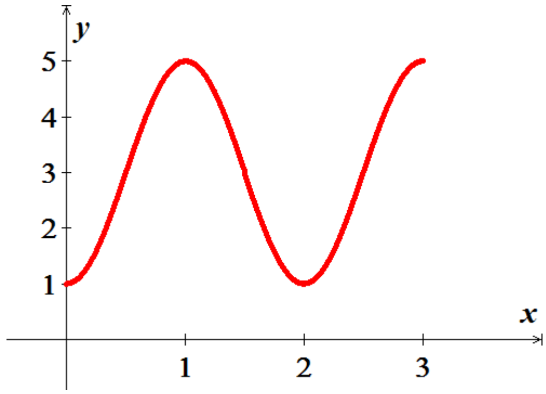
Solution

$P = \frac{3\pi}{4} - \frac{\pi}{12} = \frac{2\pi}{3}$	$B = \frac{2\pi}{P} = \frac{2\pi}{\frac{2\pi}{3}} = 3$	$D = 0$
$\phi = \frac{\pi}{12} \Rightarrow C = -B\phi = -3\frac{\pi}{12} = -\frac{\pi}{4}$	***Amplitude*** $= 2$ $\boxed{A = -2}$	

$\big|\underline{y = -2\cos\left(3x - \frac{\pi}{4}\right)}$ $\quad \frac{\pi}{12} \le x \le \frac{3\pi}{4}$

Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



Solution

Amplitude = 2 $\boxed{A = -2}$

$P = 2$

$B = \frac{2\pi}{P}$

$= \frac{2\pi}{2}$

$= \pi$

$\phi = 0$

$D = 3$

$y = 3 - 2\cos(\pi x)$ $0 \le x \le 3$

Exercise

The diameter of the Ferris wheel is 250 *feet*, the distance from the ground to the bottom of the wheel is 14 *feet*. We found the height of a rider on that Ferris wheel was given by the function:

$$H = 139 - 125\cos\left(\frac{\pi}{10}t\right)$$

Where t is the number of minutes from the beginning of a ride. Graph a complete cycle of this function.

Solution

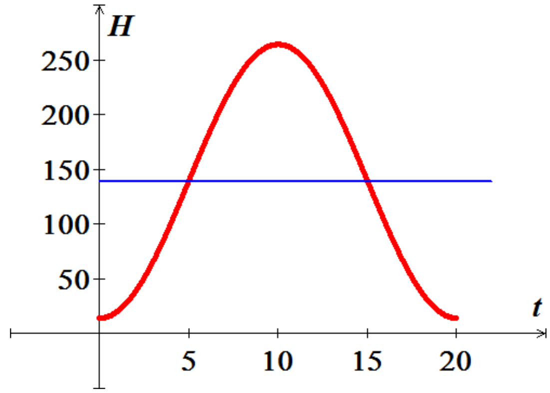
Amplitude: $A = 125$

Period: $P = \frac{2\pi}{\frac{\pi}{10}} = 20$

Phase Shift: $\phi = 0$

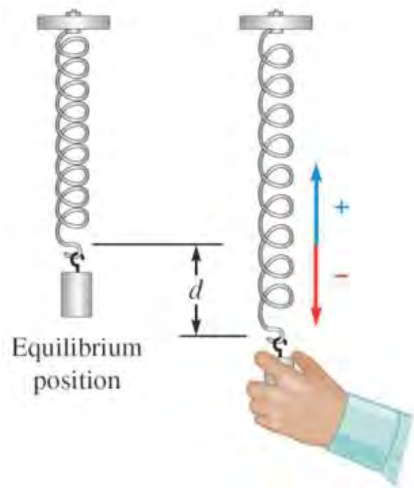
VT: $H = 139$

t	$H = 139 - 125\cos\left(\frac{\pi}{10}t\right)$
0	139-125=14
5	139
10	139+125=264
15	139
20	14



Exercise

A mass attached to a spring oscillates upward and downward. The length L of the spring after t seconds is given by the function $L = 15 - 3.5\cos(2\pi t)$, where L is measured in *cm*.



a) Sketch the graph of this function for $0 \le t \le 5$

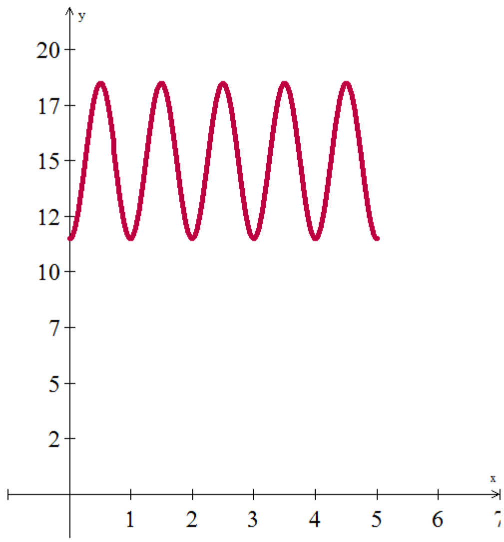
b) What is the length the spring when it is at equilibrium?

c) What is the length the spring when it is shortest?

d) What is the length the spring when it is longest?

Solution

a)



b) The length the spring when it is at equilibrium $L = 15\ cm$

c) $L = 15 - 3.5$

$\underline{= 11.5\ cm}$

d) $L = 15 + 3.5$

$\underline{= 18.5\ cm}$

Exercise

Based on years of weather data, the expected low temperature T (in °F) in Fairbanks, Alaska, can be approximated by

$$T = 36\sin\left(\frac{2\pi}{365}(t - 101)\right) + 14$$

a) Sketch the graph T for $0 \le t \le 365$

b) Predict when the coldest day of the year will occur.

Solution

a)

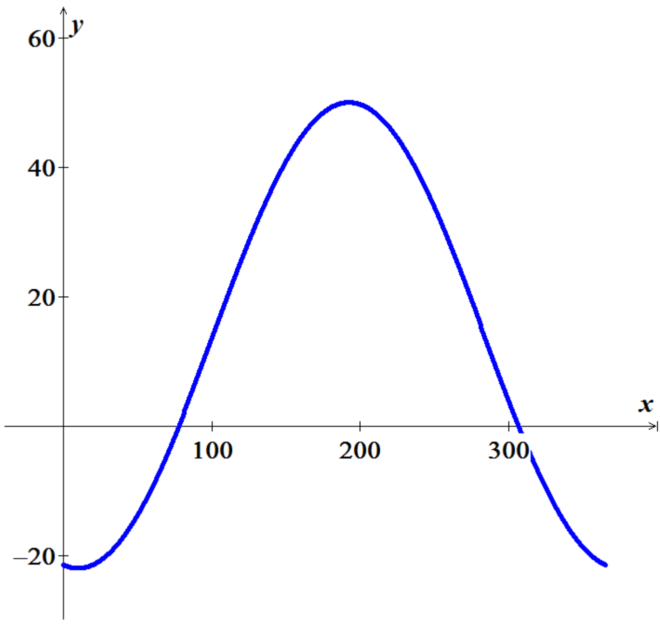
Amplitude: $|A| = 36$

Period: $P = 2\pi \frac{365}{2\pi} = 365$

Phase Shift: $\phi = 101$

VT: $y = 14$

x	***y***
101	14
$\frac{365}{4} + 101 = \frac{769}{4}$	50
$\frac{365}{2} + 101 = \frac{567}{2}$	14
$\frac{1095}{4} + 101 = \frac{1,499}{4}$	-22
$365 + 101 = 466$	14



b) From the table the coldest temperature is –22 °F at $t = \dfrac{1499}{4} = 374.75 > 365$

$t = 374.75 - 365$

$\quad \boxed{= 9.75 \ \ days}$

Exercise

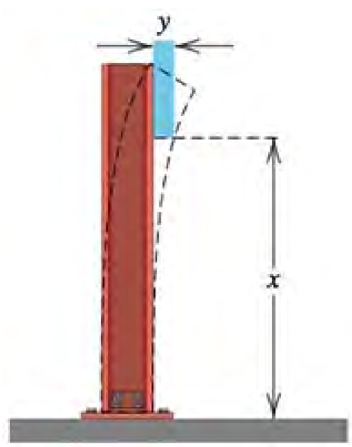
To simulate the response of a structure to an earthquake, an engineer must choose a shape for the initial displacement of the beams in the building. When the beam has length *L feet* and the maximum displacement is *a feet*, the equation

$$y = a - a\cos\frac{\pi}{2L}x$$

Has been used by engineers to estimate the displacement *y*. if $a = 1$ and $L = 10$, sketch the graph of the equation for $0 \le x \le 10$.

Solution

Given: $a = 1$ & $L = 10$



$y = a - a\cos\frac{\pi}{2L}x$

$\quad \boxed{= 1 - \cos\left(\frac{\pi}{20}x\right)}$

Amplitude: $|A| = 1$

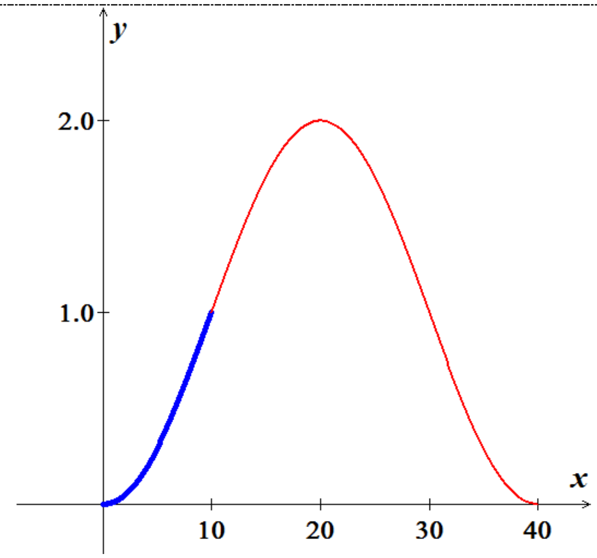
Period: $P = \frac{2\pi}{\frac{\pi}{20}} = 40$

Phase Shift: $\phi = 0$

VT: $y = 1$

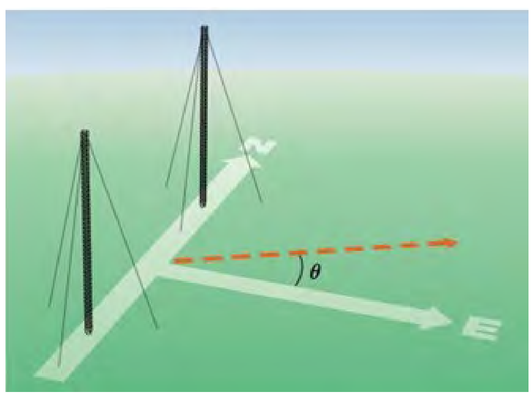
x	***y***
0	0
10	1
20	2
30	1
40	0

$0 \le x \le 10$



Exercise

Radio stations often have more than one broadcasting tower because federal guidelines do not usually permit a radio station to broadcast its signal in all directions with equal power. Since radio waves cam travel over long distances, it is important to control their directional patterns so that radio stations do not interfere with one another. Suppose that a radio station has two broadcasting towers located along a north–south line.



If the radio station is broadcasting at a wavelength λ and the distance between the two radio towers is equal to $\frac{1}{2}\lambda$, then the intensity I of the signal in the direction θ is given by

$$I = \frac{1}{2} I_0 \left[1 + \cos(\pi \sin\theta)\right]$$

where I_0 is the maximum intensity.

a) Approximate I in terms of I_0 for each θ.

i. $\theta = 0$ **ii.** $\theta = \frac{\pi}{3}$ **iii.** $\theta = \frac{\pi}{7}$

b) Determine the direction in which I has maximum or minimum values.

c) Graph I on the interval $[0,\ 2\pi)$. Graphically approximate θ to three decimal places, when I is equal to $\frac{1}{3}I_0$. (Hint: let $I_0 = 1$)

Solution

a)

θ	$I = \frac{1}{2}I_0\left[1+\cos(\pi\sin\theta)\right]$
0	$I = \frac{1}{2}I_0\left[1+\cos(\pi\sin 0)\right] = \frac{1}{2}I_0\left[1+\cos(0)\right] = \frac{1}{2}I_0(2)\ \underline{= I_0}$
$\frac{\pi}{3}$	$I = \frac{1}{2}I_0\left[1+\cos\left(\pi\sin\frac{\pi}{3}\right)\right] = \frac{1}{2}I_0\left[1+\cos\left(\frac{\sqrt{3}}{2}\pi\right)\right]\ \underline{\approx 0.044 I_0}$
$\frac{\pi}{7}$	$I = \frac{1}{2}I_0\left[1+\cos\left(\pi\sin\frac{\pi}{7}\right)\right]\ \underline{\approx I_0}$

b) I to have a *maximum* when $\cos(0) = 1$

$\therefore I$ has *maximum* at $\underline{\theta = 0}$

I to have a *minimum* when $-1 = \cos(\pi)$

$\cos(\pi\sin\theta) = \cos(\pi)$

$\sin\theta = 1 \quad \to \quad \theta = \frac{\pi}{2}$

I has *minimum* at $\underline{\theta = \frac{\pi}{2}}$

c) $I_0 = 1$

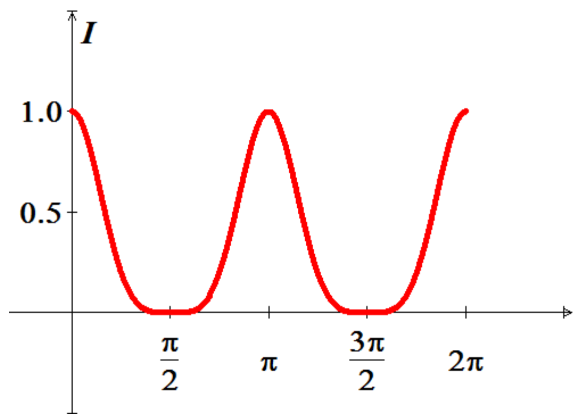
$I = \frac{1}{2} + \frac{1}{2}\cos(\pi\sin\theta)$

Amplitude: $|A| = \frac{1}{2}$

Period: $P = \frac{2\pi}{\pi} = 2$

Phase Shift: $\varphi = 0$

VT: $y = \frac{1}{2}$



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \tan\left(x - \frac{\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

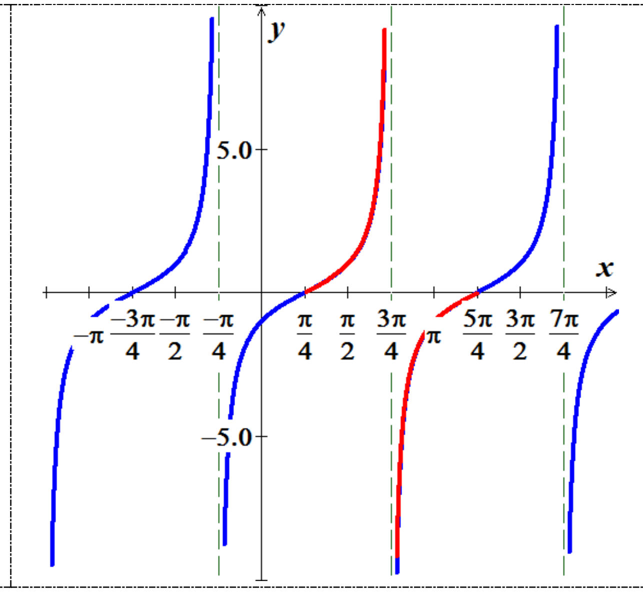
VT: $y = 0$

Asymptotes:

$$x = -\frac{\pi}{4} + n\pi$$

$$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$$

	x	y
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	1
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	∞
$\frac{3\pi}{4} + \frac{\pi}{4}$	π	-1
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\tan\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \frac{\pi}{2}$

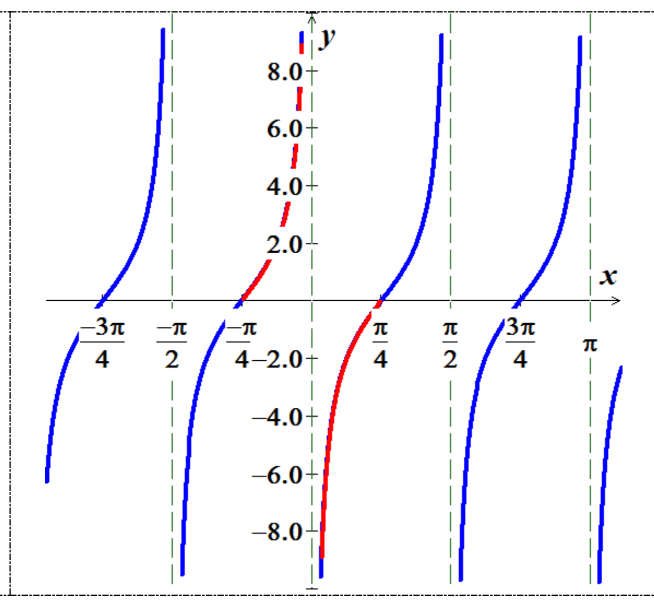
Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

Asymptotes: $x = n\frac{\pi}{2}$

$$x = \ldots, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \ldots$$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	0
$\frac{\pi}{8} - \frac{\pi}{4}$	$-\frac{\pi}{8}$	2
$\frac{\pi}{4} - \frac{\pi}{4}$	0	∞
$\frac{3\pi}{8} - \frac{\pi}{4}$	$\frac{\pi}{8}$	-2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{4}\tan\left(\frac{1}{2}x + \frac{\pi}{3}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = -\frac{2\pi}{3}$

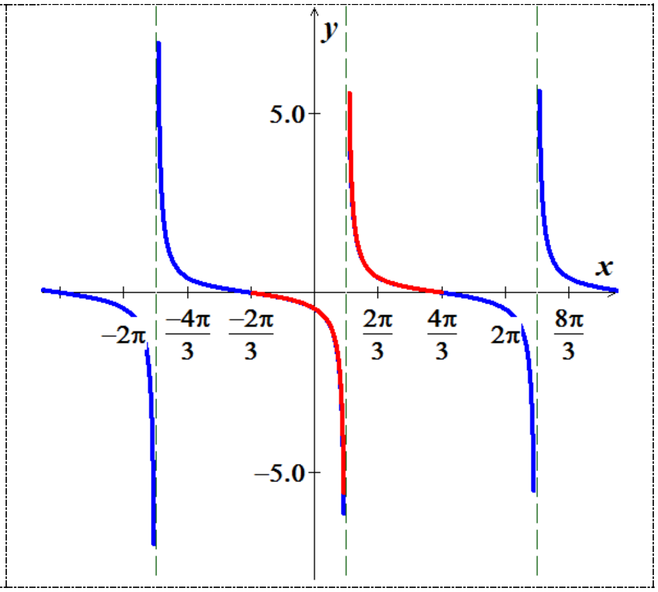
VT: $y = 0$

Asymptotes:

$$x = -\frac{5\pi}{3} + 2n\pi$$

$$x = \ldots, -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3}, \ldots$$

	x	y
$0 - \frac{2\pi}{3}$	$-\frac{2\pi}{3}$	0
$\frac{\pi}{2} - \frac{2\pi}{3}$	$-\frac{\pi}{6}$	$-\frac{1}{4}$
$\pi - \frac{2\pi}{3}$	$\frac{\pi}{3}$	∞
$\frac{3\pi}{2} - \frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{1}{4}$
$2\pi - \frac{2\pi}{3}$	$\frac{4\pi}{3}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \cot\left(x + \frac{\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

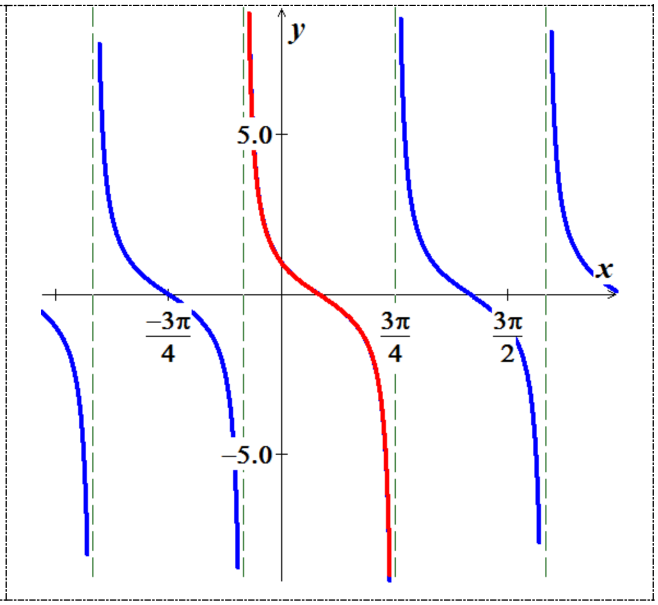
Asymptotes:

$$x = 2n\pi + \pi - \frac{\pi}{4}$$

$$x = 2n\pi + \frac{3\pi}{4}$$

$$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	∞
$\frac{\pi}{4} - \frac{\pi}{4}$	0	1
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{3\pi}{4} - \frac{\pi}{4}$	$\frac{\pi}{2}$	-1
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	∞



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\cot\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \frac{\pi}{2}$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

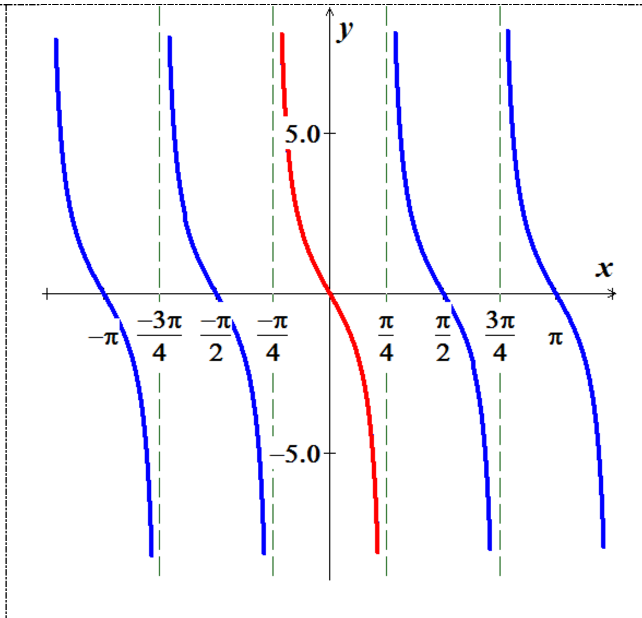
Asymptotes:

$$2x + \frac{\pi}{2} = (2n+1)\pi$$

$$2x = 2n\pi + \frac{\pi}{2}$$

$$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	∞
$\frac{\pi}{8} - \frac{\pi}{4}$	$-\frac{\pi}{8}$	2
$\frac{\pi}{4} - \frac{\pi}{4}$	0	0
$\frac{3\pi}{8} - \frac{\pi}{4}$	$\frac{\pi}{8}$	-2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	∞



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{2}\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = -\frac{\pi}{2}$

VT: $y = 0$

Asymptotes:

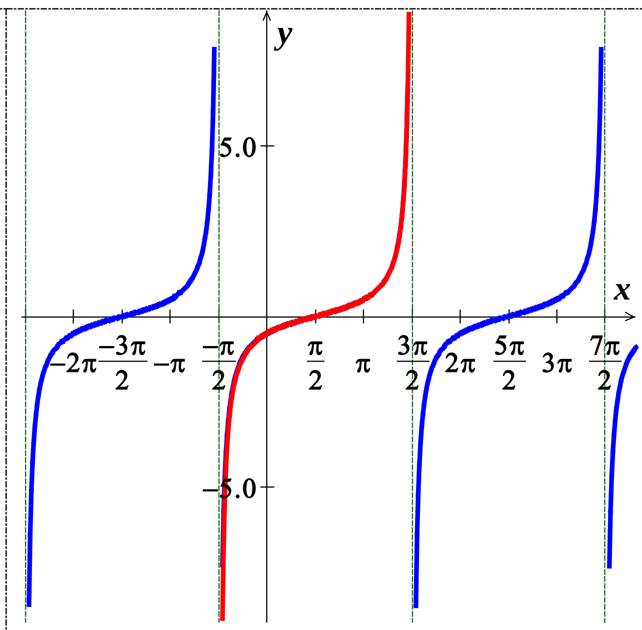
$$\frac{1}{2}x + \frac{\pi}{4} = 2n\pi + \pi$$

$$\frac{1}{2}x = 2n\pi + \frac{3\pi}{4}$$

$$x = 4n\pi + \frac{3\pi}{2}$$

$$x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{11\pi}{2}, \ldots$$

	x	y
$0 - \frac{\pi}{2}$	$-\frac{\pi}{2}$	∞
$\frac{\pi}{2} - \frac{\pi}{2}$	0	$-\frac{1}{2}$
$\pi - \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{3\pi}{2} - \frac{\pi}{2}$	π	$\frac{1}{2}$
$2\pi - \frac{\pi}{2}$	$\frac{3\pi}{2}$	∞



Exercise

Graph over a 1-period interval $y = 1 - 2\cot 2\left(x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \frac{\pi}{2}$

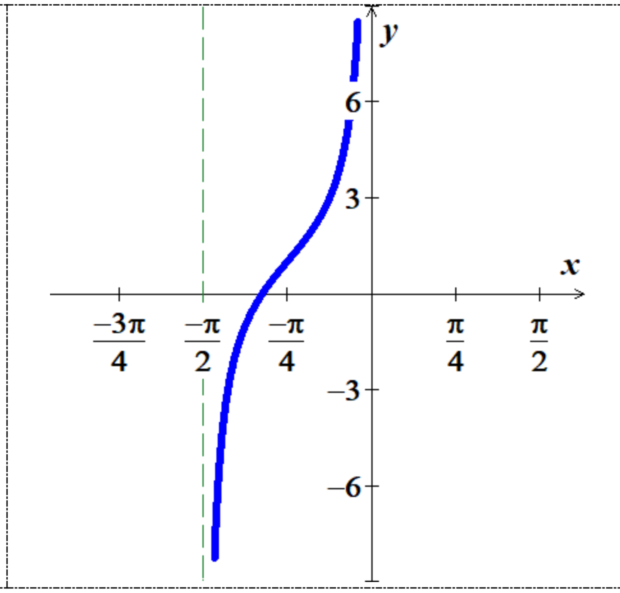
Phase Shift: $\varphi = -\frac{\pi}{2}$

VT: $y = 1$

Asymptotes:

$-\frac{\pi}{2} + n\pi$

	x	y
$0 - \frac{\pi}{2}$	$-\frac{\pi}{2}$	∞
$\frac{\pi}{8} - \frac{\pi}{2}$	$-\frac{3\pi}{8}$	-1
$\frac{\pi}{4} - \frac{\pi}{2}$	$-\frac{\pi}{4}$	1
$\frac{3\pi}{8} - \frac{\pi}{2}$	$-\frac{\pi}{8}$	3
$\frac{\pi}{2} - \frac{\pi}{2}$	0	∞



Exercise

Graph over a 1-period interval $y = \frac{2}{3}\tan\left(\frac{3}{4}x - \pi\right) - 2$

Solution

Amplitude: n/a

Period: $P = \frac{4\pi}{3}$

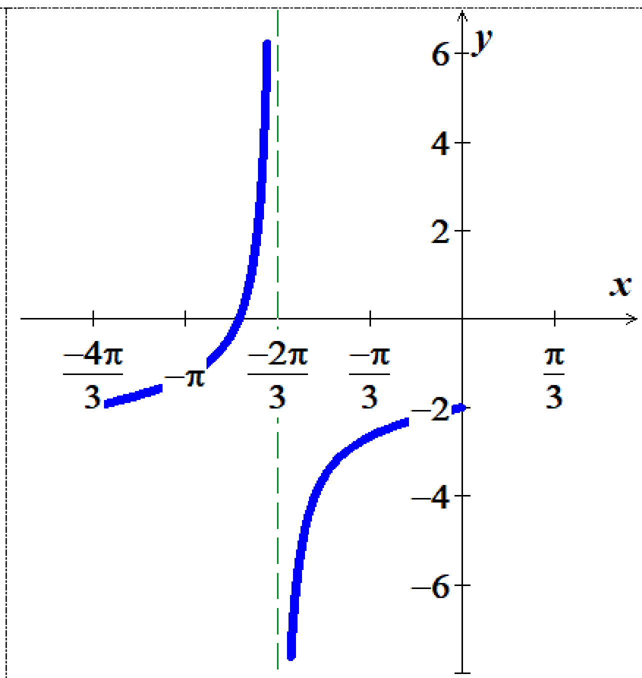
Phase Shift: $\varphi = -\frac{4\pi}{3}$

VT: $y = -2$

Asymptotes:

$x = \frac{3\pi}{4} + n\pi$

	x	y
$0 - \frac{4\pi}{3}$	$-\frac{4\pi}{3}$	-2
$\frac{\pi}{3} - \frac{4\pi}{3}$	$-\pi$	$-\frac{4}{3}$
$\frac{2\pi}{3} - \frac{4\pi}{3}$	$-\frac{2\pi}{3}$	∞
$\pi - \frac{4\pi}{3}$	$-\frac{\pi}{3}$	$-\frac{8}{3}$
$\frac{4\pi}{3} - \frac{4\pi}{3}$	0	-2



Exercise

Graph one complete cycle $y = 3 + 2\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)$

Solution

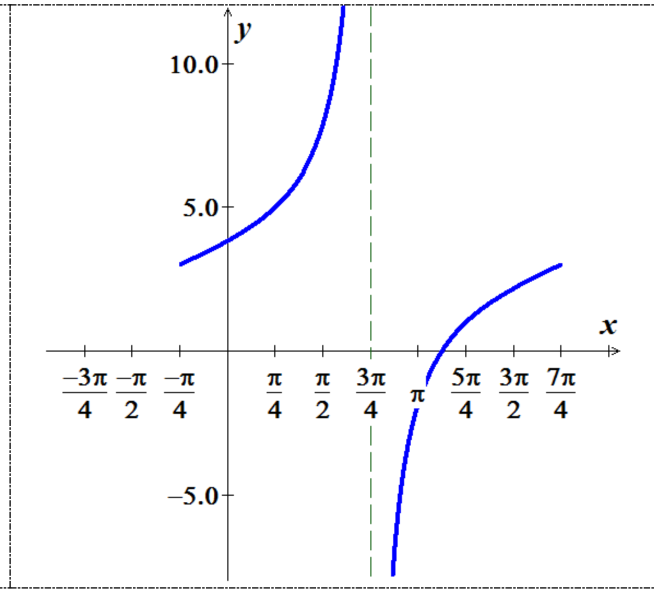
Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 3$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	3
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	5
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	∞
$\frac{3\pi}{2} - \frac{\pi}{4}$	$\frac{5\pi}{4}$	1
$2\pi - \frac{\pi}{4}$	$\frac{7\pi}{4}$	3



Exercise

Graph one complete cycles $y = -2 - \cot\left(x - \frac{\pi}{4}\right)$

Solution

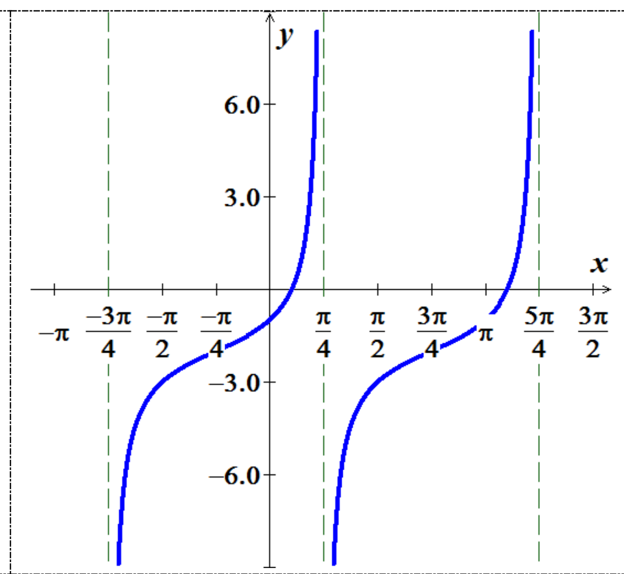
Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

VT: $y = -2$

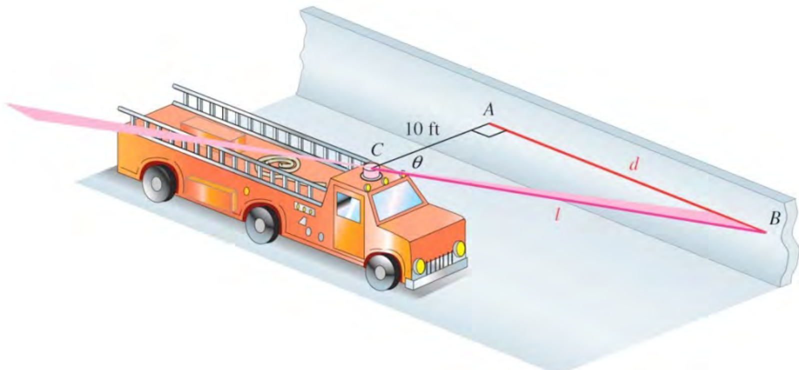
	x	y
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	∞
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	-3
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	-2
$\frac{3\pi}{4} + \frac{\pi}{4}$	π	-1
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	∞



Exercise

A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top is 10 *feet* from the wall and rotates through one complete revolution every 2 *seconds*. Graph the function that gives the length *d* in terms of time *t* from *t* = 0 to *t* = 2.

Solution



$$\omega = \frac{\theta}{t} = \frac{2\pi}{2} = \pi \ rad / sec$$

$$\tan\theta = \frac{d}{10} \quad \rightarrow \ d = 10\tan\theta$$

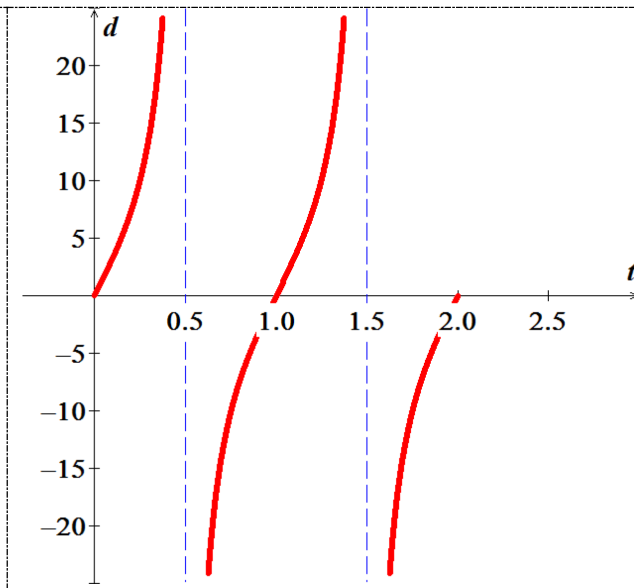
$$d(t) = 10\tan\pi t$$

$$\textbf{Period} = \frac{\pi}{\pi} = 1$$

One cycle: $0 \le \pi t \le \pi$

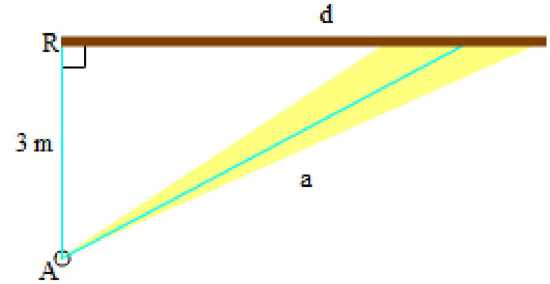
$0 \le t \le 1$

t	$d = 10\tan\pi t$
0	0
$\frac{1}{4}$	10
$\frac{1}{2}$	∞
$\frac{3}{4}$	−10
1	0



Exercise

A rotating beacon is located 3 *m* south of point *R* on an east-west wall. *d*, the length of the light display along the wall from *R*, is given by $d = 3\tan 2\pi t$, where *t* is time measured in seconds since the beacon started rotating. (When $t = 0$, the beacon is aimed at point *R*. When the beacon is aimed to the right of *R*, the value of *d* is positive; *d* is negative if the beacon is aimed to the left of *R*.) Find a for $t = 0.8$



Solution

$$d = 3\tan\left(2\pi(0.8)\right)$$

$$\approx -9.23\ m$$

Exercise

Let a person whose eyes are h_1 feet from the ground stand ***d*** feet from an object h_1 feet tall, where $h_2 > h_1$ feet. Let θ be the angle of elevation to the top of the object.

a) Show that $d = \left(h_2 - h_1\right)\cot\theta$

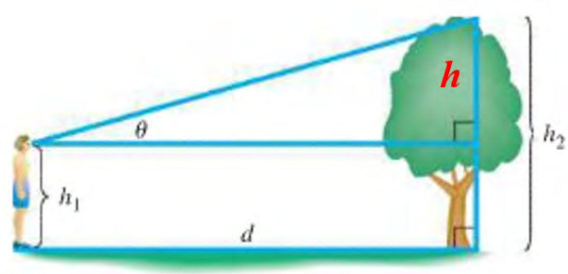
b) Let $h_2 = 55$ and $h_1 = 5$. Graph ***d*** for the interval $0 < \theta \le \frac{\pi}{2}$

Solution

a) $h = h_2 - h_1$

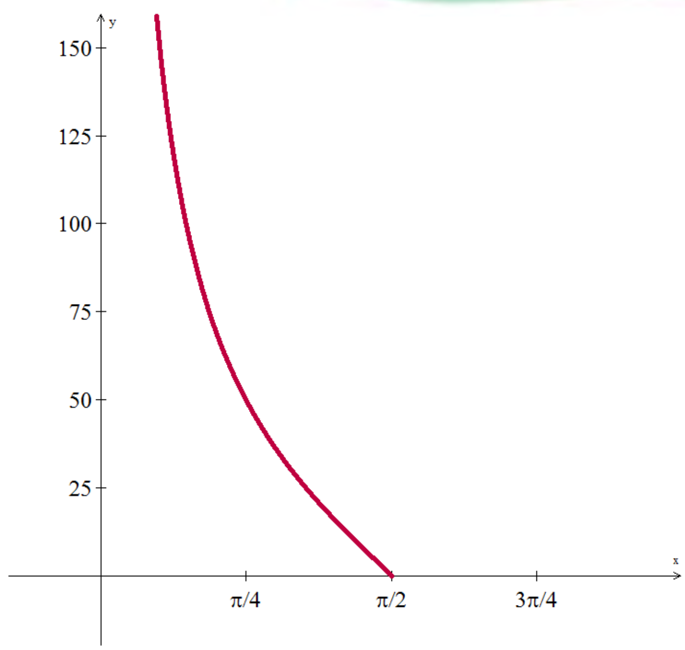
$$\cot\theta = \frac{d}{h}$$

$$d = \left(h_2 - h_1\right)\cot\theta$$



b) $d = (55 - 5)\cot\theta$

$$d = 50\cot\theta \quad 0 < \theta \le \frac{\pi}{2}$$



Solution   *Section* 7.3 – Graphing Secant & Cosecant

Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \sec\left(x - \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = \frac{\pi}{2}$

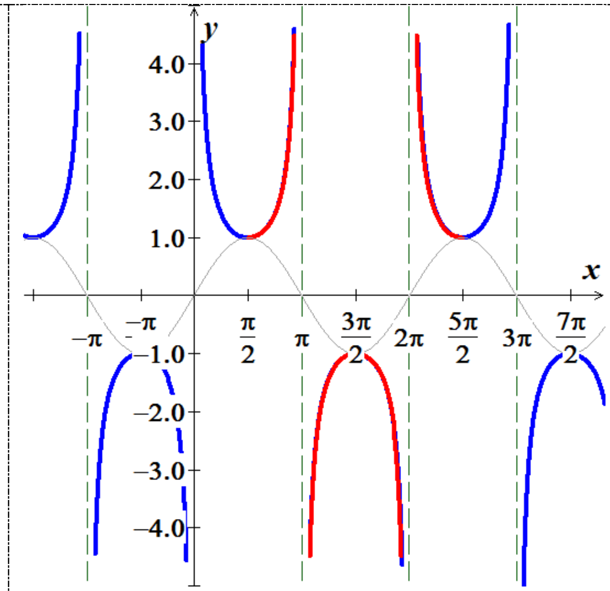
VT: $y = 0$

Asymptotes:

$x - \frac{\pi}{2} = n\pi + \frac{\pi}{2}$

$x = n\pi$

	x	$\cos\left(x - \frac{\pi}{2}\right)$
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	1
$\frac{\pi}{2} + \frac{\pi}{2}$	π	0
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	-1
$\frac{3\pi}{2} + \frac{\pi}{2}$	2π	0
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	1



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\sec\left(2x - \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

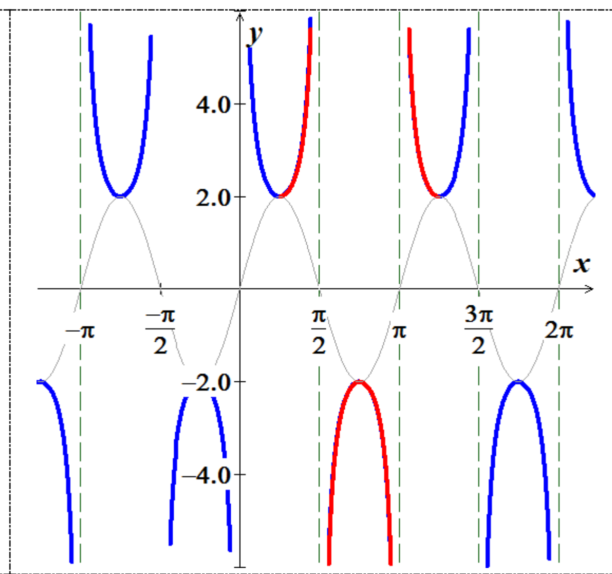
VT: $y = 0$

Asymptotes:

$2x - \frac{\pi}{2} = n\pi + \frac{\pi}{2}$

$x = \left(\frac{n+1}{2}\right)\pi$

	x	$2\cos\left(2x - \frac{\pi}{2}\right)$
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	2
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	-2
$\frac{3\pi}{4} + \frac{\pi}{4}$	2π	0
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	2



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -3\sec\left(\frac{1}{3}x + \frac{\pi}{3}\right)$

Solution

Amplitude: n / a

Period: $P = 6\pi$

Phase Shift: $\varphi = -\pi$

VT: $y = 0$

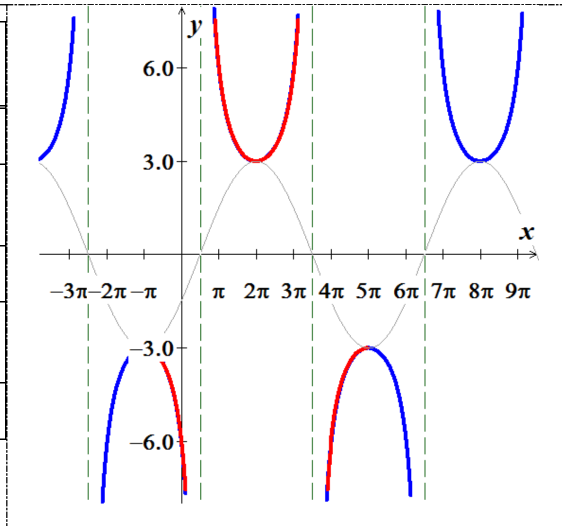
Asymptotes:

$$\frac{1}{3}x + \frac{\pi}{3} = n\pi + \frac{\pi}{2}$$

$$x = 3n\pi + \frac{\pi}{2}$$

$$x = \dots,\ -\frac{5\pi}{2},\ \frac{\pi}{2},\ \frac{7\pi}{2},\ \dots$$

	x	$-3\cos\left(\frac{1}{3}x+\frac{\pi}{3}\right)$
$0-\pi$	$-\pi$	-3
$\frac{3\pi}{2}-\pi$	$\frac{\pi}{2}$	0
$3\pi-\pi$	2π	3
$\frac{9\pi}{2}-\pi$	$\frac{7\pi}{2}$	0
$6\pi-\pi$	5π	-3



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \csc\left(x - \frac{\pi}{2}\right)$

Solution

Amplitude: n / a

Period: $P = 2\pi$

Phase Shift: $\varphi = \frac{\pi}{2}$

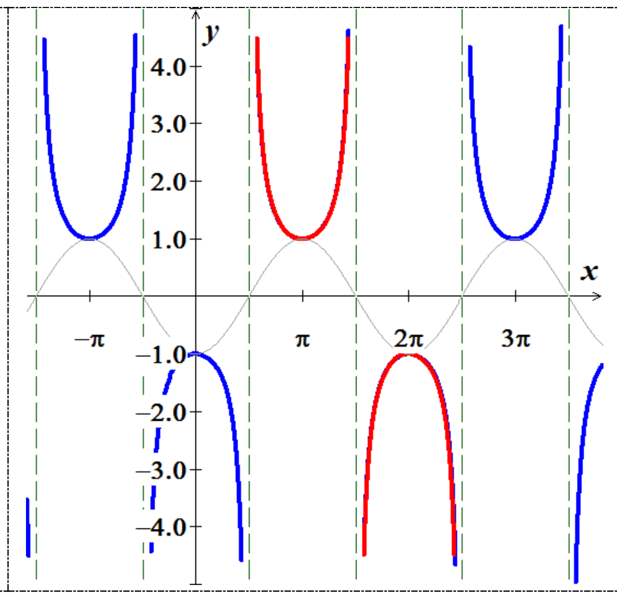
VT: $y = 0$

Asymptotes:

$$x - \frac{\pi}{2} = n\pi$$

$$x = n\pi + \frac{\pi}{2}$$

	x	$\sin\left(x-\frac{\pi}{3}\right)$
$0+\frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}+\frac{\pi}{2}$	π	1
$\pi+\frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$\frac{3\pi}{2}+\frac{\pi}{2}$	2π	-1
$2\pi+\frac{\pi}{2}$	$\frac{5\pi}{2}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\csc\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \pi$

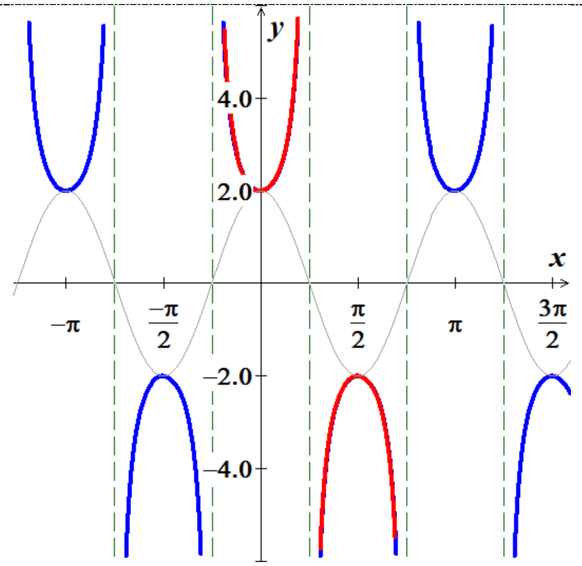
Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

Asymptotes:

$$x = -\frac{\pi}{4} + n\frac{\pi}{2}$$

	x	$2\sin\left(2x+\frac{\pi}{2}\right)$
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	0
$\frac{\pi}{4} - \frac{\pi}{4}$	0	2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{3\pi}{4} - \frac{\pi}{4}$	$\frac{\pi}{2}$	-2
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 4\csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = 4\pi$

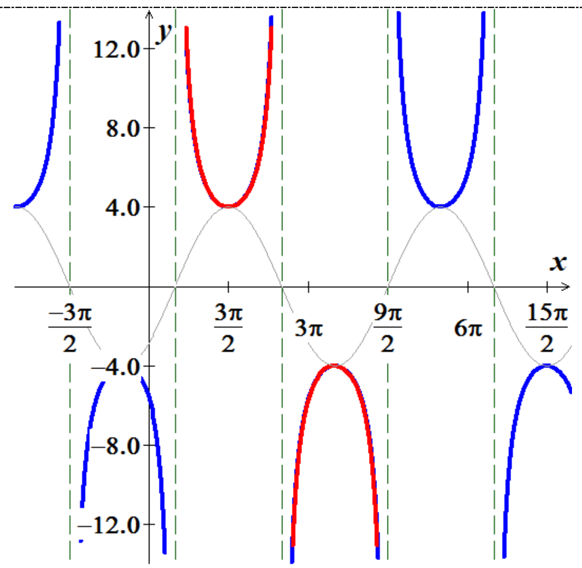
Phase Shift: $\varphi = \frac{\pi}{2}$

VT: $y = 0$

Asymptotes:

$$x = \frac{\pi}{2} + 2\pi n$$

	x	$4\sin\left(\frac{1}{2}x-\frac{\pi}{4}\right)$
$0 + \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	4
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	0
$3\pi + \frac{\pi}{2}$	$\frac{7\pi}{2}$	-4
$4\pi + \frac{\pi}{2}$	$\frac{9\pi}{2}$	0



Exercise

Graph over a one-period interval $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

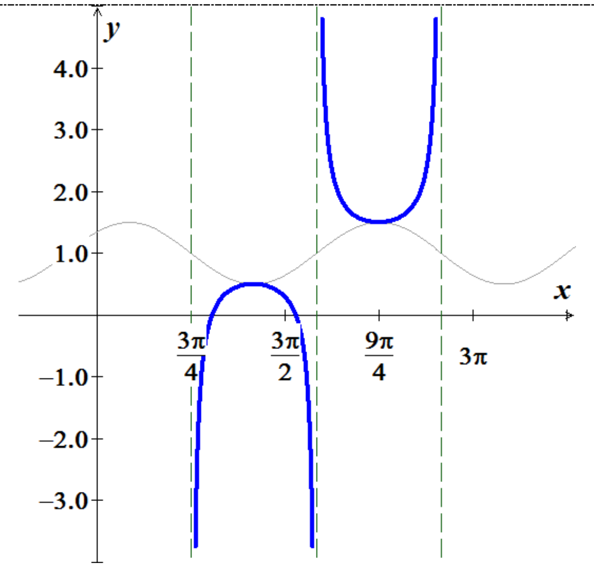
Phase Shift: $\varphi = \frac{3\pi}{4}$

VT: $y = 1$

Asymptotes:

$$x = \frac{3\pi}{4} + 2\pi n$$

	x	$4\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$
$0 + \frac{3\pi}{4}$	$\frac{3\pi}{4}$	1
$\frac{\pi}{2} + \frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{1}{2}$
$\pi + \frac{3\pi}{4}$	$\frac{7\pi}{4}$	1
$\frac{3\pi}{2} + \frac{3\pi}{4}$	$\frac{9\pi}{4}$	$-\frac{1}{2}$
$2\pi + \frac{3\pi}{4}$	$\frac{11\pi}{4}$	1



Exercise

Graph over a one-period interval $y = 2 + \frac{1}{4}\sec\left(\frac{1}{2}x - \pi\right)$

Solution

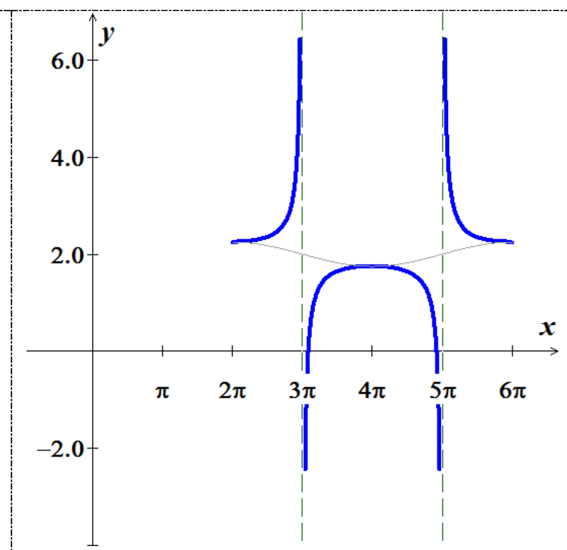
Amplitude: n/a

Period: $P = 4\pi$

Phase Shift: $\varphi = 2\pi$

VT: $y = 2$

	x	$2 + \frac{1}{4}\cos\left(\frac{1}{2}x - \pi\right)$
$0 + 2\pi$	2π	$\frac{9}{4}$
$\pi + 2\pi$	3π	2
$2\pi + 2\pi$	4π	$\frac{7}{4}$
$3\pi + 2\pi$	5π	2
$4\pi + 2\pi$	6π	$\frac{9}{4}$



Exercise

Graph $y = \frac{1}{3}\sec 2x$ *for* $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$

Solution

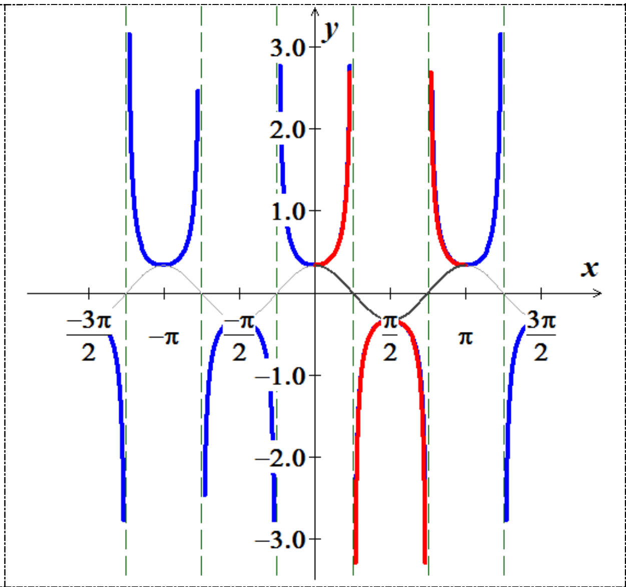
Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = 0$

VT: $y = 0$

x	$\frac{1}{3}\cos 2x$
0	$\frac{1}{3}$
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	$-\frac{1}{3}$
$\frac{3\pi}{4}$	0
π	$\frac{1}{3}$



Exercise

Graph one complete cycle $y = -1 - 3\csc\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$

Solution

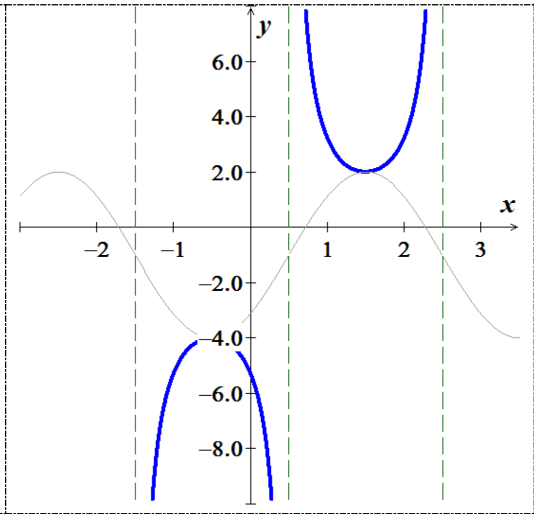
Amplitude: n/a

Period: $P = 4$

Phase Shift: $\varphi = -\frac{3}{2}$

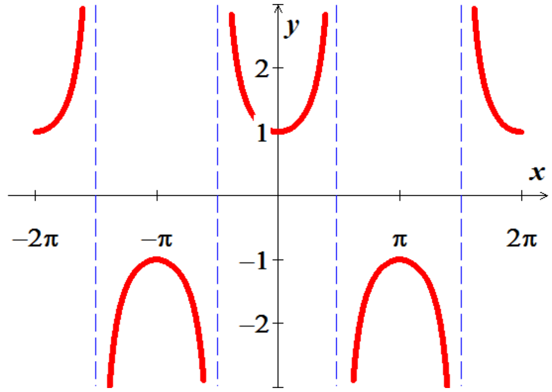
VT: $y = -1$

	x	$-1 - 3\sin\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$
$0 - \frac{3}{2}$	$-\frac{3}{2}$	-1
$1 - \frac{3}{2}$	$-\frac{1}{2}$	-4
$2 - \frac{3}{2}$	$\frac{1}{2}$	-1
$3 - \frac{3}{2}$	$\frac{3}{2}$	2
$4 - \frac{3}{2}$	$\frac{5}{2}$	-1



Exercise

Find an equation to match the graph



Solution

$P = 2\pi$

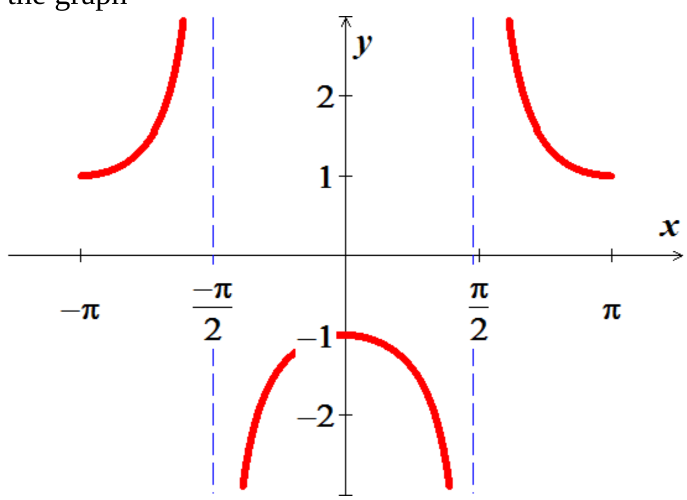
$\phi = 0$

$A = \frac{1+1}{2} = 1$

$y = \sec x \quad -2\pi \le x \le 2\pi$

Exercise

Find an equation to match the graph



Solution

$B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$

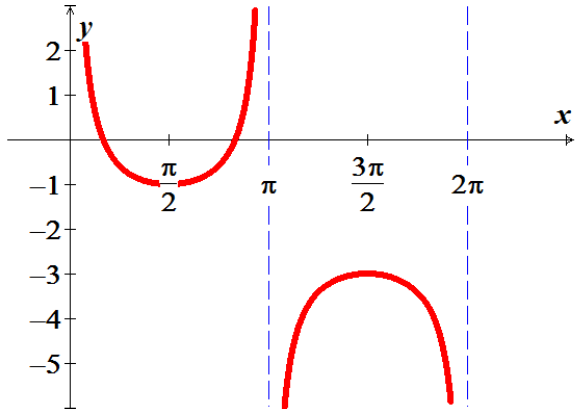
$\phi = 0 \to C = 0$

$A = \frac{1+1}{2} = 1$

$y = -\sec(x) \quad -\pi \le x \le \pi$

Exercise

Find an equation to match the graph



Solution

$$B=\frac{2\pi}{P}=\frac{2\pi}{2\pi}=1$$

$$\phi=0\to C=0$$

$$A=\frac{-3-1}{2}=-2$$

$$y=-2+\csc(x)\quad -2\pi\le x\le 2\pi$$

Exercise

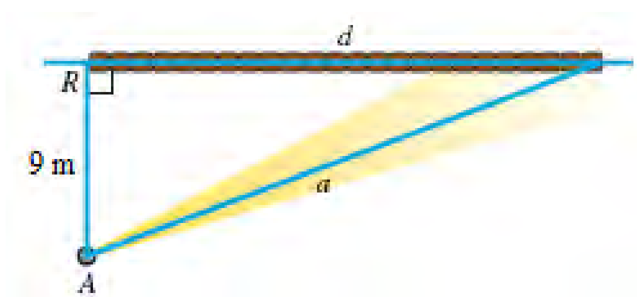
A rotating beacon is located at point A next to a long wall. The beacon is 9 m from the wall. The distance $\boldsymbol{a}$ is given by $a=9\left|\sec 2\pi t\right|$, where t is time measured in seconds since the beacon started rotating. (When $t=0$, the beacon is aimed at point R.) Find $\boldsymbol{a}$ for $t=0.45$

Solution

$$a=9\left|\sec\left(2\pi(0.45)\right)\right|$$

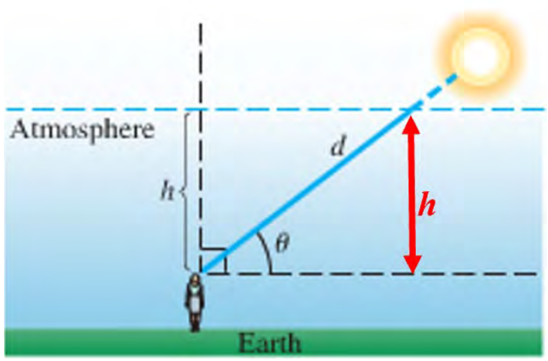
$$=\frac{9}{\left|\cos\left(2\pi(0.45)\right)\right|}$$

$$\approx 9.5\ m$$



Exercise

The shortest path for the sun's rays through Earth's atmosphere occurs when the sun is directly overhead. Disregarding the curvature of Earth, as the sun moves lower on the horizon, the distance that sunlight passes through the atmosphere increases by a factor of $\csc\theta$, where θ is the angle of elevation of the sun. This increased distance reduces both the intensity of the sun and the amount of ultraviolet light that reached Earth's surface.



a) Verify that $d = h\csc\theta$

b) Determine θ when $d = 2h$

c) The atmosphere filters out the ultraviolet light that causes skin to burn, Compare the difference between sunbathing when $\theta = \frac{\pi}{2}$ and when $\theta = \frac{\pi}{3}$. Which measure gives less ultraviolet light?

Solution

a) $\sin\theta = \frac{h}{d}$

$$= \frac{1}{\csc\theta}$$

$$\boxed{d = h\csc\theta}$$ (*cross-multiplication*)

b) $\sin\theta = \frac{h}{d}$

$$= \frac{h}{2h}$$

$$\boxed{= \frac{1}{2}}$$

$$\theta = \sin^{-1}\frac{1}{2}$$

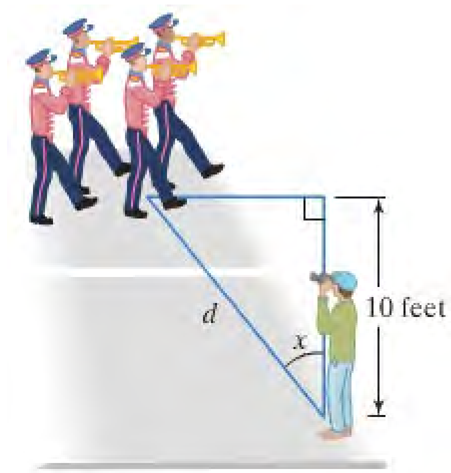
$$\boxed{= \frac{\pi}{6}}$$

c) $\begin{cases} \csc\frac{\pi}{2} = 1 \\ \csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3} \approx 1.15 \end{cases}$

When the distance to the sun is lager $\left(\theta = \frac{\pi}{3}\right)$, there is less ultraviolet light reaching the earth's surface. In this case, sunlight passes through 15% more atmosphere.

Exercise

Your friend is marching with a band and has asked you to film him. You have set yourself up 10 *feet* from the street where your friend will be passing from left to right. If d represents your distance, in feet, from your friend and x is the radian measure of the angle.



a) Express d in terms of a trigonometric function of x.

b) Graph the function for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Solution

a) $\cos x = \frac{10}{d}$

$\underline{d = 10\sec x}$

b)

$y = 10\cos x$

Amplitude: $|A| = 10$

Period: $P = 2\pi$

Phase Shift: $\phi = 0$

VT: $y = 0$

x	y
0	10
$\frac{\pi}{2}$	0
π	-10
$\frac{3\pi}{2}$	0
2π	10