

Solution

Section 2.6 – Exponential & Logarithmic Functions

Exercise

Solve $4^{2x-1} = 64$

Solution

$$4^{2x-1} = 4^3$$

$$2x-1 = 3$$

$$2x = 4$$

$$x = 2$$

Exercise

Solve $3^{1-x} = \frac{1}{27}$

Solution

$$3^{1-x} = \frac{1}{3^3}$$

$$3^{1-x} = 3^{-3}$$

$$1 - x = -3$$

$$-x = -4$$

$$x = 4$$

Exercise

Solve $9^x = \frac{1}{\sqrt[3]{3}}$

Solution

$$(3^2)^x = \frac{1}{3^{1/3}}$$

$$3^{2x} = 3^{-1/3}$$

$$2x = -\frac{1}{3}$$

$$x = -\frac{1}{6}$$

Exercise

Solve $5^{3x-6} = 125$

Solution

$$5^{3x-6} = 5^3$$

$$\Rightarrow 3x - 6 = 3$$

$$\Rightarrow 3x = 9$$

$$\Rightarrow x = 3$$

Exercise

Solve $8^{x+2} = 4^{x-3}$

Solution

$$(2^3)^{x+2} = (2^2)^{x-3}$$

$$2^{3(x+2)} = 2^{2(x-3)}$$

$$3(x+2) = 2(x-3)$$

$$3x + 6 = 2x - 6$$

$$3x - 2x = -6 - 6$$

$$x = -12$$

Exercise

Solve: $7e^{2x} - 5 = 58$

Solution

$$7e^{2x} = 63$$

$$e^{2x} = \frac{63}{7} = 9$$

$$\ln e^{2x} = \ln 9$$

$$2x \ln e = \ln 9$$

$$2x = \ln 9$$

$$x = \frac{\ln 9}{2} \approx 1.1$$

Exercise

Solve: $4\ln(3x) = 8$

Solution

$$\ln(3x) = 2$$

$$3x = e^2$$

$$\underline{x = \frac{e^2}{3}}$$

Exercise

Solve: $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

Solution

$$\ln(x-3) = \ln\left(\frac{7x-23}{x+1}\right)$$

Quotient Rule

$$x-3 = \frac{7x-23}{x+1}$$

Cross multiply

$$(x-3)(x+1) = 7x-23$$

$$x^2 - 2x - 3 = 7x - 23$$

$$x^2 - 9x + 20 = 0$$

Solve for x

$$\Rightarrow x = 4, 5$$

$$\text{Check: } x = 4 \Rightarrow \ln(4-3) = \ln(7(4)-23) - \ln(4+1)$$

$$x = 5 \Rightarrow \ln(5-3) = \ln(7(5)-23) - \ln(5+1)$$

Exercise

Use the properties of logarithms to rewrite $\log_b \left(\frac{x^3 y}{z^2} \right)$

Solution

$$\log_b \left(\frac{x^3 y}{z^2} \right) = \log_b (x^3 y) - \log_b z^2$$

$$= \log_b x^3 + \log_b y - \log_b z^2$$

$$= 3\log_b x + \log_b y - 2\log_b z$$

Exercise

Use the properties of logarithms to rewrite $\log_b \left(\frac{\sqrt[3]{xy^4}}{z^5} \right)$

Solution

$$\begin{aligned}\log_b \left(\frac{\sqrt[3]{xy^4}}{z^5} \right) &= \log_b \left(\sqrt[3]{xy^4} \right) - \log_b \left(z^5 \right) \\ &= \log_b \left(x^{1/3} \right) + \log_b \left(y^4 \right) - \log_b \left(z^5 \right) \\ &= \frac{1}{3} \log_b (x) + 4 \log_b (y) - 5 \log_b (z)\end{aligned}$$

Exercise

Use the properties of logarithms to rewrite $\log_b \sqrt[n]{\frac{x^3 y^5}{z^m}}$

Solution

$$\begin{aligned}\log_b \sqrt[n]{\frac{x^3 y^5}{z^m}} &= \log_b \left(\frac{x^3 y^5}{z^m} \right)^{1/n} \\ &= \frac{1}{n} \log_b \left(\frac{x^3 y^5}{z^m} \right) && \text{Power Rule} \\ &= \frac{1}{n} \left(\log_b x^3 y^5 - \log_b z^m \right) && \text{Quotient Rule} \\ &= \frac{1}{n} \left(\log_b x^3 + \log_b y^5 - \log_b z^m \right) && \text{Product Rule} \\ &= \frac{1}{n} \left(3 \log_b x + 5 \log_b y - m \log_b z \right) && \text{Power Rule} \\ &= \frac{3}{n} \log_b x + \frac{5}{n} \log_b y - \frac{m}{n} \log_b z\end{aligned}$$

Exercise

Use the properties of logarithms to rewrite $\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}}$

Solution

$$\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}} = \log_p \left(\frac{m^5 n^4}{t^2} \right)^{1/3}$$

$$\begin{aligned}
&= \frac{1}{3} \log_p \left(\frac{m^5 n^4}{t^2} \right) \\
&= \frac{1}{3} \left(\log_p m^5 n^4 - \log_p t^2 \right) \\
&= \frac{1}{3} \left(\log_p m^5 + \log_p n^4 - \log_p t^2 \right) \\
&= \frac{1}{3} \left(5 \log_p m + 4 \log_p n - 2 \log_p t \right) \\
&= \frac{5}{3} \log_p m + \frac{4}{3} \log_p n - \frac{2}{3} \log_p t
\end{aligned}$$

Exercise

Use the properties of logarithms to rewrite $\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}}$

Solution

$$\begin{aligned}
\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}} &= \log_a \left(\frac{m^8 n^{12}}{a^3 b^5} \right)^{1/4} \\
&= \frac{1}{4} \log_a \left(\frac{m^8 n^{12}}{a^3 b^5} \right) \\
&= \frac{1}{4} \left[\log_a m^8 n^{12} - \log_a a^3 b^5 \right] \\
&= \frac{1}{4} [\log_a m^8 + \log_a n^{12} - (\log_a a^3 + \log_a b^5)] \\
&= \frac{1}{4} [\log_a m^8 + \log_a n^{12} - \log_a a^3 - \log_a b^5] \\
&= \frac{1}{4} [8 \log_a m + 12 \log_a n - 3 \log_a a - 5 \log_a b] \\
&= 2 \log_a m + 3 \log_a n - \frac{3}{4} - \frac{5}{4} \log_a b
\end{aligned}$$

Exercise

Solve $\log_x \frac{8}{27} = 3$

Solution

$$\log_x \frac{8}{27} = 3$$

Write in exponential form

$$\frac{8}{27} = x^3$$

$$\begin{aligned}
 x &= \sqrt[3]{\frac{8}{27}} \\
 &= \frac{\sqrt[3]{8}}{\sqrt[3]{27}} \\
 &= \frac{2}{3}
 \end{aligned}$$

Exercise

Solve $\log_3 \frac{1}{9} = x$

Solution

$$\begin{aligned}
 \frac{1}{3^2} &= 3^x \\
 3^{-2} &= 3^x \\
 x &= -2
 \end{aligned}$$

Exercise

Solve $3^x = 5$

Solution

$$\begin{aligned}
 \ln 3^x &= \ln 5 \\
 x \ln 3 &= \ln 5 \\
 x &= \frac{\ln 5}{\ln 3} \\
 &\approx 1.465
 \end{aligned}$$