Section 1.6 – Minimization Problems $\geq$ (*Duality*)

The simplex method can only be used if the problem is as a standard maximization problem (problem constraints of the form ≤). With standard minimization problems, you must be minimizing the objective function, subject to constraints that are ≥ any real number. {You don't have the nonnegative restriction for the constants as you did with standard maximization.}

Formation of the Dual Problem

$$\textit{Minimize}: \; C = 16x_1 + 45x_2$$

$$\textit{Subject to}: \begin{cases} 2x_1 + 5x_2 \geq 50 \\ x_1 + 3x_2 \geq 27 \\ x_1, x_2 \geq 0 \end{cases}$$

The Coefficient Matrix

$$\begin{array}{cc} & \begin{array}{ccc} x_1 & x_2 & \end{array} \\ A = & \left[\begin{array}{cc|c} 2 & 5 & 50 \\ 1 & 3 & 27 \\ \hline 16 & 45 & 1 \end{array}\right] \end{array}$$

Transpose

$$\begin{array}{cc} & \begin{array}{ccc} y_1 & y_2 & \end{array} \\ A^T = & \left[\begin{array}{cc|c} 2 & 1 & 16 \\ 5 & 3 & 45 \\ \hline 50 & 27 & 1 \end{array}\right] \end{array}$$

$$\begin{aligned} 2y_1 + y_2 &\leq 16 \\ 5y_1 + 3y_2 &\leq 45 \\ 50y_1 + 27y_2 &= P \end{aligned}$$

Dual Problem

$$\begin{array}{ll} \textit{Maximize} & P = 50y_1 + 27y_2 \\ \textit{Subject to} & 2y_1 + y_2 \leq 16 \\ & 5y_1 + 3y_2 \leq 45 \\ & y_1, y_2 \geq 0 \end{array}$$

Formation of the Dual Problem

Given a minimization problem with $\geq$ problem constraints,

- *Use the coefficients and constants in the problem constraints and the objective function to form a matrix A with the coefficients of the objective function in the last row*
- *Interchange the rows and columns of matrix A to form the matrix* A^T *, the transpose of A.*
- *Use the rows of* A^T *to form a maximization problem with* $\leq$ *problem constraints.*

Example

Solve the following minimization problem by maximizing the dual:

$$\begin{aligned} &\textit{Minimize}: \ C = 16x_1 + 9x_2 + 21x_3 \\ &\textit{Subject to}: \ x_1 + x_2 + 3x_3 \geq 12 \\ &\qquad 2x_1 + x_2 + x_3 \geq 16 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

<u>Solution</u>

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 12 \\ 2 & 1 & 1 & 16 \\ \hline 16 & 9 & 21 & 1 \end{array}\right] \qquad A^T = \left[\begin{array}{cc|c} 1 & 2 & 16 \\ 1 & 1 & 9 \\ 3 & 1 & 21 \\ \hline 12 & 16 & 1 \end{array}\right]$$

The dual problem is:

$$\textit{Maximize}: \ P = 12y_1 + 16y_2$$

$$\textit{Subject to}: \begin{cases} y_1 + 2y_2 \leq 16 \\ y_1 + y_2 \leq 9 \\ 3y_1 + y_2 \leq 21 \\ y_1, y_2 \geq 0 \end{cases}$$

$$\begin{array}{c} \begin{array}{cccccccc} y_1 & y_2 & x_1 & x_2 & x_3 & P & & \end{array} \\ \left[\begin{array}{cccccc|c} 1 & 2 & 1 & 0 & 0 & 0 & 16 \\ 1 & 1 & 0 & 1 & 0 & 0 & 9 \\ 3 & 1 & 0 & 0 & 1 & 0 & 21 \\ \hline -12 & \langle -16 \rangle & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \quad \frac{1}{2}R_1$$

$$\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & x_3 & P \end{array} \\ \left[\begin{array}{cccccc|c} .5 & 1 & .5 & 0 & 0 & 0 & 8 \\ 1 & 1 & 0 & 1 & 0 & 0 & 9 \\ 3 & 1 & 0 & 0 & 1 & 0 & 21 \\ \hline -12 & -16 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \quad R_2 - R_1$$

$$\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 0 & 0 & 9 \\ -.5 & -1 & -.5 & 0 & 0 & 0 & -8 \\ \hline .5 & 0 & -.5 & 1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & x_3 & P \end{array} \\ \left[\begin{array}{cccccc|c} .5 & 1 & .5 & 0 & 0 & 0 & 8 \\ .5 & 0 & -.5 & 1 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 1 & 0 & 21 \\ \hline -12 & -16 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \quad R_3 - R_1$$

$$\begin{array}{ccccccc} 3 & 1 & 0 & 0 & 1 & 0 & 21 \\ -.5 & -1 & -.5 & 0 & 0 & 0 & -8 \\ \hline 2.5 & 0 & -.5 & 0 & 1 & 0 & 13 \end{array}$$

$$\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & x_3 & P \end{array} \\ \left[\begin{array}{cccccc|c} .5 & 1 & .5 & 0 & 0 & 0 & 8 \\ .5 & 0 & -.5 & 1 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 1 & 0 & 21 \\ \hline -12 & -16 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \quad R_4 + 16R_1$$

$$\begin{array}{ccccccc} -12 & -16 & 0 & 0 & 0 & 1 & 0 \\ 8 & 16 & 8 & 0 & 0 & 0 & 128 \\ \hline -4 & 0 & 8 & 0 & 0 & 1 & 128 \end{array}$$

$$\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & x_3 & P \end{array} \\ \left[\begin{array}{cccccc|c} .5 & 1 & .5 & 0 & 0 & 0 & 8 \\ \langle .5 \rangle & 0 & -.5 & 1 & 0 & 0 & 1 \\ 2.5 & 0 & -.5 & 0 & 1 & 0 & 13 \\ \hline -4 & 0 & 8 & 0 & 0 & 1 & 128 \end{array}\right] \end{array} \quad \frac{1}{.5}R_2$$

$$\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & x_3 & P \end{array} \\ \left[\begin{array}{cccccc|c} .5 & 1 & .5 & 0 & 0 & 0 & 8 \\ 1 & 0 & -1 & 2 & 0 & 0 & 2 \\ 2.5 & 0 & -.5 & 0 & 1 & 0 & 13 \\ \hline -4 & 0 & 8 & 0 & 0 & 1 & 128 \end{array}\right] \end{array} \quad \begin{array}{l} R_1 - .5R_2 \\ \\ R_3 - 2.5R_2 \\ R_4 + 4R_2 \end{array}$$

$$\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & x_3 & P \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 1 & 1 & -1 & 0 & 0 & 7 \\ 1 & 0 & -1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 2 & -5 & 1 & 0 & 8 \\ \hline 0 & 0 & 4 & 8 & 0 & 1 & 136 \end{array}\right] \end{array}$$

Min $C = 136$ @ $x_1 = 4,\ x_2 = 8,\ x_3 = 0$

Example

Solve the following minimization problem by maximizing the dual:

$$Minimize: \ C = 2x_1 + 3x_2$$

$$Subject\ to: \begin{cases} x_1 - 2x_2 \ge 2 \\ -x_1 + x_2 \ge 1 \\ x_1, x_2 \ge 0 \end{cases}$$

<u>Solution</u>

Coefficient Matrix ***Transpose***

$$A = \left[\begin{array}{cc|c} 1 & -2 & 2 \\ -1 & 1 & 1 \\ \hline 2 & 3 & 1 \end{array}\right] \qquad A^T = \left[\begin{array}{cc|c} 1 & -1 & 2 \\ -2 & 1 & 3 \\ \hline 2 & 1 & 1 \end{array}\right]$$

Dual Problem

$$Maximize: \ P = 2y_1 + y_2$$

$$Subject\ to: \begin{cases} y_1 - y_2 \le 2 \\ -2y_1 + y_2 \le 3 \\ y_1, y_2 \ge 0 \end{cases}$$

$$\begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & x_1 & x_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & 0 & 2 \\ (-2) & 1 & 0 & 1 & 0 & 3 \\ \hline \langle -2 \rangle & -1 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \ -\tfrac{1}{2}R_2$$

$$\begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & x_1 & x_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & 0 & 2 \\ 1 & -.5 & 0 & -.5 & 0 & -1.5 \\ \hline -2 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \ R_1 - R_2$$

$$\begin{array}{cccccc} 1 & -1 & 1 & 0 & 0 & 2 \\ -1 & .5 & 0 & .5 & 0 & 1.5 \\ \hline 0 & -.5 & 1 & .5 & 0 & 3.5 \end{array}$$

$$\begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & x_1 & x_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} 0 & -.5 & 1 & .5 & 0 & 3.5 \\ 1 & -.5 & 0 & -.5 & 0 & -1.5 \\ \hline -2 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \ R_3 + 2R_2$$

$$\begin{array}{cccccc} -2 & -1 & 0 & 0 & 1 & 0 \\ 2 & -1 & 0 & -1 & 0 & -3 \\ \hline 0 & -2 & 0 & -1 & 1 & -3 \end{array}$$

$$\begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & x_1 & x_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} 0 & -.5 & 1 & .5 & 0 & 3.5 \\ 1 & -.5 & 0 & -.5 & 0 & -1.5 \\ \hline 0 & -2 & 0 & -1 & 1 & -3 \end{array}\right] \end{array} \ 2R_1$$

$$\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & P & \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & -2 & -1 & 0 & -7 \\ 1 & -.5 & 0 & -.5 & 0 & -1.5 \\ \hline 0 & -2 & 0 & -1 & 1 & -3 \end{array}\right] \begin{array}{l} \\ R_2 + .5R_1 \\ R_3 + 2R_1 \end{array} \end{array}$$

$$\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & P & \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & -2 & -1 & 0 & -7 \\ 1 & 0 & -1 & -1 & 0 & -5 \\ \hline 0 & 0 & -4 & -3 & 1 & -17 \end{array}\right] \end{array}$$

No optimal solution

Caution: For the row associated with the objective function, copy the coefficients exactly as written. You derive the dual problem from the transpose matrix in reverse order, from the way you got the coefficient matrix.

Caution: When writing the dual problem, be sure to do the following:

1) Change Minimize to Maximize and change C to P.
2) Change the x-variables to y-variables.
3) Change $\le$ constraints to $\ge$ constraints.
4) Don't forget the nonnegative constraints.

Example

A computer manufacturing company has two assembly plants, plant *A* and plant *B*, and two distribution outlets, outlet **I** and outlet **II**. Plant A can assemble <u>at most</u> 700 computers a month, and plant B can assemble at most 900 computers a month. Outlet I must have <u>at least</u> 500 computers a month, and outlet II must have at least 1,000 computers a month. Transportation costs for shipping one computer from each plant to each outlet are as follows: \$7 from plant *A* to outlet I, \$5 from plant *A* to outlet II, \$4 from plant *B* to outlet I, \$3 from plant *B* to outlet II. Find a shipping schedule that will minimize the total cost of shipping the computers from the assembly plants to the distribution outlets. What is this minimum cost?

<u>*Solution*</u>

	I	**II**		***Assembly***
Plant A	\$7	\$5		
			$\le$	700
Plant B	\$4	\$3		
			$\le$	900
Min	500	1000		

Number Shipped:	From Plant A:	$x_1 + x_2 \le 700$
	From Plant B:	$x_3 + x_4 \le 900$
	From outlet I:	$x_1 + x_3 \ge 500$
	From outlet II:	$x_2 + x_4 \ge 1000$
Total Shipping Cost:		$C = 7x_1 + 5x_2 + 4x_3 + 3x_4$

$$\textit{Minimize}: \quad C = 7x_1 + 5x_2 + 4x_3 + 3x_4$$

$$\textit{Subject to} \quad \begin{cases} x_1 + x_2 \le 700 \\ x_3 + x_4 \le 900 \\ x_1 + x_3 \ge 500 \\ x_2 + x_4 \ge 1000 \\ x_1, x_2, x_3, x_4 \ge 0 \end{cases}$$

$$\textit{Minimize}: \quad C = 7x_1 + 5x_2 + 4x_3 + 3x_4$$

$$\textit{Subject to} \quad \begin{cases} -x_1 - x_2 \ge -700 \\ -x_3 - x_4 \ge -900 \\ x_1 + x_3 \ge 500 \\ x_2 + x_4 \ge 1000 \\ x_1, x_2, x_3, x_4 \ge 0 \end{cases}$$

$$A = \left[\begin{array}{cccc|c} -1 & -1 & 0 & 0 & -700 \\ 0 & 0 & -1 & -1 & -900 \\ 1 & 0 & 1 & 0 & 500 \\ 0 & 1 & 0 & 1 & 1{,}000 \\ \hline 7 & 5 & 4 & 3 & 1 \end{array}\right] \qquad A^T = \left[\begin{array}{cccc|c} -1 & 0 & 1 & 0 & 7 \\ -1 & 0 & 0 & 1 & 5 \\ 0 & -1 & 1 & 0 & 4 \\ 0 & -1 & 0 & 1 & 3 \\ \hline -700 & -900 & 500 & 1000 & 1 \end{array}\right]$$

The dual problem is:

$$\textit{Maximize}: \quad C = -700y_1 - 900y_2 + 500y_3 + 1000y_4$$

$$\textit{Subject to} \quad \begin{cases} -y_1 + y_3 \le 7 \\ -y_1 - y_4 \le 5 \\ -y_2 + y_3 \le 4 \\ -y_2 + y_4 \le 3 \\ y_1, y_2, y_3, y_4 \ge 0 \end{cases}$$

$$\begin{array}{c@{}c} & \begin{array}{ccccccccc|c} y_1 & y_2 & y_3 & y_4 & x_1 & x_2 & x_3 & x_4 & P & \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ P \end{array} & \left[\begin{array}{ccccccccc|c} -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 7 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & -1 & 0 & (1) & 0 & 0 & 0 & 1 & 0 & 3 \\ \hline 700 & 900 & -500 & \langle -1000 \rangle & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \quad \begin{array}{l} R_2 - R_4 \\ \\ \\ \\ R_5 + 1000R_4 \end{array}$$

$$\begin{array}{c@{}c} & \begin{array}{ccccccccc|c} y_1 & y_2 & y_3 & y_4 & x_1 & x_2 & x_3 & x_4 & P & \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ y_4 \\ P \end{array} & \left[\begin{array}{ccccccccc|c} -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 7 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 2 \\ 0 & -1 & (1) & 0 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 3 \\ \hline 700 & -100 & \langle -500 \rangle & 0 & 0 & 0 & 0 & 1000 & 1 & 3000 \end{array}\right] \end{array} \quad \begin{array}{l} R_1 - R_3 \\ \\ \\ \\ R_5 + 500R_3 \end{array}$$

$$\begin{array}{c@{}c} & \begin{array}{ccccccccc|c} y_1 & y_2 & y_3 & y_4 & x_1 & x_2 & x_3 & x_4 & P & \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ y_4 \\ P \end{array} & \left[\begin{array}{ccccccccc|c} -1 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 3 \\ -1 & (1) & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 2 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 3 \\ \hline 700 & \langle -600 \rangle & 0 & 0 & 0 & 0 & 500 & 1000 & 1 & 5000 \end{array}\right] \end{array} \quad \begin{array}{l} R_1 - R_2 \\ \\ R_3 + R_2 \\ R_4 + R_2 \\ R_5 + 600R_2 \end{array}$$

$$\begin{array}{c@{}c} & \begin{array}{ccccccccc|c} y_1 & y_2 & y_3 & y_4 & x_1 & x_2 & x_3 & x_4 & P & \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ y_4 \\ P \end{array} & \left[\begin{array}{ccccccccc|c} 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 3 \\ -1 & 0 & 1 & 0 & 0 & 1 & 1 & -1 & 0 & 6 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 5 \\ \hline 100 & 0 & 0 & 0 & 0 & 600 & 500 & 400 & 1 & 6200 \end{array}\right] \end{array}$$

$Min\ C = \$6,200 \qquad x_1 = 0,\ x_2 = 600,\ x_3 = 500,\ x_4 = 400$

Plant *A*:	outlet I = 0	outlet II = 600
Plant *B*:	outlet I = 500	outlet II = 400

Rewriting a linear programming problem so that it is in standard maximization or standard minimization form.

When a problem is not in one of standard forms, you may be able to rewrite it by multiplying all of the terms of the inequality by –1. {This reverses the direction of the inequality}

Example:

$$\begin{aligned} &\text{Maximize}: \ P = 3x_1 + 5x_2 \\ &\qquad\qquad 2x_1 - x_2 \geq -10 \\ &\text{Subject To}: \ x_1 + 2x_2 \leq 8 \\ &\qquad\qquad x_1, x_2 \geq 0 \end{aligned}$$

Rewritten:

$$\begin{aligned} &\text{Maximize}: \ P = 3x_1 + 5x_2 \\ &\qquad\qquad -2x_1 + x_2 \leq 10 \\ &\text{Subject To}: \ x_1 + 2x_2 \leq 8 \\ &\qquad\qquad x_1, x_2 \geq 0 \end{aligned}$$

Example:

$$\begin{aligned} &\text{Minimize}: \ C = 21x_1 + 50x_2 \\ &\qquad\qquad x_1 - 5x_2 \leq 2 \\ &\text{Subject To}: \ 3x_1 + 7x_2 \geq 17 \\ &\qquad\qquad x_1, x_2 \geq 0 \end{aligned}$$

Rewritten:

$$\begin{aligned} &\text{Minimize}: \ C = 21x_1 + 50x_2 \\ &\qquad\qquad -x_1 + 5x_2 \geq -2 \\ &\text{Subject To}: \ 3x_1 + 7x_2 \geq 17 \\ &\qquad\qquad x_1, x_2 \geq 0 \end{aligned}$$

The following problem can't be "fixed".

$$\begin{aligned} &\text{Maximize}: \ P = 3x_1 + 5x_2 \\ &\qquad\qquad 2x_1 - x_2 \geq 10 \\ &\text{Subject To}: \ x_1 + 2x_2 \leq 8 \\ &\qquad\qquad x_1, x_2 \geq 0 \end{aligned}$$

$$\begin{aligned} &\text{Maximize}: \ P = 3x_1 + 5x_2 \\ &\qquad\qquad -2x_1 + x_2 \leq -10 \\ &\text{Subject To}: \ x_1 + 2x_2 \leq 8 \\ &\qquad\qquad x_1, x_2 \geq 0 \end{aligned}$$

Even though you can fix the inequality; in standard max problems the constants can't be negative.
This example is what we call a mixed constraint problem.

Exercises **Section 1.6 – Minimization Problems ≥ (*Duality*)**

1. Find the transpose of the matrix

$a)\ \begin{bmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 4 & 4 & 4 \end{bmatrix}$ $\qquad b)\ \begin{bmatrix} -3 & -8 & 1 \\ 5 & -2 & 5 \\ 9 & 6 & -2 \\ 4 & 5 & 8 \end{bmatrix}$

2. Solve the following minimization problem by maximizing the dual:

$$Maximize:\ P = 12y_1 + 17y_2$$

$$Subject\ To: \begin{cases} 2y_1 + 3y_2 \le 21 \\ 5y_1 + 7y_2 \le 50 \\ y_1, y_2 \ge 0 \end{cases}$$

3. Solve the following minimization problem by maximizing the dual:

$$Minimize:\ C = 16x_1 + 8x_2 + 4x_3$$

$$Subject\ to \begin{cases} 3x_1 + 2x_2 + 2x_3 \ge 16 \\ 4x_1 + 3x_2 + x_3 \ge 14 \\ 5x_1 + 3x_2 + x_3 \ge 12 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

4. Customers buy 14 units of regular beer and 20 units of light beer monthly. The brewery decides to produce extra beer, beyond that needed to satisfy the customers. The cost per unit of regular beer is \$33,000 and the cost per unit of light beer is \$44,000. Every unit of regular beer brings in \$200,000 in revenue, while every unit of light beer brings in \$400,000 in revenue. The brewery wants at least \$16,000,000 in revenue. At least 18 additional units of beer can be sold. How much of each beer type should be made so as to minimize total production costs? What is the minimum cost?

5. Acme Micros markets computers with single-sided and double-sided drives. The disk driers are supplied by two other companies, Associated Electronics and Digital Drives. Associated Electronics charges \$250 for a single-sided disk drive and \$350 for a double-sided disk drive. Digital Drives charges \$290 for a single-sided disk drive and \$320 for a double-sided disk drive. Associated Electronics can supply at most 1,000 disk drives in any combination of single-sided and double-sided drives. The combined monthly total supplied by Digital Drives cannot exceed 2,000 disk drives. Acme Micros needs at least 1,200 single-sided drives and at least 1,600 double-sided drives each month. How many disk drives of each type should Acme Micros order from each supplier in order to meet its monthly demand and minimize the purchase cost? What is the minimum purchase cost?

6. A farmer can buy three types of plant food, mix A, mix B, and mix C. Each cubic yard of mix A contains 20 pounds of phosphoric acid, 10 pounds of nitrogen, and 10 pound of potash. Each cubic yard of mix B contains 10 pounds of phosphoric acid, 10 pounds of nitrogen, and 15 pound of potash. . Each cubic yard of mix C contains 20 pounds of phosphoric acid, 20 pounds of nitrogen, and 5 pound of potash. The minimum monthly requirements are 480 pounds of phosphoric acid, 320 pounds of nitrogen, and 225 pound of potash. If mix A costs $30 per cubic yard, nix B costs $36 per cubic yard, and mix C $39 per cubic yard, how many cubic yards of each mix should the farmer blend to meet the minimum monthly requirements at a minimal cost? What is the minimum cost?

7. Mark, who is ill, takes vitamin pills. Each day he must have at least 16 units of vitamin A, 5 units of vitamin B, and 20 units of vitamin C. he can choose between pill #1, which contains 8 units of A, 1 of B, and 2 of C; and pill #2, which contains 2 units of A, 1 of B, and 7 of C. Pill 1 costs 15¢, and pill 2 costs 30¢.

a) How many of each pill should be buy in order to minimize his cost?
b) What is the minimum cost?
c) For the solution in part a, Mark is receiving more than he needs of at least one vitamin. Identify that vitamin, and tell how much surplus he is receiving. Is there any ways he can avoid receiving that surplus while still meeting the other constraints and minimizing the cost?

8. One gram of soybean meal provides at least 2.5 units of vitamins and 5 calories. One gram of meat byproducts provides at least 4.5 units of vitamins and 3 calories. One gram of grain provides at least 5 units of vitamins and 10 calories. If a gram of soybean meal costs 6 cents, a gram of meat byproducts 8 cents, and a gram of grain 9 cents, what mixture of these three ingredients will provide at least 54 units of vitamins and 60 calories per serving at minimum cost? What will be the minimum cost?

9. A metropolitan school district has two high-schools that are overcrowded and two that are underenrolled. In order to balance the enrollment, the school board has decided to bus students from the crowded schools to the underenrolled schools. North Division High School has 300 more students than it should have, and South Division High School has 500 more students than it should have. Central High School can accommodate 400 additional students and Washington High School can accommodate 500 additional students. The weekly cost of busing a student from North Division to Central is $5, from North Division to Washington is $2, from South Division to Central is $3, and from South Division to Washington is $4. Determine the number of students that should be bused from each of the overcrowded schools to each of the underenrolled schools in order to balance the enrollment and minimize the cost of busing the students. What is the minimum cost?