Solution   *Section* **2.3 – Composition Functions**

Exercise

Given that $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 8$, find $(f \circ g)(x)$, $(g \circ f)(x)$ and their domain then find $(f \circ g)(7)$

Solution

$f(g(x)) = f\left(x^2 - 3x + 8\right)$ *Domain:* $(-\infty, \infty)$

$\qquad = 2(\text{-----------}) - 5$

$\qquad = 2\left(2x^2 - 3x + 8\right) - 5$

$\qquad = 2x^2 - 6x + 16 - 5$

$\qquad = 2x^2 - 6x + 11$ *Domain:* $(-\infty, \infty)$

Domain: $\mathbb{R}$

$g(f(x)) = g(2x - 5)$ *Domain:* $(-\infty, \infty)$

$\qquad = (\text{---})^2 - 3(\text{---}) + 8$

$\qquad = (2x - 5)^2 - 3(2x - 5) + 8$

$\qquad = 4x^2 - 20x + 25 - 6x + 15 + 8$

$\qquad = 4x^2 - 26x + 48$ *Domain:* $(-\infty, \infty)$

Domain: $\mathbb{R}$

$f(g(7)) = 2(7)^2 - 6(7) + 11 = 67$

Exercise

Given that $f(x) = \sqrt{x}$ and $g(x) = x - 1$, find

   a) $(f \circ g)(x) = f(g(x))$

   b) $(g \circ f)(x) = g(f(x))$

   c) $(f \circ g)(2) = f(g(2))$

Solution

   a) $(f \circ g)(x) = f(g(x))$

$\qquad\qquad = f(x - 1)$

$\qquad\qquad = \sqrt{x - 1}$

b) $(g \circ f)(x) = g(f(x))$

$$= g\left(\sqrt{x}\right)$$

$$\underline{= \sqrt{x} - 1}$$

c) $(f \circ g)(2) = f(g(2))$

$$= \sqrt{x} - 1$$

$$= \sqrt{2} - 1$$

$$\underline{= 1}$$

Exercise

Given that $f(x) = \dfrac{x}{x+5}$ and $g(x) = \dfrac{6}{x}$, find

a) $(f \circ g)(x) = f(g(x))$
b) $(g \circ f)(x) = g(f(x))$
c) $(f \circ g)(2) = f(g(2))$

Solution

a) $(f \circ g)(x) = f(g(x))$

$$= f\left(\frac{6}{x}\right)$$

$$= \frac{\frac{6}{x}}{\frac{6}{x} + 5}$$

$$= \frac{\frac{6}{x}}{\frac{6+5x}{x}}$$

$$\underline{= \frac{6}{6+5x}}$$

b) $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{x}{x+5}\right)$$

$$= \frac{6}{\frac{x}{x+5}}$$

$$= \frac{6(x+5)}{x}$$

$$\underline{= \frac{6x+30}{x}}$$

c) $(f \circ g)(2) = f(g(2))$

$$= \frac{6}{6+5(2)}$$

$$= \frac{6}{16}$$

$$= \frac{3}{8}$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x) = 2x^2 + 3x - 4$, $g(x) = 2x - 1$

Solution

$$f(g(x)) = f(2x - 1)$$

$$= 2(2x - 1)^2 + 3(2x - 1) - 4$$

$$= 2\left(4x^2 - 4x + 1\right) + 6x - 3 - 4$$

$$= 8x^2 - 8x + 2 + 6x - 7$$

$$= 8x^2 - 2x - 5$$

$$g(f(x)) = g\left(2x^2 + 3x - 4\right)$$

$$= 2\left(2x^2 + 3x - 4\right) - 1$$

$$= 4x^2 + 6x - 8 - 1$$

$$= 4x^2 + 6x - 9$$

$$f(g(-2)) = 8(-2)^2 - 2(-2) - 5$$

$$= 31$$

$$g(f(3)) = 4(3)^2 + 6(3) - 9$$

$$= 45$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x) = x^3 + 2x^2$, $g(x) = 3x$

Solution

$$f(g(x)) = f(3x)$$

$$= (3x)^3 + 2(3x)^2$$

$$= 27x^3 + 18x^2 \mid$$

$$g(f(x)) = g\left(x^3 + 2x^2\right)$$

$$= 3\left(x^3 + 2x^2\right)$$

$$= 3x^3 + 6x^2 \mid$$

$$f(g(-2)) = 27(-2)^3 + 18(-2)^2$$

$$= 288 \mid$$

$$g(f(3)) = 3(3)^3 + 6(3)^2$$

$$= 135 \mid$$

Exercise

Find $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x) = |x|$, $g(x) = -7$

Solution

$$f(g(x)) = f(-7)$$

$$= |-7|$$

$$= 7$$

$$g(f(x)) = g(|x|)$$

$$= -7$$

$$f(g(-2)) = 7\mid$$

$$g(f(3)) = -7\mid$$

Exercise

Given $f(x) = x - 3$ *and* $g(x) = x + 3$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(x + 3)$ *Domain*: $\mathbb{R}$

$$= (x - 3) + 3$$

$$= x \mid$$ *Domain*: $\mathbb{R}$

Domain: $\mathbb{R}$

b) $g(f(x)) = g(x-3)$ *Domain*: $\mathbb{R}$

            $= (x+3) - 3$

            $= x$ *Domain*: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = \dfrac{2}{3}x$ *and* $g(x) = \dfrac{3}{2}x$

  a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

  b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{3}{2}x\right)$ *Domain*: $\mathbb{R}$

            $= \dfrac{2}{3}\left(\dfrac{3}{2}x\right)$

            $= x$ *Domain*: $\mathbb{R}$

Domain: $\mathbb{R}$

b) $g(f(x)) = g\left(\dfrac{2}{3}x\right)$ *Domain*: $\mathbb{R}$

            $= \dfrac{3}{2}\left(\dfrac{2}{3}x\right)$

            $= x$ *Domain*: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = x - 1$ *and* $g(x) = 3x^2 - 2x - 1$

  a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

  b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(3x^2 - 2x - 1\right)$ *Domain*: $\mathbb{R}$

            $= 3(x-1)^2 - 2(x-1) - 1$

$$= 3\left(x^2 - 2x + 1\right) - 2x + 2 - 1$$

$$= 3x^2 - 6x + 3 - 2x + 1$$

$$\underline{= 3x^2 - 8x + 4} \,|\qquad\qquad \textbf{\textit{Domain}}: \mathbb{R}$$

Domain: $\underline{\mathbb{R}}$ |

b) $g\left(f\left(x\right)\right) = g\left(x - 1\right)$ $\qquad\qquad$ ***Domain***: $\mathbb{R}$

$$= 3x^2 - 2x - 1 - 1$$

$$\underline{= 3x^2 - 2x - 2} \,|\qquad\qquad \textbf{\textit{Domain}}: \mathbb{R}$$

Domain: $\underline{\mathbb{R}}$ |

Exercise

Given $\quad f(x) = 3x - 2 \quad and \quad g(x) = x^2 - 5$

   a) Find $\left(f \circ g\right)(x)$ and the domain of $f \circ g$

   b) Find $\left(g \circ f\right)(x)$ and the domain of $g \circ f$

Solution

a) $f\left(g\left(x\right)\right) = f\left(x^2 - 5\right)$ $\qquad\qquad$ ***Domain***: $\mathbb{R}$

$$= 3\left(x^2 - 5\right) - 2$$

$$= 3x^2 - 15 - 2$$

$$\underline{= 3x^2 - 17} \,|\qquad\qquad \textbf{\textit{Domain}}: \mathbb{R}$$

Domain: $\underline{\mathbb{R}}$ |

b) $g\left(f\left(x\right)\right) = g\left(3x - 2\right)$ $\qquad\qquad$ ***Domain***: $\mathbb{R}$

$$= \left(3x - 2\right)^2 - 5$$

$$= 9x^2 - 12x + 4 - 5$$

$$\underline{= 9x^2 - 12x - 1} \,|\qquad\qquad \textbf{\textit{Domain}}: \mathbb{R}$$

Domain: $\underline{\mathbb{R}}$ |

Exercise

Given $f(x) = x^2 - 2$ *and* $g(x) = 4x - 3$

   a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

   b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

   a) $f(g(x)) = f(4x - 3)$ ***Domain***: $\mathbb{R}$

$$= (4x - 3)^2 - 2$$

$$= 16x^2 - 24x + 9 - 2$$

$$\underline{= 16x^2 - 24x + 7} \quad\quad\quad \textbf{\textit{Domain}}: \mathbb{R}$$

    Domain: $\underline{\mathbb{R}}$

   b) $g(f(x)) = g\left(x^2 - 2\right)$ ***Domain***: $\mathbb{R}$

$$= 4\left(x^2 - 2\right) - 3$$

$$= 4x^2 - 8 - 3$$

$$\underline{= 4x^2 - 11} \quad\quad\quad\quad \textbf{\textit{Domain}}: \mathbb{R}$$

    Domain: $\underline{\mathbb{R}}$

Exercise

Given $f(x) = 4x^2 - x + 10$ *and* $g(x) = 2x - 7$

   a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

   b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

   a) $f(g(x)) = f(2x - 7)$ ***Domain***: $\mathbb{R}$

$$= 4(2x - 7)^2 - (2x - 7) + 10$$

$$= 4\left(4x^2 - 28x + 49\right) - 2x + 7 + 10$$

$$= 16x^2 - 112x + 196 - 2x + 17$$

$$\underline{= 16x^2 - 114x + 213} \quad\quad\quad \textbf{\textit{Domain}}: \mathbb{R}$$

    Domain: $\underline{\mathbb{R}}$

   b) $g(f(x)) = g\left(4x^2 - x + 10\right)$ ***Domain***: $\mathbb{R}$

$$= 2\left(4x^2 - x + 10\right) - 7$$

$$= 8x^2 - 2x + 20 - 7$$

$$\underline{= 8x^2 - 2x + 13}$$ ***Domain*: ℝ**

Domain: ℝ

Exercise

Given $f(x) = \sqrt{x}$ *and* $g(x) = x + 3$

 a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

 b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

 a) $f(g(x)) = f(x + 3)$ ***Domain*: ℝ**

$$\underline{= \sqrt{x + 3}}$$ ***Domain*: $x \geq -3$**

 Domain: $x \geq -3$

 b) $g(f(x)) = g(\sqrt{x})$ ***Domain*: $x \geq 0$**

$$\underline{= \sqrt{x} + 3}$$ ***Domain*: $x \geq 0$**

 Domain: $x \geq 0$

Exercise

Given $f(x) = \sqrt{x}$ *and* $g(x) = 2 - 3x$

 a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

 b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

 a) $f(g(x)) = f(2 - 3x)$ ***Domain*: ℝ**

$$\underline{= \sqrt{2 - 3x}}$$ ***Domain*: $x \leq \dfrac{2}{3}$**

 Domain: $x \leq \dfrac{2}{3}$

 b) $g(f(x)) = g(\sqrt{x})$ ***Domain*: $x \geq 0$**

$$\underline{= 2 - 3\sqrt{x}}$$ ***Domain*: $x \geq 0$**

 Domain: $x \geq 0$

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Exercise

Given $f(x) = 3x + 2$ *and* $g(x) = \sqrt{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(\sqrt{x})$ **Domain**: $x \geq 0$

$\qquad\qquad = 3\sqrt{x} + 2$ **Domain**: $x \geq 0$

    Domain: $x \geq 0$

b) $g(f(x)) = g(3x + 2)$ **Domain**: $\mathbb{R}$

$\qquad\qquad = \sqrt{3x + 2}$ **Domain**: $x \geq -\dfrac{2}{3}$

    Domain: $x \geq -\dfrac{2}{3}$

Exercise

Given $f(x) = x^4$ *and* $g(x) = \sqrt[4]{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f(\sqrt[4]{x})$ **Domain**: $x \geq 0$

$\qquad\qquad = \left(\sqrt[4]{x}\right)^4$

$\qquad\qquad = x$ **Domain**: $\mathbb{R}$

    Domain: $x \geq 0$

b) $g(f(x)) = g(x^4)$ **Domain**: $\mathbb{R}$

$\qquad\qquad = \sqrt[4]{x^4}$

$\qquad\qquad = x$ **Domain**: $\mathbb{R}$

    Domain: $\mathbb{R}$

Exercise

Given $f(x) = x^n$ and $g(x) = \sqrt[n]{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt[n]{x}\right)$ Domain: $\begin{cases} \text{If } n \text{ is } even & x \geq 0 \\ \text{If } n \text{ is } odd & \mathbb{R} \end{cases}$

$\qquad = \left(\sqrt[n]{x}\right)^n$

$\qquad = x$ Domain: $\mathbb{R}$

Domain: $\begin{cases} \text{If } n \text{ is } even & x \geq 0 \\ \text{If } n \text{ is } odd & \mathbb{R} \end{cases}$

b) $g(f(x)) = g\left(x^n\right)$ Domain: $\mathbb{R}$

$\qquad = \sqrt[n]{x^n}$

$\qquad = x$ Domain: $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = x^2 - 3x$ and $g(x) = \sqrt{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt{x+2}\right)$ $x + 2 \geq 0 \Rightarrow x \geq -2$

$\qquad = \left(\sqrt{x+2}\right)^2 - 3\sqrt{x+2}$

$\qquad = x + 2 - 3\sqrt{x+2}$ $x + 2 \geq 0 \Rightarrow x \geq -2$

Domain: $\{x \mid x \geq -2\}$

b) $g(f(x)) = g\left(x^2 - 3x\right)$ $\mathbb{R}$

$\qquad = \sqrt{x^2 - 3x + 2}$ $x^2 - 3x + 2 \geq 0 \Rightarrow (x = 1,\ 2) \leftrightarrow x \leq 1,\ x \geq 2$

Domain: $\{x \mid x \leq 1,\ x \geq 2\}$

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Exercise

Given $f(x) = \sqrt{x-2}$ *and* $g(x) = \sqrt{x+5}$

   a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

   b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

   a) $f(g(x)) = f(\sqrt{x+5})$ $x + 5 \geq 0 \Rightarrow x \geq -5$

            $= \sqrt{\sqrt{x+5} - 2}$ $\sqrt{x+5} - 2 \geq 0 \Rightarrow \sqrt{x+5} \geq 2$

                                        $x + 5 \geq 4$

                                        $x \geq -1$

      Domain: $\{x \mid x \geq -1\}$

   b) $g(f(x)) = g(\sqrt{x-2})$ $x - 2 \geq 0 \Rightarrow x \geq 2$

            $= \sqrt{\sqrt{x-2} + 5}$ $\sqrt{x-2} + 5 \geq 0 \Rightarrow \sqrt{x-2} \geq -5$ *Always true when* $x \geq 2$

      Domain: $\{x \mid x \geq 2\}$

Exercise

Given $f(x) = x^2 + 2$ *and* $g(x) = \sqrt{3-x}$

   a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

   b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

   a) $f(g(x)) = f(\sqrt{3-x})$ *Domain*: $x \leq 3$

            $= (\sqrt{3-x})^2 + 2$

            $= 3 - x + 2$

            $\underline{= 5 - x}$ *Domain*: $\mathbb{R}$

      Domain: $\underline{x \leq 3}$

   b) $g(f(x)) = g(x^2 + 2)$ *Domain*: $\mathbb{R}$

            $= \sqrt{3 - x^2 - 2}$

            $\underline{= \sqrt{1 - x^2}}$ *Domain*: $-1 \leq x \leq 1$

      Domain: $\underline{-1 \leq x \leq 1}$

Exercise

Given $f(x) = x^5 - 2$ *and* $g(x) = \sqrt[5]{x+2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt[5]{x+2}\right)$ ***Domain***: $\mathbb{R}$

$\qquad\quad = \left(\sqrt[5]{x+2}\right)^5 - 2$

$\qquad\quad = x + 2 - 2$

$\qquad\quad = x$ ***Domain***: $\mathbb{R}$

$\quad$ ***Domain***: $\mathbb{R}$

b) $g(f(x)) = g\left(x^5 - 2\right)$ ***Domain***: $\mathbb{R}$

$\qquad\quad = \sqrt[5]{x^5 - 2 + 2}$

$\qquad\quad = \sqrt[5]{x^5}$

$\qquad\quad = x$ ***Domain***: $\mathbb{R}$

$\quad$ ***Domain***: $\mathbb{R}$

Exercise

Given $f(x) = 1 - x^2$ *and* $g(x) = \sqrt{x^2 - 25}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\sqrt{x^2 - 25}\right)$ ***Domain***: $x \le -5 \quad x \ge 5$

$\qquad\quad = 1 - \left(\sqrt{x^2 - 25}\right)^2$

$\qquad\quad = 1 - \left(x^2 - 25\right)$

$\qquad\quad = 1 - x^2 + 25$

$\qquad\quad = 26 - x^2$ ***Domain***: $\mathbb{R}$

$\quad$ ***Domain***: $x \le -5 \quad x \ge 5$

b) $g(f(x)) = g(1 - x^2)$ *Domain*: $\mathbb{R}$

$$= \sqrt{(1 - x^2)^2 - 25}$$

$$= \sqrt{1 - 2x^2 + x^4 - 25}$$

$$\underline{= \sqrt{x^4 - 2x^2 - 24}}$$

$$x^2 = \frac{2 \pm \sqrt{4 + 96}}{2}$$

$$= \begin{cases} \dfrac{2 - 10}{2} = -4\textcolor{red}{\times} \\ \dfrac{2 + 10}{2} = 6 \end{cases}$$

$$x^2 = 6 \quad \rightarrow \quad \underline{x = \pm\sqrt{6}} \qquad\qquad \textit{Domain: } x \le -\sqrt{6} \quad x \ge \sqrt{6}$$

Domain: $\underline{x \le -\sqrt{6} \quad x \ge \sqrt{6}}$

Exercise

Given $f(x) = 2x + 3$ *and* $g(x) = \dfrac{x - 3}{2}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{x - 3}{2}\right)$ *Domain*: $\mathbb{R}$

$$= 2\left(\frac{x - 3}{2}\right) + 3$$

$$= x - 3 + 3$$

$$\underline{= x} \qquad\qquad\qquad\qquad \textit{Domain: } \mathbb{R}$$

    Domain: $\underline{\mathbb{R}}$

b) $g(f(x)) = g(2x + 3)$ *Domain*: $\mathbb{R}$

$$= \frac{1}{2}(2x + 3 - 3)$$

$$\underline{= x} \qquad\qquad\qquad\qquad \textit{Domain: } \mathbb{R}$$

    Domain: $\underline{\mathbb{R}}$

Exercise

Given $f(x) = 4x - 5$ *and* $g(x) = \dfrac{x+5}{4}$

  a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

  b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

  a) $\quad f(g(x)) = f\left(\dfrac{x+5}{4}\right)$ *Domain:* $\mathbb{R}$

$$= 4\left(\dfrac{x+5}{4}\right) - 5$$

$$= x + 5 - 5$$

$$\underline{= x}\,\big|$$ *Domain:* $\mathbb{R}$

    Domain: $\underline{\mathbb{R}}\,\big|$

  b) $\quad g(f(x)) = g(4x-5)$ *Domain:* $\mathbb{R}$

$$= \dfrac{1}{4}(4x - 5 + 5)$$

$$\underline{= x}\,\big|$$ *Domain:* $\mathbb{R}$

    Domain: $\underline{\mathbb{R}}\,\big|$

Exercise

Given $f(x) = \dfrac{4}{1-5x}$ *and* $g(x) = \dfrac{1}{x}$

  a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

  b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

  a) $\quad f(g(x)) = f\left(\dfrac{1}{x}\right)$ *Domain:* $x \neq 0$

$$= \dfrac{4}{1 - 5\dfrac{1}{x}}$$

$$= \underline{\dfrac{4x}{x-5}}\,\big|$$ *Domain:* $x \neq 5$

    Domain: $\underline{x \neq 0,\ 5}\,\big|$

  b) $\quad g(f(x)) = g\left(\dfrac{4}{1-5x}\right)$ *Domain:* $x \neq \dfrac{1}{5}$

$$= \underline{\dfrac{1 - 5x}{4}}\,\big|$$ *Domain:* $\mathbb{R}$

    Domain: $\underline{x \neq \dfrac{1}{5}}\,\big|$

Exercise

Given $f(x) = \dfrac{1}{x-2}$ and $g(x) = \dfrac{x+2}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{x+2}{x}\right)$ **Domain**: $x \neq 0$

$$= \dfrac{1}{\dfrac{x+2}{x} - 2}$$

$$= \dfrac{1}{\dfrac{x+2-2x}{x}}$$

$$= \underline{\dfrac{x}{2-x}}\,\bigg|$$ **Domain**: $x \neq 2$

Domain: $\underline{x \neq 0,\ 2}\,\big|$ $(-\infty,\ 0) \cup (0,\ 2) \cup (2,\ \infty)$

b) $g(f(x)) = g\left(\dfrac{1}{x-2}\right)$ **Domain**: $x \neq 2$

$$= \dfrac{\dfrac{1}{x-2} + 2}{\dfrac{1}{x-2}}$$

$$= \dfrac{\dfrac{1+2x-4}{x-2}}{\dfrac{1}{x-2}}$$

$$= \underline{2x-3}\,\big|$$ **Domain**: $\mathbb{R}$

Domain: $\underline{x \neq 2}\,\big|$ $(-\infty,\ 2) \cup (2,\ \infty)$

Exercise

Given $f(x) = \dfrac{3x+5}{2}$ and $g(x) = \dfrac{2x-5}{3}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{2x-5}{3}\right)$ **Domain**: $\mathbb{R}$

$$= \dfrac{3\dfrac{2x-5}{3} + 5}{2}$$

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$$= \frac{2x - 5 + 5}{2}$$

$$= \frac{2x}{2}$$

$$= x$$

Domain: $\mathbb{R}$

b) $g(f(x)) = g\left(\dfrac{3x+5}{2}\right)$ *Domain:* $\mathbb{R}$

$$= \frac{2\frac{3x+5}{2} - 5}{3}$$

$$= \frac{3x + 5 - 5}{3}$$

$$= \frac{3x}{3}$$

$$= x$$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = \dfrac{1}{1+x}$ *and* $g(x) = \dfrac{1-x}{x}$

 a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

 b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{1-x}{x}\right)$ *Domain:* $x \neq 0$

$$= \frac{1}{1 + \dfrac{1-x}{x}}$$

$$= \frac{x}{x + 1 - x}$$

$$= x$$ *Domain:* $\mathbb{R}$

Domain: $x \neq 0$

b) $g(f(x)) = g\left(\dfrac{1}{x+1}\right)$ *Domain:* $x \neq -1$

$$= \frac{1 - \dfrac{1}{x+1}}{\dfrac{1}{x+1}}$$

$$= x + 1 - 1$$

$$= x$$ *Domain:* $\mathbb{R}$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = \dfrac{x-1}{x-2}$ and $g(x) = \dfrac{x-3}{x-4}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{x-3}{x-4}\right)$ *Domain*: $x \neq 4$

$$= \dfrac{\dfrac{x-3}{x-4} - 1}{\dfrac{x-3}{x-4} - 2}$$

$$= \dfrac{\dfrac{x-3-(x-4)}{x-4}}{\dfrac{x-3-2(x-4)}{x-4}}$$

$$= \dfrac{x-3+x+4}{x-3-2x+8}$$

$$= \dfrac{2x+1}{-x+5}$$ *Domain*: $x \neq 5$

Domain: $\{x \mid x \neq 4,\ 5\}$

b) $g(f(x)) = g\left(\dfrac{x-1}{x-2}\right)$ *Domain*: $x \neq 2$

$$= \dfrac{\dfrac{x-1}{x-2} - 3}{\dfrac{x-1}{x-2} - 4}$$

$$= \dfrac{x-1-3(x-2)}{x-1-4(x-2)}$$

$$= \dfrac{x-1-3x+6}{x-1-4x+8}$$

$$= \dfrac{-2x+5}{-3x+7}$$ *Domain*: $x \neq \dfrac{7}{3}$

Domain: $\left\{x \mid x \neq 2,\ \dfrac{7}{3}\right\}$

Exercise

Given $f(x) = \dfrac{6}{x-3}$ and $g(x) = \dfrac{1}{x}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $(f \circ g)(x)$

$$f(g(x)) = f\left(\frac{1}{x}\right) \qquad\qquad \textit{Domain: } x \neq 0$$

$$= \frac{6}{\dfrac{1}{x} - 3}$$

$$= \frac{6}{\dfrac{1-3x}{x}}$$

$$= \frac{6x}{1-3x} \qquad\qquad \textit{Domain: } x \neq \frac{1}{3}$$

$$\textit{Domain: } \underline{x \neq 0, \ \frac{1}{3}} \qquad \underline{(-\infty, 0) \cup \left(0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)}$$

b) $(g \circ f)(x)$

$$g(f(x)) = g\left(\frac{6}{x-3}\right) \qquad\qquad \textit{Domain: } x \neq 3$$

$$= \frac{1}{\dfrac{6}{x-3}}$$

$$= \frac{x-3}{6} \qquad\qquad \textit{Domain: } (-\infty, \infty)$$

$$\textit{Domain: } \underline{x \neq 3} \qquad (-\infty, 3) \cup (3, \infty)$$

Exercise

Given $f(x) = \dfrac{6}{x}$ and $g(x) = \dfrac{1}{2x+1}$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{1}{2x+1}\right) \qquad\qquad \textit{Domain: } x \neq -\dfrac{1}{2}$

$$= \frac{6}{\dfrac{1}{2x+1}}$$

$$= \underline{12x + 6} \qquad\qquad \textit{Domain: } \mathbb{R}$$

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Domain: $x \neq -\dfrac{1}{2}$

b) $g(f(x)) = g\left(\dfrac{6}{x}\right)$ *Domain:* $x \neq 0$

$\qquad\qquad = \dfrac{1}{2\dfrac{6}{x}+1}$

$\qquad\qquad = \dfrac{x}{12+x}$ *Domain:* $x \neq -12$

Domain: $x \neq -12, \ 0$

Exercise

Given $f(x) = 3x - 7$ *and* $g(x) = \dfrac{x+7}{3}$

  a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

  b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

<u>Solution</u>

  a) $f(g(x)) = f\left(\dfrac{x+7}{3}\right)$ *Domain:* $\mathbb{R}$

$\qquad\qquad = 3\dfrac{x+7}{3} - 7$

$\qquad\qquad = x + 7 - 7$

$\qquad\qquad = x$ *Domain:* $\mathbb{R}$

    Domain: $\mathbb{R}$

  b) $g(f(x)) = g(3x - 7)$ *Domain:* $\mathbb{R}$

$\qquad\qquad = \dfrac{3x - 7 + 7}{3}$

$\qquad\qquad = x$ *Domain:* $\mathbb{R}$

    Domain: $\mathbb{R}$

Exercise

Given $f(x) = \dfrac{2x+3}{x-4}$ *and* $g(x) = \dfrac{4x+3}{x-2}$

  a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

  b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

<u>Solution</u>

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a) $f(g(x)) = f\left(\dfrac{4x+3}{x-2}\right)$ *Domain*: $x \neq 2$

$$= \dfrac{2\dfrac{4x+3}{x-2}+3}{\dfrac{4x+3}{x-2}-4}$$

$$= \dfrac{8x+6+3x-6}{4x+3-4x+8}$$

$$= \dfrac{11x}{11}$$

$$= x \,\rvert$$ *Domain*: $\mathbb{R}$

Domain: $x \neq 2 \,\rvert$

b) $g(f(x)) = g\left(\dfrac{2x+3}{x-4}\right)$ *Domain*: $x \neq 4$

$$= \dfrac{4\dfrac{2x+3}{x-4}+3}{\dfrac{2x+3}{x-4}-2}$$

$$= \dfrac{8x+12+3x-4}{2x+3-2x+8}$$

$$= \dfrac{11x}{11}$$

$$= x \,\rvert$$ *Domain*: $\mathbb{R}$

Domain: $x \neq 4 \,\rvert$

Exercise

Given $f(x) = \dfrac{2x+3}{x+4}$ *and* $g(x) = \dfrac{-4x+3}{x-2}$

    a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

    b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(\dfrac{-4x+3}{x-2}\right)$ *Domain*: $x \neq 2$

$$= \dfrac{2\dfrac{-4x+3}{x-2}+3}{\dfrac{4x+3}{x-2}+4}$$

$$= \dfrac{-8x+6+3x-6}{4x+3+4x-8}$$

$$= \dfrac{-5x}{-5}$$

$$= x \,\rvert$$ *Domain*: $\mathbb{R}$

Domain: $x \neq 2$

b) $g(f(x)) = g\left(\dfrac{2x+3}{x+4}\right)$ $\qquad$ *Domain*: $x \neq -4$

$$= \frac{-4\dfrac{2x+3}{x+4}+3}{\dfrac{2x+3}{x+4}-2}$$

$$= \frac{-8x-12+3x+12}{2x+3-2x-8}$$

$$= \frac{-5x}{-5}$$

$$= x \qquad\qquad \textit{Domain: } \mathbb{R}$$

Domain: $x \neq -4$

Exercise

Given $f(x) = x + 1$ $\quad$ *and* $\quad g(x) = x^3 - 5x^2 + 3x + 7$

a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

a) $f(g(x)) = f\left(x^3 - 5x^2 + 3x + 7\right)$ $\qquad$ *Domain*: $\mathbb{R}$

$$= x^3 - 5x^2 + 3x + 7 + 1$$

$$= x^3 - 5x^2 + 3x + 8 \qquad \textit{Domain: } \mathbb{R}$$

Domain: $\mathbb{R}$

b) $g(f(x)) = g(x+1)$ $\qquad\qquad\qquad$ *Domain*: $\mathbb{R}$

$$= (x+1)^3 - 5(x+1)^2 + 3(x+1) + 7$$

$$= x^3 + 3x^2 + 3x + 1 - 5\left(x^2 + 2x + 1\right) + 3x + 3 + 7$$

$$= x^3 + 3x^2 + 6x + 11 - 5x^2 - 10x - 5$$

$$= x^3 - 2x^2 - 4x + 6 \qquad \textit{Domain: } \mathbb{R}$$

Domain: $\mathbb{R}$

Exercise

Given $f(x) = x - 1$ $\quad$ and $\quad$ $g(x) = x^3 + 2x^2 - 3x - 9$

$\quad$ a) Find $(f \circ g)(x)$ and the domain of $f \circ g$

$\quad$ b) Find $(g \circ f)(x)$ and the domain of $g \circ f$

Solution

$\quad$ **a)** $f(g(x)) = f\left(x^3 + 2x^2 - 3x - 9\right)$ $\qquad$ ***Domain***: $\mathbb{R}$

$\qquad\qquad = x^3 + 2x^2 - 3x - 9 - 1$

$\qquad\qquad \underline{= x^3 + 2x^2 - 3x - 10}$ $\qquad$ ***Domain***: $\mathbb{R}$

$\qquad$ ***Domain***: $\underline{\mathbb{R}}$

$\quad$ **b)** $g(f(x)) = g(x - 1)$ $\qquad\qquad\qquad$ ***Domain***: $\mathbb{R}$

$\qquad\qquad = (x-1)^3 + 2(x-1)^2 - (x-1) - 9$

$\qquad\qquad = x^3 - 3x^2 + 3x - 1 + 2\left(x^2 - 2x + 1\right) - 3x + 3 - 9$

$\qquad\qquad = x^3 - 3x^2 - 7 + 2x^2 - 4x + 2$

$\qquad\qquad \underline{= x^3 - x^2 - 4x - 5}$ $\qquad$ ***Domain***: $\mathbb{R}$

$\qquad$ ***Domain***: $\underline{\mathbb{R}}$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ $\quad$ and $\quad$ $g(x) = x^2 - 5x$: $\quad$ $(f \circ g)(4)$

Solution

$\qquad (f \circ g)(4) = f(g(4))$

$\qquad\qquad\qquad = f(16 - 20)$

$\qquad\qquad\qquad = f(-4)$

$\qquad\qquad\qquad = -8 - 3$

$\qquad\qquad\qquad \underline{= -11}$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ $\quad$ and $\quad$ $g(x) = x^2 - 5x$: $\quad$ $(g \circ f)(4)$

Solution

$\qquad (g \circ f)(4) = g(f(4))$

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$$= g(8-3)$$
$$= g(5)$$
$$= 25 - 25$$
$$\underline{= 0}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(f \circ g)(-2)$

Solution

$$(f \circ g)(-2) = f(g(-2))$$
$$= f(4 + 10)$$
$$= f(14)$$
$$= 28 - 3$$
$$\underline{= 25}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(g \circ f)(-2)$

Solution

$$(g \circ f)(-2) = g(f(-2))$$
$$= g(-4 - 3)$$
$$= g(-7)$$
$$= 49 + 35$$
$$\underline{= 84}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(f \circ f)(-3)$

Solution

$$(f \circ f)(-3) = f(f(-3))$$
$$= f(-6 - 3)$$
$$= f(-9)$$
$$= -18 - 3$$
$$\underline{= -21}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(g \circ g)(7)$

Solution

$$(g \circ g)(7) = g(g(7))$$
$$= g(49 - 35)$$
$$= g(14)$$
$$= 196 - 70$$
$$= 126$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(f \circ g)(\sqrt{2})$

Solution

$$(f \circ g)(\sqrt{2}) = f(g(\sqrt{2}))$$
$$= f(2 - 5\sqrt{2})$$
$$= 2(2 - 5\sqrt{2}) - 3$$
$$= 4 - 10\sqrt{2} - 3$$
$$= 1 - 10\sqrt{2}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(g \circ f)(\sqrt{3})$

Solution

$$(g \circ f)(\sqrt{3}) = g(f(\sqrt{3}))$$
$$= g(2\sqrt{3} - 3)$$
$$= (2\sqrt{3} - 3)^2 - 5(2\sqrt{3} - 3)$$
$$= 12 - 12\sqrt{3} + 9 - 10\sqrt{3} + 15$$
$$= 36 - 22\sqrt{3}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(f \circ g)(2a)$

Solution

$$(f \circ g)(2a) = f\big(g(2a)\big)$$

$$= f\left(4a^2 - 10a\right)$$

$$= 2\left(4a^2 - 10a\right) - 3$$

$$\underline{= 8a^2 - 20a - 3}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(g \circ f)(3b)$

Solution

$$(g \circ f)(3b) = g\big(f(3b)\big)$$

$$= g(6b - 3)$$

$$= (6b - 3)^2 - 5(6b - 3)$$

$$= 36b^2 - 36b + 9 - 30b + 15$$

$$\underline{= 36b^2 - 66b + 24}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$: $(f \circ g)(k + 1)$

Solution

$$(f \circ g)(k + 1) = f\big(g(k + 1)\big)$$

$$= f\left((k + 1)^2 - 5k - 5\right)$$

$$= 2\left((k + 1)^2 - 5k - 5\right) - 3$$

$$= 2\left(k^2 + 2k + 1\right) - 10k - 10 - 3$$

$$= 2k^2 + 4k + 2 - 10k - 13$$

$$\underline{= 2k^2 - 6k - 11}$$

Exercise

Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x :$ $(g \circ f)(k-1)$

Solution

$$(g \circ f)(k-1) = g\big(f(k-1)\big)$$
$$= g(2k - 2 - 3)$$
$$= g(2k - 5)$$
$$= (2k - 5)^2 - 5(2k - 5)$$
$$= 4k^2 - 20k + 25 - 10k + 25$$
$$\underline{= 4k^2 - 30k + 50}$$