$$\frac{dy}{dx} = y^2 - 4$$

$$\int \frac{dy}{y^2-4} = \int dx \longrightarrow y = \pm 2 \text{ C.N}$$

$$\int \left(\frac{1/4}{y-2} - \frac{1/4}{y+2}\right) dy = \int dx \qquad \int \frac{dy}{y+a} = \int \frac{d(y+a)}{y+a}$$

$$\frac{1}{4}\left(\ln|y-2| - \ln|y+2|\right) = x + C_1$$

$$\ln\left|\frac{y-2}{y+2}\right| = 4x + 4C_1 \qquad C_2 = 4C_1$$

$$\frac{y-2}{y+2} = e^{4x+C_2} = e^{4x} e^{C_2} \quad (= C)$$

e^{C_2}: constant

$$= Ce^{4x}$$

$$y - 2 = Ce^{4x} y + 2Ce^{4x}$$

$$y(1 - Ce^{4x}) = 2 + 2Ce^{4x}$$

$$y(x) = \frac{2 + 2Ce^{4x}}{1 - Ce^{4x}}$$

$y = 2 \Rightarrow$

$$2 = \frac{2(1 + Ce^{4x})}{1 - Ce^{4x}}$$

$$2 - 2Ce^{4x} = 2 + 2Ce^{4x}$$

$$-2Ce^{4x} = 2Ce^{4x} \quad \text{if } C = 0 \checkmark$$

$y = -2 =$

$$\frac{2 + 2Ce^{4x}}{1 - Ce^{4x}}$$

$$-2 + 2Ce^{4x} = 2 + 2Ce^{4x}$$

$$-2 \neq 2 \qquad y = -2 \text{ impossible}$$

Air Resistance.

$$R(x, v) = -r(x, y) \cdot v$$

$$F = -mg + R(x, v)$$

$$m\frac{dv}{dt} = -mg - rv$$

$$\frac{dv}{dt} = -g - \frac{r}{m}v \quad -\left(g + \frac{r}{m}v\right)$$

$$\int \frac{m\,dv}{mg + rv} = -\int dt$$

$$\frac{m}{r}\int \frac{d(mg + rv)}{mg + rv} = -t + C_1$$

$$\frac{m}{r}\ln(mg + rv) = -t + C_1$$

$$\ln(mg + rv) = -\frac{r}{m}t + C_2$$

$$mg + rv = e^{-\frac{r}{m}t + C_2}$$

$$rv = C_3 e^{-rt/m} - mg$$

$$v = C e^{-rt/m} - \frac{mg}{r}$$

Linear Eqns

$y' + p(x)y = f(x)$

If $f(x) = 0 \Rightarrow y' + p(x)y = 0$ Homogeneous
→ use Separable

If $f(x) \neq 0$ non homogeneous
inhomogeneous

$y' + py = 0$

$\frac{dy}{dx} = y' = -py$

$\frac{dy}{y} = -\int p(x)\,dx$

$\ln y = -\int p\,dx$

$y = e^{-\int p(x)\,dx}$ sol'n of an homogeneous.

$y' + py = f(x)$

$y(x) = y_h + y_p$.

$y_h = e^{-\int p(x)\,dx}$

$y_p = u(x)\,y_h$

$y_p' + p y_p = f$

$(u y_h)' + p(u y_h) = f$

$u' y_h + u y_h' + p u y_h = f$

$u' y_h + u(y_h' + p y_h) = f$

$u' y_h + u(0) = f$

$u' y_h = f$

$\frac{du}{dx} y_h = f$

$$\frac{du}{dx} = \frac{f}{y_h} = \frac{f}{e^{-\int p\,dx}}$$

$$\int du = \int \frac{f(x)}{e^{-\int p(x)\,dx}}\,dx$$

$$u = \int e^{\int p(x)\,dx} f(x)\,dx$$

$$y_p = u\, e^{-\int p\,dx}$$

$$= e^{-\int p\,dx} \int f(x)\, e^{\int p\,dx}\,dx$$

$$y(x) = y_h + y_p$$

$$= (e^{-\int p\,dx} + e^{-\int p\,dx} \int f\, e^{\int p\,dx}$$

$$= e^{-\int p\,dx} \left(C + \int f(x)\, e^{\int p(x)\,dx}\right)$$

Ex $x' = x\sin t + 2te^{-\cos t}$ $\quad x(0)=1$

$x' - (\sin t)x = 2te^{-\cos t}$

(H) $e^{\int -\sin t\,dt} = e^{\cos t}$

$\int e^{\cos t}(2te^{-\cos t})dt = \int 2t\,dt = t^2$

$x(t) = \frac{1}{e^{\cos t}}(t^2 + C)$

$x(0) = 1$

$1 = \frac{1}{e}(0 + C) \Rightarrow C = e$

$x(t) = e^{-\cos t}(t^2 + e)$

Ex $x' = x\tan t + \sin t$

$x' - (\tan t)x = \sin t$

$e^{-\int \tan t\,dt} = e^{\ln\cos t} = \cos t$

$\int \cos t \sin t\,dt = -\int \cos t\,d(\cos t)$

$= -\frac{1}{2}\cos^2 t$

$x(t) = \frac{1}{\cos t}\left(-\frac{1}{2}\cos^2 t + C\right)$

$= -\frac{1}{2}\cos t + \frac{C}{\cos t}$

$x(0) = 2$ $\quad \cos 0 = 1$

$2 = -\frac{1}{2} + C$

$C = 5/2$

$x(t) = -\frac{1}{2}\cos t + \frac{5}{2}\sec t$

29/ $y' - 2y = t^2 e^{2t}$

$e^{\int -2\,dt} = e^{-2t}$

$\int e^{-2t} t^2 e^{2t}\,dt = \int t^2\,dt = \frac{1}{3}t^3$

$y(t) = \underbrace{e^{2t}}_{\frac{1}{e^{-2t}}} \left(\frac{1}{3}t^3 + C\right)$