17/

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

[13 19 10] [-1 -33 -77 / 3 -2 -14 / 4 1 -9 / -5 -25 -47 /

4 1 -9

$$\begin{bmatrix} 13 & 19 & 10 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -33 & -77 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & -14 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & -9 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -25 & -47 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & -9 \end{bmatrix} \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 4 \end{bmatrix}$$

9 3 5 2 5 18 7 0 4 5 1 4 0 1 8 5 1 4

I C E B E R G _ D E A D _ A H E A D

Sec 1-8

16/ $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ [11 21] [64 112] 25 50 29 53

23 46 40 75 55 92

$A^{-1} = \frac{1}{5-6}\begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}$

$= \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$

$[11 \quad 21]\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = [8 \quad 1]$

$[64 \quad 112]\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = [16 \quad 16]$

$[25 \quad 50]\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = [25 \quad 0]$

$[29 \quad 53]\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = [14 \quad 5]$

$[23 \quad 46]\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = [23 \quad 0]$

$[40 \quad 75]\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = [25 \quad 5]$

$[55 \quad 92]\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = [1 \quad 18]$

8 1 16 16 25 0 14 5 23 0 25 5 1 18
H A P P Y - N E W - Y E A R.

$$\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \quad R_2 - R_1$$

$$\begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} \quad R_1 + R_2 \qquad E_1 = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} \quad -\tfrac{1}{2} R_2 \qquad E_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad E_3 = \begin{pmatrix} 1 & 0 \\ 0 & -\tfrac{1}{2} \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & -\tfrac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

or

$$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \quad R_1 \leftrightarrow R_2 \qquad E_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$R_2 - R_1$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad \tfrac{1}{2} R_2 \qquad E_2 = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad E_3 = \begin{pmatrix} 1 & 0 \\ 0 & \tfrac{1}{2} \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & \tfrac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(solns are not unique)

14/

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

15/

$$\begin{bmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 : \ell_{21} = 2 \\ R_3 + R_1 : \ell_{31} = -1 \end{matrix}$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \quad R_3 - R_2 \qquad \ell_{32} = 1$$

$$\begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix} = U \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

$$\underset{A}{\begin{pmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{pmatrix}} \underset{=}{=} \underset{L}{\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}} \underset{U}{\begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}}$$

13/ $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

a) show $A^2 - 2A + 5I = 0$.

$A^2 = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

$= \begin{pmatrix} -3 & 4 \\ -4 & -3 \end{pmatrix}$

$A^2 - 2A + 5I = \begin{pmatrix} -3 & 4 \\ -4 & -3 \end{pmatrix} - 2\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$= 0$ ✓.

b) $A^{-1} = \frac{1}{5}(2I - A)$

$A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

$2I - A = 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

$A^{-1} = \frac{1}{5}(2I - A)$ ✓

c) $A\left(\frac{1}{5}(2I - A)\right) = \frac{1}{5}(2IA - A^2)$

$A^2 - 2A + 5I = 0$

$5I = 2IA - A^2$

$A\left(\frac{1}{5}(2I - A)\right) = \frac{1}{5}(5I)$

$= I \Rightarrow A^{-1} = \frac{1}{5}(2I - A)$

11. A is symmetric $\overset{?}{\Rightarrow}$ A^{-1} is symmetric

A is nonsingular $\Rightarrow$ A^{-1} is nonsingular too: $AA^{-1} = A^{-1}A = I$

A is symmetric $\Rightarrow$ $A^T = A$.

is $(A^{-1})^T = A^{-1}$?

$$A^{-1} = (A)^{-1} = (A^T)^{-1} = (A^{-1})^T \checkmark \qquad (A^T)^{-1} = (A^{-1})^T$$

$\Rightarrow$ A^{-1} is symmetric.

12. A, B, C square matrices

prove $ABC = I \Rightarrow B^{-1} = CA$?

$ABC = I \Rightarrow A$ is invertible $\Rightarrow AA^{-1} = I$

$I = AA^{-1} = ABC$

$\Rightarrow A^{-1} = BC$

$A^{-1}A = BCA$

$I = BCA \Rightarrow B$ is invertible $BB^{-1} = I$

$I = BB^{-1} = B(CA)$

$\Rightarrow B^{-1} = CA \checkmark$.

8/ A is invertible $\Rightarrow AA^{-1} = A^{-1}A = I$

$\Rightarrow (A^T)^{-1} \stackrel{?}{=} (A^{-1})^T$

$A^T(A^{-1})^T = (A^{-1}A)^T$
$= I^T$
$= I$

$(A^{-1})^T A^T = (AA^{-1})^T$
$= I^T$
$= I$ $\rightarrow$ A^T is the inverse of $(A^{-1})^T$

$\Rightarrow (A^T)^{-1} = (A^{-1})^T$

9/ C is invertible $\Rightarrow CC^{-1} = C^{-1}C = I$

if $CA = CB \Rightarrow A = B$

$CA = CB$
$C^{-1}(CA) = C^{-1}(CB)$
$(C^{-1}C)A = (C^{-1}C)B$
$IA = IB$
$A = B$ ✓

10/ if $A^2 = A \Rightarrow I - 2A = (I - 2A)^{-1}$

$(I-2A)(I-2A) = I - 2IA - 2IA + 4A^2$ $(A^2 = A)$
$= I - 4IA + 4A$ $(A = IA)$
$= I - 4IA + 4IA$
$= I$

$\rightarrow I - 2A = (I - 2A)^{-1}$

6 $\begin{pmatrix} -4 & -6 \\ 2 & 3 \end{pmatrix} = \frac{1}{-12+12} \cancel{\exists} \left(\frac{1}{0}\right).$

7 $A = \begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$ $\quad \sin^2\theta + \cos^2\theta = 1.$

$\sin^2\theta - (-\cos^2\theta) = 1 \neq 0$

$A^{-1} = \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}$

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$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 4 & 0 & 1 & 0 \\ 3 & 6 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -3 & 1 & 0 \\ 0 & 3 & 2 & -3 & 0 & 1 \end{array}\right] \begin{array}{l} 2R_1 - R_2 \\ \\ 2R_3 - 3R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 5 & -1 & 0 \\ 0 & 2 & 1 & -3 & 1 & 0 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right] \begin{array}{l} R_1 - R_3 \\ R_2 - R_3 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 2 & 2 & -2 \\ 0 & 2 & 0 & -6 & 4 & -2 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right] \begin{array}{l} \frac{1}{2} R_1 \\ \frac{1}{2} R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right]$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{pmatrix}$$

Sec 1.d | | | 1

#1 $A=\begin{pmatrix}2 & 1\\ 5 & 3\end{pmatrix}$ $B=\begin{pmatrix}3 & -1\\ -5 & 2\end{pmatrix}$

$$AB=\begin{pmatrix}2 & 1\\ 5 & 3\end{pmatrix}\begin{pmatrix}3 & -1\\ -5 & 2\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$= I \checkmark$$

$$BA=\begin{pmatrix}3 & -1\\ -5 & 2\end{pmatrix}\begin{pmatrix}2 & 1\\ 5 & 3\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$= I$$

$\therefore$ B is the inverse of A

#2/ $$\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}^{-1} = \frac{1}{6}\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}$$

$$=\begin{pmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{3}\end{pmatrix}$$

#3/ $$\begin{pmatrix}1 & 2\\ 3 & 7\end{pmatrix}^{-1} = \frac{1}{7-6}\begin{pmatrix}7 & -2\\ -3 & -1\end{pmatrix}$$

$$=\begin{pmatrix}7 & -2\\ -3 & -1\end{pmatrix}$$

#4/ $$\begin{pmatrix}-1 & 1\\ 3 & -3\end{pmatrix}^{-1} = \frac{1}{3-3}\ \cancel{A}\ \left(\frac{1}{0}\right).$$