

Series

$$\sum_{k=1}^n c = nc \qquad \sum_{k=m}^n c = (n-m+1)c$$

Arithmetic

$$\sum_{k=1}^n k = 1 + 2 + 3 + \dots + n = \frac{1}{2}n(n+1)$$

$$\sum_{k=1}^n (2k-1) = 1 + 3 + 5 + \dots + (2n-1) = n^2$$

Geometric

$$\sum_{k=1}^n ar^k = a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \dots = \frac{a}{1-r} \quad \text{if } -1 < r < 1$$

$$\sum_{k=1}^{\infty} ar^{-k} = \frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \dots = \frac{a}{r-1}$$

$$\sum_{k=1}^n r^k = r + r^2 + r^3 + \dots + r^n = \frac{r(1-r^n)}{1-r}$$

$$\sum_{k=1}^{\infty} r^k = r + r^2 + r^3 + \dots = \frac{r}{1-r}$$

$$\begin{aligned} \sum_{k=0}^n (a+kd)r^k &= a + (a+d)r + (a+2d)r^2 + \dots + (a+(n-1)d)r^{n-1} \\ &= \frac{a(1-r^n)}{1-r} + \frac{rd[1-nr^{n-1} + (n-1)r^n]}{(1-r)^2} \end{aligned}$$

$$\sum_{k=0}^{\infty} (a+kd)r^k = a + (a+d)r + (a+2d)r^2 + \dots = \frac{a}{1-r} + \frac{rd}{(1-r)^2} \quad \text{if } -1 < r < 1$$

$$\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} = (1+2+3+\dots+n)^2$$

$$\sum_{k=1}^n k^4 = 1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\begin{aligned} \cos^{-1} x &= \frac{\pi}{2} - x - \frac{1}{6}x^3 - \frac{3}{40}x^5 - \frac{5}{112}x^7 - \frac{35}{1,152}x^9 - \frac{63}{2,816}x^{11} - \frac{231}{13,312}x^{13} \\ &= \frac{\pi}{2} - x - \frac{1}{2} \frac{x^3}{3} - \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} - \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \frac{x^9}{9} - \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \frac{x^{11}}{11} - \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12} \frac{x^{13}}{13} \\ &= \frac{\pi}{2} - \sum_{n=1}^{\infty} \frac{\left(\frac{1}{2}\right)_{n-1}}{(n-1)!(2n-1)} x^{2n-1} \end{aligned}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\sin^{-1} x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9 + \dots$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \dots$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}, \quad |x| \leq 1$$

$$\cot x = x^{-1} - \frac{1}{3}x - \frac{1}{45}x^3 - \frac{2}{945}x^5 - \frac{1}{4,725}x^7 - \dots$$

$$\cot^{-1} x = \frac{\pi}{2} + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1} x^{2n+1} = \frac{\pi}{2} - x + \frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}x^7 - \frac{1}{9}x^9 + \dots$$

$$\cot^{-1}\left(\frac{1}{x}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 + \dots$$

$$\sec x = 1 + \frac{1}{2}x^2 + \frac{5}{25}x^4 + \frac{61}{720}x^6 + \frac{277}{8,064}x^8 + \dots$$

$$\csc^{-1} x = \sum_{n=0}^{\infty} \frac{(2n)!}{n!n!} \frac{1}{4^n(2n+1)} x^{-(2n+1)} = x^{-1} + \frac{1}{6}x + \frac{7}{360}x^3 + \frac{31}{15,120}x^5 + \dots$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$$

$$\cosh^{-1}(x+1) = \sqrt{2x} \left(1 - \frac{1}{12}x + \frac{3}{160}x^2 - \frac{5}{896}x^3 + \dots \right)$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

$$\sinh x = x - \frac{1}{6}x^3 + \frac{3}{40}x^5 - \frac{5}{112}x^7 + \frac{35}{1,152}x^9 - \dots$$

$$\tanh x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2,835}x^9 - \dots$$

$$\tanh^{-1} x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)} = x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \frac{1}{9}x^9 + \dots$$

$$\coth x = x^{-1} + \frac{1}{3}x - \frac{1}{45}x^3 + \frac{2}{945}x^5 - \frac{1}{4,725}x^7 + \dots$$

$$\coth^{-1}(1+x) = \frac{1}{2}\ln 2 - \frac{1}{2}\ln x + \frac{1}{4}x - \frac{1}{16}x^2 + \dots$$

$$\operatorname{sech} x = 1 - \frac{1}{2}x^2 + \frac{5}{24}x^4 - \frac{61}{720}x^6 + \frac{277}{8,064}x^8 - \dots$$

$$\operatorname{sech}^{-1} x = \ln 2 - \ln x - \frac{1}{4}x^2 - \frac{3}{32}x^4 - \dots$$

$$\operatorname{csch} x = x^{-1} - \frac{1}{6}x + \frac{7}{360}x^3 + \frac{31}{15,120}x^5 + \dots$$

$$\operatorname{csch}^{-1} x = \ln 2 - \ln x + \frac{1}{4}x^2 - \frac{3}{32}x^4 + \frac{5}{96}x^6 - \dots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\ln x = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} (x-1)^{n+1} = x-1 - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \dots \quad 0 < x \leq 2$$

$$\ln x = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} (x-1)^{n+1} = 2 \left[\frac{x-1}{x+1} + \frac{1}{3} \left(\frac{x-1}{x+1} \right)^3 + \frac{1}{5} \left(\frac{x-1}{x+1} \right)^5 + \dots \right] \quad x > 2$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots \quad -1 \leq x < 1$$

$$\frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - \dots + (-1)^n (x-1)^n$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \quad |x| < 1$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \dots \quad |x| < 1$$

$$(1+x)^\alpha = 1 + \alpha x + \alpha(\alpha-1) \frac{x^2}{2!} + \alpha(\alpha-1)(\alpha-2) \frac{x^3}{3!} + \dots$$

$$\sqrt{x} = 1 + \frac{1}{2}(x-1) - \frac{1}{2!} \frac{1}{2^2} (x-1)^2 + \frac{1}{3!} \frac{1 \cdot 3}{2^3} (x-1)^3 - \frac{1}{4!} \frac{1 \cdot 3 \cdot 5}{2^4} (x-1)^4 + \frac{1}{5!} \frac{1 \cdot 3 \cdot 5 \cdot 7}{2^5} (x-1)^5 - \dots$$

$$= 1 + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!!}{n! 2^n} (x-1)^n$$

$$\sqrt{1-x} = \sum_{n=0}^{\infty} (-1)^n \binom{1/2}{n} x^n$$

$$\sqrt{1+x} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(1-2n)(n!)^2 \binom{4n}{n}} x^n$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \dots \quad -\infty < x < \infty$$

$$a^x = e^{x \ln a} = 1 + \frac{x \ln a}{1!} + \frac{(x \ln a)^2}{2!} + \frac{(x \ln a)^3}{3!} + \dots \quad -\infty < x < \infty$$

$$e^{\sin x} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^5}{15} + \dots \quad -\infty < x < \infty$$

$$e^{\cos x} = e \left(1 - \frac{x^2}{2} + \frac{x^4}{6} - \frac{31x^6}{720} + \dots \right) \quad -\infty < x < \infty$$

$$e^{\tan x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{3x^4}{8} + \dots \quad |x| < \frac{\pi}{2}$$

$$e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} + \dots + \frac{(\sqrt{2})^n \sin\left(\frac{n\pi}{4}\right) x^n}{n!} + \dots \quad -\infty < x < \infty$$

$$e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \dots + \frac{(\sqrt{2})^n \cos\left(\frac{n\pi}{4}\right) x^n}{n!} + \dots \quad -\infty < x < \infty$$

Legendre Polynomial

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

$$P_n(x) = \frac{1}{2^n} \sum_{k=0}^{n/2} \frac{(-1)^k (2n-2k)!}{k!(n-k)!(n-2k)!} x^{n-2k}$$

The n th term of an arithmetic sequence: $a_n = a_1 + (n-1)d$

$$n = \frac{a_n - a_1}{d} + 1$$

Sum of a certain number of terms of an arithmetic sequence $S_n = \frac{n}{2}(a_1 + a_n)$

The **arithmetic mean** of two numbers a and b is defined as $\frac{a+b}{2}$

The n th term of a geometric sequence: $a_n = a_1 r^{n-1}$ $r = \frac{a_n}{a_{n-1}}$

Sum of n th term of a geometric sequence: $S_n = a_1 \frac{1-r^n}{1-r}$

Sum of infinite term of a geometric sequence: $S_n = \frac{a_1}{1-r}$