5.7 Hwk
#1

$4+8+\cdots+4n = 2n(n+1)$

For $n=1 \Rightarrow \quad 4 \overset{?}{=} 2(1+1)$

$4 = 4$ ✓

P_1 is True

P_k is true : $4+\cdots+4k = 2k(k+1)$

is P_{k+1} : $4+\cdots+4k+4(k+1) \overset{?}{=} 2(k+1)(k+2)$

$4+\cdots+4k+4(k+1) = 2k(k+1) + 4(k+1)$

$= 2(k+1)(k+2)$ ✓

P_{k+1} is also true.

∴ By the mathematical induction, the given proof is completed

#12) $1+5+9+\cdots+(4n-3)=n(2n-1)$

For $n=1 \Rightarrow \quad 1 = 1(2-1)$

$1 = 1 \checkmark$ P_1 is true.

P_k is true: $1+\cdots+(4k-3)=k(2k-1)$

is P_{k+1}:

$1+\cdots+(4k-3)+(4(k+1)-3)\stackrel{?}{=}(k+1)(2(k+1)-1)$

$1+\cdots+(4k-3)+(4k+1)\stackrel{?}{=}(k+1)(2k+1)$

$1+\cdots+(4k-3)+(4k+1)=k(2k-1)+4k+1$

$=2k^2-k+4k+1$

$=2k^2+3k+1$

$=(k+1)(2k+1) \checkmark$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the given proof is completed

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

For $n=1 \Rightarrow 1^2 = \frac{1(2)(3)}{6}$

$1 = 1$ ✓ P_1 is true.

P_k is true: $1^2 + \cdots + k^2 = \frac{1}{6}k(k+1)(2k+1)$

is P_{k+1}: $1^2 + \cdots + k^2 + (k+1)^2 = \frac{1}{6}(k+1)(k+2)(2k+3)$?

$$1^2 + \cdots + k^2 + (k+1)^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2 \frac{6}{6}$$

$$= \frac{1}{6}(k+1)\left(k(2k+1) + 6(k+1)\right)$$

$$= \frac{1}{6}(k+1)\left(2k^2 + k + 6k + 6\right)$$

$$= \frac{1}{6}(k+1)(k+2)(2k+3)$$

P_{k+1} is also true

$\therefore$ By the mathematical induction, the given proof is completed.

1) $\sum_{k=0}^{19} \frac{k-3}{4} = \frac{1}{4}\left(\sum_{k=0}^{19} k - \sum_{k=0}^{19} 3\right)$

$$= \frac{1}{4}\left(\frac{1}{2}\,19\,(20) - 3\,(19-0+1)\right)$$

$$= \frac{1}{4}\left(190 - 60\right)$$

$$= \frac{130}{4}$$

$$= \boxed{\frac{65}{2}}$$

$$\frac{1}{4}\left(\frac{19}{2}(1+19) - 60\right)$$

#2/ $\sum_{k=2}^{50}(2000-3k) = \sum_{k=2}^{50} 2\,10^3 - 3\sum_{k=2}^{50} k$

$$= 2\times10^3\,(\underbrace{50-2+1}_{49}) - 3\,\frac{49}{2}(2+50)$$

$$= 98000 - 3822$$

$$= 94,178$$

$$\sum = \frac{49}{2}\left(\overbrace{2000-6}^{1994} + \overbrace{2000-3(50)}^{1850}\right)$$

$$= \frac{49}{2}\left(4,000 - 156\right)$$

$$= \boxed{94,178}$$

$\cdot\overline{78} = .787878\ldots$

$= \underbrace{.78}_{a_1} + .0078 + .000078 + \ldots$

$r = \dfrac{78\times10^{-4}}{78\,10^{-2}} = 10^{-2} = .0$

Given

a_{20} : $a_2 = 8$ $a_7 = -7$ arithm

$d = \boxed{\dfrac{-7-8}{7-2}} = \dfrac{-15}{5} = -3$ point

$a_n = a_1 + (n-1)d$

$a_2 = a_1 + (-3) = 8$

$a_1 = 11$ point

$a_{20} = 11 + 19(-3)$

$= 11 - 57$

$= -46$

a_9: $a_2 = 4$ $a_5 = 32$ Geom

$$\frac{32}{4}^{\frac{1}{5-2}}$$

$$r = (8)^{1/3}$$
$$= (2^3)^{1/3}$$
$$\boxed{= 2}$$

$$a_n = a_1 r^{n-1}$$
$$a_2 = a_1 (2)^1 = 4$$
$$\boxed{a_1 = 2}$$

$$a_9 = 2 (2)^8$$
$$\boxed{= 2^9}$$

$$\sum_{k=1}^{50} 3 =$$

$$\sum_{k=20}^{45} 4 = 4(45 - 20 + 1)$$
$$\boxed{= 104}$$

$$\sum_{k=1}^{5} (2k-3) = (2-3) + (4-3) + (6-3) + (8-3) + (10-3)$$
$$= -1 + 1 + 3 + 5 + 7$$
$$\boxed{= 15}$$

$$\sum_{n=1}^{\infty} 2\left(\frac{3}{2}\right)^{n-1} = \infty \qquad \left|\frac{3}{2}\right| \geq 1$$

$$\sum_{k=1}^{\infty} 3\left(\frac{2}{3}\right)^{n-1} = \frac{3}{1-\frac{2}{3}} \qquad \left|\frac{2}{3}\right| < 1$$

$$= 9 \qquad \frac{3}{\frac{1}{3}}$$

$$\frac{x}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$x = A(x+3) + B(x-1)$$

$$x^1 \quad A + B = 1 \longrightarrow B = 1 - \frac{1}{4} = \frac{3}{4}$$

$$x^0 \quad 3A - B = 0$$

$$4A = 1$$

$$A = \frac{1}{4}$$

$$\frac{x}{x^2+2x-3} = \frac{1/4}{x-1} + \frac{3/4}{x+3}$$

$$\frac{1}{(2x+3)(4x-1)} = \frac{A}{2x+3} + \frac{B}{4x-1}$$

$$1 = A(4x-1) + B(2x+3)$$

$$x^1 \quad 4A + 2B = 0 \Rightarrow (1)$$

$$x^0 \quad -{}^{4}A + \overset{12}{\cancel{3}}B = \cancel{1}^{\,4}$$

$$14B = 4$$

$$\boxed{B = \frac{4}{14} = \frac{2}{7}}$$

$$(1) \quad 4A = -2\left(\frac{2}{7}\right)$$

$$A = -\frac{4}{7(4)}$$

$$\boxed{= -\frac{1}{7}}$$

$$\boxed{\frac{1}{(2x+3)(4x-1)} = \frac{-1/7}{2x+3} + \frac{2/7}{4x-1}}$$

$a_n = (-1)^{n+1} \frac{n}{n+1}$ 1st 4 & a_9

$a_1 = (-1)^2 \frac{1}{2} = \frac{1}{2}$

$a_2 = (-1)^3 \frac{2}{3} = -\frac{2}{3}$

$a_3 = (-1)^4 \frac{3}{4} = \frac{3}{4}$

$a_4 = (-1)^5 \frac{4}{5} = -\frac{4}{5}$

$a_9 = (-1)^{10} \frac{9}{10} = \frac{9}{10}$

2

$2 + 5 + 8 + \dots + 17 = \sum_{n=1}^{6} 3n-1$

$d = 3$ $(5-2)$

$a_n = 2 + (n-1)(3)$

$= 3n - 1$

$\frac{17-2}{3} = 5 + 1$

$h = 15 \quad w = 10$

$x = 5$

$$\frac{x^2}{25^2} + \frac{y^2}{20^2} = 1$$

$$\frac{y^2}{20^2} = 1 - \left(\frac{5}{25}\right)^2 \qquad \left(\frac{5}{25}\right)^2$$

$$\frac{y^2}{20^2} = 1 - \frac{1}{25}$$

$$y^2 = \left(\frac{20}{5}\right)^2 (25-1)$$

$$15^2 \; ? \; 16(24)$$

$$225 < 384$$

clear.