$\int x^5 e^{4x} dx$

	$\int e^{4x} dx$
$+\ x^5$	$\frac{1}{4} e^{4x}$
$-\ 5x^4$	$\frac{1}{2^4} e^{4x}$
$+\ 20x^3$	$\frac{1}{2^6} e^{4x}$
$-\ 60x^2$	$\frac{1}{2^8} e^{4x}$
$+\ 120x$	$\frac{1}{2^{10}} e^{4x}$
$-\ 120$	$\frac{1}{2^{12}} e^{4x}$

$$\int x^5 e^{4x} dx = e^{4x}\Big(\frac{1}{4}x^5 - \frac{5}{2^4}x^4 + \frac{5}{2^4}x^3 - \frac{15}{2^6}x^2$$
$$+ \frac{15}{2^7}x - \frac{15}{2^9}\Big) + C$$

$\int \cos 2x\, e^{3x} dx$

	$\int \cos 2x$
$+\ e^{3x}$	$\frac{1}{2}\sin 2x$
$-\ 3e^{3x}$	$-\frac{1}{4}\cos 2x$
$+\ 9e^{3x}$	integration

$$\int \cos 2x\, e^{3x} dx = e^{3x}\Big(\frac{1}{2}\sin 2x + \frac{3}{4}\cos 2x\Big) - \frac{9}{4}\int \cos 2x\, e^{3x} dx$$

$$\Big(1+\frac{9}{4}\Big)\int \cos 2x\, e^{3x} dx = \frac{e^{3x}}{4}(2\sin 2x + 3\cos 2x)$$

$$\frac{13}{4}\int \cos 2x\, e^{3x} dx = \frac{1}{4}e^{3x}(2\sin 2x + 3\cos 2x)$$

$$\int \cos 2x\, e^{3x} dx = \frac{1}{13}e^{3x}(2\sin 2x + 3\cos 2x) + C$$

$\int x^6 \cos 4x \, dx$

		$\int \cos 4x \, dx$
$+$	x^6	$\frac{1}{4} \sin 4x$
$-$	$6x^5$	$-\frac{1}{2^4} \cos 4x$
$+$	$30x^4$	$-\frac{1}{2^6} \sin 4x$
$-$	$120x^3$	$\frac{1}{2^8} \cos 4x$
$+$	$360x^2$	$\frac{1}{2^{10}} \sin 4x$
$-$	$720x$	$-\frac{1}{2^{12}} \cos 4x$
$+$	720	$-\frac{1}{2^{14}} \sin 4x$

$$\int x^6 \cos 4x \, dx = \left(\frac{1}{4} x^6 - \frac{15}{2^5} x^4 + \frac{45}{2^7} x^2 - \frac{45}{2^{10}}\right) \sin 4x + \left(\frac{3}{2^3} x^5 - \frac{15}{2^5} x^3 + \frac{15}{2^8}\right) \cos 4x + C$$

$$\begin{cases} x^n e^{mx} \\ x^n \cos mx \\ e^{nx} \cos mx \\ \quad\;\, \sin \end{cases}$$

$$\int \sin^4 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1-2\cos 2x+\cos^2 2x)\,dx$$

$$= \frac{1}{4}\int (\underbrace{1-2\cos 2x+\tfrac{1}{2}}+\tfrac{1}{2}\cos 4x)\,dx$$

$$= \frac{1}{4}\int \left(\frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x\right) dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x-\sin 2x+\frac{1}{8}\sin 4x\right)$$

$$= \frac{3}{8}x-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$$

$$\int \cos^7 x\,dx = \int \cos^6 x\cos x\,dx \quad (\cos^2 x)^3$$

$$= \int (1-\sin^2 x)^3\,d(\sin x)$$

$$= \int (1-3\sin^2 x+3\sin^4 x-\sin^6 x)\,d(\sin x)$$

$$= \sin x-\sin^3 x+\frac{3}{5}\sin^5 x-\frac{1}{7}\sin^7 x+C$$

$$\int_0^{\pi/2} \cos^{10} x\,dx = \frac{\pi}{2}\,\frac{9}{10}\,\frac{7}{8}\,\frac{5}{6}\,\frac{3}{4}\,\frac{1}{2}$$

$$= \frac{63\pi}{2^9}$$

$$\int_0^{\pi/2} \cos^9 x\,dx = \frac{8}{9}\,\frac{6}{7}\,\frac{4}{5}\,\frac{2}{3}$$

$$= \frac{2^7}{315}$$

$$\int \sqrt{9-4x^2}\,dx \qquad 2x = 3\sin\theta \qquad \sqrt{9-4x^2} = 3\cos\theta$$

$$dx = \frac{3}{2}\cos\theta\,d\theta$$

$$\int \sqrt{9-4x^2}\,dx = \int 3\cos\theta \,\frac{3}{2}\cos\theta\,d\theta$$

$$= \frac{9}{2}\int \cos^2\theta\,d\theta$$

$$= \frac{9}{4}\int (1+\cos 2\theta)\,d\theta$$

$$= \frac{9}{4}\left(\theta + \frac{1}{2}\sin 2\theta\right)$$

$$= \frac{9}{4}\left(\theta + \sin\theta\cos\theta\right)$$

$$= \frac{9}{4}\left(\sin^{-1}\frac{2x}{3} + \frac{2x}{3}\,\frac{\sqrt{9-4x^2}}{3}\right)$$

$$= \frac{9}{4}\sin^{-1}\frac{2x}{3} + \frac{1}{2}x\sqrt{9-4x^2} + C$$

$$\int \frac{dx}{\sqrt{x^2-25}} \qquad x = 5\sec\theta \qquad \sqrt{x^2-25} = 5\tan\theta$$

$$dx = 5\sec\theta\tan\theta\,d\theta$$

$$\int \frac{dx}{\sqrt{x^2-25}} = \int \frac{5\sec\theta\tan\theta\,d\theta}{5\tan\theta}$$

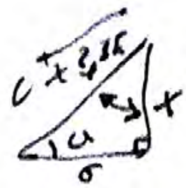
$$= \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta|$$

$$= \ln\left|\frac{x}{5} + \frac{\sqrt{x^2-25}}{5}\right| + C$$

$$\int \frac{dx}{x^2\sqrt{x^2+36}} \qquad x = 6\tan\theta \qquad \sqrt{x^2+36} = 6\sec\theta$$
$$dx = 6\sec^2\theta\, d\theta$$

$$\int \frac{dx}{x^2\sqrt{x^2+36}} = \int \frac{6\sec^2\theta\, d\theta}{36\tan^2\theta\,(6\sec\theta)}$$
$$= \frac{1}{36}\int \frac{\sec\theta\, d\theta}{\tan^2\theta}$$
$$= \frac{1}{36}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$
$$= \frac{1}{36}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$
$$= \frac{1}{36}\int \frac{d(\sin\theta)}{\sin^2\theta}$$
$$= -\frac{1}{36\sin\theta}$$
$$= -\frac{1}{36}\,\frac{\sqrt{x^2+36}}{x} + C$$



$$\int \frac{dx}{x^2-5x+6}$$
$$\frac{1}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$$
$$1 = Ax - 3A + Bx - 2B$$
$$x \quad {}^{2}A + {}^{3}B = 0$$
$$x^0 \quad -3A - 2B = 1$$
$$B = 1 \rightarrow A = -1$$

$$0A + B \quad 0 \quad 1$$
$$-3A - 2B \quad 1 \quad -2$$
$$B = 1$$
$$A = -1$$

$$\int \frac{dx}{x^2-5x+6} = -\int \frac{dx}{x-2} + \int \frac{d(x-3)}{x-3}$$
$$= -\ln|x-2| + \ln|x-3| + C$$

$$\int \frac{8x+5}{2x^2+3x+1}\,dx$$

$$\frac{8x+5}{2x^2+3x+1} = \frac{A}{x+1} + \frac{B}{2x+1}$$

$$8x+5 = 2Ax + A + Bx + B \qquad B(x+1)$$

$$\begin{array}{ll} x^1 & 2A + B = 8 \rightarrow \boxed{B = 8-6 = 2} \\ x^0 & A + B = 5 \\ \hline & \boxed{A = 3} \end{array}$$

$$\int \frac{8x+5}{2x^2+3x+1}\,dx = 3\int \frac{dx}{x+1} + \int \frac{d(2x+1)}{2x+1}$$

$$= 3\ln|x+1| + \ln|2x+1| + C$$

$$\int_0^\infty \frac{dx}{x^2+1} = \tan^{-1}x \Big|_0^\infty \qquad \rightarrow \tan = \frac{\sin}{\cos} = \infty$$

$$= \tan^{-1}\infty - \tan^{-1}0 \qquad \infty = \frac{1}{0^+}$$

$$= \frac{\pi}{2}$$

$$\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+4)^{3/2}} = \frac{1}{2}\int_{-\infty}^{\infty} (x^2+4)^{-3/2}\,d(x^2+4)$$

$$= -(x^2+4)^{-1/2}\Big|_{-\infty}^{\infty}$$

$$= -\frac{1}{\sqrt{x^2+4}}\Big|_{-\infty}^{\infty}$$

$$= 0$$

$$\int_{-\infty}^{0} x e^{-4x} dx$$

$$\begin{array}{r|l} & \int e^{-4x} \\ +x & -\frac{1}{4} e^{-4x} \\ -1 & \frac{1}{16} e^{-4x} \end{array}$$

$$\int_{-\infty}^{0} x e^{-4x} dx = \left(-\frac{1}{4}x - \frac{1}{16}\right) e^{-4x} \Big|_{-\infty}^{0}$$

$$= -\frac{1}{16} - \infty$$

$$= -\infty$$

$$y' = (1+y^2) e^x = \frac{dy}{dx}$$

$$\int \frac{dy}{1+y^2} = \int e^x dx$$

$$\tan^{-1} y = e^x + C$$

$$y = \tan(e^x + C)$$

$$\tan\theta \cdot \frac{dr}{d\theta} + r = \sin^2\theta$$

$$\frac{dr}{d\theta} + \frac{1}{\tan\theta} r = \sin^2\theta \cdot \frac{\cos\theta}{\sin\theta}$$

$$= \sin\theta \cos\theta$$

$$e^{\int \frac{\cos\theta}{\sin\theta} d\theta} = e^{\int \frac{d(\sin\theta)}{\sin\theta}}$$

$$= e^{\ln(\sin\theta)}$$

$$= \sin\theta$$

$$\int \sin^2\theta \cos\theta d\theta = \int \sin^2\theta \, d(\sin\theta)$$

$$= \frac{1}{3} \sin^3\theta$$

$$r(\theta) = \frac{1}{\sin\theta}\left(\frac{1}{3}\sin^3\theta + C\right)$$

$$= \frac{1}{3}\sin^2\theta + \frac{C}{\sin\theta}$$

$$\frac{dy}{dt} = \frac{t+2}{y} \qquad y(0)=2$$

$$\int y\,dy = \int (t+2)\,dt$$

$$\frac{1}{2}y^2 = \frac{1}{2}t^2 + 2t + C$$

$y(0)=2$

$$\boxed{2 = C}$$

$$\frac{1}{2}y^2 = \frac{1}{2}t^2 + 2t + 2$$

$$\boxed{y^2 = t^2 + 4t + 4}$$

$$y' + 4xy = x^3 e^{x^2} \qquad y(0) = -1$$

$$e^{\int 4x\,dx} = \boxed{e^{2x^2}}$$

$$\int x^3 e^{x^2} e^{2x^2}\,dx = \int x^3 e^{3x^2}\,dx$$

$$= \frac{1}{6}\int x^2 e^{3x^2}\,d(3x^2)$$

$$= \frac{1}{18}\int (3x^2)\, e^{3x^2}\,d(3x^2)$$

u	$\int e^u$
$+u$	e^u
-1	e^u

$u = 3x^2$

$$= \frac{1}{18}\int u e^u\,du$$

$$= \frac{1}{18}(u-1)e^u$$

$$= \boxed{\frac{1}{18}(3x^2-1)e^{3x^2}}$$

$$y(x) = \frac{1}{e^{2x^2}}\left(\frac{1}{18}(3x^2-1)e^{3x^2} + C\right)$$

$$= \frac{1}{18}(3x^2-1)e^{x^2} + \frac{C}{e^{2x^2}} \qquad \frac{1}{18} - 1$$

$$-1 = -\frac{1}{18} + C \rightarrow C = -\frac{17}{18}$$

$$\boxed{y(x) = \frac{1}{18}(3x^2-1)e^{x^2} - \frac{17}{18e^{2x^2}}}$$