Cal III 1/21

Ex $\vec{u} = \langle -1, 3, 1 \rangle \quad \vec{v} = \langle 4, 7, 0 \rangle$

a) $2\vec{u} + 3\vec{v} = 2\langle -1, 3, 1 \rangle + 3\langle 4, 7, 0 \rangle$

$= \langle -2, 6, 2 \rangle + \langle 12, 21, 0 \rangle$

$= \langle 10, 27, 2 \rangle$

b) $\vec{u} - \vec{v} = \langle -1, 3, 1 \rangle - \langle 4, 7, 0 \rangle$

$= \langle -5, -4, 1 \rangle$

c) $\left| \frac{1}{2}\vec{u} \right| = \frac{1}{2}|\vec{u}| \qquad \langle -1, 3, 1 \rangle$

$= \frac{1}{2}\sqrt{1+9+1}$

$= \frac{1}{2}\sqrt{11}$

$\vec{v} = \langle v_1, v_2 \rangle$ 2-dim.

$\vec{v} = \langle v_1, v_2, v_3 \rangle$ 3-dim

$|\vec{v}| = \sqrt{v_1^2 + v_2^2}$

$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^3}$

$\vec{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$\vec{0} = \langle 0, 0, 0 \rangle = 0$

0 zero

Ex $P(-3, 4, 1)$ $Q(-5, 2, 2)$

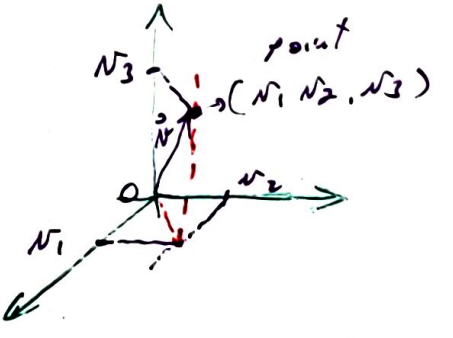
$$\vec{PQ} = \langle -5+3, 2-4, 2-1 \rangle$$
$$= \langle -2, -2, 1 \rangle$$

$$\|\vec{PQ}\| = \sqrt{4+4+1}$$
$$= 3$$

Unit vectors. (standard)

$$\hat{i} = \langle 1, 0, 0 \rangle$$
$$\hat{j} = \langle 0, 1, 0 \rangle$$
$$\hat{k} = \langle 0, 0, 1 \rangle$$

$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$
$$= v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$$




Unit Vectors (magnitude = 1)

direction.

unit vector of $\vec{u} = \dfrac{\vec{u}}{\|\vec{u}\|}$ $\quad |\vec{u}| \ \|\vec{u}\|$

Ex $P_1(1,0,1) \to P_2(3,2,0)$

$\overrightarrow{P_1P_2} = \langle 2, 2, -1 \rangle$

unit vector $= \dfrac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|}$

$= \dfrac{\langle 2, 2, -1 \rangle}{\sqrt{4+4+1}}$

$= \langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \rangle$

or $= \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$

check ☺

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Ex $\vec{v} = 3\hat{i} - 4\hat{j}$

$|\vec{v}| = \sqrt{9+16}$

$= 5$

$\vec{u} = \dfrac{3\hat{i} - 4\hat{j}}{5}$

$= \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$

$\vec{v} = 3\hat{i} - 4\hat{j}$

$= 5\left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)$

5 → magnitude or length.

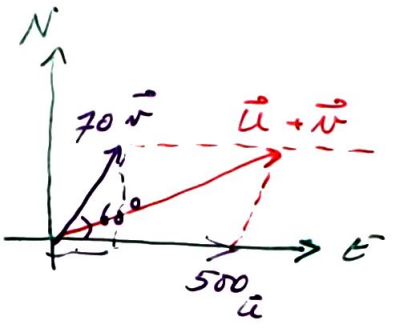
$\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$ → direction of motion

Ex

$\vec{u} = \langle 500, 0 \rangle$

$\vec{v} = \langle 70\cos 60^\circ, 70 \sin 60^\circ \rangle$

$= \langle 35, 35\sqrt{3} \rangle$




$\vec{u} + \vec{v} = \langle 535, 35\sqrt{3} \rangle$

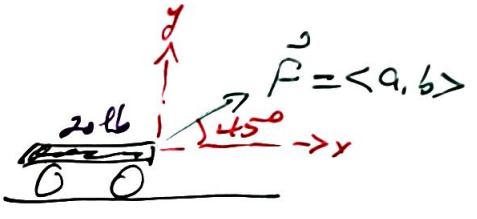
$|\vec{u} + \vec{v}| = \sqrt{535^2 + 3(35^2)}$

$\approx 538.4$ | ground speed grd

direction:

$\theta = \tan^{-1}\left(\frac{35\sqrt{3}}{535}\right) \approx 6.5^\circ$ | direction

Ex effective force = a.




$a = |\vec{F}| \cos 45^\circ$

$= 20 \frac{\sqrt{2}}{2}$

$= 10\sqrt{2}$ lb