

Reduced Row Echelon Form



main diagonal ↗ ↘
/ diagonal

$$\begin{cases} 3x + y + 2z = 31 \\ x + y + 2z = 19 \\ x + 3y + 2z = 25 \end{cases}$$

matrix

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determinant

$$\begin{array}{c} x \quad y \quad z = \\ \left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ \textcircled{3} & 1 & 2 & 31 \\ \textcircled{1} & 3 & 2 & 25 \end{array} \right] \rightarrow \begin{array}{l} R_2 - 3R_1 \leftarrow \\ R_3 - R_1 \end{array} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & -2 & -4 & -26 \\ 0 & \underline{2} & 0 & 6 \end{array} \right] \begin{array}{l} x + y + 2z = 19 \text{ (2)} \\ -2y - 4z = -26 \text{ (1)} \\ 2y = 6 \rightarrow y = 3 \end{array}$$

$$\textcircled{1} \rightarrow 4z = -2(3) + 26 \rightarrow \underline{z = 5}$$

$$\textcircled{2} \quad x = 19 - 3 - 10 \\ = 6$$

$$\therefore \underline{(6, 3, 5)}$$

$$x_1 + 3x_2 - 2x_3 + 2x_5 = 0$$

$$2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1$$

$$5x_3 + 10x_4 + 15x_6 = 5$$

$$2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6$$

$$\rightarrow \left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ \textcircled{2} & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_4 - 2R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & +1 & +2 & 0 & +3 & +1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{array} \right] \begin{array}{l} R_3 - 5R_2 \\ R_4 - 4R_2 \end{array}$$

$$\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \end{array} \right] \begin{array}{l} \rightarrow \textcircled{2} \\ \rightarrow \textcircled{1} \\ 6x_6 = 2 \end{array}$$

$$x_6 = \frac{1}{3}$$

$$\textcircled{1} \quad x_3 + 2x_4 + 3x_6 = 1$$

$$\left[\begin{array}{l} x_3 = 1 - 2x_4 - 1 \\ \quad = -2x_4 \end{array} \right]$$

$$x_3 + 3x_2 - 2x_3 + 2x_4 = 0$$

$$\textcircled{2} \quad x_1 = -3x_2 + 2x_3 - 2x_5$$

$$= -3x_2 - 4x_4 - 2x_5$$

$$\therefore (-3x_2 - 4x_4 - 2x_5, x_2, -2x_4, x_4, x_5, \frac{1}{3})$$

$$\begin{cases} 2x + 8y - z + w = 0 \\ 4x + 16y - 3z - w = -10 \\ -2x + 4y - z + 3w = -6 \\ -6x + 2y + 5z + w = 3 \end{cases}$$

$$\left[\begin{array}{cccc|c} 2 & 8 & -1 & 1 & 0 \\ 4 & 16 & -3 & -1 & -10 \\ -2 & 4 & -1 & 3 & -6 \\ -6 & 2 & 5 & 1 & 3 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_3 + R_1 \\ R_4 + 3R_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 2 & 8 & -1 & 1 & 0 \\ 0 & 0 & -1 & -3 & -10 \\ 0 & 12 & -2 & 4 & -6 \\ 0 & 26 & 2 & 4 & 3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 2 & 8 & -1 & 1 & 0 \\ 0 & 12 & -2 & 4 & -6 \\ 0 & 0 & -1 & -3 & -10 \\ 0 & 26 & 2 & 4 & 3 \end{array} \right] \begin{array}{l} 6R_4 - 13R_3 \\ R_4 - \frac{13}{6}R_2 \\ 6R_4 - 13R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 2 & 8 & -1 & 1 & 0 \\ 0 & 12 & -2 & 4 & -6 \end{array} \right]$$

$$\begin{bmatrix} 0 & 0 & -1 & +3 & +10 \\ 0 & 0 & 38 & -28 & 96 \end{bmatrix} R_4 - 38R_3$$

$$\begin{bmatrix} 2 & 8 & -1 & 1 & 0 \\ 0 & 12 & -2 & 4 & -6 \\ 0 & 0 & 1 & 3 & 10 \\ 0 & 0 & 0 & -142 & -284 \end{bmatrix} \begin{matrix} \textcircled{4} \\ \textcircled{3} \\ \textcircled{2} \\ \textcircled{1} \end{matrix}$$

$x \quad y \quad z \quad w$

$$\textcircled{1} \rightarrow w = \frac{-284}{-142} = 2$$

$$\textcircled{2} \rightarrow z = 10 - 6 = 4$$

$$\textcircled{3} \rightarrow y = \frac{1}{12} (8 - 8 - 6) = -\frac{1}{2}$$

$$\textcircled{4} \rightarrow x = \frac{1}{2} (4 + 4 - 2) = 3$$

Solution: $(3, -\frac{1}{2}, 4, 2)$

$$AX = B \Rightarrow AX = 0$$

1.3 Algebra of Matrices

$+$, $-$, $*$ (product)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} \leftarrow \text{row 1 } R_1 \\ \leftarrow R_2 \\ \leftarrow R_3 \end{matrix}$$

Column \uparrow \uparrow \uparrow
 C_1 C_2 C_3
 $A = [a_{ij}] \quad i=j=3$

- # rows = # columns
 matrix is square matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \#R & \#C \\ 2 & \times & 3 \end{matrix} \rightarrow \left. \begin{matrix} \text{size} \\ \text{order} \end{matrix} \right\}$$

Zero $\neq \neq$

Zero matrix: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array} \right] \quad \text{Augmented matrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$AX = B \quad \text{matrix form}$$

Equality of Matrices

$$a_{ij} = b_{ij} \quad \text{"Same Size" or order}$$

Ex

$$\begin{pmatrix} 2 & 1 \\ p & q \end{pmatrix}_{2 \times 2} = \begin{pmatrix} x & y \\ -1 & 0 \end{pmatrix}_{2 \times 2}$$

$$x=2, y=1, p=-1, q=0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} \quad \text{can't be true}$$

$2 \times 1 \neq 3 \times 1$

Addition & Subtraction

Same (Size) order $m \times n$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 4 & 4 \\ 9 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 7 & 8 \\ 9 & 9 \end{bmatrix}$$

scalar multiplication

$$k \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{pmatrix}$$

Multiplication (product)