#1 Find the length of the curve

a) $r(t) = (2\cos t)i + (2\sin t)j + t^2 k \quad 0 \le t \le \pi/4$

$v = -2\sin t\, i + 2\cos t\, j + 2t\, k$

$|v| = \sqrt{4\sin^2 t + 4\cos^2 t + 4t^2}$

$= \sqrt{4 + 4t^2}$

$= 2\sqrt{1+t^2}$

$$\text{Length} = \int_0^{\pi/4} 2\sqrt{1+t^2}\, dt$$

$= \left[t\sqrt{1+t^2} + \ln \left| t + \sqrt{1+t^2} \right| \right]_0^{\pi/4}$

$= \frac{\pi}{4}\sqrt{1+\frac{\pi^2}{16}} + \ln \left| \frac{\pi}{4} + \sqrt{1+\frac{\pi^2}{16}} \right| - 0 - \ln 1$

$= \frac{\pi}{4}\sqrt{1+\frac{\pi^2}{16}} + \ln \left(\frac{\pi}{4} + \sqrt{1+\frac{\pi^2}{16}} \right)$

b) $r(t) = 3\cos t\, i + 3\sin t\, j + 2t^{3/2} k \quad 0 \le t \le 3$

$v' = -3\sin t\, i + 3\cos t\, j + 3t^{1/2} k$

$|v| = \sqrt{9\sin^2 t + 9\cos^2 t + 9t}$

$= \sqrt{9 + 9t}$

$= 3\sqrt{1+t}$

$$L = \int_0^3 3\sqrt{1+t}\, dt = 3 \cdot \frac{2}{3}(1+t)^{3/2} \Big|_0^3$$

$= 2(1+t)^{3/2} \Big|_0^3$

$= 2\left[(1+3)^{3/2} - 1^{3/2} \right]$

$= 2(4^{3/2} - 1)$

$\boxed{= 14}$

#2 a) $r(t) = \frac{4}{9}(1+t)^{3/2} i + \frac{4}{9}(1-t)^{3/2} + \frac{1}{3}tk \, , \, t=0$

$$v = \frac{2}{3}(1+t)^{1/2} i - \frac{2}{3}(1-t)^{1/2} + \frac{1}{3}k$$

$$|v| = \sqrt{\frac{4}{9}(1+t) + \frac{4}{9}(1-t) + \frac{1}{9}}$$

$$= \sqrt{\frac{4}{9}(1+t+1-t) + 1/9}$$

$$= \sqrt{\frac{4}{9}(2) + \frac{1}{9}}$$

$$= \sqrt{\frac{8}{9} + 1/9}$$

$$= 1$$

$$T = \frac{v}{|v|} = \frac{2}{3}(1+t)^{1/2} i - \frac{2}{3}(1-t)^{1/2} j + \frac{1}{3}k$$

$$T(0) = \frac{2}{3}i - \frac{2}{3}j + \frac{1}{3}k$$

$$\frac{dT}{dt} = \frac{1}{3}(1+t)^{-1/2} i + \frac{1}{3}(1-t)^{-1/2} j -$$

$$\frac{dT}{dt}(0) = \frac{1}{3}i + \frac{1}{3}j \Rightarrow \left|\frac{dT}{dt}(0)\right| = \sqrt{\frac{1}{9}+\frac{1}{9}} = \frac{\sqrt{2}}{3}$$

$$N(0) = \frac{dT/dt}{|\ \ |}(0) = \frac{3}{\sqrt{2}}\left(\frac{1}{3}i + \frac{1}{3}j\right) = \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$$

$$B(0) = T \times N(0) = \begin{vmatrix} i & j & k \\ 2/3 & -2/3 & 1/3 \\ 1/\sqrt{2} & \frac{1}{\sqrt{2}} & 0 \end{vmatrix}$$

$$= -\frac{1}{3\sqrt{2}}i + \frac{1}{3\sqrt{2}}j + \frac{4}{3\sqrt{2}}k$$

$$a(t) = \frac{1}{3}(1+t)^{-1/2} i + \frac{1}{3}(1-t)^{-1/2} j \Rightarrow a(0) = \frac{1}{3}i + \frac{1}{3}j$$

$$v(0) \times a(0) = \begin{vmatrix} i & j & k \\ 2/3 & -2/3 & 1/3 \\ 1/3 & 1/3 & 0 \end{vmatrix} = -\frac{1}{9}i + \frac{1}{9}j + \frac{4}{9}k$$

$$|v(0) \times a(0)| = \sqrt{\frac{1}{81}+\frac{1}{81}+\frac{16}{81}} = \frac{\sqrt{18}}{9} = \frac{3\sqrt{2}}{9} = \frac{\sqrt{2}}{3}$$

$$\kappa(0) = \frac{|v \times a|}{|v|} = \frac{\sqrt{2}/3}{1} = \frac{\sqrt{2}}{3}$$

$$a' = -\frac{1}{6}(1+t)^{-3/2} i + \frac{1}{6}(1-t)^{-3/2} j$$

$$a'(0) = -\frac{1}{6} i + \frac{1}{6} j$$

$$\tau(0) = \frac{\begin{vmatrix} 2/3 & -2/3 & 1/3 \\ 1/3 & 1/3 & 0 \\ -1/6 & 1/6 & 0 \end{vmatrix}}{\left(\frac{\sqrt{2}}{3}\right)^2} = \frac{\frac{1}{3}\left(2\frac{1}{18}\right)}{2/9} = \frac{1}{6}$$

b) $r(t) = (e^t \sin 2t) i + (e^t \cos 2t) j + 2e^t k \qquad t=0$

$$v = (e^t \sin 2t + 2e^t \cos 2t) i + (e^t \cos 2t - 2e^t \sin 2t) j + 2e^t k$$

$$|v| = e^t \sqrt{(\sin 2t + 2\cos 2t)^2 + (\cos 2t - 2\sin 2t)^2 + 4}$$

$$= e^t \sqrt{\sin^2 2t + 4\sin 2t \cos 2t + 4\cos^2 2t + \cos^2 2t - 4\cos 2t \sin 2t + 4\sin^2 2t + 4}$$

$$= e^t \sqrt{1+4+4} = \underline{3e^t}$$

$$T = \frac{1}{3e^t}\left[(e^t \sin 2t + 2e^t \cos 2t) i + (e^t \cos 2t - 2e^t \sin 2t) j + 2e^t k\right]$$

$$= \frac{1}{3}(\sin 2t + 2\cos 2t) i + \frac{1}{3}(\cos 2t - 2\sin 2t) j + \frac{2}{3} k$$

$$T(0) = \frac{2}{3} i + \frac{1}{3} j + \frac{2}{3} k$$

$$\frac{dT}{dt} = \frac{1}{3}(2\cos 2t - 4\sin 2t) i + \frac{1}{3}(-2\sin 2t - 4\cos 2t) j$$

$$\frac{dT}{dt}(0) = \frac{2}{3} i - \frac{4}{3} j \Rightarrow \left|\frac{dT}{dt}(0)\right| = \sqrt{\frac{4}{9} + \frac{16}{9}} = \frac{2}{3}\sqrt{5}$$

$$N(0) = \frac{3}{2\sqrt{5}}\left(\frac{2}{3} i - \frac{4}{3} j\right) = \frac{1}{\sqrt{5}} i - \frac{2}{\sqrt{5}} j$$

$$B(0) = T(0) \times N(0) = \begin{vmatrix} i & j & k \\ 2/3 & 1/3 & 2/3 \\ 1/\sqrt{5} & -2/\sqrt{5} & 0 \end{vmatrix}$$

$$= \frac{4}{3\sqrt{5}} i + \frac{2}{3\sqrt{5}} j - \frac{5}{3\sqrt{5}} k$$

$$a = (4e^t \cos 2t - 3e^t \sin 2t) i + (-3e^t \cos 2t - 4e^t \sin 2t) + 2e^t k$$

$$a(0) = 4i - 3j + 2k$$

$$v(0) = 2i + j + 2k \quad \Rightarrow \quad |v(0)| = 3$$

$$\cancel{v(0) \times (a}$$

$$v(0) \times a(0) = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 4 & -3 & 2 \end{vmatrix} = 8i + 4j - 10k$$

$$|v \times a| = \sqrt{64 + 16 + 100} = 6\sqrt{5}$$

$$\kappa(0) = \frac{6\sqrt{5}}{3^3} = \frac{2\sqrt{5}}{3}$$

$$a' = (4e^t \cos 2t - 8e^t \sin 2t - 3e^t \sin 2t - 6e^t \cos 2t) i + [e^t(-3\cos 2t - 4\sin 2t) + e^t(6\sin 2t - 8\cos 2t)] j + 2e^t k$$

$$a'(0) = -2i - 11j + 2k$$

$$\tau(0) = \frac{1}{180} \begin{vmatrix} 2 & 1 & 2 \\ 4 & -3 & 2 \\ 2 & -11 & 2 \end{vmatrix} = \frac{-80}{180} = -4/9$$

c) $r(t) = t\,i + \frac{1}{2}e^{2t}j \qquad t = \ln 2$

$v = i + e^{2t}j \qquad v(\ln 2) = i + e^{2\ln 2}j$

$|v| = \sqrt{1+e^{4t}} \qquad\qquad = i + e^{\ln 4}j$

$\qquad\qquad\qquad\qquad = i + 4j$

$T = \dfrac{i + e^{2t}j}{\sqrt{1+e^{4t}}}$

$T(\ln 2) = \dfrac{1}{\sqrt{1+e^{4\ln 2}}}i + \dfrac{e^{2\ln 2}}{\sqrt{1+e^{4\ln 2}}}j$

$= \dfrac{1}{\sqrt{1+e^{\ln 16}}}i + \dfrac{e^{\ln 4}}{\sqrt{1+e^{\ln 16}}}j$

$= \dfrac{1}{\sqrt{17}}i + \dfrac{4}{\sqrt{17}}j$

$\dfrac{dT}{dt} = \dfrac{-2e^{4t}}{(1+e^{4t})^{3/2}}i + \dfrac{2e^{2t}}{(1+e^{4t})^{3/2}}j$

$\Big|_{\ln 2} = \dfrac{-2e^{4\ln 2}}{(1+16)^{3/2}}i + \dfrac{2(4)}{(1+16)^{3/2}}j = \dfrac{-32}{17^{3/2}}i + \dfrac{8}{17^{3/2}}j$

~~$N(\ln 2)$~~ $\left|\dfrac{dT}{dt}(\ln 2)\right| = \sqrt{\dfrac{32^2}{17^3} + \dfrac{64}{17^3}} = \dfrac{8\sqrt{17}}{17\sqrt{17}} = 8/17$

$N(\ln 2) = \dfrac{17}{8}\left(\dfrac{-32}{17\sqrt{17}}i + \dfrac{8}{17\sqrt{17}}j\right) = -\dfrac{4}{\sqrt{17}}i + \dfrac{1}{\sqrt{17}}j$

$B(\ln 2) = T \times N(\ln 2) = \begin{vmatrix} i & j & k \\ \frac{1}{\sqrt{17}} & 4/\sqrt{17} & 0 \\ -\frac{4}{\sqrt{17}} & 1/\sqrt{17} & 0 \end{vmatrix} = k$

$a = 2e^{2t}j \Rightarrow a(\ln 2) = 2e^{2\ln 2}j = 8j$

$v \times a(\ln 2) = \begin{vmatrix} i & j & k \\ 1 & 4 & 0 \\ 0 & 8 & 0 \end{vmatrix} = 8k \Rightarrow |v \times a| = 8.$

$|v(\ln 2)| = \sqrt{1+16} = \sqrt{17}$

$K(\ln 2) = \frac{8}{\sqrt{17}}$

$a' = 4e^{2t}j \longrightarrow a'(\ln 2) = 4e^{2\ln 2}j = 16j$

$$\tau(\ln 2) = \frac{\begin{vmatrix} 1 & 4 & 0 \\ 0 & 8 & 0 \\ 0 & 16 & 0 \end{vmatrix}}{8} = 0$$

#3 a) $r(t) = (2+3t+3t^2)i + (4t+4t^2)j - (6\cos t)k$

$v = (3+6t)i + (4+8t)j + 6\sin t\, k$

~~$v(0) =$~~

$|v| = \sqrt{(3+6t)^2 + (4+8t)^2 + 36\sin^2 t}$

$= \sqrt{9+36t+36t^2+16+64t+64t^2+36\sin^2 t}$

$= \sqrt{25+100t+100t^2+36\sin^2 t}$

$\frac{d|v|}{dt} = \frac{1}{2}(100+200t+72\sin t\cos t)(25+100t+100t^2+36\sin^2 t)^{-1/2}$

$a_T = \left.\frac{d|v|}{dt}\right|_{t=0} = \frac{1}{2}(100)(25)^{-1/2} = \frac{100}{2\sqrt{25}} = \frac{50}{5} = 10$

$a = v' = 6i + 8j + 6\cos t\, k$

$|a| = \sqrt{36+64+36\cos^2 t} = \sqrt{100+36\cos^2 t}$

$|a|_{t=0} = \sqrt{136}$

$a_N = \sqrt{|a|^2 - a_T^2} = \sqrt{136-100} = 6$

$a = 10T + 6N$

b) $r(t) = (2+t)i + (t+2t^2)j + (1+t^2)k$

$$v = i + (1+4t)j + 2t\,k$$

$$|v| = \sqrt{1 + (1+4t)^2 + 4t^2}$$

$$= \sqrt{1+1+8t+16t^2+4t^2}$$

$$= \sqrt{2+8t+20t^2}$$

$$\frac{d}{dt}|v| = \frac{1}{2}(8+40t)(2+8t+20t^2)^{-1/2}$$

$$a_T = \left.\frac{d|v|}{dt}\right|_{t=0} = \frac{1}{2} 8 (2)^{-1/2} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$a = v' = 4j + 2k$$

$$|a| = \sqrt{16+4} = 2\sqrt{5}$$

$$a_N = \sqrt{|a|^2 - a_T^2} = \sqrt{20-8} = \sqrt{12} = 2\sqrt{3}$$

$$a(0) = 2\sqrt{2}\,T + 2\sqrt{3}\,N$$

$\cancel{3}\, 2\sin t\cos t - 4\cos t\sin t$

#4 a) $r(t) = (4\cos t)i + (\sqrt{2}\sin t)j \quad t = 0, \pi/4$

$x = 4\cos t \qquad y = \sqrt{2}\sin t$

$\cos t = \frac{x}{4} \qquad \sin t = \frac{y}{\sqrt{2}}$

$\cos^2 t + \sin^2 t = 1$

$\frac{x^2}{16} + \frac{y^2}{2} = 1 \Rightarrow$ (ellipse)

$v = -4\sin t\, i + \sqrt{2}\cos t\, j \Rightarrow v(0) = \sqrt{2}\, j$

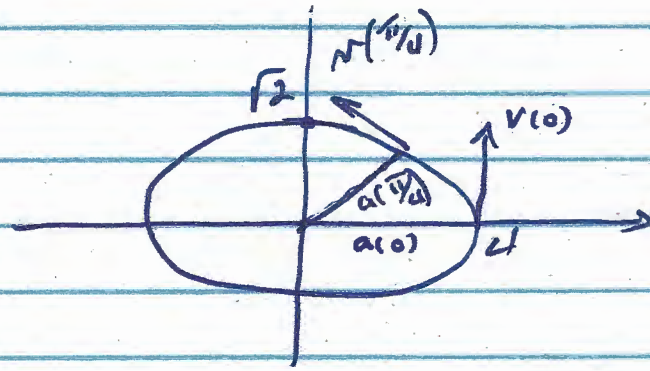
$a = -4\cos t\, i - \sqrt{2}\sin t\, j \Rightarrow a(0) = -4i$

$r\left(\frac{\pi}{4}\right) = 2\sqrt{2}\, i + j$

$v\left(\frac{\pi}{4}\right) = -2\sqrt{2}\, i + j \quad , \quad a\left(\frac{\pi}{4}\right) = -2\sqrt{2}\, i - j$

$|\ \ | = \sqrt{8+1} = 3$

$|v| = \sqrt{16\sin^2 t + 2\cos^2 t}$



$a_T = \frac{d}{dt}|v| =$

$= \frac{32\sin t\cos t - 4\cos t\sin t}{2\sqrt{16\sin^2 t + 2\cos^2 t}}$

$= \frac{14\sin t\cos t}{\sqrt{16\sin^2 t + 2\cos^2 t}}$

$t = 0$

$a_T = 0$

$a_N = \sqrt{|a|^2 - a_T^2} = 4$

$a = 0T + 4N = 4N$

$\kappa = \frac{a_N}{|v|^2} = \frac{4}{2} = 2$

$t = \pi/4$

$a_T = \frac{14\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{\sqrt{16\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right)}} = \frac{7}{3}$

$a_N = \sqrt{9 - \frac{49}{9}} = \frac{4\sqrt{2}}{3}$

$a = \frac{7}{3}T + \frac{4\sqrt{2}}{3}N$

$K = \frac{4\sqrt{2}}{3} \cdot \frac{1}{9} = \frac{4\sqrt{2}}{27}$

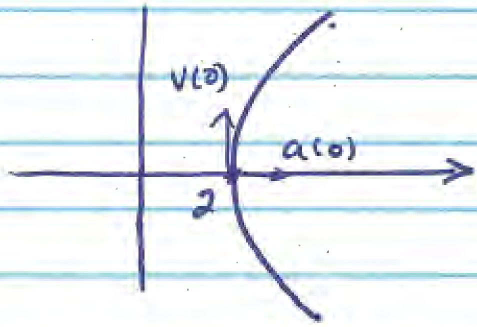
b) $r(t) = (\sqrt{3}\sec t)\,i + (\sqrt{3}\tan t)\,j \quad t = 0$

$x = \sqrt{3}\sec t \qquad y = \sqrt{3}\tan t$

$\sec t = \frac{x}{\sqrt{3}} \qquad \tan t = \frac{y}{\sqrt{3}}$

$\sec^2 - \tan^2 t = 1 \Rightarrow \frac{x^2}{3} - \frac{y^2}{3} = 1$ (hyperbolic)

$x^2 - y^2 = 3$



$v = \sqrt{3}\sec t\tan t\; i + \sqrt{3}\sec^2 t\; j$

$a = (\sqrt{3}\sec t\tan^2 t + \sqrt{3}\sec^3 t)\,i + 2\sqrt{3}\sec^2 t\tan t\; j$

$r(0) = \sqrt{3}\,i, \quad v(0) = \sqrt{3}\,j \qquad a(0) = \sqrt{3}\,i$

$|v(0)| = \sqrt{3}$

$|v| = \sqrt{3\sec^2 t\tan^2 t + 3\sec^4 t} \quad \cancel{= \sqrt{3\sec^2 t(\tan^2 t + \sec^2 t)}}$

$a_T = \frac{d}{dt}|v| = \frac{6\sec^2 t\tan^3 t + 6\sec^4 t\tan t + 12\sec^4 t\tan t}{2\sqrt{3\sec^2 t\tan^2 t + 3\sec^4 t}}$

$= \frac{3\sec^2 t\tan^3 t + 9\sec^4 t\tan t}{\sqrt{}}$

$a_T\big|_{t=0} = 0$

$a_N = \sqrt{|a|^2 - 0} = \sqrt{3}$

$a = \sqrt{3}\,N.$

$K = \frac{\sqrt{3}}{3}$

#5 $r = \frac{1}{\sqrt{1+t^2}}\, i + \frac{t}{\sqrt{1+t^2}}\, j$

$$v = -\frac{1}{2}\,\frac{2t}{(1+t^2)^{3/2}}\, i + \frac{\sqrt{1+t^2} - t\left(\frac{1}{2}\right)(2t)(1+t^2)^{-1/2}}{(1+t^2)}\, j$$

$$= -\frac{t}{(1+t^2)^{3/2}}\, i + \frac{1+t^2-t^2}{(1+t^2)^{3/2}}\, j$$

$$= -\frac{t}{(1+t^2)^{3/2}}\, i + \frac{1}{(1+t^2)^{3/2}}\, j$$

$$|v| = \sqrt{\frac{t^2}{(1+t^2)^3} + \frac{1}{(1+t^2)^3}} = \sqrt{\frac{t^2+1}{(1+t^2)^3}} =$$

$$= \frac{1}{1+t^2}$$

To maximize speed $(|v|)$

$$\frac{d|v|}{dt} = \frac{-2t}{(1+t^2)^2} = 0 \Rightarrow \boxed{t=0}$$

$$|V|_{max\ t=0} = \frac{1}{1+0} = 1$$

#6 $a = 2i + j + k = \frac{dv}{dt}$

$v = \int (2i + j + k)\,dt = 2ti + tj + tk + C_1$

particles travels in the direction:

$(3-1)i + (0+1)j + (3-2)k = 2i + j + k$

@ $t = 0 \Rightarrow |v| = 2$

$v(0) = \frac{|v|_0}{|v|} (2i + j + k) = \frac{2}{\sqrt{4+1+1}} (2i + j + k) = C$

$= \frac{2}{\sqrt{6}} (2i + j + k) = C_1$

$v = \left(2t + \frac{4}{\sqrt{6}}\right) i + \left(t + \frac{2}{\sqrt{6}}\right) j + \left(t + \frac{2}{\sqrt{6}}\right) k$

$r(t) = \int v\,dt$

$= \left(t^2 + \frac{4}{\sqrt{6}} t\right) i + \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{6}}t\right) j + \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{6}}t\right) k + C_2$

$r(0) = i - j + 2k = C_2$

$r(t) = \left(t^2 + \frac{4}{\sqrt{6}} t + 1\right) i + \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{6}} t - 1\right) j$

$+ \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{6}} t + 2\right) k$

or $\left(\frac{1}{2}t^2 + \frac{2t}{\sqrt{6}}\right)(2i + j + k) + (i - j + 2k)$

#7 $v \times a = \begin{vmatrix} i & j & k \\ 3 & 4 & 0 \\ 5 & 15 & 0 \end{vmatrix} = 25k$

$|v \times a| = 25$

$|v| = \sqrt{9+16} = 5$

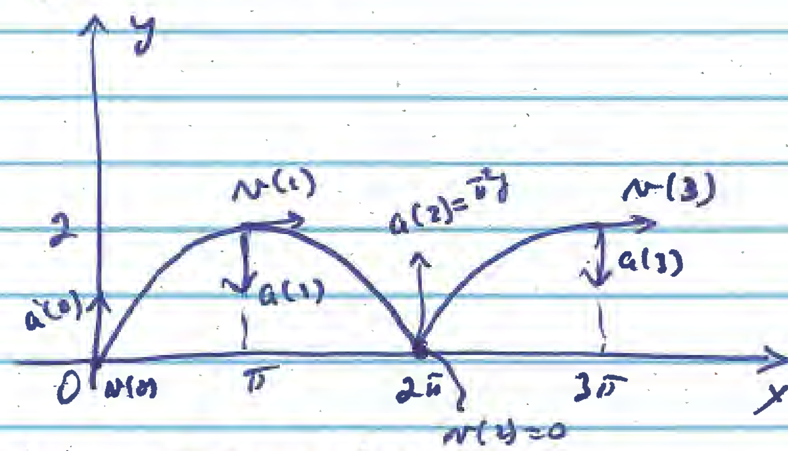
$\boxed{\cancel{K = \frac{25}{5^3} = \frac{1}{5}}}$

$K = \frac{25}{5^3} = 1/5$

#8) $r(t) = (\pi t - \sin \pi t)\,i + (1 - \cos \pi t)\,j$

$x = \pi t - \sin \pi t \quad , \quad y = 1 - \cos \pi t$

t	x	y
0	0	0
1/2	$\pi/2 - 1$	1
1	π	2
2	2π	0
3	3π	2



b) $v = (\pi - \pi \cos \pi t)\,i + \pi \sin \pi t\, j$

$a = \pi^2 \sin \pi t\, i + \pi^2 \cos \pi t\, j$

$v(0) = 0$

v

t	v	a
0	0	$\pi^2 j$
1	$2\pi i$	$-\pi^2 j$
2	0	$\pi^2 j$
3	$2\pi i$	$-\pi^2 j$

c) Forward speed at the most point $|v(1)| = |v(3)| = 2\pi$
since the circles makes $\frac{1}{2}$ rev/s, the center moves π ft // to x-axis each sec. $\Rightarrow$ forward speed of C is π ft/sec.

#9 Given: $r(0) = 6.5$ ft $= y_0$
$\alpha = 45°$
$v(0) = 44$ ft/sec.

$$y = y_0 + (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$
$$= 6.5 + (44 \sin 45°)t - 16t^2$$
$$= 6.5 + 22\sqrt{2}\,t - 16t^2$$

$$y\big|_{t=3} = 6.5 + 22\sqrt{2}(3) + 16(9) \approx -44.16 \text{ ft.}$$

The shot is on the ground at $t = 3$ sec

$\therefore y = 0 = -16t^2 + 22\sqrt{2}\,t + 6.5$
solve for $t \Rightarrow t = 2.13$ sec.
$x \approx (v_0 \cos\alpha)t = 44\,\frac{\sqrt{2}}{2}(2.13) \approx 66.27$ ft.

#10 $r(t) = ti + t^2 j + t^3 k$
$v = i + 2tj + 3t^2 k$
$|v| = \sqrt{1 + 4t^2 + 9t^4}$
$|v(1)| = \sqrt{1+4+9} = \sqrt{14}$
$$T(1) = \frac{v}{|v|} = \frac{1}{\sqrt{14}}i + \frac{2t}{\sqrt{14}}j + \frac{3t^2}{\sqrt{14}}k$$
$$= \frac{1}{\sqrt{14}}i + \frac{2}{\sqrt{14}}j + \frac{3}{\sqrt{14}}k$$ (normal to the normal plane)

$$\frac{1}{\sqrt{14}}(x-1) + \frac{2}{\sqrt{14}}(y-1) + \frac{3}{\sqrt{14}}(z-1) = 0$$

$x - 1 + 2y - 2 + 3z - 3 = 0$
$x + 2y + 3z = 6$ (eqn. of the normal plane)

#10 cont.

$$a = 2j + 6tk \Rightarrow a(1) = 2j + 6k$$

$$v \times a(1) = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 0 & 2 & 6 \end{vmatrix} = 6i - 6j + 2k$$

$$|v \times a| = \sqrt{36+36+4} = \sqrt{76}$$

$$\kappa(1) = \frac{\sqrt{76}}{(\sqrt{14})^3} = \frac{2\sqrt{19}}{14\sqrt{14}} = \frac{\sqrt{19}}{7\sqrt{14}}$$

$$\frac{ds}{dt} = |v(t)| = \sqrt{1+4t^2+9t^4}$$

$$\left.\frac{d^2s}{dt^2}\right|_{t=1} = \frac{1}{2}(8t+36t^3)(1+4t^2+9t^4)^{-1/2}\Big|_{t=1}$$

$$= \frac{1}{2}\frac{8+36}{\sqrt{1+4+9}} = \frac{22}{\sqrt{14}}$$

$$a = \frac{d^2s}{dt^2}T + \kappa\left(\frac{ds}{dt}\right)^2 N$$

$$2j+6k = \frac{22}{\sqrt{14}}\left(\frac{i+2j+3k}{\sqrt{14}}\right) + \frac{\sqrt{19}}{7\sqrt{14}}\cdot(\sqrt{14})^2 N$$

$$2j+6k = \frac{22}{14}i + \frac{44}{14}j + \frac{66}{14}k + 2\frac{\sqrt{19}}{\sqrt{14}}N$$

$$2\frac{\sqrt{19}}{\sqrt{14}}N = -\frac{22}{14}i - \frac{16}{14}j + \frac{18}{14}k$$ 18

$$N = \frac{\sqrt{14}}{2\sqrt{19}}\,\frac{-22i-16j+18k}{14} = \frac{\sqrt{14}}{2\sqrt{19}}\left(-\frac{11}{7}i - \frac{8}{7}j + \frac{9}{7}k\right)$$

$$-\frac{11}{7}(x-1) - \frac{8}{7}(y-1) + \frac{9}{7}(z-1) = 0$$

$$-11x+11-8y+8+9z-9=0$$

$$11x+8y+9z = 10$$

#10 Cont

$B(1) = T(1) \times N(1)$

$$= \frac{1}{\sqrt{14}} \cdot \frac{\sqrt{14}}{2\sqrt{19}} \cdot \frac{1}{7} \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -11 & -8 & 9 \end{vmatrix}$$

$$= \frac{1}{14\sqrt{19}} (42i - 42j + 14k)$$

$$= \frac{1}{\sqrt{19}} (3i - 3j + k)$$

$3(x-1) - 3(y-1) + z - 1 = 0$

$3x - 3 - 3y + 3 + z - 1 = 0$

$3x - 3y + z = 1$

is an eqn. of the osculating plane.

#11 $r(t) = (\cos t)i + (2\cos t)j + (\sqrt{5}\sin t)k$

$0 \le t \le 2\pi$

a) $v = r' = (-\sin t)i - (2\sin t)j + \sqrt{5}\cos t\, k$

$|v| = \sqrt{\sin^2 t + 4\sin^2 t + 5\cos^2 t} = \sqrt{5}$

$T = \frac{1}{\sqrt{5}}(-\sin t\, i - 2\sin t\, j + \sqrt{5}\cos t\, k)$

b) $r'' = -\cos t\, i - 2\cos t\, j - \sqrt{5}\sin t\, k$

$$r'' \times r' = \begin{vmatrix} i & j & k \\ -\cos t & -2\cos t & -\sqrt{5}\sin t \\ -\sin t & -2\sin t & \sqrt{5}\cos t \end{vmatrix}$$

$= (-2\sqrt{5}\cos^2 t - 2\sqrt{5}\sin^2 t)i$

$-(-\sqrt{5}\cos^2 t - \sqrt{5}\sin^2 t)j$

$+(2\sin t\cos t - 2\sin t\cos t)k$

$= -2\sqrt{5}\, i + \sqrt{5}\, j$

$|r'' \times r'| = \sqrt{20 + 5} = 5$

$K = \frac{|r'' \times r'|}{|v|^3} = \frac{5}{(\sqrt{5})^3} = \frac{1}{\sqrt{5}}$

c) $N = \frac{dT}{dt} = \frac{1}{\sqrt{5}}(-\cos t\, i - 2\cos t\, j - \sqrt{5}\sin t\, k)$

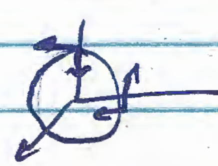
d) $|N| = \frac{1}{\sqrt{5}}\sqrt{\cos^2 t + 4\cos^2 t + 5\sin^2 t}$

$= \frac{1}{\sqrt{5}}\sqrt{5\cos^2 t + 5\sin^2 t} = \frac{\sqrt{5}}{\sqrt{5}} = 1$

$T \cdot N = \frac{1}{5}(-\sin t\, i - 2\sin t\, j + \sqrt{5}\cos t\, k) \cdot (-\cos t\, i - 2\cos t\, j - \sqrt{5}\sin t\, k)$

$= \frac{1}{5}(+\sin t\cos t + 4\sin t\cos t - 5\sin t\cos t)$

$= 0$



#12 $r(t) = (t^2+1)i + (2t)j \quad t \geq 0$

a) $v = 2ti + 2j \qquad |v| =$

$a = 2i + \cancel{2j}$

$|v| = \sqrt{4t^2+4} = 2\sqrt{t^2+1}$

$T = \frac{t}{\sqrt{t^2+1}} i + \frac{1}{\sqrt{t^2+1}} j$

$a_T = \frac{d}{dt}|v| = \frac{2t}{\sqrt{t^2+1}}$

$|v \times a| = \begin{vmatrix} i & j & k \\ 2t & 2 & 0 \\ 2 & 0 & 0 \end{vmatrix} = 4k$

$\kappa = \frac{|v \times a|}{|v|^3} = \frac{4}{8(t^2+1)^{3/2}} = \frac{1}{2(t^2+1)^{3/2}}$ (No need)

$a_N = \kappa |v|^2 = \frac{|v \times a|}{|v|} = \frac{4}{2\sqrt{t^2+1}} = \frac{2}{\sqrt{t^2+1}}$

$a = \frac{2t}{\sqrt{t^2+1}} T + \frac{2}{\sqrt{t^2+1}} N$

b) @ $t = 1 \Rightarrow a = \frac{2}{\sqrt{2}} T + \frac{2}{\sqrt{2}} N$

$= \frac{2}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j\right) + \frac{2}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j\right)$

$2i = 2i$

$t = 2 \quad \cancel{\langle 2, 0\rangle = \frac{2}{\sqrt{5}}\left\langle \frac{2}{\sqrt{5}} i + \frac{1}{\sqrt{5}} j\right\rangle + \frac{2}{\sqrt{5}}\left\langle \frac{2}{\sqrt{5}}\right.}$

$\langle 2, 0\rangle = \frac{2}{\sqrt{5}}\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle + \frac{2}{\sqrt{5}}\left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right\rangle$