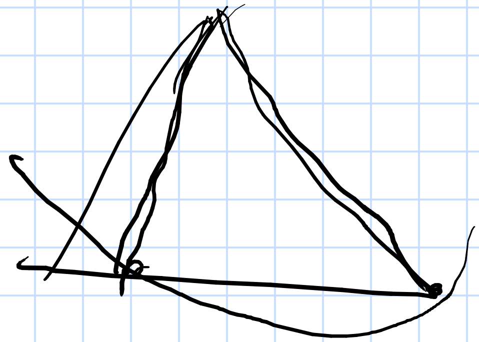
$$\begin{cases} 0 \le \sin e \le 1 \\ \text{angle} > 0 \Rightarrow \quad QI \ \& \ QII \rightarrow 180^\circ - \theta \end{cases}$$

Ex $\underline{a} = 54$ cm $\quad b = 62 \quad \underline{A} = 40^\circ \quad \triangle ABC$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{62 \sin 40^\circ}{54}$$

$$\hat{B} = \sin^{-1}\left(\frac{62 \sin 40^\circ}{54}\right) \approx 48^\circ$$



$B \approx 48^\circ$

$C = 180^\circ - 48^\circ - 40^\circ$

$\approx 92^\circ$

$$c = \frac{54 \sin 92^\circ}{\sin 40^\circ}$$

≈ 84 cm

$B \approx 180^\circ - 48^\circ$

$= 132^\circ$

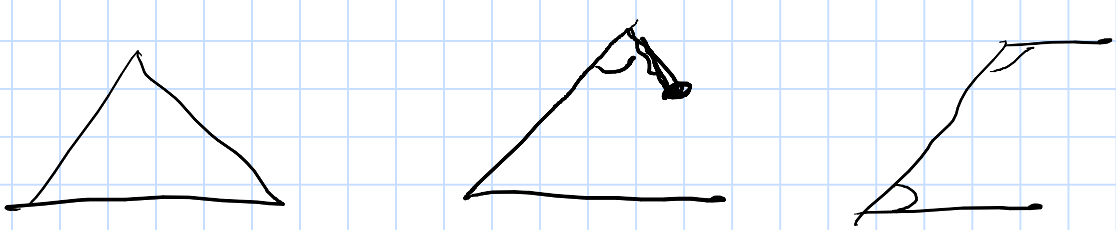
$C = 180^\circ - 40^\circ - 132^\circ$

$= 8^\circ$

$$c = \frac{a \sin C}{\sin A} = \frac{54 \sin 8^\circ}{\sin 40^\circ}$$

≈ 12

$$\begin{cases} 2 - \text{triangles} \\ 1 - \text{triangle} \\ 0 - \text{triangle} \quad \text{sine} > 1 \end{cases}$$



Area of a Triangle

$$K \text{ or Area} = \frac{1}{2} a b \sin C \leftarrow$$
$$= \frac{1}{2} a c \sin B$$
$$= \frac{1}{2} b c \sin A$$

<u>Ex</u> $\triangle$ Area? $a = 34$ $c = 42$ $B = 55°10'$

$$\text{Area} = \frac{1}{2} a c \sin B$$
$$= \frac{1}{2}(34)(42) \sin\left(55 + \frac{10}{60}\right)$$
$$\approx 586 \text{ ft}^2$$

---

4) $b = 4$ $c = 1$ $A = 120°$
$(60°)$

$$\text{Area} = \frac{1}{2} b c \sin A$$
$$= \frac{1}{2}(4)(1) \sin 120°$$
$$= 2\left(\frac{\sqrt{3}}{2}\right)$$
$$= \sqrt{3} \text{ unit}^2$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$(b-c)^2 = b^2 + c^2 - 2bc$$

$$a = \sqrt{b^2 + c^2 - 2bc\cos A}$$

$$b = \sqrt{a^2 + c^2 - 2ac\cos B}$$

$$c = \sqrt{a^2 + b^2 - 2ab\cos C}$$

Ex $A = 60^\circ \quad b = 20\text{ in} \quad c = 30\text{ in} \quad a?$

$$a = \sqrt{b^2 + c^2 - 2bc\cos A}$$

$$= \sqrt{400 + 900 - 2(20)(30)\cos 60^\circ}$$

$$= \sqrt{1300 - 1200\left(\tfrac{1}{2}\right)}$$

$$= \sqrt{700}$$

$$= 10\sqrt{7}\text{ in}$$

Ex $b = AC = 259\text{ m} \quad BC = 423\text{ m} = a$

$$ACB = 132^\circ 40' = C$$

$$c = \sqrt{(259)^2 + (423)^2 - 2(259)(423)\cos\left(132^\circ + \tfrac{40}{60}^\circ\right)}$$

$$\approx 628\text{ m}$$

3 sides: $\Rightarrow$ missing 3 angles?

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

$$B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

Ex $a = 34 \quad b = 20 \quad c = 18$

$$A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

$$= \cos^{-1}\left(\frac{20^2+18^2-34^2}{2(20)(18)}\right)$$

$$\approx 127^\circ$$

Area (3 sides SSS) Heron's Area Formula

$$s = \frac{1}{2}(a+b+c)$$

K or

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

47) $a = 4 \quad b = 5 \quad c = 7$ Area?

$$s = \frac{1}{2}(4+5+7)$$

$$= 8$$

$8(8-4)(8-7)(8-7)$

$$\text{Area} = \sqrt{8(4)(3)(1)}$$

2.4

$$= 4\sqrt{6} \text{ unit}^2$$

A ? $a = 3,\ b = 3,\ c = 2$

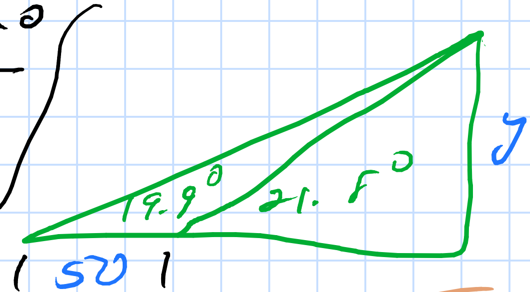
$$A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2(b)(c)}\right)$$

$$= \cos^{-1}\left(\frac{9+4-9}{2(3)(2)}\right)$$

$$= \cos^{-1}\left(\frac{1}{3}\right)$$

$$\frac{\cancel{4}}{\cancel{4}(3)}$$

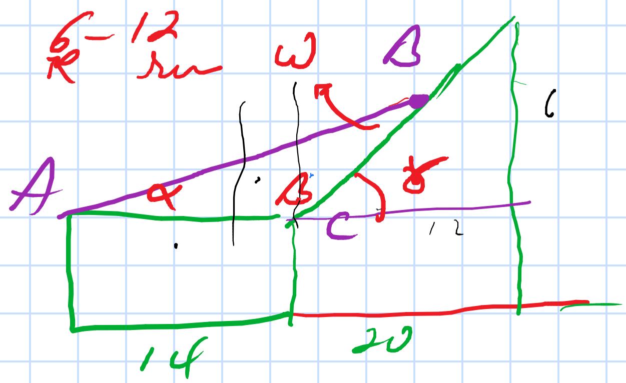
18/ $$y = \frac{50\tan 19.9^\circ \tan 21.8^\circ}{\tan 21.8^\circ - \tan 19.9^\circ}$$



19/ $\tan\gamma = \frac{6}{12} = \frac{1}{2}$ (1)

$\tan\alpha = \frac{3}{12} = \frac{1}{4}$ (2)

$AC = 14$



$\overline{AB}$

(1) $\gamma = \tan^{-1}\frac{1}{2}$

(2) $\alpha = \tan^{-1}\frac{1}{4}$

$$\angle\beta = 180^\circ - \gamma = 180^\circ - \tan^{-1}\frac{1}{2}$$

$$\frac{\overline{AB}}{\sin\beta} = \frac{14}{\sin\omega}$$

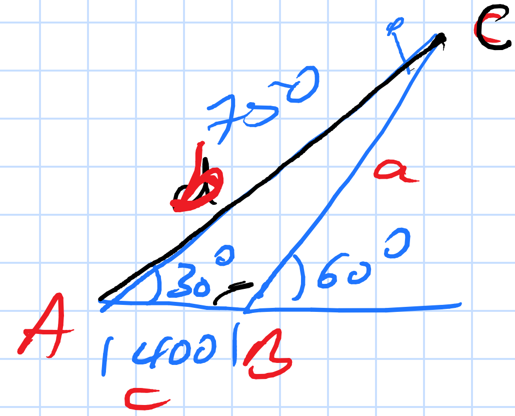
$$\omega = 180^\circ - \alpha - \beta$$
$$= 180^\circ - \tan^{-1}\frac{1}{4} - 180^\circ + \tan^{-1}\frac{1}{2}$$
$$= \tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{4}$$

$$\overline{AB} = \frac{14\sin\left(180^\circ - \tan^{-1}\frac{1}{2}\right)}{\sin\left(\tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{4}\right)} \approx 28.9$$

21/ $b \overset{?}{\ } 750$

$B = 180^\circ - 60^\circ$
$= \boxed{120^\circ}$

$C = 180^\circ - 30^\circ - 120^\circ$
$= \boxed{30^\circ}$



$$\frac{b}{\sin B} = \frac{400}{\sin C}$$

$$b = \frac{400 \sin 120^\circ}{\sin 30^\circ}$$

$$= \frac{400 \frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}}$$

$$= 40\cancel{0}\sqrt{3} \ \ ? \ \ 75\cancel{0}$$

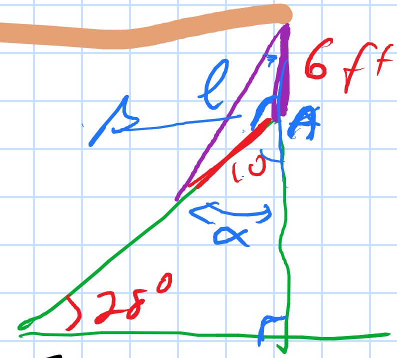
$$(40)^2 \ ? \ \left(\frac{75}{\sqrt{3}}\right)^2$$

$$1600 < 1875$$

tree will not make

31/ $\alpha = 90^\circ - 28^\circ$
$= \boxed{62^\circ}$

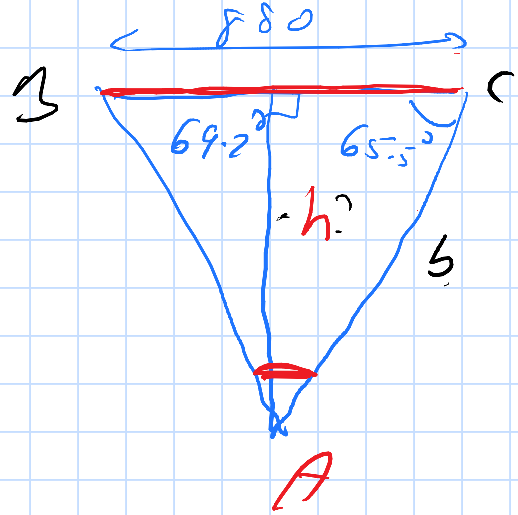
$\beta = 180^\circ - 62^\circ$
$= \boxed{118^\circ}$



$$\ell = \sqrt{6^2 + 10^2 - 2(6)(10)\cos 118^\circ}$$

$$= \boxed{\sqrt{136 - 120\cos 118^\circ}}$$

89/



$$A = 180^\circ - 69.2^\circ - 65.5^\circ$$
$$= \underline{45.3^\circ}$$

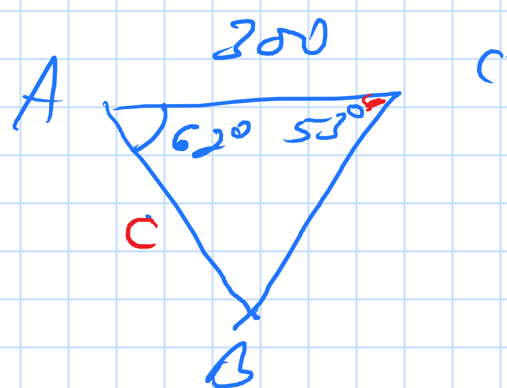
$$\frac{b}{\sin 69.2^\circ} = \frac{880}{\sin 45.3^\circ}$$

$$b = \frac{880 \sin 69.2^\circ}{\sin 45.3^\circ}$$

$$\sin 65.5^\circ = \frac{h}{b}$$

$$h = \sin 65.5^\circ \cdot \frac{880 \sin 69.2^\circ}{\sin 45.3^\circ}$$

91/

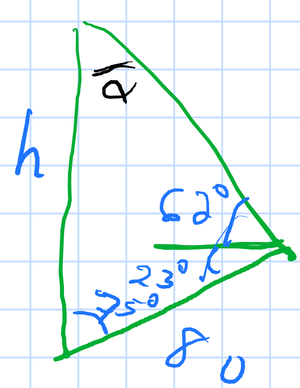


$$B = 180^\circ - 62^\circ - 53^\circ$$
$$= \underline{65^\circ}$$

$$AB = \frac{300 \sin 53^\circ}{\sin 65^\circ}$$

$$\frac{c}{\sin 53^\circ} = \frac{300}{\sin B}$$

92/



$$\frac{h}{\sin(62^\circ + 23^\circ)} = \frac{80}{\sin \alpha}$$

$$\alpha = 180^\circ - 75^\circ - 62^\circ - 23^\circ$$
$$= 20^\circ$$

$$h = \frac{80 \sin(85^\circ)}{\sin 20^\circ}$$

$$b = \sqrt{140^2 + 160^2 - 2(140)(160)\cos 80^\circ}$$

$$= 10\sqrt{14^2 + 16^2 - 28(16)\cos 80^\circ}$$

140