Inverse Functions

$$f(x)=\frac{ax+b}{cx+d} \quad \Rightarrow \quad f^{-1}(x)=\frac{-dx+b}{cx-a}$$

Proof

$$y=\frac{ax+b}{cx+d}$$

$$x=\frac{ay+b}{cy+d}$$

$$cxy+dx=ay+b$$

$$cxy-ay=-dx+b$$

$$(cx-a)y=-dx+b$$

$$y=\frac{-dx+b}{cx-a}$$

$$\boxed{f^{-1}(x)=\frac{-dx+b}{cx-a}} \quad \surd$$

Interchange ***a*** and ***d*** and change there signs.

Example

Find the inverse function of: $f(x)=\frac{1}{3x-2}$

Solution

$$f^{-1}(x)=\frac{2x+1}{3x} \qquad f(x)=\frac{0x+1}{3x-2}$$

Example

Find the inverse function of: $f(x)=\frac{3x+2}{2x-5}$

Solution

$$f^{-1}(x)=\frac{5x+2}{2x-3} \qquad f(x)=\frac{3x+2}{2x-5}$$

Example

Find the inverse function of: $f(x)=\frac{4x}{x+2}$

Solution

$$f^{-1}(x)=\frac{-2x}{x-4} \qquad f(x)=\frac{4x}{x+2}$$