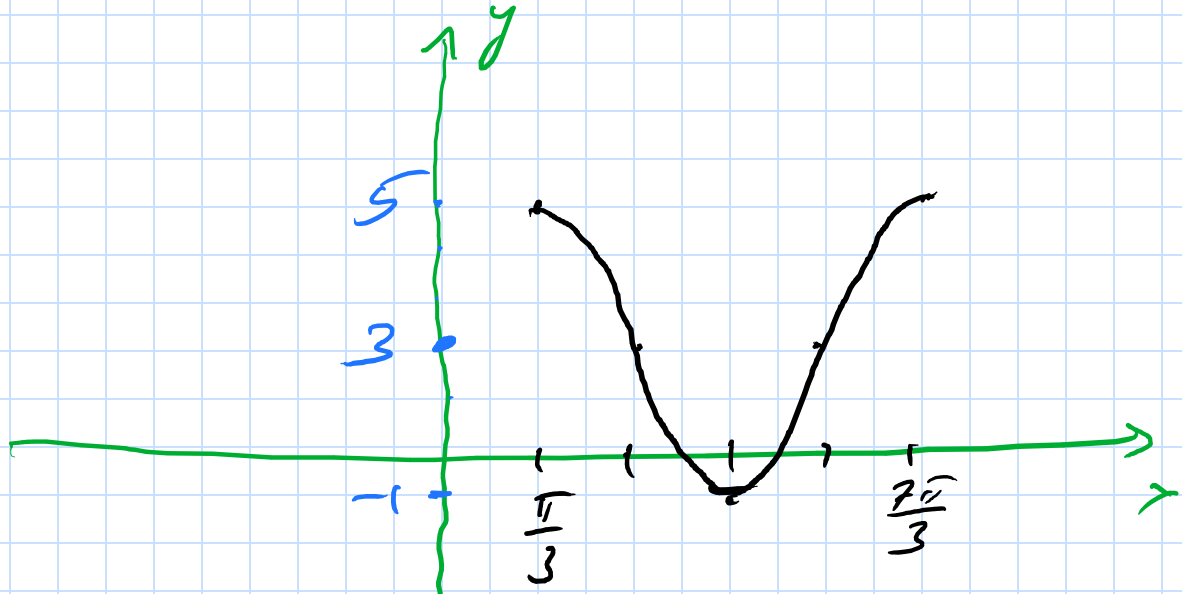
$y = 3\cos(x - \frac{\pi}{3}) + 2$ ± 1



$3, 4 \to 5$ $7, 24 \to 25$

$5, 12 \to 13$ $20, 21 \to 29$

$8, 15 \to 17$

$\csc\theta = \frac{1}{\sin\theta}$ $\sin\theta \csc\theta = 1$

$\sec\theta = \frac{1}{\cos\theta}$ $\cos\theta \sec\theta = 1$

$\tan\theta = \frac{1}{\cot\theta}$ $\tan\theta \cot\theta = 1$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cos^2\theta + \sin^2\theta = 1$ $\begin{cases} \cos^2\theta = 1 - \sin^2\theta \\ \sin^2\theta = 1 - \cos^2\theta \end{cases}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Ex $\sec\theta \tan\theta = \dfrac{1}{\cos\theta}\,\dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos^2\theta}$ $\quad \cos^2\theta + \sin^2\theta = 1$

$= \dfrac{\sin\theta}{1-\sin^2\theta}$

Ex $\dfrac{1}{\sin\theta} + \dfrac{1}{\cos\theta} = \dfrac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}$

Ex $\tan\alpha + \cot\alpha = \dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\cos\alpha}{\sin\alpha}$

$= \dfrac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha\sin\alpha}$

$= \dfrac{1}{\cos\alpha\sin\alpha}$

Prove $\tan x + \cos x = \sin x(\sec x + \cot x)$

$\sin x(\sec x + \cot x) = \sin x \dfrac{1}{\cos x} + \sin x \dfrac{\cos x}{\sin x}$

$= \tan x + \cos x$ ✓

$\tan x + \cos x = \dfrac{\sin x}{\cos x} + \cos x \dfrac{\sin x}{\sin x}$

$= \sin x\left(\dfrac{1}{\cos x} + \dfrac{\cos x}{\sin x}\right)$

$= \sin x(\sec x + \cot x)$ ✓

Ex Prove $\cot\alpha + 1 = \csc\alpha\,(\cos\alpha + \sin\alpha)$

$$\csc\alpha\,(\cos\alpha + \sin\alpha) = \frac{1}{\sin\alpha}\cos\alpha + \frac{1}{\sin\alpha}\sin\alpha$$

$$= \cot\alpha + 1 \quad \checkmark$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$a^3 + b^3 = (\quad + \quad)(\quad - \quad + \quad)$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^4 - b^4 = (a^2-b^2)(a^2+b^2) = (a-b)(a+b)(a^2+b^2)$$

$$a^2 + b^2 = \cancel{C}\ \mathbb{C}$$

$$\frac{\cos^4 t - \sin^4 t}{\cos^2 t} = 1 - \tan^2 t$$

$$\frac{\cos^4 t - \sin^4 t}{\cos^2 t} = \frac{(\cos^2 t - \sin^2 t)\overbrace{(\cos^2 t + \sin^2 t)}^{=1}}{\cos^2 t}$$

$$= \frac{\cos^2 t - \sin^2 t}{\cos^2 t}$$

$$= \frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t} \qquad \left(\frac{\sin t}{\cos t}\right)^2$$

$$= 1 - \tan^2 t \quad \checkmark$$

Prove $1+\cos\theta = \dfrac{\sin^2\theta}{1-\cos\theta}$

$$\frac{\sin^2\theta}{1-\cos\theta} = \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$

$$= 1+\cos\theta \checkmark$$

$$1+\cos\theta = (1+\cos\theta)\frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta}$$

$$1+\cos\theta \stackrel{?}{=} \frac{\sin^2\theta}{1-\cos\theta}$$

$$(1+\cos\theta)(1-\cos\theta) \stackrel{?}{=} \sin^2\theta$$

$$1-\cos^2\theta \stackrel{?}{=} \sin^2\theta$$

$$\sin^2\theta = \sin^2\theta \checkmark$$

Prove $\tan^2\alpha\,(1+\cot^2\alpha) = \dfrac{1}{1-\sin^2\alpha}$

$$\begin{aligned}
\tan^2\alpha\,(1+\cot^2\alpha) &= \tan^2\alpha + \tan^2\alpha\cot^2\alpha \\
&= \tan^2\alpha + 1 \\
&= \sec^2\alpha \\
&= \frac{1}{\cos^2\alpha} \\
&= \frac{1}{1-\sin^2\alpha} \quad \checkmark
\end{aligned}$$

$\dfrac{\sin\alpha}{1+\cos\alpha} + \dfrac{1+\cos\alpha}{\sin\alpha} = 2\csc\alpha$

$$\begin{aligned}
\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} &= \frac{\sin^2\alpha + (1+\cos\alpha)^2}{\sin\alpha\,(1+\cos\alpha)} \\
&= \frac{\sin^2\alpha + 1 + 2\cos\alpha + \cos^2\alpha}{\sin\alpha\,(1+\cos\alpha)} \\
&= \frac{2+2\cos\alpha}{\sin\alpha\,(1+\cos\alpha)} \\
&= \frac{2\,(1+\cos\alpha)}{\sin\alpha\,(1+\cos\alpha)} \\
&= \frac{2}{\sin\alpha} \\
&= 2\csc\alpha \quad \checkmark
\end{aligned}$$

$$\frac{1+\sin t}{\cos t} = \frac{\cos t}{1-\sin t}$$

$$\frac{1+\sin t}{\cos t} = \frac{1+\sin t}{\cos t}\cdot\frac{1-\sin t}{1-\sin t}$$

$$= \frac{1-\sin^2 t}{\cos t(1-\sin t)}$$

$$= \frac{\cos^2 t}{\cos t(1-\sin t)}$$

$$= \frac{\cos t}{1-\sin t} \quad \checkmark$$

$$\cot^2\theta + \cos^2\theta \neq \cot^2\theta\cos^2\theta$$

$\theta = \frac{\pi}{4}$

$$\cot^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} = 1 + \left(\frac{1}{\sqrt{2}}\right)^2 = \underbrace{1+\frac{1}{2}}$$

$$= \frac{3}{2}$$

$$\cot^2\frac{\pi}{4}\cos^2\frac{\pi}{4} = 1\cdot\frac{1}{2}$$

$$= \frac{1}{2}$$

$$\frac{1}{2} \neq \frac{3}{2}$$

$$\therefore \cot^2\theta + \cos^2\theta \neq \cot^2\theta\cos^2\theta$$

25. $7\csc^2 x - 5\cot^2 x = 2\csc^2 x + 5$

$$7\csc^2 x - 5\cot^2 x = 7\csc^2 x - 5(\csc^2 x - 1)$$
$$= 7\csc^2 x - 5\csc^2 x + 5$$
$$= 2\csc^2 x + 5 \checkmark$$

33. $(\sec x + \tan x)^2 = \dfrac{1+\sin x}{1-\sin x}$

$$(\sec x + \tan x)^2 = \sec^2 x + 2\sec x\tan x + \tan^2 x$$
$$= \sec^2 x + 2\frac{1}{\cos x}\frac{\sin x}{\cos x} + \sec^2 x - 1$$
$$= \frac{2}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} - 1$$
$$= \frac{2 + 2\sin x - \cos^2 x}{\cos^2 x}$$
$$= \frac{2 + 2\sin x - (1 - \sin^2 x)}{1 - \sin^2 x}$$
$$= \frac{\sin^2 x + 2\sin x + 1}{(1-\sin x)(1+\sin x)}$$
$$= \frac{(1+\sin x)^2}{(1-\sin x)(1+\sin x)}$$
$$= \frac{1+\sin x}{1-\sin x} \checkmark$$

6.4/ $\cot^3 x = \cot x\,(\csc^2 x - 1)$

$\cot x\,(\csc^2 x - 1) = \cot x\,(\cot^2 x)$

$= \cot^3 x$ ✓

Cosine squared θ $\cos^2\theta$

" double θ $\cos 2\theta$

$\cos\theta + 1 \xrightarrow{\text{means}} (\cos\theta) + 1$

$\cos(\theta + 1) \rightarrow \cos(\theta + 1)$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

2 cosines → 2 ≠

2 sines → 2 =

$$\cos(75^\circ) = \cos(45^\circ + 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\,\frac{1}{2}$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$\cos(x + 2\pi) = \cos x \,\cancel{\cos 2\pi}^{=1} - \sin x\, \cancel{\sin 2\pi}^{0}$$

$$= \cos x \checkmark$$

$$\cos 3x \cos 2x - \sin 3x \sin 2x$$

$$= \cos(3x + 2x)$$

$$= \cos 5x$$

$$\sin(90^\circ - A) = \cos A$$

$$\sin(90^\circ - A) = \cancel{\sin 90^\circ} \cos A - \cos 90^\circ \sin A$$

$$= \cos A \checkmark$$