Section 7.2 – Graphing Tangent & Cotangent

Vertical Asymptote

A ***vertical asymptote*** is a vertical line that the graph approaches but does not intersect, while function values increase or decrease without bound as x-values get closer and closer to the line.

Graphing the *Tangent* Functions

The graphs of $y = A\tan(Bx + C) + D$ will have the following characteristics:

Domain: $\left\{x \mid x \neq (2n+1)\frac{\pi}{2},\ where\ n \in \mathbb{Z}\right\}$

Range: $(-\infty,\ \infty)$

- The graph is discontinuous at values of x of the form $x = (2n+1)\frac{\pi}{2}$ and has ***vertical asymptotes*** at these values.
- Its ***x-intercepts*** are of the form $x = n\pi$.
- Its period is π.
- Its graph has no amplitude, since there are no minimum or maximum values.
- The graph is symmetric with respect to the origin, so the function is an odd function. For all x in the domain, $\tan(-x) = -\tan(x)$.

No *Amplitude*

Period : $P = \frac{\pi}{|B|}$

Phase Shift : $\phi = -\frac{C}{B}$

Vertical translation: $y = D$

Vertical Asymptote (*VA*): $bx + c = (2n+1)\frac{\pi}{2}$

One cycle: $0 \le argument \le \pi$ $\boxed{or}$ $-\frac{\pi}{2} < argument \le \frac{\pi}{2}$

Example

Find the period, and the phase shift and sketch the graph of $y = \frac{1}{2}\tan\left(x + \frac{\pi}{4}\right)$

Solution

Period: $P = \frac{\pi}{|B|} = \pi$

Phase shift: $\phi = -\frac{C}{B} = -\frac{\frac{\pi}{4}}{1} = -\frac{\pi}{4}$

Vertical translation: $y = 0$

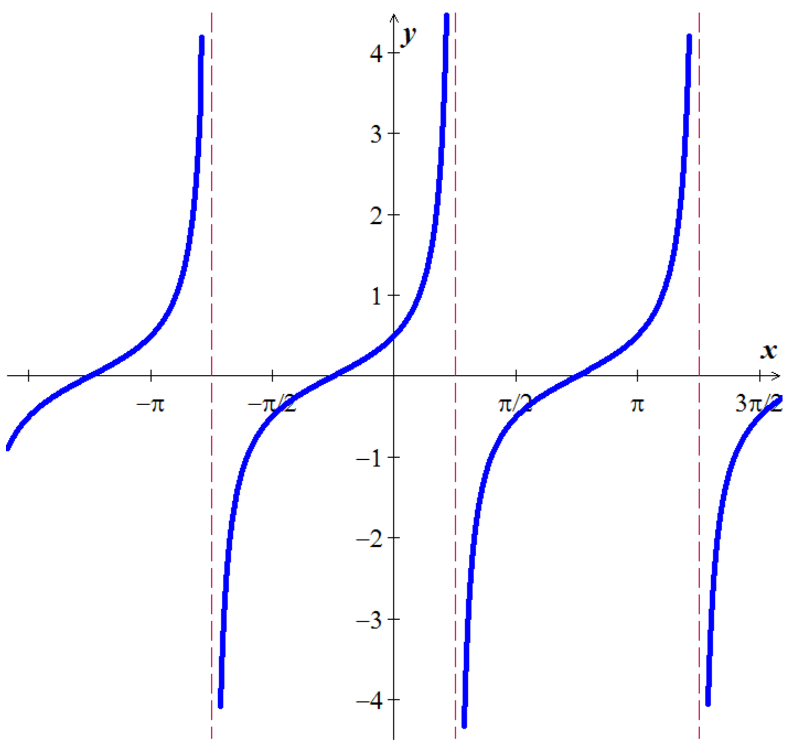
Vertical Asymptote: $x + \frac{\pi}{4} = (2n+1)\frac{\pi}{2}$

$$x + \frac{\pi}{4} = \pi n + \frac{\pi}{2}$$

$$x + \frac{\pi}{4} - \frac{\pi}{4} = \pi n + \frac{\pi}{2} - \frac{\pi}{4}$$

$$x = \pi n + \frac{\pi}{4}$$

	x	$y = \frac{1}{2}\tan\left(x + \frac{\pi}{4}\right)$
$-\frac{\pi}{4} + 0$	$-\frac{\pi}{4}$	0
$-\frac{\pi}{4} + \frac{1}{4}\pi$	0	0.5
$-\frac{\pi}{4} + \frac{1}{2}\pi$	$\frac{\pi}{4}$	∞
$-\frac{\pi}{4} + \frac{3}{4}\pi$	$\frac{\pi}{2}$	-0.5
$-\frac{\pi}{4} + \pi$	$\frac{3\pi}{4}$	0



One Complete cycle can be determined by:

$$-\frac{\pi}{2} \le x + \frac{\pi}{4} \le \frac{\pi}{2}$$

$$-\frac{\pi}{2} - \frac{\pi}{4} \le x + \frac{\pi}{4} - \frac{\pi}{4} \le \frac{\pi}{2} - \frac{\pi}{4}$$

$$-\frac{3\pi}{4} \le x \le \frac{\pi}{4}$$

Cotangent Functions

Domain: $\{x \mid x \neq n\pi, \text{ where } n \in \mathbb{Z}\}$

Range: $(-\infty, \infty)$

- The graph is discontinuous at values of x of the form $x = n\pi$ and has ***vertical asymptotes*** at these values.
- Its ***x-intercepts*** are of the form $x = (2n+1)\frac{\pi}{2}$.
- Its period is π.
- Its graph has no amplitude, since there are no minimum or maximum values.
- The graph is symmetric with respect to the origin, so the function is an odd function. For all x in the domain, $\cot(-x) = -\cot(x)$.

Example

Find the period, and the phase shift and sketch the graph of $y = \cot\left(2x - \frac{\pi}{2}\right)$

Solution

Period: $P = \frac{\pi}{|B|} = \frac{\pi}{2}$

Phase shift: $\phi = -\frac{C}{B} = -\frac{-\frac{\pi}{2}}{2} = \frac{\pi}{4}$

One cycle: $0 \le 2x - \frac{\pi}{2} \le \pi$

$$\frac{\pi}{2} \le 2x \le \frac{3\pi}{2}$$

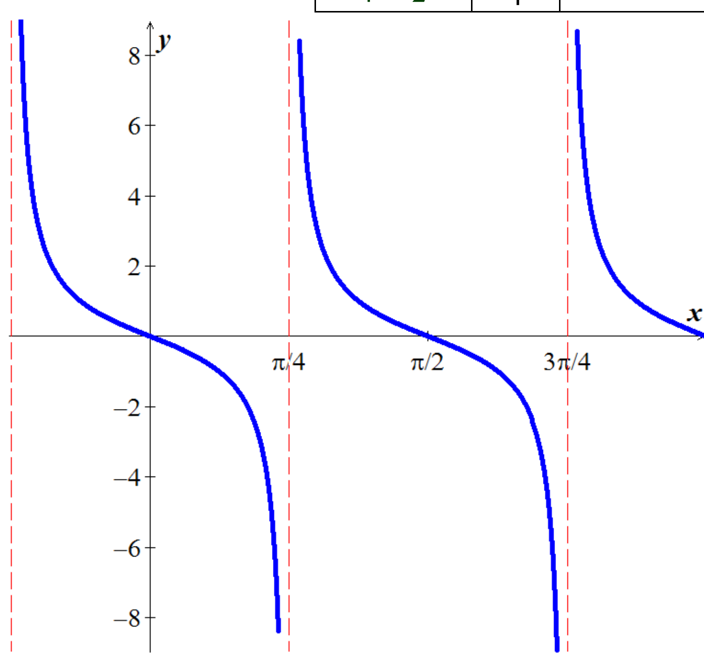
$$\frac{\pi}{4} \le x \le \frac{3\pi}{4}$$

	x	$y = \cot\left(2x - \frac{\pi}{2}\right)$
$\frac{\pi}{4} + 0$	$\frac{\pi}{4}$	∞
$\frac{\pi}{4} + \frac{\pi}{8}$	$\frac{3\pi}{8}$	1
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{4} + \frac{3\pi}{8}$	$\frac{5\pi}{8}$	-1
$\frac{\pi}{4} + \frac{\pi}{2}$	$\frac{3\pi}{4}$	∞

$V.A$: $2x - \frac{\pi}{2} = n\pi$

$$2x = n\pi + \frac{\pi}{2}$$

$$x = \frac{\pi}{2}n + \frac{\pi}{4}$$



Exercises *Section* 7.2 – Graphing Tangent & Cotangent

(**1 – 6**) Find the period, show the asymptotes, and sketch the graph of

1. $y = \tan\left(x - \frac{\pi}{4}\right)$

2. $y = 2\tan\left(2x + \frac{\pi}{2}\right)$

3. $y = -\frac{1}{4}\tan\left(\frac{1}{2}x + \frac{\pi}{3}\right)$

4. $y = \cot\left(x + \frac{\pi}{4}\right)$

5. $y = 2\cot\left(2x + \frac{\pi}{2}\right)$

6. $y = -\frac{1}{2}\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

(**7 – 10**) Graph over a **1-*period*** interval

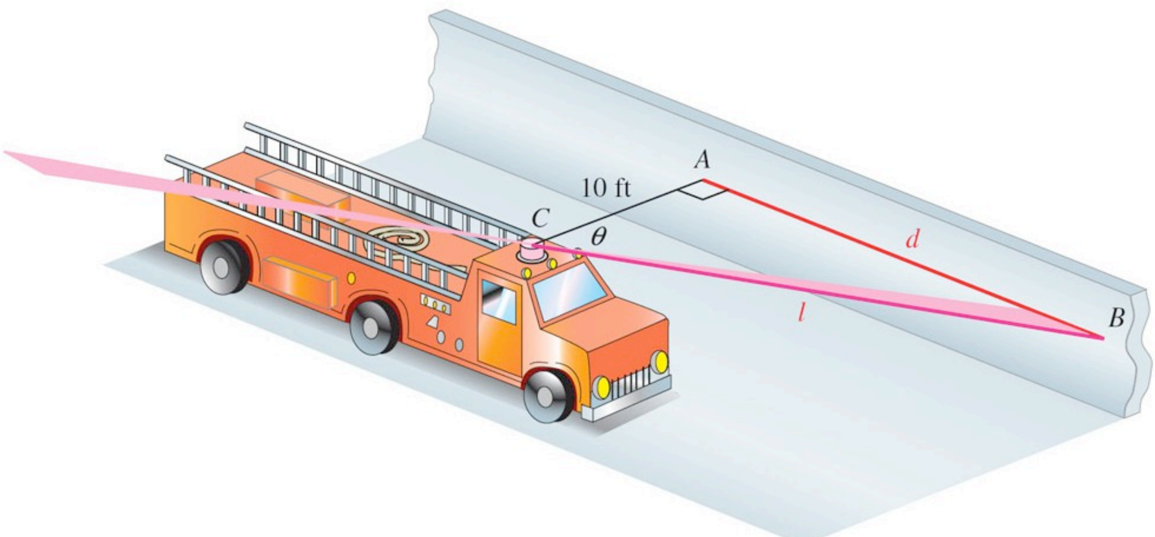
7. $y = 1 - 2\cot 2\left(x + \frac{\pi}{2}\right)$

8. $y = \frac{2}{3}\tan\left(\frac{3}{4}x - \pi\right) - 2$

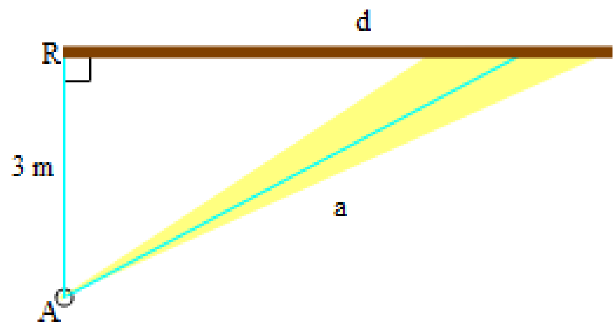
9. $y = -2 - \cot\left(x - \frac{\pi}{4}\right)$

10. $y = 3 + 2\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)$

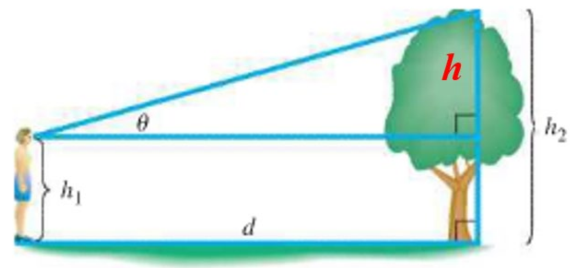
11. A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top is 10 *feet* from the wall and rotates through one complete revolution every 2 *seconds*. Graph the function that gives the length ***d*** in terms of time *t* from *t* = 0 to *t* = 2.



12. A rotating beacon is located 3 *m* south of point *R* on an east-west wall. *d*, the length of the light display along the wall from *R*, is given by $d = 3\tan 2\pi t$, where *t* is time measured in seconds since the beacon started rotating. (When *t* = 0, the beacon is aimed at point *R*. When the beacon is aimed to the right of *R*, the value of *d* is positive; *d* is negative if the beacon is aimed to the left of *R*.) Find a for *t* = 0.8



13. Let a person whose eyes are h_1 feet from the ground stand ***d*** feet from an object h_1 feet tall, where $h_2 > h_1$ feet. Let θ be the angle of elevation to the top of the object.



a) Show that $d = \left(h_2 - h_1\right)\cot\theta$

b) Let $h_2 = 55$ and $h_1 = 5$. Graph ***d*** for the interval $0 < \theta \le \frac{\pi}{2}$