Solution   *Section* 4.2 - Law of Cosines

Exercise

If $a = 13\ yd$, $b = 14\ yd$, and c = 15 yd, find the largest angle.

Solution

$$C = \cos^{-1}\frac{a^2+b^2-c^2}{2ab}$$

$$= \cos^{-1}\left(\frac{13^2+14^2-15^2}{2(13)(14)}\right)$$

$$\approx 67^\circ$$

Exercise

Solve triangle ABC if $b = 63.4$ km, and $c = 75.2$ km, $A = 124^\circ\ 40'$

Solution

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= (63.4)^2 + (75.2)^2 - 2(63.4)(75.2)\cos\left(124^\circ + \frac{40^\circ}{60}\right)$$

$$\approx 15098$$

$$a \approx 122.9\ km$$

$$\sin B = \frac{b\sin A}{a}$$

$$= \frac{63.4\sin 124.67^\circ}{122.9}$$

$$B = \sin^{-1}\left(\frac{63.4\sin 124.67^\circ}{122.9}\right)$$

$$\approx 25.1^\circ$$

$$C = 180^\circ - A - B$$

$$= 180^\circ - 124.67^\circ - 25.1^\circ$$

$$\approx 30.23^\circ$$

Exercise

Solve triangle *ABC* if *a* = 832 ft, *b* = 623 ft, and c = 345 ft

Solution

$$\underline{|C} = \cos^{-1}\frac{a^2+b^2-c^2}{2ab}$$

$$= \cos^{-1}\left(\frac{832^2+623^2-345^2}{2(832)(623)}\right)$$

$$\underline{\approx 22^\circ|}$$

$$\sin B = \frac{b\sin C}{c}$$

$$= \frac{623\sin 22^\circ}{345}$$

$$\underline{|B} = \sin^{-1}\left(\frac{623\sin 22^\circ}{345}\right)$$

$$\underline{\approx 43^\circ|}$$

$$\underline{|A} = 180^\circ - 22^\circ - 43^\circ \ \underline{= 115^\circ|}$$

Exercise

Solve triangle *ABC* if $A = 42.3^\circ,\ b = 12.9m,\ and\ c = 15.4m$

Solution

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= 12.9^2 + 15.4^2 - 2(12.9)(15.4)\cos 42.3^\circ$$

$$\approx 109.7$$

$$\boxed{a = 10.47\ m}$$

$$\sin B = \frac{b\sin A}{a} = \frac{12.9\sin 42.3^\circ}{10.47}$$

$$\underline{|B} = \sin^{-1}\left(\frac{12.9\sin 42.3^\circ}{10.47}\right) \ \underline{\approx 56.0^\circ|}$$

$$\underline{|C} = 180^\circ - 42.3^\circ - 56^\circ \ \underline{= 81.7^\circ|}$$

Exercise

Solve triangle ABC if $a=9.47\,ft,\ b=15.9\,ft,\ and\ c=21.1\,ft$

Solution

$$\underline{|C} = \cos^{-1}\frac{a^2+b^2-c^2}{2ab}$$

$$= \cos^{-1}\left(\frac{9.47^2+15.9^2-21.1^2}{2(9.47)(15.9)}\right)$$

$$\approx 109.9^\circ$$

$$\sin B = \frac{b\sin C}{c}$$

$$= \frac{15.9\sin 109.9^\circ}{21.1}$$

$$\underline{|B} = \sin^{-1}\left(\frac{15.9\sin 109.9^\circ}{21.1}\right)$$

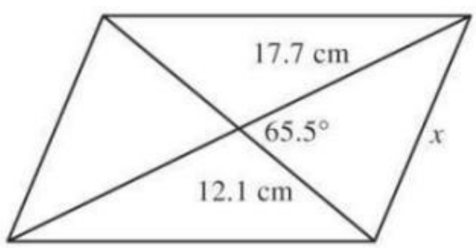
$$\approx 25.0^\circ$$

$$\underline{|A} = 180^\circ - 25^\circ - 109.9^\circ = 45.1^\circ$$

Exercise

The diagonals of a parallelogram are 24.2 cm and 35.4 cm and intersect at an angle of 65.5°. Find the length of the shorter side of the parallelogram

Solution



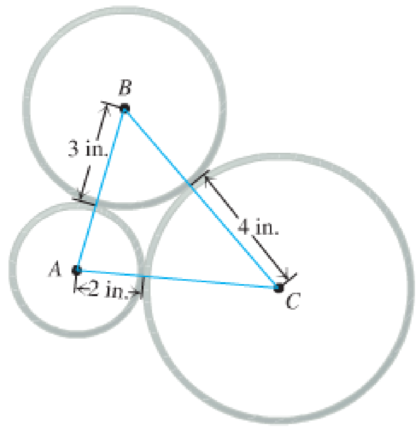
$$x^2 = 17.7^2 + 12.1^2 - 2(17.7)(12.1)\cos 65.5^\circ$$

$$= 282.07$$

$$\boxed{x = 16.8\ cm}$$

Exercise

An engineer wants to position three pipes at the vertices of a triangle. If the pipes A, B, and C have radii 2 in, 3 in, and 4 in, respectively, then what are the measures of the angles of the triangle ABC?



Solution

$$AC = 6 \quad AB = 5 \quad BC = 7$$

$$\underline{|A} = \cos^{-1}\left(\frac{5^2+6^2-7^2}{2(5)(6)}\right) \underline{\approx 78.5^\circ|}$$

$$\frac{6}{\sin B} = \frac{7}{\sin 78.5^\circ}$$

$$\sin B = \frac{6\sin 78.5^\circ}{7}$$

$$\underline{|B =} \sin^{-1}\left(\frac{6\sin 78.5^\circ}{7}\right) \underline{\approx 57.1^\circ|}$$

$$\underline{|C} = 180^\circ - 78.5^\circ - 57.1^\circ \underline{= 44.4^\circ|}$$

Exercise

Andrea and Steve left the airport at the same time. Andrea flew at 180 mph on a course with bearing 80°, and Steve flew at 240 mph on a course with bearing 210°. How far apart were they after 3 hr.?

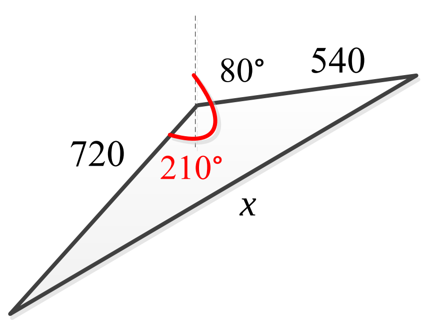
Solution

After 3 hrs., Steve flew: 3(240) = 720 mph
Andrea flew: 3(180) = 540 mph

$$x^2 = 720^2 + 540^2 - 2(720)(540)\cos 210^\circ$$

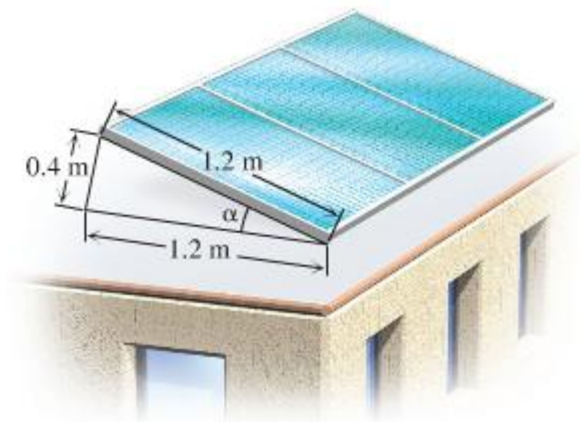
$$\underline{|x} = \sqrt{720^2 + 540^2 - 2(720)(540)\cos 210^\circ}$$

$$\underline{\approx 1144.5 \ miles|}$$



Exercise

A solar panel with a width of 1.2 m is positioned on a flat roof. What is the angle of elevation α of the solar panel?



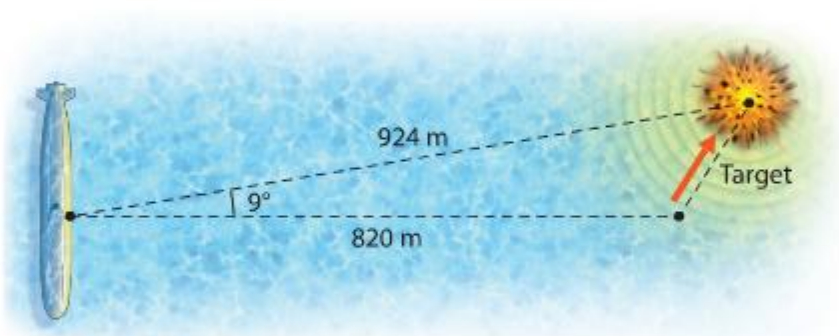
Solution

$$\alpha = \cos^{-1}\frac{1.2^2 + 1.2^2 - 0.4^2}{2(1.2)(1.2)} \approx 19.2°$$

or $\quad \alpha = \frac{s}{r} = \frac{0.4}{1.2}$

Exercise

A submarine sights a moving target at a distance of 820 m. A torpedo is fired 9° ahead of the target, and travels 924 m in a straight line to hit the target. How far has the target moved from the time the torpedo is fired to the time of the hit?

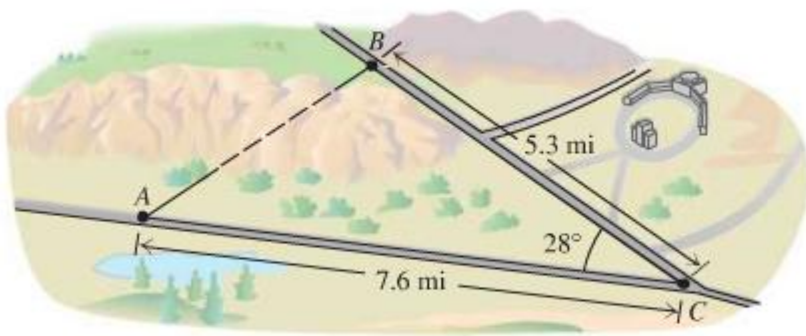


Solution

$$x = \sqrt{820^2 + 924^2 - 2(820)(924)\cos 9°} \approx 171.7 \text{ m}$$

Exercise

A tunnel is planned through a mountain to connect points *A* and *B* on two existing roads. If the angle between the roads at point *C* is 28°, what is the distance from point *A* to *B*? Find $\angle CBA$ and $\angle CAB$ to the nearest tenth of a degree.



Solution

By the cosine law,

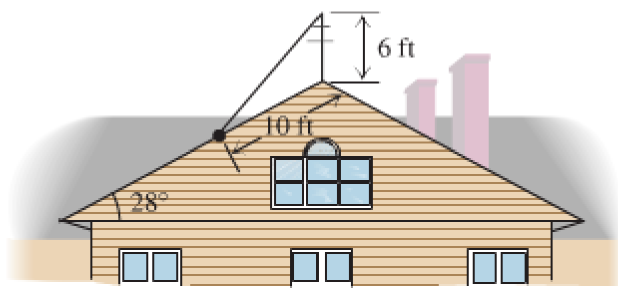
$$AB = \sqrt{5.3^2 + 7.6^2 - 2(5.3)(7.6)\cos 28^\circ} \approx 3.8$$

$$\angle CBA = \cos^{-1}\frac{3.8^2 + 5.3^2 - 7.6^2}{2(3.8)(5.3)} \approx 112^\circ$$

$$\angle BAC = 180^\circ - 112^\circ - 28^\circ \approx 40^\circ$$

Exercise

A 6-ft antenna is installed at the top of a roof. A guy wire is to be attached to the top of the antenna and to a point 10 ft down the roof. If the angle of elevation of the roof is 28°, then what length guy wire is needed?



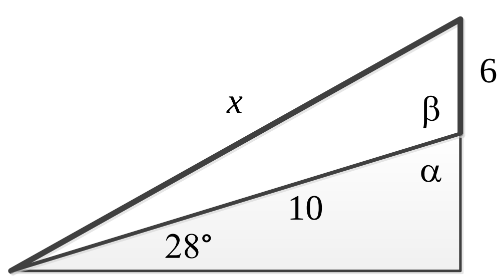
Solution

$$\alpha = 90^\circ - 28^\circ = 62^\circ$$

$$\beta = 180^\circ - 62^\circ = 118^\circ$$

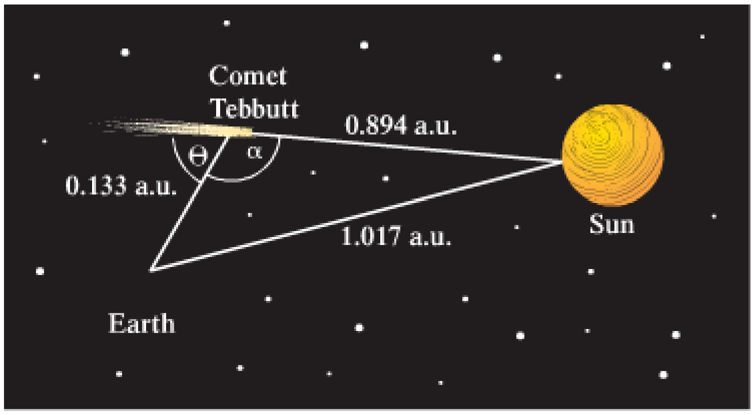
By the cosine law,

$$x = \sqrt{6^2 + 100^2 - 2(6)(10)\cos 118^\circ} \approx 13.9\ ft$$



Exercise

On June 30, 1861, Comet Tebutt, one of the greatest comets, was visible even before sunset. One of the factors that causes a comet to be extra bright is a small scattering angle θ. When Comet Tebutt was at its brightest, it was 0.133 a.u. from the earth, 0.894 a.u. from the sun, and the earth was 1.017 a.u. from the sun. Find the phase angle α and the scattering angle θ for Comet Tebutt on June 30, 1861. (One astronomical unit (a.u) is the average distance between the earth and the sub.)



Solution

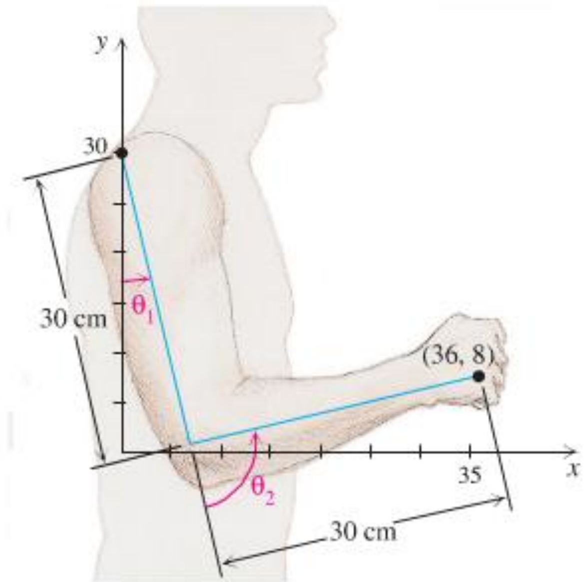
By the cosine law:

$$\alpha = \cos^{-1}\frac{0.133^2 + 0.894^2 - 1.017^2}{2(0.133)(0.891)} \approx 156°$$

$$\theta = 180° - \alpha = 180° - 156° = 24°$$

Exercise

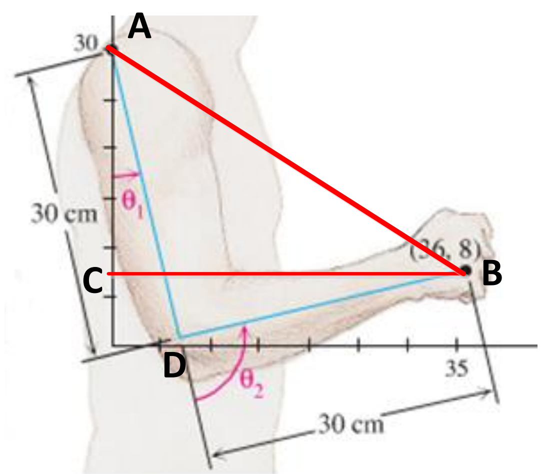
A human arm consists of an upper arm of 30 cm and a lower arm of 30 cm. To move the hand to the point (36, 8), the human brain chooses angle θ_1 *and* θ_2 to the nearest tenth of a degree.



Solution

$AC = 30 - 8 = 22 \qquad\qquad BC = 36$

$$AB = \sqrt{AC^2 + CB^2}$$
$$= \sqrt{22^2 + 36^2}$$
$$\approx 42.19$$



By the cosine law:

$$\angle ADB = \cos^{-1}\frac{AD^2 + DB^2 - AB^2}{2(AD)(DB)}$$

$$\angle ADB = \cos^{-1}\frac{30^2 + 30^2 - 42.19^2}{2(30)(30)} \quad \angle ADB \approx 89.4°$$

$$\theta_2 = 180° - 89.4° = 90.6°$$

$$\tan(\angle CAB) = \frac{BC}{AC} = \frac{36}{22}$$

$$\Rightarrow \angle CAB = \tan^{-1}\frac{36}{22} \approx 58.57°$$

$$\frac{\sin DAB}{30} = \frac{\sin 89.4°}{42.19} \Rightarrow \sin DAB = \frac{30\sin 89.4°}{42.19}$$

$$\angle DAB = \sin^{-1}\frac{30\sin 89.4°}{42.19} \approx 45.32°$$

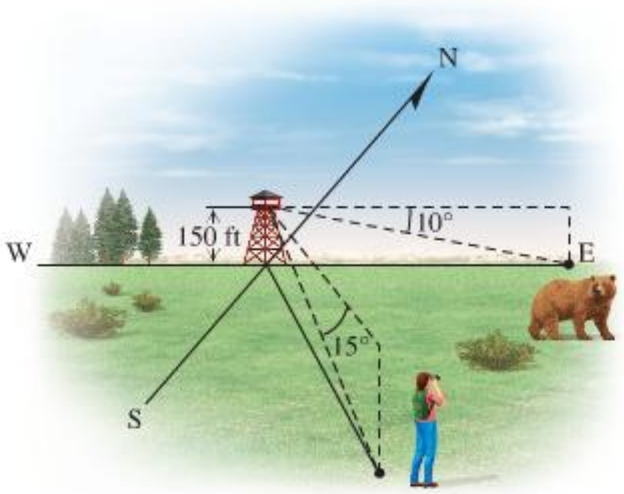
$$\boxed{\theta_1} = \angle CAB - \angle DAB$$

$$= 58.57° - 45.32°$$

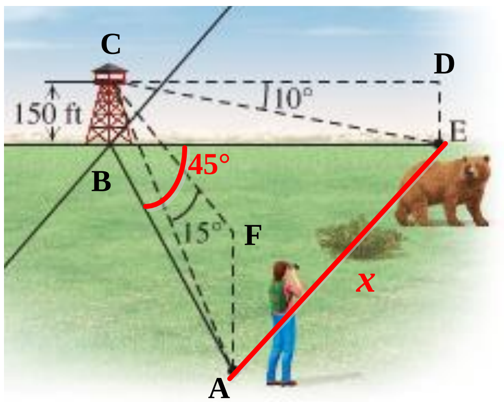
$$\underline{= 13.25°}$$

Exercise

A forest ranger is 150 ft above the ground in a fire tower when she spots an angry grizzly bear east of the tower with an angle of depression of 10°. Southeast of the tower she spots a hiker with an angle of depression of 15°. Find the distance between the hiker and the angry bear.



Solution



$\angle BEC = \angle ECD = 10°$

From triangle EBC:

$$\tan 10° = \frac{150}{BE}$$

$$\Rightarrow BE = \frac{150}{\tan 10°} \approx 850.692$$

$\angle BAC = \angle ACF = 15°$

From triangle ABC:

$$\tan 15° = \frac{150}{AB}$$

$$\Rightarrow AB = \frac{150}{\tan 15°} \approx 559.808$$

$$x = \sqrt{AB^2 + BE^2 - 2(AB)(BE)\cos 45°}$$

$$= \sqrt{559.808^2 + 850.692^2 - 2(559.808)(850.692)\cos 45°}$$

$$\underline{\approx 603\, ft}$$