

Exam 4 Review

#2.a

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ 2 & -3 & 5 & -1 & 0 \\ 1 & 0 & 3 & 1 & -4 \\ -4 & 3 & 2 & -1 & 3 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - R_1 \\ R_4 + 4R_1 \end{array}$$

$$R_2 - 2R_1 \left\{ \begin{array}{ccccc} 2 & -3 & 5 & -1 & 0 \\ -2 & 4 & -2 & -6 & 4 \\ \hline 0 & 1 & 3 & -7 & 4 \end{array} \right.$$

$$R_3 - R_1 \left\{ \begin{array}{ccccc} 1 & 0 & 3 & 1 & -4 \\ -1 & 2 & -1 & -3 & 2 \\ \hline 0 & 2 & 2 & -2 & -2 \end{array} \right.$$

$$R_4 - 4R_1 \left\{ \begin{array}{ccccc} -4 & 3 & 2 & -1 & 3 \\ -4 & 8 & -4 & -12 & 8 \\ \hline 0 & 11 & -2 & -13 & 11 \end{array} \right.$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ 0 & 1 & 3 & -7 & 4 \\ 0 & 2 & 2 & -2 & -2 \\ 0 & 11 & -2 & -13 & 11 \end{array} \right]$$

$$2x + y + 2z = 4$$

$$2x + 2y = 5$$

$$2x - z + 6z = 2$$

Write in Augmented matrix form.

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{array} \right] \checkmark$$

$$\begin{array}{c} \text{6/} \\ \left[\begin{array}{ccc|c} 9 & 1 & 1 & 1 \\ 0 & 6 & 6 & -1 \\ 3 & 1 & 9 & 7 \end{array} \right] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix} \end{array} \quad \left(\begin{array}{ccc|c} 9 & 1 & 1 & 1 \\ 0 & 6 & 6 & -1 \\ 3 & 1 & 9 & 7 \end{array} \right)$$

$$\begin{cases} 9x + y + z = 1 \\ 6y + 6z = -1 \\ 3x + y + 9z = 7 \end{cases}$$

#3.b

$$\begin{pmatrix} a & 8 & -6 \\ 5 & 4 & f \end{pmatrix} + \begin{pmatrix} 6 & b & -1 \\ d & 8 & f \end{pmatrix} = \begin{pmatrix} 8 & 1 & c \\ 3 & e & -5 \end{pmatrix}$$

$$\begin{pmatrix} a+6 & 8+b & -7 \\ 5+d & 12 & f+8 \end{pmatrix} = \begin{pmatrix} 8 & 1 & c \\ 3 & e & -5 \end{pmatrix}$$

$$a+6=8 \rightarrow \underline{a=2}$$

$$8+b=1 \rightarrow \underline{b=-7}$$

$$\underline{-7=c}$$

$$5+d=3 \rightarrow \underline{d=-2} \quad 3-5=-2$$

$$\underline{12=e}$$

$$f+8=-5 \rightarrow \underline{f=-5-8=-13}$$

$$\begin{aligned}
 \text{g) } \underbrace{B}_{2 \times 3} \underbrace{C}_{3 \times 2} &= \begin{bmatrix} -1 & 0 & 5 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} -1+20 & 1+30 \\ 2+6+16 & -2+9+24 \end{bmatrix} \\
 &= \begin{bmatrix} 19 & 31 \\ 24 & 31 \end{bmatrix}
 \end{aligned}$$

$$\#5/\text{a) } A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\begin{aligned}
 A^{-1} &= \frac{1}{-3+4} \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}
 \end{aligned}$$

$$\text{b) } B = \begin{pmatrix} 1 & -3 \\ -2 & 6 \end{pmatrix}$$

$$\begin{aligned}
 B^{-1} &= \frac{1}{6-6} = \\
 &= \frac{1}{0}
 \end{aligned}$$

B^{-1} doesn't exist
 or $B^{-1} \nexists$

$$\text{c) } C = \begin{pmatrix} 2 & -4 \\ a & b \end{pmatrix}$$

$$C^{-1} = \frac{1}{2b+4a} \begin{pmatrix} b & 4 \\ -a & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{b}{2(b+2a)} & \frac{4}{2(b+2a)} \\ \frac{-a}{2(b+2a)} & \frac{2}{2(b+2a)} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{b}{2(b+2a)} & \frac{2}{b+2a} \\ \frac{-a}{2(b+2a)} & \frac{1}{b+2a} \end{pmatrix}$$

$$9/ a) \begin{vmatrix} 3 & 1 & 2 \\ -2 & 3 & -1 \\ 3 & 4 & -6 \end{vmatrix} \begin{vmatrix} 3 & 1 \\ -2 & 3 \\ 3 & 4 \end{vmatrix} = -54 - 3 - 16 - 18 + 12 + 12$$

$$= \underline{-67}$$

$$b) \begin{vmatrix} x & 1 & -1 \\ x^2 & x & x \\ 0 & x & 1 \end{vmatrix} \begin{vmatrix} x & 1 \\ x^2 & x \\ 0 & x \end{vmatrix} = x^2 - x^3 - x^3 - x^2$$

$$= \underline{-2x^3}$$

$$c) \begin{vmatrix} 1 & x & x \\ 2 & x^2 & 2x \\ x & 0 & -1 \end{vmatrix} \begin{vmatrix} 1 & x \\ 2 & x^2 \\ x & 0 \end{vmatrix} = -x^2 + 2x^3 - x^4 + 2x$$

$$d) \begin{vmatrix} a & c \\ -2 & -4 \end{vmatrix} = -4a - (-2c)$$

$$= \underline{-4a + 2c}$$

$$\begin{vmatrix} x^2 & x \\ 9 & x \end{vmatrix} = \underline{x^3 - 9x}$$

$$\begin{vmatrix} 2 & 3 \\ 5 & 5 \end{vmatrix} = 10 - 15$$

$$= \underline{-5}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ -3 & 6 & -3 & -9 & 6 \\ 2 & 1 & 2 & 3 & 4 \\ 5 & 3 & 2 & -1 & -7 \end{array} \right] \begin{array}{l} R_2 + 3R_1 \\ R_3 - 2R_1 \\ R_4 - 5R_1 \end{array}$$

$$R_2 + 3R_1 \left\{ \begin{array}{ccccc} -3 & 6 & -3 & -9 & 6 \\ 3 & -6 & 3 & 9 & -6 \end{array} \right.$$

$$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R_3 - 2R_1 \left\{ \begin{array}{ccccc} 2 & 1 & 2 & 3 & 4 \\ -2 & 4 & -2 & -6 & 4 \end{array} \right.$$

$$\begin{array}{ccccc} 0 & 5 & 0 & -3 & 8 \end{array}$$

$$R_4 - 5R_1 \left\{ \begin{array}{ccccc} 5 & 3 & 2 & -1 & -7 \\ -5 & 10 & -5 & -15 & 10 \end{array} \right.$$

$$\begin{array}{ccccc} 0 & 13 & -3 & -16 & 3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & -3 & 8 \\ 0 & 13 & -3 & -16 & 3 \end{array} \right]$$

$$A = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix}$$

$$\begin{aligned} 4A - 2B &= 4 \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} - 2 \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 16 \\ 12 & -4 \end{pmatrix} - \begin{pmatrix} -2 & 6 \\ 4 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 10 \\ 8 & -2 \end{pmatrix} \end{aligned}$$

b) $3A + C$ = doesn't exist 2×2 3×3
not the same size.

$$\begin{aligned} \text{c) } AB &= \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -2+8 & 6-4 \\ -3-2 & 9-1 \end{pmatrix} \\ &= \begin{pmatrix} 6 & 2 \\ -6 & 10 \end{pmatrix} \end{aligned}$$

$$\begin{array}{cc} & \swarrow \searrow \\ 0 & 0 \end{array}$$

$$\begin{aligned} 2(-1) &= -2 \\ 4(2) &= 8 \\ \hline &6 \end{aligned}$$

$$\begin{aligned} \text{d) } BA &= \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -2+9 & -4-3 \\ 4-3 & 8-1 \end{pmatrix} \\ &= \begin{pmatrix} 7 & -7 \\ 1 & 9 \end{pmatrix} \end{aligned}$$

$$\begin{array}{r} -1(2) \\ 3(3) \end{array} \quad \begin{array}{r} -1(4) \\ 3(-1) \end{array}$$

$$A^2 = AA$$

$$= \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 4 \\ 3 & 13 \end{pmatrix}$$

$$\begin{array}{r|l} 2 \times 2 & 4 \\ 4 \times 3 & 7 \\ \hline & 11 \end{array}$$

$$\begin{array}{r} 2(4) = 8 \\ 4(-1) = -4 \\ \hline \end{array}$$

$$\begin{array}{l} 3(2) = 6 \\ -1(3) = -3 \\ 3(4) = 12 \\ -1(-1) = 1 \end{array}$$

$$\underbrace{AC}_{2 \times 2 \quad 3 \times 3} = \text{can't be calculated.}$$

$$A = \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{5+6} \begin{pmatrix} 5 & -3 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{11} & -\frac{3}{11} \\ \frac{2}{11} & \frac{1}{11} \end{pmatrix}$$

$$A = \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-4+4}$$

$$A^{-1} = \text{undefined}$$

$$A = \begin{pmatrix} a & 2 \\ 2 & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{a^2 - 4} \begin{pmatrix} a & -2 \\ -2 & a \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a}{a^2 - 4} & \frac{-2}{a^2 - 4} \\ \frac{-2}{a^2 - 4} & \frac{a}{a^2 - 4} \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & a \\ 3 & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{a-3a} \begin{pmatrix} a & -a \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a}{-2a} & \frac{-a}{-2a} \\ \frac{-3}{-2a} & \frac{1}{-2a} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{2a} & -\frac{1}{2a} \end{pmatrix}$$

$$A = \begin{pmatrix} 4 & -16 \\ 1 & -4 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-16+16} () = \frac{1}{0}$$

$$A^{-1} \nexists$$

$$\left| \begin{array}{ccc|cc} 1 & x & -2 & 1 & x \\ 3 & 1 & 1 & 3 & 1 \\ 0 & -2 & 2 & 0 & -2 \end{array} \right| = 2 + 12 + 2 - 6x$$

$$= \underline{16 - 6x}$$

$$\left| \begin{array}{ccc|cc} 2 & x & 1 & 2 & x \\ -3 & 1 & 0 & -3 & 1 \\ 2 & 1 & 4 & 2 & 1 \end{array} \right| = 8 - 3 - 2 + 12x$$

$$= \underline{3 + 12x}$$

$$\begin{vmatrix} x+1 & -6 \\ x+3 & -3 \end{vmatrix} = -3(x+1) - (-6)(x+3) \\ = -3x - 3 + 6x + 18 \\ = \underline{3x + 15}$$

$$\begin{vmatrix} x+2 & 6 \\ x-2 & 4 \end{vmatrix} = 4x+8 - (6x-12) \\ = 4x+8-6x+12 \\ = \underline{-2x+20}$$

$$\begin{vmatrix} x & x^2 \\ 4 & x \end{vmatrix} = x^2 - 4x^2 \\ = \underline{-3x^2}$$

$$\begin{vmatrix} \sqrt{5} & 3 \\ -2 & 2 \end{vmatrix} = \underline{2\sqrt{5} + 6}$$

$$\begin{vmatrix} \sqrt{3} & -2 \\ -3 & \sqrt{3} \end{vmatrix} = 3 - 6 = \underline{-3}$$

$$\begin{vmatrix} -4 & -1 \\ 5 & 6 \end{vmatrix} = -24 + 5 \\ = \underline{-19}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & -4 & 2 & -7 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$R_2 - 2R_1 \left\{ \begin{array}{cccc} 2 & 1 & 1 & 3 \\ -2 & -2 & -2 & -4 \\ \hline 0 & -1 & -1 & -1 \end{array} \right.$$

$$R_3 - 3R_1 \left\{ \begin{array}{cccc} 3 & -4 & 2 & -7 \\ -3 & -3 & -3 & -6 \\ \hline 0 & -7 & -1 & -13 \end{array} \right.$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -1 & -1 \\ 0 & -7 & -1 & -13 \end{array} \right]$$

$$A = \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\underbrace{AB}_{3 \times 2 \quad 2 \times 3} = \begin{bmatrix} -1 & 2+3 & -3+6 \\ 2 & -4+1 & 6+2 \\ -3 & 6+2 & -9+4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 5 & 3 \\ 2 & -3 & 8 \\ -3 & 8 & -5 \end{bmatrix}$$

$$\underbrace{BA}_{2 \times 3 \quad 3 \times 2} = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -1-4-9 & 3-2+6 \\ 2-6 & 1+4 \end{pmatrix}$$

$$= \begin{pmatrix} -14 & 7 \\ -4 & 5 \end{pmatrix}$$