Limit

$\lim_{x\to c} b = b$	$\lim_{x\to c} x = c$	$\lim_{x\to c} x^n = c^n$

$\lim_{x\to c} \sqrt[n]{x} = \sqrt[n]{c}$ n *is even* $c>0$

$$\lim_{x\to\infty}\left[\frac{p(x)}{q(x)}=\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_1x+b_0}\right]=\frac{a_n}{b_n}$$

$$\lim_{x\to c}[bf(x)] = b\lim_{x\to c} f(x)$$

$$\lim_{x\to c}[f(x)\pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$$

$$\lim_{x\to c}[f(x)\cdot g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$$

$$\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$

$$\lim_{x\to c}[f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$$

$$\lim_{x\to c}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)}$$

$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ $(\theta$ *in rad.*$)$

$\lim_{x\to\infty}\frac{1}{x^n}=0$ $n>0$

n even: $\lim_{x\to\pm\infty} x^n = \infty$	*n odd*: $\lim_{x\to\infty} x^n = \infty$ $\lim_{x\to-\infty} x^n = -\infty$

$\lim_{x\to 1^-}\frac{\lvert x-1\rvert}{x-1}=-1$	$\lim_{x\to 1^+}\frac{\lvert x-1\rvert}{x-1}=1$

$\lim_{x\to\infty} e^x=\infty$	$\lim_{x\to-\infty} e^x=0$	$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$

$\lim_{x\to\infty}\ln(x)=\infty$	$\lim_{x\to 0^+}\ln(x)=0$

End Behavior and *Asymptotes* of Rational Functions

Let $f(x) = \dfrac{p(x)}{q(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \ldots + b_1 x + b_0} = \dfrac{a_n x^n}{b_m x^m}$ be a rational function.

1. If the degree of numerator is less than of denominator $(n < m) \Rightarrow y = 0$
   Horizontal Asymptote (**HA**)
2. If the degree of numerator is equal of denominator $(n = m) \Rightarrow y = \dfrac{a_n}{b_m}$ ***Horizontal Asymptote*** (**HA**)
3. If the degree of numerator is greater than of denominator $(n > m) \Rightarrow$ No ***Horizontal Asymptote***

Vertical Asymptote - ***Think Domain***

Average rate of change: $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+h) - f(x)}{h}, \quad h \neq 0$

Sandwich Theorem $g(x) \le f(x) \le h(x) \Rightarrow \lim\limits_{x \to c} g(x) = \lim\limits_{x \to c} h(x) = L$ *then* $\lim\limits_{x \to c} f(x) = L$

Precise Definition of a Limit

Let $f(x)$ be defined on an open interval about x_0, except possibly at x_0 itself. We say that **the limit of** $f(x)$ **as *x* approaches** x_0 **is the number *L***, and write: $\lim\limits_{x \to x_0} f(x) = L$

If, for every number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all *x*,

$0 < |x - x_0| < \delta \quad \Rightarrow \quad |f(x) - L| < \varepsilon$

One-Sided Limits

If the approach is from the *right*, the limit is a ***right-hand limit***. $\lim\limits_{x \to c^+} f(x) = L$

If for every number $\varepsilon > 0$ there exists a corresponding number $\delta > 0$ such that for all *x*

$$x_0 < x < x_0 + \delta \quad \Rightarrow \quad |f(x) - L| < \varepsilon$$

If the approach is from the *left*, the limit is a ***left-hand limit***. $\lim\limits_{x \to c^-} f(x) = M$

If for every number $\varepsilon > 0$ there exists a corresponding number $\delta > 0$ such that for all *x*

$$x_0 - \delta < x < x_0 \quad \Rightarrow \quad |f(x) - L| < \varepsilon$$

***Continuity*:**

Let c be a number in the interval (a, b), and let f be a function whose domain contains the interval (a, b). The function f is continuous at the point c if the following conditions are true.

1. $f(c)$ is defined
2. $\lim\limits_{x\to c} f(x)$ *exists*
3. $\lim\limits_{x\to c} f(x) = f(c)$

Interior point: A function $y = f(x)$ is **continuous at an interior point *c*** of its domain if

$$\lim_{x\to c} f(x) = f(c)$$

Endpoint: A function $y = f(x)$ is **continuous at a left point *a*** or is **continuous at a right point *b*** of its domain if

$$\lim_{x\to a^+} f(x) = f(a) \quad \textit{or} \quad \lim_{x\to b^-} f(x) = f(b), \quad \textit{respectively}$$

Intermediate Value Theorem

We call a solution of the equation $f(x) = 0$ a ***root*** of the equation or zero of the function *f*. The Intermediate Value Theorem said that if ***f*** is continuous, then any interval on which *f* changes sign contains a zero of the function.