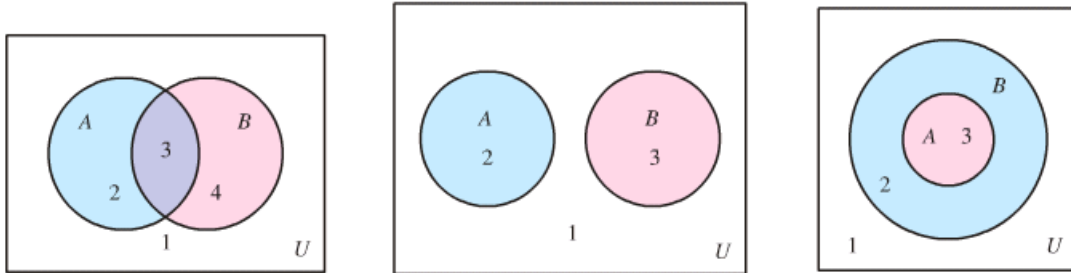


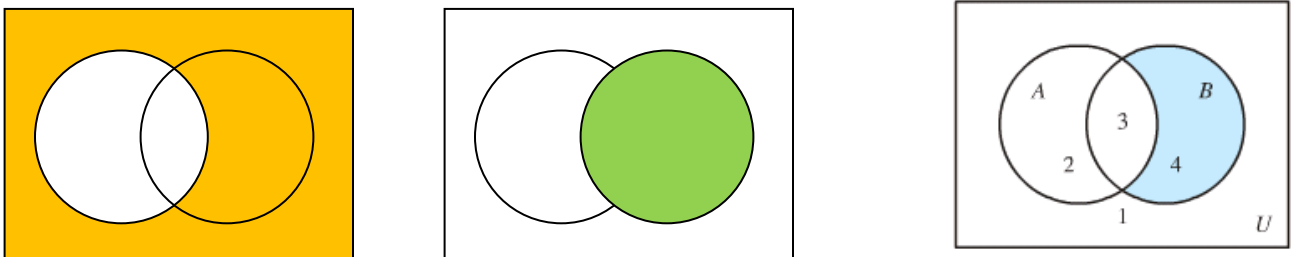
Section 3.2 – Applications of Venn Diagrams

The event might be; winning the lottery, guessing the correct answer on a test question, selecting an Ace from a deck of cards



Example

Draw Venn diagrams and shade the regions representing $A' \cap B$



Example

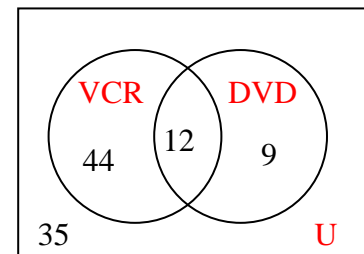
A researcher collecting data on 100 households finds that

- ✓ 21 have a DVD player;
- ✓ 56 have a videocassette recorder (VCR); and
- ✓ 12 have both

- a) How many do not have a VCR?
- b) How many have neither a DVD player nor a VCR?
- c) How many have a DVD player but not a VCR?

Solution

- a) How many do not have a VCR?
 $35 + 9 = 44$
- b) How many have neither a DVD player nor a VCR?
 35
- c) How many have a DVD player but not a VCR?
 9



Example

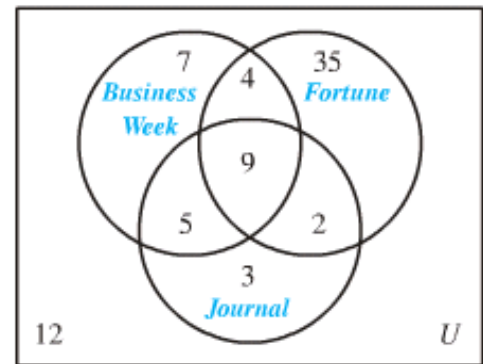
A survey of 77 freshman business students at a large university produced the following results

- ✓ 25 of the students read Business Week;
- ✓ 19 read the Wall Street Journal;
- ✓ 27 do not read Fortune
 - 11 read Business Week but not Wall Street Journal
 - 11 read the Wall Street Journal and Fortune
 - 13 read Business Week and Fortune;
 - 9 read all three

- a) How many students read none of the publications?
- b) How many read only Fortune?
- c) How many read Business Week and the Wall Street Journal, but not Fortune?

Solution

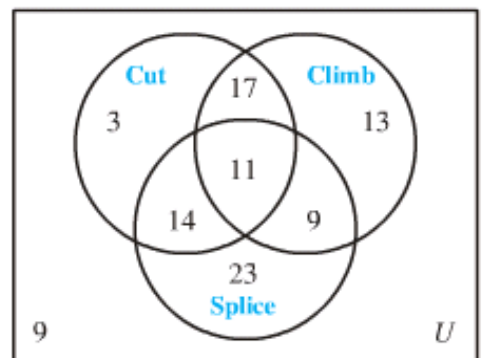
- a) **12** students read none of the publications
- b) **35** read only Fortune
- c) **5** read Business Week and the Wall Street Journal, but not Fortune $(B \cap J) \cap F'$



Example

Jeff is a section chief of an electric utility company. The employees in his section cut down trees, climb poles, and splice wire. Jeff reported the following information to the management of the utility

- “Of the 100 employees in my section,
45 can cut trees;
50 can climb poles;
57 can splice wire;
22 can climb poles but can’t cut trees;
20 can climb poles but splice wire;
25 can cut trees and splice wire;
14 can cut trees and splice wire but can’t climb poles;
9 can’t do any of the three (management trainees).”



Solution

Jeff claimed to have 100 employees, but his information indicate only 99.

Management decided that Jeff didn’t qualify as a section chief.

Moral: *He should have taken this course.*

Counting Techniques: A method that gives us a way to count how many elements are in a set, without having to actually count them.

Union Rule for Sets - Addition Principle (For Counting)

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cap B) = 0 \Rightarrow A \& B \text{ are disjoint}$$

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) \\ &\quad - n(A \cap B) - n(A \cap C) - n(B \cap C) \\ &\quad + n(A \cap B \cap C) \end{aligned}$$

Example

A group of 10 students meet to plan a school function. All are majoring in accounting or economics or both. Five of the students are economics majors and 7 are majors in accounting. How many major in both subjects?

Solution

$$n(A \cup B) = 10$$

$$n(A) = 5$$

$$n(B) = 7$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$10 = 5 + 7 - n(A \cap B)$$

$$10 - 12 = -n(A \cap B)$$

$$-2 = -n(A \cap B)$$

$$n(A \cap B) = 2$$

Exercises Section 3.2 – Applications of Venn Diagrams

1. Use the union rule to answer the following
 - a) If $n(A)=5$; $n(B)=12$, and $n(A \cap B)=4$ what is $n(A \cup B)$?
 - b) If $n(A)=15$; $n(B)=30$, and $n(A \cup B)=33$ what is $n(A \cap B)$?
 - c) $n(B)=9$; $n(A \cap B)=5$, and $n(A \cup B)=22$ what is $n(A)$?
2. Draw a Venn diagram and use the given information to fill in the number of elements
 - a) $n(U)=41$; $n(A)=16$, $n(A \cap B)=12$, $n(B')=20$
 - b) $n(A)=28$; $n(B)=12$, $n(A \cup B)=32$, $n(A')=19$
 - c) $n(A)=11$; $n(A \cap B)=6$, $n(A \cup B)=24$, $n(A' \cup B')=25$
 - d) $n(A)=28$, $n(B)=34$, $n(C)=25$, $n(A \cap B)=14$, $n(B \cap C)=15$
 $n(A \cap C)=11$; $n(A \cap B \cap C)=9$, $n(U)=59$
 - e) $n(A)=54$, $n(B')=63$, $n(C)=44$, $n(A \cap B)=22$, $n(B \cap C)=16$,
 $n(A \cap C)=15$; $n(A \cap B \cap C)=4$, $n(A \cup B)=85$
 - f) $n(A \cap C')=11$, $n(B \cap C')=8$, $n(C)=15$, $n(A \cap B)=6$, $n(B \cap C)=4$
 $n(A \cap C)=7$; $n(A \cap B \cap C)=4$, $n(A' \cap B' \cap C')=5$
3. Toward the middle of the harvesting season peaches for canning come in three types, early, late, and extra late, depending on the expected date of ripening. During a certain week, the following data were recorded at a fruit delivery station:
 - 34 trucks went out carrying early peaches;
 - 61 carried late peaches;
 - 50 carried extra late;
 - 25 carried early and late;
 - 30 carried late and extra late;
 - 8 carried early and extra late;
 - 6 carried all three
 - 9 carried only figs (no peaches at all).
 - a) How many trucks carried only variety peaches?
 - b) How many carried only extra late?
 - c) How many carried only one type of peach?
 - d) How many trucks (in all) went during the week?

4. In a survey of 100 randomly chosen students, a marketing questionnaire included the following:
- ✓ 75 own a TV
 - ✓ 45 own a car
 - ✓ 35 own a TV and a car
- a) How many students owned a car but not a TV set?
- b) How many students did not own both a car and a TV set?
5. A small town has two radio stations, an AM station and an FM station. A survey of 100 residents of the town produced the following results: In the last 30 days, 65 people have listened to the AM station, 45 have listened to the FM station, and 30 have listened to both stations.
6. In a class of 35 students, 19 are married and 20 are blondes. Given that there are 7 students that are both married and blonde, answer the following questions.
- a) How many are married, but not blonde?
- b) How many are blonde but not married?
- c) How many are blonde or married?
- d) How many are neither blonde nor married?
- e) How many are not blonde?
7. In a survey of 500 businesses it was found that 250 had copiers and 300 had fax machines. It was also determined that 100 businesses had both copiers and fax machines.
- a) How many had either a copier or a fax machine?
- b) How many had neither a copier nor a fax machine?
- c) How many had a copier, but no fax machine?
- d) How many had a fax machine, but no copier?
- e) How many had no fax machines?
8. Given: $n(U) = 105$, $n(A) = 50$, $n(B) = 75$, $n(A \cup B) = 105$, find the following:
- a) $n(A \cap B)$ b) $n(A' \cap B)$ c) $n(A' \cap B')$
- d) $n(A \cup B')$ e) $n(B')$
9. Fred interviewed 140 people in a shopping center to discover some of their cooking habits. He obtained the following results:
- 58 use microwave ovens
 - 63 use electric ranges
 - 58 use gas ranges
 - 19 use microwave ovens and electric ranges
 - 17 use microwave ovens and gas ranges
 - 4 use both gas and electric ranges
 - 1 uses all three
 - 2 use none of the three

Should he be reassigned one more time? Why or why not?

- 10.** Toward the middle of the harvesting season, peaches for canning come in three types, early, late, and extra late, depending on the expected date of ripening. During a certain week, the following data were recorded at a fruit delivery station:
- 34 trucks went out carrying early peaches
 - 61 carried late peaches
 - 50 carried extra late
 - 25 carried early and late
 - 30 carried late and extra late
 - 8 carried early and extra late
 - 6 carried all three
 - 9 carried only figs (no peaches at all)
- a) How many trucks carried only late variety peaches?
 - b) How many carried only extra late?
 - c) How many carried only one type of peach?
 - d) How many trucks (in all) went out during the week?
- 11.** Most mathematics professors love to invest their hard earned money. A recent survey of 150 math professors revealed that
- 111 invested in stocks
 - 98 invested in bonds
 - 100 invested in certificates of deposit
 - 80 invested in stocks and bonds
 - 83 invested in bonds and certificates of deposit
 - 85 invested in stocks and certificates of deposit
 - 9 did not invest in any of three
- How many mathematics professors invested in stocks and bonds and certificates of deposit?
- 12.** Suppose that a group of 150 students have joined at least one of three chat rooms; one on auto-racing, one on bicycling, and one for college students. For simplicity, we will call these rooms *A*, *B*, and *C*. In addition,
- 90 students joined room *A*;
 - 50 students joined room *B*;
 - 70 students joined room *C*;
 - 15 students joined room *A* and *C*;
 - 12 students joined room *B* and *C*;
 - 10 students joined all three rooms;
- Determine how many students joined both chat rooms *A* and *B*.