Calc Review

C.N., Inc + Dec.

#1/ $f(x) = x^3 + 3x^2 - 9x + 4$

$f'(x) = 3x^2 + 6x - 9 = 0$

$\underline{\text{C.N. } x = 1, -3}$

-3	0	1
$+$	$-$	$+$

Incr. $(-\infty, -3) \cup (1, \infty)$ Decr. $(-3, 1)$

#5/ $f(x) = -\dfrac{x}{x^2+1}$ D: $\mathbb{R}$

$f'(x) = \dfrac{-x^2+1}{(x^2+1)^2} = 0$

$x^2 = 1 \Rightarrow \underline{\text{C.N.: } x = \pm 1}$

-1	0	1
$-$	$+$	$-$

Incr: $(-1, 1)$ Decr. $(-\infty, -1)$ $(1, \infty)$

Ex: CN: Extreme

ex 16 $f(x) = 2x^3 - 6x + 1$

$f'(x) = 6x^2 - 6 = 0$

$x^2 = 1 \Rightarrow \boxed{CN: x = \pm 1}$

x	$f(x)$
-1	5
1	-3

RMIN: $(1, -3)$

RMAX: $(-1, 5)$

\# 20 $y = \sqrt{4 - x^2}$ D: $-2 \leq x \leq 2$

$$y' = \frac{-x}{\sqrt{4-x^2}} = 0$$

$(u^n)' = n u' u^{n-1}$

CN: $x = 0$, ± 2

x	$f(x)$
-2	0
0	2
2	0

LMAX: $(0, 2)$

LMIN: $(-2, 0)$ $(2, 0)$

Concavity   Pt of infl.

#127 $f(x) = -4x^3 - 8x^2 + 12$

$f'(x) = -12x^2 - 16x$

$f''(x) = -24x - 16 = 0$

$\left(x = -\frac{16}{24} = -\frac{2}{3} \right)$ Pt. of infl.

$$\begin{array}{c|c} & -2/3 \quad 0 \\ + & - \end{array}$$

Concave up: $(0, -2/3)$

" down: $(-\frac{2}{3}, 0)$

$f(x) = x^3 - 9x^2 + 24x - 16$

$f'(x) = 3x^2 - 18x + 24$

$f''(x) = 6x - 18 = 0$

Pt of infl: $\underline{x = 3}$

$$\begin{array}{c|c} 0 & 3 \\ - & + \end{array}$$

Concave up: $(3, \infty)$

" down: $(-\infty, 3)$

L'Hôpital Rule

#18) $\lim\limits_{x\to 1} \dfrac{x^n-1}{x-1} = \dfrac{0}{0}$

$= \lim\limits_{x\to 1} \dfrac{n x^{n-1}}{1}$

$= n$

#21) $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\tan x - \cot x}{x - \frac{\pi}{4}} = \dfrac{1-1}{\frac{\pi}{4}-\frac{\pi}{4}} = \dfrac{0}{0}$

$= \lim\limits_{x\to \frac{\pi}{4}} \dfrac{\sec^2 x + \csc^2 x}{1}$

$= 2 + 2$

$= 4$

#115. $\lim\limits_{x\to \infty} \left(1+\frac{a}{x}\right)^x = 1^\infty$

$\lim\limits_{x\to\infty} \ln\left(1+\frac{a}{x}\right)^x = \lim\limits_{x\to\infty} \dfrac{\ln\left(1+\frac{a}{x}\right)}{\frac{1}{x}} = \dfrac{0}{0}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{-a/x^2}{1+\frac{a}{x}}}{-\frac{1}{x^2}}$

$= \lim\limits_{x\to\infty} \dfrac{a}{1+\frac{a}{x}}$

$= a$

$\lim\limits_{x\to\infty} \left(1+\frac{a}{x}\right)^x = e^a$

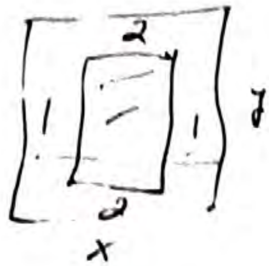
#86 $\lim_{x \to \frac{\pi}{2}} \frac{\ln \sin x}{\cos x} = \frac{\ln 1}{0} = \frac{0}{0}$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{\cos x}{\sin x}}{-\sin x}$$

$$= -\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\sin^2 x}$$

$$= 0$$

#26 (3.3)



$A_{in} = 30 \text{ in}^2$

$(x-2)(y-4) = 30$ ①

$A_p = xy$ ②

① $y = \frac{30}{x-2} + 4$ ③ $\qquad y - 4 = \frac{30}{x-2}$

$A_p = x\left(\frac{30}{x-2} + 4\right)$

$= \frac{30x}{x-2} + 4x$

$A_p' = \frac{-60}{(x-2)^2} + 4 = 0$

$\frac{-60}{(x-2)^2} = -4 \Rightarrow (x-2)^2 = 15$

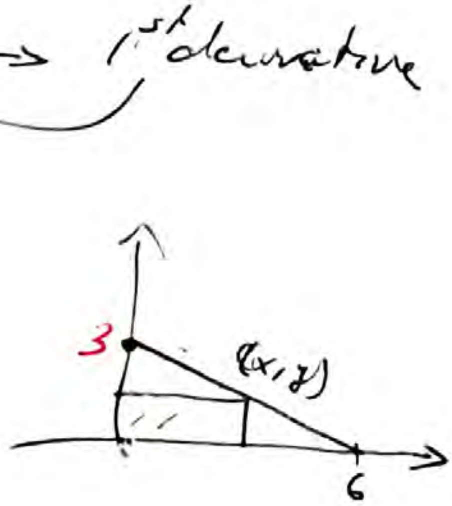
$x - 2 = -\sqrt{15} \rightarrow x = 2 - \sqrt{15} < 0$ #

$x - 2 = \sqrt{15} \rightarrow \boxed{x = 2 + \sqrt{15}}$

③ $y = \frac{30}{2+\sqrt{15}-2} + 4 = \boxed{2\sqrt{15} + 4}$

$\left.\begin{array}{l}\text{Minimize}\\ \text{Maximize}\end{array}\right\} \rightarrow$ CN $\leftrightarrow$ 1st derivative

#31 $y = \frac{1}{2}(6-x)$



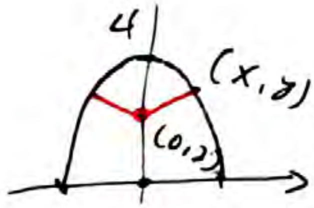
$A = xy$

$= \frac{1}{2}(6x - x^2)$

$A' = \frac{1}{2}(6 - 2x) = 0$ CN: $x = 3$

$y = \frac{3}{2}$ dim: $(3, 3/2)$

#30 $y = 4 - x^2$ closest $(0, 2)$



$d = \sqrt{x^2 + (y-2)^2}$

$f(x) = \sqrt{x^2 + (2 - x^2)^2}$

$= \sqrt{x^2 + 4 - 4x^2 + x^4}$

$= (x^4 - 3x^2 + 4)^{1/2}$

$f' = \frac{1}{2} \frac{4x^3 - 6x}{\sqrt{x^4 - 3x^2 + 4}} = 0$

$x = 0, \ x^2 = \frac{6}{4} \Rightarrow x = \pm \frac{\sqrt{6}}{2}$

x	y
0	4
$\pm\frac{\sqrt{6}}{2}$	$4 - \frac{6}{4}$

$\left(\pm \frac{\sqrt{6}}{2}, \frac{5}{2}\right)$

#3c1

max V?

$V = \frac{\pi}{3} r^2 h$

$r^2 + h^2 = 3$

$r^2 = 3 - h^2$ (1)

$V = \frac{\pi}{3}(3h - h^3)$

$h = -1$ #

$V' = \frac{\pi}{3}(3 - 3h^2) \Rightarrow \underline{h = 1}$ CN

(1) $r = \sqrt{3-1} = \sqrt{2}$

$V = \frac{\pi}{3}\, 2(1) = \underline{\frac{2\pi}{3}\ m^3}$

#48

$s = 10\cos \pi t$ $\qquad s(0) = 10$

a) speed, $|v|$

$v = s' = -10\pi \sin \pi t.$

$|\sin \pi t| = 1$

$\underline{|v| = 10\pi \quad \text{Max speed}}$

$a = v' = -10\pi^2 \cos \pi t = 0$

$\pi t = \frac{\pi}{2}, \frac{(2n+1)\pi}{2}$

$\underline{t = \frac{2n+1}{2}}$

$a = -10\pi^2 \cos \pi t.$

n	t	$a(t)$
0	$\frac{1}{2}$	0
1	$\frac{3}{2}$	0
2	$\frac{5}{2}$	0
3, 4		

$|a| = 0$

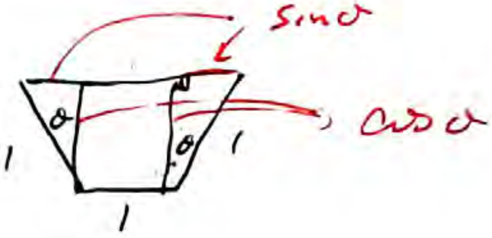
b) $|a| = 10\pi^2 |\cos \pi t|$

when $|\cos \pi t| = 1$ $\qquad \pi t = n\pi$, $t = n$

$t = 0, 1, 2, 3, 4$ sec

Speed $= 10\pi\, |\sin \pi t|$ $\quad (t = 0, 1, 2, 3, 4)$

$\underline{= 0 \text{ cm/sec}}$



$$A = 2\left(\frac{1}{2}\sin\theta\cos\theta\right) + \cos\theta$$

$$= \sin\theta\cos\theta + \cos\theta = \frac{1}{2}\sin 2\theta + \cos\theta$$

$$V = 20\left(\frac{1}{2}\sin 2\theta + \cos\theta\right)$$

$$= 10\sin 2\theta + 20\cos\theta$$

$$\theta' = \cos 2\theta - \sin\theta = 0$$

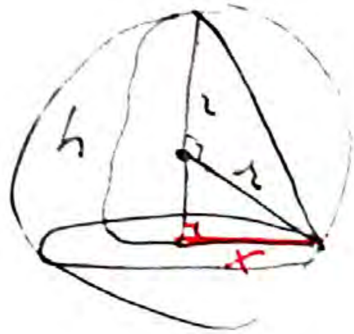
$$1 - 2\sin^2\theta - \sin\theta = 0$$

$$-2\sin^2\theta - \sin\theta + 1 = 0$$

$\sin\theta = -1$ $\qquad\qquad$ $\sin\theta = \frac{1}{2}$

$\theta = \frac{3\pi}{2}$ # $\qquad\qquad$ $\theta = \frac{\pi}{6}$

#40



$V = \frac{\pi}{3} x^2 h$

$(h-r)^2 = r^2 - x^2$

$h - r = \sqrt{r^2 - x^2}$

$h = r + \sqrt{r^2 - x^2}$

$r^2 = x^2 + (h-r)^2$

$V = \frac{\pi}{3} x^2 \left(r + \sqrt{r^2 - x^2}\right)$

$$\frac{dV}{dx} = \frac{\pi}{3} 2x \left(r + \sqrt{r^2 - x^2}\right) + \frac{\pi}{3} x^2 \frac{-x}{\sqrt{r^2 - x^2}}$$

$$= \frac{\pi}{3} x \left(2r + 2\sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}}\right) = 0$$

$2r + 2\sqrt{r^2 - x^2} = \frac{x^2}{\sqrt{r^2 - x^2}}$

$r\sqrt{r^2 - x^2} + r^2 - x^2 = \frac{1}{2}x^2$

$\left(r\sqrt{r^2 - x^2}\right)^2 = \left(\frac{3}{2}x^2 - r^2\right)^2$

$r^4 - r^2x^2 = \frac{9}{4}x^4 - 3r^2x^2 + r^4$

$\frac{9}{4}x^4 - 2r^2x^2 = 0$

$x^2\left(\frac{9}{4}x^2 - 2r^2\right) = 0$

$$x^2 = \frac{8r^2}{9} \Rightarrow x = \boxed{\frac{2\sqrt{2}}{3} r}$$

$$V' = \frac{\pi}{3} \frac{8r^2}{9} \left(r + \sqrt{r^2 - \frac{8r^2}{9}} \right)$$

$$= \frac{8\pi r^2}{27} \left(r + \frac{1}{3} r \right)$$

$$= \frac{32\pi r^3}{81} ($$

$A = x^2 + \pi r^2$ (1) $\quad 4 = 4x + 2\pi r$ (2)

(2) $\to \quad x = 1 - \frac{1}{2}\pi r$

$$A = \left(1 - \frac{1}{2}\pi r\right)^2 + \pi r^2$$

$$= 1 - \pi r + \frac{\pi^2}{4} r^2 + \pi r^2$$

$$A = \left(\frac{\pi^2}{4} + \pi\right) r^2 - \pi r + 1$$

$$A' = \left(\frac{\pi^2}{2} + 2\pi\right) r - \pi = 0$$

$$\frac{\pi}{2}(\pi + 4) r = \pi$$

$$r = \frac{2}{\pi + 4}$$

$$x = 1 - \frac{1}{2} \cdot \pi \left(\frac{2}{\pi + 4}\right)$$

$$= 1 - \frac{\pi}{\pi + 4}$$

$$= \frac{4}{\pi + 4}$$

$$A = \frac{16}{(\pi + 4)^2} + \frac{4\pi}{(\pi + 4)^2}$$

$$= \frac{4(4 + \pi)}{(\pi + 4)^2}$$

$$= \frac{4}{\pi + 4}$$

$A_{sq} = x^2 \quad P = 4x = 4 \Rightarrow x = 1$

$A(1) = 1$

$P = 2\pi r = 4 \Rightarrow r = \frac{2}{\pi}$, $x = 0$

$\boxed{A_c = \pi\left(\frac{4}{\pi^2}\right) = \frac{4}{\pi}}$ Max

$x = 0$
$r = \frac{2}{\pi}$

$A_s = 1$

$A = \frac{4}{4+\pi}$