Solution Section 1.3 – Fractions and Rationalization

Exercise

Perform each indicated operation & simplify $\frac{A}{x+1}-\frac{B}{x-1}+\frac{C}{x+2}$

Solution

$$\frac{A}{x+1}-\frac{B}{x-1}+\frac{C}{x+2}=\frac{A(x-1)(x+2)-B(x+1)(x+2)+C(x+1)(x-1)}{(x+1)(x-1)(x+2)}$$

$$=\frac{A(x^2+2x-x-2)-B(x^2+2x+x+2)+C(x^2-1)}{(x+1)(x-1)(x+2)}$$

$$=\frac{Ax^2+Ax-2A-Bx^2-3Bx-2B+Cx^2-C}{(x+1)(x-1)(x+2)}$$

$$=\frac{(A-B-C)x^2+(A-3B)x-2A-2B-C}{(x+1)(x-1)(x+2)}$$

Exercise

Perform the operation and simplify: $-\frac{\sqrt{x^2+1}}{x^2}-\frac{1}{\sqrt{x^2+1}}$

Solution

$$-\frac{\sqrt{x^2+1}}{x^2}-\frac{1}{\sqrt{x^2+1}}=\frac{-\sqrt{x^2+1}\sqrt{x^2+1}-x^2}{x^2\sqrt{x^2+1}}$$

$$-\frac{\sqrt{x^2+1}}{x^2}\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}\frac{x^2}{x^2}$$

$$=\frac{-\left(x^2+1\right)-x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{-x^2-1-x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{-2x^2-1}{x^2\sqrt{x^2+1}}$$

$$=-\frac{2x^2+1}{x^2\sqrt{x^2+1}}$$

Exercise

Perform the operation and simplify: $\left(\sqrt{x^2+1}-\dfrac{3x^3}{2\sqrt{x^2+1}}\right)\div\left(x^3+1\right)$

Solution

$$\left(\sqrt{x^2+1}-\frac{3x^3}{2\sqrt{x^2+1}}\right)\div\left(x^3+1\right)=\left(\frac{\sqrt{x^2+1}\left(2\sqrt{x^2+1}\right)-3x^3}{2\sqrt{x^2+1}}\right)\cdot\frac{1}{x^3+1}$$

$$=\frac{2\left(x^2+1\right)-3x^3}{2\left(x^3+1\right)\sqrt{x^2+1}}$$

$$=\frac{-3x^3+2x^2+2}{2\left(x^3+1\right)\sqrt{x^2+1}}$$

Exercise

Perform the operation and simplify: $\dfrac{6}{x(3x-2)}+\dfrac{5}{3x-2}-\dfrac{2}{x^2}$

Solution

$$\frac{6}{x(3x-2)}+\frac{5}{3x-2}-\frac{2}{x^2}=\frac{6}{x(3x-2)}\frac{x}{x}+\frac{5}{3x-2}\frac{x^2}{x^2}-\frac{2}{x^2}\frac{3x-2}{3x-2}$$

$x \quad 3x-2 \quad x^2$

$$=\frac{6x+5x^2-2(3x-2)}{x^2(3x-2)}$$

$$=\frac{6x+5x^2-6x+4}{x^2(3x-2)}$$

$$=\frac{5x^2+4}{x^2(3x-2)}$$

Exercise

Simplify the fraction: $\dfrac{\frac{2}{x+3}-\frac{2}{a+3}}{x-a}$

Solution

$$\frac{\frac{2}{x+3}-\frac{2}{a+3}}{x-a}=\frac{\frac{2(a+3)-2(x+3)}{(x+3)(a+3)}}{x-a}$$

$$=\frac{2a+6-2x-6}{(x+3)(a+3)}\cdot\frac{1}{x-a}$$

$$=\frac{2a-2x}{(x+3)(a+3)(x-a)}$$

$$=\frac{2(a-x)}{(x+3)(a+3)(x-a)}$$

$$=\frac{-2(x-a)}{(x+3)(a+3)(x-a)} \qquad if\ \ x\neq a$$

$$=-\frac{2}{(x+3)(a+3)}$$

Exercise

Simplify: $\dfrac{3x^2(2x+5)^{1/2}-x^3\left(\frac{1}{2}\right)(2x+5)^{-1/2}(2)}{\left[(2x+5)^{1/2}\right]^2}$

Solution

$$\frac{3x^2(2x+5)^{1/2}-x^3\left(\frac{1}{2}\right)(2x+5)^{-1/2}(2)}{\left[(2x+5)^{1/2}\right]^2}=\frac{3x^2(2x+5)^{1/2}-x^3(2x+5)^{-1/2}}{(2x+5)}$$

$$=\frac{3x^2(2x+5)^{1/2}-x^3(2x+5)^{-1/2}}{(2x+5)}\bullet\frac{(2x+5)^{1/2}}{(2x+5)^{1/2}}$$

$$=\frac{3x^2(2x+5)-x^3}{(2x+5)^{3/2}}$$

$$=\frac{6x^3+15x^2-x^3}{(2x+5)^{3/2}}$$

$$=\frac{5x^3+15x^2}{(2x+5)^{3/2}}$$

$$=\frac{5x^2(x+3)}{(2x+5)^{3/2}}$$

Exercise

Simplify the expression: $\dfrac{\left(4x^2+9\right)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left[\left(4x^2+9\right)^{1/2}\right]^2}$

Solution

$$\frac{\left(4x^2+9\right)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left[\left(4x^2+9\right)^{1/2}\right]^2}=\frac{2\left(4x^2+9\right)^{1/2}-4x(2x+3)\left(4x^2+9\right)^{-1/2}}{4x^2+9}$$

$$=\frac{2\left(4x^2+9\right)^{1/2}-4x(2x+3)\left(4x^2+9\right)^{-1/2}}{4x^2+9}\cdot\frac{\left(4x^2+9\right)^{1/2}}{\left(4x^2+9\right)^{1/2}}$$

$$=\frac{2\left(4x^2+9\right)-4x(2x+3)}{\left(4x^2+9\right)^{3/2}}$$

$$=\frac{8x^2+18-8x^2-12x}{\left(4x^2+9\right)^{3/2}}$$

$$=\frac{18-12x}{\left(4x^2+9\right)^{3/2}}$$

$$=\frac{6(3-2x)}{\left(4x^2+9\right)^{3/2}}$$

Exercise

Simplify the expression: $\dfrac{\left(1-x^2\right)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)}{\left[\left(1-x^2\right)^{1/2}\right]^2}$

Solution

$$\frac{\left(1-x^2\right)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)}{\left[\left(1-x^2\right)^{1/2}\right]^2}=\frac{2x\left(1-x^2\right)^{1/2}+x^3\left(1-x^2\right)^{-1/2}}{1-x^2}\frac{\left(1-x^2\right)^{1/2}}{\left(1-x^2\right)^{1/2}}$$

$$=\frac{2x\left(1-x^2\right)+x^3}{\left(1-x^2\right)^{3/2}}$$

$$=\frac{2x-2x^3+x^3}{\left(1-x^2\right)^{3/2}}$$

$$=\frac{2x-x^3}{\left(1-x^2\right)^{3/2}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2+4\right)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)\left(x^2+4\right)^{-2/3}(2x)}{\left[\left(x^2+4\right)^{1/3}\right]^2}$

Solution

$$\frac{\left(x^2+4\right)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)\left(x^2+4\right)^{-2/3}(2x)}{\left[\left(x^2+4\right)^{1/3}\right]^2}=\frac{3\left(x^2+4\right)^{1/3}-6x^2\left(x^2+4\right)^{-2/3}}{\left(x^2+4\right)^{2/3}}\frac{\left(x^2+4\right)^{2/3}}{\left(x^2+4\right)^{2/3}}$$

$$=\frac{3\left(x^2+4\right)-6x^2}{\left(x^2+4\right)^{4/3}}$$

$$= \frac{3x^2 + 12 - 6x^2}{\left(x^2+4\right)^{4/3}}$$

$$= \frac{-3x^2 + 12}{\left(x^2+4\right)^{4/3}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2-5\right)^4 (3x^2) - x^3 (4)\left(x^2-5\right)^3 (2x)}{\left[\left(x^2-5\right)^4\right]^2}$

Solution

$$\frac{\left(x^2-5\right)^4 (3x^2) - x^3 (4)\left(x^2-5\right)^3 (2x)}{\left[\left(x^2-5\right)^4\right]^2} = \frac{\left(x^2-5\right)^3 \left[3x^2\left(x^2-5\right) - 8x^4\right]}{\left(x^2-5\right)^8}$$

$$= \frac{\left(x^2-5\right)^3 \left[3x^4 - 15x^2 - 8x^4\right]}{\left(x^2-5\right)^8}$$

$$= \frac{\left(-5x^4 - 15x^2\right)}{\left(x^2-5\right)^5}$$

$$= \frac{-5x^2\left(x^2+3\right)}{\left(x^2-5\right)^5}$$

Exercise

Simplify the expression: $\dfrac{(3x+2)^{1/2}\left(\frac{1}{3}\right)(2x+3)^{-2/3}(2)-(2x+3)^{1/3}\left(\frac{1}{2}\right)(3x+2)^{-1/2}(3)}{\left[(3x+2)^{1/2}\right]^2}$

Solution

$$=\frac{\frac{2}{3}(3x+2)^{1/2}(2x+3)^{-2/3}-\frac{3}{2}(2x+3)^{1/3}(3x+2)^{-1/2}}{3x+2}\;\frac{6(2x+3)^{2/3}}{6(2x+3)^{2/3}}\;\frac{(3x+2)^{1/2}}{(3x+2)^{1/2}}$$

$$=\frac{4(3x+2)-9(2x+3)}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$=\frac{4(3x+2)-9(2x+3)}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$=\frac{12x+8-18x-27}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$=\frac{-6x-19}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2+2\right)^3(2x)-x^2(3)\left(x^2+2\right)^2(2x)}{\left[\left(x^2+2\right)^3\right]^2}$

Solution

$$\frac{\left(x^2+2\right)^3(2x)-x^2(3)\left(x^2+2\right)^2(2x)}{\left[\left(x^2+2\right)^3\right]^2}=\frac{2x\left(x^2+2\right)^2\left[\left(x^2+2\right)-3x^2\right]}{\left(x^2+2\right)^6}$$

$$=\frac{2x\left[x^2+2-3x^2\right]}{\left(x^2+2\right)^4}$$

$$= \frac{2x\left[-2x^2 + 2\right]}{\left(x^2 + 2\right)^4}$$

$$= \frac{4x\left[-x^2 + 1\right]}{\left(x^2 + 2\right)^4}$$

Solution Section 1.4 – Equations and Application

Exercise

Suppose that Greg, manager of a giant supermarket chain, has studied the supply and demand for watermelons. He has noticed that the demand increases as the price decreases. He has determined that the quantity (in thousands) demanded weekly q, and the price (in dollars) per watermelons, p, are related by the linear function.

$p = D(q) = 9 - 0.75q$ Demand function

a) Find the demand at a price of \$5.25 per watermelon and at a price of \$3.75 per watermelon.

b) Greg also noticed that the supply of watermelons decreased as the price decreased. Price p and supply q are related by the linear function

$p = S(q) = 0.75q$ Demand function

Find the supply at a price of \$5.25 per watermelon and at a price of \$3.00 per watermelon.

c) Use the algebra to find the equilibrium quantity for the watermelon in example 2

Solution

Demand at a price of \$5.25 per watermelon = p

$$5.25 = 9 - 0.75q$$
$$5.25 - 9 = -0.75q$$
$$-3.75 = -0.75q$$
$$\frac{-3.75}{-0.75} = q$$
$$q = 5$$ At the price of \$5.25, the demand is 5000 watermelons

Demand at a price of \$3.75 per watermelon = p

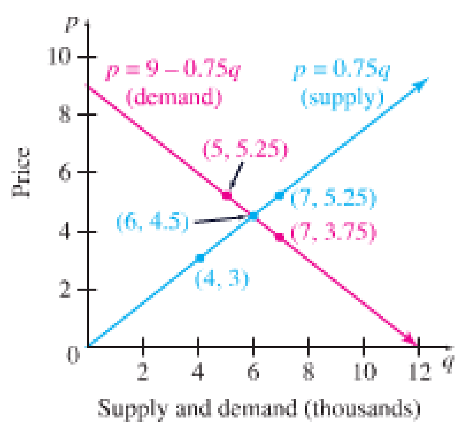
$$3.75 = 9 - 0.75q$$
$$-5.25 = -0.75q$$
$$q = 7$$ At the price of \$3.75, the demand is 7000 watermelons

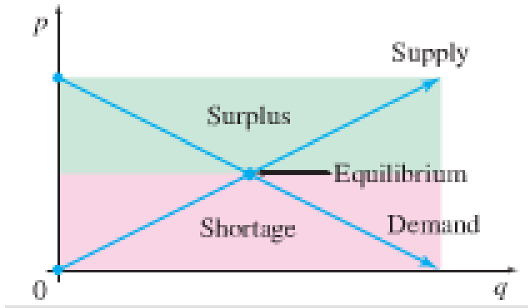
Supply at a price of \$5.25 per watermelon

$$5.25 = 0.75q$$
$q = 7$ 7000 watermelons

Supply at a price of \$3.00 per watermelon

$$3 = 0.75q$$
$q = 4$ 4000 watermelons





Equilibrium quantity $\rightarrow$ *Demand* function = *Supply* function

$$9-0.75q=0.75q$$
$$9=1.5q$$
$$6=q$$

The equilibrium quantity is 6000 watermelons.

Exercise

In Recent years, the percentage of the U.S. population age 18 and older who smoke has decreased at a roughly constant rate, from 23.3% in 2000 to 20.9% in 2004.

a) Find the equation describing this linear relationship.

b) One of the objectives of Healthy People 2010 (a campaign of the U.S. Department of Health and Human Services) is to reduce the percentage of U.S. adults to smoke to 12% or less by the year 2010. If this decline in smoking continues at the same rate, will they meet this objective

Solution

	x	***y***
2000	0	23.3
2004	4	20.9

a) $m=\dfrac{20.9-23.3}{4-0}$

$=\dfrac{-2.4}{4}$

$=-0.6$

$$y-23.3=-0.6(x-0)$$
$$y-23.3=-0.6x$$
$$y=-0.6x+23.3$$

b) In 2010 $\Rightarrow x=10$

$$y=-0.6(10)+23.3$$
$$=17.3$$

In 2010, the rate is estimated 17.3% will still smoke and the objective will not met.

Exercise

The number of African Americans earning doctorate degrees has risen at an approximately constant rate from 1987 to 2005. The linear equation $y = 63.6x + 787$, where x represents the number of years since 1987, can be used to estimate the annual number of African Americans earning doctorate degrees. Determine this number in 2006.

Solution

In 2006 $\rightarrow x = 2006 - 1987 = 19$

$$\begin{aligned} y &= 63.6x + 787 \\ &= 63.6(19) + 787 \\ &\approx 1995 \end{aligned}$$

Solution *Section* 1.5 – Limits and Asymptotes

Exercise

Find the limit: $\lim\limits_{x\to 1}(2x^2 - x + 4)$

Solution

$$\lim_{x\to 1}(2x^2 - x + 4) = 2(1)^2 - (1) + 4 \ \underline{= 5}$$

Exercise

Find the limit: $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}$

Solution

$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \frac{2^2-4}{2-2} = \frac{0}{0}$$

$$\lim_{x\to 1.9}\frac{x^2-4}{x-2} = \frac{1.9^2-4}{1.9-2} = 3.9$$

$$\lim_{x\to 2.1}\frac{x^2-4}{x-2} = \frac{2.1^2-4}{2.1-2} = 4.1$$

$$\lim_{x\to 2}\frac{x^2-4}{x-2} = 4$$

Exercise

Find the limit: $\lim\limits_{x\to 2} f(x)$ $\qquad f(x) = \begin{cases} x^2 & ,x \ne 2 \\ 0 & ,x = 2 \end{cases}$

Solution

$$\lim_{x\to 1.99} \mathrm{x}^2 = 3.96$$

$$\lim_{x\to 2} f(x) = 4$$

$$\lim_{x\to 2.01} \mathrm{x}^2 = 4.04$$

Exercise

Find the limit: $\lim_{x\to 1}(2x^2 - x + 4)$

Solution

$$\lim_{x\to 1}(2x^2 - x + 4) = 2(1^2) - 1 + 4$$

$$= 5$$

Exercise

Find the limit: $\lim_{x\to 2}\frac{x^3-8}{x-2}$

Solution

$$\lim_{x\to 2}\frac{x^3-8}{x-2} = \frac{0}{0}$$

$$\lim_{x\to 2}\frac{x^3-8}{x-2} = \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x-2}$$

$$= \lim_{x\to 2} x^2 + 2x + 4$$

$$= 2^2 + 2(2) + 4$$

$$= 12$$

Exercise

Find the limit: $\lim_{x\to 3}\frac{x^2+x-12}{x-3}$

Solution

$$\lim_{x\to 3}\frac{x^2+x-12}{x-3} = \frac{0}{0}$$

$$\lim_{x\to 3}\frac{(x-3)(x+4)}{x-3} = \lim_{x\to 3}(x+4)$$

$$= 3 + 4$$

$$= 7$$

Exercise

Find the limit: $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$

Solution

$$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{4}-2}{0} = \frac{0}{0}$$

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} &= \lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}\,\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} \\
&= \lim_{x\to 0} \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)} \\
&= \lim_{x\to 0} \frac{x}{x\left(\sqrt{x+4}+2\right)} \\
&= \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} \\
&= \frac{1}{\sqrt{4}+2} \\
&= \frac{1}{4}
\end{aligned}$$

Exercise

Find the limit: $\lim\limits_{x\to 0} f(x) = \begin{cases} x^2+1 & x<0 \\ 2x+1 & x>0 \end{cases}$

Solution

$$\lim_{x\to 0^-} x^2+1 = 1$$

$$\lim_{x\to 0+} 2x+1 = 1$$

$$\lim_{x\to 0} f(\mathrm{x}) = 1$$

Exercise

Find the limit: $\lim\limits_{x\to -2} \dfrac{5}{x+2}$

Solution

$\lim\limits_{x\to -2} \dfrac{5}{x+2} = \dfrac{5}{0} = \infty$ (***Doesn't exist***)

Exercise

Find the limit: $\lim\limits_{x\to 2^-} \dfrac{|x-2|}{x-2}$

Solution

$$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \frac{(x-2)}{-(x-2)} = -1$$

Exercise

Find the limit: $\lim\limits_{x\to 2+} \dfrac{|x-2|}{x-2}$

Solution

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \frac{(x-2)}{(x-2)} = 1$$

Exercise

$y = \dfrac{3x}{1-x}$

Solution

$VA: x = 1$

$HA: y = -3$

Exercise

$y = \dfrac{x^2}{x^2+9}$

Solution

$HA: y = 1$

Exercise

$y = \dfrac{x-2}{x^2-4x+3}$

Solution

$VA: x = 1, 3$

$HA: y = 0$

Exercise

$$y = \frac{3}{x-5}$$

Solution

VA: $x = 5$

HA: $y = 0$

Exercise

$$y = \frac{x^3 - 1}{x^2 + 1}$$

Solution

VA: *n/a*

HA: *n/a*

Exercise

$$y = \frac{3x^2 - 27}{(x+3)(2x+1)}$$

Solution

VA: $x = -3,\ -\frac{1}{2}$

HA: $y = \frac{3}{2}$

Exercise

$$y = \frac{x^3 + 3x^2 - 2}{x^2 - 4}$$

Solution

VA: $x = \pm 2$

HA: *n/a*

Exercise

$$y = \frac{x-3}{x^2-9}$$

Solution

VA: $x = -3$,

hole $x = 3$

Exercise

$$y = \frac{6}{\sqrt{x^2-4x}}$$

Solution

VA: $x = 0, 4$

Exercise

$$y = \frac{5x-1}{1-3x}$$

Solution

VA: $x = \frac{1}{3}$

HA: $y = \frac{5}{3}$

Solution Section 1.6 – Continuity and Rates of Change

Exercise

Determine whether $f(x)$ is continuous on the entire number line. Explain your reasoning.

$$f(x)=\frac{x}{x^2-1}$$

Solution

$$x^2-1=0 \to x^2=1 \Rightarrow \boxed{x=\pm 1}$$

The function is continuous on $(-\infty,\ -1)\cup(-1,\ 1)\cup(1,\ \infty)$

Exercise

Determine whether $f(x)$ is continuous on the entire number line. Explain your reasoning.

$$f(x)=\frac{x-5}{x^2-9x+20}$$

Solution

$$x^2-9x+20=0 \Rightarrow \boxed{x=4,\ 5}$$

The function is continuous on $(-\infty,\ 4)\cup(4,\ 5)\cup(5,\ \infty)$

Exercise

Find the slope of the graph of $f(x) = 2x + 5$

Solution

$$\begin{aligned}
m &= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} \\
&= \lim_{\Delta x\to 0}\frac{2(x+\Delta x)+5-(2x+5)}{\Delta x} \\
&= \lim_{\Delta x\to 0}\frac{2x+2\Delta x+5-2x-5}{\Delta x} \\
&= \lim_{\Delta x\to 0}\frac{2\Delta x}{\Delta x} \\
&= \lim_{\Delta x\to 0} 2 \\
&= 2
\end{aligned}$$

Exercise

Find the slope of the graph of $f(x) = \sqrt{x}$

Solution

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

$$m = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$