

Review

12/9 Exam 4
No class 12/7

$$\cos A = \frac{15}{17} \quad A \in QI \quad \cos B = -\frac{12}{13} \quad B \in QII$$

$$\sin A = \frac{8}{17}$$

$$\sin B = \frac{5}{13}$$

$$\begin{aligned} a) \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \left(\frac{8}{17}\right)\left(-\frac{12}{13}\right) + \left(\frac{15}{17}\right)\left(\frac{5}{13}\right) \\ &= \frac{-96 + 75}{221} \\ &= -\frac{21}{221} \end{aligned}$$

$$\begin{aligned} b) \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \frac{15}{17}\left(-\frac{12}{13}\right) - \left(\frac{8}{17}\right)\frac{5}{13} \\ &= \frac{-180 - 40}{221} \\ &= -\frac{220}{221} \end{aligned}$$

$$c) \tan(A+B) = \frac{21}{220}$$

$$\begin{aligned}
 d) \sin(A-B) &= \sin A \cos B - \cos A \sin B \\
 &= \frac{8}{17} \left(\frac{-12}{13} \right) - \frac{15}{17} \frac{5}{13} \\
 &= \frac{-96 - 75}{221} \\
 &= -\frac{171}{221}
 \end{aligned}$$

$$\begin{aligned}
 e) \cos(A-B) &= \cos A \cos B + \sin A \sin B \\
 &= \frac{15}{17} \left(\frac{-12}{13} \right) + \frac{8}{17} \frac{5}{13} \\
 &= \frac{-180 + 40}{221} \\
 &= -\frac{140}{221}
 \end{aligned}$$

$$f) \tan(A-B) = \frac{171}{140}$$

#2/

$$\sin A = \frac{3}{5} \quad A \in \text{QII}$$

$$90^\circ \leq A \leq 180^\circ$$

$$\frac{45^\circ}{2} \leq \frac{A}{2} \leq 90^\circ$$

$$\cos A = -\frac{4}{5}$$

$$\begin{aligned} \text{a) } \sin 2A &= 2 \sin A \cos A \\ &= 2 \left(\frac{3}{5} \right) \left(-\frac{4}{5} \right) \\ &= -\frac{24}{25} \end{aligned}$$

$$\begin{aligned} \text{b) } \cos 2A &= \cos^2 A - \sin^2 A \\ &= \frac{16}{25} - \frac{9}{25} \\ &= \frac{7}{25} \end{aligned}$$

$$\text{c) } \tan 2A = -\frac{24}{7}$$

$$\begin{aligned} \text{d) } \sin \frac{A}{2} &= \sqrt{\frac{1}{2} (1 - \cos A)} \\ &= \sqrt{\frac{1}{2} \left(1 + \frac{4}{5} \right)} \\ &= \frac{3}{\sqrt{10}} \end{aligned}$$

$$\begin{aligned} \text{e) } \cos \frac{A}{2} &= \sqrt{\frac{1}{2} (1 + \cos A)} \\ &= \sqrt{\frac{1}{2} \left(1 - \frac{4}{5} \right)} \\ &= \frac{1}{\sqrt{10}} \end{aligned}$$

$$\text{f) } \tan \frac{A}{2} = 3$$



$$(1 - \sin x) = \sqrt{3} \cos x \quad [0, 2\pi)$$

$$\sqrt{3} \cos x + \sin x = 1$$

$$\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{1}{2}$$

$$\cos \frac{\pi}{6} \cos x + \sin \frac{\pi}{6} \sin x = \frac{1}{2}$$

$$\cos \left(x - \frac{\pi}{6} \right) = \frac{1}{2}$$

$$x - \frac{\pi}{6} = \frac{\pi}{3}$$

$$x = \frac{\pi}{3} + \frac{\pi}{6}$$

$$= \frac{\pi}{2}$$

$$x - \frac{\pi}{6} = \frac{5\pi}{3}$$

$$x = \frac{5\pi}{3} + \frac{\pi}{6}$$

$$= \frac{11\pi}{6}$$

$$2 \tan x \csc x + 2 \csc x + \tan x + 1 = 0 \quad [0, 2\pi)$$

$$2 \csc x (\tan x + 1) + (\tan x + 1) = 0$$

$$(\tan x + 1)(2 \csc x + 1) = 0$$

$$\tan x = -1$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\csc x = -\frac{1}{2}$$

$$\sin x = -2 \quad \#$$

$$2 \sin^2 x - \cos x - 1 = 0$$

$$[0, 2\pi)$$

$$2(1 - \cos^2 x) - \cos x - 1 = 0$$

$$2 - 2\cos^2 x - \cos x - 1 = 0$$

$$-2\cos^2 x - \cos x + 1 = 0$$

$$\cos x = -1$$

$$\cos x = \frac{1}{2}$$

$$x = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\sin x \tan x = \sin x$$

$$[0, 2\pi)$$

$$\sin x \tan x - \sin x = 0$$

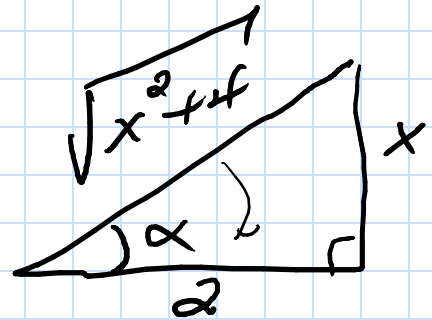
$$\sin x (\tan x - 1) = 0$$

$$\sin x = 0 \quad \tan x = 1$$

$$x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\sec \left(\sin^{-1} \frac{x}{\sqrt{x^2+4}} \right)$$

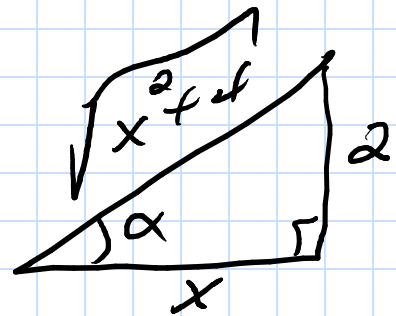
$$\sin \alpha = \frac{x}{\sqrt{x^2+4}}$$



$$\sec \alpha = \frac{\sqrt{x^2+4}}{2}$$

$$\sin \left(\cos^{-1} \frac{x}{\sqrt{x^2+4}} \right)$$

$$\cos \alpha = \frac{x}{\sqrt{x^2+4}}$$



$$\sin \alpha = \frac{2}{\sqrt{x^2+4}}$$

$$(4, 30^\circ)$$

$$\begin{aligned}x &= r \cos \theta \\&= 4 \cos 30^\circ \\&= 4 \left(\frac{\sqrt{3}}{2} \right) \\&= 2\sqrt{3}\end{aligned}$$

$$(x, y)?$$

$$\begin{aligned}y &= r \sin \theta \\&= 4 \sin 30^\circ \\&= 4 \left(\frac{1}{2} \right) \\&= 2\end{aligned}$$

$$(x, y) = (2\sqrt{3}, 2)$$

→ 4 all no need.

$$(3, -3)$$

$$(r, \theta)?$$

$$\begin{aligned}r &= \sqrt{x^2 + y^2} \\&= \sqrt{9 + 9} \\&= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}\hat{\theta} &= \tan^{-1} \left| \frac{-3}{3} \right| \\&= \tan^{-1} 1 \\&= \frac{\pi}{4}\end{aligned}$$

$$(r, \theta) = (3\sqrt{2}, \frac{7\pi}{4})$$

$$r(\sin \theta - 2 \cos \theta) = 6$$

$$r \sin \theta - 2r \cos \theta = 6$$

$$\underline{y - 2x = 6}$$

$$y^2 = x$$

$$(r \sin \theta)^2 = r \cos \theta$$

$$r^2 \sin^2 \theta = r \cos \theta$$

$$(r \neq 0)$$

$$r \sin^2 \theta = \cos \theta$$

$$\underline{r = \frac{\cos \theta}{\sin^2 \theta}}$$

$$-\sqrt{3} + i$$

$$x = -\sqrt{3}, \quad y = 1$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{3 + 1} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \hat{\theta} &= \tan^{-1} \left| \frac{1}{-\sqrt{3}} \right| \\ &= \frac{\pi}{6} \end{aligned}$$

$$-\sqrt{3} + i = 2 \operatorname{cis} \frac{5\pi}{6}$$

$$\begin{aligned} \sqrt{2} \operatorname{cis} \frac{7\pi}{4} &= \sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) \\ &= \sqrt{2} \left(\frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2} \right) \\ &= 1 - i \end{aligned}$$

$$\begin{aligned} \frac{20 \operatorname{cis}(75^\circ)}{4 \operatorname{cis}(40^\circ)} &= 5 \operatorname{cis}(75^\circ - 40^\circ) \\ &= 5 \operatorname{cis}(35^\circ) \end{aligned}$$

$$\begin{aligned} (20 \operatorname{cis} 75^\circ)(4 \operatorname{cis} 40^\circ) &= 80 \operatorname{cis}(75^\circ + 40^\circ) \\ &= 80 \operatorname{cis}(115^\circ) \end{aligned}$$