Math 2413 – Calculus I ***Exam 2*** ***Review***

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1. Find the derivative

a) $f(t)=\sqrt{t-4}$

b) $f(x)=\dfrac{1}{x+2}$

c) $f(x)=3x^4-3x^3+6x^2-x+5$

d) $y=\dfrac{2}{\sqrt[3]{x^2}}$

e) $g(t)=\dfrac{t^2-1}{t+4}$

f) $y=\left(x^5-3x\right)\left(\dfrac{1}{x^2}\right)$

g) $f(x)=(x+3)\left(1-\dfrac{2}{x-3}\right)$

h) $R(s)=\dfrac{s^3-2s^2+3}{\sqrt{s-2}}$

i) $y=\left(\dfrac{x+3}{x-4}\right)(x+5)$

j) $f(x)=\sqrt{x^2-3x+5}$

k) $h(t)=\left(2t^2-3t+4\right)^3\sqrt{t^2-3}$

l) $y=\sqrt{x}\,(x+2)^2$

m) $Q(w)=\dfrac{w+1}{\sqrt{2w+3}}$

n) $y=x^7+\sqrt{7}x-\dfrac{1}{\pi+1}$

o) $f(t)=\dfrac{\sqrt{t}}{1+\sqrt{t}}$

p) $f(x)=\left(\dfrac{2\sqrt{x}}{1+2\sqrt{x}}\right)^2$

q) $y=\sqrt{\dfrac{x^2+x}{x^2}}$

r) $y=(2x+1)\sqrt{2x+1}$

2. Find the derivative

a) $y=2\tan^2 x-\sec^2 x$

b) $y=\dfrac{1}{\sin^2 x}-\dfrac{2}{\sin x}$

c) $y=(\sec x+\tan x)^5$

d) $r=\sqrt{2\theta\sin\theta}$

e) $r=\sin\left(\theta+\sqrt{\theta+1}\right)$

f) $y=2\sqrt{x}\sin\sqrt{x}$

g) $y=x^2\sin^2\left(2x^2\right)$

h) $r=\left(\dfrac{\sin\theta}{\cos\theta-1}\right)^2$

i) $y=\left(3+\cos^3 3x\right)^{-1/3}$

3. Find the following corresponding derivative

a) $f(x)=3x^4-3x^3+6x^2-x+5 \quad ; \quad f^{(4)}(x)$

b) $f(x)=6x^5-3x^4-2x+e \quad ; \quad f^{(5)}(x)$

c) $y=\dfrac{x^2+7}{x} \quad ; \quad y'''(x)$

4. Find the derivative of

a) $y = \sqrt{2}e^{\sqrt{2}x}$

b) $y = \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$

c) $y = \ln\left(\sec^2\theta\right)$

d) $y = \log_5(3x-7)$

e) $y = (x+2)^{x+2}$

f) $y = \sin^{-1}\left(\frac{1}{\sqrt{x}}\right), \quad x > 1$

g) $y = z\cos^{-1}z - \sqrt{1-z^2}$

h) $y = t\tan^{-1}t - \frac{1}{2}\ln t$

i) $y = 10\sqrt{\frac{3x+4}{2x-4}}$

j) $y = \left(\frac{(t+1)(t-1)}{(t-2)(t+3)}\right)^5 \quad t > 2$

k) $y = (\sin\theta)^{\sqrt{\theta}}$

5. Find $\frac{dy}{dx}$ by implicit differentiation

a) $xy + 2x + 3y = 1$

b) $x^3 + 4xy - 3y^{4/3} = 2x$

c) $x^2y^2 = 1$

d) $y^2 = \sqrt{\frac{1+x}{1-x}}$

6. Find $\frac{d^2y}{dx^2}$ by implicit differentiation $x^3 + y^3 = 1$

7. The parabola $y = x^2 + C$ is to be tangent to the line $y = x$. Find *C*.

8. Find equations for the lines that are tangent and normal to the curve $x^2 + 2y^2 = 9$ at the point (1, 2).

9. Carlos is blowing air into a soap bubble at the rate of 8 cm^3/sec. Assume that the bubble is spherical $\left(V = \frac{4}{3}\pi r^3\right)$. How fast is the radius changing at the instant of time when the radius is 10 *cm*?

10. The position function for an amusement ride moving on a horizontal track is $x = -0.01t^4 + 0.3t^3 + 0.4t^2 + 12t$ where *x* is in feet and *t* is in seconds. What is the velocity at 20 seconds?

11. The population of Americans age 55 and older as a percent of the total population is approximated by the function

$$f(t) = 10.72(0.9t+10)^{0.3} \qquad (0 \le t \le 20)$$

where t is measured in years, with $t = 0$ corresponding to the year 2000. At what rate will the percent of Americans age 55 and older be changing in 2010?

12. The total surface area *S* of a right circle cylinder is related to the base radius *r* and height *h* by the equation $S = 2\pi r^2 + 2\pi rh$

a) How is $\frac{dS}{dt}$ related to $\frac{dr}{dt}$ if h is constant?

b) How is $\frac{dS}{dt}$ related to $\frac{dh}{dt}$ if r is constant?

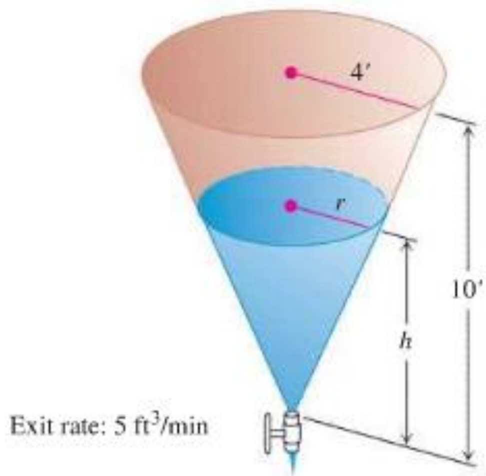
c) How is $\frac{ds}{dt}$ related to $\frac{dr}{dt}$ and $\frac{dh}{dt}$ if neither r nor h is constant?

13. A particle moves along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. Find $\frac{dx}{dt}$ when $x = 3$.

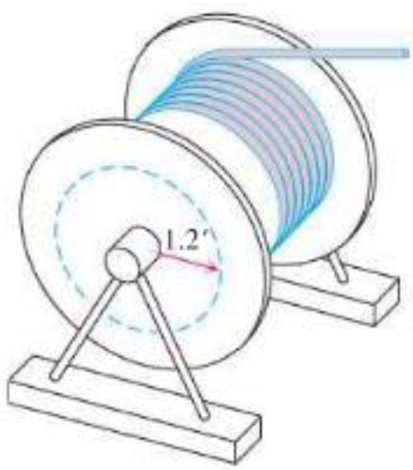
14. Water drains from the conical tank at the rate of 5 $ft^3 / \min$.

a) What is the relation between the variables h and r in the figure?

b) How fast is the water level dropping when $h = 6$ *ft*?



15. As television cable is pulled from a large spool to be in layers of constant radius. If the truck pulling the cable moves at a steady 6 *ft/sec* (a touch over 4 *mph*), use the equation $s = r\theta$ to find how fast (*rad./sec.*) the spool is turning when the layer of radius 1.2 *ft* is being unwound.



Answers

1.

a) $\dfrac{1}{2\sqrt{t-4}}$

b) $-\dfrac{1}{(x+2)^2}$

c) $12x^3-9x^2+12x-1$

d) $\dfrac{dy}{dx}=\dfrac{-4}{3\sqrt[3]{x^5}}$

e) $g'(t)=\dfrac{t^2+8t+1}{(t+4)^2}$

f) $\dfrac{dy}{dx}=3x^2+\dfrac{3}{x^2}$

g) $f'(x)=\dfrac{x^2-6x+21}{(x-3)^2}$

h) $\dfrac{5s^3-18s^2+16s-3}{2(s-2)^{3/2}}$

i) $\dfrac{dy}{dx}=\dfrac{x^2-8x-47}{(x-4)^2}$

j) $\dfrac{2x-3}{2\sqrt{x^2-3x+5}}$

k) $\dfrac{\left(2t^2-3t+4\right)^2\left[14t^3-12t^2-32t+27\right]}{\left(t^2-3\right)^{1/2}}$

l) $\dfrac{dy}{dx}=\dfrac{5x^2+12x+4}{2\sqrt{x}}$

m) $Q'(w)=\dfrac{w+2}{(2w+3)^{3/2}}$

n) $y'=7x^6+\sqrt{7}$

o) $f'(t)=\dfrac{1}{2\sqrt{t}\left(1+\sqrt{t}\right)^2}$

p) $f'(x)=\dfrac{4}{\left(1+2\sqrt{x}\right)^3}$

q) $y'=\dfrac{-1}{2x^2\cdot\sqrt{1+\dfrac{1}{x}}}$

r) $y'=3\sqrt{2x+1}$

2.

a) $y'=2\sec^2 x\tan x$

b) $y'=\left(2\csc x\cot x\right)\left(1-\csc x\right)$

c) $y'=5\sec x\left(\sec x+\tan x\right)^5$

d) $r'=\dfrac{\theta\cos\theta+\sin\theta}{\sqrt{2\theta\sin\theta}}$

e) $r'=\dfrac{2\sqrt{\theta+1}+1}{2\sqrt{\theta+1}}\cos\left(\theta+\sqrt{\theta+1}\right)$

f) $y'=\cos\sqrt{x}+\dfrac{\sin\sqrt{x}}{\sqrt{x}}$

g) $y'=8x^3\sin\left(2x^2\right)\cos\left(2x^2\right)+2x\sin^2\left(2x^2\right)$

h) $r'=\dfrac{-2\sin\theta}{(\cos\theta-1)^2}$

i) $y'=\dfrac{3\cos^2 3x\cdot\sin 3x}{\left(3+\cos^3 3x\right)^{4/3}}$

3. *a)* $f^{(4)}(x)=72$ *b)* $f^{(5)}(x)=720$ *c)* $y'''(x)=-\dfrac{42}{x^4}$

4. ***a***) $y' = 2e^{\sqrt{2}x}$ ***b***) $y' = xe^{4x}$ ***c***) $y' = 2\tan\theta$ ***d***) $y' = \dfrac{3}{(\ln 5)(3x-7)}$

e) $y' = (x+2)^{x+2}\left[\ln(x+2)+1\right]$ ***f***) $y' = \dfrac{-1}{2x\sqrt{x-1}}$ ***g***) $y' = \cos^{-1} z$

h) $y' = \tan^{-1} t + \dfrac{t}{1+t^2} - \dfrac{1}{2t}$ ***i***) $y' = 10\sqrt[]{\dfrac{3x+4}{2x-4}}\left(\dfrac{1}{10}\right)\left(\dfrac{3}{3x+4} - \dfrac{1}{x-2}\right)$

j) $y' = 5\left(\dfrac{(t+1)(t-1)}{(t-2)(t+3)}\right)^5\left(\dfrac{1}{t+1} + \dfrac{1}{t-1} - \dfrac{1}{t-2} - \dfrac{1}{t+3}\right)$

k) $y' = (\sin\theta)^{\sqrt{\theta}}\left(\sqrt{\theta}\cot\theta + \dfrac{\ln(\sin\theta)}{2\sqrt{\theta}}\right)$

5. ***a***) $\dfrac{dy}{dx} = -\dfrac{y+2}{x+3}$ ***b***) $\dfrac{dy}{dx} = \dfrac{2-3x^2-4y}{4x-4y^{1/3}}$ ***c***) $\dfrac{dy}{dx} = -\dfrac{y}{x}$ ***d***) $\dfrac{dy}{dx} = \dfrac{1}{2y^3(1-x)^2}$

6. $\dfrac{d^2y}{dx^2} = \dfrac{-2xy^3 - 2x^4}{y^5}$

7. $C = \dfrac{1}{4}$

8. $y = 4x - 2$

9. $\dfrac{1}{50\pi} \approx .0064\ cm/\sec$

10. $v(t) = -0.04t^3 + 0.9t^2 + 0.8t + 12$ $v(20) = 68\ \ ft/\sec$

11. $f\,'(t) = 2.8944(0.9t+10)^{-.7}$ $f\,'(10) = .3685$

12. $a)\ \dfrac{dS}{dt} = (4\pi r + 2\pi h)\dfrac{dr}{dt}$

$b)\ \dfrac{dS}{dt} = 2\pi r\dfrac{dh}{dt}$

$c)\ \dfrac{dS}{dt} = (4\pi r + 2\pi h)\dfrac{dr}{dt} + 2\pi r\dfrac{dh}{dt}$

13. $\dfrac{dx}{dt} = 4\ \ units/\sec$

14. $a)\ r = \dfrac{2}{5}h$ $b)\ \dfrac{dh}{dt} = -\dfrac{125}{144\pi}\ \ ft/\min$

15. $\dfrac{d\theta}{dt} = 5\ \ rad/\sec$