Single Variables

Volume of Solid of Revolution

Solid: $V = \int_a^b A(x)\,dx$

Disk: $V = \pi \int_a^b f(x)^2\,dx \quad (about\ \ x - axis)$ $\qquad V = \pi \int_c^d \left[R(y)\right]^2 dy \quad (about\ \ y - axis)$

Washer: $V = \pi \int_a^b \left(\left[R(x)\right]^2 - \left[r(x)\right]^2 \right) dx \quad (about\ \ x - axis)$

$$V = \pi \int_c^d \left(\left[R(y)\right]^2 - \left[r(y)\right]^2 \right) dy \quad (about\ \ y - axis)$$

Shell: $V = 2\pi \int_a^b y \cdot g(y)\,dy \quad (about\ \ x - axis)$ $\qquad V = 2\pi \int_a^b x \cdot f(x)\,dx \quad (about\ \ y - axis)$

$$V = \int_a^b 2\pi \begin{pmatrix} shell \\ radius \end{pmatrix} \begin{pmatrix} shell \\ height \end{pmatrix} dx$$

Length of Curves

Length $y = f(x) \quad L = \int_a^b \sqrt{1 + f'(x)^2}\ dx$ $\qquad x = g(y) \quad L = \int_c^d \sqrt{1 + g'(y)^2}\ dy$

Parametric Curve: $L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2}\ dt \qquad \begin{cases} x = f(t) \\ y = g(t) \end{cases}$

Area of the Surface $S = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$ $\qquad S = 2\pi \int_c^d x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$

Trapezoid Rule: $\int_a^b f(x)\,dx \approx T(n) = \left(\frac{1}{2} f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2} f(x_n) \right) \Delta x$

where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x \quad for \quad k = 0,\ 1,\ \ldots,\ n$

Simpson's Rule:

$$\int_a^b f(x)\,dx \approx S(n) = \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n) \right] \frac{\Delta x}{3}$$

where $\Delta x = \dfrac{b-a}{n}$, n *is even* and $x_k = a + k\Delta x$ *for* $k = 0, 1, \ldots, n$

Polar Coordinates

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \begin{cases} r^2 = x^2 + y^2 \\ \theta = \tan^{-1}\left(\dfrac{y}{x}\right) \end{cases}$$

Slope of the Curve $r = f(\theta)$

$$\left.\frac{dy}{dx}\right|_{(f(\theta_0),\theta)} = \frac{f'(\theta_0)\sin\theta + f(\theta_0)\cos\theta_0}{f'(\theta_0)\cos\theta_0 - f(\theta_0)\sin\theta_0}$$

Length of Polar Curve: $L = \displaystyle\int_\alpha^\beta \sqrt{f(\theta)^2 + g(\theta)^2}\, d\theta$

Area of Region between Polar Curves:

$$A = \frac{1}{2}\int_\alpha^\beta \left(f(\theta)^2 - g(\theta)^2 \right) d\theta \qquad f(\theta) \ge g(\theta) \ge 0$$