Section 1.3 – Evaluating Trigonometry Functions

$$\sin A = \frac{Opposite\ A}{Hypotenuse} = \frac{opp}{hyp} = \frac{a}{c} = \cos B$$

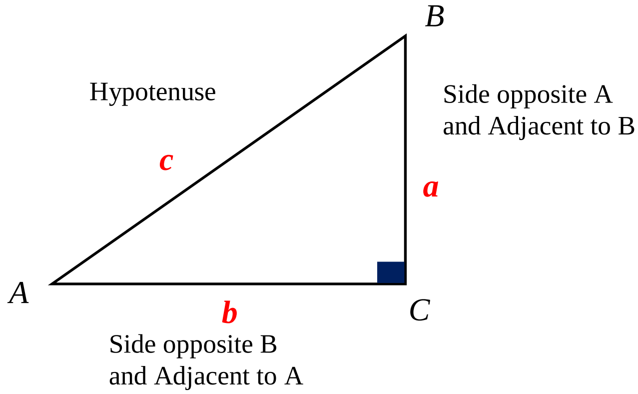
$$\cos A = \frac{Adjacent\ A}{Hypotenuse} = \frac{adj}{hyp} = \frac{b}{c} = \sin B$$

$$\tan A = \frac{opp\ A}{adj\ A} = \frac{a}{b} = \cot B$$

$$\cot A = \frac{adj\ A}{opp\ A} = \frac{b}{a} = \tan B$$

$$\sec A = \frac{hyp}{adj\ A} = \frac{c}{b} = \csc B$$

$$\csc A = \frac{hyp}{opp\ A} = \frac{c}{a} = \sec B$$



Example

Triangle ABC is a right triangle with C = 90°. If a = 6 and c = 10, find the six trigonometric functions of A.

$$\begin{aligned} b &= \sqrt{c^2 - a^2} \\ &= \sqrt{10^2 - 6^2} \\ &= 8 \end{aligned}$$

$$\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$$

$$\cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$$

$$\tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$$

$$\cot A = \frac{b}{a} = \frac{8}{6} = \frac{4}{3}$$

$$\sec A = \frac{c}{b} = \frac{10}{8} = \frac{5}{4}$$

$$\csc A = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$$

$$if\ \ A + B = 90° \Rightarrow \begin{cases} \sin A = \cos B \\ \sec A = \csc B \\ \tan A = \cot B \end{cases}$$

Cofunction Theorem

A trigonometric function of an angle is always equal to the cofunction of the complement of the angle.

Example

Fill in the blanks

a. $\sin(----) = \cos 30°$

Solution

$\sin 60° = \cos 30°$

b. $\tan y = \cot(----)$

Solution

$\tan y = \cot(90° - y)$

Example

Write each function in terms of its cofunction

a) $\cos 52°$

Solution

$\cos 52° = \sin(90° - 52°) = \sin 38°$

b) $\tan 71°$

Solution

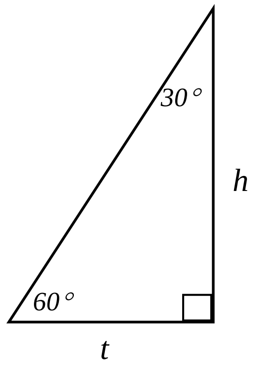
$\tan 71° = \cot(90° - 71°) = \cot 19°$

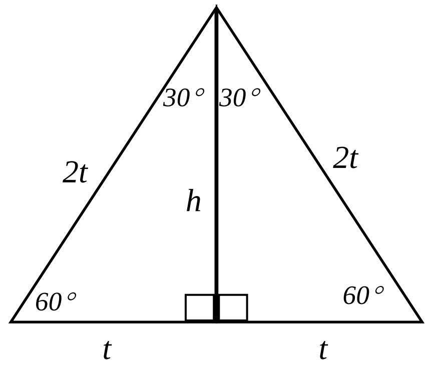
c) $\sec 24°$

Solution

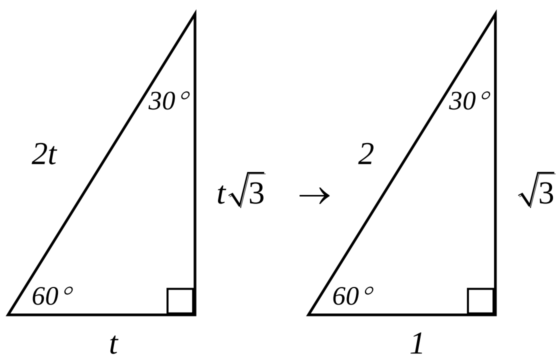
$\sec 24° = \csc(90° - 24°) = \csc 66°$

The 30° - 60° - 90° *Triangle*





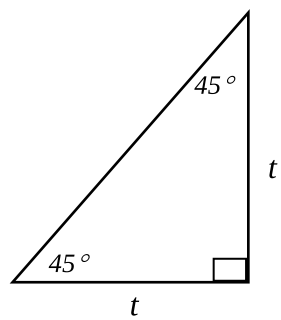
$$t^2 + h^2 = (2t)^2$$
$$t^2 + h^2 = 4t^2$$
$$h^2 = 4t^2 - t^2$$
$$h^2 = 3t^2$$
$$h = t\sqrt{3}$$

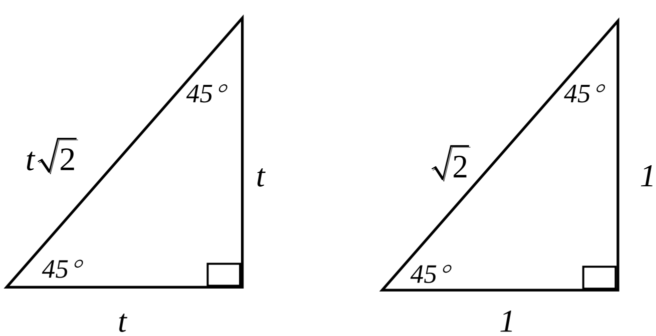


$$\Rightarrow \boxed{\sin 60° = \frac{\sqrt{3}}{2}}$$

The 45° - 45° - 90° *Triangle*

$$hypotenuse^2 = t^2 + t^2$$
$$hypotenuse = \sqrt{2t^2}$$
$$hypotenuse = t\sqrt{2}$$





$$\Rightarrow \boxed{\cos 45° = \frac{1}{\sqrt{2}}}$$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Example

Show that the following are true

a. $\cos^2 30° + \sin^2 30° = 1$

$$\cos^2 30° + \sin^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$$

b. $\cos^2 45° + \sin^2 45° = 1$

$$\cos^2 45° + \sin^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

Example

Let x = 30° and y = 45° in each of the expressions that follow, and then simplify each expression as much as possible

a. $2\sin x$

$$2\sin 30° = 2\left(\frac{1}{2}\right) = 1$$

b. $\sin 2y$

$$\sin 2\times 45° = \sin 90° = 1$$

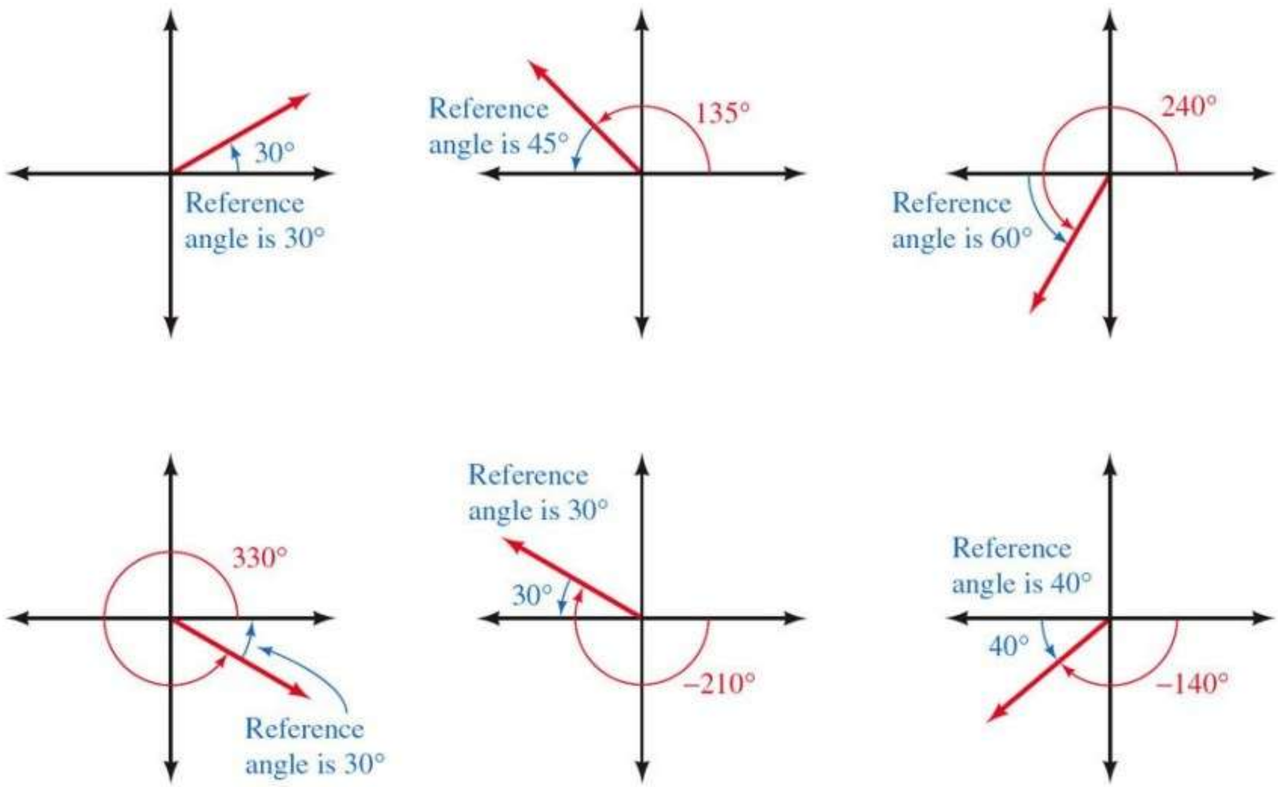
c. $4\sin(3x - 90°)$

$$4\sin(3(30°) - 90°) = 4\sin(0°) = 0$$

Reference Angle

Definition

The reference angle or related angle for any angle θ in standard position ifs the positive acute angle between the terminal side of θ and the *x*-axis, and it is denoted $\hat{\theta}$



$f\ \ \theta \in QI$ then $\hat{\theta} = \theta \quad \leftrightarrow \quad \theta = \hat{\theta}$

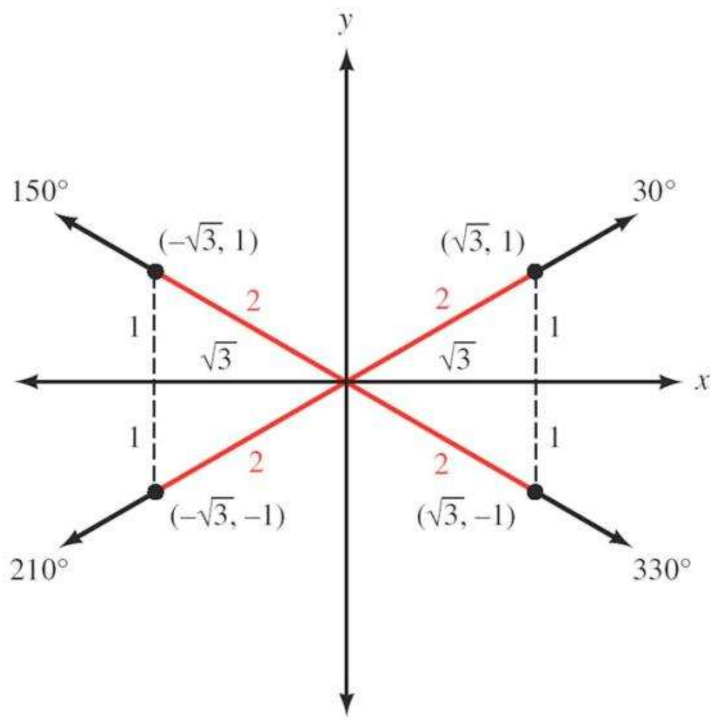
$f\ \ \theta \in QII$ then $\hat{\theta} = 180° - \theta \quad \leftrightarrow \quad \theta = 180° - \hat{\theta}$

$f\ \ \theta \in QIII$ then $\hat{\theta} = \theta - 180° \quad \leftrightarrow \quad \theta = \hat{\theta} + 180°$

$f\ \ \theta \in QIV$ then $\hat{\theta} = 360° - \theta \quad \leftrightarrow \quad \theta = 360° - \hat{\theta}$

Reference Angle Theorem

A trigonometric function of an angle and its reference angle are the same, except difference in sign.



$$\sin 150° = \sin 30° = \frac{1}{2}$$

$$\sin 210° = -\sin 30° = -\frac{1}{2}$$

Example

Find the exact value of $\sin 240°$

<u>*Solution*</u>

$$\hat{\theta} = 240° - 180° = 60° \qquad \to 240° \in QIII$$

$$\sin 240° = -\sin 60°$$

$$= -\frac{\sqrt{3}}{2}$$

Example

Find the exact value of $\tan 315°$

<u>*Solution*</u>

$$\hat{\theta} = 360° - 315° = 45° \qquad \to 315° \in QIV$$

$$\tan 315° = -\tan 45°$$

$$= -1$$

The trigonometry function of an angle and any coterminal to it are always equals.

$$\sin(\theta + 360°k) = \sin\theta$$

$$\cos(\theta + 360°k) = \cos\theta$$

Example

Find the exact value of $\cos 495°$

<u>*Solution*</u>

$495° - 360° = 135°$

$\rightarrow 135° \in QII$

$\hat{\theta} = 180° - 135° = 45°$

$\cos 495° = -\cos 45° = -\frac{1}{\sqrt{2}}$

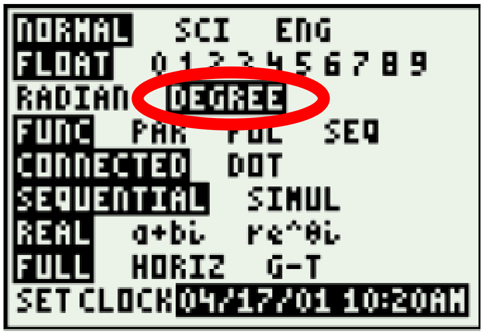
Approximation- Simply using calculator

$\sin 250° \approx -0.9397$

$\cos 250° \approx -0.3420$

$\tan 250° \approx 2.7475$

$\csc 250° = \frac{1}{\sin 250°} \approx -1.0642$



To find the angle by using the inverse trigonometry functions, always enter a ***positive*** *value.*

Example

Find θ if $\sin\theta = -0.5592$ and θ terminates in QIII with $0° \le \theta < 360°$.

<u>*Solution*</u>

$\hat{\theta} = \sin^{-1} 0.5592 \approx 34°$

$\theta \in$ QIII

$\Rightarrow \theta = 180° + 34° = 214°$

Example

Find θ to the nearest degree if $\cot\theta = -1.6003$ and θ terminates in QII with $0° \le \theta < 360°$.

<u>*Solution*</u>

$$\cot\theta = -1.6003 = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{1}{-1.6003}$$

$$\hat{\theta} = \tan^{-1}\frac{1}{1.6003}$$

$$= 32°$$

$\theta \in$ QII

$$\Rightarrow \theta = 180° - 32° = 148°$$

$$= 148°$$

Angle θ in *degree*	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0°	0	1	0	$\infty(undefined)$	1	$\infty(undefined)$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90°	1	0	$\pm\infty$	0	$\pm\infty$	1
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
135°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
180°	0	-1	0	$\pm\infty$	-1	$\pm\infty$

Exercise *Section* 1.3 – Evaluating Trigonometry Functions

1. Simplify by using the table. $5\sin^2 30°$

2. Simplify by using the table $\sin^2 60° + \cos^2 60°$

3. Simplify by using the table $(\tan 45° + \tan 60°)^2$

4. Find the exact value of $\csc 300°$

5. Find θ if $\sin\theta = -\frac{1}{2}$ and θ terminates in QIII with $0° \le \theta \le 360°$.

6. Find θ to the nearest degree if $\sec\theta = 3.8637$ and θ terminates in QIV with $0° \le \theta < 360°$.

7. Find the exact value of $\cos 225°$

8. Find the exact value of $\tan 315°$

9. Find the exact value of $\cos 420°$

10. Find the exact value of $\cot 480°$

11. Use the calculator to find the value of $\csc 166.7°$

12. Use the calculator to find the value of $\sec 590.9°$

13. Use the calculator to find the value of $\tan 195°\, 10'$

14. Use the calculator to find θ to the nearest degree if $\sin\theta = -0.3090$ with $\theta \in$ QIV with $0° \le \theta < 360°$

15. Use the calculator to find θ to the nearest degree if $\cos\theta = -0.7660$ with $\theta \in$ QIII with $0° \le \theta < 360°$

16. Use the calculator to find θ to the nearest degree if $\sec\theta = -3.4159$ with $\theta \in$ QII with $0° \le \theta < 360°$

17. Find θ to the nearest tenth of a degree if $\tan\theta = -0.8541$ and θ terminates in QIV with $0° \le \theta < 360°$

18. Use the calculator to find θ to the nearest degree if $\sin\theta = 0.49368329$ with $\theta \in$ QII with $0° \le \theta < 360°$