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8.3 Double angle

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1 \quad (1)$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A \cos A$$

(1) $2\cos^2 A = 1 + \cos 2A$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\cos^2 \frac{A}{2} = \frac{1 + \cos A}{2}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}} \quad (2)$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

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ex $\sin A = \frac{3}{5}$ $A \in$ QII 3, 4 → 5

$$\cos A = -\frac{4}{5}$$

a) $\sin 2A = 2\sin A \cos A$
$$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$$
$$= -\frac{24}{25}$$

b) $\cos 2A = \cos^2 A - \sin^2 A$
$$= \frac{16}{25} - \frac{9}{25}$$
$$= \frac{7}{25}$$

$\frac{25}{}$

c) $\tan 2A = -\frac{24}{7}$

$\frac{90^\circ}{2} < \frac{A}{2} < \frac{180^\circ}{2}$

$45^\circ < \frac{A}{2} < 90^\circ \Rightarrow \frac{A}{2} \in QI$ ←

d) $\sin\frac{A}{2} = \sqrt{\frac{1}{2}\left(1-\left(-\frac{4}{5}\right)\right)}$ ☺

$= \sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)}$

$= \sqrt{\frac{1}{2}\,\frac{9}{5}}$

$= \frac{3}{\sqrt{10}}$

$\sqrt{\frac{1}{2}(1-\cos A)}$

$\sqrt{\frac{1}{2}(1+\cos A)}$

e) $\cos\frac{A}{2} = \sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)}$

$= \sqrt{\frac{1}{2}\,\frac{1}{5}}$

$= \frac{1}{\sqrt{10}}$

f) $\tan\frac{A}{2} = 3$ $\left(\frac{3}{1}\right)$

Prove: $(\sin\theta+\cos\theta)^2 = 1+\sin 2\theta$

$(\sin\theta+\cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= 1 + \sin 2\theta$ ✓

Prove

Prove

$$\sin 2x = \frac{2\cot x}{1+\cot^2 x}$$ ✓

$$\frac{2\cot x}{1+\cot^2 x} = 2\,\frac{\frac{\cos x}{\sin x}}{1+\frac{\cos^2 x}{\sin^2 x}}$$

$\csc^2 x$

$$= 2\,\frac{\frac{\cos x}{\sin x}}{\frac{\sin^2 x+\cos^2 x}{\sin^2 x}} \div$$

$$= 2\,\frac{\cos x}{\sin x}\,\frac{\sin^2 x}{1}$$

$$= 2\cos x \sin x$$
$$= \sin 2x$$ ✓

Prove: $\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$$\cos 4x = \cos(2(2x))$$
$$= 2\cos^2(2x) - 1$$
$$= 2(\cos 2x)^2 - 1$$
$$= 2(2\cos^2 x - 1)^2 - 1$$
$$= 2(4\cos^4 x - 4\cos^2 x + 1) - 1$$
$$= 8\cos^4 x - 8\cos^2 x + 2 - 1$$
$$= 8\cos^4 x - 8\cos^2 x + 1$$ ✓

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$ A+B $\frac{\tan A+\tan B}{1-\tan\tan}$

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$$ ☺

Prove: $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta}$

$$\frac{1-\cos 2A}{\sin 2\theta} = \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$$

$$= \frac{1-1+2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta \checkmark$$

Prove: $\sin^2\frac{x}{2} = \dfrac{\tan x - \sin x}{2\tan x}$

$$\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)\cdot\frac{\tan x}{\tan x}$$

$$= \frac{\tan x - \cos x\tan x}{2\tan x}$$

$$= \frac{\tan x - \cos x\,\frac{\sin x}{\cos x}}{2\tan x}$$

$$= \frac{\tan x - \sin x}{2\tan x} \checkmark$$

Ex Prove: $\sin 3x = \sin x(3\cos^2 x - \sin^2 x)$

$$\begin{aligned}\sin 3x &= \sin(x+2x)\\ &= \sin x \cos 2x + \cos x \sin 2x\\ &= \sin x(\cos^2 x - \sin^2 x) + \cos x(2\sin x \cos x)\\ &= \sin x \cos^2 x - \sin^3 x + 2\sin x \cos^2 x\\ &= 3\sin x \cos^2 x - \sin^3 x\\ &= \sin x(3\cos^2 x - \sin^2 x)\ \checkmark\end{aligned}$$

8.4 Solving Trig

[trig fcn] a variable

Cosine + Sine → 1 angle for 1

1 → $\cos 0$, $\sin \frac{\pi}{2}$

−1 → $\cos \pi$, $\sin \frac{3\pi}{2}$

otherwise each value → 2 angles

$\frac{\pi}{n}$ Q I

$\frac{(n-1)\pi}{n}$ Q II

$\frac{(n+1)\pi}{n}$ Q III

$\frac{(2n-1)\pi}{n}$ Q IV

Ex $\sin\theta = \frac{1}{2}$ Q I, II

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad [0, 2\pi)$

$\theta = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$

Solve: $\sin x \tan x = \sin x \quad [0, 2\pi)$

$\sin x \tan x - \sin x = 0$

$\sin x (\tan x - 1) = 0$

$\sin x = 0 \qquad \tan x - 1 = 0$

$\tan x = 1$

$x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$

$x = n\pi, \frac{\pi}{4} + n\pi$

Solve: $2\sin^2 t - \cos t - 1 = 0 \quad [0, 2\pi]$

$2(1 - \cos^2 t) - \cos t - 1 = 0$

$2 - 2\cos^2 t - \cos t - 1 = 0$

$-2\cos^2 t - \cos t + 1 = 0$

$\cos t = \frac{1 \pm \sqrt{1+8}}{2(-2)} = \frac{1 \pm 3}{-4}$

$\cos t = -1 \qquad \cos t = \frac{1}{2}$

$t = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

Solve $4\sin^2 x \tan x - \tan x = 0 \quad [0, 2\pi]$

$\tan x(4\sin^2 x - 1) = 0$

$\tan x = 0$ $\frac{\sin x}{\cos x}$ $4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Solve $\csc^4 2u - 4 = 0$

$(\csc^2 2u - 2)(\csc^2 2u + 2) = 0$

$\csc^2 2u = 2$ $\quad$ $\csc^2 2u = -2$ #

$\csc 2u = \pm\sqrt{2}$

$\frac{1}{\sin 2u} = \pm\sqrt{2}$

$\sin 2u = \pm\frac{1}{\sqrt{2}}$ $\quad$ $\frac{\pi}{4}$

$2u = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$u = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

$\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ ✓

Solve:

$5\sin\theta\tan\theta - 10\tan\theta + 3\sin\theta - 6 = 0$

$5\tan\theta(\sin\theta - 2) + 3(\sin\theta - 2) = 0$

$(\sin\theta - 2)(5\tan\theta + 3) = 0$

$\sin\theta = 2$ ✗ $\quad \tan\theta = -\frac{3}{5}$

$\theta = \tan^{-1}\left(\frac{3}{5}\right)$

(−) II, IV

$\theta = \pi - \tan^{-1}\left(\frac{3}{5}\right)$

$\theta = 2\pi - \tan^{-1}\left(\frac{3}{5}\right)$

#10 $2\sin^2 x = 1 - \sin x$

$a - b + c = 0$, $x = -1, -\frac{c}{a}$

$2\sin^2 x + \sin x - 1 = 0$

$\sin x = -1 \qquad \sin x = \frac{1}{2}$

$x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$ax^2 + bx + c = 0$

$a + b + c = 0 \Rightarrow x = 1, \frac{c}{a}$

$a - b + c = 0 \qquad x = -1, -c/a$

#11 $2\sin^3 x + \sin^2 x - 2\sin x - 1 = 0$

$\sin^2 x(2\sin x + 1) - (2\sin x + 1) = 0$

$(2\sin x + 1)(\sin^2 x - 1) = 0$

$\sin x = -\frac{1}{2} \qquad \sin^2 x = 1$, $\sin x = \pm 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$

24/ $\sin\theta - \cos\theta = 1$

$(\sin\theta)^2 = (1 + \cos\theta)^2$ (method) check!

$\frac{\sqrt{2}}{2}\sin\theta - \frac{\sqrt{2}}{2}\cos\theta = \frac{\sqrt{2}}{2}$

$\sin\frac{\pi}{4}\sin\theta - \cos\frac{\pi}{4}\cos\theta = \frac{\sqrt{2}}{2}$ } $\cos\frac{\pi}{4}\sin\theta =$ [illegible]

$-(\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta) = \frac{\sqrt{2}}{2}$

$\cos(\theta + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$ } $\cos(\frac{3\pi}{4})$, $\cos(\frac{5\pi}{4})$

$\theta + \frac{\pi}{4} = \frac{3\pi}{4}$

$\theta = \frac{3\pi}{4} - \frac{\pi}{4}$

$= \frac{\pi}{2}$

$\theta + \frac{\pi}{4} = \frac{5\pi}{4}$

$\theta = \frac{5\pi}{4} - \frac{\pi}{4}$

$= \pi$

- By grouping

- $\sin^2 \rightarrow$, cosine$^2 \rightarrow$ $\underbrace{\sin^2}_{1-\cos^2 x} + \cos$

$a\sin^2 x + b\sin x + c = 0$

$(\ \ )(\ \ ) = 0$, $\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$[0, 2\pi)$

#27 $2\sin^2\theta - 2\sin\theta - 1 = 0$ $12 = 4 \cdot 3$

$\sin\theta = \frac{2 \pm \sqrt{4 + 8}}{4}$

$$= \frac{2 \pm 2\sqrt{3}}{4}$$

$$= \frac{1 \pm \sqrt{3}}{4}$$

$\hat{\theta} = \sin^{-1} \frac{1 \pm \sqrt{3}}{4}$ 4 Quad

$\theta = \hat{\theta}$

$\theta = \pi - \hat{\theta}, \quad \pi + \hat{\theta}, \quad 2\pi - \hat{\theta}$