2π (radians) $\equiv 360° \equiv 1$ revolution $\qquad \theta = \dfrac{s}{r}$ (radians) $\qquad v = \dfrac{s}{t} = r\omega = r\dfrac{\theta}{t} \qquad \omega = \dfrac{\theta}{t} = \dfrac{v}{r} = \dfrac{s}{rt} = \dfrac{v\theta}{s}$

$$3600\ rev/minute = \frac{3600\ rev}{1\ min}\,\frac{2\pi\ (radians)}{1\ rev}\,\frac{1\ min}{60\ sec} = \frac{120\pi\ (radians)}{1\ sec}$$

$$r = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} \qquad\qquad (x-h)^2 + (y-k)^2 = r^2$$

SOHCAHTOA

A way of remembering how to compute the **sine**, **cosine**, and **tangent** of an angle.

SOH stands for **S**ine equals **O**pposite over **H**ypotenuse.

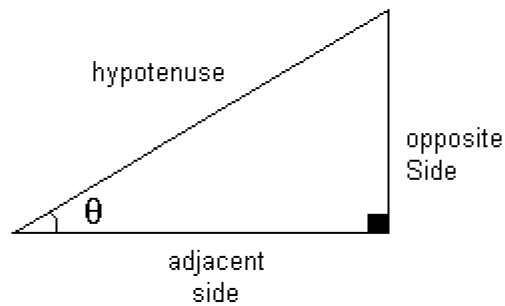
CAH stands for **C**osine equals **A**djacent over **H**ypotenuse.

TOA stands for **T**angent equals **O**pposite over **A**djacent.

SOH $\quad \sin\theta = \dfrac{Opposite}{Hypotenuse} = \dfrac{opp}{hyp}$

CAH $\quad \cos\theta = \dfrac{Adjacent}{Hypotenuse} = \dfrac{adj}{hyp}$

TOA $\quad \tan\theta = \dfrac{opposite}{adjacent} = \dfrac{opp}{adj} = \dfrac{\sin\theta}{\cos\theta}$



$$\cot\theta = \frac{adj}{opp} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$$

Angle θ in degree	Angle θ in radian	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0°	0	0	1	0	∞ (*undefined*)	1	∞ (*undefined*)
30°	$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90°	$\pi/2$	1	0	$\pm\infty$	0	$\pm\infty$	1
120°	$2\pi/3$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
135°	$3\pi/4$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	$5\pi/6$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
180°	π	0	-1	0	$\pm\infty$	-1	$\pm\infty$

Function	Domain $(n \in Z)$	Range	I	II	III	IV
$y = \sin t$	$\{t \mid -\infty < t < \infty\}$	$-1 \le y \le 1$	+	+	-	-
$y = \cos t$	$\{t \mid -\infty < t < \infty\}$	$-1 \le y \le 1$	+	-	-	+
$y = \tan t$	$\{t \mid -\infty < t < \infty,\ t \ne (2n+1)\,\pi/2\}$	$-\infty < y < \infty$	+	-	+	-
$y = \cot t$	$\{t \mid -\infty < t < \infty,\ t \ne n\pi\}$	$-\infty < y < \infty$	+	-	+	-
$y = \csc t$	$\{t \mid -\infty < t < \infty,\ t \ne n\pi\}$	$y \le -1,\ y \ge 1$	+	+	-	-
$y = \sec t$	$\{t \mid -\infty < t < \infty,\ t \ne (2n+1)\,\pi/2\}$	$y \le -1,\ y \ge 1$	+	-	-	+

$\cos^2\alpha + \sin^2\alpha = 1$ $\qquad$ $1 + \tan^2\alpha = \sec^2\alpha$ $\qquad$ $1 + \cot^2\alpha = \csc^2\alpha$

$\cos(-\alpha) = \cos\alpha$ $\qquad$ $\sin(-\alpha) = -\sin\alpha$ $\qquad$ $\tan(-\alpha) = -\tan\alpha$

$\cos(90° - \alpha) = \sin\alpha$ $\qquad$ $\sin(90° - \alpha) = \cos\alpha$ $\qquad$ $\tan(90° - \alpha) = \cot\alpha$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $\qquad$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\qquad$ $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} \qquad \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Double-Angle

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha$ $= 2\cos^2\alpha - 1$	$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$
$\cos^2\alpha = \dfrac{1 + \cos 2\alpha}{2}$	$\sin^2\alpha = \dfrac{1 - \cos 2\alpha}{2}$	$\tan^2\alpha = \dfrac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$

Half-Angle:

$\cos\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1 + \cos\alpha}{2}}$	$\sin\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1 - \cos\alpha}{2}}$	$\tan\left(\dfrac{\alpha}{2}\right) = \dfrac{\sin\alpha}{1 + \cos\alpha} = \dfrac{1 - \cos\alpha}{\sin\alpha}$

Product-to-Sum:

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$	$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$
$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha + \beta) + \cos(\alpha - \beta)\right]$	$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$

Sum-to-Product:

$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$	$\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
$\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$	$\sin x-\sin y=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$a\sin x+b\cos x=k\sin(x+\alpha)$ where $k=\sqrt{a^2+b^2}$, $\sin\alpha=\frac{b}{\sqrt{a^2+b^2}}$, and $\cos\alpha=\frac{a}{\sqrt{a^2+b^2}}$

$y=\cos^{-1}x$ *iff* $x=\cos y$ where $-1\le x\le 1$ and $0\le y\le\pi$

$y=\sin^{-1}x$ *iff* $x=\sin y$ where $-1\le x\le 1$ and $-\pi/2\le y\le\pi/2$

Law of Sines: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $\qquad$ $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$

Law of Cosines:

$a^2=b^2+c^2-2bc\cos A$	$\cos A=\frac{b^2+c^2-a^2}{2bc}$	$A=\cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$
$b^2=a^2+c^2-2ac\cos B$	$\cos B=\frac{a^2+c^2-b^2}{2ac}$	$B=\cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right)$
$c^2=a^2+b^2-2ab\cos C$	$\cos C=\frac{a^2+b^2-c^2}{2ab}$	$C=\cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right)$

Vectors:

Magnitude: $|V|=\sqrt{a^2+b^2}$

Dot Product: $U\bullet V=(ai+bj)\bullet(ci+dj)=ac+bd$

Angle: $\cos\theta=\frac{U\bullet V}{|U||V|}$

$z=r(\cos\theta+i\sin\theta)=r\,cis\theta$ $\qquad$ $r=\sqrt{x^2+y^2}$ $\qquad$ $\cos\theta=\frac{x}{r}$, $\sin\theta=\frac{y}{r}$, *and* $\tan\theta=\frac{y}{x}$

$(r_1 cis\theta_1)(r_2 cis\theta_2)=r_1r_2 cis(\theta_1+\theta_2)$ $\qquad$ $\frac{r_1 cis\theta_1}{r_2 cis\theta_2}=\frac{r_1}{r_2}cis(\theta_1-\theta_2)$

De Moivre's *Theorem*: $[r cis\theta]^n=r^n(cis n\theta)$

$[r cis\theta]^{1/n}=\sqrt[n]{r}\,cis\alpha$ $\qquad$ $\alpha=\frac{\theta}{n}+\frac{360°k}{n}$

The graphs of $y = k + A\sin(Bx + C)$ and $y = k + A\cos(Bx + C)$, where B > 0, will have the following characteristics:

Amplitude = $|A|$ Period = $\frac{2\pi}{B}$ Phase Shift $= \phi = -\frac{C}{B}$ One cycle: $0 \le argument \le 2\pi$

Vertical Shift: $y = k$

To graph "Sine or Cosine"

1- Find the Amplitude
2- Find the Period
3- Construct a table

x	$y = k + A\cos(Bx + C)$	$y = k + A\sin(Bx + C)$
ϕ	$k + A$	k
$\phi + \frac{P}{4}$	k	$k + A$
$\phi + \frac{P}{2}$	$k - A$	k
$\phi + \frac{3P}{4}$	k	$k - A$
$\phi + P$	$k + A$	k

4- Graph *One Cycle*
5- Extend the graph, if necessary

The graphs of $y = k + A\tan(Bx + C)$ and $y = k + A\cot(Bx + C)$, where B > 0, will have the following characteristics:

No Amplitude Period = $\frac{\pi}{B}$ Phase Shift $= -\frac{C}{B}$ One cycle: $0 \le argument \le \pi$

Vertical Shift: $y = k$

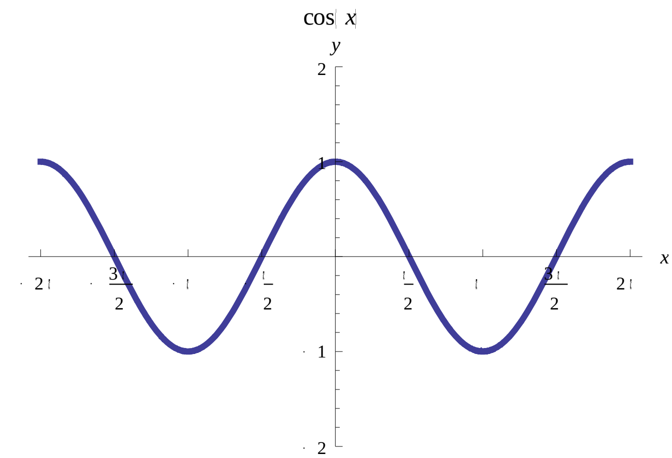
x	$y = k + A\tan(Bx + C)$	$y = k + A\cot(Bx + C)$
ϕ	k	∞
$\phi + \frac{P}{4}$	$k + A$	$k + A$
$\phi + \frac{P}{2}$	∞	k
$\phi + \frac{3P}{4}$	$k - A$	$k - A$
$\phi + P$	k	∞

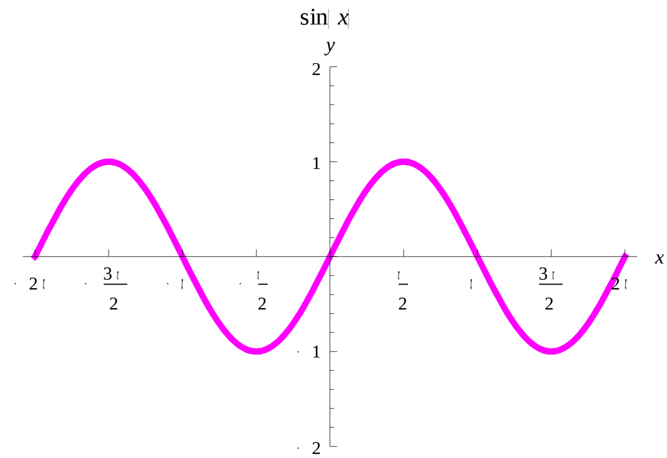
	sin	*cos*
0°	0	4
30°	1	3
45°	2	2
60°	3	1
90°	4	0

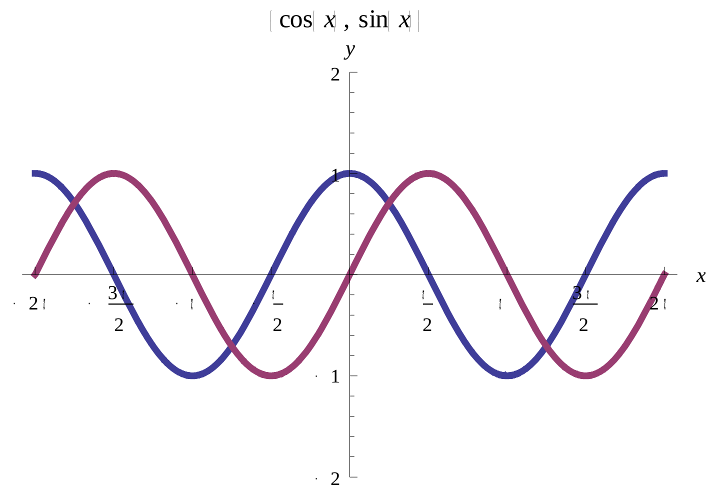
0°	$\frac{0}{4}$	$\frac{4}{4}$
30°	$\frac{1}{4}$	$\frac{3}{4}$
45°	$\frac{2}{4}$	$\frac{2}{4}$
60°	$\frac{3}{4}$	$\frac{1}{4}$
90°	$\frac{4}{4}$	$\frac{0}{4}$

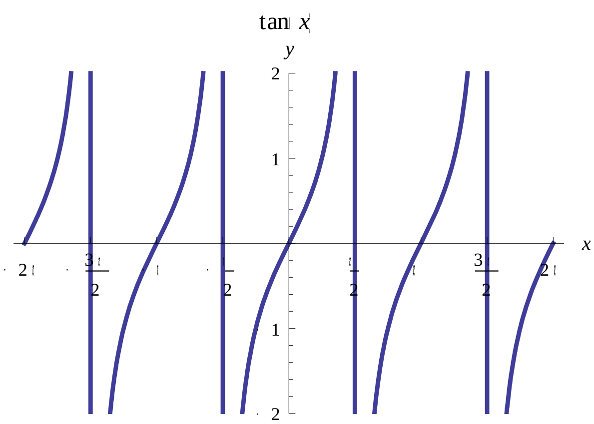
0°	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{4}{4}}$
30°	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{3}{4}}$
45°	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{2}{4}}$
60°	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{4}}$
90°	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{0}{4}}$

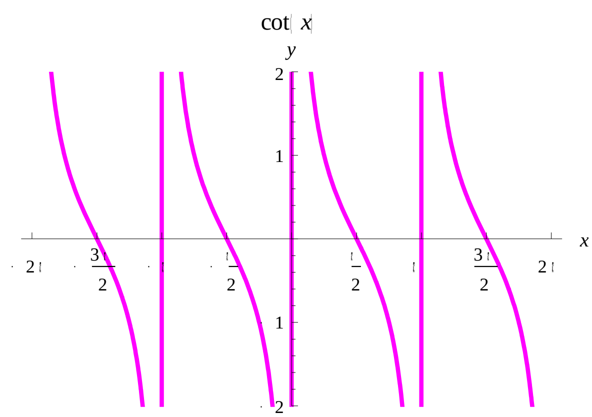
	sin	*cos*
0°	0	1
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
90°	1	0

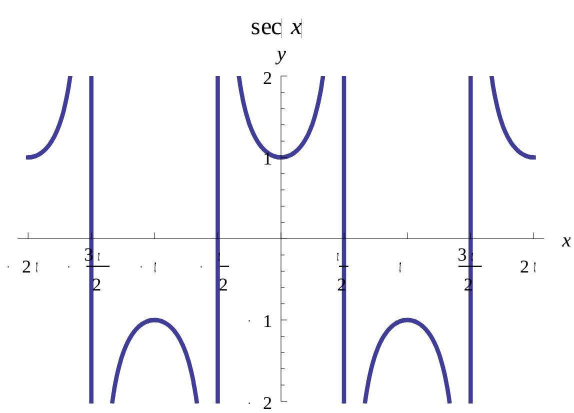

cos x
y
x

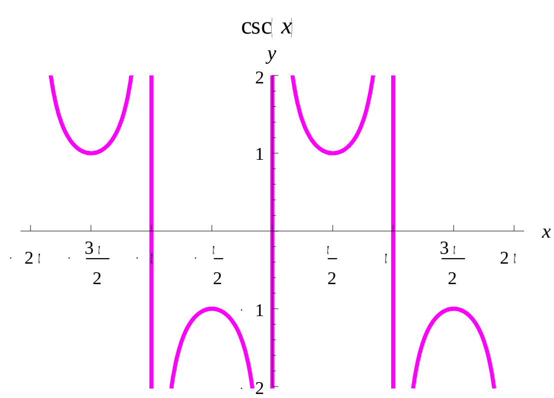

sin x
y
x

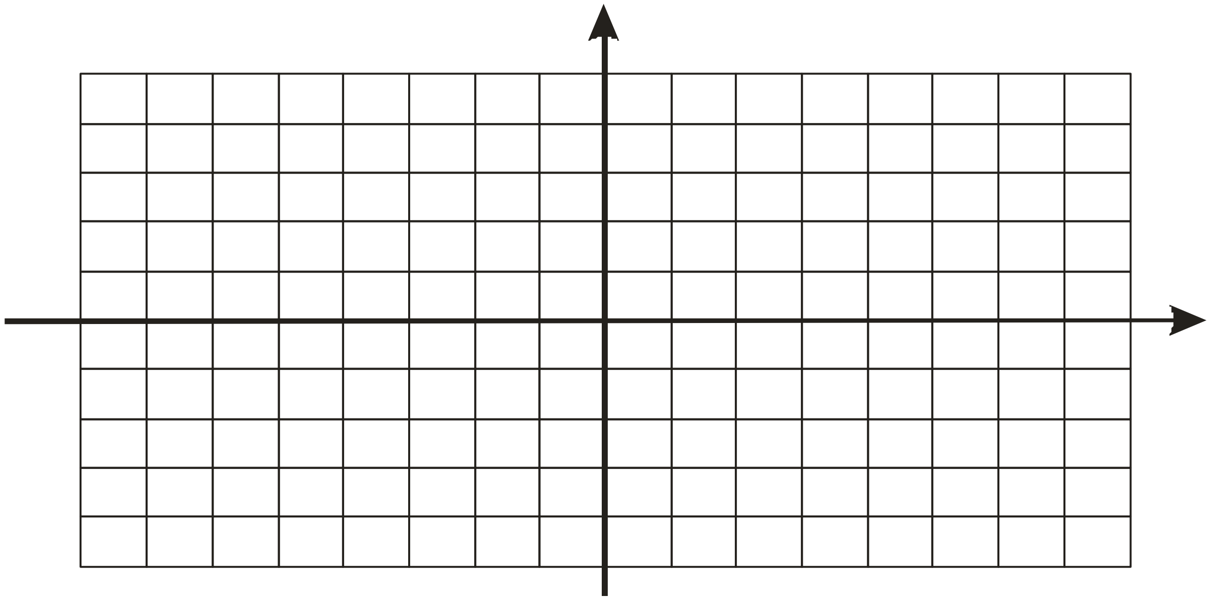

cos x , sin x
y
x

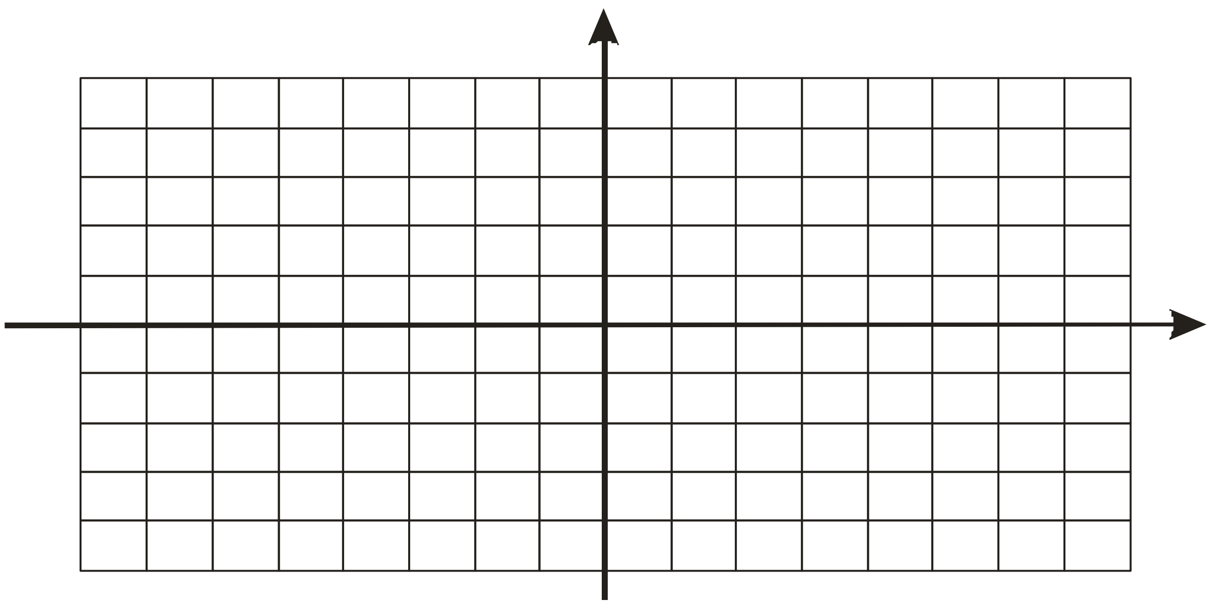

tan x
y
x

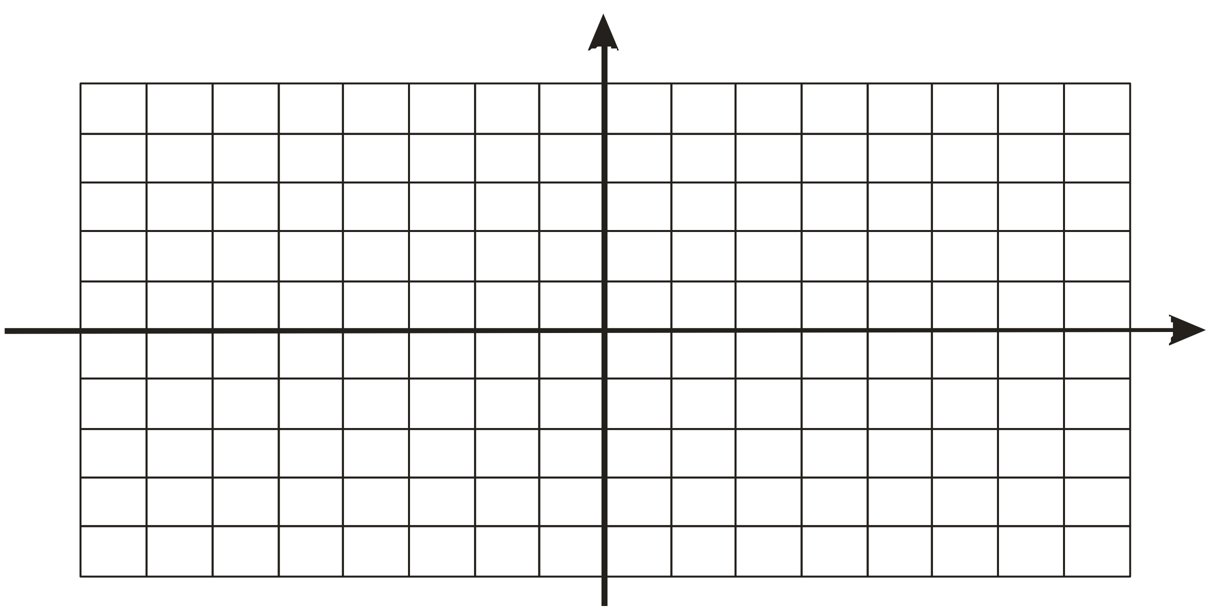

cot x
y
x

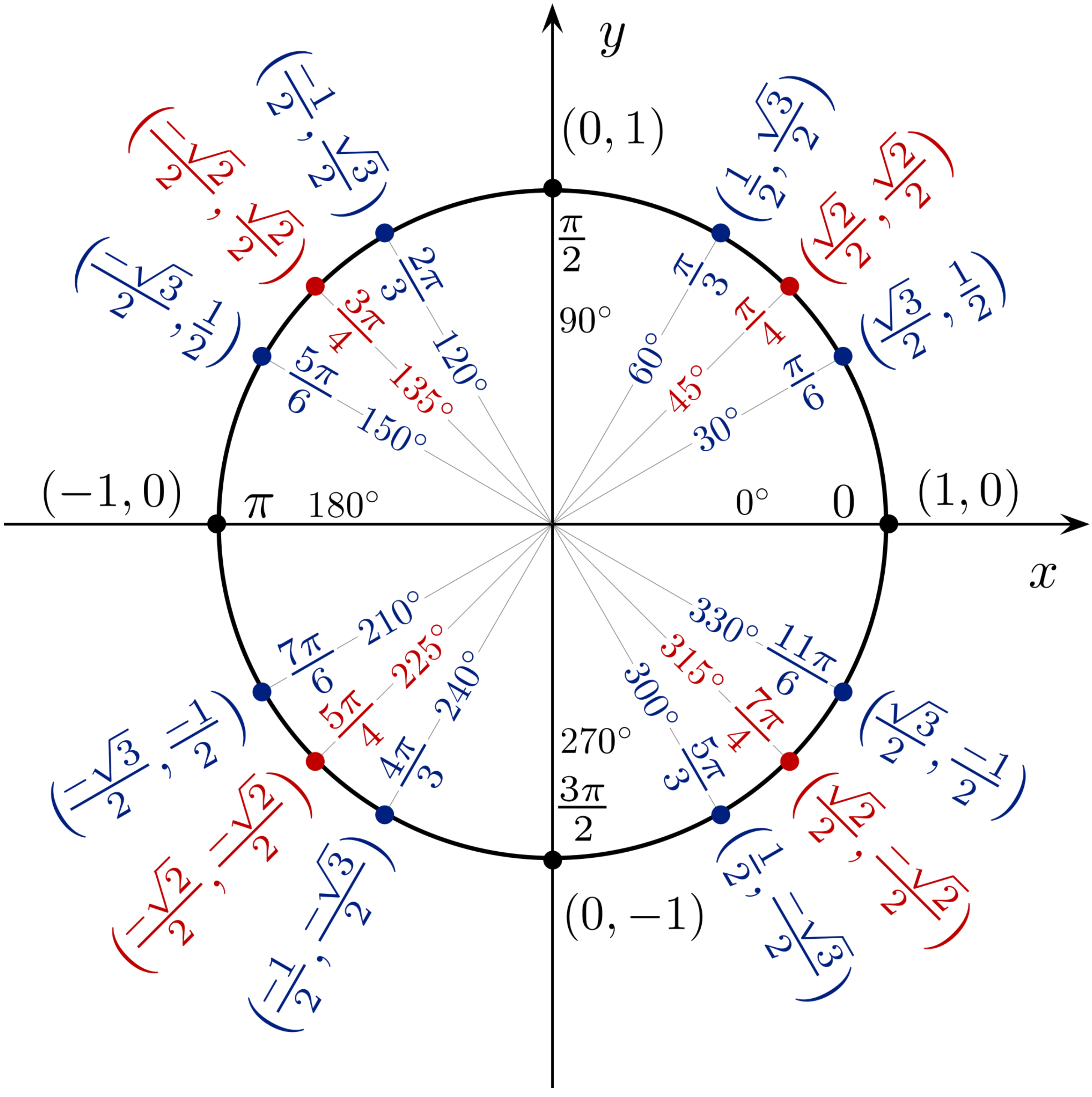

sec x
y
x


csc x
y
x









y
x
$(0, 1)$
$(1, 0)$
$(-1, 0)$
$(0, -1)$
$\frac{\pi}{2}$
90°
0
0°
π
180°
$\frac{3\pi}{2}$
270°
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{3}$
60°
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{\pi}{4}$
45°
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{\pi}{6}$
30°
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{2\pi}{3}$
120°
$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{3\pi}{4}$
135°
$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{5\pi}{6}$
150°
$\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$
$\frac{7\pi}{6}$
210°
$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
$\frac{5\pi}{4}$
225°
$\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$
$\frac{4\pi}{3}$
240°
$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$
$\frac{5\pi}{3}$
300°
$\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
$\frac{7\pi}{4}$
315°
$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$
$\frac{11\pi}{6}$
330°