Section 2.2 – Function Operations

The *Domain* of a Function

1. ***Rational*** function: $\frac{f(x)}{h(x)}$ $\Rightarrow$ ***Domain***: $\boxed{h(x) \neq 0}$

Example: $f(x) = \frac{1}{x-3}$

Domain: $x \neq 3$ $\{x \mid x \neq 3\}$

Or $(-\infty, 3) \cup (3, \infty)$ *Interval Notation*

Or $\mathbb{R} - \{3\}$

2. ***Irrational*** function: $\sqrt{g(x)}$ $\Rightarrow$ ***Domain***: $\boxed{g(x) \geq 0}$

Example: $g(x) = \sqrt{3-x} + 5$

$$3 - x \geq 0$$
$$-x \geq -3$$

Domain: $x < 3$ $(-\infty, 3]$

3. ***Otherwise***: Domain all real numbers $(-\infty, \infty)$

Example: $f(x) = x^3 + |x|$

Domain: All real numbers $\mathbb{R}$

$(-\infty, \infty)$

(1) & (2) $\rightarrow$ Find the domain: $f(x) = \frac{x+1}{\sqrt{x-3}}$

$x > 3$

Domain: $(3, \infty)$

Example

Find the domain

a) $f(x) = x^2 + 3x - 17$

Domain: $\underline{\mathbb{R}}$

b) $g(x) = \frac{5x}{x^2 - 49}$

$x^2 \neq 49$

$\underline{x \neq \pm 7}$

Domain: $\begin{cases} \{x \mid x \neq \pm 7\} & \text{or} \\ (-\infty, -7) \cup (-7, 7) \cup (7, \infty) \end{cases}$

c) $h(x) = \sqrt{9x - 27}$

$9x - 27 \geq 0$

$9x \geq 27$

Domain: $\underline{x \geq 3}$ $\quad [3, \infty)$

The *Algebra* of Functions

$$(f+g)(x)=f(x)+g(x)$$

$$(f-g)(x)=f(x)-g(x)$$

$$(f\cdot g)(x)=f(x)\cdot g(x)$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$$

Example

Let $f(x)=x^2+1$ and $g(x)=3x+5$. Find each of the following $(f+g)(1)$, $(f-g)(-3)$, $(fg)(5)$, and $\left(\frac{f}{g}\right)(0)$

Solution

$$\begin{aligned}(f+g)(1)&=f(1)+g(1)\\&=1^2+1+3(1)+5\\&=1+1+3+5\\&\underline{=10}\end{aligned}$$

$$\begin{aligned}(f-g)(-3)&=f(-3)-g(-3)\\&=(-3)^2+1-\big(3(-3)+5\big)\\&\underline{=14}\end{aligned}$$

$$\begin{aligned}(fg)(5)&=f(5)\cdot g(5)\\&=\left(5^2+1\right)\cdot\big(3(5)+5\big)\\&=(26)\cdot(20)\\&\underline{=520}\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(0)&=\frac{f(0)}{g(0)}\\&=\frac{0^2+1}{3(0)+5}\\&\underline{=\frac{1}{5}}\end{aligned}$$

Example

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x-1}$. Find each of the following and give the domain

$(f+g)(x), \quad (f-g)(x), \quad (fg)(x), \left(\frac{f}{g}\right)(x)$

Solution

Domain of f: $(-\infty, \infty)$

Domain of g: $\left[\frac{1}{2}, \infty\right)$ $\qquad \sqrt{2x-1 \geq 0} \ \rightarrow 2x \geq 1 \ \Rightarrow x \geq \frac{1}{2}$

a) $(f+g)(x) = 8x - 9 + \sqrt{2x-1}$

Domain: $x \geq \frac{1}{2}$ $\qquad \left[\frac{1}{2}, \infty\right)$

b) $(f-g)(x) = 8x - 9 - \sqrt{2x-1}$

Domain: $x \geq \frac{1}{2}$ $\qquad \left[\frac{1}{2}, \infty\right)$

c) $(fg)(x) = (8x-9)\sqrt{2x-1}$

Domain: $x \geq \frac{1}{2}$ $\qquad \left[\frac{1}{2}, \infty\right)$

d) $\left(\frac{f}{g}\right)(x) = \frac{8x-9}{\sqrt{2x-1}}$

Domain: $x > \frac{1}{2}$ $\qquad \left(\frac{1}{2}, \infty\right)$

Example

Let $f(x) = \sqrt{x-3}$ and $g(x) = \sqrt{x+1}$

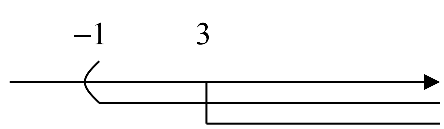
Find $(f+g)(x)$ and its domain, $\left(\frac{f}{g}\right)(x)$ and its domain

Solution

Domain $f(x)$: $x \geq 3$ $\quad$ *and* $\quad$ **Domain** $g(x)$: $x \geq -1$

a) $(f+g)(x) = \sqrt{x-3} + \sqrt{x+1}$

b) $x \geq 3$ *and* $x \geq -1$ $\Rightarrow$ **Domain**: $x \geq 3$



c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-3}}{\sqrt{x+1}}$

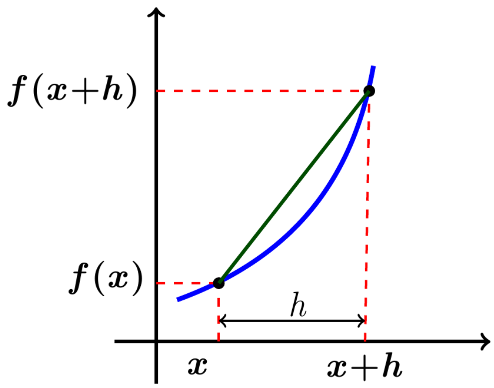
$$\rightarrow \begin{cases} x-3 \ge 0 & \Rightarrow \quad \underline{x \ge 3} \\ x+1 > 0 & \Rightarrow \quad \underline{x > -1} \end{cases}$$

Domain: $x \ge 3 \qquad [3, \infty)$

Difference Quotients

$$\frac{f(x+h)-f(x)}{(x+h)-x}$$

The difference quotient is given by: $\dfrac{f(x+h)-f(x)}{h}$



Example

For the function *f* given by $f(x)=2x-3$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

Solution

$$\begin{aligned} f(x+h) &= 2(\text{——})-3 \\ &= 2(x+h)-3 \\ &= 2x+2h-3 \end{aligned}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\underbrace{2x+2h-3}_{\text{}}\overset{f(x+h)}{} - \overset{f(x)}{(2x-3)}}{h} \\ &= \frac{2x+2h-3-2x+3}{h} \\ &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

Example

For the function *f* given by $f(x) = -2x^2 + x + 5$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$

Solution

$$f(\mathbf{x}+\mathbf{h}) = -2(\mathbf{x}+\mathbf{h})^2 + (\mathbf{x}+\mathbf{h}) + 5 \qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$f(x+h) = -2\left(x^2 + 2hx + h^2\right) + x + h + 5$$

$$f(x+h) = -2x^2 - 4hx - 2h^2 + x + h + 5$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-2x^2 - 4hx - 2h^2 + x + h + 5 - (-2x^2 + x + 5)}{h}$$

$$= \frac{-2x^2 - 4hx - 2h^2 + x + h + 5 + 2x^2 - x - 5}{h}$$

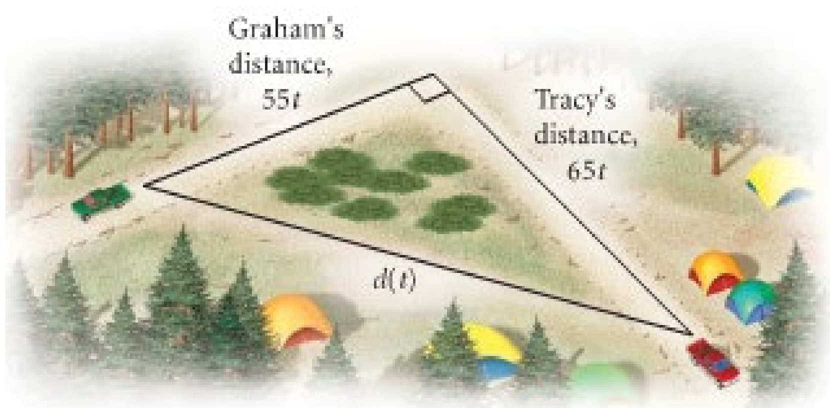
$$= \frac{-4hx - 2h^2 + h}{h}$$

$$= \frac{-4hx}{h} - \frac{2h^2}{h} + \frac{h}{h}$$

$$= -4x - 2h + 1$$

Example

Tracy and Graham drive away from a camp-ground at right angles to each other. Tracy's speed is 65 mph and Graham's is 55 *mph*.



a) Express the distance between the cars as a function of time.

b) Find the domain of the function.

Solution

a) $Distance = velocity * time$

Use Pythagorean Theorem:

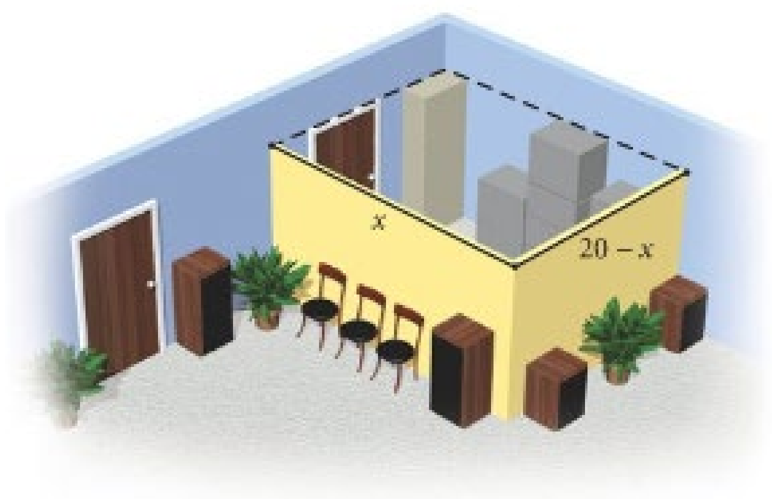
$$d^2(t) = (65t)^2 + (55t)^2$$

$$d^2 = 4225t^2 + 3025t^2$$
$$= 7250t^2$$
$$d(t) = \sqrt{7250t^2}$$
$$= \sqrt{7250}\sqrt{t^2}$$
$$\approx 85.15|t|$$
$$\underline{= 85.15\ t}$$

b) ***Domain***: $t \ge 0$

Example: (storage area)

The sound Shop has 20 *feet*. of dividers with which to set off a rectangular area for the storage of overstock. If a corner of the store is used for the storage area, the partition need only form two sides of a rectangle.



a) Express the floor area of the storage space as a function of the length of the partition.

b) Find the domain of the function.

Solution

Let x = the length

$width + length = 20$

$width = 20 - length$

a) ***Area*** $= length * width$

$$= x(20 - x)$$
$$\underline{= 20x - x^2}$$

b) ***Domain***: x value varies from 0 to 20 $\Rightarrow$ (0, 20)

Exercises *Section* 2.2 – Function Operations

(**1 – 80**) Find the Domain

1. $f(x)=7x+4$

2. $f(x)=|3x-2|$

3. $f(x)=3x+\pi$

4. $f(x)=\sqrt{7}x+\frac{1}{2}$

5. $f(x)=-2x^2+3x-5$

6. $f(x)=x^3-2x^2+x-3$

7. $f(x)=x^2-2x-15$

8. $f(x)=4-\frac{2}{x}$

9. $f(x)=\frac{1}{x^4}$

10. $g(x)=\frac{3}{x-4}$

11. $y=\frac{2}{x-3}$

12. $y=\frac{-7}{x-5}$

13. $f(x)=\frac{x+5}{2-x}$

14. $f(x)=\frac{8}{x+4}$

15. $f(x)=\frac{1}{x+4}$

16. $f(x)=\frac{1}{x-4}$

17. $f(x)=\frac{3x}{x+2}$

18. $f(x)=x-\frac{2}{x-3}$

19. $f(x)=x+\frac{3}{x-5}$

20. $f(x)=\frac{1}{2}x-\frac{8}{x+7}$

21. $f(x)=\frac{1}{x-3}-\frac{8}{x+7}$

22. $f(x)=\frac{1}{x+4}-\frac{2x}{x-4}$

23. $f(x)=\frac{3x^2}{x+3}-\frac{4x}{x-2}$

24. $f(x)=\frac{1}{x^2-2x+1}$

25. $f(x)=\frac{x}{x^2+3x+2}$

26. $f(x)=\frac{x^2}{x^2-5x+4}$

27. $f(x)=\frac{1}{x^2-4x-5}$

28. $g(x)=\frac{2}{x^2+x-12}$

29. $h(x)=\frac{5}{\frac{4}{x}-1}$

30. $y=\sqrt{x}$

31. $f(x)=\sqrt{8-3x}$

32. $y=\sqrt{4x+1}$

33. $y=\sqrt{7-2x}$

34. $f(x)=\sqrt{8-x}$

35. $f(x)=\sqrt{3-2x}$

36. $f(x)=\sqrt{3+2x}$

37. $f(x)=\sqrt{5-x}$

38. $f(x)=\sqrt{x-5}$

39. $f(x)=\sqrt{6-3x}$

40. $f(x)=\sqrt{3x-6}$

41. $f(x)=\sqrt{2x+7}$

42. $f(x)=\sqrt{x^2-16}$

43. $f(x)=\sqrt{16-x^2}$

44. $f(x)=\sqrt{9-x^2}$

45. $f(x)=\sqrt{x^2-25}$

46. $f(x)=\sqrt{x^2-5x+4}$

47. $f(x)=\sqrt{x^2+5x+4}$

48. $f(x)=\sqrt{x^2+3x+2}$

49. $f(x)=\sqrt{x^2-3x+2}$

50. $f(x)=\sqrt{x-4}+\sqrt{x+1}$

51. $f(x)=\sqrt{3-x}+\sqrt{x-2}$

52. $f(x)=\sqrt{1-x}+\sqrt{4-x}$

53. $f(x)=\sqrt{1-x}-\sqrt{x-3}$

54. $f(x)=\sqrt{x+4}-\sqrt{x-1}$

55. $f(x)=\frac{\sqrt{x+1}}{x}$

56. $g(x)=\frac{\sqrt{x-3}}{x-6}$

57. $f(x)=\frac{\sqrt{x+4}}{\sqrt{x-1}}$

58. $f(x)=\frac{\sqrt{5-x}}{x}$

59. $f(x)=\frac{x}{\sqrt{5-x}}$

60. $f(x)=\dfrac{1}{x\sqrt{5-x}}$

61. $f(x)=\dfrac{x+1}{x^3-4x}$

62. $f(x)=\dfrac{\sqrt{x+5}}{x}$

63. $f(x)=\dfrac{x}{\sqrt{x+5}}$

64. $f(x)=\dfrac{1}{x\sqrt{x+5}}$

65. $f(x)=\dfrac{x+3}{\sqrt{x-3}}$

66. $f(x)=\dfrac{\sqrt{x+3}}{\sqrt{x-3}}$

67. $f(x)=\dfrac{\sqrt{x-2}}{\sqrt{x+2}}$

68. $f(x)=\dfrac{\sqrt{2-x}}{\sqrt{x+2}}$

69. $f(x)=\dfrac{x-4}{\sqrt{x-2}}$

70. $f(x)=\dfrac{1}{(x-3)\sqrt{x+3}}$

71. $f(x)=\sqrt{x+2}+\sqrt{2-x}$

72. $f(x)=\sqrt{(x-2)(x-6)}$

73. $f(x)=\sqrt{x+3}-\sqrt{4-x}$

74. $f(x)=\dfrac{\sqrt{4x-3}}{x^2-4}$

75. $f(x)=\dfrac{4x}{6x^2+13x-5}$

76. $f(x)=\dfrac{\sqrt{2x-3}}{x^2-5x+4}$

77. $f(x)=\dfrac{x^2}{\sqrt{x^2-5x+4}}$

78. $f(x)=\dfrac{x+2}{\sqrt{x^2+5x+4}}$

79. $f(x)=\dfrac{\sqrt{x+2}}{\sqrt{x^2+3x+2}}$

80. $f(x)=\dfrac{\sqrt{2x+3}}{x^2-6x+5}$

81. Let $f(x)=4x-3$ and $g(x)=5x+7$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\dfrac{f}{g}\right)(x)$

82. Let $f(x)=2x^2+3$ and $g(x)=3x-4$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\dfrac{f}{g}\right)(x)$

83. Let $f(x)=x^2-2x-3$ and $g(x)=x^2+3x-2$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\dfrac{f}{g}\right)(x)$

84. Let $f(x)=\sqrt{4x-1}$ and $g(x)=\dfrac{1}{x}$. Find each of the following and give the domain

a) $(f+g)(x)$ *b)* $(f-g)(x)$ *c)* $(fg)(x)$ *d)* $\left(\dfrac{f}{g}\right)(x)$

85. Given that $f(x)=x+1$ and $g(x)=\sqrt{x+3}$

a) Find $(f+g)(x)$

b) Find the domain of $(f+g)(x)$

c) Find: $(f+g)(6)$

86. Given that $f(x)=x^2-4$ and $g(x)=x+2$

a) Find $(f+g)(x)$ and its domain

b) Find $(f/g)(x)$ and its domain

87. Let $f(x)=x^2+1$ and $g(x)=3x+5$. Find $(f+g)(1)$, $(f-g)(-3)$, $(fg)(5)$, and $\left(\frac{f}{g}\right)(0)$

88. Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, *and* $(f/g)(x)$ and the domain of

$f(x)=\sqrt{3-2x}, \quad g(x)=\sqrt{x+4}$

89. Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, *and* $(f/g)(x)$ and the domain of

$f(x)=\dfrac{2x}{x-4}, \quad g(x)=\dfrac{x}{x+5}$

90. Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, *and* $(f/g)(x)$ of $f(x)=x-5$ and $g(x)=x^2-1$

(**88 – 103**) Find and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for the given function

91. $f(x)=9x+5$

92. $f(x)=6x+2$

93. $f(x)=4x+11$

94. $f(x)=3x-5$

95. $f(x)=-2x-3$

96. $f(x)=-4x+3$

97. $f(x)=3x-6$

98. $f(x)=-5x-7$

99. $f(x)=2x^2$

100. $f(x)=5x^2$

101. $f(x)=3x^2-4x$

102. $f(x)=2x^2-3x$

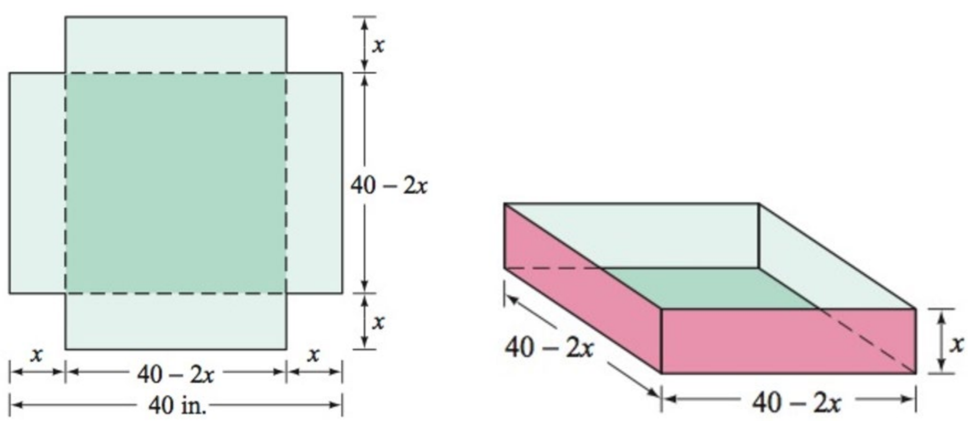
103. $f(x)=2x^2-x-3$

104. $f(x)=x^2-2x+5$

105. $f(x)=3x^2-2x+5$

106. $f(x)=-2x^2-3x+7$

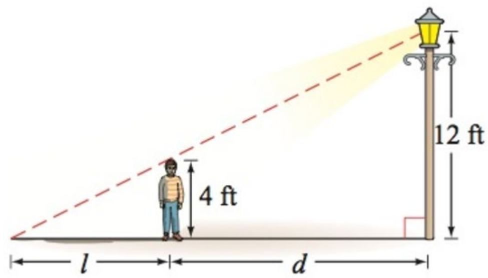
107. An open box is to be made from a square piece of cardboard that measures 40 *inches* on each side, to construct the box, squares that measure x *inches* on each side are cut from each corner of the cardboard.



a) Express the volume V of the box as a function of x.

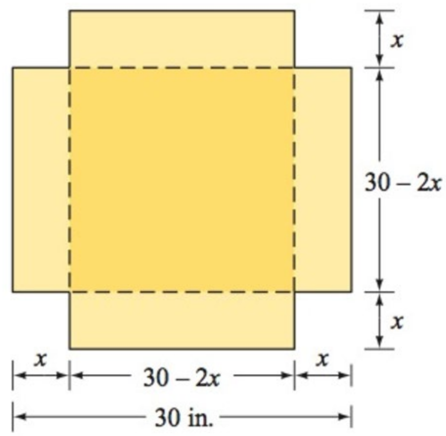
b) Determine the domain of V.

108. A child 4 *feet* tall is standing near a street lamp that is 12 *feet* high. The light from the lamp casts a shadow.



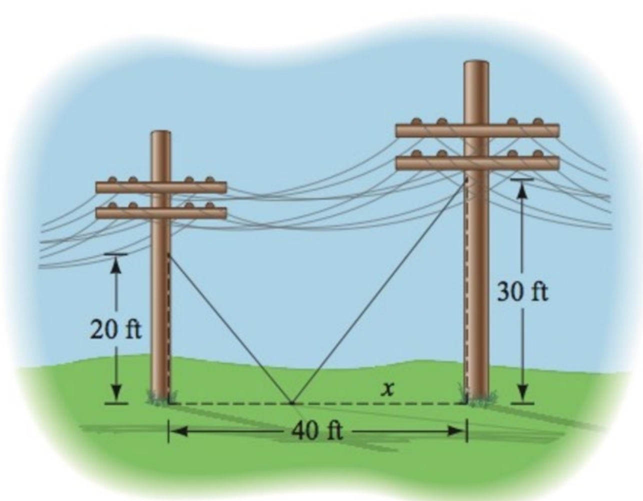
a) Find the length l of the shadow as a function of the distance d of the child from the lamppost.
b) What is the domain of the function?
c) What is the length of the shadow when the child is 8 *feet* from the base of the lamppost?

109. An open box is to be made from a square piece of cardboard with the dimensions 30 *inches* by 30 *inches* by cutting out squares of area x^2 from each corner.



a) Express the volume V of the box as a function of x.
b) Determine the domain of V.

110. Two guy wires are attached to utility poles that are 40 *feet* apart.

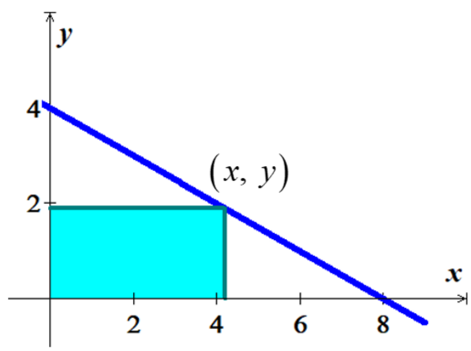


a) Find the total length of the two guy wires as a function of x.
b) What is the domain of this function?

111. A rancher has 360 *yard*s. of fencing with which to enclose two adjacent rectangular corrals, one for sheep and one for cattle. A river forms one side of the corrals. Suppose the width of each corral is x *yards*.

a) Express the total area of the two corrals as a function of x.
b) Find the domain of the function.

112. A rectangle is bounded by the x- and *y-axis* of $y=-\frac{1}{2}x+4$



a) Find the area of the rectangle as a function of x.
b) What is the domain of this function.