Sec. 5.7

1) Prove: $4+8+12+\cdots+4n = 2n(n+1)$

For $n=1 \Rightarrow 4 \stackrel{?}{=} 2(1)(2)$

$4 = 4$ ✓ P_1 is true.

Assume P_k is true: $4+8+\cdots+4k = 2k(k+1)$

is P_{k+1} $4+\cdots+4k+4(k+1) \stackrel{?}{=} 2(k+1)(k+2)$

$4+\cdots+4k+4(k+1) = 2k(k+1)+4(k+1)$

$= 2(k+1)(k+2)$ ✓

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the proof is completed

2) $1+5+9+\cdots+(4n-3) = n(2n-1)$

For $n=1 \Rightarrow 1 \stackrel{?}{=} 1(2-1)$

$1=1$ ✓ P_1 is true

Assume P_k is true: $1+5+\cdots+(4k-3) = k(2k-1)$

Is P_{k+1} also true: $1+\cdots+(4k-3)+[4(k+1)-3] = (k+1)(2k+1)$

$1+\cdots+(4k-3)+(4k+1) = k(2k-1)+(4k+1)$

$= 2k^2-k+4k+1$

$= 2k^2+3k+1$

$= (k+1)(2k+1)$ ✓

P_{k+1} is also true

$\therefore$ By the mathematical induction, the proof is completed.

3/ $2+4+\cdots+2^n = 2(2^n-1)$

For $n=1 \Rightarrow 2 = 2(2^1-1)$

$2 = 2$ ✓ P_1 is true.

Assume P_k is true, $2+\cdots+2^k = 2(2^k-1)$

Is P_{k+1}: $2+\cdots+2^k+2^{k+1} = 2(2^{k+1}-1)$ true?

$$\begin{aligned}2+\cdots+2^k+2^{k+1} &= 2(2^k-1)+2\cdot2^k \\ &= 2(2^k-1+2^k) \\ &= 2(2\cdot2^k-1) \\ &= 2(2^{k+1}-1)\ ✓\end{aligned}$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the proof is completed.

4/ $1^4+2^4+3^4+\cdots+n^4 = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$

For $n=1 \Rightarrow 1^4 = \frac{1}{30}(1)(2)(3)(5)$

$1 = 1$ ✓ P_1 is true.

Assume P_k is true: $1^4+\cdots+k^4 = \frac{1}{30}k(k+1)(2k+1)(3k^2+3k-1)$

Is P_{k+1}: $1^4+\cdots+k^4+(k+1)^4 = \frac{1}{30}(k+1)(k+2)(2k+3)(3(k+1)(k+2)-1)$

$$\begin{aligned}1^4+\cdots+k^4+(k+1)^4 &= \frac{1}{30}k(k+1)(2k+1)(3k^2+3k-1)+(k+1)^4 \\ &= \frac{(k+1)}{30}\left[k(2k+1)(3k^2+3k-1)+30(k+1)^3\right] \\ &= \frac{1}{30}(k+1)(6k^4+6k^3-2k^2+3k^3+3k^2-k+30k^3+90k^2+90k+30) \\ &= \frac{1}{30}(k+1)(6k^4+39k^3+91k^2+89k+30) \\ &= \frac{1}{30}(k+1)(k+2)(6k^3+27k^2+37k+15) \\ &= \frac{1}{30}(k+1)(k+2)(2k+3)(3k^2+9k+5)\ ✓\end{aligned}$$

$$\begin{array}{r|rrrrr} -2 & 6 & 39 & 91 & 89 & 30 \\ & & -12 & -54 & -74 & -30 \\ \hline & 6 & 27 & 37 & 15 & \boxed{0} \end{array}$$

P_{k+1} is also true

$\therefore$ By the mathematical method, proof is completed.