#1 $\lim_{x \to 4} (x^2 - 4x + 1) = 16 - 16 + 1 = \underline{1}$

#2 $\lim_{x \to 1} \frac{x+3}{x+6} = \frac{4}{7}$

#3 $\lim_{x \to 1} \frac{x^2-1}{x+1} = \frac{0}{2} = 0$

#4 $\lim_{x \to 3} \frac{x^2-6x+9}{x^2-9} = \frac{9-18+9}{9-9} = \frac{0}{0}$

$$= \lim_{x \to 3} \frac{(x-3)(x-3)}{(x-3)(x+3)}$$

$$= \lim_{x \to 3} \frac{x-3}{x+3}$$

$$= \frac{0}{6}$$

$$= 0$$

#5 $\lim_{x \to 2} \frac{1}{4-x^2} = \frac{1}{0} = \infty$

#6 $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \frac{0}{0}$

$$\lim_{x \to 9} \frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$$

$$\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x \to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)}$$

$$= \lim_{x \to 9} \frac{1}{\sqrt{x}+3}$$

$$= \frac{1}{6}$$

7/ $\lim\limits_{x \to \pi} \dfrac{(x-\pi)^2}{\pi x} = \dfrac{0}{\pi^2} = 0$

8/ $\lim\limits_{x \to 2} \dfrac{\sqrt{4-4x+x^2}}{x-2} = \dfrac{\sqrt{4-8+4}}{0} = \dfrac{0}{0}$

$= \lim\limits_{x \to 2} \dfrac{\sqrt{(x-2)^2}}{x-2}$

$= \lim\limits_{x \to 2} \dfrac{x-2}{x-2}$

$= 1$

9/ $\lim\limits_{x \to 2} \dfrac{(x-2)^2}{x^2-4} = \dfrac{0}{0}$

$= \lim\limits_{x \to 2} \dfrac{(x-2)^2}{(x-2)(x+2)}$

$= \lim\limits_{x \to 2} \dfrac{x-2}{x+2}$

$= \dfrac{0}{4}$

$= 0$

10/ $\lim\limits_{x \to 3} \dfrac{x-3}{x^2-9} = \dfrac{0}{0}$

$= \lim\limits_{x \to 3} \dfrac{x-3}{(x-3)(x+3)}$

$= \lim\limits_{x \to 3} \dfrac{1}{x+3}$

$= \dfrac{1}{6}$

11/ $\lim\limits_{x \to \frac{2\pi}{3}} \sin x = \sin \dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$

12/ $\lim\limits_{x \to \frac{5\pi}{4}} \cos x = \cos \dfrac{5\pi}{4} = -\dfrac{\sqrt{2}}{2}$

13/ $\lim_{x\to 0} \frac{\sin 2\pi x}{\sin 3\pi x} = \frac{0}{0}$

$= \frac{2}{3}$

$= \lim_{x\to 0} \frac{\sin 2\pi x}{2\pi x} \cdot \frac{1}{\frac{\sin 3\pi x}{3\pi x}} \cdot \frac{2\pi x}{3\pi x}$

$= \frac{2}{3} \lim_{2\pi x\to 0} \frac{\sin 2\pi x}{2\pi x} \cdot \frac{1}{\lim_{3\pi x\to 0} \frac{\sin 3\pi x}{3\pi x}}$

$= \frac{2}{3}(1)(1)$

$= \frac{2}{3}$

14/ $\lim_{x\to 0^+} \frac{x-\sqrt{x}}{\sqrt{\sin x}} = \frac{0}{0}$

$\sqrt{x} \cdot \frac{\sqrt{\sin x}}{\sqrt{x}}$

$= \lim_{x\to 0^+} \frac{x-\sqrt{x}}{\sqrt{x}\sqrt{\frac{\sin x}{x}}}$

$= \lim_{x\to 0^+} \frac{x-\sqrt{x}}{\sqrt{x}} \frac{1}{\sqrt{\frac{\sin x}{x}}} \to 1$

$= \lim_{x\to 0^+} \left(\frac{x}{\sqrt{x}} - \frac{\sqrt{x}}{\sqrt{x}}\right)$

$= \lim_{x\to 0^+} (\sqrt{x} - 1)$

$= -1$

$\lim_{x\to 0^+} \frac{x-\sqrt{x}}{\sqrt{\sin x}} = \lim_{x\to 0} \frac{\sqrt{x}-1}{\frac{\sqrt{\sin x}}{\sqrt{x}}} = -1$

#15/ $\lim_{x\to 0^+} \frac{\sin\sqrt{1-x}}{\sqrt{1-x^2}} = \sin 1$

16/ $\lim_{x\to 0} e^{x^2} = e^0 = 1$

17/ $\lim_{x\to 1} e^{x^2-1} = e^0 = 1$

18/ $\lim_{x \to 1} \ln x = \ln 1 = 0$

19/ $\lim_{x \to 2} (e^x - \ln x) = \underline{e^2 - \ln 2}$

20/ $\lim_{x \to 1} \frac{1}{\ln x} = \frac{1}{0} = \underline{\infty}$

1.4/ $x \to +/- \infty$ $\begin{cases} x \to \infty \\ x \to -\infty \end{cases}$

Horizontal Asymptotes (HA)

$$f(x) = \frac{\boxed{a_n x^n} + a_{n-1} x^{n-1} + --- + a_0}{\boxed{b_m x^m} + b_{m-1} x^{m-1} + --- + b_0}$$

1- $n < m \Rightarrow HA: y = 0$

$\lim_{x \to \infty} f(x) = 0$

Ex $y = \frac{2x+1}{4x^2+5} \rightarrow HA: y = 0$

$$\begin{cases} \lim_{x \to \infty} \frac{2x+1}{4x^2+5} = \lim_{x \to \infty} \frac{2x}{4x^2} \\ = \lim_{x \to \infty} \frac{1}{2x} \\ = \frac{1}{\infty} = \underline{0} \end{cases}$$

$\lim_{x \to \infty} \frac{2x+1}{4x^2+5} = \underline{0}$

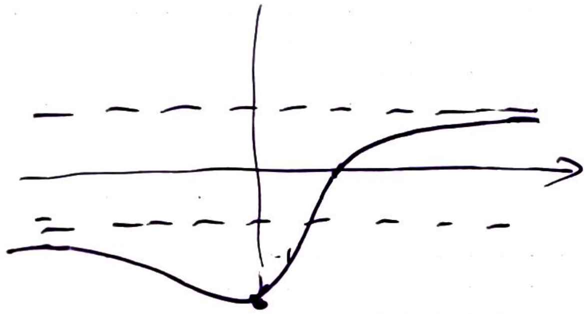
2- $n = m \Rightarrow HA: y = \frac{a_n}{b_m}$

$\lim_{x \to \infty} \frac{2x^2+1}{4x^2+5} = \frac{1}{2}$

3- $n > m \Rightarrow$ No HA $\lim_{x \to \infty} \frac{2x^4+1}{4x^2+5} = \infty$

Ex $\lim_{x\to\infty} \frac{x^3-2}{|x|^3+1} = \lim_{x\to\infty} \frac{x^3}{x^3} = 1$

$\lim_{x\to-\infty} \frac{x^3-2}{|x|^3+1} = \lim_{x\to-\infty} \frac{x^3}{-x^3} = -1$



$\lim_{x\to\infty} \sin\frac{1}{x} = \sin 0 = 0$

$x \to \pm\infty$

$\frac{1}{x} \to$

Ex $\lim_{x\to\pm\infty} x \sin\frac{1}{x} = \lim_{\frac{1}{x}\to 0} \frac{\sin\frac{1}{x}}{1/x}$

$= 1$

Ex $\lim_{x\to\pm\infty} \left(2 + \frac{\sin x}{x}\right) = 2 + \lim_{x\to\pm\infty} \frac{\sin x}{x}$

$\frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$ $\quad \lim_{x\to\pm\infty} \pm\frac{1}{x} = 0$

$\therefore \lim_{x\to\pm\infty} \frac{\sin x}{x} = 0$

$\lim_{x\to\pm\infty} \left(2 + \frac{\sin x}{x}\right) = 2$

Ex $\lim_{x\to\infty} \left(x - \sqrt{x^2+16}\right) = \infty - \infty$

$$\lim_{x\to\infty} \left(x-\sqrt{x^2+16}\right)\frac{x+\sqrt{x^2+16}}{x+\sqrt{x^2+16}}$$

$$= \lim_{x\to\infty} \frac{x^2-(x^2+16)}{x+\sqrt{x^2+16}}$$

$$= \lim_{x\to\infty}\left(-\frac{16}{x+\sqrt{x^2+16}}\right)$$

$$= \frac{-16}{\infty}$$

$$= 0$$

#24 $\lim_{x\to\infty}\left(\sqrt{x^2+3x} - \sqrt{x^2-2x}\right) = \infty - \infty$

$$= \lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-2x}\right)\frac{\sqrt{x^2+3x}+\sqrt{x^2-2x}}{\sqrt{x^2+3x}+\sqrt{x^2-2x}}$$

$$= \lim_{x\to\infty}\frac{x^2+3x-x^2+2x}{\sqrt{x^2+3x}+\sqrt{x^2-2x}}$$

$$= \lim_{x\to\infty}\frac{5x}{\underbrace{\sqrt{x^2}}_{x}+3x+\underbrace{\sqrt{x^2}}_{x}-2x}$$

$$= \lim_{x\to\infty}\frac{5x}{x+x}$$

$$= \lim_{x\to\infty}\frac{5x}{2x}$$

$$= \frac{5}{2}$$

$\infty + \infty = \infty$

$\infty - \infty = ?$

$$\lim_{x \to 2^+} \frac{x-3}{x^2-4} = \frac{-1}{0^+} = -\infty$$

$$\lim_{x \to 2^-} \frac{x-3}{x^2-4} = \frac{-1}{0^-} = \infty$$

$$\lim_{x \to 2} \frac{x-3}{x^2-4} = \boxed{\frac{-1}{0} = -\infty} \quad \nexists$$

#46 $$\lim_{x \to -\infty} \frac{\sqrt{16x^{\cancel{4}} + 64x^2} + x^2}{2x^2-4}$$ copy

$$= \lim_{x \to -\infty} \frac{4x^2 + x^2}{2x^2}$$

$$= \lim_{x \to -\infty} \frac{5x^2}{2x^2}$$

$$= \frac{5}{2}$$

58 $$\lim_{x \to \infty} \frac{2e^{x} + 10e^{-x}}{e^{x} + e^{-x}} = 2 = \lim_{x \to \infty} \frac{2e^x}{e^x}$$

1.5 Continuity.

$f(x) = \frac{1}{x}$ Domain $x \neq 0$

$\{x \mid x \neq 0\}$

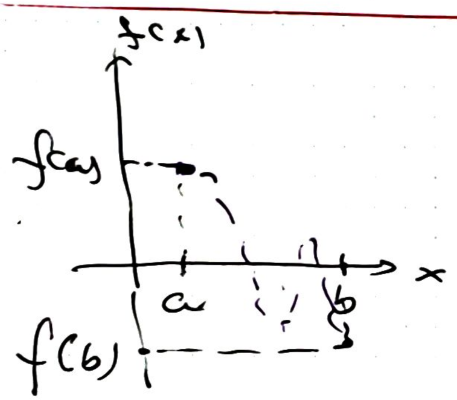
$\mathbb{R} - \{0\}$

$f(x)$ is continuous everywhere except $x = 0$

$$f(x) = \frac{ax}{(x-2)(x+3)}$$

$f(x)$ is continuous everywhere except $x = 2, -3$

$x \in \mathbb{R} - \{2, -3\}$



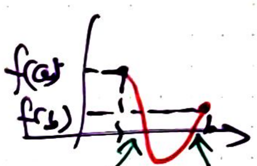
Intermediate Value Theorem.

$[a, b]$

if $f(a)$ & $f(b)$ are opposite signs $\Rightarrow$

$f(x)$ has a real value (zero) at least one

otherwise we can't be determined.



#16 $x^3 - 15x + 1 = 0$ $[-4, 4]$ 3 soln

-4	$-$
-3	$+$
-2	$+$
-1	$+$
0	$+$
1	$-$
2	$-$
3	$-$
4	$+$

$(-4, -3)$,

$(0, 1)$

$(3, 4)$