Series

$$\sum_{k=1}^{n} c = nc \qquad \sum_{k=m}^{n} c = (n-m+1)c$$

Arithmetic

$$\sum_{k=1}^{n} k = 1+2+3+\cdots+n = \frac{1}{2}n(n+1) \qquad \sum_{k=1}^{n} (2k-1) = 1+3+5+\cdots+(2n-1) = n^2$$

Geometric

$$\sum_{k=1}^{n} ar^k = a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a\left(1-r^n\right)}{1-r}$$

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r} \quad \textit{if } -1 < \boldsymbol{r} < 1$$

$$\sum_{k=1}^{\infty} ar^{-k} = \frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \ldots = \frac{a}{r-1}$$

$$\sum_{k=1}^{n} r^k = r + r^2 + r^3 + \ldots + r^n = \frac{r\left(1-r^n\right)}{1-r} \qquad \sum_{k=1}^{\infty} r^k = r + r^2 + r^3 + \ldots = \frac{r}{1-r}$$

$$\sum_{k=0}^{n} (a+kd)r^k = a + (a+d)r + (a+2d)r^2 + \ldots + \left(a+(n-1)d\right)r^{n-1}$$

$$= \frac{a\left(1-r^n\right)}{1-r} + \frac{rd\left[1 - nr^{n-1} + (n-1)r^n\right]}{(1-r)^2}$$

$$\sum_{k=0}^{\infty} (a+kd)r^k = a + (a+d)r + (a+2d)r^2 + \ldots = \frac{a}{1-r} + \frac{rd}{(1-r)^2} \quad \textit{if } -1 < r < 1$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4} = (1+2+3+\cdots+n)^2$$

$$\sum_{k=1}^{n} k^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)\left(3n^2+3n-1\right)}{30}$$

$$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots$$

$$\cos^{-1}x=\frac{\pi}{2}-x-\frac{1}{6}x^3-\frac{3}{40}x^5-\frac{5}{112}x^7 \qquad -1\le x\le 1$$

$$\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$$

$$\sin^{-1}x=x+\frac{1}{6}x^3+\frac{3}{40}x^5+\frac{5}{112}x^7+\frac{35}{1152}x^9+\cdots$$

$$\tan x=x+\frac{1}{3}x^3+\frac{2}{15}x^5+\frac{17}{315}x^7+\frac{62}{2835}x^9+\cdots$$

$$\tan^{-1}(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}, \quad |x|\le 1$$

$$\cot x=x^{-1}-\frac{1}{3}x-\frac{1}{45}x^3-\frac{2}{945}x^5-\frac{1}{4{,}725}x^7-\cdots$$

$$\cot^{-1}x=\frac{\pi}{2}+\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2n+1}x^{2n+1}=\frac{\pi}{2}-x+\frac{1}{3}x^3-\frac{1}{5}x^5+\frac{1}{7}x^7-\frac{1}{9}x^9+\cdots$$

$$\cot^{-1}\left(\frac{1}{x}\right)=\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}=x-\frac{1}{3}x^3+\frac{1}{5}x^5-\frac{1}{7}x^7+\frac{1}{9}x^9+\cdots$$

$$\sec x=1+\frac{1}{2}x^2+\frac{5}{25}x^4+\frac{61}{720}x^6+\frac{277}{8{,}064}x^8+\cdots$$

$$\csc^{-1}x=\sum_{n=0}^{\infty}\frac{(2n)!}{n!n!}\,\frac{1}{4^n(2n+1)}\,x^{-(2n+1)}=x^{-1}+\frac{1}{6}x+\frac{7}{360}x^3+\frac{31}{15{,}120}x^5+\cdots$$

$$\cosh x=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$

$$\cosh^{-1}(x+1)=\sqrt{2x}\left(1-\frac{1}{12}x+\frac{3}{160}x^2-\frac{5}{896}x^3+\cdots\right)$$

$$\sinh x=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$$

$$\sinh x = x - \frac{1}{6}x^3 + \frac{3}{40}x^5 - \frac{5}{112}x^7 + \frac{35}{1{,}152}x^9 - \cdots$$

$$\tanh x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2{,}835}x^9 - \cdots$$

$$\tanh^{-1} x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)} = x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \frac{1}{9}x^9 + \cdots$$

$$\coth x = x^{-1} + \frac{1}{3}x - \frac{1}{45}x^3 + \frac{2}{945}x^5 - \frac{1}{4{,}725}x^7 + \cdots$$

$$\coth^{-1}(1+x) = \frac{1}{2}\ln 2 - \frac{1}{2}\ln x + \frac{1}{4}x - \frac{1}{16}x^2 + \cdots$$

$$\operatorname{sech} x = 1 - \frac{1}{2}x^2 + \frac{5}{24}x^4 - \frac{61}{720}x^6 + \frac{277}{8{,}064}x^8 - \cdots$$

$$\operatorname{sech}^{-1} x = \ln 2 - \ln x - \frac{1}{4}x^2 - \frac{3}{32}x^4 - \cdots$$

$$\operatorname{csch} x = x^{-1} - \frac{1}{6}x + \frac{7}{360}x^3 + \frac{31}{15{,}120}x^5 + \cdots$$

$$\operatorname{csch}^{-1} x = \ln 2 - \ln x + \frac{1}{4}x^2 - \frac{3}{32}x^4 + \frac{5}{96}x^6 - \cdots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\ln x = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}(x-1)^{n+1} = x - 1 - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots \qquad 0 < x \le 2$$

$$\ln x = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}(x-1)^{n+1} = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \cdots\right] \qquad x > 2$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}x^{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots \qquad -1 \le x < 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \cdots \qquad |x| < 1$$

$$(1+x)^{\alpha} = 1 + \alpha x + \alpha(\alpha-1)\frac{x^2}{2!} + + \alpha(\alpha-1)(\alpha-2)\frac{x^3}{3!} + \cdots$$

Legendre Polynomial

$$\left(1-x^2\right) y'' - 2xy' + n(n+1)y = 0 \qquad P_n(x) = \frac{1}{2^n} \sum_{k=0}^{n/2} \frac{(-1)^k (2n-2k)!}{k!(n-k)!(n-2k)!} x^{n-2k}$$

The nth term of an arithmetic sequence: $a_n = a_1 + (n-1)d$

$$n = \frac{a_n - a_1}{d} + 1$$

Sum of a certain number of terms of an arithmetic sequence $S_n = \frac{n}{2}\left(a_1 + a_n\right)$

The ***arithmetic mean*** of two numbers a and b is defined as $\frac{a+b}{2}$

The nth term of a geometric sequence: $a_n = a_1 r^{n-1}$ $\qquad r = \frac{a_n}{a_{n-1}}$

Sum of nth term of a geometric sequence: $S_n = a_1 \frac{1-r^n}{1-r}$

Sum of infinite term of a geometric sequence: $S_n = \frac{a_1}{1-r}$