Solution *Section* 2.5 – Polynomial Functions

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^3 + 7x^2 - x + 9$

Solution

Leading term: $5x^3$ with 3^{rd} degree (n is ***odd***)

$x \to -\infty \;\Rightarrow\; f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \;\Rightarrow\; f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 11x^3 - 6x^2 + x + 3$

Solution

Leading term: $11x^3$ with 3^{rd} degree (n is ***odd***)

$x \to -\infty \;\Rightarrow\; f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \;\Rightarrow\; f(x) \to \infty$ $\qquad$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -11x^3 - 6x^2 + x + 3$

Solution

Leading term: $-11x^3$ with 3^{rd} degree (n is ***odd***)

$x \to -\infty \;\Rightarrow\; f(x) \to \infty$ $\qquad f(x)$ rises left

$x \to \infty \;\Rightarrow\; f(x) \to -\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 2x^3 + 3x^2 - 23x - 42$

Solution

Leading term: $2x^3$ with 3^{rd} degree (n is ***odd***)

$x \to -\infty \;\Rightarrow\; f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \;\Rightarrow\; f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=5x^4+7x^2-x+9$

Solution

Leading term: $5x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \Rightarrow f(x)\to\infty$ $\quad f(x)$ rises left

$x\to\infty \Rightarrow f(x)\to\infty$ $\quad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=11x^4-6x^2+x+3$

Solution

Leading term: $11x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \Rightarrow f(x)\to\infty$ $\quad f(x)$ rises left

$x\to\infty \Rightarrow f(x)\to\infty$ $\quad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-5x^4+7x^2-x+9$

Solution

Leading term: $-5x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \Rightarrow f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \Rightarrow f(x)\to-\infty$ $\quad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-11x^4-6x^2+x+3$

Solution

Leading term: $-11x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \Rightarrow f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \Rightarrow f(x)\to-\infty$ $\quad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^5 - 16x^2 - 20x + 64$

Solution

Leading term: $5x^5$ with 5^{th} degree (n is ***odd***)

$x \to -\infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls left

$x \to \infty \Rightarrow f(x) \to \infty$ $f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -5x^5 - 16x^2 - 20x + 64$

Solution

Leading term: $-5x^5$ with 5^{th} degree (n is ***odd***)

$x \to -\infty \Rightarrow f(x) \to \infty$ $f(x)$ rises left

$x \to \infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -3x^6 - 16x^3 + 64$

Solution

Leading term: $-3x^6$ with 6^{th} degree (n is ***even***)

$x \to -\infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls left

$x \to \infty \Rightarrow f(x) \to -\infty$ $f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 3x^6 - 16x^3 + 4$

Solution

Leading term: $3x^6$ with 6^{th} degree (n is ***even***)

$x \to -\infty \Rightarrow f(x) \to \infty$ $f(x)$ rises left

$x \to \infty \Rightarrow f(x) \to \infty$ $f(x)$ rises right

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - x - 1; \quad between \ \ 1 \ and \ 2$

Solution

$$f(1) = (1)^3 - (1) - 1$$
$$= -1$$
$$f(2) = (2)^3 - (2) - 1$$
$$= 5$$

Since $f(1) \ and \ f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - 4x^2 + 2; \quad between \ \ 0 \ and \ 1$

Solution

$$f(0) = (0)^3 - 4(0)^2 + 2$$
$$= 2$$
$$f(1) = (1)^3 - 4(1)^2 + 2$$
$$= -1$$

Since $f(0) \ and \ f(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 2x^4 - 4x^2 + 1; \quad between \ \ -1 \ and \ 0$

Solution

$$f(-1) = 2(-1)^4 - 4(-1)^2 + 1$$
$$= -1$$
$$f(0) = 2(0)^4 - 4(0)^2 + 1$$
$$= 1$$

Since $f(0)$ *and* $f(-1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between -1 and 0.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^4+6x^3-18x^2;\quad between\ \ 2\ and\ 3$

Solution

$$f(2)=(2)^4+6(2)^3-18(2)^2$$
$$\underline{=-8}$$

$$f(3)=(3)^4+6(3)^3-18(3)^2$$
$$\underline{=81}$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^3+x^2-2x+1;\quad between\ \ -3\ and\ -2$

Solution

$$f(-3)=(-3)^3+(-3)^2-2(-3)+1$$
$$\underline{=-11}$$

$$f(-2)=(-2)^3+(-2)^2-2(-2)+1$$
$$\underline{=1}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between -2 and -3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^5-x^3-1;\quad between\ \ 1\ and\ 2$

Solution

$$f(1) = (1)^5 - (1)^3 - 1$$
$$= -1$$

$$f(2) = (2)^5 - (2)^3 - 1$$
$$= 23$$

Since $f(1)$ *and* $f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 10x + 9;$ *between* -3 *and* -2

Solution

$$f(-3) = 3(-3)^3 - 10(-3) + 9$$
$$= -42$$

$$f(-2) = 3(-2)^3 - 10(-2) + 9$$
$$= 5$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between –3 and –2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 3x^3 - 8x^2 + x + 2;$ *between* 2 *and* 3

Solution

$$f(2) = 3(2)^3 - 8(2)^2 + (2) + 2$$
$$= -4$$

$$f(3) = 3(3)^3 - 8(3)^2 + (3) + 2$$
$$= 14$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-8x^2+x+2;\quad between\ \ 1\ and\ \ 2$

Solution

$$f(1)=3(1)^3-8(1)^2+(1)+2$$
$$\underline{=-2}$$

$$f(2)=3(2)^3-8(2)^2+(2)+2$$
$$\underline{=-4}$$

Since $f(1)\ and\ f(2)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^5+2x^4-6x^3+2x-3;\quad between\ \ 0\ and\ \ 1$

Solution

$$f(0)=(0)^5+2(0)^4-6(0)^3+2(0)-3$$
$$\underline{=-3}$$

$$f(1)=(1)^5+2(1)^4-6(1)^3+2(1)-3$$
$$\underline{=-4}$$

Since $f(0)\ and\ f(1)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=2x^3+3x^2-23x-42,\quad a=3,\quad b=4$

Solution

$$P(3)=54+27-69-42$$
$$\underline{=-30}$$

$$P(4)=128+48-92-42$$
$$\underline{=90}$$

Since $P(3)$ *and* $P(4)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and 4.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 4x^3 - x^2 - 6x + 1, \quad a = 0, \quad b = 1$

Solution

$P(0) \underline{=1}$

$P(1) = 4 - 1 - 6 + 1$

$\underline{= -2}$

Since $P(0)$ *and* $P(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 3x^3 + 7x^2 + 3x + 7, \quad a = -3, \quad b = -2$

Solution

$P(-3) = -81 + 63 - 9 + 7$

$\underline{= -20}$

$P(-2) = -24 + 28 - 6 + 7$

$\underline{= 5}$

Since $P(-3)$ *and* $P(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between −3 and −2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 2x^3 - 21x^2 - 2x + 25, \quad a = 1, \quad b = 2$

Solution

$P(1) = 2 - 21 - 2 + 25$

$\underline{= 4}$

$P(2) = 16 - 84 - 4 + 25$

$\underline{= -47}\ |$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 4x^4 + 7x^3 - 11x^2 + 7x - 15, \quad a = 1, \quad b = \frac{3}{2}$

Solution

$P(1) = 4 + 7 - 11 + 7 - 15$

$\underline{= -8}\ |$

$$P\left(\frac{3}{2}\right) = 81 + \frac{189}{8} - \frac{99}{4} + \frac{21}{2} - 15$$

$$= 66 + \frac{189 - 198 + 84}{8}$$

$$= 66 + \frac{75}{8}$$

$$\underline{= \frac{603}{8}}\ |$$

Since $P(1)$ *and* $P\left(\frac{3}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and $\frac{3}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 5x^3 - 16x^2 - 20x + 64, \quad a = 3, \quad b = \frac{7}{2}$

Solution

$P(3) = 135 - 144 - 60 + 64$

$\underline{= -5}\ |$

$$P\left(\frac{7}{2}\right) = \frac{1715}{8} - 196 - 70 + 64$$

$$= \frac{1715}{8} - 202$$

$$\underline{= \frac{99}{8}}\ |$$

Since $P(3)$ *and* $P\left(\frac{7}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and $\frac{7}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^4 - x^2 - x - 4, \quad a = 1, \quad b = 2$

Solution

$P(1) = 1 - 1 - 1 - 4$

$\quad = -5$

$P(2) = 16 - 4 - 2 - 4$

$\quad = 6$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^3 - x - 8, \quad a = 2, \quad b = 3$

Solution

$P(2) = 8 - 2 - 8$

$\quad = -2$

$P(3) = 27 - 3 - 8$

$\quad = 16$

Since $P(2)$ *and* $P(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^3 - x - 8, \quad a = 0, \quad b = 1$

Solution

$P(0) = -8$

$P(1) = 1 - 1 - 8$

$= -8$

Since $P(0)$ *and* $P(1)$ have same sign.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^3 - x - 8, \quad a = 2.1, \quad b = 2.2$

Solution

$$P(2.1) = P\left(\frac{21}{10}\right)$$

$$= \frac{9261}{1000} - \frac{21}{10} - 8$$

$$= \frac{9261 - 2100 - 8000}{1000}$$

$$= -\frac{839}{1,000}$$

$$P(2.2) = P\left(\frac{2.2}{10}\right)$$

$$= \frac{10,648}{1000} - \frac{22}{10} - 8$$

$$= \frac{10,648 - 2,200 - 8,000}{1000}$$

$$= \frac{448}{1,000}$$

$$= 0.448$$

Since $P(2.1)$ *and* $P(2.2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2.1 and 2.2.