Lecture Two ☺

$\vec{v}$ $\quad$ $\vec{v}$ $\quad$ $\rightarrow$ $\quad$ $\rightarrow$

vector

$\overrightarrow{AB}$

v : letter

$$\vec{v} = (v_1, v_2, \dots, v_n) = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$\vec{v}, \vec{w}$ $\qquad$ $\vec{v} + \vec{w} = \vec{w} + \vec{v}$

$$\vec{v} = (v_1, v_2, \dots, v_n)$$
$$\vec{w} = (w_1, w_2, \dots, w_n)$$

$$\begin{aligned}
\vec{v} + \vec{w} &= (v_1, v_2, \dots, v_n) + (w_1, w_2, \dots, w_n) \\
&= (v_1 + w_1, v_2 + w_2, \dots, v_n + w_n) \\
&= (w_1 + v_1, w_2 + v_2, \dots, w_n + v_n) \\
&= (w_1, w_2, \dots, w_n) + (v_1, v_2, \dots, v_n) \\
&= \vec{w} + \vec{v} \quad ✓
\end{aligned}$$

$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$?

let: $\vec{u} = (u_1, u_2, \dots, u_n)$

$\vec{v} = (v_1, v_2, \dots, v_n)$

$\vec{w} = (w_1, w_2, \dots, w_n)$

$$(\vec{u} + \vec{v}) + \vec{w} = [(u_1, u_2, \dots, u_n) + (v_1, v_2, \dots, v_n)] + (w_1, w_2, \dots, w_n)$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) + (w_1, w_2, \dots, w_n)$$

$$= ((u_1 + v_1) + w_1, (u_2 + v_2) + w_2, \dots, (u_n + v_n) + w_n)$$

$$= (u_1 + (v_1 + w_1), u_2 + (v_2 + w_2), \dots, u_n + (v_n + w_n))$$

$$= (u_1, u_2, \dots, u_n) + (v_1 + w_1, v_2 + w_2, \dots, v_n + w_n)$$

$$= \vec{u} + ((v_1, v_2, \dots, v_n) + (w_1, w_2, \dots, w_n))$$

$$= \vec{u} + (\vec{v} + \vec{w})$$ ✓

2.2 Norm, Dot Product & dist R^n

Norm vector (length, magnitude)

$$\text{length} = \|\vec{v}\|$$
$$= \sqrt{\vec{v} \cdot \vec{v}}$$
$$= \sqrt{x^2 + y^2} \quad \text{2-dim}$$
$$= \sqrt{x^2 + y^2 + z^2} \quad \text{3-dim}$$
$$= \sqrt{v_1^2 + v_2^2 + \dots + v_n^2} \quad \text{n-dim}$$

$v(x, y)\ (x, y)$

Ex $\vec{v} = (1, 2, 3)$

$$\|\vec{v}\| = \sqrt{1 + 4 + 9}$$
$$= \sqrt{14}$$

Theorem $\vec{v}$ R^n k any scalar

a) $\|\vec{v}\| \geq 0$

b) $\|\vec{v}\| = 0$ iff $\vec{v} = \vec{0}$

c) $\|k\vec{v}\| = |k|\,\|\vec{v}\|$

Unit vectors $\|\vec{v}\| = 1$

direction

$\vec{v} \cdot \vec{v} = 1$

unit vectors $= \vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|}$

Ex $\vec{v} = (2, 2, 1)$

$\|\vec{v}\| = \sqrt{4+4+1}$
$= 3$

unit vector: $\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|}$

$= \dfrac{(2,2,1)}{3}$

$= \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

$\vec{v} = (2, 2, 1)$

$\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|}$

$= \dfrac{(2,2,1)}{\sqrt{4+4+1}}$

$= \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$