#13

$$y = \frac{\sin x - 3x}{x} = \frac{\sin x}{x} - 3$$

$$y' = \frac{x(\cos x - 3) - \sin x + 3x}{x^2}$$

$$= \frac{x\cos x - 3x - \sin x + 3x}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

$$\boxed{y' = \frac{x\cos x - \sin x}{x^2}}$$

#14 $f(\theta) = \dfrac{\sin\theta}{1-\cos\theta}$

$$f'(\theta) = \frac{\cos\theta(1-\cos\theta) - \sin^2\theta}{(1-\cos\theta)^2}$$

$$= \frac{\cos\theta - (\cos^2\theta + \sin^2\theta)}{(1-\cos\theta)^2}$$

$$= \frac{\cos\theta - 1}{(\cos\theta - 1)^2}$$

$$= \boxed{\frac{1}{\cos\theta - 1}}$$

#16. $y = \dfrac{3(1-\sin x)}{2\cos x}$

$$y' = \frac{3}{2}\,\frac{-\cos^2 x + \sin x(1-\sin x)}{\cos^2 x}$$

$$= \frac{3}{2}\,\frac{-\cos^2 x + \sin x - \sin^2 x}{\cos^2 x}$$

$$= \frac{3}{2}\,\frac{\sin x - 1}{1-\sin^2 x}$$

$$= -\frac{3}{2}\cdot\frac{1-\sin x}{(1-\sin x)(1+\sin x)}$$

$$= \boxed{-\frac{3}{2}\,\frac{1}{1+\sin x}}$$

2.7 Implicit

$x^3 + y^3 - 9xy = 0 \quad , \quad x^2 + y^2 = 25$

$\frac{dy}{dx} \quad , \quad \frac{dx}{dy}$

Ex $y^2 = x \qquad \frac{dy}{dx}\,?$

$2y \frac{d}{dx} y = 1 \qquad\qquad \frac{dx}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{1}{2y}}$$

Ex $x^2 + y^2 = 25 \qquad \frac{dy}{dx}\,? = y'$

$2x + 2yy' = 0$

$2yy' = -2x$

$y' = -\frac{2x}{2y}$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

Ex $y^2 = x^2 + \sin xy \qquad \frac{dy}{dx}\,? \qquad (\sin u)' = u'\cos u$

$2yy' = 2x + (y + xy')\cos xy$

$2yy' = 2x + y\cos xy + (x\cos xy)\,y'$

$(2y - x\cos xy)\,y' = 2x + y\cos xy$

$$\boxed{y' = \frac{dy}{dx} = \frac{2x + y\cos xy}{2y - x\cos xy}}$$

= equal

$\simeq$

$\rightarrow$ | approach
imply

Ex $2x^3 - 3y^2 = 8$ $\quad \frac{dy}{dx}$? $\quad \frac{dx}{dx} = 1$

$\frac{d^2y}{dx^2}$?

$$6x^2 - 6yy' = 0$$

$$yy' = x^2$$

$$y' = \frac{x^2}{y} = \frac{dy}{dx}$$

$$(y')^2 + yy'' = 2x$$

$$yy'' = 2x - \left(\frac{x^2}{y}\right)^2$$

$$= 2x - \frac{x^4}{y^2}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{2x}{y} - \frac{x^4}{y^3}$$

$$= \frac{2xy^2 - x^4}{y^3}$$

Ex $x^3 + y^3 - 9xy = 0$ $\quad (2, 4)$ tangent $m = y'$

$$\frac{d}{dx}(x^3 + y^3 - 9xy) = 0$$

$$3x^2 + 3y^2y' - 9(y + xy') = 0$$

$$3x^2 + 3y^2y' - 9y - 9xy' = 0$$

$$(3y^2 - 9x)y' = 9y - 3x^2$$

$$m = \frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x}\Bigg|_{(2,4)}$$

$$= \frac{36 - 12}{48 - 18}$$

$$= \frac{24}{30}$$

$$= \frac{4}{5}$$

$$y = \frac{4}{5}(x - 2) + 4$$

#1 $y^2+x^2-2y-4x=4$ $\qquad \frac{dy}{dx}$?

$$2yy'+2x-2y'-\cancel{4}^{\,2}=0$$

$$(y-1)y'=2-x$$

$$\boxed{y'=\frac{dy}{dx}=\frac{2-x}{y-1}}$$

#3 $x+\sqrt{x}\sqrt{y}=y^2$ $\qquad \sqrt{u}'=\frac{u'}{2\sqrt{u}}$

$$1+\frac{1}{2\sqrt{x}}\sqrt{y}+\frac{\sqrt{x}}{2\sqrt{y}}y'=2yy'$$

$$1+\frac{\sqrt{y}}{2\sqrt{x}}=\left(2y-\frac{1}{2}\frac{\sqrt{x}}{\sqrt{y}}\right)y'$$

$$\left(\frac{4y\sqrt{y}-\sqrt{x}}{2\sqrt{y}}\right)y'=\frac{2\sqrt{x}+\sqrt{y}}{2\sqrt{x}}$$

$$y'=\frac{dy}{dx}=\frac{\sqrt{y}}{\sqrt{x}}\left(\frac{2\sqrt{x}+\sqrt{y}}{4y\sqrt{y}-\sqrt{x}}\right)$$

$$\boxed{=\frac{2\sqrt{xy}+y}{4y\sqrt{xy}-x}}$$

#10 $x\cos(2x+3y)=y\sin x$ $\qquad (uv)'=u'v+v'u$ $\qquad (\cos u)'=-u'\sin u$

$$\cos(2x+3y)-(2+3y')\sin(2x+3y)=y'\sin x+y\cos x$$

$$\cos(2x+3y)-2\sin(3x+3y)+3y'\sin(2x+3y)-y'\sin x=y\cos x$$

$$y'(3\sin(2x+3y)-\sin x)=y\cos x-\cos(2x+3y)+2\sin(3x+2y)$$

$$\boxed{y'=\frac{dy}{dx}=\frac{y\cos x-\cos(2x+3y)+2\sin(3x+2y)}{3\sin(2x+3y)-\sin x}}$$

2.8 Exp. Log fctns

$(\ln|x|)' = \frac{1}{x}$ $\quad (\ln u)' = \frac{u'}{u}$

$(\ln 2x)' = \frac{2}{2x} = \frac{1}{x}$

$(\ln(x^2+3))' = \frac{2x}{x^2+3}$

$\ln MN = \ln M + \ln N$

$\ln \frac{M}{N} = \ln M - \ln N$

$\ln x^p = p \ln x$

$\ln \frac{1}{x} = -\ln x \qquad (\ln \overset{0}{1} - \ln x)$

Ex $y = \frac{(x^2+1)(x+3)^{1/2}}{x-1} \qquad x > 1 \qquad (u^n v^m w^p)'$

$\ln y = \ln \frac{(x^2+1)(x+3)^{1/2}}{x-1}$

$\ln y = \ln(x^2+1) + \frac{1}{2}\ln(x+3) - \ln(x-1)$

$\frac{y'}{y} = \frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x+3} - \frac{1}{x-1}$

$y' = y\left(\frac{2x}{x^2+1} + \frac{1}{2(x+3)} - \frac{1}{x-1}\right)$

$= \frac{(x^2+1)(x+3)^{1/2}}{x-1}\left(\frac{2x}{x^2+1} + \frac{1}{2(x+3)} - \frac{1}{x-1}\right)$

$(e^u)' = u'e^u$

$(e^x)' = e^x$

Ex $\frac{d}{dx}(5e^x) = 5e^x$

$\frac{d}{dx}(e^{\sin x}) = \cos x \, e^{\sin x}$

$\frac{d}{dx}\left(e^{\sqrt{3x+1}}\right) = \frac{3}{2\sqrt{3x+1}} e^{\sqrt{3x+1}}$

$\ln 1 = 0 \qquad \ln e = 1$

$e^{x \ln a} = a^x \qquad e^{\ln u} = u$

$x^n = e^{n \ln x}$

$\frac{d}{dx}(x^n) = \frac{d}{dx}(e^{n \ln x})$

$= \frac{n}{x} e^{n \ln x}$

$= \frac{n}{x} x^n$

$= n x^{n-1}$

$(x^x)'$?

$f(x) = x^x$

$= e^{x \ln x}$

$f'(x) = e^{\overline{x \ln x}^{\,u}}$

$= (\ln x + 1) e^{x \ln x}$

$= (\ln x + 1) x^n$

$(a^u)' = a^u u' \ln a$

$(3^x)' = 3^x \ln 3$

$$(\log_a u)' = \frac{u'}{u}\,\frac{1}{\ln a}$$

$$(\ln u)' = \frac{u'}{u}\,\frac{1}{\ln e} = 1$$

2.8

#1 $y = \ln\sqrt{x+5}$ $\qquad \ln(x+5)^{1/2}$

$= \frac{1}{2}\ln(x+5)$

$\boxed{y' = \frac{1}{2}\,\frac{1}{x+5}}$

$\ln(ax+b) = \frac{a}{ax+b}$

$\ln(x+b) = \frac{1}{x+b}$

#6 $y = \ln\dfrac{x^2(x+1)^3}{(x+3)^{1/2}}$

$= 2\ln x + 3\ln(x+1) - \frac{1}{2}\ln(x+3)$

$\boxed{y' = \frac{2}{x} + \frac{3}{x+1} - \frac{1}{2}\,\frac{1}{x+3}}$

#34 $y = \ln\sqrt{\dfrac{(x+1)^5}{(x+2)^{20}}}$

$= \frac{5}{2}\ln(x+1) - 10\ln(x+2)$

$\boxed{y' = \frac{5}{2}\,\frac{1}{x+1} - \frac{10}{x+2}}$

#36 $f(x) = e^{-2x^3}$

$\boxed{f'(x) = -6x^2 e^{-2x^3}}$

#38 $f(x) = 2x^3 e^x$ $\qquad (uv)'$

$f'(x) = 6x^2 e^x + 2x e^x$

$= (6x^2 + 2x)\,e^x$

#39 $f(x) = \frac{3e^x}{1+e^x}$ $\left(\frac{u}{v}\right)'$

$$f'(x) = \frac{3e^x(1+e^x) - e^x(3e^x)}{(1+e^x)^2}$$

$$\left(\frac{\alpha+b}{\beta+d}\right)' = \frac{(\alpha'\beta - \beta'\alpha) + (\alpha'd - b\beta')}{(\beta+d)^2}$$

$$= \frac{3e^x + 3e^{2x} - 3e^{2x}}{(1+e^x)^2}$$

$$= \frac{3e^x}{(1+e^x)^2}$$

$$3(e^x)^2 - e^x(3e^x)$$

$$\left(\frac{3e^x}{e^x+1}\right)' = \boxed{\frac{3e^x}{(e^x+1)^2}}$$

$$\frac{ae^\alpha}{be^\alpha} \quad 0$$

#49 $f(x) = \frac{1-e^x}{1+e^x} = \frac{-e^x+1}{e^x+1}$

$$f'(x) = \frac{-e^x(1+e^x) - e^x(1-e^x)}{(1+e^x)^2}$$

$$= \frac{-e^x - e^{2x} - e^x + e^{2x}}{(1+e^x)^2}$$

$$= -\frac{2e^x}{(1+e^x)^2}$$

$$\boxed{f'(x) = \frac{-2e^x}{(1+e^x)}}$$

#51 $y = \sqrt{e^{2x^2} + e^{-2x^2}}$

$$y' = \frac{4xe^{2x^2} - 4xe^{-2x^2}}{2\sqrt{e^{2x^2} + e^{-2x^2}}}$$

$$= \frac{2x(e^{2x^2} - e^{-2x^2})}{\sqrt{e^{2x^2} + e^{-2x^2}}}$$

#67 $f(x) = e^{2x} \ln(xe^x+1)$

$$f'(x) = 2e^{2x}\ln(xe^x+1) + e^{2x}\frac{e^x+xe^x}{xe^x+1}$$

$$= \boxed{2e^{2x}\ln(xe^x+1) + \frac{(x+1)e^{3x}}{xe^x+1}}$$

#68 $f(x) = \dfrac{xe^x}{\ln(x^2+1)}$ $\left(\dfrac{u}{v}\right)'$

$$f'(x) = \frac{(e^x+xe^x)\ln(x^2+1) - \frac{2x}{x^2+1}xe^x}{(\ln(x^2+1))^2} \qquad \frac{x^2+1}{x^2+1}$$

$$= \frac{e^x(x+1)[\ln(x^2+1)](x^2+1) - 2x^2e^x}{(x^2+1)(\ln(x^2+1))^2}$$

$$= \frac{(x+1)(x^2+1)\ln(x+1) - 2x^2}{(x^2+1)(\ln(x^2+1))^2}e^x$$

#65 $y = \ln\dfrac{1+e^x}{1-e^x}$

$$= \ln(e^x+1) - \ln(1-e^x)$$

$$y' = \frac{e^x}{e^x+1} + \frac{e^x}{1-e^x}$$

$$= \left(\frac{1}{1+e^x} + \frac{1}{1-e^x}\right)e^x \quad ** $$

$$= \boxed{\frac{2}{1-e^{2x}}e^x}$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$f(x) = \sin^{-1} u$

$$= \frac{u'}{\cos(\sin^{-1} u)}$$

$$= \frac{u'}{\sqrt{1 - \sin^2(\sin^{-1} u)}} \qquad (\sin(\sin^{-1} u))^2 = u^2$$

$$(\sin^{-1} u)' = \frac{u'}{\sqrt{1 - u^2}} \qquad (\sec u)' = \frac{u'}{|u|\sqrt{u^2 - 1}}$$

$$(\tan^{-1} u)' = \frac{u'}{1 + u^2}$$

$\sqrt{1 - x^2}$ $\quad -1 \le x \le 1$

$\sqrt{x^2 - 1}$ $\quad x \le -1 \quad x \ge 1$

---

implicit fctn: 1

$f(x) = a x^n$

$f^{n}(x) = a\, n!\, x^n$

$f^{(n+1)}(x) = 0$