2.3

Product $(u \cdot v)' = u' \cdot v + v' u$

Ex $f(x) = (2x+3)(3x^2)$

$u = 2x+3 \qquad v = 3x^2$

$u' = 2 \qquad v' = 6x$

$f'(x) = 2(3x^2) + 6x(2x+3)$

$= 6x^2 + 12x^2 + 18x$

$= \underline{18x^2 + 18x}$

$f(x) = 6x^3 + 9x^2$

$\underline{f'(x) = 18x^2 + 18x}$

Ex $f(x) = (3x^2+1)(x^3+3) \qquad (uv)' = u'v + v'u$

$f'(x) = 6x(x^3+3) + (3x^2)(3x^2+1)$

$= 6x^4 + 18x + 9x^4 + 3x^2$

$= \underline{15x^4 + 3x^2 + 18x}$

Ex $y = (3x^3+2x+5)(x^2-2x+4)$

$y' = (9x^2+2)(x^2-2x+4) + (2x-2)(3x^3+2x+5)$

x^4	x^3	x^2	x^1	x^0
9	-18	36	-4	8
	-6	2	10	-10
6		4	-4	

$= \underline{15x^4 - 24x^3 + 42x^2 + 2x - 2}$

Quotient: $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

$$\left(\frac{ax+b}{cx+d}\right)' = \frac{ad-bc}{(cx+d)^2} \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\left(\frac{ax^n+b}{cx^n+d}\right)' = \frac{n(ad-bc)x^{n-1}}{(cx^n+d)^2}$$

$$\left(\frac{ax^2+bx+c}{dx^2+ex+f}\right)' = \frac{(ae-bd)x^2 + 2\begin{vmatrix} a & c \\ d & f \end{vmatrix}x + \begin{vmatrix} b & c \\ e & f \end{vmatrix}}{(dx^2+ex+f)^2}$$

Ex $\quad f(x) = \frac{2x-1}{4x+3} \quad \begin{matrix} u \\ v \end{matrix}$

$u = 2x-1 \qquad v = 4x+3$

$u' = 2 \qquad v' = 4$

$$f'(x) = \frac{2(4x+3) - 4(2x-1)}{(4x+3)^2}$$

$$= \frac{8x+6-8x+4}{(4x+3)^2}$$

$$= \boxed{\frac{10}{(4x+3)^2}}$$

$f'(x) = \frac{10}{(4x+3)^2}$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Ex $\quad y = \frac{(x-1)(x^2-2x)}{x^4}$

$$= \frac{x^3 - 3x^2 + 2x}{x^4}$$

$$= \frac{1}{x} - 3x^{-2} + 2x^{-3}$$

$$\boxed{y' = -\frac{1}{x^2} + \frac{6}{x^3} - \frac{6}{x^4}}$$

$$y = \frac{(1+x)(2x-1)}{x-1}$$

$$= \frac{2x^2+x-1}{x-1}$$

$$y' = \frac{2x^2-4x}{(x-1)^2}$$

$$\begin{array}{cccc} 2 & 1 & -1 \\ 0 & 1 & -1 \end{array}$$

x

$2(-2)$

30/ $y = \frac{x-4}{5x-2} \to \boxed{y' = \frac{18}{(5x-2)^2}}$

$$\begin{vmatrix} 1 & -4 \\ 5 & -2 \end{vmatrix}$$

$-2-(-20)$

31/ $y = \frac{3x-4}{2x-1}$

$$\begin{vmatrix} 3 & -4 \\ 2 & -1 \end{vmatrix}$$

$y' = \frac{5}{(2x-1)^2}$

$-3-(-8)$

32/ $y = \frac{3x+4}{2x+1}$

$3-(8)$

$\boxed{y' = \frac{-5}{(2x+1)^2}}$

33/ $y = \frac{-3x+4}{2x+1}$ $\quad y' = \frac{-11}{(2x+1)^2}$

$-3-(8)$

34/ $y = \frac{-3x-4}{2x-1} \Rightarrow y' = \frac{11}{(2x-1)^2}$

$3-(-8)$

52 $\quad y = \dfrac{x^2-4x+1}{5x^2-2x-1}$ $\qquad \begin{matrix} 1 & -4 & 1 \\ 5 & -2 & -1 \end{matrix}$

$$y' = \frac{18x^2-12x+6}{(5x^2-2x-1)^2} \qquad \begin{vmatrix} -4 & 1 \\ -2 & -1 \end{vmatrix}$$

$$x:\ 2\begin{vmatrix} 1 & 1 \\ 5 & -1 \end{vmatrix}$$

53 $\quad y = \dfrac{3x^2-4x+2}{2x^2+x-1}$ $\qquad \begin{matrix} 3 & -4 & 2 \\ 2 & 1 & -1 \end{matrix}$

$$\boxed{y' = \frac{11x^2-14x+3}{(2x^2+x-1)^2}} \qquad 2\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}$$

2.4 Trig.

$(\sin x)' = \cos x \qquad (\tan x)' = \sec^2 x$

$(\cos x)' = -\sin x \qquad (\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc x)' = -\csc x \cot x$

Ex $\quad y = x^2 - \sin x$

$y' = 2x - \cos x$

Ex $\quad y = x^2 \cdot \sin x \qquad (uv)' = u'v + v'u$

$y' = 2x \sin x + x^2 \cos x$

Ex: $\quad y = \dfrac{\sin x}{x} \qquad \left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$$y = \frac{x\cos x - \sin x}{x^2}$$

$y = 5x + \cos x$

$\boxed{y' = 5 - \sin x}$

$y = \sin x \cos x$ $\qquad\qquad$ $y = \frac{1}{2} \sin 2x$

$y' = \cos^2 x - \sin^2 x$

$= \cos 2x \leftarrow$

$$y = \frac{\cos x}{1 - \sin x}$$

$$y' = \frac{-\sin x(1 - \sin x) - (-\cos x)\cos x}{(1 - \sin x)^2}$$

$$= \frac{-\sin x + \overbrace{\sin^2 x + \cos^2 x}}{(1 - \sin x)^2}$$

$$= \frac{1 - \sin x}{(1 - \sin x)^2}$$

$$\boxed{= \frac{1}{1 - \sin x}}$$

$y = \tan x$ $\qquad\qquad$ $y' = \sec^2 x$

$$= \frac{\sin x}{\cos x}$$

$$y' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$= \sec^2 x$

$y = \sec x \qquad y''$

$y' = \underbrace{\sec x}_{u} \underbrace{\tan x}_{v}$

$y'' = \sec x \tan x (\tan x) + \sec^2 x \sec x$

$= \boxed{\sec x \tan^2 x + \sec^3 x}$

#11 $y = 10x + 3\cos x$

$\boxed{y' = 10 - 3\sin x}$

#7 $y = \dfrac{\cos x}{x} + \dfrac{x}{\cos x}$

$= \dfrac{\cos^2 x + x^2}{x \cos x}$

$y' = \dfrac{(-2\cos x \sin x + 2x)(x\cos x) - (\cos^2 x + x^2)(\cos x - x\sin x)}{(x\cos x)^2}$

$= \dfrac{-2x\cos^2 x \sin x + 2x^2 \cos x - \cos^3 x + x\cos^2 x \sin x - x^2 \cos x + x^3 \sin x}{x^2 \cos^2 x}$

$= \dfrac{-x\cos^2 \sin x + x^2 \cos x - \cos^3 x + x^3 \sin x}{x^2 \cos^2 x}$

$y' = \left(\dfrac{\cos x}{x}\right)' + \left(\dfrac{x}{\cos x}\right)'$

$= \dfrac{-(\sin x)(x) - \cos x}{x^2} + \dfrac{\cos x + x\sin x}{\cos^2 x}$

#8 $y = \underbrace{x^2\cos x}_{(uv)'} - 2x\sin x - 2\cos x$

$y' = 2x\cos x - x^2\sin x - (2\sin x + 2x\cos x) + 2\sin x$

$= 2x\cos x + x^2\sin x - 2\sin x - 2x\cos x + 2\sin x$

$= \boxed{x^2\sin x}$

#16 $f(x) = \dfrac{\sin x + 2x}{x}$

$f'(x) = \dfrac{(\cos x + 2)x - \sin x - 2x}{x^2}$

$= \dfrac{x\cos x + 2x - \sin x - 2x}{x^2}$

$= \boxed{\dfrac{x\cos x - \sin x}{x^2}}$

$(u^n)' = n u' u^{n-1}$ $\qquad (x^n)' = n x^{n-1}$,

$(x)' = 1$.

Ex $y = (3x^2+1)^2$

$y' = 2(6x)(3x^2+1)$

$= \boxed{12x(3x^2+1)}$

Ex $x(t) = \cos(t^2+1)$ $\qquad (\cos u)' = -u'\sin u$

$x'(t) = -2t\sin(t^2+1)$ $\qquad \dfrac{d}{dt}(t^2+1) = 2t$

Ex $\frac{d}{dx}(5x^3 - x^4)^7$ $\quad (u^n)' = n u' u^{n-1}$

$$= \underbrace{7}_{n}\,\underbrace{(15x^2 - 4x^3)}_{u'}\,(5x^3 - x^4)^6$$

Ex $\left(\frac{1}{3x-2}\right)' = \frac{-3}{(3x-2)^2}$ $\quad \begin{vmatrix} 0 & 1 \\ 3 & -2 \end{vmatrix}$

$$\left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$\left(\frac{1}{u^n}\right)' = -\frac{n u'}{u^{n+1}}$$

$(\sin x)^5$

Ex $\frac{d}{dx}(\sin^5 x) = 5\cos x \sin^4 x$

Ex $g(t) = \tan(5 - \sin 2t)$ $\quad (\tan u)' = u' \sec^2 u$

$g'(t) = -2\cos 2t \sec^2(5 - \sin 2t)$

$(-\sin 2t)'$

$-2\cos 2t$

Ex $y = \frac{1}{(1-2x)^3}$ $\quad \left(\frac{1}{u^n}\right)' = -\frac{n u'}{u^{n+1}}$

$$y' = -\frac{3(-2)}{(1-2x)^4}$$

$$= \boxed{\frac{6}{(1-2x)^4}}$$

$$(u^n v^m w^p)' = u^{n-1} v^{m-1} w^{p-1} (n u' v w + m u v' w + p u v w')$$

$$(u^n v^m)' = u^{n-1} v^{m-1} (n u' v + m u v')$$

58/ $f(x) = \underbrace{(x^2 + 2x - 3)}_{u}^{5\,(n)} \underbrace{(2x+3)}_{v}^{6\,(m)}$

$$f'(x) = (x^2+2x-3)^4 (2x+3)^5 \left(5(2x+2)(2x+3) + 6(2)(x^2+2x-3) \right)$$

x^2	x^1	x^0
20	30	30
12	20	-36
	24	

$$f'(x) = (x^2+2x-3)^4 (2x+3)^5 (32x^2 + 74x - 6)$$

67/ $f(x) = \dfrac{(x^2-3x)^3(x^2+3x-3)^4}{(x^2-3x+2)^2}$

$= (x^2-3x)^3(x^2+3x-3)^4(x^2-3x+2)^{-2}$

$f'(x) = \dfrac{(x^2-3x)^2(x^2+3x-3)^3}{(x^2-3x+2)^3}$

$\Big(3(2x-3)(x^2+3x-3)(x^2-3x+2)$

$+4(2x+3)(x^2-3x)(x^2-3x+2)$

$-2(2x-3)(x^2-3x)(x^2+3x-3)\Big)$

$(2x^3+\cancel{6x^2}-\cancel{6x}-\cancel{3x^2}-9x+9)(3x^2-9x+6)$ [3, $-15x$]

$+(8x+12)(x^4-6x^3+11x^2-6x)$

$+(-4x+6)(x^4-12x^2+9x)$

x^5	x^4	x^3	x^2	x^1	x^0
6	-18	12	18	-90	54
8	9	-27	135	-81	
-4	-48	-45	27	72	
	12	88	-48	54	
	6	-72	132		
		48	-36		
			-72		

$f'(x) = \dfrac{(x^2-3x)^2(x^2+3x-3)^3}{(x^2-3x+2)^3}\left(10x^5-39x^4+4x^3+156x^2-45x+54\right)$

-171
126