

5.2 Partial Fraction

(i) $G(x) = (x-x_1)(x-x_2) \dots (x-x_n)$

$$\frac{P}{Q} = \frac{A_1}{x-x_1} + \frac{A_2}{x-x_2} + \dots + \frac{A_n}{x-x_n}$$

Ex

$$\frac{x}{x^2-5x+6}$$

$$x^2-5x+6 = (x-2)(x-3)$$

$$\frac{x}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$\frac{A(x-3) + B(x-2)}{(x-2)(x-3)}$$

$$x = A(x-3) + B(x-2) \quad \text{--- (1)}$$

$$x^1 \quad A + B = 1$$

$$x^0 \quad -3A - 2B = 0$$

$$D = \begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} = -2 + 3 = 1$$

$$D_A = \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} = -2$$

$$A = \frac{D_A}{D} = -2$$

$$D_B = \begin{vmatrix} 1 & 1 \\ -3 & 0 \end{vmatrix} = +3$$

$$B = 3$$

$$\frac{x}{x^2-5x+6} = \frac{-2}{x-2} + \frac{3}{x-3}$$

$$3A + 2B = -9$$

$$-3A - 2B = 0$$

$$\underline{\hspace{1cm}} \quad B = 3$$

$$A = -3 - 3 = -2$$

$$Q: (x-a)^n$$

$$\frac{P}{Q} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \dots + \frac{A_n}{(x-a)^n}$$

Ex

$$\frac{x+2}{x^3-2x^2+x}$$

$$x(x^2-2x+1) = x(x-1)^2$$

$$\frac{x+2}{x^3-2x^2+x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$x+2 = A(x^2-2x+1) + Bx(x-1) + Cx$$

$$x^2 \quad A + B = 0 \rightarrow \underline{B = -2}$$

$$x^1 \quad -2A - B + C = 1 \rightarrow \underline{C = 1 + 2(2) + (-2) = 3}$$

$$x^0 \quad A = 2$$

$$\frac{x+2}{x^3-2x^2+x} = \frac{2}{x} - \frac{2}{x-1} + \frac{3}{(x-1)^2}$$



$$\frac{x^3 - 8}{x^2(x-1)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}$$

$$x^3 - 8 = Ax(x^2 - 2x^2 + 2x - 1) + B(x^3 - 3x^2 + 3x - 1) + Cx^2(x^2 - 2x + 1) + Dx^2(x-1) + Ex^2$$

$$x^4 \quad A + C = 0 \quad (2)$$

$$x^3 \quad -3A + B - 2C + D = 1 \quad (3)$$

$$x^2 \quad 3A - 3B + C - D + E = 0 \quad (4)$$

$$x^1 \quad -A + 3B = 0 \quad (1)$$

$$x^0 \quad -B = -8 \Rightarrow \underline{B = 8}$$

$$(1) \quad \underline{A = 3B = 24}$$

$$(2) \quad \underline{C = -24}$$

$$(3) \quad D = 1 + 3(24) - 8 + 2(-24) \\ = \underline{17}$$

$$(4) \quad E = -3(24) + 3(8) + 24 + 17 \\ = \underline{-7}$$

$$\frac{x^3 - 8}{x^2(x-1)^3} = \frac{24}{x} + \frac{8}{x^2} - \frac{24}{x-1} + \frac{17}{(x-1)^2} - \frac{7}{(x-1)^3}$$

21

22

$Ax + B$
 $ax^2 + bx + c$ can't be factorable

Ex: $\frac{3x-5}{x^3-1}$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\frac{3x-5}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$3x-5 = A(x^2+x+1) + (Bx+C)(x-1)$$

$$x^2 \quad A + B = 0 \Rightarrow B = -A = \frac{2}{3}$$

$$x^1 \quad A - B + C = 3 \quad (1)$$

$$x^0 \quad A - C = -5 \Rightarrow C = A + 5$$
$$= \frac{-2}{3} + 5 = \frac{13}{3}$$

$$(1) \quad A + A + A + 5 = 3$$

$$3A = -2$$

$$A = \frac{-2}{3}$$

$$\frac{3x-5}{x^3-1} = \frac{-2/3}{x-1} + \frac{\frac{2}{3}x + \frac{13}{3}}{x^2+x+1}$$

$$= -\frac{2}{3} \frac{1}{x-1} + \frac{1}{3} \frac{2x+13}{x^2+x+1}$$

$$\underline{Ex}: \frac{x^3 + x^2}{(x^2 + 4)^2} = \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2}$$

$$x^3 + x^2 = (Ax + B)(x^2 + 4) + Cx + D$$

$$x^3 \quad A = 1$$

$$x^2 \quad B = 1$$

$$x^1 \quad 4A + C = 0 \rightarrow C = -4$$

$$x^0 \quad 4B + D = 0 \rightarrow D = -4$$

$$\frac{x^3 + x^2}{(x^2 + 4)^2} = \frac{x + 1}{x^2 + 4} + \frac{-4x - 4}{(x^2 + 4)^2}$$

$$\underbrace{\quad}_{\text{imply}} \quad \underbrace{(1x)}_{\text{equal}} = x$$

$$\# 5.2-1 \quad \frac{4}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$4 = A(x-1) + Bx$$

$$x^1 \quad A + B = 0 \Rightarrow B = 4$$

$$x^0 \quad -A = 4 \Rightarrow A = -4$$

$$\frac{4}{x(x-1)} = \frac{-4}{x} + \frac{4}{x-1}$$

#18

$$\frac{4}{2x^2 - 5x + 3} = \frac{A}{x-1} + \frac{B}{2x-3}$$

$$4 = A(2x-3) + B(x-1)$$

$$x^1 \quad 2A + B = 0$$

$$x^0 \quad -3A - B = 4$$

$$\underline{-A = 4 \Rightarrow A = -4}$$

$$[B = -2(-4) = 8]$$

$$\underline{\frac{4}{2x^2 - 5x + 3} = \frac{-4}{x-1} + \frac{8}{2x-3}}$$

Sec. 5.3 Ellipse

$$\text{if } a=b \Rightarrow x^2 + y^2 = a^2$$

Circle

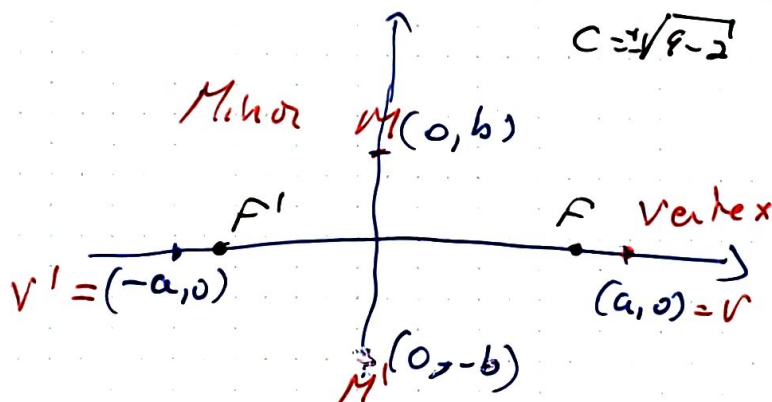
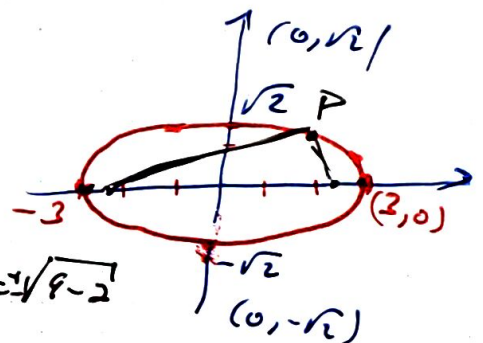


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ex

$$\frac{2x^2}{18} + \frac{9y^2}{18} = \frac{18}{18}$$

$$\frac{x^2}{9} + \frac{y^2}{2} = 1$$



$$c^2 = a^2 - b^2$$

foci (focus)

Ex $\left(\frac{9x^2}{25} + \frac{4y^2}{25} = \frac{25}{25} \right)$

$$\frac{x^2}{\frac{25}{9}} + \frac{y^2}{\frac{25}{4}} = 1$$

$$9 > 4$$

$$\frac{1}{9} < \frac{1}{4}$$

$$a^2 = \frac{25}{4} \rightarrow a = \pm \frac{5}{2}$$

$$b^2 = \frac{25}{9} \rightarrow b = \pm \frac{5}{3}$$

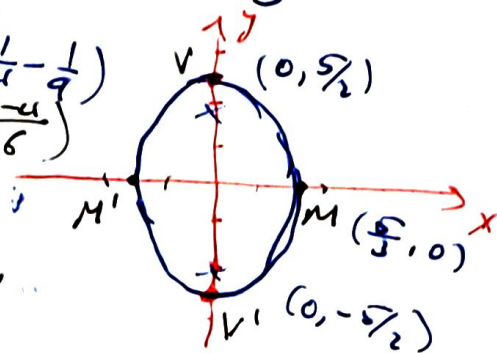
$$c^2 = 25 \left(\frac{1}{4} - \frac{1}{9} \right)$$

$$= 25 \left(\frac{9-4}{36} \right)$$

$$= \frac{125}{36}$$

$$c = \pm \frac{5\sqrt{5}}{6}$$

$$\text{Foci: } (0, \pm \frac{5\sqrt{5}}{6})$$



Ex

$$w_T = 10 \text{ ft.}$$

$$h_T = 9$$

$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

$$\frac{y^2}{10^2} = 1 - \frac{x^2}{20^2}$$

$$y^2 = 10^2 \left(\frac{20^2 - 5^2}{20^2} \right)$$

$$= \frac{10^2}{20^2} (400 - 25)$$

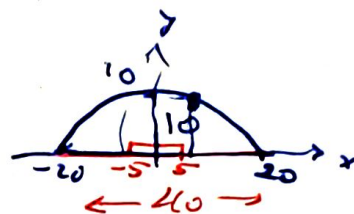
$$= \left(\frac{1}{2} \right)^2 (375)$$

$$= \frac{375}{4}$$

$$81 > 98$$

$$81 < \frac{375}{4}$$

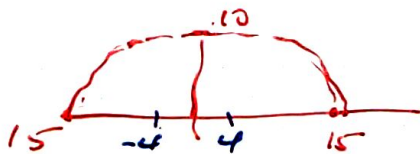
Truck will clear.



$$\left(\frac{10}{20} \right)^2$$

$$\begin{array}{r} 93 \\ 4 \overline{) 375} \\ 15 \end{array}$$

35



$$w=8, h=7$$

$$\frac{x^2}{15^2} + \frac{y^2}{10^2} = 1$$

$$\frac{y^2}{10^2} = 1 - \frac{x^2}{15^2}$$

$$= \frac{15^2 - x^2}{15^2}$$

$$y^2 = \frac{10^2}{15^2} (15^2 - 4^2)$$

$$= \left(\frac{10}{15}\right)^2 (209)$$

$$49? = \left(\frac{2}{3}\right)^2 (209)$$

$$= \frac{4}{9} (209)$$

$$441 < 836$$

clear

$$\begin{array}{r} 15 \\ 16 \\ \hline 75 \\ 15 \\ \hline 225 \\ 16 \end{array}$$

$$\frac{10}{15} =$$