

Lecture One

Section 1.1 – Introduction to System of Linear Equations

Given the linear equations

$$\begin{cases} x - 2y = 1 \\ 3x + 2y = 11 \end{cases}$$

The solution to this system is $(3, 1)$, which means that 2 lines meeting at a single point.

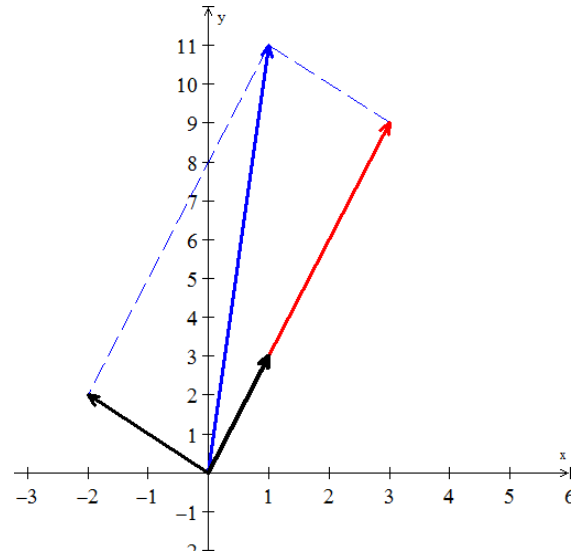
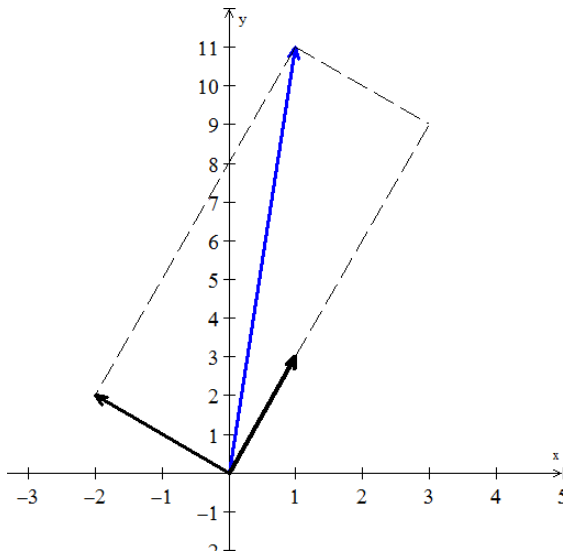
We can rewrite the system equation as linear combination:

$$x \begin{bmatrix} 1 \\ 3 \end{bmatrix} + y \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

$$x.v_1 + y.v_2 = v$$

$$\begin{bmatrix} 1+x \\ 3+y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} x=3 \\ y=9 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 9 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$



Therefore, the side vectors are $\begin{bmatrix} 3 \\ 9 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$

The diagonal sum is $\begin{bmatrix} 3-2 \\ 9+2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$

The linear combination is given by:

$$3 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

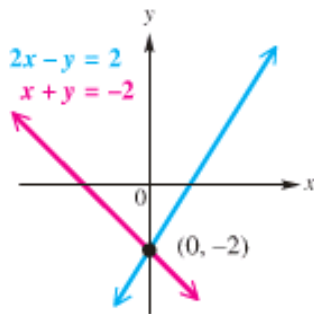
Thus, the solution is $x = 3$ $y = 1$

Note

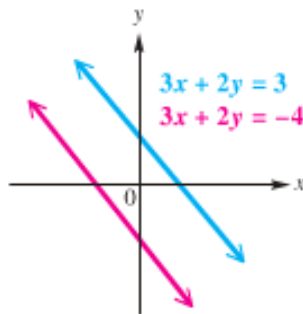
$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$ is called the “*coefficient matrix*”

The matrix form of the system is written as $Ax = b$

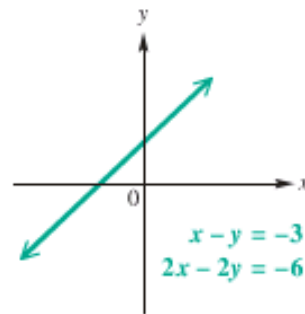
$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$



One solution (lines intersect)
Consistent
Independent



No Solution (lines //)
Inconsistent
Independent



Unique Infinite solution
Consistent
Dependent

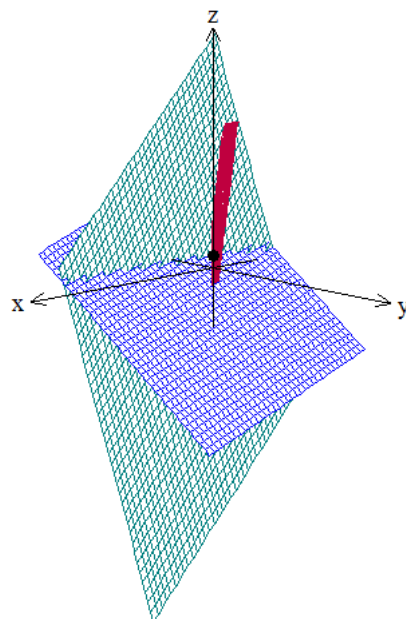
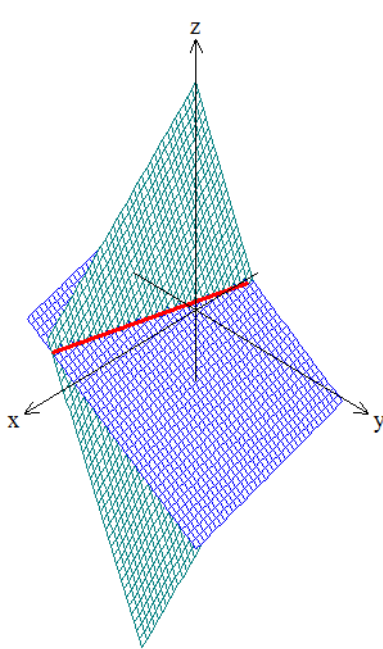
Three Equations in 3 Unknowns

Given the system equations

$$x + 2y + 3z = 6$$

$$2x + 5y + 2z = 4$$

$$6x - 3y + z = 2$$

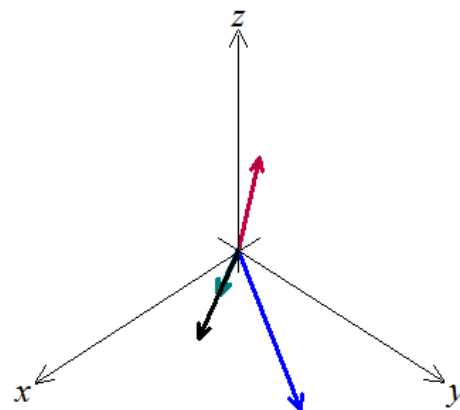


This system can be written as linear combination:

$$x \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} + y \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix} + z \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

Let $\mathbf{b} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

We want to multiply the three column vectors by x , y , z to produce \mathbf{b} .

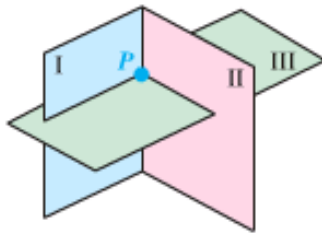


The combination of the three vectors that produces vector \mathbf{b} is 2 times the third vector.

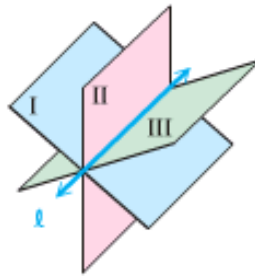
$$2(3, 2, 1) = (6, 4, 2) = \mathbf{b}$$

Therefore, the coefficients that we need are $x = 0$, $y = 0$, and $z = 2$.

$$0 \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$



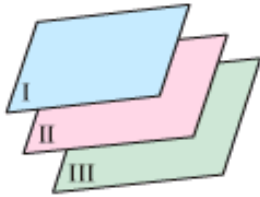
A single solution



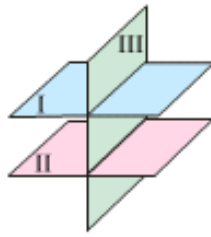
Points of a line in common



All points in common



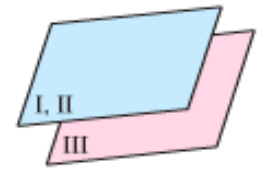
No points in common



No points in common



No points in common



No points in common

Exercises Section 1.1 – Introduction to System of Linear Equations

1. Find a solution for x, y, z to the system of equations

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3e + 2\sqrt{2} + \pi \\ 6e + 5\sqrt{2} + 4\pi \\ 9e + 8\sqrt{2} + 7\pi \end{pmatrix}$$

2. Draw the two pictures in two planes for the equations: $x - 2y = 0$, $x + y = 6$
3. Normally 4 planes in 4-dimensional space meet at a _____. Normally 4 column vectors in 4-dimensional space can combine to produce b . what combinations of $(1, 0, 0, 0)$, $(1, 1, 0, 0)$, $(1, 1, 1, 0)$, $(1, 1, 1, 1)$ produces $b = (3, 3, 3, 2)$?
What 4 equations for x, y, z, w are you solving?

4. What 2 by 2 matrix A rotates every vector through 45° ?

The vector $(1, 0)$ goes to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. The vector $(0, 1)$ goes to $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

Those determine the matrix. Draw these particular vectors in the xy -plane and find A .

5. What two vectors are obtained by rotating the plane vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ by 30° (cw)?

Write a matrix A such that for every vector \vec{v} in the plane, $A\vec{v}$ is the vector obtained by rotating \vec{v} clockwise by 30° .

Find a matrix B such that for every 3-dimensional vector \vec{v} , the vector $B\vec{v}$ is the reflection of \vec{v} through the plane $x + y + z = 0$. *Hint*: $v = (1, 0, 0)$

6. In each part, find a system of linear equation corresponding to the given augmented matrix

a) $\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 5 & 2 & 0 & -3 & -6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \\ 5 & -6 & 1 & 1 \\ -8 & 0 & 0 & 3 \end{bmatrix}$

7. Find the augmented matrix for the given system of linear equations.

$$a) \begin{cases} -2x_1 = 6 \\ 3x_1 = 8 \\ 9x_1 = -3 \end{cases}$$

$$c) \begin{cases} 2x_1 + 2x_3 = 1 \\ 3x_1 - x_2 + 4x_3 = 7 \\ 6x_1 + x_2 - x_3 = 0 \end{cases}$$

$$b) \begin{cases} 3x_1 - 2x_2 = -1 \\ 4x_1 + 5x_2 = 3 \\ 7x_1 + 3x_2 = 2 \end{cases}$$