$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

3, 4 → 5
5, 12 → 13
8, 15 → 17

20, 21 → 29
7, 24 → 25

$$\boxed{\cos^2\theta + \sin^2\theta = 1} \to \begin{cases} \cos^2\theta = 1 - \sin^2\theta \\ \sin^2\theta = 1 - \cos^2\theta \end{cases}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

$$\left\{ \cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2} \right.$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1}{2}(1 + \cos x)}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1}{2}(1 - \cos x)}$$

t

$\frac{8.2}{4}$

$\sin A = \frac{3}{5}$ $A \in QII$ $\cos B = -\frac{12}{13}$ $B \in QIII$

$\cos A = -\frac{4}{5}$ $\sin B = -\frac{5}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{-36+20}{65}$

$= \boxed{-\frac{16}{65}}$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \left(-\frac{4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{48+15}{65}$

$= \boxed{\frac{63}{65}}$

c) $\tan(A+B) = \boxed{-\frac{16}{63}}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \frac{-36-20}{65}$

$= \boxed{-\frac{56}{65}}$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{48-15}{65}$

$= \boxed{\frac{33}{65}}$

e) $\tan(A-B) = -\frac{56}{33}$ ✓

#9

Ex

$$\frac{\cos(x-y)}{\sin x \sin y} = \cot x \cot y + 1$$

$$\frac{\cos(x-y)}{\sin x \sin y} = \frac{\cos x \cos y + \sin x \sin y}{\sin x \sin y}$$

$$= \frac{\cos x \cos y}{\sin x \sin y} + \frac{\sin x \sin y}{\sin x \sin y}$$

$$= \cot x \cot y + 1$$ ✓

12, 17, 18, 19

Ex

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

$$\cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)}$$

$$= \boxed{\frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y}}$$

$$= \frac{\frac{\cos x \cos y}{\sin x \sin y} - \frac{\sin x \sin y}{\sin x \sin y}}{\frac{\sin x \cos y}{\sin x \sin y} + \frac{\cos x \sin y}{\sin x \sin y}}$$

$$= \frac{\cot x \cot y - 1}{\cot y + \cot x}$$ ✓

15, 16
26
29
30

Ex

$$\sec(x-y) = \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \sin^2 y}$$

$$\sec(x-y) = \frac{1}{\cos(x-y)} \cdot \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\cos(x+y)}{(\cos x \cos y + \sin x \sin y)(\cos x \cos y - \sin x \sin y)}$$

$$= \frac{\cos(x+y)}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}$$

$$= \frac{\cos(x+y)}{\cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y}$$

$$= \frac{\cos(x+y)}{\cos^2 x \underline{- \cos^2 y \sin^2 y} - \sin^2 y \underline{+ \cos^2 x \sin^2 y}}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \sin^2 y} \quad \checkmark$$

13, 27, 28

30) $$\frac{\cos(x-y)}{\cos(x+y)} = \frac{1+\tan x\tan y}{1-\tan x\tan y}$$

$$\frac{\cos(x-y)}{\cos(x+y)} = \frac{\dfrac{\cos x\cos y}{\cos x\cos y} + \dfrac{\sin x\sin y}{\cos x\cos y}}{\dfrac{\cos x\cos y}{\cos x\cos y} - \dfrac{\sin x\sin y}{\cos x\cos y}}$$

$$= \frac{1+\tan x\tan y}{1-\tan x\tan y} \checkmark$$

---

17) $$\frac{\sin(x-y)}{\sin x\cos y} = 1-\cot x\tan y$$

$$\frac{\sin(x-y)}{\sin x\cos y} = \frac{\sin x\cos y - \cos x\sin y}{\sin x\cos y}$$

$$= \frac{\cancel{\sin x}\cancel{\cos y}}{\cancel{\sin x}\cancel{\cos y}} - \frac{\cos x\sin y}{\sin x\cos y}$$

$$= 1-\cot x\tan y \checkmark$$

8.3 Double & half-angle

$\sin 2A = \sin(2A)$ double-angle

$\sin^2 A = (\sin A)^2$ square

$\sin x^2 = \sin(x^2)$

$\sin 2A \cancel{= 2} \sin A$

$$\boxed{\sin 2A} = \sin(A+A)$$
$$= \sin A \cos A + \cos A \sin A$$
$$= \boxed{2 \sin A \cos A}$$

$$\boxed{\cos 2A} = \cos(A+A)$$
$$= \cos A \cos A - \sin A \sin A$$
$$= \boxed{\cos^2 A - \sin^2 A}$$
$$= 1 - \sin^2 A - \sin^2 A$$
$$= \boxed{1 - 2\sin^2 A}$$
$$= \cos^2 A - (1 - \cos^2 A)$$
$$= \boxed{2\cos^2 A - 1}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$2\cos^2 x - 1 = \cos 2x$$

$$2\cos^2 x = 1 + \cos 2x$$

$$\begin{cases} \cos^2 \cancel{x}_{\frac{x}{2}} = \dfrac{1+\cos \cancel{2x}_{x}}{2} \\[2ex] \sin^2 x = \dfrac{1-\cos 2x}{2} \end{cases} \quad \begin{cases} \frac{1}{2} + \frac{1}{2}\cos 2x \\[2ex] \frac{1}{2}(\quad) \end{cases}$$

$$\cos^2 \frac{x}{2} = \frac{1}{2}(1+\cos x)$$

$$\sin^2 \frac{x}{2} = \frac{1}{2}(1-\cos x)$$

$$\left\{ \cos \frac{x}{2} = \underbrace{\pm}_{Q??} \sqrt{\frac{1}{2}(1+\cos x)} \right.$$

$$\tan \frac{A}{2} = \underbrace{\frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}}_{?}$$

$\sin A = \frac{3}{5} \quad A \in QII$

$\cos A = -\frac{4}{5}$

a) $\sin 2A = 2 \sin A \cos A$

$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= \boxed{-\frac{24}{25}}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{16}{25} - \frac{9}{25}$

$= \boxed{\frac{7}{25}}$

c) $\tan 2A = \boxed{-\frac{24}{7}}$

$\frac{90^\circ}{2} < \frac{A}{2} < \frac{180^\circ}{2} \quad \frac{A}{2} \in QI$

d) $\sin \frac{A}{2} = \sqrt{\frac{1}{2}(1 - \cos A)}$

$= \sqrt{\frac{1}{2}\left(1 + \frac{4}{5}\right)}$

$= \boxed{\frac{3}{\sqrt{10}}}$

e) $\cos \frac{A}{2} = +\sqrt{\frac{1}{2}(1 + \cos A)}$

$= \sqrt{\frac{1}{2}\left(1 - \frac{4}{5}\right)}$

$= \frac{1}{\sqrt{10}}$

f) $\tan \frac{A}{2} = 3$