

Solution **Section 3.5 - Curve Sketching (*Summary*)**

Exercise

Given $f(x) = x^4 - 4x^3 + 5$

Solution

$$f'(x) = 4x^3 - 12x^2 = 0$$

$$4x^2(x - 3) = 0$$

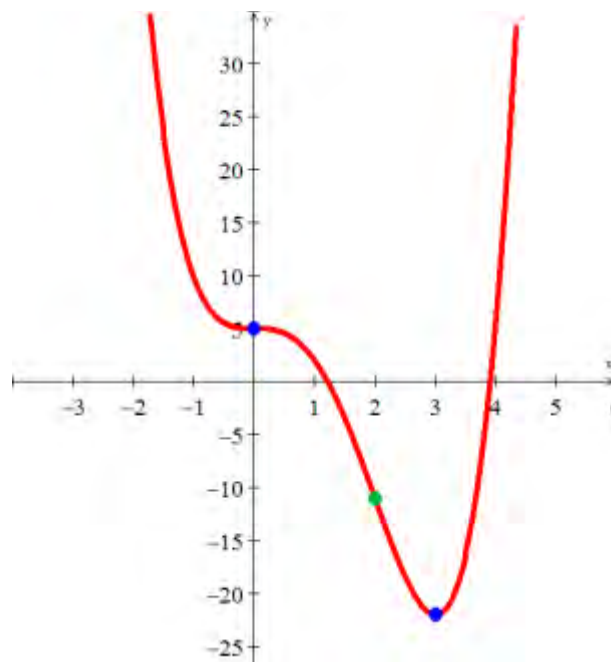
$$\Rightarrow x = 0, 0, 3$$

$$f''(x) = 12x^2 - 24x = 0$$

$$12x(x - 2) = 0$$

$$\Rightarrow x = 0, 2$$

	f	f'	f''	
$(-\infty, 0)$		-	+	Decreasing, Concave up
$x = 0$	5	0	0	RMAX
$(0, 2)$		-	-	Decreasing, Concave down
$x = 2$	-11	-	0	Point of Inflection
$(2, 3)$		-	+	Decreasing, Concave up
$x = 3$	-22	0	+	RMIN
$(3, \infty)$		+	+	Increasing, Concave up



Exercise

Given $f(x) = \frac{x^2+1}{x^2-1}$

Solution

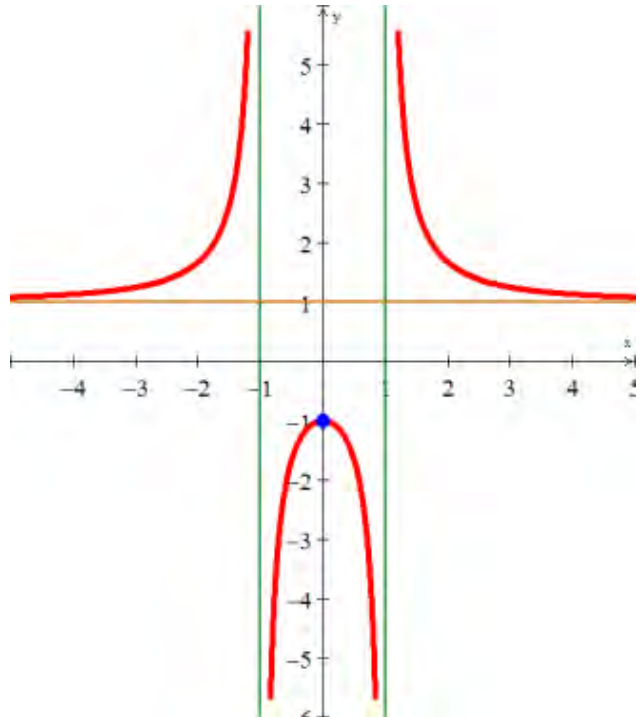
VA: $x = \pm 1$

HA: $y = 1$

$$\begin{aligned} f'(x) &= \frac{(2x)(x^2-1) - (x^2+1)(2x)}{(x^2-1)^2} \\ &= \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2} \\ &= \frac{-4x}{(x^2-1)^2} = 0 \\ &\Rightarrow x = 0 \end{aligned}$$

$$\begin{aligned} f'' &= \left(\frac{-4x}{(x^2-1)^2} \right)' \\ &= \frac{-4(x^2-1)^2 - (-4x)(x^2-1)(2x)}{(x^2-1)^4} \\ &= \frac{(x^2-1)[-4(x^2-1) - (-4x)(2x)]}{(x^2-1)^4} \\ &= \frac{-4x^2 + 4 + 8x^2}{(x^2-1)^3} \\ &= \frac{4x^2 + 4}{(x^2-1)^3} \\ &= \frac{4(x^2+1)}{(x^2-1)^3} = 0 \end{aligned}$$

$\Rightarrow x^2 + 1 = 0 \Rightarrow x^2 = -1$ (no zeros)



	f	f'	f''	
$(-\infty, -1)$		+	-	Increasing, Concave up
$x = -1$	Undef.	Undef.	Undef.	Vertical Asymptote
$(-1, 0)$		+	-	Increasing, Concave down
$x = 0$	-1	0	-	RMAX
$(0, 1)$		-	-	Decreasing, Concave down
$x = 1$	Undef.	Undef.	Undef.	Vertical Asymptote
$(1, \infty)$		-	+	Decreasing, Concave up

Exercise

Given $f(x) = 2x^{3/2} - 6x^{1/2}$

Solution

$$f'(x) = 3x^{1/2} - 3x^{-1/2} = 0$$

$$x^{1/2} (3x^{1/2} - 3x^{-1/2}) = 0$$

$$3x - 3 = 0$$

$$\Rightarrow x = 1$$

$$f''(x) = \frac{3}{2}x^{-1/2} + \frac{3}{2}x^{-3/2} = 0$$

$$\frac{2}{3}x^{3/2} \left(\frac{3}{2}x^{-1/2} + \frac{3}{2}x^{-3/2} \right) = 0$$

$$x + 1 = 0$$

$$\rightarrow x = -1 < 0$$

x	f	f'	f''	
$(0, 1)$		-	+	Decreasing, Concave up
$x = 1$	-4	0	+	RMIN
$(1, \infty)$		+	+	Increasing, Concave up

