

2.1

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\sin \theta \csc \theta = 1$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta \cos \theta = 1$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta \tan \theta = 1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\underline{\text{Ex}} \quad \sec \theta \tan \theta = \frac{1}{\cos \theta} \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\sin \theta}{\cos^2 \theta} (\cos \theta)^2$$

$$\underline{\text{Ex}} \quad \frac{1}{\sin \theta} + \frac{1}{\cos \theta} = \frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta}$$

Ex: $\frac{1}{\tan x} + \cot x$ in terms $\sin x + \cos x$

$$\begin{aligned}\tan x + \cot x &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \rightarrow = 1 \\ &= \frac{1}{\cos x \sin x}\end{aligned}$$

Prove: $\tan x + \cot x = \sin x (\sec x + \csc x)$

$$\begin{aligned}\sin x (\sec x + \csc x) &= \sin x \sec x + \sin x \csc x \\ &= \sin x \frac{1}{\cos x} + \sin x \frac{\cos x}{\sin x} \\ &= \tan x + \cos x \checkmark\end{aligned}$$

$$\tan x + \cot x = \frac{\sin x}{\cos x} + \cot x$$

$$= \frac{\sin x}{\cos x} + \cos x \frac{\sin x}{\sin x}$$

$$= \sin x \left(\frac{1}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$= \sin x (\sec x + \csc x) \checkmark$$

Prove! $\csc \alpha + 1 = \csc \alpha (\cos \alpha + \sin \alpha)$

$$\begin{aligned}\csc \alpha (\cos \alpha + \sin \alpha) &= \frac{1}{\sin \alpha} \cos \alpha + \frac{1}{\sin \alpha} \sin \alpha \\ &= \cot \alpha + 1 \quad \checkmark\end{aligned}$$

Prove! $\frac{\cos^4 t - \sin^4 t}{\cos^2 t} = 1 - \tan^2 t.$

$$\begin{aligned}\frac{\cos^4 t - \sin^4 t}{\cos^2 t} &= \frac{(\cos^2 t - \sin^2 t) (\overbrace{\cos^2 t + \sin^2 t}^{=1})}{\cos^2 t} \\ &= \frac{\cos^2 t - \sin^2 t}{\cos^2 t} \\ &= \frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t} \\ &= 1 - \tan^2 t. \quad \checkmark\end{aligned}$$

$$1 + \cos \theta = \frac{\sin^2 \theta}{1 - \cos \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\frac{\sin^2 \theta}{1 - \cos \theta} = \frac{1 - \cos^2 \theta}{1 - \cos \theta}$$

$$= \frac{(1 - \cos \theta)(1 + \cos \theta)}{1 - \cos \theta}$$

$$= 1 + \cos \theta \quad \checkmark$$

$$\star (1 + \cos \theta)(1 - \cos \theta) \stackrel{?}{=} \sin^2 \theta$$

$$1 - \cos^2 \theta \stackrel{?}{=} \sin^2 \theta$$

$$\sin^2 \theta = \sin^2 \theta \quad \checkmark$$

$$\begin{aligned}
 1 + \cos \theta &= (1 + \cos \theta) \frac{1 - \cos \theta}{1 - \cos \theta} \\
 &= \frac{1 - \cos^2 \theta}{1 - \cos \theta} \\
 &= \frac{\sin^2 \theta}{1 - \cos \theta} \quad \checkmark
 \end{aligned}$$

Prove: $\tan^2 \alpha (1 + \cot^2 \alpha) = \frac{1}{1 - \sin^2 \alpha}$

$$\begin{aligned}
 \tan^2 \alpha (1 + \cot^2 \alpha) &= \tan^2 \alpha + \tan^2 \alpha \cot^2 \alpha \\
 &= \frac{\sin^2 \alpha}{\cos^2 \alpha} + 1 \\
 &= \frac{\sin^2 \alpha + \cos^2 \alpha}{\cos^2 \alpha} \\
 &= \frac{1}{1 - \sin^2 \alpha} \quad \checkmark
 \end{aligned}$$

$$\frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} = 2 \csc \alpha$$

$$\begin{aligned}
 \frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} &= \frac{\sin^2 \alpha + (1 + \cos \alpha)^2}{\sin \alpha (1 + \cos \alpha)} \\
 &= \frac{\sin^2 \alpha + 1 + 2 \cos \alpha + \cos^2 \alpha}{\sin \alpha (1 + \cos \alpha)} \\
 &= \frac{2 + 2 \cos \alpha}{\sin \alpha (1 + \cos \alpha)} \\
 &= \frac{2(1 + \cos \alpha)}{\sin \alpha (1 + \cos \alpha)} \\
 &= 2 \csc \alpha \quad \checkmark
 \end{aligned}$$

prove $\frac{1 + \sin t}{\cos t} = \frac{\cos t}{1 - \sin t}$

$$\frac{\cos t}{1 - \sin t} = \frac{\cos t}{1 - \sin t} \cdot \frac{1 + \sin t}{1 + \sin t}$$

$$= \frac{\cos t (1 + \sin t)}{1 - \sin^2 t}$$

$$= \frac{\cos t (1 + \sin t)}{\cos^2 t}$$

$$= \frac{1 + \sin t}{\cos t} \quad \checkmark$$

$\cot^2 \theta + \cos^2 \theta \neq \cot^2 \theta \cos^2 \theta$

$\theta = \frac{\pi}{4}$ $\cot^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} \stackrel{?}{=} \cot^2 \frac{\pi}{4} \cos^2 \frac{\pi}{4}$

$$1 + \left(\frac{1}{\sqrt{2}}\right)^2 \stackrel{?}{=} 1 \left(\frac{1}{2}\right)$$

$$\frac{3}{2} \neq \frac{1}{2} \quad \checkmark$$

#1/ $\cos \theta \cot \theta + \sin \theta = \csc \theta$

$$\begin{aligned}\cos \theta \cot \theta + \sin \theta &= \cos \theta \frac{\cos \theta}{\sin \theta} + \sin \theta \\&= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta} \\&= \frac{1}{\sin \theta} \\&= \csc \theta \checkmark.\end{aligned}$$

#2/ $\sec \theta \cot \theta - \sin \theta = \frac{\cos^2 \theta}{\sin \theta}$

$$\begin{aligned}\sec \theta \cot \theta - \sin \theta &= \frac{1}{\cos \theta} \frac{\cos \theta}{\sin \theta} - \sin \theta \\&= \frac{1}{\sin \theta} - \sin \theta \\&= \frac{1 - \sin^2 \theta}{\sin \theta} \\&= \frac{\cos^2 \theta}{\sin \theta} \checkmark.\end{aligned}$$

#21/ $\frac{\cot^2 \theta + 3 \cot \theta - 4}{\cot \theta + 4} = \cot \theta - 1$

$$\begin{aligned}\frac{\cot^2 \theta + 3 \cot \theta - 4}{\cot \theta + 4} &= \frac{(\cot \theta + 4)(\cot \theta - 1)}{(\cot \theta + 4)} \\&= \cot \theta - 1 \checkmark.\end{aligned}$$

42d $\sin x (\tan x \cos x - \cot x \cos x) = 1 - 2 \cos^2 x$

$$\sin x (\tan x \cos x - \cot x \cos x)$$

$$= \sin x \frac{\sin x}{\cos x} \cos x - \sin x \frac{\cos x}{\sin x} \cos x$$

$$= \sin^2 x - \cos^2 x$$

$$= 1 - \cos^2 x - \cos^2 x$$

$$= 1 - 2 \cos^2 x \quad \checkmark$$

28 $7 \csc^2 x - 5 \cot^2 x = 2 \csc^2 x + 5$

$$7 \csc^2 x - 5 \cot^2 x = 7 \csc^2 x - 5 (\csc^2 x - 1)$$

$$= 7 \csc^2 x - 5 \csc^2 x + 5$$

$$= 2 \csc^2 x + 5 \quad \checkmark$$

$\csc^2 x - 1$

35 $\frac{\cot x + \csc x - 1}{\cot x - \csc x + 1} = \csc x + \cot x \quad \frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$$\frac{\cot x + \csc x - 1}{\cot x - \csc x + 1} = \frac{\frac{\cos x}{\sin x} + \frac{1}{\sin x} - 1}{\frac{\cos x}{\sin x} - \frac{1}{\sin x} + 1}$$

$$= \frac{\frac{\cos x + 1 - \sin x}{\sin x}}{\frac{\cos x - 1 + \sin x}{\sin x}}$$

$$= \frac{\cos x + 1 - \sin x}{\cos x + \sin x - 1} \cdot \frac{(\cos x + \sin x) + 1}{(\cos x + \sin x) + 1}$$

$$= \frac{\cos^2 x + \cos x \sin x + \cos x + \cos x \sin x + 1 - \sin x \cos x - \sin^2 x - \sin x}{\cos^2 x + 2\cos x \sin x + \sin^2 x - 1}$$

$$= \frac{\cos^2 x + 2\cos x + \cos^2 x}{2\cos x \sin x}$$

$$= \frac{2\cos^2 x + 2\cos x}{2\cos x \sin x}$$

$$= \frac{\cos x + 1}{\sin x}$$

$$= \frac{\cos x}{\sin x} + \frac{1}{\sin x}$$

$$= \cot x + \csc x \quad \checkmark$$

$$\begin{aligned} \star \cot x + \csc x - 1 &\stackrel{?}{=} (\cot x - \csc x + 1)(\csc x + \cot x) \\ &\stackrel{?}{=} \cot x \csc x + \cot^2 x - \csc^2 x - \csc x \cot x \\ &\quad + \csc x + \cot x \\ &= -1 + \csc x + \cot x \quad \checkmark \end{aligned}$$

$$1 + \cot^2 x = \csc^2 x$$

$$\cot^2 x - \csc^2 x = -1$$

$$39/ \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 1 + \sin x \cos x$$

$$\frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = \frac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\sin x - \cos x}$$

$$= 1 + \sin x \cos x \checkmark$$

$$71/ \frac{\csc x - 1}{\csc x + 1} = \frac{\cot^2 x}{\csc^2 x + 2\csc x + 1}$$

$$\frac{\csc x - 1}{\csc x + 1} = \frac{\csc x - 1}{\csc x + 1} \cdot \frac{\csc x + 1}{\csc x + 1}$$

$$= \frac{\csc^2 x - 1}{\csc^2 x + 2\csc x + 1}$$

$$= \frac{\cot^2 x}{\csc^2 x + 2\csc x + 1} \checkmark$$

$$75/ \csc^2 x - \cos^2 x \csc^2 x = 1$$

$$\csc^2 x - \cos^2 x \csc^2 x = \csc^2 x (1 - \cos^2 x)$$

$$= \frac{1}{\sin^2 x} \cdot (\sin^2 x)$$

$$= 1 \checkmark$$

$$77/ \quad \csc^2 x - \cos x \sec x = \cot^2 x$$

$$\begin{aligned} \csc^2 x - \cos x \sec x &= \frac{1}{\sin^2 x} - \cos x \frac{1}{\cos x} \\ &= \frac{1}{\sin^2 x} - 1 \\ &= \frac{1 - \sin^2 x}{\sin^2 x} \\ &= \frac{\cos^2 x}{\sin^2 x} \\ &= \cot^2 x \quad \checkmark \end{aligned}$$

$$78/ \quad (\sec x - \tan x)(\sec x + \tan x) = 1$$

$$\begin{aligned} (\sec x - \tan x)(\sec x + \tan x) &= \sec^2 x - \tan^2 x \\ &= \underline{1} \quad \checkmark \end{aligned}$$

$$1 + \tan^2 x = \sec^2 x$$

$$= \sec^2 x - (1 + \sec^2 x)$$

$$79/ \quad \frac{3 \csc^2 x - 5 \csc x - 28}{\csc x - 4} = \frac{3}{\sin x} + 7$$

$$\begin{aligned} \frac{3 \csc^2 x - 5 \csc x - 28}{\csc x - 4} &= \frac{(\csc x - 4)(3 \csc x + 7)}{\csc x - 4} \\ &= 3 \frac{1}{\sin x} + 7 \quad \checkmark \end{aligned}$$

Pro $10 \csc^2 x - 6 \cot^2 x = 4 \csc^2 x + 6$

$$10 \csc^2 x - 6 \cot^2 x = 10 \csc^2 x - 6 (\csc^2 x - 1)$$

$$= 10 \csc^2 x - 6 \csc^2 x + 6$$

$$= 4 \csc^2 x + 6 \checkmark$$