Definite Integral

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$

$$= F(b) - F(a)$$

1) Evalue $\int_a^b$ any $\mathbb{R}$.

2) Area, Vol, L, S > 0 $\mathbb{R}^+$

EX/ $\int_0^{\pi} \cos x\,dx = \sin x\Big|_0^{\pi}$

$\boxed{\begin{array}{l} \sin x + C\Big|_0^{\pi} \\ \sin \pi + C - \sin 0 - C \end{array}}$

$= \sin \pi - \sin 0$

$\underline{= 0}$
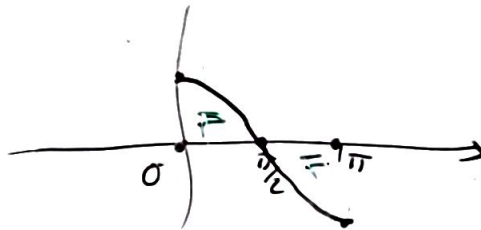
EX $\int_{-\frac{\pi}{4}}^{0} \sec x \tan x\,dx = \sec x\Big|_{-\frac{\pi}{4}}^{0}$

$= \sec 0 - \sec(-\frac{\pi}{4})$

$\underline{= 1 - \sqrt{2}}$

$$\int_1^4 \left(\frac{3}{2}\sqrt{x} - \frac{4}{x^2}\right)dx = x^{3/2} + \frac{4}{x}\Big|_1^4$$

$$\left(2^{2 \cdot \frac{3}{2}}\right) = 4^{3/2}$$

$$= 8 + 1 - (1 + 4)$$

$$= \underline{4}$$

✗ $8 + 1 - 1 - 4$

<u>Ex</u> $v(t) = 160 - 32t$ $v(t) = s'(t)$

$$s(t) = \int v(t)\,dt \qquad 0 \le t \le 8$$

$$S(t) = \int_0^8 (160 - 32t)\,dt$$

$$= 160t - 16t^2\Big|_0^8$$

$$= 1280 - 1024$$

$$= \underline{256}$$

$$v(t) = 160 - 32t = 0 \Rightarrow \underline{t = 5}$$

$$S(t) = \int_0^5 (160 - 32t)\,dt - \int_5^8 (160 - 32t)\,dt$$

$$= 160t - 16t^2\Big|_0^5 - \left(160t - 16t^2\Big|_5^8\right)$$

$$= \underline{800} - 16(25) - \left(1280 - 1024 - (\underline{800} - 16(5^2))\right)$$

$$= 400 - 256 + 400$$

$$= \underline{544}$$

$$\frac{64}{16}$$
$$\frac{384}{64}$$

6x $f(x) = x^2 - 4$ $g(x) = 4 - x^2$

$$[-2, 2]$$

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{2} (x^2 - 4)\,dx$$

$$= \frac{1}{3} x^3 - 4x \Big|_{-2}^{2}$$

$$= \frac{8}{3} - 8 - \left(-\frac{8}{3} + 8\right)$$

$$= \frac{16}{3} - 16 \longrightarrow 16\left(\frac{1}{3} - 1\right)$$

$$= -\frac{32}{3} \Big]\longleftarrow$$

$$\int_{-2}^{2} g(x)\,dx = \int_{-2}^{2} (4 - x^2)\,dx$$

$$= 4x - \frac{1}{3} x^3 \Big|_{-2}^{2}$$

$$= 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)$$

$$= 16 - \frac{16}{3}$$

$$= 16\left(1 - \frac{1}{3}\right) \longrightarrow$$

$$= \frac{32}{3} \Big]$$

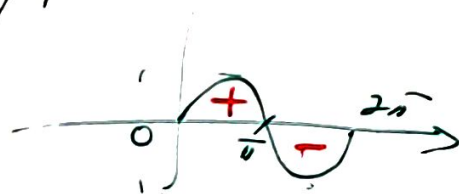$$\text{Area } (f x) = \left|-\frac{32}{3}\right| = \frac{32}{3}$$

Ex

$f(x) = \operatorname{sen} x$ $\qquad$ $x = 0$ y $x = 2\pi$

a) $\displaystyle\int_0^{2\pi} \operatorname{sen} x \, dx = -\cos x \Big|_0^{2\pi}$

$$= -(1-1)$$
$$= \underline{0}$$

b) area. $\quad f(x) = \operatorname{sen} x = 0 \Rightarrow x = 0, \pi, 2\pi$

$\text{Area} = \displaystyle\int_0^{\pi} \operatorname{sen} x \, dx - \int_\pi^{2\pi} \operatorname{sen} x \, dx$

$$= -\cos x \Big|_0^{\pi} + \cos x \Big|_\pi^{2\pi}$$
$$= -(-1-1) + 1 + 1$$
$$= \underline{4 \ \text{unit}^2}$$

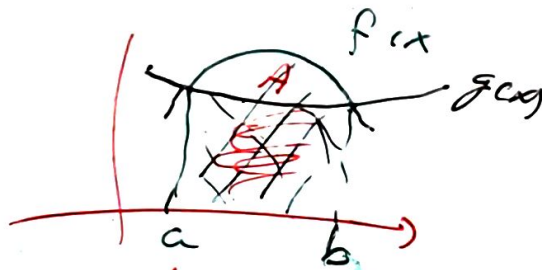**Ex** $f(x) = x^3 - x^2 - 2x$ $-1 \leq x \leq 2$

$A?$

$f(x) = x(x^2 - x - 2) = 0$ $x = 0, -1, 2$

$$\text{Area} = \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx - \int_{0}^{2} (x^3 - x^2 - 2x)\,dx$$

$$= \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 \Big|_{-1}^{0} - \left(\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right)\Big|_{0}^{2}$$

$$= -\left(\frac{1}{4} + \frac{1}{3} - 1\right) - \left(4 - \frac{8}{3} - 4\right)$$

$$= -\frac{7}{12} + 1 + \frac{8}{3}$$

$$= \frac{-7 + 12 + 32}{12}$$

$$= \frac{37}{12} \text{ unit}^2$$

Area Between curves.

$$A = \int_{a}^{b} (f(x) - g(x))\,dx$$

Ex A? $y = 2 - x^2$ & $y = -x$

Soln $y = 2 - x^2 = -x$

$$x^2 - x - 2 = 0 \Rightarrow \underline{x = -1, 2}$$

$$\text{Area} = \int_{-1}^{2} (2 - x^2 + x)\, dx$$
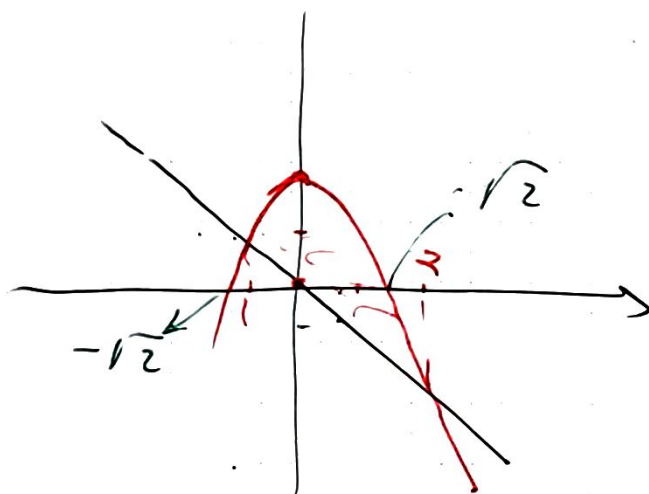
$$= 2x - \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_{-1}^{2}$$

$$= 4 - \frac{8}{3} + 2 - \left(-2 + \underbrace{\frac{1}{3} + \frac{1}{2}}_{5/6} \right)$$

$$= \frac{10}{3} - \left(-\frac{7}{6} \right)$$

$$= \frac{10}{3} + \frac{7}{6}$$

$$= \frac{27}{6}$$

$$= \frac{9}{2} \text{ unit}^2$$

$y = 2 - x^2$

$-2x = 0$

Ex Area? QI $y=\sqrt{x}$, x-axis, $y=x-2$

① ② ③ (with ① under √x, y=0 labeled ②, x-2 labeled ③)

① ∧ ② ⇒ $\boxed{(0,0)}$

① ∧ ③ ⇒ $\sqrt{x} = x-2$

$$x = (x-2)^2$$
$$x = x^2 - 4x + 4$$
$$x^2 - 5x + 4 \Rightarrow x = \begin{cases} 1 \to y = -1 \text{ ⊄} \\ 4 \Rightarrow y = 2 \end{cases}$$

(QI) circled

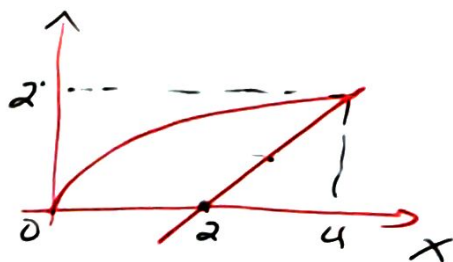$$\boxed{(4,2)}$$

② ∧ ③ → $x - 2 = 0 \Rightarrow x = 2 \Rightarrow (2,0)$

$x: \quad 0 \to 2 \to 4$

$$\text{Area} = \int_0^2 x^{1/2}\, dx + \int_2^4 (x^{1/2} - x + 2)\, dx$$
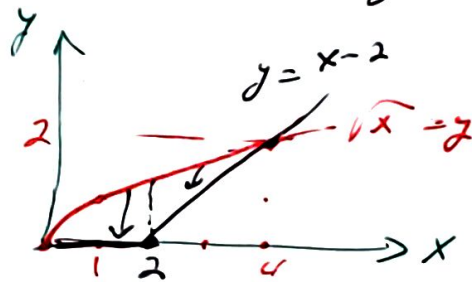
$$A = \frac{2}{3} x^{3/2}\Big|_0^2 + \left(\frac{2}{3} x^{3/2} - \frac{1}{2} x^2 + 2x \right)\Big|_2^4$$

$$= \frac{2}{3} 2^{3/2} + \frac{16}{3} - 8 + 8 - \left(\frac{2}{3} 2^{3/2} - 2 + 4 \right)$$

$$= \frac{16}{3} - 2$$

$$= \frac{10}{3} \text{ unit}^2$$

$$y = x - 2$$
$$x = y + 2$$
$$y = \sqrt{x}$$
$$x = y^2$$

$$\text{Area} = \int_0^2 (y + 2 - y^2)\, dy$$

$$= \frac{1}{2} y^2 + 2y - \frac{1}{3} y^3 \Big|_0^2$$

$$= 2 + 4 - \frac{8}{3}$$

$$= \frac{10}{3} \text{ unit}^2$$

#1 $\displaystyle\int_0^3 (2x + 1)\, dx = x^2 + x \Big|_0^3$

$$= 9 + 3$$
$$= 12$$

$$\frac{4^4}{4^2}$$

#3 $\displaystyle\int_0^4 \left(3x - \frac{1}{4} x^3\right) dx = \frac{3}{2} x^2 - \frac{1}{16} x^4 \Big|_0^4$

$$= 24 - 16$$
$$= 8$$

#2 $\displaystyle\int_1^7 \frac{dx}{x} = \ln x \Big|_1^7$ $\qquad \ln|x|$

$$= \ln 7 - \ln 1$$
$$= \ln 7$$

$$\ln 1 = 0$$
$$\ln e = 1$$

#29 $\displaystyle\int_{-2}^{-1}\left(3e^{3x}+\frac{2}{x}\right)dx = e^{3x}+2\ln|x|\ \Big|_{-2}^{-1}$

$= e^{-3}+2\ln 1 - \left(e^{-6}+2\ln 2\right)$

$= e^{-3}-e^{-6}+2\ln 2$

$= \dfrac{1}{e^3}-\dfrac{1}{e^6}+\ln 4$

#31 $A?$ $y=-x^2-2x$ $-3 \le x \le 2$

$y = -x(x-2)=0 \implies \underline{x=0,2}$

$A = \displaystyle\int_{-3}^{0}(-x^2-2x)dx - \int_{0}^{2}(-x^2-2x)dx$

$= -\dfrac{1}{3}x^3-x^2\ \Big|_{-3}^{0} - \left(-\dfrac{1}{3}x^3-x^2\right)\Big|_{0}^{2}$

$= -(9-9) - \left[-\dfrac{8}{3}-4\right]$

$= \dfrac{20}{3}\ \text{unit}^2$

#43 A? $f(x) = x^2 - 4x + 3$ $0 \le x \le 3$

$$x^2 - 4x + 3 = 0 \implies x = 1, 3$$

$$A = \int_0^1 (x^2 - 4x + 3)\, dx - \int_1^3 (x^2 - 4x + 3)\, dx$$

$$= \frac{1}{3}x^3 - 2x^2 + 3x \Big|_0^1 - \left(\frac{1}{3}x^3 - 2x^2 + 3x \Big|_1^3 \right)$$

$$= \frac{1}{3} - 2 + 3 - \left[9 - 18 + 9 - \left(\frac{1}{3} - 2 + 3 \right) \right]$$

$$= \frac{4}{3} + \frac{4}{3}$$

$$= \frac{8}{3} \text{ unit}^2$$

#48 $f(x) = 2x^2 + 4x + 2$ $-1 \le x \le 1$

$$2x^2 + 4x + 2 = 0 \implies x = -1, -1$$

$$A = \int_{-1}^1 (2x^2 + 4x + 2)\, dx$$

$$= \frac{2}{3}x^3 + 2x^2 + 2x \Big|_{-1}^1$$

$$= \frac{2}{3} + 2 + 2 - \left(-\frac{2}{3} + 2 - 2 \right)$$

$$= \frac{4}{3} + 4$$

$$= \frac{16}{3} \text{ unit}^2$$

function is odd or even
↓
power odd

↘ all power are even
$\mathbb{R} \longrightarrow$ fctn

$f(x)$
even

$$\int_{-a}^{a} f(x)\, dx = 2 \int_{0}^{a} f(x)\, dx$$

odd

$$\int_{-a}^{a} f(x)\, dx = 0$$