Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \tan\left(x - \frac{\pi}{4}\right)$

Solution

Amplitude: n / a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

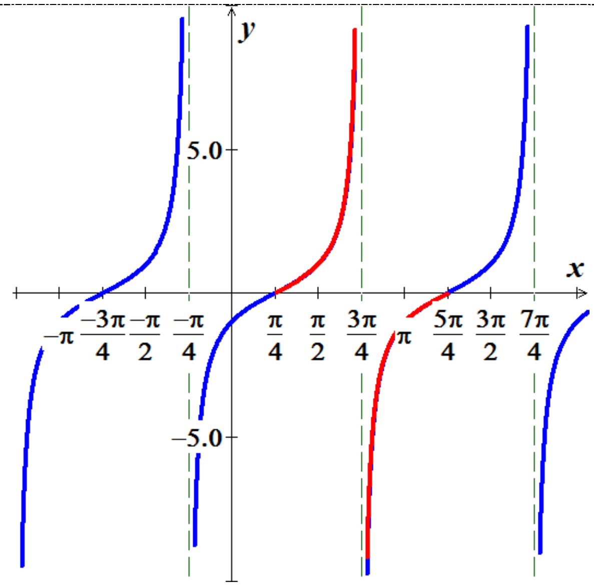
VT: $y = 0$

Asymptotes:

$$x = -\frac{\pi}{4} + n\pi$$

$$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$$

	x	y
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	1
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	∞
$\frac{3\pi}{4} + \frac{\pi}{4}$	π	-1
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\tan\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n / a

Period: $P = \frac{\pi}{2}$

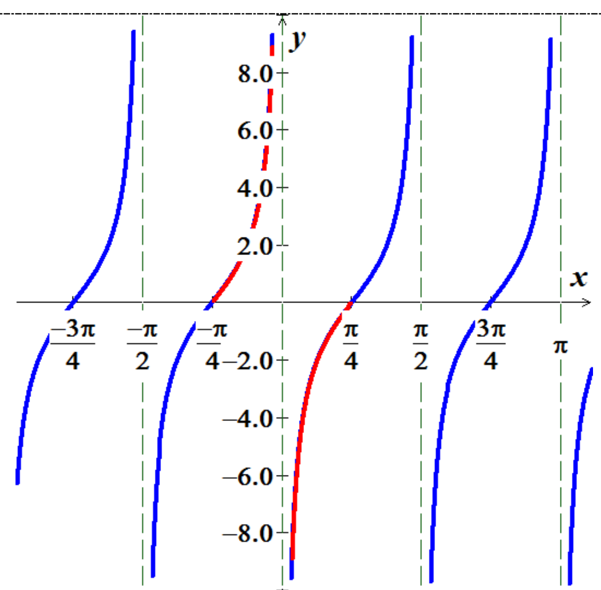
Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

Asymptotes: $x = n\frac{\pi}{2}$

$$x = \ldots, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \ldots$$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	0
$\frac{\pi}{8} - \frac{\pi}{4}$	$-\frac{\pi}{8}$	2
$\frac{\pi}{4} - \frac{\pi}{4}$	0	∞
$\frac{3\pi}{8} - \frac{\pi}{4}$	$\frac{\pi}{8}$	-2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y=-\frac{1}{4}\tan\left(\frac{1}{2}x+\frac{\pi}{3}\right)$

Solution

Amplitude: n/a

Period: $P=2\pi$

Phase Shift: $\varphi=-\frac{2\pi}{3}$

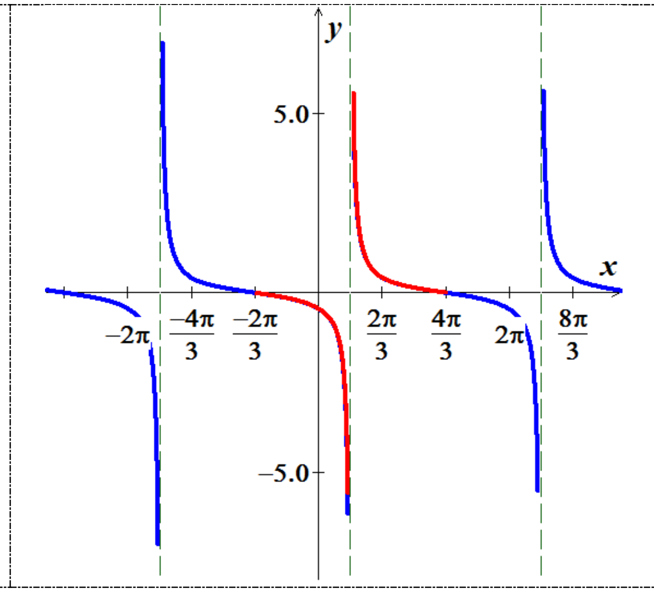
VT: $y=0$

Asymptotes:

$$x=-\frac{5\pi}{3}+2n\pi$$

$$x=\ldots,-\frac{5\pi}{3},\ \frac{\pi}{3},\ \frac{7\pi}{3},\ldots$$

	x	y
$0-\frac{2\pi}{3}$	$-\frac{2\pi}{3}$	0
$\frac{\pi}{2}-\frac{2\pi}{3}$	$-\frac{\pi}{6}$	$-\frac{1}{4}$
$\pi-\frac{2\pi}{3}$	$\frac{\pi}{3}$	∞
$\frac{3\pi}{2}-\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{1}{4}$
$2\pi-\frac{2\pi}{3}$	$\frac{4\pi}{3}$	0



Exercise

Find the period, show the asymptotes, and sketch the graph of $y=\cot\left(x+\frac{\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P=\pi$

Phase Shift: $\varphi=-\frac{\pi}{4}$

VT: $y=0$

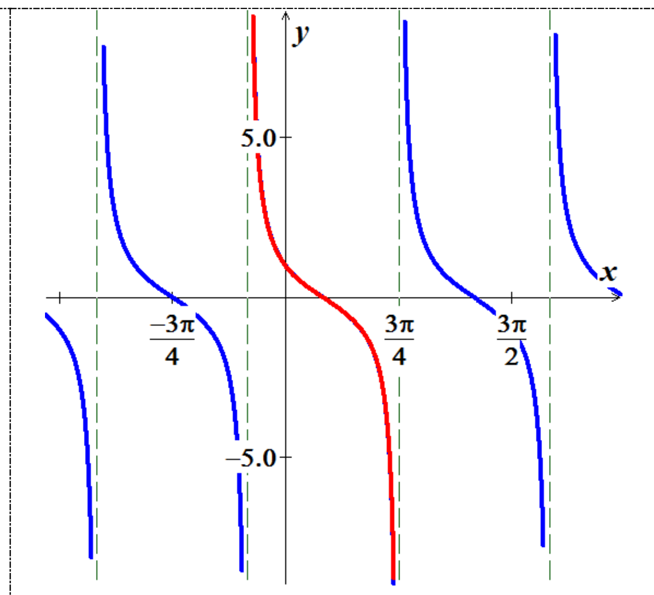
Asymptotes:

$$x=2n\pi+\pi-\frac{\pi}{4}$$

$$x=2n\pi+\frac{3\pi}{4}$$

$$x=\ldots,-\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{7\pi}{4},\ldots$$

	x	y
$0-\frac{\pi}{4}$	$-\frac{\pi}{4}$	∞
$\frac{\pi}{4}-\frac{\pi}{4}$	0	1
$\frac{\pi}{2}-\frac{\pi}{4}$	$\frac{\pi}{4}$	0
$\frac{3\pi}{4}-\frac{\pi}{4}$	$\frac{\pi}{2}$	-1
$\pi-\frac{\pi}{4}$	$\frac{3\pi}{4}$	∞



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\cot\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \frac{\pi}{2}$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 0$

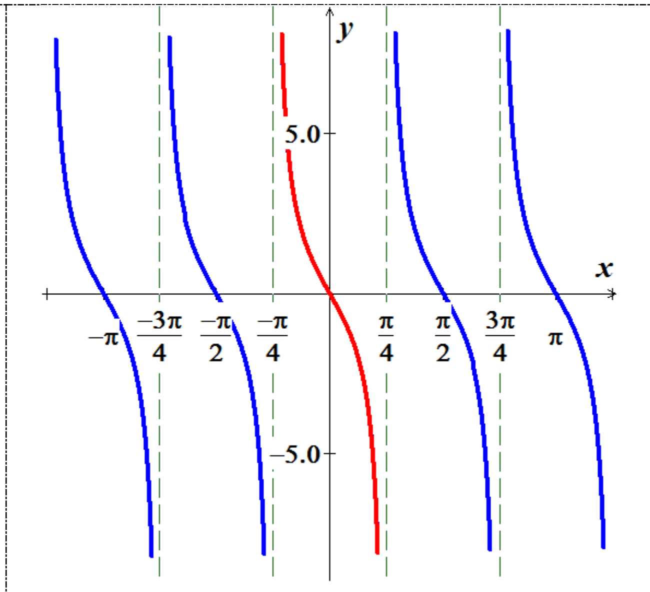
Asymptotes:

$$2x + \frac{\pi}{2} = (2n+1)\pi$$

$$2x = 2n\pi + \frac{\pi}{2}$$

$$x = \dots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \dots$$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	∞
$\frac{\pi}{8} - \frac{\pi}{4}$	$-\frac{\pi}{8}$	2
$\frac{\pi}{4} - \frac{\pi}{4}$	0	0
$\frac{3\pi}{8} - \frac{\pi}{4}$	$\frac{\pi}{8}$	-2
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	∞



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{2}\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

Solution

Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = -\frac{\pi}{2}$

VT: $y = 0$

Asymptotes:

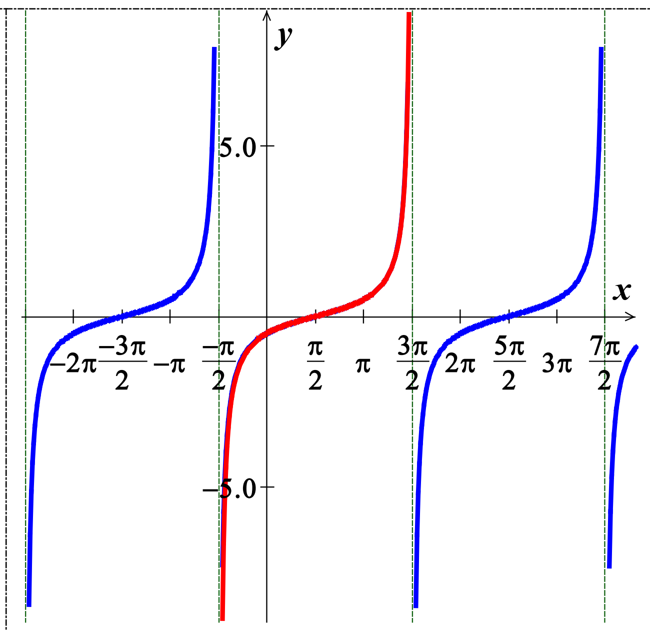
$$\frac{1}{2}x + \frac{\pi}{4} = 2n\pi + \pi$$

$$\frac{1}{2}x = 2n\pi + \frac{3\pi}{4}$$

$$x = 4n\pi + \frac{3\pi}{2}$$

$$x = \dots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{11\pi}{2}, \dots$$

	x	y
$0 - \frac{\pi}{2}$	$-\frac{\pi}{2}$	∞
$\frac{\pi}{2} - \frac{\pi}{2}$	0	$-\frac{1}{2}$
$\pi - \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{3\pi}{2} - \frac{\pi}{2}$	π	$\frac{1}{2}$
$2\pi - \frac{\pi}{2}$	$\frac{3\pi}{2}$	∞



Exercise

Graph over a 1-period interval $y = 1 - 2\cot 2\left(x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a

Period: $P = \frac{\pi}{2}$

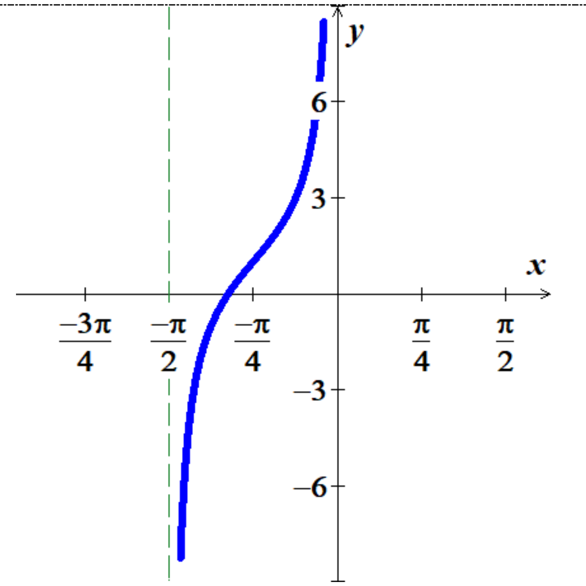
Phase Shift: $\varphi = -\frac{\pi}{2}$

VT: $y = 1$

Asymptotes:

$-\frac{\pi}{2} + n\pi$

	x	y
$0 - \frac{\pi}{2}$	$-\frac{\pi}{2}$	∞
$\frac{\pi}{8} - \frac{\pi}{2}$	$-\frac{3\pi}{8}$	-1
$\frac{\pi}{4} - \frac{\pi}{2}$	$-\frac{\pi}{4}$	1
$\frac{3\pi}{8} - \frac{\pi}{2}$	$-\frac{\pi}{8}$	3
$\frac{\pi}{2} - \frac{\pi}{2}$	0	∞



Exercise

Graph over a 1-period interval $y = \frac{2}{3}\tan\left(\frac{3}{4}x - \pi\right) - 2$

Solution

Amplitude: n/a

Period: $P = \frac{4\pi}{3}$

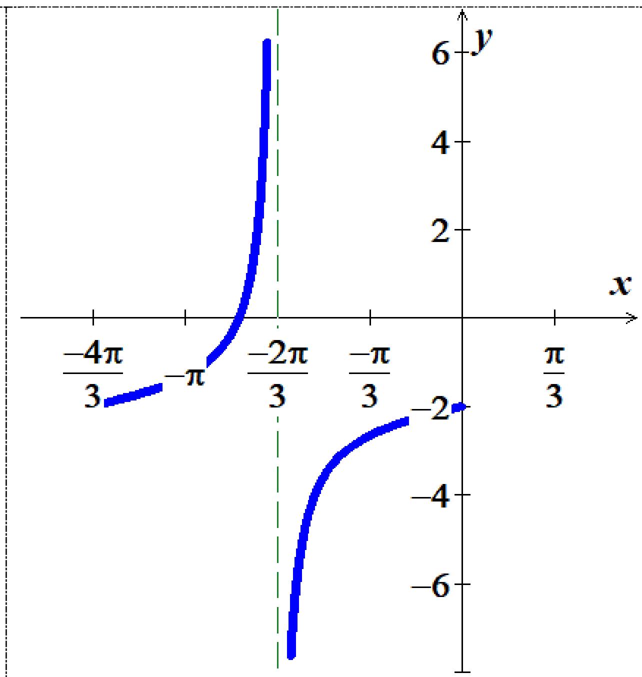
Phase Shift: $\varphi = -\frac{4\pi}{3}$

VT: $y = -2$

Asymptotes:

$x = \frac{3\pi}{4} + n\pi$

	x	y
$0 - \frac{4\pi}{3}$	$-\frac{4\pi}{3}$	-2
$\frac{\pi}{3} - \frac{4\pi}{3}$	$-\pi$	$-\frac{4}{3}$
$\frac{2\pi}{3} - \frac{4\pi}{3}$	$-\frac{2\pi}{3}$	∞
$\pi - \frac{4\pi}{3}$	$-\frac{\pi}{3}$	$-\frac{8}{3}$
$\frac{4\pi}{3} - \frac{4\pi}{3}$	0	-2



Exercise

Graph one complete cycle $y = 3 + 2\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)$

Solution

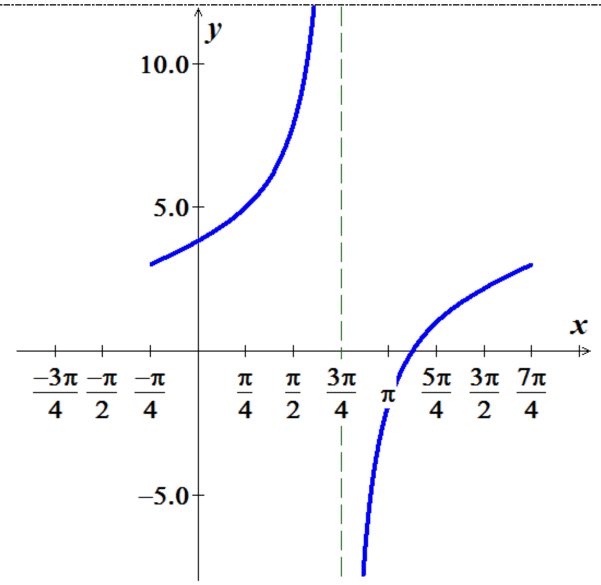
Amplitude: n/a

Period: $P = 2\pi$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: $y = 3$

	x	y
$0 - \frac{\pi}{4}$	$-\frac{\pi}{4}$	3
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	5
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	∞
$\frac{3\pi}{2} - \frac{\pi}{4}$	$\frac{5\pi}{4}$	1
$2\pi - \frac{\pi}{4}$	$\frac{7\pi}{4}$	3



Exercise

Graph one complete cycles $y = -2 - \cot\left(x - \frac{\pi}{4}\right)$

Solution

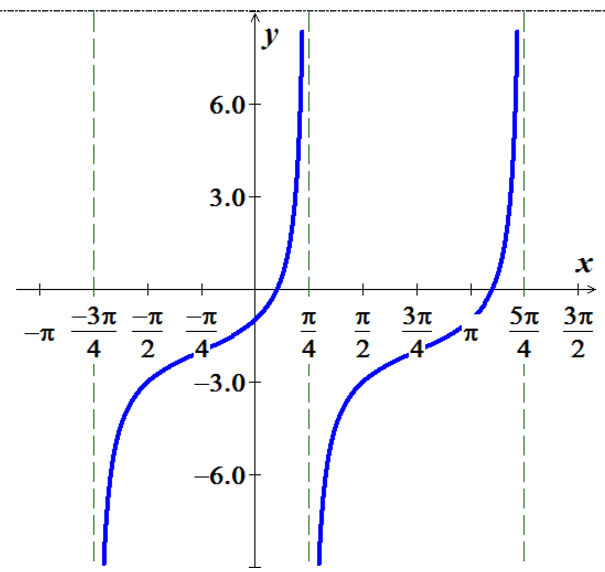
Amplitude: n/a

Period: $P = \pi$

Phase Shift: $\varphi = \frac{\pi}{4}$

VT: $y = -2$

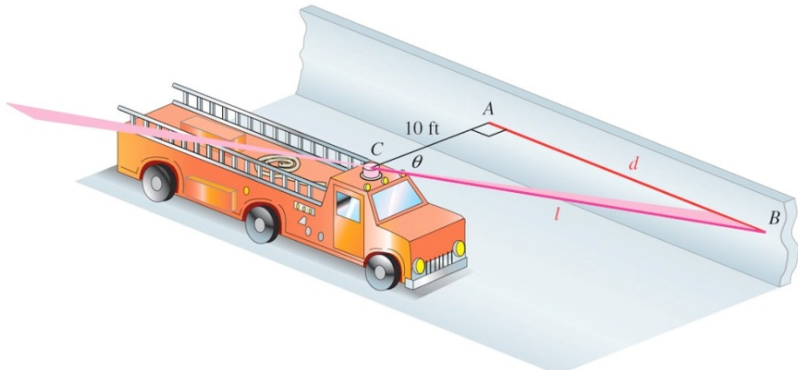
	x	y
$0 + \frac{\pi}{4}$	$\frac{\pi}{4}$	∞
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	-3
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	-2
$\frac{3\pi}{4} + \frac{\pi}{4}$	π	-1
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	∞



Exercise

A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top is 10 *feet* from the wall and rotates through one complete revolution every 2 *seconds*. Graph the function that gives the length d in terms of time t from $t = 0$ to $t = 2$.

Solution



$$\omega = \frac{\theta}{t} = \frac{2\pi}{2} = \pi \ rad / sec$$

$$\tan\theta = \frac{d}{10} \quad \rightarrow \ d = 10\tan\theta$$

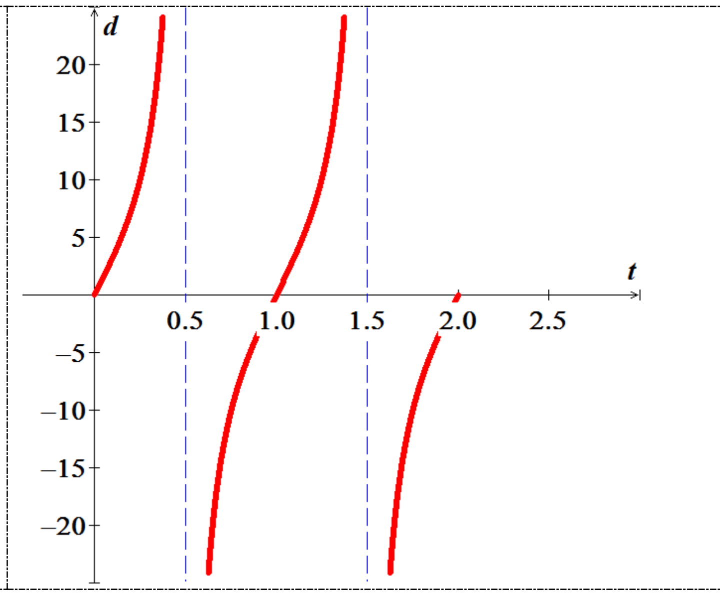
$$d(t) = 10\tan \pi t$$

Period $= \frac{\pi}{\pi} = 1$

One cycle: $0 \le \pi t \le \pi$

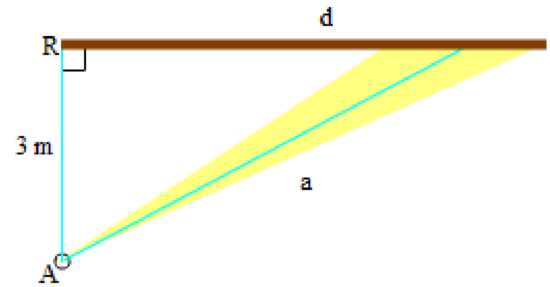
$0 \le t \le 1$

t	$d = 10\tan \pi t$
0	0
$\frac{1}{4}$	10
$\frac{1}{2}$	∞
$\frac{3}{4}$	-10
1	0



Exercise

A rotating beacon is located 3 *m* south of point *R* on an east-west wall. *d*, the length of the light display along the wall from *R*, is given by $d = 3\tan 2\pi t$, where *t* is time measured in seconds since the beacon started rotating. (When $t = 0$, the beacon is aimed at point *R*. When the beacon is aimed to the right of *R*, the value of *d* is positive; *d* is negative if the beacon is aimed to the left of *R*.) Find a for $t = 0.8$



Solution

$$d = 3\tan\left(2\pi(0.8)\right)$$

$$\approx -9.23\ m$$

Exercise

Let a person whose eyes are h_1 feet from the ground stand ***d*** feet from an object h_1 feet tall, where $h_2 > h_1$ feet. Let θ be the angle of elevation to the top of the object.

a) Show that $d = \left(h_2 - h_1\right)\cot\theta$

b) Let $h_2 = 55$ and $h_1 = 5$. Graph ***d*** for the interval $0 < \theta \le \frac{\pi}{2}$

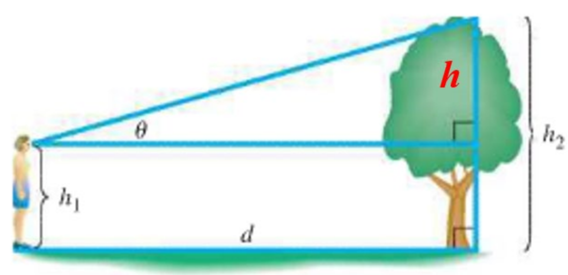
Solution

a)

$$h = h_2 - h_1$$

$$\cot\theta = \frac{d}{h}$$

$$d = \left(h_2 - h_1\right)\cot\theta$$



b)

$$d = (55 - 5)\cot\theta$$

$$d = 50\cot\theta \quad 0 < \theta \le \frac{\pi}{2}$$