

Series

$$\sum_{k=1}^n c = nc \qquad \sum_{k=m}^n c = (n-m+1)c$$

Arithmetic

$$\sum_{k=1}^n k = 1 + 2 + 3 + \dots + n = \frac{1}{2}n(n+1)$$

$$\sum_{k=1}^n (2k-1) = 1 + 3 + 5 + \dots + (2n-1) = n^2$$

Geometric

$$\sum_{k=1}^n ar^k = a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \dots = \frac{a}{1-r} \quad \text{if } -1 < r < 1$$

$$\sum_{k=1}^{\infty} ar^{-k} = \frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \dots = \frac{a}{r-1}$$

$$\sum_{k=1}^n r^k = r + r^2 + r^3 + \dots + r^n = \frac{r(1-r^n)}{1-r}$$

$$\sum_{k=1}^{\infty} r^k = r + r^2 + r^3 + \dots = \frac{r}{1-r}$$

$$\sum_{k=0}^n (a+kd)r^k = a + (a+d)r + (a+2d)r^2 + \dots + (a+(n-1)d)r^{n-1}$$

$$= \frac{a(1-r^n)}{1-r} + \frac{rd[1-nr^{n-1} + (n-1)r^n]}{(1-r)^2}$$

$$\sum_{k=0}^{\infty} (a+kd)r^k = a + (a+d)r + (a+2d)r^2 + \dots = \frac{a}{1-r} + \frac{rd}{(1-r)^2} \quad \text{if } -1 < r < 1$$

$$\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} = (1+2+3+\dots+n)^2$$

$$\sum_{k=1}^n k^4 = 1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}, \quad |x| \leq 1$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

$$\ln x = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} (x-1)^{n+1} = x-1 - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \dots$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots \quad -1 \leq x < 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \quad |x| < 1$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \dots \quad |x| < 1$$

$$(1+x)^\alpha = 1 + \alpha x + \alpha(\alpha-1) \frac{x^2}{2!} + \alpha(\alpha-1)(\alpha-2) \frac{x^3}{3!} + \dots$$

Legendre Polynomial

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

$$P_n(x) = \frac{1}{2^n} \sum_{k=0}^{n/2} \frac{(-1)^k (2n-2k)!}{k!(n-k)!(n-2k)!} x^{n-2k}$$

The n th term of an arithmetic sequence: $a_n = a_1 + (n-1)d$

$$n = \frac{a_n - a_1}{d} + 1$$

Sum of a certain number of terms of an arithmetic sequence $S_n = \frac{n}{2}(a_1 + a_n)$

The **arithmetic mean** of two numbers a and b is defined as $\frac{a+b}{2}$

The n th term of a geometric sequence: $a_n = a_1 r^{n-1}$ $r = \frac{a_n}{a_{n-1}}$

Sum of n th term of a geometric sequence: $S_n = a_1 \frac{1-r^n}{1-r}$

Sum of infinite term of a geometric sequence: $S_n = \frac{a_1}{1-r}$

SERIES CONVERGENCE/DIVERGENCE FLOW CHART

TEST FOR DIVERGENCE

