

## ***Solution***

## **Section 2.2 – Function Operations**

### ***Exercise***

Find the domain:  $f(x) = 7x + 4$

### **Solution**

**Domain:**  $(-\infty, \infty)$

### ***Exercise***

Find the domain:  $f(x) = |3x - 2|$

### **Solution**

**Domain:**  $\mathbb{R}$

### ***Exercise***

Find the domain:  $f(x) = 3x + \pi$

### **Solution**

**Domain:**  $\mathbb{R}$

### ***Exercise***

Find the domain:  $f(x) = \sqrt{7}x + \frac{1}{2}$

### **Solution**

**Domain:**  $\mathbb{R}$

### ***Exercise***

Find the domain:  $f(x) = -2x^2 + 3x - 5$

### **Solution**

**Domain:**  $\mathbb{R}$

### ***Exercise***

Find the domain:  $f(x) = x^3 - 2x^2 + x - 3$

#### **Solution**

*Domain:*  $\mathbb{R}$

### ***Exercise***

Find the domain:  $f(x) = x^2 - 2x - 15$

#### **Solution**

*Domain:*  $\mathbb{R}$

### ***Exercise***

Find the domain  $f(x) = 4 - \frac{2}{x}$

#### **Solution**

*Domain:*  $x \neq 0$

### ***Exercise***

Find the domain  $f(x) = \frac{1}{x^4}$

#### **Solution**

*Domain:*  $x \neq 0$

### ***Exercise***

Find the domain:  $g(x) = \frac{3}{x-4}$

#### **Solution**

*Domain:*  $x \neq 4$

### ***Exercise***

Find the domain  $y = \frac{2}{x-3}$

#### **Solution**

*Domain:*  $x \neq 3$

### ***Exercise***

Find the domain  $y = \frac{-7}{x-5}$

#### **Solution**

**Domain:**  $x \neq 5$

### ***Exercise***

Find the domain  $f(x) = \frac{x+5}{2-x}$

#### **Solution**

$$2 - x \neq 0$$

**Domain:**  $x \neq 2$

### ***Exercise***

Find the domain  $f(x) = \frac{8}{x+4}$

#### **Solution**

$$x + 4 \neq 0$$

**Domain:**  $x \neq -4$

### ***Exercise***

Find the domain  $f(x) = \frac{1}{x+4}$

#### **Solution**

**Domain:**  $x \neq -4$

### ***Exercise***

Find the domain  $f(x) = \frac{1}{x-4}$

#### **Solution**

**Domain:**  $x \neq 4$

### ***Exercise***

Find the domain  $f(x) = \frac{3x}{x+2}$

#### **Solution**

**Domain:**  $x \neq -2$

### ***Exercise***

Find the domain  $f(x) = x - \frac{2}{x-3}$

#### **Solution**

**Domain:**  $x \neq 3$

### ***Exercise***

Find the domain  $f(x) = x + \frac{3}{x-5}$

#### **Solution**

**Domain:**  $x \neq 5$

### ***Exercise***

Find the domain  $f(x) = \frac{1}{2}x - \frac{8}{x+7}$

#### **Solution**

**Domain:**  $x \neq -7$

### ***Exercise***

Find the domain  $f(x) = \frac{1}{x-3} - \frac{8}{x+7}$

#### **Solution**

**Domain:**  $x \neq -7, 3$

### ***Exercise***

Find the domain  $f(x) = \frac{1}{x+4} - \frac{2x}{x-4}$

#### **Solution**

**Domain:**  $x \neq \pm 4$

### Exercise

Find the domain  $f(x) = \frac{3x^2}{x+3} - \frac{4x}{x-2}$

#### Solution

Domain:  $x \neq -3, 2$

### Exercise

Find the domain  $f(x) = \frac{1}{x^2 - 2x + 1}$

#### Solution

$$x^2 - 2x + 1 \neq 0 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

Domain:  $x \neq 1$

### Exercise

Find the domain  $f(x) = \frac{x}{x^2 + 3x + 2}$

#### Solution

$$x^2 + 3x + 2 \neq 0 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

Domain:  $x \neq -1, -2$

### Exercise

Find the domain  $f(x) = \frac{x^2}{x^2 - 5x + 4}$

#### Solution

$$x^2 - 5x + 4 \neq 0 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

Domain:  $x \neq -1, -2$

### Exercise

Find the domain  $f(x) = \frac{1}{x^2 - 4x - 5}$

#### Solution

$$x^2 - 4x - 5 \neq 0 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

Domain:  $x \neq -1, 5$

### Exercise

Find the domain  $g(x) = \frac{2}{x^2 + x - 12}$

#### Solution

$$x^2 + x - 12 \neq 0$$
$$(x + 4)(x - 3) \neq 0$$

**Domain:**  $\underline{x \neq -4, 3} \mid \underline{(-\infty, -4) \cup (-4, 3) \cup (3, \infty)}$

### Exercise

Find the domain  $h(x) = \frac{5}{\frac{4}{x} - 1}$

#### Solution

$$x \neq 0 \quad \frac{4}{x} - 1 \neq 0$$
$$\frac{4 - x}{x} \neq 0$$
$$4 - x \neq 0$$
$$x \neq 4$$

**Domain:**  $\underline{x \neq 0, 4} \mid \underline{(-\infty, 0) \cup (0, 4) \cup (4, \infty)}$

### Exercise

Find the domain  $y = \sqrt{x}$

#### Solution

$$x \geq 0$$

**Domain:**  $\underline{x \geq 0} \mid \underline{[0, \infty)}$

### Exercise

Find the domain  $f(x) = \sqrt{8 - 3x}$

#### Solution

$$8 - 3x \geq 0$$

$$8 \geq 3x$$

**Domain:**  $\underline{x \leq \frac{8}{3}} \mid \underline{(-\infty, \frac{8}{3}]}$

### Exercise

Find the domain  $y = \sqrt{4x+1}$

#### Solution

$$4x+1 \geq 0 \Rightarrow x \geq -\frac{1}{4}$$

$$\text{Domain: } \underline{x \geq -\frac{1}{4}} \quad \left[ -\frac{1}{4}, \infty \right)$$

### Exercise

Find the domain  $y = \sqrt{7-2x}$

#### Solution

$$7-2x \geq 0$$

$$-2x \geq -7$$

$$\text{Domain: } \underline{x \leq \frac{7}{2}} \quad \left( -\infty, \frac{7}{2} \right]$$

### Exercise

Find the domain  $f(x) = \sqrt{8-x}$

#### Solution

$$8-x \geq 0$$

$$\text{Domain: } \underline{x \leq 8} \quad (-\infty, 8]$$

### Exercise

Find the domain  $f(x) = \sqrt{3-2x}$

#### Solution

$$\text{Domain: } \underline{x \leq \frac{3}{2}} \quad \left( -\infty, \frac{3}{2} \right]$$

### Exercise

Find the domain  $f(x) = \sqrt{3+2x}$

#### Solution

$$\text{Domain: } \underline{x \geq -\frac{3}{2}} \quad \left[ -\frac{3}{2}, \infty \right)$$

### ***Exercise***

Find the domain  $f(x) = \sqrt{5-x}$

### **Solution**

Domain:  $x \leq 5$

### ***Exercise***

Find the domain  $f(x) = \sqrt{x-5}$

### **Solution**

Domain:  $x \geq 5$

### ***Exercise***

Find the domain  $f(x) = \sqrt{6-3x}$

### **Solution**

Domain:  $x \leq 2$

### ***Exercise***

Find the domain  $f(x) = \sqrt{3x-6}$

### **Solution**

Domain:  $x \geq 2$

### ***Exercise***

Find the domain  $f(x) = \sqrt{2x+7}$

### **Solution**

Domain:  $x \geq -\frac{7}{2}$

### ***Exercise***

Find the domain  $f(x) = \sqrt{x^2-16}$

### **Solution**

$$x^2 - 16 = 0$$



$$x^2 = 16$$

$$x = \pm 4$$

$$\text{Domain: } \underline{x \leq -4 \quad x \geq 4}$$

### Exercise

Find the domain  $f(x) = \sqrt{16 - x^2}$

#### Solution

$$x = \pm 4$$

$$\text{Domain: } \underline{-4 \leq x \leq 4}$$

### Exercise

Find the domain  $f(x) = \sqrt{9 - x^2}$

#### Solution

$$x = \pm 3$$

$$\text{Domain: } \underline{-3 \leq x \leq 3}$$

### Exercise

Find the domain  $f(x) = \sqrt{x^2 - 25}$

#### Solution

$$x = \pm 5$$

$$\text{Domain: } \underline{-5 \leq x \leq 5}$$

### Exercise

Find the domain  $f(x) = \sqrt{x^2 - 5x + 4}$

#### Solution

$$x^2 - 5x + 4 = 0 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

$$x = 1, 4$$

$$\text{Domain: } \underline{x \leq 1 \quad x \geq 4}$$

### ***Exercise***

Find the domain  $f(x) = \sqrt{x^2 + 5x + 4}$

#### **Solution**

$$x^2 + 5x + 4 \qquad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x = -1, -4$$

$$\text{Domain: } \underline{x \leq -4 \quad x \geq -1}$$

### ***Exercise***

Find the domain  $f(x) = \sqrt{x^2 + 3x + 2}$

#### **Solution**

$$x^2 + 3x + 2 \qquad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x = -1, -2$$

$$\text{Domain: } \underline{x \leq -2 \quad x \geq -1}$$

### ***Exercise***

Find the domain  $f(x) = \sqrt{x^2 - 3x + 2}$

#### **Solution**

$$x^2 - 3x + 2 \qquad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

$$x = 1, 2$$

$$\text{Domain: } \underline{x \leq 1 \quad x \geq 2}$$

### ***Exercise***

Find the domain  $f(x) = \sqrt{x-4} + \sqrt{x+1}$

#### **Solution**

$$x \geq 4 \quad x \geq -1$$

$$\text{Domain: } \underline{x \geq 4}$$

**Exercise**

Find the domain  $f(x) = \sqrt{3-x} + \sqrt{x-2}$

**Solution**

$$x \leq 3 \quad x \geq 2$$

$$\text{Domain: } \underline{2 \leq x \leq 3}$$

**Exercise**

Find the domain  $f(x) = \sqrt{1-x} + \sqrt{4-x}$

**Solution**

$$x \leq 1 \quad x \leq 4$$

$$\text{Domain: } \underline{x \leq 1}$$

**Exercise**

Find the domain  $f(x) = \sqrt{1-x} - \sqrt{x-3}$

**Solution**

$$x \leq 1 \quad x \geq 3$$

$$\text{Domain: } \underline{\emptyset}$$

**Exercise**

Find the domain  $f(x) = \sqrt{x+4} - \sqrt{x-1}$

**Solution**

$$x \geq -4 \quad x \geq 1$$

$$\text{Domain: } \underline{x \geq 1}$$

**Exercise**

Find the domain  $f(x) = \frac{\sqrt{x+1}}{x}$

**Solution**

$$x+1 \geq 0$$

$$x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \underline{x \geq -1 \quad x \neq 0} \quad \underline{[-1, 0) \cup (0, \infty)}$$

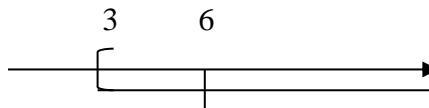
### Exercise

Find the domain  $g(x) = \frac{\sqrt{x-3}}{x-6}$

#### Solution

$$\rightarrow \begin{cases} x \geq 3 \\ x \neq 6 \end{cases}$$

Domain:  $\underline{x \geq 3 \quad x \neq 6} \quad \underline{[3, 6) \cup (6, \infty)}$



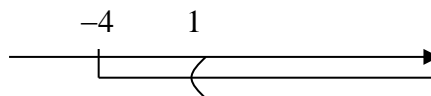
### Exercise

Find the domain  $f(x) = \frac{\sqrt{x+4}}{\sqrt{x-1}}$

#### Solution

$$\rightarrow \begin{cases} x \geq -4 \\ x > 1 \end{cases}$$

Domain:  $\underline{x > 1} \quad \underline{(1, \infty)}$



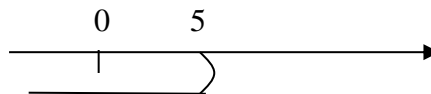
### Exercise

Find the domain  $f(x) = \frac{\sqrt{5-x}}{x}$

#### Solution

$$x \leq 5 \quad x \neq 0$$

Domain:  $\underline{x \leq 5 \quad x \neq 0} \quad \underline{(-\infty, 0) \cup (0, 5]}$



### Exercise

Find the domain  $f(x) = \frac{x}{\sqrt{5-x}}$

#### Solution

Domain:  $\underline{x < 5} \quad \underline{(-\infty, 5)}$

### ***Exercise***

Find the domain  $f(x) = \frac{1}{x\sqrt{5-x}}$

#### **Solution**

$$x < 5 \quad x \neq 0$$

$$\text{Domain: } \underline{x < 5 \quad x \neq 0} \mid \underline{(-\infty, 0) \cup (0, 5)}$$

### ***Exercise***

Find the domain  $f(x) = \frac{x+1}{x^3-4x}$

#### **Solution**

$$x^3 - 4x \neq 0$$

$$x(x^2 - 4) \neq 0$$

$$\text{Domain: } \underline{x \neq 0, \pm 2} \mid (-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$$

### ***Exercise***

Find the domain  $f(x) = \frac{\sqrt{x+5}}{x}$

#### **Solution**

$$x \geq -5 \quad x \neq 0$$

$$\text{Domain: } \underline{x \geq -5 \quad x \neq 0}$$

### ***Exercise***

Find the domain  $f(x) = \frac{x}{\sqrt{x+5}}$

#### **Solution**

$$x > -5$$

$$\text{Domain: } \underline{x > -5}$$

### ***Exercise***

Find the domain  $f(x) = \frac{1}{x\sqrt{x+5}}$

### **Solution**

$$x > -5 \quad x \neq 0$$

$$\text{Domain: } \underline{x > -5 \quad x \neq 0}$$

### ***Exercise***

Find the domain  $f(x) = \frac{x+3}{\sqrt{x-3}}$

### **Solution**

$$\text{Domain: } \underline{x > 3}$$

### ***Exercise***

Find the domain  $f(x) = \frac{\sqrt{x+3}}{\sqrt{x-3}}$

### **Solution**

$$x \geq -3 \quad x > 3$$

$$\text{Domain: } \underline{x > 3}$$

### ***Exercise***

Find the domain  $f(x) = \frac{\sqrt{x-2}}{\sqrt{x+2}}$

### **Solution**

$$x \geq 2 \quad x > -2$$

$$\text{Domain: } \underline{x \geq 2}$$

### ***Exercise***

Find the domain  $f(x) = \frac{\sqrt{2-x}}{\sqrt{x+2}}$

### **Solution**

$$x \leq 2 \quad x > -2$$

$$\text{Domain: } \underline{-2 < x \leq 2}$$

### Exercise

Find the domain  $f(x) = \frac{x-4}{\sqrt{x-2}}$

### Solution

Domain:  $x > 2$

### Exercise

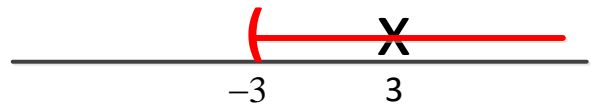
Find the domain of  $f(x) = \frac{1}{(x-3)\sqrt{x+3}}$

### Solution

$$x-3 \neq 0 \quad x+3 > 0$$

$$x \neq 3 \quad x > -3$$

Domain:  $\{x \mid x > -3 \text{ and } x \neq 3\}$   
 $(-3, 3) \cup (3, \infty)$



### Exercise

Find the domain of  $f(x) = \sqrt{x+2} + \sqrt{2-x}$

### Solution

$$x+2 \geq 0 \quad 2-x \geq 0$$

$$x \geq -2 \quad -x \geq -2 \rightarrow x \leq 2$$

Domain:  $\{x \mid -2 \leq x \leq 2\}$



### Exercise

Find the domain of  $f(x) = \sqrt{(x-2)(x-6)}$

### Solution

$$x-2 \geq 0 \quad x-6 \geq 0$$

$$x \geq 2 \quad x \geq 6$$

Domain:  $\{x \mid x \leq 2, x \geq 6\}$

	2	6
-	+	+
-	-	+
+	-	+

### Exercise

Find the domain of  $f(x) = \sqrt{x+3} - \sqrt{4-x}$

#### Solution

$$x \geq -3 \quad x \leq 4$$

$$\text{Domain: } \underline{-3 \leq x \leq 4}$$

### Exercise

Find the domain of  $f(x) = \frac{\sqrt{4x-3}}{x^2-4}$

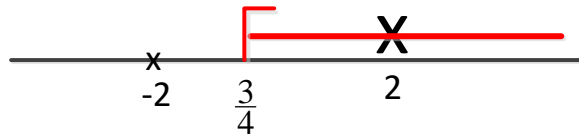
#### Solution

$$4x-3 \geq 0 \quad x^2-4 \neq 0$$

$$4x \geq 3 \quad x \neq \pm 2$$

$$x \geq \frac{3}{4}$$

$$\text{Domain: } \left[ \frac{3}{4}, 2 \right) \cup (2, \infty)$$



### Exercise

Find the domain of  $f(x) = \frac{4x}{6x^2+13x-5}$

#### Solution

$$6x^2+13x-5 \neq 0$$

$$x = \frac{-13 \pm \sqrt{169+120}}{12}$$

$$= \begin{cases} \frac{-13-17}{12} = -\frac{5}{2} \\ \frac{-13+17}{12} = \frac{1}{3} \end{cases}$$

$$\text{Domain: } \underline{x \neq -\frac{5}{2}, \frac{1}{3}}$$

### Exercise

Find the domain of  $f(x) = \frac{\sqrt{2x-3}}{x^2-5x+4}$

#### Solution

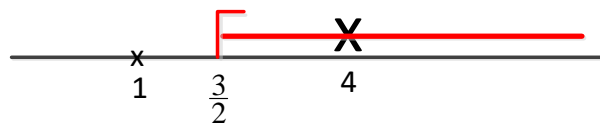
$$2x-3 \geq 0 \quad x^2-5x+4 \neq 0$$



$$2x \geq 3 \quad x \neq 1, 4$$

$$x \geq \frac{3}{2}$$

$$\text{Domain: } \underline{x \geq \frac{3}{2}, x \neq 4} \quad \left[ \frac{3}{2}, 4 \right) \cup (4, \infty)$$



### Exercise

Find the domain of  $f(x) = \frac{x^2}{\sqrt{x^2 - 5x + 4}}$

#### Solution

$$x^2 - 5x + 4 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

$$x = 1, 4$$

$$\text{Domain: } \underline{x < 1 \quad x > 4}$$

### Exercise

Find the domain of  $f(x) = \frac{x+2}{\sqrt{x^2 + 5x + 4}}$

#### Solution

$$x^2 + 5x + 4 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x = -1, -4$$

$$\text{Domain: } \underline{x < -4 \quad x > -1}$$

### Exercise

Find the domain of  $f(x) = \frac{\sqrt{x+2}}{\sqrt{x^2 + 3x + 2}}$

#### Solution

$$x^2 + 3x + 2 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x < -2 \quad x > -1$$

$$\sqrt{x+2} \rightarrow x \geq -2$$

$$\text{Domain: } \underline{x > -1}$$

### Exercise

Find the domain of  $f(x) = \frac{\sqrt{2x+3}}{x^2 - 6x + 5}$

### Solution

$$x^2 - 6x + 5 \qquad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

$$x \neq 1, 5$$

$$\sqrt{2x+3} \rightarrow x \geq -\frac{3}{2}$$

$$\text{Domain: } \underline{x \geq -\frac{3}{2} \quad x \neq 1, 5}$$

### Exercise

Let  $f(x) = 4x - 3$  and  $g(x) = 5x + 7$ . Find each of the following and give the domain

$$a) (f+g)(x) \qquad b) (f-g)(x) \qquad c) (fg)(x) \qquad d) \left(\frac{f}{g}\right)(x)$$

### Solution

$$a) (f+g)(x) = 4x - 3 + 5x + 7 \\ \underline{= 9x + 4}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

$$b) (f-g)(x) = 4x - 3 - (5x + 7) \\ = 4x - 3 - 5x - 7 \\ \underline{= -x - 10}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

$$c) (fg)(x) = (4x - 3)(5x + 7) \\ \underline{= 20x^2 + 13x - 21}$$

$$\text{Domain: } \underline{\mathbb{R}}$$

$$d) \left(\frac{f}{g}\right)(x) = \underline{\frac{4x-3}{5x+7}}$$

$$\text{Domain: } \underline{x \neq -\frac{7}{5}}$$

### Exercise

Let  $f(x) = 2x^2 + 3$  and  $g(x) = 3x - 4$ . Find each of the following and give the domain

$$a) (f + g)(x) \quad b) (f - g)(x) \quad c) (fg)(x) \quad d) \left(\frac{f}{g}\right)(x)$$

### Solution

$$a) (f + g)(x) = 2x^2 + 3 + 3x - 4 \\ = 2x^2 + 3x - 1$$

$$\text{Domain: } \mathbb{R}$$

$$b) (f - g)(x) = 2x^2 + 3 - (3x - 4) \\ = 2x^2 + 3 - 3x + 4 \\ = 2x^2 - 3x + 7$$

$$\text{Domain: } \mathbb{R}$$

$$c) (fg)(x) = (2x^2 + 3)(3x - 4) \\ = 6x^2 + x - 12$$

$$\text{Domain: } \mathbb{R}$$

$$d) \left(\frac{f}{g}\right)(x) = \frac{2x^2 + 3}{3x - 4}$$

$$\text{Domain: } x \neq -\frac{4}{3}$$

### Exercise

Let  $f(x) = x^2 - 2x - 3$  and  $g(x) = x^2 + 3x - 2$ . Find each of the following and give the domain

$$a) (f + g)(x) \quad b) (f - g)(x) \quad c) (fg)(x) \quad d) \left(\frac{f}{g}\right)(x)$$

### Solution

$$a) (f + g)(x) = x^2 - 2x - 3 + x^2 + 3x - 2 \\ = 2x^2 + x - 5$$

$$\text{Domain: } \mathbb{R}$$

$$b) (f - g)(x) = x^2 - 2x - 3 - x^2 - 3x + 2 \\ = -5x - 1$$

**Domain:**  $\mathbb{R}$

$$\begin{aligned} c) \quad (fg)(x) &= (x^2 - 2x - 3)(x^2 + 3x - 2) \\ &= x^4 + 3x^3 - 2x^2 - 2x^3 - 6x^2 + 4x - 3x^2 - 9x + 6 \\ &= x^4 + x^3 - 11x^2 - 5x + 6 \end{aligned}$$

**Domain:**  $\mathbb{R}$

$$d) \quad \left( \frac{f}{g} \right)(x) = \frac{2x^2 + 3}{3x - 4}$$

**Domain:**  $x \neq -\frac{4}{3}$

### Exercise

Let  $f(x) = \sqrt{4x-1}$  and  $g(x) = \frac{1}{x}$ . Find each of the following and give the domain

$$a) \quad (f+g)(x) \quad b) \quad (f-g)(x) \quad c) \quad (fg)(x) \quad d) \quad \left( \frac{f}{g} \right)(x)$$

### Solution

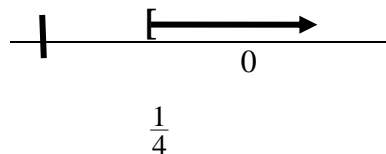
$$a) \quad (f+g)(x)$$

$$(f+g)(x) = \sqrt{4x-1} + \frac{1}{x}$$

$$4x-1 \geq 0 \quad x \neq 0$$

$$x \geq \frac{1}{4}$$

**Domain:**  $\left[ \frac{1}{4}, \infty \right)$



$$b) \quad (f-g)(x)$$

$$(f-g)(x) = \sqrt{4x-1} - \frac{1}{x}$$

$$4x-1 \geq 0 \quad x \neq 0$$

$$x \geq \frac{1}{4}$$

**Domain:**  $\left[ \frac{1}{4}, \infty \right)$

$$\begin{aligned} c) \quad (fg)(x) &= \sqrt{4x-1} \left( \frac{1}{x} \right) \\ &= \frac{\sqrt{4x-1}}{x} \end{aligned}$$

$$4x - 1 \geq 0 \quad x \neq 0$$

$$x \geq \frac{1}{4}$$

$$\text{Domain: } \left[ \frac{1}{4}, \infty \right)$$

$$d) \left( \frac{f}{g} \right)(x) = \frac{\sqrt{4x-1}}{\frac{1}{x}} \quad \text{Domain: } x \neq 0$$

$$= x\sqrt{4x-1}$$

$$4x - 1 \geq 0$$

$$x \geq \frac{1}{4}$$

$$\text{Domain: } \left[ \frac{1}{4}, \infty \right)$$

### Exercise

Given that  $f(x) = x + 1$  and  $g(x) = \sqrt{x + 3}$

a) Find  $(f + g)(x)$

b) Find the domain of  $(f + g)(x)$

c) Find:  $(f + g)(6)$

### Solution

$$a) (f + g)(x) = f(x) + g(x) \\ = x + 1 + \sqrt{x + 3}$$

$$b) x + 3 \geq 0 \rightarrow x \geq -3$$

$$\text{Domain} = [-3, \infty)$$

$$c) (f + g)(6) = 6 + 1 + \sqrt{6 + 3} \\ = 10$$

### Exercise

Given that  $f(x) = x^2 - 4$  and  $g(x) = x + 2$

a) Find  $(f + g)(x)$  and its domain

b) Find  $(f / g)(x)$  and its domain

### Solution

$$\begin{aligned} a) \quad (f + g)(x) &= x^2 - 4 + x + 2 \\ &= x^2 + x - 2 \end{aligned}$$

**Domain:**  $\mathbb{R}$

$$b) \quad \frac{f(x)}{g(x)} = \frac{x^2 - 4}{x + 2}$$

$$x \neq -2$$

**Domain:**  $(-\infty, -2) \cup (-2, \infty)$

### Exercise

Let  $f(x) = x^2 + 1$  and  $g(x) = 3x + 5$ . Find  $(f + g)(1)$ ,  $(f - g)(-3)$ ,  $(fg)(5)$ , and  $\left(\frac{f}{g}\right)(0)$

### Solution

$$\begin{aligned} a) \quad (f + g)(1) &= f(1) + g(1) \\ &= 1^2 + 1 + 3(1) + 5 \\ &= 10 \end{aligned}$$

$$\begin{aligned} b) \quad (f - g)(-3) &= f(-3) - g(-3) \\ &= (-3)^2 + 1 - (3(-3) + 5) \\ &= 10 \end{aligned}$$

$$\begin{aligned} c) \quad (fg)(5) &= f(5) \cdot g(5) \\ &= (5^2 + 1) \cdot (3(5) + 5) \\ &= (26) \cdot (20) \\ &= 520 \end{aligned}$$

$$\begin{aligned} d) \quad \left(\frac{f}{g}\right)(0) &= \frac{f(0)}{g(0)} \\ &= \frac{0^2 + 1}{3(0) + 5} \\ &= \frac{1}{5} \end{aligned}$$

### Exercise

Find  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(f \cdot g)(x)$ , and  $(f / g)(x)$  and the domain of

$$f(x) = \sqrt{3 - 2x}, \quad g(x) = \sqrt{x + 4}$$

### Solution

$$f(x) + g(x) = \sqrt{3 - 2x} + \sqrt{x + 4}$$

$$3 - 2x \geq 0 \quad x + 4 \geq 0$$

$$-2x \geq -3 \quad x \geq -4$$

$$x \leq \frac{3}{2}$$

$$\text{Domain: } \left\{ x \mid -4 \leq x \leq \frac{3}{2} \right\}$$



$$f(x) - g(x) = \sqrt{3 - 2x} - \sqrt{x + 4}$$

$$3 - 2x \geq 0 \quad x + 4 \geq 0$$

$$-2x \geq -3 \quad x \geq -4$$

$$x \leq \frac{3}{2}$$

$$\text{Domain: } \left\{ x \mid -4 \leq x \leq \frac{3}{2} \right\}$$

$$(f \cdot g)(x) = (\sqrt{3 - 2x})(\sqrt{x + 4})$$

$$= \sqrt{(3 - 2x)(x + 4)}$$

$$= \sqrt{-2x^2 - 5x + 12}$$

$$3 - 2x \geq 0 \quad x + 4 \geq 0$$

$$-2x \geq -3 \quad x \geq -4$$

$$x \leq \frac{3}{2}$$

$$\text{Domain: } \left\{ x \mid -4 \leq x \leq \frac{3}{2} \right\}$$

$$(f / g)(x) = \frac{\sqrt{3 - 2x}}{\sqrt{x + 4}} \cdot \frac{\sqrt{x + 4}}{\sqrt{x + 4}}$$

$$= \frac{\sqrt{-2x^2 - 5x + 12}}{x + 4}$$

$$3 - 2x \geq 0 \quad x + 4 > 0$$

$$-2x \geq -3 \quad x > -4$$

$$x \leq \frac{3}{2}$$

$$\text{Domain: } \left\{ x \mid -4 < x \leq \frac{3}{2} \right\}$$

$$\left( -4, \frac{3}{2} \right]$$

### Exercise

Find  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(f \cdot g)(x)$ , and  $(f / g)(x)$  and the domain of

$$f(x) = \frac{2x}{x-4}, \quad g(x) = \frac{x}{x+5}$$

### Solution

$$\begin{aligned}(f + g)(x) &= \frac{2x}{x-4} + \frac{x}{x+5} \\&= \frac{2x(x+5) + x(x-4)}{(x-4)(x+5)} \\&= \frac{2x^2 + 10x + x^2 - 4x}{(x-4)(x+5)} \\&= \frac{3x^2 + 6x}{(x-4)(x+5)}\end{aligned}$$

$$x-4 \neq 0 \quad x+5 \neq 0$$

$$x \neq 4 \quad x \neq -5$$

$$\text{Domain: } \{x \mid x \neq -5, 4\} \quad (-\infty, -5) \cup (-5, 4) \cup (4, \infty)$$

$$\begin{aligned}(f - g)(x) &= \frac{2x}{x-4} - \frac{x}{x+5} \\&= \frac{2x(x+5) - x(x-4)}{(x-4)(x+5)} \\&= \frac{2x^2 + 10x - x^2 + 4x}{(x-4)(x+5)} \\&= \frac{x^2 + 14x}{(x-4)(x+5)}\end{aligned}$$

$$x \neq 4 \quad x \neq -5$$

$$\text{Domain: } \{x \mid x \neq -5, 4\}$$

$$\begin{aligned}(f \cdot g)(x) &= \frac{2x}{x-4} \cdot \frac{x}{x+5} \\&= \frac{2x^2}{(x-4)(x+5)}\end{aligned}$$

$$x \neq 4 \quad x \neq -5$$

$$\text{Domain: } \{x \mid x \neq -5, 4\}$$

$$\begin{aligned}(f / g)(x) &= \frac{2x}{x-4} \div \frac{x}{x+5} \\&= \frac{2x}{x-4} \cdot \frac{x+5}{x}\end{aligned}$$



$$= 2 \frac{x+5}{x-4}$$

$$x \neq 4 \quad x \neq -5$$

**Domain:**  $\{x \mid x \neq -5, 4\}$

### Exercise

Find  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(f \cdot g)(x)$ , and  $(f / g)(x)$  of  $f(x) = x - 5$  and  $g(x) = x^2 - 1$

### Solution

$$\begin{aligned} a) \quad (f + g)(x) &= f(x) + g(x) \\ &= x - 5 + x^2 - 1 \\ &= x^2 + x - 6 \end{aligned}$$

$$\begin{aligned} b) \quad (f - g)(x) &= f(x) - g(x) \\ &= x - 5 - (x^2 - 1) \\ &= x - 5 - x^2 + 1 \\ &= -x^2 + x - 4 \end{aligned}$$

$$\begin{aligned} c) \quad (fg)(x) &= f(x)g(x) \\ &= (x - 5)(x^2 - 1) \\ &= x^3 - x - 5x^2 + 5 \\ &= x^3 - 5x^2 - x + 5 \end{aligned}$$

$$\begin{aligned} d) \quad \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x-5}{x^2-1} \end{aligned}$$

### Exercise

For the function  $f$  given by  $f(x) = 9x + 5$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$f(x+h) = 9(x+h) + 5 = 9x + 9h + 5$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\overbrace{9x+9h+5}^{f(x+h)} - \overbrace{(9x+5)}^{f(x)}}{h} \\ &= \frac{9x+9h+5 - 9x-5}{h} \end{aligned}$$

$$= \frac{9h}{h}$$

$$= 9$$

### ***Exercise***

For the function  $f$  given by  $f(x) = 6x + 2$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### **Solution**

$$\frac{f(x+h) - f(x)}{h} = \frac{6(x+h) + 2 - (6x + 2)}{h}$$

$$= \frac{6x + 6h + 2 - 6x - 2}{h}$$

$$= \frac{6h}{h}$$

$$= 6$$

### ***Exercise***

For the function  $f$  given by  $f(x) = 4x + 11$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### **Solution**

$$\frac{f(x+h) - f(x)}{h} = \frac{4(x+h) + 11 - (4x + 11)}{h}$$

$$= \frac{4x + 4h + 11 - 4x - 11}{h}$$

$$= \frac{4h}{h}$$

$$= 4$$

### ***Exercise***

For the function  $f$  given by  $f(x) = 3x - 5$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### **Solution**

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h) - 5 - 3x + 5}{h}$$

$$= \frac{3x + 3h - 5 - 3x + 5}{h}$$

$$= \frac{3h}{h}$$

$$= 3$$

### Exercise

For the function  $f$  given by  $f(x) = -2x - 3$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-2(x+h) - 3 + 2x + 3}{h} \\ &= \frac{-2x - 2h - 3 + 2x + 3}{h} \\ &= \frac{-2h}{h} \\ &= -2\end{aligned}$$

### Exercise

For the function  $f$  given by  $f(x) = -4x + 3$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-4(x+h) + 3 + 4x - 3}{h} \\ &= \frac{-4x - 4h + 3 + 4x - 3}{h} \\ &= \frac{-4h}{h} \\ &= -4\end{aligned}$$

### Exercise

For the function  $f$  given by  $f(x) = 3x - 6$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{3(x+h) - 6 - 3x + 6}{h} \\ &= \frac{3x + 3h - 6 - 3x + 6}{h} \\ &= \frac{3h}{h} \\ &= 3\end{aligned}$$

### Exercise

For the function  $f$  given by  $f(x) = -5x - 7$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{-5(x+h) - 7 + 5x + 7}{h} \\ &= \frac{-5x - 5h - 7 + 5x + 7}{h} \\ &= \frac{-5h}{h} \\ &= \underline{-5}\end{aligned}$$

### Exercise

Given the function:  $f(x) = 2x^2$ . Find and simplify the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}f(x+h) &= 2(x+h)^2 \\ &= 2(x^2 + 2hx + h^2) \\ &= 2x^2 + 4hx + 2h^2 \\ \frac{f(x+h) - f(x)}{h} &= \frac{2x^2 + 4hx + 2h^2 - 2x^2}{h} \\ &= \frac{4hx + 2h^2}{h} \\ &= \frac{4hx}{h} + \frac{2h^2}{h} \\ &= \underline{4x + 2h}\end{aligned}$$

### Exercise

For the function  $f$  given by  $f(x) = 5x^2$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{5(x+h)^2 - 5x^2}{h} \\ &= \frac{5(x^2 + 2hx + h^2) - 5x^2}{h} \\ &= \frac{5x^2 + 10hx + 5h^2 - 5x^2}{h}\end{aligned}$$

$$\begin{aligned}
 &= \frac{10hx + 5h^2}{h} \\
 &= \underline{10x + 5h}
 \end{aligned}$$

### Exercise

For the function  $f$  given by  $f(x) = 3x^2 - 4x$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}
 \frac{f(x+h) - f(x)}{h} &= \frac{3(x+h)^2 - 4x - 3x^2 + 4x}{h} \\
 &= \frac{3(x^2 + 2hx + h^2) - 4x - 3x^2 + 4x}{h} \\
 &= \frac{3x^2 + 6hx + 3h^2 - 4x - 3x^2 + 4x}{h} \\
 &= \frac{6hx + 3h^2}{h} \\
 &= \underline{6x + 3h}
 \end{aligned}$$

### Exercise

For the function  $f$  given by  $f(x) = 2x^2 - 3x$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}
 f(x+h) &= 2(\text{---})^2 - 3(\text{---}) \\
 &= 2(x+h)^2 - 3(x+h) & (a+b)^2 &= a^2 + 2ab + b^2 \\
 &= 2(x^2 + 2xh + h^2) - 3x - 3h \\
 &= 2x^2 + 4xh + 2h^2 - 3x - 3h \\
 \frac{f(x+h) - f(x)}{h} &= \frac{\overbrace{2x^2 + 4xh + 2h^2 - 3x - 3h}^{f(x+h)} - \overbrace{(2x^2 - 3x)}^{f(x)}}{h} \\
 &= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h} \\
 &= \frac{4xh + 2h^2 - 3h}{h} \\
 &= \frac{4xh}{h} + \frac{2h^2}{h} - \frac{3h}{h} \\
 &= \underline{4x + 2h - 3}
 \end{aligned}$$

### Exercise

For the function  $f$  given by  $f(x) = 2x^2 - x - 3$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}f(x+h) &= 2(x+h)^2 - (x+h) - 3 \\&= 2(x^2 + 2hx + h^2) - x - h - 3 \\&= 2x^2 + 4hx + 2h^2 - x - h - 3\end{aligned}$$

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{2x^2 + 2h^2 + 4hx - x - h - 3 - (2x^2 - x - 3)}{h} \\&= \frac{2x^2 + 2h^2 + 4hx - x - h - 3 - 2x^2 + x + 3}{h} \\&= \frac{2h^2 + 4hx - h}{h} \\&= \frac{2h^2}{h} + \frac{4hx}{h} - \frac{h}{h} \\&= \underline{2h + 4x - 1}\end{aligned}$$

### Exercise

For the given function  $f(x) = 2x^2 - x - 3$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{2(x+h)^2 - (x+h) - 3 - 2x^2 + x + 3}{h} \\&= \frac{2(x^2 + 2hx + h^2) - x - h - 3 - 2x^2 + x + 3}{h} \\&= \frac{2x^2 + 4hx + 2h^2 - h - 2x^2}{h} \\&= \frac{4hx + 2h^2 - h}{h} \\&= \underline{4x + 2h - 1}\end{aligned}$$

### Exercise

For the given function  $f(x) = x^2 - 2x + 5$ , find the difference quotient  $\frac{f(x+h) - f(x)}{h}$

### Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^2 - 2(x+h) + 5 - x^2 + 2x - 5}{h} \\
&= \frac{x^2 + 2hx + h^2 - 2x - 2h - x^2 + 2x}{h} \\
&= \frac{2hx + h^2 - 2h}{h} \\
&= \underline{2x + h - 2}
\end{aligned}$$

### Exercise

For the given function  $f(x) = 3x^2 - 2x + 5$ , find the difference quotient  $\frac{f(x+h)-f(x)}{h}$

### Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)^2 - 2(x+h) + 5 - 3x^2 + 2x - 5}{h} \\
&= \frac{3(x^2 + 2hx + h^2) - 2x - 2h - 3x^2 + 2x}{h} \\
&= \frac{3x^2 + 6hx + 3h^2 - 2h - 3x^2}{h} \\
&= \frac{6hx + 3h^2 - 2h}{h} \\
&= \underline{6x + 3h - 2}
\end{aligned}$$

### Exercise

For the given function  $f(x) = -2x^2 - 3x + 7$ , find the difference quotient  $\frac{f(x+h)-f(x)}{h}$

### Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{-2(x+h)^2 - 3(x+h) + 7 + 2x^2 + 3x - 7}{h} \\
&= \frac{-2(x^2 + 2hx + h^2) - 3x - 3h + 2x^2 + 3x}{h} \\
&= \frac{-2x^2 - 4hx - 2h^2 - 3h + 2x^2}{h} \\
&= \frac{-4hx - 2h^2 - 3h}{h} \\
&= \underline{-4x - 2h - 3}
\end{aligned}$$

### Exercise

An open box is to be made from a square piece of cardboard that measures 40 inches on each side, to construct the box, squares that measure  $x$  inches on each side are cut from each corner of the cardboard.

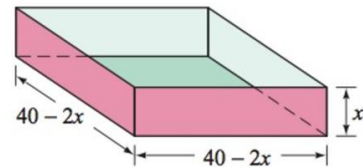
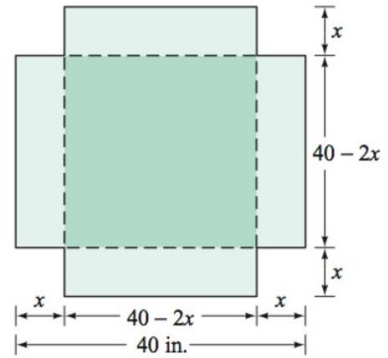
- Express the volume  $V$  of the box as a function of  $x$ .
- Determine the domain of  $V$ .

### Solution

$$\begin{aligned} a) \quad V(x) &= x(40 - 2x)^2 \\ &= x(1600 - 160x + 4x^2) \\ &= 4x^3 - 160x^2 + 1600x \end{aligned}$$

$$\begin{aligned} b) \quad 40 - 2x &= 0 \\ x &= 20 \end{aligned}$$

$$\text{Domain: } \{x \mid 0 < x < 20\}$$



### Exercise

A child 4 feet tall is standing near a street lamp that is 12 feet high. The light from the lamp casts a shadow.

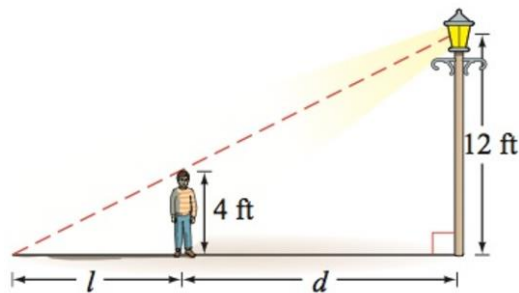
- Find the length  $l$  of the shadow as a function of the distance  $d$  of the child from the lamppost.
- What is the domain of the function?
- What is the length of the shadow when the child is 8 feet from the base of the lamppost?

### Solution

$$\begin{aligned} a) \quad \frac{l + d}{12} &= \frac{l}{4} \\ l + d &= 3l \\ 2l &= d \\ l(d) &= \frac{1}{2}d \end{aligned}$$

$$b) \quad \text{Domain: } \{x \mid 0 \leq d < \infty\}$$

$$\begin{aligned} c) \quad \text{Given: } d &= 8 \\ l &= 4 \text{ feet} \end{aligned}$$





### Exercise

An open box is to be made from a square piece of cardboard with the dimensions 30 inches by 30 inches by cutting out squares of area  $x^2$  from each corner.

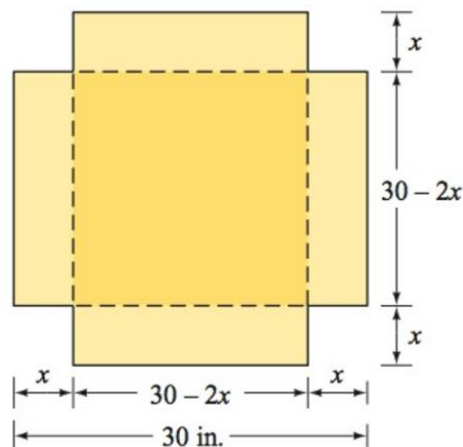
- Express the volume  $V$  of the box as a function of  $x$ .
- Determine the domain of  $V$ .

### Solution

$$\begin{aligned} a) \quad V(x) &= x(30 - 2x)^2 \\ &= x(900 - 120x + 4x^2) \\ &= 4x^3 - 120x^2 + 900x \end{aligned}$$

$$\begin{aligned} b) \quad 30 - 2x &= 0 \\ x &= 15 \end{aligned}$$

$$\text{Domain: } \{x \mid 0 < x < 15\}$$



### Exercise

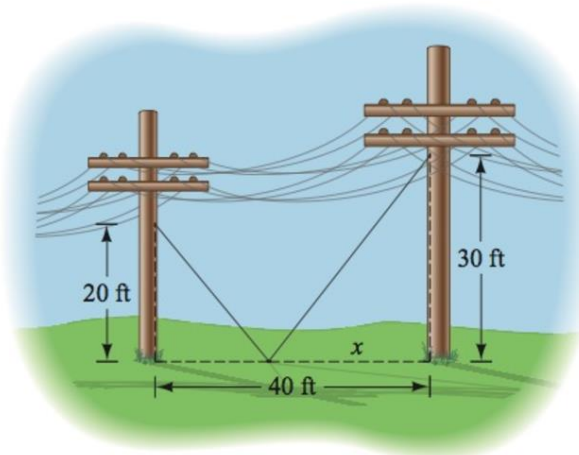
Two guy wires are attached to utility poles that are 40 feet apart.

- Find the total length of the two guy wires as a function of  $x$ .
- What is the domain of this function?

### Solution

$$\begin{aligned} a) \quad \ell_1 &= \sqrt{(40 - x)^2 + 20^2} \\ &= \sqrt{1,600 - 80x + x^2 + 400} \\ &= \sqrt{2,000 - 80x + x^2} \\ \ell_2 &= \sqrt{x^2 + 30^2} \\ &= \sqrt{x^2 + 900} \\ \ell(x) &= \sqrt{2,000 - 80x + x^2} + \sqrt{x^2 + 900} \end{aligned}$$

$$b) \quad \text{Domain: } [0, 40]$$



### Exercise

A rancher has 360 yards of fencing with which to enclose two adjacent rectangular corrals, one for sheep and one for cattle. A river forms one side of the corrals. Suppose the width of each corral is  $x$  yards.

- Express the total area of the two corrals as a function of  $x$ .
- Find the domain of the function.

### Solution

a)  $P = 3x + l = 360$

$$l = 360 - 3x$$

$$A = xl$$

$$= x(360 - 3x)$$

$$\underline{A(x) = 360x - 3x^2}$$

b)  $x(360 - 3x) = 0$

$$\underline{x = 0}$$

$$360 - 3x = 0$$

$$3x = 360$$

$$\Rightarrow \underline{x = 120}$$

**Domain:**  $\underline{0 < x < 120}$



### Exercise

A rectangle is bounded by the  $x$ - and  $y$ -axis of  $y = -\frac{1}{2}x + 4$

- Find the area of the rectangle as a function of  $x$ .
- What is the domain of this function.

### Solution

a)  $\text{Area} = xy$

$$\begin{aligned} A(x) &= x\left(-\frac{1}{2}x + 4\right) \\ &= \underline{-\frac{1}{2}x^2 + 4x} \end{aligned}$$

b)  $x\left(-\frac{1}{2}x + 4\right) = 0$

$$x = 0 \quad x = 8$$

**Domain:**  $\underline{0 < x < 8}$

