Lecture One

Section 1.1 – Introduction to System of Linear Equations

Given the linear equations

$$\begin{cases} x - 2y = 1 \\ 3x + 2y = 11 \end{cases}$$

The solution to this system is $(3,\ 1)$, which means that 2 lines meeting at a single point.

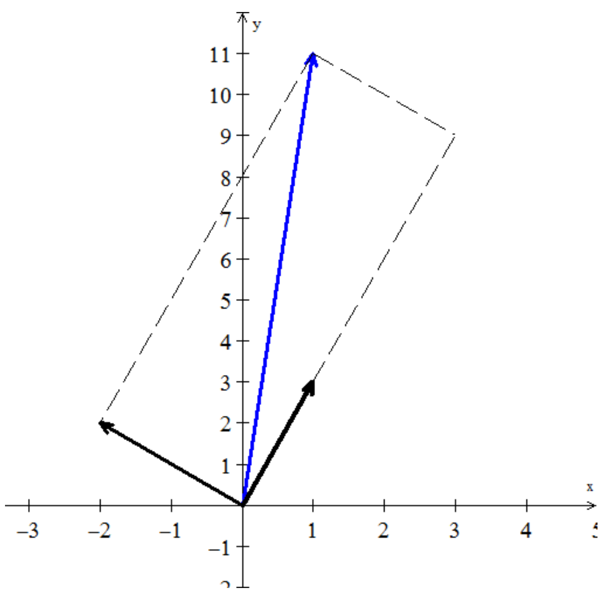
We can rewrite the system equation as linear combination:

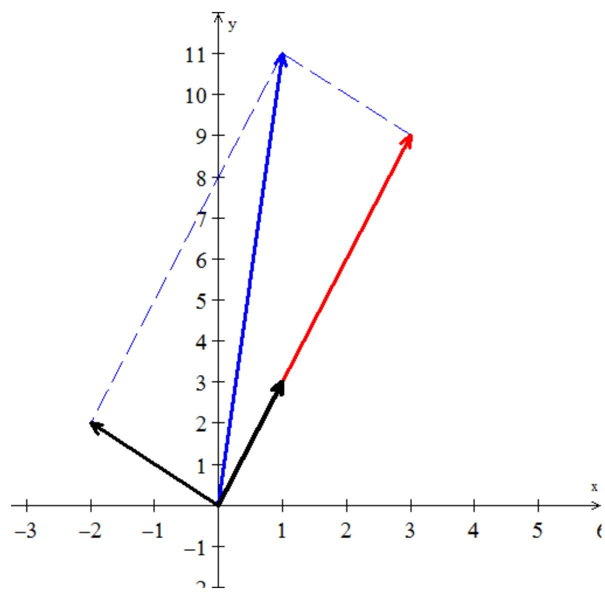
$$x\begin{bmatrix} 1 \\ 3 \end{bmatrix} + y\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

$$x.v_1 + y.v_2 = v$$

$$\begin{bmatrix} 1+x \\ 3+y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} x=3 \\ y=9 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 9 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$





Therefore, the side vectors are $\begin{bmatrix} 3 \\ 9 \end{bmatrix}$ *and* $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$

The diagonal sum is $\begin{bmatrix} 3-2 \\ 9+2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$

The linear combination is given by:

$$3\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

Thus, the solution is $x = 3 \quad y = 1$

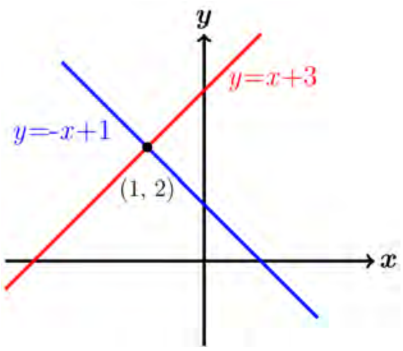
Note

$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$ is called the "***coefficient matrix***"

The matrix form of the system is written as $\qquad Ax = b$

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

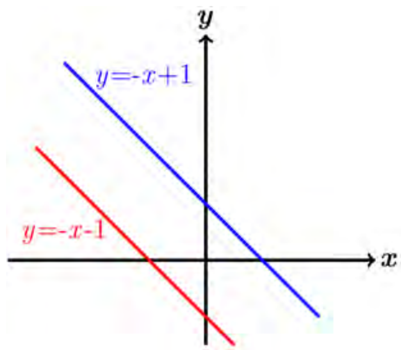
Graphically



One solution (lines intersect)

Consistent

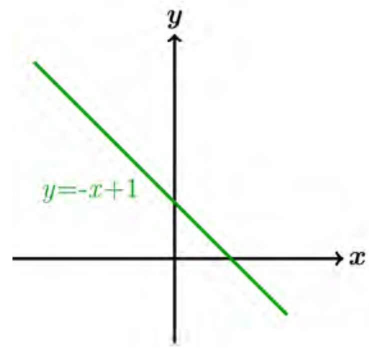
Independent



No Solution (lines //)

Inconsistent

Independent



Infinite solution

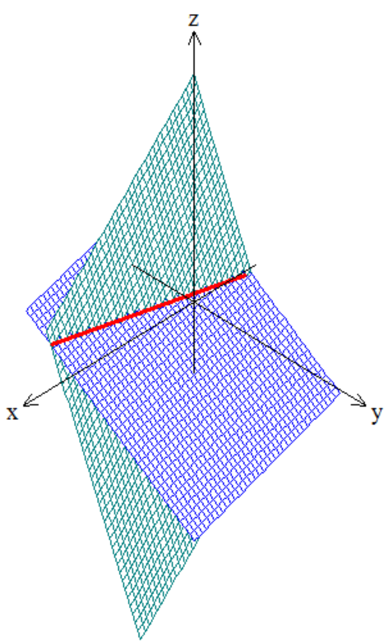
Consistent

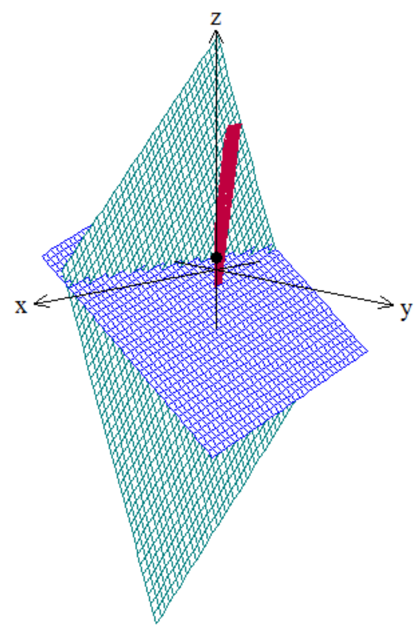
Dependent

Three Equations in 3 Unknowns

Given the system equations

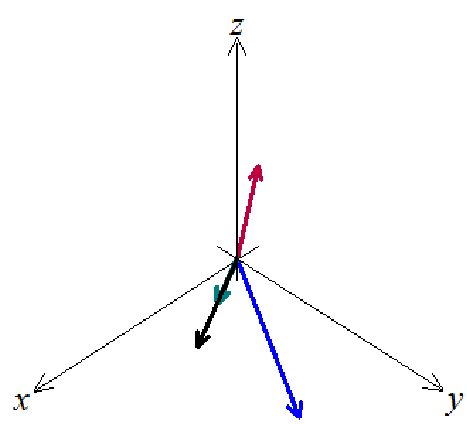
$$x + 2y + 3z = 6$$
$$2x + 5y + 2z = 4$$
$$6x - 3y + z = 2$$





This system can be written as linear combination:

$$x\begin{bmatrix}1\\2\\6\end{bmatrix} + y\begin{bmatrix}2\\5\\-3\end{bmatrix} + z\begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{bmatrix}6\\4\\2\end{bmatrix}$$



Let $b = \begin{bmatrix}6\\4\\2\end{bmatrix}$

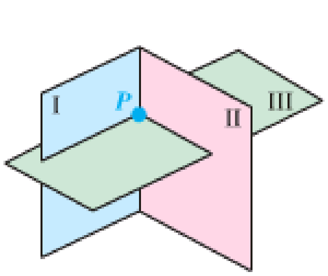
We want to multiply the three column vectors by $x,\ y,\ z$ to produce ***b***.

The combination of the three vectors that produces vector b is 2 times the third vector.

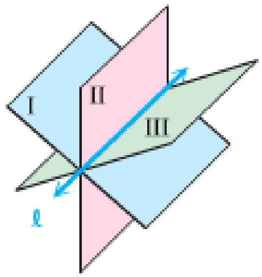
$$2(3,\ 2,\ 1) = (6,\ 4,\ 2) = b$$

Therefore, the coefficients that we need are $x = 0, \quad y = 0, \quad and\ z = 2$.

$$0\begin{bmatrix}1\\2\\6\end{bmatrix} + 0\begin{bmatrix}2\\5\\-3\end{bmatrix} + 2\begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{bmatrix}6\\4\\2\end{bmatrix}$$

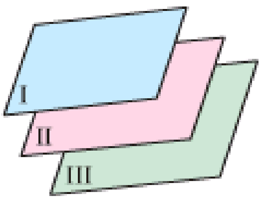


A single solution

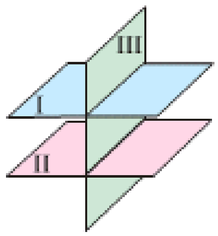


Points of a line in common

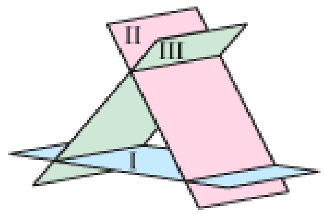
All points in common



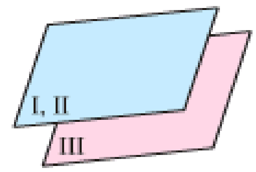
No points in common



No points in common



No points in common



No points in common

Exercises *Section* 1.1 – Introduction to System of Linear Equations

1. Find a solution for x, y, z to the system of equations

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3e+2\sqrt{2}+\pi \\ 6e+5\sqrt{2}+4\pi \\ 9e+8\sqrt{2}+7\pi \end{pmatrix}$$

2. Draw the two pictures in two planes for the equations: $x-2y=0, \quad x+y=6$

3. Normally 4 planes in 4-dimensional space meet at a ________. Normally 4 column vectors in 4-deimensional space can combine to produce b. what combinations of
$(1,\ 0,\ 0,\ 0), \quad (1,\ 1,\ 0,\ 0), \quad (1,\ 1,\ 1,\ 0), \quad (1,\ 1,\ 1,\ 1)$ produces $b=(3,\ 3,\ 3,\ 2)$?
What 4 equations for $x,\ y,\ z,\ w$ are you solving?

4. What 2 by 2 matrix A rotates every vector through 45° ?
The vector (1, 0) goes to $\left(\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right)$. The vector (0, 1) goes to $\left(-\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right)$.
Those determine the matrix. Draw these particular vectors is the xy-plane and find A.

5. What two vectors are obtained by rotating the plane vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ by 30° (*cw*) ?
Write a matrix A such that for every vector $\vec{v}$ in the plane, $A\vec{v}$ is the vector obtained by rotating $\vec{v}$ clockwise by 30°.
Find a matrix B such that for every 3-dimensional vector $\vec{v}$, the vector $B\vec{v}$ is the reflection of $\vec{v}$ through the plane $x+y+z=0$. $\mathit{Hint}: v=(1,\ 0,\ 0)$

6. In each part, find a system of linear equation corresponding to the given augmented matrix

a) $\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 5 & 2 & 0 & -3 & -6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \\ 5 & -6 & 1 & 1 \\ -8 & 0 & 0 & 3 \end{bmatrix}$

7. Find the augmented matrix for the given system of linear equations.

a) $\begin{cases} -2x_1 = 6 \\ 3x_1 = 8 \\ 9x_1 = -3 \end{cases}$

c) $\begin{cases} 2x_1 \quad + 2x_3 = 1 \\ 3x_1 - x_2 + 4x_3 = 7 \\ 6x_1 + x_2 - x_3 = 0 \end{cases}$

b) $\begin{cases} 3x_1 - 2x_2 = -1 \\ 4x_1 + 5x_2 = 3 \\ 7x_1 + 3x_2 = 2 \end{cases}$