

Section 2.2 – Homogeneous Equations with Constant Coefficients

The homogeneous equations of the form:

$$y'' + py' + qy = 0 \quad (H : \text{Homogeneous})$$

Where p and q are continuous functions on some interval I .

The zero function, $y(x) = 0$ for all $x \in I$, ($y \equiv 0$) is a solution of the equation (given above).

The zero solution is called **trivial solution**. Any other solution is a **nontrivial** solution.

This is a class of equations that we can solve easily.

Theorem

If $y = y(x)$ is a solution of $y'' + py' + qy = 0$ and if C is any real number, then $u(x) = Cy(x)$ is also solution of $y'' + py' + qy = 0$.

Any constant multiple of a solution of $y'' + py' + qy = 0$ is also a solution of $y'' + py' + qy = 0$

Proof

$$u'(x) = Cy'(x)$$

$$u''(x) = Cy''(x)$$

$$u'' + pu' + qu = 0$$

$$Cy'' + pCy' + qCy = 0$$

$$\Rightarrow C(y'' + py' + qy) = 0$$

Theorem

If $y = y_1(x)$ and $y = y_2(x)$ are any two solutions of $y'' + py' + qy = 0$ the $u(x) = y_1(x) + y_2(x)$ is also a solution of $y'' + py' + qy = 0$.

Any linear combination of solutions of $y'' + py' + qy = 0$ is also a solution of $y'' + py' + qy = 0$.

Definition

Let $f = f(x)$ and $g = g(x)$ be functions defined on some interval I , and let C_1 and C_2 be real numbers. The expression

$$C_1 f(x) + C_2 g(x)$$

Is called a linear combination of f and g .

Definition

Let $y = y_1(x)$ and $y = y_2(x)$ are any two solutions of $y'' + py' + qy = 0$ (H). The function W defined by

$$W[y_1, y_2] = y_1(x)y_2'(x) - y_2(x)y_1'(x)$$

Is called the Wronskian of y_1, y_2

Determinant notation:

$$\begin{aligned} W(x) &= y_1(x)y_2'(x) - y_2(x)y_1'(x) \\ &= \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix} \end{aligned}$$

Theorem

Let $y = y_1(x)$ and $y = y_2(x)$ be solutions of equation (H), and let $W(x)$ be their Wronskian. Exactly one of the following holds

1. $W(x) = 0$ for all $x \in I$ and y_1 is a constant multiple of y_2

Example:

$$W(x) = \begin{vmatrix} x^2 & 2x^2 \\ 2x & 4x \end{vmatrix} = 4x^3 - 4x^3 = \underline{0}$$

2. $W(x) \neq 0$ for all $x \in I$ and $y = C_1 y_1(x) + C_2 y_2(x)$ is the general solution of (H)

Definition Fundamental Set – Solution basis

A pair of solutions $y = y_1(x)$ and $y = y_2(x)$ of equation (H) forms a **fundamental set of solutions** (also called a **solution basis**) if $W(x) \neq 0$ for all $x \in I$.

The analogous first-order, linear, homogeneous equation:

$$y' + p(t)y = 0$$

It is separable and easily solved, its general solution is

$$y(t) = Ce^{-Pt}$$

Let look for a solution of the type

$$y(t) = e^{\lambda t}$$

$$y' = \lambda e^{\lambda t}$$

$$y'' = \lambda^2 e^{\lambda t}$$

$$\begin{aligned} y'' + py' + qy &= \lambda^2 e^{\lambda t} + p\lambda e^{\lambda t} + qe^{\lambda t} \\ &= (\lambda^2 + p\lambda + q)e^{\lambda t} \\ &= 0 \end{aligned}$$

$$\lambda^2 + p\lambda + q = 0 \quad \text{This is called the **characteristic equation**}$$

We can rewrite the differential equation and its characteristic equations

$$y'' + py' + qy = 0$$

$$\lambda^2 + p\lambda + q = 0$$

The roots are: $\lambda_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$

$$\text{If } p^2 - 4q > 0 \Rightarrow \text{Two distinct real roots}$$

$$\text{If } p^2 - 4q < 0 \Rightarrow \text{Two distinct complex roots}$$

$$\text{If } p^2 - 4q = 0 \Rightarrow \text{One repeated real root}$$

Case 1: Distinct Real Root

If $\lambda^2 + p\lambda + q = 0$ has two distinct real roots: $\lambda_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$

Then:

$$y_1 = C_1 e^{\lambda_1 t} \text{ and } y_2 = C_2 e^{\lambda_2 t} \text{ are both solutions.}$$

Proposition

If the characteristic equations $\lambda^2 + p\lambda + q = 0$ has two distinct real roots λ_1 and λ_2 , then the **general solution** to $y'' + py' + qy = 0$ is

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Where C_1 and C_2 are arbitrary constants.

Example

Find the general solution to the equation $y'' - 3y' + 2y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = 1$

Solution

The characteristic equation:

$$y'' - 3y' + 2y = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

The solution: $\lambda_{1,2} = 1, 2$

The general solution

$$y(t) = C_1 e^t + C_2 e^{2t}$$

$$y' = C_1 e^t + 2C_2 e^{2t}$$

$$y(0) = 2 \quad y(0) = C_1 e^0 + C_2 e^{2(0)}$$

$$2 = C_1 + C_2$$

$$y'(0) = 1 \quad y'(0) = C_1 e^0 + 2C_2 e^{2(0)}$$

$$1 = C_1 + 2C_2$$

$$C_1 + C_2 = 2$$

$$C_1 + 2C_2 = 1 \Rightarrow C_2 = -1 \quad C_1 = 3$$

The unique solution is: $y(t) = 3e^t - e^{2t}$

Case 2: Complex Roots

Proposition

If the characteristic equation $\lambda^2 + p\lambda + q = 0$ has two complex conjugate roots $\lambda = a + ib$ and $\bar{\lambda} = a - ib$.

1. The functions

$$z = e^{(a+ib)t} \text{ and } \bar{z} = e^{(a-ib)t}$$

So, the general solution is

$$w(t) = C_1 e^{(a+ib)t} + C_2 e^{(a-ib)t}$$

Where C_1 and C_2 are arbitrary complex constants.

2. The functions

$$y_1(t) = e^{at} \cos(bt) \text{ and } y_2(t) = e^{at} \sin(bt)$$

So, the general solution is

$$y(t) = e^{at} (A_1 \cos bt + A_2 \sin bt)$$

Where A_1 and A_2 are constants.

Example

Find the general solution to the equation $y'' + 2y' + 2y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = 3$

Solution

The characteristic equation: $\lambda^2 + 2\lambda + 2 = 0$

The solution: $\lambda_{1,2} = -1 \pm i = a \pm ib$

$$a = -1; \quad b = 1$$

The general solution

$$y(t) = e^{-t} (C_1 \cos t + C_2 \sin t)$$

$$y(0) = e^{-(0)} (C_1 \cos(0) + C_2 \sin(0))$$

$$2 = 1(C_1 + C_2(0)) \Rightarrow \boxed{C_1 = 2}$$

$$y' = -e^{-t} (C_1 \cos t + C_2 \sin t) + e^{-t} (-C_1 \sin t + C_2 \cos t)$$

$$y'(0) = -e^{-(0)} (C_1 \cos(0) + C_2 \sin(0)) + e^{-(0)} (-C_1 \sin(0) + C_2 \cos(0))$$

$$3 = -(C_1) + (C_2)$$

$$C_2 - C_1 = 3$$

$$C_2 = 3 + 2 = 5$$

$$y(t) = e^{-t} (2 \cos t + 5 \sin t)$$

Example

Find the general solution to the equation $y'' - 4y' + 13y = 0$

Solution

The characteristic equation: $\lambda^2 - 4\lambda + 13 = 0$

The solutions: $\lambda_{1,2} = \frac{4 \pm \sqrt{16 - 52}}{2} = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$

$a = 2$; $b = 3$

The general solution: $y(x) = e^{2x} (C_1 \cos 3x + C_2 \sin 3x)$

Case 3: Repeated Roots

If the roots of the characteristic equations are repeated and the solutions are independent.

$$\lambda^2 + p\lambda + q = 0$$

$$(\lambda - \lambda_1)^2 = 0$$

$$\lambda_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$$

$$p^2 - 4q = 0 \Rightarrow q = \frac{p^2}{4}$$

$$\lambda_{1,2} = -\frac{p}{2}$$

$$\begin{aligned} y_1 &= C_1 e^{\lambda_1 t} \\ &= C_1 e^{-pt/2} \end{aligned}$$

$$\begin{aligned} y_2 &= v(t)y_1(t) \\ &= v(t)e^{-pt/2} \end{aligned}$$

$$y'' + py' + qy = 0$$

$$y'' + py' + \frac{p^2}{4}y = 0$$

$$y_2' = v'e^{-pt/2} - \frac{p}{2}ve^{-pt/2}$$

$$y_2'' = v''e^{-pt/2} - \frac{p}{2}v'e^{-pt/2} - \frac{p}{2}v'e^{-pt/2} + \frac{p^2}{4}ve^{-pt/2}$$

$$v''e^{-pt/2} - \frac{p}{2}v'e^{-pt/2} - \frac{p}{2}v'e^{-pt/2} + \frac{p^2}{4}ve^{-pt/2} + p\left(v'e^{-pt/2} - \frac{p}{2}ve^{-pt/2}\right) + \frac{p^2}{4}ve^{-pt/2} = 0$$

$$v''e^{-pt/2} = 0$$

$$v'' = 0$$

$$\Rightarrow v' = a$$

$$\Rightarrow v = at + b$$

$$v = t$$

$$\underline{y_2 = te^{-pt/2}}$$

Proposition

If the characteristic equations $\lambda^2 + p\lambda + q = 0$ has one double root λ_1 , then the **general solution** to $y'' + py' + qy = 0$ is

$$\begin{aligned}y(t) &= C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t} \\&= (C_1 + C_2 t) e^{\lambda_1 t}\end{aligned}$$

Where C_1 and C_2 are arbitrary constants.

Example

Find the general solution to the equation $y'' - 2y' + y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = -1$

Solution

The characteristic equation: $\lambda^2 - 2\lambda + 1 = 0$

The solutions are: $\lambda_{1,2} = 1$

$$\begin{aligned}y(t) &= C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t} \\&= C_1 e^t + C_2 t e^t\end{aligned}$$

$$y(0) = C_1 e^{(0)} + C_2 (0) e^{(0)} \Rightarrow 2 = C_1$$

$$\begin{aligned}y' &= C_1 e^t + C_2 e^t + C_2 t e^t \\y'(0) &= 2e^{(0)} + C_2 e^{(0)} + C_2 (0) e^{(0)} \\-1 &= 2 + C_2 \Rightarrow C_2 = -3\end{aligned}$$

$$\underline{y(t) = 2e^t - 3te^t}$$

Example

Find the general solution to the equation $y'' - 10y' + 25y = 0$

Solution

The characteristic equation: $\lambda^2 - 10\lambda + 25 = (\lambda - 5)^2 = 0$

The solutions are: $\lambda_{1,2} = 5$

The general solution: $\underline{y(t) = C_1 e^{5t} + C_2 t e^{5t}}$

Higher-Order Equations

In general, to solve an n th-order differential equation, we must solve an n th degree characteristic polynomial equation

$$a_n \lambda^n + a_{n-1} \lambda^{n-1} + \dots + a_2 \lambda^2 + a_1 \lambda + a_0 = 0$$

If all roots are real and distinct, then the general solution is

$$y(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + \dots + C_n e^{\lambda_n x}$$

If all roots are equal to λ , then the general solution is

$$y(x) = C_1 e^{\lambda x} + C_2 x e^{\lambda x} + C_3 x^2 e^{\lambda x} + \dots + C_n x^{n-1} e^{\lambda x}$$

Example

Find the general solution of $y''' + 3y'' - 4y = 0$

Solution

$$\lambda^3 + 3\lambda^2 - 4 = 0 \quad \text{Solve for } \lambda$$

$$\lambda_1 = 1, \quad \lambda_{2,3} = -2$$

$$\underline{y(x) = C_1 e^x + (C_2 + C_3 x) e^{-2x}}$$

$$\text{Rational zero theorem: } \pm \left\{ \frac{4}{1} \right\} = \pm \{1, 2, 4\}$$

$$\lambda_1 = 1, \quad \lambda_2 = -2$$

$$(\lambda - 1)(\lambda + 2)(\lambda - a) = 0$$

$$(-1)(2)(-a) = -4 \Rightarrow a = -2$$

Example

Find the general solution of $\lambda^4(\lambda + 1)(\lambda + 2)^2(\lambda^2 + 4) = 0$

Solution

$$\lambda^2 + 4 = 0 \Rightarrow \lambda^2 = -4 \rightarrow \lambda = \pm 2i$$

The solution: $\lambda = 0, 0, 0, 0, -1, -2, -2, \pm 2i$

$$\underline{y(x) = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + C_5 e^{-x} + (C_6 + C_7 x) e^{-2x} + C_8 \cos 2x + C_9 \sin 2x}$$

Summary

The equation: $y'' + py' + qy = 0$

The characteristic equations $\lambda^2 + p\lambda + q = 0$

If $p^2 - 4q > 0$	$y_1(t) = C_1 e^{\lambda_1 t}$ and $y_2(t) = C_2 e^{\lambda_2 t}$	$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$
If $p^2 - 4q < 0$	$y_1(t) = e^{at} \cos bt$ and $y_2(t) = e^{at} \sin bt$	$y(t) = e^{at} (A_1 \cos bt + A_2 \sin bt)$
If $p^2 - 4q = 0$	$y_1 = e^{\lambda t}$ and $y_2 = t e^{\lambda t}$	$y(t) = (C_1 + C_2 t) e^{\lambda_1 t}$

Exercises **Section 2.2 – Linear, Homogeneous Equations with Constant Coefficients**

(Exercises 1- 4) Decide whether the equation is linear or nonlinear. For the linear equation, state whether the equation is homogenous or inhomogeneous.

35. $t^2 y'' = 4y' - \sin t$

37. $t^2 y'' + 4yy' = 0$

36. $ty'' + (\sin t)y' = 4y - \cos 5t$

38. $y'' + 4y' + 7y = 3e^{-t} \sin t$

(Exercises 5- 6) Show by direct substitution that the given functions $y_1(t)$ and $y_2(t)$ are solutions of the given differential equation. Then verify by direct substitution, that any linear combination $C_1 y_1(t) + C_2 y_2(t)$ of the 2 given solutions is also a solution.

39. $y'' + 4y = 0$; $y_1(t) = \cos 2t$ $y_2(t) = \sin 2t$

40. $y'' - 2y' + 2y = 0$; $y_1(t) = e^t \cos t$ $y_2(t) = e^t \sin t$

41. Explain why $y_1(t)$ and $y_2(t)$ are linearly independent solutions. Calculate Wronskian and use it to explain the independence of the given solutions.

$$y'' + 9y = 0; \quad y_1(t) = \cos 3t \quad y_2(t) = \sin 3t$$

42. Show that $y_1(t) = e^t$ and $y_2(t) = e^{-3t}$ form a fundamental set of solutions for $y'' + 2y' - 3y = 0$, then find a solution satisfying $y(0) = 1$ and $y'(0) = -2$.

Find the general solution of the second order differential equation

43. $y'' + y' = 0$

56. $y'' + y' - 6y = 0$

69. $y'' + 3y' - 4y = 0$

44. $y'' - 4y = 0$

57. $y'' - y' - 11y = 0$

70. $y'' + 4y' - y = 0$

45. $y'' + 8y = 0$

58. $y'' - y' - 12y = 0$

71. $y'' - 4y' + 4y = 0$

46. $y'' - 36y = 0$

59. $y'' + 2y' + y = 0$

72. $y'' + 4y' + 4y = 0$

47. $y'' + 9y = 0$

60. $y'' + 2y' + 3y = 0$

73. $y'' - 4y' + 5y = 0$

48. $y'' - 9y = 0$

61. $y'' + 2y' - 3y = 0$

74. $y'' + 4y' + 5y = 0$

49. $y'' + 16y = 0$

62. $y'' - 2y' - 3y = 0$

75. $y'' + 4y' - 5y = 0$

50. $y'' + 25y = 0$

63. $y'' - 2y' + 3y = 0$

76. $y'' + 4y' + 7y = 0$

51. $y'' - 64y = 0$

64. $y'' + 2y' + 4y = 0$

77. $y'' + 4y' + 9y = 0$

52. $y'' + y' + y = 0$

65. $y'' - 2y' + 5y = 0$

78. $y'' + 5y' = 0$

53. $y'' + y' - y = 0$

66. $y'' + 2y' - 15y = 0$

79. $y'' + 5y' + 6y = 0$

54. $y'' - y' - 2y = 0$

67. $y'' + 2y' + 17y = 0$

80. $y'' + 6y' + 9y = 0$

55. $y'' - y' - 6y = 0$

68. $y'' - 3y' + 2y = 0$

81. $y'' - 6y' + 9y = 0$

- | | | |
|----------------------------|------------------------------|------------------------------|
| 82. $y'' - 6y' + 25y = 0$ | 98. $3y'' + y = 0$ | 114. $6y'' + 13y' - 5y = 0$ |
| 83. $y'' + 8y' + 16y = 0$ | 99. $3y'' - y' = 0$ | 115. $6y'' + 13y' + 7y = 0$ |
| 84. $y'' + 8y' - 16y = 0$ | 100. $3y'' + 2y' + y = 0$ | 116. $6y'' - 13y' + 7y = 0$ |
| 85. $y'' - 9y' + 20y = 0$ | 101. $3y'' + 11y' - 7y = 0$ | 117. $8y'' - 10y' - 3y = 0$ |
| 86. $y'' - 10y' + 25y = 0$ | 102. $3y'' - 20y' + 12y = 0$ | 118. $9y'' - y = 0$ |
| 87. $y'' + 14y' + 49y = 0$ | 103. $4y'' + y' = 0$ | 119. $9y'' + 6y' + y = 0$ |
| 88. $2y'' - y' - 3y = 0$ | 104. $4y'' + 4y' + y = 0$ | 120. $9y'' - 12y' + 4y = 0$ |
| 89. $2y'' + y' - y = 0$ | 105. $4y'' - 4y' + y = 0$ | 121. $9y'' + 24y' + 16y = 0$ |
| 90. $2y'' + 2y' + y = 0$ | 106. $4y'' + 4y' + 2y = 0$ | 122. $12y'' - 5y' - 2y = 0$ |
| 91. $2y'' + 2y' + 3y = 0$ | 107. $4y'' - 4y' + 13y = 0$ | 123. $16y'' - 8y' + 7y = 0$ |
| 92. $2y'' - 3y' - 2y = 0$ | 108. $4y'' - 8y' + 7y = 0$ | 124. $16y'' - 12y' - 4y = 0$ |
| 93. $2y'' - 3y' + 4y = 0$ | 109. $4y'' - 12y' + 9y = 0$ | 125. $16y'' - 24y' + 9y = 0$ |
| 94. $2y'' - 4y' + 8y = 0$ | 110. $4y'' + 20y' + 25y = 0$ | 126. $25y'' + 10y' + y = 0$ |
| 95. $2y'' + 5y' = 0$ | 111. $6y'' + 5y' - 6y = 0$ | 127. $25y'' - 10y' + y = 0$ |
| 96. $2y'' - 5y' - 3y = 0$ | 112. $6y'' + y' - 2y = 0$ | 128. $35y'' - y' - 12y = 0$ |
| 97. $2y'' + 7y' - 4y = 0$ | 113. $6y'' - 7y' - 20y = 0$ | |

Find the general solution of the given higher-order differential equation

- | | |
|---------------------------------------|---|
| 129. $y''' + 3y'' + 3y' + y = 0$ | 149. $y^{(4)} + y''' + y'' = 0$ |
| 130. $y''' + 3y'' - y' - 3y = 0$ | 150. $y^{(4)} - 2y'' + y = 0$ |
| 131. $y^{(3)} + 3y'' - 4y = 0$ | 151. $16y^{(4)} + 24y'' + 9y = 0$ |
| 132. $3y''' - 19y'' + 36y' - 10y = 0$ | 152. $y^{(4)} - 7y'' - 18y = 0$ |
| 133. $y''' - 6y'' + 12y' - 8y = 0$ | 153. $y^{(4)} + 2y'' + y = 0$ |
| 134. $y''' + 5y'' + 7y' + 3y = 0$ | 154. $y^{(4)} + y''' + y'' = 0$ |
| 135. $y^{(3)} + y' - 10y = 0$ | 155. $y^{(4)} + 4y = 0$ |
| 136. $y''' + y'' - 6y' + 4y = 0$ | 156. $y^{(4)} + 2y''' + 9y'' - 2y' - 10y = 0$ |
| 137. $y''' - 6y'' - y' + 6y = 0$ | 157. $x^{(4)} - 4x^{(3)} + 7x'' - 4x' + 6x = 0$ |
| 138. $y''' + 2y'' - 4y' - 8y = 0$ | 158. $x^{(4)} + 8x^{(3)} + 24x'' + 32x' + 16x = 0$ |
| 139. $y''' - 7y'' + 7y' + 15y = 0$ | 159. $x^{(4)} - 4x'' + 16x' + 32x = 0$ |
| 140. $y''' + 3y'' - 4y' - 12y = 0$ | 160. $x^{(4)} + 4x^{(3)} + 6x'' + 4x' + x = 0$ |
| 141. $y''' - 4y'' - 5y' = 0$ | 161. $y^{(4)} - y^{(3)} + y'' - 3y' - 6y = 0$ |
| 142. $y''' - y = 0$ | 162. $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$ |
| 143. $y''' - 5y'' + 3y' + 9y = 0$ | 163. $x^{(5)} - x^{(4)} - 2x^{(3)} + 2x'' + x' - x = 0$ |
| 144. $y''' + 3y'' - 4y' - 12y = 0$ | 164. $x^{(5)} + 5x^{(4)} + 10x^{(3)} + 10x'' + 5x' + x = 0$ |
| 145. $y''' + y'' - 2y = 0$ | |
| 146. $y''' - y'' - 4y = 0$ | |
| 147. $y''' + 3y'' + 3y' + y = 0$ | |
| 148. $y''' - 6y'' + 12y' - 8y = 0$ | |

$$165. y^{(5)} + 5y^{(4)} - 2y''' - 10y'' + y' + 5y = 0$$

$$166. 2y^{(5)} - 7y^{(4)} + 12y''' + 8y'' = 0$$

$$167. y^{(5)} - 2y^{(4)} + 17y''' = 0$$

$$168. x^{(6)} - 5x^{(4)} + 16x^{(3)} + 36x'' - 16x' - 32x = 0$$

$$169. (D^2 + 6D + 13)^2 y = 0$$

$$170. \lambda^3(\lambda - 1)(\lambda - 2)^3(\lambda^2 + 9) = 0$$

Find the solution of the given initial value problem.

$$171. y'' + y = 0; \quad y\left(\frac{\pi}{3}\right) = 0, \quad y'\left(\frac{\pi}{3}\right) = 2$$

$$172. y'' + y = 0; \quad y(0) = 0, \quad y'\left(\frac{\pi}{2}\right) = 0$$

$$173. y'' + y' = 0; \quad y(0) = 2, \quad y'(0) = 1$$

$$174. y'' - y' - 2y = 0; \quad y(0) = -1, \quad y'(0) = 2$$

$$175. y'' + y' + 2y = 0; \quad y(0) = 0, \quad y'(0) = 0$$

$$176. y'' + 2y' + y = 0; \quad y(0) = 1, \quad y'(0) = -3$$

$$177. y'' - 2y' + y = 0, \quad y(0) = 5, \quad y'(0) = 10$$

$$178. y'' - 2y' - 2y = 0; \quad y(0) = 0, \quad y'(0) = 3$$

$$179. y'' - 2y' + 2y = 0; \quad y(0) = 1, \quad y(\pi) = 1$$

$$180. y'' - 2y' - 3y = 0; \quad y(0) = 2, \quad y'(0) = -3$$

$$181. y'' + 2y' - 8y = 0; \quad y(0) = 3, \quad y'(0) = -12$$

$$182. y'' - 2y' + 17y = 0; \quad y(0) = -2, \quad y'(0) = 3$$

$$183. y'' + 2\sqrt{2}y' + 2y = 0; \quad y(0) = 1, \quad y'(0) = 0$$

$$184. y'' + 3y' - 10y = 0; \quad y(0) = 4, \quad y'(0) = -2$$

$$185. y'' + 4y = 0; \quad y(0) = 0, \quad y(\pi) = 0$$

$$186. y'' + 4y = 0; \quad y\left(\frac{\pi}{4}\right) = -2, \quad y\left(\frac{\pi}{4}\right) = 1$$

$$187. y'' + 4y' + 2y = 0; \quad y(0) = -1, \quad y'(0) = 2$$

$$188. y'' - 4y' + 3y = 0; \quad y(0) = 1, \quad y'(0) = \frac{1}{3}$$

$$189. y'' - 4y' + 4y = 0, \quad y(1) = 1, \quad y'(1) = 1$$

$$190. y'' + 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 3$$

$$191. y'' - 4y' + 5y = 0; \quad y(0) = 1, \quad y'(0) = 5$$

$$192. y'' + 4y' + 5y = 0; \quad y(0) = 1, \quad y'(0) = 0$$

$$193. y'' + 4y' + 5y = 0; \quad y\left(\frac{\pi}{2}\right) = \frac{1}{2}, \quad y'\left(\frac{\pi}{2}\right) = -2$$

$$194. y'' - 4y' - 5y = 0, \quad y(1) = 0, \quad y'(1) = 2$$

$$195. y'' - 4y' - 5y = 0, \quad y(-1) = 3, \quad y'(-1) = 9$$

$$196. y'' - 4y' + 9y = 0, \quad y(0) = 0, \quad y'(0) = -8$$

$$197. y'' - 4y' + 13y = 0; \quad y(0) = -1, \quad y'(0) = 2$$

$$198. y'' - 5y' + 6y = 0; \quad y(1) = e^2, \quad y'(1) = 3e^2$$

$$199. y'' + 6y' + 9y = 0; \quad y(0) = 2, \quad y'(0) = -2$$

$$200. y'' + 6y' + 5y = 0, \quad y(1) = 0, \quad y'(0) = 3$$

$$201. y'' - 6y' + 5y = 0; \quad y(0) = 3, \quad y'(0) = 11$$

$$202. y'' - 6y' + 9y = 0, \quad y(0) = 2, \quad y'(0) = \frac{25}{3}$$

$$203. y'' - 6y' + 9y = 0; \quad y(0) = 0, \quad y'(0) = 5$$

$$204. y'' + 8y' - 9y = 0; \quad y(1) = 2, \quad y'(1) = 0$$

$$205. y'' - 8y' + 17y = 0; \quad y(0) = 4, \quad y'(0) = -1$$

$$206. y'' - 9y = 0, \quad y(0) = 2, \quad y'(0) = -1$$

$$207. y'' - 10y' + 25y = 0, \quad y(0) = 1, \quad y'(1) = 0$$

$$208. y'' + 10y' + 25y = 0; \quad y(0) = 2, \quad y'(0) = -1$$

$$209. y'' + 11y' + 24y = 0; \quad y(0) = 0, \quad y'(0) = -7$$

$$210. y'' + 12y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

$$211. y'' + 16y = 0, \quad y(0) = 2, \quad y'(0) = -2$$

$$212. y'' + 16y = 0, \quad y(\pi) = 2, \quad y'(0) = -2$$

$$213. y'' + 16y = 0, \quad y\left(\frac{\pi}{2}\right) = -10, \quad y'\left(\frac{\pi}{2}\right) = 3$$

$$214. y'' + 25y = 0; \quad y(0) = 1, \quad y'(0) = -1$$

$$215. 2y'' - 2y' + y = 0; \quad y(-\pi) = 1, \quad y'(-\pi) = -1$$

$$216. 3y'' + y' - 14y = 0, \quad y(0) = 2, \quad y'(0) = -1$$

$$217. 3y'' + 2y' - 8y = 0, \quad y(0) = -6, \quad y'(0) = -18$$

$$218. 4y'' - 4y' + y = 0, \quad y(0) = 4, \quad y'(0) = 4$$

$$219. 4y'' - 4y' + y = 0, \quad y(1) = -4, \quad y'(1) = 0$$

220. $4y'' - 4y' - 3y = 0$, $y(0) = 1$, $y'(0) = 5$
221. $4y'' + 4y' + 5y = 0$, $y(\pi) = 1$, $y'(\pi) = 0$
222. $4y'' + 4y' + 17y = 0$, $y(0) = -1$, $y'(0) = 2$
223. $4y'' - 5y' = 0$, $y(-2) = 0$, $y'(-2) = 7$
224. $4y'' + 12y' + 9y = 0$, $y(0) = 2$, $y'(0) = 1$
225. $4y'' + 24y' + 37y = 0$, $y(\pi) = 1$, $y'(\pi) = 0$
226. $9y'' + y = 0$; $y\left(\frac{\pi}{2}\right) = 4$, $y'\left(\frac{\pi}{2}\right) = 0$
227. $9y'' + \pi^2 y = 0$; $y(3) = 2$, $y'(3) = -\pi$
228. $9y'' - 6y' + y = 0$; $y(3) = -2$, $y'(3) = -\frac{5}{3}$
229. $9y'' + 6y' + 2y = 0$; $y(3\pi) = 0$, $y'(3\pi) = \frac{1}{3}$
230. $9y'' - 12y' + 4y = 0$, $y(0) = -1$, $y'(0) = 1$
231. $12y'' + 5y' - 2y = 0$, $y(0) = 1$, $y'(0) = -1$
232. $16y'' - 8y' + y = 0$; $y(0) = -4$, $y'(0) = 3$
233. $25y'' + 20y' + 4y = 0$; $y(5) = 4e^{-2}$, $y'(5) = -\frac{3}{5}e^{-2}$
234. $y''' + 12y'' + 36y' = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -7$
235. $y''' + 2y'' - 5y' - 6y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$

236. The roots of the characteristic equation of a certain differential equation are:

$$3, -5, 0, 0, 0, 0, -5, 2 \pm 3i \text{ and } 2 \pm 3i$$

Write a general solution of this homogeneous differential equation.

237. $y(x) = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos 2x + C_4 \sin 2x$ is the general solution of a homogeneous equation. What is the equation?

238. Show that the second differential equation $y'' + 4y = 0$

a) Has no solution to the boundary value $y(0) = 0$, $y(\pi) = 1$

b) There are infinitely many solutions to the boundary value $y(0) = 0$, $y(\pi) = 0$

239. Show that the general solution of the equation

$$y'' + Py' + Qy = 0$$

(where P and Q are constant) approaches 0 as $x \rightarrow \infty$ if and only if P and Q are both positive.