

Prime number Peter  
 $P_i(\pi)$  vlada  
 exponential(e)  $\Rightarrow$  Alex.

gravitation ( $g_{32.2}^{9.8}$ )

Integration  $\Rightarrow$  William

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$$\forall x \in \mathbb{Z}^+, \forall y \in \mathbb{Z}^+, x+y > 0$$

$\mathbb{R}$ : real numbers

$\mathbb{R}^+$ : a positive #

$\mathbb{R} - \{0\}$

$$\boxed{x \neq 0, x \in \mathbb{R}}$$

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$$\forall x (C(x) \vee \exists y (C(y) \wedge F(x,y)))$$

$C(x)$ :  $x$  has a computer

$F(x,y)$   $x$  &  $y$  are friends

$x, y$  students in school.

For every student  $x$  in your school,  $(\forall x)$   
 $x$  has a computer or there is a

student  $y$  such that  $y$  has a computer  
and  $x$  and  $y$  are friends.

every student has a computer or  
has a friend who has a computer.

Everyone has exactly one best friend.

$\forall x$  (person has <sup>exactly</sup> 1 friend)  
 $y$  is best friend of  $x \iff B(x, y)$

$z$  is not best friend of  $x$ :

$\exists y (B(x, y) \wedge \forall z (z \neq y \rightarrow \neg B(x, z)))$

$\forall x \exists y (B(x, y))$

$$\neg \forall (x) P(x) = \exists x \neg P(x)$$

$$\neg \exists (x) P(x) = \forall x \neg P(x)$$

not  
conjugate  $\{ \forall = \neg \exists \}$

$$\begin{aligned} \neg \forall x \exists y (xy = 1) &= \exists x \neg (\exists y (xy = 1)) \\ &= \exists x \forall y \neg (xy = 1) \\ &= \exists x \forall y (xy \neq 1) \end{aligned}$$

$$\neg (\forall x \exists y \forall z P(x, y, z))$$

$$= \exists x \forall y \exists z \neg P(x, y, z)$$


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## Intro to Proofs

Theorem  $\rightarrow$  statement is true

Proof  $\nearrow$  is true.

axioms.

Propositions

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Direct proofs

$$P \rightarrow Q$$

$$F \rightarrow F \quad \text{F}$$

$$F \rightarrow T \quad \text{T}$$

P is false  $\Rightarrow$  statement always True

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if  $n$  is an odd integer, then  $n^2$  is odd.

Proof  $n$  is an odd  $\Rightarrow n = 2k + 1$   
 $(\forall k \in \mathbb{Z})$

$$n^2 = (2k + 1)^2$$

$$= 4k^2 + 4k + 1$$

$$= 2(2k^2 + 2k) + 1$$

$$\text{let } k: n = 2k + 1$$

$$= 2k + 1 \text{ is odd.}$$

By the defn  $n$  is an odd integer  
then  $n^2$  is an odd.

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$$p \rightarrow q \quad \text{contraposition}$$

$$\neg q \rightarrow \neg p$$

Ex  $n \in \mathbb{Z}, 3n+2 \text{ is odd} \Rightarrow n \text{ odd}$

$$3n+2 = 2k+1$$

$$3n = 2k - 1$$

$$n = \frac{2}{3}k - \frac{1}{3} \quad ??$$

$$n \text{ even} \Rightarrow 3n+2 \text{ is even.}$$

$$n \text{ even} \Rightarrow n = 2k$$

$$3n+2 = 3(2k)+2$$

$$= 6k+2$$

$$= 2(3k+1) \text{ even.}$$

$$\Rightarrow 3n+2 \text{ is odd} \Rightarrow n \text{ is odd}$$


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