

## ***Solution***      **Section 1.5 – Inequalities**

### ***Exercise***

Find:  $(-3, 0) \cap [-1, 2]$

### **Solution**

$$(-3, 0) \cap [-1, 2] = \underline{[-1, 0]}$$

### ***Exercise***

Find:  $(-3, 0) \cup [-1, 2]$

### **Solution**

$$(-3, 0) \cup [-1, 2] = \underline{(-3, 2]}$$

### ***Exercise***

Find:  $(-4, 0) \cap [-2, 1]$

### **Solution**

$$(-4, 0) \cap [-2, 1] = \underline{[-2, 0]}$$

### ***Exercise***

Find:  $(-4, 0) \cup [-2, 1]$

### **Solution**

$$(-4, 0) \cup [-2, 1] = \underline{(-4, 1]}$$

### ***Exercise***

Find:  $(-\infty, 5) \cap [1, 8)$

### **Solution**

$$(-\infty, 5) \cap [1, 8) = \underline{[1, 5]}$$

### Exercise

Find:  $(-\infty, 5) \cup [1, 8)$

#### Solution

$$(-\infty, 5) \cup [1, 8) = \underline{(-\infty, 8)}$$

### Exercise

Solve  $-3x + 5 > -7$

#### Solution

$$-3x > -7 - 5$$

$$-3x > -12$$

$$\frac{-3}{-3}x < \frac{-12}{-3}$$

$$\therefore \text{Solution: } \underline{x < 4} \quad (-\infty, 4)$$

### Exercise

Solve  $2 - 3x \leq 5$

#### Solution

$$-3x \leq 3 \quad \text{Divide by } -3 \text{ both sides}$$

$$\frac{-3}{-3}x \geq \frac{3}{-3}$$

$$\therefore \text{Solution: } \underline{x \geq -1} \quad \text{or } [-1, \infty)$$

### Exercise

Solve  $4 - 3x \leq 7 + 2x$

#### Solution

$$4 - 3x - 4 \leq 7 + 2x - 4$$

$$-3x \leq 3 + 2x$$

$$-3x - 2x \leq 3 + 2x - 2x$$

$$-5x \leq 3$$

$$\therefore \text{Solution: } \underline{x \geq -\frac{3}{5}} \quad \text{or } \left[-\frac{3}{5}, \infty\right)$$

### ***Exercise***

Solve the inequality equation  $5x + 11 < 26$

#### **Solution**

$$5x < 15$$

$$\therefore \text{Solution: } \underline{x < 3} \mid \text{ or } (-\infty, 3)$$

### ***Exercise***

Solve the inequality equation  $3x - 8 \geq 13$

#### **Solution**

$$3x \geq 21$$

$$\therefore \text{Solution: } \underline{x \geq 7} \mid \text{ or } [7, \infty)$$

### ***Exercise***

Solve the inequality equation  $-9x \geq 36$

#### **Solution**

$$-\frac{9x}{9} \geq \frac{36}{9}$$

$$-x \geq 4$$

$$\therefore \text{Solution: } \underline{x \leq -4} \mid \text{ or } (-\infty, -4]$$

### ***Exercise***

Solve the inequality equation  $-4x \leq 64$

#### **Solution**

$$-\frac{4x}{4} \leq \frac{64}{4}$$

$$-x \leq 16$$

$$\therefore \text{Solution: } \underline{x \geq -16} \mid \text{ or } [-16, \infty)$$

### ***Exercise***

Solve the inequality equation  $8x - 11 \leq 3x - 13$

#### **Solution**

$$5x \leq -2$$

$$\therefore \text{Solution: } \underline{x \leq -\frac{2}{5}} \quad \text{or} \quad \left(-\infty, -\frac{2}{5}\right]$$

### ***Exercise***

Solve the inequality equation  $18x + 45 \leq 12x - 8$

#### **Solution**

$$6x \leq -53$$

$$\therefore \text{Solution: } \underline{x \leq -\frac{53}{6}} \quad \text{or} \quad \left(-\infty, -\frac{53}{6}\right]$$

### ***Exercise***

Solve the inequality equation  $4(x+1) + 2 \geq 3x + 6$

#### **Solution**

$$4x + 4 + 2 \geq 3x + 6$$

$$\therefore \text{Solution: } \underline{x \geq 0} \quad \text{or} \quad [0, \infty)$$

### ***Exercise***

Solve the inequality equation  $8x + 3 > 3(2x + 1) + x + 5$

#### **Solution**

$$8x + 3 > 6x + 3 + x + 5$$

$$8x + 3 > 7x + 8$$

$$\therefore \text{Solution: } \underline{x > 5} \quad \text{or} \quad (5, \infty)$$

### ***Exercise***

Solve the inequality equation  $2x - 11 < -3(x + 2)$

#### **Solution**

$$2x - 11 < -3x - 6$$

$$5x < 5$$

$$\therefore \text{Solution: } \underline{x < 1} \quad \text{or} \quad (-\infty, 1)$$

### ***Exercise***

Solve the inequality equation  $-4(x + 2) > 3x + 20$

#### **Solution**

$$-4x - 8 > 3x + 20$$

$$-7x > 28$$

$$-x > 4$$

$$\therefore \text{Solution: } \underline{x < -4} \mid \text{ or } (-\infty, -4)$$

### ***Exercise***

Solve the inequality equation  $1 - (x + 3) \geq 4 - 2x$

#### **Solution**

$$1 - x - 3 \geq 4 - 2x$$

$$-x - 2 \geq 4 - 2x$$

$$x \geq 6$$

$$\therefore \text{Solution: } \underline{x \geq 6} \mid \text{ or } [6, \infty)$$

### ***Exercise***

Solve the inequality equation  $5(3 - x) \leq 3x - 1$

#### **Solution**

$$15 - 5x \leq 3x - 1$$

$$-8x \leq -16$$

$$-x \leq -2$$

$$\therefore \text{Solution: } \underline{x \geq 2} \mid \text{ or } [2, \infty)$$

### ***Exercise***

Solve the inequality equation  $\frac{x}{4} - \frac{1}{2} \leq \frac{x}{2} + 1$

#### **Solution**

$$4 \times \frac{x}{4} - \frac{1}{2} \leq \frac{x}{2} + 1$$

$$x - 2 \leq 2x + 4$$

$$-x \leq 6$$

$$\therefore \text{Solution: } \underline{x \geq -6} \mid \text{ or } [-6, \infty)$$

### Exercise

Solve the inequality equation  $\frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$

#### Solution

$$10 \times \frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$$

$$3x + 10 \geq 2 - x$$

$$4x \geq -8$$

$$\therefore \text{Solution: } \underline{x \geq -2} \mid \text{ or } [-2, \infty)$$

### Exercise

Solve  $6x - (2x + 3) \geq 4x - 5$

#### Solution

$$6x - 2x - 3 \geq 4x - 5$$

$$4x - 3 \geq 4x - 5$$

$$4x - 4x \geq 3 - 5$$

$$0 \geq -2 \quad (\text{true})$$

$$\therefore \text{Solution: } \mathbb{R} \quad (-\infty, \infty)$$

### Exercise

Solve  $\frac{2x-5}{-8} \leq 1-x$

#### Solution

$$(-8) \frac{2x-5}{-8} \geq (-8)(1-x)$$

$$2x - 5 \geq -8 + 8x$$

$$2x - 8x \geq -8 + 5$$

$$-6x \geq -3$$

$$\frac{-6}{-6} x \leq \frac{-3}{-6}$$

$$\therefore \text{Solution: } \underline{x \leq \frac{1}{2}} \mid \quad \left(-\infty, \frac{1}{2}\right]$$

### Exercise

Solve the inequality equation  $1 - \frac{x}{2} > 4$

#### Solution

$$2 \times 1 - \frac{x}{2} > 4$$

$$2 - x > 8$$

$$-x > 6$$

$$\therefore \text{Solution: } \underline{x < -6} \mid \text{ or } (-\infty, -6)$$

### Exercise

Solve the inequality equation  $7 - \frac{4}{5}x < \frac{3}{5}$

#### Solution

$$5 \times 7 - \frac{4}{5}x < \frac{3}{5}$$

$$35 - 4x < 3$$

$$-4x > -32$$

$$\therefore \text{Solution: } \underline{x < 8} \mid \text{ or } (-\infty, 8)$$

### Exercise

Solve the inequality equation  $\frac{x-4}{6} \geq \frac{x-2}{9} + \frac{5}{18}$

#### Solution

$$18 \times \frac{x-4}{6} \geq \frac{x-2}{9} + \frac{5}{18}$$

$$3x - 12 \geq 2x - 4 + 5$$

$$3x - 12 \geq 2x + 1$$

$$\therefore \text{Solution: } \underline{x \geq -13} \mid \text{ or } [-13, \infty)$$

### Exercise

Solve the inequality equation  $\frac{4x-3}{6} + 2 \geq \frac{2x-1}{12}$

#### Solution

$$12 \times \frac{4x-3}{6} + 2 \geq \frac{2x-1}{12}$$

$$8x - 6 + 24 \geq 2x - 1$$

$$8x + 18 \geq 2x - 1$$

$$6x \geq -19$$

$$\therefore \text{Solution: } \underline{x \geq -\frac{19}{6}} \mid \text{ or } \left[-\frac{19}{6}, \infty\right)$$

### ***Exercise***

Solve the inequality equation  $4(3x - 2) - 3x < 3(1 + 3x) - 7$

#### **Solution**

$$12x - 8 - 3x < 3 + 9x - 7$$

$$9x - 8 < 9x - 4$$

$$-8 < -4 \quad \text{True}$$

$$\therefore \text{Solution: } \mathbb{R}$$

### ***Exercise***

Solve the inequality equation  $3(x - 8) - 2(10 - x) < 5(x - 1)$

#### **Solution**

$$3x - 24 - 20 + 2x < 5x - 5$$

$$5x - 44 < 5x - 5$$

$$-44 < -5 \quad \text{True}$$

$$\therefore \text{Solution: } \mathbb{R}$$

### ***Exercise***

Solve the inequality equation  $8(x + 1) \leq 7(x + 5) + x$

#### **Solution**

$$8x + 8 \leq 7x + 35 + x$$

$$8x + 8 \leq 8x + 35$$

$$8 \leq 35$$

$$\therefore \text{Solution: } \mathbb{R}$$

### ***Exercise***

Solve the inequality equation  $4(x - 1) \geq 3(x - 2) + x$

#### **Solution**

$$4x - 4 \geq 3x - 6 + x$$

$$4x - 4 \geq 4x - 6$$

$$-4 \geq -6 \quad \text{True}$$

$$\therefore \text{Solution: } \mathbb{R}$$



### Exercise

Solve the inequality equation  $7(x+4)-13 > 12+13(3+x)$

#### Solution

$$7x+28-13 > 12+39+13x$$

$$7x+15 > 51+13x$$

$$-6x > 36$$

$$\therefore \text{Solution: } \underline{x < -9} \mid \text{ or } (-\infty, -9)$$

### Exercise

Solve the inequality equation  $-2[7x-(2x-3)] < -2(x+1)$

#### Solution

$$-2(7x-2x+3) < -2x-2$$

$$-2(5x+3) < -2x-2$$

$$-10x-6 < -2x-2$$

$$-8x < 4$$

$$\therefore \text{Solution: } \underline{x > \frac{1}{2}} \mid \text{ or } \left(\frac{1}{2}, \infty\right)$$

### Exercise

Solve the inequality equation  $6-\frac{2}{3}(3x-12) \leq \frac{2}{5}(10x+50)$

#### Solution

$$15 \times \quad 6-\frac{2}{3}(3x-12) \leq \frac{2}{5}(10x+50)$$

$$90-10(3x-12) \leq 6(10x+50)$$

$$90-30x+120 \leq 60x+300$$

$$210-30x \leq 60x+300$$

$$-90x \leq 90$$

$$\therefore \text{Solution: } \underline{x \geq -1} \mid \text{ or } [-1, \infty)$$

### Exercise

Solve the inequality equation  $\frac{2}{7}(7-21x)-4 < 10-\frac{3}{11}(11x-11)$

#### Solution

$$77 \times \frac{2}{7}(7 - 21x) - 4 < 10 - \frac{3}{11}(11x - 11)$$

$$22(7 - 21x) - 308 < 770 - 21(11x - 11)$$

$$154 - 462x - 308 < 770 - 231x + 231$$

$$-462x - 154 < -231x + 1,001$$

$$-231x < 1,155$$

$$-x < 5$$

$$\therefore \text{Solution: } \underline{x > 5} \mid \text{ or } (5, \infty)$$

### Exercise

Solve the inequality equation  $3[3(x + 5) + 8x + 7] + 5[3(x - 6) - 2(3x - 5)] < 2(4x + 3)$

#### Solution

$$3(3x + 15 + 8x + 7) + 5(3x - 18 - 6x + 10) < 8x + 6$$

$$3(11x + 22) + 5(-3x - 8) < 8x + 6$$

$$33x + 66 - 15x - 40 < 8x + 6$$

$$18x + 26 < 8x + 6$$

$$10x < -20$$

$$\therefore \text{Solution: } \underline{x < -2} \mid \text{ or } (-\infty, -2)$$

### Exercise

Solve the inequality equation  $5[3(2 - 3x) - 2(5 - x)] - 6[5(x - 2) - 2(4x - 3)] < 3x + 19$

#### Solution

$$5(6 - 9x - 10 + 2x) - 6(5x - 10 - 8x + 6) < 3x + 19$$

$$5(-7x - 4) - 6(-3x - 4) < 3x + 19$$

$$-35x - 20 + 18x + 24 < 3x + 19$$

$$-17x + 4 < 3x + 19$$

$$-20x < 15$$

$$\therefore \text{Solution: } \underline{x > -\frac{3}{4}} \mid \text{ or } \left(-\frac{3}{4}, \infty\right)$$

### Exercise

Solve the inequality equation  $0 \leq 3x - 1 \leq 10$

#### Solution

$$0 + 1 \leq 3x - 1 + 1 \leq 10 + 1$$

$$1 \leq 3x \leq 11$$

$$\therefore \text{Solution: } \underline{\frac{1}{3} \leq x \leq \frac{11}{3}}$$

### ***Exercise***

Solve the inequality equation  $0 \leq 1 - 3x \leq 10$

#### **Solution**

$$-1 \leq -3x \leq 9$$

$$-\frac{1}{3} \leq -x \leq 3$$

$$\therefore \text{Solution: } \underline{-3 \leq x \leq \frac{1}{3}}$$

### ***Exercise***

Solve the inequality equation  $0 \leq 2x + 6 \leq 54$

#### **Solution**

$$-6 \leq 2x \leq 48$$

$$\therefore \text{Solution: } \underline{-3 \leq x \leq 24}$$

### ***Exercise***

Solve the inequality equation  $-3 \leq \frac{2}{3}x - 5 \leq -1$

#### **Solution**

$$-3 \leq \frac{2}{3}x - 5 \leq -1$$

$$-3 + 5 \leq \frac{2}{3}x - 5 + 5 \leq -1 + 5$$

$$2 \leq \frac{2}{3}x \leq 4$$

$$2 \cdot \frac{3}{2} \leq \frac{3}{2} \cdot \frac{2}{3}x \leq \frac{3}{2} \cdot 4$$

$$\therefore \text{Solution: } \underline{3 \leq x \leq 6}$$

### Exercise

Solve  $-6 \leq 6x + 3 \leq 21$

#### Solution

$$-6 - 3 \leq 6x + 3 - 3 \leq 21 - 3$$

$$-9 \leq 6x \leq 18$$

$$-\frac{9}{6} \leq \frac{6}{6}x \leq \frac{18}{6}$$

$$\therefore \text{Solution: } \underline{-\frac{3}{2} \leq x \leq 3} \quad \left[ -\frac{3}{2}, 3 \right]$$

### Exercise

Solve the inequality equation:  $1 \leq 2x + 3 < 11$

#### Solution

$$1 - 3 \leq 2x + 3 - 3 < 11 - 3$$

$$-2 \leq 2x < 8$$

$$-\frac{2}{2} \leq \frac{2}{2}x < \frac{8}{2}$$

$$\therefore \text{Solution: } \underline{-1 \leq x < 4}$$

### Exercise

Solve the inequality equation  $|x| < 2$

#### Solution

$$\therefore \text{Solution: } \underline{-2 < x < 2}$$

### Exercise

Solve the inequality equation  $|x| \geq 2$

#### Solution

$$\therefore \text{Solution: } \underline{x \leq -2 \quad x \geq 2}$$

### Exercise

Solve the inequality equation  $|x - 2| < 1$

#### Solution

$$-1 < x - 2 < 1$$

$$\therefore \text{Solution: } \underline{1 < x < 3}$$

### ***Exercise***

Solve the inequality equation  $|x - 1| < 4$

#### **Solution**

$$-4 < x - 1 < 4$$

$$\therefore \text{Solution: } \underline{-3 < x < 5}$$

### ***Exercise***

Solve the inequality equation  $|x + 2| \geq 1$

#### **Solution**

$$x + 2 \leq -1 \quad x + 2 \geq 1$$

$$x \leq -3 \quad x \geq -1$$

$$\therefore \text{Solution: } \underline{x \leq -3 \quad x \geq -1}$$

### ***Exercise***

Solve the inequality equation  $|x + 1| \geq 4$

#### **Solution**

$$x + 1 \leq -4 \quad x + 1 \geq 4$$

$$x \leq -5 \quad x \geq 3$$

$$\therefore \text{Solution: } \underline{x \leq -5 \quad x \geq 3}$$

### ***Exercise***

Solve the inequality equation  $|3x + 5| < 17$

#### **Solution**

$$-17 < 3x + 5 < 17$$

$$-22 < 3x < 12$$

$$\therefore \text{Solution: } \underline{-11 < x < 4}$$

### ***Exercise***

Solve the inequality equation  $|5x - 2| < 13$

#### **Solution**

$$-13 < 5x - 2 < 13$$

$$-11 < 5x < 15$$

$$\therefore \text{Solution: } \underline{-\frac{11}{5} < x < 3}$$

### ***Exercise***

Solve the inequality equation  $|5x - 2| \geq 13$

#### **Solution**

$$5x - 2 \leq -13 \qquad 5x - 2 \geq 13$$

$$5x \leq -11 \qquad 5x \geq 15$$

$$x \leq -\frac{11}{5} \qquad x \geq 3$$

$$\therefore \text{Solution: } \underline{x \leq -\frac{11}{5} \quad x \geq 3}$$

### ***Exercise***

Solve the inequality equation  $|2(x - 1) + 4| \leq 8$

#### **Solution**

$$-8 \leq 2x - 2 + 4 \leq 8$$

$$-8 \leq 2x + 2 \leq 8$$

$$-10 \leq 2x \leq 6$$

$$\therefore \text{Solution: } \underline{-5 \leq x \leq 3}$$

### ***Exercise***

Solve the inequality equation  $|3(x - 1) + 2| \leq 20$

#### **Solution**

$$-20 \leq 3x - 3 + 2 \leq 20$$

$$-20 \leq 3x - 1 \leq 20$$

$$-19 \leq 3x \leq 21$$

$$\therefore \text{Solution: } \underline{-\frac{19}{3} \leq x \leq 7}$$

### ***Exercise***

Solve the inequality equation  $\left| \frac{2x+6}{3} \right| > 2$

#### **Solution**

$$|2x+6| > 6$$

$$2x+6 < -6 \qquad 2x+6 > 6$$

$$2x < -12 \qquad 2x > 0$$

$$x < -6 \qquad x > 0$$

$$\therefore \text{Solution: } \underline{x < -6 \quad x > 0}$$

### ***Exercise***

Solve the inequality equation  $\left| \frac{3x-3}{4} \right| < 6$

#### **Solution**

$$\frac{3}{4}|x-1| < 6$$

$$|x-1| < 8$$

$$-8 < x-1 < 8$$

$$\therefore \text{Solution: } \underline{-7 < x < 9}$$

### ***Exercise***

Solve the inequality equation  $\left| \frac{2x+2}{4} \right| \geq 2$

#### **Solution**

$$\frac{1}{2}|x+1| \geq 2$$

$$|x+1| \geq 4$$

$$x+1 \leq -4 \qquad x \geq 3$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x \geq 3}$$

### ***Exercise***

Solve the inequality equation  $\left| \frac{3x-3}{9} \right| \leq 1$

#### **Solution**

$$\frac{1}{3}|x-1| \leq 1$$

$$|x-1| \leq 3$$

$$-3 \leq x-1 \leq 3$$

$$\therefore \text{Solution: } \underline{-2 \leq x \leq 4}$$

### Exercise

Solve the inequality equation  $\left|3 - \frac{2x}{3}\right| > 5$

#### Solution

$$\frac{1}{3}|9-2x| > 5$$

$$|9-2x| > 15$$

$$9-2x < -15 \quad 9-2x > 15$$

$$-2x < -24 \quad -2x > 6$$

$$x > 12 \quad x < -3$$

$$\therefore \text{Solution: } \underline{x < -3 \quad x > 12}$$

### Exercise

Solve the inequality equation  $\left|3 - \frac{3x}{4}\right| < 9$

#### Solution

$$\frac{3}{4}|4-x| < 9$$

$$|4-x| < 12$$

$$-12 < 4-x < 12$$

$$-16 < -x < 8$$

$$16 > x > -8$$

$$\therefore \text{Solution: } \underline{-8 < x < 16}$$

### Exercise

Solve the inequality equation  $|x-2| < -1$

#### Solution

Impossible, since Absolute value can't be negative.

$\therefore$  No Solution



### ***Exercise***

Solve the inequality equation  $|x + 2| < -3$

#### **Solution**

*Impossible*, since Absolute value can't be negative.

$\therefore$  No ***Solution***

### ***Exercise***

Solve the inequality equation  $|x + 6| > -10$

#### **Solution**

$\therefore$  ***Solution:***  $\mathbb{R}$

### ***Exercise***

Solve the inequality equation  $|x + 2| > -8$

#### **Solution**

$\therefore$  ***Solution:***  $\mathbb{R}$

### ***Exercise***

Solve the inequality equation  $|x + 2| + 9 \leq 16$

#### **Solution**

$$|x + 2| \leq 5$$

$$-5 \leq x + 2 \leq 5$$

$\therefore$  ***Solution:***  $-7 \leq x \leq 3$

### ***Exercise***

Solve the inequality equation  $|x - 2| + 4 \geq 5$

#### **Solution**

$$|x - 2| \geq 1$$

$$x - 2 \leq -1$$

$$x \leq 1$$

$$x - 2 \geq 1$$

$$x \geq 3$$

$\therefore$  ***Solution:***  $x \leq 1 \quad x \geq 3$

### ***Exercise***

Solve the inequality equation  $2|2x - 3| + 10 > 12$

#### **Solution**

$$2|2x - 3| > 2$$

$$|2x - 3| > 1$$

$$2x - 3 < -1 \qquad 2x - 3 > 1$$

$$2x < 2 \qquad 2x > 4$$

$$x < 1 \qquad x > 2$$

$$\therefore \text{Solution: } \underline{x < 1 \quad x > 3}$$

### ***Exercise***

Solve the inequality equation  $3|2x - 1| + 2 < 8$

#### **Solution**

$$3|2x - 1| < 6$$

$$|2x - 1| < 2$$

$$-2 < 2x - 1 < 2$$

$$-1 < 2x < 3$$

$$\therefore \text{Solution: } \underline{-\frac{1}{2} < x < \frac{3}{2}}$$

### ***Exercise***

Solve the inequality equation  $-4|1 - x| < -16$

#### **Solution**

$$|1 - x| > 4$$

$$1 - x < -4 \qquad 1 - x > 4$$

$$-x < -5 \qquad -x > 3$$

$$x > 5 \qquad x < -3$$

$$\therefore \text{Solution: } \underline{x < -3 \quad x > 5}$$

### ***Exercise***

Solve the inequality equation  $-2|5 - x| < -6$

#### **Solution**

$$|5 - x| > 3$$

$$5 - x < -3$$

$$5 - x > 3$$

$$-x < -8$$

$$-x > -2$$

$$x > 8$$

$$x < 2$$

$$\therefore \text{Solution: } \underline{x < 2 \quad x > 8}$$

### Exercise

Solve the inequality equation  $3 \leq |2x - 1|$

#### Solution

$$|2x - 1| \geq 3$$

$$2x - 1 \leq -3$$

$$2x - 1 \geq 3$$

$$2x \leq -2$$

$$2x \geq 4$$

$$x \leq -1$$

$$x \geq 2$$

$$\therefore \text{Solution: } \underline{x \leq -1 \quad x \geq 2}$$

### Exercise

Solve the inequality equation  $9 \leq |4x + 7|$

#### Solution

$$|4x + 7| \geq 9$$

$$4x + 7 \leq -9$$

$$4x + 7 \geq 9$$

$$4x \leq -16$$

$$4x \geq 2$$

$$x \leq -4$$

$$x \geq \frac{1}{2}$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x \geq \frac{1}{2}}$$

### Exercise

Solve the inequality equation:  $12 < \left| -2x + \frac{6}{7} \right| + \frac{3}{7}$

#### Solution

$$(7)12 < \left| -(7)2x + (7)\frac{6}{7} \right| + (7)\frac{3}{7}$$

*Multiply by 7 both sides*

$$84 < \left| -14x + 6 \right| + 3$$

$$81 < \left| -14x + 6 \right|$$

$$|-14x+6| > 81$$

$$-14x+6 < -81 \qquad -14x+6 > 81$$

$$-14x < -81 - 6 \qquad -14x > 81 - 6$$

$$-14x < -87 \qquad -14x > 75$$

$$x > \frac{87}{14} \qquad x < -\frac{75}{14}$$

$$\therefore \text{Solution: } \underline{x < -\frac{75}{14} \quad x > \frac{87}{14}} \quad \left(-\infty, -\frac{75}{14}\right) \cup \left(\frac{87}{14}, \infty\right)$$

### Exercise

Solve the inequality equation:  $4 + \left|3 - \frac{x}{3}\right| \geq 9$

#### Solution

$$\left|3 - \frac{x}{3}\right| \geq 9 - 4$$

$$\left|3 - \frac{x}{3}\right| \geq 5$$

$$\left|(3)3 - (3)\frac{x}{3}\right| \geq (3)5$$

$$|9 - x| \geq 15$$

$$9 - x \leq -15 \qquad 9 - x \geq 15$$

$$-x \leq -24 \qquad -x \geq 6$$

$$x \geq 24 \qquad x \leq -6$$

$$\therefore \text{Solution: } \underline{x \leq -6 \quad x \geq 24} \quad (-\infty, -6] \cup [24, \infty)$$

### Exercise

Solve the inequality equation:  $|x - 2| < 5$

#### Solution

$$-5 < x - 2 < 5$$

$$\therefore \text{Solution: } \underline{-3 < x < 7}$$

### Exercise

Solve the inequality equation:  $|2x + 1| < 7$

#### Solution

$$-7 < 2x + 1 < 7$$

$$-7 - 1 < 2x + 1 - 1 < 7 - 1$$

$$-8 < 2x < 6$$

$$-\frac{8}{2} < \frac{2}{2}x < \frac{6}{2}$$

$$\therefore \text{Solution: } \underline{-4 < x < 3}$$

### Exercise

Solve the inequality equation:  $|5x + 2| - 2 < 3$

#### Solution

$$|5x + 2| < 5$$

$$-5 < 5x + 2 < 5$$

$$-7 < 5x < 3$$

$$\therefore \text{Solution: } \underline{-\frac{7}{5} < x < \frac{3}{5}} \quad \left(-\frac{7}{5}, \frac{3}{5}\right)$$

### Exercise

Solve the inequality equation:  $|2 - 7x| - 1 > 4$

#### Solution

$$|2 - 7x| > 5$$

$$2 - 7x < -5$$

$$2 - 7x > 5$$

$$-7x < -7$$

$$-7x > 3$$

$$x > 1$$

$$x < -\frac{3}{7}$$

$$\therefore \text{Solution: } \underline{x < -\frac{3}{7} \quad x > \frac{1}{14}} \quad \left(-\infty, -\frac{3}{7}\right) \cup \left(\frac{1}{14}, \infty\right)$$

### Exercise

Solve the inequality equation:  $|3x - 4| < 2$

#### Solution

$$-2 < 3x - 4 < 2$$

$$-2 + 4 < 3x - 4 + 4 < 2 + 4$$

$$2 < 3x < 6$$

$$\therefore \text{Solution: } \underline{\frac{2}{3} < x < 2} \quad \left(\frac{2}{3}, 2\right)$$

### Exercise

Solve the inequality equation:  $|2x + 5| \geq 3$

#### Solution

$$2x + 5 \leq -3 \qquad 2x + 5 \geq 3$$

$$2x \leq -8 \qquad 2x \geq -2$$

$$x \leq -4 \qquad x \geq -1$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x \geq -1} \quad (-\infty, -4] \cup [-1, \infty)$$

### Exercise

Solve  $|12 - 9x| \geq -12$

#### Solution

$\therefore$  **Solution** set:  $(-\infty, \infty)$  because the absolute value always greater than any negative number.

### Exercise

Solve  $|6 - 3x| < -11$

#### Solution

$\therefore$  **No solution**, because the absolute value cannot be less than any negative number

### Exercise

Solve the inequality equation  $|7 + 2x| < 0$

#### Solution

$\therefore$  **No solution**, because the absolute value cannot be any negative number

### Exercise

Solve:  $x^2 - 7x + 10 > 0$

#### Solution

$$x^2 - 7x + 10 > 0$$

$$(x - 5)(x - 2) > 0$$

$$\underline{x = 2, 5}$$

$$\therefore \text{Solution: } \underline{x < 2 \quad x > 5} \quad (-\infty, 2) \cup (5, \infty)$$

0	2		5
+		-	+

### Exercise

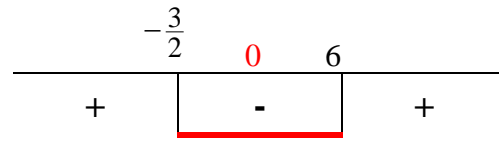
Solve:  $2x^2 - 9x \leq 18$

#### Solution

$$2x^2 - 9x - 18 \leq 0$$

$$(2x + 3)(x - 6) \leq 0$$

$$\therefore \text{Solution: } \underline{-\frac{3}{2} \leq x \leq 6} \quad \left[ -\frac{3}{2}, 6 \right]$$



### Exercise

Solve the inequality:  $x^2 - 5x + 4 > 0$

#### Solution

$$x^2 - 5x + 4 > 0$$

$$x = 1, 4$$

$$\therefore \text{Solution: } \underline{x < 1 \quad x > 4} \quad \underline{(-\infty, 1) \cup (4, \infty)}$$

### Exercise

Solve  $x^2 + x - 2 > 0$

#### Solution

$$x^2 + x - 2 = 0 \rightarrow x = -2, 1$$

$$\therefore \text{Solution: } \underline{x < -2 \quad x > 1} \quad \underline{(-\infty, -2) \cup (1, \infty)}$$

### Exercise

Solve  $x^2 - 4x + 12 < 0$

#### Solution

$$x^2 - 4x + 12 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 48}}{2}$$

$$= \frac{4 \pm \sqrt{-32}}{2} \quad \text{Complex number}$$

$\therefore$  No Solution

### ***Exercise***

Solve the inequality equation  $x^2 + 7x > 0$

#### **Solution**

$$x^2 + 7x = 0$$

$$x(x + 7) = 0$$

$$\underline{x = 0, -7}$$

$$\therefore \text{Solution: } \underline{x < -7 \quad x > 0}$$

### ***Exercise***

Solve the inequality equation  $x^2 - 49 < 0$

#### **Solution**

$$x^2 - 49 = 0$$

$$\underline{x = \pm 7}$$

$$\therefore \text{Solution: } \underline{-7 < x < 7}$$

### ***Exercise***

Solve the inequality equation  $x^2 - 5x \geq 0$

#### **Solution**

$$x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$$\underline{x = 0, 5}$$

$$\therefore \text{Solution: } \underline{x \leq 0 \quad x \geq 5}$$

### ***Exercise***

Solve the inequality equation  $x^2 - 16 \leq 0$

#### **Solution**

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$\underline{x = \pm 4}$$

$$\therefore \text{Solution: } \underline{-4 \leq x \leq 4}$$



### ***Exercise***

Solve the inequality equation  $x^2 + 7x + 10 < 0$

#### **Solution**

$$x^2 + 7x + 10 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{2}$$

$$= \frac{-7 \pm 3}{2}$$

$$= \begin{cases} \frac{-7-3}{2} = -5 \\ \frac{-7+3}{2} = -2 \end{cases}$$

$$\therefore \text{Solution: } \underline{-5 < x < -2}$$

### ***Exercise***

Solve the inequality equation  $x^2 - 3x \geq 28$

#### **Solution**

$$x^2 - 3x - 28 = 0$$

$$x = \frac{3 \pm \sqrt{121}}{2}$$

$$= \begin{cases} \frac{3-11}{2} = -4 \\ \frac{3+11}{2} = 7 \end{cases}$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x \geq 7}$$

### ***Exercise***

Solve the inequality equation  $x^2 + 5x + 6 < 0$

#### **Solution**

$$x^2 + 5x + 6 = 0$$

$$x = \frac{-5 \pm 1}{2}$$

$$= \begin{cases} \frac{-5-1}{2} = -3 \\ \frac{-5+1}{2} = -2 \end{cases}$$

$$\therefore \text{Solution: } \underline{-3 < x < -2}$$

### Exercise

Solve the inequality equation  $x^2 < -x + 30$

#### Solution

$$x^2 + x - 30 = 0$$

$$x = \frac{-1 \pm 11}{2}$$

$$= \begin{cases} \frac{-1-11}{2} = -6 \\ \frac{-1+11}{2} = 5 \end{cases}$$

$$\therefore \text{Solution: } \underline{-6 < x < 5}$$

### Exercise

Solve:  $x^3 - 3x^2 - 9x + 27 < 0$

#### Solution

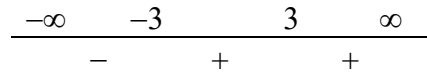
$$x^3 - 3x^2 - 9x + 27 = 0$$

$$x^2(x-3) - 9(x-3) = 0$$

$$(x-3)(x^2-9) = 0$$

$$\rightarrow \begin{cases} x-3=0 \rightarrow \underline{x=3} \\ x^2-9=0 \rightarrow x^2=9 \rightarrow \underline{x=\pm 3} \end{cases}$$

$$\therefore \text{Solution: } \underline{x < -3} \quad \underline{(-\infty, -3)}$$



### Exercise

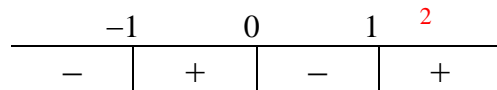
Solve  $x^3 - x > 0$

#### Solution

$$x(x^2 - 1) = 0$$

$$\rightarrow \begin{cases} x = 0 \\ x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1 \end{cases}$$

$$\therefore \text{Solution: } \underline{-1 < x < 0 \quad x > 1} \quad \underline{(-1, 0) \cup (1, \infty)}$$



### Exercise

$$x^3 + 3x^2 \leq x + 3$$

#### Solution

$$x^3 + 3x^2 - x - 3 = 0$$

$$x^2(x+3) - (x+3) = 0$$

$$(x+3)(x^2 - 1) = 0$$

$$\begin{cases} x+3=0 \rightarrow x=-3 \\ x^2-1=0 \rightarrow x^2=1 \rightarrow x=\pm 1 \end{cases}$$

	-3		-1	0	1	
	-	+	-	+		

$$\therefore \text{Solution: } \underline{-1 < x < 0 \quad x > 1} \quad \underline{(-\infty, -3] \cup [-1, 1]}$$

### Exercise

$$\text{Solve } x^3 + x^2 \geq 48x$$

#### Solution

$$x^3 + x^2 - 48x = 0$$

$$x(x^2 + x - 48) = 0$$

$$x = 0 \quad x^2 + x - 48 = 0$$

$$x = \frac{-1 \pm \sqrt{1+192}}{2}$$

	$\frac{-1-\sqrt{193}}{2}$		0	1	$\frac{-1+\sqrt{193}}{2}$	
-	+	-	+			

$$\therefore \text{Solution: } \underline{\frac{-1-\sqrt{193}}{2} < x < 0 \quad x > \frac{-1+\sqrt{193}}{2}} \quad \underline{\left[ \frac{-1-\sqrt{193}}{2}, 0 \right] \cup \left[ \frac{-1+\sqrt{193}}{2}, \infty \right)}$$

### Exercise

$$\text{Solve the inequality equation } x^3 - x^2 - 16x + 16 < 0$$

#### Solution

$$x^2(x-1) - 16(x-1) = 0$$

$$(x-1)(x^2 - 16) = 0$$

$$x = 1, \pm 4$$

	-4	0	1	4	
-	+	-	+		

$$\therefore \text{Solution: } \underline{x < -4 \quad 1 < x < 4}$$

### Exercise

Solve the inequality equation  $x^3 + x^2 - 9x - 9 > 0$

#### Solution

$$x^2(x+1) - 9(x+1) = 0$$

$$(x+1)(x^2 - 9) = 0$$

$$x = -1, \pm 3$$

$$\therefore \text{Solution: } \underline{-3 < x < -1 \quad x > 3}$$

	-3	-1	0	3	
-		+	-	+	

### Exercise

Solve the inequality equation  $x^3 + 3x^2 - 4x - 12 \geq 0$

#### Solution

$$x^2(x+3) - 4(x+3) = 0$$

$$(x+3)(x^2 - 4) = 0$$

$$x = -3, \pm 2$$

$$\therefore \text{Solution: } \underline{-3 < x < -2 \quad x > 2}$$

	-3	-2	0	2	
-		+	-	+	

### Exercise

Solve the inequality equation  $x^4 - 20x^2 + 64 \leq 0$

#### Solution

$$x^4 - 20x^2 + 64 = 0$$

$$x^2 = \frac{20 \pm \sqrt{400 - 256}}{2}$$

$$= \begin{cases} \frac{20-12}{2} = 4 \\ \frac{20+12}{2} = 16 \end{cases}$$

$$\begin{cases} x^2 = 4 \rightarrow x = \pm 2 \\ x^2 = 16 \rightarrow x = \pm 4 \end{cases}$$

$$\therefore \text{Solution: } \underline{-4 \leq x \leq -2 \quad 2 \leq x \leq 4}$$

	-4	-2	0	2	4	
+		-	+	-	+	

### Exercise

Solve the inequality equation  $x^4 - 10x^2 + 9 \geq 0$

#### Solution

$$x^4 - 10x^2 + 9 = 0$$

$$\begin{cases} x^2 = 1 \rightarrow x = \pm 1 \\ x^2 = 9 \rightarrow x = \pm 3 \end{cases}$$

$$\therefore \text{Solution: } \underline{x \leq -3 \quad -1 \leq x \leq 1 \quad x \geq 3}$$



### Exercise

Solve the inequality equation  $\frac{x+4}{x-1} < 0$

#### Solution

Restriction:  $x \neq 1$

$$\frac{x+4}{x-1} = 0$$

$$\underline{x = -4}$$

$$\therefore \text{Solution: } \underline{-4 < x < 1}$$



### Exercise

Solve the inequality equation  $\frac{x-2}{x+3} > 0$

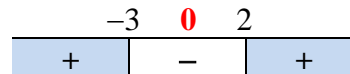
#### Solution

Restriction:  $x \neq -3$

$$\frac{x-2}{x+3} = 0$$

$$\underline{x = 2}$$

$$\therefore \text{Solution: } \underline{x < -3 \quad x > 2}$$



### Exercise

Solve the inequality equation  $\frac{x-5}{x+8} \geq 3$

#### Solution

Restriction:  $x \neq -8$

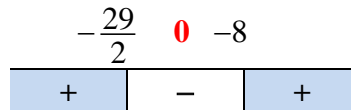
$$\frac{x-5}{x+8} - 3 = 0$$

$$x-5-3x-24=0$$

$$-2x=29$$

$$x = -\frac{29}{2}$$

$$\therefore \text{Solution: } \underline{x \leq -\frac{29}{2} \quad x > -8}$$



### Exercise

Solve the inequality equation  $\frac{x-4}{x+6} \leq 1$

#### Solution

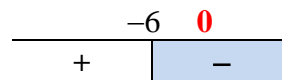
Restriction:  $x \neq -6$

$$\frac{x-4}{x+6} - 1 = 0$$

$$x-4-x-6=0$$

$$-10=0 \quad \times$$

$$\therefore \text{Solution: } \underline{x > -6}$$



### Exercise

Solve the inequality equation  $\frac{x}{2x+7} \geq 4$

#### Solution

Restriction:  $x \neq -\frac{7}{2}$

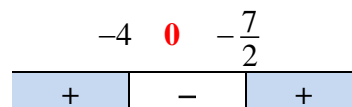
$$\frac{x}{2x+7} - 4 = 0$$

$$x-8x-28=0$$

$$7x=-28$$

$$x = -4$$

$$\therefore \text{Solution: } \underline{x \leq -4 \quad x > -\frac{7}{2}}$$



### Exercise

Solve the inequality equation  $\frac{x}{3x-5} \leq -5$

#### Solution

Restriction:  $x \neq \frac{5}{3}$

$$\frac{x}{3x-5} + 5 = 0$$

$$x + 15x - 25 = 0$$

$$16x = 25$$

$$x = \frac{25}{16}$$

$$\therefore \text{Solution: } \frac{25}{16} \leq x < \frac{5}{3}$$

$0$	$\frac{25}{16}$	$\frac{5}{3}$
+	-	+

### Exercise

Solve the inequality equation  $\frac{x+2}{x-5} \leq 2$

#### Solution

Restriction:  $x \neq 5$

$$\frac{x+2}{x-5} - 2 = 0$$

$$x + 2 - 2x + 10 = 0$$

$$x = 12$$

$$\therefore \text{Solution: } 5 \leq x < 12$$

5	$0$	12
+	-	+

### Exercise

Solve the inequality equation  $\frac{3x+1}{x-2} \geq 4$

#### Solution

Restriction:  $x \neq 2$

$$\frac{3x+1}{x-2} - 4 = 0$$

$$3x + 1 - 4x + 8 = 0$$

$$x = 9$$

$$\therefore \text{Solution: } 2 \leq x < 9$$

$0$	2	9
-	+	-

### Exercise

Solve the inequality equation  $\frac{x}{x-3} > 0$

#### Solution

Restriction:  $x \neq 3$

$$\frac{x}{x-3} = 0$$

$$x = 0$$

$$\therefore \text{Solution: } \underline{x < 0 \quad x > 3} \quad \underline{(-\infty, 0) \cup (3, \infty)}$$

	0		3
+		-	+

### Exercise

Solve the inequality equation  $\frac{x-3}{x+2} \geq 0$

#### Solution

Restriction:  $x \neq -2$

$$\frac{x-3}{x+2} = 0$$

$$x = 3$$

$$\therefore \text{Solution: } \underline{x < -2 \quad x \geq 3}$$

	-2	0	3
+		-	+

### Exercise

Solve the inequality equation  $\frac{x-2}{x+2} \leq 2$

#### Solution

Restriction:  $x \neq -2$

$$\frac{x-2}{x+2} - 2 = 0$$

$$x-2-2x-4=0$$

$$-x-6=0$$

$$x = -6$$

$$\therefore \text{Solution: } \underline{x \leq -6 \quad x > -2} \quad \underline{(-\infty, -6] \cup (-2, \infty)}$$

	-6		-2
-		+	-

### Exercise

Solve the inequality equation  $\frac{x+2}{x-2} \geq 2$

#### Solution

Restriction:  $x \neq 2$

$$\frac{x+2}{x-2} - 2 = 0$$

$$x+2-2x+4=0$$



$$\underline{x = 6}$$

$$\therefore \text{Solution: } \underline{2 < x \leq 6}$$



### Exercise

Solve the inequality equation  $\frac{x+2}{3+2x} \leq 5$

### Solution

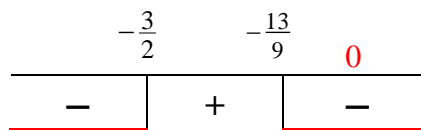
Restriction:  $x \neq -\frac{3}{2}$

$$\frac{x+2}{3+2x} - 5 = 0$$

$$x+2-15-10x=0$$

$$-9x-13=0$$

$$\underline{x = -\frac{13}{9}}$$



$$\therefore \text{Solution: } \underline{x < -\frac{3}{2} \quad x > -\frac{13}{9}} \quad \left( -\infty, -\frac{3}{2} \right) \cup \left[ -\frac{13}{9}, \infty \right)$$

### Exercise

Solve the inequality  $\frac{x+6}{x-14} \geq 1$

### Solution

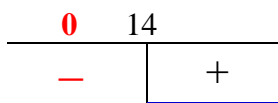
Restriction:  $x-14 \neq 0 \Rightarrow \boxed{x \neq 14}$

$$\frac{x+6}{x-14} - 1 = 0$$

$$x+6-x+14=0$$

$$20=0 \quad (\text{Impossible}) \quad \text{No Solution}$$

$$\therefore \text{Solution: } \underline{x > 14} \quad \underline{(14, \infty)}$$



### Exercise

Solve:  $\frac{x-3}{x+4} \geq \frac{x+2}{x-5}$

### Solution

Conditions:  $x+4 \neq 0 \rightarrow x \neq -4$  and  $x-5 \neq 0 \rightarrow x \neq 5$

$$\frac{x-3}{x+4} - \frac{x+2}{x-5} = 0$$

$$\frac{0-3}{0+4} - \frac{0+2}{0-5} = \frac{-3}{4} - \frac{2}{-5} = \frac{-3}{4} + \frac{2}{5} = -$$

$$\begin{aligned}
 (x+4)(x-5) \left[ \frac{x-3}{x+4} - \frac{x+2}{x-5} \right] &= 0 \\
 (x-5)(x-3) - (x+4)(x+2) &= 0 \\
 x^2 - 3x - 5x + 15 - (x^2 + 2x + 4x + 8) &= 0 \\
 x^2 - 3x - 5x + 15 - x^2 - 2x - 4x - 8 &= 0 \\
 -14x + 7 &= 0 \\
 -14x &= -7 \\
 x = \frac{-7}{-14} &= \frac{1}{2}
 \end{aligned}$$

	-4	0	1/2	5	
	+	-	+	-	

$$\therefore \text{Solution: } \underline{x < -4 \quad \frac{1}{2} \leq x < 5} \quad \underline{(-\infty, -4) \cup \left[ \frac{1}{2}, 5 \right)}$$

### Exercise

Solve:  $\frac{x-4}{x+3} - \frac{x+2}{x-1} \leq 0$

### Solution

Conditions:  $x \neq -3$  and  $x \neq 1$

$$\frac{x-4}{x+3} - \frac{x+2}{x-1} = 0 \qquad \frac{0-4}{0+3} - \frac{0+2}{0-1} = -\frac{4}{3} + 2 > 0$$

$$(x+3)(x-1) \left[ \frac{x-4}{x+3} - \frac{x+2}{x-1} \right] = 0$$

$$(x-1)(x-4) - (x+3)(x+2) = 0$$

$$x^2 - 5x + 4 - (x^2 + 5x + 6) = 0$$

$$x^2 - 5x + 4 - x^2 - 5x - 6 = 0$$

$$-10x - 2 = 0$$

$$\underline{x = -\frac{1}{5}}$$

	-3	0	$-\frac{1}{5}$	1	
	+	-	+	-	

$$\therefore \text{Solution: } \underline{-3 < x \leq -\frac{1}{5} \quad x > 1} \quad \underline{\left( -3, -\frac{1}{5} \right] \cup (1, \infty)}$$

### Exercise

Solve:  $\frac{2x-1}{x+3} \geq \frac{x+1}{3x+1}$

### Solution

Conditions:  $x \neq -3$  and  $x \neq -\frac{1}{3}$

$$\frac{2x-1}{x+3} - \frac{x+1}{3x+1} \geq 0$$

$$(x+3)(3x+1) \frac{2x-1}{x+3} - (x+3)(3x+1) \frac{x+1}{3x+1} = 0$$

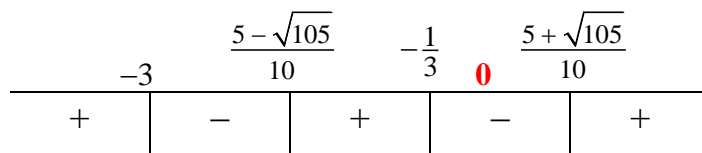
$$(3x+1)(2x-1) - (x+3)(x+1) = 0$$

$$6x^2 - 3x + 2x - 1 - (x^2 + x + 3x + 3) = 0$$

$$6x^2 - x - 1 - x^2 - 4x - 3 = 0$$

$$5x^2 - 5x - 4 = 0$$

$$x = \frac{5 \pm \sqrt{105}}{10} \rightarrow -0.5 \quad 1.5$$



$$\therefore \text{Solution: } \underline{x < -3 \quad \frac{5-\sqrt{105}}{10} \leq x < -\frac{1}{3} \quad x \geq \frac{5+\sqrt{105}}{10}}$$

$$(-\infty, -3) \cup \left[ \frac{5-\sqrt{105}}{10}, -\frac{1}{3} \right) \cup \left[ \frac{5+\sqrt{105}}{10}, \infty \right)$$

### Exercise

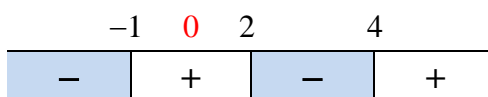
Solve the inequality equation  $\frac{(x+1)(x-4)}{x-2} < 0$

#### Solution

**Restriction:**  $x \neq 2$

$$\frac{(x+1)(x-4)}{x-2} = 0$$

$$x = -1, 4$$



$$\therefore \text{Solution: } \underline{x < -1 \quad 2 < x < 4}$$

### Exercise

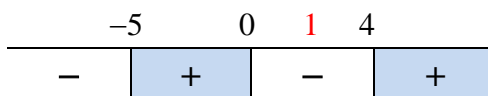
Solve the inequality equation  $\frac{x(x-4)}{x+5} > 0$

#### Solution

**Restriction:**  $x \neq -5$

$$\frac{x(x-4)}{x+5} = 0$$

$$x = 0, 4$$



$$\therefore \text{Solution: } \underline{-5 < x < 0 \quad x > 4}$$

### Exercise

Solve the inequality equation  $\frac{6x^2 - 11x - 10}{x} > 0$

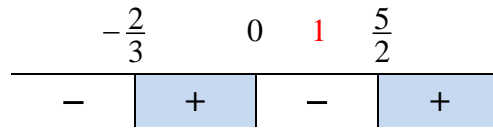
#### Solution

**Restriction:**  $x \neq 0$

$$6x^2 - 11x - 10 = 0$$

$$x = \frac{11 \pm \sqrt{121 + 240}}{12}$$

$$= \left\{ \begin{array}{l} \frac{11-19}{12} = -\frac{2}{3} \\ \frac{11+19}{12} = \frac{5}{2} \end{array} \right\}$$



$$\therefore \text{Solution: } \underline{-\frac{2}{3} < x < 0 \quad x > \frac{5}{2}}$$

### Exercise

Solve the inequality equation  $\frac{3x^2 - 2x - 8}{x - 1} \geq 0$

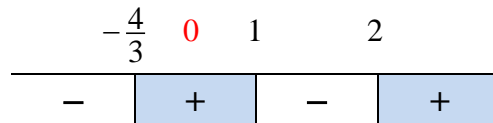
#### Solution

**Restriction:**  $x \neq 1$

$$3x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 96}}{6}$$

$$= \left\{ \begin{array}{l} \frac{2-10}{6} = -\frac{4}{3} \\ \frac{2+10}{6} = 2 \end{array} \right\}$$



$$\therefore \text{Solution: } \underline{-\frac{4}{3} \leq x < 1 \quad x \geq 2}$$

### Exercise

Solve the inequality equation  $\frac{x^2 - 6x + 9}{x - 5} \leq 0$

#### Solution

**Restriction:**  $x \neq 5$

$$x^2 - 6x + 9 = 0$$

$$(x - 3)^2 = 0$$

$$\underline{x = 3}$$

$$\therefore \text{Solution: } \underline{x < 5}$$

	0	3	5
	-	-	+

### Exercise

Solve the inequality equation  $\frac{x^2 + 10x + 25}{x + 1} < 0$

### Solution

**Restriction:**  $x \neq -1$

$$x^2 + 10x + 25 = 0$$

$$(x + 5)^2 = 0$$

$$\underline{x = -5}$$

$$\therefore \text{Solution: } \underline{x < -1}$$

	-5	-1	0
-	-	+	

### Exercise

A car can be rented from Basic Rental for \$260 per week with no extra charge for mileage. Continental charges \$80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

### Solution

$x$ : number of miles driven

For Continental, cost:  $80 + .25x$

Basic Rental a better deal than Continental's

$$260 < 80 + 0.25x$$

$$260 - 80 < 0.25x$$

$$180 < .25x$$

$$720 < x$$

**Solution:** more than 720 miles per week.

### Exercise

If a projectile is launched from ground level with an initial velocity of 96 ft per sec, its height in feet  $t$  seconds after launching is  $s$  feet, where

$$s = -16t^2 + 96t$$

When will the projectile be greater than 80 ft above the ground?

### Solution

Projectile be greater than 80 ft above the ground

$$s > 80$$

$$-16t^2 + 96t > 80$$

$$-16t^2 + 96t - 80 > 0$$

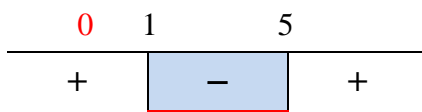
$$\frac{-16}{-16}t^2 + \frac{96}{-16}t - \frac{80}{-16} < 0$$

$$t^2 - 6t + 5 < 0$$

$$t^2 - 6t + 5 = 0$$

$$(t-1)(t-5) = 0$$

$$t = 1, 5$$



**Solution** (1, 5)

### Exercise

A projectile is fired straight up from ground level. After  $t$  seconds, its height above the ground is  $s$  ft, where

$$s = -16t^2 + 220t$$

For what time period is the projectile at least 624 ft above the ground?

### Solution

Projectile at least 624 ft.

$$s \geq 624$$

$$-16t^2 + 220t \geq 624$$

$$-16t^2 + 220t - 624 \geq 0 \quad \text{Divide by -4}$$

$$4t^2 - 55t + 156 \leq 0$$

$$t = \frac{-(-55) \pm \sqrt{(-55)^2 - 4(4)(156)}}{2(4)} = \frac{55 \pm 23}{16}$$

$$t = \frac{55+23}{16} \quad t = \frac{55-23}{16}$$

$$= \frac{78}{16} \quad = \frac{32}{16}$$

$$= \frac{39}{8} \quad = 2$$

**Solution:**  $\left[2, \frac{39}{8}\right]$

### ***Exercise***

Your test scores of 70 and 81 in your math class. To receive a *C* grade, you must obtain an average greater than or equal to 72 but less than 82. What range of test scores on the one remaining test will enable you to get a *C* for the course.

#### **Solution**

$$72 \leq \frac{70 + 81 + x}{3} < 82$$

$$216 \leq 151 + x < 246$$

$$65 \leq x < 95$$

∴ The range of test scores on the one remaining test will enable you to get a *C* for the course is

$$65 \leq x < 95$$

### ***Exercise***

A truck can be rented from Basic Rental for \$50 a day plus \$0.20 per *mile*. Continental charges \$20 per day plus \$0.50 per *mile* to rent the same truck. How many miles must be driven in a day to make the rental cost for Basic Rental a better deal than Constiental's?

#### **Solution**

$$\text{Basic Rental: } BR = 50 + 0.2x$$

$$\text{Continental: } C = 20 + 0.5x$$

$$BR > C$$

$$50 + 0.2x > 20 + 0.5x$$

$$30 > 0.3x$$

$$x < 100 \mid$$

∴ **100** miles must be driven in a day to make the rental cost for Basic Rental a better deal than Constiental's.

### ***Exercise***

You are choosing between two telephone plans. Plan **A** has a monthly fee of \$15 with a charge of \$0.08 per *minute* for all calls. Plan **B** has a monthly fee of \$3 with a charge of \$0.12 per *minute* for all calls. How many calling minutes in a month make plan **A** the better deal?

#### **Solution**

$$\text{Plan A: } 15 + 0.08x$$

$$\text{Plan B: } 3 + .12x$$

$$A < B$$

$$15 + 0.08x < 3 + 0.12x$$

$$12 < 0.04x$$

$$x > 300 \mid$$

∴ Plan **A** is a better deal when more than 300 minutes.

### ***Exercise***

A City commission has proposed two tax bills. The first bill requires that a homeowner pay \$1,800 plus 3% of the assessed home value in taxes. The second bill requires taxes of \$200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal for the homeowner?

### **Solution**

$$\text{First bill: } B_1 = 1,800 + 0.03x$$

$$\text{Second bill: } B_2 = 200 + 0.08x$$

$$B_1 < B_2$$

$$1,800 + 0.03x < 200 + 0.08x$$

$$1,600 < 0.05x$$

$$0.05x > 1,600$$

$$x > \frac{1,600}{0.05}$$
$$= \underline{32,000}$$

∴ The first bill is a better deal for the homeowner when greater than **\$32,000**

### ***Exercise***

A local bank charges \$8 per month plus \$0.05 per check. The credit union charges \$2 per month \$0.08 per check. How many checks should be written each month to make the credit union a better deal?

### **Solution**

$$\text{Local bank: } C_1 = 8 + .05x$$

$$\text{Credit union: } C_2 = 2 + .08x$$

$$C_1 > C_2$$

$$8 + .05x > 2 + .08x$$

$$6 > .03x$$

$$.03x < 6$$

$$x < \frac{6}{0.03}$$
$$= \underline{200}$$

∴ The credit union make less than **200** checks for a better deal.



### ***Exercise***

A company manufactures and sells blank audiocassette tapes. The weekly fixed cost is \$10,000 and it costs \$0.40 to produce each tape. The selling price is \$2.00 per tape. How many tapes must be produced and sold each week for the company to have a profit?

#### **Solution**

$$\text{Cost: } C = 10,000 + .4x$$

$$\text{Revenue: } R = 2x$$

$$C < R$$

$$10,000 + .4x < 2x$$

$$10,000 < 1.6x$$

$$1.6x > 10,000$$

$$x > \frac{10,000}{1.6}$$

$$= 6,250 \mid$$

∴ For the company to have a profit, they must sell more than **6,250** tapes.

### ***Exercise***

A company manufactures and sells stationery. The weekly fixed cost is \$3,000 and it costs \$3.00 to produce each package of stationery. The selling price is \$5.50 per package. How many packages of stationery must be produced and sold each week for the company to have a profit?

#### **Solution**

$$\text{Cost: } C = 3,000 + 3x$$

$$\text{Revenue: } R = 5.5x$$

$$C < R$$

$$3,000 + 3x < 5.5x$$

$$3,000 < 2.5x$$

$$2.5x > 3,000$$

$$x > \frac{3,000}{2.5}$$

$$= 1,200 \mid$$

∴ For the company to have a profit when it produces more than **1,200** packages each week.

### ***Exercise***

An elevator at a construction site has a maximum capacity of 3,000 *pounds*. If the elevator operator weighs 200 *pounds* and each cement bag weighs 70 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

#### **Solution**

The weight inside the elevator:  $200 + 70x$

$$200 + 70x \leq 3,000$$

$$70x \leq 2,800$$

$$x \leq \frac{2,800}{70}$$

$$= 40 \mid$$

$\therefore$  **50** bags of cement or less.

### ***Exercise***

An elevator at a construction site has a maximum capacity of 2,500 *pounds*. If the elevator operator weighs 160 *pounds* and each cement bag weighs 60 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

### **Solution**

The weight inside the elevator:  $160 + 60x$

$$160 + 60x \leq 2,500$$

$$60x \leq 2,340$$

$$x \leq \frac{2,340}{60}$$

$$= 39 \mid$$

$\therefore$  **39** bags of cement or less.

#56

### ***Exercise***

You can rent a car for the day from Company **A** for \$29.00 plus \$0.12 a *mile*. Company **B** charges \$22.00 plus \$0.21 a *mile*. Find the number of miles  $m$  per day for which it is cheaper to rent from Company **A**.

### **Solution**

Plan **A**:  $29 + 0.12x$

Plan **B**:  $22 + 0.21x$

$$A < B$$

$$29 + 0.12x < 22 + 0.21x$$

$$7 < 0.09x$$

$$0.09x > 7$$

$$x > \frac{7}{.09}$$

$$= \frac{700}{9} \mid \approx 78$$

$\therefore$  Plan **A** is a better deal when more than 78 days.

### Exercise

UPS will only ship packages for which the length is less than or equal to 108 *inches* and the length plus the girth is less than or equal to 130 *inches*. The length of a package is defined as the length of the longest side. The girth is defined as twice the width plus twice the height of the package. If a box has a length of 34 *inches* and a width of 22 *inches*, determine the possible range of heights  $h$  for this package if you wish to ship it by UPS.

### Solution

**Given:**  $\ell \leq 108$

$$\ell + 2w + 2h \leq 130$$

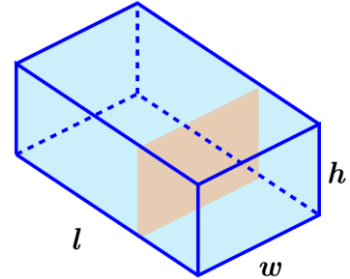
$$34 + 2(22) + 2h \leq 130$$

$$2h \leq 130 - 34 - 44$$

$$2h \leq 52$$

$$h \leq 26$$

$\therefore$  The possible range of heights  $h$  for this package  $0 < h \leq 26$



### Exercise

The sum of three consecutive odd integers is between 63 and 81. Find all possible sets of integers that satisfy these conditions.

### Solution

Let the first odd number is given by:  $2n + 1$

$$63 < (2n + 1) + (2n + 3) + (2n + 5) < 81$$

$$63 < 6n + 9 < 81$$

$$54 < 6n < 72$$

$$9 < n < 12$$

For  $n = 10 \rightarrow 21, 23, 25$

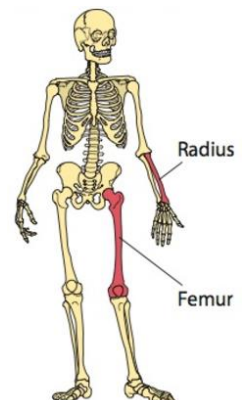
For  $n = 11 \rightarrow 23, 25, 27$

### Exercise

Forensic specialists can estimate the height of a deceased person from the lengths of the person's bones. For instance, an inequality that relates the height  $h$ , in *cm*, of an adult female and the length  $f$ , in *cm*, of her femur is  $|h - (2.47f + 54.10)| \leq 3.72$ . Use the inequalities to estimate the possible range of heights for an adult female whose measures 32.24 *cm*.

### Solution

**Given:**  $f = 32.24$



$$|h - 2.47(32.24) + 54.10| \leq 3.72$$

$$-3.72 \leq h - 79.6328 - 54.10 \leq 3.72$$

$$-3.72 \leq h - 133.7328 \leq 3.72$$

$$133.7328 - 3.72 \leq h \leq 3.72 + 133.7328$$

$$\underline{130.01 \leq h \leq 137.45 \text{ cm}}$$

### ***Exercise***

An inequality that is used to calculate the height  $h$  of an adult male from the length  $r$  of his radius is

$$|h - (3.32r + 85.43)| \leq 4.57$$

Where  $h$  and  $r$  are both in  $cm$ . Use this inequality to estimate the possible range of heights for an adult male whose radius measures  $26.36 \text{ cm}$ .

### **Solution**

**Given:**  $r = 26.36$

$$|h - (3.32(26.36) + 85.43)| \leq 4.57$$

$$-4.57 \leq h - 87.5152 - 85.43 \leq 4.57$$

$$-4.57 \leq h - 172.9452 \leq 4.57$$

$$172.9452 - 4.57 \leq h \leq 172.9452 + 4.57$$

$$\underline{168.4 \leq h \leq 177.5 \text{ cm}}$$