

SOLUTION

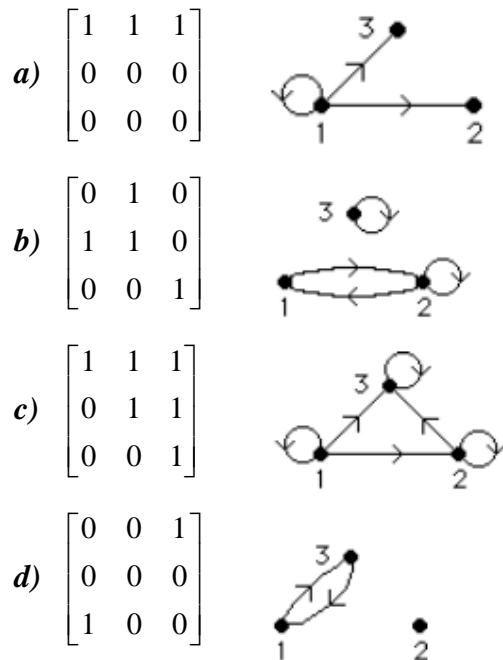
Section 4.2 – Representing Relations

Exercise

Represent each of these relations on $\{1, 2, 3\}$ with a matrix (with the elements of this set listed in increasing order). Then draw the directed graphs representing each relation

- a) $\{(1, 1), (1, 2), (1, 3)\}$
- b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$
- c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$
- d) $\{(1, 3), (3, 1)\}$

Solution



Exercise

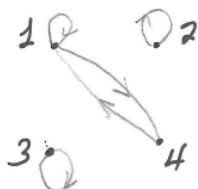
Represent each of these relations on $\{1, 2, 3, 4\}$ with a matrix (with the elements of this set listed in increasing order). Then draw the directed graphs representing each relation

- a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$
- b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$
- c) $\{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$
- d) $\{(2, 4), (3, 1), (3, 2), (3, 4)\}$

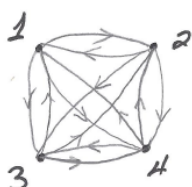
Solution



$$b) \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$



$$c) \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$



$$d) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



Exercise

List the ordered pairs in the relations on $\{1, 2, 3\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order). Then draw the directed graphs representing each relation

$$a) \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$b) \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

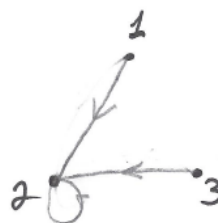
$$c) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Solution

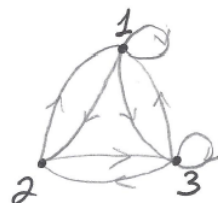
$$a) \{(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)\}$$



$$b) \{(1, 2), (2, 2), (3, 2)\}$$



$$c) \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (3, 3)\}$$



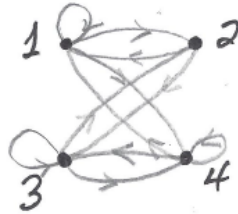
Exercise

List the ordered pairs in the relations on $\{1, 2, 3, 4\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order). Then draw the directed graphs representing each relation

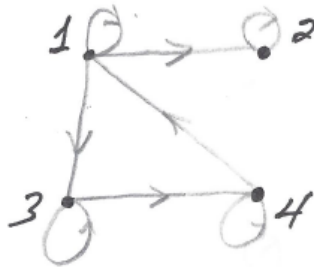
$$a) \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad b) \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad c) \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Solution

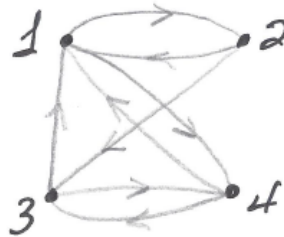
$$a) \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 3), (3, 2), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4)\}$$



$$b) \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 3), (3, 4), (4, 1), (4, 4)\}$$



$$c) \{(1, 2), (1, 4), (2, 1), (2, 3), (3, 1), (3, 4), (4, 1), (4, 3)\}$$



Exercise

Let R be the relation represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Find: a) R^2 b) R^3 c) R^4

Solution

$$a) \quad M_{R^2} = M_R^2 = M_R \odot M_R$$

$$R^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$b) \quad M_{R^3} = M_R^3 = M_R \odot M_{R^2}$$

$$R^3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

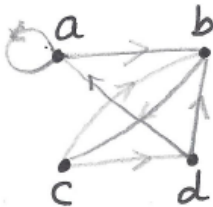
$$c) \quad M_{R^4} = M_R^{(4)} = M_R \odot M_{R^3}$$

$$R^4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Exercise

Draw the directed graph that represents the relation $\{(a, a), (a, b), (b, c), (c, b), (c, d), (d, a), (d, b)\}$

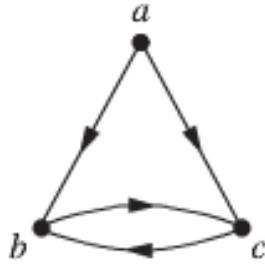
Solution



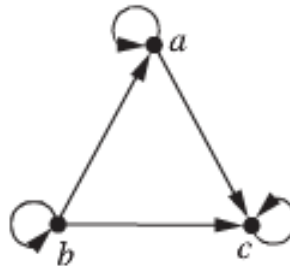
Exercise

Determine whether the relations represented by the directed graphs are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive

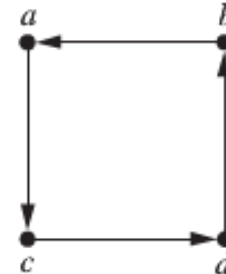
a)



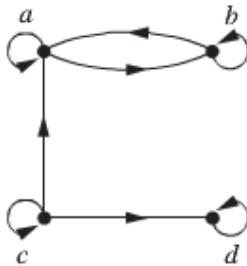
b)



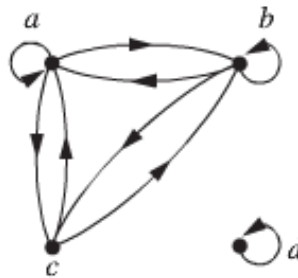
c)



d)



e)



f)



Solution

a) $\{(a, b), (a, c), (b, c), (c, b)\}$

It is not reflexive since (a, a) doesn't exist

It is not symmetric

It is transitive since $(a, b), (b, c) \Rightarrow (a, c)$

b) $\{(a, a), (a, c), (b, a), (b, b), (b, c), (c, c)\}$

It is reflexive

It is not symmetric

It is transitive since $(b, a), (a, c) \Rightarrow (b, c)$

c) $\{(a, c), (b, a), (c, d), (d, b)\}$

It is not reflexive; it is not symmetric, and not transitive since

d) $\{(a, a), (a, b), (b, a), (b, b), (c, a), (c, c), (c, d), (d, d)\}$

It is reflexive, not symmetric (no (a, c)), and not transitive $(c, a), (a, b)$ but no (c, b)

e) $\{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (d, d)\}$

It is not reflexive; it is symmetric and transitive

f) $\{(a, a), (a, b), (b, a), (b, b), (c, c), (c, d), (d, d), (d, c)\}$

It is reflexive; it is symmetric and transitive