$$\int \frac{dx}{(1-x^2)^{3/2}} = \int \frac{\cos\theta\, d\theta}{\cos^3\theta}$$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\sqrt{1-x^2} = \cos\theta$

$$= \int \sec^2\theta\, d\theta$$

$$= \tan\theta + C$$

$$= \frac{\sin\theta}{\cos\theta} + C$$

$$\boxed{= \frac{x}{\sqrt{1-x^2}} + C}$$

$$\int \frac{dx}{(1+x^2)^{3/2}} = \int \frac{\sec^2\theta\, d\theta}{\sec^3\theta}$$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$\sqrt{1+x^2} = \sec\theta$

$\sin\theta = \frac{\tan\theta}{\sec\theta}$

$$= \int \frac{d\theta}{\sec\theta}$$

$$= \int \cos\theta\, d\theta$$

$$= \sin\theta + C$$

$$\boxed{= \frac{x}{\sqrt{x^2+1}} + C}$$

$$\int \frac{dx}{x^2\sqrt{x^2+9}} = \int \frac{3\sec^2\theta\, d\theta}{(9\tan^2\theta)(3\sec\theta)}$$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$\sqrt{x^2+9} = 3\sec\theta$

$\sin\theta = \frac{\tan\theta}{\sec\theta}$

$$= \frac{1}{9}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$= \frac{1}{9}\int \frac{1}{\cos\theta}\frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$= \frac{1}{9}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$$= +\frac{1}{9}\int \frac{d(\sin\theta)}{\sin^2\theta}$$

$$= -\frac{1}{9}\frac{1}{\sin\theta} + C$$

$$= -\frac{1}{9}\frac{\sqrt{x^2+9}}{x} + C$$

$$\int \frac{dx}{x^2\sqrt{9-x^2}} = \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta\,(3\cos\theta)}$$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$\sqrt{9-x^2} = 3\cos\theta$

$$= \frac{1}{9}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{9}\cot\theta + C$$

$$= -\frac{1}{9}\,\frac{\sqrt{9-x^2}}{x} + C$$

$$= \boxed{-\frac{\sqrt{9-x^2}}{9x} + C}$$

$$\int \frac{dx}{\sqrt{36-x^2}} = \int \frac{6\cos\theta\, d\theta}{6\cos\theta}$$

$x = 6\sin\theta$

$dx = 6\cos\theta\, d\theta$

$\sqrt{36-x^2} = 6\cos\theta$

$\sin\theta = x/6$

$$= \int d\theta$$

$$= \theta + C$$

$$= \boxed{\sin^{-1}\frac{x}{6} + C}$$

$$\int \frac{dx}{\sqrt{16+4x^2}} = \int \frac{2\sec^2\theta\, d\theta}{4\sec\theta}$$

$2x = 4\tan\theta$

$x = 2\tan\theta$

$\sqrt{16+4x^2} = 4\sec\theta$

$16 + 16\tan^2\theta$

$dx = 2\sec^2\theta\, d\theta$

$$= \frac{1}{2}\int \sec\theta\, d\theta$$

$$= \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2-81}} = \int \frac{9\sec\theta\tan\theta\, d\theta}{9\tan\theta}$$

$x = 9\sec\theta$

$dx = 9\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-81} = 9\tan\theta$

$$= \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \boxed{\ln\left|\frac{x}{9} + \frac{\sqrt{x^2-81}}{9}\right| + C}$$ or $\ln\left(x+\sqrt{x^2-81}\right) + C_1$

$C_1 = C - \ln 9.$

$$\int \frac{dx}{\sqrt{1-2x^2}} = \frac{1}{\sqrt{2}}\int \frac{\cos\theta\, d\theta}{\cos\theta}$$

$$= \frac{1}{\sqrt{2}}\theta + C$$

$$= \boxed{\frac{1}{\sqrt{2}}\sin^{-1}(\sqrt{2}x) + C}$$

$\sqrt{2}x = \sin\theta$

$dx = \frac{1}{\sqrt{2}}\cos\theta\, d\theta$

$\sqrt{1-2x^2} = \cos\theta$

$$\int \frac{dx}{(1+4x^2)^{3/2}} = \frac{1}{2}\int \frac{\sec^2\theta\, d\theta}{\sec^3\theta}$$

$$= \frac{1}{2}\int \cos\theta\, d\theta$$

$$= \frac{1}{2}\sin\theta + C$$

$$= \frac{1}{2}\frac{2x}{\sqrt{1+4x^2}} + C$$

$$= \boxed{\frac{x}{\sqrt{1+4x^2}} + C}$$

$2x = \tan\theta$

$dx = \frac{1}{2}\sec^2\theta\, d\theta$

$\sqrt{1+4x^2} = \sec\theta$

$\sin\theta = \frac{\sin\theta}{\cos\theta}\cos\theta = \frac{\tan\theta}{\sec\theta}$

$$\int \frac{dx}{(x^2-36)^{3/2}} = \int \frac{6\sec\theta\tan\theta}{6^3\tan^3\theta}\, d\theta$$

$$= \frac{1}{36}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$= \frac{1}{36}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$$= \frac{1}{36}\int \frac{d(\sin\theta)}{\sin^2\theta}$$

$$= -\frac{1}{36}\frac{1}{\sin\theta} + C$$

$$= -\frac{1}{36}\frac{x/6}{\sqrt{x^2-36}/6} + C$$

$$= \boxed{\frac{-1}{36\sqrt{x^2-36}} + C}$$

$x = 6\sec\theta$

$dx = 6\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-36} = 6\tan\theta$

$\sin\theta = \frac{\tan\theta}{\sec\theta}$

$$\int \frac{x^2}{\sqrt{16-x^2}}\,dx = \int \frac{16\sin^2\theta}{4\cos\theta}\,4\cos\theta\,d\theta$$

$x = 4\sin\theta$
$dx = 4\cos\theta\,d\theta$
$\sqrt{16-x^2} = 4\cos\theta$

$$= 8\int (1-\cos 2\theta)\,d\theta$$
$$= 8\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$
$$= 8\sin^{-1}\frac{x}{4} - 8\sin\theta\cos\theta + C$$
$$= 8\sin^{-1}\frac{x}{4} - 8\,\frac{x}{4}\cdot\frac{\sqrt{16-x^2}}{4} + C$$
$$\boxed{= 8\sin^{-1}\frac{x}{4} - \frac{1}{2}x\sqrt{16-x^2} + C}$$

$$\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{3\tan\theta}{3\sec\theta}\,3\sec\theta\tan\theta\,d\theta$$

$x = 3\sec\theta$
$dx = 3\sec\theta\tan\theta\,d\theta$
$\sqrt{x^2-9} = 3\tan\theta$

$$= 3\int \tan^2\theta\,d\theta$$
$$= 3\int (\sec^2\theta - 1)\,d\theta$$
$$= 3(\tan\theta - \theta) + C$$
$$\boxed{= \sqrt{x^2-9} - 3\sec^{-1}\frac{x}{3} + C}$$

$$\int \frac{dx}{\sqrt{3-2x-x^2}} = \int \frac{dx}{\sqrt{4-(x+1)^2}}$$

$3-2x-x^2 = 3+1-1-2x-x^2$
$= 4-(x+1)^2$

$x+1 = 2\sin\theta$
$dx = 2\cos\theta\,d\theta$
$\sqrt{4-(x+1)^2} = 2\cos\theta$

$$= \int \frac{2\cos\theta\,d\theta}{2\cos\theta}$$
$$= \int d\theta$$
$$= \theta + C$$
$$\boxed{= \sin^{-1}\left(\frac{x+1}{2}\right) + C}$$

$x = \tan\theta$
$dx = \sec^2\theta \, d\theta$
$1 + x^2 = \sec^2\theta$

$$\int \frac{x^4}{1+x^2}\,dx = \int \frac{\tan^4\theta}{\sec^2\theta}\sec^2\theta\,d\theta$$
$$= \int \tan^4\theta\,d\theta$$
$$= \int \tan^2\theta(\sec^2\theta - 1)\,d\theta$$
$$= \int \tan^2\theta\sec^2\theta\,d\theta - \int \tan^2\theta\,d\theta$$
$$= \int \tan^2\theta\,d(\tan\theta) - \int(\sec^2\theta - 1)\,d\theta$$
$$= \frac{1}{3}\tan^3\theta - \tan\theta + \theta + C$$
$$= \boxed{\frac{x^3}{3} - x + \tan^{-1}x + C}$$

$3x = \sec\theta$
$dx = \frac{1}{3}\sec\theta\tan\theta\,d\theta$
$\sqrt{9x^2-1} = \tan\theta$
$\sin\theta = \frac{\tan\theta}{\sec\theta}$

$$\int \frac{dx}{x^2\sqrt{9x^2-1}} = \int \frac{\frac{1}{3}\sec\theta\tan\theta\,d\theta}{\frac{1}{9}\sec^2\theta\tan\theta}$$
$$= 3\int \frac{d\theta}{\sec\theta}$$
$$= 3\int \cos\theta\,d\theta$$
$$= 3\sin\theta + C$$
$$= \boxed{\frac{\sqrt{9x^2-1}}{x} + C}$$

$x = 10\sin\theta$
$dx = 10\cos\theta\,d\theta$
$\sqrt{100-x^2} = 10\cos\theta$

$$\int \frac{x^2}{(100-x^2)^{3/2}}\,dx = \int \frac{10^2\sin^2\theta}{10^3\cos^3\theta}10\cos\theta\,d\theta$$
$$= \int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$$
$$= \int \tan^2\theta\,d\theta$$
$$= \int(\sec^2\theta - 1)\,d\theta$$
$$= \tan\theta - \theta + C$$
$$= \boxed{\frac{x}{\sqrt{100-x^2}} - \sin^{-1}\frac{x}{10} + C}$$

$$\int \frac{dx}{x^3\sqrt{x^2-100}} = \int \frac{10\sec\theta\tan\theta\,d\theta}{10^3\sec^3\theta\,(10\tan\theta)}$$

$x = 10\sec\theta$
$dx = 10\sec\theta\tan\theta\,d\theta$
$\sqrt{x^2-100} = 10\tan\theta$

$$= \frac{1}{10^3}\int \frac{d\theta}{\sec^2\theta}$$

$$= \frac{1}{10^3}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{2000}\int (1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{2000}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{1}{2000}\left(\tan^{-1}\frac{\sqrt{x^2-100}}{10} + \sin\theta\cos\theta\right) + C$$

$$= \frac{1}{2000}\left(\tan^{-1}\frac{\sqrt{x^2-100}}{10} + \frac{\sqrt{x^2-100}}{x}\,\frac{10}{x}\right) + C$$

$$\boxed{= \frac{1}{2000}\tan^{-1}\frac{\sqrt{x^2-100}}{10} + \frac{\sqrt{x^2-100}}{200x^2} + C}$$

$$\int_{8\sqrt{2}}^{16} \frac{dx}{\sqrt{x^2-64}} = \int_{8\sqrt{2}}^{16} \frac{8\sec\theta\tan\theta\,d\theta}{8\tan\theta}$$

$x = 8\sec\theta$
$dx = 8\sec\theta\tan\theta\,d\theta$
$\sqrt{x^2-64} = 8\tan\theta$

$$= \int_{8\sqrt{2}}^{16} \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta|\Big|_{8\sqrt{2}}^{16}$$

$16^2 - 64 = 8^2(4-1)$

$$= \ln\left|\frac{x}{8} + \frac{\sqrt{x^2-64}}{8}\right|\Bigg|_{8\sqrt{2}}^{16}$$

$$= \ln(2+\sqrt{3}) - \ln(\sqrt{2}+1)$$

$$\boxed{= \ln\left(\frac{2+\sqrt{3}}{\sqrt{2}-1}\right)}$$