$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} \qquad x^2$$

$$= \sum_{n=1}^{\infty} n(n+1) a_{n+1} x^{\cancel{n}-1} \qquad x^1$$

$$= \sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n$$

Ex $y' - 2xy = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

$$y' - 2xy = 0$$

$$\sum_{n=1}^{\infty} n a_n x^{n-1} - 2x \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1) a_{n+1} x^n - \sum_{n=0}^{\infty} 2a_n x^{n+1} = 0$$

$$a_1 + \sum_{n=0}^{\infty} (n+2) a_{n+2} x^{n+1} - \sum_{n=0}^{\infty} 2a_n x^{n+1} = 0$$

$$a_1 + \sum_{n=0}^{\infty} \left((n+2) a_{n+2} - 2a_n\right) x^{n+1} = 0.$$

$a_1 = 0$ $\qquad (n+2) a_{n+2} - 2a_n = 0$

$$\boxed{a_{n+2} = \frac{2}{n+2} a_n}$$

let $y(0) = a_0$

a_0 $\qquad a_1 = 0$

$a_2 = \frac{2}{2} a_0 = a_0$ $\qquad a_3 = \frac{2}{3} a_1 = 0$

$a_4 = \frac{2}{4} a_2 = \frac{1}{2} a_0$ $\qquad a_5 = 0$

$a_6 = \frac{2}{6} a_4 = \frac{1}{2 \cdot 3} a_0$

$\vdots$

$a_{2k} = \frac{1}{k!} a_0$

$$f(x) = \sum_{k=0}^{\infty} a_{2k} x^{2k}$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} a_0 x^{2k}$$

$$= a_0 \sum_{k=0}^{\infty} \frac{x^{2k}}{k!}$$

#13 $xy' + y = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$xy' + y = 0$$

$$x \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} ((n+1) a_n) x^n = 0$$

$$(n+1) a_n = 0 \Rightarrow a_n = 0$$

$$y(x) = 0$$

12) $(x-3)y' + 2y = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

$$xy' - 3y' + 2y = 0$$

$$x\sum_{n=1}^{\infty} n a_n x^{n-1} - 3\sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + 2\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} 3(n+1) a_{n+1} x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left(-3(n+1)a_{n+1} + (n+2)a_n\right) x^n = 0$$

$$3(n+1)a_{n+1} = (n+2)a_n$$

$$a_{n+1} = \frac{n+2}{3(n+1)} a_n$$

a_0

$n=0$ $\quad a_1 = \frac{2}{3} a_0$

1 $\quad a_2 = \frac{3}{3\cdot 2} a_1 = \frac{1}{3} a_0$

2 $\quad a_3 = \frac{2^2}{3\cdot 3} a_2 = \frac{2^2}{3^3} a_0$

3 $\quad a_4 = \frac{5}{3^2\cdot 4} a_3 = \frac{5}{3^4} a_0$

$\vdots$

$$y(x) = a_0\left(1 + \frac{2}{3}x + \frac{1}{3^2}x^2 + \frac{4}{9}x^3 + \dots\right)$$

$$= a_0 \sum_{n=0}^{\infty} \frac{n+1}{3^n} x^n$$

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$$n! = 1\cdot 2\cdot 3 \cdots (n-1)\, n$$

$$0! = 1$$

$$(2n-1)!! = 1\cdot 3\cdot 5\cdot 7 \cdots (n-2)\, n$$

$$n!! = 2\cdot 4\cdot 6\cdot 8\, \cdot$$

$$n!!! = n\,(n-3)\,(n-6)$$

$$5!!! = 5\cdot 2$$

$$6!!! = 6\cdot 3\cdot 0!$$

22) $y'' + xy' + y = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n$$

$$y'' + xy' + y = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n + x \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left((n+1)(n+2) a_{n+2} + a_n \right) x^n + \sum_{n=0}^{\infty} n a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[(n+1)(n+2) a_{n+2} + (n+1) a_n \right] x^n = 0$$

$$(n+1)(n+2) a_{n+2} = -(n+1) a_n$$

$$\boxed{a_{n+2} = -\frac{1}{n+2} a_n}$$

	a_0	n	a_1
$n=0$	$a_2 = -\frac{1}{2} a_0$	1	$a_3 = -\frac{1}{3} a_1$
2	$a_4 = -\frac{1}{4} a_2 = \frac{1}{2^3} a_0$	3	$a_5 = -\frac{1}{5} a_3 = \frac{1}{3.5} a_1$
4	$a_6 = -\frac{1}{6} a_4 = \frac{-1}{2.4.6} a_0$		$a_7 = -\frac{1}{7} a_5 = \frac{-1}{3.5.7} a_1$
	⋮		⋮

$$y(x) = a_0 \left(1 - \frac{1}{2} x^2 + \frac{1}{2.4} x^4 - \frac{1}{2.4.6} x^6 + \cdots \right)$$
$$+ a_1 \left(x - \frac{1}{3} x^3 + \frac{1}{3.5} x^5 - \cdots \right)$$

$y'' + xy' + y = 0 \qquad y(0)=0 \qquad y'(0)=2$

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n$$

$$y'' + xy' + y = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + x \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n + \sum_{n=0}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[(n+1)(n+2) a_{n+2} + (n+1) a_n \right] x^n = 0$$

$$(n+1)(n+2) a_{n+2} = -(n+1) a_n$$

$$a_{n+2} = -\frac{1}{n+2} a_n$$

$a_0 = y(0) = 0 \qquad a_1 = y'(0) = 2$

$n=0$ $\quad a_2 = -\frac{1}{2} a_0 = 0 \qquad a_3 = -\frac{1}{3} a_1 = -\frac{2}{3}$

$a_4 = +\frac{1}{8} a_0 = 0 \qquad a_5 = -\frac{1}{5} a_3 = \frac{1}{15} a_1 = -\frac{2}{15}$

$a_6 = -\frac{1}{6} a_4 = -\frac{1}{2.4.6} a_0 = 0 \qquad a_7 = \frac{-1}{7} a_5 = -\frac{1}{3.5.7} a_1 = \frac{-2}{3.5.7}$

$$y(x) = (a_0 + a_2 x^2 + a_4 x^4 + \cdots) + (a_1 x + a_3 x^3 + a_5 x^5 + \cdots)$$

$$= a_0 \left(1 - \frac{1}{2} x^2 + \frac{1}{2.4} x^4 - \frac{1}{2.4.6} x^6 + \cdots \right)$$

$$+ a_1 \left(x - \frac{1}{3} x^3 + \frac{1}{3.5} x^5 - \cdots \right)$$

$$= 2x - \frac{2}{3} x^3 - \frac{2}{15} x^5 - \frac{1}{3.5.7} x^7 - \cdots$$