3.2

$$\mathcal{L}(y')(s) = s\,\mathcal{L}(y(s)) - y(0)$$
$$= s\,Y(s) - y(0)$$

$$\mathcal{L}(y'')(s) = s^2 Y(s) - s\,y(0) - y'(0)$$

Ex

$3\sin 2t - 4t + 5e^{3t}$

$$\mathcal{L}\{3\sin 2t - 4t + 5e^{3t}\}$$
$$= \mathcal{L}\{3\sin 2t\} - 4\,\mathcal{L}\{t\} + 5\mathcal{L}\{e^{3t}\}$$
$$= 3\,\frac{2}{s^2+4} - 4\,\frac{1}{s^2} + \frac{5}{s-3}$$
$$= \frac{6}{s^2+4} - \frac{4}{s^2} + \frac{5}{s-3}$$

Ex. $y'' - y = e^{2t}$ $\quad y(0)=0$, $y'(0)=1$

$$\mathcal{L}\{y''-y\}(s) = \mathcal{L}\{e^{2t}\}(s)$$

$$s^2 Y(s) - s\,\underbrace{y(0)}_{0} - \underbrace{y'(0)}_{1} - Y(s) = \frac{1}{s-2}$$

$$(s^2-1)\,Y(s) = \frac{1}{s-2} + 1$$

$$= \frac{s-1}{s-2}$$

$$Y(s) = \frac{s-1}{(s-2)(s-1)(s+1)}$$

$$= \frac{1}{(s-2)(s+1)}$$

$y(t) = e^{2t}\sin 3t$

$$\mathcal{L}\{e^{2t}\sin 3t\}(s) = \frac{3}{(s-2)^2+9}$$

$$f(t) \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}} F(s)$$

Ex. $F(s) = \frac{1}{s-2} - \frac{16}{s^2+4}$

$$y(t) = \mathcal{L}^{-1}\{F(s)\}$$
$$= \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \mathcal{L}^{-1}\left\{\frac{16}{s^2+4}\right\} \quad 8\,\frac{2}{s^2+4}$$
$$= e^{2t} - 8\sin 2t$$

Ex. $F(s) = \frac{1}{s^2-2s-3}$

$$\frac{1}{s^2-2s-3} = \frac{A}{s+1} + \frac{B}{s-3}$$
$$\rightarrow 1 = As - 3A + Bs + B$$

$s^1 \quad A + B = 0$

$s^0 \quad -3A + B = 1$

$$\boxed{-4A = 1 \rightarrow} \; A = -\frac{1}{4} \rightarrow B = \frac{1}{4}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{-1/4}{s+1} + \frac{1}{4}\frac{1}{s-3}\right\}$$
$$= -\frac{1}{4}e^{-t} + \frac{1}{4}e^{3t}$$

$$F(s) = \frac{1}{s^2+4s+13} \rightarrow ① \rightarrow \left(\tfrac{1}{2}4\right)^2$$

$$s^2+4s+13 = s^2+4s+4+9$$
$$= (s+2)^2+9$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+9}\right\}$$
$$= \frac{1}{3}e^{-2t}\sin 3t$$

$$F(s) = \frac{2s^2+s+13}{(s-1)\left((s+1)^2+4\right)}$$

$$\frac{2s^2+s+13}{(s-1)((s+1)^2+4)} = \frac{A}{s-1} + \frac{Bs+C}{(s+1)^2+4}$$

$$As^2+2As+A+4A+Bs^2-Bs+Cs-C$$
$$= 2s^2+s+13$$

$$s^2 \quad A+B = 2$$
$$s^1 \quad 2A-B+C = 1$$
$$s^0 \quad 5A-C = 13$$

$$\Delta = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 5 & 0 & -1 \end{vmatrix} = 8 \qquad \Delta_A = \begin{vmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ 13 & 0 & -1 \end{vmatrix}$$
$$= 16$$

$$A = 2 \rightarrow B = 0$$
$$C = 10-13 = -3$$

$$f(t) = 2\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+4}\right\}$$
$$= 2e^{t} - \frac{3}{2}e^{-t}\sin 2t$$

$y'' - 2y' - 3y = 0 \qquad y(0) = 1, \; y'(0) = 0$

$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} - 3\mathcal{L}\{y\} = 0$

$s^2 Y(s) - s y(0) - y'(0) - 2sY(s) + 2y(0) - 3Y(s) = 0$

$(s^2 - 2s - 3) Y(s) = s - 2$

$$Y(s) = \frac{s-2}{(s+1)(s-3)}$$

$$\frac{s-2}{(s+1)(s-3)} = \frac{A}{s+1} + \frac{B}{s-3}$$

$A(s-3) + B(s+1) = s - 2$

$s^1 \quad A + B = 1$

$s^0 \quad -3A + B = -2 \qquad A = \frac{3}{4}, \; B = \frac{1}{4}$

$$y(t) = \frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$= \frac{3}{4} e^{-t} + \frac{1}{4} e^{3t}$$

$2\frac{1}{2}$