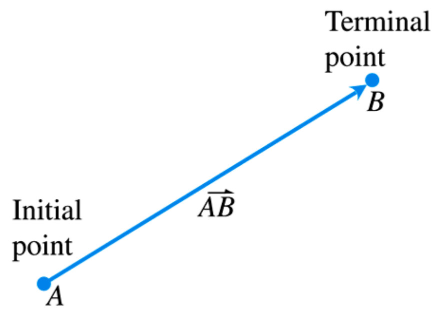
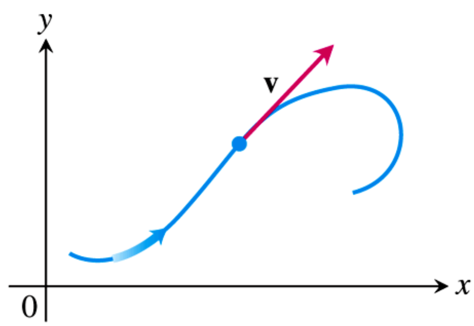
Lecture One – Vectors and Vector-Values Functions

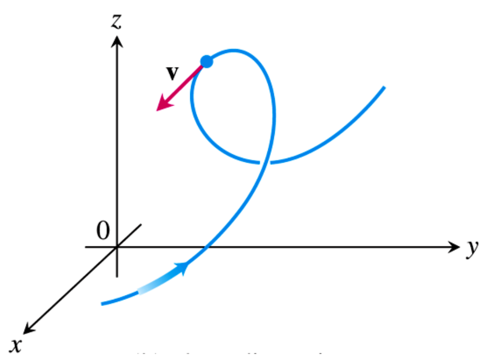
Section 1.1 – Vectors

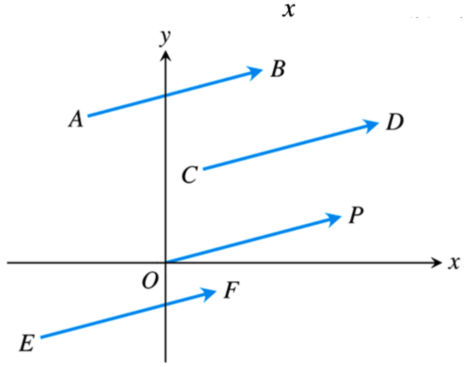


Component Form

A quantity such as force, or velocity is called a vector and is represented by a directed line segment.

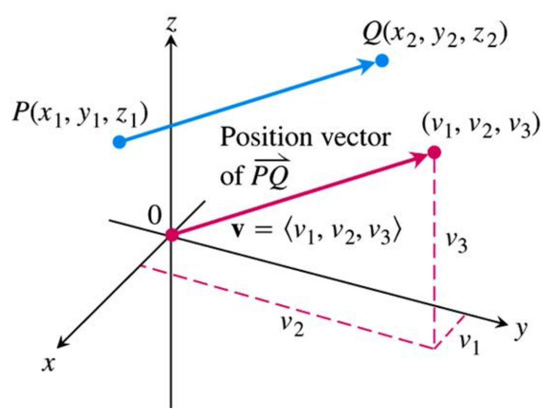






Definition

The vector represented by the direected line segment $\overrightarrow{PQ}$ has initial point P and terminal point Q and its length is denoted by $\left|\overrightarrow{PQ}\right|$



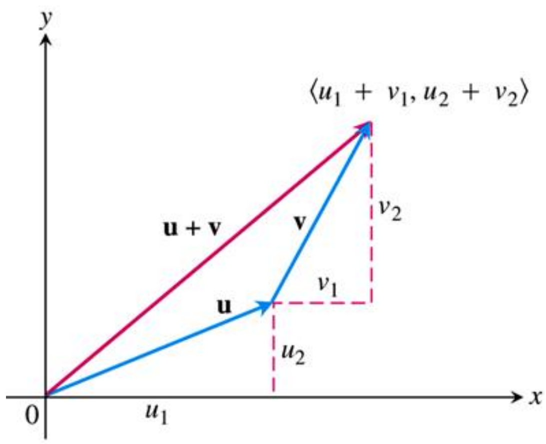
Vector Algebra Operations

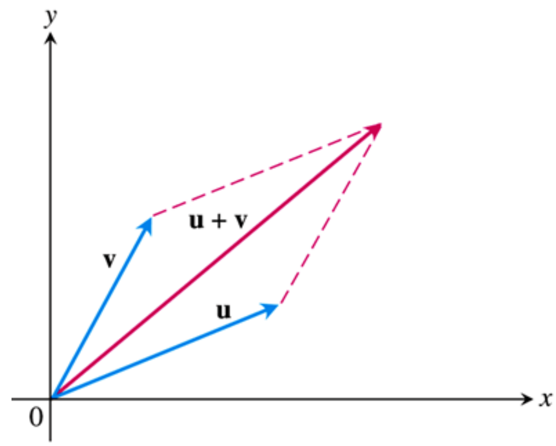
Definitions

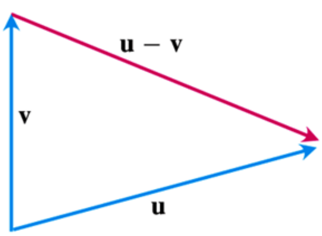
Let $\boldsymbol{u} = \langle u_1,\ u_2,\ u_3 \rangle$ and $\boldsymbol{v} = \langle v_1,\ v_2,\ v_3 \rangle$ be vectors with $\boldsymbol{k}$ a scalar

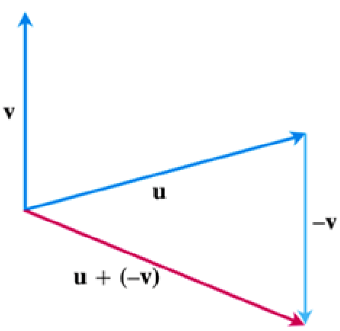
Addition: $\boldsymbol{u} + \boldsymbol{v} = \langle u_1 + v_1,\ u_2 + v_2,\ u_3 + v_3 \rangle$

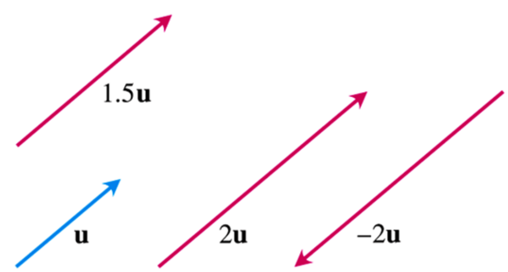
Scalar multiplication: $k\boldsymbol{u} = \langle ku_1,\ ku_2,\ ku_3 \rangle$











Example

Let $\boldsymbol{u} = \langle -1,\ 3,\ 1 \rangle$ and $\boldsymbol{v} = \langle 4,\ 7,\ 0 \rangle$. Find the components of

a) $2\boldsymbol{u} + 3\boldsymbol{v}$ *b)* $\boldsymbol{u} - \boldsymbol{v}$ *c)* $\left| \frac{1}{2}\boldsymbol{u} \right|$

Solution

a) $2\boldsymbol{u} + 3\boldsymbol{v} = 2\langle -1,\ 3,\ 1 \rangle + 3\langle 4,\ 7,\ 0 \rangle$

$$= \langle -2,\ 6,\ 2 \rangle + \langle 12,\ 21,\ 0 \rangle$$

$$\boxed{= \langle 10,\ 27,\ 2 \rangle}$$

b) $\boldsymbol{u} - \boldsymbol{v} = \langle -1,\ 3,\ 1 \rangle - \langle 4,\ 7,\ 0 \rangle$

$$\boxed{= \langle -5,\ -4,\ 1 \rangle}$$

c) $\left|\frac{1}{2}\boldsymbol{u}\right| = \left|\left\langle -\frac{1}{2},\ \frac{3}{2},\ \frac{1}{2}\right\rangle\right|$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{9}{4} + \frac{1}{4}}$$

$$= \sqrt{\frac{11}{4}}$$

$$= \frac{\sqrt{11}}{2}$$

Proporties of Vector Operations

Let ***u, v, w*** be vectors and *a, b* be scalars

1. $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$
2. $(\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w} = \boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w})$
3. $\boldsymbol{u} + \mathbf{0} = \boldsymbol{u}$
4. $\boldsymbol{u} + (-\boldsymbol{u}) = \mathbf{0}$
5. $0\boldsymbol{u} = \mathbf{0}$
6. $1\boldsymbol{u} = \boldsymbol{u}$
7. $a(b\boldsymbol{u}) = (ab)\boldsymbol{u}$
8. $(a+b)\boldsymbol{u} = a\boldsymbol{u} + b\boldsymbol{u}$
9. $a(\boldsymbol{u} + \boldsymbol{v}) = a\boldsymbol{u} + a\boldsymbol{v}$

Definition

If $\boldsymbol{v}$ is a **two-dimensional** vector in the plane equal to the vector with initial point at the origin and terminal point $(v_1,\ v_2)$, then the ***component form*** of $\boldsymbol{v}$ is

$$\boldsymbol{v} = \langle v_1,\ v_2\rangle$$

If $\boldsymbol{v}$ is a **three-dimensional** vector in the plane equal to the vector with initial point at the origin and terminal point $(v_1,\ v_2,\ v_3)$, then the ***component form*** of $\boldsymbol{v}$ is

$$\boldsymbol{v} = \langle v_1,\ v_2,\ v_3\rangle$$

The magnitude or length of the vector $\boldsymbol{v} = \overrightarrow{PQ}$ is the nonegative number

$$|\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The only vector with length 0 is the ***zero vector*** $\mathbf{0} = \langle 0,\ 0,\ 0\rangle$

Example

Find the component form and the length of the vector with initial point $P(-3,\ 4,\ 1)$ and terminal point $Q(-5,\ 2,\ 2)$

Solution

The component form of $\overrightarrow{PQ}$ is $\langle -5-(-3),\ 2-4,\ 2-1\rangle = \boxed{\langle -2,\ -2,\ 1\rangle}$

The length is $\left|\overrightarrow{PQ}\right| = \sqrt{(-2)^2+(-2)^2+1^2}\ \underline{=3|}$

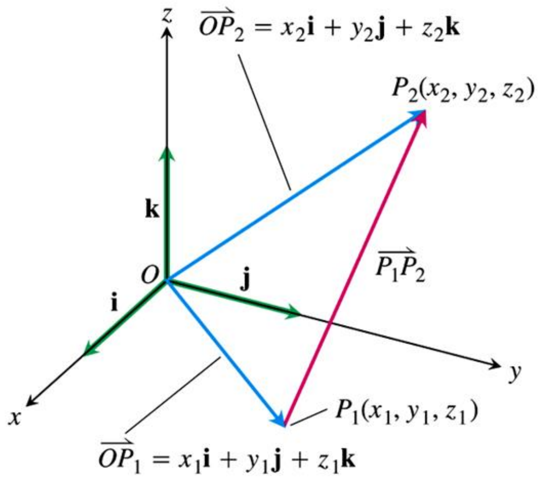
Unit Vectors

A vector $\boldsymbol{v}$ of length 1 is called a ***unit vector***. The ***standard unit vectors*** are

$$\hat{\boldsymbol{i}} = \langle 1,\ 0,\ 0\rangle, \quad \hat{\boldsymbol{j}} = \langle 0,\ 1,\ 0\rangle, \quad and \quad \hat{\boldsymbol{k}} = \langle 0,\ 0,\ 1\rangle$$

Any vector $\boldsymbol{v} = \left\langle v_1,\ v_2,\ v_3\right\rangle$ can be written as a linear combination of the standard unit vectors as follows:

$$\begin{aligned}\boldsymbol{v} &= \left\langle v_1,\ v_2,\ v_3\right\rangle \\ &= \left\langle v_1,\ 0,\ 0\right\rangle + \left\langle 0,\ v_2,\ 0\right\rangle + \left\langle 0,\ 0,\ v_3\right\rangle \\ &= v_1\langle 1,\ 0,\ 0\rangle + v_2\langle 0,\ 1,\ 0\rangle + v_3\langle 0,\ 0,\ 1\rangle \\ &= v_1\hat{\boldsymbol{i}} + v_2\hat{\boldsymbol{j}} + v_3\hat{\boldsymbol{k}}\end{aligned}$$



Example

Find a unit vector $\boldsymbol{u}$ in the direction of the vector from $P_1(1,\ 0,\ 1)$ to $P_2(3,\ 2,\ 0)$.

Solution

$$\overrightarrow{P_1P_2} = (3-1)\boldsymbol{i} + (2-0)\boldsymbol{j} + (0-1)\boldsymbol{k} \ = 2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$$

$$\left|\overrightarrow{P_1P_2}\right| = \sqrt{2^2+2^2+(-1)^2} = \sqrt{9}\ \underline{=3|}$$

$$\boldsymbol{u} = \frac{\overrightarrow{P_1P_2}}{\left|\overrightarrow{P_1P_2}\right|} = \frac{2\boldsymbol{i}+2\boldsymbol{j}-\boldsymbol{k}}{3} \quad \underline{=\frac{2}{3}\boldsymbol{i} + \frac{2}{3}\boldsymbol{j} - \frac{1}{3}\boldsymbol{k}|}$$

Example

If $\boldsymbol{v} = 3\hat{\boldsymbol{i}} - 4\hat{\boldsymbol{j}}$ is a velocity vector, express $\boldsymbol{v}$ as a product of its speed times a unit vector in the direction of motion.

Solution

Speed is the magnitude (length) of $\boldsymbol{v}$: $\quad |\boldsymbol{v}| = \sqrt{3^2 + (-4)^2} = \boxed{5}$

The unit vector has the same direction as $\boldsymbol{v}$: $\quad \dfrac{\boldsymbol{v}}{|\boldsymbol{v}|} = \dfrac{3\hat{\boldsymbol{i}} - 4\hat{\boldsymbol{j}}}{5} \boxed{= \dfrac{3}{5}\hat{\boldsymbol{i}} - \dfrac{4}{5}\hat{\boldsymbol{j}}}$

$$\boldsymbol{v} = 3\hat{\boldsymbol{i}} - 4\hat{\boldsymbol{j}} = \underset{\substack{\text{Length}\\\text{(speed)}}}{5}\underbrace{\left(\frac{3}{5}\hat{\boldsymbol{i}} - \frac{4}{5}\hat{\boldsymbol{j}}\right)}_{\substack{\text{Direction}\\\text{of motion}}}$$

Note:

If $v \neq 0$, then

1. $\dfrac{\boldsymbol{v}}{|\boldsymbol{v}|}$ is a unit vector in the direction of $\boldsymbol{v}$;

2. The equation $\boldsymbol{v} = |\boldsymbol{v}|\dfrac{\boldsymbol{v}}{|\boldsymbol{v}|}$ expresses $\boldsymbol{v}$ as its length times its direction.

Example

A force of 6 Newton is applied in the direction of the vector $\boldsymbol{v} = 2\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}$. Express the force $\boldsymbol{F}$ as a product of its magnitude and direction.

Solution

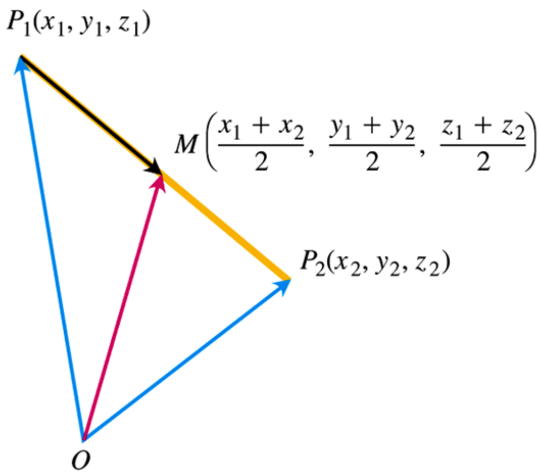
$$|v| = \sqrt{2^2 + 2^2 + (-1)^2} = 3$$

$$\begin{aligned}\boldsymbol{F} &= |\boldsymbol{v}|\frac{\boldsymbol{v}}{|\boldsymbol{v}|}\\ &= 3\frac{2\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}}{3}\\ &\boxed{= 3\left(\frac{2}{3}\hat{\boldsymbol{i}} + \frac{2}{3}\hat{\boldsymbol{j}} - \frac{1}{3}\hat{\boldsymbol{k}}\right)}\end{aligned}$$

Midpoint of a Line Segment

The midpoint M of the line segment joining points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is the point

$$\boxed{\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)}$$



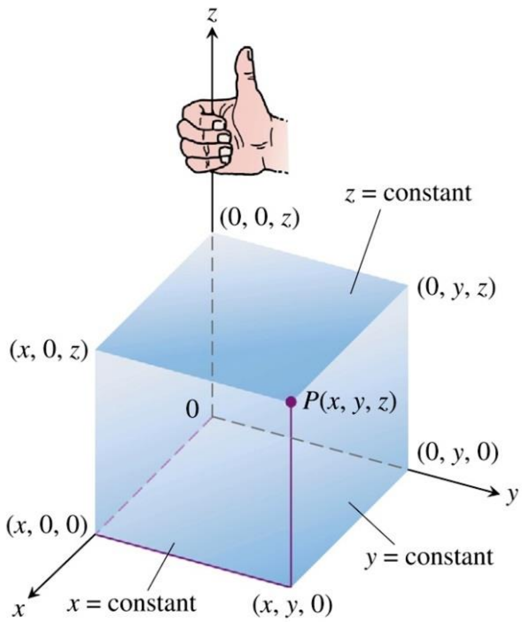
Example

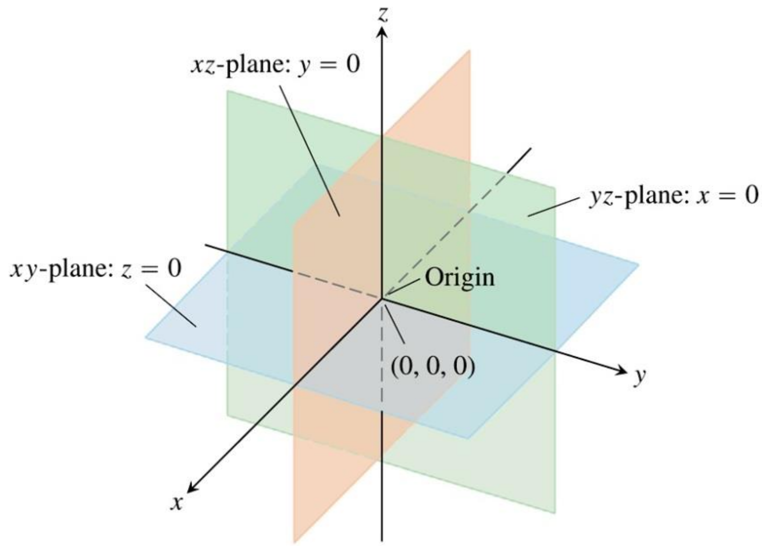
Find the midpoint of the segment $P_1(3, -2, 0)$ and $P_2(7, 4, 4)$

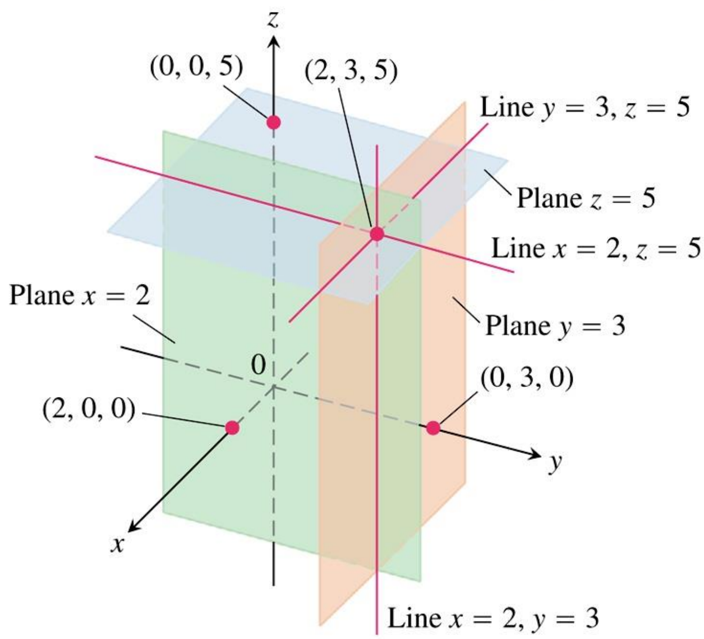
Solution

$$M = \left(\frac{3+7}{2}, \frac{-2+4}{2}, \frac{0+4}{2}\right)$$

$$= (5, 1, 2)$$







Example

What points $P(x, y, z)$ satisfy the equations

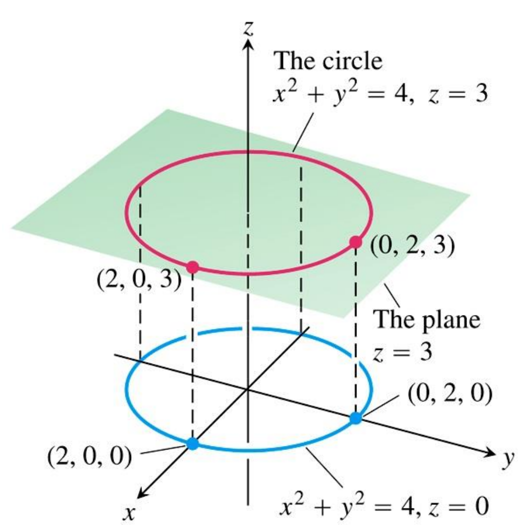
$$x^2 + y^2 = 4 \quad \textit{and} \quad z = 3$$

Solution

The point lie in the horizontal plane $z = 3$ and the circle $x^2 + y^2 = 4$.

The solution is the set of points:

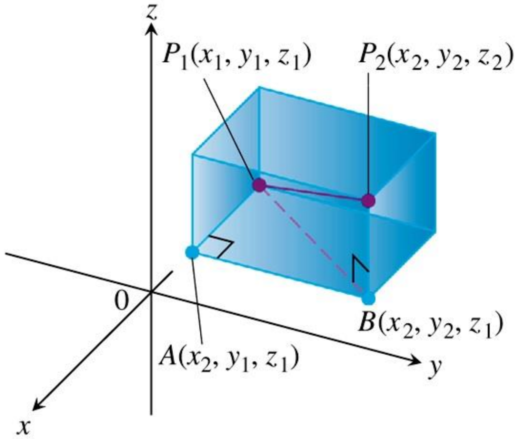
"the circle $x^2 + y^2 = 4$ in the plane $z = 3$"



Distance in Space

The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$\left|P_1P_2\right| = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$$



Proof

$$\left|P_1A\right| = \left|x_2 - x_1\right|, \quad \left|AB\right| = \left|y_2 - y_1\right|, \quad \left|BP_2\right| = \left|z_2 - z_1\right|$$

From the right triangles P_1AB and P_1BP_2 :

$$\left|P_1B\right|^2 = \left|P_1A\right|^2 + \left|AB\right|^2 \quad \textit{and} \quad \left|P_1P_2\right|^2 = \left|P_1B\right|^2 + \left|BP_2\right|^2$$

$$\begin{aligned}
\left|P_1P_2\right|^2 &= \left|P_1B\right|^2 + \left|BP_2\right|^2 \\
&= \left|P_1A\right|^2 + \left|AB\right|^2 + \left|BP_2\right|^2 \\
&= \left|x_2 - x_1\right|^2 + \left|y_2 - y_1\right|^2 + \left|z_2 - z_1\right|^2 \\
&= \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2 \quad \checkmark
\end{aligned}$$

Example

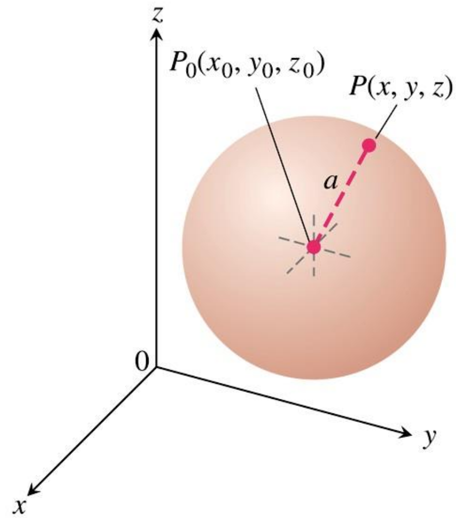
Find the distance between $P_1(2, 1, 5)$ and $P_2(-2, 3, 0)$

Solution

$$\begin{aligned}
\left|P_1P_2\right| &= \sqrt{(-2-2)^2 + (3-1)^2 + (0-5)^2} \\
&= \sqrt{16 + 4 + 25} \\
&= \boxed{\sqrt{45} \ \ or \ \approx 6.708}
\end{aligned}$$

The Standard Equation for the Sphere of Radius a and Center (x_0, y_0, z_0)

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=a^2$$



Example

Find the center and radius of the sphere $x^2+y^2+z^2+3x-4z+1=0$

Solution

$$\left(x^2+3x\right)+y^2+\left(z^2-4z\right)=-1$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+y^2+\left(z^2-4z+\left(\frac{-4}{2}\right)^2\right)=-1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+y^2+(z-2)^2=-1+\frac{9}{4}+4$$

$$\left(x+\frac{3}{2}\right)^2+y^2+(z-2)^2=\frac{21}{4}$$

Therefore; the center is $\left(-\frac{3}{2},\ 0,\ 2\right)$ and the radius is $\boxed{\frac{\sqrt{21}}{2}}$

Applications

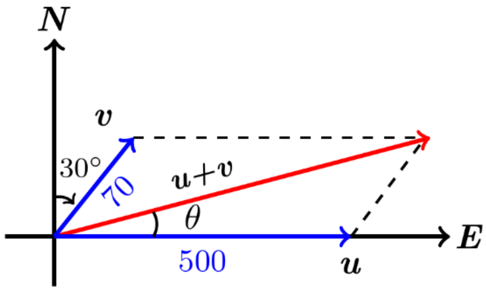
Example

A jet airliner, flying due east at 500 *mph* in still air, encounters a 70-*mph* tailwind blowing in the direction 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?

Solution

$\boldsymbol{u}$ = the velocity of the airplane
$\boldsymbol{v}$ = the velocity of the tailwind

Given: $|u| = 500 \quad |v| = 70$



$$\boldsymbol{u} = \langle 500,\ 0 \rangle$$

$$\boldsymbol{v} = \langle 70\cos 60°,\ 70\sin 60° \rangle$$

$$= \left\langle 35,\ 35\sqrt{3} \right\rangle$$

$$\boldsymbol{u} + \boldsymbol{v} = \left\langle 535,\ 35\sqrt{3} \right\rangle = 535\boldsymbol{i} + 35\sqrt{3}\boldsymbol{j}$$

$$|\boldsymbol{u} + \boldsymbol{v}| = \sqrt{535^2 + \left(35\sqrt{3}\right)^2}$$

$$\approx 538.4$$

$$\theta = \tan^{-1}\frac{35\sqrt{3}}{535}$$

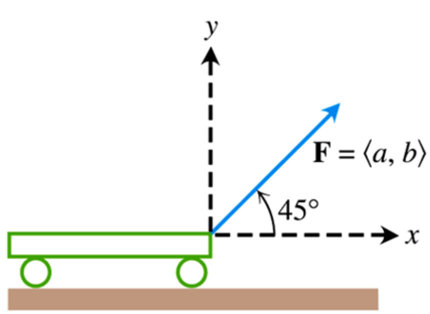
$$\approx 6.5°$$

The ground speed of the airplane is about 538.4 *mph*, and its direction is about 6.5° north of east.

Example

A small cart is being pulled along a 20-*lb* smooth horizontal floor with a force **F** making a 45° angle to the floor. What is the effective force moving the cart forward?

Solution



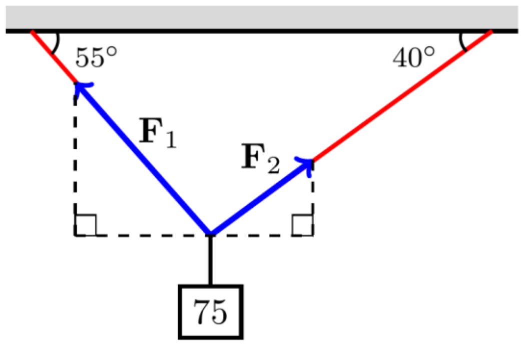
$$a = |F|\cos 45°$$

$$= (20)\left(\frac{\sqrt{2}}{2}\right)$$

$$\approx 14.14\ \ lb$$

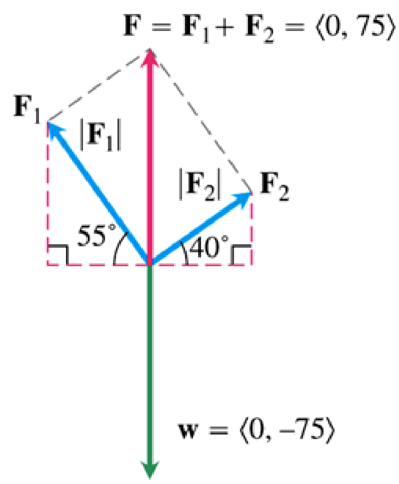
Example

A 75-*N* weight is suspended by two wires.



Find the forces $\vec{F}_1$ and $\vec{F}_2$ acting both wires

Solution



$$\vec{F}_1 = \left\langle -\left|\vec{F}_1\right|\cos 55°,\ \left|\vec{F}_1\right|\sin 55° \right\rangle$$

$$\vec{F}_2 = \left\langle \left|\vec{F}_2\right|\cos 40°,\ \left|\vec{F}_2\right|\sin 40° \right\rangle$$

$$\vec{F}_1 + \vec{F}_2 = \langle 0,\ 75\rangle$$

$$-\left|\vec{F}_1\right|\cos 55° + \left|\vec{F}_2\right|\cos 40° = 0$$

$$\Rightarrow \quad \left|\vec{F}_2\right| = \left|\vec{F}_1\right|\frac{\cos 55°}{\cos 40°}$$

$$\left|\vec{F}_1\right|\sin 55° + \left|\vec{F}_2\right|\sin 40° = 75$$

$$\left|\vec{F}_1\right|\sin 55° + \left|\vec{F}_1\right|\frac{\cos 55°}{\cos 40°}\sin 40° = 75$$

$$\left|\vec{F}_1\right|\left(\sin 55° + \cos 55° \tan 40°\right) = 75$$

$$\left|\vec{F}_1\right| = \frac{75}{\sin 55° + \cos 55° \tan 40°}$$

$$\underline{\approx 57.67\ N|}$$

$$\left|\vec{F}_2\right| = 57.67\frac{\cos 55°}{\cos 40°}$$

$$\underline{\approx 43.18\ N|}$$

The force vectors are then:

$$\vec{F}_1 = \left\langle -\left|\vec{F}_1\right|\cos 55°,\ \left|\vec{F}_1\right|\sin 55° \right\rangle$$

$$= \langle -57.67\cos 55°,\ 57.67\sin 55° \rangle$$

$$= \boxed{\langle -33.08,\ 47.24 \rangle}$$

$$\vec{F}_2 = \left\langle \left|\vec{F}_2\right| \cos 40°,\ \left|\vec{F}_2\right| \sin 40° \right\rangle$$

$$= \langle 43.18 \cos 40°,\ 43.18 \sin 40° \rangle$$

$$= \boxed{\langle 33.08,\ 27.76 \rangle}$$

Exercises *Section* 1.1 – Vectors

Give a geometric description of the set of points in space whose coordinates satisfy the given pairs of equations

1. $x^2 + z^2 = 4, \quad y = 0$

2. $x^2 + y^2 = 4, \quad z = -2$

3. $x^2 + y^2 + z^2 = 1, \quad x = 0$

4. $x^2 + (y-1)^2 + z^2 = 4, \quad y = 0$

5. $x^2 + y^2 + z^2 = 4, \quad y = x$

Find the distance between points P_1 *and* P_2

6. $P_1(1,\ 1,\ 1), \quad P_2(3,\ 3,\ 0)$

7. $P_1(-1,\ 1,\ 5), \quad P_2(2,\ 5,\ 0)$

8. $P_1(1,\ 4,\ 5), \quad P_2(4,\ -2,\ 7)$

9. $P_1(3,\ 4,\ 5), \quad P_2(2,\ 3,\ 4)$

Find the center and radii of the spheres

10. $x^2 + y^2 + z^2 + 4x - 4z = 0$

11. $x^2 + y^2 + z^2 - 6y + 8z = 0$

12. $2x^2 + 2y^2 + 2z^2 + x + y + z = 9$

13. Find a formula for the distance from the point $P(x,\ y,\ z)$ to x-axis

14. Find a formula for the distance from the point $P(x,\ y,\ z)$ to xz-plane.

15. Let $\boldsymbol{u} = \langle -3,\ 4 \rangle$ *and* $\boldsymbol{v} = \langle 2,\ -5 \rangle$. Find the component form and the magnitude if the vector

a) $3\boldsymbol{u} - 4\boldsymbol{v}$ ***b)*** $-2\boldsymbol{u}$ ***c)*** $\boldsymbol{u} + \boldsymbol{v}$

16. Let $\boldsymbol{u} = \langle 3,\ -2 \rangle$ *and* $\boldsymbol{v} = \langle -2,\ 5 \rangle$. Find the component form and the magnitude if the vector

a) $3\boldsymbol{u}$

b) $\boldsymbol{u} - \boldsymbol{v}$

c) $2\boldsymbol{u} - 3\boldsymbol{v}$

d) $-2\boldsymbol{u} + 5\boldsymbol{v}$

e) $-\frac{5}{13}\boldsymbol{u} + \frac{12}{13}\boldsymbol{v}$

17. Find scalars a, b, and c such that $\langle 2,\ 2,\ 2 \rangle = a\langle 1,\ 1,\ 0 \rangle + b\langle 0,\ 1,\ 1 \rangle + c\langle 1,\ 0,\ 1 \rangle$

18. Find the component form of the vector: The sum of $\overrightarrow{AB}$ and $\overrightarrow{CD}$ where

$A = (1,-1), \quad B = (2,0), \quad C = (-1,3), \quad \text{and} \quad D = (-2,2)$

Find the component form of the vector:

19. The unit vector that makes an angle $\theta = \frac{2\pi}{3}$ with the positive x-axis

20. The unit vector obtained by rotating the vector $\langle 0,\ 1 \rangle$ $120°$ counterclockwise about the origin.

21. The unit vector obtained by rotating the vector $\langle 1,\ 0\rangle$ $135°$ counterclockwise about the origin.

22. The unit vector that makes an angle $\theta = \frac{\pi}{6}$ with the positive x-axis

23. The vector 5 units long in the direction opposite to the direction of $\frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{j}$

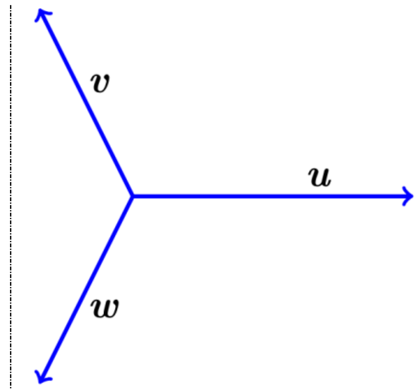
24. Express the velocity vector $\boldsymbol{v} = \left(e^t \cos t - e^t \sin t\right)\boldsymbol{i} + \left(e^t \cos t + e^t \sin t\right)\boldsymbol{j}$ *when* $t = \ln 2$ in terms of its length and direction.

25. Sketch the indicated vector

a) $\boldsymbol{u} - \boldsymbol{v}$

b) $2\boldsymbol{u} - \boldsymbol{v}$

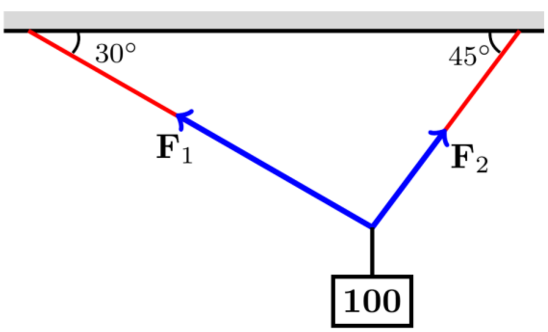
c) $\boldsymbol{u} - \boldsymbol{v} + \boldsymbol{w}$



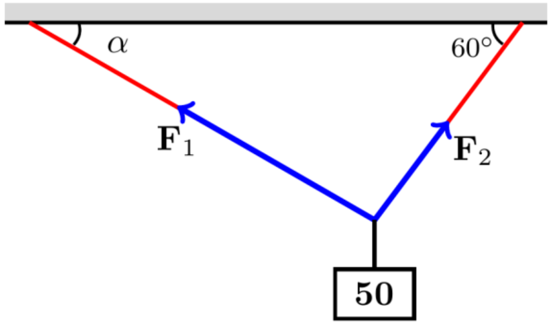
26. An Airplane is flying in the direction 25° west of north at 800 *km/h*. Find the component form of the velocity of the airplane, assuming that the positive x-axis represents due east and the positive y-axis represents due north.

27. A jet airliner, flying due east at 500 *mph* in still air, encounters a 70-*mph* tailwind blowing in the direction 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What speed and direction should the jetliner have in order for the resultant vector to be 500 *mph* due east?

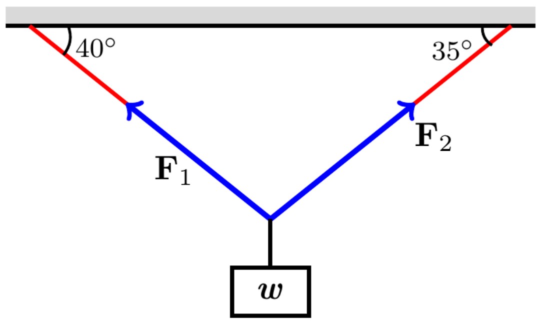
28. Consider a 100-N weight suspended by two wires. Find the magnitudes and components of the force vectors F_1 *and* F_2



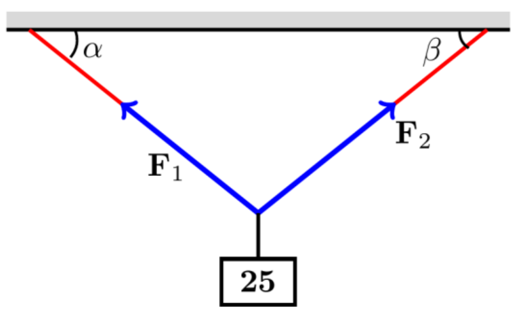
29. Consider a 50-N weight suspended by two wires, If the magnitude of vector $F_1 = 35\ N$, find the angle α and the magnitude of vector F_2



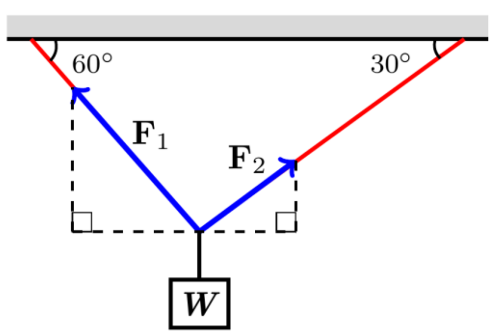
30. Consider a ***w***-N weight suspended by two wires, If the magnitude of vector $F_2 = 100\ N$, find ***w*** and the magnitude of vector F_1



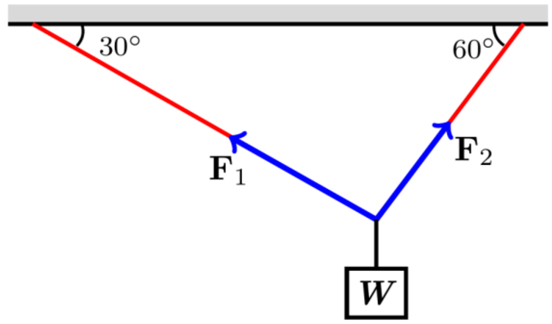
31. Consider a 25-N weight suspended by two wires, If the magnitude of vector F_1 *and* F_2 are both 75 N, then angles α and β are equal. Find α.



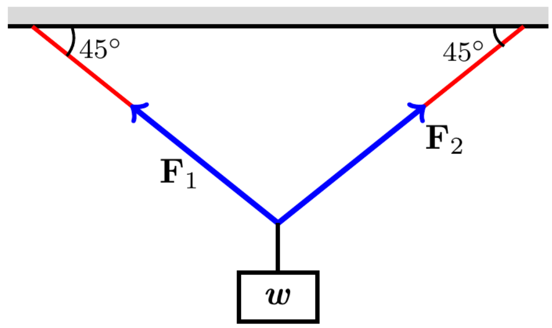
32. Consider a $W = 100\ N$ weight suspended by two wires. Find the magnitudes and components of the force vectors F_1 *and* F_2



33. Consider a $W = 50\ N$ weight suspended by two wires. Find the magnitudes and components of the force vectors F_1 *and* F_2



34. Consider a $W = 100\ N$ weight suspended by two wires. Find the magnitudes and components of the force vectors F_1 *and* F_2

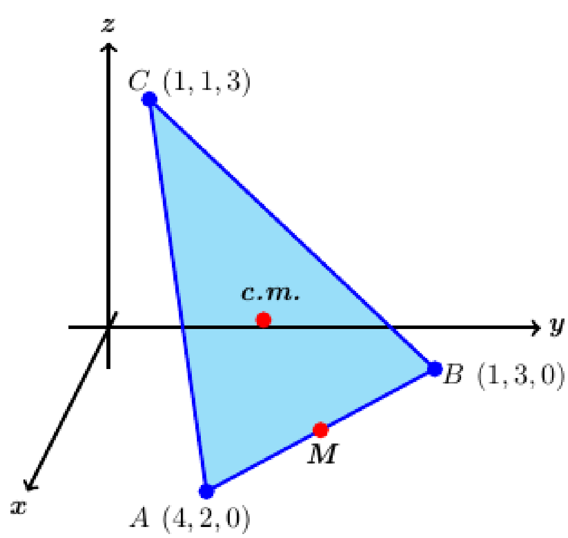


35. A bird flies from its nest 5 km in the direction 60° north east, where it stops to rest on a tree. It then flies 10 km in the direction due southeast and lands atop a telephone pole. Place an *xy*-coordinate system so that the origin is the bird's nest, the *x*-axis points east, and the *y*-axis points north.

a) At what point is the tree located?

b) At what point is the telephone pole?

36. Suppose that *A*, *B*, and *C* are the corner points of the thin triangular plate of constant density.



a) Find the vector from *C* to the midpoint *M* of side *AB*.

b) Find the vector from *C* to the point that lies two-thirds of the way from *C* to *M* on the median *CM*.

c) Find the coordinates of the point in which the medians of $\triangle ABC$ intersect (this point is the plate's center of mass).

37. Show that a unit vector in the plane can be expressed as $\boldsymbol{u} = (\cos\theta)\boldsymbol{i} + (\sin\theta)\boldsymbol{j}$, obtained by rotating $\boldsymbol{i}$ through an angle θ in the counterclockwise direction. Explain why this form gives ***every unit vector*** in the plane.

38. Assume the positive *x-axis* points east and the positive *y-axis* points north.

a) An airliner flies northeast at a constant altitude at 550 *mi/hr* in calm air. Find *a* and *b* such that it velocity may be expressed in the form $\boldsymbol{v} = a\hat{\boldsymbol{i}} + b\hat{\boldsymbol{j}}$

b) An airliner flies northeast at a constant altitude at 550 *mi/hr* relative to the air in a southerly crosswind $\boldsymbol{w} = \langle 0,\ 40 \rangle$. Find the velocity of the airliner relative to the ground.

39. Let $\overrightarrow{PQ}$ extended from $P(2,\ 0,\ 6)$ to $Q(2,\ -8,\ 5)$

a) Find the position vector equal to $\overrightarrow{PQ}$.

b) Find the midpoint *M* of the line segment PQ. Then find the magnitude of $\overrightarrow{PM}$

c) Find a vector of length 8 with direction opposite that of $\overrightarrow{PQ}$

40. An object at the origin is acted on by the forces $\boldsymbol{F}_1 = -10\hat{i} + 20\hat{k}$, $\boldsymbol{F}_2 = 40\hat{j} + 10\hat{k}$, and $\boldsymbol{F}_3 = -50\hat{i} + 20\hat{j}$. Find the magnitude of the combined force and use a sketch to illustrate the direction of the combined force.

41. A remote sensing probe falls vertically with a terminal of 60 *m/s* when it encounters a horizontal crosswind blowing north at 4 *m/s* and an updraft blowing vertically at 10 *m/s*. find the magnitude and direction of the resulting velocity relative to the ground.

42. A small plane is flying north in calm air at 250 *mi/hr* when it is hit by a horizontal crosswind blowing northeast at 40 *mi/hr* and a 25 *mi/hr* downdraft. Find the resulting velocity and speed of the plane.