Solution   *Section* 2.5 – Counting Techniques

Exercise

Decide whether the situation involves ***permutations*** or ***combinations***

a) A batting order for 9 players for a baseball game
b) An arrangement of 8 people for a picture
c) A committee of 7 delegates chosen from a class of 30 students to bring a petition to the administration
d) A selection of a chairman and a secretary from a committee of 14 people
e) A sample of 5 items taken from 71 items on an assembly line
f) A blend of 3 spices taken from 7 spices on a spice rack
g) From the 7 male and 10 female sales representatives for an insurance company, team of 8 will be selected to attend a national conference on insurance fraud.
h) Marbles are being drawn without replacement from a bag containing 15 marbles.
i) The new university president named 3 new officers a vice-president of finance, a vice-president of academic affairs, and a vice-president of student affairs.
j) A student checked out 4 novels from the library to read over the holiday.
k) A father ordered an ice cream cone (chocolate, vanilla, or strawberry) for each of his 4 children.

Solution

a) Permutation
b) Permutation
c) Combination
d) Permutation
e) Combination
f) Combination
g) Combination
h) Combination
i) Permutation
j) Combination
k) Neither

Exercise

Find the number of different ways that five test questions can be arranged in order by evaluating 5!

Solution

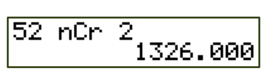
$$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$$

Exercise

In the game of blackjack played with one deck, a player is initially dealt 2 cards. Find the number of different two-card initial hands by evaluating ${}_{52}C_2$

Solution

$${}_{52}C_2 = \frac{52!}{50!\cdot 2!} = \frac{52\cdot 51}{2} = 1326$$

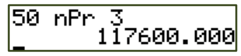


Exercise

A political strategist must visit state capitols, but she has time to visit only 3 of them. Find the number of different possible routes by evaluating ${}_{50}P_3$

Solution

$$ {}_{50}P_3 = \frac{50!}{(50-3)!} = \frac{50!}{45!} = 50 \cdot 49 \cdot 48 \; \boxed{= 117,600} $$



Exercise

Select the six winning numbers from 1, 2, …, 54. Find the probability of winning lottery by buying one ticket. $\left(of \;\; winning \; this \; lottery \quad \frac{1}{575,757} \right)$

Solution

$$ {}_{54}C_6 = \frac{54!}{48! \cdot 6!} = 25,827,165 \;\; possibilities $$

Since only one combination wins

$$ P(winning \;\; with \; a \; single \; selection) \; \boxed{= \frac{1}{25,827,165}} $$

Exercise

Select the five winning numbers from 1, 2, …, 36. Find the probability of winning lottery by buying one ticket. $\left(of \;\; winning \; this \; lottery \quad \frac{1}{575,757} \right)$

Solution

$$ {}_{36}C_5 = \frac{36!}{31! \cdot 5!} = 376,992 \;\; possibilities $$

Since only one combination wins

$$ P(winning \;\; with \; a \; single \; selection) \; \boxed{= \frac{1}{376,992}} $$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be chosen that have

a) All men? *b)* All women? *c)* 3 men and 2 women?

Solution

a) $C(9,5) = 126$

b) $C(11,5) = 462$

c) $C(9,3).C(11,2) = (84)(55) = 4620$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be selected that have

a) At least 4 women? *b)* No more than 2 men?

Solution

a) $C(11,4)C(9,1)+C(11,5)C(9,0)=3432$

b) $C(9,0)C(11,5)+C(9,1)C(11,4)+C(9,2)C(11,3)=9372$

Exercise

In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?

Solution

$P_{9,5}=15,120$

Exercise

From a pool of 8 secretaries, 3 are selected to be assigned to 3 managers, one per manager. In how many ways can they be selected and assigned?

Solution

$P_{8,3}=336$

Exercise

A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.

a) How many delegations are possible?
b) If it is decided that a particular worker must be in the delegation, how many different delegations are possible?
c) If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?

Solution

a) $C_{9,3}=84$

b) $1.C_{8,2}=28$

c) $C_{4,1}C_{5,2}+C_{4,2}C_{5,1}+C_{4,3}=74$

Exercise

Hamburger Hut sells regular hamburgers as well as a larger burger. Either type can include cheese, relish, lettuce, tomato, mustard, or catsup.

a) How many different hamburgers can be ordered with exactly three extras?
b) How many different regular hamburgers can be ordered with exactly three extras?
c) How many different regular hamburgers can be ordered with at least five extras?

Solution

a) $C_{2,1}C_{6,3} = 40$

b) $C_{6,3} = 20$

c) $C_{6,5} + C_{6,6} = 7$

Exercise

In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.

a) In how many ways can this be done?
b) In how many ways can this be done if exactly 2 wheat plants must be included?

Solution

a) $C_{11,4} = 330$

b) $C_{6,2}C_{5,2} = 150$

Exercise

A legislative committee consists of 5 Democrats and 4 Republicans. A delegation of 3 is to be selected to visit a small Pacific island republic.

a) How many different delegations are possible?
b) How many delegations would have all Democrats?
c) How many delegations would have 2 Democrats and 1 Republican?
d) How many delegations would have at least 1 Republican?

Solution

a) $C_{9,3} = 84$

b) $C_{5,3} = 10$

c) $C_{5,2}C_{4,1} = 40$

d) $C_{9,3} - C_{5,3} = 84 - 10 = 74$

Exercise

Five cards are chosen from an ordinary deck to form a hand in poker. In how many ways is it possible to get the following results?

a) 4 queens
b) No face card
c) Exactly 2 face cards
d) At least 2 face cards
e) 1 heart, 2 diamonds, and 2 clubs

Solution

a) $C_{4,4}C_{48,1} = 48$

b) $C_{40,5} = 658,008$

c) $C_{12,2}C_{40,3} = 652,080$

d) $C_{12,2}C_{40,3} + C_{12,3}C_{40,2} + C_{12,4}C_{40,1} + C_{12,5} = 844,272$

e) $C_{13,1}C_{13,2}C_{13,2} = 79,092$

Exercise

The student sitting next to you in class concludes that the probability of the ceiling falling down on both of you before class ends is 1/2, because there are two possible outcomes - the ceiling will fall or not fall. What is wrong with this reasoning?

Solution

The outcomes are not equally likely.

Exercise

Identity theft often begins by someone discovering your 9-digit social security number. Answer each of the following. Express probabilities as fractions.

a) What is the probability of randomly generating 9 digits and getting your social security number?
b) In the past, many teachers posted grades along with the last 4 digits of your social security number, what is the probability that if they randomly generated the order digits, they would match yours? Is that something to worry about?

Solution

a) Let G = generating a given social security number in a single trial.

Total number of possible sequences $= 10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10$

$= 1,000,000,000$

Since only one sequence is correct: $P(G) = \boxed{\dfrac{1}{1,000,000,000}}$

b) Let F = generating first 5 digits of a given social security number in a single trial.

Total number of possible sequences $= 10\cdot10\cdot10\cdot10\cdot10$

$= 100,000$

Since only one sequence is correct: $P(F) = \boxed{\dfrac{1}{100,000}}$

Since this probability is so small, need not worry about the given scenario

Exercise

Credit card numbers typically have 16 digits, but not all of them are random. Answer the following and express probabilities as fractions.

a) What is the probability of randomly generating 16 digits and getting your MasterCard number?

b) Receipts often show the last 4 digits of a credit card number. If those last 4 digits are known, what is the probability of randomly generating the order digits of your MasterCard number?

c) Discover cards begin with the digits 6011. If you also know the last 4 digits, what is the probability of randomly generating the other digits and getting all of them correct? Is this something to worry about?

Solution

a) Let G = generating a given credit card number in a single trial.

Total number of possible sequences $= 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10$
$= 10,000,000,000,000,000$

Since only one sequence is correct: $P(G) = \boxed{\dfrac{1}{10,000,000,000,000,000}}$

b) Let F = generating first 12 digits of a given credit card number in a single trial.

Total number of possible sequences $= 10^{12}$
$= 1,000,000,000,000$

Since only one sequence is correct: $P(F) = \boxed{\dfrac{1}{1,000,000,000,000}}$

c) Let M = generating the middle digits of a given credit card number in a single trial.

Total number of possible sequences $= 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10$
$= 100,000,000$

Since only one sequence is correct: $P(M) = \boxed{\dfrac{1}{100,000,000}}$

This is not something to worry about.

Exercise

When testing for current in a cable with five color-coded wires, the author used a meter to test two wires at a time. How many different tests are required for every possible pairing of two wires?

Solution

Since the order of the 2 wires being tested is irrelevant:

$${}_5C_2 = \frac{5!}{3!\cdot 2!} \boxed{= 10 \ \textit{different tests}}$$

Exercise

The starting 4 players for the Boston Celtics basketball team have agreed to make charity appearances tomorrow night. If you must send three players to the United Way event and the other 2 to a Heart Fund event, how many different ways can you make the assignments?

Solution

Since the order in which the 3 are picked makes no difference.

$${}_5C_3 = \frac{5!}{2!\cdot 3!} \underline{=10 \;\; \textit{different ways}|}$$

Exercise

In phase I of a clinical trial with gene therapy used for treating HIV, 5 subjects were treated (based on data from Medical News Today). If 20 people were eligible for the Phase I treatment and a simple random of 5 is selected, how many different simple random samples are possible? What is the probability of each simple random sample?

Solution

Since the order in which the subjects are placed in the groups is not relevant.

$${}_{20}C_5 = \frac{20!}{15!\cdot 5!} \underline{=15,504 \;\; \textit{possibilities}|}$$

$$P(\textit{any one combination}) \underline{= \frac{1}{15,504}|}$$

Exercise

Many newspapers carry "Jumble" a puzzle in which the reader must unscramble letters to form words. The letters BUJOM were included in newspapers. How many ways can the letters if BUJOM be arranged? Identify the correct unscrambling and then determine the probability of getting that result by randomly selecting one arrangement of the given letters.

Solution

The number of possible sequences: $5! \underline{=120 \; \textit{sequences}|}$

The unscrambled sequence word is JUMBO. $\frac{1}{120}$

Since there is only 1 correct sequence; the probability of finding it with one random arrangement is

Exercise

There are 11 members on the board of directors for the Coca Cola Company.

a) If they must select a chairperson, first vice chairperson, second vice chairperson, and secretary, how many different slates of candidates are possible?

b) If they must form an ethics subcommittee of 4 members, how many different subcommittees are possible?

Solution

a) Since order makes a difference, there are 4 different offices ${}_{11}P_4 = \frac{11!}{7!} \underline{= 7920|}$

b) Since the order in which the 4 are picked makes no differences ${}_{11}C_4 = \frac{11!}{7!4!} \underline{= 330|}$

Exercise

The author owns a safe in which he stores his book. The safe combination consists of 4 numbers between 0 and 99. If another author breaks in and tries to steal this book, what is the probability that he or she will get the correct combination on the first attempt? Assume that the numbers are randomly selected. Given the number of possibilities, does it seem feasible to try opening the safe by making random guesses for the combination?

Solution

There are 4 tasks to perform, and each task can be performed in any of 100 ways.

Total number of possible sequence is $100 \cdot 100 \cdot 100 \cdot 100 \underline{= 100,000,000 \ \ possibilities|}$

Since there is only one correct sequence, the probability of finding is $\frac{1}{100,000,000}$

Since there are so many possibilities, it would not be feasible to try opening the safe by making random guesses.

Exercise

In a preliminary test of the MicroSort gender selection method, 14 babies were born and 13 of them were girls

a) Find the number of different possible sequences of genders that are possible when 14 babies are born.

b) How many ways can 13 girls and 1 boy be arranged in a sequence?

c) If 14 babies are randomly selected, what is the probability that they consist of 13 girls and 1 boy?

d) Does the gender-selection method appear to yield a result that is significantly different from a result that might be expected by random chance?

Solution

a) There are 14 tasks to perform, and each task can be performed in either of 2 ways

Total number of possible sequences is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{14}$

$\underline{= 16,384 \ \ possibilities|}$

b) The number of possible sequences of n objects is when some are alike is

$\frac{n!}{n_1! \cdot n_2! \cdots n_k!} = \frac{14!}{13!1!} \underline{= 14 \ \ possibilities|}$

c) $P(13G,\ 1B) = \dfrac{\#of\ \ ways\ to\ get\ 13G}{Total\ \#\ of\ ways} = \dfrac{14}{16,384}$ $\underline{= 0.000854|}$

d) Yes, since $P(13G,\ 1B)$ is so small, and since 13G, 1B so far (only the 14G, 0B result is more extreme) from the expected 7G, 7B result, the gender-selection method appears to yield results significantly different from those of chance alone.

Exercise

You become suspicious when a genetics researcher randomly selects groups of 20 newborn babies and seems to consistently get 10 girls and 10 boys. The researchers claims that it is common to get 10 girls and 10 boys in such cases,

a) If 20 newborn babies are randomly selected, how many different gender sequences are possible?
b) How many different ways can 10 girls and 10 boys be arranged in sequence?
c) What is the probability of getting 10 girls and 10 boys when 10 babies are born?
d) Based on the preceding results, do you agree with the researcher's explanation that ir is common to get 10 girls and 10 boys when 20 babies are randomly selected?

Solution

a) There are 20 tasks to perform, and each task can be performed in either of 2 ways

Total number of possible sequences is $2 \cdot 2 \cdot 2 \cdot \cdots \cdot 2 = 2^{20}$

$\underline{= 1,048,576\ \ possibilities|}$

b) The number of possible sequences of n objects is when some are alike is

$\dfrac{n!}{n_1! \cdot n_2! \cdots n_k!} : \dfrac{20!}{10!10!}$ $\underline{= 184,756\ \ possibilities|}$

c) $P(10G,\ 10B) = \dfrac{184,756}{1,048,576}$ $\underline{= 0.176|}$

d) It is not unusual for an event with probability 0.176 to occur once, but repeated occurrences should be considered unusual – as the probability of the event occurring twice in a row, for example, is $(.176)(.176) = 0.0310$.

Exercise

The Powerball lottery is run in 29 states. Winning the jackpot requires that you select the correct five numbers between 1 and 55 and, in a separate drawing, you must also select the correct single number between 1 and 42. Find the probability of winning the jackpot.

Solution

Let A = selecting the correct 5 numbers from 1 to 55
Let B = selecting the correct winning number from 1 to 42

The number of possible selection: ${}_{55}C_5 = \dfrac{55!}{50! \cdot 5!}$ $\underline{= 3,478,761\ \ possibilities|}$

Since there is only one winning number: $P(A)=\frac{1}{3,478,761}$

There are 42 possible selections.

Since there is only one winning number: $P(B)=\frac{1}{42}$

$$\begin{aligned} P(\textit{winning Powerball}) &= P(A \textit{ and } B) \\ &= P(A)P(B) \\ &= \frac{1}{3,478,761}\cdot\frac{1}{42} \\ &= \frac{1}{146,107,962} \\ &= 0.00000000684 \end{aligned}$$

Exercise

The Mega Millions lottery is run in 12 states. Winning the jackpot requires that you select the correct 5 numbers between 1 and 56 and, in a separate drawing, you must also select the correct single number between 1 and 46. Find the probability of winning the jackpot.

Solution

Let A = selecting the correct 5 numbers from 1 to 56
Let B = selecting the correct winning number from 1 to 46

The number of possible selection: ${}_{56}C_5=\frac{56!}{51!\cdot 5!}=3,819,816\ \textit{possibilities}$

Since there is only one winning number: $P(A)=\frac{1}{3,478,761}$

There are 46 possible selections.

Since there is only one winning number: $P(B)=\frac{1}{46}$

$$\begin{aligned} P(\textit{winning Mega Millions}) &= P(A \textit{ and } B) \\ &= P(A)P(B) \\ &= \frac{1}{3,819,816}\cdot\frac{1}{46} \quad = \frac{1}{175,711,536} \\ &= 0.00000000569 \end{aligned}$$

Exercise

A state lottery involves the random selection of six different numbers between 1 and 31. If you select one six number combination, what is the probability that it will be the winning combination?

Solution

$$P(\textit{winning})=\frac{1}{\binom{31}{6}}=\frac{1}{736,281}$$

Exercise

How many ways can 6 people be chosen and arranged in a straight line if there are 8 people to choose from?

Solution

$$8\times7\times6\times5\times4\times3 = 20,160 \ \ ways$$

Exercise

12 wrestlers compete in a competition. If each wrestler wrestles one match with each other wrestler, what are the total numbers of matches?

Solution

$$11+10+9+8+7+6+5+4+3+2+1 = 66$$

Exercise

Wing has different books to arrange on a shelf: 4 blue, 3 green, and 2 red.

a) In how many ways can the books be arranged on a shelf?
b) If books of the same color are to be grouped together, how many arrangements are possible?
c) In how many distinguishable ways can the books be arranged if books of the same color are identical but need not be grouped together?
d) In how many ways can you select 3 books, one of each color, if the order in which the books are selected does not matter?
e) In how many ways can you select 3 books, one of each color, if the order in which the books are selected matters?

Solution

a) $P(9,9) = 362,880$ *ways*

b) $4!.3!.2!.3! = 1728$ *possibilities*

c) $\dfrac{9!}{4!3!2!} = 1260$

d) $4.3.2 = 24$

e) $24.3! = 144$ ***(24 from part-d)***

Exercise

A child has a set of differently shaped plastic objects. There are 3 pyramids, 4 cubes, and 7 spheres.

a) In how many ways can she arrange the objects in a row if each is a different color?
b) How many arrangements are possible if objects of the same shape must be grouped together and each object is a different color?
c) In how many distinguishable ways can the objects be arranged in a row if objects of the same shape are also the same color, but need not be grouped together?

d) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected does not matter and each object is a different color?

e) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected matters and each object is a different color?

Solution

a) $P(14,14) = 8.7178291 \times 10^{10}$

b) $3!4!7!3! = 4,354,560$ ($3!$ ***number of ways to arrange the order of 3 groups***)

c) $\dfrac{14!}{3!4!7!} = 120,120$

d) $3.4.7 = 84$

e) $84.3! = 504$

Exercise

In a club with 16 members, how many ways can a slate of 3 officers consisting of president, vice-president, and secretary/treasurer be chosen?

Solution

$$P(16,3) = \underline{3360|}$$

Exercise

Twelve drugs have been found to be effective in the treatment of a disease. It is believed that the sequence in which the drugs are administered is important in the effectiveness of the treatment. In how many different sequences can 5 of the 12 drugs be administered?

Solution

$$P(12,5) = \underline{95,040|}$$

Exercise

In how many ways can 7 of 11 monkeys be arranged in a row for a genetics experiment?

Solution

$$P(11,7) = \underline{1,663,200|}$$

Exercise

In an experiment on social interaction, 6 people will sit in 6 seats in a row. In how many ways can this be done?

Solution

$$P(6,6) = \underline{720|}$$

Exercise

In an election with 3 candidates for one office and 6 candidates for another office, how many different ballots may be printed?

Solution

Office 1: $P(3,3)$

Office2: $P(6,6)$

Multiplication principle: $2.P(3,3)P(6,6) \ \underline{=8640|}$

Exercise

A business school gives courses in typing, shorthand, transcription, business English, technical writing, and accounting. In how many ways can a student arrange a schedule if 3 courses are taken? assume that the order in which courses are schedules matters.

Solution

$P(6,3) \ \underline{=120|}$

Exercise

If your college offers 400 courses, 25 of which are in mathematics, and your counselor arranges your schedule of 4 courses by random selection, how many schedules are possible that do not include a math course? Assume that the order in which courses are scheduled matters.

Solution

$$P(nonmath) = P(375,4) = \underline{1.946\times 10^{10}|}$$

Exercise

A baseball team has 19 players. How many 9-player batting orders are possible?

Solution

$$P(19,9) = \underline{3.352\times 10^{10}|}$$

Exercise

A chapter of union Local 715 has 35 members. In how many different ways can the chapter select a president, a vice-president, a treasurer, and a secretary?

Solution

$$P(35,4) = \underline{1,256,640|}$$

Exercise

A concert to raise money for an economics prize is to consist of 5 works; 2 overtures, 2 sonatas, and a piano concerto.

a) In how many ways can the program be arranged?
b) In how many ways can the program be arranged if an overture must come first?

Solution

a) $P(5,5) = \underline{120|}$

b) $P(2,1).P(4,4) = \underline{48|}$

Exercise

A zydeco band from Louisiana will play 5 traditional and 3 original Cajun compositions at a concert. In how many ways can they arrange the program if

a) The begin with a traditional piece?
b) An original piece will be played last?

Solution

a) $P(5,1).P(7,7) = \underline{25,200|}$

b) $P(7,7).P(3,1) = \underline{15,120|}$

Exercise

Given the set $\{A, B, C, D\}$, how many permutations are there of this set of 4 object taken 2 at a time?

a) Using the multiplication principle
b) Using the Permutation

Solution

a) 4.3 = 12

b) $P_{4,2} = \frac{4!}{2!} = \underline{12|}$

Exercise

Find the number of permutations of 30 objects taken 4 at a time.

Solution

$$P_{30,4} = \frac{30!}{(30-4)!} \underline{= 657,720|}$$

Exercise

Five cards are marked with the numbers 1, 2, 3, 4, and 5, then shuffled, and 2 cards are drawn.

a) How many different 2-card combinations are possible?
b) How many 2-card hands contain a number less than 3?

Solution

a) $C_{5,2} = 10$

b) $\left\{ \begin{array}{l} \{1,2\},\ \{1,3\},\ \{1,4\},\ \{1,5\},\ \{2,3\} \\ \{2,4\},\ \{2,5\},\ \{3,4\},\ \{3,5\},\ \{4,5\} \end{array} \right\}$

7 contain a card numbered less than 3.

Exercise

An economics club has 31 members.

a) If a committee of 4 is to be selected, in how many ways can the selection be made?
b) In how many ways can a committee of at least 1 and at most 3 be selected?

Solution

a) $C_{31,4} = 31,465$

b) $$\begin{aligned} P(at\ least\ 1\ and\ at\ most\ 3\ be\ selected) &= C_{31,1} + C_{31,2} + C_{31,3} \\ &= 31 + 465 + 4495 \\ &= 4991 \end{aligned}$$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be chosen that have

a) All men? *b)* All women? *c)* 3 men and 2 women?

Solution

a) $C(9,5) = 126$

b) $C(11,5) = 462$

c) $C(9,3).C(11,2) = (84)(55) = 4620$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be selected that have

a) At least 4 women? *b)* No more than 2 men?

Solution

a) $C(11,4)C(9,1)+C(11,5)C(9,0)=\boxed{3432}$

b) $C(9,0)C(11,5)+C(9,1)C(11,4)+C(9,2)C(11,3)=\boxed{9372}$

Exercise

Use a tree diagram for the following

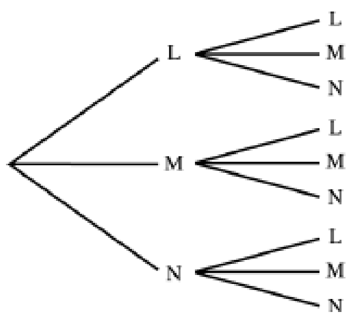
a) Find the number of ways 2 letters can be chosen from the set $\{L, M, N\}$ if order is important and repetition is allowed.

b) Reconsider part a if no repeats are allowed

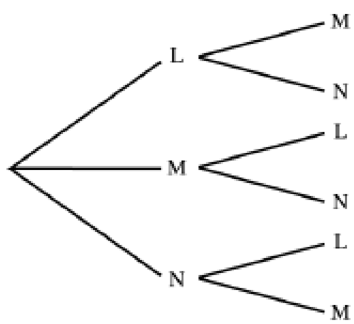
c) Find the number of combinations of 3 elements taken 2 at a time. Does this answer differ from part *a* or *b*?

Solution

a) There are 9 ways to choose 2 letters if repetition is allowed



b) There are 9 ways to choose 2 letters if repetition is allowed



c) /The number of 3 elements taken 2 at a time is: $C_{3,2}=\boxed{3}$

Exercise

In a game of musical chairs, 12 children will sit in 11 chairs arranged in a row (one will be left out). In how many ways can this happen, if we count rearrangements of the children in the chairs as different outcomes?

Solution

$P(12,11)=\boxed{479,001,600}$

Exercise

A group of 3 students is to be selected from a group of 14 students to take part in a class in cell biology.

a) In how many ways can this be done?
b) In how many ways can the group who will not take part be chosen?

Solution

a) $\binom{14}{3} = 364$

b) $\binom{14}{11} = 364$

Exercise

Marbles are being drawn without replacement from a bag containing 16 marbles.

a) How many samples of 2 marbles can be drawn?
b) How many samples of 2 marbles can be drawn?
c) If the bag contains 3 yellow, 4 white, and 9 blue marbles, how many samples of 2 marbles can be drawn in which both marbles are blue?

Solution

a) $C(16,2) = 120$

b) $C(16,4) = 1820$

c) $C(9,2) = 36$

Exercise

There are 7 rotten apples in a crate of 26 apples

a) How many samples of 3 apples can be drawn from the crate?
b) How many samples of 3 could be drawn in which all 3 are rotten?
c) How many samples of 3 could be drawn in which there are two good apples and one rotten one?

Solution

a) $C_{26,3} = 2600$

b) $C_{7,3} = 35$

c) $C_{26,3} = 2600$

Exercise

A bag contains 5 black, 1 red, and 3 yellow jelly beans; you take 3 at random. How many samples are possible in which the jelly beans are

a) All black?
b) All red?
c) All yellow?
d) 2 black and 1 red?
e) 2 black and 1 yellow?
f) 2 yellow and 1 black?
g) 2 red and 1 yellow?

Solution

a) $C_{5,3} = 10$

b) No 3 red. $C_{1,3} = 0$

c) $C_{3,3} = 1$

d) $C_{5,2}C_{1,1} = 10$

e) $C_{5,2}C_{3,1} = 30$

f) $C_{3,2}C_{5,1} = 15$

g) *There is only 1 red.*

Exercise

In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?

Solution

$P_{9,5} = \underline{15,120|}$

Exercise

From a pool of 8 secretaries, 3 are selected to be assigned to 3 managers, one per manager. In how many ways can they be selected and assigned?

Solution

$P_{8,3} = \underline{336|}$

Exercise

A salesperson has the names of 6 prospects.

a) In how many ways can she arrange her schedule if she calls on all 6?
b) In how many ways can she arrange her schedule if she can call on only 4 of the 6?

Solution

a) $P_{6,6} = \underline{720|}$

b) $P_{6,4} = 360$

Exercise

Five items are to be randomly selected from the first 50 items on an assembly line to determine the defect rate. How many different samples of 5 items can be chosen?

Solution

$$C_{50,5} = 2{,}118{,}760$$

Exercise

A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.

a) How many delegations are possible?
b) If it is decided that a particular worker must be in the delegation, how many different delegations are possible?
c) If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?

Solution

a) $C_{9,3} = 84$

b) $1.C_{8,2} = 28$

c) $C_{4,1}C_{5,2} + C_{4,2}C_{5,1} + C_{4,3} = 74$

Exercise

From a group of 16 smokers and 22 nonsmokers, a researcher wants to randomly select 8 smokers and 8 nonsmokers for a study. in how many ways can the study group be selected?

Solution

$$C_{16,8}C_{22,8} = 4{,}115{,}439{,}900$$

Exercise

Hamburger Hut sells regular hamburgers as well as a larger burger. Either type can include cheese, relish, lettuce, tomato, mustard, or catsup.

a) How many different hamburgers can be ordered with exactly three extras?
b) How many different regular hamburgers can be ordered with exactly three extras?
c) How many different regular hamburgers can be ordered with at least five extras?

Solution

a) $C_{2,1}C_{6,3} = 40$

b) $C_{6,3} = 20$

c) $C_{6,5} + C_{6,6} = 7$

Exercise

In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.

c) In how many ways can this be done?
d) In how many ways can this be done if exactly 2 wheat plants must be included?

Solution

c) $C_{11,4} = 330$

d) $C_{6,2}C_{5,2} = 150$

Exercise

A legislative committee consists of 5 Democrats and 4 Republicans. A delegation of 3 is to be selected to visit a small Pacific island republic.

a) How many different delegations are possible?
b) How many delegations would have all Democrats?
c) How many delegations would have 2 Democrats and 1 Republican?
d) How many delegations would have at least 1 Republican?

Solution

a) $C_{9,3} = 84$

b) $C_{5,3} = 10$

c) $C_{5,2}C_{4,1} = 40$

d) $C_{9,3} - C_{5,3} = 84 - 10 = 74$

Exercise

From 10 names on a ballot, 4 will be elected to a political party committee. in how many ways can the committee of 4 be formed if each person will have a different responsibility, and different assignments of responsibility are considered different committees?

Solution

$P_{10,4} = 5040$

Exercise

How many different 13-card bridge hands can be selected from an ordinary deck?

Solution

$$C_{52,13} = \underline{635,013,559,600|}$$

Exercise

Five cards are chosen from an ordinary deck to form a hand in poker. In how many ways is it possible to get the following results?

a) 4 queens
b) No face card
c) Exactly 2 face cards
d) At least 2 face cards
e) 1 heart, 2 diamonds, and 2 clubs

Solution

a) $C_{4,4}C_{48,1} = \underline{48|}$

b) $C_{40,5} = \underline{658,008|}$

c) $C_{12,2}C_{40,3} = \underline{652,080|}$

d) $C_{12,2}C_{40,3} + C_{12,3}C_{40,2} + C_{12,4}C_{40,1} + C_{12,5} = \underline{844,272|}$

e) $C_{13,1}C_{13,2}C_{13,2} = \underline{79,092|}$

Exercise

In poker, a flush consists of 5 cards with the same suit, such as 5 diamonds.

a) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by listing all the possibilities.
b) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by using combinations

Solution

a) $\{(5,6,7,8,9);(5,6,7,8,10);(5,7,8,9,10);(5,6,8,9,10);(5,7,8,9,10);(6,7,8,9,10)\}$

There are 6 possibilities for each suit and there are 4 suits: 4.6 = 24

b) $4C_{6,5} = \underline{24|}$

Exercise

If a baseball coach has 5 good hitters and 4 poor hitters on the bench and chooses 3 players at random, in how many ways can be choose at least 2 good hitters?

Solution

$$C_{5,2}C_{4,1} + C_{5,3}C_{4,0} \underline{=50|}$$

Exercise

The coach of a softball team has 6 good hitters and 8 poor hitters. He chooses 3 hitters at random.

a) In how many ways can he choose 2 good hitters and 1 poor hitter?
b) In how many ways can he choose 3 good hitters?
c) In how many ways can he choose at least 3 good hitters?

Solution

a) $C_{6,2}C_{8,1} \underline{=120|}$
b) $C_{6,3} \underline{=20|}$
c) $C_{6,2}C_{8,1} + C_{6,3} \underline{=140|}$

Exercise

How many 5 card hands will have 3 aces and 2 kings?

Solution

$$\text{Number of hands } = C_{4,3} \cdot C_{4,2} \underline{=24|}$$

Exercise

How many 5 card hands will have 3 hearts and 2 spades?

Solution

$$\text{Number of hands } = C_{13,3} \cdot C_{13,2} \underline{=22,308|}$$

Exercise

2 letters follow by 3 numbers; 2 letters out of 8 & 3 numbers out of 10

Solution

$$\text{Number } = P_{8,2} \cdot P_{10,3} \underline{=40320|}$$

Exercise

Serial numbers for a product are to be made using 3 letters follow by 2 digits (0 – 9 no repeats). If the letters are to be taken from the first 8 letters of the alphabet with no repeats, how many serial numbers are possible?

Solution

$$\text{Possible } = P_{8,3} \cdot P_{10,2} \underline{= 30,240}$$

Exercise

A company has 7 senior and 5 junior officers. An ad hoc legislative committee is to be formed.

a) How many 4-officer committees with 1 senior officer and 3 junior officers can be formed?
b) How many 4-officer committees with 4 junior officers can be formed?
c) How many 4-officer committees with at least 2 junior officers can be formed?

Solution

a) $C_{7,1} \cdot C_{5,3} \underline{= 70}$

b) $C_{5,4} \underline{= 5}$

c) $C_{7,2} . C_{5,2} + C_{7,1} . C_{5,3} + C_{7,0} . C_{5,4} \underline{= 285}$

Exercise

From a committee of 12 people,

a) In how many ways can we choose a chairperson, a vice-chairperson, a secretary, and a treasurer, assuming that one person can't hold more than on position
b) In how many ways can we choose a subcommittee of 4 people?

Solution

a) $P_{12,4} = \underline{11,880 \ \textit{ways}}$

b) $C_{12,4} \underline{= 495 \ \ \textit{ways}}$

Exercise

Find the number of combinations of 30 objects taken 4 at a time.

Solution

$$C_{30,4} = \frac{30!}{4!(30-4)!} \underline{= 27,405}$$

Exercise

How many different permutations are the of the set $\{a, b, c, d, e, f, g\}$?

Solution

$P(7,\ 7)\ \underline{=5040}$

Exercise

How many permutations of $\{a, b, c, d, e, f, g\}$ end with a?

Solution

To find the permutation to with a, then we may forget about the a, and leave us $\{b, c, d, e, f, g\}$

$P(6,\ 6)\ \underline{=720}$

Exercise

Find the number of 5-permutations of a set with nine elements

Solution

$P(9,\ 5)\ \underline{=15{,}120}$

Exercise

In how many different orders can five runners finish a race if no ties are allowed?

Solution

$P(5,\ 5)\ \underline{=120}$

Exercise

A coin flipped eight times where each flip comes up either heads or tails. How many possible outcomes

a) Are there in total?

b) Contain exactly three heads?

c) Contain at least three heads?

d) Contain the same number of heads and tails?

Solution

a) Each flip can be either heads or tails: There are $2^8 \underline{= 256\ \ possible\ coutcomes}$

b) $C(8,\ 3)\ \underline{=56\ \ outcomes}$

c) At least three heads means: 3, 4, 5, 6, 7, 8 heads.

$C(8,\ 3)+C(8,\ 4)+C(8,\ 5)+C(8,\ 6)+C(8,\ 7)+C(8,\ 8)\ \boxed{=219\ \ outcomes}$

OR

$256-C(8,\ 0)-C(8,\ 1)-C(8,\ 2)=256-28-8-1\ \boxed{=219\ \ outcomes}$

d) To have an equal number of heads and tails means 4 heads and 4 tails.

Therefore; $C(8,\ 4)\ \boxed{=70\ \ outcomes}$

Exercise

In how many ways can a set of two positive integers less than 100 be chosen?

Solution

$C_{99,\ 2}\ \boxed{=4851\ \ ways}$

Exercise

In how many ways can a set of five letters be selected from the English alphabet?

Solution

$C_{26,\ 5}\ \boxed{=65,780\ \ ways}$