3.4 Functions

Calculus is an area of mathematics in which you study functions of one or more real variables in a variety of ways. The topics below will help you to enter functions into your calculator and to analyze their values and graphs. First, make sure that your calculator is set to Function Mode, (that is, `FUNCTION` should be displayed for the `Graph` mode). (See the Section 3.2 in Part II of this manual.)

3.4.1 Entering Functions

The TI-89 allows you to store functions into its memory. Press $\boxed{\diamond}$ `[Y=]` to access the `Y= Editor`. Figure 72 shows you the result of entering the functions $y_1 = 2x - 3$ and $y_2 = y_1(x) - x^2 = 2x - 3 - x^2$. To type in the expression for y_2, use the key sequence $\boxed{\text{Y}}$ $\boxed{1}$ $\boxed{(}$ $\boxed{\text{X}}$ $\boxed{)}$ $\boxed{-}$ $\boxed{\text{X}}$ $\boxed{\wedge}$ $\boxed{2}$.

Use the arrow keys to scroll up and down to select a function; to edit a function select it and press $\boxed{\text{ENTER}}$, this will copy the definition of the function to the entry line where you can scroll left and right to edit. The $\boxed{\text{CLEAR}}$ key erases an entire line. In function mode, the $\boxed{\text{X}}$ key produces `x`, which is used as the independent variable.

When a function is selected a check mark will appear to the left of the definition of the function. If you wish to deselect a function, highlight the definition of the function and press $\boxed{\text{F4}}$. One nice thing about the TI-89 is that you can use numbers, variables, matrices, lists, and other functions to define new functions. These features can be particularly useful when studying calculus.

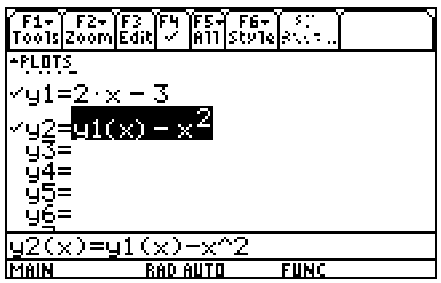

Figure 72: The `Y= Editor`

3.4.2 Graph Style

Functions can be graphed in several different styles. Two such styles and the necessary keystrokes to display them are described in this section. For additional information see the guidebook that came with your calculator.

The standard style for drawing graphs is called `Line`. This is the default style setting. With this setting the calculator plots certain points of the graph and then joins them with tiny line segments creating a continuous-looking graph. In `Dot` style, the calculator simply plots certain points on the graph of the function. To change the style of the graph you must be in the `Y= Editor`, and must have a function highlighted in order to see the `Style` menu. Press the $\boxed{\texttt{2nd}}$ `[F6]` keys, to see the different styles that are available. `Line` and `Dot` styles are the first two. Use the arrow keys to scroll down to the desired style, press $\boxed{\texttt{ENTER}}$, and the new style will be selected (Figure 73).

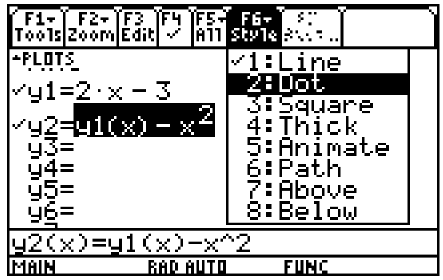

Figure 73: The `Dot` style selected

3.4.3 Viewing Window

The viewing window represents a portion of the Cartesian plane. The standard viewing window is within the bounds $-10 \leq x \leq 10$, and $-10 \leq y \leq 10$. In many cases you will need to draw graphs of functions that are outside this range, but this is not a problem if you are using a TI-89, since you can set the viewing window as needed. Press $\boxed{\Diamond}$ `[WINDOW]` to access the window settings (Figure 74).

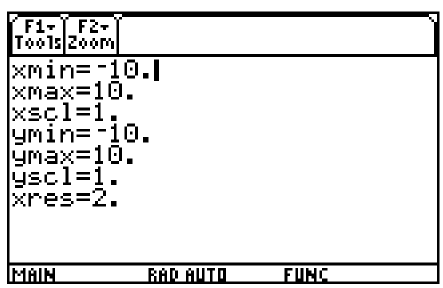

Figure 74: `WINDOW`

The values of `xmin`, `xmax`, `ymin`, and `ymax` determine the portion of the Cartesian plane that will be shown. You must enter values that satisfy `xmin<xmax`, and `ymin<ymax`. The numbers `xscl` and `yscl` determine the distance between tickmarks. Setting these numbers equal to ten will result in a tickmark at every ten units; setting these numbers equal to zero will result in no tickmarks. The number `xres` sets pixel resolution.

3.4.4 Graphing a Function

Press the $\boxed{\Diamond}$ `[GRAPH]` to display the graphs of the functions that you have selected. Your calculator allows you to analyze graphs in a variety of ways. The remainder of the section contains descriptions of several of the features connected to functions and their graphs. See Section 3.5 for additional topics.

3.4.5 Zoom

The `Zoom` item in the `GRAPH` menu allows you to change the viewing window in specific ways. Select the first item (`ZoomBox`) by entering $\boxed{1}$. Move the cursor to a position that will become one corner of the viewing window, then press $\boxed{\texttt{ENTER}}$. Next, move the cursor to determine the opposite corner of the window, then press $\boxed{\texttt{ENTER}}$. The graph will be redrawn within the boundaries of the selected window.

The `ZoomIn` and `ZoomOut` features allow you to look at the graph from closer or further away, respectively. Select again the `ZOOM` menu from the graph window, and select one of these items by pressing $\boxed{2}$ and $\boxed{3}$, respectively, or by highlighting a specific command and pressing $\boxed{\texttt{ENTER}}$. A cursor will appear on the graph, which will determine the center of the new viewing window. Move the cursor to the desired center and press $\boxed{\texttt{ENTER}}$. The graph will be redrawn.

The viewing window `xmin=-10, xmax=10, xscl=1, ymin=-10, ymax=10, yscl=1` is the default set at the factory. You can restore this window by selecting `ZoomStd` . `ZoomSqr` sets the dimensions of the viewing window so that a circle will look like a circle. `ZoomData` is convenient when plotting statistical data points; it sets the viewing window so that all data points are visible. `ZoomFit` resizes the window by changing only the `Y` values in such a way that the graph is displayed within the prespecified values of `X`. The other items in the `Zoom` menu are discussed the guidebook that came with your calculator.

3.4.6 Trace

The `Trace` feature allows you to move the cursor along the graph of a function as the calculator displays the values of the coordinates of the points on the graph. Select `Trace` from the $\boxed{\texttt{GRAPH}}$ menu, and you will see your graph displayed and the trace cursor appear on the graph. Use the left and right arrow keys to move the cursor along the graph. You can also move the cursor to a specific point by entering the x-value of the point and pressing the $\boxed{\texttt{ENTER}}$ key. If the values of x and y are within the viewing window, the cursor will immediately move to the point on the graph that has the given x-coordinate and the calculator will display both coordinates. Figure 75 shows the cursor on the graph of the function $y_2 = x^3 - 5x^2 + 4x + 1$ and the coordinates of the point where the cursor is positioned. Use the up and down arrows to move from function to function.

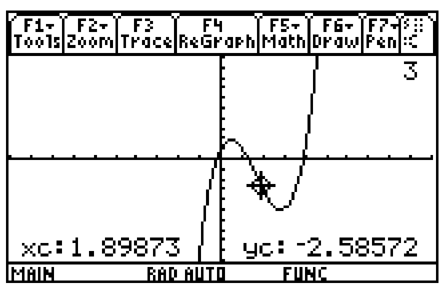

Figure 75: `Trace`

3.4.7 Table

If you have entered a function into `y1` (or any other dependent variable) the table feature will allow you to compute values for this function for many values of the independent variable. For example, from the `Y= Editor`, enter the function $y_1 = x^3 - 7x^2 + 3x + 9$. To set up the table, press $\boxed{\Diamond}$ `[TblSet]` to set the starting value of `x` (`tblStart=1`) and the increment of `x` (`Δtbl=.1`). Set `Independent` to `AUTO`, and press $\boxed{\texttt{ENTER}}$ to save the values (Figure 76).

Press $\boxed{\Diamond}$ `[TABLE]` to view the table in which the values for `y1` are computed automatically. Figure 77 displays a table of values for the function $x^3 - 7x^2 + 3x + 9$. You can scroll through the table of values using the up and down arrow keys.

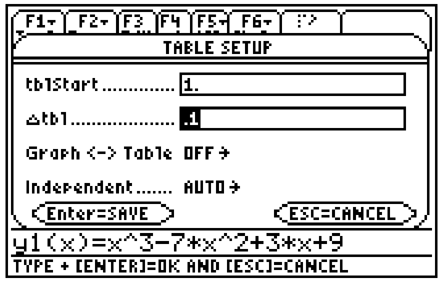

Figure 76: `TblSet`

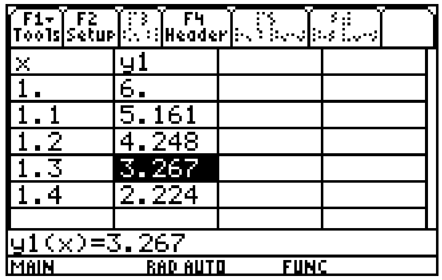

Figure 77: `TABLE`

You can also set `Independent` to `ASK`. Press ENTER to save these options, then press ◊ `[TABLE]`. Enter a value for `x`, press ENTER and the corresponding value for `y1` will be computed. For more information on tables, see the guidebook that came with your calculator.

3.4.8 Finding Zeros of Functions

This section contains methods for finding zeros of functions, that is, points where the graph of the function crosses the x-axis. Your calculator has built-in algorithms that make use of graphs and tables for finding zeros of functions. The values obtained with these methods may be very rough approximations, depending on your calculator. (See Section 3.3 for other methods of finding zeros of functions.)

Trace. Enter and graph the function $y_3 = x^3 + \frac{1}{2}x^2 - 4x - 2$, use the viewing window `xmin=-4, xmax=4, xscl=1, ymin=-5, ymax=5, yscl=1`. Select `Trace` from the menu and use the arrow keys to move the cursor to the point where the graph meets the x-axis. Once you establish an x-value that gives you a y-value close to zero, you can experiment by zooming in to reach other x-values that may give a y-value closer to zero (Figure 78). Often you will not arrive at an x-value that lies exactly on the x-axis.

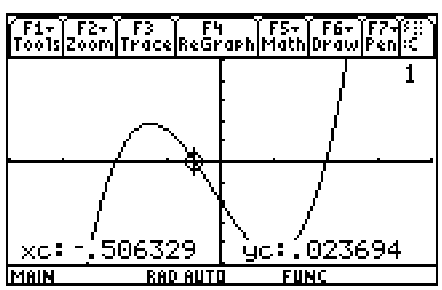

Figure 78: Finding the zero of a function with `Trace`

Table. Enter $y_3 = x^3 + \frac{1}{2}x^2 - 4x - 2$, and construct a table of values for this function. You may want to take a peek at the graph to see if there is a solution between -1 and 0. If this is the case, it's a good idea to set `tblStart=-1` and Δ`tbl=0.01`, and `Independent` to `AUTO`. Scroll through the values in the table to find values of the dependent variable close to zero (Figure 79). Once you establish an x-value that gives you a y-value close to zero, you can experiment with other values of `tblStart` and Δ`tbl` to see if you can achieve a y-value closer to zero. Often you will not arrive at an x-value that yields exactly zero.

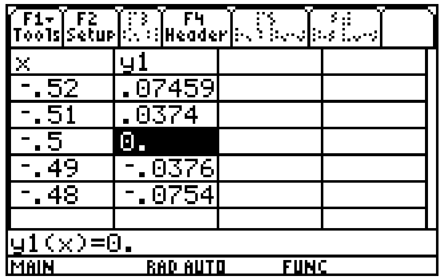

Figure 79: Using `TABLE` to find zeros of a function

Zero. Enter the function $y_3 = x^3 + \frac{1}{2}x^2 - 4x - 2$ and graph it in the viewing window `xmin=-4, xmax=4, xscl=1, ymin=-5, ymax=5, yscl=1`. Select `Zero` from the `Math` menu ($\boxed{\text{F5}}$), and press $\boxed{\text{ENTER}}$. Use the arrows to move the cursor to select the lower bound and upper bound, as prompted by the calculator. Press $\boxed{\text{ENTER}}$ to save each of your selections. The cursor will move to the zero of the function and the calculator will display the values of x and y at that point (Figures 80–82).

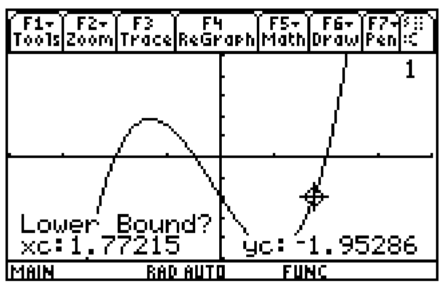

Figure 80: Lower bound

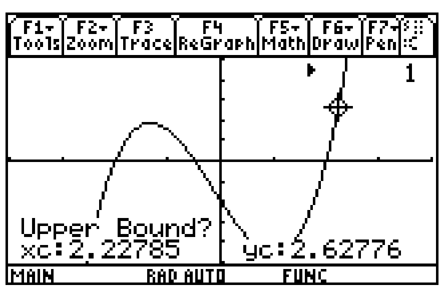

Figure 81: Upper bound

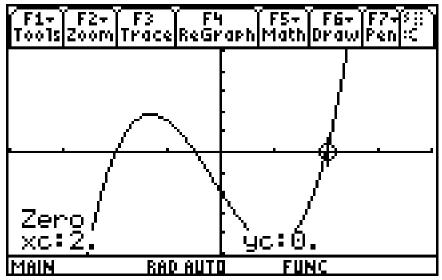

Figure 82: The zero

Intersection. Suppose you want to solve $e^{3x} - 5x - 7 = 0$ for x. This problem is equivalent to finding the x-value at the point where the graphs of $y_1 = e^{3x}$ and $y_2 = 5x + 7$ meet. Enter both functions into memory and graph them, using the viewing window `xmin=-5, xmax=5, xscl=1, ymin=-3, ymax=15, yscl=1`. Select `Intersection` from the `Math` menu (F5), and press ENTER. Use the arrows to move the cursor to select the first curve, the second curve, a lower bound and an upper bound as prompted by the calculator. Press ENTER to save each of your selections. The cursor will move to the point of intersection of the curves and the calculator will display the values of x and y at that point (Figures 83–85).

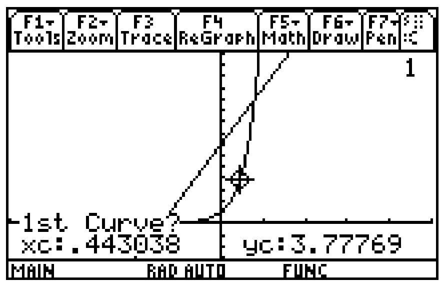

Figure 83: First curve

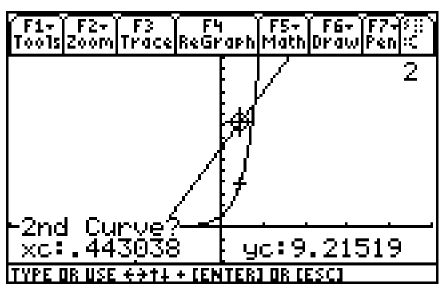

Figure 84: Second curve

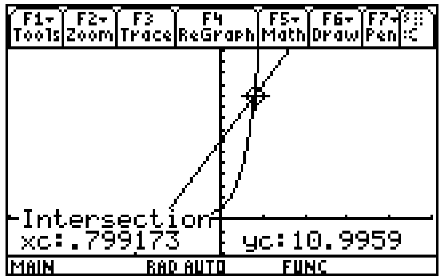

Figure 85: The intersection

3.4.9 Composition of Functions

Functions defined in the TI-89 can be combined to form new functions. One such combination is the composition of two functions. Enter the functions $y_1 = 1 - x$ and $y_2 = e^x$ into your calculator. Both functions have domain equal to the set of real numbers, therefore the compositions $y_1(y_2(x))$, and $y_2(y_1(x))$ can both be formed without restrictions. Enter $\mathtt{y_3} = \mathtt{y_1}(\mathtt{y_2}(\mathtt{x}))$ into the `Y= Editor` by using the key sequence $\boxed{\text{Y}}$ $\boxed{\text{1}}$ $\boxed{\text{(}}$ $\boxed{\text{Y}}$ $\boxed{\text{2}}$ $\boxed{\text{(}}$ $\boxed{\text{X}}$ $\boxed{\text{)}}$ $\boxed{\text{)}}$ as shown in Figure 86.

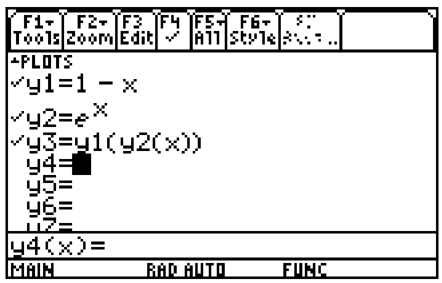

Figure 86: The graph of $y_3 = 1 - e^x$

This is the function $y_3 = 1 - e^x$; its graph is shown in Figure 87 in the viewing window to `xmin=-5, xmax=5, xscl=1, ymin=-5, ymax=5, yscl=1`. Notice that the functions y_1 and y_2 have been deselected by unchecking them (use $\boxed{\text{F4}}$ while in the `Y= Editor`) so that only the graph of y_3 is generated. Enter $\mathtt{y_4} = \mathtt{y_2}(\mathtt{y_1}(\mathtt{x}))$. This is the function $y_4 = e^{1-x}$; its graph is shown in Figure 88.

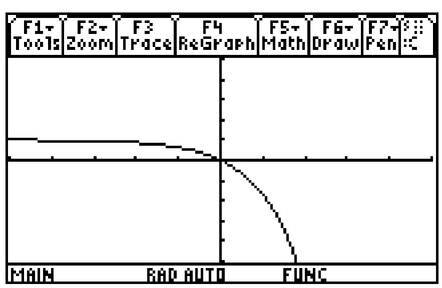

Figure 87: The graph of $y_3 = 1 - e^x$

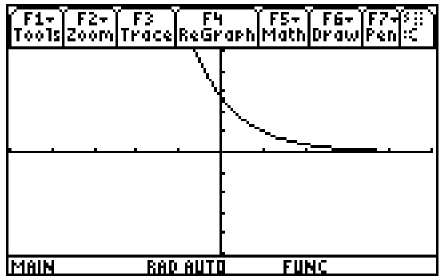

Figure 88: The graph of $y_4 = e^{1-x}$

3.4.10 Piecewise-defined functions

In many applications, functions cannot be given by one unique formula. Instead, functions related to applications are often given in parts. Such functions are called *piecewise-defined functions*. The TI-89 allows you to enter and graph piecewise-defined functions. Consider the function

$$f(x) = \begin{cases} e^x + 1 & \text{if } -2 \le x \le 0, \\ x^2 - 2x + 2 & \text{if } 0 < x \le \frac{3}{2} \end{cases}$$

Enter this function as (see Figure 89)

```
when(-2≤x and x≤0,e^(x)+1,when(0<x and x≤3/2,x^2-2x+2,{})).
```

You can type this expression in directly by using the `alpha` keys and the sequence [<] [=] to make '≤'. The symbol '<' is obtained by pressing [2nd] [0]. Alternatively, the symbol '≤' is in the `MATH Test` menu, or the `CHAR Math` menu (scroll down to `C`).

The format for the '`when`' command is: `when(condition, trueExpression, falseExpression)`. If you want, you can find the '`when`' command in the [CATALOG]. The definition of the function above requires two nested '`when`' commands. Also notice that `falseExpression` for the second command is empty (Figure 89).

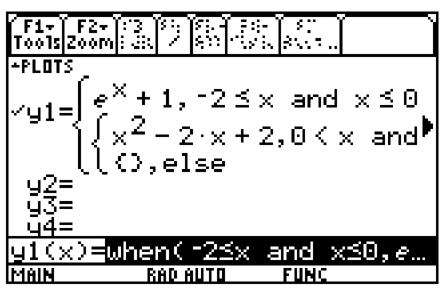

Figure 89: Entering a piecewise-defined function

To graph, change the Graph Style to `Dot` (see Section 3.4.2) to avoid any vertical lines. Set the viewing window to `xmin=-2.5, xmax=2, xscl=1, ymin=-1, ymax=4, yscl=1` and press [◊] `[GRAPH]`. The graph of the piecewise-defined function is shown in Figure 90. Notice that the graph is limited to the interval $[-2, \frac{3}{2}]$.

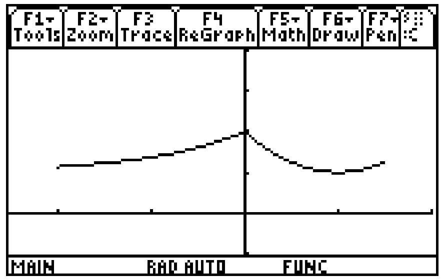

Figure 90: The graph of a piecewise-defined function

3.4.11 Polar Graphing

Polar graphing is used to plot graphs whose equation is given in polar coordinates, that is $r = r(\theta)$. Polar coordinates are used to describe some interesting classical geometric figures whose Cartesian representation would result in extremely complicated equations. Polar coordinates are also used in complex analysis and in engineering applications.

Polar graphing is illustrated below with the equation $r = 2\cos(\theta) - 1$. To use polar graphing on the TI-89 you need to change mode settings. Press the $\boxed{\texttt{MODE}}$ key. Use the arrow keys to select `Graph POLAR` as shown in Figure 91, and press $\boxed{\text{ENTER}}$ twice. Press $\boxed{\diamond}$ `[Y=]` to access the `Y= Editor` and enter the equation (Figure 92). To enter the variable θ press $\boxed{\diamond}$ `[`θ`]`, found above the $\boxed{\wedge}$ key.

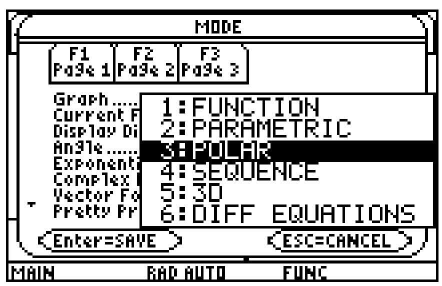

Figure 91: Mode Settings on the TI-89

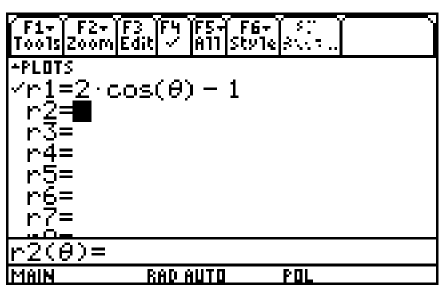

Figure 92: `Y= Editor` in polar mode

Press $\boxed{\diamond}$ `[WINDOW]` to change the window settings. In polar mode you need to specify values for θ. Let $0 \le \theta \le 2\pi$, that is, θ`min=0`, θ`max=`2π; then set θ`step=`$\frac{\pi}{24}$`, xmin=-3, xmax=6, xscl=1, ymin=-2, ymax=2, yscl=1` (Figure 93).

Press $\boxed{\diamond}$ `[GRAPH]` to view the graph (Figure 94). You can also use the `TABLE`, `ZOOM` and `TRACE` features when graphing in polar mode, in the latter case, the values of θ, x and y are displayed on the screen.

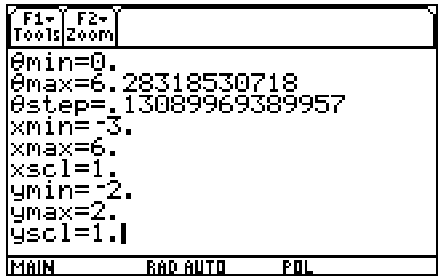

Figure 93: `Window` in polar mode

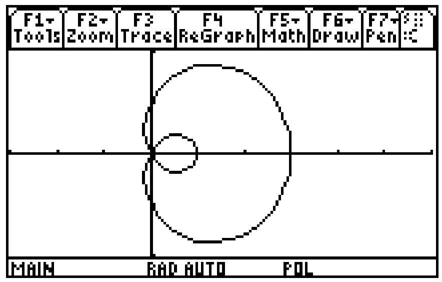

Figure 94: Polar graph

3.4.12 Parametric Graphing

Parametric graphing allows you to plot functions given by two equations $x = x(t)$, and $y = y(t)$, that is, both the x- and y-variables are given in terms of a *parameter* t. Parametric graphing arises in applications, where the variables in question are dependent on time. One particularly useful example is when x and y represent the position of an object at time t.

Parametric graphing is illustrated below with the equations $x = \cos(t-1)$, and $y = \sin(t)$. For parametric graphing on the TI-89 you need to change mode settings. Press the `MODE` key, and use the arrow keys to select `Graph PARAMETRIC` as shown in Figure 95. Press `ENTER` twice to save your settings and exit. Press ◊ `[Y=]` to access the `Y= Editor` and enter the equations (Figure 96). To enter the variable `t` press `T`.

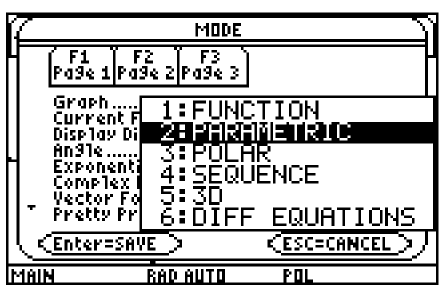

Figure 95: Mode Settings on the TI-89

Press ◊ `[WINDOW]` to change window settings. In parametric mode you will also need to specify values for `t`. In this case you have trigonometric functions, so let $0 \le \theta \le 2\pi$, that is, `tmin=0, tmax=`2π; then set `tstep=`$\frac{\pi}{24}$`, xmin=-2, xmax=2, xscl=1, ymin=-2, ymax=2, yscl=1` (Figure 97). Press ◊ `[GRAPH]` to view the graph (Figure 98). You can also use the `TABLE`, `ZOOM` and `TRACE` features when graphing in parametric mode, in the latter case, the values of t, x and y are displayed on the screen.

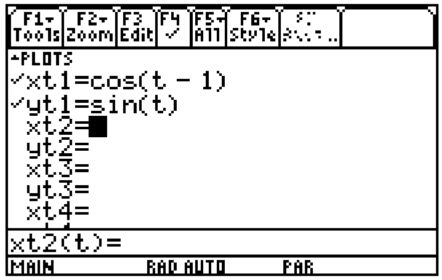

Figure 96: `Y= Editor` in parametric mode

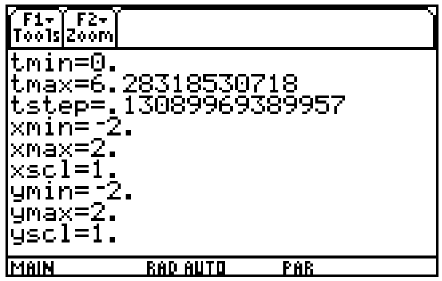

Figure 97: `Window` in parametric mode

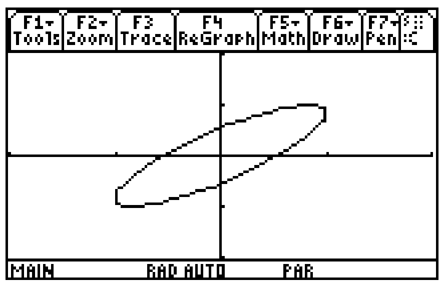

Figure 98: Parametric graph

3.4.13 Split Screen

The TI-89 allows you to view two screens at a time. For example, you can look at the graph of a function while computing its values in an adjacent table. To set up a split screen, press the MODE F2 keys, move the cursor to `Split Screen`, press the right arrow key to view the options as in Figure 99, then select the `LEFT-RIGHT` option by pressing ENTER. Press ENTER again to change the mode setting. You can display any of the applications in the `APPS` menu on either side of the screen.

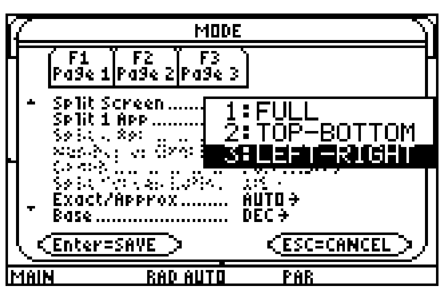

Figure 99: Selecting a spilt screen

Figure 100 shows a graph on the left hand side of the screen and a table on the right hand side. The key sequence 2nd APPS deactivates the current half of the screen and activates the other half. Split screens can be used when graphing in polar and parametric mode.

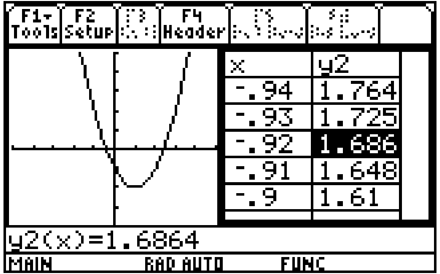

Figure 100: Split screen