Section 4.2 – Matrix operations and Their Applications

Matrix Notation

The Matrix:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix} \text{ is called the coefficient matrix of the system}$$

The matrix is said to be of order *m x n*

m: numbers of rows,

n: number of columns

A matrix A with m rows and n columns can be written in a general form

$$A = \left[a_{\text{ij}} \right] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

The matrix A above has 3 rows and 3 columns; therefore, the order of the matrix A is (3 *x* 3)

When $m = n$, then matrix is said to be ***square***.

The numbers in a matrix are called ***entries.***

Example

$$Let \quad A = \begin{bmatrix} 5 & -2 \\ -3 & \pi \\ 1 & 6 \end{bmatrix}$$

a. What is the order of A?

3 rows and 2 columns $\Rightarrow A$ is 3 *x* 2

b. $a_{12} = -2 \qquad\qquad a_{31} = 1$

Equality of Matrices

Definition of Equality of Matrices

Two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are equal if and only if they have the same order (size) $m \times n$ and if each pair corresponding elements is equal

$$a_{ij} = b_{ij} \text{ for } i = 1, 2, \ldots, m \textbf{ and } j = 1, 2, \ldots, n$$

Example

Find the values of the variables for which each statement is true, if possible.

a) $\begin{bmatrix} 2 & 1 \\ p & q \end{bmatrix} = \begin{bmatrix} x & y \\ -1 & 0 \end{bmatrix}$

$$x = 2, \; y = 1, \; p = -1, \; q = 0$$

b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$

can't be true

c) $\begin{bmatrix} w & x \\ 8 & -12 \end{bmatrix} = \begin{bmatrix} 9 & 17 \\ y & z \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} w = 9 & x = 17 \\ 8 = y & -12 = z \end{bmatrix}$$

Matrix Addition and Subtraction

Given two $m \times n$ matrices $A = \left[a_{ij}\right]$ and $B = \left[b_{ij}\right]$ their sum is $A + B = \left[a_{ij} + b_{ij}\right]$

And their difference is $A - B = \left[a_{ij} - b_{ij}\right]$

The matrices have to be the ***same order***

Example

Find $\begin{bmatrix} -4 & 3 \\ 7 & -6 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & -4 \end{bmatrix}$

Solution

$$\begin{bmatrix} -4 & 3 \\ 7 & -6 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} -4+6 & 3+(-3) \\ 7+2 & -6+(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 9 & -10 \end{bmatrix}$$

Example

Find $\begin{bmatrix} 5 & 4 \\ -3 & 7 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 8 \\ 6 & 0 \\ -5 & 3 \end{bmatrix}$

Solution

$$\begin{bmatrix} 5 & 4 \\ -3 & 7 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 8 \\ 6 & 0 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 5-(-4) & 4-8 \\ -3-6 & 7-0 \\ 0-(-5) & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -4 \\ -9 & 7 \\ 5 & -2 \end{bmatrix}$$

Example

Find $\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix}$

Solution

$$\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 5-4 & -6+6 \\ 8+8 & 9-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 16 & 6 \end{bmatrix}$$

Scalar Multiplication

The scalar product of a number k and a matrix A is denoted by $\boldsymbol{kA}$.

$$kA = k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

Example

Find $5\begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}$

Solution

$$5\begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2(5) & -3(5) \\ 0(5) & 4(5) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -15 \\ 0 & 20 \end{bmatrix}$$

Example

Find $\frac{3}{4}\begin{bmatrix} 20 & 36 \\ 12 & -16 \end{bmatrix}$

Solution

$$\frac{3}{4}\begin{bmatrix} 20 & 36 \\ 12 & -16 \end{bmatrix} = \begin{bmatrix} 15 & 27 \\ 9 & -12 \end{bmatrix}$$

Example

Given: $A = \begin{bmatrix} -4 & 1 \\ 3 & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix}$

Find:

a) $-6B$

b) $3A + 2B$

Solution

a) $-6B = -6\begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix}$

$$= \begin{bmatrix} -1(-6) & -2(-6) \\ 8(-6) & 5(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ -48 & -30 \end{bmatrix}$$

b) $3A + 2B = 3\begin{bmatrix} -4 & 1 \\ 3 & 0 \end{bmatrix} + 2\begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix}$

$$= \begin{bmatrix} -4(3) & 1(3) \\ 3(3) & 0(3) \end{bmatrix} + \begin{bmatrix} -1(2) & -2(2) \\ 8(2) & 5(2) \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 3 \\ 9 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ 16 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} -12-2 & 3-4 \\ 9+16 & 0+10 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & -1 \\ 25 & 10 \end{bmatrix}$$

Matrix Multiplication

Product of Two Matrices

Let A be an $m \times n$ matrix and let B be an $n \times k$ matrix. To find the element in the i^{th} row and j^{th} column of the product matrix AB, multiply each element in the i^{th} row of A by the corresponding element in the j^{th} column of B, and then add these products. The product matrix AB is an $m \times k$ matrix.

Matrix A *Matrix B*

$m \times n$ $n \times k$

Inner *must be equal*

Outer: Order of AB is $m \times k$

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$2x2 \quad 2x2 \quad \rightarrow \quad 2x2$

$$a_{11} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & \_ \\ \_ & \_ \end{bmatrix}$$

$$a_{12} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} \_ & af + bh \\ \_ & \_ \end{bmatrix}$$

$$a_{21} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} \_ & \_ \\ ce + dg & \_ \end{bmatrix}$$

$$a_{22} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} \_ & \_ \\ \_ & cf + dh \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Example

Given: $A = \begin{bmatrix} 1 & -3 \\ 7 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 4 & -1 \end{bmatrix}$

Find $\boldsymbol{AB}$ and $\boldsymbol{BA}$.

Solution

$$\boldsymbol{AB} = \begin{bmatrix} 1 & -3 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+(-3)3 & 1(0)+(-3)1 & 1(-1)+(-3)4 & 1(2)+(-3)(-1) \\ 7(1)+2(3) & 7(0)+2(1) & 7(-1)+2(4) & 7(2)+2(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -3 & -13 & 5 \\ 13 & 2 & 1 & 12 \end{bmatrix}$$

$\boldsymbol{BA}$ can be found since: B: 2x4 and A: 2x2

Note: $AB \neq BA$

Example

Given: $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$

Find $\boldsymbol{AB}$.

Solution

$$AB = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(4)+3(1) & 1(6)+3(0) \\ 2(4)+5(1) & 2(6)+5(0) \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 6 \\ 13 & 12 \end{bmatrix}$$

Example

Given: $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}$ Find $\boldsymbol{AB}$.

Solution

$$AB = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3(1)+1(3)-1(-2) & 3(6)+1(-5)-1(4) \\ 2(1)+0(3)+3(-2) & 2(6)+0(-5)+3(4) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 9 \\ -4 & 24 \end{bmatrix}$$

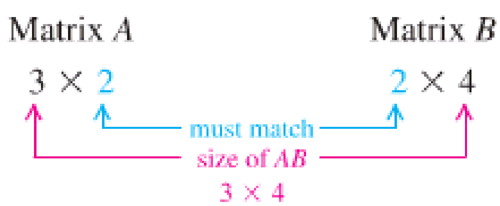
Example

Suppose A is a 3x2 matrix, while B is a 2x4 matrix.

a) Can the product AB be calculated?
b) If AB can be calculated, what size is it?
c) Can BA be calculated?
d) If BA can be calculated, what size is it?

Solution

a)



b) The product AB size is $3\ x\ 4$

c)

d) Can't be calculated

Example

Given: $A_{1x3} = \begin{bmatrix} 2 & 0 & 4 \end{bmatrix}$ $\quad B_{3x1} = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$ $\quad$ Find ***AB*** and ***BA***.

Solution

$$\begin{array}{ccc} \boldsymbol{A} & & \boldsymbol{B} \\ 1\ x\ 3 & & 3\ x\ 1 \end{array}$$

Inner: Are equal

Outer: Order of *AB* is 1x1

AB = [2(1) + 0(3) + 4(7)]

= [30]

BA : 3x1 ---- 1x3

$$\boldsymbol{BA} = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix} \begin{bmatrix} 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(0) & 1(4) \\ 3(2) & 3(0) & 3(4) \\ 7(2) & 7(0) & 7(4) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 4 \\ 6 & 0 & 12 \\ 14 & 0 & 28 \end{bmatrix}$$

Example

Given: $\quad A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix}$ $\quad$ Find ***AB*** and ***BA***.

Solution

a) $\boldsymbol{AB} = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix}$ $\qquad$ 2x2 --- 2x4

$$= \begin{bmatrix} 1(2)+3(0) & 1(3)+3(5) & 1(-1)+3(4) & 1(6)+3(1) \\ 0(2)+2(0) & 0(3)+2(5) & 0(-1)+2(4) & 0(6)+2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 18 & 11 & 9 \\ 0 & 10 & 8 & 2 \end{bmatrix}$$

b) $\boldsymbol{BA} = \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ + Undefined $\quad$ 2x4 --- 2x2 (*Inner order are not equal* 2, 4)

Properties of Matrix

Addition and Scalar Multiplication

$A+B=B+A$	*Commutative Property of Addition*
$A+(B+C)=(A+B)+C$	*Associative Property of Addition*
$(kl)A=k(lA)$	*Associative Property of Scalar Multiplication*
$k(A+B)=kA+kB$	*Distributive Property*
$(k+l)A=kA+lA$	*Distributive Property*
$A+0=0+A=A$	*Additive Identity Property*
$A+(-A)=(-A)+A=0$	*Additive Inverse Property*

Multiplication

$A(BC)=(AB)C$	*Associative Property of Multiplication*
$A(B+C)=AB+AC$	*Distributive Property*
$(B+C)A=BA+CA$	*Distributive Property*

Exercises *Section* 4.2 – Matrix operations and Their Applications

(1 – 7) Find values for the variables so that the matrices are equal.

1. $\begin{bmatrix} w & x \\ 8 & -12 \end{bmatrix} = \begin{bmatrix} 9 & 17 \\ y & z \end{bmatrix}$

2. $\begin{bmatrix} x & y+3 \\ 2z & 8 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 6 & 8 \end{bmatrix}$

3. $\begin{bmatrix} 5 & x-4 & 9 \\ 2 & -3 & 8 \\ 6 & 0 & 5 \end{bmatrix} = \begin{bmatrix} y+3 & 2 & 9 \\ z+4 & -3 & 8 \\ 6 & 0 & w \end{bmatrix}$

4. $\begin{bmatrix} a+2 & 3b & 4c \\ d & 7f & 8 \end{bmatrix} + \begin{bmatrix} -7 & 2b & 6 \\ -3d & -6 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 25 & 6 \\ -8 & 1 & 6 \end{bmatrix}$

5. $\begin{bmatrix} a+11 & 12z+1 & 5m \\ 11k & 3 & 1 \end{bmatrix} + \begin{bmatrix} 9a & 9z & 4m \\ 12k & 5 & 3 \end{bmatrix} = \begin{bmatrix} 41 & -62 & 72 \\ 92 & 8 & 4 \end{bmatrix}$

6. $\begin{bmatrix} x+2 & 3y+1 & 5z \\ 8w & 2 & 3 \end{bmatrix} + \begin{bmatrix} 3x & 2y & 5z \\ 2w & 5 & -5 \end{bmatrix} = \begin{bmatrix} 10 & -14 & 80 \\ 10 & 7 & -2 \end{bmatrix}$

7. $\begin{bmatrix} 2x-3 & y-2 & 2z+1 \\ 5 & 2w & 7 \end{bmatrix} + \begin{bmatrix} 3x-3 & y+2 & z-1 \\ -5 & 5w+1 & 3 \end{bmatrix} = \begin{bmatrix} 20 & 8 & 9 \\ 0 & 8 & 10 \end{bmatrix}$

8. Given $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix}$ Find: $A-B,\ \ 3A+2B$

9. Given $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $F = \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$ Find: $3F+2A$

(10 – 22) Evaluate

10. $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 12 \end{bmatrix}$

11. $\begin{bmatrix} 5 & 8 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 & 1 \\ 4 & 2 & 5 \end{bmatrix}$

12. $\begin{bmatrix} -5 & 0 \\ 4 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix}$

13. $\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix}$

14. $\begin{bmatrix} -5 & 6 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 5 & -8 \end{bmatrix}$

15. $\begin{bmatrix} 8 & 6 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -8 \end{bmatrix}$

16. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$

17. $\begin{bmatrix} -3 & 4 & 2 \\ 5 & 0 & 4 \end{bmatrix}\begin{bmatrix} -6 & 4 \\ 2 & 3 \\ 3 & -2 \end{bmatrix}$

18. $\begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} . \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

19. $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} . \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}$

20. $\begin{bmatrix} -2 & -3 & -4 \\ 2 & -1 & 0 \\ 4 & -2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix}$

21. $\begin{bmatrix} \sqrt{2} & \sqrt{2} & -\sqrt{18} \\ \sqrt{3} & \sqrt{27} & 0 \end{bmatrix} \begin{bmatrix} 8 & -10 \\ 9 & 12 \\ 0 & 2 \end{bmatrix}$

22. $\begin{bmatrix} x & 2x+1 & 4 \\ 5 & x-1 & 8 \\ -2 & 3x & 2x+1 \end{bmatrix} + \begin{bmatrix} 2x-1 & -2x-1 & 4x \\ -5 & 6 & x+1 \\ -5 & 2 & -2x \end{bmatrix}$

(**23 – 33**) Find AB and BA, if possible

23. $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 7 \\ 0 & 2 \end{bmatrix}$

24. $A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} \quad B = \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix}$

25. $A = \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -1 & -1 \\ 0 & 4 \end{pmatrix}$

26. $A = \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 1 \\ 2 & -3 \end{pmatrix}$

27. $A = \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix}$

28. $A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \end{pmatrix}$

29. $A = \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$

30. $A = \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ -3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 0 & -2 \\ -2 & 6 & 2 \end{pmatrix}$

31. $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$

32. $A = \begin{pmatrix} -1 & 2 & 0 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$

33. $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & -2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} -3 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix}$

34. Given $A = \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}$, Find

a) $A+B$ *b)* $A-B$ *c)* $3A$ *d)* $-2B$ *e)* $2A+3B$ *f)* A^2 *g)* AB *h)* BA

35. Given $A=\begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix}$ $B=\begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix}$, Find

a) $A+B$ *b)* $A-B$ *c)* $3A$ *d)* $-2B$ *e)* $2A+3B$ *f)* A^2 *g)* AB *h)* BA

36. Given $A=\begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix}$ $B=\begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$, Find

a) $A+B$ *b)* $A-B$ *c)* $3A$ *d)* $-2B$ *e)* $2A+3B$ *f)* A^2 *g)* AB *h)* BA

37. Given $A=\begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix}$ $B=\begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix}$, Find

a) $A+B$ *b)* $A-B$ *c)* $3A$ *d)* $-2B$ *e)* $2A+3B$ *f)* A^2 *g)* AB *h)* BA

38. Given $A=\begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$ $B=\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ $C=\begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix}$ $D=\begin{pmatrix} -2 & 3 \\ 2 & -1 \\ 3 & 2 \end{pmatrix}$, Find

a) $4A-2B$ *b)* $3A+C$ *c)* $3A+B$ *d)* $2A-3B$ *e)* AB *f)* BA *g)* A^2 *h)* B^3 *i)* AC *j)* CA *k)* CD *l)* DC

39. Given $A=\begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$ $B=\begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$ $C=\begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix}$ $D=\begin{pmatrix} 2 & 4 & -2 \\ 0 & 3 & 5 \\ -3 & 1 & 1 \end{pmatrix}$, Find

a) $4A-2B$ *b)* $3A+C$ *c)* $3A+B$ *d)* $2A-3B$ *e)* AB *f)* BA *g)* A^2 *h)* B^3 *i)* AC *j)* CB *k)* CD *l)* DC

40. A contractor builds three kinds of houses, models *A, B*, and *C*, with a choice of two styles, Spanish and contemporary. Matrix *P* shows the number of each kind of house planned for a new 100-home subdivision. The amounts for each of the exterior materials depend primarily on the style of the house. These amounts are shown in matrix *Q*. (concrete is in *cubic yards*, lumber in units of 1000 board *feet*, brick in 1000*s*, and shingles in units of 100 ft^2.) Matrix *R* gives the cost in dollars for each kind of material.

a) What is the total cost of these materials for each model?
b) How much of each of four kinds of material must be ordered
c) What is the total cost for exterior materials?

41. Mitchell Fabricators manufactures three styles of bicycle frames in its two plants. The following table shows the number of each style produced at each plant

	Mountain Bike	***Racing Bike***	***Touring Bike***
North Plant	150	120	100
South Plant	180	90	130

a) Write a 2 *x* 3 matrix ***A*** that represents the information in the table
b) The manufacturer increased production of each style by 20%. Find a Matrix ***M*** that represents the increased production figures.
c) Find the matrix ***A* + *M*** and tell what it represents

42. Sal's Shoes and Fred's Footwear both have outlets in California and Arizona. Sal's sells shoes for \$80, sandals for \$40, and boots for \$120. Fred's prices are \$60, \$30, and \$150 for shoes, sandals and boots, respectively. Half of all sales in California stores are shoes, 1/4 are *sandals*, and 1/4 are *boots*. In Arizona, the fractions are 1/5 *shoes*, 1/5 are *sandals*, and 3/5 are *boots*.
a) Write a 2 x 3 matrix called *P* representing prices for the two stores and three types of footwear.
b) Write a 2 x 3 matrix called *F* representing fraction of each type of footwear sold in each state.
c) Only one of the two products *PF* and *FP* is meaningful. Determine which one it is, calculate the product, and describe what the entries represent.