$$\sum_{k=1}^{\infty} \frac{|\sin k|}{k^2} \qquad |\sin k| \le 1 \Rightarrow \frac{|\sin k|}{k^2} \le \frac{1}{k^2}$$

$\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges by p-series ($p=2>1$)

$\therefore$ By Comparison test, the given series converges.

$$\sum_{k=1}^{\infty} \frac{\sin^2 k}{k^2} \qquad \sin^2 k \le 1 \Rightarrow \frac{\sin^2 k}{k^2} \le \frac{1}{k^2}$$

$\sum \frac{1}{k^2}$ converges by p-series ($p=2>1$)

$\therefore$ The given series converges by Comparison Test

$$\sum_{k=1}^{\infty} \sin^2 \frac{1}{k}$$

let $b_k = \frac{1}{k^2}$ converges by p-series ($p=2>1$)

$a_k = \sin^2 \frac{1}{k}$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{\sin^2 \frac{1}{k}}{1/k^2} = \lim_{k\to\infty} \left(\frac{\sin 1/k}{1/k}\right)^2 = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 1^2 = \boxed{1}$$

$\therefore$ the given series converges by the Limit Comparison Test

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$$\sum_{k=1}^{\infty} \sin \frac{1}{k}$$

let $b_k = \frac{1}{k}$ diverges (p-series $p \le 1$)

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{\sin 1/k}{1/k} = 1$$

$\therefore$ the given series also diverges by Limit comparison test.

$\sum_{k=1}^{\infty} \frac{1}{k} \sin \frac{1}{k}$ let $b_k = \frac{1}{k^2} \to$ converges p-series $(p=2>1)$

$\lim_{k\to\infty} \frac{1/k \sin 1/k}{1/k^2} = \lim_{k\to\infty} \frac{\sin 1/k}{1/k} = 1$

$\therefore$ the given series also converges by the Limit Comparison Test.

$\sum_{n=1}^{\infty} \tan \frac{1}{n}$ let $b_n = \frac{1}{n}$ which diverges.

$\lim_{n\to\infty} \frac{\tan 1/n}{1/n} = 1$ $\lim_{x\to 0} \frac{\tan x}{x} = 1$

$\therefore$ The given series also diverges by the Limit Comparison Test.

$\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ $\cos \pi n = (-1)^n$

$\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ alternating series

$\{ \frac{1}{n^2} \to$ converges by p-series $(p=2>1) \}$

or $\frac{1}{n^2} > 0$

$n^2 < (n+1)^2 \Rightarrow \frac{1}{n^2} > \frac{1}{(n+1)^2}$

$\lim_{n\to\infty} \frac{1}{n^2} = 0$

$\therefore$ the given series converges by alternating series Test

$$\sum_{k=1}^{\infty} \frac{\sin(1/k)}{k^2} \qquad -1 < \sin\left(\frac{1}{k}\right) < 1$$

$$-\frac{1}{k^2} < \frac{\sin(1/k)}{k^2} < \frac{1}{k^2}$$

Since $\sum \frac{1}{k^2}$ converges by p-series $(p = 2 > 1)$

$\therefore$ The given series converges by Comparison Test.

$$\sum_{k=1}^{\infty} (-1)^k k \sin\frac{1}{k}$$

$$\lim_{k\to\infty} k \sin\frac{1}{k} = \lim_{k\to\infty} \frac{\sin 1/k}{1/k} = 1 \neq 0$$

$\therefore$ The given series diverges by alternating series Test

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$$\sum_{k=1}^{\infty} \frac{\cos k}{k^3} \qquad |\cos k| < 1 \quad \text{absolute}$$

$$\frac{|\cos k|}{k^3} < \frac{1}{k^3}$$

Since $\frac{1}{k^3}$ converges by p-series $(p = 3 > 1)$

$\therefore$ The given series converges absolutely by Comparison Test.

$$\sum_{k=1}^{\infty} (-1)^k \tan^{-1} k$$

$$\lim_{k\to\infty} \tan^{-1} k = \frac{\pi}{2} \neq 0$$

$\therefore$ The given series diverges by Divergence Test.

$$\sum_{k=2}^{\infty} \frac{k}{\ln k} \qquad \lim_{k\to\infty} \frac{k}{\ln k} = \frac{\infty}{\infty}$$

$$= \lim_{k\to\infty} \frac{1}{1/k}$$

$$= \lim_{k\to\infty} k$$

$$= \underline{\infty}$$

∴ series diverges.

$$\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$$

$$\int_2^{\infty} \frac{dx}{x(\ln x)^2} = \int_2^{\infty} \frac{d(\ln x)}{(\ln x)^2}$$

$$= -\frac{1}{\ln x}\Big|_2^{\infty}$$

$$= \underline{\frac{1}{\ln 2}}$$

∴ The given series converges by Integral Test

$$\sum_{k=1}^{\infty} \left(\frac{1}{\ln(k+1)}\right)^k$$

$$\lim_{k\to\infty} \sqrt[k]{\left(\frac{1}{\ln(k+1)}\right)^k} = \lim_{k\to\infty} \frac{1}{\ln(k+1)} = \frac{1}{\infty} = \underline{0}$$

∴ The given series converges by the Root Test.

$$\sum_{k=2}^{\infty} \frac{1}{k^2(\ln k)^2} \qquad (k \ln k)^2 > k^2$$

$$\frac{1}{(k \ln k)^2} < \frac{1}{k^2} \leftarrow \text{converges } p\text{-series } p=2>1$$

∴ The given series also converges by comparison Test

$$\sum_{k=3}^{\infty} \frac{1}{\ln k} \qquad \text{let } b_k = \frac{1}{k}$$

$$\lim_{k\to\infty} \frac{1/\ln k}{1/k} = \lim_{k\to\infty} \frac{k}{\ln k} = \frac{\infty}{\infty}$$

$$= \lim_{k\to\infty} \frac{1}{1/k}$$

$$= \lim_{k\to\infty} k$$

$$= \infty$$

$\therefore$ The given series diverges by Limit Comparison Test

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$$\sum_{k=2}^{\infty} \frac{5 \ln k}{k}$$

$$\int_2^{\infty} \frac{5 \ln x}{x} dx = 5 \int_2^{\infty} \ln x \, d(\ln x)$$

$$= \frac{5}{2} (\ln x)^2 \Big|_2^{\infty}$$

$$= \infty$$

$\therefore$ The given series diverges by Integral Test

$b_k = \frac{5}{k}$ diverges

$$\lim_{k\to\infty} \frac{\frac{5 \ln k}{k}}{5/k} = \lim_{k\to\infty} \ln k$$

$$= \infty$$

Comparison Test

$$\sum_{k=1}^{\infty} \ln\left(\frac{k+2}{k+1}\right)$$

$$\ln\left(\frac{k+2}{k+1}\right) = \underbrace{\ln(k+2)}_{\text{last}} - \underbrace{\ln(k+1)}_{1^{st}} \qquad \therefore \text{ series telescopes}$$

$$\sum_{k=1}^{\infty} \ln\left(\frac{k+2}{k+1}\right) = \ln(n+2) - \ln 2$$

$$\lim_{n\to\infty} [\ln(n+2) - \ln 2] = \infty$$

$\therefore$ The given series diverges.

$$\sum_{k=2}^{\infty} \frac{1}{k^2 \ln k}$$

$$k^2 \ln k > k^2$$

$$\frac{1}{k^2 \ln k} < \frac{1}{k^2} \qquad \frac{1}{k^2} \to \text{converges } p\text{-series } p = 2 > 1$$

$\therefore$ The given series also converges by Comparison Test

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$$\sum_{k=2}^{\infty} 1/k^{\ln k} \qquad \text{as } k \text{ gets bigger } \ln k > 2$$

$$k^{\ln k} > k^2$$

$$\Rightarrow \frac{1}{k^{\ln k}} < \frac{1}{k^2} \Rightarrow \frac{1}{k^2} \text{ converges}$$

series also converges by Comparison Test

$$\sum_{k=2}^{\infty} (-1)^k \frac{\ln k}{k^2} \qquad f(x) = \frac{\ln x}{x^2} \to f'(x) = \frac{x - 2x\ln x}{x^4} = \frac{1 - 2\ln x}{x^3}$$

as x get larger $\Rightarrow f'(x) < 0. \Rightarrow \Sigma$ decreases.

$$\lim_{k\to\infty} \frac{\ln k}{k^2} = \frac{\infty}{\infty} = \lim_{k\to\infty} \frac{1/k}{2k}$$

$$= \lim_{k\to\infty} \frac{1}{2k^2} = 0 \checkmark.$$

$\therefore$ The given series converges by alternating Series Test

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$$\sum_{k=2}^{\infty} \frac{(-1)^k}{k \ln^2 k} \qquad k\ln^2 k < (k+1)\ln^2(k+1)$$

$$\frac{1}{k\ln^2 k} > \frac{1}{(k+1)\ln^2(k+1)}$$

$$\lim_{k\to\infty} \frac{1}{k\ln^2 k} = 0$$

$\therefore$ The given series converges by alternating Test

$$\int_2^{\infty} \frac{dx}{x\ln^2 x} = \int_2^{\infty} \frac{d(\ln x)}{\ln^2 x} = -\frac{1}{\ln x}\Big|_2^{\infty} = \frac{1}{\ln 2} \text{ converges by integral Test}$$

$$\sum_{k=2}^{\infty} \frac{(-1)^k}{\ln k}$$

$\frac{1}{\ln k} > \frac{1}{\ln(k+1)}$

$\lim_{k=\infty} \frac{1}{\ln k} = 0$ ∴ series converges by alternating

however. $\ln k < k$

$\frac{1}{\ln k} > \frac{1}{k} \Rightarrow \Sigma \frac{1}{k}$ diverges by p-series

∴ the given series converges Conditionally.

$$\sum_{k=2}^{\infty} 3e^{-k} = 3\sum_{k=2}^{\infty} \left(\frac{1}{e}\right)^k \qquad r = \frac{1}{e}, \quad a_2 = \frac{3}{e^2} = a$$

$$S = \frac{3/e^2}{1-\frac{1}{e}} = \frac{3}{e(e-1)}$$

∴ Given Series converges by Geometric series

$$\sum_{k=1}^{\infty} \frac{2^k}{e^k - 1} \qquad a_k = \frac{2^k}{e^k-1} \qquad b_n = \frac{2^k}{e^k} = \left(\frac{2}{e}\right)^k$$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{2^k}{e^k - 1} \cdot \frac{e^k}{2^k}$$

$$= \lim_{k\to\infty} \frac{e^k}{e^k-1} = 1 \checkmark$$

$\Sigma \left(\frac{2}{e}\right)^k$ converges. it's a geometric series $r = \frac{2}{e}$

((or) Root test $\sqrt[k]{\left(\frac{2}{e}\right)^k} = \frac{2}{e}$)

∴ The given series converges by Limit Comparison Test

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} e^k}{(k+1)!} \Rightarrow \text{Absolute value is } \sum_{k=1}^{\infty} \frac{e^k}{(k+1)!}$$

Ratio Test, $\frac{a_{k+1}}{a_k} = \frac{e^{k+1}}{(k+2)!} \cdot \frac{(k+1)!}{e^k} = \frac{e}{k+2} \longrightarrow 0$

∴ The given series converges absolutely.

$$\sum_{k=1}^{\infty} \frac{k}{(k^2+1)^3}$$

$$\int_1^{\infty} \frac{x\,dx}{(x^2+1)^3} = \frac{1}{2}\int_1^{\infty} (x^2+1)^{-3}\, d(x^2+1)$$

$$= -\frac{1}{4}\,(x^2+1)^{-2}\Big|_1^{\infty}$$

$$= -\frac{1}{4}\left(0 - \frac{1}{4}\right)$$

$$= \boxed{\frac{1}{16}}$$

$\therefore$ The given series converges by integral Test.

$$\sum_{k=2}^{\infty} \frac{k^e}{k^{\pi}} = \sum_{k=2}^{\infty} \frac{1}{k^{\pi-e}} \qquad \pi - e \cong 3.141 - 2.718 \approx .423 < 1$$

$\therefore$ The given series diverges by p-series $p<1$.

$$\sum_{k=3}^{\infty} \frac{1}{(k-2)^4} = \sum_{k=1}^{\infty} \frac{1}{k^4}$$

$\therefore$ series converges by p-series $p=4>1$

$$\sum_{k=1}^{\infty} \left(\frac{2k}{k+1}\right)^k \qquad \sqrt[k]{\left(\frac{2k}{k+1}\right)^k} = \frac{2k}{k+1} \longrightarrow 2 > 1$$

$\therefore$ Given Series diverges by Root Test.

$$\sum_{k=1}^{\infty} \frac{k^2}{2^k} \qquad \sqrt[k]{\frac{k^2}{2^k}} = \frac{k^{2/k}}{2} \longrightarrow \frac{1}{2} < 1$$

$\therefore$ The given series converges by Root Test.

$$\sum_{k=1}^{\infty} \frac{k^2-1}{k^3+4} \qquad a_k = \frac{k^2-1}{k^3+4} \quad b_k = \frac{k^2}{k^3} = \frac{1}{k}$$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{k^2-1}{k^3+4}\cdot k = \lim_{k\to\infty} \frac{k^3}{k^3} = 1$$

$\therefore$ the given series diverges by the Limit Comparison Test