Section **1.1** – *Polynomials and Factoring*

Polynomials

Adding and Subtracting Polynomials

Properties of Real numbers

For all real numbers a, b, and c:

$a+b=b+a$ ***Commutative properties***

$ab=ba$

$(a+b)+c=a+(b+c)$ ***Associative properties***

$(ab)c=a(bc)$

$a(b+c)=ab+ac$ ***Distributive properties***

Add or subtract as indicated

a) $\left(8x^3-4x^2+6x\right)+\left(3x^3+5x^2-9x+8\right)$

$$\begin{aligned}\left(8x^3-4x^2+6x\right)+\left(3x^3+5x^2-9x+8\right)&=8x^3-4x^2+6x+3x^3+5x^2-9x+8\\&=\left(8x^3+3x^3\right)+\left(-4x^2+5x^2\right)+(6x-9x)+8\\&=11x^3+x^2-3x+8\end{aligned}$$

b) $\left(-4x^4+6x^3-9x^2-12\right)+\left(-3x^3+8x^2-11x+7\right)$

$$\begin{aligned}\left(-4x^4+6x^3-9x^2-12\right)+\left(-3x^3+8x^2-11x+7\right)&=-4x^4+6x^3-3x^3-9x^2+8x^2-11x-12+7\\&=-4x^4+3x^3-x^2-11x-5\end{aligned}$$

c) $\left(2x^2-11x+8\right)-\left(7x^2-6x+2\right)$

$$\begin{aligned}\left(2x^2-11x+8\right)-\left(7x^2-6x+2\right)&=2x^2-11x+8-7x^2+6x-2\\&=-5x^2-5x+6\end{aligned}$$

Multiply

a) $8x(6x-4)$

$$\begin{aligned} 8x(6x-4) &= 8x(6x)-8x(4) \\ &= 48x^2-32x \end{aligned}$$

b) $(3p-2)\left(p^2+5p-1\right)$

$$\begin{aligned} (3p-2)\left(p^2+5p-1\right) &= 3p^3+15p^2-3p-2p^2-10p+2 \\ &= 3p^3+13p^2-13p+2 \end{aligned}$$

c) $(x+2)(x+3)(x-4)$

$$\begin{aligned} (x+2)(x+3)(x-4) &= \left(x^2+3x+2x+6\right)(x-4) \\ &= \left(x^2+5x+6\right)(x-4) \\ &= x^3+5x^2+6x-4x^2-20x-24 \\ &= x^3+x^2-14x-24 \end{aligned}$$

Find $(2m-5)(m+4)$

$$\begin{aligned} (2m-5)(m+4) &= 2mm+2m(4)-5m-5(4) \\ &= 2m^2+8m-5m-20 \\ &= 2m^2+3m-20 \end{aligned}$$

Find $(2k-5)^2$

$$\begin{aligned} (2k-5)^2 &= (2k-5)(2k-5) \\ &= 4k^2-10k-10k+25 \\ &= 4k^2-20k+25 \end{aligned}$$

$$(a-b)^2 = a^2-2ab+b^2$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$(a-b)(a+b) = a^2-b^2$$

Perform the indicated operations: $2\left(3x^2+4x+2\right)-3\left(-x^2+4x-5\right)$

$$\begin{aligned}2\left(3x^2+4x+2\right)-3\left(-x^2+4x-5\right)&=6x^2+8x+4+3x^2-12x+15\\&=9x^2-4x+19\end{aligned}$$

Perform the indicated operations: $(3t-2y)(3t+5y)$

$$\begin{aligned}(3t-2y)(3t+5y)&=9t^2+15ty-6yt-10y^2\\&=9t^2+9yt-10y^2\end{aligned}$$

Perform the indicated operations: $(2a-4b)^2$

$$\begin{aligned}(2a-4b)^2&=(2a)^2-2(2a)(4b)+(4b)^2\\&=4a^2-16ab+16b^2\end{aligned}$$

$(a-b)^2=a^2-2ab+b^2$

Factoring

Prime Factorization

A process that allows us to write a composite number as a product of two or more prime numbers.

Tree

10

2 / \ 5 $10 = 2 \times 5$

$72 = 2.\ 36$
$= 2.\ 6.\ 6$
$= 2.\ 2.\ 3.\ 2.\ 3$
$= 2^3\ 3^2$

The Greatest Common Factor (GCF)

The largest factor that two or more numbers (or terms) have in common

Find GCF (18, 36)

18: 2. 9
2. 3. 3

36: 2. 18
2. 2. 3. 3

$18:\ 2\ 3^2 \rightarrow 1, 2, 3, 6, 9,$ $\underline{\mathbf{18}}$

$36:\ 2^2\ 3^2 \rightarrow 1, 2, 3, 4, 6, 9, 12,$ $\underline{\mathbf{18}}$, 36 GCF (18, 36) = 18 (is the greatest common factor)

Find GCF (27, 45)

$27 = 3^3$
$45 = \underline{3^2\ 5}$
3^2 GCF (27, 45) = 9

Find GCF (40, 56)

$40 = 2^3\ 5$
$56 = \underline{2^3\ 7}$
2^3 GCF (40, 56) = 8

Find GCF (80, 60)

$80 = 2^4\ \ 5$
$60 = \underline{2^2\ 3\ 5}$
$2^2\ \ 5$ GCF (80, 60) = 20

Factor out the greatest common factor

12	2 . 2 . 3
18	2 . . 3 . 3
	2 . 3

a) $12p-18q$

$$12p-18q=6(2p-3q)$$

b) $8x^3-9x^2+15x$

$$8x^3-9x^2+15x=x\left(8x^2-9x+15\right)$$

Factoring Trinomial

Factor $y^2+8y+15$

Product 15	***Sum*** 8
15 *x* 1	15 + 1
3 *x* 5	3 + 5

$$y^2+8y+15=(y+3)(y+5)$$

Factor $4x^2+8xy-5y^2$

$$4x^2+8xy-5y^2=(2x-y)(2x+5y)$$

Special Factorization

$$a^2-b^2=(a-b)(a+b)$$
$$a^2+2ab+b^2=(a+b)^2$$
$$a^2-2ab+b^2=(a-b)^2$$
$$a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$$
$$a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$$

Factor

a) $64p^2 - 49q^2$

$$\begin{aligned} 64p^2 - 49q^2 &= (8p)^2 - (7q)^2 \\ &= (8p - 7q)(8p + 7q) \end{aligned}$$

b) $x^2 + 36$

$x^2 + 36$ *can't be factored (in real number) it is prime.*

c) $x^2 + 12x + 36$

$$x^2 + 12x + 36 = (x+6)^2$$

d) $9y^2 - 24yz + 16z^2$

$$\begin{aligned} 9y^2 - 24yz + 16z^2 &= (3y)^2 - 2(3y)(4z) + (4z)^2 \\ &= (3y - 4z)^2 \end{aligned}$$

e) $y^3 - 8$

$$\begin{aligned} y^3 - 8 &= y^3 - 2^3 \\ &= (y-2)\left(y^2 + 2y + 4\right) \end{aligned}$$

f) $m^3 + 125$

$$m^3 + 125 = (m+5)\left(m^2 - 5m + 25\right)$$

g) $8k^3 - 27z^3$

$$\begin{aligned} 8k^3 - 27z^3 &= (2k)^3 - (3z)^3 \\ &= (2k - 3z)\left((2k)^2 + 6kz + (3z)^2\right) \\ &= (2k - 3z)\left(4k^2 + 6kz + 9z^2\right) \end{aligned}$$

h) $p^4 - 1$

$$\begin{aligned} p^4 - 1 &= \left(p^2\right)^2 - (1)^2 \\ &= \left(p^2 - 1\right)\left(p^2 + 1\right) \\ &= (p-1)(p+1)\left(p^2 + 1\right) \end{aligned}$$

Factor: $60m^4 - 120m^3n + 50m^2n^2$

$$60m^4 - 120m^3n + 50m^2n^2 = 10m^2\left(6m^2 - 12mn + 5n^2\right)$$

Factor: $y^2 - 4yz - 21z^2$

$$y^2 - 4yz - 21z^2 = (y+3z)(y-7z)$$

Factor: $4a^2 + 10a + 6$

$$\begin{aligned} 4a^2 + 10a + 6 &= 2\left(2a^2 + 5a + 3\right) \\ &= 2(2a+3)(a+1) \end{aligned}$$

Factor: $16a^4 - 81b^4$

$$\begin{aligned} 16a^4 - 81b^4 &= \left(4a^2\right)^2 - \left(9b^2\right)^2 \\ &= \left(4a^2 - 9b^2\right)\left(4a^2 + 9b^2\right) \\ &= \left((2a)^2 - (3b)^2\right)\left(4a^2 + 9b^2\right) \\ &= (2a-3b)(2a+3b)\left(4a^2 + 9b^2\right) \end{aligned}$$

Section 1.2 – *Exponents*

Integer Exponents

Definition of exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdots\cdots a}_{n-times}$$

a appears as a factor n times

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

$$a^m \cdot a^n = a^{m+n} \qquad \left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

$$\left(a^m\right)^n = a^{mn} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad (ab)^m = a^m b^m$$

a) 6^0

$$6^0 = 1$$

b) $(-9)^0$

$$(-9)^0 = 1$$

c) 3^{-2}

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

d) $\left(\frac{3}{4}\right)^{-1}$

$$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$$

a) $7^4.7^6$

$$7^4.7^6 = 7^{4+6} = 7^{10}$$

b) $\dfrac{9^{14}}{9^6}$

$$\frac{9^{14}}{9^6} = 9^{14-6} = 9^8$$

c) $\dfrac{r^9}{r^{17}}$

$$\frac{r^9}{r^{17}} = \frac{1}{r^{17-9}} = \frac{1}{r^8}$$

d) $\left(2m^3\right)^4$

$$\left(2m^3\right)^4 = (2)^4\left(m^3\right)^4$$

$$= 16m^{12}$$

e) $\left(\dfrac{x^2}{y^3}\right)^6$

$$\left(\frac{x^2}{y^3}\right)^6 = \frac{\left(x^2\right)^6}{\left(y^3\right)^6}$$

$$= \frac{x^{2.6}}{y^{3.6}}$$

$$= \frac{x^{12}}{y^{18}}$$

f) $\dfrac{a^{-3}b^5}{a^4b^{-7}}$

$$\frac{a^{-3}b^5}{a^4b^{-7}} = \frac{b^5b^7}{a^3a^4}$$

$$= \frac{b^{5+7}}{a^{4+3}}$$

$$= \frac{b^{12}}{a^7}$$

g) $p^{-1}+q^{-1}$

$$p^{-1}+q^{-1}=\frac{1}{p}+\frac{1}{q}$$
$$=\frac{1}{p}\frac{q}{q}+\frac{1}{q}\frac{p}{p}$$
$$=\frac{q+p}{pq}$$

h) $\dfrac{x^{-2}-y^{-2}}{x^{-1}-y^{-1}}$

$$\frac{x^{-2}-y^{-2}}{x^{-1}-y^{-1}}=\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}-\frac{1}{y}}$$
$$=\frac{\frac{y^2-x^2}{x^2y^2}}{\frac{y-x}{xy}}$$
$$=\frac{y^2-x^2}{x^2y^2}\cdot\frac{xy}{y-x}$$
$$=\frac{(y-x)(y+x)}{(xy)^2}\cdot\frac{xy}{y-x}$$
$$=\frac{y+x}{xy}$$

Calculations with exponents

a) $121^{1/2}=11$

b) $625^{1/4}=5$

c) $(-32)^{1/5}=-2$

d) $(-49)^{1/2}$ *is not a real number*

Rational Exponents

$$a^{m/n} = \left(a^{1/n}\right)^m$$

Calculations with Exponents

a) $27^{2/3}$ $\qquad$ 27^(2/3)

$$\begin{aligned} 27^{2/3} &= \left(27^{1/3}\right)^2 \\ &= \left(\left(3^3\right)^{1/3}\right)^2 \\ &= \left(3^{3.\frac{1}{3}}\right)^2 \\ &= (3)^2 \\ &= 9 \end{aligned}$$

b) $32^{2/5}$ $\qquad$ 32^(2/5)

$$\begin{aligned} 32^{2/5} &= \left(\left(2^5\right)^{1/5}\right)^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

c) $64^{4/3}$ $\qquad$ 64^(4/3)

$$\begin{aligned} 64^{4/3} &= \left(\left(4^3\right)^{1/3}\right)^4 \\ &= (4)^4 \\ &= 256 \end{aligned}$$

Simplify

a) $\dfrac{y^{1/3}y^{5/3}}{y^3}$

$$\begin{aligned}\frac{y^{1/3}y^{5/3}}{y^3} &= \frac{y^{\frac{1}{3}+\frac{5}{3}}}{y^3} \\ &= \frac{y^{\frac{6}{3}}}{y^3} \\ &= \frac{y^2}{y^3} \\ &= \frac{1}{y^{3-2}} \\ &= \frac{1}{y}\end{aligned}$$

b) $m^{2/3}\left(m^{7/3}+7m^{1/3}\right)$

$$\begin{aligned}m^{2/3}\left(m^{7/3}+7m^{1/3}\right) &= m^{2/3}m^{7/3}+7m^{2/3}m^{1/3} \\ &= m^{\frac{2}{3}+\frac{7}{3}}+7m^{\frac{2}{3}+\frac{1}{3}} \\ &= m^{\frac{9}{3}}+7m^{\frac{3}{3}} \\ &= m^3+7m\end{aligned}$$

c) $\left(\dfrac{m^7n^{-2}}{m^{-5}n^2}\right)^{1/4}$

$$\begin{aligned}\left(\frac{m^7n^{-2}}{m^{-5}n^2}\right)^{1/4} &= \left(\frac{m^{7+5}}{n^{2+2}}\right)^{1/4} \\ &= \left(\frac{m^{12}}{n^4}\right)^{1/4} \\ &= \frac{\left(m^{12}\right)^{1/4}}{\left(n^4\right)^{1/4}} \\ &= \frac{m^{12/4}}{n^{4/4}} \\ &= \frac{m^3}{n}\end{aligned}$$

Simplify

a) $4m^{1/2}+3m^{3/2}$

$$4m^{1/2}+3m^{3/2}=m^{1/2}\left(4m^{1/2-1/2}+3m^{3/2-1/2}\right)$$
$$=m^{1/2}(4+3m)$$

b) $9x^{-2}-6x^{-3}$

$$9x^{-2}-6x^{-3}=3x^{-3}(3x-2)$$

c) $2\left(x^2+5\right)(3x-1)^{-1/2}+(3x-1)^{1/2}(2x)$

$$2\left(x^2+5\right)(3x-1)^{-1/2}+(3x-1)^{1/2}(2x)=2(3x-1)^{-1/2}\left[x^2+5+x(3x-1)\right]$$
$$=2(3x-1)^{-1/2}\left[x^2+5+3x^2-x\right]$$
$$=2(3x-1)^{-1/2}\left(4x^2-x+5\right)$$

Radicals

$$a^{1/n}=\sqrt[n]{a}$$

a) $\sqrt[4]{16}$

$$\sqrt[4]{16}=16^{1/4}=2$$

b) $\sqrt[5]{-32}=-2$

c) $\sqrt[3]{1000}$

$$\sqrt[3]{1000}=1000^{1/3}=10$$

d) $\sqrt[6]{\frac{64}{729}}$

$$\sqrt[6]{\frac{64}{729}}=\frac{\sqrt[6]{64}}{\sqrt[6]{729}}=\frac{2}{3}$$

Properties

$$\left(\sqrt[n]{a}\right)^n = a$$

$$\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n \text{ is even} \\ a & \text{if } n \text{ is odd} \end{cases}$$

$$\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

Simplify

a) $\sqrt{1000}$

$$\sqrt{1000} = \sqrt{100(10)}$$

$$= \sqrt{100}\sqrt{10}$$

$$= 10\sqrt{10}$$

b) $\sqrt{128}$

$$\sqrt{128} = \sqrt{64(2)}$$

$$= 8\sqrt{2}$$

c) $\sqrt{2}\sqrt{18}$

$$\sqrt{2}\sqrt{18} = \sqrt{2(18)}$$

$$= \sqrt{36}$$

$$= 6$$

d) $\sqrt[3]{54}$

$$\sqrt[3]{54} = \sqrt[3]{27(2)}$$

$$= 3\sqrt[3]{2}$$

e) $\sqrt{288m^5}$

$$\sqrt{288m^5} = \sqrt{144(2)m^4 m}$$

$$= 12m^2\sqrt{2m}$$

f) $2\sqrt{18}-5\sqrt{32}$

$$\begin{aligned} 2\sqrt{18}-5\sqrt{32} &= 2\sqrt{9(2)}-5\sqrt{16(2)} \\ &= 6\sqrt{2}-20\sqrt{2} \\ &= -14\sqrt{2} \end{aligned}$$

Section 1.3 – Fractions and Rationalization

Fraction (*Basic*)

$\frac{a}{b} = \frac{numerator}{denominator}$

$\frac{a}{b} = \frac{c}{d} \iff ad = bc$ *Cross multiplication*

$\frac{a}{b} = \frac{na}{nb} = \frac{an}{bn}$

a) $\frac{5}{6} = \frac{25}{30}?$

$\frac{5}{6} = \frac{5}{6}\frac{5}{5} = \frac{25}{30}$

b) $\frac{16}{48} = \frac{1}{3}$

$\frac{16}{48} = \frac{1}{3} \iff (16)(3) = (1)(48)$

$48 = 48$

Simplify: $\frac{12}{18} = \frac{2.6}{2.9}$

$= \frac{2.2.3}{2.3.3}$

$= \frac{2}{3}$

Simplify: $\frac{36}{56} = \frac{2.18}{2.28}$

$= \frac{18}{28}$

$= \frac{2.9}{2.14}$

$= \frac{9}{14}$

If the denominators are the same $\Rightarrow$ add the numerators

$$\frac{3}{5}+\frac{4}{5}=\frac{3+4}{5}=\frac{7}{5}$$

If the denominators are the same $\Rightarrow$ subtract the numerators

$$\frac{4}{9}-\frac{2}{9}=\frac{4-2}{9}=\frac{2}{9}$$

If the denominators are not the same

- $\Rightarrow$ Find Least Common Denominator (LCD) and convert so that the fractions have the same denominators

LCD: is the smallest whole number that is a multiple of each

$\frac{5}{8}+\frac{1}{12}$ LCD (8, 12)

$$\begin{aligned} 8 &= 2^3 \\ 12 &= \underline{2^2\ 3} \\ & \quad 2^3\ 3 = 24 \end{aligned}$$

LCD (8, 12) = 24

$$\begin{aligned} \frac{5}{8}+\frac{1}{12} &= \frac{5}{8}\frac{3}{3}+\frac{1}{12}\frac{2}{2} \\ &= \frac{15}{24}+\frac{2}{24} \\ &= \frac{15+2}{24} \\ &= \frac{17}{24} \end{aligned}$$

$\frac{69}{75}-\frac{1}{50}$

LCD (75, 50)

$$\begin{aligned} 75 &= \ 5^3 \\ 50 &= \underline{2\ 5^2} \\ & \quad 2\ 5^3 = 150 \end{aligned}$$

LCD (75, 50) = 150

$$\begin{aligned} \frac{69}{75}-\frac{1}{50} &= \frac{(69)(2)-(1)(3)}{150} \\ &= \frac{138-3}{150} \\ &= \frac{135}{150} \\ &= \frac{9}{10} \end{aligned}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

$$\begin{aligned}\frac{2}{7}+\frac{3}{5}&=\frac{2(5)+3(7)}{7(5)}\\&=\frac{10+21}{35}\\&=\frac{31}{35}\end{aligned}$$

or $$\begin{aligned}\frac{2}{7}\frac{5}{5}+\frac{3}{5}\frac{7}{7}&=\frac{10}{35}+\frac{21}{35}\\&=\frac{10+21}{35}\\&=\frac{31}{35}\end{aligned}$$

$$\begin{aligned}\frac{5}{9}+\frac{3}{4}&=\frac{5(4)+3(9)}{9(4)}\\&=\frac{20+27}{36}\\&=\frac{47}{36}\end{aligned}$$

$$\begin{aligned}\frac{17}{15}+\frac{5}{12}&=\frac{17(12)+5(15)}{15(12)}\\&=\frac{204+75}{180}\\&=\frac{279}{180}\\&=\frac{31(9)}{20(9)}\\&=\frac{31}{20}\end{aligned}$$

$$\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}=\frac{5(7)(9)+(3)(7)(9)+(3)(5)(9)+(3)(5)(7)}{(3)(5)(7)(9)}$$

$$=\frac{315+189+135+105}{945}$$

$$=\frac{744}{945}$$

$$=\frac{248}{315}\frac{3}{3}$$

$$=\frac{248}{315}$$

$$\frac{8}{9}+\frac{1}{12}+\frac{3}{16}$$

$$\frac{8}{9}+\frac{1}{12}+\frac{3}{16}=\frac{8(16)+1(12)+3(9)}{144}$$

$$\begin{cases} 9 = \quad\quad 3^2 \\ 12 = 2^2 \quad 3 \\ 16 = 2^4 \end{cases}$$

$$\text{LCD} \quad 2^4 \quad 3^2 = 144$$

$$=\frac{128+12+27}{144}$$

$$=\frac{167}{144}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

$$\frac{2}{7}-\frac{3}{5}=\frac{2(5)-3(7)}{7(5)}=\frac{10-21}{35}=-\frac{11}{35}$$

$$\frac{a}{c}\frac{b}{d}=\frac{ab}{cd}$$

$$\frac{2}{7}\frac{3}{5}=\frac{6}{35}$$

$$\frac{a}{c}\div\frac{b}{d}=\frac{a}{c}\times\frac{d}{b}=\frac{ad}{cb}$$

$$\frac{2}{7}\div\frac{3}{5}=\frac{2}{7}\frac{5}{3}=\frac{10}{21}$$

Find:

1. $\frac{13}{21}+\frac{5}{21}=\frac{13+5}{21}=\frac{6}{7}$

2. $\frac{7}{12}-\frac{4}{15}=\frac{7(5)-4(4)}{60}=\frac{35-16}{60}=\frac{19}{60}$

3. $\frac{5}{8}+\frac{1}{2}=\frac{5+4}{8}=\frac{9}{8}$

4. $\frac{5}{8}+\frac{1}{2}+\frac{2}{3}=\frac{5(3)+1(12)+2(8)}{24}=\frac{43}{24}$

5. $\frac{7}{8}-\frac{1}{10}=\frac{7(5)-1(4)}{40}=\frac{31}{40}$

6. $\frac{11}{5}-\frac{31}{7}=-\frac{78}{35}$

7. $\frac{3}{4}\bullet\frac{3}{2}=\frac{9}{8}$

8. $\frac{3}{4}\bullet\frac{4}{3}\bullet\frac{2}{3}=\frac{2}{3}$

9. $\frac{3}{4}\div\frac{3}{2}=\frac{3}{4}\bullet\frac{2}{3}=\frac{2}{4}=\frac{1}{2}$

10. $\frac{14}{15}\div\frac{14}{3}=\frac{14}{15}\bullet\frac{3}{14}=\frac{1}{5}$

Operations with Fractions

A rational expression is proper if the degree of numerator is less than the degree of denominator
A rational expression is improper if the degrees of numerator is greater than or equal the degree of denominator

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)=\frac{ac}{bd}$$

$$\frac{a/b}{c/d}=\frac{a}{b}\frac{d}{c}=\frac{ad}{bc}$$

$$\frac{a/b}{c}=\frac{a}{b}\frac{1}{c}=\frac{a}{bc}$$

$$\frac{ab}{ac}=\frac{b}{c}$$

$$\frac{ad+ac}{ad}=\frac{a(d+c)}{ad}=\frac{b+c}{d}$$

$$\frac{ab+cd}{ad} \quad stay$$

Example

Perform each indicated operation & simplify

a) $x+\frac{2}{x}=\frac{x^2+2}{x}$

b) $\frac{2}{x+1}-\frac{1}{2x+1}=\frac{2(2x+1)-1(x+1)}{(x+1)(2x+1)}$

$$=\frac{4x+2-x-1}{(x+1)(2x+1)}$$

$$=\frac{3x+1}{(x+1)(2x+1)}$$

Example

Perform each indicated operation & simplify

a) $\dfrac{x}{x^2-4}-\dfrac{1}{x-2}=\dfrac{x-1(x+2)}{(x-2)(x+2)}$

$x^2-4=(x-2)(x+2)$

$$=\frac{x-x-2}{(x-2)(x+2)}$$

$$=\frac{-2}{(x-2)(x+2)}$$

b) $\dfrac{1}{3(x^2+2x)}-\dfrac{1}{3x}=\dfrac{1-1(x+2)}{3x(x+2)}$

$3(x^2+2x)=3x(x+2)$

$$=\frac{1-x-2}{3x(x+2)}$$

$$=\frac{-x-1}{3x(x+2)}$$

Example

Perform each indicated operation & simplify

a) $\dfrac{\sqrt{x+2}-\frac{x}{4\sqrt{x+2}}}{x+2}=\left(\sqrt{x+2}-\dfrac{x}{4\sqrt{x+2}}\right)\div(x+2)$

$$=\left(\frac{4\sqrt{x+2}\sqrt{x+2}-x}{4\sqrt{x+2}}\right)\left(\frac{1}{x+2}\right)$$

$$=\frac{4(x+2)-x}{4(x+2)\sqrt{x+2}}$$

$$=\frac{4x+8-x}{4(x+2)\sqrt{x+2}}$$

$$=\frac{3x+8}{4(x+2)\sqrt{x+2}}$$

b) $\left(\dfrac{1}{x+\sqrt{x^2+4}}\right)\left(1+\dfrac{x}{\sqrt{x^2+4}}\right)=\dfrac{1}{x+\sqrt{x^2+4}}\,\dfrac{\sqrt{x^2+4}+x}{\sqrt{x^2+4}}$

$$=\frac{1}{\sqrt{x^2+4}}$$

Example

Perform each indicated operation & simplify

$$\frac{-x\left(\frac{3x}{3\sqrt{x^2+4}}\right)+\sqrt{x^2+4}}{x^2}+\left(\frac{1}{x+\sqrt{x^2+4}}\right)\left(1+\frac{3x}{3\sqrt{x^2+4}}\right)$$

$$=\left(-\frac{3x^2}{3\sqrt{x^2+4}}+\sqrt{x^2+4}\right)\frac{1}{x^2}+\left(\frac{1}{x+\sqrt{x^2+4}}\right)\left(\frac{3\sqrt{x^2+4}+3x}{3\sqrt{x^2+4}}\right)$$

$$=\left(\frac{-3x^2+3\left(\sqrt{x^2+4}\right)^2}{3\sqrt{x^2+4}}\right)\frac{1}{x^2}+\left(\frac{1}{x+\sqrt{x^2+4}}\right)\left(\frac{3(\sqrt{x^2+4}+x)}{3\sqrt{x^2+4}}\right)$$

$$=\left(\frac{-3x^2+3(x^2+4)}{3\sqrt{x^2+4}}\right)\frac{1}{x^2}+\frac{3}{3\sqrt{x^2+4}}$$

$$=\frac{-3x^2+3x^2+12}{3\sqrt{x^2+4}}\frac{1}{x^2}+\frac{3}{3\sqrt{x^2+4}}$$

$$=\frac{12}{3\sqrt{x^2+4}}\frac{1}{x^2}+\frac{3}{3\sqrt{x^2+4}}$$

$$=\frac{12+3x^2}{3x^2\sqrt{x^2+4}}$$

$$=\frac{3(x^2+4)}{3x^2\left(x^2+4\right)^{1/2}}$$

$$=\frac{\sqrt{x^2+4}}{x^2}$$

Rationalization Techniques

1. If the denominator is $\sqrt{a}$, multiply by $\frac{\sqrt{a}}{\sqrt{a}}$

2. If the denominator is $\sqrt{a}-\sqrt{b}$, multiply by $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

3. If the denominator is $\sqrt{a}+\sqrt{b}$, multiply by $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

$$\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)=a-b$$

Example

Simplify by rationalizing the denominator

a) $\frac{4}{\sqrt{3}}=\frac{4}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{4\sqrt{3}}{3}$

b) $\frac{2}{\sqrt[3]{x}}=\frac{2}{\sqrt[3]{x}}\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

$=\frac{2\sqrt[3]{x^2}}{x}$

c) $\frac{1}{1-\sqrt{2}}=\frac{1}{1-\sqrt{2}}\frac{1+\sqrt{2}}{1+\sqrt{2}}$

$=\frac{1+\sqrt{2}}{1-2}$

$=\frac{1+\sqrt{2}}{-1}$

$=-1-\sqrt{2}$

Example

Simplify $\sqrt{27}\sqrt{3}$

$$\begin{aligned}\sqrt{27}\sqrt{3} &= \sqrt{27(3)}\\ &= \sqrt{81}\\ &= 9\end{aligned}$$

Example

Simplify $\sqrt[4]{x^8y^7z^{11}}$

$$\sqrt[4]{x^8y^7z^{11}} = x^2yz^2\ \sqrt[4]{y^3z^3}$$

Example

Simplify $\frac{5}{\sqrt{10}}$

$$\begin{aligned}\frac{5}{\sqrt{10}} &= \frac{5}{\sqrt{10}}\frac{\sqrt{10}}{\sqrt{10}}\\ &= \frac{5\sqrt{10}}{10}\\ &= \frac{\sqrt{10}}{2}\end{aligned}$$

Example

Simplify $\frac{5}{2-\sqrt{6}}$

$$\begin{aligned}\frac{5}{2-\sqrt{6}} &= \frac{5}{2-\sqrt{6}}\frac{2+\sqrt{6}}{2+\sqrt{6}}\\ &= \frac{5\left(2+\sqrt{6}\right)}{4-6}\\ &= -\frac{5}{2}\left(2+\sqrt{6}\right)\end{aligned}$$

Example

Simplify $\frac{1}{\sqrt{r}-\sqrt{3}}$

$$\begin{aligned}\frac{1}{\sqrt{r}-\sqrt{3}} &= \frac{1}{\sqrt{r}-\sqrt{3}}\frac{\sqrt{r}+\sqrt{3}}{\sqrt{r}+\sqrt{3}}\\ &= \frac{\sqrt{r}+\sqrt{3}}{r-3}\end{aligned}$$

Example

Rationalize the denominator or numerator

a) $\dfrac{5}{\sqrt{8}}$

$$= \frac{5}{\sqrt{8}}\frac{\sqrt{8}}{\sqrt{8}}$$

$$= \frac{5\sqrt{8}}{8}$$

b) $\dfrac{1}{\sqrt{6}-\sqrt{3}}$

$$= \frac{1}{\sqrt{6}-\sqrt{3}}\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}$$

$$= \frac{\sqrt{6}+\sqrt{3}}{\left(\sqrt{6}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{6}+\sqrt{3}}{6-3} = \frac{\sqrt{6}+\sqrt{3}}{3}$$

$$\boxed{= \frac{\sqrt{6}+\sqrt{3}}{3}}$$

c) $\dfrac{1}{\sqrt{x}+\sqrt{x+2}}$

$$= \frac{1}{\sqrt{x}+\sqrt{x+2}}\frac{\sqrt{x}-\sqrt{x+2}}{\sqrt{x}-\sqrt{x+2}}$$

$$= \frac{\sqrt{x}-\sqrt{x+2}}{x-(x+2)}$$

$$= \frac{\sqrt{x}-\sqrt{x+2}}{x-x-2}$$

$$= \frac{\sqrt{x}-\sqrt{x+2}}{-2}$$

$$\boxed{= \frac{\sqrt{x+2}-\sqrt{x}}{2}}$$

Example

$$\frac{2}{x^2-4} - \frac{1}{x-2} = \frac{2-(x+2)}{(x-2)(x+2)}$$

$$= \frac{2-x-2}{(x-2)(x+2)}$$

$$\boxed{= -\frac{x}{(x-2)(x+2)}}$$

Example

$$-\frac{\sqrt{x^2+1}}{x^2}-\frac{1}{\sqrt{x^2+1}}=\frac{-\sqrt{x^2+1}\sqrt{x^2+1}-x^2}{x^2\sqrt{x^2+1}}$$

$$-\frac{\sqrt{x^2+1}}{x^2}\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}\frac{x^2}{x^2}$$

$$=\frac{-\left(x^2+1\right)-x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{-x^2-1-x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{-2x^2-1}{x^2\sqrt{x^2+1}}$$

$$\boxed{=-\frac{2x^2+1}{x^2\sqrt{x^2+1}}}$$

Example

$$\left(\sqrt{x^2+1}-\frac{3x^3}{2\sqrt{x^2+1}}\right)\div\left(x^3+1\right)=\left(\frac{\sqrt{x^2+1}\left(2\sqrt{x^2+1}\right)-3x^3}{2\sqrt{x^2+1}}\right)\cdot\frac{1}{x^3+1}$$

$$=\frac{2\left(x^2+1\right)-3x^3}{2\left(x^3+1\right)\sqrt{x^2+1}}$$

$$\boxed{=\frac{-3x^3+2x^2+2}{2\left(x^3+1\right)\sqrt{x^2+1}}}$$

Exercise

Perform each indicated operation & simplify $\frac{A}{x+1}-\frac{B}{x-1}+\frac{C}{x+2}$

Solution

$$\frac{A}{x+1}-\frac{B}{x-1}+\frac{C}{x+2}=\frac{A(x-1)(x+2)-B(x+1)(x+2)+C(x+1)(x-1)}{(x+1)(x-1)(x+2)}$$

$$=\frac{A(x^2+2x-x-2)-B(x^2+2x+x+2)+C(x^2-1)}{(x+1)(x-1)(x+2)}$$

$$=\frac{Ax^2+Ax-2A-Bx^2-3Bx-2B+Cx^2-C}{(x+1)(x-1)(x+2)}$$

$$\boxed{=\frac{(A-B-C)x^2+(A-3B)x-2A-2B-C}{(x+1)(x-1)(x+2)}}$$

Exercise

Perform the operation and simplify: $-\frac{\sqrt{x^2+1}}{x^2}-\frac{1}{\sqrt{x^2+1}}$

Solution

$$-\frac{\sqrt{x^2+1}}{x^2}-\frac{1}{\sqrt{x^2+1}}=\frac{-\sqrt{x^2+1}\sqrt{x^2+1}-x^2}{x^2\sqrt{x^2+1}}$$

$$-\frac{\sqrt{x^2+1}}{x^2}\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}\frac{x^2}{x^2}$$

$$=\frac{-\left(x^2+1\right)-x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{-x^2-1-x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{-2x^2-1}{x^2\sqrt{x^2+1}}$$

$$\boxed{=-\frac{2x^2+1}{x^2\sqrt{x^2+1}}}$$

Exercise

Perform the operation and simplify: $\left(\sqrt{x^2+1}-\dfrac{3x^3}{2\sqrt{x^2+1}}\right)\div\left(x^3+1\right)$

Solution

$$\left(\sqrt{x^2+1}-\frac{3x^3}{2\sqrt{x^2+1}}\right)\div\left(x^3+1\right)=\left(\frac{\sqrt{x^2+1}\left(2\sqrt{x^2+1}\right)-3x^3}{2\sqrt{x^2+1}}\right)\cdot\frac{1}{x^3+1}$$

$$=\frac{2\left(x^2+1\right)-3x^3}{2\left(x^3+1\right)\sqrt{x^2+1}}$$

$$=\boxed{\frac{-3x^3+2x^2+2}{2\left(x^3+1\right)\sqrt{x^2+1}}}$$

Exercise

Perform the operation and simplify: $\dfrac{6}{x(3x-2)}+\dfrac{5}{3x-2}-\dfrac{2}{x^2}$

Solution

$$\frac{6}{x(3x-2)}+\frac{5}{3x-2}-\frac{2}{x^2}=\frac{6}{x(3x-2)}\frac{x}{x}+\frac{5}{3x-2}\frac{x^2}{x^2}-\frac{2}{x^2}\frac{3x-2}{3x-2}$$

$x \quad 3x-2 \quad x^2$

$$=\frac{6x+5x^2-2(3x-2)}{x^2(3x-2)}$$

$$=\frac{6x+5x^2-6x+4}{x^2(3x-2)}$$

$$=\boxed{\frac{5x^2+4}{x^2(3x-2)}}$$

Exercise

Simplify the fraction: $\dfrac{\frac{2}{x+3}-\frac{2}{a+3}}{x-a}$

Solution

$$\frac{\frac{2}{x+3}-\frac{2}{a+3}}{x-a}=\frac{\frac{2(a+3)-2(x+3)}{(x+3)(a+3)}}{x-a}$$

$$=\frac{2a+6-2x-6}{(x+3)(a+3)}\cdot\frac{1}{x-a}$$

$$=\frac{2a-2x}{(x+3)(a+3)(x-a)}$$

$$=\frac{2(a-x)}{(x+3)(a+3)(x-a)}$$

$$=\frac{-2(x-a)}{(x+3)(a+3)(x-a)} \qquad \text{if } x\neq a$$

$$\boxed{=-\frac{2}{(x+3)(a+3)}}$$

Exercise

Simplify: $\dfrac{3x^2(2x+5)^{1/2}-x^3\left(\frac{1}{2}\right)(2x+5)^{-1/2}(2)}{\left[(2x+5)^{1/2}\right]^2}$

Solution

$$\frac{3x^2(2x+5)^{1/2}-x^3\left(\frac{1}{2}\right)(2x+5)^{-1/2}(2)}{\left[(2x+5)^{1/2}\right]^2}=\frac{3x^2(2x+5)^{1/2}-x^3(2x+5)^{-1/2}}{(2x+5)}$$

$$=\frac{3x^2(2x+5)^{1/2}-x^3(2x+5)^{-1/2}}{(2x+5)}\bullet\frac{(2x+5)^{1/2}}{(2x+5)^{1/2}}$$

$$=\frac{3x^2(2x+5)-x^3}{(2x+5)^{3/2}}$$

$$=\frac{6x^3+15x^2-x^3}{(2x+5)^{3/2}}$$

$$=\frac{5x^3+15x^2}{(2x+5)^{3/2}}$$

$$=\boxed{\frac{5x^2(x+3)}{(2x+5)^{3/2}}}$$

Exercise

Simplify the expression: $\dfrac{\left(4x^2+9\right)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left[\left(4x^2+9\right)^{1/2}\right]^2}$

Solution

$$\frac{\left(4x^2+9\right)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left[\left(4x^2+9\right)^{1/2}\right]^2}=\frac{2\left(4x^2+9\right)^{1/2}-4x(2x+3)\left(4x^2+9\right)^{-1/2}}{4x^2+9}$$

$$=\frac{2\left(4x^2+9\right)^{1/2}-4x(2x+3)\left(4x^2+9\right)^{-1/2}}{4x^2+9}\cdot\frac{\left(4x^2+9\right)^{1/2}}{\left(4x^2+9\right)^{1/2}}$$

$$=\frac{2\left(4x^2+9\right)-4x(2x+3)}{\left(4x^2+9\right)^{3/2}}$$

$$=\frac{8x^2+18-8x^2-12x}{\left(4x^2+9\right)^{3/2}}$$

$$=\frac{18-12x}{\left(4x^2+9\right)^{3/2}}$$

$$=\boxed{\frac{6(3-2x)}{\left(4x^2+9\right)^{3/2}}}$$

Exercise

Simplify the expression: $\dfrac{\left(1-x^2\right)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)}{\left[\left(1-x^2\right)^{1/2}\right]^2}$

Solution

$$\frac{\left(1-x^2\right)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)}{\left[\left(1-x^2\right)^{1/2}\right]^2}=\frac{2x\left(1-x^2\right)^{1/2}+x^3\left(1-x^2\right)^{-1/2}}{1-x^2}\,\frac{\left(1-x^2\right)^{1/2}}{\left(1-x^2\right)^{1/2}}$$

$$=\frac{2x\left(1-x^2\right)+x^3}{\left(1-x^2\right)^{3/2}}$$

$$=\frac{2x-2x^3+x^3}{\left(1-x^2\right)^{3/2}}$$

$$=\frac{2x-x^3}{\left(1-x^2\right)^{3/2}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2+4\right)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)\left(x^2+4\right)^{-2/3}(2x)}{\left[\left(x^2+4\right)^{1/3}\right]^2}$

Solution

$$\frac{\left(x^2+4\right)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)\left(x^2+4\right)^{-2/3}(2x)}{\left[\left(x^2+4\right)^{1/3}\right]^2}=\frac{3\left(x^2+4\right)^{1/3}-6x^2\left(x^2+4\right)^{-2/3}}{\left(x^2+4\right)^{2/3}}\,\frac{\left(x^2+4\right)^{2/3}}{\left(x^2+4\right)^{2/3}}$$

$$=\frac{3\left(x^2+4\right)-6x^2}{\left(x^2+4\right)^{4/3}}$$

$$=\frac{3x^2+12-6x^2}{\left(x^2+4\right)^{4/3}}$$

$$=\boxed{\frac{-3x^2+12}{\left(x^2+4\right)^{4/3}}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2-5\right)^4(3x^2)-x^3(4)\left(x^2-5\right)^3(2x)}{\left[\left(x^2-5\right)^4\right]^2}$

Solution

$$\frac{\left(x^2-5\right)^4(3x^2)-x^3(4)\left(x^2-5\right)^3(2x)}{\left[\left(x^2-5\right)^4\right]^2}=\frac{\left(x^2-5\right)^3\left[3x^2\left(x^2-5\right)-8x^4\right]}{\left(x^2-5\right)^8}$$

$$=\frac{\left(x^2-5\right)^3\left[3x^4-15x^2-8x^4\right]}{\left(x^2-5\right)^8}$$

$$=\frac{\left(-5x^4-15x^2\right)}{\left(x^2-5\right)^5}$$

$$=\boxed{\frac{-5x^2\left(x^2+3\right)}{\left(x^2-5\right)^5}}$$

Exercise

Simplify the expression: $\dfrac{(3x+2)^{1/2}\left(\frac{1}{3}\right)(2x+3)^{-2/3}(2)-(2x+3)^{1/3}\left(\frac{1}{2}\right)(3x+2)^{-1/2}(3)}{\left[(3x+2)^{1/2}\right]^2}$

Solution

$$=\frac{\frac{2}{3}(3x+2)^{1/2}(2x+3)^{-2/3}-\frac{3}{2}(2x+3)^{1/3}(3x+2)^{-1/2}}{3x+2}\,\frac{6(2x+3)^{2/3}}{6(2x+3)^{2/3}}\,\frac{(3x+2)^{1/2}}{(3x+2)^{1/2}}$$

$$=\frac{4(3x+2)-9(2x+3)}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$= \frac{4(3x+2)-9(2x+3)}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$= \frac{12x+8-18x-27}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$\boxed{= \frac{-6x-19}{6(3x+2)^{3/2}(2x+3)^{2/3}}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2+2\right)^3(2x)-x^2(3)\left(x^2+2\right)^2(2x)}{\left[\left(x^2+2\right)^3\right]^2}$

Solution

$$\frac{\left(x^2+2\right)^3(2x)-x^2(3)\left(x^2+2\right)^2(2x)}{\left[\left(x^2+2\right)^3\right]^2} = \frac{2x\left(x^2+2\right)^2\left[\left(x^2+2\right)-3x^2\right]}{\left(x^2+2\right)^6}$$

$$= \frac{2x\left[x^2+2-3x^2\right]}{\left(x^2+2\right)^4}$$

$$= \frac{2x\left[-2x^2+2\right]}{\left(x^2+2\right)^4}$$

$$\boxed{= \frac{4x\left[-x^2+1\right]}{\left(x^2+2\right)^4}}$$

Section 1.4 – Equations and Application

Linear Equations

A ***linear equation*** in one variable is an equation that is equivalent to one of the form $mx+b=0$

Equation-Solving Principles

<u>Addition Principle</u>: If $a = b$ is true $\Rightarrow a+c=b+c$

<u>Multiplication Principle</u>: If $a = b$ is true $\Rightarrow ac=bc$

Solve the following equations

a) $x-2=3$

$$x-2+2=3+2$$
$$x=5$$

b) $\frac{x}{2}=3$

$$2\frac{x}{2}=(2)3$$
$$x=6$$

Solve: $2x-5+8=3x+2(2-3x)$

$$2x-5+8=3x+4-6x$$
$$2x+3=4-3x$$
$$2x+3-3+3x=4-3x-3+3x$$
$$5x=1$$

Divide both sides by 5

$$x=\frac{1}{5}$$

The Zero-Product Principle:

If $ab = 0$. then $a = 0$ or $b = 0$.

Solve $6x^2+7x=3$

$$6x^2+7x-3=0$$
$$(3x-1)(2x+3)=0$$

$$3x-1=0 \qquad 2x+3=0$$
$$3x=1 \qquad 2x=-3$$
$$x=\frac{1}{3} \qquad x=-\frac{3}{2}$$

Quadratic Formula

$$ax^2+bx+c=0 \quad \Rightarrow \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Solve $x^2-4x-5=0 \qquad \Rightarrow \ a=1, \ \ b=-4, \ \ c=-5$

$$\begin{aligned}
x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\
&=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-5)}}{2(1)}\\
&=\frac{4\pm\sqrt{16+20}}{2}\\
&=\frac{4\pm\sqrt{36}}{2}\\
&=\frac{4\pm 6}{2}
\end{aligned}$$

$$\begin{aligned}
x&=\frac{4+6}{2} & \qquad x&=\frac{4-6}{2}\\
&=\frac{10}{2} & &=\frac{-2}{2}\\
&=5 & &=-1
\end{aligned}$$

Solve $x^2+1=4x$

$$x^2-4x+1=0$$

$$\begin{aligned}
x&=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}\\
&=\frac{4\pm\sqrt{16-4}}{2}\\
&=\frac{4\pm\sqrt{12}}{2}\\
&=\frac{4\pm 2\sqrt{3}}{2}\\
&=\frac{2\left(2\pm\sqrt{3}\right)}{2}\\
&=2\pm\sqrt{3}
\end{aligned}$$

Equations with Fractions

Solve $\frac{r}{10}-\frac{2}{15}=\frac{3r}{20}-\frac{1}{5}$

10	2 5
15	3 5
20	2 2 5
5	5
	2 2 3 5 = 60

$(60)\frac{r}{10}-(60)\frac{2}{15}=(60)\frac{3r}{20}-(60)\frac{1}{5}$

$6r-8=9r-12$

$6r-8+8-9r=9r-12+8-9r$

$-3r=-4$

$r=\frac{-4}{-3}=\frac{4}{3}$

Solve $\frac{2}{x-3}+\frac{1}{x}=\frac{6}{x(x-3)}$ $\qquad x-3\neq 0$

Conditions: $x\neq 0,\ 3$

$x(x-3)\frac{2}{x-3}+x(x-3)\frac{1}{x}=x(x-3)\frac{6}{x(x-3)}$

$2x+x-3=6$

$3x=9$

$x=3$

Solve $\frac{1}{x-2}-\frac{3x}{x-1}=\frac{2x+1}{x^2-3x+2}$ $\qquad cond.\ x\neq 1,\ 2$

$(x-2)(x-1)\frac{1}{x-2}-(x-2)(x-1)\frac{3x}{x-1}=(x-2)(x-1)\frac{2x+1}{x^2-3x+2}$

$x-1-3x(x-2)=2x+1$

$x-1-3x^2+6x-2x-1=0$

$-3x^2+5x-2=0$

$3x^2-5x+2=0$

$(x-1)(3x-2)=0$

$x-1=0 \qquad 3x-2=0$

$x=1 \qquad x=\frac{2}{3}$

Solution: $x=\frac{2}{3}$

Slopes and Equations of Lines

Slope of a line *(Definition)*

The slope of a line is defined as the vertical change (the *rise*) over the horizontal change (the *run*) as one travels along the line.

$$slope: \quad m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of the line through each pair point

a) $(7, 6)$ *and* $(-4, 5)$

$m = \dfrac{5-6}{-4-7}$

$= \dfrac{-1}{-11}$

$= \dfrac{1}{11}$

b) $(5, -3)$ *and* $(-2, -3)$

$m = \dfrac{-3+3}{-2-5}$

$= \dfrac{0}{-7}$

$= 0$

c) $(2, -4)$ *and* $(2, 3)$

$m = \dfrac{3+4}{2-2}$

$= \dfrac{7}{0}$ Which is undefined $\rightarrow$ line is vertical.

Equations of a Line

$$y = mx + b$$

This *linear equation* is called the *slope-intercept form* of the equation of a line.

Point-Slope Form

$$y - y_1 = m\left(x - x_1\right)$$

Example

Find the equation of the line through $(0,\ -3)$ with slope $\frac{3}{4}$

Solution

$$y - y_1 = m\left(x - x_1\right)$$

$$y + 3 = \frac{3}{4}(x - 0)$$

$$y + 3 = \frac{3}{4}x$$

$$y = \frac{3}{4}x - 3$$

$$(4)y = (4)\frac{3}{4}x - (4)3$$

$$4y = 3x - 12$$

$$4y - 3x = -12$$

$$3x - 4y = 12$$

Example

Find the equation of the line that passes through the point $(3,\ -7)$ and has slope $\frac{5}{4}$

Solution

$$y - y_1 = m\left(x - x_1\right)$$

$$y + 7 = \frac{5}{4}(x - 3)$$

$$y + 7 = \frac{5}{4}x - \frac{15}{4}$$

$$y + 7 - 7 = \frac{5}{4}x - \frac{15}{4} - 7 \qquad -\frac{15}{4} - 7 = -\frac{15}{4} - 7\frac{4}{4} = -\frac{15}{4} - \frac{28}{4} = -\frac{43}{4}$$

$$y = \frac{5}{4}x - \frac{43}{4}$$

Parallel Lines (//)

Two lines are parallel if and only if they have the same slope, or they are both vertical. $m_1 = m_2$

Example

Find the equation of the line that passes through the point $(3,\ 5)$ and is parallel to the line $2x + 5y = 4$

Solution

$$2x + 5y = 4$$

$$5y = -2x + 4$$

$$y = -\frac{2}{5}x + \frac{4}{5}$$

$$m_1 = m_2$$

$$Slope: m = -\frac{2}{5}$$

$$y - y_1 = m\left(x - x_1\right)$$

$$y - 5 = -\frac{2}{5}(x - 3)$$

$$y - 5 = -\frac{2}{5}x + \frac{6}{5}$$

$$y - 5 + 5 = -\frac{2}{5}x + \frac{6}{5} + 5$$

$$y = -\frac{2}{5}x + \frac{31}{5}$$

Perpendicular Lines (⊥)

Two lines are perpendicular if and only if the product of their slope is -1. $m_1 \cdot m_2 = -1$

Example

Find the slope of the line L perpendicular to the line having the equation $5x - y = 4$

Solution

$$5x - y = 4$$

$$5x - 4 = y \rightarrow \quad \text{Slope} = 5$$

Slope of the line L = $-\frac{1}{5}$

Linear Functions and Applications

Linear Function

A relationship f defined by

$$y = f(x) = mx + b$$

For real numbers m and b, is a ***linear function***

Example

Let $g(x) = -4x + 5$***. Find*** $g(3)$***,*** $g(0)$***,*** $g(-2)$***, and*** $g(b)$

Solution

$$g(x) = -4x + 5$$
$$g(\text{—}) = -4(\text{—}) + 5$$

$$\begin{aligned} g(3) &= -4(3) + 5 \\ &= -7 \end{aligned}$$

$$\begin{aligned} g(0) &= -4(0) + 5 \\ &= 5 \end{aligned}$$

$$\begin{aligned} g(-2) &= -4(-2) + 5 \\ &= 13 \end{aligned}$$

$$g(b) = -4b + 5$$