Solution *Section* 3.6 – Hypothesis Tests for a Population Proportion

Exercise

In a Harris poll, adults were asked if they are in favor of abolishing the penny. Among the responses, 1261 answered "no", and 491 answered "yes", and 384 had no opinion. What is the sample proportion of *yes* responses, and what notation is used to represent it?

Solution

Total Responses: $1261+491+384 \ \underline{=2136}$

The sample proportion of yes responses is $\hat{p}=\frac{x}{n}=\frac{491}{2136} \ \underline{=0.230}$

$\hat{p}$: is used to represent a sample proportion.

Exercise

A recent study showed that 53% of college applications were submitted online. Assume that this result is based on a simple random sample of 1000 college applications, with 530 submitted online. Use a 0.01 significance level to test the claim that among all college applications the percentage submitted online is equal to 50%

a) What is the test statistic?
b) What are the critical values?
c) What is the *P*-Value?
d) What is the conclusion?
e) Can a hypothesis test be used to "prove" that the percentage of college applications submitted online is equal to 50% as claimed?

Solution

a) $\hat{p}=\frac{x}{n}=\frac{530}{1000} \ \underline{=0.530}$

$$z_{\hat{p}}=\frac{\hat{p}-\mu_{\hat{p}}}{\sigma_{\hat{p}}}=\frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$$

$$=\frac{0.53-0.5}{\sqrt{\frac{(.5)(.5)}{1000}}}$$

$$\underline{=1.90}$$

z score	Area
1.645	0.9500
$\boxed{2.575}$	$\boxed{0.9950}$

b) $\frac{\alpha}{2}=\frac{0.01}{2}=0.005 \quad \Rightarrow \quad A=1-0.005=0.995$

$$z=\pm z_{\alpha/2}=\pm z_{0.01/2}=\pm z_{0.005} \ \underline{=\pm 2.575}$$

c) $P-value=2\cdot P(z>1.90)$

$$=2\cdot(1-0.9713)$$

z	.00	.01

$= 0.0574$

1.9	.9713	.9719

d) Do not reject H_0; there is not sufficient evidence to reject the claim that the percentage of all college applications that are submitted online is 50%.

e) No. A hypothesis test will either "reject" or "fail to reject" a claim that a population parameter is equal to a specified value.

Exercise

In a survey, 1864 out of 2246 randomly selected adults in the U.S. said that texting while driving should be illegal. Consider a hypothesis test that uses a 0.05 significance level to test the claim that more than 80% of adults believe that testing while driving should be illegal

a) What is the test statistic?

b) What are the critical values?

c) What is the *P*-Value?

d) What is the conclusion?

Solution

a) $\hat{p} = \frac{x}{n} = \frac{1864}{2246} = 0.830$

$$z_{\hat{p}} = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.83 - 0.80}{\sqrt{\frac{(0.8)(0.2)}{2246}}}$$

$$= 3.54$$

b) $\alpha = 0.05 \quad \Rightarrow \quad A = 1 - 0.05 = 0.95$

$z = z_{\alpha} = z_{0.05} = 1.645$

z score	Area
1.645	0.9500
2.575	0.9950

c) $P - value = P(z > 3.54)$

$= 1 - 0.9999$

$= 0.0001$

3.50 and up	.9999

d) Reject H_0; there is sufficient evidence to conclude that the proportion of adults who believe that texting while driving should be illegal is greater than 80%.

Exercise

In a Pew Research Center poll of 745 randomly selected adults, 589 said that it is morally wrong to not report all income on tax returns. Use a 0.01 significance level to test the claim that 75% of adults say that it is morally wrong to not report all income on tax returns.
Identify the null hypothesis, alternative hypothesis, test statistic, *P*-value or critical value(s), conclusion about the null hypothesis, and final conclusion that address the original claim.

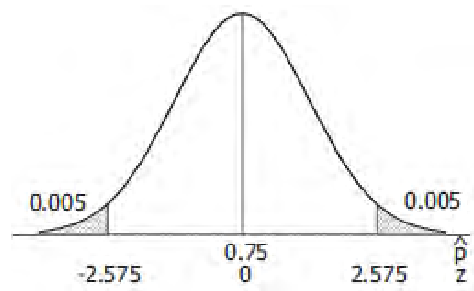
Solution

Original claim: $p = 0.75$

$\hat{p} = \frac{x}{n} = \frac{589}{745} \underline{= 0.791}$

$H_0 : p = 0.75$

$H_1 : p \neq 0.75$



Assume: $\alpha = 0.01$

$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005 \quad \Rightarrow \quad A = 1 - 0.005 = 0.995$

Critical value: $z = \pm z_{\alpha/2} = \pm z_{0.005} = \pm 2.575$

z score	Area
1.645	0.9500
2.575	0.9950

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.791 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{745}}}$$

$$\underline{= 2.56}$$

$P - value = 2 \cdot P(z > 2.56)$

$= 2 \cdot (1 - 0.9948)$

$\underline{= 0.0104}$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952

Conclusion

Reject H_0; there is sufficient evidence to conclude that the proportion of adults who believe that texting while driving should be illegal is greater than 80%.

Exercise

308 out of 611 voters surveyed said that they voted for the candidate who won. Use a 0.01 significance level to test the claim that among all voters, the percentage who believe that they voted for the winning candidate is equal to 43%, which is the actual percentage of votes for the winning candidate. What does the result suggest about voter perceptions?
Identify the null hypothesis, alternative hypothesis, test statistic, *P*-value or critical value(s), conclusion about the null hypothesis, and final conclusion that address the original claim.

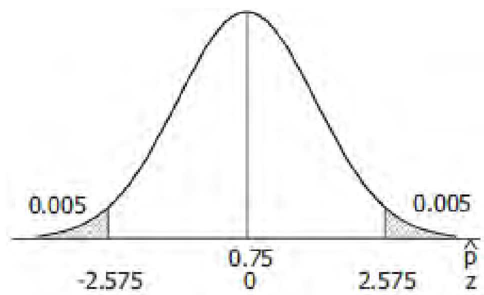
Solution

Original claim: $p = 0.43$

$$\hat{p} = \frac{x}{n} = \frac{308}{611} \underline{= 0.504|}$$

$$H_0 : p = 0.43$$

$$H_1 : p \neq 0.43$$



Assume: $\alpha = 0.01$

$$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005 \quad \Rightarrow \quad A = 1 - 0.005 = 0.995$$

Critical value: $z = \pm z_{\alpha/2} = \pm z_{0.005} = \pm 2.575$

z score	Area
1.645	0.9500
2.575	0.9950

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.504 - 0.43}{\sqrt{\frac{(0.43)(0.57)}{611}}}$$

$$\underline{= 3.70|}$$

$$P - value = 2 \cdot P(z > 3.70)$$

$$= 2 \cdot (1 - 0.9999)$$

$$\underline{= 0.0002|}$$

3.50 and up	.9999

Conclusion

Reject H_0; there is sufficient evidence to reject the claim that $p = 0.43$ and conclude that $p \neq 0.43$ (in fact, that $p > 0.43$). There is sufficient evidence to reject the claim that the proportion of adults who believe they voted for the winning candidate is 43%. Either the voters are deliberately not telling the truth, or they have faulty memories about how they actually voted.

Exercise

The company Drug Test Success provides a "1-Panel-THC" test for marijuana usage. Among 300 tested subjects, results from 27 subjects were wrong (either a false positive or a false negative). Use a 0.05 significance level to test the claim that less than 10% of the test results are wrong. Does the test appear to be good for most purposes?

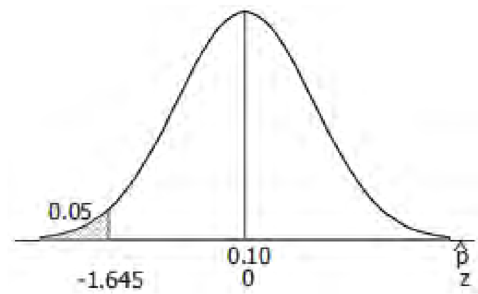
Solution

Original claim: $p < 0.10$

$\hat{p} = \frac{x}{n} = \frac{27}{300} = 0.090$

$H_0 : p = 0.10$

$H_1 : p < 0.10$



$\alpha = 0.05$

Critical Value: $z = -z_\alpha = -z_{0.05} = -1.645$

z score	Area
1.645	0.9500
2.575	0.9950

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.09 - 0.1}{\sqrt{\frac{(0.1)(0.9)}{300}}}$$

$$= -0.58$$

$P - value = P(z < -0.58)$

$= 0.2810$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776

Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $p < 0.10$. There is not sufficient evidence to support the claim that the proportion of test results that are incorrect is less than 10%.

No; the test appears to have too high of an error rate to be considered reliable for most purposes.

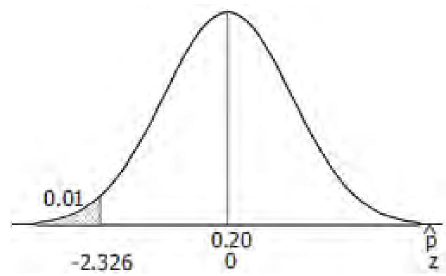
Exercise

When testing gas pumps in Michigan for accuracy, fuel-quality enforcement specialists tested pumps and found that 1299 of them were not pumping accurately (within 3.3 oz. when 5 gal. is pumped), and 5686 pumps were accurate. Use a 0.01 significance level to test the claim of an industry representative that less than 20% of Michigan gas pumps are inaccurate. From the perspective of the consumer, does that rate appear to be low enough?

Solution

Original claim: $p < 0.20$

$$\hat{p} = \frac{x}{n} = \frac{1299}{1299 + 5686} = 0.186$$



$H_0 : p = 0.20$

$H_1 : p < 0.20$

$\alpha = 0.01$

Critical Value:

$z = -z_{\alpha} = -z_{0.01}$

$\underline{= -2.326}$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.186 - 0.2}{\sqrt{\frac{(0.2)(0.8)}{6985}}}$$

$\underline{= -2.93}$

$P - value = P(z < -2.93)$

$\underline{= 0.0017}$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014

Conclusion

Reject H_0; there is sufficient evidence to conclude that $p < 0.20$. There is sufficient evidence to support the claim that less than 20% of Michigan gas pumps are inaccurate.

No; from the perspective of the consumer, the rate does not appear to be low enough. While the point estimate of 0.186 indicates the rate is lower than 20%. It should probably be about $\frac{1}{10}$ of that.

Exercise

Trials in an experiment with a polygraph include 98 results that include 24 cases of wrong results and 74 cases of correct results. Use a 0.05 significance level to test the claim that such polygraph results are correct less than 80% of the time. Based on the results, should polygraph test results be prohibited as evidence in trials?

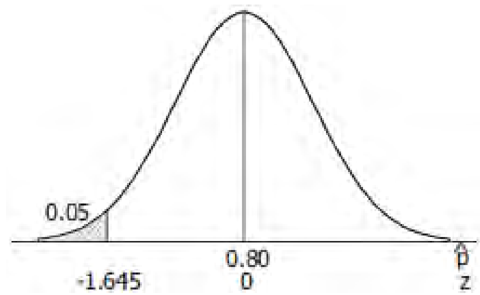
Solution

Original claim: $p < 0.80$

$$\hat{p} = \frac{x}{n} = \frac{74}{98} = 0.755$$

$$H_0 : p = 0.80$$

$$H_1 : p < 0.80$$



$\alpha = 0.05$

z score	Area
1.645	0.9500
2.575	0.9950

Critical Value: $z = -z_{\alpha} = -z_{0.05} = -1.645$

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.755 - 0.8}{\sqrt{\frac{(0.8)(0.2)}{98}}}$$

$$= -1.11$$

$$P - value = P(z < -1.11)$$

$$= 0.1335$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.1	.1357	$\boxed{.1335}$	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170

Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $p < 0.80$. There is not sufficient evidence to support the claim polygraph tests are correct less than 80% of the time.

Yes; based on these results, polygraph test results should probably be prohibited as evidence in trials. Even though the point estimate of 75.5% accuracy does not support the less than 80% claim, the accuracy rate is still far too small to make conclusions beyond a reasonable doubt.

Exercise

In recent years, the Town of Newport experienced an arrest rate of 25% for robberies. The new sheriff compiles records showing that among 30 recent robberies, the arrest rate is 30%, so she claims that her arrest rate is greater than the 25% rate in the past. Is there sufficient evidence to support her claim that the arrest rate is greater than 25%?

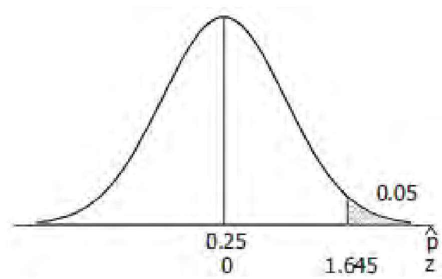
Solution

Original claim: $p > 0.25$

$$\hat{p} = \frac{x}{n} = \frac{(0.3)(30)}{30} = 0.300$$

$$H_0 : p = 0.25$$

$$H_1 : p > 0.25$$



Assumed: $\alpha = 0.05$

Critical Value: $z = z_\alpha = z_{0.05} = 1.645$

z score	Area
1.645	0.9500
2.575	0.9950

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.3 - 0.25}{\sqrt{\frac{(0.25)(0.75)}{30}}}$$

$$= 0.63$$

$$P - value = P(z > 0.63)$$

$$= 1 - .7357$$

$$= 0.2643$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549

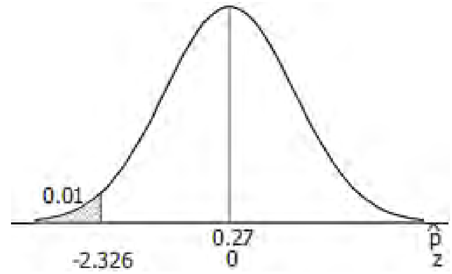
Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $p > 0.30$. There is not sufficient evidence to support the new sheriff's claim that the new arrest rate is greater than 25%.

Exercise

A survey showed that among 785 randomly selected subjects who completed 4 years of college, 18.3 % smoke and 81.7% do not smoke. Use a 0.01 significance level to test the claim that the rate of smoking among those with 4 years of college is less than the 27% rate for the general population. Why would college graduates smoke at a lower rate than others?

Solution



Original claim: $p < 0.27$

$$\hat{p} = \frac{x}{n} = \frac{(0.183)(785)}{785} = 0.183$$

$H_0 : p = 0.27$

$H_1 : p < 0.27$

$\alpha = 0.01$

Critical Value:

$$z = -z_\alpha = -z_{0.01}$$
$$= -2.326$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.183 - 0.27}{\sqrt{\frac{(0.27)(0.73)}{785}}}$$

$$= -5.46$$

−3.50 and lower	.0001

$$P - value = P(z < -5.46) = 0.0001$$

Conclusion

Reject H_0; there is sufficient evidence to conclude that $p < 0.27$. There is sufficient evidence to support the claim that the proportion of smokers among those with four years of college is less than the general rate of 27%. College graduates would smoke at a lower rate others because those with better education tend to make wiser decisions and are more likely to recognize the various disadvantages of smoking.

Exercise

When 3011 adults were surveyed, 73% said that they use the Internet. Is it okay for a newspaper reporter to write that “3/4 of all adults use the internet”? Why or Why not?

Solution

The value for x is not given,

$$(72.5\%)(3011) < x < (73.5\%)(3011)$$

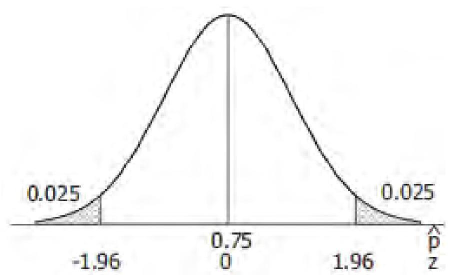
$$2183 < x < 2213$$

Original claim: $p = 0.75$

$$\hat{p} = \frac{x}{n} = \frac{x}{3011} = 0.73$$

$$H_0 : p = 0.75$$

$$H_1 : p \neq 0.75$$



Assumed: $\alpha = 0.05$

Critical Value:

$$z = \pm z_{\alpha/2} = \pm z_{0.025}$$

$$= \pm 1.96$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.73 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{3011}}}$$

$$= -2.53$$

$$P-value = 2 \cdot P(z < -2.53)$$

$$= 2(0.0057)$$

$$= 0.0114$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	* .0049	.0048

Conclusion

Reject H_0; there is sufficient evidence to conclude that $p = 0.75$ and conclude that $p \neq 0.75$ (in fact, that $p < 0.75$). There is sufficient evidence to reject the claim that the proportion of adults who use the internet is $\frac{3}{4}$. While the difference between 0.73 and 0.75 may be of little practical significance, in the interest of accuracy the reporter should not write that $\frac{3}{4}$ of all adults use the internet.

Exercise

A hypothesis test is performed to test the claim that a population proportion is greater than 0.7. Find the probability of a type II error, β, given that the true value of the population proportion is 0.72. The sample size is 50 and the significance level is 0.05.

Solution

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \Rightarrow \hat{p} - p = z\sqrt{\frac{pq}{n}} \Rightarrow \hat{p} = p + z\sqrt{\frac{pq}{n}}$$

The rejection region for the test is any sample proportion greater than:

z score	Area
1.645	0.9500

$n = 50; \quad p = 0.7; \quad q = 0.3; \quad z = 1.645$

$$\begin{aligned}\hat{p} &= p + z\sqrt{\frac{pq}{n}} \\ &= 0.7 + 1.645\sqrt{\frac{0.7(0.3)}{50}} \\ &\approx 0.81\end{aligned}$$

The probability for the true value of 0.72

$$P\left(Z > \frac{0.81 - 0.72}{\sqrt{\frac{0.72(0.28)}{50}}} \right) = P(Z > 1.42) \approx 0.9222$$

Because for a type II error, failure to reject the null when it is false, requires that the true value be in alternative space. Since the in the first set up, the value of 0.72 is in the null space, the null hypothesis is true and β does not exists, because a Type II error is impossible.

Exercise

In a sample of 88 children selected randomly from one town, it is found that 8 of them suffer asthma. Find the *P*-value for a test of the claim that the proportion of all children in the town who suffer from asthma is equal to 11%.

Solution

Original claim: $p = 0.11$ & $\hat{p} = \frac{x}{n} = \frac{8}{88} = 0.091$

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.091 - .11}{\sqrt{\frac{(.11)(0.89)}{88}}} = -0.57$$

$$\begin{aligned}P - value &= 2 \cdot P(z = -0.57) \\ &= 2(.2843) \\ &= 0.5686\end{aligned}$$

Exercise

An airline claims that the no-show rate for passengers booked on its flights is less than 6%. Of 380 randomly selected reservation, 18 were no-shows. Find the P-value for a test of the airline's claim.

Solution

Original claim: $p = 0.06$

$$z_{\hat{p}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{18}{380} - .06}{\sqrt{\frac{(.06)(.94)}{380}}} \underline{= -1.04}$$

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(18/380-.06)/√(.0
              -1.0368
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$$P - value = P(z < -1.04)$$

$$\underline{= 0.1492}$$

Exercise

In 1997, 46% of Americans said they did not trust the media "when it comes to reporting the news fully, accurately and fairly". In a 2007 poll of 1010 adults nationwide, 525 stated they did not trust the media. At the $\alpha = 0.05$ level of significance, is there evidence to support the claim that the percentage of Americans that do not trust the media to report fully and accurately has increased since 1997?

Solution

We want to know if $p > 0.46$. First, we must verify the requirements to perform the hypothesis test:

1. This is a simple random sample.
2. $np_0(1 - p_0) = 1010(0.46)(1 - 0.46) \underline{= 250.9 > 10}$
3. Since the sample size is less than 5% of the population size, the assumption of independence is met.

Step 1: $H_0 : p = 0.46 \quad vs \quad H_1 : p > 0.46$

Step 2: The level of significance is $\alpha = 0.05$.

Step 3: The sample proportion is $\hat{p} = \frac{525}{1010} = 0.52$. The test statistic is then

$$z_0 = \frac{0.52 - 0.46}{\sqrt{\frac{0.46(1 - 0.46)}{1010}}} \underline{= 3.83}$$

Classical Approach

Step 4: Since this is a right-tailed test, we determine the critical value at the $\alpha = 0.05$ level of significance to be $z_{0.05} = 1.645$

Step 5: Since the test statistic $z_0 = 3.83$, is greater than the critical value 1.645, we reject the null hypothesis.

P-Value Approach

Step 4: Since this is a right-tailed test, the *P*–value is the area under the standard normal distribution to the right of the test statistic $z_0 = 3.83$. That is, *P*–value = $P(Z > 3.83) \approx 0$.

Step 5: Since the *P*-value is less than the level of significance, we reject the null hypothesis.

Step 6: There is sufficient evidence at the $\alpha = 0.05$ level of significance to conclude that the percentage of Americans that do not trust the media to report fully and accurately has increased since 1997.

Exercise

In 2006, 10.5% of all live births in the United States were to mothers under 20 years of age. A sociologist claims that births to mothers under 20 years of age is decreasing. She conducts a simple random sample of 34 births and finds that 3 of them were to mothers under 20 years of age. Test the sociologist's claim at the $\alpha = 0.01$ level of significance.

Solution

Step 1: $H_0 : p = 0.105 \quad vs \quad H_1 : p > 0.105$

Step 2: From the null hypothesis, we have $p_0 = 0.105$. There were 34 mothers sampled, so

$$np_0\left(1 - p_0\right) = 32(0.105)(1 - 0.105) \underline{= 3.20 < 10}$$

Thus, the sampling distribution of $\hat{p}$ is not approximately normal.

Step 3: Let *X* represent the number of live births in the United States to mothers under 20 years of age. We have *x* = 3 successes in *n* = 34 trials so $\hat{p} = \frac{3}{34} = 0.088$. If the population mean is truly 0.105. Thus,

$$P - value = P\left(X \le 3 \text{ assuming } p = 0.105\right)$$

$$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$\underline{= 0.51}$$

Step 4: The *P*-value = 0.51 is greater than the level of significance so we do not reject H_0. There is insufficient evidence to conclude that the percentage of live births in the United States to mothers under the age of 20 has decreased below the 2006 level of 10.5%.