

(1)

#1 $y = x^2, y = \sqrt{x} \quad 0 \leq x \leq 1$

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (\sqrt{x} - x^2)^2 = \frac{\pi}{4} (x - 2\sqrt{x}x^2 + x^4)$$

$$V = \int_0^1 A(x) dx = \frac{\pi}{4} \int_0^1 (x - 2x^2 x^{1/2} + x^4) dx$$

$$= \frac{\pi}{4} \left[\frac{1}{2} x^2 - 2 \frac{2}{7} x^{5/2} + \frac{1}{5} x^5 \right]_0^1$$

$$= \frac{\pi}{4} \left[\frac{1}{2} - \frac{4}{7} + \frac{1}{5} \right] = \frac{9\pi}{280}$$

#2 $y = x; y = 2\sqrt{x}$

$$y = \underline{x = 2\sqrt{x}} \Rightarrow x^2 = 4x$$

$$x^2 - 4x = 0 \rightarrow x(x - 4) = 0$$

$$\Rightarrow x = \underset{a}{0}, x = \underset{b}{4}$$

$$\text{Area (equilateral } \Delta) = \frac{1}{2} (\text{side})^2 (\sin \pi/3)$$

$$= \frac{1}{2} (2\sqrt{x} - x)^2 \left(\frac{\sqrt{3}}{2} \right)$$

$$= \frac{\sqrt{3}}{4} (4x - 4x\sqrt{x} + x^2)$$



$$V = \int_0^4 A(x) dx = \int_0^4 \frac{\sqrt{3}}{4} (4x - 4x^{3/2} + x^2) dx$$

$$= \frac{\sqrt{3}}{4} \left[2x^2 - 4 \frac{2}{5} x^{5/2} + \frac{1}{3} x^3 \right]_0^4$$

$$= \frac{\sqrt{3}}{4} \left[2(4)^2 - \frac{8}{5} 4^{5/2} + \frac{1}{3} 4^3 \right]$$

$$= \frac{8\sqrt{3}}{15}$$

(2)

#3 $x^{1/2} + y^{1/2} = \sqrt{6} \Rightarrow 0 \leq x \leq 6$

$$\begin{aligned} A(x) &= (\text{edge})^2 = ((\sqrt{6} - \sqrt{x})^2 - 0)^2 \\ &= (\sqrt{6} - \sqrt{x})^4 \\ &= 36 - 24\sqrt{6}\sqrt{x} + 36x - 4\sqrt{6}x^{3/2} + x^2 \end{aligned}$$

$$\begin{aligned} V &= \int_0^6 (36 - 24\sqrt{6}x^{1/2} + 36x - 4\sqrt{6}x^{3/2} + x^2) dx \\ &= 36x - 24\sqrt{6} \frac{2}{3} x^{3/2} + 18x^2 - 4\sqrt{6} \frac{2}{5} x^{5/2} + \frac{1}{3} x^3 \Big|_0^6 \\ &= 36(6) - 16\sqrt{6}(6)^{3/2} + 18(6)^2 - \frac{8}{5}\sqrt{6}(6)^{5/2} + \frac{1}{3}(6)^3 \\ &= \frac{72}{5} \end{aligned}$$

#4 $y = \frac{4}{x^3}$ $x=1$ $y = \frac{1}{2}$ $\left\{ \frac{4}{x^3} = \frac{1}{2} \Rightarrow x^3 = 8 \right.$ $\boxed{x=2=b}$

a) x-axis \rightarrow washer method.

$$R(x) = \frac{4}{x^3}, \quad r(x) = \frac{1}{2}$$

$$\begin{aligned} V &= \pi \int_1^2 [R^2(x) - r^2(x)] dx = \pi \int_1^2 \left[\left(\frac{4}{x^3} \right)^2 - \left(\frac{1}{2} \right)^2 \right] dx \\ &= \pi \int_1^2 \left(16x^{-6} - \frac{1}{4} \right) dx \\ &= \pi \left[\frac{16x^{-5}}{-5} - \frac{1}{4}x \right]_1^2 = \pi \left[\left(-\frac{16}{5} \cdot 2^{-5} - \frac{1}{4} \right) - \left(-\frac{16}{5} - \frac{1}{4} \right) \right] \\ &= \frac{57\pi}{20} \end{aligned}$$

4 cont: b) shell method

$$\begin{aligned} V &= 2\pi \int_1^2 x \left(\frac{4}{x^3} - \frac{1}{2} \right) dx = 2\pi \int_1^2 \left(4x^{-2} - \frac{1}{2}x \right) dx \\ &= 2\pi \left[\frac{4x^{-1}}{-1} - \frac{1}{4}x^2 \right]_1^2 \\ &= 2\pi \left(-\frac{4}{2} - \frac{1}{4}2^2 \right) - \left(-\frac{4}{1} - \frac{1}{4} \right) \\ &= 2\pi \left(-\frac{5}{2} + \frac{17}{4} \right) \\ &= \frac{3\pi}{2} \end{aligned}$$

c) shell method: $x=2 \Rightarrow$ shell radius $= 2-x$

$$\begin{aligned} V &= 2\pi \int_1^2 (2-x) \left(\frac{4}{x^3} - \frac{1}{2} \right) dx \\ &= 2\pi \int_1^2 \left(\frac{8}{x^3} - 1 - 4x^{-2} + \frac{1}{2}x \right) dx \\ &= 2\pi \left[\frac{8x^{-2}}{-2} - x - \frac{4x^{-1}}{-1} + \frac{1}{4}x^2 \right]_1^2 \\ &= 2\pi \left[-4(2)^{-2} - 2 + \frac{4}{3}2^{-3} + \frac{1}{4}2^2 \right] - \left[-4 - 1 + \frac{4}{3} + \frac{1}{4} \right] \\ &= \frac{3\pi}{2} \end{aligned}$$

d) washer method. $y=4$ $R(x) = 4 - \frac{1}{2} = \frac{8-1}{2} = \frac{7}{2}$
 $r(x) = 4 - \frac{4}{x^2}$

$$\begin{aligned} V &= \pi \int_1^2 \left[\left(\frac{7}{2} \right)^2 - \left(4 - \frac{4}{x^2} \right)^2 \right] dx \\ &= \pi \int_1^2 \left[\frac{49}{4} - \left(16 - 32x^{-2} + 16x^{-4} \right) \right] dx \\ &= \pi \int_1^2 \left(\frac{49}{4} - 16 + 32x^{-2} - 16x^{-4} \right) dx \end{aligned}$$

4-d
cont.

(4)

$$V = \pi \int_1^2 \left(-\frac{15}{4} + 32x^{-2} - 16x^{-4} \right) dx$$

$$= \pi \left[-\frac{15}{4}x + 32 \frac{x^{-1}}{-1} - 16 \frac{x^{-3}}{-3} \right]_1^2$$

$$= \pi \left[-\frac{15}{4}x - 32x^{-1} + \frac{16}{3}x^{-3} \right]_1^2$$

$$= \pi \left[-\frac{15}{4} \cdot 2 - \frac{32}{2} + \frac{16}{3} \cdot 2^{-3} - \left(-\frac{15}{4} - 32 + \frac{16}{3} \right) \right]$$

$$= \frac{103}{20} \pi$$

#5

$$y = \sin x \quad x=0, \pi \text{ and } y=2 \text{ about } y=2$$

$$V = \pi \int_0^\pi (2 - \sin x)^2 dx = \pi \int_0^\pi (4 - 4\sin x + \sin^2 x) dx$$

$$= \pi \left[4x + 4\cos x + \frac{x}{2} - \frac{1}{4}\sin 2x \right]_0^\pi$$

$$= \pi \left[4\pi + 4\cos \pi + \frac{\pi}{2} - \frac{1}{4}\sin 2\pi - (4) \right]$$

$$= \pi \left[\frac{9}{2}\pi - 4 - 4 \right]$$

$$= \pi \left(\frac{9}{2}\pi - 8 \right)$$

$$= \frac{\pi}{2} (9\pi - 16)$$

#6

$$\frac{4x^2}{121} + \frac{y^2}{12} = 1 \Rightarrow \frac{y^2}{12} = 1 - \frac{4x^2}{121}$$

$$y^2 = 12 \left(1 - \frac{4x^2}{121}\right) \Rightarrow y = \sqrt{12 \left(1 - \frac{4x^2}{121}\right)}$$

$$V = \pi \int_{-11/2}^{11/2} R^2(x) dx = \pi \int_{-11/2}^{11/2} \left(\sqrt{12 \left(1 - \frac{4x^2}{121}\right)} \right)^2 dx$$

$$= \pi \int_{-11/2}^{11/2} 12 \left(1 - \frac{4}{121} x^2\right) dx = 12\pi \left[x - \frac{4}{121} \frac{x^3}{3} \right]_{-11/2}^{11/2}$$

$$= 24\pi \left(x - \frac{4}{363} x^3 \right) \Big|_0^{11/2}$$

$$= 24\pi \left(\frac{11}{2} - \frac{4}{363} \left(\frac{11}{2}\right)^3 \right) = 88\pi \approx 276. \text{ in}^3$$

#7 a) Shell Method.

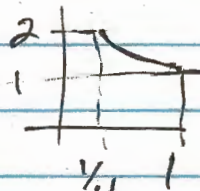
$$y = \frac{1}{\sqrt{x}} \Rightarrow \sqrt{x} = \frac{1}{y} \Rightarrow x = \frac{1}{y^2}$$

$$V = \pi \int_1^2 [R(y)^2 - r(y)^2] dy = \pi \int_1^2 \left(\frac{1}{y^4} - \frac{1}{16} \right) dy$$

$$= \pi \left[\frac{y^{-3}}{-3} - \frac{1}{16} y \right]_1^2 = \pi$$

$$= \pi \left[-\frac{1}{3} 2^{-3} - \frac{1}{16} (2) - \left(-\frac{1}{3} - \frac{1}{16} \right) \right]$$

$$= \frac{11\pi}{48}$$



b) Washer Method, $V = 2\pi \int_{1/4}^1 x \left(\frac{1}{\sqrt{x}} - 1 \right) dx$

$$V = 2\pi \int_{1/4}^1 (x^{1/2} - x) dx = 2\pi \left[\frac{2}{3} x^{3/2} - \frac{1}{2} x^2 \right]_{1/4}^1$$

$$= 2\pi \left[\frac{2}{3} - \frac{1}{2} - \left(\frac{2}{3} \left(\frac{1}{4}\right)^{3/2} - \frac{1}{2} \left(\frac{1}{4}\right)^2 \right) \right]$$

$$= \frac{11\pi}{48}$$

#8

$$y = x^{1/2} - \frac{1}{3} x^{3/2} \quad 1 \leq x \leq 4$$

$$y' = \frac{1}{2} x^{-1/2} - \frac{1}{3} \cdot \frac{3}{2} x^{1/2} = \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{1/2}$$

$$(y')^2 = \frac{1}{4} (x^{-1/2} - x^{1/2})^2 = \frac{1}{4} (x^{-1} - 2 + x)$$

$$L = \int_1^4 \sqrt{1 + \frac{1}{4} \left(\frac{1}{x} - 2 + x \right)^2} dx$$

$$= \int_1^4 \sqrt{1 + \frac{1}{4} \frac{1}{x} - \frac{1}{2} + \frac{1}{4} x} dx$$

$$= \int_1^4 \sqrt{\frac{1}{2} + \frac{1}{4} \frac{1}{x} + \frac{1}{4} x} dx = \int_1^4 \sqrt{\frac{1}{4} \left(\frac{1}{x} + 2 + x \right)} dx$$

$$= \frac{1}{2} \int_1^4 \sqrt{\left(\frac{1}{x} + x \right)^2} dx = \frac{1}{2} \int_1^4 \left(\frac{1}{x} + x \right) dx$$

$$= \frac{1}{2} \int_1^4 \left(x^{-1/2} + x^{1/2} \right)^2 dx = \frac{1}{2} \int_1^4 \left(x^{-1/2} + x^{1/2} \right) dx$$

$$= \frac{1}{2} \left[2x^{1/2} + \frac{2}{3} x^{3/2} \right]_1^4$$

$$= \frac{1}{2} \left[2\sqrt{4} + \frac{2}{3} 4^{3/2} - \left(2 + \frac{2}{3} \right) \right]$$

$$= \frac{10}{3}$$

#9

$$x = y^{2/3} \quad 1 \leq y \leq 8$$

$$x' = \frac{2}{3} y^{-1/3} \rightarrow (x')^2 = \frac{4}{9} y^{-2/3}$$

$$L = \int_1^8 \sqrt{1 + \frac{4}{9} y^{-2/3}} dy = \int_1^8 \sqrt{1 + \frac{4}{9 y^{2/3}}} dy$$

$$= \int_1^8 \sqrt{\frac{9 y^{2/3} + 4}{9 y^{2/3}}} dy = \frac{1}{3} \int_1^8 \sqrt{9 y^{2/3} + 4} (y^{-2/3}) dy$$

$$(\sqrt{y^{2/3}} = y^{1/3})$$

$$L = \frac{1}{3} \int_1^8 (9 y^{2/3} + 4)^{1/2} \frac{1}{6} d(9 y^{2/3} + 4)$$

$$= \frac{1}{18} \frac{2}{3} (9 y^{2/3} + 4)^{3/2} \Big|_1^8$$

$$= \frac{1}{27} \left[(9 \underbrace{(8)^{2/3}}_4 + 4)^{3/2} - (9 + 4)^{3/2} \right]$$

$$= \frac{1}{27} [40^{3/2} - 13^{3/2}]$$

$$\approx 7.634$$

$$u = 9 y^{2/3} + 4$$

$$du = 9 \cdot \frac{2}{3} y^{-1/3} dy$$

$$du = 6 y^{-1/3} dy$$

$$\frac{1}{6} du = y^{-1/3} dy$$

#10

Area of the surface: $y = \frac{1}{3}x^3$ $0 \leq x \leq 1$ x -axis

$$\Rightarrow y' = x^2 \Rightarrow (y')^2 = x^4$$

$$S = 2\pi \int_0^1 \sqrt{1+(y')^2} dx = 2\pi \int_0^1 \frac{1}{3}x^3 \sqrt{1+x^4} dx$$

$$= \frac{2}{3}\pi \int_0^1 x^3 (1+x^4)^{1/2} dx \quad d(1+x^4) = 4x^3 dx$$

$$= \frac{2}{3}\pi \int_0^1 (1+x^4)^{1/2} \frac{1}{4} d(1+x^4) \quad \frac{1}{4} d(1+x^4) = x^3 dx$$

$$= \frac{\pi}{6} \int_0^1 (1+x^4)^{1/2} d(1+x^4)$$

$$= \frac{\pi}{6} \frac{2}{3} (1+x^4)^{3/2} \Big|_0^1$$

$$= \frac{\pi}{9} [2^{3/2} - 1^{3/2}]$$

$$= \frac{\pi}{9} (2\sqrt{2} - 1)$$

#11

$$x = \sqrt{4y - y^2} \quad 1 \leq y \leq 2 \quad y\text{-axis}$$

$$x' = \frac{1}{2} (4y - y^2)^{-1/2} (4 - 2y) = (2 - y) (4y - y^2)^{-1/2}$$

$$(x')^2 = (2 - y)^2 (4y - y^2)^{-1} = \frac{4 - 4y + y^2}{4y - y^2}$$

$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{4 - 4y + y^2}{4y - y^2} = \frac{4y - y^2 + 4 - 4y + y^2}{4y - y^2} \\ = \frac{4}{4y - y^2}$$

$$S = 2\pi \int_1^2 x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$= 2\pi \int_1^2 (4y - y^2)^{1/2} \frac{2}{\sqrt{4y - y^2}} dy$$

$$= 4\pi \int_1^2 dy = 4\pi y \Big|_1^2 = 4\pi (2 - 1)$$

$$= 4\pi$$

$$\#12 \quad y = 2\sqrt{x} \Rightarrow y' = 2 \cdot \frac{1}{2} x^{-1/2} = x^{-1/2} = \frac{1}{\sqrt{x}}$$

$$(y')^2 = \frac{1}{x} \Rightarrow ds = \sqrt{(y')^2 + 1} dx = \sqrt{\frac{1}{x} + 1} dx$$

$$A = \int_0^3 y ds$$

$$= \int_0^3 2\sqrt{x} \sqrt{\frac{1+x}{x}} dx$$

$$d(1+x) = dx$$

$$= 2 \int_0^3 \sqrt{1+x} dx = 2 \int_0^3 (1+x)^{1/2} d(x+1)$$

$$= 2 \left[\frac{2}{3} (1+x)^{3/2} \right]_0^3$$

$$= \frac{4}{3} [4^{3/2} - 1^{3/2}]$$

$$= \frac{28}{3}$$

$$\#13 \quad \text{Equipment alone: } F_1 = 100 \text{ N.}$$

$$W_1 = \int_0^{40} F_1 dx = \int_0^{40} 100 dx = 100x \Big|_0^{40} = 4000 \text{ J.}$$

$$\text{Rope: } F_2 = kx = .8(40-x)$$

$$W_2 = \int_0^{40} F_2 dx = \int_0^{40} .8(40-x) dx$$

$$= .8 \left(40x - \frac{1}{2} x^2 \right) \Big|_0^{40} = .8 \left[40^2 - \frac{1}{2} 40^2 - 0 \right]$$

$$= 640 \text{ J.}$$

$$\text{Total work} = 4000 + 640 = 4640 \text{ J}$$

#14

800 \rightarrow 400 gal.

$$8 \frac{\text{lb}}{\text{gal}} \cdot 800 \text{ gal} \rightarrow 8 \frac{\text{lb}}{\text{gal}} \cdot 400 \text{ gal}$$

$$6400 \text{ lb} \rightarrow 3200 \text{ lb}$$

$$P(x) = 6400 \left(\frac{2(4750) - x}{2(4750)} \right) = 6400 \left(1 - \frac{x}{9500} \right) \text{ lb}$$

$$\begin{aligned} W &= \int_0^{4750} 6400 \left(1 - \frac{x}{9500} \right) dx = 6400 \left[x - \frac{x^2}{19000} \right]_0^{4750} \\ &= 6400 \left[4750 - \frac{4750^2}{19000} \right] = 6400 \left(4750 - \frac{4750}{3} \right) \\ &= 22,800,000 \text{ ft} \cdot \text{lb} \end{aligned}$$

#15

$$F = kx \rightarrow 200 = k(0.8) \rightarrow k = \frac{200}{0.8} = 250 \text{ N/m}$$

$$300 = 250x \rightarrow \boxed{x = \frac{300}{250} = 1.2 \text{ m}}$$

$$W = \int_0^{1.2} F(x) dx = \int_0^{1.2} 250x dx = 125x^2 \Big|_0^{1.2} = 125(1.2)^2$$

$$\boxed{2160 \text{ J}}$$

#16

$$a) y = \sqrt{x} e^{\sqrt{x}} \rightarrow y' = \frac{1}{2\sqrt{x}} e^{\sqrt{x}} + \sqrt{x} e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2} e^{\sqrt{x}} + e^{\sqrt{x}} = \frac{3}{2} e^{\sqrt{x}}$$

$$b) y = \frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x} \rightarrow y' = \frac{1}{4} e^{4x} + \frac{1}{4} x (4e^{4x}) - \frac{1}{4} e^{4x}$$

$$\begin{aligned} y' &= \frac{1}{4} e^{4x} + x e^{4x} - \frac{1}{4} e^{4x} \\ &= x e^{4x} \end{aligned}$$

#16 cont c) $y = \ln(\sec^2 \theta) \Rightarrow y' = \frac{(\sec^2 \theta)'}{\sec^2 \theta} = \frac{2 \sec \theta (\sec \theta)'}{\sec^2 \theta}$
 $= \frac{2 \sec \theta \sec \theta \tan \theta}{\sec^2 \theta}$
 $= \underline{2 \tan \theta}$

d) $y = \log_5(3x-7) = \frac{\ln(3x-7)}{\ln 5}$

$$y' = \frac{1}{\ln 5} \frac{(3x-7)'}{3x-7} = \frac{3}{(\ln 5)(3x-7)}$$

e) $y = (x+2)^{x+2}$

$$\ln y = \ln (x+2)^{x+2} = (x+2) \ln (x+2)$$

$$(\ln y)' = \left[\underbrace{(x+2)}_u \underbrace{\ln(x+2)}_v \right]' \quad u'v + v'u$$

$$\frac{y'}{y} = \ln(x+2) + (x+2) \frac{(x+2)'}{x+2}$$

$$= \ln(x+2) + 1$$

$$y' = y [\ln(x+2) + 1]$$

$$= (x+2)^{x+2} [\ln(x+2) + 1]$$

f) $y = \sin^{-1}\left(\frac{1}{\sqrt{x}}\right) \quad x > 1$

$$y' = \frac{(x^{-1/2})'}{\sqrt{1 - \left(\frac{1}{\sqrt{x}}\right)^2}} = \frac{-\frac{1}{2} x^{-3/2}}{\sqrt{1 - \frac{1}{x}}} = -\frac{1}{2} \frac{1}{x^{3/2} \sqrt{\frac{x-1}{x}}}$$

$$= -\frac{1}{2 x^{3/2} \sqrt{\frac{x-1}{x}}} = -\frac{1}{2 x \sqrt{x-1}}$$

16 cont

$$g) y = z \cos^{-1} z - \sqrt{1-z^2}$$

$$\begin{aligned} y' &= \cos^{-1} z + z \left(\frac{-1}{\sqrt{1-z^2}} \right) - \frac{1}{2} (1-z^2)^{-1/2} (-2z) \\ &= \cos^{-1} z - \frac{z}{\sqrt{1-z^2}} + \frac{z}{\sqrt{1-z^2}} \\ &= \cos^{-1} z \end{aligned}$$

$$h) y = t \tan^{-1} t - \frac{1}{2} \ln t$$

$$\begin{aligned} y' &= \tan^{-1} t + t \left(\frac{1}{1+t^2} \right) - \frac{1}{2} \left(\frac{1}{t} \right) \\ &= \tan^{-1} t + \frac{t}{1+t^2} - \frac{1}{2t} \end{aligned}$$

$$i) y = \sqrt[10]{\frac{3x+4}{2x-4}}$$

$$\begin{aligned} \ln y &= \ln \left(\frac{3x+4}{2x-4} \right)^{1/10} = \frac{1}{10} \ln \frac{3x+4}{2x-4} \\ &= \frac{1}{10} [\ln(3x+4) - \ln(2x-4)] \end{aligned}$$

derivative:

$$\frac{y'}{y} = \frac{1}{10} \left[\frac{3}{3x+4} - \frac{2}{2x-4} \right]$$

$$y' = y \cdot \frac{1}{10} \left(\frac{3}{3x+4} - \frac{2}{2x-4} \right)$$

$$= \frac{1}{10} \sqrt[10]{\frac{3x+4}{2x-4}} \left(\frac{3}{3x+4} - \frac{2}{2x-4} \right)$$

16 cont. d) $y = \left(\frac{(t+1)(t-1)}{(t-2)(t+3)} \right)^5 \quad t > 2.$

$$\ln y = 5 \ln \frac{(t+1)(t-1)}{(t-2)(t+3)}$$

$$= 5 \left[\ln(t+1) + \ln(t-1) - \ln(t-2) - \ln(t+3) \right]$$

$$\frac{y'}{y} = 5 \left(\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t+3} \right)$$

$$y' = 5 y \left(\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t+3} \right)$$

$$y' = 5 \left(\frac{(t+1)(t-1)}{(t-2)(t+3)} \right)^5 \left(\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t+3} \right)$$

k) $y = (\sin \theta)^{\sqrt{\theta}}$
 $\ln y = \sqrt{\theta} \ln \sin \theta$

$$\frac{y'}{y} = \frac{1}{2} \theta^{-1/2} \ln \sin \theta + \sqrt{\theta} \frac{\cos \theta}{\sin \theta}$$

$$y' = y \left(\frac{1}{2\sqrt{\theta}} \ln \sin \theta + \sqrt{\theta} \cot \theta \right)$$

$$= (\sin \theta)^{\sqrt{\theta}} \left(\frac{1}{2\sqrt{\theta}} \ln \sin \theta + \sqrt{\theta} \cot \theta \right)$$

#17 a) $\int e^t \cos(3e^t - 2) dt$. $d(3e^t - 2) = 3e^t dt$
 $\frac{1}{3} d(3e^t - 2) = e^t dt$.

$$\int e^t \cos(3e^t - 2) dt = \frac{1}{3} \int \cos(3e^t - 2) d(3e^t - 2)$$

$$= \frac{1}{3} \sin(3e^t - 2) + C.$$

b) $\int e^y \csc(e^y + 1) \cot(e^y + 1) dy$. $u = e^y + 1$
 $du = e^y dy$

$$\int e^y \csc(e^y + 1) \cot(e^y + 1) dy = \int \csc u \cot u du$$

$$= -\csc u + C$$

$$= -\csc(e^y + 1) + C.$$

c) $\int (\csc^2 x) e^{\cot x} dx$ $u = \cot x \rightarrow du = -\csc^2 x dx$

$$= \int e^u (-du) = -e^u + C = -e^{\cot x} + C$$

d) $\int_1^e \frac{\sqrt{\ln x}}{x} dx$ $u = \ln x \rightarrow du = \frac{1}{x} dx$

$$= \int_1^e u^{1/2} du = \frac{2}{3} u^{3/2} \Big|_1^e$$

$$= \frac{2}{3} (\ln x)^{3/2} \Big|_1^e$$

$$= \frac{2}{3} [(\ln e)^{3/2} - (\ln 1)^{3/2}] = \frac{2}{3} (1 - 0)$$

$$= \frac{2}{3}$$

#17 cont.

$$e) \int_{-\pi/2}^{\pi/6} \frac{\cos t}{1 - \sin t} dt$$

$$u = 1 - \sin t \rightarrow du = -\cos t dt$$

$$-du = \cos t dt$$

$$= \int_{-\pi/2}^{\pi/6} \frac{-du}{u} = -\ln|u| \Big|_{-\pi/2}^{\pi/6}$$

$$= -\ln|1 - \sin t| \Big|_{-\pi/2}^{\pi/6} = -\left[\ln|1 - \sin \frac{\pi}{6}| - \ln|1 - \sin(-\frac{\pi}{2})| \right]$$

$$= -\left(\ln\left|\frac{1}{2}\right| - \ln|2| \right)$$

$$= -(-\ln 2 - \ln 2)$$

$$= 2 \ln 2 = \ln 2^2$$

$$= \ln 4$$

$$f) \int 2^{\tan x} \sec^2 x dx$$

$$u = \tan x \rightarrow du = \sec^2 x dx$$

$$= \int 2^u du = \frac{1}{\ln 2} 2^u + C = \frac{1}{\ln 2} 2^{\tan x} + C.$$

$$g) \int_1^8 \left(\frac{2}{3x} - \frac{8}{x^2} \right) dx = \int_1^8 \left(\frac{2}{3} \frac{1}{x} - 8x^{-2} \right) dx$$

$$= \frac{2}{3} \ln|x| - 8 \frac{x^{-1}}{-1} \Big|_1^8$$

$$= \frac{2}{3} \left[\ln|x| + \frac{8}{x} \right]_1^8$$

$$= \frac{2}{3} \left[\ln 8 + \frac{8}{8} - \left(\ln 1 + \frac{8}{1} \right) \right]$$

$$= \frac{2}{3} (\ln 8 + 1 - (0 + 8))$$

$$= \frac{2}{3} (\ln 8 + 1 - 8)$$

$$= \frac{2}{3} (\ln 8 - 7) = \frac{2}{3} \ln 8 - \frac{14}{3} = \ln 8^{2/3} - \frac{14}{3}$$

$$\ln 8 - 7$$

#17. cont. b) $\int_0^{\ln 9} e^{\theta} (e^{\theta} - 1)^{1/2} d\theta$ $u = e^{\theta} - 1 \Rightarrow du = e^{\theta} d\theta$

$$= \int_0^{\ln 9} u^{1/2} du = \frac{2}{3} u^{3/2} \Big|_0^{\ln 9}$$

$$= \frac{2}{3} (e^{\theta} - 1)^{3/2} \Big|_0^{\ln 9}$$

$$= \frac{2}{3} [(e^{\ln 9} - 1)^{3/2} - (e^0 - 1)^{3/2}]$$

$$= \frac{2}{3} (9 - 1)^{3/2} = \frac{2}{3} (8^{3/2}) = \frac{2}{3} (2^3 \sqrt{2})$$

$$= \frac{2}{3} \sqrt{(2^3)^3} = \frac{2}{3} \sqrt{2^9} = \frac{2}{3} 2^4 \sqrt{2}$$

$$= \frac{32\sqrt{2}}{3}$$

c) $\int_{-1/5}^{1/5} \frac{6 dx}{\sqrt{4 - 25x^2}}$ $\begin{cases} a^2 = 4 \Rightarrow a = 2 \\ u^2 = 25x^2 \Rightarrow u = 5x \\ du = 5 dx \\ \frac{1}{5} du = dx \end{cases}$

$$= \int_{-1/5}^{1/5} \frac{6}{\sqrt{a^2 - u^2}} \cdot \frac{1}{5} du$$

$$= \frac{6}{5} \left[\sin^{-1} \left(\frac{u}{a} \right) \right]_{-1/5}^{1/5} = \frac{6}{5} \left[\sin^{-1} \left(\frac{5x}{2} \right) \right]_{-1/5}^{1/5}$$

$$= \frac{6}{5} \left(\sin^{-1} \left(\frac{5}{2} \cdot \frac{1}{5} \right) - \sin^{-1} \left(\frac{5}{2} \cdot \left(-\frac{1}{5} \right) \right) \right)$$

$$= \frac{6}{5} \left(\sin^{-1} \frac{1}{2} + \sin^{-1} \left(-\frac{1}{2} \right) \right)$$

$$= \frac{6}{5} \left(\frac{\pi}{6} - \left(-\frac{\pi}{6} \right) \right) = \frac{6}{5} \left(\frac{\pi}{3} \right)$$

$$= \frac{2\pi}{5}$$

17 cont.

$$j) \int \frac{24 dy}{y \sqrt{y^2 - 16}} = 24 \left(\frac{1}{4} \sec^{-1} \left| \frac{y}{4} \right| \right) + C \quad \begin{matrix} a^2 = 16 \\ a = 4 \end{matrix}$$

$$= 6 \sec^{-1} \left| \frac{y}{4} \right| + C$$

$$k) \int \frac{dx}{\sqrt{-x^2 + 4x - 1}}$$

$$= \int \frac{du}{\sqrt{a^2 - u^2}}$$

$$= \sin^{-1} \left(\frac{x-2}{\sqrt{3}} \right) + C$$

$$\begin{aligned} -x^2 + 4x - 1 &= -(x^2 - 4x) - 1 \\ &= -(x^2 - 4x) - 1 + 4 - 4 \\ &= -(x^2 - 4x + 4) - 1 + 4 \\ &= -(x-2)^2 + 3 \\ &= 3 - (x-2)^2 \end{aligned}$$

$$\begin{aligned} a^2 &= 3 \Rightarrow a = \sqrt{3} \\ u^2 &= (x-2)^2 \Rightarrow u = (x-2) \\ du &= dx \end{aligned}$$

$$l) \int_{-1}^1 \frac{3 dv}{4v^2 + 4v + 4}$$

$$= \int_{-1}^1 \frac{3 dv}{4(v^2 + v + 1)}$$

$$= \frac{3}{4} \int_{-1}^1 \frac{dv}{\frac{3}{4} + (v + \frac{1}{2})^2}$$

$$= \frac{3}{4} \int_{-1}^1 \frac{du}{a^2 + u^2}$$

$$= \frac{3}{4} \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \frac{2u}{\sqrt{3}} \Big|_{-1}^1$$

$$\begin{aligned} v^2 + v + 1 &= (v^2 + v) + \frac{1}{4} - \frac{1}{4} + 1 \\ &= (v^2 + v + \frac{1}{4}) + \frac{3}{4} \\ &= (v + \frac{1}{2})^2 + \frac{3}{4} \end{aligned}$$

$$\begin{aligned} a &= \frac{3}{4} \Rightarrow a = \frac{\sqrt{3}}{2} \\ u^2 &= (v + \frac{1}{2})^2 \\ u &= v + \frac{1}{2} \\ du &= dv \end{aligned}$$

$$\begin{aligned} &= \frac{3}{2\sqrt{3}} \tan^{-1} \frac{2}{\sqrt{3}} (v + \frac{1}{2}) \Big|_{-1}^1 = \frac{\sqrt{3}}{2} \left[\tan^{-1} \left(\frac{2}{\sqrt{3}} \cdot \frac{3}{2} \right) - \tan^{-1} \left(\frac{2}{\sqrt{3}} \cdot (-\frac{1}{2}) \right) \right] \\ &= \frac{\sqrt{3}}{2} \left(\tan^{-1} \frac{3}{\sqrt{3}} - \tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) \right) = \frac{\sqrt{3}}{2} \left(\frac{\pi}{3} - (-\frac{\pi}{6}) \right) = \frac{\sqrt{3}}{4} \pi \end{aligned}$$

#17 cont m)
$$\int \frac{dt}{(3t+1)\sqrt{9t^2+6t}}$$

$$= \int \frac{(1/3) du}{u \sqrt{u^2-1}}$$

$$= \frac{1}{3} \sec^{-1}|u| + C$$

$$= \frac{1}{3} \sec^{-1}|3t+1| + C.$$

$$9t^2+6t = 9t^2+6t+1-1$$

$$= (3t+1)^2 - 1$$

$$\left. \begin{array}{l} u = 3t+1 \\ du = 3 dt \Rightarrow \frac{1}{3} du = dt \end{array} \right\}$$

#18 a)
$$\lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1} = \frac{1^a - 1}{1^b - 1} = \frac{1-1}{1-1} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 1} \frac{a x^{a-1}}{b x^{b-1}} = \boxed{\frac{a}{b}}$$

b)
$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \cos 0 = \boxed{1}$$

c)
$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx} = \lim_{x \rightarrow 0} \frac{m \cos mx}{n \cos nx} = \frac{m \cos 0}{n \cos 0} = \boxed{\frac{m}{n}}$$

d)
$$\lim_{x \rightarrow \infty} \left(\frac{x^3}{x^2-1} - \frac{x^3}{x^2+1} \right) = \infty - \infty$$

$$= \lim_{x \rightarrow \infty} \left(\frac{x^3}{x^2-1} - \frac{x^3}{x^2+1} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{x^3(x^2+1) - x^3(x^2-1)}{(x^2-1)(x^2+1)} = \lim_{x \rightarrow \infty} \frac{x^5 + x^3 - x^5 + x^3}{x^4 - 1}$$

$$= \lim_{x \rightarrow \infty} \frac{2x^3}{x^4} = \lim_{x \rightarrow \infty} \frac{2}{x} = \boxed{0}$$

#18 cont.

$$g) \lim_{x \rightarrow 0} \frac{2^{-\sin x} - 1}{e^x - 1} = \lim_{x \rightarrow 0} \frac{2^{-\sin x} (1 \ln 2)(-\cos x)}{e^x} = -\ln 2$$

$$f) \lim_{x \rightarrow 4} \frac{\sin^2 \pi x}{e^{x-4} + 3 - x} = \lim_{x \rightarrow 4} \frac{2(\sin \pi x)(\pi \cos \pi x)}{e^{x-4} - 1}$$

$$= \lim_{x \rightarrow 4} \frac{\pi \sin 2\pi x}{e^{x-4} - 1} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 4} \frac{\pi (2\pi) \cos 2\pi x}{e^{x-4}} = \boxed{2\pi^2}$$

$$g) \lim_{x \rightarrow \infty} \left(\frac{e^x + 1}{e^x - 1} \right)^{\ln x}$$

$$f(x) = \left(\frac{e^x + 1}{e^x - 1} \right)^{\ln x} \Rightarrow \ln f(x) = \ln x \ln \left(\frac{e^x + 1}{e^x - 1} \right)$$

$$\left\{ \frac{e^x + 1}{e^x - 1} \cdot \frac{e^{-x/2}}{e^{-x/2}} = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} = \coth \left(\frac{x}{2} \right) \right\}$$

$$\lim_{x \rightarrow \infty} \ln x \ln \left(\frac{e^x + 1}{e^x - 1} \right) = \lim_{x \rightarrow \infty} \ln x \ln \coth \left(\frac{x}{2} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{\ln \coth \left(\frac{x}{2} \right)}{\frac{1}{\ln x}} = \frac{0}{\infty}$$

$$= \lim_{x \rightarrow \infty} \left(\frac{\frac{\operatorname{csch}^2(x/2)}{\coth(x/2)} (-1/2)}{-\frac{1}{(\ln x)^2} \frac{1}{x}} \right) = \lim_{x \rightarrow \infty} \frac{x(\ln x)^2}{2 \sinh \frac{x}{2} \cosh \frac{x}{2}}$$

$$= \lim_{x \rightarrow \infty} \frac{x(\ln x)^2}{\sinh x} = \lim_{x \rightarrow \infty} \frac{(\ln x)^2 + 2x \ln x \left(\frac{1}{x} \right)}{\cosh x}$$

$$= \lim_{x \rightarrow \infty} \frac{(\ln x)^2 + 2 \ln x}{\cosh x} = \lim_{x \rightarrow \infty} \left(\frac{2 \left(\frac{1}{x} \right) + 2(\ln x) \left(\frac{1}{x} \right)}{\sinh x} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{2 + 2 \ln x}{x \sinh x} = \lim_{x \rightarrow \infty} 2 \frac{1/x}{x \cosh x + \sinh x}$$

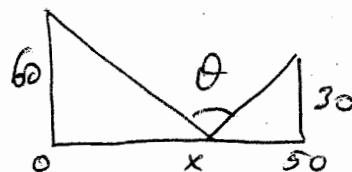
$$= \lim_{x \rightarrow \infty} \frac{2}{x^2 \cosh x + x \sinh x} = 0 \Rightarrow$$

$$\ln f(x) = 0 \Rightarrow \boxed{f(x) = 1}$$

#19

$$\theta = \pi - \cot^{-1}\left(\frac{x}{60}\right) - \cot^{-1}\left(\frac{50-x}{30}\right)$$

$$0 < x < 50$$



$$\theta' = \frac{\frac{1}{60}}{1 + \left(\frac{x}{60}\right)^2} + \frac{-\frac{1}{30}}{1 + \left(\frac{50-x}{30}\right)^2} = \frac{1}{60} \frac{1}{1 + \frac{x^2}{60^2}} - \frac{1}{30} \frac{1}{1 + \frac{(50-x)^2}{30^2}}$$

$$= \frac{1}{60} \frac{1}{\frac{60^2 + x^2}{60^2}} - \frac{1}{30} \frac{1}{\frac{30^2 + (50-x)^2}{30^2}}$$

$$= 60 \frac{1}{60^2 + x^2} - 30 \frac{1}{30^2 + (50-x)^2}$$

$$= 30 \left(\frac{2}{60^2 + x^2} - \frac{1}{30^2 + (50-x)^2} \right)$$

$$= 30 \left[\frac{2(30^2 + 50^2 - 100x + x^2) - 60^2 - x^2}{(60^2 + x^2)(30^2 + (50-x)^2)} \right] = 0$$

$$2(900 + 2500 - 100x + x^2) - 3600 - x^2 = 0$$

$$1800 + 5000 - 200x + 2x^2 - 3600 - x^2 = 0$$

$$x^2 - 200x + 3200 = 0$$

$$\Rightarrow x = 100 \pm 20\sqrt{7} \Rightarrow \begin{cases} x = 80.46 > 50 \\ x = 17.54 \end{cases}$$

to maximize $\theta \Rightarrow \underline{x = 17.54 \text{ m}}$

20

$$R = x^2 \ln\left(\frac{1}{x}\right) = x^2 (\ln 1 - \ln x) = -x^2 \ln x$$

$$R' = -2x \ln x - x^2 \left(\frac{1}{x}\right)$$

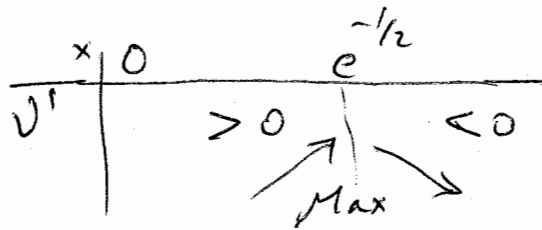
$$= -2x \ln x - x$$

$$= -x (2 \ln x + 1) = 0$$

$$\left\{ \begin{array}{l} x = 0 \\ 2 \ln x + 1 = 0 \end{array} \right.$$

$$2 \ln x + 1 = 0 \Rightarrow 2 \ln x = -1 \Rightarrow \ln x = -\frac{1}{2}$$

$$\ln x = -\frac{1}{2} \Rightarrow x = e^{-1/2}$$



$$R_{MAX} @ x = e^{-1/2}$$

$$x = \frac{\lambda}{h} \Rightarrow \left[h = \frac{\lambda}{x} = \frac{1}{e^{-1/2}} = e^{1/2} = \sqrt{e} \approx 1.65 \text{ nm} \right]$$