1/ $y = x^3$, $y = 8$, $x = 0$ about y-axis.

$$V = \pi \int_0^8 (y^{1/3})^2 dy$$

$$= \frac{3\pi}{5} y^{5/3} \Big|_0^8 \qquad 8 = 2^3$$

$$= \frac{3\pi}{5}(2^5 - 0)$$

$$= \frac{96\pi}{5} \text{ unit}^2$$

2/ $y = x^2$, $y = x$ about x-axis

$y = x^2 = x \rightarrow x = 0, 1$

$$V = \pi \int_0^1 (x^2 - x^4) dx \qquad (x)^2 \quad (x^2)^2$$

$$= \pi \left(\frac{1}{3}x^3 - \frac{1}{5}x^5 \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{5} \right)$$

$$= \frac{2\pi}{15} \text{ unit}^2$$

3/ $y = x^2$ $y = x$ line: $y = 2$.

$y = x^2 = x \rightarrow x = 0, 1.$

$$V = \pi \int_0^1 \left((2 - x^2)^2 - (2 - x)^2\right) dx$$

$$= \pi \int_0^1 (4 - 4x^2 + x^4 - 4 + 4x - x^2)\, dx$$

$$= \int_0^1 (x^4 - 5x^2 + 4x)\, dx$$

$$= \pi \left(\frac{1}{5}x^5 - \frac{5}{3}x^3 + 2x^2\right)\Big|_0^1$$

$$= \pi \left(\frac{1}{5} - \frac{5}{3} + 2\right)$$

$$= \frac{8\pi}{15} \text{ unit}^2$$

4/ $y = \sqrt{x}$ $\quad y^2 = x$ $\quad 0 < y < 1$ $\quad$ x-axis

$$V = 2\pi \int_0^1 y(1 - y^2)\, dy$$

$$= 2\pi \int_0^1 (y - y^3)\, dy$$

$$= 2\pi \left(\frac{1}{2}y^2 - \frac{1}{4}y^4\right)\Big|_0^1$$

$$= 2\pi \left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{\pi}{2} \text{ unit}^2$$

5/ $y = x - x^2 \geq 0$, $y = 0$ about $x = 2$.
$x \geq 0, 1$

$$V = 2\pi \int_0^1 (2-x)(x-x^2)\,dx$$

$$= 2\pi \int_0^1 (2x - 3x^2 + x^3)\,dx$$

$$= 2\pi \left(x^2 - x^3 + \frac{1}{4}x^4\right)\Big|_0^1$$

$$= 2\pi \left(1 - 1 + \frac{1}{4}\right)$$

$$= \frac{\pi}{2} \text{ unit}^2$$

6/ $y = -x^2 + 6x - 8 = 0$ $\quad y = 0$ $\quad \approx y\text{-axis}$
$x = 2, 4$

$$V = 2\pi \int_2^4 x(-x^2 + 6x - 8)\,dx \qquad \text{shell-method}$$

$$= 2\pi \int_2^4 (-x^3 + 6x^2 - 8x)\,dx$$

$$= 2\pi \left(-\frac{1}{4}x^4 + 2x^3 - 4x^2\right)\Big|_2^4$$

$$= 2\pi \left(-64 + 128 - 64 - (-4 + 16 - 16)\right)$$

$$= 8\pi \text{ unit}^2$$

7/ $y = -x^2 + 6x - 8 = 0$ $\quad y = 0$ $\quad$ – x-axis

$x = 2, 4$

$$V = \pi \int_2^4 (-x^2+6x-8)^2 \, dx$$

$$= \pi \int_2^4 (x^4 - 12x^3 + 16x^2 + 36x^2 + 64 - 96x)\, dx$$

$$= \pi \int_2^4 (x^4 - 12x^3 + 52x^2 - 96x + 64)\, dx$$

$$= \pi \left(\frac{1}{5}x^5 - 3x^4 + \frac{52}{3}x^3 - 48x^2 + 64x \right)\Big|_2^4$$

$$= \pi \left(\frac{1024}{5} - 768 + \frac{3,328}{3} - 768 + 256 \right.$$
$$\left. - \frac{32}{5} + 48 - \frac{416}{3} + 192 - 128 \right)$$

$$= \pi \left(\frac{992}{5} + \frac{2,912}{3} - 1168 \right)$$

$$= \frac{16\pi}{15} \text{ unit}^2$$

8/ $x = (y-3)^2$ $\quad x = 4$ $\quad \sim y = 1$

$x = (y-3)^2 = 4 \rightarrow \; y - 3 = \pm 2 \;\Big\}\; y = 1, 5$

$$V = 2\pi \int_1^5 (y-1)(4-(y-3)^2)\, dy$$

$$= 2\pi \int_1^5 (y-1)(-5 - y^2 + 6y)\, dy$$

$$= 2\pi \int_1^5 (-y^3 - 11y + 7y^2 + 5)\, dy$$

$$= 2\pi \left(-\frac{1}{4}y^4 - \frac{11}{2}y^2 + \frac{7}{3}y^3 + 5y \right)\Big|_1^5$$

$$= 2\pi \left(-\frac{625}{4} - \frac{275}{2} + \frac{875}{3} + 25 + \frac{1}{4} + \frac{11}{2} - \frac{7}{3} - 5 \right)$$

$$= 2\pi \left(-288 + 20 + \frac{856}{3} \right) \rightarrow \frac{64}{3}$$

$$= \frac{128\pi}{3} \text{ unit}^2$$