

Ex Given: $y = x^2 + 1$ $y = -x + 3$

rev. x -axis $\rightarrow dx$

Find: Vol?

Soln $y = x^2 + 1 = -x + 3$

$$x^2 + x - 2 = 0 \Rightarrow x = 1, -2$$

$$a + b + c = 0$$

$$x = 1, c/a$$

$$a - b + c = 0$$

$$x = -1$$

$$-c/a$$

$$V = \pi \int_{-2}^1 [(-x+3)^2 - (x^2+1)^2] dx$$

$$= \pi \int_{-2}^1 (x^2 - 6x + 9 - x^4 - 2x^2 - 1) dx$$

$$= \pi \int_{-2}^1 (-x^4 - x^2 - 6x + 8) dx$$

$$= \pi \left(-\frac{1}{5}x^5 - \frac{1}{3}x^3 - 3x^2 + 8x \right) \Big|_{-2}^1$$

$$= \pi \left(\underline{-\frac{1}{5}} - \underline{\frac{1}{3}} - 3 + 8 - \left(\underline{\frac{32}{5}} + \underline{\frac{8}{3}} - 12 - 16 \right) \right)$$

$$= \pi \left(-\frac{33}{5} + 2 + 28 \right)$$

$$= \underline{\underline{\frac{117\pi}{5} \text{ unit}^3}}$$

Ex Given: $y = x^2$, $y = 2x$ QI y -axis
Find: Vol?

Soln $y = x^2 \rightarrow x = \sqrt{y}$ (QI)

$$y = 2x \rightarrow x = \frac{1}{2}y$$

$$y = x^2 = 2x \Rightarrow \begin{cases} x = 0 \rightarrow y = 0 \\ x = 2 \rightarrow y = 4 \end{cases}$$

\therefore 1 4 2 2 1

$$V = \pi \int_0^4 \left[(\sqrt{y})^2 - \left(\frac{1}{2}y\right)^2 \right] dy$$

$$= \pi \int_0^4 \left(y - \frac{1}{4}y^2 \right) dy$$

$$= \pi \left(\frac{1}{2}y^2 - \frac{1}{12}y^3 \right) \Big|_0^4$$

$$= \pi \left(8 - \frac{16}{3} \right)$$

$$= \frac{8\pi}{3} \text{ unit}^3$$

$$\frac{4^2}{3 \cdot 4}$$

Boundary: $0 \rightarrow 4$ dy

select $1 \rightarrow 2$
 $x = 1^2 - \left(\frac{1}{2}\right)^2$

Ex Given: $f(x) = \sqrt{x}$ $g(x) = x^2$ $0 \leq x \leq 1$

Find: Vol: x -axis $\rightarrow dx$

soln

$$V = \pi \int_0^1 (x - x^4) dx$$

$$= \pi \left(\frac{1}{2}x^2 - \frac{1}{5}x^5 \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{2} - \frac{1}{5} \right)$$

$$= \frac{3\pi}{10} \text{ unit}^3$$

$$\frac{1}{4} > \frac{1}{16}$$

$$(\sqrt{x})^2 - (x^2)^2$$

1.4 Shell Method vs Washer Method.

$$V = 2\pi \int_a^b (\text{shell Rad}) (\text{shell height}) dx$$

$$= 2\pi \int_a^b x f(x) dx \quad \text{y-axis}$$

$$= 2\pi \int_c^a y g(y) dy \quad \text{--- } x\text{-axis}$$

Ex $f(x) = \sin x^2$ $y=0$, $x = \sqrt{\frac{\pi}{2}}$
 Vol? $y\text{-axis} \rightarrow dx$

Soln

$$V = 2\pi \int_0^{\sqrt{\pi/2}} x \sin x^2 dx$$

$d(x^2) = 2x dx$

$$= \pi \int_0^{\sqrt{\pi/2}} \sin(x^2) d(x^2)$$

$$= -\pi \cos x^2 \Big|_0^{\sqrt{\pi/2}}$$

$$= -\pi (0 - 1)$$

$$= \pi \text{ unit}^3$$

Ex $y = \sqrt{x-2}$ $y=2$

a) $x\text{-axis} \rightarrow dy$ shell method.

$$y = \sqrt{x-2} \Rightarrow y^2 = x-2$$

$$x = y^2 + 2$$

$$V = 2\pi \int_0^2 y (y^2 + 2) dy$$

$$= 2\pi \int_0^2 (y^3 + 2y) dy$$

$$= 2\pi \left(\frac{1}{4} y^4 + y^2 \right) \Big|_0^2$$

$$= 2\pi (4 + 4)$$

$$= \underline{16\pi \text{ unit}^3}$$

b) $y = -2$

$$V = 2\pi \int_0^2 (y+2)(y^2+2) dy$$

$$= 2\pi \int_0^2 (y^3 + 2y^2 + 2y + 4) dy$$

$$= 2\pi \left(\frac{1}{4} y^4 + \frac{2}{3} y^3 + y^2 + 4y \right) \Big|_0^2$$

$$= 2\pi \left(4 + \frac{16}{3} + 4 + 8 \right)$$

$$= 82\pi \left(1 + \frac{1}{3} \right)$$

$$= \underline{\underline{\frac{128\pi}{3} \text{ unit}^3}}$$

Ex $f(x) = 2x - x^2$ $g(x) = x$ $[0, 1]$
Soln $1 - \frac{1}{2}, \frac{1}{2}$ V? x-axis

$$\left. \begin{aligned} 2x - x^2 &= x \Rightarrow x = 0 \\ 2 - x &= 0 \Rightarrow x = 1 \end{aligned} \right\}$$

Washer Method (dx)

$$V = \pi \int_0^1 \left[(2x - x^2)^2 - x^2 \right] dx$$

$$= \pi \int_0^1 (3x^2 - 4x^3 + x^4) dx$$

$$= \pi \left(x^3 - x^4 + \frac{1}{5} x^5 \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{5} \right)$$

$$= \frac{\pi}{5} \text{ unit}^3$$

Shell Method $\rightarrow dy$ $0 < x \leq 1$

$$y = 2x - x^2$$

$$y = x \quad \text{for } 1 + \sqrt{1-y} > 1$$

$$x^2 - 2x + y = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4y}}{2} = 1 \pm \sqrt{1-y}$$

$$x=0 \rightarrow y=0$$

$$x=1 \rightarrow y=1$$

$$x = 1 - \sqrt{1-y}$$

$$V = 2\pi \int_0^1 y [y - (1 - \sqrt{1-y})] dy$$

$$= 2\pi \int_0^1 [y^2 - y + y(1-y)^{1/2}] dy$$

$$\int y(1-y)^{1/2} dy \quad \begin{cases} u = 1-y \\ du = -dy \\ y = 1-u \end{cases}$$

$$= \int (1-u) u^{1/2} (-du)$$

$$= - \int (u^{1/2} - u^{3/2}) du$$

$$= - \left(\frac{2}{3} (1-y)^{3/2} - \frac{2}{5} (1-y)^{5/2} \right)$$

$$= 2\pi \left(\frac{1}{3} y^3 - \frac{1}{2} y^2 - \frac{2}{3} (1-y)^{3/2} + \frac{2}{5} (1-y)^{5/2} \right) \Big|_0^1$$

$$= 2\pi \left[\frac{1}{3} - \frac{1}{2} - \left(-\frac{2}{3} + \frac{2}{5} \right) \right]$$

$$= 2\pi \left(1 - \frac{1}{2} - \frac{2}{5} \right)$$

$$= 2\pi \left(\frac{1}{2} - \frac{2}{5} \right)$$

$$= \frac{\pi}{5} \text{ unit}^3$$

$$\frac{5-4}{10}$$

$$f(x) = 2x - x^2$$

$$g(x) = x$$

$dx \rightarrow$ $\left\{ \begin{array}{l} x\text{-axis} \rightarrow \text{washer} \\ y\text{-axis} \rightarrow \text{shell} \end{array} \right.$

$dy \rightarrow$ $\left\{ \begin{array}{l} x\text{-axis} \rightarrow \text{shell} \\ y\text{-axis} \rightarrow \text{washer} \end{array} \right.$

1.5 Length (2D) (x-axis)

$$L = \int_a^b \sqrt{1 + (f')^2} dx$$

Ex $L?$ $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$ $0 \leq x \leq 1$

$$\frac{dy}{dx} = 2\sqrt{2} x^{1/2}$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + 8x}$$

$$L = \int_0^1 (1 + 8x)^{1/2} dx$$

$$d(1 + 8x) = 8dx$$

$$\begin{aligned}
 &= \frac{1}{8} \int_0^1 (1+8x)^{1/2} d(1+8x) \\
 &= \frac{1}{12} (1+8x)^{3/2} \Big|_0^1 \\
 &= \frac{1}{12} (27 - 1) \\
 &= \frac{13\pi}{6} \text{ unit}
 \end{aligned}$$

$$y = ax^m + bx^n$$

$$\begin{cases}
 \textcircled{1} & m+n=2 \\
 \textcircled{2} & abmn = -\frac{1}{4}
 \end{cases}$$

$$\begin{aligned}
 L &= \int_c^d \sqrt{1+(y')^2} dx \\
 &= ax^m - bx^n \Big|_c^d
 \end{aligned}$$

$$y = ae^{mx} + be^{nx}$$

$$\begin{cases}
 \textcircled{1} & m=-n \\
 \textcircled{2} & ambn = -\frac{1}{4}
 \end{cases}$$

$$L = ae^{mx} - be^{nx} \Big|_c^d$$

Ex
c.1.

$$f(x) = \frac{x^3}{12} + \frac{1}{x} \quad \text{on } x', 1 \leq x \leq 4$$

soln $\begin{cases} m+1 = 3-1 = 2 \checkmark \\ a m b n = \frac{1}{12}(3)(1)(-1) = -\frac{1}{4} \checkmark \end{cases}$

$$\begin{aligned} L &= \left. \frac{x^3}{12} - \frac{1}{x} \right|_1^4 \\ &= \frac{16}{3} - \frac{1}{4} - \left(\frac{1}{12} - 1 \right) \\ &= \frac{61}{12} - \frac{1}{12} + 1 \\ &= 6 \text{ unit} \end{aligned}$$

Ex #9 $y = 2e^x + \frac{1}{8}e^{-x} \quad 0 \leq x \leq \ln 2$

soln $\begin{cases} m = -n \checkmark \\ a b m n = 2(1)\left(\frac{1}{8}\right)(-1) = -\frac{1}{4} \checkmark \end{cases}$

$$L = \left. 2e^x - \frac{1}{8}e^{-x} \right|_0^{\ln 2}$$

$$= 2e^{\ln 2} - \frac{1}{8}e^{-\ln 2} - \left(2 - \frac{1}{8} \right)$$

$$= 2(2) - \frac{1}{16} - 2 + \frac{1}{8}$$

$$= \frac{63}{16} - \frac{15}{8}$$

$$= \frac{33}{16} \text{ unit}$$

$$\begin{aligned} e^{\ln x} &= x \\ e^{-\ln x} &= \frac{1}{x} \end{aligned}$$

Ex $y = \ln(x + \sqrt{x^2 + 1}) \quad [1, \sqrt{2}]$

1 $\int_1^{\sqrt{2}} \sqrt{1+1^2+1^2} dx$

$$L = \int_a^b \int_c^d \sqrt{1+(x')^2} dy$$

$$y = \ln(x + \sqrt{x^2 - 1}) \quad \leftarrow$$

$$x + \sqrt{x^2 - 1} = e^y$$

$$(\sqrt{x^2 - 1})^2 = (e^y - x)^2$$

$$x^2 - 1 = e^{2y} - 2xe^y + x^2$$

$$-1 = e^{2y} - 2xe^y$$

$$2xe^y = e^{2y} + 1$$

$$x = \frac{1}{2}(e^y + e^{-y})$$

$$= \frac{1}{2}e^y + \frac{1}{2}e^{-y}$$

$$m = -n \quad \checkmark$$

$$am b n = \frac{1}{2}(1)\left(\frac{1}{2}\right)(-1) = -\frac{1}{4} \quad \checkmark$$

$$L = \frac{1}{2}e^y - \frac{1}{2}e^{-y} \Bigg|_0^{\ln(\sqrt{2}+1)} \quad \begin{array}{l} x=1 \rightarrow y=0 \\ x=\sqrt{2} \rightarrow y=\ln(\sqrt{2}+1) \end{array}$$

$$= \frac{1}{2}(\sqrt{2}+1) - \frac{1}{2}\left(\frac{1}{(\sqrt{2}+1)}\right) - \left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{1}{2(1+\sqrt{2})}$$

$$= \frac{\sqrt{2}+1}{2} - \frac{1}{2(\sqrt{2}+1)}$$

$$= \frac{1}{2} \left(\frac{2+2\sqrt{2}+1-1}{\sqrt{2}+1} \right)$$

$$= \frac{1}{2} (2)$$

$$= \underline{1 \text{ unit}}$$

Ex $f(x) = \frac{1}{12}x^3 + \frac{1}{x}$ $A = (1, \frac{13}{12})$
initial

soln $\left\{ \begin{array}{l} m+n = 3-1 = 2 \checkmark \\ \text{ambn} = \frac{1}{12}(3)(1)(-1) = -\frac{1}{4} \checkmark \end{array} \right.$

$$L = \frac{1}{12}x^3 - \frac{1}{x} \Big|_1^x$$

$$= \frac{1}{12}x^3 - \frac{1}{x} - \left(\frac{1}{12} - 1 \right)$$

$$= \frac{1}{12}x^3 - \frac{1}{x} + \frac{11}{12} \quad \left. \vphantom{\frac{1}{12}x^3} \right\} \text{unit}$$

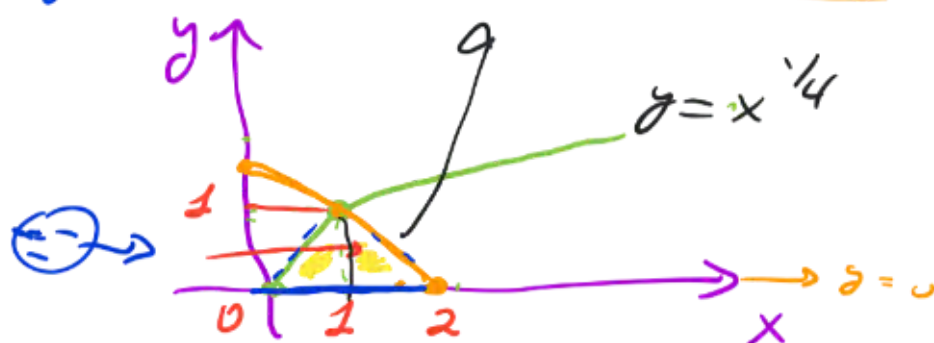
HWk 3

$$y^4 = x$$

$$y = \sqrt{2-x}$$

$$y = 0$$

$$\begin{aligned} 2-x &= y^2 \\ x &= 2-y^2 \end{aligned}$$



1 1 1/4 1 2

$$H = \int_0^1 x^{-1} dx + \int_1^2 \sqrt{2-x^2} dx$$

or

$$A = \int_0^1 (2 - y^2 - y^4) dy.$$

3 fctns Plot
