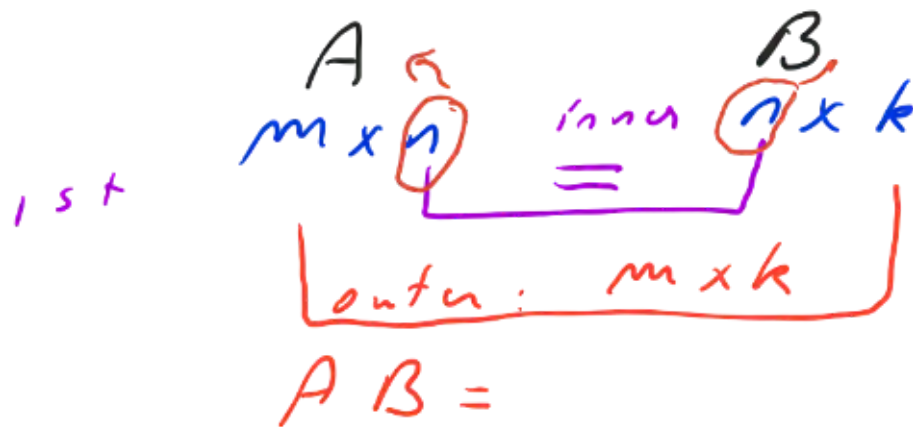
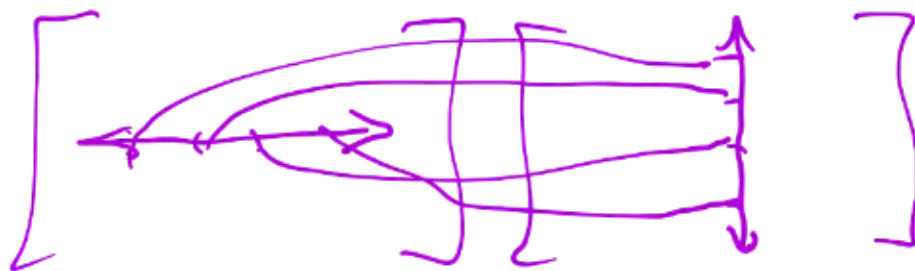


Matrix Multiplication



- AB is dot product
 $A: n$ col $B: n$ rows.
- Square matrices can be multiplied
 iff have same size
- (row i of A) (col j of B)
 $\sum a_{ik} b_{kj}$



Ex $AB = \begin{pmatrix} \underline{a} & \underline{b} \\ \underline{c} & \underline{d} \end{pmatrix} \begin{pmatrix} \underline{e} & \underline{f} \\ \underline{g} & \underline{h} \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$= \begin{pmatrix} \underline{ae + bg} & \underline{af + bh} \\ \underline{ce + dg} & \underline{cf + dh} \end{pmatrix}$$

$$\text{ex } \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 5+1 & 6+0 \\ 10-1 & 12-0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 6 \\ 9 & 12 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 4 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix} = \begin{matrix} 2 \times 1 & 1 \times 2 \end{matrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 5 & 0 \end{bmatrix}$$

Identity matrix I $n \times n$ (square) matrix

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow IA = A$$

\therefore note matrix I 's: $n \times n$ all one

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$A+B = B+A$$

$$A+(B+C) = (A+B)+C$$

$$k(lA) = (kl)A$$

$$k(A+B) = kA + kB$$

$$(k+l)A = kA + lA$$

$$\begin{matrix} \textcircled{A^k} & A^k \\ \boxed{kA} & k \text{ times } A \end{matrix}$$

$$A + 0 = 0 + A = A$$

$$A + (-A) = (-A) + A = 0 \rightarrow \text{matrix}$$

$$AB \neq BA$$

Ex $A_{2 \times 3} \quad B_{3 \times 3}$

$$AB = 2 \times 3$$

$$BA \quad \text{not possible}$$

$$\vec{u} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \neq (1, -1, 0) \leftarrow$$

$$\# \begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$$

1.4 A^{-1}

Defn matrix A is invertible, $A^{-1} \exists$

$$A \text{ invertible} \Rightarrow AA^{-1} = A^{-1}A = I$$

$$a \cdot a^{-1} = a \frac{1}{a} = 1$$

$\rightarrow A$ has to be square matrix

Not All matrices have inverses.

* Inverse \exists iff n pivots

* A cannot have 2 different inverses.

* A invertible $\Rightarrow AX = B$

$$X = A^{-1}B$$

Proof

$$AX = B$$

$$A^{-1}(AX) = A^{-1}(B)$$

$$(A^{-1}A)X = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B \quad \checkmark$$

Given

A is invertible

$$\Rightarrow AA^{-1} = A^{-1}A = I$$

A invertible

$$\Rightarrow AA^{-1} = A^{-1}A = I$$

$$\text{if } \underline{AX=0} \Rightarrow A^{-1} \nexists$$

homogeneous.

$$2x + y = 5$$

$$\rightarrow 2x + y = 0$$

Finding A^{-1} ?? $AA^{-1} = I$

only 2×2

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\therefore \text{if } \underline{ad-bc=0} \Rightarrow A^{-1} \nexists$$

determinant.

Can't have 2 different inverses.

Inverse(A) is unique(!)

Suppose A has A^{-1} and $B \ni BA = I$

Proof $\begin{aligned} B &= BI & A \text{ inv} \Rightarrow I &= AA^{-1} \\ &= B(AA^{-1}) \\ &= (BA)A^{-1} \\ &= IA^{-1} \\ &= A^{-1} \end{aligned}$

\therefore The inverse is unique.

never use numbers to prove equality

$(AB)^{-1} = B^{-1}A^{-1}$ $A \& B$ are invertible

Given: $A \text{ inv} \Rightarrow AA^{-1} = A^{-1}A = I$
 $B \dots$

$$\begin{matrix} (AB) & (AB)^{-1} \\ M & M^{-1} \end{matrix} = I$$

$$\begin{aligned} (AB)(AB)^{-1} &= (AB)(B^{-1}A^{-1}) \\ &= A(BB^{-1})A^{-1} && \begin{matrix} B \text{ invertible} \\ BB^{-1} = I \end{matrix} \\ &= (AI)A^{-1} && AI = A \\ &= AA^{-1} && A \text{ invertible} \\ &= I \quad \checkmark \end{aligned}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$

A is invertible:

$$(A^{-1})^{-1} = A$$

$$(A^{-1})^T = A^{-n} = (A^n)$$

$$(kA)^{-1} = k^{-1} A^{-1} = \frac{1}{k} A^{-1}$$

$$(kA)(kA)^{-1} = (kA)(k^{-1}A^{-1}) =$$

$$\begin{aligned} (kA)(k^{-1}A^{-1}) &= k^{-1}(kA)A^{-1} \\ &= (k^{-1}k) \underline{A} \underline{A^{-1}} \\ &= 1(I) \\ &= I \quad \checkmark \end{aligned}$$

$$AA^T = I$$

$$A^T A = I?$$

$$A^{-1}?$$

$$[A | I] \xrightarrow[\text{Jordan}]{\text{Gauss-}} [I | A^{-1}]$$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{pmatrix}$$

$$|A| = 0 \nexists A^{-1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & -2 & -1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -2 & -3 & -2 & 1 & 0 \\ 0 & 0 & -3 & -3 & 0 & 1 \end{array} \right) -\frac{1}{2} R_2$$

$$\begin{array}{ccc} 0 & 0 & 0 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -3 & -3 & 0 & 1 \end{array} \right) -\frac{1}{3} R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 1 & 0 & -\frac{1}{3} \end{array} \right) \begin{array}{l} R_1 - R_3 \\ R_2 - \frac{3}{2} R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 & 0 & -\frac{1}{3} \end{array} \right]$$

$$A^{-1} = \begin{pmatrix} 0 & 0 & \frac{1}{3} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & -\frac{1}{3} \end{pmatrix}$$

$A^{-1} \exists$

A $n \times n$

n -pivots (not)

row \rightarrow zero row

matrix is invertible \Rightarrow $\left. \begin{array}{l} \text{zero row} \\ n\text{-pivots} \end{array} \right\}$

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Elementary Matrices

$E = e^T$

$$\begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix} \quad R_2 \text{ of } I \text{ by } -3$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow -5 \times R_3 \text{ of } I$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \text{add } -3 R_1 + R_2 = R_2$$

A & B are equivalent. ($A \neq B$)

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 3 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \quad \begin{matrix} R_1 + R_2 \\ R_2 - 2R_1 \end{matrix}$$

$$= \begin{pmatrix} 3 & 0 & 1 \\ 0 & 3 & 1 \end{pmatrix}$$

$$= B$$

$$A_{2 \times 2} \rightarrow A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A \begin{pmatrix} a & b \\ c & d \end{pmatrix} \leftarrow \text{only } 2 \times 2$$

$$\text{if } ad-bc=0 \Rightarrow A^{-1} \nexists$$

$$[A]^{-1} = [A^T]^T$$

$$[I | A] \rightarrow [I | 0]$$

$$\downarrow \Rightarrow A^{-1}$$

\hookrightarrow zero row

$$A \text{ is invertible} \Rightarrow AA^{-1} = A^{-1}A = I$$

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$