Ex $y = (3x^2+1)^2$

$y' = 2(6x)(3x^2+1)$

$= 12x(3x^2+1)$

Ex $\frac{d}{dx}(5x^3-x^4)^7 = 7(15x^2-4x^3)(5x^3-x^4)^6$

$\left(\frac{1}{u^n}\right)' = (u^{-n})'$

$= -n u' u^{-n-1}$

$= \frac{-n u'}{u^{n+1}}$

Ex $\frac{d}{dx}\left(\frac{1}{3x-2}\right) = -\frac{3}{(3x-2)^2}$ $\begin{vmatrix} 0 & 1 \\ 3 & -2 \end{vmatrix}$

Ex $\frac{d}{dx}(\sin^5 x) = 5\cos x \sin^4 x$ $(\sin x)^5$

Ex $g(t) = \tan(5-\sin 2t)$

$g'(t) = (-2\cos 2t)\sec^2(5-\sin 2t)$ (u ↙)

Ex $y = \frac{1}{(1-2x)^3}$

$y' = \frac{-3(-2)}{(1-2x)^4}$

$= \frac{6}{(1-2x)^4}$

59 $f(x) = (2x^2-4x+3)^4 (3x-5)^5$

$f'(x) = (2x^2-4x+3)^3 (3x-5)^4$ $\left[4(4x-4)(3x-5) + 5(3)(2x^2-4x+3) \right]$

x^2	x^1	x^0
48	-80	80
30	-48	45
	-60	

$f'(x) = (2x^2-4x+3)^3 (3x-5)^4 (78x^2-188x+125)$

67 $f(x) = \dfrac{(x^2-3x)^3 (x^2+3x-3)^4}{(x^2-3x+2)^2}$ $\rightarrow (x^2-3x+2)^{-2-1}$

$f'(x) = \dfrac{(x^2-3x)^2 (x^2+3x-3)^3}{(x^2-3x+2)^3}$

$\left[3(2x-3)(x^2+3x-3)(x^2-3x+2) + 4(2x+3)(x^2-3x)(x^2-3x+2) - 2(2x-3)(x^2-3x)(x^2+3x-3) \right]$

$= \dfrac{(x^2-3x)^2 (x^2+3x-3)^3}{(x^2-3x+2)^3}$

$\left[(6x^3+18x^2-18x-9x^2-27x+27)(x^2-3x+2) + (8x^3-12x^2-36x)(x^2-3x+2) - (4x^3-18x^2+18x)(x^2+3x-3) \right]$

x^5	x^4	x^3	x^2	x^1	x^0
6	-18	12	36	-36	54
8	18	-54	54	-54	54
-4	4	-18	-18	-81	
	-24	27	81	-72	
	-12	-27	27	-54	
	-12	6	-24		
	18	36	108		
		-36	-54		
		12			

$$f'(x) = \frac{(x^2+3x)^2(x^2+3x-3)^3}{(x^2-3x+2)^3}$$

$$(10x^5 - 21x^4 - 6x^3 + 190x^2 - 297x + 108)$$

$$y = \left(\frac{2x^4-3}{2x^4+1}\right)^5$$

$$y' = 20(8)x^3 \frac{(2x^4-3)^4}{(2x^4+1)^5}$$

$$= \frac{160x^3(2x^4-3)^4}{(2x^4+1)^5}$$

2.7 Implicit

$x^3 + y^3 - 9xy = 0$ · $\frac{dy}{dx}$ (y')

$$\frac{d}{dx}(x^3 + y^3 - 9xy) = 0$$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) - 9\frac{d}{dx}(xy) = 0$$

$$3x^2\frac{dx}{dx} + 3y^2\frac{dy}{dx} - 9\left(y\frac{dx}{dx} + x\frac{dy}{dx}\right)$$

$$3x^2 + 3y^2\frac{dy}{dx} - 9\left(y + x\frac{dy}{dx}\right) = 0$$

$$(3y^2 - 9x)\frac{dy}{dx} = -3x^2 + 9y$$

$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x}$$

$$= \frac{3y - x^2}{y^2 - x}$$

$$y^2 = x^2 + \sin xy$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin xy)$$

$$2y\frac{dy}{dx} = 2x + \left(\frac{d}{dx}(xy)\right)\cos xy$$

$$2y\frac{dy}{dx} = 2x + \left(y + x\frac{dy}{dx}\right)\cos xy$$

$$2y\frac{dy}{dx} = 2x + y\cos xy + x\cos xy\frac{dy}{dx}$$

$$(2y - x\cos xy)\frac{dy}{dx} = 2x + y\cos xy$$

$$\frac{dy}{dx} = \frac{2x + y\cos xy}{2y - x\cos xy}$$

$$y^2 = x^2 + \sin xy$$

$$2yy' = 2x + (y + xy')\cos xy$$

$$(2y - x\cos xy)y' = 2x + y\cos xy$$

$$\frac{dy}{dx} = \frac{2x + y\cos xy}{2y - x\cos xy}$$

Ex $2x^3 - 3y^2 = 8$ $\frac{d^2y}{dx^2}$

$6x^2 - 6yy' = 0$ $\quad x^2 = yy'$

$y' = \frac{x^2}{y}$

$x^2 - yy' = 0$

$2x - ((y')^2 + yy'') = 0$

$2x - \frac{x^4}{y^2} - yy'' = 0$

$yy'' = 2x - \frac{x^4}{y^2} = \frac{2xy^2 - x^4}{y^2}$

$y'' = \frac{2xy^2 - x^4}{y^3}$

#1 $y^2 + x^2 - 2y - 4x = 4$ $\frac{dy}{dx}$

$2yy' + 2x - 2y' - 4 = 0$

$(y-1)y' = 2 - x$

$\frac{dy}{dx} = \frac{2-x}{y-1}$

#3 $x + \sqrt{x}\sqrt{y} = y^2$

$$1 + \frac{1}{2\sqrt{x}}\sqrt{y} + \frac{\sqrt{x}}{2\sqrt{y}}y' = 2yy'$$

$$1 + \frac{\sqrt{y}}{2\sqrt{x}} = \left(2y - \frac{1}{2}\frac{\sqrt{x}}{\sqrt{y}}\right)y'$$

$$\frac{2\sqrt{x} + \sqrt{y}}{2\sqrt{x}} = \left(\frac{4y\sqrt{y} - \sqrt{x}}{2\sqrt{y}}\right)y'$$

$$\frac{dy}{dx} = \sqrt{\frac{y}{x}} \cdot \frac{2\sqrt{x} + \sqrt{y}}{4y\sqrt{y} - \sqrt{x}}$$

$$= \frac{2\sqrt{xy} + y}{4y\sqrt{xy} - x}$$

#4 $x^2y + xy^2 = 6$

$$2xy + x^2y' + y^2 + 2xyy' = 0$$

$$(x^2 + 2xy)y' = -y^2 - 2xy$$

$$\frac{dy}{dx} = -\frac{y(1+2x)}{x(x+2y)} \qquad \frac{-y^2 - 2xy}{x^2 + 2xy}$$

#6 $y^2 = \frac{x-1}{x+1}$

$2yy' = \frac{2}{(x+1)^2}$

$\frac{dy}{dx} = \frac{1}{y(x+1)^2}$

$xy^2 + y^2 = x - 1$

$y^2 + 2yxy' + 2yy' = 1$

$(2xy + 2y)y' = 1 - y^2$

$y' = \frac{1-y^2}{2y(x+1)}$

$$\boxed{y = \pm\left(\frac{x-1}{x+1}\right)^{1/2}}$$

$y^3 = \frac{x-1}{x+1}$

$y = \left(\frac{x-1}{x+1}\right)^{1/3}$

$y' = \frac{1}{3}\left(\frac{2}{(x+1)^2}\right)\left(\frac{x-1}{x+1}\right)^{-2/3}$

$= \frac{2}{3}\frac{1}{(x+1)^2}\frac{(x+1)^{2/3}}{(x-1)^{2/3}}$

$= \frac{2}{3}\frac{1}{(x-1)^{2/3}(x+1)^{4/3}}$