$\cos^2\theta + \sin^2\theta = 1$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1 \Rightarrow \cos^2 A = \frac{1+\cos 2A}{2}$

$= 1 - 2\sin^2 A \Rightarrow \sin^2 A = \frac{1-\cos 2A}{2}$

$\cos^2 A = \frac{1+\cos 2A}{2}$ $\qquad A = \frac{x}{2}$

$\cos^2 \frac{x}{2} = \frac{1+\cos x}{2} \Rightarrow \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\sin^2 \frac{A}{2} = \frac{1-\cos A}{2}$ $\Big\}$ $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$\pm$? G ??

Ex $\sin A = \frac{3}{5}$ $A \in Q\,II$ $\frac{90^\circ}{2} < \frac{A}{2} < \frac{180^\circ}{2}$

$\cos A = -\frac{4}{5}$ $\frac{A}{2} \in Q\,I$

a) $\sin 2A = 2\sin A \cos A$

$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= -\frac{24}{25}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{16}{25} - \frac{9}{25}$ $\frac{16-9}{25}$

$= \frac{7}{25}$

c) $\tan 2A = -\frac{24}{7}$ $A/2 \in Q\,I$

d) $\sin \frac{A}{2} = \sqrt{\frac{1}{2}(1-\cos A)}$

$= \sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)} = \sqrt{\frac{9}{10}}$ $\frac{9}{2 \mid 10}$

$= \frac{3}{\sqrt{10}}$

e) $\cos \frac{A}{2} = \sqrt{\frac{1}{2}(1+\cos A)}$

$= \sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)}$

$= \frac{1}{\sqrt{10}}$

f) $\tan A/2 = 3$

3 #4 $\cos A = \frac{5}{13}$ $A \in Q\text{IV}$ $\frac{270^\circ}{2} < \frac{A}{2} < \frac{360^\circ}{2}$

$\sin A = -\frac{12}{13}$ $\frac{A}{2} \in Q\text{II}$

a) $\sin 2A = 2\sin A\cos A$

$= 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right)$

$= -\frac{120}{169}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{25}{169} - \frac{144}{169}$

$= -\frac{119}{169}$

c) $\tan 2A = \frac{120}{119}$

d) $\sin\frac{A}{2} = \sqrt{\frac{1}{2}(1-\cos A)}$

$= \sqrt{\frac{1}{2}\left(1-\frac{5}{13}\right)}$ $\sqrt{\frac{8}{2(13)}}$

$= \frac{2}{\sqrt{13}}$

e) $\cos\frac{A}{2} = -\sqrt{\frac{1}{2}(1+\cos A)}$

$= -\sqrt{\frac{1}{2}\left(1+\frac{5}{13}\right)}$ $\sqrt{\frac{18}{2\ 13}}$

$= \frac{-3}{\sqrt{13}}$

f) $\tan\frac{A}{2} = -\frac{2}{3}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$\tan \frac{A}{2} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$$

$$\tan 15^\circ = \tan\left(\frac{30}{2}\right)^\circ$$

$$= \frac{1-\cos 30^\circ}{\sin 30^\circ}$$

$$= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{\frac{2-\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \boxed{2-\sqrt{3}}$$

Prove: $\sin^2\frac{x}{2} = \frac{\tan x - \sin x}{2\tan x}$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}\cdot\frac{\tan x}{\tan x}$$

$$= \frac{\tan x - \cos x\,\frac{\sin x}{\cos x}}{2\tan x}$$

$$= \frac{\tan x - \sin x}{2\tan x} \quad ✓$$

#21 $\cos 3x = \cos^3 x - 3\cos x \sin^2 x$

$$\cos 3x = \cos(x + 2x)$$
$$= \cos x \cos 2x - \sin x \sin 2x$$
$$= \cos x(\cos^2 x - \sin^2 x) - \sin x(2\sin x \cos x)$$
$$= \cos^3 x - \cos x \sin^2 x - 2\cos x \sin^2 x$$
$$= \cos^3 x - 3\cos x \sin^2 x \checkmark$$

#30 $\cos 4x = \cos^4 x - 6\sin^2 x \cos^2 x + \sin^4 x$

$$\cos 4x = \cos 2(2x)$$
$$= \cos^2 2x - \sin^2 2x$$
$$\boxed{= (\cos 2x)^2 - (\sin 2x)^2}$$
$$= (\cos^2 x - \sin^2 x)^2 - (2\sin x \cos x)^2$$
$$= \cos^4 x - 2\sin^2 x \cos^2 x + \sin^4 x - 4\sin^2 x \cos^2 x$$
$$= \cos^4 x - 6\sin^2 x \cos^2 x + \sin^4 x \checkmark$$

#44 $\sec^2\frac{x}{2} = \dfrac{2\sec x + 2}{\sec x + 2 + \cos x}$

$$\sec^2\frac{x}{2} = \frac{1}{\cos^2\frac{x}{2}}$$

$$= \frac{1}{\frac{1+\cos x}{2}}$$

$$= \frac{2}{1+\cos x} \cdot \frac{1+\sec x}{1+\sec x}$$

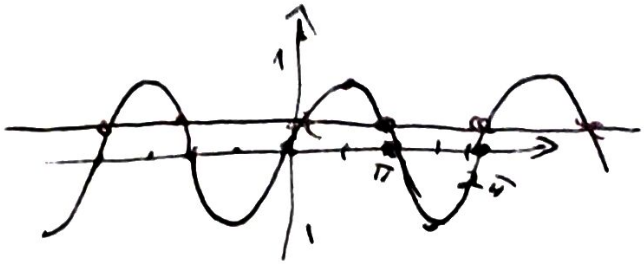
$$= \frac{2+2\sec x}{1+\sec x+\cos x+\underbrace{\cos x\sec x}_{=1}}$$

$$= \frac{2+2\sec x}{2+\sec x+\cos x}$$

5.4 Solving Trig. eqns

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$



$\theta = \frac{\pi}{6} + 2n\pi$

$\theta = \frac{5\pi}{6} + 2n\pi$

		$\frac{\pi}{2}$
Q I	$\frac{\pi}{n}$	
Q II	$\frac{(n-1)\pi}{n}$	$\frac{11\pi}{12}$
Q III	$\frac{(n+1)\pi}{n}$	$\frac{13\pi}{12}$
Q IV	$\frac{(2n-1)\pi}{n}$	$\frac{23\pi}{12}$

$\sin x \tan x = \sin x$

$\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0 \qquad \tan x = 1$

$\sin x = 0$

$\tan x = 1$

$x = 0, \pm\pi, \pm 2\pi, \ldots$

$= n\pi$

$x = \pm\frac{\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{9\pi}{4}, \ldots$

$x = \frac{\pi}{4} + n\pi$

Ex Solve $2\sin^2 t - \cos t - 1 = 0$

$2(1-\cos^2 t) - \cos t - 1 = 0$

$2 - 2\cos^2 t - \cos t - 1 = 0$

$-2\cos^2 t - \cos t + 1 = 0$

$\cos t = -1 \qquad \cos t = \frac{1}{2}$

$t = \pi \qquad t = \pm\frac{\pi}{3}, \pm\frac{5\pi}{3}$

$t = \pi + 2n\pi, \; t = \frac{\pi}{3} + 2n\pi, \; t = \frac{5\pi}{3} + 2n\pi$

$[0, 2\pi)$ $\quad t = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

Ex $4\sin^2 x \tan x - \tan x = 0 \qquad [0, 2\pi)$

$\tan x(4\sin^2 x - 1) = 0$

$\tan x = 0 \qquad 4\sin^2 x - 1 = 0$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4} \Rightarrow$

$\sin x = \pm\frac{1}{2}$

$x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Ex Solve: $\csc^4 2u - 4 = 0 \quad [0, 2\pi)$

$(\csc^2 2u - 2)(\csc^2 2u + 2) = 0$

$\csc^2 2u = 2 \qquad \csc^2 2u = -2$ #

$\csc 2u = \pm\sqrt{2} = \dfrac{1}{\sin 2u}$

$\sin 2u = \pm \dfrac{1}{\sqrt{2}}$

$2u = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

$u = \dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{9\pi}{8}, \dfrac{11\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}$.

Ex $5\sin\theta\tan\theta - 10\tan\theta + 3\sin\theta - 6 = 0$

$5\tan\theta(\sin\theta - 2) + 3(\sin\theta - 2) = 0$

$(5\tan\theta + 3)(\sin\theta - 2) = 0$

$\tan\theta = -\dfrac{3}{5} \qquad \sin\theta = 2 > 1$ #

$\theta = \tan^{-1}\dfrac{3}{5}$

$\begin{cases} \theta = \pi - \tan^{-1}\dfrac{3}{5} \\ \theta = 2\pi - \tan^{-1}\dfrac{3}{5} \end{cases}$

#10 $2\sin^2 x = 1 - \sin x \quad [0, 2\pi)$

$2\sin^2 x + \sin x - 1 = 0$

$\sin x = -1 \qquad \sin x = \frac{1}{2}$

$\boxed{x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$

12 $(1 - \sin x)^2 = (\sqrt{3}\cos x)^2 \quad [0, 2\pi)$

$1 - 2\sin x + \sin^2 x = 3\cos^2 x$

$\sin^2 x - 2\sin x + 1 - 3(1 - \sin^2 x) = 0$

$4\sin^2 x - 2\sin x - 2 = 0$

$\sin x = 1 \qquad \sin x = \frac{-1}{2}$

$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Check

$1 - \sin x = \sqrt{3}\cos x$

$1 = \sqrt{3}\cos x + \sin x$

$\frac{1}{2} = \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$\cos\frac{5\pi}{3} = \cos x \cos\frac{\pi}{6} + \sin x \sin\frac{\pi}{6}$

$\cos\frac{\pi}{3} = \cos\left(x - \frac{\pi}{6}\right)$

$x - \frac{\pi}{6} = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{2}$

$x - \frac{\pi}{6} = \frac{5\pi}{3} \Rightarrow x = \frac{11\pi}{6}$

2d $\sin\theta - \cos\theta = 1$

$1 \quad 0$

$0 \quad -(-1)$

$\theta = \frac{\pi}{2}, \pi$

$$\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\cos\frac{\pi}{4}\sin\theta - \sin\frac{\pi}{4}\cos\theta = \sin\frac{\pi}{4}, \frac{3\pi}{4}$$

$$\sin\left(\theta - \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$$

$$\sin\frac{3\pi}{4}$$

$$\theta - \frac{\pi}{4} = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}$$

$$\theta - \frac{\pi}{4} = \frac{3\pi}{4} \Rightarrow \theta = \pi$$