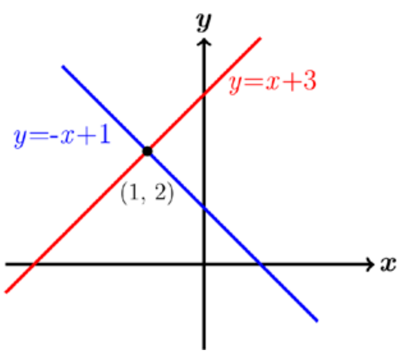
# *Lecture Four*

## *Section* 4.1 – System of linear Equations

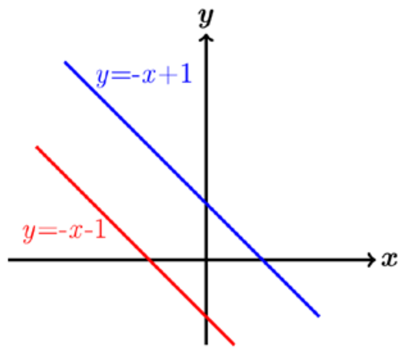
### Solving Systems of Equations

1. Graphically
2. Substitution Method
3. Elimination Method

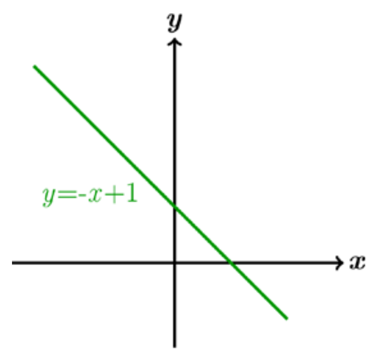
### *Graphically*




***One solution (lines intersect)***
***Consistent***
***Independent***




***No Solution (lines // )***
***Inconsistent***
***Independent***




***Infinite solution***
***Consistent***
***Dependent***

### *Substitution Method*

Solve: $\begin{cases} 3x + 2y = 11 & (1) \\ -x + y = 3 & (2) \end{cases}$

***Solution***

*From* $(2) \rightarrow y = x + 3 \quad (3)$

$(1) \Rightarrow 3x + 2(x + 3) = 11$

$$3x + 2x + 6 = 11$$

$$5x + 6 = 11$$

$$5x + 6 - 6 = 11 - 6$$

$$5x = 5$$

$$x = 1$$

$From\ (3) \to y = 1 + 3 = 4$

$Solution: \boxed{(1,\ 4)}$

## *Elimination* Method

Solve: $\begin{cases} 3x - 4y = 1 & (1) \\ 2x + 3y = 12 & (2) \end{cases}$

***Solution***

$$\begin{aligned} -2\times)\quad & 3x - 4y = 1 \\ 3\times)\quad & 2x + 3y = 12 \end{aligned}$$

$$\begin{array}{r} -6x + 8y = -2 \\ 6x + 9y = 36 \\ \hline 17y = 34 \end{array}$$

$y = \frac{34}{17} = 2$

$$\begin{aligned} From\ (1) \Rightarrow 3x &= 1 + 4y \\ 3x &= 1 + 4(2) \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

$Solution: \boxed{(3,\ 2)}$

# *Matrices*

$$\begin{array}{cc} & \begin{array}{ccc} & \textit{Column} & \\ C_1 & C_2 & C_3 \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{array}{r} Row\ 1 \rightarrow R_1 \\ Row\ 2 \rightarrow R_2 \\ Row\ 3 \rightarrow R_3 \end{array} & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \end{array}$$

This is called Matrix (*Matrices*)

Each number in the array is an ***element*** or ***entry***

The matrix is said to be of order $m$ x $n$

$m$: numbers of rows,

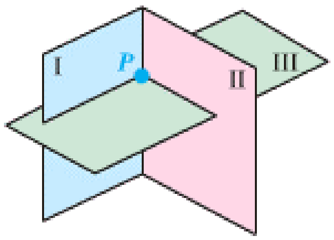
$n$: number of columns

When $m = n$, then matrix is said to be ***square***.

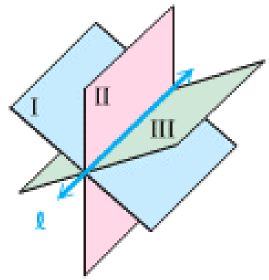
*Given the system equations*

$$\begin{aligned} 3x + \ \ y + 2z &= 31 \\ x + \ \ y + 2z &= 19 \\ x + 3y + 2z &= 25 \end{aligned}$$

The ***augmented matrix*** form is: $\left[\begin{array}{ccc|c} 3 & 1 & 2 & 31 \\ 1 & 1 & 2 & 19 \\ 1 & 3 & 2 & 25 \end{array}\right]$

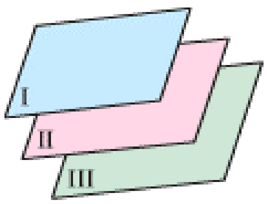



A single solution

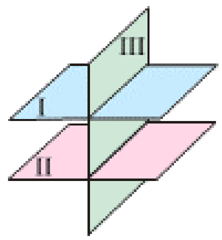



Points of a line in common

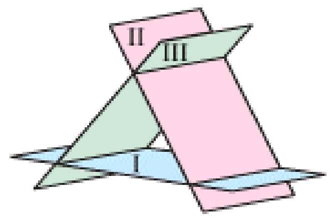

All points in common



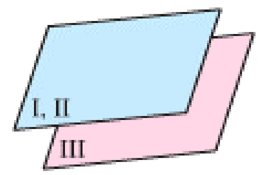

No points in common




No points in common




No points in common




No points in common

## *Gaussian* Elimination

### *Example*

Use the Gaussian elimination method to solve the system

$$\begin{aligned} 3x + \ \ y + 2z &= 31 \\ x + \ \ y + 2z &= 19 \\ x + 3y + 2z &= 25 \end{aligned}$$

***Solution***

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 3 & 1 & 2 & 31 \\ 1 & 3 & 2 & 25 \end{array}\right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - R_1 \end{array} \qquad \begin{array}{cccc} 3 & 1 & 2 & 31 \\ -3 & -3 & -6 & -57 \\ \hline 0 & -2 & -4 & -26 \end{array} \qquad \begin{array}{cccc} 1 & 3 & 2 & 25 \\ -1 & -1 & -2 & -19 \\ \hline 0 & 2 & 0 & 6 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & -2 & -4 & -26 \\ 0 & 2 & 0 & 6 \end{array}\right] \begin{array}{l} \\ -\frac{1}{2}R_2 \\ \\ \end{array} \qquad \begin{array}{cccc} 0 & 1 & 2 & 13 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 2 & 0 & 6 \end{array}\right] \begin{array}{l} \\ \\ R_3 - 2R_2 \end{array} \qquad \begin{array}{cccc} 0 & 2 & 0 & 6 \\ 0 & -2 & -4 & -26 \\ \hline 0 & 0 & -4 & -20 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & -4 & -20 \end{array}\right] \begin{array}{l} \\ \\ -\frac{1}{4}R_3 \end{array} \qquad \begin{array}{cccc} 0 & 0 & 1 & 5 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 1 & 5 \end{array}\right] \Rightarrow \begin{array}{rl} x + y + 2z = 19 & (3) \\ y + 2z = 13 & (2) \\ z = 5 & (1) \end{array}$$

$(2) \Rightarrow y = 13 - 2z = 13 - 2(5) = 3$

$(3) \Rightarrow \ x = 19 - y - 2z = 19 - 3 - 10 = 6$

$\Rightarrow$ **(6, 3, 5)**

# *Gauss-Jordan* Elimination

## *Example*

Use the Gauss-Jordan method to solve the system

$$\begin{aligned} 3x + \ \ y + 2z &= 31 \\ x + \ \ y + 2z &= 19 \\ x + 3y + 2z &= 25 \end{aligned}$$

***Solution***

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 3 & 1 & 2 & 31 \\ 1 & 3 & 2 & 25 \end{array}\right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - R_1 \end{array}$$

$$\begin{array}{cccc} 3 & 1 & 2 & 31 \\ -3 & -3 & -6 & -57 \\ \hline 0 & -2 & -4 & -26 \end{array} \qquad \begin{array}{cccc} 1 & 3 & 2 & 25 \\ -1 & -1 & -2 & -19 \\ \hline 0 & 2 & 0 & 6 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & -2 & -4 & -26 \\ 0 & 2 & 0 & 6 \end{array}\right] \ -\tfrac{1}{2}R_2 \qquad 0 \quad 1 \quad 2 \quad 13$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 2 & 0 & 6 \end{array}\right] \begin{array}{l} R_1 - R_2 \\ \\ R_3 - 2R_2 \end{array}$$

$$\begin{array}{cccc} 0 & 2 & 0 & 6 \\ 0 & -2 & -4 & -26 \\ \hline 0 & 0 & -4 & -20 \end{array} \qquad \begin{array}{cccc} 1 & 1 & 2 & 19 \\ 0 & -1 & -2 & -13 \\ \hline 1 & 0 & 0 & 6 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & -4 & -20 \end{array}\right] \ -\tfrac{1}{4}R_3 \qquad 0 \quad 0 \quad 1 \quad 5$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 1 & 5 \end{array}\right] \ R_2 - 2R_3 \qquad \begin{array}{cccc} 0 & 1 & 2 & 13 \\ 0 & 0 & -2 & -10 \\ \hline 0 & 1 & 0 & 3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

***Solution*: (6, 3, 5)**

### *Example*

Use the Gaussian elimination method to solve the system

$$\begin{aligned} 2x + \ y + 2z &= 4 \\ 2x + 2y \quad &= 5 \\ 2x - \ y + 6z &= 2 \end{aligned}$$

***Solution***

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{array}\right] \ \tfrac{1}{2}R_1 \qquad\qquad 1 \quad \tfrac{1}{2} \quad 1 \quad 2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 2 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 2R_1 \end{array} \qquad \begin{array}{cccc} 2 & 2 & 0 & 5 \\ -2 & -1 & -2 & -4 \\ \hline 0 & 1 & -2 & 1 \end{array} \qquad \begin{array}{cccc} 2 & -1 & 6 & 2 \\ -2 & -1 & -2 & -4 \\ \hline 0 & -2 & 4 & -2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \end{array}\right] \begin{array}{l} \\ \\ R_3 + 2R_2 \end{array} \qquad \begin{array}{cccc} 0 & -2 & 4 & -2 \\ 0 & 2 & -4 & 2 \\ \hline 0 & 0 & 0 & 0 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{rl} x + \frac{1}{2}y + z = 2 & (3) \\ y - 2z = 1 & (2) \\ 0 = 0 & (1) \end{array}$$

From (1): $0 = 0$ is a true statement. Let ***z*** be the variable.

From (2): $y = 1 + 2z$

From (3):

$$x = -\frac{1}{2}y - z + 2$$

$$x = -\frac{1}{2}(1 + 2z) - z + 2$$

$$x = -\frac{1}{2} - z - z + 2$$

$$x = -2z + \frac{3}{2}$$

***Solution***: $\left(-2z + \frac{3}{2},\ 2z + 1,\ z\right)$

### *Example*

Use the Gaussian elimination method to solve the system

$$\begin{aligned} x+2y-5z &= -1 \\ 2x+3y-2z &= 2 \\ 3x+5y-7z &= 4 \end{aligned}$$

***Solution***

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & -1 \\ 2 & 3 & -2 & 2 \\ 3 & 5 & -7 & 4 \end{array}\right] \begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \end{array}$$

$$\begin{array}{cccc} 2 & 3 & -2 & 2 \\ -2 & -4 & 10 & 2 \\ \hline 0 & -1 & 8 & 4 \end{array} \qquad \begin{array}{cccc} 3 & 5 & -7 & 4 \\ -3 & -6 & 15 & 3 \\ \hline 0 & -1 & 8 & 7 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & -1 \\ 0 & -1 & 8 & 4 \\ 0 & -1 & 8 & 7 \end{array}\right] \begin{array}{l} \\ -R_2 \\ \\ \end{array} \qquad 0 \quad 1 \quad -8 \quad -4$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & -1 \\ 0 & 1 & -8 & -4 \\ 0 & -1 & 8 & 7 \end{array}\right] \begin{array}{l} \\ \\ R_3+R_2 \end{array} \qquad \begin{array}{cccc} 0 & -1 & 8 & 7 \\ 0 & 1 & -8 & -4 \\ \hline 0 & 0 & 0 & 3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & -1 \\ 0 & 1 & -8 & -4 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

From Row 3: $0 = 3$ is a False statement.

***No Solution*** or ***Inconsistent***

# *Exercises* *Section* 4.1 – System of linear Equations

(**1 – 15**) Use any method to solve the system equation (***elimination*** or ***substitution*** method)

**1.** $\begin{cases} 3x+2y=-4 \\ 2x-y=-5 \end{cases}$

**2.** $\begin{cases} 2x+5y=7 \\ 5x-2y=-3 \end{cases}$

**3.** $\begin{cases} 4x-7y=-16 \\ 2x+5y=9 \end{cases}$

**4.** $\begin{cases} 3x+2y=4 \\ 2x+y=1 \end{cases}$

**5.** $\begin{cases} 3x+4y=2 \\ 2x+5y=-1 \end{cases}$

**6.** $\begin{cases} 5x-2y=4 \\ -10x+4y=7 \end{cases}$

**7.** $\begin{cases} x-4y=-8 \\ 5x-20y=-40 \end{cases}$

**8.** $\begin{cases} 2x+y=3 \\ x-y=3 \end{cases}$

**9.** $\begin{cases} 2x+10y=-14 \\ 7x-2y=-16 \end{cases}$

**10.** $\begin{cases} 4x-3y=24 \\ -3x+9y=-1 \end{cases}$

**11.** $\begin{cases} 4x+2y=12 \\ 3x-2y=16 \end{cases}$

**12.** $\begin{cases} x+2y=-1 \\ 4x-2y=6 \end{cases}$

**13.** $\begin{cases} x-2y=5 \\ -10x+2y=4 \end{cases}$

**14.** $\begin{cases} 12x+15y=-27 \\ 30x-15y=-15 \end{cases}$

**15.** $\begin{cases} 4x-4y=-12 \\ 4x+4y=-20 \end{cases}$

(**16 – 27**) Perform the matrix row operation (or operations) and write the new matrix.

**16.** $\left[\begin{array}{cc|c} 1 & 4 & 7 \\ 3 & 5 & 0 \end{array}\right]$ $R_2-3R_1$

**17.** $\left[\begin{array}{cc|c} 1 & -3 & 1 \\ 2 & 1 & -5 \end{array}\right]$ $R_2-2R_1$

**18.** $\left[\begin{array}{cc|c} 1 & -3 & 3 \\ 5 & 2 & 19 \end{array}\right]$ $R_2-5R_1$

**19.** $\left[\begin{array}{cc|c} 2 & -3 & 8 \\ -6 & 9 & 4 \end{array}\right]$ $R_2+3R_1$

**20.** $\left[\begin{array}{cc|c} 2 & 3 & 11 \\ 1 & 2 & 8 \end{array}\right]$ $2R_2-R_1$

**21.** $\left[\begin{array}{cc|c} 3 & 5 & -13 \\ 2 & 3 & -9 \end{array}\right]$ $3R_2-2R_1$

**22.** $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 5 & 4 & 1 \end{array}\right]$ $R_3-5R_2$

**23.** $\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array}\right]$ $\begin{array}{l} R_2-3R_1 \\ R_3-R_1 \end{array}$

**24.** $\left[\begin{array}{ccc|c} 3 & 2 & 1 & 1 \\ 2 & 4 & 4 & 22 \\ -1 & -2 & 3 & 15 \end{array}\right]$ $\begin{array}{l} 3R_2-2R_1 \\ 3R_3+R_1 \end{array}$

**25.** $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & -4 & 2 & -7 \end{array}\right]$ $\begin{array}{l} R_2-2R_1 \\ R_3-3R_1 \end{array}$

**26.** $\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ 2 & -3 & 5 & -1 & 0 \\ 1 & 0 & 3 & 1 & -4 \\ -4 & 3 & 2 & -1 & 3 \end{array}\right]$ $\begin{array}{l} R_2-2R_1 \\ R_3-R_1 \\ R_4+4R_1 \end{array}$

**27.** $\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ -3 & 6 & -3 & -9 & 6 \\ 2 & 1 & 2 & 3 & 4 \\ 5 & 3 & 2 & -1 & -7 \end{array}\right]$ $\begin{array}{l} R_2+3R_1 \\ R_3-2R_1 \\ R_4-5R_1 \end{array}$

**(28 – 34)** Use the Gauss-Jordan method to solve the system

**28.** $\begin{cases} x - y + 5z = -6 \\ 3x + 3y - z = 10 \\ x + 3y + 2z = 5 \end{cases}$

**29.** $\begin{cases} 2x - y + 4z = -3 \\ x - 2y - 10z = -6 \\ 3x + 4z = 7 \end{cases}$

**30.** $\begin{cases} 4x + 3y - 5z = -29 \\ 3x - 7y - z = -19 \\ 2x + 5y + 2z = -10 \end{cases}$

**31.** $\begin{cases} x + 2y - 3z = -15 \\ 2x - 3y + 4z = 18 \\ -3x + y + z = 1 \end{cases}$

**32.** $\begin{cases} x + 2y + 3z = 10 \\ 4x + 5y + 6z = 11 \\ 7x + 8y + 9z = 12 \end{cases}$

**33.** $\begin{cases} 2x + y + 2z = 4 \\ 2x + 2y = 5 \\ 2x - y + 6z = 2 \end{cases}$

**34.** $\begin{cases} x_1 + x_2 + 2x_3 = 8 \\ -x_1 - 2x_2 + 3x_3 = 1 \\ 3x_1 - 7x_2 + 4x_3 = 10 \end{cases}$

**(35 – 69)** Use augmented elimination to solve linear system

**35.** $\begin{cases} 2x - 5y + 3z = 1 \\ x - 2y - 2z = 8 \end{cases}$

**36.** $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$

**37.** $\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \\ 3x - y + z = 9 \end{cases}$

**38.** $\begin{cases} 3y - z = -1 \\ x + 5y - z = -4 \\ -3x + 6y + 2z = 11 \end{cases}$

**39.** $\begin{cases} x + 3y + 4z = 14 \\ 2x - 3y + 2z = 10 \\ 3x - y + z = 9 \end{cases}$

**40.** $\begin{cases} x + 4y - z = 20 \\ 3x + 2y + z = 8 \\ 2x - 3y + 2z = -16 \end{cases}$

**41.** $\begin{cases} 2y - z = 7 \\ x + 2y + z = 17 \\ 2x - 3y + 2z = -1 \end{cases}$

**42.** $\begin{cases} -2x + 6y + 7z = 3 \\ -4x + 5y + 3z = 7 \\ -6x + 3y + 5z = -4 \end{cases}$

**43.** $\begin{cases} 2x - y + z = 1 \\ 3x - 3y + 4z = 5 \\ 4x - 2y + 3z = 4 \end{cases}$

**44.** $\begin{cases} 3x - 4y + 4z = 7 \\ x - y - 2z = 2 \\ 2x - 3y + 6z = 5 \end{cases}$

**45.** $\begin{cases} x - 2y - z = 2 \\ 2x - y + z = 4 \\ -x + y + z = 4 \end{cases}$

**46.** $\begin{cases} x + y + z = 3 \\ -y + 2z = 1 \\ -x + z = 0 \end{cases}$

**47.** $\begin{cases} 3x + y + 3z = 14 \\ 7x + 5y + 8z = 37 \\ x + 3y + 2z = 9 \end{cases}$

**48.** $\begin{cases} 4x - 2y + z = 7 \\ x + y + z = -2 \\ 4x + 2y + z = 3 \end{cases}$

**49.** $\begin{cases} 2x - 2y + z = -4 \\ 6x + 4y - 3z = -24 \\ x - 2y + 2z = 1 \end{cases}$

**50.** $\begin{cases} 9x + 3y + z = 4 \\ 16x + 4y + z = 2 \\ 25x + 5y + z = 2 \end{cases}$

**51.** $\begin{cases} 2x - y + 2z = -8 \\ x + 2y - 3z = 9 \\ 3x - y - 4z = 3 \end{cases}$

**52.** $\begin{cases} x - 3z = -5 \\ 2x - y + 2z = 16 \\ 7x - 3y - 5z = 19 \end{cases}$

**53.** $\begin{cases} x + 2y - z = 5 \\ 2x - y + 3z = 0 \\ 2y + z = 1 \end{cases}$

**54.** $\begin{cases} x + y + z = 6 \\ 3x + 4y - 7z = 1 \\ 2x - y + 3z = 5 \end{cases}$

**55.** $\begin{cases} 3x + 2y + 3z = 3 \\ 4x - 5y + 7z = 1 \\ 2x + 3y - 2z = 6 \end{cases}$

**56.** $\begin{cases} x-3y+z=2 \\ 4x-12y+4z=8 \\ -2x+6y-2z=-4 \end{cases}$

**57.** $\begin{cases} 2x-2y+z=-1 \\ x+2y-z=2 \\ 6x+4y+3z=5 \end{cases}$

**58.** $\begin{cases} x_1-5x_2+2x_3-2x_4=4 \\ x_2-3x_3-x_4=0 \\ 3x_1+2x_3-x_4=6 \\ -4x_1+x_2+4x_3+2x_4=-3 \end{cases}$

**59.** $\begin{cases} x_1+x_2+x_3+x_4=5 \\ x_1+2x_2-x_3-2x_4=-1 \\ x_1-3x_2-3x_3-x_4=-1 \\ 2x_1-x_2+2x_3-x_4=-2 \end{cases}$

**60.** $\begin{cases} 2x+8y-z+w=0 \\ 4x+16y-3z-w=-10 \\ -2x+4y-z+3w=-6 \\ -6x+2y+5z+w=3 \end{cases}$

**61.** $\begin{cases} 2x_1+x_2+3x_3=0 \\ x_1+2x_2=0 \\ x_2+x_3=0 \end{cases}$

**62.** $\begin{cases} 2x+2y+4z=0 \\ -y-3z+w=0 \\ 3x+y+z+2w=0 \\ x+3y-2z-2w=0 \end{cases}$

**63.** $\begin{cases} 2x+z+w=5 \\ y-w=-1 \\ 3x-z-w=0 \\ 4x+y+2z+w=9 \end{cases}$

**64.** $\begin{cases} 4y+z=20 \\ 2x-2y+z=0 \\ x+z=5 \\ x+y-z=10 \end{cases}$

**65.** $\begin{cases} x-y+2z-w=-1 \\ 2x+y-2z-2w=-2 \\ -x+2y-4z+w=1 \\ 3x-3w=-3 \end{cases}$

**66.** $\begin{cases} 2u-3v+w-x+y=0 \\ 4u-6v+2w-3x-y=-5 \\ -2u+3v-2w+2x-y=3 \end{cases}$

**67.** $\begin{cases} 6x_3+2x_4-4x_5-8x_6=8 \\ 3x_3+x_4-2x_5-4x_6=4 \\ 2x_1-3x_2+x_3+4x_4-7x_5+x_6=2 \\ 6x_1-9x_2+11x_4-19x_5+3x_6=1 \end{cases}$

**68.** $\begin{cases} 3x_1+2x_2-x_3=-15 \\ 5x_1+3x_2+2x_3=0 \\ 3x_1+x_2+3x_3=11 \\ -6x_1-4x_2+2x_3=30 \end{cases}$

**69.** $\begin{cases} x_1+3x_2-2x_3+2x_5=0 \\ 2x_1+6x_2-5x_3-2x_4+4x_5-3x_6=-1 \\ 5x_3+10x_4+15x_6=5 \\ 2x_1+6x_2+8x_4+4x_5+18x_6=6 \end{cases}$

**70.** At Snack Mix, caramel corn worth \$2.50 per *pound* is mixed with honey roasted missed nuts worth \$7.50 per *pound* in order to get 20 *lbs*. of a mixture worth \$4.50 per *pound*. How much of each snack is used?