

## Section 6.4 – Solving Right Triangle Trigonometry

### Example

In the right triangle  $ABC$  ( $C = 90^\circ$ ),  $A = 40^\circ$  and  $c = 12$  cm. Find  $a$ ,  $b$ , and  $B$ .

### Solution

$$\sin 40^\circ = \frac{a}{c} = \frac{a}{12}$$

$$a = 12 \sin 40^\circ$$

$$\approx 7.7 \text{ cm}$$

$$\cos 40^\circ = \frac{b}{c}$$

$$= \frac{b}{12}$$

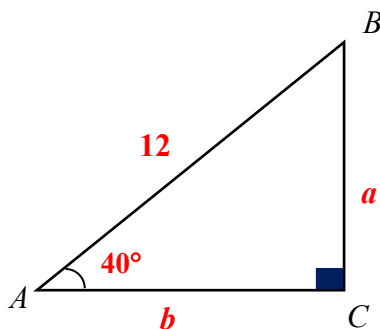
$$b = 12 \cos 40^\circ$$

$$\approx 9.2 \text{ cm}$$

$$B = 90^\circ - A$$

$$= 90^\circ - 40^\circ$$

$$= 50^\circ$$



### Example

A circle has its center at  $C$  and a radius of 18 inches. If triangle  $ADC$  is a right triangle and  $A = 35^\circ$ . Find  $x$ , the distance from  $A$  to  $B$ .

### Solution

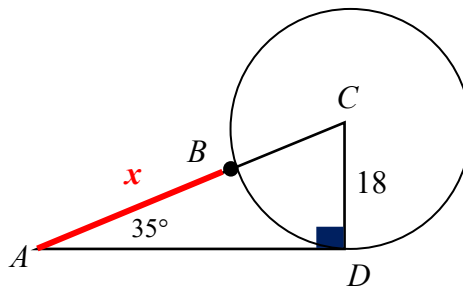
$$\sin 35^\circ = \frac{18}{x+18}$$

$$(x+18)\sin 35^\circ = 18$$

$$x+18 = \frac{18}{\sin 35^\circ}$$

$$x = \frac{18}{\sin 35^\circ} - 18$$

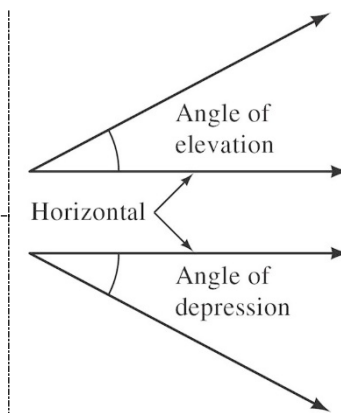
$$\approx 13 \text{ in}$$



## Definitions

An angle measured from the horizontal up is called an *angle of elevation*.

An angle measured from the horizontal down is called an *angle of depression*.



## Example

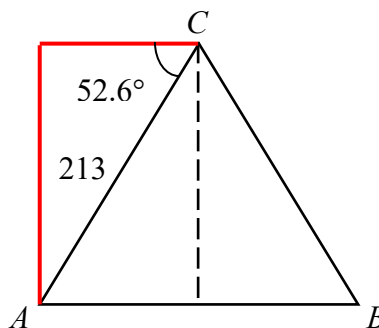
A man climbs 213 *meters* up the side of a pyramid. Find that the angle of depression to his starting point is  $52.6^\circ$ . How high off of the ground is he?

### Solution

$$\sin 52.6^\circ = \frac{h}{213}$$

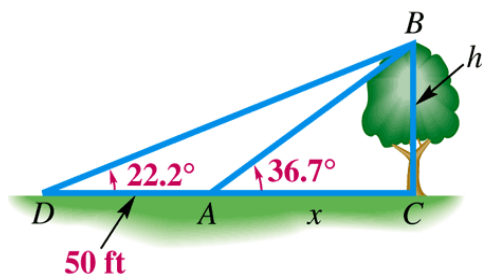
$$h = 213 \sin 52.6^\circ$$

$$h \approx 169 \text{ m}$$



### Example

From a given point on the ground, the angle of elevation to the top of a tree is  $36.7^\circ$ . From a second point, 50 feet back, the angle of elevation to the top of the tree is  $22.2^\circ$ . Find the height of the tree to the nearest foot.



### Solution

Triangle **DCB**

$$\Rightarrow \tan 22.2^\circ = \frac{h}{50 + x}$$

$$h = (50 + x) \tan 22.2^\circ$$

Triangle **ACB**

$$\Rightarrow \tan 36.7^\circ = \frac{h}{x}$$

$$h = x \tan 36.7^\circ$$

$$x \tan 36.7^\circ = (50 + x) \tan 22.2^\circ$$

$$x \tan 36.7^\circ = 50 \tan 22.2^\circ + x \tan 22.2^\circ$$

$$x \tan 36.7^\circ - x \tan 22.2^\circ = 50 \tan 22.2^\circ$$

$$x(\tan 36.7^\circ - \tan 22.2^\circ) = 50 \tan 22.2^\circ$$

$$x = \frac{50 \tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}$$

$$h = x \tan 36.7^\circ$$

$$= \left( \frac{50 \tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ} \right) \tan 36.7^\circ$$

$$\approx 45 \text{ ft}$$

The tree is about 45 feet tall.

OR

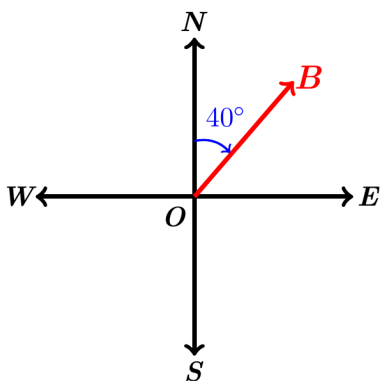
$$h = \frac{x \tan \alpha \tan \beta}{\tan \beta - \tan \alpha} = \frac{50 \tan 22.2^\circ \tan 36.7^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}$$

## Bearing

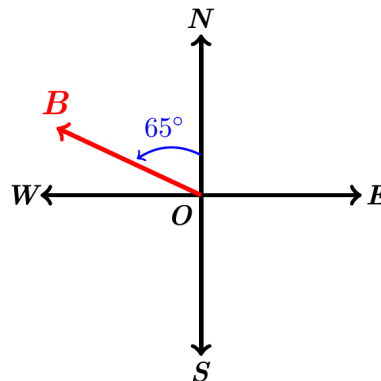
### Definition

The *bearing* of a line  $\ell$  is the acute angle formed by the *north-south* line and the line  $\ell$ .

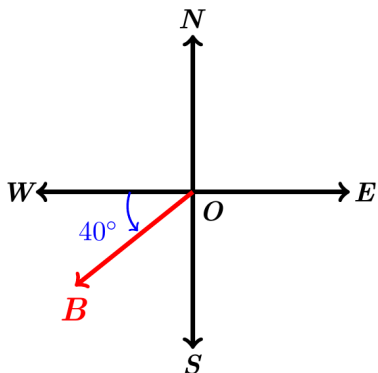
The notation used to designate the bearing of a line begins with *N* (for **north**) or *S* (for **south**), followed by the number of degrees in the angle, and ends with *E* (for **east**) or *W* (for **west**).



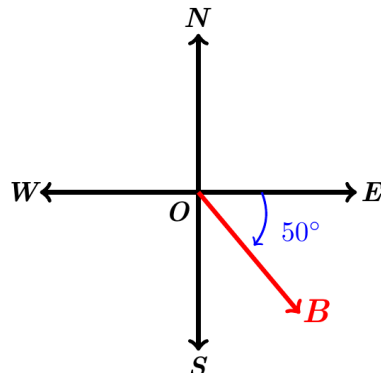
The bearing of  $B$  from  $O$  is  $N\ 40^\circ\ E$



The bearing of  $B$  from  $O$  is  $N\ 65^\circ\ W$



The bearing of  $B$  from  $O$  is  $W\ 40^\circ\ S$



The bearing of  $B$  from  $O$  is  $E\ 50^\circ\ S$

### Example

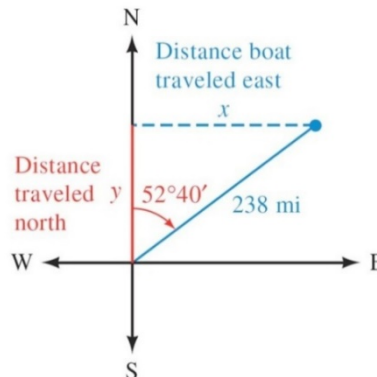
A boat travels on a course of bearing  $N\ 52^\circ\ 40'\ E$  for distance of 238 *miles*. How many miles north and how many miles east have the boat traveled?

### Solution

$$\begin{aligned} 52^\circ 40' &= 52^\circ + 40' \frac{1^\circ}{60'} \\ &\approx 52.6667^\circ \end{aligned}$$

$$\sin 52.6667^\circ = \frac{x}{238}$$

$$x = 238 \sin 52.6667^\circ$$



$$\approx 189 \text{ mi}$$

$$\cos 52.6667^\circ = \frac{y}{238}$$

$$y = 238 \cos 52.6667^\circ$$

$$\approx 144 \text{ mi}$$

### Example

A helicopter is hovering over the desert when it develops mechanical problems and is forced to land. After landing, the pilot radios his position to a pair of radar station located 25 miles apart along a straight road running north and south. The bearing of the helicopter from one station is N  $13^\circ$  E, and from the other it is S  $19^\circ$  E. After doing a few trigonometric calculations, one of the stations instructs the pilot to walk due west for 3.5 miles to reach the road. Is this information correct?

### Solution

In triangle  $AFC$

$$\tan 13^\circ = \frac{y}{x}$$

$$y = x \tan 13^\circ$$

In triangle  $BFC$

$$\tan 19^\circ = \frac{y}{25 - x}$$

$$y = (25 - x) \tan 19^\circ$$

$$y = y$$

$$(25 - x) \tan 19^\circ = x \tan 13^\circ$$

$$25 \tan 19^\circ - x \tan 19^\circ = x \tan 13^\circ$$

$$25 \tan 19^\circ = x \tan 13^\circ + x \tan 19^\circ$$

$$25 \tan 19^\circ = x(\tan 13^\circ + \tan 19^\circ)$$

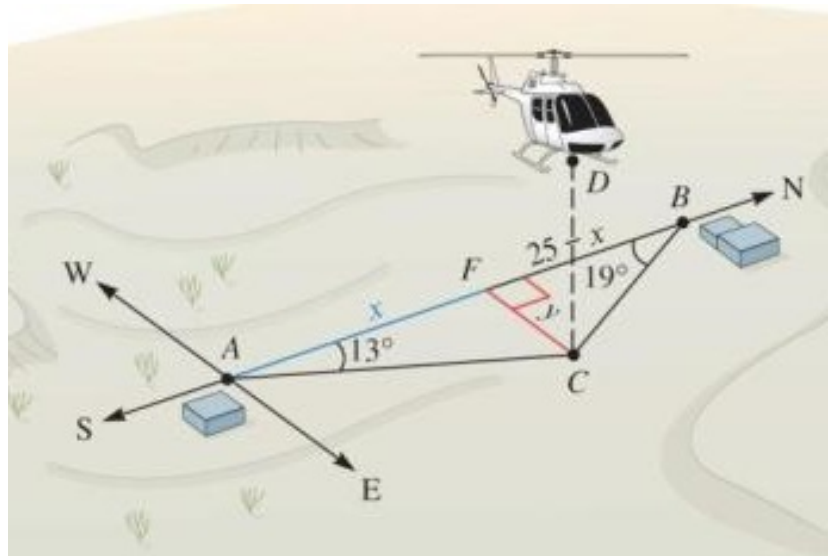
$$\frac{25 \tan 19^\circ}{\tan 13^\circ + \tan 19^\circ} = x$$

$$x = 14.966$$

$$y = x \tan 13^\circ$$

$$= 14.966 \tan 13^\circ$$

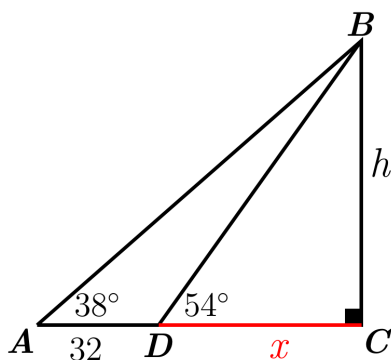
$$\approx 3.5 \text{ mi}$$



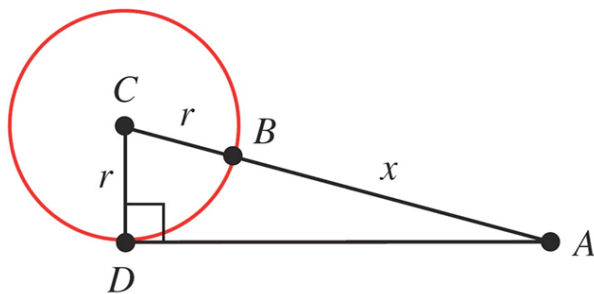
# Exercises

## Section 6.4 – Solving Right Triangle Trigonometry

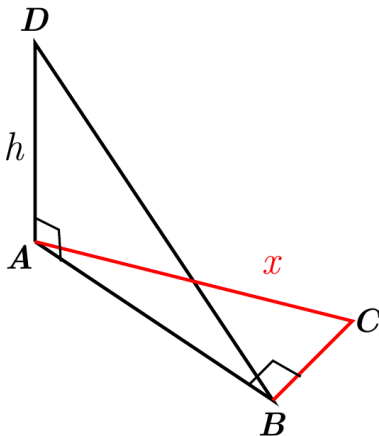
1. In the right triangle  $ABC$  ( $C = 90^\circ$ ),  $a = 29.43$  and  $c = 53.58$ . Find the remaining side and angles.
2. In the right triangle  $ABC$  ( $C = 90^\circ$ ),  $a = 2.73$  and  $b = 3.41$ . Find the remaining side and angles..
3. The two equal sides of an isosceles triangle are each  $24\text{ cm}$ . If each of the two equal angles measures  $52^\circ$ , find the length of the base and the altitude.
4. The distance from  $A$  to  $D$  is  $32\text{ feet}$ . Use the information in figure to solve  $x$ , the distance between  $D$  and  $C$ .



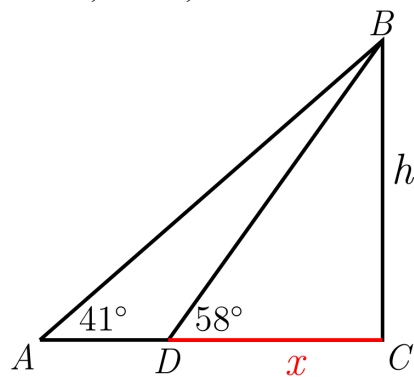
- (5 – 6) Find  $x$ .



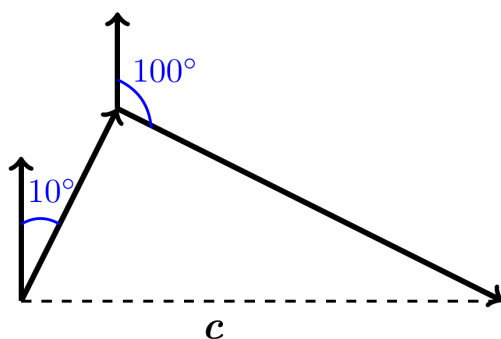
5. If  $C = 26^\circ$  and  $r = 19$
6. If  $C = 30^\circ$  and  $r = 15$
7. If  $\angle ABD = 53^\circ$ ,  $C = 48^\circ$ , and  $BC = 42$ , find  $x$  and then find  $h$ .



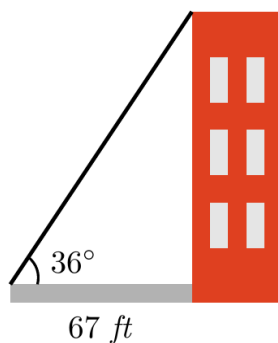
8. If  $A = 41^\circ$ ,  $\angle BDC = 58^\circ$ , and  $AB = 28$ , find  $h$ , then  $x$ .



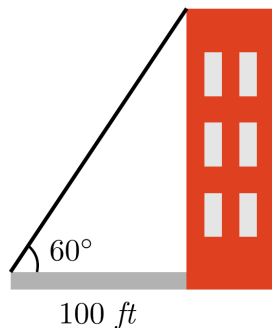
9. A plane flies 1.7 hours at 120 mph on a bearing of  $10^\circ$ . It then turns and flies 9.6 hours at the same speed on a bearing of  $100^\circ$ . How far is the plane from its starting point?



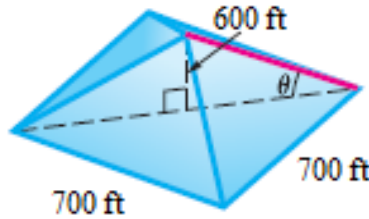
10. The shadow of a vertical tower is 67.0 feet long when the angle of elevation of the sun is  $36.0^\circ$ . Find the height of the tower.



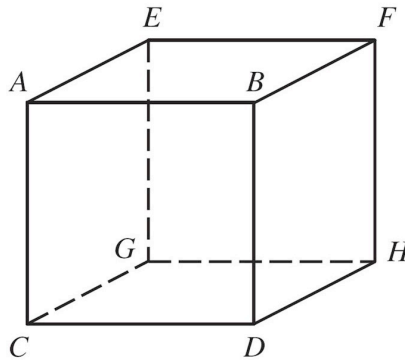
11. The shadow of a vertical tower is 100 feet long when the angle of elevation of the sun is  $60^\circ$ . Find the height of the tower.



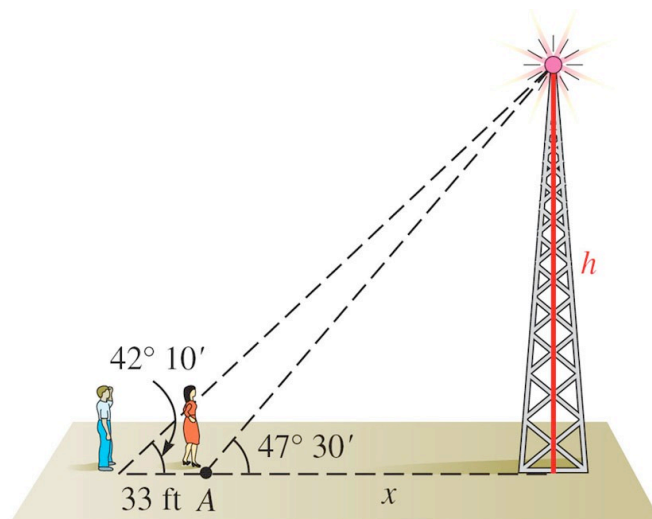
12. The base of a pyramid is square with sides  $700\text{ feet}$  long, and the height of the pyramid is  $600\text{ feet}$ . Find the angle of elevation of the edge indicated in the figure to two significant digits. (Hint: The base of the triangle in the figure is half the diagonal of the square base of the pyramid.)



13. If a  $73\text{-foot}$  flagpole casts a shadow  $51\text{ feet}$  long, what is the angle of elevation of the sun (to the nearest tenth of a degree)?
14. If a  $75\text{-foot}$  flagpole casts a shadow  $43\text{ feet}$  long, to the nearest  $10\text{ minutes}$  what is the angle of elevation of the sun from the tip of the shadow?
15. Suppose each edge of the cube is  $3.00\text{ inches}$  long. Find the measure of the angle formed by diagonals  $DE$  and  $DG$ . Round your answer to the nearest tenth of a degree.



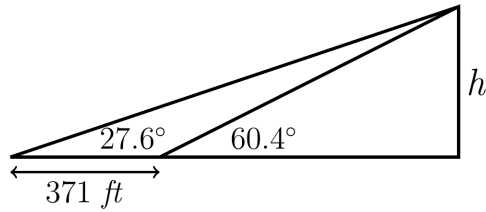
16. A person standing at point  $A$  notices that the angle of elevation to the top of the antenna is  $47^\circ 30'$ . A second person standing  $33.0\text{ feet}$  farther from the antenna than the person at  $A$  finds the angle of elevation to the top of the antenna to be  $42^\circ 10'$ . How far is the person at  $A$  from the base of the antenna?



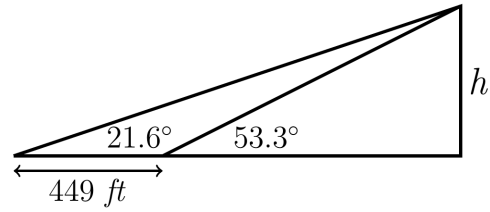


(17 – 22) Find  $h$  as indicated in the figure.

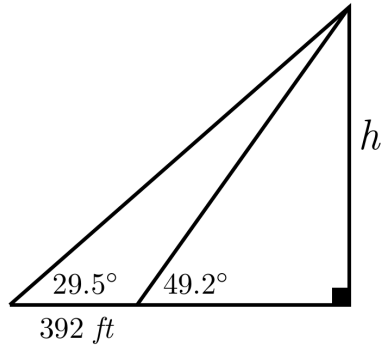
17.



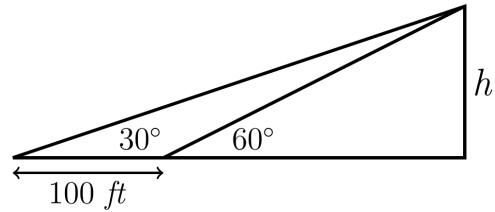
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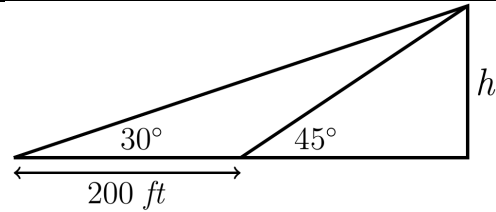
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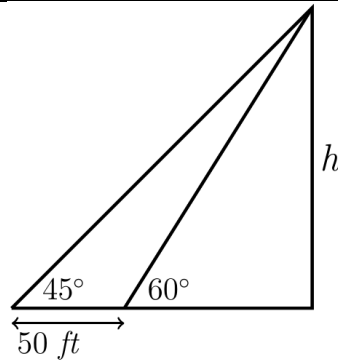
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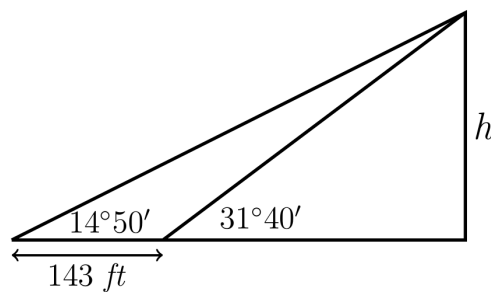
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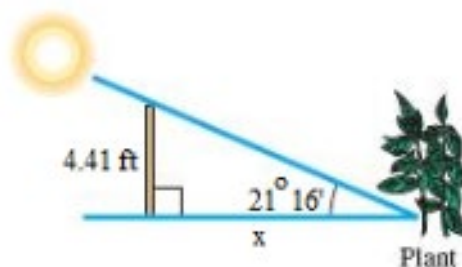
22.



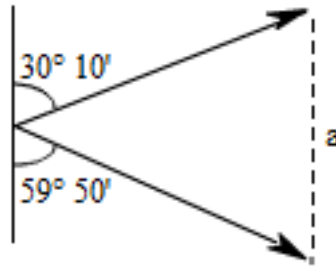
23. The angle of elevation from a point on the ground to the top of a pyramid is  $31^\circ 40'$ . The angle of elevation from a point 143 feet farther back to the top of the pyramid is  $14^\circ 50'$ . Find the height of the pyramid.



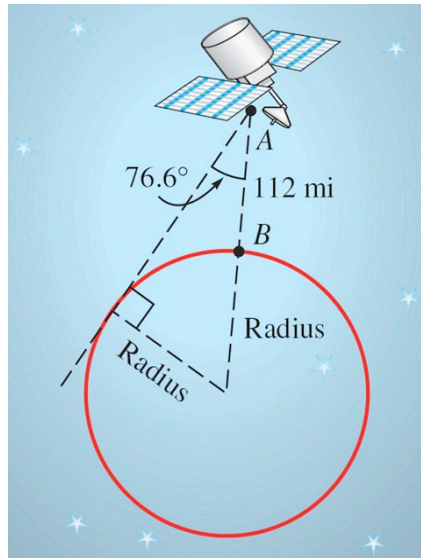
24. In one area, the lowest angle of elevation of the sun in winter is  $21^\circ 16'$ . Find the minimum distance,  $x$ , that a plant needing full sun can be placed from a fence 4.41 feet high.



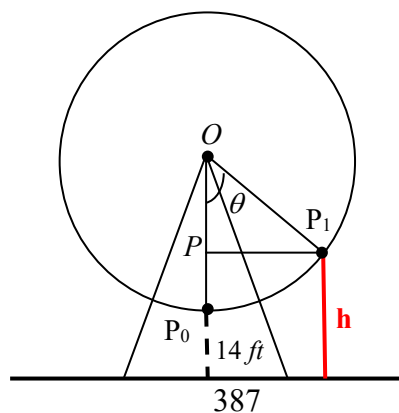
25. A ship leaves its port and sails on a bearing of  $N 30^\circ 10' E$ , at speed  $29.4 \text{ mph}$ . Another ship leaves the same port at the same time and sails on a bearing of  $S 59^\circ 50' E$ , at speed  $17.1 \text{ mph}$ . Find the distance between the two ships after  $2 \text{ hrs}$ .



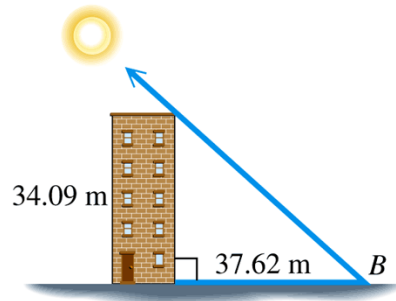
26. Radar stations  $A$  and  $B$  are on the east-west line,  $3.7 \text{ km}$  apart. Station  $A$  detects a plane at  $C$ , on a bearing of  $61^\circ$ . Station  $B$  simultaneously detects the same plane, on a bearing of  $331^\circ$ . Find the distance from  $A$  to  $C$ .
27. Suppose the figure below is exaggerated diagram of a plane flying above the earth. If the plane is  $4.55 \text{ miles}$  above the earth and the radius of the earth is  $3,960 \text{ miles}$ , how far is it from the plane to the horizon? What is the measure of angle  $A$ ?



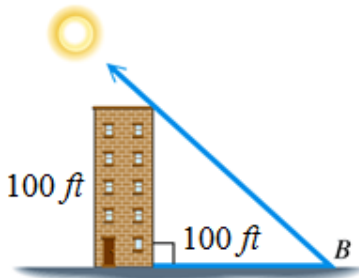
28. The Ferry wheel has a  $250 \text{ feet}$  diameter and  $14 \text{ feet}$  above the ground. If  $\theta$  is the central angle formed as a rider moves from position  $P_0$  to position  $P_1$ , find the rider's height above the ground  $h$  when  $\theta$  is  $45^\circ$ .



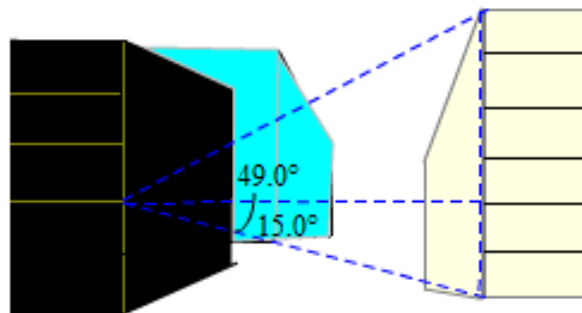
29. The length of the shadow of a building 34.09 *m* tall is 37.62 *m*. Find the angle of the elevation of the sun.



30. The length of the shadow of a building 100 *feet* tall is 100 *feet*. Find the angle of the elevation of the sun.

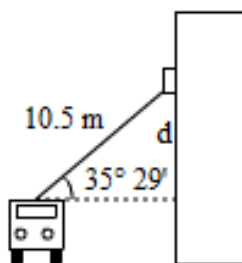


31. San Luis Obispo, California is 12 *miles* due north of Grover Beach. If Arroyo Grande is 4.6 *miles* due east of Grover Beach, what is the bearing of San Luis Obispo from Arroyo Grande?
32. The bearing from *A* to *C* is S  $52^\circ$  E. The bearing from *A* to *B* is N  $84^\circ$  E. The bearing from *B* to *C* is S  $38^\circ$  W. A plane flying at 250 *mph* takes 2.4 hours to go from *A* to *B*. Find the distance from *A* to *C*.
33. From a window 31.0 *feet*. above the street, the angle of elevation to the top of the building across the street is  $49.0^\circ$  and the angle of depression to the base of this building is  $15.0^\circ$ . Find the height of the building across the street.

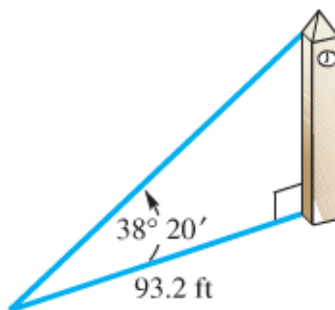


34. A man wondering in the desert walks 2.3 *miles* in the direction S  $31^\circ$  W. He then turns  $90^\circ$  and walks 3.5 *miles* in the direction N  $59^\circ$  W. At that time, how far is he from his starting point, and what is his bearing from his starting point?

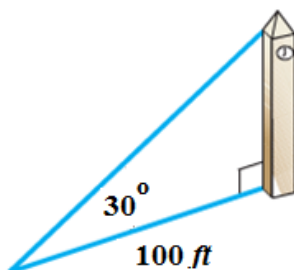
35. A 10.5-*m* fire truck ladder is leaning against a wall. Find the distance ***d*** the ladder goes up the wall (above the fire truck) if the ladder makes an angle of  $35^\circ 29'$  with the horizontal.



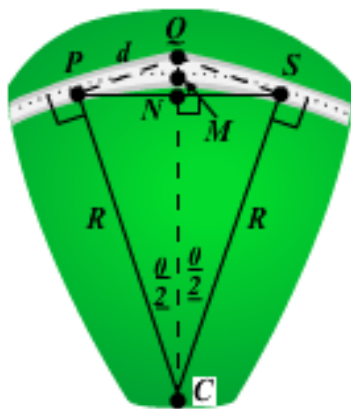
36. The angle of elevation from a point 93.2 *feet* from the base of a tower to the top of the tower is  $38^\circ 20'$ . Find the height of the tower.



37. The angle of elevation from a point 100 *feet* from the base of a tower to the top of the tower is  $30^\circ$ . Find the height of the tower.



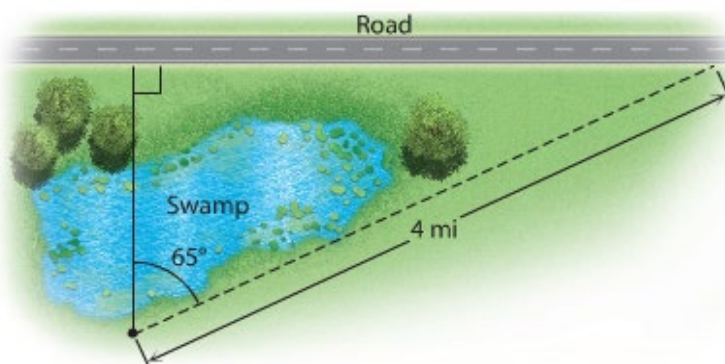
38. A basic curve connecting two straight sections of road is often circular. In the figure, the points ***P*** and ***S*** mark the beginning and end of the curve. Let ***Q*** be the point of intersection where the two straight sections of highway leading into the curve would meet if extended. The radius of the curve is ***R***, and the central angle denotes how many degrees the curve turns.



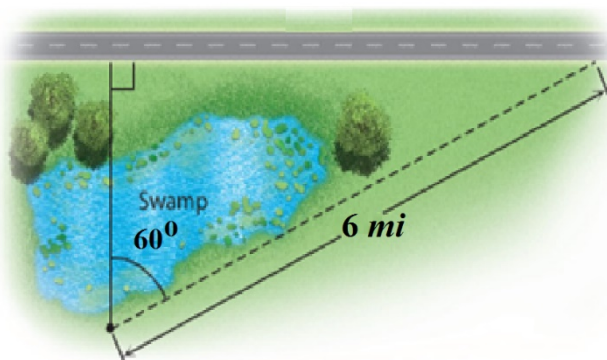
a) If  $R = 965 \text{ ft.}$  and  $\theta = 37^\circ$ , find the distance  $d$  between  $P$  and  $Q$ .

b) Find an expression in terms of  $R$  and  $\theta$  for the distance between points  $M$  and  $N$ .

39. Jane was hiking directly toward a long straight road when she encountered a swamp. She turned  $65^\circ$  to the right and hiked  $4 \text{ mi}$  in that direction to reach the road. How far was she from the road when she encountered the swamp?

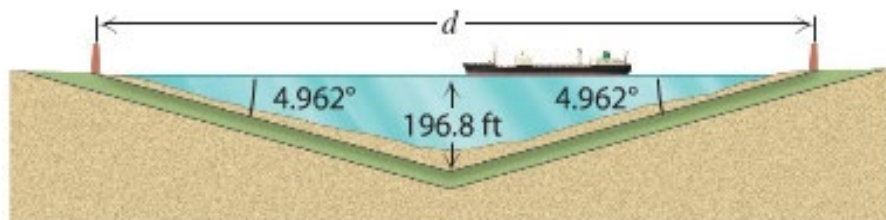


40. You were hiking directly toward a long straight road when you encountered a swamp. you turned  $60^\circ$  to the right and hiked  $6 \text{ mi}$  in that direction to reach the road. How far were you from the road when you encountered the swamp?

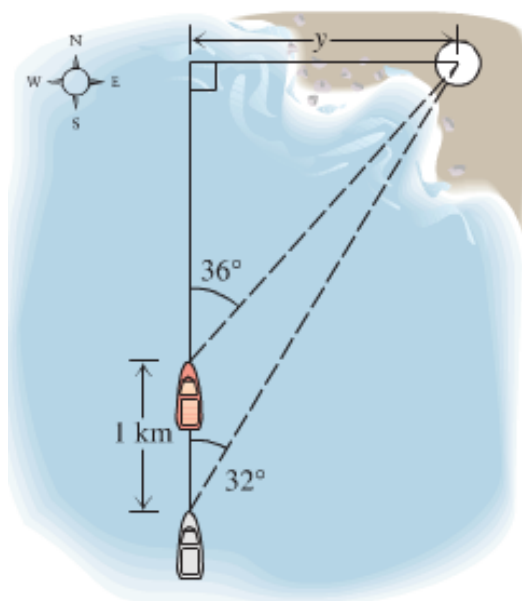


41. From a highway overpass,  $14.3 \text{ m}$  above the road, the angle of depression of an oncoming car is measured at  $18.3^\circ$ . How far is the car from a point on the highway directly below the observer?

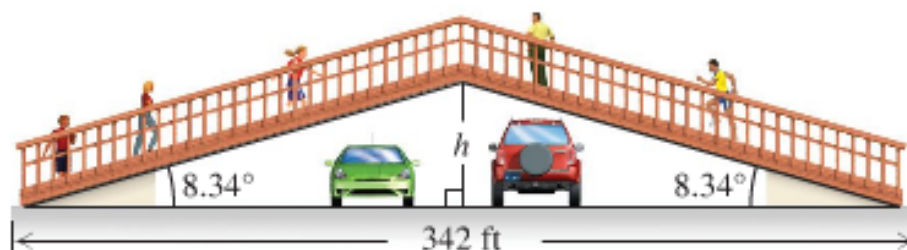
42. A tunnel under a river is  $196.8 \text{ feet}$  below the surface at its lowest point. If the angle of depression of the tunnel is  $4.962^\circ$ , then how far apart on the surface are the entrances to the tunnel? How long is the tunnel?



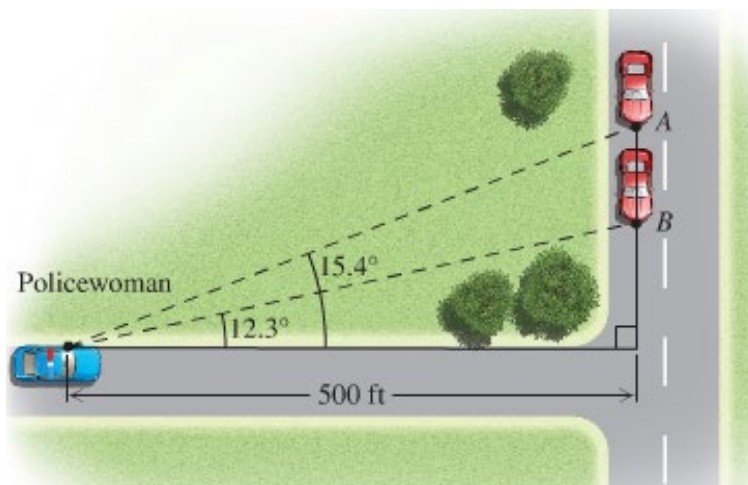
43. A boat sailing north sights a lighthouse to the east at an angle of  $32^\circ$  from the north. After the boat travels one more *kilometer*, the angle of the lighthouse from the north is  $36^\circ$ . If the boat continues to sail north, then how close will the boat come to the lighthouse?



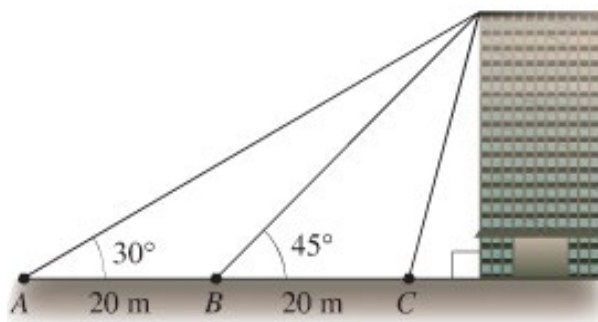
44. The angle of elevation of a pedestrian crosswalk over a busy highway is  $8.34^\circ$ , as shown in the drawing. If the distance between the ends of the crosswalk measured on the ground is  $342 \text{ feet}$ , then what is the height  $h$  of the crosswalk at the center?



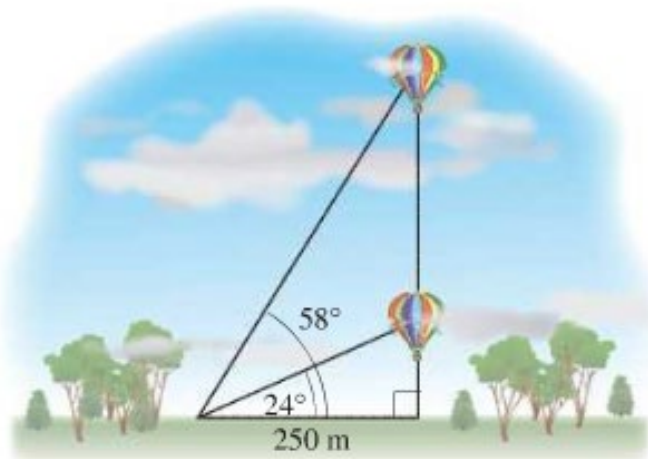
45. A policewoman has positioned herself 500 *feet*. from the intersection of two roads. She has carefully measured the angles of the lines of sight to points  $A$  and  $B$ . If a car passes from  $A$  to  $B$  is 1.75 *sec* and the speed limit is 55 *mph*, is the car speeding? (Hint: Find the distance from  $B$  to  $A$  and use  $R = D/T$ )



36. From point  $A$  the angle of elevation to the top of the building is  $30^\circ$ . From point  $B$ , 20 *meters* closer to the building, the angle of elevation is  $45^\circ$ . Find the angle of elevation of the building from point  $C$ , which is another 20 *meters* closer to the building.

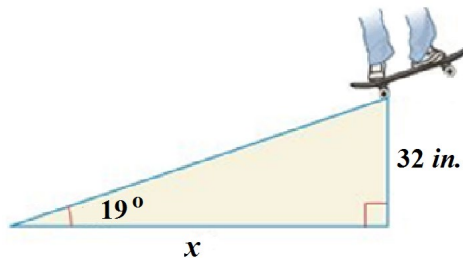


37. A hot air balloon is rising upward from the earth at a constant rate. An observer 250 *m* away spots the balloon at an angle of elevation of  $24^\circ$ . Two minutes later the angle of elevation of the balloon is  $58^\circ$ . At what rate is the balloon ascending?

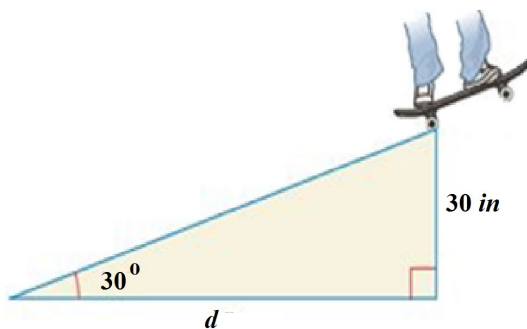




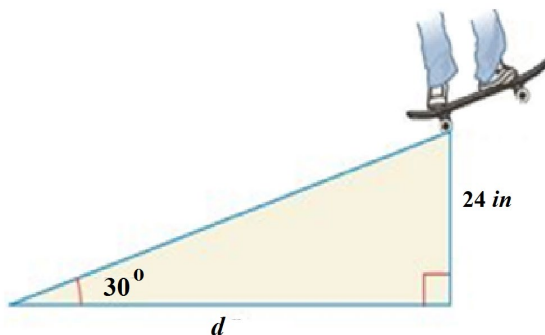
38. A skateboarder wishes to build a jump ramp that is inclined at a  $19^\circ$  angle and that has a maximum height of 32.0 *inches*. Find the horizontal width  $x$  of the ramp.



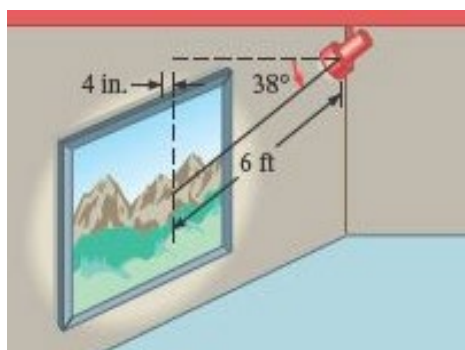
39. A skateboarder wishes to build a jump ramp that is inclined at a  $30^\circ$  angle and that has a maximum height of 30 *inches*. Find the horizontal width  $d$  of the ramp.



40. A skateboarder wishes to build a jump ramp that is inclined at a  $30^\circ$  angle and that has a maximum height of 24 *inches*. Find the horizontal width  $d$  of the ramp.

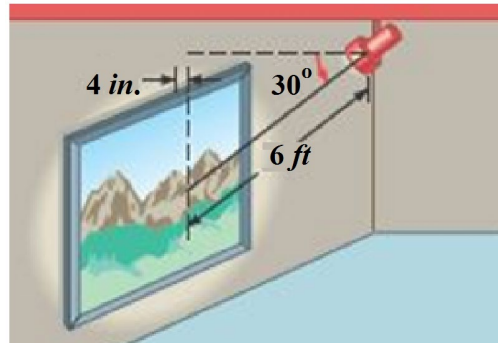


41. For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 *feet* from the piece of art and that the angle of depression of the light be  $38^\circ$ . How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 *inches* from the wall.

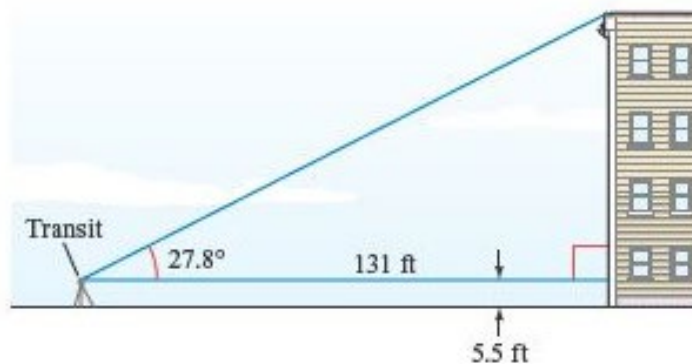




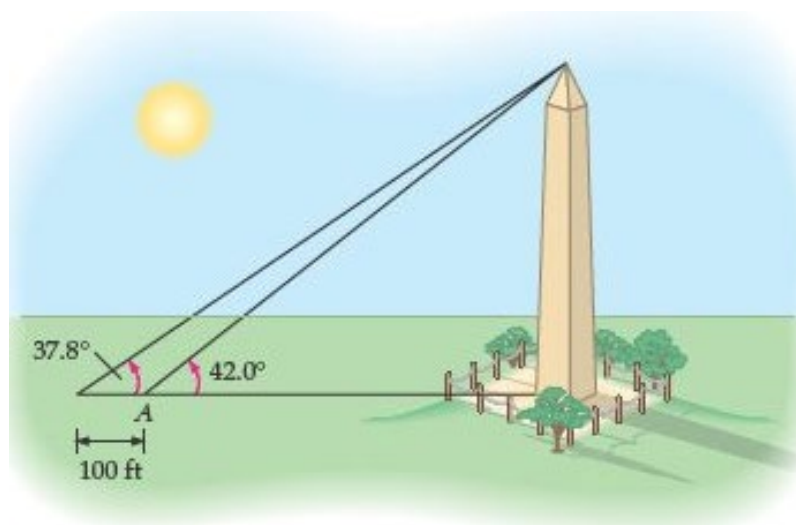
42. For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 feet from the piece of art and that the angle of depression of the light be  $30^\circ$ . How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 inches from the wall.



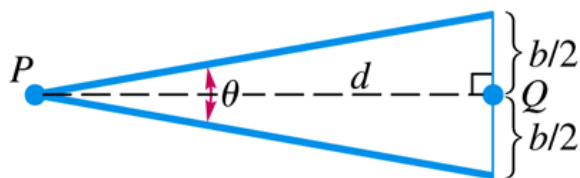
43. A surveyor determines that the angle of elevation from a transit to the top of a building is  $27.8^\circ$ . The transit is positioned 5.5 feet above ground level and 131 feet from the building. Find the height of the building to the nearest tenth of a foot.



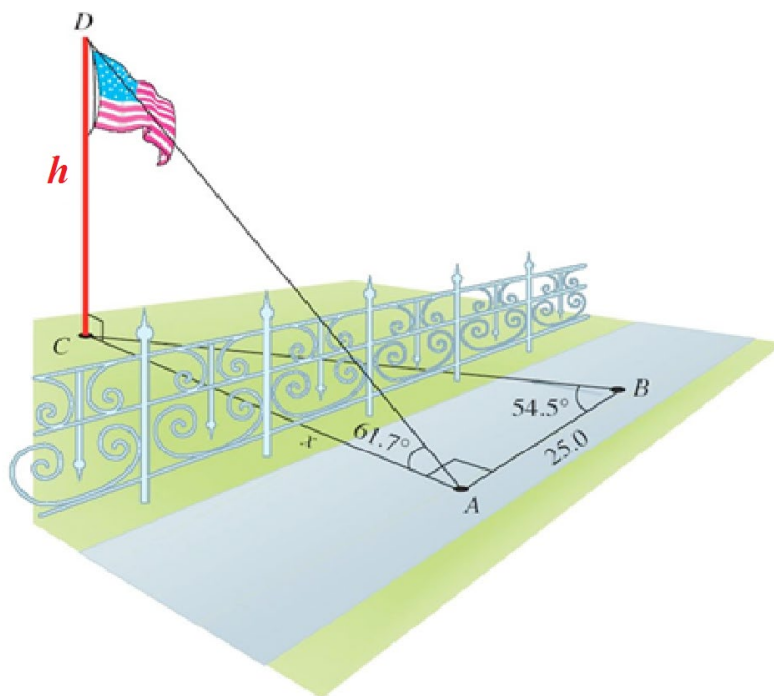
44. From a point  $A$  on a line from the base of the Washington Monument, the angle of elevation to the top of the monument is  $42.0^\circ$ . From a point 100 feet away from  $A$  and on the same line, the angle to the top is  $37.8^\circ$ . Find the height, to the nearest foot, of the Monument.



45. A method that surveyors use to determine a small distance  $d$  between two points  $P$  and  $Q$  is called the **subtense bar method**. The subtense bar with length  $b$  is centered at  $Q$  and situated perpendicular to the line of sight between  $P$  and  $Q$ . Angle  $\theta$  is measured, then the distance  $d$  can be determined.

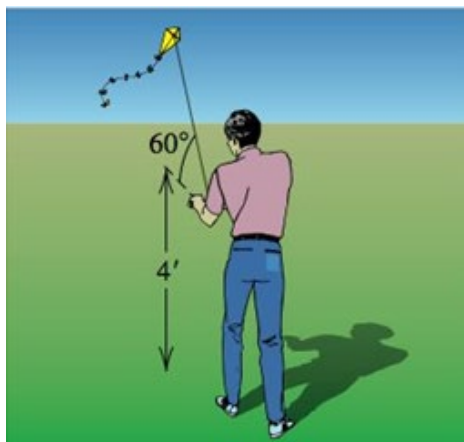


- a) Find  $d$  with  $\theta = 1^\circ 23' 12''$  and  $b = 2.000 \text{ cm}$
- b) Angle  $\theta$  usually cannot be measured more accurately than to the nearest  $1''$ . How much change would there be in the value of  $d$  if  $\theta$  were measured  $1''$  larger?
46. A diagram that shows how Diane estimates the height of a flagpole. She can't measure the distance between herself and the flagpole directly because there is a fence in the way. So, she stands at point  $A$  facing the pole and finds the angle of elevation from point  $A$  to the top of the pole to be  $61.7^\circ$ . Then she turns  $90^\circ$  and walks  $25.0 \text{ feet}$  to point  $B$ , where she measures the angle between her path and a line from  $B$  to the base of the pole. She finds that angle is  $54.5^\circ$ . Use this information to find the height of the pole.

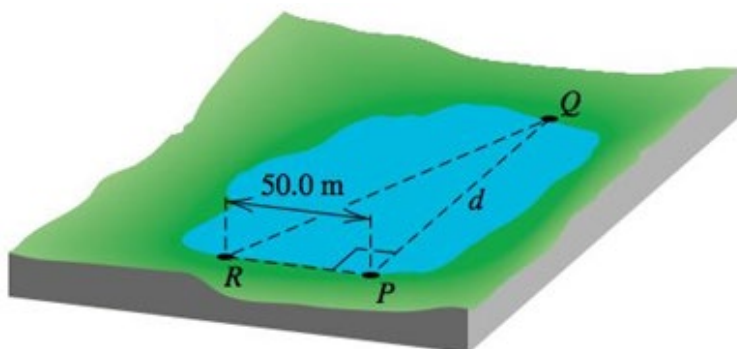


47. From a point  $15 \text{ feet}$  above level ground, a surveyor measures the angle of depression of an object on the ground at  $68^\circ$ . Approximate the distance from the object to the point on the ground directly beneath the surveyor.
48. A pilot, flying at an altitude of  $5,000 \text{ feet}$  wishes to approach the numbers on a runway at an angle of  $10^\circ$ . Approximate, to the nearest  $100 \text{ feet}$ , the distance from the airplane to the numbers at the beginning of the descent.

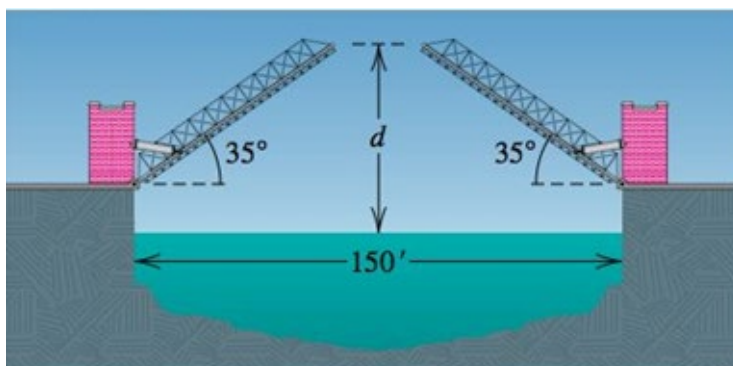
49. A person flying a kite holds the string 4 feet above ground level. The string of the kite is taut and makes an angle of  $60^\circ$  with the horizontal. Approximate the height of the kite above level ground if 500 feet of string is paved out.



50. To find the distance  $d$  between two points  $P$  and  $Q$  on opposite shores of a lake, a surveyor locates a point  $R$  that is 50.0 meters from  $P$  such that  $RP$  is perpendicular to  $PQ$ . Next, using a transit, the surveyor measures angle  $PRQ$  as  $72^\circ 40'$ . Find  $d$ .

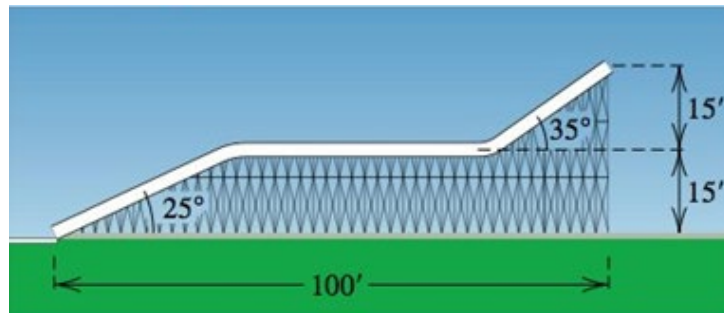


51. A drawbridge is 150 feet long when stretched across a river. The two sections of the bridge can be rotated upward through an angle of  $35^\circ$ .

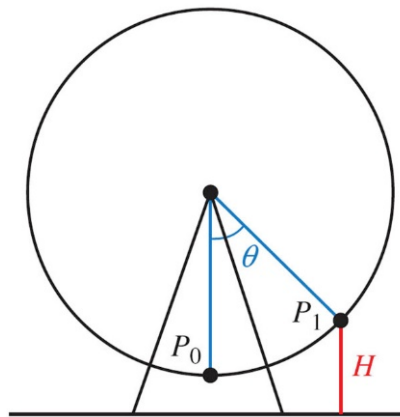


- If the water level is 15 feet below the closed bridge, find the distance  $d$  between the end of a section and the water level when the bridge is fully open.
- Approximately how far apart are the ends of the two sections when the bridge is fully open?

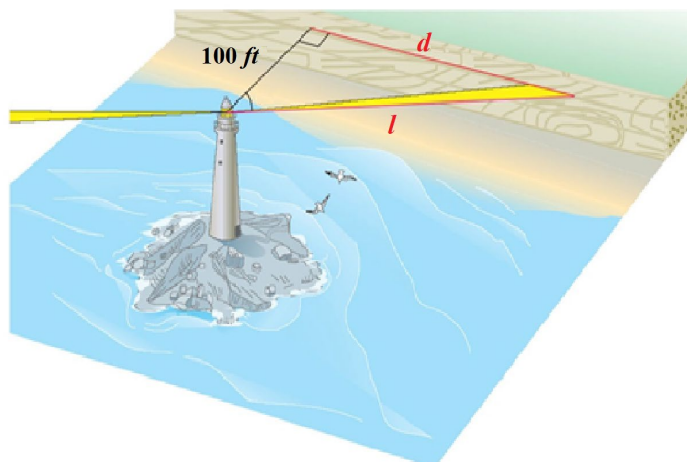
52. Find the total length of a design for a water slide to the nearest foot.



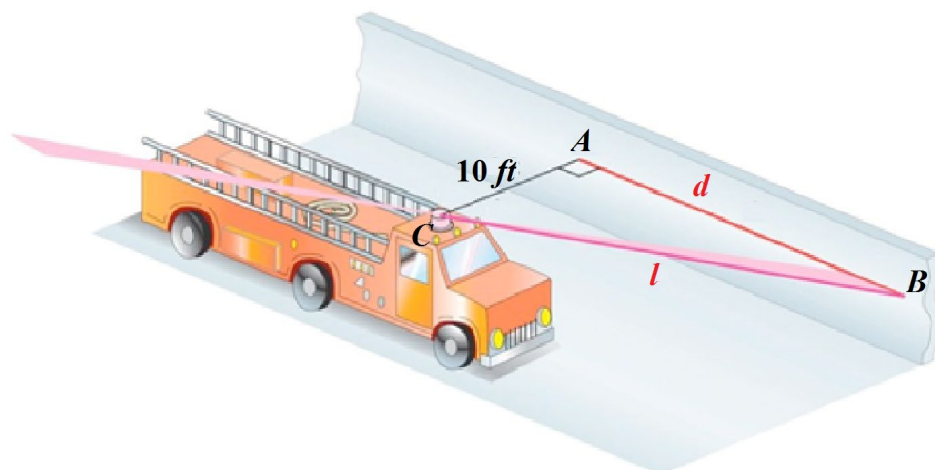
53. A Ferris wheel has radius 50.0 *feet*. A person takes a seat and then the wheel turns  $\frac{2\pi}{3}$  *rad*.
- How far is the person above the ground?
  - If it takes 30 *sec* for the wheel to turn  $\frac{2\pi}{3}$  *rad*, what is the angular speed of the wheel?
54. The diameter of the Ferris wheel is 250 *feet*, the distance from the ground to the bottom of the wheel is 14 *feet*, and one complete revolution takes 20 *minutes*, find



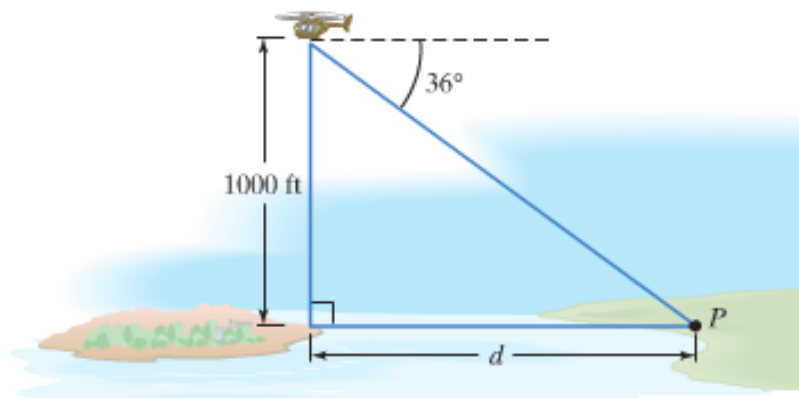
- The linear velocity, in miles per hour, of a person riding on the wheel.
  - The height of the rider in terms of the time  $t$ , where  $t$  is measured in minutes.
55. Find an equation that expresses  $l$  in terms of time  $t$ . Find  $l$  when  $t$  is 0.5 *sec*, 1.0 *sec*, and 1.5 *sec*. (assume the light goes through one rotation every 4 *seconds*.)



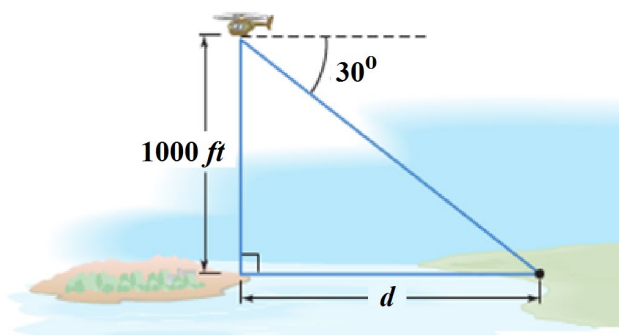
56. A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top of the truck is 10 feet from the wall and rotates through a complete revolution every 2 seconds. Find the equations that give the lengths  $d$  and  $\ell$  in terms of time.



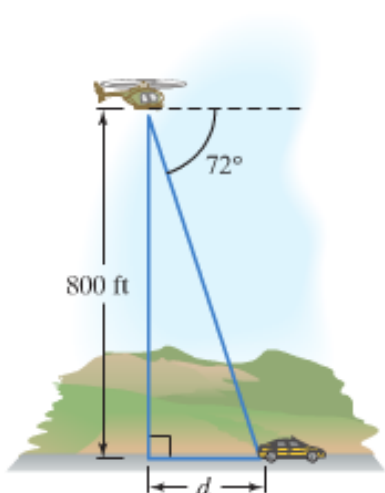
57. A helicopter hovers 1,000 feet above a small island. The angle of depression from the helicopter to point  $P$  on the coast is  $36^\circ$ . How far off the coast is the island?



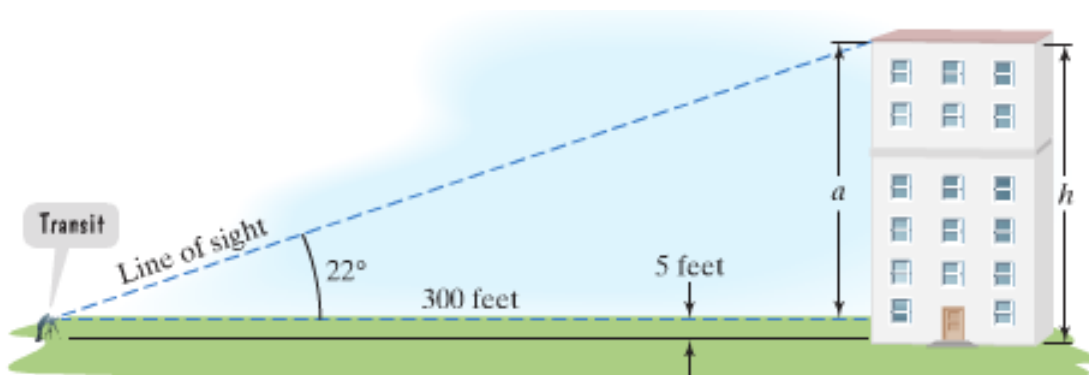
58. A helicopter hovers 1,000 feet above a small island. The angle of depression from the helicopter to point  $P$  on the coast is  $30^\circ$ . How far off the coast is the island?



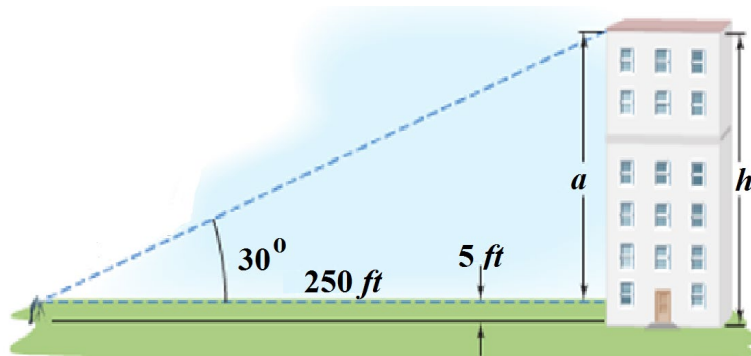
59. A police helicopter is flying at 800 feet. A stolen car is sighted at an angle of depression of  $72^\circ$ . Find the distance of the stolen car from a point directly below the helicopter.



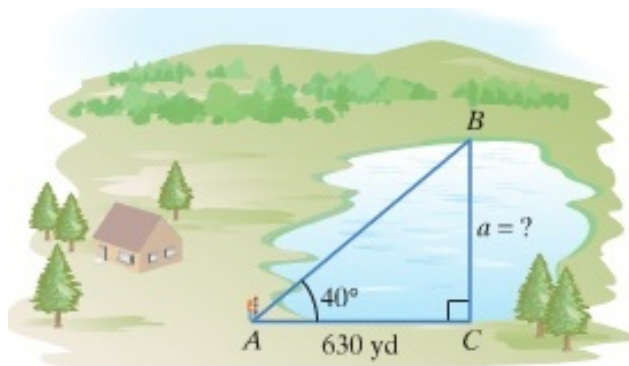
60. Sighting the top of a building a surveyor measured the angle of elevation to be  $22^\circ$ . The transit is 5 feet above the ground and 300 feet from the building. Find the building's height.



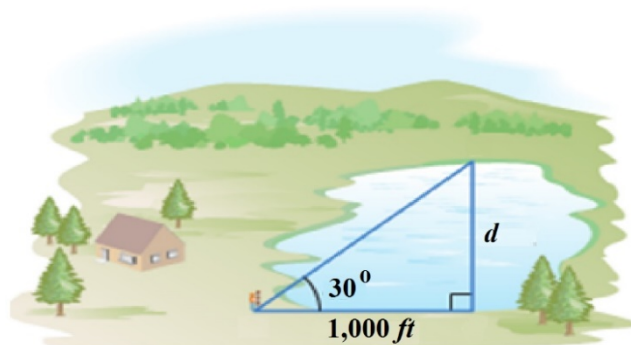
61. Sighting the top of a building a surveyor measured the angle of elevation to be  $30^\circ$ . The transit is 5 feet above the ground and 250 feet from the building. Find the building's height.



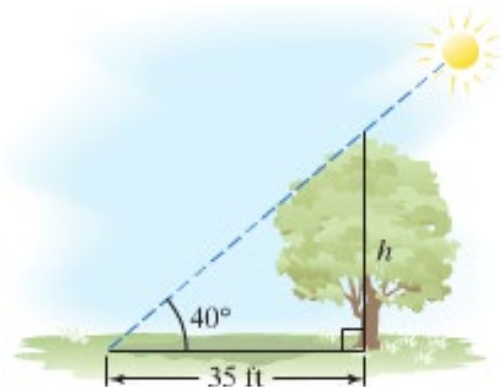
62. Determine how far it is across the lake.



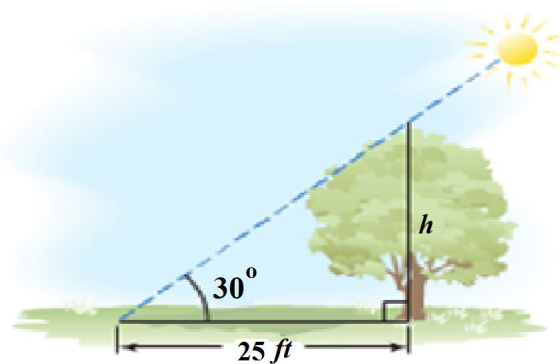
63. Determine how far it is across the lake.



64. At a certain time of day, the angle of elevation of the sun is  $40^\circ$ . Find the height of a tree whose shadow is 35 feet long.

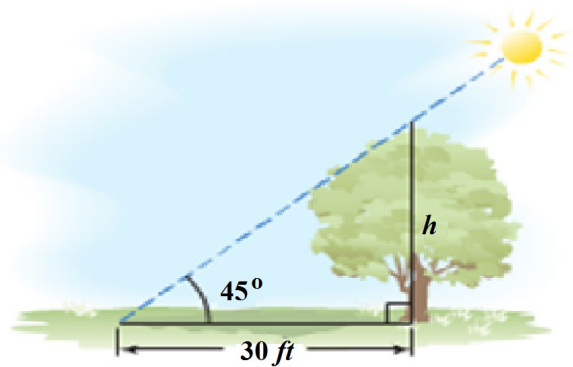


65. At a certain time of day, the angle of elevation of the sun is  $30^\circ$ . Find the height of a tree whose shadow is 25 feet long.

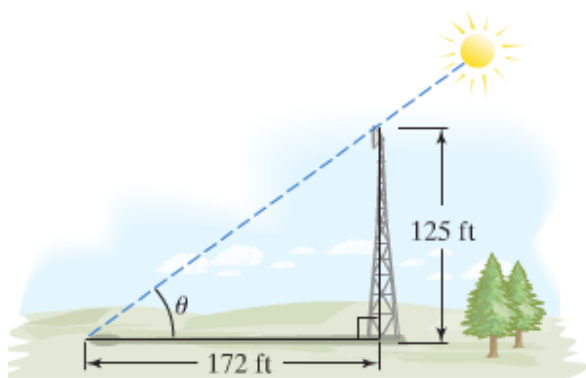




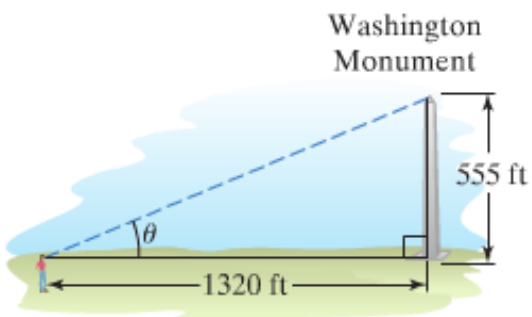
66. At a certain time of day, the angle of elevation of the sun is  $45^\circ$ . Find the height of a tree whose shadow is 30 *feet* long.



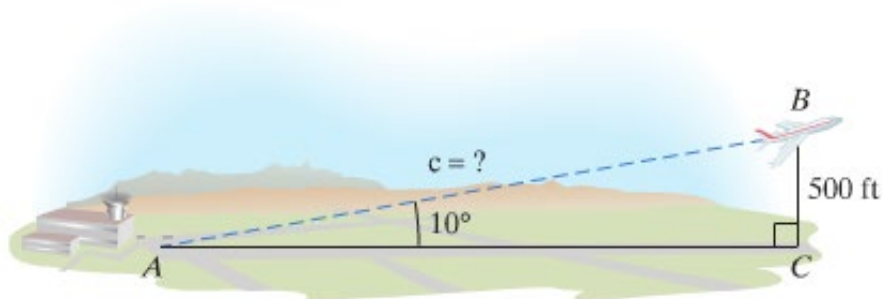
67. A tower that is 125 *feet* casts a shadow 172 *feet* long. Find the angle of elevation of the sun.



68. The Washington Monument is 555 *feet* high. If you are standing one quarter of a mile, or 1,320 *feet*, from the base of the monument and looking to the top, find the angle of elevation.

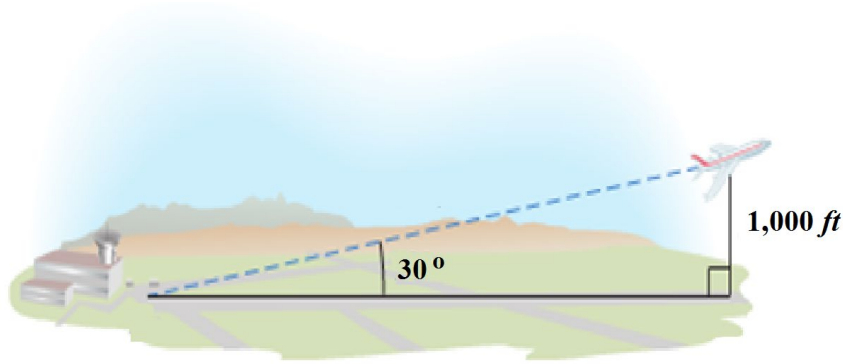


69. A plane rises from take-off and flies at an angle of  $10^\circ$  with the horizontal runway. When it has gained 500 *feet*, find the distance the plane has flown.

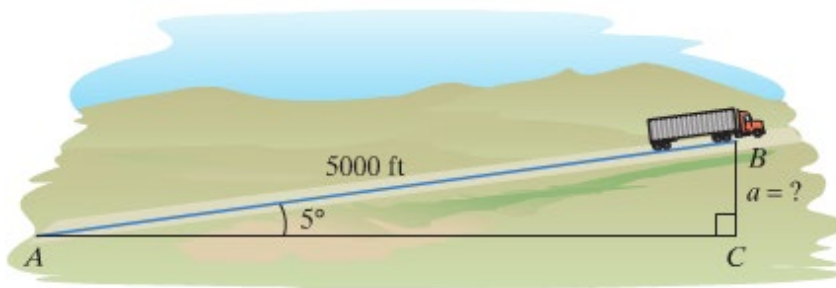




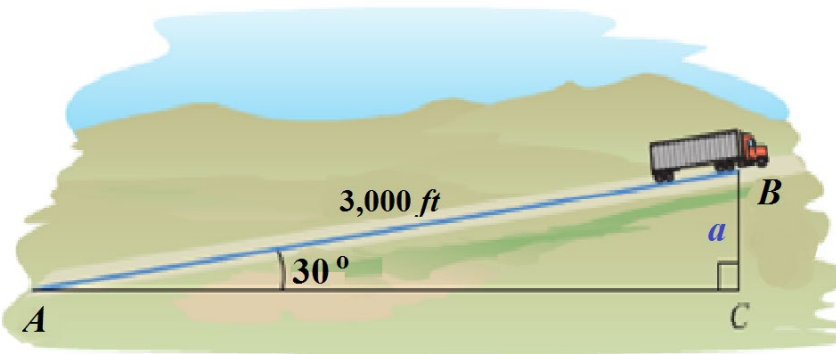
70. A plane rises from take-off and flies at an angle of  $30^\circ$  with the horizontal runway. When it has gained 1,000 *feet*, find the distance the plane has flown.



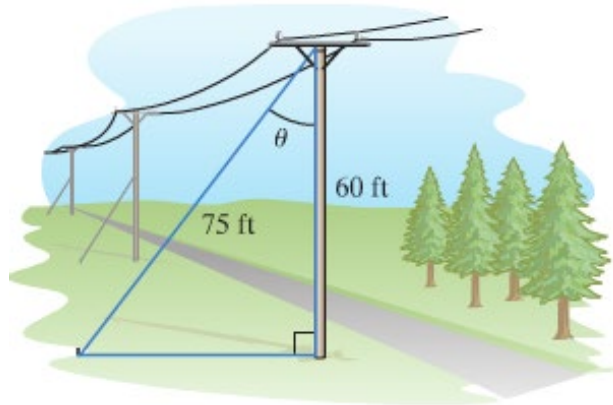
71. A road is inclined at an angle of  $5^\circ$ . After driving 5,000 *feet* along this road, find the driver's increase in altitude.



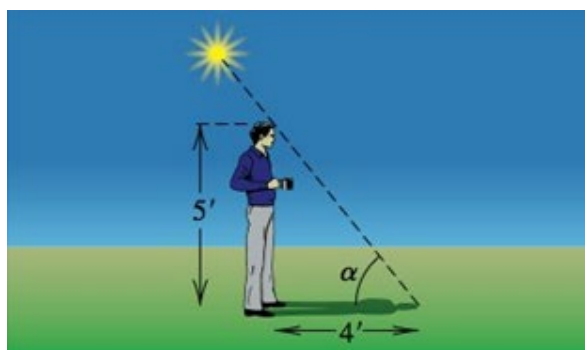
72. A road is inclined at an angle of  $30^\circ$ . After driving 3,000 *feet* along this road, find the driver's increase in altitude.



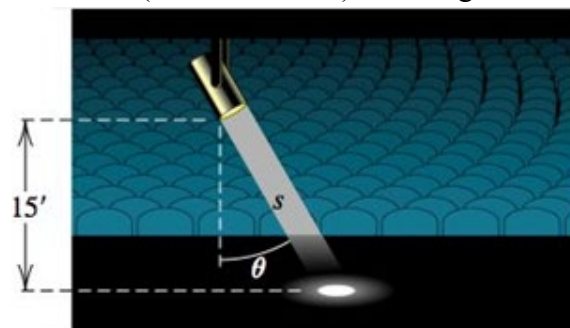
73. A telephone pole is 60 *feet* tall. A guy wire 75 *feet* long is attached from the ground to the top of the pole. Find the angle between the wire and the pole.



74. Approximate the angle of elevation  $\alpha$  of the sun if a person 5.0 *feet* tall casts a shadow 4.0 *feet* long on level ground.



75. A spotlight with intensity 5000 candles is located 15 *feet* above a stage. If the spotlight is rotated through an angle  $\theta$ , the illuminance  $E$  (in foot-candles) in the lighted area of the stage is given by

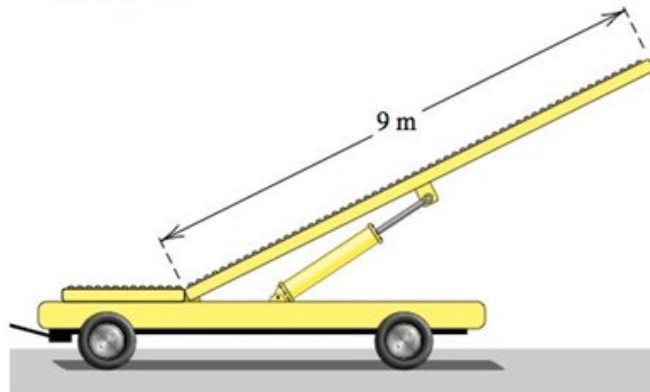


$$E = \frac{5,000 \cos \theta}{s^2}$$

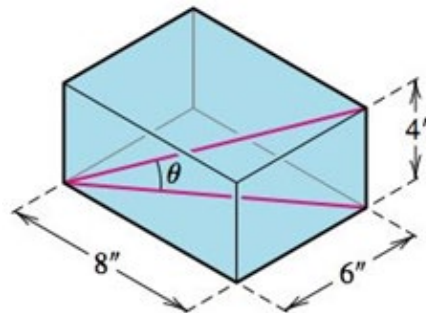
Where  $s$  is the distance (in *feet*) that the light must travel.

- Find the illuminance if the spotlight is rotated through an angle of  $30^\circ$ .
- The maximum illuminance occurs when  $\theta = 0^\circ$ . For what value of  $\theta$  is the illuminance one-half the maximum value.

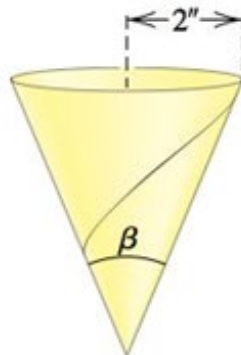
76. A conveyor belt 9 *meters* long can be hydraulically rotated up to an angle of  $40^\circ$  to unload cargo from airplanes.



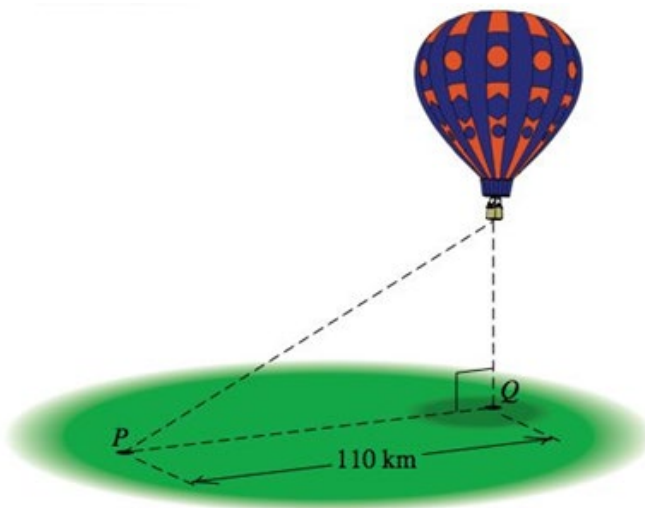
- Find, to the nearest degree, the angle through which the conveyor belt should be rotated up to reach a door that is 4 *meters* above the platform supporting the belt.
  - Approximate the maximum height above the platform that the belt can reach.
77. A rectangular box has dimensions  $8'' \times 6'' \times 4''$ . Approximate, to the nearest tenth of a degree, the angle  $\theta$  formed by a diagonal of the base and the diagonal of the box.



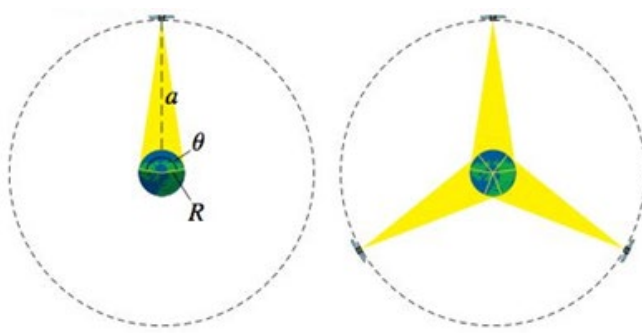
78. A conical paper cup has a radius of 2 *inches*, approximate, to the nearest degree, the angle  $\beta$  so that the cone will have a volume of  $20 \text{ in}^3$ .



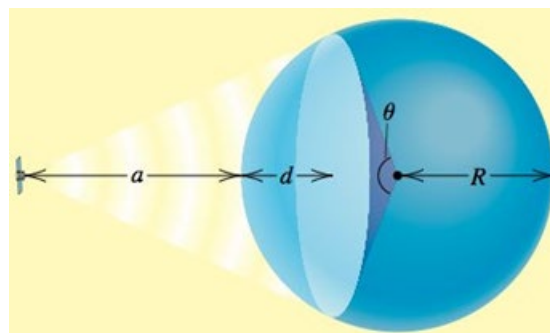
79. As a hot-air balloon rises vertically, its angle of elevation from a point  $P$  on level ground  $110 \text{ km}$  from the point  $Q$  directly underneath the balloon changes from  $19^\circ 20'$  to  $31^\circ 50'$ . Approximately how far does the balloon rise during this period?



80. Shown in the left part of the figure is a communications satellite with an equatorial orbit—that is, a nearly circular orbit in the plane determined by Earth's equator. If the satellite circles Earth at an altitude of  $a = 22,300 \text{ mi}$ , its speed is the same as the rotational speed of Earth; to an observer on the equator, the satellite appears to be stationary—that is, its orbit is synchronous.



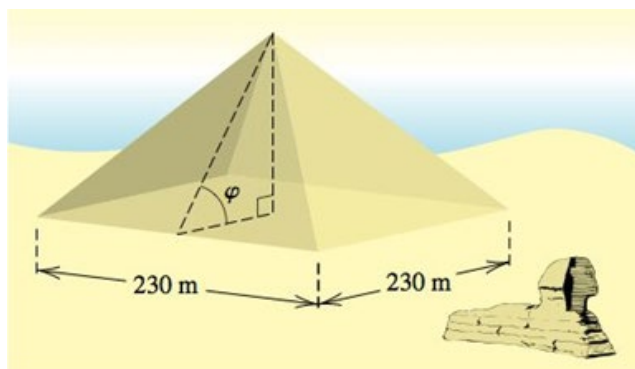
**a**



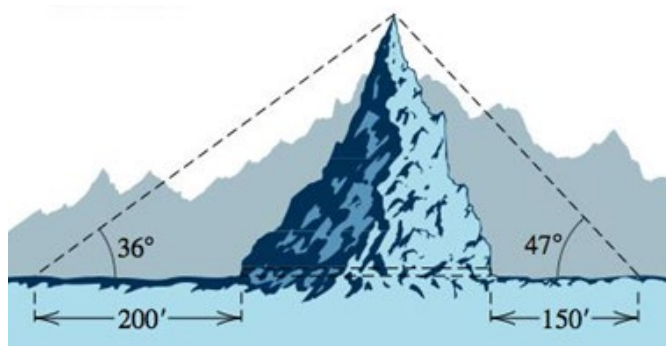
**b**

- Using  $R = 4,000 \text{ mi}$  for the radius of Earth, determine the percentage of the equator that is within signal range of such a satellite.
- As shown in the right part of the figure (**a**), three satellites are equally spaced in equatorial synchronous orbits. Use the value of  $\theta$  obtained in part (a) to explain why all points on the equator are within signal range of at least one of the three satellites.
- The figure (**b**) shows the area served by a communication satellite circling a planet of radius  $R$  at an altitude  $a$ . The portion of the planet's surface within range of the satellite is a spherical cap of depth  $d$  and surface area  $A = 2\pi R d$ . Express  $d$  in terms of  $R$  and  $\theta$ .
- Estimate the percentage of the planet's surface that is within signal range of a single satellite in equatorial synchronous orbit.

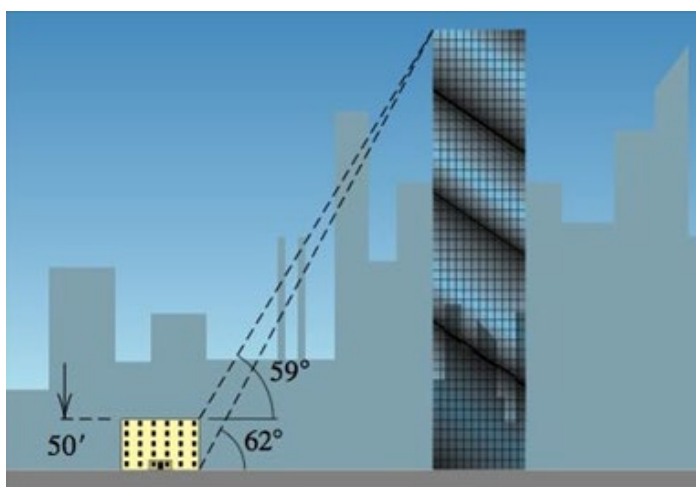
81. The great Pyramid of Egypt is 147 *meters* high, with a square base of side 230 *meters*. Approximate, to the nearest degree, the angle  $\varphi$  formed when an observer stands at the midpoint of one the sides and views the apex of the pyramid.



82. A tunnel for a new highway is to be cut through a mountain that is 260 *feet* high. At a distance of 200 *feet* from the base of the mountain, the angle of elevation is  $36^\circ$ . From a distance of 150 *feet* on the other side, the angle of elevation is  $47^\circ$ . Approximate the length of the tunnel to the nearest foot.

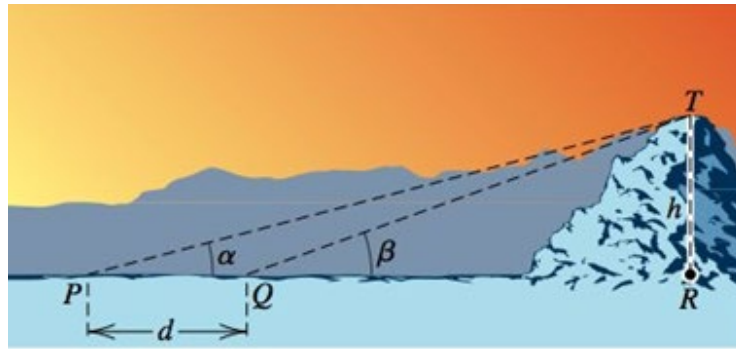


83. When a certain skyscraper is viewed from the top of a building 50 *feet* tall, the angle of elevation is  $59^\circ$ . When viewed from the street next to the shorter building, the angle of elevation is  $62^\circ$ .

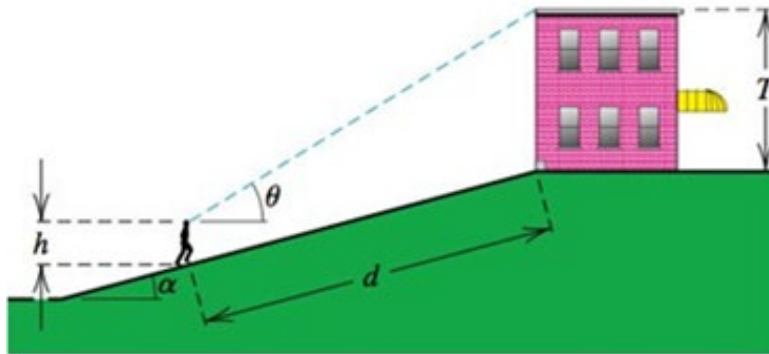


- Approximately how far apart are the two structures?
- Approximate the height of the skyscraper to the nearest tenth of a *foot*.

84. When a mountaintop is viewed from the point  $P$ , the angle of elevation is  $\alpha$ . From a point  $Q$ , which is  $d$  miles closer to the mountain, the angle of elevation increases to  $\beta$ .



- a) Show that the height  $h$  of the mountain is given by:  $h = \frac{d}{\cot \alpha - \cot \beta}$ .
- b) If  $d = 2\text{mi}$ ,  $\alpha = 15^\circ$ , and  $\beta = 20^\circ$ , approximate the height of the mountain.
85. An observer of height  $h$  stands on an incline at a distance  $d$  from the base of a building of height  $T$ . The angle of elevation from the observer to the top of the building is  $\theta$ , and the incline makes an angle of  $\alpha$  with the horizontal.



- a) Express  $T$  in terms of  $h$ ,  $d$ ,  $\alpha$ , and  $\theta$ .
- b) If  $d = 50\text{ ft}$ ,  $h = 6\text{ ft}$ ,  $\alpha = 15^\circ$ , and  $\theta = 31.4^\circ$ , estimate the height of the building.