Section 2.2 - Linear, Homogeneous Equations with Constant Coefficients

The equations of the form: $y'' + py' + qy = 0$

This is a class of equations that we can solve easily.

The analogous first-order, linear, homogeneous equation:

$$y' + py = 0$$

It is separable and easily solved, its general solution is

$$y(t) = Ce^{-pt}$$

Let look for a solution of the type

$$y(t) = e^{\lambda t}$$

$$y' = \lambda e^{\lambda t}$$

$$y'' = \lambda^2 e^{\lambda t}$$

$$\begin{aligned} y'' + py' + qy &= \lambda^2 e^{\lambda t} + p\lambda e^{\lambda t} + qe^{\lambda t} \\ &= \left(\lambda^2 + p\lambda + q\right)e^{\lambda t} \\ &= 0 \end{aligned}$$

$\lambda^2 + p\lambda + q = 0$ *This is called the **characteristic equation***

We can rewrite the differential equation and its characteristic equations

$$y'' + py' + qy = 0$$

$$\lambda^2 + p\lambda + q = 0$$

The roots are: $\lambda_{1,2} = \dfrac{-p \pm \sqrt{p^2 - 4q}}{2}$

$$\begin{aligned} &\text{If } p^2 - 4q > 0 \Rightarrow \textit{Two distinct real roots} \\ &\text{If } p^2 - 4q < 0 \Rightarrow \textit{Two distinct complex roots} \\ &\text{If } p^2 - 4q = 0 \Rightarrow \text{One } \textit{repeated real root} \end{aligned}$$

Case 1: Distinct Real Root

$y_1 = C_1 e^{\lambda_1 t}$ *and* $y_2 = C_2 e^{\lambda_2 t}$ are both solutions.

Proposition

If the characteristic equations $\lambda^2 + p\lambda + q = 0$ has two distinct real roots λ_1 and λ_2, then the ***general solution*** to $y'' + py' + qy = 0$ is

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Where C_1 and C_2 are arbitrary constants.

Example

Find the general solution to the equation $y'' - 3y' + 2y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = 1$

Solution

The characteristic equation:

$$y'' - 3y' + 2y = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

The solution: $\lambda_{1,2} = 1,\ 2$

The general solution

$$y(t) = C_1 e^t + C_2 e^{2t}$$

$$y' = C_1 e^t + 2C_2 e^{2t}$$

$y(0) = 2 \quad y(0) = C_1 e^0 + C_2 e^{2(0)}$

$$2 = C_1 + C_2$$

$y'(0) = 1 \quad y'(0) = C_1 e^0 + 2C_2 e^{2(0)}$

$$1 = C_1 + 2C_2$$

$$\begin{matrix} C_1 + C_2 = 2 \\ C_1 + 2C_2 = 1 \end{matrix} \Rightarrow C_2 = -1 \quad C_1 = 3$$

The unique solution is: $\boxed{y(t) = 3e^t - e^{2t}}$

Case 2: Complex Roots

Proposition

If the characteristic equations $\lambda^2 + p\lambda + q = 0$ has two complex conjugate roots $\lambda = a + ib$ and $\bar{\lambda} = a - ib$.

1. The functions

$$z = e^{(a+ib)t} \text{ and } \bar{z} = e^{(a-ib)t}$$

So the general solution is

$$w(t) = C_1 e^{(a+ib)t} + C_2 e^{(a-ib)t}$$

Where C_1 and C_2 are arbitrary complex constants.

2. The functions

$$y_1(t) = e^{at}\cos(bt) \text{ and } y_2(t) = e^{at}\sin(bt)$$

So the general solution is

$$y(t) = e^{at}\left(A_1 \cos bt + A_2 \sin bt\right)$$

Where A_1 and A_2 are constants.

Example

Find the general solution to the equation $y'' + 2y' + 2y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = 3$

Solution

The characteristic equation:

$$y'' + 2y' + 2y = 0$$

$$\lambda^2 + 2\lambda + 2 = 0$$

The solution: $\lambda_{1,2} = -1 \pm i = a \pm ib$

$a = -1;\quad b = 1$

The general solution

$$y(t) = e^{-t}\left(C_1 \cos t + C_2 \sin t\right)$$

$$y(0) = e^{-(0)}\left(C_1 \cos(0) + C_2 \sin(0)\right)$$

$$2 = 1\left(C_1 + C_2(0)\right)$$

$$\Rightarrow \boxed{C_1 = 2}$$

$$y' = -e^{-t}\left(C_1 \cos t + C_2 \sin t\right) + e^{-t}\left(-C_1 \sin t + C_2 \cos t\right)$$

$$y'(0) = -e^{-(0)}\left(C_1 \cos(0) + C_2 \sin(0)\right) + e^{-(0)}\left(-C_1 \sin(0) + C_2 \cos(0)\right)$$

$$3 = -\left(C_1\right) + \left(C_2\right)$$

$$C_2 - C_1 = 3$$

$$C_2 = 3 + 2 = 5$$

$$\underline{y(t) = e^{-t}\left(2\cos t + 5\sin t\right)}$$

Example

Find the general solution to the equation $y'' - 4y' + 13y = 0$

Solution

The characteristic equation: $\lambda^2 - 4\lambda + 13 = 0$

The solutions: $\lambda_{1,2} = \dfrac{4 \pm \sqrt{16 - 52}}{2} = \dfrac{4 \pm \sqrt{-36}}{2} = \dfrac{4 \pm 6i}{2} = 2 \pm 3i$

$a = 2; \quad b = 3$

The general solution: $\underline{y(x) = e^{2x}\left(C_1 \cos 3x + C_2 \sin 3x\right)}$

Case 3: Repeated Roots

If the roots of the characteristic equations are repeated

$$\lambda^2 + p\lambda + q = 0$$

$$\left(\lambda - \lambda_1\right)^2 = 0$$

$$\lambda_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$$

$$p^2 - 4q = 0 \;\Rightarrow\; q = \frac{p^2}{4}$$

$$\lambda_{1,2} = -\frac{p}{2}$$

$$y_1 = C_1 e^{\lambda_1 t}$$
$$= C_1 e^{-pt/2}$$

$$y_2 = v(t) y_1(t)$$
$$= v(t) e^{-pt/2}$$

$$y'' + py' + qy = 0$$

$$y'' + py' + \frac{p^2}{4} y = 0$$

$$y_2{}' = v' e^{-pt/2} - \frac{P}{2} v e^{-pt/2}$$

$$y_2{}'' = v'' e^{-pt/2} - \frac{P}{2} v' e^{-pt/2} - \frac{P}{2} v' e^{-pt/2} + \frac{P^2}{4} v e^{-pt/2}$$

$$v'' e^{-pt/2} - \frac{P}{2} v' e^{-pt/2} - \frac{P}{2} v' e^{-pt/2} + \frac{P^2}{4} v e^{-pt/2} + p\left(v' e^{-pt/2} - \frac{P}{2} v e^{-pt/2}\right) + \frac{P^2}{4} v e^{-pt/2} = 0$$

$$v'' e^{-pt/2} = 0$$

$$v'' = 0$$

$$\Rightarrow v' = a$$

$$\Rightarrow v = at + b$$

$$v = t$$

$$y_2 = t e^{-pt/2}$$

Proposition

If the characteristic equations $\lambda^2 + p\lambda + q = 0$ has one double root λ_1, then the ***general solution*** to $y'' + py' + qy = 0$ is

$$\begin{aligned} y(t) &= C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t} \\ &= \left(C_1 + C_2 t\right) e^{\lambda_1 t} \end{aligned}$$

Where C_1 and C_2 are arbitrary constants.

Example

Find the general solution to the equation $y'' - 2y' + y = 0$

Find the unique solution corresponding to the initial conditions $y(0) = 2$ and $y'(0) = -1$

Solution

The characteristic equation:

$$\lambda^2 - 2\lambda + 1 = 0$$

The solution: $\lambda_{1,2} = 1$

$$\begin{aligned} y(t) &= C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t} \\ &= C_1 e^{t} + C_2 t e^{t} \end{aligned}$$

$$y(0) = C_1 e^{(0)} + C_2 (0) e^{(0)} \quad \Rightarrow \quad 2 = C_1$$

$$y' = C_1 e^{t} + C_2 e^{t} + C_2 t e^{t}$$

$$y'(0) = 2e^{(0)} + C_2 e^{(0)} + C_2 (0) e^{(0)}$$

$$-1 = 2 + C_2 \quad \Rightarrow \quad \boxed{C_2 = -3}$$

$$\boxed{y(t) = 2e^{t} - 3te^{t}}$$

Example

Find the general solution to the equation $y'' - 10y' + 25y = 0$

Solution

The characteristic equation: $\lambda^2 - 10\lambda + 25 = (\lambda - 5)^2 = 0$

The solutions are: $\lambda_{1,2} = 5$

The general solution: $\boxed{y(t) = C_1 e^{5t} + C_2 t e^{5t}}$

Higher-Order Equations

In general, to solve an *n*th-order differential equation, we must solve an *n*th degree characteristic polynomial equation

$$a_n\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

If all roots are real and distinct, then the general solution is

$$y(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + \cdots + C_n e^{\lambda_n x}$$

If all roots are equal to λ, then the general solution is

$$y(x) = C_1 e^{\lambda x} + C_2 x e^{\lambda x} + C_3 x^2 e^{\lambda x} + \cdots + C_n x^{n-1} e^{\lambda x}$$

Example

Find the general solution of $\quad y''' + 3y'' - 4y = 0$

Solution

$\lambda^3 + 3\lambda^2 - 4 = 0 \qquad$ ***Solve for*** $\boldsymbol{\lambda}$

$\lambda_1 = 1, \quad \lambda_{2,\,3} = -2$

$$y(x) = C_1 e^{x} + \left(C_2 + C_3 x\right)e^{-2x}$$

Rational zero theorem: $\pm\left\{\frac{4}{1}\right\} = \pm\{1,\ 2,\ 4\}$

$\lambda_1 = 1, \quad \lambda_2 = -2$

$(\lambda - 1)(\lambda + 2)(\lambda - a) = 0$

$(-1)(2)(-a) = -4 \quad \Rightarrow \quad a = -2$

Example

Find the general solution of $\quad \lambda^4(\lambda + 1)(\lambda + 2)^2\left(\lambda^2 + 4\right) = 0$

Solution

$\lambda^2 + 4 = 0 \quad \Rightarrow \quad \lambda^2 = -4 \ \rightarrow \ \lambda = \pm 2i$

The solution: $\lambda = 0,\ 0,\ 0,\ 0,\ -1,\ -2,\ -2,\ \pm 2i$

$$y(x) = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + C_5 e^{-x} + \left(C_6 + C_7 x\right)e^{-2x} + C_8 \cos 2x + C_9 \sin 2x$$

Summary

The equation: $y'' + py' + qy = 0$

The characteristic equations $\lambda^2 + p\lambda + q = 0$

If $p^2 - 4q > 0$	$y_1(t) = C_1 e^{\lambda_1 t}$ *and* $y_1(t) = C_2 e^{\lambda_2 t}$	$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$
If $p^2 - 4q < 0$	$y_1(t) = e^{at}\cos bt$ *and* $y_1(t) = e^{at}\sin bt$	$y(t) = e^{at}\left(A_1 \cos bt + A_2 \sin bt\right)$
If $p^2 - 4q = 0$	$y_1 = e^{\lambda t}$ *and* $y_1 = te^{\lambda t}$	$y(t) = \left(C_1 + C_2 t\right)e^{\lambda_1 t}$

Exercises *Section* 2.2 – Linear, Homogeneous Equations with Constant Coefficients

Find the general solution of the second order differential equation

1. $y'' + y' = 0$
2. $y'' - 4y = 0$
3. $y'' + 8y = 0$
4. $y'' - 36y = 0$
5. $y'' + 9y = 0$
6. $y'' - 9y = 0$
7. $y'' + 16y = 0$
8. $y'' + 25y = 0$
9. $y'' - 64y = 0$
10. $y'' + y' + y = 0$
11. $y'' + y' - y = 0$
12. $y'' - y' - 2y = 0$
13. $y'' - y' - 6y = 0$
14. $y'' + y' - 6y = 0$
15. $y'' - y' - 11y = 0$
16. $y'' - y' - 12y = 0$
17. $y'' + 2y' + y = 0$
18. $y'' + 2y' + 3y = 0$
19. $y'' + 2y' - 3y = 0$
20. $y'' - 2y' - 3y = 0$
21. $y'' - 2y' + 3y = 0$
22. $y'' + 2y' + 4y = 0$
23. $y'' - 2y' + 5y = 0$
24. $y'' + 2y' - 15y = 0$
25. $y'' + 2y' + 17y = 0$
26. $y'' - 3y' + 2y = 0$
27. $y'' + 3y' - 4y = 0$
28. $y'' + 4y' - y = 0$
29. $y'' - 4y' + 4y = 0$
30. $y'' + 4y' + 4y = 0$
31. $y'' - 4y' + 5y = 0$
32. $y'' + 4y' + 5y = 0$
33. $y'' + 4y' - 5y = 0$
34. $y'' + 4y' + 7y = 0$
35. $y'' + 4y' + 9y = 0$
36. $y'' + 5y' = 0$
37. $y'' + 5y' + 6y = 0$
38. $y'' + 6y' + 9y = 0$
39. $y'' - 6y' + 9y = 0$
40. $y'' - 6y' + 25y = 0$
41. $y'' + 8y' + 16y = 0$
42. $y'' + 8y' - 16y = 0$
43. $y'' - 9y' + 20y = 0$
44. $y'' - 10y' + 25y = 0$
45. $y'' + 14y' + 49y = 0$
46. $2y'' - y' - 3y = 0$
47. $2y'' + y' - y = 0$
48. $2y'' + 2y' + y = 0$
49. $2y'' + 2y' + 3y = 0$
50. $2y'' - 3y' - 2y = 0$
51. $2y'' - 3y' + 4y = 0$
52. $2y'' - 4y' + 8y = 0$
53. $2y'' + 5y' = 0$
54. $2y'' - 5y' - 3y = 0$
55. $2y'' + 7y' - 4y = 0$
56. $3y'' + y = 0$
57. $3y'' - y' = 0$
58. $3y'' + 2y' + y = 0$
59. $3y'' + 11y' - 7y = 0$
60. $3y'' - 20y' + 12y = 0$
61. $4y'' + y' = 0$
62. $4y'' + 4y' + y = 0$
63. $4y'' - 4y' + y = 0$
64. $4y'' + 4y' + 2y = 0$
65. $4y'' - 4y' + 13y = 0$
66. $4y'' - 8y' + 7y = 0$
67. $4y'' - 12y' + 9y = 0$
68. $4y'' + 20y' + 25y = 0$
69. $6y'' + 5y' - 6y = 0$
70. $6y'' + y' - 2y = 0$
71. $6y'' - 7y' - 20y = 0$
72. $6y'' + 13y' - 5y = 0$
73. $6y'' + 13y' + 7y = 0$
74. $6y'' - 13y' + 7y = 0$
75. $8y'' - 10y' - 3y = 0$
76. $9y'' - y = 0$
77. $9y'' + 6y' + y = 0$
78. $9y'' - 12y' + 4y = 0$
79. $9y'' + 24y' + 16y = 0$
80. $12y'' - 5y' - 2y = 0$
81. $16y'' - 8y' + 7y = 0$
82. $16y'' - 12y' - 4y = 0$
83. $16y'' - 24y' + 9y = 0$
84. $25y'' + 10y' + y = 0$
85. $25y'' - 10y' + y = 0$
86. $35y'' - y' - 12y = 0$

Find the general solution of the given higher-order differential equation

87. $y''' + 3y'' + 3y' + y = 0$

88. $y''' + 3y'' - y' - 3y = 0$

89. $y^{(3)} + 3y'' - 4y = 0$

90. $3y''' - 19y'' + 36y' - 10y = 0$

91. $y''' - 6y'' + 12y' - 8y = 0$

92. $y''' + 5y'' + 7y' + 3y = 0$

93. $y^{(3)} + y' - 10y = 0$

94. $y''' + y'' - 6y' + 4y = 0$

95. $y''' - 6y'' - y' + 6y = 0$

96. $y''' + 2y'' - 4y' - 8y = 0$

97. $y''' - 7y'' + 7y' + 15y = 0$

98. $y''' + 3y'' - 4y' - 12y = 0$

99. $y''' - 4y'' - 5y' = 0$

100. $y''' - y = 0$

101. $y''' - 5y'' + 3y' + 9y = 0$

102. $y''' + 3y'' - 4y' - 12y = 0$

103. $y''' + y'' - 2y = 0$

104. $y''' - y'' - 4y = 0$

105. $y''' + 3y'' + 3y' + y = 0$

106. $y''' - 6y'' + 12y' - 8y = 0$

107. $y^{(4)} + y''' + y'' = 0$

108. $y^{(4)} - 2y'' + y = 0$

109. $16y^{(4)} + 24y'' + 9y = 0$

110. $y^{(4)} - 7y'' - 18y = 0$

111. $y^{(4)} + 2y'' + y = 0$

112. $y^{(4)} + y''' + y'' = 0$

113. $y^{(4)} + 4y = 0$

114. $y^{(4)} + 2y''' + 9y'' - 2y' - 10y = 0$

115. $x^{(4)} - 4x^{(3)} + 7x'' - 4x' + 6x = 0$

116. $x^{(4)} + 8x^{(3)} + 24x'' + 32x' + 16x = 0$

117. $x^{(4)} - 4x'' + 16x' + 32x = 0$

118. $x^{(4)} + 4x^{(3)} + 6x'' + 4x' + x = 0$

119. $y^{(4)} - y^{(3)} + y'' - 3y' - 6y = 0$

120. $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$

121. $x^{(5)} - x^{(4)} - 2x^{(3)} + 2x'' + x' - x = 0$

122. $x^{(5)} + 5x^{(4)} + 10x^{(3)} + 10x'' + 5x' + x = 0$

123. $y^{(5)} + 5y^{(4)} - 2y''' - 10y'' + y' + 5y = 0$

124. $2y^{(5)} - 7y^{(4)} + 12y''' + 8y'' = 0$

125. $y^{(5)} - 2y^{(4)} + 17y''' = 0$

126. $x^{(6)} - 5x^{(4)} + 16x^{(3)} + 36x'' - 16x' - 32x = 0$

127. $\left(D^2 + 6D + 13\right)^2 y = 0$

128. $\lambda^3(\lambda - 1)(\lambda - 2)^3\left(\lambda^2 + 9\right) = 0$

Find the solution of the given initial value problem.

129. $y'' + y = 0$; $y\left(\frac{\pi}{3}\right) = 0,\quad y'\left(\frac{\pi}{3}\right) = 2$

130. $y'' + y = 0$; $y(0) = 0,\quad y'\left(\frac{\pi}{2}\right) = 0$

131. $y'' + y' = 0$; $y(0) = 2,\quad y'(0) = 1$

132. $y'' - y' - 2y = 0$; $y(0) = -1,\quad y'(0) = 2$

133. $y'' + y' + 2y = 0$; $y(0) = 0,\quad y'(0) = 0$

134. $y'' + 2y' + y = 0$; $y(0) = 1,\quad y'(0) = -3$

135. $y'' - 2y' + y = 0,\quad y(0) = 5,\quad y'(0) = 10$

136. $y'' - 2y' - 2y = 0$; $y(0) = 0,\quad y'(0) = 3$

137. $y'' - 2y' + 2y = 0$; $y(0) = 1,\quad y(\pi) = 1$

138. $y'' - 2y' - 3y = 0$; $y(0) = 2,\quad y'(0) = -3$

139. $y'' + 2y' - 8y = 0$; $y(0) = 3,\quad y'(0) = -12$

140. $y'' - 2y' + 17y = 0$; $y(0) = -2,\quad y'(0) = 3$

141. $y'' + 2\sqrt{2}y' + 2y = 0$; $y(0) = 1,\quad y'(0) = 0$

142. $y'' + 3y' - 10y = 0$; $y(0) = 4,\quad y'(0) = -2$

143. $y'' + 4y = 0$; $y(0) = 0,\quad y(\pi) = 0$

144. $y'' + 4y = 0$; $y\left(\frac{\pi}{4}\right) = -2,\quad y\left(\frac{\pi}{4}\right) = 1$

145. $y'' + 4y' + 2y = 0$; $y(0) = -1,\quad y'(0) = 2$

146. $y'' - 4y' + 3y = 0$; $y(0) = 1,\quad y'(0) = \frac{1}{3}$

147. $y'' - 4y' + 4y = 0,\quad y(1) = 1,\quad y'(1) = 1$

148. $y'' + 4y' + 4y = 0,\quad y(0) = 1,\quad y'(0) = 3$

149. $y'' - 4y' + 5y = 0;\quad y(0) = 1,\quad y'(0) = 5$

150. $y'' + 4y' + 5y = 0;\quad y(0) = 1,\quad y'(0) = 0$

151. $y'' + 4y' + 5y = 0;\quad y\left(\frac{\pi}{2}\right) = \frac{1}{2},\quad y'\left(\frac{\pi}{2}\right) = -2$

152. $y'' - 4y' - 5y = 0,\quad y(1) = 0,\quad y'(1) = 2$

153. $y'' - 4y' - 5y = 0,\quad y(-1) = 3,\quad y'(-1) = 9$

154. $y'' - 4y' + 9y = 0,\quad y(0) = 0,\quad y'(0) = -8$

155. $y'' - 4y' + 13y = 0;\quad y(0) = -1,\quad y'(0) = 2$

156. $y'' - 5y' + 6y = 0;\quad y(1) = e^2,\quad y'(1) = 3e^2$

157. $y'' + 6y' + 9y = 0;\quad y(0) = 2,\quad y'(0) = -2$

158. $y'' + 6y' + 5y = 0,\quad y(1) = 0,\quad y'(0) = 3$

159. $y'' - 6y' + 5y = 0;\quad y(0) = 3,\quad y'(0) = 11$

160. $y'' - 6y' + 9y = 0,\quad y(0) = 2,\quad y'(0) = \frac{25}{3}$

161. $y'' - 6y' + 9y = 0;\quad y(0) = 0,\quad y'(0) = 5$

162. $y'' + 8y' - 9y = 0;\quad y(1) = 2,\quad y'(1) = 0$

163. $y'' - 8y' + 17y = 0;\quad y(0) = 4,\quad y'(0) = -1$

164. $y'' - 9y = 0,\quad y(0) = 2,\quad y'(0) = -1$

165. $y'' - 10y' + 25y = 0,\quad y(0) = 1,\quad y'(1) = 0$

166. $y'' + 10y' + 25y = 0;\quad y(0) = 2,\quad y'(0) = -1$

167. $y'' + 11y' + 24y = 0;\quad y(0) = 0,\quad y'(0) = -7$

168. $y'' + 12y = 0,\quad y(0) = 0,\quad y'(0) = 1$

169. $y'' + 16y = 0,\quad y(0) = 2,\quad y'(0) = -2$

170. $y'' + 16y = 0,\quad y(\pi) = 2,\quad y'(0) = -2$

171. $y'' + 16y = 0,\quad y\left(\frac{\pi}{2}\right) = -10,\quad y'\left(\frac{\pi}{2}\right) = 3$

172. $y'' + 25y = 0;\quad y(0) = 1,\quad y'(0) = -1$

173. $2y'' - 2y' + y = 0;\quad y(-\pi) = 1,\quad y'(-\pi) = -1$

174. $3y'' + y' - 14y = 0,\quad y(0) = 2,\quad y'(0) = -1$

175. $3y'' + 2y' - 8y = 0,\quad y(0) = -6,\quad y'(0) = -18$

176. $4y'' - 4y' + y = 0,\quad y(0) = 4,\quad y'(0) = 4$

177. $4y'' - 4y' + y = 0,\quad y(1) = -4,\quad y'(1) = 0$

178. $4y'' - 4y' - 3y = 0,\quad y(0) = 1,\quad y'(0) = 5$

179. $4y'' + 4y' + 5y = 0,\quad y(\pi) = 1,\quad y'(\pi) = 0$

180. $4y'' + 4y' + 17y = 0,\quad y(0) = -1,\quad y'(0) = 2$

181. $4y'' - 5y' = 0,\quad y(-2) = 0,\quad y'(-2) = 7$

182. $4y'' + 12y' + 9y = 0,\quad y(0) = 2,\quad y'(0) = 1$

183. $4y'' + 24y' + 37y = 0,\quad y(\pi) = 1,\quad y'(\pi) = 0$

184. $9y'' + y = 0;\quad y\left(\frac{\pi}{2}\right) = 4,\quad y'\left(\frac{\pi}{2}\right) = 0$

185. $9y'' + \pi^2 y = 0;\quad y(3) = 2,\quad y'(3) = -\pi$

186. $9y'' - 6y' + y = 0;\quad y(3) = -2,\quad y'(3) = -\frac{5}{3}$

187. $9y'' + 6y' + 2y = 0$; $y(3\pi) = 0,\quad y'(3\pi) = \frac{1}{3}$

188. $9y'' - 12y' + 4y = 0,\quad y(0) = -1,\quad y'(0) = 1$

189. $12y'' + 5y' - 2y = 0,\quad y(0) = 1,\quad y'(0) = -1$

190. $16y'' - 8y' + y = 0$; $y(0) = -4,\quad y'(0) = 3$

191. $25y'' + 20y' + 4y = 0$; $y(5) = 4e^{-2},\quad y'(5) = -\frac{3}{5}e^{-2}$

192. $y''' + 12y'' + 36y' = 0,\quad y(0) = 0,\quad y'(0) = 1,\quad y''(0) = -7$

193. $y''' + 2y'' - 5y' - 6y = 0,\quad y(0) = 0,\quad y'(0) = 0,\quad y''(0) = 1$

194. The roots of the characteristic equation of a certain differential equation are:

$$3, -5, 0, 0, 0, 0, -5,\ 2 \pm 3i \text{ and } 2 \pm 3i$$

Write a general solution of this homogeneous differential equation.

195. $y(x) = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos 2x + C_4 \sin 2x$ is the general solution of a homogeneous equation. What is the equation?

196. Show that the second differential equation $y'' + 4y = 0$

a) Has no solution to the boundary value $y(0) = 0, \quad y(\pi) = 1$

b) There are infinitely many solutions to the boundary value $y(0) = 0, \quad y(\pi) = 0$

197. Show that the general solution of the equation

$$y'' + Py' + Qy = 0$$

(where P and Q are constant) approaches 0 as $x \to \infty$ if and only if P and Q are both positive.