Exercise

Find indefinite integral $\int v^2 dv$

Solution

$$\int v^2 dv = \boxed{\frac{v^3}{3} + C}$$

Exercise

Find indefinite integral $\int x^{1/2} dx$

Solution

$$\int x^{1/2} dx = \boxed{\frac{2}{3} x^{3/2} + C}$$

Exercise

Find indefinite integral $\int 4y^{-3} dy$

Solution

$$\int 4y^{-3} dy = 4\frac{y^{-2}}{-2} + C$$

$$= \boxed{-\frac{2}{y^2} + C}$$

Exercise

Find indefinite integral $\int (x^3 - 4x + 2) dx$

Solution

$$\int (x^3 - 4x + 2) dx = \frac{x^4}{4} - 4\frac{x^2}{2} + 2x + C$$

$$= \boxed{\frac{1}{4} x^4 - 2x^2 + 2x + C}$$

Exercise

Find indefinite integral $\int \left(3z^2 - 4z + 5\right) dz$

Solution

$$\int \left(3z^2 - 4z + 5\right) dz = 3\frac{z^3}{3} - 4\frac{z^2}{2} + 5z + C$$

$$= z^3 - 2z^2 + 5z + C$$

Exercise

Find indefinite integral $\int \left(x^2 - 1\right)^2 dx$

Solution

$$\int \left(x^2 - 1\right)^2 dx = \int \left(x^4 - 2x^2 + 1\right) dx \qquad (a-b)^2 = a^2 - 2ab + b^2$$

$$= \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + C$$

Exercise

Find indefinite integral $\int \frac{x^2 + 1}{\sqrt{x}} dx$

Solution

$$\int \frac{x^2 + 1}{\sqrt{x}} dx = \int \left(\frac{x^2}{x^{1/2}} + \frac{1}{x^{1/2}}\right) dx$$

$$= \int \left(x^{3/2} + x^{-1/2}\right) dx$$

$$= \frac{x^{5/2}}{5/2} - \frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{5}x^{5/2} - 2x^{1/2} + C$$

Exercise

Find indefinite integral $\int \left(\sqrt[4]{x^3} + 1\right) dx$

Solution

$$\int \left(\sqrt[4]{x^3} + 1 \right) dx = \int \left(x^{3/4} + 1 \right) dx$$
$$= \underline{\left| \frac{4}{7} x^{7/4} + x + C \right|}$$

Exercise

Find indefinite integral $\int \sqrt{x}\,(x+1)\,dx$

Solution

$$\int \sqrt{x}\,(x+1)\,dx = \int x^{1/2}(x+1)\,dx$$
$$= \int \left(x^{3/2} + x^{1/2} \right) dx$$
$$= \underline{\left| \frac{2}{5} x^{5/2} + \frac{2}{3} x^{3/2} + C \right|}$$

Exercise

Find indefinite integral $\int (1+3t)\,t^2\,dt$

Solution

$$\int (1+3t)\,t^2\,dt = \int \left(t^2 + 3t^3 \right) dt$$
$$= \underline{\left| \frac{1}{3} t^3 + \frac{3}{4} t^4 + C \right|}$$

Exercise

Find indefinite integral $\int \frac{x^2 - 5}{x^2} dx$

Solution

$$\int \frac{x^2 - 5}{x^2}\,dx = \int \left(1 - \frac{5}{x^2} \right) dx$$
$$= \int \left(1 - 5x^{-2} \right) dx$$
$$= \underline{\left| x + \frac{5}{x} + C \right|}$$

Exercise

Find indefinite integral $\int (-40x + 250)dx$

Solution

$$\int (-40x + 250)dx = \underline{\boxed{-20x^2 + 250x + C}}$$

Exercise

Find indefinite integral $\int \frac{x+2}{\sqrt{x}}\, dx$

Solution

$$\int \frac{x+2}{\sqrt{x}}dx = \int \left[\frac{x}{x^{1/2}} + \frac{2}{x^{1/2}}\right]dx$$

$$= \int \frac{x}{x^{1/2}}dx + \int \frac{2}{x^{1/2}}dx$$

$$= \int x^{1/2}dx + 2\int x^{-1/2}dx$$

$$= \frac{x^{3/2}}{3/2} + 2\frac{x^{1/2}}{1/2} + C$$

$$\underline{\boxed{= \frac{2}{3}x^{3/2} + 4x^{1/2} + C}}$$

Exercise

Find indefinite integral $\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x\right)dx$

Solution

$$\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x\right)dx = \int \frac{1}{5}dx - \int 2x^{-3}dx + \int 2xdx$$

$$= \frac{x}{5} - 2\frac{x^{-2}}{-2} + x^2 + C$$

$$\underline{\boxed{= \frac{x}{5} + \frac{1}{x^2} + x^2 + C}}$$

Exercise

Find indefinite integral $\int \left(\sqrt{x}+\sqrt[3]{x}\right)dx$

Solution

$$\int \left(\sqrt{x}+\sqrt[3]{x}\right)dx = \int \left(x^{1/2}+x^{1/3}\right)dx$$

$$= \frac{x^{3/2}}{3/2}+\frac{x^{4/3}}{4/3}+C$$

$$= \frac{2}{3}x^{3/2}+\frac{3}{4}x^{4/3}+C$$

Exercise

Find indefinite integral $\int 2x\left(1-x^{-3}\right)dx$

Solution

$$\int 2x\left(1-x^{-3}\right)dx = \int \left(2x-2x^{-2}\right)dx$$

$$= x^2-2\frac{x^{-1}}{-1}+C$$

$$= x^2+\frac{2}{x}+C$$

Exercise

Find indefinite integral $\int \left(\frac{4+\sqrt{t}}{t^3}\right)dt$

Solution

$$\int \left(\frac{4+\sqrt{t}}{t^3}\right)dt = \int \left(\frac{4}{t^3}+\frac{t^{1/2}}{t^3}\right)dt$$

$$= \int \left(4t^{-3}+t^{-5/2}\right)dt$$

$$= 4\frac{t^{-2}}{-2}+\frac{t^{-3/2}}{-3/2}+C$$

$$= -\frac{2}{t^2}-\frac{2}{3t^{3/2}}+C$$

Exercise

Find ech indefinite integral $\int (-2\cos t)\, dt$

Solution

$$\int (-2\cos t)\, dt = \underline{-2\sin t + C}$$

Exercise

Find indefinite integral $\int 7\sin\frac{\theta}{3} d\theta$

Solution

$$\int 7\sin\frac{\theta}{3}\, d\theta = 7\frac{-\cos\left(\frac{\theta}{3}\right)}{\frac{1}{3}} + C$$

$$= \underline{-21\cos\left(\frac{\theta}{3}\right) + C}$$

Exercise

Find indefinite integral $\int \frac{2}{5}\sec\theta\tan\theta\, d\theta$

Solution

$$\int \frac{2}{5}\sec\theta\tan\theta\, d\theta = \underline{\frac{2}{5}\sec\theta + C}$$

Exercise

Find indefinite integral $\int \left(4\sec x\tan x - 2\sec^2 x\right)dx$

Solution

$$\int \left(4\sec x\tan x - 2\sec^2 x\right)dx = 4\int (\sec x\tan x)dx - 2\int \left(\sec^2 x\right)dx$$

$$= \underline{4\sec x - 2\tan x + C}$$

Exercise

Find indefinite integral $\int (2\cos 2x - 3\sin 3x)dx$

Solution

$$\int (2\cos 2x - 3\sin 3x)dx = \sin 2x + \cos 3x + C$$

Exercise

Find indefinite integral $\int \left(1+\tan^2\theta\right)d\theta$

Solution

$$\int \left(1+\tan^2\theta\right)d\theta = \int \left(\sec^2\theta\right)d\theta$$
$$= \tan\theta + C$$

Exercise

Find indefinite integral $\int \frac{\csc\theta}{\csc\theta - \sin\theta}d\theta$

Solution

$$\int \frac{\csc\theta}{\csc\theta - \sin\theta}d\theta = \int \frac{1}{1-\frac{\sin\theta}{\csc\theta}}d\theta \qquad \textit{divide by } \csc\theta \ \& \ \csc\theta = \frac{1}{\sin\theta}$$
$$= \int \frac{1}{1-\sin^2\theta}d\theta \qquad \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1-\sin^2\theta = \cos^2\theta$$
$$= \int \frac{1}{\cos^2\theta}d\theta$$
$$= \int \sec^2\theta d\theta$$
$$= \tan\theta + C$$

Exercise

Evaluate the integral $\int \left(2e^x - 3e^{-2x}\right)dx$

Solution

$$\int \left(2e^{x} - 3e^{-2x}\right) dx = \underline{\left| 2e^{x} + \frac{3}{2}e^{-2x} + C \right|}$$

Exercise

Evaluate $\int \frac{dx}{\sqrt{9 - x^2}}$

Solution

$$\int \frac{dx}{\sqrt{9 - x^2}} = \underline{\left| \sin^{-1}\left(\frac{x}{3}\right) + C \right|}$$

Exercise

Evaluate $\int \frac{dx}{9 + 3x^2}$

Solution

$$\int \frac{dx}{9 + 3x^2} = \frac{1}{3}\int \frac{dx}{3 + x^2} \qquad a^2 = 3 \quad \rightarrow a = \sqrt{3}$$

$$= \frac{1}{3}\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$= \underline{\left| \frac{\sqrt{3}}{9}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C \right|}$$

Exercise

Find the integral $\int \frac{4x^2 - 3x + 2}{x^2} dx$

Solution

$$\int \frac{4x^2 - 3x + 2}{x^2} dx = \int \left(\frac{4x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2}\right) dx$$

$$= \int \left(4 - \frac{3}{x} + 2x^{-2}\right) dx$$

$$= 4x - 3\ln|x| - 2x^{-1} + C$$

$$= \underline{\left| 4x - 3\ln|x| - \frac{2}{x} + C \right|}$$

Exercise

Find the integral $\int \left(x^8 - 3x^3 + 1\right) dx$

Solution

$$\int \left(x^8 - 3x^3 + 1\right) dx = \frac{1}{9}x^9 - \frac{3}{4}x^4 + x + C$$

Exercise

Find the integral $\int (2x+1)^2 \, dx$

Solution

$$\int (2x+1)^2 \, dx = \int \left(4x^2 + 4x + 1\right) dx$$
$$= \frac{4}{3}x^3 + 2x^2 + x + C$$

Exercise

Find the integral $\int \frac{x+1}{x} dx$

Solution

$$\int \frac{x+1}{x} dx = \int \left(1 + \frac{1}{x}\right) dx$$
$$= x + \ln|x| + C$$

Exercise

Find the integral $\int \left(\frac{1}{x^2} - \frac{2}{x^{5/2}}\right) dx$

Solution

$$\int \left(\frac{1}{x^2} - \frac{2}{x^{5/2}}\right) dx = \int \left(\frac{1}{x^2} - 2x^{-5/2}\right) dx$$
$$= -\frac{1}{x} + \frac{4}{3}x^{-3/2} + C$$
$$= -\frac{1}{x} + \frac{4}{3x^{3/2}} + C$$

Exercise

Find the integral $\int \frac{x^4 - 2\sqrt{x} + 2}{x^2} dx$

Solution

$$\int \frac{x^4 - 2\sqrt{x} + 2}{x^2} dx = \int \left(x^2 - 2x^{-3/2} + 2x^{-2} \right) dx$$

$$= \frac{1}{3}x^3 + 4x^{-1/2} - \frac{2}{x} + C$$

Exercise

Find the integral $\int (1 + \cos 3\theta) d\theta$

Solution

$$\int (1 + \cos 3\theta) d\theta = \theta + \frac{1}{3}\sin 3\theta + C$$

Exercise

Find the integral $\int 2\sec^2 \theta \, d\theta$

Solution

$$\int 2\sec^2 \theta \, d\theta = 2\tan\theta + C$$

Exercise

Find the integral $\int \sec 2x \tan 2x \, dx$

Solution

$$\int \sec 2x \tan 2x \, dx = \frac{1}{2}\sec 2x + C$$

Exercise

Find the integral $\int 2e^{2x}dx$

Solution

$$\int 2e^{2x}dx = e^{2x} + C$$

Exercise

Find the integral $\int \frac{12}{x}dx$

Solution

$$\int \frac{12}{x}dx = 12\ln|x| + C$$

Exercise

Find the integral $\int \frac{dx}{\sqrt{1-x^2}}$

Solution

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$$

Exercise

Find the integral $\int \frac{dx}{x^2+1}$

Solution

$$\int \frac{dx}{x^2+1} = \tan^{-1}x + C$$

Exercise

Find the integral $\int \frac{1+\tan\theta}{\sec\theta}d\theta$

Solution

$$\int \frac{1+\tan\theta}{\sec\theta}\,d\theta = \int \left(\frac{1}{\sec\theta}+\frac{\tan\theta}{\sec\theta}\right)d\theta$$

$$= \int \left(\cos\theta+\sin\theta\right)d\theta$$

$$\underline{= \sin\theta - \cos\theta + C}$$

Exercise

Find the integral $\int \left(\sqrt[4]{x^3}+\sqrt{x^5}\right)dx$

Solution

$$\int \left(\sqrt[4]{x^3}+\sqrt{x^5}\right)dx = \int \left(x^{3/4}+x^{5/2}\right)dx$$

$$= \frac{4}{7}x^{7/4}+\frac{2}{7}x^{7/2}+C$$

$$\underline{= \frac{4}{7}x\sqrt[4]{x^3}+\frac{2}{7}x^3\sqrt{x}+C}$$

Exercise

Find the integral $\int \left(x^{-3}+7e^{5x}+\frac{4}{x}\right)dx$

Solution

$$\int \left(x^{-3}+7e^{5x}+\frac{4}{x}\right)dx = \frac{x^{-2}}{-2}+\frac{7}{5}e^{5x}+4\ln|x|+C$$

$$\underline{= -\frac{1}{2x^2}+\frac{7}{5}e^{5x}+4\ln|x|+C}$$

Exercise

Find the integral $\int \left(\frac{2}{x}+\frac{x}{2}\right)dx$

Solution

$$\int \left(\frac{2}{x}+\frac{x}{2}\right)dx \underline{= 2\ln|x|+\frac{1}{4}x^2+C}$$

Exercise

Find the integral $\int \frac{1}{ax}dx$

Solution

$$\int \frac{1}{ax}dx = \underline{\left| \frac{1}{a}\ln|x| + C \right|}$$

Exercise

Find the integral $\int x\sqrt{x}\ dx$

Solution

$$\int x\sqrt{x}\ dx = \int x^{3/2}\ dx$$
$$= \underline{\left| \frac{2}{5}x^{5/2} + C \right|}$$

Exercise

Find the integral $\int \left(\frac{2}{\sqrt{x}} + 2\sqrt{x} \right) dx$

Solution

$$\int \left(\frac{2}{\sqrt{x}} + 2\sqrt{x} \right) dx = \int \left(2x^{-1/2} + 2x^{1/2} \right) dx$$
$$= 4x^{1/2} + \frac{4}{3}x^{3/2} + C$$
$$= \underline{\left| 4\sqrt{x} + \frac{4}{3}x\sqrt{x} + C \right|}$$

Exercise

Find the integral $\int \left(x - 2x^2 + \frac{1}{2x} \right) dx$

Solution

$$\int \left(x - 2x^2 + \frac{1}{2x} \right) dx = \underline{\left| \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{2}\ln|x| + C \right|}$$

Exercise

Find the integral $\int\left(\frac{7}{2x^3}-\sqrt[3]{x}\right)dx$

Solution

$$\int\left(\frac{7}{2x^3}-\sqrt[3]{x}\right)dx=\int\left(\frac{7}{2}x^{-3}-x^{1/3}\right)dx$$

$$=-\frac{7}{4}x^{-2}-\frac{3}{4}x^{4/3}+C$$

$$=\underline{\left|-\frac{7}{4x^2}-\frac{3}{4}x\sqrt[3]{x}+C\right|}$$

Exercise

Find the integral $\int 3e^{-2x}dx$

Solution

$$\int 3e^{-2x}dx=\underline{\left|-\frac{3}{2}e^{-2x}+C\right|}$$

Exercise

Find the integral $\int e^{-x}dx$

Solution

$$\underline{\left|\int e^{-x}dx=-e^{-x}+C\right|}$$

Exercise

Find the integral $\int e\ dx$

Solution

$$\underline{\left|\int e\ dx=ex+C\right|}$$

Exercise

Find the integral $\int \frac{7}{2e^{2x}}dx$

Solution

$$\int \frac{7}{2e^{2x}}dx = \frac{7}{2}\int e^{-2x}\,dx$$
$$= -\frac{7}{4}e^{-2x} + C$$
$$\underline{= -\frac{7}{4e^{2x}} + C}$$

Exercise

Find the integral $\int -3\left(e^{2x}+1\right)dx$

Solution

$$\int -3\left(e^{2x}+1\right)dx \underline{= -3\left(\frac{1}{2}e^{2x}+x\right)+C}$$

Exercise

Find the integral $\int \left(-3e^{-x}+2x-\frac{1}{2}e^{5x}\right)dx$

Solution

$$\int \left(-3e^{-x}+2x-\frac{1}{2}e^{5x}\right)dx \underline{= 3e^{-x}+x^2-\frac{1}{10}e^{5x}+C}$$

Exercise

Find the integral $\int \left(\sqrt[4]{x^3}+1\right)dx$

Solution

$$\int \left(\sqrt[4]{x^3}+1\right)dx = \int \left(x^{3/4}+1\right)dx$$
$$\underline{= \frac{4}{7}x^{7/4}+x+C}$$

Exercise

Find the integral $\int \left(5x^4 + 3x^2 + 2x + 5\right)\, dx$

Solution

$$\int \left(5x^4 + 3x^2 + 2x + 5\right)\, dx = x^5 + x^3 + x^2 + 5x + C$$

Exercise

Find the integral $\int \left(5x^{4/3} + 3x^{2/3} + 2x^{1/3}\right)\, dx$

Solution

$$\int \left(5x^{4/3} + 3x^{2/3} + 2x^{1/3}\right)\, dx = \frac{15}{7}x^{7/3} + \frac{9}{5}x^{5/3} + \frac{3}{2}x^{4/3} + C$$

Exercise

Find the integral $\int \left(5x^{-4/3} + 3x^{-2/3} + 2x^{-1/3}\right)\, dx$

Solution

$$\int \left(5x^{-4/3} + 3x^{-2/3} + 2x^{-1/3}\right)\, dx = -15x^{-1/3} + 9x^{1/3} + 3x^{2/3} + C$$

Exercise

Find the integral $\int \frac{x^4 - 3x^2 + 5}{x^4}\, dx$

Solution

$$\int \frac{x^4 - 3x^2 + 5}{x^4}\, dx = \int \left(1 - \frac{3}{x^2} + 5x^{-4}\right)\, dx$$

$$= x + \frac{3}{x} - \frac{5}{3x^3} + C$$

Exercise

Find the integral $\int \left(\frac{3}{x^7}-\frac{5}{x^6}\right) dx$

Solution

$$\int \left(\frac{3}{x^7}-\frac{5}{x^6}\right) dx = \int \left(3x^{-7}-5x^{-6}\right) dx$$
$$= -\frac{1}{2}x^{-6}+x^{-5}+C$$
$$= \underline{\left| -\frac{1}{2x^6}+\frac{1}{x^5}+C \right|}$$

Exercise

Find the integral $\int \frac{x+8}{\sqrt{x}}\, dx$

Solution

$$\int \frac{x+8}{\sqrt{x}}\, dx = \int \left(x^{1/2}+8x^{-1/2}\right) dx$$
$$= \underline{\left| \frac{2}{3}x^{3/2}+16x^{1/2}+C \right|}$$

Exercise

Find the integral $\int \frac{x^2+8}{\sqrt[3]{x}}\, dx$

Solution

$$\int \frac{x^2+8}{\sqrt[3]{x}}\, dx = \int \left(x^{5/3}+8x^{-1/3}\right) dx$$
$$= \underline{\left| \frac{3}{8}x^{8/3}+12x^{2/3}+C \right|}$$

Exercise

Find the integral $\int \cos\left(\frac{5\pi}{3}x\right) dx$

Solution

$$\int \cos\left(\frac{5\pi}{3}x\right) dx = \underline{\left| \frac{3}{5\pi}\sin\frac{5\pi}{3}x+C \right|}$$

Exercise

Find the integral $\int \sin\left(\frac{2x}{3}\right)\,dx$

Solution

$$\int \sin\left(\frac{2x}{3}\right)\,dx = -\frac{3}{2}\cos\frac{2x}{3} + C$$

Exercise

Find the integral $\int \left(5\cos x + 4\sin x + 3\sec^2 x\right)\,dx$

Solution

$$\int \left(5\cos x + 4\sin x + 3\sec^2 x\right)\,dx = 5\sin x - 4\cos x + 3\tan x + C$$

Exercise

Find the integral $\int \sec\theta\left(\sec\theta + \tan\theta\right)\,d\theta$

Solution

$$\int \sec\theta\left(\sec\theta + \tan\theta\right)\,d\theta = \int \left(\sec^2\theta + \sec\theta\tan\theta\right)\,d\theta$$
$$= \tan\theta + \sec\theta + C$$

Exercise

Find the integral $\int \left(\tan^2\theta + 1\right)\,d\theta$

Solution

$$\int \left(\tan^2\theta + 1\right)\,d\theta = \int \sec^2\theta\,d\theta$$
$$= \tan\theta + C$$

Exercise

Find the integral $\int \left(\cos^4\theta - \sin^4\theta\right)\, d\theta$

Solution

$$\int \left(\cos^4\theta - \sin^4\theta\right)\, d\theta = \int \left(\cos^2\theta - \sin^2\theta\right)\left(\cos^2\theta + \sin^2\theta\right)\, d\theta$$

$$= \int \cos 2\theta\, d\theta \qquad \cos^2\theta - \sin^2\theta = \cos 2\theta \quad \cos^2\theta + \sin^2\theta = 1$$

$$= \frac{1}{2}\sin 2\theta + C$$

Exercise

Find the integral $\int \left(\cos^2\theta - \sin^2\theta\right)\, d\theta$

Solution

$$\int \left(\cos^2\theta - \sin^2\theta\right)\, d\theta = \int \cos 2\theta\, d\theta \qquad \cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$= \frac{1}{2}\sin 2\theta + C$$

Exercise

Find the integral $\int \left(\cos^2\theta + \sin^2\theta\right)\, d\theta$

Solution

$$\int \left(\cos^2\theta + \sin^2\theta\right)\, d\theta = \int (1)\, d\theta$$

$$= \theta + C$$

Exercise

Find the integral $\int \left(\cos 2x \cos 4x - \sin 2x \sin 4x\right)\, dx$

Solution

$$\int \left(\cos 2x \cos 4x - \sin 2x \sin 4x\right)\, dx = \int \cos 6x\, dx$$

$$= \frac{1}{6}\sin 6x + C$$

Exercise

Find the integral $\int \left(\sin 2x \cos 4x - \cos 2x \sin 4x\right)\, dx$

Solution

$$\int \left(\sin 2x \cos 4x - \cos 2x \sin 4x\right)\, dx = \int \sin(-2x)\, dx$$

$$= -\int \sin 2x\, dx$$

$$= \frac{1}{2}\cos 2x + C$$

Exercise

Find the integral $\int \left(\sin 3x \cos 2x + \cos 3x \sin 2x\right)\, dx$

Solution

$$\int \left(\sin 3x \cos 2x + \cos 3x \sin 2x\right)\, dx = \int \sin 5x\, dx$$

$$= -\frac{1}{5}\cos 5x + C$$

Exercise

Find the integral $\int \cos 2x \sin 2x\, dx$

Solution

$$\int \cos 2x \sin 2x\, dx = \frac{1}{2}\int \sin 4x\, dx \qquad \sin\alpha = 2\sin\alpha\cos\alpha$$

$$= -\frac{1}{8}\cos 4x + C$$

Exercise

Find the integral $\int \left(2\cos^2 x - 1\right)\, dx$

Solution

$$\int \left(2\cos^2 x - 1\right)\, dx = \int \cos 2x\, dx \qquad \cos 2x = 2\cos^2 x - 1$$

$$= \frac{1}{2}\sin 2x + C$$

Exercise

Find the integral $\int \left(1 - 2\sin^2 x\right)\, dx$

Solution

$$\int \left(1 - 2\sin^2 x\right)\, dx = \int \cos 2x\, dx \qquad \cos 2x = 1 - 2\sin^2 x$$

$$= \frac{1}{2}\sin 2x + C$$

Exercise

Find the integral $\int e^{-5x}\, dx$

Solution

$$\int e^{-5x}\, dx = -\frac{1}{5}e^{-5x} + C$$

Exercise

Find the integral $\int 4e^{4x}\, dx$

Solution

$$\int 4e^{4x}\, dx = e^{4x} + C$$

Exercise

Find the integral $\int \left(2\sin\theta - 5e^{\theta}\right)\, d\theta$

Solution

$$\int \left(2\sin\theta - 5e^{\theta}\right)\, d\theta = -2\cos\theta - 5e^{\theta} + C$$

Exercise

Find the integral $\int \left(\frac{3}{x}+\sec^2 x\right)\,dx$

Solution

$$\int \left(\frac{3}{x}+\sec^2 x\right)\,dx = 3\ln|x|+\tan x+C$$

Exercise

Find the integral $\int \left(\sin x+2^x\right)\,dx$

Solution

$$\int \left(\sin x+2^x\right)\,dx = -\cos x+\frac{2^x}{\ln 2}+C \qquad \left(a^x\right)'=a^x\ln a$$

Exercise

Find the integral $\int \left(2x-3^x\right)\,dx$

Solution

$$\int \left(2x-3^x\right)\,dx = x^2-\frac{3^x}{\ln 3}+C \qquad \left(a^x\right)'=a^x\ln a$$

Exercise

Find the integral $\int \left(4x-\frac{3}{x}-\csc^2 x\right)\,dx$

Solution

$$\int \left(4x-\frac{3}{x}-\csc^2 x\right)\,dx = 2x^2-3\ln|x|+\cot x+C$$

Exercise

Find the integral $\int \left(e^{4x}-\frac{3}{x}+2\csc x\cot x\right)\,dx$

Solution

$$\int \left(e^{4x}-\frac{3}{x}+2\csc x\cot x\right)\,dx = \frac{1}{4}e^{4x}-3\ln|x|-2\csc x+C$$

Exercise

Find the integral $\int (a+b)e^{(a+b)x}\,dx$

Solution

$$\int (a+b)e^{(a+b)x}\,dx = \underline{e^{(a+b)x}+C}$$

Exercise

Find the integral $\int \left(a^2-b^2\right)e^{(a-b)x}\,dx$

Solution

$$\int \left(a^2-b^2\right)e^{(a-b)x}\,dx = \frac{a^2-b^2}{a-b}e^{(a-b)x}+C$$

$$= \underline{(a+b)e^{(a-b)x}+C}$$

Exercise

Find the function with the following property: $\frac{dy}{dx} = 2x-7, \quad y(2)=0$

Solution

$$\frac{dy}{dx} = 2x-7$$

$$dy = (2x-7)\,dx$$

$$\int dy = \int (2x-7)\,dx$$

$$y = x^2-7x+C$$

At point (2, 0):

$$0 = 2^2-7(2)+C$$

$$0 = 4-14+C$$

$$\rightarrow 0 = -10+C$$

$$\underline{C=10}$$

$$\underline{y(x) = x^2-7x+10}$$

Exercise

Find the function with the following property: $\frac{dy}{dx}=\frac{1}{x^2}+x,\quad y(2)=1;\ \ x>0$

Solution

$$\frac{dy}{dx}=\frac{1}{x^2}+x$$

$$dy=\left(x^{-2}+x\right)dx$$

$$\int dy=\int\left(x^{-2}+x\right)dx$$

$$y=-x^{-1}+\frac{1}{2}x^2+C$$

$$1=-(2)^{-1}+\frac{1}{2}(2)^2+C$$

$$1+\frac{1}{2}-2=C$$

$$\underline{C=-\frac{1}{2}}$$

$$\underline{y(x)=-\frac{1}{x}+\frac{1}{2}x^2-\frac{1}{2}}$$

Exercise

Find the function with the following property: $\frac{ds}{dt}=1+\cos t,\quad s(0)=4$

Solution

$$\frac{ds}{dt}=1+\cos t$$

$$ds=(1+\cos t)\,dt$$

$$\int ds=\int(1+\cos t)\,dt$$

$$s=t+\sin t+C$$

$$4=0+\sin(0)+C$$

$$\underline{C=4}$$

$$\underline{s(t)=t+\sin t+4}$$

Exercise

Find the function with the following property: $\frac{ds}{dt} = \cos t + \sin t, \quad s(\pi) = 1$

Solution

$$\frac{ds}{dt} = \cos t + \sin t$$

$$ds = (\cos t + \sin t)\,dt$$

$$\int ds = \int (\cos t + \sin t)\,dt$$

$$s = \sin t - \cos t + C$$

$$1 = \sin \pi - \cos \pi + C$$

$$1 = 0 - (-1) + C$$

$$1 = 1 + C$$

$$\underline{C = 0}$$

$$\underline{s(t) = \sin t - \cos t}$$

Exercise

Find the function with the following property: $f'(x) = 3x^2 - 1 \quad \& \quad f(0) = 10$

Solution

$$f(x) = \int (3x^2 - 1)\,dx$$

$$= x^3 - x + C$$

$$f(0) = \underline{C = 10}$$

$$\underline{f(x) = x^3 - x + 10}$$

Exercise

Find the function with the following property: $f'(t) = \sin t + 2t \quad \& \quad f(0) = 5$

Solution

$$f(t) = \int (\sin t + 2t)\,dt$$

$$= -\cos t + t^2 + C$$

$$f(0) = -1 + C = 5$$

$$C = 6$$

$$f(t) = -\cos t + t^2 + 6$$

Exercise

Find the function with the following property: $f'(x) = x^2 + x^{-2}$ & $f(1) = 1$

Solution

$$f(x) = \int \left(x^2 + x^{-2}\right) dx$$

$$= \frac{1}{3}x^3 - \frac{1}{x} + C$$

$$f(1) = \frac{1}{3} - 1 + C = 1$$

$$C = \frac{5}{3}$$

$$f(x) = \frac{1}{3}x^3 - \frac{1}{x} + \frac{5}{3}$$

Exercise

Find the function with the following property: $f'(x) = \sin^2 x$ & $f(1) = 1$

Solution

$$f(x) = \int \sin^2 x \, dx \qquad \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$= \frac{1}{2}\int (1 - \cos 2x)\, dx$$

$$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$$

$$f(1) = \frac{1}{2} - \frac{1}{4}\sin 2 + C = 1$$

$$C = \frac{1}{2} + \frac{1}{4}\sin 2$$

$$f(x) = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + \frac{1}{2} + \frac{1}{4}\sin 2$$

Exercise

Derive the position function if a ball is thrown upward with initial velocity of 32 *feet* per second from an initial height of 48 *feet*. When does the ball hit the ground? With what velocity does the ball hit the ground?

Solution

$$s(t) = -16t^2 + 32t + 48$$

$$s(0) = 48$$

$$s'(0) = 32$$

$$s''(t) = -32$$

$$s'(t) = \int -32\,dt$$

$$= -32t + C_1$$

$$s'(0) = -32(0) + C_1 = 32$$

$$\Rightarrow \underline{C_1 = 32}$$

$$s'(t) = -32t + 32$$

$$s(t) = \int (-32t + 32)\,dt$$

$$= -32\frac{t^2}{2} + 32t + C_2$$

$$s(0) = -32\frac{0^2}{2} + 32(0) + C_2 = 48$$

$$\Rightarrow \underline{C_2 = 48}$$

$$s(t) = -16t^2 + 32t + 48 = 0$$

$$-t^2 + 2t + 3 = 0$$

$$\underline{t_{1,2} = -1,\ 3}$$

The ball hits the ground in 3 *seconds*

The velocity: $v(t) = s'(t) = -32t + 32$

$$v(t = 3) = -32(3) + 32$$

$$\underline{= -64\ \ ft/\sec^2}$$

Exercise

Suppose a publishing company has found that the marginal cost at a level of production of x thousand books is given by

$$\frac{dC}{dx} = \frac{50}{\sqrt{x}}$$

And that the fixed cost (the cost before the first book can be produced) is a \$25,000. Find the cost function $C(x)$.

Solution

$$\frac{dC}{dx} = \frac{50}{\sqrt{x}} = 50x^{-1/2}$$

$$dC = 50x^{-1/2}dx$$

$$\int dC = \int 50x^{-1/2}dx$$

$$C(x) = 50\frac{x^{1/2}}{1/2} + C$$

$$= 50(2)x^{1/2} + C$$

$$= 100\sqrt{x} + C$$

$25000 = 100\sqrt{0} + C$ Before the first (x = 0) costs 25,000

$25,000 = C$

$C(x) = 100\sqrt{x} + 25,000$

Exercise

Find the general solution of $F'(x) = 4x + 2$, and find the particular solution that satisfies the initial condition $F(1) = 8$.

Solution

$$F(x) = \int (4x+2)\,dx$$

$$= 4\frac{x^2}{2} + 2x + C$$

$$= 2x^2 + 2x + C$$

$$F(x) = 2(1)^2 + 2(1) + C = 8$$

$$2 + 2 + C = 8$$

$$4 + C = 8$$

$C = 4$

$F(x) = 2x^2 + 2x + 4$

Exercise

The marginal cost function for producing x units of a product is modeled by

$$\frac{dC}{dx} = 28 - 0.02x$$

It costs \$40 to produce one unit. Find the cost of producing 200 units.

Solution

$$C = \int (28 - 0.02x)\,dx$$

$$= 28x - 0.02\frac{x^2}{2} + K$$

Cost \$40 for one unit

$$C(x = 1) = 40$$

$$C(x = 1) = 28(1) - 0.01(1)^2 + K = 40$$

$$K = 12.01$$

$$C(x) = -0.01x^2 + 28x + 12.01$$

$$C(200) = -0.01(200)^2 + 28(200) + 12.01$$

$$= \$5212.01$$