Cal II 9/14

1.6 Surface

$$S = 2\pi \int_a^b y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$y \downarrow f(x)$

Ex S? $y = 2\sqrt{x}$ $1 \le x \le 3$ x-axis

soln

$$\sqrt{1+\left[\frac{dy}{dx}\right]^2} = \sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}$$

$$= \sqrt{1+\frac{1}{x}}$$

$$= \sqrt{\frac{x+1}{x}}$$

$$S = 2\pi \int_1^3 2\sqrt{x}\, \frac{\sqrt{x+1}}{\sqrt{x}}\, dx$$

$$= 4\pi \int_1^3 (x+1)^{1/2}\, d(x+1)$$

$$= \frac{8}{3}\pi \left.(x+1)^{3/2}\right|_1^3$$

$$= \frac{8}{3}\pi \left[8 - 2^{3/2}\right]$$

$$= \frac{16\pi}{3}\left(4-\sqrt{2}\right) \text{ unit}^2$$

Ex $a \le b \le 2a$ ... a b

$S = 2\pi a h$?

soln $x^2 + y^2 = a^2$

$y = \sqrt{a^2 - x^2}$

$(u^n)' = n u' u^{n-1}$

$$\sqrt{1 + (y')^2} = \sqrt{1 + \left(\frac{-x}{\sqrt{a^2 - x^2}}\right)^2}$$

$$= \sqrt{1 + \frac{x^2}{a^2 - x^2}}$$

$$= \frac{a}{\sqrt{a^2 - x^2}}$$

$$S = 2\pi \int_{a-h}^{a} \sqrt{a^2 - x^2} \, \frac{a}{\sqrt{a^2 - x^2}} \, dx$$

$$= 2\pi a \int_{a-h}^{a} dx$$

$$= 2\pi a \, x \Big|_{a-h}^{a}$$

$$= 2\pi a (a - a + h)$$

$$= 2\pi a h \text{ unit}^2$$

Ex $x = 1 - y$ $\quad 0 \le y \le 1$ $\quad$ y-axis

line

soln $S = \frac{2\pi}{2} \cdot \sqrt{2}$

$= \pi \sqrt{2}$

$$\sqrt{1+(x')^2} = \sqrt{1+1}$$
$$= \sqrt{2}$$

$$S = 2\pi \int_0^1 (1-y)\sqrt{2}\, dy$$
$$= 2\pi\sqrt{2} \left(y - \tfrac{1}{2}y^2 \right)\Big|_0^1$$
$$= 2\pi\sqrt{2}\left(1-\tfrac{1}{2}\right)$$
$$= \pi\sqrt{2} \text{ unit}^2$$

$$f(x) = ax^m + bx^n \qquad ae^m + be^n$$
$$\begin{cases} m+n = 2 \\ ambn = -\tfrac{1}{4} \end{cases} \qquad \{ n = -m$$

$$\sqrt{1+(f')^2} = f'$$

Ex
$$y = \ln\left(\frac{x+\sqrt{x^2-1}}{2}\right)$$
rev y-axis

$$\left(\tfrac{5}{4}, 0\right) \ \& \ \left(\tfrac{17}{18}, \ln 2\right)$$

Soln

$$0 \le y \le \ln 2$$
$$\frac{x+\sqrt{x^2-1}}{2} = e^y$$
$$x+\sqrt{x^2-1} = 2e^y$$
$$\left(\sqrt{x^2-1}\right)^2 = \left(2e^y - x\right)^2$$
$$x^2 - 1 = 4e^{2y} - 4xe^y + x^2$$

$x^2 - 1 = 4e^{2y} - 4xe^y + 1$

$4xe^y = 4e^{2y} + 1$

$x = e^y + \frac{1}{4}e^{-y}$

$\left\{\begin{array}{l} l = -1 \checkmark \\ a m b n = 2(1)\left(\frac{1}{4}\right)(-1) = \frac{-1}{4} \checkmark \end{array}\right.$

$x' = e^y - \frac{1}{4}e^{-y}$

$$S = 2\pi \int_0^{\ln 2} \left(e^y + \frac{1}{4}e^{-y}\right)\left(e^y + \frac{1}{4}e^{-y}\right) dy$$

$$= 2\pi \int_0^{\ln 2} \left(e^{2y} + \frac{1}{2} + \frac{1}{16}e^{-2y}\right) dy$$

$$= 2\pi \left(\frac{1}{2}e^{2y} + \frac{1}{2}y - \frac{1}{32}e^{-2y}\right)\Big|_0^{\ln 2}$$

$$= 2\pi \left[\frac{1}{2}e^{2\ln 2} + \frac{1}{2}\ln 2 - \frac{1}{32}e^{-2\ln 2} - \left(\frac{1}{2} - \frac{1}{32}\right)\right]$$

$$= 2\pi \left(2 + \frac{1}{2}\ln 2 - \frac{1}{128} - \frac{15}{32}\right)$$

$$\begin{array}{r} 256 \\ 61 \\ \hline 195 \end{array}$$

$$= 2\pi \left(\frac{195}{128} + \frac{1}{2}\ln 2\right)$$

$$= \pi \left(\frac{195}{64} + \ln 2\right) \text{ unit}^2$$

$$\sqrt{1 + (f'(x))^2} = \sqrt{\quad}$$

$f'(x)$ ✓

1.7 Physical

mass = density . volume.

$$m = \int_a^b \rho(x)\,dx$$

Ex $0 \le x \le 2$ $\quad \rho(x) = 1+x^2$ $\quad m$?

soln

$$m = \int_0^2 (1+x^2)\,dx$$

$$= x + \frac{1}{3}x^3 \Big|_0^2$$

$$= 2 + \frac{8}{3}$$

$$= \frac{14}{3}\ kg$$

$$W = F \cdot D$$

Force Distance

$$= \int_a^b F(x)\,dx$$

Hooke's law: $f = kx$

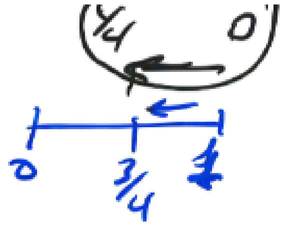
k: spring constant

Force

Ex $W?$ $k = 16$

$$F = kx = 16x$$

$$W = \int_0^{1/4} 16x\,dx = 8x^2\Big|_0^{1/4} = 8\frac{1}{16} = \frac{1}{2} \text{ ft-lb}$$



$F = (-)\int_{1/4}^{0}$

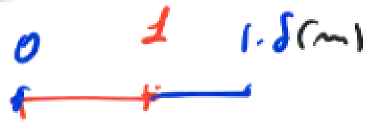
Ex Given $F = 24\ N$

a) k?

$$F = kx$$

$$24 = k(1.8 - 1)$$

$$k = \frac{24}{.8} = \frac{240}{8} = 30\ N/m$$



1.8 m

$1.8\ (m)^r$

b) $W?$ $0 \le x \le 2$

$$F = kx = 30x$$

$$W = \int_0^2 30x\,dx = 15x^2\Big|_0^2 = 60\ J$$

c) x? $F = 45\ N.$

$$F = 30x = 45$$

$$x = \frac{45}{30} = \frac{9}{6}$$

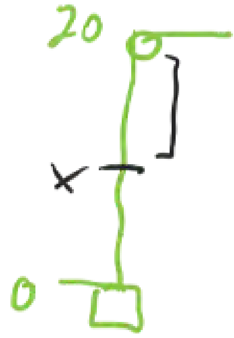
$$\boxed{= \frac{3}{2}\ m}$$

15)

$W_b = 5\ lb$

$w_r = .05\ lb/ft$

W?



soln

$$\boxed{W_B = 5(20) = 100}$$

$$W = \int_0^{20} .05(20-x)\,dx$$

$$= \frac{5}{100}\left(20x - \frac{1}{2}x^2\right|_0^{20}$$

$$= \frac{2}{25}(400 - 200)$$

$$= 16$$

$$\boxed{W_T = 100 + 16 = 116\ ft\text{-}lb}$$

$g = 9.8 \ m/sec$

$= 32.2 \ ft/sec^2$

$\frac{1}{2}g = 16$ only

$F = mg$ Lifting

$W = F \cdot D$

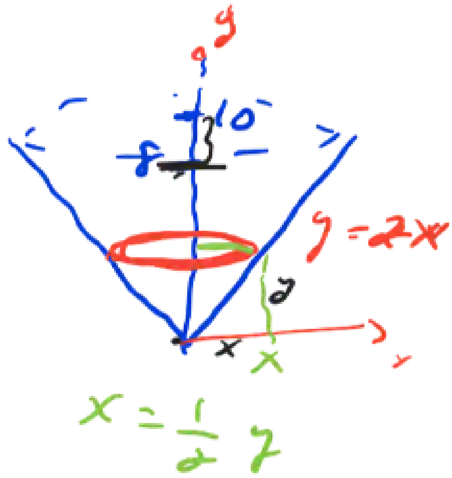
$= mgy$

$$W = \rho g \int_a^b A(y) \overbrace{(h-y)}^{D(y)} dy$$

$$= \rho g \int_a^b A(y) D(y) dy$$

Ex $\rho g = 57 \ lb/ft^3$

$W = ?$



Soln

$A(y) = \pi \left(\frac{1}{2}y\right)^2$

$= \frac{\pi}{4} y^2$

$D(y) = 10 - y$

$$W = 57 \int_0^8 \frac{\pi}{4} y^2 (10-y) dy$$

$$= \frac{57\pi}{4} \int_0^8 (10y^2 - y^3) dy$$

$$= \frac{57\pi}{4} \left(\frac{10}{3} y^3 - \frac{1}{4} y^4 \right|_0^8$$

$$= \frac{57\pi}{4} \left(\frac{5{,}120}{3} - 1{,}024 \right)$$

$$= \frac{57\pi}{4} \cdot \frac{2048}{3}$$

$$= \boxed{9728\pi \ \text{ft-lb}}$$

$$F = m g$$

$$= \text{Vol} \cdot \text{density} \cdot g$$

$$= A \cdot h \cdot \underbrace{\rho g}$$

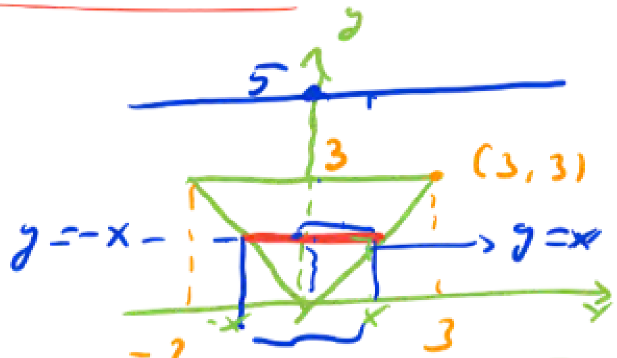
$$F = \rho g \int_0^a (a - y) w(y)\, dy$$

$L(y)$ same $w(y)$

Ex $\rho = 62.4$

F?

$h = 5 - y$



$y = x$

$$F = 62.4 \int_0^3 (5 - y)(2x)\, dy$$

$$= 62.4 \int_0 (5-y)(2y)\,dy$$

$$= \frac{1248}{10} \int_0^3 (5y - y^2)\,dy$$

$$= \frac{624}{5} \left(\frac{5}{2} y^2 - \frac{1}{3} y^3 \right|_0^3$$

$$= \frac{624}{5} \left(\frac{45}{2} - 9 \right)$$

$$= \frac{624}{5} \; \frac{27}{2}$$

$$= \frac{8,424}{5} \quad lb$$

$$\rho g = 1000 \quad kg/m^3$$

$$= 10^3$$