







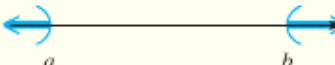



Section 1.7 – Inequalities

Notation

Type of Interval	Set	Interval Notation	Graph
Open interval	$\{x \mid x > a\}$	(a, ∞)	
	$\{x \mid a < x < b\}$	(a, b)	
	$\{x \mid x < b\}$	$(-\infty, b)$	
Other intervals	$\{x \mid x \geq a\}$	$[a, \infty)$	
	$\{x \mid a < x \leq b\}$	$(a, b]$	
	$\{x \mid a \leq x < b\}$	$[a, b)$	
	$\{x \mid x \leq b\}$	$(-\infty, b]$	
Closed interval	$\{x \mid a \leq x \leq b\}$	$[a, b]$	
Disjoint interval	$\{x \mid x < a \text{ or } x > b\}$	$(-\infty, a) \cup (b, \infty)$	
All real numbers	$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	

Properties of inequality

1. If $a < b$, then $a + c < b + c$
2. If $a < b$ and if $c > 0$, then $ac < bc$
3. If $a < b$ and if $c < 0$, then $ac > bc$

Example

Solve $3x + 1 > 7x - 15$

Solution

$$3x - 7x > -1 - 15$$

$$-4x > -16$$

Divide by -4 both sides

$$\underline{x < 4} \quad \text{or} \quad (-\infty, 4) \quad \text{or} \quad \{x \mid x < 4\}$$

Example

Solve $\frac{x-4}{2} \geq \frac{x-2}{3} + \frac{5}{6}$ LCD: 2, 3, 6

Solution

$$(6)\frac{x-4}{2} \geq (6)\frac{x-2}{3} + (6)\frac{5}{6}$$

$$3(x-4) \geq 2(x-2) + 5$$

$$3x - 12 \geq 2x - 4 + 5$$

$$3x - 12 \geq 2x + 1$$

$$3x - 2x \geq 12 + 1$$

$$\underline{x \geq 13}$$

Example

a) $3(x+1) > 3x+2$

$$3x + 3 > 3x + 2$$

$$3x - 3x > -3 + 1$$

$$0 > -1 \text{ (True statement)}$$

$$\text{Sol.: } \mathbb{R} \text{ or } \{x | \text{All Real numbers}\} \text{ or } (-\infty, \infty)$$

b) $x+1 \leq x-1$

$$x - x \leq -1 - 1$$

$$0 \leq -2$$

$$\text{Sol.: } \emptyset$$

Example

Solve $-2 < 5 + 3x < 20$ Give the solution set in interval notation and graph it.

Solution

$$-2 - 5 < 5 + 3x - 5 < 20 - 5$$

$$-7 < 3x < 15$$

$$-\frac{7}{3} < \frac{3}{3}x < \frac{15}{3}$$

$$-\frac{7}{3} < x < 5$$

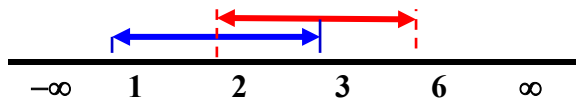
$$\text{Solution: } \left(-\frac{7}{3}, 5\right)$$

Intersections of Interval \cap

To find the intersection, take the portion of the number line that the two graphs have in *common*

Example

$$[1, 3] \cap (2, 6) = (2, 3]$$

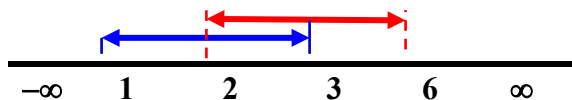


Unions of Interval \cup

To find the union, take the portion of the number line representing the total *collection* of numbers in the two graphs.

Example

$$[1, 3] \cup (2, 6) = [1, 6)$$



Solving an *Absolute Value* Inequality:

If X is an algebraic expression and c is a positive number,

1. The solutions of $|X| < c$ are the numbers that satisfy $-c < X < c$.
2. The solutions of $|X| > c$ are the numbers that satisfy $X < -c$ or $X > c$.

Example

Solve: $-3|5x - 2| + 20 \geq -19$

Solution

$$-3|5x - 2| \geq -39$$

$$-|5x - 2| \geq -13$$

$$|5x - 2| \leq 13$$

$$-13 \leq 5x - 2 \leq 13$$

$$-11 \leq 5x \leq 15$$

$$\underline{-\frac{11}{5} \leq x \leq 3} \quad \text{or} \quad \underline{\left[-\frac{11}{5}, 3\right]}$$

Example

Solve: $18 < |6 - 3x|$

Solution

$$|6 - 3x| > 18$$

$$6 - 3x < -18$$

$$-3x < -18 - 6$$

$$-3x < -24$$

$$\frac{-3}{-3}x > \frac{-24}{-3}$$

$$x > 8$$

$$6 - 3x > 18$$

$$-3x > 18 - 6$$

$$-3x > 12$$

$$\frac{-3}{-3}x < \frac{12}{-3}$$

$$x < -4$$

$$\text{Solution: } \underline{(-\infty, -4) \cup (8, \infty)}$$

Special Cases

Example

Solve the inequality $|2 - 5x| \geq -4$

Solution

$$|2 - 5x| \geq -4$$

It is always **true**

\therefore The solution set is: \mathbb{R} All real numbers $(-\infty, \infty)$

Example

Solve the inequality $|4x - 7| < -3$

Solution

$$|4x - 7| < -3$$

Any absolute value can't be less than any negative number.

\therefore No solution or \emptyset

Example

Solve the inequality $|5x + 15| = 0$

Solution

$$|5x + 15| = 0$$

$$5x + 15 = 0$$

$$5x = -15$$

\therefore Solution: $x = -3$

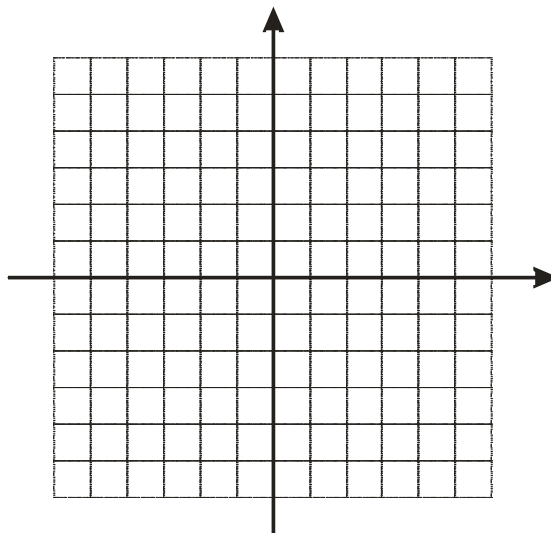
Definition of a Polynomial Inequality

A polynomial inequality is any inequality that can be put into one of the forms

$$f(x) < 0 \quad f(x) > 0 \quad f(x) \leq 0 \quad f(x) \geq 0$$

Where f is a polynomial function.

$$f(x) = x^2 - 5x + 4 \quad (x = 1, 4)$$



Procedure for Solving Polynomial Inequalities

Example

1. Express the inequality in the form $f(x) ? 0$	$x^2 - x < 12$ $x^2 - x - 12 < 0$
2. Solve $f(x) = 0$	$x^2 - x - 12 = 0$ $x = -3, 4$
3. Locate the boundary	$-3 \quad 0 \quad 4$
4. Choose one test value	$+$ $-$ $+$
5. Write the solution set	$(-3, 4)$

$$\checkmark \quad ax^2 + bx + c \geq 0 \rightarrow \text{if } a > 0 \Rightarrow x \leq x_1, x \geq x_2$$

$$\checkmark \quad ax^2 + bx + c \leq 0 \rightarrow \text{if } a > 0 \Rightarrow x_1 \leq x \leq x_2$$

Example

Solve $2x^2 + 5x - 12 \geq 0$

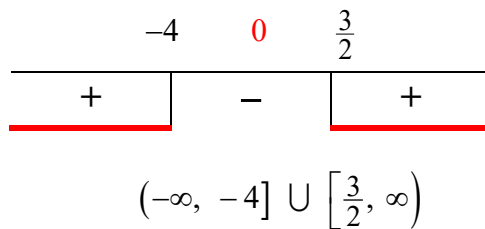
Solution

$$2x^2 + 5x - 12 = 0$$

$$(2x - 3)(x + 4) = 0$$

$$x = -4, \frac{3}{2}$$

$$\text{Solution: } \underline{x \leq -4 \quad x \geq \frac{3}{2}} \quad |$$



Example

Solve: $x^3 + 3x^2 \leq x + 3$

Solution

$$x^3 + 3x^2 - x - 3 = 0$$

$$x^2(x + 3) - (x + 3) = 0$$

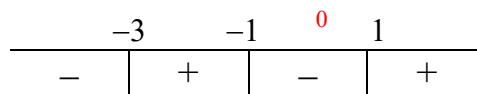
$$(x + 3)(x^2 - 1) = 0$$

$$x + 3 = 0 \quad x^2 - 1 = 0$$

$$x = -3 \quad x^2 = 1$$

$$\underline{x = -3} \quad \underline{x = \pm 1} \quad |$$

$$\text{Solution: } \underline{x \leq -3 \quad -1 \leq x \leq 1} \quad |$$



$$\underline{(-\infty, -3] \cup [-1, 1]} \quad |$$

Rational Inequality

Example

Solve: $\frac{2x}{x+1} \geq 1$

Solution

$$\frac{2x}{x+1} = 1 \rightarrow \text{Cond.: } x+1 \neq 0 \Rightarrow \underline{x \neq -1}$$

$$(x+1) \frac{2x}{x+1} - 1(x+1) = 0$$

$$2x - x - 1 = 0$$

$$x - 1 = 0$$

$$x = 1$$

$$\text{Solution: } \underline{x \leq -1 \quad x \geq 1} \quad | \quad \underline{(-\infty, -1) \cup [1, \infty)} |$$

-1	0	1
+	-	+

Example

Solve $\frac{5}{x+4} \geq 1$

Solution

$$\frac{5}{x+4} - 1 = 0 \quad \text{Exception: } x+4 \neq 0 \Rightarrow \underline{x \neq -4}$$

$$(x+4) \frac{5}{x+4} - 1(x+4) = 0$$

$$5 - x - 4 = 0$$

$$\underline{x = 1}$$

$$\text{Solution: } \underline{-4 < x \leq 1} \quad | \quad \underline{(-4, 1]} |$$

-4	0	1
-	+	-

Example

Solve $\frac{2x-1}{3x+4} < 5$

Solution

$$\frac{2x-1}{3x+4} - 5 = 0 \quad \text{Restriction: } 3x+4 \neq 0 \Rightarrow \underline{x \neq -\frac{4}{3}}$$

$$(3x+4) \frac{2x-1}{3x+4} - 5(3x+4) = 0$$

$$2x - 1 - 15x - 20 = 0$$

$$-13x - 21 = 0$$

$$\underline{x = -\frac{21}{13}}$$

$$\text{Solution: } \underline{x < -\frac{21}{13} \quad x > -\frac{4}{3}} \quad | \quad \underline{\left(-\infty, -\frac{21}{13}\right) \cup \left(-\frac{4}{3}, \infty\right)} |$$

$-\frac{21}{13}$	$-\frac{4}{3}$	0
-	+	-

Exercises Section 1.7 – Inequalities

(1 – 6) Find:

- | | | |
|---------------------------|---------------------------|-------------------------------|
| 1. $(-3, 0) \cap [-1, 2]$ | 3. $(-4, 0) \cap [-2, 1]$ | 5. $(-\infty, 5) \cap [1, 8]$ |
| 2. $(-3, 0) \cup [-1, 2]$ | 4. $(-4, 0) \cup [-2, 1]$ | 6. $(-\infty, 5) \cup [1, 8]$ |

(7 – 45) Solve the inequality equation

- | | |
|---|--|
| 7. $-3x + 5 > -7$ | 28. $\frac{x-4}{6} \geq \frac{x-2}{9} + \frac{5}{18}$ |
| 8. $2 - 3x \leq 5$ | 29. $\frac{4x-3}{6} + 2 \geq \frac{2x-1}{12}$ |
| 9. $4 - 3x \leq 7 + 2x$ | 30. $4(3x-2) - 3x < 3(1+3x) - 7$ |
| 10. $5x + 11 < 26$ | 31. $3(x-8) - 2(10-x) < 5(x-1)$ |
| 11. $3x - 8 \geq 13$ | 32. $8(x+1) \leq 7(x+5) + x$ |
| 12. $-9x \geq 36$ | 33. $4(x-1) \geq 3(x-2) + x$ |
| 13. $-4x \leq 64$ | 34. $7(x+4) - 13 > 12 + 13(3+x)$ |
| 14. $8x - 11 \leq 3x - 13$ | 35. $-2[7x - (2x-3)] < -2(x+1)$ |
| 15. $18x + 45 \leq 12x - 8$ | 36. $6 - \frac{2}{3}(3x-12) \leq \frac{2}{5}(10x+50)$ |
| 16. $4(x+1) + 2 \geq 3x + 6$ | 37. $\frac{2}{7}(7-21x) - 4 < 10 - \frac{3}{11}(11x-11)$ |
| 17. $8x + 3 > 3(2x+1) + x + 5$ | 38. $3[3(x+5) + 8x+7] + 5[3(x-6) - 2(3x-5)] < 2(4x+3)$ |
| 18. $2x - 11 < -3(x+2)$ | 39. $5[3(2-3x) - 2(5-x)] - 6[5(x-2) - 2(4x-3)] < 3x+19$ |
| 19. $-4(x+2) > 3x+20$ | 40. $0 \leq 3x-1 \leq 10$ |
| 20. $1 - (x+3) \geq 4 - 2x$ | 41. $0 \leq 1-3x \leq 10$ |
| 21. $5(3-x) \leq 3x-1$ | 42. $0 \leq 2x+6 \leq 54$ |
| 22. $\frac{x}{4} - \frac{1}{2} \leq \frac{x}{2} + 1$ | 43. $-3 \leq \frac{2}{3}x - 5 \leq -1$ |
| 23. $\frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$ | 44. $-6 \leq 6x+3 \leq 21$ |
| 24. $6x - (2x+3) \geq 4x-5$ | 45. $1 \leq 2x+3 \leq 11$ |
| 25. $\frac{2x-5}{-8} \leq 1-x$ | |
| 26. $1 - \frac{x}{2} > 4$ | |
| 27. $7 - \frac{4}{5}x < \frac{3}{5}$ | |

(46 – 85) Solve the inequality equation

46. $|x| < 2$

47. $|x| \geq 2$

48. $|x - 2| < 1$

49. $|x - 1| < 4$

50. $|x + 2| \geq 1$

51. $|x + 1| \geq 4$

52. $|3x + 5| < 17$

53. $|5x - 2| < 13$

54. $|5x - 2| \geq 13$

55. $|2(x - 1) + 4| \leq 8$

56. $|3(x - 1) + 2| \leq 20$

57. $\left| \frac{2x + 6}{3} \right| > 2$

58. $\left| \frac{3x - 3}{4} \right| < 6$

59. $\left| \frac{2x + 2}{4} \right| \geq 2$

60. $\left| \frac{3x - 3}{9} \right| \leq 1$

61. $\left| 3 - \frac{2x}{3} \right| > 5$

62. $\left| 3 - \frac{3x}{4} \right| < 9$

63. $|x - 2| < -1$

64. $|x + 2| < -3$

65. $|x + 6| > -10$

66. $|x + 2| > -8$

67. $|x + 2| + 9 \leq 16$

68. $|x - 2| + 4 \geq 5$

69. $2|2x - 3| + 10 > 12$

70. $3|2x - 1| + 2 < 8$

71. $-4|1 - x| < -16$

72. $-2|5 - x| < -6$

73. $3 \leq |2x - 1|$

74. $9 \leq |4x + 7|$

75. $12 < \left| -2x + \frac{6}{7} \right| + \frac{3}{7}$

76. $4 + \left| 3 - \frac{x}{3} \right| \geq 9$

77. $|x - 2| < 5$

78. $|2x + 1| < 7$

79. $|5x + 2| - 2 < 3$

80. $|2 - 7x| - 1 > 4$

81. $|3x - 4| < 2$

82. $|2x + 5| \geq 3$

83. $|12 - 9x| \geq -12$

84. $|6 - 3x| < -11$

85. $|7 + 2x| < 0$

(86 – 107) Solve the inequality equation

86. $x^2 - 7x + 10 > 0$

87. $2x^2 - 9x \leq 18$

88. $x^2 - 5x + 4 > 0$

89. $x^2 + x - 2 > 0$

90. $x^2 - 4x + 12 < 0$

91. $x^2 + 7x > 0$

92. $x^2 - 49 < 0$

93. $x^2 - 5x \geq 0$

94. $x^2 - 16 \leq 0$

95. $x^2 + 7x + 10 < 0$

96. $x^2 - 3x \geq 28$

97. $x^2 + 5x + 6 < 0$

98. $x^2 < -x + 30$

99. $x^3 - 3x^2 - 9x + 27 < 0$

100. $x^3 - x > 0$

101. $x^3 + 3x^2 \leq x + 3$

102. $x^3 + x^2 \geq 48x$

103. $x^3 - x^2 - 16x + 16 < 0$

104. $x^3 + x^2 - 9x - 9 > 0$

105. $x^3 + 3x^2 - 4x - 12 \geq 0$

106. $x^4 - 20x^2 + 64 \leq 0$

107. $x^4 - 10x^2 + 9 \geq 0$

(108 – 130) Solve the inequality equation

108. $\frac{x+4}{x-1} < 0$

116. $\frac{x}{x-3} > 0$

124. $\frac{2x-1}{x+3} \geq \frac{x+1}{3x+1}$

109. $\frac{x-2}{x+3} > 0$

117. $\frac{x-3}{x+2} \geq 0$

125. $\frac{(x+1)(x-4)}{x-2} < 0$

110. $\frac{x-5}{x+8} \geq 3$

118. $\frac{x-2}{x+2} \leq 2$

126. $\frac{x(x-4)}{x+5} > 0$

111. $\frac{x-4}{x+6} \leq 1$

119. $\frac{x+2}{x-2} \geq 2$

127. $\frac{6x^2-11x-10}{x} > 0$

112. $\frac{x}{2x+7} \geq 4$

120. $\frac{x+2}{3+2x} \leq 5$

128. $\frac{3x^2-2x-8}{x-1} \geq 0$

113. $\frac{x}{3x-5} \leq -5$

121. $\frac{x+6}{x-14} \geq 1$

129. $\frac{x^2-6x+9}{x-5} \leq 0$

114. $\frac{x+2}{x-5} \leq 2$

122. $\frac{x-3}{x+4} \geq \frac{x+2}{x-5}$

130. $\frac{x^2+10x+25}{x+1} \leq 0$

115. $\frac{3x+1}{x-2} \geq 4$

123. $\frac{x-4}{x+3} - \frac{x+2}{x-1} \leq 0$