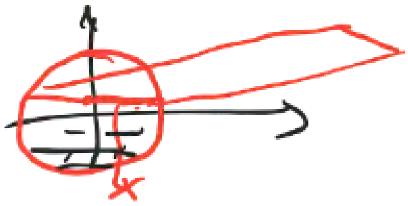
Ex $\quad x^2+y^2=25 \qquad \rho \approx 737$

$l=10$



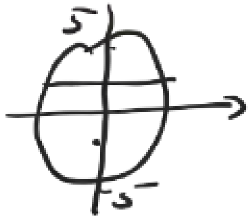
soln

$$x=\pm\sqrt{25-y^2}$$

$$\text{Area} = l\cdot w = l(2x) = 20\sqrt{25-y^2}$$

$$D = 5-y$$

$$W=\rho g\int_a^b A(y)\,D\,dy$$



$$W = 737(9.8)\int_{-5}^{0} 20\sqrt{25-y^2}\,(5-y)\,dy$$

$$= 144{,}452\int_{-5}^{0} 5(25-y^2)^{1/2}\,dy \quad \Big\} \ \tfrac{1}{4}\text{circle}$$

$$-\ 144{,}452\int_{-5}^{0} y(25-y^2)^{1/2}\,dy$$

$$= 144{,}452\left[5\cdot\tfrac{1}{4}(25\pi)+\tfrac{1}{2}\int_{-5}^{0}(25-y^2)^{1/2}\,d(25-y^2)\right]$$

$$= 144{,}452\left[\frac{125\pi}{4}+\frac{1}{3}(25-y^2)^{3/2}\Big|_{-5}^{0}\right]$$

$$= 144{,}452\left(\frac{125\pi}{4}+\frac{1}{3}(125)\right)$$

$$= 144{,}452(125)\left(\frac{\pi}{4}+\frac{1}{3}\right) \quad J$$

1.5/ Exponential (application)

CA.

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{du}{u} = \ln|u| + C$$

only $\int$ use u sub. $x\underbrace{(x)}_{=u}^{n}$

$\ln e = 1$

$\ln 1 = 0$

$\ln e^{x} = x$

$\ln e^{-x} = \frac{1}{x}$

$$\boxed{\ln 0 = -\infty}$$

$\ln a^{p} = p\ln a$

Ex
$$\int_0^4 \frac{x}{x^2+9}\,dx = \frac{1}{2}\int_0^4 \frac{d(x^2+9)}{x^2+9}$$
$$= \frac{1}{2}\ln(x^2+9)\Big|_0^4$$
$$= \frac{1}{2}(\ln 5^2 - \ln 3^2)$$
$$= \frac{1}{2}(2\ln 5 - 2\ln 3)$$
$$= \ln 5 - \ln 3$$
$$= \ln\frac{5}{3}$$

Ex

Ex $\int \frac{e^x}{1+e^x}\,dx = \int \frac{d(1+e^x)}{1+e^x}$ $\quad (e^x)' = e^x$

$$= \ln(1+e^x) + C$$

$$\frac{d}{dx} a^u = a^u \ln a \, \frac{du}{dx}$$
$$= u' a^u \ln a$$

$$(\log_a u)' = \frac{u'}{u} \, \frac{1}{\ln a}$$

$$\int a^u \, du = \frac{a^u}{\ln a} + C$$

Ex $\int x\, 3^{x^2} dx = \frac{1}{2} \int 3^{x^2} d(x^2)$

$$= \frac{1}{2} \, \frac{3^{x^2}}{\ln 3} + C$$

$$e^{n \ln x} = e^{\ln x^n} = x^n$$
$$x^n = e^{n \ln x}$$

Ex $f(x) = x^{2x}$ $\quad f'$??

$$\frac{d}{dx}\left(x^{2x}\right) = \frac{d}{dx}\left(e^{2x \ln x}\right)$$
$$= (2x \ln x)' \, e^{2x \ln x}$$
$$= 2(\ln x + 1)\, e^{2x \ln x}$$

$= 2(\ln x + 1) e$

$$= 2(\ln x + 1)\, x^{2x}$$

$\cancel{x^{x-1}}$

$$y = x^x$$

$$\ln y = \ln x^x$$

$$= x \ln x$$

$$\frac{y'}{y} = \ln x + 1$$

$$y' = x^x(\ln x + 1)$$

$$y(t) = y_0(t)\, e^{kt}$$

$k > 0 \to$ growth

$k < 0 \to$ decay

$t =$ time

y_0 : initial @ t=0

$y(t)$ @ certain time.

$$\boxed{kT = \ln \frac{A}{A_0}}$$

ln 2 ← double

$\ln \frac{1}{2}$ ← $\frac{1}{2}$ life

finance: continuous $P = P_0 e^{rt}$

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

d.

exp. e^x

Total Energy $= \int_a E(t)\,dt$ — Energy

$$= \int_a^b P(t)\,dt \to \text{Power}$$

#d) $f(x) = \dfrac{e^{\sqrt{x}}}{\ln(\sqrt{x}+1)}$ $\qquad \left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$$f'(x) = \frac{1}{(\ln(\sqrt{x}+1))^2}\left[\frac{1}{2\sqrt{x}}e^{\sqrt{x}}\ln(\sqrt{x}+1) - \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}+1}e^{\sqrt{x}}\right]$$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}\,(\ln(\sqrt{x}+1))^2}\left(\ln(\sqrt{x}+1) - \frac{1}{\sqrt{x}+1}\right)$$

39) $\displaystyle\int \frac{e^{2x}}{\sqrt{e^{2x}+4}}\,dx = \frac{1}{2}\int (e^{2x}+4)^{-1/2}\,d(e^{2x}+4)$

$$= \boxed{\sqrt{e^{2x}+4} + C}$$

1.9)

$$\boxed{\begin{aligned}\cosh x &= \frac{e^x + e^{-x}}{2}\\ \sinh x &= \frac{e^x - e^{-x}}{2}\end{aligned}}$$

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$\frac{d}{dt}\left(\tanh\sqrt{1+t^2}\right) = \frac{t}{\sqrt{1+t^2}}\operatorname{sech}^2\sqrt{1+t^2}$$

$$(\operatorname{sech} 3x)' = -3\operatorname{sech} 3x \tanh 3x$$

$$(\operatorname{sech} 3x)'' = -3\left(-3\operatorname{sech} 3x \tanh^2 3x + 3\operatorname{sech}^3 3x\right)$$

$$= 9\operatorname{sech}(3x)\left(\tanh^2 3x - \operatorname{sech}^2 3x\right)$$

$$\int \coth 5x\,dx = \frac{1}{5}\int \frac{\cosh 5x}{\sinh 5x}\,d(5x)$$

$$= \frac{1}{5}\int \frac{d(\sinh 5x)}{\sinh 5x}$$

$$= \frac{1}{5}\ln|\sinh 5x| + C$$

$$\int_0^{\ln 2} 4e^x \sinh x\,dx = 4\int_0^{\ln 2} e^x \left(\frac{1}{2}\right)(e^x - e^{-x})\,dx$$

$$= 2\int_0^{\ln 2} (e^{2x} - 1)\,dx$$

$$= 2\left(\frac{1}{2}e^{2x} - x\right|_0^{\ln 2}$$

$$= 2\left(2 - \ln 2 - \frac{1}{2}\right)$$

$$= 3 - 2\ln 2$$

$y = \sinh^{-1} x \Rightarrow$

$y = \ln\left(x + \sqrt{x^2+1}\right)$

$y = \cosh^{-1} x = \ln\left(x \pm \sqrt{x^2-1}\right)$

Conclude Lect 1

Exam 1 $\Rightarrow$ 9/23

(1) Area

(3) or (2) Volume { any method

(2) length { $ax^n + bx^m$ $\to$ $m+n=2$, $abmn = -\frac{1}{4}$
{ $ae^{nx} + be^{mx} \to m = -n$

(1) Surface $\to$ $\sqrt{1+f'(x)} = f'(x)$

(1) mass $= \int_a^b \rho\, dx$

(1) ~~work~~ F

(1) der. ln & e

(2) integral $\int \ln e$, $\int e$ & hyp.

10 $\quad$ ~~300~~ 10^3

Hwk 1.4 # 8

$x = (y-3)^2 \ , \ x = 4 \quad \sim y = 1$ ¢
x-axis
opp

$(y-3)^2 = 4$

$y - 3 = \pm 2 \Rightarrow \underline{y = 1, 5}$

$V = 2\pi \int_1^5 (y-1)(4-(y-3)^2)\, dy$

$= 2\pi \int_1^5 (y-1)(-5 - y^2 + 6y)\, dy$

$= 2\pi \int_1^5 (-11y - y^3 + 7y^2 + \cancel{6}5)\, dy$

$= 2\pi \left(-\frac{11}{2}y^2 - \frac{1}{4}y^4 + \frac{7}{3}y^3 + \cancel{6}5y \right|_1^5$

$= 2\pi \left(-\frac{275}{2} - \frac{625}{4} + \frac{875}{3} + 25 + \frac{11}{2} + \frac{1}{4} - \frac{7}{3} - 5 \right)$

$= 2\pi \left(-132 - 156 + \frac{868}{3} + 20\right)$

$= 2\pi \left(\frac{868}{3} - 268\right)$

$= \boxed{\frac{128\pi}{3} \text{ unit}^3}$

#7 $y = -x^2 + 6x - 8 = 0 \sim x\text{-axis}$

$y = 0$

$x = 2, 4$

$$V = \pi \int_2^4 (-x^2 + 6x - 8)^2 \, dx$$

$$= \pi \int_2^4 (x^4 - 6x^3 + 8x^2 - 6x^3 + 36x^2 - 48x$$
$$8x^2 - 48x + 64) \, dx$$

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b)

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10.a

10.b → space

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Conversation → type Im done