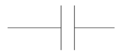
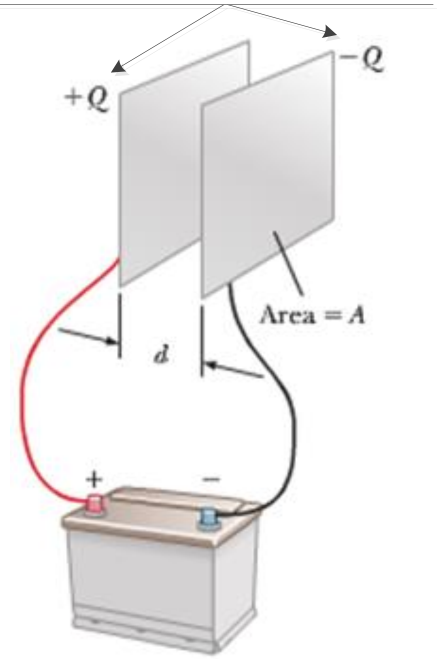
2.4 – *Capacitance and Dielectric*

A Capacitor is two conductors (*or more*) separated by an insulator. A capacitor is used to store charges or electrical energy. The circuit symbol of a capacitor is





The plates carry equal & opposite charges.

When a capacitor is connected to a potential difference (*such as a battery*), charges are transferred from one of the conductors to the other conductor, and both conductors acquire equal but opposite charge. The charge accumulated is directly proportional to the potential difference between the conductors. That is

$$\frac{Q}{\Delta V} = constant$$

Where Q is charge accumulated by the conductors and ΔV is the potential difference between the conductors. The constant of proportionality is called the capacitance of the capacitor and denoted by C.

$$\frac{Q}{\Delta V} = C$$

or

$$Q = C\Delta V$$

The unit of measurement for capacitance is ***coulomb/volt*** which is defined to be the ***Fared*** abbreviated as ***F***.

Example

Calculate the capacitance of a capacitor that accumulates a charge of $2mC$ when connected to a potential difference of $8V$.

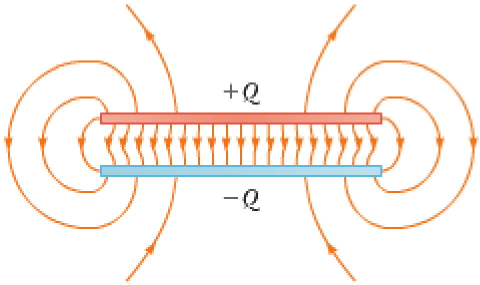
<u>Solution</u>

Given: $\Delta V = 8V \quad Q = 2mC$

$$C = \frac{Q}{\Delta V} = \frac{2mC}{8V} = 0.25\ mF$$

Parallel Plate Capacitor

Is two parallel plates separated by an insulator. From symmetry, the electric field is uniform and perpendicular to the plate inside (*between*) the plates and approximately zero outside the plates.
If the charge density is δ, an expression for the electric field can be obtained by using Gauss's law by taking the Gaussian surface to be the cylindrical surface shown.



The electric flux crossing part of the cylinder outside the plates is zero because the electric field outside the plates is approximately zero. The electric flux on the curved surface inside plates is zero because the area vector and the electric field perpendicular to each other. The only contribution to the electric flux comes from the end face inside the plates. If the area of the end face is *A* then

$$\oint \vec{E} \cdot d\vec{A} = EA$$

where is the magnitude of the electric field inside. From Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = 4\pi kq$$

where *q* is the charge enclosed by the Gaussian surface.

$$\therefore\ q = \sigma A$$

$$\oint \vec{E} \cdot d\vec{A} = EA = 4\pi kq = 4\pi k\sigma A$$

$$\Rightarrow \underline{E = 4\pi k\sigma|}$$

Coulomb's constant *k* is related with another constant called ***electrical permittivity*** in ***vacuum*** (*denoted by* ε_0) as

$$\varepsilon_0 = \frac{1}{4\pi k} \quad or \quad k = \frac{1}{4\pi\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

E: Electric field inside parallel plate capacitor
δ: Charge density of the plates

ε_0 : Electrical permittivity of vacuum $\left(\varepsilon_0 = 8.85\times10^{-12}\,\frac{C^2}{N^2m^2}\right)$

Now that we have an expression for the electric field inside, we can also obtain an expression for the potential difference between the plates. Since the electric field is a constant

$$\left|\Delta V\right| = Ed \quad but \quad E = \frac{\sigma}{\varepsilon_0}$$

$$\left|\Delta V\right| = \frac{\sigma}{\varepsilon_0} d$$

The capacitance of the capacitor is the ratio between the charge and the potential difference.

$$C_{||} = \frac{Q}{\Delta V} \quad but \quad Q = \sigma A$$

(where *A is the area of one of the plates*)

$$C_{||} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} \quad \Rightarrow \quad \boxed{C_{||} = \frac{\varepsilon_0 A}{d}}$$

Capacitance of a parallel plate capacitor of area A and separation d, (*if they are separated by vacuum or air*)

Example

The plates of a parallel plate capacitor have an area of $2\ cm^2$ and are separated by a distance of $0.5cm$. Each has a charge of $5C$.

a) Calculate the capacitance of the capacitor.
b) Calculate the potential difference between the plates.
c) Calculate the strength of the electric field between the plates.

Solution

a) Calculate the capacitance of the capacitor.

Given: $A = 2cm^2 = 2\times10^{-4} m^2 \quad d = 0.5cm = 5\times10^{-3} m$

$$C_{||} = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-2}\right)\left(2\times10^{-4}\right)}{5\times10^{-3}}$$

$$= 3.54\times10^{-13}\ F$$

b) Calculate the potential difference between the plates.

Given: $Q = 5\times10^{-12} C$

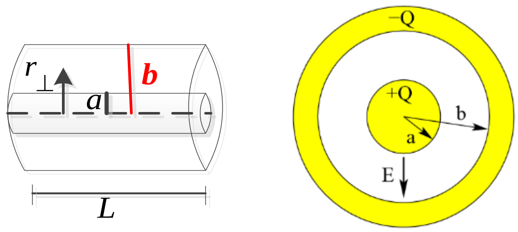
$$\Delta V = \frac{Q}{C_{||}} = \frac{5\times10^{-12}}{3.54\times10^{-13}} = 14.1\ V$$

c) Calculate the strength of the electric field between the plates.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{5\times10^{-12}}{\left(2\times10^{-4}\right)\left(8.85\times10^{-12}\right)} = 2.8\times10^{3}\ N/C$$

Concentric Cylindrical Capacitor

Concentric Cylindrical Capacitor is a capacitor made up of two concentric cylinders. Consider two concentric cylinders of radii a and b. Let the linear charge density of the cylinders be $\lambda = \frac{Q}{L}$ where Q is the total charge and L is the length of the cylinders.



If the cylinders are long enough, it can be shown from Gauss's law that the elastic field between the cylinders is given by

$$\vec{E} = \frac{2k\lambda}{r_\perp}\overrightarrow{e_{r_\perp}}$$

Where $r_\perp$ *is the* $\perp$ *distance between points and the axis of the cylinders*

$$\vec{e}_{r_\perp} = \frac{\vec{r}_\perp}{r_\perp}$$

a unit vector in the direction of a radial position vector

$\left(\vec{r}_\perp\right)$ perpendicular to the axis. The potential difference between the two cylinders is given by

$$\Delta V = -\int \vec{E}\cdot d\vec{r} \qquad \text{Taking the integral along } \vec{r}_\perp \text{ (\textit{the integral is path independent})}$$

$$d\vec{r} = dr_\perp \vec{e}_r$$

$$\vec{E}\cdot d\vec{r} = \frac{2k\lambda}{r_\perp}\vec{e}_{r_\perp}\cdot dr_\perp \vec{e}_{r_\perp}$$

$$= \frac{2k\lambda dr_\perp}{r_\perp}$$

$$\Delta V = -\int_{r_\perp = a}^{r_\perp = b} \frac{2k\lambda}{r_\perp} dr_\perp$$

$$= -2k\lambda \int_a^b \frac{dr_\perp}{r_\perp}$$

$$= -2k\lambda \left[\ln r_\perp\right]_a^b$$

$$= -2k\lambda\left(\ln b - \ln a\right)$$

$$\boxed{\Delta V = -2k\lambda \ln\frac{b}{a}}$$

The capacitance of the concentric cylinders is obtained as the ratio between the total and potential difference.

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{2k\lambda \ln \frac{b}{a}} \quad but \quad \lambda = \frac{Q}{L}$$

$$\boxed{C = \frac{L}{2k\ln\left(\frac{b}{a}\right)}} \qquad \textit{Capacitance of concentric cylinders of radii } a \textit{ and } b \ (a < b) \textit{ and length } L$$

Example

Calculate the capacitance of a concentrically cylinders of radii 4*cm*, 10*cm* and 100*cm* long.

Solution

Given: $a = 0.04m, \quad b = 0.1m, \quad L = 1m$

$$C = \frac{L}{2k\ln\left(\frac{b}{a}\right)}$$

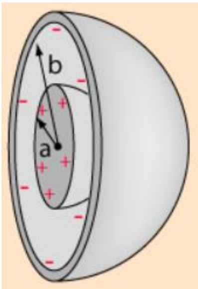
$$= \frac{1}{2\times 9\times 10^9 \ln\left(\frac{.1}{.04}\right)}$$

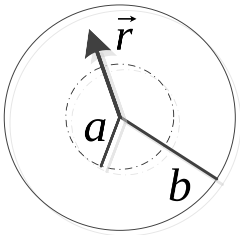
$$= 0.0606\times 10^{-9}$$

$$\underline{= 60.6\ pF|}$$

Concentrically Spherical Capacitor

Concentrically Spherical Capacitor is a capacitor formed by two concentric spheres. Consider two concentric spherical surfaces of radii a and b (with $a < b$). Let the total charge of the capacitor be Q.





From Gauss's law, it can be shown that the electric field between the spherical surfaces is given by

$$E = \frac{KQ}{r^2}\overrightarrow{e_r} \quad where \quad \frac{\vec{r}}{r} = \overrightarrow{e_r}$$

$\vec{r}$ *is a position vector with respect to the center of the sphere*

The potential difference between the two spheres is given by

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

If the integral is taken along the position vector $\vec{r}$, $d\vec{r} = dr\ \vec{e}_r$

$$\vec{E} \cdot d\vec{r} = \frac{kQ}{r^2}\vec{e}_r \cdot dr\vec{e}_r = \frac{kQ}{r^2}dr$$

$$\Delta V = -\int_{r=a}^{r=b} \frac{kQ}{r^2}dr$$

$$= -kQ\left[-\frac{1}{r}\right]_a^b$$

$$= -kQ\left(-\frac{1}{b} + \frac{1}{a}\right)$$

$$= -kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= -kQ\left(\frac{b-a}{ab}\right)$$

$$|\Delta V| = kQ\left(\frac{b-a}{ab}\right)$$

Therefore the capacitance of the concentric spheres is given by

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{kQ\left(\frac{b-a}{ab}\right)}$$

$$\boxed{C = \frac{ab}{k(b-a)}}$$

Capacitance of two concentric spheres of radii a and b

Example

Calculate the capacitance of two concentric spheres of radii 2cm and 10*cm*.

<u>Solution</u>

Given: $a = 0.02m, \quad b = 0.1m$

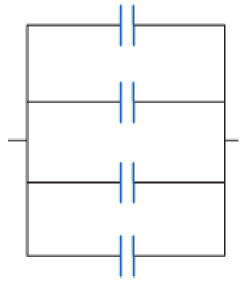
$$C = \frac{ab}{k(b-a)} = \frac{(.02)(.1)}{9\times10^9(0.1-0.02)}$$

$$= 2.8\times10^{-12}F$$

$$= 2.8\ pF$$

Parallel Combination of Capacitors

Capacitors are said to be connected in parallel if they are connected in a branched connection as shown



Capacitors connected in parallel have the same potential difference because the conductors on the same side are connected by conductors and are at the same potential but the charge will be divided among the capacitors according to their capacitances. So the total charge is equal to the sum of the charges of each capacitor.

If capacitors $C_1,\ C_2,\ C_3,\ \dots$ are connected in parallel and then connected to a potential difference ΔV, then

$$\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3 = \dots$$

And

$$Q = Q_1 + Q_2 + Q_3 + \dots$$

Where Q is the total charge.

The Equivalent Capacitance

The equivalent capacitance of a group of capacitors is defined to be the ratio between the total charge (Q) and the total potential difference (ΔV)

$$C_{eq} = \frac{Q}{\Delta V}$$

For parallel combination

$$Q = Q_1 + Q_2 + Q_3 + \dots$$

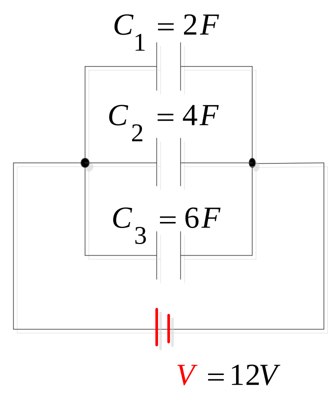
But $Q = C_{eq}\Delta V,\ \ Q_1 = C_1\Delta V_1 = C_1\Delta V,\ \ Q_2 = C_2\Delta V,\ \ Q_3 + = C_3\Delta V, \dots$

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \dots}$$

The equivalent capacitance of capacitors connected parallel is obtained by adding the individual capacitors.

Example

A $2F$, a $4F$ and a $6F$ capacitor are connected in parallel and then connected to a potential difference of $12V$.



a) Calculate the potential difference across each capacitor.
b) Calculate the charge stored by each capacitor
c) Calculate the total charge stored by the capacitors
d) Calculate the equivalent capacitance of the capacitors.

<u>Solution</u>

a) ***Calculate the potential difference across each capacitor***.

For parallel connection

$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V = 12\ V$

b) Calculate the charge stored by each capacitor

$Q_1 = C_1 \Delta V_1 = 2(12)\ = 24\ C$

$Q_2 = C_2 \Delta V_2 = 4(12)\ = 48\ C$

$Q_3 = C_3 \Delta V_3 = 6(12)\ = 72\ C$

c) Calculate the total charge stored by the capacitors

$Q = Q_1 + Q_2 + Q_3$

$= 24 + 48 + 72$

$= 144\ C$

d) Calculate the equivalent capacitance of the capacitors.

$C_{eq} = C_1 + C_2 + C_3 = 2F + 4F + 6F\ = 12\ F$

Or $C_{eq} = \dfrac{Q}{\Delta V} = \dfrac{144}{12}\ = 12\ F$

Series Combination of Capacitors

Capacitors are said to be connected in series when they are connected in one line.

When capacitors are connected in series are connected to a potential difference, electrons are taken from one of the outermost conductors and taken to the outermost conductor on the other side. All the conductors in between are charges by induction. Thus, in a series combination all of the capacitor will acquire the same charge which is also equal to the total charge acquired by the capacitors. The total potential difference across the combination will be divided into the capacitors according to their capacitance.

If capacitors $C_1, C_2, C_3, \dots$ are connected in series and then connected to a potential difference ΔV

$$Q = Q_1 = Q_2 + Q_3 = \dots$$ Q: is the total charge

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \dots$$

The equivalent capacitance of the combination is defined to be the ratio between the total charge & the total potential difference.

$$C_{eq} = \frac{Q}{\Delta V}$$

$$\Delta V = \frac{Q}{C_{eq}}, \quad \Delta V_1 = \frac{Q_1}{C_1} = \frac{Q}{C_1}, \quad \Delta V_2 = \frac{Q}{C_2}, \ \dots$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \ \cdots$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ \cdots}$$ Equivalent capacitance of capacitors connected in series

$$\color{red}{or,\ alternative}\quad C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

This expression can be simplified if only two capacitors are involved.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 C_2}$$

$$\boxed{C_{eq} = \frac{C_1 C_2}{C_2 + C_1}}$$

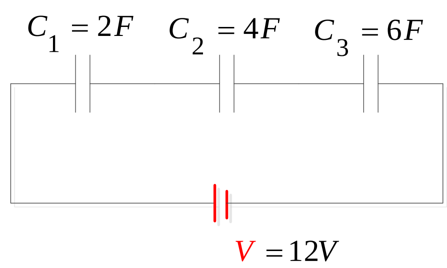
Note: This formula is valid only for two capacitors in series.

Example

A $2F$, a $4F$ and a $6F$ capacitor are connected in series and then connected to a potential difference of $12V$.

a) Calculate the equivalent capacitance of the capacitors.
b) Calculate the total charge accumulated by the capacitors
c) Calculate the charge accumulated by each capacitor
d) Calculate the potential difference across each capacitor.

Solution



a) Calculate the equivalent capacitance of the capacitors.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{6}$$

$$= \frac{6+3+2}{12}$$

$$= \frac{11}{12}$$

$$C_{eq} \boxed{= \frac{12}{11}\ F}$$

b) Calculate the total charge accumulated by the capacitors

$$Q = C_{eq}\Delta V = \frac{12}{11}(12) \boxed{= \frac{144}{11}\ F}$$

c) Calculate the charge accumulated by each capacitor

$$Q_1 = Q_2 = Q_3 = Q \boxed{= \frac{144}{11}\ F}$$

d) Calculate the potential difference across each capacitor.

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{Q}{C_1} = \frac{\frac{144}{11}}{2} \boxed{= \frac{72}{11}\ C}$$

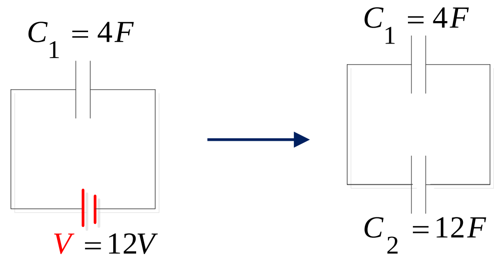
$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{Q}{C_2} = \frac{144}{11}\frac{1}{4} \boxed{= \frac{36}{11}\ C}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{Q}{C_3} = \frac{144}{11}\frac{1}{6} \boxed{= \frac{18}{11}\ C}$$

Example

A $4F$ capacitor is connected to a $12V$ battery. Then it is disconnected from the battery & then connected to a $12F$ capacitor.
Calculate the charge transferred to the $12F$ capacitor.

Solution



When the $4F$ capacitor is connected to the $12V$ battery, it will accumulate $(4)(12)c = 48c$ of charge. When it is disconnected and from battery & then connected to the $12F$ capacitor, charge will flow from the $4F$ capacitor to the $12F$ capacitor until both of them have the same potential. Let the charge transferred be Q, then

$$\Delta V_1 = \Delta V_2$$

$$\frac{48 - Q}{4} = \frac{Q}{12}$$

$$48 - Q = \frac{Q}{3}$$

$$\frac{4}{3}Q = 48$$

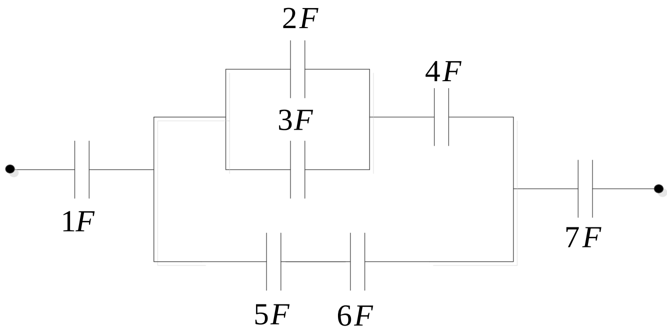
$$\boxed{Q = 36\ C}$$

Series – Parallel Combination

Series–Parallel combination can be simplified by replacing each series or parallel combination by its equivalent capacitance.

Example

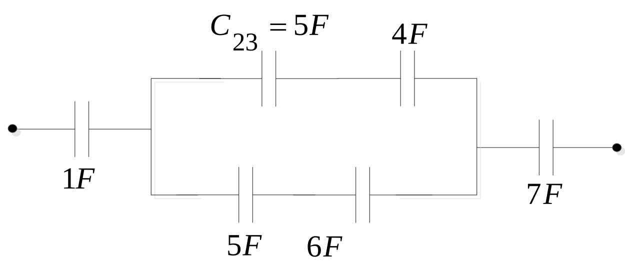
Calculate the equivalent capacitance of the combination shown below



<u>Solution</u>

First let's replace the parallel combination of the 2F & 3F capacitor by their equivalent capacitance.

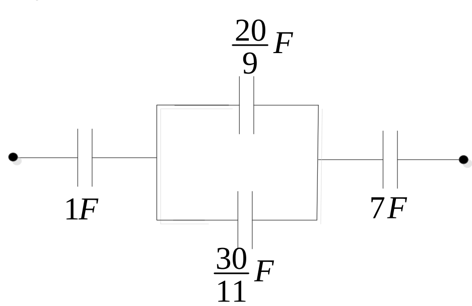
$$C_{23} = C_2 + C_3 = 2 + 3 = \underline{5F}$$



For the series combinations:

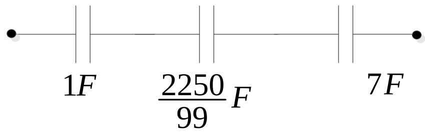
C_{23} & C_4 $\qquad C_{234} = \dfrac{C_{23}C_4}{C_{23} + C_4} = \dfrac{5 \cdot 4}{5+4} = \underline{\dfrac{20}{9}F}$

C_5 & C_6 $\qquad C_{56} = \dfrac{C_5 C_6}{C_5 + C_6} = \dfrac{5 \cdot 6}{5+6} = \underline{\dfrac{30}{11}F}$



C_{234} & C_{56} are in parallel. Therefore

$$C_{23456} = C_{234} + C_{56} = \frac{20}{9} + \frac{30}{11} = \underline{\frac{2250}{99}F}$$



C_1, C_{23456} & C_7 are in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{232456}} + \frac{1}{C_7}$$

$$= \frac{1}{1} + \frac{1}{\frac{2250}{99}} + \frac{1}{7}$$

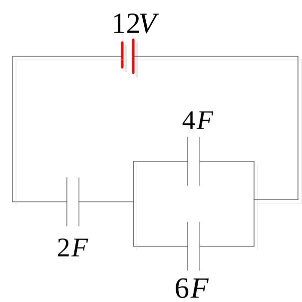
$$= 1 + \frac{99}{2250} + \frac{1}{7}$$

$$= \frac{15750 + 693 + 2250}{15750}$$

$$\boxed{= \frac{18693}{15750}\ F}$$

Example

Calculate the circuit shown



a) Find the equivalent capacitance of the combination
b) Calculate the total charge accumulated by the combination
c) Find the charge accumulated by the $2F$ capacitor
d) Calculate the potential difference across the $2F$ capacitor
e) Calculate the potential difference across the $4F$ & $6F$ capacitors
f) Calculate the charges accumulated by the $4F$ & $6F$ capacitors

<u>Solution</u>

a) Find the equivalent capacitance of the combination

C_4 & C_6 are in parallel. Therefore

$$C_{46} = C_4 + C_6 = 4 + 6 \boxed{= 10\ F}$$

C_{46} & C_2 are in series. Therefore

$$C_{eq} = \frac{C_2 C_{46}}{C_2 + C_{46}} = \frac{2 \cdot 10}{2 + 10} = \frac{20}{12} \boxed{= \frac{5}{3} F}$$

b) Calculate the total charge accumulated by the combination

$$Q = C_{eq} \Delta V = \frac{5}{3}(12) \boxed{= 20\ C}$$

c) Find the charge accumulated by the 2F capacitor

Since C_2 & C_{46} are in series

$$Q_2 = Q_{46} = Q \boxed{= 20\ C}$$

d) Calculate the potential difference across the 2F capacitor

$$V_2 = \frac{Q_2}{C_2} = \frac{20}{2} \boxed{= 10\ V}$$

e) Calculate the potential difference across the 4F & 6F capacitors

Since C_4 & C_6 are in parallel.

$$V_4 = V_6 = V_{46}$$

But $V_{46} = \frac{Q_{46}}{C_{46}} = \frac{20}{10} \boxed{= 2\ V}$

$$V_4 = V_6 \boxed{= 2\ V}$$

f) Calculate the charges accumulated by the 4F & 6F capacitors

$$Q_4 = C_4 V_4 = (4)(2) \boxed{= 8\ C}$$

$$Q_6 = C_6 V_6 = (6)(2) \boxed{= 12\ C}$$

Energy Stored by a Capacitor

As a charge q is stored in a capacitor through a potential difference, the electrical energy stored in the capacitor (du) is given as

$du = qdV$ ***i.e.*** change in potential energy (du) equals charge (q) times the potential difference (dV)

As the potential difference is increased from zero to a certain value, say dV, the total electrical potential energy is obtained by integrating this equation

$$u = \int du = \int_0^{\Delta V} q dV' \qquad \text{but } q = C\Delta V'$$

$$= C \int_0^{\Delta V} V' dV'$$

$$= \frac{1}{2} C \left[\Delta V'^2 \right]_0^{\Delta V}$$

$$\boxed{= \frac{1}{2} C \Delta V^2}$$

Electrical potential energy stored by a capacitor of capacitance C & potential difference ΔV

$$u = \frac{1}{2} C \Delta V^2 = \frac{1}{2}(C\Delta V)\Delta V \qquad \textit{but} \quad C\Delta V = Q$$

$\boxed{u = \frac{1}{2} Q \Delta V}$ also, since $\Delta V = \frac{Q}{C}$

$\boxed{u = \frac{1}{2} \frac{Q}{C}}$

Example

Calculate the electric potential energy stored by a $2 \mu F$ capacitor when connected to a potential difference of 10V.

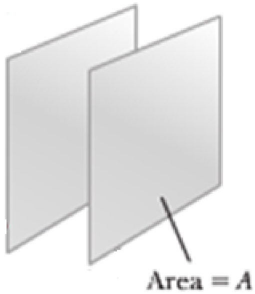
<u>Solution</u>

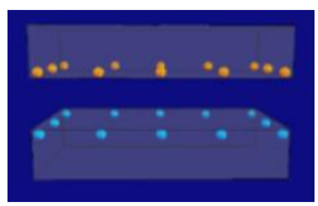
Given: $C = 2 \times 10^{-6} F, \quad \Delta V = 10V$

$u = \frac{1}{2} C \Delta V^2 = \frac{1}{2}\left(2 \times 10^{-6}\right)(10) \underline{= 10^{-4}\ J}$

Energy Density of a parallel plate capacitor

If the area of the plates is A and the separation between the plates is d, then the volume of the parallel plate capacitor is Ad .





The energy density $\left(U_E\right)$ is given by

$$U_E = \frac{U}{Ad} \qquad but \quad u = \frac{1}{2} C \Delta V^2$$

For a parallel plate capacitor

$\Delta V = Ed$ Where E is the electric field strength

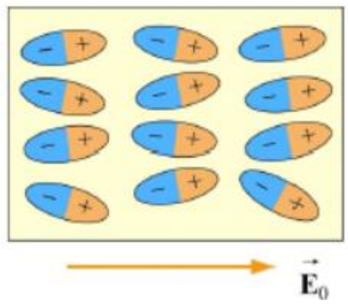
$$\& \ C = \frac{\varepsilon_0 A}{d}$$

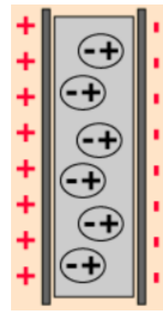
$$U_E = \frac{\frac{1}{2} C \Delta V^2}{Ad} = \frac{\frac{1}{2} \frac{\varepsilon_0 A}{d} (Ed)^2}{Ad}$$

$\boxed{U_E = \frac{1}{2} \varepsilon_0 E^2}$ Energy density of a parallel plate capacitor.

Capacitors with Dielectrics

A dielectric is an insulator placed between the conductors of a capacitor. A dielectric is used to increase the capacitance of a capacitor consider the parallel plate capacitor shown





The charges in the plates of the capacitor will set up an electric field directed from the positive plate towards the negative plate. Let the strength of this field be E_0. This is also the field between the capacitors when the plates are separated by vacuum (or air). This electric field will exert force on the modules of the dielectric. The negative part of a molecule will be pulled towards the positive plate and the positive part of the molecule will be pulled towards the negative plate. Thus the molecules of the dielectric will set up their own electric field directed from their positive end towards their negative and which is opposite to the electric field set up by the charges on the plates. The net electric field between the plates is the field due to the charges in the plates minus the field due to the molecules of the dielectric.

Thus the electric field strength with a dielectric is less than the field strength without a dielectric. The ratio between the field strength without a dielectric (E_0) and the field strength with a dielectric is defined to be dielectric constant (κ) of the dielectric

$$\boxed{\kappa = \frac{E_0}{E}}$$

κ : Dielectric constant

E_0 : Electric field without the dielectric

E: Electric field with the dielectric

$$\kappa > 1 \quad \left(because \;\; E_0 > E\right)$$

The potential difference between the plates (V_0) when there is to dielectric (i.e. vacuum) is given by

$$\Delta V_0 = E_0 d$$ Where d is the separation between the plates

& the potential difference (ΔV) with the dielectric is given by

$$\Delta V = Ed \qquad but \;\; E = \frac{E_0}{\kappa}$$

$$\boxed{\Delta V = \frac{E_0 d}{\kappa} = \frac{\Delta V_0}{\kappa}}$$

ΔV : The potential difference with a dielectric of dielectric constant κ

ΔV_0 : The potential difference without a dielectric

The capacitance of the capacitor without a dielectric is given by

$$C_0 = \frac{Q_0}{\Delta V_0}$$ Where Q is the charge on the plates

& the capacitance with a dielectric of dielectric constant κ is

$$C = \frac{Q}{\Delta V}$$ *but* $\Delta V = \frac{\Delta V_0}{\kappa}$ & $Q_0 = Q$ (the charge remains the same)

$$C = \frac{\kappa Q_0}{\Delta V_0}$$

$$\boxed{C = \kappa C_0}$$

C : Capacitance with a dielectric of dielectric constant κ

C_0 : Capacitance without a dielectric

When a dielectric is inserted between the conductors of a capacitor, the electric field decreases by a factor of $\frac{1}{\kappa}$, the potential difference decreases by a factor of $\frac{1}{\kappa}$ and the capacitance increases by a factor of κ.

Example

A parallel capacitor has an area of $2\ cm^2$ and a separation of 4 *mm*.

a) When the plates are separated by vacuum (air)

i. Calculate the capacitance of the capacitor

ii. Now suppose the capacitor is connected to a 10 V potential difference. Calculate the charge on the plates and the electric field between the plates

b) Now the capacitor is disconnected from the battery on a dielectric of dielectric constant 15 is inserted between the plates.

i. Calculate its capacitance

ii. Calculate the potential difference between the plates

iii. Calculate the strength of the electric field

iv. Calculate the charge on the plates

Solution

a) i. *Given*: $A = 2cm^2 = 2\times10^{-4} m^2, \quad d = 4mm = 4\times10^{-3} m, \quad \varepsilon_0 = 8.85\times10^{-12}\frac{C^2}{Nm^2}$

$$C_0 = \frac{\varepsilon_0 A}{d}$$

$$= \frac{\left(8.85\times10^{-12}\right)\left(2\times10^{-4}\right)}{4\times10^{-3}}$$

$$= 4.425 \times 10^{-13}\ F$$
$$= 0.4425\ pF$$

ii. $Q_0 = C_0 \Delta V_0 = (0.4425\ pF)(10V) = 4.425\ pC$

$$E_0 = \frac{\Delta V_0}{d} = \frac{10\ V}{4 \times 10^{-3} m} = 2.5 \times 10^3\ V$$

b) i. $C = \kappa C_0 = (1.5)(0.4425\ pF) = 0.66\ pF$

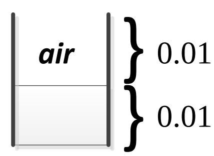
ii. $\Delta V = \frac{\Delta V_0}{\kappa} = \frac{10\ V}{1.5} = 6.67\ V$

iii. $E = \frac{E_0}{\kappa} = \frac{2.5 \times 10^3\ V}{1.5} = 1.6 \times 10^3\ V$

iv. $Q = Q_0 = 4.425\ pC$

Example

A parallel capacitor has an area of $0.02\ cm^2$ and a separation of 0.1 m. the bottom half of the capacitor is filled with a dielectric of dielectric constant 2 as shown.



Calculate the capacitance of the capacitor.

Solution

This can be treated as the parallel combination of two capacitor whose area is half of the total area

$C = C_a + C_d$

Where C_a Capacitance of the upper half with air

C_d Capacitance of the lower half with dielectric

$$C_a = \frac{\varepsilon_0 \frac{A}{2}}{d} = \frac{8.85 \times 10^{-12}\left(\frac{0.02}{2}\right)}{0.1} = 8.85 \times 10^{-11}\ F$$

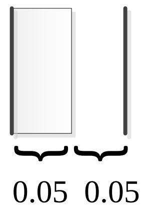
$$C_d = \frac{\kappa \varepsilon_0 \frac{A}{2}}{d} = \frac{2\left(8.85 \times 10^{-12}\right)\left(\frac{0.02}{2}\right)}{0.1} = 17.7 \times 10^{-11}\ F$$

$$C = C_a + C_d = (8.85 + 17.7) \times 10^{-11} = 26.55 \times 10^{-11}\ F$$

Example

A parallel capacitor has an area of $0.02\ cm^2$ and a separation of 0.1 m.

Suppose now the left half is filled with dielectric of dielectric constant 2 as shown



Calculate the capacitance of the capacitor.

Solution

This can be treated as the series combination of two capacitor whose area is half of the separation

$$C = \frac{C_a C_d}{C_a + C_d}$$

Where C_a Capacitance of the one with air

C_d Capacitance of the one with dielectric

$$C_a = \frac{\varepsilon_0}{d/2} = \frac{8.85\times10^{-12}(0.02)}{0.01/2} = 25.4\times10^{-11}\ F$$

$$C_d = \frac{\kappa\varepsilon_0 A}{d/2} = \frac{2\left(8.85\times10^{-12}\right)(0.02)}{0.05} = 70.8\times10^{-11}\ F$$

$$C = \frac{\left(35.4\times10^{-11}\right)\left(70.8\times10^{-11}\right)}{35.4\times10^{-11} + 70.8\times10^{-11}} = 23.6\times10^{-11}\ F$$