4.1 #9

$x = \frac{t}{t-1}$ $\quad y = \frac{t-2}{t+1}$ $\quad -1 < t < 1$

$t = tx - x$

$t(1-x) = -x$

$t = \frac{x}{x-1}\,;\ x \neq 1$

$-\frac{2.9}{.1}$ $\quad \frac{-.9}{1.9} = \frac{9}{19}$

$$y = \frac{\frac{x}{x-1} - 2}{\frac{x}{x-1} + 1}$$

$$= \frac{x - 2x + 2}{x + x - 1}$$

$$= \boxed{\frac{-x+2}{2x-1}}$$

t	x	y
0	0	-2
$-.9$	$9/19$	-29
$.9$	-9	$11/19$

$\frac{-.9}{-.1}$

$t\,\frac{1}{2} < x$

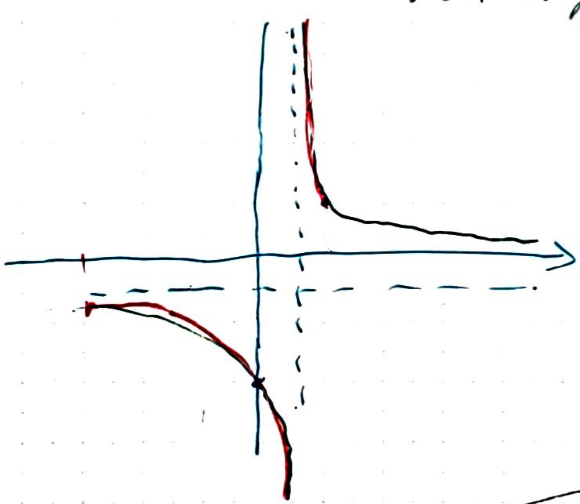
$y \neq -\frac{1}{2}$

$x \neq \frac{1}{2}$

$\frac{-1.1}{1.9}$

$t = -1 \Rightarrow x = \frac{1}{2}$

$t = 1 \Rightarrow y = -\frac{1}{2}$



$x = \lim_{t \to +1} \frac{t}{t-1} = +\infty$ $\quad \Rightarrow \lim_{t} \frac{t}{t-1}$

$x = \lim_{t \to -1} \frac{t}{t-1} = \frac{-1}{-2} = \frac{1}{2}$

$y = \lim_{t = -1} \frac{t-2}{t+1} = \frac{-3}{0} = -\infty$

$\frac{t-2}{t+1} \xrightarrow[t \to 1]{} -\frac{1}{2}$

4.2 #28

$$\begin{cases} x = \theta - \sin\theta = t - \sin t \\ y = 1 - \cos\theta = 1 - \cos t \end{cases} \qquad \theta = \pi$$

Soln $\dfrac{d^2y}{dx^2}$?

$$\frac{dx}{d\theta} = 1 - \cos\theta \qquad \frac{dy}{d\theta} = \sin\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\sin\theta}{1-\cos\theta}$$

$\sin\theta$ 0
$-\cos\theta$ 1

$$\frac{dy'}{d\theta} = \frac{\cos\theta - \cos^2\theta - \sin^2\theta}{(1-\cos\theta)^2} =$$

$$= \frac{\cos\theta - 1}{(1-\cos\theta)^2} \; \Bigg| = \Bigg| - \frac{(1-\cos\theta)}{(\quad)^2}$$

$$= - \frac{1}{1-\cos\theta}$$

$$\frac{d^2y}{dx^2} = \frac{dy'/d\theta}{dx/d\theta}$$

$$= - \frac{1}{1-\cos\theta} \cdot \frac{1}{1-\cos\theta}$$

$$= \frac{-1}{(1-\cos\theta)^2}\Bigg|_{\theta=\pi}$$

$$= \frac{-1}{4}$$

4.2 #35

$$\begin{cases} x = 2 - \pi \cos t \\ y = 2t - \pi \sin t \end{cases}$$

Soln $\frac{dx}{dt} = \pi \sin t$ $\qquad \frac{dy}{dt} = 2 - \pi \cos t = x$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2 - \pi \cos t}{\pi \sin t} \qquad \sin t \neq 0 \ (t \neq 0, \pi)$$

$x = y \Rightarrow 2 - \pi \cos t = 2t - \pi \sin t$

$$\begin{cases} x = 2 - \pi \cos t = 0 \to \cos t = \frac{2}{\pi} \\ y = 2t - \pi \sin t = 0 \to 2t = -\pi \sin t \end{cases}$$

$x = 2 = 2 - \pi \cos t \Rightarrow \cos t = 0 \Rightarrow t = \frac{\pi}{2}$

4.2 #50 A? one arch: $x = a(t - \sin t)$ $y = a(1-\cos t)$

$$\text{Area} = \int y\,dx \qquad dx = d(at - a\sin t) = a(1-\cos t)\,dt$$

$$= a^2\int_0^{2\pi}(1-\cos t)^2\,dt$$

$$= a^2\int_0^{2\pi}(1 - 2\cos t + \cos^2 t)\,dt \qquad \cos^2 t \to \tfrac{1}{2} + \tfrac{1}{2}\cos 2t$$

$$= a^2\int_0^{2\pi}\left(\tfrac{3}{2} - 2\cos t + \tfrac{1}{2}\cos 2t\right)dt$$

$$= a^2\left(\tfrac{3}{2}t - 2\sin t + \tfrac{1}{4}\sin 2t\right)\Big|_0^{2\pi}$$

$$= 3\pi a^2$$

4.2 #61 L? $\begin{cases} x = 8\cos t + 8t\sin t \\ y = 8\sin t - 8t\cos t \end{cases}$ $0 \le t \le \frac{\pi}{2}$

Soln

$$\frac{dx}{dt} = -8\sin t + 8\sin t + 8t\cos t = 8t\cos t$$

$$\frac{dy}{dt} = 8\cos t - 8\cos t + 8t\sin t = 8t\sin t$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{64t^2\cos^2 t + 64t^2\sin^2 t} = 8t$$

$$L = \int_0^{\pi/2} 8t\,dt$$

$$= 4t^2\Big|_0^{\pi/2}$$

$$= \pi^2 \text{ unit}$$

4.2 # 71

$$\begin{cases} x = t + \sqrt{2} \\ y = \frac{1}{2}t^2 + \sqrt{2}\,t \end{cases} \qquad -\sqrt{2} \le t \le \sqrt{2}$$

Soln $\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = t + \sqrt{2}$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{1 + t^2 + 2\sqrt{2}\,t + 2}$$

$$= \sqrt{t^2 + 2\sqrt{2}\,t + 3}$$

$$A = 2\pi \int_{-\sqrt{2}}^{\sqrt{2}} (t + \sqrt{2})\left(t^2 + 2\sqrt{2}\,t + 3\right)^{1/2} dt.$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left(t^2 + 2\sqrt{2}\,t + 3\right)^{1/2} d\left(t^2 + 2\sqrt{2}\,t + 3\right)$$

$$= \frac{2\pi}{3}\left(t^2 + 2\sqrt{2}\,t + 3\right)^{3/2} \Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= \frac{2\pi}{3}\left[27 - (1)\right]$$

$$= \frac{52\pi}{3} \text{ unit}^2$$

4.4 # 20

$r = 2\cos 2\theta$ given $0 \le \theta \le \pi$

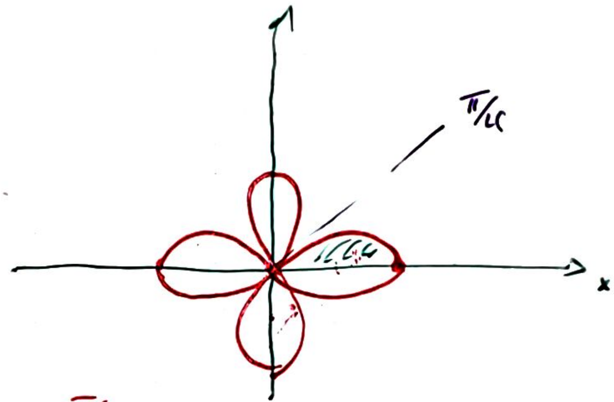
$A = 2\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$ $\qquad 2 \to$ (twice

$= 2\frac{1}{2}\int_0^{\pi} 4\cos^2 2\theta \, d\theta$

$= 2 \cdot 2 \int_0^{\pi} \frac{1}{2}(1+\cos 4\theta)\, d\theta$

$= 2\left(\theta + \frac{1}{4}\sin 4\theta\right)\Big|_0^{\pi}$

$= 2\pi \text{ unit}^2$



$A = 8\int_0^{\pi/4} \frac{1}{2} r^2 \, d\theta$

$= 8\left(\theta + \frac{1}{4}\sin 4\theta\right)\Big|_0^{\pi/4}$

$= 8\left(\frac{\pi}{4}\right.$

$= 2\pi$

4.4 #13 $\qquad r = 4 + 4\sin\theta = 4(1+\sin\theta)$

$$A = \frac{1}{2}\int_0^{2\pi} 16\,(1+\sin\theta)^2\,d\theta$$

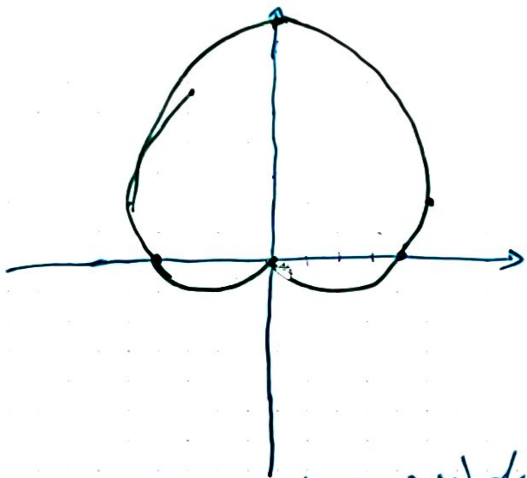
$$= 8\int_0^{2\pi}\left(1 + 2\sin\theta + \overbrace{\frac{1}{2} - \frac{\cos 2\theta}{2}}^{\sin^2\theta}\right)d\theta$$

$$= 8\int_0^{2\pi}\left(\frac{3}{2} + 2\sin\theta - \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= 8\left(\frac{3}{2} - 2\cos\theta - \frac{1}{4}\sin 2\theta\,\right|_0^{2\pi}$$

$$= 8(3\pi - 2 + 2)$$

$$\boxed{= 24\pi \text{ unit}^2}$$



$$A = 2\,\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 16\,(1+\sin\theta)^2\,d\theta$$

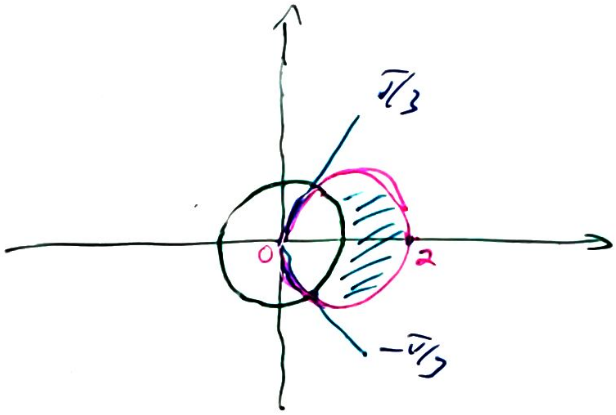
$$A = 16\int_{-\pi/2}^{\pi/2}\left(\frac{3}{2} + 2\sin\theta - \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= 16\left(\frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta\,\right|_{-\pi/2}^{\pi/2}$$

$$= 16\left(\frac{3\pi}{4} + \frac{3\pi}{4}\right)$$

$$\boxed{= 24\pi \text{ unit}^2}$$

4.4 # 49 Inside $r = 2\cos\theta$
outside $r = 1$



$r = 2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}, -\frac{\pi}{3}}$

$$A = \frac{1}{2}\int_{-\pi/3}^{\pi/3} \left((2\cos\theta)^2 - 1\right) d\theta$$

$$= 2\left(\tfrac{1}{2}\right)\int_0^{\pi/3} \left(4\,\frac{1+\cos 2\theta}{2} - 1\right) d\theta$$

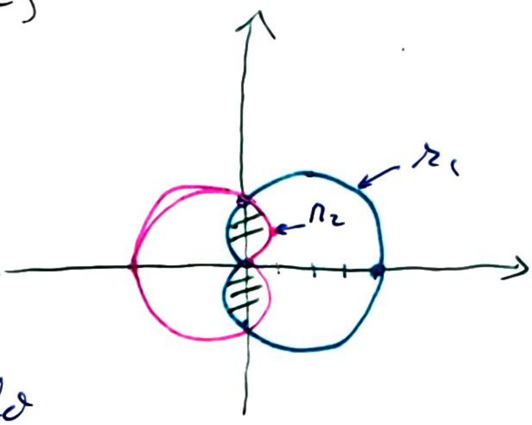
$$= \int_0^{\pi/3} (1 + 2\cos 2\theta)\, d\theta$$

$$= \theta + \sin 2\theta \Big|_0^{\pi/3}$$

$$\boxed{= \frac{\pi}{3} + \frac{\sqrt{3}}{2} \text{ unit}^2}$$

4.4 #49 Common interior

$$\begin{cases} r = 2(1+\cos\theta) \\ r = 2(1-\cos\theta) \end{cases}$$



$$A = 4\left(\tfrac{1}{2}\right)\int_0^{\pi/2} 4(1-\cos\theta)^2\,d\theta$$

$$= 8\int_0^{\pi/2}\Big(1 - 2\cos\theta + \overbrace{\tfrac{1}{2}+\tfrac{1}{2}\cos 2\theta}^{\cos^2\theta}\Big)\,d\theta$$

$$= 8\int_0^{\pi/2}\left(\tfrac{3}{2} - 2\cos\theta + \tfrac{1}{2}\cos 2\theta\right)d\theta$$

$$= 8\left(\tfrac{3}{2}\theta - 2\sin\theta + \tfrac{1}{4}\sin 4\theta\right)\Big|_0^{\pi/2}$$

$$= 8\left(\tfrac{3\pi}{4} - 2\right)$$

$$= \underline{6\pi - 16 \text{ unit}^2}$$

4.4 #68 $r = \frac{1}{\sqrt{2}} e^{\theta} \quad 0 \le \theta \le \pi \quad L?$

$$\sqrt{r^2 + (r')^2} = \sqrt{\frac{1}{2}e^{2\theta} + \frac{1}{2}e^{2\theta}}$$
$$= e^{\theta}$$

$$L = \int_0^{\pi} e^{\theta}\, d\theta$$
$$= e^{\theta}\Big|_0^{\pi}$$
$$= e^{\pi} - 1 \text{ unit}$$

#4.4 #80

$r = \sec\theta \qquad 0 \le \theta \le \pi/3$

$$\sqrt{r^2 + (r')^2} = \sqrt{\sec^2\theta + \sec^2\theta \tan^2\theta}$$
$$= \sec\theta \sqrt{1 + \tan^2\theta} \rightarrow \sec^2\theta$$
$$= \sec^2\theta$$

$$L = \int_0^{\pi/3} \sec^2\theta\, d\theta$$
$$= \tan\theta \Big|_0^{\pi/3}$$
$$= \sqrt{3} \text{ unit}$$

4.4 # 44

$r = 1+4\cos\theta \qquad 0 \le \theta \le \frac{\pi}{2} \qquad$ Polar axis

$S = 2\pi \int_\alpha^\beta f(\theta)\sin\theta \sqrt{f^2+(f')^2}\, d\theta$

$$\sqrt{(1+4\cos\theta)^2 + 16\sin^2\theta} = \sqrt{1+8\cos\theta + \underbrace{16\cos^2\theta + 16\sin^2\theta}_{=16}}$$

$$= \sqrt{17+8\cos\theta}$$

$$S = 2\pi \int_0^{\pi/2} (1+4\cos\theta)\sin\theta \sqrt{17+8\cos\theta}\, d\theta$$

$$= 2\pi \int_0^{\pi/2} \sin\theta(17+8\cos\theta)^{1/2} d\theta + 8\pi \int_0^{\pi/2} \cos\theta \sin\theta (17+8\cos\theta)^{1/2} d\theta$$

$$= -\frac{2\pi}{8} \int_0^{\pi/2} (17+8\cos\theta)^{1/2} d(17+8\cos\theta) + S_1$$

$u = 17+8\cos\theta \Rightarrow \cos\theta = \frac{u-17}{8}$

$du = -8\underline{\sin\theta\, d\theta}$ $\quad \sin\theta\, d\theta$

$$S_1 = 8\pi \int_0^{\pi/2} \frac{1}{8}(u-17)\, u^{1/2}\left(-\frac{1}{8}du\right)$$

$$= -\frac{\pi}{8} \int_0^{\pi/2} (u^{3/2} - 17u^{1/2})\, du$$

$$= -\frac{\pi}{8}\left(\frac{2}{5}u^{5/2} - \frac{34}{3}u^{3/2}\right|_0^{\pi/2}$$

$$= -\frac{\pi}{8}\left(\frac{2}{5}(17+8\cos\theta)^{5/2} - \frac{34}{3}(17+8\cos\theta)^{3/2}\right|_0^{\pi/2}$$

$$= -\frac{\pi}{8}\left(\frac{2}{5}(17)^{5/2} - \frac{34}{3}(17)^{3/2} - \frac{2}{5}5^5 + \frac{34}{3}5^3\right)$$

$$-\frac{2\pi}{8}\,\frac{2}{3}\,(17+8\cos\theta)^{3/2}\Big|_0^{\pi/2}$$

$$= -\frac{\pi}{6}\left(17^{3/2} - 5^3\right)$$

$$S = -\frac{17\pi}{6}\sqrt{17} + \frac{125\pi}{6} - \frac{\pi}{20}(17)^2\sqrt{17} + \frac{\pi}{12}(17)^2\sqrt{17} - 2(5)^4 + \frac{34}{3}5^3$$

4.4 # 95 $\quad r = 2\sin\theta \quad 0 \le \theta \le \frac{\pi}{2} \qquad \theta = \frac{\pi}{2}$

$$\sqrt{r^2 + r'^2} = \sqrt{4\sin^2\theta + 4\cos^2\theta}$$
$$= 2$$

$$S = 2\pi \int_0^{\pi/2} \overbrace{2\sin\theta\cos\theta}^{\sin 2\theta}\,(2)\,d\theta$$

$$= 4\left(-\frac{1}{2}\right)\cos 2\theta \Big|_0^{\pi/2}$$

$$= -2\,(-1-1)$$

$$\boxed{= 4 \text{ unit}^2}$$