Exercise

Find the second derivative: $f(x)=3\left(2-x^2\right)^3$

Solution

$$f'(x)=9(-2x)\left(2-x^2\right)^2 \qquad U=2-x^2 \Rightarrow U'=-2x$$

$$=-18x\left(2-x^2\right)^2$$

$$f=x \qquad f'=1$$

$$g=\left(2-x^2\right)^2 \qquad g'=-2x\left(2-x^2\right)$$

$$f''(x)=-18\left[\left(2-x^2\right)^2+x(-2x)\left(2-x^2\right)\right]$$

$$=-18\left(2-x^2\right)\left[2-x^2-2x^2\right]$$

$$=-18\left(2-x^2\right)\left(2-3x^2\right)$$

Exercise

Find the third derivative: $f(x)=5x(x+4)^3$

Solution

$$f'(x)=5\left[(x+4)^3+3x(x+4)^2\right]$$

$$=5(x+4)^2\left[(x+4)+3x\right]$$

$$=5(x+4)^2(4x+4)$$

$$=20(x+4)^2(x+1)$$

$$f''(x)=20\left[2(x+4)(x+1)+(x+4)^2\right]$$

$$=20(x+4)(2x+2+x+4)$$

$$=20(x+4)(3x+6)$$

$$=60(x+4)(x+2)$$

OR

$$f(x)=5x\left(x^3+12x^2+48x+64\right)$$

$$f(x)=5x^4+60x^3+240x^2+320x$$

$$f'(x)=20x^3+180x^2+480x$$

$$f''(x)=60x^2+360x$$

$$f'''(x)=120x+360$$

$$f'''(x) = 60\left[(x+2)+(x+4)\right]$$

$$= 60(2x+6)$$

$$= 120(x+3)$$

Exercise

Find the given value: $f(x) = \sqrt{4-x}$; $f'''(-5)$

Solution

$$f(x) = (4-x)^{1/2}$$

$$f'(x) = -\frac{1}{2}(4-x)^{-1/2}$$

$$f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(-1)(4-x)^{-1/2}$$

$$= -\frac{1}{4}(4-x)^{-3/2}$$

$$f''(x) = -\frac{1}{4}\left(-\frac{3}{2}\right)(-1)(4-x)^{-5/2}$$

$$= -\frac{3}{8}(4-x)^{-5/2}$$

$$f'''(-5) = -\frac{3}{8}(4-(-5))^{-5/2}$$

$$= -\frac{3}{8}(9)^{-5/2}$$

$$= -0.013889$$

Exercise

Find the 4th derivative of $f(x) = x^4 + 2x^3 + 3x^2 - 5x + 7$

Solution

$$f'(x) = 4x^3 + 6x^2 + 6x - 5$$

$$f''(x) = 12x^2 + 12x + 6$$

$$f'''(x) = 24x + 12$$

$$f^{(4)}(x) = 24$$

Exercise

Find the second derivative of $f(x)=\left(x^2-1\right)^2$

Solution

$$f'(x)=2(2x)\left(x^2-1\right)$$
$$=4x\left(x^2-1\right)$$
$$=4x^3-4x$$
$$f''(x)=12x^2-4$$

Exercise

Find $f''(x)$ for $f(x)=\sqrt{x^2+36}$, then find $f''(0)$ and $f''(9)$

Solution

$$f(x)=\sqrt{x^2+36}$$
$$=\left(x^2+36\right)^{1/2} \qquad \left(U^{1/2}\right)'=\tfrac{1}{2}U'U^{1-1/2}$$

$$f'(x)=\tfrac{1}{2}(2x)\left(x^2+36\right)^{-1/2}$$
$$=x\left(x^2+36\right)^{-1/2}$$

$$f=x \qquad g=\left(x^2+36\right)^{-1/2}$$

$$f'=1 \qquad g'=-\tfrac{1}{2}(2x)\left(x^2+36\right)^{-3/2}=-x\left(x^2+36\right)^{-3/2}$$

$$f''(x)=(1)\left(x^2+36\right)^{-1/2}+(-x)\left(x^2+36\right)^{-3/2}(x)$$
$$=\left(x^2+36\right)^{-1/2}-x^2\left(x^2+36\right)^{-3/2}$$
$$=\left[\left(x^2+36\right)^{-1/2}-x^2\left(x^2+36\right)^{-3/2}\right]\frac{\left(x^2+36\right)^{3/2}}{\left(x^2+36\right)^{3/2}}$$
$$=\left[\left(x^2+36\right)^{-1/2}\frac{\left(x^2+36\right)^{3/2}}{\left(x^2+36\right)^{3/2}}-x^2\left(x^2+36\right)^{-3/2}\frac{\left(x^2+36\right)^{3/2}}{\left(x^2+36\right)^{3/2}}\right]$$

$$= \left[\frac{\left(x^2+36\right)-x^2}{\left(x^2+36\right)^{3/2}} \right]$$

$$= \frac{x^2+36-x^2}{\left(x^2+36\right)^{3/2}}$$

$$= \underline{\left| \frac{36}{\left(x^2+36\right)^{3/2}} \right|}$$

$$f''(0) = \frac{36}{\left(0^2+36\right)^{3/2}} = \frac{36}{36^{3/2}} = \frac{1}{36^{1/2}} = \frac{1}{\sqrt{36}} = \underline{\left| \frac{1}{6} \right|}$$

$$f''(9) = \frac{36}{\left(9^2+36\right)^{3/2}} = \frac{36}{117^{3/2}} = 0.028446 \approx \underline{\left| 0.03 \right|}$$ ***Use the Calculator***

Exercise

Find $f''(x)$ for $f(x) = \sqrt{x^2+81}$, then find $f''(0)$ and $f''(2)$

Solution

$$f(x) = \sqrt{x^2+81} = \left(x^2+81\right)^{1/2} \qquad \left(U^{1/2}\right)' = \tfrac{1}{2}U'U^{1-1/2}$$

$$f'(x) = \tfrac{1}{2}(2x)\left(x^2+81\right)^{-1/2}$$

$$= x\left(x^2+81\right)^{-1/2}$$

$$u = x \qquad v = \left(x^2+81\right)^{-1/2}$$

$$u' = 1 \qquad v' = -\tfrac{1}{2}(2x)\left(x^2+81\right)^{-3/2}$$

$$= -x\left(x^2+81\right)^{-3/2}$$

$$f''(x) = (1)\left(x^2+81\right)^{-1/2} + (-x)\left(x^2+81\right)^{-3/2}(x)$$

$$= \left(x^2+81\right)^{-1/2} - x^2\left(x^2+81\right)^{-3/2}$$

$$= \left[\left(x^2+81\right)^{-1/2} \frac{\left(x^2+81\right)^{3/2}}{\left(x^2+81\right)^{3/2}} - x^2\left(x^2+81\right)^{-3/2} \frac{\left(x^2+81\right)^{3/2}}{\left(x^2+81\right)^{3/2}} \right]$$

$$= \frac{x^2+81-x^2}{\left(x^2+81\right)^{3/2}}$$

$$= \boxed{\frac{81}{\left(x^2+81\right)^{3/2}}}$$

$$f''(0) = \frac{81}{\left(0^2+81\right)^{3/2}} = \frac{81}{81^{3/2}} = \boxed{\frac{1}{9}}$$

$$f''(2) = \frac{81}{\left(2^2+81\right)^{3/2}} = 0.10336 \approx \boxed{0.10}$$ *Use the Calculator*

Exercise

The position function on Earth, where s is measured in meters, t is measured in seconds, v_0 is the initial velocity in meters per second, and h_0 is the initial height in meters, is

$$s = -4.9t^2 + v_0 t + h_0$$

If the initial velocity is 2.2 and the initial height is 3.6, what is the acceleration due to gravity on Earth in meters per second per second?

Solution

$$s = -4.9t^2 + 2.2t + 3.6$$

$$s' = -9.8t + 2.2$$

$$\boxed{a(t) = s'' = -9.8}$$