Lwk 1.2

1) $\lim_{x \to 4} (x^2 - 4x + 1) = 16 - 16 + 1$
$= \underline{1}$

2) $\lim_{x \to 1} \frac{x+3}{x+6} = \underline{\frac{4}{7}}$

3) $\lim_{x \to 1} \frac{x^2 - 1}{x+1} = \frac{0}{2}$
$= \underline{0}$

4) $\lim_{x \to 3} \frac{x^2 - 6x + 9}{x^2 - 9} = \frac{9 - 18 + 9}{0} = \frac{0}{0}$
$= \lim_{x \to 3} \frac{(x-3)(x-3)}{(x-3)(x+3)}$
$= \lim_{x \to 3} \frac{x-3}{x+3}$
$= \frac{0}{6}$
$= \underline{0}$

5) $\lim_{x \to 2} \frac{1}{4 - x^2} = \frac{1}{0} = \underline{\infty}$

6) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \frac{3 - 3}{9 - 9} = \frac{0}{0}$
$= \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$
$= \lim_{x \to 9} \frac{(x - 9)}{(x - 9)(\sqrt{x} + 3)}$
$= \lim_{x \to 9} \frac{1}{\sqrt{x} + 3}$
$= \underline{\frac{1}{6}}$

$\lim_{x \to 9} \frac{\sqrt{x} - 3}{(\sqrt{x} - 3)(\sqrt{x} + 3)}$

7/ $\lim\limits_{x \to \pi} \dfrac{(x-\pi)^2}{\pi x} = \dfrac{0}{\pi^2} = 0$

8/ $\lim\limits_{x \to 2} \dfrac{\sqrt{4-4x+x^2}}{x-2} = \dfrac{\sqrt{4-8+4}}{0} = \dfrac{0}{0}$

$= \lim\limits_{x \to 2} \dfrac{\sqrt{(x-2)(x-2)}}{x-2}$ $\sqrt{(x-2)^2}$

$= \lim\limits_{x \to 2} \dfrac{x-2}{x-2}$

$= 1$

9/ $\lim\limits_{x \to 2} \dfrac{(x-2)^2}{x^2-4} = \dfrac{0}{0}$

$= \lim\limits_{x \to 2} \dfrac{(x-2)^2}{(x-2)(x+2)}$

$= \lim\limits_{x \to 2} \dfrac{x-2}{x+2}$

$= \dfrac{0}{4}$

$= 0$

10/ $\lim\limits_{x \to 3} \dfrac{x-3}{x^2-9} = \dfrac{0}{0}$

$= \lim\limits_{x \to 3} \dfrac{x-3}{(x-3)(x+3)}$

$= \lim\limits_{x \to 3} \dfrac{1}{x+3}$

$= \dfrac{1}{6}$

11/ $\lim\limits_{x \to \frac{2\pi}{3}} \sin x = \sin \dfrac{2\pi}{3}$

$= \dfrac{\sqrt{3}}{2}$

12/ $\lim\limits_{x \to \frac{5\pi}{4}} \cos x = \cos \dfrac{5\pi}{4}$

$= -\dfrac{\sqrt{2}}{2}$

13/ $\lim_{x\to 0} \frac{\sin 2\pi x}{\sin 3\pi x} = \frac{0}{0}$

$= \lim_{x\to 0} \frac{2\pi x}{3\pi x} \frac{\sin 2\pi x}{2\pi x} \frac{1}{\frac{\sin 3\pi x}{3\pi x}} \to 1$

$= \frac{2}{3}$

14/ $\lim_{x\to 0^+} \frac{x-\sqrt{x}}{\sqrt{\sin x}} = \frac{0}{0}$

$\lim_{\alpha\to 0} \frac{\sin\alpha}{\alpha} = 1$

$= \lim_{x\to 0^+} \frac{x-\sqrt{x}}{\sqrt{x}\sqrt{\frac{\sin x}{x}}}$

$\sqrt{x} \cdot \frac{\sqrt{\sin x}}{\sqrt{x}}$

$= \lim_{x\to 0^+} \frac{\sqrt{x}-1}{\sqrt{\frac{\sin x}{x}}} \to 1$

$= \frac{-1}{1}$

$= -1$

15/ $\lim_{x\to 0^+} \frac{\sin\sqrt{1-x}}{\sqrt{1-x^2}} = \frac{\sin 1}{1} = \sin 1$

$\lim_{x\to 1^+} \frac{\sin\sqrt{1-x}}{\sqrt{1-x^2}} = \frac{0}{0}$

$= \lim_{\sqrt{1-x}\to 0} \frac{\sin\sqrt{1-x}}{\sqrt{1-x}} \cdot \frac{1}{\sqrt{1+x}}$

$= \frac{1}{\sqrt{2}}$

16/ $\lim_{x\to 0} e^{x^2} = e^0 = 1$

17/ $\lim_{x\to 1} e^{x^2-1} = 1$

18/ $\lim_{x\to 1} \ln x = \ln 1 = 0$

$\ln 0^+ = -\infty$
$\ln \infty = \infty$

19/ $\lim_{x\to 2} (e^x - \ln x) = e^2 - \ln 2$

20/ $\lim_{x\to 1} \frac{1}{\ln x} = \frac{1}{0} = \infty$

Oblique (slant) Asymptote

$$y = \frac{3x^2-1}{x-2}$$

$$\begin{array}{r}
3x+6 \\
x-2\,\big)\,\overline{3x^2-1} \\
-3x^2+6x \\
\hline
6x-1 \\
-6x+12 \\
\hline
11
\end{array}$$

$$y = \underbrace{3x+6}_{O.A.} + \frac{11}{3x+6}$$

$$\lim_{x\to 2} \frac{(x-2)^2}{x^2-4} = \frac{0}{0}$$

$$= \lim_{x\to 2} \frac{(x-2)^2}{(x-2)(x+2)}$$

$$= \lim_{x\to 2} \frac{x-2}{x+2}$$

$$= \frac{0}{4}$$

$$= 0$$

$$\lim_{x\to 2} \frac{x-2}{x^2-4} = \frac{0}{0}$$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)(x+2)}$$

$$= \lim_{x\to 2} \frac{1}{x+2}$$

$$= \frac{1}{4}$$

$$\lim_{x\to 2^+} \frac{x-3}{x^2-4} = \frac{-1}{0^+}$$

$$= -\infty$$

$$\lim_{x\to 2^-} \frac{x-3}{x^2-4} = \frac{-1}{0^-}$$

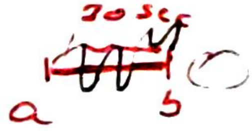
$$= \infty$$

$$\lim_{x\to 2} \frac{x-3}{x^2-4} = \frac{1}{0} = \infty$$ I love

Continuity

Defn

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x) \ \exists$
3. $\lim_{x \to c} f(x) = f(c)$



Ex. $f(x) = \sqrt{4 - x^2}$ is continuous?

$f(x)$ is continuous on $[-2, -2]$

or $-2 \le x \le 2$

Ex $f(x) = \frac{1}{x}$ is continuous?

$f(x)$ is continuous everywhere except $x = 0$

$D: x \in \mathbb{R} - \{0\}$

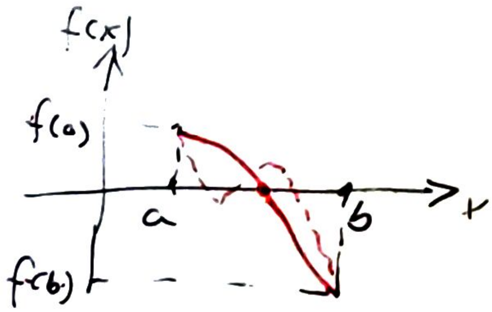
$x \neq 0$

$\{x \mid x \neq 0\}$

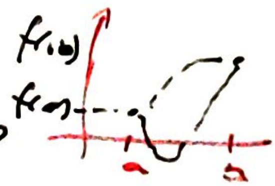
Ex $f(x) = \dfrac{1}{2x^2 + 3x + 1}$ is cont.?

$f(x)$ is continuous everywhere except $x = -1, -\frac{1}{2}$

Intermediate Value Theorem



$f(a)$ and $f(b)$ are the same sign?



$f(x) = x^3 - x - 1 \qquad 1 < x < 2.$

$f(1) = 1 - 1 - 1 = -1$
$f(2) = 8 - 2 - 1 = 5$ } $\exists$ at least ~~1~~ zero

#16. $f(x) = x^3 - 15x + 1 = 0 \qquad [-4, 4]$

x	$f(x)$
-4	$-$
-3	$+$
-2	$+$
-1	$+$
0	$+$
1	$-$
2	$-$
3	$-$
4	$+$

solutions are between
$(-4, -3)$
$(0, 1)$
$(3, 4)$

1.6 Precise Defn of a Limit

Defn $\lim_{x \to x_0} f(x) = L$

for every number (+) $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x

Defn of limit $\lim_{x \to x_0} f(x) = L$ { $\forall\, \varepsilon > 0,\ \exists\, \delta > 0 \ni \forall x$
$|x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$-\delta < x - x_0 < \delta \qquad\qquad -\varepsilon < f(x) - L < \varepsilon$

Ex $\lim_{x \to 1} (5x-3) = 2$

$x_0 = 1$, $f(x) = 5x-3$, $L = 2$

$0 < \boxed{|x-1| < \delta} \Rightarrow |f(x) - 2| < \varepsilon$

$-\delta < x-1 < \delta$

$|5x-3-2| < \varepsilon$

$|5x-5| < \varepsilon$

$5|x-1| < \varepsilon$

$\boxed{|x-1| < \frac{\varepsilon}{5}}$

$\delta = \frac{\varepsilon}{5}$

Ex $f(x) = \sqrt{x-1}$ $x_0 = 5$, $L = 2$, $\varepsilon = 1$

Find δ?

$-\delta < x-5 < \delta$

$5-\delta < x < \delta+5 \leftarrow$

$|f(x) - L| < \varepsilon$

$|\sqrt{x-1} - 2| < 1$

$-1 < \sqrt{x-1} - 2 < +1$

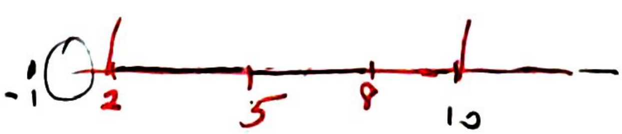
$1 < \sqrt{x-1} < 3$

$1 < x-1 < 9$

$2 < x < 10$

$5-\delta < x < \delta+5$

$\begin{cases} 5-\delta = 2 \Rightarrow \boxed{\delta = 3} \\ 5+\delta = 10 \Rightarrow \delta = 5 \end{cases}$



#4. $f(x) = x+1 \quad L = 5 \quad x_0 = 4 \quad \varepsilon = .01 = \frac{1}{100}$

$-\delta < x - 4 < \delta$

$4 - \delta < x < 4 + \delta$

$|f(x) - L| < \varepsilon$

$-\varepsilon < x + 1 - 5 < \varepsilon$

$-.01 < x - 4 < .01$

$4 - .01 < x < 4 + .01$

$3.99 < x < 4.01$

$4 - \delta = 3.99 \Rightarrow \delta = .01$

$4 + \delta = 4.01 \Rightarrow \delta = .01$

$\therefore \delta = 0.01$

$4 - \frac{1}{100} < x < 4 + \frac{1}{100}$

$\frac{399}{100} < x < \frac{401}{100}$

$4 - \delta = \frac{399}{100}$ ①

$4 + \delta = \frac{401}{100}$ ②

① $\delta = 4 - \frac{399}{100} = \frac{1}{100}$

② $\delta = \frac{401}{100} - 4 = \frac{1}{100}$

$\delta = \frac{1}{100}$

#6. $f(x) = \sqrt{x-7} \quad L = 4 \quad x_0 = 23 \quad \varepsilon = 1$

$-\delta < x - 23 < \delta$

$23 - \delta < x < 23 + \delta$

$-1 < \sqrt{x-7} - 4 < 1$

$3 < \sqrt{x-7} < 5$

$9 < x - 7 < 25$

$16 < x < 32$

$23 - \delta = 16 \Rightarrow \delta = 7$

$23 + \delta = 32 \Rightarrow \delta = 9$

$\delta = 7$