37 $\begin{cases} x_1 - x_2 = 0 & (1) \\ 3x_1 - 2x_2 = -1 & (2) \end{cases}$

(1) $\rightarrow x_1 = x_2$

(2) $3x_1 - 2x_1 = -1$

$x_1 = -1 = x_2$

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39 $\begin{cases} 3u + v = 240 \\ u + 3v = 240 & (1) \end{cases}$ $-3\times$

$\begin{cases} 3u + v = 240 \\ -3u - 9v = -720 \end{cases}$

$-8v = -480$

$v = 60$

(1) $u = 240 - 3(60)$

$= 60$

#49 $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 1 & (1) \\ x_1 + x_2 - 2x_3 = 3 & (2) \\ 2x_1 - 3x_2 + 6x_3 = 8 & (3) \end{cases}$

$3(2) - (1)$, $3(3) - 2(1)$ $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 1 \\ x_2 - 2x_3 = 8 & (4) \\ -5x_2 + 10x_3 = 22 & (5) \end{cases}$

$(5) + 5(4)$ $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 1 \\ x_2 - 2x_3 = 8 \\ 0 = 42 \Rightarrow \text{(impossible)} \end{cases}$

No solution.

#1 $\begin{bmatrix} 1 & 2 & -4 \\ 3 & -4 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ size: 3×3 (R C)

#3 $\begin{bmatrix} 2 & -1 & -1 & 1 \\ -6 & 2 & 0 & 1 \end{bmatrix}$ size 2×4 (R. C.)

#11 $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 2 \end{array}\right] \rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = 2 \end{array}$

19 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Reduced

#31
$$\begin{cases} x_1 - 3x_3 = -2 \\ 3x_1 + x_2 - 2x_3 = 5 \\ 2x_1 + 2x_2 + x_3 = 4 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{array}\right] \begin{array}{l} R_2 - 3R_1 \\ R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{array}\right] R_3 - 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{array}\right] \begin{array}{l} \rightarrow x_1 - 3x_3 = -2 \quad (2) \\ \rightarrow x_2 + 7x_3 = 11 \quad (1) \\ \rightarrow -7x_3 = -14 \end{array}$$

$x_3 = 2$

(1) $x_2 = 11 - 14 = -3$

(2) $x_1 = -2 + 6 = 4$

$\therefore (4, -3, 2)$

#33
$$\begin{cases} 2x_1 + 3x_3 = 3 \\ 4x_1 - 3x_2 + 7x_3 = 5 \\ 8x_1 - 9x_2 + 15x_3 = 10 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 4 & -3 & 7 & 5 \\ 8 & -9 & 15 & 10 \end{array}\right] \begin{matrix} R_2 - 2R_1 \\ R_3 - 4R_1 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & -9 & 3 & -2 \end{array}\right] R_3 - 3R_2$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right] \rightarrow 0 = 1 \text{ (imp)}$$

$\therefore$ No solution

#43
$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right] \rightarrow x_1 = 0$$
$$\rightarrow x_2 + x_3 = 0.$$
$$x_2 = -x_3$$

let $x_3 = t \Rightarrow x_2 = -t$

$\therefore (0, -t, t)$

#49 $A = \left[\begin{array}{cc|c} 1 & k & 2 \\ -3 & 4 & 1 \end{array}\right]$ Augmented matrix

a) 2 eqns w 2 variables.

b) $\left[\begin{array}{cc|c} 1 & k & 2 \\ -3 & 4 & 1 \end{array}\right] R_2 + 3R_1$

$$\left[\begin{array}{cc|c} 1 & k & 2 \\ 0 & 4+3k & 7 \end{array}\right]$$

For the system to be consistent: $4 + 3k \neq 0$

$\Rightarrow k \neq -\frac{4}{3}$ $\quad k \in \mathbb{R} - \{-\frac{4}{3}\}$

$\rightarrow$

#49 cont System is homogeneous.

a) $\left[\begin{array}{ccc|c} 1 & k & 2 & 0 \\ -3 & 4 & 1 & 0 \end{array}\right]$ 2 eqns w/ 3 variables

b) $R_2 + 3R_1$

$$\left[\begin{array}{ccc|c} 1 & k & 2 & 0 \\ 0 & 4+3k & 7 & 0 \end{array}\right] \begin{array}{l} \rightarrow x_1 + kx_2 + 2x_3 = 0 \quad (2) \\ \rightarrow (4+3k)x_2 + 7x_3 = 0 \quad (1) \end{array}$$

$k \in \mathbb{R}$

#1 (2, 5) (3, 2) (4, 5)

a) $p(x) = a_0 + a_1 x + a_2 x^2$

$$p(2) = a_0 + 2a_1 + 4a_2 = 5$$
$$p(3) = a_0 + 3a_1 + 9a_2 = 2$$
$$p(4) = a_0 + 4a_1 + 16a_2 = 5$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 1 & 3 & 9 & 2 \\ 1 & 4 & 16 & 5 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 0 & 2 & 12 & 0 \end{array}\right] R_3 - 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

$a_0 + 2a_1 + 4a_2 = 5$ (2)

$a_1 + 5a_2 = -3$ (1)

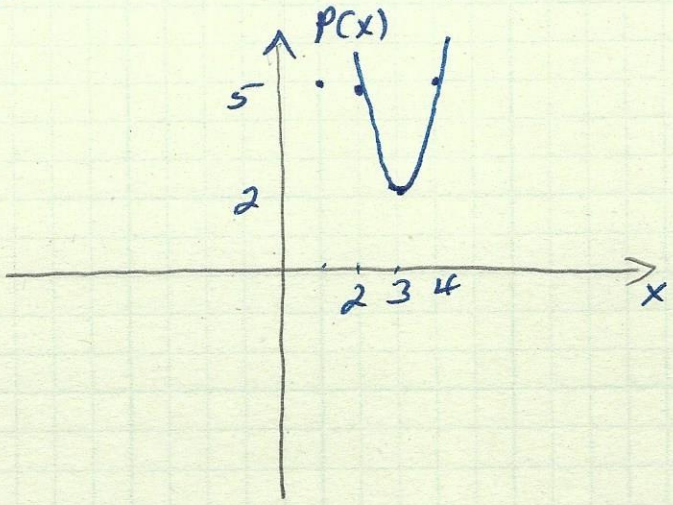
$2a_2 = 6 \Rightarrow \underline{a_2 = 3}$

(1) $\rightarrow$ $a_1 = -3 - 15 = \underline{-18}$

(2) $\rightarrow$ $a_0 = 5 + 36 - 12 = 29$

$$p(x) = 29 - 18x + 3x^2$$

b)



#5 $(-1,3)$ $(0,0)$ $(1,1)$ $(4,58)$

a) $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$p(-1)$ $a_0 - a_1 + a_2 - a_3 = 3$ ①

$p(0)$ $a_0 = 0$

$p(1)$ $a_0 + a_1 + a_2 + a_3 = 1$ ②

$p(4)$ $a_0 + 4a_1 + 16a_2 + 64a_3 = 58$ ③

① → $-a_1 + a_2 - a_3 = 3$ +

② → $a_1 + a_2 + a_3 = 1$

$2a_2 = 4 \Rightarrow a_2 = 2$

② ⇒ $a_1 + a_3 = -1$

③ → $4a_1 + 64a_3 = 58 - 32$

$= 26$

$-2\times$ (times) $\begin{cases} a_1 + a_3 = -1 \\ 2a_1 + 32a_3 = 13 \end{cases}$

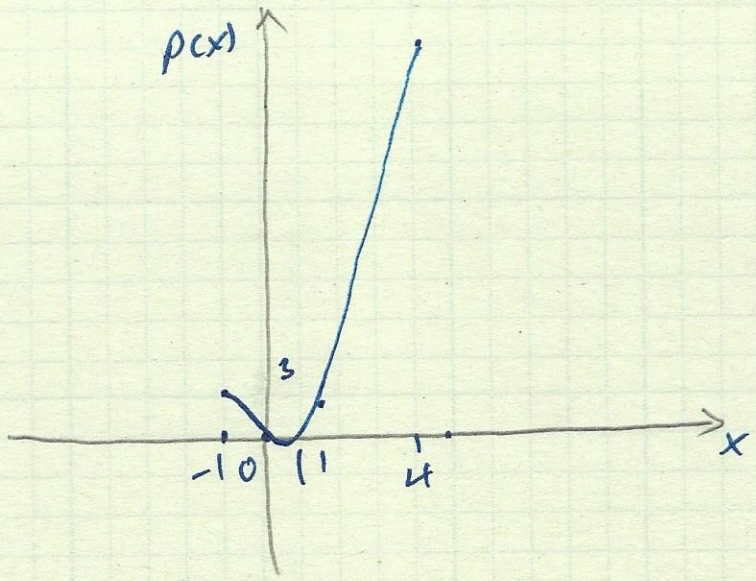
$\begin{cases} -2a_1 - 2a_3 = 2 \\ 2a_1 + 32a_3 = 13 \end{cases}$

$30a_3 = 15 \Rightarrow a_3 = \frac{1}{2}$

$a_1 = -1 - \frac{1}{2} = -\frac{3}{2}$

$\therefore p(x) = -\frac{3}{2}x + 2x^2 + \frac{1}{2}x^3$

b)



#17 Let assume $x=0$ @ year 1990
$\Rightarrow (0, 249)$

2000 $\rightarrow$ $(10, 282)$
2010 $\rightarrow$ $(20, 309)$

$P(x) = a_0 + a_1 x + a_2 x^2$

$P(0)$: $\boxed{a_0 = 249}$

$P(10)$: $a_0 + 10a_1 + 100a_2 = 282$
$P(20)$: $a_0 + 20a_1 + 400a_2 = 309$

$\times(-2)$ $10a_1 + 100a_2 = 33$ (1)
$20a_1 + 400a_2 = 60$

$-20a_1 - 200a_2 = -66$
$20a_1 + 400a_2 = 60$

$200a_2 = -6 \Rightarrow \boxed{a_2 = -\frac{3}{100}}$

(1) $\rightarrow 10a_1 = 33 + 3$
$\boxed{a_1 = \frac{36}{10} = \frac{18}{5}}$

$p(x) = 249 + \frac{18}{5}x - \frac{3}{100}x^2$

year: 2020 $\rightarrow x = 30$

$p(30) = 249 + 18(6) - 27 = \underline{330 \text{ mill}}$ in 2020

year 2030 $\rightarrow x = 40$

$p(40) = 249 + 18(8) - 3(16)$
$= \underline{345 \text{ million}}$ in 2030

#23 a)
$$\begin{cases} x_3 = x_1 + 100 \\ x_1 = 200 + x_2 \\ x_4 = 100 + x_2 \\ x_3 = 200 + x_4 \end{cases} \qquad \begin{cases} -x_1 + x_3 = 100 \\ x_1 - x_2 = 200 \\ -x_2 + x_4 = 100 \\ x_3 - x_4 = 200 \end{cases}$$

$$\left[\begin{array}{cccc|c} -1 & 0 & 1 & 0 & 100 \\ 1 & -1 & 0 & 0 & 200 \\ 0 & -1 & 0 & 1 & 100 \\ 0 & 0 & 1 & -1 & 200 \end{array}\right] \begin{matrix} \\ R_2 + R_1 \\ \\ \\ \end{matrix} \qquad \begin{matrix} -R_1 \\ -R_2 \end{matrix}$$

$$\left[\begin{array}{cccc|c} +1 & 0 & -1 & 0 & 100 \\ 0 & +1 & -1 & 0 & 300 \\ 0 & -1 & 0 & 1 & 100 \\ 0 & 0 & 1 & -1 & 200 \end{array}\right] \begin{matrix} \\ \\ R_3 + R_2 \\ \\ \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & -300 \\ 0 & 0 & -1 & 1 & -200 \\ 0 & 0 & 1 & -1 & 200 \end{array}\right] \begin{matrix} \\ \\ \\ R_4 + R_3 \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & -300 \\ 0 & 0 & -1 & 1 & -200 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{cases} x_1 - x_3 = -100 \\ x_2 - x_3 = -300 \\ -x_3 + x_4 = -200 \end{cases} \quad \text{or}$$

Let $x_4 = t$

$x_3 = t + 200$

$x_2 = t + 200 - 300 = t - 100$

$x_1 = -100 + t + 200$

$$\begin{cases} x_1 = t + 100 \\ x_2 = t - 100 \\ x_3 = t + 200 \\ x_4 = t \end{cases}$$

b) $t = 0 \Rightarrow \begin{cases} x_1 = 100 \\ x_2 = -100 \\ x_3 = 200 \\ x_4 = 0 \end{cases}$

#23. cont.

c) $x_4 = 100 = t$.

$$\begin{cases} x_1 = 200 \\ x_2 = 0 \\ x_3 = 300 \\ x_4 = 100 \end{cases}$$

d) $x_1 = 2x_2$

$t + 100 = 2(t - 100)$

$t + 100 = 2t - 200$

$t = 300$

$$\begin{cases} x_1 = 400 \\ x_2 = 200 \\ x_3 = 500 \\ x_4 = 300 \end{cases}$$