Solution *Section R.1* – Derivative

Exercise

Find the derivative of $f(t) = -3t^2 + 2t - 4$

Solution

$$f'(t) = -6t + 2$$

Exercise

Find the derivative of $g(x) = 4\sqrt[3]{x} + 2$

Solution

$$g(x) = 4x^{1/3} + 2$$

$$g'(x) = \frac{4}{3}x^{-2/3}$$

$$= \frac{4}{3x^{2/3}}$$

$$= \frac{4}{3\sqrt[3]{x^2}}$$

Exercise

Find the derivative of $f(x) = x\left(x^2 + 1\right)$

Solution

$$f(x) = x^3 + x$$

$$f'(x) = 3x^2 + 1$$

Exercise

Find the derivative of $f(x) = \frac{2x^2 - 3x + 1}{x}$

Solution

$$f(x) = \frac{2x^2}{x} - \frac{3x}{x} + \frac{1}{x}$$

$$= 2x - 3 + \frac{1}{x}$$

$$f'(x) = 2 - \frac{1}{x^2}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

Exercise

Find the derivative of $f(x)=\frac{4x^3-3x^2+2x+5}{x^2}$

Solution

$$f(x)=4x-3+\frac{2}{x}+5x^{-2} \qquad \left(\frac{1}{x}\right)'=-\frac{1}{x^2}$$

$$f'(x)=4-\frac{2}{x^2}-10x^{-3}$$

$$\underline{\boxed{=4-\frac{2}{x^2}-\frac{10}{x^3}}}$$

Exercise

Find the derivative of $f(x)=\frac{-6x^3+3x^2-2x+1}{x}$

Solution

$$f(x)=-6x^2+3x-2+\frac{1}{x} \qquad \left(\frac{1}{x}\right)'=-\frac{1}{x^2}$$

$$\underline{\boxed{f'(x)=-12x+3-\frac{1}{x^2}}}$$

Exercise

Find the derivative of $f(x)=x\left(1-\frac{2}{x+1}\right)$

Solution

$$f(x)=x-\frac{2x}{x+1}$$

$$\left(\frac{2x}{x+1}\right)' \Rightarrow \quad \begin{matrix} f=2x & f'=2 \\ g=x+1 & g'=1 \end{matrix}$$

$$f'(x)=1-\frac{2(x+1)-2x}{(x+1)^2}$$

$$=1-\frac{2x+2-2x}{(x+1)^2}$$

$$\underline{\boxed{=1-\frac{2}{(x+1)^2}}}$$

Exercise

Find the derivative of $g(s)=\dfrac{s^2-2s+5}{\sqrt{s}}$

Solution

$$g(s)=\frac{s^2}{s^{1/2}}-2\frac{s}{s^{1/2}}+\frac{5}{s^{1/2}}$$

$$=s^{3/2}-2s^{1/2}+5s^{-1/2}$$

$$g'(s)=\frac{3}{2}s^{1/2}-2\frac{1}{2}s^{-1/2}+5\left(-\frac{1}{2}\right)s^{-3/2}$$

$$=\frac{3}{2}s^{1/2}-s^{-1/2}-\frac{5}{2}s^{-3/2}$$

$$=\frac{3}{2}\sqrt{s}-\frac{1}{\sqrt{s}}-\frac{5}{2s^{3/2}}$$

$$\underline{\left|=\frac{3}{2}\sqrt{s}-\frac{1}{\sqrt{s}}-\frac{5}{2s\sqrt{s}}\right|}$$

Exercise

Find the derivative of $f(x)=\dfrac{x+1}{\sqrt{x}}$

Solution

$$f(x)=\frac{x}{x^{1/2}}+\frac{1}{x^{1/2}}$$

$$=x^{1/2}+x^{-1/2}$$

$$f'(x)=\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{-3/2}$$

$$\underline{\left|=\frac{1}{2x^{1/2}}-\frac{1}{2x^{3/2}}\right|}$$

Exercise

Find the derivative to the following functions $y=3x\left(2x^2+5x\right)$

Solution

$$y=6x^3+15x^2$$

$$\underline{\left|y'=18x^2+30x\right|}$$

Exercise

Find the derivative to the following functions $y = 3\left(2x^2 + 5x\right)$

Solution

$$y = 6x^2 + 15x$$
$$\underline{y' = 12x + 15}$$

Exercise

Find the derivative to the following functions $y = \frac{x^2 + 4x}{5}$

Solution

$$y = \frac{1}{5}\left(x^2 + 4x\right)$$
$$\underline{y' = \frac{1}{5}(2x + 4)}$$

Exercise

Find the derivative to the following functions $y = \frac{3x^4}{5}$

Solution

$$y = \frac{3}{5}x^4$$
$$\underline{y' = \frac{12}{5}x^3}$$

Exercise

Find the derivative to the following functions $y = \frac{x^2 - 4}{2x + 5}$

Solution

$$\underline{y' = \frac{2x^2 + 10x + 8}{(2x + 5)^2}}$$

$$\begin{matrix} 1 & 0 & -4 \\ 0 & 2 & 5 \end{matrix}$$

$$\left(\frac{ax^2 + bx + c}{dx^2 + ex + f}\right)' = \frac{(ae - bd)x^2 + 2(af - cd)x + bf - ce}{\left(dx^2 + ex + f\right)^2}$$

Exercise

Find the derivative to the following functions $y=\dfrac{(1+x)(2x-1)}{x-1}$

Solution

$$y=\frac{(1+x)(2x-1)}{x-1}$$

$$=\frac{2x^2+x-1}{x-1}$$

$$\boxed{y'=\frac{2x^2-4x}{(x-1)^2}}$$

$2 \quad 1 \quad -1$

$0 \quad 1 \quad -1$

$$\left(\frac{ax^2+bx+c}{dx^2+ex+f}\right)'=\frac{(ae-bd)x^2+2(af-cd)x+bf-ce}{\left(dx^2+ex+f\right)^2}$$

Exercise

Find the derivative to the following functions $y=\dfrac{4}{2x+1}$

Solution

$$\boxed{y'=-\frac{8}{(2x+1)^2}}$$

$$\left(\frac{ax+b}{cx+d}\right)'=\frac{ad-bc}{(cx+d)^2}$$

Exercise

Find the derivative to the following functions $y=\dfrac{2}{(x-1)^3}$

Solution

$$\boxed{y'=-\frac{6}{(x-1)^4}}$$

$$\left(\frac{1}{U^n}\right)'=-\frac{nU'}{U^{n+1}}$$

Exercise

Find the derivative to the following functions $y=\sqrt[3]{(x+4)^2}$

Solution

$$y=(x+4)^{2/3}$$

$$\boxed{y'=\frac{2}{3\sqrt[3]{x+4}}}$$

$$\left(U^n\right)'=nU'\,U^{n-1}$$

Exercise

Find the derivative of $f(x)=\sqrt{2t^2+5t+2}$

Solution

$$f(t)=\left(2t^2+5t+2\right)^{1/2} \qquad U=2t^2+5t+2 \;\rightarrow\; U'=4t+5$$

$$f'(t)=\frac{1}{2}(4t+5)\left(2t^2+5t+2\right)^{-1/2} \qquad \left(U^n\right)'=nU'\,U^{n-1}$$

$$=\underline{\left|\frac{1}{2}\frac{4t+5}{\sqrt{2t^2+5t+2}}\right|}$$

Exercise

Find the derivative of $f(x)=\dfrac{1}{\left(x^2-3x\right)^2}$

Solution

$$\underline{\left|f'(x)=-\frac{2(2x-3)}{\left(x^2-3x\right)^3}\right|} \qquad \left(U^n\right)'=nU'\,U^{n-1}$$

Exercise

Find the derivative of $y=t^2\sqrt{t-2}$

Solution

$$y'=\frac{t\left(2t-4+\frac{1}{2}t\right)}{(t-2)^{1/2}} \qquad \left(U^nV^m\right)'=U^{n-1}V^{m-1}\left(nU'V+mUV'\right)$$

$$=\underline{\left|\frac{5t^2-4t}{2\sqrt{t-2}}\right|}$$

Exercise

Find the derivative of $y=\left(\dfrac{6-5x}{x^2-1}\right)^2$

Solution

$$\underline{\left|y'=\frac{2\left(5x^2-12x+5\right)(6-5x)}{\left(x^2-1\right)^3}\right|} \qquad \frac{d}{dx}\left(\frac{ax^n+b}{cx^n+d}\right)^m=mn(ad-bc)x^{n-1}\frac{\left(ax^n+b\right)^{m-1}}{\left(cx^n+d\right)^{m+1}}$$

Exercise

Find the derivative to the following functions $y = x^2\sqrt{x^2+1}$

Solution

$$y' = \frac{x\left(2\left(x^2+1\right)+\frac{1}{2}(2x)x\right)}{\sqrt{x^2+1}}$$

$$= \boxed{\frac{x\left(3x^2+2\right)}{\sqrt{x^2+1}}}$$

$$\left(U^nV^m\right)' = U^{n-1}V^{m-1}\left(nU'V+mUV'\right)$$

Exercise

Find the derivative to the following functions $y = \left(\frac{x+1}{x-5}\right)^2$

Solution

$$\boxed{y' = -\frac{12(x+1)}{(x-5)^3}}$$

$$\frac{d}{dx}\left(\frac{ax^n+b}{cx^n+d}\right)^m = mn(ad-bc)x^{n-1}\frac{\left(ax^n+b\right)^{m-1}}{\left(cx^n+d\right)^{m+1}}$$

Exercise

Find the derivative to the following functions $y = x^2\sin x$

Solution

$$\boxed{y' = 2x\sin x + x^2\cos x}$$

$$u = x^2 \quad v = \sin x$$
$$u' = 2x \quad v' = \cos x$$

Exercise

Find the derivative to the following functions $y = \frac{\sin x}{x}$

Solution

$$\boxed{y' = \frac{x\cos x - \sin x}{x^2}}$$

$$u = \sin x \quad v = x$$
$$u' = \cos x \quad v' = 1$$

Exercise

Find the derivative to the following functions $y = \frac{\cot x}{1+\cot x}$

Solution

$$u = \cot x \qquad v = 1 + \cot x$$

$$u' = -\csc^2 x \qquad v' = -\csc^2 x$$

$$y' = \frac{-\csc^2 x(1+\cot x) + \csc^2 x \cot x}{(1+\cot x)^2}$$

$$= \frac{-\csc^2 x - \csc^2 x \cot x + \csc^2 x \cot x}{(1+\cot x)^2}$$

$$= \underline{\left| \frac{-\csc^2 x}{(1+\cot x)^2} \right|}$$

Exercise

Find the derivative to the following functions $y = x^2 \sin x + 2x \cos x - 2 \sin x$

Solution

$$y' = 2x \sin x + x^2 \cos x + 2 \cos x - 2x \sin x - 2 \cos x$$

$$\underline{= x^2 \cos x}$$

Exercise

Find the derivative to the following functions $y = x^3 \sin x \cos x$

Solution

$$y' = \left(x^3\right)' \sin x \cos x + x^3 (\sin x)' \cos x + x^3 \sin x (\cos x)'$$

$$= \underline{3x^2 \sin x \cos x + x^3 \cos^2 x - x^3 \sin^2 x}$$

Exercise

Find the derivative to the following functions $y = \frac{4}{\cos x} + \frac{1}{\tan x}$

Solution

$$y' = \frac{-4 \sin x}{\cos^2 x} - \frac{\sec^2 x}{\tan^2 x} \qquad \left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$= -4 \frac{\sin x}{\cos x} \frac{1}{\cos x} - \frac{1}{\cos^2 x} \frac{\cos^2 x}{\sin^2 x}$$

$$\underline{= -4 \tan x \sec x - \csc^2 x}$$

Exercise

Find the derivative of $f(x)=\dfrac{\left(x^2-6x\right)^5}{\left(3x^2+5x-2\right)^4}$

Solution

$$f(x)=\left(x^2-6x\right)^5\left(3x^2+5x-2\right)^{-4} \qquad \left(U^mV^n\right)'=U^{m-1}V^{n-1}\left(mU'V+nUV'\right)$$

$$f'(x)=\left(x^2-6x\right)^4\left(3x^2+5x-2\right)^{-5}\left[5(2x-6)\left(3x^2+5x-2\right)-4\left(x^2-6x\right)(6x+5)\right]$$

$$=\left(x^2-6x\right)^4\left(3x^2+5x-2\right)^{-5}\left[(10x-30)\left(3x^2+5x-2\right)-4\left(6x^3-31x^2-30x\right)\right]$$

$$=\left(x^2-6x\right)^4\left(3x^2+5x-2\right)^{-5}$$

$$\begin{array}{ll} x^3 & 30-24 \\ x^2 & 50-90+124 \\ x & -20-150+120 \\ x^0 & 60 \end{array}$$

$$=\boxed{\frac{\left(x^2-6x\right)^4\left(6x^3+84x^2-50x+60\right)}{\left(3x^2+5x-2\right)^5}}$$

Exercise

Find the derivative of $y=\ln\sqrt{x+5}$

Solution

$$y=\ln(x+5)^{1/2}$$

$$=\frac{1}{2}\ln(x+5)$$

$$\boxed{y'=\frac{1}{2(x+5)}}$$

Exercise

Find the Derivatives of $y=(3x+7)\ln(2x-1)$

Solution

$$\boxed{y'=3x\ln(2x-1)+\frac{2(3x+7)}{2x-1}}$$

Exercise

Find the Derivatives of $f(x)=\ln\sqrt[3]{x+1}$

Solution

$$f(x)=\frac{1}{3}\ln(x+1)$$

$$\boxed{f'(x)=\frac{1}{3(x+1)}}$$

Exercise

Find the Derivatives of $f(x)=\ln\left(x^2\sqrt{x^2+1}\right)$

Solution

$$f(x)=2\ln x+\frac{1}{2}\ln\left(x^2+1\right)$$

$$f'(x)=2\frac{1}{x}+\frac{1}{2}\frac{2x}{x^2+1}$$

$$\boxed{=\frac{2}{x}+\frac{x}{x^2+1}}$$

Exercise

Find the Derivatives of $y=\ln\dfrac{x^2}{x^2+1}$

Solution

$$y=\ln x^2-\ln\left(x^2+1\right)$$

$$y'=\frac{2x}{x^2}-\frac{2x}{x^2+1}$$

$$\boxed{=\frac{2}{x}-\frac{2x}{x^2+1}}$$

Exercise

Find the derivative of $f(x)=e^{-2x^3}$

Solution

$$\boxed{f'(x)=-\frac{6x^2}{e^{2x^3}}}$$

Exercise

Find the derivative of $f(x) = 4e^{x^2}$

Solution

$$f'(x) = 8xe^{x^2}$$

Exercise

Find the derivative of $f(x) = x^2 e^x$

Solution

$$f'(x) = (2x)e^x + x^2 e^x$$
$$= xe^x(2+x)$$

Exercise

Find the derivative $f(x) = 2x^3 e^x$

Solution

$$f'(x) = 6x^2 e^x + 2x^3 e^x$$
$$= 2x^2 e^x (3+x)$$

Exercise

Find the derivative $f(x) = \dfrac{3e^x}{1+e^x}$

Solution

$$f'(x) = \frac{3e^x}{(1+e^x)^2}$$

$$\left(\frac{\alpha+b}{\beta+d}\right)' = \frac{\alpha' d - \beta' b}{(\beta+d)^2}$$

Exercise

Find the derivative $f(x) = 5e^x + 3x + 1$

Solution

$$f'(x) = 5e^x + 3$$

Exercise

Find the derivative of $f(x)=\frac{e^x+e^{-x}}{2}$

Solution

$$f'(x)=\frac{1}{2}e^x-e^{-x}$$

Exercise

Find the derivative of $f(x)=\frac{e^x}{x^2}$

Solution

$$f'(x)=\frac{x^2e^x-2xe^x}{x^4}$$

$$=\frac{xe^x(x-2)}{x^4}$$

$$=\frac{e^x(x-2)}{x^3}$$

Exercise

Find the derivative of $f(x)=x^2e^x-e^x$

Solution

$$f'(x)=e^x(2x)+x^2e^x-e^x$$

$$=e^x\left(x^2+2x-1\right)$$

Exercise

Find the derivative of $f(x)=(1+2x)e^{4x}$

Solution

$$f'(x)=(2)e^{4x}+(1+2x)\left(4e^{4x}\right)$$

$$=2e^{4x}(1+2+4x)$$

$$=2e^{4x}(3+4x)$$

Exercise

Find the derivative of $y = x^2 e^{5x}$

Solution

$$y' = x^2\left(5e^{5x}\right) + 2x\left(e^{5x}\right)$$
$$= xe^{5x}\left(5x+2\right)$$

Exercise

Find the derivative of $y = x^2 e^{-2x}$

Solution

$$y' = 2xe^{-2x} - 2x^3 e^{-2x}$$
$$= 2xe^{-2x}\left(1-x^2\right)$$

Exercise

Find the derivative $f(x) = \dfrac{e^x}{x^2+1}$

Solution

$u = e^x \quad v = x^2+1$

$u' = e^x \quad v' = 2x$

$$f'(x) = \frac{e^x\left(x^2+1\right) - 2xe^x}{\left(x^2+1\right)^2}$$
$$= \frac{\left(x^2+1-2x\right)e^x}{\left(x^2+1\right)^2}$$

Exercise

Find the derivative $f(x) = \dfrac{1-e^x}{1+e^x}$

Solution

$u = 1-e^x \quad v = 1+e^x$

$u' = -e^x \quad v' = e^x$

$$f'(x)=\frac{-e^x\left(1+e^x\right)-e^x\left(1-e^x\right)}{\left(1+e^x\right)^2}$$

$$=\frac{-e^x-e^{2x}-e^x+e^{2x}}{\left(1+e^x\right)^2}$$

$$=\underline{\left|-\frac{2e^x}{\left(1+e^x\right)^2}\right|}$$

Exercise

Find the Derivatives of $y=\dfrac{\ln x}{e^{2x}}$

Solution

$$y'=\frac{e^{2x}\left(\frac{1}{x}\right)-\left(2e^{2x}\right)\ln x}{e^{4x}}$$

$$=\frac{e^{2x}-2x\,e^{2x}\ln x}{e^{4x}}$$

$$=\underline{\left|\frac{1-2x\ln x}{e^{2x}}\right|}$$

Exercise

Find the Derivatives of $f(x)=e^{2x}\ln\left(xe^x+1\right)$

Solution

$f=e^{2x}$ $\quad U=2x\rightarrow U'=2$ $\quad f'=2e^{2x}$

$g=\ln\left(xe^x+1\right)$ $\quad U=xe^x+1\rightarrow U'=e^x+xe^x$ $\quad g'=\dfrac{e^x+xe^x}{xe^x+1}$

$$f'(x)=2e^{2x}\ln\left(xe^x+1\right)+e^{2x}\frac{e^x+xe^x}{xe^x+1}$$

$$=\underline{\left|e^{2x}\left(2\ln\left(xe^x+1\right)+\frac{e^x(1+x)}{xe^x+1}\right)\right|}$$

Exercise

Find the Derivatives of $f(x)=\dfrac{xe^x}{\ln\left(x^2+1\right)}$

Solution

$$u=xe^x \qquad u'=e^x+xe^x$$

$$v=\ln\left(x^2+1\right) \qquad v'=\frac{2x}{x^2+1}$$

$$f'(x)=\frac{e^x(1+x)\ln\left(x^2+1\right)-\dfrac{2x}{x^2+1}xe^x}{\left(\ln\left(x^2+1\right)\right)^2}$$

$$=\frac{e^x\left[(1+x)\ln\left(x^2+1\right)-\dfrac{2x^2}{x^2+1}\right]}{\left[\ln\left(x^2+1\right)\right]^2}$$

$$=\frac{e^x\dfrac{\left(x^2+1\right)(1+x)\ln\left(x^2+1\right)-2x^2}{x^2+1}}{\ln^2\left(x^2+1\right)}$$

$$=\underline{\left|\frac{e^x\left[\left(x^2+1\right)(1+x)\ln\left(x^2+1\right)-2x^2\right]}{\left(x^2+1\right)\left(\ln\left(x^2+1\right)\right)^2}\right|}$$

Exercise

Find the derivative $y=\cos^{-1}\left(\dfrac{1}{x}\right)$

Solution

$$y=\cos^{-1}\left(\frac{1}{x}\right)$$

$$=\sec^{-1}(x)$$

$$y'=\underline{\left|\frac{1}{|x|\cdot\sqrt{x^2-1}}\right|}$$

Exercise

Find the derivative $y = \sin^{-1}\left(\sqrt{2}\,t\right)$

Solution

$$y' = \frac{\sqrt{2}}{\sqrt{1-\left(\sqrt{2}t\right)^2}}$$

$$= \underline{\left| \frac{\sqrt{2}}{\sqrt{1-2t^2}} \right|}$$

Exercise

Find the derivative $y = \sec^{-1}(5s)$

Solution

$$y' = \frac{5s}{|5s|\sqrt{(5s)^2-1}}$$

$$= \underline{\left| \frac{s}{|s|\sqrt{25s^2-1}} \right|}$$

Exercise

Find the derivative $y = \cot^{-1}\sqrt{t-1}$

Solution

$$y' = -\frac{\frac{1}{2}(t-1)^{-1/2}}{1+\left[(t-1)^{1/2}\right]^2}$$

$$= -\frac{1}{2(t-1)^{1/2}(1+t-1)}$$

$$= \underline{\left| -\frac{1}{2t\sqrt{t-1}} \right|}$$

Exercise

Find the derivative $y = \ln\left(\tan^{-1}x\right)$

Solution

$$y' = \frac{\frac{1}{1+x^2}}{\tan^{-1} x}$$

$$= \underline{\boxed{\frac{1}{\left(1+x^2\right)\tan^{-1} x}}}$$

Exercise

Find the derivative $y = \tan^{-1}(\ln x)$

Solution

$$y' = \frac{\frac{1}{x}}{1+(\ln x)^2}$$

$$= \underline{\boxed{\frac{1}{x\left(1+(\ln x)^2\right)}}}$$

$$\left(\tan^{-1} u\right)' = \frac{u'}{1+u^2}$$

Solution *Section R.2* – Integration

Exercise

Find each indefinite integral. $\int \frac{x+2}{\sqrt{x}}\,dx$

Solution

$$\int \frac{x+2}{\sqrt{x}}\,dx = \int \left(\frac{x}{x^{1/2}} + \frac{2}{x^{1/2}}\right)dx$$

$$= \int \left(x^{1/2} + 2x^{-1/2}\right)dx$$

$$= \frac{x^{3/2}}{3/2} + 2\frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{3}x^{3/2} + 4x^{1/2} + C$$

Exercise

Find each indefinite integral $\int 4y^{-3}\,dy$

Solution

$$\int 4y^{-3}dy = -\frac{2}{y^2} + C$$

Exercise

Find each indefinite integral $\int \left(x^3 - 4x + 2\right)dx$

Solution

$$\int \left(x^3 - 4x + 2\right)dx = \frac{1}{4}x^4 - 2x^2 + 2x + C$$

Exercise

Find each indefinite integral $\int \left(\sqrt[4]{x^3} + 1\right)dx$

Solution

$$\int \left(x^{3/4}+1\right) dx = \frac{4}{7}x^{7/4} + x + C$$

Exercise

Find each indefinite integral $\int \sqrt{x}\,(x+1)\,dx$

Solution

$$\int x^{1/2}\,(x+1)\,dx = \int \left(x^{3/2} + x^{1/2}\right) dx$$

$$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$$

Exercise

Find each indefinite integral $\int (1+3t)t^2\,dt$

Solution

$$\int \left(t^2 + 3t^3\right) dt = \frac{1}{3}t^3 + \frac{3}{4}t^4 + C$$

Exercise

Find each indefinite integral $\int \frac{x^2-5}{x^2}\,dx$

Solution

$$\int \frac{x^2-5}{x^2}\,dx = \int \left(1 - \frac{5}{x^2}\right) dx$$

$$= \int \left(1 - 5x^{-2}\right) dx$$

$$= x + 5x^{-1} + C$$

$$= x + \frac{5}{x} + C$$

Exercise

Find each indefinite integral $\int (-40x+250)\,dx$

Solution

$$\int (-40x+250)\,dx = -20x^2+250x+C$$

Exercise

Find each indefinite integral $\int \left(7-3x-3x^2\right)(2x+1)\,dx$

Solution

$$\int \left(7-3x-3x^2\right)(2x+1)\,dx = \int \left(14x+7-6x^2-3x-6x^3-3x^2\right)dx$$

$$= \int \left(-6x^3-9x^2+11x+7\right)dx$$

$$= -\frac{3}{2}x^4-3x^3+\frac{11}{2}x^2+7x+C$$

Exercise

Find the integral $\int (1+\cos 3\theta)\,d\theta$

Solution

$$\int (1+\cos 3\theta)\,d\theta = \theta+\frac{1}{3}\sin 3\theta+C$$

Exercise

Find the integral $\int 2\sec^2\theta\,d\theta$

Solution

$$\int 2\sec^2\theta\,d\theta = 2\tan\theta+C$$

Exercise

Find the integral $\int \sec 2x \tan 2x \; dx$

Solution

$$\int \sec 2x \tan 2x \; dx = \frac{1}{2}\sec 2x + C$$

Exercise

Find the integral $\int 2e^{2x} dx$

Solution

$$\int 2e^{2x} dx = e^{2x} + C$$

Exercise

Find the integral $\int \frac{12}{x} \, dx$

Solution

$$\int \frac{12}{x} \, dx = 12 \ln|x| + C$$

Exercise

Find the integral $\int \frac{dx}{\sqrt{1-x^2}}$

Solution

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

Exercise

Find the integral $\int \frac{dx}{x^2+1}$

Solution

$$\boxed{\int \frac{dx}{x^2+1} = \tan^{-1} x + C}$$

Exercise

Find the integral $\int \frac{1+\tan\theta}{\sec\theta}\, d\theta$

Solution

$$\int \frac{1+\tan\theta}{\sec\theta}\, d\theta = \int \left(\frac{1}{\sec\theta} + \frac{\tan\theta}{\sec\theta}\right) d\theta$$

$$= \int (\cos\theta + \sin\theta)\, d\theta$$

$$\boxed{= \sin\theta - \cos\theta + C}$$

Exercise

Find the general solution of the differential equation $y' = 2t + 3$

Solution

$$dy = (2t+3)\, dt$$

$$\int dy = \int (2t+3)\, dt$$

$$\boxed{y = t^2 + 3t + C}$$

Exercise

Find the general solution of the differential equation $y' = 3t^2 + 2t + 3$

Solution

$$\int dy = \int \left(3t^2 + 2t + 3\right) dt$$

$$\boxed{y = t^3 + t^2 + 3t + C}$$

Exercise

Find the general solution of the differential equation $y' = \sin 2t + 2\cos 3t$

Solution

$$\int dy = \int \left(\sin 2t + 2\cos 3t\right)\, dt$$

$$\boxed{y(t) = -\frac{1}{2}\cos 2t + \frac{2}{3}\sin 3t + C}$$

Exercise

Find the general solution of the differential equation: $y' = x^3\left(3x^4+1\right)^2$

Solution

$$d\left(3x^4+1\right) = 12x^3 dx$$

$$\int x^3\left(3x^4+1\right)^2 dx = \frac{1}{12}\int \left(3x^4+1\right)^2 d\left(3x^4+1\right)$$

$$= \frac{1}{36}\left(3x^4+1\right)^3 + C$$

$$\boxed{y(x) = \frac{1}{36}\left(3x^4+1\right)^3 + C}$$

Exercise

Find the general solution of the differential equation: $y' = 5x\sqrt{x^2-1}$

Solution

$$d\left(x^2-1\right) = 2x\, dx$$

$$\int 5x\left(x^2-1\right)^{1/2} dx = \frac{5}{2}\int \left(x^2-1\right)^{1/2} d\left(x^2-1\right)$$

$$\boxed{= \frac{5}{3}\left(x^2-1\right)^{3/2} + C}$$

Exercise

Find the general solution of the differential equation: $y' = x\sqrt{x^2+4}$

Solution

$$\int x\sqrt{x^2+4}\, dx = \frac{1}{2}\int \left(x^2+4\right)^{1/2} d\left(x^2+4\right)$$

$$= \frac{1}{3}\left(x^2+4\right)^{3/2} + C$$

Exercise

Evaluate the integrals $\int_{-2}^{2}\left(x^3-2x+3\right)dx$

Solution

$$\int_{-2}^{2}\left(x^3-2x+3\right)dx = \left(\frac{x^4}{4}-x^2+3x\right)\Big|_{-2}^{2}$$

$$= \left(\frac{(2)^4}{4}-(2)^2+3(2)\right)-\left(\frac{(-2)^4}{4}-(-2)^2+3(-2)\right)$$

$$= 12$$

Exercise

Evaluate the integrals $\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx$

Solution

$$\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx = \left(\frac{x^3}{3}+\frac{2}{3}x^{3/2}\right)\Big|_{0}^{1}$$

$$= \left(\frac{(1)^3}{3}+\frac{2}{3}(1)^{3/2}\right)-0$$

$$= 1$$

Exercise

Evaluate the integrals $\int_{0}^{\pi/3} 4\sec u\tan u\ du$

Solution

$$\int_{0}^{\pi/3} 4\sec u\tan u\ du = 4\sec u\Big|_{0}^{\pi/3}$$

$$= 4\left(\sec\frac{\pi}{3}-\sec 0\right)$$

$$= 4(2-1)$$

$$= 4$$

Exercise

Evaluate the integrals $\int_{\pi/4}^{3\pi/4} \csc\theta \cot\theta \, d\theta$

Solution

$$\int_{\pi/4}^{3\pi/4} \csc\theta \cot\theta \, d\theta = -\csc\theta \Big|_{\pi/4}^{3\pi/4}$$

$$= -\left(\csc\frac{3\pi}{4} - \csc\frac{\pi}{4}\right)$$

$$= -\left(\sqrt{2} - \sqrt{2}\right)$$

$$= 0$$

Exercise

Evaluate the integrals $\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt$

Solution

$$\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt = \int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \pi t^{-2}\right) dt$$

$$= \left(4\tan t - \pi t^{-1}\right|_{-\pi/3}^{-\pi/4}$$

$$= \left(4\tan\left(-\frac{\pi}{4}\right) - \pi\left(-\frac{4}{\pi}\right)\right) - \left(4\tan\left(-\frac{\pi}{3}\right) - \pi\left(-\frac{3}{\pi}\right)\right)$$

$$= \left(4(-1) + 4\right) - \left(4\left(-\sqrt{3}\right) + 3\right)$$

$$= -\left(-4\sqrt{3} + 3\right)$$

$$= 4\sqrt{3} - 3$$

Exercise

Evaluate the integrals $\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} \, dy$

Solution

$$\int_{-3}^{-1} \frac{y^5 - 2y}{y^3}\,dy = \int_{-3}^{-1} \left(y^2 - 2y^{-2}\right) dy$$

$$= \frac{1}{3}y^3 + 2y^{-1}\Big|_{-3}^{-1}$$

$$= \left(\frac{1}{3}(-1)^3 + \frac{2}{-1}\right) - \left(\frac{1}{3}(-3)^3 + \frac{2}{-3}\right)$$

$$= \frac{22}{3}$$

Exercise

Evaluate the integrals $\displaystyle\int_1^8 \frac{\left(x^{1/3}+1\right)\left(2-x^{2/3}\right)}{x^{1/3}}\,dx$

Solution

$$\int_1^8 \frac{\left(x^{1/3}+1\right)\left(2-x^{2/3}\right)}{x^{1/3}}\,dx = \int_1^8 \frac{2x^{1/3} - x + 2 - x^{2/3}}{x^{1/3}}\,dx$$

$$= \int_1^8 \left(2 - x^{2/3} + 2x^{-1/3} - x^{1/3}\right) dx$$

$$= 2x - \frac{3}{5}x^{5/3} + 3x^{2/3} - \frac{3}{4}x^{4/3}\Big|_1^8$$

$$= \left(2(8) - \frac{3}{5}(8)^{5/3} + 3(8)^{2/3} - \frac{3}{4}(8)^{4/3}\right) - \left(2(1) - \frac{3}{5}(1)^{5/3} + 3(1)^{2/3} - \frac{3}{4}(1)^{4/3}\right)$$

$$= \left(-\frac{16}{5}\right) - \left(\frac{73}{20}\right)$$

$$= -\frac{137}{20}$$

Exercise

Evaluate: $\displaystyle\int_0^1 (2t+3)^3\,dt$

Solution

$d(2t+3) = 2dt$

$$\int_0^1 (2t+3)^3\,dt = \frac{1}{2}\int_0^1 (2t+3)^3\,d(2t+3)$$

$$= \frac{1}{8}\,(2t+3)^4 \,\Big|_0^1$$

$$= \frac{1}{8}\left(5^4 - 3^4\right)$$

$$= 68$$

Exercise

Evaluate the integral $\displaystyle\int_{-1}^{1} r\sqrt{1-r^2}\; dr$

Solution

$$\int_{-1}^{1} r\sqrt{1-r^2}\; dr = -\frac{1}{2}\int_{-1}^{1} \left(1-r^2\right)^{1/2}\; d\left(1-r^2\right)$$

$$= -\frac{1}{3}\,\left(1-r^2\right)^{3/2} \,\Big|_{-1}^{1}$$

$$= -\frac{1}{3}(0-0)$$

$$= 0$$