

$$\frac{x}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$x = A(x-1) + B(x+3)$$

$$x^1 \quad A + B = 1$$

$$x^0 \quad -A + 3B = 0 \rightarrow A = 3\left(\frac{1}{4}\right)$$

$$\frac{4B}{4} = 1$$

$$B = \frac{1}{4}$$

$$\frac{x}{x^2+2x-3} = \frac{3}{4} \frac{1}{x+3} + \frac{1/4}{x-1}$$

$$\frac{1}{(2x+3)(4x-1)} = \frac{A}{2x+3} + \frac{B}{4x-1}$$

$$1 = A(4x-1) + B(2x+3)$$

$$x^1 \quad 4A + 2B = 0 \rightarrow \textcircled{1}$$

$$x^0 \quad -A + 3B = 1$$

$$14B = 4 \Rightarrow B = \frac{2}{7}$$

$$\textcircled{1} \quad 4A = -2\left(\frac{2}{7}\right)$$

$$A = \frac{-4}{4(7)} = -\frac{1}{7}$$

$$\frac{1}{(2x+3)(4x-1)} = \frac{-1/7}{2x+3} + \frac{2/7}{4x-1}$$

1st 4 terms? a_8 ? \rightarrow

$$a_n = (-1)^{n+1} \frac{n}{n+1}$$

$$a_1 = (-1)^2 \frac{1}{2} = \underline{\underline{\frac{1}{2}}}$$

$$a_2 = (-1)^3 \frac{2}{3} = \underline{\underline{-\frac{2}{3}}}$$

$$a_3 = (-1)^4 \frac{3}{4} = \underline{\underline{\frac{3}{4}}}$$

$$a_4 = (-1)^5 \frac{4}{5} = \underline{\underline{-\frac{4}{5}}}$$

$$a_8 = (-1)^9 \frac{8}{9} = \underline{\underline{-\frac{8}{9}}}$$

$$2 + 5 + 8 + \dots + 17 = \sum_{n=1}^6 (3n-1)$$

$$d = 5 - 2 = 3$$

$$a_n = 2 + (n-1)(3)$$

$$= 2 + 3n - 3$$

$$= 3n - 1$$

$$\frac{17 - 2}{3} = 5 + 1$$

Arithm. a_{12} : $a_8 = 4$ $a_{18} = -96$

$$d = \frac{-96 - 4}{18 - 8} = -\frac{100}{10} = -10$$

$$a_n = a_1 + (n-1)d$$

$$a_8 = a_1 + 7(-10)$$

$$4 = a_1 - 70$$

$$a_1 = 74$$

$$\begin{aligned} a_{12} &= 74 + 11(-10) \\ &= 74 - 110 \\ &= -36 \end{aligned}$$

Geom

a_9 : $a_2 = 4$

$a_5 = 32$

$$r = \left(\frac{32}{4}\right)^{\frac{1}{5-2}}$$

$$= (8)^{\frac{1}{3}}$$

$$= 2$$

$$a_2 = a_1 \cdot 2^1 = 4$$

$$a_1 = 2$$

$$\begin{aligned} a_9 &= 2(2)^8 \\ &= 2^9 \end{aligned}$$

$$(2^3)^{\frac{1}{3}}$$

$$a_n = a_1 r^{n-1}$$

$$\sum_{n=1}^{25} 5 = 5(25) = \underline{125}$$

$$\sum_{k=15}^{45} 8 = 8(45 - 15 + 1) = 8(31) = \underline{248}$$

you can avoid this step

$$\sum_{k=1}^5 (2k+1) = (2+1) + (4+1) + (6+1) + (8+1) + 11 = 3 + 5 + 7 + 9 + 11 = \underline{35}$$

$$\sum_{n=1}^{\infty} 5 \left(\frac{3}{2}\right)^n = \infty$$

$$\left|\frac{3}{2}\right| \geq 1$$

$$\sum_{n=1}^{\infty} 3 \left(\frac{2}{3}\right)^n = \frac{3}{1 - \frac{2}{3}} = \underline{9} \quad \text{---} \quad \frac{3}{\frac{1}{3}}$$

$$\left|\frac{2}{3}\right| < 1$$

$$\frac{a}{1-r}$$

$$4 + 8 + 12 + \dots + 4n = 2n(n+1)$$

$$n=1 \Rightarrow 4 = 2(1+1)$$

$$4 = 4 \checkmark \quad P_1 \text{ is true.}$$

Assume P_k is true: $4 + \dots + 4k = 2k(k+1)$

is P_{k+1} : $4 + \dots + 4k + 4(k+1) = 2(k+1)(k+2)$?

$$4 + \dots + 4k + 4(k+1) = 2k(k+1) + 4(k+1)$$

$$= 2(k+1)(k+2) \checkmark$$

P_{k+1} is also true.

\therefore By the mathematical model, the given proof is completed.

$$h = 15 \quad w = 10$$

$$x = 5$$

$$\frac{x^2}{25^2} + \frac{y^2}{20^2} = 1$$

$$\frac{y^2}{20^2} = 1 - \left(\frac{5}{25}\right)^2$$

$$\left(\frac{15}{20}\right)^2 = 1 - \frac{1}{25}$$

$$\frac{225}{400} = \frac{24}{25}$$

$$\frac{225}{400} \cdot \frac{25}{25} = \frac{24}{25}$$

$$\frac{225 \cdot 25}{400 \cdot 25} = \frac{24 \cdot 25}{25 \cdot 25}$$

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will clear

$$625y^2 - 400x^2 = 25 \times 10^4.$$

$$\rightarrow \frac{y^2}{400} - \frac{x^2}{625} = 1$$

$$a^2 = 400$$

$$a = 20$$

The closest point: $2a = \underline{40}$ yards.

$$1 + 5 + 9 + \dots + (4n-3) = n(2n-1)$$

For $n=1 \Rightarrow 1 = 1(2-1)$
 $1 = 1 \checkmark$ P_1 is true

Assume P_k is true: $1 + \dots + (4k-3) = k(2k-1)$

is P_{k+1} : $1 + \dots + (4k-3) + (4(k+1)-3) =$
 $(k+1)(2(k+1)-1)$

$$1 + \dots + (4k-3) + (4k+1) \stackrel{?}{=} (k+1)(2k+1)$$

$$1 + \dots + (4k-3) + (4k+1) = k(2k-1) + (4k+1)$$

$$= 2k^2 - k + 4k + 1$$

$$= 2k^2 + 3k + 1$$

$$= (k+1)(2k+1) \checkmark$$

P_{k+1} is also true

\therefore By the mathematical induction, the given proof is complete

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

For $n=1 \Rightarrow 1^2 = \frac{1(2)(3)}{6}$

$$1 = 1 \checkmark P_1 \text{ is true.}$$

Assume: P_k is true: $1^2 + \dots + k^2 = \frac{1}{6} k(k+1)(2k+1)$

is P_{k+1} : $1^2 + \dots + k^2 + (k+1)^2 = \frac{1}{6} (k+1)(k+2)(2k+3)$

$$\begin{aligned} 1^2 + \dots + k^2 + (k+1)^2 &= \frac{1}{6} k(k+1)(2k+1) + \frac{(k+1)^2}{6} \\ &= \frac{1}{6} (k+1) [k(2k+1) + 6(k+1)] \\ &= \frac{1}{6} (k+1) (2k^2 + 7k + 6) \\ &= \frac{1}{6} (k+1) (k+2)(2k+3) \checkmark \end{aligned}$$

P_{k+1} is also true

\therefore By the mathematical induction,
the given proof is completed.

14 to 15 questions
on Exam 1

Proof always on separate (only)
paper

$$\frac{3x-1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$3x-1 = A(x+2) + B(x-1)$$

$$x^1 \quad A + B = 3 \rightarrow \underline{B = 3 - \frac{2}{3}}$$

$$x^0 \quad \underline{2A - B = -1} \quad = \underline{\frac{7}{3}}$$

$$3A = 2$$

$$\underline{A = \frac{2}{3}}$$

$$\underline{\frac{3x-1}{(x-1)(x+2)} = \frac{2/3}{x-1} + \frac{7/3}{x+2}}$$

$$\frac{2x+1}{2x^2+x-3} = \frac{A}{2x+3} + \frac{B}{x-1}$$

$$2x+1 = A(x-1) + B(2x+3)$$

$$x^1 \quad A + 2B = 2 \rightarrow \underline{A = 2 - 2\left(\frac{3}{5}\right)}$$

$$x^0 \quad \underline{-A + 3B = 1}$$

$$5B = 3$$

$$\underline{B = \frac{3}{5}}$$

$$\underline{\frac{2x+1}{2x^2+x-3} = \frac{4/5}{2x+3} + \frac{3/5}{x-1}}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \quad \left(\frac{x}{a}\right)^2$$

$$= \frac{a^2 - x^2}{a^2}$$

$$y^2 = \frac{b^2}{a^2} (a^2 - x^2)$$

$$y^2 ? \left(\frac{b}{a}\right)^2 (a^2 - x^2)$$

↓
simplify

$$w = 10 \rightarrow x = 5$$

$$h = y = 9$$

$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

$$\frac{y^2}{10^2} = 1 - \left(\frac{5}{20}\right)^2$$

$$= 1 - \frac{1}{16}$$

$$= \frac{15}{16}$$

$$9^2 ? \left(\frac{10}{4}\right)^2 (15)$$

$$81 ? \frac{25}{4} (15)$$

$$81 \times 4 ? (15)(25)$$

$$324 < 375$$

clear ✓