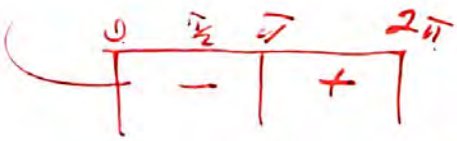
$y = 3 + \sin x \qquad x \in [0, 2\pi]$

Concavity:

$y' = \cos x$

$y'' = -\sin x = 0$

$x = 0, \pi, 2\pi$ } point of inflection
pt. infl.



concave up: $(\pi, 2\pi)$

" down: $(0, \pi)$

5. [illegible]

$s'(t) =$ [illegible]

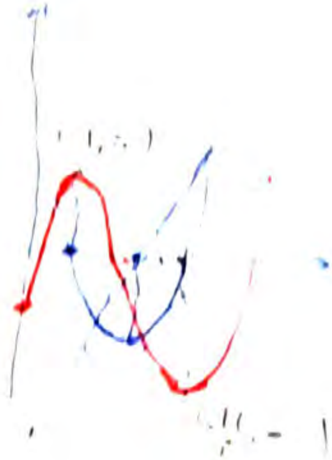
$= 6t^2 - 2$ [illegible]

Crit: $t = 1, \frac{11}{3}$

t	$s(t)$
1	5
$\frac{11}{3}$	−

[illegible]

$(1, \ldots)$, $(\frac{11}{3}, \ldots)$



	0		1	2	$\frac{11}{3}$		5
s'		+		−		+	

Inc: $(0, 1)$ $(\frac{11}{3}, \infty)$

Dec: $(1, \frac{11}{3})$

$s''(t) = $ [illegible]

$= 12t - 28 = 0$

point of infl: $t = \frac{7}{3}$

0	1		$\frac{7}{3}$	
	−			+

concave down: $(0, \frac{7}{3})$

" up: $(\frac{7}{3}, \infty)$

21. $f(x) = x^4 - 4x^3 + 10$

$f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

C.N.: $x = 0, 0, 3$

Incr: $(3, \infty)$

Dec: $(-\infty, 0)\ (0, 3)$

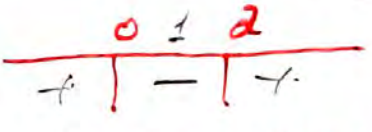
$f(3) = -17$

L.P.: $(3, -17)$

Concavity:

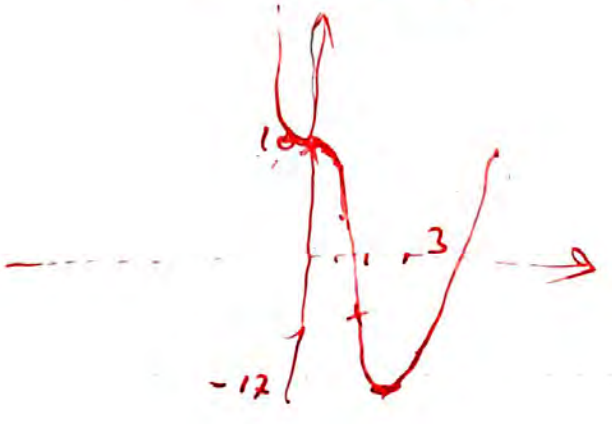
$f''(x) = 12x^2 - 24x = 0 \quad 12x(x-2) = 0$

Point of inf: $x = 0, 2$



concave up: $(-\infty, 0)\ (2, \infty)$

" down: $(0, 2)$



$f(x) = \dfrac{(x+1)^2}{1-x^2}$ $\quad (x+1)^2\ (1+x^2)^{-1}$

$(u^n)' = u^{n-1} \cdot u'$ $\quad (uv)' = u'v + uv'$

$f'(x) = \dfrac{x+1}{(1+x^2)^2}\left(-2(1+x^2) - 2x(x+1)\right)$

$= \dfrac{(x+1)(2+2x^2-2x^2-2x)}{(1+x^2)^2}$

$= \dfrac{(x+1)(2-2x)}{(1+x^2)^2} = 0$

CV: $x = -1, 1$

	−1		0		1	
f'	−	0	+	0	−	

Incr: $(-1, 1)$

Decr: $(-\infty, -1)\ (1, \infty)$

x	$f(x)$	
−1	0	RMIN $(-1, 0)$
1	2	RMAX $(1, 2)$

$f'(x) = 2\dfrac{1-x^2}{(1+x^2)^2}$ $\quad (1-x^2)^1 (1+x^2)^{-2}$

$f''(x) = \dfrac{2}{(1+x^2)^3}\left(-2x(1+x^2) - 2(2x)(1-x^2)\right)$

$= \dfrac{2(-2x-2x^3-4x+4x^3)}{(1+x^2)^3}$

$= \dfrac{2(2x^3-6x)}{(1+x^2)^2}$

$= \dfrac{4x(x^2-3)}{(1+x^2)^2} = 0$

$x = 0, \pm\sqrt{3}$ point of infl.

$-\sqrt{3}$		0		$\sqrt{3}$	
$-$	$+$		$-$		$+$

Concave up: $(-\sqrt{3}, 0)\ (\sqrt{3}, \infty)$

" down: $(-\infty, -\sqrt{3})\ (0, \sqrt{3})$

3.3 Optimization { Max / Minimize } 1st deriv.

{ point of inf: changing } 2nd deriv = 0

$y = ax^2 + bx + c$

$y' = 2ax + b = 0 \quad x = -\frac{b}{2a}$

x ?

$V = (12-2x)^2(x)$

$= x(144 - 48x + 4x^2)$

$= 4x^3 - 48x^2 + 144x$

$\frac{dV}{dx} = 12x^2 - 96x + 144 = 0$

$x^2 - 8x + 12 = 0$

CN: $x = 2, 6$

$V(2) = 2(64) = 128$

$V(6) = 6(0)$ #

$x = 2$ w/ Largest Volume of 128 $unit^3$

Ex $V = 1\,\ell = 10^3\,cm^3 = \pi r^2 h$ (1)

Material (surface)

$$\text{Surface} = 2\pi r^2 + 2\pi r h = A \quad (2)$$

(1) $\Rightarrow h = \frac{10^3}{\pi r^2}$ (3)

(2) $A(r) = 2\pi r^2 + 2\pi r \cdot \frac{10^3}{\pi r^2}$

$$= 2\pi r^2 + 2\frac{10^3}{r}$$

$$\frac{dA}{dr} = 4\pi r - \frac{2\times 10^3}{r^2} = 0$$

$$2\pi r = \frac{10^3}{r^2}$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}} = \sqrt[3]{\frac{10^3}{2\pi}} = \frac{10}{(2\pi)^{1/3}}$$

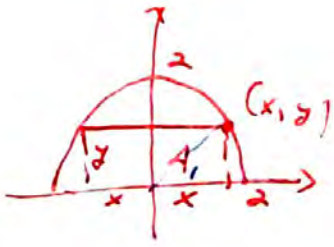
(3) $\Rightarrow h = \frac{10^3}{\pi \frac{100}{(2\pi)^{2/3}}}$

$$= \frac{10\times 2^{2/3}\pi^{2/3}}{\pi}$$

$$= \frac{10\,(2^{2/3})}{\pi^{1/3}}$$

$x^2 + y^2 = 4$

A_{max} ? dim



$y = \sqrt{4 - x^2}$

$A = xy$

$A(x) = x(4 - x^2)^{1/2}$

$\frac{dA}{dx} = \frac{1}{\sqrt{4-x^2}}\left(4 - x^2 + \frac{1}{2}(-2x)x\right)$

$= \frac{4 - 2x^2}{\sqrt{4-x^2}} = 0 \Rightarrow x^2 = 2$

$x = \pm\sqrt{2}$

CN: $x = \sqrt{2}$

$y = \sqrt{2}$] $\left(\sqrt{2}, \sqrt{2}\right)$

$\ell = 2\sqrt{2}$, $w = \sqrt{2}$

$A_{rect} = \ell w$

$= 2\sqrt{2}(\sqrt{2})$

$= 4$ unit^2