Solution   *Section* 1.1 - Linear Equation and Slope

Exercise

Find an equation of the line through $(-4,1)$ having slope -3.

Solution

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -3(x - (-4))$$
$$y - 1 = -3(x + 4)$$
$$y - 1 = -3x - 12$$
$$y - 1 + 1 = -3x - 12 + 1$$
$$\boxed{y = -3x - 11}$$

Exercise

Find an equation of the line containing the given pair of points (3, 2) and (9, 7)

Solution

$$Slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-2}{9-3} = \frac{5}{6}$$
$$y = m(x - x_1) + y_1$$
$$y = \frac{5}{6}(x - 3) + 2$$
$$= \frac{5}{6}x - \frac{5}{2} + 2$$
$$= \frac{5}{6}x - \frac{1}{2}$$

Exercise

Write a slope-intercept equation for a line passing through the point $(3,-2)$ that is parallel to the line $2x - y = 5$

Solution

$$2x - 5 = y \Rightarrow m = 2$$
$$y - y_1 = m(x - x_1)$$
$$y + 2 = 2(x - 3)$$
$$y + 2 = 2x - 6 \qquad \boxed{y = 2x - 8}$$

Exercise

Write a slope-intercept equation for a line passing through the point $(3,-2)$ that is perpendicular to the line $2x-y=5$

Solution

$$2x-y=5 \quad\Rightarrow\quad y=2x-5 \text{ Slope: } m=2$$

$$\Rightarrow m=-\frac{1}{2}$$

$$y-y_1=m\left(x-x_1\right)$$

$$y+2=-\frac{1}{2}(x-3)$$

$$y+2=-\frac{1}{2}x+\frac{3}{2}$$

$$y=-\frac{1}{2}x+\frac{3}{2}-2$$

$$\boxed{y=-\frac{1}{2}x-\frac{1}{2}}$$

Exercise

Write a slope-intercept equation for a line passing through the point (3, 5) that is parallel to the line $y=\frac{2}{7}x+1$

Solution

$$y=\frac{2}{7}x+1 \qquad \Rightarrow m_1=\frac{2}{7}$$

$$m_2=\frac{2}{7}$$

$$y-5=\frac{2}{7}(x-3)$$

$$y-5=\frac{2}{7}x-\frac{6}{7}$$

$$y=\frac{2}{7}x-\frac{6}{7}+5$$

$$y=\frac{2}{7}x+\frac{29}{7}$$

Exercise

Write a slope-intercept equation for a line passing through the point (3, 5) that is perpendicular to the line $y=\frac{2}{7}x+1$

Solution

$$m_2=-\frac{7}{2}$$

$$\Rightarrow\ y-5=-\frac{7}{2}(x-3)$$

$$y-5=-\frac{7}{2}x+\frac{21}{2}$$

$$y=-\frac{7}{2}x+\frac{21}{2}+5$$

$$\boxed{y=-\frac{7}{2}x+\frac{31}{2}}$$

Exercise

Write a slope-intercept equation for a line passing through the point (3, –2) that is parallel to the line $3x + 4y = 5$

Solution

$$4y=-3x+5$$

$$y=-\frac{3}{4}x+\frac{5}{4}$$

$$m=-\frac{3}{4}$$

$$\Rightarrow\ y-(-2)=-\frac{3}{4}(x-3)$$

$$y+2=-\frac{3}{4}x+\frac{9}{4}$$

$$y=-\frac{3}{4}x+\frac{9}{4}-2$$

$$\boxed{y=-\frac{3}{4}x+\frac{1}{4}}$$

Exercise

Write a slope-intercept equation for a line passing through the point (3, –2) that is perpendicular to the line $3x + 4y = 5$

Solution

$$4y = 5 - 3x \Rightarrow\ y=-\frac{3}{4}x+5$$

$$\rightarrow\ m=\frac{4}{3}$$

$$y+2=\frac{4}{3}(x-3)$$

$$\boxed{y=\frac{4}{3}x-6}$$

Exercise

Write a slope-intercept equation for a line passing through the point $(6,-3)$ that is parallel to the line $8x+5y=3$

Solution

$$8x+5y=3 \implies 5y=-8x+3$$

$$y=-\frac{8}{5}x+\frac{3}{5} \rightarrow m=-\frac{8}{5}$$

Parallel $\boxed{m=-\frac{8}{5}}$

$$y=m\left(x-x_1\right)+y_1$$

$$y=-\frac{8}{5}(x-6)+(-3)$$

$$=-\frac{8}{5}x+\frac{48}{5}-3$$

$$=-\frac{8}{5}x+\frac{48-15}{5}$$

$$\boxed{y=-\frac{8}{5}x+\frac{33}{5}}$$

Exercise

Write a slope-intercept equation for a line passing through the point $(6,-3)$ that is perpendicular to the line $8x+5y=3$

Solution

$$8x+5y=3 \implies 5y=-8x+3$$

$$y=-\frac{8}{5}x+\frac{3}{5} \rightarrow m=-\frac{8}{5}$$

Perpendicular $\boxed{m=\frac{5}{8}}$

$$y=m\left(x-x_1\right)+y_1$$

$$y=\frac{5}{8}(x-6)-3$$

$$=\frac{5}{8}x-\frac{15}{4}-3$$

$$\boxed{y=\frac{5}{8}x-\frac{27}{4}}$$

Solution *Section* **1.2 - Linear Equations and Rational Equations**

Exercise

Solve: $\frac{1}{14}(3x-2)=\frac{x+10}{10}$

Solution

$$\frac{1}{14}(3x-2)=\frac{x+10}{10}$$

$$(70)\frac{1}{14}(3x-2)=(70)\frac{x+10}{10}$$

$$5(3x-2)=7(x+10)$$

$$15x-10=7x+70$$

$$15x-7x=10+70$$

$$8x=80$$

$$\boxed{x=10}$$

Exercise

Solve: $4(x+7)=2(x+12)+2(x+1)$

Solution

$$4x+28=2x+24+2x+2$$

$$4x+28=4x+26$$

$$4x-4x=26-28$$

$$0=-2 \quad \textit{(False)}$$

Solution: $\{\varnothing\}$

Exercise

Solve: $2x-\left\{x-\left[3x-(8x+6)\right]\right\}=2x-2$

Solution

$2x-x+\left[3x-(8x+6)\right]=2x-2$ — ***Distribute the minus***

$2x-x+3x-(8x+6)=2x-2$ — ***Distribute the plus***

$2x-x+3x-8x-6=2x-2$ — ***Distribute the minus***

$-4x-6=2x-2$

$-4x-2x-6+6=2x-2x-2+6$

$-6x=4$ — ***Divide by –6***

$$\boxed{x=\frac{4}{-6}=-\frac{2}{3}}$$

Exercise

Solve: $45-[4-2y-4(y+7)]=-4(1+3y)-[4-3(y+2)-2(2y-5)]$

Solution

$$45-[4-2y-4y-28]=-4-12y-[4-3y-6-4y+10]$$
$$45-4+2y+4y+28=-4-12y-4+3y+6+4y-10$$
$$69+6y=-12-5y$$
$$6y+5y=-12-69$$
$$11y=-81$$
$$\boxed{y=-\frac{81}{11}}$$

Exercise

Solve: $\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$

Solution

$$(28)\frac{x-3}{4}=(28)\frac{5}{14}-(28)\frac{x+5}{7} \qquad LCD: 4 \quad 14 \quad 7 \to 28$$

$7(x-3) = 2(5) - 4(x + 5)$

$7x - 21 = 10 - 4x - 20$

$7x + 4x = 21 - 10$

$11x = 11$

$\Rightarrow x = 1$

Exercise

Solve: $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

Solution

$4 \quad 6 \quad 3 \quad \to 12(\textit{common denominator})$

$$12\frac{x+1}{4}=12\frac{1}{6}+12\frac{2-x}{3}$$

$$3(x+1)=2+4(2-x)$$
$$3x+3=2+8-4x$$
$$3x+4x=2+8-3$$
$$7x=7$$
$$\boxed{x=1}$$

Exercise

Solve $\frac{x-8}{3}+\frac{x-3}{2}=0$

Solution

$$(6)\frac{x-8}{3}+(6)\frac{x-3}{2}=0(6) \qquad \textit{LCD: 2, 3: } \mathbf{6}$$

$$2(x-8)+3(x-3)=0$$

$$2x-16+3x-9=0$$

$$5x-25=0$$

$$5x=25$$

$$\boxed{x=\frac{25}{5}=5}$$

Exercise

Solve: $\frac{5}{2x}-\frac{8}{9}=\frac{1}{18}-\frac{1}{3x}$

Solution

Restriction: $x \neq 0$

$$18x\frac{5}{2x}-18x\frac{8}{9}=18x\frac{1}{18}-18x\frac{1}{3x} \qquad 2x \quad 9 \quad 18 \quad 3x \quad \to 18x$$

$$9(5)-2x(8)=x-6$$

$$45-16x=x-6$$

$$45+6=x+16x$$

$$51=17x$$

$$\boxed{x=\frac{51}{17}=3}$$

Exercise

Solve: $\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{x^2-16}$

Solution

Restrictions: $x \neq \pm 4$

$$(x+4)(x-4)\frac{1}{x+4}+(x+4)(x-4)\frac{1}{x-4}=(x+4)(x-4)\frac{22}{x^2-16}$$

$$x-4+x+4=22$$

$$2x=22$$

$$\Rightarrow x=11$$

Exercise

Solve: $\frac{3x-1}{3}-\frac{2x}{x-1}=x$

Solution

Condition (Restriction): $x-1\neq 0 \Rightarrow x\neq 1$

$$3(x-1)\frac{3x-1}{3}-3(x-1)\frac{2x}{x-1}=3(x-1)x$$

$$3x^2-x-3x+1-6x=3x^2-3x$$

$$3x^2-x-3x+1-6x-3x^2+3x=0$$

$$-7x+1=0$$

$$-7x=-1$$

$$\boxed{x=\frac{1}{7}}$$

Exercise

Solve: $\frac{x}{x-2}=\frac{2}{x-2}+2$

Solution

Restriction: $x-2\neq 0 \Rightarrow x\neq 2$

$$(x-2)\frac{x}{x-2}=(x-2)\frac{2}{x-2}+2(x-2)$$

$$x=2+2x-4$$

$$-x=-2$$

$x=2$ $\Rightarrow$ No *Solution* or $\{\varnothing\}$ because of the restriction.

Exercise

Solve the equation $\frac{x}{x-7}=\frac{7}{x-7}+8$

Solution

Restriction: $x-7\neq 0 \;\Rightarrow\; \boxed{x\neq 7}$

$$(x-7)\frac{x}{x-7}=(x-7)\frac{7}{x-7}+8(x-7)$$

$$x=7+8x-56$$

$$x-8x=-49$$

$$-7x=-49$$

$$x=\frac{-49}{-7}=7$$

But $x\neq 7$ (restriction), therefore there is ***no solution***

Exercise

Solve: $\frac{3x+2}{x-2}+\frac{1}{x}=\frac{-2}{x^2-2x}$

Solution

Restriction: $\begin{cases} x-2\neq 0 \Rightarrow x\neq 2 \\ \qquad\qquad\quad x\neq 0 \end{cases}$

$$x(x-2)\frac{3x+2}{x-2}+x(x-2)\frac{1}{x}=x(x-2)\frac{-2}{x^2-2x}$$

$$3x^2+2x+x-2=-2$$

$$3x^2+3x=0$$

$$3x(x+1)=0$$

$3x=0 \qquad\qquad x+1=0$

$x=0 \qquad\qquad x=-1$

$\boxed{x=-1}$ is the only solution

Exercise

Solve: $\frac{-4x}{x-1}+\frac{4}{x+1}=\frac{-8}{x^2-1}$

Solution

Restriction: $x\neq \pm 1$

$$(x-1)(x+1)\frac{-4x}{x-1}+(x-1)(x+1)\frac{4}{x+1}=(x-1)(x+1)\frac{-8}{x^2-1}$$

$$-4x(x+1)+4(x-1)=-8$$

$$-4x^2-4x+4x-4=-8$$

$$-4x^2=-4$$

$$\frac{-4}{-4}x^2=\frac{-4}{-4}$$

$$x^2=1$$

$x=\pm 1$ The solution is $\{\varnothing\}$

Exercise

Solve: $\dfrac{4x+3}{x+1}+\dfrac{2}{x}=\dfrac{1}{x^2+x}$

Solution

Restriction: $x+1\neq 0 \rightarrow \boxed{x\neq -1}\quad \boxed{x\neq 0}$

$$x(x+1)\frac{4x+3}{x+1}+x(x+1)\frac{2}{x}=x(x+1)\frac{1}{x^2+x}$$

$$x(4x+3)+2(x+1)=1$$

$$4x^2+3x+2x+2=1$$

$$4x^2+5x+2-1=1-1$$

$$4x^2+5x+1=0$$

$$(4x+1)(x+1)=0$$

$$4x+1=0 \qquad x+1=0$$

$$\boxed{x=-\frac{1}{4}} \qquad \boxed{x=-1}$$

Exercise

Solve: $\dfrac{6}{x+3}-\dfrac{5}{x-2}=\dfrac{-20}{x^2+x-6}$

Solution

$$\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{x^2+x-6}$$

Restriction: $x\neq -3,\ 2$

$$(x+3)(x-2)\frac{6}{x+3}-(x+3)(x-2)\frac{5}{x-2}=(x+3)(x-2)\frac{-20}{x^2+x-6}$$

$$6(x-2)-5(x+3)=-20$$

$$6x-12-5x-15=-20$$

$$x=-20+12+15$$

$$\boxed{x=7}$$

Exercise

Solve: $A=\frac{1}{2}h\left(B+b\right)$ *for* B

Solution

$2A=h\left(B+b\right)$ *Multiply both sides by 2*

$\dfrac{2A}{h}=B+b$ *Divide both sides by h*

$$\frac{2A}{h}-b=B$$

Exercise

Solve: $A = \frac{1}{2}h(a+b)$ for a

Solution

$$2A = 2\frac{1}{2}h(a+b)$$

$$2A = h(a+b)$$

$$\frac{2A}{h} = \frac{h}{h}(a+b)$$

$$\frac{2A}{h} = a+b$$

$$\frac{2A}{h} - b = a$$

$$a = \frac{2A}{h} - b \qquad or \qquad a = \frac{2A-bh}{h}$$

Exercise

Solve: $A = \frac{1}{2}h\left(b_1 + b_2\right)$, *for* h

Solution

$$2A = h\left(b_1 + b_2\right)$$

$$\boxed{\frac{2A}{b_1+b_2} = h}$$

Exercise

Solve: $A = \frac{1}{2}h\left(b_1 + b_2\right)$, *for* b_2

Solution

$$2A = h(b_1 + b_2)$$

$$\frac{2A}{h} = b_1 + b_2$$

$$\frac{2A}{h} - b_1 = b_2$$

Exercise

Solve: $S = P + \Pr t$ *for t.*

Solution

$S - P = \Pr t$

$\frac{S-P}{\Pr} = \frac{\Pr}{\Pr}t$

$\boxed{\frac{S-P}{\Pr}} = t$

Exercise

Solve: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ *for* R_1

Solution

1st Method

	Multiply by the common denominator RR_1R_2
$RR_1R_2\frac{1}{R} = RR_1R_2\frac{1}{R_1} + RR_1R_2\frac{1}{R_2}$	***Simplify***
$R_1R_2 = RR_2 + RR_1$	***Move*** R_1 ***to one side***
$R_1R_2 - RR_1 = RR_2$	***Factor*** R_1
$R_1\left(R_2 - R\right) = RR_2$	***Divide by*** $R_2 - R$
$R_1 = \frac{RR_2}{R_2 - R}$	

2nd Method

$\frac{1}{R} - \frac{1}{R_2} = \frac{1}{R_1}$	***Common denominator on one side of the equality***
$\frac{R_2 - R}{RR_2} = \frac{1}{R_1}$	***Cross multiplication***
$R_1R_2 - RR_1 = RR_2$	***Factor*** R_1
$R_1\left(R_2 - R\right) = RR_2$	***Divide by*** $R_2 - R$
$R_1 = \frac{RR_2}{R_2 - R}$	

3rd Method

$\frac{1}{R} - \frac{1}{R_2} = \frac{1}{R_1}$	
$\frac{R_2 - R}{RR_2} = \frac{1}{R_1}$	***Cross multiplication***
$\frac{RR_2}{R_2 - R} = R_1$	***Flip***

Exercise

Solve the formula for the indicated variable $S = 3\pi rk + 3\pi r^2 \quad for \quad k$

Solution

$$S = 3\pi rk + 3\pi r^2$$

$$S - 3\pi r^2 = 3\pi rk + 3\pi r^2 - 3\pi r^2$$

$$S - 3\pi r^2 = 3\pi rk$$

$$\frac{S - 3\pi r^2}{3\pi r} = \frac{3\pi r}{3\pi r} k$$

$$\boxed{\frac{S - 3\pi r^2}{3\pi r} = k}$$

Exercise

A sewage treatment plant has two inlet pipes to its settling pond. One can fill the pond in 10 hr, the other in 12 hr. If the first pipe is open for 5 hr and then the second pipe us opened, how long will it take to fill the pond?

Solution

	Rate	**Time**	**Job:** $A = rt$
One	$\frac{1}{10}$	x	$\frac{x}{10}$
Other	$\frac{1}{12}$	x - 5	$\frac{x-5}{12}$

$$\frac{x}{10} + \frac{x-5}{12} = 1$$

$$(60)\frac{x}{10} + (60)\frac{x-5}{12} = 1(60)$$

$$6x + 5(x-5) = 60$$

$$6x + 5x - 25 = 60$$

$$11x = 85$$

$$x = \frac{85}{11} \approx 7.3$$

The other $\approx 7.3 - 5 \approx 2.3$

Solution   Section 1.3 - Applications

Exercise

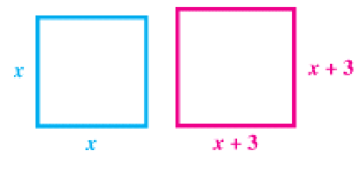
If the length of each side of a square is increased by 3 cm, the perimeter of the new square is 40 cm more than twice of each side of the original square. Find the dimensions of the origin square.

Solution

New perimeter: $P = 4(x+3)$

New square is 40 cm more than twice of each side of the original square.

$$P = 40 + 2x$$



$$40 + 2x = 4(x+3)$$
$$40 + 2x = 4x + 12$$
$$28 = 2x$$
$$14 = x$$

Exercise

When a number is decreased by 30% of itself, the result is 28. What is the number?

Solution

$$n - 0.3n = 28$$
$$0.7n = 28$$
$$n = \frac{28}{0.7}$$
$$= 40$$

Exercise

When 80% a number is added to the number, the result is 252. What is the number?

Solution

$$0.8n + n = 252$$
$$1.8n = 252$$
$$n = \frac{252}{1.8}$$
$$= 140$$

Exercise

The length of a rectangular label is 2.5cm less than twice the width. The perimeter is 40.6 cm. Find the width.

Solution

$l = 2w - 2.5$

$P = 2w + 2l$

$$\begin{aligned} 40.6 &= 2w + 2(2w - 2.5) \\ &= 2w + 4w - 5 \\ &= 6w - 5 \end{aligned}$$

$40.6 + 5 = 6w$

$6w = 45.6$

$w = \frac{45.6}{6} = 7.6\ cm$

Exercise

An Automobile repair shop charged a customer \$448, listing \$63 for parts and the remainder for labor. If the cost of labor is \$35 per hour, how many hours of labor did it take to repair the car?

Solution

$448 = 63 + 35x$

$448 - 63 = 35x$

$385 = 35x$

$x = \frac{385}{35} = 11$

Exercise

In the morning, Margaret drove to a business appointment at 50 *mph*. Her average speed on the return trip in the afternoon was 40 *mph*. The return trip took $\frac{1}{4}$ *hr* longer because of heavy traffic. How far did she travel to the appointment?

Solution

	r	***t***	***d***
Morning	*50*	x	50x
Afternoon	40	$x + \frac{1}{4}$	$40\left(x + \frac{1}{4}\right)$

The same distance

$50x = 40\left(x + \frac{1}{4}\right)$

$50x = 40x + 10$

$$10x = 10$$
$$x = 1\ hr$$

Distance = $50x$ = 50(1) = 50

Exercise

Latoya borrowed \$5240 for new furniture. She will pay it off in 11 months at an annual simple interest rate of 4.5%. How much interest will she pay?

Solution

Given: P = 5240

$$r = 4.5\% = \frac{4.5}{100} = 0.045$$
$$t = \frac{11}{12}$$

$$\begin{aligned} I &= \Pr t \\ &= (5240)(0.045)\left(\frac{11}{12}\right) \\ &= \$216.15 \end{aligned}$$

Exercise

One of the most effective ways of removing contaminants such as carbon monoxide and nitrogen dioxide from the air while cooking is to use a vented range hood. If a range hood removes contaminants at a rate of *F* liters of air per second, then the percent *P* of contaminants that are also removed from the surrounding air can be modeled by the linear equation

$$P = 1.06F + 7.18$$

Where $10 \le F \le 75$. What flow *F* must a range hood have to remove 50% of the contaminants from the air?

Solution

$$50 = 1.06F + 7.18$$

$$50 - 7.18 = 1.06F$$

$$42.82 = 1.06F$$

$$\begin{aligned} F &= \frac{42.82}{1.06} \\ &\approx 40.40 \end{aligned}$$

Exercise

Americans spent about $511 billion dining out in 2006. This was a 5.1% increase over the amount spent in 2005. How much was spent dining out in 2005?

Solution

Let x be the amount spent in 2005

511 is the amount spent in 2006

$5.1\ \% = \frac{5.1}{100} = 0.051$

$$x + .051x = 511$$

$$1.051x = 511$$

$$x = \frac{511}{1.051} = \$486 \text{ billion}$$

Exercise

For households with at least one credit card, the average U.S. credit-card debt per household was $9312 in 2004. This was $6346 more than the average credit-card debt in 1990. What was the average credit-card debt per household in 1990?

Solution

$1990: \ x$

$2004: \ y = 9312$

$y = x + 6346$

$9312 - 6346 = x$

$2966 = x$

Exercise

Morgan's Seeds has a rectangular test plot with a perimeter of 322 m. The length is 25 m more than the width. Find the dimensions of the plot?

Solution

$P = 322 = 2l + 2w$ *Divide by 2*

$l + w = 161$ **(1)**

$l = w + 25$ **(2)**

From (2) $\rightarrow$ (1) $\Rightarrow w + 25 + w = 161$

$2w = 161 - 25$

$w = \frac{136}{2} = 68m$

$l = w + 25 = 68 + 25 = 93m$

Exercise

Together, a dog owner and a cat owner spend an average of $376 annually for veterinary-related expenses. A dog owner spends $150 more per year than a cat owner. Find the average annual veterinary-related expenses of a dog owner and of a cat owner.

Solution

A dog owner and a cat owner spend an average of $376

$d + c = 376 \qquad (1)$

A dog owner spends $150 more per year than a cat owner

$d = c + 150 \qquad (2)$

From (2); (1) $d + c = 376$

$$\Rightarrow c + 150 + c = 376$$

$$\Rightarrow 2c = 376 - 150$$

$$\Rightarrow 2c = 226$$

$$\Rightarrow c = \frac{226}{2} = \$113$$

(2) $d = c + 150$

$= 113 + 150$

$= \$263$

Exercise

America West Airlines fleet includes Boeing, each with a cruising speed of 500 mph, and Bombardier Dash each with a cruising speed of 302 mph. Suppose that a Dash takes off and travels at its cruising speed. One hour later, a Boeing takes off and follows the same route, traveling at its cruising speed. How long will it take the Boeing to overtake the Dash?

Solution

	Distance	Rate	Time
Boeing	d	500	t
Dash	d	302	$t + 1$

$d = 500t$

$d = 302(t + 1)$

$$500t = 302t + 302$$

$$500t - 302t = 302$$

$$198t = 302$$

$$t = \frac{302}{198} \approx 1.53$$

Exercise

Jared's two student loans total $12,000. One loan is at 5% simple interest and the other is at 8% simple interest. After 1 yr. Jared owes $750 in interest. What is the amount of each loan?

Solution

	Amount Borrowed	*Interest Rate*	*Time*	*Amount of Interest*
5%	x	0.05	1	$0.05\,x$
8%	$12{,}000 - x$	0.08	1	$0.08(12000 - x)$
Total	12,000			750

$0.05\,x + 0.08(12000 - x) = 750$

$0.05x + 960 - 0.08\,x = 750$

$-\,0.03x = 750 - 960$

$-\,0.03x = -\,210$

$x = \frac{-210}{-0.03} = 7000$

5% loan: $7000

8% loan: 12000-7000= $5000

Exercise

Cody wishes to sell a piece of property for $240,000. He wants the money to be paid off in two ways – a short-term note at 6% interest and a long-term note at 5%. Find the amount of each note if the total annual interest paid is $13,000.

Solution

Amount	*Rate*	*Year*	*I = Prt*
x	.06	1	$.06\,x$
$240{,}000 - x$	.05	1	$.05(240{,}000 - x)$
240,000			$13,000

$.06x + .05(240,000 - x) = 13000$

$.06x + 12000 - .05x = 13000$

$.01x = 1000$

$x = \frac{1000}{.01} = \$100,000$

Cody should invest $100,000 at 6% and $2410,000 - $100,000 = $140,000 for 5%

Exercise

An artist has sold a painting for $410,000. He needs some of the money in 6 months and the rest in 1 yr. He can get a treasury bond for 6 months at 4.65% and for one year at 4.91%. His broker tells him the two investments will earn a total of $14,961. How much should be invested at each rate to obtain that amount of interest?

Solution

Amount	***Rate***	***Year***	***I = Prt***
x	.0465	0.5	$.0465(0.5)\ x$
$410,000 - x$	.0491	1	$.0491(1)\ (410,000 - x)$
410,000			$14,961

$.0465(0.5)\ \boldsymbol{x} + .0491(1)\ (410,000 - \boldsymbol{x}) = 14,961$

$.02325\ \boldsymbol{x} + 20,131 - 0.0491\ \boldsymbol{x} = 14,961$

$-\ 0.025851\ \boldsymbol{x} = 14,961 - 20,131$

$-\ 0.025851\ \boldsymbol{x} = -\ 5170$

$\boldsymbol{x} = \$200,000$

The artist should invest $200,000 at 4.65% and $410,000 - $200,000 = $210,000 for 4.91%

Exercise

The number of steps needed to burn off a Cheeseburger exceeds the number needed to burn off a 12-ounce Soda by 4140. The number needed to burn off a Doughnut exceeds the number needed to burn off 12 ounce soda by 2300. If you chow down a cheeseburger, doughnut, and 12-ounce soda, a 16790 step walk is needed to burn off the calories (and perhaps alleviate the guilt).Determine the number of steps it takes to burn off a cheeseburger, a doughnut, and a 12-ounce soda.

Solution

$C = S + 4140$

$D = S + 2300$

$C + D + S = 16790$

$S + 4140 + S + 2300 + S = 16790$

$3S = 16790 - 4140 - 2300$

$3S = 10350$

$S = 3450$

$C = S + 4140 = 7590$

$D = S + 2300 = 5750$

Exercise

Although organic milk accounts for only 12% of the market, consumption is increasing. In 2004, Americans purchased 40.7 million gallons of organic milk, increasing at a rate of 5.6 million gallons per year. If this trend continues, when will Americans purchase 79.9 million gallons of organic milk?

Solution

$79.9 = 40.7 + 5.6x$

$5.6x = 39.2$

$x = 7$ years

$\Rightarrow$ Year = 2004 + 7 = 2011

Exercise

How many gallons of a 5% acid solution must be fixed with 5 gal of a 10% solution to obtain 7% solution?

Solution

Strength	***Liters of Solution***	***Liters of Pure Alcohol***
5%	x	$.05\,x$
10%	5	.10 (5)
7%	$x + 5$	$.07(x + 5)$

$$\begin{aligned} &.05x + .5 = .07(x+5) \\ &.05x + .5 = .07x + .35 \\ &.05x - .07x = .35 - .5 \\ &-.02x = -.15 \\ &x = \frac{-.15}{-.02} = 7.5\ L \end{aligned}$$

Exercise

In 1969, 88% of the women considered this objective essential or very important. Since then, this percentage has decreased by approximately 1.1 each year. If this trend continues, by which year will only 33% of female freshmen consider "developing a meaningful philosophy of life" essential or very important?

Solution

$$\begin{aligned} &88 - 1.1x = 33 \\ &88 - 1.1x - 88 = 33 - 88 \\ &-1.1x = -55 \\ &x = \frac{-55}{-1.1} \\ &x = 50\ years \quad \rightarrow year = 1969 + 50 = 2019 \end{aligned}$$

Exercise

Charlotte is a chemist. She needs a 20% solution of alcohol. She has a 15% solution on hand, as well as a 30% solution. How many liters of the 15% solution should she add to 3 L of the 30% solution to obtain her 20% solution?

Solution

Strength	***Liters of Solution***	***Liters of Pure Alcohol***
15%	x	$.15\,x$
30%	3	$.30\,(3)$
20%	$3 + x$	$.20(3 + x)$

Liters in 15%	+	Liters in 30%	=	Liters in 20%
$.15x$	+	$.30(3)$	=	$.20(3 + x)$

$$.15x + .9 = .6 + .2x$$

$$.15x - .2x = .6 - .9$$

$$-.05x = -.3$$

$$x = \frac{-.3}{-.05} = 6$$

Solution   *Section* 1.4 - Quadratic Equations

Exercise

Solve: $x^2 = -25$

Solution

$$x = \pm\sqrt{-25}$$
$$= \pm 5i$$

Exercise

Solve: $(x-4)^2 = 12$

Solution

$$x - 4 = \pm\sqrt{12}$$
$$x = 4 \pm \sqrt{12} \qquad \sqrt{12} = \sqrt{4(3)} = 2\sqrt{3}$$
$$x = 4 \pm 2\sqrt{3}$$

Exercise

Solve: $5x^2 - 45 = 0$

Solution

$$5x^2 = 45$$
$$x = \frac{45}{5}$$
$$x^2 = 9$$
$$\Rightarrow x = \pm 3$$

Exercise

Solve: $(4x+1)^2 = 20$

Solution

$$4x + 1 = \pm\sqrt{20}$$
$$4x = -1 \pm 2\sqrt{5}$$
$$x = \frac{-1 \pm 2\sqrt{5}}{4}$$

Exercise

Solve $x^2 - 6x = -7$

Solution

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)} \\ &= \frac{6 \pm \sqrt{8}}{2} \\ &= \frac{6 \pm 2\sqrt{2}}{2} \\ &= \frac{2\left(3 \pm \sqrt{2}\right)}{2} \\ &= 3 \pm \sqrt{2} \end{aligned}$$

Exercise

Solve $-6x^2 = 3x + 2$

Solution

$$-6x^2 - 3x - 2 = 0$$

$$6x^2 + 3x + 2 = 0$$

$$\begin{aligned} x &= \frac{-3 \pm \sqrt{3^2 - 4(6)(2)}}{2(6)} \\ &= \frac{-3 \pm \sqrt{-39}}{12} \\ &= \frac{-3}{12} \pm i\frac{\sqrt{39}}{12} \\ &= -\frac{1}{4} \pm i\frac{\sqrt{39}}{12} \end{aligned}$$

Exercise

Solve: $3x^2 + 2x = 7$

Solution

$3x^2 + 2x - 7 = 0 \qquad \Rightarrow \ a = 3, \ b = 2, \ \mathbf{c = -7}$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-7)}}{2(3)} \end{aligned}$$

$$= \frac{-2 \pm \sqrt{88}}{6}$$

$$= \frac{-2 \pm \sqrt{4(22)}}{6}$$

$$= \frac{-2 \pm 2\sqrt{22}}{6}$$

$$= \frac{2\left(-1 \pm \sqrt{22}\right)}{6}$$

$$= \frac{-1 \pm \sqrt{22}}{3} \qquad x = -\frac{1}{3} \pm \frac{\sqrt{22}}{3}$$

Exercise

$3x^2 + 6 = 10x$

Solution

$$3x^2 - 10x + 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(6)}}{2(3)}$$

$$= \frac{10 \pm \sqrt{100 - 72}}{6}$$

$$= \frac{10}{6} \pm \frac{\sqrt{28}}{6}$$

$$= \frac{5}{3} \pm \frac{2\sqrt{7}}{6}$$

$$= \frac{5}{3} \pm \frac{\sqrt{7}}{3}$$

Exercise

$5x^2 + 2 = x$

Solution

$$5x^2 - x + 2 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(2)}}{2(5)}$$

$$= \frac{1 \pm \sqrt{1-40}}{10}$$

$$= \frac{1 \pm \sqrt{-39}}{10}$$

$$= \frac{1 \pm i\sqrt{39}}{10}$$

$$= \frac{1}{10} \pm i\frac{\sqrt{39}}{10}$$

Exercise

Solve: $5x^2 = 2x - 3$

Solution

$$5x^2 - 2x + 3 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(3)}}{2(5)}$$

$$= \frac{2 \pm \sqrt{4-60}}{10}$$

$$= \frac{2 \pm \sqrt{-56}}{10}$$

$$= \frac{2 \pm i\sqrt{4(14)}}{10}$$

$$= \frac{2 \pm i2\sqrt{14}}{10}$$

$$= \frac{2}{10} \pm i\frac{2\sqrt{14}}{10}$$

$$= \frac{1}{5} \pm i\frac{\sqrt{14}}{5}$$

Exercise

Solve: $x^2 + 8x + 15 = 0$

Solution

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{64 - 60}}{2}$$

$$= \frac{-8 \pm \sqrt{4}}{2}$$

$$= \frac{-8 \pm 2}{2}$$

$$= \begin{cases} \frac{-8+2}{2} = \frac{-6}{2} = -3 \\ \frac{-8-2}{2} = \frac{-10}{2} = -5 \end{cases}$$

Exercise

Solve: $x^2 + 5x + 2 = 0$

Solution

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$= \frac{-5 \pm \sqrt{17}}{2}$$

$$= \frac{-5}{2} \pm \frac{\sqrt{17}}{2}$$

Exercise

Solve: $x^2 - 6x - 10 = 0$

Solution

$$x^2 - 6x = 10$$

$$x^2 - 6x + \left(\frac{-6}{2}\right)^2 = 10 + \left(\frac{-6}{2}\right)^2$$

$$x^2 - 2(3)x + (3)^2 = 10 + 9$$

$$(x-3)^2 = 19$$

$$x - 3 = \pm\sqrt{19}$$

$$x = 3 \pm \sqrt{19}$$

Exercise

Solve: $2x^2 + 3x - 4 = 0$

Solution

$$\Rightarrow 2x^2 + 3x = 4$$

$$x^2 + \frac{3}{2}x \quad = 2$$

$$x^2 + \frac{3}{2}x + \left(\frac{1}{2}\frac{3}{2}\right)^2 = 2 + \left(\frac{1}{2}\frac{3}{2}\right)^2$$

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = 2 + \frac{9}{16}$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{41}{16}$$

$$x + \frac{3}{4} = \pm\sqrt{\frac{41}{16}}$$

$$x = -\frac{3}{4} \pm \frac{\sqrt{41}}{4}$$

Exercise

Solve $x^2 - x + 8 = 0$

Solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(8)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1-32}}{2}$$

$$= \frac{1 \pm \sqrt{-31}}{2}$$

$$\boxed{= \frac{1 \pm i\sqrt{31}}{2}}$$

Exercise

Solve $2x^2 - 13x = 1$

Solution

$$2x^2 - 13x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{13 \pm \sqrt{169+8}}{4}$$

$$\boxed{= \frac{13 \pm \sqrt{177}}{4}}$$

Exercise

Solve $r^2 + 3r - 3 = 0$

Solution

$$r = \frac{-3 \pm \sqrt{3^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9+12}}{2}$$

$$\boxed{= \frac{-3 \pm \sqrt{21}}{2}}$$

Exercise

Solve: $x^3 + 8 = 0$

Solution

$$(x+2)\left(x^2 - 2x + 4\right) = 0 \qquad\qquad a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

$$x + 2 = 0 \qquad\qquad x^2 - 2x + 4 = 0$$

$$x = -2 \qquad\qquad x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm 2i\sqrt{3}}{2}$$

$$= \frac{2\left(1 \pm i\sqrt{3}\right)}{2}$$

$$= 1 \pm i\sqrt{3}$$

The solution set is $\left\{-2,\ 1 \pm i\sqrt{3}\right\}$

Exercise

Solve for the specified variable $A = \frac{\pi d^2}{4}$, *for d*

Solution

$$\frac{4}{\pi} A = \frac{4}{\pi} \frac{\pi d^2}{4} \qquad\qquad 4A = 4\frac{\pi d^2}{4}$$

$$\frac{4A}{\pi} = d^2 \qquad\qquad 4A = \pi d^2$$

$$d^2 = \frac{4A}{\pi} \qquad\qquad \frac{4A}{\pi} = \frac{\pi}{\pi} d^2$$

$$d = \pm\sqrt{\frac{4A}{\pi}}$$

$$= \pm 2\frac{\sqrt{A}}{\sqrt{\pi}}$$

$$= \pm 2 \frac{\sqrt{A}}{\sqrt{\pi}} \frac{\sqrt{\pi}}{\sqrt{\pi}}$$

$$= \pm \frac{2\sqrt{\pi A}}{\pi}$$

Exercise

Solve for the specified variable $rt^2 - st - k = 0 \ \ (r \neq 0), \quad for\ t$

Solution

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

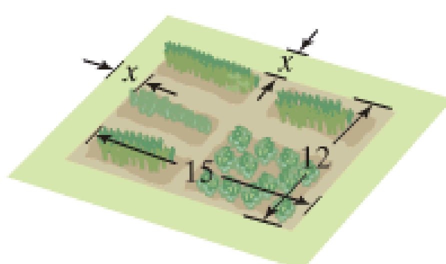
$$t = \frac{-(-s) \pm \sqrt{(-s)^2 - 4(r)(-k)}}{2(r)}$$

$$t = \frac{s \pm \sqrt{s^2 + 4rk}}{2r}$$

Exercise

A vacant rectangular lot is being turned into a community vegetable garden measuring 15 *meters* by 12 *meters*. A path of uniform width is to surround the garden. If the area of the garden and path combined is 378 *square meters*, find the width of the path.

Solution



$$Area = (15 + 2x)(12 + 2x)$$

$$378 = (15 + 2x)(12 + 2x)$$

$$378 = 180 + 30x + 24x + 4x^2$$

$$0 = 180 + 54x + 4x^2 - 378$$

$$0 = 4x^2 + 54x - 198$$

$$4x^2 + 54x - 198 = 0$$

$$x = \frac{-(54) \pm \sqrt{(54)^2 - 4(4)(-198)}}{2(4)}$$

$$= \frac{-54 \pm \sqrt{6084}}{8}$$

$$= \frac{-54 \pm 78}{8} \rightarrow \begin{cases} x = \dfrac{-54+78}{8} = 3 \\ x = \dfrac{-54-78}{8} = -16.5 \end{cases}$$

Exercise

A rectangular park is 6 *miles* long and 2 *miles* wide. How long is a pedestrian route that runs diagonally across the park?

Solution

$$d^2 = 6^2 + 2^2$$

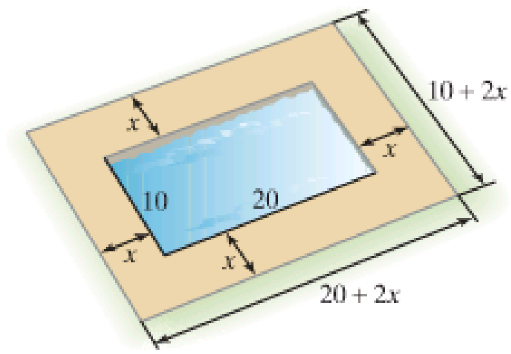
$$d^2 = 40$$

$$d = \sqrt{40} \approx 6.32 \ miles$$

Exercise

A pool measuring 10 m by 20 m is surrounded by a path of uniform width. If the area of the pool and the path combined is 600 m^2, what is the width of the path?

Solution



$$A = lw$$

$$600 = (20+2x)(10+2x)$$

$$600 = 200 + 40x + 20x + 4x^2$$

$$0 = -600 + 200 + 60x + 4x^2$$

$$0 = -400 + 60x + 4x^2$$

$$0 = -100 + 15x + x^2$$

$$x^2 + 15x - 100 = 0$$

$$x = \frac{-15 \pm \sqrt{15^2 - 4(1)(-100)}}{2(1)}$$

$$= \frac{-15 \pm \sqrt{625}}{2}$$

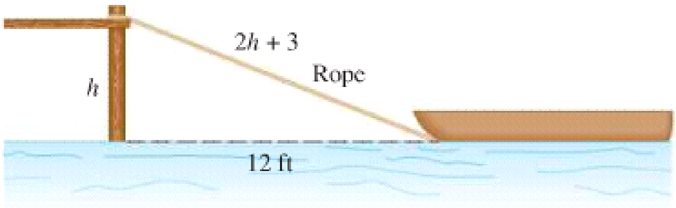
$$= \begin{cases} \frac{-15-25}{2} = -20 \\ \frac{-15+25}{2} = 5 \end{cases}$$

The width of the path is 5 *m*.

Exercise

A boat is being pulled into a dock with a rope attached to the boat at water level. Where the boat is 12 ft from the dock, the length of the rope from the boat to the dock is 3 ft longer than twice the height of the dock above the water. Find the height of the dock.

Solution



$$(2h+3)^2 = h^2 + 12^2$$

$$4h^2 + 12h + 9 = h^2 + 144$$

$$4h^2 + 12h + 9 - h^2 - 144 = 0$$

$$3h^2 + 12h - 135 = 0$$

$$h^2 + 4h - 45 = 0$$

$$(h+9)(h-5) = 0$$

$$h = -9, \quad 5$$

Height = 5 *ft*.

Exercise

What is the width of a 25-*inch* television set whose height is 15 *inches*?

Solution

$$w^2 + 15^2 = 25^2$$

$$\Rightarrow w^2 = 25^2 - 15^2$$

$$\Rightarrow w = \sqrt{25^2 - 15^2}$$

$$\Rightarrow w \approx 20 \;\; in$$

Exercise

Logan and Cassidy leave a campsite, Logan biking due north and Cassidy biking due east. Logan bikes 7 km/h slower than Cassidy. After 4 hr, they are 68 km apart. Find the speed of each bicyclist.

Solution

$$4r^2 + [4(r-7)]^2 = 68^2$$

$$16r^2 + 16(r^2 - 14r + 49) = 4624$$

$$16r^2 + 16r^2 - 224r + 784 = 4624$$

$$32r^2 - 224r + 784 - 4624 = 0$$

$$32r^2 - 224r - 3840 = 0$$

$$r^2 - 7r - 120 = 0$$

$$\rightarrow r = -8, \quad 15$$

$$\Rightarrow Cassidy's = 15\ km/h$$

$$\Rightarrow Logan's = 8\ km/h$$

Exercise

Towers are 1482 ft tall. How long would it take an object dropped from the top to reach the ground? Given $s = 16t^2$

Solution

$$1482 = 16t^2$$

$$\frac{1482}{16} = t^2$$

$$\Rightarrow \sqrt{\frac{1482}{16}} = t$$

$$\Rightarrow \boldsymbol{t} \approx 9.624 \text{ sec}$$

Exercise

The formula $P = 0.01A^2 + 0.05A + 107$ models a woman's normal Point systolic blood pressure, P, an age A. Use this formula to find the age, to the nearest year, of a woman whose normal systolic blood pressure is 115 mm Hg.

Solution

$$0.01A^2 + 0.05A + 107 = 115$$

$$\Rightarrow 0.01A^2 + 0.05A - 8 = 0$$

$$A = \frac{-.05 \pm \sqrt{.05^2 - 4(.01)(-8)}}{2(.01)}$$

$$= \frac{-.05 \pm \sqrt{.0025 + .32}}{.02}$$

$$= \frac{-.05 \pm .567}{.02}$$

$$= \begin{cases} \frac{-.05 - .567}{.02} = -31 \ (\text{Not a Solution}) \\ \frac{-.05 + .567}{.02} = 25.89 \approx 26 \end{cases}$$

Exercise

A rectangular piece of metal is 10 in. longer than it is wide. Squares with sides 2 in. long are cut from the four corners, and the flaps folded upward to form an open box. If the volume of the box is 832 in^3, what were the original dimensions of the piece of metal?

Solution

$l = w + 10$

Bottom width: $w - 4$

Bottom length: $l - 4 = w + 10 - 4 = w + 6$

$$V = lwh = (w+6)(w-4)2$$

$$= 2(w^2 - 4w + 6w - 24)$$

$$= 2w^2 + 4w - 48$$

$$2w^2 + 4w - 48 = 832$$

$$2w^2 + 4w - 880 = 0$$

$$w^2 + 2w - 440 = 0$$

$$(w+22)(w-20) = 0$$

$$w + 22 = 0 \qquad w - 20 = 0$$

$$w = -22 \qquad w = 20$$

Width of the metal is 20 in by the length (20+10) 30 in.

Exercise

An astronaut on the moon throws a baseball upward. The astronaut is 6 ft., 6 in., tall, and the initial velocity of the ball is 30 ft. per sec. The height s of the ball in feet is given by the equation

$s = -2.7t^2 + 30t + 6.5$

Where ***t*** is the number of seconds after the ball was thrown.

a) After how many seconds is the ball 12 ft above the moon's surface?

b) How many seconds will it take for the ball to return to the surface?

Solution

a) After how many seconds is the ball 12 ft above the moon's surface?

$$12 = -2.7t^2 + 30t + 6.5$$

$$0 = -2.7t^2 + 30t + 6.5 - 12$$

$$0 = -2.7t^2 + 30t - 5.5$$

$$t = \frac{-30 \pm \sqrt{(30)^2 - 4(-2.7)(-5.5)}}{2(-2.7)} \approx \frac{-30 \pm 29}{-5.4}$$

$$t \approx \frac{-30-29}{-5.4} \qquad t \approx \frac{-30+29}{-5.4}$$

$$t \approx 10.9 \text{ sec} \qquad t \approx .12 \text{ sec}$$

b) How many seconds will it take for the ball to return to the surface?

$$0 = -2.7t^2 + 30t + 6.5$$

$$t = \frac{-30 \pm \sqrt{(30)^2 - 4(-2.7)(6.5)}}{2(-2.7)} \approx \frac{-30 \pm 31.15}{-5.4}$$

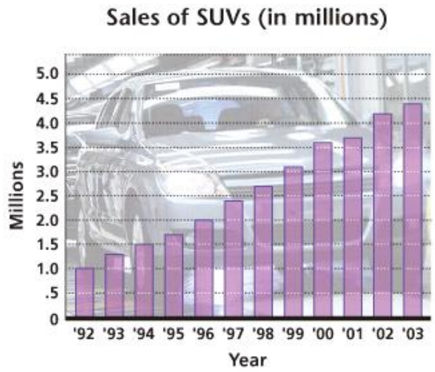
$$t \approx \frac{-30-31.15}{-5.4} \qquad t \approx \frac{-30+31.15}{-5.4}$$

$$t \approx 11.32 \qquad t \approx -0.212$$

It will take 11.32 sec.

Exercise

The bar graph shows of SUVs (sport utility vehicles0 in the US, in millions. The quadratic equation $S = .00579x^2 + .2579x + .9703$ models sales of SUVs from 1992 to 2003, where S represents sales in millions, and x = 0 represents 1992, x = 1 represents 1993 and so on.



a) Use the model to determine sales in 2002 and 2003. Compare the results to the actual figures of 4.2 million and 4.4 million from the graph.

b) According to the model, in what year do sales reach 3.5 million? Is the result accurate?

Solution

a) For 2002 $\Rightarrow x = 10$

$$S = .00579(10)^2 + .2579(10) + .9703$$
$$\approx 4.1 \ \ million$$

For 2003 $\Rightarrow x = 11$

$$S = .00579(11)^2 + .2579(11) + .9703$$
$$\approx 4.5 \ \ million$$

b) $3.5 = .00579x^2 + .2579x + .9703$

$$0 = .00579x^2 + .2579x + .9703 - 3.5$$
$$0 = .00579x^2 + .2579x - 2.5297$$

$$x = \frac{-.2579 \pm \sqrt{(.2579)^2 - 4(.00579)(-2.5297)}}{2(.00579)}$$

$$= \frac{-.2579 \pm \sqrt{.1251}}{.01158}$$

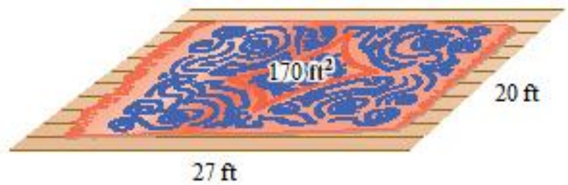
$$x = \frac{-.2579 - .3537}{.01158} \qquad x = \frac{-.2579 + .3537}{.01158}$$

$$x \approx -52.8 \qquad x \approx 8.3$$

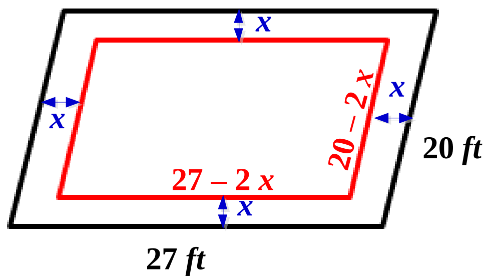
According to the model, the number reached 3.5 million in the year 2000. The model closely matches the graph, so it is accurate

Exercise

Cynthia wants to buy a rug for a room that is 20 ft. wide and 27 ft. long. She wants to leave a uniform strip of floor around the rug. She can afford to buy 170 square feet of carpeting. What dimension should the rug have?



Solution



The area of the rug is:

$$(27-2x)(20-2x)=170$$

$$540-54x-40x+4x^2=170$$

$$540-94x+4x^2-170=0$$

$$4x^2-94x+370=0 \qquad \textit{Solve for } x.$$

$$\cancel{x=18.5} \quad or \quad x=5$$

$$20-2x=20-2(5)=10 \quad and \quad 27-2x=27-2(5)=17$$

Therefore, the dimensions are: 10, 20 *ft*.

Exercise

Erik finds a piece of property in the shape of a right triangle. He finds that the longer leg is 20 m longer than twice the length of the shorter leg. The hypotenuse is 10 m longer than the length of the longer leg. Find the lengths of the sides of the triangular lot.

Solution

$l:$ *longer leg*

$s:$ *shorter leg*

Longer leg is 20 m longer than twice the length of the shorter leg

$$l=2s+20$$

The hypotenuse is 10 m longer than the length of the longer leg

$$\begin{aligned} h &= l+10 \\ &= 2s+20+10 \\ &= 2s+30 \end{aligned}$$

$$l^2 + s^2 = h^2$$
$$(2s+20)^2 + s^2 = (2s+30)^2$$
$$4s^2 + 80s + 400 + s^2 = 4s^2 + 120s + 900$$
$$4s^2 + 80s + 400 + s^2 - 4s^2 - 120s - 900 = 0$$
$$s^2 - 40s - 500 = 0$$
$$(s+10)(s-50) = 0$$
$$s+10=0 \qquad\qquad s-50=0$$
$$s=-10 \qquad\qquad s=50$$

The shorter length is 50 m.

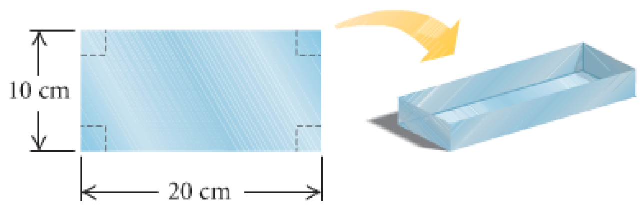
The longer length is $l = 2s+20 = 2(50)+20 = 120$

$h = l + 10 = 120 + 10 = 130\ m$

Exercise

An open box is made from a 10-cm by 20-cm piece of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2. What is the length of the sides of the squares?

Solution



Area of the base = $(10 - 2x)(20 - 2x)$

$$= 200 - 20x - 40x + 4x^2$$
$$= 4x^2 - 60x + 200$$

$$4x^2 - 60x + 200 = 96$$
$$4x^2 - 60x + 104 = 0$$
$$x^2 - 15x + 26 = 0$$
$$(x-13)(x-2) = 0$$

$$\begin{cases} x-13=0 \rightarrow x=13 \\ x-2=0 \rightarrow x=2 \end{cases} \quad \Rightarrow x = 2\ (only)$$

Solution Section 1.5 - Other Types of Equations

Exercise

Solve $3x^3 + 2x^2 = 12x + 8$

Solution

$$3x^3 + 2x^2 - (12x + 8) = 0$$

$$x^2(3x + 2) - 4(3x + 2) = 0$$

$$(3x + 2)\left(x^2 - 4\right) = 0$$

$3x + 2 = 0 \qquad x^2 - 4 = 0$

$3x = -2 \qquad x^2 = 4$

$x = -\frac{2}{3} \qquad x = \pm\sqrt{4} = \pm 2$

Exercise

Solve $x^4 + 3x^2 = 10$

Solution

$$x^4 + 3x^2 - 10 = 0$$

$$\left(x^2 + 5\right)\left(x^2 - 2\right) = 0$$

$x^2 + 5 = 0 \qquad x^2 - 2 = 0$

$x^2 = -5 \qquad x^2 = 2$

$x = \pm\sqrt{-5} \qquad x = \pm\sqrt{2}$

$x = \pm i\sqrt{5}$

Exercise

Solve: $5x^4 = 40x$

Solution

$$5x^4 - 40x = 0$$

$$5x\left(x^3 - 8\right) = 0$$

$$5x\left(x^3 - 2^3\right) = 0$$

$$5x(x-2)\left(x^2+2x+2^2\right)=0$$

$$5x(x-2)\left(x^2+2x+4\right)=0$$

$$\boxed{x=0} \qquad x-2=0 \qquad x^2+2x+4=0$$

$$\boxed{x=2} \qquad x=\frac{-2\pm\sqrt{2^2-4(1)(4)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{-12}}{2}$$

$$=\frac{-2\pm 2i\sqrt{3}}{2}$$

$$=\frac{2\left(-1\pm i\sqrt{3}\right)}{2}$$

$$\boxed{x=-1\pm i\sqrt{3}}$$

Exercise

Solve $9x^4-9x^2+2=0$

Solution

Assume $x^2=u \quad\Rightarrow\quad x^4=u^2$

Then we can rewrite the equation in a quadratic form: $9u^2-9u+2=0$

Solve for u using the quadratic formula.

$$u=\frac{-(-9)\pm\sqrt{(-9)^2-4(9)(2)}}{2(9)} \qquad\qquad \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{9\pm\sqrt{9}}{18}$$

$$=\frac{9\pm 3}{18}$$

$$\rightarrow\begin{cases} u=\frac{9-3}{18}=\frac{6}{18}=\frac{1}{3} \\ u=\frac{9+3}{18}=\frac{12}{18}=\frac{2}{3} \end{cases}$$

Since $u=x^2$

$$\begin{cases} x^2=u=\frac{1}{3} \quad\Rightarrow \boxed{x=\pm\frac{1}{\sqrt{3}}} \\ x^2=u=\frac{2}{3} \quad\Rightarrow \boxed{x=\pm\frac{\sqrt{2}}{\sqrt{3}}} \end{cases} \qquad \begin{cases} x=\pm\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}=\pm\frac{\sqrt{3}}{3} \\ x=\pm\frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}=\pm\frac{\sqrt{6}}{3} \end{cases}$$

Exercise

Solve: $x^4 + 720 = 89x^2$

Solution

$$x^4 - 89x^2 + 720 = 0$$

$$\left(x^2\right)^2 - 89\left(x^2\right)^1 + 720 = 0$$

Assume: $u = x^2$

$$u^2 - 89u + 720 = 0$$

$$u = \frac{-(-89) \pm \sqrt{(-89)^2 - 4(1)(720)}}{2(1)}$$

$$= \frac{89 \pm \sqrt{5041}}{2}$$

$$= \frac{89 \pm 71}{2}$$

$$\rightarrow \begin{cases} u = \frac{89 - 71}{2} = 9 & = x^2 \\ u = \frac{89 + 71}{2} = 80 & = x^2 \end{cases}$$

$$x^2 = 9 \quad \Rightarrow \quad \boxed{x = \pm 3}$$

$$x^2 = 80 \quad \Rightarrow x = \pm\sqrt{80} = \pm\sqrt{(16)(5)} \quad \boxed{x = \pm 4\sqrt{5}}$$

Exercise

Solve $\quad x - \sqrt{2x+3} = 0$

Solution

$$x = \sqrt{2x+3}$$

$$(x)^2 = \left(\sqrt{2x+3}\right)^2$$

$$x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x + 1 = 0 \qquad x - 3 = 0$$

$$x = -1 \qquad x = 3$$

Check

$x = -1$ $\qquad$ $x = 3$

$(-1) - \sqrt{2(-1)+3} = 0$ $\qquad$ $(3) - \sqrt{2(3)+3} = 0$

$-1-\sqrt{1}=0$ $\qquad\qquad$ $3-\sqrt{9}=0$

False $\qquad\qquad$ *True*

$x=3$ is the only solution

Exercise

Solve: $\sqrt{x+3}+3=x$

Solution

$$\sqrt{x+3}=x-3$$

$$\left(\sqrt{x+3}\right)^2=(x-3)^2$$

$$x+3=x^2-6x+9$$

$$x^2-7x+6=0$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(1)(6)}}{2(1)}$$

$$=\frac{7\pm\sqrt{49-24}}{2}$$

$$=\frac{7\pm\sqrt{25}}{2}$$

$$=\frac{7\pm 5}{2}$$

$$=\begin{cases}\frac{7+5}{2}=\frac{12}{2}=6\\ \frac{7-5}{2}=\frac{2}{2}=1\end{cases} \qquad \Rightarrow x=1,\ 6$$

Check:

$x=1 \quad \Rightarrow \sqrt{1+3}+3=1 \Rightarrow 5=1$ (*Not a solution*)

$x=6 \quad \Rightarrow \sqrt{6+3}+3=6 \Rightarrow 6=6 \rightarrow x=6$ is the only solution

Exercise

Solve $x-\sqrt{x+11}=1$

Solution

$$-\sqrt{x+11}=1-x$$

Square both side

$$\left(-\sqrt{x+11}\right)^2=(1-x)^2$$

$$(a-b)^2=a^2-2ab+b^2$$

$$x+11=1-2x+x^2$$

$$0=x^2-2x+1-x-11$$

$0 = x^2 - 3x - 10$

$x^2 - 3x - 10 = 0$ *Solve for* x

$x = 5, -2$

Check

$x = 5 \Rightarrow\ 5 - \sqrt{5+11} = 1\ \Rightarrow 5 - \sqrt{16} = 1\ \Rightarrow 5 - 4 = 1\ \Rightarrow 1 = 1$

$x = -2 \Rightarrow\ -2 - \sqrt{-2+11} = 1\ \Rightarrow -2 - \sqrt{9} = 1\ \Rightarrow -2 - 3 = 1\ \Rightarrow -5 = 1$ (*False*)

Solution: $x = 5$

Exercise

Solve: $\sqrt{2x-3} + \sqrt{x-2} = 1$

Solution

$$\sqrt{2x-3} = 1 - \sqrt{x-2}$$

$$\left(\sqrt{2x-3}\right)^2 = \left(1 - \sqrt{x-2}\right)^2$$

$$2x - 3 = 1 - 2\sqrt{x-2} + \left(\sqrt{x-2}\right)^2$$

$$2x - 3 - 1 = -2\sqrt{x-2} + x - 2$$

$$2x - 4 - x + 2 = -2\sqrt{x-2}$$

$$x - 2 = -2\sqrt{x-2}$$

$$(x-2)^2 = \left(-2\sqrt{x-2}\right)^2$$

$$x^2 - 4x + 4 = 4(x-2)$$

$$x^2 - 4x + 4 = 4x - 8$$

$$x^2 - 4x + 4 - 4x + 8 = 0$$

$$x^2 - 8x + 12 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$$

$x = 2, 6$

Check

$x = 2 \Rightarrow \sqrt{2(2)-3} + \sqrt{2-2} = 1 \Rightarrow 1 + 0 = 1$

$x = 6 \Rightarrow \sqrt{2(6)-3} + \sqrt{6-2} = 1 \Rightarrow 3 + 2 = 1 \Rightarrow 5 \neq 1$

Solution: $x = 2$

Exercise

Solve: $\sqrt{x+5}-\sqrt{x-3}=2$

Solution

$$\sqrt{x+5}=2+\sqrt{x-3}$$

$$\left(\sqrt{x+5}\right)^2=\left(2+\sqrt{x-3}\right)^2$$

$$x+5=4+4\sqrt{x-3}+\left(\sqrt{x-3}\right)^2$$

$$x+5=4+4\sqrt{x-3}+x-3$$

$$x-x+5-4-3=4\sqrt{x-3}$$

$$4=4\sqrt{x-3}$$

$$1=\sqrt{x-3}$$

$$1=x-3$$

$$\Rightarrow x=4$$

Check: $\sqrt{4+5}-\sqrt{4-3}=2$

$3-1=2$ (True statement)

$x=4$ is a solution

Exercise

Solve: $\sqrt{2x+3} = 1+\sqrt{x+1}$

Solution

$$\left(\sqrt{2x+3}\right)^2 = \left(1+\sqrt{x+1}\right)^2 \qquad (a+b)^2 = a^2+2ab+b^2$$

$$2x+3 = 1+2\sqrt{x+1}+x+1$$

$$2x+3 = 2\sqrt{x+1}+x+2$$

$$x+1 = 2\sqrt{x+1}$$

$$(x+1)^2 = \left(2\sqrt{x+1}\right)^2$$

$$x^2+2x+1 = 4(x+1)$$

$$x^2+2x+1 = 4x+4$$

$$x^2-2x-3 = 0$$

$$(x-3)(x+1) = 0$$

$$x-3=0 \qquad x+1=0$$

$$x=3 \qquad x=-1$$

Check

$x=3$

$\sqrt{2(3)+3} = 1+\sqrt{(3)+1}$

$\sqrt{9} = 1+\sqrt{4}$

$3=3$ $(true)$

$x=-1$

$\sqrt{2(-1)+3} = 1+\sqrt{(-1)+1}$

$\sqrt{1} = 1+\sqrt{0}$

$1=1$ $(true)$

$x=3$ *and* -1 are solutions

Exercise

Solve: $\sqrt{x+5}-\sqrt{x-3} = 2$

Solution

$$\sqrt{x+5} = 2+\sqrt{x-3}$$

$$\left(\sqrt{x+5}\right)^2 = \left(2+\sqrt{x-3}\right)^2$$

$$x+5 = 4+4\sqrt{x-3}+\left(\sqrt{x-3}\right)^2$$

$$x+5 = 4+4\sqrt{x-3}+x-3$$

$$x-x+5-4-3 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$1 = \sqrt{x-3}$

$1 = x - 3$

$\Rightarrow x = 4$

Check: $\sqrt{4+5} - \sqrt{4-3} = 2$

$3 - 1 = 2$ (True statement)

$x = 4$ is a solution

Exercise

Solve: $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$

Solution

$$\left(\sqrt[3]{4x^2 - 4x + 1}\right)^3 = \left(\sqrt[3]{x}\right)^3$$

$$4x^2 - 4x + 1 = x$$

$$4x^2 - 5x + 1 = 0$$

$$(4x-1)(x-1) = 0$$

$$4x - 1 = 0 \qquad x - 1 = 0$$

$$x = \frac{1}{4} \qquad x = 1$$

Check

$x = \frac{1}{4}$

$$\sqrt[3]{4\left(\frac{1}{4}\right)^2 - 4\frac{1}{4} + 1} - \sqrt[3]{\frac{1}{4}} = 0$$

$$\sqrt[3]{\frac{1}{4}} - \sqrt[3]{\frac{1}{4}} = 0$$

$0 = 0$ (*true*)

$x = 1$

$$\sqrt[3]{4(1)^2 - 4(1) + 1} - \sqrt[3]{1} = 0$$

$$\sqrt[3]{4 - 4 + 1} - \sqrt[3]{1} = 0$$

$0 = 0$ (*true*)

Solutions: $x = -1, \ \frac{1}{4}$

Exercise

Solve $12x^4 - 11x^2 + 2 = 0$

Solution

$$12\left(x^2\right)^2 - 11\left(x^2\right) + 2 = 0$$

let $u = x^2$

$$12u^2 - 11u + 2 = 0$$

$$u = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(12)(2)}}{2(12)}$$

$$= \frac{11 \pm \sqrt{25}}{24}$$

$$= \frac{11 \pm 5}{24}$$

$$x^2 = \frac{11+5}{24} = \frac{16}{24} = \frac{2}{3} \qquad x^2 = \frac{11-5}{24} = \frac{8}{24} = \frac{1}{4}$$

$$(3u - 2)(4u - 1) = 0$$

$$\left(3x^2 - 2\right)\left(4x^2 - 1\right) = 0$$

$3u - 2 = 0$ | $4u - 1 = 0$

$u = \frac{2}{3} = x^2$ | $u = \frac{1}{4} = x^2$

$x = \pm\frac{\sqrt{2}}{\sqrt{3}}$ | $x = \pm\sqrt{\frac{1}{4}}$

$x = \pm\frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$ | $x = \pm\frac{1}{2}$

$x = \pm\frac{\sqrt{6}}{3}$

The solution set is $\boxed{\left\{\pm\frac{\sqrt{6}}{3},\ \pm\frac{1}{2}\right\}}$

Exercise

Solve $2x^4 - 7x^2 + 5 = 0$

Solution

$$\left(2x^2 - 5\right)\left(x^2 - 1\right) = 0$$

$2x^2 - 5 = 0$ | $x^2 - 1 = 0$

$x^2 = \frac{5}{2}$ | $x^2 = 1$

$x = \pm\frac{\sqrt{5}}{\sqrt{2}}$ | $x = \pm 1$

$x = \pm\frac{\sqrt{5}}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}$

$$\boxed{x = \pm\frac{\sqrt{10}}{2}}$$

Exercise

Solve $x^4 - 5x^2 + 4 = 0$

Solution

$$\left(x^2\right)^2 - 5\left(x^2\right) + 4 = 0$$

$$U^2 - 5U + 4 = 0$$

Solve for $U \Rightarrow U = \frac{-(-5)\pm\sqrt{5^2-4(1)(4)}}{2(1)}$

$$= \frac{5\pm\sqrt{9}}{2}$$

$$= \frac{5\pm 3}{2}$$

$$x^2 = U \to \begin{cases} x^2 = \frac{5-3}{2} = 1 \to x = \pm 1 \\ x^2 = \frac{5+3}{2} = 4 \to x = \pm 2 \end{cases}$$

$$\left(x^2 - 1\right)\left(x^2 - 4\right) = 0$$

$x^2 - 1 = 0 \qquad x^2 - 4 = 0$

$x^2 = 1 \qquad x^2 = 4$

$x = \pm 1 \qquad x = \pm 2$

Exercise

Solve $x^4 + 3x^2 = 10$

Solution

$$x^4 + 3x^2 - 10 = 0$$

$$\left(x^2 + 5\right)\left(x^2 - 2\right) = 0$$

$x^2 + 5 = 0 \qquad x^2 - 2 = 0$

$x^2 = -5 \qquad x^2 = 2$

$x = \pm\sqrt{-5} \qquad x = \pm\sqrt{2}$

$x = \pm i\sqrt{5}$

Exercise

Solve $x - 3\sqrt{x} - 4 = 0$

Solution

$$\left(\sqrt{x} - 4\right)\left(\sqrt{x} + 1\right) = 0$$

$\sqrt{x} - 4 = 0 \qquad \sqrt{x} + 1 = 0$

$\sqrt{x} = 4 \qquad \sqrt{x} = -1$ *Impossible*

$x = 16$

Exercise

Solve $\left(5x^2-6\right)^{1/4}=x$

Solution

$$\left[\left(5x^2-6\right)^{1/4}\right]^4=x^4$$

$$5x^2-6=x^4$$

$$x^4-5x^2+6=0$$

$$\left(x^2-3\right)\left(x^2-2\right)=0$$

$$x^2-3=0 \qquad\qquad x^2-2=0$$

$$x^2=3 \qquad\qquad x^2=2$$

$$x=\pm\sqrt{3} \qquad\qquad x=\pm\sqrt{2}$$

Exercise

Solve $\left(x^2+24x\right)^{1/4}=3$

Solution

$$\left[\left(x^2+24x\right)^{1/4}\right]^4=3^4$$

$$x^2+24x=81$$

$$x^2+24x-81=0$$

$$(x+27)(x-3)=0$$

$$x+27=0 \qquad\qquad x-3=0$$

$$x=-27 \qquad\qquad x=3$$

Exercise

Solve: $x^{5/2} = 32$

Solution

$x = 32^{2/5}$ *Reciprocal* or **use the calculator: 32^(2/5) = 4**

$x = \sqrt[5]{32^2}$

$= \sqrt[5]{1024}$

$\underline{= 4}$

Exercise

Solve $7|5x| + 2 = 16$

Solution

$7|5x| = 16 - 2$

$7|5x| = 14$

$|5x| = \frac{14}{7}$

$|5x| = 2$

$5x = 2$	$5x = -2$
$x = \frac{2}{5}$	$x = -\frac{2}{5}$

Solutions: $x = \pm\frac{2}{5}$

Exercise

Solve $|x+7| + 6 = 2$

Solution

$|x+7| = 2 - 6$

$|x+7| = -4$

$\Rightarrow$ No solution or $\varnothing$, since the absolute value can't be a negative

Exercise

Solve $4\left|1-\frac{3}{4}x\right|+7=10$

Solution

$$4\left|1-\frac{3}{4}x\right|=10-7 \qquad\qquad |4-3x|=10-7 \quad \textit{Distribute 4}$$

$$4\left|1-\frac{3}{4}x\right|=3 \qquad\qquad |4-3x|=3$$

$$\left|1-\frac{3}{4}x\right|=\frac{3}{4}$$

$1-\frac{3}{4}x=\frac{3}{4}$	$1-\frac{3}{4}x=-\frac{3}{4}$	$4-3x=-3$	$4-3x=3$
$-\frac{3}{4}x=\frac{3}{4}-1$	$-\frac{3}{4}x=-\frac{3}{4}-1$	$-3x=-7$	$-3x=-1$
$-\frac{3}{4}x=-\frac{1}{4}$	$-\frac{3}{4}x=-\frac{7}{4}$	$x=\frac{7}{3}$	$x=\frac{1}{3}$
$x=-\frac{1}{4}\left(-\frac{4}{3}\right)$	$x=-\frac{7}{4}\left(-\frac{4}{3}\right)$		
$x=\frac{1}{3}$	$x=\frac{7}{3}$		

Solutions: $\boxed{x=\frac{1}{3},\frac{7}{3}}$

Exercise

Solve equation: $|5-3x|=12$

Solution

$$\begin{array}{ll} 5-3x=12 & 5-3x=-12 \\ 5-3x-5=12-5 & 5-3x-5=-12-5 \\ -3x=7 & -3x=-17 \\ \boxed{x=-\frac{7}{3}} & \boxed{x=\frac{17}{3}} \end{array}$$

Exercise

Solve equation: $|4x+2|=5$

Solution

$$\begin{array}{ll} 4x+2=-5 & 4x+2=5 \\ 4x=-7 & 4x=3 \\ \boxed{x=-\frac{7}{4}} & \boxed{x=\frac{3}{4}} \end{array}$$

Exercise

Solve equation: $\left|\dfrac{6x+1}{x-1}\right|=3$

Solution

$$\frac{6x+1}{x-1}=-3 \qquad\qquad \frac{6x+1}{x-1}=3$$

$$(x-1)\frac{6x+1}{x-1}=-3(x-1) \qquad\qquad 6x+1=3(x-1)$$

$$6x+1=-3x+3 \qquad\qquad 6x+1=3x-3$$

$$6x+1+3x=-3x+3+3x \qquad\qquad 6x+1-3x=3x-3-3x$$

$$9x+1=3 \qquad\qquad 3x+1=-3$$

$$9x=2 \qquad\qquad 3x=-4$$

$$x=\frac{2}{9} \qquad\qquad x=-\frac{4}{3}$$

Exercise

Solve equation: $|x+1|=|1-3x|$

Solution

$$x+1=-(1-3x) \qquad\qquad x+1=1-3x$$

$$x+1=-1+3x \qquad\qquad x+3x=1-1$$

$$x-3x=-1-1 \qquad\qquad 4x=0$$

$$-2x=-2 \qquad\qquad x=0$$

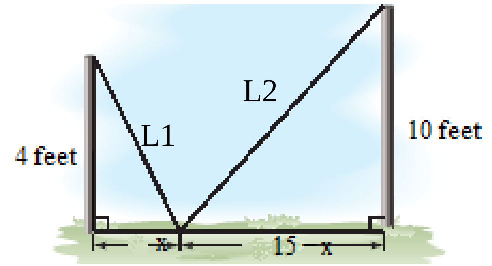
$$x=1$$

Solution: $\{0,\ 1\}$

Exercise

Two vertical poles of lengths 4 feet and 10 feet stand 15 feet apart. A cable reaches from the top of one pole to some point on the ground between the poles and then to the top of the other pole. Where should this point be located to use 24 feet of cable?

Solution



$$l_1^2 = x^2 + 4^2 \qquad l_1 = \sqrt{x^2+16}$$

$$l_2^2 = (15-x)^2 + 10^2 \qquad l_2 = \sqrt{(15-x)^2+100}$$

$$l_1 + l_2 = 24$$

$$\sqrt{x^2+16} + \sqrt{(15-x)^2+100} = 24$$

$$\sqrt{(15-x)^2+100} = 24 - \sqrt{x^2+16}$$

Square both sides

$$\left(\sqrt{(15-x)^2+100}\right)^2 = \left(24-\sqrt{x^2+16}\right)^2$$

$$x^2 - 30x + 225 + 100 = 576 - 48\sqrt{x^2+16} + x^2 + 16$$

$$x^2 - 30x + 325 - x^2 - 576 - 16 = -48\sqrt{x^2+16}$$

$$-30x - 267 = -48\sqrt{x^2+16}$$

$$30x + 267 = 48\sqrt{x^2+16}$$

$$(30x+267)^2 = 48^2\left(x^2+16\right)$$

$$900x^2 + 16020x + 71289 = 2304\left(x^2+16\right)$$

$$900x^2 + 16020x + 71289 = 2304x^2 + 36864$$

$$900x^2 + 16020x + 71289 - 2304x^2 - 36864 = 0$$

$$-1404x^2 + 16020x + 34425 = 0$$

Solve for x:

$$x = 13.259$$

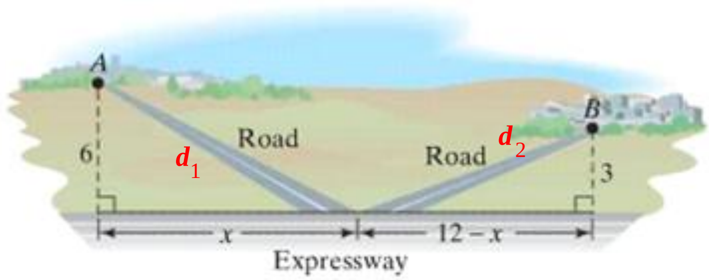
Exercise

Towns A and B are located 6 miles and 3 miles, respectively, from a major expressway. The point on the expressway closet to town A is 12 miles from the point on the expressway closet to town B. Two new roads are to be built from A to the expressway and then to B.

a. Express the combined lengths of the new road in terms of *x*.

b. If the combined lengths of the new roads is 15 miles, what distance does *x* represent?

Solution



a) $d_1^2 = x^2 + 6^2 \quad \to \quad d_1 = \sqrt{x^2+36}$

$d_2^2 = (12-x)^2 + 3^2 \quad \to \quad d_2 = \sqrt{(12-x)^2+9}$

$d_1 + d_2 = \sqrt{x^2+36} + \sqrt{(12-x)^2+9}$

b) $\sqrt{x^2+36} + \sqrt{(12-x)^2+9} = 15$

$\sqrt{x^2+36} = 15 - \sqrt{144-24x+x^2+9}$

$\left(\sqrt{x^2+36}\right)^2 = \left(15-\sqrt{x^2-24x+153}\right)^2$

$x^2+36 = 225 - 30\sqrt{x^2-24x+153} + x^2 - 24x + 153$

$30\sqrt{x^2-24x+153} = -24x+342$

$\left(30\sqrt{x^2-24x+153}\right)^2 = (-24x+342)^2$

$900\left(x^2-24x+153\right) = 576x^2 - 16416x + 116964$

$900x^2 - 21600x + 137700 = 576x^2 - 16416x + 116964$

$324x^2 - 5184x + 20736 = 0$

Solve for *x*: $\boxed{x = 8}$

Solution *Section* 1.6 – Inequalities

Exercise

Find: $(-3,0)\cap[-1,2]$

Solution

$$(-3,0)\cap[-1,2]=[-1,0)$$

Exercise

Find: $(-3,0)\cup[-1,2]$

Solution

$$(-3,0)\cup[-1,2]=(-3,2]$$

Exercise

Find: $(-4,0)\cap[-2,1]$

Solution

$$(-4,0)\cap[-2,1]=[-2,0)$$

Exercise

Find: $(-4,0)\cup[-2,1]$

Solution

$$(-4,0)\cup[-2,1]=(-4,1]$$

Exercise

Find: $(-\infty,5)\cap[1,8)$

Solution

$$(-\infty,5)\cap[1,8)=[1,5)$$

Exercise

Find: $(-\infty, 5) \cup [1, 8)$

Solution

$$(-\infty, 5) \cup [1, 8) = (-\infty, 8)$$

Exercise

Solve $-3x + 5 > -7$ Give the solution set in interval notation.

Solution

$$-3x > -7 - 5$$

$$-3x > -12$$

$$\frac{-3}{-3}x < \frac{-12}{-3}$$

$$x < 4 \qquad \textit{Solution}: (-\infty,\ 4)$$

Exercise

Solve $2 - 3x \le 5$

Solution

$$-3x \le 5 - 2$$

$$-3x \le 3 \qquad \textit{Divide by } -3 \textit{ both sides}$$

$$\frac{-3}{-3}x \ge \frac{3}{-3}$$

$$x \ge -1 \qquad \textit{or } [-1, \infty)$$

Exercise

Solve $4 - 3x \le 7 + 2x$ Give the solution set in interval notation and graph it.

Solution

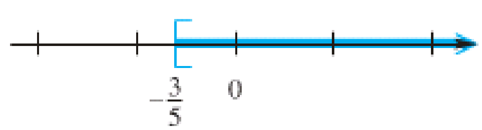
$$4 - 3x - 4 \le 7 + 2x - 4$$

$$-3x \le 3 + 2x$$

$$-3x - 2x \le 3 + 2x - 2x$$

$$-5x \le 3$$

$$x \ge -\frac{3}{5}$$



In interval notation the solution: $\left[-\frac{3}{5},\ \infty\right)$

Exercise

Solve $-3 \le \frac{2}{3}x - 5 \le -1$

Solution

$$-3 \le \frac{2}{3}x - 5 \le -1$$

$$-3 + 5 \le \frac{2}{3}x - 5 + 5 \le -1 + 5$$

$$2 \le \frac{2}{3}x \le 4$$

$$2\frac{3}{2} \le \frac{3}{2}\frac{2}{3}x \le \frac{3}{2}4$$

$$3 \le x \le 6$$

Exercise

Solve $6x - (2x+3) \ge 4x - 5$ Give the solution set in interval notation and graph it.

Solution

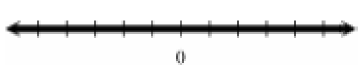
$$6x - 2x - 3 \ge 4x - 5$$

$$4x - 3 \ge 4x - 5$$

$$4x - 4x \ge 3 - 5$$

$$0 \ge -2 \quad (true)$$

Solution: $(-\infty,\ \infty)$

Graph: 

Exercise

Solve $\frac{2x-5}{-8} \le 1 - x$ Give the solution set in interval notation and graph it.

Solution

$$(-8)\frac{2x-5}{-8} \ge (-8)(1-x)$$

$$2x - 5 \ge -8 + 8x$$

$$2x - 8x \ge -8 + 5$$

$$-6x \ge -3$$

$$\frac{-6}{-6}x \le \frac{-3}{-6}$$

$x \le \frac{1}{2}$ $\left(-\infty,\ \frac{1}{2}\right]$

Graph: 

Exercise

Solve $-6 \le 6x+3 \le 21$ Give the solution set in interval notation and graph it.

Solution

$$-6-3 \le 6x+3-3 \le 21-3$$

$$-9 \le 6x \le 18$$

$$-\frac{9}{6} \le \frac{6}{6}x \le \frac{18}{6}$$

$$-\frac{3}{2} \le x \le 3 \qquad \left[-\frac{3}{2},\ 3\right]$$

Exercise

Solve the inequality equation: $1 \le 2x + 3 < 11$

Solution

$$1 - 3 \le 2x + 3 - 3 < 11 - 3$$

$$-2 \le 2x < 8$$

$$-\frac{2}{2} \le \frac{2}{2}x < \frac{8}{2}$$

$$-1 \le x < 4$$

Sol.: [-1, 4)

Exercise

Solve the inequality equation: $12 < \left|-2x+\frac{6}{7}\right| + \frac{3}{7}$

Solution

$$(7)12 < \left|-(7)2x+(7)\frac{6}{7}\right| + (7)\frac{3}{7}$$ *Multiply by 7 both sides*

$$84 < |-14x+6| + 3$$

$$81 < |-14x+6|$$

$$|-14x+6| > 81$$

$$-14x+6 < -81 \qquad -14x+6 > 81$$

$$-14x < -81-6 \qquad -14x > 81-6$$

$$-14x < -87 \qquad -14x > 75$$

$$x > \frac{87}{14} \qquad x < -\frac{75}{14}$$

$$\left(-\infty, -\frac{75}{14}\right) \cup \left(\frac{87}{14}, \infty\right)$$

Exercise

Solve the inequality equation: $4+\left|3-\frac{x}{3}\right|\ge 9$

Solution

$$\left|3-\frac{x}{3}\right|\ge 9-4$$

$$\left|3-\frac{x}{3}\right|\ge 5$$

$$\left|(3)3-(3)\frac{x}{3}\right|\ge (3)5$$

$$|9-x|\ge 15$$

$9-x\le -15$	$9-x\ge 15$
$-x\le -24$	$-x\ge 6$
$x\ge 24$	$x\le -6$

$$(-\infty,-6]\cup[24,\infty)$$

Exercise

Solve the inequality equation: $|x-2|<5$

Solution

$$-5 < x - 2 < 5$$

$$-5 + 2 < x < 5 + 2$$

$$-3 < x < 7$$

Exercise

Solve the inequality equation: $|2x+1|<7$

Solution

$$-7<2x+1<7$$

$$-7-1<2x+1-1<7-1$$

$$-8<2x<6$$

$$-\frac{8}{2}<\frac{2}{2}x<\frac{6}{2}$$

$$-4<x<3$$

Exercise

Solve the inequality equation: $|2-7x|-1>4$

Solution

$$|5x+2|<5$$

$$-5<5x+2<5$$

$$-7<5x<3$$

$$-\frac{7}{5}<x<\frac{3}{5}$$

Solution set: $\left(-\frac{7}{5},\ \frac{3}{5}\right)$

Exercise

Solve the inequality equation: $|2-7x|-1>4$

Solution

$$|2-7x|>5$$

$$2-7x<-5 \qquad 2-7x>5$$

$$-7x<-7 \qquad -7x>3$$

$$x>1 \qquad x<-\frac{3}{7}$$

The solution set: $\left(-\infty,\ -\frac{3}{7}\right)\cup(1,\ \infty)$

Exercise

Solve the inequality equation: $|3x-4|<2$

Solution

$$-2<3x-4<2$$

$$-2+4<3x-4+4<2+4$$

$$2<3x<6$$

$$\frac{2}{3}<x<2$$

Solution set: $\left(\frac{2}{3},\ 2\right)$

Exercise

Solve the inequality equation: $|2x+5| \ge 3$

Solution

$$2x+5 \le -3 \qquad\qquad 2x+5 \ge 3$$

$$2x \le -8 \qquad\qquad 2x \ge -2$$

$$x \le -4 \qquad\qquad x \ge -1$$

Solution set: $(-\infty,\ -4] \cup [-1,\ \infty)$

Exercise

Solve $|12-9x| \ge -12$

Solution

Solution set: $(-\infty,\ \infty)$ because the absolute value always greater than any negative number.

Exercise

Solve $|6-3x| < -11$

Solution

No solution, because the absolute value cannot be less than any negative number

Exercise

Solve $|7+2x| = 0$

Solution

$$7+2x=0$$

$$2x=-7$$

$$x=-\frac{7}{2}$$