1. Find the inverse of the given relation?

a) {(2, 1), (–2, 3), (3, 4), (–3, 2), (1, 5)}

b) {(–7, 3), (–2, 1), (–2, 4), (0, 7)}

2. For the following functions:

a) $f(x)=\sqrt{x+5}+1$

b) $f(x)=\frac{x+4}{x-3}$

i) Is *f*(*x*) one-to-one?

ii) Find the inverse, if exists.

iii) Determine the Domain for the inverse function

3. Use the calculator to find the following. Round to 4 decimal places

a) $e^{-2.458}$ *b)* $\left(\frac{1}{e^3}\right)^2$ *c)* $ln(0.00037)$ *d)* $2\,e^{-\pi}$

e) $\log_5 4.43$ *f)* $4^{\sqrt{3}}$ *g)* $\log(-2.3)$ *h)* $\ln(2+e)$

4. Write each equation in its logarithmic form

a) $5^{-3}=\frac{1}{125}$ *c)* $e^{-1}=0.368$ *e)* $e^x=z$

b) $4^{2y}=24.5$ *d)* $15^{0.457}=3$

5. Write each equation in its exponential form

a) $6=\log_2 64$ *c)* $y=\ln 2^{\pi}$ *e)* $\log y=x$

b) $2=\log_3 x$ *d)* $6.2=\ln x$ *f)* $\log_3 x=\frac{1}{3}$

6. Graph and determine its ***asymptote*** (**label** the graph).

a) $f(x)=log(x+2)$ *c)* $f(x)=\ln(2x-4)$

b) $f(x)=\left(\frac{1}{3}\right)^{x-3}$ *d)* $f(x)=e^{2x}-4$

7. Find the ***domain***, ***range*** and the ***asymptote*** of each logarithmic function

a) $f(x) = 2 + \ln(2x - 4)$ *b)* $f(x) = \ln(7 - x)$ *c)* $f(x) = \ln\left(x^2 - 4x - 5\right)$

d) $f(x) = \ln(x-3)^2$ *e)* $f(x) = \log\left(\frac{x-7}{x+5}\right)$ *f)* $f(x) = 5 + e^{2x+3}$

g) $f(x) = 2 - 3e^{x+1}$ *h)* $f(x) = 2^{3x+1}$

8. Express in terms of sums and differences of logarithms

a) $\log_3\left(\frac{x^3y^2}{z}\right)$ *b)* $\log\left(\frac{x^3y^2}{\sqrt[3]{(z+1)^2}}\right)$ *c)* $\log_b\left(\frac{x^3y^2}{a^4b^5}\right)$

9. Write each expression as a single logarithm

a) $\frac{1}{3}\left(\log_4 x - \log_4 y\right)$

b) $2\ln(x-3) - \frac{1}{2}\ln(x+2) + 4\ln x - \ln y$

c) $\frac{2}{3}\left[\ln\left(x^2-4\right) - \ln(x+2)\right] + \ln(x+y)$

10. Solve the exponential equation

a) $2^{2x+1} = 64$ *c)* $3^{x+4} = 2^{2x+5}$

b) $5^{x+3} = 25^{x-5}$ *d)* $e^{1-8x} = 7957$

11. Solve the Logarithmic equation

a) $\log_3(x+2) + \log_3 x = 1$

b) $\ln\sqrt{x+4} = 1$

c) $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

d) $\log_2 3x + \log_2 3 = \log_2(2x+15)$

12. A sum of $1000 is invested at an annual interest rate of 5.9%. Find the account balance after 5 years if interest is compounded

a) Quarterly *b)* Monthly

13. Find the accumulated value of an investment of $2500 for 5 years at an interest rate of 4.5% if the money is compounded

a) Semiannually *b)* Quarterly *c)* Monthly

14. The population of the United States is about 300 million. If it is growing at a rate of 2.1% per year, how long to the nearest tenth of a year, will it take for the population to triple?

15. An endangered species of fish has a population that is decreasing exponentially according to the equation $A(t) = 14000e^{kt}$ where *A* is the fish population *t* years after 1990. The fish population was 14,000 in 1990, and nine years later it was 12,000. Use this information to find ***k*** to 4 decimal places.

16. The value of a particular investment follows a pattern of exponential growth. In the year 2000, you invested money in a money market account. The value of your investment *t* years after 2000 is given by the exponential growth model $A(t) = 4000e^{0.055t}$. When will the account be worth $6210?

17. In 2000, the population of China was about 1.3 billion. In 2003, the population was 1.33 billion.

a) Find the exponential growth rate

b) Find the exponential growth function

c) Estimate the population in 2009

d) After how long will the population be double what it was in 2000?

Solution

1. *a*. {(1, 2), (2, –2), (4, 3), (2, –3), (5, 1)}

b. {(3, –7), (1, –2), (4, –2), (7, 0)}

2. *a*) $f(x)=\sqrt{x+5}+1$

i) $f(a)=f(b)$

$\Rightarrow \sqrt{a+5}+1=\sqrt{b+5}+1$

$\Rightarrow \sqrt{a+5}=\sqrt{b+5}$ (square both side)

$\Rightarrow a+5=b+5$

$\Rightarrow a = b \rightarrow f(x)$ is one-to-one

ii) $y=\sqrt{x+5}+1$

$\Rightarrow x=\sqrt{y+5}+1$

$\Rightarrow x-1=\sqrt{y+5}$

$\Rightarrow (x-1)^2=y+5$

$\Rightarrow y=(x-1)^2-5=f^{-1}(x)$

iii) Domain: $x \geq 1$

b) $f(x)=\frac{x+4}{x-3}$

i) $f(a)=f(b)$

$\Rightarrow \frac{a+4}{a-3}=\frac{b+4}{b-3}$

$\Rightarrow (a+4)(b-3)=(a-3)(b+4)$

$\Rightarrow ab-3a+4b-12=ab+4a-3b-12$

$\Rightarrow -3a=4a-7b$

$\Rightarrow -7a=-7b$

$\Rightarrow a=b$

$\rightarrow f(x)$ is one-to-one

ii) $y=\frac{x+4}{x-3}$

$\Rightarrow x=\frac{y+4}{y-3}$

$\Rightarrow x(y-3)=y+4$

$\Rightarrow xy-3x=y+4$

$\Rightarrow xy-y=3x+4$

$\Rightarrow y(x-1)=3x+4$

$\Rightarrow y=\frac{3x+4}{x-1}=f^{-1}(x)$

iii) Domain of $f^{-1}(x)$: $\{x \mid x \neq 1\}$

3. *a*) 0.0856 *b*) 0.0025 *c*) −7.9020 *d*) 0.0864 *e*) 0.9248
f) 11.0357 *g*) *doesn't exist* *h*) 1.5514

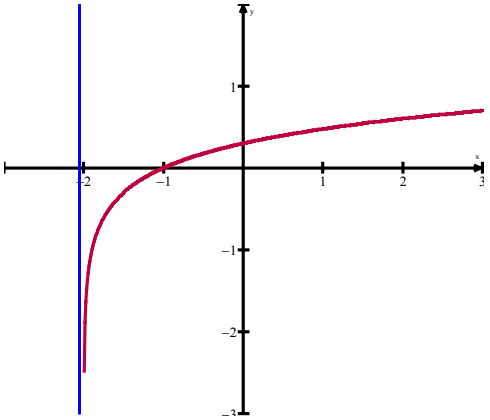
4. *a*) $\log_5 \frac{1}{125} = -3$ *b*) $2y = \log_4 24.5$ *c*) $\ln(0.3679) = -1$
d) $0.4057 = \log_{15} 3$ *e*) $x = \ln z$

5. *a*) $2^6 = 64$ *b*) $3^2 = x$ *c*) $e^y = 2^\pi$ *d*) $e^{6.2} = x$ *e*) $y = 10^x$ *f*) $x = 3^{\frac{1}{3}}$

6. *a*) $f(x) = \log(x+2)$

Asymptote: $x = -2$

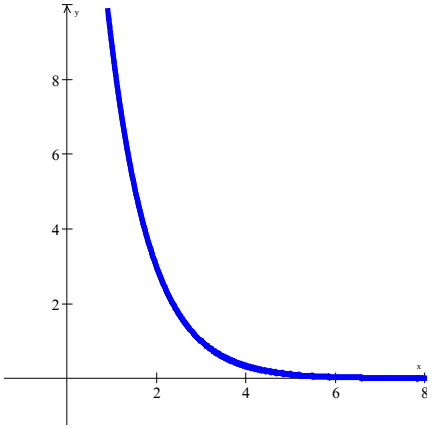
x	y
~~−2~~	
-1.5	-.3
-1	1
0	.3

Shifted  left 2 units

b) $f(x) = \left(\frac{1}{3}\right)^{x-3}$

Asymptote: $y = 0$

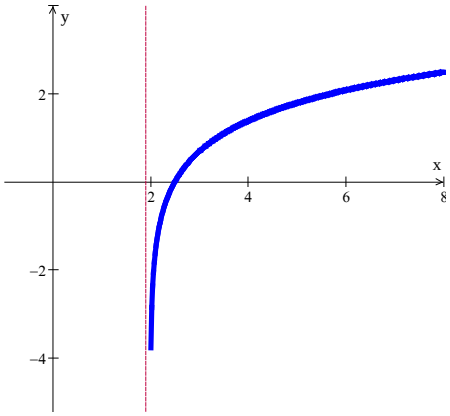
x	y
2	~~3~~
3	1
4	.33
5	.1



c) $f(x) = \ln(2x-4)$

Asymptote: $x = 2$

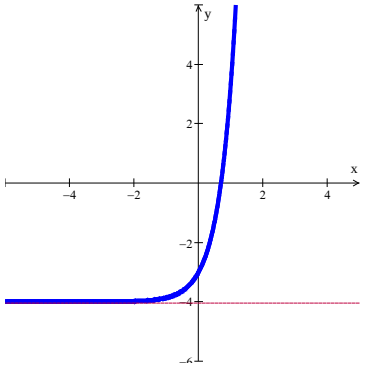
x	y
~~2~~	
2.5	0
3	.7
4	1.4



d) $f(x) = e^{2x} - 4$

Asymptote: $y = -4$

x	y
-1	-3.9
0	-3
1	3.4
2	51



7. *a)* *Domain*: $(2, \infty)$; *Range*: $(-\infty, \infty)$; *Asymptote*: $x = 2$

b) *Domain*: $(-\infty, 7)$; *Range*: $(-\infty, \infty)$; *Asymptote*: $x = 7$

c) *Domain*: $(-\infty, -1) \cup (5, \infty)$; *Range*: $(-\infty, \infty)$; *Asymptote*: $x = -1,\ x = 5$

d) *Domain*: $(-\infty, 3) \cup (3, \infty)$; *Range*: $(-\infty, \infty)$; *Asymptote*: $x = 3$

e) *Domain*: $(-\infty, -5) \cup (7, \infty)$; *Range*: $(-\infty, 0) \cup (0, \infty)$; *Asymptote*: $x = -5,\ x = 7$

f) *Domain*: $(-\infty, \infty)$; *Range*: $(5, \infty)$; *Asymptote*: $y = 5$

g) *Domain*: $(-\infty, \infty)$; *Range*: $(-\infty, 2)$; *Asymptote*: $y = 2$

h) *Domain*: $(-\infty, \infty)$; *Range*: $(0, \infty)$; *Asymptote*: $y = 0$

8. *a)* $3\log_3 x + 2\log_3 y - \log_3 z$

b) $3\log x + 2\log y - \frac{2}{3}\log(z+1)$

c) $3\log_b x + 2\log_b y - 4\log_b a - 5$

9. *a)* $\log_4\left(\sqrt[3]{\frac{x}{y}}\right)$ *b)* $ln\left(\frac{x^4(x-3)^2}{y\sqrt{x+2}}\right)$ *c)* $ln(x-2)^{2/3}(x+y)$ *or* $\ln\sqrt[3]{(x-2)^2}(x+y)$

10. *a)* $\frac{5}{2}$ *b)* $x = 13$ *c)* $\frac{5\ln 2 - 4\ln 3}{\ln 3 - 2\ln 2} \approx 3.23$ *d)* ≈ -0.9977

11. *a)* 1 *b)* 3.389 *c)* 4, 5 *d)* $\frac{15}{7}$

12. *a)* \$1,340.24 *b)* \$1,342.16

13. *a)* \$3,123.01 *b)* \$3,126.88 *c)* \$3,129.49

14. $\frac{1000}{21}\ln 3$ *years*

15. $k = -0.0171$

16. $\frac{200}{11}\ln\left(\frac{621}{400}\right)$ *years*

17. *a)* $k \approx 0.0076$ *b)* $A(t) = 1.3e^{.0076t}$ *c)* 1.392 billion *d)* 91.2 years