# Solving Exponential Function with different bases

$$a^{mx+n} = b^{px+q} \implies x = \frac{q \ln b - n \ln a}{m \ln a - p \ln b}$$
 coefficient  $\frac{no \ x's}{x's}$ 

**Numerator**: multiply q with  $\ln b$  minus multiply n with  $\ln a$  multiply n with  $\ln a$  multiply p with  $\ln b$ 

## **Proof**

$$\ln a^{mx+n} = \ln b^{px+q}$$

$$(mx+n)\ln a = (px+q)\ln b$$

$$mx\ln a + n\ln a = px\ln b + q\ln b$$

$$mx\ln a - px\ln b = q\ln b - n\ln a$$

$$x(m\ln a - p\ln b) = q\ln b - n\ln a$$

$$x = \frac{q\ln b - n\ln a}{m\ln a - p\ln b}$$

 $mx \ln a + n \ln a = px \ln b + q \ln b$ 

### Example

Solve: 
$$\frac{2x-1}{3} = \frac{x+1}{7}$$

#### **Solution**

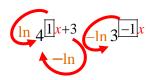
$$x = \frac{\ln 3 + \ln 7}{2\ln 3 - \ln 7}$$

## Example

Solve: 
$$4^{x+3} = 3^{-x}$$

### **Solution**

$$x = \frac{-3\ln 4}{\ln 4 + \ln 3}$$



## Example

Solve: 
$$4^{-x} = 3^{x+3}$$

### **Solution**

$$x = \frac{3\ln 3}{\ln 3 - \ln 4}$$

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