

Solution **Section 1.3 – Infinite Limits**

Exercise

Find $\lim_{x \rightarrow 5} \frac{x-7}{x(x-5)^2}$

Solution

$$\lim_{x \rightarrow 5} \frac{x-7}{x(x-5)^2} = \frac{-2}{0} \\ = \infty$$

Exercise

Find $\lim_{x \rightarrow -5^+} \frac{x-5}{x+5}$

Solution

$$\lim_{x \rightarrow -5^+} \frac{x-5}{x+5} = \frac{-10}{0^+} \\ = -\infty$$

Exercise

Find $\lim_{x \rightarrow 3^-} \frac{x-4}{x^2-3x}$

Solution

$$\lim_{x \rightarrow 3^-} \frac{x-4}{x^2-3x} = \frac{-1}{0^-} \\ = \infty$$

Exercise

Find $\lim_{x \rightarrow 0^+} \frac{1}{3x}$

Solution

$$\lim_{x \rightarrow 0^+} \frac{1}{3x} = \frac{1}{0^+} \\ = \infty$$

Exercise

Find $\lim_{x \rightarrow -5^-} \frac{3x}{2x+10}$

Solution

$$\lim_{x \rightarrow -5^-} \frac{3x}{2x+10} = \lim_{x \rightarrow -5^-} \frac{3}{2 + \frac{10}{x}}$$

$$\underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow 0} \frac{1}{x^{2/3}}$

Solution

$$\lim_{x \rightarrow 0} \frac{1}{x^{2/3}} = \lim_{x \rightarrow 0} \frac{1}{\left(x^{1/3}\right)^2}$$

$$\underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow 0^-} \frac{1}{3x^{1/3}}$

Solution

$$\lim_{x \rightarrow 0^-} \frac{1}{3x^{1/3}} = \frac{1}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim_{x \rightarrow \left(-\frac{\pi}{2}\right)^+} \sec x$

Solution

$$\lim_{x \rightarrow \left(-\frac{\pi}{2}\right)^+} \sec x \underline{= \infty}$$

Exercise

Find $\lim_{\theta \rightarrow 0^-} (1 + \csc \theta)$

Solution

$$\lim_{\theta \rightarrow 0^-} (1 + \csc \theta) = \lim_{\theta \rightarrow 0^-} \left(1 + \frac{1}{\sin \theta}\right)$$

$$\underline{\underline{= -\infty}}$$

Exercise

Find $\lim_{\theta \rightarrow 0^+} \csc \theta$

Solution

$$\lim_{\theta \rightarrow 0^+} \csc \theta = \lim_{\theta \rightarrow 0^+} \frac{1}{\sin \theta}$$

$$\underline{\underline{= +\infty}}$$

As $\theta \rightarrow 0^+$ $\sin \theta > 0$

Exercise

Find $\lim_{x \rightarrow 0^+} (-10 \cot x)$

Solution

$$\lim_{x \rightarrow 0^+} (-10 \cot x) = -10 \lim_{x \rightarrow 0^+} \frac{\cos \theta}{\sin \theta} = -10 \left(\frac{1}{0}\right)$$

$$\underline{\underline{= -\infty}}$$

As $x \rightarrow 0^+$ $\cos \theta > 0$; $\sin \theta > 0$

Exercise

Find $\lim_{\theta \rightarrow \frac{\pi}{2}^+} \frac{1}{3} \tan \theta$

Solution

$$\lim_{\theta \rightarrow \frac{\pi}{2}^+} \frac{1}{3} \tan \theta = \frac{1}{3} \lim_{\theta \rightarrow \frac{\pi}{2}^+} \frac{\sin \theta}{\cos \theta} = \frac{1}{3} \left(-\frac{1}{0}\right)$$

$$\underline{\underline{= -\infty}}$$

As $\theta \rightarrow \frac{\pi}{2}^+$ $\cos \theta < 0$; $\sin \theta > 0$

Exercise

Find $\lim_{x \rightarrow 2^+} \frac{1}{x-2}$

Solution

$$\lim_{x \rightarrow 2^+} \frac{1}{x-2} = \frac{1}{2^+ - 2} = \frac{1}{0^+}$$

$$\underline{\underline{= \infty}}$$

Exercise

Find $\lim_{x \rightarrow 2^-} \frac{1}{x-2}$

Solution

$$\lim_{x \rightarrow 2^-} \frac{1}{x-2} = \frac{1}{2^- - 2} = \frac{1}{0^-} \\ \underline{= -\infty}$$

Exercise

Find $\lim_{x \rightarrow 2} \frac{1}{x-2}$

Solution

$$\lim_{x \rightarrow 2} \frac{1}{x-2} = \frac{1}{0} \\ \underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow 3^+} \frac{2}{(x-3)^3}$

Solution

$$\lim_{x \rightarrow 3^+} \frac{2}{(x-3)^3} = \frac{2}{0^+} \\ \underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow 3^-} \frac{2}{(x-3)^3}$

Solution

$$\lim_{x \rightarrow 3^-} \frac{2}{(x-3)^3} = \frac{2}{0^-} \\ \underline{= -\infty}$$

Exercise

Find $\lim_{x \rightarrow 3} \frac{2}{(x-3)^3}$

Solution

$$\lim_{x \rightarrow 3} \frac{2}{(x-3)^3} = \frac{2}{0}$$

$$\underline{\underline{= \infty}}$$

Exercise

Find $\lim_{x \rightarrow 4^+} \frac{x-5}{(x-4)^2}$

Solution

$$\lim_{x \rightarrow 4^+} \frac{x-5}{(x-4)^2} = \frac{-1}{0}$$

$$\underline{\underline{= -\infty}}$$

Exercise

Find $\lim_{x \rightarrow 4^-} \frac{x-5}{(x-4)^2}$

Solution

$$\lim_{x \rightarrow 4^-} \frac{x-5}{(x-4)^2} = \frac{-1}{0}$$

$$\underline{\underline{= -\infty}}$$

Exercise

Find $\lim_{x \rightarrow 4} \frac{x-5}{(x-4)^2}$

Solution

$$\lim_{x \rightarrow 4^-} \frac{x-5}{(x-4)^2} = \frac{-1}{0}$$

$$\underline{\underline{= -\infty}}$$

Exercise

Find $\lim_{x \rightarrow 1^+} \frac{x-2}{(x-1)^3}$

Solution

$$\lim_{x \rightarrow 1^+} \frac{x-2}{(x-1)^3} = \frac{-1}{0^+}$$

$$\underline{\underline{= -\infty}}$$

Exercise

Find $\lim_{x \rightarrow 1^-} \frac{x-2}{(x-1)^3}$

Solution

$$\lim_{x \rightarrow 1^-} \frac{x-2}{(x-1)^3} = \frac{-1}{0^-}$$
$$\underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow 1} \frac{x-2}{(x-1)^3}$

Solution

$$\lim_{x \rightarrow 1} \frac{x-2}{(x-1)^3} = \frac{-1}{0^+}$$
$$\underline{= \cancel{\infty}}$$

Exercise

Find $\lim_{x \rightarrow 3^+} \frac{(x-1)(x-2)}{x-3}$

Solution

$$\lim_{x \rightarrow 3^+} \frac{(x-1)(x-2)}{x-3} = \frac{2}{0}$$
$$\underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow 3^-} \frac{(x-1)(x-2)}{x-3}$

Solution

$$\lim_{x \rightarrow 3^-} \frac{(x-1)(x-2)}{x-3} = \frac{2}{0^-}$$
$$\underline{= -\infty}$$

Exercise

Find $\lim_{x \rightarrow 3} \frac{(x-1)(x-2)}{x-3}$

Solution

$$\lim_{x \rightarrow 3^-} \frac{(x-1)(x-2)}{x-3} = \frac{2}{0^-} \\ = -\infty$$

$$\lim_{x \rightarrow 3^+} \frac{(x-1)(x-2)}{x-3} = \frac{2}{0^+} \\ = \infty$$

$$\lim_{x \rightarrow 3} \frac{(x-1)(x-2)}{x-3} = \text{DNE}$$

Exercise

Find $\lim_{x \rightarrow -2^+} \frac{x-4}{x(x+2)}$

Solution

$$\lim_{x \rightarrow -2^+} \frac{x-4}{x(x+2)} = \frac{-6}{0^+} \\ = \infty$$

Exercise

Find $\lim_{x \rightarrow -2^-} \frac{x-4}{x(x+2)}$

Solution

$$\lim_{x \rightarrow -2^-} \frac{x-4}{x(x+2)} = \frac{-6}{0^-} \\ = -\infty$$

Exercise

Find $\lim_{x \rightarrow -2} \frac{x-4}{x(x+2)}$

Solution

$$\lim_{x \rightarrow -2} \frac{x-4}{x(x+2)} = \infty$$

$$\lim_{x \rightarrow -2^-} \frac{x-4}{x(x+2)} = -\infty$$

$$\lim_{x \rightarrow -2} \frac{x-4}{x(x+2)} = \underline{\underline{\text{not defined}}}$$

Exercise

Find $\lim_{x \rightarrow 2^+} \frac{x^2 - 4x + 3}{(x-2)^2}$

Solution

$$\lim_{x \rightarrow 2^+} \frac{x^2 - 4x + 3}{(x-2)^2} = \frac{-1}{0^+} = \underline{\underline{-\infty}}$$

Exercise

Find $\lim_{x \rightarrow 2^-} \frac{x^2 - 4x + 3}{(x-2)^2}$

Solution

$$\lim_{x \rightarrow 2^-} \frac{x^2 - 4x + 3}{(x-2)^2} = \frac{-1}{0^+} = \underline{\underline{-\infty}}$$

Exercise

Find $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 3}{(x-2)^2}$

Solution

$$\lim_{x \rightarrow 2} \frac{x^2 - 4x + 3}{(x-2)^2} = \frac{-1}{0} = \underline{\underline{-\infty}}$$

Exercise

Find $\lim_{x \rightarrow -2^+} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2}$

Solution

$$\lim_{x \rightarrow -2^+} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2} = \lim_{x \rightarrow -2^+} \frac{x(x-2)(x-3)}{x^2(x-2)(x+2)}$$

$$= \lim_{x \rightarrow -2^+} \frac{x-3}{x(x+2)} \quad \frac{-}{-(+)} \\ = \infty$$

Exercise

Find $\lim_{x \rightarrow -2^-} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2}$

Solution

$$\lim_{x \rightarrow -2^-} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2} = \lim_{x \rightarrow -2^-} \frac{x(x-2)(x-3)}{x^2(x-2)(x+2)} \\ = \lim_{x \rightarrow -2^-} \frac{x-3}{x(x+2)} \quad \frac{-}{-(-)} \\ = -\infty$$

Exercise

Find $\lim_{x \rightarrow -2} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2}$

Solution

$$\lim_{x \rightarrow -2} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2} = \frac{-8 - 20 - 12}{16 - 16} \\ = \frac{-40}{0} \\ = -\infty$$

Exercise

Find $\lim_{u \rightarrow 0^+} \frac{u-1}{\sin u}$

Solution

$$\lim_{u \rightarrow 0^+} \frac{u-1}{\sin u} = \frac{-1}{0^+} \\ = -\infty$$

Exercise

Find $\lim_{x \rightarrow 0^-} \frac{2}{\tan x}$

Solution

$$\lim_{x \rightarrow 0^-} \frac{2}{\tan x} = \frac{2}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim_{x \rightarrow 1^+} \frac{x^2 - 5x + 6}{x - 1}$

Solution

$$\lim_{x \rightarrow 1^+} \frac{x^2 - 5x + 6}{x - 1} = \frac{2}{0^+}$$

$$\underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow 2\pi^-} \csc x$

Solution

$$\lim_{x \rightarrow 2\pi^-} \csc x = \frac{1}{\sin(2\pi^-)} = \frac{1}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim_{x \rightarrow 0^+} e^{\sqrt{x}}$

Solution

$$\lim_{x \rightarrow 0^+} e^{\sqrt{x}} \underline{= 1}$$

Exercise

Find $\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{1 + \sin x}{\cos x}$

Solution

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{1 + \sin x}{\cos x} = \frac{2}{0^+}$$

$$\underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{1 + \sin x}{\cos x}$

Solution

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{1 + \sin x}{\cos x} = \frac{2}{0^-}$$
$$\underline{\underline{= -\infty}}$$

Exercise

Find $\lim_{x \rightarrow 0^-} \frac{e^x}{1 + e^x}$

Solution

$$\lim_{x \rightarrow 0^-} \frac{e^x}{1 - e^x} = \frac{1}{0^+}$$
$$\underline{\underline{= \infty}}$$

Exercise

Find $\lim_{x \rightarrow 0^+} \frac{e^x}{1 - e^x}$

Solution

$$\lim_{x \rightarrow 0^+} \frac{e^x}{1 - e^x} = \frac{1}{0^-}$$
$$\underline{\underline{= -\infty}}$$

Exercise

Find $\lim_{x \rightarrow 1^-} \frac{x}{\ln x}$

Solution

$$\lim_{x \rightarrow 1^-} \frac{x}{\ln x} = \frac{1}{0^-}$$
$$\underline{\underline{= -\infty}}$$

Exercise

Find $\lim_{x \rightarrow 0^+} \frac{x}{\ln x}$

Solution

$$\lim_{x \rightarrow 0^+} \frac{x}{\ln x} = \frac{0}{-\infty}$$

$$\underline{= 0}$$

Exercise

Find $\lim_{x \rightarrow 0^-} \frac{2e^x + 5e^{3x}}{e^{2x} - e^{3x}}$

Solution

$$\lim_{x \rightarrow 0^-} \frac{2e^x + 5e^{3x}}{e^{2x} - e^{3x}} = \lim_{x \rightarrow 0^-} \frac{2e^x + 5e^{3x}}{e^{2x}(1 - e^x)}$$

$$= \frac{7}{0}$$

$$\underline{= \infty}$$

Exercise

Find $\lim_{x \rightarrow 0^+} \frac{2e^x + 5e^{3x}}{e^{2x} - e^{3x}}$

Solution

$$\lim_{x \rightarrow 0^+} \frac{2e^x + 5e^{3x}}{e^{2x} - e^{3x}} = \lim_{x \rightarrow 0^+} \frac{2e^x + 5e^{3x}}{e^{2x}(1 - e^x)}$$

$$= \frac{7}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim_{x \rightarrow 1^-} \frac{\ln x}{\sin^{-1} x}$

Solution

$$\lim_{x \rightarrow 1^-} \frac{\ln x}{\sin^{-1} x} = \frac{\ln 1}{\sin^{-1} 1}$$

$$= \frac{0}{\frac{\pi}{2}}$$

$$\underline{= 0}$$

Exercise

Let $f(x) = \frac{x^2 - 7x + 12}{x - a}$

- a) For what values of a , if any, does $\lim_{x \rightarrow a^+} f(x)$ equal a finite number?
- b) For what values of a , if any, does $\lim_{x \rightarrow a^+} f(x) = \infty$?
- c) For what values of a , if any, does $\lim_{x \rightarrow a^+} f(x) = -\infty$?

Solution

$$f(x) = \frac{x^2 - 7x + 12}{x - a} = \frac{(x-3)(x-4)}{x - a}$$

a) If $a = 3$, then

$$\lim_{x \rightarrow 3} \frac{(x-3)(x-4)}{x-3} = \lim_{x \rightarrow 3} (x-4)$$

$$\underline{= -1}$$

If $a = 4$, then

$$\lim_{x \rightarrow 4} \frac{(x-3)(x-4)}{x-4} = \lim_{x \rightarrow 4} (x-3)$$

$$\underline{= 1}$$

b) $\lim_{x \rightarrow a^+} f(x) = \infty$ for any number other than 3 or 4.

As $x \rightarrow a^+$, then $(x - a)$ is always positive.

$$(x-3)(x-4) > 0 \Rightarrow (-\infty, 3) \cup (4, \infty)$$

c) $\lim_{x \rightarrow a^+} f(x) = -\infty$ for any number other than 3 or 4.

As $x \rightarrow a^+$, then $(x - a)$ is always positive, and $(3, 4)$

Exercise

Analyze $\lim_{x \rightarrow 1^+} \sqrt{\frac{x-1}{x-3}}$ and $\lim_{x \rightarrow 1^-} \sqrt{\frac{x-1}{x-3}}$

Solution

$$\lim_{x \rightarrow 1^+} \sqrt{\frac{x-1}{x-3}} = \sqrt{\frac{0^+}{-2}} \quad \nexists$$

$$\lim_{x \rightarrow 1^-} \sqrt{\frac{x-1}{x-3}} = \sqrt{\frac{0^-}{-2}} \\ = 0$$