Exercise

In building the space shuttle, NASA contracts for certain guidance components to be supplied by three different companies: 41% by company *A*, 25% by company *B*, and 34% by company *C*. It has been found that 1%, 1.75%, and 2% of the components from companies *A*, *B*, and *C*, respectively, are defective. If one of these guidance components is selected at random, what is the probability that it is defective?

Solution

D = *defective;* ***A*** = *company A;* ***B*** = *company B;* ***C*** = *company C*

$$P(D) = P(A \cap D) \cup P(B \cap D) \cup P(C \cap D)$$
$$= 0.41(.01) + 0.25(.0175) + .34(.02)$$
$$= .0153$$

Exercise

Suppose the probability of *A* is $P(A) = \frac{1}{4}$ and the probability of *B* is $P(B) = \frac{2}{3}$. What would the probability of *A* intersect *B* need to be for *A* and *B* to be independent events?

Solution

Since *A* and *B* to be independent events:

$$P(A \cap B) = P(A)P(B)$$
$$= \frac{1}{4}\frac{2}{3}$$
$$= \frac{1}{6}$$

Exercise

In 2 throws of a fair die, what is the probability that you will get at least 5 on each throw? At least 5 on the first or second throw?

Solution

Let *A* = ″ At least 5 on the first throw″. $\{5,6\} \to P(A) = \frac{2}{6} = \frac{1}{3}$

B = ″ At least 5 on the second throw″. $\{5,6\} \to P(B) = \frac{2}{6} = \frac{1}{3}$

Since the events A and B are independent: $P(A \cap B) = P(A) \cdot P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{3} + \frac{1}{3} - \frac{1}{3}\frac{1}{3}$$
$$= \frac{5}{9}$$

Exercise

2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that the second ball was red, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

Solution

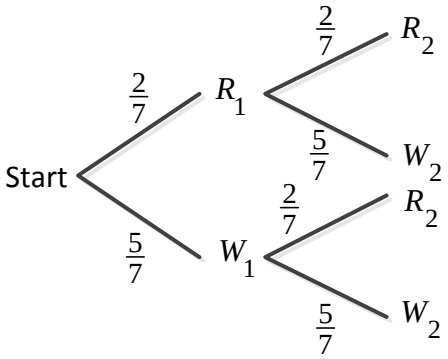
a) $P(R_2) = P(R_1 \cap R_2) + P(W_1 \cap R_2)$

$= P(R_1)P(R_2 | R_1) + P(W_1)P(R_2 | W_1)$

$= \frac{2}{7}\frac{2}{7} + \frac{5}{7}\frac{2}{7}$

$= \frac{14}{49}$

$= \boxed{\frac{2}{7}}$



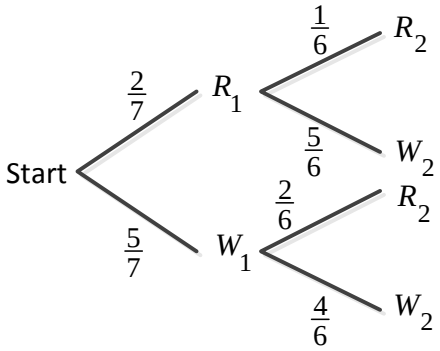
b) $P(R_2) = P(R_1 \cap R_2) + P(W_1 \cap R_2)$

$= P(R_1)P(R_2 | R_1) + P(W_1)P(R_2 | W_1)$

$= \frac{2}{7}\frac{1}{6} + \frac{5}{7}\frac{2}{6}$

$= \frac{12}{42}$

$= \boxed{\frac{2}{7}}$



Exercise

2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that at least 1 ball was red, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

Solution

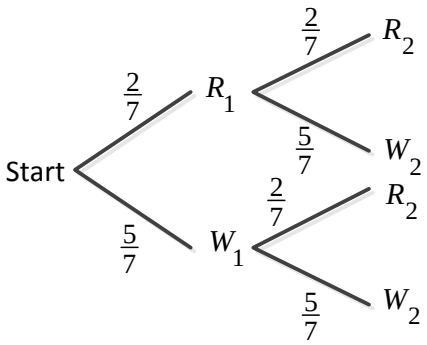
Let E = ″ At least 1 ball was red″.

a) With replacement:

$P(E) = P(R_1 \cap R_2) + P(R_1 \cap W_2) + P(W_1 \cap R_2)$

$= \frac{2}{7}\frac{2}{7} + \frac{2}{7}\frac{5}{7} + \frac{5}{7}\frac{2}{7}$

$= \boxed{\frac{24}{49}}$

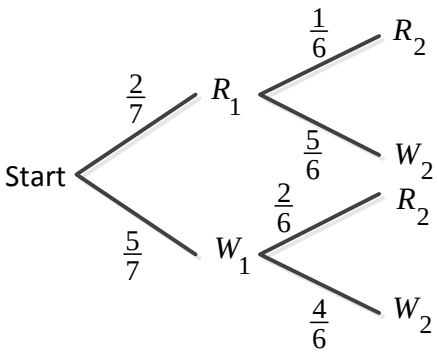


b) $P(E) = P(R_1 \cap R_2) + P(R_1 \cap W_2) + P(W_1 \cap R_2)$

$$= \frac{2}{7}\frac{1}{6} + \frac{2}{7}\frac{5}{6} + \frac{5}{7}\frac{2}{6}$$

$$= \frac{22}{42}$$

$$= \boxed{\frac{11}{21}}$$



Exercise

2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that both balls were the same color, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

Solution

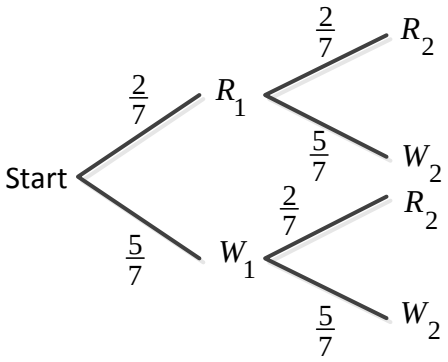
Let E = ″ both balls were the same color″.

a) With replacement:

$$P(E) = P(R_1 \cap R_2) + P(W_1 \cap W_2)$$

$$= \frac{2}{7}\frac{2}{7} + \frac{5}{7}\frac{5}{7}$$

$$= \boxed{\frac{29}{49}}$$



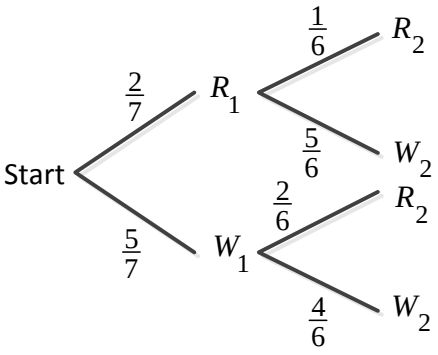
b) Without replacement:

$$P(E) = P(R_1 \cap R_2) + P(W_1 \cap W_2)$$

$$= \frac{2}{7}\frac{1}{6} + \frac{5}{7}\frac{4}{6}$$

$$= \frac{22}{42}$$

$$= \boxed{\frac{11}{21}}$$



Exercise

An automobile manufacturer produces 37% of its cars at plant A. If 5% of the cars manufactured at plant A have defective emission control devices, what is the probability that one of this manufacturer's cars was manufactured at plant A and has a defective emission control device?

Solution

Let A = "car is produced at plant A".
B = "car is defective".

$$P(A) = .37, \quad P(B|A) = .05$$

$$P(A \cap B) = P(A)P(B|A)$$
$$= (.37)(.05)$$
$$= .0185$$

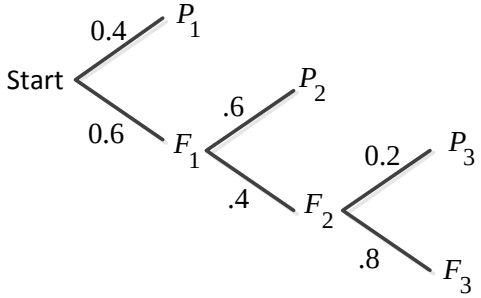
Exercise

To transfer into a particular department, a company requires an employee to pass a screening test. A maximum of 3 attempts are allowed at 6-month intervals between trials. From past records it is found that 40% pass on the first trial; of those that fail the first trial and take the test a second time, 60% pass; and of those that fail on the second trial and take the test a third time, 20% pass. For an employee wishing to transfer:

a) What is the probability of passing the test on the first or second try?
b) What is the probability of failing on the first 2 trials and passing on the third?
c) What is the probability of failing on all 3 attempts?

Solution

a) $P(\text{passing } 1^{st} \text{ or } 2^{nd} \text{ try}) = P(P_1) + P(F_1 \cap P_2)$
$= .4 + (.6)(.6)$
$= .76$



b) $P(\text{failing } 1^{st} \text{ 2 trials or passing } 3^{nd})$
$= P(F_1 \cap F_2 \cap P_3)$
$= (.6)(.4)(.2)$
$= .048$

c) $P(\text{failing all trials}) = P(F_1 \cap F_2 \cap F_3)$
$= (.6)(.4)(.8)$
$= .192$

Exercise

A survey of the residents of a precinct in a large city revealed that 55% of the residents where members of the Democratic Party and that 60% of the Democratic Party members voted in the last election. What is the probability that a person selected at random from the residents of this precinct is a member of the Democratic Party and voted in the last election?

Solution

Let D = ″member of Democratic Party″.

V = ″voted in the last election″.

Then $P(D) = .55, \quad P(V|D) = .6$

$$P(D \cap V) = P(D)\,P(V|D)$$
$$= (.55)(.6)$$
$$= 0.33$$