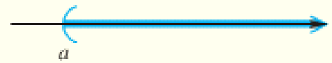
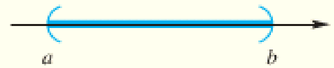
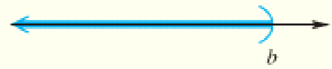
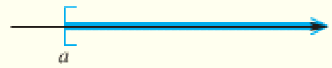
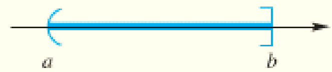
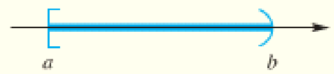
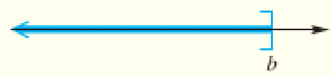
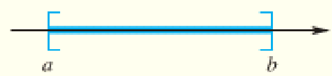
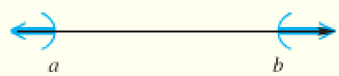
Section 1.5 – Inequalities

Notation

Type of Interval	Set	Interval Notation	Graph
Open interval	$\{x \mid x > a\}$	(a, ∞)	
	$\{x \mid a < x < b\}$	(a, b)	
	$\{x \mid x < b\}$	$(-\infty, b)$	
Other intervals	$\{x \mid x \geq a\}$	$[a, \infty)$	
	$\{x \mid a < x \leq b\}$	$(a, b]$	
	$\{x \mid a \leq x < b\}$	$[a, b)$	
	$\{x \mid x \leq b\}$	$(-\infty, b]$	
Closed interval	$\{x \mid a \leq x \leq b\}$	$[a, b]$	
Disjoint interval	$\{x \mid x < a \text{ or } x > b\}$	$(-\infty, a) \cup (b, \infty)$	
All real numbers	$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	

Properties of inequality

1. ***If a < b, then a + c < b + c***
2. ***If a < b and if c > 0, then ac < bc***
3. ***If a < b and if c < 0, then ac > bc***

Example

Solve $3x + 1 > 7x - 15$

Solution

$3x - 7x > -1 - 15$

$-4x > -16$ *Divide by – 4 both sides*

$x < 4$ *or* $(-\infty, 4)$ *or* $\{x \mid x < 4\}$

Example

Solve $\frac{x-4}{2} \ge \frac{x-2}{3} + \frac{5}{6}$ *LCD*: 2, 3, 6

Solution

$$(6)\frac{x-4}{2} \ge (6)\frac{x-2}{3} + (6)\frac{5}{6}$$

$$3(x-4) \ge 2(x-2) + 5$$

$$3x - 12 \ge 2x - 4 + 5$$

$$3x - 12 \ge 2x + 1$$

$$3x - 2x \ge 12 + 1$$

$$\underline{x \ge 13\,}|$$

Example

a) $3(x + 1) > 3x + 2$

$$3x + 3 > 3x + 2$$

$$3x - 3x > -3 + 1$$

$$0 > -1 \ (\textit{True statement})$$

Sol.: $\mathbb{R}$ ***or*** $\{x|$ *All Real numbers*$\}$ ***or*** $(-\infty, \infty)$

b) $x + 1 \le x - 1$

$$x - x \le -1 - 1$$

$$0 \le -2$$

Sol.: $\varnothing$

Example

Solve $-2 < 5 + 3x < 20$ Give the solution set in interval notation and graph it.

Solution

$$-2 - 5 < 5 + 3x - 5 < 20 - 5$$

$$-7 < 3x < 15$$

$$-\frac{7}{3} < \frac{3}{3}x < \frac{15}{3}$$

$$-\frac{7}{3} < x < 5$$

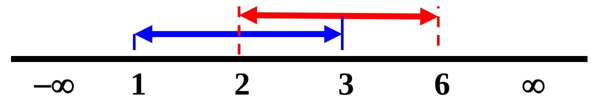
Solution: $\left(-\frac{7}{3},\ 5\right)$

Intersections of Interval $\cap$

To find the intersection, take the portion of the number line that the two graphs have in ***common***

Example

$[1, 3] \cap (2, 6) = (2, 3]$

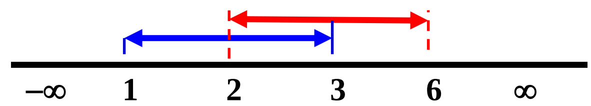


Unions of Interval $\cup$

To find the union, take the portion of the number line representing the total ***collection*** of numbers in the two graphs.

Example

$[1, 3] \cup (2, 6) = [1, 6)$



Solving an ***Absolute Value*** Inequality:

If X is an algebraic expression and c is a positive number,

1. The solutions of $|X| < c$ are the numbers that satisfy $-c < X < c$.
2. The solutions of $|X| > c$ are the numbers that satisfy $X < -c$ or $X > c$.

Example

Solve: $-3|5x-2| + 20 \ge -19$

Solution

$-3|5x-2| \ge -39$

$-|5x-2| \ge -13$

$|5x-2| \le 13$

$-13 \le 5x - 2 \le 13$

$-11 \le 5x \le 15$

$\boxed{-\frac{11}{5} \le x \le 3}$ or $\boxed{\left[-\frac{11}{5}, 3\right]}$

Example

Solve: $18 < |6-3x|$

Solution

$|6-3x|>18$

$$\begin{array}{ll} 6-3x<-18 & 6-3x>18 \\ -3x<-18-6 & -3x>18-6 \\ -3x<-24 & -3x>12 \\ \frac{-3}{-3}x>-\frac{24}{-3} & \frac{-3}{-3}x<\frac{12}{-3} \\ x>8 & x<-4 \end{array}$$

Solution: $\underline{(-\infty,-4)\cup(8,\infty)}$

Special Cases

Example

Solve the inequality $|2-5x|\ge -4$

Solution

$|2-5x|\ge -4$

It is always ***true***

$\therefore$ The solution set is: $\mathbb{R}$ All real numbers $(-\infty,\ \infty)$

Example

Solve the inequality $|4x-7|<-3$

Solution

$|4x-7|<-3$

Any absolute value can't be less than any negative number.

$\therefore$ No solution or $\varnothing$

Example

Solve the inequality $|5x+15|=0$

Solution

$|5x+15|=0$

$$5x+15=0$$
$$5x=-15$$

$\therefore$ Solution: $\underline{x=-3}$

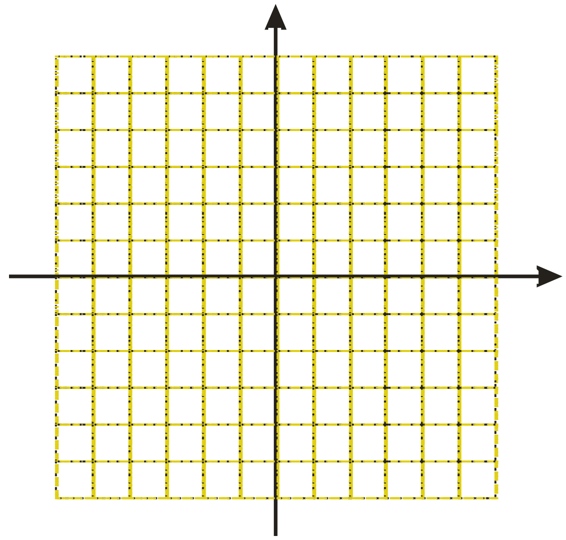
Definition of a Polynomial Inequality

A polynomial inequality is any inequality that can be put into one of the forms

$$f(x) < 0 \qquad f(x) > 0 \qquad f(x) \le 0 \qquad f(x) \ge 0$$

Where f is a polynomial function.

$f(x) = x^2 - 5x + 4 \quad (x = 1, 4)$



Procedure for Solving Polynomial Inequalities — *Example*

1. Express the inequality in the form $f(x)$? 0	$x^2 - x < 12$ $x^2 - x - 12 < 0$
2. Solve $f(x) = 0$	$x^2 - x - 12 = 0$ $x = -3,\ 4$
3. Locate the boundary 4. Choose one test value	$-3 \quad 0 \quad 4$ $+ \quad - \quad +$
5. Write the solution set	$(-3,\ 4)$

$$\checkmark \quad ax^2 + bx + c \ge 0 \ \rightarrow \ \text{if } a > 0 \ \Rightarrow \ x \le x_1, \ \ x \ge x_2$$

$$\checkmark \quad ax^2 + bx + c \le 0 \ \rightarrow \ \text{if } a > 0 \ \Rightarrow \ x_1 \le x \le x_2$$

Example

Solve $2x^2+5x-12\ge 0$

Solution

$$2x^2+5x-12=0$$
$$(2x-3)(x+4)=0$$

$$x=-4,\ \frac{3}{2}$$

	-4	0	$\frac{3}{2}$	
$+$		$-$		$+$

$$(-\infty,\ -4]\ \cup\ \left[\frac{3}{2},\ \infty\right)$$

Solution: $x\le -4 \quad x\ge \frac{3}{2}$

Example

Solve: $x^3+3x^2\le x+3$

Solution

$$x^3+3x^2-x-3=0$$
$$x^2(x+3)-(x+3)=0$$
$$(x+3)(x^2-1)=0$$

$x+3=0 \qquad x^2-1=0$

$x=-3 \qquad x^2=1$

$x=-3 \qquad x=\pm 1$

	-3		-1	0	1	
$-$		$+$		$-$		$+$

Solution: $(-\infty,\ -3]\cup[-1,\ 1]$

Rational Inequality

Example

Solve: $\dfrac{2x}{x+1} \ge 1$

Solution

$$\frac{2x}{x+1} = 1 \qquad \to \ \textit{Cond.}: x+1 \ne 0 \Rightarrow \ \boxed{x \ne -1}$$

$$(x+1)\frac{2x}{x+1} - 1(x+1) = 0$$

$$2x - x - 1 = 0$$

$$x - 1 = 0$$

$$x = 1$$

-1	0	1
$+$	$-$	$+$

Solution: $\boxed{x \le -1 \quad x \ge 1}$ $\qquad \boxed{(-\infty,\ -1) \cup [1,\ \infty)}$

Example

Solve $\dfrac{5}{x+4} \ge 1$

Solution

$$\frac{5}{x+4} - 1 = 0 \qquad \textit{Exception}: \ x+4 \ne 0 \ \Rightarrow x \ne -4$$

$$(x+4)\frac{5}{x+4} - 1(x+4) = 0$$

$$5 - x - 4 = 0$$

$$1 - x = 0$$

$$\boxed{x = 1}$$

-4	0	1
$-$	$+$	$-$

Solution: $\boxed{-4 < x \le 1}$ $\qquad \boxed{(-4,\ 1]}$

Example

Solve $\dfrac{2x-1}{3x+4} < 5$

Solution

$$\frac{2x-1}{3x+4} - 5 = 0 \qquad \textit{Restriction}: \ 3x+4 \ne 0 \ \Rightarrow \ \boxed{x \ne -\frac{4}{3}}$$

$$(3x+4)\frac{2x-1}{3x+4} - 5(3x+4) = 0$$

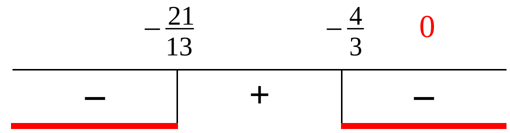
$$2x - 1 - 15x - 20 = 0$$

$$-13x - 21 = 0$$

$x = -\frac{21}{13}$

Solution: $x < -\frac{21}{13} \quad x > -\frac{4}{3}$

$\left(-\infty,\ -\frac{21}{13}\right) \cup \left(-\frac{4}{3},\ \infty\right)$



Exercises *Section* 1.5 – Inequalities

Find:

1. $(-3,0)\cap[-1,2]$

2. $(-3,0)\cup[-1,2]$

3. $(-4,0)\cap[-2,1]$

4. $(-4,0)\cup[-2,1]$

5. $(-\infty,5)\cap[1,8)$

6. $(-\infty,5)\cup[1,8)$

Solve the inequality equation

7. $-3x+5>-7$

8. $2-3x\le 5$

9. $4-3x\le 7+2x$

10. $5x+11<26$

11. $3x-8\ge 13$

12. $-9x\ge 36$

13. $-4x\le 64$

14. $8x-11\le 3x-13$

15. $18x+45\le 12x-8$

16. $4(x+1)+2\ge 3x+6$

17. $8x+3>3(2x+1)+x+5$

18. $2x-11<-3(x+2)$

19. $-4(x+2)>3x+20$

20. $1-(x+3)\ge 4-2x$

21. $5(3-x)\le 3x-1$

22. $\frac{x}{4}-\frac{1}{2}\le\frac{x}{2}+1$

23. $\frac{3x}{10}+1\ge\frac{1}{5}-\frac{x}{10}$

24. $6x-(2x+3)\ge 4x-5$

25. $\frac{2x-5}{-8}\le 1-x$

26. $1-\frac{x}{2}>4$

27. $7-\frac{4}{5}x<\frac{3}{5}$

28. $\frac{x-4}{6}\ge\frac{x-2}{9}+\frac{5}{18}$

29. $\frac{4x-3}{6}+2\ge\frac{2x-1}{12}$

30. $4(3x-2)-3x<3(1+3x)-7$

31. $3(x-8)-2(10-x)<5(x-1)$

32. $8(x+1)\le 7(x+5)+x$

33. $4(x-1)\ge 3(x-2)+x$

34. $7(x+4)-13>12+13(3+x)$

35. $-2[7x-(2x-3)]<-2(x+1)$

36. $6-\frac{2}{3}(3x-12)\le\frac{2}{5}(10x+50)$

37. $\frac{2}{7}(7-21x)-4<10-\frac{3}{11}(11x-11)$

38. $3[3(x+5)+8x+7]+5[3(x-6)-2(3x-5)]<2(4x+3)$

39. $5[3(2-3x)-2(5-x)]-6[5(x-2)-2(4x-3)]<3x+19$

40. $0\le 3x-1\le 10$

41. $0\le 1-3x\le 10$

42. $0\le 2x+6\le 54$

43. $-3\le\frac{2}{3}x-5\le -1$

44. $-6\le 6x+3\le 21$

45. $1\le 2x+3\le 11$

Solve the inequality equation

46. $|x| < 2$

47. $|x| \ge 2$

48. $|x-2| < 1$

49. $|x-1| < 4$

50. $|x+2| \ge 1$

51. $|x+1| \ge 4$

52. $|3x+5| < 17$

53. $|5x-2| < 13$

54. $|5x-2| \ge 13$

55. $|2(x-1)+4| \le 8$

56. $|3(x-1)+2| \le 20$

57. $\left|\frac{2x+6}{3}\right| > 2$

58. $\left|\frac{3x-3}{4}\right| < 6$

59. $\left|\frac{2x+2}{4}\right| \ge 2$

60. $\left|\frac{3x-3}{9}\right| \le 1$

61. $\left|3-\frac{2x}{3}\right| > 5$

62. $\left|3-\frac{3x}{4}\right| < 9$

63. $|x-2| < -1$

64. $|x+2| < -3$

65. $|x+6| > -10$

66. $|x+2| > -8$

67. $|x+2|+9 \le 16$

68. $|x-2|+4 \ge 5$

69. $2|2x-3|+10 > 12$

70. $3|2x-1|+2 < 8$

71. $-4|1-x| < -16$

72. $-2|5-x| < -6$

73. $3 \le |2x-1|$

74. $9 \le |4x+7|$

75. $12 < \left|-2x+\frac{6}{7}\right|+\frac{3}{7}$

76. $4+\left|3-\frac{x}{3}\right| \ge 9$

77. $|x-2| < 5$

78. $|2x+1| < 7$

79. $|5x+2|-2 < 3$

80. $|2-7x|-1 > 4$

81. $|3x-4| < 2$

82. $|2x+5| \ge 3$

83. $|12-9x| \ge -12$

84. $|6-3x| < -11$

85. $|7+2x| < 0$

Solve the inequality equation

86. $x^2-7x+10 > 0$

87. $2x^2-9x \le 18$

88. $x^2-5x+4 > 0$

89. $x^2+x-2 > 0$

90. $x^2-4x+12 < 0$

91. $x^2+7x > 0$

92. $x^2-49 < 0$

93. $x^2-5x \ge 0$

94. $x^2-16 \le 0$

95. $x^2+7x+10 < 0$

96. $x^2-3x \ge 28$

97. $x^2+5x+6 < 0$

98. $x^2 < -x+30$

99. $x^3-3x^2-9x+27 < 0$

100. $x^3-x > 0$

101. $x^3+3x^2 \le x+3$

102. $x^3+x^2 \ge 48x$

103. $x^3-x^2-16x+16 < 0$

104. $x^3+x^2-9x-9 > 0$

105. $x^3+3x^2-4x-12 \ge 0$

106. $x^4-20x^2+64 \le 0$

107. $x^4-10x^2+9 \ge 0$

Solve the inequality equation

108. $\frac{x+4}{x-1}<0$

109. $\frac{x-2}{x+3}>0$

110. $\frac{x-5}{x+8}\ge 3$

111. $\frac{x-4}{x+6}\le 1$

112. $\frac{x}{2x+7}\ge 4$

113. $\frac{x}{3x-5}\le -5$

114. $\frac{x+2}{x-5}\le 2$

115. $\frac{3x+1}{x-2}\ge 4$

116. $\frac{x}{x-3}>0$

117. $\frac{x-3}{x+2}\ge 0$

118. $\frac{x-2}{x+2}\le 2$

119. $\frac{x+2}{x-2}\ge 2$

120. $\frac{x+2}{3+2x}\le 5$

121. $\frac{x+6}{x-14}\ge 1$

122. $\frac{x-3}{x+4}\ge \frac{x+2}{x-5}$

123. $\frac{x-4}{x+3}-\frac{x+2}{x-1}\le 0$

124. $\frac{2x-1}{x+3}\ge \frac{x+1}{3x+1}$

125. $\frac{(x+1)(x-4)}{x-2}<0$

126. $\frac{x(x-4)}{x+5}>0$

127. $\frac{6x^2-11x-10}{x}>0$

128. $\frac{3x^2-2x-8}{x-1}\ge 0$

129. $\frac{x^2-6x+9}{x-5}\le 0$

130. $\frac{x^2+10x+25}{x+1}\ll 0$

131. A car can be rented from Basic Rental for \$260 per week with no extra charge for mileage. Continental charges \$80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

132. If a projectile is launched from ground level with an initial velocity of 96 *ft.* per *sec*, its height in feet ***t*** seconds after launching is ***s*** feet, where

$$s=-16t^2+96t$$

When will the projectile be greater than 80 *ft*. above the ground?

133. A projectile is fired straight up from ground level. After ***t*** seconds, its height above the ground is ***s*** *ft*., where

$$s=-16t^2+220t$$

For what time period is the projectile at least 624 *ft*. above the ground?

134. Your test scores of 70 and 81 in your math class. To receive a *C* grade, you must obtain an average greater than or equal to 72 but less than 82. What range of test scores on the one remaining test will enable you to get a *C* for the course.

135. A truck can be rented from Basic Rental for \$50 a day plus \$0.20 per *mile*. Continental charges \$20 per day plus \$0.50 per *mile* to rent the same truck. How many miles must be driven in a day to make the rental cost for Basic Rental a better deal than Constiental's?

136. You are choosing between two telephone plans. Plan ***A*** has a monthly fee of \$15 with a charge of \$0.08 per *minute* for all calls. Plan ***B*** has a monthly fee of \$3 with a charge of \$0.12 per *minute* for all calls. How many calling minutes in a month make plan ***A*** the better deal?

137. A City commission has proposed two tax bills. The first bill requires that a homeowner pay $1,800 plus 3% of the assesses home value in taxes. The second bill requires taxes of $200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal for the homeowner?

138. A local bank charges $8 per month plus $0.05 per check. The credit union charges $2 per month $0.08 per check. How many checks should be written each month to make the credit union a better deal?

139. A company manufactures and sells blank audiocassette tapes. The weekly fixed cost is $10,000 and it costs $0.40 to produce each tape. The selling price is $2.00 per tape. How many tapes must be produced and sold each week for the company to have a profit?

140. A company manufactures and sells stationery. The weekly fixed cost is $3,000 and it costs $3.00 to produce each package of stationery. The selling price is $5.50 per package. How many packages of stationery must be produced and sold each week for the company to have a profit?

141. An elevator at a construction site has a maximum capacity of 3,000 *pounds*. If the elevator operator weighs 200 *pounds* and each cement bag weighs 70 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

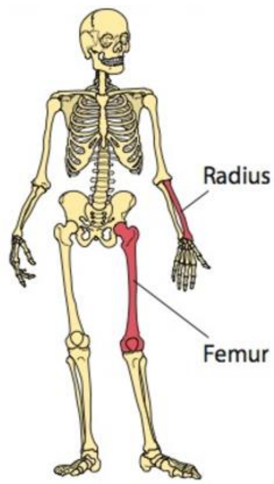
142. An elevator at a construction site has a maximum capacity of 2,500 *pounds*. If the elevator operator weighs 160 *pounds* and each cement bag weighs 60 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

143. You can rent a car for the day from Company ***A*** for $29.00 plus $0.12 a *mile*. Company ***B*** charges $22.00 plus $0.21 a *mile*. Find the number of miles m per day for which it is cheaper to rent from Company ***A***.

144. UPS will only ship packages for which the length is less than or equal to 108 *inches* and the length plus the girth is less than or equal to 130 *inches*. The length of a package is defined as the length of the longest side. The girth is defined as twice the width plus twice the height of the package. If a box has a length of 34 *inches* and a width of 22 *inches,* determine the possible range of heights h for this package if you wish to ship it by UPS.

145. The sum of three consecutive odd integers is between 63 and 81. Find all possible sets of integers that satisfy these conditions.

146. Forensic specialists can estimate the height of a deceased person from the lengths of the person's bones. For instance, an inequality that relates the height h, in *cm*, of an adult female and the length f, in *cm*, of her femur is $\left|h-(2.47f+54.10)\right| \le 3.72$. Use the inequalities to estimate the possible range of heights for an adult female whose measures 32.24 *cm*.



147. An inequality that is used to calculate the height h of an adult male from the length r of his radius is

$$\left|h-(3.32r+85.43)\right| \leq 4.57$$

Where h and r are both in *cm*. Use this inequality to estimate the possible range of heights for an adult male whose radius measures 26.36 *cm*.