Section 3.3 – Absolute Extrema

Absolute Extrema

Let f be defined on an interval I containing c.

1. $f(c)$ is an ***absolute minimum*** of f on I if $f(c) \le f(x)$ for every x in I.

2. $f(c)$ is an ***absolute maximum*** of f on I if $f(c) \ge f(x)$ for every x in I.

The *absolute minimum* and *absolute maximum* values of a function on an interval are sometimes called the minimum and maximum of f on I.

Example

Find the minimum and maximum values of $f(x) = x^2 - 8x + 10$ on the interval [0, 7].

<u>Solution</u>

$$f'(x) = 2x - 8 = 0$$

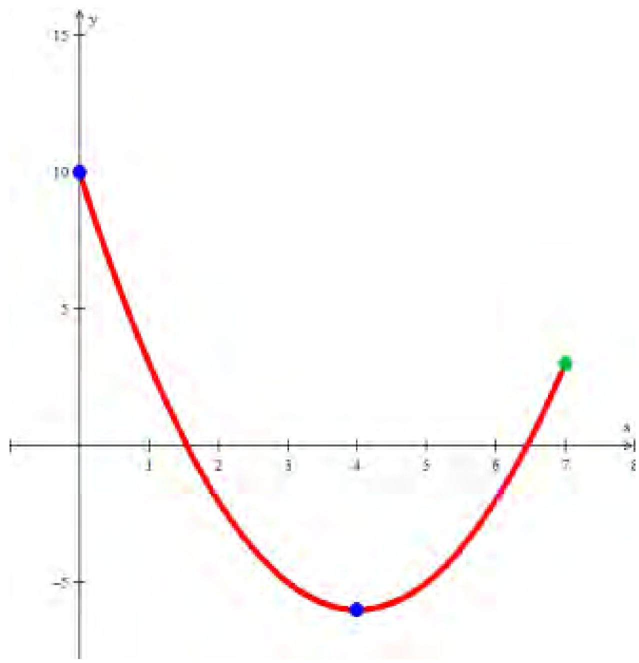
$$\Rightarrow x = 4 \quad (CN)$$

$$\rightarrow y = 16 - 32 + 10 = -6$$

$$\begin{cases} x = 0 \rightarrow y = 10 \\ x = 7 \rightarrow y = 3 \end{cases}$$

Absolute Maximum (0, 10)

Absolute Minimum (4, -6)



Example

Find the absolute extrema of $f(x) = x^{8/3} - 16x^{2/3}$ on the interval $[-1,\ 8]$.

<u>*Solution*</u>

$$f'(x) = \frac{8}{3}x^{5/3} - \frac{32}{3}x^{-1/3}$$

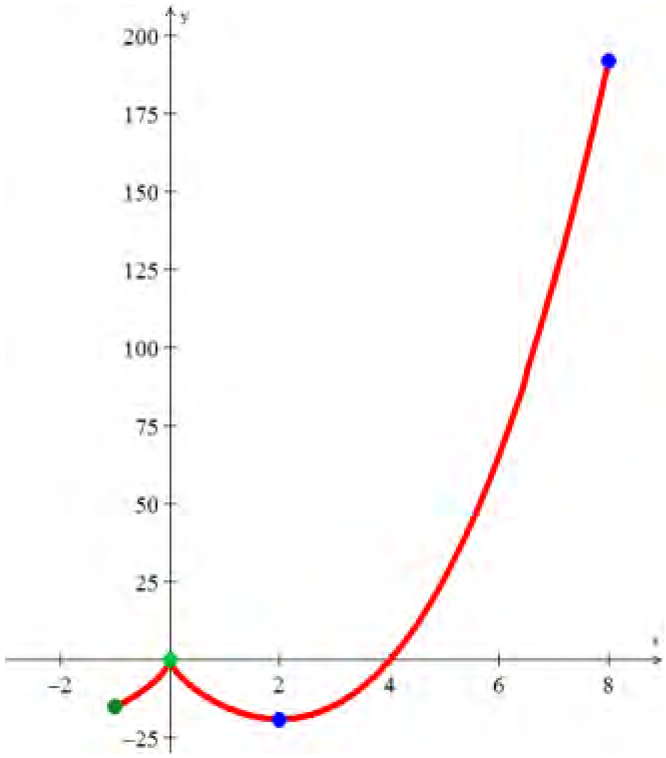
$$= \frac{8}{3}\left(x^{5/3} - \frac{4}{x^{1/3}}\right)$$

$$= \frac{8}{3}\left(\frac{x^2-4}{x^{1/3}}\right) = 0$$

$$CN: \boxed{x = \pm 2}$$

$x \neq -2 \notin [-1,\ 8]$

The derivative is undefined at $\boxed{x=0}$



x	$f(x)$
-1	-15
0	0
2	-19.05
8	192

Absolute Maximum (8, 192)

Absolute Minimum (2, -19.05)

Example

Based on data from the U.S. Census Bureau, the number of people (in millions) in the US below poverty level between 1999 and 2006 can be approximated by the function

$$p(t) = -0.0982t^3 + 1.210t^2 - 3.322t + 34.596$$

Where t is the number of years since March 1999. Based on this approximation,

In what year during this period did the number of people living below the poverty level reach its absolute maximum? What was the maximum number of people living below the poverty level during that period?

<u>Solution</u>

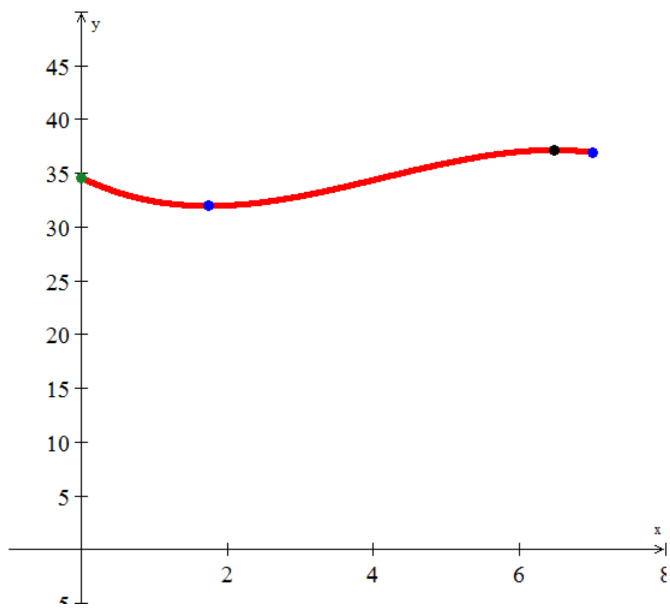
$$p'(t) = -0.2946t^2 + 2.42t - 3.322 = 0$$

$$\Rightarrow \boxed{t = 1.74,\ \ 6.47}$$

Based on the interval year from 1999 – 2006 $\Rightarrow [0,\ 7]$

t	$p(t)$
0	34.6
1.74	32
6.47	**37.2**
7	36.9

About 6.47 years after March 1999. And about 37.2 million people were below poverty level.



Exercise Section 3.3 – Absolute Extrema

1. Find the absolute extrema of the function on the closed interval $f(x) = 2(3 - x), \quad [-1, 2]$

2. Find the absolute extrema of the function on the closed interval $f(x) = x^3 - 3x^2, \quad [0, 4]$

3. Find the absolute extrema of the function on the closed interval

$$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x - 4, \quad [-2, 5]$$

4. Find the absolute extrema of the function on the closed interval $f(x) = \frac{1}{x+2}, \quad [-4, 1]$

5. Find the absolute extrema of the function on the closed interval $f(x) = \left(x^2 + 4\right)^{2/3}, \quad [-2, 2]$

6. $P(x) = -x^3 + \frac{27}{2}x^2 - 60x + 100, \quad x \ge 5$ is an approximation to the total profit (in thousands of dollars) from the sale of x hundred thousand tires. Find the number of hundred thousands of tires that must be sold to maximize profit.

7. $P(x) = -x^3 + 12x^2 - 36x + 400, \quad x \ge 3$ is an approximation to the total profit (in thousands of dollars) from the sale of x hundred thousand tires. Find the number of hundred thousands of tires that must be sold to maximize profit.

8. Researchers have discovered that by controlling both the temperature and the relative humidity in a building, the growth of a certain fungus can be limited. The relationship between temperature and relative humidity, which limits growth, can be described by

$$R(T) = -0.00008T^3 + 0.386T^2 - 1.6573T + 97.086, \quad 0 \le T \le 46$$

where $R(T)$ is the relative humidity (in %) and T is the temperature (in °C). Find the temperature at which the relative humidity is minimized.