

# HWK 1.3-4

## Soln

$$1. \lim_{x \rightarrow 1^+} \frac{1}{x-1} = \frac{1}{0^+} = \infty$$

$$2. \lim_{x \rightarrow 1^-} \frac{1}{x-1} = \frac{1}{0^-} = -\infty$$

$$3. \lim_{x \rightarrow \infty} (x + \sqrt{x^2 - 4x + 1}) = \infty + \infty = \infty$$

$$4. \lim_{x \rightarrow -\infty} (x + \sqrt{x^2 - 4x + 2}) = -\infty + \infty$$

$$= \lim_{x \rightarrow -\infty} (x + \sqrt{x^2 - 4x + 2}) \cdot \frac{x - \sqrt{x^2 - 4x + 2}}{x - \sqrt{x^2 - 4x + 2}}$$

$$= \lim_{x \rightarrow -\infty} \frac{x^2 - x^2 + 4x - 2}{x - \sqrt{x^2 - 4x + 2}}$$

$$= \lim_{x \rightarrow -\infty} \frac{4x - 2}{x - |x|} \quad (\text{leading Term})$$

$$= \lim_{x \rightarrow -\infty} \frac{4x}{-2|x|}$$

$$= -2$$

$$5. \lim_{x \rightarrow \infty} \frac{2x + 100}{x^2 + 3} = 0$$

$$6. \lim_{x \rightarrow \infty} \frac{1 - x^2}{3x^2 - x - 1} = -\frac{1}{3}$$

$$7. \lim_{x \rightarrow \infty} \frac{x^3 - 2}{x^2 + 5} = \infty \quad \lim_{x \rightarrow \infty} \lim_{x \rightarrow \infty} \neq x$$

$$8. \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x^2 + 1}} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0 = \frac{1}{\infty}$$

$$9. \lim_{x \rightarrow \infty} \sqrt{x} = \infty$$

$$\frac{1}{x} \rightarrow \frac{1}{\infty} = 0$$

$$10. \lim_{x \rightarrow \infty} \frac{\cos x}{x}$$

$$0 \begin{cases} -1 \leq \cos x \leq 1 \\ \frac{-1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x} \end{cases}$$

$$\lim_{x \rightarrow \infty} \left| \frac{1}{x} \right| = 0$$

$$\lim_{x \rightarrow \infty} \frac{\cos x}{x} = 0$$

$$11. \lim_{x \rightarrow 0} \sin\left(\frac{1}{x^2}\right) = \sin \frac{1}{0} \\ = \sin \infty \\ = \text{DNE}$$

$$12. \lim_{x \rightarrow \infty} \sin x = \sin \infty \\ = \text{DNE}$$

$$13. \lim_{x \rightarrow -\infty} e^{x^2} = e^{\infty} = \infty$$

$$14. \lim_{x \rightarrow -\infty} e^{x^3} = e^{-\infty} = 0 \quad \left( \frac{1}{e^{\infty}} \right)$$

$$15. \lim_{x \rightarrow -\infty} \ln|x| = \ln \infty = \infty$$

Lect 2.

$$2.1 \quad m = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{x - x - h}{x(x+h)} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{-h}{x(x+h)} \right)$$

$$= - \lim_{h \rightarrow 0} \frac{1}{x^2 + hx}$$

$$= \left[ -\frac{1}{x^2} \right]$$

$$\frac{d}{dx} \left( \frac{1}{x} \right) = -\frac{1}{x^2}$$

$f(x)$  derivative:  $f'(x)$ ,  $f'$ ,  $\frac{df}{dx}$   
 $\frac{dy}{dx}$ ,  $y'$ ,  $\underbrace{D_x(y)}$

$$f(x) = \frac{x}{x-1} -$$

determinant 2x2

$$f'(x) = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{x+h}{x+h-1} - \frac{x}{x-1} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \frac{x^2 - x + hx - h - x^2 - hx + x}{(x-1)(x+h-1)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \frac{-h}{(x-1)(x+h-1)}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{(x-1)(x+h-1)}$$

$$= \frac{-1}{(x-1)(x-1)}$$

$$= \frac{-1}{(x-1)^2}$$



1) def of limit.  $\lim_{x \rightarrow x_0} f(x) = L$   
 $\forall \varepsilon > 0, \exists \delta > 0 \exists \forall x$   
 $|x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$f(x) = \sqrt{x-7} \quad L=4 \quad x_0=23 \quad \varepsilon=1$$

$$-5 < x-23 < 5$$

$$23-5 < x < 23+5$$

$$-1 < \sqrt{x-7} - 4 < 1$$

$$3 < \sqrt{x-7} < 5$$

$$9 < x-7 < 25$$

$$16 < x < 32$$

$$\begin{cases} 23-5=16 \rightarrow \delta=7 \\ 23+5=32 \rightarrow \delta=9 \end{cases}$$

$$\therefore \delta = 7$$

$$4. \lim_{x \rightarrow 0} \frac{\sin 5x}{3x} = \frac{5}{3}$$

$$5. \lim_{\theta \rightarrow 0} \frac{\theta \cot 4\theta}{\sin^2 \theta \cot^2 \theta} = \frac{0}{0}$$

$$= \lim_{\theta \rightarrow 0} \frac{\theta}{\sin \theta} \cdot \frac{1}{\sin \theta} \cdot \frac{\cos 4\theta}{\sin 4\theta} \cdot \frac{1}{\frac{\cos 2\theta}{\sin 2\theta}}$$

$$= \lim_{\theta \rightarrow 0} \frac{\theta}{\sin 4\theta} = \frac{0}{0}$$

$$= \lim_{\theta \rightarrow 0} \frac{\theta}{2 \sin 2\theta \cos 2\theta}$$

$$= \frac{1}{2} \lim_{\theta \rightarrow 0} \frac{1}{\cos 2\theta} \lim_{\theta \rightarrow 0} \frac{1}{2 \frac{\sin 2\theta}{2\theta}} = \frac{1}{1}$$

$$\begin{aligned}
 6 - \lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4} &= \frac{a^2 - a^2}{a^4 - a^4} = \frac{0}{0} \\
 &= \lim_{x \rightarrow a} \frac{x^2 - a^2}{(x^2 - a^2)(x^2 + a^2)} \\
 &= \lim_{x \rightarrow a} \frac{1}{x^2 + a^2} \\
 &= \frac{1}{2a^2}
 \end{aligned}$$

$$\begin{aligned}
 11 \lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{-x} &= \frac{\frac{1}{2} - \frac{1}{2}}{0} = \frac{0}{0} \\
 &= \lim_{x \rightarrow 0} \frac{1}{x} \left( \frac{2 - 2 - x}{2(x+2)} \right) \\
 &= \lim_{x \rightarrow 0} \frac{-x}{2x(x+2)} \\
 &= -\frac{1}{2} \lim_{x \rightarrow 0} \frac{1}{x+2} \\
 &= -\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 12 \lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8} - 3}{x+1} &= \frac{3-3}{0} = \frac{0}{0} \\
 &= \lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8} - 3}{x+1} \cdot \frac{\sqrt{x^2 + 8} + 3}{\sqrt{x^2 + 8} + 3} \\
 &= \lim_{x \rightarrow -1} \frac{x^2 + 8 - 9}{(x+1)(\sqrt{x^2 + 8} + 3)} \\
 &= \lim_{x \rightarrow -1} \frac{(x+1)(x-1)}{(x+1)(\sqrt{x^2 + 8} + 3)} \\
 &= \lim_{x \rightarrow -1} \frac{x-1}{\sqrt{x^2 + 8} + 3} \\
 &= \frac{-2}{6} \\
 &= -\frac{1}{3}
 \end{aligned}$$

$$\underline{15} \quad \lim_{x \rightarrow \infty} \frac{x + \sin x + 2\sqrt{x}}{x + \sin x} = \lim_{x \rightarrow \infty} \frac{x}{x} = \underline{1}$$

$$\underline{\# 16} \quad \lim_{x \rightarrow -\infty} \left( \frac{x^2 + x - 1}{8x^2 - 3} \right)^{1/3} = \left( \frac{1}{8} \right)^{1/3} = \underline{\frac{1}{2}} \quad 8^{1/3} = 2^{3^{1/3}}$$

$$\underline{\# 17} \quad \lim_{x \rightarrow -\infty} \frac{4 - 3x^3}{\sqrt{x^6 + 9}} = \lim_{x \rightarrow -\infty} \frac{-3x^3}{x^3} = \underline{-3}$$

$$f = |x-1| + \sin x$$

$f$  is continuous everywhere

$$f = \frac{x}{x^2 - 3x + 2} \quad (x \neq 1, 2)$$

$f$  is continuous everywhere except when  $x = 1, 2$ .

$$\underline{\# 18} \quad \lim_{x \rightarrow -\infty} \frac{x^2 - 4x + 5}{3x^3} = \underline{0}$$

$$\underline{\# 19} \quad \lim_{x \rightarrow \infty} \frac{2x^2 + 3}{5x^2 + 7} = \underline{\frac{2}{5}}$$

$$\underline{20} \quad \lim_{x \rightarrow \infty} \frac{x^4 + x^3}{12x^3 + 126} = \underline{\infty}$$

$$1.2 \quad \lim_{x \rightarrow -2} (x^3 - 2x^2 + 4x + 8) = -8 - 8 - 8 + 8 = \underline{-16}$$

$$\frac{0}{0} \quad \left\{ \begin{array}{l} \text{factor} \\ \text{conjugate } \frac{+}{+} \\ \text{conjugate } \frac{-}{-} \end{array} \right.$$

$$\lim_{x \rightarrow 4} \frac{x-1}{\sqrt{x+3}+2} = \frac{0}{4} = \underline{0}$$

$$\frac{\neq 0}{0} = \infty$$

$$\text{sign } \infty \rightarrow a^+ \text{ or } a^-$$

$$1.3 \rightarrow 1.2 \quad \lim_{x \rightarrow 3^-} \frac{2}{(x-3)^3} = \frac{2}{0^-} = -\infty$$

$$\lim_{x \rightarrow 3^+} \frac{2}{(x-3)^3} = \frac{2}{0^+} = \infty$$

