Solution Section 2.1 – Vectors in 2-Space, 3-Space, and *n*-Space

Exercise

Sketch the following vectors with initial points located at the origin

a) $P_1(4,\ 8),\quad P_2(3,\ 7)$

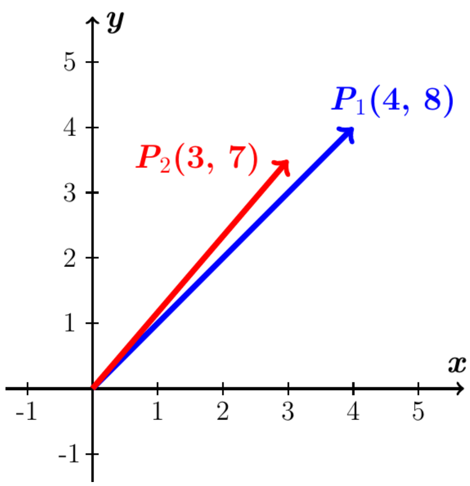
b) $P_1(0,\ -2),\quad P_2(-3,\ 5)$

c) $P_1(-1,\ 0,\ 2),\quad P_2(0,\ -1,\ 0)$

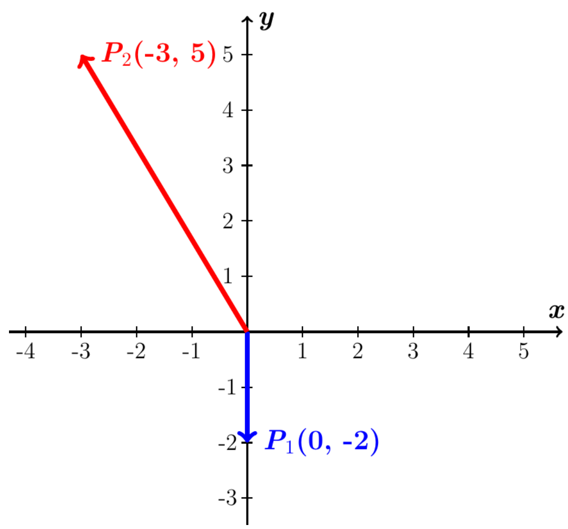
d) $P_1(3,\ -7,\ 2),\quad P_2(-2,\ 5,\ -4)$

Solution

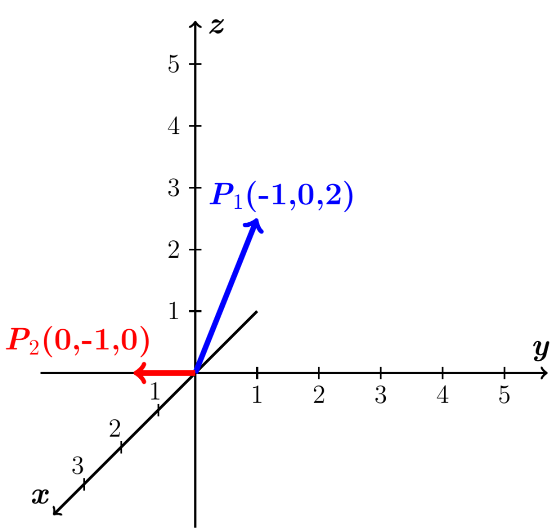
a)



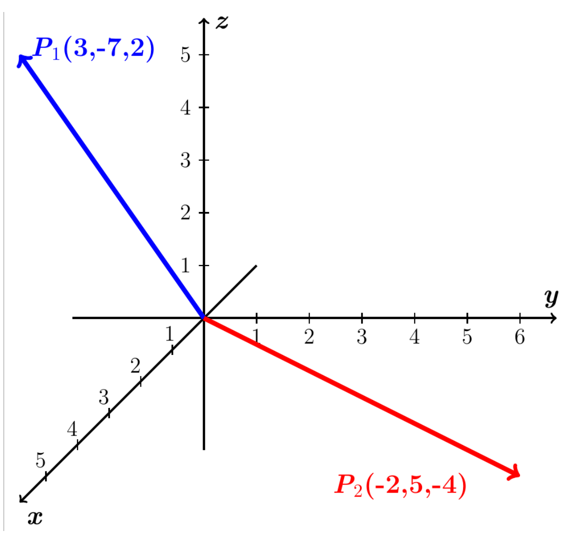
b)



c)



d)



Exercise

Find the components of the vector $\overrightarrow{P_1P_2}$

a) $P_1(3,\ 5)\quad P_2(2,\ 8)$

b) $P_1(-3,\ 2),\quad P_2(4,\ -5)$

c) $P_1(5,\ -2,\ 1)\quad P_2(2,\ 4,\ 2)$

d) $P_1(0,\ 0,\ 0)\quad P_2(-1,\ 6,\ 1)$

Solution

a) $\overrightarrow{P_1P_2} = (2-3,\ 8-5)$

$= (-1,\ 3)$

b) $\overrightarrow{P_1P_2} = (4+3,\ -5-2)$

$= (7,\ -7)$

c) $\overrightarrow{P_1P_2} = (2-5,\ 4-(-2),\ 2-1)$

$= (-3,\ 6,\ 1)$

d) $\overrightarrow{P_1P_2} = (-1-0,\ 6-0,\ 1-0)$

$= (-1,\ 6,\ 1)$

Exercise

Find the terminal point of the vector that is equivalent to $\vec{u} = (1, 2)$ and whose initial point is $A(1,\ 1)$

Solution

The terminal point: $B(b_1,\ b_2)$

$(b_1 - 1,\ b_2 - 1) = (1,\ 2)$

$$\begin{cases} b_1 - 1 = 1 & \Rightarrow b_1 = 2 \\ b_2 - 1 = 2 & \Rightarrow b_2 = 3 \end{cases}$$

The terminal point: $B(2,\ 3)$

Exercise

Find the initial point of the vector that is equivalent to $\vec{u} = (1, 1, 3)$ and whose terminal point is $B(-1, -1, 2)$

Solution

The initial point: $A(x, y, z)$

$$(-1-x,\ -1-y,\ 2-z) = (1,\ 1,\ 3)$$

$$\begin{cases} -1-x=1 & \Rightarrow x=-2 \\ -1-y=1 & \Rightarrow y=-2 \\ 2-z=3 & \Rightarrow z=-1 \end{cases}$$

The initial point: $\underline{A(-2,\ -2,\ -1)}$

Exercise

Find a nonzero vector $\vec{u}$ with initial point $P(-1, 3, -5)$ such that

a) $\vec{u}$ has the same direction as $\vec{v} = (6, 7, -3)$

b) $\vec{u}$ is oppositely directed as $\vec{v} = (6, 7, -3)$

Solution

a) $\vec{u}$ has the same direction as $\vec{v}$

$\boldsymbol{u} = \vec{v} = (6, 7, -3)$

The initial point $P(-1, 3, -5)$ then the terminal point:

$$(-1+6,\ 3+7,\ -5-3) = \underline{(5,\ 10,\ -8)}$$

b) $\vec{u}$ is oppositely directed as $\vec{v} = (6, 7, -3)$

$\vec{u} = -\vec{v} = (-6, -7, 3)$

The initial point $P(-1, 3, -5)$ then the terminal point:

$$(-1-6,\ 3-7,\ -5+3) = \underline{(-7,\ -4,\ -2)}$$

Exercise

Let $\vec{u} = (-3, 1, 2)$, $\vec{v} = (4, 0, -8)$, and $\vec{w} = (6, -1, -4)$. Find the components

a) $\vec{v} - \vec{w}$

b) $6\vec{u} + 2\vec{v}$

c) $5(\vec{v} - 4\vec{u})$

d) $-3(\vec{v} - 8\vec{w})$

e) $(2\vec{u} - 7\vec{w}) - (8\vec{v} + \vec{u})$

f) $-\vec{u} + (\vec{v} - 4\vec{w})$

Solution

a) $\vec{v}-\vec{w}=(4-6,\ 0-(-1),\ -8-(-4))$

$=(-2,\ 1,\ -4)$

b) $6\vec{u}+2\vec{v}=(-18,6,12)+(8,0,-16)$

$=(-10,\ 6,\ -4)$

c) $5(\vec{v}-4\vec{u})=5(4-(-12),\ 0-4,\ -8-8)$

$=5(16,\ -4,\ -16)$

$=(80,\ -20,\ -80)$

d) $-3(\vec{v}-8\vec{w})=-3(4-48,\ 0-(-8),\ -8-(-32))$

$=-3(-44,\ 8,\ 24)$

$=(32,\ -24,\ -72)$

e) $(2\vec{u}-7\vec{w})-(8\vec{v}+\vec{u})=[(-6,2,\ 4)-(42,\ -7,\ -28)]-[(32,\ 0,\ -64)+(-3,\ 1,\ 2)]$

$=(-48,\ 9,\ 32)-(29,\ 1,\ -62)$

$=(-77,\ 8,\ 94)$

f) $-u+(v-4w)=(3,-1,-2)+[(4,0,-8)-(24,-4,-16)]$

$=(3,\ -1,\ -2)+(-20,\ 4,\ 8)$

$=(-17,\ 3,\ 6)$

Exercise

Let $\vec{u}=(4,\ -1,\ 3)$, $\vec{v}=(-4,\ 5,\ 2)$, and $\vec{w}=(-5,\ 0,\ -3)$.Find the components

a) $\vec{v}+\vec{w}$

b) $6\vec{u}-2\vec{v}$

c) $4(\vec{v}-3\vec{u})$

d) $-5(\vec{v}-6\vec{w})$

e) $(2\vec{u}-7\vec{w})-(8\vec{v}+\vec{u})$

f) $-\vec{u}+(\vec{v}-4\vec{w})$

Solution

a) $\vec{v}+\vec{w}=(-4,\ 5,\ 2)+(-5,\ 0,\ -3)$

$=(-9,\ 5,\ -1)$

b) $6\vec{u}-2\vec{v}=6(4,\ -1,\ 3)-2(-4,\ 5,\ 2)$

$=(24,\ -6,\ 18)-(-8,\ 10,\ 4)$

$=(32,\ 4,\ 14)$

c) $4(\vec{v}-3\vec{u})=4((-4,\ 5,\ 2)-3(4,\ -1,\ 3))$

$$= 4\left((-4,\ 5,\ 2) - (12,\ -3,\ 9)\right)$$
$$= 4(-16,\ 8,\ -7)$$
$$\boxed{= (-64,\ 32,\ -28)}$$

d) $-5(\vec{v} - 6\vec{w}) = -5\left((-4,\ 5,\ 2) - 6(4,\ 0,\ -3)\right)$

$$= -5\left((-4,\ 5,\ 2) - (24,\ 0,\ -18)\right)$$
$$= -5(-28,\ 5,\ 20)$$
$$\boxed{= (140,\ -25,\ -100)}$$

e) $(2\vec{u} - 7\vec{w}) - (8\vec{v} + \vec{u}) = \left(2(4,\ -1,\ 3) - 7(-5,\ 0,\ -3)\right) - \left((-4,\ 5,\ 2) + (4,\ -1,\ 3)\right)$

$$= (8,\ -2,\ 6) - (-35,\ 0,\ -21) - (0,\ 4,\ 5)$$
$$\boxed{= (43,\ -6,\ 22)}$$

f) $-\vec{u} + (\vec{v} - 4\vec{w}) = -(4,\ -1,\ 3) + (-4,\ 5,\ 2) - 4(-5,\ 0,\ -3)$

$$= (-4,\ 1,\ -3) + (-4,\ 5,\ 2) - (-20,\ 0,\ -12)$$
$$\boxed{= (12,\ 6,\ 11)}$$

Exercise

Let $\vec{u}$ = (2, 1, 0, 1, −1) and $\vec{v}$ = (−2, 3, 1, 0, 2). Find scalars a and b so that $a\vec{u} + b\vec{v} = (-8,\ 8,\ 3,\ -1,\ 7)$

Solution

$$a\vec{u} + b\vec{v} = a(2,\ 1,\ 0,\ 1,\ -1) + b(-2,\ 3,\ 1,\ 0,\ 2)$$
$$= (a - 2b,\ a + 3b,\ b,\ a,\ -a + 2b)$$
$$\boxed{= (-8,\ 8,\ 3,\ -1,\ 7)}$$

$$\begin{cases} a - 2b = -8 \\ a + 3b = 8 \\ b = 3 \\ a = -1 \\ -a + 2b = 7 \end{cases}$$

$\rightarrow$ $a = -1 \quad b = 3$ ***Unique solution***

Exercise

Find all scalars $c_1,\ c_2,\ and\ c_3$ such that $c_1(1,\ 2,\ 0)+c_2(2,\ 1,\ 1)+c_3(0,\ 3,\ 1)=(0,\ 0,\ 0)$

Solution

$$\begin{aligned} c_1(1,\ 2,\ 0)+c_2(2,\ 1,\ 1)+c_3(0,\ 3,\ 1) &= \left(c_1+2c_2,\ 2c_1+c_2+3c_3,\ c_2+c_3\right) \\ &= (0,\ 0,\ 0) \end{aligned}$$

$$\begin{cases} c_1+2c_2 \qquad\quad = 0 \\ 2c_1+c_2+3c_3 = 0 \\ \qquad\quad c_2+c_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \quad R_2-2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \quad -\tfrac{1}{3}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_1-2R_2 \\ \\ R_3-R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \quad \tfrac{1}{2}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_2+R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$c_1=c_2=c_3=0$$

Exercise

Find the distance between the given points $\begin{bmatrix}5 & 1 & 8 & -1 & 2 & 9\end{bmatrix}$, $\begin{bmatrix}4 & 1 & 4 & 3 & 2 & 8\end{bmatrix}$

Solution

$$d = \sqrt{(4-5)^2 + (1-1)^2 + (4-8)^2 + (3+1)^2 + (2-2)^2 + (8-9)^2}$$

$$= \sqrt{1+0+16+16+0+1}$$

$$= \sqrt{34}$$

Exercise

Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operation on $\vec{u} = (u_1, u_2)$ $\vec{v} = (v_1, v_2)$

$$\vec{u} + \vec{v} = (u_1 + v_1 + 1,\ u_2 + v_2 + 1) \qquad k\vec{u} = (ku_1,\ ku_2)$$

a) Compute $\vec{u} + \vec{v}$ and $k\vec{u}$ for $\vec{u}$ = (0, 4), $\vec{v}$ = (1, –3), and $k = 2$.

b) Show that (0, 0) $\neq \vec{0}$.

c) Show that (–1, –1) = 0.

d) Show that $\vec{u} + (-\vec{u}) = 0$ for $\vec{u} = (u_1, u_2)$

e) Find two vector space axioms that fail to hold.

Solution

a) $\vec{u} + \vec{v} = (0+1+1,\ 4-3+1)$

$$= (2,\ 2)$$

$$k\vec{u} = (ku_1,\ ku_2)$$

$$= (2(0),\ 2(4))$$

$$= (0,\ 8)$$

b) $(0,\ 0) + (u_1, u_2) = (0 + u_1 + 1,\ 0 + u_2 + 1)$

$$= (u_1 + 1,\ u_2 + 1)$$

$$\neq (u_1, u_2)$$

Therefore (0, 0) is not the zero vector **0** required (by Axiom).

c) $(-1,\ -1) + (u_1, u_2) = (-1 + u_1 + 1,\ -1 + u_2 + 1)$

$$= (u_1, u_2)$$

$$\left(u_1, u_2\right)+(-1,\ -1)=\left(u_1-1+1,\ u_2-1+1\right)$$
$$=\left(u_1, u_2\right)$$

Therefore (–1, –1) = **0** holds.

d) Let $\vec{u}=\left(u_1, u_2\right)$ &

$$-\vec{u}=\left(-2-u_1,\ -2-u_2\right)$$

$$\vec{u}+(-\vec{u})=\left(u_1+\left(-2-u_1\right)+1,\ u_2+\left(-2-u_2\right)+1\right)$$
$$=(-1,\ -1)$$
$$\underline{=\vec{0}}$$

$\vec{u}+(-\vec{u})=0$ holds

e) Axiom 7: $k(\vec{u}+\vec{v})\overset{?}{=}k\vec{u}+k\vec{v}$

$$k(\vec{u}+\vec{v})=k\left(u_1+v_1+1,\ u_2+v_2+1\right)$$
$$=\left(ku_1+kv_1+k,\ ku_2+kv_2+k\right)$$

$$k\vec{u}+k\vec{v}=\left(ku_1, ku_2\right)+\left(kv_1, kv_2\right)$$
$$=\left(ku_1+kv_1+1,\ ku_2+kv_2+1\right)$$

Therefore, $k(\vec{u}+\vec{v})\neq k\vec{u}+k\vec{v}$;

∴ Axiom 7 fails to hold

Axiom 8: $(k+m)\vec{u}\overset{?}{=}k\vec{u}+m\vec{u}$

$$(k+m)\vec{u}=\left((k+m)u_1,\ (k+m)u_2\right)$$
$$=\left(ku_1+mu_1,\ ku_2+mu_2\right)$$

$$k\vec{u}+m\vec{u}=\left(ku_1, ku_2\right)+\left(mu_1, mu_2\right)$$
$$=\left(ku_1+mu_1+1,\ ku_2+mu_2+1\right)$$

Therefore, $(k+m)\vec{u}\neq k\vec{u}+m\vec{u}$;

∴ Axiom 8 fails to hold

Exercise

Find $\vec{w}$ given that $10\vec{u}+3\vec{w}=4\vec{v}-2\vec{w}$, $\vec{u}=\begin{pmatrix}1\\-6\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}-20\\5\end{pmatrix}$.

Solution

$$-10\vec{u}+10\vec{u}+3\vec{w}+2\vec{w}=-10\vec{u}+4\vec{v}-2\vec{w}+2\vec{w}$$

$$5\vec{w}=-10\vec{u}+4\vec{v}$$

$$\vec{w}=-2\vec{u}+\frac{4}{5}\vec{v}$$

$$=-2\begin{pmatrix}1\\-6\end{pmatrix}+\frac{4}{5}\begin{pmatrix}-20\\5\end{pmatrix}$$

$$=\begin{pmatrix}-2\\12\end{pmatrix}+\begin{pmatrix}-16\\4\end{pmatrix}$$

$$=\begin{pmatrix}-18\\16\end{pmatrix}$$

Exercise

Find $\vec{w}$ given that $\vec{u}+3\vec{v}-2\vec{w}=5\vec{u}+\vec{v}-4\vec{w}$, $\vec{u}=\begin{pmatrix}1\\-1\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}-2\\3\end{pmatrix}$

Solution

$$\vec{u}-\vec{u}+3\vec{v}-3\vec{v}-2\vec{w}+4\vec{w}=5\vec{u}-\vec{u}+\vec{v}-3\vec{v}-4\vec{w}+4\vec{w}$$

$$2\vec{w}=4\vec{u}-2\vec{v}$$

$$\vec{w}=2\vec{u}-\vec{v}$$

$$=2\begin{pmatrix}1\\-1\end{pmatrix}+\begin{pmatrix}-2\\3\end{pmatrix}$$

$$=\begin{pmatrix}2\\-2\end{pmatrix}+\begin{pmatrix}-2\\3\end{pmatrix}$$

$$=\begin{pmatrix}0\\1\end{pmatrix}$$

Exercise

Find $\vec{w}$ given that $2\vec{u}+\vec{v}-3\vec{w}=5\vec{u}+7\vec{v}+3\vec{w}$, $\vec{u}=\begin{pmatrix}1\\-1\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}-2\\3\end{pmatrix}$

Solution

$$2\vec{u}-2\vec{u}+\vec{v}-\vec{v}-3\vec{w}-3\vec{w}=5\vec{u}-2\vec{u}+7\vec{v}-\vec{v}+3\vec{w}-3\vec{w}$$

$$-6\vec{w}=3\vec{u}+6\vec{v}$$

$$\vec{w} = -\frac{1}{2}\vec{u} - \vec{v}$$

$$= -\frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{2} \\ -\frac{5}{2} \end{pmatrix}$$

Exercise

Find $\vec{w}$ given that $\vec{u} - 2\vec{v} + 3\vec{w} = 5\vec{u} + 7\vec{v} - 2\vec{w}$, $\vec{u} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ *and* $\vec{v} = \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix}$

Solution

$$\vec{u} - 2\vec{v} + 3\vec{w} = 5\vec{u} + 7\vec{v} - 2\vec{w}$$

$$5\vec{w} = 4\vec{u} + 9\vec{v}$$

$$\vec{w} = \frac{1}{5}\left(4\vec{u} + 9\vec{v}\right)$$

$$\vec{w} = \frac{1}{5}\left(4\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 9\begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix}\right)$$

$$= \frac{1}{5}\left(\begin{pmatrix} 8 \\ -2 \\ 12 \end{pmatrix} + \begin{pmatrix} -18 \\ 45 \\ 36 \end{pmatrix}\right)$$

$$= \frac{1}{5}\begin{pmatrix} -10 \\ 43 \\ 48 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ \frac{43}{5} \\ \frac{48}{5} \end{pmatrix}$$

Exercise

Draw $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, and $\vec{u}+2\vec{v}$ $\quad \vec{u}=\begin{pmatrix}1\\-1\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}-2\\3\end{pmatrix}$

Solution

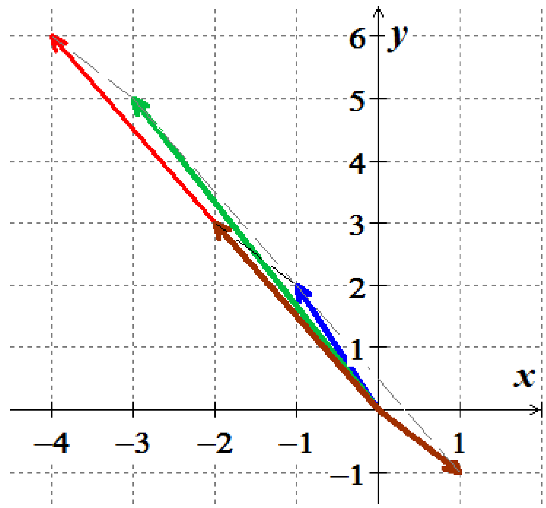
$$\vec{u}+\vec{v}=\begin{pmatrix}1\\-1\end{pmatrix}+\begin{pmatrix}-2\\3\end{pmatrix}$$

$$=\begin{pmatrix}-1\\2\end{pmatrix}$$

$$\vec{u}+2\vec{v}=\begin{pmatrix}1\\-1\end{pmatrix}+2\begin{pmatrix}-2\\3\end{pmatrix}$$

$$=\begin{pmatrix}1\\-1\end{pmatrix}+\begin{pmatrix}-4\\6\end{pmatrix}$$

$$=\begin{pmatrix}-3\\5\end{pmatrix}$$



Exercise

Draw $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, and $\vec{u}+2\vec{v}$ $\quad \vec{u}=\begin{pmatrix}-1\\1\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}2\\1\end{pmatrix}$

Solution

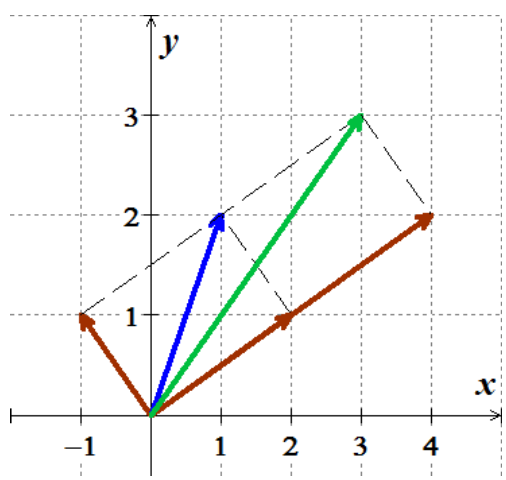
$$\vec{u}+\vec{v}=\begin{pmatrix}-1\\1\end{pmatrix}+\begin{pmatrix}2\\1\end{pmatrix}$$

$$=\begin{pmatrix}1\\2\end{pmatrix}$$

$$\vec{u}+2\vec{v}=\begin{pmatrix}-1\\1\end{pmatrix}+2\begin{pmatrix}2\\1\end{pmatrix}$$

$$=\begin{pmatrix}-1\\1\end{pmatrix}+\begin{pmatrix}4\\2\end{pmatrix}$$

$$=\begin{pmatrix}3\\3\end{pmatrix}$$



Exercise

Draw $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, and $\vec{u}+2\vec{v}$ $\quad \vec{u}=\begin{pmatrix}1\\0\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}1\\2\end{pmatrix}$

Solution

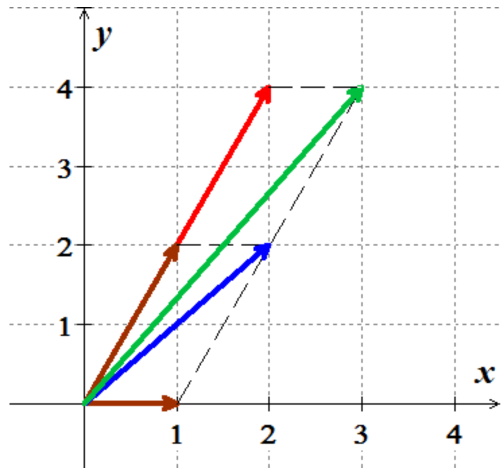
$$\vec{u}+\vec{v}=\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}1\\2\end{pmatrix}$$

$$=\boxed{\begin{pmatrix}2\\2\end{pmatrix}}$$

$$\vec{u}+2\vec{v}=\begin{pmatrix}1\\0\end{pmatrix}+2\begin{pmatrix}1\\2\end{pmatrix}$$

$$=\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}2\\4\end{pmatrix}$$

$$=\boxed{\begin{pmatrix}3\\4\end{pmatrix}}$$



Exercise

Draw $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, and $\vec{u}+2\vec{v}$ $\qquad \vec{u}=\begin{pmatrix}-1\\0\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}1\\-1\end{pmatrix}$

Solution

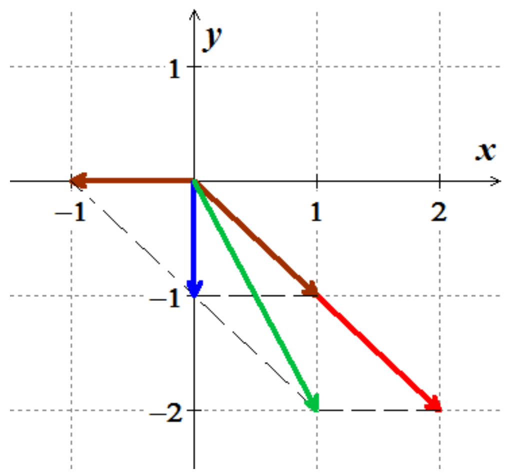
$$\vec{u}+\vec{v}=\begin{pmatrix}-1\\0\end{pmatrix}+\begin{pmatrix}1\\-1\end{pmatrix}$$

$$=\boxed{\begin{pmatrix}0\\-1\end{pmatrix}}$$

$$\vec{u}+2\vec{v}=\begin{pmatrix}-1\\0\end{pmatrix}+2\begin{pmatrix}1\\-1\end{pmatrix}$$

$$=\begin{pmatrix}-1\\0\end{pmatrix}+\begin{pmatrix}2\\-2\end{pmatrix}$$

$$=\boxed{\begin{pmatrix}1\\-2\end{pmatrix}}$$



Exercise

Draw $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, and $\vec{u}+2\vec{v}$ $\qquad \vec{u}=\begin{pmatrix}2\\3\\2\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}1\\-1\\-2\end{pmatrix}$

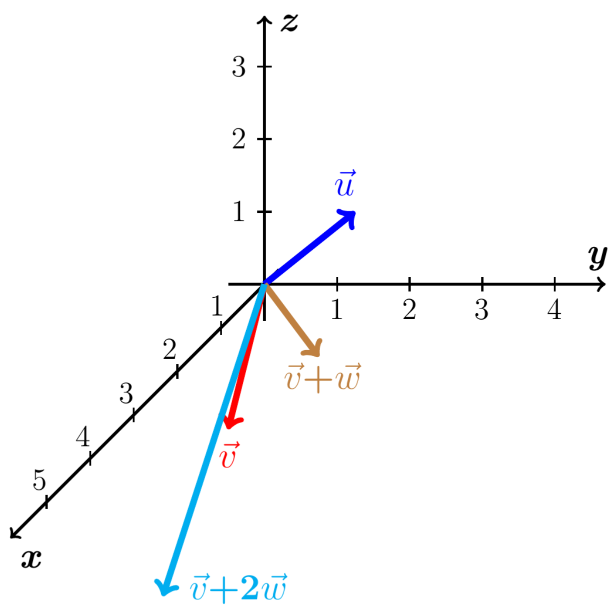
Solution

$$\vec{u}+\vec{v}=\begin{pmatrix}2\\3\\2\end{pmatrix}+\begin{pmatrix}1\\-1\\-2\end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$$

$$\vec{u} + 2\vec{v} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$$



Exercise

Draw $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, and $\vec{u}+2\vec{v}$ $\qquad \vec{u} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ *and* $\vec{v} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

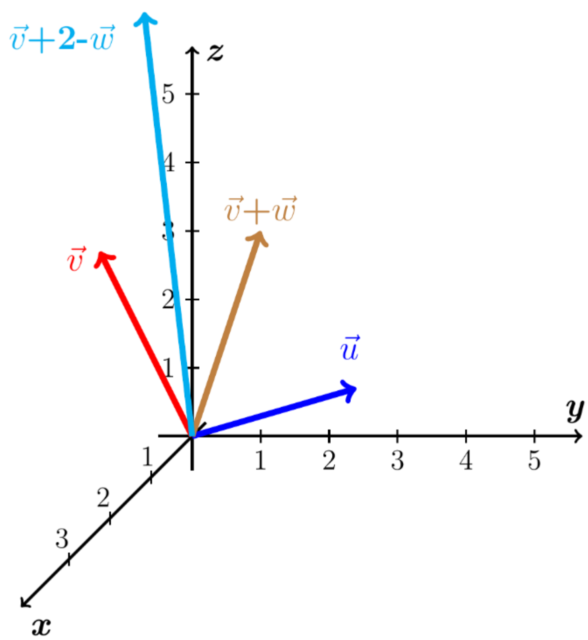
Solution

$$\vec{u} + \vec{v} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

$$\vec{u}+2\vec{v}=\begin{pmatrix}-1\\2\\0\end{pmatrix}+2\begin{pmatrix}1\\-1\\3\end{pmatrix}$$

$$=\begin{pmatrix}1\\0\\6\end{pmatrix}$$



Exercise

Draw $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, and $\vec{u}+2\vec{v}$ $\quad \vec{u}=\begin{pmatrix}1\\-2\\0\end{pmatrix}$ *and* $\vec{v}=\begin{pmatrix}-1\\1\\-3\end{pmatrix}$

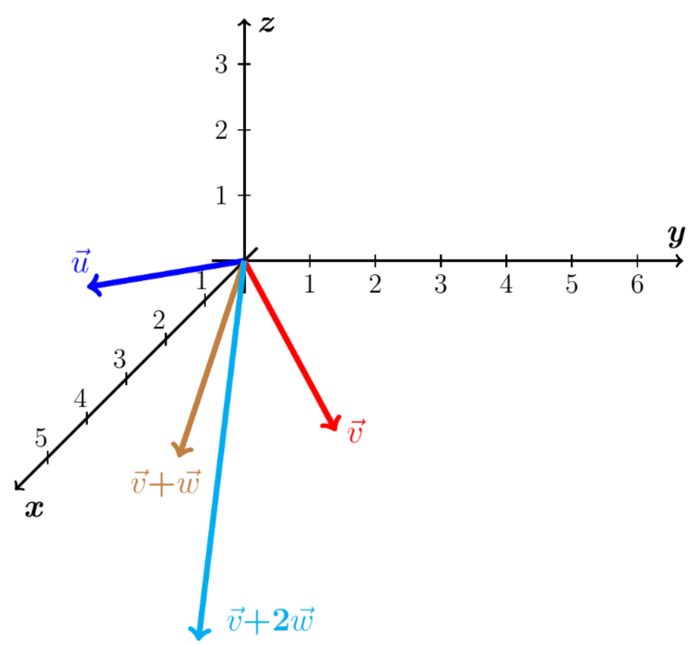
Solution

$$\vec{u}+\vec{v}=\begin{pmatrix}1\\-2\\0\end{pmatrix}+\begin{pmatrix}-1\\1\\-3\end{pmatrix}$$

$$=\begin{pmatrix}0\\-1\\-3\end{pmatrix}$$

$$\vec{u}+2\vec{v}=\begin{pmatrix}1\\-2\\0\end{pmatrix}+2\begin{pmatrix}-1\\1\\-3\end{pmatrix}$$

$$=\begin{pmatrix}-1\\0\\-6\end{pmatrix}$$



Exercise

Prove that $\vec{u}+\vec{v}=\vec{v}+\vec{u}$

Solution

Let $\vec{u}=\left(u_1,\ u_2,\ \ldots,\ u_n\right)$

$\vec{v}=\left(v_1,\ v_2,\ \ldots,\ v_n\right)$

$$\begin{aligned}\vec{u}+\vec{v}&=\left(u_1,\ u_2,\ \ldots,\ u_n\right)+\left(v_1,\ v_2,\ \ldots,\ v_n\right)\\&=\left(u_1+v_1,\ u_2+v_2,\ \ldots,\ u_n+v_n\right)\\&=\left(v_1+u_1,\ v_2+u_2,\ \ldots,\ v_n+u_n\right)\\&=\left(v_1,\ v_2,\ \ldots,\ v_n\right)+\left(u_1,\ u_2,\ \ldots,\ u_n\right)\\&=\vec{v}+\vec{u}\quad\checkmark\end{aligned}$$

Exercise

Prove that $k\left(\vec{u}+\vec{v}\right)=k\vec{u}+k\vec{v}$

Solution

Let $\vec{u}=\left(u_1,\ u_2,\ \ldots,\ u_n\right)$

$\vec{v}=\left(v_1,\ v_2,\ \ldots,\ v_n\right)$

$$\begin{aligned}k\left(\vec{u}+\vec{v}\right)&=k\left(\left(u_1,\ u_2,\ \ldots,\ u_n\right)+\left(v_1,\ v_2,\ \ldots,\ v_n\right)\right)\\&=k\left(u_1+v_1,\ u_2+v_2,\ \ldots,\ u_n+v_n\right)\\&=\left(k\left(u_1+v_1\right),\ k\left(u_2+v_2\right),\ \ldots,\ k\left(u_n+v_n\right)\right)\\&=\left(ku_1+kv_1,\ ku_2+kv_2,\ \ldots,\ ku_n+kv_n\right)\\&=\left(kv_1+ku_1,\ kv_2+ku_2,\ \ldots,\ kv_n+ku_n\right)\\&=\left(kv_1,\ kv_2,\ \ldots,\ kv_n\right)+\left(ku_1,\ ku_2,\ \ldots,\ ku_n\right)\\&=k\left(v_1,\ v_2,\ \ldots,\ v_n\right)+k\left(u_1,\ u_2,\ \ldots,\ u_n\right)\\&=k\vec{v}+k\vec{u}\quad\checkmark\end{aligned}$$

Exercise

Prove that $(k+m)\vec{u} = k\vec{u} + m\vec{u}$

Solution

Let $\vec{u} = \left(u_1,\ u_2,\ \dots,\ u_n\right)$

$$\begin{aligned}
(k+m)\vec{u} &= (k+m)\left(u_1,\ u_2,\ \dots,\ u_n\right) \\
&= \left((k+m)u_1,\ (k+m)u_2,\ \dots,\ (k+m)u_n\right) \\
&= \left(ku_1 + mu_1,\ ku_2 + mu_2,\ \dots,\ ku_n + mu_n\right) \\
&= \left(ku_1,\ ku_2,\ \dots,\ ku_n\right) + \left(mu_1,\ mu_2,\ \dots,\ mu_n\right) \\
&= k\left(u_1,\ u_2,\ \dots,\ u_n\right) + m\left(u_1,\ u_2,\ \dots,\ u_n\right) \\
&= k\vec{u} + m\vec{u} \qquad \checkmark
\end{aligned}$$