Lecture 3.

3.1 Mathematical Induction

1- $n=1$, P_1 is true

1^{st} term = ?

2- P_k is true (assume) → substitute n w/ k

P_{k+1} is also true

add 1 term $(k+1)$

$= k$ w/ $k+1$

Ex $\quad 1+2+\dots+n = \dfrac{n(n+1)}{2}$

1. $n=1 \Rightarrow 1 = \dfrac{1(2)}{2}$?

$1 = 1$ ✓ $\quad P_1$ is true

2 Assume, P_k : $1+2+\dots+k = \frac{1}{2}k(k+1)$ is true

Is P_{k+1} : $1+\dots+k+(k+1) = \frac{1}{2}(k+1)(k+2)$? true

Copy

$$1+\dots+k+(k+1) = \frac{1}{2}k(k+1)+(k+1)\frac{2}{2}$$

$$= \frac{1}{2}(k+1)(k+2) \checkmark$$

P_{k+1} is also true

∴ By the mathematical induction, the proof is completed.

Ex $1+2+2^2+\cdots+2^n=2^{n+1}-1$ $\boxed{1=2^0}$

n nonnegative : $n \in \mathbb{Z}^+ + \{0\}$

1. For $n=0 \Rightarrow 1 \stackrel{?}{=} 2^1-1$

$1=1$ ✓

P_0 is true

Let P_k : $1+2+\cdots+2^k=2^{k+1}-1$ is true

Is, P_{k+1}: $1+\cdots+2^k+2^{k+1} \stackrel{?}{=} 2^{k+2}-1$

$$\underbrace{1+\cdots+2^k}+2^{k+1} = 2^{k+1}-1+2^{k+1}$$

$$=2^1 2^{k+1}-1$$

$$=2^{k+2}-1 \checkmark$$

P_{k+1} is also true

$\therefore$ By the mathematical induction, the proof is completed.

$$a + ar + ar^2 + \cdots + ar^n = \frac{ar^{n+1} - a}{r-1} \qquad r \neq 1$$

$$n = 0 \Rightarrow \quad a \stackrel{?}{=} \frac{ar - a}{r-1}$$

$$a \stackrel{?}{=} \frac{a(r-1)}{r-1} \qquad r \neq 1$$

$$a = a \checkmark \qquad P_0 \text{ is true}$$

let P_k, $a + ar + \cdots + ar^k = \dfrac{ar^{k+1} - a}{r-1}$ true

is P_{k+1}: $a + \cdots + ar^k + ar^{k+1} = \dfrac{ar^{k+2} - a}{r-1}$?

$$a + \cdots + ar^k + ar^{k+1} = \frac{ar^{k+1} - a}{r-1} + r^{k+1}$$

$$= a\left(\frac{r^{k+1} - 1}{r-1} + r^{k+1}\right)$$

$$= a\,\frac{r^{k+1} - 1 + r^{k+2} - r^{k+1}}{r-1}$$

$$= a\left(\frac{r^{k+2} - 1}{r-1}\right) \checkmark$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the given proof is completed.

$(n \in \mathbb{Z}^+)$ $\qquad n < 2^n$

1. For $n = 1 \Rightarrow \quad 1 < 2^1$?

$1 < 2$ ✓ $\quad P_1$ is true.

let P_k : $k < 2^k$ is true

Is P_{k+1} : $(k+1) < 2^{k+1}$?

$$k+1 < 2^k + 1 \qquad (k < 2^k)$$

$$2^k + 1 < 2.2^k \qquad 2^k \qquad 2\,2^k$$

$$< 2.2^k \qquad\qquad 2$$

$$= 2^{k+1} \;✓$$

P_{k+1} is also true

$\therefore$ By the mathematical induction, the given proof is completed

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$$k+1 < k+k$$

$$< 2^k + 2^k$$

$$= 2\,2^k$$

$$= 2^{k+1}$$

$2^n < n!$ $\quad n \geq 4$

For $n = 4 \Rightarrow$ $2^4 \overset{?}{<} 4!$

$16 < 24$ ✓

P_1 is true

let P_k: $2^k < k!$ is true

Is P_{k+1}: $2^{k+1} < (k+1)!$?

$$
\begin{aligned}
2^{k+1} &= 2^k 2 \\
&< k!\,(2) \\
&< (k+1)\,k! \qquad 2 < k+1 \\
&= (k+1)! \checkmark
\end{aligned}
$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the given proof is completed

$n^3 - n$ is divisible by 3 $\quad n \in \mathbb{Z}^+$

For $n = 1$ $\qquad 1^3 - 1 = 0$ is divisible by 3

, is true

let P_k : $k^3 - k$ is divisible by 3

Is P_{k+1} : $(k+1)^3 - (k+1)$ is ? divisible by 3

$$(k+1)^3 - (k+1) = k^3 + \underbrace{3k^2 + 3k}_{} + 1 \underline{- k} - 1$$

$$= k^3 - k + 3(k^2 + k)$$

$k^3 - k$ is divisible by 3

& $3(\ \ )$ " " 3 ✓

P_{k+1} is also true

∴ By the mathematical induction, the proof is completed

$7^{n+2} + 8^{2n+1}$ is divisible by 57

n: nonnegative

For $n = 0 \Rightarrow 7^2 + 8^1 = 49 + 8$
$= 57$ ✓

P_0 is true

let P_k: $7^{k+2} + 8^{2k+1}$ is divisible by 57

Is P_{k+1}: $7^{k+3} + 8^{2k+3}$ is? divisible by 57

$$
\begin{aligned}
7^{k+3} + 8^{2k+3} &= 7\,7^{k+2} + 8^2\,8^{2k+1} \\
&= 7\,7^{k+2} + 64 \cdot 8^{2k+1} \\
&= 7 \cdot 7^{k+2} + (57+7)\,8^{2k+1} \\
&= 7 \cdot 7^{k+2} + 7 \cdot 8^{2k+1} + 57 \cdot 8^{2k+1} \\
&= 7\,(\underline{7^{k+1} + 8^{2k+1}}) + 57 \cdot 8^{2k+1}
\end{aligned}
$$

$7^{k+1} + 8^{2k+1}$ is divisible by 57
57 " " itself

P_{k+1} is also true

∴ By the mathematical induction, the given proof is completed.