#11 a) $\vec{AB} = \langle -1, 1, -5 \rangle$

$\vec{DC} = \langle 1-x, 2-y, 3-z \rangle$

$$\vec{AB} = \vec{DC} \Rightarrow \begin{cases} 1-x = -1 \Rightarrow x = 2 \\ 2-y = 1 \Rightarrow y = 1 \\ 3-z = -5 \Rightarrow z = 8 \end{cases} \Rightarrow D\,(2, 1, 8)$$

b) $\vec{BC} = \langle 0, 2, 4 \rangle$

$\vec{BA} = \langle 1, -1, 5 \rangle$

$$\cos B = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}|\,|\vec{BC}|}$$

$$= \frac{0 - 2 + 20}{\sqrt{4+16}\,\sqrt{1+1+25}}$$

$$= \frac{18}{\sqrt{20}\,\sqrt{27}}$$

$$= \frac{18}{2\sqrt{5}\;3\sqrt{3}}$$

$$= \boxed{\frac{3}{\sqrt{15}}}$$

c) $$\text{proj}_{\vec{BC}}\vec{BA} = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BC}|^2}\vec{BC}$$

$$= \frac{18}{20}\,(2\hat{j} + 4\hat{k})$$

$$= \frac{9}{5}\hat{j} + \frac{18}{5}\hat{k}.$$

d) $$\text{Area} = |\vec{BA} \times \vec{BC}|$$

$$= \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 5 \\ 0 & 2 & 4 \end{vmatrix} \right| \begin{matrix} 1 & -1 \\ 0 & 2 \end{matrix}$$

$$= |-14\hat{i} - 4\hat{j} + 2\hat{k}|$$

$$= \sqrt{14^2 + 16 + 4}$$

$$= \sqrt{216}$$

$$= \boxed{6\sqrt{6}}$$

#11 cont e) $\vec{BA} \times \vec{BC} = -14\hat{i} - 4\hat{j} + 2\hat{k} = \vec{n}$

$$-14(x-1) - 4(y) + 2(z+1) = 0$$

$$-14x + 14 - 4y + 2z + 2 = 0$$

$$-14x - 4y + 2z = -16$$

$$\underline{7x + 2y + z = 8}$$

f) $\vec{n} = -14\hat{i} - 4\hat{j} + 2\hat{k}$

area of the projection on yz plane $= |\vec{n} \cdot \hat{i}| = 14$

xz plane $= |\vec{n} \cdot \hat{j}| = 4$

xy plane $= |\vec{n} \cdot \hat{k}| = 2.$

#12 a) $|\vec{n}| = \sqrt{A^2 + B^2 + C^2}$ if $y = z = 0 \Rightarrow P = (\frac{D}{A}, 0, 0)$

$d = \left| \vec{PP_0} \cdot \frac{\vec{n}}{|\vec{n}|} \right|$; $\vec{PP_0} = (x_0 - D/A)\hat{i} + y_0\hat{j} + z_0\hat{k}$

$$= \frac{\left[(x_0 - \frac{D}{A})\hat{i} + y_0\hat{j} + z_0\hat{k}\right] \cdot (A\hat{i} + B\hat{j} + C\hat{k})}{|\vec{n}|}$$

$$= \frac{Ax_0 + By_0 + Cz_0 - D}{\sqrt{A^2 + B^2 + C^2}}$$

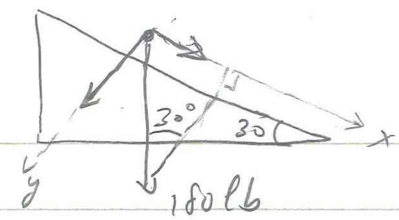
b) $d = \frac{|2(2) - 2(-1) - 1(3) - 9|}{\sqrt{4 + 4 + 1}}$

$$= \frac{|-6|}{3}$$

$$= 2 \text{ units}$$

#13 a) $|F_{\parallel}| = 180\sin 30^\circ = 90$ lb $(=F_x)$

$|F_{\perp}| = 180\cos 30^\circ = 90\sqrt{3}$ lb $(=F_y)$



b) $\text{Work} = d \cdot F_x$

$= 10\,(90)$

$= 900$ ft-lbs.

#14 $T(\theta) = |r|\,|F| \sin\theta$

$= (.4\text{ m})(98)\sin\theta$

$= 39.2 \sin\theta \quad (N\text{-}m)$

Max. Torque $= 39.2$ when $\sin\theta = 1 \Rightarrow \theta = 90^\circ$

Min " $= 0$ $\quad \sin\theta = 0 \Rightarrow \theta = 0^\circ$

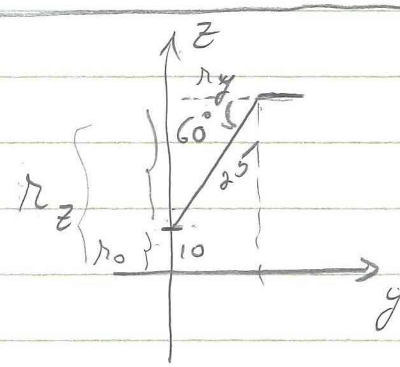
The direction of the torque does not change as the knee is lifted.

#15) $r_0 = 10k$

$r = r_y\hat{j} + r_z\hat{k}$

$= 25\cos 60^\circ\,\hat{j} + (10 + 25\sin 60^\circ)\vec{k}$

$\approx 12.5\hat{j} + 31.65\vec{k}$



$T = (r - r_0) \times F$

$\approx (12.5\hat{j} + 21.65\hat{k}) \times 500\hat{i}$

$= (21.65)(500)\hat{j} - (12.5)(500)\hat{k}$

$= 10{,}825\hat{j} - 6{,}250\hat{k}$

$$\begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & & \\ 0 & 12.5 & 21.65 & 0 & 12.5 \\ 500 & 0 & 0 & 500 & 0 \end{array}$$

Since the torque is effective in turning horizontally $\Rightarrow$

$T = 10{,}825$ N-cm (or 108.25 N-m)

The effective torque $(r_0 = 0) \Rightarrow 31.65k \times 500\hat{i} = 15{,}825\hat{j}$

≈ 158.25 N.m