Newton's Laws of Motion (Chapter 5 Lecture 1)

Newton's laws of motion are laws that establish the relationship between force and motion. In layman terms, force is a push, a pull, a squeeze and so on. Scientifically, force is something that causes change of motion or change of shape. The SI unit of measurement force is the Newton (N).

Newton's First Law: states that an object will remain at rest or continue to move in a straight line with a constant sped unless acted upon by a net force.

$$\text{If } \vec{F}_{net} = 0, \text{ then } \vec{a} = 0$$

Newton's Second Law: states that the force acting on an object is directly proportional to the acceleration produced

$$\vec{F}_{net} \propto \vec{a}$$

The constant of proportionality (ratio between $\vec{F}_{net}$ & $\vec{a}$) is defined to be the mass of the object. Mass is a measure of the amount of matter an object has.

$$\boxed{\vec{F}_{net} = m\vec{a}}$$

$\vec{F}_{net} \longrightarrow$ net force acting on the object
$m \longrightarrow$ mass of the object
$\vec{a} \longrightarrow$ acceleration

Unit of mass = $\frac{F}{a} = \frac{N}{\frac{m}{s^2}}$ which is defined to be the kilogram (kg).

Newton's 2nd law may be written in component form as

$$\boxed{\begin{aligned} F_{net\ x} &= m\, a_x \\ F_{net\ y} &= m\, a_y \end{aligned}}$$

If there are a number of forces acting on an object, the net force is the vector sum of all the forces acting on the object.

$$\vec{F}_{net} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots$$
$$\vec{F}_{net\ x} = \sum \vec{F}_x = \vec{F}_{1x} + \vec{F}_{2x} + \cdots$$
$$\vec{F}_{net\ y} = \sum \vec{F}_y = \vec{F}_{1y} + \vec{F}_{2y} + \cdots$$

Types of Forces: Forces may be classified as contact & non-contact forces. Contact forces are forces exerted by objects which are in direct contact with the object; while non-contact forces are forces exerted by objects which are not in direct contact with the object. Examples of non-contact forces are gravitational and electromagnetic forces. In this course only gravitational force will be dealt with. While contact forces are self-explanatory, one kind of contact force, surface force, requires special attention.

Gravitational Force: Any two objects in the universe exert gravitational forces on each other. The gravitational force exerted on an object by the massive object in its vicinity is called the weight of the object. For example, the weight of an object on the surface of Earth is the gravitational force exerted by earth on the object. From Newton's 2nd law, weight is equal to the product of mass and gravitational acceleration.

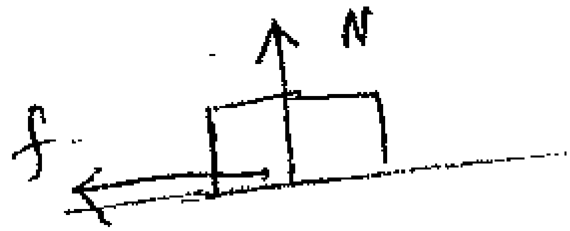
$$\boxed{\begin{aligned} \vec{w} &= m\vec{g} \\ w &= m|g| \end{aligned}}$$

$\vec{w} \longrightarrow$ weight
$m \longrightarrow$ mass
$\vec{g} \longrightarrow$ gravitational force

The direction of weight (gravitational acceleration) is downward. In the $\hat{\imath} - \hat{\jmath}$ notation, weight may be written as $\vec{w} = -m\,|g|\hat{\jmath}$

The weight of an object varies from planet to planet because it depends on the masses of the planets; while mass is the same everywhere because it is a measure of the amount of matter the object has.

Surface Forces are forces exerted between two surfaces in contact. Surface force can be decomposed into a force perpendicular to the surfaces (pressing force) which is called a normal force (N) and a force parallel to the surface which is called friction (f). Friction and normal force are directly proportional. The greater the pressing (Normal) force, the greater the force of friction.



$$f \propto N$$

$$\frac{f}{N} = \text{constant}$$

This constant is called the coefficient of friction between the surfaces and is denoted by mu (μ).

Force of friction depends on the kinds of materials in contact. But it does not depend on the area of contact surface or the relative sliding speed between the surfaces.

Experimentation shows that the force requires to just get an object sliding is greater than the force required to get it sliding once it has started sliding. The friction that exists when an object is just starting to slide is called static friction (f_s) and the friction that exists once the object has started is called kinetic friction (f_k)

$$f_s > f_k$$

Thus, there are two kinds of coefficients if friction:

Static coefficient of friction (μ_s): $\boxed{f_s = \mu_s N}$

Kinetic Coefficient of friction (μ_k): $\boxed{f_k = \mu_k N}$

And $\mu_s > \mu k$

Newton's Third Law: states that for any action there is an equal but opposite reaction. If object A exerts force $\vec{F}_{AB}$ on object B, then object B also will exert force on object A such that

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Action reaction forces act on different objects & thus they cannot cancel each other. An object cannot exert force on itself, but it can use the principle of action reaction to move itself. For example, if a person wants to move forward, he has to push on the ground backward so that the reaction force moves him forward. For a rocket to move upward it has to push on the gases downward so that the reaction force of the gas propels it upward.

Solving Force Problems: The most important task in solving force problems is identifying the forces acting on the object. These forces can be contact or non-contact forces. For an object on the surface of earth there is always force of gravity which is its weight. The direction of weight is always vertically downward. To identify the contact forces examine all the objects in contact with the object to determine the force exerted. If the surface of the object is in contact with another surface, remember to include surface force which is usually decomposed into friction (parallel to the surface) & normal force (perpendicular to the surface).

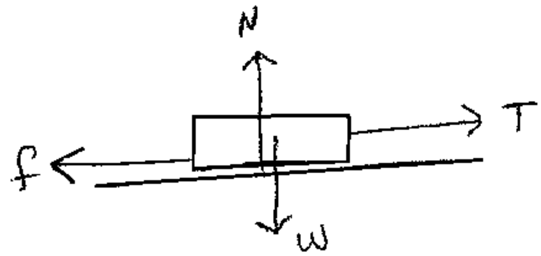
5.1 <u>Statics</u> is the study of objects in equilibrium. An object is said to be in equilibrium (translational) if it is either at rest or moving in a straight line with a constant speed.
<u>Condition of translational equilibrium:</u> An object will be in translational equilibrium if the net force acting on it is zero.

$$\boxed{\vec{F}_{net} = \sum F = 0}$$

In component form

$$\boxed{\begin{array}{l} \vec{F}_{net\,x} = \sum F_x = 0 \\ \vec{F}_{net\,y} = \sum F_y = 0 \end{array}}$$

<u>Example:</u> A 10kg object is sliding on a horizontal surface with uniform speed by means of a horizontal string. The coefficient of friction between the object & the ground is 0.2. Calculate the normal force, the force of friction and the tension in the string (T).



<u>Solution:</u>

Since the object is moving with a uniform speed in a straight line, it is in equilibrium

$\therefore\ \vec{F}_{net} = 0$

Forces acting on the object:

1) Force exerted by the Earth-weight($\vec{w}$)
2) Force exerted by the string ($\vec{T}$)
3) Normal force (perpendicular component of surface force) $(\vec{N})$
4) Friction (horizontal component of surface force) ($\vec{f}$)

$$\therefore\ \vec{F}_{net} = \vec{w} + \vec{T} + \vec{N} + \vec{f} = 0$$

or

$$\vec{F}_{net\,x} = w_x + T_x + N_x + f_x = 0$$
$$\vec{F}_{net\,y} = w_y + T_y + N_y + f_y = 0$$

$$\vec{w} = -m|g|\hat{\jmath} = -10(10)N\,\hat{\jmath}$$
$$= -100\,N\,\hat{\jmath}$$
$$w_x = 0 \qquad w_y = -100$$

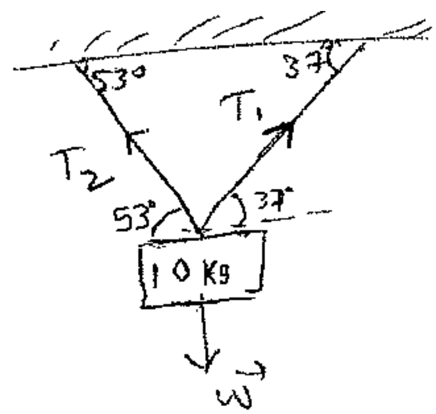
$$\vec{T} = T\,\hat{\imath} \qquad T_z = T \qquad T_y = 0$$
$$\vec{N} \qquad N_x = 0 \qquad N_y = N$$
$$\vec{f} = -f\,\hat{\imath} \qquad f_x = -f \qquad f_y = 0$$

$\therefore w_x + T_x + N_x + f_x = 0$
$\Longrightarrow 0 + T + 0 - f = 0$
$T = f$ ... ... (1)

$w_y + T_y + N_y + f_y = 0$
$-100 + 0 + N + 0 = 0$
$N = +100\ N$

$$f = \mu N \quad \& \quad \mu = 0.2$$
$$\therefore \quad f = (0.2)(100) = 20\ N$$
$$eq\ (1) \quad \Longrightarrow \quad T = f = 20\ N$$

Example: A 10 kg object is hanging from a ceiling by means of two strings as shown. Calculate the tensions in the strings.



Solution

The 10 kg object is at rest.

$\therefore \vec{F}_{net} = 0$

Forces Acting

1) The weight of the object ($\vec{w}$)
2) The force exerted by string 1 ($\vec{T}_1$)
3) The force exerted by string 2 ($\vec{T}_2$)

$$\therefore \quad \vec{F}_{net} = \vec{w} + \vec{T}_1 + \vec{T}_2 = 0$$

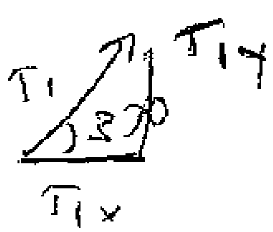
or

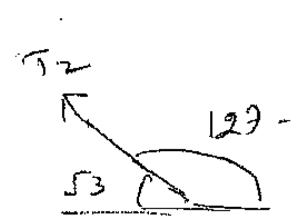
$$\vec{F}_{net\ x} = \vec{w}_x + \vec{T}_{1x} + \vec{T}_{2x} = 0$$
$$\vec{F}_{net\ y} = \vec{w}_y + \vec{T}_{1y} + \vec{T}_{2y} = 0$$

$\vec{w} = -m|g|\ \hat{j} = -(10)(10)\hat{j}$

$= -100\ N\ \hat{j}$

$\Longrightarrow w_x = 0 \quad\quad w_y = -100\ N$





$\vec{T}_1 = T_1 \cos 37\ \hat{\imath} + T_1 \sin 37\, \hat{j}$

$= 0.8\ T_1 \hat{\imath} + 0.6 T_1\ \hat{j}$

$\boxed{T_{1x} = 0.8\ T_1 \hat{\imath}}$ $\quad$ $\boxed{T_{1y} = 0.6 T_1}$

$\vec{T}_2 = T_2 \cos 127\, \hat{\imath} + T_2 \sin 127\, \hat{j}$

$= -0.6\ T_2 \hat{\imath} + 0.8\ T_2\ \hat{j}$

$\boxed{T_{2x} = -0.6\ T_2}$ $\quad$ $\boxed{T_{2y} = 0.8\ T_2}$

$\vec{w}_x + \vec{T}_{1x} + \vec{T}_{2x} = 0$

$0 + 0.8T_1 - 0.8T_2 = 0$

$0.8T_1 = 0.6T_2$

$T_1 = \frac{3}{4} T_2 \ldots \ldots \ldots \ldots\ \boldsymbol{eq(1)}$

$\vec{w}_y + \vec{T}_{1y} + \vec{T}_{2y} = 0$

$-100N + 0.6T_1 + 0.8T_2 = 0$

$0.6T_1 + 0.8T_2 = 100N \ldots \ldots \ldots\ \boldsymbol{eq(2)}$

Substituting for T_1 in $eq(2)$ from $eq(2)$

$$0.6T_1 + 0.8T_2 = 100 \quad \text{but } T_1 = \tfrac{3}{4}T_2$$

$$0.6\left(\frac{3}{4}T_2\right) + 0.8T_2 = 100$$
$$.45T_2 + 0.8T_2 = 100$$
$$1.25T_2 = 100$$
$$T_2 = \frac{100}{1.25} = 80\ N$$

$$T_1 = \frac{3}{4}T_2$$
$$= \frac{3}{4}(80N) = 60$$
$$\boxed{T_1 = 60N} \qquad \boxed{T_2 = 80N}$$

Dynamics (Chapter 5 Lecture 2)

Dynamics is the study of accelerated systems. From Newton's 2nd Law

$$\vec{F}_{net} = m\vec{a}$$

In component form

$$\boxed{\begin{aligned}\vec{F}_{net\ x} &= m\ a_x \\ \vec{F}_{net\ y} &= m\ a_y\end{aligned}}$$

Also since $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2}$$

$$\boxed{\vec{F}_{net\ x} = m\frac{dv_x}{dt} = m\frac{d^2x}{dt^2}} \qquad \boxed{\vec{F}_{net\ y} = m\frac{dv_y}{dt} = m\frac{d^2y}{dt^2}}$$

Example: The position vector of a certain particle of mass 0.2 kg varies with time according to the equation $\vec{r} = (3t^2 - 4t)\hat{\imath} + (4t^3 + 8)\hat{\jmath}$.

Calculate the force acting on the particle after 2 seconds

Solution

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}$$

$$\vec{r} = (3t^2 - 4t)\hat{\imath} + (4t^3 + 8)\hat{\jmath}$$

$$\frac{d\vec{r}}{dt} = (6t - 4)\hat{\imath} + (12t^2)\hat{\jmath}$$

$$\frac{d^2\vec{r}}{dt^2} = 6\hat{\imath} + 24t\hat{\jmath}$$

$$\vec{F}|_{t=2} = m\frac{d^2\vec{r}}{dt^2}|_{t=2}$$

$$= (0.2)(6\hat{\imath} + 24t\hat{\jmath})|_{t=2}$$

$$= 0.2(6\hat{\imath} + 48\hat{\jmath})$$

$$\underline{= (1.2\hat{\imath} + 9.6\hat{\jmath})N}$$

Example: A mass of 4 kg is being acted on by 3 forces. Two of the are equal to $(2\hat{\imath} + 3\hat{\jmath})$N & $(-4\hat{\imath} + 5\hat{\jmath})N$, respectively. The third force is unknown. If the object is moving with an acceleration of $(3\hat{\imath} + 6\hat{\jmath})$ m/s^2, calculate the unknown force.

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\ \vec{a}$

$m = 4\ kg$ $\quad$ $\vec{a} = (3\hat{\imath} + 6\hat{\jmath})$ m/s^2

$\vec{F}_1 = (2\hat{\imath} + 3\hat{\jmath})N$ $\quad$ $\vec{F}_3 = ??$

$\vec{F}_2 = (-4\hat{\imath} + 5\hat{\jmath})N$

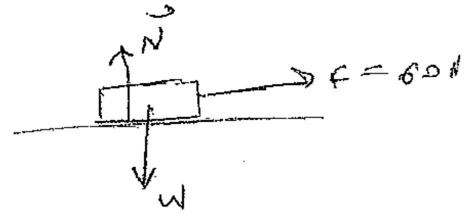
$\therefore\ [2\hat{\imath} + 3\hat{\jmath}] + [-4\hat{\imath} + 5\hat{\jmath}] + \vec{F}_3 = 4(3\hat{\imath} + 6\hat{\jmath})$

$$-2\hat{\imath} + 8\hat{\jmath} + \vec{F}_3 = 12\hat{\imath} + 24\hat{\jmath}$$

$$\vec{F}_3 = [14\hat{\imath} + 16\hat{\jmath}]N$$

Example: A 10 kg object is being pulled horizontally by a force of 60N

a) Assuming no friction, calculate the acceleration of the object



Solution

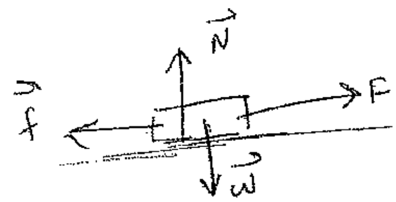
Forces Acting:

1) The horizontal force $(\vec{F})$
2) Normal force $(\vec{N})$
3) Weight $(\vec{w})$

$\vec{N}$ & $\vec{w}$ do not contribute to the horizontal motion because they are vertical forces.

$$\vec{F} = 60\ N\ \hat{\imath} \ \Longrightarrow F_x = 60N$$
$$\vec{F}_{net\ x} = F_x = m\ a_x \qquad m = 10kg$$
$$60N = (10kg)a_x$$
$$a_x = \underline{6\ \text{m}/s^2}$$

b) Assuming the coefficient of friction between the surfaces is 0.2, calculate the acceleration of the object.



Solution

Forces Acting:

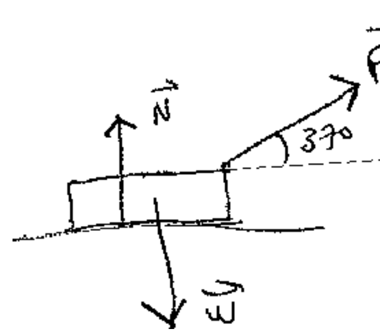
1) Horizontal force $(\vec{F})$
2) Normal force $(\vec{N})$
3) Weight $(\vec{w})$
4) Friction $(\vec{f})$

$\vec{F} = 60N\ \hat{\imath} \ \Longrightarrow$	$F_x = 60N$	$F_y = 0$
$\vec{N} = N\ \hat{\jmath} \ \Longrightarrow$	$N_x = 0$	$N_y = N$
$\vec{w} = -m\lvert g\rvert\hat{\jmath}$ $= -(10)(10)\hat{\jmath} \quad = -100\hat{\jmath} \ \Longrightarrow$	$w_x = 0$	$w_y = -100$

$$\vec{f} = -f\ \hat{\imath} \ \ \Longrightarrow \qquad f_x = -f \qquad f_y = 0$$

$\sum F_y = F_y + N_y + w_y + f_y = m\left(a_y\right)$ but $a_y = 0$ b/c it is moving horizontally $\therefore\ \ 0 + N - 100 + 0 = 0$ $N = 100N$	$\sum F_x = F_x + N_x + w_x + f_x = m(a_x)$ $60 + 0 + 0 - f = 10a_x$ $60 + 0 + 0 - 20 = 10a_x$ $40 = 10a_x$ $a_x = \underline{4\ \text{m/s}^2}$	But $f = \mu N$ $= (0.2)(100)$ $f = 20N$

Example: A 10kg object is being pulled horizontally by a 100N force that makes an angle of 37° with the horizontal right.



a) Assuming no friction, calculate its acceleration and the normal

force.
Solution

Forces Acting:

1) Pulling Force $(\vec{F})$
2) Normal Force $(\vec{N})$
3) Weight $(\vec{w})$

$$\vec{F} = 100N \text{ at } 37°$$
$$\vec{F} = [100\cos 37\ \hat{\imath} + 100\sin 37\, \hat{\jmath}]N$$
$$= 80N\hat{\imath} + 60N\hat{\jmath} \implies F_x = 80N \quad F_y = 60N$$

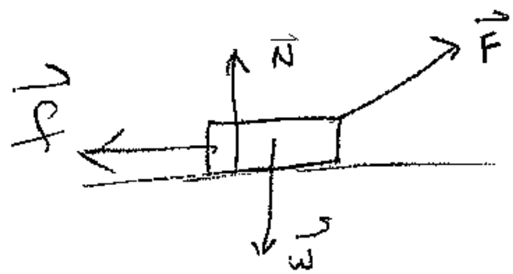
$$\vec{N} = N\hat{\jmath} \implies N_x = 0 \quad N_y = N$$

$$\vec{w} = -m|g|\hat{\jmath}$$
$$= -10(10)\hat{\jmath}$$
$$= (-100\hat{\jmath})N$$
$$N_x = 0 \quad N_y = -100N$$

$$\sum F_y = F_y + N_y + w_y = m\,(a_y)$$
but $a_y = 0$ b/c it is moving horizontally
$$\therefore \quad 60 + N - 100 = 0$$
$$N = 40N$$

$$\sum F_x = F_x + N_x + w_x = m\,(a_x) \qquad m = 10kg$$
$$80 + 0 + 0 = 10\,a_x$$
$$a_x = 8 \text{ m/s}^2$$

b) Assuming the coefficient of friction between the surfaces is 0.2, calculate the normal force and the acceleration.



Solution

Force Acting:

1) Pulling Force $(\vec{F})$
2) Normal Force $(\vec{N})$
3) Weight $(\vec{w})$
4) Friction $(\vec{f})$

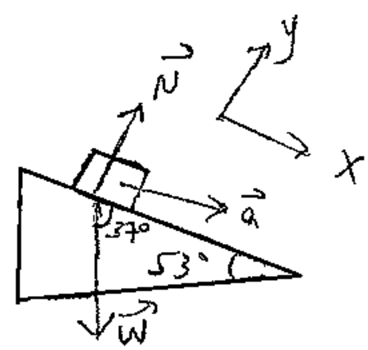
$\vec{F} = 80N\ \hat{\imath} + 60\ N\ \hat{\jmath}$	$F_x = 80\ N$	$F_y = 60\ N$
$\vec{N} = N\hat{\jmath}$	$N_x = 0$	$N_y = N$
$\vec{f} = -f\ \hat{\imath}$	$f_x = -f$	$f_y = 0$
$\vec{w} = -100\hat{\jmath}$	$w_x = 0$	$w_y = -100$

$$\sum F_y = F_y + N_y + w_y + f_y = m\,(a_y)$$
but $a_y = 0$ b/c it is moving horizontally
$$\therefore \quad 60 + N + 0 - 100 = 0$$
$$N = 40N$$

$$\sum F_x = F_x + N_x + w_x + f_x = m\,(a_x) \qquad m = 10kg$$
$$80 + 0 - f + 0 = 10\,a_x$$
$$80 - f = 10\,a_x$$
$$80 - 8 = 10a_x$$
$$a_x = \underline{7.2 \text{ m/s}^2}$$

but $f = \mu N$
$= (0.2)(40)$
$= 8N$

Example: A 10 kg object is sliding down a 53° inclined plane.
a) Assuming no friction, calculate the normal force & its acceleration.



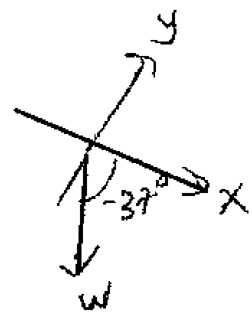
Solution

Forces Acting

1) Normal Force ($\vec{N}$)

2) Weight ($\vec{w}$)

Using the coordinate system to show where the x-axis is parallel to the plane.



$\vec{N} = N\,\hat{j}$ $\quad N_x = 0$ $\quad N_y = N$

$$\vec{w} = m|g|\cos -37\,\hat{\imath} + m|g|\sin -37\,\hat{j}$$
$$= 80N\hat{\imath} - 60N\hat{j}$$
$$w_x = 80\,N \quad w_y = -60N$$

$\sum F_y = N_y + w_y = m\left(a_y\right)$

but $a_y = 0$ b/c it is sliding parallel to the plane

$N - (60N) = 0$

$N = 60N$

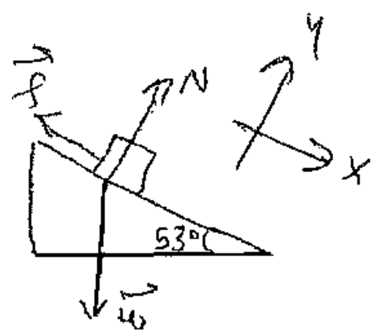
$\sum F_x = N_x + w_x = m\,(a_x)$

$0 + 80N = 10a_x$

$a_x = \underline{8\text{m}/s^2}$

$m = 10kg$

b) Assuming the coefficient of friction between the surfaces is 0.2, calculate the normal force and acceleration.



Solution

Force Acting:

1) Normal Force ($\vec{N}$)
2) Weight ($\vec{w}$)
3) Friction ($\vec{f}$)

Using the coordinate system shown

$\vec{N} = N\hat{j}$	$N_x = 0$	$N_y = N$
$\vec{f} = -f\hat{\imath}$	$f_x = -f$	$f_y = 0$

$$\vec{w} = m|g|\cos -37\,\hat{\imath} + m|g|\sin -37\,\hat{j}$$
$$= 80N\hat{\imath} - 60N\hat{j}$$
$$w_X = 80n \quad w_y = -60N$$

$\sum F_y = N_y + w_y + f_y = m\left(a_y\right)$

but $a_y = 0$ b/c no motion $\perp$ to the plane

$N - 60 + 0 = 0$

$N = 60N$

$\sum F_x = N_x + f_x + w_x = m\,(a_x)$

$0 - f + 80 = 10a_x$

$-12 + 80 = 10a\text{-}_x$

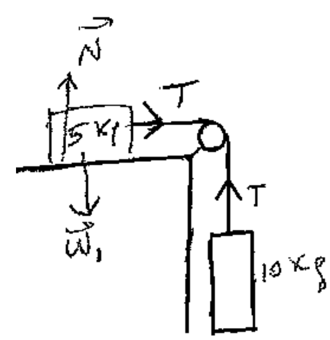
$a_x = \underline{6.8\text{ m/s}^2}$

but $f = \mu N$

$= (0.2)(60)$

$= 12N$

Example: Consider the arrangement shown. Assuming no friction, calculate the tension in the string and the acceleration of the system.



Solution

5kg Object|

Forces Acting:			
1) Tension ($\vec{T}$)	$\vec{T} = T\hat{\imath}$	$T_x = T$	$T_y = 0$
2) Its weight ($\vec{w}_1$)	$\vec{w}_1 = -50\hat{\jmath}$	$w_{ix} = 0$	$w_{1y} = -50\hat{\jmath}$
3) Normal Force ($\vec{N}$)	$\vec{N}_1 = N_1\hat{\jmath}$	$N_{ix} = 0$	$N_{1y} = N$

$\sum F_x = N_x + w_x + T_x = m\,(a_x)$ $0 + 0 + T = 5a_1$ $T = 5a_1 \ldots\ldots\ldots eq(1)$	$m = 5kg$ $a_x = a$ (a is their common acceleration)

<u>10 kg Object</u>

Forces Acting

1) Tension in String $(\vec{T})$
2) Its weight ($\vec{w}_2$)

$\vec{T} = T\hat{\jmath}$ (pulling upwards)

$T_x = 0 \quad T_y = T$

$\vec{w}_2 = -100\hat{\jmath}$

$w_{2x} = 0 \quad w_{2y} = -100$

$$\sum F_y = T_y + w_{2y} = m\,(a_y) = -10a_2 \qquad a_y = -a_2 = -a$$

$T - 100 = -10a_2$ (negative b/c acc. is downward)

$a_1 = a_2$ since both objects have the same acceleration

$a_1 = a$

$$\therefore \quad T = 5a$$

Substituting for T

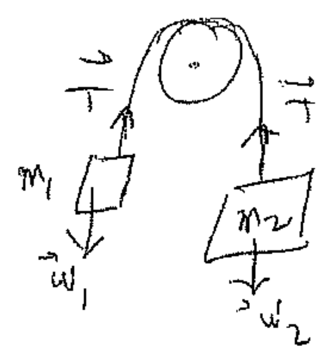
$$T - 100 = -10a$$
$$5a - 100 = -10a$$
$$-100 = -15a$$

$$a = \frac{100}{15} = 6.667 m/s^2$$

<u>Example:</u> Consider the arrangement shown. Assuming no friction, obtain an expression for the acceleration of the system. (Assume $m_2 > m_1$)

<u>Solution</u>

Let the magnitude of their common acceleration be <u>a</u>



$\mathbf{m_1}$

<u>Forces Acting</u>

1) Tension in string ($\vec{T}$)	$\vec{T} = T\hat{\jmath}$	$T_y = T$	$T_x = 0$
2) Its weight ($\vec{w}_1$)	$\vec{w}_1 = -m_1\lvert g\rvert\hat{\jmath}$	$w_{1y} = -m_1\lvert g\rvert$	$w_{1x} = 0$

$$\sum F_y = T_y + w_{1y} = m_1\,(a_{1y}) = m_1 a \qquad a_{1y} = a$$

$$T - m_1|g| \quad = m_1 a \dots \dots eq(1)$$

$\mathbf{m_2}$

Forces Acting

1) Tension in string ($\vec{T}$)
2) Its weight ($\vec{w}_2$)

$\vec{T} = T\hat{j}$ $\vec{w}_2 = -m_2|g|\hat{j}$	$T_y = T$ $w_{2y} = -m_2|g|$	$T_x = 0$ $w_{2x} = 0$

$\sum F_y = T_y + w_{2y}$ $T - m_2|g|$	$= m_2\left(a_{2y}\right)$ $= m_2 a$ $= m_2 a \dots \dots eq(2)$	$a_{2y} = -a$ (negative b/c it is downward)

Substituting for T on $eq(2)$ from $eq(1)$

(1) $T - m_1|g| = m_1 a$

$T = m_1 a + m_1|g|$

(2) $T - m_2|g| = m_2 a$

$m_1 a + m_1|g| = m_2|g| - m_2 a$

$$\boxed{a = \frac{(m_2 - m_1)|g|}{m_1 + m_2}}$$