Math 2312 – Pre-Calculus **_Exam 4_** **_Review_**

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1. If $\cos A = -\frac{5}{13}$ with A in QII and $\tan B = \frac{15}{8}$ with B in QIII, find the exact value of each of the following:

a) $\cos(A-B)$ *b)* $\cos 2B$ *c)* $\sin(A-B)$ *d)* $\tan(A-B)$

2. If $\sin\theta = \frac{15}{17}$, $0 < \theta < \frac{\pi}{2}$, find

a) $\cos 2\theta$ *b)* $\sin 2\theta$ *c)* $\tan 2\theta$ *d)* $\sin\frac{\theta}{2}$ *e)* $\cos\frac{\theta}{2}$

3. Find the solution of the equation that are in the interval $[0, 2\pi)$

a) $2\cos\alpha + \tan\alpha = \sec\alpha$

b) $\csc^5 x - 4\csc x = 0$

c) $2\cos^3 x + \cos^2 x - 2\cos x - 1 = 0$

d) $2\sec x\sin x + 2 = 4\sin x + \sec x$

e) $\sin x\cos 2x + \cos x\sin 2x = 0$

f) $\cos\pi x + \sin\pi x = 0$

4. Find the exact value of the expression whenever it is defined:

a) $\sin\left[\arccos\left(-\frac{\sqrt{3}}{2}\right)\right]$

b) $\cos\left(\sin^{-1}\frac{15}{17} - \sin^{-1}\frac{8}{17}\right)$

c) $\sin\left(\sin^{-1}\frac{5}{13} - \cos^{-1}\left(-\frac{3}{5}\right)\right)$

d) $\tan\left[\cos^{-1}\frac{1}{2} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

e) $\tan(\arccos x)$

f) $\sec\left(\sin^{-1}\frac{x}{\sqrt{x^2+4}}\right)$

g) $\tan\left(2\arcsin\frac{2}{5}\right)$

h) $\sec\left(\arctan\frac{x-2}{2}\right)$

i) $\tan\left(\operatorname{arcsec}\frac{\sqrt{x^2+25}}{x}\right)$

j) $\sec\left(\tan^{-1}\frac{\sqrt{x^2-9}}{3}\right)$

5. Sketch the graph of the equation

a) $y = \cos^{-1} 3x$

b) $y = 1 - \sin^{-1} x$

6. Convert to rectangular coordinates $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$

7. Convert to polar coordinates $(-1, \sqrt{3})$

8. Convert to polar coordinates $(-3,\ -3)$ $r \geq 0$ $0° \leq \theta < 360°$

9. Write the equation in rectangular coordinates

a) $r^2 = 4\cos 2\theta$ *b*) $r(\cos\theta - \sin\theta) = 2$

10. Find a polar equation that has the same graph as the equation in *x* and *y*.

a) $y^2 = 6x$ *b*) $(x+2)^2 + (y-3)^2 = 13$

11. Write complex form in trigonometric form

a) $11 + 2i$ *b*) $-2 + 3i$

12. Write $4cis\frac{\pi}{2}$ in standard form.

13. Find the quotient $\frac{20cis(75°)}{4cis(40°)}$. Write the result in rectangular form.

14. Find $(1+i)^{10}$ and express the result in rectangular form.

15. Find ***fifth*** roots of $z = 1 + i\sqrt{3}$ and express the result in rectangular form.

Solution

1. *a*) $-\frac{140}{221}$ *b*) $-\frac{161}{289}$ *c*) $-\frac{171}{221}$ *d*) $\frac{171}{140}$

2. *a*) $-\frac{161}{289}$ *b*) $\frac{240}{289}$ *c*) $-\frac{240}{161}$ *d*) $\frac{3\sqrt{34}}{34}$ *e*) $\frac{5\sqrt{34}}{34}$

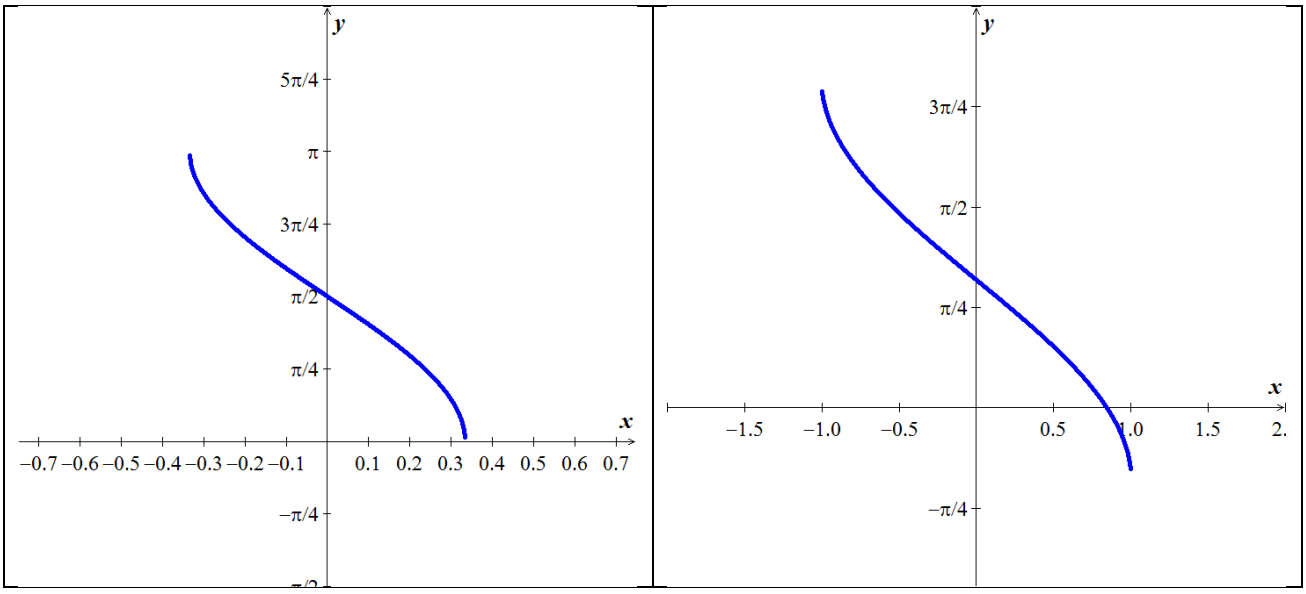
3. ***a***) $\alpha = \frac{7\pi}{6}, \frac{11\pi}{6}$ ***b***) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ ***c***) $x = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

d) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{3}$ ***e***) $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$ ***f***) $x = \frac{3}{4}, \frac{7}{4}, \frac{11}{4}, \frac{15}{4}, \frac{19}{4}, \frac{23}{4}$

4. ***a***) $\frac{1}{2}$ ***b***) $\frac{240}{289}$ ***c***) $-\frac{63}{65}$ ***d***) *not defined* ***e***) $\frac{\sqrt{1-x^2}}{x}$ ***f***) $\frac{\sqrt{x^2+4}}{2}$

g) $\frac{4\sqrt{21}}{17}$ ***h***) $\frac{1}{2}\sqrt{x^2-4x+8}$ ***i***) $\frac{5}{x}$ ***j***) $\frac{x}{3}$

5.



6. $(1,\ -1)$

7. $\left(2,\ 120°\right)$

8. $\left(3\sqrt{2},\ 225°\right)$

9. *a*) $\left(x^2+y^2\right)^4 = 4x^2 - 4y^2$ *b*) $x - y = 2$

10. *a*) $r = 6\frac{\cos\theta}{\sin^2\theta}$ *b*) $r = 6\sin\theta - 4\cos\theta$

11. *a*) $5\sqrt{5}\ cis10.3°$ *b*) $\sqrt{13}\ cis123.69°$

12. $4i$

13. $4.1+2.87i$

14. $32i$

15. $\sqrt[5]{2}\ cis12°$ $\sqrt[5]{2}\ cis84°$ $\sqrt[5]{2}\ cis156°$ $\sqrt[5]{2}\ cis228°$ $\sqrt[5]{2}\ cis300°$