Solution *Section* 1.3 – Fractions and Rationalization

Exercise

Perform the operation and simplify $\frac{2}{x^2-4}-\frac{1}{x-2}$

Solution

$$\frac{2}{x^2-4}-\frac{1}{x-2}=\frac{2-(x+2)}{(x-2)(x+2)}$$
$$=\frac{2-x-2}{(x-2)(x+2)}$$
$$=\underline{\left|-\frac{x}{(x-2)(x+2)}\right|}$$

Exercise

Perform each indicated operation & simplify $\frac{A}{x+1}-\frac{B}{x-1}+\frac{C}{x+2}$

Solution

$$\frac{A}{x+1}-\frac{B}{x-1}+\frac{C}{x+2}=\frac{A(x-1)(x+2)-B(x+1)(x+2)+C(x+1)(x-1)}{(x+1)(x-1)(x+2)}$$
$$=\frac{A(x^2+2x-x-2)-B(x^2+2x+x+2)+C(x^2-1)}{(x+1)(x-1)(x+2)}$$
$$=\frac{Ax^2+Ax-2A-Bx^2-3Bx-2B+Cx^2-C}{(x+1)(x-1)(x+2)}$$
$$=\underline{\left|\frac{(A-B-C)x^2+(A-3B)x-2A-2B-C}{(x+1)(x-1)(x+2)}\right|}$$

Exercise

Perform the operation and simplify: $-\frac{\sqrt{x^2+1}}{x^2}-\frac{1}{\sqrt{x^2+1}}$

Solution

$$-\frac{\sqrt{x^2+1}}{x^2}-\frac{1}{\sqrt{x^2+1}}=\frac{-\sqrt{x^2+1}\sqrt{x^2+1}-x^2}{x^2\sqrt{x^2+1}}$$
$$=\frac{-\left(x^2+1\right)-x^2}{x^2\sqrt{x^2+1}}$$

$$-\frac{\sqrt{x^2+1}}{x^2}\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}\frac{x^2}{x^2}$$

$$= \frac{-x^2 - 1 - x^2}{x^2\sqrt{x^2+1}}$$

$$= \frac{-2x^2 - 1}{x^2\sqrt{x^2+1}}$$

$$= \boxed{-\frac{2x^2+1}{x^2\sqrt{x^2+1}}}$$

Exercise

Perform the operation and simplify: $\left(\sqrt{x^2+1} - \frac{3x^3}{2\sqrt{x^2+1}}\right) \div \left(x^3+1\right)$

Solution

$$\left(\sqrt{x^2+1} - \frac{3x^3}{2\sqrt{x^2+1}}\right) \div \left(x^3+1\right) = \left(\frac{\sqrt{x^2+1}\left(2\sqrt{x^2+1}\right) - 3x^3}{2\sqrt{x^2+1}}\right) \cdot \frac{1}{x^3+1}$$

$$= \frac{2\left(x^2+1\right) - 3x^3}{2\left(x^3+1\right)\sqrt{x^2+1}}$$

$$= \boxed{\frac{-3x^3 + 2x^2 + 2}{2\left(x^3+1\right)\sqrt{x^2+1}}}$$

Exercise

Perform the operation and simplify: $\frac{6}{x(3x-2)} + \frac{5}{3x-2} - \frac{2}{x^2}$

Solution

$$\frac{6}{x(3x-2)} + \frac{5}{3x-2} - \frac{2}{x^2} = \frac{6}{x(3x-2)}\frac{x}{x} + \frac{5}{3x-2}\frac{x^2}{x^2} - \frac{2}{x^2}\frac{3x-2}{3x-2}$$ x $3x-2$ x^2

$$= \frac{6x + 5x^2 - 2(3x-2)}{x^2(3x-2)}$$

$$= \frac{6x + 5x^2 - 6x + 4}{x^2(3x-2)}$$

$$= \boxed{\frac{5x^2+4}{x^2(3x-2)}}$$

Exercise

Simplify the fraction: $\dfrac{\dfrac{2}{x+3}-\dfrac{2}{a+3}}{x-a}$

Solution

$$\frac{\frac{2}{x+3}-\frac{2}{a+3}}{x-a}=\frac{\frac{2(a+3)-2(x+3)}{(x+3)(a+3)}}{x-a}$$

$$=\frac{2a+6-2x-6}{(x+3)(a+3)}\cdot\frac{1}{x-a}$$

$$=\frac{2a-2x}{(x+3)(a+3)(x-a)}$$

$$=\frac{2(a-x)}{(x+3)(a+3)(x-a)}$$

$$=\frac{-2(x-a)}{(x+3)(a+3)(x-a)} \qquad \text{if } x\neq a$$

$$=\boxed{-\frac{2}{(x+3)(a+3)}}$$

Exercise

Simplify: $\dfrac{3x^2(2x+5)^{1/2}-x^3\left(\frac{1}{2}\right)(2x+5)^{-1/2}(2)}{\left[(2x+5)^{1/2}\right]^2}$

Solution

$$\frac{3x^2(2x+5)^{1/2}-x^3\left(\frac{1}{2}\right)(2x+5)^{-1/2}(2)}{\left[(2x+5)^{1/2}\right]^2}=\frac{3x^2(2x+5)^{1/2}-x^3(2x+5)^{-1/2}}{(2x+5)}$$

$$=\frac{3x^2(2x+5)^{1/2}-x^3(2x+5)^{-1/2}}{(2x+5)}\cdot\frac{(2x+5)^{1/2}}{(2x+5)^{1/2}}$$

$$=\frac{3x^2(2x+5)-x^3}{(2x+5)^{3/2}}$$

$$=\frac{6x^3+15x^2-x^3}{(2x+5)^{3/2}}$$

$$= \frac{5x^3 + 15x^2}{(2x+5)^{3/2}}$$

$$= \boxed{\frac{5x^2(x+3)}{(2x+5)^{3/2}}}$$

Exercise

Simplify the expression: $\dfrac{\left(4x^2+9\right)^{1/2}(2) - (2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left[\left(4x^2+9\right)^{1/2}\right]^2}$

Solution

$$\frac{\left(4x^2+9\right)^{1/2}(2) - (2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left[\left(4x^2+9\right)^{1/2}\right]^2} = \frac{2\left(4x^2+9\right)^{1/2} - 4x(2x+3)\left(4x^2+9\right)^{-1/2}}{4x^2+9}$$

$$= \frac{2\left(4x^2+9\right)^{1/2} - 4x(2x+3)\left(4x^2+9\right)^{-1/2}}{4x^2+9} \cdot \frac{\left(4x^2+9\right)^{1/2}}{\left(4x^2+9\right)^{1/2}}$$

$$= \frac{2\left(4x^2+9\right) - 4x(2x+3)}{\left(4x^2+9\right)^{3/2}}$$

$$= \frac{8x^2 + 18 - 8x^2 - 12x}{\left(4x^2+9\right)^{3/2}}$$

$$= \frac{18 - 12x}{\left(4x^2+9\right)^{3/2}}$$

$$= \boxed{\frac{6(3-2x)}{\left(4x^2+9\right)^{3/2}}}$$

Exercise

Simplify the expression: $\dfrac{\left(1-x^2\right)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)}{\left[\left(1-x^2\right)^{1/2}\right]^2}$

Solution

$$\frac{\left(1-x^2\right)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)}{\left[\left(1-x^2\right)^{1/2}\right]^2}=\frac{2x\left(1-x^2\right)^{1/2}+x^3\left(1-x^2\right)^{-1/2}}{1-x^2}\frac{\left(1-x^2\right)^{1/2}}{\left(1-x^2\right)^{1/2}}$$

$$=\frac{2x\left(1-x^2\right)+x^3}{\left(1-x^2\right)^{3/2}}$$

$$=\frac{2x-2x^3+x^3}{\left(1-x^2\right)^{3/2}}$$

$$=\boxed{\frac{2x-x^3}{\left(1-x^2\right)^{3/2}}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2+4\right)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)\left(x^2+4\right)^{-2/3}(2x)}{\left[\left(x^2+4\right)^{1/3}\right]^2}$

Solution

$$\frac{\left(x^2+4\right)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)\left(x^2+4\right)^{-2/3}(2x)}{\left[\left(x^2+4\right)^{1/3}\right]^2}=\frac{3\left(x^2+4\right)^{1/3}-6x^2\left(x^2+4\right)^{-2/3}}{\left(x^2+4\right)^{2/3}}\frac{\left(x^2+4\right)^{2/3}}{\left(x^2+4\right)^{2/3}}$$

$$=\frac{3\left(x^2+4\right)-6x^2}{\left(x^2+4\right)^{4/3}}$$

$$=\frac{3x^2+12-6x^2}{\left(x^2+4\right)^{4/3}}$$

$$= \boxed{\frac{-3x^2+12}{\left(x^2+4\right)^{4/3}}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2-5\right)^4(3x^2)-x^3(4)\left(x^2-5\right)^3(2x)}{\left[\left(x^2-5\right)^4\right]^2}$

Solution

$$\frac{\left(x^2-5\right)^4(3x^2)-x^3(4)\left(x^2-5\right)^3(2x)}{\left[\left(x^2-5\right)^4\right]^2} = \frac{\left(x^2-5\right)^3\left[3x^2\left(x^2-5\right)-8x^4\right]}{\left(x^2-5\right)^8}$$

$$= \frac{\left(x^2-5\right)^3\left[3x^4-15x^2-8x^4\right]}{\left(x^2-5\right)^8}$$

$$= \frac{\left(-5x^4-15x^2\right)}{\left(x^2-5\right)^5}$$

$$= \boxed{\frac{-5x^2\left(x^2+3\right)}{\left(x^2-5\right)^5}}$$

Exercise

Simplify the expression: $\dfrac{(3x+2)^{1/2}\left(\frac{1}{3}\right)(2x+3)^{-2/3}(2)-(2x+3)^{1/3}\left(\frac{1}{2}\right)(3x+2)^{-1/2}(3)}{\left[(3x+2)^{1/2}\right]^2}$

Solution

$$=\frac{\frac{2}{3}(3x+2)^{1/2}(2x+3)^{-2/3}-\frac{3}{2}(2x+3)^{1/3}(3x+2)^{-1/2}}{3x+2}\,\frac{6(2x+3)^{2/3}}{6(2x+3)^{2/3}}\,\frac{(3x+2)^{1/2}}{(3x+2)^{1/2}}$$

$$=\frac{4(3x+2)-9(2x+3)}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$=\frac{4(3x+2)-9(2x+3)}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$=\frac{12x+8-18x-27}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

$$=\boxed{\frac{-6x-19}{6(3x+2)^{3/2}(2x+3)^{2/3}}}$$

Exercise

Simplify the expression: $\dfrac{\left(x^2+2\right)^3(2x)-x^2(3)\left(x^2+2\right)^2(2x)}{\left[\left(x^2+2\right)^3\right]^2}$

Solution

$$\frac{\left(x^2+2\right)^3(2x)-x^2(3)\left(x^2+2\right)^2(2x)}{\left[\left(x^2+2\right)^3\right]^2}=\frac{2x\left(x^2+2\right)^2\left[\left(x^2+2\right)-3x^2\right]}{\left(x^2+2\right)^6}$$

$$=\frac{2x\left[x^2+2-3x^2\right]}{\left(x^2+2\right)^4}$$

$$= \frac{2x\left[-2x^2+2\right]}{\left(x^2+2\right)^4}$$

$$= \boxed{\frac{4x\left[-x^2+1\right]}{\left(x^2+2\right)^4}}$$