Solution — *Section* 2.11 – Piecewise Continuous with Nonhomogeneous Terms

Exercise

Find the inverse Laplace transform $f(t)$ of each function given $F(s)=\dfrac{e^{-3s}}{s^2}$

Solution

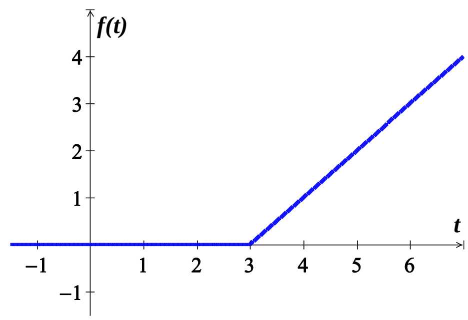
$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}=t$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{e^{-3s}\frac{1}{s^2}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-as}F(s)\right\}=u(t-a)f(t-a)$$

$$f(t)=u(t-3)\cdot(t-3)$$

$$=\begin{cases}0 & if \quad t<3\\ t-3 & if \quad t\ge 3\end{cases}$$



Exercise

Find the inverse Laplace transform $f(t)$ of each function given $F(s)=\dfrac{e^{-s}-e^{-3s}}{s^2}$

Solution

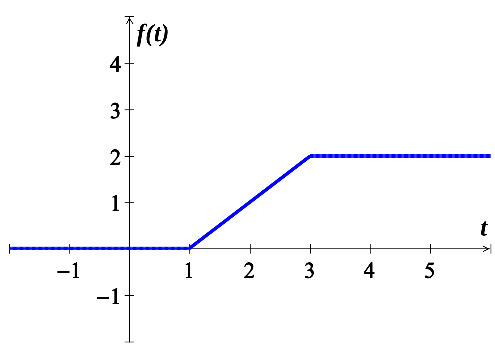
$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}=t$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{e^{-s}\frac{1}{s^2}-e^{-3s}\frac{1}{s^2}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-as}F(s)\right\}=u(t-a)f(t-a)$$

$$f(t)=(t-1)\cdot u(t-1)-(t-3)\cdot u(t-3)$$

$$=\begin{cases}0 & if \quad t<1\\ t-1 & if \quad 1\le t<3\\ 2 & if \quad t\ge 3\end{cases}$$



Exercise

Find the inverse Laplace transform $f(t)$ of each function given $F(s)=\dfrac{e^{-s}}{s+2}$

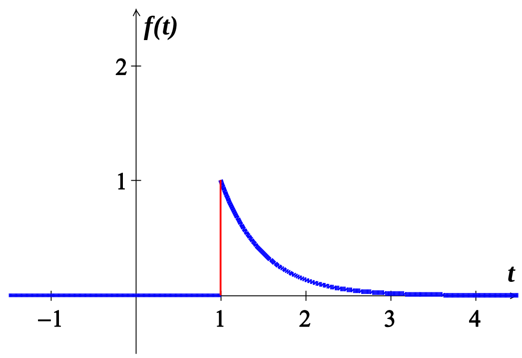
Solution

$$\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}=e^{-2t}$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{e^{-s}\frac{1}{s+2}\right\} \qquad \mathcal{L}^{-1}\left\{e^{-as}F(s)\right\}=u(t-a)\,f(t-a)$$

$$f(t)=e^{-2(t-1)}\cdot u(t-1)$$

$$=\begin{cases} 0 & \text{if} \quad t<1 \\ e^{-2(t-1)} & \text{if} \quad t\ge 1 \end{cases}$$



Exercise

Find the inverse Laplace transform $f(t)$ of each function given $F(s)=\dfrac{e^{-s}-e^{2-2s}}{s-1}$

Solution

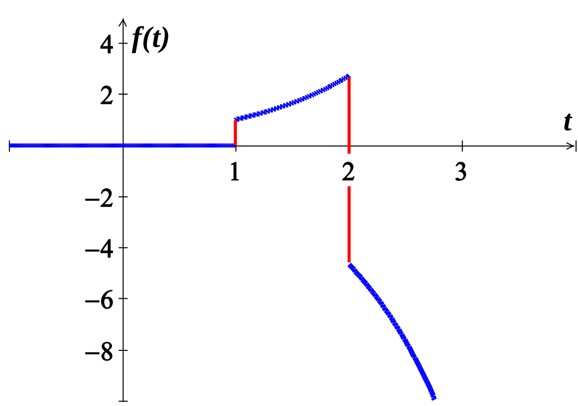
$$\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}=e^{t}$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{e^{-s}\frac{1}{s-1}-e^{2}e^{-2s}\frac{1}{s-1}\right\} \qquad \mathcal{L}^{-1}\left\{e^{-as}F(s)\right\}=u(t-a)\,f(t-a)$$

$$f(t)=e^{t-1}\cdot u(t-1)-e^{2}e^{t-2}\cdot u(t-2)$$

$$=e^{t-1}\cdot u(t-1)-e^{t}\cdot u(t-2)$$

$$=\begin{cases} 0 & \text{if} \quad t<1 \\ e^{t-1} & \text{if} \quad 1\le t<2 \\ e^{t-1}-e^{t} & \text{if} \quad t\ge 2 \end{cases}$$



Exercise

Find the inverse Laplace transform $f(t)$ of each function given $\quad F(s)=\dfrac{e^{-\pi s}}{s^2+1}$

Solution

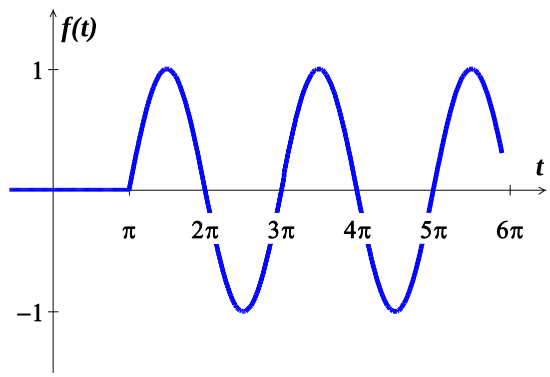
$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}=\sin t$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{e^{-\pi s}\frac{1}{s^2+1}\right\} \qquad \mathcal{L}^{-1}\left\{e^{-as}F(s)\right\}=u(t-a)f(t-a)$$

$$f(t)=\sin(t-\pi)\cdot u(t-\pi)$$

$$=-u(t-\pi)\sin t$$

$$=\begin{cases} 0 & if \quad t<\pi \\ -\sin t & if \quad t\geq\pi \end{cases}$$



Exercise

Find the inverse Laplace transform $f(t)$ of each function given $\quad F(s)=\dfrac{1-e^{-2\pi s}}{s^2+1}$

Solution

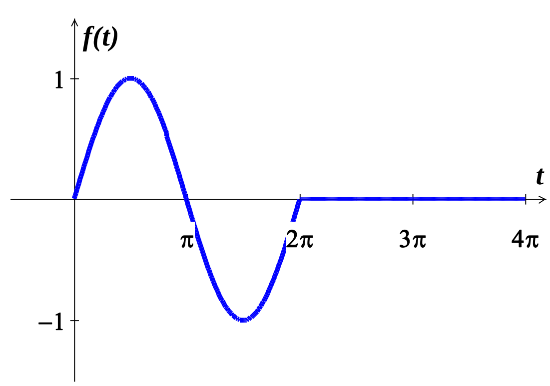
$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}=\sin t$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}-e^{-2\pi s}\frac{1}{s^2+1}\right\} \qquad \mathcal{L}^{-1}\left\{e^{-as}F(s)\right\}=u(t-a)f(t-a)$$

$$f(t)=\sin t-\sin(t-2\pi)\cdot u(t-2\pi)$$

$$=\sin t-u(t-2\pi)\sin t$$

$$=\begin{cases} \sin t & if \quad t<2\pi \\ 0 & if \quad t\geq 2\pi \end{cases}$$



Exercise

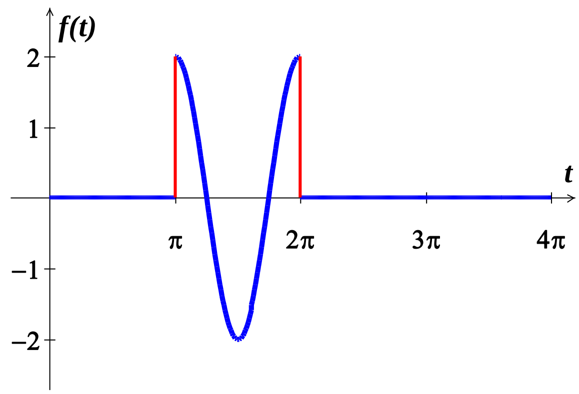
Find the inverse Laplace transform $f(t)$ of each function given $\quad F(s)=\dfrac{2s\left(e^{-\pi s}-e^{-2\pi s}\right)}{s^2+4}$

Solution

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}=\cos 2t$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{2e^{-\pi s}\frac{1}{s^2+4}-2e^{-2\pi s}\frac{1}{s^2+4}\right\} \qquad \mathcal{L}^{-1}\left\{e^{-as}F(s)\right\}=u(t-a)\,f(t-a)$$

$$\begin{aligned} f(t)&=2\cos 2(t-\pi)\cdot u(t-\pi)-2\cos 2(t-2\pi)\cdot u(t-2\pi)\\ &=2\cos 2t\cdot u(t-\pi)-2\cos 2t\cdot u(t-2\pi)\\ &=2\left[u(t-\pi)-u(t-2\pi)\right]\cos 2t\\ &=\begin{cases} 0 & \text{if} \quad t<\pi \text{ or } t\ge 2\pi\\ 2\cos 2t & \text{if} \quad \pi\le t<2\pi \end{cases} \end{aligned}$$



Exercise

Find the Laplace transforms of the given functions $\quad f(t)=\begin{cases} 2 & \text{if } 0\le t<3\\ 0 & \text{if } t\ge 3\end{cases}$

Solution

For $t\ge 3\to f(t)=2u(t-3)$

For $0\le t<3$, $f(t)=2-2\cdot u(t-3)$

$$\mathcal{L}\{f(t)\}=\mathcal{L}\{2-u(t-3)\cdot 2\} \qquad \mathcal{L}\{2\}=\frac{2}{s};\quad \mathcal{L}\{u(t)-u(t-a)\}=\frac{1}{s}\left(1-e^{-as}\right)$$

$$F(s)=\frac{2}{s}\left(1-e^{-3s}\right)$$

Exercise

Find the Laplace transforms of the given functions $f(t)=\begin{cases}1 & if\ \ 1\le t\le 4\\ 0 & if\ \ t<1,\ or\ t>4\end{cases}$

Solution

For $t<1 \to f(t)=u(t-1)$

For $t>4 \to f(t)=u(t-4)$

For $1\le t\le 4,\ f(t)=1\cdot u(t-1)-1\cdot u(t-4)$

$$\mathcal{L}\{f(t)\}=\mathcal{L}\{u(t-1)-u(t-4)\} \qquad \mathcal{L}\{u(t-a)\}=\frac{e^{-as}}{s}$$

$$F(s)=\boxed{\frac{e^{-s}}{s}-\frac{e^{-4s}}{s}}$$

Exercise

Find the Laplace transforms of the given functions $f(t)=\begin{cases}\sin t & if\ \ 0\le t\le 2\pi\\ 0 & if\ \ t>2\pi\end{cases}$

Solution

For $t>2\pi \to f(t)=u(t-2\pi)$

For $0\le t\le 2\pi \quad \to \quad f(t)=\sin t-u(t-2\pi)\cdot\sin t$

$\Rightarrow \ f(t)=\sin t-u(t-2\pi)\cdot\sin(t-2\pi)$

$$F(s)=\frac{1}{s^2+1}-\frac{e^{-2\pi s}}{s^2+1}$$

$$=\boxed{\frac{1-e^{-2\pi s}}{s^2+1}}$$

Exercise

Find the Laplace transforms of the given functions $f(t)=\begin{cases}\cos \pi t & if\ \ 0\le t\le 2\\ 0 & if\ \ t>2\end{cases}$

Solution

For $t>2 \to f(t)=u(t-2)$

For $0\le t\le 2 \quad \to \quad f(t)=\cos\pi t-u(t-2)\cdot\cos\pi t$

$\Rightarrow \ f(t)=\cos\pi t-u(t-2)\cdot\cos\pi(t-2)$

$$F(s)=\frac{s}{s^2+\pi^2}-e^{-2s}\frac{s}{s^2+\pi^2}$$

$$=\boxed{\frac{s\left(1-e^{-2s}\right)}{s^2+\pi^2}}$$

Exercise

Find the Laplace transforms of the given functions $f(t)=\begin{cases}\sin 2t & \text{if } \pi \le t \le 2\pi \\ 0 & \text{if } t<\pi \text{ or } t>2\pi\end{cases}$

Solution

For $\pi \le t \le 2\pi \quad \rightarrow \quad f(t)=u(t-\pi)\cdot\sin 2(t-\pi)-u(t-2\pi)\cdot\sin 2(t-2\pi)$

$$F(s)=\frac{2e^{-\pi s}}{s^2+4}-\frac{2e^{-2\pi s}}{s^2+4}$$

$$=\frac{2\left(e^{-\pi s}-e^{-2\pi s}\right)}{s^2+4}$$

Exercise

Use the Laplace transform method to solve the initial-value problem

$$y'+2y=f(x);\quad y(0)=1 \quad \text{where} \quad f(x)=\begin{cases}x & \text{if } 0\le x<3 \\ 1 & \text{if } x\ge 3\end{cases}$$

Solution

$$f(x)=x-x\cdot u(x-3)+u(x-3)$$

$$=x-(x-3+3)\cdot u(x-3)+u(x-3)$$

$$\mathcal{L}\{y'+2y\}=\mathcal{L}\{f(x)\}$$

$$sY(s)-y(0)+2Y(s)=\frac{1}{s^2}-e^{-3s}\frac{1}{s^2}+e^{-3s}\frac{1}{s}$$

$$(s+2)Y(s)-1=\frac{1}{s^2}-\frac{e^{-3s}}{s^2}+\frac{e^{-3s}}{s}$$

$$(s+2)Y(s)-1=\frac{1}{s^2}-\frac{e^{-3s}}{s^2}+\frac{e^{-3s}}{s}$$

$$(s+2)Y(s)=\frac{1}{s^2}-\frac{e^{-3s}}{s^2}+\frac{e^{-3s}}{s}+1$$

$$Y(s)=\frac{1}{s^2(s+2)}-\frac{e^{-3s}}{s^2(s+2)}+\frac{e^{-3s}}{s(s+2)}+\frac{1}{s+2}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+2)}+\frac{1}{s+2}\right\}=\frac{1}{2}x-\frac{1}{4}+\frac{1}{4}e^{-2x}+e^{-2x}=\frac{1}{2}x-\frac{1}{4}+\frac{5}{4}e^{-2x}$$

$$\mathcal{L}^{-1}\left\{-e^{-3s}\frac{1}{s^2(s+2)}+e^{-3s}\frac{1}{s(s+2)}\right\}=-\frac{1}{4}\left(2(x-3)-1+e^{-2(x-3)}\right)+\frac{1}{2}-\frac{1}{2}e^{-2(x-3)}$$

$$=-\frac{1}{2}x+\frac{3}{2}+\frac{1}{4}-\frac{1}{4}e^{-2(x-3)}+\frac{1}{2}-\frac{1}{2}e^{-2(x-3)}$$

$$= -\frac{1}{2}x + \frac{9}{4} - \frac{3}{4}e^{-2(x-3)}$$

For $x \ge 3$

$$y(x) = -\frac{1}{2}x + \frac{9}{4} - \frac{3}{4}e^{-2(x-3)} + \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}$$

$$= 2 + \frac{5}{4}e^{-2x} - \frac{3}{4}e^{-2(x-3)}$$

$$y(x) = \begin{cases} \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x} & 0 \le x < 3 \\ 2 + \frac{5}{4}e^{-2x} - \frac{3}{4}e^{-2(x-3)} & x \ge 3 \end{cases}$$

Exercise

Use the Laplace transform method to solve the initial-value problem

$$y'' + 2y' + y = f(x); \quad y(0) = y'(0) = 0 \quad \textit{where} \quad f(x) = \begin{cases} 1 & \textit{if } 0 \le x < 2 \\ x+1 & \textit{if } x \ge 2 \end{cases}$$

Solution

$$f(x) = 1 - 1 \cdot u(x-2) + (x+1) \cdot u(x-2)$$

$$= 1 - u(x-2) + (x - 2 + 2 + 1) \cdot u(x-2)$$

$$= 1 - u(x-2) + (x-2) \cdot u(x-2) + 3 \cdot u(x-2)$$

$$= 1 + 2 \cdot u(x-2) + (x-2) \cdot u(x-2)$$

$$\mathcal{L}\{y'' + 2y' + y\} = \mathcal{L}\{f(x)\}$$

$$s^2Y(s) - sy(0) - y'(0) + 2sY(s) - 2y(0) + Y(s) = \frac{1}{s} + \frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2}$$

$$\left(s^2 + 2s + 1\right)Y(s) = \frac{1}{s} + \frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2}$$

$$Y(s) = \frac{1}{s(s+1)^2} + \frac{2e^{-2s}}{s(s+1)^2} + \frac{e^{-2s}}{s^2(s+1)^2}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)^2}\right\} = 1 - (x+1)e^{-x} \qquad \mathcal{L}^{-1}\left\{\frac{a^2}{s(s+a)^2}\right\} = 1 - (at+1)e^{-at}$$

$$\mathcal{L}^{-1}\left\{\frac{2e^{-2s}}{s(s+1)^2} + \frac{e^{-2s}}{s^2(s+1)^2}\right\} = 2\left(1 - (x-2+1)e^{-(x-2)}\right) + x - 2 + 2e^{-(x-2)} + (x-2)e^{-(x-2)}$$

$$= 2 - 2xe^{-(x-2)} + 2e^{-(x-2)} + x - 2 + xe^{-(x-2)}$$

$$= x + (x-2)e^{-(x-2)}$$

For $x \ge 3$

$$y(x) = x + (x-2)e^{-(x-2)} + 1 - (x+1)e^{-x}$$
$$= x - 1 - (x+1)e^{-x} - (x-2)e^{-(x-2)}$$

$$y(x) = \begin{cases} 1-(x+1)e^{-x}, & 0 \le x < 2 \\ x-1-(x+1)e^{-x}-(x-2)e^{-(x-2)}, & x \ge 2 \end{cases}$$

Exercise

The values of mass *m*, spring constant *k*, dashpot resistance *c*, and force $f(t)$ are given for a mass-spring-dashpot system with external forcing function. Solve the initial value problem, and construct the graph position $x(t)$ $\quad mx'' + cx' + kx = f(t); \quad x(0) = x'(0) = 0$

$$m = 1, \quad k = 4, \quad c = 0, \quad f(t) = \begin{cases} 1 & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}$$

Solution

$$f(t) = 1 - 1 \cdot u(t-\pi) \quad \Rightarrow \quad \mathcal{L}\{f(t)\} = \frac{1}{s} - \frac{e^{-\pi s}}{s}$$

The initial value problem is: $\quad x'' + 4x = f(t); \quad x(0) = x'(0) = 0$

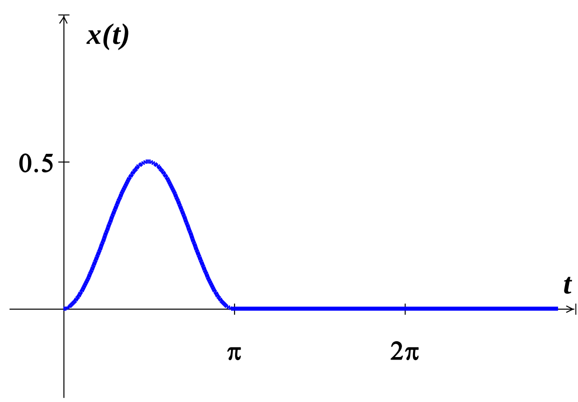
$$\mathcal{L}\{x'' + 4x\} = \mathcal{L}\{f(t)\}$$

$$\left(s^2+4\right)X(x) = \frac{1}{s} - \frac{e^{-\pi s}}{s}$$

$$X(x) = \frac{1}{s\left(s^2+4\right)} - \frac{e^{-\pi s}}{s\left(s^2+4\right)}$$

$$x(t) = \mathcal{L}^{-1}\{X(x)\}$$
$$= \frac{1}{2}\sin^2 t - u(t-\pi)\cdot\frac{1}{2}\sin^2(t-\pi)$$
$$= \frac{1}{2}\sin^2 t - u(t-\pi)\cdot\frac{1}{2}\sin^2 t$$
$$= \frac{1}{2}\left[1-u(t-\pi)\right]\sin^2 t$$

$$x(t) = \begin{cases} \frac{1}{2}\sin^2 t & \text{if } t < \pi \\ 0 & \text{if } t \ge \pi \end{cases}$$



Exercise

The values of mass m, spring constant k, dashpot resistance c, and force $f(t)$ are given for a mass-spring-dashpot system with external forcing function. Solve the initial value problem, and construct the graph position $x(t)$ $\quad mx'' + cx' + kx = f(t); \quad x(0) = x'(0) = 0$

$$m = 1, \quad k = 4, \quad c = 5, \quad f(t) = \begin{cases} 1 & 0 \le t < 2 \\ 0 & t \ge 2 \end{cases}$$

Solution

$$f(t) = 1 - 1 \cdot u(t-2) \quad \Rightarrow \quad \mathcal{L}\{f(t)\} = \frac{1}{s} - \frac{e^{-2s}}{s}$$

The initial value problem is: $\quad x'' + 5x' + 4x = f(t); \quad x(0) = x'(0) = 0$

$$\mathcal{L}\{x'' + 5x' + 4x\} = \mathcal{L}\{f(t)\}$$

$$\left(s^2 + 5s + 4\right)X(x) = \frac{1}{s} - \frac{e^{-2s}}{s}$$

$$X(x) = \frac{1}{s(s+1)(s+4)} - \frac{e^{-2s}}{s(s+1)(s+4)}$$

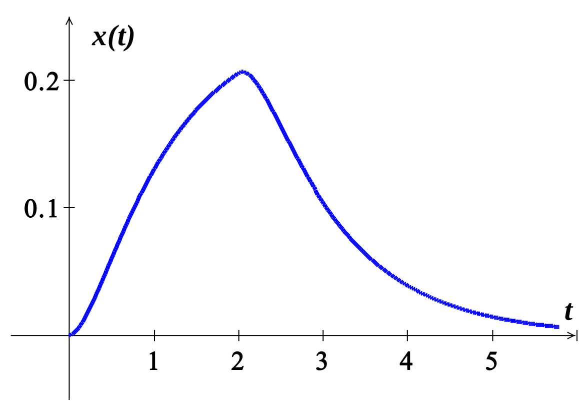
$$x(t) = \mathcal{L}^{-1}\{X(x)\} \qquad\qquad \mathcal{L}^{-1}\left\{\frac{1}{s(s+a)(s+b)}\right\} = \frac{1}{ab}\left(1 - \frac{b}{b-a}e^{-at} + \frac{a}{b-a}e^{-bt}\right)$$

$$= \frac{1}{4}\left(1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t}\right) - \frac{1}{4}\left(1 - \frac{4}{3}e^{-(t-2)} + \frac{1}{3}e^{-4(t-2)}\right)$$

$$= \frac{1}{12}\left(3 - 4e^{-t} + e^{-4t}\right) - \frac{1}{12}\left(3 - 4e^{-(t-2)} + e^{-4(t-2)}\right)$$

$$= \frac{1}{12}\left(-4e^{-t} + e^{-4t} + 4e^{-(t-2)} - e^{-4(t-2)}\right)$$

$$x(t) = \begin{cases} \frac{1}{12}\left(3 - 4e^{-t} + e^{-4t}\right) & \text{if } t < 2 \\ \frac{1}{12}\left(-4e^{-t} + e^{-4t} + 4e^{-(t-2)} - e^{-4(t-2)}\right) & \text{if } t \ge 2 \end{cases}$$



Exercise

The values of mass *m*, spring constant *k*, dashpot resistance *c*, and force $f(t)$ are given for a mass-spring-dashpot system with external forcing function. Solve the initial value problem, and construct the graph position $x(t)$ $mx'' + cx' + kx = f(t); \quad x(0) = x'(0) = 0$

$$m = 1, \quad k = 9, \quad c = 0, \quad f(t) = \begin{cases} \sin t & 0 \le t \le 2\pi \\ 0 & t > 2\pi \end{cases}$$

Solution

$$f(t) = \sin t - \sin t \cdot u(t - 2\pi) \quad \Rightarrow \quad \mathcal{L}\{f(t)\} = \frac{1}{s^2+1} - \frac{e^{-2\pi s}}{s^2+1}$$

The initial value problem is: $x'' + 9x = f(t); \quad x(0) = x'(0) = 0$

$$\mathcal{L}\{x'' + 9x\} = \mathcal{L}\{f(t)\}$$

$$\left(s^2+9\right)X(x) = \frac{1}{s^2+1} - \frac{e^{-2\pi s}}{s^2+1}$$

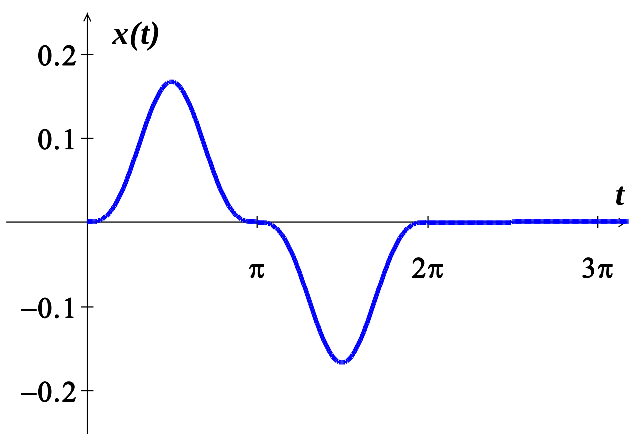
$$X(x) = \left(1 - e^{-2\pi s}\right)\frac{1}{\left(s^2+1\right)\left(s^2+9\right)}$$

$$= \frac{1}{8}\left(1 - e^{-2\pi s}\right)\left(\frac{1}{s^2+1} - \frac{1}{s^2+9}\right)$$

$$x(t) = \mathcal{L}^{-1}\{X(x)\}$$

$$= \frac{1}{8}\left(1 - u(t-2\pi)\right)\left(\sin t - \frac{1}{3}\sin 3t\right)$$

$$x(t) = \begin{cases} \frac{1}{8}\left(\sin t - \frac{1}{3}\sin 3t\right) & \text{if } t < 2\pi \\ \frac{1}{8}\left(\sin t - \frac{1}{3}\sin 3t - \sin(t-2\pi) + \frac{1}{3}\sin 3(t-2\pi)\right) & \text{if } t \ge 2\pi \end{cases}$$



Exercise

The values of mass m, spring constant k, dashpot resistance c, and force $f(t)$ are given for a mass-spring-dashpot system with external forcing function. Solve the initial value problem, and construct the graph position $x(t)$ $\quad mx'' + cx' + kx = f(t); \quad x(0) = x'(0) = 0$

$$m = 1, \quad k = 4, \quad c = 4, \quad f(t) = \begin{cases} t & 0 \le t \le 2 \\ 0 & t > 2 \end{cases}$$

Solution

$$f(t) = t - t \cdot u(t-2) = t - ((t-2)+2) \cdot u(t-2) \quad \Rightarrow \quad \mathcal{L}\{f(t)\} = \frac{1}{s^2} - e^{-2s}\left(\frac{1}{s^2} + \frac{2}{s}\right)$$

The initial value problem is: $\quad x'' + 4x' + 4x = f(t); \quad x(0) = x'(0) = 0$

$$\mathcal{L}\{x'' + 4x' + 4x\} = \mathcal{L}\{f(t)\}$$

$$\left(s^2 + 4s + 4\right) X(x) = \frac{1}{s^2} - e^{-2s}\left(\frac{1}{s^2} + \frac{2}{s}\right)$$

$$X(x) = \frac{1}{s^2(s+2)^2} - e^{-2s}\frac{1}{(s+2)^2}\left(\frac{1+2s}{s^2}\right)$$

$$= \frac{1}{s^2(s+2)^2} - e^{-2s}\frac{2s+1}{s^2(s+2)^2}$$

$$\frac{2s+1}{s^2(s+2)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{D}{(s+2)^2}$$

$$2s + 1 = As\left(s^2 + 4s + 4\right) + B\left(s^2 + 4s + 4\right) + Cs^2(s+2) + Ds^2$$

$$\begin{cases} s^3 & A + C = 0 \\ s^2 & 4A + B + 2C + D = 0 \\ s & 4A + 4B = 2 \\ s^0 & 4B = 1 \to \boxed{B = \frac{1}{4}} \end{cases} \to \boxed{A = \frac{1}{4}} \quad \boxed{C = -\frac{1}{4}} \quad \boxed{D = -\frac{3}{4}}$$

$$\frac{2s+1}{s^2(s+2)^2} = \frac{1}{4}\left(\frac{1}{s} + \frac{1}{s^2} - \frac{1}{s+2} - \frac{3}{(s+2)^2}\right)$$

$$\mathcal{L}^{-1}\frac{1}{4}\left(\frac{1}{s} + \frac{1}{s^2} - \frac{1}{s+2} - \frac{3}{(s+2)^2}\right) = \frac{1}{4}\left(1 + t - e^{-2t} - 3te^{-2t}\right)$$

$$x(t) = \mathcal{L}^{-1}\{X(x)\} \qquad\qquad \mathcal{L}^{-1}\frac{1}{s^2(s+a)^2} = \frac{1}{a^3}\left(-2 + at + (2+at)e^{-at}\right)$$

$$= \frac{1}{8}\left(-2 + 2t + (2+2t)e^{-2t}\right) - \frac{1}{4}\left(1 + t - 2 - (3t - 6 + 1)e^{-2(t-2)}\right)$$

$$= \frac{1}{4}\left(-1+t+(1+t)e^{-2t}\right) - \frac{1}{4}\left(t-1-(3t-5)e^{-2(t-2)}\right)$$

$$= \frac{1}{4}\left((1+t)e^{-2t} + (3t-5)e^{-2(t-2)}\right)$$

$$x(t) = \begin{cases} \frac{1}{4}\left(-1+t+(1+t)e^{-2t}\right) & \text{if } t < 2 \\ \frac{1}{4}\left((1+t)e^{-2t} + (3t-5)e^{-2(t-2)}\right) & \text{if } t \geq 2 \end{cases}$$