Exercise

Write $-\sqrt{3}+i$ in trigonometric form. (Use radian measure)

Solution

$$-\sqrt{3}+i \;\Rightarrow\; \begin{cases} x=-\sqrt{3} \\ y=1 \end{cases}$$

$$r=\sqrt{\left(-\sqrt{3}\right)^2+1^2}$$

$$=2$$

$$\hat{\theta}=\tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) \qquad \theta=\tan^{-1}\frac{y}{x}$$

$$=\frac{\pi}{6}$$

The angle is in quadrant *II*, therefore;

$$\theta=\pi-\frac{\pi}{6}$$

$$=\frac{5\pi}{6}$$

$$-\sqrt{3}+i=2\ cis\frac{5\pi}{6}$$

Exercise

Write $3-4i$ in trigonometric form.

Solution

$$3-4i \;\Rightarrow\; \begin{cases} x=3 \\ y=-4 \end{cases}$$

$$r=\sqrt{3^2+(-4)^2}$$

$$=2$$

$$\hat{\theta}=\tan^{-1}\left(\frac{4}{3}\right) \qquad \theta=\tan^{-1}\frac{y}{x}$$

$$\approx 53°$$

The angle is in quadrant *II*, therefore;

$$\theta=180°-53°$$

$$=127°$$

$$3-4i=5\ cis127°$$

Exercise

Write $-21-20i$ in trigonometric form.

Solution

$$-21-20i \Rightarrow \begin{cases} x=-21 \\ y=-20 \end{cases}$$

$$r=\sqrt{(-21)^2+(-20)^2}$$
$$=29$$

$$\hat{\theta}=\tan^{-1}\left(\frac{20}{21}\right) \qquad \theta=\tan^{-1}\frac{y}{x}$$
$$\approx 43.6°$$

The angle is in quadrant *III*, therefore;

$$\theta=180°+43.6°$$
$$=223.6°$$

$$-21-20i=29\ cis223.6°$$

Exercise

Write $11+2i$ in trigonometric form.

Solution

$$11+2i \Rightarrow \begin{cases} x=11 \\ y=2 \end{cases}$$

$$r=\sqrt{11^2+2^2}$$
$$=\sqrt{125}$$
$$=5\sqrt{5}$$

$$\theta=\tan^{-1}\left(\frac{2}{11}\right) \qquad \theta=\tan^{-1}\frac{y}{x}$$
$$\approx 10.3°$$

$$11+2i=5\sqrt{5}\ cis10.3°$$

Exercise

Write $\sqrt{3}-i$ in trigonometric form.

Solution

$$\sqrt{3}-i \Rightarrow \begin{cases} x=\sqrt{3} \\ y=-1 \end{cases}$$

$$r=\sqrt{3+1}$$
$$=2$$

$$\hat{\theta}=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad \theta=\tan^{-1}\frac{y}{x}$$
$$\approx 30°$$

The angle is in quadrant IV, therefore;

$$\theta=360°-30°$$
$$=330°$$

$$\sqrt{3}-i=2\ cis330°$$

Exercise

Write $1-\sqrt{3}i$ in trigonometric form.

Solution

$$1-\sqrt{3}i \Rightarrow \begin{cases} x=1 \\ y=-\sqrt{3} \end{cases}$$

$$r=\sqrt{1+3}$$
$$=2$$

$$\hat{\theta}=\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) \qquad \theta=\tan^{-1}\frac{y}{x}$$
$$=60°$$

The angle is in quadrant IV, therefore;

$$\theta=360°-60°$$
$$=300°$$

$$1-\sqrt{3}i=2\ cis300°$$

Exercise

Write $9\sqrt{3}+9i$ in trigonometric form.

Solution

$$9\sqrt{3}+9i \Rightarrow \begin{cases} x = 9\sqrt{3} \\ y = 9 \end{cases}$$

$$r = 9\sqrt{3+1}$$

$$= 18$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= 30°$$

$$9\sqrt{3}+9i = 18\ cis30°$$

Exercise

Write $-2+3i$ in trigonometric form.

Solution

$$-2+3i \Rightarrow \begin{cases} x = -2 \\ y = 3 \end{cases}$$

$$r = \sqrt{4+9}$$

$$= \sqrt{13}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{3}{2}\right) \qquad \theta = \tan^{-1}\frac{y}{x}$$

$$\approx 56.31°$$

The angle is in quadrant *II*, therefore;

$$\theta = 180° - 56.31°$$

$$\approx 123.69°$$

$$-2+3i = \sqrt{13}\ cis123.69°$$

Exercise

Write $4\left(\cos 30° + i\sin 30°\right)$ in standard form.

Solution

$$4\left(\cos 30° + i\sin 30°\right) = 4\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$= 2\sqrt{3} + 2i$$

Exercise

Write $\sqrt{2}\ cis\frac{7\pi}{4}$ in standard form.

Solution

$$\sqrt{2}\ cis\frac{7\pi}{4} = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$$

$$\underline{= 1 - i}$$

Exercise

Write $3cis210°$ in standard form.

Solution

$$3cis210° = 3\left(\cos 210° + i\sin 210°\right)$$

$$\underline{= -\frac{3\sqrt{3}}{2} - \frac{3}{2}i}$$

Exercise

Write $4\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ in standard form.

Solution

$$4\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = 4\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$$

$$\underline{= 2\sqrt{2} - 2i\sqrt{2}}$$

Exercise

Write $4cis\frac{\pi}{2}$ in standard form.

Solution

$$4cis\frac{\pi}{2} = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$\underline{= 4i}$$

Exercise

Find the quotient $\frac{20cis(75°)}{4cis(40°)}$. Write the result in rectangular form.

Solution

$$\frac{20cis(75°)}{4cis(40°)} = \frac{20}{4}cis(75° - 40°)$$

$$= 5cis(35°)$$

$$= 5(\cos 35° + i\sin 35°)$$

$$\underline{= 4.1 + 2.87i}$$

Exercise

Divide $z_1 = 1 + i\sqrt{3}$ *by* $z_2 = \sqrt{3} + i$. Write the result in rectangular form.

Solution

$$\frac{z_1}{z_2} = \frac{1 + i\sqrt{3}}{\sqrt{3} + i}$$

$$= \frac{1 + i\sqrt{3}}{\sqrt{3} + i}\frac{\sqrt{3} - i}{\sqrt{3} - i}$$

$$= \frac{\sqrt{3} - i + 3i - \sqrt{3}\ i^2}{3 + 1}$$

$$= \frac{2\sqrt{3} + 2i}{4}$$

$$= \frac{2\sqrt{3}}{4} + \frac{2i}{4}$$

$$\underline{= \frac{\sqrt{3}}{2} + \frac{i}{2}}$$

or

$$1 + i\sqrt{3} \rightarrow \begin{cases} r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} \\ \theta = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3} \end{cases}$$

$$\sqrt{3} + i \rightarrow \begin{cases} r = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} \\ \theta = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{cases}$$

$$\frac{z_1}{z_2} = \frac{2cis\frac{\pi}{3}}{2cis\frac{\pi}{6}}$$

$$= \frac{2}{2}cis\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{2}{2}cis\left(\frac{\pi}{6}\right)$$

$$= cis\left(\frac{\pi}{6}\right)$$

$$\underline{= \frac{\sqrt{3}}{2} + \frac{i}{2}}$$

Exercise

Find $(1+i)^8$ and express the result in rectangular form.

Solution

$$1+i \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$$

$$r = \sqrt{1^2 + 1^2}$$

$$\underline{= \sqrt{2}}$$

$$\theta = \tan^{-1}(1)$$

$$\underline{= \frac{\pi}{4}}$$

$$\underline{1+i = \sqrt{2}cis\frac{\pi}{4}}$$

$$(1+i)^8 = \left(\sqrt{2}cis\frac{\pi}{4}\right)^8$$

$$= \left(\sqrt{2}\right)^8 cis\left[8\left(\frac{\pi}{4}\right)\right]$$

$$= 16cis2\pi$$

$$= 16(\cos 2\pi + i\sin 2\pi)$$

$$= 16(1+i0)$$

$$\underline{= 16}$$

Exercise

Find $(1+i)^{10}$ and express the result in rectangular form.

Solution

$$1+i \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$$

$$r = \sqrt{1^2 + 1^2}$$

$$\underline{= \sqrt{2}}$$

$$\theta = \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

$$1+i = \sqrt{2}cis\frac{\pi}{4}$$

$$(1+i)^{10} = \left(\sqrt{2}cis\frac{\pi}{4}\right)^{10}$$

$$= \left(\sqrt{2}\right)^{10} cis\left[10\left(\frac{\pi}{4}\right)\right]$$

$$= 32cis\frac{5\pi}{2}$$

$$= 32cis\frac{\pi}{2}$$

$$= 32\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$= 32(0+i)$$

$$= 32i$$

Exercise

Find and express the result in rectangular form $(1-i)^5$

Solution

$$1-i \Rightarrow \begin{cases} x = 1 \\ y = -1 \end{cases}$$

$$r = \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$\hat{\theta} = \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

The angle is in quadrant *IV*, therefore;

$$\theta = 2\pi - \frac{\pi}{4}$$

$$= \frac{7\pi}{4}$$

$$1-i = \sqrt{2}\ cis\frac{7\pi}{4}$$

$$(1-i)^5 = \left(\sqrt{2}\ cis\frac{7\pi}{4}\right)^5$$

$$= 4\sqrt{2}\left(\ cis\left(5\times\frac{7\pi}{4}\right)\right)$$

$$= 4\sqrt{2}\left(cis\frac{35\pi}{4}\right) \qquad 35\pi - 8\times 2\pi = 3\pi$$

$$= 4\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 4\sqrt{2}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$$

$$\underline{= -4 + 4i\ \big|}$$

Exercise

Find and express the result in rectangular form $\left(1-\sqrt{5}i\right)^8$

Solution

$$1-\sqrt{5}i \ \Rightarrow\ \begin{cases} x = 1 \\ y = -\sqrt{5} \end{cases}$$

$$r = \sqrt{1+5}$$

$$\underline{= \sqrt{6}\ \big|}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{\sqrt{5}}{1}\right)$$

$$\underline{\approx 66^\circ\ \big|}$$

The angle is in quadrant *IV*, therefore;

$$\theta = 360^\circ - 66^\circ$$

$$\underline{= 294^\circ\ \big|}$$

$$\underline{1-\sqrt{5}i = \sqrt{6}\ cis294^\circ\ \big|}$$

$$\left(1-\sqrt{5}i\right)^8 = \left(\sqrt{6}cis294^\circ\right)^8$$

$$= \left(\sqrt{6}\right)^8 \left(cis2352^\circ\right) \qquad 2352^\circ - 6\times 360^\circ = 192^\circ$$

$$= 1296\left(\cos 192^\circ + i\sin 192^\circ\right)$$

$$= 1296\left(-.978 - 0.208i\right)$$

$$\underline{= -1267.488 - 269.568\ i\ \big|}$$

Exercise

Find and express the result in rectangular form $(3cis80°)^3$

Solution

$$(3cis80°)^3 = 3^3(cis240°)$$
$$= 27(\cos 240° + i\sin 240°)$$
$$= 27\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$
$$\underline{= -\frac{27}{2} - i\frac{27\sqrt{3}}{2}}$$

Exercise

Find and express the result in rectangular form $\left(\sqrt{3}cis10°\right)^6$

Solution

$$\left(\sqrt{3}cis10°\right)^6 = 27(cis60°)$$
$$= 27(\cos 60° + i\sin 60°)$$
$$\underline{= \frac{27}{2} + i\frac{27\sqrt{3}}{2}}$$

Exercise

Find and express the result in rectangular form $\left(\sqrt{2} - i\right)^6$

Solution

$$\sqrt{2} - i \Rightarrow \begin{cases} x = \sqrt{2} \\ y = -1 \end{cases}$$
$$r = \sqrt{2+1}$$
$$\underline{= \sqrt{3}}$$
$$\hat{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$\underline{\approx 35.26°}$$

The angle is in quadrant *IV*, therefore;

$$\theta = 360° - 35.26°$$
$$\underline{= 324.74°}$$
$$\underline{\sqrt{2} - i = \sqrt{3}\ cis324.74°}$$

$$\left(\sqrt{2}-i\right)^6=\left(\sqrt{3}\ cis324.74°\right)^6$$

$$=27\left(cis1948.44°\right) \qquad 1948.44°-5\times360°=148.44°$$

$$=27\left(\cos148.44°+i\sin148.44°\right)$$

$$=-23+14.142i$$

Exercise

Find and express the result in rectangular form $\left(4cis40°\right)^6$

Solution

$$\left(4cis40°\right)^6=4^6\left(cis\left(6\times40°\right)\right)$$

$$=4^6\left(\cos240°+i\sin240°\right)$$

$$=4096\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$$

$$=-2048+2048\ i\sqrt{3}$$

Exercise

Find and express the result in rectangular form $\left(2cis30°\right)^5$

Solution

$$\left(2cis30°\right)^5=2^5cis\left(5(30°)\right)$$

$$=32\left(\cos150°+i\sin150°\right)$$

$$=32\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$$

$$=-16\sqrt{3}+16i$$

Exercise

Find and express the result in rectangular form $\left(\frac{1}{2}cis72°\right)^5$

Solution

$$\left(\frac{1}{2}cis72°\right)^5=\frac{1}{2^5}cis\left(5\times72°\right)$$

$$=\frac{1}{32}cis\left(\cos360°+i\sin360°\right)$$

$$=\frac{1}{32}$$

Exercise

Find ***fifth*** roots of $z = 1 + i\sqrt{3}$ and express the result in rectangular form.

Solution

$$1 + i\sqrt{3} \Rightarrow \begin{cases} x = 1 \\ y = \sqrt{3} \end{cases}$$

$$r = \sqrt{1+3}$$

$$= 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$= 60°$$

$$1 + i\sqrt{3} = 2\ cis60°$$

$$\left(1 + i\sqrt{3}\right)^{1/5} = \left(2\ cis60°\right)^{1/5}$$

$$= \sqrt[5]{2}\left(cis\frac{60°}{5} + \frac{360°k}{5}\right)$$

$$= \sqrt[5]{2}\ cis\left(12° + 72°k\right) \qquad k = 0,\ 1,\ 2,\ 3,\ 4$$

For $k = 0$

$$\sqrt[5]{2}\ cis\left(12° + 72°0\right) = \sqrt[5]{2}\ cis12°$$

For $k = 1$

$$\sqrt[5]{2}\ cis\left(12° + 72°.(1)\right) = \sqrt[5]{2}\ cis84°$$

For $k = 2$

$$\sqrt[5]{2}\ cis\left(12° + 72°.(2)\right) = \sqrt[5]{2}\ cis156°$$

For $k = 3$

$$\sqrt[5]{2}\ cis\left(12° + 72°.(3)\right) = \sqrt[5]{2}\ cis228°$$

For $k = 4$

$$\sqrt[5]{2}\ cis\left(12° + 72°.(4)\right) = \sqrt[5]{2}\ cis300°$$

Exercise

Find the ***fourth*** roots of $z = 16cis60°$

Solution

$$\sqrt[4]{z} = \sqrt[4]{16}\ cis\left(\frac{60°}{4} + \frac{360°}{4}k\right)$$

$$= \underline{2cis(15° + 90°k)} \qquad k = 0,\ 1,\ 2,\ 3$$

For $k = 0$

$$2\ cis(15° + 90°(0)) \ \underline{= 2cis15°}$$

For $k = 1$

$$2\ cis(15° + 90°(1)) \ \underline{= 2cis105°}$$

For $k = 2$

$$2\ cis(15° + 90°(2)) \ \underline{= 2cis195°}$$

For $k = 3$

$$2\ cis(15° + 90°(3)) \ \underline{= 2cis285°}$$

Exercise

Find the ***fourth*** roots of $\sqrt{3} - i$

Solution

$$\sqrt{3} - i \ \Rightarrow \ \begin{cases} x = \sqrt{3} \\ y = -1 \end{cases}$$

$$r = \sqrt{3+1}$$

$$\underline{= 2}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\underline{= \frac{\pi}{6}}$$

The angle is in quadrant *IV*, therefore;

$$\theta = 2\pi - \frac{\pi}{6}$$

$$\underline{= \frac{11\pi}{6}}$$

$$\underline{\sqrt{3} - i = 2\ cis\frac{11\pi}{6}}$$

$$\sqrt[4]{\sqrt{3} - i} = \sqrt[4]{2\ cis\frac{11\pi}{6}}$$

$$= \sqrt[4]{2}\ cis\left(\frac{1}{4}\frac{11\pi}{6} + \frac{2\pi k}{4}\right)$$

$$= \underline{\sqrt[4]{2}\ cis\left(\frac{11\pi}{24} + \frac{\pi k}{2}\right)} \qquad k = 0,\ 1,\ 2,\ 3$$

For $k = 0$

$$\sqrt[4]{2}\ cis\left(\frac{11\pi}{24} + 0\right) \underline{= \sqrt[4]{2}\ cis\frac{11\pi}{24}}$$

For $k = 1$

$$\sqrt[4]{2}\ cis\left(\frac{11\pi}{24} + \frac{\pi}{2}\right) \underline{= \sqrt[4]{2}\ cis\frac{23\pi}{24}}$$

For $k = 2$

$$\sqrt[4]{2}\ cis\left(\frac{11\pi}{24} + \pi\right) \underline{= \sqrt[4]{2}\ cis\frac{35\pi}{24}}$$

For $k = 3$

$$\sqrt[4]{2}\ cis\left(\frac{11\pi}{24} + \frac{3\pi}{2}\right) \underline{= \sqrt[4]{2}\ cis\frac{47\pi}{24}}$$

Exercise

Find the ***fourth*** roots of $4 - 4\sqrt{3}i$

Solution

$$4 - 4\sqrt{3}i \Rightarrow \begin{cases} x = 4 \\ y = -4\sqrt{3} \end{cases}$$

$$r = 4\sqrt{3+1}$$

$$\underline{= 8}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$$

$$= \tan^{-1}\left(\sqrt{3}\right)$$

$$\underline{= \frac{\pi}{3}}$$

The angle is in quadrant *IV*, therefore;

$$\theta = 2\pi - \frac{\pi}{3}$$

$$\underline{= \frac{5\pi}{3}}$$

$$\underline{4 - 4\sqrt{3}i = 8\ cis\frac{5\pi}{3}}$$

$$\sqrt[4]{4 - 4\sqrt{3}i} = \sqrt[4]{8\ cis\frac{5\pi}{3}}$$

$$= \sqrt[4]{8}\ cis\left(\frac{5\pi}{12} + \frac{\pi k}{2}\right) \qquad k = 0,\ 1,\ 2,\ 3$$

For $k = 0$

$$\sqrt[4]{8}\ cis\left(\frac{5\pi}{12} + 0\right) = \sqrt[4]{8}\ cis\frac{5\pi}{12}$$

For $k = 1$

$$\sqrt[4]{8}\ cis\left(\frac{5\pi}{12} + \frac{\pi}{2}\right) = \sqrt[4]{8}\ cis\frac{11\pi}{12}$$

For $k = 2$

$$\sqrt[4]{8}\ cis\left(\frac{5\pi}{12} + \pi\right) = \sqrt[4]{8}\ cis\frac{17\pi}{12}$$

For $k = 3$

$$\sqrt[4]{8}\ cis\left(\frac{5\pi}{12} + \frac{3\pi}{2}\right) = \sqrt[4]{8}\ cis\frac{23\pi}{12}$$

Exercise

Find the ***fourth*** roots of $-16i$

Solution

$$-16i \Rightarrow \begin{cases} x = 0 \\ y = -16 \end{cases}$$

$$r = 16$$

$$\theta = \frac{3\pi}{2}$$

$$-16i = 16\ cis\frac{3\pi}{2}$$

$$\sqrt[4]{-16i} = \sqrt[4]{16\ cis\frac{3\pi}{2}}$$

$$= 2\ cis\left(\frac{3\pi}{8} + \frac{\pi k}{2}\right) \qquad k = 0,\ 1,\ 2,\ 3$$

For $k = 0$

$$2\ cis\left(\frac{3\pi}{8} + 0\right) = 2\ cis\frac{3\pi}{8}$$

For $k = 1$

$$2\ cis\left(\frac{3\pi}{8} + \frac{\pi}{2}\right) = 2\ cis\frac{7\pi}{8}$$

For $k = 2$

$$2\ cis\left(\frac{3\pi}{8} + \pi\right) = 2\ cis\frac{11\pi}{8}$$

For $k=3$

$$2\ cis\left(\frac{3\pi}{8}+\frac{3\pi}{2}\right) = \underline{2\ cis\frac{15\pi}{8}\ \Big|}$$

Exercise

Find the ***cube*** roots of 27.

Solution

$$\sqrt[3]{27} = \left(27 cis 0°\right)^{1/3}$$

$$= \sqrt[3]{27}\ cis\left(\frac{0°}{3}+\frac{360°}{3}k\right)$$

$$= \underline{3\ cis\left(0°+120°k\right)\ \Big|} \qquad k=0,\ 1,\ 2$$

For $k=0$

$$z = 3\ cis\left(0°+120°(0)\right) = \underline{2\ cis 0°\ \Big|}$$

For $k=1$

$$z = 3\ cis\left(0°+120°(1)\right) = \underline{2\ cis 120°\ \Big|}$$

For $k=2$

$$z = 3\ cis\left(0°+120°(2)\right) = \underline{2\ cis 240°\ \Big|}$$

Exercise

Find the ***cube*** roots of $8-8i$

Solution

$$8-8i \ \Rightarrow\ \begin{cases} x=8 \\ y=-8 \end{cases}$$

$$r = 8\sqrt{1+1}$$

$$= \underline{8\sqrt{2}\ \Big|}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{8}{8}\right)$$

$$= \tan^{-1}(1)$$

$$= \underline{\frac{\pi}{4}\ \Big|}$$

The angle is in quadrant *IV*, therefore;

$$\theta = 2\pi - \frac{\pi}{4}$$

$$= \underline{\frac{7\pi}{4}\ \Big|}$$

$$\sqrt[3]{8-8i} = \sqrt[3]{8\sqrt{2}\ cis\frac{7\pi}{4}}$$

$$= \underline{2\sqrt[3]{2}\ cis\left(\frac{7\pi}{12}+\frac{2\pi k}{3}\right)} \qquad k = 0,\ 1,\ 2$$

For $k = 0$

$$z = 2\sqrt[3]{2}\ cis\left(\frac{7\pi}{12}+0\right) \underline{= 2\sqrt[3]{2}\ cis\frac{7\pi}{12}}$$

For $k = 1$

$$z = 2\sqrt[3]{2}\ cis\left(\frac{7\pi}{12}+\frac{2\pi}{3}\right) \underline{= 2\sqrt[3]{2}\ cis\frac{15\pi}{12}}$$

For $k = 2$

$$z = 2\sqrt[3]{2}\ cis\left(\frac{7\pi}{12}+\frac{4\pi}{3}\right) \underline{= 2\sqrt[3]{2}\ cis\frac{23\pi}{12}}$$

Exercise

Find the ***cube*** roots of -8

Solution

$\underline{r = 8}$

$\underline{\theta = \frac{3\pi}{2}}$

$$\sqrt[3]{-8} = \sqrt[3]{8\ cis\frac{3\pi}{2}}$$

$$\underline{= 2\ cis\left(\frac{\pi}{2}+\frac{2\pi k}{3}\right)} \qquad k = 0,\ 1,\ 2$$

For $k = 0$

$$z = 2\ cis\left(\frac{\pi}{2}+0\right) \underline{= 2\ cis\frac{\pi}{2}}$$

For $k = 1$

$$z = 2\ cis\left(\frac{\pi}{2}+\frac{2\pi}{3}\right) \underline{= 2\ cis\frac{7\pi}{6}}$$

For $k = 2$

$$z = 2\ cis\left(\frac{\pi}{2}+\frac{4\pi}{3}\right) \underline{= 2\ cis\frac{11\pi}{6}}$$

Exercise

Find all complex number solutions of $x^3+1=0$.

Solution

$$x^3+1=0$$
$$x^3=-1$$
$$\underline{r=1}$$

$$\theta=\tan^{-1}\left(\frac{0}{-1}\right)$$
$$\underline{=\pi}$$

$$x^3=1\ cis(\pi)$$

$$x=(1\ cis\pi)^{1/3}$$
$$=(1)^{1/3}\ \ cis\left(\frac{\pi}{3}+\frac{2\pi}{3}k\right)$$
$$\underline{=cis\left(\frac{\pi}{3}+\frac{2\pi}{3}k\right)} \qquad k=0,\ 1,\ 2$$

For $k=0$

$$x=cis\left(\frac{\pi}{3}+\frac{2\pi}{3}(0)\right)$$
$$=cis\frac{\pi}{3}$$
$$=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$$
$$\underline{=\frac{1}{2}+i\frac{\sqrt{3}}{2}}$$

For $k=1$

$$x=cis\left(\frac{\pi}{3}+\frac{2\pi}{3}(1)\right)$$
$$=cis\left(\frac{3\pi}{3}\right)$$
$$=cis\pi$$
$$=\cos\pi+i\sin\pi$$
$$\underline{=-1}$$

For $k=2$

$$x=cis\left(\frac{\pi}{3}+\frac{2\pi}{3}(2)\right)$$
$$=cis\frac{5\pi}{3}$$

$$= \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$$

$$\boxed{= \frac{1}{2} - i\frac{\sqrt{3}}{2}}$$