2.3 #1 17

a) $357 = 21 \times 17$ yes

b) $35 = 2 \times 17 \pm 1$ No

#3 if $a|b$ and $b|a \Rightarrow a = b$ or $a = -b$

if $a|b \Rightarrow b = at$

if $b|a \Rightarrow a = bs$

$a = ats \Rightarrow ts = 1, \quad a \neq 0$

$s, t \in \mathbb{Z} \Rightarrow s = t = \pm 1$

$b = a$ or $b = -a$ ✓

$\frac{b}{a}$

$$a|b \Leftrightarrow b = at \quad (t \in \mathbb{Z})$$

#6 a) 12-hr 80 hours after 11:00?

$80 \bmod 12 = 8$ hrs.

7:00

$80 \bmod 11$.

$80 = 8(11) - 8$

$(80 + 11) \bmod 12$

$91 = 12(7) + 7$

$-17 \bmod 2$

$-17 \bmod 2 = 1$

(or) $= -1$

$-17 = 2(-9) + 1$

$= 2(-8) - 1$

15-a $a \equiv -15 \pmod{27}$ $-26 < a < 0$

$a = -15 - 0\ (23)$ $\quad a = -15$

$-15 = x\ (27) + a$

$-15 - (1)(23) = a$

16-a $(-133 \bmod 23 + 261 \bmod 23) \bmod 23$

$((-133 + 261) \bmod 23) \bmod 23$

$(128 \bmod 23) \bmod 23$

$128 = (23) \times 5 \neq 13$

$13 \bmod 23 = 13$

2.cl $(10101111)_2$ binary base 2

$= 1.2^8 + 0.2^7 + 1.2^6 + 0.2^5 + 1.2^4 + 1.2^3 + 1.2^2$
$+ 1.2^1 + 1.2^0$
$= 256 + 64 + 16 + 8 + 4 + 2 + 1$
$= 351$

Base 8 → octal
16 → Hexadecimal

Ex: $(7016)_8 = 6\times8^0 + 1\times8^1 + 0\cdot8^2 + 7\times8^3$
$= 6 + 8 + 3584$
$= 3598$

Ex $(2AE\emptyset B)_{16} = 11\times16^0 + 0\cdot16^1 + 14\times16^2 + 10\times16^3 + 2\times16^4$
$= 175,627$

Ex $(12345)_{10}$ - octal

$12345 = 8(1543) + 1$
$1543 = 8(192) + 7$
$192 = 8(24) + 0$
$24 = 8(3) + 0$
3

$(12345)_{10} = (30071)_8$

Ex: $(241)_{10} = (\quad)_2$

$241 = 2(120) + 1$
$120 = 2(60) + 0$
$60 = 2(30) + 0$
$30 = 2(15) + 0$
$15 = 2(7) + 1$
$7 = 2(3) + 1$
$3 = 2(1) + 1$
1

$(241)_{10} = (11110001)_2$

$(11,111,010,11\ 1,100)_2$
$(011\ 111\ 010\ 111\ 100)_2$
$= (3\ 7\ \ 2\ \ 7\ \ 4)_8$

$(0011\ 1110\ 1011\ 1100)_2 = (3EBC)_{16}$

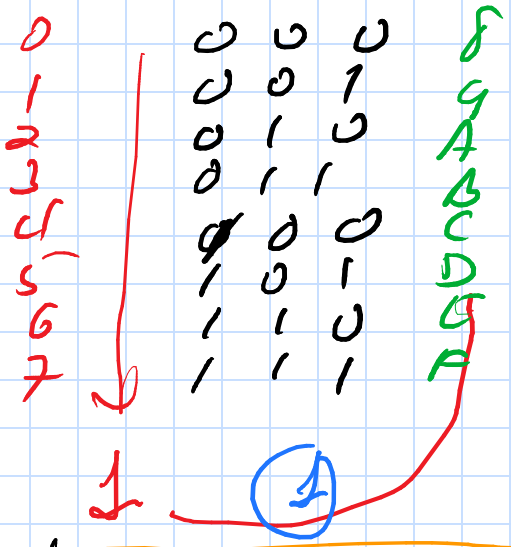
1	0	0	0	1	8
2	0	0	1	0	9
3	0	0	1	1	10
4	0	1	0	0	11
5	0	1	0	1	12
6	0	1	1	0	13
7	0	1	1	1	14
8	1	0	0	0	15
9	1	0	0	1	

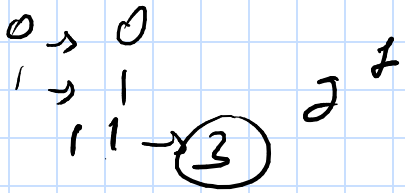
1	0	0	0	1	8
2	0	0	1	0	9
3					A
4					B
					C
					D
					E
					F

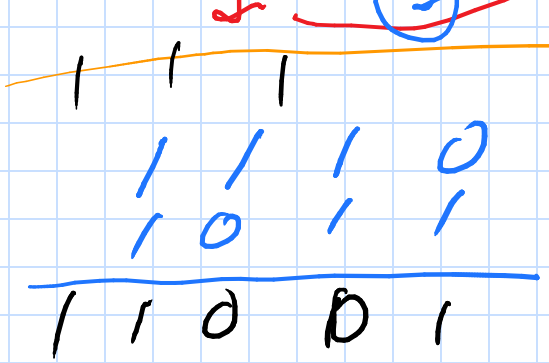
$(765^{-})_8 = (111\ 110\ 101)_2$

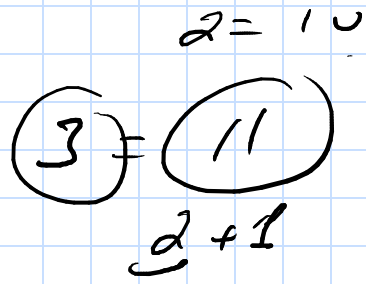
$(A3D)_{16} = (1010\ 0011\ 1101)_2$

0	000	0
1	001	1
2	010	A
3	011	3
4	100	C
5	101	5
6	110	E
7	111	F









2.5 Prime numbers (nb)

divide 1 or itself ←

Factor 1 or itself

1, 2, 3, 5, 7, 11, 13

$100 = 2^2 \cdot 5^2$ $\quad 4 \times 25$

6 d 1 is prime

$999 = 3^3 \times 37$ $\quad 9 \times 111$, 3^2 3 $\times 37$

$1024 = 2^{10}$

Trial Division

If n is a composite $\mathbb{Z}$, n has a prime divisor $\leq \sqrt{n}$

n : composite ⇒ n ⇒ has factor a

$n = ab$ $\quad b > 1$

$a > \sqrt{n}$, $b > \sqrt{n}$

$ab > \sqrt{n}\sqrt{n}$

$= n$ #

$a \leq \sqrt{n}$ $\quad b \leq \sqrt{n}$

$n = ab \leq n$

---

101 is prime

$\sqrt{101} > 10$

2, 3, 5, 7

$7007 = 7^2 \times 11 \times 13$

1001 | 143
30 | 13
21 | 11
1

$\sqrt{7007}$

Mersenne Primes: $2^p - 1$

$p = 1$ $\quad 2^1 - 1 = 1$
$p = 2$ $\quad 2^2 - 1 = 3$
3 $\quad 2^3 - 1 = 7$
5 $\quad 2^5 - 1 = 31$
2 $\quad 2^2 - 1 =$

$2^{11} - 1 = 2047 = 23 \times 89$

$\frac{x}{\ln x}$

GCD $\quad \gcd(24, 26) = 2$

$24 = 2^3 \times 3$
$26 = 2 \times 13$

$\gcd(24, 36) = 12$

$24 = 2^3 3$
$36 = 2^2 3^2$
$2^2 3$

10, 17, 21

$\gcd(10, 17) = 1$
$\gcd(10, 21) = 1$
$\gcd(17, 21) = 1$

$10 = 2 \times 5$
$17 = 17$
$21 = 3 \times 7$

LCM : Least Common Multiply

$LCM(24, 36) = 2^3 \times 3^2$

$= 8 \times 9$

$= 72$

$24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$