2.3 Composition $\circ$

input   input

$$\begin{tikzcd} x \arrow[r, "f(x)"] \arrow[rr, bend right, "g(f(x))=(g\circ f)(x)"'] & y=f(x) \arrow[r, "g(y)"] & z \end{tikzcd}$$

$$g\,(f(x)) = (g\circ f)(x)$$

<u>Ex</u> $f(x) = 5x+6$ $g(x) = 2x^2 - x - 1$

substitute.

$$\begin{aligned}
(f\circ g)(x) &= f(g(x)) \\
&= f(2x^2 - x - 1) \quad \xrightarrow{1^{st}\ \text{sub.}} \quad \mathbb{R} \\
&= 5(2x^2 - x - 1) + 6 \\
&= 10x^2 - 5x - 5 + 6 \\
&= \boxed{10x^2 - 5x + 1} \quad \xrightarrow{2nd} \quad \mathbb{R}
\end{aligned}$$

$\boxed{\text{Domain: } \mathbb{R}.}$

$$\begin{aligned}
(g\circ f)(x) &= g\,(f(x)) \\
&= g\,(5x+6) \quad \rightarrow \quad \mathbb{R} \\
&= 2\,(5x+6)^2 - (5x+6) - 1 \\
&= 2(25x^2 + 60x + 36) - 5x - 6 - 1 \\
&= 50x^2 + 120x + 72 - 5x - 7 \\
&= \boxed{50x^2 + 115x + 65} \quad \rightarrow \quad \mathbb{R}
\end{aligned}$$

$\boxed{\text{Domain: } \mathbb{R}}$

Ex $f(x) = \sqrt{x}$ $\quad g(x) = 4x+2$

$(f \circ g)(x) = f(g(x))$

$= f(4x+2)$ $\quad \mathbb{R}$

$= \sqrt{4x+2}$ $\quad x \geq -\frac{1}{2}$ $\quad 4x = -2$

Domain: $x \geq -\frac{1}{2}$

$(g \circ f)(x) = g(f(x))$

$= g(\sqrt{x}) \rightarrow x \geq 0$

$= 4\sqrt{x} + 2 \rightarrow x \geq 0$

Domain: $x \geq 0$

Ex $f(x) = 2x-1$ $\quad g(x) = \frac{4}{x-1}$

a) $(f \circ g)(2) = f(g(2))$ $\quad g(2) = \frac{4}{2-1}$

$= f\left(\frac{4}{2-1}\right)$

$= f(4)$

$= 2(4) - 1$

$= 8 - 1$

$= 7$

b) $(g \circ f)(-3) = g(f(-3))$

$= g(-6-1)$

$= g(-7)$

$= \frac{4}{-7-1}$ $\quad \boxed{= -\frac{4}{8}}$

$= -\frac{1}{2}$

Ex $f(x) = \frac{4}{x+2}$ $\quad g(x) = \frac{1}{x}$

$(f \circ g)(x) = f(g(x))$

$= f\left(\frac{1}{x}\right) \longrightarrow x \neq 0$

$= \frac{4}{\frac{1}{x}+2}$

$= \frac{4}{\frac{1+2x}{x}}$

$= \frac{4x}{1+2x}$ $\quad x \neq -\frac{1}{2}$

Domain: $x \neq 0, -\frac{1}{2}$ $\quad \mathbb{R} - \{0, -\frac{1}{2}\}$

#11 $f(x) = x^2+2$ $\quad g(x) = 4x-3$

$(f \circ g)(x) = f(g(x))$

$= f(4x-3) \longrightarrow \mathbb{R}$

$= (4x-3)^2+2$

$= 16x^2-24x+9+2$

$= 16x^2-24x+11 \rightarrow \mathbb{R}$

Domain: $\mathbb{R}$

$(g \circ f)(x) = g(f(x))$

$= g(x^2+2)$

$= 4(x^2+2)-3$

$= 4x^2+8-3$

$= 4x^2+5$

Domain: $\mathbb{R}$

#18) $f(x) = x^2 - 3x$ $\quad g(x) = \sqrt{x+2}$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x+2})$ $\quad x \geq -2$

$= x+2 - 3\sqrt{x+2}$

Domain: $x \geq -2$

$(g \circ f)(x) = g(f(x))$

$= g(x^2 - 3x)$ $\quad \mathbb{R}$

$= \sqrt{x^2 - 3x + 2}$ $\quad x \leq \quad x \geq$

Domain $x \leq 1 \quad x \geq 2$

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23) $f(x) = 2x + 3$ $\quad g(x) = \dfrac{x-3}{2}$

$(f \circ g)(x) = f(g(x))$

$= f\left(\dfrac{x-3}{2}\right)$

$= 2\left(\dfrac{x-3}{2}\right) + 3$

$= x - 3 + 3$

$= x$

Domain: $\mathbb{R}$

33) $f(x) = \dfrac{2x+3}{x-4}$ $\qquad g(x) = \dfrac{4x+3}{x-2}$

$(f \circ g)(x) = f(g(x))$

$= f\left(\dfrac{4x+3}{x-2}\right)$ $\qquad x \neq 2$

$= \dfrac{2\,\dfrac{4x+3}{x-2} + 3}{\dfrac{4x+3}{x-2} - 4}$ $\qquad 3(x-2)$

$= \dfrac{\dfrac{8x+6+3x-6}{x-2}}{\dfrac{4x+3-4x+8}{x-2}}$

$= \dfrac{11x}{11}$

$= \boxed{x}$ $\longrightarrow \mathbb{R}$

Domain: $\boxed{x \neq 2}$

$(g \circ f)(x) = g(f(x))$

$= g\left(\dfrac{2x+3}{x-4}\right) \longrightarrow \boxed{x \neq 4}$

$= \dfrac{4\,\dfrac{2x+3}{x-4} + 3}{\dfrac{2x+3}{x-4} - 2}$ $\qquad 4(2x+3) + 3(x-4)$

$= \dfrac{\dfrac{8x+12+3x-12}{x-4}}{\dfrac{2x+3-2x+8}{x-4}}$

$= \dfrac{11x}{11}$

$= \boxed{x}$ $\qquad \mathbb{R}$

Domain: $\boxed{x \neq 4}$

$$\frac{x^3+2x^2-5x-6}{x+1}$$

$$\begin{array}{r l}
 & x^2+x-6 \quad \leftarrow \text{Quotient} \\
x+1 \,\big) & x^3+2x^2-5x-6 \\
 & \underline{-x^3 \mp x^2} \\
 & \quad x^2-5x \\
 & \quad \underline{-x^2 \mp x} \\
 & \qquad -6x-6 \\
 & \qquad \underline{+6x+6} \\
 & \qquad\quad 0 \quad \leftarrow \text{remainder}
\end{array}$$

$$\frac{x^3}{x} = x^{3-1}$$

$Q(x) = x^2+x-6$

$R(x) = 0$

Ex $(x^4-16) \div (x^2+3x+1)$

$$\begin{array}{r l}
 & x^2-3x+8 \\
x^2+3x+1 \,\big) & x^4+0x^3+0x^2+0x-16 \\
 & \underline{-x^4 \mp 3x^3 \mp x^2} \\
 & \quad -3x^3-x^2 \\
 & \quad \underline{\pm 3x^3 \pm 9x^2 \pm 3x} \\
 & \qquad 8x^2+3x-16 \\
 & \qquad \underline{-8x^2 \mp 24x \mp 8} \\
 & \qquad\quad -21x-24
\end{array}$$

$+ \quad \ominus$

use $(-)$

$Q(x) = x^2-3x+8$

$R(x) = -21x-24$

$$\frac{x^4-16}{x^2+3x+1} = x^2-3x+8 + \frac{-21x-24}{x^2+3x+1}$$

$\overline{x+c}$

$f(-c) = $ Remainder

Synthetic Division

$\overline{ax^1 + b} = 0 \Rightarrow x\,?$

EX $(4x^3 - 3x^2 + x + 7) \div (x - 2)$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x^1 & x^0 \\ 2 & 4 & -3 & 1 & 7 \\ & \downarrow & 8 & 10 & 22 \\ \hline & 4 & 5 & 11 & \boxed{29} \\ & x^2 & x^1 & x^0 & \end{array}$$

← Remainder

$\boxed{Q(x) = 4x^2 + 5x + 11}$

$\boxed{R(x) = 29}$

$f(x) = x^3 + 8x^2 - 29x + 44$ $\quad -11$ is a zero of f

$$\begin{array}{r|rrrr} & x^3 & x^2 & x^1 & x^0 \\ -11 & 1 & 8 & -29 & 44 \\ & & -11 & 33 & -44 \\ \hline & 1 & -3 & 4 & \boxed{0} \end{array}$$ ✓

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \text{—} + a_1 x + a_0$$

Theorem Rational Zero Theorem

possibilities: $\pm\left\{\frac{a_0}{a_n}\right\}$

Ex $3x^4 + 14x^3 + 14x^2 - 8x - 8 = 0$

+	−
48	112
56	−8
16	
120	120

a) possibilities: $\pm\left\{\frac{8}{3}\right\} = \pm\left\{\frac{1,2,4,8}{1,3}\right\}$

$= \pm\left\{1, 2, 4, 8, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}\right\}$

x^4

$$\begin{array}{r|rrrrr} -2 & 3 & 14 & 14 & -8 & -8 \\ & \downarrow & -6 & -16 & 4 & 8 \\ \hline -\frac{2}{3} & 3 & 8 & -2 & -4 & \boxed{0} \\ & \downarrow & -2 & -4 & 4 & \\ \hline & 3 & 6 & -6 & \boxed{0} & \end{array}$$

✓ ✓

$3x^3 + 8x^2 - 2x - 4$

$\left\{\frac{4}{3}\right\} = \pm\left\{\frac{1,2,4}{1,3}\right\}$

$= \pm\left\{1, 2, 4, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}\right\}$

$3x^2 + 6x - 6 = 0$ (divide by 3)

$x^2 + 2x - 2 = 0$

$$x = \frac{-2 \pm \sqrt{4 + 8}}{2}$$

$12 = 4(3)$

$$= \frac{-2 \pm 2\sqrt{3}}{2}$$

$\therefore$ Zeros: $x = -2, -\frac{2}{3}, -1 \pm \sqrt{3}$

#17 $x^3 - x^2 - 10x - 8 = 0$

a) possibilities. $\pm\{\frac{8}{1}\} = \pm\{1, 2, 4, 8\}$

b)
$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & \downarrow & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & \boxed{0} \end{array}$$

$x^2 - 2x - 8 = 0$ $\qquad$ $x = \frac{2 \pm \sqrt{4+32}}{2}$ (6)

$x = -2, 4$ $\qquad$ $\frac{2-6}{2}$ $\quad$ $\frac{2+6}{2}$

$\therefore \; \boxed{x = -1, -2, 4}$

Zero functions

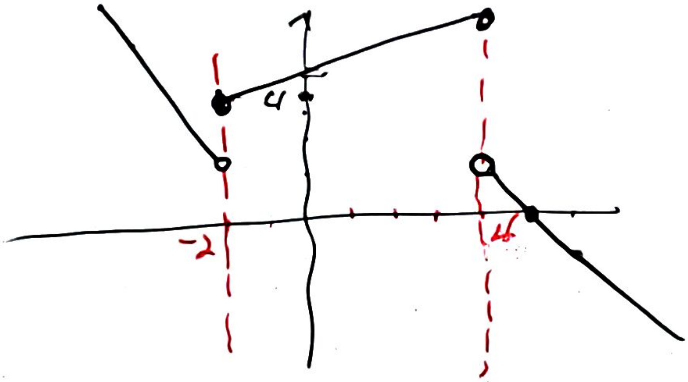
Due Tomorrow $\quad$ Ex2 - Review

6, 7, 11, 15

Hwk #1

$$f(x) = \begin{cases} -5x - 8 & ① \quad x < -2 \\ \frac{1}{2}x + 5 & ② \quad -2 \le x \le 4 \\ 10 - 2x & ③ \quad x > 4 \end{cases}$$

a)



b) $f(-1) = \frac{1}{2}(-1) + 5$ ②

$= -\frac{1}{2} + 5$

$= \boxed{\frac{9}{2}}$

#5 $f(x) = \sqrt{x+3}$ $\quad g(x) = \frac{x+2}{x-1}$ $\quad h(x) = x-5$

a) Domain $f(x)$: $\boxed{x \geq -3}$

b) " $g(x)$: $\boxed{x \neq 1}$

c) " $h(x) = \mathbb{R}$

d) $\frac{f}{h} = \frac{\sqrt{x+3}}{x-5}$ $\rightarrow x \geq -3$ $\rightarrow x \neq 5$

Domain: $\boxed{x \geq -3,\ x \neq 5}$

f) $\frac{g}{f} = \frac{x+2}{x-1} \cdot \frac{1}{\sqrt{x+3}}$ $\qquad g(x) \cdot \frac{1}{f(x)}$

$= \frac{x+2}{(x-1)\sqrt{x+3}}$ $\rightarrow \mathbb{R}$ $\rightarrow x \neq 1, x > -3$

Domain: $\boxed{x > -3,\ x \neq 1}$

#8 d) $f(x) = 2x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left(2(x+h)^2 - 2x^2\right)$$

$$= \frac{1}{h}\left(2(x^2+2hx+h^2) - 2x^2\right)$$

$$= \frac{1}{h}\left(2x^2 + 4hx + 2h^2 - 2x^2\right)$$

$$= \frac{4hx}{h} + \frac{2h^2}{h}$$

$$= \boxed{4x + 2h}$$