

# Polynomials

## Adding and Subtracting Polynomials

### Properties of Real numbers

For all real numbers  $a$ ,  $b$ , and  $c$ :

$$a + b = b + a \quad \text{Commutative properties}$$

$$ab = ba$$

$$(a + b) + c = a + (b + c) \quad \text{Associative properties}$$

$$(ab)c = a(bc)$$

$$a(b + c) = ab + ac \quad \text{Distributive properties}$$

### Add or subtract as indicated

$$\begin{aligned} a) \quad & (8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8) \\ & (8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8) = 8x^3 - 4x^2 + 6x + 3x^3 + 5x^2 - 9x + 8 \\ & = (8x^3 + 3x^3) + (-4x^2 + 5x^2) + (6x - 9x) + 8 \\ & = 11x^3 + x^2 - 3x + 8 \end{aligned}$$

$$\begin{aligned} b) \quad & (-4x^4 + 6x^3 - 9x^2 - 12) + (-3x^3 + 8x^2 - 11x + 7) \\ & (-4x^4 + 6x^3 - 9x^2 - 12) + (-3x^3 + 8x^2 - 11x + 7) = -4x^4 + 6x^3 - 3x^3 - 9x^2 + 8x^2 - 11x - 12 + 7 \\ & = -4x^4 + 3x^3 - x^2 - 11x - 5 \end{aligned}$$

$$\begin{aligned} c) \quad & (2x^2 - 11x + 8) - (7x^2 - 6x + 2) \\ & (2x^2 - 11x + 8) - (7x^2 - 6x + 2) = 2x^2 - 11x + 8 - 7x^2 + 6x - 2 \\ & = -5x^2 - 5x + 6 \end{aligned}$$

Multiply

a)  $8x(6x-4)$

$$\begin{aligned}8x(6x-4) &= 8x(6x) - 8x(4) \\ &= 48x^2 - 32x\end{aligned}$$

b)  $(3p-2)(p^2+5p-1)$

$$\begin{aligned}(3p-2)(p^2+5p-1) &= 3p^3 + 15p^2 - 3p - 2p^2 - 10p + 2 \\ &= 3p^3 + 13p^2 - 13p + 2\end{aligned}$$

c)  $(x+2)(x+3)(x-4)$

$$\begin{aligned}(x+2)(x+3)(x-4) &= (x^2 + 3x + 2x + 6)(x-4) \\ &= (x^2 + 5x + 6)(x-4) \\ &= x^3 + 5x^2 + 6x - 4x^2 - 20x - 24 \\ &= x^3 + x^2 - 14x - 24\end{aligned}$$

Find  $(2m-5)(m+4)$

$$\begin{aligned}(2m-5)(m+4) &= 2mm + 2m(4) - 5m - 5(4) \\ &= 2m^2 + 8m - 5m - 20 \\ &= 2m^2 + 3m - 20\end{aligned}$$

Find  $(2k-5)^2$

$$\begin{aligned}(2k-5)^2 &= (2k-5)(2k-5) \\ &= 4k^2 - 10k - 10k + 25 \\ &= 4k^2 - 20k + 25\end{aligned}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)(a+b) = a^2 - b^2$$

Perform the indicated operations:  $2(3x^2 + 4x + 2) - 3(-x^2 + 4x - 5)$

$$\begin{aligned} 2(3x^2 + 4x + 2) - 3(-x^2 + 4x - 5) &= 6x^2 + 8x + 4 + 3x^2 - 12x + 15 \\ &= 9x^2 - 4x + 19 \end{aligned}$$

Perform the indicated operations:  $(3t - 2y)(3t + 5y)$

$$\begin{aligned} (3t - 2y)(3t + 5y) &= 9t^2 + 15ty - 6yt - 10y^2 \\ &= 9t^2 + 9yt - 10y^2 \end{aligned}$$

Perform the indicated operations:  $(2a - 4b)^2$

$$\begin{aligned} (2a - 4b)^2 &= (2a)^2 - 2(2a)(4b) + (4b)^2 \\ &= 4a^2 - 16ab + 16b^2 \end{aligned}$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

# Factoring

## Prime Factorization

A process that allows us to write a composite number as a product of two or more prime numbers.

$$\begin{array}{c} \text{Tree} \\ 2 \swarrow 10 \searrow 5 \\ 10 = 2 \times 5 \end{array}$$

$$\begin{aligned} 72 &= 2 \cdot 36 \\ &= 2 \cdot 6 \cdot 6 \\ &= 2 \cdot 2 \cdot 3 \cdot 2 \cdot 3 \\ &= 2^3 3^2 \end{aligned}$$

## The Greatest Common Factor (GCF)

The largest factor that two or more numbers (or terms) have in common

**Find GCF** (18, 36)

$$\begin{aligned} 18: & 2 \cdot 9 \\ & 2 \cdot 3 \cdot 3 \end{aligned}$$

$$\begin{aligned} 36: & 2 \cdot 18 \\ & 2 \cdot 2 \cdot 3 \cdot 3 \end{aligned}$$

$$18: 2 \cdot 3^2 \rightarrow 1, 2, 3, 6, 9, \underline{18}$$

$$36: 2^2 \cdot 3^2 \rightarrow 1, 2, 3, 4, 6, 9, 12, \underline{18}, 36$$

$$\text{GCF}(18, 36) = 18 \text{ (is the greatest common factor)}$$

**Find GCF** (27, 45)

$$\begin{aligned} 27 &= 3^3 \\ 45 &= \frac{3^2}{3^2} 5 \end{aligned}$$

$$\text{GCF}(27, 45) = 9$$

**Find GCF** (40, 56)

$$\begin{aligned} 40 &= 2^3 5 \\ 56 &= \frac{2^3}{2^3} 7 \end{aligned}$$

$$\text{GCF}(40, 56) = 8$$

**Find GCF** (80, 60)

$$\begin{aligned} 80 &= 2^4 5 \\ 60 &= \frac{2^2}{2^2} 3 5 \end{aligned}$$

$$\text{GCF}(80, 60) = 20$$

### Factor out the greatest common factor

a)  $12p - 18q$

$$12p - 18q = 6(2p - 3q)$$

<b>12</b>	2 . 2 . 3
<b>18</b>	2 . . 3 . 3
	2 . . 3

b)  $8x^3 - 9x^2 + 15x$

$$8x^3 - 9x^2 + 15x = x(8x^2 - 9x + 15)$$

### Factoring Trinomial

**Factor**  $y^2 + 8y + 15$

<i>Product</i> 15	<i>Sum</i> 8
15 x 1	15 + 1
3 x 5	3 + 5

$$y^2 + 8y + 15 = (y + 3)(y + 5)$$

**Factor**  $4x^2 + 8xy - 5y^2$

$$4x^2 + 8xy - 5y^2 = (2x - y)(2x + 5y)$$

### Special Factorization

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

**Factor**

a)  $64p^2 - 49q^2$

$$\begin{aligned} 64p^2 - 49q^2 &= (8p)^2 - (7q)^2 \\ &= (8p - 7q)(8p + 7q) \end{aligned}$$

b)  $x^2 + 36$

$x^2 + 36$  can't be factored (in real number) it is prime.

c)  $x^2 + 12x + 36$

$$x^2 + 12x + 36 = (x + 6)^2$$

d)  $9y^2 - 24yz + 16z^2$

$$\begin{aligned} 9y^2 - 24yz + 16z^2 &= (3y)^2 - 2(3y)(4z) + (4z)^2 \\ &= (3y - 4z)^2 \end{aligned}$$

e)  $y^3 - 8$

$$\begin{aligned} y^3 - 8 &= y^3 - 2^3 \\ &= (y - 2)(y^2 + 2y + 4) \end{aligned}$$

f)  $m^3 + 125$

$$m^3 + 125 = (m + 5)(m^2 - 5m + 25)$$

g)  $8k^3 - 27z^3$

$$\begin{aligned} 8k^3 - 27z^3 &= (2k)^3 - (3z)^3 \\ &= (2k - 3z)\left((2k)^2 + 6kz + (3z)^2\right) \\ &= (2k - 3z)(4k^2 + 6kz + 9z^2) \end{aligned}$$

h)  $p^4 - 1$

$$\begin{aligned} p^4 - 1 &= (p^2)^2 - (1)^2 \\ &= (p^2 - 1)(p^2 + 1) \\ &= (p - 1)(p + 1)(p^2 + 1) \end{aligned}$$

**Factor:**  $60m^4 - 120m^3n + 50m^2n^2$

$$60m^4 - 120m^3n + 50m^2n^2 = 10m^2(6m^2 - 12mn + 5n^2)$$

**Factor:**  $y^2 - 4yz - 21z^2$

$$y^2 - 4yz - 21z^2 = (y + 3z)(y - 7z)$$

**Factor:**  $4a^2 + 10a + 6$

$$\begin{aligned} 4a^2 + 10a + 6 &= 2(2a^2 + 5a + 3) \\ &= 2(2a + 3)(a + 1) \end{aligned}$$

**Factor:**  $16a^4 - 81b^4$

$$\begin{aligned} 16a^4 - 81b^4 &= (4a^2)^2 - (9b^2)^2 \\ &= (4a^2 - 9b^2)(4a^2 + 9b^2) \\ &= ((2a)^2 - (3b)^2)(4a^2 + 9b^2) \\ &= (2a - 3b)(2a + 3b)(4a^2 + 9b^2) \end{aligned}$$

## ***Fraction***

$$\frac{a}{b} = \frac{\text{numerator}}{\text{denominator}}$$

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc \quad \text{Cross multiplication}$$

$$\frac{a}{b} = \frac{na}{nb} = \frac{an}{bn}$$

$$a) \quad \frac{5}{6} = \frac{25}{30}?$$

$$\frac{5}{6} = \frac{5}{6} \frac{5}{5} = \frac{25}{30}$$

$$b) \quad \frac{16}{48} = \frac{1}{3}$$

$$\frac{16}{48} = \frac{1}{3} \Leftrightarrow (16)(3) = (1)(48)$$

$$48 = 48$$

$$\begin{aligned} \text{Simplify: } \frac{12}{18} &= \frac{2.6}{2.9} \\ &= \frac{2.2.3}{2.3.3} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{Simplify: } \frac{36}{56} &= \frac{2.18}{2.28} \\ &= \frac{18}{28} \\ &= \frac{2.9}{2.14} \\ &= \frac{9}{14} \end{aligned}$$

If the denominators are the same  $\Rightarrow$  add the numerators

$$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5}$$

If the denominators are the same  $\Rightarrow$  subtract the numerators

$$\frac{4}{9} - \frac{2}{9} = \frac{4-2}{9} = \frac{2}{9}$$



If the denominators are not the same

⇒ Find Least Common Denominator (LCD) and convert so that the fractions have the same denominators

**LCD:** is the smallest whole number that is a multiple of each

$$\frac{5}{8} + \frac{1}{12}$$

LCD (8, 12)

$$8 = 2^3$$

$$12 = 2^2 \cdot 3$$

$$2^3 \cdot 3 = 24$$

$$\text{LCD}(8, 12) = 24$$

$$\frac{5}{8} + \frac{1}{12} = \frac{5 \cdot 3}{8 \cdot 3} + \frac{1 \cdot 2}{12 \cdot 2}$$

$$= \frac{15}{24} + \frac{2}{24}$$

$$= \frac{15+2}{24}$$

$$= \frac{17}{24}$$

$$\frac{69}{75} - \frac{1}{50}$$

LCD (75, 50)

$$75 = 5^3$$

$$50 = 2 \cdot 5^2$$

$$2 \cdot 5^3 = 150$$

$$\text{LCD}(75, 50) = 150$$

$$\frac{69}{75} - \frac{1}{50} = \frac{(69)(2) - (1)(3)}{150}$$

$$= \frac{138-3}{150}$$

$$= \frac{135}{150}$$

$$= \frac{9}{10}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\begin{aligned}\frac{2}{7} + \frac{3}{5} &= \frac{2(5) + 3(7)}{7(5)} \\ &= \frac{10 + 21}{35} \\ &= \frac{31}{35}\end{aligned}$$

$$\begin{aligned}\text{or } \frac{2}{7} \frac{5}{5} + \frac{3}{5} \frac{7}{7} &= \frac{10}{35} + \frac{21}{35} \\ &= \frac{10 + 21}{35} \\ &= \frac{31}{35}\end{aligned}$$

$$\begin{aligned}\frac{5}{9} + \frac{3}{4} &= \frac{5(4) + 3(9)}{9(4)} \\ &= \frac{20 + 27}{36} \\ &= \frac{47}{36}\end{aligned}$$

$$\begin{aligned}\frac{17}{15} + \frac{5}{12} &= \frac{17(12) + 5(15)}{15(12)} \\ &= \frac{204 + 75}{180} \\ &= \frac{279}{180} \\ &= \frac{31(9)}{20(9)} \\ &= \frac{31}{20}\end{aligned}$$

$$\begin{aligned}\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} &= \frac{5(7)(9) + (3)(7)(9) + (3)(5)(9) + (3)(5)(7)}{(3)(5)(7)(9)} \\ &= \frac{315 + 189 + 135 + 105}{945} \\ &= \frac{744}{945} \\ &= \frac{248}{315} \frac{3}{3} \\ &= \frac{248}{315}\end{aligned}$$

$$\frac{8}{9} + \frac{1}{12} + \frac{3}{16}$$

$$\frac{8}{9} + \frac{1}{12} + \frac{3}{16} = \frac{8(16) + 1(12) + 3(9)}{144}$$

$$= \frac{128 + 12 + 27}{144}$$

$$= \frac{167}{144}$$

$$\begin{cases} 9 = 3^2 \\ 12 = 2^2 \cdot 3 \\ 16 = 2^4 \end{cases}$$

$$\text{LCD } 2^4 \cdot 3^2 = 144$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

$$\frac{2}{7} - \frac{3}{5} = \frac{2(5) - 3(7)}{7(5)} = \frac{10 - 21}{35} = -\frac{11}{35}$$

$$\frac{a}{c} \frac{b}{d} = \frac{ab}{cd}$$

$$\frac{2}{7} \frac{3}{5} = \frac{6}{35}$$

$$\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \times \frac{d}{b} = \frac{ad}{cb}$$

$$\frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \frac{5}{3} = \frac{10}{21}$$

A. Find:

$$1. \quad \frac{13}{21} + \frac{5}{21} = \frac{13+5}{21} = \frac{6}{7}$$

$$2. \quad \frac{7}{12} - \frac{4}{15} = \frac{7(5) - 4(4)}{60} = \frac{35-16}{60} = \frac{19}{60}$$

$$3. \quad \frac{5}{8} + \frac{1}{2} = \frac{5+4}{8} = \frac{9}{8}$$

$$4. \quad \frac{5}{8} + \frac{1}{2} + \frac{2}{3} = \frac{5(3) + 1(12) + 2(8)}{24} = \frac{43}{24}$$

$$5. \quad \frac{7}{8} - \frac{1}{10} = \frac{7(5) - 1(4)}{40} = \frac{31}{40}$$

$$6. \quad \frac{11}{5} - \frac{31}{7} = -\frac{78}{35}$$

$$7. \quad \frac{3}{4} \cdot \frac{3}{2} = \frac{9}{8}$$

$$8. \quad \frac{3}{4} \cdot \frac{4}{3} \cdot \frac{2}{3} = \frac{2}{3}$$

$$9. \quad \frac{3}{4} \div \frac{3}{2} = \frac{3}{4} \cdot \frac{2}{3} = \frac{2}{4} = \frac{1}{2}$$

$$10. \quad \frac{14}{15} \div \frac{14}{3} = \frac{14}{15} \cdot \frac{3}{14} = \frac{1}{5}$$

# Exponents

## Integer Exponents

Definition of exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n\text{-times}}$$

a appears as a factor n times

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^m \cdot a^n = a^{m+n}$$

$$\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

$$\left(a^m\right)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$(ab)^m = a^m b^m$$

a)  $6^0$

$$6^0 = 1$$

b)  $(-9)^0$

$$(-9)^0 = 1$$

c)  $3^{-2}$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

d)  $\left(\frac{3}{4}\right)^{-1}$

$$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$$

$$a) 7^4 \cdot 7^6$$

$$7^4 \cdot 7^6 = 7^{4+6} = 7^{10}$$

$$b) \frac{9^{14}}{9^6}$$

$$\frac{9^{14}}{9^6} = 9^{14-6} = 9^8$$

$$c) \frac{r^9}{r^{17}}$$

$$\frac{r^9}{r^{17}} = \frac{1}{r^{17-9}} = \frac{1}{r^8}$$

$$d) (2m^3)^4$$

$$\begin{aligned} (2m^3)^4 &= (2)^4 (m^3)^4 \\ &= 16m^{12} \end{aligned}$$

$$e) \left( \frac{x^2}{y^3} \right)^6$$

$$\begin{aligned} \left( \frac{x^2}{y^3} \right)^6 &= \frac{(x^2)^6}{(y^3)^6} \\ &= \frac{x^{2 \cdot 6}}{y^{3 \cdot 6}} \\ &= \frac{x^{12}}{y^{18}} \end{aligned}$$

$$f) \frac{a^{-3}b^5}{a^4b^{-7}}$$

$$\begin{aligned} \frac{a^{-3}b^5}{a^4b^{-7}} &= \frac{b^5 b^7}{a^3 a^4} \\ &= \frac{b^{5+7}}{a^{4+3}} \\ &= \frac{b^{12}}{a^7} \end{aligned}$$

$$g) \quad p^{-1} + q^{-1}$$

$$\begin{aligned} p^{-1} + q^{-1} &= \frac{1}{p} + \frac{1}{q} \\ &= \frac{1}{p} \frac{q}{q} + \frac{1}{q} \frac{p}{p} \\ &= \frac{q+p}{pq} \end{aligned}$$

$$h) \quad \frac{x^{-2} - y^{-2}}{x^{-1} - y^{-1}}$$

$$\begin{aligned} \frac{x^{-2} - y^{-2}}{x^{-1} - y^{-1}} &= \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} - \frac{1}{y}} \\ &= \frac{\frac{y^2 - x^2}{x^2 y^2}}{\frac{y-x}{xy}} \\ &= \frac{y^2 - x^2}{x^2 y^2} \cdot \frac{xy}{y-x} \\ &= \frac{(y-x)(y+x)}{(xy)^2} \cdot \frac{xy}{y-x} \\ &= \frac{y+x}{xy} \end{aligned}$$

Calculations with exponents

$$a) \quad 121^{1/2} = 11$$

$$b) \quad 625^{1/4} = 5$$

$$c) \quad (-32)^{1/5} = -2$$

$$d) \quad (-49)^{1/2} \text{ is not a real number}$$



## Rational Exponents

$$a^{m/n} = \left(a^{1/n}\right)^m$$

### *Calculations with Exponents*

a)  $27^{2/3}$

$$27^{2/3}$$

$$\begin{aligned} 27^{2/3} &= \left(27^{1/3}\right)^2 \\ &= \left(3^3\right)^{1/3}^2 \\ &= \left(3^{3 \cdot \frac{1}{3}}\right)^2 \\ &= (3)^2 \\ &= 9 \end{aligned}$$

b)  $32^{2/5}$

$$32^{2/5}$$

$$\begin{aligned} 32^{2/5} &= \left(2^5\right)^{1/5}^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

c)  $64^{4/3}$

$$64^{4/3}$$

$$\begin{aligned} 64^{4/3} &= \left(4^3\right)^{1/3}^4 \\ &= (4)^4 \\ &= 256 \end{aligned}$$

Simplify

$$a) \frac{y^{1/3} y^{5/3}}{y^3}$$

$$\frac{y^{1/3} y^{5/3}}{y^3} = \frac{y^{\frac{1}{3} + \frac{5}{3}}}{y^3}$$

$$= \frac{y^{\frac{6}{3}}}{y^3}$$

$$= \frac{y^2}{y^3}$$

$$= \frac{1}{y^{3-2}}$$

$$= \frac{1}{y}$$

$$b) m^{2/3} (m^{7/3} + 7m^{1/3})$$

$$m^{2/3} (m^{7/3} + 7m^{1/3}) = m^{2/3} m^{7/3} + 7m^{2/3} m^{1/3}$$

$$= m^{\frac{2}{3} + \frac{7}{3}} + 7m^{\frac{2}{3} + \frac{1}{3}}$$

$$= m^{\frac{9}{3}} + 7m^{\frac{3}{3}}$$

$$= m^3 + 7m$$

$$c) \left( \frac{m^7 n^{-2}}{m^{-5} n^2} \right)^{1/4}$$

$$\left( \frac{m^7 n^{-2}}{m^{-5} n^2} \right)^{1/4} = \left( \frac{m^{7+5}}{n^{2+2}} \right)^{1/4}$$

$$= \left( \frac{m^{12}}{n^4} \right)^{1/4}$$

$$= \frac{(m^{12})^{1/4}}{(n^4)^{1/4}}$$

$$= \frac{m^{12/4}}{n^{4/4}}$$

$$= \frac{m^3}{n}$$

## Simplify

a)  $4m^{1/2} + 3m^{3/2}$

$$\begin{aligned}4m^{1/2} + 3m^{3/2} &= m^{1/2} \left( 4m^{1/2 - 1/2} + 3m^{3/2 - 1/2} \right) \\&= m^{1/2} (4 + 3m)\end{aligned}$$

b)  $9x^{-2} - 6x^{-3}$

$$9x^{-2} - 6x^{-3} = 3x^{-3} (3x - 2)$$

c)  $2(x^2 + 5)(3x - 1)^{-1/2} + (3x - 1)^{1/2}(2x)$

$$\begin{aligned}2(x^2 + 5)(3x - 1)^{-1/2} + (3x - 1)^{1/2}(2x) &= 2(3x - 1)^{-1/2} \left[ x^2 + 5 + x(3x - 1) \right] \\&= 2(3x - 1)^{-1/2} \left[ x^2 + 5 + 3x^2 - x \right] \\&= 2(3x - 1)^{-1/2} (4x^2 - x + 5)\end{aligned}$$

## Radicals

$$a^{1/n} = \sqrt[n]{a}$$

a)  $\sqrt[4]{16}$

$$\sqrt[4]{16} = 16^{1/4} = 2$$

b)  $\sqrt[5]{-32} = -2$

c)  $\sqrt[3]{1000}$

$$\sqrt[3]{1000} = 1000^{1/3} = 10$$

d)  $\sqrt[6]{\frac{64}{729}}$

$$\sqrt[6]{\frac{64}{729}} = \frac{\sqrt[6]{64}}{\sqrt[6]{729}} = \frac{2}{3}$$

## Properties

$$\left(\sqrt[n]{a}\right)^n = a$$

$$\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n \text{ is even} \\ a & \text{if } n \text{ is odd} \end{cases}$$

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

Simplify

a)  $\sqrt{1000}$

$$\sqrt{1000} = \sqrt{100(10)}$$

$$= \sqrt{100} \sqrt{10}$$

$$= 10\sqrt{10}$$

b)  $\sqrt{128}$

$$\sqrt{128} = \sqrt{64(2)}$$

$$= 8\sqrt{2}$$

c)  $\sqrt{2}\sqrt{18}$

$$\sqrt{2}\sqrt{18} = \sqrt{2(18)}$$

$$= \sqrt{36}$$

$$= 6$$

d)  $\sqrt[3]{54}$

$$\sqrt[3]{54} = \sqrt[3]{27(2)}$$

$$= 3\sqrt[3]{2}$$

e)  $\sqrt{288m^5}$

$$\sqrt{288m^5} = \sqrt{144(2)m^4m}$$

$$= 12m^2\sqrt{2m}$$

$$\begin{aligned}
 f) \quad & 2\sqrt{18} - 5\sqrt{32} \\
 & 2\sqrt{18} - 5\sqrt{32} = 2\sqrt{9(2)} - 5\sqrt{16(2)} \\
 & = 6\sqrt{2} - 20\sqrt{2} \\
 & = -14\sqrt{2}
 \end{aligned}$$

## Rationalizing Denominators

Simplify by rationalizing the denominator

$$\begin{aligned}
 a) \quad & \frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} \\
 & = \frac{4\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{2}{\sqrt[3]{x}} = \frac{2}{\sqrt[3]{x}} \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} \\
 & = \frac{2\sqrt[3]{x^2}}{x}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{1}{1-\sqrt{2}} = \frac{1}{1-\sqrt{2}} \frac{1+\sqrt{2}}{1+\sqrt{2}} \\
 & = \frac{1+\sqrt{2}}{1-2} \\
 & = \frac{1+\sqrt{2}}{-1} \\
 & = -1-\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{Simplify } & \sqrt{27}\sqrt{3} \\
 & \sqrt{27}\sqrt{3} = \sqrt{27(3)} \\
 & = \sqrt{81} \\
 & = 9
 \end{aligned}$$

$$\begin{aligned}
 \text{Simplify } & \sqrt[4]{x^8 y^7 z^{11}} \\
 & \sqrt[4]{x^8 y^7 z^{11}} = x^2 yz^2 \sqrt[4]{y^3 z^3}
 \end{aligned}$$

Simplify  $\frac{5}{\sqrt{10}}$

$$\begin{aligned}\frac{5}{\sqrt{10}} &= \frac{5}{\sqrt{10}} \frac{\sqrt{10}}{\sqrt{10}} \\ &= \frac{5\sqrt{10}}{10} \\ &= \frac{\sqrt{10}}{2}\end{aligned}$$

Simplify  $\frac{5}{2-\sqrt{6}}$

$$\begin{aligned}\frac{5}{2-\sqrt{6}} &= \frac{5}{2-\sqrt{6}} \frac{2+\sqrt{6}}{2+\sqrt{6}} \\ &= \frac{5(2+\sqrt{6})}{4-6} \\ &= -\frac{5}{2}(2+\sqrt{6})\end{aligned}$$

Simplify  $\frac{1}{\sqrt{r}-\sqrt{3}}$

$$\begin{aligned}\frac{1}{\sqrt{r}-\sqrt{3}} &= \frac{1}{\sqrt{r}-\sqrt{3}} \frac{\sqrt{r}+\sqrt{3}}{\sqrt{r}+\sqrt{3}} \\ &= \frac{\sqrt{r}+\sqrt{3}}{r-3}\end{aligned}$$