Solution   *Section* 4.4 – Trigonometric Form of Complex Numbers

Exercise

Write $-\sqrt{3}+i$ in trigonometric form. (Use radian measure)

Solution

$$-\sqrt{3}+i \Rightarrow \begin{cases} x=-\sqrt{3} \\ y=1 \end{cases}$$

$$r=\sqrt{\left(-\sqrt{3}\right)^2+1^2}=2$$

$$\tan\theta=\frac{y}{x}=\frac{1}{-\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

The reference angle for θ is $\frac{\pi}{6}$ and the angle is in quadrant II.

Therefore, $\theta=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$

$$\boxed{-\sqrt{3}+i=2\ cis\frac{5\pi}{6}}$$

Exercise

Write $3-4i$ in trigonometric form.

Solution

$$3-4i \Rightarrow \begin{cases} r=\sqrt{3^2+(-4)^2}=5 \\ \hat{\theta}=\tan^{-1}\left(\frac{4}{3}\right)\approx 53° \end{cases}$$

The angle is in quadrant II; therefore, $\theta=180°-53°=127°$

$$\boxed{3-4i=5\ cis127°}$$

Exercise

Write $-21-20i$ in trigonometric form.

Solution

$$-21-20i \Rightarrow \begin{cases} r = \sqrt{(-21)^2+(-20)^2} = 29 \\ \hat{\theta} = \tan^{-1}\left(\frac{20}{21}\right) \approx 43.6^\circ \end{cases}$$

The angle is in quadrant III; therefore, $\underline{\theta =} \ 180^\circ + 43.6^\circ \ \underline{= 223.6^\circ}$

$$\boxed{-21-20i = 29\ cis 223.6^\circ}$$

Exercise

Write $11+2i$ in trigonometric form.

Solution

$$11+2i \Rightarrow \begin{cases} r = \sqrt{11^2+2^2} = \sqrt{125} = 5\sqrt{5} \\ \hat{\theta} = \tan^{-1}\left(\frac{2}{11}\right) \approx 10.3^\circ \end{cases}$$

The angle is in quadrant I; therefore, $\underline{\theta} \ \underline{= 10.3^\circ}$

$$\boxed{11+2i = 5\sqrt{5}\ cis 10.3^\circ}$$

Exercise

Write $4(\cos 30^\circ + i\sin 30^\circ)$ in standard form.

Solution

$$\begin{aligned} 4(\cos 30^\circ + i\sin 30^\circ) &= 4\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) \\ &= 2\sqrt{3} + 2i \end{aligned}$$

Exercise

Write $\sqrt{2}\ cis\frac{7\pi}{4}$ in standard form.

Solution

$$\begin{aligned}\sqrt{2}\ cis\frac{7\pi}{4} &= \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\\ &= \sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)\\ &= 1 - i\end{aligned}$$

Exercise

Find the quotient $\frac{20cis(75°)}{4cis(40°)}$. Write the result in rectangular form.

Solution

$$\begin{aligned}\frac{20cis(75°)}{4cis(40°)} &= \frac{20}{4}cis(75° - 40°)\\ &= 5cis(35°)\\ &= 5(\cos 35° + i\sin 35°)\\ &= 4.1 + 2.87i\end{aligned}$$

Exercise

Divide $z_1 = 1 + i\sqrt{3}$ *by* $z_2 = \sqrt{3} + i$. Write the result in rectangular form.

Solution

$$\frac{z_1}{z_2} = \frac{1+i\sqrt{3}}{\sqrt{3}+i}$$

$$= \frac{1+i\sqrt{3}}{\sqrt{3}+i}\,\frac{\sqrt{3}-i}{\sqrt{3}-i}$$

$$= \frac{\sqrt{3}-i+3i-\sqrt{3}\,i^2}{3+1}$$

$$= \frac{2\sqrt{3}+2i}{4}$$

$$= \frac{2\sqrt{3}}{4} + \frac{2i}{4}$$

$$= \frac{\sqrt{3}}{2} + \frac{i}{2}$$

$\boxed{or}$

$$1+i\sqrt{3}: \begin{cases} r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} \\ \theta = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3} \end{cases}$$

$$\sqrt{3}+i: \begin{cases} r = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} \\ \theta = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{cases}$$

$$\frac{z_1}{z_2} = \frac{2cis\frac{\pi}{3}}{2cis\frac{\pi}{6}}$$

$$= \frac{2}{2}cis\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{2}{2}cis\left(\frac{\pi}{6}\right)$$

$$= cis\left(\frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)$$