7.5
#5

$\cos x + 1 = 2\sin^2 x \qquad [0, 2\pi)$

$= 2(1 - \cos^2 x)$

$= 2 - 2\cos^2 x$

$2\cos^2 x + \cos x - 1 = 0$

$\cos x = -1 \qquad \cos x = \frac{1}{2}$

$x = \pi, \ \frac{\pi}{3}, \frac{5\pi}{3}$

#9 $\cos(\ln x) = 0$

$\ln x = \frac{\pi}{2} \qquad \ln x = \frac{3\pi}{2}$

$x = e^{\pi/2}, \ e^{3\pi/2}$

#10 $2\sin^2 x = 1 - \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$\sin x = -1 \qquad \sin x = \frac{1}{2}$

$x = \frac{3\pi}{2}, \ \frac{\pi}{6}, \frac{5\pi}{6}$

#11 $\tan^2 x \sin x = \sin x$ $[0, 2\pi)$

$\tan^2 x \sin x - \sin x = 0$

$\sin x (\tan^2 x - 1) = 0$

$\tan^2 x - 1 = 0$

$\tan^2 x = 1$

$\sin x = 0 \qquad \tan x = \pm 1$

$x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

#12 $1 - \sin x = \sqrt{3}\cos x$

$(\ )^2 = (\ )^2$

☺ Check

$(-1)^2 = (1)^2$

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{1}{2}$

$\cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x = \frac{1}{2}$

$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

$x - \frac{\pi}{6} = \frac{\pi}{3}$

$x = \frac{\pi}{6} + \frac{\pi}{3}$

$= \frac{\pi}{2}$

$x - \frac{\pi}{6} = \frac{5\pi}{3}$

$x = \frac{5\pi}{3} + \frac{\pi}{6}$

$= \frac{11\pi}{6}$

#14 $2\sin^3 x + \sin^2 x - 2\sin x - 1 = 0$

$\sin^2 x(2\sin x + 1) - (2\sin x + 1) = 0$

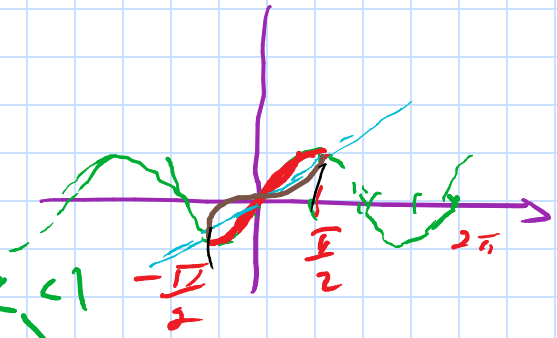
$(2\sin x + 1)(\sin^2 x - 1) = 0$

$\sin x = -\frac{1}{2}$ $\quad$ $\sin x = \pm 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$

8.3 Inverse Trig. fcns

Inverse Sine: $\sin^{-1}$ arcsin



angle

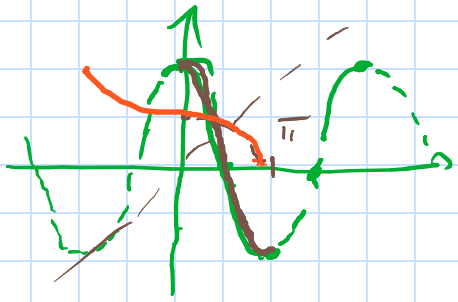
$y = \sin^{-1}(x)$ or $y = \arcsin x$

iff $x = \sin(y)$ angle

$$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$$
$$-1 \le x \le 1$$

$\sin(\sin^{-1} x) = x$

Inverse Cosine



$y = \cos^{-1} x$ or $\arccos x$

$$\begin{cases} 0 \le y \le \pi \\ -1 \le x \le 1 \end{cases}$$

Inverse tangent.

$y = \tan^{-1} x$ or $y = \arctan x$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ $\qquad x \in \mathbb{R}$

Ex $\sin\left(\arccos\left(-\frac{2}{3}\right)\right)$

$\alpha = \arccos\left(-\frac{2}{3}\right)$

$\cos\alpha = -\frac{2}{3}$

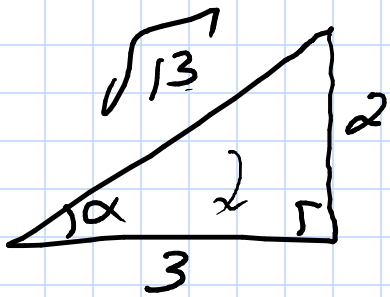
$\sin\alpha = \frac{\sqrt{5}}{3}$

Ex $\csc\left(\arctan\frac{2}{3}\right)$

$\alpha = \arctan\frac{2}{3}$

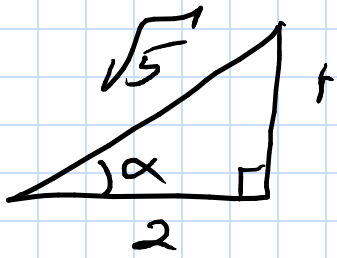
$\tan\alpha = \frac{2}{3}$

$\csc\alpha = \frac{\sqrt{13}}{3}$



5x $\sin\left(\underbrace{\arctan\frac{1}{2}}_{\alpha} - \underbrace{\arccos\frac{4}{5}}_{\beta}\right)$

$\tan\alpha = \frac{1}{2}$ $\Big\}$ $\sin\alpha = \frac{1}{\sqrt{5}}$, $\cos\alpha = \frac{2}{\sqrt{5}}$



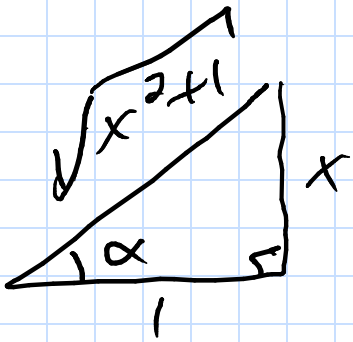
$\cos\beta = \frac{4}{5} \Rightarrow \sin\beta = \frac{3}{5}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$= \frac{1}{\sqrt{5}}\frac{4}{5} - \frac{3}{5}\frac{2}{\sqrt{5}}$

$= \frac{-2}{5\sqrt{5}}$

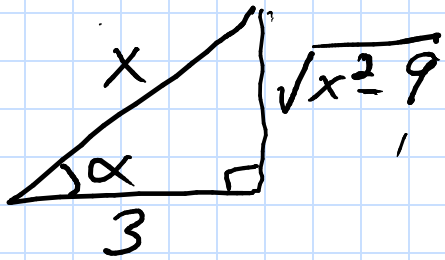
6x $\cos\left(\underbrace{\sin^{-1}x}_{\alpha}\right)$



$\sin\alpha = \frac{x}{1}$

$\cos\alpha = \frac{1}{\sqrt{x^2+1}}$

34) $\cot\left(\underbrace{\sin^{-1}\frac{\sqrt{x^2-9}}{x}}_{\alpha}\right)$



$$\sin\alpha = \frac{\sqrt{x^2-9}}{x}$$

$$\cot\alpha = \frac{3}{\sqrt{x^2-9}}$$

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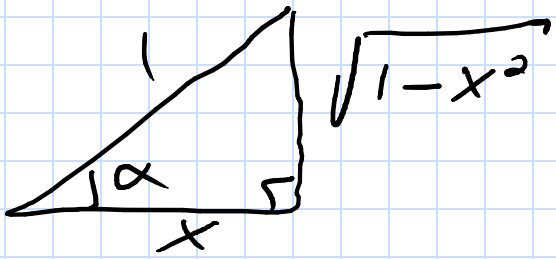
35) $\tan\left(\underbrace{\cos^{-1}x}_{\alpha}\right)$



$$\cos\alpha = \frac{x}{1}$$

$$\tan\alpha = \frac{\sqrt{1-x^2}}{x}$$

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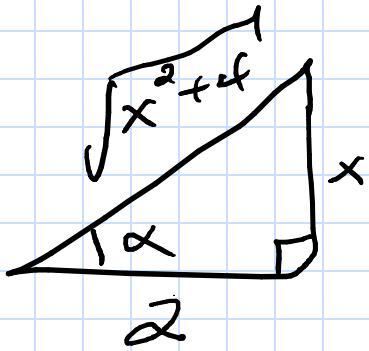
33) $\sec\left(\underbrace{\sin^{-1}\frac{x}{\sqrt{x^2+4}}}_{\alpha}\right)$

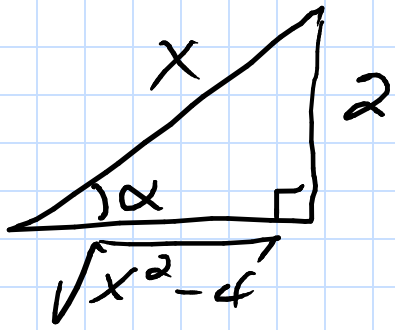


$$\sin\alpha = \frac{x}{\sqrt{x^2+4}}$$

$$\sec\alpha = \frac{\sqrt{x^2+4}}{2}$$

39/ $\sec\left(\underbrace{\tan^{-1}\frac{2}{\sqrt{x^2-4}}}_{\alpha}\right)$

$$\tan\alpha = \frac{2}{\sqrt{x^2-4}}$$



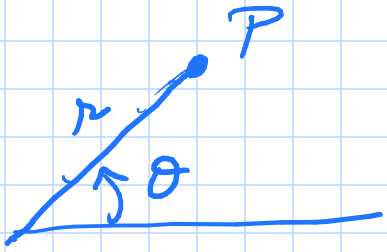
$$\sec\alpha = \frac{x}{\sqrt{x^2-4}}$$

8.6 Polar Coordinates

$P(r, \theta)$

r: distance / radius

θ: deg or rad.



$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1}\frac{y}{x} \end{cases}$$

$(r, \theta) = \left(4, \frac{7\pi}{6}\right)$ $\qquad (x, y)$?

$$x = r\cos\theta = 4\cos\frac{7\pi}{6} = 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$$

$$y = r\sin\theta = 4\sin\frac{7\pi}{6} = 4\left(-\frac{1}{2}\right) = -2$$

$$(x, y) = (-2\sqrt{3}, -2)$$

Ex $(x, y) = (-1, \sqrt{3})$ (r, θ) ?

$r = \sqrt{x^2 + y^2}$

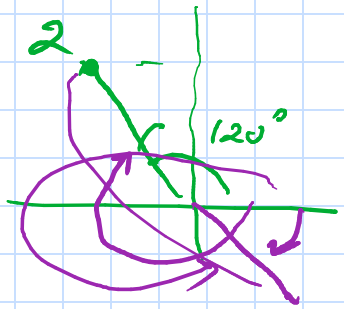
$= \sqrt{1 + 3}$

$= 2$

$\theta = \tan^{-1} \frac{\sqrt{3}}{1}$

$= 60^\circ$ or $\frac{\pi}{3}$

$(r, \theta) = \left(2, \frac{2\pi}{3}\right)$



$(-2, -60^\circ)$

$(2, -240^\circ)$

Ex $ax + by = c$ ← line

$ar\cos\theta + br\sin\theta = c$

$r(a\cos\theta + b\sin\theta) = c$

$r = \frac{c}{a\cos\theta + b\sin\theta}$

Ex $x^2 - y^2 = 16$

$$(r\cos\theta)^2 - (r\sin\theta)^2 = 16$$
$$r^2\cos^2\theta - r^2\sin^2\theta = 16$$
$$r^2(\cos^2\theta - \sin^2\theta) = 16$$
$$r^2(\cos 2\theta) = 16$$
$$r^2 = \frac{16}{\cos 2\theta} = 16\sec 2\theta$$

---

46 $r = 4\cos\theta + 2\sin\theta$

$$r^2 = 4r\cos\theta + 2r\sin\theta$$
$$x^2 + y^2 = 4x + 4y$$

$$x^2 - 4x + (-2)^2 + y^2 - 4y + (-2)^2 = 4 + 4$$
$$(x-2)^2 + (y-2)^2 = 8$$

---

50 $r^2 = 4\cos 2\theta$

$$= 4\cos^2\theta - 4\sin^2\theta$$
$$r^4 = 4r^2\cos^2\theta - 4r^2\sin^2\theta$$
$$(x^2+y^2)^2 = 4x^2 - 4y^2$$