Solution Section 3.8 – Hypothesis Tests for a Population Standard Deviation

Exercise

There is a claim that the lengths of men's hands have a standard deviation less than 200 mm. You plan to test that claim with a 0.01 significance level by constructing a confidence interval. What level of confidence should be used for the confidence interval? Will the conclusion based on the confidence interval be the same as the conclusion based on a hypothesis test that uses the traditional method or the *P*-value method?

Solution

A one-tailed test at the 0.01 level of significance rejects the null hypothesis if the sample statistic falls into the extreme 1% of the sampling distribution in the appropriate tail. The corresponding (two-sided) confidence interval test that places 1% each tail would be a 98% confidence interval. When testing claims about a standard deviation, the confidence interval method gives the same results as test using the traditional method or the *P*-value method.

Exercise

There is a claim that daily rainfall amounts in Boston have a standard deviation equal to 0.25 in. Sample data show that daily rainfall amounts are from a population with a distribution that is very far from normal. Can the use of a very large sample compensate for the lack of normality, so that the methods of this section can be used for the hypothesis test?

Solution

No. Unlike tests and confidence intervals involving the mean, which do not require normality when $n > 30$, test and confidence intervals involving standard deviations require approximate normality for all sample sizes.

Exercise

There is a claim that men have foot breaths with a variance equal to 36 mm^2. Is a hypothesis test of the claim that the variance is equal to 36 mm^2 equivalent to a test of the claim that the standard deviation is equal to 6 *mm*.

Solution

Yes. The claim that the variance is equal to 36 mm^2 and the claim that the standard deviation is equal to 6 *mm* are equivalent claims, and their corresponding tests are equivalent.

Exercise

Given: $H_1 : \sigma \neq 696\ g, \quad \alpha = 0.05, \quad n = 25, \quad s = 645\ g$, Find

a) Find the test statistic

b) Find critical value(s)

c) Find *P*-value limits

d) Determine whether there is sufficient evidence to support the given alternative hypothesis.

Solution

a) Test statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(24)(645)^2}{(696)^2} \underline{= 20.612|}$

b) Critical values for $\alpha = 0.05 \quad and \quad df = 24$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559

$\chi^2 = \chi^2_{1-\alpha/2} = \chi^2_{0.975} \underline{= 12.401|}$

$\chi^2 = \chi^2_{\alpha/2} = \chi^2_{0.025} \underline{= 39.364|}$

c) *P*-value limits: $15.659 < 20.612$ (part *a*)

P-value > 0.20

P-value exact: $2 \bullet \chi^2\ cdf\ (0,\ 20.612,\ 24) \underline{= 0.6770|}$

d) Conclusion: Do not reject H_0; there is not sufficient to conclude that $\sigma \neq 696$

Exercise

Given: $H_1 : \sigma < 29\ lb, \quad \alpha = 0.05, \quad n = 8, \quad s = 7.5\ lb$, Find

a) Find the test statistic

b) Find critical value(s)

c) Find *P*-value limits

d) Determine whether there is sufficient evidence to support the given alternative hypothesis.

Solution

a) Test statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(7)(7.5)^2}{(29)^2} \underline{= 0.468|}$

b) Critical values for $\alpha = 0.05 \quad and \quad df = 7$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278

$\chi^2 = \chi^2_{1-\alpha} = \chi^2_{0.95} \underline{= 2.167|}$

c) P-value limits: (part a) $0.468 < 0.989$

P-value < 0.005

P-value exact: $\chi^2\ cdf\,(0,\ 0.468,\ 7) = 4.451E-4\ \underline{= 0.0004}$

d) Conclusion: Reject H_0; there is sufficient to conclude that $\sigma < 29$

Exercise

Given: $H_1:\ \sigma > 3.5\ min, \quad \alpha = 0.01, \quad n = 15, \quad s = 4.8\ min$, Find

a) Find the test statistic
b) Find critical value(s)
c) Find P-value limits
d) Determine whether there is sufficient evidence to support the given alternative hypothesis.

Solution

a) Test statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(14)(4.8)^2}{(3.5)^2}\ \underline{= 26.331}$

b) Critical values for $\alpha = 0.01 \quad and \quad df = 14$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319

$\chi^2 = \chi^2_\alpha = \chi^2_{0.01}\ \underline{= 29.141}$

c) P-value limits: $26.119 < 26.331 < 29.141$

$0.01 < P\text{-value} < 0.025$

P-value exact: $\chi^2\ cdf\,(26,\ 999,\ 14)\ \underline{= 0.0235}$

d) Conclusion: Do not reject H_0; there is not sufficient evidence to conclude that $\sigma > 3.5$

Exercise

Given: $H_1:\ \sigma \ne 0.25, \quad \alpha = 0.01, \quad n = 26, \quad s = 0.18$, Find

a) Find the test statistic
b) Find critical value(s)
c) Find P-value limits
d) Determine whether there is sufficient evidence to support the given alternative hypothesis.

Solution

a) Test statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(25)(0.18)^2}{(0.25)^2}\ \underline{= 12.960}$

b) Critical values for $\alpha = 0.01 \quad and \quad df = 25$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928

$$\chi^2 = \chi^2_{1-\alpha/2} = \chi^2_{0.995} \underline{= 10.520|}$$

$$\chi^2 = \chi^2_{\alpha/2} = \chi^2_{0.005} \underline{= 46.928|}$$

c) *P*-value limits: $12.198 < 12.960 < 13.844$

$0.02 < P\text{-value} < 0.05$

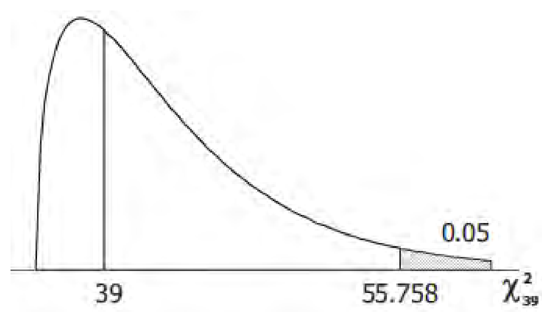
P-value exact: $2 \cdot \chi^2 \ cdf\ (0,\ 12.960,\ 25) \underline{= 0.0460|}$

d) Conclusion: Do not reject H_0; there is not sufficient to conclude that $\sigma \neq 0.25$

Exercise

A simple random sample of 40 men results in a standard deviation of 11.3 beats per minute. The normal range of pulse rates of adults is typically given as 60 to 100 beats per minute. If the range rule of thumb is applied to that normal range, the result is a standard deviation of 10 beats per minute. Use the sample results with a 0.05 significance level to test the claim that rates of men have a standard deviation greater than 10 beats per minute.

Solution



Original claim: $\sigma > 10$ *beats/min*

H_0 : $\sigma = 10$ *beats/min*

H_1 : $\sigma > 10$ *beats/min*

Given: $\alpha = 0.05$ *and* $df = 39$

Critical value: $\chi^2 = \chi^2_\alpha = \chi^2_{0.05} \underline{= 55.758|}$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(39)(11.3)^2}{(10)^2} \underline{= 49.799|}$$

P-value exact: $\chi^2 \ cdf\ (49.799,\ 999,\ 39) \underline{= 0.1152|}$

Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $\sigma > 10$. There is not sufficient evidence to support the claim that the pulse rate of men have a standard deviation greater than 10 *beats/min*.

Exercise

A simple random sample of 25 filtered 100 mm cigarettes is obtained, and the tar content of each cigarette is measured. The sample has a standard deviation of 3.7 mg. Use a 0.05 significance level to test the claim that the tar content of filtered 100 mm cigarettes has a standard deviation different from 3.2 mg, which is the standard deviation for unfiltered king size cigarettes.

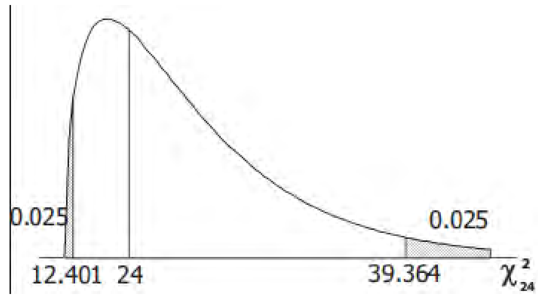
Solution

Original claim: $\sigma \neq 3.2\ mg$

$H_0 : \sigma = 3.2\ mg$

$H_1 : \sigma \neq 3.2\ mg$

Given: $\alpha = 0.05$ *and* $df = 24$



Critical value:

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559

$$\chi^2 = \chi^2_{1-\alpha/2} = \chi^2_{0.975} = 12.401$$

$$\chi^2 = \chi^2_{\alpha/2} = \chi^2_{0.025} = 39.364$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24)(3.7)^2}{(3.2)^2} = 32.086$$

P-value exact: $2 \cdot \chi^2\ cdf\,(32.086,\ 999,\ 24) = 0.2498$

Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $\sigma \neq 3.2$. There is not sufficient evidence to support the claim that the tar content of such cigarettes has a standard deviation different from 3.2 *mg*.

Exercise

When 40 people used the Weight Watchers diet for one year, their weight losses had a standard deviation of 4.9 lb. Use 0.01 significance level to test the claim that the amounts of weight loss have a standard deviation equal to 6.0 lb., which appears to be the standard deviation for the amounts of weight loss with the Zone diet.

Solution

Original claim: $\sigma = 6.2\ lbs$

$H_0 : \sigma = 6.2\ lbs$

$H_1 : \sigma \neq 6.2\ lbs$

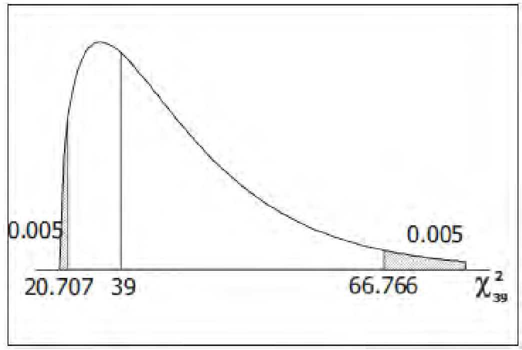
Given: $\alpha = 0.01$ *and* $df = 39$

Critical value:

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766

$$\chi^2 = \chi^2_{1-\alpha/2} = \chi^2_{0.995} \underline{=20.707}$$

$$\chi^2 = \chi^2_{\alpha/2} = \chi^2_{0.005} \underline{=66.766}$$



$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(39)(4.9)^2}{(6.0)^2} \underline{=26.011}$$

P-value exact:

$$2 \cdot \chi^2\ cdf\ (0,\ 26.011,\ 39) \underline{=0.1101}$$

Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $\sigma = 6.0$. There is not sufficient evidence to support the claim that the weight losses from this diet have a standard deviation different of 6.0 *lbs*.

Exercise

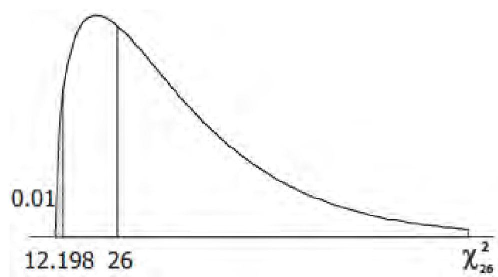
Tests in the statistic classes have scores with a standard deviation equal to 14.1. One of the last classes has 27 test scores with a standard deviation of 9.3. Use a 0.01 significance level to test the claim that this class has less variation than other past classes. Does a lower standard deviation suggest that this last class is doing better?

Solution

Original claim: $\sigma < 14.1$

$H_0 : \sigma = 14.1$

$H_1 : \sigma < 14.1$



Given: $\alpha = 0.01$ *and* $df = 26$

Critical value:

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290

$$\chi^2 = \chi^2_{1-\alpha} = \chi^2_{0.99} \underline{=12.198}$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(26)(9.3)^2}{(14.1)^2} \underline{=11.311}$$

P-value exact: $\chi^2\ cdf\ (0,\ 11.311,\ 26)\ \underline{=0.0056}$

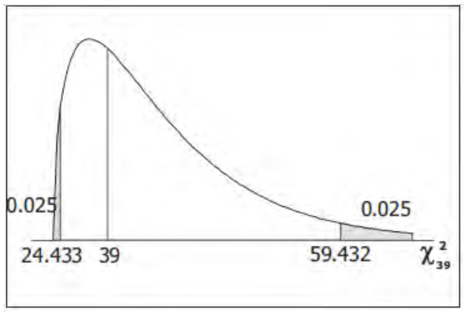
Conclusion

Reject H_0; there is sufficient evidence to conclude that $\sigma < 14.1$. There is sufficient evidence to support the claim that this class has less variation than other past classes.

No; a lower standard deviation means that the scores are closer together, but it says nothing about whether they are higher or lower.

Exercise

A simple random sample of pulse rates of 40 women results in a standard deviation of 12.5 *beats/min*. The normal range of pulse rates of adults is typically given as 60 to 100 *beats/min*. If the range rule of thumb is applied to that normal range, the result is a standard deviation of 10 *beats/min*. Use the sample results with a 0.05 significance level to test the claim that pulse rates of women have a standard deviation equal to 10 *beats/min*.



Solution

Original claim: $\sigma = 10$ *beats/min*

$H_0:\ \sigma = 10$ *beats/min*

$H_1:\ \sigma \ne 10$ *beats/min*

Given: $\alpha = 0.05$ *and* $df = 39$

Critical value:

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766

$$\chi^2 = \chi^2_{1-\alpha/2} = \chi^2_{0.975}\ \underline{=24.433}$$

$$\chi^2 = \chi^2_{\alpha/2} = \chi^2_{0.025}\ \underline{=59.432}$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(39)(12.5)^2}{(10)^2}\ \underline{=60.9375}$$

P-value exact: $2\cdot\chi^2\ cdf\ (60.9375,\ 999,\ 39)\ \underline{=0.0277}$

Conclusion

Reject H_0; there is sufficient evidence to reject the claim that $\sigma = 10$ conclude that $\sigma \ne 10$. There is sufficient evidence to reject the claim that the pulse rates of women have a standard deviation equal to 10 *beats/min*.

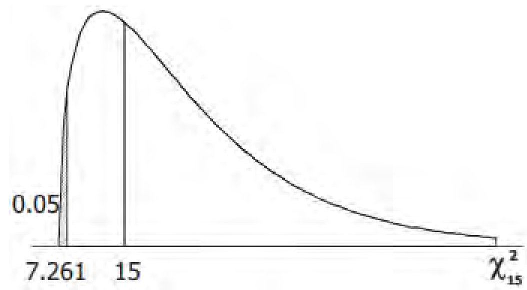
Exercise

Listed below are the playing times (in seconds) of songs that were popular at the time of this writing. Use a 0.05 significance level to test the claim that the songs are from a population with a standard deviation less than on minute.

448 242 231 246 246 293 280 227 244 213 262 239 213 258 255 257

Solution

$n = 16, \quad \bar{x} = 259.3, \quad s = 54.549$



Original claim: $\sigma < 60\ sec$

$H_0 : \sigma = 60\ sec$

$H_1 : \sigma < 60\ sec$

Given: $\alpha = 0.05 \quad and \quad df = 15$

Critical value

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801

$$\chi^2 = \chi^2_{1-\alpha} = \chi^2_{0.95} = 7.261$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(15)(54.549)^2}{(60)^2} = 12.398$$

P-value exact: $\chi^2\ cdf\,(0,\ 12.398,\ 15) = 0.3513$

Conclusion

Do not reject H_0; there is not sufficient evidence to conclude that $\sigma < 60$. There is not sufficient evidence to support the claim that the songs are from a population with a standard deviation less than one minute.

Exercise

Find the critical value or values of χ^2 based on the given information

a) $H_0 : \sigma = 8.0, \quad \alpha = 0.01, \quad n = 10$

b) $H_1 : \sigma > 3.5, \quad \alpha = 0.05, \quad n = 14$

c) $H_1 : \sigma < 0.14, \quad \alpha = 0.10, \quad n = 23$

d) $H_1 : \sigma \ne 9.3, \quad \alpha = 0.05, \quad n = 28$

Solution

Using Chi-Square $\left(\chi^2\right)$ Distribution Table

a) $\alpha = 0.01 \quad and \quad df = 10 - 1 = 9$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589

Critical value: $\chi^2_{1-\alpha/2} = \chi^2_{0.995} \underline{= 1.735|}$

$\chi^2 = \chi^2_{\alpha/2} = \chi^2_{0.005} \underline{= 23.589|}$

b) $\alpha = 0.05 \quad and \quad df = 14 - 1 = 13$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819

Since $H_1 : \sigma > 3.5$, then $\quad \chi^2 = \chi^2_{\alpha} = \chi^2_{0.05} \underline{= 22.362|}$

c) $\alpha = 0.10 \quad and \quad df = 23 - 1 = 22$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796

Since $H_1 : \sigma < 0.14$, then $\quad \chi^2 = \chi^2_{1-\alpha} = \chi^2_{0.90} \underline{= 14.042|}$

d) $\alpha = 0.05 \quad and \quad df = 28 - 1 = 27$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645

Critical value: $\chi^2_{1-\alpha/2} = \chi^2_{0.975} \underline{= 14.573|}$

$\chi^2 = \chi^2_{\alpha/2} = \chi^2_{0.025} \underline{= 43.194|}$