

Solution **Section 1.2 – Dot Products**

Exercise

Find for $\vec{v} = 2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}$, $\vec{u} = -2\hat{i} + 4\hat{j} - \sqrt{5}\hat{k}$

- a) $\vec{v} \cdot \vec{u}$, $|\vec{v}|$, $|\vec{u}|$
- b) The cosine of the angle between \vec{v} and \vec{u}
- c) The scalar component of \vec{u} in the direction of \vec{v}
- d) The vector $\text{proj}_{\vec{v}} \vec{u}$

Solution

$$\begin{aligned} \text{a) } \vec{v} \cdot \vec{u} &= (2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}) \cdot (-2\hat{i} + 4\hat{j} - \sqrt{5}\hat{k}) \\ &= -4 - 16 - 5 \\ &= \underline{-25} \end{aligned}$$

$$\begin{aligned} |\vec{v}| &= \sqrt{2^2 + (-4)^2 + (\sqrt{5})^2} \\ &= \sqrt{4 + 16 + 5} \\ &= \sqrt{25} \\ &= \underline{5} \end{aligned}$$

$$\begin{aligned} |\vec{u}| &= \sqrt{(-2)^2 + 4^2 + (-\sqrt{5})^2} \\ &= \sqrt{25} \\ &= \underline{5} \end{aligned}$$

$$\begin{aligned} \text{b) } \cos \theta &= \frac{-25}{(5)(5)} & \cos \theta &= \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \\ &= \underline{-1} \end{aligned}$$

$$\begin{aligned} \text{c) } |\vec{u}| \cos \theta &= (5)(-1) \\ &= \underline{-5} \end{aligned}$$

$$\begin{aligned} \text{d) } \text{proj}_{\vec{v}} \vec{u} &= \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} \\ &= \left(\frac{-25}{5^2} \right) (2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}) \\ &= -(2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}) \\ &= \underline{-2\hat{i} + 4\hat{j} - \sqrt{5}\hat{k}} \end{aligned}$$

Exercise

Find for $\vec{v} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}$, $\vec{u} = 5\hat{i} + 12\hat{j}$

- a) $\vec{v} \cdot \vec{u}$, $|\vec{v}|$, $|\vec{u}|$
- b) The cosine of the angle between \vec{v} and \vec{u}
- c) The scalar component of \vec{u} in the direction of \vec{v}
- d) The vector $\text{proj}_{\vec{v}} \vec{u}$

Solution

$$\begin{aligned} \text{a) } \vec{v} \cdot \vec{u} &= \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{k} \right) \cdot (5\hat{i} + 12\hat{j}) \\ &= \underline{3} \end{aligned}$$

$$\begin{aligned} |\vec{v}| &= \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} \\ &= \sqrt{\frac{9}{25} + \frac{16}{25}} \\ &= \sqrt{\frac{25}{25}} \\ &= \underline{1} \end{aligned}$$

$$\begin{aligned} |\vec{u}| &= \sqrt{5^2 + 12^2} \\ &= \underline{13} \end{aligned}$$

$$\begin{aligned} \text{b) } \cos \theta &= \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \\ &= \frac{3}{(1)(13)} \\ &= \underline{\frac{3}{13}} \end{aligned}$$

$$\text{c) } |\vec{u}| \cos \theta = (13) \left(\frac{3}{13} \right) = \underline{3}$$

$$\begin{aligned} \text{d) } \text{proj}_{\vec{v}} \vec{u} &= \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} \\ &= \left(\frac{3}{1^2} \right) \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{k} \right) \\ &= \underline{\frac{9}{5}\hat{i} + \frac{12}{5}\hat{k}} \end{aligned}$$

Exercise

Find for $\vec{v} = 2\hat{i} + 10\hat{j} - 11\hat{k}$, $\vec{u} = 2\hat{i} + 2\hat{j} + \hat{k}$

- a) $\vec{v} \cdot \vec{u}$, $|\vec{v}|$, $|\vec{u}|$
- b) The cosine of the angle between \vec{v} and \vec{u}
- c) The scalar component of \vec{u} in the direction of \vec{v}
- d) The vector $\text{proj}_{\vec{v}} \vec{u}$

Solution

$$\begin{aligned} a) \quad \vec{v} \cdot \vec{u} &= (2\hat{i} + 10\hat{j} - 11\hat{k}) \cdot (2\hat{i} + 2\hat{j} + \hat{k}) \\ &= 4 + 20 - 11 \\ &= 13 \end{aligned}$$

$$\begin{aligned} |\vec{v}| &= \sqrt{2^2 + 10^2 + (-11)^2} \\ &= \sqrt{4 + 100 + 121} \\ &= \sqrt{225} \\ &= 15 \end{aligned}$$

$$\begin{aligned} |\vec{u}| &= \sqrt{2^2 + 2^2 + 1^2} \\ &= 3 \end{aligned}$$

$$\begin{aligned} b) \quad \cos \theta &= \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \\ &= \frac{13}{(3)(15)} \\ &= \frac{13}{45} \end{aligned}$$

$$\begin{aligned} c) \quad |\vec{u}| \cos \theta &= (3) \left(\frac{13}{45} \right) \\ &= \frac{13}{15} \end{aligned}$$

$$\begin{aligned} d) \quad \text{proj}_{\vec{v}} \vec{u} &= \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} \\ &= \left(\frac{13}{15^2} \right) (2\hat{i} + 10\hat{j} - 11\hat{k}) \\ &= \frac{13}{225} (2\hat{i} + 10\hat{j} - 11\hat{k}) \end{aligned}$$

Exercise

Find for $\vec{v} = -\hat{i} + \hat{j}$, $\vec{u} = 2\hat{i} + \sqrt{17}\hat{j}$

- a) $\vec{v} \cdot \vec{u}$, $|\vec{v}|$, $|\vec{u}|$
- b) The cosine of the angle between \vec{v} and \vec{u}
- c) The scalar component of \vec{u} in the direction of \vec{v}
- d) The vector $\text{proj}_{\vec{v}} \vec{u}$

Solution

$$a) \quad \vec{v} \cdot \vec{u} = (5\hat{i} + \hat{j}) \cdot (2\hat{i} + \sqrt{17}\hat{j})$$

$$= 10 + \sqrt{17}$$

$$|\vec{v}| = \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$$|\vec{u}| = \sqrt{4 + 17}$$

$$= \sqrt{21}$$

$$b) \quad \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

$$= \frac{10 + \sqrt{17}}{\sqrt{21}\sqrt{26}}$$

$$= \frac{10 + \sqrt{17}}{\sqrt{546}}$$

$$c) \quad |\vec{u}| \cos \theta = (\sqrt{21}) \left(\frac{10 + \sqrt{17}}{\sqrt{546}} \right)$$

$$= \frac{10 + \sqrt{17}}{\sqrt{26}}$$

$$d) \quad \text{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v}$$

$$= \left(\frac{10 + \sqrt{17}}{26} \right) (5\hat{i} + \hat{j})$$

Exercise

Find for $\vec{v} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} \right\rangle$, $\vec{u} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}} \right\rangle$

- a) $\vec{v} \cdot \vec{u}$, $|\vec{v}|$, $|\vec{u}|$
- b) The cosine of the angle between \vec{v} and \vec{u}
- c) The scalar component of \vec{u} in the direction of \vec{v}
- d) The vector $\text{proj}_{\vec{v}} \vec{u}$

Solution

$$\begin{aligned} \text{a) } \vec{v} \cdot \vec{u} &= \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} \right\rangle \cdot \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}} \right\rangle \\ &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

$$\begin{aligned} |\vec{v}| &= \sqrt{\frac{1}{2} + \frac{1}{3}} \\ &= \frac{\sqrt{5}}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{\sqrt{30}}{6} \end{aligned}$$

$$\begin{aligned} |\vec{u}| &= \sqrt{\frac{1}{2} + \frac{1}{3}} \\ &= \frac{\sqrt{5}}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{\sqrt{30}}{6} \end{aligned}$$

$$\begin{aligned} \text{b) } \cos \theta &= \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \\ &= \frac{\frac{1}{6}}{\frac{\sqrt{30}}{6} \frac{\sqrt{30}}{6}} \\ &= \frac{1}{6} \left(\frac{36}{30} \right) \\ &= \frac{1}{5} \end{aligned}$$

$$\text{c) } |\vec{u}| \cos \theta = \left(\frac{\sqrt{30}}{6} \right) \left(\frac{1}{5} \right)$$

$$= \frac{\sqrt{30}}{30}$$

$$= \frac{1}{\sqrt{30}} \quad \Big|$$

$$d) \quad \text{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v}$$

$$= \frac{1}{6} \left(\frac{36}{30} \right) \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} \right\rangle$$

$$= \frac{1}{5} \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} \right\rangle \quad \Big|$$

Exercise

Find the angles between the vectors $\vec{u} = 2\hat{i} + \hat{j}$, $\vec{v} = \hat{i} + 2\hat{j} - \hat{k}$

Solution

$$\theta = \cos^{-1} \left(\frac{2+2+0}{\sqrt{4+1}\sqrt{1+4+1}} \right) \quad \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

$$= \cos^{-1} \left(\frac{4}{\sqrt{5}\sqrt{6}} \right)$$

$$= \cos^{-1} \left(\frac{4}{\sqrt{30}} \right)$$

$$\approx 0.84 \text{ rad} \quad \Big|$$

Exercise

Find the angles between the vectors $\vec{u} = \sqrt{3}\hat{i} - 7\hat{j}$, $\vec{v} = \sqrt{3}\hat{i} + \hat{j} + \hat{k}$

Solution

$$\theta = \cos^{-1} \left(\frac{3-7+0}{\sqrt{3+49}\sqrt{3+1+1}} \right) \quad \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

$$= \cos^{-1} \left(\frac{-4}{\sqrt{52}\sqrt{5}} \right)$$

$$= \cos^{-1} \left(-\frac{4}{\sqrt{260}} \right)$$

$$\approx 1.82 \text{ rad} \quad \Big|$$

Exercise

Find the angles between the vectors $\vec{u} = \hat{i} + \sqrt{2}\hat{j} - \sqrt{2}\hat{k}$, $\vec{v} = -\hat{i} + \hat{j} + \hat{k}$

Solution

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{-1 + \sqrt{2} - \sqrt{2}}{\sqrt{1+2+2}\sqrt{1+1+1}}\right) & \cos \theta &= \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \\ &= \cos^{-1}\left(\frac{-1}{\sqrt{5}\sqrt{3}}\right) \\ &= \cos^{-1}\left(-\frac{1}{\sqrt{15}}\right) \\ &\approx 1.83 \text{ rad}\end{aligned}$$

Exercise

Consider $\vec{u} = -3\hat{j} + 4\hat{k}$, $\vec{v} = -4\hat{i} + \hat{j} + 5\hat{k}$

- Find the angle between \vec{u} and \vec{v} .
- Compute $\text{proj}_{\vec{v}} \vec{u}$ and $\text{scal}_{\vec{v}} \vec{u}$
- Compute $\text{proj}_{\vec{u}} \vec{v}$ and $\text{scal}_{\vec{u}} \vec{v}$

Solution

$$\begin{aligned}a) \quad \theta &= \cos^{-1} \frac{(-3\hat{j} + 4\hat{k}) \cdot (-4\hat{i} + \hat{j} + 5\hat{k})}{\sqrt{9+16} \sqrt{16+1+25}} & \cos \theta &= \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \\ &= \cos^{-1} \frac{-3+20}{\sqrt{25} \sqrt{42}} \\ &= \cos^{-1} \frac{17}{5\sqrt{42}} \\ &\approx 1.02 \text{ rad}\end{aligned}$$
$$\begin{aligned}b) \quad \text{proj}_{\vec{v}} \vec{u} &= \frac{17}{42}(-4\hat{i} + \hat{j} + 5\hat{k}) & \text{proj}_{\vec{v}} \vec{u} &= \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \vec{v} \\ &= \frac{17}{42} \langle -4, 1, 5 \rangle \\ \text{scal}_{\vec{v}} \vec{u} &= \frac{17}{\sqrt{42}} & \text{scal}_{\vec{v}} \vec{u} &= \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}\end{aligned}$$
$$\begin{aligned}c) \quad \text{proj}_{\vec{u}} \vec{v} &= \frac{17}{25}(-3\hat{j} + 4\hat{k}) & \text{proj}_{\vec{u}} \vec{v} &= \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2} \vec{u} \\ &= \frac{17}{25} \langle 0, -3, 4 \rangle\end{aligned}$$

$$\underline{\text{scal}_{\vec{u}} \vec{v} = \frac{17}{5}} \quad |$$

$$\text{scal}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$$

Exercise

Consider $\vec{u} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{v} = 3\hat{i} + 6\hat{j} + 6\hat{k}$

- Find the angle between \vec{u} and \vec{v} .
- Compute $\text{proj}_{\vec{v}} \vec{u}$ and $\text{scal}_{\vec{v}} \vec{u}$
- Compute $\text{proj}_{\vec{u}} \vec{v}$ and $\text{scal}_{\vec{u}} \vec{v}$

Solution

$$\begin{aligned} a) \quad \theta &= \cos^{-1} \frac{(-\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 6\hat{j} + 6\hat{k})}{\sqrt{1+4+4} \sqrt{9+36+36}} \\ &= \cos^{-1} \frac{-3+12+12}{3(9)} \\ &= \cos^{-1} \frac{21}{27} \\ &= \cos^{-1} \frac{7}{9} \quad | \\ &\approx 0.68 \text{ rad} \quad | \end{aligned}$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

$$\begin{aligned} b) \quad \text{proj}_{\vec{v}} \vec{u} &= \frac{21}{81} \langle 3, 6, 6 \rangle \\ &= \frac{7}{9} \langle 1, 2, 2 \rangle \quad | \end{aligned}$$

$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \vec{v}$$

$$\begin{aligned} \text{scal}_{\vec{v}} \vec{u} &= \frac{21}{9} \\ &= \frac{7}{3} \quad | \end{aligned}$$

$$\text{scal}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$$

$$\begin{aligned} c) \quad \text{proj}_{\vec{u}} \vec{v} &= \frac{21}{9} \langle -1, 2, 2 \rangle \\ &= \frac{7}{3} \langle -1, 2, 2 \rangle \quad | \end{aligned}$$

$$\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2} \vec{u}$$

$$\begin{aligned} \text{scal}_{\vec{u}} \vec{v} &= \frac{21}{3} \\ &= 7 \quad | \end{aligned}$$

$$\text{scal}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$$

Exercise

The direction angles α , β , and γ of a vector $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ are defined as follows:

is the angle between \vec{v} and the positive x -axis ($0 \leq \alpha \leq \pi$)

is the angle between \vec{v} and the positive y -axis ($0 \leq \beta \leq \pi$)

is the angle between \vec{v} and the positive z -axis ($0 \leq \gamma \leq \pi$)

a) Show that $\cos \alpha = \frac{a}{|\vec{v}|}$, $\cos \beta = \frac{b}{|\vec{v}|}$, $\cos \gamma = \frac{c}{|\vec{v}|}$, and $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$. These cosines

are called the direction cosines of \vec{v} .

b) Show that if $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ is a unit vector, then a , b , and c are the direction cosines of \vec{v} .

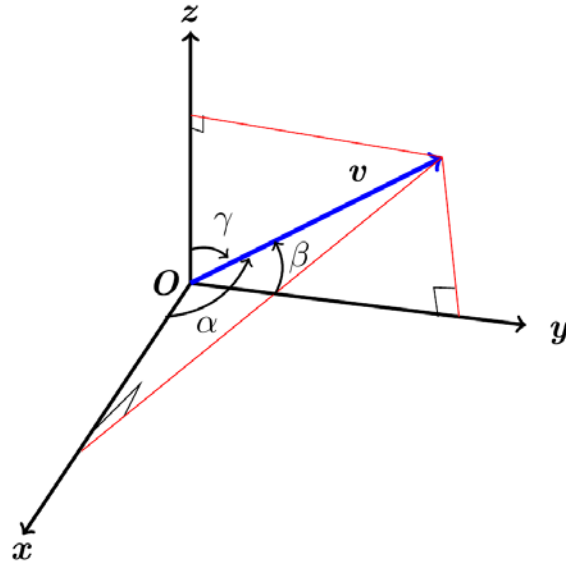
Solution

$$\begin{aligned} \text{a) } \cos \alpha &= \frac{\hat{i} \cdot \vec{v}}{|\hat{i}| |\vec{v}|} \\ &= \frac{\hat{i} \cdot (a\hat{i} + b\hat{j} + c\hat{k})}{|\vec{v}|} \\ &= \frac{a}{|\vec{v}|} \end{aligned}$$

$$\begin{aligned} \cos \beta &= \frac{\hat{j} \cdot \vec{v}}{|\hat{j}| |\vec{v}|} \\ &= \frac{\hat{j} \cdot (a\hat{i} + b\hat{j} + c\hat{k})}{|\vec{v}|} \\ &= \frac{b}{|\vec{v}|} \end{aligned}$$

$$\begin{aligned} \cos \gamma &= \frac{\hat{k} \cdot \vec{v}}{|\hat{k}| |\vec{v}|} \\ &= \frac{\hat{k} \cdot (a\hat{i} + b\hat{j} + c\hat{k})}{|\vec{v}|} \\ &= \frac{c}{|\vec{v}|} \end{aligned}$$

$$\begin{aligned} \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= \left(\frac{a}{|\vec{v}|} \right)^2 + \left(\frac{b}{|\vec{v}|} \right)^2 + \left(\frac{c}{|\vec{v}|} \right)^2 \\ &= \frac{a^2}{|\vec{v}|^2} + \frac{b^2}{|\vec{v}|^2} + \frac{c^2}{|\vec{v}|^2} \end{aligned}$$



$$\begin{aligned}
 &= \frac{a^2 + b^2 + c^2}{|\vec{v}|^2} \\
 &= \frac{a^2 + b^2 + c^2}{a^2 + b^2 + c^2} \\
 &= 1
 \end{aligned}$$

b) If $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ is a unit vector $\Rightarrow |\vec{v}| = 1$

$\cos \alpha = \frac{a}{|\vec{v}|} = a$, $\cos \beta = \frac{b}{|\vec{v}|} = b$, $\cos \gamma = \frac{c}{|\vec{v}|} = c$ are the direction cosines of \vec{v} .

Exercise

A water main is to be constructed with 20% grade in the north direction and a 10% grade in the east direction. Determine the angle θ required in the water main for the turn from north to east.

Solution

20% grade in the north direction

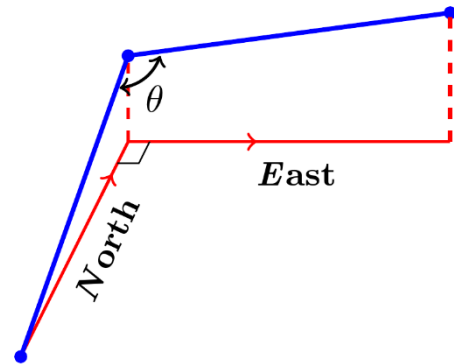
$$z\hat{k} = 20\% x\hat{i} = .2x\hat{i}$$

$$\rightarrow \text{If } x = 10 \quad z = 2$$

Let $\vec{u} = 10\hat{i} + 2\hat{k}$ be parallel to the pipe in the north direction.

$\vec{v} = 10\hat{j} + \hat{k}$ be parallel to the pipe in the east direction.

$$\begin{aligned}
 \theta &= \cos^{-1} \frac{0 + 0 + 2}{\sqrt{100 + 4} \sqrt{100 + 1}} & \theta &= \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \\
 &= \cos^{-1} \frac{2}{\sqrt{104} \sqrt{101}} \\
 &\approx 88.88^\circ
 \end{aligned}$$



Exercise

A gun with muzzle velocity of 1200 ft/sec is fired at an angle of 8° above the horizontal. Find the horizontal and vertical components of the velocity.

Solution

$$\text{Horizontal component: } 1200 \cos 8^\circ \approx 1188 \text{ ft/s}$$

$$\text{Vertical component: } 1200 \sin 8^\circ \approx 167 \text{ ft/s}$$

Exercise

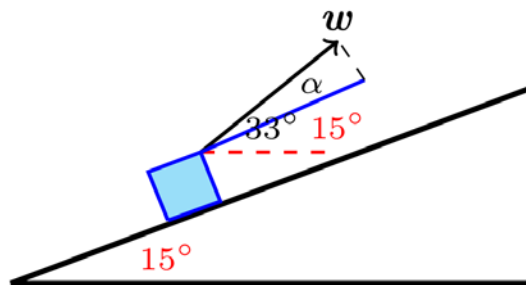
Suppose that a box is being towed up an inclined plane. Find the force w needed to make the component of the force parallel to the indicated plane equal to 2.5 lb .

Solution

$$2.5 = |w| \cos \alpha$$

$$|w| = \frac{2.5}{\cos(33^\circ - 15^\circ)}$$
$$= \frac{2.5}{\cos 18^\circ}$$

$$\vec{w} = \frac{2.5}{\cos 18^\circ} \langle \cos 33^\circ, \sin 33^\circ \rangle$$
$$= \langle 2.205, 1.432 \rangle$$



Exercise

Find the work done by a force $\vec{F} = 5\hat{i}$ (magnitude 5 N) in moving an object along the line from the origin to the point $(1, 1)$ (distance in meters)

Solution

$$P(1, 1) \Rightarrow \vec{OP} = \hat{i} + \hat{j}$$

$$W = \vec{F} \cdot \vec{OP}$$
$$= 5\hat{i} \cdot (\hat{i} + \hat{j})$$
$$= 5 \text{ J}$$

Exercise

How much work does it take to slide a crate 20 m along a loading dock by pulling on it with a 200 N force at an angle of 30° from the horizontal?

Solution

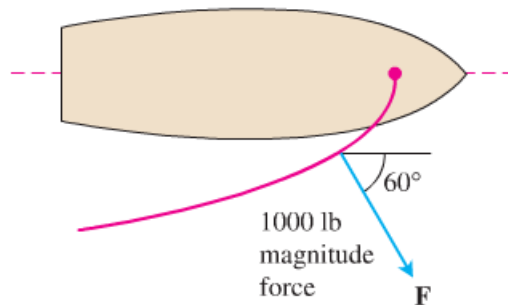
$$W = |F| |\vec{PQ}| \cos \theta$$
$$= (200)(20) \cos 30^\circ$$
$$= 3464.10 \text{ J}$$

Exercise

The wind passing over a boat's sail exerted a 1000-*lb* magnitude force F . How much work did the wind perform in moving the boat forward 1 *mi*? Answer in foot-pounds.

Solution

$$\begin{aligned} W &= |F| |\overrightarrow{PQ}| \cos \theta \\ &= (1000N) \left(1 \text{ mi} \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \cos 60^\circ \\ &= \underline{2,640,000 \text{ ft} \cdot \text{lb}} \end{aligned}$$



Exercise

Use a dot product to find an equation of the line in the xy -plane passing through the point (x_0, y_0) perpendicular to the vector $\langle a, b \rangle$.

Solution

$$\begin{aligned} \langle x - x_0, y - y_0 \rangle \cdot \langle a, b \rangle &= 0 \\ \underline{a(x - x_0) + b(y - y_0) = 0} \end{aligned}$$

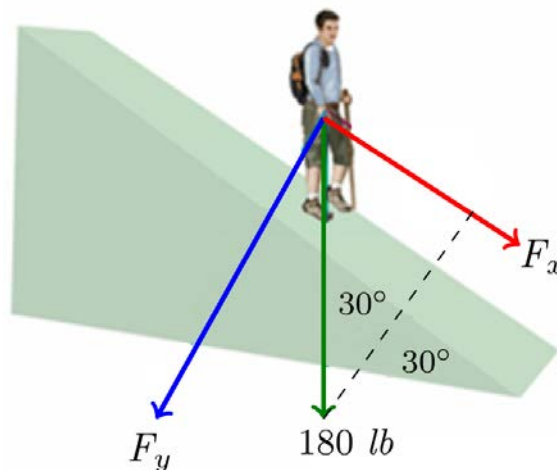
Exercise

A 180-*lb* man stands on a hillside that makes an angle of 30° with the horizontal, producing a force of $W = \langle 0, -180 \rangle$ *lbs*.

- Find the component of his weight in the downward direction perpendicular to the hillside and in the downward parallel to the hillside.
- How much work is done when the man moves 10 *ft* up the hillside?

Solution

$$\begin{aligned} a) \quad |F_\perp| &= |F_y| = 180 \cos 30^\circ \\ &= 180 \left(\frac{\sqrt{3}}{2} \right) \\ &= \underline{90\sqrt{3} \text{ lb}} \\ |F_\parallel| &= |F_x| = 180 \sin 30^\circ \\ &= 180 \left(\frac{1}{2} \right) \\ &= \underline{90 \text{ lb}} \end{aligned}$$



$$\begin{aligned}
 \text{b) } \textit{Work} &= d \cdot F_x \\
 &= 10(90) \\
 &= \underline{900 \text{ ft-lbs}}
 \end{aligned}$$