Solution *Section* 2.1 – Scatter Diagrams and Correlation

Exercise

For each of several randomly selected years, the total number of points scored in the Super Bowl football game and the total number of new cars sold in The U.S. are recorded. For this sample of paired data

a) What does r represent?

b) What does ρ represent?

c) With our doing any research or calculations, estimate the value of r.

Solution

a) r = the correlation in the sample. In this context, r is the linear correlation coefficient computed using the chosen paired (points in Super Bowl, number of new cars sold) values for the randomly selected years in the sample.

b) ρ = the correlation in the population. In this context, ρ is the linear correlation coefficient computed using the paired (points in Super Bowl, number of new cars sold) values for every year there has been a Super Bowl.

c) Since there is no relationship between the number of points scored in a Super Bowl and the number of new cars sold that year, the estimated value of r is 0.

Exercise

The heights (in inches) of a sample of eight mother/daughter pairs of subjects measured. Using Excel with the paired mother/daughter heights, the linear correlation coefficient is found to be 0.693. Is there sufficient evidence to support the claim that there is a linear correlation between the heights of mothers and the heights of their daughters? Explain.

Solution

From the table for $n = 8, \quad C.V. = \pm 0.707$. Therefore r = 0.693 indicates a significant (positive) linear correlation. Yes; there is sufficient evidence to support the claim that there is a linear correlation between the heights of mothers and the heights of their daughters,

Exercise

The heights and weights of a sample of 9 supermodels were measured. Using a TI calculator, the linear correlation coefficient is found to be 0.360. Is there sufficient evidence to support the claim that there is a linear correlation between the heights and weights of supermodels? Explain.

Solution

From the table for $n = 9, \quad C.V. = \pm 0.666$. Therefore r = 0.360 does not indicate a significant linear correlation. No; there is not sufficient evidence to support the claim that there is a linear correlation between the heights and weights of supermodels.

Exercise

Given the table below

x	10	8	13	9	11	14	6	4	12	7	5
y	9.14	8.14	8.74	8.77	9.26	8.10	6.13	3.10	9.13	7.26	4.74

a) Construct a scatterplot

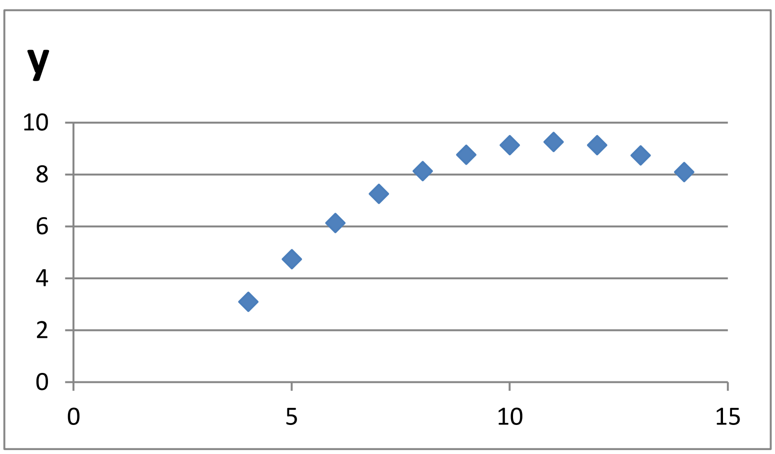
b) Find the value of linear correlation coefficient r and then determine whether there is sufficient evidence to support the claim of a linear correlation between the 2 variables.

c) Identify the feature of the data that would be missed if part (b) was completed without constructing the scatterplot.

Solution

a) Excel produces the following

x	y	xy	x^2	y^2
10	9.14	91.4	100	83.5396
8	8.14	65.12	64	66.2596
13	8.74	113.62	169	76.3876
9	8.77	78.93	81	76.9129
11	9.26	101.86	121	85.7476
14	8.1	113.4	196	65.6100
6	6.13	36.78	36	37.5769
4	3.1	12.4	16	9.6100
12	9.13	109.56	144	83.3569
7	7.26	50.82	49	52.7076
5	4.74	23.7	25	22.4676
99	82.51	797.59	1001	660.176



b) $$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{(11)(797.59)-(99)(82.51)}{\sqrt{(11)(1001)-(99)^2}\cdot\sqrt{(11)(660.1763)-(82.51)^2}}$$

$$\underline{= 0.816}$$

From table A-5; $n = 11, \quad C.V. = \pm 0.602$

Therefore r = 0.816 indicates a significant (positive) linear correlation. Yes; there is sufficient evidence to support the claim that there is a linear correlation between the 2 variables.

c) The scatterplot indicates that the relationship between the variables is quadratic, not linear.

Exercise

Given the table below

x	10	8	13	9	11	14	6	4	12	7	5
y	7.46	6.77	12.74	7.11	7.81	8.84	6.08	5.39	8.15	6.42	5.73

a) Construct a scatterplot

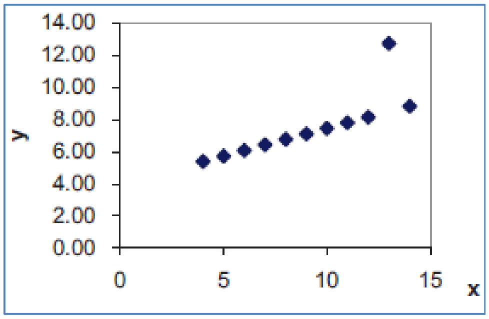
b) Find the value of linear correlation coefficient r and then determine whether there is sufficient evidence to support the claim of a linear correlation between the 2 variables.

c) Identify the feature of the data that would be missed if part (b) was completed without constructing the scatterplot.

Solution

a) Excel produces the following

x	**y**	**xy**	x^2	y^2
10	7.46	74.60	100	55.6516
8	6.77	54.16	64	45.8329
13	12.74	165.62	169	162.3076
9	7.11	63.99	81	50.5521
11	7.81	85.91	121	60.9961
14	8.84	123.76	196	78.1456
6	6.08	36.48	36	36.9664
4	5.39	21.56	16	29.0521
12	8.15	97.80	144	66.4225
7	6.42	44.94	49	41.2164
5	5.73	28.65	25	32.8329
99	82.50	797.47	1001	659.9762



$$\textbf{\textit{b)}}\quad r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right)-\left(\sum x\right)^2}\cdot\sqrt{n\left(\sum y^2\right)-\left(\sum y\right)^2}}$$

$$= \frac{(11)(797.59)-(99)(82.5)}{\sqrt{(11)(1001)-(99)^2}\cdot\sqrt{(11)(659.9762)-(82.5)^2}}$$

$$\underline{= 0.816}$$

From table A-5; $n = 11$, $C.V. = \pm 0.602$

Therefore r = 0.816 indicates a significant (positive) linear correlation. Yes; there is sufficient evidence to support the claim that there is a linear correlation between the 2 variables.

c) The scatterplot indicates that the relationship between the variables is essentially a perfect straight line except for one point, which is likely an error or an oulier.

Exercise

The paired values of the Consumer Price Index (CPI) and the cost of a slice of pizza are shown below

CPI	30.2	48.3	112.3	162.2	191.9	197.8
Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00

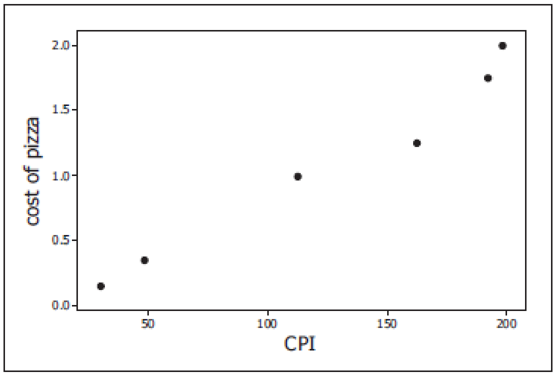
a) Construct a scatterplot

b) Find the value of linear correlation coefficient r.

Solution

a) Excel produces the following

x	y	xy	x^2	y^2
30.2	0.15	4.53	912.04	0.0225
48.3	0.35	16.905	2332.89	0.1225
112.3	1.00	112.3	12611.29	1.00
162.2	1.25	202.75	26308.84	1.5625
191.9	1.75	335.825	36825.61	3.0625
197.8	2.00	395.60	39124.84	4.00
742.7	6.50	1067.91	118115.5	9.77



b) $$r = \frac{n\sum xy-\left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right)-\left(\sum x\right)^2}\cdot\sqrt{n\left(\sum y^2\right)-\left(\sum y\right)^2}}$$

$$= \frac{(6)(1067.91)-(742.7)(6.5)}{\sqrt{(6)(118115.5)-(742.7)^2}\cdot\sqrt{(6)(9.77)-(6.5)^2}}$$

$$\underline{= 0.985}$$

Exercise

Listed below are systolic blood pressure measurements (in mm HG) obtained from the same woman.

Right Arm	102	101	94	79	79
Left Arm	175	169	182	146	144

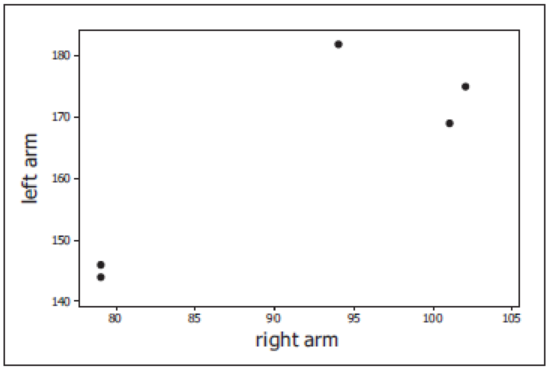
a) Construct a scatterplot

b) Find the value of linear correlation coefficient r.

Solution

a) Excel produces the following

x	y	xy	x^2	y^2
102	175	17850	10404	30625
101	169	17069	10201	28561
94	182	17108	8836	33124
79	146	11534	6241	21316
79	144	11376	6241	20736
455	816	74937	41923	134362



b) $$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{(5)(74937) - (455)(816)}{\sqrt{(5)(41923) - (255)^2} \cdot \sqrt{(5)(134362) - (816)^2}}$$

$$= 0.867$$

Exercise

Listed below are costs (in dollars) of air fares for different airlines from NY to San Francisco. The costs are based on tickets purchased 30 days in advance and one day in advance.

30 Days	244	260	264	264	278	318	280
One Day	456	614	567	943	628	1088	536

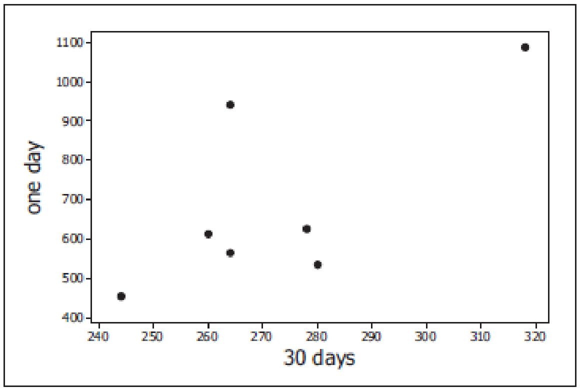
a) Construct a scatterplot

b) Find the value of linear correlation coefficient r.

Solution

a) Excel produces the following

x	y	xy	x^2	y^2
244	456	111264	59536	207936
260	614	159640	67600	376996
264	567	149688	69696	889249
264	943	248952	69696	889249
278	628	174584	77284	394384
318	1088	345984	101124	1183744
280	536	150080	78400	287296
1908	4832	1340192	523336	3661094



b) $$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{(7)(1340192) - (1908)(4832)}{\sqrt{(7)(523336) - (1908)^2} \cdot \sqrt{(7)(3661094) - (4832)^2}}$$

$$= 0.709$$

Exercise

Listed below are repair costs (in dollars) for cars crashed at 6 mi/h in full-front crash tests and the same cars crashed at 6 mi/f in full-rear crash tests.

Front	936	978	2252	1032	3911	4312	3469
Rear	1480	1202	802	3191	1122	739	2767

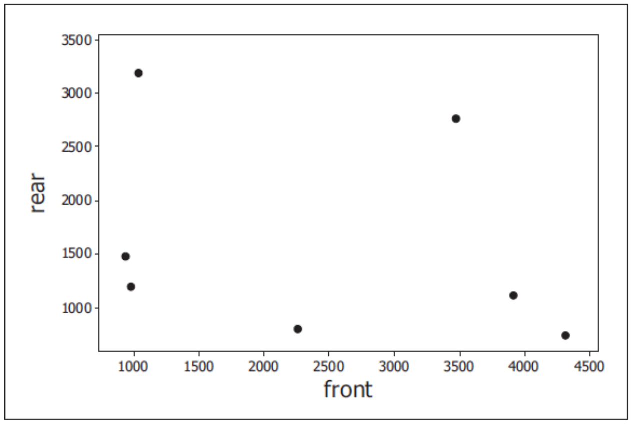
a) Construct a scatterplot

b) Find the value of linear correlation coefficient *r*.

Solution

a) Excel produces the following

x	y	xy	x^2	y^2
936	1480	1385280	876096	2190400
978	1202	1175556	956484	1444804
2252	802	1806104	5071504	643204
1032	3191	3293112	1065024	10182481
3911	1122	4388142	15295921	1258884
4312	739	3186568	18593344	546121
3469	2767	9598723	12033961	7656289
16890	11303	24833485	53892334	23922183



b) $r = \dfrac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$

$= \dfrac{(7)(24833485) - (16890)(11303)}{\sqrt{(7)(53892344) - (16890)^2} \cdot \sqrt{(7)(23922183) - (11303)^2}}$

$\underline{= -0.283}$

Exercise

For the following data:

x	2	4	7	7	9
y	1.6	2.1	2.4	2.6	3.2

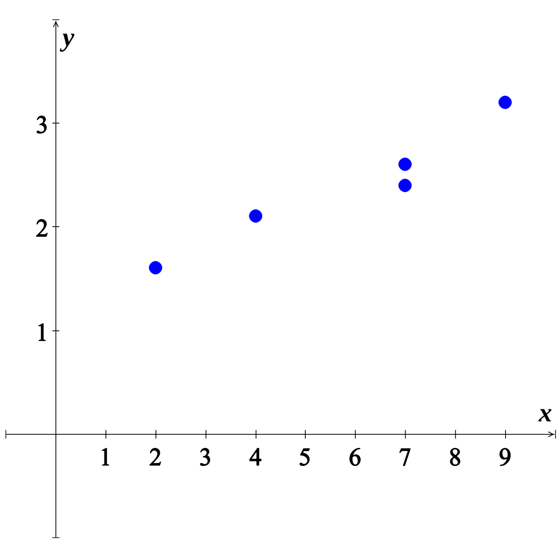
a) Draw a scatter diagram

b) Compute the correlation coefficient

c) Comment on the type of the relation that appears to exist between *x* and *y*.

Solution

a)



b) $r \approx 0.968$

c) The linear correlation is close to **1**, so a positive linear relation exists between *x* and *y*.

Exercise

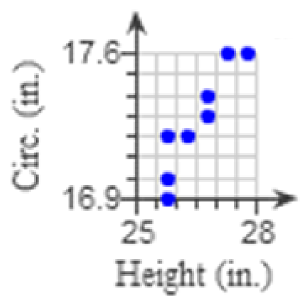
A pediatrician wants to determine the relation that may exist between a child's height and head circumference. She randomly selects 8 children, measures their height and head cieecumference, and obtains the data shown in the table.

***Height* (in.)**	27.25	25.75	26.25	25.75	27.75	26.75	25.75	26.75
***Head Circumference* (in.)**	17.6	17	17.2	16.9	17.6	17.3	17.2	17.4

a) Draw a scatter diagram
b) Compute the correlation coefficient
c) If the pediatrician wants to use height to predict head circumference, determine which variable is the explanatory variable and which is the response variable.
d) Does a linear relation exist between height and head circumference?

Solution

a)



b) $r \approx 0.968$

c) The explanatory variable is height and the response is head circumference.

d) Yes, there appears to be a positive linear association because r is positive and is greater than the critical value.

Exercise

An engineer wanted to determine how the weight of a car affects gas mileage. The accompanying data represent the weight of various domestic cars and their mileages in the city for the 2008 model year. Suppose that we add Car 12 to the original data. Car 12 weights 3,305 *lbs*. and gets 19 miles per gallon.

Car	*Weight (lbs)*	*Miles / Gal*
1	3,775	21
2	3,964	17
3	3,470	21
4	3,175	22
5	2,580	27
6	3,730	18
7	2,605	26
8	3,772	17
9	3,310	20
10	2,991	25
11	2,752	26

a) Draw a scatter diagram

b) Compute the correlation coefficient.

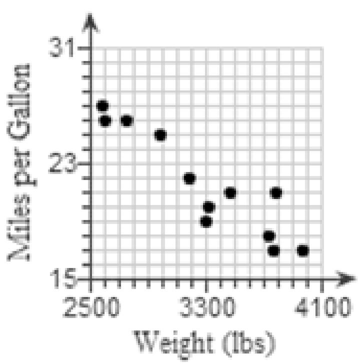
c) Compute the correlation coefficient with Car 12 included

d) Compare the correlation coefficient in part (*b*) & (*c*), and why are the results reasonable.

e) Suppose that we add Car 13 (a hybrid car) to the original data (remove the Car 12). Car 13 weighs 2,890 *lbs*. and gets 60 miles per gallon. Compute the linear coefficient with Car 13 included,

Solution

a)



b) $r \approx -0.946$

c) $r \approx -0.925$

d) The results are reasonable because the Car 12 follows the overall pattern of the data.

e) $r \approx -0.502$

Solution   *Section* 2.2 – Least-Squares Regression

Exercise

A physician measured the weights and cholesterol levels of a random sample of men. The regression equation is $\hat{y} = -116 + 2.44x$, where x represents weight (in pounds). What does the symbol $\hat{y}$ represent? What does the predictor variable represent? What does the response variable represent?

Solution

The symbol $\hat{y}$ represents the predicted cholesterol level. The predictor variable x represents weight. The response variable represents cholesterol level.

Exercise

In what sense is the regression line the straight line that "best" fits the points in a scatterplot?

Solution

The regression line is the best fit for the points of a scatterplot in the sense that it minimizes the sum of the squared differences between the observed y values and the y values predicted by the regression line.

Exercise

In a study, the total weight (in pounds) of garbage discarded in one week and the household size were recorded for 62 households. The linear correlation coefficient is $r = 0.759$ and the regression equation $\hat{y} = 0.445 + 0.119x$, where x represents the total weight of discarded garbage. The mean of the 62 garbage weights is 27.4 lb. and the 62 households have a mean size of 3.71 people. What is the best predicted number of people in a household that discards 50 lb. of garbage?

Solution

For n = 62, the critical value = ± 0.254.

Since r = 0.759 > 0.254, use the regression line for prediction.

$$\hat{y}\Big|_{x=50} = 0.445 + 0.119(50) \backslash$$

$$\underline{= 6.4 \textit{ people}}$$

Exercise

A sample of 8 mother/daughter pairs of subjects was obtained, and their heights (in inches) were measured. The linear correlation coefficient is 0.693 and the regression equation $\hat{y} = 69 - 0.0849x$, where x represents the height of the mother. The mean height of the mothers is 63.1 in. and the mean height of the daughters is 63.3 in. Find the best predicted height of a daughter given that the mother has a height of 60 in.

Solution

For n = 8, the critical value = ±0.707.

Since r = 0.693 < 0.707, use the regression line for prediction. $\hat{y} = \bar{y}$

$$\left.\hat{y}\right|_{x=60} = \bar{y} = \underline{= 63.3\ in}$$

Exercise

A sample of 40 women is obtained, and their heights (in inches) and pulse rates (in beats per minute) are measured. The linear correlation coefficient is 0.202 and the equation of the regression line is $\hat{y} = 18.2 + 0.920x$, where x represents height. The mean of the 40 heights is 63.2 in. and the mean of the 40 pulse rates is 76.3 beats per minute. Find the best predicted pulse rate of a woman who is 70 in. tall.

Solution

For n = 40, the critical value = ±0.312.

Since r = 0.202 < 0.312, use the regression line for prediction. $\hat{y} = \bar{y}$

$$\left.\hat{y}\right|_{x=70} = \bar{y}\ \underline{= 76.3\ \ beats / \min}$$

Exercise

Heights (in inches) and weights (in pounds) are obtained from a random sample of 9 supermodels. The linear correlation coefficient is 0.360 and the equation of the regression line is $\hat{y} = 31.8 + 1.23x$, where x represents height. The mean of the 9 heights is 69.3 in. and the mean of the 9 weights is 117 lb. Find the best predicted weight of a supermodel with a height of 72 in.?

Solution

For n = 9, the critical value = ±0.666.

Since r = 0.360 < 0.666, use the regression line for prediction. $\hat{y} = \bar{y}$

$$\left.\hat{y}\right|_{x=72} \underline{= 117\ \ lbs}$$

Exercise

Find the equation of the regression line for the given data below

x	10	8	13	9	11	14	6	4	12	7	5
y	9.14	8.14	8.74	8.77	9.26	8.10	6.13	3.10	9.13	7.26	4.74

Examine the scatterplot and identify a characteristic of the data that is ignored by the regression line

Solution

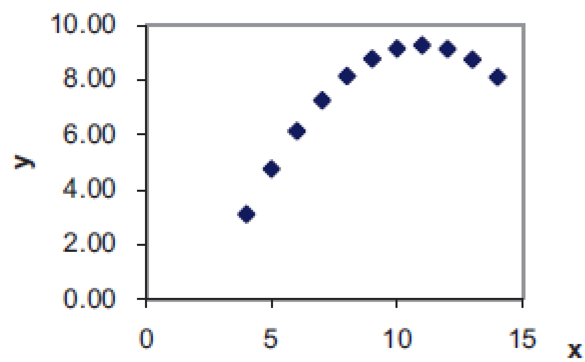
x	**y**	**xy**	x^2	y^2
10	9.14	91.40	100	83.5396
8	8.14	65.12	64	66.2596
13	8.74	113.62	169	76.3876
9	8.77	78.93	81	76.9129
11	9.26	101.86	121	85.7476
14	8.10	113.40	196	65.61
6	6.13	36.78	36	37.5769
4	3.10	12.40	16	9.61
12	9.13	109.56	144	83.3569
7	7.26	50.82	49	52.7076
5	4.74	23.70	25	22.4676
99	82.51	797.59	1001	660.1763

$$\bar{x} = \frac{\sum x}{n} = \frac{99}{11} = 9.0 \qquad \bar{y} = \frac{\sum y}{n} = \frac{82.52}{11} = 7.5$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{11(797.59) - (99)(82.51)}{11(1001) - (99)^2}$$

$$\underline{= 0.50}$$



$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 7.50 - 0.5(9)$$

$$= 3$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= 3.0 + 0.5x}$$

The scatterplot indicates that the relationship between the variables is quadratic, not linear.

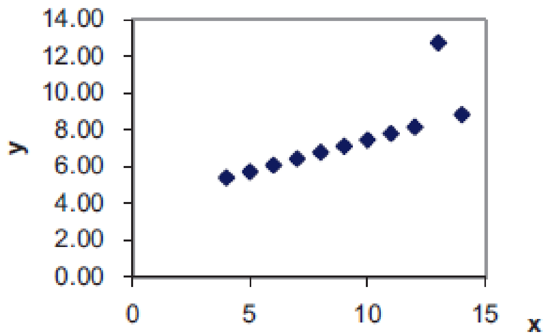
Exercise

Find the equation of the regression line for the given data below

x	10	8	13	9	11	14	6	4	12	7	5
y	7.46	6.77	12.74	7.11	7.81	8.84	6.08	5.39	8.15	6.42	5.73

Examine the scatterplot and identify a characteristic of the data that is ignored by the regression line

Solution



x	***y***	***xy***	x^2	y^2
10	7.46	74.60	100	55.6516
8	6.77	54.16	64	45.8329
13	112.74	165.62	169	162.3076
9	7.11	63.99	81	50.5521
11	7.81	85.91	121	60.9961
14	8.84	123.76	196	78.1456
6	6.08	36.48	36	36.9664
4	5.39	21.56	16	29.0521
12	8.15	97.80	144	66.4225
7	6.42	44.94	49	41.2164
5	5.73	28.65	25	32.8329
99	82.50	797.47	1001	659.9762

$$\overline{x}=\frac{\sum x}{n}=\frac{99}{11}=9.0 \qquad \overline{y}=\frac{\sum y}{n}=\frac{82.52}{11}=7.5$$

$$b_1=\frac{n\left(\sum xy\right)-\left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right)-\left(\sum x\right)^2}$$

$$=\frac{11(797.47)-(99)(82.50)}{11(1001)-(99)^2}$$

$$=0.50$$

$$b_0=\overline{y}-b_1\overline{x}$$

$$=7.50-0.5(9)$$

$$=3$$

$$\hat{y}=b_0+b_1x$$

$$=3.0+0.5x$$

The scatterplot indicates that the relationship between the variables is essentially a perfect straight line except for one point, which is likely an error or an outlier.

Exercise

Find the equation of the regression line for the given data below

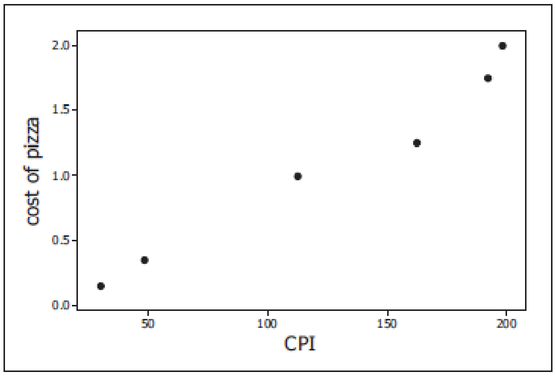
CPI	30.2	48.3	112.3	162.2	191.9	197.8
Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00

Let the first variable be the predictor (x) variable. Find the best indicated predicted cost of a slice of pizza when the Consumer Price Index (CPI) is 182.5 (in the year 2000).

Solution

Excel produces the following

x	y	xy	x^2	y^2
30.2	0.15	4.53	912.04	0.0225
48.3	0.35	16.905	2332.89	0.1225
112.3	1.00	112.3	12611.29	1.00
162.2	1.25	202.75	26308.84	1.5625
191.9	1.75	335.825	36825.61	3.0625
197.8	2.00	395.60	39124.84	4.00
742.7	6.50	1067.91	118115.5	9.77



$$\bar{x} = \frac{\sum x}{n} = \frac{742.7}{6} = 123.78 \qquad \bar{y} = \frac{\sum y}{n} = \frac{6.50}{6} = 1.08$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{6(1067.91) - (742.7)(6.5)}{6(118115.5) - (742.7)^2}$$

$$\underline{= 0.01005}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 1.08 - 0.0101(123.78)$$

$$\underline{= -0.1616}$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= -0.162 + 0.0101x}$$

$$\hat{y}_{182.5} = -0.162 + 0.0101(182.5)$$

$$\underline{= \$1.67}$$

Exercise

Find the equation of the regression line for the given data below

CPI	30.2	48.3	112.3	162.2	191.9	197.8
Subway fare	0.15	0.35	1.00	1.35	1.5	2.00

Let the first variable be the predictor (x) variable. Find the best indicated predicted cost of a slice of pizza when the Consumer Price Index (CPI) is 182.5 (in the year 2000).

Solution

x	y	xy	x^2	y^2
30.2	0.15	4.53	912.04	0.0225
48.3	0.35	16.905	2332.89	0.1225
112.3	1.00	112.3	12611.29	1.00
162.2	1.35	218.97	26308.84	1.8225
191.9	1.50	287.85	36825.61	2.25
197.8	2.00	395.60	39124.84	4.00
742.7	6.35	1036.155	118115.51	9.2175

$$\bar{x} = \frac{\sum x}{n} = \frac{742.7}{6} = 123.78 \qquad \bar{y} = \frac{\sum y}{n} = \frac{6.35}{6} = 1.06$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{6(1036.155) - (742.7)(6.35)}{6(118115.51) - (742.7)^2}$$

$$\underline{= 0.00955}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 1.06 - 0.00955(123.78)$$

$$\underline{= -0.124}$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= -0.124 + 0.00955x}$$

$$\hat{y}_{182.5} = -0.124 + 0.00955(182.5)$$

$$\underline{= \$1.62}$$

Exercise

Listed below are systolic blood pressure measurements (in mm HG) obtained from the same woman.

Right Arm	102	101	94	79	79
Left Arm	175	169	182	146	144

Find the best predicted systolic blood pressure in the left arm given that the systolic blood pressure in the right arm is 100 mm Hg.

Solution

x	y	xy	x^2	y^2
102	175	17850	10404	30625
101	169	17069	10201	28561
94	182	17108	8836	33124
79	146	11534	6241	21316
79	144	11376	6241	20736
455	816	74937	41923	134362

$$\bar{x} = \frac{\sum x}{n} = \frac{455}{5} = 91.0 \qquad \bar{y} = \frac{\sum y}{n} = \frac{816}{5} = 163.2$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{5(74937) - (455)(816)}{5(41923) - (455)^2}$$

$$\underline{= 1.315}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 163.2 - 1.315(91)$$

$$\underline{= 43.56}$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= 43.6 + 1.31x}$$

$$\hat{y}_{182.5} = \bar{y} \ \underline{= 163.2 \ \ mmHg}$$

No significant correlation

Exercise

Find the best predicted height of runner-up Goldwater, given that the height of the winning presidential candidate is 75 in. Is the predicted height of Goldwater close to his actual height of 72 in.?

Winner	69.5	73	73	74	74.5	74.5	71	71
Runner-Up	72	69.5	70	68	74	74	73	76

Solution

x	y	xy	x^2	y^2
69.5	72	5004	4830.25	5184
73	69.5	5073.5	5329	4830.25
73	70	5110	5329	4900
74	68	5032	5476	4624
74.5	74	5513	5550.25	5476
74.5	74	5513	5550.25	5476
71	76	5183	5041	5329
71	76	5396	5041	5776
580.5	576.5	41824.5	42146.75	41595.25

$$\bar{x} = \frac{\sum x}{n} = \frac{580.5}{8} = 72.56 \qquad \bar{y} = \frac{\sum y}{n} = \frac{576.5}{8} = 72.06$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{8(41824.5) - (580.5)(576.5)}{8(42146.75) - (580.5)^2}$$

$$\underline{= -0.321}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 72.06 - (-0.321)(72.56)$$

$$\underline{= 95.38}$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= 95.4 - 0.321x}$$

$$\hat{y}_{182.5} = \bar{y} \ \underline{= 72.1 \ \ in.}$$

No significant correlation

Exercise

Find the best predicted amount of revenue (in millions of dollars), given that the amount has a size 87 thousand ft^2. How does the result compare to the actual revenue of \$65.1 million?

Size	160	227	140	144	161	147	141
Revenue	189	157	140	127	123	106	101

Solution

x	y	xy	x^2	y^2
160	189	30240	25600	35721
227	157	35639	51529	24649
140	140	19600	19600	19600
144	127	18288	20736	16129
161	123	19803	25921	15129
147	106	15582	21609	11236
141	101	14241	19881	10201
1120	943	153393	184876	132665

$$\bar{x} = \frac{\sum x}{n} = \frac{1120}{7} = 160.0 \qquad \bar{y} = \frac{\sum y}{n} = \frac{943}{7} = 134.71$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{7(153393) - (1120)(943)}{7(184876) - (1120)^2}$$

$$\underline{= 0.443}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 134.71 - (0.443)(160)$$

$$\underline{= 63.87}$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= 63.9 + 0.443x}$$

$$\hat{y}_{182.5} = \bar{y} \ \underline{= 134.7 \ \ million\ \$}$$ ***No significant correlation***

The predicted value is far from the actual value. Since there is no significant correlation, the mean is used for all predictions – but the x = 87 thousand ft^2 is well outside the range of x values used to construct the predictive regression equation.

Exercise

Find the best predicted new mileage rating of a jeep given that old rating is 19 mi/gal. Is the predicted value close to the actual value of 17 mi/gal?

Old	16	27	17	33	28	24	18	22	20	29	21
New	15	24	15	29	25	22	16	20	18	26	19

Solution

x	y	xy	x^2	y^2
16	15	240	256	225
27	24	648	729	576
17	16	272	289	256
33	29	957	1089	841
28	25	700	784	625
24	22	528	576	484
18	16	288	324	256
22	20	440	484	400
20	18	360	400	324
29	26	754	841	676
21	19	399	441	361
255	230	5586	6213	5024

$$\bar{x} = \frac{\sum x}{n} = \frac{255}{11} = 23.18 \qquad \bar{y} = \frac{\sum y}{n} = \frac{230}{11} = 20.82$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{11(5586) - (255)(230)}{11(6213) - (255)^2}$$

$$\underline{= 0.863}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$= 20.82 - (0.863)(23.18)$$

$$\underline{= 0.808}$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= 0.808 + 0.863x}$$

$$\hat{y}_{182.5} = 0.808 + 0.863(19) \ \underline{= 17.2 \ \ mpg}$$

Yes; the predicted value is close to the actual value of 17 mpg.

Exercise

Find the best predicted temperature for a recent year in which the concentration (in parts per million) of CO_2 is 370.9. Is the predicted temperature close to the actual temperature of 14.5° C??

CO_2	314	317	320	326	331	339	346	354	361	369
Temperature	13.9	14.0	13.9	14.1	14.0	14.3	14.1	14.5	14.5	14.4

Solution

x	y	xy	x^2	y^2
314	13.9	4364.6	985696	193.21
317	14	4438	100489	196
320	13.9	4448	102400	193.21
326	14.1	4596.6	106276	198.81
331	14	4634	109561	196
339	14.3	4847.7	114921	204.49
346	14.1	4878.6	119716	198.81
354	14.5	5133	125316	210.25
361	14.5	5234.5	130321	210.25
369	14.4	5313.6	136161	207.36
3377	141.7	47888.6	1143757	2008.39

$$\bar{x} = \frac{\sum x}{n} = \frac{3377}{10} = 337.7 \qquad \bar{y} = \frac{\sum y}{n} = \frac{141.7}{10} = 14.17$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{10(47888.6) - (3377)(141.7)}{10(1143757) - (3377)^2}$$

$$\underline{= 0.0109}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 14.17 - (0.0109)(337.7)$$

$$\underline{= 10.48}$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= 0.10.5 + 0.0109x}$$

$$\hat{y}_{182.5} = 10.5 + 0.0109(370.9)\ \underline{= 14.5\ \ °C}$$

Yes; the predicted temperature is equal to the actual temperature of 14.5 °C..

Exercise

Find the best predicted IQ score of someone with a brain size of 1275 cm^3

Brain Size	965	1029	1030	1285	1049	1077	1037	1068	1176	1105
IQ	90	85	86	102	103	97	124	125	102	114

Solution

x	***y***	***xy***	x^2	y^2
965	90	86850	931225	8100
1029	85	87465	1058841	7225
1030	86	88580	1060900	7396
1285	102	131070	1651225	10404
1049	103	108047	1100401	10609
1077	97	104469	1159929	9409
1037	124	128588	1075369	15376
1068	125	133500	1140624	15625
1176	102	119952	1382976	10404
1105	114	125970	1221025	12996
10821	1028	1114491	11782515	107544

$$\bar{x} = \frac{\sum x}{n} = \frac{10821}{10} = 1082.1 \qquad \bar{y} = \frac{\sum y}{n} = \frac{1028}{10} = 102.8$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{10(1114491) - (10821)(1028)}{10(1178251) - (10821)^2}$$

$$\underline{= 0.0286}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 102.80 - (0.0286)(1082.1)$$

$$\underline{= 71.83}$$

$$\hat{y} = b_0 + b_1 x$$

$$\underline{= 71.8 - 0.0286x}$$

$$\hat{y}_{182.5} = \bar{y} \ \underline{= 102.8}$$

No significant correlation

Exercise

Listed below are the word counts for men and women.

Male

27531	15684	5638	27997	25433	8077	21319	17572	26429	21966	11680	10818
12650	21683	19153	1411	20242	10117	20206	16874	16135	20734	7771	6792
26194	10671	13462	12474	13560	18876	13825	9274	20547	17190	10578	14821
15477	10483	19377	11767	13793	5908	18821	14069	16072	16414	19017	37649
17427	46978	25835	10302	15686	10072	6885	20848				

Female

20737	24625	5198	18712	12002	15702	11661	19624	13397	18776	15863	12549
17014	23511	6017	18338	23020	18602	16518	13770	29940	8419	17791	5596
11467	18372	13657	21420	21261	12964	33789	8709	10508	11909	29730	20981
16937	19049	20224	15872	18717	12685	17646	16255	28838	38154	25510	34869
24480	31553	18667	7059	25168	16143	14730	28117				

Find the best predicted word count of a woman given that her male partner speaks 6,000 words in a day.

Solution

Using Excel spread sheet - ***Regression***

	Coefficients
Intercept	13438.884
X Variable 1	0.302

$\hat{y} = 13439 + 0.302x$

$\hat{y}\Big|_{6000} = 13439 + 0.302(6000)$

$= 15,248 \ \textit{words per week}$

Exercise

According the least-squares property, the regression line minimizes the sum of the squares of the residuals. Listed below are the paired data consisting of the first 6 pulse and the first systolic blood pressures of males.

Pulse (x)	68	64	88	72	64	110
Systolic (y)	125	107	126	110	72	107

a) Find the equation of the regression line.

b) Identify the residuals, and find the sum of squares of the residuals.

c) Show that the equation $\hat{y} = 70 + 0.5x$ results in a larger sum of squares of residuals.

Solution

x = pulse rate

y = systolic blood pressures

a) Using Excel spread sheet ***– Data Analysis*** - ***Regression***

	Coefficients
Intercept	71.678
X Variable 1	0.5956

The equation of the regression line: $\hat{y} = 71.678 + 0.5956x$

b) $y - \hat{y} =$ residuals for the regression line

x	y	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$
68	125	112.208	12.792	163.635
64	107	109.824	-2.824	7.975
88	126	124.128	1.872	3.504
72	110	114.592	-4.592	21.086
64	110	109.824	0.176	0.031
72	107	114.592	-7.592	57.638
428	685	684.997	0.003	253.866

The table indicates that the sum of the squares of the residuals is 253.866

c) $y-v=$ residuals for the regression line where $v=70+0.5x$

x	y	v	$y-v$	$(y-v)^2$
68	125	104.000	21.000	441.000
64	107	102.000	5.000	25.000
88	126	114.000	12.000	144.000
72	110	106.000	4.000	16.000
64	110	102.000	8.000	64.000
72	107	106.000	1.000	1.000
428	685	634.0	51.0	691.0

The table indicates that the sum of the squares of the residuals is 691, which is greater the the 253.866 of the least squares regression equation.

Exercise

The scatter diagram for the data set below

x	0	2	3	5	5	5
y	7.3	5.1	6	4	5.3	3.6

Given that $\bar{x}=3.333$, $s_x=2.0655911$, $\bar{y}=5.217$, $s_y=1.3467244$, and $r=-0.8363944$, determine the least squares regression line.

Solution

$$b_1=r\cdot\frac{s_y}{s_x}=-0.8363944\frac{1.3467244}{2.0655911}\approx \underline{-.54531}$$

$$b_0=\bar{y}-b_1\bar{x}=5.217-(-.54531)(3.333)\ \underline{=7.0345}$$

$$\hat{y}=b_0+b_1x$$

$$\underline{\hat{y}=-0.5453x+7.0345}$$

Exercise

The scatter diagram for the data set below

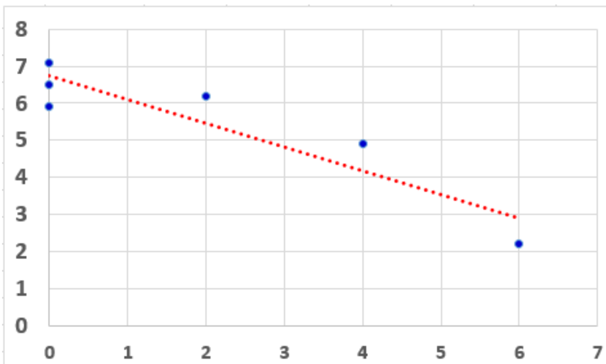
x	0	0	0	2	4	6
y	7.1	5.9	6.5	6.2	4.9	2.2

a) Determine the least squares regression line.
b) Graph the least-squares regression line on the scatter diagram

Solution

a) $\hat{y} = -0.6375x + 6.7417$ (***using excel***)

b)



Exercise

The scatter diagram for the data set below

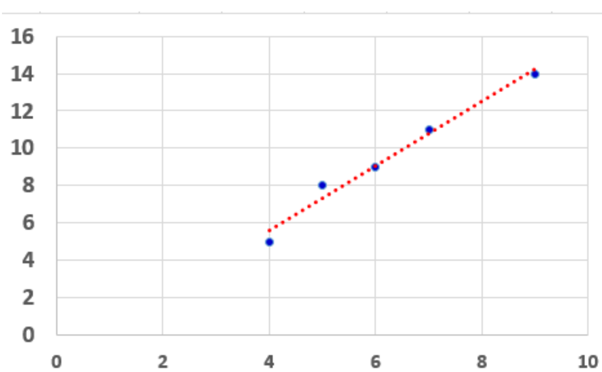
x	4	5	6	7	9
y	5	8	9	11	14

a) Determine the least squares regression line.
b) Graph the least-squares regression line on the scatter diagram.
c) Compute the sum of the squared residuals for the least-squares regression line found in part (*a*).

Solution

a) $\hat{y} = 1.73x - 1.324$

b)



c) Using excel regression

ANOVA		
	df	*SS*
Regressio	1	44.28108
Residual	3	0.918919

The sum of the squared residuals for the least-squares regression line is **0.919**.

Exercise

A student at a junior college conducted a survey of 20 randomly selected full-time students to determine the relation between the number of hours of video game playing each week, *x*, and grade-point average, *y*. She found that a linear relation exists between the two variables. The least-squares regression line that describes this relation is $\hat{y} = -0.0531x + 2.9213$.

a) Predict the grade-point average of a student who plays video games 8 hours per week.

b) Interpret the slope

c) Interpret the appropriate *y*-intercept.

d) A student who plays video games 7 hours per week has a grade-point average of 2.67. Is the student grade-point average above or below average among all students who play video games 7 hours per week.

Solution

a) $\hat{y} = -0.0531(8) + 2.9213 \approx 2.50$

b) For each additional hour that a student spends playing video games in a week, the grade-point average will decrease by 0.0531 points, on average.

c) The grade-point average of a student who does not play video games in 2.9213

d) $\hat{y} = -0.0531(7) + 2.9213 \approx 2.55$

The student's grade-point average is above average for those who play video games 7 hours per week.

Solution Section 2.3 – Probability Rules, Addition Rule and Complements

Exercise

Based on recent results, the probability of someone in the U.S. being injured while using sports or recreation equipment is $\frac{1}{500}$ (based on data from Statistical Abstract of the U.S.). What does it mean when we say that the probability is $\frac{1}{500}$? Is such an injury unusual?

Solution

The probability of being injured while using recreation equipment in $\frac{1}{500}$ means that approximately one injury occurs for every 500 times that recreation equipment is used. The probability is $\frac{1}{500}$ $\underline{= 0.002|}$ is small; such an injury is considered unusual.

Exercise

When a baby is born, there is approximately a 50–50 chance that the baby is a girl. Indicate the degree of likelihood as a probability value between 0 and 1.

Solution

$$"50-50\ \textit{chance}" = 50\% = \frac{50}{100}\ \underline{= 0.50|}$$

Exercise

When a rolling a single die, there are 6 chances in 36 that the outcome is a 7. Indicate the degree of likelihood as a probability value between 0 and 1.

Solution

$$\frac{6}{36}\ \underline{= 0.167|}$$

Exercise

Identify probability values

a) What is the probability of an event that is certain to occur?
b) What is the probability of an impossible event?
c) A sample space consists of 10 separate events that are equally likely. What is the probability of each?
d) On a true/false test, what is the probability of answering a question correctly if you make a random guess?

e) On a multiple-choice test with five possible answers for each question, what is the probability of answering a question correctly if you make a random guess?

Solution

a) If event E is certain to occur, then $P(E)=1$

b) If it is not possible for event E to occur, then $P(E)=0$

c) A sample space consists of 10 separate events that are equally likely, then

$P(\text{of any one of them})=\frac{1}{10}\ \underline{=0.10|}$

d) $P(\text{answering correctly})=\frac{1}{2}\ \underline{=0.5|}$

e) $P(\text{answering correctly})=\frac{1}{5}\ \underline{=0.2|}$

Exercise

When a couple has 3 children, find the probability of each event.

a) There is exactly one girl. *b)* There are exactly 2 girls. *c)* All are girls

Solution

$$S=\{ggg,\ ggb,\ gbg,\ bgg,\ bbg,\ bgb,\ gbb,\ bbb\}$$

a) $A=\{bbg,\ bgb,\ gbb\}$

$P(\text{exactly } 1 \text{ girl})=\frac{3}{8}\ \underline{=0.375|}$

b) $B=\{ggb,\ gbg,\ bgg\}$

$P(\text{exactly } 2 \text{ girls})=\frac{3}{8}\ \underline{=0.375|}$

c) $C=\{ggg\}$ $\qquad P(\text{All girls})=\frac{1}{8}\ \underline{=0.125|}$

Exercise

The 110th Congress of the U.S. included 84 male Senators and 16 female Senators. If one of these Senators is randomly selected, what is the probability that a woman is selected? Does this probability agree with a claim that men and women have the same chance of being elected as Senators?

Solution

Total senators: 84 + 16 = 100 senators.

$P(\text{selecting women})=\frac{16}{100}\ \underline{=0.16|}$

No; this probability is too far below 0.50 to agree with the claim that men and women have equal opportunities to become a senator.

Exercise

When Mendel conducted his famous genetics experiments with peas, one sample of offspring consisted of 428 green peas and 152 yellow peas. Based on those results, estimate the probability of getting an offspring pea that is green, Is the result reasonably close to the expected value of $\frac{3}{4}$, as claimed by Mendel?

Solution

Total plants: 428 + 152 = 580 plants.
Let G = getting an offspring pea that is green.

$$P(G) = \frac{428}{580} \underline{= 0.738|}$$

The result is very close to the $\frac{3}{4} = 0.75$ expected by Mendel

Exercise

A single fair die is rolled. Find the probability of each event

a) Getting a 2
b) Getting an odd number
c) Getting a number less than 5
d) Getting a number greater than 2
e) Getting any number except 3

Solution

a) $P = \underline{\frac{1}{6}|}$

b) $P(Odd) = \frac{3}{6} \underline{= \frac{1}{2}|}$

c) $P(<5) = \frac{4}{6} \underline{= \frac{2}{3}|}$

d) $P(>2) = \frac{4}{6} \underline{= \frac{2}{3}|}$

e) $P(no\ 3) \underline{= \frac{5}{6}|}$

Exercise

A jar contains 3 white, 4 orange, 5 yellow, and 8 black marbles. If a marble is drawn at random, find the probability that it is the following.

a) White *b)* Orange *c)* Yellow *d)* Black *e)* Not black

Solution

a) $P(white) \underline{= \frac{3}{20}|}$

b) $P(orange) = \frac{4}{20} \ \boxed{= \frac{1}{5}}$

c) $P(yellow) = \frac{5}{20} \ \boxed{= \frac{1}{4}}$

d) $P(black) = \frac{8}{20} \ \boxed{= \frac{2}{5}}$

e) $P(no\ black) = \frac{12}{20} \ \boxed{= \frac{3}{5}}$ $\qquad 1 - P(black)$

Exercise

The student sitting next to you in class concludes that the probability of the ceiling falling down on both of you before class ends is 1/2, because there are two possible outcomes - the ceiling will fall or not fall. What is wrong with this reasoning?

Solution

The outcomes are not equally likely.

Exercise

Let consider rolling 2 dice. Find the probabilities of the following events

a) E = Sum of 5 turns up

b) F = a sum that is a prime number greater than 7 turns up

Solution

1 – 1	1 – 2	1 – 3	1 – 4	1 – 5	1 – 6
2 – 1	2 – 2	2 – 3	2 – 4	2 – 5	2 – 6
3 – 1	3 – 2	3 – 3	3 – 4	3 – 5	3 – 6
4 – 1	4 – 2	4 – 3	4 – 4	4 – 5	4 – 6
5 – 1	5 – 2	5 – 3	5 – 4	5 – 5	5 – 6
6 – 1	6 – 2	6 – 3	6 – 4	6 – 5	6 – 6

a) $P(E) = \frac{4}{36} \ \boxed{= \frac{1}{9}}$

b) $P(F) = \frac{2}{36} \ \boxed{= \frac{1}{18}}$

Exercise

A poll was conducted preceding an election to determine the relationship between voter persuasion concerning a controversial issue and the area of the city in which the voter lives. Five hundred registered voters were interviewed from three areas of the city. The data are shown below. Compute the probability of having no opinion on the issue or living in the inner city.

Area of city	***Favor***	***Oppose***	***No Opinion***
East	30	40	55
North	25	45	50
Inner	95	65	85

Solution

$$\mathrm{P}(event) = \frac{Total\ Inner + No\ Opinion\ East + No\ Opinion\ North}{500}$$

$$= \frac{95 + 65 + 85 + 55 + 50}{500}$$

$$= \frac{350}{500}$$

$$\boxed{\approx 0.7}$$

Exercise

Suppose a single fair die is rolled. Use the sample space $S = \{1, 2, 3, 4, 5, 6\}$ and give the probability of each event.

a) E: the die shows an even number
b) F: the die show a number less than 10
c) G: the die shows an 8

Solution

a) Even number: $E = \{2, 4, 6\}$

$$P(E) = \frac{n(S)}{n(E)}$$

$$= \frac{3}{6}$$

$$\boxed{= \frac{1}{2}}$$

b) Number less than 10

$F = \{1, 2, 3, 4, 5, 6\}$

$P(F) = \frac{6}{6} \ \boxed{= 1}$

c) Die shows an 8

$\boxed{P(G) = 0}$ ***Impossible***

Exercise

A solitaire game was played 500 times. Among the 500 trials, the game was won 77 times. (The results are from the solitaire game, and the Vegas rules of "draw 3" with \$52 bet and a return of \$5 per card are used). Based on these results, find the odds against winning.

Solution

Odd against winning are: $\dfrac{P(\text{not winning})}{P(\text{winning})} = \dfrac{\frac{423}{500}}{\frac{77}{500}} = \dfrac{423}{77}$ *or* $423:77$

Which is approximately 5.5:1 ***or*** 11:2

Exercise

A roulette wheel has 38 slots. One slot is 0, another is 00, and the others are numbered 1 through 36, respectively. You place a bet that the outcome is an odd number.

a) What is your probability of winning?

b) What are the actual odds against winning?

c) When you bet that the outcome is an odd number, the payoff odds are 1:1. How much profit do you make if you bet $18 and win?

d) How much profit would you make on the $18 bet if you could somehow convince the casino to change its payoff odds so that they are the same as the actual odds against winning?

Solution

Out of 38 slots, 18 numbers are odd.

Let W = outcome is an odd number occurs

a) $P(W)=\frac{18}{38}\ \underline{=0.474}$

b) Odds against: $W=\dfrac{P(\text{not } W)}{P(W)}=\dfrac{\frac{20}{38}}{\frac{18}{38}}=\dfrac{20}{18}=\dfrac{10}{9}$ *or* $10:9$

c) If you payoff odds are 1:1, if you bet $18 and win, you get back 18 + 18 = $36 and your profit is $18.

d) If you payoff odds are 10:9 (odds against), a win get back your bet $10 for every $9 bet. If you bet $18 and win, you get back 18 + 2(10) = $38 and your profit is $20.

Exercise

Women have a 0.25% rate of red/green color blindness. If a woman is randomly selected, what is the probability that she does not have red/green color blindness?

Solution

Let W = a selected woman has red/green color blindness.

$P(W)=0.25\%\ \underline{=0.0025}$

$$P(\text{does not } W)=P(\overline{W})$$

$$=1-0.0025$$

$$\underline{=0.9975}$$

Exercise

A pew Research center poll showed that 79% of Americans believe that it is morally wrong to not report all income on tax returns. What is the probability that an American does not have that belief?

Solution

Let A = a American believes is morally wrong to not report all income.

$$P(A) = 0.79$$

$$\begin{aligned} P(\text{does not } A) &= P(\bar{A}) \\ &= 1 - 0.79 \\ &= \underline{0.21} \end{aligned}$$

Exercise

When the author observed a sobriety checkpoint conducted by the Dutchess County Sheriff Department, he saw that 676 drivers were screened and 6 were arrested for driving while intoxicated. Based on those results, we can estimate that $P(I) = 0.00888$, where *I* denotes the event of screening a driver and getting someone who is intoxicated. What does $P(\bar{I})$ denote and what is its value?

Solution

$P(\bar{I})$ is the probability that a screened driver is not intoxicated

$$\begin{aligned} P(\bar{I}) &= 1 - P(I) \\ &= 1 - 0.00888 \\ &= \underline{0.99112} \end{aligned}$$

Exercise

Use the polygraph test data

	No (Did Not Lie)	***Yes*** (Lied)
Positive test result	15 (*false positive*)	42 (*true positive*)
Negative test result	32 (*true negative*)	9 (*false negative*)

a) If one of the test subjects is randomly selected, find the probability that the subject had a positive test result or did not lie

b) If one of the test subjects is randomly selected, find the probability that the subject did not lie

c) If one of the test subjects is randomly selected, find the probability that the subject had a true negative test result

d) If one of the test subjects is randomly selected, find the probability that the subject had a negative test result or lied.

Solution

From the table:

$$P(\text{Positive}) = \frac{57}{98} \quad \rightarrow P(\bar{P}) = \frac{41}{98}$$

$$P(\text{Lie}) = \frac{51}{98} \quad \rightarrow P(\bar{L}) = \frac{47}{98}$$

a) $P(P \text{ or } \bar{L}) = P(P) + P(\bar{L}) - P(P \text{ and } \bar{L})$

$$= \frac{57}{98} + \frac{47}{98} - \frac{15}{98}$$

$$= \frac{89}{98}$$

$$\underline{= 0.908|}$$

b) $P(\bar{L}) = \frac{47}{98}\ \underline{= 0.480|}$

c) $P(\bar{L} \text{ and } \bar{P}) = \frac{32}{98}\ \underline{= 0.327|}$

d) $P(\bar{P} \text{ or } L) = P(\bar{P}) + P(L) - P(\bar{P} \text{ and } L)$

$$= \frac{41}{98} + \frac{51}{98} - \frac{9}{98}$$

$$= \frac{83}{98}$$

$$\underline{= 0.847|}$$

Exercise

Use the data

Was the challenge to the call successful?		
	Yes	***No***
Men	201	288
Women	126	224

a) If *S* denotes the event of selecting a successful challenge, find $P(\bar{S})$

b) If *M* denotes the event of selecting a challenge made by a man, find $P(\bar{M})$

c) Find the probability that the selected challenge was made by a man or was successful.

d) Find the probability that the selected challenge was made by a woman or was successful.

e) Find $P(\textit{challenge was made by a man or was not successful})$

f) Find $P(\textit{challenge was made by a woman or was not successful})$

Solution

Total people = 201 + 126 + 288 + 224 = 839

a) $P(\bar{S}) = \frac{288+224}{839} = \frac{512}{839}\ \underline{= 0.610|}$

b) $P(\bar{M}) = \frac{126+224}{839} = \frac{350}{839}\ \underline{= 0.417|}$

c) $P(M \text{ or } S) = P(M) + P(S) - P(M \text{ and } S)$

$$= \frac{489}{839} + \frac{327}{839} - \frac{201}{839}$$

$$= 0.733$$

d) $P(\bar{M} \text{ or } S) = P(\bar{M}) + P(S) - P(\bar{M} \text{ and } S)$

$$= \frac{350}{839} + \frac{327}{839} - \frac{126}{839}$$

$$= 0.657$$

e) $P(M \text{ or } \bar{S}) = P(M) + P(\bar{S}) - P(M \text{ and } \bar{S})$

$$= \frac{489}{839} + \frac{512}{839} - \frac{288}{839}$$

$$= 0.850$$

f) $P(\bar{M} \text{ or } \bar{S}) = P(\bar{M}) + P(\bar{S}) - P(\bar{M} \text{ and } \bar{S})$

$$= \frac{350}{839} + \frac{512}{839} - \frac{224}{839}$$

$$= 0.760$$

Exercise

Refer to the table below

Age

	18 – 21	22 – 29	30 – 39	40 – 49	50 – 59	60 and over
Responded	73	255	245	136	138	202
Refused	11	20	33	16	27	49

a) What is the probability that the selected person refused to answer? Does that probability value suggest that refusals are a problem for pollsters? Why or why not?

b) A pharmaceutical company is interested in opinions of the elderly, because they are either receiving Medicare or will receive it soon. What is the probability that the selected subject is someone 60 and over who responded?

c) What is the probability that the selected person responded or is in the 18–21 age bracket?

d) What is the probability that the selected person refused or is over 59 years of age?

e) A market researcher is interested in responses, especially from those between the ages of 22 and 39, because they are the people more likely to make purchases. Find the probability that a selected responds or is aged between the ages of 22 and 39.

f) A market researcher is not interested in refusals or subjects below 22 years of age or over 59. Find the probability that the selected person refused to answer or is below 22 or is older than 59.

Solution

Let: Y = there is a response =73+255+245+136+138+202=1049 $\quad$ N = there is a refusal =11+20+33+16+27+49=156

A = age is 18 – 21 $\quad$ B = age is 22 – 29 $\quad$ C = age is 30 – 39

D = age is 40 – 49 $\quad$ E = age is 50 – 59 $\quad$ F = age is 60+

a) $P(N)=\frac{156}{1205}\ \underline{=0.129}$

b) $P(F\ and\ Y)=\frac{202}{1205}\ \underline{=0.168}$

c) $P(Y\ or\ A)=P(Y)+P(A)-P(Y\ and\ S)$

$$=\frac{1049}{1205}+\frac{84}{1205}-\frac{73}{1205}$$

$$\underline{=0.880}$$

d) $P(N\ or\ F)=P(N)+P(F)-P(N\ and\ F)$

$$=\frac{156}{1205}+\frac{251}{1205}-\frac{49}{1205}\quad=\frac{358}{1205}$$

$$\underline{=0.297}$$

e) $P(responds\ or\ the\ ages\ between\ 22\ and\ 39)=P(Y\,or\ B\ or\ C)$

Use the intuitive approach rather than the formal addition rule

$P(Y\,or\ B\ or\ C)=P(Y)+P(B\ and\ N)+P(C\ and\ N)$

$$=\frac{1049}{1205}+\frac{20}{1205}+\frac{33}{1205}\quad=\frac{1102}{1205}$$

$$\underline{=0.915}$$

f) Use the intuitive approach rather than the formal addition rule

$P(N\,or\ A\,or\ F)=P(N)+P(A\,and\ Y)+P(F\ and\ Y)$

$$=\frac{156}{1205}+\frac{73}{1205}+\frac{202}{1205}\quad=\frac{431}{1205}$$

$$\underline{=0.368}$$

Exercise

Two dice are rolled. Find the probabilities of the following events.

a) The first die is 3 or the sum is 8 *b)* The second die is 5 or the sum is 10.

Solution

a) $P(3\ or\ sum\ is\ 8)=P(3)+P(sum\ 8)-P(3\ and\ sum\ 8)$

$$=\frac{6}{36}+\frac{5}{36}-\frac{1}{36}\quad=\frac{10}{36}$$

$$\underline{=\frac{5}{18}}$$

b) $P(5\ or\ sum\ 10)=P(5)+P(sum\ 10)-P(5\ and\ sum\ 10)$

$$=\frac{6}{36}+\frac{3}{36}-\frac{1}{36}\quad=\frac{8}{36}$$

$$\underline{=\frac{2}{9}}$$

Exercise

One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) A 9 or 10
b) A red card or a 3
c) A 9 or a black 10
d) A heart or a black card
e) A face card or a diamond

Solution

a) $P(9\ or\ 10)=\frac{8}{52}\ =\frac{2}{13}$

b) $P(red\ \ or\ \ 3)=\frac{28}{52}\ =\frac{7}{13}$

c) $P(9\ or\ black\text{-}10)=\frac{6}{52}\ =\frac{3}{26}$

d) $P(heart\ \ or\ \ black)=\frac{39}{52}\ =\frac{3}{4}$

e) $P(\ face\ \ or\ \ diamond)=\frac{22}{52}\ =\frac{11}{26}$

Exercise

One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) Less than a 4 (count aces as ones)
b) A diamond or a 7
c) A black card or an ace
d) A heart or a jack
e) A red card or a face card

Solution

a) $P(<4)=P\left(ace,2,3\right)=\frac{12}{52}\ =\frac{3}{13}$

b) $P(diamond\ \ or\ \ 7)=\frac{16}{52}\ =\frac{4}{13}$

c) $P(black\ \ or\ \ ace)=\frac{28}{52}\ =\frac{7}{13}$

d) $P(heart\ \ or\ \ jack)=\frac{16}{52}\ =\frac{4}{13}$

e) $P(red\ \ or\ \ face)=\frac{32}{52}\ =\frac{8}{13}$

Exercise

Pam invites relatives to a party: her mother, 2 aunts, 3 uncles, 2 brothers, 1 male cousin, and 4 female cousins. If the chances of any one guest first equally likely, find the probabilities that the first guest to arrive is as follows.

a) A brother or an uncle
b) A brother or a cousin
c) A brother or her mother
d) An uncle or a cousin
e) A male or a cousin
f) A female or a cousin

Solution

a) $P(brother\ or\ uncle) = \boxed{\frac{5}{13}}$

b) $P(brother\ or\ cousin) = \boxed{\frac{7}{13}}$

c) $P(brother\ or\ mother) = \boxed{\frac{3}{13}}$

d) $P(uncle\ or\ cousin) = \boxed{\frac{8}{13}}$

e) $P(male\ or\ cousin) = \boxed{\frac{10}{13}}$

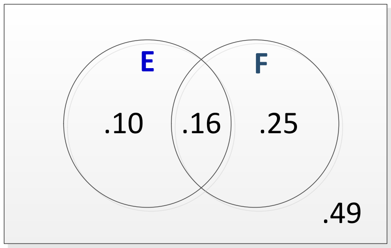
f) $P(brother\ or\ cousin) = \boxed{\frac{8}{13}}$

Exercise

Suppose $P(E) = 0.26$, $P(F) = 0.41$, and $P(E \cap F) = 0.16$. Find the following

a) $P(E \cup F)$
b) $P(E' \cap F)$
c) $P(E \cap F')$
d) $P(E' \cup F')$

Solution



a) $P(E \cup F) = .1 + .16 + .25 = .51$

b) $P(E' \cap F) = .25$

c) $P(E \cap F') = .10$

d) $P(E' \cup F') = .74 + .59 - .49 = .84$

Exercise

Suppose $P(E)=0.42$, $P(F)=0.35$, and $P(E\cup F)=0.59$. Find the following

a) $P(E'\cap F')$

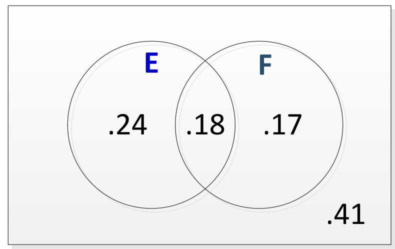
b) $P(E'\cup F')$

c) $P(E'\cup F)$

d) $P(E\cap F')$

Solution

$$P(E\cap F)=P(E)+P(F)-P(E\cup F)$$
$$=.42+.35-.59$$
$$=.18$$



a) $P(E'\cap F')\ \underline{=.41}$

b) $P(E'\cup F')=1-.18\ \underline{=.82}$

c) $P(E'\cup F)=.17+.41+.18\ \underline{=.76}$

d) $P(E\cap F')\ \underline{=.24}$

Exercise

From survey involving 1,000 people in the certain city, it was found that 500 people had tried a certain brand of diet cola, 600 had tried a certain brand of regular cola, and 200 had tried both types of cola. If a resident of the city is selected at random, what is the empirical probability that

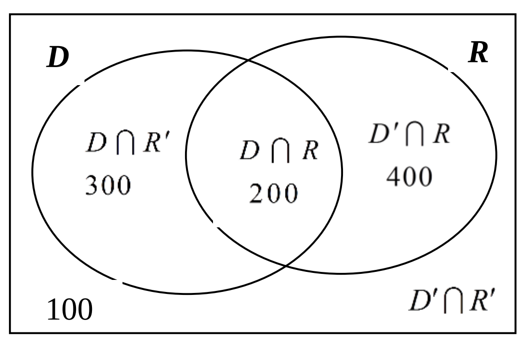
a) The resident has not tried either cola? What are the empirical odds for this event?

b) The resident has tried the diet or has not tried the regular cola? What are the empirical odds against this event?

Solution

a) $n(S) = 1000 \qquad D\cap R=200$

$D\cap R'=300 \qquad D'\cap R=400$



$$P(\text{neither D or } R)=P(D'\cap R')$$
$$=\frac{100}{1000}$$
$$\underline{=.1}$$

$$P(E')=1-.1=0.9$$

Odds for $\boldsymbol{E}$: $\dfrac{P(E)}{P(E')}=\dfrac{.1}{.9}=\dfrac{1}{9}$ *or* $\boxed{1:9}$

b) $P(E)=P(D\cup R')$

$$=P(D)+P(R')-P(D\cap R')$$

$$= \frac{500}{1000} + \frac{400}{1000} - \frac{300}{1000}$$

$$\underline{= .6|}$$

$$\Rightarrow P(E') = 1 - .6 = .4$$

Against Odds for P(E): $\frac{P(E)}{P(E')} = \frac{.4}{.6} = \frac{2}{3} \quad or \quad \boxed{2:3}$

Exercise

In a poll, respondents were asked whether they had ever been in a car accident. 329 respondents indicated that they had been in a car accident and 322 respondents said that they had not been in a car accident. If one of these respondents is randomly selected, what is the probability of getting someone who has been in a car accident?

Solution

$$\frac{329}{329+322} \underline{= 0.505|}$$

Exercise

Refer to the table which summarizes the results of testing for a certain disease

	Positive Test Result	***Negative Test Result***
Subject has the disease	114	5
Subject does not have the disease	12	177

If one of the results is randomly selected, what is the probability that it is a false negative (test indicates the person does not have the disease when in fact they do)?. What is the probability suggest about the accuracy of the test?

Solution

Person does not have the disease when in fact they do $\Rightarrow$ Person has the disease & Negative

$$P = \frac{5}{114+5+12+177} \underline{= 0.016|}$$

The probability of this error is low so the test is fairly accurate.

Exercise

In a certain town, 2% of people commute to work by bicycle. If a person is selected randomly from the town, what are the odds against selecting someone who commutes by bicycle?

Solution

$$P(Event) = 2\% = 0.02; \quad P\left(\bar{E}\right) = .98$$

$$\frac{P(\bar{E})}{P(E)} = \frac{.98}{.02} = 49$$

$odds\ Against \quad \boxed{49:1}$

Exercise

Suppose you are playing a game of chance, if you bet \$4 on a certain event, you will collect \$176 (including your \$4 bet) if you win. Find the odds used for determining the payoff.

Solution

The amount that you win: $P(E) = 176 - 4 = 172$

You loose: $P(\bar{E}) = 4$

$$\frac{P(E)}{P(\bar{E})} = \frac{172}{4} = 43$$

$odds \quad \boxed{43:1}$

Exercise

The odds in favor of a particular horse winning a race are 4:5.

a) Find the probability of the horse winning.

b) Find the odds against the horse winning.

Solution

a) $P(E) = \dfrac{a}{a+b} = \dfrac{4}{4+5} \boxed{= \dfrac{4}{9}}$

b) The odds against the horse winning $\boxed{5:4}$

Exercise

Consider the sample space of equally likely events for the rolling of a single fair die.

a) What is the probability of rolling an odd number **and** a prime number?

b) What is the probability of rolling an odd number **or** a prime number?

Solution

a) $odd = \{1,\ 3,\ 5\} \quad prime = \{3,\ 5\}$

$$P(odd \cap prime) = \frac{n(A \cap B)}{n(S)} = \frac{2}{6} \boxed{= \frac{1}{3}}$$

b) $P(odd \cup prime) = \dfrac{n(A \cup B)}{n(S)} = \dfrac{4}{6} \boxed{= \dfrac{2}{3}}$

Exercise

Suppose that 2 fair Dice are rolled

a) What is the probability of that a sum of 2 or 3 turns up?

b) What is the probability of that both dice turn up the same or that a sum greater than 8 turns up?

Solution

a)
$$\begin{aligned} P\left(\sum=2 \text{ or } 3 \text{ turns up}\right) &= P\left(\sum=2\right)+P\left(\sum=3\right) \\ &= \frac{1}{36}+\frac{2}{36} \\ &= \frac{3}{36} \\ &= \boxed{\frac{1}{12}} \end{aligned}$$

b)
$$\begin{aligned} P\left(\text{same or } \sum>8\right) &= P\left(\text{same } \cup \sum>8\right) \\ &= P(\text{same})+P\left(\sum>8\right)-P\left(\text{same } \cap \sum>8\right) \\ &= \frac{6}{36}+\frac{10}{36}-\frac{2}{36} \\ &= \boxed{\frac{7}{18}} \end{aligned}$$

Exercise

A single card is drawn from an ordinary of 52 cards. Calculate the probabilities of and odds for each event

a) A face card or a club is drawn

b) A king or a heart is drawn

c) A black card or an ace is drawn

d) A heart or a number less than 7 (count an ace as 1) is drawn.

Solution

a)
$$\begin{aligned} \Pr\left(\text{Face or Club}\right) &= \Pr\left(F \cup C\right) \\ &= P\left(F\right)+P\left(C\right)-P\left(F \cap C\right) \\ &= \frac{12}{52}+\frac{13}{52}-\frac{3}{52} \\ &= \boxed{\frac{11}{26}} \end{aligned}$$

$$P\left[\left(F \cup C\right)'\right] = 1-\frac{11}{26} = \boxed{\frac{15}{26}}$$

$$\text{Odds for } F \cup C = \frac{\frac{11}{26}}{\frac{15}{26}} = \boxed{\frac{11}{15}} \quad \boxed{11:15}$$

b) $\Pr\left(\text{King or Heart}\right) = \text{P}\left(K \cup H\right)$

$$
\begin{aligned}
&= P(K) + P(H) - P(K \cap H) \\
&= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\
&= \frac{16}{52} \\
&\boxed{= \frac{4}{13}}
\end{aligned}
$$

$$P\left[(K \cup H)'\right] = 1 - \frac{4}{13} \boxed{= \frac{9}{13}}$$

$$\text{Odds for } K \cup H = \frac{\frac{4}{13}}{\frac{9}{13}} \boxed{= \frac{4}{9}} \quad \boxed{4:9}$$

c) $\Pr(\textit{Black card or Ace}) = \mathrm{P}(B \cup A)$

$$
\begin{aligned}
&= P(B) + P(A) - P(B \cap A) \\
&= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} \\
&= \frac{28}{52} \\
&\boxed{= \frac{7}{13}}
\end{aligned}
$$

$$P\left[(B \cup A)'\right] = 1 - \frac{7}{13} \boxed{= \frac{6}{13}}$$

$$\text{Odds for } B \cup A = \frac{\frac{7}{13}}{\frac{6}{13}} \boxed{= \frac{7}{6}} \quad \boxed{7:6}$$

d) $\Pr(\textit{Heart or } < 7) = \mathrm{P}(H \cup < 7)$

$$
\begin{aligned}
&= P(B) + P(A) - P(B \cap A) \\
&= \frac{13}{52} + \frac{6*4}{52} - \frac{6}{52} \\
&\boxed{= \frac{31}{52}}
\end{aligned}
$$

$$P\left[(H \cup \# < 7)'\right] = 1 - \frac{31}{52} \boxed{= \frac{21}{52}}$$

$$\text{Odds for } B \cup A = \frac{\frac{31}{52}}{\frac{21}{52}} \boxed{= \frac{31}{21}} \quad \boxed{31:21}$$

Exercise

What is the probability of getting at least 1 black card in a 7-card hand dealt from a standard 52-card deck?

Solution

There are 26 black cards.

Let A = "at least 1 black card in a 7-card hand dealt"

A' = "0 black cards in a 7-card hand dealt"

$$n(A') = C_{26,7}$$

$$n(S) = C_{52,7}$$

$$\begin{aligned} P(A) &= 1 - P(A') \\ &= 1 - \frac{n(A')}{n(S)} \\ &= 1 - \frac{C_{26,7}}{C_{52,7}} \\ &= 1 - .005 \\ &= .995 \end{aligned}$$

Exercise

What is the probability that a number selected at random from the first 600 positive integers is (exactly) divisible by 6 or 9?

Solution

Let A = "Number divisible by 6"

B = "Number divisible by 9"

A number divisible by 6 $\Rightarrow n(A) = \frac{600}{6} = 100$

A number divisible by 9 $\Rightarrow n(B) = \frac{600}{9} \approx 66$

A number divisible by 6 and by 9 $\rightarrow 18k \quad \Rightarrow n(A \cap B) = \frac{600}{18} \approx 33$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{100}{600} + \frac{66}{600} - \frac{33}{600} \\ &= \frac{133}{600} \\ &\approx 0.2217 \end{aligned}$$

Exercise

What is the probability that a number selected at random from the first 1,000 positive integers is (exactly) divisible by 6 or 8?

Solution

Let A = ″Number divisible by 6″

B = ″Number divisible by 8″

A number divisible by 6 $\Rightarrow n(A) = \frac{1{,}000}{6} = 166$

A number divisible by 8 $\Rightarrow n(B) = \frac{1{,}000}{8} \approx 125$

A number divisible by 6 and by 8 $\rightarrow 24k \quad \Rightarrow n(A \cap B) = \frac{1{,}000}{24} \approx 41$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{166}{1000} + \frac{125}{1000} - \frac{41}{1000}$$

$$= \frac{250}{1000}$$

$$\approx 0.25$$

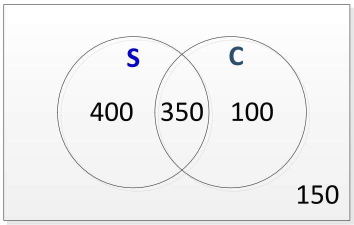
Exercise

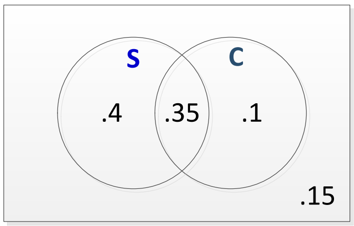
From a survey involving 1,000 students at a large university, a market research company found that 750 students owned stereos, 450 owned cars, and 350 owned cars and stereos. If a student at the university is selected at random, what is the (empirical) probability that

a) The student owns either a car or a stereo?

b) The student owns neither a car nor a stereo?

Solution





Let S = ″Number of stereos ″

C = ″Number of cars″

a) $P(S \cup C) = P(S) + P(C) - P(S \cap C)$

$$= \frac{750}{1000} + \frac{450}{1000} - \frac{350}{1000}$$

$$= \frac{850}{1000}$$

$$\approx 0.85$$

b) $P(S' \cap C') = .15$

Exercise

In order to test a new car, an automobile manufacturer wants to select 4 employees to test drive the car for 1 year. If 12 management and 8 union employees volunteer to be test drivers and the selection is made at random, what is the probability that at least 1 union employee is selected.

Solution

Let A = "at least 1 union employee is selected"

A' = "no union employee is selected"

$$\Rightarrow n(A') = C_{12,4}, \quad n(S) = C_{20,4}$$

$$P(A) = 1 - P(A')$$

$$= 1 - \frac{C_{12,4}}{C_{20,4}}$$

$$\approx 0.90$$

Exercise

A shipment of 60 inexpensive digital watches, including 9 that are defective, is sent to a department store. The receiving department selects 10 at random for testing and rejects the whole shipment if 1 or more in the sample are found defective. What is the probability that the shipment will be rejected?

Solution

The sample space: $S = C_{60,10}$

Let E = "Event that contains at least 1 defective watch".

E' = "Event that contains no defective watches".

$$n(E') = C_{51,10}$$

Probability that the shipment will be rejected:

$$P(E) = 1 - P(E')$$

$$= 1 - \frac{n(E')}{n(S)}$$

$$= 1 - \frac{C_{51,10}}{C_{60,10}}$$

$$= .83$$

Exercise

If you bet $5 on the number 13 in roulette, your probability of winning is $\frac{1}{38}$ and the payoff odds are given by the casino as 35:1.

a) Find the actual odds against the outcome of 13.

b) How much net profit would you make if you win by betting on 13?

c) If the casino was not operating for profit, and the payoff odds were changed to match the actual odds against 13, how much would you win if the outcome were 13?

Solution

a) With odds: $P(13)=\frac{1}{38}$ and $P(\text{not } 13)=\frac{37}{38}$

Actual odds against 13 $\frac{P(\text{not } 13)}{P(13)}=\frac{37/38}{1/38}=\frac{37}{1}$ or 37:1

b) Because the payoffs odds against 13 are 35:1, we have:

35:1 = (*net profit*) : (*amount bet*)

So there is a $35 profit for each $1 bet. For $5 bet, the net profit is $5\times 35=\$175$.
The winning bettor would collect $175 plus the original $5 bet. That is, the total amount collected would be $180, for the net profit of $175.

c) If the casino were not operating for profit, the payoff odds would be equal to the actual odds against the outcome of 13, or 37:1. So there is a net profit of $37 for each $1 bet.
For a $5 bet the net profit would be $185. (The casino makes its profit by paying only $175 instead of the $185 that would be paid with a roulette game that is fair instead of favoring the casino.)

Solution Section 2.4 – Multiplication Rule and Conditional

Exercise

Use the data below:

	No (Did Not Lie)	***Yes*** (Lied)
Positive test result	15 (*false positive*)	42 (*true positive*)
Negative test result	32 (*true negative*)	9 (*false negative*)

a) If 2 of the 98 test subjects are randomly selected without replacement find the probability that they both had false positive results. Is it unusual to randomly select 2 subjects without replacement and get 2 results that are both false positive results? Explain.

b) If 3 of the 98 test subjects are randomly selected without replacement, find the probability that all had false positive results. Is it unusual to randomly select 3 subjects without replacement and get 3 results that are all false positive results? Explain.

c) If 4 of the test subjects are randomly selected without replacement find the probability that, in each case, the polygraph indicated that the subject lied. Is such an event unusual?

d) If 4 of the test subjects are randomly selected without replacement find the probability that they all had incorrect test result (either false positive or false negative). Is such an event Likely?

e) Assume that 1 of the 98 test subjects is randomly selected. Find the probability of selecting a subject with a negative test result, given that the subject lied. What does this result suggest about the polygraph test?

f) Find $P(\textit{negative test result} \mid \textit{subject did not lie})$

g) Find $P(\textit{subject did not lie} \mid \textit{negative test result})$

Solution

	$\bar{L}$	L	
P	15	42	57
N	32	9	41
	47	51	**98**

Let F = selected person had false positive results

Let P = selected person tested positive

a) $P(F_1 \text{ and } F_2) = P(F_1) \cdot P(F_2 | F_1)$

$$= \frac{15}{98} \cdot \frac{14}{97}$$

$$= 0.0221$$

Yes; since $0.0221 \le 0.05$, getting 2 subjects who had false positives would be unusual.

b) $P(F_1 \text{ and } F_2 \text{ and } F_3) = P(F_1) \cdot P(F_2 | F_1) \cdot P(F_3 | F_1 \text{ and } F_2)$

$$= \frac{15}{98} \cdot \frac{14}{97} \cdot \frac{13}{96}$$

$$= 0.00299$$

Yes; since $0.00229 \le 0.05$, getting 3 subjects who had false positives would be unusual.

c) $P\left(P_1 \text{ and } P_2 \text{ and } P_3 \text{ and } P_4\right) = P\left(P_1\right)\cdot P\left(P_2|P_1\right)\cdot P\left(P_3|P_1 \text{ and } P_2\right)\cdot P\left(P_4|P_1 \text{ and } P_2 \text{ and } P_3\right)$

$$= \frac{57}{98}\cdot\frac{56}{97}\cdot\frac{55}{96}\cdot\frac{54}{95}$$

$$= 0.109$$

No; since 0.109 > 0.05, getting 4 subjects who had false positives would not be unusual.

d) Let I = selected person had incorrect results.

$$P\left(I_1\right) = \frac{15}{98} + \frac{9}{98} = \frac{24}{98}$$

$$P\left(I_1 \text{ and } I_2 \text{ and } I_3 \text{ and } I_4\right) = P\left(I_1\right)\cdot P\left(I_2|I_1\right)\cdot P\left(I_3|I_1 \text{ and } I_2\right)\cdot P\left(I_4|I_1 \text{ and } I_2 \text{ and } I_3\right)$$

$$= \frac{24}{98}\cdot\frac{23}{97}\cdot\frac{22}{96}\cdot\frac{21}{95}$$

$$= 0.00294$$

Yes; since 0.00294 ≤ 0.05, getting 4 subjects who had incorrect results would be unusual.

e) $P\left(\overline{P}|Y\right) = \frac{9}{51}$ $= 0.176$

This result suggests that the polygraph is not very reliable because 17.6% of the time it fails to catch a person who really is lying.

f) $P\left(\text{negative test result}|\text{subject did not lie}\right) = P\left(\overline{P}|\overline{Y}\right) = \frac{32}{47}$ $= 0.681$

g) $P\left(\text{subject did not lie}|\text{negative test result}\right) = P\left(\overline{Y}|\overline{P}\right) = \frac{32}{41}$ $= 0.780$

Exercise

Use the data in the table below

	Group			
Type	***O***	***A***	***B***	***AB***
Rh^+	39	35	8	4
Rh^-	6	5	2	1

a) If 2 of the 100 subjects are randomly selected, find the probability that they are both group O and type Rh^+

i. Assume that the selections are made with replacement.

ii. Assume that the selections are made without replacement.

b) If 3 of the 100 subjects are randomly selected, find the probability that they are both group B and type Rh^-

i. Assume that the selections are made with replacement.

ii. Assume that the selections are made without replacement.

c) People with blood that is group O and type Rh^{-} are considered to be universal donors, because they can give blood to anyone. If 4 of the 100 subjects are randomly selected, find the probability that they are all universal donors.

i. Assume that the selections are made with replacement.

ii. Assume that the selections are made without replacement.

d) People with blood that is group AB and type Rh^{+} are considered to be universal donors, because they can give blood to anyone. If 3 of the 100 subjects are randomly selected, find the probability that they are all universal recipients.

i. Assume that the selections are made with replacement.

ii. Assume that the selections are made without replacement.

Solution

a) $P(O\ and\ Rh+) = \frac{39}{100}$

i. $P(With\ replacement) = \frac{39}{100} \cdot \frac{39}{100} \ \underline{= 0.152}|$

ii. $P(Without\ replacement) = \frac{39}{100} \cdot \frac{38}{99} \ \underline{= 0.150}|$

b) $P(B\ and\ Rh-) = \frac{2}{100}$

iii. $P(With\ replacement) = \frac{2}{100} \cdot \frac{2}{100} \cdot \frac{2}{100} \ \underline{= 0.000008}|$

iv. $P(Without\ replacement) = \frac{39}{100} \cdot \frac{1}{99} \cdot \frac{0}{98} \ \underline{= 0}|$

c) $P(O\ and\ Rh-) = \frac{6}{100}$

v. $P(With\ replacement) = \frac{6}{100} \cdot \frac{6}{100} \cdot \frac{6}{100} \cdot \frac{6}{100} \ \underline{= 0.0000130}|$

vi. $P(Without\ replacement) = \frac{6}{100} \cdot \frac{5}{99} \cdot \frac{4}{98} \cdot \frac{3}{97} \underline{= 0.00000383}|$

d) $P(AB\ and\ Rh+) = \frac{4}{100}$

vii. $P(With\ replacement) = \frac{4}{100} \cdot \frac{4}{100} \cdot \frac{4}{100} \ \underline{= 0.000064}|$

viii. $P(Without\ replacement) = \frac{4}{100} \cdot \frac{3}{99} \cdot \frac{2}{98} \ \underline{= 0.0000247}|$

Exercise

With one method of a procedure called *acceptance sampling*, a sample of items is randomly selected without replacement and the entire batch is accepted if every item in the sample is okay. The Telektronics Company manufactured a batch of 400 backup power supply units for computers, and 8 of them are defective. If 3 of the units are randomly selected for testing, what is the probability that the entire batch will be accepted?

Solution

Let P = power supply unit is OK.

$$\begin{aligned} P(\text{entire batch is accepted}) &= P\left(P_1 \text{ and } P_2 \text{ and } P_3\right) \\ &= P\left(P_1\right)\cdot P\left(P_2 | P_1\right)\cdot P\left(P_3 | P_1 \text{ and } P_2\right) \\ &= \frac{392}{400}\cdot\frac{391}{399}\cdot\frac{390}{398} \\ &= 0.941 \end{aligned}$$

Exercise

It is common for public opinion polls to have a "confidence level" of 95% meaning that there is a 095 probability that the poll results are accurate within the claimed margins of error. If each of the following organizations conducts an independent poll, find the probability that all of them are accurate within the claim margins of error: Gallup, Roper, Yankelovich, Harris, CNN, ABC, CBS, and NBC, New York Times. Does the result suggest that with a confidence level of 95%, we can expect that almost all polls will be within the claimed margin of error?

Solution

Let A = public opinion poll is accurate within its margin of error.

Each polling organization: $P(A) = 0.95$

$$\begin{aligned} P(\text{all } 9 \text{ are accurate}) &= P\left(A_1 \text{ and } A_2 \text{ and } \cdots \text{ and } A_9\right) \\ &= P\left(A_1\right)\cdot P\left(A_2\right)\cdots P\left(A_9\right) \\ &= (.95)^9 \\ &= 0.630 \end{aligned}$$

No, with 9 independent polls the probability that at least one of them is not accurate within its margin of error is

$$\begin{aligned} P(\text{all accurate}) &= 1 - 0.630 \\ &= 0.370 \end{aligned}$$

Exercise

The principle of redundancy is used when system reliability is improved through redundant or back up components. Assume that your alarm clock has a 0.9 probability of working on any given morning.

a) What is the probability that your alarm clock will not work on the morning of an important final exam?

b) If you have 2 such alarm clocks, what is the probability that they both fail on the morning of an important final exam?

c) With one alarm clock, you have a 0.9 probability of being awakened. What is the probability of being awakened if you use two alarm clocks?

d) Does a second alarm clock result in greatly improved reliability?

Solution

Let A = alarm works.

For each alarm: $P(A)=0.9$

a) $P(\bar{A})=1-P(A)=1-0.9 \underline{=0.1}$

b) $P(\bar{A}_1 \text{ and } \bar{A}_2)=P(\bar{A}_1)\cdot P(\bar{A}_2)$

$$=(0.1)(.1)$$

$$\underline{=0.01}$$

c) $P(\text{being awakened})=P\left(\overline{\bar{A}_1 \text{ and } \bar{A}_2}\right)$

$$=1-P(\bar{A}_1 \text{ and } \bar{A}_2)$$

$$=1-.01$$

$$\underline{=0.99}$$

d) From parts (*b*) and (*c*) assume that the alarm clocks work independently of each other. This would not be true if they are both electric alarm clocks.

Exercise

The wheeling Tire Company produced a batch of 5,000 tires that includes exactly 200 that are defective.

a) If 4 tires are randomly selected for installation on a car, what is the probability that they are all good?

b) If 100 tires are randomly selected for shipment to an outlet, what is the probability that they are all good? Should this outlet plan to deal with defective tires returned by consumers?

Solution

Let G = getting a good tire.

$$P(A)=\frac{4800}{5000}=0.96$$

a) $P(G_1 \text{and } G_2 \text{and } G_3 \text{and } G_4)=P(G_1)\cdot P(G_2|G_1)\cdot P(G_3|G_1 \text{ and } G_2)\cdot P(G_4|G_1 \text{ and } G_2 \text{and } G_3)$

$$= \frac{4800}{5000} \cdot \frac{4799}{4999} \cdot \frac{4798}{4998} \cdot \frac{4797}{4997}$$

$$\underline{= 0.849}$$

b) Since n = 100 represents $\frac{100}{5000} = 0.02 \le 0.05$ of the population, use the 5% guideline and treat the repeated selections as being independent.

$$P\left(G_1 \text{ and } G_2 \text{ and } \ldots \text{and } G_4\right) = P\left(G_1\right) \cdot P\left(G_2\right) \cdot \cdots \cdot P\left(G_{100}\right)$$

$$= (0.96)(0.96) \cdots (0.96)$$

$$= (0.96)^{100}$$

$$\underline{= 0.0169}$$

Yes; since 0.0169 ≤ 0.05, getting 100 good tires would be unusual event and the outlet should plan on dealing with returns of defective tires.

Exercise

When the 15 players on the LA Lakers basketball team are tested for steroids, at least one of them tests positive. Provide a written description of the complement of this event.

Solution

If it is not true that at least one of the 15 tests positive, then all 15 of them test negative.

Exercise

If a couple plans to have 6 children, what is the probability that they will have at least one girl? Is that probability high enough for the couple to be very confident that they will get at least one girl in six children?

Solution

$$P(\text{at least one girl}) = 1 - P(\text{all boys})$$

$$= 1 - P\left(B_1 \text{ and } B_2 \text{ and } B_3 \text{ and } B_4 \text{ and } B_5 \text{ and } B_6\right)$$

$$= 1 - P\left(B_1\right) \cdot P\left(B_2\right) \cdot P\left(B_3\right) \cdot P\left(B_4\right) \cdot P\left(B_5\right) \cdot P\left(B_6\right)$$

$$= 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$\underline{= 0.984}$$

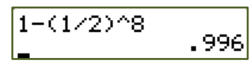
Yes, the probability is high enough for the couple to be very confident that they will get at least one girl in 6 children.

Exercise

If a couple plans to have 8 children (it could happen), what is the probability that they will have at least one girl? Is the couple eventually has 8 children and they are all boys, what can the couple conclude?

Solution

$$\begin{aligned} P(\text{at least one girl}) &= 1 - P(\text{all boys}) \\ &= 1 - P\left(B_1 \text{ and } B_2 \cdots \text{and } B_8\right) \\ &= 1 - P\left(B_1\right)\cdot P\left(B_2\right)\cdot P\left(B_3\right)\cdot P\left(B_4\right)\cdot P\left(B_5\right)\cdot P\left(B_6\right)\cdot P\left(B_7\right)\cdot P\left(B_8\right) \\ &= 1 - \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} \\ &= 0.996 \end{aligned}$$



If the couple has 8 boys, either a very rare event has occurred or there is some environmental or genetic factor that makes boys more likely for this couple.

Exercise

If you make guesses for 4 multiple-choice test questions (each with 5 possible answers), what is the probability of getting at least one correct? If a very lenient instructor says that passing test occurs if there is at least one correct answer, can you reasonably expect to pass by guessing?

Solution

$$\begin{aligned} P(\text{at least one correct}) &= 1 - P(\text{all wrong}) \\ &= 1 - P\left(W_1 \text{ and } W_2 \text{ and } W_3 \text{ and } W_4\right) \\ &= 1 - P\left(W_1\right)\cdot P\left(W_2\right)\cdot P\left(W_3\right)\cdot P\left(W_4\right) \\ &= 1 - \frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5} \\ &= 0.590 \end{aligned}$$

Since there is a greater chance of passing than of failing, the expectation is that such a strategy would lead to passing. In that sense, one can reasonably expect to pass by guess. Nut while the expectation for a single test may be pass, such a strategy can be expected to lead to failing about 4 times every 10 times it is applied.

Exercise

Find the probability of a couple having a baby girl when their fourth child is born, given that the first 3 children were all girls. Is the result the same as the probability of getting 4 girls among 4 children?

Solution

Sample Space:

$$S = \{BBBB,\ BBBG,\ BBGB,\ BGBB,\ GBBB,\ BBGG,\ BGBG,\ BGGB,$$
$$GBBG,\ GBGB,\ GGBB,\ BGGG,\ GBGG,\ GGBG,\ GGGB,\ GGGG\}$$

Let F = first 3 children are girls.

Let G_4 = fourth child is a girl.

$P(F) = \frac{2}{16}$ $\qquad\qquad$ $P(G_4) = \frac{8}{16} = \frac{1}{2}$

$P(F \text{ and } G_4) = \frac{1}{16}$

$$P(G_4 | F) = \frac{P(G_4 \text{ and } F)}{P(F)}$$
$$= \frac{\frac{1}{16}}{\frac{2}{16}}$$
$$\boxed{= \frac{1}{2}}$$

$P(G_4) = \frac{1}{2} = P(G)$ independent (occurred during the first 3 births)

This probability is not true the same as $P(GGGG) = \frac{1}{16}$

Exercise

In China, the probability of a baby being a boy is 0.5845. Couples are allowed to have only one child. If relatives give birth to 5 babies, what is the probability that there is at least one girl? Can that system continue to work indefinitely?

Solution

$$P(\text{at least one girl}) = 1 - P(\text{all boys})$$
$$= 1 - P(B_1 \text{ and } B_2 \text{ and } B_3 \text{ and } B_4 \text{ and } B_5)$$
$$= 1 - P(B_1) \cdot P(B_2) \cdot P(B_3) \cdot P(B_4) \cdot P(B_5)$$
$$= 1 - (0.5845) \cdot (0.5845) \cdot (0.5845) \cdot (0.5845) \cdot (0.5845)$$

```
1-.5845^5
            .932
```

$$\boxed{= 0.932}$$

Probably not; the system will produce such a shortage of females that baby girls will become a valuable asset and parents will take appropriate measures to change the probabilities.

Exercise

An experiment with fruit flies involves one parent with normal wings and one parent with vestigial wings. When these parents have an offspring, there is a $\frac{3}{4}$ probability that the offspring has normal wings and a $\frac{1}{4}$ probability of vestigial wings. If the parents give birth to 10 offspring, what is the probability that at least 1 of the offspring has vestigial wings? If researchers need at least one offspring with vestigial wings, can they be reasonably confident of getting one?

Solution

$$\begin{aligned} P(\text{at least } 1 \text{ w/ vestigial wings}) &= 1 - P(\text{all have normal wings}) \\ &= 1 - P\left(N_1 \text{ and } N_2 \text{ and } \ldots \text{and } N_{10}\right) \\ &= 1 - P\left(N_1\right) \cdot P\left(N_2\right) \cdot P\left(N_3\right) \cdot \cdots \cdot P\left(N_{10}\right) \\ &= 1 - \left(\frac{3}{4}\right) \cdot \left(\frac{3}{4}\right) \cdot \left(\frac{3}{4}\right) \cdot \cdots \cdot \left(\frac{3}{4}\right) \\ &= 1 - \left(\frac{3}{4}\right)^{10} \\ &= 0.944 \end{aligned}$$

```
1-(3/4)^10
          .944
```

Yes, the researchers can be 94.4% certain of getting at least one such offspring.

Exercise

According to FBI data, 24.9% of robberies are cleared with arrests. A new detective is assigned to 10 different robberies.

a) What is the probability that at least 1 of them is cleared with an arrest?

b) What is the probability that the detective clears all 10 robberies with arrests?

c) What should we conclude if the detective clears all 10 robberies with arrests?

Solution

Let $P(C) = P(\text{cleared with arrest}) = (24.9\%) = 0.249$

$P(N) = P(\text{not cleared with arrest}) = 1 - .249 = 0.751$

a)
$$\begin{aligned} P(\text{at least } 1 \text{ cleared}) &= 1 - P(\text{all not cleared}) \\ &= 1 - P\left(N_1 \text{ and } N_2 \text{ and } \ldots \text{and } N_{10}\right) \\ &= 1 - P\left(N_1\right) \cdot P\left(N_2\right) \cdot P\left(N_3\right) \cdot \cdots \cdot P\left(N_{10}\right) \\ &= 1 - (.751) \cdot (.751) \cdot \cdots \cdot (.751) \\ &= 1 - (.751)^{10} \\ &= 0.943 \end{aligned}$$

```
1-.751^10
          .943
```

b) $P(\text{cleared all 10}) = P\left(C_1 \text{ and } C_2 \text{ and } \ldots \text{and } C_{10}\right)$

$$= P\left(C_1\right) \cdot P\left(C_2\right) \cdot \cdots \cdot P\left(C_{10}\right)$$

$$= (.249) \cdot (.249) \cdot \cdots \cdot (.249)$$

$$= (.249)^{10}$$

$$\underline{= 0.000000916}$$

c) If the detective clears all 10 cases with arrests, we should conclude that the $P(C) = 0.249$ rate does not apply to this detective – The probability he clears a case with an arrest is much higher than 0.249.

Exercise

A statistics student wants to ensure that she is not late for an early statistics class because of a malfunctioning alarm clock. Instead of using one alarm clock, she decides to use three. What is the probability that at least one of her alarm clocks works correctly if each individual alarm clock has a 90% chance of working correctly? Does the student really gain much by using three alarm clocks instead on only one? How are the results affected if all of the alarm clocks run on electricity instead of batteries?

Solution

Let $P(F) = P(\text{alarm clock fails}) = 1 - .9 = 0.1$

$P(\text{at least 1 works}) = 1 - P(\text{all fail})$

$$= 1 - P\left(F_1 \text{ and } F_2 \text{ and } F_3\right)$$

$$= 1 - P\left(F_1\right) \cdot P\left(F_2\right) \cdot P\left(F_3\right)$$

$$= 1 - (0.1) \cdot (0.1) \cdot (0.1)$$

$$\underline{= 0.999}$$

Yes, the probability of a working clock rises from 90% with just one clock to 99.9% with 3 clocks? If the alarm clocks run on electricity instead of batteries, then the clocks do not operate independently and the failure of one could be the result of a power failure or interruption and may be related to the failure of another $P\left(F_2 \middle| F_1\right)$ no longer $P(F) = 0.90$

Exercise

In a batch of 8,000 clock radios 8% are defective. A sample of 5 clock radios is randomly selected without replacement from the 8,000 and tested. The entire batch will be rejected if at least one of those tested is defective. Find the probability that the entire batch will be rejected.

Solution

Number of defective radios: $8000 \times .08 = 640$

$$P(at\ \ least\ \ 1) = 1 - P(none\ \ defective)$$

$$= 1 - \frac{\binom{7,360}{5}\binom{640}{0}}{\binom{8,000}{5}}$$

$$= 1 - .659$$

$$\approx 0.341$$

Exercise

In a blood testing procedure, blood samples from 3 people are combined into one mixture. The mixture will only test negative if all the individual samples are negative. If the probability that an individual sample tests positive is 0.1, find the probability that the mixture will test positive.

Solution

Probability that the mixture will test positive = 1 – (probability all negative)

$$P(all\ -) = 1 - 0.1 = 0.9$$

$$P = 1 - .9^3 \ = 0.271$$

Exercise

A sample of 4 different calculators is randomly selected from a group containing 16 that are defective and 36 that have no effects. Find the probability that at least one of the calculator is defective.

Solution

$$P(at\ \ least\ \ 1) = 1 - P(none\ \ defective)$$

$$= 1 - \frac{\binom{36}{4}\binom{16}{0}}{\binom{52}{4}}$$

$$= 1 - .218$$

$$\approx 0.782$$

Exercise

Among the contestants in a competition are 46 women and 29 men. If 5 winners are randomly selected, find the probability that they are all men?

Solution

$$P(All\ men) = \left(\frac{29}{29+46}\right)^5 \underline{\left.= 0.009\right|}$$

Exercise

A bin contains 60 lights bubs of which 7 are defective. If 4 light bulbs are randomly selected from the bin with replacement, find the probability that all the bulbs selected are good ones.

Solution

$$P(All\ good\ ones) = \left(\frac{53}{60}\right)^4 \underline{\left.= 0.609\right|}$$

Exercise

You are dealt two cards successively (without replacement) from a shuffled deck of 52 playing cards. Find the probability that both cards are black. Express your answer as a simplified fraction.

Solution

$$P(2\ cards\ Black) = \frac{C_{26,2}}{C_{52,2}} = \frac{325}{1326} \underline{\left.= \frac{25}{102}\right|}$$

Solution Section 2.5 – Counting Techniques

Exercise

Decide whether the situation involves ***permutations*** or ***combinations***

a) A batting order for 9 players for a baseball game
b) An arrangement of 8 people for a picture
c) A committee of 7 delegates chosen from a class of 30 students to bring a petition to the administration
d) A selection of a chairman and a secretary from a committee of 14 people
e) A sample of 5 items taken from 71 items on an assembly line
f) A blend of 3 spices taken from 7 spices on a spice rack
g) From the 7 male and 10 female sales representatives for an insurance company, team of 8 will be selected to attend a national conference on insurance fraud.
h) Marbles are being drawn without replacement from a bag containing 15 marbles.
i) The new university president named 3 new officers a vice-president of finance, a vice-president of academic affairs, and a vice-president of student affairs.
j) A student checked out 4 novels from the library to read over the holiday.
k) A father ordered an ice cream cone (chocolate, vanilla, or strawberry) for each of his 4 children.

Solution

a) Permutation
b) Permutation
c) Combination
d) Permutation
e) Combination
f) Combination
g) Combination
h) Combination
i) Permutation
j) Combination
k) Neither

Exercise

Find the number of different ways that five test questions can be arranged in order by evaluating 5!

Solution

$$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$$

Exercise

In the game of blackjack played with one deck, a player is initially dealt 2 cards. Find the number of different two-card initial hands by evaluating ${}_{52}C_2$

Solution

$${}_{52}C_2 = \frac{52!}{50!\cdot 2!} = \frac{52\cdot 51}{2} = 1326$$

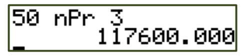
```
52 nCr 2
        1326.000
```

Exercise

A political strategist must visit state capitols, but she has time to visit only 3 of them. Find the number of different possible routes by evaluating ${}_{50}P_3$

Solution

$${}_{50}P_3 = \frac{50!}{(50-3)!} = \frac{50!}{45!} = 50\cdot 49\cdot 48 \ \underline{= 117,600|}$$



Exercise

Select the six winning numbers from 1, 2, …, 54. Find the probability of winning lottery by buying one ticket. $\left(\textit{of winning this lottery } \frac{1}{575,757}\right)$

Solution

$${}_{54}C_6 = \frac{54!}{48!\cdot 6!} = 25,827,165 \ \ \textit{possibilities}$$

Since only one combination wins

$$P(\textit{winning with a single selection}) \ \underline{= \frac{1}{25,827,165}|}$$

Exercise

Select the five winning numbers from 1, 2, …, 36. Find the probability of winning lottery by buying one ticket. $\left(\textit{of winning this lottery } \frac{1}{575,757}\right)$

Solution

$${}_{36}C_5 = \frac{36!}{31!\cdot 5!} = 376,992 \ \ \textit{possibilities}$$

Since only one combination wins

$$P(\textit{winning with a single selection}) \ \underline{= \frac{1}{376,992}|}$$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be chosen that have

a) All men? *b)* All women? *c)* 3 men and 2 women?

Solution

a) $C(9,5) = 126$

b) $C(11,5) = 462$

c) $C(9,3).C(11,2) = (84)(55) = 4620$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be selected that have

a) At least 4 women? *b)* No more than 2 men?

Solution

a) $C(11,4)C(9,1)+C(11,5)C(9,0)=3432$

b) $C(9,0)C(11,5)+C(9,1)C(11,4)+C(9,2)C(11,3)=9372$

Exercise

In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?

Solution

$$P_{9,5}=15,120$$

Exercise

From a pool of 8 secretaries, 3 are selected to be assigned to 3 managers, one per manager. In how many ways can they be selected and assigned?

Solution

$$P_{8,3}=336$$

Exercise

A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.

a) How many delegations are possible?
b) If it is decided that a particular worker must be in the delegation, how many different delegations are possible?
c) If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?

Solution

a) $C_{9,3}\underline{=84|}$

b) $1.C_{8,2}\underline{=28|}$

c) $C_{4,1}C_{5,2}+C_{4,2}C_{5,1}+C_{4,3}\underline{=74|}$

Exercise

Hamburger Hut sells regular hamburgers as well as a larger burger. Either type can include cheese, relish, lettuce, tomato, mustard, or catsup.

a) How many different hamburgers can be ordered with exactly three extras?
b) How many different regular hamburgers can be ordered with exactly three extras?
c) How many different regular hamburgers can be ordered with at least five extras?

Solution

a) $C_{2,1}C_{6,3} = 40$

b) $C_{6,3} = 20$

c) $C_{6,5} + C_{6,6} = 7$

Exercise

In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.

a) In how many ways can this be done?
b) In how many ways can this be done if exactly 2 wheat plants must be included?

Solution

a) $C_{11,4} = 330$

b) $C_{6,2}C_{5,2} = 150$

Exercise

A legislative committee consists of 5 Democrats and 4 Republicans. A delegation of 3 is to be selected to visit a small Pacific island republic.

a) How many different delegations are possible?
b) How many delegations would have all Democrats?
c) How many delegations would have 2 Democrats and 1 Republican?
d) How many delegations would have at least 1 Republican?

Solution

a) $C_{9,3} = 84$

b) $C_{5,3} = 10$

c) $C_{5,2}C_{4,1} = 40$

d) $C_{9,3} - C_{5,3} = 84 - 10 = 74$

Exercise

Five cards are chosen from an ordinary deck to form a hand in poker. In how many ways is it possible to get the following results?

a) 4 queens
b) No face card
c) Exactly 2 face cards
d) At least 2 face cards
e) 1 heart, 2 diamonds, and 2 clubs

Solution

a) $C_{4,4}C_{48,1} = 48$

b) $C_{40,5} = 658,008$

c) $C_{12,2}C_{40,3} = 652,080$

d) $C_{12,2}C_{40,3} + C_{12,3}C_{40,2} + C_{12,4}C_{40,1} + C_{12,5} = 844,272$

e) $C_{13,1}C_{13,2}C_{13,2} = 79,092$

Exercise

The student sitting next to you in class concludes that the probability of the ceiling falling down on both of you before class ends is 1/2, because there are two possible outcomes - the ceiling will fall or not fall. What is wrong with this reasoning?

Solution

The outcomes are not equally likely.

Exercise

Identity theft often begins by someone discovering your 9-digit social security number. Answer each of the following. Express probabilities as fractions.

a) What is the probability of randomly generating 9 digits and getting your social security number?
b) In the past, many teachers posted grades along with the last 4 digits of your social security number, what is the probability that if they randomly generated the order digits, they would match yours? Is that something to worry about?

Solution

a) Let G = generating a given social security number in a single trial.

Total number of possible sequences $= 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
$= 1,000,000,000$

Since only one sequence is correct: $P(G) = \boxed{\dfrac{1}{1,000,000,000}}$

b) Let F = generating first 5 digits of a given social security number in a single trial.

Total number of possible sequences $= 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
$= 100,000$

Since only one sequence is correct: $P(F) = \boxed{\frac{1}{100,000}}$

Since this probability is so small, need not worry about the given scenario

Exercise

Credit card numbers typically have 16 digits, but not all of them are random. Answer the following and express probabilities as fractions.

a) What is the probability of randomly generating 16 digits and getting your MasterCard number?

b) Receipts often show the last 4 digits of a credit card number. If those last 4 digits are known, what is the probability of randomly generating the order digits of your MasterCard number?

c) Discover cards begin with the digits 6011. If you also know the last 4 digits, what is the probability of randomly generating the other digits and getting all of them correct? Is this something to worry about?

Solution

a) Let G = generating a given credit card number in a single trial.

Total number of possible sequences $= 10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10$

$= 10,000,000,000,000,000$

Since only one sequence is correct: $P(G) = \boxed{\frac{1}{10,000,000,000,000,000}}$

b) Let F = generating first 12 digits of a given credit card number in a single trial.

Total number of possible sequences $= 10^{12}$

$= 1,000,000,000,000$

Since only one sequence is correct: $P(F) = \boxed{\frac{1}{1,000,000,000,000}}$

c) Let M = generating the middle digits of a given credit card number in a single trial.

Total number of possible sequences $= 10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10$

$= 100,000,000$

Since only one sequence is correct: $P(M) = \boxed{\frac{1}{100,000,000}}$

This is not something to worry about.

Exercise

When testing for current in a cable with five color-coded wires, the author used a meter to test two wires at a time. How many different tests are required for every possible pairing of two wires?

Solution

Since the order of the 2 wires being tested is irrelevant:

$${}_5C_2 = \frac{5!}{3!\cdot 2!} \boxed{= 10 \ \ \textit{different tests}}$$

Exercise

The starting 4 players for the Boston Celtics basketball team have agreed to make charity appearances tomorrow night. If you must send three players to the United Way event and the other 2 to a Heart Fund event, how many different ways can you make the assignments?

Solution

Since the order in which the 3 are picked makes no difference.

$${}_5C_3 = \frac{5!}{2!\cdot 3!} = \boxed{10 \ \text{different ways}}$$

Exercise

In phase I of a clinical trial with gene therapy used for treating HIV, 5 subjects were treated (based on data from Medical News Today). If 20 people were eligible for the Phase I treatment and a simple random of 5 is selected, how many different simple random samples are possible? What is the probability of each simple random sample?

Solution

Since the order in which the subjects are placed in the groups is not relevant.

$${}_{20}C_5 = \frac{20!}{15!\cdot 5!} = \boxed{15{,}504 \ \text{possibilities}}$$

$$P(\text{any one combination}) = \boxed{\frac{1}{15{,}504}}$$

Exercise

Many newspapers carry "Jumble" a puzzle in which the reader must unscramble letters to form words. The letters BUJOM were included in newspapers. How many ways can the letters if BUJOM be arranged? Identify the correct unscrambling and then determine the probability of getting that result by randomly selecting one arrangement of the given letters.

Solution

The number of possible sequences: $5! = \boxed{120 \ \text{sequences}}$

The unscrambled sequence word is JUMBO. $\frac{1}{120}$

Since there is only 1 correct sequence; the probability of finding it with one random arrangement is

Exercise

There are 11 members on the board of directors for the Coca Cola Company.

a) If they must select a chairperson, first vice chairperson, second vice chairperson, and secretary, how many different slates of candidates are possible?

b) If they must form an ethics subcommittee of 4 members, how many different subcommittees are possible?

Solution

a) Since order makes a difference, there are 4 different offices ${}_{11}P_4 = \frac{11!}{7!}\ \underline{= 7920|}$

b) Since the order in which the 4 are picked makes no differences ${}_{11}C_4 = \frac{11!}{7!4!}\ \underline{= 330|}$

Exercise

The author owns a safe in which he stores his book. The safe combination consists of 4 numbers between 0 and 99. If another author breaks in and tries to steal this book, what is the probability that he or she will get the correct combination on the first attempt? Assume that the numbers are randomly selected. Given the number of possibilities, does it seem feasible to try opening the safe by making random guesses for the combination?

Solution

There are 4 tasks to perform, and each task can be performed in any of 100 ways.

Total number of possible sequence is $100 \cdot 100 \cdot 100 \cdot 100\ \underline{= 100,000,000\ \ possibilities|}$

Since there is only one correct sequence, the probability of finding is $\frac{1}{100,000,000}$

Since there are so many possibilities, it would not be feasible to try opening the safe by making random guesses.

Exercise

In a preliminary test of the MicroSort gender selection method, 14 babies were born and 13 of them were girls

a) Find the number of different possible sequences of genders that are possible when 14 babies are born.

b) How many ways can 13 girls and 1 boy be arranged in a sequence?

c) If 14 babies are randomly selected, what is the probability that they consist of 13 girls and 1 boy?

d) Does the gender-selection method appear to yield a result that is significantly different from a result that might be expected by random chance?

Solution

a) There are 14 tasks to perform, and each task can be performed in either of 2 ways

Total number of possible sequences is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{14}$

$\underline{= 16,384\ \ possibilities|}$

b) The number of possible sequences of n objects is when some are alike is

$\frac{n!}{n_1! \cdot n_2! \cdots n_k!} = \frac{14!}{13!1!}\ \underline{= 14\ \ possibilities|}$

c) $P(13G,\ 1B) = \dfrac{\#\text{of ways to get } 13G}{\text{Total \# of ways}} = \dfrac{14}{16,384}$ $\underline{= 0.000854|}$

d) Yes, since $P(13G,\ 1B)$ is so small, and since 13G, 1B so far (only the 14G, 0B result is more extreme) from the expected 7G, 7B result, the gender-selection method appears to yield results significantly different from those of chance alone.

Exercise

You become suspicious when a genetics researcher randomly selects groups of 20 newborn babies and seems to consistently get 10 girls and 10 boys. The researchers claims that it is common to get 10 girls and 10 boys in such cases,

a) If 20 newborn babies are randomly selected, how many different gender sequences are possible?

b) How many different ways can 10 girls and 10 boys be arranged in sequence?

c) What is the probability of getting 10 girls and 10 boys when 10 babies are born?

d) Based on the preceding results, do you agree with the researcher's explanation that ir is common to get 10 girls and 10 boys when 20 babies are randomly selected?

Solution

a) There are 20 tasks to perform, and each task can be performed in either of 2 ways

Total number of possible sequences is $2 \cdot 2 \cdot 2 \cdot \cdots \cdot 2 = 2^{20}$

$\underline{= 1,048,576 \ \ \textit{possibilities}|}$

b) The number of possible sequences of *n* objects is when some are alike is

$\dfrac{n!}{n_1! \cdot n_2! \cdots n_k!} : \dfrac{20!}{10!10!}$ $\underline{= 184,756 \ \ \textit{possibilities}|}$

c) $P(10G,\ 10B) = \dfrac{184,756}{1,048,576}$ $\underline{= 0.176|}$

d) It is not unusual for an event with probability 0.176 to occur once, but repeated occurrences should be considered unusual – as the probability of the event occurring twice in a row, for example, is $(.176)(.176) = 0.0310$.

Exercise

The Powerball lottery is run in 29 states. Winning the jackpot requires that you select the correct five numbers between 1 and 55 and, in a separate drawing, you must also select the correct single number between 1 and 42. Find the probability of winning the jackpot.

Solution

Let *A* = selecting the correct 5 numbers from 1 to 55

Let B = selecting the correct winning number from 1 to 42

The number of possible selection: ${}_{55}C_5 = \dfrac{55!}{50! \cdot 5!}$ $\underline{= 3,478,761 \ \ \textit{possibilities}|}$

Since there is only one winning number: $P(A)=\dfrac{1}{3,478,761}$

There are 42 possible selections.

Since there is only one winning number: $P(B)=\dfrac{1}{42}$

$$\begin{aligned} P(\textit{winning Powerball}) &= P(A \textit{ and } B) \\ &= P(A)P(B) \\ &= \frac{1}{3,478,761}\cdot\frac{1}{42} \\ &= \frac{1}{146,107,962} \\ &\underline{= 0.00000000684} \end{aligned}$$

Exercise

The Mega Millions lottery is run in 12 states. Winning the jackpot requires that you select the correct 5 numbers between 1 and 56 and, in a separate drawing, you must also select the correct single number between 1 and 46. Find the probability of winning the jackpot.

Solution

Let A = selecting the correct 5 numbers from 1 to 56
Let B = selecting the correct winning number from 1 to 46

The number of possible selection: ${}_{56}C_5 = \dfrac{56!}{51!\cdot 5!} \underline{= 3,819,816 \ \textit{possibilities}}$

Since there is only one winning number: $P(A)=\dfrac{1}{3,478,761}$

There are 46 possible selections.

Since there is only one winning number: $P(B)=\dfrac{1}{46}$

$$\begin{aligned} P(\textit{winning Mega Millions}) &= P(A \textit{ and } B) \\ &= P(A)P(B) \\ &= \frac{1}{3,819,816}\cdot\frac{1}{46} \quad = \frac{1}{175,711,536} \\ &\underline{= 0.00000000569} \end{aligned}$$

Exercise

A state lottery involves the random selection of six different numbers between 1 and 31. If you select one six number combination, what is the probability that it will be the winning combination?

Solution

$$P(\textit{winning}) = \frac{1}{\binom{31}{6}} \underline{= \frac{1}{736,281}}$$

Exercise

How many ways can 6 people be chosen and arranged in a straight line if there are 8 people to choose from?

Solution

$$8\times7\times6\times5\times4\times3 = 20{,}160\ \ ways$$

Exercise

12 wrestlers compete in a competition. If each wrestler wrestles one match with each other wrestler, what are the total numbers of matches?

Solution

$$11+10+9+8+7+6+5+4+3+2+1 = 66$$

Exercise

Wing has different books to arrange on a shelf: 4 blue, 3 green, and 2 red.

a) In how many ways can the books be arranged on a shelf?
b) If books of the same color are to be grouped together, how many arrangements are possible?
c) In how many distinguishable ways can the books be arranged if books of the same color are identical but need not be grouped together?
d) In how many ways can you select 3 books, one of each color, if the order in which the books are selected does not matter?
e) In how many ways can you select 3 books, one of each color, if the order in which the books are selected matters?

Solution

a) $P(9,9) = 362{,}880\ \ ways$

b) $4!.3!.2!.3! = 1728\ \ possibilities$

c) $\dfrac{9!}{4!3!2!} = 1260$

d) $4.3.2 = 24$

e) $24.3! = 144$ ***(24 from part-d)***

Exercise

A child has a set of differently shaped plastic objects. There are 3 pyramids, 4 cubes, and 7 spheres.

a) In how many ways can she arrange the objects in a row if each is a different color?
b) How many arrangements are possible if objects of the same shape must be grouped together and each object is a different color?
c) In how many distinguishable ways can the objects be arranged in a row if objects of the same shape are also the same color, but need not be grouped together?

d) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected does not matter and each object is a different color?

e) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected matters and each object is a different color?

Solution

a) $P(14,14) = 8.7178291 \times 10^{10}$

b) $3!4!7!3! = 4,354,560$ (*3! number of ways to arrange the order of 3 groups*)

c) $\dfrac{14!}{3!4!7!} = 120,120$

d) $3.4.7 = 84$

e) $84.3! = 504$

Exercise

In a club with 16 members, how many ways can a slate of 3 officers consisting of president, vice-president, and secretary/treasurer be chosen?

Solution

$$P(16,3) = \underline{3360}$$

Exercise

Twelve drugs have been found to be effective in the treatment of a disease. It is believed that the sequence in which the drugs are administered is important in the effectiveness of the treatment. In how many different sequences can 5 of the 12 drugs be administered?

Solution

$$P(12,5) = \underline{95,040}$$

Exercise

In how many ways can 7 of 11 monkeys be arranged in a row for a genetics experiment?

Solution

$$P(11,7) = \underline{1,663,200}$$

Exercise

In an experiment on social interaction, 6 people will sit in 6 seats in a row. In how many ways can this be done?

Solution

$$P(6,6) = \underline{720}$$

Exercise

In an election with 3 candidates for one office and 6 candidates for another office, how many different ballots may be printed?

Solution

Office 1: $P(3,3)$

Office2: $P(6,6)$

Multiplication principle: $2.P(3,3)P(6,6)$ $\underline{=8640}|$

Exercise

A business school gives courses in typing, shorthand, transcription, business English, technical writing, and accounting. In how many ways can a student arrange a schedule if 3 courses are taken? assume that the order in which courses are schedules matters.

Solution

$P(6,3)$ $\underline{=120}|$

Exercise

If your college offers 400 courses, 25 of which are in mathematics, and your counselor arranges your schedule of 4 courses by random selection, how many schedules are possible that do not include a math course? Assume that the order in which courses are scheduled matters.

Solution

$$P(nonmath)=P(375,4)=\underline{1.946\times 10^{10}}|$$

Exercise

A baseball team has 19 players. How many 9-player batting orders are possible?

Solution

$$P(19,9)=\underline{3.352\times 10^{10}}|$$

Exercise

A chapter of union Local 715 has 35 members. In how many different ways can the chapter select a president, a vice-president, a treasurer, and a secretary?

Solution

$$P(35,4)=\underline{1,256,640}|$$

Exercise

A concert to raise money for an economics prize is to consist of 5 works; 2 overtures, 2 sonatas, and a piano concerto.

a) In how many ways can the program be arranged?
b) In how many ways can the program be arranged if an overture must come first?

Solution

a) $P(5,5) = \underline{120}$

b) $P(2,1).P(4,4) = \underline{48}$

Exercise

A zydeco band from Louisiana will play 5 traditional and 3 original Cajun compositions at a concert. In how many ways can they arrange the program if

a) The begin with a traditional piece?
b) An original piece will be played last?

Solution

a) $P(5,1).P(7,7) = \underline{25,200}$

b) $P(7,7).P(3,1) = \underline{15,120}$

Exercise

Given the set $\{A, B, C, D\}$, how many permutations are there of this set of 4 object taken 2 at a time?

a) Using the multiplication principle
b) Using the Permutation

Solution

a) 4.3 = 12

b) $P_{4,2} = \frac{4!}{2!} = \underline{12}$

Exercise

Find the number of permutations of 30 objects taken 4 at a time.

Solution

$$P_{30,4} = \frac{30!}{(30-4)!} = \underline{657,720}$$

Exercise

Five cards are marked with the numbers 1, 2, 3, 4, and 5, then shuffled, and 2 cards are drawn.

a) How many different 2-card combinations are possible?
b) How many 2-card hands contain a number less than 3?

Solution

a) $C_{5,2} = 10$

b) $\left\{ \begin{array}{l} \{1,2\},\ \{1,3\},\ \{1,4\},\ \{1,5\},\ \{2,3\} \\ \{2,4\},\ \{2,5\},\ \{3,4\},\ \{3,5\},\ \{4,5\} \end{array} \right\}$

7 contain a card numbered less than 3.

Exercise

An economics club has 31 members.

a) If a committee of 4 is to be selected, in how many ways can the selection be made?
b) In how many ways can a committee of at least 1 and at most 3 be selected?

Solution

a) $C_{31,4} = 31,465$

b) $$\begin{aligned} P(at\ least\ 1\ and\ at\ most\ 3\ be\ selected) &= C_{31,1} + C_{31,2} + C_{31,3} \\ &= 31 + 465 + 4495 \\ &= 4991 \end{aligned}$$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be chosen that have

a) All men? *b)* All women? *c)* 3 men and 2 women?

Solution

a) $C(9,5) = 126$

b) $C(11,5) = 462$

c) $C(9,3).C(11,2) = (84)(55) = 4620$

Exercise

In a club with 9 male and 11 female members, how many 5-member committees can be selected that have

a) At least 4 women? *b)* No more than 2 men?

Solution

a) $C(11,4)C(9,1)+C(11,5)C(9,0)=\boxed{3432}$

b) $C(9,0)C(11,5)+C(9,1)C(11,4)+C(9,2)C(11,3)=\boxed{9372}$

Exercise

Use a tree diagram for the following

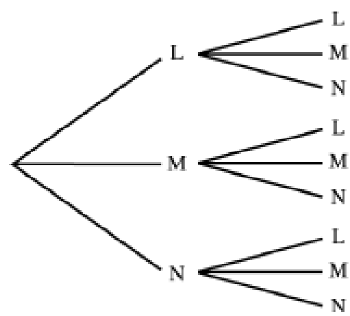
a) Find the number of ways 2 letters can be chosen from the set $\{L, M, N\}$ if order is important and repetition is allowed.

b) Reconsider part a if no repeats are allowed

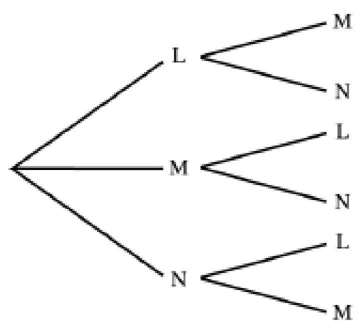
c) Find the number of combinations of 3 elements taken 2 at a time. Does this answer differ from part *a* or *b*?

Solution

a) There are 9 ways to choose 2 letters if repetition is allowed



b) There are 9 ways to choose 2 letters if repetition is allowed



c) /The number of 3 elements taken 2 at a time is: $C_{3,2}=\boxed{3}$

Exercise

In a game of musical chairs, 12 children will sit in 11 chairs arranged in a row (one will be left out). In how many ways can this happen, if we count rearrangements of the children in the chairs as different outcomes?

Solution

$P(12,11)=\boxed{479,001,600}$

Exercise

A group of 3 students is to be selected from a group of 14 students to take part in a class in cell biology.

a) In how many ways can this be done?
b) In how many ways can the group who will not take part be chosen?

Solution

a) $\binom{14}{3} = 364$

b) $\binom{14}{11} = 364$

Exercise

Marbles are being drawn without replacement from a bag containing 16 marbles.

a) How many samples of 2 marbles can be drawn?
b) How many samples of 2 marbles can be drawn?
c) If the bag contains 3 yellow, 4 white, and 9 blue marbles, how many samples of 2 marbles can be drawn in which both marbles are blue?

Solution

a) $C(16,2) = 120$

b) $C(16,4) = 1820$

c) $C(9,2) = 36$

Exercise

There are 7 rotten apples in a crate of 26 apples

a) How many samples of 3 apples can be drawn from the crate?
b) How many samples of 3 could be drawn in which all 3 are rotten?
c) How many samples of 3 could be drawn in which there are two good apples and one rotten one?

Solution

a) $C_{26,3} = 2600$

b) $C_{7,3} = 35$

c) $C_{26,3} = 2600$

Exercise

A bag contains 5 black, 1 red, and 3 yellow jelly beans; you take 3 at random. How many samples are possible in which the jelly beans are

a) All black?
b) All red?
c) All yellow?
d) 2 black and 1 red?
e) 2 black and 1 yellow?
f) 2 yellow and 1 black?
g) 2 red and 1 yellow?

Solution

a) $C_{5,3} = 10$

b) No 3 red. $C_{1,3} = 0$

c) $C_{3,3} = 1$

d) $C_{5,2}C_{1,1} = 10$

e) $C_{5,2}C_{3,1} = 30$

f) $C_{3,2}C_{5,1} = 15$

g) *There is only 1 red.*

Exercise

In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?

Solution

$P_{9,5} = \underline{15,120}$

Exercise

From a pool of 8 secretaries, 3 are selected to be assigned to 3 managers, one per manager. In how many ways can they be selected and assigned?

Solution

$P_{8,3} = \underline{336}$

Exercise

A salesperson has the names of 6 prospects.

a) In how many ways can she arrange her schedule if she calls on all 6?
b) In how many ways can she arrange her schedule if she can call on only 4 of the 6?

Solution

a) $P_{6,6} = \underline{720}$

b) $P_{6,4} = 360$

Exercise

Five items are to be randomly selected from the first 50 items on an assembly line to determine the defect rate. How many different samples of 5 items can be chosen?

Solution

$$C_{50,5} = \underline{2,118,760}$$

Exercise

A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.

a) How many delegations are possible?

b) If it is decided that a particular worker must be in the delegation, how many different delegations are possible?

c) If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?

Solution

a) $C_{9,3} = \underline{84}$

b) $1.C_{8,2} = \underline{28}$

c) $C_{4,1}C_{5,2} + C_{4,2}C_{5,1} + C_{4,3} = \underline{74}$

Exercise

From a group of 16 smokers and 22 nonsmokers, a researcher wants to randomly select 8 smokers and 8 nonsmokers for a study. in how many ways can the study group be selected?

Solution

$$C_{16,8}C_{22,8} = \underline{4,115,439,900}$$

Exercise

Hamburger Hut sells regular hamburgers as well as a larger burger. Either type can include cheese, relish, lettuce, tomato, mustard, or catsup.

a) How many different hamburgers can be ordered with exactly three extras?

b) How many different regular hamburgers can be ordered with exactly three extras?

c) How many different regular hamburgers can be ordered with at least five extras?

Solution

a) $C_{2,1}C_{6,3} = \underline{40}$

b) $C_{6,3} = 20$

c) $C_{6,5} + C_{6,6} = 7$

Exercise

In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.

c) In how many ways can this be done?
d) In how many ways can this be done if exactly 2 wheat plants must be included?

Solution

c) $C_{11,4} = 330$

d) $C_{6,2}C_{5,2} = 150$

Exercise

A legislative committee consists of 5 Democrats and 4 Republicans. A delegation of 3 is to be selected to visit a small Pacific island republic.

a) How many different delegations are possible?
b) How many delegations would have all Democrats?
c) How many delegations would have 2 Democrats and 1 Republican?
d) How many delegations would have at least 1 Republican?

Solution

a) $C_{9,3} = 84$

b) $C_{5,3} = 10$

c) $C_{5,2}C_{4,1} = 40$

d) $C_{9,3} - C_{5,3} = 84 - 10 = 74$

Exercise

From 10 names on a ballot, 4 will be elected to a political party committee. in how many ways can the committee of 4 be formed if each person will have a different responsibility, and different assignments of responsibility are considered different committees?

Solution

$P_{10,4} = 5040$

Exercise

How many different 13-card bridge hands can be selected from an ordinary deck?

Solution

$$C_{52,13} = \underline{635,013,559,600}$$

Exercise

Five cards are chosen from an ordinary deck to form a hand in poker. In how many ways is it possible to get the following results?

a) 4 queens
b) No face card
c) Exactly 2 face cards
d) At least 2 face cards
e) 1 heart, 2 diamonds, and 2 clubs

Solution

a) $C_{4,4}C_{48,1} = \underline{48}$

b) $C_{40,5} = \underline{658,008}$

c) $C_{12,2}C_{40,3} = \underline{652,080}$

d) $C_{12,2}C_{40,3} + C_{12,3}C_{40,2} + C_{12,4}C_{40,1} + C_{12,5} = \underline{844,272}$

e) $C_{13,1}C_{13,2}C_{13,2} = \underline{79,092}$

Exercise

In poker, a flush consists of 5 cards with the same suit, such as 5 diamonds.

a) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by listing all the possibilities.
b) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by using combinations

Solution

a) $\{(5,6,7,8,9);(5,6,7,8,10);(5,7,8,9,10);(5,6,8,9,10);(5,7,8,9,10);(6,7,8,9,10)\}$

There are 6 possibilities for each suit and there are 4 suits: 4.6 = 24

b) $4C_{6,5} = \underline{24}$

Exercise

If a baseball coach has 5 good hitters and 4 poor hitters on the bench and chooses 3 players at random, in how many ways can be choose at least 2 good hitters?

Solution

$$C_{5,2}C_{4,1}+C_{5,3}C_{4,0}\ \underline{=50|}$$

Exercise

The coach of a softball team has 6 good hitters and 8 poor hitters. He chooses 3 hitters at random.

a) In how many ways can he choose 2 good hitters and 1 poor hitter?
b) In how many ways can he choose 3 good hitters?
c) In how many ways can he choose at least 3 good hitters?

Solution

a) $C_{6,2}C_{8,1}\ \underline{=120|}$

b) $C_{6,3}\ \underline{=20|}$

c) $C_{6,2}C_{8,1}+C_{6,3}\ \underline{=140|}$

Exercise

How many 5 card hands will have 3 aces and 2 kings?

Solution

Number of hands $=C_{4,3}\cdot C_{4,2}\ \underline{=24|}$

Exercise

How many 5 card hands will have 3 hearts and 2 spades?

Solution

Number of hands $=C_{13,3}\cdot C_{13,2}\ \underline{=22,308|}$

Exercise

2 letters follow by 3 numbers; 2 letters out of 8 & 3 numbers out of 10

Solution

Number $=P_{8,2}\cdot P_{10,3}\ \underline{=40320|}$

Exercise

Serial numbers for a product are to be made using 3 letters follow by 2 digits (0 – 9 no repeats). If the letters are to be taken from the first 8 letters of the alphabet with no repeats, how many serial numbers are possible?

Solution

$$\text{Possible } = P_{8,3} \cdot P_{10,2} \ \underline{= 30,240}$$

Exercise

A company has 7 senior and 5 junior officers. An ad hoc legislative committee is to be formed.

a) How many 4-officer committees with 1 senior officer and 3 junior officers can be formed?

b) How many 4-officer committees with 4 junior officers can be formed?

c) How many 4-officer committees with at least 2 junior officers can be formed?

Solution

a) $C_{7,1} \cdot C_{5,3} \ \underline{= 70}$

b) $C_{5,4} \ \underline{= 5}$

c) $C_{7,2} . C_{5,2} + C_{7,1} . C_{5,3} + C_{7,0} . C_{5,4} \ \underline{= 285}$

Exercise

From a committee of 12 people,

a) In how many ways can we choose a chairperson, a vice-chairperson, a secretary, and a treasurer, assuming that one person can't hold more than on position

b) In how many ways can we choose a subcommittee of 4 people?

Solution

a) $P_{12,4} \ \underline{= 11,880 \ ways}$

b) $C_{12,4} \ \underline{= 495 \ \ ways}$

Exercise

Find the number of combinations of 30 objects taken 4 at a time.

Solution

$$C_{30,4} = \frac{30!}{4!(30-4)!} \ \underline{= 27,405}$$

Exercise

How many different permutations are the of the set $\{a, b, c, d, e, f, g\}$?

Solution

$P(7,\ 7)\ \underline{=5040}$

Exercise

How many permutations of $\{a, b, c, d, e, f, g\}$ end with a?

Solution

To find the permutation to with a, then we may forget about the a, and leave us $\{b, c, d, e, f, g\}$

$P(6,\ 6)\ \underline{=720}$

Exercise

Find the number of 5-permutations of a set with nine elements

Solution

$P(9,\ 5)\ \underline{=15{,}120}$

Exercise

In how many different orders can five runners finish a race if no ties are allowed?

Solution

$P(5,\ 5)\ \underline{=120}$

Exercise

A coin flipped eight times where each flip comes up either heads or tails. How many possible outcomes

a) Are there in total?
b) Contain exactly three heads?
c) Contain at least three heads?
d) Contain the same number of heads and tails?

Solution

a) Each flip can be either heads or tails: There are $2^8 \underline{= 256\ \ possible\ coutcomes}$

b) $C(8,\ 3)\ \underline{=56\ \ outcomes}$

c) At least three heads means: 3, 4, 5, 6, 7, 8 heads.

$C(8,\ 3)+C(8,\ 4)+C(8,\ 5)+C(8,\ 6)+C(8,\ 7)+C(8,\ 8)\ \underline{=219\ \ outcomes|}$

OR

$256-C(8,\ 0)-C(8,\ 1)-C(8,\ 2)=256-28-8-1\ \underline{=219\ \ outcomes|}$

d) To have an equal number of heads and tails means 4 heads and 4 tails.
Therefore; $C(8,\ 4)\ \underline{=70\ \ outcomes|}$

Exercise

In how many ways can a set of two positive integers less than 100 be chosen?

Solution

$C_{99,\ 2}\ \underline{=4851\ \ ways|}$

Exercise

In how many ways can a set of five letters be selected from the English alphabet?

Solution

$C_{26,\ 5}\ \underline{=65,780\ \ ways|}$

Solution Section 2.6 – Discrete Random Variables

Exercise

Determine whether or not a probability distribution is given. If a probability is given, find its mean and standard deviation. If the probability is not given, identify the requirements that are not satisfied.

a)

x	$P(x)$
0	0.125
1	0.375
2	0.375
3	0.125

b)

x	$P(x)$
0	0.22
1	0.16
2	0.21
3	0.16

c)

x	$P(x)$
0	0.528
1	0.360
2	0.098
3	0.013
4	0.001
5	0^+

d)

x	$P(x)$
0	0.02
1	0.15
2	0.29
3	0.26
4	0.16
5	0.12

0^+ denotes a positive probability value that is very small.

Solution

a) Since $0 \le P(x) \le 1;\quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.125	0	0	0
1	0.375	0.375	1	0.375
2	0.375	0.750	4	1.500
3	0.125	0.375	9	1.125
	1.000	1.500		3.000

Mean: $\mu = \sum\left[x \cdot P(x)\right] = 1.50$

Variance: $\sigma^2 = \sum\left[x^2 \cdot P(x)\right] - \mu^2 = 3.0 - (1.5)^2 = 0.75$

Standard deviation: $\sigma = \sqrt{.75} = .866$

b) Since $0 \le P(x) \le 1;$

$\sum P(x) = 0.22 + 0.16 + 0.21 + 0.16 = 0.75 \ne 1$

This given table is not a probability distribution.

c) Since $0 \le P(x) \le 1;\quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.528	0	0	0
1	0.360	0.360	1	0.360
2	0.098	0.196	4	0.392
3	0.013	0.039	9	0.117
4	0.001	.004	16	0.016
5	0+	0	25	0
	1.000	0.599		0.885

Mean: $\mu = \sum \left[x \cdot P(x) \right] \underline{= 0.599}$

Variance: $\sigma^2 = \sum \left[x^2 \cdot P(x) \right] - \mu^2 = 0.885 - (0.599)^2 \underline{= 0.526}$

Standard deviation: $\sigma = \sqrt{0.526} \underline{= 0.725}$

d) Since $0 \le P(x) \le 1;\quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.02	0	0	0
1	0.15	0.15	1	0.15
2	0.29	0.58	4	1.16
3	0.26	0.78	9	2.34
4	0.16	0.64	16	2.56
5	0.12	0.60	25	3.00
	1.000	2.75		9.21

Mean: $\mu = \sum \left[x \cdot P(x) \right] \underline{= 2.75}$

Variance: $\sigma^2 = \sum \left[x^2 \cdot P(x) \right] - \mu^2 = 9.21 - (2.75)^2 \underline{= 1.6475}$

Standard deviation: $\sigma = \sqrt{1.6475} \underline{= 1.284}$

Exercise

Based on past results found in the *Information Please Almanac*, there is a 0.1919 probability that a baseball World Series context will last 4 games, is a 0.2121 probability that it will last 5 games, a 0.2222 probability that it will last 6 games, a 0.3737 probability that us will last 7 games.

a) Does the given information describe a probability distribution?

b) Assuming that the given information describes a probability distribution, find the mean and standard deviation for the numbers of games in World Series contests.

c) Is it unusual for a team to "sweep" by winning in four games? Why or why not?

Solution

a) Since $0 \le P(x) \le 1;\quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
4	0.1919	0.7676	16	3.0704
5	0.2121	1.0605	25	5.3025
6	0.2222	1.3332	36	7.9992
7	0.3737	2.6159	49	18.3113
	0.9999	5.7772		34.6834

Mean: $\mu = \sum [x \cdot P(x)] \underline{= 5.7772}$

Variance: $\sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2 = 34.6834 - (5.7772)^2 \underline{= 1.3074}$

Standard deviation: $\sigma = \sqrt{1.3074} \underline{= 1.1434}$

```
1-Var Stats
 x̄=5.778
 Σx=5.777
 Σx²=34.683
 Sx=
 σx=1.142
↓n=1.000
```

b) $\mu = \underline{5.8\ \text{games}}$ *and* $\sigma = \underline{1.1\ \text{games}}$

c) No, since $P(x=4) = 0.1919 > 0.05$, winning in 4 games is not an unusual event.

Exercise

Based on information from MRI Network, some job applicants are required to have several interviews before a decision is made. The number of required interviews and the corresponding probabilities are: 1 (0.09); 2 (0.31); 3 (0.37); 4 (0.12); 5 (0.05); 6 (0.05).

a) Does the given information describe a probability distribution?
b) Assuming that a probability distribution is described, find its mean and standard deviation.
c) Use the range rule of thumb to identify the range of values for usual numbers of interviews.
d) Is it unusual to have a decision after just one interview? Explain?

Solution

a) Since $0 \le P(x) \le 1;\quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
1	0.09	0.09	1	0.09
2	0.31	0.62	4	1.24
3	0.37	1.11	9	3.33
4	0.12	0.48	16	1.92
5	0.05	0.25	25	1.25
6	0.05	0.30	36	1.80
	0.99	2.85		9.63

Mean: $\mu = \sum [x \cdot P(x)] \underline{= 2.85}$

Variance: $\sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2 = 9.63 - (2.85)^2 \underline{= 1.5075}$

Standard deviation: $\sigma = \sqrt{1.5075} \underline{= 1.228}$

b) $\mu = \underline{2.9\ \text{interviews}}$ *and* $\sigma = \underline{1.2\ \text{interviews}}$

c) The range rule of thumb suggests that "usual" values are those within two standard deviations of the mean.

$$\textit{Minimum usual value } = \mu - 2\sigma = 2.9 - 2(1.2) \ \underline{= 0.5}$$

$$\textit{Maximum usual value } = \mu + 2\sigma = 2.9 + 2(1.2) \ \underline{= 5.3}$$

The range of values for usual numbers of interviews is from 0.5 to 5.3.

d) No, since $0.05 < 1 < 5.3$, it is not unusual to have a decision after just one interview.

Exercise

Based on information from Car dealer, when a car is randomly selected the number of bumper stickers and the corresponding probabilities are: 0 (0.824); 1 (0.083); 2 (0.039); 3 (0.014); 4 (0.012); 5 (0.008); 6 (0.008); 7 (0.004); 8 (0.004); 9 (0.004).

a) Does the given information describe a probability distribution?

b) Assuming that a probability distribution is described, find its mean and standard deviation.

c) Use the range rule of thumb to identify the range of values for usual numbers of bumper stickers.

d) Is it unusual for a car to have more than one bumper sticker? Explain?

Solution

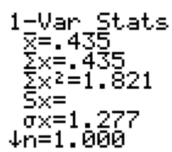
a) Since $0 \le P(x) \le 1; \quad \sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.824	0	0	0
1	0.083	0.083	1	0.83
2	0.039	0.078	4	0.156
3	0.014	0.042	9	0.126
4	0.012	0.048	16	0.192
5	0.008	0.040	25	0.288
6	0.008	0.048	36	0.288
7	0.004	0.028	49	0.196
8	0.004	0.032	64	0.256
9	0.004	0.036	81	0.324
	1.000	0.435		1.821

Mean: $\mu = \sum \left[x \cdot P(x) \right] \underline{= 0.435}$

Variance: $\sigma^2 = \sum \left[x^2 \cdot P(x) \right] - \mu^2 = 1.821 - (0.435)^2 \underline{= 1.635}$

Standard deviation: $\sigma = \sqrt{1.635} \ \underline{= 1.279}$



b) $\mu = \underline{0.4 \ \textit{bumper stickers}}$ *and* $\sigma = \underline{1.3 \ \textit{bumper stickers}}$

c) The range rule of thumb suggests that "*usual*" values are those within two standard deviations of the mean.

$$\textit{Minimum usual value } = \mu - 2\sigma = 0.4 - 2(1.3) \ \underline{= -2.2}$$

$$\textit{Maximum usual value } = \mu + 2\sigma = 0.4 + 2(1.3) \ \underline{= 3.0}$$

The range of values for usual numbers of interviews is from 0 to 3.0.

d) No, since $0 < 1 < 3.0$, it is not unusual to have more than 1 bumper sticker.

Exercise

A Company hired 8 employees from a large pool of applicants with an equal numbers of males and females. If the hiring is done without regard to sex, the numbers of females hired and the corresponding probabilities are: 0 (0.004); 1 (0.0313); 2 (0.109); 3 (0.219); 4 (0.273); 5 (0.219); 6 (0.109); 7 (0.031); 8 (0.004).

a) Does the given information describe a probability distribution?

b) Assuming that a probability distribution is described, find its mean and standard deviation.

c) Use the range rule of thumb to identify the range of values for usual numbers of females hired in such groups of 8.

d) If the most recent group of 8 newly hired employees does not include any females, does there appear to be discrimination based on sex? Explain?

Solution

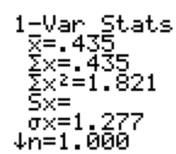
a) Since $0 \le P(x) \le 1$; $\sum P(x) = 1$

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.004	0	0	0
1	0.031	0.031	1	0.031
2	0.109	0.218	4	0.436
3	0.219	0.657	9	1.971
4	0.273	1.092	16	4.368
5	0.219	1.095	25	5.475
6	0.109	0.654	36	3.924
7	0.031	0.217	49	1.519
8	0.004	0.032	64	0.256
	0.999	3.996		17.980

Mean: $\mu = \sum [x \cdot P(x)] = 3.996$

Variance: $\sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2 = 17.980 - (3.996)^2 = 2.012$

Standard deviation: $\sigma = \sqrt{2.02} = 1.418$



b) $\mu = 4$ *females* and $\sigma = 1.4$ *females*

c) The range rule of thumb suggests that "*usual*" values are those within two standard deviations of the mean.

Minimum usual value $= \mu - 2\sigma = 4 - 2(1.4) = 1.2$

Maximum usual value $= \mu + 2\sigma = 4 + 2(1.4) = 6.8$

The range of values for usual numbers of interviews is from 1.2 to 6.8.

d) Yes, since $0 < 1.2$, it would be unusual to hire no females if only factors were in operation.

Exercise

Let the random variable x represent the number of girls in a family of 4 children. Construct a table describing the probability distribution; then find the mean and the standard deviation. (Hint: List the different possible outcomes.) Is it unusual for a family of 3 children to consist of 3 girls?

Solution

The sample space is: $S = \{GGG,\ GGB,\ GBG,\ BGG,\ BBG,\ BGB,\ GBB,\ BBB\}$

Therefore, there are 8 equally likely possible outcomes.

x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
0	0.125	0	0	0
1	0.375	0.375	1	0.375
2	0.375	0.750	4	1.500
3	0.125	0.375	9	1.125
	1.000	1.500		3.000

Mean: $\mu = \sum \left[x \cdot P(x) \right] = 1.50$

Variance: $\sigma^2 = \sum \left[x^2 \cdot P(x) \right] - \mu^2 = 3.0 - (1.5)^2 = 0.75$

Standard deviation: $\sigma = \sqrt{.75} = .866$

$\mu = 1.5\ girls$ *and* $\sigma = 0.9\ girls$

No, since $P(x=3) = 0.125 > 0.05$, it is not unusual for a family to have all girls.

Exercise

In 4 lottery game, you pay 50¢ to select a sequence of 4 digits, such 1332. If you select the same sequence of 4 digits that are drawn, you win and collect \$2788.

a) How many different selections are possible?

b) What is the probability of winning?

c) If you win, what is your net profit?

d) Find the expected value.

Solution

a) Since each of the 4 positions could be filled with replacement by any of the 10 digits
$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000\ possibilities$

b) Since only one possible selection: $P(winning) = \frac{1}{10{,}000} = 0.0001$

c) The net profit is the payoff minus the original bet. \$2,788.00 – \$0.50 = \$2,787.50

d) The expected value is –22.1¢

Event	x	$P(x)$	$x \cdot P(x)$
Lose	–\$0.50	0.9999	–\$.49995
Gain	\$2787.50	0.0001	\$0.27875
Total			–\$0.2212

Exercise

When playing roulette at casino, a gambler is trying to decide whether to bet \$5 on the number 13 or bet \$5 that the outcomes any one of these 5 possibilities: 0 or 00 or 1 or 2 or 3.the expected value of the \$5 bet for a single number is –26¢. For the \$5 bet that the outcome 0 or 00 or 1 or 2 or 3, there is a probability of $\frac{5}{38}$ of making a net profit of \$30 and a $\frac{33}{38}$ probability of losing \$5.

a) Find the expected value for the \$5 bet that the outcome is 0 or 00 or 1 or 2 or 3.

b) Which bet is better: A \$5 bet on the number 13 or a \$5 bet the outcome is 0 or 00 or 1 or 2 or 3? Why?

Solution

a)

Event	x	$P(x)$	$x \cdot P(x)$
Lose	–5	$\frac{33}{38}$	$-\frac{165}{38}$
Gain	30	$\frac{5}{38}$	$\frac{150}{38}$
Total		1	–0.3947

The expected value is –39.5¢

b) Since $-26 > -39.5$, wagering \$5 on the number 13 is the better bet.

Exercise

There is a 0.9986 probability that a randomly selected 30-year-old male lives through the year. As insurance company charges \$161 for insuring that the male will live through the year. If the male does not survive the year, the policy pays out \$100,000 as a death benefit.

a) From the perspective of the 30-year-old male, what are the values corresponding to the 2 events of surviving the year and not surviving?

b) If a 30-year-old male purchases the policy, what is his expected value?

c) Can the insurance company expect to make a profit from many such policies? Why?

Solution

a) From the 30-year-old male's perspective, the 2 possible outcome values are –\$161, if he lives $100,000 - 161 = \$99,839$ is he dies.

b)

Event	x	$P(x)$	$x \cdot P(x)$
Lose	–161	0.9986	–160.7446
Gain	99,839	0.0014	139.7746
Total		1.0000	–21

The expected value is –\$21.0

c) Yes; the insurance company can expect to make an average of \$21.00 per policy.

Exercise

An insurance company charges a 21-year-old male a premium of \$500 for a one-year \$100,000 life insurance policy. A 21-year-old male has a 0.9985 probability of living for a year.

a) From the perspective of a 21-year-old male (or estate), what are the values of the two different outcomes?

b) What is the expected value for a 21-year-old male who buys the insurance?

c) What would be the cost of the insurance if the company just breaks even (in the long run with many such policies), instead of making a profit?

d) Given that the expected value is negative (so the insurance company can make a profit), why

Solution

a) The value if he lives is $-\$500$

The value if he dies is $=100,000-500 \ \underline{=\$99,500}$

b) The expected value is:

$(.9985)(500)+(1-.9985)(99500) \ \underline{=-\$350}$

c) $(.9985)x-(1-.9985)(100,000-x)=0$

$.9985x-150+.0015x=0$

$\underline{x=\$150}$

d) Insuring the financial security of loved ones compensates for the negative expected value.

Solution Section 2.7 – Binomial Probability Distribution

Exercise

20 different Senators are randomly selected from the 100 Senators in the current Congress, and each was asked whether he or she is in favor of abolishing estate taxes. Does this procedure result in a binomial distribution, if it is not binomial, identify at least one requirement that is not satisfied?

Solution

NO; requirements (2) and (4) are not met.

Since $\frac{20}{100} = 0.20 > 0.05$ and the selections are done without replacement, the trials cannot be considered independent. The value of p of obtaining a success changes from trial to trial as each selection without replacement changes the population from which the next selection is made.

Exercise

15 different Governors are randomly selected from the 50 Governors in the currently office and the sex of each Governor is recorded. Does this procedure result in a binomial distribution, if it is not binomial, identify at least one requirement that is not satisfied?

Solution

NO; requirements (2) and (4) are not met.

Since $\frac{15}{50} = 0.30 > 0.05$ and the selections are done without replacement, the trials cannot be considered independent. The value of p of obtaining a success changes from trial to trial as each selection without replacement changes the population from which the next selection is made.

Exercise

200 statistics students are randomly selected and each asked if he or she owns a TI-84 Plus calculator. Does this procedure result in a binomial distribution, if it is not binomial, identify at least one requirement that is not satisfied?

Solution

Yes; all requirements are met. Since 200 statistics students are assumed to be less than 5% of the population of all statistics students, the selections can considered to be independent – even though ther are made without replacement.

Exercise

Multiple choice questions on the SAT test have 5 possible answers (*a*, *b*, *c*, *d*, *e*), 1 of which is correct. Assume that you guess the answers to 3 such questions.

a) Use the multiplication rule to find the probability that the first 2 guesses are wrong and the third is correct. That is, find $P(WWC)$, where C denotes a correct answer and W denotes a wrong answer.

b) Beginning with WWC, make a complete list of the different possible arrangements of 2 wrong answers and 1 correct answer, then find the probability for each entry in the list.

c) Based on the proceeding results, what is the probability of getting exactly 1 correct answer when 3 guesses are made?

Solution

$$P(C:correct)=\frac{1}{5}, \quad P(W:wrong)=\frac{4}{5}$$

a) $P(WWC)=P(W)\cdot P(W)\cdot P(C)$

$$=\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{1}{5}$$

$$\underline{=0.128|}$$

b) There are: *WWC*, *WCW*, *CWW* – 3 possible arrangements

$$P(WWC)=P(W)\cdot P(W)\cdot P(C)=\frac{16}{125}$$

$$P(WCW)=P(W)\cdot P(C)\cdot P(W)=\frac{4}{5}\cdot\frac{1}{5}\cdot\frac{4}{5}=\frac{16}{125}$$

$$P(WCW)=P(C)\cdot P(W)\cdot P(W)=\frac{1}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}=\frac{16}{125}$$

c) $P(\text{exactly one correct})=P(WWC \text{ or } WCW \text{ or } CWW)$

$$=P(WWC)+P(WCW)+P(CWW)$$

$$=\frac{16}{125}+\frac{16}{125}+\frac{16}{125}$$

$$=\frac{48}{125}$$

$$\underline{=0.384|}$$

Exercise

A psychology test consists of multiple choice questions, each having 4 possible answers (*a*, *b*, *c*, *d*), 1 of which is correct. Assume that you guess the answers to 6 such questions.

a) Use the multiplication rule to find the probability that the first 2 guesses are wrong and the last 4 guesses are correct. That is, find $P(WWCCCC)$, where C denotes a correct answer and W denotes a wrong answer.

b) Beginning with $WWCCCC$, make a complete list of the different possible arrangements of 2 wrong answers and 4 correct answers, then find the probability for each entry in the list.

c) Based on the proceeding results, what is the probability of getting exactly 4 correct answers when 6 guesses are made?

Solution

$$P(C:correct)=\frac{1}{4}, \quad P(W:wrong)=\frac{3}{4}$$

a) $P(WWCCCC) = P(W) \cdot P(W) \cdot P(C) \cdot P(C) \cdot P(C) \cdot P(C)$

$$= \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$$

$$= \frac{9}{4096}$$

$$\underline{= 0.00220}$$

b) There are ${}_6C_2 = 15$ possible arrangements.

WWCCCC WCWCCC WCCWCC WCCCWC WCCCCW CWCCCW CCWCCW
CCCWCW CCCCWW CWWCCC CCWWCC CCCWWC CCCWWC CWCWCC CCWCWC

$P(each\ arrangement) = \frac{9}{4096}$ (part *a*)

c) $P(exactly\ 4\ correct) = P(WWCCCC\ or\ WCWCCC\ or\ \cdots\ or\ CCWCWC)$

$$= P(WWCCCC) + P(WCWCCC) + \ldots + P(CCWCWC)$$

$$= \frac{9}{4096} + \frac{9}{4096} + \cdots + \frac{9}{4096}$$

$$= 15 \cdot \frac{9}{4096}$$

$$\underline{= 0.0330}$$

Exercise

Use the Binomial Probability Table to find the probability of *x* success given the probability *p* of success on a single trial

$a)\quad n = 2,\quad x = 1,\quad p = .30$ $\qquad b)\quad n = 5,\quad x = 1,\quad p = 0.95$

$c)\quad n = 15,\quad x = 11,\quad p = 0.99$ $\qquad d)\quad n = 14,\quad x = 4,\quad p = 0.60$

$e)\quad n = 10,\quad x = 2,\quad p = 0.05$ $\qquad f)\quad n = 12,\quad x = 12,\quad p = 0.70$

Solution

a) $n = 2,\quad x = 1,\quad p = .30$

		p												
n	*x*	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
2	0	.980	.902	.810	.640	.490	.360	.250	.160	.090	.040	.010	.002	0+
	1	.020	.095	.180	.320	.420	.480	.500	.480	.420	.320	.180	.095	.020

$P(x = 1)\ \underline{= 0.420}$

b) $n = 5,\quad x = 1,\quad p = 0.95$

From the Table: $P(x = 1)\ \underline{= 0+}$

c) $n = 15,\quad x = 11,\quad p = 0.99$

15	0+	0+	0+	0+	0+	0+	0+	0+	.005	.035	.206	.463	.860

From the Table: $P(x = 11)\ \underline{= 0+}$

d) $n=14, \quad x=4, \quad p=0.60$

From the Table: $P(x=4)\ \underline{=0.014}$

e) $n=10, \quad x=2, \quad p=0.05$

From the Table: $P(x=2)\ \underline{=0.075}$

f) $n=12, \quad x=12, \quad p=0.70$

From the Table: $P(x=12)\ \underline{=0.014}$

Exercise

Use the Binomial Probability Formula to find the probability of *x* success given the probability *p* of success on a single trial

a) $n=12, \quad x=10, \quad p=\frac{3}{4}$

b) $n=9, \quad x=2, \quad p=0.35$

c) $n=20, \quad x=4, \quad p=0.15$

d) $n=15, \quad x=13, \quad p=\frac{1}{3}$

Solution

a) $n=12, \quad x=10, \quad p=\frac{3}{4} \rightarrow q=1-\frac{3}{4}=\frac{1}{4}$

$$P(x)=\frac{n!}{(n-x)!\ x!}\cdot p^x\cdot q^{n-x}$$

$$=\frac{12!}{(12-10)!\ 10!}\cdot\left(\frac{3}{4}\right)^{10}\left(\frac{1}{4}\right)^{12-10}$$

$$=\frac{12!}{2!\ 10!}\cdot\left(\frac{3}{4}\right)^{10}\left(\frac{1}{4}\right)^{2}$$

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12!(3/4)^10*(1/4
)^2/(2!10!
                .232
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$\underline{=0.232}$

b) $n=9, \quad x=2, \quad p=0.35 \rightarrow q=1-0.35=.65$

$$P(x)=\frac{n!}{(n-x)!\ x!}\cdot p^x\cdot q^{n-x}$$

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9!(.35²)*.65^7/(
7!2!
                .216
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$$=\frac{9!}{(9-2)!\ 2!}\cdot(.35)^2(.65)^{9-2}$$

$$=\frac{9!}{7!\ 2!}\cdot(.35)^2(.65)^7$$

9! .35 ^ 2 * .65 ^ 7 / (7!* 2!)

$\underline{=0.216}$

c) $n=20, \quad x=4, \quad p=0.15 \rightarrow q=1-0.15=.85$

$$P(x)=\frac{n!}{(n-x)!\ x!}\cdot p^x\cdot q^{n-x}$$

$$=\frac{20!}{16!\ 4!}\cdot(.15)^4(.85)^{16}$$

20! .15 ^ 4 * .85 ^ 4 / (16! 4!)

$\underline{=0.182}$

$d)\quad n=15,\quad x=13,\quad p=\frac{1}{3}\to q=1-\frac{1}{3}=\frac{2}{3}$

$$P(x)=\frac{n!}{(n-x)!\,x!}\cdot p^{x}\cdot q^{n-x}$$

$$=\frac{15!}{2!\,13!}\cdot\left(\frac{1}{3}\right)^{13}\left(\frac{2}{3}\right)^{2}$$

$15!\ (1/3)\wedge13*(2/3)\wedge2/(2!\ 13!)$

$$\underline{=0.0000293}$$

Exercise

In the US, 40% of the population have brown eyes. If 14 people are randomly selected, find the probability that at least 12 of them have brown eyes. Is it unusual to randomly select 14 people and find that at least 12 of them have brown eyes? Why or why not?

Solution

Let x = number of people with brown eyes.

Binomial: $n=14;\quad p=0.4$

From the Binomial Probability Table:

$$P(x\ge12)=P(x=12)+P(x=13)+P(x=14)$$

$$=0.001+0^{+}+0^{+}$$

$$\underline{=0.001}$$

Yes, since $0.001\le0.05$, getting at least 12 persons with brown eyes would be unusual.

Exercise

When blood donors were randomly selected, 45% of them had blood that is Group *O*. The display shows that the probabilities obtained by entering the values of n = 5 and p = 0.45.

a) Find the probability that at least 1 of the 5 donors has Group *O* blood. If at least 1 Group *O* donor is needed, is it reasonable to expect that at least 1 will be obtained?

b) Find the probability that at least 3 of the 5 donors have Group *O* blood. If at least 3 Group *O* donors are needed, is it very likely to expect that at least 3 will be obtained?

c) Find the probability that all donors have Group *O* blood. Is it unusual to get 5 Group *O* donors from 5 randomly selected donors? Why or Why not?

d) Find the probability that at most 2 of the 5 donors have Group *O* blood.

Solution

x	*P(x)*
0	0.050328
1	0.205889
2	0.336909
3	0.275653
4	0.112767
5	0.018453

a) $P(x\ge1)=1-P(x=0)$

$$=1-0.050328$$

$$\underline{\approx0.95}$$

Yes, it is reasonable to expect that at least one group O donor will be obtained.

b) $P(x \ge 3) = P(x=3) + P(x=4) + P(x=5)$

$$= 0.275653 + 0.112767 + 0.018453$$
$$= 0.406873$$
$$\approx 0.407$$

No; it is not *very likely* that at least 3 group of O donors will be obtained.
Since $0.407 > 0.05$ getting at least 3 such donors would not be an unusual event – but it would not be considered *very likely*.

c) $P(x=5) = 0.018453 \approx 0.018$

Yes, since $0.018 \le 0.05$ getting all 5 donors from group O would be considered unusual event.

d) $P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$

$$= 0.050328 + 0.205889 + 0.336909$$
$$= 0.593126$$
$$\approx 0.593$$

Exercise

There is 1% delinquency rate for consumers with FICO credit rating scores above 800. If a bank provides large loans 12 people with FICL scores above 800, what is the probability that at least one of them becomes delinquent? Based on that probability, should the bank plan on dealing with a delinquency?

Solution

Let *x* = number of delinquencies
Binomial: $n = 12$; $p = 0.01$; from the Binomial Probability Table:

$$P(x \ge 1) = 1 - P(x=0)$$
$$= 1 - .886$$
$$= 0.114$$

Yes, since $0.114 > 0.05$, it would be unusual for at least one of the people to become delinquent. The bank should make plans for dealing with a delinquency.

Exercise

Ten peas are generated from parents having the green/yellow pair of genes, so there is a 0.75 probability that an individual pea will have a green pod. Find the probability that among the 10 offspring peas, at least 9 have green pods. Is it unusual to get at least 9 peas with green pods when 10 offspring peas are generated? Why or why not?

Solution

Let x = number of delinquencies

Binomial: $n=10; \quad p=0.75$; using the binomial formula:

$$P(x)=\frac{n!}{(n-x)!\ x!}\cdot p^x\cdot q^{n-x}$$

$$P(x\ge 1)=1-P(x=0)$$

$$=1-\frac{10!}{10!\ 0!}\cdot(.75)^0\cdot(.25)^{10}$$

$$\underline{=0.999999046}$$

The usual rounded rule (to 3 significant digits) is not satisfactory in this case since applying that rule would suggest $P(x\ge 1)=1.00$, which is a certainly.

In this case, 6 significant digits are necessary to differentiate the probability of this very likely event from the probability of an event that is a certainly.

Exercise

You purchased a slot machine configured so that there is a $\frac{1}{2,000}$ probability of winning the jackpot on any individual trial. Although no one would seriously consider tricking the author, suppose that a guest claims that she played the slot machine 5 times and hit the jackpot twice

a) Find the probability of exactly 2 jackpots in 5 trials.
b) Find the probability of at least 2 jackpots in 5 trials.
c) Does the guest's claim of hitting 2 jackpots in 5 trials seem valid? Explain.

Solution

Let x = number of jackpots hit.

Binomial: $n=5; \quad p=\frac{1}{2000}=0.0005$; using the binomial formula: $P(x)=\frac{n!}{(n-x)!\ x!}\cdot p^x\cdot q^{n-x}$

a) $P(x=2)=\frac{5!}{3!\ 2!}\cdot(.0005)^2\cdot(.9995)^3$

$$\underline{=0.00002496}$$

b) $P(x\ge 2)=1-P(x<2)$

$$=1-\left[P(x=0)+P(x=1)\right]$$

$$=1-\left[\frac{5!}{5!\ 0!}\cdot(.0005)^0\cdot(.9995)^5+\frac{5!}{4!\ 1!}\cdot(.0005)^1\cdot(.9995)^4\right]$$

$$=1-[.997502499+.002495004]$$

$= 0.000002497$

c) No; since $0.00002497 \leq 0.05$, it would be unusual to hit 2 jackpots. If the machine is functioning as it is supposed to, either the guest is not telling the truth or an extremely rare event has occurred.

Exercise

In a survey of 320 college graduates, 36% reported that they stayed on their first full-time job less than one year.

a) If 15 of those survey subjects are randomly selected without replacement for a follow-up survey, find the probability that 5 of them stayed on their first full-time job less than one year.

b) If part (a) is changed so that 20 different survey subjects are selected, explain why the binomial probability formula *cannot* be used.

Solution

Let x = number who stayed less than one year.

Binomial: $n = 15$; $p = 0.36$; using the binomial formula: $P(x) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x}$

a) $P(x=5) = \frac{15!}{10!\;5!} \cdot (0.36)^5 \cdot (.64)^{10}$ $= 0.209$

b) The binomial distribution requires that the repeated selections be independent. Since these persons are selected from the original group of 320 without replacement, the repeated selections are not independent and the binomial distribution should not be used.

The sample size is $\frac{15}{320} = .046 \leq .05$ of the population and the repeated samples may be treated as though they are independent.

If the sample size is increased to 20, the sample is $\frac{20}{320} = .0625 > .05$ of the population and the criteria for using independence to get an approximate probability is no longer met.

Exercise

In a survey of 150 senior executives, 47% said that the most common job interview mistake is to have little or no knowledge of the company.

a) If 6 of those surveyed executives are randomly selected without replacement for a follow-up survey, find the probability that 3 of them said that the most common job interview mistake is to have little or no knowledge of the company.

b) If part (a) is changed so that 9 different surveyed executives are selected, explain why the binomial probability formula *cannot* be used.

Solution

Let x = number who said the most common mistake is not to know the company

Binomial: $n = 6$; $p = 0.47$; using the binomial formula: $P(x) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x}$

a) $P(x=3)=\frac{6!}{3!\ 3!}\cdot(0.47)^3\cdot(0.53)^3\ \boxed{=0.309}$

b) The binomial distribution requires that the repeated selections be independent. Since these persons are selected from the original group of 150 without replacement, the repeated selections are not independent and the binomial distribution should not be used. In part (*a*), however, the sample size is $\frac{6}{150}=0.04\le 0.05$ of the population and the repeated samples may be treated as though they are independent. If the sample size is increased to 9, the sample is $\frac{9}{150}=0.06>0.05$ of the population and the criteria for using independence to get an approximate probability is no longer met.

Exercise

In a Gallup poll of 1236 adults, it was found that 5% of those polled said that bad luck occurs after breaking a mirror. Based on these results, such randomly selected groups of 1236 adults will have a mean of 61.8 people with that belief, and a standard deviation of 7.7 people. What is the variance?

Solution

Exact formula: $\sigma^2=npq=1236(0.05)(0.95)\ \boxed{=58.71\ \ people^2}$

Using the rounded value: $\sigma^2=(\sigma)^2=(7.7)^2\ \boxed{=59.29\ \ people^2}$

Exercise

Random guesses are made for 50 SAT multiple choice questions, so $n=50$ and $p=0.2$.

a) Find the mean μ and standard deviation σ.

b) Use the range rule of thumb to find the minimum usual number and the maximum usual number.

Solution

a) *Mean*: $\mu=np=(50)(0.2)\ \boxed{=10.0}$

Standard deviation: $\sigma=\sqrt{npq}=\sqrt{50(0.2)(0.8)}\ \boxed{=2.8}$

b) *Minimum usual value* $=\mu-2\sigma=10-2(2.828)\ \boxed{=4.3}$

Maximum usual value $=\mu+2\sigma=10+2(2.828)\ \boxed{=15.7}$

Exercise

In an analysis of test result from the YSORT gender selection method, 152 babies are born and it is assumed that boys and girls are equally likely, so $n=152$ and $p=0.5$.

a) Find the mean μ and standard deviation σ.

b) Use the range rule of thumb to find the minimum usual number and the maximum usual number.

Solution

a) *Mean*: $\mu = np = (152)(0.5) \; \underline{=76.0}$

Standard deviation: $\sigma = \sqrt{npq} = \sqrt{152(0.5)(0.5)} \; \underline{=6.164}$

b) *Minimum usual value* $= \mu - 2\sigma = 76 - 2(6.164) \; \underline{=63.7}$

Maximum usual value $= \mu + 2\sigma = 76 + 2(6.164) \; \underline{=88.3}$

Exercise

In a Gallup poll of 1236 adults, it showed that 145% believe that bad luck follows if your path is crossed by a black car, so $n = 1236$ and $p = 0.14$.

a) Find the mean μ and standard deviation σ.

b) Use the range rule of thumb to find the minimum usual number and the maximum usual number.

Solution

a) *Mean*: $\mu = np = (1236)(0.14) \; \underline{=173.04}$

Standard deviation: $\sigma = \sqrt{npq} = \sqrt{1236(0.14)(0.86)} \; \underline{=12.199}$

b) *Minimum usual value* $= \mu - 2\sigma = 173.04 - 2(12.199) \; \underline{=148.6}$

Maximum usual value $= \mu + 2\sigma = 173.04 + 2(12.199) \; \underline{=197.4}$

Exercise

The midterm exam in a nursing course consists of 75 true/false questions. Assume that an unprepared student makes random guesses for each of the answers.

a) Find the mean and standard deviation for the number of correct answers for such students.

b) Would it be unusual for a student to pass this exam by guessing and getting at least 45 correct answers? Why or why not?

Solution

Since the questions are T/F, then $p = \frac{1}{2} = 0.5 = q$ and $n = 75$

a) *Mean*: $\mu = np = (75)(0.5) \; \underline{=37.5}$

Standard deviation: $\sigma = \sqrt{npq} = \sqrt{75(0.5)(0.5)} \; \underline{=4.33}$

b) *Minimum usual value* $= \mu - 2\sigma = 37.5 - 2(4.33) \; \underline{=28.8}$

Maximum usual value $= \mu + 2\sigma = 37.5 + 2(4.33) \; \underline{=46.2}$

No, Since 45 is within the above limits, it would not be unusual for a student to pass by getting at least 45 correct answers.

Exercise

The final exam in a nursing course consists of 100 multiple-choice questions. Each question has 5 possible answers, and only 1 of them is correct. An unprepared student makes random guesses for all of the answers.

a) Find the mean and standard deviation for the number of correct answers for such students.

b) Would it be unusual for a student to pass this exam by guessing and getting at least 60 correct answers? Why or why not?

Solution

Since there are 5 questions, and only 1 of them is correct $p = \frac{1}{5} = 0.2; \quad q = 0.8$ and $n = 100$

a) *Mean*: $\mu = np = (100)(0.2)\ \underline{= 20.0}$

Standard deviation: $\sigma = \sqrt{npq} = \sqrt{100(0.2)(0.8)}\ \underline{= 4.0}$

b) *Minimum usual value* $= \mu - 2\sigma = 20 - 2(4)\ \underline{= 12.0}$

Maximum usual value $= \mu + 2\sigma = 20 + 2(4)\ \underline{= 28.0}$

Yes, Since 60 is not within the above limits, it would be unusual for a student to pass by getting at least 60 correct answers.

Exercise

In a test of the XSORT method of gender selection, 574 babies are born to couples trying to have baby girls, and 525 of those babies are girls.

a) If the gender-selection method has no effect and boys and girls are equally likely, find the mean and standard deviation for the numbers of girls born in groups of 574.

b) Is the result of 525 girls unusual? Does it suggest that the gender-selection method appears to be effective?

Solution

Binomial problem: since 2 genders, $p = \frac{1}{2} = 0.5; \quad q = 0.5$ and $n = 574$

a) *Mean*: $\mu = np = (574)(0.5)\ \underline{= 287.0}$

Standard deviation: $\sigma = \sqrt{npq} = \sqrt{574(0.5)(0.5)}\ \underline{= 11.979}$

b) *Minimum usual value* $= \mu - 2\sigma = 287.0 - 2(11.979)\ \underline{= 263.0}$

Maximum usual value $= \mu + 2\sigma = 287.0 + 2(11.979)\ \underline{= 311.0}$

Yes, Since 525 is not within the above limits, it would be unusual for 574 births to include 525 girls. The results suggest that the gender selection method is effective.

Exercise

In a test of the YSORT method of gender selection, 152 babies are born to couples trying to have baby boys, and 127 of those babies are boys.

a) If the gender-selection method has no effect and boys and girls are equally likely, find the mean and standard deviation for the numbers of boys born in groups of 152.

b) Is the result of 127 boys unusual? Does it suggest that the gender-selection method appears to be effective?

Solution

Binomial problem; and since 2 genders, $p = \frac{1}{2} = 0.5; \quad q = 0.5$ and $n = 152$

a) *Mean*: $\mu = np = (152)(0.5) \ \underline{= 76.0}$

Standard deviation: $\sigma = \sqrt{npq} = \sqrt{152(0.5)(0.5)} \ \underline{= 6.164}$

b) *Minimum usual value* $= \mu - 2\sigma = 76.0 - 2(6.164) \ \underline{= 63.7}$

Maximum usual value $= \mu + 2\sigma = 76.0 + 2(6.164) \ \underline{= 86.3}$

Yes, Since 127 is not within the above limits, it would be unusual for 152 births to include 127 boys. The results suggest that the gender selection method is effective.

Exercise

A headline in USA Today states that "most stay at first job less than 2 years." That headline is based on a poll of 320 college graduates. Among those polled, 78% stayed at their full-time job less than 2 years.

a) Assuming that 50% is the true percentage of graduates who stay at their first job less than 2 years, find the mean and the standard deviation of the numbers of such graduates in randomly selected groups of 320 graduates.

b) Assuming that the 50% rate in part (*a*) is correct; find the range of usual values for the numbers of graduates among 320 who stay at their first job less than 2 years.

c) Find the actual number of surveyed who stayed at their first job less 2 years. Use the range of values from part (*b*) to determine whether that number is unusual. Does the result suggest that the headline is not justified?

d) This statement was given as part of the description of the survey methods used: "Alumni who opted-in to receive communications from Experience were invited to participate in the online poll, and 320 of them completed the survey." What does that statement suggest about the result?

Solution

Let x = the number who stay at their job less than 2 years.

Binomial problem, $p = 0.5; \quad q = 0.5$ and $n = 320$

a) *Mean*: $\mu = np = (320)(0.5) \ \underline{= 160.0}$

Standard deviation: $\sigma = \sqrt{npq} = \sqrt{320(0.5)(0.5)} \ \underline{= 8.944}$

b) *Minimum unusual value* $= \mu - 2\sigma = 160.0 - 2(8.944) \ \underline{= 142.1}$

$Maximum\ unusual\ value\ = \mu + 2\sigma = 160.0 + 2(8.944)\ \underline{=177.9}|$

c) $x = (.78)(320) \approx 250$

Since 250 is not within the above limits, it would be unusual for 320 graduates to include 250 persons who stayed at their job less than 2 years if the true proportion were 50%.
Since 250 is greater than above limits, the true proportion is most likely greater than 50%. The result suggests that the headline is justified.

d) The statement suggests that the 320 participants were a voluntary response sample, and so the results might not be representative of the target population.

Exercise

In a study of 420,095 cell phone users in Denmark, it was found that 135 developed cancer of the brain or nervous system. If we assume that the use of cell phones has no effect on developing such cancer, then the probability of a person having such cancer is 0.000340.

a) Assuming that the cell phones have no effect on developing cancer, find the mean and the standard deviation of the numbers of people in groups of 420,095 that can be expected to have cancer of the brain or nervous system.

b) Based on the result from part (*a*), is it unusual to find that among 420,095 people, there are 135 cases of cancer of the brain or nervous system? Why or why not?

c) What do these results suggest about the publicized concern that cell phones are a health danger because they increase the risk of cancer of the brain or nervous system?

Solution

Binomial problem, $p = 0.00034;\quad q = 0.99966$ and $n = 420{,}095$

a) *Mean*: $\mu = np = (420{,}095)(0.00034)\ \underline{=142.8}|$

Standard deviation: $\sigma = \sqrt{npq} = \sqrt{(420{,}095)(0.00034)(0.99966)}\ \underline{=11.949}|$

b) $Minimum\ unusual\ value\ = \mu - 2\sigma = 142.8 - 2(11.949)\ \underline{=118.9}|$

$Maximum\ unusual\ value\ = \mu + 2\sigma = 142.8 + 2(11.949)\ \underline{=166.7}|$

No, since 135 is within the above limits, it is not an unusual result.

c) These results do not provide evidence that cell phone use increases the risk of such cancers.

Exercise

Mario's Pizza Parlor has just opened. Due to a lack of employee training, there is only a 0.8 probability that a pizza will be edible. An order for 5 pizzas has just been placed. What is the minimum number of pizzas that must be made in order to be at least 99% sure that there will be 5 that are edible?

Solution

Let x = the number of edible pizzas.

Binomial problem, $p = 0.8;\quad \to q = 0.2$ and $n = unknown$

Find: $P(x \ge 5) \ge 0.99$

Using *Binomial Probability Table*:

For $n=5,\quad P(x\geq 5)=P(x=5)$

$$=0.328$$

For $n=6,\quad P(x\geq 5)=P(x=5)+P(x=6)$

$$=0.393+0.262$$
$$=0.655$$

For $n=7,\quad P(x\geq 5)=P(x=5)+P(x=6)+P(x=7)$

$$=0.275+0.367+0.210$$
$$=0.852$$

For $n=8,\quad P(x\geq 5)=P(x=5)+P(x=6)+P(x=7)+P(x=8)$

$$=0.147+0.294+0.336+0.168$$
$$=0.945$$

For $n=9,\quad P(x\geq 5)=P(x=5)+P(x=6)+P(x=7)+P(x=8)+P(x=9)$

$$=0.066+0.176+0.302+0.302+0.134$$
$$=0.980$$

For $n=10,\quad P(x\geq 5)=P(x=5)+P(x=6)+P(x=7)+P(x=8)+P(x=9)+P(x=10)$

$$=0.026+0.088+0.201+0.302+0.268+0.107$$
$$=0.992$$

The minimum number of pizza necessary to be at least 99% sure that there will be 5 edible pizzas available is n = 10.

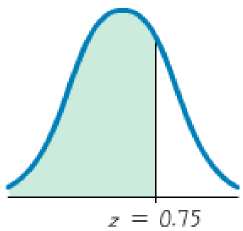
This procedure may not be the most efficient, but it is easy to follow and promotes better understanding of the concepts involved.

Solution Section 2.8 – Properties of the Normal Distribution

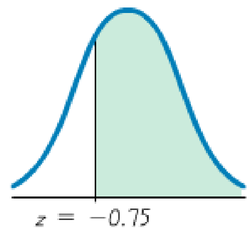
Exercise

Find the area shaded region. The graph depicts the standard distribution with mean 0 and standard deviation 1.

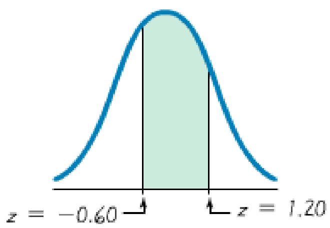
a)



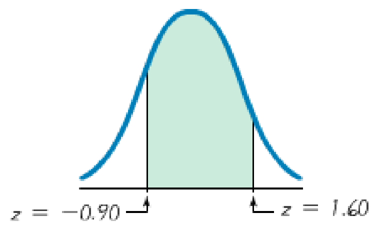
b)



c)



d)



Solution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852

a) $P(z < 0.75) \underline{= 0.7734}$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148

b) $P(z > -0.75) = 1 - P(z < -0.75)$

$$= 1 - 0.2266$$

$$\underline{= 0.7734}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451

c) $P(-0.60 < z < 1.20) = P(z < 1.20) - P(z < -0.60)$

$$= 0.8849 - 0.2743$$

$$\underline{= 0.6106}$$

d) $P(-0.90 < z < 1.60) = P(z < 1.60) - P(z < -0.90)$

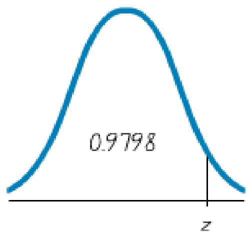
$$= 0.9452 - 0.1841$$

$$\underline{= 0.7611}$$

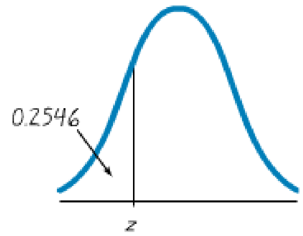
Exercise

Find the indicated z-score. The graph depicts the standard distribution with mean 0 and standard deviation 1.0.

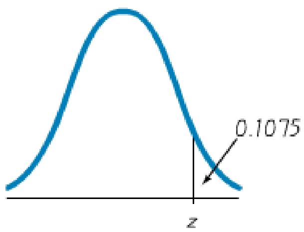
a)



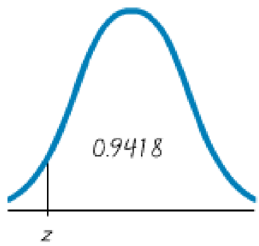
b)



c)



d)



<u>**Solution**</u>

Using Normal Distribution Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

a) For $A = 0.9798 \quad \Rightarrow \quad \underline{z = 2.05}$

b) For $A = 0.2546 \quad \Rightarrow \quad \underline{z = -0.66}$

c) Area to the right of z, then: $A = 1 - 0.1075 = 0.8925$

For $A = 0.8925 \quad \Rightarrow \quad \underline{z = 1.24}$

d) Area to the right of z, then: $A = 1 - 0.9418 = 0.0582$

For $A = 0.0582 \quad \Rightarrow \quad \underline{z = -1.57}$

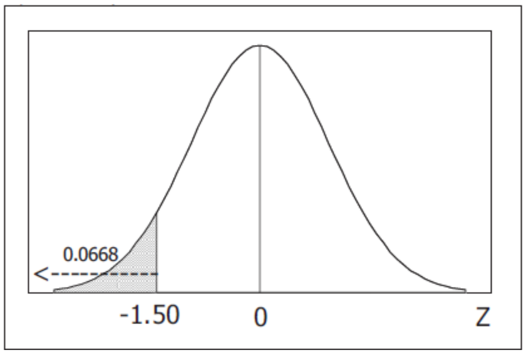
Exercise

Assume that thermometer readings are normally distributed with a mean of 0ºC and the standard deviation of the readings is 1.00ºC. A thermometer is randomly selected and tested. In each case, draw a sketch, and find the probability of each reading.

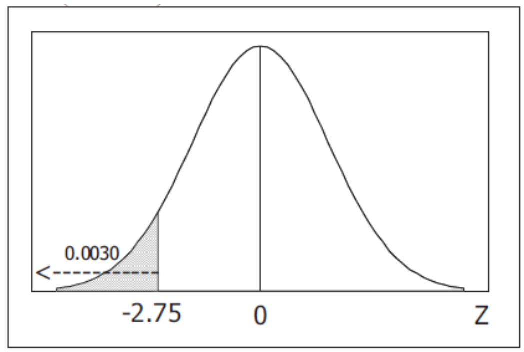
a) Less than –1.50
b) Less than –2.75
c) Less than 1.23
d) Greater than 2.22
e) Greater than 2.33
f) Greater than –1.75
g) Between 0.50 and 1.00
h) Between –3.00 and –1.00
i) Between –1.20 and 1.95
j) Between –2.50 and 5.00
k) Greater than 0
l) Less than 0

Solution

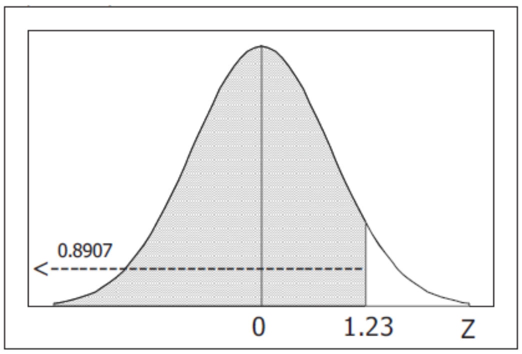
a) $P(z<-1.50)\ \underline{=0.0668|}$



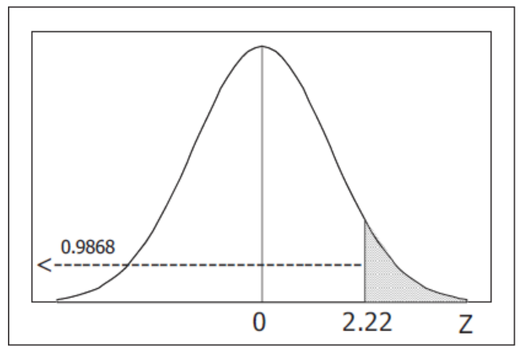
b) $P(z<-2.75)\ \underline{=0.0030|}$



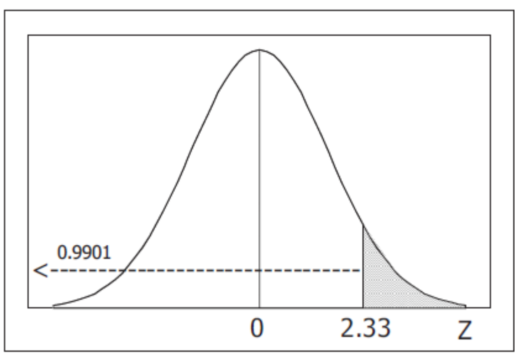
c) $P(z<1.23)\ \underline{=0.8907|}$



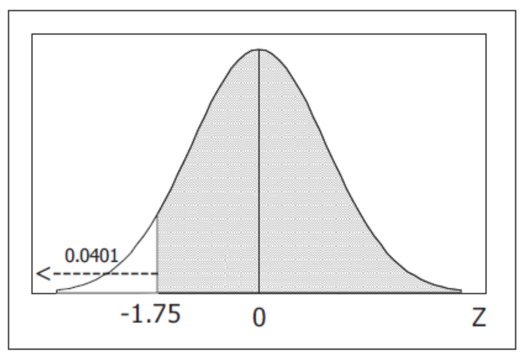
d) $P(z>2.22)=1-0.9868\ \underline{=0.0132|}$



e) $P(z>2.33)=1-0.9901\ \underline{=0.0099|}$



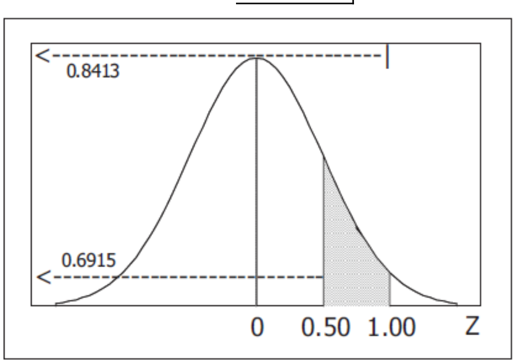
f) $P(z>-1.75)=1-0.0401\ \underline{=0.9599|}$



g) $P(0.50<z<1.00)=P(z<1)-P(z<0.50)$

$$=0.8413-0.6915$$

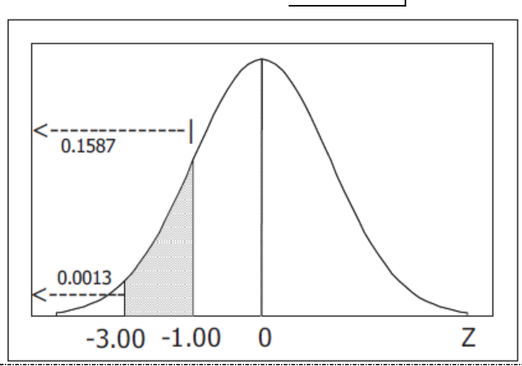
$$\underline{=0.1498|}$$



h) $P(-3.00<z<-1.00)=P(z<-1)-P(z<-3)$

$$=0.1587-0.0013$$

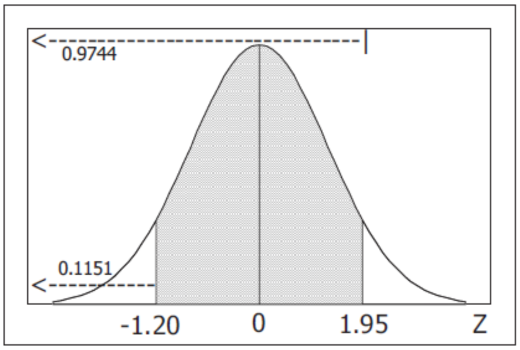
$$\underline{=0.1574|}$$



i) $P(-1.2 < z < 1.95) = P(z < 1.95) - P(z < -1.2)$

$= 0.9744 - 0.1151$

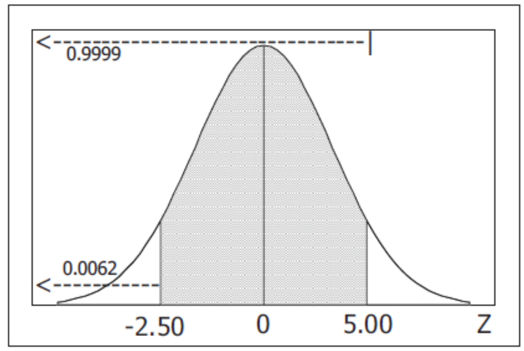
$= 0.8593$



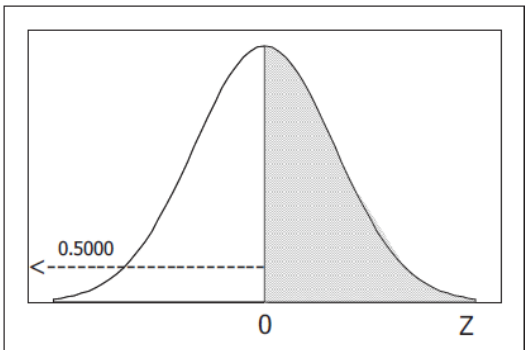
j) $P(-2.5 < z < 5) = P(z < 5) - P(z < -2.5)$

$= 0.9999 - 0.0062$

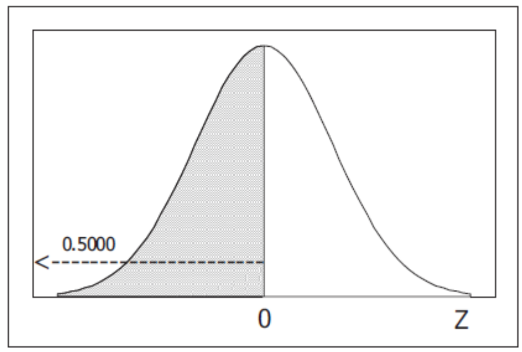
$= 0.9937$



k) $P(z > 0) = 1 - 0.5000 = 0.5000$



l) $P(z < 0) = 0.5000$



Exercise

Assume that thermometer readings are normally distributed with a mean of 0°C and the standard deviation of the readings is 1.00°*C*. A thermometer is randomly selected and tested. In each case, draw a sketch, and find the temperature reading corresponding to the given information.

a) Find P_{95}, the 95th percentile. This is the temperature separating the bottom 95% from the top 5%.

b) Find P_1, the 1st percentile. This is the temperature separating the bottom 1% from the top 99%.

c) If 2.5% of the thermometers are rejected because they have readings that are too high and another 2.5% are rejected because they have readings that are too low, find the 2 readings that are cutoff values separating the rejected thermometers from the others.

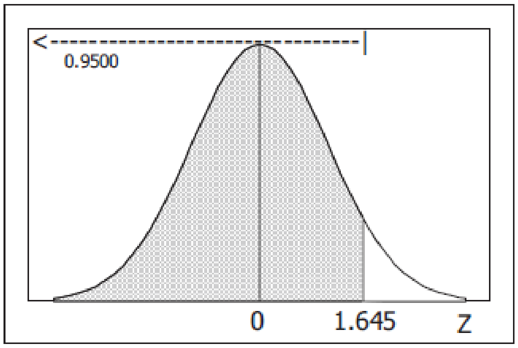
d) If 0.5% of the thermometers are rejected because they have readings that are too high and another 0.5% are rejected because they have readings that are too low, find the 2 readings that are cutoff values separating the rejected thermometers from the others.

Solution

a) For P_{95}, the cumulative area is 0.95000.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

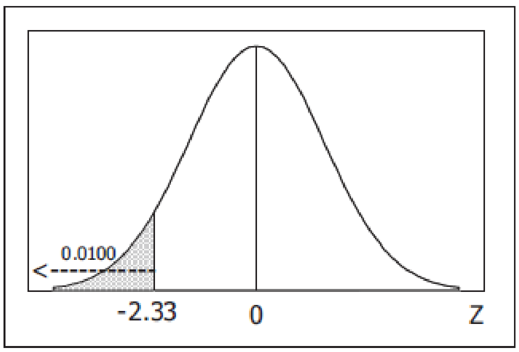
$A = 0.9500 \quad \Rightarrow \quad \boxed{z = 1.645}$



b) For P_1, the cumulative area is 0.0100.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084

$A = 0.0100 \quad \Rightarrow \quad \boxed{z = -2.33}$

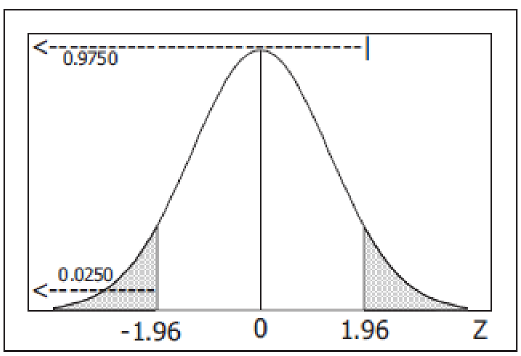


c) For the lowest 2.5%, the cumulative area is 0.0250.

$A = 0.0250 \quad \Rightarrow \quad \boxed{z = -1.96}$

For the highest 2.5%, the cumulative area is 1 – 0.0250 = 0.9750

$A = 0.9750 \quad \Rightarrow \quad \boxed{z = 1.96}$

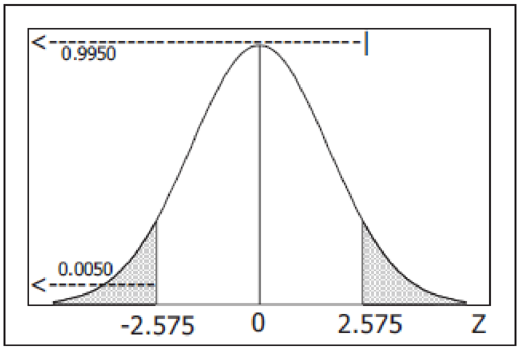


d) For the lowest 0.5%, the cumulative area is 0.0050.

$A = 0.0050 \quad \Rightarrow \quad \boxed{z = -2.575}$

For the highest 0.5%, the cumulative area is 1 – 0.0050 = 0.9950

$A = 0.9950 \quad \Rightarrow \quad \boxed{z = 2.575}$



Exercise

For a standard normal distribution, find the percentage of data that are

a) Within 2 standard deviations of the mean.

b) More than 1 standard deviation away from the mean.

c) More than 1.96 standard deviations away from the mean.

d) Between $\mu - 3\sigma$ and $\mu + 3\sigma$.

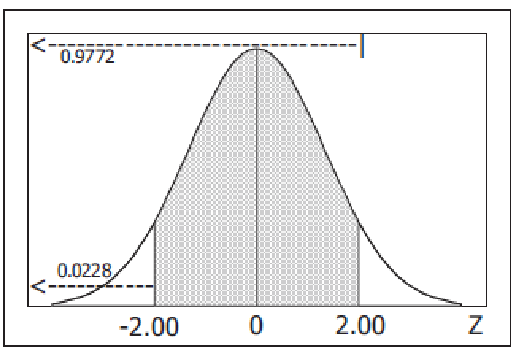
e) More than 3 standard deviations away from the mean.

Solution

a) $P(-2 < z < 2) = P(z < 2) - P(z < -2)$

$$= 0.9772 - 0.0228$$

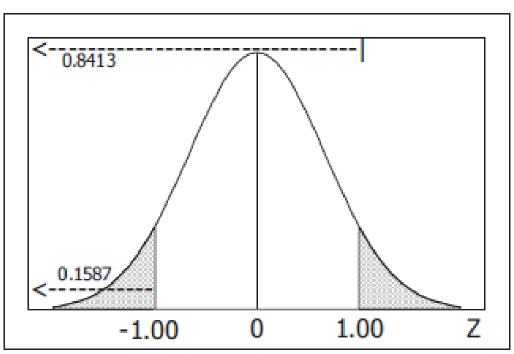
$$\underline{= 0.9544} \quad \textit{or} \quad 95.44\%$$



b) $P(z < -1 \text{ or } z > 1) = P(z < -1) + P(z > 1)$

$$= 0.1587 - (1 - .8413)$$

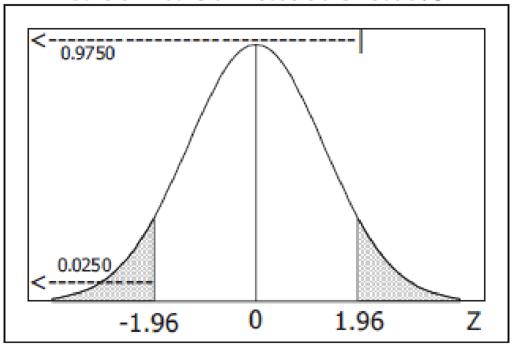
$$\underline{= 0.3174} \quad \textit{or} \quad 31.74\%$$



c) $P(z<-1.96 \text{ or } z>1.96) = P(z<-1.96) + P(z>1.96)$

$$= 0.0250 - (1 - .9750)$$

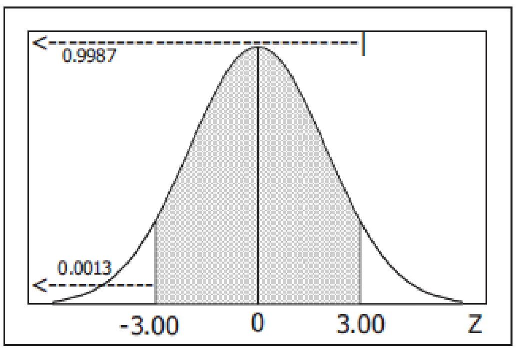
$$\underline{= 0.0500|} \quad \textit{or} \quad 5.00\%$$



d) $P(-3<z<3) = P(z=3) - P(z=-3)$

$$= 0.9987 - 0.0013$$

$$\underline{= 0.9974|} \quad \textit{or} \quad 99.74\%$$



Solution Section 2.9 – Applications of the Normal Distribution

Exercise

The distribution of IQ scores is a nonstandard normal distribution with mean of 100 and standard deviation of 15. What are the values of the mean and standard deviation after all IQ scores have been standardized by converting them to *z*-scores using $z = \dfrac{x-\mu}{\sigma}$?

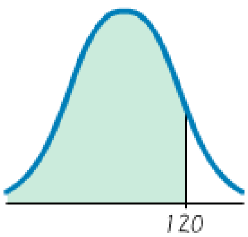
Solution

For any distribution, converting to *z* scores using the formula $z = \dfrac{x-\mu}{\sigma}$ produces a same-shaped distribution with mean 0 and standard deviation 1.

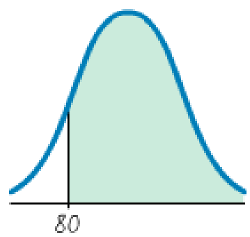
Exercise

Find the area of the shaded region. The graphs depict IQ scores adults, and those scores are normally distributed with mean of 100 and standard deviation of 15.

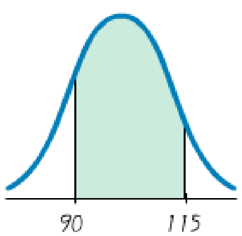
a)



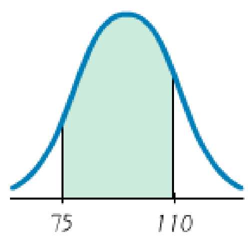
b)



c)



d)



Solution

a) $z = \dfrac{x-\mu}{\sigma} = \dfrac{120-100}{15} = \dfrac{20}{15} = 1.33$

$P(x<120) = P(z=1.33) = 0.9082$

b) $z = \dfrac{x-\mu}{\sigma} = \dfrac{80-100}{15} = -\dfrac{20}{15} = -1.33$

$$\begin{aligned} P(x>80) &= P(z>-1.33) \\ &= 1 - P(z<-1.33) \\ &= 1 - 0.0918 \\ &= 0.9082 \end{aligned}$$

c) $x = 90 \quad \rightarrow \quad z = \dfrac{x-\mu}{\sigma} = \dfrac{90-100}{15} = -\dfrac{10}{15} \; \underline{= -0.67|}$

$x = 115 \quad \rightarrow \quad z = \dfrac{x-\mu}{\sigma} = \dfrac{115-100}{15} \; \underline{= 1|}$

$$\begin{aligned} P(90 < x < 115) &= P(-0.67 < z < 1.00) \\ &= P(z < 1.00) - P(z < -0.67) \\ &= 0.8413 - 0.0475 \\ &\underline{= 0.5899|} \end{aligned}$$

d) $x = 75 \quad \rightarrow \quad z = \dfrac{x-\mu}{\sigma} = \dfrac{75-100}{15} = -\dfrac{25}{15} \; \underline{= -1.67|}$

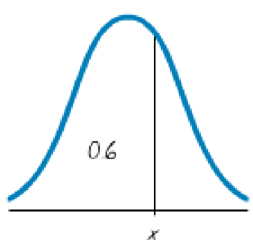
$x = 110 \quad \rightarrow \quad z = \dfrac{x-\mu}{\sigma} = \dfrac{110-100}{15} \; \underline{= 0.67|}$

$$\begin{aligned} P(75 < x < 110) &= P(-1.67 < z < 0.67) \\ &= P(z < 067) - P(z < -1.67) \\ &= 0.7846 - 0.0475 \\ &\underline{= 0.7011|} \end{aligned}$$

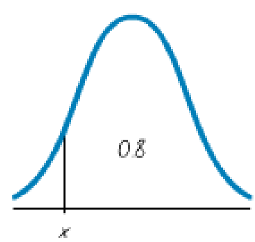
Exercise

Find the Indicated IQ scores. The graphs depict IQ scores adults, and those scores are normally distributed with mean of 100 and standard deviation of 15.

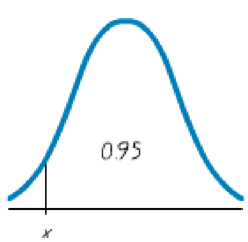
a)



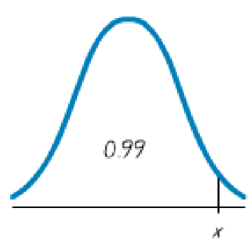
b)



c)



d)



Solution

a) The z score with A = 0.6 below it is: z = 0.25 (~0.5987)

$$\begin{aligned} x &= \mu + z\sigma \\ &= 100 + (0.25)(15) \\ &= 103.75 \\ &\underline{= 103.8|} \end{aligned}$$

b) The z score with $A = 0.8$ above is the z score with $A = 0.2$ below; it is: $z = -0.84$

$$\begin{aligned} x &= \mu + z\sigma \\ &= 100 + (-0.84)(15) \\ &= 87.4 \end{aligned}$$

c) The z score with $A = 0.95$ above is the z score with $A = 0.05$ below; it is: $z = -1.645$

$$\begin{aligned} x &= \mu + z\sigma \\ &= 100 + (-1.645)(15) \\ &= 75.3 \end{aligned}$$

d) The z score with $A = 0.99$ below; it is: $z = 2.33$

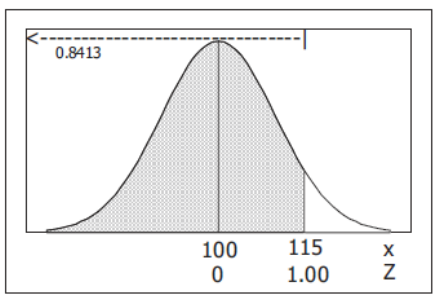
$$\begin{aligned} x &= \mu + z\sigma \\ &= 100 + (2.33)(15) \\ &= 135.0 \end{aligned}$$

Exercise

Assume that adults have IQ scores that are normally distributed with mean of 100 and standard deviation of 15

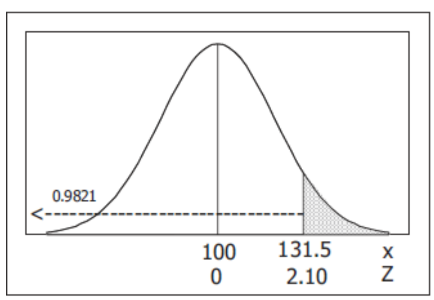
a) Find the probability that a randomly selected adult has an IQ that is less than 115.
b) Find the probability that a randomly selected adult has an IQ that is greater than 131.5.
c) Find the probability that a randomly selected adult has an IQ that is between 90 and 110.
d) Find the probability that a randomly selected adult has an IQ that is between 110 and 120.
e) Find P_{30} which is the IQ score separating the bottom 30% from the top 70%.
f) Find the first quartile Q_1 which is the IQ score separating the bottom 25% from the top 75%.

Solution



a) $x = 115 \quad \rightarrow \quad z = \dfrac{x-\mu}{\sigma} = \dfrac{115-100}{15} = \dfrac{15}{15} = 1$

$P(x < 115) = P(z < 1) = 0.8413$



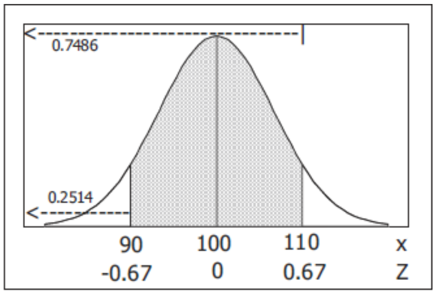
b) $x = 131.5 \quad \rightarrow \quad z = \dfrac{x-\mu}{\sigma} = \dfrac{131.5-100}{15} = 2.10$

$$\begin{aligned} P(x > 131.5) = P(z > 2.10) &\\ &= 1 - P(z < 2.10) \\ &= 1 - 0.9821 \\ &= 0.0179 \end{aligned}$$

c) $x = 90 \rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{90-100}{15} = \underline{-0.67|}$

$x = 110 \rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{110-100}{15} = \underline{0.67|}$

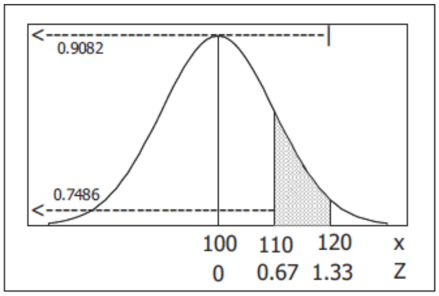
$$\begin{aligned} P(90 < x < 110) &= P(-0.67 < z < 0.67) \\ &= P(z < 0.67) - P(z < -0.67) \\ &= 0.7486 - 0.2514 \\ &\underline{= 0.4972|} \end{aligned}$$



d) $x = 120 \rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{120-100}{15} = \underline{1.33|}$

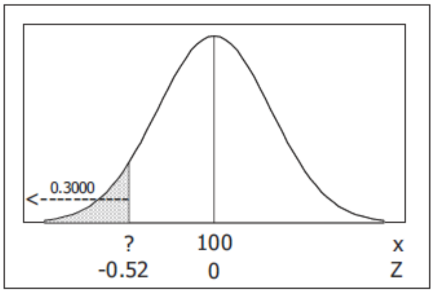
$x = 110 \rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{110-100}{15} = \underline{0.67|}$

$$\begin{aligned} P(110 < x < 120) &= P(0.67 < z < 1.33) \\ &= P(z < 1.33) - P(z < 0.67) \\ &= 0.9082 - 0.7486 \\ &\underline{= 0.1596|} \end{aligned}$$



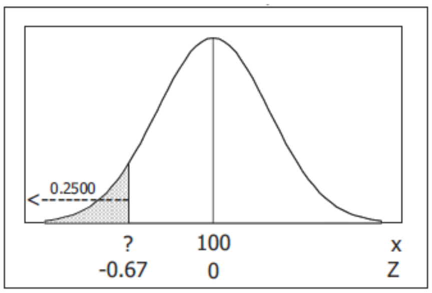
e) For P_{30}, $A = 0.300$ $[0.3015] \rightarrow z = -0.52$

$$\begin{aligned} x &= \mu + z\sigma \\ &= 100 + (-0.52)(15) \\ &\underline{= 92.2|} \end{aligned}$$



f) For Q_1, $A = 0.2500$ $[0.2514] \rightarrow z = -0.67$

$$\begin{aligned} x &= \mu + z\sigma \\ &= 100 + (-0.67)(15) \\ &\underline{\approx 90|} \end{aligned}$$



Exercise

The Gulfstream 100 is an executive jet that seats six, and it has a doorway height of 51.6

- ***Men's*** heights are normally distributed with mean 69.0 in. and standard deviation 2.8 in.
- ***Women's*** heights are normally distributed with mean 63.6 in. and standard deviation 2.5 in.

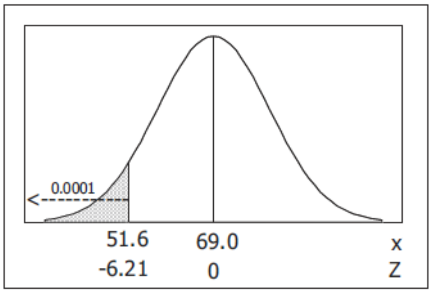
a) What percentage of adult men can fit through the door without bending?

b) What percentage of adult women can fit through the door without bending?

c) Does the door design with a height of 51.6 in. appear to be adequate? Why didn't the engineers design larder door?

d) What doorway height would allow 60% of men to fit without bending?

Solution

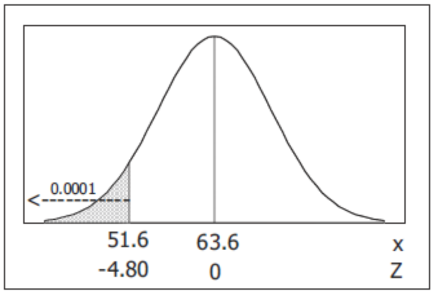


a) Normal distribution with: $\mu = 69.0, \quad \sigma = 2.8$

$$x = 51.6 \quad \rightarrow \quad z = \frac{x-\mu}{\sigma} = \frac{51.6 - 69.0}{2.8} \underline{= -6.21|}$$

$$P(x < 51.6) = P(z < -6.21)$$

$$\underline{= 0.0001|} \quad or \quad 0.01\%$$



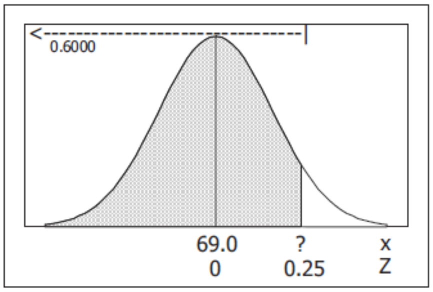
b) Normal distribution with: $\mu = 63.6, \quad \sigma = 2.5$

$$x = 51.6 \quad \rightarrow \quad z = \frac{x-\mu}{\sigma} = \frac{51.6 - 63.6}{2.5} \underline{= -4.80|}$$

$$P(x < 51.6) = P(z < -4.80)$$

$$\underline{= 0.0001|} \quad or \quad 0.01\%$$

c) Maybe. While it may not be convenient, it presents no danger of injury because of the obvious need for everyone to bend. Considering the small size of the plane, the door is probably as large as possible.



d) For $A = 0.6000 \ [0.5987] \rightarrow z = 0.25$

$$x = \mu + z\sigma$$

$$= 69 + (0.25)(2.8)$$

$$\underline{= 69.7 \ \ inches|}$$

Exercise

Assume that human body temperatures are normally distributed with a mean of 98.20°F and a standard deviation of 0.62°F.

a) A Hospital uses 100.6°F as the lowest temperature considered to be a fever. What percentage of normal and healthy persons would be considered to have fever? Does this percentage suggest that a cutoff of 100.6°F is appropriate?

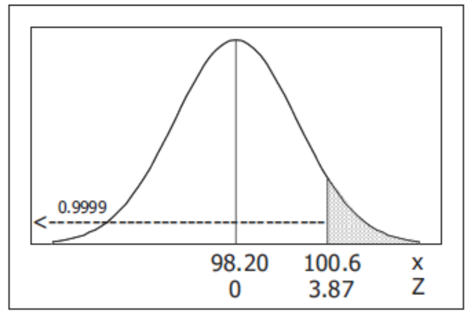
b) Physicians want to select a minimum temperature for requiring further medical tests. What should that temperature be, if we want only 5.0% of healthy people to exceed it? (Such a result is a false positive, meaning that the test result is positive, but the subject is not really sick.)

Solution

a) Normal distribution with: $\mu = 98.20, \quad \sigma = 0.62$

$$x = 100.6 \quad \rightarrow \quad z = \frac{x-\mu}{\sigma} = \frac{100.6 - 98.2}{0.62} \underline{=3.87}$$

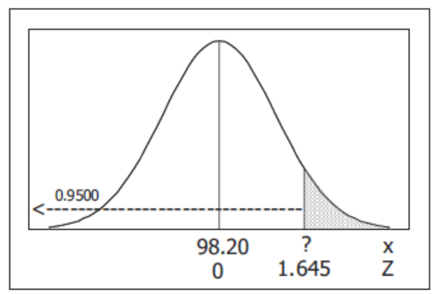
$$\begin{aligned} P(x > 100.6) = P(z > 3.87) &= 1 - P(z < 3.87) \\ &= 1 - 0.9999 \\ &\underline{= 0.0001} \end{aligned}$$



Yes. The cut-off is appropriate in that there is a small probability of saying that a healthy person has a fever, but many with low grade fevers may erroneously be labeled healthy.

b) For the highest 5%: $A = 0.9500 \quad \rightarrow \quad z = 1.645$

$$\begin{aligned} x &= \mu + z\sigma \\ &= 98.2 + (1.645)(0.62) \\ &\underline{= 99.22\ °F} \end{aligned}$$



Exercise

The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days.

a) One classical use of the normal distribution is inspired by a letter to "Dear Abby" in which a wife claimed to have given birth 308 days after a brief visit from her husband. Given this information, find the probability of a pregnancy lasting 308 days or longer. What does the result suggest?

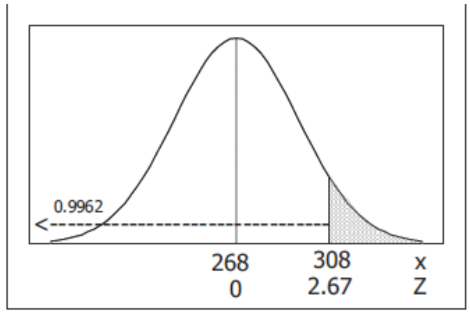
b) If we stipulate that a baby is premature if the length of pregnancy is the lowest 4%, find the length that separates premature babies from those who are not premature. Premature babies often require special care, and this result could be helpful to hospital administrators in planning for that care.

Solution

Normal distribution with: $\mu = 268, \quad \sigma = 15$

a) $P(x > 308) = P(z > 2.67)$

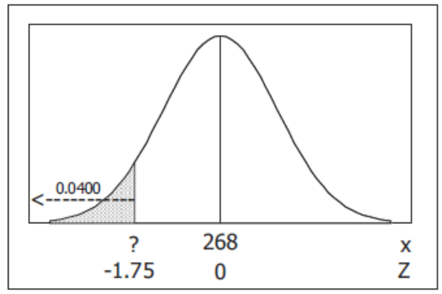
$$= 1 - P(z < 2.67)$$
$$= 1 - 0.9962$$
$$\underline{= 0.0038}$$



The result suggests that an unusual event has occurred – but certainly not an impossible one, as about 38 of every 10,000 pregnancies can be expected to last as long.

b) For the lowest 4%: $A = 0.0400\ \ [0.0401] \ \rightarrow \ z = -1.75$

$$x = \mu + z\sigma$$
$$= 268 + (-1.75)(15)$$
$$\underline{= 242 \ \ days}$$



Exercise

A statistics professor gives a test and finds that the scores are normally distributed with a mean of 25 and a standard deviation of 5. She plans to curve the scores.

a) If the curves by adding 50 to each grade, what is the new mean? What is the new standard deviation?

b) Is it fair to curve by adding 50 to each grade? Why or why not?

c) If the grades are curved according to the following scheme (instead of adding 50), find the numerical limits for each letter grade.

A: Top 10%
B: Scores above the bottom 70% and below the top 10%.
C: Scores above the bottom 30% and below the top 30%.
D: Scores above the bottom 10% and below the top 70%.
F: Bottom 10%.

d) Which method of curving the grades is fairer: Adding 50 to each grade or using the scheme given in part (*c*)? Explain.

Solution

Normal distribution with: $\mu = 25, \quad \sigma = 5$

a) For a population of size N, $\mu = \dfrac{\sum x}{N}, \quad \sigma^2 = \dfrac{\sum (x-\mu)^2}{N}$

Adding a constant to each score increases the mean by that amount but does not affect the standard deviation.

In non-statistical terms, shifting everything by k units does not affect the spread of the scores. This is true for any set of scores – regardless of the shape of the original distribution.

Let $y = x + k$

$$\begin{aligned}\mu_y &= \frac{\sum(x+k)}{N}\\ &= \frac{\sum x + \sum k}{N}\\ &= \frac{\sum x}{N} + \frac{\sum k}{N}\\ &= \frac{\sum x}{N} + \frac{Nk}{N}\\ &= \mu_x + k\end{aligned}$$

$$\begin{aligned}\sigma_y^2 &= \frac{\sum\left(y-\mu_y\right)^2}{N}\\ &= \frac{\sum\left((x+k)-\left(\mu_x+k\right)\right)^2}{N}\\ &= \frac{\sum\left(x-\mu_x\right)^2}{N}\\ &= \sigma_x^2\end{aligned}$$

If the teacher adds 50 to each grade,

New mean = 25 + 50 = 75

New standard deviation = 5

b) No; curving should consider the variation. Had the test been more appropriately constructed, it is not likely that every student would score exactly 50 points higher, If the typical student score increased by 50, we would expect the better students to increase by more than 50 and the poorer students to increase by less than 50. This would make the scores spread out and would increase the standard deviation.

c) For the top 10%: $A = 1 - 0.1 = 0.9000 \quad [0.8997] \quad \rightarrow \quad z = 1.28$

$$\begin{aligned}x &= \mu + z\sigma\\ &= 25 + (1.28)(5)\\ &= 31.4\end{aligned}$$

For the bottom 70%: $A = 0.7000 \quad [0.6985] \quad \rightarrow \quad z = 0.52$

$$\begin{aligned}x &= \mu + z\sigma\\ &= 25 + (0.52)(5)\\ &= 27.6\end{aligned}$$

For the bottom 30%: $A = 0.3000 \quad [0.3015] \quad \rightarrow \quad z = -0.52$

$$\begin{aligned}x &= \mu + z\sigma\\ &= 25 + (-0.52)(5)\end{aligned}$$

$\underline{= 22.4}$

For the bottom 10%: $A = 0.1000 \quad [0.1003] \quad \rightarrow \quad z = -1.28$

$$x = \mu + z\sigma$$
$$= 25 + (-1.28)(5)$$
$$\underline{= 18.6}$$

A	Higher than 31.4
B	27.6 to 31.4
C	22.4 to 27.6
D	18.6 to 22.4
E	Less than 18.6

d) The curving scheme in part (*c*) is fairer because it takes into account the variation as discussed in part (*b*). Assuming the usual 90-80-7060 letter grade cut-offs, for example, the percentage of *A*'s under the scheme in part (a) with $\mu = 25$ *and* $\sigma = 5$ is

$$P(x > 90) = 1 - P(x < 90)$$
$$= 1 - P(z < 3.00)$$
$$= 1 - .9987$$
$$\underline{= 0.0013} \quad or \quad 0.13\%$$

This is considerably less than the 10% *A* under the scheme in part (*c*) and reflects the fact that the variation in part (a) in unrealistically small.