$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\qquad r = \sqrt{x^2+y^2}$, $\quad \theta = \tan^{-1}\frac{y}{x}$

Ex $(r,\theta) = \left(4, \frac{7\pi}{6}\right)$ $\qquad (x?, y?)$

$$\begin{aligned} x &= r\cos\theta \\ &= 4\cos\frac{7\pi}{6} \\ &= -4\left(\frac{\sqrt{3}}{2}\right) \\ &= -2\sqrt{3} \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= 4\sin\frac{7\pi}{6} \\ &= 4\left(-\frac{1}{2}\right) \\ &= -2 \end{aligned}$$

$(x,y) = (-2\sqrt{3}, -2)$

Ex $(x,y) = (-1, \sqrt{3})$ $\qquad (r,\theta)?$

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ &= \sqrt{1+3} \\ &= 2 \end{aligned} \qquad \begin{aligned} \hat{\theta} &= \tan^{-1}\frac{\sqrt{3}}{1} \\ &= \frac{\pi}{3} \end{aligned}$$

$(r,\theta) = \left(2, \frac{2\pi}{3}\right)$

$\left(2, -\frac{4\pi}{3}\right)$

$\left(-2, -\frac{\pi}{3}\right)$ $\quad \left(-2, \frac{5\pi}{3}\right)$

Ex

$ax + by = c$

$a r\cos\theta + b r\sin\theta = c$

$r(a\cos\theta + b\sin\theta) = c$

$r = \dfrac{c}{a\cos\theta + b\sin\theta}$

Ex $x^2 - y^2 = 16$

$r^2\cos^2\theta - r^2\sin^2\theta = 16$

$r^2(\cos^2\theta - \sin^2\theta) = 16$

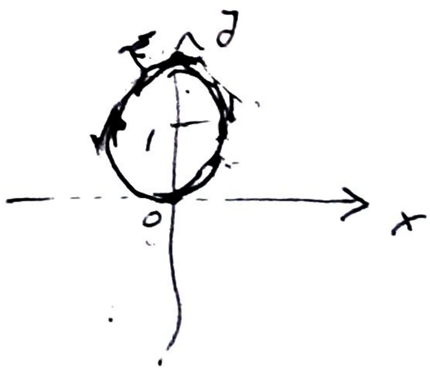
$r^2 \cos 2\theta = 16$

$r^2 = \dfrac{16}{\cos 2\theta}$

$= 16\sec 2\theta$

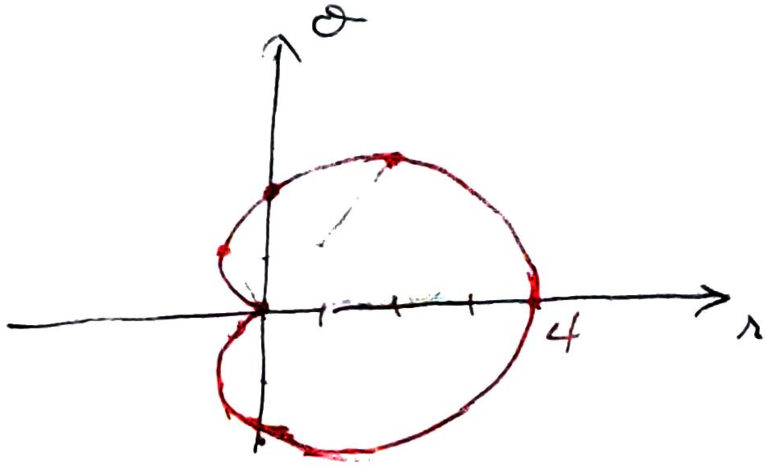
$r = 2\sin\theta$

θ	r
0	0
$\pi/6$	1
$\pi/2$	2
$5\pi/6$	1
π	0
$7\pi/6$	-1



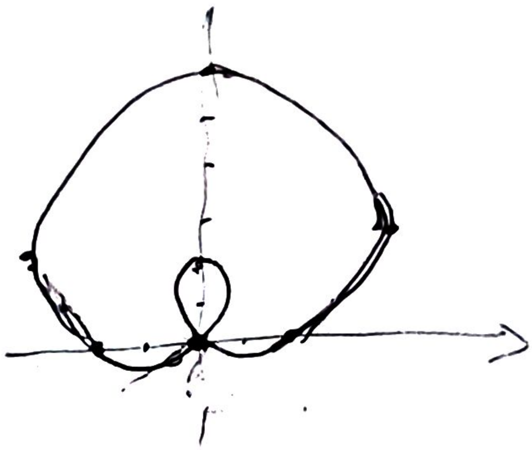
Graph $r = 2 + 2\cos\theta$

θ	r
0	4
$\pi/3$	3
$\pi/2$	2
$2\pi/3$	1
π	0



Cardioid

$r = 2 + 4\sin\theta$



8.6 6. $\left(4\sqrt{3}, -\frac{\pi}{6}\right)$

$x = r\cos\theta$
$= 4\sqrt{3}\cos\left(-\frac{\pi}{6}\right)$
$= 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$
$= 6$

$y = r\sin\theta$
$= 4\sqrt{3}\sin\left(-\frac{\pi}{6}\right)$
$= -4\sqrt{3}\left(\frac{1}{2}\right)$
$= -2\sqrt{3}$

$(x, y) = (6, -2\sqrt{3})$

12. $(-1, \sqrt{3})$ (r, θ)?

$r = \sqrt{x^2+y^2}$
$= \sqrt{1+3}$
$= 2$

$\hat{\theta} = \tan^{-1}\frac{\sqrt{3}}{1}$
$= \frac{\pi}{3}$
$\theta \in Q\,II$

$(r, \theta) = \left(2, \frac{2\pi}{3}\right)$

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2+y^2$

24. $r^2 = 4\cos 2\theta$
$= 4(\cos^2\theta - \sin^2\theta)$

$r^2 r^2 = 4(r^2\cos^2\theta - r^2\sin^2\theta)$

$(x^2+y^2)^2 = 4(x^2-y^2)$

$x^4 + 2x^2y^2 + y^4 = 4x^2 - 4y^2$

41/ $x^2+y^2=4x$

$$r^2 = 4r\cos\theta \quad (r \neq 0)$$

$$r = 4\cos\theta$$

38/ $y^2 - x^2 = 4$

$$r^2\sin^2\theta - r^2\cos^2\theta = 4$$

$$r^2(\underbrace{\sin^2\theta - \cos^2\theta}_{-\cos 2\theta}) = 4$$

$$r^2 = -\frac{4}{\cos 2\theta}$$

8.7 Complex

$\sqrt{-1} = i$

$$\sqrt{-1}\sqrt{-1} = i \cdot i = i^2 = -1$$

you can Not

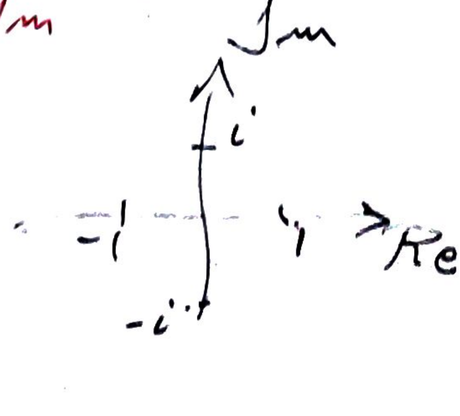
$$\sqrt{-1}\sqrt{-1} \neq \sqrt{(-1)(-1)}$$

$z = a + ib$

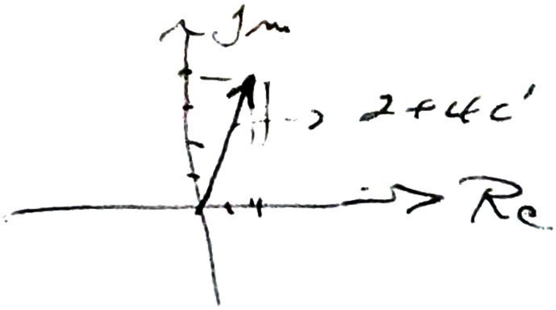
Real part Re

Imaginary part Im

$a, b \in \mathbb{R}$



$z = 2 + 4i$



$z = x + iy$

$r = \sqrt{x^2 + y^2}$ modulus

$\theta = \tan^{-1} \dfrac{y}{x}$ Argument

$z = x + iy$

$$\boxed{\begin{aligned} &= r\cos\theta + i\, r\sin\theta \\ &= r \operatorname{cis} \theta \end{aligned}}$$

Trig Form

$z = -1 + i$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{1 + 1}$

$= \sqrt{2}$

$\hat{\theta} = \tan^{-1} \dfrac{1}{1} = \dfrac{\pi}{4}$

$\theta = \dfrac{3\pi}{4}$

$-1 + i = \sqrt{2} \operatorname{cis}\left(\dfrac{3\pi}{4}\right)$

$= \sqrt{2} \cos \dfrac{3\pi}{4} + i \sqrt{2} \sin \dfrac{3\pi}{4}$

$z = 2 \operatorname{cis} 60^0$

$= 2\cos 60^0 + i\, 2\sin 60^0$

$= \underline{1 + i\sqrt{3}}$

$z = 7 \Rightarrow r = 7 \quad \theta = 0$

$= 7 + i0$

$5i \Rightarrow r = 5 \ , \ \theta = \frac{\pi}{2}$

Product.

$a^2 + b^2 = (a+ib)(a-ib)$

$a + b = (\sqrt{a} + i\sqrt{b})(\sqrt{a} - i\sqrt{b})$

$z_1 z_2 = (r_1 \operatorname{cis}\theta_1)(r_2 \operatorname{cis}\theta_2)$

$= r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$

$(3 \operatorname{cis} 45^0)(2 \operatorname{cis} 135^0)$

$45^0 + 135^0$

$= 6 \operatorname{cis}(180^0)$

$= 6\cos 180^0 + i6 \sin 180^0$

$= \underline{-6}$

Quotient
$$\frac{r_1 \operatorname{cis}\theta_1}{r_2 \operatorname{cis}\theta_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$\frac{10 \operatorname{cis}(-60^\circ)}{5 \operatorname{cis}(150^\circ)} = \frac{10}{5} \operatorname{cis}(-60-150)$$

$$= 2 \operatorname{cis}(-210^\circ)$$
$$= 2\cos(-210^\circ) + i\, 2\sin(-210^\circ)$$
$$= 2\left(-\frac{\sqrt{3}}{2}\right) - i\, 2\left(-\frac{1}{2}\right)$$
$$= \underline{-\sqrt{3} + i}$$

De Moivre's Theorem
$$(r \operatorname{cis}\theta)^n = r^n \operatorname{cis} n\theta$$

$$(1 + i\sqrt{3})^8$$
$$r = \sqrt{1+3} \qquad \theta = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{6}$$
$$= \underline{2}$$

$$(1 + i\sqrt{3})^8 = \left(2 \operatorname{cis}\frac{\pi}{6}\right)^8$$
$$= 2^8 \operatorname{cis}\frac{8\pi}{6} \qquad \frac{4\pi}{3}$$
$$= 256\cos\frac{4\pi}{3} + i\, 256\sin\frac{4\pi}{3}$$
$$= -\frac{256}{2} + i\,\frac{256\sqrt{3}}{2}$$
$$= \underline{-128 + i\, 128\sqrt{3}}$$

n^{th} root

$\sqrt[n]{}$

$$(r \operatorname{cis} \theta)^{1/n} = \sqrt[n]{r} \operatorname{cis} \alpha$$

$$\alpha = \frac{\theta + 2\pi k}{n}$$

Info

4^{th} root $-8+8i\sqrt{3}$

$$r = \sqrt{64 + 64(3)} \qquad 8(2)$$

$$= \boxed{16}$$

$$\hat{\theta} = \tan^{-1} \frac{8\sqrt{3}}{8} = -\frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

$$\boxed{\sqrt[4]{16} = 2}$$

$$\alpha = \frac{\theta + 2\pi k}{n} =$$

$$= \frac{1}{4}\left(\frac{2\pi}{3} + 2\pi k\right) \qquad \left(\frac{1}{3} + k\right)$$

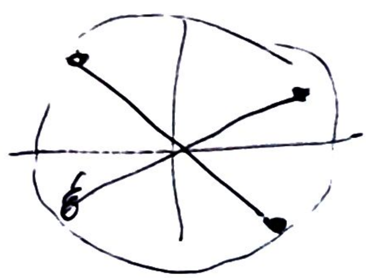
$$= \frac{\pi}{2}\left(\frac{1}{3} + k\right)$$

$k=0 \rightarrow \alpha = \pi/6$

$k=1 \rightarrow \alpha = 2\pi/3$

$k=2 \rightarrow \alpha = 7\pi/6$

$k=3 \rightarrow \alpha = 5\pi/3$



$$2 \operatorname{cis} \frac{\pi}{6},\ 2 \operatorname{cis} \frac{2\pi}{3},\ 2 \operatorname{cis} \frac{7\pi}{6},\ 2 \operatorname{cis} \frac{5\pi}{3}$$

8.6 ② $\left(-\sqrt{2}, \frac{3\pi}{4}\right)$

$x = r\cos\theta$
$= -\sqrt{2}\cos\frac{3\pi}{4}$
$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$
$= 1$

$y = r\sin\theta$
$= -\sqrt{2}\sin\frac{3\pi}{4}$
$= -\sqrt{2}\,\frac{1}{\sqrt{2}}$
$= -1$

$(x, y) = (1, -1)$

$(2, -2\sqrt{3})$ $\quad r \geq 0 \quad 0^\circ \leq \theta \leq 360^\circ.$

$r = \sqrt{x^2 + y^2}$
$= \sqrt{4 + 12}$
$= 4$

$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)$
$= 60^\circ$
$\theta = 300^\circ.$

$(r, \theta) = (4, 300^\circ)$

31/ $r(\sin\theta - 2\cos\theta) = 6$
$\underline{r\sin\theta} - 2\underline{r\cos\theta} = 6$
$y - 2x = 6$

32/ $r = 8\sin\theta - 2\cos\theta$
$r^2 = 8r\sin\theta - 2r\cos\theta \qquad r \neq 0$
$x^2 + y^2 = 8y - 2x$

37) $(x+2)^2 + (y-3)^2 = 13$

$$(r\cos\theta + 2)^2 + (r\sin\theta - 3)^2 = 13$$

$$r^2\cos^2\theta + 4r\cos\theta + 4 + r^2\sin^2\theta - 6r\sin\theta + 9 = 13$$

$$r^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1} + r(4\cos\theta - 6\sin\theta) = 0$$

$$r^2 + r(4\cos\theta - 6\sin\theta) = 0 \quad (r \neq 0)$$

$$r + 4\cos\theta - 6\sin\theta = 0$$

$$\boxed{r = 6\sin\theta - 4\cos\theta}$$

$x^2 + y^2 = r^2$

Exam 4

May 6

5/6/20