Critical Points CP (x, y)

$f'(c) = 0$

* $f'(x) = 0$, then solve for x (critical #) C.N

Ex. abs. (absolute) extreme.

$f(x) = x^2 \qquad [-2, 1]$

Sol'n. $f'(x) = 2x$

$x = 0$ C.N

x	f(x)	
−2	4	→ abs. Max @ (−2, 4)
0	0	→ abs. Min @ (0, 0)
1	1	

Ex $g(t) = 8t - t^4 \qquad [-2, 1]$ abs. Extr.

Sol'n. $g'(t) = 8 - 4t^3 = 0$ C.N

$t^3 = 2 \Rightarrow t = \sqrt[3]{2} > 1$

t	g(t)	
−2	−32	→ abs. Min (−2, −32)
1	7	→ abs. Max (1, 7)

Ex $f(x) = x^{2/3}$ $[-2, 3]$ extrem.

Sol'n $f'(x) = \frac{2}{3} x^{-1/3}$ $\frac{2}{3x^{1/3}} \neq 0$

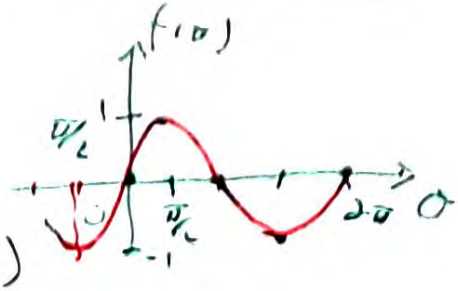
C.N: $x = 0$

x	$f(x)$	
-2	$\sqrt[3]{4}$	
0	0	→ abs. Min $(0, 0)$
3	$\sqrt[3]{9}$	→ abs. Max $(3, \sqrt[3]{9})$

#3: $f(\theta) = \sin\theta$ $-\frac{\pi}{2} \leq \theta \leq \frac{5\pi}{2}$, QII

$f'(\theta) = \cos\theta = 0$

$\theta = \pm\frac{\pi}{2}, \frac{3\pi}{2}$,



θ	$f(\theta)$	
$-\frac{\pi}{2}$	-1	→ abs. Min $(-\frac{\pi}{2}, -1)$
$\frac{\pi}{2}$	1	→ abs. Max $(\frac{\pi}{2}, 1)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	

3.2 Increasing Decreasing
(Incr) Decr.

Ex $f(x) = x^2 - 12x - 5$ Inc? Dec?

$f'(x) = 2x - 12 = 0$

CN: $x = 6$

f' → ? — | + ← $f'(7)$ (sign chart with 6 marked, 5 tested on left)

Incr: $(6, \infty)$ Decr: $(-\infty, 6)$

$f(x) = x^3 - 12x - 5$ deg 3 | 3 − 1 = 2 extr.
| 3 → I + D.

$f'(x) = 3x^2 - 12 = 0$

$x^2 = 4$ → CN: $x = \pm 2$

(sign chart: −2, 0, 2 with + | − | +)

Critical: use f' sign → ↗ and ↘

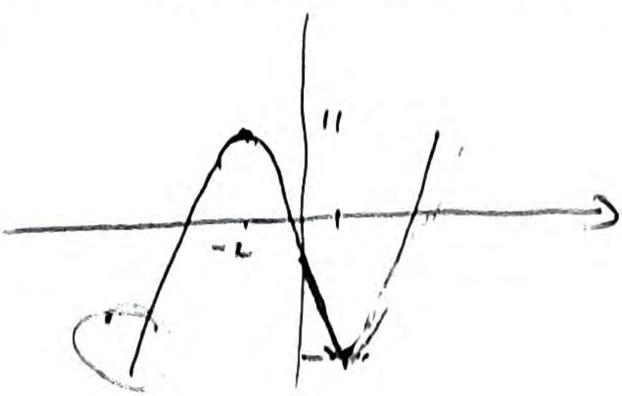
Incr: $(-\infty, -2), (2, \infty)$

Decr: $(-2, 2)$

x	f(x)
−2	11
2	−21

→ Relative Max (−2, 11)

Rel Min (2, −21)



→ CN ← Abs. Ext, Ext, (Inc, Dec)

Ex $f(x) = x^{1/3}(x-4)$

$= x^{4/3} - 4x^{1/3}$

$f'(x) = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3}$ → $x^{-2/3} = \frac{1}{x^{2/3}}$ $\boxed{x \neq 0}$

$= \frac{4}{3}\left(x^{1/3} - x^{-2/3}\right) = 0$

$x^{1/3} = x^{-2/3}$ CN $\begin{cases} x = 0 \\ x = 1 \end{cases}$

	0		1	8
−		−		+

8 → 2 − $\frac{1}{4}$

Inc: $(1, \infty)$ Dec: $(-\infty, 1)$

LMIN. $(1, -3)$ → Min. Point

Concavity { Concave Upward (up) +
" downward (down) −

→ Point of Inflection
Pt. Infl.

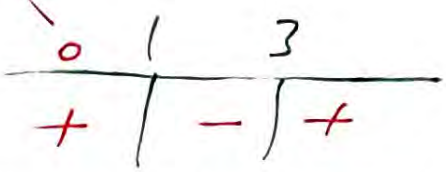
To find Pt. Infl
$f''(x) = 0$ →

Ex $f(x) = x^4 - 8x^3 + 18x^2$

$f'(x) = 4x^3 - 24x^2 + 36x$

$f''(x) = 12x^2 - 48x + 36 = 0$

pt. of Infl: $x = 1, 3$



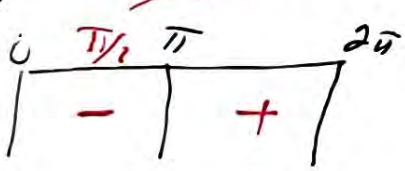
Concave up: $(-\infty, 1)$ $(3, \infty)$

" down: $(1, 3)$

Ex $y = 3 + \sin x$ $[0, 2\pi]$ Concavities??

$y' = \cos x$

$y'' = -\sin x = 0 \Rightarrow$ pt. Infl: $x = 0, \pi, 2\pi$



Concave up: $(\pi, 2\pi)$

" down: $(0, \pi)$

Ex $S(t) = 2t^3 - 14t^2 + 22t - 5 \quad t \geq 0$

$S' = 6t^2 - 28t + 22 = 0$

CN: $t = 1, \frac{11}{3}$

$S''(t) = 12t - 28 = 0$ Pt. Infl. $t = \frac{7}{3}$

t	$S(t)$
1	5
$\frac{11}{3}$	-14

0	1	$\frac{11}{3}$
+	−	+

0	$\frac{7}{3}$
−	+

$t \geq 0$

LMIN $(\frac{11}{3}, -14)$ LMAX: $(1, 5]$

Inc: $(0, 1) \; (\frac{11}{3}, \infty)$

Dec: $(1, \frac{11}{3})$

concave up: $(\frac{7}{3}, \infty)$

" down: $(0, \frac{7}{3})$

(2)

- CN
  - abs. ext.
  - ext. → critical point (CP)
  - Inc Dec
    - Pt of Infl. ?
    - Concavity

$f(x) = x^4 - 4x^3 + 10$

$f'(x) = 4x^3 - 12x^2$

$= 4x^2(x-3) = 0$

CN: $x = 0, 0, 3$

$f''(x) = 12x^2 - 24x$

$= 12x(x-2) = 0$

Pt. Infl: $x = 0, 2$

0	1	3
−	−	+

0	1	2
+	−	+

LMIN: $(3, -17)$

Inc: $(3, \infty)$

Dec: $(-\infty, 3)$

Concave up: $(-\infty, 0)$ $(2, \infty)$

" down: $(0, 2)$

---

or Maximize / Minimize.

$f' = 0$ C.N