Section 3.7 – Probability Applications of Counting Principles

$$P(E) = \frac{n(E)}{n(S)}$$

Example

The Environment Protection Agency is considering inspecting 6 plants for environment compliance: 3 in Chicago, 2 in Los Angeles, and 1 NY. Due to a lack of inspectors, they decide to inspect 2 plants selected at random, 1 this month and 1 next month, with each plant equally likely to be selected, but no plant is selected twice. What is the probability that 1 Chicago plant and 1 Los Angeles plant are selected?

Solution

Chicago plant can be selected from 3 in $\binom{3}{1}$ ways.

Los Angeles plant can be selected from 2 in $\binom{2}{1}$ ways.

$n(S)$ = (# ways selected 1st month) (# ways selected 2nd month)

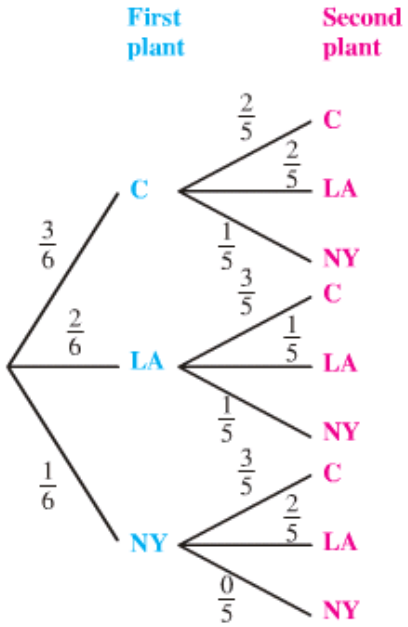
= 6 . 5

= 30 ways

$$P(1C \text{ and } 1LA) = \frac{\binom{3}{1}\binom{2}{1} + \binom{2}{1}\binom{3}{1}}{30} = 0.4$$

OR

$$P(1C \text{ and } 1LA) = \frac{3}{6}\frac{2}{5} + \frac{2}{6}\frac{3}{5} = 0.4$$



Example

From a group of 22 nurses, 4 are to be selected to present a list of grievances to management.

a) In how many ways can this be done?

b) One of the nurses is Julie. Find the probability that Julie will be among the 4 selected.

c) Find the probability that Julie will not be selected

Solution

a) $C_{22,4}$ $= 7,315$

b) $P(\text{Julie is chosen}) = \frac{n(E)}{n(S)} = \frac{\binom{1}{1}\binom{21}{3}}{\binom{22}{4}} \approx 0.1818$

c) $P(\text{Julie is not chosen}) = 1 - 0.1818 = 0.8182$

Example

When shipping diesel engines abroad, it is common to pack 12 engines in one container that is then loaded on a rail car and sent to a port. Suppose that a company has received complaints from its customers that many of the engines arrive in nonworking condition. To help solve this problem, the company decides to make a spot check of containers after loading. The company will test 3 engines from a container at random; if any of the 3 is nonworking, the container will not be shipped until each engine in it is checked. Suppose a given container has 2 nonworking engines. Find the probability that the container will not be shipped.

Solution

$$\Pr(\textit{not shipping}) = \Pr(1\ \textit{defective}) + \Pr(2\ \textit{defective})$$

$$= \frac{\binom{2}{1}\binom{10}{2}}{\binom{12}{3}} + \frac{\binom{2}{2}\binom{10}{1}}{\binom{12}{3}}$$

$$= 0.4545$$

OR

$$\Pr(\textit{not shipping}) = 1 - \Pr(0\ \textit{defective})$$

$$= 1 - \frac{\binom{2}{0}\binom{10}{3}}{\binom{12}{3}}$$

$$= 0.4545$$

Example

In a common form of the card game poker, a hand of 5 cards is dealt to each player from a deck of 52 cards. Find the probability of getting each of the following hands.

a) A hand containing only hearts, called a heart flush

$$P(\textit{heart flush}) = \frac{\binom{13}{5}\binom{39}{0}}{\binom{52}{5}} \approx 0.0004952$$

b) A flush of any suit (5 cards of the same suit)

$$P(\textit{flush}) = 4(0.0004952) = 0.001981$$

c) A full house of aces and eights (3 aces & 2 eights)

$$P(3\ \textit{aces},\ 2\ \textit{eights}) = \frac{\binom{4}{3}\binom{4}{2}\binom{44}{0}}{\binom{52}{5}} \approx 0.000009234$$

Example

A music teacher has 3 violin pupils, Fred, Carl, and Helen. For a recital, the teacher selects a first violinist and a second violinist. The third pupil will play with the others, but not solo. If the teacher selects randomly, what is the probability that Helen is the first violinist, Carl is second violinist, and Fred does not solo?

Solution

$P(3,3) = 6$

$P = \frac{1}{6}$

Example

Ray and Nate are arranging a row of fruit at random on a table. They have 5 apples, 6 oranges, and 7 lemons. What is the probability that all fruit of the same kind are together?

Solution

To arrange all 18 pieces of fruit $= 18!$ *ways*

To arrange 3 kinds of fruit $= 3!$ *ways*

$P(\textit{all fruit of the same kind are together}) = \frac{3!5!6!7!}{18!} = 0.4081 \times 10^{-7}$

Exercises *Section* 3.7 – Probability Applications of Counting Principles

1. A basket contains 7 red apples and 4 yellow apples. A sample of 3 apples is drawn. Find the probabilities that the sample contains the following.

a) All red apples
b) All yellow apples
c) 2 yellow and 1 red apple
d) More red than yellow apples

2. Two cards are drawn at random from an ordinary deck of 52. How many 2-card hands are possible?

3. Find the probability that the 2-card hand contains the following.

a) 2 aces
b) At least 1 ace
c) All spades
d) 2 cards of the same suit
e) Only face cards
f) No face cards
g) No card higher than 8 (count ace as 1)

4. A reader wrote to the "Ask Marilyn" column in a magazine. "You have six envelopes to pick from. Two-thirds (= 4) are empty. One-third (= 2) contain a $100 bill. You're told to choose 2 envelopes at random. Which is more likely: **(1)** that you'll get at least one $100 bill, or **(2)** that you'll get no $100 bill at all?" Find the two probabilities.

5. After studying all night for a final exam, a bleary-eyed student randomly grabs 2 socks from a drawer containing 9 black, 6 brown, and 2 blue socks, all mixed together. What is the probability that she grabs a matched pair?

6. At a conference of writers, special-edition books were selected to be given away in contests. There were 9 books written by Hughes, 5 books by Baldwin, and 7 books by Morrison. The judge of one contest selected 6 books at random for prizes. Find the probabilities that he selection consisted of the following.

a) 3 Hughes and 3 Morrison books
b) Exactly 4 Baldwin books
c) 2 Hughes, 3 Baldwin, and 1 Morrison book
d) At least 4 Hughes books
e) Exactly 4 books written by males (Morrison is female)
f) No more than 2 books written by Baldwin

7. A school in Bangkok requires that students take an entrance examination. After the examination, there is a drawing in which 5 students are randomly selected from each group of 40 for automatic acceptance into the school, regardless of their performance on the examination. The drawing consists of pacing 35 red and 5 green pieces of paper into a box. Each student picks a piece of paper from the box and then does not return the piece of paper to the box. The 5 lucky students who pick the green pieces are automatically accepted into the school.

a) What is the probability that the first person wins automatic acceptance?
b) What is the probability that the last person wins automatic acceptance?
c) If the students are chosen by the order of their seating does this give the student who goes first a better chance of winning than the second, third… person?

(*Hint*: Imagine that the 40 pieces of paper have been mixed up and laid in a row so that the first student picks the first piece of paper, the second student picks the second piece of paper, and so on.)

8. A controversy arose in 1992 over the Teen Talk Barbie doll, each of which was programmed with four saying randomly picked from a set of 270 sayings. The controversy was over the saying, "Math class is tough," which some felt gave a negative message toward girls doing well in math. In an interview with Science, a spokeswoman for Mattel, the makers of Barbie, said that "There is a less than 1% chance you're going to get a doll that says math class is tough". Is this figure correct? If not, give the correct figure.

9. Bingo has become popular in the U.S., and it is an efficient way for many organizations to raise money. The bingo card has 5 rows and 5 columns of numbers from 1 to 75, with the center given as a free cell. Balls showing one of the 75 numbers are picked at random from a container. If the drawn number appears on a player's card, then the player covers the number. In general, the winner is the person who first has a card with an entire row, column, or diagonal covered.

a) Find the probability that a person will win bingo after just four numbers are called.
b) An L occurs when the first column and the bottom row are both covered. Find the probability that an L will occur in the fewest number of calls.
c) An X-out occurs when both diagonals are covered. Find the probability than an X-out occurs in the fewest number of calls.
d) If bingo cards are constructed so that column one has 5 of the numbers from 1 to 15, column two has 5 of the numbers from 16 to 30, column three has 4 of the numbers from 31 to 45, column four has 5 of the numbers from 46 to 60, column five has 5 of the numbers from 61 to 75, how many different bingo cards could be constructed? (*Hint*: Order matters!)