Review

16 questions

(3) $x^n e^{ax}$ $\qquad$ $x^n \begin{cases} \text{cosine} \\ \text{sine} \end{cases}$ $\qquad$ $e^{ax} \begin{cases} \text{cosine} \\ \text{sine} \end{cases}$

Jose

$$\int x^4 e^{5x}\,dx$$

		$\int e^{5x}\,dx$
$+$	x^4	$\frac{1}{5}e^{5x}$
$-$	$4x^3$	$\frac{1}{5^2}e^{5x}$
$+$	$12x^2$	$\frac{1}{5^3}e^{5x}$
$-$	$24x$	$\frac{1}{5^4}e^{5x}$
$+$	24	$\frac{1}{5^5}e^{5x}$

$$\int x^4 e^{5x}\,dx = \left(\frac{x^4}{5} - \frac{4}{25}x^3 + \frac{12}{5^3}x^2 - \frac{24}{5^4} + \frac{24}{5^5}\right)e^{5x} + C$$

$$\int x^3 \sin 4x\,dx$$

		$\int \sin 4x$
$+$	x^3	$-\frac{1}{4}\cos 4x$
$-$	$3x^2$	$-\frac{1}{16}\sin 4x$
$+$	$6x$	$\frac{1}{64}\cos 4x$
$-$	6	$\frac{1}{256}\sin 4x$

$$\int x^3 \sin 4x\,dx = \left(-\frac{x^3}{4} + \frac{3}{32}x\right)\cos 4x + \left(\frac{3}{16}x^2 - \frac{3}{128}\right)\sin 4x + C$$

$\int e^{3x}\cos 2x\,dx$

		$\int \cos 2x\,dx$
$+$	e^{3x}	$\frac{1}{2}\sin 2x$
$-$	$3e^{3x}$	$-\frac{1}{4}\cos 2x$
$+$	$9e^{3x}$	

copy

$$\int e^{3x}\cos 2x\,dx = e^{3x}\left(\frac{1}{2}\sin 2x + \frac{3}{4}\cos 2x\right) - \frac{9}{4}\int e^{3x}\cos 2x\,dx$$

$$\frac{13}{4}\int e^{3x}\cos 2x\,dx = \frac{1}{4}(2\sin 2x + 3\cos 2x)e^{3x}$$

$$\int e^{3x}\cos 2x\,dx = \frac{1}{13}(2\sin 2x + 3\cos 2x)e^{3x} + C$$

$$\int \cos^3 x\,dx = \int \cos^2 x\cos x\,dx$$

$$= \int (1-\sin^2 x)\,d(\sin x)$$

$$= \sin x - \frac{1}{3}\sin^3 x + C$$

$$\int \cos^2 x\sin^2 x\,dx = \frac{1}{4}\int (1+\cos 2x)(1-\cos 2x)\,dx$$

$$= \frac{1}{4}\int (1-\cos^2 2x)\,dx$$

$$= \frac{1}{4}\int \left(\underbrace{1-\frac{1}{2}} - \frac{1}{2}\cos 4x\right)dx$$

$$= \frac{1}{4}\left(\frac{1}{2}x - \frac{1}{8}\sin 4x\right) + C$$

$$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$$

$$\int_0^{\pi/2} \cos^9 x\,dx = \frac{8}{9}\cdot\frac{\cancel{6}^{\,2}}{7}\cdot\frac{4}{5}\cdot\frac{2}{\cancel{3}}$$

$$= \boxed{\frac{2^7}{315}}$$

$$\int \frac{dx}{x\sqrt{4x^2+9}}$$

$2x = 3\tan\theta$

$dx = \frac{3}{2}\sec^2\theta\, d\theta$

$\sqrt{4x^2+9} = 3\sec\theta$

$\sec\theta = \frac{\sqrt{\;}}{3}$

$$\int \frac{dx}{x\sqrt{4x^2+9}} = \frac{3}{2}\int \frac{\sec^2\theta\, d\theta}{\frac{3}{2}\tan\theta\,(3\sec\theta)}$$

$\frac{1}{\cos\theta} \cdot \frac{\cos}{\sin}$

$$= \frac{1}{3}\int \frac{\sec\theta}{\tan\theta}\, d\theta$$

$$= \frac{1}{3}\int \frac{1}{\sin\theta}\, d\theta$$

$\frac{1}{\sin} = \frac{\sec}{\tan}$

$$= \frac{1}{3}\int \csc\theta\, d\theta$$

$$= -\frac{1}{3}\ln\left|\csc\theta + \cot\theta\right|$$

$$= -\frac{1}{3}\ln\left|\frac{\sqrt{4x^2+9}}{2x} + \frac{3}{2x}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2-81}}$$

$x = 9\sec\theta$

$dx = 9\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-81} = 9\tan\theta$

$$\int \frac{dx}{\sqrt{x^2-81}} = \int \frac{9\sec\theta\tan\theta\, d\theta}{9\tan\theta}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln\left|\sec\theta + \tan\theta\right|$$

$$= \ln\left|\frac{x}{9} + \frac{\sqrt{x^2-81}}{9}\right| + C$$

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{3}\Big|_0^3$$

$$= \sin^{-1} 1 - \sin^{-1} 0$$

$$= \boxed{\frac{\pi}{2}}$$

$\boxed{x = 3\sin\theta}$ $\sqrt{9-x^2} = 3\cos\theta$

$dx = 3\cos\theta\, d\theta$

$$\int \frac{3\cos\theta\, d\theta}{3\cos\theta}$$

$$= \int d\theta$$

$$= \theta$$

$$\int \frac{x+3}{2x^3-8x}\,dx$$

$$\frac{1}{2}\int \frac{x+3}{x^3-4x} = \frac{1}{2}\left(\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}\right)$$

$$x+3 = A(x^2-4) + Bx(x+2) + Cx(x-2)$$

x^2 $\quad A + B + C = 0$ ①

x^1 $\quad 2B - 2C = 1$ ②

x^0 $\quad -4A = 3 \Rightarrow \boxed{A = -\frac{3}{4}}$

$$\begin{cases} B + C = \frac{3}{4} \\ 2B - 2C = 1 \end{cases}$$

$$\begin{cases} 2B + 2C = \frac{3}{2} \\ 2B - 2C = 1 \end{cases}$$

$$4B = \frac{5}{2}$$

$$\boxed{B = \frac{5}{8}}$$

$$\boxed{C = \frac{3}{4} - \frac{5}{8} = \frac{1}{8}}$$

$$\int \frac{x+3}{2x^3-8x}\,dx = \frac{1}{2}\left(-\frac{3}{4}\int \frac{dx}{x} + \frac{5}{8}\int \frac{dx}{x-2} + \frac{1}{8}\int \frac{dx}{x+2}\right)$$

$$\boxed{= -\frac{3}{8}\ln|x| + \frac{5}{16}\ln|x-2| + \frac{1}{16}\ln|x+2| + K}$$

$$\int \frac{5x+8}{2x^2+3x+1}\,dx$$

$$\frac{5x+8}{2x^2+3x+1} = \frac{A}{2x+1} + \frac{B}{x+1}$$

$$5x+8 = A(x+1) + B(2x+1)$$

$$\begin{array}{lrcl} x^1 & A + 2B &=& 5 \\ x^0 & -A + B &=& 8 \\ \hline & B &=& -3 \end{array} \quad \rightarrow \boxed{A = 8+3 = 11}$$

$$\boxed{B=-3}$$

$$\int \frac{5x+8}{2x^2+3x+1}\,dx = \frac{11}{2}\int \frac{d(2x+1)}{2x+1} - 3\int \frac{dx}{x+1}$$

$$= \boxed{\frac{11}{2}\ln|2x+1| - 3\ln|x+1| + C}$$

$$\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+4)^{5/2}} = \frac{1}{2}\int_{\infty}^{\infty} (x^2+4)^{-5/2}\, d(x^2+4)$$

$$= -\frac{1}{3}(x^2+4)^{-3/2}\Big|_{-\infty}^{\infty}$$

$$= -\frac{1}{3}(0-0)$$

$$= \boxed{0}$$

$$\int_0^{\infty} x^2 e^{-x}\,dx = (-x^2-2x-2)e^{-x}\Big|_0^{\infty}$$

$$= 0-(-2)$$

$$= \boxed{2}$$

		$\int e^{-x}$
+	x^2	$-e^{-x}$
−	$2x$	e^{-x}
+	2	$-e^{-x}$

$+$	x	$\frac{1}{2}e^{2x}$
$-$	1	$\frac{1}{4}e^{2x}$

$$y' + 2y = e^{-x} + x + 1$$

$$e^{\int 2\,dx} = e^{2x}$$

$$\int e^{2x}(e^{-x} + x + 1)\,dx = \int \left(e^{x} + \underbrace{xe^{2x}} + e^{2x}\right)dx$$

$$= e^{x} + \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x} + \frac{1}{2}e^{2x}$$

$$= e^{x} + \left(\frac{x}{2} + \frac{1}{4}\right)e^{2x}$$

$$y(x) = \frac{1}{e^{2x}}\left(e^{x} + \frac{1}{4}(2x+1)e^{2x} + C\right)$$

$$= \frac{1}{e^{x}} + \frac{1}{2}x + \frac{1}{4} + Ce^{-2x}$$

$$xy' + y = 3xy \qquad y(1) = 0$$

$$x\frac{dy}{dx} = (3x - 1)y$$

$$\int \frac{dy}{y} = \int \left(3 - \frac{1}{x}\right)dx$$

$$\ln|y| = 3x - \ln|x| + C$$

$$0 = 3 + C$$

$$C = -3$$

$$\ln y = 3x - \ln x - 3 \qquad y(x) = e^{3x - \ln x - 3}$$

$$e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$$

$$= (1 + e^{-2x}) e^{-y}$$

$$\int y e^y \, dy = \int (e^{-x} + e^{-3x}) \, dx$$

$$\boxed{(y-1)e^y = -e^{-x} - \frac{1}{3}e^{-3x} + C}$$

$$t y' + 2y = t^2 - t + 1 \qquad y(1) = \frac{1}{2}$$

$$y' + \frac{2}{t} y = t - 1 + \frac{1}{t}$$

$$e^{\int \frac{2}{t} dt} = e^{2 \ln|t|} = t^2$$

$$\int t^2 \left(t - 1 + \frac{1}{t}\right) dt = \int (t^3 - t^2 + t) \, dt$$

$$= \frac{1}{4}t^4 - \frac{1}{3}t^3 + \frac{1}{2}t^2$$

$$y(t) = \frac{1}{t^2}\left(\frac{1}{4}t^4 - \frac{1}{3}t^3 + \frac{1}{2}t^2 + C\right)$$

$$= \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{C}{t^2}$$

$$\frac{1}{2} = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + C$$

$$\boxed{C = \frac{1}{12}}$$

$$\boxed{y(t) = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{1}{12t^2}}$$

$$\int \sqrt{x}\sqrt{1+\sqrt{x}}\,dx$$

$\cancel{u = 1+\sqrt{x}}$

$(u-1)^2$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$x = u^2$

$$\int \sqrt{x}\sqrt{1+\sqrt{x}}\,dx = \int u\,(1+u)^{1/2}(2u\,du)$$

$$= 2\int u^2(1+u)^{1/2}\,du$$

		$\int (1+u)^{1/2}\,d(u+1)$
$+$	u^2	$\frac{2}{3}(1+u)^{3/2}$
$-$	$2u$	$\frac{4}{15}(1+u)^{5/2}$
$+$	2	$\frac{8}{105}(1+u)^{7/2}$

$$\int \sqrt{x}\sqrt{1+\sqrt{x}}\,dx = 2\Big(\frac{2}{3}u^2(1+u)^{3/2} - \frac{8u}{15}(1+u)^{5/2} + \frac{16}{105}(1+u)^{7/2}\Big)$$

$$= \frac{4}{3}x(1+\sqrt{x})^{3/2} - \frac{16\sqrt{x}}{15}(1+\sqrt{x})^{5/2} + \frac{32}{105}(1+\sqrt{x})^{7/2} + C$$

$$\int_0^{\ln 3} \frac{e^x}{(e^x-1)^{2/3}}\,dx = \int_0^{\ln 3} (e^x-1)^{-2/3}\, d(e^x-1)$$

$$= 3\,(e^x-1)^{1/3}\Big|_0^{\ln 3}$$

$$= 3\left[2^{1/3} - 0\right)$$

$$= \boxed{3\sqrt[3]{2}}$$

$$\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+5} = \int_{-\infty}^{\infty} \frac{d(x+1)}{(x+1)^2+4}$$

$$\frac{du}{u^2+a^2}$$

$$= \frac{1}{2}\tan^{-1}\frac{x+1}{2}\Big|_{-\infty}^{\infty}$$

$$= \frac{1}{2}\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$\tan(-\infty)$

$$= \boxed{\frac{\pi}{2}}$$

$$\int_0^{\infty} \frac{1}{e^x+e^{-x}}\,dx = \int_0^{\infty} \frac{1}{e^x+e^{-x}}\,\frac{e^x}{e^x}\,dx$$

$$= \int_0^{\infty} \frac{e^x\,dx}{e^{2x}+1}$$

$$= \int_0^{\infty} \frac{d(e^x)}{(e^x)^2+1}$$

$$= \tan^{-1} e^x \Big|_0^{\infty}$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \boxed{\frac{\pi}{4}}$$

$$\frac{1}{2\cosh x}$$

$$\frac{1}{2}\int \operatorname{sech} x$$

$$\int \frac{2}{x(x^2+1)^2}\,dx$$

$$\frac{2}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$2 = A(x^4+2x^2+1) + (Bx+C)(x^3+x) + Dx^2+Ex$$

x^4 $\quad A+B=0 \rightarrow \underline{B=-2}$

x^3 $\quad \underline{C=0}$

x^2 $\quad 2A+B+D=0 \rightarrow \underline{D=-2}$

x^1 $\quad C+E=0 \rightarrow \underline{E=0}$

x^0 $\quad \underline{A=2}$

$$\int \frac{2\,dx}{x(x^2+1)^2} = 2\int\frac{dx}{x} - \int\frac{2x}{x^2+1}dx - \int\frac{2x\,dx}{(x^2+1)^2}$$

$$= 2\ln|x| - \int\frac{d(x^2+1)}{x^2+1} - \int\frac{d(x^2+1)}{(x^2+1)^2}$$

$$\underline{= 2\ln|x| - \ln(x^2+1) + \frac{1}{x^2+1} + K}$$

$$\int_{8\sqrt{2}}^{16} \frac{dx}{\sqrt{x^2-64}}$$

$x = 8\sec\theta$ $\quad$ $\sqrt{x^2-64} = 8\tan\theta$

$dx = 8\sec\theta\tan\theta\, d\theta$

$$\int_{8\sqrt{2}}^{16} \frac{dx}{\sqrt{x^2-64}} = \int_{8\sqrt{2}}^{16} \frac{8\sec\theta\tan\theta}{8\tan\theta}\, d\theta$$

$$= \int_{8\sqrt{2}}^{16} \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| \Big|_{8\sqrt{2}}^{16}$$

$$= \ln\left|\frac{x}{8} + \frac{\sqrt{x^2-64}}{8}\right| \Big|_{8\sqrt{2}}^{16}$$

$$= \ln\left(2 + \frac{4\sqrt{12}}{8}\right) - \ln(\sqrt{2}+1)$$

$$= \ln(2+\sqrt{3}) - \ln(\sqrt{2}+1)$$

$$\int \frac{\cos^3\theta}{\sqrt{\sin\theta}}\, d\theta = \int (\sin\theta)^{-1/2}\cos^2\theta\,\cos\theta\, d\theta$$

$$= \int (\sin\theta)^{-1/2}(1-\sin^2\theta)\, d(\sin\theta)$$

$$= \int \left[(\sin\theta)^{-1/2} - \sin^{3/2}\theta\right] d(\sin\theta)$$

$$= 2\sqrt{\sin\theta} - \frac{2}{5}\sin^{5/2}\theta + C$$

$$\int \cos^4 x\,dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1+2\cos 2x+\cos^2 2x)\,dx$$

$$= \frac{1}{4}\int \left(1+2\cos 2x+\frac{1}{2}+\frac{1}{2}\cos 4x\right)dx$$

$$= \frac{1}{4}\int \left(\frac{3}{2}+2\cos 2x+\frac{1}{2}\cos 4x\right)dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x+\sin 2x+\frac{1}{8}\sin 4x\right)+C$$

$$= \frac{3}{8}x+\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$$

$$\int e^{-3x}\sin 5x\,dx$$

		$\int \sin 5x\,dx$
$+$	e^{-3x}	$-\frac{1}{5}\cos 5x$
$-$	$-3e^{-3x}$	$-\frac{1}{25}\sin 5x$
$+$	$9e^{-3x}$	

$$\int e^{-3x}\sin 5x\,dx = e^{-3x}\left(-\frac{1}{5}\cos 5x-\frac{3}{25}\sin 5x\right)-\frac{9}{25}\int e^{-3x}\sin 5x\,dx$$

$$\frac{34}{25}\int e^{-3x}\sin 5x\,dx = \frac{1}{25}(-5\cos 5x-3\sin 5x)\,e^{-3x}$$

$$\int e^{-3x}\sin 5x\,dx = \frac{1}{34}(-5\cos 5x-3\sin 5x)+C$$

$$\int x^3 \cos 2x \, dx$$

		$\int \cos 2x \, dx$
$+$	x^3	$\frac{1}{2} \sin 2x$
$-$	$3x^2$	$-\frac{1}{4} \cos 2x$
$+$	$6x$	$-\frac{1}{8} \sin 2x$
$-$	6	$\frac{1}{16} \cos 2x$

$$\int x^3 \cos 2x \, dx = \left(\frac{1}{2}x^3 - \frac{3}{4}x\right)\sin 2x + \left(\frac{3}{4}x^2 - \frac{3}{8}\right)\cos 2x + C$$

$$\int x^3 e^{3x} \, dx$$

		$\int e^{3x}$
$+$	x^3	$\frac{1}{3} e^{3x}$
$-$	$3x^2$	$\frac{1}{9} e^{3x}$
$+$	$6x$	$\frac{1}{27} e^{3x}$
$-$	6	$\frac{1}{81} e^{3x}$

$$\int x^3 e^{3x} \, dx = \left(\frac{1}{3}x^3 - \frac{1}{3}x^2 + \frac{2}{9}x - \frac{2}{27}\right)e^{3x} + C$$