

Section 3.4 – Permutations and Combinations

Permutation

A permutation of a set of distinct objects is an arrangement of the objects is a *specific Order Without* repetition. An ordered arrangement of r elements of a set is called an *r -permutation*.

$$P_{n,r} = \frac{n!}{(n-r)!}$$

n Math \rightarrow Prob \rightarrow nPr r

$$P_{n,n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

$$P_{7,7} = 7!$$

Example

In mid 2007, eight candidates sought the Democratic nomination for president. In how many ways could voters rank their first, second, and third choices?

Solution

$$P_{8,3} = 336$$

8 Math \rightarrow Prob \rightarrow nPr 3

Requirements:

1. There are n items available, and some items are identical to others.
2. We select all of the n items (without replacement).
3. We consider rearrangements of distinct items to be different sequences.

If the preceding requirements are satisfied, and if there are n_1 alike, n_2 alike, \dots , n_k alike, the number of permutations (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1! n_2! \cdots n_k!}$$

Example

In how many ways can the letters in the word *Mississippi* be arranged?

Solution

$$\frac{11!}{1!4!4!2!} = 34,650 \text{ ways}$$

m	i	s	p
1	4	4	2

Example

A student buys 3 cherry yogurts, 2 raspberry yogurts, and 2 blueberry yogurts. She puts them in her dormitory refrigerator to eat one a day for the next week. Assuming yogurts of the same flavor are indistinguishable, in how many ways can she select yogurts to eat for the next week?

Solution

$$\frac{7!}{3!2!2!} = 210 \text{ ways}$$

Example

A televised show will include 4 women and 3 men as panelists

- a) In how many ways can the panelists be seated in a row of 7 chairs?
- b) In how many ways can the panelists be seated if the men and women are to be alternated?
- c) In how many ways can the panelists be seated if the men must sit together, and the women must also sit together?
- d) In how many ways can one woman and one man from the panel be selected?

Solution

a) $P_{7,7} = 5040 \text{ ways}$

b) 1st seat: 4 (4 women and not enough men 3)

2nd seat: 3

3rd seat: 3

4.3.3.2.2.1.1 = 144 ways

W	M	W	M	W	M	W
4	3	3	2	2	1	1

c) Arrange the 2 groups: 2!

Arranging women: 4!

Arranging men: 3!

Total: $2! \cdot 4! \cdot 3! = 288 \text{ ways}$

d) $4 \cdot 3 = 12 \text{ ways}$

Example

Suppose that there are eight runners in a race. The winner receives a gold medal, the second-place finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

Solution

There are: $P(8,3) = 8 \cdot 7 \cdot 6 = 336 \text{ ways}$

Combination

Combination of a set of n distinct objects taken r @ a time **without** repetition is an r element subset of the set of n objects.

The arrangement of the elements **doesn't matter**.

$$C_{n,r} = \binom{n}{r} = \frac{P_{n,r}}{r!} = \frac{n!}{r!(n-r)!}$$

n Math \rightarrow Prob $\rightarrow 3(nCr)$ r

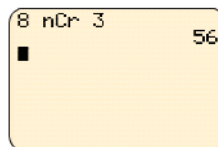
Example

How many committees of 3 people can be formed from a group of 8 people?

Solution

$$C_{8,3} = \binom{8}{3} = 56$$

8 Math \rightarrow Prob $\rightarrow 3(nCr)$ 3



Example

Three lawyers are to be selected from a group of 30 to work on a special project.

a) In how many different ways can the lawyers be selected?

$$C_{30,3} = \underline{4060 \text{ ways}}$$

b) In how many ways can the group of 3 be selected if a certain lawyer must work on the project?

1 already selected which left 29 to select from

$$C_{29,2} = \binom{29}{2} = \underline{406 \text{ ways}}$$

c) In how many ways can a nonempty group of at most 3 lawyers be selected from these 30 lawyers?

At most 3 = "1 or 2 or 3"

$$C_{30,1} + C_{30,2} + C_{30,3} = 30 + 435 + 4060 = \underline{4525 \text{ ways}}$$

Example

A salesman has 10 accounts in a certain city

- a) In how many ways can he select 3 accounts to call on?
- b) In how many ways can he select at least 8 of the accounts to use in preparing a report?

Solution

- a) In how many ways can he select 3 accounts to call on?

$$C_{10,3} = \binom{10}{3} = \underline{120 \text{ ways}}$$

- b) In how many ways can he select at least 8 of the accounts to use in preparing a report?

At least 8 = “8 or 9 or 10”

$$C_{10,8} + C_{10,9} + C_{10,10} = 45 + 10 + 1 = \underline{56 \text{ ways}}$$

Permutation: *order matter.*

Combination: *Order doesn't matter.*

Examples

For each problem, tell whether permutations or combinations should be used to solve the problem.

- a) How many 4-digit code numbers are possible if no digits are repeated?

Permutation

- b) A sample of 3 light bulbs is randomly selected from batch of 15. How many different samples are possible?

Combination

- c) In a baseball conference with 8 teams, how many games must be played so that each team plays every other team exactly once?

Combination

- d) In how many ways can 4 patients be assigned to 6 different hospital rooms so that each patient has a private room?

Permutation

Example

A manager must select 4 employees for promotion; 12 employees are eligible.

a) In how many ways can the 4 be chosen?

$$C_{12,4} = 495 \text{ ways}$$

b) In how many ways can 4 employees be chosen (from 12) to be placed in 4 different jobs?

$$P_{12,4} = 11,880 \text{ ways}$$

Example

In how many ways can a full house of aces and eights (3 aces and 2 eights) occur in 5-card poker?

Solution

$$C_{4,3} : \text{get 3 aces out of 4}$$

$$C_{4,2} : \text{get 2 eights out of 4}$$

$$C_{4,3} C_{4,2} = 24$$

Example

Five cards are dealt from a standard 52-card deck

a) How many hands have only face cards?

$$C_{12,5} = 792 \text{ hands}$$

b) How many such hands have exactly 2 hearts?

2 hearts will be selected from 13 cards, the other 3 cards will be selected from 39 remaining cards.

$$C_{13,2} C_{39,3} = 712,842 \text{ hands}$$

c) How many such hands have cards of a single suit?

There are 4 different suits

$$4 \cdot C_{13,5} = 5148 \text{ hands}$$

Exercises **Section 3.4 – Permutations and Combinations**

1. Decide whether the situation involves *permutations* or *combinations*
 - a) A batting order for 9 players for a baseball game
 - b) An arrangement of 8 people for a picture
 - c) A committee of 7 delegates chosen from a class of 30 students to bring a petition to the administration
 - d) A selection of a chairman and a secretary from a committee of 14 people
 - e) A sample of 5 items taken from 71 items on an assembly line
 - f) A blend of 3 spices taken from 7 spices on a spice rack
 - g) From the 7 male and 10 female sales representatives for an insurance company, team of 8 will be selected to attend a national conference on insurance fraud.
 - h) Marbles are being drawn without replacement from a bag containing 15 marbles.
 - i) The new university president named 3 new officers a vice-president of finance, a vice-president of academic affairs, and a vice-president of student affairs.
 - j) A student checked out 4 novels from the library to read over the holiday.
 - k) A father ordered an ice cream cone (chocolate, vanilla, or strawberry) for each of his 4 children.
2. Wing has different books to arrange on a shelf: 4 blue, 3 green, and 2 red.
 - a) In how many ways can the books be arranged on a shelf?
 - b) If books of the same color are to be grouped together, how many arrangements are possible?
 - c) In how many distinguishable ways can the books be arranged if books of the same color are identical but need not be grouped together?
 - d) In how many ways can you select 3 books, one of each color, if the order in which the books are selected does not matter?
 - e) In how many ways can you select 3 books, one of each color, if the order in which the books are selected matters?
3. A child has a set of differently shaped plastic objects. There are 3 pyramids, 4 cubes, and 7 spheres.
 - a) In how many ways can she arrange the objects in a row if each is a different color?
 - b) How many arrangements are possible if objects of the same shape must be grouped together and each object is a different color?
 - c) In how many distinguishable ways can the objects be arranged in a row if objects of the same shape are also the same color, but need not be grouped together?
 - d) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected does not matter and each object is a different color?
 - e) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected matters and each object is a different color?
4. Twelve drugs have been found to be effective in the treatment of a disease. It is believed that the sequence in which the drugs are administered is important in the effectiveness of the treatment. In how many different sequences can 5 of the 12 drugs be administered?

5. In a club with 16 members, how many ways can a slate of 3 officers consisting of president, vice-president, and secretary/treasurer be chosen?
6. In how many ways can 7 of 11 monkeys be arranged in a row for a genetics experiment?
7. In an experiment on social interaction, 6 people will sit in 6 seats in a row. In how many ways can this be done?
8. In an election with 3 candidates for one office and 6 candidates for another office, how many different ballots may be printed?
9. A business school gives courses in typing, shorthand, transcription, business English, technical writing, and accounting. In how many ways can a student arrange a schedule if 3 courses are taken? Assume that the order in which courses are scheduled matters.
10. If your college offers 400 courses, 25 of which are in mathematics, and your counselor arranges your schedule of 4 courses by random selection, how many schedules are possible that do not include a math course? Assume that the order in which courses are scheduled matters.
11. A baseball team has 19 players. How many 9-player batting orders are possible?
12. A chapter of union Local 715 has 35 members. In how many different ways can the chapter select a president, a vice-president, a treasurer, and a secretary?
13. A concert to raise money for an economics prize is to consist of 5 works; 2 overtures, 2 sonatas, and a piano concerto.
 - a) In how many ways can the program be arranged?
 - b) In how many ways can the program be arranged if an overture must come first?
14. A zydeco band from Louisiana will play 5 traditional and 3 original Cajun compositions at a concert. In how many ways can they arrange the program if
 - a) it begins with a traditional piece?
 - b) An original piece will be played last?
15. Given the set $\{A, B, C, D\}$, how many permutations are there of this set of 4 objects taken 2 at a time?
 - a) Using the multiplication principle
 - b) Using the Permutation
16. Find the number of permutations of 30 objects taken 4 at a time.
17. Five cards are marked with the numbers 1, 2, 3, 4, and 5, then shuffled, and 2 cards are drawn.
 - a) How many different 2-card combinations are possible?
 - b) How many 2-card hands contain a number less than 3?
18. An economics club has 31 members.
 - a) If a committee of 4 is to be selected, in how many ways can the selection be made?
 - b) In how many ways can a committee of at least 1 and at most 3 be selected?

19. Use a tree diagram for the following

- a) Find the number of ways 2 letters can be chosen from the set $\{L, M, N\}$ if order is important and repetition is allowed.
- b) Reconsider part a if no repeats are allowed
- c) Find the number of combinations of 3 elements taken 2 at a time. Does this answer differ from part a or b?

For each problem, decide whether permutations or combinations should be used to solve the problem.

20. In a club with 9 male and 11 female members, how many 5-member committees can be chosen that have

- a) All men?
- b) All women?
- c) 3 men and 2 women?

21. In a club with 9 male and 11 female members, how many 5-member committees can be selected that have

- a) At least 4 women?
- b) No more than 2 men?

22. In a game of musical chairs, 12 children will sit in 11 chairs arranged in a row (one will be left out). In how many ways can this happen, if we count rearrangements of the children in the chairs as different outcomes?

23. A group of 3 students is to be selected from a group of 14 students to take part in a class in cell biology.

- a) In how many ways can this be done?
- b) In how many ways can the group who will not take part be chosen?

24. Marbles are being drawn without replacement from a bag containing 16 marbles.

- a) How many samples of 2 marbles can be drawn?
- b) How many samples of 2 marbles can be drawn?
- c) If the bag contains 3 yellow, 4 white, and 9 blue marbles, how many samples of 2 marbles can be drawn in which both marbles are blue?

25. There are 7 rotten apples in a crate of 26 apples

- a) How many samples of 3 apples can be drawn from the crate?
- b) How many samples of 3 could be drawn in which all 3 are rotten?
- c) How many samples of 3 could be drawn in which there are two good apples and one rotten one?

26. A bag contains 5 black, 1 red, and 3 yellow jelly beans; you take 3 at random. How many samples are possible in which the jelly beans are

- a) All black?
- b) All red?
- c) All yellow?

- d) 2 black and 1 red?
 - e) 2 black and 1 yellow?
 - f) 2 yellow and 1 black?
 - g) 2 red and 1 yellow?
27. In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?
28. From a pool of 8 secretaries, 3 are selected to be assigned to 3 managers, one per manager. In how many ways can they be selected and assigned?
29. A salesperson has the names of 6 prospects.
- a) In how many ways can she arrange her schedule if she calls on all 6?
 - b) In how many ways can she arrange her schedule if she can call on only 4 of the 6?
30. A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.
- a) How many delegations are possible?
 - b) If it is decided that a particular worker must be in the delegation, how many different delegations are possible?
 - c) If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?
31. Hamburger Hut sells regular hamburgers as well as a larger burger. Either type can include cheese, relish, lettuce, tomato, mustard, or catsup.
- a) How many different hamburgers can be ordered with exactly three extras?
 - b) How many different regular hamburgers can be ordered with exactly three extras?
 - c) How many different regular hamburgers can be ordered with at least five extras?
32. Five items are to be randomly selected from the first 50 items on an assembly line to determine the defect rate. How many different samples of 5 items can be chosen?
33. From a group of 16 smokers and 22 nonsmokers, a researcher wants to randomly select 8 smokers and 8 nonsmokers for a study. In how many ways can the study group be selected?
34. In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.
- a) In how many ways can this be done?
 - b) In how many ways can this be done if exactly 2 wheat plants must be included?
35. A legislative committee consists of 5 Democrats and 4 Republicans. A delegation of 3 is to be selected to visit a small Pacific island republic.
- a) How many different delegations are possible?
 - b) How many delegations would have all Democrats?
 - c) How many delegations would have 2 Democrats and 1 Republican?
 - d) How many delegations would have at least 1 Republican?

36. From 10 names on a ballot, 4 will be elected to a political party committee. In how many ways can the committee of 4 be formed if each person will have a different responsibility, and different assignments of responsibility are considered different committees?
37. How many different 13-card bridge hands can be selected from an ordinary deck?
38. Five cards are chosen from an ordinary deck to form a hand in poker. In how many ways is it possible to get the following results?
- a) 4 queens
 - b) No face card
 - c) Exactly 2 face cards
 - d) At least 2 face cards
 - e) 1 heart, 2 diamonds, and 2 clubs
39. In poker, a flush consists of 5 cards with the same suit, such as 5 diamonds.
- a) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by listing all the possibilities.
 - b) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by using combinations
40. If a baseball coach has 5 good hitters and 4 poor hitters on the bench and chooses 3 players at random, in how many ways can he choose at least 2 good hitters?
41. The coach of a softball team has 6 good hitters and 8 poor hitters. He chooses 3 hitters at random.
- a) In how many ways can he choose 2 good hitters and 1 poor hitter?
 - b) In how many ways can he choose 3 good hitters?
 - c) In how many ways can he choose at least 3 good hitters?
42. How many 5 card hands will have 3 aces and 2 kings?
43. How many 5 card hands will have 3 hearts and 2 spades?
44. 2 letters follow by 3 numbers; 2 letters out of 8 & 3 numbers out of 10
45. Serial numbers for a product are to be made using 3 letters followed by 2 digits (0 – 9 no repeats). If the letters are to be taken from the first 8 letters of the alphabet with no repeats, how many serial numbers are possible?
46. A company has 7 senior and 5 junior officers. An ad hoc legislative committee is to be formed.
- a) How many 4-officer committees with 1 senior officer and 3 junior officers can be formed?
 - b) How many 4-officer committees with 4 junior officers can be formed?
 - c) How many 4-officer committees with at least 2 junior officers can be formed?
47. From a committee of 12 people,
- a) In how many ways can we choose a chairperson, a vice-chairperson, a secretary, and a treasurer, assuming that one person can't hold more than one position
 - b) In how many ways can we choose a subcommittee of 4 people?
48. Find the number of combinations of 30 objects taken 4 at a time.

49. How many different permutations are there of the set $\{a, b, c, d, e, f, g\}$?
50. How many permutations of $\{a, b, c, d, e, f, g\}$ end with a ?
51. Find the number of 5-permutations of a set with nine elements
52. In how many different orders can five runners finish a race if no ties are allowed?
53. A coin flipped eight times where each flip comes up either heads or tails. How many possible outcomes
- a) Are there in total?
 - b) Contain exactly three heads?
 - c) Contain at least three heads?
 - d) Contain the same number of heads and tails?
54. In how many ways can a set of two positive integers less than 100 be chosen?
55. In how many ways can a set of five letters be selected from the English alphabet?