4.1

$$\int x^n dx = \frac{x^{(n+1)}}{n+1} + C \qquad n \neq -1$$

$$\int x^{-1} dx = \int \frac{dx}{x}$$
$$= \ln|x| + C$$

$$\int \frac{dx}{x^2} = -\frac{1}{x} + C$$

Ex $\int (4x^3 - 5x + 2)\,dx = x^4 - \frac{5}{2}x^2 + 2x + C$

Ex $\int (x^2 - 2x + 5)\,dx = \frac{1}{3}x^3 - x^2 + 5x + C$

Ex $\int \sin x\,dx = -\cos x + C$

Ex $\int \cos 3x\,dx = \frac{1}{3}\sin 3x + C$

$$\int e^x dx = e^x + C \qquad \int \frac{dx}{x} = \ln|x| + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\left.\begin{aligned} \int \frac{dx}{\sqrt{a^2 - x^2}} &= \sin^{-1}\frac{x}{a} + C \\ \int \frac{dx}{x^2 + a^2} &= \frac{1}{a}\tan^{-1}\frac{x}{a} + C \end{aligned}\right\} \leftarrow$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$$

Ex. $\int e^{-10x}\,dx = -\frac{1}{10}e^{-10x} + C$

Ex $\int \frac{5}{x}\,dx = 5\ln|x| + C$

Ex $\int \frac{4\,dx}{\sqrt{9-x^2}} = 4\sin^{-1}\frac{x}{3} + C$

Ex $\int \frac{dx}{16x^2+1} = \frac{1}{16}\int \frac{dx}{x^2+\frac{1}{16}}$

$= \frac{1}{16}(4)\tan^{-1}\left(\frac{x}{1/4}\right) + C$

$= \frac{1}{4}\tan^{-1}(4x) + C$

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#1. $\int v^2\,dv = \frac{1}{3}v^3 + C$

#2 $\int x^{1/2}\,dx = \frac{2}{3}x^{3/2} + C$

#3 $\int 4y^{-3}\,dy = -2y^{-2} + C$

#4 $\int (x^3 - 4x + 2)\,dx = \frac{1}{4}x^4 - 2x^2 + 2x + C$

#7 $\int \left(\frac{x^2+1}{\sqrt{x}}\right)dx = \int (x^{3/2} + x^{-1/2})\,dx$

$= \frac{2}{5}x^{5/2} + 2x^{1/2} + C$

#15 $\int (\sqrt{x} + \sqrt[3]{x})\,dx = \int (x^{1/2} + x^{1/3})\,dx$

$= \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

#16. $\int 2x(1-x^{-3})\,dx = \int (2x - 2x^{-2})\,dx$

$= x^2 + x^{-1} + C$

$= x^2 + \frac{1}{x} + C$

#18 $\int (-2\cos t)\,dt = -2\sin t + C$

#19 $\int 7\sin\frac{\theta}{3}\,d\theta = -21\cos\frac{\theta}{3} + C$

#20 $\int \frac{2}{5}\sec\theta\tan\theta\,d\theta = \frac{2}{5}\sec\theta + C$

#21 $\int (4\sec x\tan x - 2\sec^2 x)\,dx = 4\sec x - 2\tan x + C$

#37 $\int 2e^{2x}\,dx = e^{2x} + C$

#41 $\int \frac{1+\tan\theta}{\sec\theta}\,d\theta = \int \left(\frac{1}{\sec\theta} + \frac{\tan\theta}{\sec\theta}\right)d\theta$

$= \int (\cos\theta + \sin\theta)\,d\theta$

$= \sin\theta - \cos\theta + C$

4.4

Fundamental Theorem of Calculus I : P2

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Ex $\int_0^{\pi} \cos x\,dx = \sin x \Big|_0^{\pi}$ $\qquad -\Big|_a^b$

$= \sin\pi - \sin 0$

$= 0$

Ex $\int_{-\pi/4}^{0} \sec x \tan x\,dx = \sec x \Big|_{-\frac{\pi}{4}}^{0}$ $\qquad \sec\ \frac{1}{\cos}$

$= \sec 0 - \sec\left(-\frac{\pi}{4}\right)$

$= 1 - \sqrt{2}$

Ex $\int_1^4 \left(\frac{3}{2}\sqrt{x} - \frac{4}{x^2}\right)dx = x^{3/2} + \frac{4}{x}\Big|_1^4$

$= (2^2)^{3/2} + 1 - (1 + 4)$

$= 8 + 1 - 5$

$= 4$

Ex. Given $v(t) = 160 - 32t$ ft/sec

a) displacement? $s(t)$? $0 \le t \le 8$

$$s(t) = \int_0^8 (160 - 32t)\,dt$$

$$= 160t - 16t^2 \Big|_0^8 \qquad 16(10t - t^2)$$

$$= 16[80 - 64 - (0)]$$

$$= 256 \text{ ft}$$

b) Total distance

$$v(t) = 160 - 32t = 0$$

$$t = \frac{160}{32} = 5$$

$$D = \int_0^5 (160-32t)\,dt - \int_5^8 (160-32t)\,dt$$

$$= 16(10t - t^2)\Big|_0^5 - 16(10t - t^2)\Big|_5^8$$

$$= 16[50 - 25] - 16[80 - 64 - (50 - 25)]$$

$$= 400 - 16(16 - 25)$$

$$= 400 + 144$$

$$= 544 \text{ ft}$$

2. $f(x) = x^2 - 4$

a) $[-2, 2]$

$$\int_{-2}^{2} (x^2 - 4)\,dx = \frac{1}{3}x^3 - 4x \Big|_{-2}^{2}$$

$$= \frac{8}{3} - 8 - \left(-\frac{8}{3} + 8\right)$$

$$= \frac{8}{3} - 8 + \frac{8}{3} - 8$$

$$= \frac{16}{3} - 16$$

$$= -\frac{32}{3}$$

Area between $[-2, 2]$

$$f(x) = x^2 - 4 = 0$$

$$x = \pm 2$$

$$\text{Area} = -\int_{-2}^{2} (x^2 - 4)\,dx$$

$$= \frac{32}{3} \text{ unit}^2$$

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$g(x) = 4 - x^2 = 0 \quad x = \pm 2$ (0)

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\,dx$$

$$= 4x - \frac{1}{3}x^3 \Big|_{-2}^{2}$$

$$= 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)$$

$$= 16 - \frac{16}{3}$$

$$= \frac{32}{3} \text{ unit}^2$$

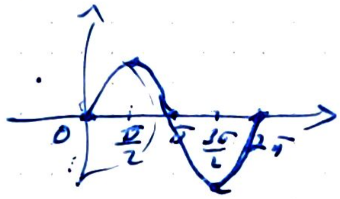
Ex $f(x) = \sin x \quad x \in [0, 2\pi]$

a) $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, dx = -\cos x \Big|_0^{2\pi}$$

$$= -(\cos 2\pi - \cos 0)$$

$$= 0$$




b) Area?

$\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

$$\text{Area} = \int_0^{\pi} \sin x \, dx - \int_{\pi}^{2\pi} \sin x \, dx$$

$$= -\cos x \Big|_0^{\pi} + \cos x \Big|_{\pi}^{2\pi}$$

$$= -(\cos \pi - \cos 0) + \cos 2\pi - \cos \pi$$

$$= 2 + 1 + 1$$

$$= 4 \text{ unit}^2$$

Ex. $f(x) = x^3 - x^2 - 2x \qquad -1 \le x \le 2$

x-axis : $y = 0$

$x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

$\underline{x = 0, -1, 2}$

$\frac{2^4}{2^2}$

$$\text{Area} = \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx - \int_{0}^{2} (x^3 - x^2 - 2x)\,dx$$

$$= \left(\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 - x^2\right)\Big|_{-1}^{0} - \left(\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 - x^2\right)\Big|_{0}^{2}$$

$$= 0 - \left(\tfrac{1}{4} + \tfrac{1}{3} - 1\right) - \left(4 - \tfrac{8}{3} - 4\right)$$

$$= -\frac{3+4-12}{12} + \frac{8}{3}$$

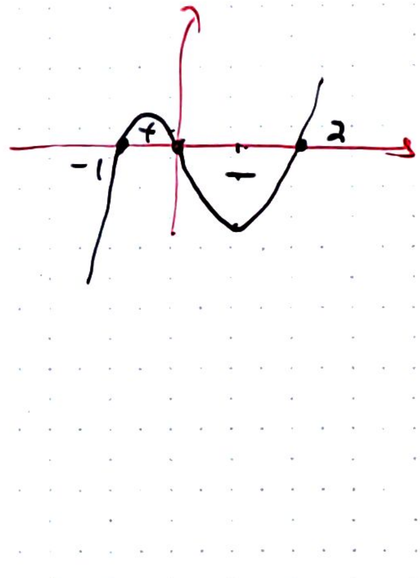
$$= \frac{5}{12} + \frac{8}{3}$$

$$= \underline{\frac{37}{12} \text{ unit}^2}$$

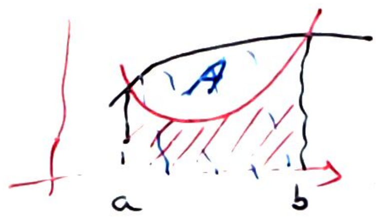
$x^3 - x^2 - 2x$

$3x^2 - 2x - 2 = 0$

$\frac{2 \pm \sqrt{28}}{6}$



# Area between 2 curves




Ex $A$? $y = 2 - x^2$ & $y = -x$

soln $y = 2 - x^2 = -x$

$x^2 - x - 2 = 0 \Rightarrow \underline{x = -1, 2}$

$$\text{Area} = \int_{-1}^{2} (2 - x^2 + x)\, dx$$

$$= 2x - \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_{-1}^{2}$$

$$= 4 - \frac{8}{3} + 2 - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$$

$$= 6 - \frac{8}{3} + 2 - \frac{1}{3} - \frac{1}{2}$$

$$= 5 - \frac{1}{2}$$

$$= \underline{\frac{9}{2}\ \text{unit}^2}$$