$$\int 2x\, e^{3x}dx = 2e^{3x}\left(\frac{x}{3}-\frac{1}{9}\right)+C$$
$$= \boxed{\frac{2}{9}e^{3x}(3x-1)+C}$$

$\int e^{3x}$

$+\ x \quad \frac{1}{3}e^{3x}$

$-\ 1 \quad \frac{1}{9}e^{3x}$

$$\int \frac{x\,dx}{\sqrt{x+1}} = \boxed{2x\sqrt{x+1}-\frac{4}{3}(x+1)^{3/2}+C}$$

$\int (x+1)^{-1/2}\frac{dx}{d(x+1)}$

$+\ X \quad 2(x+1)^{1/2}$

$-\ 1 \quad \frac{4}{3}(x+1)^{3/2}$

$$\int \frac{\ln x}{x^{10}}dx = -\frac{\ln x}{9x^9}+\frac{1}{9}\int x^{-10}dx$$
$$= \boxed{-\frac{\ln x}{9x^9}-\frac{1}{81x^9}+C}$$

$u=\ln x \quad v=\int x^{-10}dx$

$du=\frac{dx}{x} \quad = -\frac{x^{-9}}{9}$

$$\int \sin^{-1}x\,dx = x\sin^{-1}x-\int\frac{x\,dx}{\sqrt{1-x^2}}$$
$$= x\sin^{-1}x+\frac{1}{2}\int\frac{d(1-x^2)}{\sqrt{1-x^2}}$$
$$= \boxed{x\sin^{-1}x+\sqrt{1-x^2}+C}$$

$u=\sin^{-1}x \quad v=\int dx = x$

$du=\frac{dx}{\sqrt{1-x^2}}$

$$\int x\sec^{-1}x\,dx = \frac{1}{2}x^2\sec^{-1}x-\frac{1}{2}\int\frac{x\,dx}{\sqrt{x^2-1}}$$

$\boxed{x\geq 1}$

$$= \frac{1}{2}x^2\sec^{-1}x-\frac{1}{4}\int\frac{d(x^2-1)}{\sqrt{x^2+1}}$$
$$= \boxed{\frac{1}{2}x^2\sec^{-1}x-\frac{1}{2}\sqrt{x^2+1}+c}$$

$u=\sec^{-1}x \to du=\frac{dx}{|x|\sqrt{x^2-1}}$

$v=\int x\,dx=\frac{1}{2}x^2$

$$\int x\sin x\cos x\,dx = \frac{1}{2}\int x\sin 2x\,dx$$
$$= \frac{1}{2}\left(\frac{1}{2}x\cos 2x-\frac{1}{4}\sin 2x\right)+C$$
$$= \boxed{\frac{1}{4}x\cos 2x-\frac{1}{8}\sin 2x+C}$$

$\int \sin 2x$

$-\ X \quad -\frac{1}{2}\cos 2x$

$+\ 1 \quad -\frac{1}{4}\sin 2x$

$$\int e^{3x}\cos 2x\,dx = \frac{1}{2}e^{3x}\sin 2x + \frac{e^{3x}}{12}\cos 2x - \frac{1}{36}\int e^{3x}\cos 2x\,dx$$

		$\int \cos 2x$
$+$	e^{3x}	$\frac{1}{2}\sin 2x$
$-$	$\frac{1}{3}e^{3x}$	$-\frac{1}{4}\cos 2x$
$+$	$\frac{1}{9}e^{3x}$	$\int -\frac{1}{4}\cos 2x$

$$\frac{37}{36}\int e^{3x}\cos 2x\,dx = \frac{e^{3x}}{12}(6\sin 2x + \cos 2x)$$

$$\boxed{\int e^{3x}\cos 2x\,dx = \frac{3}{37}(6\sin 2x + \cos 2x) + C}$$

$$\int_0^{1/\sqrt{2}} y\tan^{-1}y^2\,dy = \frac{1}{2}\int_0^{1/\sqrt{2}} \tan^{-1}y^2\,d(y^2) \qquad x = y^2$$

$$= \frac{1}{2}\int_0^{1/\sqrt{2}} \tan^{-1}x\,dx \qquad u = \tan^{-1}x \quad du = \frac{dx}{1+x^2} \quad v = \int dx = x$$

$$= \frac{1}{2}\left[x\tan^{-1}x \Big|_0^{1/\sqrt{2}} - \int_0^{1/\sqrt{2}} \frac{x\,dx}{1+x^2}\right.$$

$$= \frac{1}{2}\left[\left(y^2\tan^{-1}y^2\right)\Big|_0^{1/\sqrt{2}} - \frac{1}{2}\int_0 \frac{d(x^2+1)}{1+x^2}\right]$$

$$= \frac{1}{4}\tan^{-1}\frac{1}{2} - \frac{1}{4}\ln(1+y^4)\Big|_0^{1/\sqrt{2}}$$

$$= \frac{1}{4}\tan^{-1}\frac{1}{2} - \frac{1}{4}\ln\frac{5}{4}$$

$$\int x^2\ln^2 x\,dx = \frac{1}{3}x^3\ln^2 x - \frac{2}{3}\int x^2\ln x\,dx \qquad u = \ln^2 x \quad du = 2\frac{\ln x}{x} \quad v = \int x^2 = \frac{1}{3}x^3$$

$$= \frac{1}{3}x^3\ln^2 x - \frac{2}{3}\left[\frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx\right] \qquad u = \ln x \to du = \frac{dx}{x}$$

$$v = \int x^2\,dx = \frac{1}{3}x^3$$

$$= \frac{1}{3}x^3\ln^2 x - \frac{2}{9}x^3\ln x + \frac{2}{27}x^3 + C$$

$$= \frac{x^3}{27}(9\ln^2 x - 6\ln x + 2) + C$$

$$\boxed{\int x^4 e^x\,dx = e^x(x^4 - 4x^3 + 12x^2 - 24x + 24) + C}$$

		e^x
x^4	$+$	e^x
$4x^3$	$-$	e^x
$12x^2$	$+$	e^x
$24x$	$-$	e^x
24	$+$	e^x

$$\int_{-1}^{0} 2x^2\sqrt{x+1}\,dx = \frac{4x^2}{3}(x+1)^{3/2} - \frac{16x}{15}(x+1)^{5/2} + \frac{32}{105}(x+1)^{7/2}\Big|_{-1}^{0}$$

$$= \boxed{\frac{32}{105}}$$

$\int (x+1)^{1/2}\,d(x+1)$

$+$	$2x^2$	$\frac{2}{3}(x+1)^{3/2}$
$-$	$4x$	$\frac{4}{15}(x+1)^{5/2}$
$+$	4	$\frac{8}{105}(x+1)^{7/2}$

$$\int (x^3-2x)\sin 2x\,dx = -\frac{1}{2}(x^3-2x)\cos 2x + \frac{1}{4}(3x^2-2)\sin 2x + \frac{3x}{4}\cos 2x - \frac{3}{8}\sin 2x + C$$

$\int \sin 2x$

$+$	x^3-2x	$-\frac{1}{2}\cos 2x$
$-$	$3x^2-2$	$-\frac{1}{4}\sin 2x$
$+$	$6x$	$\frac{1}{8}\cos 2x$
$-$	6	$\frac{1}{16}\sin 2x$

$$\int \frac{2x^2-3x}{(x-1)^3}\,dx$$

$$= -\frac{1}{2}(2x^2-3x)(x-1)^{-2} - \frac{1}{2}(4x-3)(x-1)^{-1} + 2\ln|x-1| + C$$

$\int (x-1)^{-3}\,d(x-1)$

$+$	$2x^2-3x$	$-\frac{1}{2}(x-1)^{-2}$
$-$	$4x-3$	$\frac{1}{2}(x-1)^{-1}$
$+$	4	$\frac{1}{2}\ln\|x-1\|$

$$\int \frac{x^2+3x+4}{\sqrt[3]{2x+1}}\,dx$$

$\frac{1}{2}\int (2x+1)^{-1/3}\,d(2x+1)$

	x^2+3x+4	$\frac{3}{4}(2x+1)^{2/3}$
$-$	$2x+3$	$\frac{1}{2}\frac{9}{20}(2x+1)^{5/3}$
$+$	2	$\frac{1}{2}\frac{27}{320}(2x+1)^{8/3}$

$$\int \frac{x^2+3x+4}{\sqrt[3]{2x+1}}\,dx = \frac{3}{4}(x^2+3x+4)(2x+1)^{2/3} - \frac{9}{40}(2x+3)(2x+1)^{5/3} + \frac{27}{320}(2x+1)^{8/3} + C$$

Find the volume of the solid that is generated by the region bounded by $f(x) = x\ln x$, and the x-axis on $[1, e^2]$ is revolved about the y-axis.

Using Disk method

$$V = \pi\int_1^{e^2}(x\ln x)^2\,dx$$

$$= \pi\int_1^{e^2} x^2\ln^2 x\,dx$$

$$= \pi\left[\frac{1}{3}x^3\ln^2 x - \frac{2}{9}x^3\ln x + \frac{2}{27}x^3\right|_1^{e^2}$$

$$= \frac{\pi}{27}\left[e^6\left(9(\ln e^2)^2 - 6\ln e^2 + 2\right) - 2\right)$$

$$= \frac{\pi}{27}\left(e^6(36 - 12 + 2) - 2\right)$$

$$= \frac{\pi}{27}\left(26e^6 - 2\right)$$

Find the volume of the solid that is generated by the region bounded by $f(x) = e^{-x}$, x-axis on $[0, \ln 2]$ is revolved about the line $x = \ln 2$.

Shells

$$\int e^{-x}$$

x	$-e^{-x}$
1	e^{-x}
$-c$	

$$V = 2\pi\int_0^{\ln 2}(\ln 2 - x)e^{-x}\,dx$$

$$= 2\pi\int_0^{\ln 2}\ln 2\, e^{-x} - 2\pi\int_0^{\ln 2} x e^{-x}\,dx$$

$$= 2\pi\left[-\ln 2\, e^{-x} + xe^{-x} + e^{-x}\right|_0^{\ln 2}$$

$$= 2\pi\left(-\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 + \frac{1}{2} + \ln 2 - 1\right)$$

$$= 2\pi\left(\ln 2 - \frac{1}{2}\right)$$

$$= \pi(2\ln 2 - 1)$$

$$= \pi(\ln 4 - 1)$$