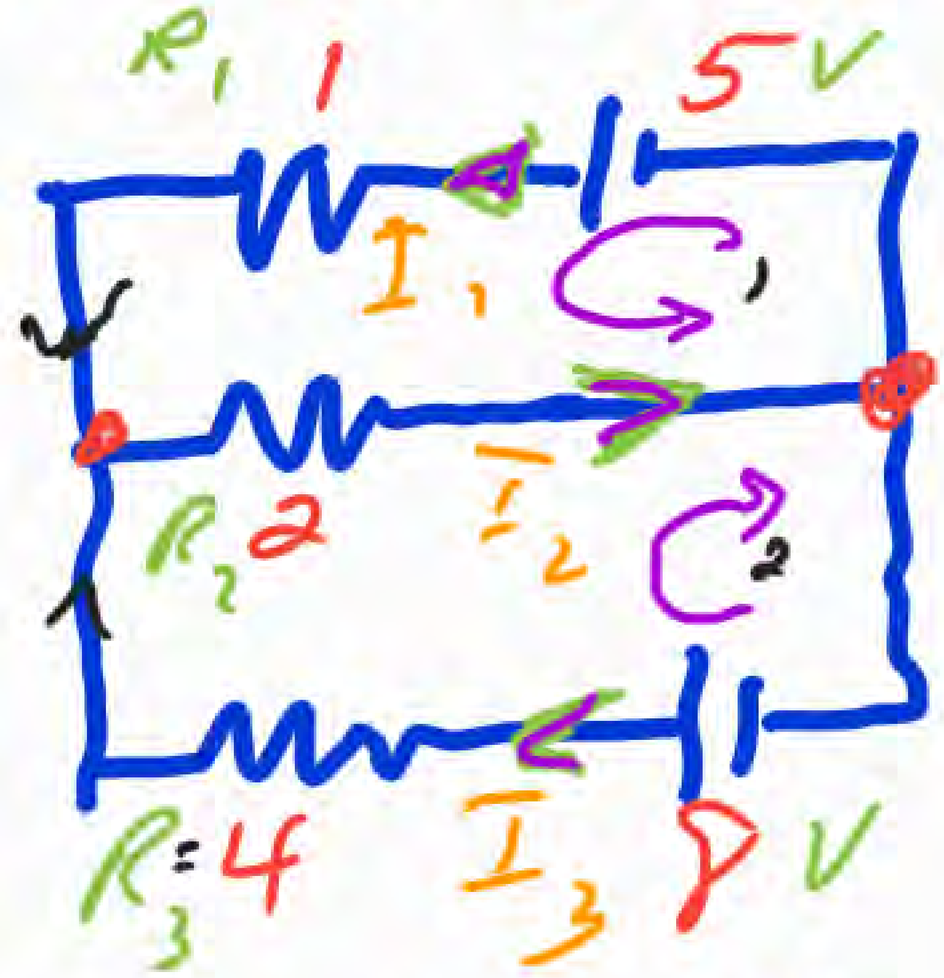
Electrical network



Ω

$V = RI$

$\sum I_i = 0$

Kirkchhoff's

I_1, I_2, I_3

$$\begin{cases} I_1 + I_3 = I_2 \\ I_1 + 2I_2 = 5 \\ 4I_3 + 2I_2 = 8 \end{cases}$$

$$\begin{cases} I_1 - I_2 + I_3 = 0 \\ I_1 + 2I_2 \qquad = 5 \\ \qquad I_2 + 2I_3 = 4 \end{cases}$$

$$D = \begin{vmatrix} 1 & -1 & +1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 7 \qquad D_{I_1} = \begin{vmatrix} 0 & -1 & 1 \\ 5 & 2 & 0 \\ 4 & 1 & 2 \end{vmatrix} = 7$$

$$D_2 = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 5 & 0 \\ 0 & 4 & 2 \end{vmatrix} = 14 \qquad D_3 = \begin{vmatrix} 1 & -1 & 0 \\ 1 & 2 & 5 \\ 0 & 1 & 4 \end{vmatrix} = 7$$

$$\boxed{I_1 = 1A, \quad I_2 = 2A, \quad I_3 = 1A}$$

Ex $A = \begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix}$ (3×3)

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$[13 \ 5 \ 5][20 \ 0 \ 13][5 \ 0 \ 13][15 \ 14 \ 4][1 \ 25 \ 0]$

$[13 \ 5 \ 5] \ 20 \ 0 \ 13 \ 5 \ 0 \ 13 \ 15 \ 14 \ 4 \ 1 \ 25 \ 0$

$$[13 \ 5 \ 5]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [13 \ -26 \ 21]$$

$$[20 \ 0 \ 13]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [33 \ -53 \ -12]$$

$$[5 \ 0 \ 13]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [18 \ -23 \ -42]$$

$$[15 \ 14 \ 4]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [5 \ -20 \ 56]$$

$$[1 \ 25 \ 0]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [-24 \ 23 \ 77]$$

cryptogram:

$[13 \ -26 \ 21][33 \ -53 \ -12][18 \ -23 \ -42]$

$[5 \ -20 \ 56]\,[-24 \ 23 \ 77]$

$$A = \begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} \qquad |A| = 1$$

$$A^{-1} = \begin{pmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{pmatrix}$$

$$[13 \ -26 \ 21]\begin{pmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{pmatrix} = [13 \ 5 \ 5]$$

$$[33 \ -53 \ -12]\begin{pmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{pmatrix} = [20 \ 0 \ 13]$$

$$[18 \ -23 \ -42]\begin{pmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{pmatrix} = [5 \ 0 \ 13]$$

$$[5 \ -20 \ 56]\begin{pmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{pmatrix} = [15 \ 14 \ 4]$$

$$[-24 \ 23 \ 77]\begin{pmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{pmatrix} = [1 \ 25 \ 0]$$

13 5 5 20 0 13 5 0 13 15 14 4 1 25 0

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on Exam Table $\begin{matrix} A & B & C & D & - - \\ 0 & 1 & 2 & 3 \end{matrix}$

key word $\underline{1}\ \underline{2}\ \underline{3}\ \underline{4}$ $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

Crypto is given.

$[a_1\ a_2]\ a_3\ - -$

$A^{-1} = \checkmark$

$[a_1\ a_2] A^{-1} = [b_1\ b_2]$

$\vdots \qquad \vdots$

$b_1\ b_2\ b_3$

$\underline{\ }\ \underline{\ }\ \underline{\ }$

Exam 1

A^{-1} 2×2, 3×3

det (2)

$\underline{5 \times 5}$ A^{-1} A^k diagonal

ask question → answer

- Cramer's 3×3

=

Prove (out)

A : invertible $\Rightarrow AA^{-1} = A^{-1}A = I$

A symmetric $\Rightarrow A = A^T$

$tr(A+B) = tr(A) + tr(B)$ $n \times n$

$A = [a_{ij}]$ $B = [b_{ij}]$

$tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$

$tr(B) = b_{11} + b_{22} + \cdots + b_{nn}$

$A + B = [a_{ij} + b_{ij}]$ $[a_{ij}] + [b_{ij}]$

$tr(A+B) = (a_{11} + b_{11}) + (a_{22} + b_{22}) + \cdots + (a_{nn} + b_{nn})$

$= (a_{11} + a_{22} + \cdots + a_{nn}) + (b_{11} + b_{22} + \cdots + b_{nn})$

$= tr(A) + tr(B)$ ✓

Prove: A is invertible $\Rightarrow AA^{-1} = A^{-1}A = I$

$(A^T)^{-1} = (A^{-1})^T$?

$A^T (A^{-1})^T = (A^{-1}A)^T$

$= I^T$

$= I$ ✓

$(A^{-1})^T A^T = (A A^{-1})^T$

$= I^T$

$= I$ ✓

$=$

1.8 #9 C is inv $CC^{-1} = C^{-1}C = I$

$$CA = CB \Rightarrow A = B$$

$$CA = CB$$
$$C^{-1}(CA) = C^{-1}(CB)$$
$$(C^{-1}C)A = (C^{-1}C)B$$
$$IA = IB$$
$$A = A \checkmark.$$

#10 $A = A^T$ $B = B^T$ $C = C^T$

$$ABC = I$$

B inv: $BB^{-1} = B^{-1}B = I$

$B^{-1} = CA$?

$B^{-1}A = AB^{-1}$

$$I = ABC$$
$$\boxed{BB^{-1} = ABC}$$
$$A^{-1}I = A^{-1}ABC$$
$$A^{-1} = BC$$
$$A^{-1}A = BCA$$
$$I = BCA$$
$$B^{-1}I = B^{-1}BCA$$
$$B^{-1} = ICA$$
$$= CA$$

A

$$A^2 - 2A + 5I = 0$$

$$A^2 = \begin{pmatrix} & \\ & \end{pmatrix}$$

$$\begin{pmatrix} & \\ & \end{pmatrix} - 2\begin{pmatrix} & \\ & \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \checkmark$$
