

Solution

Section 2.2 – Trigonometric Functions

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-2, 3)$ is on the terminal side of θ .

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

$$\sin \theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$$

$$\cos \theta = \frac{x}{r} = -\frac{2}{\sqrt{13}}$$

$$\tan \theta = \frac{y}{x} = -\frac{3}{2}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{r}{x} = -\frac{\sqrt{13}}{2}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

$$\cot \theta = \frac{x}{y} = -\frac{2}{3}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-3, -4)$ is on the terminal side of θ .

Solution

$$3, 4 \rightarrow 5$$

$$\sin \theta = -\frac{4}{5}$$

$$\cos \theta = -\frac{3}{5}$$

$$\tan \theta = \frac{-4}{-3} = \frac{4}{3}$$

$$\csc \theta = -\frac{5}{4}$$

$$\sec \theta = -\frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

Exercise

Find the six trigonometry functions of θ in standard position with terminal side through the point $(-3, 0)$.

Solution

$$r = \sqrt{(-3)^2 + 0^2} = 3$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin \theta = \frac{0}{3} = 0$$

$$\cos \theta = \frac{-3}{3} = -1$$

$$\tan \theta = \frac{0}{-3} = 0$$

$$\csc \theta = \frac{1}{0} \rightarrow \infty$$

$$\sec \theta = \frac{1}{-1} = -1$$

$$\cot \theta = \frac{1}{0} = \infty$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(12, -5)$ is on the terminal side of θ .

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{12^2 + (-5)^2} = \underline{13}$$

$$\sin \theta = -\frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$\tan \theta = -\frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = \frac{13}{12}$$

$$\cot \theta = -\frac{12}{5}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(5, -12)$ is on the terminal side of θ .

Solution

$$5 \quad 12 \rightarrow 13$$

$$\sin \theta = -\frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = -\frac{12}{5}$$

$$\csc \theta = -\frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = -\frac{5}{12}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(9, -12)$ is on the terminal side of θ .

Solution

$$(9, -12) = 3(3, -4) \Rightarrow 3 \quad 4 \rightarrow 5$$

$$\sin \theta = -\frac{4}{5}$$

$$\cos \theta = \frac{3}{5}$$

$$\tan \theta = -\frac{4}{3}$$

$$\csc \theta = -\frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = -\frac{3}{4}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(16, -12)$ is on the terminal side of θ .

Solution

$$(16, -12) = 4(4, -3) \Rightarrow 4 \quad 3 \rightarrow 5$$

$$\sin \theta = -\frac{3}{5} \qquad \cos \theta = \frac{4}{5} \qquad \tan \theta = -\frac{3}{4}$$

$$\csc \theta = -\frac{5}{3} \qquad \sec \theta = \frac{5}{4} \qquad \cot \theta = -\frac{4}{3}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(15, -8)$ is on the terminal side of θ .

Solution

$$15 \quad 8 \rightarrow 17$$

$$\sin \theta = -\frac{8}{17} \qquad \cos \theta = \frac{15}{17} \qquad \tan \theta = -\frac{8}{15}$$

$$\csc \theta = -\frac{17}{8} \qquad \sec \theta = \frac{17}{15} \qquad \cot \theta = -\frac{15}{8}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-6, 8)$ is on the terminal side of θ .

Solution

$$(-6, 8) = 2(-3, 4) \Rightarrow 3 \quad 4 \rightarrow 5$$

$$\sin \theta = \frac{4}{5} \qquad \cos \theta = -\frac{3}{5} \qquad \tan \theta = -\frac{4}{3}$$

$$\csc \theta = \frac{5}{4} \qquad \sec \theta = -\frac{5}{3} \qquad \cot \theta = -\frac{3}{4}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-15, 8)$ is on the terminal side of θ .

Solution

$$15 \quad 8 \rightarrow 17$$

$$\sin \theta = \frac{8}{17}$$

$$\cos \theta = -\frac{15}{17}$$

$$\tan \theta = -\frac{8}{15}$$

$$\csc \theta = \frac{17}{8}$$

$$\sec \theta = -\frac{17}{15}$$

$$\cot \theta = -\frac{15}{8}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-7, 24)$ is on the terminal side of θ .

Solution

$$7 \quad 24 \rightarrow 25$$

$$\sin \theta = \frac{24}{25}$$

$$\cos \theta = -\frac{7}{25}$$

$$\tan \theta = -\frac{24}{7}$$

$$\csc \theta = \frac{25}{24}$$

$$\sec \theta = -\frac{25}{7}$$

$$\cot \theta = -\frac{7}{24}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(10, -24)$ is on the terminal side of θ .

Solution

$$(10, -24) = 2(5, -12) \Rightarrow 5 \quad 12 \rightarrow 13$$

$$\sin \theta = -\frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = -\frac{12}{5}$$

$$\csc \theta = -\frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = -\frac{5}{12}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(7, 24)$ is on the terminal side of θ .

Solution

$$7 \quad 24 \rightarrow 25$$

$$\sin \theta = \frac{24}{25}$$

$$\cos \theta = \frac{7}{25}$$

$$\tan \theta = \frac{24}{7}$$

$$\csc \theta = \frac{25}{24}$$

$$\sec \theta = \frac{25}{7}$$

$$\cot \theta = \frac{7}{24}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-7, -24)$ is on the terminal side of θ .

Solution

$$7 \quad 24 \rightarrow 25$$

$$\sin \theta = -\frac{24}{25}$$

$$\cos \theta = -\frac{7}{25}$$

$$\tan \theta = \frac{24}{7}$$

$$\csc \theta = -\frac{25}{24}$$

$$\sec \theta = -\frac{25}{7}$$

$$\cot \theta = \frac{7}{24}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(-24, -7)$ is on the terminal side of θ .

Solution

$$24 \quad 7 \rightarrow 25$$

$$\sin \theta = -\frac{7}{25}$$

$$\cos \theta = -\frac{24}{25}$$

$$\tan \theta = \frac{7}{24}$$

$$\csc \theta = -\frac{25}{7}$$

$$\sec \theta = -\frac{25}{24}$$

$$\cot \theta = \frac{24}{7}$$

Exercise

Find the six trigonometry functions of θ if θ is in the standard position and the point $(24, -10)$ is on the terminal side of θ .

Solution

$$(24, -10) = 2(12, -5) \Rightarrow 12 \quad 5 \rightarrow 13$$

$$\sin \theta = -\frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$\tan \theta = -\frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = \frac{13}{12}$$

$$\cot \theta = -\frac{12}{5}$$

Exercise

Find the values of the six trigonometric functions for an angle of 90° .

Solution

$$\sin 90^\circ = 1$$

$$\tan 90^\circ = \infty$$

$$\csc 90^\circ = 1$$

$$\cos 90^\circ = 0$$

$$\cot 90^\circ = 0$$

$$\sec 90^\circ = \infty$$

Exercise

Indicate the two quadrants θ could terminate in if $\cos \theta = \frac{1}{2}$

Solution

$$\cos \theta = \frac{1}{2} \rightarrow QI \text{ \& } QIV$$

Exercise

Indicate the two quadrants θ could terminate in if $\csc \theta = -2.45$

Solution

$$\csc \theta = -2.45 = \frac{1}{\sin \theta} \rightarrow QIII \text{ \& } QIV$$

Exercise

Find the remaining trigonometric function of θ if $\sin \theta = \frac{12}{13}$ and θ terminates in QI

Solution

$$5 \quad 12 \rightarrow 13$$

$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = \frac{12}{5}$$

$$\csc \theta = \frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = \frac{5}{12}$$

Exercise

Find the remaining trigonometric function of θ if $\cot \theta = -2$ and θ terminates in QII .

Solution

$$\cot \theta = -2 = \frac{x}{y} \quad (\theta \in QII) \Rightarrow \boxed{x = -2, y = 1}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (1)^2}$$

$$= \sqrt{5}$$

$$-2 \quad 1 \rightarrow \sqrt{5}$$

$$\sin \theta = \frac{1}{\sqrt{5}}$$

$$\cos \theta = -\frac{2}{\sqrt{5}}$$

$$\tan \theta = -\frac{1}{2}$$

$$\csc \theta = \sqrt{5}$$

$$\sec \theta = -\frac{\sqrt{5}}{2}$$

Exercise

Find the remaining trigonometric function of θ if $\tan \theta = \frac{3}{4}$ and θ terminates in **QIII**.

Solution

$$\tan \theta = \frac{3}{4} = \frac{y}{x} \quad (\theta \in \text{QIII}) \Rightarrow \boxed{x = -4, y = -3}$$

$$4 \quad 3 \rightarrow 5$$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = -\frac{4}{5}$$

$$\csc \theta = -\frac{5}{3}$$

$$\sec \theta = -\frac{5}{4}$$

$$\cot \theta = \frac{4}{3}$$

Exercise

Find the remaining trigonometric function of θ if $\cos \theta = \frac{24}{25}$ and θ terminates in **QIV**.

Solution

$$24 \quad 7 \rightarrow 25$$

$$\cos \theta = \frac{24}{25} \quad \theta \in \text{QIV} \Rightarrow y = -7$$

$$\sin \theta = -\frac{7}{25}$$

$$\cos \theta = \frac{24}{25}$$

$$\tan \theta = -\frac{7}{24}$$

$$\csc \theta = -\frac{25}{7}$$

$$\sec \theta = \frac{25}{24}$$

$$\cot \theta = -\frac{24}{7}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = \frac{\sqrt{3}}{2}$ and θ is terminates in **QIV**.

Solution

$$\cos \theta = \frac{\sqrt{3}}{2} = \frac{x}{r} \Rightarrow x = \sqrt{3}, r = 2$$

$$\text{Since } \theta \text{ is QIV} \Rightarrow y = -\sqrt{2^2 - \sqrt{3}^2} = -\sqrt{4-3} = -1$$

$$\sqrt{3} \quad 1 \rightarrow 2$$

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{1}{\sqrt{3}}$$

$$\csc \theta = -2$$

$$\sec \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot \theta = -\sqrt{3}$$

Exercise

Find the remaining trigonometric functions of θ if $\tan \theta = -\frac{1}{2}$ and $\cos \theta > 0$.

Solution

$$\tan \theta = \frac{\sin \theta}{\cos \theta} < 0 \quad \& \quad \cos \theta > 0 \Rightarrow \sin \theta < 0 \Rightarrow \theta \text{ in } QIV$$

$$\Rightarrow y = 1, \quad x = 2 \rightarrow r = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$2 \quad -1 \rightarrow \sqrt{5}$$

$$\sin \theta = -\frac{1}{\sqrt{5}}$$

$$\cos \theta = \frac{2}{\sqrt{5}}$$

$$\tan \theta = -\frac{1}{2}$$

$$\csc \theta = -\frac{\sqrt{5}}{1}$$

$$\sec \theta = \frac{\sqrt{5}}{2}$$

$$\cot \theta = -2$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = \frac{3}{5}$ & $\theta \in QI$

Solution

$$3 \quad 4 \rightarrow 5$$

$$\sin \theta = \frac{4}{5}$$

$$\cos \theta = \frac{3}{5}$$

$$\tan \theta = \frac{4}{3}$$

$$\csc \theta = \frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = -\frac{4}{5}$ & $\theta \in QII$

Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-4 \quad 3 \rightarrow 5$$

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = -\frac{4}{5}$$

$$\tan \theta = -\frac{3}{4}$$

$$\csc \theta = \frac{5}{3}$$

$$\sec \theta = -\frac{5}{4}$$

$$\cot \theta = -\frac{4}{3}$$

Exercise

Find the remaining trigonometric functions of θ if $\sin \theta = -\frac{3}{5}$ & $\theta \in QIII$

Solution

$$\theta \in QIII \quad \& \quad \cos \theta < 0$$

$$-4 \quad -3 \rightarrow 5$$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = -\frac{4}{5}$$

$$\tan \theta = \frac{3}{4}$$

$$\csc \theta = -\frac{5}{3}$$

$$\sec \theta = -\frac{5}{4}$$

$$\cot \theta = \frac{4}{3}$$

Exercise

Find the remaining trigonometric functions of θ if $\sin \theta = -\frac{3}{5}$ & $\theta \in QIV$

Solution

$$\theta \in QIV \quad \& \quad \cos \theta > 0$$

$$4 \quad -3 \rightarrow 5$$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = -\frac{3}{4}$$

$$\csc \theta = -\frac{5}{3}$$

$$\sec \theta = \frac{5}{4}$$

$$\cot \theta = -\frac{4}{3}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = -\frac{12}{13}$ & $\theta \in QIII$

Solution

$$\theta \in QIII \quad \& \quad \sin \theta < 0$$

$$-12 \quad -5 \rightarrow 13$$

$$\sin \theta = -\frac{5}{13}$$

$$\cos \theta = -\frac{12}{13}$$

$$\tan \theta = \frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = -\frac{13}{12}$$

$$\cot \theta = \frac{12}{5}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = -\frac{5}{13}$ & $\theta \in QII$

Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-5 \quad 12 \rightarrow 13$$

$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = -\frac{5}{13}$$

$$\tan \theta = -\frac{12}{5}$$

$$\csc \theta = \frac{13}{12}$$

$$\sec \theta = -\frac{13}{5}$$

$$\cot \theta = -\frac{5}{12}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = \frac{12}{13}$ & $\theta \in QIV$

Solution

$$\theta \in QIV \quad \& \quad \sin \theta < 0$$

$$12 \quad -5 \rightarrow 13$$

$$\sin \theta = -\frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$\tan \theta = -\frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = \frac{13}{12}$$

$$\cot \theta = -\frac{12}{5}$$

Exercise

Find the remaining trigonometric functions of θ if $\sin \theta = -\frac{8}{17}$ & $\theta \in QIII$

Solution

$$\theta \in QIII \quad \& \quad \cos \theta < 0$$

$$-15 \quad -8 \rightarrow 17$$

$$\sin \theta = -\frac{8}{17}$$

$$\cos \theta = -\frac{15}{17}$$

$$\tan \theta = \frac{8}{15}$$

$$\csc \theta = -\frac{17}{8}$$

$$\sec \theta = -\frac{17}{15}$$

$$\cot \theta = \frac{15}{8}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = -\frac{15}{17}$ & $\theta \in QII$

Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-15 \quad 8 \rightarrow 17$$

$$\sin \theta = \frac{8}{17}$$

$$\cos \theta = -\frac{15}{17}$$

$$\tan \theta = -\frac{8}{15}$$

$$\csc \theta = \frac{17}{8}$$

$$\sec \theta = -\frac{17}{15}$$

$$\cot \theta = -\frac{15}{8}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = -\frac{8}{17}$ & $\theta \in QII$

Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-8 \quad 15 \rightarrow 17$$

$$\sin \theta = \frac{15}{17}$$

$$\cos \theta = -\frac{8}{17}$$

$$\tan \theta = -\frac{15}{8}$$

$$\csc \theta = \frac{17}{15}$$

$$\sec \theta = -\frac{17}{8}$$

$$\cot \theta = -\frac{8}{15}$$

Exercise

Find the remaining trigonometric functions of θ if $\cos \theta = -\frac{7}{25}$ & $\theta \in QII$

Solution

$$\theta \in QII \quad \& \quad \sin \theta > 0$$

$$-7 \quad 24 \rightarrow 25$$

$$\sin \theta = \frac{24}{25}$$

$$\cos \theta = -\frac{7}{25}$$

$$\tan \theta = -\frac{24}{7}$$

$$\csc \theta = \frac{25}{24}$$

$$\sec \theta = -\frac{25}{7}$$

$$\cot \theta = -\frac{7}{24}$$

Exercise

Find the remaining trigonometric functions of θ if $\sin \theta = -\frac{7}{25}$ & $\theta \in QIII$

Solution

$$\theta \in QIII \quad \& \quad \cos \theta < 0$$

$$\textcolor{red}{-24} \quad \textcolor{blue}{-7} \rightarrow \textcolor{blue}{25}$$

$$\sin \theta = -\frac{7}{25} \qquad \cos \theta = -\frac{24}{25} \qquad \tan \theta = \frac{7}{24}$$

$$\csc \theta = -\frac{25}{7} \qquad \sec \theta = -\frac{25}{24} \qquad \cot \theta = \frac{24}{7}$$

Exercise

Find the remaining trigonometric functions of θ if $\sin \theta = -\frac{24}{25}$ & $\theta \in QIV$

Solution

$$\theta \in QIV \quad \& \quad \cos \theta > 0$$

$$\textcolor{red}{7} \quad \textcolor{blue}{-24} \rightarrow \textcolor{blue}{25}$$

$$\sin \theta = -\frac{24}{25} \qquad \cos \theta = \frac{7}{25} \qquad \tan \theta = -\frac{24}{7}$$

$$\csc \theta = -\frac{25}{24} \qquad \sec \theta = \frac{25}{7} \qquad \cot \theta = -\frac{7}{24}$$

Exercise

If $\sin \theta = -\frac{5}{13}$, and θ is $QIII$, find $\cos \theta$ and $\tan \theta$.

Solution

$$\sin \theta = -\frac{5}{13} = \frac{y}{r} \rightarrow y = -5, \quad r = 13$$

$$\Rightarrow x = \pm \sqrt{13^2 - 5^2} = \pm 12 \quad \text{Since } \theta \text{ is } QIII \Rightarrow x = -12 \qquad x = \pm \sqrt{r^2 - y^2}$$

$$\cos \theta = -\frac{12}{13}$$

$$\tan \theta = \frac{5}{12}$$

Exercise

If $\cos \theta = \frac{3}{5}$, and θ is *QIV*, find $\sin \theta$ and $\tan \theta$.

Solution

$$\cos \theta = \frac{3}{5} = \frac{x}{r} \quad (\theta \in \text{QIV}) \Rightarrow \boxed{x=3} \quad y = \underline{-4}$$

$$\sin \theta = \underline{-\frac{4}{5}}, \quad \tan \theta = \underline{-\frac{4}{3}}$$

Exercise

Use the reciprocal identities if $\cos \theta = \frac{\sqrt{3}}{2}$ find $\sec \theta$

Solution

$$\begin{aligned} \sec \theta &= \frac{1}{\cos \theta} \\ &= \frac{2}{\sqrt{3}} \\ &= \underline{\frac{2\sqrt{3}}{3}} \end{aligned}$$

Exercise

Find $\cos \theta$, given that $\sec \theta = \frac{5}{3}$

Solution

$$\begin{aligned} \cos \theta &= \frac{1}{\sec \theta} \\ &= \frac{1}{\frac{5}{3}} \\ &= \underline{\frac{3}{5}} \end{aligned}$$

Exercise

Find $\sin \theta$, given that $\csc \theta = -\frac{\sqrt{12}}{2}$

Solution

$$\begin{aligned} \sin \theta &= \frac{1}{\csc \theta} \\ &= -\frac{2}{\sqrt{12}} \frac{\sqrt{12}}{\sqrt{12}} \end{aligned}$$

$$= -\frac{2\sqrt{12}}{12}$$

$$= -\frac{\sqrt{12}}{6}$$

Exercise

Use a ratio identity to find $\tan \theta$ if $\sin \theta = \frac{3}{5}$ and $\cos \theta = -\frac{4}{5}$

Solution

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{3}{5}}{-\frac{4}{5}}$$

$$= -\frac{3}{4}$$

Exercise

If $\cos \theta = -\frac{1}{2}$ and θ terminates in QII , find $\sin \theta$

Solution

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Exercise

If $\sin \theta = \frac{3}{5}$ and θ terminated in QII , find $\cos \theta$ and $\tan \theta$.

Solution

$$\cos \theta = -\frac{4}{5} \quad (3, 4 \rightarrow 5)$$

$$\tan \theta = -\frac{3}{4}$$

Exercise

Find $\tan \theta$ if $\sin \theta = \frac{1}{3}$ and θ terminates in QI

Solution

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$= \sqrt{1 - \frac{1}{9}}$$

$$= \sqrt{\frac{8}{9}}$$

$$= \frac{2\sqrt{2}}{3}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}$$

$$= \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}$$

Exercise

Find the remaining trigonometric ratios of θ , if $\sec \theta = -3$ and $\theta \in QIII$

Solution

$$\sec \theta = \frac{1}{\cos \theta} = -3$$

$$\cos \theta = -\frac{1}{3}$$

$$\sin \theta = -\sqrt{1 - \cos^2 \theta}$$

$$= -\sqrt{1 - \frac{1}{9}}$$

$$= -\sqrt{\frac{8}{9}}$$

$$= -\frac{2\sqrt{2}}{3}$$

$$\tan \theta = 2\sqrt{2}$$

$$\cot \theta = \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}$$

$$\csc \theta = -\frac{3}{2\sqrt{2}}$$

$$= -\frac{3\sqrt{2}}{4}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

Exercise

Using the calculator and rounding your answer to the nearest hundredth, find the remaining trigonometric ratios of θ if $\csc \theta = -2.45$ and $\theta \in QIII$

Solution

$$\sin \theta = \frac{1}{-2.45}$$

$$= -\frac{100}{245}$$

$$= -\frac{20}{49}$$

$$= -0.41$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = -\sqrt{1 - \sin^2 \theta}$$

$$= -\sqrt{1 - .41^2}$$

$$= -0.91$$

$$\tan \theta = \frac{-0.41}{-0.91}$$

$$= \frac{41}{91}$$

$$= 0.45$$

$$\cot \theta = \frac{1}{0.45}$$

$$= \frac{100}{45}$$

$$= \frac{20}{9}$$

$$= 2.22$$

$$\sec \theta = \frac{1}{-0.91}$$

$$= -\frac{100}{91}$$

$$= -1.1$$

Exercise

Write $\frac{\sec \theta}{\csc \theta}$ in terms of $\sin \theta$ and $\cos \theta$, and then simplify if possible.

Solution

$$\begin{aligned}\frac{\sec \theta}{\csc \theta} &= \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta}} \\ &= \frac{1}{\cos \theta} \frac{\sin \theta}{1} \\ &= \frac{\sin \theta}{\cos \theta}\end{aligned}$$

Exercise

Write $\cot \theta - \csc \theta$ in terms of $\sin \theta$ and $\cos \theta$, and then simplify if possible.

Solution

$$\begin{aligned}\cot \theta - \csc \theta &= \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} \\ &= \frac{\cos \theta - 1}{\sin \theta}\end{aligned}$$

Exercise

Write $\frac{\sin \theta}{\cos \theta} + \frac{1}{\sin \theta}$ in terms of $\sin \theta$ and/or $\cos \theta$, and then simplify if possible.

Solution

$$\frac{\sin \theta}{\cos \theta} + \frac{1}{\sin \theta} = \frac{\sin^2 \theta + \cos \theta}{\cos \theta \sin \theta}$$

Exercise

Write $\sin \theta \cot \theta + \cos \theta$ in terms of $\sin \theta$ and $\cos \theta$, and then simplify if possible.

Solution

$$\begin{aligned}\sin \theta \cot \theta + \cos \theta &= \sin \theta \frac{\cos \theta}{\sin \theta} + \cos \theta \\ &= \cos \theta + \cos \theta \\ &= 2 \cos \theta\end{aligned}$$

Exercise

Multiply $(1 - \cos \theta)(1 + \cos \theta)$

Solution

$$\begin{aligned}(1 - \cos \theta)(1 + \cos \theta) &= 1 - \cos^2 \theta \\ &= \sin^2 \theta \quad | \end{aligned}$$

Exercise

Multiply $(\sin \theta + 2)(\sin \theta - 5)$

Solution

$$(\sin \theta + 2)(\sin \theta - 5) = \sin^2 \theta - 3\sin \theta - 10$$

Exercise

Simplify the expression $\sqrt{25 - x^2}$ as much as possible after substituting $5\sin \theta$ for x .

Solution

$$\begin{aligned}\sqrt{25 - x^2} &= \sqrt{25 - (5\sin \theta)^2} \\ &= \sqrt{25 - 25\sin^2 \theta} \\ &= \sqrt{25(1 - \sin^2 \theta)} \\ &= \sqrt{25}\sqrt{\cos^2 \theta} \\ &= 5\cos \theta \quad | \end{aligned}$$

Exercise

Simplify the expression $\sqrt{4x^2 + 16}$ as much as possible after substituting $2\tan \theta$ for x

Solution

$$\begin{aligned}\sqrt{4x^2 + 16} &= \sqrt{4(2\tan \theta)^2 + 16} \\ &= \sqrt{16\tan^2 \theta + 16} \\ &= \sqrt{16(\tan^2 \theta + 1)} \\ &= 4\sqrt{\tan^2 \theta + 1} \\ &= 4\sqrt{\sec^2 \theta} \\ &= 4\sec \theta \quad | \end{aligned}$$

Exercise

Simplify by using the table. $5 \sin^2 30^\circ$

Solution

$$\begin{aligned} 5 \sin^2 30^\circ &= 5 \left(\frac{1}{2} \right)^2 \\ &= \frac{5}{4} \end{aligned}$$

Exercise

Simplify by using the table. $\sin^2 60^\circ + \cos^2 60^\circ$

Solution

$$\begin{aligned} \sin^2 60^\circ + \cos^2 60^\circ &= \left(\frac{\sqrt{3}}{2} \right)^2 + \left(\frac{1}{2} \right)^2 \\ &= \frac{3}{4} + \frac{1}{4} \\ &= 1 \end{aligned}$$

Exercise

Simplify by using the table. $(\tan 45^\circ + \tan 60^\circ)^2$

Solution

$$\begin{aligned} (\tan 45^\circ + \tan 60^\circ)^2 &= (1 + \sqrt{3})^2 \\ &= 1 + 3 + 2\sqrt{3} \\ &= 4 + 2\sqrt{3} \end{aligned}$$

Exercise

Find the exact value of $\csc 300^\circ$

Solution

$$\hat{\theta} = 360^\circ - 300^\circ = 60^\circ \rightarrow 300^\circ \in QIV$$

$$\begin{aligned} \csc 300^\circ &= -\frac{1}{\sin 60^\circ} \\ &= -\frac{1}{\frac{\sqrt{3}}{2}} \\ &= -\frac{2}{\sqrt{3}} \end{aligned}$$

Exercise

Find θ if $\sin \theta = -\frac{1}{2}$ and θ terminates in **QIII** with $0^\circ \leq \theta \leq 360^\circ$.

Solution

$$\hat{\theta} = \sin^{-1} \frac{1}{2} = 30^\circ$$

$$\theta \in \text{QIII}$$

$$\Rightarrow \theta = 180^\circ + 30^\circ \\ = 210^\circ$$

Exercise

Find θ to the nearest degree if $\sec \theta = 3.8637$ and θ terminates in QIV with $0^\circ \leq \theta \leq 360^\circ$.

Solution

$$\sec \theta = 3.8637 = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{3.8637}$$

$$\hat{\theta} = \cos^{-1} \frac{1}{3.8637} \\ = 75^\circ$$

$$\text{Calculator: } \cos^{-1}(1 / 3.8637)$$

$$\theta \in \text{QIV}$$

$$\Rightarrow \theta = 360^\circ - 75^\circ \\ = 285^\circ$$

Exercise

Find the exact value of $\cos 225^\circ$

Solution

$$\hat{\theta} = 225^\circ - 180^\circ = 45^\circ$$

$$\rightarrow 225^\circ \in \text{QIII}$$

$$\cos 225^\circ = -\cos 45^\circ$$

$$= -\frac{\sqrt{2}}{2}$$

Exercise

Find the exact value of $\tan 315^\circ$

Solution

$$\hat{\theta} = 360^\circ - 315^\circ = 45^\circ \quad \rightarrow 315^\circ \in QIV$$

$$\begin{aligned}\tan 315^\circ &= -\tan 45^\circ \\ &= -1\end{aligned}$$

Exercise

Find the exact value of $\cos 420^\circ$

Solution

$$\hat{\theta} = 420^\circ - 360^\circ = 60^\circ \quad \rightarrow 420^\circ \in QI$$

$$\begin{aligned}\cos 420^\circ &= \cos 60^\circ \\ &= \frac{1}{2}\end{aligned}$$

Exercise

Find the exact value of $\cot 480^\circ$

Solution

$$\hat{\theta} = 480^\circ - 360^\circ = 120^\circ$$

$$\hat{\theta} = 180^\circ - 120^\circ = 60^\circ \quad \rightarrow 480^\circ \in QII$$

$$\begin{aligned}\cot 480^\circ &= -\frac{\cos 60^\circ}{\sin 60^\circ} \\ &= -\frac{1/2}{\sqrt{3}/2} \\ &= -\frac{1}{\sqrt{3}}\end{aligned}$$

Exercise

Use the calculator to find the value of $\csc 166.7^\circ$

Solution

$$\begin{aligned}\csc 166.7^\circ &= \frac{1}{\sin 166.7^\circ} \\ &\approx 4.3469\end{aligned}$$

Exercise

Use the calculator to find the value of $\sec 590.9^\circ$

Solution

$$\begin{aligned}\sec 590.9^\circ &= \frac{1}{\cos 590.9^\circ} \\ &\approx -1.5856 \quad | \end{aligned}$$

Exercise

Use the calculator to find the value of $\tan 195^\circ 10'$

Solution

$$\begin{aligned}\tan(195^\circ 10') &= \tan\left(195^\circ + \frac{10}{60}\right) \\ &= \tan 195.1667^\circ \\ &\approx 0.271 \quad | \end{aligned}$$

Exercise

Use the calculator to find θ to the nearest degree if $\sin \theta = -0.3090$ with $\theta \in \text{QIV}$ with $0^\circ \leq \theta \leq 360^\circ$

Solution

$$\begin{aligned}\hat{\theta} &= \sin^{-1}(0.3090) && \text{Since } \theta \in \text{QIV} \\ &\approx 18.0^\circ \quad | \\ \theta &= 180^\circ + 40.0^\circ \\ &= 220.0^\circ \quad | \end{aligned}$$

Exercise

Use the calculator to find θ to the nearest degree if $\cos \theta = -0.7660$ with $\theta \in \text{QIII}$ with $0^\circ \leq \theta \leq 360^\circ$

Solution

$$\begin{aligned}\hat{\theta} &= \cos^{-1}(0.7660) && \text{Since } \theta \in \text{QIII} \\ &\approx 40.0^\circ \\ \theta &= 180^\circ + 40.0^\circ \\ &= 220.0^\circ \quad | \end{aligned}$$

Exercise

Use the calculator to find θ to the nearest degree if $\sec \theta = -3.4159$ with $\theta \in QII$ with $0^\circ \leq \theta \leq 360^\circ$

Solution

$$\sec \theta = -3.4159$$

$$\cos \theta = -\frac{1}{3.4159}$$

$$\hat{\theta} = \cos^{-1}\left(\frac{1}{3.4159}\right) \quad \text{Since } \theta \in QII$$

$$\approx 73.0^\circ$$

$$\theta \approx 180^\circ - 73.0^\circ$$

$$\underline{= 107.0^\circ}$$

Exercise

Find θ to the nearest tenth of a degree if $\tan \theta = -0.8541$ and θ terminates in QIV with $0^\circ \leq \theta \leq 360^\circ$.

Solution

$$\hat{\theta} = \tan^{-1} 0.8541 \quad \theta \in QIV$$

$$\approx 40.5^\circ$$

$$\Rightarrow \theta = 360^\circ - 40.5^\circ$$

$$\underline{\approx 319.5^\circ}$$

Exercise

Use the calculator to find θ to the nearest degree if $\sin \theta = 0.49368329$ with $\theta \in QII$ with $0^\circ \leq \theta < 360^\circ$

Solution

$$\hat{\theta} = \sin^{-1} 0.49368329 \quad \theta \in QII$$

$$= 29.6^\circ$$

$$\Rightarrow \theta = 180^\circ - 29.6^\circ$$

$$\underline{= 150.4^\circ}$$