Solution *Section* 3.2 – Applications of Venn Diagrams

Exercise

Use the union rule to answer the following

a) If $n(A)=5;\quad n(B)=12,\quad$ *and* $\quad n(A\cap B)=4$ what is $n(A\cup B)$?

b) If $n(A)=15;\quad n(B)=30,\quad$ *and* $\quad n(A\cup B)=33$ what is $n(A\cap B)$?

c) $n(B)=9;\quad n(A\cap B)=5,\quad$ *and* $\quad n(A\cup B)=22$ what is $n(A)$?

Solution

a) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$=5+12-4$

$=13$

b) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$33=15+30-n(A\cap B)$

$33=45-n(A\cap B)$

$n(A\cap B)=45-33$ $=12$

c) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$22=n(A)+9-5$

$22=n(A)+4$

$n(A)=22-4$ $=18$

Exercise

Draw a Venn diagram and use the given information to fill in the number of elements

a) $n(U)=41;\quad n(A)=16,\quad n(A\cap B)=12,\quad n(B')=20$

b) $n(A)=28;\quad n(B)=12,\quad n(A\cup B)=32,\quad n(A')=19$

c) $n(A)=11;\quad n(A\cap B)=6,\quad n(A\cup B)=24,\quad n(A'\cup B')=25$

d) $n(A)=28,\quad n(B)=34,\quad n(C)=25,\quad n(A\cap B)=14,\quad n(B\cap C)=15$
$n(A\cap C)=11;\quad n(A\cap B\cap C)=9,\quad n(U)=59$

e) $n(A)=54,\quad n(B')=63,\quad n(C)=44,\quad n(A\cap B)=22,\quad n(B\cap C)=16,$
$n(A\cap C)=15;\quad n(A\cap B\cap C)=4,\quad n(A\cup B)=85$

f) $n(A\cap C')=11,\quad n(B\cap C')=8,\quad n(C)=15,\quad n(A\cap B)=6,\quad n(B\cap C)=4$
$n(A\cap C)=7;\quad n(A\cap B\cap C)=4,\quad n(A'\cap B'\cap C')=5$

Solution

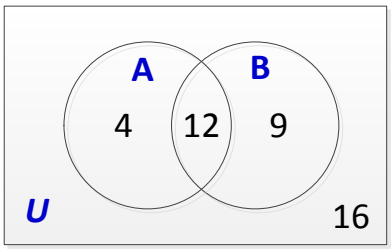
a) $n(U)=41;\quad n(A)=16,\quad n(A\cap B)=12,\quad n(B')=20$

Start with $n(A\cap B)=12$

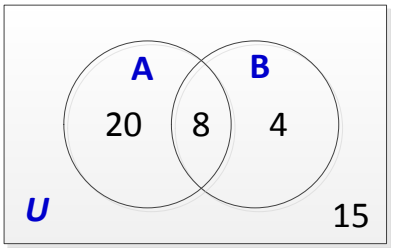
Since $n(A)=16$ and 12 in $A\cap B$, there must be $(16-12=4)$ 4 elements in A but not in $A\cap B$.

$n(B')=20$, so there are 20 not in B, 4 already in A which leave us with $20-4=16$.

$n(U)=41\rightarrow 41-4-12-16=9$ in B but not in $A\cap B$



b) $n(A)=28;\quad n(B)=12,\quad n(A\cup B)=32,\quad n(A')=19$



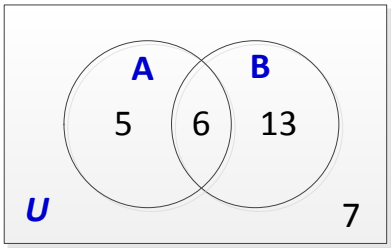
c) $n(A)=11;\quad n(A\cap B)=6,\quad n(A\cup B)=24,\quad n(A'\cup B')=25$

Start with $n(A\cap B)=6$

Since $n(A)=11$ and 6 in $A\cap B$, there must be $11-6=5$ elements in A but not in $A\cap B$.

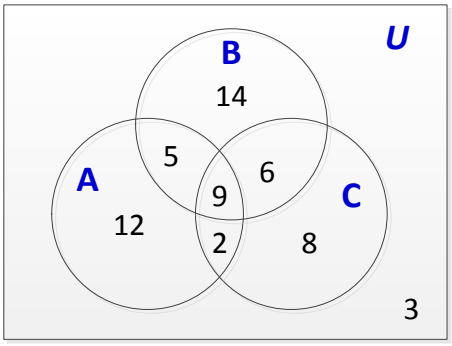
$n(A\cup B)=24$, we already have 11 so $24-11=13$ in B but not in $A\cap B$

$A'\cup B'=U-(A\cap B)\rightarrow 25-5-13=7$

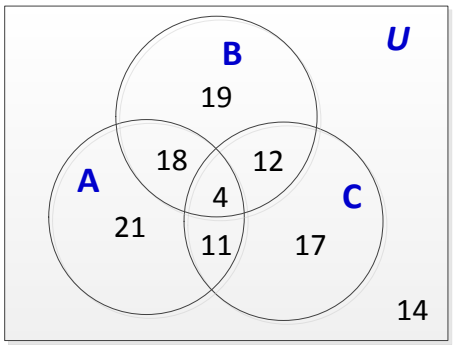


d) $n(A)=28,\quad n(B)=34,\quad n(C)=25,\quad n(A\cap B)=14,\quad n(B\cap C)=15$

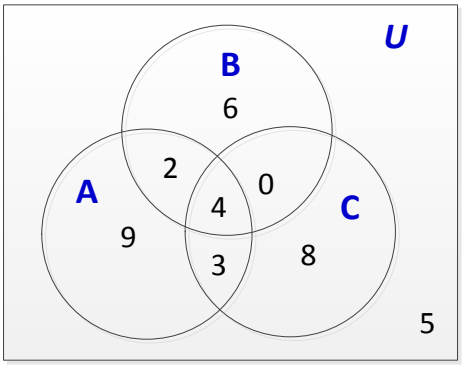
$n(A\cap C)=11;\quad n(A\cap B\cap C)=9,\quad n(U)=59$



e) $n(A)=54,\quad n(B')=63,\quad n(C)=44,\quad n(A\cap B)=22,\quad n(B\cap C)=16,$
$n(A\cap C)=15;\quad n(A\cap B\cap C)=4,\quad n(A\cup B)=85$



f) $n(A\cap C')=11,\quad n(B\cap C')=8,\quad n(C)=15,\quad n(A\cap B)=6,\quad n(B\cap C)=4$
$n(A\cap C)=7;\quad n(A\cap B\cap C)=4,\quad n(A'\cap B'\cap C')=5$



Exercise

Toward the middle of the harvesting season peaches for canning come in three types, early, late, and extra late, depending on the expected date of ripening. During a certain week, the following data were recorded at a fruit delivery station:

34 trucks went out carrying early peaches;
61 carried late peaches;
50 carried extra late;
25 carried early and late;
30 carried late and extra late;
8 carried early and extra late;
6 carried all three
9 carried only figs (no peaches at all).

a) How many trucks carried only variety peaches?
b) How many carried only extra late?
c) How many carried only one type of peach?
d) How many trucks (in all) went during the week?

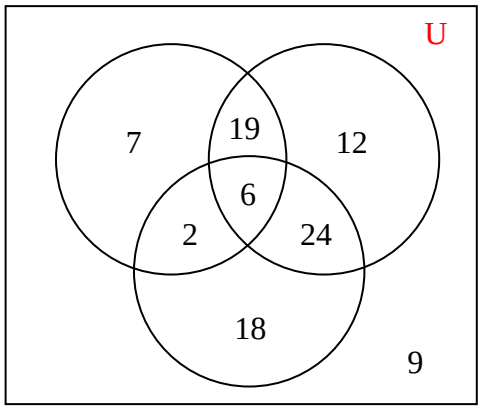
Solution

a) 12

b) 18

c) $7+12+18=37$

d) $7+2+6+19+12+24+18+9=97$



Exercise

In a survey of 100 randomly chosen students, a marketing questionnaire included the following:

- ✓ 75 own a TV
- ✓ 45 own a car
- ✓ 35 own a TV and a car

a) How many students owned a car but not a TV set?
b) How many students did not own both a car and a TV set?

Solution

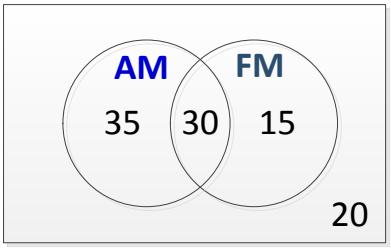
a) 10

b) 65

Exercise

A small town has two radio stations, an AM station and an FM station. A survey of 100 residents of the town produced the following results: In the last 30 days, 65 people have listened to the AM station, 45 have listened to the FM station, and 30 have listened to both stations.

Solution



Total 100, $AM = 65$, $FM = 45$, $AM \cap FM = 30$

Exercise

In a class of 35 students, 19 are married and 20 are blondes. Given that there are 7 students that are both married and blonde, answer the following questions.

a) How many are married, but not blonde?
b) How many are blonde but not married?
c) How many are blonde or married?
d) How many are neither blonde nor married?
e) How many are not blonde?

Solution

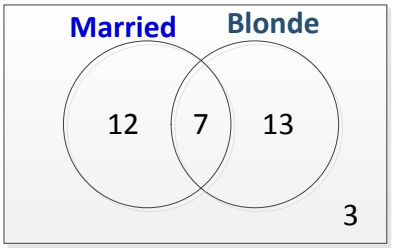
a) 12

b) 13

c) 12 + 7 + 13 = 32

d) 35 – 32 = 3

e) 12 + 3 = 15



Exercise

In a survey of 500 businesses it was found that 250 had copiers and 300 had fax machines. It was also determined that 100 businesses had both copiers and fax machines.

a) How many had either a copier or a fax machine?
b) How many had neither a copier nor a fax machine?
c) How many had a copier, but no fax machine?
d) How many had a fax machine, but no copier?
e) How many had no fax machines?

Solution

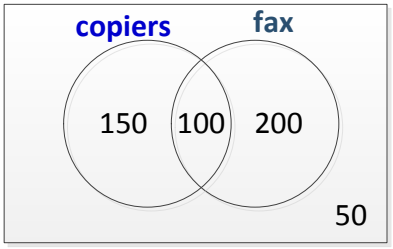
a) 450

b) 50

c) 150

d) 200

e) 200



Exercise

Given: $n(U)=105,\quad n(A)=50,\quad n(B)=75,\quad n(A\cup B)=105$, find the following:

Solution

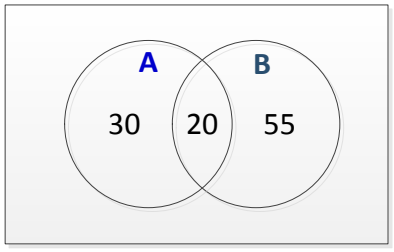
a) $n(A\cap B)=20$

b) $n(A'\cap B)=55$

c) $n(A'\cap B')=5$

d) $n(A\cup B')=55$

e) $n(B')=35$



Exercise

Fred interviewed 140 people in a shopping center to discover some of their cooking habits. He obtained the following results:

58 use microwave ovens
63 use electric ranges
58 use gas ranges
19 use microwave ovens and electric ranges
17 use microwave ovens and gas ranges
4 use both gas and electric ranges
1 uses all three
2 use none of the three

Should he be reassigned one more time? Why or why not?

Solution

Let *M*: use microwave ovens
E: use electric ranges
G: use gas ranges

$n(U)=140$

2 use none of the three $\Rightarrow n(M'\cap E'\cap G')=2$

1 uses all three $\Rightarrow n(M\cap E\cap G)=1$

4 use both gas and electric ranges $=n(E\cap G)$

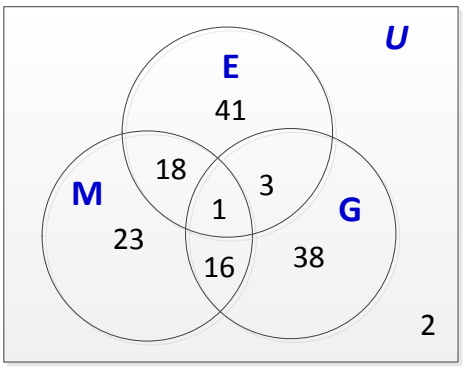
17 use microwave ovens and gas ranges $\Rightarrow n(M\cap G)=17$

19 use microwave ovens and electric ranges $\Rightarrow n(M\cap E)=19$

58 use gas ranges $\Rightarrow n(G)=58$

63 use electric ranges $\Rightarrow n(E)=63$

58 use microwave ovens $\Rightarrow n(M)=58$



Exercise

Toward the middle of the harvesting season, peaches for canning come in three types, early, late, and extra late, depending on the expected date of ripening. During a certain week, the following data were recorded at a fruit delivery station:

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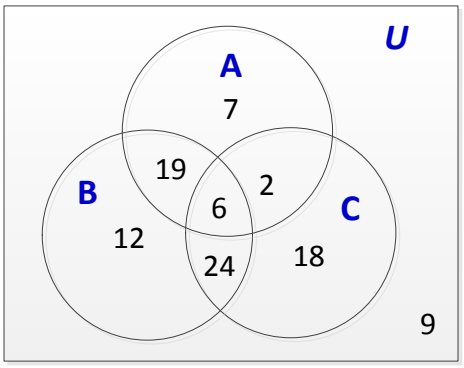
a) How many trucks carried only late variety peaches?
b) How many carried only extra late?
c) How many carried only one type of peach?
d) How many trucks (in all) went out during the week?

Solution



Let A: use early peaches
B: use late peaches
C: use extra late peaches

9 carried only figs $\Rightarrow n(A' \cap B' \cap C') = 9$

6 carried all three $\Rightarrow n(A \cap B \cap C) = 6$

8 carried early and extra late $\Rightarrow n(A \cap C) = 8$

30 carried late and extra late $\Rightarrow n(B \cap C) = 30$

25 carried early and late $\Rightarrow n(A \cap B) = 25$

50 carried extra late $\Rightarrow n(C) = 50$

61 carried late peaches $\Rightarrow n(B) = 61$

34 trucks went out carrying early peaches $\Rightarrow n(A) = 34$

Exercise

Most mathematics professors love to invest their hard earned money. A recent survey of 150 math professors revealed that

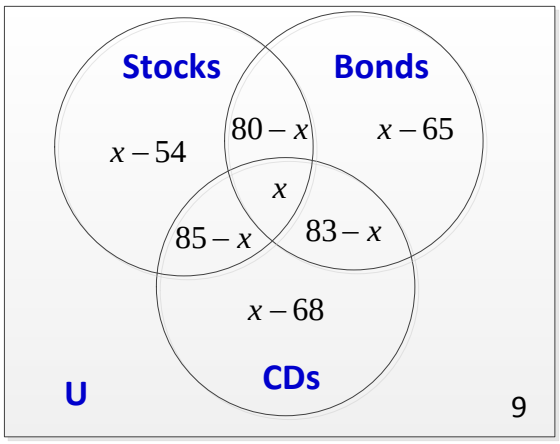
111 invested in stocks
98 invested in bonds
100 invested in certificates of deposit
80 invested in stocks and bonds
83 invested in bonds and certificates of deposit
85 invested in stocks and certificates of deposit
9 did not invest in any of three

How many mathematics professors invested in stocks and bonds and certificates of deposit?

Solution

Let A: Stocks
B: Bonds
C: CDs

$n(A \cap B \cap C) = x$



9 did not invest in any of three
$\Rightarrow n(A' \cap B' \cap C') = 9$

85 invested in stocks and CDs
$\Rightarrow n(A \cap C) = 85$
$85 - x$ invested in only stocks and CDs

83 invested in bonds and CDs
$\Rightarrow n(B \cap C) = 83$
$83 - x$ invested in only bonds and CDs

80 invested in stocks and bonds $\Rightarrow n(A \cap B) = 80$
$80 - x$ invested in only stocks and bonds

100 invested in CDs $\Rightarrow n(C) = 100$
$100 - [(85-x) + (83-x) + x] = 100 - 85 + x - 83 + x - x = x - 68$ invested in only CDs

98 invested in bonds $\Rightarrow n(B) = 98$
$98 - [(80-x) + (83-x) + x] = 111 - 80 + x - 83 + x - x = x - 65$ invested in only bonds

111 invested in stocks $\Rightarrow n(A) = 111$
$111 - [(80-x) + (85-x) + x] = 111 - 80 + x - 85 + x - x = x - 54$ invested in only stocks

$n(U) = 150$
$150 = x - 54 + 80 - x + x - 65 + 85 - x + 83 - x + x - 68 + x + 9$

$150 = 70 + x$

$\boxed{x = 80}$

80 professors invested in stocks, bonds and CDs

Exercise

Suppose that a group of 150 students have joined at least one of three chat rooms; one on auto-racing, one on bicycling, and one for college students. For simplicity, we will call these rooms *A*, *B*, and *C*. In addition,

90 students joined room *A*;
50 students joined room *B*;
70 students joined room *C*;
15 students joined room *A* and *C*;
12 students joined room *B* and *C*;
10 students joined all three rooms;

Determine how many students joined both chat rooms *A* and *B*.

Solution

$(75 - x) + 5 + x + 10 + (38 - x) + 2 + 53 = 150$

$75 - x + 5 + x + 10 + 38 - x + 2 + 53 = 150$

$183 - x = 150$

$183 - 150 = x$

$\boxed{x = 33}$