Lecture Four – Statistics

Section 4.1 – *Random Variable & probability*

A ***random variable*** is a function that assigns a numerical value to each simple event in a sample space ***S***.

The probability distribution for a random variable *X*

The probability distribution for a random variable *X*, denoted P(*X* = *x*) = p(*x*) satisfies

1. $0 \le p(x) \le 1, \quad x \in \{x_1, x_2, \cdots, x_n\}$
2. $p(x_1) + p(x_2) + \ldots + p(x_n) = 1$

where $\{x_1, x_2, \cdots, x_n\}$ are the (range) values of *X*.

Expected value of the random variable *X*

The expected value of the random variable *X* is the sum of the products of the values of *X* and their respective probabilities.

x_i	x_1	x_2	…	x_n
p_i	p_1	p_2	…	p_n

$$E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

Example

x_i	1	2	3	4	5	6
p_i	.14	.13	.18	.20	.11	.24

What is the expected value of *X*?

Solution

E(X) = 1(.14) + 2(.13) + 3(.18) + 4(.2) + 5(.11) + 6(.24)

= 3.72

Example

A carton of 20 watch batteries contains 2 dead ones. A random sample of 4 is selected from 20 and tested. Let X be the random variable associated with the number of dead batteries found in a sample.

a) Find the probability distribution of X.
b) Find the expected number of dead batteries in a sample.

Solution

a) $P(0 \text{ dead battery}) = \dfrac{C_{18,4}}{C_{20,4}} = .632$

$P(1 \text{ dead battery}) = \dfrac{C_{2,1}C_{18,3}}{C_{20,4}} = .337$

$P(2 \text{ dead battery}) = \dfrac{C_{2,2}C_{18,2}}{C_{20,4}} = .032$

X	0	1	2	
P	.632	.337	.032	$\rightarrow \approx 1$

b) E(X) = 0(.632) + 1(.337) + 2(.032) = .4

Example

A spinner device is numbered from 0 to 5, and each of the 6 numbers is as likely to come up as any other. A player who bets \$1 on any given number wins \$4 (gets the bet back) if the pointer comes to rest on the chosen number, otherwise, the \$1 bet is lost. What is the expected value of the game?

Solution

S = {0, 1, 2, 3, 4, 5}

$E(X) = 4\left(\frac{1}{6}\right) + (-1)\left(\frac{5}{6}\right) \approx -17$ per game

X	\$4	-\$1
P	$\frac{1}{6}$	$\frac{5}{6}$

The player will lose an average of about 17¢ per game

Example

Suppose you are interested in insuring a car stereo system for \$500 against theft. An insurance company charges a premium of \$60 for coverage 1 year, claiming an empirically determining probability of 0.1 that the stereo will be stolen some time during the year. What is your expected return from the insurance company if you take out this insurance?

Solution

We pay premium \$60 to get \$500

$\rightarrow$ 500 – 60 = \$440 with P = .1

E(X) = .1(440) + .9(-60) = -\$10

X	\$440	-\$60
P	.1	.9

Exercises *Section* 4.1 – *Random Variable & probability*

1. Suppose a random sample of 2 light bulbs is selected from a group of 8 bulbs that contain 3 defective bulbs.

a) What is the expected value of the number of defective bulbs in the sample?

b) Probability Distribution Table

c) What is your expected return?

2. Suppose 1000 raffle tickets are sold at a price of $10 each. Two first place tickets will be drawn, 5 second place tickets will be drawn and 10 third place tickets will be drawn. The first place prize is a $200 VCR, the second place prize is a $100 printer, and the third place prize is a $50 gift certificate.

3. Find the expected value of each random variable.

a)

x	2	3	4	5
$P(x)$	0.1	0.4	0.3	0.2

b)

x	4	6	8	10
$P(x)$	0.4	0.4	0.05	0.15

c)

x	9	12	15	18	21
$P(x)$	0.14	0.22	0.38	0.19	0.07

d)

x	30	32	36	38	44
$P(x)$	0.31	0.29	0.26	0.09	0.05

4. A delegation of 3 selected from a city council made up of 5 liberals and 6 conservatives.

a) What is the expected number of liberals in the delegation?

b) What is the expected number of conservatives in the delegation?

5. From a group of 3 women and 5 men, a delegation of 2 is selected. Find the expected number of women in the delegation.

6. In a club with 20 senior and 10 junior members, what is the expected number of junior members on a 4-member committee?

7. If 2 cards are drawn at one time from a deck of 52 cards, what is the expected number of diamonds?

8. A local used-car dealer gets complaints about his car as shown in the table below. Find the expected number of complaints per day

Number of Complaints per day	0	1	2	3	4	5	6
Probability	0.02	0.06	0.16	0.25	0.32	0.13	0.06

9. An insurance company has written 100 policies for $100,000, 500 policies for $50,000, and 100 policies for $10,000 for people of age 20. If experience shows that the probability that a person will die at age 20 is 0.0012, how much can the company expect to pay out during the ear policies were written?

10. An insurance policy on an electrical device pays a benefit of $4,000 if the device fails during the first year. The amount of the benefit decreases by $1,000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure that year is 0.4. What the expected benefit under the policy? (Choose the appropriate answer)

a. $2,234 *b.* $2,400 *c.* $2,500 *d.* $2,667 *e.* $2,694

11. A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs $50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay $100 (ticket cost + $50 penalty) to the tourist. What is the expected revenue of the tour operator? (Choose the appropriate answer)

a. $935 *b.* $950 *c.* $967 *d.* $976 *e.* $985