Ex $\quad y \dfrac{dy}{dx} = x(y^2+1)$

$$\frac{y\,dy}{y^2+1} = x\,dx$$

$$\frac{1}{2}\int \frac{d(y^2+1)}{y^2+1} = \int x\,dx$$

$$\frac{1}{2}\ln(y^2+1) = \frac{1}{2}x^2$$

$$\ln(y^2+1) = x^2 + C$$

$$y^2+1 = Ae^{x^2}$$

Ex $\quad \dfrac{dy}{dt} = y\cos t + y \qquad y(0) = 2$

$$\int \frac{dy}{y} = \int(\cos t + 1)\,dt$$

$$\ln|y| = \sin t + t + C$$

$$\ln 2 = C$$

$$\ln|y| = \sin t + t + \ln 2$$

$$y' + 4\cot 2x \, y = 6\cos 2x \qquad y\left(\frac{\pi}{4}\right) = 2$$

$$e^{4\int \cot 2x \, dx} = e^{2\ln \sin 2x} = e^{\ln \sin^2 2x}$$

$$= \sin^2 2x$$

$$\int 6\cos 2x \sin^2 2x \, dx = 3\int \sin^2 2x \, d(\sin 2x)$$

$$= \sin^3 2x$$

$$y(x) = \frac{1}{\sin^2 2x}\left(\sin^3 2x + C\right)$$

$$= \sin 2x + \frac{C}{\sin^2 2x}$$

$$y\left(\frac{\pi}{4}\right) = 2 = 1 + C \Rightarrow C = 1$$

$$\boxed{y(x) = \sin 2x + \frac{1}{\sin^2 2x}}$$

$$y' - \frac{1}{2}y = 2\sin 3t$$

$$e^{\int -\frac{1}{2}dt} = e^{-t/2}$$

$$\int 2(\sin 3t)e^{-t/2}dt$$

		$\sin 3t$
$+$	$2e^{-t/2}$	$-\frac{1}{3}\cos 3t$
$-$	$-e^{-t/2}$	$-\frac{1}{9}\sin 3t$
$+$	$\frac{1}{2}e^{-t/2}$	

$$\int 2e^{-t/2}\sin 3t\,dt = e^{-t/2}\left(-\frac{2}{3}\cos 3t - \frac{1}{9}\sin 3t\right) - \frac{1}{18}\int e^{-t/2}\sin 3t\,dt$$

$$\frac{37}{18}$$

$$\left(2+\frac{1}{18}\right)\int e^{-t/2}\sin 3t\,dt = \frac{1}{9}(-6\cos 3t - \sin 3t)e^{-t/2}$$

$$2\int e^{-3t/2}\sin 3t\,dt = \frac{\cancel{2}\,4}{37}(-6\cos 3t - \sin 3t)e^{-t/2}$$

$$y(t) = e^{t/2}\left(\frac{-4}{37}(6\cos 3t + \sin 3t)e^{-t/2} + C\right)$$

$$= -\frac{24}{37}\cos 3t - \frac{4}{37}\sin 3t + Ce^{t/2}$$

$$2xy - 9x^2 + (2y + x^2 + 1)y' = 0$$

$M = 2xy - 9x^2$ $\quad\quad$ $N = 2y + x^2 + 1$

$M_y = 2x$ $\quad\quad$ $N_x = 2x$

$M_y = N_x = 2x$ ✓

$$\Psi_y = \int (2xy - 9x^2)\,dx$$

$$= x^2y - 3x^3 + h(y)$$

$$\Psi_y' = x^2 + h'(y) = 2y + x^2 + 1$$

$$h'(y) = 2y + 1$$

$$h(y) = \int (2y+1)\,dy$$

$$= y^2 + y$$

$$x^2y - 3x^3 + y^2 + y = C$$

$(x+y)^2 + (2xy + x^2 - 1)\,dy = 0 \quad y(1) = 1$

$M = x^2 + 2xy + y^2 \qquad N = 2xy + x^2 - 1$

$M_y = 2x + 2y \qquad N_x = 2y + 2x$

$M_y = N_x$ ✓

$\varphi = \int (x^2 + 2xy + y^2)\,dx$

$= \frac{1}{3}x^3 + x^2 y + y^2 x + h(y)$

$\varphi_y = x^2 + 2yx + h'(y) = 2xy + x^2 - 1$

$h'(y) = -1 \Rightarrow h(y) = -y$

$\frac{1}{3}x^3 + x^2 y + xy^2 - y = C$

$\frac{1}{3} + 1 + 1 - 1 = C \Rightarrow C = \frac{4}{3}$

$\frac{1}{3}x^3 + x^2 y + xy^2 - y = \frac{4}{3}$