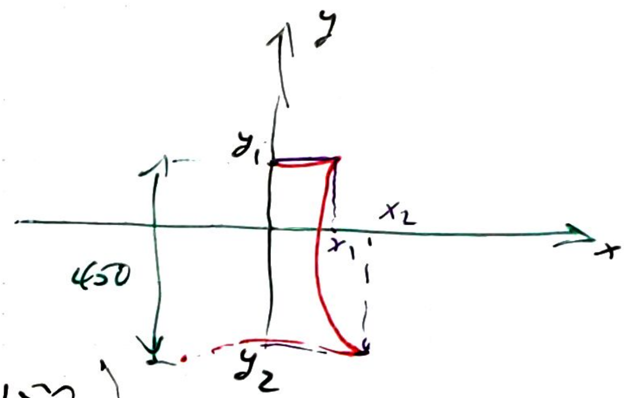
$$\frac{x^2}{90^2} - \frac{y^2}{130^2} = 1$$



$$y_1 = \frac{1}{2} y_2$$

$$y_2 = 2y_1$$

$$y_1 + y_2 = 450$$

$$3y_1 = 450 \Rightarrow y_1 = 150$$

$$y_2 = 300$$

$$\frac{x^2}{90^2} = 1 + \frac{y^2}{130^2}$$

$$x^2 = 90^2 \left(\frac{130^2 + y^2}{130^2} \right)$$

$$x = \frac{90}{130} \sqrt{130^2 + y^2}$$

$$= \frac{9}{13} \sqrt{y^2 + 130^2}$$

@ $y_1 = 150$

$$x_1 = \frac{9}{13} \sqrt{150^2 + 130^2}$$

$$= \frac{90}{13} \sqrt{394}$$

Top diameter $= 2x_1 = \frac{180}{13} \sqrt{394}$

@ $y_2 = 300$

$$x_2 = \frac{9}{13} \sqrt{300^2 + 130^2}$$

$$= \frac{90}{13} \sqrt{1069}$$

Bottom diameter $= \frac{180}{13} \sqrt{1069}$

$\approx$

$S_n = a_1 + \cdots + a_n$ (Geometric)

$$S_n = a_1 \frac{1-r^n}{1-r}$$

Ex $1, .3, .09, .0027, \ldots$ 1st 5 terms

$$S_5 = 1 \frac{1-\left(\frac{3}{10}\right)^5}{1-\frac{3}{10}}$$

$$r = \frac{.3}{1} = .3 = \frac{3}{10}$$

$$= \frac{1-\frac{3^5}{10^5}}{\frac{7}{10}}$$

$$= \boxed{\frac{10^5-3^5}{7(10^4)}}$$

Infinite Geometric Series

$$S = \frac{a_1}{1-r}$$

iff (if and only If) $|r| < 1$

otherwise, ∞

$$\sum_{n=1}^{\infty} 3\left(\overbrace{-\frac{2}{3}}^{r}\right)^{n-1} = \frac{3}{1+\frac{2}{3}} = \boxed{\frac{9}{5}}$$

$$\boxed{|r| = \frac{2}{3} < 1}$$

$$\sum_{n=1}^{\infty} 3\left(-\frac{3}{2}\right)^{n-1} = \infty$$

$$|r| = \frac{3}{2} \geq 1$$

$$5.4\overline{27} = 5.4272727\ldots$$

$$= 5.4 + .027 + .00027 + \ldots$$

$$= \frac{54}{10} + 27(.001 + .00001 + \ldots$$

$$27\times10^{-3} + 27\times10^{-5} + \ldots$$

$$r = \frac{10^{-5}}{10^{-3}} = 10^{-2} = \frac{1}{10^2}$$

$$5.4\overline{27} = \frac{54}{10} + 27\,\frac{.001}{1-\frac{1}{10^2}}$$

$$\frac{}{\frac{100-1}{100}} = \frac{100}{99}$$

$$= \frac{54}{10} + \frac{27}{99}$$

$$\frac{54}{10} + \frac{27}{990}$$

$$\frac{54}{10} + \frac{3}{110}$$

$$= \frac{54(99)+27}{990}$$

$$= \frac{597}{110}$$

$a_1 = 18$

$a_2 = .98(18)$

$a_3 = .98^2(18)$

$\vdots$

$a_{10} = (.98)^9(18) \approx \boxed{15.007 \text{ in}}$

$a_n = 18(.98)^{n-1}$

$T = \frac{18}{1-.98} = \frac{18}{1-\frac{98}{100}}$

$= \frac{1800}{2}$

$\boxed{= 900}$

5.7 Mathematical Induction

P_1 is true ✓.
Assume P_k is true ⇒ we need to prove P_{k+1} is also true

Ex sum of 1's $n \in \mathbb{Z}^+$

$$\underbrace{\frac{n(n+1)}{2}}_{n=1} = 1+2+3+\cdots+n$$

Soln

For $n=1 \Rightarrow 1 \overset{?}{=} \frac{1(2)}{2}$

$1 = 1$ ✓ P_1 is true.

Let P_k : $1+2+\cdots+k = \frac{k(k+1)}{2}$ is true

(substitute $n \to t$)

Is P_{k+1} : $1+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$? $k \to k+1$

copy

$$1+\cdots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

compare

$$= (k+1)\left(\frac{k}{2}+1\right)$$

$$= \frac{(k+1)(k+2)}{2} \checkmark$$

P_{k+1} is also true.

Therefore, By the mathematical induction, the proof is completed

Ex $1^2+3^2+\cdots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

For $n=1 \Rightarrow \quad 1^2 \overset{?}{=} \frac{1(1)(3)}{3}$

$1=1$ ✓ P_1 is true

Let P_k: $1^2+\cdots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}$ is true

Is P_{k+1}: $1^2+\cdots+(2k-1)^2+(2(k+1)-1)^2$

$$\overset{?}{=} \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$$

$$1^2+\cdots+(2k-1)^2+(2k+1)^2 \overset{?}{=} \frac{1}{3}(k+1)(2k+1)(2k+3)$$

$$1^2+\cdots+(2k-1)^2+(2k+1)^2=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$$

$$=(2k+1)\left(\frac{1}{3}k(2k-1)+2k+1\right)$$

$$=(2k+1)\,\frac{2k^2-k+6k+3}{3}$$

$$=\frac{1}{3}(2k+1)(k+1)(2k+3)$$

$$\frac{2k^2+5k+3}{3}$$

P_{k+1} is also true

$\therefore$ By the mathematical induction,

the proof is completed

Ex 2 is a factor of n^2+5n (n: positive integer)
$\mathbb{Z}$

For $n=1 \Rightarrow 1^2+5(1)=6$
$=2(3)$ ✓ P_1 is true.

P_k: is true 2 is a factor of k^2+5k
$k^2+5k = 2K$

is P_{k+1} $(k+1)^2+5(k+1)$?

$$(k+1)^2+5(k+1) = k^2+2k+1+5k+5$$
$$= k^2+5k+2k+6$$
$$= 2K+2k+6$$
$$= 2(K+k+3)$ ✓.$$

2 is a factor $\rightarrow$ P_{k+1} is also true

$\therefore$ By the mathematical induction, the proof is completed

Ex a nonzero $\mathbb{R}$ $a > -1$

nonzero $\mathbb{R} = \mathbb{R} - \{0\}$

Prove $(1+a)^n > 1+na$ $\quad n \geq 2$

$$\boxed{\begin{aligned} n=1 \Rightarrow (1+a)^1 &\overset{?}{>} 1+a \\ 1+a &> 1+a \end{aligned}}$$

$n=2 \Rightarrow (1+a)^2 \overset{?}{>} 1+2a$

$1+2a+a^2 \overset{?}{>} 1+2a$

$a > -1 \Rightarrow a^2 > 1$

$(1+a)^2 > 1+2a \Rightarrow P_2$ is true

$P_k : (1+a)^k > 1+ka$ is true.

is P_{k+1} $(1+a)^{k+1} > 1+(k+1)a$?

$$\begin{aligned} (1+a)^{k+1} &= (1+a)(1+a)^k \\ &> (1+a)(1+ka) \\ &= 1+ka+a+ka^2 \\ &= 1+(k+1)a+ka^2 \\ &> 1+(k+1)a \checkmark \end{aligned}$$

P_{k+1} is also true

$\therefore$ By the mathematical induction,

the proof is completed

11) $1^3+2^3+\dots+n^3=\dfrac{n^2(n+1)^2}{4}$

For $n=1 \Rightarrow 1^3 \overset{?}{=} \dfrac{1^2(2)^2}{4}$

$1=1$ ✓ P_1 is true.

$P_k: 1^3+\dots+k^3=\dfrac{k^2(k+1)^2}{4}$ is true.

Is $P_{k+1}: 1^3+\dots+k^3+(k+1)^3=\frac{1}{4}(k+1)^2(k+2)^2$?

$$1^3+\dots+k^3+(k+1)^3=\frac{1}{4}k^2(k+1)^2+(k+1)^3$$
$$=(k+1)^2\left(\frac{1}{4}k^2+k+1\right)$$
$$=(k+1)^2\,\frac{k^2+4k+4}{4}$$
$$=\frac{1}{4}(k+1)^2(k+2)^2$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the proof is completed.