P-2

$$\cos(A \underset{-}{+} B) = \cos A \cos B \underset{+}{-} \sin A \sin B$$

$$\sin(A \underset{-}{\pm} B) = \sin A \cos B \underset{-}{\pm} \cos A \sin B$$

$$\cos 75^\circ = \cos(45^\circ + 30^\circ)$$
$$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\,\frac{1}{2}$$
$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

show $\cos(x + 2\pi) = \cos x$

$$\cos(x+2\pi) = \cos x \cos 2\pi - \sin x \underbrace{\sin 2\pi}_{=0}$$
$$= \cos x$$

$$\cos 3x \cos 2x - \sin 3x \sin 2x = \cos(3x + 2x)$$
$$= \cos 5x$$

$$\cos(90^\circ - x) = \sin x$$

$$\cos(90^\circ - x) = \underbrace{\cos 90^\circ}_{=0} \cos x + \overbrace{\sin 90^\circ}^{=1} \sin x$$
$$= \boxed{\sin x}$$

Ex $\cos 15^\circ = ?$

$$\cos 15^\circ = \cos(45^\circ - 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2}$$

$$= \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

$$\boxed{\begin{aligned} \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \end{aligned}}$$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

Ex $\sin A = \frac{3}{5}$, $A \in QI$ / $\cos B = -\frac{5}{13}$ $B \in QIII$

$\cos A = \frac{4}{5}$ $\sin B = -\frac{12}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{3}{5}\left(\frac{-5}{13}\right) + \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right)$

$= \frac{-15-48}{65}$

$= -\frac{63}{65}$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{4}{5}\left(\frac{-5}{13}\right) - \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right)$

$= \frac{-20+36}{65}$

$= \frac{16}{65}$

c) $\tan(A+B) = -\frac{63}{16}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \frac{3}{5}\left(\frac{-5}{13}\right) - \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right)$

$= \frac{-15+48}{65}$

$= \frac{33}{65}$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{-20-36}{65}$

$= \frac{-56}{65}$

f) $\tan(A-B) = -\frac{33}{56}$

Ex. $\dfrac{\cos(x-y)}{\sin x \sin y} = \cot x \cot y + 1$

$$\frac{\cos(x-y)}{\sin x \sin y} = \frac{\cos x \cos y + \sin x \sin y}{\sin x \sin y}$$

$$= \frac{\cos x}{\sin x}\frac{\cos y}{\sin y} + \frac{\sin x \sin y}{\sin x \sin y}$$

$$= \cot x \cot y + 1 \checkmark$$

Ex Prove: $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

$$\cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y}$$

$$= \frac{\dfrac{\cos x \cos y}{\sin x \sin y} - \dfrac{\sin x \sin y}{\sin x \sin y}}{\dfrac{\sin x \cos y}{\sin x \sin y} + \dfrac{\cos x \sin y}{\sin x \sin y}}$$

$$= \frac{\cot x \cot y - 1}{\cot y + \cot x} \checkmark$$

$$\sec(x-y) = \frac{\cos x\cos y - \sin x \sin y}{\cos^2 x - \sin^2 y}$$

$$\sec(x-y) = \frac{1}{\cos(x-y)} \cdot \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{(\cos x \cos y + \sin x \sin y)(\cos x \cos y - \sin x \sin y)}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x(1-\sin^2 y) - (1-\cos^2 x)\sin^2 y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \sin^2 y} \checkmark$$

#9 $\cos A = \frac{15}{17}$, $A \in Q\,I$ $\quad \cos B = -\frac{12}{13}$ $B \in Q\,II$

$\sin A = \frac{8}{17}$ $\quad \sin B = \frac{5}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{8}{17}\,\frac{-12}{13} + \frac{15}{17}\cdot\frac{5}{13}$

$= \frac{-96+75}{221}$

$= \boxed{-\frac{21}{221}}$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{15}{17}\,\frac{-12}{13} - \frac{8}{17}\,\frac{5}{13}$

$= \frac{-180-40}{221}$

$= \boxed{-\frac{220}{221}}$

c) $\tan(A+B) = \boxed{\frac{21}{220}}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \frac{-96-75}{221}$

$= -\frac{171}{221}$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{-180+40}{221}$

$= \boxed{-\frac{140}{221}}$

f) $\tan(A-B) = \boxed{\frac{171}{140}}$

#12 $\dfrac{\sin(A-B)}{\cos A\cos B} = \tan A - \tan B$

$$\frac{\sin(A-B)}{\cos A\cos B} = \frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}$$

$$= \tan A - \tan B$$ ✓.

#15 $\dfrac{\cos(x+y)}{\cos(x-y)} = \dfrac{\cot y - \tan x}{\cot y + \tan x}$

$$\frac{\cos(x+y)}{\cos(x-y)} = \frac{\cos x\cos y - \sin x\sin y}{\cos x\cos y + \sin x\sin y}$$

$$= \frac{\dfrac{\cos x\cos y}{\cos x\sin y} - \dfrac{\sin x\sin y}{\cos x\sin y}}{\dfrac{\cos x\cos y}{\cos x\sin y} + \dfrac{\sin x\sin y}{\cos x\sin y}}$$

$$= \frac{\cot y - \tan x}{\cot y + \tan x}$$ ✓

#25 $\csc(x-y) = \dfrac{\sin x\cos y + \cos x\sin y}{\sin^2 x - \sin^2 y}$

$$\csc(x-y) = \frac{1}{\sin(x-y)} \cdot \frac{\sin x\cos y + \cos x\sin y}{\sin x\cos y + \cos x\sin y}$$

$$= \frac{\sin x\cos y + \cos x\sin y}{(\sin x\cos y - \cos x\sin y)(\sin x\cos y + \cos x\sin y)}$$

$$= \frac{\sin x\cos y + \cos x\sin y}{\sin^2 x\cos^2 y - \cos^2 x\sin^2 y}$$

$$= \frac{\sin x\cos y + \cos x\sin y}{\sin^2 x(1-\sin^2 y) - (1-\sin^2 x)\sin^2 y}$$

$$= \frac{\sin x\cos y + \cos x\sin y}{\sin^2 x - \underline{\sin^2 x\sin^2 y} - \sin^2 y + \underline{\sin^2 x\sin^2 y}}$$

$$= \frac{\sin x\cos y + \cos x\sin y}{\sin^2 x - \sin^2 y} \checkmark$$

7.3

$\sin 2A \neq 2\sin A$

double angle $\neq \sin^2 A$

$$\boxed{\sin 2A} = \sin(A+A)$$
$$= \sin A\cos A + \cos A \sin A$$
$$= \boxed{2\sin A\cos A}$$

$$\cos 2A = \cos(A+A)$$
$$= \cos A\cos A - \sin A\sin A$$
$$= \boxed{\cos^2 A - \sin^2 A}$$

$$= \cos^2 A - (1-\cos^2 A)$$
$$= \boxed{2\cos^2 A - 1}$$

$$= 1-\sin^2 A - \sin^2 A$$
$$= \boxed{1-2\sin^2 A}$$

$$\cos 2A = 2\cos^2 A - 1$$
$$\cos 2A + 1 = 2\cos^2 A \Rightarrow \boxed{\cos^2 A = \frac{1+\cos 2A}{2}}$$

$$\boxed{\sin^2 A = \frac{1-\cos 2A}{2}}$$

$$(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$$

$$(\sin\theta + \cos\theta)^2 = \underbrace{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}$$

$$= 1 + \sin 2\theta \checkmark$$

Prove

$$\sin 2x = \frac{2\cot x}{1+\cot^2 x}$$

$$\frac{2\cot x}{1+\cot^2 x} = 2\,\frac{\frac{\cos x}{\sin x}}{1+\frac{\cos^2 x}{\sin^2 x}}$$

$$= 2\,\frac{\frac{\cos x}{\sin x}}{\frac{\{\sin^2 x+\cos^2 x\} \to 1}{\sin^2 x}}$$

$$= 2\,\frac{\cos x}{\sin x}\cdot \sin^2 x$$

$$= 2\cos x \sin x$$

$$= \sin 2x \checkmark.$$

$$\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$$

$$\begin{aligned}
\cos(4x) &= \cos 2(2x) \\
&= 2\cos^2 2x - 1 \qquad (\cos 2x)^2 \\
&= 2\left(2\cos^2 x - 1\right)^2 - 1 \\
&= 2\left(4\cos^4 x - 4\cos^2 x + 1\right) - 1 \\
&= 8\cos^4 x - 8\cos^2 x + 1 \qquad 2-1
\end{aligned}$$

$$\tan 2A = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$= \boxed{\frac{2\tan A}{1 - \tan^2 A}}$$