Solution *Section* 4.1 – Inferences about Two Portions

Exercise

A Student surveyed her friends and found that among 20 males, 4 smoke and among 30 female, 6 smoke. Give two reasons why these results should not be used for a hypothesis test of the claim that the proportions of male smokers and female smokers are equal.

Solution

There are two requirements for using the methods of this section, and each of them is violated.

i. The samples should be 2 sample random samples that are independent. These samples are convenience samples, not simple random samples. These samples are likely not independent. Since she surveyed her friends, she may well have males and females that are dating each other (or least that associate with each other) – and people tend to associate with those that have similar behaviors.

ii. The number of successes for each sample should be at least 5, and the number of failures for each sample be at least 5. This is not true for the males, for which $x = 4$.

Using $\hat{p} = \frac{x}{n}$ to estimate p and $\hat{q} = 1 - \frac{x}{n} = \frac{n-x}{n}$ to estimate q.

$$n\hat{p} \geq 5 \qquad n\hat{q} \geq 5$$

$$n\left(\frac{x}{n}\right) \geq 5 \qquad n\left(\frac{n-x}{n}\right) \geq 5$$

$$x \geq 5 \qquad (n-x) \geq 5$$

These inequalities state that the number of successes must be greater than 5, and the number of failures must be greater than 5.

Exercise

In clinical trials of the drug Zocor, some subjects were treated with Zocor and other were given a placebo. The 95% confidence interval estimate of the difference between the proportions of subjects who experienced headaches is $-0.0518 < p_1 - p_2 < 0.0194$. Write a statement interpreting that confidence interval.

Solution

We have 95% confidence that the limits of –0.0518 and 0.01094 contain the true difference between the population proportions of subjects who experience headaches. Repeating the trials many times would result in confidence limits that would include the true difference between the population proportions 95% of the time. Since the interval includes the value 0, there is no significant difference between the two population proportions.

Exercise

Among 8834 malfunctioning pacemakers, in 15.8% the malfunctions were due to batteries. Find the number of successes x.

Solution

$$x = (0.158)(8834)$$
$$\approx 1396$$

Exercise

Among 129 subjects who took Chantix as an aid to stop smoking, 12.4% experienced nausea. Find the number of successes x.

Solution

$$x = (0.124)(129)$$
$$\approx 16$$

Exercise

Among 610 adults selected randomly from among the residents of one town, 26.1% said that they have favor stronger gun-control laws. Find the number of successes x.

Solution

$$x = (610)(0.261)$$
$$\approx 159$$

Exercise

A computer manufacturer randomly selects 2,410 of its computers for quality assurance and finds that 3.13% of these computer are found defective. Find the number of successes x.

Solution

$$x = (2,410)(0.0313)$$
$$\approx 67$$

Exercise

Assume that you plan to use a significance level of $\alpha = 0.05$ to test the claim that $p_1 = p_2$. Use the given sample sizes and number of successes to find the pooled estimate $\bar{p}$

a) $n_1 = 288, \quad n_2 = 252, \quad x_1 = 75, \quad x_2 = 70$

b) $n_1 = 100, \quad n_2 = 100, \quad \hat{p}_1 = 0.2, \quad \hat{p}_2 = 0.18$

Solution

a) $\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{75}{288} = 0.26 \qquad \hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{70}{252} = 0.278$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{75 + 70}{288 + 252} = 0.269$$

b) $x_1 = n_1 \hat{p}_1 = (100)(0.2) = 20 \qquad x_2 = n_2 \hat{p}_2 = (100)(0.18) = 18$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{20 + 18}{100 + 100} = 0.19$$

Exercise

The numbers of online applications from simple random samples of college applications for 2003 and for the current year are given below.

	2003	Current Year
Number of application in sample	36	27
Number of online applications in sample	13	14

Assume that you plan to use a significance level of $\alpha = 0.05$ to test the claim that $p_1 = p_2$. Find

a) The pooled estimate $\bar{p}$

b) The *x* test statistic

c) The critical *z* values

d) The *P*–value

Assume 95% confidence interval

e) The margin of error E

f) The 95% confidence interval.

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{13}{36} = 0.361 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{14}{27} = 0.519$$

$$\hat{p}_1 - \hat{p}_2 = 0.361 - 0.519 = -0.157$$

a) $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{13 + 14}{36 + 27} = \dfrac{27}{63} = 0.429$

b) $z_{\hat{p}_1-\hat{p}_2} = \dfrac{\left(\hat{p}_1-\hat{p}_2\right)-\left(p_1-p_2\right)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1}+\dfrac{\bar{p}\bar{q}}{n_2}}}$

$= \dfrac{-0.157-0}{\sqrt{\dfrac{(0.429)(0.571)}{36}+\dfrac{(0.429)(0.571)}{27}}}$

$= -1.25$

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575

c) For $\alpha = 0.05$, the critical values are

$z = \pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$

d) $P-value = 2 \cdot P(z < -1.25)$

$= 2(0.1056)$

$= 0.2112$

z	.00	.01	.02	.03	.04	.05
−1.2	.1151	.1131	.1112	.1093	.1075	.1056

e) $E = z_{\alpha/2}\sqrt{\dfrac{\hat{P}_1\hat{q}_1}{n_1}+\dfrac{\hat{P}_2\hat{q}_2}{n_2}}$

$= 1.96\sqrt{\dfrac{(0.361)(0.639)}{36}+\dfrac{(0.519)(0.481)}{27}}$

$= 0.2452$

f) $\left(\hat{p}_1-\hat{p}_2\right)-E < \left(p_1-p_2\right) < \left(\hat{p}_1-\hat{p}_2\right)+E$

$-0.1574-0.2452 < p_1-p_2 < -0.1574+0.2452$

$-0.4026 < p_1-p_2 < 0.0878$

Exercise

Chantix is a drug used as an aid to stop smoking. The numbers of subjects experiencing insomnia for each of two treatment groups in a clinical trial of the drug Chantix are given below:

	Chantix Treatment	Placebo
Number in group	129	805
Number experiencing insomnia	19	13

Assume that you plan to use a significance level of $\alpha = 0.05$ to test the claim that $p_1 = p_2$. Find

a) The pooled estimate $\bar{p}$

b) The *x* test statistic

c) The critical *z* values

d) The *P*–value

Assume 95% confidence interval

e) The margin of error E

f) The 95% confidence interval.

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{19}{129} = 0.147 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{13}{805} = 0.016$$

$$\hat{p}_1 - \hat{p}_2 = 0.147 - 0.016 = 0.131$$

a) $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{19+13}{129+805} = \dfrac{32}{934} = 0.0343$

b) $$z_{\hat{p}_1 - \hat{p}_2} = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{0.131 - 0}{\sqrt{\frac{(0.0343)(0.9657)}{129} + \frac{(0.0343)(0.9657)}{805}}}$$

$$= 7.60$$

c) For $\alpha = 0.05$, the critical values are

$$\frac{\alpha}{2} = \frac{1-0.05}{2} = 0.025 \quad \Rightarrow A = 1 - 0.025 = 0.975$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$$z = \pm z_{\alpha/2} = \pm z_{0.025}$$

$$= \pm 1.96$$

d) $P - value = 2 \cdot P(z > 7.60)$

$$= 2(1 - 0.9999)$$

$$= 0.0002$$

3.50 and up	.9999

e) $$E = z_{\alpha/2}\sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}$$

$$= 1.96\sqrt{\frac{(0.147)(0.853)}{129} + \frac{(0.147)(0.853)}{805}}$$

$$= 0.0618$$

f) $\left(\hat{p}_1 - \hat{p}_2\right) - E < \left(p_1 - p_2\right) < \left(\hat{p}_1 - \hat{p}_2\right) + E$

$$0.1311 - 0.0618 < p_1 - p_2 < 0.1311 + 0.0618$$

$$0.0694 < p_1 - p_2 < 0.1929$$

Exercise

In a 1993 survey of 560 college students, 171 said that they used illegal drugs during the previous year. In a recent survey of 720 college students, 263 said that they used illegal drugs during the previous year. Use a 0.05 significance level to test the claim that the proportion of college students using illegal drugs in 1993 was less than it is now.

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{171}{560} = 0.305 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{263}{720} = 0.365$$

$$\hat{p}_1 - \hat{p}_2 = 0.305 - 0.365 = -0.60$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$= \frac{170 + 263}{560 + 720} \qquad (170 + 263) / (560 + 720)$$

$$= 0.339$$

Original Claim: $p_1 - p_2 < 0$

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 < 0$$

z score	Area
1.645	0.9500
2.575	0.9950

$\alpha = 0.05$, the critical value

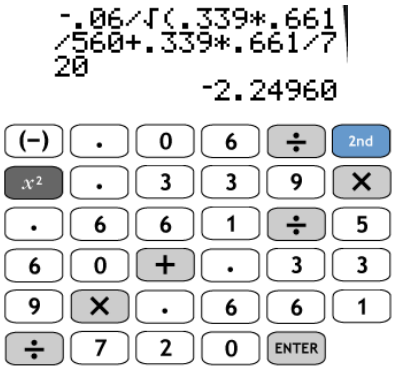
$$z = -z_\alpha = -z_{0.05}$$

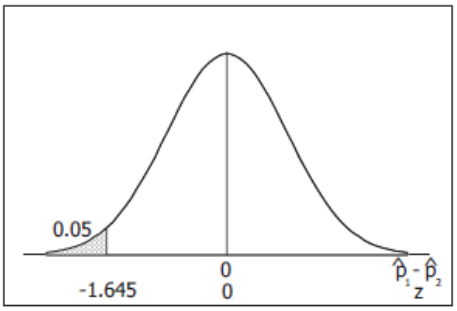
$$= -1.645$$

$$z_{\hat{p}_1 - \hat{p}_2} = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{-0.060 - 0}{\sqrt{\frac{(0.339)(0.661)}{560} + \frac{(0.339)(0.661)}{720}}}$$

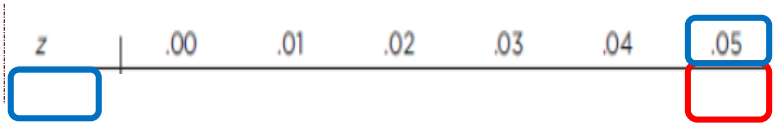
$$= -2.25$$





$$P - value = P(z < -2.52)$$

$$= 0.0122$$



−2.2	.0139	.0136	.0132	.0129	.0125	.0122

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 < 0$. There is sufficient evidence to support the claim that the proportion of college students using illegal drugs in 1993 was less than it is now.

Exercise

In a 1993 survey of 560 college students, 171 said that they used illegal drugs during the previous year. In a recent survey of 720 college students, 263 said that they used illegal drugs during the previous year. Construct the confidence interval corresponding to the hypothesis test conducted with a 0.05 significance level. What conclusion does the confidence interval suggest?

Solution

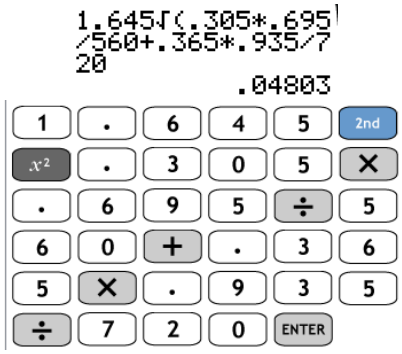
$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{171}{560} = 0.305 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{263}{720} = 0.365$$

$$\hat{p}_1 - \hat{p}_2 = 0.305 - 0.365 = -0.60$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sigma_{\hat{p}_1 - \hat{p}_2}$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1\bar{q}_1}{n_1} + \frac{\bar{p}_2\bar{q}_2}{n_2}}$$

$$-0.0599 \pm 1.645\sqrt{\frac{(0.305)(.695)}{560} + \frac{(0.365)(.935)}{720}}$$



$$-0.0599 \pm 0.0480$$

$$-0.0599 - 0.0480 < p_1 - p_2 < -0.0599 + 0.0480$$

$$-0.1079 < p_1 - p_2 < -0.0119$$

Since the confidence interval does not include the value 0, it suggests that the two population proportions are not equal and that the proportion of college students using illegal drugs in 1993 was less than it is now.

Exercise

A simple random sample of front-seat occupants involved in car crashes is obtained. Among 2823 occupants not wearing seat belts, 31 were killed. Among 7765 occupants wearing seat belts, 16 were killed. Construct a 90% confidence interval estimate of the difference between the fatality rates for those not wearing seat belts and those wearing seat belts. What does the result suggest about the effectiveness of seat belts?

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{31}{2823} = 0.01098 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{16}{7765} = 0.00206$$

$$\hat{p}_1 - \hat{p}_2 = 0.01098 - 0.00206 = 0.00892$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sigma_{\hat{p}_1 - \hat{p}_2}$$

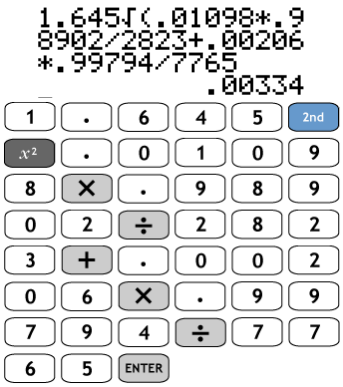
$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1\bar{q}_1}{n_1} + \frac{\bar{p}_2\bar{q}_2}{n_2}}$$

$$-0.00892 \pm 1.645\sqrt{\frac{(.01098)(.98902)}{2823} + \frac{(.00206)(.99794)}{7765}}$$

$$-0.00892 \pm 0.00334$$

$$-0.00892 - 0.00334 < p_1 - p_2 < -0.00892 + 0.00334$$

$$0.00558 < p_1 - p_2 < 0.01226$$



Since the confidence interval does not include the value 0, it suggests that the two population proportions are not equal and that seat belts are effective because the proportion of non-users who killed is greater than the proportion of users who are killed.

Exercise

A Pew Research Center poll asked randomly selected subjects if they agreed with the statement that "It is morally wrong fir married people to have an affair" Among the 386 women surveyed, 347 agrees with the statement. Among the 359 men surveyed, 305 agreed with the statement.

a) Use a 0.05 significance level to test the claim that the percentage of women who agree is difference from the percentage of men who agree. Does there appear to be a difference in the way women and men feel about this issue?

b) Construct the confidence interval corresponding to the hypothesis test conducted with a 0.05 significance level. What conclusion does the confidence interval suggest?

Solution

a) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{347}{386} = 0.899 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{305}{359} = 0.850$

$$\hat{p}_1 - \hat{p}_2 = 0.899 - 0.85 = 0.049$$

$$\bar{p} = \frac{437 + 305}{386 + 359} = 0.875$$

Original Claim: $p_1 - p_2 \neq 0$

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 \neq 0$$

$\alpha = 0.05$, $\frac{\alpha}{2} = \frac{1-0.05}{2} = 0.025 \quad \Rightarrow A = 1 - 0.025 = 0.975$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

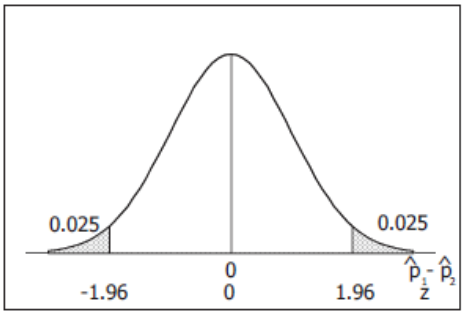
$$z = \pm z_{\alpha/2} = \pm z_{0.025}$$

$$= \pm 1.96$$

$$z_{\hat{p}_1 - \hat{p}_2} = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{0.049 - 0}{\sqrt{\frac{(.875)(.125)}{386} + \frac{(.875)(.125)}{359}}}$$

$$= 2.04$$



$$P - value = 2 \cdot P(z > 2.04)$$

$$= 2 \cdot (1 - 0.9793)$$

$$= 0.0414$$

z	.00	.01	.02	.03	.04	.05
2.0	.9772	.9778	.9783	.9788	.9793	

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 \neq 0$ (in fact, that $p_1 - p_2 > 0$. There is sufficient evidence to support the claim that the percentage of women who agree is different from the percentage of men who agree. Yes; there does appear to be a difference in the way that women and men feel about the issue.

b) $\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sigma_{\hat{p}_1 - \hat{p}_2}$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1\bar{q}_1}{n_1} + \frac{\bar{p}_2\bar{q}_2}{n_2}}$$

$$0.04938 \pm 1.96\sqrt{\frac{(.899)(.101)}{386} + \frac{(.85)(.15)}{359}}$$

$$0.04938 \pm 0.04766$$

$$0.04938 - 0.04766 < p_1 - p_2 < 0.04938 + 0.04766$$

$$0.00172 < p_1 - p_2 < 0.09704$$

Since the confidence interval does not include the value 0, it suggests that the two population proportions are not equal and the percentage of women who agree is different from the percentage of men who agree. Since the interval includes only positive values, conclude that the percentage of women who agree is greater than the percentage of men who agree.

Exercise

Tax returns include an option of designating $3 for presidential election campaigns, and it does not cost the taxpayer anything to make that designation. In a simple random sample of 250 tax returns from 1976, 27.6% of the returns designated the $3 for the campaign. In a simple random sample of 300 recent tax returns, 7.3% of the returns designated the $3 for the campaign. Use a 0.05 significance level to test the claim that the percentage of returns designating the $3 for the campaign was greater in 1973 than it is now.

Solution

$x_1 = (.276)(250) = 69 \quad x_2 = (.073)(300) = 22$

$\hat{p}_1 = \frac{x_1}{n_1} = \frac{69}{250} = 0.276 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{22}{300} = 0.073$

$\hat{p}_1 - \hat{p}_2 = 0.276 - 0.073 = 0.203$

$\bar{p} = \frac{69 + 22}{250 + 300} = 0.165$

Original Claim: $p_1 - p_2 > 0$

$H_0 : p_1 - p_2 = 0$

$H_1 : p_1 - p_2 > 0$

$\alpha = 0.01$

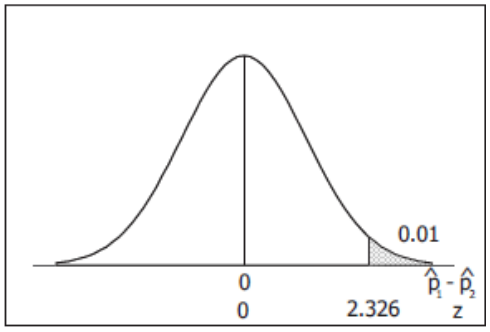
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.9893	.9896	.9898	.9901	.9904					

$z = z_\alpha = z_{0.01} = 2.326$

$$z_{\hat{p}_1 - \hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{0.203 - 0}{\sqrt{\frac{(.165)(.835)}{250} + \frac{(.165)(.835)}{300}}}$$

$= 6.37$



3.50 and up	.9999

$P - value = P(z > 6.37)$

$= 1 - 0.9999$

$= 0.0001$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 > 0$. There is sufficient evidence to support the claim that the percentage of returns designated funds for campaigns was greater on 1976 than it is now.

Exercise

In an experiment, 16% of 734 subjects treated with Viagra experienced headaches. In the same experiment, 4% of 725 subjects given a placebo experienced headaches.

a) Use a 0.01 significance level to test the claim that the proportion of headaches is greater for those treated with Viagra. Do headaches appear to be a concern for those who take Viagra?

b) Construct the confidence interval corresponding to the hypothesis test conducted with a 0.01 significance level. What conclusion does the confidence interval suggest?

Solution

a) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{x_1}{734} = 0.16$ $\hat{p}_2 = \frac{x_2}{n_2} = \frac{x_2}{725} = 0.04$

$\hat{p}_1 - \hat{p}_2 = 0.16 - 0.04 = 0.12$

$\bar{p} = \frac{(.16)(734) + (.04)(725)}{734 + 725} = 0.100$

Original Claim: $p_1 - p_2 > 0$

$H_0 : p_1 - p_2 = 0$

$H_1 : p_1 - p_2 > 0$

$\alpha = 0.01$

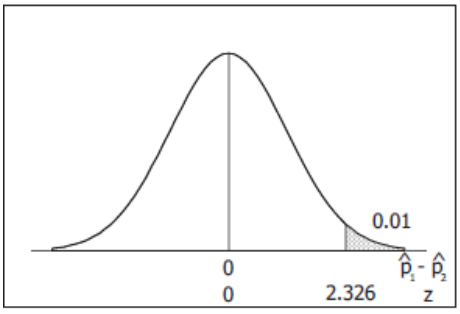
z	.00	.01	.02	.03	.04	.05
2.3	.9893	.9896	.9898	.9901	.9904	

$z = z_\alpha = z_{0.01} = 2.326$

$$z_{\hat{p}_1 - \hat{p}_2} = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{0.12 - 0}{\sqrt{\frac{(0.1)(0.9)}{734} + \frac{(0.1)(0.9)}{725}}}$$

$= 7.63$



3.50 and up	.9999

$P - value = P(z > 7.63)$

$= 1 - 0.9999$

$= 0.0001$

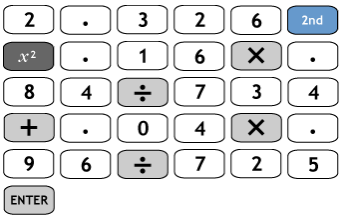
Conclusion:

Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 > 0$. There is sufficient evidence to support the claim that the proportion of persons experiencing headaches is greater for those treated with Viagra. Yes; headaches do appear to be a concern for those who take Viagra.

b) $\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\dfrac{\bar{p}_1\bar{q}_1}{n_1} + \dfrac{\bar{p}_2\bar{q}_2}{n_2}}$

$0.12 \pm 2.326\sqrt{\dfrac{(.16)(.84)}{734} + \dfrac{(.04)(.96)}{725}}$

2.326√(.16*.84/734+.04*.96/725)
.0357



0.12 ± 0.0357

$0.12 - 0.0357 < p_1 - p_2 < 0.12 + 0.0357$

$0.0843 < p_1 - p_2 < 0.1557$

Since the confidence interval does not include the value 0, there is a significant difference the two proportions. Since the confidence interval includes only positive values, the proportion of persons experiencing headaches is greater for those treated with Viagra.

Exercise

Two different simple random samples are drawn from two different populations. The first sample consists of 20 people with 10 having a common attribute. The second sample consists of 2000 people with 1404 of them having the same common attribute. Compare the results from a hypothesis test of $p_1 = p_2$ (with a 0.05 significance level) and a 95% confidence interval estimate of $p_1 - p_2$.

Solution

$\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{10}{20} = 0.5$ $\qquad$ $\hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{1404}{2000} = 0.702$

$\hat{p}_1 - \hat{p}_2 = 0.5 - 0.702 = -0.202$

$\bar{p} = \dfrac{10 + 1404}{20 + 2000} = 0.70$

Original Claim: $p_1 - p_2 \neq 0$

$H_0 : p_1 - p_2 = 0$

$H_1 : p_1 - p_2 \neq 0$

$\alpha = 0.05$, $\dfrac{\alpha}{2} = \dfrac{1 - 0.05}{2} = 0.025 \quad \Rightarrow A = 1 - 0.025 = 0.975$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

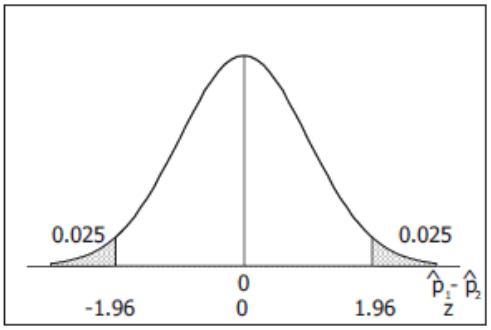
$z = \pm z_{\alpha/2} = \pm z_{0.025}$

$= \pm 1.96$

$$z_{\hat{p}_1-\hat{p}_2} = \frac{\left(\hat{p}_1-\hat{p}_2\right)-\left(p_1-p_2\right)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{-0.202-0}{\sqrt{\frac{(0.7)(0.3)}{20}+\frac{(0.7)(0.3)}{2000}}}$$

$$= -1.9615$$



$$P-value = 2\cdot P(z<-1.96)$$
$$= 2\cdot(.025)$$
$$\approx 0.05$$

z	.00	.01	.02	.03	.04	.05	.06
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250

Conclusion:

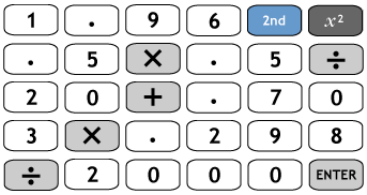
Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 = 0$ and conclude that $p_1 - p_2 \le 0$ (in fact, that $p_1 - p_2 < 0$).

The confidence interval is:

$$\left(\hat{p}_1-\hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1\bar{q}_1}{n_1}+\frac{\bar{p}_2\bar{q}_2}{n_2}}$$

$$-0.202 \pm 1.96\sqrt{\frac{(0.5)(0.5)}{20}+\frac{(0.702)(0.298)}{2000}}$$

1.96√(.5*.5/20+.703*.298/2000
.2201



$$-0.202 \pm 0.220$$
$$-0.202-0.220 < p_1 - p_2 < -0.202+0.220$$
$$-0.422 < p_1 - p_2 < 0.018$$

Since the confidence interval includes the value 0, p_1 *and* p_2 could have the same values and one should not reject the claim that $p_1 - p_2 = 0$.

The test of hypothesis and the confidence interval lead to different conclusions. In this instance, they are not equivalent.

Exercise

A report on the nightly news broadcast stated that 11 out of 142 households with pet dogs were burglarized and 21 out of 217 without pet dogs were burglarized. Find the *z* test statistic for the hypothesis test.

Assume that you plan to use a significance level of $\alpha = 0.05$ to test the claim that $p_1 = p_2$.

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{11}{142} = 0.0775 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{21}{217} = 0.0968$$

$$\hat{p}_1 - \hat{p}_2 = .0775 - .0968 = -0.0193$$

$$\bar{p} = \frac{11+21}{142+217} = 0.089$$

$$z = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{-.0193 - 0}{\sqrt{\frac{(.089)(0.911)}{142} + \frac{(.089)(0.911)}{217}}}$$

$$= -0.62$$

Exercise

Assume that the samples are independent and that they have been randomly selected. Construct a 90% confidence interval for the difference between population proportions $p_1 = p_2$

$n_1 = 39, \quad n_2 = 50, \quad x_1 = 13, \quad x_2 = 28$

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{13}{39} = 0.3333 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{28}{50} = 0.56$$

$$\hat{p}_1 - \hat{p}_2 = 0.33 - 0.56 = -0.23$$

$$\bar{p} = \frac{13+28}{39+50} = 0.461$$

$$A = 0.9 \implies z = 1.645$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1\bar{q}_1}{n_1} + \frac{\bar{p}_2\bar{q}_2}{n_2}} = -0.23 \pm 1.645\sqrt{\frac{(0.461)(0.539)}{39} + \frac{(0.461)(0.539)}{50}}$$

$$-0.23 \pm 0.175$$

$$-0.23 - 0.175 < p_1 - p_2 < -0.23 + 0.175$$

$$-0.405 < p_1 - p_2 < -0.055$$

Exercise

The sample size needed to estimate the difference between two population proportions ti within a margin of error E with a confidence level of $1 - \alpha$ can be found as follows:

$$E = z_{\alpha/2}\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}\,.$$

In this expression, replace n_1 and n_2 by n (assuming both samples have the same size) and replace each of p_1, q_1, p_2 *and* q_2 by 0.5 (because their values are not known). Then solve for n.

Use this approach to find the size pf each sample of you want to estimate the difference between the proportions of men and women who plan to vote in the next presidential election. Assume that you want 99% confidence that your error is no more than 0.05.

Solution

$$A = 99\% = 0.99 \quad \Rightarrow \quad z = 2.575$$

$$E = z_{\alpha/2}\sqrt{2\frac{p_1 q_1}{n}} \qquad p_1 = q_1 = p_2 = q_2 = 0.5$$

$$0.05 = 2.575\sqrt{2\frac{(0.5)^2}{n}}$$

$$\frac{0.05}{2.575} = \sqrt{\frac{2(0.5)^2}{n}}$$

$$\left(\frac{0.05}{2.575}\right)^2 = \frac{2(0.5)^2}{n}$$

$$n = 2(0.5)^2\left(\frac{2.575}{0.05}\right)^2 \approx 1{,}327$$

Exercise

If the pulse rates of men and women shown in the data below

Women:

76	72	88	60	72	68	80	64	68	68	80	76	68	72	96	72	68	72	64	80
64	80	76	76	76	80	104	88	60	76	72	72	88	80	60	72	88	88	124	64

Men:

68	64	88	72	64	72	60	88	76	60	96	72	56	64	60	64	84	76	84	88
72	56	68	64	60	68	60	60	56	84	72	84	88	56	64	56	56	60	64	72

These data are used to construct 95% confidence interval for the difference between the two population means, the result is $-12.2 < \mu_1 - \mu_2 < -1.6$, where pulse rates of men correspond to population 1 and pulse rates of women correspond to population 2. Express the confidence interval with pulse rates of women being population 1 and pulse rates of men being population 2.

Solution

Reversing the designation of which sample is considered group 1 and which sample is considered group 2 changes the sign of the point estimate and the signs of the endpoints of the interval estimate. The confidence interval using the new designation is $1.6 < \mu_1 - \mu_2 < 12.2$

Exercise

Assume that you want to use a 0.01 significance level to test the claim that the mean pulse rate of men is less than the mean pulse rate of women. What confidence level should be used if you want to test that claim using a confidence interval?

Solution

A one-tailed test of hypothesis at the 0.01 level of significance corresponds to a two-sided confidence interval at the 2(0.01) = 0.02 level of significance –i.e., to an interval with a confidence level of 98%

Exercise

To test the effectiveness of Lipitor, cholesterol levels are measured in 250 subjects before and after Lipitor treatments. Determine whether this sample is independent or dependent.

Solution

Dependent, since cholesterol levels and determined by many factors that the Lipitor treatment cannot change. Treatments to lower cholesterol typically reduce everyone's levels by a certain amount, by persons who were high compared to the others before the treatment, for example, will likely still be high compared to the others after the treatment.

Exercise

On each of 40 different days, you measured the voltage supplied to your home and you also measured the voltage produced by the gasoline-powered generator. One sample consists of the voltages in the house and the second sample consists of the voltages produced by the generator. Determine whether this sample is independent or dependent.

Solution

Independent, since there is no relationship between the voltage supplied to the house by the power company and the voltage generated by a completely separate gasoline-powered generator.

Exercise

In a randomized controlled trial conducted with children suffering from viral croup, 46 children were treated with low humidity while 46 other children were treated with high humidity. Researchers used the Westley Croup Score to assess the results after one hour. The low humidity group had a mean score of 0.98 with standard deviation of 1.22 while the high humidity group had a mean score of 1.09 with standard deviation of 1.11.

a) Use a 0.05 significance level to test the claim that the two groups are from populations with the same mean. What does the result suggest about the common treatment of humidity?
Assume that the two samples are independent simple random samples selected from normally distributed populations.

b) Assume that $\sigma_1 = \sigma_2$, how are the results affected by this additional assumption?

Solution

a) *Original Claim*: $\mu_1 - \mu_2 = 0$

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 \neq 0$$

$\alpha = 0.05$ *and* $df = 45$

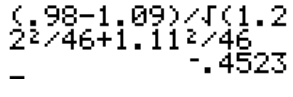
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
45	2.690	2.412	2.014	1.679	1.301

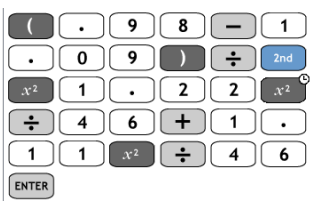
Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.014$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(.98 - 1.09) - (0)}{\sqrt{\frac{1.22^2}{46} + \frac{1.11^2}{46}}}$$

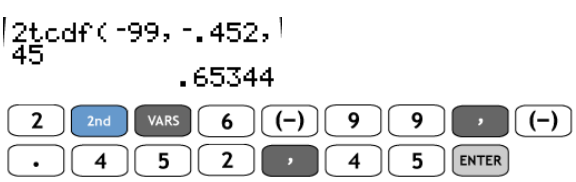
$$= -0.452$$

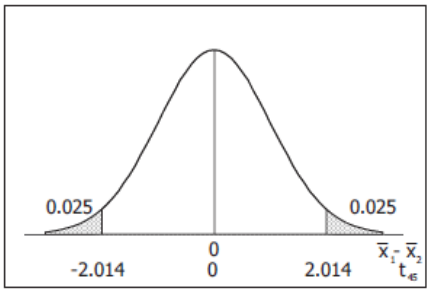




$$P-value = 2 \cdot tcdf\left(-99, -0.452, 45\right)$$

$$= \underline{0.6534}$$





Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim that $\mu_1 - \mu_2 = 0$. There is not sufficient evidence to reject the claim that the two groups are from populations with the same mean. The results suggest that increasing the humidity does not have a significant effect on the treatment of croup.

b) $df = df_1 + df_2$

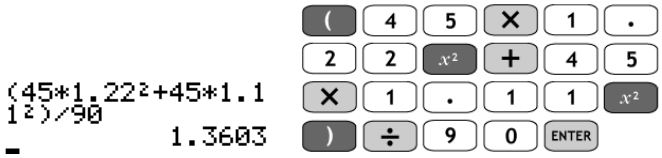
$= 45 + 45$

$= 90$

$$s_p^2 = \frac{\left(df_1\right)s_1^2 + \left(df_2\right)s_2^2}{df}$$

$$= \frac{(45)(1.22)^2 + (45)(1.11)^2}{90}$$

$$= \underline{1.3603}$$

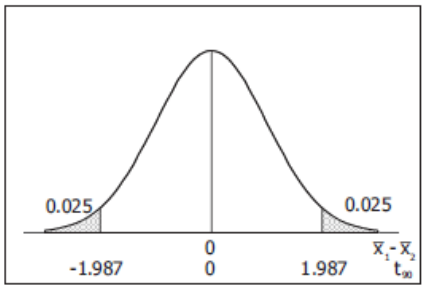


Original Claim: $\mu_1 - \mu_2 = 0$

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

$\alpha = 0.05 \quad and \quad df = 90$



Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \underline{\pm 1.987}$

Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(.98 - 1.09) - (0)}{\sqrt{\frac{1.3603}{46} + \frac{1.3603}{46}}}$$

$$= \underline{-0.452}$$

$$P-value = 2 \cdot tcdf\left(-99,\ -0.452,\ 90\right)$$

$$= 0.6521$$

Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim that $\mu_1 - \mu_2 = 0$. There is not sufficient evidence to reject the claim that the two groups are from populations with the same mean. The results suggest that increasing the humidity does not have a significant effect on the treatment of croup.

When $n_1 - n_2 = 0$, the calculated *t* statistic does not change at all. The only difference the assumption of equal standard deviations makes in this instance is to change the df from 45 to 90 and the P-value from 0.6532 to 0.6521. The conclusion is unaffected.

Exercise

The mean tar content of a simple random sample of 25 unfiltered king size cigarettes is 21.1 mg, with a standard deviation of 3.2 mg. The mean tar content of a simple random sample of 25 filtered 100 mm cigarettes is 13.2 mg, with a standard deviation of 3.7 mg.

Assume that the two samples are independent simple random samples selected from normally distributed populations in part a and b.

a) Construct a 90% confidence interval estimate of the difference between the mean tar content of unfiltered king size cigarettes and the mean tar content of filtered 100 mm cigarettes. Does the result suggest that 100 mm filtered cigarettes have less tar than unfiltered king size cigarettes?

b) Use a 0.05 significance level to test the claim that unfiltered king size cigarettes have a mean tar content greater than that of filtered 100 mm cigarettes. What does the result suggest about the effectiveness of cigarette filters?

c) Assume that $\sigma_1 = \sigma_2$, how are the results affected by this additional assumption?

Solution

a) Let the unfiltered cigarettes be group 1.

$\alpha = 0.1 \quad and \quad df = 24$

Critical value: $t = t_{0.05} = 1.711$

Degrees of Freedom	Area in One Tail 0.005	0.01	0.025	0.05	0.10
24	2.797	2.492	2.064	1.711	1.318

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\left(21.1-13.2\right) - 1.711\sqrt{\frac{3.2^2}{25} + \frac{3.7^2}{25}} < \mu_1 - \mu_2 < \left(21.1-13.2\right) + 1.711\sqrt{\frac{3.2^2}{25} + \frac{3.7^2}{25}}$$

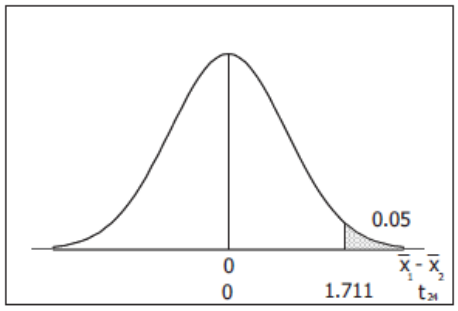
$$6.2 < \mu_1 - \mu_2 < 9.6$$

Yes; since the confidence interval includes only positive values, the results suggest that the filtered cigarettes have less tar than the unfiltered ones.

b) *Original Claim*: $\mu_1 - \mu_2 > 0$

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 > 0$$



$\alpha = 0.1 \quad and \quad df = 24$

Critical value: $t = t_{0.05} = 1.711$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(21.1 - 13.2) - 0}{\sqrt{\frac{3.2^2}{25} + \frac{3.7^2}{25}}}$$

$$= 8.075$$

$$P - value = tcdf\,(8.075,\ 99,\ 24)$$

$$= 1.338E - 8 \qquad \approx 0.00000001$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 > 0$. There is sufficient evidence to support the claim that unfiltered king size cigarettes have a mean tar content greater than that of filtered 100 mm cigarettes. The results suggest that filters are effective in reducing the tar content cigarettes.

c) $df = df_1 + df_2$

$$= 24 + 24$$

$$= 48$$

$$s_p^2 = \frac{(df_1)s_1^2 + (df_2)s_2^2}{df}$$

$$= \frac{(24)(3.2)^2 + (24)(3.7)^2}{48}$$

$$= 11.965$$

$\alpha = 0.10 \quad and \quad df = 48$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(21.1-13.2)-1.676\sqrt{\frac{11.965}{25}+\frac{11.965}{25}}<\mu_1-\mu_2<(21.1-13.2)+1.676\sqrt{\frac{11.965}{25}+\frac{11.965}{25}}$$

$$\boxed{6.3<\mu_1-\mu_2<9.5}$$

Yes; since the confidence interval includes only positive values, the results suggest that the filtered cigarettes have less tar than the unfiltered ones.

When $n_1=n_2$ the value of is unchanged. The only difference the assumption of equal standard deviations makes in this instance is to change the *df* from 24 to 48 and the from 1.711 to 1.676. This makes the interval slightly narrower, but the conclusion is unaffected.

Exercise

The heights are measured for the simple random sample of supermodels Crawford, Bundchen, Pestova, Christenson, Hume, Moss, Campbell, Schiffer, and Taylor. They have a mean of 70.0 in. and a standard deviation of 1.5 in. 40 women who are not supermodels, listed below and they have heights with means of 63.2 in. and a standard deviation of 2.7 in.

64.3	66.4	62.3	62.3	59.6	63.6	59.8	63.3	67.9	61.4	66.7	64.8	63.1	66.7	66.8
64.7	65.1	61.9	64.3	63.4	60.7	63.4	62.6	60.6	63.5	58.6	60.2	67.6	63.4	64.1
62.7	61.3	58.2	63.2	60.5	65.0	61.8	68.0	67.0	57.0					

a) Use a 0.01 significance level to test the claim that the mean height of supermodels is greater than the mean height of women who are not supermodels

b) Construct a 98% confidence interval level for the difference between the mean height of supermodels and the mean height of women who are not supermodels. What does the result suggest about those two means?

Solution

a) Let the supermodels be group 1. For which $n_1=9$

Original Claim: $\mu_1-\mu_2>0$ $(inches)$

$$H_0:\mu_1-\mu_2=0$$

$$H_1:\mu_1-\mu_2>0$$

$\alpha=0.01$ *and* $df=8$

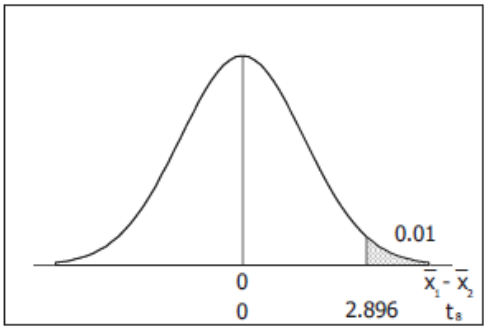
Critical value: $t=t_\alpha=t_{0.01}=\boxed{2.896}$

Degrees of Freedom	Area in One Tail 0.005	0.01	0.025	0.05	0.10
8	3.355	2.896	2.306	1.860	1.397

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(70.0 - 63.2) - 0}{\sqrt{\frac{1.5^2}{9} + \frac{2.7^2}{40}}}$$

$$= 10.343$$



$$P - value = tcdf\,(10.343,\ 99,\ 8)$$

$$= 3.29E-6 \qquad \approx 0.000003$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 > 0$. There is sufficient evidence to support the claim that the mean height of supermodels is greater than the mean height of women who are not supermodels.

b) Let the supermodels be group 1. For which $n_1 = 9$.

$\alpha = 1 - .98 = 0.2$ *and* $df = 8$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(70 - 63.2) - 2.896\sqrt{\frac{1.5^2}{9} + \frac{2.7^2}{40}} < \mu_1 - \mu_2 < (70 - 63.2) + 2.896\sqrt{\frac{1.5^2}{9} + \frac{2.7^2}{40}}$$

$$4.9 < \mu_1 - \mu_2 < 8.7 \quad (inches)$$

Since the confidence interval includes only positive values, the results suggest that the mean height of supermodels is greater than the mean height of women who are not supermodels.

Exercise

Many studies have been conducted to test the effects of marijuana use on mental abilities. In one such study, groups of light and heavy users of marijuana in college were tested for memory recall, with the results given below. Use a 0.01 significance level to test the claim that the population of heavy marijuana users has a lower mean than the light users. Should marijuana use be of concern to college students?

Items sorted correctly by light marijuana users: $n = 64,\quad \bar{x} = 53.3,\quad s = 3.6$

Items sorted correctly by heavy marijuana users: $n = 65,\quad \bar{x} = 51.3,\quad s = 4.5$

Solution

Original Claim: $\mu_1 - \mu_2 > 0$

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 > 0$

$\alpha = 0.01 \quad and \quad df = 63$

Critical value: $t = t_\alpha = t_{0.01} = 2.390$

Degrees of Freedom	Area in One Tail 0.005	0.01	0.025	0.05	0.10
60	2.660	2.390	2.000	1.671	1.296

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(53.3 - 51.3) - (0)}{\sqrt{\frac{3.6^2}{64} + \frac{4.5^2}{65}}}$$

$= 2.790$

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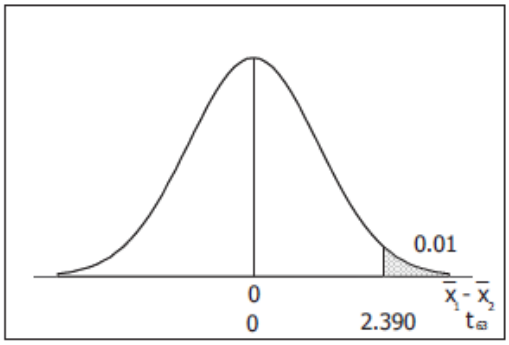
(53.3-51.3)/√(3.
6²/64+4.5²/65
                2.78954
```



$P - value = tcdf\,(2.790,\ 99,\ 63)$

$= 0.0035$

```
tcdf(2.790,99,63
                .00348
```

2nd VARS 6 2 . 7 9 0 , 9 9 , 6 3 ENTER

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 > 0$. There is sufficient evidence to support the claim that heavy marijuana users have a lower mean number of recalled items than do light users.

Yes; marijuana use should be of concern to college students – and an even more valuable study might one comparing light users to those who do not use marijuana at all.

Exercise

The trend of thinner Miss America winners has generated charges that the contest encourages unhealthy diet habits among young women. Listed below are body mass indexes (BMI) for Miss America winners from two different time periods. Consider the listed values to be simple random samples selected from larger populations.

a) Use a 0.05 significance level to test the claim that recent winners have a lower mean BMI than winners from the 1920s and 1930s.

b) Construct a 90% Confidence interval for the difference between the mean BMI of recent winners and the mean BMI of winners from the 1920s and 1930s.

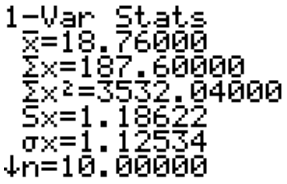
BMI (from recent winners):	19.5	20.3	19.6	20.2	17.8	17.9	19.1	18.8	17.6	16.8
BMI (from 1920s and 1930s):	20.4	21.9	22.1	22.3	20.3	18.8	18.9	19.4	18.4	19.1

Solution

Group 1: recent (n = 10)

$$\bar{x}_1 = \frac{\sum x}{n_1} = \frac{187.6}{10} = 18.76$$

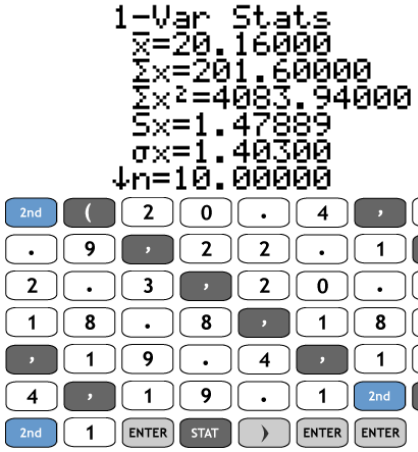
$$s_1 = 1.186$$



Group 2: 1920,1930 (n = 10)

$$\bar{x}_2 = \frac{\sum x}{n_2} = \frac{201.6}{10} = 20.16$$

$$s_2 = 1.479$$



a) *Original Claim*: $\mu_1 - \mu_2 < 0$

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 < 0$$

$\alpha = 0.05$ *and* $df = 9$

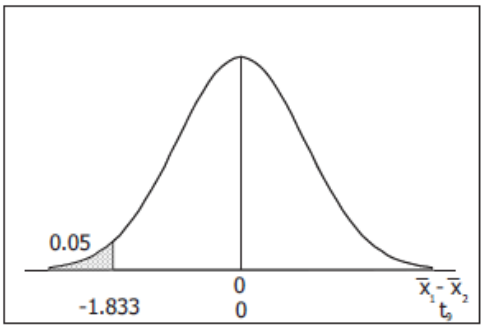
Critical value: $t = -t_\alpha = -t_{0.05} = -1.833$

Degrees of Freedom	Area in One Tail 0.005	0.01	0.025	0.05	0.10
9	3.250	2.821	2.262	1.833	1.383

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(18.76 - 20.16) - (0)}{\sqrt{\frac{1.186^2}{10} + \frac{1.479^2}{10}}}$$

$$= -2.335$$



$$P - value = tcdf\,(-99,\ -2.335,\ 9)$$

$$= 0.0222$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 < 0$. There is sufficient evidence to support the claim that recent winners have a lower mean BMI than winners from the 1920s and 1930s.

b) $\alpha = 0.1$ *and* $df = 9$.

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(18.76 - 20.16) - 1.833\sqrt{\frac{1.186^2}{10} + \frac{1.479^2}{10}} < \mu_1 - \mu_2 < (18.76 - 20.16) + 1.833\sqrt{\frac{1.186^2}{10} + \frac{1.479^2}{10}}$$

$$-2.50 < \mu_1 - \mu_2 < -0.30$$

Exercise

Listed below are amounts of strontium-90 (in millibecquerels or mBq per gram of calcium) in a simple random sample of baby teeth obtained from Pennsylvania residents and New York residents born after 1979.

a) Use a 0.05 significance level to test the claim that the mean amount of strontium-90 from Pennsylvania residents is greater than the mean amount from New York residents.

b) Construct a 90% Confidence interval for the difference between the mean amount of strontium-90 from Pennsylvania residents and the mean amount from New York residents.

Pennsylvania:	155	142	149	130	151	163	151	142	156	133	138	161
New York:	133	140	142	131	134	129	128	140	140	140	137	143

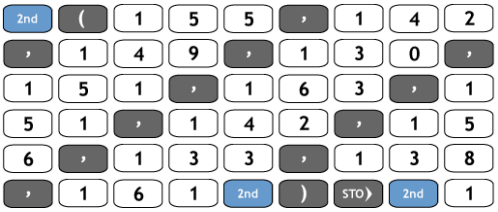
Solution

Group 1: PA ($n = 12$)

$$\bar{x}_1 = \frac{\sum x}{n_1} = \frac{1771}{12} = \underline{147.58}$$

$$s_1 = 10.64$$

```
1-Var Stats
 x̄=147.58333
 Σx=1771.00000
 Σx²=262615.000
 Sx=10.63834
 σx=10.18543
↓n=12.00000
```

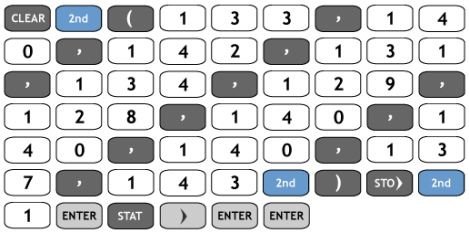


Group 2: NY ($n = 12$)

$$\bar{x}_2 = \frac{\sum x}{n_2} = \frac{1637}{12} = \underline{136.42}$$

$$s_2 = 5.21$$

```
1-Var Stats
 x̄=136.41667
 Σx=1637.00000
 Σx²=223613.000
 Sx=5.21289
 σx=4.99096
↓n=12.00000
```



a) *Original Claim*: $\mu_1 - \mu_2 > 0$

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 > 0$$

$\alpha = 0.05$ *and* $df = 11$

Critical value: $t = t_\alpha = t_{0.05} = \underline{1.796}$

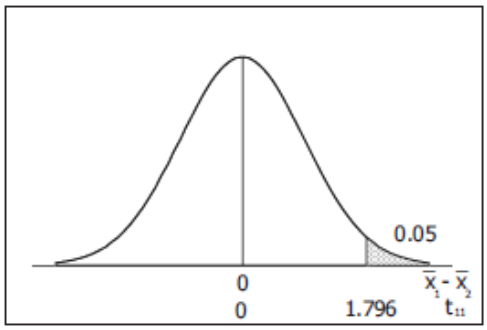
Degrees of Freedom	Area in One Tail 0.005	0.01	0.025	0.05	0.10
11	3.106	2.718	2.201	1.796	1.363

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(147.58 - 136.42) - (0)}{\sqrt{\frac{10.64^2}{12} + \frac{5.21^2}{12}}}$$

$$= \underline{3.263}$$

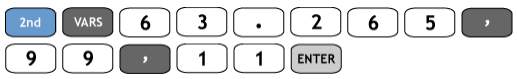
```
(147.58-136.42)/
√(10.64²/12+5.21
²/12
          3.26319
```



$$P - value = tcdf\,(3.265,\ 99,\ 11)$$

$$= \underline{0.0038}$$

```
tcdf(3.265,99,11
          .0038
```



Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 > 0$. There is sufficient evidence to support the claim that the mean amount of Strontium-90 from Pennsylvania residents is greater than the mean amount from N.Y. residents.

c) $\alpha = 0.1$ *and* $df = 11$.

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(147.58 - 136.42) - 1.796\sqrt{\frac{10.64^2}{12} + \frac{5.21^2}{12}} < \mu_1 - \mu_2 < (147.58 - 136.42) + 1.796\sqrt{\frac{10.64^2}{12} + \frac{5.21^2}{12}}$$

$$\underline{5.0 < \mu_1 - \mu_2 < 17.3} \quad (mBq)$$

Exercise

Listed below are the word counts for male and female psychology students.

a) Use a 0.05 significance level to test the claim that male and female psychology students speak the same mean number of words in a day.

b) Construct a 95% Confidence interval estimate of the difference between the mean number of words spoken in a day by male and female psychology students. Do the confidence interval limits include 0, and what does that suggest about the two means?

Male	21143	17791	36571	6724	15430	11552	11748	12169	15581	23858	5269
	12384	11576	17707	15229	18160	22482	18626	1118	5319		

Female	6705	21613	11935	15790	17865	13035	24834	7747	3852	11648	25862
	17183	11010	11156	11351	25693	13383	19992	14926	14128	10345	13516
	12831	9671	17011	28575	23557	13656	8231	10601	8124		

Assume that the two samples are independent simple random samples selected from normally distributed populations. Do not assume that the population standard deviations are equal.

Solution

Group 1: Males (n = 20)

$\bar{x}_1 = \underline{15021.9}$

$s_1 = \underline{7863.87}$

Group 2: Females (n = 31)

$\bar{x}_2 = \underline{14704.1}$

$s_2 = \underline{6215.35}$

a) *Original Claim*: $\mu_1 - \mu_2 = 0$ ***words/day***

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

$\alpha = 0.05$ *and* $df = 19$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \underline{\pm 2.093}$

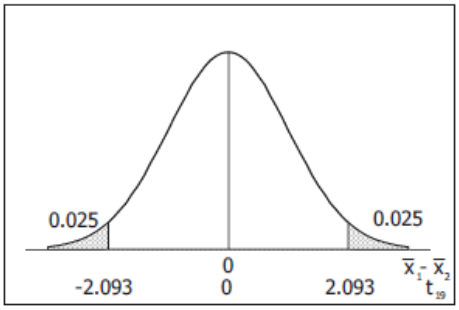
Degrees of Freedom	Area in Two Tails 0.01	0.02	0.05	0.10	0.20
19	2.861	2.539	2.093	1.729	1.328

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(15021.9 - 14704.1) - (0)}{\sqrt{\frac{7863.87^2}{20} + \frac{6215.35^2}{31}}}$$

$$= 0.153$$

```
(15021.9-14704.1
)/√(7863.87²/20+
6215.35²/31
                .1526
```



$$P - value = 2 \cdot tcdf\left(0.1526,\ 99,\ 19\right)$$

$$= 0.8803$$

```
2tcdf(.1526,99,1
9)
                .8803
```

Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim $\mu_1 - \mu_2 = 0$. There is not sufficient evidence to reject the claim that the male and female students speak the same mean number of words per day.

d) $\alpha = 0.05$ *and* $df = 19$.

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$317.8 - 2.093\sqrt{\frac{7863.87^2}{20} + \frac{6215.35^2}{31}} < \mu_1 - \mu_2 < 317.8 + 2.093\sqrt{\frac{7863.87^2}{20} + \frac{6215.35^2}{31}}$$

$$-4041.6 < \mu_1 - \mu_2 < 4677.1 \quad (words/day)$$

```
317.8-2.093√(786
3.87²/20+6215.35
²/31
              -4041.5578
```

Yes; since the confidence interval includes zero, there does not appear to be significant difference between the mean number of words spoken by the male and female students.

Exercise

Refer to the tables below and test the claim that they contain the same amount of cola, the mean weight of cola cans of regular Coke is the same as the mean weight of cola in cans of Diet Coke. If there is a difference in the mean weights, identify the most likely explanation for that difference.

Coke										
	0.8192	0.815	0.8163	0.8211	0.8181	0.8247	0.8062	0.8128	0.8172	0.811
	0.8251	0.8264	0.7901	0.8244	0.8073	0.8079	0.8044	0.817	0.8161	0.8194
	0.8189	0.8194	0.8176	0.8284	0.8165	0.8143	0.8229	0.815	0.8152	0.8244
	0.8207	0.8152	0.8126	0.8295	0.8161	0.8192				

Diet										
	0.7773	0.7758	0.7896	0.7868	0.7844	0.7861	0.7806	0.783	0.7852	0.7879
	0.7881	0.7826	0.7923	0.7852	0.7872	0.7813	0.7885	0.776	0.7822	0.7874
	0.7822	0.7839	0.7802	0.7892	0.7874	0.7907	0.7771	0.787	0.7833	0.7822
	0.7837	0.791	0.7879	0.7923	0.7859	0.7811				

Assume that the two samples are independent simple random samples selected from normally distributed populations. Do not assume that the population standard deviations are equal.

Solution

Group 1: Regular Coke (n = 36)

$\bar{x}_1 = 0.8168222$

$s_1 = 0.0075074$

Group 2: Diet Coke (n = 36)

$\bar{x}_2 = 0.7847944$

$s_2 = 0.0043909$

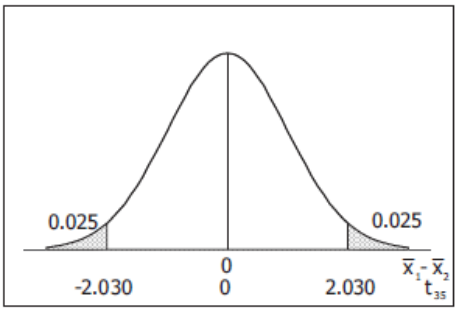
Original Claim: $\mu_1 - \mu_2 = 0$ ***lbs***

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

$\alpha = 0.05$ *and* $df = 35$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.030$



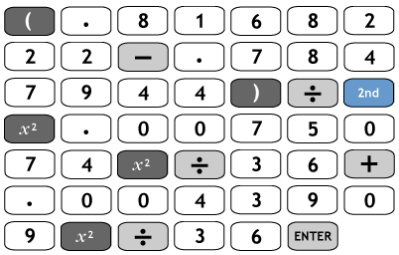
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
35	2.724	2.438	2.030	1.690	1.306

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(0.8168222 - 0.7847944) - (0)}{\sqrt{\frac{.0075074^2}{36} + \frac{.0043909^2}{36}}}$$

$= 22.095$

(.8168222-.78479
44)/√(.0075074²/
36+.0043909²/36
22.0953



$P - value = 2 \cdot tcdf\,(22.095,\ 99,\ 35)$

$= 0.8803$

Conclusion:

Reject H_0; there is sufficient evidence to reject the claim that $\mu_1 - \mu_2 = 0$ and conclude that $\mu_1 - \mu_2 \neq 0$ (in fact, that $\mu_1 - \mu_2 > 0$). There is sufficient evidence to reject the claim that the mean weight of cola in cans of regular Coke is the same as the mean weight of cola in cans of Diet Coke. The regular Coke may weigh more because it contains sugar.

Exercise

An Experiment was conducted to test the effects of alcohol. Researchers measured the breath alcohol levels for a treatment group of people who drank ethanol and another group given a placebo. The results are given in the accompanying table. Use a 0.05 significance level to test the claim that the two sample groups come from populations with the same mean.

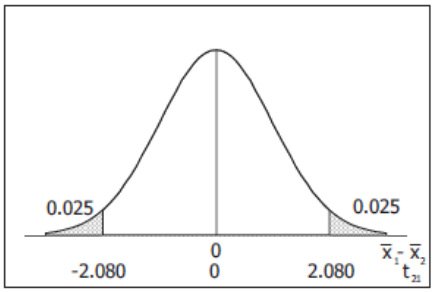
Treatment Group:	$n_1 = 22$	$\bar{x}_1 = 0.049$	$s_1 = 0.015$
Placebo Group:	$n_2 = 22$	$\bar{x}_2 = 0.000$	$s_2 = 0.000$

Solution

Original Claim: $\mu_1 - \mu_2 = 0$ ***lbs***

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$



$\alpha = 0.05$ *and* $df = 21$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.080$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(0.049 - 0.0) - (0)}{\sqrt{\frac{.015^2}{22} + \frac{0^2}{22}}}$$

$$= 15.322$$

$$P - value = 2 \cdot tcdf\,(15.322,\ 99,\ 21)$$

$$= 7.14E-13 \approx 0$$

Conclusion:

Reject H_0; there is sufficient evidence to reject the claim that $\mu_1 - \mu_2 = 0$ and conclude that $\mu_1 - \mu_2 \neq 0$ (in fact, that $\mu_1 - \mu_2 > 0$). There is sufficient evidence to reject the claim that the two sample groups come from populations with the same mean.

The fact that there was no variation in the second sample did not affect the calculations or present any special problems. Since there is no variation in x_2, it is really equivalent to the constant value zero – and the test is mathematically equivalent to the one-sample test

$H_0 : \mu_1 = 0$ for which $t = \dfrac{\bar{x}_1 - 0}{s_{\bar{x}_1}}$

Exercise

A researcher was interested in comparing the GPAs of students at two different colleges. Independent simple populations. Do samples of 8 students from college *A* and 13 students from college *B* yielding the following GPAs.

College *A*	3.7	3.2	3.0	2.5	2.7	3.6	2.8	3.4					
College *B*	3.8	3.2	3.0	3.9	3.8	2.5	3.9	2.8	4.0	3.6	2.6	4.0	3.6

Construct a 95% confidence interval for $\mu_1 - \mu_2$. The difference between the mean GPA of college *A* students and the mean GPA of college *B* students.

$\left(Note: \ \bar{x}_1 = 3.1125, \ \bar{x}_2 = 3.4385, \ s_1 = 0.4357, \ s_2 = 0.5485\right)$

Solution

$\alpha = 0.05 \quad and \quad df = 21$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.080$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\left(3.1125 - 3.4385\right) - 2.08\sqrt{\frac{0.4357^2}{8} + \frac{0.5485^2}{13}} \approx -0.78$$

$$\left(3.1125 - 3.4385\right) + 2.08\sqrt{\frac{0.4357^2}{8} + \frac{0.5485^2}{13}} \approx 0.13$$

$$-0.78 < \mu_1 - \mu_2 < 0.13$$

Exercise

Assume that the two samples are independent simple random samples selected from normal distributed populations. Do not assume that the population standard deviations are equal.
A researcher was interested in comparing the heights of women in two different countries. Independent simple random samples of 9 women from country ***A*** and 9 women from ***B*** yielded to the following heights (in inches).

Country A	64.1	66.4	61.7	62.0	67.3	64.9	64.7	68.0	63.6
Country B	65.3	60.2	61.7	65.8	61.0	64.6	60.0	65.4	59.0

Construct a 90% confidence interval for $\mu_1 - \mu_2$ the difference between the mean height of women in country ***A*** and the mean height of women in country ***B***. Round to two decimal places.

$\left(Note: \bar{x}_1 = 64.744\ in,\ \bar{x}_2 = 62.556\ in,\ s_1 = 2.192\ in,\ s_2 = 2.697\ in\right)$

Solution

$\bar{x}_1 = 64.744 \quad \bar{x}_2 = 62.556 \qquad s_1 = 2.192 \quad s_2 = 2.697$

$$A = \frac{s_1^2}{n_1} = 0.53 \quad B = \frac{s_2^2}{n_2} = 0.81$$

$$df = \frac{(A+B)^2}{\frac{A^2}{n_1 - 1} + \frac{B^2}{n_2 - 1}} \approx 15$$

Degrees of Freedom	Area in One Tail 0.005	0.01	0.025	0.05	0.10
15	**2.947**	2.602	**2.131**	1.753	1.341

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(64.744 - 62.556) - 1.753\sqrt{\frac{2.192^2}{9} + \frac{2.697^2}{9}} < \mu_1 - \mu_2 < (64.744 - 62.556) + 1.753\sqrt{\frac{2.192^2}{9} + \frac{2.697^2}{9}}$$

$$0.16 < \mu_1 - \mu_2 < 4.22$$

Exercise

Listed below are the time intervals (in minutes) before and after eruptions of the Old Faithful geyser. Find the values of $\bar{d}$ and s_d. In general, what does μ_d represent?

Time interval before eruption	98	92	95	87	96
Time interval after eruption	92	95	92	100	90

Solution

The difference values are:

	98	92	95	87	96
	92	95	92	100	90
Difference = d	6	–3	3	–13	6
d^2	36	9	9	169	36

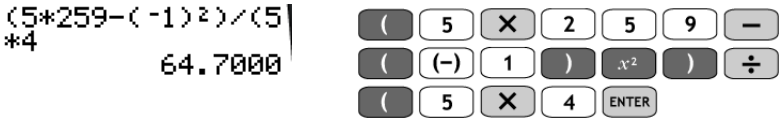
$n = 5; \quad \sum d = 6 - 3 + 3 - 13 + 6 = -1; \quad \sum d^2 = 36 + 9 + 9 + 169 + 36 = 259$

$$\bar{d} = \frac{\sum d}{n} = \frac{-1}{5} = -0.2 \text{ min}$$

$$s_d^2 = \frac{n\sum d^2 - (\sum d)^2}{n(n-1)}$$

$$= \frac{5(259) - (-1)^2}{5(4)}$$

$$= 64.7$$



$$s_d = \sqrt{64.7} = 8.0 \text{ min}$$

In general, μ_d represents the true mean of the differences from the population of matched pairs (which is mathematically equivalent to the true of the difference between the means of the two populations).

Exercise

Listed below are measured fuel consumption amount (in miles/gal) from a sample of cars.

City fuel consumption	18	22	21	21
Highway fuel consumption	26	31	29	29

Assume that wou want to use a 0.05 significance level to test the claim that the paired sample data come from a population for which the mean differecne is $\mu_d = 0$. Find

a) $\bar{d}$

b) s_d

c) The *t* test statistic

d) The critical values.

Solution

The difference values are:

	18	22	21	21
	26	31	29	29
Difference = d	-8	-9	-8	-8
d^2	64	81	64	64

$n = 4; \quad \sum d = -8-9-8-8 = -33; \quad \sum d^2 = 64+81+64+64 = 273$

a) $\bar{d} = \dfrac{\sum d}{n} = \dfrac{-33}{4} = -8.3\ mpg$

b) $s_d^2 = \dfrac{n\sum d^2 - (\sum d)^2}{n(n-1)} = \dfrac{4(273) - (-33)^2}{4(3)} = 0.25$

$s_d = \sqrt{0.25} = .5\ mpg$

c) $t_{\bar{d}} = \dfrac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{-8.25 - 0}{\frac{0.5}{\sqrt{4}}} = -33.00$

d) With $df = 3$ *and* $\alpha = 0.05$, the critical values are $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 3.182$

Exercise

Listed below are predicted high temperatures that were forecast different days.

Predicted high temperatures forecast 3 *days ahead*	79	86	79	83	80
Predicted high temperatures forecast 5 *days ahead*	80	80	79	80	79

Assume that wou want to use a 0.05 significance level to test the claim that the paired sample data come from a population for which the mean differecne is $\mu_d = 0$. Find

a) $\bar{d}$

b) s_d

c) The *t* test statistic

d) The critical values.

Solution

The difference values are:

	79	83	79	83	80
	80	80	79	80	79
Difference = d	–1	6	0	3	1
d^2	1	36	0	9	1

$n = 4; \quad \sum d = -1+6+0+3+1 = 9; \quad \sum d^2 = 1+36+0+9+1 = 47$

a) $\bar{d} = \dfrac{\sum d}{n} = \dfrac{9}{5} = 1.8\ °F$

b) $s_d^2 = \dfrac{n\sum d^2 - (\sum d)^2}{n(n-1)} = \dfrac{5(47)-(9)^2}{5(4)} = 7.7$

$s_d = \sqrt{7.7} = 2.8\ °F$

c) $t_{\bar{d}} = \dfrac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{1.8-0}{\frac{2.7749}{\sqrt{5}}} = 1.45$

d) With $df = 4$ *and* $\alpha = 0.05$, the critical values are $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.776$

Exercise

Listed below are body mass indices (BMI). The BMI of each student was measured in September and April of the freshman year.

a) Use a 0.05 significance level to tet the claim that the mean change in BMI for all students is equal to 0. Does BMI appear to change during freshman year?

b) Construct a 95% confidence interval estimate of the change in BMI during freshman year. Does the confidence interval include 0, and what does that suggest about BMI during freshman year?

April BMI	20.15	19.24	20.77	23.85	21.32
September BMI	20.68	19.48	19.59	24.57	20.96

Solution

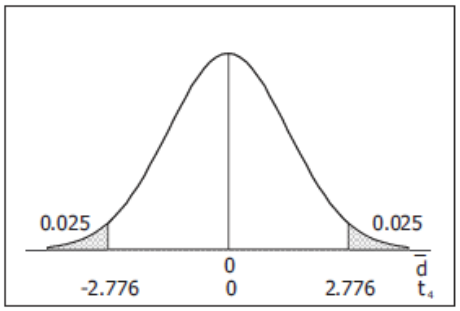
	20.15	19.24	20.77	23.85	21.32
	20.68	19.48	19.59	24.57	20.96
Difference = d	–0.53	–0.24	1.18	–0.72	0.36
d^2	.2809	.0576	1.3924	.5184	.1296

$n = 5; \quad \sum d = -53 - .24 + 1.18 - .72 + .36 = .05; \quad \sum d^2 = 2.3789$

a) Original claim: $\mu_d = 0$

$H_0 : \mu_d = 0$

$H_1 : \mu_d \neq 0$



$df = 4 \quad and \quad \alpha = 0.05$

$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.776$

$$s_d = \sqrt{\frac{n\sum d^2 - (\sum d)^2}{n(n-1)}}$$

$$= \sqrt{\frac{5(2.3789) - (0.05)^2}{5(4)}}$$

$$= 0.7711$$

$$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{0.01 - 0}{\frac{0.7711}{\sqrt{5}}} = 0.029$$

$P - value = 2 \cdot tcdf\,(0.029,\ 99,\ 4) = 0.9783$

Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim $\mu_d = 0$. There is not sufficient evidence to reject the claim that the mean change in BMI for all students is equal to 0.

No; BMI does not appear to change during the freshman year.

b) $\bar{d} - E < \mu_d < \bar{d} + E$

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

$$0.01 - 2.776\left(\frac{0.771}{\sqrt{5}}\right) < \mu_d < 0.01 + 2.776\left(\frac{0.771}{\sqrt{5}}\right)$$

$$-0.947 < \mu_d < 0.967$$

Yes; the confidence interval includes 0, which suggests that the mean of the differences could be 0 and that there is no change in BMI during the freshman year

Exercise

Listed below are body temperature (in °F) of subjects measured at 8:00 AM and at 12:00 AM. Construct a 95% confidence interval estimate of the difference between the 8:00 AM temperatures and the 12:00 AM temperatures. Is body temperature basically the same at both times?

8:00 AM	97.0	96.2	97.6	96.4	97.8	99.2
12:00 AM	98.0	98.6	98.8	98.0	98.6	97.6

Solution

	97.0	96.2	97.6	96.4	97.8	99.2
	98.0	98.6	98.8	98.0	98.6	97.6
Difference = d	–1.0	–2.4	–1.2	–1.6	–0.8	1.6
d^2	1	5.76	1.44	2.56	.64	2.56

$$n = 6; \quad \sum d = -5.4; \quad \sum d^2 = 13.96 \quad \bar{d} = \frac{\sum d}{n} = \frac{-5.4}{6} = .9$$

$$s_d^2 = \frac{n\sum d^2 - (\sum d)^2}{n(n-1)} = \frac{6(13.96) - (-5.4)^2}{6(5)} = 1.82$$

$$s_d = \sqrt{1.82} = 1.349$$

$$df = 5 \quad and \quad \alpha = 0.05$$

$$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.571$$

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

$$-0.90 - 2.571\left(\frac{1.349}{\sqrt{6}}\right) < \mu_d < -0.90 + 2.571\left(\frac{1.349}{\sqrt{6}}\right)$$

$$-2.32\ °F < \mu_d < 0.52\ °F$$

Yes; since the confidence intervals includes 0, body temperature is basically the same at both times.

Exercise

Listed below are systolic blood pressure measuremetns (mm Hg) taking rom the right and left arms of the same woman. Use a 0.05 significance level to test for a difference the measurements from the two arms. What do you conclude?

Right arm	102	101	94	79	79
Left arm	175	169	182	146	144

Solution

	102	101	94	79	79
	175	169	182	146	144
Difference = d	–73	–68	–88	–67	–65
d^2	5329	4624	7744	4489	4225

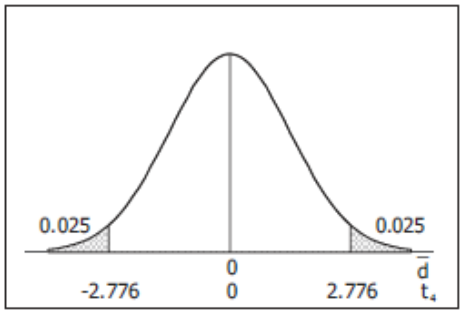
$$n = 5; \quad \sum d = -361; \quad \sum d^2 = 26,411 \quad \bar{d} = \frac{\sum d}{n} = \frac{-361}{5} = -72.2$$

$$s_d^2 = \frac{n\sum d^2 - (\sum d)^2}{n(n-1)} = \frac{5(26,411) - (-361)^2}{5(4)} = 86.6947$$

$$s_d = \sqrt{86.6947} = 9.311$$

$$df = 4 \quad and \quad \alpha = 0.05$$

$$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.776$$



$$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$= \frac{-72.2 - 0}{\frac{9.311}{\sqrt{5}}}$$

$$= -17.339$$

$$P - value = 2 \cdot tcdf\,(-99,\ -17.338,\ 4) = 6.488E-5 = 0.00006$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_d \neq 0$ (in fact, that $\mu_d < 0$). There is sufficient evidence to support the claim that there is a difference in measurements between the two arms. The statistical conclusion is that the right arm. Since the right and left arms should yield the same measurements, the practical conclusion is that a mistake has been made. The most obvious explanation is that diastolic (and not the systolic) values were mistakenly recorded for the right arm. Further investigation is definitely in order.

Exercise

As part of the National Health and Nutrition Examination Survey, the Department of Health and Human Services obtained self-reported heights and measured heights for males ages 12 – 16. All measurement are in inches. Listed below are sample results

Reported height	68	71	63	70	71	60	65	64	54	63	66	72
Measured height	67.9	69.9	64.9	68.3	70.3	60.6	64.5	67.0	55.6	74.2	65.0	70.8

a) Is there sufficient evidence to support the claim that there is a difference between self-reported heights and measured heights of males? Use a 0.05 significance level.

b) Construct a 95% confidence interval estimate of the man difference between reported heights and measured heights. Interpret the resulting confidence interval, and comment on the implications of whether the confidence interval limits contain 0.

Solution

a)

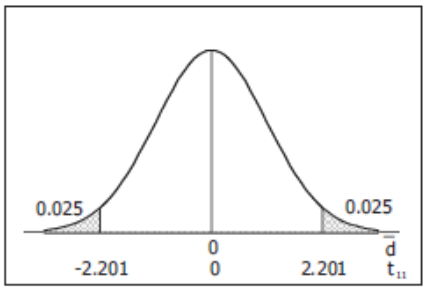
	68	71	63	70	71	60	65	64	54	63	66	72
	67.9	69.9	64.9	68.3	70.3	60.6	64.5	67.0	55.6	74.2	65.0	70.8
Difference = d	0.1	1.1	–1.9	1.7	0.7	–0.6	0.5	–3.0	–1.6	–11.2	1.0	1.2
d^2	.01	1.21	3.61	2.89	.49	.36	.25	9	2.56	125.44	1	1.44

$n = 12; \quad \sum d = -12.0; \quad \sum d^2 = 148.26 \quad \bar{d} = \dfrac{\sum d}{n} = \dfrac{-12}{12} = -1.0$

$$s_d = \sqrt{\frac{n\sum d^2 - (\sum d)^2}{n(n-1)}}$$

$$= \sqrt{\frac{12(148.26) - (-12)^2}{12(11)}}$$

$$= 3.52$$



Original claim: $\mu_d \neq 0$ *inches*

$$H_0 : \mu_d = 0$$

$$H_1 : \mu_d \neq 0$$

$df = 11 \quad and \quad \alpha = 0.05$

$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.201$

$$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$= \frac{-1.0 - 0}{\frac{3.52}{\sqrt{12}}}$$

$$= -0.984$$

$$P-value = 2 \cdot tcdf\left(-99,\ -0.984,\ 11\right)$$

$$= 0.3461$$

Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim $\mu_d \neq 0$. There is not sufficient evidence to support the claim that there is a difference between self-reported heights and measured height of such males.

b) $df = 11 \quad and \quad \alpha = 0.05$

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

$$-1.0 - 2.201\left(\frac{3.52}{\sqrt{12}}\right) < \mu_d < -1.0 + 2.201\left(\frac{3.52}{\sqrt{12}}\right)$$

$$-3.2\ in < \mu_d < 1.2\ in$$

Since the confidence interval contains 0, there is no significant difference between the reported and measured heights.

Exercise

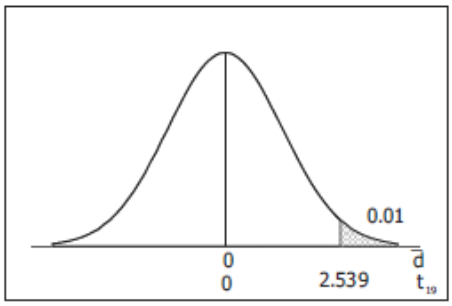
Listed below are combined city – highway fuel consumption ratings (in miles/gal) for different cars measured under both the old rating system and a new rating system introducing in 2008. The new ratings were implemented in response to complaints that the old ratings were too high. Use a 0.01 significance level to test the claim the old ratings are higher than the new ratings.

Old rating	16	18	27	17	33	28	33	18	24	19	18	27	22	18	20	29	19	27	20	21
New rating	15	16	24	15	29	25	29	16	22	17	16	24	20	16	18	26	17	25	18	19

Solution

	16	18	27	17	33	28	33	18	24	19	18	27	22	18	20	29	19	27	20	21
	15	16	24	15	29	25	29	16	22	17	16	24	20	16	18	26	17	25	18	19
Diff = d	1	2	3	2	4	3	4	2	2	2	2	3	2	2	2	3	2	2	2	2
d^2	1	4	9	4	16	9	16	4	4	4	4	9	4	4	4	9	4	4	4	4

$n = 20; \quad \sum d = 47; \quad \sum d^2 = 121 \quad \bar{d} = \dfrac{\sum d}{n} = \dfrac{47}{20} = 2.35$



$$s_d = \sqrt{\frac{n\sum d^2 - (\sum d)^2}{n(n-1)}} = \sqrt{\frac{20(121) - (47)^2}{20(19)}}$$

$$= 0.745$$

Original claim: $\mu_d > 0$ *mpg*

$$H_0 : \mu_d = 0$$

$$H_1 : \mu_d > 0$$

$df = 19 \quad and \quad \alpha = 0.01$

$t = t_\alpha = t_{0.01} = 2.539$

$$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$= \frac{2.35 - 0}{\frac{0.745}{\sqrt{20}}}$$

$$= 14.104$$

$P - value = tcdf\,(14.104,\ 99,\ 19)$

$$= 9.093E - 12 \approx 0$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_d > 0$. There is sufficient evidence to support the claim that the old ratings are higher than the new ratings.

Exercise

Listed below are 2 tables. Construct a 95% confidence interval estimate of the mean of the differences between weights of discarded paper and weights of discarded plastic. Which seems to weigh more: discarded paper or discarded plastic?

Paper

2.41	7.57	9.55	8.82	8.72	6.96	6.83	11.42	16.08	6.38	13.05	11.36	15.09
2.80	6.44	5.86	11.08	12.43	6.05	13.61	6.98	14.33	13.31	3.27	6.67	17.65
12.73	9.83	16.39	6.33	9.19	9.41	9.45	12.32	20.12	7.72	6.16	7.98	9.64
8.08	10.99	13.11	3.26	1.65	10.00	8.96	9.46	5.88	8.26	12.45	10.58	5.87
8.78	11.03	12.29	20.58	12.56	9.92	3.45	9.09	3.69	2.61			

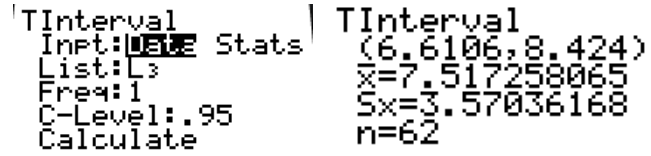
Plastic

0.27	1.41	2.19	2.83	2.19	1.81	0.85	3.05	3.42	2.10	2.93	2.44	2.17
1.41	2.00	0.93	2.97	2.04	0.65	2.13	0.63	1.53	4.69	0.15	1.45	2.68
3.53	1.49	2.31	0.92	0.89	0.80	0.72	2.66	4.37	0.92	1.40	1.45	1.68
1.53	1.44	1.44	1.36	0.38	1.74	2.35	2.30	1.14	2.88	2.13	5.28	1.48
3.36	2.83	2.87	2.96	1.61	1.58	1.15	1.28	0.58	0.74			

Solution

Using Ti-84, store paper into List 1 and plastic in List 2

To create list 3: **[2nd] 1** (L1) – **[2nd] 2** (L2) [***STO→***] **[2nd] 3** (L3)



$6.6107\ lbs < \mu_d < 8.424\ lbs$

Since the confidence interval includes only positive values, there discarded paper appears to weigh more than the discarded plastic.

Exercise

Suppose you wish to test the claim that μ_d, the mean value of the differences d for a population of paired data, is different from 0. Given a sample of $n = 23$ and a significance level of $\alpha = 0.05$, what criterion would be used for rejecting the null hypothesis?

Solution

Given: $n = 23 \Rightarrow df = 23 - 1 = 22$ and $\alpha = 0.05$

Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
22	**2.819**	2.508	**2.074**	1.717	1.321

To reject null hypothesis if test statistic is: $|t| > 2.074$ or < -2.074 *or* > 2.074

Exercise

Assume that he paired data came from a population that is normally distributed. Using a 0.05 significance level, find $\bar{d}$, s_d , the *t* test statistic, and the critical values to test the claim that $\mu_d = 0$

x	14	8	4	14	3	12	4	13
y	15	8	7	13	5	11	6	15

Solution

	14	8	4	14	3	12	4	13	
	15	8	7	13	5	11	6	15	
Difference = d	−1	0	−3	1	−2	1	−2	−2	
$(d-\bar{d})^2$	0	1	4	4	1	4	1	1	16

$$\bar{d} = \frac{-1+0-3+1-2+1-2-2}{8} = -1$$

$$s_d = \sqrt{\frac{\sum(d-\bar{d})^2}{n-1}} = \sqrt{\frac{16}{7}} \approx 1.5119$$

$$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$= \frac{-1-0}{\frac{1.5119}{\sqrt{8}}}$$

$$= -1.8708$$

$df = 7$ *and* $\alpha = 0.05$

$$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.365$$

Exercise

Assume that he paired data came from a population that is normally distributed. Using a 0.05 significance level, find $\bar{d}$, s_d , the *t* test statistic, and the critical values to test the claim that $\mu_d = 0$

x	12	5	1	20	3	16	12	8
y	7	10	5	15	7	14	10	13

Solution

	12	5	1	20	3	16	12	8
	7	10	5	15	7	14	10	13
Difference = d	5	−5	−4	5	−4	2	2	−5
$(d-\bar{d})^2$	25	25	16	25	16	4	4	25

$$\bar{d} = \frac{5-5-4+5-4+2+2-5}{8} = -0.5$$

$$s_d = \sqrt{\frac{n\sum d^2 - (\sum d)^2}{n(n-1)}} = \sqrt{\frac{8(140) - (-0.5)^2}{7}} \approx 4.440$$

$$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$= \frac{-0.5 - 0}{\frac{4.440}{\sqrt{8}}}$$

$$= -0.319$$

$df = 7 \quad and \quad \alpha = 0.05$

$$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.365$$

Solution *Section* 4.4 – Correlation

Exercise

For each of several randomly selected years, the total number of points scored in the Super Bowl football game and the total number of new cars sold in The U.S. are recorded. For this sample of paired data

a) What does r represent?

b) What does ρ represent?

c) With our doing any research or calculations, estimate the value of r.

Solution

a) r = the correlation in the sample. In this context, r is the linear correlation coefficient computed using the chosen paired (points in Super Bowl, number of new cars sold) values for the randomly selected years in the sample.

b) ρ = the correlation in the population. In this context, ρ is the linear correlation coefficient computed using the paired (points in Super Bowl, number of new cars sold) values for every year there has been a Super Bowl.

c) Since there is no relationship between the number of points scored in a Super Bowl and the number of new cars sold that year, the estimated value of r is 0.

Exercise

The heights (in inches) of a sample of eight mother/daughter pairs of subjects measured. Using Excel with the paired mother/daughter heights, the linear correlation coefficient is found to be 0.693. Is there sufficient evidence to support the claim that there is a linear correlation between the heights of mothers and the heights of their daughters? Explain.

Solution

From the table for $n = 8, \quad C.V. = \pm 0.707$. Therefore r = 0.693 indicates a significant (positive) linear correlation. Yes; there is sufficient evidence to support the claim that there is a linear correlation between the heights of mothers and the heights of their daughters,

Exercise

The heights and weights of a sample of 9 supermodels were measured. Using a TI calculator, the linear correlation coefficient is found to be 0.360. Is there sufficient evidence to support the claim that there is a linear correlation between the heights and weights of supermodels? Explain.

Solution

From the table for $n = 9, \quad C.V. = \pm 0.666$. Therefore r = 0.360 does not indicate a significant linear correlation. No; there is not sufficient evidence to support the claim that there is a linear correlation between the heights and weights of supermodels.

Exercise

Given the table below

x	10	8	13	9	11	14	6	4	12	7	5
y	9.14	8.14	8.74	8.77	9.26	8.10	6.13	3.10	9.13	7.26	4.74

a) Construct a scatterplot

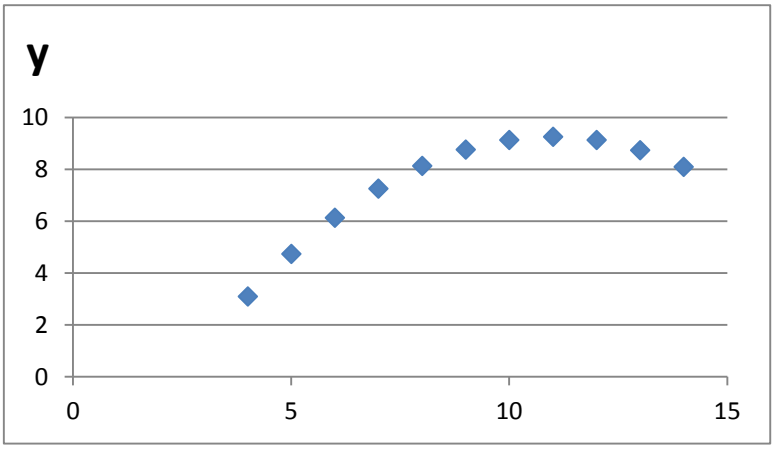
b) Find the value of linear correlation coefficient *r* and then determine whether there is sufficient evidence to support the claim of a linear correlation between the 2 variables.

c) Identify the feature of the data that would be missed if part (*b*) was completed without constructing the scatterplot.

Solution

a) Excel produces the following

x	***y***	***xy***	x^2	y^2
10	9.14	91.4	100	83.5396
8	8.14	65.12	64	66.2596
13	8.74	113.62	169	76.3876
9	8.77	78.93	81	76.9129
11	9.26	101.86	121	85.7476
14	8.1	113.4	196	65.6100
6	6.13	36.78	36	37.5769
4	3.1	12.4	16	9.6100
12	9.13	109.56	144	83.3569
7	7.26	50.82	49	52.7076
5	4.74	23.7	25	22.4676
99	82.51	797.59	1001	660.176



b) $$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{(11)(797.59) - (99)(82.51)}{\sqrt{(11)(1001) - (99)^2} \cdot \sqrt{(11)(660.1763) - (82.51)^2}}$$

$= 0.816$

From table *A*-5; $n = 11$, $C.V. = \pm 0.602$

Therefore r = 0.816 indicates a significant (positive) linear correlation. Yes; there is sufficient evidence to support the claim that there is a linear correlation between the 2 variables.

c) The scatterplot indicates that the relationship between the variables is quadratic, not linear.

Exercise

Given the table below

x	10	8	13	9	11	14	6	4	12	7	5
y	7.46	6.77	12.74	7.11	7.81	8.84	6.08	5.39	8.15	6.42	5.73

a) Construct a scatterplot

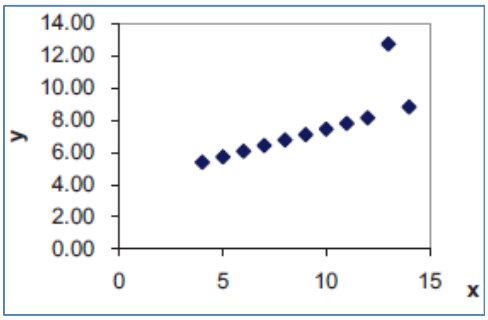
b) Find the value of linear correlation coefficient r and then determine whether there is sufficient evidence to support the claim of a linear correlation between the 2 variables.

c) Identify the feature of the data that would be missed if part (*b*) was completed without constructing the scatterplot.

Solution

a) Excel produces the following

x	***y***	***xy***	x^2	y^2
10	7.46	74.60	100	55.6516
8	6.77	54.16	64	45.8329
13	12.74	165.62	169	162.3076
9	7.11	63.99	81	50.5521
11	7.81	85.91	121	60.9961
14	8.84	123.76	196	78.1456
6	6.08	36.48	36	36.9664
4	5.39	21.56	16	29.0521
12	8.15	97.80	144	66.4225
7	6.42	44.94	49	41.2164
5	5.73	28.65	25	32.8329
99	82.50	797.47	1001	659.9762



b) $$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{(11)(797.59) - (99)(82.5)}{\sqrt{(11)(1001) - (99)^2} \cdot \sqrt{(11)(659.9762) - (82.5)^2}}$$

$$= 0.816$$

From table *A*-5; $n = 11$, $C.V. = \pm 0.602$

Therefore r = 0.816 indicates a significant (positive) linear correlation. Yes; there is sufficient evidence to support the claim that there is a linear correlation between the 2 variables.

c) The scatterplot indicates that the relationship between the variables is essentially a perfect straight line except for one point, which is likely an error or an oulier.

Exercise

The paired values of the Consumer Price Index (CPI) and the cost of a slice of pizza are shown below

CPI	30.2	48.3	112.3	162.2	191.9	197.8
Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00

a) Construct a scatterplot

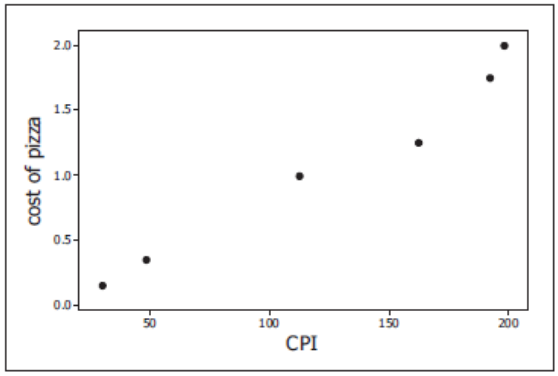
b) Find the value of linear correlation coefficient r and find the critical values if r, using α = 0.05.

c) Determine whether there is sufficient evidence to support the claim of a linear correlation between the CPI and the cost of a slice of pizza?

Solution

a) Excel produces the following

x	***y***	***xy***	x^2	y^2
30.2	0.15	4.53	912.04	0.0225
48.3	0.35	16.905	2332.89	0.1225
112.3	1.00	112.3	12611.29	1.00
162.2	1.25	202.75	26308.84	1.5625
191.9	1.75	335.825	36825.61	3.0625
197.8	2.00	395.60	39124.84	4.00
742.7	6.50	1067.91	118115.5	9.77



b) $$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{(6)(1067.91) - (742.7)(6.5)}{\sqrt{(6)(118115.5) - (742.7)^2} \cdot \sqrt{(6)(9.77) - (6.5)^2}}$$

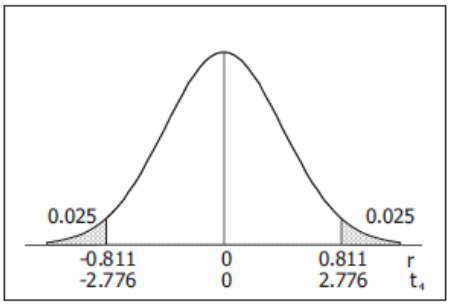
$$= 0.985$$

c) $H_0 : \rho = 0$

$H_1 : \rho \neq 0$

$\alpha = 0.05 \quad and \quad df = 4$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.776$



$$t_r = \frac{r - \mu_r}{s_r}$$

$$= \frac{0.985 - 0}{\sqrt{\frac{1-(0.985)^2}{4}}}$$

$$= 11.504$$

$$P - value = 2 \cdot tcdf\left(11.504,\ 99,\ 4\right) = 0.0003$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\rho \neq 0$ (in fact, that $\rho > 0$).

Yes; there is sufficient evidence to support the claim of a linear correlation between the CPI and the cost of a slice of pizza.

Exercise

Listed below are systolic blood pressure measurements (in mm HG) obtained from the same woman.

Right Arm	102	101	94	79	79
Left Arm	175	169	182	146	144

a) Construct a scatterplot

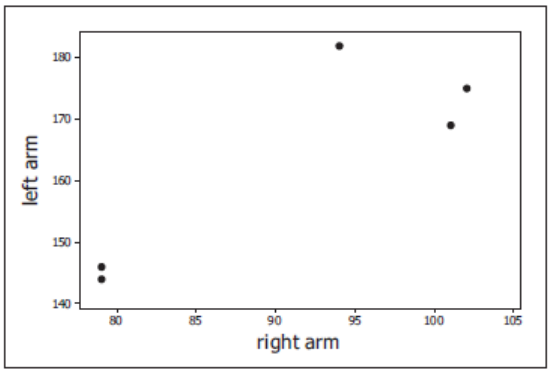
b) Find the value of linear correlation coefficient *r* and find the critical values if *r*, using α = 0.05.

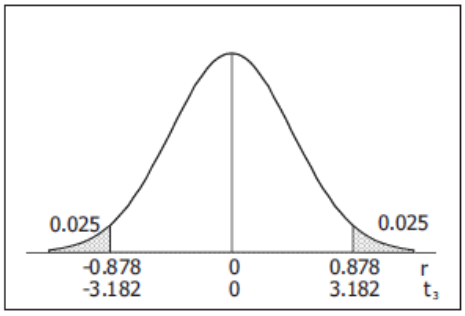
c) Determine whether there is sufficient evidence to support the claim of a linear correlation between the right and left arm systolic blood pressure measurements?

Solution

a) Excel produces the following

x	***y***	***xy***	x^2	y^2
102	175	17850	10404	30625
101	169	17069	10201	28561
94	182	17108	8836	33124
79	146	11534	6241	21316
79	144	11376	6241	20736
455	816	74937	41923	134362





b) $r = \dfrac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$

$= \dfrac{(5)(74937) - (455)(816)}{\sqrt{(5)(41923) - (255)^2} \cdot \sqrt{(5)(134362) - (816)^2}}$

$= 0.867$

c) $H_0 : \rho = 0$

$H_1 : \rho \neq 0$

$\alpha = 0.05 \quad and \quad df = 3$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 3.182$

or $r = \pm 0.878$

$t_r = \dfrac{r - \mu_r}{s_r} = \dfrac{0.867 - 0}{\sqrt{\frac{1-(0.867)^2}{3}}} = 3.015$

$P - value = 2 \cdot tcdf\,(3.015,\ 99,\ 3) = 0.0570$

Conclusion:

Do not reject H_0 ; there is no sufficient evidence to conclude that $\rho \neq 0$.

No; there is not sufficient evidence to support the claim of a linear correlation between the right and left arm systolic blood pressure measurements.

Exercise

Listed below are costs (in dollars) of air fares for different airlines from NY to San Francisco. The costs are based on tickets purchased 30 days in advance and one day in advance.

30 Days	244	260	264	264	278	318	280
One Day	456	614	567	943	628	1088	536

a) Construct a scatterplot

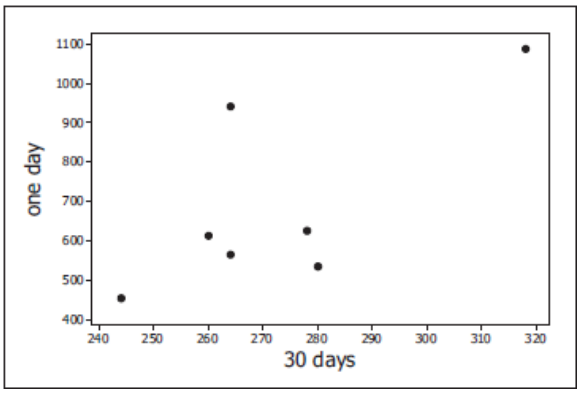
b) Find the value of linear correlation coefficient r and find the critical values if r, using $\alpha = 0.05$.

c) Determine whether there is sufficient evidence to support the claim of a linear correlation between costs of tickets purchased 30 days in advance and those purchased one day in advance?

Solution

a) Excel produces the following

x	y	xy	x^2	y^2
244	456	111264	59536	207936
260	614	159640	67600	376996
264	567	149688	69696	889249
264	943	248952	69696	889249
278	628	174584	77284	394384
318	1088	345984	101124	1183744
280	536	150080	78400	287296
1908	4832	1340192	523336	3661094



b) $$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{(7)(1340192) - (1908)(4832)}{\sqrt{(7)(523336) - (1908)^2} \cdot \sqrt{(7)(3661094) - (4832)^2}}$$

$$= 0.709$$

c) $H_0 : \rho = 0$

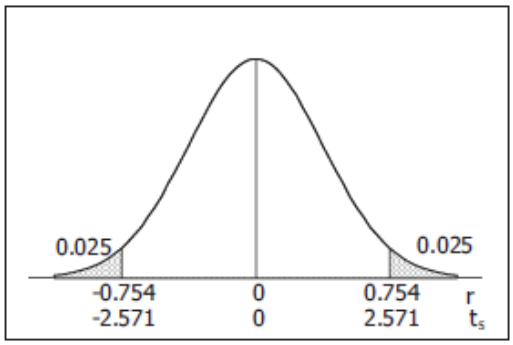
$H_1 : \rho \neq 0$

$\alpha = 0.05 \quad and \quad df = 5$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.571$

or $r = \pm 0.754$

$$t_r = \frac{r - \mu_r}{s_r} = \frac{0.709 - 0}{\sqrt{\frac{1-(0.709)^2}{5}}} = 2.247$$



$$P-value = 2 \cdot tcdf\left(2.247,\ 99,\ 5\right) = 0.0746$$

Conclusion:

Do not reject H_0; there is no sufficient evidence to conclude that $\rho \neq 0$.

No; there is not sufficient evidence to support the claim of a linear correlation between the costs of tickets purchased 30 days in advance and those purchased one day in advance.

Exercise

Listed below are repair costs (in dollars) for cars crashed at 6 mi/h in full-front crash tests and the same cars crashed at 6 mi/f in full-rear crash tests.

Front	936	978	2252	1032	3911	4312	3469
Rear	1480	1202	802	3191	1122	739	2767

a) Construct a scatterplot

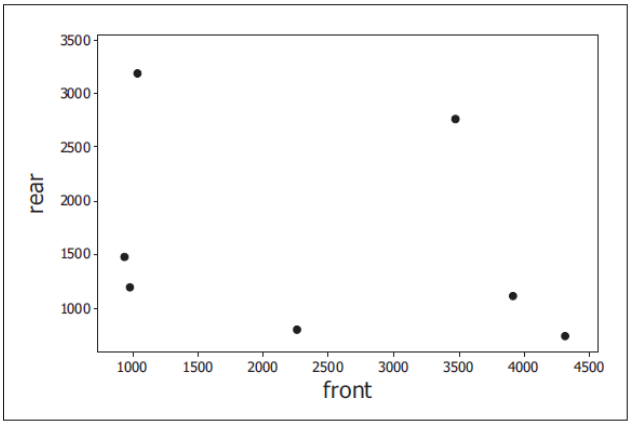
b) Find the value of linear correlation coefficient *r* and find the critical values if *r*, using $\alpha = 0.05$.

c) Determine whether there is sufficient evidence to support the claim of a linear correlation between costs from full-front crashes and full-rear crashes?

Solution

a) Excel produces the following

x	***y***	***xy***	x^2	y^2
936	1480	1385280	876096	2190400
978	1202	1175556	956484	1444804
2252	802	1806104	5071504	643204
1032	3191	3293112	1065024	10182481
3911	1122	4388142	15295921	1258884
4312	739	3186568	18593344	546121
3469	2767	9598723	12033961	7656289
16890	11303	24833485	53892334	23922183



b) $$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{(7)(24833485) - (16890)(11303)}{\sqrt{(7)(53892344) - (16890)^2} \cdot \sqrt{(7)(23922183) - (11303)^2}}$$

$= -0.283$

c) $H_0 : \rho = 0$

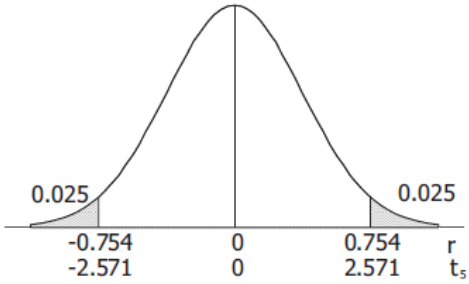
$H_1 : \rho \neq 0$

$\alpha = 0.05 \quad and \quad df = 5$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.571$

or $r = \pm 0.754$

$$t_r = \frac{r - \mu_r}{s_r} = \frac{-0.283 - 0}{\sqrt{\frac{1-(-.283)^2}{5}}} = -0.659$$



$P - value = 2 \cdot tcdf\left(-99,\ -0.659,\ 5\right) = 0.5392$

Conclusion:

Do not reject H_0; there is no sufficient evidence to conclude that $\rho \neq 0$.

No; there is not sufficient evidence to support the claim of a linear correlation between the costs from full-front crashes and full-rear crashes.

Solution *Section* 4.5 – Regression

Exercise

A physician measured the weights and cholesterol levels of a random sample of men. The regression equation is $\hat{y} = -116 + 2.44x$, where x represents weight (in pounds). What does the symbol $\hat{y}$ represent? What does the predictor variable represent? What does the response variable represent?

Solution

The symbol $\hat{y}$ represents the predicted cholesterol level. The predictor variable x represents weight. The response variable represents cholesterol level.

Exercise

In what sense is the regression line the straight line that "best" fits the points in a scatterplot?

Solution

The regression line is the best fit for the points of a scatterplot in the sense that it minimizes the sum of the squared differences between the observed y values and the y values predicted by the regression line.

Exercise

In a study, the total weight (in pounds) of garbage discarded in one week and the household size were recorded for 62 households. The linear correlation coefficient is $r = 0.759$ and the regression equation $\hat{y} = 0.445 + 0.119x$, where x represents the total weight of discarded garbage. The mean of the 62 garbage weights is 27.4 lb. and the 62 households have a mean size of 3.71 people. What is the best predicted number of people in a household that discards 50 lb. of garbage?

Solution

For n = 62, the critical value = ± 0.254.

Since r = 0.759 > 0.254, use the regression line for prediction.

$$\hat{y}\Big|_{x=50} = 0.445 + 0.119(50) \backslash$$

$$= 6.4 \text{ people}$$

Exercise

A sample of 8 mother/daughter pairs of subjects was obtained, and their heights (in inches) were measured. The linear correlation coefficient is 0.693 and the regression equation $\hat{y} = 69 - 0.0849x$, where x represents the height of the mother. The mean height of the mothers is 63.1 in. and the mean height of the daughters is 63.3 in. Find the best predicted height of a daughter given that the mother has a height of 60 in.

Solution

For n = 8, the critical value = ±0.707.

Since r = 0.693 < 0.707, use the regression line for prediction. $\hat{y} = \bar{y}$

$\hat{y}\big|_{x=60} = \bar{y} =$ $= 63.3\ in$

Exercise

A sample of 40 women is obtained, and their heights (in inches) and pulse rates (in beats per minute) are measured. The linear correlation coefficient is 0.202 and the equation of the regression line is $\hat{y} = 18.2 + 0.920x$, where x represents height. The mean of the 40 heights is 63.2 in. and the mean of the 40 pulse rates is 76.3 beats per minute. Find the best predicted pulse rate of a woman who is 70 in. tall.

Solution

For n = 40, the critical value = ±0.312.

Since r = 0.202 < 0.312, use the regression line for prediction. $\hat{y} = \bar{y}$

$\hat{y}\big|_{x=70} = \bar{y}$ $= 76.3\ \ beats / \min$

Exercise

Heights (in inches) and weights (in pounds) are obtained from a random sample of 9 supermodels. The linear correlation coefficient is 0.360 and the equation of the regression line is $\hat{y} = 31.8 + 1.23x$, where x represents height. The mean of the 9 heights is 69.3 in. and the mean of the 9 weights is 117 lb. Find the best predicted weight of a supermodel with a height of 72 in.?

Solution

For n = 9, the critical value = ±0.666.

Since r = 0.360 < 0.666, use the regression line for prediction. $\hat{y} = \bar{y}$

$\hat{y}\big|_{x=72}$ $= 117\ \ lbs$

Exercise

Find the equation of the regression line for the given data below

x	10	8	13	9	11	14	6	4	12	7	5
y	9.14	8.14	8.74	8.77	9.26	8.10	6.13	3.10	9.13	7.26	4.74

Examine the scatterplot and identify a characteristic of the data that is ignored by the regression line

Solution

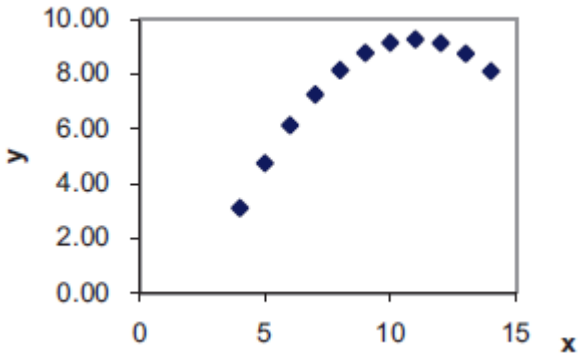
x	***y***	***xy***	x^2	y^2
10	9.14	91.40	100	83.5396
8	8.14	65.12	64	66.2596
13	8.74	113.62	169	76.3876
9	8.77	78.93	81	76.9129
11	9.26	101.86	121	85.7476
14	8.10	113.40	196	65.61
6	6.13	36.78	36	37.5769
4	3.10	12.40	16	9.61
12	9.13	109.56	144	83.3569
7	7.26	50.82	49	52.7076
5	4.74	23.70	25	22.4676
99	82.51	797.59	1001	660.1763

$$\bar{x} = \frac{\sum x}{n} = \frac{99}{11} = 9.0 \qquad \bar{y} = \frac{\sum y}{n} = \frac{82.52}{11} = 7.5$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{11(797.59) - (99)(82.51)}{11(1001) - (99)^2}$$

$$= 0.50$$



$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 7.50 - 0.5(9)$$

$$= 3$$

$$\hat{y} = b_0 + b_1 x$$

$$= 3.0 + 0.5x$$

The scatterplot indicates that the relationship between the variables is quadratic, not linear.

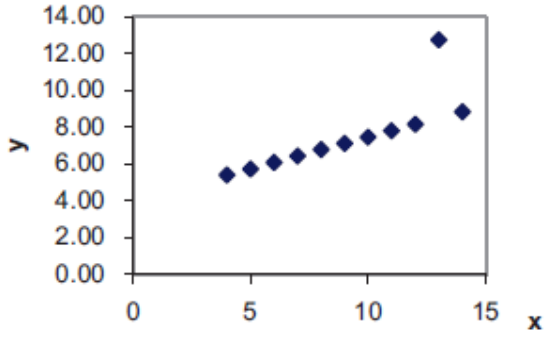
Exercise

Find the equation of the regression line for the given data below

x	10	8	13	9	11	14	6	4	12	7	5
y	7.46	6.77	12.74	7.11	7.81	8.84	6.08	5.39	8.15	6.42	5.73

Examine the scatterplot and identify a characteristic of the data that is ignored by the regression line

Solution



x	***y***	***xy***	x^2	y^2
10	7.46	74.60	100	55.6516
8	6.77	54.16	64	45.8329
13	112.74	165.62	169	162.3076
9	7.11	63.99	81	50.5521
11	7.81	85.91	121	60.9961
14	8.84	123.76	196	78.1456
6	6.08	36.48	36	36.9664
4	5.39	21.56	16	29.0521
12	8.15	97.80	144	66.4225
7	6.42	44.94	49	41.2164
5	5.73	28.65	25	32.8329
99	82.50	797.47	1001	659.9762

$$\bar{x} = \frac{\sum x}{n} = \frac{99}{11} = 9.0 \qquad \bar{y} = \frac{\sum y}{n} = \frac{82.52}{11} = 7.5$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{11(797.47) - (99)(82.50)}{11(1001) - (99)^2}$$

$$= 0.50$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 7.50 - 0.5(9)$$

$$= 3$$

$$\hat{y} = b_0 + b_1 x$$

$$= 3.0 + 0.5x$$

The scatterplot indicates that the relationship between the variables is essentially a perfect straight line except for one point, which is likely an error or an outlier.

Exercise

Find the equation of the regression line for the given data below

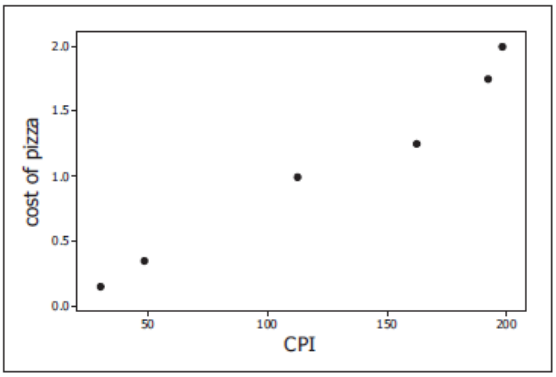
CPI	30.2	48.3	112.3	162.2	191.9	197.8
Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00

Let the first variable be the predictor (x) variable. Find the best indicated predicted cost of a slice of pizza when the Consumer Price Index (CPI) is 182.5 (in the year 2000).

Solution

Excel produces the following

x	y	xy	x^2	y^2
30.2	0.15	4.53	912.04	0.0225
48.3	0.35	16.905	2332.89	0.1225
112.3	1.00	112.3	12611.29	1.00
162.2	1.25	202.75	26308.84	1.5625
191.9	1.75	335.825	36825.61	3.0625
197.8	2.00	395.60	39124.84	4.00
742.7	6.50	1067.91	118115.5	9.77



$$\bar{x} = \frac{\sum x}{n} = \frac{742.7}{6} = 123.78 \qquad \bar{y} = \frac{\sum y}{n} = \frac{6.50}{6} = 1.08$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{6(1067.91) - (742.7)(6.5)}{6(118115.5) - (742.7)^2}$$

$$= \underline{0.01005}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 1.08 - 0.0101(123.78)$$

$$= \underline{-0.1616}$$

$$\hat{y} = b_0 + b_1 x$$

$$= \underline{-0.162 + 0.0101x}$$

$$\hat{y}_{182.5} = -0.162 + 0.0101(182.5)$$

$$= \underline{\$1.67}$$

Exercise

Find the equation of the regression line for the given data below

CPI	30.2	48.3	112.3	162.2	191.9	197.8
Subway fare	0.15	0.35	1.00	1.35	1.5	2.00

Let the first variable be the predictor (x) variable. Find the best indicated predicted cost of a slice of pizza when the Consumer Price Index (CPI) is 182.5 (in the year 2000).

Solution

x	y	xy	x^2	y^2
30.2	0.15	4.53	912.04	0.0225
48.3	0.35	16.905	2332.89	0.1225
112.3	1.00	112.3	12611.29	1.00
162.2	1.35	218.97	26308.84	1.8225
191.9	1.50	287.85	36825.61	2.25
197.8	2.00	395.60	39124.84	4.00
742.7	6.35	1036.155	118115.51	9.2175

$$\bar{x} = \frac{\sum x}{n} = \frac{742.7}{6} = 123.78 \qquad \bar{y} = \frac{\sum y}{n} = \frac{6.35}{6} = 1.06$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{6(1036.155) - (742.7)(6.35)}{6(118115.51) - (742.7)^2}$$

$$= 0.00955$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 1.06 - 0.00955(123.78)$$

$$= -0.124$$

$$\hat{y} = b_0 + b_1 x$$

$$= -0.124 + 0.00955x$$

$$\hat{y}_{182.5} = -0.124 + 0.00955(182.5)$$

$$= \$1.62$$

Exercise

Listed below are systolic blood pressure measurements (in mm HG) obtained from the same woman.

Right Arm	102	101	94	79	79
Left Arm	175	169	182	146	144

Find the best predicted systolic blood pressure in the left arm given that the systolic blood pressure in the right arm is 100 mm Hg.

Solution

x	y	xy	x^2	y^2
102	175	17850	10404	30625
101	169	17069	10201	28561
94	182	17108	8836	33124
79	146	11534	6241	21316
79	144	11376	6241	20736
455	816	74937	41923	134362

$$\bar{x} = \frac{\sum x}{n} = \frac{455}{5} = 91.0 \qquad \bar{y} = \frac{\sum y}{n} = \frac{816}{5} = 163.2$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{5(74937) - (455)(816)}{5(41923) - (455)^2}$$

$$= \underline{1.315}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 163.2 - 1.315(91)$$

$$= \underline{43.56}$$

$$\hat{y} = b_0 + b_1 x$$

$$= \underline{43.6 + 1.31x}$$

$\hat{y}_{182.5} = \bar{y} = \underline{163.2 \ \ mmHg}$ ***No significant correlation***

Exercise

Find the best predicted height of runner-up Goldwater, given that the height of the winning presidential candidate is 75 in. Is the predicted height of Goldwater close to his actual height of 72 in.?

Winner	69.5	73	73	74	74.5	74.5	71	71
Runner-Up	72	69.5	70	68	74	74	73	76

Solution

x	***y***	***xy***	x^2	y^2
69.5	72	5004	4830.25	5184
73	69.5	5073.5	5329	4830.25
73	70	5110	5329	4900
74	68	5032	5476	4624
74.5	74	5513	5550.25	5476
74.5	74	5513	5550.25	5476
71	76	5183	5041	5329
71	76	5396	5041	5776
580.5	576.5	41824.5	42146.75	41595.25

$$\bar{x} = \frac{\sum x}{n} = \frac{580.5}{8} = 72.56 \qquad \bar{y} = \frac{\sum y}{n} = \frac{576.5}{8} = 72.06$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{8(41824.5) - (580.5)(576.5)}{8(42146.75) - (580.5)^2}$$

$$= \underline{-0.321}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 72.06 - (-0.321)(72.56)$$

$$= \underline{95.38}$$

$$\hat{y} = b_0 + b_1 x$$

$$= \underline{95.4 - 0.321x}$$

$\hat{y}_{182.5} = \bar{y} = \underline{72.1 \text{ in.}}$ ***No significant correlation***

Exercise

Find the best predicted amount of revenue (in millions of dollars), given that the amount has a size 87 thousand ft^2. How does the result compare to the actual revenue of \$65.1 million?

Size	160	227	140	144	161	147	141
Revenue	189	157	140	127	123	106	101

Solution

x	***y***	***xy***	x^2	y^2
160	189	30240	25600	35721
227	157	35639	51529	24649
140	140	19600	19600	19600
144	127	18288	20736	16129
161	123	19803	25921	15129
147	106	15582	21609	11236
141	101	14241	19881	10201
1120	943	153393	184876	132665

$$\bar{x} = \frac{\sum x}{n} = \frac{1120}{7} = 160.0 \qquad \bar{y} = \frac{\sum y}{n} = \frac{943}{7} = 134.71$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{7(153393) - (1120)(943)}{7(184876) - (1120)^2}$$

$$= 0.443$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 134.71 - (0.443)(160)$$

$$= 63.87$$

$$\hat{y} = b_0 + b_1 x$$

$$= 63.9 + 0.443x$$

$$\hat{y}_{182.5} = \bar{y} = 134.7 \text{ million \$}$$ ***No significant correlation***

The predicted value is far from the actual value. Since there is no significant correlation, the mean is used for all predictions – but the x = 87 thousand ft^2 is well outside the range of x values used to construct the predictive regression equation.

Exercise

Find the best predicted new mileage rating of a jeep given that old rating is 19 mi/gal. Is the predicted value close to the actual value of 17 mi/gal?

Old	16	27	17	33	28	24	18	22	20	29	21
New	15	24	15	29	25	22	16	20	18	26	19

Solution

x	***y***	***xy***	x^2	y^2
16	15	240	256	225
27	24	648	729	576
17	16	272	289	256
33	29	957	1089	841
28	25	700	784	625
24	22	528	576	484
18	16	288	324	256
22	20	440	484	400
20	18	360	400	324
29	26	754	841	676
21	19	399	441	361
255	230	5586	6213	5024

$$\bar{x} = \frac{\sum x}{n} = \frac{255}{11} = 23.18 \qquad \bar{y} = \frac{\sum y}{n} = \frac{230}{11} = 20.82$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{11(5586) - (255)(230)}{11(6213) - (255)^2}$$

$$= 0.863$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 20.82 - (0.863)(23.18)$$

$$= 0.808$$

$$\hat{y} = b_0 + b_1 x$$

$$= 0.808 + 0.863x$$

$$\hat{y}_{182.5} = 0.808 + 0.863(19) = 17.2 \ mpg$$

Yes; the predicted value is close to the actual value of 17 mpg.

Exercise

Find the best predicted temperature for a recent year in which the concentration (in parts per million) of CO_2 is 370.9. Is the predicted temperature close to the actual temperature of 14.5° C??

CO_2	314	317	320	326	331	339	346	354	361	369
Temperature	13.9	14.0	13.9	14.1	14.0	14.3	14.1	14.5	14.5	14.4

Solution

x	***y***	***xy***	x^2	y^2
314	13.9	4364.6	985696	193.21
317	14	4438	100489	196
320	13.9	4448	102400	193.21
326	14.1	4596.6	106276	198.81
331	14	4634	109561	196
339	14.3	4847.7	114921	204.49
346	14.1	4878.6	119716	198.81
354	14.5	5133	125316	210.25
361	14.5	5234.5	130321	210.25
369	14.4	5313.6	136161	207.36
3377	141.7	47888.6	1143757	2008.39

$$\bar{x} = \frac{\sum x}{n} = \frac{3377}{10} = 337.7 \qquad \bar{y} = \frac{\sum y}{n} = \frac{141.7}{10} = 14.17$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{10(47888.6) - (3377)(141.7)}{10(1143757) - (3377)^2}$$

$$= \underline{0.0109|}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 14.17 - (0.0109)(337.7)$$

$$= \underline{10.48|}$$

$$\hat{y} = b_0 + b_1 x$$

$$= \underline{0.10.5 + 0.0109x|}$$

$$\hat{y}_{182.5} = 10.5 + 0.0109(370.9) = \underline{14.5\ \ ^\circ C|}$$

Yes; the predicted temperature is equal to the actual temperature of 14.5 °C..

Exercise

Find the best predicted IQ score of someone with a brain size of 1275 cm^3

Brain Size	965	1029	1030	1285	1049	1077	1037	1068	1176	1105
IQ	90	85	86	102	103	97	124	125	102	114

Solution

x	***y***	***xy***	x^2	y^2
965	90	86850	931225	8100
1029	85	87465	1058841	7225
1030	86	88580	1060900	7396
1285	102	131070	1651225	10404
1049	103	108047	1100401	10609
1077	97	104469	1159929	9409
1037	124	128588	1075369	15376
1068	125	133500	1140624	15625
1176	102	119952	1382976	10404
1105	114	125970	1221025	12996
10821	1028	1114491	11782515	107544

$$\bar{x} = \frac{\sum x}{n} = \frac{10821}{10} = 1082.1 \qquad \bar{y} = \frac{\sum y}{n} = \frac{1028}{10} = 102.8$$

$$b_1 = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{10(1114491) - (10821)(1028)}{10(1178251) - (10821)^2}$$

$$= \underline{0.0286}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= 102.80 - (0.0286)(1082.1)$$

$$= \underline{71.83}$$

$$\hat{y} = b_0 + b_1 x$$

$$= \underline{71.8 - 0.0286x}$$

$$\hat{y}_{182.5} = \bar{y} = \underline{102.8}$$

No significant correlation

Exercise

Listed below are the word counts for men and women.

Male

27531	15684	5638	27997	25433	8077	21319	17572	26429	21966	11680	10818
12650	21683	19153	1411	20242	10117	20206	16874	16135	20734	7771	6792
26194	10671	13462	12474	13560	18876	13825	9274	20547	17190	10578	14821
15477	10483	19377	11767	13793	5908	18821	14069	16072	16414	19017	37649
17427	46978	25835	10302	15686	10072	6885	20848				

Female

20737	24625	5198	18712	12002	15702	11661	19624	13397	18776	15863	12549
17014	23511	6017	18338	23020	18602	16518	13770	29940	8419	17791	5596
11467	18372	13657	21420	21261	12964	33789	8709	10508	11909	29730	20981
16937	19049	20224	15872	18717	12685	17646	16255	28838	38154	25510	34869
24480	31553	18667	7059	25168	16143	14730	28117				

Find the best predicted word count of a woman given that her male partner speaks 6,000 words in a day.

Solution

Using Excel spread sheet - ***Regression***

	Coefficients
Intercept	13438.884
X Variable 1	0.302

$$\hat{y} = 13439 + 0.302x$$

$$\hat{y}\Big|_{6000} = 13439 + 0.302(6000)$$

$$= 15,248 \ \ words\ per\ week$$

Exercise

According the least-squares property, the regression line minimizes the sum of the squares of the residuals. Listed below are the paired data consisting of the first 6 pulse and the first systolic blood pressures of males.

Pulse **(*x*)**	68	64	88	72	64	110
Systolic **(*y*)**	125	107	126	110	72	107

a) Find the equation of the regression line.
b) Identify the residuals, and find the sum of squares of the residuals.
c) Show that the equation $\hat{y} = 70 + 0.5x$ results in a larger sum of squares of residuals.

Solution

x = pulse rate
y = systolic blood pressures

a) Using Excel spread sheet ***– Data Analysis*** - ***Regression***

	Coefficients
Intercept	71.678
X Variable 1	0.5956

The equation of the regression line: $\hat{y} = 71.678 + 0.5956x$

b) $y - \hat{y}$ = residuals for the regression line

x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
68	125	112.208	12.792	163.635
64	107	109.824	-2.824	7.975
88	126	124.128	1.872	3.504
72	110	114.592	-4.592	21.086
64	110	109.824	0.176	0.031
72	107	114.592	-7.592	57.638
428	685	684.997	0.003	253.866

The table indicates that the sum of the squares of the residuals is 253.866

c) $y - v$ = residuals for the regression line where $v = 70 + 0.5x$

x	y	v	$y - v$	$(y - v)^2$
68	125	104.000	21.000	441.000
64	107	102.000	5.000	25.000
88	126	114.000	12.000	144.000
72	110	106.000	4.000	16.000
64	110	102.000	8.000	64.000
72	107	106.000	1.000	1.000
428	685	634.0	51.0	691.0

The table indicates that the sum of the squares of the residuals is 691, which is greater the the 253.866 of the least squares regression equation.

Solution Section 4.6 – Variation and Prediction Intervals

Exercise

A height of 70 in. is used to find the predicted weight is 180 lb. In your own words, describe a prediction interval in this situation.

Solution

A prediction interval is an interval estimate for a predicted value. In this situation it will be a range of weights centered at the prediction's point estimate of 180 lbs.

Exercise

A height of 70 in. is used to find the predicted weight is 180 lb. What is the major advantage of using a prediction interval instead of the predicted weight of 180 lb.? Why is the terminology of prediction interval used instead of confidence interval?

Solution

By providing a range of values instead of a single point, a prediction interval gives an indication of the accuracy of the prediction. A confidence interval is an internal estimate of a parameter – i.e., of a conceptually fixed, although unknown, value. A prediction interval is an interval estimate of a random variable – i.e., of a value from a distribution of values.

Exercise

Use the value of the linear correlation $r = 0.873$ to find the coefficient of determination and the percentage of the total variation that can be explained by the linear relationship between the 2 variables

$x = tar\ in\ menthol\ cigarettes$

16	13	16	9	14	13	12	14	14	13	13	16	13	13	18
9	19	2	13	14	14	15	16	6	8					

$y = nicotine\ in\ menthol\ cigarettes$

1.1	0.8	1	0.9	0.8	0.8	0.8	0.8	0.9	0.8	0.8	1.2	0.8	0.8	1.3
0.7	1.4	0.2	0.8	1	0.8	0.8	1.2	0.6	0.7					

Solution

The coefficient of determination is $r = (0.873)^2 = 0.762$

The portion of the total variation in y explained by the regression is $r^2 = 0.762$ = 76.2%

Regression Statistics	
Multiple R	0.873034386
R Square	0.762189039
Adjusted R Square	0.751849432
Standard Error	0.120760017
Observations	25

Exercise

Use the value of the linear correlation $r = 0.744$ to find the coefficient of determination and the percentage of the total variation that can be explained by the linear relationship between the 2 variables

$x =$ *movie budget*

41	20	116	70	75	52	120	65	6.5	60	125	20	5	150
4.5	7	100	30	225	70	80	40	70	50	74	200	113	68
72	160	68	29	132	40								

$y =$ *movie gross*

117	5	103	66	121	116	101	100	55	104	213	34	12	290
47	10	111	100	322	19	117	48	228	47	17	373	380	118
114	120	101	120	234	209								

Solution

The coefficient of determination is $r = (0.744)^2 = 0.554$

The portion of the total variation in *y* explained by the regression is $r^2 = 0.554$ = 55.4%

Exercise

Use the value of the linear correlation $r = -0.865$ to find the coefficient of determination and the percentage of the total variation that can be explained by the linear relationship between the 2 variables

$x =$ *car weight,* $y =$ *city fuel consumption in mi/gal*

Solution

The coefficient of determination is $r = (-0.865)^2 = 0.748$

The portion of the total variation in *y* explained by the regression is $r^2 = 0.748$ = 74.8%

Exercise

Use the value of the linear correlation $r = -0.488$ to find the coefficient of determination and the percentage of the total variation that can be explained by the linear relationship between the 2 variables

$x =$ *age of home,* $y =$ *home selling price*

Solution

The coefficient of determination is $r = (-0.488)^2 = 0.238$

The portion of the total variation in *y* explained by the regression is $r^2 = 0.238$ = 23.8%

Exercise

Refer to the display obtained by using the paired data consisting of weights (in *lb*.) of 32 cars and their highway fuel consumption amounts (in *mi/gal*). A car weight of 4000 *lb*. to be used for predicting the highway fuel consumption amount

```
The regression equation is
Highway = 50.5 - 0.00587 Weight

Predictor         Coef     SE Coef       T      P
Constant        50.502       2.860   17.66  0.000
Weight     -0.0058685   0.0007859   -7.47  0.000

S = 2.19498   R-Sq = 65.0%   R-Sq(adj) = 63.9%

Predicted Values for New Observations

New
Obs     Fit  SE Fit        95% CI              95% PI
  1  27.028   0.497  (26.013, 28.042)  (22.431, 31.624)

Values of Predictors for New Observations
New
Obs  Weight
  1    4000
```

a) What percentage of the total variation in highway fuel consumption can be explained by the linear correlation between weight and highway fuel consumption?

b) If a car weighs 4000 *lb*., what is the single value that is the best predicted amount of highway fuel consumption? (Assume that there is a linear correlation between weight and highway fuel consumption.)

Solution

a) $R - squared = 65.0\%$

b) The given point estimate is $\hat{y} = 27.028 \ \ mpg$

Exercise

The paired values of the Consumer Price Index (CPI) and the cost of a slice of pizza are shown below

CPI	30.2	48.3	112.3	162.2	191.9	197.8
Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00

a) Find the explained variation
b) Find the unexplained variation
c) Find the total variation
d) Find the coefficient of determination
e) Find the standard error of estimate s_e
f) Find the predicted cost of a slice of pizza for the year 2001, when the CPI was 187.1.
g) Find a 95% prediction interval estimate of the cost of a slice of pizza when the CPI was 187.1

In each case, there is sufficient evidence to support a claim of a linear correlation so that it is reasonable to use the regression equation when making predictions.

Solution

The predicted values:

	Coefficients
Intercept	-0.161601
X Variable	0.0100574

$$\hat{y} = -0.161601 + 0.0100574x$$

x	***y***	$\hat{y}$	$\bar{y}$	$\hat{y}-\bar{y}$	$(\hat{y}-\bar{y})^2$	$y-\hat{y}$	$(y-\hat{y})^2$	$y-\bar{y}$	$(y-\bar{y})^2$
30.2	0.15	0.142	1.083	–0.940	0.886	0.008	0.000	–0.930	0.871
48.3	0.35	0.324	1.083	–0.760	0.576	0.023	0.001	–0.730	0.538
112.3	1.00	0.968	1.083	–0.120	0.013	0.032	0.001	–0.080	0.007
162.2	1.25	1.470	1.083	0.386	0.149	–0.220	0.048	0.167	0.028
191.9	1.75	1.768	1.083	0.685	0.469	–0.018	0.000	0.667	0.444
197.8	2.00	1.828	1.083	0.744	0.554	0.172	0.030	0.917	0.840
742.7	6.50	6.50	6.50	0.0	2.648	0.0	0.08	0.0	2.728

a) The explained variation is $\sum(\hat{y}-\bar{y})^2 = 2.648$

b) The unexplained variation is $\sum(y-\hat{y})^2 = 0.080$

c) The total variation is $\sum(y-\bar{y})^2 = 2.728$

d) $r^2 = \dfrac{\sum(\hat{y}-\bar{y})^2}{\sum(y-\bar{y})^2} = \dfrac{2.648}{2.728} = 0.971$

e) $s_e^2 = \dfrac{\sum (y-\hat{y})^2}{n-2} = \dfrac{0.08}{4} = 0.02$

$s_e = \sqrt{0.02} = 0.141$

f) $\hat{y}\big|_{187.1} = -0.161601 + 0.0100574(187.1)$

$= 1.7201$

$= \$1.72$

x	y	x^2
30.2	0.15	912.04
48.3	0.35	2332.89
112.3	1.00	12611.29
162.2	1.25	26308.84
191.9	1.75	36825.61
197.8	2.00	39124.84
742.7	6.50	118115.5

g) Preliminary calculations for $n = 6$

$\bar{x} = \dfrac{\sum x}{n} = \dfrac{742.7}{6} = 123.783$

$\alpha = 0.05$ (2-tails)

$t_{\alpha/2} = 2.776; \quad df = 6 - 2 = 4$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
4	4.604	3.747	2.776	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\sum x^2) - (\sum x)^2}}$$

$$= (2.776)(0.141)\sqrt{1 + \frac{1}{6} + \frac{6(187.1 - 123.783)^2}{6(118115.5) - (742.7)^2}}$$

≈ 0.4450

$\hat{y} - E < y < \hat{y} + E$

$1.7201 - 0.445 < y_{187.1} < 1.7201 + 0.445$

$\$1.27 < y_{187.1} < \$2.17$

Exercise

The paired values of the Consumer Price Index (CPI) and the subway fare are shown below

CPI	30.2	48.3	112.3	162.2	191.9	197.8
Subway fare	0.15	0.35	1.00	1.35	1.5	2.00

a) Find the explained variation
b) Find the unexplained variation
c) Find the total variation
d) Find the coefficient of determination
e) Find the standard error of estimate s_e
f) Find the predicted cost of subway fare for the year 2001, when the CPI was 187.1.
g) Find a 95% prediction interval estimate of the cost of subway fare when the CPI was 187.1

In each case, there is sufficient evidence to support a claim of a linear correlation so that it is reasonable to use the regression equation when making predictions.

Solution

The predicted values (from Excel):

	Coefficients
Intercept	-0.124252712
X Variable 1	0.009553677

$\hat{y} = -0.124253 + 0.00955368x$

x	y	$\hat{y}$	$\bar{y}$	$\hat{y}-\bar{y}$	$(\hat{y}-\bar{y})^2$	$y-\hat{y}$	$(y-\hat{y})^2$	$y-\bar{y}$	$(y-\bar{y})^2$
30.2	0.15	0.164	1.058	–0.890	0.799	–0.014	0.0	–0.910	0.825
48.3	0.35	0.337	1.058	–0.720	0.520	0.013	0.0	–0.710	0.502
112.3	1.00	0.949	1.058	–0.110	0.012	0.051	0.006	0.292	0.085
162.2	1.35	1.425	1.058	0.367	0.135	–0.075	0.006	0.292	0.085
191.9	1.50	1.709	1.058	0.651	0.423	–0.209	0.044	0.442	0.195
197.8	2.00	1.765	1.58	0.707	0.500	0.235	0.055	0.942	0.887
742.7	6.35	6.350	6.350	0.0	2.930	0.0	0.104	0.0	2.497

a) The explained variation is $\sum(\hat{y}-\bar{y})^2 = 2.390$

b) The unexplained variation is $\sum(y-\hat{y})^2 = 0.107$

c) The total variation is $\sum(y-\bar{y})^2 = 2.497$

Regression Statistics	
Multiple R	0.978255696
R Square	0.956984207
Adjusted R Square	0.946230258
Standard Error	0.163870391
Observations	6

d) $r^2 = \dfrac{\sum(\hat{y}-\bar{y})^2}{\sum(y-\bar{y})^2} = \dfrac{2.648}{2.728} = 0.957$

e) $s_e^2 = \dfrac{\sum(y-\hat{y})^2}{n-2} = \dfrac{0.107}{4} = 0.02675$

$s_e = \sqrt{0.02675} = 0.164$

x	y	x^2
30.2	0.15	912.04
48.3	0.35	2332.89
112.3	1.00	12611.29
162.2	1.35	26308.84
191.9	1.50	36825.61
197.8	2.00	39124.84
742.7	6.35	118115.51

f) $\hat{y}\big|_{187.1} = -0.124253 + 0.00955368(187.1)$

$= \$1.66$

g) Preliminary calculations for $n = 6$

$\bar{x} = \dfrac{\sum x}{n} = \dfrac{742.7}{6} = 123.783$

$\alpha = 0.05$ (2-tails)

$t_{\alpha/2} = 2.776; \quad df = 6 - 2 = 4$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
4	4.604	3.747	2.776	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\sum x^2) - (\sum x)^2}}$$

$$= (2.776)(0.164)\sqrt{1 + \frac{1}{6} + \frac{6(187.1 - 123.783)^2}{6(118115.5) - (742.7)^2}}$$

≈ 0.5230

$\hat{y} - E < y < \hat{y} + E$

$1.6632 - 0.5230 < y_{187.1} < 1.6632 + 0.5230$

$\$1.14 < y_{187.1} < \$2.19$

Exercise

Find the best predicted temperature for a recent year in which the concentration (in parts per million) of CO_2 and temperature (in °C) for different years

CO_2	314	317	320	326	331	339	346	354	361	369
Temperature	13.9	14.0	13.9	14.1	14.0	14.3	14.1	14.5	14.5	14.4

a) Find the explained variation
b) Find the unexplained variation
c) Find the total variation
d) Find the coefficient of determination
e) Find the standard error of estimate s_e
f) Find the predicted temperature (in °C) when CO_2 concentration is 370.9 parts per million.
g) Find a 99% prediction interval estimate temperature (in °C) when CO_2 concentration is 370.9 parts per million

In each case, there is sufficient evidence to support a claim of a linear correlation so that it is reasonable to use the regression equation when making predictions.

Solution

The predicted values (from Excel):

	Coefficients
Intercept	10.48308065
X Variable 1	0.010917736

$\hat{y} = 10.4831 + 0.0109177x$

x	y	$\hat{y}$	$\bar{y}$	$\hat{y}-\bar{y}$	$(\hat{y}-\bar{y})^2$	$y-\hat{y}$	$(y-\hat{y})^2$	$y-\bar{y}$	$(y-\bar{y})^2$
314	13.9	13.911	14.17	–0.259	0.067	–0.011	0.0	–0.27	0.073
317	14	13.944	14.17	–0.266	0.051	0.056	0.003	–0.17	0.029
320	13.9	13.977	14.17	–0.193	0.037	–0.077	0.006	–0.27	0.073
326	14.1	14.042	14.17	–0.128	0.016	0.058	0.003	–0.07	0.005
331	14	14.097	14.17	–0.073	0.005	–0.097	0.009	–0.17	0.029
339	14.3	14.184	14.17	0.014	0.0	0.116	0.013	0.13	0.017
346	14.1	14.261	14.17	0.091	0.008	–0.161	0.026	–0.07	0.005
354	14.5	14.348	14.17	0.178	0.032	0.152	0.023	0.33	0.109
361	14.5	14.424	14.17	0.254	0.065	0.076	0.006	0.33	0.109
369	14.4	14.512	14.17	0.342	0.117	–0.112	0.012	0.23	0.053
3377	141.7	141.7	141.70	0.0	0.399	0.0	0.102	0.0	0.501

a) The explained variation is $\sum(\hat{y}-\bar{y})^2 = 0.399$

b) The unexplained variation is $\sum(y-\hat{y})^2 = 0.102$

c) The total variation is $\sum(y-\bar{y})^2 = 0.501$

Regression Statistics	
Multiple R	0.891976355
R Square	0.795621818
Adjusted R Square	0.770074545
Standard Error	0.113133477
Observations	10

d) $r^2 = \dfrac{\sum(\hat{y}-\bar{y})^2}{\sum(y-\bar{y})^2} = \dfrac{0.399}{0.501} = 0.796$

e) $s_e^2 = \dfrac{\sum(y-\hat{y})^2}{n-2} = \dfrac{0.102}{8} = 0.01275$

$s_e = \sqrt{0.01275} = 0.113$

f) $\hat{y}\big|_{370.9} = 10.4831 + 0.0109177(370.9)$

$= 14.53\ °C$

x	y	x^2
314	13.9	985696
317	14	100489
320	13.9	102400
326	14.1	106276
331	14	109561
339	14.3	114921
346	14.1	119716
354	14.5	125316
361	14.5	130321
369	14.4	136161
3377	141.7	1143757

g) Preliminary calculations for $n = 8$

$\bar{x} = \dfrac{\sum x}{n} = \dfrac{3377}{10} = 337.7$

$\alpha = 0.01 \quad and \quad df = n - 2 = 8$ (2-tails)

$t_{\alpha/2} = t_{0.005} = 3.355$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
8	3.355	2.896	2.306	1.860	1.397

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\sum x^2) - (\sum x)^2}}$$

$$= (3.355)(0.113)\sqrt{1 + \frac{1}{10} + \frac{10(370.9 - 337.7)^2}{10(1143757) - (3377)^2}}$$

≈ 0.4533

$\hat{y} - E < y < \hat{y} + E$

$14.5324 - 0.4533 < y_{370.9} < 14.5324 + 0.4533$

$14.08\ °C < y_{370.9} < 14.99\ °C$

Exercise

Find a prediction interval data listed below.

Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00
Subway Fare	0.15	0.35	1.00	1.35	1.50	2.00

Using: *Cost of a slice of pizza:* \$2.10; 99% *confidence*

Solution

The predicted values (from Excel):

	Coefficients
Intercept	0.03456017
X Variable 1	0.94502138

$$\hat{y} = 0.034560 + 0.945021x$$

$$\hat{y}\big|_{2.1} = 0.034560 + 0.945021(2.1)$$

$$= 2.019$$

$\alpha = 0.01$ *and* $df = n - 2 = 4$

$t_{\alpha/2} = t_{0.005} = 4.604$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
4	**4.604**	3.747	**2.776**	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (4.604)(0.122987)\sqrt{1 + \frac{1}{6} + \frac{6(2.1 - 1.083333)^2}{6(9.77) - (6.5)^2}}$$

$$\approx 0.704$$

x	x^2
0.15	0.0225
0.35	0.1225
1	1
1.25	1.5625
1.75	3.0625
2	4
6.5	9.77

$$\hat{y} - E < y < \hat{y} + E$$

$$2.019 - 0.704 < y_{2.1} < 2.019 + 0.704$$

$$\$1.32 < y_{2.1} < \$2.72$$

Exercise

Find a prediction interval data listed below.

Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00
Subway Fare	0.15	0.35	1.00	1.35	1.50	2.00

Using: *Cost of a slice of pizza:* \$2.10; 90% *confidence*

Solution

The predicted values (from Excel):

	Coefficients
Intercept	0.03456017
X Variable 1	0.94502138

$$\hat{y} = 0.034560 + 0.945021x$$

$$\hat{y}\big|_{2.1} = 0.034560 + 0.945021(2.1)$$

$$= 2.019$$

Regression Statistics	
Multiple R	0.98781094
R Square	0.97577045
Adjusted R Square	0.96971306
Standard Error	0.122987
Observations	6

$\alpha = 0.1 \quad and \quad df = n - 2 = 4$

$t_{\alpha/2} = t_{0.05} = 2.132$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
4	**4.604**	3.747	**2.776**	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (2.132)(0.122987)\sqrt{1 + \frac{1}{6} + \frac{6(2.10 - 1.083333)^2}{6(9.77) - (6.5)^2}}$$

$$\approx 0.326$$

x	x^2
0.15	0.0225
0.35	0.1225
1	1
1.25	1.5625
1.75	3.0625
2	4
6.5	9.77

$$\hat{y} - E < y < \hat{y} + E$$

$$2.019 - 0.326 < y_{2.1} < 2.019 + 0.326$$

$$\$1.69 < y_{2.1} < \$2.34$$

Exercise

Find a prediction interval data listed below.

Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00
Subway Fare	0.15	0.35	1.00	1.35	1.50	2.00

Using: *Cost of a slice of pizza:* \$0.50; 95% *confidence*

Solution

The predicted values (from Excel):

	Coefficients
Intercept	0.03456017
X Variable 1	0.94502138

$$\hat{y} = 0.034560 + 0.945021x$$

$$\hat{y}\Big|_{0.50} = 0.034560 + 0.945021(0.5)$$

$$= 0.507$$

Regression Statistics	
Multiple R	0.98781094
R Square	0.97577045
Adjusted R Square	0.96971306
Standard Error	0.122987
Observations	6

$\alpha = 0.05 \quad and \quad df = n - 2 = 4$

$t_{\alpha/2} = t_{0.025} = 2.776$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
4	4.604	3.747	2.776	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (2.776)(0.122987)\sqrt{1 + \frac{1}{6} + \frac{6(0.5 - 1.083333)^2}{6(9.77) - (6.5)^2}}$$

$$\approx 0.388$$

$$\hat{y} - E < y < \hat{y} + E$$

$$0.507 - 0.388 < y_{0.5} < 0.507 + 0.388$$

$$\$0.12 < y_{0.5} < \$0.89$$

Exercise

Find a prediction interval data listed below.

Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00
Subway Fare	0.15	0.35	1.00	1.35	1.50	2.00

Using: *Cost of a slice of pizza:* \$0.75; 99% *confidence*

Solution

The predicted values (from Excel):

	Coefficients
Intercept	0.03456017
X Variable 1	0.94502138

$$\hat{y} = 0.034560 + 0.945021x$$

$$\hat{y}\big|_{0.75} = 0.034560 + 0.945021(0.75)$$

$$= 0.743$$

$\alpha = 0.01$ *and* $df = n - 2 = 4$

$t_{\alpha/2} = t_{0.005} = 4.604$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
4	**4.604**	3.747	**2.776**	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (4.604)(0.122987)\sqrt{1 + \frac{1}{6} + \frac{6(0.75 - 1.083333)^2}{6(9.77) - (6.5)^2}}$$

$$\approx 0.622$$

$$\hat{y} - E < y < \hat{y} + E$$

$$0.743 - 0.622 < y_{0.75} < 0.743 + 0.622$$

$$\$0.12 < y_{0.75} < \$1.37$$

Solution Section 4.7 – Goodness-of-Fit

Exercise

A poll typically involves the selection of random digits to be used for telephone numbers. The New York Times states that "within each (telephone) exchange, random digits were added to form a complete telephone number, thus permitting access to listed and unlisted numbers. "When such digits are randomly generated, what is the distribution of those digits? Given such randomly generated digits, what is a test for "goodness-off-fit"?

Solution

When digits are randomly generated they should form a uniform distribution – i.e., a distribution in which each of the digits is equally likely. The test for goodness-to-fit is a test of the hypothesis that the sample data fit the uniform distribution.

Exercise

When generating random digits, we can test the generated digits for goodness-of-fit with the distribution in which all of the digits are equally likely. What does an exceptionally large value of the χ^2 test statistic suggest about the goodness-of-fit? What does an exceptionally small value of the χ^2 test statistic (such as 0.002) suggest about the goodness-of-fit?

Solution

The calculated χ^2 is a measure of the discrepancy between the hypothesis distribution and the sample data. An exceptionally large value of the χ^2 test statistic suggests a large discrepancy between the hypothesized distribution and the sample data – that there is not goodness-of-fit, and that the observed and expected frequencies are quite different. An exceptionally small of the χ^2 test statistic suggests an extremely good fir – that the observed and expected values are almost identical.

Exercise

You purchased a slot machine, and tested it by playing it 1197 times. There are 10 different categories of outcome, including no win, win jackpot, win with three bells, and so on. When testing the claim the observed outcomes agree with the expected frequencies, the author obtained a test statistic of $\chi^2 = 8.185$. Use a 0.05 significance level to test the claim that the actual outcomes agree with the expected frequencies. Does the slot machine appear to be functioning as expected?
Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Solution

Original claim: the actual outcomes agree with the expected frequencies
H_0 : The actual outcomes agree with the expected frequencies

H_1 : At least one outcome is not as expected

$\alpha = 0.05 \quad and \quad df = 9$

$C.V.\ \chi^2 = \chi^2_{\alpha} = \chi^2_{0.05} = 16.919$

Calculations:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 8.185$$

P-value = $\chi^2\ cdf\ (8.185,\ 99,\ 9) = 0.5156$

Conclusion

Do not reject H_0 ; there is not sufficient evidence to reject the claim that the actual outcomes agree with the expected frequencies. There is no reason to say the slot machine is not functioning as expected.

Exercise

Do "*A*" students tend to sit in a particular part of the classroom? The author recorded the locations of the students who received grades *A*, with these results: 17 sat in the front, 9 sat in the middle, and 5 sat in the back of the classroom. When testing the assumption that the "*A*" students are distributed evenly throughout the room, the author obtained the test statistic of $\chi^2 = 7.226$. If using a 0.05 significance level, is there sufficient evidence to support the claim that the "*A*" students are not evenly distributed throughout the classroom? If so, does that mean you can increase your likelihood of getting an *A* by sitting in the front of the room?
Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Solution

Original claim: "*A*" student are not evenly distributed throughout the classroom

H_0 : "*A*" students are evenly distributed throughout the classroom

H_1 : "*A*" students are not evenly distributed throughout the classroom

$\alpha = 0.05 \quad and \quad df = 2$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597

$C.V.\ \chi^2 = \chi^2_{\alpha} = \chi^2_{0.05} = 5.991$

Calculations:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 7.226$$

P-value = $\chi^2\ cdf\ (7.226,\ 99,\ 2) = 0.0270$

Conclusion

Reject H_0 ; there is sufficient evidence to support the claim "*A*" students are not evenly distributed throughout the classroom.

Exercise

Randomly selected nonfat occupational injuries and illnesses are categorized according to the day of the week that they first occurred, and the results are listed below. Use a 0.05 significance level to test the claim that such injuries and illness occur with equal frequency on the different days of the week Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Day	*Mon*	*Tues*	*Wed*	*Thurs*	*Fri*
Number	23	23	21	21	19

Solution

Original Claim: The injuries and illnesses occur with equal frequencies on the different days.

$H_0 : p_M = p_T = p_W = p_{Th} = p_F = \frac{1}{5} = 0.20$

$H_1 : at\ \text{least one}\ p_i \neq 0$

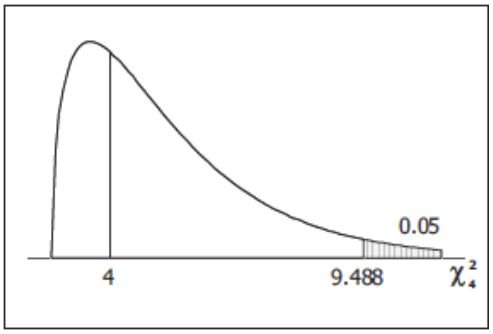
$\alpha = 0.05 \quad and \quad df = 4$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860

$C.V.\ \chi^2 = \chi^2_{\alpha} = \chi^2_{0.05} = 9.488 \quad E = \frac{1}{5}\sum O = \frac{107}{5} = 21.4$

Calculations:

Day	*O*	*E*	$\frac{(O-E)^2}{E}$
M	23	21.4	0.1196
T	23	21.4	0.1196
W	21	21.4	0.0075
Th	21	21.4	0.075
F	19	21.4	0.2693
	107	107	0.5234



$\chi^2 = \sum \frac{(O-E)^2}{E} = 0.5234$

P-value = $\chi^2\ cdf\ (0.523,\ 99,\ 4) = 0.9712$

Conclusion

Do not reject H_0 ; there is not sufficient evidence to reject the claim that $p_i \neq 0$ for each day.

There is no sufficient evidence to reject the claim that the injuries and illnesses occur with equal frequencies on the different days of the week.

Exercise

Records of randomly selected births were obtained and categorized according to the day of the week that they occurred. Because babies are unfamiliar with our schedule of weekdays, a reasonable claim is that occur on the different days with equal frequency. Use a 0.01 significance level to test that claim. Can you provide an explanation for the result?

Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Day	*Sun*	*Mon*	*Tues*	*Wed*	*Thurs*	*Fri*	*Sat*
Number of births	77	110	124	122	120	123	97

Solution

Original Claim: births occur on the different days with equal frequency.

$H_0 : p_S = p_M = p_T = p_W = p_{Th} = p_F = p_S = \frac{1}{7}$

$H_1 : at\ \text{least one}\ p_i \neq \frac{1}{7}$

$\alpha = 0.01 \quad and \quad df = 6$

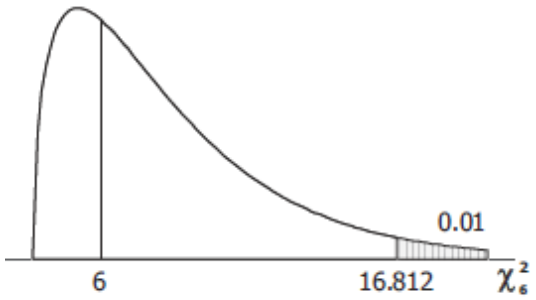
Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548

$C.V.\ \chi^2 = \chi^2_{\alpha} = \chi^2_{0.05} = 16.812$

$E = \frac{1}{7}\sum O = \frac{773}{7} = 110.43$

Calculations:

Day	*O*	*E*	$\frac{(O-E)^2}{E}$
S	77	110.43	10.119
M	110	110.43	0.0017
T	124	110.43	1.6679
W	122	110.43	1.2125
Th	120	110.43	0.8296
F	123	110.43	1.4312
S	97	110.43	1.6330
	773	773	16.8952



$\chi^2 = \sum \frac{(O-E)^2}{E} = 16.8952$

P-value = $\chi^2\ cdf\ (16.895,\ 99,\ 6) = 0.0097$

Conclusion

Reject H_0; there is sufficient evidence to support the claim that $p_i = \frac{1}{7}$ for each day. There is sufficient evidence to reject the claim that births occur on the different days with equal frequency. Births that do not occur naturally (induced, Caesarean sections) are typically not scheduled for Saturday and Sunday, accounting for the smaller than expected numbers of births on those days.

Exercise

The table below lists the frequency of wins for different post positions in the Kentucky Derby horse race. A post position of 1 is closest to the inside rail, so that horse has the shortest distance to run. (Because the number of horses varies from year to year, only the first ten post positions are included.) Use a 0.05 significance level to test the claim that the likelihood of winning is the same for the different post positions. Based on the result, should bettor consider the post position of a horse racing in the Kentucky Derby?

Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Post Position	1	2	3	4	5	6	7	8	9	10
Wins	19	14	11	14	14	7	8	11	5	11

Solution

Original Claim: The likelihood of winning is the same for all post positions.

$H_0 : p_1 = p_2 = \ldots = p_{10} = \frac{1}{10}$

$H_1 : at\ \text{least one}\ p_i \neq \frac{1}{10}$

$\alpha = 0.05 \quad and \quad df = 9$

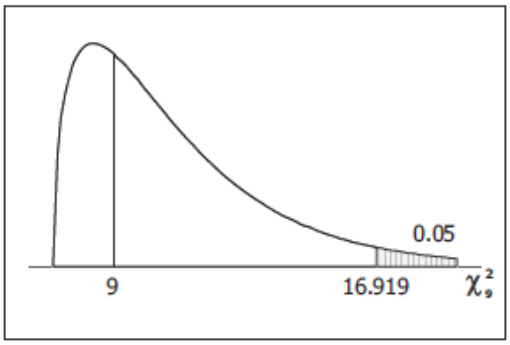
Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589

$C.V.\ \chi^2 = \chi^2_{\alpha} = \chi^2_{0.05} = 16.919$

$E = \frac{1}{10}\sum O = \frac{114}{10} = 11.4$

Calculations:

Position	*O*	*E*	$\frac{(O-E)^2}{E}$
1	19	11.4	5.0667
2	14	11.4	0.5930
3	11	11.4	0.0140
4	14	11.4	0.5930
5	14	11.4	0.5930
6	7	11.4	1.6982
7	8	11.4	1.0140
8	11	11.4	0.0140
9	5	11.4	3.5930
10	11	11.4	0.0140
	114	114.0	13.193



$\chi^2 = \sum \frac{(O-E)^2}{E} = 13.193$

P-value = $\chi^2\ cdf\ (13.193,\ 99,\ 9) = 0.1541$

Conclusion

Do not reject H_0; there is not sufficient evidence to reject the claim that $p_i = \frac{1}{10}$ for each position. There is no sufficient evidence to reject the claim that the likelihood of winning is the same for all post positions. Based on these results, post position is not a significant consideration when betting on the Kentucky Derby.

Exercise

The table below lists the cases of violent crimes are randomly selected and categorized by month.
Use a 0.01 significance level to test the claim that the rate of violent crime is the same for each month.
Can you explain the result?
Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Number	786	704	835	826	900	868	920	901	856	862	783	797

Solution

Original Claim: The occurrence of violent crime is the same for each month.

$H_0 : p_{Jan} = p_{Feb} = \ldots = p_{Dec} = \frac{1}{12}$

$H_1 : at \text{ least one } p_i \neq \frac{1}{12}$

$\alpha = 0.01 \quad and \quad df = 11$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757

$C.V.\ \chi^2 = \chi^2_{\alpha} = \chi^2_{0.01} = 24.725$

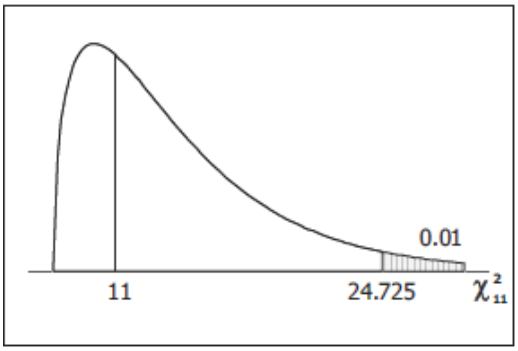
$E = \frac{1}{12}\sum O$

$= \frac{10038}{12}$

$= 836.5$

Calculations:

Month	*O*	*E*	$\frac{(O-E)^2}{E}$
Jan	786	836.5	3.0487
Feb	704	836.5	20.9877
Mar	835	836.5	0.0027
Apr	826	836.5	0.1318
May	900	836.5	4.8204
Jun	868	836.5	1.1862
Jul	920	836.5	8.3350
Aug	901	836.5	4.9734
Sep	856	836.5	0.4546
Oct	862	836.5	0.7773
Nov	783	836.5	3.4217
Dec	797	836.5	1.8652
	10038	10038.0	50.0048



$$\chi^2 = \sum \frac{(O-E)^2}{E} = 50.0048$$

P-value = $\chi^2\ cdf\ (50.005,\ 99,\ 11) = 0.0000006$

Conclusion

Reject H_0; there is sufficient evidence to support the claim that $p_i = \frac{1}{12}$ for each month. There is sufficient evidence to reject the claim that the occurrence of violent crime is the same for each month. A major factor involved in this conclusion is the large contribution of the month of February to the calculated χ^2 statistic. The comparison of frequencies for each month is not fair because not all months have the same number of days.

Exercise

The table below lists the results of the Advanced Placement Biology class conducted genetics experiments with fruit flies. Use a 0.05 significance level to test the claim that the observed frequencies agree with the proportions that were expected according to principles of genetics

Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Characteristic	***Red eye / normal wing***	***Sepia eye / normal wing***	***Red eye / vestigial wing***	***Sepia eye / vestigial wing***
Frequency	59	15	2	4
Expected proportion	$\frac{9}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

Solution

Original Claim: Observed frequencies fit the expected proportions.

$H_0: \; p_1 = \frac{9}{16}, \quad p_2 = \frac{3}{16}, \quad p_3 = \frac{3}{16}, \quad p_4 = \frac{1}{16}$

H_1 : *at least one* p_i is not as claimed

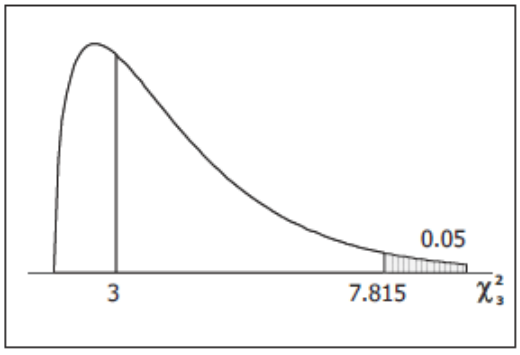
$\alpha = 0.05 \quad and \quad df = 3$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838

$C.V. \; \chi^2 = \chi^2_{\alpha} = \chi^2_{0.05} = 7.815$

Calculations:

Day	*O*	*E*	$\frac{(O-E)^2}{E}$
1	59	$80 \cdot \frac{9}{16} = 45$	4.3556
2	15	$80 \cdot \frac{3}{16} = 15$	0.00
3	2	$80 \cdot \frac{3}{16} = 15$	11.2667
4	4	$80 \cdot \frac{1}{16} = 5$	0.200
	80	80	15.8222



$\chi^2 = \sum \frac{(O-E)^2}{E} = 15.8222$

P-value = $\chi^2 \; cdf\,(15.822,\; 99,\; 3) = 0.0012$

Conclusion

Reject H_0 ; there is sufficient evidence to reject the claim that the proportions are as claimed.

There is sufficient evidence to reject the claim that observed frequencies fit the proportions that were expected according to the principles of genetics

Exercise

The table below lists the claims that its M&M plain candies are distributed with the following color percentages: 16% green, 20% orange, 14% yellow, 24% blue, 13% red, and 13% brown. Use a 0.05 significance level to test the claim that the color distribution is as claimed.

Solution

Original Claim: The color distribution is as stated

$H_0: \; p_G = .16, \quad p_O = .20, \quad p_Y = .14, \quad p_{Bl} = .24, \quad p_R = .13, \quad p_{BR} = .13$

H_1 : *at least one* p_i is not as claimed

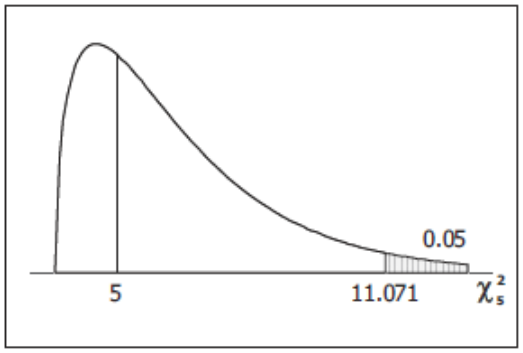
$\alpha = 0.05 \quad and \quad df = 5$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750

$C.V. \; \chi^2 = \chi^2_{\alpha} = \chi^2_{0.05} = 11.071$

Calculations:

Day	*O*	*E*	$\frac{(O-E)^2}{E}$
G	19	$100(.16) = 16$	0.5625
O	25	$100(.20) = 20$	1.2500
Y	8	$100(.14) = 14$	2.5714
Bl	27	$100(.24) = 24$	0.3750
R	13	$100(.13) = 13$	0.0
Br	8	$100(.13) = 13$	1.9231
	100	100	6.6820



$\chi^2 = \sum \frac{(O-E)^2}{E} = 6.682$

P-value = $\chi^2 \; cdf\,(6.682,\; 99,\; 5) = 0.2454$

Conclusion

Do not reject H_0 ; there is not sufficient evidence to reject the claim that the proportion are as stated. There is no sufficient evidence to reject the claim that the color distribution is as stated.