Solution   *Section* 6.1– **Introduction**

Exercise

Indicate the angle if it is an acute or obtuse. Then give the complement and the supplement of each angle.

    a) 10°   *b*) 52°   *c*) 90°   *d*) 120°   *e*) 150°

Solution

a) Acute;
Complement is $90° - 10° = 80°$;
Supplement is $180° - 10° = 170°$.

b) Acute;
Complement is $90° - 52° = 38°$;
Supplement is $180° - 52° = 128°$.

c) Neither (*right angle*);
Complement is $90° - 90° = 0°$;
Supplement is $180° - 90° = 90°$.

d) Obtuse;
Complement is $90° - 120° = -30°$;
Supplement is $180° - 120° = 60°$.

e) Obtuse;
Complement is $90° - 150° = -60°$;
Supplement is $180° - 150° = 30°$.

Exercise

Change to decimal degrees

a) 10° 45′	*c*) 274° 18′ 59″	*e*) 98° 22′ 45″	*g*) 1° 2′ 3″
b) 34° 51′ 35″	*d*) 74° 8′ 14″	*f*) 9° 9′ 9″	*h*) 73° 40′ 40″

Solution

a) $10°\ 45′ = 10° + 45′$
$$= 10° + 45'\frac{1°}{60'}$$
$$= 10° + 0.75°$$
$$\underline{= 10.75°}$$

b) $34°\ 51′\ 35″ = 34° + 51′ + 35″$
$$= 34° + 51'\cdot\frac{1°}{60'} + 35''\cdot\frac{1°}{3600''}$$
$$= 34° + 0.85° + 0.00972°$$

$$= 34.85972°$$

c) $274° \ 18' \ 59'' = 274° + 18' + 59''$

$$= 274° + 18' \cdot \frac{1°}{60'} + 59'' \cdot \frac{1°}{3600''}$$

$$= 274° + 0.3° + 0.016389°$$

$$= 274.316389°$$

d) $74° \ 8' \ 14'' = 74° + \frac{8°}{60} + \frac{14°}{3600}$

$$= 74° + 0.1333° + 0.0039°$$

$$= 74.137°$$

e) $98° \ 22' \ 45'' = 98° + 22' + 45''$

$$= 98° + 22' \cdot \frac{1°}{60'} + 45'' \cdot \frac{1°}{3600''}$$

$$= 98° + 0.36667° + 0.0125°$$

$$= 98.37917°$$

f) $9° \ 9' \ 9'' = 9° + 9' + 9''$

$$= 9° + 9' \cdot \frac{1°}{60'} + 9'' \cdot \frac{1°}{3600''}$$

$$= 9° + 0.15° + 0.0025°$$

$$= 9.1525°$$

g) $1° \ 2' \ 3'' = 1° + 2' + 3''$

$$= 1° + 2' \cdot \frac{1°}{60'} + 3'' \cdot \frac{1°}{3600''}$$

$$= 1° + 0.03333° + 0.000833°$$

$$= 1.034163°$$

h) $73° \ 40' \ 40'' = 73° + 40' + 40''$

$$= 73° + 40' \cdot \frac{1°}{60'} + 40'' \cdot \frac{1°}{3600''}$$

$$= 73° + 0.6667° + 0.0111°$$

$$= 73.67778°$$

Exercise

Convert to degrees, minutes, and seconds.

a) 89.9004°   c) 122.6853°   e) 44.01°   g) 29.411°

b) 34.817°   d) 178.5994°   f) 19.99°   h) 18.255°

Solution

a)
$$89.9004° = 89° + 0.9004°$$
$$= 89° + 0.9004° \cdot (60')$$
$$= 89° \quad 54.024'$$
$$= 89° \quad 54' + 0.024'$$
$$= 89° \quad 54' \quad 0.024' \cdot (60'')$$
$$= 89° \quad 54' \quad 1.44''$$

b)
$$34.817° = 34° + 0.817°$$
$$= 34° + 0.817(60')$$
$$= 34° + 49.02'$$
$$= 34° + 49' + .02(60'')$$
$$= 34° + 49' + 1.2''$$
$$= 34° \quad 49' \quad 1.2''$$

c)
$$122.6853° = 122° + .6853°$$
$$= 122° + \quad 0.6853 \cdot (60')$$
$$= 122° \quad 41.118'$$
$$= 122° \quad 41' + 0.118'$$
$$= 122° \quad 41' \quad 0.118 \cdot (60'')$$
$$= 122° \quad 41' \quad 7.1''$$

d)
$$178.5994° = 178° + .5994°$$
$$= 178° + \quad .5994 \cdot (60')$$
$$= 178° \quad 35.964'$$
$$= 178° \quad 35' + .964'$$
$$= 178° \quad 35' \quad 0.964 \cdot (60'')$$
$$= 178° \quad 35' \quad 57.84''$$

e)
$$44.01° = 44° + .01°$$
$$= 44° + \quad .01 \cdot (60')$$
$$= 44° \quad 0.6'$$

$$= 44° \quad 0.6 \cdot (60'')$$
$$\underline{= 44° \quad 36''}$$

f) $19.99° = 19° + .99°$
$$= 19° + \quad .99 \cdot (60')$$
$$= 19° \quad 59.4'$$
$$= 19° \quad 59' + 0.4'$$
$$= 19° \quad 59' \quad 0.4 \cdot (60'')$$
$$\underline{= 19° \quad 59' \quad 24''}$$

g) $29.411° = 29° + 0.411°$
$$= 29° + \quad 0.411 \cdot (60')$$
$$= 29° \quad 24.66'$$
$$= 29° \quad 24' + 0.66'$$
$$= 29° \quad 24' \quad 0.66 \cdot (60'')$$
$$\underline{= 29° \quad 24' \quad 39.6''}$$

h) $18.255° = 18° + 0.255°$
$$= 18° + \quad 0.255 \cdot (60')$$
$$= 18° \quad 15.3'$$
$$= 18° \quad 15' + 0.3'$$
$$= 18° \quad 15' \quad 0.3 \cdot (60'')$$
$$\underline{= 18° \quad 15' \quad 18''}$$

Exercise

Perform each calculation

a) $51° \ 29' + 32° \ 46'$ *b)* $90° - 73°12'$ *c)* $90° - 36° \ 18' \ 47''$ *d)* $75° \ 15' + 83° \ 32'$

Solution

a) $51°29' + 32°46'$

$$\begin{array}{r} 51° \quad 29' \\ +32° \quad 46' \\ \hline 83° \quad 75' \end{array}$$

$$83° \quad 75' = 1°15' \qquad \boxed{84° \quad 15'}$$

b) $90° - 73°12'$

$$\begin{array}{rr} 89° & 60' \\ -73° & 12' \\ \hline 16° & 48' \end{array}$$

c) $90° - 36° \ 18' \ 47''$

$$\begin{array}{rrr} 90° & & \\ -36° & 18' & 47'' \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{rrr} 89° & 59' & 60'' \\ -36° & 18' & 47'' \\ \hline 53° & 41' & 13'' \end{array}$$

d) $75° \ 15' + 83° \ 32'$

$$\begin{array}{rr} 75° & 15' \\ 83° & 32' \\ \hline 158° & 47' \end{array}$$

Exercise

Find the angle of least possible positive measure coterminal with an angle of

 a) $-75°$ b) $-800°$ c) $270°$

Solution

 a) $360° - 75° = 285°$

 b) $3(360°) - 800° = 280°$

 c) $360° + 270° = 630°$

Exercise

A vertical rise of the Forest Double chair lift 1,170 *feet* and the length of the chair lift as 5,570 *feet*. To the nearest foot, find the horizontal distance covered by a person riding this lift.
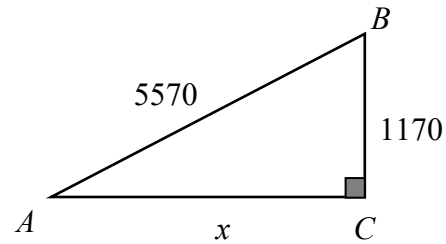
Solution

$$x^2 + 1170^2 = 5570^2$$

$$x^2 = 5570^2 - 1170^2$$

$$x = \sqrt{5570^2 - 1170^2}$$

$$\underline{x = 5,445.73 \ ft}$$

Right triangle with vertices A, B, C. Hypotenuse $AB = 5570$, vertical side $BC = 1170$, horizontal side $AC = x$, right angle at C.

Exercise

A tire is rotating 600 times per minute. Through how many degrees does a point of the edge of the tire move in $\frac{1}{2}$ *second*?

Solution

$$\frac{1}{2} \, 600 \frac{rev}{min} \cdot \frac{1min}{60\,sec} \cdot \frac{360°}{1rev} = \underline{1800 \ deg/sec}$$

Exercise

A windmill makes 90 *revolutions* per *minute*. How many revolutions does it make per *second*?

Solution

$$90 \frac{rev}{min} \cdot \frac{1min}{60\,sec} = \underline{1.5 \ rev/sec}$$

Exercise

Convert to radians

a) 256° 20′   *b)* −78.4°   *c)* 330°   *d)* −60°   *e)* −225°

Solution

a) $256° \ 20′ = 256° + \frac{20°}{60}$

$$= 256° + \frac{2°}{6}$$

$$= \frac{1538°}{6} = \left(\frac{769}{3}\right)^{°}$$

$$\frac{769°}{3} \cdot \frac{\pi}{180°} = \underline{\frac{769\pi}{540} \ rad} \approx \underline{4.47 \ rad}$$

b) $-78.4° = -78.4°\left(\dfrac{\pi}{180°}\right)rad$

$\approx -1.37\ rad$

c) $330° = 330°\left(\dfrac{\pi}{180°}\right)rad$

$= \dfrac{11\pi}{6}\ rad$

d) $-60° = -60°\left(\dfrac{\pi}{180°}\right)rad$

$= -\dfrac{\pi}{3}\ rad$

e) $-225° = -225°\left(\dfrac{\pi}{180°}\right)rad$

$= -\dfrac{5\pi}{4}\ rad$

Exercise

Convert to degrees

a) $\dfrac{11\pi}{6}$

b) $-\dfrac{5\pi}{3}$

c) $\dfrac{\pi}{6}$

d) 2.4

e) $\dfrac{\pi}{3}$

f) $-\dfrac{5\pi}{12}$

g) -4π

h) $\dfrac{7\pi}{13}$

Solution

a) $\dfrac{11\pi}{6}\ (rad) = \dfrac{11\pi}{6}\cdot\dfrac{180°}{\pi}$

$= 330°$

b) $-\dfrac{5\pi}{3}(rad) = -\dfrac{5\pi}{3}\cdot\dfrac{180°}{\pi}$

$= -300°$

c) $\dfrac{\pi}{6}(rad) = \dfrac{\pi}{6}\left(\dfrac{180}{\pi}\right)^{\circ}$

$= 30^{\circ}$

d) $2.4\ rad = 2.4\cdot\dfrac{180°}{\pi}$

$= \dfrac{432°}{\pi}$

$\approx 137.5°$

e) $\frac{\pi}{3}(rad) = \frac{\pi}{3}\left(\frac{180}{\pi}\right)^{\circ}$

$\underline{= 60}^{\circ}$

f) $-\frac{5\pi}{12}(rad) = -\frac{5\pi}{12}\left(\frac{180}{\pi}\right)^{\circ}$

$\underline{= -75}^{\circ}$

g) $-4\pi(rad) = -4\pi\left(\frac{180}{\pi}\right)^{\circ}$

$\underline{= -720}^{\circ}$

h) $\frac{7\pi}{13}(rad) = \frac{7\pi}{13}\left(\frac{180}{\pi}\right)^{\circ}$

$\underline{\approx 96.923}^{\circ}$