

# Lecture One

## Section 1.1 – Polynomials and Factoring

### Polynomials

#### Adding and Subtracting Polynomials

##### Properties of Real numbers

For all real numbers  $a$ ,  $b$ , and  $c$ :

$$a + b = b + a \quad \text{Commutative properties}$$

$$ab = ba$$

$$(a + b) + c = a + (b + c) \quad \text{Associative properties}$$

$$(ab)c = a(bc)$$

$$a(b + c) = ab + ac \quad \text{Distributive properties}$$

##### Add or subtract as indicated

$$\begin{aligned} a) \quad & (8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8) \\ & (8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8) = 8x^3 - 4x^2 + 6x + 3x^3 + 5x^2 - 9x + 8 \\ & = (8x^3 + 3x^3) + (-4x^2 + 5x^2) + (6x - 9x) + 8 \\ & = 11x^3 + x^2 - 3x + 8 \end{aligned}$$

$$\begin{aligned} b) \quad & (-4x^4 + 6x^3 - 9x^2 - 12) + (-3x^3 + 8x^2 - 11x + 7) \\ & (-4x^4 + 6x^3 - 9x^2 - 12) + (-3x^3 + 8x^2 - 11x + 7) = -4x^4 + 6x^3 - 3x^3 - 9x^2 + 8x^2 - 11x - 12 + 7 \\ & = -4x^4 + 3x^3 - x^2 - 11x - 5 \end{aligned}$$

$$\begin{aligned} c) \quad & (2x^2 - 11x + 8) - (7x^2 - 6x + 2) \\ & (2x^2 - 11x + 8) - (7x^2 - 6x + 2) = 2x^2 - 11x + 8 - 7x^2 + 6x - 2 \\ & = -5x^2 - 5x + 6 \end{aligned}$$

Multiply

a)  $8x(6x-4)$

$$\begin{aligned}8x(6x-4) &= 8x(6x) - 8x(4) \\ &= 48x^2 - 32x\end{aligned}$$

b)  $(3p-2)(p^2+5p-1)$

$$\begin{aligned}(3p-2)(p^2+5p-1) &= 3p^3 + 15p^2 - 3p - 2p^2 - 10p + 2 \\ &= 3p^3 + 13p^2 - 13p + 2\end{aligned}$$

c)  $(x+2)(x+3)(x-4)$

$$\begin{aligned}(x+2)(x+3)(x-4) &= (x^2 + 3x + 2x + 6)(x-4) \\ &= (x^2 + 5x + 6)(x-4) \\ &= x^3 + 5x^2 + 6x - 4x^2 - 20x - 24 \\ &= x^3 + x^2 - 14x - 24\end{aligned}$$

Find  $(2m-5)(m+4)$

$$\begin{aligned}(2m-5)(m+4) &= 2mm + 2m(4) - 5m - 5(4) \\ &= 2m^2 + 8m - 5m - 20 \\ &= 2m^2 + 3m - 20\end{aligned}$$

Find  $(2k-5)^2$

$$\begin{aligned}(2k-5)^2 &= (2k-5)(2k-5) \\ &= 4k^2 - 10k - 10k + 25 \\ &= 4k^2 - 20k + 25\end{aligned}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)(a+b) = a^2 - b^2$$

Perform the indicated operations:  $2(3x^2 + 4x + 2) - 3(-x^2 + 4x - 5)$

$$\begin{aligned} 2(3x^2 + 4x + 2) - 3(-x^2 + 4x - 5) &= 6x^2 + 8x + 4 + 3x^2 - 12x + 15 \\ &= 9x^2 - 4x + 19 \end{aligned}$$

Perform the indicated operations:  $(3t - 2y)(3t + 5y)$

$$\begin{aligned} (3t - 2y)(3t + 5y) &= 9t^2 + 15ty - 6yt - 10y^2 \\ &= 9t^2 + 9yt - 10y^2 \end{aligned}$$

Perform the indicated operations:  $(2a - 4b)^2$

$$\begin{aligned} (2a - 4b)^2 &= (2a)^2 - 2(2a)(4b) + (4b)^2 \\ &= 4a^2 - 16ab + 16b^2 \end{aligned}$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

## Factoring

### Prime Factorization

A process that allows us to write a composite number as a product of two or more prime numbers.

$$\begin{array}{c} \text{Tree} \\ 2 \swarrow 10 \searrow 5 \\ 10 = 2 \times 5 \end{array}$$

$$\begin{aligned} 72 &= 2 \cdot 36 \\ &= 2 \cdot 6 \cdot 6 \\ &= 2 \cdot 2 \cdot 3 \cdot 2 \cdot 3 \\ &= 2^3 \cdot 3^2 \end{aligned}$$

### The Greatest Common Factor (GCF)

The largest factor that two or more numbers (or terms) have in common

**Find GCF** (18, 36)

$$\begin{aligned} 18 &: 2 \cdot 9 \\ &2 \cdot 3 \cdot 3 \end{aligned}$$

$$\begin{aligned} 36 &: 2 \cdot 18 \\ &2 \cdot 2 \cdot 3 \cdot 3 \end{aligned}$$

$$18: 2 \cdot 3^2 \rightarrow 1, 2, 3, 6, 9, \underline{18}$$

$$36: 2^2 \cdot 3^2 \rightarrow 1, 2, 3, 4, 6, 9, 12, \underline{18}, 36$$

$$\text{GCF}(18, 36) = 18 \text{ (is the greatest common factor)}$$

**Find GCF** (27, 45)

$$27 = 3^3$$

$$45 = \frac{3^2 \cdot 5}{3^2}$$

$$\text{GCF}(27, 45) = 9$$

**Find GCF** (40, 56)

$$40 = 2^3 \cdot 5$$

$$56 = \frac{2^3 \cdot 7}{2^3}$$

$$\text{GCF}(40, 56) = 8$$

**Find GCF** (80, 60)

$$80 = 2^4 \cdot 5$$

$$60 = \frac{2^2 \cdot 3 \cdot 5}{2^2 \cdot 5}$$

$$\text{GCF}(80, 60) = 20$$

**Factor out the greatest common factor**

a)  $12p - 18q$

$$12p - 18q = 6(2p - 3q)$$

<b>12</b>	2 . 2 . 3
<b>18</b>	2 . . 3 . 3
	2 . 3

b)  $8x^3 - 9x^2 + 15x$

$$8x^3 - 9x^2 + 15x = x(8x^2 - 9x + 15)$$

## Factoring Trinomial

**Factor**  $y^2 + 8y + 15$

<i>Product</i> 15	<i>Sum</i> 8
15 x 1	15 + 1
3 x 5	3 + 5

$$y^2 + 8y + 15 = (y + 3)(y + 5)$$

**Factor**  $4x^2 + 8xy - 5y^2$

$$4x^2 + 8xy - 5y^2 = (2x - y)(2x + 5y)$$

## Special Factorization

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

**Factor**

a)  $64p^2 - 49q^2$

$$\begin{aligned} 64p^2 - 49q^2 &= (8p)^2 - (7q)^2 \\ &= (8p - 7q)(8p + 7q) \end{aligned}$$

b)  $x^2 + 36$

$x^2 + 36$  can't be factored (in real number) it is prime.

c)  $x^2 + 12x + 36$

$$x^2 + 12x + 36 = (x + 6)^2$$

d)  $9y^2 - 24yz + 16z^2$

$$\begin{aligned} 9y^2 - 24yz + 16z^2 &= (3y)^2 - 2(3y)(4z) + (4z)^2 \\ &= (3y - 4z)^2 \end{aligned}$$

e)  $y^3 - 8$

$$\begin{aligned} y^3 - 8 &= y^3 - 2^3 \\ &= (y - 2)(y^2 + 2y + 4) \end{aligned}$$

f)  $m^3 + 125$

$$m^3 + 125 = (m + 5)(m^2 - 5m + 25)$$

g)  $8k^3 - 27z^3$

$$\begin{aligned} 8k^3 - 27z^3 &= (2k)^3 - (3z)^3 \\ &= (2k - 3z)((2k)^2 + 6kz + (3z)^2) \\ &= (2k - 3z)(4k^2 + 6kz + 9z^2) \end{aligned}$$

h)  $p^4 - 1$

$$\begin{aligned} p^4 - 1 &= (p^2)^2 - (1)^2 \\ &= (p^2 - 1)(p^2 + 1) \\ &= (p - 1)(p + 1)(p^2 + 1) \end{aligned}$$

**Factor:**  $60m^4 - 120m^3n + 50m^2n^2$

$$60m^4 - 120m^3n + 50m^2n^2 = 10m^2(6m^2 - 12mn + 5n^2)$$

**Factor:**  $y^2 - 4yz - 21z^2$

$$y^2 - 4yz - 21z^2 = (y + 3z)(y - 7z)$$

**Factor:**  $4a^2 + 10a + 6$

$$\begin{aligned} 4a^2 + 10a + 6 &= 2(2a^2 + 5a + 3) \\ &= 2(2a + 3)(a + 1) \end{aligned}$$

**Factor:**  $16a^4 - 81b^4$

$$\begin{aligned} 16a^4 - 81b^4 &= (4a^2)^2 - (9b^2)^2 \\ &= (4a^2 - 9b^2)(4a^2 + 9b^2) \\ &= ((2a)^2 - (3b)^2)(4a^2 + 9b^2) \\ &= (2a - 3b)(2a + 3b)(4a^2 + 9b^2) \end{aligned}$$