## $\boldsymbol{A}$ is $\boldsymbol{n}$ by $\boldsymbol{n}$

## Nonsingular

**A** is invertible

The columns are independent

The rows are independent

The determinant is not zero

Ax = 0 has one solution x = 0

Ax = b has one solution  $x = A^{-1}b$ 

 $\mathbf{A}$  has n (nonzero) pivots

**A** has full rank r = n

The reduced row echelon form is R = I

The column space is all of

The row space is all of

All eigenvalues are nonzero

Is symmetric positive definite

*A* has *n* (positive) singular values

## Singular

A is not invertible

The columns are dependent

The rows are dependent

The determinant is zero

Ax = 0 has infinitely many solutions

Ax = b has no solution or infinitely many

**A** has r < n pivots

**A** has rank r < n

R has at least one zero row

The column space has dimension r < n

The row space has dimension r < n

Zero is an eigenvalue of A.

is only semidefinite

 $\boldsymbol{A}$  has r < n singular values