Lecture One

Section 1.1 – *Polynomials and Factoring*

Polynomials

Adding and Subtracting Polynomials

Properties of Real numbers

For all real numbers a, b, and c:

$a+b=b+a$ ***Commutative properties***

$ab=ba$

$(a+b)+c=a+(b+c)$ ***Associative properties***

$(ab)c=a(bc)$

$a(b+c)=ab+ac$ ***Distributive properties***

Add or subtract as indicated

a) $\left(8x^3-4x^2+6x\right)+\left(3x^3+5x^2-9x+8\right)$

$$\left(8x^3-4x^2+6x\right)+\left(3x^3+5x^2-9x+8\right)=8x^3-4x^2+6x+3x^3+5x^2-9x+8$$
$$=\left(8x^3+3x^3\right)+\left(-4x^2+5x^2\right)+\left(6x-9x\right)+8$$
$$=11x^3+x^2-3x+8$$

b) $\left(-4x^4+6x^3-9x^2-12\right)+\left(-3x^3+8x^2-11x+7\right)$

$$\left(-4x^4+6x^3-9x^2-12\right)+\left(-3x^3+8x^2-11x+7\right)=-4x^4+6x^3-3x^3-9x^2+8x^2-11x-12+7$$
$$=-4x^4+3x^3-x^2-11x-5$$

c) $\left(2x^2-11x+8\right)-\left(7x^2-6x+2\right)$

$$\left(2x^2-11x+8\right)-\left(7x^2-6x+2\right)=2x^2-11x+8-7x^2+6x-2$$
$$=-5x^2-5x+6$$

Multiply

a) $8x(6x-4)$

$$8x(6x-4)=8x(6x)-8x(4)$$
$$=48x^2-32x$$

b) $(3p-2)\left(p^2+5p-1\right)$

$$(3p-2)\left(p^2+5p-1\right)=3p^3+15p^2-3p-2p^2-10p+2$$
$$=3p^3+13p^2-13p+2$$

c) $(x+2)(x+3)(x-4)$

$$(x+2)(x+3)(x-4)=\left(x^2+3x+2x+6\right)(x-4)$$
$$=\left(x^2+5x+6\right)(x-4)$$
$$=x^3+5x^2+6x-4x^2-20x-24$$
$$=x^3+x^2-14x-24$$

Find $(2m-5)(m+4)$

$$(2m-5)(m+4)=2mm+2m(4)-5m-5(4)$$
$$=2m^2+8m-5m-20$$
$$=2m^2+3m-20$$

Find $(2k-5)^2$

$$(2k-5)^2=(2k-5)(2k-5)$$
$$=4k^2-10k-10k+25$$
$$=4k^2-20k+25$$

$$(a-b)^2=a^2-2ab+b^2$$
$$(a+b)^2=a^2+2ab+b^2$$
$$(a-b)(a+b)=a^2-b^2$$

Perform the indicated operations: $2\left(3x^2+4x+2\right)-3\left(-x^2+4x-5\right)$

$$2\left(3x^2+4x+2\right)-3\left(-x^2+4x-5\right)=6x^2+8x+4+3x^2-12x+15$$
$$=9x^2-4x+19$$

Perform the indicated operations: $\left(3t-2y\right)\left(3t+5y\right)$

$$\left(3t-2y\right)\left(3t+5y\right)=9t^2+15ty-6yt-10y^2$$
$$=9t^2+9yt-10y^2$$

Perform the indicated operations: $\left(2a-4b\right)^2$

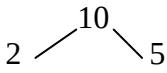
$$\left(2a-4b\right)^2=\left(2a\right)^2-2\left(2a\right)\left(4b\right)+\left(4b\right)^2 \qquad \left(a-b\right)^2=a^2-2ab+b^2$$
$$=4a^2-16ab+16b^2$$

Factoring

Prime Factorization

A process that allows us to write a composite number as a product of two or more prime numbers.

Tree



$10 = 2 \times 5$

$72 = 2.\ 36$
$= 2.\ 6.\ 6$
$= 2.\ 2.\ 3.\ 2.\ 3$
$= 2^3\ 3^2$

The Greatest Common Factor (GCF)

The largest factor that two or more numbers (or terms) have in common

Find GCF (18, 36)

18: 2. 9
2. 3. 3

36: 2. 18
2. 2. 3. 3

18: $2\ 3^2 \rightarrow$ 1, 2, 3, 6, 9, **18**
36: $2^2\ 3^2 \rightarrow$ 1, 2, 3, 4, 6, 9, 12, **18**, 36

GCF (18, 36) = 18 (is the greatest common factor)

Find GCF (27, 45)

$27 = 3^3$
$45 = \underline{3^2\ 5}$
3^2

GCF (27, 45) = 9

Find GCF (40, 56)

$40 = 2^3\ 5$
$56 = \underline{2^3\ 7}$
2^3

GCF (40, 56) = 8

Find GCF (80, 60)

$80 = 2^4\ \ 5$
$60 = \underline{2^2\ 3\ 5}$
$2^2\ \ 5$

GCF (80, 60) = 20

Factor out the greatest common factor

a) $12p-18q$

$12p-18q=6(2p-3q)$

12	2 . 2 . 3
18	2 . . 3 . 3
	2 . 3

b) $8x^3-9x^2+15x$

$8x^3-9x^2+15x=x\left(8x^2-9x+15\right)$

Factoring Trinomial

Factor $y^2+8y+15$

Product 15	***Sum*** 8
15 *x* 1	15 + 1
3 *x* 5	3 + 5

$y^2+8y+15=(y+3)(y+5)$

Factor $4x^2+8xy-5y^2$

$4x^2+8xy-5y^2=(2x-y)(2x+5y)$

Special Factorization

$a^2-b^2=(a-b)(a+b)$

$a^2+2ab+b^2=(a+b)^2$

$a^2-2ab+b^2=(a-b)^2$

$a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

$a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$

Factor

a) $64p^2 - 49q^2$

$$64p^2 - 49q^2 = (8p)^2 - (7q)^2$$
$$= (8p - 7q)(8p + 7q)$$

b) $x^2 + 36$

$x^2 + 36$ *can't be factored (in real number) it is prime.*

c) $x^2 + 12x + 36$

$$x^2 + 12x + 36 = (x + 6)^2$$

d) $9y^2 - 24yz + 16z^2$

$$9y^2 - 24yz + 16z^2 = (3y)^2 - 2(3y)(4z) + (4z)^2$$
$$= (3y - 4z)^2$$

e) $y^3 - 8$

$$y^3 - 8 = y^3 - 2^3$$
$$= (y - 2)\left(y^2 + 2y + 4\right)$$

f) $m^3 + 125$

$$m^3 + 125 = (m + 5)\left(m^2 - 5m + 25\right)$$

g) $8k^3 - 27z^3$

$$8k^3 - 27z^3 = (2k)^3 - (3z)^3$$
$$= (2k - 3z)\left((2k)^2 + 6kz + (3z)^2\right)$$
$$= (2k - 3z)\left(4k^2 + 6kz + 9z^2\right)$$

h) $p^4 - 1$

$$p^4 - 1 = \left(p^2\right)^2 - (1)^2$$
$$= \left(p^2 - 1\right)\left(p^2 + 1\right)$$
$$= (p - 1)(p + 1)\left(p^2 + 1\right)$$

Factor: $60m^4 - 120m^3n + 50m^2n^2$

$$60m^4 - 120m^3n + 50m^2n^2 = 10m^2\left(6m^2 - 12mn + 5n^2\right)$$

Factor: $y^2 - 4yz - 21z^2$

$$y^2 - 4yz - 21z^2 = (y + 3z)(y - 7z)$$

Factor: $4a^2 + 10a + 6$

$$\begin{aligned} 4a^2 + 10a + 6 &= 2\left(2a^2 + 5a + 3\right) \\ &= 2(2a + 3)(a + 1) \end{aligned}$$

Factor: $16a^4 - 81b^4$

$$\begin{aligned} 16a^4 - 81b^4 &= \left(4a^2\right)^2 - \left(9b^2\right)^2 \\ &= \left(4a^2 - 9b^2\right)\left(4a^2 + 9b^2\right) \\ &= \left((2a)^2 - (3b)^2\right)\left(4a^2 + 9b^2\right) \\ &= (2a - 3b)(2a + 3b)\left(4a^2 + 9b^2\right) \end{aligned}$$