Math 2413 – Calculus I *Exam* **1** ***Review***

Instructor: Fred Khoury

1. Find the slope of the parabola $y = x^2 + 3$ at the point $P(3,\ 12)$. Write an equation for the tangent to the parabola at this point.

2. Prove that $\lim\limits_{x \to 0} \dfrac{x}{\sin x} = 1$

Find

3. $\lim\limits_{x \to 0^+} \dfrac{\sqrt{x^2 + 4x + 5} - \sqrt{5}}{x}$

4. $\lim\limits_{x \to 0} \dfrac{\sin 5x}{3x}$

5. $\lim\limits_{\theta \to 0} \dfrac{\theta \cot 4\theta}{\sin^2 \theta \cot^2 2\theta}$

6. $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$

7. $\lim\limits_{x \to 1} \dfrac{1 - \sqrt{x}}{1 - x}$

8. $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

9. $\lim\limits_{x \to 2} \dfrac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x}$

10. $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x - 2}$

11. $\lim\limits_{x \to 0} \dfrac{\frac{1}{2 + x} - \frac{1}{2}}{x}$

12. $\lim\limits_{x \to -1} \dfrac{\sqrt{x^2 + 8} - 3}{x + 1}$

13. $\lim\limits_{x \to 0} \dfrac{\tan(2x)}{\tan(\pi x)}$

14. $\lim\limits_{x \to 0} \dfrac{\cos 2x - 1}{\sin x}$

15. $\lim\limits_{x \to \infty} \dfrac{x + \sin x + 2\sqrt{x}}{x + \sin x}$

16. $\lim\limits_{x \to -\infty} \left(\dfrac{x^2 + x - 1}{8x^2 - 3} \right)^{1/3}$

17. $\lim\limits_{x \to -\infty} \dfrac{4 - 3x^3}{\sqrt{x^6 + 9}}$

18. $\lim\limits_{x \to -\infty} \dfrac{x^2 - 4x + 8}{3x^3}$

19. $\lim\limits_{x \to -\infty} \dfrac{2x^2 + 3}{5x^2 + 7}$

20. $\lim\limits_{x \to \infty} \dfrac{x^4 + x^3}{12x^3 + 128}$

21. If $\lim\limits_{x \to 4} \dfrac{f(x) - 5}{x - 2} = 1$, find $\lim\limits_{x \to 4} f(x)$

22. Find the limit as $x \to \infty$ and as $x \to -\infty$ of $h(x) = \dfrac{-5 + \frac{7}{x}}{3 - \frac{1}{x^2}}$

23. Find an open interval about x_0 on which the inequality $\left|f(x)-L\right|<\varepsilon$ holds. Then give a value for δ > 0 such that for all x satisfying $0<\left|x-x_0\right|<\delta$ the inequality $\left|f(x)-L\right|<\varepsilon$ holds.

$$f(x)=\sqrt{x-7}, \quad L=4, \quad x_0=23, \quad \varepsilon=1$$

24. At what points is the function $y=\left|x-1\right|+\sin x$ continuous?

25. Show that the equation $x^3-15x+1=0$ has three solutions in the interval [–4, 4]

Find the vertical, horizontal, hole and oblique asymptotes (if any) of

26. $y=\dfrac{x-2}{x^2-4x+3}$

27. $f(x)=\dfrac{x^2-x-2}{x^2-2x+1}$

28. $f(x)=\dfrac{x^3+3x^2-2}{x^2-4}$

29. $f(x)=\dfrac{x^3-2x^2-4x+8}{x-2}$

30. $f(x)=\dfrac{x^2+4}{x-3}$

31. $y=\dfrac{\sqrt{x^2+4}}{x}$

Answers

1. $y = 6x - 6$

2. $\lim_{x \to 0} \frac{x}{\sin x} = \frac{0}{0}$

$$\lim_{x \to 0} \frac{x}{\sin x} = \lim_{x \to 0} \frac{\frac{x}{x}}{\frac{\sin x}{x}}$$
$$= \lim_{x \to 0} \frac{1}{\frac{\sin x}{x}}$$
$$= \frac{1}{\lim_{x \to 0} \frac{\sin x}{x}}$$
$$= \frac{1}{1}$$
$$= 1$$

3. $\lim_{x \to 0^+} \frac{\sqrt{x^2 + 4x + 5} - \sqrt{5}}{x} = \frac{2}{\sqrt{5}}$

4. $\lim_{x \to 0} \frac{\sin 5x}{3x} = \frac{5}{3}$

5. $\lim_{\theta \to 0} \frac{\theta \cot 4\theta}{\sin^2 \theta \cot^2 2\theta} = 1$

6. $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4} = \frac{1}{2a^2}$

7. $\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x} = \frac{1}{2}$

8. $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = 5$

9. $\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x} = 0$

10. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = 12$

11. $\lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = -\frac{1}{4}$

12. $\lim_{x \to -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1} = -\frac{1}{3}$

13. $\lim\limits_{x\to 0} \dfrac{\tan(2x)}{\tan(\pi x)} = \dfrac{2}{\pi}$

14. $\lim\limits_{x\to 0} \dfrac{\cos 2x - 1}{\sin x} = 0$

15. $\lim\limits_{x\to \infty} \dfrac{x + \sin x + 2\sqrt{x}}{x + \sin x} = 1$

16. $\lim\limits_{x\to -\infty} \left(\dfrac{x^2 + x - 1}{8x^2 - 3}\right)^{1/3} = \dfrac{1}{2}$

17. $\lim\limits_{x\to -\infty} \dfrac{4 - 3x^3}{\sqrt{x^6 + 9}} = -3$

18. $\lim\limits_{x\to -\infty} \dfrac{x^2 - 4x + 8}{3x^3} = 0$

19. $\lim\limits_{x\to -\infty} \dfrac{2x^2 + 3}{5x^2 + 7} = \dfrac{2}{5}$

20. $\lim\limits_{x\to \infty} \dfrac{x^4 + x^3}{12x^3 + 128} = \infty$

21. $\lim\limits_{x\to 4} f(x) = 7$

22. $\lim\limits_{x\to \pm\infty} h(x) = -\dfrac{5}{3}$

23. $\left|\sqrt{x-7} - 4\right| < 1 \quad \Rightarrow \quad -1 < \sqrt{x-7} - 4 < 1$

$$3 < \sqrt{x-7} < 5$$

$$(3)^2 < \left(\sqrt{x-7}\right)^2 < (5)^2$$

$$9 < x - 7 < 25$$

$$9 + 7 < x - 7 + 7 < 25 + 7$$

$$16 < x < 32$$

$$|x - 23| < \delta \quad \Rightarrow \quad -\delta < x - 23 < \delta$$

$$-\delta + 23 < x < \delta + 23$$

$$\left.\begin{array}{l} -\delta + 23 = 16 \ \Rightarrow \ \lfloor \delta = 23 - 16 = 7 \rfloor \\ \delta + 23 = 32 \ \Rightarrow \ \lfloor \delta = 32 - 23 = 9 \rfloor \end{array}\right\} \to \boxed{\delta = 7}$$

24. The function is continuous everywhere

25. By the Intermediate Value Theorem, $f(x) = 0$ for some x in each of the intervals $-4 < x < -3$,

$0 < x < 1$, and $3 < x < 4$

26. $VA: x = 1, 3 \quad HA: y = 0 \quad OA: n/a$

27. $VA: x = 1 \quad HA: y = 1 \quad OA: n/a$

28. $VA: x = \pm 2 \quad HA: n/a \quad OA: y = x + 3$

29. $VA: n/a \quad HA: n/a \quad OA: n/a$ ***hole*: (2, 0)**

30. $VA: x = 3 \quad HA: n/a \quad OA: y = x + 3$

31. $VA: x = 0 \quad HA: y = \pm 1 \quad OA: n/a$