Solution *Section* 2.1 – Simple and Compound Interest

Exercise

If you want to earn an annual rate of 10% on your investments, how much should you pay for a note that will be worth $5,000 in 6 month?

Solution

$$A = P(1+rt)$$

$$5000 = P\left(1+.1\left(\tfrac{6}{12}\right)\right)$$

$$5000 = P\left(1+\tfrac{.1}{2}\right)$$

$$P = \frac{5000}{\left(1+\frac{.1}{2}\right)} = \$4761.90$$

5000 / (1 + .1 / 2)

Exercise

a) How much should you deposit initially in an account paying 10% compounded semiannually in order to have $1,000,000 in 30 years?

b) Compounded monthly?

c) Compounded daily?

Solution

$$A = P\left(1+\frac{r}{m}\right)^{mt} \Rightarrow P = \frac{A}{\left(1+\frac{r}{m}\right)^{mt}} = or = A\left(1+\frac{r}{m}\right)^{-mt}$$

a) $P = \dfrac{1000000}{(1+0.10/2)^{60}} = \$53,535.52$

$$or \quad 1000000(1+0.10/2)^{-60} = \$53,535.52$$

b) $P = 1000000(1+0.10/12)^{-360} = \$50,409.83$

c) $P = 1000000(1+0.10/365)^{-10950} = \$49,807.53$

Exercise

You have \$7,000 toward the purchase of a \$10,000 automobile. How long will it take the \$7000 to grow to the \$10,000 if it is invested at 9% compounded quarterly? (Round up to the next highest quarter if not exact.)

Solution

$$10,000 = 7,000(1+0.09/4)^n$$

$$10/7 = 1.0225^n$$

$$\rightarrow \ln(10/7) = \ln 1.0225^n$$

Log Power Property: $\ln a^x = x\ln a$

$$\rightarrow \ln(10/7) = n\ln 1.0225$$

$$n = \frac{\ln(10/7)}{\ln 1.0225} = 16.03 \ \ or \ \ \underline{17 \ quarters}$$

Exercise

How long, to the nearest tenth of a year, will it take \$1000 to grow to \$3600 at 8% annual interest compounded quarterly?

Solution

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$3600 = 1000\left(1+\frac{0.08}{4}\right)^{4t}$$

$$3.6 = (1.02)^{4t}$$

$$\ln 3.6 = \ln(1.02)^{4t}$$

$$\ln 3.6 = 4t\ln(1.02)$$

$$\frac{\ln 3.6}{4\ln 1.02} = t$$

$$\Rightarrow \boxed{t \approx 16.2\,yr}$$

Exercise

Jennifer invested \$4,000 in her savings account for 4 years. When she withdrew it, she had \$4,350.52. Interest was compounded continuously. What was the interest rate on the account? Round to the nearest tenth of a percent.

Solution

$$A = Pe^{rt}$$

$$4350.52 = 4000e^{r4}$$

$$\frac{4350.52}{4000} = e^{4r}$$

$$\ln\left(\frac{4350.52}{4000}\right) = \ln e^{4r}$$

Inverse Property: $\ln e^{x} = x$

$$\ln\left(\frac{4350.52}{4000}\right) = 4r$$

$$\to r = \frac{1}{4}\ln\left(\frac{4350.52}{4000}\right) \approx .021$$

$$\Rightarrow \boxed{r = 2.1\%}$$

Exercise

An actuary for a pension fund need to have \$14.6 million grow to \$22 million in 6 years. What interest rate compounded annually does he need for this investment to growth as specified. Round your answer to the nearest hundredth of a percent.

Solution

$$22 = 14.6\left(1 + \frac{r}{1}\right)^{(1)(6)}$$

$$22 = 14.6(1 + r)^{6}$$

$$\frac{22}{14.6} = (1 + r)^{6}$$

$$\left(\frac{22}{14.6}\right)^{1/6} = 1 + r$$

$$r = \left(\frac{22}{14.6}\right)^{1/6} - 1$$

$$\approx .0707$$

$$\to \boxed{r \approx 7.07\%}$$

Exercise

Which is the better investment: 9% compounded monthly or 9.1% compounded quarterly?

Solution

For 9%: $APY = r_e = \left(1+\frac{0.09}{12}\right)^{12} - 1 = 9.38\%$

For 9.1%: $r_e = \left(1+\frac{0.091}{4}\right)^{4} - 1 = 9.42\%$

9.1% is better

Exercise

Sun Kang borrowed \$5200 from his friend to pay for remodeling work on his house. He repaid the loan 10 months later with simple interest at 7%. His friend then invested the proceeds in a 5-year CD paying 6.3% compounded quarterly. How much will his friend have at the end of the 5 years?

Solution

For 7%: $A_1 = P(1+rt)$

$$A_1 = 5200\left(1+0.07\frac{10}{12}\right) = \$5503.33$$

For 6.3%: $A_2 = 5503.33\left(1+\frac{0.063}{4}\right)^{20} = \$7,522.50$

Exercise

The consumption of electricity has increased historically at 6% per year. If it continues to increase at this rate indefinitely, find the number of years before the electric utilities will need to double their generating capacity. {Round up to the next highest year}

Solution

$$2P = P\left(1+\frac{0.06}{1}\right)^n$$

$\Rightarrow 2 = 1.06^n$ *ln both sides*

$$\ln 2 = \ln 1.06^n$$

$$\ln 2 = n\ln 1.06$$

$$n = \frac{\ln 2}{\ln 1.06} = 12 \text{ years}$$

Exercise

In the New Testament, Jesus commends a widow who contributed 2 mites (roughly ¼ cent) to the temple treasury. Suppose the temple invested those mites at 4% compounded quarterly. How much would the money be worth 2000 years later?

Solution

Given: $P = \frac{1}{4}¢ = .25¢\frac{\$1}{100¢} = \$0.0025$

$r = 0.04 \quad m = 4 \quad t = 2000$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$A = 0.0025\left(1+\frac{0.04}{4}\right)^{4(2000)}$$ 0.0025(1+0.04/4)^(4*2000)

$$\underline{= \$9.3x10^{31}|}$$

Exercise

If \$1,000 is invested in an account that earns 9.75% compounded annually for 6 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

Solution

Given: $P = 1,000 \quad r = .0975 \quad m = 1 \quad t = 6$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

1^{st} year: $t = 1 \Rightarrow \quad A_1 = 1000\left(1+\frac{.0975}{1}\right)^{1(1)} = \$1,097.50$

$Interest = \$1,097.50 - \$1,000 = \$97.50$

2^{nd} year: $t = 2 \Rightarrow \quad A_2 = 1000(1+.0975)^2 = \$1,204.51$

$Interest = \$1,204.51 - \$1,097.50 = \$107.01$

3^{rd} year: $t = 3 \Rightarrow \quad A_3 = 1000(1+.0975)^3 = \$1,321.95$

4^{th} year: $t = 4 \Rightarrow \quad A_4 = 1000(1+.0975)^4 = \$1,450.84$

5^{th} year: $t = 5 \Rightarrow \quad A_5 = 1000(1+.0975)^5 = \$1,592.29$

6^{th} year: $t = 6 \Rightarrow \quad A_6 = 1000(1+.0975)^6 = \$1,747.54$

Period	*Amount*	*Interest*
0	\$1,000.00	
1	\$1,097.50	\$97.50
2	\$1,204.51	\$107.01
3	\$1,321.95	\$117.44
4	\$1,450.84	\$128.89
5	\$1,582.29	\$141.46
6	\$1,747.54	\$155.25

Exercise

If $2,000 is invested in an account that earns 8.25% compounded annually for 5 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

Solution

Given: $P = 2,000 \quad r = 8.25\% = .08.25 \quad m = 1 \quad t = 5$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

1^{st} year: $t = 1 \Rightarrow \quad A_1 = 1000\left(1 + \frac{.0825}{1}\right)^{1(1)} = \$2,165.00$

$Interest = \$2,165.00 - \$2,000 = \$165.00$

2^{nd} year: $t = 2 \Rightarrow \quad A_2 = 2000(1 + .0825)^2 = \$2,343.61$

$Interest = \$2,343.61 - \$2,165.00 = \$178.61$

3^{rd} year: $t = 3 \Rightarrow \quad A_3 = 2000(1 + .0825)^3 = \$2,536.96$

4^{th} year: $t = 4 \Rightarrow \quad A_4 = 2000(1 + .0825)^4 = \$2,746.26$

5^{th} year: $t = 5 \Rightarrow \quad A_5 = 2000(1 + .0825)^5 = \$2,972.83$

Period	Amount	Interest
0	$2,000.00	
1	$2,165.00	$165.00
2	$2,343.61	$178.61
3	$2,536.96	$193.35
4	$2,746.26	$209.30
5	$2,972.83	$226.57

Exercise

If an investment company pays 6% compounded semiannually, how much you should deposit now to have $10,000

a) 5 years from now?

b) 10 years from now?

Solution

Given: $A = 10,000 \quad r = .06 \quad m = 2$

a) $t = 5 \quad A = P\left(1 + \frac{r}{m}\right)^{mt}$

$$10,000 = P\left(1 + \frac{.06}{2}\right)^{2(5)}$$

$$10,000 = P(1.03)^{10}$$

$$\underline{|P} = \frac{10,000}{(1.03)^{10}} \underline{= \$7,440.94|}$$

b) $t = 10$

$$10,000 = P(1.03)^{2(10)}$$

$$\underline{|P} = \frac{10,000}{(1.03)^{20}} \underline{= \$5,536.76|}$$

Exercise

If an investment company pays 8% compounded quarterly, how much you should deposit now to have $6,000

a) 3 years from now?

b) 6 years from now?

Solution

Given: $A = 6,000 \quad r = 8\% = .05 \quad m = 4$

a) $t = 3 \quad A = P\left(1 + \frac{r}{m}\right)^{mt}$

$$6,000 = P\left(1 + \frac{.08}{4}\right)^{4(3)}$$

$$6,000 = P(1.02)^{12}$$

$$\underline{|P} = \frac{6,000}{(1.02)^{12}} \underline{= \$4,730.96|}$$

6000 / 1.02 ^12

b) $t = 6$

$$6,000 = P\left(1 + \frac{.08}{4}\right)^{4(6)}$$

$$6,000 = P(1.02)^{24}$$

$$\underline{|P} = \frac{6,000}{(1.02)^{24}} \underline{= \$3,730.33|}$$

Exercise

What is the annual percentage yield (*APY*) for money invested at:

a) 4.5% compounded monthly?

b) 5.8% compounded quarterly?

Solution

a) ***Given***: $r = 0.045 \quad m = 12$

$$APY = \left(1 + \frac{.045}{12}\right)^{12} - 1 \approx 0.04594 \qquad (1 + .045/12)\wedge 12 - 1$$

$\boxed{APY: \ 4.594\%}$

b) ***Given***: $r = 0.058 \quad m = 4$

$$APY = \left(1 + \frac{.058}{4}\right)^{4} - 1 \approx 0.05927 \qquad (1 + .058/4)\wedge 4 - 1$$

$\boxed{APY: \ 5.927\%}$

Exercise

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What is the annual percentage yield (APY) for money invested at

a) 6.2% compounded semiannually?

b) 7.1% compounded monthly?

Solution

a) ***Given***: $r = 6.2\% = 0.062 \quad m = 2$

$$APY = \left(1 + \frac{.062}{2}\right)^{2} - 1 \approx 0.06296 \qquad (1 + .062/2)\wedge 2 - 1$$

$\boxed{APY: \ 6.296\%}$

b) ***Given***: $r = 7.1\% = 0.071 \quad m = 12$

$$APY = \left(1 + \frac{.071}{12}\right)^{12} - 1 \approx 0.07336 \qquad (1 + .071/12)\wedge 12 - 1$$

$\boxed{APY: \ 7.336\%}$

Exercise

A newborn child receives a $20,000 gift toward a college education from her grandparents. How much will the $20,000 be worth in 17 years if it is invested at 7% compounded quarterly?

Solution

Given: $P = 20,000 \quad r = 7\% = .07 \quad m = 4 \quad t = 17$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$= 20,000\left(1+\frac{.07}{4}\right)^{4(17)} \qquad 20000(1+.07/4)\wedge(4*17)$$

$$= \$65,068.44$$

Exercise

A person with $14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy more expensive car. How much will be available for the purchase of a car at the end of 3 years?

Solution

Given: $P = 14,000 \quad r = 6.5\% = .065 \quad m = 2 \quad t = 3$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$= 14,000\left(1+\frac{.065}{2}\right)^{2(3)} \qquad 14000(1+.065/2)\wedge(2*3)$$

$$= \$16,961.66$$

Exercise

You borrowed $7200 from a bank to buy a car. You repaid the bank after 9 months at an annual interest rate of 6.2%. Find the total amount you repaid. How much of this amount is interest?

Solution

Given: $P = 7,200 \quad r = 6.2\% = .062 \quad t = \frac{9}{12}$

$$A = P(1+rt)$$

$$= 7200\left[1+.062\left(\frac{9}{12}\right)\right] \qquad 7200(1+.07(9/12))$$

$$= \$7,534.80$$

Exercise

An account for a corporation forgot to pay the firm's income tax of \$321,812.85 on time. The government changed a penalty based on an annual interest rate of 13.4% for the 29 days the money was late. Find the total amount (tax and penalty) that was paid. (Use 365 days a year.)

Solution

Given: $P = 321,812.85 \quad r = 13.4\% = .13.4 \quad t = \frac{29}{365}$

$$A = P(1+rt)$$

$$= 321,812.85\left[1+.134\left(\frac{29}{365}\right)\right]$$

$7200(1+.07(9/12))$

$$\underline{= \$325,239.05}$$

Exercise

A bond with a face value of \$10,000 in 10 years can be purchased now for \$5,988.02. What is the simple interest rate?

Solution

The interest earned is: \$10,000 – \$5,988.02 = \$4011.98

Given: $P = 5988.02 \quad I = 4011.98 \quad t = 10$

$$I = \Pr t$$

$$4011.98 = 5988.02\, \boldsymbol{r}\ (10)$$

$$\boldsymbol{r} = \frac{4011.98}{5988.02(10)} \approx 0.067$$

$4011.98/(5988.02*10)$

The interest rate was about $\underline{6.7\%}$

Exercise

A stock that sold for \$22 at the beginning of the year was selling for \$24 at the end of the year. If the stock paid a dividend of \$0.50 per share, what is the simple interest rate on an investment in this stock?

Solution

The interest earned is: $(\$24 - \$22) + \$0.50 = \$2.50$

Given: $P = 22 \quad I = 2.50 \quad t = 1$

$$I = \Pr t$$

$$2.50 = 22\, \boldsymbol{r}\ (1)$$

$$\boldsymbol{r} = \frac{2.5}{22} \approx 0.11364$$

The interest rate was about $\underline{11.36\%}$

Exercise

The Frank Russell Company is an investment fund that tracks the average performance of various groups of stocks. On average, a \$10,000 investment in midcap growth funds over a recent 10-year period would have grown to \$63,000. What annual nominal rate would produce the same growth if

a) Annually

b) Continuously

Solution

a) Annually: $m = 1$

$$63{,}000 = 10{,}000\left(1+\frac{r}{1}\right)^{1(10)}$$

$$\frac{63000}{10000} = (1+r)^{10}$$

$$6.3 = (1+r)^{10}$$

$$\sqrt[10]{6.3} = 1+r$$

$$r = \sqrt[10]{6.3} - 1 \approx 0.20208 \quad or \quad \underline{20.208\%}$$

b) Continuously : $A = Pe^{rt}$

$$63{,}000 = 10{,}000e^{r(10)}$$

$$6.3 = e^{10r}$$

$$ln 6.3 = ln e^{10r} \qquad\qquad \ln e^{x} = x$$

$$ln 6.3 = 10r$$

$$r = \frac{ln 6.3}{10 ln e} \approx 0.18405 \quad or \quad \underline{18.405\%}$$