

8.1

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

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$$\sec \theta \tan \theta = \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta}$$
$$= \frac{\sin \theta}{\cos^2 \theta}$$

$$\frac{1}{\sin \theta} + \frac{1}{\cos \theta} = \frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta}$$

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$$\tan \alpha + \cot \alpha = \frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha}$$
$$= \frac{\sin^2 \alpha + \cos^2 \alpha}{\cos \alpha \sin \alpha}$$
$$= \frac{1}{\cos \alpha \sin \alpha}$$

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Prove,  $\tan x + \cot x = \sin x (\sec x + \csc x)$

$$\sin x (\sec x + \csc x) = \sin x \left( \frac{1}{\cos x} + \frac{\cos x}{\sin x} \right)$$
$$= \frac{\sin x}{\cos x} + \cos x$$
$$= \tan x + \cot x \quad \checkmark$$

Prove  $\cot \alpha + 1 = \csc \alpha (\cos \alpha + \sin \alpha)$

$$\begin{aligned}\csc \alpha (\cos \alpha + \sin \alpha) &= \frac{1}{\sin \alpha} (\cos \alpha + \sin \alpha) \\ &= \frac{\cos \alpha}{\sin \alpha} + 1 \\ &= \cot \alpha + 1 \quad \checkmark\end{aligned}$$

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Prove  $\frac{\cos^4 t - \sin^4 t}{\cos^2 t} = 1 - \tan^2 t$   $\cos^4 t = (\cos^2 t)^2$

$$\begin{aligned}\frac{\cos^4 t - \sin^4 t}{\cos^2 t} &= \frac{(\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t)}{\cos^2 t} \\ &= \frac{\cos^2 t - \sin^2 t}{\cos^2 t} \\ &= \frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t} \\ &= 1 - \tan^2 t \quad \checkmark\end{aligned}$$

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$1 + \cos \theta = \frac{\sin^2 \theta}{1 - \cos \theta} ?$   $\cos^2 \theta + \sin^2 \theta = 1$

$$\begin{aligned}\frac{\sin^2 \theta}{1 - \cos \theta} &= \frac{1 - \cos^2 \theta}{1 - \cos \theta} \\ &= \frac{(1 - \cos \theta)(1 + \cos \theta)}{1 - \cos \theta} \\ &= 1 + \cos \theta \quad \checkmark\end{aligned}$$

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$$\begin{aligned}1 + \cos \theta &= (1 + \cos \theta) \frac{1 - \cos \theta}{1 - \cos \theta} \\ &= \frac{1 - \cos^2 \theta}{1 - \cos \theta} \\ &= \frac{\sin^2 \theta}{1 - \cos \theta} \quad \checkmark\end{aligned}$$

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$$\begin{aligned}(1 + \cos \theta)(1 - \cos \theta) &\stackrel{P}{=} \sin^2 \theta \\ 1 - \cos^2 \theta &\stackrel{P}{=} \sin^2 \theta \\ \sin^2 \theta &= \sin^2 \theta \quad \checkmark\end{aligned}$$

$$\tan^2 \alpha (1 + \cot^2 \alpha) \stackrel{?}{=} \frac{1}{1 - \sin^2 \alpha}$$

$$\tan^2 \alpha (1 + \cot^2 \alpha) = \tan^2 \alpha + \overbrace{\tan^2 \alpha \cot^2 \alpha}^{=1}$$

$$= \tan^2 \alpha + 1$$

$$= \sec^2 \alpha$$

$$= \frac{1}{\cos^2 \alpha}$$

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$= \frac{1}{1 - \sin^2 \alpha} \quad \checkmark$$

$$\frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} \stackrel{?}{=} 2 \csc \alpha$$

$$\frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} = \frac{\sin^2 \alpha + (1 + \cos \alpha)^2}{\sin \alpha (1 + \cos \alpha)}$$

$$= \frac{\sin^2 \alpha + 1 + 2\cos \alpha + \cos^2 \alpha}{\sin \alpha (1 + \cos \alpha)}$$

$$= \frac{2 + 2\cos \alpha}{\sin \alpha (1 + \cos \alpha)}$$

$$= \frac{2(1 + \cos \alpha)}{\sin \alpha (1 + \cos \alpha)}$$

$$= \frac{2}{\sin \alpha}$$

$$= 2 \csc \alpha \quad \checkmark$$



$$\frac{1 + \sin t}{\cos t} = \frac{\cos t}{1 - \sin t}$$

$$\begin{aligned} \frac{1 + \sin t}{\cos t} &= \frac{1 + \sin t}{\cos t} \cdot \frac{1 - \sin t}{1 - \sin t} \\ &= \frac{1 - \sin^2 t}{\cos t (1 - \sin t)} \end{aligned}$$

$$\cos^2 t + \sin^2 t = 1$$

$$= \frac{\cos^2 t}{\cos t (1 - \sin t)}$$

$$= \frac{\cos t}{1 - \sin t} \quad \checkmark$$

$$\cot^2 \theta + \cos^2 \theta \neq \cot^2 \theta \cos^2 \theta$$

$$\theta = \frac{\pi}{4} \rightarrow \cot^2 \frac{\pi}{4} = 1, \quad \cos^2 \frac{\pi}{4} = \frac{1}{2}$$

$$\cot^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} \stackrel{?}{=} \cot^2 \theta \cos^2 \theta$$

$$1 + \frac{1}{2} \stackrel{?}{=} 1 \cdot \frac{1}{2}$$

$$\frac{3}{2} \neq \frac{1}{2} \quad \checkmark$$

$$\#21 \quad \frac{\cot^2 \theta + 3 \cot \theta - 4}{\cot \theta + 4} = \cot \theta - 1$$

$$\frac{\cot^2 \theta + 3 \cot \theta - 4}{\cot \theta + 4} = \frac{(\cot \theta + 4)(\cot \theta - 1)}{(\cot \theta + 4)}$$

$$= \cot \theta - 1 \quad \checkmark$$

$$\#24 \quad \sin \alpha (\tan \alpha \cos \alpha - \cot \alpha \cos \alpha) = 1 - 2 \cos^2 \alpha$$

$$\sin \alpha (\tan \alpha \cos \alpha - \cot \alpha \cos \alpha) = \sin \alpha \frac{\sin \alpha}{\cos \alpha} \cos \alpha - \sin \alpha \frac{\cos \alpha}{\sin \alpha} \cos \alpha$$

$$= \sin^2 \alpha - \cos^2 \alpha$$

$$= 1 - \cos^2 \alpha - \cos^2 \alpha$$

$$= 1 - 2 \cos^2 \alpha \quad \checkmark$$