$$\begin{cases} 3x + y + 2z = 31 \\ x + y + 2z = 19 \\ x + 3y + 2z = 25 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \end{array}\right] \begin{array}{l} x + \dots = \\ y + \quad = \\ \leftarrow z = \end{array}$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 1 & 2 & 31 \\ 1 & 1 & 2 & 19 \\ 1 & 3 & 2 & 25 \end{array}\right]$$

$1 \rightarrow$ { recip / divide by itself

$$\frac{a}{b}\frac{b}{a} = 1 \qquad \frac{a}{a} = 1$$

$0 \rightarrow R - R$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 3 & 1 & 2 & 31 \\ 1 & 3 & 2 & 25 \end{array}\right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - R_1 \end{array}$$

$$\begin{array}{cccc} 3 & 1 & 2 & 31 \\ -3 & -3 & -6 & -57 \\ \hline 0 & -2 & -4 & -26 \end{array}$$

$$\begin{array}{cccc} 1 & 3 & 2 & 25 \\ -1 & -1 & -2 & -19 \\ \hline 0 & 2 & 0 & 6 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & \boxed{-2} & -4 & -26 \\ 0 & 2 & 0 & 6 \end{array}\right] \frac{1}{-2}R_2 \qquad 0 \quad 1 \quad 2 \quad 13$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 2 & 0 & 6 \end{array}\right] R_3 - 2R_2$$

$$\begin{array}{cccc} 0 & 2 & 0 & 6 \\ 0 & -2 & -4 & -26 \\ \hline 0 & 0 & -4 & -20 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & -4 & -20 \end{array}\right] \begin{array}{l} \rightarrow x + y + 2z = 19 \quad (3) \\ \rightarrow y + 2z = 13 \quad (2) \\ \rightarrow -4z = -20 \quad (1) \end{array}$$

solve ↑

(1) $z = \frac{-20}{-4} = 5$

(2) $y = 13 - 2(5) = 3$

(3) $x = 19 - 3 - 10 = 6$

$\therefore (6, 3, 5)$

$$\begin{cases} 2x + y + 2z = 4 \\ 2x + 2y = 5 \\ 2x - y + 6z = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{array}\right] \frac{1}{2}R_1 \qquad 1 \quad \frac{1}{2} \quad 1 \quad 2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 2 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 2R_1 \end{array}$$

$$\begin{array}{cccc} 2 & 2 & 0 & 5 \\ -2 & -1 & -2 & -4 \\ \hline 0 & 1 & -2 & 1 \end{array}$$

$$\begin{array}{cccc} 2 & -1 & 6 & 2 \\ -2 & -1 & -2 & -4 \\ \hline 0 & -2 & 4 & -2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \end{array}\right] R_3 + 2R_2$$

$$\begin{array}{ccc|c} 0 & -2 & 4 & -2 \\ 0 & 2 & -4 & 2 \\ \hline 0 & 0 & 0 & 0 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \to x + \frac{1}{2}y + z = 2 \quad (2) \\ \to y - 2z = 1 \quad (1) \\ \to 0 = 0 \checkmark \end{array}$$

$(1) \to y = 1 + 2z$

$(2) \to x = 2 - \frac{1}{2}(1 + 2z) - z$

$= 2 - \frac{1}{2} - z - z$

$= \frac{3}{2} - 2z$

$\therefore \left(\frac{3}{2} - 2z,\ 1 + 2z,\ z\right)$

$$\begin{cases} x + 2y - 5z = -1 \\ 2x + 3y - 2z = 2 \\ 3x + 5y - 7z = 4 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & -1 \\ 2 & 3 & -2 & 2 \\ 3 & 5 & -7 & 4 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\begin{array}{cccc} 2 & 3 & -2 & 2 \\ -2 & -4 & 10 & 2 \\ \hline 0 & -1 & 8 & 4 \end{array}$$

$$\begin{array}{cccc} 3 & 5 & -7 & 4 \\ -3 & -6 & 15 & 3 \\ \hline 0 & -1 & 8 & 7 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & -1 \\ 0 & \cancel{-}1 & \cancel{-}8 & \cancel{-}4 \\ 0 & -1 & 8 & 7 \end{array}\right] \begin{array}{l} \\ -R_2 \\ R_3 + R_2 \end{array}$$

$$\begin{array}{cccc} 0 & -1 & 8 & 7 \\ 0 & 1 & -8 & -4 \\ \hline 0 & 0 & 0 & 3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & -1 \\ 0 & 1 & -8 & -4 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

$0 = 3$ impossible (False)

$\therefore$ No solution

#26 -

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ 2 & -3 & 5 & -1 & 0 \\ 1 & 0 & 3 & 1 & -4 \\ -4 & 3 & 2 & -1 & 3 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - R_1 \\ R_4 + 4R_1 \end{array}$$

$$\begin{array}{ccccc} 2 & -3 & 5 & -1 & 0 \\ -2 & 4 & -2 & -6 & 4 \\ \hline 0 & 1 & 3 & -7 & 4 \end{array} \qquad \begin{array}{ccccc} 1 & 0 & 3 & 1 & -4 \\ -1 & 2 & -1 & -3 & +2 \\ \hline 0 & 2 & 2 & -2 & -2 \end{array}$$

$$\begin{array}{ccccc} -4 & 3 & 2 & -1 & 3 \\ 4 & -8 & 4 & 12 & -8 \\ \hline 0 & -5 & 6 & 11 & -5 \end{array}$$

$$\begin{array}{ccccc} x & y & z & w & cr \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ 0 & 1 & 3 & -7 & 4 \\ 0 & 2 & 2 & -2 & -2 \\ 0 & 1 & 3 & -7 & 4 \end{array}\right]$$

$$A = \begin{bmatrix} 5 & -2 \\ -3 & \pi \\ 1 & 6 \end{bmatrix}$$

a) what is the order (or size) A?

3×2

rows # columns

b) $a_{12} = -2$ $\qquad a_{31} = 1$ $\qquad a_{33} =$ doesn't exist $\nexists$

$A = B$

same size $(n \times m)$

$$\underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ p & q \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} x & y \\ -1 & 0 \end{bmatrix}}$$

$x = 2$ $y = 1$
$p = -1$ $q = 0$

$$\underset{2\times 1}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}}$$

can't be true

$$\begin{pmatrix} w & x \\ 8 & -12 \end{pmatrix} = \begin{pmatrix} 9 & 17 \\ y & z \end{pmatrix}$$

$w = 9$ $x = 17$
$y = 8$ $z = -12$

[ ] or ( ) matrix

| | ← determinant
= value, no