Section 2.5 – Higher Order Derivatives

Higher Derivatives

$\frac{d}{dx}[f'(x)] = f''(x)$ *Second derivative*

$\frac{d}{dx}[f''(x)] = f'''(x)$ *Third derivative*

Notation for Higher-Order Derivatives						
1.	***1st derivative***	y'	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}[f(x)]$	$D_x[y]$
2.	***2nd derivative***	y''	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}[f(x)]$	$D_x^2[y]$
3.	***3rd derivative***	y'''	$f'''(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}[f(x)]$	$D_x^3[y]$
4.	***4th derivative***	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}[f(x)]$	$D_x^4[y]$
5.	***nth derivative***	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}[f(x)]$	$D_x^n[y]$

Example

Find the first four derivatives of $f(x) = 6x^3 - 2x^2 + 1$

Solution

$f'(x) = 18x^2 - 4x$

$f''(x) = 36x - 4$

$f'''(x) = 36$

$f^{(4)}(x) = 0$

Example

Find the value of $g'''(1)$ for $g(x) = x^4 - x^3 + 2x$

Solution

$$g'(x) = 4x^3 - 3x^2 + 2$$

$$g''(x) = 12x^2 - 6x$$

$$g'''(x) = 24x - 6$$

$$\Rightarrow g'''(1) = 24 - 6 = 18$$

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \qquad \Rightarrow \boxed{f^{(n)}(x) = n! a_n}$$

Example

Find the fourth derivative of $y = \frac{1}{x^2}$

Solution

$$\Rightarrow y = \frac{1}{x^2} = x^{-2}$$

$$y' = -2x^{-3} = -\frac{2}{x^3}$$

$$y'' = 6x^{-4} = \frac{6}{x^4}$$

$$y''' = -24x^{-5} = -\frac{24}{x^5}$$

$$y^{(4)} = 120x^{-6} = \frac{120}{x^6}$$

Acceleration

$s = f(t)$ ***P***osition function

$\frac{ds}{dt} = f'(t)$ ***V***elocity function

$\frac{d^2s}{dt^2} = f'''(t)$ ***A***cceleration function

Example

A ball is thrown upward from the top of an 80-foot cliff with an initial velocity of 64 feet per second. Give the position function. Then find the velocity and acceleration functions.

Solution

$$s(t) = -16t^2 + 64t + 80$$

$$Velocity: v(t) = s'(t) = -32t + 64$$

$$Acceleration: a(t) = s''(t) = -32$$

Exercises *Section* 2.5 – Higher Order Derivatives

1. Find the second derivative: $f(x) = 3\left(2 - x^2\right)^3$

2. Find the third derivative: $f(x) = 5x\left(x + 4\right)^3$

3. Find $f'''(-5)$ the given value: $f(x) = \sqrt{4 - x}$

4. Find the 4th derivative of $f(x) = x^4 + 2x^3 + 3x^2 - 5x + 7$

5. Find the second derivative of $f(x) = \left(x^2 - 1\right)^2$

6. Find $f''(x)$ for $f(x) = \sqrt{x^2 + 36}$, then find $f''(0)$ and $f''(9)$

7. Find $f''(x)$ for $f(x) = \sqrt{x^2 + 81}$, then find $f''(0)$ and $f''(2)$

8. The position function on Earth, where s is measured in meters, t is measured in seconds, v_0 is the initial velocity in meters per second, and h_0 is the initial height in meters, is

$$s = -4.9t^2 + v_0 t + h_0$$

If the initial velocity is 2.2 and the initial height is 3.6, what is the acceleration due to gravity on Earth in meters per second per second?