

$\Delta ABC$

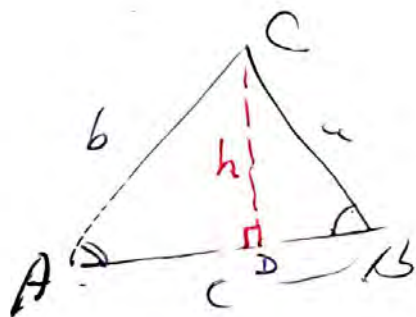
$$\sin A = \frac{h}{b} \rightarrow h = b \sin A$$

$\Delta ABC$

$$\sin B = \frac{h}{a} \rightarrow h = a \sin B$$

$$b \sin A = a \sin B$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \underline{\text{Law of Sines}}$$



Ex

$$A = 30^\circ \quad B = 70^\circ \quad a = 80 \quad c = ?$$

$$C = 180^\circ - 30^\circ - 70^\circ \\ = 80^\circ$$

$$\frac{c}{\sin 80^\circ} = \frac{80}{\sin 30^\circ}$$

$$c = \frac{2(80) \sin 80^\circ}{160 \sin 80^\circ}$$

$$\frac{\sin 70^\circ}{b} = \frac{\sin 30^\circ}{80}$$

$$b = \frac{80 \sin 70^\circ}{\frac{1}{2}} \\ = 160 \sin 70^\circ$$

~~Ex~~  $A = 32^\circ \quad C = 81.8^\circ \quad a = 42.9$

$$B = 180^\circ - 32^\circ - 81.8^\circ = 66.2^\circ$$

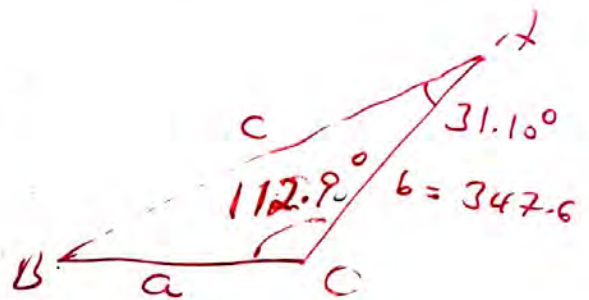
$$b = \frac{42.9 \sin 66.2^\circ}{\sin 32^\circ}$$

$$c = \frac{a \sin C}{\sin A} = \frac{42.9 \sin 81.8^\circ}{\sin 32^\circ}$$

$a??$

$$B = 180^\circ - 112.9^\circ - 31.1^\circ = 36^\circ$$

$$a = \frac{347.6 \sin 31.1^\circ}{\sin 36^\circ} \approx 305.5 \text{ ft}$$



$$a = \frac{b \sin A}{\sin B}$$

$$a = 562 \quad B = 5.7^\circ$$

$$A = 85.3^\circ$$

$$b = \frac{a \sin B}{\sin A} = \frac{562 \sin 5.7^\circ}{\sin 85.3^\circ}$$



$$\left. \begin{array}{l} 0 \leq \sin \theta \leq 1 \\ \theta \in Q I + Q II \end{array} \right\}$$

Ambiguous Case

(1)  $a = 2, b = 6, A = 30^\circ$

$$\begin{aligned} \sin B &= \frac{b \sin A}{a} \\ &= \frac{6}{2} \sin 30^\circ \\ &= \frac{3}{2} > 1 \quad \# \end{aligned}$$

$\therefore$  No Triangle

$C = 35.4^\circ \quad a = 205 \quad c = 314$

$$\begin{aligned} \sin A &= \frac{a \sin C}{c} \\ &= \frac{205}{314} \sin 35.4^\circ \end{aligned}$$

$$A = \sin^{-1} \left( \frac{205}{314} \sin 35.4^\circ \right)$$

$$\approx 22.2^\circ$$

$$A \approx 22.2^\circ$$

$$\begin{aligned} A &= 180^\circ - 22.2^\circ \\ &\approx 157.8^\circ \end{aligned}$$

$$\begin{aligned} B &= 180^\circ - 22.2^\circ - 35.4^\circ \\ &\approx 122.4^\circ \end{aligned}$$

$$\begin{aligned} B &= 180^\circ - 157.8^\circ - 35.4^\circ \\ &= - \quad \# \end{aligned}$$

$$\begin{aligned} b &= \frac{c \sin B}{\sin C} = \frac{314 \sin 122.4^\circ}{\sin 35.4^\circ} \\ &\approx 458 \quad \# \end{aligned}$$

Ex      $a = 54$       $b = 62$       $A = 40^\circ$

$$B = \sin^{-1}\left(\frac{62 \sin 40^\circ}{54}\right) \quad \frac{b \sin A}{a}$$

$$\approx 48^\circ$$

$$B \approx 48^\circ$$

$$B = 180^\circ - 40^\circ - 48^\circ$$

$$\approx 132^\circ$$

$$C = 180^\circ - 48^\circ - 40^\circ$$

$$= 92^\circ$$

$$C = 180^\circ - 132^\circ - 40^\circ$$

$$= 8^\circ$$

$$c = \frac{54 \sin 92^\circ}{\sin 40^\circ}$$

$$\approx 84$$

$$c = \frac{54 \sin 8^\circ}{\sin 40^\circ}$$

$$\approx 12$$

SAS

$$K = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} ac \sin B$$

Ex      $A = 24^\circ 40'$       $b = 27.3$       $C = 52^\circ 40'$

$$B = 180^\circ - 24^\circ 40' - 52^\circ 40'$$

$$\approx 102.667^\circ$$

$$77^\circ 20'$$

$$77 + \frac{20 \times 60}{60}$$

$$K_a = \frac{27.3 \sin 24^\circ 40'}{\sin 102.667^\circ} \approx 11.7$$

$$K = \frac{1}{2} (\overset{a}{11.7}) (\overset{b}{27.3}) \sin \overset{C}{52^\circ 40'}$$

$$\approx 127 \text{ cm}^2$$

11.241       $b=4$        $c=1$        $A=120^\circ$

$$K = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} (4)(1) \sin(120^\circ)$$

$$= 2 \frac{\sqrt{3}}{2}$$

$$= \sqrt{3} \text{ unit}^2$$

$b, c, A.$

Law of cosine

$$a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$b = \sqrt{a^2 + c^2 - 2ac \cos B}$$



Ex  $A = 60^\circ$   $b = 20$   $c = 30$  in

$$\begin{aligned} a &= \sqrt{b^2 + c^2 - 2bc \cos A} \\ &= \sqrt{20^2 + 30^2 - 2(20)(30) \cos 60^\circ} \\ &= 10 \sqrt{4 + 9 - 12\left(\frac{1}{2}\right)} \\ &= 10\sqrt{7} \text{ in} \end{aligned}$$

---

$a, b, c$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \rightarrow A = \cos^{-1} \frac{b^2 + c^2 - a^2}{2bc}$$

$$B = \cos^{-1} \frac{a^2 + c^2 - b^2}{2ac}$$

---

Ex  $a = 34$   $b = 20$   $c = 18$

$$A = \cos^{-1} \frac{20^2 + 18^2 - 34^2}{720}$$

S.S.S.

$s = \frac{1}{2}(a+b+c)$

Heron's Area

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

---