Ex $\dfrac{x^3+x^2}{(x^2+4)^2} = \dfrac{Ax+B}{x^2+4} + \dfrac{Cx+D}{(x^2+4)^2}$

$x^3+x^2 = (Ax+B)(x^2+4) + Cx+D$

x^3 $A = 1$

x^2 $B = 1$

x^1 $4A + C = 0 \Rightarrow C = -4$

x^0 $4B + D = 0 \Rightarrow D = -4$

$$\frac{x^3+x^2}{(x^2+4)^2} = \frac{x+1}{x^2+4} + \frac{-4x-4}{(x^2+4)^2}$$

#21 $\dfrac{y}{y^2-2y-3} = \dfrac{A}{y+1} + \dfrac{B}{y-3}$

$y = A(y-3) + B(y+1)$

y^1 $A + B = 1 \Rightarrow B = 1 - \frac{1}{4} = \frac{3}{4}$

y^0 $-3A + B = 0$

$4A = 1$

$A = \frac{1}{4}$

$$\frac{y}{y^2-2y-3} = \frac{1/4}{y+1} + \frac{3/4}{y-3}$$

$$= \frac{1}{4}\frac{1}{y+1} + \frac{3}{4}\frac{1}{y-3}$$

skip # avoid

distribution ⇒ table

$(y^n)^m$

y^{nm-1} : $+A+B=1$

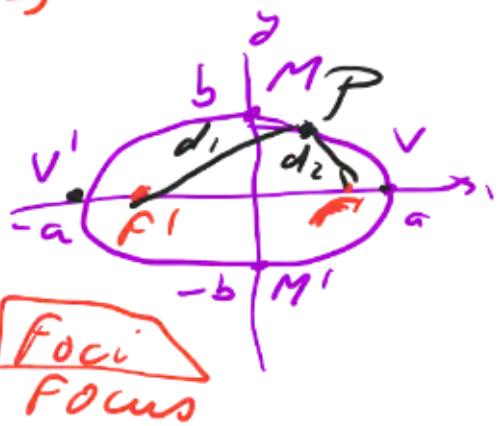
y^1 : $-3A+B=0$

y^0 : $4A+0=1$

$A=\frac{1}{4}$

$-3A+B=0$ $\quad +$

5.3 Ellipse (s)



$d_1+d_2=2a$

$F(\pm c, 0)$

$V(\pm a, 0)$

$M(0, \pm b)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$x^2 + y^2 = a^2$ (circle) $a = b$

$a > b$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Major

Ex $2x^2 + 9y^2 = 18$

$$\frac{x^2}{9} + \frac{y^2}{2} = 1$$

$a^2 = 9 \Rightarrow a = \pm 3$

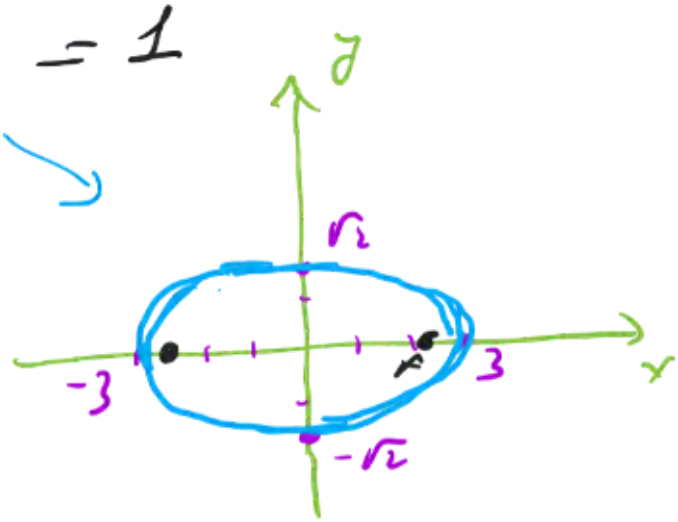
$b^2 = 2 \Rightarrow b = \pm\sqrt{2}$

$c^2 = a^2 - b^2$

$= 9 - 2$

$= 7$

$c = \pm\sqrt{7}$



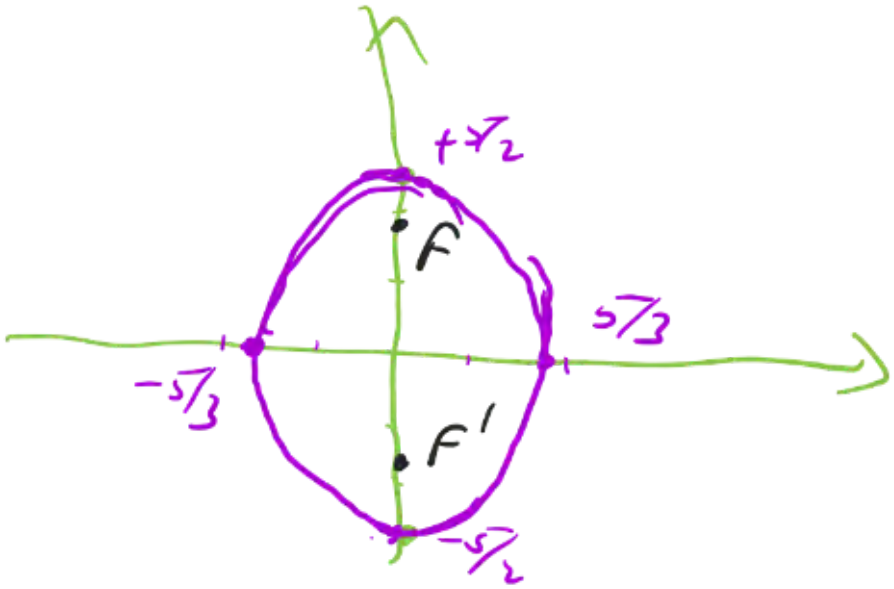
$F(\pm\sqrt{7}, 0)$ $V(\pm 3, 0)$ $M(0, \pm\sqrt{2})$

Ex $9x^2 + 4y^2 = 25$

$$\frac{9x^2}{25} + \frac{4y^2}{25} = \frac{25}{25}$$

$\left(\frac{1}{2}\right)^2 > \frac{1}{4}$

$$\frac{x^2}{\frac{25}{9}} + \frac{y^2}{\frac{25}{4}} = 1$$



$V = \left(0, \pm\frac{5}{2}\right)$ $M = \left(\pm\frac{5}{3}, 0\right)$

$c^2 = \frac{25}{4} - \frac{25}{9}$

$= 25\left(\frac{1}{4} - \frac{1}{9}\right)$

$= 25\left(\frac{5}{36}\right)$

$= \frac{125}{36}$

$c = \pm\frac{5\sqrt{5}}{6}$ $F\left(0, \pm\frac{5\sqrt{5}}{6}\right)$

$16x^2 + 9y^2 + 64x - 18y - 71 = 0$

$16x^2 + 64x + 9y^2 - 18y = 71$

$16(x^2 + 4x + 4) + 9(y^2 - 2y + 1) = 144 + 16 + 9$

$\frac{4}{2}$ $(\frac{4}{2})^2$ $(\frac{-2}{2})^2$

$16(x+2)^2 + 9(y-1)^2 = 144$

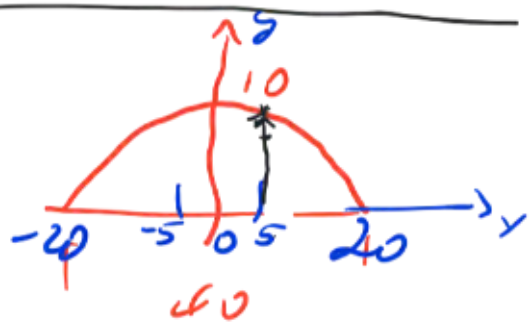
$$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{16} = 1$$

Center $(-2, 1)$ $a = \pm 4$, $b = \pm 3$

$V(-2, 1 \pm 4) \Rightarrow V'(-2, -3)$ $V(-2, 5)$

$M(-2 \pm 3, 1) \Rightarrow M(1, 1), (-5, 1)$

Ex Given: $a = 20$
$b = 10$



$W = 10$ ft, $h = 9$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

$$\frac{y^2}{10^2} = 1 - \frac{x^2}{400} \quad |_{x=5}$$

$$\frac{9^2}{10^2} \; ? \; 1 - \frac{25}{400}$$

$$\frac{81}{100} \; ? \; \frac{400 - 25}{400}$$

100 : 400

$\frac{81}{100}$? $\frac{375}{400}$ 15

81 ? $\frac{100}{400}$ 375

$81 < 93\ldots$

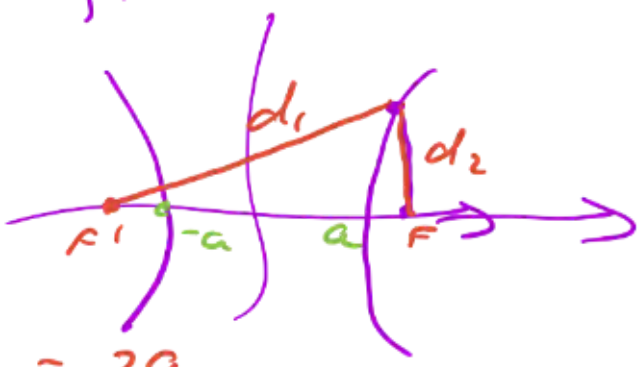
truck will clear

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipse

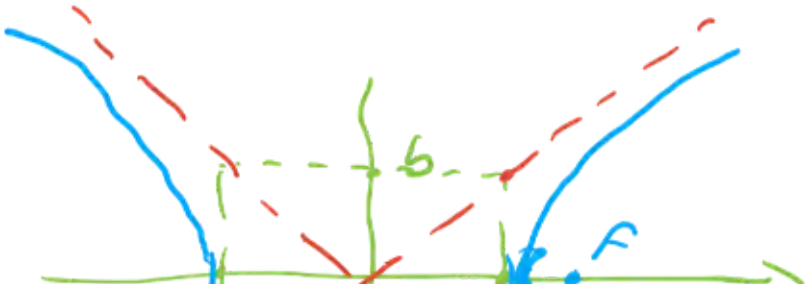
$x^2 + y^2 = a^2$ circle

5.4 Hyperbolas.

$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \end{cases}$$



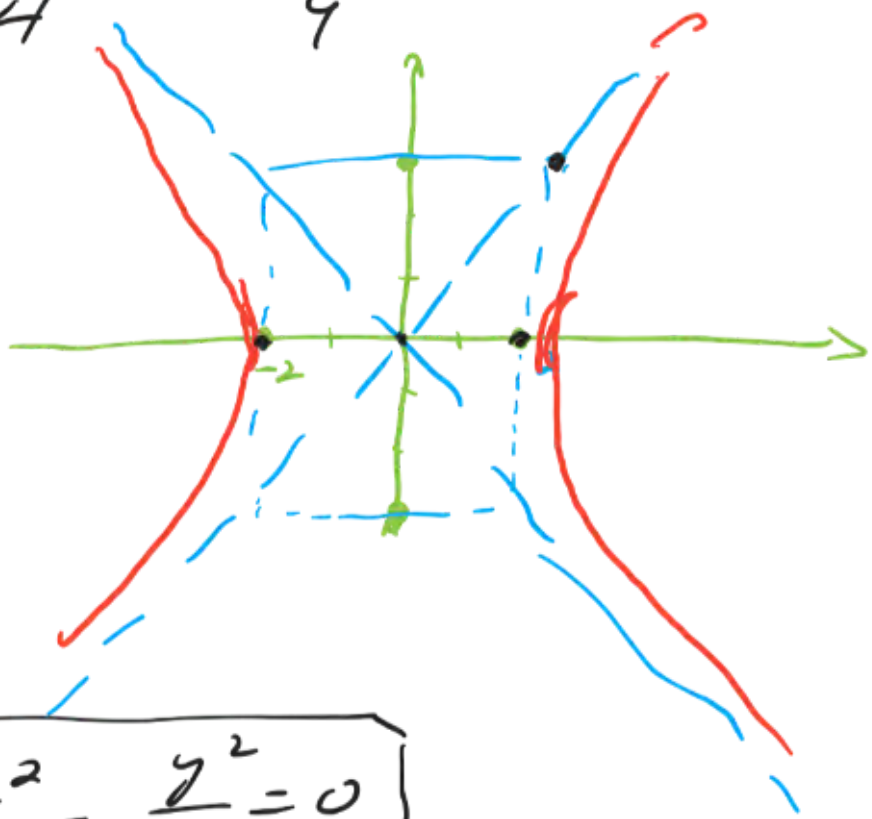
$d_1 - d_2 = 2a$



Parabola

Ex $9x^2 - 4y^2 = 36$

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$



$$\frac{x^2}{4} - \frac{y^2}{9} = 0$$

$$\frac{y^2}{9} = \frac{x^2}{4} \rightarrow$$

$$y^2 = \frac{9}{4}x^2$$

$$y = \pm\frac{3}{2}x$$

$V(\pm 2, 0)$ $W = (0, \pm 3)$

endpoints

$c^2 = a^2 + b^2$

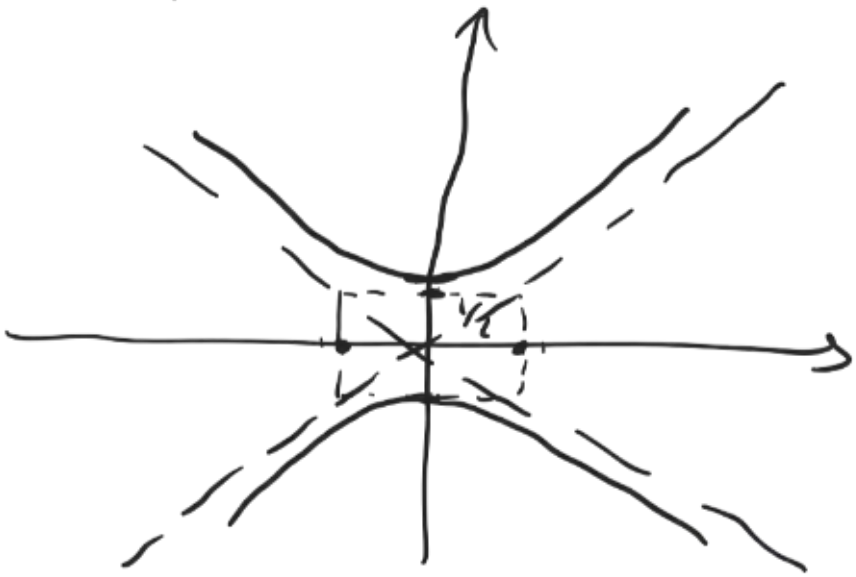
$= 9 + 4$

$= 13$

$F(\pm\sqrt{13}, 0)$

Ex $4y^2 - 2x^2 = 1$

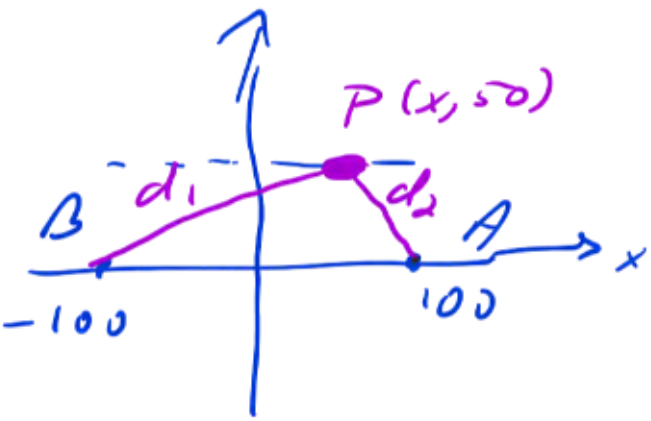
$$\frac{y^2}{\frac{1}{4}} - \frac{x^2}{\frac{1}{2}} = 1$$



Ex

$v = 980$ ft/msec

$t = 400$ msec



$d_1 - d_2 = 2a$

$= vt$

$= (980)(400)\ \frac{1\ \text{mi}}{3250\ \text{ft}}$

msec $= \frac{1}{10^3}$ sec

$= 10^{-3}$ sec

$$= \frac{39200}{328}$$

$$a = \frac{1}{2} \frac{19600}{164}$$

$$= \frac{9800}{164} \cdot \frac{2450}{41} \approx 37.\overline{12}$$

$$a^2 \approx 1378$$

$$c = 100$$

$$b^2 = c^2 - a^2$$

$$= 10{,}000 - 1378$$

$$= \underline{8{,}622}$$

$$\frac{x^2}{1378} - \frac{y^2}{8{,}622} = 1$$

$$\frac{x^2}{1378} = 1 + \frac{50^2}{8{,}622}$$

$$x^2 = 1378 \left(1 + \frac{50^2}{8{,}622}\right)$$

$$x \approx \underline{42.16}$$