#19

$$x = 1 - 3\sin 4\pi t \implies \sin 4\pi t = \frac{1-x}{3}$$
$$y = 2 + 3\cos 4\pi t \implies \cos 4\pi t = \frac{y-2}{3}$$

$$\sin^2 4\pi t + \cos^2 4\pi t = 1$$
$$\left(\frac{1-x}{3}\right)^2 + \left(\frac{y-2}{3}\right)^2 = 1$$
$$\frac{(1-x)^2}{9} + \frac{(y-2)^2}{9} = 1$$
$$(x-1)^2 + (y-2)^2 = 9$$

circle w/ center @ $(1,2)$, radius of 3

#27
$$x = 2\sin t - 3 \implies \sin t = \frac{x+3}{2}$$
$$y = 5 + \cos 2t$$
$$\quad = 5 + 2\cos^2 t - 1$$
$$\cos^2 t = \frac{y-4}{2}$$

$$\cos^2 t + \sin^2 t = 1$$
$$\frac{1}{2}y - 2 + \frac{(x+3)^2}{4} = 1$$
$$\frac{1}{2}y = 3 - \frac{1}{4}(x+3)^2$$
$$y = 6 - \frac{1}{2}(x+3)^2 \quad \text{Parabola}$$

#16
$$x = e^{2t} = (e^t)^2$$
$$y = e^t + 1 \implies e^t = y - 1$$
$$x = (y-1)^2$$
$$\quad = y^2 - 2y + 1 \quad \text{Parabola.}$$

4.2

#13 $\begin{cases} x = \sin 2\pi t \\ y = \cos 2\pi t \end{cases}$ @ $t = -\frac{1}{6}$ $\quad \dfrac{d^2 y}{dx^2}$

$$\frac{dx}{dt} = 2\pi \cos 2\pi t$$

$$\frac{dy}{dt} = -2\pi \sin 2\pi t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\pi \sin 2\pi t}{2\pi \cos 2\pi t}$$

$$= -\tan 2\pi t$$

$$\frac{d^2 y}{dx^2} = \frac{-2\pi \sec^2 2\pi t}{2\pi \cos 2\pi t} \qquad\qquad \frac{dy'/dt}{dx/dt}$$

$$= -\frac{1}{\cos^3 2\pi t}\bigg|_{t=-\frac{1}{6}}$$

$$= -\frac{1}{\left(\cos \frac{\pi}{3}\right)^3} \qquad\qquad \cos(-\alpha) = \cos\alpha$$

$$= -8 \;\rfloor$$

#20 $\begin{cases} x = \sqrt{t} \\ y = 3t - 1 \end{cases}$ $t = 1$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t}} \qquad \frac{dy}{dt} = 3$$

$$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= 3(2\sqrt{t}) \Big|_{t=1}$$

$$= 6$$

@ $t=1$ $x = 1$, $y = 2$ $(1, 2)$

$$y = 6(x-1) + 2$$
$$= 6x - 4$$

$$\frac{dy'}{dt} = \frac{d}{dt}(6\sqrt{t})$$

$$= \frac{3}{\sqrt{t}}$$

$$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$$

$$= \frac{3}{\sqrt{t}} \cdot 2\sqrt{t}$$

$$= 6$$

2nd method $x = \sqrt{t} \implies t = x^2 \Big|_{t=1}^{x=1}$

$$y = 3t - 1$$
$$= 3x^2 - 1$$

$$m = y' = 6x = 6$$

$$y'' = 6$$

#18

$$\begin{cases} x = t + e^t = 1 \\ y = 1 - e^t = 0 \end{cases} \qquad \underline{t = 0}$$

$$\frac{dx}{dt} = 1 + e^t \qquad\qquad \frac{dy}{dt} = -e^t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{-e^t}{1 + e^t} \bigg|_{t=0}$$

$$m = \frac{-1}{2}$$

$$y = -\frac{1}{2}(x - 1)$$

$$= -\frac{1}{2} x + \frac{1}{2}$$

$$\frac{dy'}{dt} = \frac{d}{dt}\left(\frac{-e^t}{1 + e^t}\right)$$

$$= -\frac{e^t}{(1 + e^t)^2}$$

$$\frac{d y^2}{dx^2} = \frac{dy'/dt}{dx/dt}$$

$$= -\frac{e^t}{(1 + e^t)^2} \cdot \frac{1}{1 + e^t}$$

$$= -\frac{e^t}{(1 + e^t)^3} \bigg|_{t=0}$$

$$= -\frac{1}{8}$$

$$\frac{e^t + 0}{e^t + 1}$$

$$\underline{e^t(1 + e^t) - e^t e^t}$$

#51) A? $\begin{cases} x = t - t^2 \\ y = 1 + e^{-t} \end{cases}$ y-axis
$x = 0$

$x = y$
$t - t^2 = 1 + e^{-t}$
$x = 0 = t - t^2 \implies \underline{t = 0, 1}$

$A = \displaystyle\int_0^1 x \, dy$ $\dfrac{dy}{dt} = \dfrac{d(1 + e^{-t})}{dt} = -e^{-t}$

$= \displaystyle\int_0^1 (t - t^2)(-e^{-t}) \, dt$

		$\int e^{-t}$
$-$	$t - t^2$	$-e^{-t}$
$+$	$1 - 2t$	e^{-t}
$-$	-2	$-e^{-t}$

$= \left. (t - t^2 + 1 - 2t - 2) e^{-t} \right|_0^1$

$= (1 - 1 + 1 - 2 - 2) e^{-1} - (1 - 2)$

$= -\dfrac{3}{e} + 1 \quad \text{unit}^2$

#60 L? $\begin{cases} x = t^3 \\ y = \frac{3}{2} t^2 \end{cases}$ $0 \le t \le \sqrt{3}$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(3t^2)^2 + (3t)^2}$$

$$= 3\sqrt{t^4 + t^2}$$

$$= 3t\sqrt{t^2 + 1}$$

$$L = \int_0^{\sqrt{3}} 3t(t^2 + 1) \, dt$$

$$= \frac{3}{2} \int_0^{\sqrt{3}} (t^2 + 1)^{1/2} \, d(t^2 + 1)$$

$$= (t^2 + 1)^{3/2} \Big|_0^{\sqrt{3}}$$

$$= 8 - 1$$

$$= 7 \text{ unit}$$

#62
$$\begin{cases} x = \ln(\sec t + \tan t) - \sin t \\ y = \cos t \end{cases} \qquad 0 \le t \le \frac{\pi}{3}$$

$$\frac{dx}{dt} = \frac{\sec t \tan t + \sec^2 t}{\sec t + \tan t} - \cos t$$

$$= \frac{\sec t \tan t + \sec^2 t}{\sec t}$$

$$= \frac{\sec t(\tan t + \sec t)}{\sec t + \tan t} - \cos t$$

$$= \sec t - \cos t$$

$$\frac{dy}{dt} = -\sin t$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\sec^2 t - 2 + \underbrace{\cos^2 t + \sin^2 t}_{=1}}$$

$$= \sqrt{\sec^2 t - 1}$$

$$= \sqrt{\tan^2 t}$$

$$= \tan t$$

$$L = \int_0^{\pi/3} \tan t \, dt$$

$$= -\ln|\cos t| \Big|_0^{\pi/3}$$

$$= -\left(\ln \frac{1}{2} - \ln 1\right)$$

$$= -\ln \frac{1}{2}$$

$$= \underline{\ln 2} \quad \text{unit}$$

$$\ln \frac{1}{x} = -\ln x$$

#70

$$\begin{cases} x = \dfrac{2}{3} t^{3/2} \\ y = 2\sqrt{t} \end{cases} \qquad 0 \le t \le \sqrt{3} \qquad \text{x-axis}$$

$$\frac{dx}{dt} = t^{1/2} \qquad\qquad \frac{dy}{dt} = \frac{1}{\sqrt{t}} = t^{-1/2}$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{t + \frac{1}{t}}$$

$$= \sqrt{\frac{t^2+1}{t}}$$

$$S = 2\pi \int_0^{\sqrt{3}} 2\sqrt{t} \; \frac{\sqrt{t^2+1}}{\sqrt{t}} \; dt .$$

$$\begin{aligned} t &= \tan\theta \\ dt &= \sec^2\theta \, d\theta \\ \sqrt{1+t^2} &= \sec\theta \end{aligned}$$

$$= 4\pi \int_0^{\sqrt{3}} \sqrt{t^2+1} \; dt$$

$$= 4\pi \int_0^{\sqrt{3}} \sec^3\theta \, d\theta$$

$$= 4\pi \, \frac{1}{2} \left(\sec\theta \tan\theta + \ln(\sec\theta + \tan\theta) \right)$$

$$= 2\pi \left(t\sqrt{t^2+1} + \ln\left(\sqrt{t^2+1} + t\right) \right) \Big|_0^{\sqrt{3}}$$

$$= 2\pi \left(2\sqrt{3} + \ln(2+\sqrt{3}) \right) \text{ unt}^2$$

#76

$$\begin{cases} x = 5\cos\theta \\ y = 5\sin\theta \end{cases} \qquad \begin{array}{l} y\text{-axis} \\ 0 \le \theta \le \dfrac{\pi}{2} \end{array}$$

$$\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} = \sqrt{25\sin^2\theta + 25\cos^2\theta}$$

$$25(\sin^2 t + \cos^2 t)$$

$$= 5$$

$$S = 2\pi \int_0^{\pi/2} 5\cos\theta \,(5) \; d\theta$$

$$= 50\pi \; \sin\theta \Big|_0^{\pi/2}$$
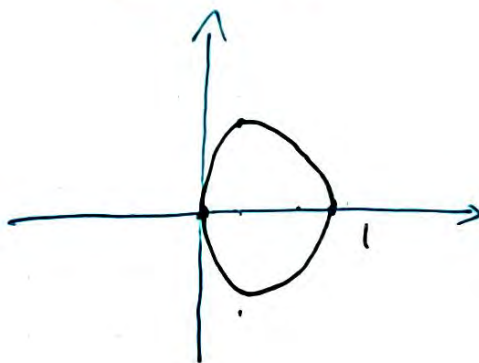
$$= \underline{50\pi} \; unt^2$$

4.4 😊

#11 inside: $r = \sqrt{\cos\theta} \rightarrow$

$$\cos\theta \ge 0 \rightarrow \text{even fctn}$$
$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$A = \frac{1}{2} \int_\alpha^\beta r^2 \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} 2\cos\theta \, d\theta$$

$$= \sin\theta \Big|_0^{\pi/2}$$

$$= \underline{1 \; unit^2}$$
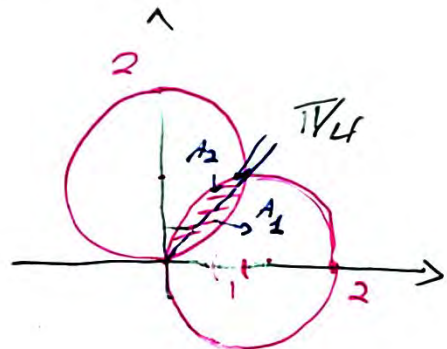
#14 inside Limaçon $r = 2 + \cos\theta$: even fctn

$$A = \frac{1}{2} \, 2 \int_0^\pi (2 + \cos\theta)^2 \, d\theta$$

$$= \int_0^\pi \left(4 + 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta \right) d\theta \quad \overset{\cos^2\theta}{}$$

$$= \int_0^\pi \left(\frac{9}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta \right) d\theta$$

$$= \frac{9}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta \Big|_0^\pi$$

$$= \frac{9\pi}{2} \quad \text{unit}^2$$

#18 shared circles : $r = 2\cos\theta$, $r = 2\sin\theta$

$$r = 2\cos\theta = 2\sin\theta$$

$$\theta = \frac{\pi}{4} \,\Big] \quad \frac{5\pi}{4}$$

$$A = \frac{1}{2} \int_0^{\pi/4} 2\,(2\sin\theta)^2 \, d\theta$$

$$= 4 \int_0^{\pi/4} \frac{1 - \cos 2\theta}{2} \, d\theta$$

$$= 2 \left(\theta - \frac{1}{2}\sin 2\theta \right) \Big|_0^{\pi/4}$$

$$= 2 \left(\frac{\pi}{4} - \frac{1}{2} \right)$$

$$= \frac{\pi}{2} - 1$$

$$A_2 = \frac{1}{2} \int_{\pi/4}^{\pi/2} 2\,(2\cos\theta)^2$$