



Proof

Triangle DCB : $\tan \alpha = \frac{h}{d+x} \Rightarrow h = (d+x) \tan \alpha$

Triangle ACB : $\tan \beta = \frac{h}{d} \Rightarrow h = d \tan \beta$

$$h = d \tan \beta = (d+x) \tan \alpha$$

$$d \tan \beta = d \tan \alpha + x \tan \alpha$$

$$d \tan \beta - d \tan \alpha = x \tan \alpha$$

$$d (\tan \beta - \tan \alpha) = x \tan \alpha$$

$$d = \frac{x \tan \alpha}{\tan \beta - \tan \alpha}$$

$$h = \frac{x \tan \alpha \tan \beta}{\tan \beta - \tan \alpha}$$

Height is equal to distance times (tan tan) divides by the (tan(larger angle) – tan) (difference between tangents)

Example

From a given point on the ground, the angle of elevation to the top of a tree is 36.7° . From a second point, 50 feet back, the angle of elevation to the top of the tree is 22.2° . Find the height of the tree to the nearest foot.

Solution

$$h = 50 \frac{\tan 22.2^\circ \tan 36.7^\circ}{\tan 36.7^\circ - \tan 22.2^\circ} \approx 45 \text{ ft}$$

