# *Section* 3.3 – Counting; Multiplication Principle

## *Basic Counting Principle*

### *The Product Rule*

A procedure can be broken down into a sequence of two tasks. There are $n_1$ ways to do the first task and $n_2$ ways to do the second task. Then there are $n_1 \cdot n_2$ ways to do the procedure

***Example***

How many bit strings of length seven are there?

*Solution*

Since each of the seven bits is either a 0 or a 1, the answer is $2^7 = 128$.

***Example***

A new company with just two employees rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

*Solution*

The procedure of assigning offices to these 2 employees consists of assigning an office to one employee, which can be done in 12 ways, then assigning an office to the second different from the office assigned to the first, which can be done in 11 ways.

By the product rule, there are $12 \cdot 11 \ \underline{= 132 \ \ ways}|$ to assign offices to these 2 employees.

***Example***

There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a computer in the center are there?

*Solution*

$32 \cdot 24 \ \underline{= 768 \ \ ports}|$

## *Multiplication Principle*

Sequence of operations $\Rightarrow$ set multiplication to count numbers

> Suppose $n$ choices must be made, with $m_1$ ways to make choice 1,
>
> *and* for each of these ways, with $m_2$ ways to make choice 2,
>
> *and* so on, with $m_n$ ways to make choice $n$.
>
> Then there are: $m_1.m_2 \ldots m_n$

## *Theorem*

- If two operations $O_1$ $O_2$ are performed in $O$ order with $N_1$ possible outcomes for 1st $O_1$ & $N_2$ possible outcomes for $O_2$ $\Rightarrow$ $N_1.N_2$
- $O_1, O_2, \cdots, O_n \quad \Rightarrow N_1.N_2 \; ..... \; N_3$

### *Example*

A certain combination lock can be set to open to any 3-letter sequence.

*a)* How many sequences are possible?

*b)* How many sequences are possible if no letter is repeated?

<u>*Solution*</u>

*a)* Possible sequences: $26.26.26 = 17,576$ *different sequences*

*b)* Possibility if no letter is repeated: $26.25.24 = 15,600$ *possibilities*

### *Example*

Each question on a multiple-choice test has 5 choices. If there are 5 such question on a test, how many different response sheets are possible if only 1 choice is marked for each question?

<u>*Solution*</u>

$$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^5 = 3125 \text{ different responses}$$

***Example***

Morse code uses a sequence of dots and dashes to represent letters and words. How many sequences are possible with at most 3 symbols?

<u>*Solution*</u>

At most 3 means "1 *or* 2 *or* 3"
Sequence: *dot and dash*

| Number of Symbols | Number of Sequences |
|---|---|
| 1 | **2** |
| 2 | $\mathbf{2} \cdot \mathbf{2} = 4$ |
| 3 | $\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} = 8$ |

*Possibilities*: 2 + 4 +8 = 14

***Example***

A teacher has 5 different books that he wishes to arrange side by side. How many different arrangements are possible?

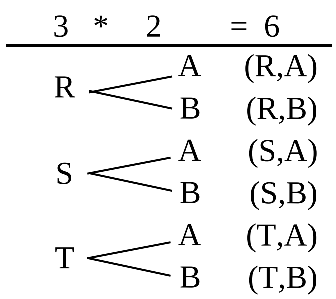
<u>*Solution*</u>

*Possibilities*: 5.4.3.2.1 = 120

***Example***

A company offers its employee's health plans from three different companies *R*, *S*, and *T*. Each company offers two levels of coverage, *A* and *B*, with one level requirement additional employee contributions. What are the combined choices, and how many choices are?

<u>*Solution*</u>




***Count = 6***

***Tree Diagram***

## *Factorial*

$n! = n(n-1)(n-2)\cdots(3).(2).(1)$ (*n factorial*)

*Calculators*: **Math → Prob → 4**

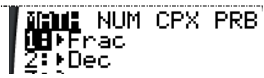
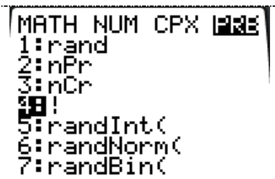
5.4.3.2.1 = 5!

$\frac{7!}{6!} = 7$

0! = 1

$\frac{8!}{5!} = \frac{(8)(7)(6)(5!)}{5!} = 8(7)(6) = 336$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

| **4** | **Math** | **PRB** | **Type 4** | |
|---|---|---|---|---|
| |   |   |   |   |

***Example***

During the summer, you are planning to visit these 6 national parks: Glacier, Yellowstone, Yosemite, Arches, Zion, and Grand Canyon. You would like to plan the most efficient route and you decide to list all of the possible routes. How many different routes are possible?

<u>*Solution*</u>

There 6 different parks can be arranged in order 6! different ways.

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \ \underline{= 720 \ \textit{ways}|}$

***Example***

A teacher has 5 different books that he wishes to place only 3 of the 5 books on his desk. How many arrangements of 3 books are possible?

<u>*Solution*</u>

*Possibilities*: 5.4.3 = 60 ***arrangements***

# *Exercises* *Section* 3.3 – Counting; Multiplication Principle

**1.** How many different types of homes are available if a builder offers a choice of 6 basic plans, 3 roof styles, and 2 exterior finishes?

**2.** A menu offers a choice of 3 salads, 8 main dishes, and 7 desserts. How many different meals consisting of one salad, one main dish, and one dessert are possible?

**3.** A couple has narrowed down the choice of a name for their new baby to 4 first names and 5 middle names. How many different first- and middle-name arrangements are possible?

**4.** An automobile manufacturer produces 8 models, each available in 7 different exterior colors, with 4 different upholstery fabrics and 5 interior colors. How many varieties of automobile are available?

**5.** A biologist is attempting to classify 52,000 species of insects by assigning 3 initials to each species. Is it possible to classify all the species in this way? If not, how many initials should be used?

**6.** How many 4-letter code words are possible using the first 10 letters of the alphabet under:

*a)* No letter can be repeated
*b)* Letters can be repeated
*c)* Adjacent can't be alike

**7.** How many 3 letters license plate code words are possible without repeats possible

**8.** How many ways can 2 coins turn up heads, H, or tails, T – if the combined outcome (H, T) is to be distinguished from the outcome (T, H)?

**9.** How many 2-letter code words can be formed from the first 3 letters of the alphabet if no letter can be used more than once?

**10.** A coin is tossed with possible outcomes heads, H, or tails, T. Then a single die is tossed with possible outcomes 1, 2, 3, 4, 5, or 6. How many combined outcomes are there?

**11.** In how many ways can 3 coins turn up heads, H, or tails, T – if combined outcomes such as (H,T,H), (H, H, T), and (T, H, H) are to be considered different?

**12.** An entertainment guide recommends 6 restaurants and 3 plays that appeal to a couple.
*a)* If the couple goes to dinner or to a play, how many selections are possible?
*b)* If the couple goes to dinner and then to a play, how many combined selections are possible?

**13.** There are 18 mathematics majors and 325 computer science majors at a college
*a)* In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
*b)* In how many ways can one representative be picked who either a mathematics major or a computer science major?

**14.** An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

**15.** A multiple-choice test contains 10 questions. There are four possible answers for each question

*a)* In how many ways can a student answer the questions on the test if the student answers every question?

*b)* In how many ways can a student answer the questions on the test if the student can leave answers blank?

**16.** A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of the shirts are made?

**17.** How many different three-letter initials can people have?

**18.** How many different three-letter initials with none of the letters repeated can people have?

**19.** How many different three-letter initials are there that begin with an *A*?

**20.** How many strings are there of four lowercase letters that have the letter $x$ in them?

**21.** How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

**22.** How many license plates can be made using either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters?

**23.** How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

**24.** How many strings of eight English letter are there

*a)* That contains no vowels, if letters can be repeated?

*b)* That contains no vowels, if letters cannot be repeated?

*c)* That starts with a vowel, if letters can be repeated?

*d)* That starts with a vowel, if letters cannot be repeated?

*e)* That contains at least one vowel, if letters can be repeated?

*f)* That contains at least one vowel, if letters cannot be repeated?

**25.** In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

*a)* The bride must be in the picture?

*b)* Both the bride and groom must be in the picture?

*c)* Exactly one of the bride and the groom is in the picture?