MATH 1314 – College Algebra **EXAM 2** **REVIEW**
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1. Determine whether each relation is a function and find the domain and the range.

a) $\{(1, 2), (2, 3), (3, 2), (4, 5), (5, 4), (6, 1), (8, 2)\}$

b) $\{(-1, 2), (-2, -3), (3, 2), (5, 5), (5, 4), (-2, 1), (6, 2)\}$

c) $\{(1, 2), (2, 3), (3, 2), (4, 4), (5, 4), (6, 1), (7, 2), (-1, 2)\}$

2. Given $g(x) = -2x^2 + x + 6$, find:

a) $g(0)$ *b)* $g(-4)$ *c)* $g(2)$ *d)* $g(x+1)$

3. Given $f(x) = \begin{cases} -5x - 8 & \text{for} \quad x < -2 \\ \frac{1}{2}x + 5 & \text{for} \quad -2 \le x \le 4 \\ 10 - 2x & \text{for} \quad x > 4 \end{cases}$

a. Graph $f(x)$

b. Find $f(-1)$

4. Given $f(x) = \begin{cases} x^2 - 2 & \text{if} \quad x \le 2 \\ 1 & \text{if} \quad x > 2 \end{cases}$ find:

a) $f(-1)$ b) $f(2)$ c) $f(4)$ d) $f(-2)$

5. Given $g(x) = \begin{cases} -|x + 2| & \text{if} \quad x < -1 \\ -2x + 3 & \text{if} \quad x \ge -1 \end{cases}$ find:

a) $f(-1)$ b) $f(2)$ c) $f(4)$ d) $f(-2)$

6. Determine if each function is odd, even, or neither.

a) $g(x) = |x| + 4$ *b)* $h(x) = x^4 + 2x^2 - 8$ *c)* $f(x) = -x^3 + 2x$

d) $k(x) = x^2 - x + 6$ *e)* $f(x) = \dfrac{x-1}{x-2}$

7. Let $f(x) = \sqrt{x+3}$, $g(x) = \dfrac{x+2}{x-1}$ and $h(x) = x - 5$ Find the following:

a) Domain f *b)* Domain g *c)* Domain h

d) Domain $\dfrac{f}{h}$ *e)* Domain of $g + f$, $g - f$, *and* $g \cdot f$

f) Domain $\frac{g}{f}$ *g*) Domain of $f+h,\ f-h,\ and\ f\cdot h$

8. Let $f(x)=3x+2$ and $g(x)=2x^2-1$. Find the following:

a) $(f\circ g)(4)$ b) $(g\circ f)(2)$ c) $(f\circ g)(x)$ d) $(g\circ f)(x)$

9. Let $f(x)=\sqrt{x+1}$, $g(x)=x^2-3$, and $h(x)=\frac{1}{x}$. Find the following functions, and state the domain of each:

a) $(f\circ g)(x)$ *b*) $(g\circ f)(x)$ *c*) $(h\circ f)(x)$

10. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for

a) $f(x)=4x-5$ *b*) $f(x)=3-4x$ *c*) $f(x)=3x+1$ *d*) $f(x)=2x^2$

11. An airplane is flying at an altitude of 3700 *ft*. The slanted distance directly to the airport is *d* feet. Express the horizontal distance *h* as a function of *d*.

12. *a*) How can the graph of $f(x)=-(x-8)^2$ be obtained from the graph $y=x^2$?

b) How can the graph of $f(x)=\sqrt{x+6}-5$ be obtained from the graph $y=\sqrt{x}$?

c) How can the graph of $f(x)=|x+7|+2$ be obtained from the graph $y=|x|$?

d) How can the graph of $f(x)=-(x-3)^2+4$ be obtained from the graph $y=x^2$?

e) How can the graph of $f(x)=\sqrt{-(x+6)}-5$ be obtained from the graph $y=\sqrt{x}$?

13. For $f(x)=-x^2+6x-5$, find

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a maximum or minimum value *and* find that value
d) Find the zeros of $f(x)$
e) Find the range and the domain of the function.
f) Graph the function and ***label.***
g) On what intervals is the function increasing? Decreasing?

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15. Determine the end behavior of the graph of the polynomial function.

a) $f(x) = 2x^4 - 9x^3 - 5x^2 + 57x - 45$

b) $f(x) = x(x-2)^3(x+2)^2$

c) $f(x) = -4x^5 + 16x^4 + 13x^3 - 76x^2 - 3x + 18$

d) $f(x) = (x-2)^2(x-5)$

e) $f(x) = -(x-2)^2(x-5)^2$

16. Find the quotient and the remainder:

a) $\dfrac{x^3 + x^2 - 11x - 10}{x-3}$

b) $\dfrac{3x^3 + 8x^2 + 5x + 10}{x+2}$

c) $\dfrac{2x^3 - x + 6}{x+4}$

d) $\left(x^4 + 3x^3 + 3x^2 + 3x + 2\right) \div (x+2)$

17. Use the Intermediate Value Theorem to determine whether the function has zeros between ***a*** and ***b***.

a) $f(x) = x^3 + 3x^2 - 9x - 13;\ \ a = 1,\ b = 2$

b) $f(x) = 4x^2 - 5x - 3;\ \ a = 1,\ b = 2$

c) $f(x) = x^3 - 8x^2 + x + 2;\ \ a = -1,\ b = 0$

d) $f(x) = x^3 - 8x^2 + x + 2;\ \ a = 2,\ b = 3$

18. Use synthetic division to find the indicated function value

a) $f(x) = x^3 + 2x^2 - 13x + 10;\ \ f(-2)$

b) $f(x) = x^4 - 16;\ \ f(-2)$

19. Find all solutions of the equation: $x^4 + 9x^3 + 31x^2 + 49x + 30 = 0$

20. Use the Rational Zero theorem to list all possible rational zero for each of the following:

a) $f(x) = x^3 + 3x^2 - 6x - 8$

b) $f(x) = 2x^3 + x^2 - 25x + 12$

c) $f(x) = 3x^4 + 23x^3 + 56x^2 + 52x + 16$

21. Find the vertical and horizontal asymptotes (if any) of:

a) $y = \dfrac{x-2}{x^2 - 4x + 3}$

b) $y = \dfrac{(x+2)(x-1)}{x^2 - 3x - 10}$

c) $y = \dfrac{x^3 - 2x^2 - 4x + 8}{x - 2}$

d) $y = \dfrac{-x+1}{-2x^2 + 5x - 3}$

22. A rancher has 360 *yd.* of fencing with which to enclose two adjacent rectangular corrals, one for sheep and one for cattle. A river forms one side of the corrals. Suppose the width of each corral is *x* yards.

a) Express the total area of the two corrals as a function of *x*.
b) Find the domain of the function.
c) Find the maximum area
d) Find the dimensions that maximize the corrals area

23. A projectile is fired vertically upward, and its height $s(t)$ in feet after *t* seconds is given by the function defined by $s(t) = -16t^2 + 800t + 600$

a) From what height was the projectile fired?
b) After how many seconds will it reach its maximum height?
c) What is the maximum height it will reach?

24. A ball is thrown upwards, and its height *s* at time *t* can be determined by the function $s(t) = -16t^2 + 48t + 8$, where *s* is measured in feet above the ground and *t* is the number of seconds of flight. Find:

a) The time it takes the ball to reach its maximum height.
b) The maximum height the ball attains.

SOLUTION

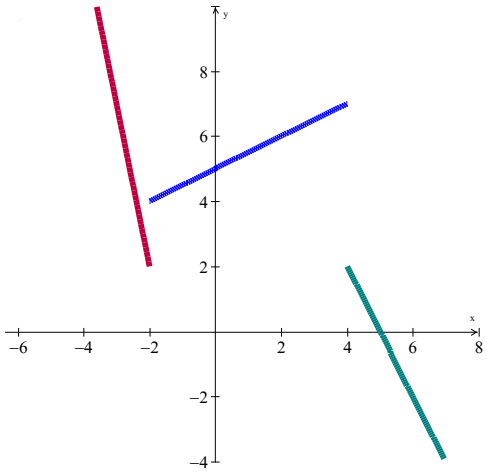
1. *a*) *Function*; *Domain* $= \{1,2,3,4,5,6,8\}$ *Range* $= \{1,2,3,4,5\}$

b) *Not a function*; *Domain* $= \{-2,-1,1,3,5,6\}$ *Range* $= \{-3,1,2,4,5\}$

c) *Function*; *Domain* $= \{-1,1,2,3,4,5,6,7\}$ *Range* $= \{1,2,3,4\}$

2. *a*) 6 *b*) -30 *c*) 0 *d*) $-2x^2 - 3x + 5$

3. *a*)



b) $f(-1) = \frac{1}{2}(-1) + 5 = \frac{9}{2}$

4. *a*) -1 *b*) 0 *c*) 1 *d*) 2

5. *a*) 5 *b*) -1 *c*) -5 *d*) 0

6. *a*) even *b*) even *c*) odd *d*) neither *e*) neither

7. *a*) $[-3, \infty)$ *b*) $(-\infty, 1) \cup (1, \infty)$ *c*) $(-\infty, \infty)$ *d*) $[-3, 1) \cup (1, \infty)$

e) $(-3, 1) \cup (1, \infty)$ *f*) $[-3, \infty)$ *g*) $[-3, 5) \cup (5, \infty)$

8. *a*) 95 *b*) 127 *c*) $6x^2 - 1$ *d*) $18x^2 + 24x + 7$

9. *a*) $(f \circ g)(x) = \sqrt{x^2 - 2}$; $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

b) $(g \circ f)(x) = x - 2$; $(-\infty, \infty)$

c) $(h \circ f)(x) = \frac{1}{\sqrt{x+1}}$; $(-1, \infty)$

10. *a*) 4 *b*) -4 *c*) 3 *d*) $4x + h$

11. $h(t) = \sqrt{d^2 - (3700)^2}$

12. *a*) Reflected across *x*-axis (or upside-down) and shifted right 8 units.
b) Shifted left 6 units and down 5 units.
c) Shifted left 7 units and up 2 units.
d) Reflected across *x*-axis (or upside-down) and shifted right 3 units and up 4 units.
e) Reflected across *y*-axis, shifted left 6 units and down 5 units.

13. Vertex: $x = -\frac{b}{2a}$ $\quad f(x) = -x^2 + 6x - 5$

$= -\frac{6}{2(-1)}$

$= 3$

$y = f(3) = -(3)^2 + 6(3) - 5$

$= 4$

Vertex point: $(3,4)$

Axis of symmetry: $x = 3$

Maximum point @ $(3,4)$

x-intercept: $x = 1,5$

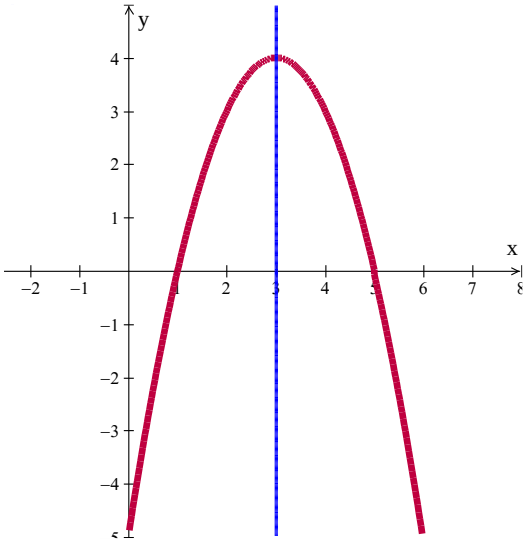
y-intercept: $y = -5$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

Increasing: $(-\infty, 3)$

Decreasing: $(3, \infty)$



14. Vertex: $x = -\frac{1}{2(1)} = -\frac{1}{2}$

$y = f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 6 = -\frac{25}{4}$

Vertex point: $\left(-\frac{1}{2}, -\frac{25}{4}\right)$

Axis of symmetry: $x = -\frac{1}{2}$

Maximum point @ $\left(-\frac{1}{2}, -\frac{25}{4}\right)$

x-intercept: $x = -3, 2$

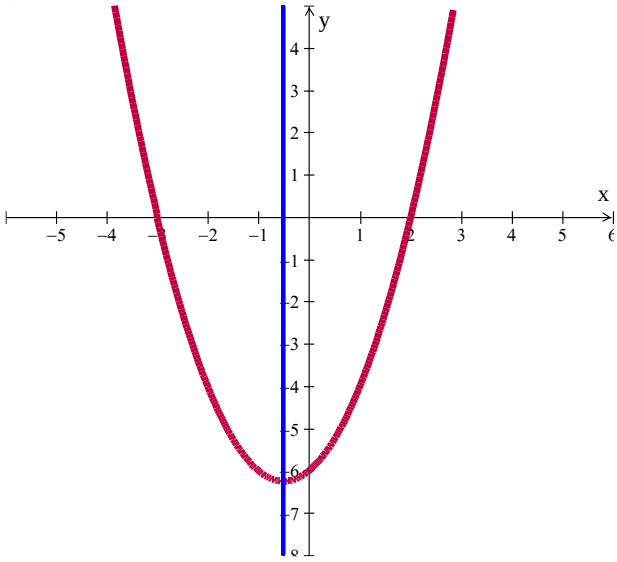
y-intercept: $y = -6$

Domain: $(-\infty, \infty)$

Range: $\left[-\frac{25}{4}, \infty\right)$

Increasing: $\left(-\frac{1}{2}, \infty\right)$

Decreasing: $\left(-\infty, -\frac{1}{2}\right)$



15. *a*) Leading Term: $2x^4$; rises left and right

b) Leading Term: x^6; rises left and right

c) Leading Term: $-4x^5$; rises left and falls right

d) Leading Term: x^3; fall left and rises right

e) Leading Term: $-x^4$; falls left and right

16. *a*) $Q(x) = x^2 + 4x + 1; \quad R(x) = -7$

b) $Q(x) = 3x^2 + 2x + 1; \quad R(x) = 8$

c) $Q(x) = 2x^2 - 8x + 31; \quad R(x) = -118$

d) $Q(x) = x^3 + x^2 + x + 1; \quad R(x) = 0$

17. *a*) Can't be determined

b) Yes

c) Yes

d) Can't be determined

18. *a*) $f(-2) = 36$ *b*) $f(-2) = 0$

19. $-3, \ -2, \ -2, \ \pm i$

20. *a*) $\pm\{1, \ 2, \ 4, \ 8\}$

b) $\pm\left\{1, \ 2, \ 4, \ 6, \ 12, \ \frac{1}{2}, \ \frac{3}{2}\right\}$

c) $\pm\left\{1, \ 2, \ 4, \ 8, \ 16, \ \frac{1}{3}, \ \frac{2}{3}, \ \frac{4}{3}, \ \frac{8}{3}, \ \frac{16}{3}\right\}$

21. *a*) $VA: \ x = 1, \ x = 3; \quad HA: \ y = 0$

b) $VA: \ x = 5; \quad HA: \ y = \frac{4}{3}$

c) $VA: n / a; \quad HA: n / a$

d) $VA: x = \frac{3}{2}; \quad HA: y = 0$

22. *a*) $A(x) = 360x - 3x^2$ *b*) Domain: $0 < x < 120$ *c*) 10800 yd^2 *d*) 60 by 180 yd.

23. *a*) Height = 600 *ft*. ($t = 0$) *b*) t = 25 sec. *c*) Max. Height: 10,600 ft.

24. *a*) t = 1.5 secs *b*) Max height is 44 feet.