Solution Section 3.1 – Definition of the Laplace Transform

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=3$

Solution

$$F(s)=\int_0^\infty 3e^{-st}dt$$

$$=\lim_{T\to\infty}\int_0^T 3e^{-st}dt$$

$$=\lim_{T\to\infty}\left(-\frac{3e^{-st}}{s}\right)_{t=0}^{T}$$

$$=\lim_{T\to\infty}\left(-\frac{3}{s}e^{-sT}+\frac{3}{s}\right) \qquad \lim_{T\to\infty}\left(e^{-sT}\right)=0$$

$$\boxed{=\frac{3}{s}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=t$

Solution

$$F(s)=\int_0^\infty te^{-st}dt$$

		$\int e^{-st}dt$
+	t	$-\frac{1}{s}e^{-st}$
–	1	$\frac{1}{s^2}e^{-st}$

$$=\lim_{T\to\infty}\left(\left(-\frac{t}{s}-\frac{1}{s^2}\right)e^{-st}\right)_{t=0}^{T}$$

$$=\lim_{T\to\infty}\left(\left(-\frac{T}{s}-\frac{1}{s^2}\right)e^{-sT}+\frac{1}{s^2}\right) \qquad \lim_{T\to\infty}\left(e^{-sT}\right)=0$$

$$\boxed{=\frac{1}{s^2}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=t^2$

Solution

$$F(s)=\int_0^\infty t^2e^{-st}dt$$

$$= \left(-\frac{t^2}{s} - \frac{2t}{s^2} - \frac{2}{s^3}\right)e^{-st}\Big|_0^\infty$$

$$\boxed{= \frac{2}{s^3}}$$

		$\int e^{-st}dt$
+	t^2	$-\frac{1}{s}e^{-st}$
−	$2t$	$\frac{1}{s^2}e^{-st}$
+	2	$-\frac{1}{s^3}e^{-st}$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{6t}$

Solution

$$F(s) = \int_0^\infty e^{6t}e^{-st}dt$$

$$= \int_0^\infty e^{-(s-6)t}dt$$

$$= -\frac{e^{-(s-6)t}}{s-6}\Big|_0^\infty$$

$$\boxed{= \frac{1}{s-6}} \qquad with: s > 6$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{-2t}$

Solution

$$F(s) = \int_0^\infty e^{-2t}e^{-st}dt$$

$$= \lim_{T\to\infty}\int_0^T e^{-(s+2)t}dt$$

$$= \lim_{T\to\infty}\left(\frac{-e^{-(s+2)t}}{s+2}\right)_{t=0}^T$$

$$= \lim_{T\to\infty}\left(-\frac{e^{-(s+2)T}}{s+2} + \frac{1}{s+2}\right) \qquad \lim_{T\to\infty}\left(e^{-(s+2)T}\right) = 0$$

$$\boxed{= \frac{1}{s+2}} \qquad with: s > -2$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = te^{-3t}$

Solution

$$F(s)=\int_0^{\infty} te^{-3t}e^{-st}dt$$

$$=\int_0^{\infty} te^{-(s+3)t}dt$$

		$\int e^{-(s+3)t}dt$
+	t	$-\frac{1}{s+3}e^{-(s+3)t}$
−	1	$\frac{1}{(s+3)^2}e^{-(s+3)t}$

$$F(s)=\left(-\frac{1}{s+3}te^{-(s+3)t}-\frac{1}{(s+3)^2}e^{-(s+3)t}\right)_0^{\infty}$$

$$=\frac{1}{(s+3)^2} \quad \text{with } s>-3 \qquad e^{-\infty}=0 \quad e^0=1$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=te^{3t}$

Solution

$$F(s)=\int_0^{\infty} te^{3t}e^{-st}dt$$

$$=\int_0^{\infty} te^{-(s-3)t}dt$$

		$\int e^{-(s-3)t}dt$
+	t	$-\frac{1}{s-3}e^{-(s-3)t}$
−	1	$\frac{1}{(s-3)^2}e^{-(s-3)t}$

$$F(s)=-\frac{1}{s-3}te^{-(s-3)t}-\frac{1}{(s-3)^2}e^{-(s-3)t}\Big|_0^{\infty}$$

$$=\frac{1}{(s-3)^2} \quad \text{with } s>3 \qquad e^{-\infty}=0 \quad e^0=1$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=e^{2t}\cos 3t$

Solution

$$F(s)=\int_0^{\infty}\left(e^{2t}\cos 3t\right)e^{-st}dt$$

$$=\int_0^{\infty} e^{-(s-2)t}\cos 3t\, dt$$

		$\int \cos 3t\, dt$
+	$e^{-(s-2)t}$	$\frac{1}{3}\sin 3t$
−	$-(s-2)e^{-(s-2)t}$	$-\frac{1}{9}\cos 3t$
+	$(s-2)^2e^{-(s-2)t}$	$-\frac{1}{9}\int \cos 3t$

$$\int e^{-(s-2)t}\cos 3t\, dt=\frac{1}{3}e^{-(s-2)t}\sin 3t-\frac{1}{9}(s-2)e^{-(s-2)t}\cos 3t-\frac{1}{9}(s-2)^2\int e^{-(s-2)t}\cos 3t\, dt$$

$$\left(1+\frac{1}{9}(s-2)^2\right)\int e^{-(s-2)t}\cos 3t\, dt=\frac{1}{3}e^{-(s-2)t}\sin 3t-\frac{1}{9}(s-2)e^{-(s-2)t}\cos 3t$$

$$\left(9+(s-2)^2\right)\int e^{-(s-2)t}\cos 3t\ dt = 3e^{-(s-2)t}\sin 3t-(s-2)e^{-(s-2)t}\cos 3t$$

$$\int e^{-(s-2)t}\cos 3t\ dt = \frac{1}{9+(s-2)^2}\left[3e^{-(s-2)t}\sin 3t-(s-2)e^{-(s-2)t}\cos 3t\right]$$

$$F(s)=\left(\frac{3}{9+(s-2)^2}e^{-(s-2)t}\sin 3t-\frac{s-2}{9+(s-2)^2}e^{-(s-2)t}\cos 3t\right)\Bigg|_0^{\infty}$$

$$=\boxed{\frac{s-2}{9+(s-2)^2}}\quad s>2$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=\sin 3t$

Solution

$$F(s)=\int_0^{\infty}(\sin 3t)e^{-st}dt$$

$$\int \sin 3t\ e^{-st}dt=-\frac{1}{3}e^{-st}\cos 3t-\frac{s}{9}e^{-st}\sin 3t+\frac{s^2}{9}\int e^{-st}\sin 3t\ dt$$

		$\int \sin 3t\ dt$
+	e^{-st}	$-\frac{1}{3}\cos 3t$
−	$-se^{-st}$	$-\frac{1}{9}\sin 3t$
+	s^2e^{-st}	$-\frac{1}{9}\int \sin 3t$

$$\int \sin 3t\ e^{-st}dt+\frac{1}{9}s^2\int \sin 3t\ e^{-st}dt=-\frac{1}{3}e^{-st}\cos 3t-\frac{1}{9}se^{-st}\sin 3t$$

$$\left(9+s^2\right)\int \sin 3t\ e^{-st}dt=-(3\cos 3t-s\sin 3t)e^{-st}$$

$$\int \sin 3t\ e^{-st}dt=-\frac{3\cos 3t-s\sin 3t}{s^2+9}e^{-st}$$

$$F(s)=-\frac{3\cos 3t-s\sin 3t}{s^2+9}e^{-st}\Big|_0^{\infty}\qquad =-0+\frac{3\cos 3(0)-s\sin 3(0)}{s^2+9}e^{-s(0)}$$

$$=\boxed{\frac{3}{s^2+9}}\quad s>0$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=\sin 2t$

Solution

$$F(s)=\int_0^{\infty}(\sin 2t)e^{-st}dt$$

$$\int \sin 2t\ e^{-st}dt=-\frac{1}{2}e^{-st}\cos 2t-\frac{s}{4}e^{-st}\sin 2t+\frac{s^2}{4}\int e^{-st}\sin 2t\ dt$$

$$\left(4+s^2\right)\int \sin 2t\ e^{-st}dt = -\left(2\cos 2t - s\sin 2t\right)e^{-st}$$

$$\int \sin 2t\ e^{-st}dt = -\frac{2\cos 2t - s\sin 2t}{s^2+4}e^{-st}$$

		$\int \sin 2t\ dt$
+	e^{-st}	$-\frac{1}{2}\cos 2t$
−	$-se^{-st}$	$-\frac{1}{4}\sin 2t$
+	s^2e^{-st}	

$$F(s) = -\frac{2\cos 2t - s\sin 2t}{s^2+4}e^{-st}\Big|_0^\infty$$

$$= -0 + \frac{2\cos 2(0) - s\sin 2(0)}{s^2+4}e^{-s(0)}$$

$$\boxed{= \frac{2}{s^2+4}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = \cos 2t$

Solution

$$F(s) = \int_0^\infty (\cos 2t)e^{-st}dt$$

$$\int \cos 2t\ e^{-st}dt = \frac{1}{2}e^{-st}\sin 2t - \frac{s}{4}e^{-st}\cos 2t - \frac{s^2}{4}\int e^{-st}\cos 2t\ dt$$

$$\left(4+s^2\right)\int \cos 2t\ e^{-st}dt = \left(2\sin 2t - s\cos 2t\right)e^{-st}$$

$$\int \cos 2t\ e^{-st}dt = \frac{2\sin 2t - s\cos 2t}{s^2+4}e^{-st}$$

		$\int \cos 2t\ dt$
+	e^{-st}	$\frac{1}{2}\sin 2t$
−	$-se^{-st}$	$-\frac{1}{4}\cos 2t$
+	s^2e^{-st}	

$$F(s) = \frac{2\sin 2t - s\cos 2t}{s^2+4}e^{-st}\Big|_0^\infty$$

$$\boxed{= \frac{s}{s^2+4}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = \cos bt$

Solution

$$F(s) = \int_0^\infty (\cos bt)e^{-st}dt$$

$$\int \cos bt\ e^{-st}dt = \frac{1}{b}e^{-st}\sin bt - \frac{s}{b^2}e^{-st}\cos bt - \frac{s^2}{b^2}\int e^{-st}\cos bt\ dt$$

$$\left(b^2+s^2\right)\int \cos bt\ e^{-st}dt = \left(b\sin bt - s\cos bt\right)e^{-st}$$

$$\int \cos bt\ e^{-st}dt = \frac{b\sin bt - s\cos bt}{s^2+b^2}e^{-st}$$

$$F(s) = \frac{b\sin bt - s\cos bt}{s^2+b^2}e^{-st}\Big|_0^\infty$$

$$\boxed{= \frac{s}{s^2+b^2}}$$

		$\int \cos bt\ dt$
+	e^{-st}	$\frac{1}{b}\sin bt$
−	$-se^{-st}$	$-\frac{1}{b^2}\cos bt$
+	s^2e^{-st}	

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{t+7}$

Solution

$$F(s) = \int_0^\infty e^{t+7}e^{-st}dt$$

$$= \int_0^\infty e^7 e^{-(s-1)t}dt$$

$$= -\frac{e^7}{s-1}e^{-(s-1)t}\Big|_0^\infty \qquad e^{-\infty} = 0 \quad e^0 = 1$$

$$\boxed{= \frac{e^7}{s-1}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{-2t-5}$

Solution

$$F(s) = \int_0^\infty e^{-2t-5}e^{-st}dt$$

$$= e^{-5}\int_0^\infty e^{-(s+2)t}dt$$

$$= -\frac{1}{e^5}\cdot\frac{1}{s+2}\left(e^{-(s+2)t}\right)_0^\infty \qquad e^{-\infty} = 0 \quad e^0 = 1$$

$$\boxed{= \frac{1}{e^5}\cdot\frac{1}{s+2}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = te^{4t}$

Solution

$$F(s) = \int_0^\infty te^{4t}e^{-st}\,dt$$

$$= \int_0^\infty te^{-(s-4)t}\,dt$$

$$= \left(-\frac{t}{s-4} - \frac{1}{(s-4)^2} \right) e^{-(s-4)t} \Big|_0^\infty$$

$$= \frac{1}{(s-4)^2}$$

	$\int e^{-(s-4)t}\,dt$
t	$-\frac{1}{s-4}e^{-(s-4)t}$
1	$\frac{1}{(s-4)^2}e^{-(s-4)t}$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = t^2e^{-2t}$

Solution

$$F(s) = \int_0^\infty t^2e^{-2t}e^{-st}\,dt = \int_0^\infty t^2e^{-(s+2)t}\,dt$$

$$= \left(-\frac{t^2}{s+2} - \frac{2t}{(s+2)^2} - \frac{2}{(s+2)^3} \right) e^{-(s+2)t} \Big|_0^\infty$$

$$= \frac{2}{(s+2)^3}$$

	$\int e^{-(s+2)t}\,dt$
t^2	$-\frac{1}{s+2}e^{-(s+2)t}$
$2t$	$\frac{1}{(s+2)^2}e^{-(s+2)t}$
2	$-\frac{1}{(s+2)^3}e^{-(s+2)t}$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{-t}\sin t$

Solution

$$F(s) = \int_0^\infty e^{-t}\sin t\; e^{-st}\,dt$$

$$= \int_0^\infty \sin t\; e^{-(s+1)t}\,dt$$

$$\int \sin t\; e^{-(s+1)t}\,dt = \left(-\cos t - (s+1)\sin t\right)e^{-(s+1)t} - (s+1)^2 \int \sin t\; e^{-(s+1)t}\,dt$$

$$\left((s+1)^2 + 1\right)\int \sin t\; e^{-(s+1)t}\,dt = \left(-\cos t - (s+1)\sin t\right)e^{-(s+1)t}$$

$$\int_0^\infty \sin t\ e^{-(s+1)t}dt = -\frac{\cos t + (s+1)\sin t}{(s+1)^2+1}e^{-(s+1)t}$$

	$\int \sin t\ dt$
$e^{-(s+1)t}$	$-\cos t$
$-(s+1)e^{-(s+1)t}$	$-\sin t$
$(s+1)^2 e^{-(s+1)t}$	$-\int \sin t\ dt$

$$F(s) = -\frac{\cos t + (s+1)\sin t}{(s+1)^2+1}e^{-(s+1)t}\Big|_0^\infty$$

$$\boxed{= \frac{1}{(s+1)^2+1}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{2t}\cos 3t$

Solution

$$F(s) = \int_0^\infty e^{2t}\cos 3t\ e^{-st}dt$$

$$= \int_0^\infty \cos 3t\ e^{-(s-2)t}dt$$

$$\int \cos 3t\ e^{-(s-2)t}dt = \left(\tfrac{1}{3}\sin 3t + \tfrac{1}{9}(s-2)\cos 3t\right)e^{-(s-2)t} - \tfrac{1}{9}(s-2)^2\int \cos 3t\ e^{-(s-2)t}dt$$

$$\left((s-2)^2+9\right)\int \sin t\ e^{-(s-2)t}dt = \left(3\sin 3t + (s-2)\cos 3t\right)e^{-(s-2)t}$$

$$\int_0^\infty \cos 3t\ e^{-(s-2)t}dt = \frac{3\sin 3t + (s-2)\cos 3t}{(s-2)^2+9}e^{-(s-2)t}$$

		$\int \cos 3t\ dt$
+	$e^{-(s-2)t}$	$\frac{1}{3}\sin 3t$
−	$-(s-2)e^{-(s-2)t}$	$-\frac{1}{9}\cos 3t$
+	$(s-2)^2 e^{-(s-2)t}$	

$$F(s) = \frac{3\sin 3t + (s-2)\cos 3t}{(s-2)^2+9}e^{-(s-2)t}\Big|_0^\infty$$

$$\boxed{= \frac{s-2}{(s-2)^2+9}}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = e^{-t}\sin 2t$

Solution

$$F(s) = \int_0^\infty e^{-t}\sin 2t\ e^{-st}dt$$

$$= \int_0^\infty \sin 2t\ e^{-(s+1)t}dt$$

$$\int \sin 2t\, e^{-(s+1)t}dt = \left(-\tfrac{1}{2}\cos 2t - \tfrac{1}{4}(s+1)\sin 2t\right)e^{-(s+1)t} - \tfrac{1}{4}(s+1)^2 \int \sin 2t\, e^{-(s+1)t}dt$$

$$\left((s+1)^2+4\right)\int \sin 2t\, e^{-(s+1)t}dt = -\left(2\cos 2t + (s+1)\sin 2t\right)e^{-(s+1)t}$$

$$\int_0^\infty \sin 2t\, e^{-(s+1)t}dt = -\frac{2\cos 2t + (s+1)\sin 2t}{(s+1)^2+4}e^{-(s+1)t}$$

	$\int \sin 2t\, dt$
$e^{-(s+1)t}$	$-\frac{1}{2}\cos t$
$-(s+1)e^{-(s+1)t}$	$-\frac{1}{4}\sin t$
$(s+1)^2 e^{-(s+1)t}$	

$$F(s) = -\frac{2\cos 2t + (s+1)\sin 2t}{(s+1)^2+4}e^{-(s+1)t}\Big|_0^\infty$$

$$= \frac{2}{(s+1)^2+4}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t) = t\sin t$

Solution

$$F(s) = \int_0^\infty t\sin t\, e^{-st}dt$$

$$\int t\sin t\, e^{-st}dt = \left(-t\cos t + (1-st)\sin t\right)e^{-st} - s^2\int t\sin t\, e^{-st}dt + 2s\int \sin t\, e^{-st}dt$$

$$\int \sin t\, e^{-st}dt = (-\cos t - s\sin t)e^{-st} - s^2\int \sin t\, e^{-st}dt$$

$$\left(s^2+1\right)\int \sin t\, e^{-st}dt = (-\cos t - s\sin t)e^{-st}$$

$$\int \sin t\, e^{-st}dt = -\frac{\cos t + s\sin t}{s^2+1}e^{-st}$$

	$\int \sin t\, dt$
te^{-st}	$-\cos t$
$(1-st)e^{-st}$	$-\sin t$
$\left(s^2t-2s\right)e^{-st}$	$-\int \sin t\, dt$

$$\left(s^2+1\right)\int t\sin t\, e^{-st}dt = \left(-t\cos t + (1-st)\sin t\right)e^{-st} - \frac{2s}{s^2+1}(\cos t + s\sin t)e^{-st}$$

$$\int t\sin t\, e^{-st}dt = \frac{1}{s^2+1}\left(-t\cos t + (1-st)\sin t\right)e^{-st} - \frac{2s}{\left(s^2+1\right)^2}(\cos t + s\sin t)e^{-st}$$

$$F(s) = \left(\frac{(1-st)\sin t - t\cos t}{s^2+1} - \frac{2s(\cos t + s\sin t)}{\left(s^2+1\right)^2}\right)e^{-st}\Big|_0^\infty$$

	$\int \sin t\, dt$
e^{-st}	$-\cos t$
$-se^{-st}$	$-\sin t$
s^2e^{-st}	$-\int \sin t\, dt$

$$= \frac{2s}{\left(s^2+1\right)^2}$$

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=t\cos t$

Solution

$$F(s)=\int_0^{\infty} t\cos t\, e^{-st}dt$$

$$\int t\cos t\, e^{-st}dt=\left(t\sin t-(1-st)\cos t\right)e^{-st}-s^2\int t\cos t\, e^{-st}dt+2s\int \cos t\, e^{-st}dt$$

$$\int \cos t\, e^{-st}dt=\left(\sin t+s\cos t\right)e^{-st}-s^2\int \cos t\, e^{-st}dt$$

$$\left(s^2+1\right)\int \cos t\, e^{-st}dt=\left(\sin t+s\cos t\right)e^{-st}$$

$$\int \cos t\, e^{-st}dt=\frac{\sin t+s\cos t}{s^2+1}e^{-st}$$

	$\int \cos t\ dt$
te^{-st}	$\sin t$
$(1-st)e^{-st}$	$-\cos t$
$\left(s^2t-2s\right)e^{-st}$	

$$\left(s^2+1\right)\int t\cos t\, e^{-st}dt=\left(t\sin t-(1-st)\cos t\right)e^{-st}+\frac{2s\left(\sin t+s\cos t\right)}{s^2+1}e^{-st}$$

$$\int t\cos t\, e^{-st}dt=\frac{1}{s^2+1}\left(t\sin t-(1-st)\cos t\right)e^{-st}+\frac{2s\left(\sin t+s\cos t\right)}{\left(s^2+1\right)^2}e^{-st}$$

$$F(s)=\left(\frac{t\sin t-(1-st)\cos t}{s^2+1}+\frac{2s\left(\sin t+s\cos t\right)}{\left(s^2+1\right)^2}\right)e^{-st}\Big|_0^{\infty}$$

$$=\frac{1}{s^2+1}+\frac{2s^2}{\left(s^2+1\right)^2}$$

$$=\frac{-s^2-1+2s^2}{\left(s^2+1\right)^2}$$

$$=\boxed{\frac{s^2-1}{\left(s^2+1\right)^2}}$$

	$\int \cos t\ dt$
e^{-st}	$\sin t$
$-se^{-st}$	$-\cos t$
s^2e^{-st}	

Exercise

Use Definition of Laplace transform to find the Laplace transform of $f(t)=2t^4$

Solution

$$F(s) = \int_0^\infty 2t^4\, e^{-st} dt$$

$$= 2\left(-\frac{t^4}{s} - \frac{4t^3}{s^2} - \frac{12t^2}{s^3} - \frac{24t}{s^4} - \frac{24}{s^5} \right) e^{-st} \Big|_0^\infty$$

$$= 2\left(0 + \frac{24}{s^5} \right)$$

$$\boxed{= \frac{48}{s^5}}$$

	$\int e^{-st} dt$
t^4	$-\frac{1}{s} e^{-st}$
$4t^3$	$\frac{1}{s^2} e^{-st}$
$12t^2$	$-\frac{1}{s^3} e^{-st}$
$24t$	$\frac{1}{s^4} e^{-st}$
24	$-\frac{1}{s^5} e^{-st}$

Exercise

Use Definition of Laplace Transform to show the Laplace transform of $f(t) = \cos \omega t$ is $F(s) = \frac{s}{s^2 + \omega^2}$

Solution

$$F(s) = \int_0^\infty (\cos \omega t) e^{-st} dt$$

$$\int \cos \omega t\ e^{-st} dt = \frac{1}{\omega} e^{-st} \sin \omega t - \frac{s}{\omega^2} e^{-st} \cos \omega t + \frac{s^2}{\omega^2} \int e^{-st} \cos \omega t\ dt$$

$$\left(1 - \frac{s^2}{\omega^2} \right) \int e^{-st} \cos \omega t\ dt = \frac{1}{\omega} e^{-st} \sin \omega t - \frac{s}{\omega^2} e^{-st} \cos \omega t$$

$$\left(\frac{\omega^2 - s^2}{\omega^2} \right) \int e^{-st} \cos \omega t\ dt = \frac{1}{\omega} \left(\sin \omega t - \frac{s}{\omega} \cos \omega t \right) e^{-st}$$

$$\int e^{-st} \cos \omega t\ dt = \frac{\omega^2}{\omega^2 - s^2} \frac{1}{\omega^2} (\omega \sin \omega t - s \cos \omega t) e^{-st}$$

$$= \frac{e^{-st}}{\omega^2 - s^2} (\omega \sin \omega t - s \cos \omega t)$$

		$\int \cos \omega t\ dt$
+	e^{-st}	$\frac{1}{\omega} \sin \omega t$
−	$-se^{-st}$	$-\frac{1}{\omega^2} \cos \omega t$
+	$s^2 e^{-st}$	$-\frac{1}{\omega^2} \int \cos \omega t$

$$F(s) = \lim_{T \to \infty} \frac{e^{-st}}{\omega^2 - s^2} (\omega \sin \omega t - s \cos \omega t) \Big|_0^T$$

$$= \lim_{T \to \infty} \left[\frac{e^{-sT}}{\omega^2 - s^2} (\omega \sin \omega T - s \cos \omega T) - \frac{1}{\omega^2 - s^2} (\omega \sin 0 - s \cos 0) \right]$$

$$= 0 - \frac{1}{\omega^2 - s^2} (-s)$$

$$\lim_{T \to \infty} e^{-sT} = \lim_{T \to \infty} \frac{1}{e^{-sT}} = 0$$

$$\boxed{= \frac{s}{s^2 + \omega^2}} \quad s > 0$$