Math 2312 – Pre-Calculus **Exam 1** **Review**

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1. Find the domain of:

   a) $f(x)=\dfrac{\sqrt{2x+3}}{x^2-6x+5}$ *b)* $f(x)=\dfrac{1}{(x+4)\sqrt{x-2}}$

2. For the function f given by $f(x)=-x^2+x+5$, find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

3. Let $f(x)=\sqrt{x+4}$

   a) Sketch the graph of f.

   b) Find the domain and range of f.

   c) Find the intervals on which f is increasing or is decreasing

4. Explain how the graph $y=-2f\left[\frac{1}{2}(x-3)\right]+5$ compares to the graph of $y=f(x)$

5. Sketch the graph $f(x)=\begin{cases} x-3 & \text{if} \quad x\le -2 \\ -x^2 & \text{if} \quad -2<x<1 \\ -x+4 & \text{if} \quad x\ge 1 \end{cases}$

6. Let $f(x)=\sqrt{3x+2}$ *and* $g(x)=\dfrac{1}{x^2}$

   a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

   b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

7. Let $f(x)=\sqrt{25-x^2}$ *and* $g(x)=\sqrt{x-3}$

   a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

   b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

8. Let $f(x)=\dfrac{x-1}{x-2}$ *and* $g(x)=\dfrac{x-3}{x-4}$

   a) Find $(f\circ g)(x)$ and the domain of $f\circ g$

   b) Find $(g\circ f)(x)$ and the domain of $g\circ f$

9. Let $f(x)=\frac{1}{15}\left(x^5-20x^3+64x\right)$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

10. Find the quotient and remainder if $f(x)=3x^5-4x^3+x+5$ is divided by $p(x)=x^3-2x+7$

11. Find the zeros of $f(x)=\left(x^2-2x+1\right)^2\left(x^2+2x-3\right)$, and state the multiplicity of each zero.

12. Find all solutions of the equation: $x^4+9x^3+31x^2+49x+30=0$

13. Find the vertical asymptotes, horizontal asymptotes, oblique asymptotes, intercepts, and holes (if any) of:

a) $y=\dfrac{x-2}{x^2-4x+3}$

b) $y=\dfrac{(x+2)(x-1)}{x^2-3x-10}$

c) $f(x)=\dfrac{x^3-2x^2-4x+8}{x-2}$

14. Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: x=-3 \\ \textit{horizontal asymptote}: y=\frac{3}{2} \\ x-\textit{intercept}: 5 \\ \textit{hole at } x=2 \end{cases}$$

15. Let $f(x)=2x^3-5$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists.

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

d) Sketch $f(x)$ and $f^{-1}(x)$

16. Let $f(x)=\dfrac{2x-7}{9x+1}$

a) Is $f(x)$ one-to-one function

b) Find $f^{-1}(x)$, if it exists.

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

d) Sketch $f(x)$ and $f^{-1}(x)$

17. Let $f(x)=3^{-x^2}$, determine the asymptote, domain, range, increasing and decreasing, and sketch $f(x)$

18. Let $f(x)=\log_2(x+3)$, determine the asymptote, domain, range, increasing and decreasing, and sketch $f(x)$

19. Solve the equations:

a) $2^{5x+3}=3^{2x+1}$

b) $8^{2x}\left(\frac{1}{4}\right)^{x-2}=4^{-x}\left(\frac{1}{2}\right)^{2-x}$

c) $3^{2x-1}=\frac{1}{3}$

d) $2\ln(x+3)-\ln(x+1)=3\ln 2$

e) $\log_4(x+1)=2+\log_4(3x-2)$

f) $x^2\left(-2xe^{-x^2}\right)+2xe^{-x^2}=0$

g) $e^x+2=8e^{-x}$

h) $\log\sqrt{x}=\sqrt{\log x}$

i) $\log\left(x^2\right)=(\log x)^2$

j) $\log_2(x+3)=\log_2(x-3)+\log_3 9+4^{\log_4 3}$

20. Solve the equation for ***x*** in terms of ***y***.

a) $y=\dfrac{1}{10^x+10^{-x}}$

b) $y=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$

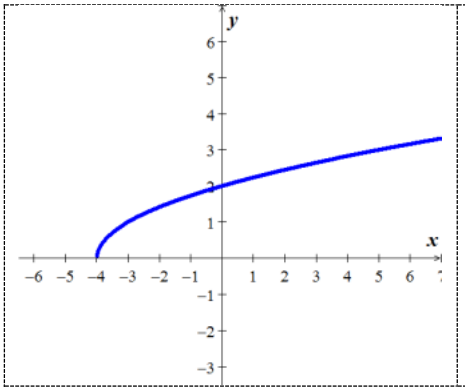
21. Express the following in terms of sums and differences of logarithms $\log_a \sqrt[4]{\dfrac{m^8 n^{12}}{a^3 b^5}}$

Solution

1. $a)\left[-\frac{3}{2},1\right)\cup(1,5)\cup(5,\infty)$ $\quad b)\{x>2\}$

2. $-2x-h+1$

3.



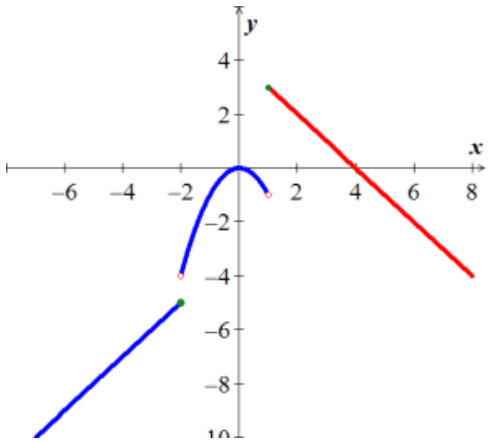
Domain:
$x+4\geq 0 \Rightarrow \boxed{x\geq -4}$

Range: $y\geq 0$

The function is increasing on $[-4,\ \infty)$

4. Reflected across x-axis, stretched vertically by 2 units, stretched horizontally by $\frac{1}{2}$ units, shifted right 3 units, and up 5 units.

5.



6. $a)\ f\left(g(x)\right)=\frac{\sqrt{2x^2+3}}{x}$ $\quad Domain:\{x\neq 0\}$

$b)\ g\left(f(x)\right)=\frac{1}{3x+2}$ $\quad Domain:\left\{x\neq -\frac{2}{3}\right\}$

7. $a)\ f\left(g(x)\right)=\sqrt{28-x}$ $\quad Domain:\{3\leq x\leq 28\}$

$b)\ f\left(g(x)\right)=\sqrt{\sqrt{25-x^2}-3}$ $\quad Domain:\{-4\leq x\leq 4\}$

8. $a)\ f\left(g(x)\right)=\frac{1}{-x+5}$ $\quad Domain:\{x\neq 4,\ 5\}$

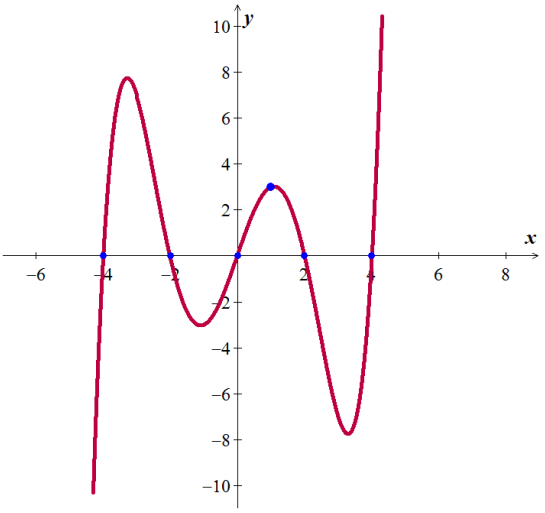
b) $f\left(g(x)\right)=\frac{-2x+5}{-3x+7}$ $\quad$ $Domain:\left\{x\neq 2,\ \frac{7}{3}\right\}$

9. $f(x)=\frac{1}{15}x\left(x^4-20x^2+64\right)\rightarrow$ $\quad Zeros: 0,\ \pm 2,\ \pm 4$

	−4		−2		0		2		4	
−		+		−		+		−		+

$f(x)>0 \quad -4<x<-2;\ 0<x<2;\ x>4$

$f(x)<0 \quad x<-4;\ -2<x<0;\ 2<x<4$



10. $Q(x)=3x^2+2$ $\quad$ $R(x)=-21x^2+5x-9$

11. $x=1$ *(multiplicity of 5)*; $x=-3$ *(multiplicity of 1)*

12. $-3,\ -2,\ -2\pm i$

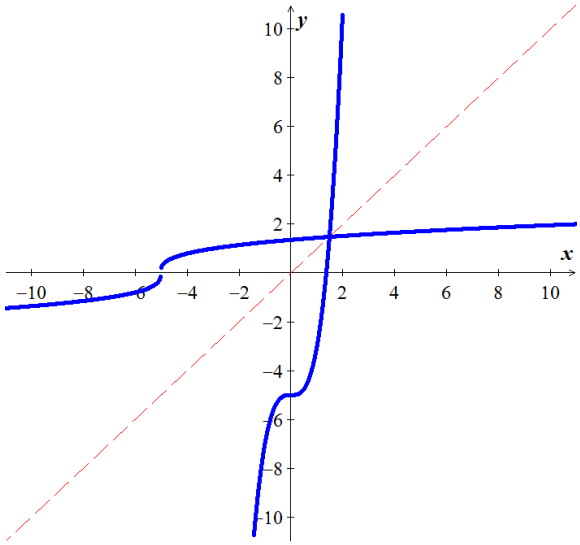
13. *a*) $VA:\ x=1,\ x=3,;$ $\quad HA:\ y=0;$ $\quad x-\text{int}:\ 2;$ $\quad y-\text{int}:\ -\frac{2}{3}$ $\quad Hole:\ n/a$ $\quad OA:\ n/a$

b) $VA:\ x=5;$ $\quad HA:\ y=\frac{4}{3};$ $\quad x-\text{int}:\ 1;$ $\quad y-\text{int}:\ \frac{4}{15};$ $\quad hole:\left(-2,\ \frac{4}{7}\right)$ $\quad OA:\ n/a$

c) $VA: n/a;$ $\quad HA: n/a$ $\quad x-\text{int}:\ -2;$ $\quad y-\text{int}:\ -4;$ $\quad hole:(2,\ 0)\ O\ A:\ y=x^2-4$

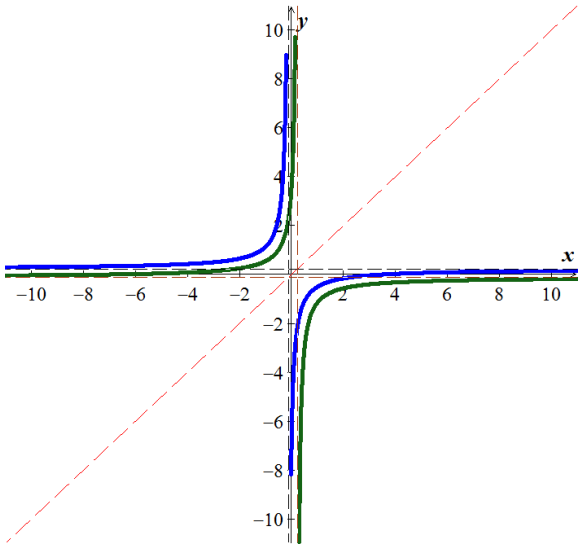
14. $f(x)=\frac{3x^2-21x+30}{2x^2+2x-12}$

15. *a*) Yes $\quad$ *b*) $f^{-1}(x)=\sqrt[3]{\frac{x+5}{2}}$ $\quad$ *c*) Domain & Range of $f(x)$ and $f^{-1}(x)$: □



16. *a*) Yes *b*) $f^{-1}(x)=\frac{-x-7}{9x-2}$ *c*) Domain of $f(x)$ = Range of $f^{-1}(x)$: $\mathbb{R}-\left\{-\frac{1}{9}\right\}$

Range of $f(x)$ = Domain of $f^{-1}(x)$: $\mathbb{R}-\left\{\frac{2}{9}\right\}$



17.

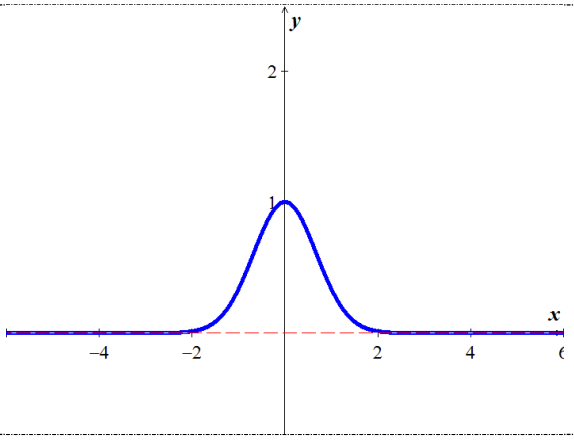
Asymptote: $y = 0$

Domain: $(-\infty,\infty)$

Range: $(0,1]$

Incr.: $(-\infty,\ 0)$

Decr.: $(0,\ \infty)$



18.

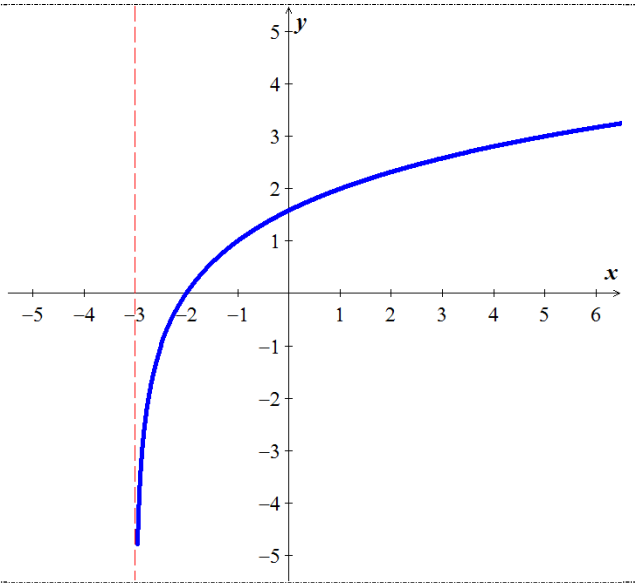
Asymptote: $x = -3$

Domain: $(-3, \infty)$

Range: $(-\infty, \infty)$

Incr.: $(-3, \infty)$

Decr.: None.



19. *a*) $x = \dfrac{\ln 3 - 3\ln 2}{5\ln 2 - 2\ln 3} = \dfrac{\ln\left(\frac{3}{8}\right)}{\ln\left(\frac{32}{9}\right)} \approx -0.7732$

b) $x = -\dfrac{6}{5}$

c) $x = 0$

d) $x = 1$

e) $x = \dfrac{33}{47}$

f) $x = 0,\ \pm 1$

g) $x = \ln 2$

h) $x = 1,\ 10{,}000$

i) $x = 1,\ 100$

j) $x = \dfrac{99}{31}$

20. *a*) $x = \log\left(\dfrac{1 \pm \sqrt{1 - 4y^2}}{2y}\right)$ *b*) $x = \dfrac{1}{2}\ln\left(\dfrac{1+y}{1-y}\right)$

21. $2\log_a m + 3\log_a n - \dfrac{3}{4} - \dfrac{5}{4}\log_a b$