Math 2414 – Calculus II ***Exam* 2** ***Review***

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1. Evaluate the following using integration by parts.

a) $\int x^2 \ln x dx$

b) $\int (x+1)^2 e^x dx$

c) $\int e^{-2x} \sin 3x dx$

d) $\int x^2 \cos x dx$

2. Evaluate the integrals.

a) $\int \sin^3 x \cos^4 x dx$

b) $\int \sin^5 x \cos^5 x dx$

c) $\int_0^{\pi/2} \cos^4 x dx$

d) $\int_0^{\pi/6} \sin^5 x dx$

e) $\int \tan^4 u du$

3. Evaluate using a trigonometric substitution

a) $\int \frac{y dy}{\sqrt{16-y^2}}$

b) $\int \frac{x dx}{\sqrt{x^2+4}}$

c) $\int \frac{x dx}{\sqrt{4x^2-1}}$

d) $\int \frac{dy}{y^2\sqrt{9-y^2}}$

4. Express the integrand as a sum of partial fractions and evaluate the integrals

a) $\int \frac{x dx}{x^2+4x+3}$

b) $\int \frac{x+1}{x^2(x-1)} dx$

c) $\int \frac{x+3}{2x^3-8x} dx$

d) $\int \frac{2x^3+x^2-21x+24}{x^2+2x-8} dx$

5. Evaluate the improper integrals

a) $\int_1^\infty \frac{dx}{(x+1)^9}$

b) $\int_0^\infty xe^{-x} dx$

c) $\int_0^1 \ln x dx$

d) $\int_1^\infty \frac{3x-1}{4x^3-x^2} dx$

e) $\int_{-\infty}^\infty \frac{4dx}{x^2+16}$

6. Evaluate the integrals.

a) $\int \theta \cos(2\theta + 1)\, d\theta$

b) $\int \frac{x+1}{x^2\left(x^2+4\right)}\, dx$

c) $\int \frac{1+x^2}{(x+1)^3}\, dx$

d) $\int \sqrt{x} \cdot \sqrt{1+\sqrt{x}}\, dx$

7. Solve the differential equation

a) $x(x-1)dy - ydx = 0$

b) $xy' + 2y = 1 - x^{-1}$

c) $xy' - y = 2x \ln x$

d) $\left(1+e^x\right)dy + \left(ye^x + e^{-x}\right)dx = 0$

e) $\left(x + 3y^2\right)dy + ydx = 0$

8. Solve the differential equation

a) $\frac{dy}{dx} + 3x^2 y = x^2, \quad y(0) = -1$

b) $xdy + (y - \cos x)dx = 0, \quad y\left(\frac{\pi}{2}\right) = 0$

c) $\frac{dy}{dt} = \frac{t+1}{2ty}, \quad y(1) = 4$

d) $\frac{dy}{dt} = \sqrt{y}\, sint, \quad y(0) = 4$

9. Find the length of the graph of the function $y = \ln\left(1 - x^2\right), \quad 0 \le x \le \frac{1}{2}$

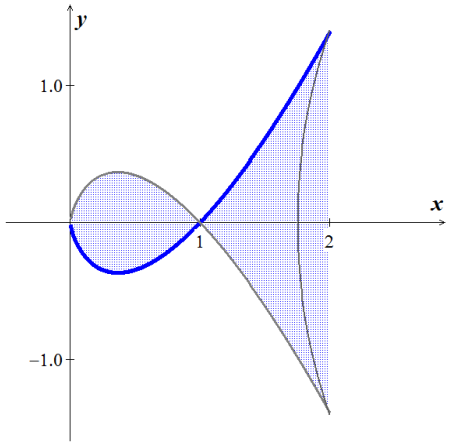
10. The region in the first quadrant that is enclosed by the *x*-axis, the curve $y = \frac{5}{x\sqrt{5-x}}$, and the lines $x = 1$ and $x = 4$ is revolved about the *x*-axis to generate a solid. Find the volume of the solid.

11. The region between the *x*-axis and the curve

$$f(x) = \begin{cases} 0, & x = 0 \\ x \ln x, & 0 < x \le 2 \end{cases}$$

is revolved about the *x*-axis to generate the solid.

Find the volume of the solid.



12. A tank with a 2,000 *gal* capacity initially contains 500 *gal* of brine containing 100 *lbs*. of salt starting at time *t* = 0, brine containing 0.1 *lb/gal* of salt is added at a rate of 60 *gal/min* and the mixed solution is drained off at a rate of 40 *gal/min*. How much salt is in the tank when it reaches the point of over flowing?

13. An object of mass ***m*** is released from a balloon. Find the distance it falls in ***t*** seconds, if the force of resistance due to the air is directly proportional to the speed of the object.

Solution

1. *a*) $\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+C$ *b*) $\left(x^2+1\right)e^x+C$

c) $-\frac{2}{13}e^{-2x}\sin 3x-\frac{3}{13}e^{-2x}\cos 3x+C$ *d*) $\left(x^2-2\right)\sin x+2x\cos x+C$

2. *a*) $-\frac{1}{5}\cos^5 x+\frac{1}{7}\cos^7 x+C$ *b*) $\frac{1}{6}\sin^6 x-\frac{1}{4}\sin^8 x+\frac{1}{10}\sin^{10} x+C$

c) $\frac{3\pi}{16}$ *d*) $\frac{256-147\sqrt{3}}{480}$ *e*) $\frac{1}{3}\tan^3 u-\tan u+u+C$

3. *a*) $-\sqrt{16-y^2}+C$ *b*) $\sqrt{4+x^2}+C$

c) $\frac{1}{4}\tan\theta+C$ *d*) $-\frac{1}{9}\frac{\sqrt{9-y^2}}{y}+C$

4. *a*) $\frac{2}{3}\ln|x+3|-\frac{1}{2}\ln|x+1|+C$ *b*) $-2\ln|x|+\frac{1}{x}+2\ln|x-1|+C$

c) $\frac{1}{2}\left[-\frac{3}{4}\ln|x|+\frac{5}{8}\ln|x-2|+\frac{1}{8}\ln|x+2|\right]+C$ *d*) $x^2-3x+\frac{2}{3}\ln|x+4|+\frac{1}{3}\ln|x-2|+C$

5. *a*) $\frac{1}{2048}$ *b*) 1 *c*) -1 *d*) $\ln\frac{1}{4}-\ln\frac{1}{3}+1=1+\ln\frac{3}{4}$ *e*) π

6. *a*) $\frac{1}{2}\theta\sin(2\theta+1)+\frac{1}{4}\cos(2\theta+1)+C$

b) $\frac{1}{4}\ln|x|-\frac{1}{4x}-\frac{1}{8}\ln\left(x^2+4\right)-\frac{1}{4}\tan^{-1}\frac{x}{2}+C$

c) $\ln|x+1|+\frac{2}{x+1}-\frac{1}{(x+1)^2}+C$

d) $\frac{4}{3}x\left(1+\sqrt{x}\right)^{3/2}-\frac{16}{15}\sqrt{x}\left(1+\sqrt{x}\right)^{5/2}+\frac{32}{105}\left(1+\sqrt{x}\right)^{7/2}+C$

7. *a*) $y(x)=C_1\frac{x-1}{x}$ *b*) $y(x)=\frac{1}{2}-\frac{1}{x}+\frac{C}{x^2}$ *c*) $y(x)=x(\ln x)^2+Cx$

d) $y(x)=\frac{e^{-x}+C}{1+e^x}$ *e*) $xy=-y^3+C$

8. *a*) $y(x)=\frac{1}{3}-\frac{4}{3e^{x^3}}$ *b*) $y(x)=\frac{\sin x-1}{x}$

c) $y(t)=\sqrt{t+\ln|t|+15}$ *d*) $y(t)=\frac{1}{4}(5-\cos t)^2$

9. $L=\ln 3-\frac{1}{2}$

10. $V=\pi\left(2\ln 4+\frac{15}{4}\right)$

11. $V=\frac{2\pi}{3}\left(4(\ln 2)^2-\frac{4}{3}\ln 2+\frac{4}{9}\right)$

12. $y(t)=\frac{1}{10}(500+20t)+\frac{12,500,000}{(500+20t)^2}$

13. $s(t)=\frac{mg}{k}t+\frac{m^2g}{k^2}e^{-kt/m}-\frac{m^2g}{k^2}$