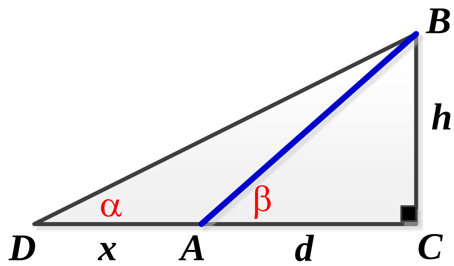


Proof

Triangle DCB: $\tan\alpha = \dfrac{h}{d+x} \quad \Rightarrow \quad h = (d+x)\tan\alpha$

Triangle ACB: $\tan\beta = \dfrac{h}{d} \quad \Rightarrow \quad h = d\tan\beta$

$$h = d\tan\beta = (d+x)\tan\alpha$$

$$d\tan\beta = d\tan\alpha + x\tan\alpha$$

$$d\tan\beta - d\tan\alpha = x\tan\alpha$$

$$d(\tan\beta - \tan\alpha) = x\tan\alpha$$

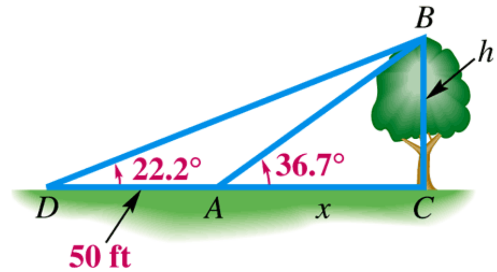
$$d = \frac{x\tan\alpha}{\tan\beta - \tan\alpha}$$

$$\boxed{h = \frac{x\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}}$$

Height is equal to distance times (tan tan) divides by the (tan(larger angle) – tan) (difference between tangents)

Example

From a given point on the ground, the angle of elevation to the top of a tree is 36.7°. From a second point, 50 feet back, the angle of elevation to the top of the tree is 22.2°. Find the height of the tree to the nearest foot.



Solution

$$h = 50\frac{\tan 22.2^\circ \ \tan 36.7^\circ}{\tan 36.7^\circ - \tan 22.2^\circ} \approx 45 \ ft$$