SOLUTION   *Section* **4.2 – Representing Relations**

Exercise

Represent each of these relations on {1, 2, 3} with a matrix (with the elements of this set listed in increasing order). Then draw the directed graphs representing each relation
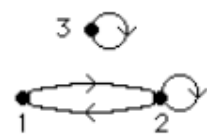
   a) {(1, 1), (1, 2), (1, 3)}
   b) {(1, 2), (2, 1), (2, 2), (3, 3)}
   c) {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}
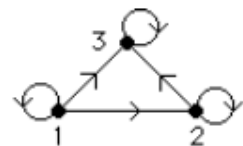   *d)* {(1, 3), (3, 1)}

Solution

a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

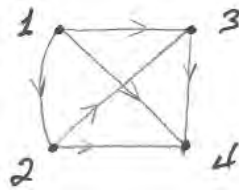*d)* $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Exercise

Represent each of these relations on {1, 2, 3, 4} with a matrix (with the elements of this set listed in increasing order). Then draw the directed graphs representing each relation
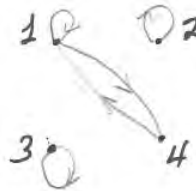
   a) {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}
   b) {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)}
   c) { (1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}
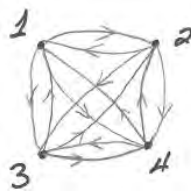   *d)* {(2, 4), (3, 1), (3, 2), (3, 4)}

Solution

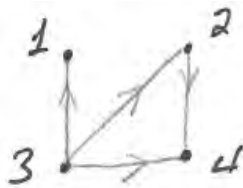a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Exercise

List the ordered pairs in the relations on $\{1, 2, 3\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order). Then draw the directed graphs representing each relation

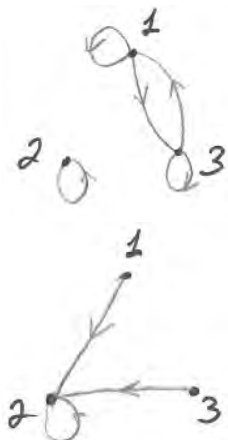a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
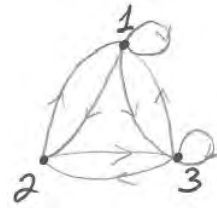c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Solution

a) $\{(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)\}$

b) $\{(1, 2), (2, 2), (3, 2)\}$

*c)* {(1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (3, 3)}

Exercise

List the ordered pairs in the relations on {1, 2, 3, 4} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order). Then draw the directed graphs representing each relation
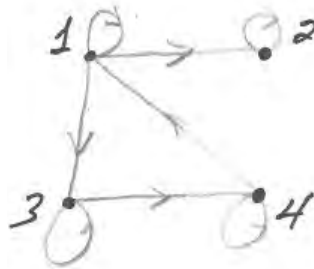
a) $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$
c) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$
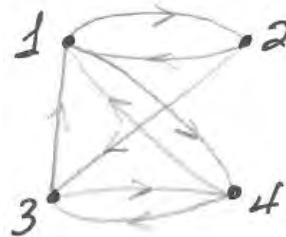
Solution

a) {(1, 1), (1, 2), (1, 4), (2, 1), (2, 3), (3, 2), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4)}
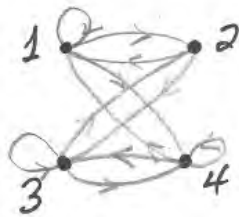
b) {(1, 1), (1, 2), (1, 3), (2, 2), (3, 3), (3, 4), (4, 1), (4, 4)}

c) {(1, 2), (1, 4), (2, 1), (2, 3), (3, 1), (3, 4), (4, 1), (4, 3)}

Exercise

Let R be the relation represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Find: a) R^2 b) R^3 c) R^4

Solution

a) $M_{R^2} = M^2_R = M_R \odot M_R$

$$R^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

b) $M_{R^3} = M^3_R = M_R \odot M^2_R$

$$R^3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

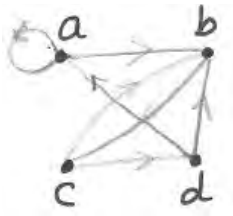$$= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

c) $M_{R^4} = M^{(4)}_R = M_R \odot M^{(3)}_R$

$$R^4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Exercise

Draw the directed graph that represents the relation $\{(a, a), (a, b), (b, c), (c, b), (c, d), (d, a), (d, b)\}$
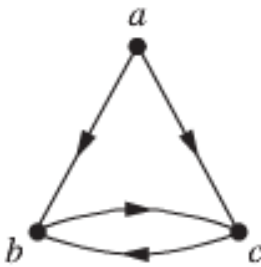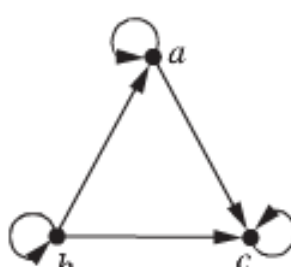
Solution

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Exercise

Determine whether the relations represented by the directed graphs are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive
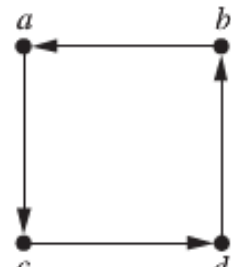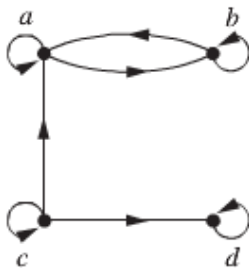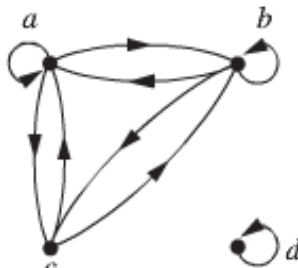
a)

b)

c)

d)

e)

f)

Solution

a) $\{(a, b), (a, c), (b, c), (c, b)\}$

It is not reflexive since (a, a) doesn't exist

It is not symmetric

It is transitive since $(a, b), (b, c) \Rightarrow (a, c)$

b) $\{(a, a), (a, c), (b, a), (b, b), (b, c), (c, c)\}$

It is reflexive

It is not symmetric

It is transitive since $(b, a), (a, c) \Rightarrow (b, c)$

c) $\{(a, c), (b, a), (c, d), (d, b)\}$

It is not reflexive; it is not symmetric, and not transitive since

d) $\{(a, a), (a, b), (b, a), (b, b), (c, a), (c, c), (c, d), (d, d)\}$

It is reflexive, not symmetric (no (a, c)), and not transitive (c, a), (a, b) but no (c, b)

e) $\{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (d, d)\}$

It is not reflexive; it is symmetric and transitive

f) $\{(a, a), (a, b), (b, a), (b, b), (c, c), (c, d), (d, d), (d, c)\}$
It is reflexive; it is symmetric and transitive