Hwk -1.1

1- $\int 5\pi\, dx = 5\pi x + C$

2- $\int (x+7)\, dx = \frac{1}{2}x^2 + 7x + C$

3- $\int (13-x)\, dx = 13x - \frac{1}{2}x^2 + C$

4- $\int (2x-3x^2)\, dx = x^2 - x^3 + C$

5- $\int (8x^3 - 9x^2 + 4)\, dx = 2x^4 - 3x^3 + 4x + C$

6- $\int (x^5 - 4)\, dx = \frac{1}{6}x^6 - 4x + C$

7- $\int (6x^{3/2} - 7x + 2)\, dx = \frac{12}{5}x^{5/2} - \frac{7}{2}x^2 + 2x + C$

8- $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int \left(x^{1/2} + \frac{1}{2}x^{-1/2}\right) dx$

$= \frac{2}{3}x^{3/2} + x^{1/2} + C$

$= \frac{2}{3}x\sqrt{x} + \sqrt{x} + C$

9- $\int (\sqrt[3]{x^2})\, dx = \int x^{2/3}\, dx$

$= \frac{3}{5}x^{5/3} + C$

10- $\int (4\sqrt[4]{x^3} - 9x^3)\, dx = \int (4x^{3/4} - 9x^3)\, dx$

$= \frac{16}{7}x^{7/4} - \frac{9}{4}x^4 + C$

11- $\int \left(\frac{x+6}{\sqrt{x}}\right) dx = \int (x^{1/2} + 6x^{-1/2})\, dx$

$= \frac{2}{3}x^{3/2} + 12x^{1/2} + C$

12- $\int \left(\frac{x^2 - 2x - 3}{x^3}\right) dx = \int \left(\frac{1}{x} - 2\frac{1}{x^2} - 3x^{-3}\right) dx$

$= \ln|x| + \frac{2}{x} + \frac{3}{2}x^{-2} + C$

$\boxed{\int \frac{dx}{x} = \ln|x| \qquad \int \frac{dx}{x^2} = -\frac{1}{x}}$

13- $\int \frac{1}{2}\frac{dx}{x} = \frac{1}{2}\ln|x| + C$

14- $\int (2x^2 - 1)^2 dx = \int (4x^4 - 4x^2 + 1) dx$

$= \frac{4}{5}x^5 - \frac{4}{3}x^3 + x + C$

15- $\int (1 + 3t) t^2 dt = \int (t^2 + 3t^3) dt$

$= \frac{1}{3}t^3 + \frac{3}{4}t^4 + C$

16- $\int t^2 \sqrt{t}\, dt = \int t^{5/2} dt \qquad t^2 t^{1/2} \quad t^{2+\frac{1}{2}}$

$= \frac{2}{7} t^{7/2} + C$

17- $\int (5\cos x + 4\sin x) dx = 5\sin x - 4\cos x + C$

18- $\int (x^2 - \cos x) dx = \frac{1}{3}x^3 - \sin x + C$

19- $\int (1 - \csc x \cot x) dx = x + \csc x + C$

20- $\int (\theta^2 + \sec^2\theta) d\theta = \frac{1}{3}\theta^3 + \tan\theta + C$

21- $\int (\sec^2\theta - \sin\theta) d\theta = \tan\theta + \cos\theta + C$

22- $\int \sec\theta(\tan\theta - \sec\theta)\,d\theta = \int (\sec\theta\tan\theta - \sec^2\theta)\,d\theta$

$= \sec\theta - \tan\theta + C$

23- $\int \frac{\cos x}{1-\cos^2 x}\,dx = \int \frac{\cos x}{\sin^2 x}\,dx$

$= \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\,dx$

$= \int \cot x \cdot \csc x\,dx$

$= -\csc x + C$

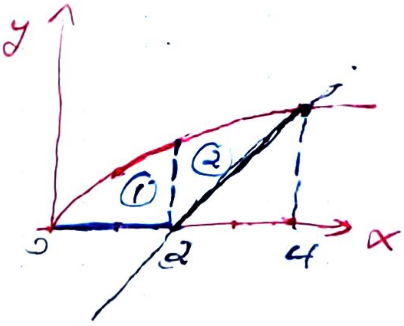
24- $\int 5e^{5x}\,dx = e^{5x} + C$

25- $\int e^{-2x}\,dx = -\frac{1}{2}e^{-2x} + C$

Sec 4.4 cont.

Ex. A? QI $y=\sqrt{x}$ x-axis $y=0$ $y=x-2$

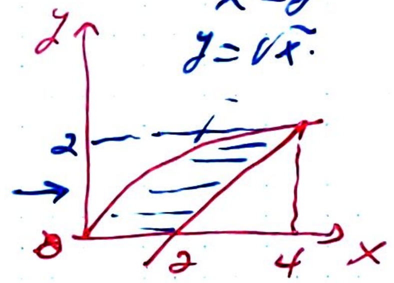
$$y=(\sqrt{x})^2=(x-2)^2$$
$$x=x^2-4x+4$$
$$x^2-5x+4=0$$
$$x=1,4$$



$$A=\int_0^2 x^{1/2}dx+\int_2^4 (x^{1/2}-x+2)dx$$
$$=\frac{2}{3}x^{3/2}\Big|_0^2+\left(\frac{2}{3}x^{3/2}-\frac{1}{2}x^2+2x\right)\Big|_2^4$$

$(4)^{3/2}$ $(2^2)^{3/2}$

$$=\frac{2}{3}2^{3/2}+\left(\frac{2}{3}(8)-8+8-\left(\frac{2}{3}2^{3/2}-2+4\right)\right)$$
$$=\frac{16}{3}-2$$
$$=\frac{10}{3}\text{ unit}^2$$



$x=y^2$
$y=\sqrt{x}$
$y=x-2$
$x=y+2$

$$\text{Area}=\int_0^2 (y+2-y^2)\,dy$$
$$=\frac{1}{2}y^2+2y-\frac{1}{3}y^3\Big|_0^2$$
$$=2+4-\frac{8}{3}$$
$$=\frac{10}{3}\text{ unit}^2$$

43 A? $f(x) = x^2 - 4x + 3 = 0 \quad 0 \le x \le 3$

$\underline{x = 1, 3}$

$$\text{Area} = \int_0^1 (x^2 - 4x + 3)\,dx - \int_1^3 (x^2 - 4x + 3)\,dx$$

$$= \frac{1}{3}x^3 - 2x^2 + 3x \Big|_0^1 - \left(\frac{1}{3}x^3 - 2x^2 + 3x\right)\Big|_1^3$$

$$= \underbrace{\frac{1}{3} - 2 + 3}_{} - \Big(\underbrace{9 - 18 + 9}_{f(3)} - \underbrace{\left(\frac{1}{3} - 2 + 3\right)}_{f(1)} \Big)$$

$$= 2\left(\frac{1}{3} + 1\right)$$

$$= \frac{8}{3} \text{ unit}^2$$

4-5

$[-a, a]$ $\quad f(x)$ is even: $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)$

$f(x)$ is odd: $\int_{-a}^{a} f(x)\,dx = 0$

Ex $\int_{-2}^{2} (x^4 - 4x^2 + 6)\,dx = 2\int_0^2 (x^4 - 4x^2 + 6)\,dx$

$$= 2\left(\frac{1}{5}x^5 - \frac{4}{3}x^3 + 6x\right)\Big|_0^2$$

$32\left(\frac{1}{5} - \frac{1}{3}\right)$

$$= 2\left(\frac{32}{5} - \frac{32}{3} + 12\right)$$

$$= 2\left(32\,\frac{-2}{15} + 12\right)$$

$$= 8\left(-\frac{16}{15} + 3\right)$$

$$= 8\left(\frac{29}{15}\right)$$

$$= \frac{232}{15}$$

$\begin{matrix}45\\ 16\end{matrix}$

#5/ $\int_{-200}^{200} 2x^5\,dx = 0$ $\quad$ odd fct.

#6/ $\int_{-\frac{\pi}{4}}^{\pi/4} \cos x\,dx = 2\int_0^{\pi/4} \cos x\,dx = 2\left(\sin x \Big|_0^{\pi/4}\right.$

$= \sqrt{2}$

#7/ $\int_{-2}^{2} (x^9 - 3x^5 + 2x^2 - 10)\,dx = \int_{-2}^{2} (x^9 - 3x^5)\,dx + \int_{-2}^{2} (2x^2 - 10)\,dx$

$= 2\int_0^2 (2x^2 - 10)\,dx$

$= 2\left(\frac{2}{3}x^3 - 10x\Big|_0^2\right.$

$= 2\left(\frac{16}{3} - 20\right)$

$= -\frac{88}{3}$

4.6 Substitution Rule

u

$$\frac{d}{dx}\left(\frac{u^{n+1}}{n+1}\right) = u^n \frac{du}{dx}$$

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

Ex $\int (x^3+x)^5 (3x^2+1)\,dx$

$u = x^3 + x$

$du = (3x^2+1)\,dx$

$$\int (x^3+x)^5 (3x^2+1)\,dx = \int u^5 \, du$$
$$= \frac{1}{6} u^6 + C$$
$$= \frac{1}{6}(x^3+x)^6 + C$$

$$\int (x^3+x)^5 (3x^2+1)\,dx = \int (x^3+x)^5 \, d(x^3+x) \qquad d(x^3+x) = (3x^2+1)\,dx$$
$$= \frac{1}{6}(x^3+x) + C$$

#7 $\int \theta \sqrt[4]{1-\theta^2}\, d\theta = -\int \frac{1}{2}(1-\theta^2)^{1/4}\, d(1-\theta^2) \qquad d(1-\theta^2) = -2\theta\, d\theta$

$$= -\frac{2}{5}(1-\theta^2)^{5/4} + C$$

Ex $\int 5\sec^2(5t+1)\,dt$ $\quad d(5t+1) = 5\,dt$

$$= \int \sec^2(5t+1)\,d(5t+1)$$

$$= \tan(5t+1) + C$$

Ex $\int_0^{\ln 2} e^{3x}\,dx = \frac{1}{3} e^{3x}\Big|_0^{\ln 2}$ $\quad d(3x) = 3\,dx$

$$= \frac{1}{3}\left(e^{3\ln 2} - e^0\right)$$

$$= \frac{1}{3}\left(e^{\ln 2^3} - 1\right)$$

$$\boxed{e^{\ln x} = x}$$

$$= \frac{1}{3}(8-1)$$

$$= \frac{7}{3}$$

Ex $\int \cos(7\theta+3)\,d\theta = \frac{1}{7}\int \cos(7\theta+3)\,d(7\theta+3)$ $\quad d(7\theta+3) = 7\,d\theta$

$$= \frac{1}{7}\sin(7\theta+3) + C$$

Ex $\int x^2 \sin(x^3)\,dx = \frac{1}{3}\int \sin(x^3)\,d(x^3)$ $\quad d(x^3) = 3x^2\,dx$

$$= -\frac{1}{3}\cos(x^3) + C$$

$$\int x\sqrt{2x+1}\,dx \qquad u = 2x+1 \to x = \frac{u-1}{2}, \quad du = 2\,dx$$

$$\int x\sqrt{2x+1}\,dx = \int \tfrac{1}{2}(u-1)u^{1/2}\left(\tfrac{1}{2}\,du\right)$$

$$= \frac{1}{4}\int (u^{3/2} - u^{1/2})\,du$$

$$= \frac{1}{4}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + C$$

<u>Ex</u> $$\int \frac{2z\,dz}{\sqrt[3]{z^2+1}} \qquad d(z^2+1) = 2z\,dz$$

$$\int \frac{2z\,dz}{(z^2+1)^{1/3}} = \int (z^2+1)^{-1/3}\,d(z^2+1)$$

$$= \frac{3}{2}(z^2+1)^{2/3} + C$$

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx \qquad d(x^3+1) = 3x^2\,dx$$

$$= \int_{-1}^{1} (x^3+1)^{1/2}\,d(x^3+1)$$

$$= \frac{2}{3}(x^3+1)^{3/2}\Big|_{-1}^{1}$$

$$= \frac{2}{3}\left(2^{3/2} - 0\right)$$

$$= \frac{2^{5/2}}{3} \qquad 2^2\sqrt{2} \qquad \frac{4\sqrt{2}}{3}$$

Ex. $\displaystyle\int_{\pi/4}^{\pi/2} \cot\theta \csc^2\theta \, d\theta = \int_{\pi/4}^{\pi/2} \underline{\cot\theta \csc\theta} \csc\theta \, \underline{d\theta}$

$d(\csc\theta) = -\overbrace{\cot\theta \csc\theta \, d\theta}$.

$$\int_{\pi/4}^{\pi/2} \cot\theta \csc^2\theta \, d\theta = -\int_{-\pi/4}^{\pi/2} \csc\theta \, d(\csc\theta)$$

$$= -\frac{1}{2}\csc^2\theta \Big|_{-\pi/4}^{\pi/2} \qquad \left(\frac{1}{\sin\theta}\right)^2$$

$$= -\frac{1}{2}\left(1 - (\sqrt{2})^2\right) \rightarrow = 2$$

$$= \frac{1}{2}$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \quad \csc$$

$$\int_0^{\pi/6} \tan 2x \, dx = \frac{1}{2}\int_0^{\pi/6} \tan 2x \, d(2x) \qquad d(2x) = 2\,dx$$

$$= \frac{1}{2} \ln|\sec 2x| \Big|_0^{\pi/6} \qquad \frac{1}{\cos\frac{\pi}{3}}$$

$$= \frac{1}{2}\left(\ln 2 - \ln 1\right)$$

$$= \frac{1}{2}\ln 2$$

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \frac{1}{2}(1-\cos 2x)$$

$$\frac{1}{2} - \frac{1}{2}\cos 2x$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \sin^2 x\, dx = \frac{1}{2}\int (1-\cos 2x)\, dx$$

$$= \underline{\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C}$$

$$\int \cos^2 x\, dx = \frac{1}{2}\int (1+\cos 2x)\, dx$$

$$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C$$

$$= \underline{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}$$