Section 3.5 - Curve Sketching (*Summary*)

Example

Given $f(x) = -x^3 + 3x^2 + 9x - 27$

Solution

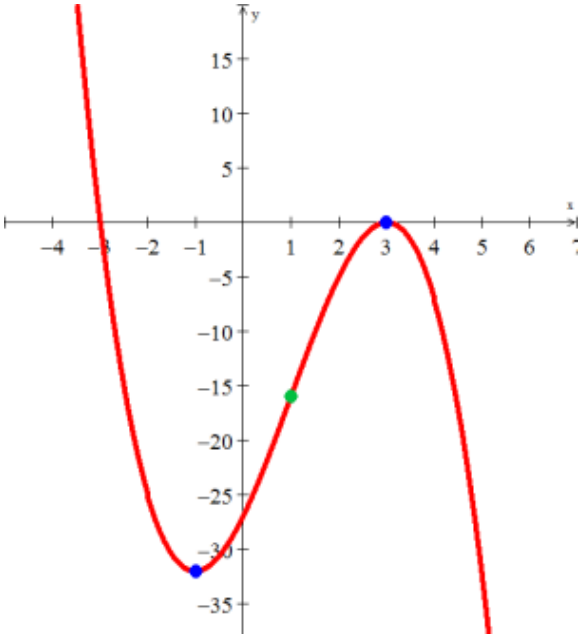
$$f'(x) = -3x^2 + 6x + 9 = 0$$

$$\Rightarrow \boxed{x = -1,\ 3}$$

$$f''(x) = -6x + 6 = 0$$

$$\Rightarrow -x + 1 = 0 \Rightarrow \boxed{x = 1}$$

	f	f'	f''	
(-∞, -1)		–	+	Decreasing, Concave up
x = -1	–32	0	+	Relative Min
(-1, 1)		+	+	Increasing, Concave up
x = 1	–16	+	0	Point of Inflection
(1, 3)		+	–	Increasing, Concave down
x = 3	0	0	–	Relative Max
(3, ∞)		–	–	Decreasing, Concave down



Example

Given $f(x)=\frac{x^2}{x-1}$

Solution

Vertical Asymptote: $x=1$

$$f'(x)=\frac{2x(x-1)-x^2}{(x-1)^2}$$

$$=\frac{2x^2-2x-x^2}{(x-1)^2}$$

$$=\frac{x^2-2x}{(x-1)^2}$$

$$=\frac{x(x-2)}{(x-1)^2}=0$$

$$\Rightarrow x=0,\ 2$$

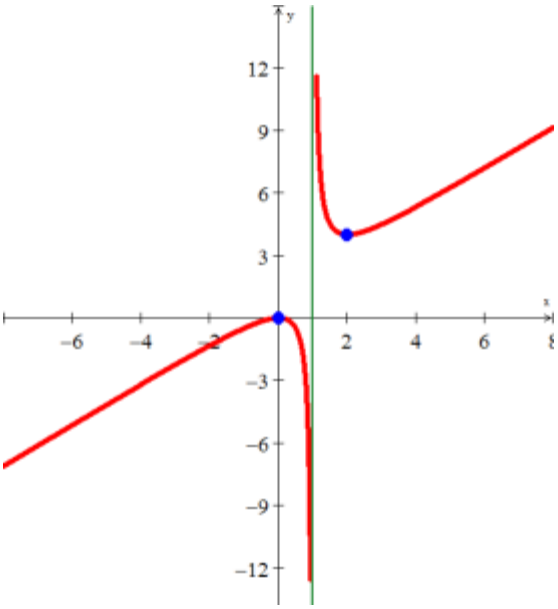
$$f''=\left(\frac{x^2-2x}{(x-1)^2}\right)'$$

$$=\frac{(2x-2)(x-1)^2-2(x^2-2x)(x-1)}{(x-1)^4}$$

$$=\frac{(x-1)\left[(2x-2)(x-1)-2(x^2-2x)\right]}{(x-1)^4}$$

$$=\frac{2x^2-2x-2x+2-2x^2+4x}{(x-1)^3}$$

$$=\frac{2}{(x-1)^3}$$



	f	f'	f''	
$(-\infty, 0)$		+	–	Increasing, Concave down
$x = 0$	0	0	–	RMAX
$(0, 1)$		–	–	Decreasing, Concave down
$x = 1$	*Undef.*	*Undef.*	*Undef.*	Vertical Asymptote
$(1, 2)$		–	+	Decreasing, Concave up
$x = 2$	4	0	+	RMIN
$(2, \infty)$		+	+	Increasing, Concave up

Example

Graph $f(x)=\dfrac{\ln x}{x^2}$

Solution

Domain: $x>0$

$$f'(x)=\frac{\frac{1}{x}x^2-2x\ln x}{x^4}$$

$$=\frac{x(1-2\ln x)}{x^4}$$

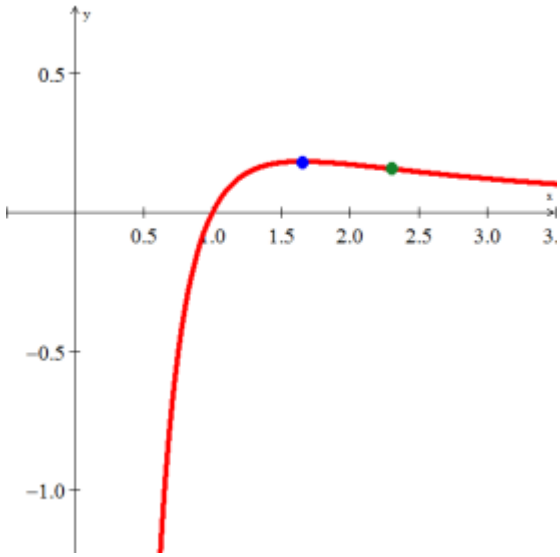
$$=\frac{1-2\ln x}{x^3}=0$$

$$\Rightarrow 1-2\ln x=0$$

$$\ln x=\frac{1}{2}\Rightarrow \boxed{x=e^{1/2}\approx 1.65}$$

$$f(1.65)=\frac{\ln 1.65}{1.65^2}=0.18$$

$(1.65,\ 0.18)$



$$f''(x)=\left(\frac{1-2\ln x}{x^3}\right)'$$

$$=\frac{-2\frac{1}{x}x^3-3x^2(1-2\ln x)}{x^6}$$

$$=\frac{x^2(-2-3+6\ln x)}{x^6}$$

$$=\frac{-5+6\ln x}{x^4}=0$$

$$-5+6\ln x=0$$

$$\ln x=\frac{5}{6}\Rightarrow \boxed{x=e^{5/6}\approx 2.3}$$

$$f(2.3)=\frac{\ln 1.65}{1.65^2}=0.16$$

$(2.3,\ 0.16)$

-∞	1.65	2.3	∞
$f'(1)>0$	$f'(2)<0$	$f'(3)<0$	
Increasing	***Decreasing***	***Decreasing***	
$f''(1)<0$	$f''(2)<0$	$f''(3)>0$	
Downward	***Downward***	***Upward***	

Exercises *Section* 3.5 - Curve Sketching

Graph

1. $f(x) = x^4 - 4x^3 + 5$

2. $f(x) = \frac{x^2+1}{x^2-1}$

3. $f(x) = 2x^{3/2} - 6x^{1/2}$