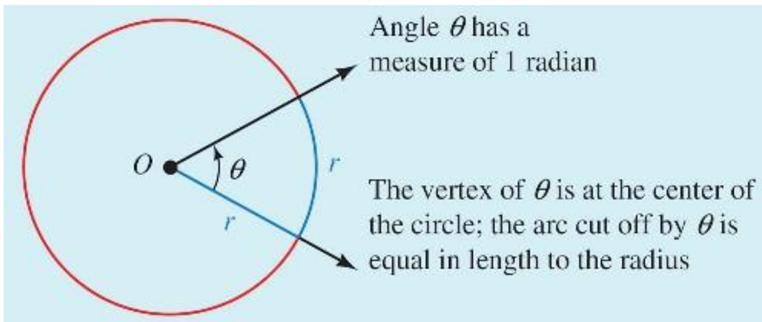
Lecture Two - Circular & Graph Functions

Section 2.1 - Radians & Degrees, Circular Functions

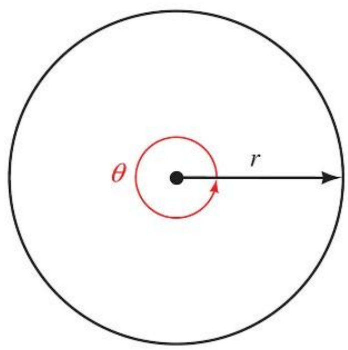
Radians

Definition

In a circle, a central angle that cuts off an arc equal in length to the radius of the circle has a measure of 1 radian (***rad***).



Degrees - Radians



θ *measures one full rotation* $\quad \theta = 2\pi \quad$ *The measure of* θ *in radians is* 2π

$$\boxed{1 = 1\ rad}$$

$$1^\circ = 1 \text{ degree}$$

If no unit of angle measure is specified, then the angle is to be measured in radians.

$$Full\ Rotation:\ 360^\circ = 2\pi\ rad$$

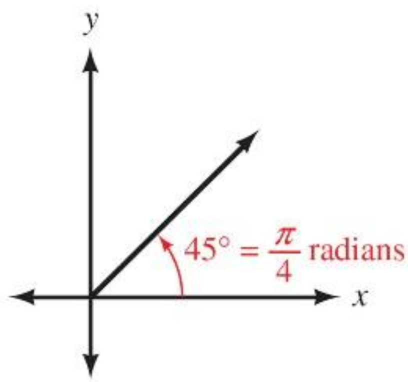
$$180^\circ = \pi\ rad$$

Converting from Degrees to Radians

$$\frac{180^\circ}{180} = \frac{\pi}{180}\ rad$$

$$\Rightarrow 1^\circ = \frac{\pi}{180}\ rad$$

Multiply a degree measure by $\frac{\pi}{180}\ rad$ and simplify to convert to radians.



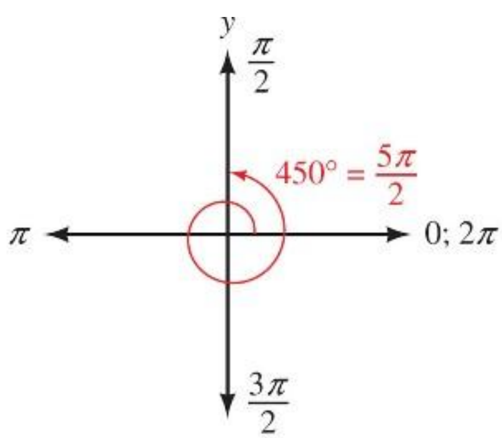
Example

Convert 45° to radians

<u>*Solution*</u>

$$45^\circ = 45\left(\frac{\pi}{180}\right) rad$$

$$= \frac{\pi}{4}\ rad$$



Example

Convert -450° to radians

<u>*Solution*</u>

$$-450^\circ = -450\left(\frac{\pi}{180}\right) rad$$

$$= -\frac{5\pi}{2}\ rad$$

Example

Convert 249.8° to radians

<u>*Solution*</u>

$$249.8^\circ = 249.8\left(\frac{\pi}{180}\right) rad$$

$$\approx 4.360\ rad$$

Converting from Radians to Degrees

Multiply a radian measure by $\frac{180^\circ}{\pi}$ radian and simplify to convert to degrees.

$$\frac{180^\circ}{\pi} = \frac{\pi}{\pi}\ rad$$

$$\left(\frac{180}{\pi}\right)^\circ = 1\ rad$$

Example

Convert 1 to degrees

Solution

$$\begin{aligned} 1\ rad &= 1\left(\frac{180}{\pi}\right)^\circ \\ &= 1\left(\frac{180}{3.14}\right)^\circ \\ &= 57.3^\circ \end{aligned}$$

Example

Convert $\frac{4\pi}{3}$ to degrees

Solution

$$\begin{aligned} \frac{4\pi}{3} &= \frac{4\pi}{3}\left(\frac{180}{\pi}\right)^\circ \\ &= 240^\circ \end{aligned}$$

Example

Convert-4.5 to degrees

Solution

$$\begin{aligned} -4.5 &= -4.5\left(\frac{180}{\pi}\right)^\circ \\ &\approx -257.8^\circ \end{aligned}$$

Equivalent Angle Measures in Degrees and Radians

Example

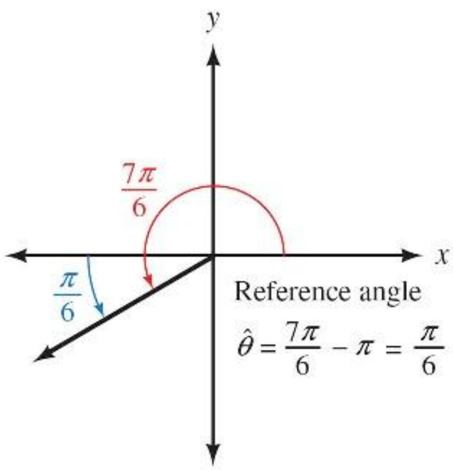
Find $\sin\frac{\pi}{6}$

Solution

$$\sin\frac{\pi}{6} = \sin 30^\circ$$
$$= \frac{1}{2}$$

Example

Find $4\sin\frac{7\pi}{6}$



Solution

$$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$$
$$4\sin\frac{7\pi}{6} = 4\left(-\sin\frac{\pi}{6}\right)$$
$$= 4\left(-\frac{1}{2}\right)$$
$$= -2$$

Example

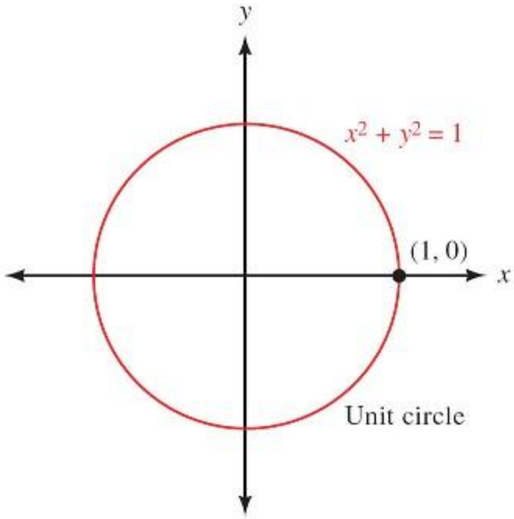
Evaluate $4\sin(2x+\pi)$ when $x = \frac{\pi}{6}$

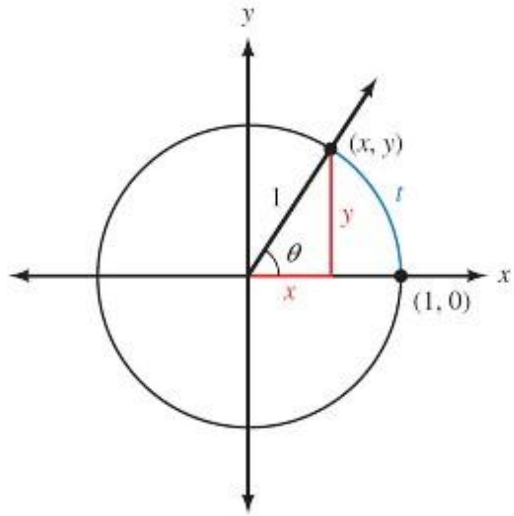
Solution

$$4\sin(2x+\pi) = 4\sin(2\frac{\pi}{6}+\pi)$$
$$= 4\sin\left(\frac{\pi}{3}+\pi\right)$$
$$= -4\sin\left(\frac{\pi}{3}\right)$$
$$= -4\left(\frac{\sqrt{3}}{2}\right)$$
$$= -2\sqrt{3}$$

Circular Functions

A unit circle has its center at the origin and a radius of 1 unit.

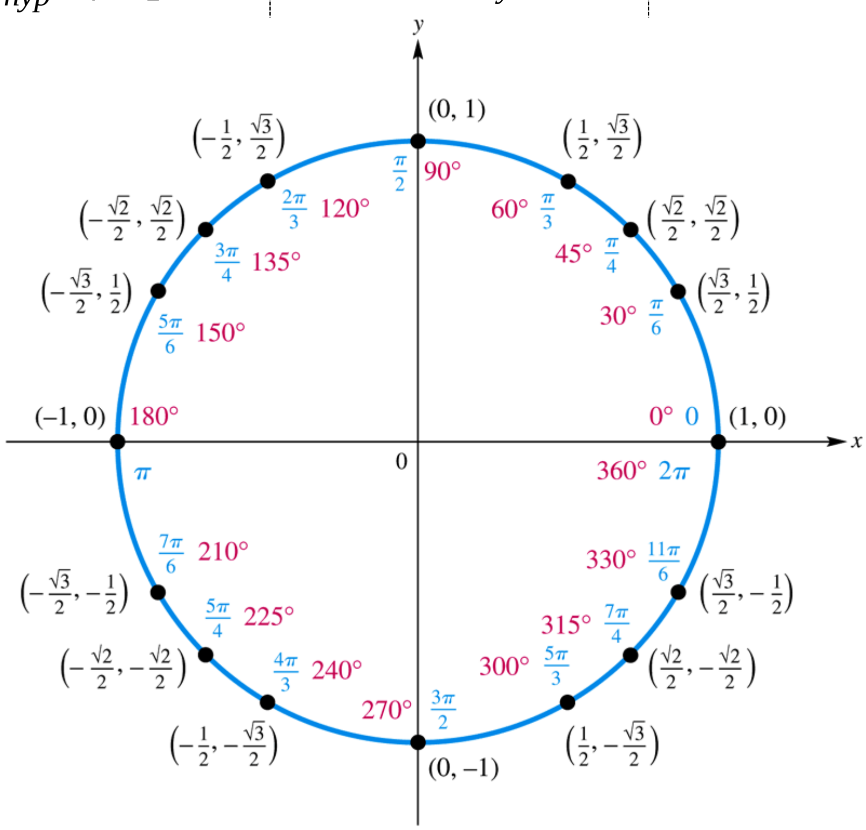




The equation of the unit circle (r = 1) is: $\boldsymbol{x^2 + y^2 = 1}$

When interpreted this way, they are called ***circular functions***.

$\sin\theta = \dfrac{opp}{hyp} = \dfrac{y}{r} = \dfrac{y}{1} = y$	$\tan\theta = \dfrac{y}{x}$	$\csc\theta = \dfrac{1}{y}$
$\cos\theta = \dfrac{adj}{hyp} = \dfrac{x}{r} = \dfrac{x}{1} = x$	$\cot\theta = \dfrac{x}{y}$	$\sec\theta = \dfrac{1}{x}$



The unit circle $x^2 + y^2 = 1$

The Unit Circle

The unit circle is symmetric with respect to the *x*-axis, *y*-axis, and the origin

Example

Find the six trigonometry functions of $\frac{5\pi}{6}$

<u>*Solution*</u>

$$\sin\frac{5\pi}{6} = y = \frac{1}{2}$$

$$\cos\frac{5\pi}{6} = x = -\frac{\sqrt{3}}{2}$$

$$\tan\frac{5\pi}{6} = \frac{\text{y}}{\text{x}} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\cot\frac{5\pi}{6} = -\frac{1}{1/\sqrt{3}} = -\sqrt{3}$$

$$\sec\frac{5\pi}{6} = \frac{1}{\cos\frac{5\pi}{6}} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$$

$$\csc\frac{5\pi}{6} = \frac{1}{\sin\frac{5\pi}{6}} = \frac{1}{-1/2} = -2$$

Example

Use the unit circle to find all values of *t* between 0 and 2π for which $\cos t = \frac{1}{2}$

<u>*Solution*</u>

The angles for $\cos t = \frac{1}{2}$ are $t = \frac{\pi}{3}$ *or* $60°$ and $t = \frac{5\pi}{3}$ *or* $300°$

Example

Find $\tan t$ if *t* corresponds to the point (-0.737, 0.675) on the unit circle.

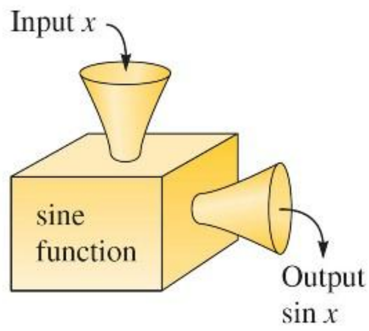
<u>*Solution*</u>

$$\tan t = \frac{y}{x}$$

$$= \frac{0.675}{-0.737}$$

$$\underline{\approx -0.916}$$

Definition of the *function* is a rule that pairs each element of the domain with exactly one element from the range.

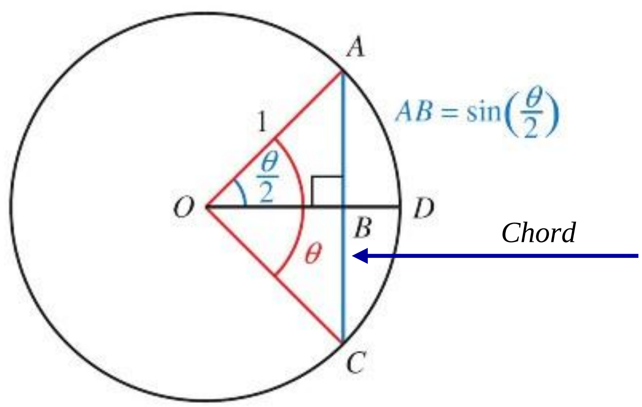


$y = \sin x \Rightarrow y = f(x) = \sin(x)$

Argument of the function = Angle

Value of the function = y

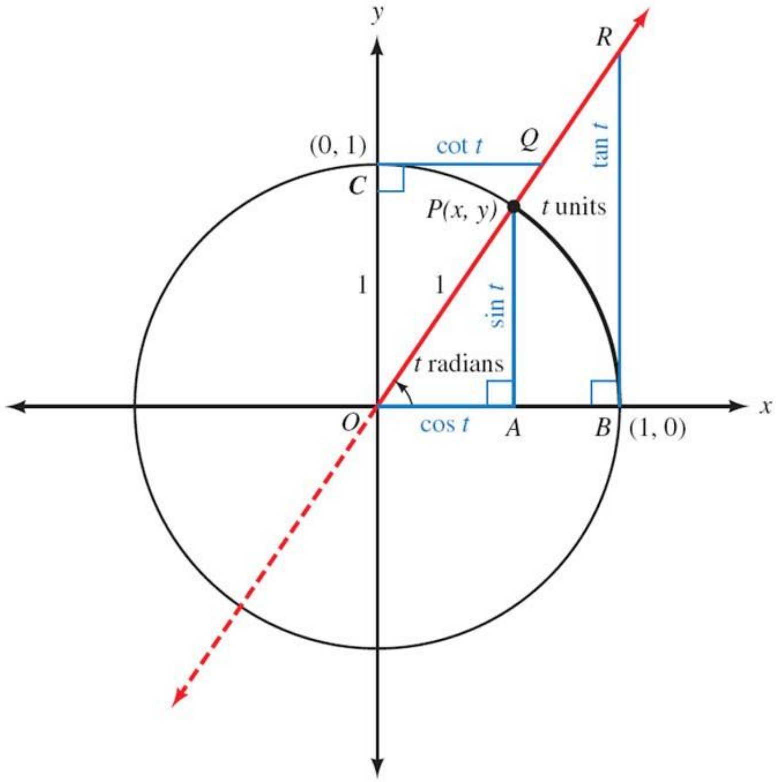
Geometric Representations



$chord(\theta) = AC = 2AB = 2\sin\left(\frac{\theta}{2}\right)$

Example

Describe how $\sec t$ varies as t increases from 0 to $\frac{\pi}{2}$



<u>*Solution*</u>

When $t=0,\ OR=1=OB$

$$\Rightarrow \sec t=\frac{1}{\frac{OB}{OR}}=\frac{OR}{OB}=1$$

$\rightarrow \sec t$ Will begin at a value of 1 as t increases

$\rightarrow \sec t$ Grows larger and larger

When $t=\frac{\pi}{2}\Rightarrow OP$ will be vertical

$\Rightarrow \sec t=OR$ will no longer be defined

Exercises Section 2.1 - Radians & Degrees, Circular Functions

1. Use a calculator to convert 256° 20′ to radians to the nearest hundredth of a radian.

2. Convert –78.4° to radians

3. Convert $\frac{11\pi}{6}$ to degrees

4. Convert $-\frac{5\pi}{3}$ to degrees

5. Convert $\frac{\pi}{6}$ to degrees

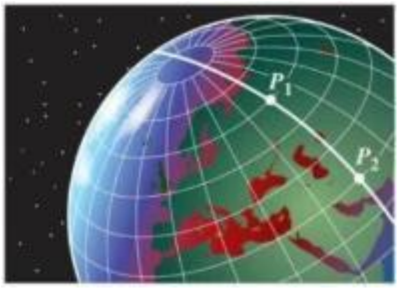
6. Use the calculator to convert 2.4 to degree measure to the nearest tenth of a degree.

7. In navigation, distance is not usually measured along a straight line, but along a great circle because the Earth is round. The formula to determine the great circle distance between two points $P_1\left(LT_1, LN_1\right)$ and $P_2\left(LT_2, LN_2\right)$ whose coordinates are given as latitudes and longitudes involves the expression

$$\sin\left(LT_1\right)\sin\left(LT_2\right)+\cos\left(LT_1\right)\cos\left(LT_2\right)\cos\left(LN_1-LN_2\right)$$

To use this formula, the latitudes and longitudes must be entered as angles in radians. However, most GPS units give these coordinates in degrees and minutes. To use this formula thus requires converting from degrees to radians.

Evaluate this expression for $P_1(N\ 32°\ 22.108', W\ 64°\ 41.178')$ and $P_2(N\ 13°\ 0.4809', W\ 59°\ 29.263')$ corresponding to Bermuda and Barbados, respectively.



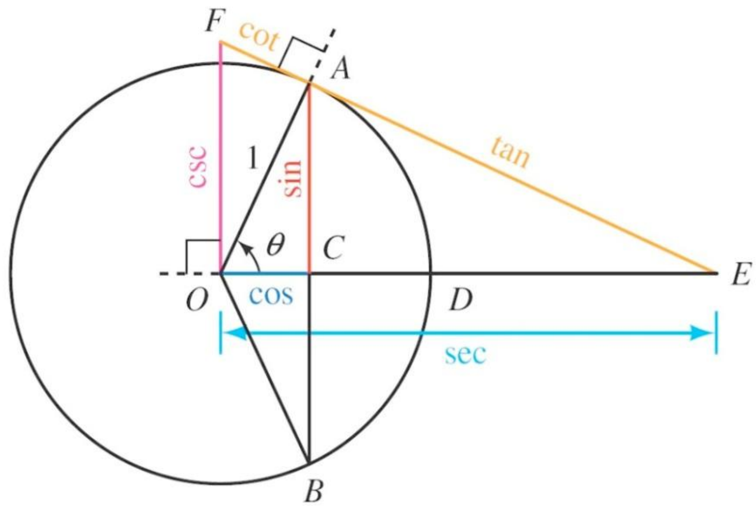
8. If the angle θ is in standard position and the terminal side of θ intersects the unit circle at the point $\left(-\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$

9. Find the exact values of $\sin\frac{3\pi}{2},\ \cos\frac{3\pi}{2},\ and\ \tan\frac{3\pi}{2}$

10. Use reference angles and degree/radian conversion to find exact value of $\cos\frac{2\pi}{3}$

11. Evaluate $\sin\frac{13\pi}{6}$. Identify the function, the argument of the function, and the function value.

12. Show why $OF = \csc\theta$



13. Evaluate $\sin\frac{9\pi}{4}$. Identify the function, the argument of the function, and the value of the function.

14. The function is the sine function, $\frac{9\pi}{4}$ is the argument, and $\frac{1}{\sqrt{2}}$ is the value of the function

15. Evaluate: $\cot 2.37$