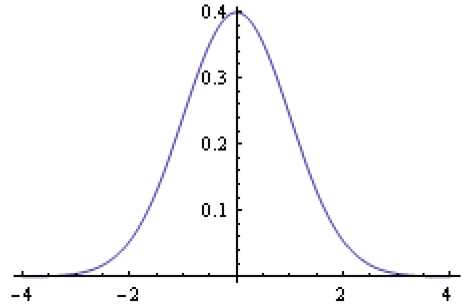
Section 4.5 – Normal Distribution

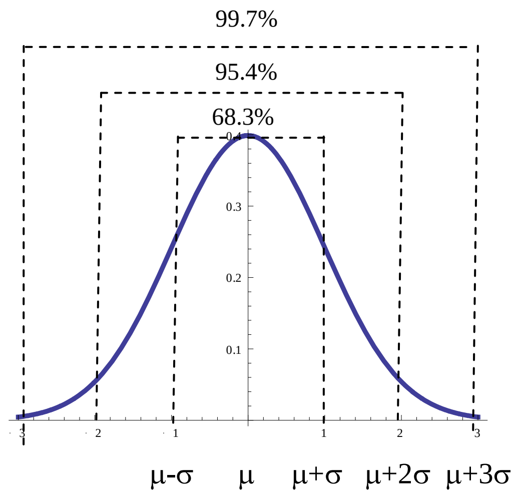
Braham De Moivre (1667 – 1754), *Laplace, Gauss*



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}$$

μ: Mean

σ: Standard deviation



Properties:

1. Symmetrical
2. μ Mean @ axis of Symmetry
3. Shape determined by μ & σ

Area under Normal Curves

Area between μ & σ: μ → μ + σ *or* μ - σ (are the same)

To Find how many standard deviations a measurement x is from a mean μ, first, determine the distance between x and μ then divide by σ.

$z-scores$

$$z = \frac{distance\ between\ x\ and\ \mu}{standard\ deviation} = \frac{x-\mu}{\sigma}$$

Properties

$P(a \le x \le b)$ = Area under the normal curve from $a \to b$

$P(-\infty < x < \infty) = 1$ = Total Area

$P(x = c) = 0$

Example

A manufacturing process produces light bulbs with life expectancies that are normally distributed with a mean of 500 hours and a standard deviation of 100 hours. What *percentage* of the light bulbs can be expected to last 500 to 750 hours?

<u>*Solution*</u>

$\mu = 500,\ \sigma = 100$

$\Rightarrow 500 \ \&\ 750$

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{750-500}{100}$$

$$= 2.5$$

$$\Rightarrow \boxed{A = 49.38\%}$$

Example

What is the *probability* of the light bulbs can be expected to last 400 to 500 hours?

<u>*Solution*</u>

$400 \to 500$

$$z = \frac{400-500}{100}$$

$$= -1$$

$$\Rightarrow \boxed{A = .3413}$$

Example

If a normal distribution has mean 50 and standard deviation 4, find the following $z-scores$.

a) The $z-scores$ for $x = 46$

b) The $z-scores$ for $x = 60$

<u>*Solution*</u>

a) $z = \dfrac{x-\mu}{\sigma}$

$$= \frac{46-50}{4}$$

$$= -1$$

b) $z = \dfrac{60-50}{4}$

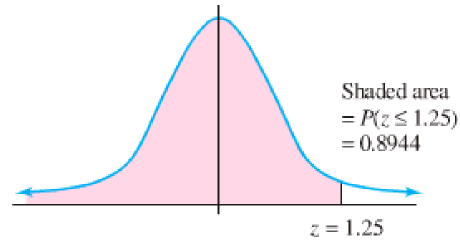
$$= 2.5$$

The area of the shaded region under the normal curve from *a* to *b* is the ***probability*** that an observed data value will be between *a* to *b*.

Example

Find the areas under the standard normal curve

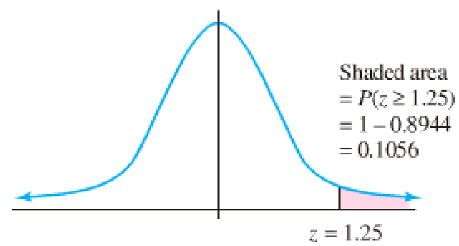
a) The area to the ***left*** of $z = 1.25$



$$A = P(z \le 1.25) \qquad \textbf{(\textit{Using left curve table})}$$
$$= 0.8944$$

$$A = P(z \le 1.25) \qquad \textbf{(\textit{Using right curve table})}$$
$$= .5 + 0.3944$$
$$= 0.8944$$

b) The area to the ***right*** of $z = 1.25$

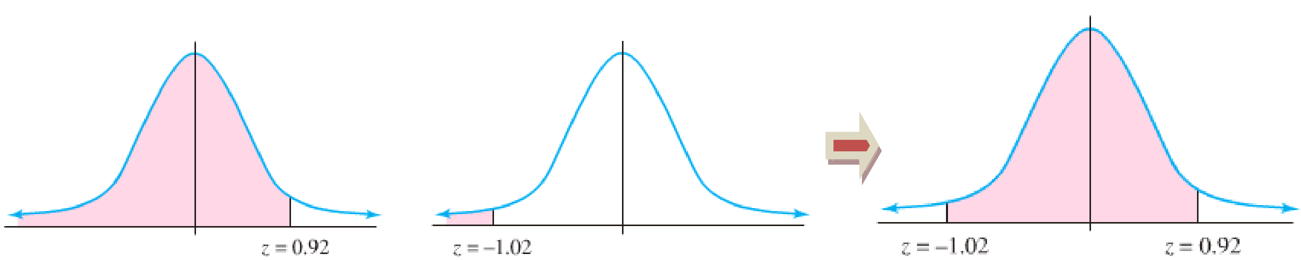


$$A = P(z \ge 1.25) \qquad \textbf{(\textit{Using left curve table})}$$
$$= 1 - 0.8944$$
$$= 0.1056$$

$$A = P(z \ge 1.25) \qquad \textbf{(\textit{Using right curve table})}$$
$$= .5 - 0.3944$$
$$= 0.1056$$

c) The area ***between*** $z = -1.02$ and $z = 0.92$

$$A = P(-1.02 \le z \le 0.92) \qquad \textbf{(\textit{Using left curve table})}$$
$$= 0.8212 - 0.1539$$
$$= 0.6673$$



$$A = P(-1.02 \le z \le 0.92) \qquad (\textit{Using right curve table})$$
$$= 0.3461 + 0.3212$$
$$= 0.6673$$

Example

Find a value of z satisfying the following conditions

a) 12.1% of the area is to the left of z.

b) 20% of the area is to the right of z.

<u>Solution</u>

a) $A = 0.121 \Rightarrow \boxed{z = -1.17}$

b) $A = 1 - .2 = .8 \Rightarrow \boxed{z = 0.84}$

Example

Dixie Office Supplies finds that it sales force drives an average of 1200 miles per month per person, with a standard deviation of 150 miles. Assume that the number of miles driven by a salesperson is closely approximated by a normal distribution.

a) Find the probability that a salesperson drives between 1200 miles and 1600 miles per month

b) Find the probability that a salesperson drives between 1000 miles and 1500 miles per month

c) Find the shortest and longest distances driven by the middle 95% of the data.

<u>Solution</u>

a) For $x_1 = 1200$

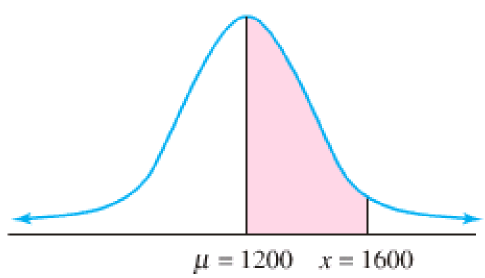
$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{1200-1200}{150} = 0$$

$\rightarrow A_1 = 0.5000$

For $x_2 = 1600$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{1600-1200}{150} \approx 2.67$$

$\rightarrow A_2 = 0.9962$



$A = 0.9962 - 0.5000 = 0.4962$

$P(1200 \le z \le 1600) \; \underline{= 0.4962}$

b) For $x_1 = 1000$

$$z_1 = \frac{1000-1200}{150} \approx -1.33$$

$\rightarrow A_1 = 0.0918$

For $x_2 = 1500$

$$z_2 = \frac{1500-1200}{150} \approx 2.00$$

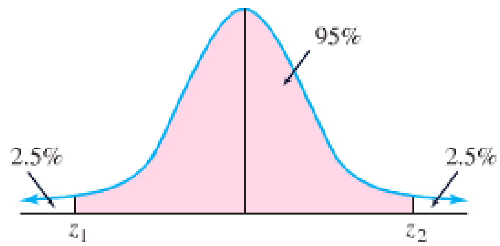
$\rightarrow A_2 = 0.9772$

$A = 0.9772 - 0.0918 = 0.8854$

$P(1000 \le z \le 1500) \; \underline{= 0.8854}$

c) The lower z value has 2.5% $\Rightarrow z_1 = -1.96$

The higher z value has 97.5% $\Rightarrow z_2 = 1.96$



Shortest distance $= \mu + z\sigma$

$= 1200 + (-1.96)(150)$

$= 906\ miles$

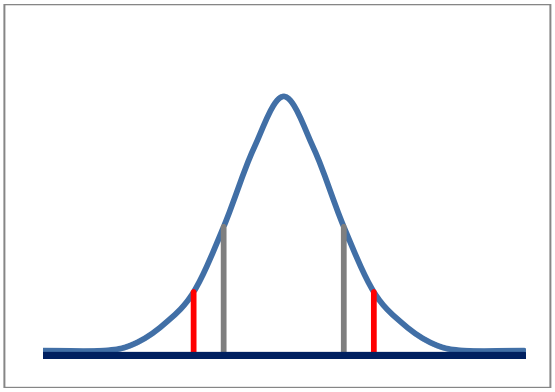
Longest distance $= \mu + z\sigma$

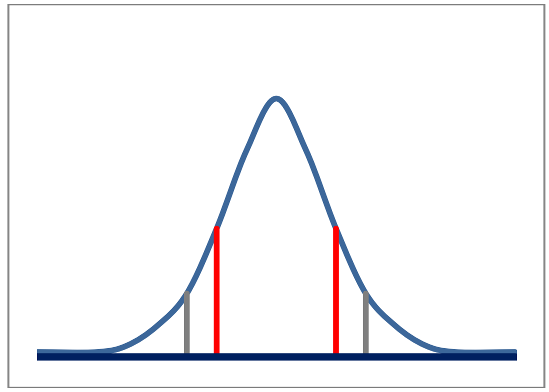
$= 1200 + (1.96)(150)$

$= 1494\ miles$

The distances driven between by the middle 95% of the sales force are between 906 and 1494 miles.

Always Add (0.5) to the line limit of the area that you are trying to find.





Example

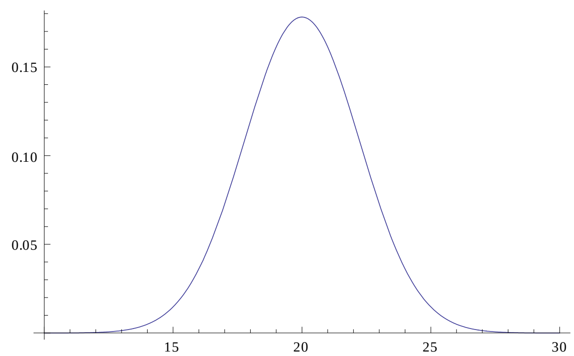
A company manufactures 50,000 ballpoint pens each day. The manufacturing process produces 40 defective pens per 1,000 on the average. A random sample of 400 pens is selected from each day's production and tested. What is the probability that the sample contains

a) At least 10 and no more than 20 defective pens?

b) 27 or more defective pens?

<u>*Solution*</u>

a) 40 defective pens per 1,000



$$\Rightarrow\ p = \frac{40}{1000} = 0.04$$

$$\begin{aligned}\mu &= np\\ &= 400(0.04)\\ &= 16\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{npq}\\ &= \sqrt{400(.04)(.96)}\\ &= 3.92\end{aligned}$$

$$z_1 = \frac{x-\mu}{\sigma} = \frac{9.5-16}{3.92} = -1.66\ \Rightarrow A_1 = .4515$$

$$z_2 = \frac{x-\mu}{\sigma} = \frac{20.5-16}{3.92} = 1.15\ \Rightarrow A_2 = .3749$$

$$\Rightarrow A = A_1 + A_2 = .4515 + .3749 \approx .8264$$

b) $z = \dfrac{26.5-16}{3.92} \approx 2.68 \Rightarrow A_1 = .4963$

$$\Rightarrow A = 0.5 - A_1 = .5 - .4963 \approx .0037$$

Exercises ***Section 4.5 – Normal Distribution***

1. A manufacturing process produces light bulbs with life expectancies that are normally distributed with a mean of 500 hours and a standard deviation of 100 hours. What percentage of the light bulbs can be expected to last 500 to 750 hours?

2. What is the probability of the light bulbs can be expected to last 400 to 500 hours?

3. The average lifetime for a car battery of a certain brand is 170 weeks, with a standard deviation of 10 weeks. If the company guarantees the battery for 3 years, what percentage of the batteries sold would be expected to be returned before the end of the warranty period? Assume a normal distribution

4. A manufacturing process produces a critical part of average length 100 millimeters, with a standard deviation of 2 millimeters. All parts deviating by more than 5 millimeters from the mean must be rejected. What percentage of the parts must be rejected, on the average? Assume a normal distribution.

5. An automated manufacturing process produces a component with an average width of 7.55 cm, with a standard deviation of 0.02 cm. All components deviating by more than 0.05 cm from the mean must be rejected. What percent of the parts must be rejected, on the average? Assume a normal distribution.

6. A company claims that 60% of the households in a given community uses its product. A competitor surveys the community, using a random sample of 40 households, and finds only 15 households out of the 40 in the sample using the product. If the company's claim is correct, what is the probability of 15 or fewer households using the product in a sample of 40? Conclusion? Approximate a binomial distribution with a normal distribution**.**

7. A union representative claims 60% of the union membership will vote in favor of a particular settlement. A random sample of 100 members is polled, and out of these, 47 favor the settlement. What is the approximate probability of 47 or fewer in a sample of 100 favoring the settlement when 60% of all the membership favor the settlement? Conclusion? Approximate a binomial distribution with a normal distribution.

8. The average healing time of a certain type of incision is 240 hours, with standard deviation of 20 hours. What percentage of the people having this incision would heal in 8 days or less? Assume a normal distribution.

9. The average height of a hay crop is 38 inches, with a standard deviation of 1.5 inches. What percentage of the crop will be 40 inches or more? Assume a normal distribution

10. In a family with 2 children, the probability that both children are girls is approximately .25. In a random sample of 1,000 families with 2 children, what is the approximate probability that 220 or fewer will have 2 girls? Approximate a binomial distribution with a normal distribution.

11. Aptitude Tests are scaled so that the mean score is 500 and the standard deviation is 100. What percentage of the students taking this test should score 700 or more? Assume a normal distribution

12. Candidate Harkins claims a private poll shows that she will receive 52% of the vote for governor. Her opponent, Mankey, secures the services of another pollster, who finds that 470 out of a random sample of 1,000 registered voters favor Harkins. If Harkin's claim is correct, what is the probability that only 470 or fewer will favor her in a random sample of 1,000? Conclusion? Approximate a binomial distribution with a normal distribution.

13. An instructor grades on a curve by assuming the grades on a test are normally distributed. If the average grade is 70 and the standard deviation is 8, find the test scores for each grade interval if the instructor wishes to assign grades as follow: 10% A's, 20% B's, 40% C's, 20% D's, and 10% F's.

14. At the discount Market, the average weekly grocery bill is $74.50, with a standard deviation of $24.30. What are the largest and smallest amounts spent by the middle 50% of this market's customers?

15. A certain type of light bulb has an average life of 500 hours, with a standard deviation of 100 hours. The length of life of the bulb can be closely approximated by a normal curve. An amusement park buys and installs 10,000 such bulbs. Find the total number that can be expected to last for each period of time. Find the shortest and longest lengths of life for the middle 60% of the bulbs.

16. A machine that fills quart milk cartons is set up to average 32.2 oz. per carton, with a standard deviation of 1.2 oz. What is the probability that a filled carton will contain less than 32 oz. of milk?

17. A machine produces bolts with an average diameter of 0.25 in. and a standard deviation of 0.02 in. What is the probability that a bolt will be produced with a diameter greater than 0.3 in.?