Review

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3,\ x_1 + 5x_2,\ x_3)$$

$$= \begin{bmatrix} x_1 + 2x_2 + x_3 \\ x_1 + 5x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

standard matrix is: $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

T- transf is Linear Domain $A \supset T = f_n$

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3,\ x_2 - 6x_3)$$

let $\vec{u} = (u_1, u_2, u_3)$

$\vec{v} = (v_1, v_2, v_3)$

$$T(\vec{u} + \vec{v}) = T(u_1 + v_1,\ u_2 + v_2,\ u_3 + v_3)$$

$$= \big(u_1 + v_1 - 5(u_2 + v_2) + 4(u_3 + v_3),\ u_2 + v_2 - 6(u_3 + v_3)\big)$$

$$= \big(u_1 + v_1 - 5u_2 - 5v_2 + 4u_3 + 4v_3,\ u_2 + v_2 - 6u_3 - 6v_3\big)$$

$$= \big((u_1 - 5u_2 + 4u_3) + (v_1 - 5v_2 + 4v_3),\ (u_2 - 6u_3) + (v_2 - 6v_3)\big)$$

$$= (u_1 - 5u_2 + 4u_3,\ u_2 - 6u_3) + (v_1 - 5v_2 + 4v_3,\ v_2 - 6v_3)$$

$$= T(u_1, u_2, u_3) + T(v_1, v_2, v_3)$$

$$= T(\vec{u}) + T(\vec{v}) \checkmark$$

it's closed under addition

$$T(r\vec{u}) = T(r(u_1, u_2, u_3))$$
$$= T(ru_1, ru_2, ru_3)$$
$$= (ru_1 - 5ru_2 + 4ru_3,\ ru_2 - 6ru_3)$$
$$= r(u_1 - 5u_2 + 4u_3,\ \underline{u_2 - 6u_3})$$
$$= r\,T(u_1, u_2, u_3)$$
$$= r\,T(\vec{u})$$

it's closed under scalar multiplication

since $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ and $T(r\vec{u}) = rT(\vec{u})$, then function T is a linear transformation

$$T: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

Domain

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3,\ x_2 - 6x_3)$$
$$= \begin{pmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{pmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{pmatrix} \quad R_3 - 3R_1 \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{pmatrix} = U$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= \lambda^2 - 4\lambda + 5$$

Characteristic eqn: $\lambda^2 - 4\lambda + 5 = 0$

$$\frac{+4 \pm \sqrt{16-20}}{2}$$

$$\frac{4 \pm 2i}{2}$$

Eigenvalues: $\lambda_{1,2} = 2 \pm i$

For $\lambda_1 = 2 - i \Rightarrow (A - \lambda_1 I) V_1 = 0$

$$\begin{pmatrix} i & -1 \\ 1 & i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$ix = y \Leftarrow$$

Eigenvectors

$$V_1 = \begin{pmatrix} 1 \\ i \end{pmatrix} \qquad V_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$ax = by \qquad \begin{pmatrix} b \\ a \end{pmatrix}$$

$$A = \begin{pmatrix} 5 & 3 \\ -6 & -4 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 5-\lambda & 3 \\ -6 & -4-\lambda \end{vmatrix}$$

$$= \lambda^2 - \lambda - 2 = 0$$

Characteristic eqn : $\lambda^2 - \lambda - 2 = 0$

Eigenvalues : $\lambda_1 = -1, \quad \lambda_2 = 2$

For $\lambda_1 = -1 \Rightarrow (A - \lambda_1 I) V_1 = 0$

$$\begin{pmatrix} 6 & 3 \\ -6 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{matrix} 6x_1 = -3y_1 \\ 2x_1 = -y_1 \end{matrix}$$

$$\boxed{V_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}}$$

For $\lambda_2 = 2 \Rightarrow (A - \lambda_2 I) V_2 = 0$

$$\begin{pmatrix} 3 & 3 \\ -6 & -6 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad x_2 = -y_2$$

$$\boxed{V_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}}$$

$$A = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -6-\lambda & 5 \\ -5 & 4-\lambda \end{vmatrix}$$

$$= \lambda^2 + 2\lambda + 1 = 0$$

Eigenvalues: $\lambda_{1,2} = -1$

For $\lambda = -1 \Rightarrow (A - \lambda_1 I) V_1 = 0$

$$\begin{pmatrix} -5 & 5 \\ -5 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow x_1 = y_1$$

$$\boxed{V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

$$A V_2 = V_1$$

$$\begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \longrightarrow$$

$$-6x_2 + 5y_2 = 1 \quad \text{(line)}$$

$$\boxed{V_2 = \begin{pmatrix} 0 \\ 1/5 \end{pmatrix}} \quad \text{or} \quad \begin{pmatrix} -1/6 \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 2 & -5 & -6 \\ -2 & 3 & 4 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & 0 & 0 \\ 2 & -5-\lambda & -6 \\ -2 & 3 & 4-\lambda \end{vmatrix}$$

$$= (-1-\lambda)(-20 + \lambda + \lambda^2 + 18)$$

$$= -(1+\lambda)(\lambda^2 + \lambda - 2) = 0 \leftarrow$$

$$\lambda_1 = -1, 1, -2$$

Characteristic eqn: $-\lambda^3 - 2\lambda^2 + \lambda + 2 = 0$