Solution Section 3.4 – Estimating a Population Standard Deviation

Exercise

Using the weights of the M&M candies. We use the standard deviation of the sample $(s = 0.05179\ g)$ to obtain the following 95% confidence interval estimate of the standard deviation of the weights of all M&Ms: $0.0455\ g < \sigma < 0.0602\ g$. Write a statement that correctly interprets that confidence interval.

Solution

We can be 95% confident that the interval from 0.0455 grams to 0.0602 grams includes the true value of the standard deviation in the weights for the population of all M&M's

Exercise

Find χ_L^2 and χ_R^2 that corresponds to: 95%; $n = 9$

Solution

$$\alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \quad and \quad df = 8$$

TABLE Chi-Square (χ^2) Distribution

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955

$$\chi_L^2 = \chi^2_{8,\ 0.0975} \underline{= 2.180} \qquad \chi_R^2 = \chi^2_{8,\ 0.025} \underline{= 17.535}$$

Exercise

Find χ_L^2 and χ_R^2 that corresponds to: 99%; $n = 81$

Solution

$$\alpha = 1 - .99 = 0.01 \rightarrow \frac{\alpha}{2} = 0.005 \quad and \quad df = 80$$

TABLE Chi-Square (χ^2) Distribution

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.32

$$\chi_L^2 = \chi^2_{80,\ 0.0995} \underline{= 51.172} \qquad \chi_R^2 = \chi^2_{80,\ 0.005} \underline{= 116.321}$$

Exercise

Find χ_L^2 and χ_R^2 that corresponds to: 90%; $n = 51$

Solution

$$\alpha = 1 - .90 = 0.1 \rightarrow \frac{\alpha}{2} = 0.05 \quad and \quad df = 50$$

TABLE Chi-Square (χ^2) Distribution

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490

$$\chi_L^2 = \chi_{50,\ 1-.05}^2 = \chi_{50,\ .95}^2 = 34.764$$

$$\chi_R^2 = \chi_{50,\ .05}^2 = 67.505$$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005

Exercise

Find a confidence interval for the population standard deviation σ

95% *confidence*; $n = 30$, $\bar{x} = 1533$, $s = 333$ (*Assume has a normal distribution*)

Solution

$$\alpha = 1 - .95 = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \quad and \quad df = 29$$

TABLE Chi-Square (χ^2) Distribution

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336

$$\chi_L^2 = \chi_{29,\ 1-.025}^2 = \chi_{29,\ .975}^2 = 16.047$$

$$\chi_R^2 = \chi_{29,\ .025}^2 = 45.722$$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(29)(333)^2}{45.722}} < \sigma < \sqrt{\frac{(29)(333)^2}{16.047}}$$

$$265 < \sigma < 448$$

Exercise

Find a confidence interval for the population standard deviation σ

95% *confidence*; $n=25$, $\bar{x}=81.0\ mi/h$, $s=2.3\ mi/h$ (*Assume has a normal distribution*)

Solution

$$\alpha = 1-.95 = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \quad and \quad df = n-1 = 24$$

TABLE Chi-Square (χ^2) Distribution										
Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559

$$\chi_L^2 = \chi_{24,\ 1-.025}^2 = \chi_{24,\ .975}^2 = 12.401 \qquad \chi_R^2 = \chi_{24,\ .025}^2 = 39.364$$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(24)(2.3)^2}{39.364}} < \sigma < \sqrt{\frac{(24)(2.3)^2}{12.401}}$$

$$1.8 < \sigma < 3.2 \quad (mph)$$

Exercise

Find a confidence interval for the population standard deviation σ

99% *confidence*; $n=7$, $\bar{x}=7.106$, $s=2.019$ (*Assume has a normal distribution*)

Solution

$$\alpha = 1-.99 = 0.01 \rightarrow \frac{\alpha}{2} = 0.005 \quad and \quad df = n-1 = 6$$

TABLE Chi-Square (χ^2) Distribution										
Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548

$$\chi_L^2 = \chi_{6,\ 1-.005}^2 = \chi_{6,\ .995}^2 = 0.676 \qquad \chi_R^2 = \chi_{6,\ .025}^2 = 18.548$$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(6)(2.019)^2}{18.548}} < \sigma < \sqrt{\frac{(6)(2.019)^2}{0.676}}$$

$$1.148 < \sigma < 6.015 \quad (cells/microliter)$$

Exercise

In a study of the effects of prenatal cocaine use on infants, the following sample data were obtained for weights at birth: $n = 190, \quad \bar{x} = 2700\ g, \quad s = 645\ g$. Use the sample data to construct a 95% confidence interval estimate of the standard deviation of all birth weights of infants born to mothers who used cocaine during pregnancy. Because from the Table, a maximum of 100 degrees of freedom while we require 189 degrees of freedom, use these critical values to obtained $\chi_L^2 = 152.8222$ and $\chi_R^2 = 228.9638$.

Based on the result, does the standard deviation appear to be different from the standard deviation of 696g for birth weights of babies born to mothers who did not use cocaine during pregnancy?

Solution

Given: $\chi_L^2 = 152.8222 \qquad \chi_R^2 = 228.9638$

$$\alpha = 1 - .95 = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \quad \text{and} \quad df = n - 1 = 189$$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(189)(645)^2}{228.9638}} < \sigma < \sqrt{\frac{(189)(645)^2}{152.8222}}$$

$$\underline{586 < \sigma < 717} \quad (g)$$

No. Since the confidence interval includes 696, it is a reasonable possibility for σ.

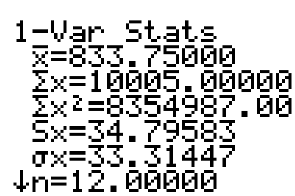
Exercise

In the course of designing theather seats, the sitting heights (in *mm*) of a simple random sample of adults women is obtained, and the results are

849 807 821 856 864 877 772 848 802 807 887 815

Use the sample data to construct a 95% confidence interval estimate of σ, the standard deviation of sitting heights of all women. Does the confidence contain the value of 35 *mm*, which is believed to be the standard deviation of sitting heights of women?

Solution



Using the calculator: $n = 12, \quad \bar{x} = 833.75 \quad s = 34.796$

$$\alpha = 1 - .95 = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \quad \text{and} \quad df = n - 1 = 11$$

TABLE Chi-Square (χ^2) Distribution										
Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757

$\chi^2_L = \chi^2_{11,\ 1-.025} = \chi^2_{11,\ .975}$ $\underline{= 3.816|}$ $\chi^2_R = \chi^2_{11,\ .025}$ $\underline{= 21.920|}$

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{\frac{(11)(34.796)^2}{21.920}} < \sigma < \sqrt{\frac{(11)(34.796)^2}{3.816}}$$

$\underline{24.7 < \sigma < 59|}$ (mm)

Yes. The interval contains the traditionally believed value of 35 *mm*.

Exercise

One way to measure the risk of a stock is through the standard deviation rate of return of the stock. The following data represent the weekly rate of return (in percent) of Microsoft for 15 randomly selected weeks. Compute the 90% confidence interval for the risk of Microsoft stock.

5.34	9.63	–2.38	3.54	–8.76	2.12	–1.95	0.27
0.15	5.84	–3.90	–3.80	2.85	–1.61	–3.31	

Solution

A normal probability plot and boxplot indicate the data is approximately normal with no outliers.

$s = 4.6974 \quad s^2 = 22.0659$

$df = 15 - 1 \underline{= 14|}$

Degrees of Freedom	Chi-Square (χ^2) Distribution Area to the Right of Critical Value 0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319

$\chi^2_{0.95} = 6.571 \qquad \chi^2_{0.05} = 23.685$

Lower bound: $\dfrac{(n-1)s^2}{\chi^2_R} = \dfrac{14(22.0659)}{23.685} \underline{= 13.04|}$

Upper bound: $\dfrac{(n-1)s^2}{\chi^2_L} = \dfrac{14(22.0659)}{6.571} \underline{= 47.01|}$

We are 90% confident that the population standard deviation rate of return of the stock is between 13.04 and 47.01.