

## ***Solution***      **Section 1.5 – Limits and Asymptotes**

### ***Exercise***

Find the limit:  $\lim_{x \rightarrow 1} (2x^2 - x + 4)$

### **Solution**

$$\lim_{x \rightarrow 1} (2x^2 - x + 4) = 2(\textcolor{red}{1})^2 - (\textcolor{red}{1}) + 4 = \underline{\textcolor{blue}{5}}$$

### ***Exercise***

Find the limit:  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

### **Solution**

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \frac{2^2 - 4}{2 - 2} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1.9} \frac{x^2 - 4}{x - 2} = \frac{1.9^2 - 4}{1.9 - 2} = 3.9$$

$$\lim_{x \rightarrow 2.1} \frac{x^2 - 4}{x - 2} = \frac{2.1^2 - 4}{2.1 - 2} = 4.1$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$$

### ***Exercise***

Find the limit:  $\lim_{x \rightarrow 2} f(x)$        $f(x) = \begin{cases} x^2, & x \neq 2 \\ 0, & x = 2 \end{cases}$

### **Solution**

$$\lim_{x \rightarrow 1.99} x^2 = 3.96$$

$$\lim_{x \rightarrow 2} f(x) = 4$$

$$\lim_{x \rightarrow 2.01} x^2 = 4.04$$

***Exercise***

Find the limit:  $\lim_{x \rightarrow 1} (2x^2 - x + 4)$

**Solution**

$$\begin{aligned}\lim_{x \rightarrow 1} (2x^2 - x + 4) &= 2(1^2) - 1 + 4 \\ &= 5\end{aligned}$$

***Exercise***

Find the limit:  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

**Solution**

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} &= \frac{0}{0} \\ \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} &= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{x - 2} \\ &= \lim_{x \rightarrow 2} x^2 + 2x + 4 \\ &= 2^2 + 2(2) + 4 \\ &= 12\end{aligned}$$

***Exercise***

Find the limit:  $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3}$

**Solution**

$$\begin{aligned}\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3} &= \frac{0}{0} \\ \lim_{x \rightarrow 3} \frac{(x-3)(x+4)}{x - 3} &= \lim_{x \rightarrow 3} (x + 4) \\ &= 3 + 4 \\ &= 7\end{aligned}$$

### Exercise

Find the limit:  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

### Solution

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} &= \frac{\sqrt{4} - 2}{0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} &= \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} \cdot \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} \\ &= \lim_{x \rightarrow 0} \frac{x+4-4}{x(\sqrt{x+4} + 2)} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+4} + 2)} \\ &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+4} + 2} \\ &= \frac{1}{\sqrt{4} + 2} \\ &= \frac{1}{4}\end{aligned}$$

### Exercise

Find the limit:  $\lim_{x \rightarrow 0} f(x) = \begin{cases} x^2 + 1 & x < 0 \\ 2x + 1 & x > 0 \end{cases}$

### Solution

$$\lim_{x \rightarrow 0^-} x^2 + 1 = 1$$

$$\lim_{x \rightarrow 0^+} 2x + 1 = 1$$

$$\lim_{x \rightarrow 0} f(x) = 1$$

### Exercise

Find the limit:  $\lim_{x \rightarrow -2} \frac{5}{x+2}$

### Solution

$$\lim_{x \rightarrow -2} \frac{5}{x+2} = \frac{5}{0} = \infty \text{ (Doesn't exist)}$$

***Exercise***

Find the limit:  $\lim_{x \rightarrow 2^-} \frac{|x-2|}{x-2}$

**Solution**

$$\lim_{x \rightarrow 2^-} \frac{|x-2|}{x-2} = \frac{(x-2)}{-(x-2)} = -1$$

***Exercise***

Find the limit:  $\lim_{x \rightarrow 2^+} \frac{|x-2|}{x-2}$

**Solution**

$$\lim_{x \rightarrow 2^+} \frac{|x-2|}{x-2} = \frac{(x-2)}{(x-2)} = 1$$

***Exercise***

$$y = \frac{3x}{1-x}$$

**Solution**

**VA:**  $x = 1$

**HA:**  $y = -3$

***Exercise***

$$y = \frac{x^2}{x^2 + 9}$$

**Solution**

**HA:**  $y = 1$

***Exercise***

$$y = \frac{x-2}{x^2 - 4x + 3}$$

**Solution**

$$VA: x = 1, 3$$

$$HA: y = 0$$

**Exercise**

$$y = \frac{3}{x-5}$$

**Solution**

$$VA: x = 5$$

$$HA: y = 0$$

**Exercise**

$$y = \frac{x^3 - 1}{x^2 + 1}$$

**Solution**

$$VA: n/a$$

$$HA: n/a$$

**Exercise**

$$y = \frac{3x^2 - 27}{(x+3)(2x+1)}$$

**Solution**

$$VA: x = -3, -\frac{1}{2}$$

$$HA: y = \frac{3}{2}$$

**Exercise**

$$y = \frac{x^3 + 3x^2 - 2}{x^2 - 4}$$

**Solution**

$$VA: x = \pm 2$$

$$HA: n/a$$

***Exercise***

$$y = \frac{x-3}{x^2-9}$$

**Solution**

VA:  $x = -3$ ,  
hole  $x = 3$

***Exercise***

$$y = \frac{6}{\sqrt{x^2-4x}}$$

**Solution**

VA:  $x = 0, 4$

***Exercise***

$$y = \frac{5x-1}{1-3x}$$

**Solution**

VA:  $x = \frac{1}{3}$

HA:  $y = \frac{5}{3}$