# 1.3 – Standing Waves

Standing waves are waves with equally spaced points of zero vibration. An example is the kind of wave that can be seen when a rubber band fixed at both sides is excited. The points of zero vibration are called nods. And the points of maximum vibration are called antinodes. A standing wave is usually formed when an incident wave and a reflected wave are superposed. Consider a wave of the form $y_i = A_1\cos(\omega t - Kx - \phi)$ reflected from a boundary between two medium (*or obstacles*) a distance of L from the source. When an incident wave and a reflected wave meet at a distance *x* from the source, the incident wave would have travelled a distance *x* and the reflected wave would have travelled a distance of $L + (L - x) = 2L - x$. As stated in the previous chapter, a reflected will have a phase change of $\pi$ on reflection if the medium from which it is reflected is denser that it's medium and there will be no phase change if reflected from a less dense medium. Let's consider both separate.

### *Case* 1: wave reflected from a more dense medium

let $y_r$ be the reflected wave.

If $y_i = A_1\cos(\omega t - Kx - \phi)$ then $y_r = A_2\cos(K(2L - x) - \omega t + \phi + \pi)$

and the net wave $y_{net}$ is given by

$$y_{net} = y_i + y_r = A_1\cos(Kx - \omega t + \phi) + A_2\cos(K(2L - x) - \omega t + \phi + \pi)$$

and multiplying the argument of the cosine by $-1$ (*remember* $\cos(-x) = \cos x$)

$$y_{net} = A_1\cos(Kx - \omega t + \phi) + A_2\cos(\omega t - K(2L - x) - \phi - \pi)$$

At a given location *x*, (*x* = *constant*) these two waves are harmonic oscillators with phase angles

$$\beta_1' = Kx + \varphi_1 \quad and \quad \beta_2' = K(2L - x) + \varphi + \pi$$

$$y_{net} = A_1\cos(Kx - \beta_1') + A_2\cos(\omega t - \beta_2')$$

$$= A\cos(\omega t - \delta)$$

Where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\beta_2' - \beta_1')}$

$$\delta = \tan^{-1}\left(\frac{A_1\sin\beta_1' + A_2\sin\beta_2'}{A_1\cos\beta_1' + A_2\cos\beta_2'}\right)$$

$$\begin{aligned}\beta_2' - \beta_1' &= K(2L - x) + \phi + \pi - (Kx + \phi)\\ &= 2KL - Kx + \phi + \pi - Kx - \phi\\ &= 2KL - 2Kx + \pi\\ &= \boxed{2K(L - x) + \pi}\end{aligned}$$

For simplicity, let's assume that the wave is reflected 100% (*even though some of it may be transmitted*) so that $A_1 = A_2$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\left(\beta_2' - \beta_1'\right)}$$
$$= \sqrt{2A_1^2 + 2A_1^2\cos\left(2K\left(L-x\right)+\pi\right)}$$
$$= A_1\sqrt{2\left(1-\cos\left(2K\left(L-x\right)\right)\right)}$$

But $\dfrac{1-\cos 2\theta}{2} = \sin^2\theta$

$$A = A_1\sqrt{4\sin^2\left(K\left(L-x\right)\right)}$$
$$\boxed{A = 2A_1\left|\sin\left(K\left(L-x\right)\right)\right|}$$

With this as an amplitude, the net wave is given by

$$y_{net} = 2A_1\left|\sin\left(K\left(L-x\right)\right)\right|\cos\left(\omega t-\delta\right)$$

This shows that the amplitude of the harmonic oscillators is a function of position, *x*. And since the amplitude varies like a sine, there are going to be points with zero amplitude or no vibration. These are the points called the nodes of the waves. And of course the points where sine has a maximum are the antinode points.

**Distance between Consecutive Nodes**

The nodes are the values of *x* for which

$$\sin\left(K\left(L-x\right)\right)=0 \quad\Rightarrow\quad K\left(L-x\right)=m\pi \quad \textit{where m is integer}$$

With $K = \dfrac{2\pi}{\lambda}$

$$\frac{2\pi}{\lambda}\left(L-x_m\right)=m\pi$$
$$L-x_m = \frac{1}{2}m\lambda$$
$$\boxed{x_m = L-\frac{1}{2}m\lambda}$$

(*Location of the m*$^{th}$ *node with* $x_0$ *represents the reflection point*)

The distance between consecutive node equal to $\left|x_{m+1}-x_m\right|$

$$\left|x_{m+1}-x_m\right| = \left|\left(L-\frac{m+1}{2}\lambda\right)-\left(L\frac{m}{2}\lambda\right)\right| = \left|\left(\frac{m+1}{2}-\frac{m}{2}\right)\lambda\right| = \frac{\lambda}{2}$$

$$\boxed{x_{m+1}-x_m = \frac{\lambda}{2}}$$

(*The distance between two consecutive nodes is half of the wave length of the wave*)

**Amplitude at Point of Reflection**

At the point of reflection $x = L$

$$A = 2A_1 \left|\sin\left(K\left(L - x\right)\right)\right|$$

$$A\Big|_{x=L} = 2A_1 \left|\sin K\left(L - L\right)\right| = 0$$

Therefore the point of refection is a node

## Boundary Conditions

Boundary conditions restrict the wavelengths of waves that can exist as standing wave in a medium we will consider two very common boundary conditions.

**1. <u>The other end ($x = 0$) also required to be a node</u>**

That is the standing wave will have nodes at both ends. An example of this is a standing wave is a string where both ends are fixed.

This condition

$$A\big|_{x=0} = 0$$

Since $$A = 2A_1 \left|\sin\left(K\left(L - x\right)\right)\right|$$

$$\sin\left(K\left(L - 0\right)\right) = 0$$

$$\sin\left(KL\right) = \sin\left(\frac{2\pi}{\lambda}L\right) = 0$$

$$\frac{2\pi}{\lambda}L = n\pi$$

$$\boxed{\lambda_n = \frac{2L}{n}}$$

(*Where n is integer (positive) and is wavelength corresponding to n*)

This means the only wavelength that can form standing wave in a standing wave of length L are only $\lambda_1 = \frac{2L}{1},\ \lambda_2 = \frac{2L}{2} = L,\ \lambda_3 = \frac{2L}{3},\ \lambda_4 = \frac{2L}{4} = \frac{L}{2},\ \ldots$ and so on.

Since the speed of a wave depends on the properties of the medium only and the frequency of the wave is given as, the frequencies that can exist as a standing wave are also restricted

$$f_n = \frac{V}{\lambda_n} = \frac{V}{\left(2L/n\right)} = n\left(\frac{V}{2L}\right) \qquad (n \textit{ positive integer})$$

$$f_n = n\left(\frac{V}{2L}\right) \qquad n = 1,\ 2,\ 3,\ \ldots$$

$f_n \to n^{th}$ frequency that can be exist as a standing wave.

The standing wave whose frequency is called the n$^{th}$ harmonic of the standing one. The first harmonic is also called the fundamental harmonic with

$$n = 1 \quad f_1 = \frac{V}{2L} \qquad \textit{fundamental frequency}$$

$$\boxed{f_n = nf_1}$$

This shows that the frequencies of all the harmonics of a standing wave are integral multiples of the fundamental frequency, $f_1$.

**Number of Loops (*N*)**

The part of the standing wave between two consecutive nodes is called a loop. The number of loops for the harmonic can be obtained by dividing the length of the standing wave ($L$) by the size of one loop $\left(\frac{\lambda_n}{2}\right)$

$$N = \frac{L}{\lambda_n/2} = \frac{L}{\left(L/n\right)} = n \qquad \left(\textit{using } \lambda_n = \frac{2L}{n}\right)$$

$\boxed{N = n}$ (*The n$^{th}$ harmonic contains n loops*)

## *Example*

A wave of the form $y = 5\cos(4\pi - 20\pi t)$ is reflected from a boundary (*obstacle*) 2m away from the point where it is initiated. If the wave is reflected 100%.

*a)* Calculate the amplitude of the harmonic oscillation of a particle located at an antinode.
*b)* Determine the size of one loop of the standing wave (*that is distance between consecutive nodes*)
*c)* Determine location of the nodes
*d)* Determine the number of loops
*e)* Calculate the amplitude of the harmonic oscillation of a particle located at x=0.4m

***Solution***

***a)*** At an antinode the amplitude is maximum. $A_1 = 5$

$$A = 2A_1\left|\sin\left(K\left(L - x\right)\right)\right|$$

The maximum value of $A$ occurs when $\sin[K(L - x)] = 1$ because maximum of a sine function is 1.

$$A_{max} = 2A_1 = 2(5) = 10m$$

***b)*** K = 4π $K = 4\pi = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda = \frac{1}{2} = 0.5$

$$\left|x_{m+1} - x_m\right| = \frac{\lambda}{2} = \frac{0.5}{2} \underline{= 0.25|}$$

***c)*** $L$ = 2m; λ = 0.5m

$$x_m = L - \frac{1}{2}m\lambda \underline{= 2 - 0.25m|}$$

$x_0 = 2m;\ \ x_1 = 1.75m;\ \ x_2 = 1.5m;\ \ x_3 = 1.25;\ \ x_4 = 1m;$

$x_5 = 0.75m;\ \ x_6 = 0.5m\ \ x_7 = 0.25m;\ \ x_8 = 0$

***d)*** Since the size of one loop is λ/2

$$N = \frac{L}{\left(\lambda/2\right)} = \frac{2L}{\lambda} = \frac{2(2)}{0.5}\ \underline{= 8\,loops|}$$

***e)*** $x = 0.4\text{m}$ $\lambda = 0.5\text{m}$ $L = 2\text{m}$ $N = 8$ $K = 4\pi$

$$A = 2A_1\left|\sin\left(K\left(L - x\right)\right)\right|$$

$$A\Big|_{x=0.4} = 2(5)\left|\sin\left(4\pi\left(2 - 0.4\right)\right)\right|$$

$$\underline{= 9.5\ m|}$$

## *Example*

Consider a standing wave formed in a string of length 4*m* fixed at both of its ends. The mass of the string is 0.02 kg. There is a tension of 100 N in the string.

*a)* Determine the wavelengths of the first 3 harmonics
*b)* Determine the speed of the wave in the string
*c)* Determine the frequencies of the first 3 harmonics
*d)* Calculate the wavelength and frequency of the 15$^{th}$ harmonic.

***<u>Solution</u>***

***a)*** L = 4m ; $\lambda_n = \frac{2L}{n}$

$$\lambda_1 = \frac{2(4)}{1} = 8m$$

$$\lambda_2 = \frac{2(4)}{2} = 4m$$

$$\lambda_3 = \frac{2(4)}{3} = {}^{8}\!/_{3}\,m$$

***b)*** T=100N l=4m m=0.02K

$$v = \sqrt{{}^{T}\!/_{\mu}}$$

$$\mu = \frac{m}{l} = \frac{0.02}{4} = 0.005\,{}^{kg}\!/_{m}$$

$$\therefore v = \sqrt{{}^{100}\!/_{0.005}} = 141\,{}^{m}\!/_{s}$$

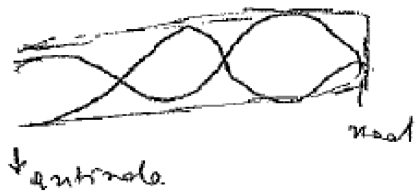
***c)*** $f_1 = \frac{v}{\lambda_1} = \frac{141}{8}\ \underline{= 17.6\ Hz|}$

***d)*** $f_{15} = 15 f_1 = 15(17.6)\ \underline{= 264\ Hz|}$

***e)*** $\lambda_{15} = \frac{v}{f_{15}} = \frac{141}{264}\ \underline{= 0.534\ m|}$

## 2. The $x$ = 0 end required to be an Antinode

Remember the other end (*the reflection end is a node*). Therefore this is a standing wave with a node on one side end and an antinode on the other end. An example of this is sound resonance in a tube closed on one end. It will have an antinode on the open end and a node on the closed end.




Since $A = 2A_1\left|\sin\left(K\left(L-x\right)\right)\right|$ for an antinode the value of the sine should be one because the maximum of sine is one.

$$A\Big|_{x=0} = A_{\max} = 2A_1$$

$$\sin\left(K\left(L-x\right)\right)\Big|_{x=0} = 1$$

$$\sin\left(KL\right) = 1 \quad \Rightarrow \quad KL = \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2},\ \ldots = \frac{2n-1}{2}\pi$$

$$K_n L = \left(\frac{2n-1}{2}\right)\pi \quad but \quad K_n = \frac{2\pi}{\lambda_n}$$

$$\frac{2\pi}{\lambda_n}L = \left(\frac{2n-1}{2}\right)\pi$$

$$\boxed{\lambda_n = \frac{4L}{2n-1}}$$ (*Allowed wavelength when one end is node and the other end is an antinode*)

This implies that the only waves that can exist in a standing wave of length $L$ with a node on one end and an antinode on the other end are waves with wavelengths and so on.

$$\lambda_1 = \frac{4L}{2(1)-1} = 4L, \quad \lambda_2 = \frac{4L}{2(2)-1} = \frac{4L}{3}, \quad \lambda_3 = \frac{4L}{2(3)-1} = \frac{4L}{5}, \quad \ldots \quad \textit{and so on.}$$

If the speed of the wave is $v$ (*the speed depends only on the properties of the medium*), then the allowed frequencies are given by

$$\boxed{f_n = \frac{V}{\lambda_n} = \frac{V}{\left(4L/(2n-1)\right)} = (2n-1)\frac{V}{4L}}$$

Since, $f_1 = \frac{V}{4L}$, this also may be written in terms of the fundamental frequency $f_1$, as

$$\boxed{f_n = (2n-1)\,f_1} \quad where \quad f_1 = \frac{V}{4L}$$

<u>Number of loops ($N$)</u>: $\boxed{N = \frac{L}{\lambda_n/2} = \frac{L}{\frac{1}{2}\frac{4L}{2n-1}} = \frac{2n-1}{2}}$

The n^th harmonic has $\frac{2n-1}{2}$ loops when one end is a node & the other antinode.

## *Example*

Consider sound resonance (*standing wave*) formed in a pipe open at one end closed at the other end. Its length is 0.5m. Assume the temperature is 20°C.

*a)* Calculate the wave lengths of the first 3 harmonics

*b)* Calculate the frequencies of the first 3 harmonics

*c)* Calculate the wavelength and frequency of the 4th harmonic

***Solution***

***a)*** The sound resonance will have a node at the closed end & antinode at the open end.

$$\lambda_n = \frac{4L}{2n-1} = \frac{4(0.5)}{2n-1} = \frac{2}{2n-1}$$

$$\lambda_1 = \frac{2}{2(1)-1} = 2m; \quad \lambda_2 = \frac{2}{2(2)-1} = \frac{2}{3}m; \quad \lambda_3 = \frac{2}{2(3)-1} = \frac{2}{5}m;$$

***b)*** $T = 20°C = 293°K$

$$V = 331\sqrt{\frac{293}{273}} = 343$$

$$f_1 = \frac{V}{\lambda_1} = \frac{243}{2} \ \underline{\approx 172\ Hz|}$$

$$f_n = (2n-1)\, f_1$$

$$f_2 = (4-1)(172) \ \underline{= 516\ Hz|}$$

$$f_3 = (6-1)(172) \ \underline{= 860\ Hz|}$$

***c)*** $f_n = (2n-1)\, f_1$

$$f_{11} = (2(11)-1)\, f_1 = (21)(172) \ \underline{= 3612\ Hz|}$$

$$\lambda_{11} = \frac{V}{f_{11}} = \frac{343}{3612} \ \underline{= 0.095\ m|}$$

## *Case* 2: Wave Reflected from a less Dense Medium

In this case there is no phase shift on reflection. Therefore if the incident wave $y_i$ is given by

$$y_i = A_1 \cos\left(Kx - \omega t + \phi\right)$$

Then the reflected wave is given by $y_r = A_2 \cos\left(K\left(2L - x\right) - \omega t + \phi\right)$

As shown before the amplitude of the net wave is given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\left(\beta_2' - \beta_1'\right)}$$

Where $\beta_1' = Kx + \phi \quad \beta_2' = K\left(2L - x\right) + \phi$

$$\beta_2' - \beta_1' = 2K\left(L - x\right)$$

And assuming the wave is reflected 100% $\left(A_1 = A_L\right)$

$$A = \sqrt{A_1^2 + A_1^2 + 2A_1^2 \cos\left(2K\left(L - x\right)\right)}$$

$$= \sqrt{2}\left|A_1\right|\sqrt{1 + \cos\left(2K\left(L - x\right)\right)}$$

$$= \sqrt{2}A_1\sqrt{2\cos^2\left(K\left(L - x\right)\right)} \qquad \left(\cos^2 x = \frac{1 + \cos 2x}{2}\right)$$

$$\boxed{A = 2A_1 \cos\left(K\left(L - x\right)\right)}$$ (*Amplitude of a standing wave for the case where the wave is reflected from a less dense medium*)

At the reflection point $X = L$ & $A\Big|_{X=L} = 2A_1 \cos\left(0\right) = 2A_1$ which is the maximum amplitude.

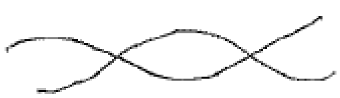
Therefore the reflection points is an antinode

## Boundary Conditions

Requiring the *x* = 0 and to be a node will result in a node in one end and an antinode in the other end which has been discussed already. So here we will discuss only the case where the *x* = 0 end is required to an antinode. That is case where both ends are antinodes. An example is sound resonance formed in a pipe open at both ends. With a similar analysis, it can be shown that the equations for the case where both ends are antinodes are identical to the case where both ends are nodes.

$$\boxed{\begin{aligned} \lambda_n &= \frac{2L}{n} \\ f_n &= n\left(\frac{V}{2L}\right) \\ f_1 &= \frac{V}{2L} \\ f_n &= nf_1 \\ N &= n \end{aligned}}$$

Equations for the case where both ends are antinodes



### *Example*

Consider sound resonance (*standing waves*) in a 2m pipe open at both ends. Assume temperature is 20°C. Calculate the wavelength and frequency of the 8th harmonic.

***Solution***

L = 2m; T = 20°C = 293°K

For a pipe open at both ends, both ends are anitnodes

$$f_1 = \frac{V}{2L}$$

$$V = \left(\sqrt{\frac{T}{273}}\right)331 = 331\sqrt{\frac{293}{273}} = 343\,{}^{m}/_{s}$$

$$\therefore f_1 = \frac{343}{2(2)} = 85.7Hz$$

$$f_n = nf_1 \Longrightarrow f_8 = 8f_1 = 8(85.7) = 426.7Hz$$

$$\lambda_8 = \frac{V}{f_8} = \frac{343}{426.7} = 0.8m$$

## A Beat

A beat is a wave with points of zero vibration which for a given location are separated by equal intervals of time, for example sound waves, at a given location, the event of no sound will be separated by equal intervals of time. The wave between two consecutive events of zero vibration is called a beat. The time taken for a beat is called the beat period and the number of beats heard per second is called the beat frequency.

A beat is formed when two waves with close frequencies interfere. Consider two interfering waves of frequencies $w_1$ and $w_2$ such that $\frac{\left|w_1 - w_2\right|}{w_1} < t$

$$y_1 = A\cos\left(K_1 x - \omega_1 t\right) \quad and \quad y_2 = A\cos\left(K_2 x - \omega_2 t\right)$$

(*for simplicity we are assuming both waves have the same amplitude*). Since the waves are travelling in the same medium they will have the same speed, $v_i$ that $v = \frac{\omega_1}{K_1} = \frac{\omega_2}{K_2}$

$$\begin{aligned} y_{net} &= y_1 + y_2 \\ &= A\cos(K_1 x - \omega_1 t) + A\cos(K_2 x - \omega_2 t) \\ &= A[\cos(K_1 x - \omega_1 t) + \cos(K_2 x - \omega_2 t)] \end{aligned}$$

Before we proceed let’s review a trignometric relationship that will help us combine the two into one term.

## Brief Review of a Trigonometric Identity

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

Adding these equations

$$\cos(a+b) + \cos(a-b) = 2\cos(a)\cos(b) \qquad \text{eqation.1}$$

Now let's transform the variables *a* and *b* into variables *x* and *y* using the equations

$x = a + b$

$y = a - b$

Sdding these equations results in

$$2a = x + y \quad or \quad a = \frac{x+y}{2}$$

Subtracting these equations results in

$$2b = x - y \quad or \quad b = \frac{x-y}{2}$$

Therefore equation 1 in terms of the variables *x* and *y* becomes

$$\boxed{\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

end of review

Using this formula, $y_{net}$ can be expressed as a simple term

$$\boxed{y_{net} = A\{\cos[K_1 x - \omega_1 t] + \cos[K_2 x - \omega_2 t]\}}$$

$$\boxed{= 2A\cos\left[\left(\frac{K_1+K_2}{2}\right)x - \left(\frac{\omega_1+\omega_2}{2}\right)t\right]\cos\left[\left(\frac{K_1-K_2}{2}\right)x - \left(\frac{\omega_1-\omega_2}{2}\right)t\right]}$$

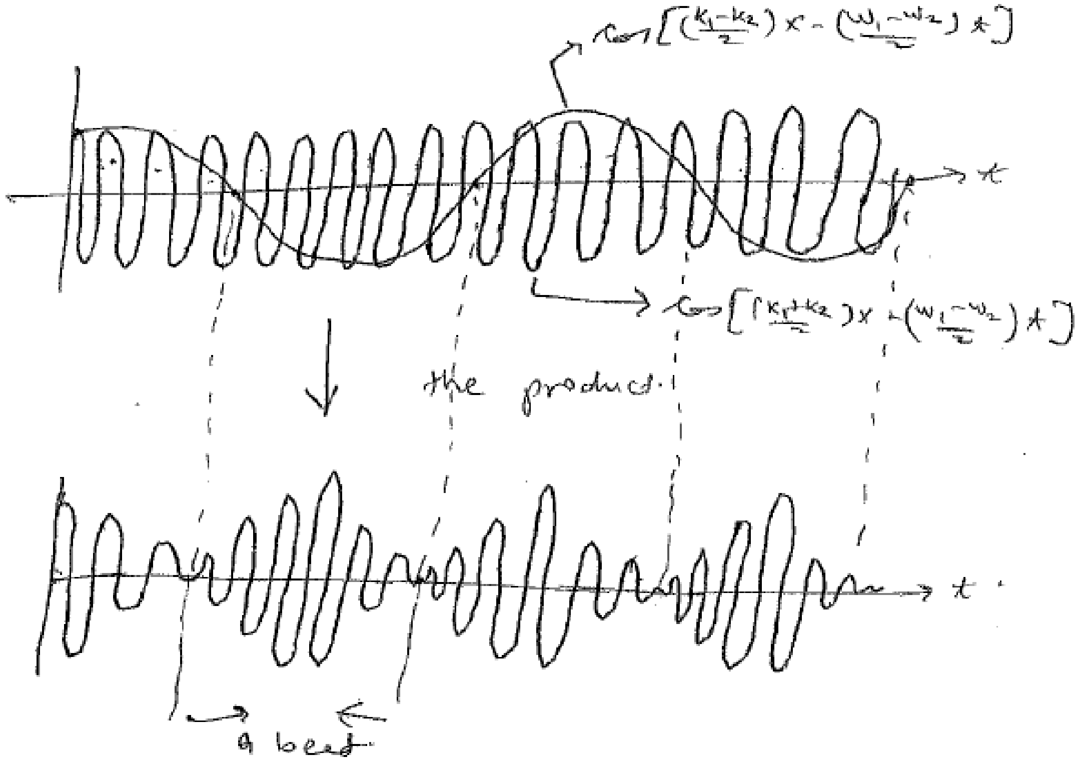
As can be seen from this equation the effect of the interference of two waves of close frequencies is the product of a high frequency wave $\left(\cos\left[\left(\frac{K_1+K_2}{2}\right)x - \left(\frac{\omega_1+\omega_2}{2}\right)t\right]\right)$ and a low frequency wave $\left(\cos\left[\left(\frac{K_1-K_2}{2}\right)x - \left(\frac{\omega_1+\omega_2}{2}\right)t\right]\right)$. The result is tracelling wave packets (*beats*) where the amplitude of the high frequency vibrations are controlled by the low frequency wave.

At a particular location, the two waves and their product (*the net wave*) may be diagrammatically represented as follows

As can be seen from here there are two beats in one cycle of the low frequency wave. That is the beat frequency $\left(f_b\right)$ is twice the frequency of the low frequency wave. Since the frequency of the law frequency wave is given by $\frac{|f_1 - f_2|}{2}$, it follows that

$$\boxed{f_b = |f_1 - f_2|}$$

(*the beat frequency, which is the number of beats per second is equal to the difference of the frequencies of the interfering wave*)




And since the beat period ($T_b$), time taken for one beat, is equal to $1/\delta_b$

$$T_b = \frac{1}{|f_1 - f_2|}$$

### *Example*

Consider the interference of the following two waves: $y_1 = 2\cos(2x - 200t)$ and $y_2 = 2\cos(2.02x - 202t)$.

*a)* Express the net wave (*beat*) as a product of two waves.
*b)* Calculate the number of beats per second
*c)* How long does one beat take?

<u>Solution</u>

***a)*** $K_1$=2, $\omega_1$=200, $K_2$=2.02, $\omega_2$=202, A=2

$$y_{net} = 2A\cos\left[\left(\frac{K_1 - K_2}{2}\right)x - \left(\frac{\omega_1 - \omega_2}{2}\right)t\right]\cos\left[\left(\frac{K_1 + K_2}{2}\right)x - \left(\frac{\omega_1 + \omega_2}{2}\right)t\right]$$

$$= 2(2)\cos\left[\left(\frac{.02}{2}\right)x + \left(\frac{2}{2}\right)t\right]\cos\left[\left(\frac{4.02}{2}\right)x - \left(\frac{402}{2}\right)t\right]$$

$$= 4\cos[.01x - t]\cos[2.01x - 201t]$$

***b)*** $f_b = |f_1 - f_2| = \left|\frac{\omega_1}{2\pi} - \frac{\omega_2}{2\pi}\right| = \frac{1}{2\pi}|\omega_1 - \omega_2| = \frac{1}{2\pi}|200 - 202| = \frac{2}{2\pi} \approx 0.318Hz$

***c)*** $T_b = \frac{1}{f_b} = \frac{1}{1/\pi} = \pi$

### *Example*

When a sound of frequency 1000Hz interferes with sound of unknown frequency 2 beats can be heard in 5 seconds.

*a)* Calculate the two possible frequencies of the unknown wave.

*b)* If both waves have amplitude of 159m and the temperature is 20°C, express the beat as a product of two waves (*just for one of the unknown frequency*)

***Solution***

***a)*** $f_1 = 1{,}000Hz; \quad f_b = \dfrac{2beats}{5\sec} = 0.4Hz$

$f_b = |f_1 - f_2|$

$f_1 - f_2 = \pm f_b$

$f_2 = f_1 \pm f_b = 1000 \pm 0.4$

***b)*** $\omega_1 = 2\pi f_1 = 2{,}000\pi \ rad/s; \quad \omega_1 = 2\pi f_2 = 200.8\pi \ rad/s; \quad A = 10^{-9}$

T=20°C=293°K

$$v = \sqrt{\frac{T}{273}} = \sqrt{\frac{293}{273}} = 343\ ^{m}/_{s}$$

$$K_1 = \frac{\omega_1}{v} = \frac{2000\pi}{343} = 5.83\pi/_{m} = 18.318/_{m}$$

$$y_{net} = 2A\cos\left[\left(\frac{K_1 - K_2}{2}\right)x - \left(\frac{\omega_1 - \omega_2}{2}\right)t\right]\cos\left[\left(\frac{K_1 + K_2}{2}\right)x - \left(\frac{\omega_1 + \omega_2}{2}\right)t\right]$$

$$y_{net} = 2 \times 10^{-9}\cos[.004x - 1.25t]\cos[18.32x - 6284t]$$