Solution   *Section* 3.3 – Properties of Division

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=2x^4-x^3+7x-12; \quad p(x)=x^2-3$

Solution

$$\begin{array}{r}
2x^2 - x + 6 \\
x^2-3\,\overline{)\,2x^4 - x^3 + 0x^2 + 7x - 12} \\
\underline{2x^4 \qquad - 6x^2 \qquad\qquad} \\
-x^3 + 6x^2 + 7x \qquad \\
\underline{-x^3 \qquad\quad + 3x} \qquad \\
6x^2 + 4x - 12 \\
\underline{6x^2 \qquad - 18} \\
4x + 6
\end{array}$$

$Q(x)=2x^2-x+6; \quad R(x)=4x+6$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=3x^3+2x-4; \quad p(x)=2x^2+1$

Solution

$$\begin{array}{r}
\frac{3}{2}x \qquad \\
2x^2+1\,\overline{)\,3x^3 + 0x^2 + 2x - 4} \\
\underline{3x^3 \qquad\quad + \frac{3}{2}x \qquad} \\
\frac{1}{2}x - 4
\end{array}$$

$Q(x)=\frac{3}{2}x; \quad R(x)=\frac{1}{2}x-4$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=7x+2; \quad p(x)=2x^2-x-4$

Solution

$Q(x)=0; \quad R(x)=7x+2$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=9x+4;\quad p(x)=2x-5$

Solution

$$\begin{array}{r} \frac{9}{2} \\ 2x-5 \overline{\big)\, 9x+4} \\ 9x-\frac{45}{2} \\ \hline -\frac{37}{2} \end{array}$$

$$Q(x)=\frac{9}{2};\quad R(x)=-\frac{37}{2}$$

Exercise

Use the remainder theorem to find $f(c)$: $f(x)=x^4-6x^2+4x-8;\quad c=-3$

Solution

$$f(-3)=(-3)^4-6(-3)^2+4(-3)-8 \ \underline{=7}|$$

Exercise

Use the remainder theorem to find $f(c)$: $f(x)=x^4+3x^2-12;\quad c=-2$

Solution

$$f(-2)=(-2)^4+3(-2)^2-12 \ \underline{=16}|$$

Exercise

Use the factor theorem to show that $x-c$ is a factor of $f(x)$: $f(x)=x^3+x^2-2x+12;\quad c=-3$

Solution

$$f(-3)=(-3)^3+(-3)^2-2(-3)+12 \ \underline{=0}|$$

From the factor theorem; $x+3$ is a factor of $f(x)$.

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second: $2x^3-3x^2+4x-5;\quad x-2$

Solution

$$\begin{array}{r|rrrr} 2 & 2 & -3 & 4 & -5 \\ & & 4 & 2 & 12 \\ \hline & 2 & 1 & 6 & \boxed{7} \end{array}$$

$Q(x) = 2x^2 + x + 6 \qquad R(x) = 7$

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second: $5x^3 - 6x^2 + 15; \quad x - 4$

Solution

$$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 15 \\ & & 20 & 56 & 224 \\ \hline & 5 & 14 & 56 & \boxed{239} \end{array}$$

$Q(x) = 5x^2 + 14x + 56 \qquad R(x) = 239$

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second: $9x^3 - 6x^2 + 3x - 4; \quad x - \frac{1}{3}$

Solution

$$\begin{array}{r|rrrr} \frac{1}{3} & 9 & -6 & 3 & -4 \\ & & 3 & -1 & \frac{2}{3} \\ \hline & 9 & -3 & 2 & \boxed{-\frac{10}{3}} \end{array}$$

$Q(x) = 9x^2 - 3x + 2 \qquad R(x) = -\frac{10}{3}$

Exercise

Use the synthetic division to find $f(c)$: $f(x) = 2x^3 + 3x^2 - 4x + 4; \quad c = 3$

Solution

$$\begin{array}{r|rrrr} 3 & 2 & 3 & -4 & 4 \\ & & 9 & 36 & 93 \\ \hline & 3 & 12 & 32 & \boxed{97} \end{array}$$

$\boxed{f(3) = 97}$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=8x^5-3x^2+7; \quad c=\frac{1}{2}$

Solution

$$\begin{array}{c|cccccc} \frac{1}{2} & 8 & 0 & 0 & -3 & 0 & 7 \\ & & 4 & 2 & 1 & -1 & -\frac{1}{2} \\ \hline & 8 & 4 & 2 & -2 & -1 & \boxed{\frac{13}{2}} \end{array}$$

$$\boxed{f\left(\frac{1}{2}\right)=\frac{13}{2}}$$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=x^3-3x^2-8; \quad c=1+\sqrt{2}$

Solution

$$\begin{array}{c|cccc} 1+\sqrt{2} & 3 & -3 & 0 & -8 \\ & & 3+3\sqrt{2} & 6+3\sqrt{2} & 12+9\sqrt{2} \\ \hline & 3 & 3\sqrt{2} & 6+3\sqrt{2} & \boxed{4+9\sqrt{2}} \end{array}$$

$$\boxed{f\left(1+\sqrt{2}\right)=4+9\sqrt{2}}$$

Exercise

Use the synthetic division to show that c is a zero of $f(x)$: $f(x)=3x^4+8x^3-2x^2-10x+4; \quad c=-2$

Solution

$$\begin{array}{c|ccccc} -2 & 3 & 8 & -2 & -10 & 4 \\ & & -6 & -4 & 12 & -4 \\ \hline & 3 & 2 & -6 & 2 & \boxed{0} \end{array}$$

$$\boxed{f(-2)=0}$$

Exercise

Use the synthetic division to show that c is a zero of $f(x)$: $f(x)=27x^4-9x^3+3x^2+6x+1; \quad c=-\frac{1}{3}$

Solution

$$\begin{array}{c|ccccc} -\frac{1}{3} & 27 & -9 & 3 & 6 & 1 \\ & & -9 & 6 & -3 & -1 \\ \hline & 27 & -18 & 9 & 3 & \boxed{0} \end{array}$$

$$\boxed{f\left(-\frac{1}{3}\right)=0}$$

Exercise

Find all values of k such that $f(x)$ is divisible by the given linear polynomial:

$f(x)=kx^3+x^2+k^2x+3k^2+11; \quad x+2$

Solution

$$\begin{array}{r|cccc} -2 & k & 1 & k^2 & 3k^2+11 \\ & & -2k & 4k-2 & -2k^2-8k+4 \\ \hline & k & 1-2k & k^2+4k-2 & k^2-8k+15 \end{array}$$

$k^2-8k+15=0 \Rightarrow \boxed{k=3,\ 5}$

Exercise

Find all solutions of the equation: $\quad x^3-x^2-10x-8=0$

Solution

possibilities for $\frac{c}{d}: \frac{\pm 1,\ \pm 2,\ \pm 4,\ \pm 8}{\pm 1}$

Using the calculator, the result will show that the solutions are: $x=-1,\ -2,\ 4$

Exercise

Find all solutions of the equation: $\quad x^3+x^2-14x-24=0$

Solution

possibilities for $\frac{c}{d}: \frac{\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 8,\ \pm 12,\ \pm 24}{\pm 1}$

Using the calculator, the result will show that the solutions are: $\boxed{x=-2}$

$$\begin{array}{r|cccc} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

We have the factorization of: $(x+2)\left(x^2-x-12\right)=0$

$x^2-x-12=0 \Rightarrow \boxed{x=-3,\ 4}$

Exercise

Find all solutions of the equation: $2x^3 - 3x^2 - 17x + 30 = 0$

Solution

possibilities for $\frac{c}{d}: \frac{\pm 1,\ \pm 2,\ \pm 3,\ \pm 5,\ \pm 6,\ \pm 10,\ \pm 15,\ \pm 30}{\pm 1,\ \pm 2}$

Using the calculator, the result will show that the solutions are: $\boxed{x = 2}$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & \boxed{0} \end{array}$$

We have the factorization of: $(x-2)\left(2x^2 + x - 15\right) = 0$

$2x^2 + x - 15 = 0 \Rightarrow \boxed{x = -3,\ \frac{5}{2}}$

Exercise

Find all solutions of the equation: $12x^3 + 8x^2 - 3x - 2 = 0$

Solution

possibilities for $\frac{c}{d}: \frac{\pm 1,\ \pm 2}{\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12}$

Using the calculator, the result will show that the solutions are: $\boxed{x = \frac{1}{2}}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 12 & 8 & -3 & -2 \\ & & 6 & 7 & 2 \\ \hline & 12 & 14 & 4 & \boxed{0} \end{array}$$

We have the factorization of: $\left(x - \frac{1}{2}\right)\left(12x^2 + 14x + 4\right) = 0$

$12x^2 + 14x + 4 = 0 \Rightarrow \boxed{x = -\frac{2}{3},\ -\frac{1}{2}}$

Exercise

Find all solutions of the equation: $x^4 + 3x^3 - 30x^2 - 6x + 56 = 0$

Solution

possibilities for $\frac{c}{d}: \frac{\pm 1,\ \pm 2,\ \pm 4,\ \pm 7,\ \pm 8,\ \pm 14, \cdots, \pm 56}{\pm 1}$

Using the calculator, the result will show that the solutions are: $\boxed{x = 4}$

$$\begin{array}{r|rrrrr} 4 & 1 & 3 & -30 & -6 & 56 \\ & & 4 & 28 & -8 & -56 \\ \hline & 1 & 7 & -2 & -14 & \boxed{0} \end{array}$$

We have the factorization of: $(x-4)\left(x^3+7x^2-2x-14\right)=0$

For $x^3+7x^2-2x-14 \quad \Rightarrow \quad \frac{c}{d}=\frac{\pm 1,\ \pm 2,\ \pm 7,\ \pm 14}{\pm 1}$

$\boxed{x=-7}$ is another solution.

$$\begin{array}{r|rrrr} -7 & 1 & 7 & -2 & -14 \\ & & -7 & 0 & 14 \\ \hline & 1 & 0 & -2 & \boxed{0} \end{array}$$

We have the factorization of: $(x+4)(x+7)\left(x^2-2\right)=0$

By applying quadratic formula to solve: $x^2-2=0 \ \Rightarrow \boxed{x=\pm\sqrt{2}}$

Exercise

Find all solutions of the equation: $3x^5-10x^4-6x^3+24x^2+11x-6=0$

Solution

$$\boxed{x=-1,\ \ -1,\ \frac{1}{3},\ 2,\ 3}$$

Exercise

Find all solutions of the equation: $6x^5+19x^4+x^3-6x^2=0$

Solution

$$x^2\left(6x^3+19x^2+x-6\right)=0$$

$$\boxed{x=0,\ 0,\ -\frac{2}{3},\ -3,\ \frac{1}{2}}$$