

Solution ***Section R.1 – Derivative***

Exercise

Find the derivative of $f(t) = -3t^2 + 2t - 4$

Solution

$$f'(t) = \underline{-6t + 2}$$

Exercise

Find the derivative of $g(x) = 4\sqrt[3]{x} + 2$

Solution

$$g(x) = 4x^{1/3} + 2$$

$$g'(x) = \frac{4}{3}x^{-2/3}$$

$$= \frac{4}{3x^{2/3}}$$

$$= \underline{\frac{4}{3\sqrt[3]{x^2}}}$$

Exercise

Find the derivative of $f(x) = x(x^2 + 1)$

Solution

$$f(x) = x^3 + x$$

$$f'(x) = \underline{3x^2 + 1}$$

Exercise

Find the derivative of $f(x) = \frac{2x^2 - 3x + 1}{x}$

Solution

$$f(x) = \frac{2x^2}{x} - \frac{3x}{x} + \frac{1}{x}$$

$$= 2x - 3 + \frac{1}{x}$$

$$f'(x) = \underline{2 - \frac{1}{x^2}}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

Exercise

Find the derivative of $f(x) = \frac{4x^3 - 3x^2 + 2x + 5}{x^2}$

Solution

$$f(x) = 4x - 3 + \frac{2}{x} + 5x^{-2} \qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\begin{aligned} f'(x) &= 4 - \frac{2}{x^2} - 10x^{-3} \\ &= 4 - \frac{2}{x^2} - \frac{10}{x^3} \end{aligned}$$

Exercise

Find the derivative of $f(x) = \frac{-6x^3 + 3x^2 - 2x + 1}{x}$

Solution

$$f(x) = -6x^2 + 3x - 2 + \frac{1}{x} \qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$f'(x) = -12x + 3 - \frac{1}{x^2}$$

Exercise

Find the derivative of $f(x) = x\left(1 - \frac{2}{x+1}\right)$

Solution

$$f(x) = x - \frac{2x}{x+1}$$

$$\left(\frac{2x}{x+1}\right)' \Rightarrow \begin{array}{ll} f = 2x & f' = 2 \\ g = x+1 & g' = 1 \end{array}$$

$$\begin{aligned} f'(x) &= 1 - \frac{2(x+1) - 2x}{(x+1)^2} \\ &= 1 - \frac{2x+2-2x}{(x+1)^2} \\ &= 1 - \frac{2}{(x+1)^2} \end{aligned}$$

Exercise

Find the derivative of $g(s) = \frac{s^2 - 2s + 5}{\sqrt{s}}$

Solution

$$\begin{aligned} g(s) &= \frac{s^2}{s^{1/2}} - 2 \frac{s}{s^{1/2}} + \frac{5}{s^{1/2}} \\ &= s^{3/2} - 2s^{1/2} + 5s^{-1/2} \end{aligned}$$

$$\begin{aligned} g'(s) &= \frac{3}{2}s^{1/2} - 2 \frac{1}{2}s^{-1/2} + 5\left(-\frac{1}{2}\right)s^{-3/2} \\ &= \frac{3}{2}s^{1/2} - s^{-1/2} - \frac{5}{2}s^{-3/2} \\ &= \frac{3}{2}\sqrt{s} - \frac{1}{\sqrt{s}} - \frac{5}{2s^{3/2}} \\ &= \frac{\frac{3}{2}\sqrt{s} - \frac{1}{\sqrt{s}} - \frac{5}{2s\sqrt{s}}}{1} \end{aligned}$$

Exercise

Find the derivative of $f(x) = \frac{x+1}{\sqrt{x}}$

Solution

$$\begin{aligned} f(x) &= \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}} \\ &= x^{1/2} + x^{-1/2} \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} \\ &= \frac{\frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}}{1} \end{aligned}$$

Exercise

Find the derivative to the following functions $y = 3x(2x^2 + 5x)$

Solution

$$\begin{aligned} y &= 6x^3 + 15x^2 \\ \Rightarrow y' &= 18x^2 + 30x \end{aligned}$$

Exercise

Find the derivative to the following functions $y = 3(2x^2 + 5x)$

Solution

$$y = 6x^2 + 15x$$

$$\Rightarrow \boxed{y' = 12x + 15}$$

Exercise

Find the derivative to the following functions $y = \frac{x^2 + 4x}{5}$

Solution

$$y = \frac{1}{5} [x^2 + 4x]$$

$$\boxed{y' = \frac{1}{5} (2x + 4)}$$

Exercise

Find the derivative to the following functions $y = \frac{3x^4}{5}$

Solution

$$y = \frac{3}{5} x^4$$

$$\boxed{y' = \frac{12}{5} x^3}$$

Exercise

Find the derivative to the following functions $y = \frac{x^2 - 4}{2x + 5}$

Solution

$$y' = \frac{(2x + 5)(2x) - (x^2 - 4)(2)}{(2x + 5)^2}$$

$$= \frac{4x^2 + 10x - 2x^2 + 8}{(2x + 5)^2}$$

$$\boxed{= \frac{2x^2 + 10x + 8}{(2x + 5)^2}}$$

Exercise

Find the derivative to the following functions $y = \frac{(1+x)(2x-1)}{x-1}$

Solution

$$\begin{aligned}
 y &= \frac{(1+x)(2x-1)}{x-1} \\
 &= \frac{2x-1+2x^2-x}{x-1} \\
 &= \frac{2x^2+x-1}{x-1} \\
 y' &= \frac{(x-1)(4x+1) - (2x^2+x-1)(1)}{(x-1)^2} \\
 &= \frac{4x^2+x-4x-1-2x^2-x+1}{(x-1)^2} \\
 &= \frac{2x^2-4x}{(x-1)^2}
 \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{4}{2x+1}$

Solution

$$\begin{aligned}
 y &= 4(2x+1)^{-1} \\
 y' &= -4(2x+1)^{-2}(2) \\
 &= -8(2x+1)^{-2} \\
 &= -\frac{8}{(2x+1)^2}
 \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{2}{(x-1)^3} = 2(x-1)^{-3}$

Solution

$$\begin{aligned}
 y &= 2(x-1)^{-3} \\
 y' &= -\frac{6}{(x-1)^4}
 \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \sqrt[3]{(x+4)^2}$

Solution

$$y = (x+4)^{2/3}$$

$$y' = \frac{2}{3}(x+4)^{-1/3}$$

$$= \frac{2}{3} \frac{1}{(x+4)^{1/3}}$$

$$= \frac{2}{3 \sqrt[3]{x+4}} \quad \Big|$$

Exercise

Find the derivative of $f(x) = \sqrt{2t^2 + 5t + 2}$

Solution

$$f(t) = (2t^2 + 5t + 2)^{1/2}$$

$$f'(t) = \frac{1}{2}(4t+5)(2t^2 + 5t + 2)^{-1/2}$$

$$= \frac{1}{2} \frac{4t+5}{\sqrt{2t^2 + 5t + 2}} \quad \Big|$$

$$U = 2t^2 + 5t + 2 \rightarrow U' = 4t + 5$$

$$(U^n)' = nU'U^{n-1}$$

Exercise

Find the derivative of $f(x) = \frac{1}{(x^2 - 3x)^2}$

Solution

$$f(x) = (x^2 - 3x)^{-2}$$

$$f'(x) = -2(2x-3)(x^2 - 3x)^{-3}$$

$$= -\frac{2(2x-3)}{(x^2 - 3x)^3} \quad \Big|$$

ExerciseFind the derivative of $y = t^2\sqrt{t-2}$ **Solution**

$$y' = 2t\sqrt{t-2} + t^2 \frac{1}{2}(t-2)^{-1/2}$$

$$= \left[2t(t-2)^{1/2} + t^2 \frac{1}{2}(t-2)^{-1/2} \right] \frac{2(t-2)^{1/2}}{2(t-2)^{1/2}}$$

$$= \frac{4t(t-2) + t^2}{2(t-2)^{1/2}}$$

$$= \frac{4t^2 - 8t + t^2}{2\sqrt{t-2}}$$

$$= \frac{5t^2 - 8t}{2\sqrt{t-2}}$$

$$f = t^2$$

$$f' = 2t$$

$$g = (t-2)^{1/2}$$

$$g' = \frac{1}{2}(t-2)^{-1/2}$$

ExerciseFind the derivative of $y = \left(\frac{6-5x}{x^2-1} \right)^2$ **Solution**

$$f = 6-5x \quad g = x^2-1$$

$$f' = -5 \quad g' = 2x$$

$$y' = 2 \frac{-5(x^2-1) - 2x(6-5x)}{(x^2-1)^2} \left(\frac{6-5x}{x^2-1} \right)$$

$$= 2 \frac{-5x^2 + 5 - 12x + 10x^2}{(x^2-1)^3} (6-5x)$$

$$= \frac{2(5x^2 - 12x + 5)(6-5x)}{(x^2-1)^3}$$

$$(U^n)' = nU'U^{n-1}$$

Exercise

Find the derivative to the following functions $y = x^2 \sqrt{x^2 + 1}$

Solution

$$\begin{aligned} y &= x^2 (x^2 + 1)^{1/2} \\ y' &= x^2 \left[\frac{1}{2} (x^2 + 1)^{-1/2} (2x) \right] + (x^2 + 1)^{1/2} [2x] \\ &= x^3 (x^2 + 1)^{-1/2} + 2x (x^2 + 1)^{1/2} \\ &= \frac{(x^2 + 1)^{1/2}}{(x^2 + 1)^{1/2}} \left[x^3 (x^2 + 1)^{-1/2} + 2x (x^2 + 1)^{1/2} \right] \\ &= \frac{x^3 (x^2 + 1)^{-1/2} (x^2 + 1)^{1/2} + 2x (x^2 + 1)^{1/2} (x^2 + 1)^{1/2}}{(x^2 + 1)^{1/2}} \\ &= \frac{x^3 + 2x(x^2 + 1)}{(x^2 + 1)^{1/2}} \\ &= \frac{x^3 + 2x^3 + 2x}{\sqrt{x^2 + 1}} \\ &= \frac{3x^3 + 2x}{\sqrt{x^2 + 1}} \\ &= \frac{x(3x^2 + 2)}{\sqrt{x^2 + 1}} \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \left(\frac{x+1}{x-5} \right)^2$

Solution

$$\begin{aligned} y' &= 2 \left(\frac{x+1}{x-5} \right) \frac{d}{dx} \left[\frac{x+1}{x-5} \right] \\ &= 2 \left(\frac{x+1}{x-5} \right) \left[\frac{(1)(x-5) - (1)(x+1)}{(x-5)^2} \right] \\ &= 2 \left(\frac{x+1}{x-5} \right) \left(\frac{x-5-x-1}{(x-5)^2} \right) \\ &= 2 \left(\frac{x+1}{x-5} \right) \left(\frac{-6}{(x-5)^2} \right) \\ &= -\frac{12(x+1)}{(x-5)^3} \end{aligned}$$

Exercise

Find the derivative to the following functions $y = x^2 \sin x$

Solution

$$y' = \underline{2x \sin x + x^2 \cos x}$$

$$\begin{aligned} u &= x^2 & v &= \sin x \\ u' &= 2x & v' &= \cos x \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{\sin x}{x}$

Solution

$$y' = \underline{\frac{x \cos x - \sin x}{x^2}}$$

$$\begin{aligned} u &= \sin x & v &= x \\ u' &= \cos x & v' &= 1 \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{\cot x}{1 + \cot x}$

Solution

$$\begin{aligned} y' &= \frac{-\csc^2 x (1 + \cot x) + \csc^2 x \cot x}{(1 + \cot x)^2} \\ &= \frac{-\csc^2 x - \csc^2 x \cot x + \csc^2 x \cot x}{(1 + \cot x)^2} \\ &= \underline{\frac{-\csc^2 x}{(1 + \cot x)^2}} \end{aligned}$$

$$\begin{aligned} u &= \cot x & v &= 1 + \cot x \\ u' &= -\csc^2 x & v' &= -\csc^2 x \end{aligned}$$

Exercise

Find the derivative to the following functions $y = x^2 \sin x + 2x \cos x - 2 \sin x$

Solution

$$\begin{aligned} y' &= 2x \sin x + x^2 \cos x + 2 \cos x - 2x \sin x - 2 \cos x \\ &= \underline{x^2 \cos x} \end{aligned}$$

Exercise

Find the derivative to the following functions $y = x^3 \sin x \cos x$

Solution

$$\begin{aligned} y' &= \left(x^3\right)' \sin x \cos x + x^3 (\sin x)' \cos x + x^3 \sin x (\cos x)' \\ &= \underline{3x^2 \sin x \cos x + x^3 \cos^2 x - x^3 \sin^2 x} \end{aligned}$$

Exercise

Find the derivative to the following functions $y = \frac{4}{\cos x} + \frac{1}{\tan x}$

Solution

$$\begin{aligned} y' &= \frac{-4\sin x}{\cos^2 x} - \frac{\sec^2 x}{\tan^2 x} & \left(\frac{1}{u}\right)' &= -\frac{u'}{u^2} \\ &= -4 \frac{\sin x}{\cos x} \frac{1}{\cos x} - \frac{1}{\cos^2 x} \frac{\cos^2 x}{\sin^2 x} \\ &= -4 \tan x \sec x - \csc^2 x \end{aligned}$$

Exercise

Find the derivative of $f(x) = \frac{(x^2 - 6x)^5}{(3x^2 + 5x - 2)^4}$

Solution

$$\begin{aligned} f(x) &= (x^2 - 6x)^5 (3x^2 + 5x - 2)^{-4} & (U^m V^n)' &= U^{m-1} V^{n-1} (mU'V + nUV') \\ f'(x) &= (x^2 - 6x)^4 (3x^2 + 5x - 2)^{-5} \left[5(2x - 6)(3x^2 + 5x - 2) - 4(x^2 - 6x)(6x + 5) \right] \\ &= (x^2 - 6x)^4 (3x^2 + 5x - 2)^{-5} \left[(10x - 30)(3x^2 + 5x - 2) - 4(6x^3 - 31x^2 - 30x) \right] \\ &= (x^2 - 6x)^4 (3x^2 + 5x - 2)^{-5} \\ &\quad \left[\begin{array}{rcl} x^3 & 30 - 24 & \\ x^2 & 50 - 90 + 124 & \\ x & -20 - 150 + 120 & \\ x^0 & 60 & \end{array} \right] \\ &= \frac{(x^2 - 6x)^4 (6x^3 + 84x^2 - 50x + 60)}{(3x^2 + 5x - 2)^5} \end{aligned}$$

Exercise

Find the derivative of $y = \ln \sqrt{x+5}$

Solution

$$\begin{aligned} y &= \ln(x+5)^{1/2} = \frac{1}{2} \ln(x+5) \\ y' &= \frac{1}{2(x+5)} \end{aligned}$$

Exercise

Find the Derivatives of $y = (3x + 7)\ln(2x - 1)$

Solution

$$f = 3x + 7 \quad f' = 3$$

$$g = \ln(2x - 1) \quad g' = \frac{2}{2x - 1}$$

$$\underline{y' = 3x\ln(2x - 1) + \frac{2(3x + 7)}{2x - 1}}$$

Exercise

Find the Derivatives of $f(x) = \ln \sqrt[3]{x + 1}$

Solution

$$\begin{aligned} f(x) &= \ln(x + 1)^{1/3} \\ &= \frac{1}{3} \ln(x + 1) \end{aligned}$$

$$u = x + 1 \Rightarrow \frac{du}{dx} = 1$$

$$\begin{aligned} f'(x) &= \frac{1}{3} \frac{1}{x + 1} \\ &= \frac{1}{3(x + 1)} \end{aligned}$$

Exercise

Find the Derivatives of $f(x) = \ln \left[x^2 \sqrt{x^2 + 1} \right]$

Solution

$$f(x) = \ln(x^2) + \ln \sqrt{x^2 + 1} \quad \text{Product Property}$$

$$f(x) = \ln(x^2) + \ln(x^2 + 1)^{1/2}$$

$$f(x) = 2 \ln x + \frac{1}{2} \ln(x^2 + 1) \quad \text{Power Property}$$

$$\begin{aligned} f'(x) &= 2 \frac{1}{x} + \frac{1}{2} \frac{2x}{x^2 + 1} \\ &= \frac{2}{x} + \frac{x}{x^2 + 1} \end{aligned} \quad \text{Differentiate}$$

Exercise

Find the Derivatives of $y = \ln \frac{x^2}{x^2 + 1}$

Solution

$$y = \ln x^2 - \ln x^2 + 1$$

$$y' = \frac{2x}{x^2} - \frac{2x}{x^2 + 1} = \frac{2}{x} - \frac{2x}{x^2 + 1}$$

Exercise

Find the derivative of $f(x) = e^{-2x^3}$

Solution

$$f'(x) = e^{-2x^3} (-6x^2) = -\frac{6x^2}{e^{2x^3}}$$

Exercise

Find the derivative of $f(x) = 4e^{x^2}$

Solution

$$f'(x) = 4e^{x^2} (2x) = 8xe^{x^2}$$

Exercise

Find the derivative of $f(x) = x^2 e^x$

Solution

$$\begin{aligned} f'(x) &= e^x \frac{d}{dx}[x^2] + x^2 \frac{d}{dx}[e^x] \\ &= e^x (2x) + x^2 e^x \\ &= xe^x (2 + x) \end{aligned}$$

Exercise

Find the derivative $f(x) = 2x^3 e^x$

Solution

$$\begin{aligned} f'(x) &= 6x^2 e^x + 2x^3 e^x \\ &= 2x^2 e^x (3 + x) \end{aligned}$$

$$\begin{aligned} u &= 2x^3 & v &= e^x \\ u' &= 6x^2 & v' &= e^x \end{aligned}$$

Exercise

Find the derivative $f(x) = \frac{3e^x}{1+e^x}$

Solution

$$\begin{aligned} f'(x) &= \frac{3e^x(1+e^x) - 3e^x e^x}{(1+e^x)^2} \\ &= \frac{3e^x + 3e^{2x} - 3e^{2x}}{(1+e^x)^2} \\ &= \frac{3e^x}{(1+e^x)^2} \end{aligned}$$

$$\begin{aligned} u &= 3e^x & v &= 1+e^x \\ u' &= 3e^x & v' &= e^x \end{aligned}$$

Exercise

Find the derivative $f(x) = 5e^x + 3x + 1$

Solution

$$f'(x) = 5e^x + 3$$

Exercise

Find the derivative of $f(x) = \frac{e^x + e^{-x}}{2}$

Solution

$$\begin{aligned} f(x) &= \frac{1}{2}(e^x + e^{-x}) \\ f'(x) &= \frac{1}{2}(e^x - e^{-x}) \end{aligned}$$

Exercise

Find the derivative of $f(x) = \frac{e^x}{x^2}$

Solution

$$\begin{aligned} f'(x) &= \frac{x^2 e^x - 2x e^x}{x^4} \\ &= \frac{x e^x (x-2)}{x^4} \\ &= \frac{e^x (x-2)}{x^3} \end{aligned}$$

Exercise

Find the derivative of $f(x) = x^2e^x - e^x$

Solution

$$\begin{aligned}f'(x) &= e^x \frac{d}{dx}[x^2] + x^2 \frac{d}{dx}[e^x] - \frac{d}{dx}[e^x] \\&= e^x(2x) + x^2e^x - e^x \\&= \underline{e^x(x^2 + 2x - 1)}\end{aligned}$$

Exercise

Find the derivative of $f(x) = (1 + 2x)e^{4x}$

Solution

$$\begin{aligned}f'(x) &= (2)e^{4x} + (1 + 2x)(4e^{4x}) \\&= 2e^{4x} + (1 + 2x)(4e^{4x}) \\&= 2e^{4x}(1 + 2(1 + 2x)) \\&= 2e^{4x}(1 + 2 + 4x) \\&= \underline{2e^{4x}(3 + 4x)}\end{aligned}$$

Exercise

Find the derivative of $y = x^2e^{5x}$

Solution

$$\begin{aligned}y' &= x^2(5e^{5x}) + 2x(e^{5x}) \\&= \underline{xe^{5x}(5x + 2)}\end{aligned}$$

Exercise

Find the derivative of $y = x^2e^{-2x}$

Solution

$$\begin{aligned}y' &= 2xe^{-2x} - 2x^3e^{-2x} \\&= \underline{2xe^{-2x}(1 - x^2)}\end{aligned}$$

Exercise

Find the derivative $f(x) = \frac{e^x}{x^2 + 1}$

Solution

$$\begin{aligned} f'(x) &= \frac{e^x(x^2 + 1) - 2xe^x}{(x^2 + 1)^2} \\ &= \frac{(x^2 + 1 - 2x)e^x}{(x^2 + 1)^2} \end{aligned}$$

$$u = e^x \quad v = x^2 + 1$$

$$u' = e^x \quad v' = 2x$$

Exercise

Find the derivative $f(x) = \frac{1 - e^x}{1 + e^x}$

Solution

$$\begin{aligned} f'(x) &= \frac{-e^x(1 + e^x) - e^x(1 - e^x)}{(1 + e^x)^2} \\ &= \frac{-e^x - e^{2x} - e^x + e^{2x}}{(1 + e^x)^2} \\ &= -\frac{2e^x}{(1 + e^x)^2} \end{aligned}$$

$$u = 1 - e^x \quad v = 1 + e^x$$

$$u' = -e^x \quad v' = e^x$$

Exercise

Find the Derivatives of $y = \frac{\ln x}{e^{2x}}$

Solution

$$\begin{aligned} y' &= \frac{e^{2x}(1/x) - \ln x(2e^{2x})}{e^{4x}} \\ &= \frac{e^{2x} - 2x \ln x(e^{2x})}{e^{4x}} \\ &= \frac{e^{2x}(1 - 2x \ln x)}{e^{4x}} \end{aligned}$$

Exercise

Find the Derivatives of $f(x) = e^{2x} \ln(xe^x + 1)$

Solution

$$\begin{aligned} f'(x) &= 2e^{2x} \ln(xe^x + 1) + e^{2x} \frac{e^x + xe^x}{xe^x + 1} \\ &= e^{2x} \left[2 \ln(xe^x + 1) + \frac{e^x(1+x)}{xe^x + 1} \right] \end{aligned}$$
$$\begin{aligned} f &= e^{2x} & U &= 2x \rightarrow U' = 2 & f' &= 2e^{2x} \\ g &= \ln(xe^x + 1) & U &= xe^x + 1 \rightarrow U' = e^x + xe^x & g' &= \frac{e^x + xe^x}{xe^x + 1} \end{aligned}$$

Exercise

Find the Derivatives of $f(x) = \frac{xe^x}{\ln(x^2 + 1)}$

Solution

$$\begin{aligned} f'(x) &= \frac{e^x(1+x) \ln(x^2 + 1) - \frac{2x}{x^2 + 1} xe^x}{\left[\ln(x^2 + 1) \right]^2} \\ &= \frac{e^x \left[(1+x) \ln(x^2 + 1) - \frac{2x^2}{x^2 + 1} \right]}{\left[\ln(x^2 + 1) \right]^2} \\ &= \frac{e^x \left[\frac{(x^2 + 1)(1+x) \ln(x^2 + 1) - 2x^2}{x^2 + 1} \right]}{\left[\ln(x^2 + 1) \right]^2} \\ &= \frac{e^x \left[(x^2 + 1)(1+x) \ln(x^2 + 1) - 2x^2 \right]}{(x^2 + 1) \left[\ln(x^2 + 1) \right]^2} \end{aligned}$$
$$\begin{aligned} u &= xe^x & u' &= e^x + xe^x \\ v &= \ln(x^2 + 1) & v' &= \frac{2x}{x^2 + 1} \end{aligned}$$

Exercise

Find the derivative $y = \cos^{-1}\left(\frac{1}{x}\right)$

Solution

$$\begin{aligned} y &= \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(x) \\ y' &= \frac{1}{|x| \cdot \sqrt{x^2 - 1}} \end{aligned}$$

Exercise

Find the derivative $y = \sin^{-1} \sqrt{2}t$

Solution

$$y' = \frac{\sqrt{2}}{\sqrt{1 - (\sqrt{2}t)^2}}$$

$$= \frac{\sqrt{2}}{\sqrt{1 - 2t^2}} \quad \Big|$$

Exercise

Find the derivative $y = \sec^{-1}(5s)$

Solution

$$y' = \frac{5s}{|5s| \sqrt{(5s)^2 - 1}}$$

$$= \frac{s}{|s| \sqrt{25s^2 - 1}} \quad \Big|$$

Exercise

Find the derivative $y = \cot^{-1} \sqrt{t-1}$

Solution

$$y' = -\frac{\frac{1}{2}(t-1)^{-1/2}}{1 + \left[(t-1)^{1/2}\right]^2}$$

$$= -\frac{1}{2(t-1)^{1/2}(1+t-1)}$$

$$= -\frac{1}{2t\sqrt{t-1}} \quad \Big|$$

Exercise

Find the derivative $y = \ln(\tan^{-1} x)$

Solution

$$y' = \frac{1}{1+x^2} \cdot \frac{1}{\tan^{-1} x} = \frac{1}{(1+x^2)\tan^{-1} x} \quad \Big|$$

Exercise

Find the derivative $y = \tan^{-1}(\ln x)$

Solution

$$\begin{aligned} y' &= \frac{\frac{1}{x}}{1 + (\ln x)^2} \\ &= \frac{1}{x[1 + (\ln x)^2]} \end{aligned}$$

$$\left(\tan^{-1} u\right)' = \frac{u'}{1 + u^2}$$

Solution Section R.2 – Integration

Exercise

Find each indefinite integral. $\int \frac{x+2}{\sqrt{x}} dx$

Solution

$$\begin{aligned}\int \frac{x+2}{\sqrt{x}} dx &= \int \left[\frac{x}{x^{1/2}} + \frac{2}{x^{1/2}} \right] dx \\&= \int \frac{x}{x^{1/2}} dx + \int \frac{2}{x^{1/2}} dx \\&= \int x^{1/2} dx + 2 \int x^{-1/2} dx \\&= \frac{x^{3/2}}{3/2} + 2 \frac{x^{1/2}}{1/2} + C \\&= \underline{\underline{\frac{2}{3}x^{3/2} + 4x^{1/2} + C}}\end{aligned}$$

Exercise

Find each indefinite integral $\int 4y^{-3} dy$

Solution

$$\begin{aligned}\int 4y^{-3} dy &= 4 \frac{y^{-2}}{-2} + C \\&= \underline{\underline{-\frac{2}{y^2} + C}}\end{aligned}$$

Exercise

Find each indefinite integral $\int (x^3 - 4x + 2) dx$

Solution

$$\int (x^3 - 4x + 2) dx = \underline{\underline{\frac{1}{4}x^4 - 2x^2 + 2x + C}}$$

Exercise

Find each indefinite integral $\int \left(\sqrt[4]{x^3} + 1 \right) dx$

Solution

$$\int \left(x^{3/4} + 1 \right) dx = \underline{\frac{4}{7} x^{7/4} + x + C}$$

Exercise

Find each indefinite integral $\int \sqrt{x}(x+1) dx$

Solution

$$\begin{aligned} \int x^{1/2}(x+1) dx &= \int \left(x^{3/2} + x^{1/2} \right) dx \\ &= \underline{\frac{2}{5} x^{5/2} + \frac{2}{3} x^{3/2} + C} \end{aligned}$$

Exercise

Find each indefinite integral $\int (1+3t)t^2 dt$

Solution

$$\int \left(t^2 + 3t^3 \right) dt = \underline{\frac{1}{3} t^3 + \frac{3}{4} t^4 + C}$$

Exercise

Find each indefinite integral $\int \frac{x^2-5}{x^2} dx$

Solution

$$\begin{aligned} \int \frac{x^2-5}{x^2} dx &= \int \left(1 - \frac{5}{x^2} \right) dx \\ &= \int \left(1 - 5x^{-2} \right) dx \\ &= x + 5x^{-1} + C \\ &= \underline{x + \frac{5}{x} + C} \end{aligned}$$

Exercise

Find each indefinite integral $\int (-40x + 250) dx$

Solution

$$\int (-40x + 250) dx = \underline{-20x^2 + 250x + C}$$

Exercise

Find each indefinite integral $\int (7 - 3x - 3x^2)(2x + 1) dx$

Solution

$$\begin{aligned} \int (7 - 3x - 3x^2)(2x + 1) dx &= \int (14x + 7 - 6x^2 - 3x - 6x^3 - 3x^2) dx \\ &= \int (-6x^3 - 9x^2 + 11x + 7) dx \\ &= \underline{-\frac{3}{2}x^4 - 3x^3 + \frac{11}{2}x^2 + 7x + C} \end{aligned}$$

Exercise

Find the integral $\int (1 + \cos 3\theta) d\theta$

Solution

$$\int (1 + \cos 3\theta) d\theta = \underline{\theta + \frac{1}{3} \sin 3\theta + C}$$

Exercise

Find the integral $\int 2 \sec^2 \theta d\theta$

Solution

$$\int 2 \sec^2 \theta d\theta = \underline{2 \tan \theta + C}$$

Exercise

Find the integral $\int \sec 2x \tan 2x \, dx$

Solution

$$\int \sec 2x \tan 2x \, dx = \underline{\frac{1}{2} \sec 2x + C}$$

Exercise

Find the integral $\int 2e^{2x} \, dx$

Solution

$$\int 2e^{2x} \, dx = \underline{e^{2x} + C}$$

Exercise

Find the integral $\int \frac{12}{x} \, dx$

Solution

$$\int \frac{12}{x} \, dx = \underline{12 \ln |x| + C}$$

Exercise

Find the integral $\int \frac{dx}{\sqrt{1-x^2}}$

Solution

$$\int \frac{dx}{\sqrt{1-x^2}} = \underline{\sin^{-1} x + C}$$

Exercise

Find the integral $\int \frac{dx}{x^2 + 1}$

Solution

$$\int \frac{dx}{x^2 + 1} = \underline{\tan^{-1} x + C}$$

Exercise

Find the integral $\int \frac{1 + \tan \theta}{\sec \theta} d\theta$

Solution

$$\begin{aligned}\int \frac{1 + \tan \theta}{\sec \theta} d\theta &= \int \left(\frac{1}{\sec \theta} + \frac{\tan \theta}{\sec \theta} \right) d\theta \\ &= \int (\cos \theta + \sin \theta) d\theta \\ &= \sin \theta - \cos \theta + C\end{aligned}$$

Exercise

Find the general solution of the differential equation $y' = 2t + 3$

Solution

$$\begin{aligned}dy &= (2t + 3) dt \\ \int dy &= \int (2t + 3) dt \\ y &= t^2 + 3t + C\end{aligned}$$

Exercise

Find the general solution of the differential equation $y' = 3t^2 + 2t + 3$

Solution

$$\begin{aligned}\int dy &= \int (3t^2 + 2t + 3) dt \\ y &= t^3 + t^2 + 3t + C\end{aligned}$$

Exercise

Find the general solution of the differential equation $y' = \sin 2t + 2 \cos 3t$

Solution

$$\begin{aligned}\int dy &= \int (\sin 2t + 2 \cos 3t) dt \\ y(t) &= -\frac{1}{2} \cos 2t + \frac{2}{3} \sin 3t + C\end{aligned}$$

Exercise

Find the general solution of the differential equation: $y' = x^3(3x^4 + 1)^2$

Solution

$$\int x^3(3x^4 + 1)^2 dx$$

$$u = 3x^4 + 1 \Rightarrow du = 12x^3 dx$$

$$\Rightarrow \frac{1}{12} du = x^3 dx$$

$$\begin{aligned}\int x^3(3x^4 + 1)^2 dx &= \int \frac{1}{12} u^2 du \\ &= \frac{1}{12} \frac{(3x^4 + 1)^3}{3} + C \\ &= \frac{1}{36} (3x^4 + 1)^3 + C\end{aligned}$$

$$\boxed{y = \frac{1}{36} (3x^4 + 1)^3 + C}$$

Exercise

Find the general solution of the differential equation: $y' = 5x\sqrt{x^2 - 1}$

Solution

$$\begin{aligned}\int 5x(x^2 - 1)^{1/2} dx &= \frac{5}{2} \int (x^2 - 1)^{1/2} d(x^2 - 1) & d(x^2 - 1) = 2x dx \\ &= \frac{5}{3} (x^2 - 1)^{3/2} + C\end{aligned}$$

Exercise

Find the general solution of the differential equation: $y' = x\sqrt{x^2 + 4}$

Solution

$$\begin{aligned}\int \sqrt{x^2 + 4} x dx &= \frac{1}{2} \int (x^2 + 4)^{1/2} d(x^2 + 4) \\ &= \frac{1}{3} (x^2 + 4)^{3/2} + C\end{aligned}$$

Exercise

Evaluate the integrals $\int_{-2}^2 (x^3 - 2x + 3) dx$

Solution

$$\begin{aligned}\int_{-2}^2 (x^3 - 2x + 3) dx &= \left[\frac{x^4}{4} - x^2 + 3x \right]_{-2}^2 \\ &= \left(\frac{(2)^4}{4} - (2)^2 + 3(2) \right) - \left(\frac{(-2)^4}{4} - (-2)^2 + 3(-2) \right) \\ &= \underline{\underline{12}}\end{aligned}$$

Exercise

Evaluate the integrals $\int_0^1 (x^2 + \sqrt{x}) dx$

Solution

$$\begin{aligned}\int_0^1 (x^2 + \sqrt{x}) dx &= \left[\frac{x^3}{3} + \frac{2}{3} x^{3/2} \right]_0^1 \\ &= \left(\frac{(1)^3}{3} + \frac{2}{3} (1)^{3/2} \right) - 0 \\ &= \underline{\underline{1}}\end{aligned}$$

Exercise

Evaluate the integrals $\int_0^{\pi/3} 4 \sec u \tan u \, du$

Solution

$$\begin{aligned}\int_0^{\pi/3} 4 \sec u \tan u \, du &= 4 \sec u \Big|_0^{\pi/3} \\ &= 4 \left(\sec \frac{\pi}{3} - \sec 0 \right) \\ &= 4(2 - 1) \\ &= \underline{\underline{4}}\end{aligned}$$

Exercise

Evaluate the integrals $\int_{\pi/4}^{3\pi/4} \csc \theta \cot \theta d\theta$

Solution

$$\begin{aligned}\int_{\pi/4}^{3\pi/4} \csc \theta \cot \theta d\theta &= -\csc \theta \Big|_{\pi/4}^{3\pi/4} \\&= -\left(\csc \frac{3\pi}{4} - \csc \frac{\pi}{4}\right) \\&= -(\sqrt{2} - \sqrt{2}) \\&= 0\end{aligned}$$

Exercise

Evaluate the integrals $\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt$

Solution

$$\begin{aligned}\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt &= \int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \pi t^{-2}\right) dt \\&= \left[4\tan t - \pi t^{-1}\right]_{-\pi/3}^{-\pi/4} \\&= \left(4\tan\left(-\frac{\pi}{4}\right) - \pi\left(-\frac{4}{\pi}\right)\right) - \left(4\tan\left(-\frac{\pi}{3}\right) - \pi\left(-\frac{3}{\pi}\right)\right) \\&= (4(-1) + 4) - (4(-\sqrt{3}) + 3) \\&= -(-4\sqrt{3} + 3) \\&= 4\sqrt{3} - 3\end{aligned}$$

Exercise

Evaluate the integrals $\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} dy$

Solution

$$\begin{aligned}\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} dy &= \int_{-3}^{-1} \left(\frac{y^5}{y^3} - \frac{2y}{y^3}\right) dy \\&= \int_{-3}^{-1} (y^2 - 2y^{-2}) dy\end{aligned}$$

$$\begin{aligned}
&= \left[\frac{1}{3} y^3 + 2y^{-1} \right]_{-3}^{-1} \\
&= \left(\frac{1}{3} (-1)^3 + \frac{2}{-1} \right) - \left(\frac{1}{3} (-3)^3 + \frac{2}{-3} \right) \\
&= \underline{\underline{\frac{22}{3}}}
\end{aligned}$$

Exercise

Evaluate the integrals $\int_1^8 \frac{(x^{1/3} + 1)(2 - x^{2/3})}{x^{1/3}} dx$

Solution

$$\begin{aligned}
\int_1^8 \frac{(x^{1/3} + 1)(2 - x^{2/3})}{x^{1/3}} dx &= \int_1^8 \frac{2x^{1/3} - x + 2 - x^{2/3}}{x^{1/3}} dx \\
&= \int_1^8 (2 - x^{2/3} + 2x^{-1/3} - x^{1/3}) dx \\
&= \left[2x - \frac{3}{5} x^{5/3} + 3x^{2/3} - \frac{3}{4} x^{4/3} \right]_1^8 \\
&= \left(2(8) - \frac{3}{5}(8)^{5/3} + 3(8)^{2/3} - \frac{3}{4}(8)^{4/3} \right) - \left(2(1) - \frac{3}{5}(1)^{5/3} + 3(1)^{2/3} - \frac{3}{4}(1)^{4/3} \right) \\
&= \left(-\frac{16}{5} \right) - \left(\frac{73}{20} \right) \\
&= \underline{\underline{-\frac{137}{20}}}
\end{aligned}$$

Exercise

Evaluate: $\int_0^1 (2t + 3)^3 dt$

Solution

$$\begin{aligned}
\int_0^1 (2t + 3)^3 dt &= \frac{1}{2} \int_0^1 (2t + 3)^3 d(2t + 3) & d(2t + 3) &= 2dt \\
&= \frac{1}{8} (2t + 3)^4 \Big|_0^1 \\
&= \frac{1}{8} [5^4 - 3^4] \\
&= \underline{\underline{68}}
\end{aligned}$$

Exercise

Evaluate the integral $\int_{-1}^1 r\sqrt{1-r^2} \, dr$

Solution

$$\begin{aligned}\int_{-1}^1 r\sqrt{1-r^2} \, dr &= -\frac{1}{2} \int_{-1}^1 (1-r^2)^{1/2} d(1-r^2) \\ &= -\frac{1}{3} \left[(1-r^2)^{3/2} \right]_{-1}^1 \\ &= -\frac{1}{3} [0-0] \\ &= \underline{0}\end{aligned}$$