

4.3.

Polar.

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$r = \sqrt{x^2 + y^2}$$

$$\hat{\theta} = \tan^{-1} \frac{y}{x}$$

$$(x, y) \rightarrow (r, \theta)$$

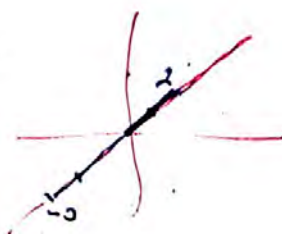
$$r, \theta \in \mathbb{R}$$

$$(r, \theta) ? \text{ no } (r, \theta + \pi)$$

$$f(\theta) = r = f(\theta)$$



$$-3 \leq r \leq 2 \quad \theta = \frac{\pi}{4}$$



$$r \cos \theta = 2 \Rightarrow x = 2$$

$$r^2 \cos \theta \sin \theta = 4$$

$$(r \cos \theta)(r \sin \theta) = 4$$

$$xy = 4$$

$$r = 1 - \cos \theta$$

$$r \neq 0$$

$$r^2 = r - r \cos \theta$$

$$x^2 + y^2 = \sqrt{x^2 + y^2} - x$$

$$x^4 + y^4 + 2x^2y^2 + 2x^3 + 2xy^2 - y^2 = 0$$

$$0, 1, \frac{1}{2} \quad \left\{ \begin{array}{l} \cos 60^\circ = \frac{1}{2} \\ \sin 60^\circ = \frac{\sqrt{3}}{2} \end{array} \right.$$

Symmetry x-axis  $(-r, \theta)$

$$\underline{(r, -\theta) \quad (-r, \pi - \theta)}$$



$$r = f(\theta) = \cos \theta$$
$$f(-\theta) = \cos(-\theta)$$
$$= \cos(\theta)$$

Symmetry y-axis

$$(-r, -\theta) \quad (r, \pi - \theta)$$



$$r = \sin \theta$$
$$f(\theta) = \sin(-\theta)$$
$$= -\sin \theta$$
$$= -r$$

Chargin:  $(-r, \theta) \quad (r, \pi + \theta)$



$$r = 1 - \cos \theta$$

$$f(\theta) = \frac{1 - \cos \theta}{r}$$



Cardioid.

# 11.4 Calculus

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$
$$= \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}$$

55  $f(\theta) = 1 - \cos \theta = r$   
 $-\pi \leq \theta \leq \pi$

$$\frac{dy}{dx} = \frac{\sin^2 \theta + (1 - \cos \theta) \cos \theta}{\sin \theta \cos \theta - (1 - \cos \theta) \sin \theta}$$
$$= \frac{\sin^2 \theta - \cos^2 \theta + \cos \theta}{2 \sin \theta \cos \theta - \sin \theta}$$
$$= \frac{1 - 2 \cos^2 \theta + \cos \theta}{\sin \theta (2 \cos \theta - 1)} = 0$$

$\sin \theta \neq 0 \Rightarrow \theta \neq 0, \pi$ ,  $\cos \theta \neq \frac{1}{2}$

$$-2 \cos^2 \theta + \cos \theta + 1 = 0$$

$$\cos \theta = 1$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \pm \frac{2\pi}{3}$$

denominator  
 $\theta = 0 \neq$

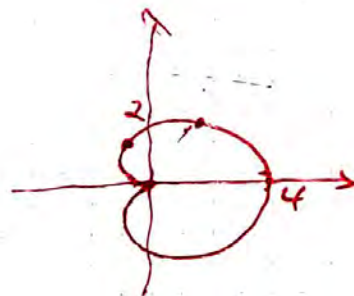
$$\theta = \pm \pi, \quad \theta = \pm \frac{\pi}{3}$$



$$I = \frac{1}{2} r^2 \theta \quad (\theta \text{ rad})$$

$$I_{\text{area}} = \frac{1}{2} \int_{\alpha}^{\beta} (f(\theta))^2 d\theta$$

$$= \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$



Ex  $r = 2(1 + \cos \theta)$

$$I = \frac{1}{2} \int_0^{2\pi} r^2 d\theta$$

$$= 2 \int_0^{2\pi} (1 + \cos \theta)^2 d\theta \quad + \int_0^{\pi}$$

$$= 2 \int_0^{2\pi} (1 + \cos^2 \theta + 2 \cos \theta) d\theta$$

$$\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$$

$$= 2 \int_0^{2\pi} \left( \frac{3}{2} + \frac{1}{2} \cos 2\theta + 2 \cos \theta \right) d\theta$$

$$= 2 \left( \frac{3}{2} \theta + \frac{1}{4} \sin 2\theta + 2 \sin \theta \right) \Big|_0^{2\pi}$$

$$= 2(3\pi)$$

$$= 6\pi \text{ unit}^2$$



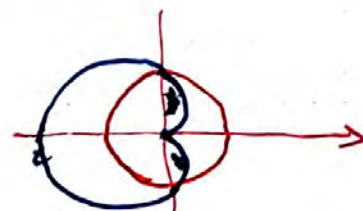
$$\text{Area} = \frac{1}{2} \int_{\alpha}^{\beta} (r_2^2 - r_1^2) d\theta$$

Ex  $\overset{\text{in}}{r} = 1$  (circle)  $\overset{\text{out}}{r} = 1 - \cos \theta$

$$1 - \cos \theta = 1$$

$$\cos \theta = 0$$

$$\theta = \pm \frac{\pi}{2}$$



$$A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} (1^2 - (1 - \cos \theta)^2) d\theta$$

$$= 2\left(\frac{1}{2}\right) \int_0^{\pi/2} (2\cos \theta - \cos^2 \theta) d\theta$$

$$= \int_0^{\pi/2} (2\cos \theta - \frac{1}{2} - \frac{1}{2} \cos 2\theta) d\theta$$

$$= 2\sin \theta - \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \Big|_0^{\pi/2}$$

$$= 2 - \frac{\pi}{4} \text{ unit}^2$$

Length

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

Ex  $L?$   $r = 1 - \cos \theta$

$$\frac{dr}{d\theta} = \sin \theta$$

$$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{1 - 2\cos \theta + \cos^2 \theta + \sin^2 \theta}$$

$$= \sqrt{2 - 2\cos \theta}$$

$$= \sqrt{4 \sin^2 \frac{\theta}{2}}$$

$$= 2 \sin \frac{\theta}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$2 \sin^2 \frac{\theta}{2} = 1 - \cos \theta$$

$$L = 2 \int_0^{2\pi} \sin \frac{\theta}{2} d\theta$$

$$= -4 \cos \frac{\theta}{2} \Big|_0^{2\pi}$$

$$= -4(-1 - 1)$$

$$= 8 \text{ unit}$$

## Surface

$$S = 2\pi \int_{\alpha}^{\beta} f(\sigma) \sin \sigma \sqrt{f(\sigma)^2 + (f'(\sigma))^2} d\sigma$$

polar axis

$$S = 2\pi \int_{\alpha}^{\alpha} f(\sigma) \cos \sigma \sqrt{f^2 + (f')^2} d\sigma$$

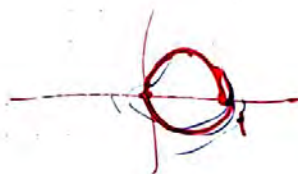
line  $\sigma = \frac{\pi}{2}$

Ex

$$f(\sigma) = \cos \sigma$$

about line  $\sigma = \frac{\pi}{2}$

$$\sqrt{r^2 + (r')^2} = \sqrt{\cos^2 \sigma + \sin^2 \sigma} = 1$$



$$S = 2\pi \int_0^{\pi} \cos^2 \sigma d\sigma$$

$$= \pi \int_0^{\pi} (1 + \cos 2\sigma) d\sigma$$

$$= \pi \left( \sigma + \frac{1}{2} \sin 2\sigma \right) \Big|_0^{\pi}$$

$$= \pi^2 \text{ unit}^2$$

11.7, A? 1 leaf  $r = \cos 3\theta$

$$A = \frac{1}{2} \left( \frac{1}{3} \right) \int_0^{2\pi} \cos^2 3\theta \, d\theta$$

$$= \frac{1}{12} \int_0^{2\pi} (1 + \cos 6\theta) \, d\theta$$

$$= \frac{1}{12} \left( \theta + \frac{1}{6} \sin 6\theta \right) \Big|_0^{2\pi}$$

$$\cos 3\theta = \frac{1}{2}$$

$$= \frac{1}{12} (2\pi)$$

$$= \frac{\pi}{6}$$

$$A = \frac{1}{2} \int_{-\pi/6}^{\pi/6} \cos^2 3\theta \, d\theta$$

$$= \frac{1}{4} (2) \int_0^{\pi/6} (1 + \cos 6\theta) \, d\theta$$

$$= \frac{1}{2} \left( \theta + \frac{1}{6} \sin 6\theta \right) \Big|_0^{\pi/6}$$

$$= \frac{1}{2} \left( \frac{\pi}{6} \right)$$

$$= \frac{\pi}{12} \text{ unit}^2$$





#8 A?  $r = 4 + 2 \sin \theta$

$$A = \frac{1}{2} \int_0^{2\pi} 4(2 + \sin \theta)^2 d\theta$$

$$= 2 \int_0^{2\pi} (4 + 4 \sin \theta + \sin^2 \theta) d\theta$$

$$= 2 \int_0^{2\pi} \left( \frac{9}{2} + 4 \sin \theta - \frac{1}{2} \cos 2\theta \right) d\theta$$

$$= 2 \left( \frac{9}{2} \theta - 4 \cos \theta - \frac{1}{4} \sin 2\theta \right) \Big|_0^{2\pi}$$

$$= 2(9\pi - 4 + 4)$$

$$= 18\pi \text{ unit}^2$$

#10

A?

$r^2 = 2 \sin 3\theta$   $\frac{\pi}{2}$

$$A = \frac{1}{2} \int_0^{\frac{\pi}{2}} r^2 d\theta$$

$$= (6) \frac{1}{2} \left( \frac{2}{3} \right) \int_0^{\frac{\pi}{2}} 2 \sin 3\theta d\theta$$

$$= -\frac{12}{3} \cos 3\theta \Big|_0^{\frac{\pi}{2}}$$

$$= -4(0 - 1)$$

$$= 4 \text{ unit}^2$$



Limaçon  $a + b \begin{cases} \cos \theta \\ \sin \theta \end{cases}$

Cardioid  $a(1 + \begin{cases} \cos \theta \\ \sin \theta \end{cases})$

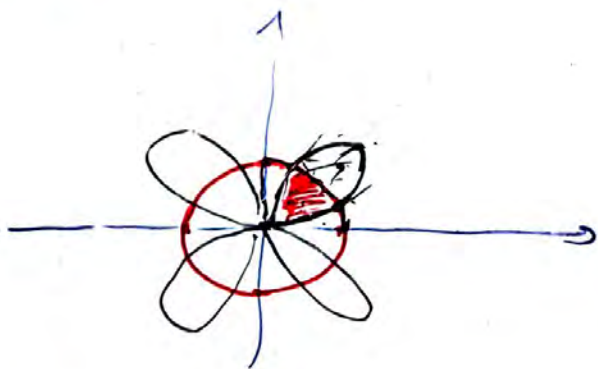
11.25-  $r = 4 \sin 2\theta$   $r = 2$

$$r = 4 \sin 2\theta = 2$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$$



$$A = \frac{1}{2} \int_{\pi/12}^{5\pi/12} (16 \sin^2 2\theta - 4) d\theta$$

$$= \frac{1}{2} \int_{\pi/12}^{5\pi/12} (8 - 8 \cos 4\theta - 4) d\theta$$

$$= 2 \int_{\pi/12}^{5\pi/12} (1 - 2 \cos 4\theta) d\theta$$

$$= 2 \left( \theta - \frac{1}{2} \sin 4\theta \right) \Big|_{\pi/12}^{5\pi/12}$$

$$= 2 \left( \frac{5\pi}{12} + \frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\sqrt{3}}{4} \right)$$

$$= \frac{2\pi}{3} + \sqrt{3}$$



$$A = \frac{1}{4} \pi r^2$$

$$= \pi$$

$$A = 4 \left( \frac{\pi}{3} + \frac{2\pi}{3} + \sqrt{3} \right)$$

$$= \frac{20\pi}{3} + 4\sqrt{3}$$

11.18    1.7)     $r = 2 \cos \theta$      $r = 2 \sin \theta$

$$r = 2 \cos \theta = 2 \sin \theta$$

$$\theta = \frac{\pi}{4}$$

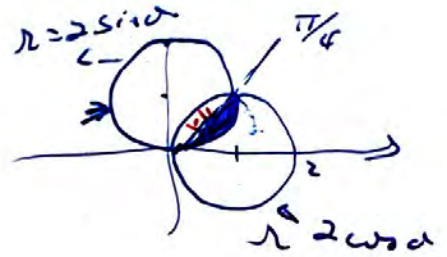
$$A = 2 \cdot \frac{1}{2} \int_0^{\pi/4} 4 \sin^2 \theta d\theta$$

$$= 2 \int_0^{\pi/4} (1 - \cos 2\theta) d\theta$$

$$= 2 \left( \theta - \frac{1}{2} \sin 2\theta \right) \Big|_0^{\pi/4}$$

$$= 2 \left( \frac{\pi}{4} - \frac{1}{2} \right)$$

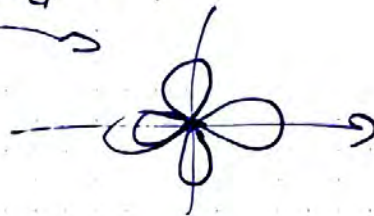
$$= \frac{\pi}{2} - 1 \text{ unit}^2$$



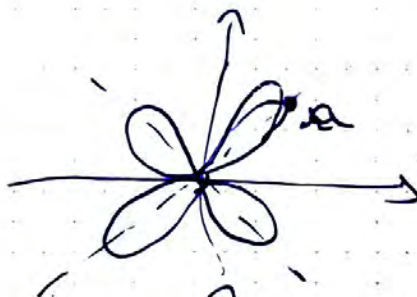
$\cos 3\theta$



$$\frac{360^\circ}{4} = 90^\circ$$



$\sin 2\theta$



$\sin 3\theta$

