Pre-Cal (9/3)

5.1 Cramer's Rule

[ ] ( )

Determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

D or Δ

det(A)

Ex

$$\begin{cases} 5x + 7y = -1 \\ 6x + 8y = 1 \end{cases}$$

$$D = \begin{vmatrix} 5 & 7 \\ 6 & 8 \end{vmatrix} = 40 - 42$$

$$= -2$$

$$D_x = \begin{vmatrix} -1 & 7 \\ 1 & 8 \end{vmatrix} = -8 - 7 = -15$$

$$D_y = \begin{vmatrix} 5 & -1 \\ 6 & 1 \end{vmatrix} = 5 + 6 = 11$$

$$x = \underbrace{\frac{D_x}{D} = \frac{-15}{-2}} = \frac{15}{2}$$

$$y = -\frac{11}{2}$$

Soln: $\left(\frac{15}{2}, -\frac{11}{2}\right)$

Diagonal method (3×3)

$3! = 6$
$2! = 2\times1$

$$\left|\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right|$$

Ex

$$x - 3y + 7z = 13$$
$$x + y + z = 1$$
$$x - 2y + 3z = 4$$

$$D = \left|\begin{array}{ccc|cc} 1 & -3 & 7 & 1 & -3 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & -2 & 3 & 1 & -2 \end{array}\right|$$

$$= 3 - 3 - 14 - 7 + 2 + 9$$
$$= -10$$

$$D_x = \left|\begin{array}{ccc|cc} 13 & -3 & 7 & 13 & -3 \\ 1 & 1 & 1 & 1 & 1 \\ 4 & -2 & 3 & 4 & -2 \end{array}\right|$$

$$= 39 - 12 - 14 - 28 + 26 + 9$$
$$= 20$$

$$D_y = \left|\begin{array}{ccc|cc} 1 & 13 & 7 & 1 & 13 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 4 & 3 & 1 & 4 \end{array}\right|$$

$$= 3 + 13 + 28 - 7 - 4 - 39$$
$$= -6$$

$$D_z = \left|\begin{matrix} 1 & -3 & 13 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{matrix}\right| \begin{matrix} 1 & -3 \\ 1 & 1 \\ 1 & -2 \end{matrix}$$

$$= 4 - 3 - 26 - 13 + 2 + 12$$
$$= \underline{-24}$$

$$\left(-2, \frac{-6}{-10}, \frac{-24}{-10}\right)$$

$$\therefore \boxed{\left(-2, \frac{3}{5}, \frac{12}{5}\right)}$$

2. Partial Fraction Decomposition

$$\frac{P(x)}{(x-a_1)\cdots(x-a_n)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \cdots + \frac{A_n}{x-a_n}$$

Ex

$$\frac{x}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$\boxed{\frac{x}{x^2-5x+6} \longleftrightarrow \frac{A(x-3)+B(x-2)}{(x-2)+(x-3)}}$$

$$x = A(x-3) + B(x-2)$$

x^1: ${}^3A + {}^3B = {}^3 1 \rightarrow A = 1-3 = \underline{-2}$

x^0: $-3A - 2B = 0$

$B = 3$

$$\frac{x}{x^2-5x+6} = \frac{-2}{x-2} + \frac{3}{x-3}$$

$$3A + 3B = 3 \Leftarrow \times 3$$

$$-3A - 2B = 0$$

$$0 \qquad B = 3$$

$$A + B = 1 \qquad A = \frac{-2}{1} = -2$$

$$-3A - 2B = 0$$

$$A = \frac{\begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix}} = \frac{-2}{1} = -2$$

$$B = \frac{\begin{vmatrix} 1 & 1 \\ -3 & 0 \end{vmatrix}}{1} = 3$$

$$\frac{}{(x-a)^n} = \frac{}{x-a} + \frac{}{(x-a)^2} + \dots + \frac{}{(x-a)^n}$$

Ex $\dfrac{x-2}{x^3-2x^2+x}$

$$x(x^2-2x+1) = x(x-1)(x-1)$$

$$\frac{x-2}{x^3-2x^2+x} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$\boxed{x-2} = A(x^2-2x+1) + Bx(x-1) + Cx$$

x^2: $A + B = 0 \Rightarrow \underline{B=2}$

x^1: $-2A - B + C = 1$ ①

x^0: $\underline{A = -2}$

① $\to$ $C = 1 + 2(-2) + 2$
$= \underline{-1}$

$$\frac{x-2}{x^3-2x^2+x} = \frac{-2}{x} + \frac{2}{x-1} - \frac{1}{(x-1)^2}$$

$$\frac{x^3-8}{x^2(x-1)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}$$

$$x^3-8 = Ax(x-1)^3 + B(x-1)^3 + Cx^2(x-1)^2 + Dx^2(x-1) + Ex^2$$

1 3 3 1

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$x^3 - 8 = Ax(x^3 - 3x^2 + 3x - 1)$$
$$+ B(x^3 - 3x^2 + 3x - 1)$$
$$+ Cx^2(x^2 - 2x + 1)$$
$$+ D(x^3 - x^2) + Ex^2$$

x^4: $A + C = 0 \rightarrow \boxed{C = -24}$

x^3: $-3A + B + D - 2C = 1$ (1)

x^2: $3A - 3B - D + E + C = 0$ (2)

x^1: $-A + 3B = 0 \rightarrow \boxed{A = 3(8) = 24}$

x^0: $-B = -8 \Rightarrow \boxed{B = 8}$

(1) $\rightarrow$ $D = 1 + \underbrace{3(24) - 8 + 2(-24)}_{24}$
$= \boxed{17}$

(2) $E = -3(24) + 3(8) + 17 + 24$
$= \boxed{-7}$

$$\boxed{\frac{x^3 - 8}{x^2(x-1)^3} = \frac{24}{x} + \frac{8}{x^2} - \frac{24}{x-1} + \frac{17}{(x-1)^2} - \frac{7}{(x-1)^3}}$$

$$\frac{Ax+B}{ax^2+bx+c} \leftarrow \text{not factorable}$$

$$(a-b)^2 = a^2-2ab+b^2$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$+ \quad + \quad -$

$$\frac{3x-5}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$\boxed{3x-5} = A(x^2+x+1)+(Bx+C)(x-1)$$

x^2 $\quad A+B=0 \longrightarrow \boxed{B=-A=\frac{2}{3}}$

x^1 $\quad A-B+C=3 \quad (1)$

x^0 $\quad A-C=-5 \longrightarrow \boxed{C=A+5 = 5-\frac{2}{3} = \frac{13}{3}}$

$(1) \quad A+A+A+5=3$

$3A=-2 \Rightarrow \boxed{A=-\frac{2}{3}}$

$$\frac{3x-5}{x^3-1} = \frac{-2/3}{x-1} + \frac{2/3\,x+13/3}{x^2+x+1}$$

$$= -\frac{2}{3}\,\frac{1}{x-1} + \frac{1}{3}\,\frac{2x+13}{x^2+x+1}$$