Section 1.2 – *Exponents*

Integer Exponents

Definition of exponent

$a^n = \underbrace{a \cdot a \cdot a \cdots\cdots a}_{n-times}$ a appears as a factor n times

$a^0 = 1$ $\qquad$ $a^{-n} = \frac{1}{a^n}$

$a^m \cdot a^n = a^{m+n}$ $\qquad$ $\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$

$\left(a^m\right)^n = a^{mn}$ $\qquad$ $\frac{a^m}{a^n} = a^{m-n}$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $\qquad$ $(ab)^m = a^m b^m$

a) 6^0

$6^0 = 1$

b) $(-9)^0$

$(-9)^0 = 1$

c) 3^{-2}

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

d) $\left(\frac{3}{4}\right)^{-1}$

$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

a) $7^4.7^6$

$7^4.7^6 = 7^{4+6} = 7^{10}$

b) $\frac{9^{14}}{9^6}$

$\frac{9^{14}}{9^6} = 9^{14-6} = 9^8$

c) $\frac{r^9}{r^{17}}$

$\frac{r^9}{r^{17}} = \frac{1}{r^{17-9}} = \frac{1}{r^8}$

d) $\left(2m^3\right)^4$

$\left(2m^3\right)^4 = (2)^4\left(m^3\right)^4$

$= 16m^{12}$

e) $\left(\frac{x^2}{y^3}\right)^6$

$\left(\frac{x^2}{y^3}\right)^6 = \frac{\left(x^2\right)^6}{\left(y^3\right)^6}$

$= \frac{x^{2.6}}{y^{3.6}}$

$= \frac{x^{12}}{y^{18}}$

f) $\frac{a^{-3}b^5}{a^4b^{-7}}$

$\frac{a^{-3}b^5}{a^4b^{-7}} = \frac{b^5b^7}{a^3a^4}$

$= \frac{b^{5+7}}{a^{4+3}}$

$= \frac{b^{12}}{a^7}$

g) $p^{-1}+q^{-1}$

$$p^{-1}+q^{-1}=\frac{1}{p}+\frac{1}{q}$$

$$=\frac{1}{p}\frac{q}{q}+\frac{1}{q}\frac{p}{p}$$

$$=\frac{q+p}{pq}$$

h) $\dfrac{x^{-2}-y^{-2}}{x^{-1}-y^{-1}}$

$$\frac{x^{-2}-y^{-2}}{x^{-1}-y^{-1}}=\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}-\frac{1}{y}}$$

$$=\frac{\frac{y^2-x^2}{x^2y^2}}{\frac{y-x}{xy}}$$

$$=\frac{y^2-x^2}{x^2y^2}\cdot\frac{xy}{y-x}$$

$$=\frac{(y-x)(y+x)}{(xy)^2}\cdot\frac{xy}{y-x}$$

$$=\frac{y+x}{xy}$$

Calculations with exponents

a) $121^{1/2}=11$

b) $625^{1/4}=5$

c) $(-32)^{1/5}=-2$

d) $(-49)^{1/2}$ *is not a real number*

Rational Exponents

$$a^{m/n} = \left(a^{1/n}\right)^m$$

Calculations with Exponents

a) $27^{2/3}$ 27^(2/3)

$$27^{2/3} = \left(27^{1/3}\right)^2$$
$$= \left(\left(3^3\right)^{1/3}\right)^2$$
$$= \left(3^{3.\frac{1}{3}}\right)^2$$
$$= (3)^2$$
$$= 9$$

b) $32^{2/5}$ 32^(2/5)

$$32^{2/5} = \left(\left(2^5\right)^{1/5}\right)^2$$
$$= 2^2$$
$$= 4$$

c) $64^{4/3}$ 64^(4/3)

$$64^{4/3} = \left(\left(4^3\right)^{1/3}\right)^4$$
$$= (4)^4$$
$$= 256$$

Simplify

a) $\dfrac{y^{1/3}y^{5/3}}{y^3}$

$$\frac{y^{1/3}y^{5/3}}{y^3} = \frac{y^{\frac{1}{3}+\frac{5}{3}}}{y^3}$$
$$= \frac{y^{\frac{6}{3}}}{y^3}$$
$$= \frac{y^2}{y^3}$$
$$= \frac{1}{y^{3-2}}$$
$$= \frac{1}{y}$$

b) $m^{2/3}\left(m^{7/3}+7m^{1/3}\right)$

$$m^{2/3}\left(m^{7/3}+7m^{1/3}\right) = m^{2/3}m^{7/3}+7m^{2/3}m^{1/3}$$
$$= m^{\frac{2}{3}+\frac{7}{3}}+7m^{\frac{2}{3}+\frac{1}{3}}$$
$$= m^{\frac{9}{3}}+7m^{\frac{3}{3}}$$
$$= m^3+7m$$

c) $\left(\dfrac{m^7n^{-2}}{m^{-5}n^2}\right)^{1/4}$

$$\left(\frac{m^7n^{-2}}{m^{-5}n^2}\right)^{1/4} = \left(\frac{m^{7+5}}{n^{2+2}}\right)^{1/4}$$
$$= \left(\frac{m^{12}}{n^4}\right)^{1/4}$$
$$= \frac{\left(m^{12}\right)^{1/4}}{\left(n^4\right)^{1/4}}$$
$$= \frac{m^{12/4}}{n^{4/4}}$$
$$= \frac{m^3}{n}$$

Simplify

a) $4m^{1/2}+3m^{3/2}$

$$4m^{1/2}+3m^{3/2}=m^{1/2}\left(4m^{1/2-1/2}+3m^{3/2-1/2}\right)$$
$$=m^{1/2}\left(4+3m\right)$$

b) $9x^{-2}-6x^{-3}$

$$9x^{-2}-6x^{-3}=3x^{-3}\left(3x-2\right)$$

c) $2\left(x^2+5\right)\left(3x-1\right)^{-1/2}+\left(3x-1\right)^{1/2}\left(2x\right)$

$$2\left(x^2+5\right)\left(3x-1\right)^{-1/2}+\left(3x-1\right)^{1/2}\left(2x\right)=2\left(3x-1\right)^{-1/2}\left[x^2+5+x\left(3x-1\right)\right]$$
$$=2\left(3x-1\right)^{-1/2}\left[x^2+5+3x^2-x\right]$$
$$=2\left(3x-1\right)^{-1/2}\left(4x^2-x+5\right)$$

Radicals

$$a^{1/n}=\sqrt[n]{a}$$

a) $\sqrt[4]{16}$

$$\sqrt[4]{16}=16^{1/4}=2$$

b) $\sqrt[5]{-32}=-2$

c) $\sqrt[3]{1000}$

$$\sqrt[3]{1000}=1000^{1/3}=10$$

d) $\sqrt[6]{\frac{64}{729}}$

$$\sqrt[6]{\frac{64}{729}}=\frac{\sqrt[6]{64}}{\sqrt[6]{729}}=\frac{2}{3}$$

Properties

$$\left(\sqrt[n]{a}\right)^n = a$$

$$\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n \text{ is even} \\ a & \text{if } n \text{ is odd} \end{cases}$$

$$\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

Simplify

a) $\sqrt{1000}$

$$\sqrt{1000} = \sqrt{100(10)}$$

$$= \sqrt{100}\sqrt{10}$$

$$= 10\sqrt{10}$$

b) $\sqrt{128}$

$$\sqrt{128} = \sqrt{64(2)}$$

$$= 8\sqrt{2}$$

c) $\sqrt{2}\sqrt{18}$

$$\sqrt{2}\sqrt{18} = \sqrt{2(18)}$$

$$= \sqrt{36}$$

$$= 6$$

d) $\sqrt[3]{54}$

$$\sqrt[3]{54} = \sqrt[3]{27(2)}$$

$$= 3\sqrt[3]{2}$$

e) $\sqrt{288m^5}$

$$\sqrt{288m^5} = \sqrt{144(2)m^4m}$$

$$= 12m^2\sqrt{2m}$$

f) $2\sqrt{18}-5\sqrt{32}$

$$2\sqrt{18}-5\sqrt{32}=2\sqrt{9(2)}-5\sqrt{16(2)}$$
$$=6\sqrt{2}-20\sqrt{2}$$
$$=-14\sqrt{2}$$