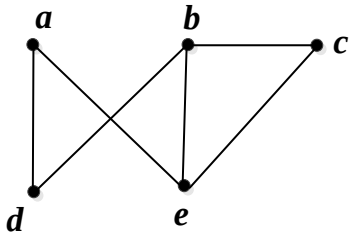
SOLUTION *Section* 4.8 – Connectivity

Exercise

Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? Which are the lengths of those that are paths?



a) *a, e, b, c, b* ***b)*** *a, e, a, d, b, c, a* ***c)*** *e, b, a, d, b, e* ***d)*** *c, b , d, a, e, c*

Solution

a) This is a path of length 4, but it is not a circuit, since it ends at a vertex other than the one at which it began. It is not a simple, since it uses an edge more than once.

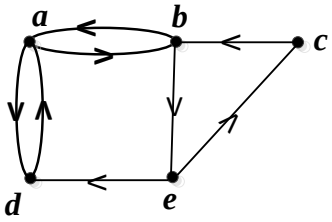
b) This is not a path, since there is no edge from *c* to *a*.

c) This is not a path, since there is no edge from *b* to *a*.

d) This is a path of length 5, which is a circuit. It is simple, since no edges are repeated.

Exercise

Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? Which are the lengths of those that are paths?



a) *a, b, e, c, b* ***b)*** *a, d, a, d, a* ***c)*** *a, d, b, e, a* ***d)*** *a, b , e, c, b, d, a*

Solution

a) This is a path of length 4, but it is not a circuit, since it ends at a vertex other than the one at which it began. It is simple, since no edges are repeated.

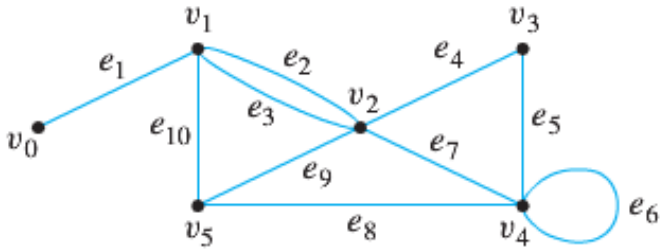
b) This is a path of length 4, which is a circuit. It is not simple, since it uses an edge more than once.

c) This is not a path, since there is no edge from *d* to *b*.

d) This is not a path, since there is no edge from *b* to *d*.

Exercise

Determine whether of the following walks are trails, paths, circuits, or simple circuits or just walk to the graph below.



a) $v_0e_1v_1e_{10}v_5e_9v_2e_2v_1$ *b)* $v_4e_7v_2e_9v_5e_{10}v_1e_3v_2e_9v_5$ *c)* v_2

d) $v_5v_2v_3v_4v_4v_5$ *e)* $v_2v_3v_4v_5v_2v_4v_3v_2$ *f)* $e_5e_8e_{10}e_3$

Solution

a) It is trail since no repeated edge.
It is not a path, repeated vertex v_1

It is not a circuit, since it ends at a vertex other than the one at which it began v_0

b) It is a walk; it is not a trail since it has a repeated edge e_9.

It is not a circuit, since it ends at a vertex other than the one at which it began v_4.

c) It is a closed walk, starts and ends at the same vertex v_2.

It is a trail since no repeated edge.
It is not a path or a circuit, since no edge.

d) It is a path and it is circuit but not a simple circuit since it has a repeated vertex v_4

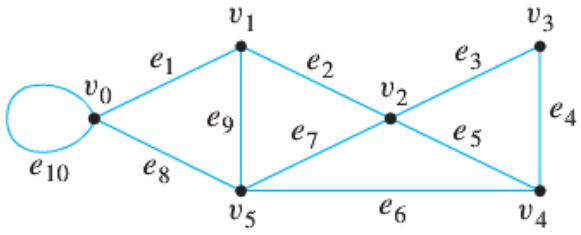
e) It is a closed walk, starts and ends at the same vertex v_2.

It is not a trail since it has repeated edges $\{v_2, v_3\}$ & $\{v_3, v_4\}$.

f) It is a path, it is not a circuit, since it ends at a vertex other than the one at which it began.

Exercise

Determine whether of the following walks are trails, paths, circuits, or simple circuits or just walk to the graph below.



a) $v_1 e_2 v_2 e_3 v_3 e_4 v_4 e_5 v_2 e_2 v_1 e_1 v_0$ *b)* $v_2 v_3 v_4 v_5 v_2$ *c)* $v_4 v_2 v_3 v_4 v_5 v_2 v_4$

d) $v_2 v_1 v_5 v_2 v_3 v_4 v_2$ *e)* $v_0 v_5 v_2 v_3 v_4 v_2 v_1$ *f)* $v_5 v_4 v_2 v_1$

Solution

a) It is not a trail since it has a repeated edge e_2 . It is not a path, repeated vertex v_1 , it is not a circuit, since it ends at a vertex other than the one at which it began v_1

b) It is a closed walk, starts and ends at the same vertex v_2 . It is a trail since no repeated edge. It is a circuit.

c) It is not a trail since it has repeated edges $\{v_2, v_4\}$.

It is a circuit, but not a simple circuit.

d) It is a path and it is circuit but not a simple circuit since it has a repeated vertex v_2

e) It is a trail since no repeated edge.
It is not a circuit, since it ends at a vertex other than the one at which it began.

f) It is a path, it is not a circuit, since it ends at a vertex other than the one at which it began.