Solution *Section* 3.5 - Curve Sketching (*Summary*)

Exercise

Given $f(x) = x^4 - 4x^3 + 5$

Solution

$$f'(x) = 4x^3 - 12x^2 = 0$$

$$4x^2(x-3) = 0$$

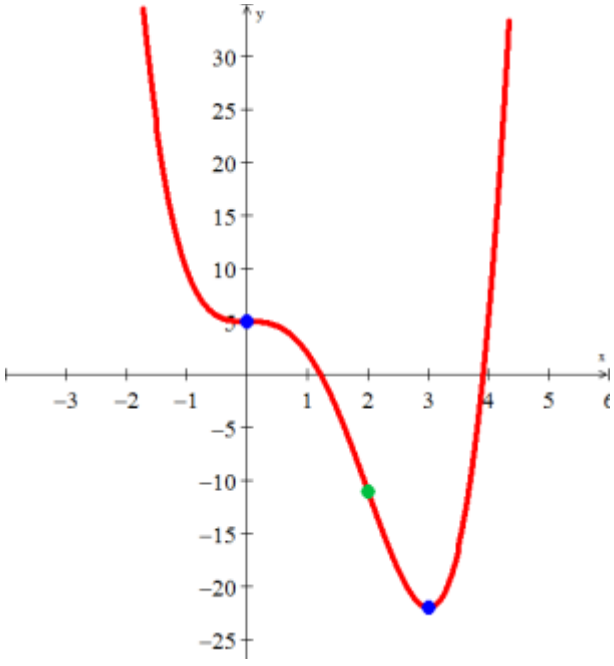
$$\Rightarrow x = 0,\ 0,\ 3$$

$$f''(x) = 12x^2 - 24x = 0$$

$$12x(x-2) = 0$$

$$\Rightarrow x = 0,\ 2$$

	f	f'	f''	
(-∞, 0)		-	+	Decreasing, Concave up
x = 0	5	0	0	RMAX
(0, 2)		-	-	Decreasing, Concave down
x = 2	-11	-	0	Point of Inflection
(2, 3)		-	+	Decreasing, Concave up
x = 3	-22	0	+	RMIN
(3, ∞)		+	+	Increasing, Concave up



Exercise

Given $f(x)=\dfrac{x^2+1}{x^2-1}$

Solution

VA: $x=\pm 1$

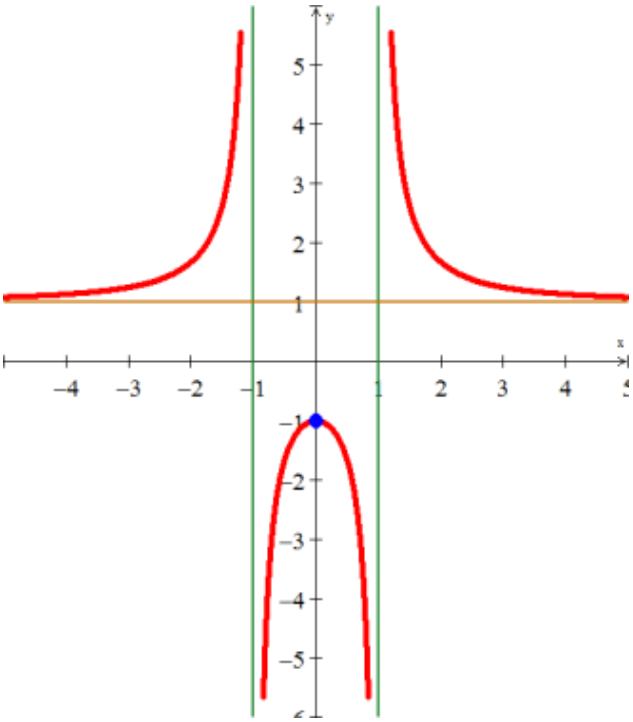
HA: $y=1$

$$f'(x)=\frac{(2x)(x^2-1)-(x^2+1)(2x)}{(x^2-1)^2}$$

$$=\frac{2x^3-2x-2x^3-2x}{(x^2-1)^2}$$

$$=\frac{-4x}{(x^2-1)^2}=0$$

$$\Rightarrow x=0$$



$$f''=\left(\frac{-4x}{(x^2-1)^2}\right)'$$

$$=\frac{-4(x^2-1)^2-(-4x)(x^2-1)(2x)}{(x^2-1)^4}$$

$$=\frac{(x^2-1)\left[-4(x^2-1)-(-4x)(2x)\right]}{(x^2-1)^4}$$

$$=\frac{-4x^2+4+8x^2}{(x^2-1)^3}$$

$$=\frac{4x^2+4}{(x^2-1)^3}$$

$$=\frac{4(x^2+1)}{(x^2-1)^3}=0$$

$\Rightarrow x^2+1=0 \Rightarrow x^2=-1$ (*no zeros*)

	f	f'	f''	
(-∞, -1)		+	-	Increasing, Concave up
x = -1	*Undef.*	*Undef.*	*Undef.*	Vertical Asymptote
(-1, 0)		+	-	Increasing, Concave down
x = 0	*-1*	*0*	-	RMAX
(0, 1)		-	-	Decreasing, Concave down
x = 1	*Undef.*	*Undef.*	*Undef.*	Vertical Asymptote
(1, ∞)		-	+	Decreasing, Concave up

Exercise

Given $f(x) = 2x^{3/2} - 6x^{1/2}$

Solution

$$f'(x) = 3x^{1/2} - 3x^{-1/2} = 0$$

$$x^{1/2}\left(3x^{1/2} - 3x^{-1/2}\right) = 0$$

$$3x - 3 = 0$$

$$\Rightarrow x = 1$$

$$f''(x) = \frac{3}{2}x^{-1/2} + \frac{3}{2}x^{-3/2} = 0$$

$$\frac{2}{3}x^{3/2}\left(\frac{3}{2}x^{-1/2} + \frac{3}{2}x^{-3/2}\right) = 0$$

$$x + 1 = 0$$

$$\rightarrow x = -1 < 0$$

x	f	f'	f''	
(0, 1)		-	+	Decreasing, Concave up
x = 1	-4	0	+	RMIN
(1, ∞)		+	+	Increasing, Concave up