# 6.5 Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$b = \frac{a \sin B}{\sin A}$$

Ex $A = 32^\circ$ $C = 81.8^\circ$ $a = 42.9$

$B = 180^\circ - 32^\circ - 81.8^\circ$

$= 66.2^\circ$

$$b = \frac{42.9^\circ \sin 66.2^\circ}{\sin 32^\circ} \approx 74.1 \text{ cm}$$

$$c = \frac{42.9^\circ \sin 81.8^\circ}{\sin 32^\circ} \approx 80.1 \text{ cm}$$

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$\sin 30^\circ = \frac{1}{2}$

$\sin 40^\circ \approx .6$

$\frac{\sqrt{2}}{2} \approx .707\ldots$

$\frac{\sqrt{3}}{2} \approx 1.732$

$.866$

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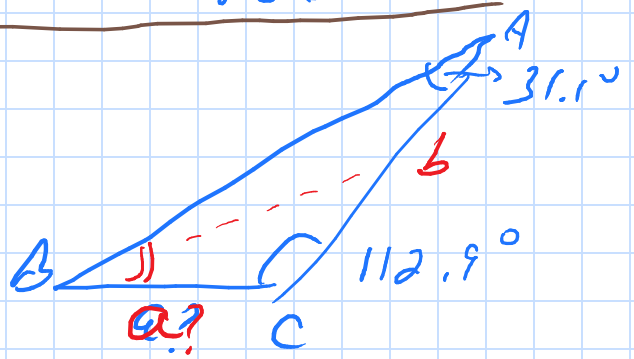
Ex $b = 347.6$




$B = 180^\circ - 31.1^\circ - 112.9^\circ$

$= 36^\circ$

$$a = \frac{347.6 \sin 31.1^\circ}{\sin 36^\circ}$$

$\approx 305.5$ ft

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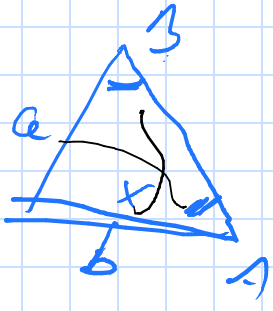
Ex $a = 562$ ft $B = 5.7^\circ$ $A = 85.3^\circ$

$x$?




$$x = \frac{562 \sin 5.7^\circ}{\sin 85.3^\circ} \approx 56.0 \text{ ft}$$

# Ambiguous Cases. Law of sines

$$-1 \le \sin\theta \le 1 \Leftarrow$$

$$\begin{cases} 0 \le \sin\theta \le 1 \\ \sin\theta\ (+) : \text{QI} \ \& \ \text{QII} \end{cases}$$

① ②

**Ex** $a = 2 \quad b = 6 \quad A = 30^\circ \quad B?$

$$\frac{\sin B}{6} = \frac{\sin A}{a}$$

$$\sin B = \frac{6 \sin 30^\circ}{2}$$

$$= 3\left(\frac{1}{2}\right)$$

$$= \frac{3}{2} > 1 \ \#$$

No triangle

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**Ex** $C = 35.4^\circ \quad a = 205 \quad c = 314 \quad A, B, b$

$$\frac{\sin A}{205} = \frac{\sin 35.4^\circ}{314}$$

$$\sin A = \frac{205 \sin 35.4^\circ}{314}$$ ~~#~~ cal ~~#~~

$$\hat{A} = \sin^{-1}\left(\frac{205 \sin 35.4^\circ}{314}\right)$$

$$\approx 22.2^\circ$$

$A \approx 22.2^\circ$

$B = 180^\circ - 22.2^\circ - 35.4^\circ$

$\approx 122.4^\circ$

$$b = \frac{314 \sin 122.4^\circ}{\sin 35.4^\circ}$$

$\approx 458$

1 triangle.

?

$A = 180^\circ - 22.2^\circ$

$= 157.8^\circ$

$B = 180^\circ - 157.8^\circ - 35.4^\circ$

$= -$ #

32 0. 1/C1 =

Ex $A = 40^\circ$ $a = 54$ $b = 62$ $\Delta ABC$

$$\sin B = \frac{b \sin A}{a} \quad \Big\} \Leftarrow \qquad c = \frac{a \sin C}{\sin A}$$

$$\hat{B} = \sin^{-1}\left(\frac{62 \sin 40^\circ}{54}\right)$$
$$\approx 48^\circ$$

$B \approx 48^\circ$

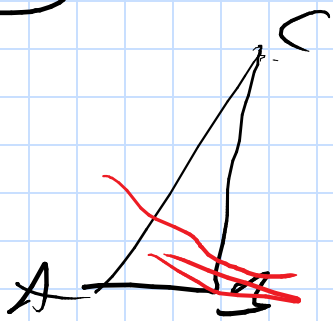
$C = 180^\circ - 48^\circ - 40^\circ$
$\approx 92^\circ$

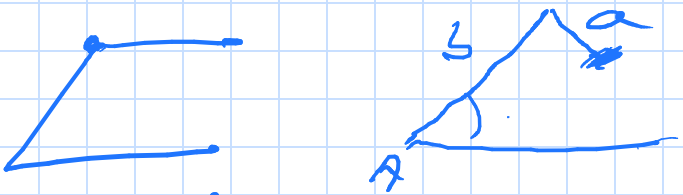
$$C = \frac{54 \sin 92^\circ}{\sin 40^\circ}$$
$\approx 84$ cm

$B \approx 180^\circ - 48^\circ$
$\approx 132^\circ$

$C = 180^\circ - 40^\circ - 132^\circ$
$\approx 8^\circ$

$$C = \frac{54 \sin 8^\circ}{\sin 40^\circ}$$
$\approx 12$ cm


no triangle

$0 \leq \sin \leq 1$

$\sin > 1 \Rightarrow$ No triangle

# Area of a Triangle ABC

Area

$$K = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}ac\sin B$$
$$= \frac{1}{2}bc\sin A$$

Ex $A = 24^\circ 40'$ $b = 27.3$ $C = 52^\circ 40'$

$= \left(24 + \frac{40}{60}\right)^\circ$

$B = 180^\circ - 24^\circ 40' - 52^\circ 40'$

$\approx 102.667^\circ$

$$a = \frac{27.3\sin(24^\circ 40')}{\sin 102.667^\circ}$$

$\approx 11.7$

$a \quad b \quad \sin C$

$$K = \frac{1}{2}(11.7)(27.3)\sin(52^\circ 40')$$

$\approx 127 \text{ cm}^2$

#1 $b = 4$ $c = 1$ $A = 120^\circ$ $60^\circ$

$$\text{Area} = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2}(4)(1)\sin 60^\circ$$
$$= 2\left(\frac{\sqrt{3}}{2}\right)$$
$$= \sqrt{3} \text{ unit}^2$$

unit$^2$

a $a = 4$ $c = 6$ $B = 45^\circ$

$$\text{Area} = \frac{1}{2}(4)(6)\sin 45^\circ$$
$$= 12\,\frac{\sqrt{2}}{2}$$
$$= 6\sqrt{2} \text{ unit}^2$$

# Law of Cosine.

$a^2 = b^2 + c^2 - 2bc\cos A$ $\quad (b-c)^2$

$a = \sqrt{b^2 + c^2 - 2bc\cos A}$

$b = \sqrt{a^2 + c^2 - 2ac\cos B}$

$c = \sqrt{a^2 + b^2 - 2ab\cos C}$

Ex $A = 60^\circ \quad b = 20 \quad c = 30$

$a = \sqrt{20^2 + (30)^2 - 2(20)(30)\cos 60^\circ}$

$= \sqrt{400 + 900 - 1200\frac{1}{2}}$

$= \sqrt{700}$

$= 10\sqrt{7} \approx 26$ in

$B = \sin^{-1}\left(\frac{20\sin 60^\circ}{26}\right)$

$\approx 42^\circ$

$C = 180^\circ - 60^\circ - 42^\circ$

$\approx 78^\circ$

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3 sides $a, b, c$

$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$

$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$

Ex $a = 34 \quad b = 20 \quad c = 18$

$$A = \cos^{-1}\left(\frac{20^2 + 18^2 - 34^2}{2(20)(18)}\right) \approx 127°$$

Ex $a = 3 \quad b = 2 \quad c = 5$

$$A = \cos^{-1}\left(\frac{4 + 25 - 9}{2(2)(5)}\right) \quad \frac{20}{20}$$

$$= \cos^{-1}(1)$$

$$= 0°$$

?

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3 sides. Area?

Heron's Area Formula

$$s = \frac{1}{2}(a+b+c)$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

Ex $a = 4, \; b = 5, \; c = 3$

$$s = \frac{1}{2}(4+5+3)$$

$$= 6$$

$$K = \sqrt{6(2)(1)(3)} \qquad (6-4)(6-5)(6-3)$$

$$= \sqrt{36}$$

$$= 6 \text{ units}^2$$

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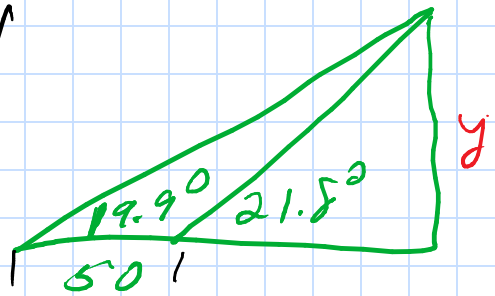
Law sine

cosine side!

Area using sine

15

$$y = \frac{50 \tan 21.8^\circ \tan 19.9^\circ}{\tan 21.8^\circ - \tan 19.9^\circ}$$



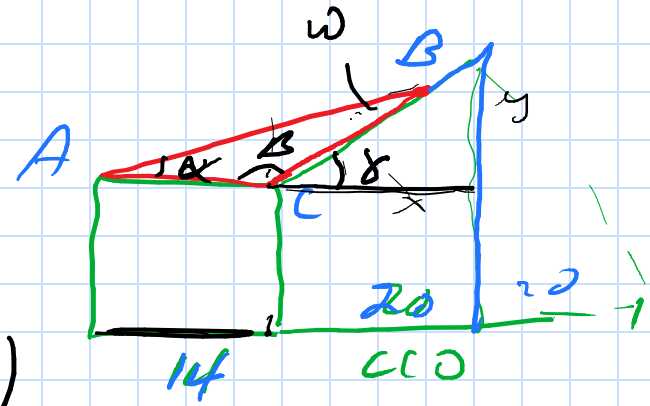

19 6-12

$$\tan\gamma = \frac{6}{12} = \frac{1}{2}$$

$$\gamma = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\tan\alpha = \frac{3}{12} \Rightarrow \alpha = \tan^{-1}\left(\frac{1}{4}\right)$$

$$\omega = 180^\circ - \tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{4}$$




$\overline{AB} \rightarrow \beta$ ?

$$\beta = 180^\circ - \gamma$$
$$= 180^\circ - \tan^{-1}\frac{1}{2}$$

$$\tan^{-1} 1 = 45^\circ$$

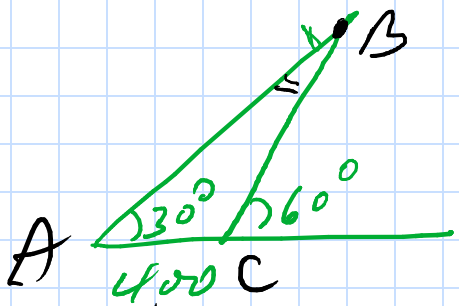
| $\theta$ | $\theta^\circ$ |
|---|---|
| $\infty$ | $\frac{\pi}{2}$ |

$$\frac{AB}{\sin\beta} = \frac{14}{\sin\omega}$$

$$\overline{AB} = \frac{14 \sin\left(180^\circ - \tan^{-1}(1/2)\right)}{\sin\left(180^\circ - \tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{4}\right)}$$

Ex 22 $d = 800$ ft

$AB \; ? \; d$




$C = 180° - 60°$

$= 120°$

$B = 180° - 30° - 120°$

$= 30°$

$$\frac{AB}{\sin C} = \frac{400}{\sin B}$$

$$AB = \frac{400 \sin 120°}{\sin 30°}$$

$$= \frac{400 \frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= 400\sqrt{3}$$

$400\sqrt{3} \; ? \; 800$

$\sqrt{3} < 2$

$>$ $<$

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