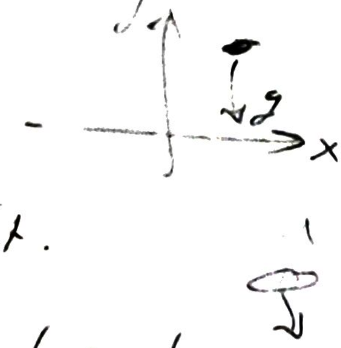
1. Limit

1.1 Idea

Ex $s(t) = -16t^2 + v_0 t + s_0$

$$= -\frac{1}{2} g t^2 + v_0 t + s_0$$

$g = 32.2 \ ft/sec^2$
$\quad\ \ 9.8 \ m/s^2$



v_0: initial value
s_0: " position (height.

(Ex) $y(t) = 16t^2$ falling object.

Average rate $= \frac{\Delta s}{\Delta t} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

a) 1^{st} 2 sec. avg?

$$\text{average speed} = \frac{y(2) - y(0)}{2 - 0}$$

$$= \frac{16(4) - 0}{2}$$

$$= 32 \ ft/sec$$

b) from $1 \to 2$ sec

$$\text{average speed} = \frac{y(2) - y(1)}{2 - 1}$$

$$= \frac{64 - 16}{1}$$

$$= 48 \ ft/sec$$

Ex. $y = 16t^2$ $\quad [t_0, t_0+h]$

$(a+b)^2$
$a^2 + 2ab + b^2$

$$\frac{\Delta y}{\Delta t} = \frac{y(t_0+h) - y(t_0)}{t_0 + h - t_0}$$

$$= \frac{16(t_0+h)^2 - 16t_0^2}{h}$$

$$= \frac{1}{h}\left(16t_0^2 + 32ht_0 + 16h^2 - 16t_0^2\right)$$

$$= \boxed{\frac{1}{h}\left(32ht_0 + 16h^2\right)}$$

$$= 32t_0 + 16h$$

if $t_0 = 1 \Rightarrow \frac{\Delta y}{\Delta t} = V_{avg.}$

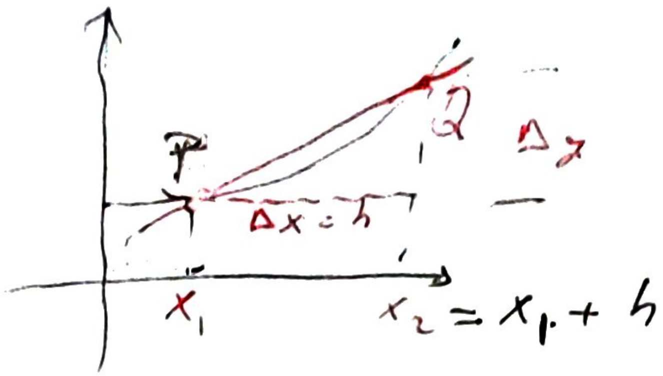
$$= 32 + 16h$$

if $\underline{h \text{ approaches } 0} \rightarrow \frac{\Delta y}{\Delta t} = 32$ ft/sec

if $t_0 = 2 \Rightarrow \frac{\Delta y}{\Delta t} = 64 + 16h.$

h approaches $0 \Rightarrow \frac{\Delta y}{\Delta t} = 64$

$$\left.\frac{\Delta y}{\Delta t}\right|_{1<t<2} = 64 - 32 = 48$$



$P(x_1, y_1)$ $\quad$ $Q(x_2, y_2)$

Line PQ (secant)

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{f(x_1+h) - f(x_1)}{h} \quad \text{(Difference Quotient)}$$

1.2 Definitions / Techniques (defn)

Limits

Defn

the limit of $f(x)$ as x approaches x_0 is L

$$\lim_{x \to x_0} f(x) = L$$

$x \to a^-$: x approaches a from the left

$x \to a^+$: right

$-1^- \to -1.1$

$-1^+ \to -.9$

Ex $f(x) = \dfrac{x^2-1}{x-1}$ behave near $x = 1$

$f(1) = \dfrac{0}{0}$ $\left\{ \begin{array}{l} 1\text{- factoring} \\ 2\text{- conjugate} \end{array} \right.$

$x = 1$

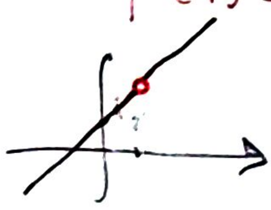
$x - 1 = 0$

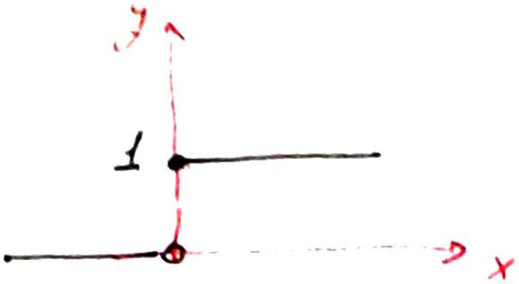
$f(x) = \dfrac{(x-1)(x+1)}{x-1}$ $\left\{ \begin{array}{l} a^2-b^2 \\ a^3-b^3 \\ a^3+b^3 \end{array} \right\}$

$= x+1 \Big|_{x=1} \to f(1) =$

$f(1) = 2$

<u>hole</u>





$$u(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$\lim_{x \to 0} u(x) =$ doesn't exist $\nexists$

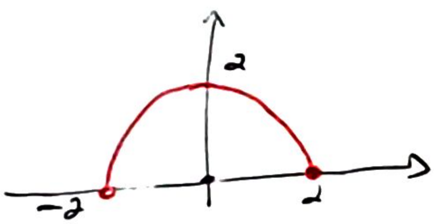
$\boxed{\begin{array}{l} \exists \text{ such that} \\ \in \text{ element.} \end{array}}$

$\lim_{x \to 0^-} u(x) = 0$

$\lim_{x \to 0^+} u(x) = 1$

$\lim_{x \to c^-} f(x) = L$ left-hand side

$\lim_{x \to c^+} f(x) = M$ right-hand side



$x^2 + y^2 = 4 \Rightarrow y = \sqrt{4 - x^2}$

$\underbrace{-2 \leq x \leq 2}$ $\underbrace{x \in [-2, 2]}$

$\lim_{x \to -2^+} y(x) = 0$

$\lim_{x \to 2^-} y(x) = 0$

(infinite ∞

doesn't exist: $\nexists$

$\frac{0}{0}$ $\begin{cases} 1- \\ 2. \end{cases}$

$\frac{0}{nb \neq 0} = 0.$

$\frac{nb \neq 0}{0} = ?\ \infty$