ODE:

1st Order.

$$y' \qquad \frac{dy}{dt} \qquad D_t y$$

$$y' = y - t \qquad \text{form} \qquad y' + p_{(x)}y = f_{(x)}$$

$$y' + 4y = e^{-3t}$$

second order, y'' $\quad \ddot{y}$

$$\frac{\partial^2 \omega}{\partial t^2} = c^2 \frac{\partial^2 \omega}{\partial x^2} \qquad \textcolor{magenta}{\text{partial derevative}}$$

$$\frac{dy}{dt} = y' = f(t, y) = 0$$

$$y'' = f(t, y, y') = 0$$

$$y^{(n)} = f(t, y, y', \cdots, y^{(n-1)})$$

Ex $y(t) = Ce^{-t^2}$ is a solution $y' = -2ty$

$$y = Ce^{-t^2}$$

$$y' = -2t \, \underline{Ce^{-t^2}}$$

$$= -2t \, y \quad \checkmark$$

Ex $y(t) = \cos t$ is a soln. $y' = 1 + y^2$

$$y = \cos t$$

$$y' = -\sin t = 1 + y^2$$

$$-\sin t = 1 + \cos^2 t$$

is not a soln.

1.2   Separable Eqns

$$y' = y(x) = \frac{dy}{dx}$$

$$\int g(x)\, dx = \int dy$$

$$\frac{dy}{dx} = y + \sin x \qquad \text{not separable.}$$

Ex $y' = t\, y^2$

$$\frac{dy}{dt} = t y^2$$

$$\int \frac{dy}{y^2} = \int t\, dt$$

$$-\frac{1}{y} = \frac{1}{2} t^2 + C$$

$$= \frac{t^2 + 2C}{2}$$

$$y(t) = -\frac{2}{t^2 + 2C}$$

Newton's Law of Cooling

$$\frac{dT}{dt} = -k(T-A)$$

$$\int_{T_0}^{T} \frac{dT}{T-A} = -\int_0^t k\,dt \qquad\qquad d(T-A) = dT$$

$$\ln|T-A|\Big|_{T_0}^{T} = -kt\Big|_0^t$$

$$\ln|T-A| - \ln|T_0-A| = -kt$$

$$\ln\frac{|T-A|}{|T_0-A|} = -kt$$

$$\left|\frac{T-A}{T_0-A}\right| = e^{-kt}$$

$$\frac{T-A}{T_0-A} = e^{-kt}$$

$$T-A = (T_0-A)e^{-kt}$$

$$\boxed{T = A + (T_0-A)e^{-kt}}$$

$$A = 70° \qquad T_0 = 40°$$

$$T(t=10) = 50°$$

$$50° = 70° + (40°-70°)e^{-10k}$$

$$+20 = +30\, e^{-10k}$$

$$e^{-10k} = \frac{2}{3} \quad\Rightarrow\quad -10k = \ln\left(\frac{2}{3}\right)$$

$$k = \frac{-1}{10}\ln\left(\frac{2}{3}\right) \qquad \text{or} \qquad \frac{1}{10}\ln\left(\frac{3}{2}\right)$$

$$.04005$$

$$T(30) = 70 - 30\, e^{-(.04005)(30)}$$

$$\approx 64.1°F$$