[illegible]

[illegible] $\sin A = \frac{3}{5}$ [illegible] $\cos B = -\frac{12}{13}$ [illegible]

$\cos A = -\frac{4}{5}$ $\qquad \sin B = -\frac{5}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{3}{5}\left(-\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)$

$= \frac{-36+20}{65}$

$= -\frac{16}{65}$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(-\frac{5}{13}\right)$

$= \frac{48+15}{65}$

$= \frac{63}{65}$

c) $\tan(A+B) = -\frac{16}{63}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \frac{-36-20}{65}$

$= \frac{-56}{65}$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{48-15}{65}$

$= \frac{33}{65}$

f) $\tan(A-B) = -\frac{56}{33}$

S:3. G $\sin A = -\frac{3}{5}$ $A \in Q\,\mathrm{III}$

$\cos A = -\frac{4}{5}$

$\frac{180^\circ}{2} < \frac{A}{2} < \frac{270^\circ}{2}$

$\boxed{\frac{A}{2} \in Q\,\mathrm{II}}$

a) $\sin 2A = 2\sin A\cos A$

$= 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= \boxed{\frac{24}{25}}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

c) $\tan 2A = \boxed{\frac{24}{7}}$

d) $\sin \frac{A}{2} = \sqrt{\frac{1}{2}(1-\cos A)}$ $\boxed{\frac{A}{2} \in Q\,\mathrm{II}}$

$= \sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)}$

$= \sqrt{\frac{9}{10}}$

$= \boxed{\frac{3}{\sqrt{10}}}$

e) $\cos \frac{A}{2} = -\sqrt{\frac{1}{2}(1+\cos A)}$

$= -\sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)}$

$= \boxed{-\frac{1}{\sqrt{10}}}$

f) $\tan A/2 = \boxed{-3}$

P.? #34

$\cos 2\theta + 3\sin\theta - 2 = 0$

$1 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$2\sin^2\theta + 3\sin\theta - 1 = 0$

$\sin\theta = 1 \qquad \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{2},\ \frac{\pi}{6},\ \frac{5\pi}{6}$

$\sin\theta\tan\theta = \sin\theta$ $\quad$ ($\sin\theta\tan\theta - \sin\theta = 0$

$\sin\theta(\tan\theta - 1) = 0$

$\sin\theta = 0 \qquad \tan\theta = 1$ $\quad$ $\tan\theta - 1 = 0$

$\tan\theta = 1$

$\theta = 0,\ \pi,\ \frac{\pi}{4},\ \frac{5\pi}{4}$

15 $2\tan x\csc x + 2\csc x + \tan x + 1 = 0$

$2\csc x(\tan x + 1) + (\tan x + 1) = 0$

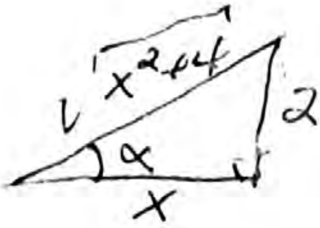
$(\tan x + 1)(2\csc x + 1) = 0$

$\tan x = -1 \qquad \csc x = -\frac{1}{2} = \frac{1}{\sin x} \Rightarrow \sin$

$\sin x = -2$ #

$x = \frac{3\pi}{4},\ \frac{7\pi}{4}$

7.5. $\sin\left(\underbrace{\cos^{-1}\frac{x}{\sqrt{x^2+4}}}_{\alpha}\right)$




$\cos\alpha = \frac{x}{\sqrt{x^2+4}}$

$\sin\alpha = \frac{2}{\sqrt{x^2+4}}$

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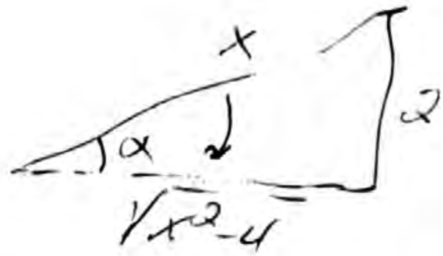
39. $\sec\left(\tan^{-1}\frac{2}{\sqrt{x^2-4}}\right)$




$\tan\alpha = \frac{2}{\sqrt{x^2-4}}$

$\sec\alpha = \frac{x}{\sqrt{x^2-4}}$

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8.6 6 $\left(4\sqrt{3}, -\frac{\pi}{6}\right)$

$x = r\cos\theta$
$= 4\sqrt{3}\cos\left(-\frac{\pi}{6}\right)$
$= 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$
$= 6$

$y = r\sin\theta$
$= 4\sqrt{3}\sin\left(-\frac{\pi}{6}\right)$
$= -4\sqrt{3}\left(\frac{1}{2}\right)$
$= -2\sqrt{3}$

$(x,y) = (6, -2\sqrt{3})$

16) $(-1, -\sqrt{3})$

$r = \sqrt{x^2+y^2}$

$= \sqrt{1+3}$

$\boxed{= 2}$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$ $\quad \frac{\sqrt{3}/2}{1/2}$

$= \frac{\pi}{3}$

$\boxed{\in \text{Q III} \quad \theta = \frac{4\pi}{3}}$

$$\boxed{(r, \theta) = \left(2, \frac{4\pi}{3}\right)}$$

32) $r = 8\sin\theta - 2\cos\theta$

$\underbrace{r(r)}_{r^2} = 8r\sin\theta - 2r\cos\theta$

$$\boxed{x^2+y^2 = 8y - 2x}$$

43) $x+y=4$

$r\cos\theta + r\sin\theta = 4$

$r(\cos\theta + \sin\theta) = 4$

$$\boxed{r = \frac{4}{\cos\theta + \sin\theta}}$$

22, 8.1 $\cos^4 x - \sin^4 x = \cos 2x$

$$\cos^4 x - \sin^4 x = \underline{(\cos^2 x - \sin^2 x)}(\cos^2 x + \sin^2 x)$$
$$= \cos 2x \quad (1)$$
$$= \cos 2x$$ ✓.

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51 $\dfrac{1-\cos^2 x}{1+\cos x} = \dfrac{\sec x - 1}{\sec x}$

$$\frac{\sec x - 1}{\sec x} = \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x}}$$
$$= \frac{1-\cos x}{\cos x} \cdot \frac{\cos x}{1}$$
$$= 1 - \cos x \ \frac{1+\cos x}{1+\cos x}$$
$$= \frac{1-\cos^2 x}{1+\cos x}$$ ✓

63 $\csc x - \sin x = \cos x \cot x$

$$\csc x - \sin x = \frac{1}{\sin x} - \sin x$$
$$= \frac{1 - \sin^2 x}{\sin x}$$
$$= \frac{\cos^2 x}{\sin x}$$
$$= \cos x \ \frac{\cos x}{\sin x}$$
$$= \cos x \cot x$$ ✓.

8.2. $\sec(A+B) = \dfrac{\cos(A-B)}{\cos^2 A - \sin^2 B}$

$$\sec(A+B) = \frac{1}{\cos(A+B)} \cdot \frac{\cos(A-B)}{\cos(A-B)}$$

$$= \frac{\cos(A-B)}{(\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)}$$

$$= \frac{\cos(A-B)}{\cos^2 A\cos^2 B - \sin^2 A\sin^2 B}$$

$$= \frac{\cos(A-B)}{\cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B}$$

$$= \frac{\cos(A-B)}{\cos^2 A \underbrace{- \cos^2 A\sin^2 B} - \sin^2 B \underbrace{+ \cos^2 A\sin^2 B}}$$

$$= \frac{\cos(A-B)}{\cos^2 A - \sin^2 B}$$

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16. $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{\cot y + \cot x}{\cot y - \cot x}$

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\dfrac{\sin x\cos y}{\sin x\sin y} + \dfrac{\cos x\sin y}{\sin x\sin y}}{\dfrac{\sin x\cos y}{\sin x\sin y} - \dfrac{\cos x\sin y}{\sin x\sin y}}$$

$$= \frac{\cot y + \cot x}{\cot y - \cot x}$$

11.17 $\frac{\sin(x-y)}{\sin x \cos y} = 1 - \cot x \tan y$

$$\frac{\sin(x-y)}{\sin x \cos y} = \frac{\sin x \cos y - \cos x \sin y}{\sin x \cos y}$$

$$= \frac{\sin x \cos y}{\sin x \cos y} - \frac{\cos x \sin y}{\sin x \cos y}$$

$$= 1 - \cot x \tan y.$$

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5.3 27 $\frac{\cos 2x}{\cos^2 x} = \sec^2 x - 2\tan^2 x$

$$\frac{\cos 2x}{\cos^2 x} = \frac{1 - 2\sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - 2\frac{\sin^2 x}{\cos^2 x}$$

$$= \sec^2 x - 2\tan^2 x$$

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42 $2\sin^2\frac{x}{2} = \frac{\sin^2 x}{1+\cos x}$

$$2\sin^2\frac{x}{2} = 2\,\frac{1-\cos x}{2}$$

$$= (1-\cos x)\frac{1+\cos x}{1+\cos x}$$

$$= \frac{1-\cos^2 x}{1+\cos x} \qquad \boxed{\cos^2 x + \sin^2 x = 1}$$

$$= \frac{\sin^2 x}{1+\cos x}$$

10. $\sin^2\frac{x}{2}\cos^2\frac{x}{2} = \frac{\sin^2 x}{4}$

$$\sin^2\frac{x}{2}\cos^2\frac{x}{2} = \left(\sin\frac{x}{2}\cos\frac{x}{2}\right)^2$$
$$= \left(\frac{1}{2}\sin x\right)^2 \quad \sin 2\left(\frac{x}{2}\right)$$
$$= \frac{1}{4}\sin^2 x \checkmark$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$\sin^2\frac{x}{2}\cos^2\frac{x}{2} = \frac{1-\cos x}{2}\cdot\frac{1+\cos x}{2}$$
$$= \frac{1}{4}(1-\cos^2 x)$$
$$= \frac{1}{4}\sin^2 x \checkmark$$

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8.4

12. $1-\sin x = \sqrt{3}\cos x \qquad (\ )^2 = (\ )^2$

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{1}{2}$$
$$\sin\frac{\pi}{6}\sin x + \cos\frac{\pi}{6}\cos x = \cos\frac{\pi}{3},\ \frac{5\pi}{3}$$
$$\cos\left(x-\frac{\pi}{6}\right) = \cos\frac{\pi}{3},\ \frac{5\pi}{3}$$

$x - \frac{\pi}{6} = \frac{\pi}{3} \qquad x - \frac{\pi}{6} = \frac{5\pi}{3}$

$x = \frac{\pi}{3} + \frac{\pi}{6} \qquad x = \frac{5\pi}{3} + \frac{\pi}{6}$

$= \frac{\pi}{2} \qquad = \frac{11\pi}{6}$

20 $\tan x + \sqrt{3} = \sec x$

$$\frac{\sin x}{\cos x} + \sqrt{3} = \frac{1}{\cos x}$$

$\cos x \neq 0$

$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x + \sqrt{3}\cos x = 1$

$x = \cancel{\frac{\pi}{2}}, \frac{11\pi}{6}$