Section 2.3 – Composition Functions

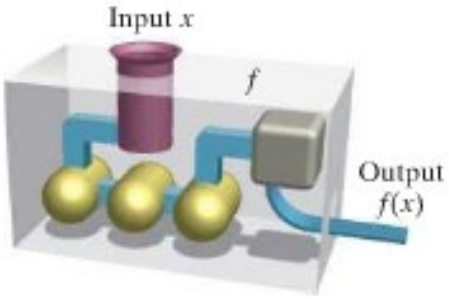
Composition of Functions

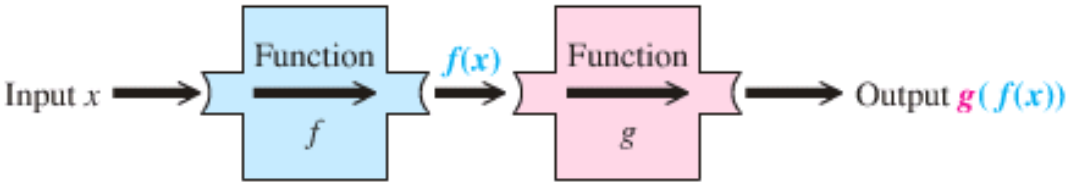
The composite function $g \circ f$, the composite of f and g, is defined as

$$(g \circ f)(x) = g\left(f(x)\right)$$

Where x is in the domain of f

and $g(x)$ is in the domain of f





Example

Given that $f(x) = 5x + 6$ and $g(x) = 2x^2 - x - 1$, find $(f \circ g)(x)$ and $(g \circ f)(x)$

Solution

$(f \circ g)(x) = f\left(g(x)\right)$

$= 5(\text{---------}) + 6$

$= 5(2x^2 - x - 1) + 6$

$= 10x^2 - 5x - 5 + 6$

$= 10x^2 - 5x + 1$

$= f(2x^2 - x - 1)$ ***Domain:*** All real numbers

Domain: All real numbers

$(g \circ f)(x) = g\left(f(x)\right)$

$= g(5x + 6)$ ***Domain:*** All real numbers

$= 2(\quad)^2 - (\quad) - 1$

$= 2(5x + 6)^2 - (5x + 6) - 1$

$= 2(25x^2 + 60x + 36) - 5x - 6 - 1$

$= 50x^2 + 120x + 72 - 5x - 7$

$= 50x^2 + 115x + 65$ ***Domain:*** All real numbers

Example

Let $f(x)=\sqrt{x}$ and $g(x)=4x+2$, find each of the following and its domain.

a) $(f\circ g)(x)$

b) $(g\circ f)(x)$

Solution

a) $(f\circ g)(x)=f\big(g(x)\big)$

$=f\big(4x+2\big)$ $\qquad (-\infty,\infty)$

$=\sqrt{4x+2}$

$4x+2\geq 0$

$4x\geq -2$

$x\geq -\frac{2}{4}$

Domain: $\underline{x\geq -\frac{1}{2}}$ $\qquad \left[-\frac{1}{2},\infty\right)$

b) $(g\circ f)(x)=g\big(f(x)\big)$

$=g\left(\sqrt{x}\right)$ $\qquad x\geq 0$

$=4\sqrt{x}+2$ $\qquad x\geq 0$

Domain: $\underline{x\geq 0}$ $\qquad [0,\infty)$

Example

Let $f(x)=2x-1$ and $g(x)=\frac{4}{x-1}$ Find:

a) $(f\circ g)(2)$

b) $(g\circ f)(-3)$

Solution

a) $(f\circ g)(2)=f\big(g(2)\big)$

$=f\left(\frac{4}{2-1}\right)$

$=f\big(4\big)$

$=2\big(4\big)-1$

$\underline{=7}$

b) $(g\circ f)(-3)=g\big(f(-3)\big)$

$=g\big(2(-3)-1\big)$

$$= g(-7)$$

$$= \frac{4}{-7-1}$$

$$= \frac{4}{-8}$$

$$\boxed{= -\frac{1}{2}}$$

Example

Given that $f(x) = \frac{4}{x+2}$ and $g(x) = \frac{1}{x}$, find

a) $(f \circ g)(x)$

b) Domain of $(f \circ g)(x)$

Solution

a) $(f \circ g)(x) = f(g(x))$

$$= f\left(\frac{1}{x}\right) \qquad \textbf{\textit{Domain:}}: x \neq 0$$

$$= \frac{4}{\frac{1}{x}+2}$$

$$= \frac{4}{\frac{1+2x}{x}}$$

$$= 4 \div \frac{1+2x}{x}$$

$$= 4\frac{x}{1+2x}$$

$$\boxed{= \frac{4x}{1+2x}} \qquad \textbf{\textit{Domain:}}: x \neq -\frac{1}{2}$$

b) Domain: $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 0\right) \cup (0, \infty)$

Exercises *Section* 2.3 – Composition Functions

1. Given that $f(x)=2x-5$ and $g(x)=x^2-3x+8$, find $(f\circ g)(x)$, $(g\circ f)(x)$ and their domain then find $(f\circ g)(7)$

2. Given that $f(x)=\sqrt{x}$ and $g(x)=x-1$, find

a) $(f\circ g)(x)=f(g(x))$

b) $(g\circ f)(x)=g(f(x))$

c) $(f\circ g)(2)=f(g(2))$

3. Given that $f(x)=\dfrac{x}{x+5}$ and $g(x)=\dfrac{6}{x}$, find

a) $(f\circ g)(x)=f(g(x))$

b) $(g\circ f)(x)=g(f(x))$

c) $(f\circ g)(2)=f(g(2))$

4. Find $(f\circ g)(x)$, $(g\circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x)=2x^2+3x-4$, $g(x)=2x-1$

5. Find $(f\circ g)(x)$, $(g\circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x)=x^3+2x^2$, $g(x)=3x$

6. Find $(f\circ g)(x)$, $(g\circ f)(x)$, $f(g(-2))$ *and* $g(f(3))$: $f(x)=|x|$, $g(x)=-7$

(7 – 36) For the given function; find:

a) Find $(f\circ g)(x)$ and the ***domain*** of $f\circ g$

b) Find $(g\circ f)(x)$ and the ***domain*** of $g\circ f$

7. $f(x)=x-3$ *and* $g(x)=x+3$

8. $f(x)=\dfrac{2}{3}x$ *and* $g(x)=\dfrac{3}{2}x$

9. $f(x)=x-1$ *and* $g(x)=3x^2-2x-1$

10. $f(x)=3x-2$ *and* $g(x)=x^2-5$

11. $f(x)=x^2-2$ *and* $g(x)=4x-3$

12. $f(x)=4x^2-x+10$ *and* $g(x)=2x-7$

13. $f(x)=\sqrt{x}$ *and* $g(x)=x+3$

14. $f(x)=\sqrt{x}$ *and* $g(x)=2-3x$

15. $f(x)=3x+2$ *and* $g(x)=\sqrt{x}$

16. $f(x)=x^4$ *and* $g(x)=\sqrt[4]{x}$

17. $f(x)=x^n$ *and* $g(x)=\sqrt[n]{x}$

18. $f(x)=x^2-3x$ *and* $g(x)=\sqrt{x+2}$

19. $f(x)=\sqrt{x-2}$ *and* $g(x)=\sqrt{x+5}$

20. $f(x)=x^2+2$ *and* $g(x)=\sqrt{3-x}$

21. $f(x)=x^5-2$ *and* $g(x)=\sqrt[5]{x+2}$

22. $f(x)=1-x^2$ *and* $g(x)=\sqrt{x^2-25}$

23. $f(x) = 2x + 3$ *and* $g(x) = \frac{x-3}{2}$

24. $f(x) = 4x - 5$ *and* $g(x) = \frac{x+5}{4}$

25. $f(x) = \frac{4}{1-5x}$ *and* $g(x) = \frac{1}{x}$

26. $f(x) = \frac{1}{x-2}$ *and* $g(x) = \frac{x+2}{x}$

27. $f(x) = \frac{1}{1+x}$ *and* $g(x) = \frac{1-x}{x}$

28. $f(x) = \frac{3x+5}{2}$ *and* $g(x) = \frac{2x-5}{3}$

29. $f(x) = \frac{x-1}{x-2}$ *and* $g(x) = \frac{x-3}{x-4}$

30. $f(x) = \frac{6}{x-3}$ *and* $g(x) = \frac{1}{x}$

31. $f(x) = \frac{6}{x}$ *and* $g(x) = \frac{1}{2x+1}$

32. $f(x) = 3x - 7$ *and* $g(x) = \frac{x+7}{3}$

33. $f(x) = \frac{2x+3}{x-4}$ *and* $g(x) = \frac{4x+3}{x-2}$

34. $f(x) = \frac{2x+3}{x+4}$ *and* $g(x) = \frac{-4x+3}{x-2}$

35. $f(x) = x + 1$ *and* $g(x) = x^3 - 5x^2 + 3x + 7$

36. $f(x) = x - 1$ *and* $g(x) = x^3 + 2x^2 - 3x - 9$

(**37 – 48**) Evaluate each composite function, where $f(x) = 2x - 3$ *and* $g(x) = x^2 - 5x$

37. $(f \circ g)(4)$

38. $(g \circ f)(4)$

39. $(f \circ g)(-2)$

40. $(g \circ f)(-2)$

41. $(f \circ f)(-3)$

42. $(g \circ g)(7)$

43. $(f \circ g)(\sqrt{2})$

44. $(g \circ f)(\sqrt{3})$

45. $(f \circ g)(2a)$

46. $(g \circ f)(3b)$

47. $(f \circ g)(k+1)$

48. $(g \circ f)(k-1)$