

Lecture Three – Trigonometric

Section 3.1 – Introduction

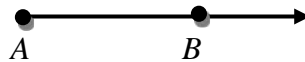
Basic Terminology

Two distinct points determine line AB .

Line segment AB : portion of the line between A and B .

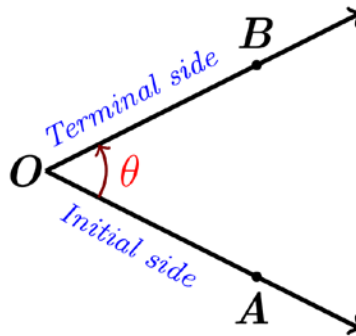


Ray AB : portion of the line AB starts at A and continues through B , and past B .



Angles in General

An angle is formed by 2 rays with the same end point.

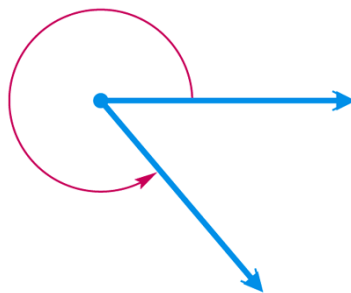


The two rays are the sides of the angle, angle $\theta = AOB$

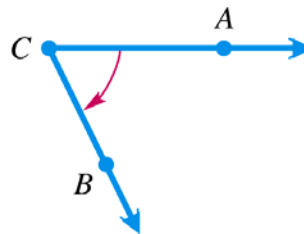
O is the common endpoint and it is called **vertex** of the angle.

An angle is in a Counterclockwise (**CCW**) direction: positive angle.

An angle is in a Clockwise (**CW**) direction: negative angle.

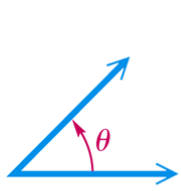


Positive angle

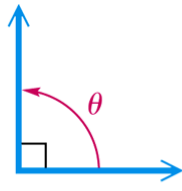


Negative angle

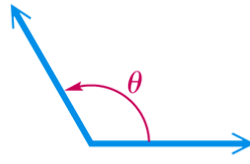
Type of Angles: *Degree*



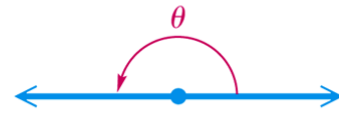
Acute angle
 $0^\circ < \theta < 90^\circ$



Right angle
 $\theta = 90^\circ$



Obtuse angle
 $90^\circ < \theta < 180^\circ$



Straight angle
 $\theta = 180^\circ$

Complementary angles: $\alpha + \beta = 90^\circ$

Supplementary angles: $\alpha + \beta = 180^\circ$

Example

Give the complement and the supplement of each angle: 40° 110°

Solution

a. 40° Complement: $90^\circ - 40^\circ = 50^\circ$ Supplement: $180^\circ - 40^\circ = 140^\circ$

b. 110° Complement: $90^\circ - 110^\circ = -20^\circ$ Supplement: $180^\circ - 110^\circ = 70^\circ$

Degrees, Minutes, Seconds

1° : 1 degree

$1'$: 1 minute

$1''$: 1 second

$$1 \text{ full Rotation or Revolution} = 360^\circ \quad 1^\circ = 60' = 3600'' \quad 1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^\circ$$

Example

Change 27.25° to degrees and minutes

Solution

$$\begin{aligned} 27.25^\circ &= 27^\circ + .25^\circ \\ &= 27^\circ + .25(60') \\ &= 27^\circ + 15' \\ &= \underline{27^\circ \quad 15'} \end{aligned}$$

Example

Add $48^\circ 49'$ and $72^\circ 26'$

Solution

$$\begin{array}{r} 48^\circ \quad 49' \\ + 72^\circ \quad 26' \\ \hline 120^\circ \quad 75' \end{array}$$

$$\begin{aligned} 120^\circ 75' &= 120^\circ 60' + 15' \\ &= \underline{121^\circ 15'} \end{aligned}$$

Example

Subtract $24^\circ 14'$ and 90°

Solution

$$\begin{array}{r} 90^\circ \\ - 24^\circ \quad 14' \\ \hline \end{array} = \begin{array}{r} 89^\circ \quad 60' \\ - 24^\circ \quad 14' \\ \hline 65^\circ \quad 46' \end{array}$$

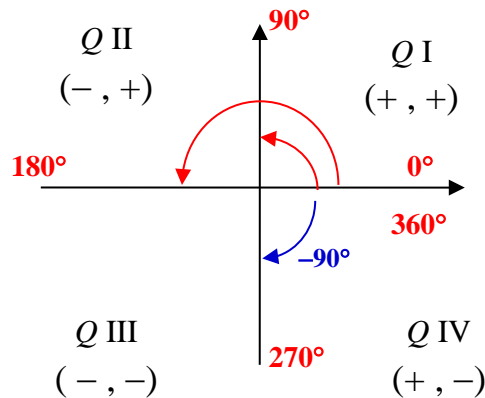
Angles in Standard Position

An angle is said to be in standard position if its initial side is along the positive x -axis and its vertex is at the origin. If angle θ is in standard position and the terminal side of θ lies in quadrant I, then we say θ lies in QI

$$\theta \in QI$$

If the terminal side of an angle in standard position lies along one of the axes (x -axis or y -axis), such as angles with measures 90° , 180° , 270° , then that called a **quadrantal angle**.

Two angles in standard position with the same terminal side are called **coterminal angles**.



Example

Find all angles that are coterminal with 120° .

Solution

$$120^\circ + 360^\circ k$$

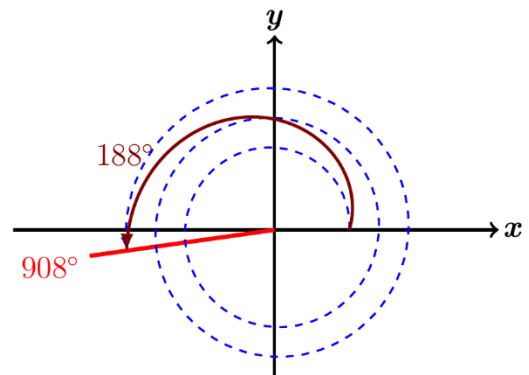
Example

Find the angle of least possible positive measure coterminal with an angle of 908° .

Solution

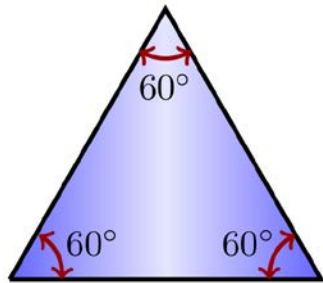
$$908^\circ - 2 \cdot 360^\circ = 188^\circ$$

An angle of 908° is coterminal with an angle of 188°

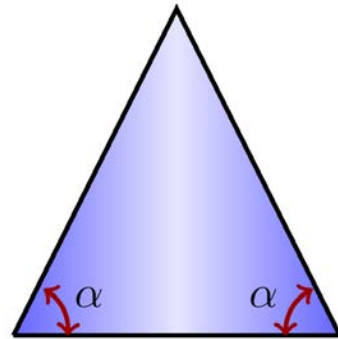


Triangles

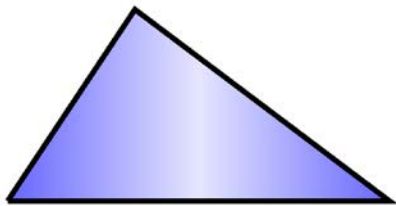
Equilateral – All angles always equal to 60° & all sides are equal



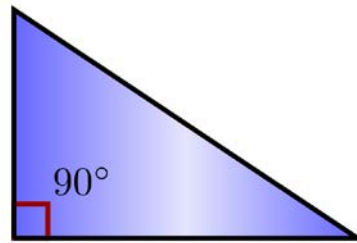
Isosceles: 2 sides and angles are equal



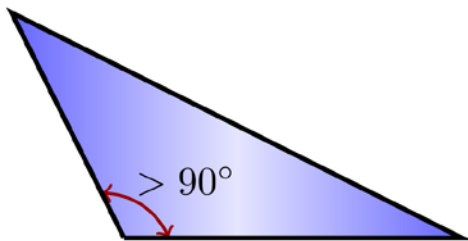
Scalene: No equal sides or angles



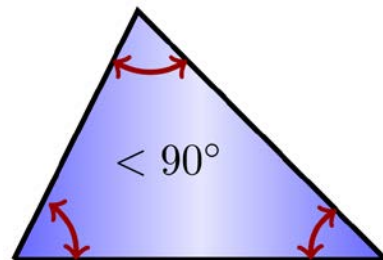
Right: Has a right angle 90° .



Obtuse: Has an angle more than 90° .

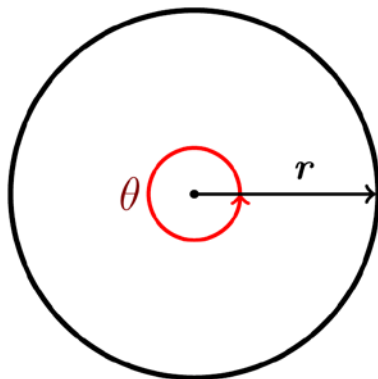


Acute: All angles are less than 90° .



Radians

Degrees - Radians



θ measures one full rotation $\theta = 2\pi$ The measure of θ in radians is 2π

$$\boxed{1 = 1 \text{ rad}}$$

$$1^\circ = 1 \text{ degree}$$

If no unit of angle measure is specified, then the angle is to be measured in radians.

$$\text{Full Rotation: } 360^\circ = 2\pi \text{ rad}$$

$$180^\circ = \pi \text{ rad}$$

Converting from Degrees to Radians

$$\frac{180^\circ}{180} = \frac{\pi}{180} \text{ rad} \quad \Rightarrow 1^\circ = \frac{\pi}{180} \text{ rad}$$

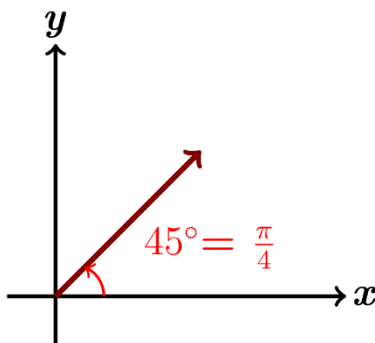
Multiply a degree measure by $\frac{\pi}{180} \text{ rad}$ and simplify to convert to radians.

Example

Convert 45° to radians

Solution

$$\begin{aligned} 45^\circ &= 45 \left(\frac{\pi}{180} \right) \text{ rad} \\ &= \frac{\pi}{4} \text{ rad} \end{aligned}$$



Example

Convert -450° to radians

Solution

$$\begin{aligned} -450^\circ &= -450 \left(\frac{\pi}{180} \right) rad \\ &= -\frac{5\pi}{2} rad \end{aligned}$$

Example

Convert 249.8° to radians

Solution

$$\begin{aligned} 249.8^\circ &= \frac{2498}{10} \left(\frac{\pi}{180} \right) rad \\ &= \frac{1,249\pi}{900} rad \\ &\approx 4.360 rad \end{aligned}$$

Converting from Radians to Degrees

Multiply a radian measure by $\frac{180^\circ}{\pi}$ radian and simplify to convert to degrees.

$$\frac{180^\circ}{\pi} = \frac{\pi}{\pi} rad$$

$$\boxed{\frac{180^\circ}{\pi} = 1 rad}$$

Example

Convert 1 to *degrees*

Solution

$$\begin{aligned} 1 rad &= 1 \left(\frac{180^\circ}{\pi} \right) \\ &= 1 \left(\frac{180^\circ}{3.14} \right) \\ &= 57.3^\circ \end{aligned}$$

Example

Convert $\frac{4\pi}{3}$ to *degrees*

Solution

$$\begin{aligned}\frac{4\pi}{3} &= \frac{4\pi}{3} \left(\frac{180^\circ}{\pi} \right) \\ &= 240^\circ\end{aligned}$$

Example

Convert -4.5 to *degrees*

Solution

$$\begin{aligned}-4.5 &= -4.5 \left(\frac{180^\circ}{\pi} \right) \\ &\approx -257.8^\circ\end{aligned}$$

Exercises **Section 3.1– Introduction**

1. Indicate the angle if it is an acute or obtuse. Then give the complement and the supplement of each angle.
a) 10° b) 52° c) 90° d) 120° e) 150°
2. Change to decimal degrees.
a) $10^\circ 45'$ c) $274^\circ 18' 59''$ e) $98^\circ 22' 45''$ g) $1^\circ 2' 3''$
b) $34^\circ 51' 35''$ d) $74^\circ 8' 14''$ f) $9^\circ 9' 9''$ h) $73^\circ 40' 40''$
3. Convert to degrees, minutes, and seconds.
a) 89.9004° c) 122.6853° e) 44.01° g) 29.411°
b) 34.817° d) 178.5994° f) 19.99° h) 18.255°
4. Perform each calculation
a) $51^\circ 29' + 32^\circ 46'$ b) $90^\circ - 73^\circ 12'$ c) $90^\circ - 36^\circ 18' 47''$ d) $75^\circ 15' + 83^\circ 32'$
5. Find the angle of least possible positive measure coterminal with an angle of
a) -75° b) -800° c) 270°
6. Convert to radians
a) $256^\circ 20'$ b) -78.4° c) 330° d) -60° e) -225°
7. Convert to degrees
a) $\frac{11\pi}{6}$ c) $\frac{\pi}{6}$ e) $\frac{\pi}{3}$ g) -4π
b) $-\frac{5\pi}{3}$ d) 2.4 f) $-\frac{5\pi}{12}$ h) $\frac{7\pi}{13}$
8. A vertical rise of the Forest Double chair lift 1,170 *feet* and the length of the chair lift as 5,570 *feet*. To the nearest foot, find the horizontal distance covered by a person riding this lift.
9. A tire is rotating 600 times per *minute*. Through how many degrees does a point of the edge of the tire move in $\frac{1}{2}$ second?
10. A windmill makes 90 *revolutions* per *minute*. How many revolutions does it make per second?