

## **Solution**    Section 3.3 – Absolute Extrema

### **Exercise**

Find the absolute extrema of the function on the closed interval  $f(x) = 2(3-x)$ ,  $[-1, 2]$

### **Solution**

$$f' = -2$$

$$f(-1) = 2(3 - (-1)) = 8$$

$$f(2) = 2(3 - 2) = 2$$

$$\text{RMAX: } (-1, 8)$$

$$\text{RMIN: } (2, 2)$$

### **Exercise**

Find the absolute extrema of the function on the closed interval  $f(x) = x^3 - 3x^2$ ,  $[0, 4]$

### **Solution**

$$f'(x) = 3x^2 - 6x = 0$$

$$3x(x-2) = 0 \rightarrow \begin{cases} x = 0 \\ x - 2 = 0 \Rightarrow x = 2 \end{cases}$$

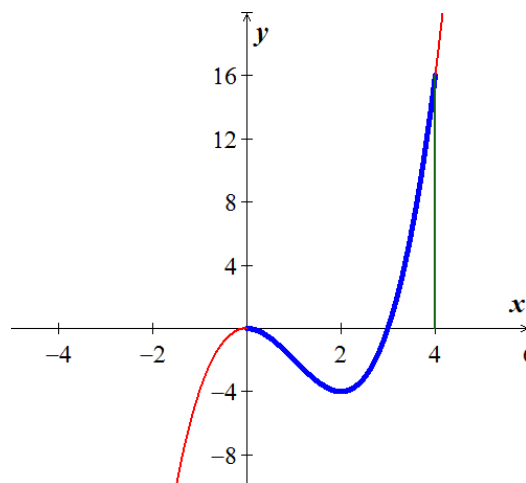
$$f(0) = 0^3 - 3(0)^2 = 0$$

$$f(2) = 2^3 - 3(2)^2 = -4$$

$$f(4) = 4^3 - 3(4)^2 = 16$$

$$\text{RMAX: } (4, 16)$$

$$\text{RMIN: } (2, -4)$$



### Exercise

Find the absolute extrema of the function on the closed interval  $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x - 4$ ,  $[-2, 5]$

### Solution

$$f'(x) = x^2 - 4x + 3 = 0$$

$$x^2 - 4x + 3 = 0 \rightarrow \begin{cases} x = 1 \\ x = 3 \end{cases}$$

$$f(-2) = \frac{1}{3}(-2)^3 - 2(-2)^2 + 3(-2) - 4 = -20.66$$

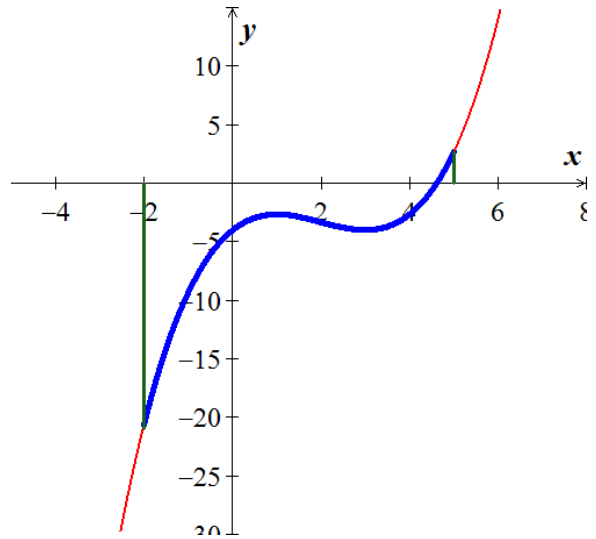
$$f(1) = \frac{1}{3}(1)^3 - 2(1)^2 + 3(1) - 4 = -2.66$$

$$f(3) = \frac{1}{3}(3)^3 - 2(3)^2 + 3(3) - 4 = -4$$

$$f(5) = \frac{1}{3}(5)^3 - 2(5)^2 + 3(5) - 4 = 2.66$$

**RMAX:** (5, 2.66)

**RMIN:** (-2, -20.66)



### Exercise

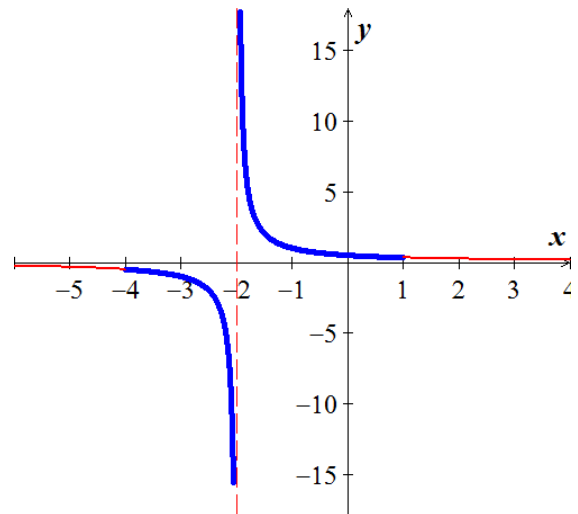
Find the absolute extrema of the function on the closed interval  $f(x) = \frac{1}{x+2}$ ,  $[-4, 1]$

### Solution

$$x + 2 \neq 0 \rightarrow x \neq -2 \quad (\text{Asymptote})$$

$$f'(x) = -\frac{1}{(x+2)^2} \neq 0$$

There is no Relative Extrema.



### Exercise

Find the absolute extrema of the function on the closed interval  $f(x) = (x^2 + 4)^{2/3}$ ,  $[-2, 2]$

### Solution

$$f'(x) = \frac{2}{3}(2x)(x^2 + 4)^{2/3-1}$$

$$= \frac{4x}{3}(x^2 + 4)^{-1/3}$$

$$f' = \frac{4x}{3}(x^2 + 4)^{-1/3} = 0; \quad x^2 + 4 \neq 0$$

$$\boxed{x = 0}$$

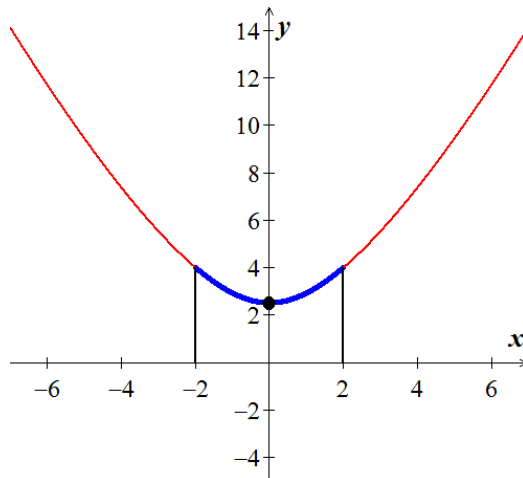
$$f(x = -2) = \left((-2)^2 + 4\right)^{2/3} = 4$$

$$f(x = 0) = \left((0)^2 + 4\right)^{2/3} = 2.52$$

$$f(x = 2) = \left((2)^2 + 4\right)^{2/3} = 4$$

**RMAX:**  $(-2, 4)$   $(2, 4)$

**RMIN:**  $(0, 2.52)$



### Exercise

$P(x) = -x^3 + \frac{27}{2}x^2 - 60x + 100$ ,  $x \geq 5$  is an approximation to the total profit (in thousands of dollars) from the sale of  $x$  hundred thousand tires. Find the number of hundred thousands of tires that must be sold to maximize profit.

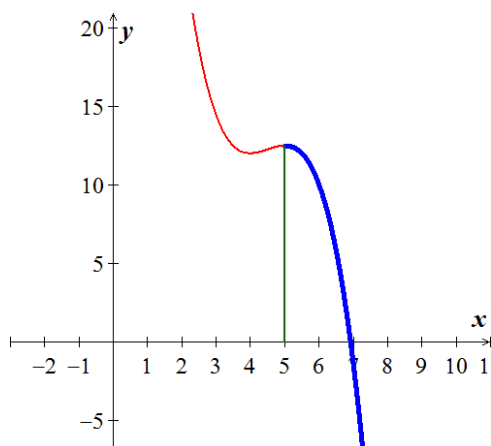
### Solution

$$P'(x) = -3x^2 + 27x - 60 = 0$$

$$x = 5, \quad 4 \text{ (not in the interval)} \quad (x \geq 5)$$

$$P(x = 5) = -(5)^3 + \frac{27}{2}(5)^2 - 60(5) + 100$$

$$\underline{\underline{= 12.5}}$$



### Exercise

$P(x) = -x^3 + 12x^2 - 36x + 400$ ,  $x \geq 3$  is an approximation to the total profit (in thousands of dollars) from the sale of  $x$  hundred thousand tires. Find the number of hundred thousands of tires that must be sold to maximize profit.

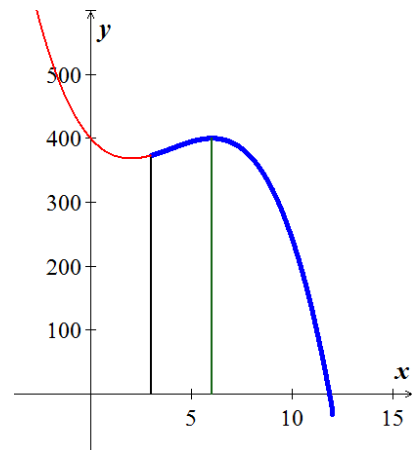
### Solution

$$P'(x) = -3x^2 + 24x - 36 = 0$$

$$x = 6, \quad 2 \text{ (not in the interval)} \quad (x \geq 3)$$

$$P(x = 6) = -(6)^3 + 12(6)^2 - 36(6) + 400$$

$$= 400$$



### Exercise

Researchers have discovered that by controlling both the temperature and the relative humidity in a building, the growth of a certain fungus can be limited. The relationship between temperature and relative humidity, which limits growth, can be described by

$$R(T) = -0.00008T^3 + 0.386T^2 - 1.6573T + 97.086, \quad 0 \leq T \leq 46$$

where  $R(T)$  is the relative humidity (in %) and  $T$  is the temperature (in °C). Find the temperature at which the relative humidity is minimized.

### Solution

$$R'(T) = -0.00024T^2 + 0.772T - 1.6573 = 0$$

$$T = 2.15, \quad 3214.52 \text{ (not in the interval)} \quad (0 \leq T \leq 46)$$

$$R(T = 0) = -0.00008(0)^3 + 0.386(0)^2 - 1.6573(0) + 97.086 = 97.1^\circ\text{C}$$

$$R(T = 2.15) = -0.00008(2.15)^3 + 0.386(2.15)^2 - 1.6573(2.15) + 97.086 = 95.3^\circ\text{C}$$

$$R(T = 46) = -0.00008(46)^3 + 0.386(46)^2 - 1.6573(46) + 97.086 = 830.5^\circ\text{C}$$

