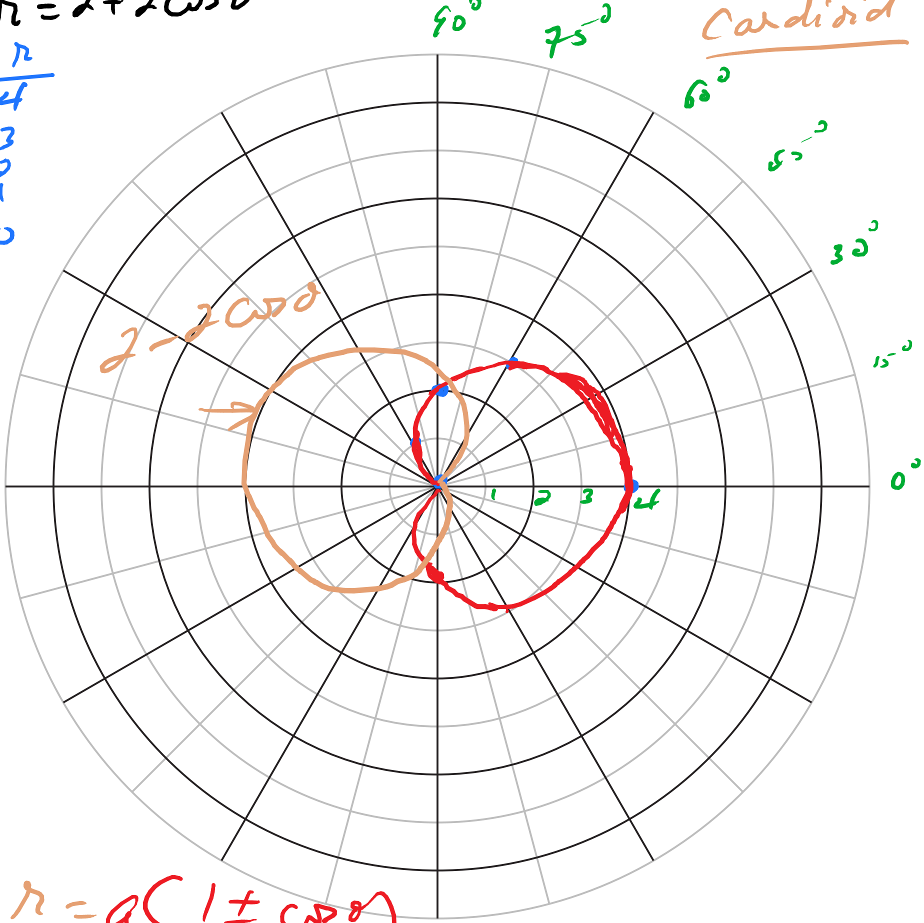
$r = 2 + 2\cos\theta$

Cardioid

θ	r
0	4
60°	3
90°	2
120°	1
180°	0



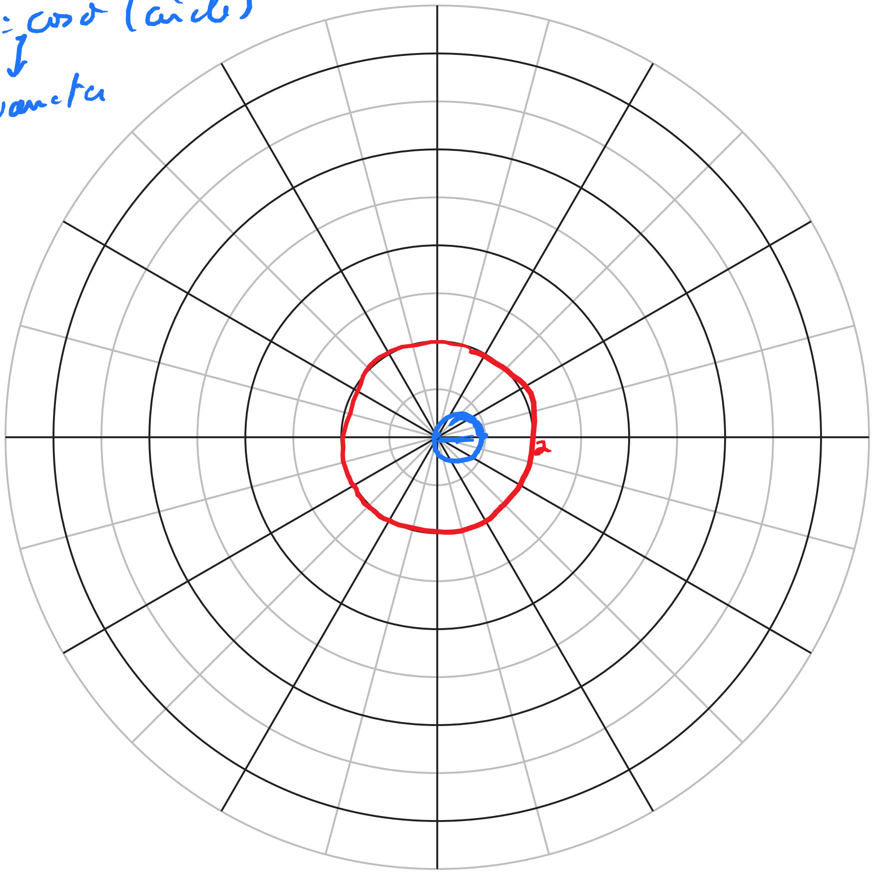
$r = a(1 \pm \cos\theta)$

$a(1 \pm \sin\theta)$

$r = 2$

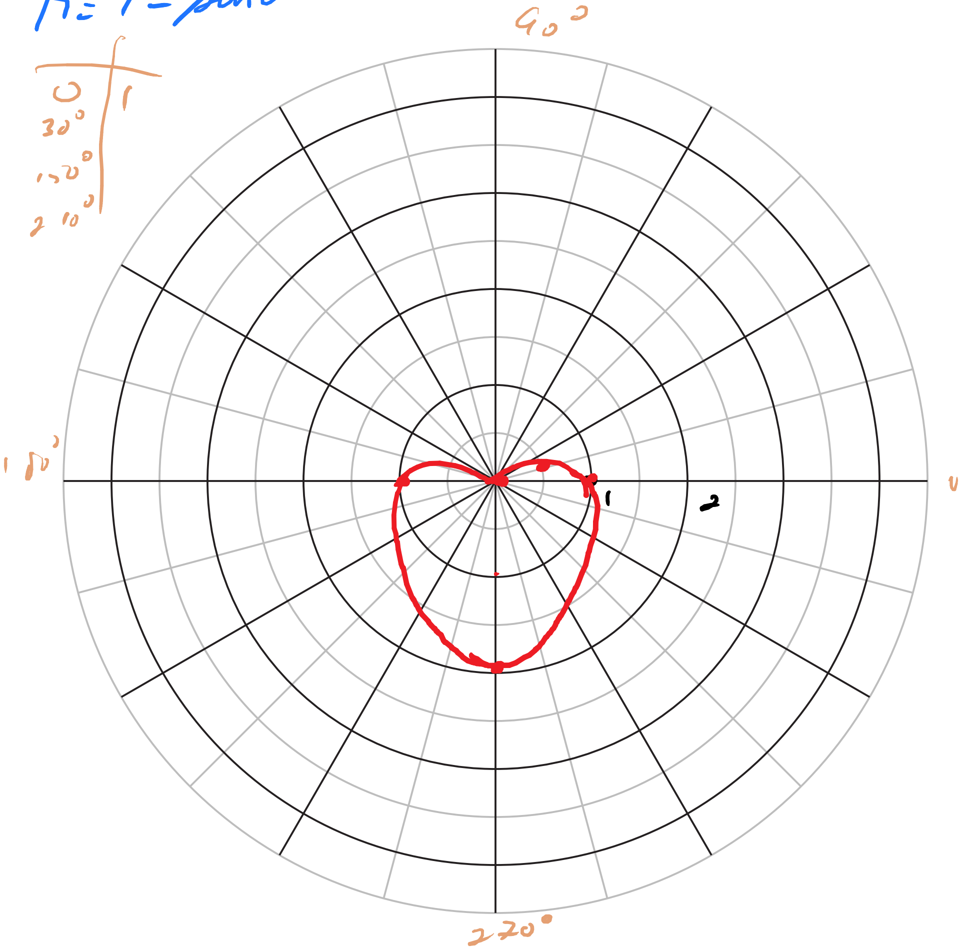
$r = \cos\theta$ (circle)

diameter



$r = 1 - \sin\theta$

0	1
30°	
150°	
210°	

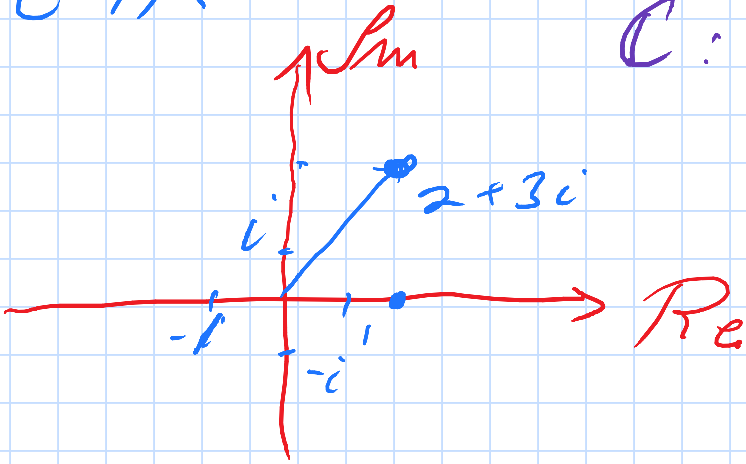


$$z = \overbrace{a}^{\text{Real part}} + \underbrace{ib}_{\text{Imaginary part}}$$

i { i unit

a & $b \in \mathbb{R}$

$\mathbb{C}$: complex nb



$2+3i$

Conjugate of $z = a + ib$
is $\bar{z} = a - ib$

$$z\bar{z} = a^2 + b^2$$

$$\sqrt{-1} = i$$

$$-1 = i^2$$

$$-i = i^3$$

$$1 = i^4$$

Defn

$z = x + yi$ or $x + iy$

modulus: $r = |z| = \sqrt{x^2 + y^2}$

Argument: $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\begin{aligned} z &= x + yi \\ &= r\cos\theta + (r\sin\theta)i \\ &= \underbrace{r(\cos\theta + i\sin\theta)}_{r\,\text{cis}\,\theta} \end{aligned}$$

Trig of z.

Same

$z = 3 - 2i\sqrt{3}$

Ex $\quad z = -1 + i$

$$\begin{cases} x = -1 \\ y = +1 \end{cases} \quad Q\,II$$

$$r = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}$$

$$\theta = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$$

$$z = \sqrt{2}\ \text{cis}\,\frac{3\pi}{4}$$

Ex $\quad z = 2\ \text{cis}\ 30^\circ$

$$= 2\left(\cos 30^\circ + i \sin 30^\circ\right)$$

$$= 2\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right)$$

$$= \sqrt{3} + i$$

Ex $\quad z = 2\ \text{cis}\ 300^\circ$

$$= 2\left(\cos 300^\circ + i \sin 300^\circ\right)$$

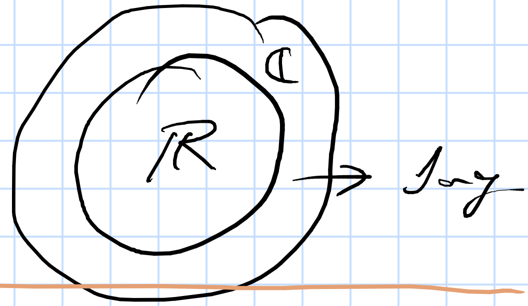
$$= 2\left(\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right)$$

$$= 1 - i\sqrt{3}$$

$z = 5i = 5\,\text{cis}\,90^\circ$

$z = 7 = 7\,\text{cis}\,0^\circ$

$$\boxed{7 + 0i}$$



Product Theorem

$$(r_1\,\text{cis}\,\theta_1)(r_2\,\text{cis}\,\theta_2) = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$$

$$a^2 + b^2 = (a - ib)(a + ib)$$

$$a + b = (\sqrt{a} - i\sqrt{b})(\sqrt{a} + i\sqrt{b})$$

ex $r_1 = 3\,\text{cis}\,45^\circ$ $\quad r_2 = 2(\cos 135^\circ + i\sin 135^\circ)$

$$\underbrace{(3\,\text{cis}\,45^\circ)(2\,\text{cis}\,135^\circ)}_{=6} = 6\,\text{cis}(180^\circ)$$

$$= 6(\cos 180^\circ + i\sin 180^\circ)$$

$$= \underline{-6}$$

Quotient Theorem

$$\frac{r_1 \operatorname{cis} \theta_1}{r_2 \operatorname{cis} \theta_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Ex

$$\frac{10 \operatorname{cis}(-60^\circ)}{5 \operatorname{cis}(150^\circ)} = 2 \operatorname{cis}(-210^\circ)$$

$$\cancel{= 2(\cos(-210^\circ) + i \sin(-210^\circ))}$$

$$= 2(\cos 210^\circ - i \sin 210^\circ)$$

$$= 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2} i\right)$$

$$= -\sqrt{3} + i$$

De Moivre's Theorem

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} n\theta$$

Ex

$$\left(1+i\sqrt{3}\right)^8$$

$$r=\sqrt{1+3}$$
$$=2$$

$$\theta=\tan^{-1}\frac{\sqrt{3}}{1}$$
$$=60^\circ \quad \boxed{\text{or } \frac{\pi}{3}}$$

$$\left(1+i\sqrt{3}\right)^8=\left(2\operatorname{cis}60^\circ\right)^8$$
$$=2^8\operatorname{cis}\left(480^\circ\right)$$
$$=256\left(\cos 120^\circ+i\sin 120^\circ\right)$$
$$=256\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$$
$$=-128+128i\sqrt{3}$$

n^{th} Root

$\sqrt[n]{\ }$

$$(r \operatorname{cis} \theta)^{1/n} = \sqrt[n]{r} \operatorname{cis} \alpha$$

$$\alpha = \frac{\theta}{n} + 360^\circ \frac{k}{n} \quad \text{or} \quad \frac{2\pi k}{n}$$

Ex/ $\sqrt{4i}$ $\quad n=2$

$4i \quad r=4 \quad \theta = 90^\circ = \frac{\pi}{2}$

$$\sqrt{4i} = \sqrt{4 \operatorname{cis} \frac{\pi}{2}}$$
$$= \sqrt{4} \operatorname{cis} \alpha$$
$$= 2 \operatorname{cis} \alpha$$

$$\alpha = \frac{1}{n}\left(\frac{\pi}{2} + 2\pi k\right)$$
$$\alpha = \frac{1}{2}\left(\frac{\pi}{2} + 2\pi k\right)$$
$$= \frac{\pi}{4} + \pi k$$

$n=2 \rightarrow 2\ k$'s $\quad k = 0, 1$

For $k=0 \Rightarrow \alpha = \frac{\pi}{4}$

$k=1 \Rightarrow \alpha = \frac{5\pi}{4}$

$$\sqrt{4i} = \begin{cases} 2 \operatorname{cis} \pi/4 \\ 2 \operatorname{cis} 5\pi/4 \end{cases}$$

$$2 \operatorname{cis} \frac{\pi}{4} = 2\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$$

$$= 2\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\sqrt{2} + i\sqrt{2}}$$

$$2 \operatorname{cis} \frac{5\pi}{4} = \boxed{-\sqrt{2} - i\sqrt{2}}$$