

Good Morning ☺

① Area (graph 3 fctns only)

② Volume any method.

② Length  $\left\{ \begin{array}{l} ax^m + bx^n \end{array} \right\} \begin{array}{l} m+n=2 \\ abmn = -\frac{1}{4} \end{array}$   
 $\left\{ \begin{array}{l} ae^{mx} + be^{nx} \end{array} \right\} \begin{array}{l} m=-n \\ abmn = -\frac{1}{4} \end{array}$   
 $L = ax^m + bx^n$   
 $\left\{ \begin{array}{l} ae^{mx} - be^{-n} \end{array} \right.$

① Surface  $f' \Rightarrow \sqrt{1+f'^2} = \bar{f}'$

① mass

① Force

① derivative ✓

② Integration ✓

(a) (b)

$$\left\{ \begin{array}{l} \cosh x = \frac{e^x + e^{-x}}{2} \\ \sinh x = \frac{e^x - e^{-x}}{2} \end{array} \right.$$

less 10  $\rightarrow$  (1  $\rightarrow$  scratch)

1st  $\rightarrow$  your name on it

Conversation: type: done

# pages

you are taking of exam

- Please wait until you get permission to be dismissed.



- if you take a picture (low resolution)

1/1 A:  $4x^2 + y = 4$

$$y = 4 - 4x^2$$

$$x^4 - y = 1$$

$$y = x^4 - 1$$

$$x^4 - 1 = 4 - 4x^2$$

$$x^4 + 4x^2 - 5 = 0$$

$$x^2 = 1 \quad \text{~~2.5~~}$$

$$\underline{x = \pm 1}$$

$$A = \int_{-1}^1 (4 - 4x^2 - x^4 + 1) dx$$

$$= \int_{-1}^1 (5 - 4x^2 - x^4) dx$$

$$= 5x - \frac{4}{3}x^3 - \frac{1}{5}x^5 \Big|_{-1}^1 \quad 2 \left( \int_0^1 \right)$$

$$= 5 - \frac{4}{3} - \frac{1}{5} - \left( -5 + \frac{4}{3} + \frac{1}{5} \right)$$

$$= 10 - \frac{8}{3} - \frac{2}{5}$$

$$= \frac{150 - 40 - 6}{15}$$

$$= \frac{104}{15} \text{ unit}^2$$



# V?  $y = 4 - x^2$   $0 \leq x \leq 2$   $x$ -axis

$$V = \pi \int_0^2 (4 - x^2)^2 dx$$

$$= \pi \int_0^2 (16 - 8x^2 + x^4) dx$$

$$= \pi \left( 16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right) \Big|_0^2$$


$$= \pi \left( 32 - \frac{64}{3} + \frac{32}{5} \right)$$

$$= \frac{32(15 - 10 + 3)}{15} \pi$$

$$= \frac{256\pi}{15} \text{ unit}^3$$

V?  $x = -y^2 + 4y = 0$   $y$ -axis

1.4  
4/ V?  $y = \frac{1}{2}x^2 + 1$   $y$ -axis



$$V = 2\pi \int_0^2 x \left( 3 - \frac{1}{2}x^2 - 1 \right) dx$$

$$= 2\pi \int_0^2 \left( 2x - \frac{1}{2}x^3 \right) dx$$

$$= 2\pi \left( x^2 - \frac{1}{8}x^4 \right) \Big|_0^2$$

$$= 2\pi (4 - 2) = 4\pi \text{ unit}^3$$





$$V: \quad y = x^2 \quad y = 4x - x^2 \quad \sim \underline{x=4}$$

// y-axis

$$x^2 = 4x - x^2$$

$$2x^2 - 4x = 0$$

$$\underline{x = 0, 2}$$

$$V = 2\pi \int_0^2 (4-x) \left( 4x - \overbrace{x^2 - x^2}^{-2x^2} \right) dx$$

$$= 2\pi \int_0^2 (16x - 12x^2 + 2x^3) dx$$

$$= 4\pi \left( 8x^2 - 2x^3 + \frac{1}{4}x^4 \right) \Big|_0^2$$

$$= 4\pi (32 - 32 + 4)$$

$$= \underline{16\pi \text{ unit}^3}$$

$$f(x) = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}$$

$$1 \leq x \leq 4 \quad L?$$

$$m+n = \frac{3}{2} + \frac{1}{2} = 2 \quad \checkmark$$

$$a \cdot m \cdot b \cdot n = \frac{2}{3} \left( \frac{3}{2} \right) \left( -\frac{1}{2} \right) \left( \frac{1}{2} \right) = -\frac{1}{4} \quad \checkmark$$

$$L = \frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} \Big|_1^4$$

$$= \frac{16}{3} + 1 - \frac{2}{3} - \frac{1}{2}$$

$$= \frac{14}{3} + \frac{1}{2}$$

$$= \underline{\frac{31}{6} \text{ unit}}$$



$$L? f(y) = 2e^{\sqrt{2}y} + \frac{1}{16}e^{-\sqrt{2}y} \quad 0 \leq y \leq \frac{\ln 2}{\sqrt{2}}$$

$$m = -n \quad \checkmark$$

$$a = 2, m = \sqrt{2}, b = \frac{1}{16}$$

$$abmn = 2(\sqrt{2})\left(\frac{1}{16}\right)(-\sqrt{2}) = -\frac{1}{4} \quad \checkmark \quad n = -\sqrt{2}$$

$$L = 2e^{\sqrt{2}y} - \frac{1}{16}e^{-\sqrt{2}y} \quad \left| \frac{\ln 2 / \sqrt{2}}{0} \right.$$

$$= 2(2) - \frac{1}{16}\left(\frac{1}{2}\right)$$

$$= 4 - \frac{1}{32}$$

$$= \frac{127}{32} \text{ unit}$$

$$e^{\ln x} = x$$

$$e^{-\ln x} = \frac{1}{x}$$

$$5? y = x^{3/2} - \frac{1}{3}x^{1/2} \quad 1 \leq x \leq 2 \quad \text{w.r.t. axis}$$

$$m + n = \frac{3}{2} + \frac{1}{2} = 2$$

$$abmn = 1\left(\frac{3}{2}\right)\left(-\frac{1}{3}\right)\left(\frac{1}{2}\right) = -\frac{1}{4} \quad \checkmark$$

$$y' = \frac{3}{2}x^{1/2} - \frac{1}{6}x^{-1/2} \quad \rightarrow$$

$$S = 2\pi \int_1^2 \left(x^{3/2} - \frac{1}{3}x^{1/2}\right) \left(\frac{3}{2}x^{1/2} + \frac{1}{6}x^{-1/2}\right) dx$$

$$= 2\pi \int_1^2 \left(\frac{3}{2}x^2 + \frac{1}{6}x - \frac{1}{2}x - \frac{1}{18}\right) dx$$

$$= 2\pi \int_1^2 \left(\frac{3}{2}x^2 - \frac{1}{3}x - \frac{1}{18}\right) dx$$



$$\begin{aligned}
 S &= 2\pi \left( \frac{1}{2}x^3 - \frac{1}{6}x^2 - \frac{1}{18}x \right) \Big|_1^2 \\
 &= 2\pi \left( 4 - \frac{2}{3} - \frac{1}{9} - \frac{1}{2} + \frac{1}{6} + \frac{1}{18} \right) \\
 &= 2\pi \left( 4 + \frac{-12 - 2 - 9 + 3 + 1}{18} \right) \\
 &= 2\pi \left( 4 - \frac{19}{18} \right) \\
 &= \frac{53\pi}{9} \text{ unit}^2
 \end{aligned}$$

$\frac{7^2}{53}$

m?  $\rho(x) = x\sqrt{2-x^2}$   $0 \leq x \leq 1$

$$\begin{aligned}
 m &= \int_0^1 x\sqrt{2-x^2} dx \\
 &= -\frac{1}{2} \int_0^1 (2-x^2)^{1/2} d(2-x^2) \\
 &= -\frac{1}{3} (2-x^2)^{3/2} \Big|_0^1 \\
 &= -\frac{1}{3} (1 - 2^{3/2}) \text{ unit}
 \end{aligned}$$



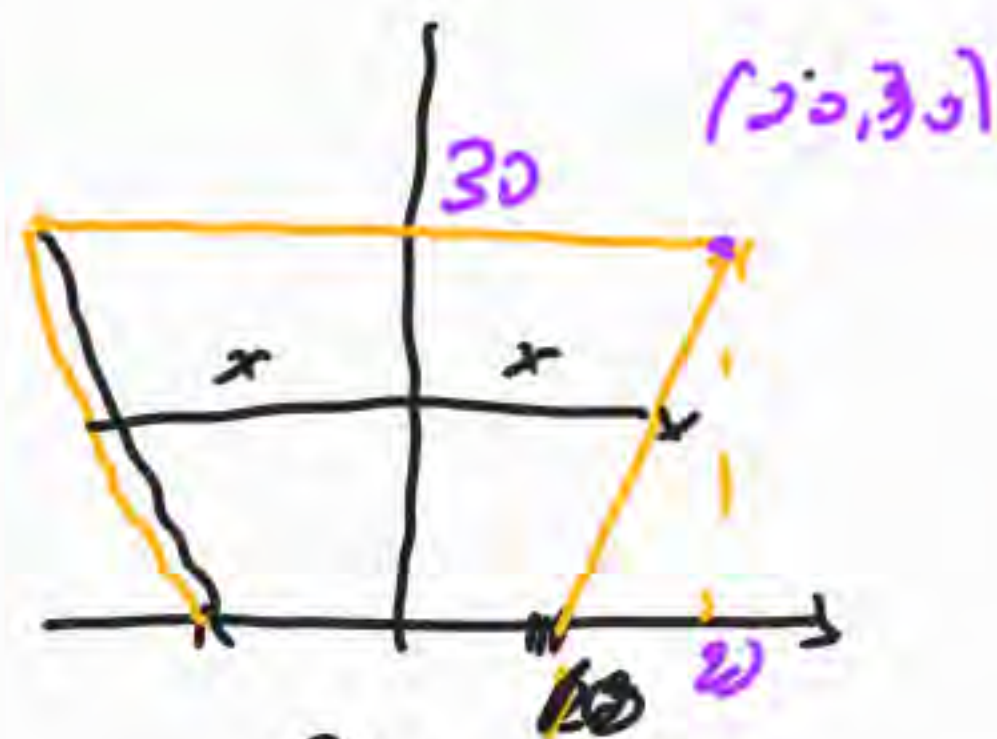
$$\rho = 1000 = 10^3$$

$$g = 9.8 = \frac{98}{10}$$

$$F = \rho g \int_0^a (a-y) w(y) dy$$

$$w(y) = 2x$$

$$= \frac{2}{3}y + 20$$



$$y = 3(x-10)$$

$$\frac{1}{3}y + 10 = x$$

$$F = 98 \times 10^2 \int_0^{30} (30-y) \left( \frac{2}{3}y + 20 \right) dy$$

$$= 98 \times 10^2 \int_0^{30} \left( 20y + 600 - \frac{2}{3}y^2 - 20y \right) dy$$

$$= 9800 \int_0^{30} \left( 600 - \frac{2}{3}y^2 \right) dy$$

$$= 9800 \left( 600y - \frac{2}{9}y^3 \right) \Big|_0^{30}$$

$$= 9800 \left( 18 \times 10^3 - 6 \times 10^3 \right)$$

$$= 98 \times 10^5 (12)$$

$$= 1176 \times 10^5 \text{ unit}$$



$$f(x) = \frac{\ln x}{\ln x + 1}$$

$$f'(x) = \frac{\frac{1}{x}(\ln x + 1) - \frac{1}{x} \ln x}{(\ln x + 1)^2}$$

$$= \frac{1}{x(\ln x + 1)^2}$$

$$\int_e^{e^2} \frac{dx}{x \ln^3 x} = \int_e^{e^2} (\ln x)^{-3} d(\ln x)$$

$$= -\frac{1}{2} \frac{1}{(\ln x)^2} \Big|_e^{e^2}$$

$$= -\frac{1}{2} \left( \frac{1}{2} - 1 \right)$$

$$= \frac{3}{8}$$



$$\begin{aligned}
 \int_0^{\ln 2} 4e^{-\theta} \sinh \theta d\theta &= 2 \int_0^{\ln 2} e^{-\theta} (e^{\theta} - e^{-\theta}) d\theta \\
 &= 2 \int_0^{\ln 2} (1 - e^{-2\theta}) d\theta \\
 &= 2 \left( \theta + \frac{1}{2} e^{-2\theta} \right) \Big|_0^{\ln 2} \\
 &= 2 \left( \ln 2 + \frac{1}{2} e^{-2\ln 2} - \frac{1}{2} \right) \\
 &= 2 \left( \ln 2 + \frac{1}{8} - \frac{1}{2} \right) \\
 &= 2 \ln 2 - \frac{3}{4}
 \end{aligned}$$

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solid washer (x-axis) disk shell  
 Surface, Force.

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Set up the Integration