Solution   Section 2.3 – Probability Rules, Addition Rule and Complements

Exercise

Based on recent results, the probability of someone in the U.S. being injured while using sports or recreation equipment is $\frac{1}{500}$ (based on data from Statistical Abstract of the U.S.). What does it mean when we say that the probability is $\frac{1}{500}$? Is such an injury unusual?

Solution

The probability of being injured while using recreation equipment in $\frac{1}{500}$ means that approximately one injury occurs for every 500 times that recreation equipment is used. The probability is $\frac{1}{500}$ $\underline{=0.002}$ is small; such an injury is considered unusual.

Exercise

When a baby is born, there is approximately a 50−50 chance that the baby is a girl. Indicate the degree of likelihood as a probability value between 0 and 1.

Solution

$$\text{"}50-50\ \textit{chance}\text{"} = 50\% = \frac{50}{100}\ \underline{=0.50}$$

Exercise

When a rolling a single die, there are 6 chances in 36 that the outcome is a 7. Indicate the degree of likelihood as a probability value between 0 and 1.

Solution

$$\frac{6}{36}\ \underline{=0.167}$$

Exercise

Identify probability values

a) What is the probability of an event that is certain to occur?
b) What is the probability of an impossible event?
c) A sample space consists of 10 separate events that are equally likely. What is the probability of each?
d) On a true/false test, what is the probability of answering a question correctly if you make a random guess?

e) On a multiple-choice test with five possible answers for each question, what is the probability of answering a question correctly if you make a random guess?

Solution

a) If event E is certain to occur, then $P(E)=1$

b) If it is not possible for event E to occur, then $P(E)=0$

c) A sample space consists of 10 separate events that are equally likely, then

$$P(\text{of any one of them})=\frac{1}{10}\ \underline{=0.10|}$$

d) $P(\text{answering correctly})=\frac{1}{2}\ \underline{=0.5|}$

e) $P(\text{answering correctly})=\frac{1}{5}\ \underline{=0.2|}$

Exercise

When a couple has 3 children, find the probability of each event.

a) There is exactly one girl. *b)* There are exactly 2 girls. *c)* All are girls

Solution

$$S=\{ggg,\ ggb,\ gbg,\ bgg,\ bbg,\ bgb,\ gbb,\ bbb\}$$

a) $A=\{bbg,\ bgb,\ gbb\}$

$$P(\text{exactly 1 girl})=\frac{3}{8}\ \underline{=0.375|}$$

b) $B=\{ggb,\ gbg,\ bgg\}$

$$P(\text{exactly 2 girls})=\frac{3}{8}\ \underline{=0.375|}$$

c) $C=\{ggg\}$ $\quad P(\text{All girls})=\frac{1}{8}\ \underline{=0.125|}$

Exercise

The 110^{th} Congress of the U.S. included 84 male Senators and 16 female Senators. If one of these Senators is randomly selected, what is the probability that a woman is selected? Does this probability agree with a claim that men and women have the same chance of being elected as Senators?

Solution

Total senators: 84 + 16 = 100 senators.

$$P(\text{selecting women})=\frac{16}{100}\ \underline{=0.16|}$$

No; this probability is too far below 0.50 to agree with the claim that men and women have equal opportunities to become a senator.

Exercise

When Mendel conducted his famous genetics experiments with peas, one sample of offspring consisted of 428 green peas and 152 yellow peas. Based on those results, estimate the probability of getting an offspring pea that is green, Is the result reasonably close to the expected value of $\frac{3}{4}$, as claimed by Mendel?

Solution

Total plants: 428 + 152 = 580 plants.

Let G = getting an offspring pea that is green.

$$P(G)=\frac{428}{580}\ \underline{=0.738|}$$

The result is very close to the $\frac{3}{4}=0.75$ expected by Mendel

Exercise

A single fair die is rolled. Find the probability of each event

a) Getting a 2

b) Getting an odd number

c) Getting a number less than 5

d) Getting a number greater than 2

e) Getting any number except 3

Solution

a) $P=\frac{1}{6}$

b) $P(Odd)=\frac{3}{6}=\frac{1}{2}$

c) $P(<5)=\frac{4}{6}=\frac{2}{3}$

d) $P(>2)=\frac{4}{6}=\frac{2}{3}$

e) $P(no\ 3)=\frac{5}{6}$

Exercise

A jar contains 3 white, 4 orange, 5 yellow, and 8 black marbles. If a marble is drawn at random, find the probability that it is the following.

a) White *b)* Orange *c)* Yellow *d)* Black *e)* Not black

Solution

a) $P(white)=\frac{3}{20}$

b) $P(orange) = \frac{4}{20} \boxed{= \frac{1}{5}}$

c) $P(yellow) = \frac{5}{20} \boxed{= \frac{1}{4}}$

d) $P(black) = \frac{8}{20} \boxed{= \frac{2}{5}}$

e) $P(no\ black) = \frac{12}{20} \boxed{= \frac{3}{5}}$ $\qquad 1 - P(black)$

Exercise

The student sitting next to you in class concludes that the probability of the ceiling falling down on both of you before class ends is 1/2, because there are two possible outcomes - the ceiling will fall or not fall. What is wrong with this reasoning?

Solution

The outcomes are not equally likely.

Exercise

Let consider rolling 2 dice. Find the probabilities of the following events

a) $E =$ Sum of 5 turns up
b) $F =$ a sum that is a prime number greater than 7 turns up

Solution

1 – 1	1 – 2	1 – 3	1 – 4	1 – 5	1 – 6
2 – 1	2 – 2	2 – 3	2 – 4	2 – 5	2 – 6
3 – 1	3 – 2	3 – 3	3 – 4	3 – 5	3 – 6
4 – 1	4 – 2	4 – 3	4 – 4	4 – 5	4 – 6
5 – 1	5 – 2	5 – 3	5 – 4	5 – 5	5 – 6
6 – 1	6 – 2	6 – 3	6 – 4	6 – 5	6 – 6

a) $P(E) = \frac{4}{36} \boxed{= \frac{1}{9}}$

b) $P(F) = \frac{2}{36} \boxed{= \frac{1}{18}}$

Exercise

A poll was conducted preceding an election to determine the relationship between voter persuasion concerning a controversial issue and the area of the city in which the voter lives. Five hundred registered voters were interviewed from three areas of the city. The data are shown below. Compute the probability of having no opinion on the issue or living in the inner city.

Area of city	***Favor***	***Oppose***	***No Opinion***
East	30	40	55
North	25	45	50
Inner	95	65	85

Solution

$$\mathrm{P}(event) = \frac{Total\ Inner + No\ Opinion\ East + No\ Opinion\ North}{500}$$

$$= \frac{95+65+85+55+50}{500}$$

$$= \frac{350}{500}$$

$$\boxed{\approx 0.7}$$

Exercise

Suppose a single fair die is rolled. Use the sample space $S = \{1,2,3,4,5,6\}$ and give the probability of each event.

a) E: the die shows an even number

b) F: the die show a number less than 10

c) G: the die shows an 8

Solution

a) Even number: $E = \{2,4,6\}$

$$P(E) = \frac{n(S)}{n(E)}$$

$$= \frac{3}{6}$$

$$\boxed{= \frac{1}{2}}$$

b) Number less than 10

$F = \{1,2,3,4,5,6\}$

$P(F) = \frac{6}{6}\ \boxed{= 1}$

c) Die shows an 8

$\boxed{P(G) = 0}$ ***Impossible***

Exercise

A solitaire game was played 500 times. Among the 500 trials, the game was won 77 times. (The results are from the solitaire game, and the Vegas rules of "draw 3" with \$52 bet and a return of \$5 per card are used). Based on these results, find the odds against winning.

Solution

Odd against winning are: $\dfrac{P(\text{not winning})}{P(\text{winning})} = \dfrac{\frac{423}{500}}{\frac{77}{500}} = \dfrac{423}{77}$ *or* $423:77$

Which is approximately 5.5:1 ***or*** 11:2

Exercise

A roulette wheel has 38 slots. One slot is 0, another is 00, and the others are numbered 1 through 36, respectively. You place a bet that the outcome is an odd number.

a) What is your probability of winning?

b) What are the actual odds against winning?

c) When you bet that the outcome is an odd number, the payoff odds are 1:1. How much profit do you make if you bet $18 and win?

d) How much profit would you make on the $18 bet if you could somehow convince the casino to change its payoff odds so that they are the same as the actual odds against winning?

Solution

Out of 38 slots, 18 numbers are odd.

Let W = outcome is an odd number occurs

a) $P(W)=\frac{18}{38}\ \underline{=0.474}$

b) Odds against: $W=\dfrac{P(not\ W)}{P(W)}=\dfrac{\frac{20}{38}}{\frac{18}{38}}=\dfrac{20}{18}=\dfrac{10}{9}$ *or* $10:9$

c) If you payoff odds are 1:1, if you bet $18 and win, you get back 18 + 18 = $36 and your profit is $18.

d) If you payoff odds are 10:9 (odds against), a win get back your bet $10 for every $9 bet. If you bet $18 and win, you get back 18 + 2(10) = $38 and your profit is $20.

Exercise

Women have a 0.25% rate of red/green color blindness. If a woman is randomly selected, what is the probability that she does not have red/green color blindness?

Solution

Let W = a selected woman has red/green color blindness.

$P(W)=0.25\%\ \underline{=0.0025}$

$$P(does\ not\ W)=P(\overline{W})$$
$$=1-0.0025$$
$$\underline{=0.9975}$$

Exercise

A pew Research center poll showed that 79% of Americans believe that it is morally wrong to not report all income on tax returns. What is the probability that an American does not have that belief?

Solution

Let A = a American believes is morally wrong to not report all income.

$P(A) = 0.79$

$$P(does\ not\ A) = P(\bar{A})$$
$$= 1 - 0.79$$
$$\underline{= 0.21}$$

Exercise

When the author observed a sobriety checkpoint conducted by the Dutchess County Sheriff Department, he saw that 676 drivers were screened and 6 were arrested for driving while intoxicated. Based on those results, we can estimate that $P(I) = 0.00888$, where *I* denotes the event of screening a driver and getting someone who is intoxicated. What does $P(\bar{I})$ denote and what is its value?

Solution

$P(\bar{I})$ is the probability that a screened driver is not intoxicated

$$P(\bar{I}) = 1 - P(I)$$
$$= 1 - 0.00888$$
$$\underline{= 0.99112}$$

Exercise

Use the polygraph test data

	No (Did Not Lie)	***Yes*** (Lied)
Positive test result	15 (*false positive*)	42 (*true positive*)
Negative test result	32 (*true negative*)	9 (*false negative*)

a) If one of the test subjects is randomly selected, find the probability that the subject had a positive test result or did not lie

b) If one of the test subjects is randomly selected, find the probability that the subject did not lie

c) If one of the test subjects is randomly selected, find the probability that the subject had a true negative test result

d) If one of the test subjects is randomly selected, find the probability that the subject had a negative test result or lied.

Solution

From the table:

$$P(Positive) = \frac{57}{98} \quad \to P(\bar{P}) = \frac{41}{98}$$

$$P(Lie) = \frac{51}{98} \quad \to P(\bar{L}) = \frac{47}{98}$$

a) $P(P \text{ or } \bar{L}) = P(P) + P(\bar{L}) - P(P \text{ and } \bar{L})$

$$= \frac{57}{98} + \frac{47}{98} - \frac{15}{98}$$

$$= \frac{89}{98}$$

$$\underline{= 0.908}$$

b) $P(\bar{L}) = \frac{47}{98} \ \underline{= 0.480}$

c) $P(\bar{L} \text{ and } \bar{P}) = \frac{32}{98} \ \underline{= 0.327}$

d) $P(\bar{P} \text{ or } L) = P(\bar{P}) + P(L) - P(\bar{P} \text{ and } L)$

$$= \frac{41}{98} + \frac{51}{98} - \frac{9}{98}$$

$$= \frac{83}{98}$$

$$\underline{= 0.847}$$

Exercise

Use the data

Was the challenge to the call successful?		
	Yes	***No***
Men	201	288
Women	126	224

a) If S denotes the event of selecting a successful challenge, find $P(\bar{S})$

b) If M denotes the event of selecting a challenge made by a man, find $P(\bar{M})$

c) Find the probability that the selected challenge was made by a man or was successful.

d) Find the probability that the selected challenge was made by a woman or was successful.

e) Find $P(\text{challenge was made by a man or was not successful})$

f) Find $P(\text{challenge was made by a woman or was not successful})$

Solution

$$\text{Total people} = 201 + 126 + 288 + 224 = 839$$

a) $P(\bar{S}) = \frac{288+224}{839} = \frac{512}{839} \ \underline{= 0.610}$

b) $P(\bar{M}) = \frac{126+224}{839} = \frac{350}{839} \ \underline{= 0.417}$

c) $P(M \text{ or } S) = P(M) + P(S) - P(M \text{ and } S)$

$$= \frac{489}{839} + \frac{327}{839} - \frac{201}{839}$$

$\underline{=0.733}$

d) $P(\bar{M} \text{ or } S) = P(\bar{M}) + P(S) - P(\bar{M} \text{ and } S)$

$$= \frac{350}{839} + \frac{327}{839} - \frac{126}{839}$$

$\underline{=0.657}$

e) $P(M \text{ or } \bar{S}) = P(M) + P(\bar{S}) - P(M \text{ and } \bar{S})$

$$= \frac{489}{839} + \frac{512}{839} - \frac{288}{839}$$

$\underline{=0.850}$

f) $P(\bar{M} \text{ or } \bar{S}) = P(\bar{M}) + P(\bar{S}) - P(\bar{M} \text{ and } \bar{S})$

$$= \frac{350}{839} + \frac{512}{839} - \frac{224}{839}$$

$\underline{=0.760}$

Exercise

Refer to the table below

Age

	18 – 21	22 – 29	30 – 39	40 – 49	50 – 59	60 and over
Responded	73	255	245	136	138	202
Refused	11	20	33	16	27	49

a) What is the probability that the selected person refused to answer? Does that probability value suggest that refusals are a problem for pollsters? Why or why not?

b) A pharmaceutical company is interested in opinions of the elderly, because they are either receiving Medicare or will receive it soon. What is the probability that the selected subject is someone 60 and over who responded?

c) What is the probability that the selected person responded or is in the 18–21 age bracket?

d) What is the probability that the selected person refused or is over 59 years of age?

e) A market researcher is interested in responses, especially from those between the ages of 22 and 39, because they are the people more likely to make purchases. Find the probability that a selected responds or is aged between the ages of 22 and 39.

f) A market researcher is not interested in refusals or subjects below 22 years of age or over 59. Find the probability that the selected person refused to answer or is below 22 or is older than 59.

Solution

Let: $Y =$ there is a response $=73+255+245+136+138+202=1049$ $\quad$ $N =$ there is a refusal $=11+20+33+16+27+49=156$

$A =$ age is 18 – 21 $\quad$ $B =$ age is 22 – 29 $\quad$ $C =$ age is 30 – 39

$D =$ age is 40 – 49 $\quad$ $E =$ age is 50 – 59 $\quad$ $F =$ age is 60+

a) $P(N)=\frac{156}{1205}$ $\boxed{=0.129}$

b) $P(F \text{ and } Y)=\frac{202}{1205}$ $\boxed{=0.168}$

c) $P(Y \text{ or } A)=P(Y)+P(A)-P(Y \text{ and } S)$

$$=\frac{1049}{1205}+\frac{84}{1205}-\frac{73}{1205}$$

$$\boxed{=0.880}$$

d) $P(N \text{ or } F)=P(N)+P(F)-P(N \text{ and } F)$

$$=\frac{156}{1205}+\frac{251}{1205}-\frac{49}{1205} \quad =\frac{358}{1205}$$

$$\boxed{=0.297}$$

e) $P(\text{responds or the ages between } 22 \text{ and } 39)=P(Y \text{ or } B \text{ or } C)$

Use the intuitive approach rather than the formal addition rule

$P(Y \text{ or } B \text{ or } C)=P(Y)+P(B \text{ and } N)+P(C \text{ and } N)$

$$=\frac{1049}{1205}+\frac{20}{1205}+\frac{33}{1205} \quad =\frac{1102}{1205}$$

$$\boxed{=0.915}$$

f) Use the intuitive approach rather than the formal addition rule

$P(N \text{ or } A \text{ or } F)=P(N)+P(A \text{ and } Y)+P(F \text{ and } Y)$

$$=\frac{156}{1205}+\frac{73}{1205}+\frac{202}{1205} \quad =\frac{431}{1205}$$

$$\boxed{=0.368}$$

Exercise

Two dice are rolled. Find the probabilities of the following events.

a) The first die is 3 or the sum is 8 *b)* The second die is 5 or the sum is 10.

Solution

a) $P(3 \text{ or sum is } 8)=P(3)+P(\text{sum } 8)-P(3 \text{ and sum } 8)$

$$=\frac{6}{36}+\frac{5}{36}-\frac{1}{36} \quad =\frac{10}{36}$$

$$\boxed{=\frac{5}{18}}$$

b) $P(5 \text{ or sum } 10)=P(5)+P(\text{sum } 10)-P(5 \text{ and sum } 10)$

$$=\frac{6}{36}+\frac{3}{36}-\frac{1}{36} \quad =\frac{8}{36}$$

$$\boxed{=\frac{2}{9}}$$

Exercise

One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) A 9 or 10
b) A red card or a 3
c) A 9 or a black 10
d) A heart or a black card
e) A face card or a diamond

Solution

a) $P(9 \text{ or } 10) = \frac{8}{52} = \frac{2}{13}$

b) $P(red \text{ or } 3) = \frac{28}{52} = \frac{7}{13}$

c) $P(9 \text{ or } black\text{-}10) = \frac{6}{52} = \frac{3}{26}$

d) $P(heart \text{ or } black) = \frac{39}{52} = \frac{3}{4}$

e) $P(face \text{ or } diamond) = \frac{22}{52} = \frac{11}{26}$

Exercise

One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) Less than a 4 (count aces as ones)
b) A diamond or a 7
c) A black card or an ace
d) A heart or a jack
e) A red card or a face card

Solution

a) $P(<4) = P(ace, 2, 3) = \frac{12}{52} = \frac{3}{13}$

b) $P(diamond \text{ or } 7) = \frac{16}{52} = \frac{4}{13}$

c) $P(black \text{ or } ace) = \frac{28}{52} = \frac{7}{13}$

d) $P(heart \text{ or } jack) = \frac{16}{52} = \frac{4}{13}$

e) $P(red \text{ or } face) = \frac{32}{52} = \frac{8}{13}$

Exercise

Pam invites relatives to a party: her mother, 2 aunts, 3 uncles, 2 brothers, 1 male cousin, and 4 female cousins. If the chances of any one guest first equally likely, find the probabilities that the first guest to arrive is as follows.

a) A brother or an uncle
b) A brother or a cousin
c) A brother or her mother
d) An uncle or a cousin
e) A male or a cousin
f) A female or a cousin

Solution

a) $P(\textit{brother or uncle}) = \boxed{\frac{5}{13}}$

b) $P(\textit{brother or cousin}) = \boxed{\frac{7}{13}}$

c) $P(\textit{brother or mother}) = \boxed{\frac{3}{13}}$

d) $P(\textit{uncle or cousin}) = \boxed{\frac{8}{13}}$

e) $P(\textit{male or cousin}) = \boxed{\frac{10}{13}}$

f) $P(\textit{brother or cousin}) = \boxed{\frac{8}{13}}$

Exercise

Suppose $P(E) = 0.26$, $P(F) = 0.41$, and $P(E \cap F) = 0.16$. Find the following

a) $P(E \cup F)$
b) $P(E' \cap F)$
c) $P(E \cap F')$
d) $P(E' \cup F')$

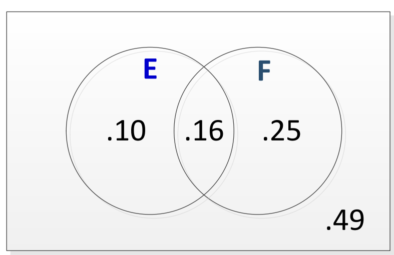
Solution

a) $P(E \cup F) = .1 + .16 + .25 = .51$

b) $P(E' \cap F) = .25$

c) $P(E \cap F') = .10$

d) $P(E' \cup F') = .74 + .59 - .49 = .84$



Exercise

Suppose $P(E)=0.42$, $P(F)=0.35$, and $P(E\cup F)=0.59$. Find the following

a) $P(E'\cap F')$

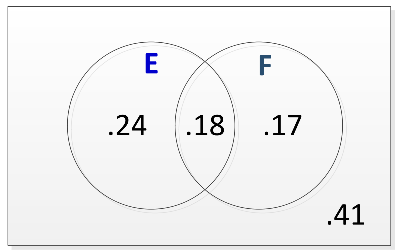
b) $P(E'\cup F')$

c) $P(E'\cup F)$

d) $P(E\cap F')$

Solution

$$P(E\cap F)=P(E)+P(F)-P(E\cup F)$$
$$=.42+.35-.59$$
$$=.18$$



a) $P(E'\cap F')\ \underline{=.41}$

b) $P(E'\cup F')=1-.18\ \underline{=.82}$

c) $P(E'\cup F)=.17+.41+.18\ \underline{=.76}$

d) $P(E\cap F')\ \underline{=.24}$

Exercise

From survey involving 1,000 people in the certain city, it was found that 500 people had tried a certain brand of diet cola, 600 had tried a certain brand of regular cola, and 200 had tried both types of cola. If a resident of the city is selected at random, what is the empirical probability that

a) The resident has not tried either cola? What are the empirical odds for this event?

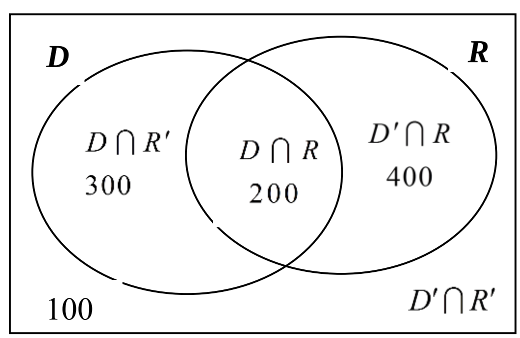
b) The resident has tried the diet or has not tried the regular cola? What are the empirical odds against this event?

Solution

a) $n(S)=1000 \qquad D\cap R=200$

$D\cap R'=300 \qquad D'\cap R=400$

$$P(neither\ \text{D}\ or\ R)=P(D'\cap R')$$
$$=\frac{100}{1000}$$
$$\underline{=.1}$$



$$P(E')=1-.1=0.9$$

Odds for $\boldsymbol{E}$: $\dfrac{P(E)}{P(E')}=\dfrac{.1}{.9}=\dfrac{1}{9}$ *or* $\boxed{1:9}$

b) $P(E)=P(D\cup R')$
$$=P(D)+P(R')-P(D\cap R')$$

$$= \frac{500}{1000} + \frac{400}{1000} - \frac{300}{1000}$$

$$\underline{= .6}$$

$$\Rightarrow P(E') = 1 - .6 = .4$$

Against Odds for P(E): $\frac{P(E)}{P(E')} = \frac{.4}{.6} = \frac{2}{3} \quad or \quad \boxed{2:3}$

Exercise

In a poll, respondents were asked whether they had ever been in a car accident. 329 respondents indicated that they had been in a car accident and 322 respondents said that they had not been in a car accident. If one of these respondents is randomly selected, what is the probability of getting someone who has been in a car accident?

Solution

$$\frac{329}{329+322} \underline{= 0.505}$$

Exercise

Refer to the table which summarizes the results of testing for a certain disease

	Positive Test Result	***Negative Test Result***
Subject has the disease	114	5
Subject does not have the disease	12	177

If one of the results is randomly selected, what is the probability that it is a false negative (test indicates the person does not have the disease when in fact they do)?. What is the probability suggest about the accuracy of the test?

Solution

Person does not have the disease when in fact they do $\Rightarrow$ Person has the disease & Negative

$$P = \frac{5}{114+5+12+177} \underline{= 0.016}$$

The probability of this error is low so the test is fairly accurate.

Exercise

In a certain town, 2% of people commute to work by bicycle. If a person is selected randomly from the town, what are the odds against selecting someone who commutes by bicycle?

Solution

$$P(Event) = 2\% = 0.02; \quad P\left(\bar{E}\right) = .98$$

$$\frac{P(\bar{E})}{P(E)} = \frac{.98}{.02} = 49$$

odds Against $\underline{49:1}$

Exercise

Suppose you are playing a game of chance, if you bet \$4 on a certain event, you will collect \$176 (including your \$4 bet) if you win. Find the odds used for determining the payoff.

Solution

The amount that you win: $P(E) = 176 - 4 = 172$

You loose: $P(\bar{E}) = 4$

$$\frac{P(E)}{P(\bar{E})} = \frac{172}{4} = 43$$

odds $\underline{43:1}$

Exercise

The odds in favor of a particular horse winning a race are 4:5.

a) Find the probability of the horse winning.

b) Find the odds against the horse winning.

Solution

a) $P(E) = \dfrac{a}{a+b} = \dfrac{4}{4+5} \underline{= \dfrac{4}{9}}$

b) The odds against the horse winning $\boxed{5:4}$

Exercise

Consider the sample space of equally likely events for the rolling of a single fair die.

a) What is the probability of rolling an odd number **and** a prime number?

b) What is the probability of rolling an odd number **or** a prime number?

Solution

a) $odd = \{1,\ 3,\ 5\} \quad prime = \{3,\ 5\}$

$$P(odd \cap prime) = \frac{n(A \cap B)}{n(S)} = \frac{2}{6} \underline{= \frac{1}{3}}$$

b) $P(odd \cup prime) = \dfrac{n(A \cup B)}{n(S)} = \dfrac{4}{6} \underline{= \dfrac{2}{3}}$

Exercise

Suppose that 2 fair Dice are rolled

a) What is the probability of that a sum of 2 or 3 turns up?

b) What is the probability of that both dice turn up the same or that a sum greater than 8 turns up?

Solution

a)
$$\begin{aligned} P(\sum=2 \text{ or } 3 \text{ turns up}) &= P(\sum=2) + P(\sum=3) \\ &= \frac{1}{36} + \frac{2}{36} \\ &= \frac{3}{36} \\ &= \frac{1}{12} \end{aligned}$$

b)
$$\begin{aligned} P(\text{same or } \sum > 8) &= P(\text{same } \cup \sum > 8) \\ &= P(\text{same}) + P(\sum > 8) - P(\text{same } \cap \sum > 8) \\ &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\ &= \frac{7}{18} \end{aligned}$$

Exercise

A single card is drawn from an ordinary of 52 cards. Calculate the probabilities of and odds for each event

a) A face card or a club is drawn

b) A king or a heart is drawn

c) A black card or an ace is drawn

d) A heart or a number less than 7 (count an ace as 1) is drawn.

Solution

a)
$$\begin{aligned} \Pr(\text{Face or Club}) &= \Pr(F \cup C) \\ &= P(F) + P(C) - P(F \cap C) \\ &= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} \\ &= \frac{11}{26} \end{aligned}$$

$$P\left[(F \cup C)'\right] = 1 - \frac{11}{26} = \frac{15}{26}$$

$$\text{Odds for } F \cup C = \frac{\frac{11}{26}}{\frac{15}{26}} = \frac{11}{15} \qquad \boxed{11:15}$$

b) $\Pr(\text{King or Heart}) = \text{P}(K \cup H)$

$$= P(K) + P(H) - P(K \cap H)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$\boxed{= \frac{4}{13}}$$

$$P\left[(K \cup H)'\right] = 1 - \frac{4}{13} \boxed{= \frac{9}{13}}$$

Odds for $K \cup H = \dfrac{\frac{4}{13}}{\frac{9}{13}} \boxed{= \frac{4}{9}} \quad \boxed{4:9}$

c) $\Pr(\textit{Black card or Ace}) = \mathrm{P}(B \cup A)$

$$= P(B) + P(A) - P(B \cap A)$$

$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$$

$$= \frac{28}{52}$$

$$\boxed{= \frac{7}{13}}$$

$$P\left[(B \cup A)'\right] = 1 - \frac{7}{13} \boxed{= \frac{6}{13}}$$

Odds for $B \cup A = \dfrac{\frac{7}{13}}{\frac{6}{13}} \boxed{= \frac{7}{6}} \quad \boxed{7:6}$

d) $\Pr(\textit{Heart or } < 7) = \mathrm{P}(H \cup < 7)$

$$= P(B) + P(A) - P(B \cap A)$$

$$= \frac{13}{52} + \frac{6*4}{52} - \frac{6}{52}$$

$$\boxed{= \frac{31}{52}}$$

$$P\left[(H \cup \# < 7)'\right] = 1 - \frac{31}{52} \boxed{= \frac{21}{52}}$$

Odds for $B \cup A = \dfrac{\frac{31}{52}}{\frac{21}{52}} \boxed{= \frac{31}{21}} \quad \boxed{31:21}$

Exercise

What is the probability of getting at least 1 black card in a 7-card hand dealt from a standard 52-card deck?

Solution

There are 26 black cards.

Let A = "at least 1 black card in a 7-card hand dealt"

A' = "0 black cards in a 7-card hand dealt"

$$n(A') = C_{26,7}$$

$$n(S) = C_{52,7}$$

$$\begin{aligned} P(A) &= 1 - P(A') \\ &= 1 - \frac{n(A')}{n(S)} \\ &= 1 - \frac{C_{26,7}}{C_{52,7}} \\ &= 1 - .005 \\ &= .995 \end{aligned}$$

Exercise

What is the probability that a number selected at random from the first 600 positive integers is (exactly) divisible by 6 or 9?

Solution

Let $A =$ "Number divisible by 6"

$B =$ "Number divisible by 9"

A number divisible by 6 $\Rightarrow n(A) = \frac{600}{6} = 100$

A number divisible by 9 $\Rightarrow n(B) = \frac{600}{9} \approx 66$

A number divisible by 6 and by 9 $\rightarrow 18k \quad \Rightarrow n(A \cap B) = \frac{600}{18} \approx 33$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{100}{600} + \frac{66}{600} - \frac{33}{600} \\ &= \frac{133}{600} \\ &\approx 0.2217 \end{aligned}$$

Exercise

What is the probability that a number selected at random from the first 1,000 positive integers is (exactly) divisible by 6 or 8?

Solution

Let $A=$ "Number divisible by 6"

$B=$ "Number divisible by 8"

A number divisible by 6 $\Rightarrow n(A)=\frac{1{,}000}{6}=166$

A number divisible by 8 $\Rightarrow n(B)=\frac{1{,}000}{8}\approx 125$

A number divisible by 6 and by 8 $\rightarrow 24k \quad \Rightarrow n(A\cap B)=\frac{1{,}000}{24}\approx 41$

$$
\begin{aligned}
P(A\cup B) &= P(A)+P(B)-P(A\cap B)\\
&= \frac{166}{1000}+\frac{125}{1000}-\frac{41}{1000}\\
&= \frac{250}{1000}\\
&\approx 0.25
\end{aligned}
$$

Exercise

From a survey involving 1,000 students at a large university, a market research company found that 750 students owned stereos, 450 owned cars, and 350 owned cars and stereos. If a student at the university is selected at random, what is the (empirical) probability that

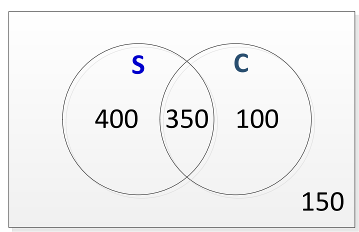
a) The student owns either a car or a stereo?

b) The student owns neither a car nor a stereo?

Solution

Let $S=$ "Number of stereos"

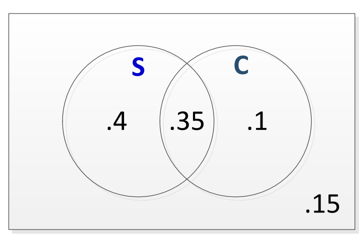
$C=$ "Number of cars"



a) $P(S\cup C)=P(S)+P(C)-P(S\cap C)$

$$
\begin{aligned}
&= \frac{750}{1000}+\frac{450}{1000}-\frac{350}{1000}\\
&= \frac{850}{1000}\\
&\approx 0.85
\end{aligned}
$$

b) $P(S'\cap C')=.15$



Exercise

In order to test a new car, an automobile manufacturer wants to select 4 employees to test drive the car for 1 year. If 12 management and 8 union employees volunteer to be test drivers and the selection is made at random, what is the probability that at least 1 union employee is selected.

Solution

Let $A =$ "at least 1 union employee is selected"

$A' =$ "no union employee is selected"

$\Rightarrow n(A') = C_{12,4}, \quad n(S) = C_{20,4}$

$$P(A) = 1 - P(A')$$

$$= 1 - \frac{C_{12,4}}{C_{20,4}}$$

$$\approx 0.90$$

Exercise

A shipment of 60 inexpensive digital watches, including 9 that are defective, is sent to a department store. The receiving department selects 10 at random for testing and rejects the whole shipment if 1 or more in the sample are found defective. What is the probability that the shipment will be rejected?

Solution

The sample space: $S = C_{60,10}$

Let $E =$ "Event that contains at least 1 defective watch".

$E' =$ "Event that contains no defective watches".

$n(E') = C_{51,10}$

Probability that the shipment will be rejected:

$$P(E) = 1 - P(E')$$

$$= 1 - \frac{n(E')}{n(S)}$$

$$= 1 - \frac{C_{51,10}}{C_{60,10}}$$

$$= .83$$

Exercise

If you bet $5 on the number 13 in roulette, your probability of winning is $\frac{1}{38}$ and the payoff odds are given by the casino as 35:1.

a) Find the actual odds against the outcome of 13.
b) How much net profit would you make if you win by betting on 13?
c) If the casino was not operating for profit, and the payoff odds were changed to match the actual odds against 13, how much would you win if the outcome were 13?

Solution

a) With odds: $P(13)=\frac{1}{38}$ and $P(not\ 13)=\frac{37}{38}$

Actual odds against 13 $\frac{P(not\ 13)}{P(13)}=\frac{37/38}{1/38}=\frac{37}{1}$ or 37:1

b) Because the payoffs odds against 13 are 35:1, we have:

35:1 = (*net profit*) : (*amount bet*)

So there is a $35 profit for each $1 bet. For $5 bet, the net profit is $5\times 35=\$175$.
The winning bettor would collect $175 plus the original $5 bet. That is, the total amount collected would be $180, for the net profit of $175.

c) If the casino were not operating for profit, the payoff odds would be equal to the actual odds against the outcome of 13, or 37:1. So there is a net profit of $37 for each $1 bet.
For a $5 bet the net profit would be $185. (The casino makes its profit by paying only $175 instead of the $185 that would be paid with a roulette game that is fair instead of favoring the casino.)