Section 3.6 – Solving Trigonometry Equations

Addition Property of Equality

For any three algebraic expressions *A*, *B*, and *C*

If $A = B$

Then $A + C = B + C$

Multiplication Property of Equality

For any three algebraic expressions *A*, *B*, and *C*, with $C \neq 0$

If $A = B$

Then $AC = BC$

Example

Solve $2\sin x - 1 = 0$

Solution

$2\sin x = 1$

$\sin x = \frac{1}{2}$

Solutions between (0° and 360°) $x = 30°$ *or* $150°$

Solutions between (0 and 2π) $x = \frac{\pi}{6}$ *or* $\frac{5\pi}{6}$

All solutions: $x = 30° + 360°k$ *or* $150° + 360°k$

$x = \frac{\pi}{6} + 2k\pi$ *or* $\frac{5\pi}{6} + 2k\pi$

Example

Solve $2\sin\theta - 3 = 0$, *if* $0° \leq \theta < 360°$

Solution

$2\sin\theta = 3$

$\sin\theta = \frac{3}{2}$

$\sin\theta$ *can't be greater than 1*

No solution

Example

Solve $\cos(A-25°) = -\frac{1}{\sqrt{2}}$

Solution

$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

$-\frac{1}{\sqrt{2}}$ *is negative* $\rightarrow$ *cosine is in QII or QIII.*

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = 135° \quad or \quad 225°$

$\cos(A-25°) = -\frac{1}{\sqrt{2}} = \cos(135°)$ $\qquad$ $\cos(A-25°) = -\frac{1}{\sqrt{2}} = \cos(225°)$

$A-25° = 135° + 360°k$ $\qquad$ $A-25° = 225° + 360°k$

$A = 25° + 135° + 360°k$ $\qquad$ $A = 25° + 225° + 360°k$

$A = 160° + 360°k$ $\qquad$ $A = 250° + 360°k$

Example

Solve $3\sin\theta - 2 = 7\sin\theta - 1 \quad if \quad 0° \le \theta < 360°$

Solution

$3\sin\theta - 7\sin\theta = 2 - 1$

$-4\sin\theta = 1$

$\sin\theta = -\frac{1}{4}$

$\hat{\theta} = \sin^{-1}\left(-\frac{1}{4}\right) = 14.5°$

Negative sign $\rightarrow$ *sine is in QIII or QIV*

$\theta = 14.5° + 180°$ $\qquad$ $\theta = 360° - 14.5°$

$\theta = 194.5°$ $\qquad$ $\theta = 345.5°$

Example

Solve $2\sin^2\theta + 2\sin\theta - 1 = 0$ *if* $0 \le \theta < 2\pi$

Solution

$$\sin\theta = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{12}}{4}$$

$$= \frac{-2 \pm 2\sqrt{3}}{4}$$

$$= \frac{2(-1 \pm \sqrt{3})}{4}$$

$$= \frac{-1 \pm \sqrt{3}}{2}$$

$\sin\theta = \frac{-1-\sqrt{3}}{2} < -1$

$\sin\theta = \frac{-1+\sqrt{3}}{2} = 0.3661$

$\theta = \sin^{-1}(0.3661)$

$\hat{\theta} = 0.37$ *(QI or QII)*

$\theta = 0.37 \qquad \theta = \pi - 0.37 = 2.77$

Example

Solve: $2\cos x - 1 = \sec x$, *if* $0 \le x < 2\pi$

Solution

$$2\cos x - 1 = \frac{1}{\cos x}$$

$$2\cos^2 x - \cos x = 1$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$2\cos x + 1 = 0$ $\qquad$ $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\qquad$ $x = 0$

The solutions are: $\boxed{0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}}$

Example

Solve: $\cos 2\theta + 3\sin\theta - 2 = 0$, *if* $0° \le \theta < 360°$

Solution

$1 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$-2\sin^2\theta + 3\sin\theta - 1 = 0$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0$	$\sin\theta - 1 = 0$
$\sin\theta = \frac{1}{2}$	$\sin\theta = 1$
$\theta = 30°, 150°$	$\theta = 90°$

The solutions are: $\boxed{\theta = 30°,\ 90°,\ 150°}$

Example

Solve: $4\cos^2 x + 4\sin x - 5 = 0$, *if* $0 \le x < 2\pi$

Solution

$4(1 - \sin^2 x) + 4\sin x - 5 = 0$

$4 - 4\sin^2 x + 4\sin x - 5 = 0$

$-4\sin^2 x + 4\sin x - 1 = 0$

$4\sin^2 x - 4\sin x + 1 = 0$

$(2\sin x - 1)^2 = 0$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

The solutions are: $\boxed{x = \frac{\pi}{6},\ \frac{5\pi}{6}}$

Example

Solve: $\sin 2\theta + \sqrt{2}\cos\theta = 0$, *if* $0° \le \theta < 360°$

Solution

$2\sin\theta\cos\theta + \sqrt{2}\cos\theta = 0$

$\cos\theta\left(2\sin\theta + \sqrt{2}\right) = 0$

$\cos\theta = 0$	$2\sin\theta + \sqrt{2} = 0$
$\cos\theta = 0$	$\sin\theta = -\frac{\sqrt{2}}{2}$
	$\hat{\theta} = \sin^{-1}\frac{1}{\sqrt{2}} = 45°$
$\theta = 90°, 270°$	$\theta = 225°, 315°$

Example

Solve: $\sin\theta - \cos\theta = 1$, *if* $0 \le \theta < 2\pi$

Solution

$\sin\theta = \cos\theta + 1$

$\sin^2\theta = (\cos\theta + 1)^2$

$1 - \cos^2\theta = \cos^2\theta + 2\cos\theta + 1$

$0 = \cos^2\theta + 2\cos\theta + 1 - 1 + \cos^2\theta$

$0 = 2\cos^2\theta + 2\cos\theta$

$2\cos^2\theta + 2\cos\theta = 0$

$2\cos\theta(\cos\theta + 1) = 0$

$2\cos\theta = 0$	$\cos\theta + 1 = 0$
$\cos\theta = 0$	$\cos\theta = -1$
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	$\theta = \pi$

Check

$\theta = \frac{\pi}{2}$	$\theta = \frac{3\pi}{2}$	$\theta = \pi$
$\sin\frac{\pi}{2} - \cos\frac{\pi}{2} \stackrel{?}{=} ?1$	$\sin\frac{3\pi}{2} - \cos\frac{3\pi}{2} = 1$	$\sin\pi - \cos\pi = ?1$
$1 - 0 = 1$	$-1 - 0 = 1$	$0 - (-1) = 1$
	(False statement)	

The solutions are: $\boxed{\frac{\pi}{2},\ \pi}$

Exercises *Section* 3.6 – Solving Trigonometry Equations

1. Solve $2\cos\theta+\sqrt{3}=0$ *if* $0°\le\theta<360°$

2. Solve $5\cos t+\sqrt{12}=\cos t$ *if* $0\le t<2\pi$

3. Solve $\tan\theta-2\cos\theta\tan\theta=0$ *if* $0°\le\theta<360°$

4. Solve $2\sin^2\theta-2\sin\theta-1=0$ *if* $0°\le\theta<360°$

5. Solve: $4\cos\theta-3\sec\theta=0$ *if* $0°\le\theta<360°$

6. Solve: $2\sin^2 x-\cos x-1=0$ *if* $0\le x<2\pi$

7. Solve: $\sin\theta-\sqrt{3}\cos\theta=1$ *if* $0°\le\theta<360°$

8. Solve: $7\sin^2\theta-9\cos 2\theta=0$ *if* $0°\le\theta<360°$

9. Solve $2\cos^2 t-9\cos t=5$ *if* $0\le t<2\pi$

10. Solve $\sin\theta\tan\theta=\sin\theta$ *if* $0°\le\theta<360°$

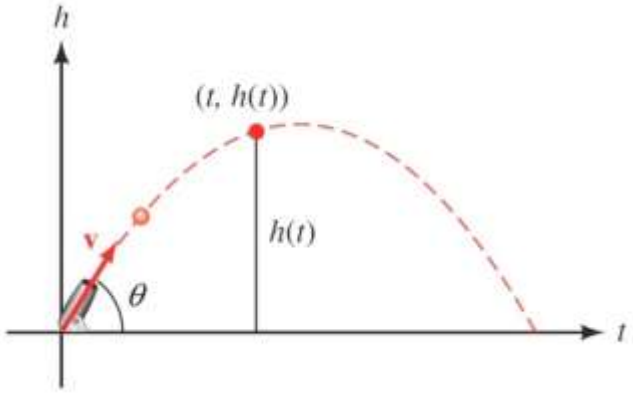
11. Solve $\tan^2 x+\tan x-2=0$ *if* $0\le x<2\pi$

12. Solve $\tan x+\sqrt{3}=\sec x$ *if* $0\le x<2\pi$

13. Solve $\cos\left(A-\frac{\pi}{9}\right)=-\frac{1}{2}$

14. If a projectile (such as a bullet) is fired into the air with an initial velocity ***v*** at an angle of elevation θ, then the height h of the projectile at time t is given by:

$$h(t)=-16t^2+vt\sin\theta$$



a) Give the equation for the height, if ***v*** is 600 *ft./sec* and θ = 45°.

b) Use the equation in part (*a*) to find the height of the object after $\sqrt{3}$ seconds.

c) Find the angle of elevation of θ of a rifle barrel, if a bullet fired at 1,500 ft./sec takes 3 seconds to reach a height of 750 feet. Give your answer in the nearest of a degree.