**Solving Special Quadratic Functions**

$$ax^2+bx+c=0$$

If $\boldsymbol{a+b+c=0} \Rightarrow x=1,\ \frac{c}{a}$

***Proof***

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad a+b+c=0 \ \rightarrow\ b=-a-c$$

$$=\frac{-(-a-c)\pm\sqrt{(-a-c)^2-4ac}}{2a}$$

$$=\frac{a+c\pm\sqrt{a^2+2ac+c^2-4ac}}{2a}$$

$$=\frac{a+c\pm\sqrt{a^2-2ac+c^2}}{2a}$$

$$=\frac{a+c\pm\sqrt{(a-c)^2}}{2a}$$

$$=\frac{a+c\pm(a-c)}{2a}$$

$$\underline{\left|x_1=\frac{a+c+(a-c)}{2a}\right.} \qquad \underline{\left|x_2=\frac{a+c-(a-c)}{2a}\right.}$$

$$=\frac{a+c+a-c}{2a} \qquad =\frac{a+c-a+c}{2a}$$

$$=\frac{2a}{2a} \qquad \frac{2c}{2a}$$

$$\underline{=1\left|\right.} \qquad \underline{=\frac{c}{a}\left|\right.}$$

***Example***

$$2x^2+x-3=0$$

$$2+1-3=0$$

$$\underline{\Rightarrow x=1,\ -\frac{3}{2}\left|\right.}$$

$$ax^2+bx+c=0$$

If $\boldsymbol{a - b + c = 0} \Rightarrow x=-1,\ -\frac{c}{a}$

***Proof***

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad a-b+c=0 \ \rightarrow\ b=a+c$$

$$=\frac{-(a+c)\pm\sqrt{(a+c)^2-4ac}}{2a}$$

$$=\frac{-a-c\pm\sqrt{a^2+2ac+c^2-4ac}}{2a}$$

$$=\frac{-a-c\pm\sqrt{a^2-2ac+c^2}}{2a}$$

$$=\frac{-a-c\pm\sqrt{(a-c)^2}}{2a}$$

$$=\frac{-a-c\pm(a-c)}{2a}$$

$$\begin{cases} \underline{\left|x_1=\frac{-a-c+(a-c)}{2a}=\frac{-a-c+a-c}{2a}=\frac{2c}{2a}\ =-\frac{c}{a}\right|} \\ \underline{\left|x_2=\frac{-a-c-(a-c)}{2a}=\frac{-a-c-a+c}{2a}=\frac{-2a}{2a}\ =-1\right|} \end{cases}$$

***Example***

$$2x^2-x-3=0$$

$$2-(-1)-3=0$$

$$\Rightarrow \underline{\left|x=-1,\ \frac{3}{2}\right|}$$