Solution *Section* 2.5 – Numerical Integration

Exercise

Find the Midpoint Rule approximations to: $\int_0^1 \sin \pi x \, dx$ *using* $n = 6$ *subintervals*

Solution

$\Delta x = \frac{1-0}{6} = \frac{1}{6}$ $\qquad \Delta x = \frac{b-a}{n}$

$x_k = a + k\Delta x$

$x_0 = 0$

$x_1 = 0 + \frac{1}{6} = \frac{1}{6}$

$x_2 = 0 + 2\left(\frac{1}{6}\right) = \frac{1}{3}$

$x_3 = 0 + 3\left(\frac{1}{6}\right) = \frac{1}{2}$

$x_4 = 0 + 4\left(\frac{1}{6}\right) = \frac{2}{3}$

$x_5 = 0 + 5\left(\frac{1}{6}\right) = \frac{5}{6}$

$x_6 = 0 + 6\left(\frac{1}{6}\right) = 1$

$m_1 = \frac{1}{2}\left(0 + \frac{1}{6}\right) = \frac{1}{12}$ $\qquad m_k = \frac{x_{k-1} + x_k}{2}$

$m_2 = \frac{1}{2}\left(\frac{1}{6} + \frac{1}{3}\right) = \frac{1}{4}$

$m_3 = \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{5}{12}$

$m_4 = \frac{1}{2}\left(\frac{1}{2} + \frac{2}{3}\right) = \frac{7}{12}$

$m_5 = \frac{1}{2}\left(\frac{2}{3} + \frac{5}{6}\right) = \frac{3}{4}$

$m_6 = \frac{1}{2}\left(\frac{5}{6} + 1\right) = \frac{11}{12}$

$M(n) = f\left(m_1\right)\Delta x + f\left(m_2\right)\Delta x + \cdots + f\left(m_n\right)\Delta x$

$M(6) = \left(\sin\left(\frac{\pi}{12}\right) + \sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{5\pi}{12}\right) + \sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{9\pi}{12}\right) + \sin\left(\frac{11\pi}{12}\right)\right)\left(\frac{1}{6}\right)$

≈ 0.6439505509

Exercise

Find the Midpoint Rule approximations to: $\displaystyle\int_0^{\pi} x^2 \sin x \, dx \quad n=8$ *subintervals*

Solution

$$\Delta x = \frac{\pi - 0}{8} = \frac{\pi}{8} \qquad\qquad \Delta x = \frac{b-a}{n}$$

$$x_0 = 0$$

$$x_1 = 0 + \frac{\pi}{8} = \frac{\pi}{8} \qquad\qquad x_k = x_0 + k\Delta x$$

$$x_2 = 0 + 2\left(\frac{\pi}{8}\right) = \frac{\pi}{4}$$

$$x_3 = 0 + 3\left(\frac{\pi}{8}\right) = \frac{3\pi}{8}$$

$$x_4 = 0 + 4\left(\frac{\pi}{8}\right) = \frac{\pi}{2}$$

$$x_5 = 0 + 5\left(\frac{\pi}{8}\right) = \frac{5\pi}{8}$$

$$x_6 = 0 + 6\left(\frac{\pi}{8}\right) = \frac{3\pi}{4}$$

$$x_7 = 0 + 7\left(\frac{\pi}{8}\right) = \frac{7\pi}{8}$$

$$x_8 = 0 + 8\left(\frac{\pi}{8}\right) = \pi$$

$$m_1 = \frac{1}{2}\left(0 + \frac{\pi}{8}\right) = \frac{\pi}{16} \qquad\qquad m_k = \frac{x_{k-1} + x_k}{2}$$

$$m_2 = \frac{1}{2}\left(\frac{\pi}{8} + \frac{\pi}{4}\right) = \frac{3\pi}{16}$$

$$m_3 = \frac{1}{2}\left(\frac{\pi}{4} + \frac{3\pi}{8}\right) = \frac{5\pi}{16}$$

$$m_4 = \frac{1}{2}\left(\frac{3\pi}{8} + \frac{\pi}{2}\right) = \frac{7\pi}{16}$$

$$m_5 = \frac{1}{2}\left(\frac{\pi}{2} + \frac{5\pi}{8}\right) = \frac{9\pi}{16}$$

$$m_6 = \frac{1}{2}\left(\frac{5\pi}{8} + \frac{3\pi}{4}\right) = \frac{11\pi}{16}$$

$$m_7 = \frac{1}{2}\left(\frac{3\pi}{4} + \frac{7\pi}{8}\right) = \frac{13\pi}{16}$$

$$m_8 = \frac{1}{2}\left(\frac{7\pi}{8} + \pi\right) = \frac{15\pi}{16}$$

$$M(n) = f\left(m_1\right)\Delta x + f\left(m_2\right)\Delta x + \cdots + f\left(m_n\right)\Delta x$$

$$M(8)=\left(\begin{array}{l}\left(\frac{\pi}{16}\right)^2\sin\frac{\pi}{16}+\left(\frac{3\pi}{16}\right)^2\sin\frac{3\pi}{16}+\left(\frac{5\pi}{16}\right)^2\sin\frac{5\pi}{16}+\left(\frac{7\pi}{16}\right)^2\sin\frac{7\pi}{16}+\left(\frac{9\pi}{16}\right)^2\sin\frac{9\pi}{16}\\+\left(\frac{11\pi}{16}\right)^2\sin\frac{11\pi}{16}+\left(\frac{13\pi}{16}\right)^2\sin\frac{13\pi}{16}+\left(\frac{15\pi}{16}\right)^2\sin\frac{15\pi}{16}\end{array}\right)\left(\frac{\pi}{8}\right)$$

$$=\left(\begin{array}{l}\sin\frac{\pi}{16}+9\sin\frac{3\pi}{16}+25\sin\frac{5\pi}{16}+49\sin\frac{7\pi}{16}+81\sin\frac{9\pi}{16}+121\sin\frac{11\pi}{16}\\+169\sin\frac{13\pi}{16}+225\sin\frac{15\pi}{16}\end{array}\right)\frac{\pi^3}{2,048}$$

$$\approx\left(\begin{array}{l}0.19509+5.000132+20.7867403+67.3478925+79.4436077\\100.607823+93.89136938+43.895322453\end{array}\right)\frac{\pi^3}{2,048}$$

$$\approx 6.22414635$$

Exercise

Find the Midpoint Rule approximations to: $\int_0^1 e^{-\sqrt{x}}dx \quad n=6 \;\; subintervals$

Solution

$\Delta x=\frac{1-0}{6}=\frac{1}{6}$ $\qquad\qquad \Delta x=\frac{b-a}{n}$

$x_0=0$

$x_1=0+\frac{1}{6}=\frac{1}{6}$ $\qquad\qquad x_k=x_0+k\Delta x$

$x_2=0+2\left(\frac{1}{6}\right)=\frac{1}{3}$

$x_3=0+3\left(\frac{1}{6}\right)=\frac{1}{2}$

$x_4=0+4\left(\frac{1}{6}\right)=\frac{2}{3}$

$x_5=0+5\left(\frac{1}{6}\right)=\frac{5}{6}$

$x_6=0+6\left(\frac{1}{6}\right)=1$

$m_1=\frac{1}{2}\left(0+\frac{1}{6}\right)=\frac{1}{12}$ $\qquad\qquad m_k=\frac{x_{k-1}+x_k}{2}$

$m_2=\frac{1}{2}\left(\frac{1}{6}+\frac{1}{3}\right)=\frac{1}{4}$

$m_3=\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{5}{12}$

$m_4=\frac{1}{2}\left(\frac{1}{2}+\frac{2}{3}\right)=\frac{7}{12}$

$$m_5 = \frac{1}{2}\left(\frac{2}{3}+\frac{5}{6}\right) = \frac{3}{4}$$

$$m_6 = \frac{1}{2}\left(\frac{5}{6}+1\right) = \frac{11}{12}$$

$$M(n) = f\left(m_1\right)\Delta x + f\left(m_2\right)\Delta x + \cdots + f\left(m_n\right)\Delta x$$

$$M(6) = \left(e^{-\sqrt{1/12}} + e^{-\sqrt{1/4}} + e^{-\sqrt{5/12}} + e^{-\sqrt{7/12}} + e^{-\sqrt{3/4}} + e^{-\sqrt{11/12}}\right)\left(\frac{1}{6}\right)$$

$$\approx (.74925557 + .6065306597 + .52440173 + .46591 + .42062 + .383879)\left(\frac{1}{6}\right)$$

$$\approx 0.67787732$$

Exercise

Find the Midpoint Rule approximations to: $\int_0^1 e^{-x}dx \quad using \quad n = 8 \;\; subintervals$

Solution

$$\Delta x = \frac{1-0}{8} = \frac{1}{8}$$

$$x_0 = 0$$

$$x_1 = 0 + \frac{1}{8} = \frac{1}{8} \qquad\qquad x_k = x_0 + k\Delta x$$

$$x_2 = 0 + 2\left(\frac{1}{8}\right) = \frac{1}{4}$$

$$x_3 = 0 + 3\left(\frac{1}{8}\right) = \frac{3}{8}$$

$$x_4 = 0 + 4\left(\frac{1}{8}\right) = \frac{1}{2}$$

$$x_5 = 0 + 5\left(\frac{1}{8}\right) = \frac{5}{8}$$

$$x_6 = 0 + 6\left(\frac{1}{8}\right) = \frac{3}{4}$$

$$x_7 = 0 + 7\left(\frac{1}{8}\right) = \frac{7}{8}$$

$$x_8 = 0 + 8\left(\frac{1}{8}\right) = 1$$

$$m_1 = \frac{1}{2}\left(0+\frac{1}{8}\right) = \frac{1}{16} \qquad\qquad m_k = \frac{x_{k-1}+x_k}{2}$$

$$m_2 = \frac{1}{2}\left(\frac{1}{8}+\frac{1}{4}\right) = \frac{3}{16}$$

$$m_3 = \frac{1}{2}\left(\frac{1}{4}+\frac{3}{8}\right) = \frac{5}{16}$$

$$m_4 = \frac{1}{2}\left(\frac{3}{8}+\frac{1}{2}\right) = \frac{7}{16}$$

$$m_5 = \frac{1}{2}\left(\frac{1}{2}+\frac{5}{8}\right) = \frac{9}{16}$$

$$m_6 = \frac{1}{2}\left(\frac{5}{8}+\frac{3}{4}\right) = \frac{11}{16}$$

$$m_7 = \frac{1}{2}\left(\frac{3}{4}+\frac{7}{8}\right) = \frac{13}{16}$$

$$m_8 = \frac{1}{2}\left(\frac{7}{8}+1\right) = \frac{15}{16}$$

$$M(8) = \frac{1}{8}\left(e^{-1/16}+e^{-3/16}+e^{-5/16}+e^{-7/16}+e^{-9/16}+e^{-11/16}+e^{-13/16}+e^{-15/16}\right)$$

$$\approx 0.6317092095$$

Exercise

Estimate the minimum number of subintervals to approximate the integrals with an error of magnitude of 10^{-4} by (***a***) the Trapezoid Rule and (***b***) Simpson's Rule. $\int_1^3 (2x-1)\,dx$

Solution

a) ***i***) $\Delta x = \frac{3-1}{4}$ $\qquad \Delta x = \frac{b-a}{n}$

$$= \frac{1}{2}$$

$$T = \frac{1}{2}\Delta x\left(m\, f\left(x_i\right)\right)$$

$$= \frac{1}{2}\frac{1}{2}(24) = 6$$

	x_i	$f\left(x_i\right) = 2x_i - 1$	m	$mf\left(x_i\right)$
x_0	1	1	1	1
x_1	$1+\frac{1}{2}=\frac{3}{2}$	2	2	4
x_2	2	3	2	6
x_3	$\frac{5}{2}$	4	2	8
x_4	3	5	1	5
				24

$$f(x) = 2x-1 \quad \Rightarrow \quad f'(x) = 2$$

$$\Rightarrow \quad f''(x) = 0 = M$$

$$\Rightarrow Error = 0$$

ii) $\int_1^3 (2x-1)\,dx = \left[x^2 - x\right]_1^3$

$$= \left(3^2-3\right)-\left(1^2-1\right)$$

$$= 6$$

iii) $Error = \dfrac{|E_T|}{True \;\; Value} \times 100$

$= 0\%$

b) ***i***) $\Delta x = \dfrac{3-1}{4}$ $\Delta x = \dfrac{b-a}{n}$

$= \dfrac{1}{2}$

$S = \frac{1}{3}\Delta x\left(\sum m\, f\left(x_i\right)\right)$

$= \frac{1}{3}\frac{1}{2}(36)$

$= 6$

	x_i	$f\left(x_i\right) = 2x_i - 1$	m	$mf\left(x_i\right)$
x_0	1	1	1	1
x_1	$\frac{3}{2}$	2	4	8
x_2	2	3	2	6
x_3	$\frac{5}{2}$	4	4	16
x_4	3	5	1	5
				36

$f(x) = 2x - 1$

$\Rightarrow \quad f^{(4)}(x) = 0 = M$

$\Rightarrow |E_s| = 0$

ii) $\displaystyle\int_1^3 (2x-1)\,dx = 6$

$|E_s| = \displaystyle\int_1^3 (2x-1)\,dx - S$

$= 6 - 6$

$= 0$

iii) $Error = \dfrac{|E_T|}{True \;\; Value} \times 100$

$= 0\%$

Exercise

Estimate the minimum number of subintervals to approximate the integrals with an error of magnitude of 10^{-4} by (***a***) the Trapezoid Rule and (***b***) Simpson's Rule. $\int_{-1}^{1}\left(x^2+1\right)dx$

Solution

a) ***i***) $\Delta x=\frac{b-a}{n}=\frac{1+1}{4}=\frac{1}{2}$

$$T=\frac{1}{2}\Delta x\left(m\,f\left(x_i\right)\right)=\frac{1}{2}\frac{1}{2}(11)\ =\frac{11}{4}$$

$$f(x)=x^2+1\ \Rightarrow\ f'(x)=2x\Rightarrow\ f''(x)=2=M$$

$$\left|E_T\right|=\frac{1-(-1)}{12}\left(\frac{1}{2}\right)^2(2)=0.0833\ldots$$

ii) $\int_{-1}^{1}\left(x^2+1\right)dx=\left[\frac{1}{3}x^3+x\right]_{-1}^{1}=\left(\frac{1}{3}+1\right)-\left(-\frac{1}{3}-1\right)=\frac{8}{3}$

$$E_T=\int_{-1}^{1}\left(x^2+1\right)dx-T=\frac{8}{3}-\frac{11}{4}=-\frac{1}{12}$$

iii) $Error=\frac{|E_T|}{True\ \ Value}\times 100=\frac{\frac{1}{12}}{\frac{8}{3}}\approx 3\%$

	x_i	$f\left(x_i\right)$	m	$m\,f\left(x_i\right)$
x_0	-1	2	1	2
x_1	$-\frac{1}{2}$	$\frac{5}{4}$	2	$\frac{5}{2}$
x_2	0	1	2	2
x_3	$\frac{1}{2}$	$\frac{5}{4}$	2	$\frac{5}{2}$
x_4	1	2	1	2
				11

b) ***i***) $\Delta x=\frac{b-a}{n}=\frac{-1-(-1)}{4}=\frac{1}{2}$

$$S=\frac{1}{3}\Delta x\left(\sum m\,f\left(x_i\right)\right)=\frac{1}{3}\frac{1}{2}(16)=\frac{8}{3}$$

$$f(x)=x^2+1\ \Rightarrow\ f^{(4)}(x)=0=M$$

$$\Rightarrow\left|E_s\right|=0$$

ii) $\int_{-1}^{1}\left(x^2+1\right)dx=\frac{8}{3}$

$$E_S=\int_{-1}^{1}\left(x^2+1\right)dx-S=\frac{8}{3}-\frac{8}{3}=0$$

iii) $Error=\frac{|E_T|}{True\ \ Value}\times 100=0\%$

	x_i	$f\left(x_i\right)$	m	$m\,f\left(x_i\right)$
x_0	-1	2	1	2
x_1	$-\frac{1}{2}$	$\frac{5}{4}$	4	5
x_2	0	1	2	2
x_3	$\frac{1}{2}$	$\frac{5}{4}$	4	5
x_4	1	2	1	2
				16

Exercise

Estimate the minimum number of subintervals to approximate the integrals with an error of magnitude of 10^{-4} by (***a***) the Trapezoid Rule and (***b***) Simpson's Rule. $\int_2^4 \frac{1}{(s-1)^2}\,ds$

Solution

a) $\Delta x = \frac{4-2}{4}$ $\qquad\qquad \Delta x = \frac{b-a}{n}$

$$= \frac{1}{2}$$

$$x_0 = 2$$

$$x_1 = 2 + \frac{1}{2} = \frac{5}{2}$$

$$x_2 = 2 + 2\left(\frac{1}{2}\right) = 3$$

$$x_3 = 2 + 3\left(\frac{1}{2}\right) = \frac{7}{2}$$

$$x_4 = 4$$

$$T = \frac{1}{2}\Delta x\left(m\,f\left(x_i\right)\right)$$

$$= \frac{1}{2}\frac{1}{2}\left(\frac{1}{(2-1)^2} + 2\frac{1}{\left(\frac{5}{2}-1\right)^2} + 2\frac{1}{(3-1)^2} + 2\frac{1}{\left(\frac{7}{2}-1\right)^2} + \frac{1}{(4-1)^2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{1} + \frac{8}{9} + \frac{1}{2} + \frac{8}{25} + \frac{1}{9}\right)$$

$$= \frac{1269}{1800}$$

$$\approx 0.705$$

$$f(s) = (s-1)^{-2} \quad \Rightarrow f'(s) = -2(s-1)^{-3}$$

$$\Rightarrow f''(s) = 6(s-1)^{-4} = \frac{6}{(s-1)^4} \Rightarrow M = 6$$

$$\int_2^4 \frac{1}{(s-1)^2}\,ds = \int_2^4 (s-1)^{-2}\,d(s-1)$$

$$= -\left[(s-1)^{-1}\right]_2^4$$

$$= -\left(3^{-1} - 1^{-1}\right)$$

$$= \frac{2}{3}$$

The percentage error: $\approx \frac{|0.705-.6667|}{.6667}$

≈ 0.0575 $\quad$ 5.75%

b) $\Delta x = \frac{4-2}{4}$ $\qquad\qquad$ $\Delta x = \frac{b-a}{n}$

$= \frac{1}{2}$

$x_0 = 2$

$x_1 = 2 + \frac{1}{2} = \frac{5}{2}$

$x_2 = 2 + 2\left(\frac{1}{2}\right) = 3$

$x_3 = 2 + 3\left(\frac{1}{2}\right) = \frac{7}{2}$

$x_4 = 4$

$$S = \frac{1}{3}\Delta x\left(m\, f\left(x_i\right)\right)$$

$$= \frac{1}{3}\frac{1}{2}\left(\frac{1}{(2-1)^2} + 4\frac{1}{\left(\frac{5}{2}-1\right)^2} + 2\frac{1}{(3-1)^2} + 4\frac{1}{\left(\frac{7}{2}-1\right)^2} + \frac{1}{(4-1)^2}\right)$$

$$= \frac{1}{6}\left(\frac{1}{1} + \frac{16}{9} + \frac{1}{2} + \frac{16}{25} + \frac{1}{9}\right)$$

$$= \frac{1813}{450}$$

$$\approx 0.67148$$

$$\int_2^4 \frac{1}{(s-1)^2}\,ds = \frac{2}{3}$$

The percentage error: $= \frac{|0.67148-.6667|}{.6667}$

≈ 0.0072 $\quad$ 0.72%

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^1 \sin \pi x \, dx \quad n = 6 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.1666666667	0.5000000000	1.0000000000
2	0.3333333333	0.8660254000	1.7320508000
3	0.5000000000	1.0000000000	2.0000000000
4	0.6666666667	0.8660254000	1.7320508000
5	0.8333333333	0.5000000000	1.0000000000
6	1.0000000000	0.0000000000	0.0000000000

Trapezoid Rule approximation ≈ 0.62200847

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.1666666667	0.5000000000	2.0000000000
2	0.3333333333	0.8660254000	1.7320508000
3	0.5000000000	1.0000000000	2.0000000000
4	0.6666666667	0.8660254000	1.7320508000
5	0.8333333333	0.5000000000	1.0000000000
6	1.0000000000	0.0000000000	0.0000000000

Simpson's Rule approximation ≈ 0.63689453

	Exact	***Trapezoid***	***Simpson***
Value:	0.63661977	0.62200847	0.63689453
Error:		2.2951 %	0.0432 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations to and error to $\int_0^1 e^{-x}dx \quad n=8 \;\; \textit{subintervals}$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.1250000000	0.8824969000	1.7649938000
2	0.2500000000	0.7788007800	1.5576015600
3	0.3750000000	0.6872892800	1.3745785600
4	0.5000000000	0.6065306600	1.2130613200
5	0.6250000000	0.5352614300	1.0705228600
6	0.7500000000	0.4723665500	0.9447331000
7	0.8750000000	0.4168620200	0.8337240400
8	1.0000000000	0.3678794400	0.3678794400

Trapezoid Rule approximation ≈ 0.63294342

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.1250000000	0.8824969000	3.5299876000
2	0.2500000000	0.7788007800	1.5576015600
3	0.3750000000	0.6872892800	2.7491571200
4	0.5000000000	0.6065306600	1.2130613200
5	0.6250000000	0.5352614300	2.1410457200
6	0.7500000000	0.4723665500	0.9447331000
7	0.8750000000	0.4168620200	1.6674480800
8	1.0000000000	0.3678794400	0.3678794400

Simpson's Rule approximation ≈ 0.63212141

	Exact	***Trapezoid***	***Simpson***
Value:	0.63212056	0.63294342	0.63212141
Error:		0.1302 %	0.0001 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error to:

$$\int_1^5 \left(3x^2 - 2x\right)dx \quad n = 8 \;\; subintervals$$

Solution

Trapezoid Rule Method

n	x_n	$f\left(x_n\right)$	$m \cdot f\left(x_n\right)$
0	0.0000000000	1.0000000000	1.0000000000
1	1.5000000000	3.7500000000	7.5000000000
2	2.0000000000	8.0000000000	16.0000000000
3	2.5000000000	13.7500000000	27.5000000000
4	3.0000000000	21.0000000000	42.0000000000
5	3.5000000000	29.7500000000	59.5000000000
6	4.0000000000	40.0000000000	80.0000000000
7	4.5000000000	51.7500000000	103.500000000
8	5.0000000000	65.0000000000	65.0000000000

Trapezoid Rule approximation ≈ 100.50000000

Simpson's Rule Method

n	x_n	$f\left(x_n\right)$	$m \cdot f\left(x_n\right)$
0	0.0000000000	1.0000000000	1.0000000000
1	1.5000000000	3.7500000000	15.0000000000
2	2.0000000000	8.0000000000	16.0000000000
3	2.5000000000	13.7500000000	55.0000000000
4	3.0000000000	21.0000000000	42.0000000000
5	3.5000000000	29.7500000000	119.000000000
6	4.0000000000	40.0000000000	80.0000000000
7	4.5000000000	51.7500000000	207.000000000
8	5.0000000000	65.0000000000	65.0000000000

Simpson's Rule approximation ≈ 100.00000000

	Exact	***Trapezoid***	***Simpson***
Value:	100.000000	100.500000	100.00000000
Error:		0.5000%	0.0000 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^{\pi/4} 3\sin 2x \, dx \quad n = 8 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.0981747704	0.5852709700	1.1705419400
2	0.1963495408	1.1480503000	2.2961006000
3	0.2945243113	1.6667107000	3.3334214000
4	0.3926990817	2.1213203400	4.2426406800
5	0.4908738521	2.4944088400	4.9888176800
6	0.5890486225	2.7716386000	5.5432772000
7	0.6872233930	2.9423558400	5.8847116800
8	0.7853981634	3.0000000000	3.0000000000

Trapezoid Rule approximation ≈ 1.49517776

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.0981747704	0.5852709700	2.3410838800
2	0.1963495408	1.1480503000	2.2961006000
3	0.2945243113	1.6667107000	6.6668428000
4	0.3926990817	2.1213203400	4.2426406800
5	0.4908738521	2.4944088400	9.9776353600
6	0.5890486225	2.7716386000	5.5432772000
7	0.6872233930	2.9423558400	11.7694233600
8	0.7853981634	3.0000000000	3.0000000000

Simpson's Rule approximation ≈ 1.50001244

	Exact	***Trapezoid***	***Simpson***
Value:	1.500000	1.49517776	1.50001244
Error:		0.3215 %	0.0008 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^8 e^{-2x}\,dx \quad n=8\ \ subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m\cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	1.0000000000	0.1353352800	0.2706705600
2	2.0000000000	0.0183156400	0.0366312800
3	3.0000000000	0.0024787500	0.0049575000
4	4.0000000000	0.0003354600	0.0006709200
5	5.0000000000	0.0000454000	0.0000908000
6	6.0000000000	0.0000061400	0.0000122800
7	7.0000000000	0.0000008300	0.0000016600
8	8.0000000000	0.0000001100	0.0000001100

Trapezoid Rule approximation ≈ 0.65651755

Simpson's Rule Method

n	x_n	$f(x_n)$	$m\cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	1.0000000000	0.1353352800	0.5413411200
2	2.0000000000	0.0183156400	0.0366312800
3	3.0000000000	0.0024787500	0.0099150000
4	4.0000000000	0.0003354600	0.0006709200
5	5.0000000000	0.0000454000	0.0001816000
6	6.0000000000	0.0000061400	0.0000122800
7	7.0000000000	0.0000008300	0.0000033200
8	8.0000000000	0.0000001100	0.0000001100

Simpson's Rule approximation ≈ 0.52958521

	Exact	***Trapezoid***	***Simpson***
Value:	0.49999994	0.65651755	0.52958521
Error:		31.3035 %	5.9171 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_{-1}^{1}\sqrt{x^2+1}\,dx \quad n=8 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f\left(x_n\right)$	$m\cdot f\left(x_n\right)$
0	–1.0000000000	1.4142135600	1.4142135600
1	–0.7500000000	1.2500000000	2.5000000000
2	–0.5000000000	1.1180339900	2.2360679800
3	–0.2500000000	1.0307764100	2.0615528200
4	0.0000000000	1.0000000000	2.0000000000
5	0.2500000000	1.0307764100	2.0615528200
6	0.5000000000	1.1180339900	2.2360679800
7	0.7500000000	1.2500000000	2.5000000000
8	1.0000000000	1.4142135600	1.4142135600

Trapezoid Rule approximation ≈ **2.30295859**

Simpson's Rule Method

n	x_n	$f\left(x_n\right)$	$m\cdot f\left(x_n\right)$
0	–1.0000000000	1.4142135600	1.4142135600
1	–0.7500000000	1.2500000000	5.0000000000
2	–0.5000000000	1.1180339900	2.2360679800
3	–0.2500000000	1.0307764100	4.1231056400
4	0.0000000000	1.0000000000	2.0000000000
5	0.2500000000	1.0307764100	4.1231056400
6	0.5000000000	1.1180339900	2.2360679800
7	0.7500000000	1.2500000000	5.0000000000
8	1.0000000000	1.4142135600	1.4142135600

Simpson's Rule approximation ≈ **2.29556453**

	Exact	***Trapezoid***	***Simpson***
Value:	2.29558715	2.30295859	2.29556453
Error:		0.3211 %	0.0010 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^{1/2} \sin\left(x^2\right)\,dx \quad n = 4$ *subintervals*

Solution

Trapezoid Rule Method

n	x_n	$f\left(x_n\right)$	$m \cdot f\left(x_n\right)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.1250000000	0.0156243642	0.0312487284
2	0.2500000000	0.0624593178	0.1249186357
3	0.3750000000	0.1401619723	0.2803239447
4	0.5000000000	0.2474039593	0.2474039593

Trapezoid Rule approximation ≈ 0.0427434543

Simpson's Rule Method

n	x_n	$f\left(x_n\right)$	$m \cdot f\left(x_n\right)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.1250000000	0.0156243642	0.0624974569
2	0.2500000000	0.0624593178	0.1249186357
3	0.3750000000	0.1401619723	0.5606478894
4	0.5000000000	0.2474039593	0.2474039593

Simpson Rule approximation ≈ 0.0414778309

	Exact	***Trapezoid***	***Simpson***
Value:	0.0414810243	0.0427434543	0.0414778309
Error:		3.04339%	0.00770 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\displaystyle\int_{\pi/2}^{\pi} \frac{\sin x}{x}\, dx \quad n = 6 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	1.5708000000	0.6366182800	0.6366182800
1	1.8323333333	0.5271932200	1.0543864400
2	2.0938666667	0.4137271600	0.8274543200
3	2.3554000000	0.3004450800	0.6008901600
4	2.6169333333	0.1914141900	0.3828283800
5	2.8784666667	0.0903606800	0.1807213600
6	3.1400000000	0.0005072100	0.0005072100

Trapezoid Rule approximation ≈ 0.48166674

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	1.5708000000	0.6366182800	0.6366182800
1	1.8323333333	0.5271932200	2.1087728800
2	2.0938666667	0.4137271600	0.8274543200
3	2.3554000000	0.3004450800	1.2017803200
4	2.6169333333	0.1914141900	0.3828283800
5	2.8784666667	0.0903606800	0.3614427200
6	3.1400000000	0.0005072100	0.0005072100

Simpson's Rule approximation ≈ 0.48116938

	Exact	***Trapezoid***	***Simpson***
Value:	0.48117214	0.48166674	0.48116938
Error:		0.1028 %	0.0006 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^{\pi/4} x\tan x\, dx \quad n = 6 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.1308996939	0.0172332716	0.0344665433
2	0.2617993878	0.0701489345	0.1402978691
3	0.3926990817	0.1626612856	0.3253225711
4	0.5235987756	0.3022998940	0.6045997881
5	0.6544984695	0.5022143392	1.0044286785
6	0.7853981634	0.7853981634	0.7853981634

Trapezoid Rule approximation ≈ 0.1894454730

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.1308996939	0.0172332716	0.0689330865
2	0.2617993878	0.0701489345	0.1402978691
3	0.3926990817	0.1626612856	0.6506451423
4	0.5235987756	0.3022998940	0.6045997881
5	0.6544984695	0.5022143392	2.0088573569
6	0.7853981634	0.7853981634	0.7853981634

Simpson Rule approximation ≈ 0.1858222125

	Exact	***Trapezoid***	***Simpson***
Value:	0.1857845357	0.1894454730	0.1858222125
Error:		1.97053%	0.02028 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^1 e^{-x^2}\,dx \quad n=10 \ \ subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m\cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.1000000000	0.9900498337	1.9800996675
2	0.2000000000	0.9607894392	1.9215788783
3	0.3000000000	0.9139311853	1.8278623705
4	0.4000000000	0.8521437890	1.7042875779
5	0.5000000000	0.7788007831	1.5576015661
6	0.6000000000	0.6976763261	1.3953526521
7	0.7000000000	0.6126263942	1.2252527884
8	0.8000000000	0.5272924240	1.0545848481
9	0.9000000000	0.4448580662	0.8897161324
10	1.0000000000	0.3678794412	0.3678794412

Trapezoid Rule approximation ≈ 0.7462107961

Simpson's Rule Method

n	x_n	$f(x_n)$	$m\cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.1000000000	0.9900498337	3.9601993350
2	0.2000000000	0.9607894392	1.9215788783
3	0.3000000000	0.9139311853	3.6557247411
4	0.4000000000	0.8521437890	1.7042875779
5	0.5000000000	0.7788007831	3.1152031323
6	0.6000000000	0.6976763261	1.3953526521
7	0.7000000000	0.6126263942	2.4505055767
8	0.8000000000	0.5272924240	1.0545848481
9	0.9000000000	0.4448580662	1.7794322649
10	1.0000000000	0.3678794412	0.3678794412

Simpson Rule approximation ≈ 0.7468249483

	Exact	***Trapezoid***	***Simpson***
Value:	0.7468241328	0.7462107961	0.7468249483
Error:		0.08213%	0.00011 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^2 \frac{1}{\sqrt{1+x^2}}\,dx \quad n=10 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.2000000000	0.9805806800	1.9611613600
2	0.4000000000	0.9284766900	1.8569533800
3	0.6000000000	0.8574929300	1.7149858600
4	0.8000000000	0.7808688100	1.5617376200
5	1.0000000000	0.7071067800	1.4142135600
6	1.2000000000	0.6401844000	1.2803688000
7	1.4000000000	0.5812381900	1.1624763800
8	1.6000000000	0.5299989400	1.0599978800
9	1.8000000000	0.4856429300	0.9712858600
10	2.0000000000	0.4472136000	0.4472136000

Trapezoid Rule approximation ≈ 1.44303943

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.2000000000	0.9805806800	3.9223227200
2	0.4000000000	0.9284766900	1.8569533800
3	0.6000000000	0.8574929300	3.4299717200
4	0.8000000000	0.7808688100	1.5617376200
5	1.0000000000	0.7071067800	2.8284271200
6	1.2000000000	0.6401844000	1.2803688000
7	1.4000000000	0.5812381900	2.3249527600
8	1.6000000000	0.5299989400	1.0599978800
9	1.8000000000	0.4856429300	1.9425717200
10	2.0000000000	0.4472136000	0.4472136000

Simpson's Rule approximation ≈ 1.44363449

	Exact	***Trapezoid***	***Simpson***
Value:	1.44363548	1.44303943	1.44363449
Error:		0.0413 %	0.0001 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^{1/2} \sin\left(e^{x/2}\right) dx \quad n = 8 \ \ subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f\left(x_n\right)$	$m \cdot f\left(x_n\right)$
0	0.0000000000	0.8414709848	0.8414709848
1	0.0625000000	0.8581952249	1.7163904498
2	0.1250000000	0.8745438796	1.7490877592
3	0.1875000000	0.8904281963	1.7808563927
4	0.2500000000	0.9057510229	1.8115020459
5	0.3125000000	0.9204063003	1.8408126006
6	0.3750000000	0.9342785616	1.8685571232
7	0.4375000000	0.9472424468	1.8944848937
8	0.5000000000	0.9591622435	0.9591622435

Trapezoid Rule approximation ≈ 0.4519476404

Simpson's Rule Method

n	x_n	$f\left(x_n\right)$	$m \cdot f\left(x_n\right)$
0	0.0000000000	0.8414709848	0.8414709848
1	0.0625000000	0.8581952249	3.4327808996
2	0.1250000000	0.8745438796	1.7490877592
3	0.1875000000	0.8904281963	3.5617127853
4	0.2500000000	0.9057510229	1.8115020459
5	0.3125000000	0.9204063003	3.6816252012
6	0.3750000000	0.9342785616	1.8685571232
7	0.4375000000	0.9472424468	3.7889697874
8	0.5000000000	0.9591622435	0.9591622435

Simpson Rule approximation ≈ 0.4519764340

	Exact	***Trapezoid***	***Simpson***
Value:	0.4519764600	0.4519476404	0.4519764340
Error:		0.00638%	0.00001%

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\displaystyle\int_2^3 \frac{1}{\ln x}\,dx \quad n=10\ \ subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m\cdot f(x_n)$
0	2.0000000000	1.4426950400	1.4426950400
1	2.1000000000	1.3478227100	2.6956454200
2	2.2000000000	1.2682994000	2.5365988000
3	2.3000000000	1.2006111700	2.4012223400
4	2.4000000000	1.1422452400	2.2844904800
5	2.5000000000	1.0913566700	2.1827133400
6	2.6000000000	1.0465599400	2.0931198800
7	2.7000000000	1.0067940700	2.0135881400
8	2.8000000000	0.9712326500	1.9424653000
9	2.9000000000	0.9392222400	1.8784444800
10	3.0000000000	0.9102392300	0.9102392300

Trapezoid Rule approximation ≈ 1.11906112

Simpson's Rule Method

n	x_n	$f(x_n)$	$m\cdot f(x_n)$
0	2.0000000000	1.4426950400	1.4426950400
1	2.1000000000	1.3478227100	5.3912908400
2	2.2000000000	1.2682994000	2.5365988000
3	2.3000000000	1.2006111700	4.8024446800
4	2.4000000000	1.1422452400	2.2844904800
5	2.5000000000	1.0913566700	4.3654266800
6	2.6000000000	1.0465599400	2.0931198800
7	2.7000000000	1.0067940700	4.0271762800
8	2.8000000000	0.9712326500	1.9424653000
9	2.9000000000	0.9392222400	3.7568889600
10	3.0000000000	0.9102392300	0.9102392300

Simpson's Rule approximation ≈ 1.11842787

	Exact	***Trapezoid***	***Simpson***
Value:	1.11842481	1.11906112	1.11842787
Error:		0.0569 %	0.0003 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_1^2 e^{1/x}\,dx \quad n=4 \ \ subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	1.0000000000	2.7182818300	2.7182818300
1	1.2500000000	2.2255409300	4.4510818600
2	1.5000000000	1.9477340400	3.8954680800
3	1.7500000000	1.7707949500	3.5415899000
4	2.0000000000	1.6487212700	1.6487212700

Trapezoid Rule approximation ≈ 2.03189287

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	1.0000000000	2.7182818300	2.7182818300
1	1.2500000000	2.2255409300	8.9021637200
2	1.5000000000	1.9477340400	3.8954680800
3	1.7500000000	1.7707949500	7.0831798000
4	2.0000000000	1.6487212700	1.6487212700

Simpson's Rule approximation ≈ 2.02065122

	Exact	***Trapezoid***	***Simpson***
Value:	2.02005862	2.03189287	2.02065122
Error:		0.5858 %	0.0293 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^1 \ln\left(1+e^x\right)\,dx \quad n=8 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f\left(x_n\right)$	$m\cdot f\left(x_n\right)$
0	0.0000000000	0.6931471800	0.6931471800
1	0.1250000000	0.7575990400	1.5151980800
2	0.2500000000	0.8259394200	1.6518788400
3	0.3750000000	0.8981232600	1.7962465200
4	0.5000000000	0.9740769800	1.9481539600
5	0.6250000000	1.0537006800	2.1074013600
6	0.7500000000	1.1368710100	2.2737420200
7	0.8750000000	1.2234445800	2.4468891600
8	1.0000000000	1.3132616900	1.3132616900

Trapezoid Rule approximation ≈ 0.98411993

Simpson's Rule Method

n	x_n	$f\left(x_n\right)$	$m\cdot f\left(x_n\right)$
0	0.0000000000	0.6931471800	0.6931471800
1	0.1250000000	0.7575990400	3.0303961600
2	0.2500000000	0.8259394200	1.6518788400
3	0.3750000000	0.8981232600	3.5924930400
4	0.5000000000	0.9740769800	1.9481539600
5	0.6250000000	1.0537006800	4.2148027200
6	0.7500000000	1.1368710100	2.2737420200
7	0.8750000000	1.2234445800	4.8937783200
8	1.0000000000	1.3132616900	1.3132616900

Simpson's Rule approximation ≈ 0.98381891

	Exact	***Trapezoid***	***Simpson***
Value:	0.98381904	0.98411993	0.98381891
Error:		0.0306 %	0.0000 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^1 x^5 e^x \, dx \quad n=10 \ \ subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.1000000000	0.0000110500	0.0000221000
2	0.2000000000	0.0003908500	0.0007817000
3	0.3000000000	0.0032801600	0.0065603200
4	0.4000000000	0.0152762800	0.0305525600
5	0.5000000000	0.0515225400	0.1030450800
6	0.6000000000	0.1416879600	0.2833759200
7	0.7000000000	0.3384514200	0.6769028400
8	0.8000000000	0.7292652500	1.4585305000
9	0.9000000000	1.4523710400	2.9047420800
10	1.0000000000	2.7182818300	2.7182818300

Trapezoid Rule approximation ≈ 0.40913975

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.1000000000	0.0000110500	0.0000442000
2	0.2000000000	0.0003908500	0.0007817000
3	0.3000000000	0.0032801600	0.0131206400
4	0.4000000000	0.0152762800	0.0305525600
5	0.5000000000	0.0515225400	0.2060901600
6	0.6000000000	0.1416879600	0.2833759200
7	0.7000000000	0.3384514200	1.3538056800
8	0.8000000000	0.7292652500	1.4585305000
9	0.9000000000	1.4523710400	5.8094841600
10	1.0000000000	2.7182818300	2.7182818300

Simpson's Rule approximation ≈ 0.39580225

	Exact	***Trapezoid***	***Simpson***
Value:	0.39559955	0.40913975	0.39580225
Error:		3.4227 %	0.0512 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^4 \sqrt{x}\ \sin x\, dx \quad n = 8\ \ subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.5000000000	0.3390050500	0.6780101000
2	1.0000000000	0.8414709800	1.6829419600
3	1.5000000000	1.2216768700	2.4433537400
4	2.0000000000	1.2859407500	2.5718815000
5	2.5000000000	0.9462675500	1.8925351000
6	3.0000000000	0.2444270200	0.4888540400
7	3.5000000000	–0.6562553300	–1.3125106600
8	4.0000000000	–1.5136049900	–1.5136049900

Trapezoid Rule approximation ≈ 1.73286520

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.0000000000	0.0000000000
1	0.5000000000	0.3390050500	1.3560202000
2	1.0000000000	0.8414709800	1.6829419600
3	1.5000000000	1.2216768700	4.8867074800
4	2.0000000000	1.2859407500	2.5718815000
5	2.5000000000	0.9462675500	3.7850702000
6	3.0000000000	0.2444270200	0.4888540400
7	3.5000000000	–0.6562553300	–2.6250213200
8	4.0000000000	–1.5136049900	–1.5136049900

Simpson's Rule approximation ≈ 1.77214151

	Exact	***Trapezoid***	***Simpson***
Value:	1.76874870	1.73286520	1.77214151
Error:		2.0288 %	0.1918 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_0^3 \frac{1}{1+x^5}\,dx \quad n=6 \ \ subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.5000000000	0.9696969700	1.9393939400
2	1.0000000000	0.5000000000	1.0000000000
3	1.5000000000	0.1163636400	0.2327272800
4	2.0000000000	0.0303030300	0.0606060600
5	2.5000000000	0.0101362100	0.0202724200
6	3.0000000000	0.0040983600	0.0040983600

Trapezoid Rule approximation ≈ 1.06427452

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.5000000000	0.9696969700	3.8787878800
2	1.0000000000	0.5000000000	1.0000000000
3	1.5000000000	0.1163636400	0.4654545600
4	2.0000000000	0.0303030300	0.0606060600
5	2.5000000000	0.0101362100	0.0405448400
6	3.0000000000	0.0040983600	0.0040983600

Simpson's Rule approximation ≈ 1.07491528

	Exact	***Trapezoid***	***Simpson***
Value:	1.06587854	1.06427452	1.07491528
Error:		0.150488%	.84782%

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_1^4 \frac{e^x}{x}\,dx \quad n=10 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f(x_n)$	$m\cdot f(x_n)$
0	1.0000000000	2.7182818300	2.7182818300
1	1.3000000000	2.8225359000	5.6450718000
2	1.6000000000	3.0956452700	6.1912905400
3	1.9000000000	3.5188918100	7.0377836200
4	2.2000000000	4.1022788600	8.2045577200
5	2.5000000000	4.8729975800	9.7459951600
6	2.8000000000	5.8730881300	11.7461762600
7	3.1000000000	7.1606294500	14.3212589000
8	3.4000000000	8.8129706000	17.6259412000
9	3.7000000000	10.9317038800	21.8634077600
10	4.0000000000	13.6495375100	13.6495375100

Trapezoid Rule approximation ≈ 17.81239534

Simpson's Rule Method

n	x_n	$f(x_n)$	$m\cdot f(x_n)$
0	1.0000000000	2.7182818300	2.7182818300
1	1.3000000000	2.8225359000	11.2901436000
2	1.6000000000	3.0956452700	6.1912905400
3	1.9000000000	3.5188918100	14.0755672400
4	2.2000000000	4.1022788600	8.2045577200
5	2.5000000000	4.8729975800	19.4919903200
6	2.8000000000	5.8730881300	11.7461762600
7	3.1000000000	7.1606294500	28.6425178000
8	3.4000000000	8.8129706000	17.6259412000
9	3.7000000000	10.9317038800	43.7268155200
10	4.0000000000	13.6495375100	13.6495375100

Simpson's Rule approximation ≈ 17.73628195

	Exact	***Trapezoid***	***Simpson***
Value:	17.73575665	17.81239534	17.73628195
Error:		0.4321 %	0.0030 %

Exercise

Find the ***Trapezoid*** & ***Simpson's*** Rule approximations and error: $\int_1^2 \frac{dx}{x} \quad n=10 \;\; subintervals$

Solution

Trapezoid Rule Method

n	x_n	$f\left(x_n\right)$	$m \cdot f\left(x_n\right)$
0	1.0000000000	1.0000000000	1.0000000000
1	1.1000000000	0.9090909100	1.8181818200
2	1.2000000000	0.8333333300	1.6666666600
3	1.3000000000	0.7692307700	1.5384615400
4	1.4000000000	0.7142857100	1.4285714200
5	1.5000000000	0.6666666700	1.3333333400
6	1.6000000000	0.6250000000	1.2500000000
7	1.7000000000	0.5882352900	1.1764705800
8	1.8000000000	0.5555555600	1.1111111200
9	1.9000000000	0.5263157900	1.0526315800
10	2.0000000000	0.5000000000	0.5000000000

Trapezoid Rule approximation ≈ 0.69377140

Simpson's Rule Method

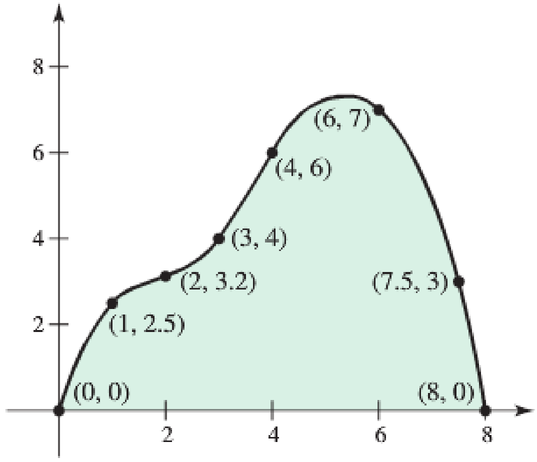
n	x_n	$f\left(x_n\right)$	$m \cdot f\left(x_n\right)$
0	1.0000000000	1.0000000000	1.0000000000
1	1.1000000000	0.9090909100	3.6363636400
2	1.2000000000	0.8333333300	1.6666666600
3	1.3000000000	0.7692307700	3.0769230800
4	1.4000000000	0.7142857100	1.4285714200
5	1.5000000000	0.6666666700	2.6666666800
6	1.6000000000	0.6250000000	1.2500000000
7	1.7000000000	0.5882352900	2.3529411600
8	1.8000000000	0.5555555600	1.1111111200
9	1.9000000000	0.5263157900	2.1052631600
10	2.0000000000	0.5000000000	0.5000000000

Simpson's Rule approximation ≈ 0.69315023

	Exact	***Trapezoid***	***Simpson***
Value:	0.69314718	0.69377140	0.69315023
Error:		0.0901%	0.0004%

Exercise

A piece of wood paneling must be cut in the shape shown below. The coordinates of several point on its curved surface are also shown (with units of inches).



a) Estimate the surface area of the paneling using the Trapezoid Rule

b) Estimate the surface area of the paneling using a left Riemann sum.

c) Could two identical pieces be cut from a 9-in by 9-in piece of wood?

Solution

a) The *trapezoid* Rule gives

$$\frac{(0+.25)\cdot 1}{2}+\frac{(2.5+3.2)\cdot 1}{2}+\frac{(3.2+4)\cdot 1}{2}+\frac{(4+6)\cdot 1}{2}+\frac{(6+7)\cdot 2}{2}+\frac{(7+5.3)\cdot 1.5}{2}+\frac{(3+0)\cdot 0.5}{2}$$

$$=35.675$$

b) The left *Riemann* sum gives

$$0\cdot 1+2.5\cdot 1+3.2\cdot 1+4\cdot 1+6\cdot 2+7\cdot 1.5+5.3\cdot 0.5 = 34.85$$

c) Although the surface area of the piece appears to be less than half of $81=9^2$ (area of 9×9 piece of wood), the shape prohibits the creation of two identical pieces.

Exercise

The region bounded by the curves $y=\dfrac{1}{1+e^{-x}}$, $x=0$ and $x=10$ is rotated about $x-axis$. Use Simpson's Rule with $n=10$ to estimate the volume of the resulting solid.

Solution

Using *Disk* method:

$$V=\pi\int_0^{10}\frac{1}{\left(1+e^{-x}\right)^2}\,dx$$

Simpson's Rule Method

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	0.2500000000	0.2500000000
1	1.0000000000	0.5344466454	2.1377865816
2	2.0000000000	0.7758034926	1.5516069851
3	3.0000000000	0.9073974671	3.6295898684
4	4.0000000000	0.9643510838	1.9287021676
5	5.0000000000	0.9866590924	3.9466363696
6	6.0000000000	0.9950608676	1.9901217351
7	7.0000000000	0.9981787276	3.9927149105
8	8.0000000000	0.9993294122	1.9986588244
9	9.0000000000	0.9997532261	3.9990129043
10	10.000000000	0.9999092063	0.9999092063

Simpson Rule approximation ≈ 8.8082465177

$$V = \pi \int_0^{10} \frac{1}{\left(1+e^{-x}\right)^2}\, dx$$

$$\approx \pi\,(8.8082465177)$$

$$\approx 27.6719226 \quad unit^3$$

Exercise

A pendulum with length L that makes a maximum angle θ_0 with the vertical. Using Newton's Second Law it can be shown that the period T (the time for one complete swing) is given by

$$T = 4\sqrt{\frac{L}{g}} \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$$

Where $k = \sin\left(\frac{1}{2}\theta_0\right)$ and g is the acceleration due to gravity. If $L = 1\ m$ and $\theta_0 = 42°$, use Simpson's Rule with $n = 10$ to find the period.

Solution

$$T = 4\sqrt{\frac{L}{g}} \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$$

$$= 4\sqrt{\frac{1}{9.8}} \int_0^{\pi/2} \frac{dx}{\sqrt{1-\sin^2\left(\frac{1}{2}42°\right)\sin^2 x}}$$

$$= \frac{4}{\sqrt{9.8}} \int_0^{\pi/2} \frac{dx}{\sqrt{1 - \sin^2(21°)\sin^2 x}}$$

Simpson's Rule Method $\displaystyle\int_0^{\pi/2} \frac{dx}{\sqrt{1 - \sin^2(21°)\sin^2 x}}$

n	x_n	$f(x_n)$	$m \cdot f(x_n)$
0	0.0000000000	1.0000000000	1.0000000000
1	0.1570796327	1.0031527554	4.0126110216
2	0.3141592654	1.0124160101	2.0248320201
3	0.4712388980	1.0271895774	4.1087583096
4	0.6283185307	1.0464308046	2.0928616093
5	0.7853981634	1.0686201540	4.2744806162
6	0.9424777961	1.0917709315	2.1835418629
7	1.0995574288	1.1135333115	4.4541332459
8	1.2566370614	1.1314314233	2.2628628466
9	1.4137166941	1.1432291699	4.5729166795
10	1.5707963268	1.1473515974	1.1473515974

Simpson Rule approximation ≈ 1.6825506215

$$T = \frac{4}{\sqrt{9.8}} \int_0^{\pi/2} \frac{dx}{\sqrt{1 - \sin^2(21°)\sin^2 x}}$$

$$\approx \frac{4}{\sqrt{9.8}}(1.6825506215)$$

$$\approx 2.149884$$