

Section 3.3 – Applied Optimization

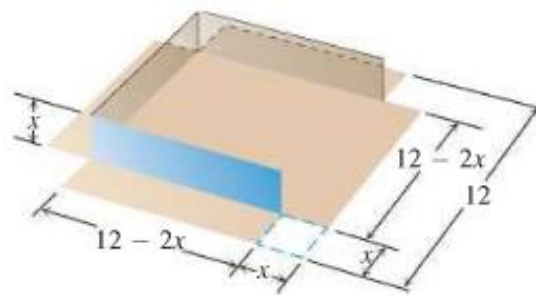
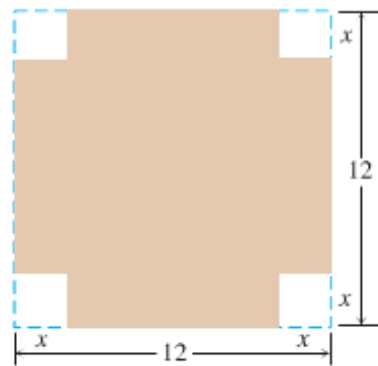
Solving Applied Optimization Problems

1. Read the problem
2. Draw a picture
3. Introduce variables
4. Write an equation for the unknown quantity
5. Test the critical points and endpoints in the domain of the unknown

Example

An open-top box is to be made by cutting small congruent squares from the corners of a 12-in. by 12-in. sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?

Solution



$$\begin{aligned} V(x) &= hlw \\ &= x(12 - 2x)^2 \\ &= x(144 - 48x + 4x^2) \\ &= 4x^3 - 48x^2 + 144x \quad 0 \leq x \leq 6 \end{aligned}$$

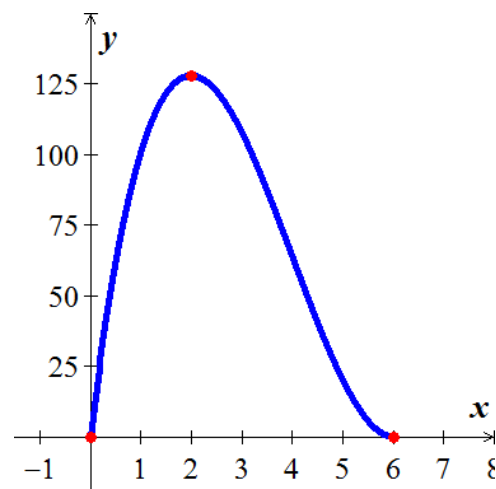
$$\frac{dV}{dx} = 12x^2 - 96x + 144 = 0$$

$$\boxed{x = 2, 6} \quad (CP)$$

$$V(2) = 4(2)^3 - 48(2)^2 + 144(2) = 128$$

$$V(6) = 4(6)^3 - 48(6)^2 + 144(6) = 0$$

The maximum volume is 128 in^3 . The cutout squares is 2 in.



Example

You have been asked to design a one-liter can shaped like a right circular cylinder.
What dimensions will use the least material?

Solution

Volume of can: $V = \pi r^2 h = 1 \text{ liter} = 1000 \text{ cm}^3$

Surface area of can: $A = 2\pi r^2 + 2\pi r h$

$$\pi r^2 h = 1000 \Rightarrow h = \frac{1000}{\pi r^2}$$

$$\begin{aligned} A &= 2\pi r^2 + 2\pi r \left(\frac{1000}{\pi r^2} \right) \\ &= 2\pi r^2 + \frac{2000}{r} \end{aligned}$$

$$\frac{dA}{dr} = 4\pi r - \frac{2000}{r^2} = 0 \quad \text{Solve for } r$$

$$4\pi r = \frac{2000}{r^2}$$

$$r^3 = \frac{500}{\pi}$$

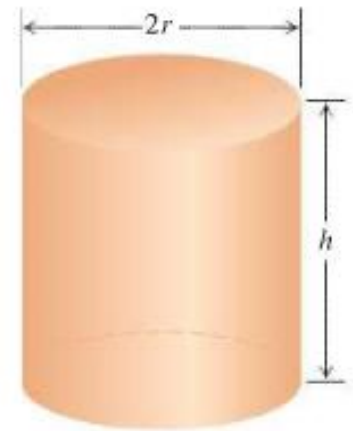
$$r = \sqrt[3]{\frac{500}{\pi}} \approx 5.42$$

$$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi (5.42)^2} \approx 10.84 \quad (h = 2r)$$

The graph is concave up on its domain.

The on 2-liter can that uses the least material has height equal to twice its radius.

$$r \approx 5.42 \text{ cm} \quad h \approx 10.84 \text{ cm}$$



0	5.42
$A'(1) < 0$	$A'(6) > 0$
<i>decreasing</i>	<i>increasing</i>



Short and wide



Tall and thin

Example

A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area of the rectangle can have, and what are its dimensions?

Solution

The equation of the circle is given by the equation: $x^2 + y^2 = 4$

Therefore, the semicircle is: $y = \sqrt{4 - x^2}$

The dimensions of semicircle: *length* : $2x$ *height* : $\sqrt{4 - x^2}$

Area: $A(x) = lh = 2x\sqrt{4 - x^2}$

$$A'(x) = 2\sqrt{4 - x^2} - \frac{2x^2}{\sqrt{4 - x^2}} \qquad \begin{aligned} u &= 2x & v &= \sqrt{4 - x^2} \\ u' &= 2 & v' &= \frac{-x}{\sqrt{4 - x^2}} \end{aligned}$$

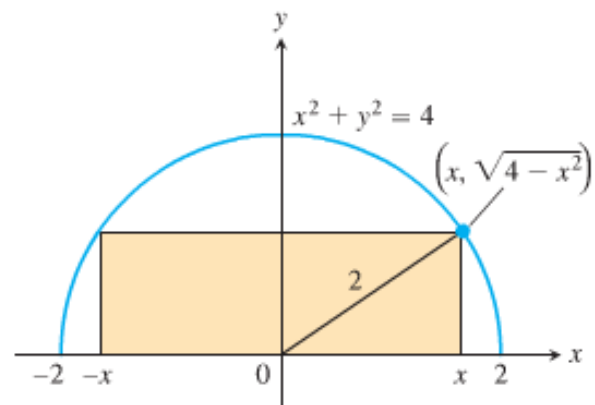
$$= \frac{8 - 2x^2 - 2x^2}{\sqrt{4 - x^2}}$$

$$= \frac{8 - 4x^2}{\sqrt{4 - x^2}} = 0 \quad \text{Solve for } x$$

$$8 - 4x^2 = 0 \Rightarrow x^2 = 2 \rightarrow \boxed{x = \pm\sqrt{2}}$$

$$A(\sqrt{2}) = 2(\sqrt{2})\sqrt{4 - (\sqrt{2})^2} = 4$$

$$A(2) = 2(2)\sqrt{4 - (2)^2} = 0$$



The area has a maximum value of 4 when the height is $x = \sqrt{2}$ *unit* and length $2x = 2\sqrt{2}$ *unit*

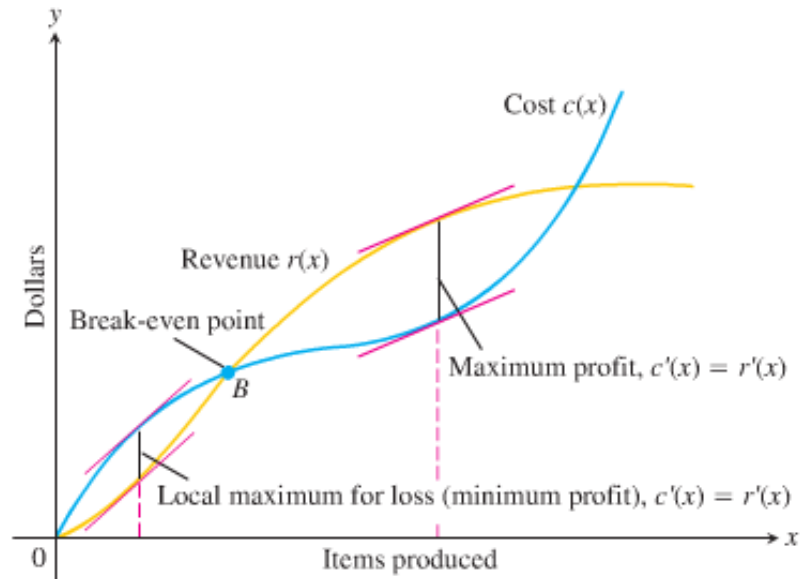
Example from *Economics*

$r(x)$ = Revenue from selling x items

$c(x)$ = Cost of producing the x items

$p(x) = r(x) - c(x)$ Profit from producing and selling x items

At a production level yielding maximum profit, marginal revenue equals to marginal cost.



Marginal Analysis

Profit = P Revenue = R Cost = C $P = R - C$

The derivatives of these quantities are called *Marginal*

$$\frac{dP}{dx} = \text{Marginal Profit}$$

$$\frac{dR}{dx} = \text{Marginal Revenue}$$

$$\frac{dC}{dx} = \text{Marginal Cost}$$

Example

Suppose that $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$, where x represents millions of MP3 players produced. Is there a production level that maximizes profit? If so, what is it?

Solution

$$r'(x) = 9 \quad c'(x) = 3x^2 - 12x + 15$$

To find the intersection between the 2 derivatives, set $r'(x) = c'(x)$

$$3x^2 - 12x + 15 = 9$$

$$3x^2 - 12x + 6 = 0 \rightarrow \begin{cases} x = 2 - \sqrt{2} \approx .586 \\ x = 2 + \sqrt{2} \approx 3.414 \end{cases}$$

The possible productions are $x \approx 0.586$ or $x \approx 3.414$ million.

$$p(x) = r(x) - c(x) = 9x - x^3 + 6x^2 - 15x$$

$$p(x) = -x^3 + 6x^2 - 6x$$

$$p'(x) = -3x^2 + 12x - 6$$

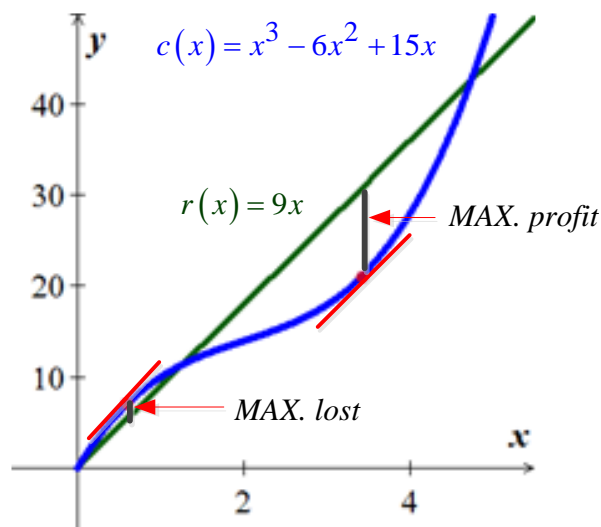
$$p''(x) = -6x + 12 = 0 \rightarrow \boxed{x = 2}$$

Concave down: $(0, 2)$

Concave up: $(2, \infty)$

The maximum profit is about $x \approx 3.414$ million

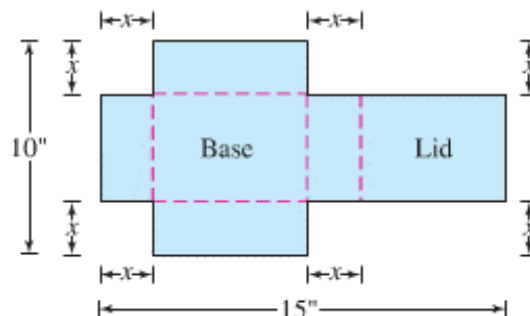
The maximum loss is about $x \approx .586$ million



Exercises Section 3.3 – Applied Optimization

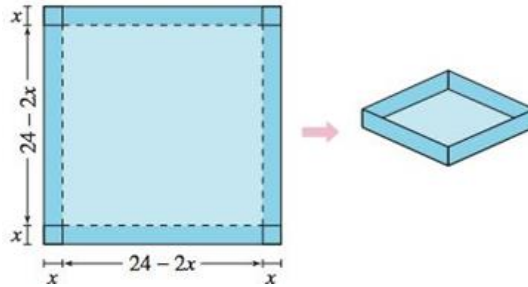
1. Find two nonnegative numbers x and y for which $2x + y = 30$, such that xy^2 is maximized.
2. A rectangular page will contain 54 in^2 of print. The margins at the top and bottom of the page are 1.5 inches wide. The margins on each side are 1 inch wide. What should the dimensions of the page be to minimize the amount of paper used?
3. A rectangular page in a textbook (with width x and length y) has an area of 98 in^2 , top and bottom margins set at 1 in. , left and right margins set at $\frac{1}{2} \text{ in.}$ The printable area of the page is the rectangle that lies within the margins. What are the dimensions of the page that maximize the printable area?
4. What point on the graph of $f(x) = \frac{5}{2} - x^2$ is closest to the origin? (*Hint:* You can minimize the square of the distance.)
5. A line segment of length 10 joins the points $(0, p)$ and $(q, 0)$ to form a triangle in the first quadrant. Find the values of p and q that maximize the area of the triangle.
6. A metal cistern in the shape of a right circular cylinder with volume $V = 50 \text{ m}^3$ needs to be painted each year to reduce corrosion. The paint is applied only to surfaces exposed to the elements (the outside cylinder wall and the circular top). Find the dimensions r and h of the cylinder that minimize the area of the painted surfaces.
7. The product of two numbers is 72. Minimize the sum of the second number and twice the first number
8. Verify the function $V = 27x - \frac{1}{4}x^3$ has an absolute maximum when $x = 6$. What is the maximum volume?
9. A net enclosure for golf practice is open at one end. The volume of the enclosure is $83\frac{1}{3}$ cubic meters. Find the dimensions that require the least amount of netting.
10. Find two numbers x and y such that their sum is 480 and x^2y is maximized.
11. If the price charged for a candy bar is $p(x)$ cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 82 - \frac{x}{20}$. How many candy bars must be sold to maximize revenue?

12. $S(x) = -x^3 + 6x^2 + 288x + 4000$; $4 \leq x \leq 20$ is an approximation to the number of salmon swimming upstream to spawn, where x represents the water temperature in degrees Celsius. Find the temperature that produces the maximum number of salmon.
13. A company wishes to manufacture a box with a volume of 52 cubic feet that is open on top and is twice as long as it is wide. Find the width of the box that can be produced using the minimum amount of material.
14. A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$3 per foot for two opposite sides, and \$4 per foot for the other two sides. Find the dimensions of the field of area 730 square feet that would be the cheapest to enclose.
15. A manufacturer wants to design an open box that has a square base and a surface area of 108 in^2 . What dimensions will produce a box with a maximum volume?
16. A company wants to manufacture cylinder aluminum can with a volume 1000 cm^3 . What should the radius and height of the can be to minimize the amount of aluminum used?
17. What is the smallest perimeter possible for a rectangle whose area is 16 in^2 , and what are its dimensions?
18. You are planning to make an open rectangular box from an 8-in. by 15-in. piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way, and what is its volume?
19. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?
20. A piece of cardboard measures 10-in. by 15-in. Two equal squares are removed from the corners of 10-in. side. Two equal rectangles are removed from the other corners so that the tabs can be folded to form a rectangular box with lid.

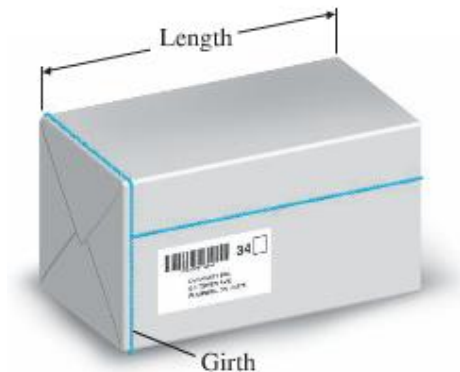


- a) Write a formula $V(x)$ for the volume of the box
- b) Find the domain of V for the problem situation and graph V over this domain
- c) Use the graphical or analytically method to find the maximum volume and the value of x that gives it.

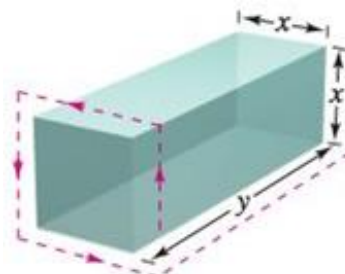
21. An open box of maximum volume is to be made from a square piece of material, 24 *inches* on a side, by cutting equal squares from the corners and turning up the sides.



- Write the volume V as a function of x .
 - Find the critical number of the function and find the maximum value.
 - Graph the function and verify the maximum volume from the graph.
22. A manufacturer wants to design an open box having square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?
23. A rectangular solid (with a square base) has a surface area of 337.5 cm^2 . Find the dimensions that will result in a solid with maximum volume.
24. A parcel delivery service will deliver a package only if the length plus girth (distance around) does not exceed 108 *inches*.
- Find the dimensions of a rectangular box with square ends that satisfies the delivery service's restriction and has maximum volume. What is the maximum volume?
 - Find the dimensions (radius and height) of a cylinder container that meets the delivery service's requirement and has maximum volume. What is the maximum volume?



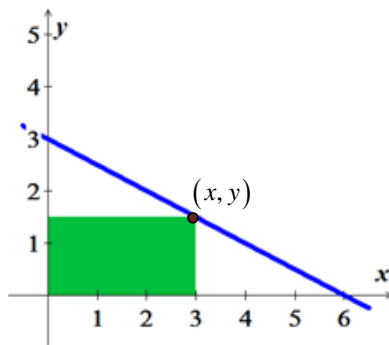
25. A rectangular package to be sent by a postal service can have a maximum combined length and girth (perimeter of a cross section) of 108 *inches*. Find the dimensions of the package of maximum volume that can be sent.



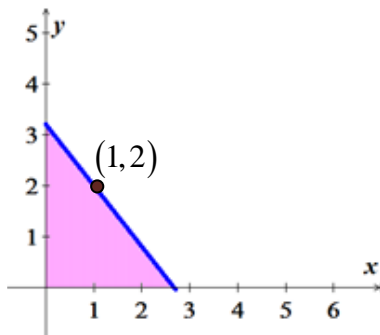
26. A page is to contain 30 *square inches* of print. The margins at the top and bottom of the page are 2 *inches* wide. The margins on the sides are 1 *inch* wide. What dimensions will minimize the amount of paper used?
27. A rectangular page is to contain 24 *squares inches* of print. The margins at the top and bottom of the page are $1\frac{1}{2}$ *inches*, and the margins on the left and right are to be 1 *inch*. What should the dimensions of the page be so that the least amount of paper used?



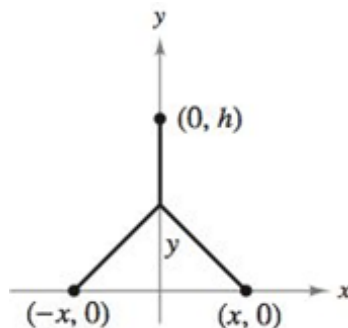
28. A rectangle has its base on the x -axis and its upper vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?
29. Find the points of $y = 4 - x^2$ that are closest to $(0, 3)$
30. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$?
31. A rectangle is bounded by the x - and y -axes and the graph of $y = \frac{1}{2}(6 - x)$. What length and width should the rectangle have so that its area is a maximum?



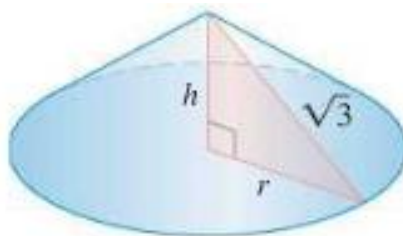
32. A right triangle is formed in the first quadrant by the x - and y -axes and a line through the point $(1, 2)$.



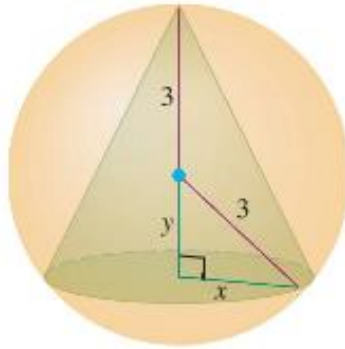
- Write the length L of the hypotenuse as a function of x .
 - Graph the function and approximate x graphically such that the length of the hypotenuse is a minimum.
 - Find the vertices of the triangle such that its area is a minimum.
33. Two factories are located at the coordinates $(-x, 0)$ and $(x, 0)$, and their power is at $(0, h)$. Find y such that the total length of power line from the power supply to the factories is a minimum.



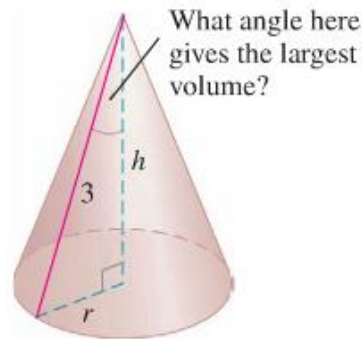
34. A right triangle whose hypotenuse is $\sqrt{3} m$ long is revolved about one of its legs to generate a right circular cone. Find the radius, height, and volume of the cone of greatest volume that can be made this way.



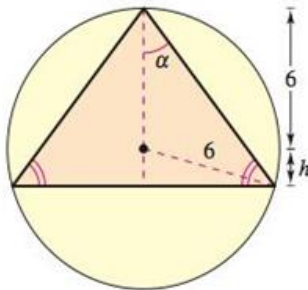
35. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.



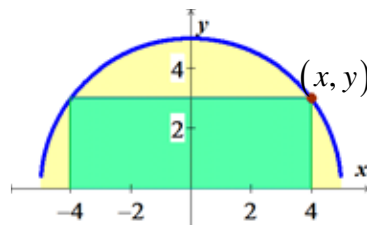
36. The slant height of the cone is 3 m. How large should the indicated angle be to maximize the cone's volume?



37. Find the area of the largest isosceles triangle that can be inscribed in a circle of radius 6.

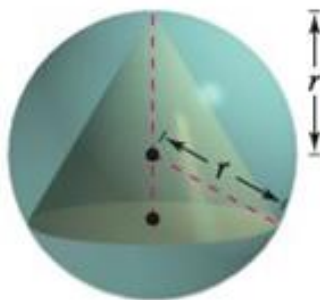


- Solve the area as a function of h .
 - Solve the area as a function of α .
 - Identify the type of triangle of maximum area.
38. A rectangle is bounded by the x -axis and the semicircle $y = \sqrt{25 - x^2}$

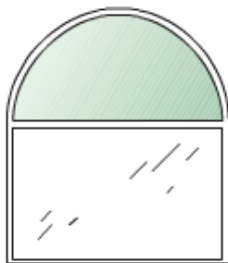


What length and width should the rectangle have so that its area is a maximum?

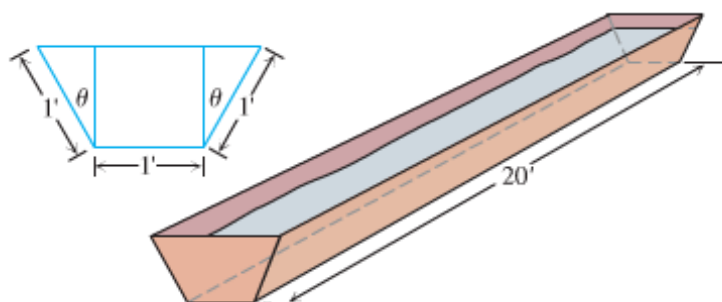
39. What are the dimensions of the lightest open-top right circular cylindrical can that will hold a volume of 1000 cm^3 ?
40. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius r .



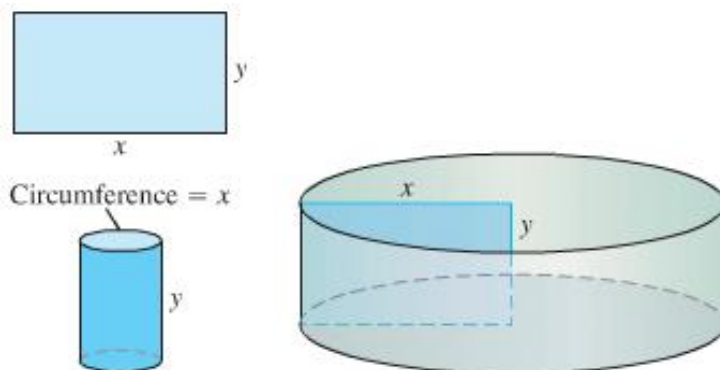
41. A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only half as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.



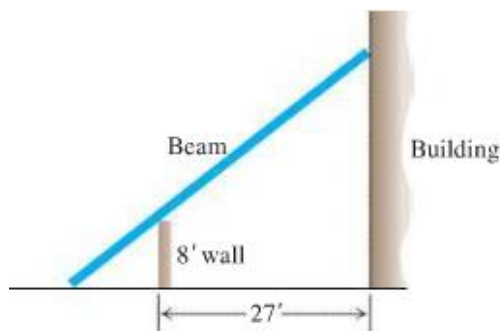
42. The cost per hour for fuel to run a train is $\frac{v^2}{4}$ dollars, where v is the speed of the train in miles per hour. (Note that the cost goes up as the square of the speed.) Other costs, including labor are \$300 per hour. How fast should the train travel on a 360-mile trip to minimize the total cost for the trip?
43. Jane is 2 mi offshore in a boat and wishes to reach a coastal village 6 mi down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?
44. The trough in the figure is to be made to the dimensions shown. Only the angle θ can be varied. What value of θ will maximize the trough's volume?



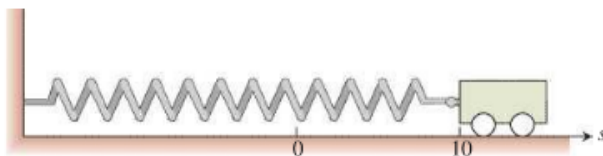
45. The height above the ground of an object moving vertically is given by $s = -16t^2 + 96t + 112$ with s in feet and t in seconds. Find
- The object's velocity when $t = 0$
 - Its maximum height and when it occurs
 - Its velocity when $s = 0$
46. Compare the answers to the following two construction problems.
- A rectangular sheet of perimeter 36 cm and dimensions x cm and y cm is to be rolled into a cylinder as shown in part (a) of the figure. What values of x and y give the largest volume?
 - The same sheet is to be revolved about one of the sides of length y to sweep out the cylinder as shown in part (b) of the figure. What values of x and y give the largest volume?



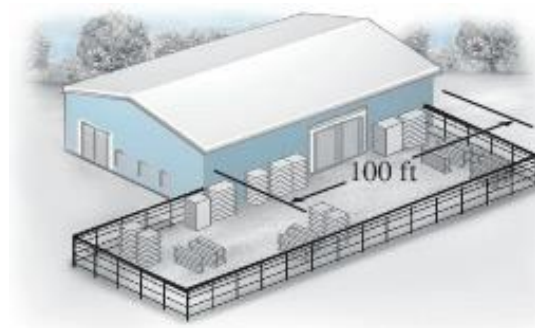
47. The 8-foot wall stands 27 feet from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.



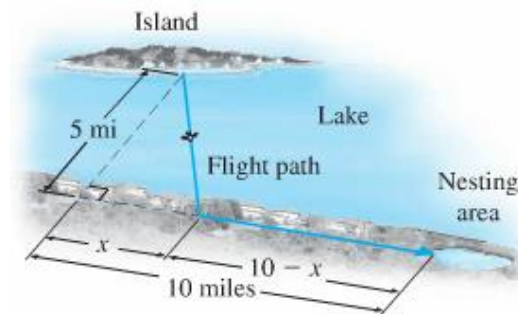
48. A small frictionless cart, attached to the wall by a spring, is pulled 10 cm from its rest position and released at time $t = 0$ to roll back and forth for 4 sec. Its position at time t is $s = 10\cos \pi t$
- What is the cart's maximum speed? When is the cart moving that fast? Where is it then? What is the magnitude of the acceleration then?
 - Where is the cart when the magnitude of the acceleration is greatest? What is the cart's speed then?



49. The owner of a retail lumber store wants to construct a fence an outdoor storage area adjacent to the store, using all of the store as part of one side of the area. Find the dimensions that will enclose the largest area if

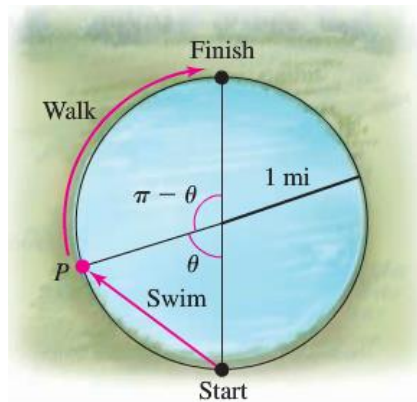


- 240 feet fencing material are used.
 - 400 feet fencing material are used.
50. Some birds tend to avoid flights over large bodies of water during daylight hours. Suppose that an adult bird with this tendency is taken from its nesting area on the edge of a large lake to an island 5 miles offshore and is then release.

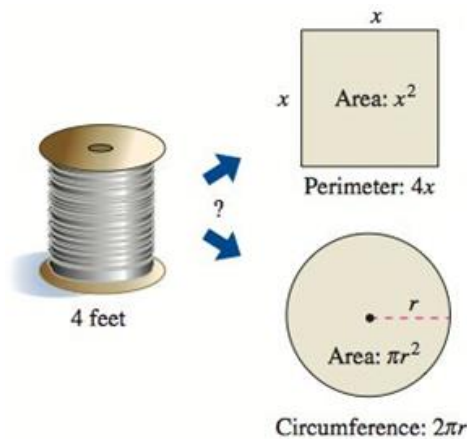


- If it takes only 1.4 times as much energy to fly over water as land, how far up the shore (x , in miles) should the bird head to minimize the total energy expended in returning to the nesting area?
 - If it takes only 1.1 times as much energy to fly over water as land, how far up the shore should the bird head to minimize the total energy expended in returning to the nesting area?
51. A pharmacy has a uniform annual demand for 200 bottles of a certain antibiotic. It costs \$10 to store one bottle for one year and \$40 to place an order. How many times during the year should the pharmacy order the antibiotic in order to minimize the total storage and reorder costs?
52. A 300-room hotel in Las Vegas is filled to capacity every night at \$80 a room. For each \$1 increase in rent, 3 fewer rooms are rented. If each rented room costs \$10 to service per day, how much should be the management charge for each room to maximize gross profit? What is the maximum gross profit?
53. A multimedia company anticipates that there will be a demand for 20,000 copies of a certain DVD during the next year. It costs the company \$0.50 to store a DVD for one year. Each time it must make additional DVDs, it costs \$200 to set up the equipment. How many DVDs should the company make during each production run to minimize its total storage and setup costs?

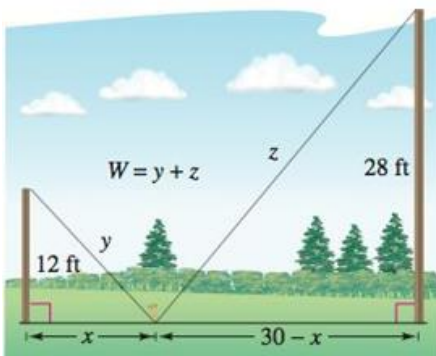
54. A university student center sells 1,600 cups of coffee per day at a price of \$2.40.
- A market survey shows that for every \$0.05 reduction in price, 50 more cups of coffee will be sold. How much should be the student center charge for a cup of coffee in order to maximize revenue?
 - A different market survey shows that for every \$0.10 reduction in the original \$2.40 price, 60 more cups of coffee will be sold. Now how much should the student center charge for a cup of coffee in order to maximize revenue?
55. Suppose you are standing on the shore of a circular pond with radius 1 *mi* and you want to get to a point on the shore directly opposite your position (on the other end of a diameter). You plan to swim at 2 *mi/hr* from your current position to another point *P* on the shore and then walk at 3 *mi/hr* along the shore to the terminal point. How should you choose *P* to minimize the total time for the trip?



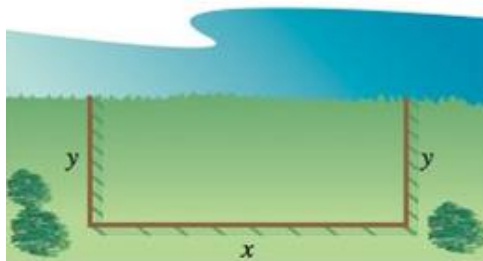
56. Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum total area?



57. Two posts, one 12 *feet* high and the other 28 *feet* high, stand 30 *feet* apart. They are to be stayed by two wires, attached to a single stake, running from ground level on the top of each post. Where should the stake be placed to use the least amount of wire?



58. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain $245,000 \text{ m}^2$ in order to provide enough grass for the herd. No fencing is needed along the river. What dimensions will require the least amount of fencing?

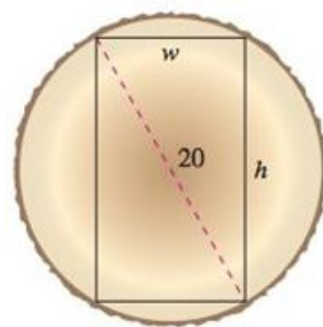


59. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area when the total perimeter is 16 feet.



60. A wooden beam has a rectangular cross section of height h and width w , the strength S of the beam is directly proportional to the width and the square of the height. What are the dimensions of the strongest beam that can be cut from a round log of diameter 20 inches?

$$S = kwh^2 \quad (k : \text{proportional constant})$$



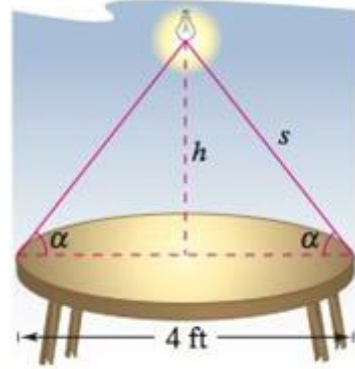
61. A light source is located over the center of a circular table of diameter 4 feet. Find the height h of the light source such that the illumination I at the perimeter of the table is maximum when

$$I = \frac{k \sin \alpha}{s^2}$$

where s is the slant height

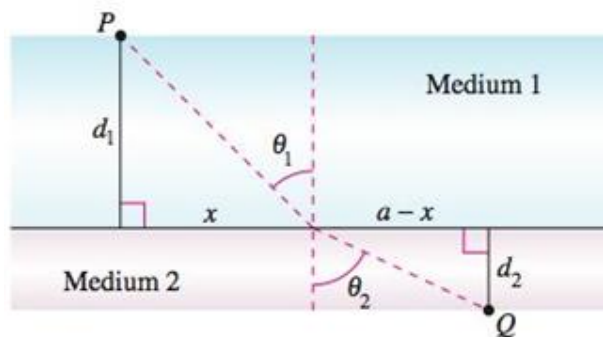
α is the angle at which the light strikes the table

k is a constant.



62. When light waves traveling in a transparent medium strike the surface of a second transparent medium, they change direction. This change of direction is called refraction and is defined by **Snell's Law of Refraction**,

$$\frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2}$$

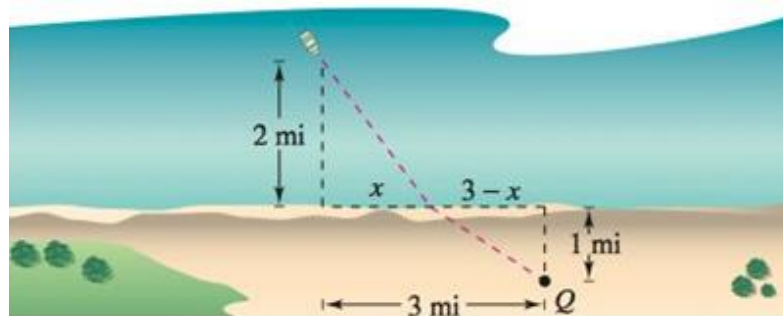


Where θ_1 and θ_2 are the magnitudes of the angles.

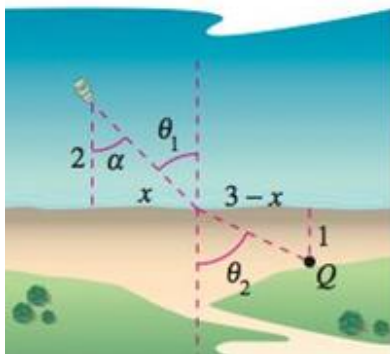
v_1 and v_2 are the velocities of light in the two media.

Show that the light waves traveling from P to Q follow the path of the minimum time.

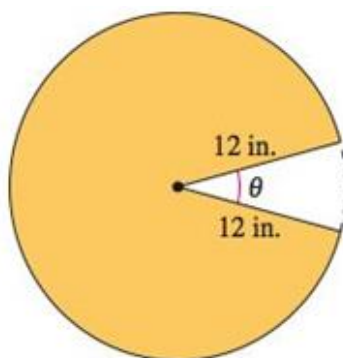
63. You are in a boat 2 miles from the nearest point on the coast. You are to go to a point Q that is 3 miles down the coast and 1 mile inland. You can row at 3 mph and walk 4 mph. Toward what point on the coast should you row in order to reach Q in the least time?



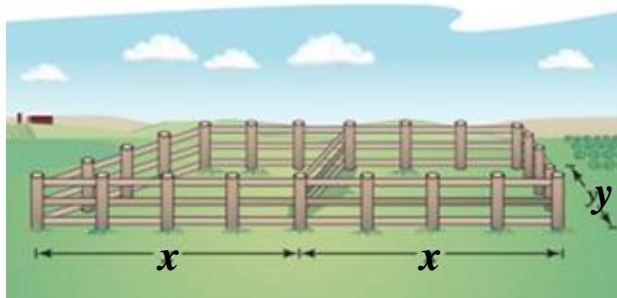
64. You are in a boat 2 miles from the nearest point on the coast. You are to go to a point Q that is 3 miles down the coast and 1 mile inland. You can row at 2 mph and walk 4 mph. Toward what point on the coast should you row in order to reach Q in the least time?



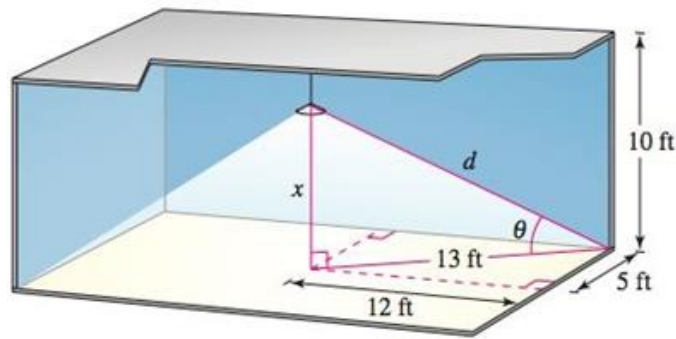
65. A sector with central angle θ is cut from a circle of radius 12 inches, and the edges of the sector are brought together to form a cone. Find the magnitude of θ such that the volume of the cone is a maximum.



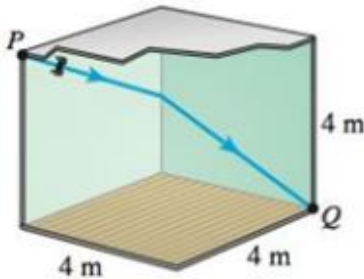
66. A rancher has 400 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?



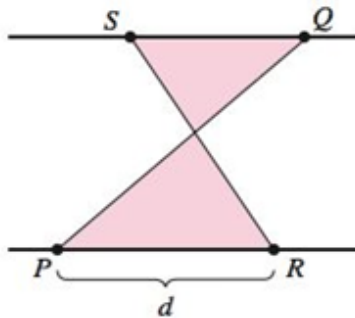
67. The amount of illumination of a surface is proportional to the intensity of the light source, inversely proportional to the square of the distance from the light source, and proportional to $\sin \theta$, where θ is the angle at which the light strikes the surface. A rectangular room measures 10 feet by 24 feet, with a 10-foot ceiling. Determine the height at which the light should be placed to allow the corners of the floor to receive as much light as possible.



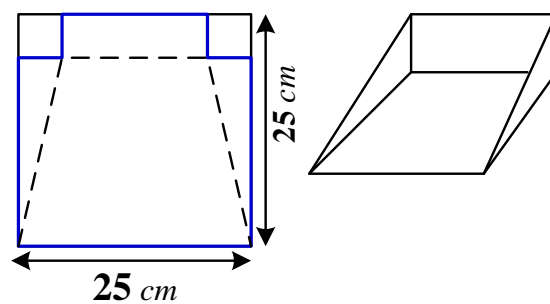
68. Consider a room in the shape of a cube, 4 meters on each side. A bug at point P wants to walk to point Q at the opposite corner. Determine the shortest path.



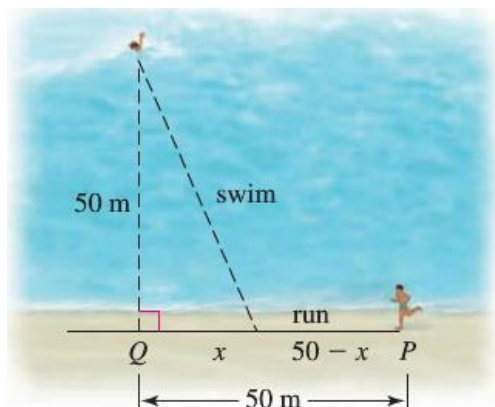
69. The line joining P and Q crosses the two parallel lines. The point R is d units from P . How far from Q should the point S be positioned so that the sum of the areas of the two shaded triangles is a minimum? So that the sum is a maximum?



70. Equal squares are cut out of two adjacent corners of a square of sheet metal having sides of length 25 cm. the three resulting flaps are bent up, to form the sides of a dustpan. Find the maximum volume of a dustpan made in this way.



71. You must get from a point P on the straight shore of a lake to a stranded swimmer who is 50 m from a point Q on the shore that is 50 m from you. If you can swim at a speed of 2 m/s and run at a speed of 4 m/s, at what point along the shore, x meters from Q , should you stop running and start swimming if you want to reach the swimmer in the minimum time?



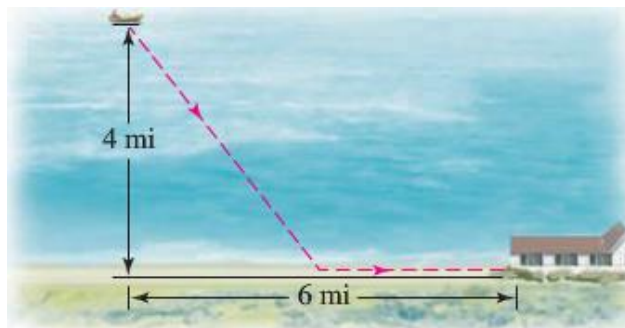
- Find the function T that gives the travel time as a function of x , where $0 \leq x \leq 50$
 - Find the critical point of T on $(0, 50)$
 - Evaluate T at the critical point and the endpoints ($x = 0$ and $x = 50$) to verify that the critical point corresponds to an absolute minimum. What is the minimum travel time?
 - Graph the function T to check your work.
72. Consider the function $f(x) = ax^2 + bx + c$ with $a \neq 0$. Explain geometrically why f has exactly one absolute extreme value on $(-\infty, \infty)$. Find the critical points to determine the value of x at which f has an extreme value.
73. An 8-foot-tall fence runs parallel to the side of a house 3 feet away. What is the length of the shortest ladder that clears the fence and reaches the house? Assume that the vertical wall of the house and the horizontal ground have infinite extent.



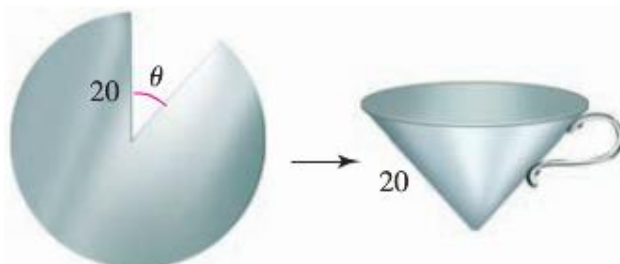
74. A man wishes to get from an initial point on the shore of a circular pond with radius 1 mi to a point on the shore directly opposite (on the other end of the diameter). He plans to swim from the initial point to another point on the shore and then walk along the shore to the terminal point.
- If he swims at 2 mi/hr. and walks at 4 mi/hr., what are the minimum and maximum times for the trip?

- b) If he swims at 2 mi/hr and walks at 1.5 mi/hr , what are the minimum and maximum times for the trip?
- c) If he swims at 2 mi/hr , what is the minimum walking speed for which it is quickest to walk the entire distance?

75. A boat on the ocean is 4 mi from the nearest point on a straight shoreline; that point is 6 mi from a restaurant on the shore. A woman plans to row the boat straight to a point on the shore and then walk along the shore to the restaurant.



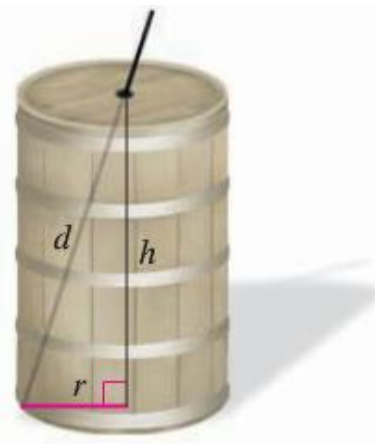
- a) If she walks at 3 mi/hr and rows at 2 mi/hr , at which point on the shore should she land to minimize the total travel time?
- b) If she walks at 3 mi/hr what is the minimum speed at which she must row so that the quickest way to the restaurant is to row directly (with no walking)?
76. A cone is constructed by cutting a sector of angle θ from a circular sheet of metal with radius 20 cm . the cut sheet is then folded up and wheeled. What angle θ maximizes the volume of the cone?



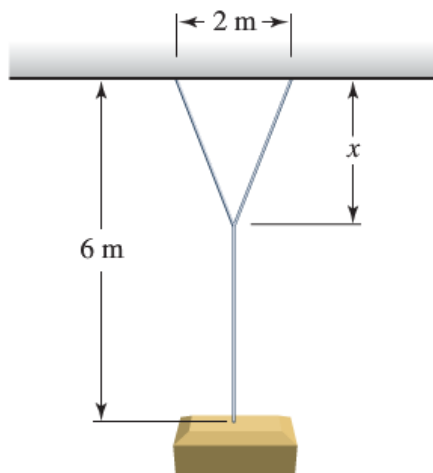
Find the radius and height of the cone with maximum volume that can be formed in this way.

77. Several mathematical stories originated with the second wedding of the mathematician and astronomer Johannes Kepler. Here is one: while shopping for wine for his wedding, Kepler noticed that the price of a barrel of wine (here assumed to be a cylinder) was determined solely by the length d of a dipstick that was inserted diagonally through a hole in the top of the barrel to the edge of the base of the barrel.

Kepler realized that this measurement does not determine the volume of the barrel and that for a fixed value of d , the volume varies the radius r and height h of the barrel. For a fixed value of d , what is the ratio r/h that maximizes the volume of the barrel?

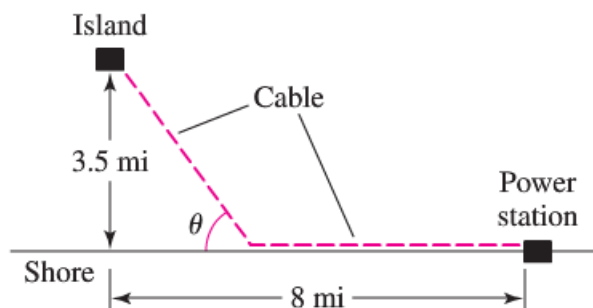


78. A load must be suspended 6 m below a high ceiling using cables attached to two supports that are 2 m apart.



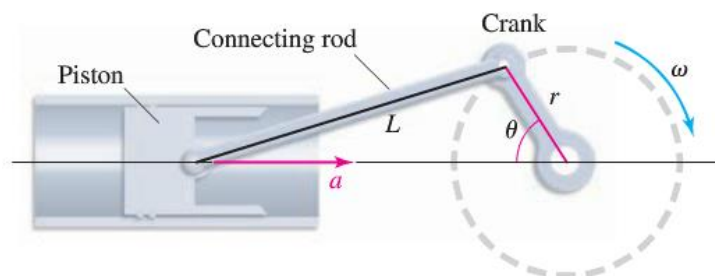
How far below the ceiling (x) should the cables be joined to minimize the total length of cable used?

79. An island is 3.5 mi from the nearest point on a straight shoreline; that point is 8 mi from a power station. A utility company plans to lay electrical cable underwater from the island to the shore and then underground along the shore to the power station. Assume that it costs $\$2,400/\text{mi}$ to lay underwater cable and $\$1,200/\text{mi}$ to lay underground cable. At what point should the underwater cable meet the shore in order to minimize the cost of the project?



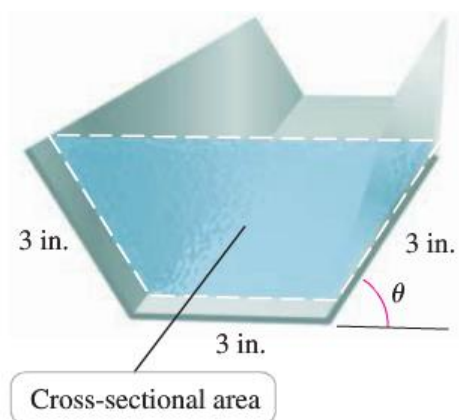
80. A crank of radius r rotates with an angular frequency ω . It is connected to a piston by a connecting rod of length L . The acceleration of the piston varies with the position of the crank according to the function

$$a(\theta) = \omega^2 r \left(\cos \theta + \frac{r \cos 2\theta}{L} \right)$$



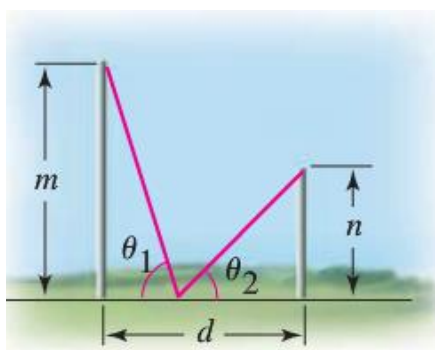
For fixed ω and r , find the values of θ , with $0 \leq \theta \leq 2\pi$, for which the acceleration of the piston is a maximum and minimum.

81. A rain gutter is made from sheets of metal 9 in wide. The gutters have a 3-in base and two 3-in sides, folded up at an angle θ .



What angle θ maximizes the cross-sectional area of the gutter?

82. Two poles of heights m and n are separated by a horizontal distance d . A rope is stretched from the top of one pole to the ground and then to the top of the other pole. Show that the configuration that requires the least amount of rope occurs when $\theta_1 = \theta_2$



83. Fermat's principle states that when light travels between two points in the same medium (at a constant speed), it travels on the path that minimizes the travel time. Show that when light from a source A reflects off of a surface and is received at point B , the angle of incidence equals the angle of reflection, or $\theta_1 = \theta_2$

