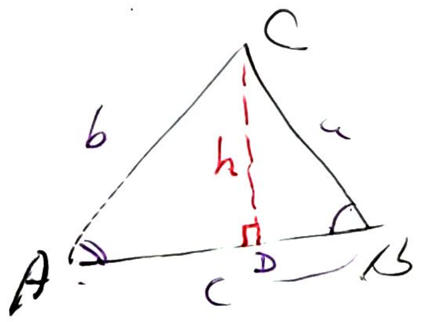
$\triangle ACD$

$$\sin A = \frac{h}{b} \Rightarrow h = b\sin A$$

$\triangle BCD$

$$\sin B = \frac{h}{a} \Rightarrow h = a\sin B$$



$$b\sin A = a\sin B$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{Law of Sines}$$

Ex $\quad A = 30^\circ \quad B = 70^\circ \quad a = 80 \quad c?$

$$C = 180^\circ - 30^\circ - 70^\circ$$
$$= 80^\circ$$

$$\frac{c}{\sin 80^\circ} = \frac{80}{\sin 30^\circ}$$

$$c = 2(80)\sin 80^\circ$$
$$= 160 \sin 80^\circ$$

$$\frac{\sin 70^\circ}{b} = \frac{\sin 30^\circ}{80}$$

$$b = \frac{80 \sin 70^\circ}{1/2}$$
$$= 160 \sin 70^\circ$$

Ex $A = 32^\circ \quad C = 81.8^\circ \quad a = 42.9$

$B = 180^\circ - 32^\circ - 81.8^\circ$

$= \boxed{66.2^\circ}$

$b = \dfrac{42.9 \sin 66.2^\circ}{\sin 32^\circ}$

$c = \dfrac{a \sin C}{\sin A} = \boxed{\dfrac{42.9 \sin 81.8^\circ}{\sin 32^\circ}}$

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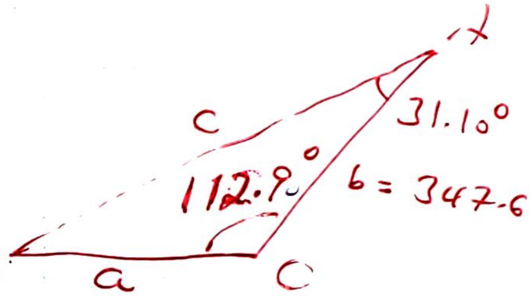
a ??

$B = 180^\circ - 112.9^\circ - 31.1^\circ$

$= \boxed{36^\circ}$



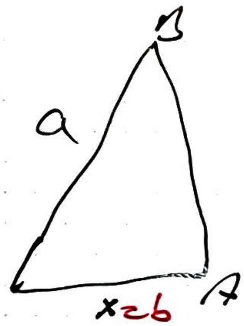
$a = \boxed{\dfrac{347.6 \sin 31.1^\circ}{\sin 36^\circ}}$

$a = \dfrac{b \sin A}{\sin B}$

≈ 305.5 ft

$a = 562 \quad B = 5.7^\circ$

$A = 85.3^\circ$



$b = \dfrac{a \sin B}{\sin A}$

$= \boxed{\dfrac{562 \sin 5.7^\circ}{\sin 85.3^\circ}}$

$0 \le \sin\theta \le 1$

$\Big\{ \theta \in QI + QII$

Ambiguous Case

1. $a = 2, \ b = 6 \quad A = 30^\circ$

$$\sin B = \frac{b \sin A}{a}$$

$$= \frac{6}{2} \sin 30^\circ$$

$$= \frac{3}{2} > 1 \ \#$$

$\therefore$ No Triangle

$C = 35.4^\circ \qquad a = 205 \qquad c = 314$

$$\sin A = \frac{a \sin C}{c}$$

$$= \frac{205}{314} \sin 35.4^\circ$$

$$A = \sin^{-1}\left(\frac{205}{314} \sin 35.4^\circ\right)$$

$$\approx 22.2^\circ$$

$A \approx 22.2^\circ$

$B = 180^\circ - 22.2^\circ - 35.4^\circ$

$\boxed{\approx 122.4^\circ}$

$A = 180^\circ - 22.2^\circ$

$\approx 157.8^\circ$

$B = 180^\circ - 157.8^\circ - 35.4^\circ$

$= -$

$\#$

$$b = \frac{c \sin B}{\sin C} = \frac{314 \sin 122.4^\circ}{\sin 35.4^\circ}$$

$\boxed{\approx 458 \text{ ft}}$

Ex $a = 54$ $b = 62$ $A = 40^\circ$

$$B = \sin^{-1}\left(\frac{62 \sin 40^\circ}{54}\right) \qquad \frac{b \sin A}{a}$$

$\approx 48^\circ$

$B \approx 48^\circ$

$C = 180^\circ - 48^\circ - 40^\circ$

$= 92^\circ$

$$c = \frac{54 \sin 92^\circ}{\sin 40^\circ}$$

≈ 84

$B = 180^\circ - 48^\circ$

$\approx 132^\circ$

$C = 180^\circ - 132^\circ - 40^\circ$

$= 8^\circ$

$$c = \frac{54 \sin 8^\circ}{\sin 40^\circ}$$

≈ 12

SAS

$K = \frac{1}{2} bc \sin A$

$= \frac{1}{2} ab \sin C$

$= \frac{1}{2} ac \sin B$

Ex $A = 24^\circ 40'$ $b = 27.3$ $C = 52^\circ 40'$

$B = 180^\circ - 24^\circ 40' - 52^\circ 40'$

$\approx 102.667^\circ$

$77^\circ 20'$

$77 + \frac{20}{60}$

$$a = \frac{27.3 \sin 24^\circ 40'}{\sin 102.667^\circ} \approx 11.7$$

$$K = \frac{1}{2}(\overset{a}{11.7})(\overset{b}{27.3}) \sin \overset{C}{52°40'}$$

$$\approx 127 \text{ cm}^2$$

11-21 $\quad b = 4 \quad c = 1 \quad A = 120°$

$$K = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2}(4)(1) \sin(120°)$$

$$= 2 \frac{\sqrt{3}}{2}$$

$$= \sqrt{3} \text{ unit}^2$$

$b, c, A.$

Law of cosine

$$a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$b = \sqrt{a^2 + c^2 - 2ac \cos B}$$

Ex $A = 60^\circ \quad b = 20 \quad c = 30$ in

$$a = \sqrt{b^2 + c^2 - 2bc\cos A}$$

$$= \sqrt{20^2 + 30^2 - 2(20)(30)\cos 60^\circ}$$

$$= 10\sqrt{4 + 9 - 12\left(\frac{1}{2}\right)}$$

$$= 10\sqrt{7} \text{ in}$$

$a, \quad b, \quad c$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow A = \cos^{-1}\frac{b^2 + c^2 - a^2}{2bc}$$

$$B = \cos^{-1}\frac{a^2 + c^2 - b^2}{2ac}$$

Ex $a = 34 \quad b = 20 \quad c = 18$

$$A = \cos^{-1}\frac{20^2 + 18^2 - 34^2}{720}$$

SSS.

Heron's Area

$s = \frac{1}{2}(a + b + c)$

$K = \sqrt{s(s-a)(s-b)(s-c)}$