$r = 1 + 2r\cos\theta$

$(\sqrt{x^2+y^2})^2 = (1+2x)^2$

$x^2 + y^2 = 1 + 4x + 4x^2$

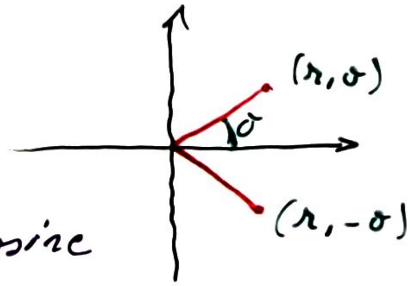
$y^2 - 3x^2 - 4x - 1 = 0$

Symmetry

1- About x-axis

$(r,\theta) \to (r,-\theta)$

$(-r, \pi+\theta)$

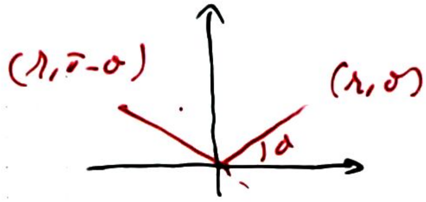


cosine

2- y-axis

$(r,\theta) \to (r, \pi-\theta)$

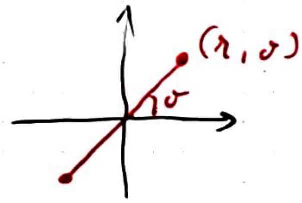
$(-r, -\theta)$



3 about the Origin

$(r,\theta) \to (r, \pi+\theta)$

$(-r, \theta)$

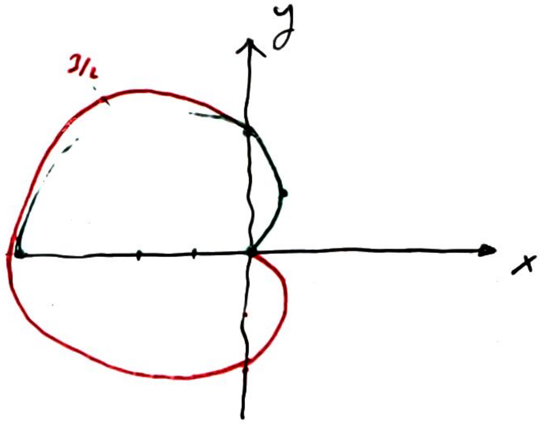


cosine & secant are even fctns

$\mathbb{R}$ are even fctn

Ex Graph $r = 1 - \cos\theta$

Soln



θ	r
0	0
60°	1/2
90	1
120	3/2
180	2

Ex Graph $r^2 = 4\cos\theta > 0$ $\quad \frac{\pi}{2} < \theta < \frac{3\pi}{2}$

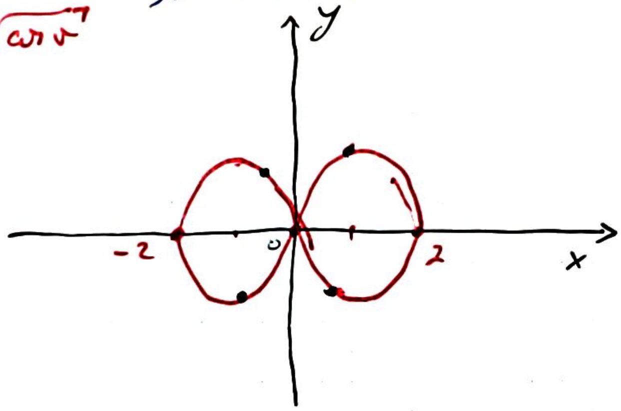
$-\theta \to 4\cos(-\theta) = 4\cos\theta = r^2 \to$ x-axis

$(-r, -\theta) \to (-r)^2 = 4\cos(-\theta) \to$ origin.

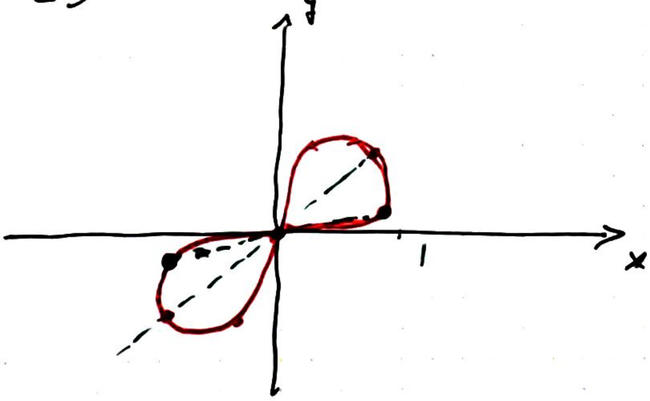
$r^2 = 4\cos\theta$ ✓

$r = \pm 2\sqrt{\cos\theta}$

θ	r
0	± 2
$-\frac{\pi}{2}$	0
$-60°$	$\pm\sqrt{2}$



Ex $r^2 = \sin 2\theta$ lemniscate $\quad \sin 2(15°) = \frac{1}{2}$



4.4 Calculus.

slope: $r = f(\theta)$

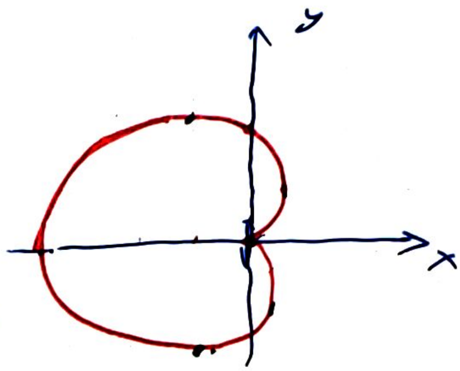
$$\begin{cases} x = r\cos\theta = f(\theta)\cos\theta \\ y = r\sin\theta = f(\theta)\sin\theta \end{cases}$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} \qquad \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

Ex $-\pi \leq \theta \leq \pi$

cardoid: $f(\theta) = 1 - \cos\theta = r$



$$\frac{dy}{dx} = \frac{\sin^2\theta + (1-\cos\theta)\cos\theta}{\sin\theta\cos\theta - (1-\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta - \cos^2\theta}{\sin\theta\cos\theta - \sin\theta + \cos\theta\sin\theta}$$

$$= \frac{1 - \cos^2\theta + \cos\theta - \cos^2\theta}{2\cos\theta\sin\theta - \sin\theta}$$

$$= -\frac{2\cos^2\theta - \cos\theta - 1}{\sin\theta(2\cos\theta - 1)}$$

$$= -\frac{(\cos\theta - 1)(2\cos\theta + 1)}{\sin\theta(2\cos\theta - 1)} = 0$$

$\cos\theta = 1$, $\theta = 0$ # $\cos\theta = \frac{-1}{2}$, $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\sin\theta \neq 0$, $\theta \neq 0, \pi$ $\cos\theta \neq \frac{1}{2}$, $\theta = \pm\frac{\pi}{3}$

$$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{0}{0}$$ L'hôpital Rule

$$= -\left.\frac{-4\cos\theta\sin\theta + \sin\theta}{2\cos 2\theta - \cos\theta}\right|_{\theta=0}$$

$$= \frac{0}{1} = 0$$ ✓

Area

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2}\int_{\alpha}^{\beta} (f(\theta))^2 \, d\theta = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

Ex A? $r = 2(1+\cos\theta)$

$$A = \frac{1}{2}\int_0^{2\pi} [2(1+\cos\theta)]^2 \, d\theta$$

$$= 2\int_0^{2\pi} (1 + 2\cos\theta + \cos^2\theta) \, d\theta$$

$$= 2\int_0^{2\pi} \left(1 + 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \int_0^{2\pi} (3 + 4\cos\theta + \cos 2\theta) \, d\theta$$

$$= 3\theta + 4\sin\theta + \frac{1}{2}\sin 2\theta \Big|_0^{2\pi}$$

$$= 6\pi \text{ unit}^2$$

Ex A? $r = 1$ & $r = 1 - \cos\theta$

inside outside

Soln $r = 1 = 1 - \cos\theta \Rightarrow \cos\theta = 0$

$\theta = \pm\frac{\pi}{2}$

$\theta = 0$

$$\text{Area} = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \Big(\overbrace{1^2}^{r_2^2} - \overbrace{(1-\cos\theta)^2}^{r_1^2}\Big)\, d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \Big(\underbrace{1-1}\, + 2\cos\theta - \underbrace{\cos^2\theta}_{\frac{1+\cos 2\theta}{2}}\Big)\, d\theta$$

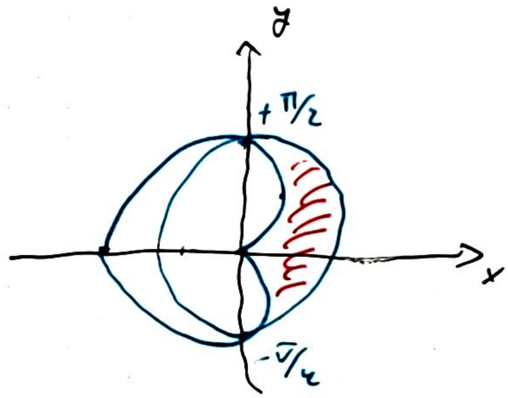
$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \Big(2\cos\theta - \frac{1}{2} - \frac{1}{2}\cos 2\theta\Big)\, d\theta$$

$$= \frac{1}{2}\Big(2\sin\theta - \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}\Big(2 - \frac{\pi}{4} + 2 - \frac{\pi}{4}\Big)$$

$$= \frac{1}{2}\Big(4 - \frac{\pi}{2}\Big)$$

$$= 2 - \frac{\pi}{4} \ \text{unit}^2$$



Length

$$L = \int_{\alpha}^{\beta} \sqrt{r^2+(r')^2}\, d\theta$$

Ex L? $r = 1-\cos\theta$

soln $0 \le \theta \le 2\pi$

$$\sqrt{r^2+(r')^2} = \sqrt{(1-\cos\theta)^2+(\sin\theta)^2}$$

$$= \sqrt{1-2\cos\theta+\cos^2\theta+\sin^2\theta}$$

$$= \sqrt{2-2\cos\theta}$$

$\sin^2\frac{\theta}{2} = \frac{1-\cos 2\theta/2}{2}$

$$= \sqrt{2(1-\cos\theta)}$$

$2\sin^2\frac{\theta}{2} = 1-\cos\theta$

$$= \sqrt{2(2\sin^2\theta/2)}$$

$$= 2\sin\theta/2$$

$$L = 2\int_0^{2\pi} \sin\frac{\theta}{2}\, d\theta$$

$$= -4\cos\frac{\theta}{2}\Big|_0^{2\pi}$$

$$= -4(-1-1)$$

$$= 8 \text{ unit}$$

Surface (revolution

About

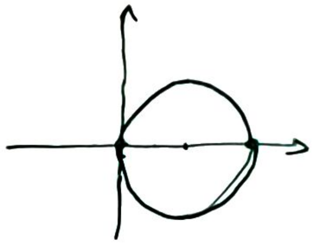
$$S = 2\pi \int_{\alpha}^{\beta} f(\theta)\sin\theta \sqrt{(f(\theta))^2 + (f'(\theta))^2}\, d\theta$$ polar axis

$$= 2\pi \int_{\alpha}^{\alpha} f(\theta)\cos\theta \sqrt{(f(\theta))^2 + (f'(\theta))^2}\, d\theta$$ line : $\theta = \frac{\pi}{2}$

Ex Area S? $f(\theta) = \cos\theta$ about line $\theta = \frac{\pi}{2}$

$0 \le \theta \le \pi$

$r = \cos\theta$



$$\sqrt{r^2 + r'^2} = \sqrt{\cos^2\theta + \sin^2\theta}$$

$$= 1$$

$$S = 2\pi \int_0^{\pi} \cos^2\theta \, d\theta$$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$$= \pi \int_0^{\pi} (1 + \cos 2\theta)\, d\theta$$

$$= \pi \left(\theta + \frac{1}{2}\sin 2\theta \right|_0^{\pi}$$

$$= \pi(\pi)$$

$$= \pi^2 \text{ unit}^2$$

#8 Inside Oval Limaçon : $r = 4 + 2\sin\theta$

soln

$$A = \frac{1}{2}\int_0^{2\pi} (4 + 2\sin\theta)^2 \, d\theta$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \frac{1}{2}\int_0^{2\pi} (16 + 16\sin\theta + 4\sin^2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} 16 + 16\sin\theta + 2 - 2\cos 2\theta)\, d\theta$$

$$= \int_0^{2\pi} (9 + 8\sin\theta - \cos 2\theta)\, d\theta$$

$$= 9\theta - 8\cos\theta - \frac{1}{2}\sin 2\theta \Big|_0^{2\pi}$$

$$= 18\pi - 8 + 8$$

$$= 18\pi \text{ unit}^2$$

\#17 Inside 1 leave of $r = \cos 3\theta$

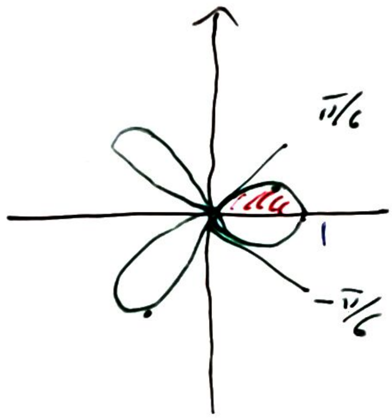
$$\text{Area} = \frac{1}{2}\int_0^{2\pi} \frac{1}{3} \cos^2 3\theta \, d\theta \qquad \cos^2 3\theta = \frac{1+\cos 6\theta}{2}$$

$$= \frac{1}{12}\int_0^{2\pi} (1+\cos 6\theta)\, d\theta$$

$$= \frac{1}{12}\left(\theta + \frac{1}{6}\sin 6\theta\right)\Big|_0^{2\pi} \quad \pi$$

$$= \frac{1}{12}(2\pi)$$

$$= \frac{\pi}{6}$$



Graphing

$$A = \frac{1}{2}\int_{-\pi/6}^{\pi/6} (\cos^2 3\theta)\, d\theta$$

$$= \frac{1}{4}\int_{-\pi/6}^{\pi/6} (1+\cos 6\theta)\, d\theta$$

$$= \frac{1}{4}\left(\theta + \frac{1}{6}\sin 6\theta\right.\Big|_{-\pi/6}^{\pi/6}$$

$$= \frac{1}{4}\left(\frac{\pi}{6}+\frac{\pi}{6}\right)$$

$$= \frac{\pi}{12} \text{ unit}^2$$