Math 2415 – Calculus III **_Exam_ 3** **_Review_**

Professor: Fred Khoury

1. Evaluate the double integrals

a) $\displaystyle\int_1^{10}\int_0^{1/y} ye^{xy}\,dxdy$

b) $\displaystyle\int_0^{1}\int_{\sqrt{y}}^{2-\sqrt{y}} xy\ dxdy$

c) $\displaystyle\int_0^{1}\int_{x^2}^{x} \sqrt{x}\ dydx$

d) $\displaystyle\int_0^{3/2}\int_{-\sqrt{9-4y^2}}^{\sqrt{9-4y^2}} y\,dxdy$

e) $\displaystyle\int_0^{2}\int_0^{4-x^2} 2x\ dydx$

f) $\displaystyle\int_0^{1}\int_{2y}^{2} 4\cos\left(x^2\right)\ dxdy$

g) $\displaystyle\int_0^{1}\int_{\sqrt[3]{y}}^{1} \frac{2\pi\sin \pi x^2}{x^2}\ dxdy$

2. Find the area of the region enclosed by the line $y = 2x + 4$ and the parabola $y = 4 - x^2$ in the *xy*-plane.

3. Find the area of the region enclosed by the lines $y = -x - 4,\ \ y = x\ \ and\ \ y = 2x - 4$

4. Find the volume under the parabolic cylinder $z = x^2$ above the region enclosed by the parabola $y = 6 - x^2$ and the line $y = x$ in the *xy*-plane

5. Evaluate the integral by changing to polar coordinates

a) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2dydx}{\left(1+x^2+y^2\right)^2}$

b) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln\left(x^2+y^2+1\right)dxdy$

6. Evaluate the integrals

a) $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \cos(x+y+z)\,dxdydz$

b) $\displaystyle\int_1^{e}\int_1^{x}\int_0^{z} \frac{2y}{z^3}\,dydzdx$

c) $\displaystyle\int_{\ln 6}^{\ln 7}\int_0^{\ln 2}\int_{\ln 4}^{\ln 5} e^{(x+y+z)}\,dzdydx$

d) $\displaystyle\int_0^{1}\int_0^{x^2}\int_0^{x+y} (2x-y-z)\,dzdydx$

7. Convert $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} 3dz\ rdrd\theta,\qquad r \ge 0$

a) Rectangular coordinates with order of integration dzd*x*d*y*.
b) Spherical coordinates
c) Evaluate one of the integrals.

8. Set up an integral in rectangular coordinates equivalent to the integral

$$\int_0^{\pi/2}\int_1^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r^3(\sin\theta\cos\theta)z^2\ dzdrd\theta$$

Arrange the order of integration to be z first, then y, then x.

9. Evaluate the spherical coordinate integral

$a)\ \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{3}\rho^2\sin\phi\ d\rho d\phi d\theta$

$c)\ \int_0^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{\sin\theta} 2\cos\phi\ \rho^2\ d\rho d\theta d\phi$

$b)\ \int_0^{2\pi}\int_0^{\pi/4}\int_0^{3}\rho^3\cos\phi\sin\phi\ d\rho d\phi d\theta$

$c)\ \int_0^{\pi}\int_0^{\frac{\pi}{4}}\int_{2\sec\varphi}^{4\sec\varphi}\sin\varphi\ \rho^2\ d\rho d\varphi d\theta$

10. Evaluate

a) $\iint_R \frac{2y}{\sqrt{x^4+1}}dA;$ *R is the region bounded by* $x=1,\quad x=2,\quad y=x^{3/2},\quad y=0$

b) $\iint_R (x+y)dA;$ *R is the disk bounded by the circle* $r=4\sin\theta$

c) $\iint_R (x^2+y^2)dA;$ *R is the region* $\{(x,y);\ 0\le x\le 2,\ 0\le y\le x\}$

11. Let R be the region bounded by the lines $x+y=1;\quad x+y=4;\quad x-2y=0;\quad x-2y=-4$

Evaluate the integral $\iint_R 3xydA$

12. Let R be the region bounded by the square with vertices (0, 1), (1, 2), (2, 1), & (1, 0).

Evaluate the integral $\iint_R (x+y)^2\sin^2(x-y)dA$

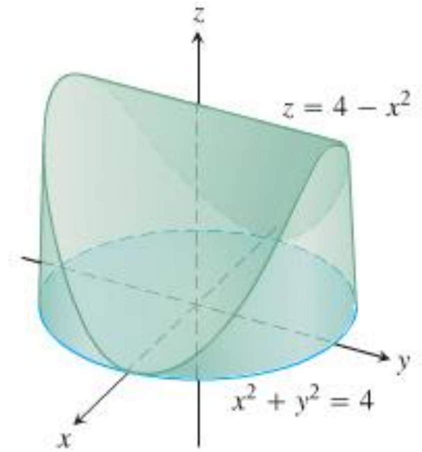
13. Evaluate $\iiint_D yzdV$ D is bounded by the planes: $x+2y=1,\ x+2y=2,\ x-z=0,\ x-z=2,$ $2y-z=0,$ and $2y-z=3$

14. Find the center of mass (centroid) of the thin constant-density plates. The region bounded by $y=\sin x$ and $y=0$ between $x=0$ and $x=\pi$

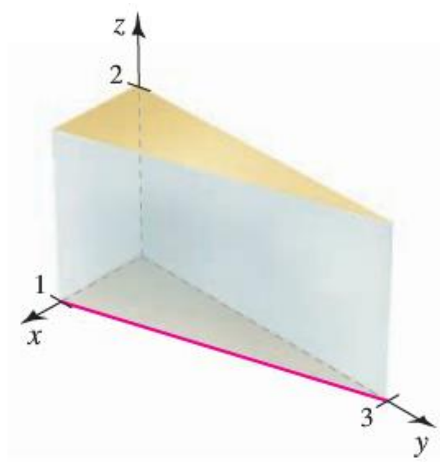
15. Find the center of mass of the solid, assuming a constant density. The paraboloid bowl bounded by $z=x^2+y^2$ and $z=36$

16. Find the coordinates of the center of mass of the solid with the upper half of a ball $\left\{(\rho,\ \varphi,\ \theta):\ 0 \le \rho \le 16,\ 0 \le \varphi \le \frac{\pi}{2},\ 0 \le \theta \le 2\pi\right\}$ with density $f(\rho,\ \varphi,\ \theta) = 1 + \frac{\rho}{4}$

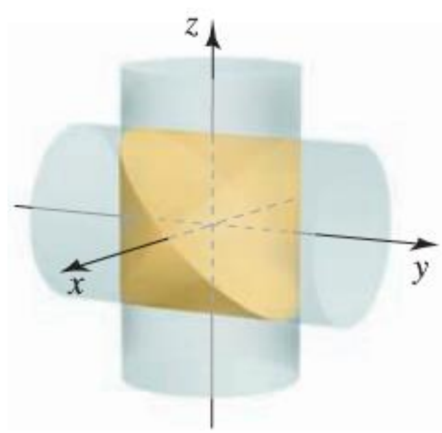
17. Find the volume of the solid that is bounded above by the cylinder $z = 4 - x^2$, on the sides by the cylinder $x^2 + y^2 = 4$, and below by the *xy*-plane.



18. Find the volume of the prism in the first octant bounded by the planes $y = 3 - 3x$ *and* $z = 2$

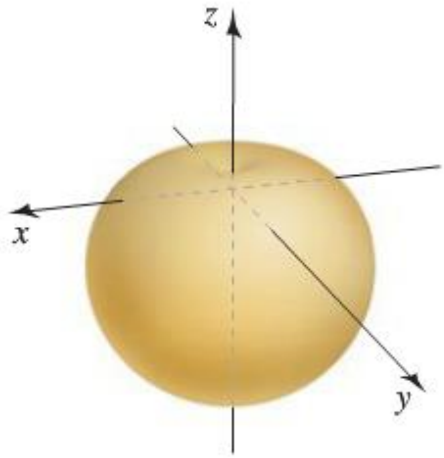


19. Find the volume of the prism in the first octant bounded by the planes $x^2 + y^2 = 4$ *and* $x^2 + z^2 = 4$



20. Find the volume of the cardioid of revolution

$$D=\left\{(\rho,\varphi,\theta):\ 0\le\rho\le\frac{1-\cos\varphi}{2},\ 0\le\varphi\le\pi,\ 0\le\theta\le 2\pi\right\}$$



21. A cake is shaped like a solid cone with radius 4 and height 2, with its base on the *xy*-plane. A wedge of the cake is removed by making two slices from the axis of the cone outward, perpendicular to the *xy*-plane separated by an angle of Q radians, where $0<Q<2\pi$

a) Use a double integral to find the volume of the slice for $Q=\frac{\pi}{4}$. Use geometry to check your answer.

b) Use a double integral to find the volume of the slice for $0<Q<2\pi$. Use geometry to check your answer.

22. A spherical fish tank with a radius of 1 *ft* is filled with water to a level 6 *in*. below the top of the tank.

a) Determine the volume and weight of the water in the fish tank. (The weight density of water is about 62.5 lb/ft^3.)

b) How much additional water must be added to completely fill the tank?

Solution

1. ***a***) $9e-9$ ***b***) $\frac{1}{5}$ ***c***) $\frac{4}{35}$ ***d***) $\frac{9}{2}$ ***e***) 8 ***f***) $\sin 4$ ***g***) 2

2. $\frac{4}{3}\ unit^2$

3. $12\ \ unit^2$

4. $\frac{125}{4}\ unit^3$

5. ***a***) π ***b***) $\pi(\ln 4-1)$

6. ***a***) 0 ***b***) 1 ***c***) 1 ***d***) $\frac{8}{35}$

7. ***a***) $\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2+y^2}} dzdxdy$ ***b***) $\int_0^{2\pi}\int_0^{\pi/4}\int_0^2 3\sin\phi d\rho d\phi d\theta$ ***c***) $2\pi\left(8-4\sqrt{2}\right)$

8. $\int_0^1\int_{\sqrt{1-x^2}}^{\sqrt{3-x^2}}\int_1^{\sqrt{4-x^2-y^2}} z^2yxdzdydx+\int_1^{\sqrt{3}}\int_0^{\sqrt{3-x^2}}\int_1^{\sqrt{4-x^2-y^2}} z^2yxdzdydx$

9. ***a***) $9\pi\left(2-\sqrt{2}\right)$ ***b***) $\frac{81\pi}{8}$ ***c***) $\frac{8}{9}$ ***d***) $\frac{28\pi}{3}$

10. ***a***) $\frac{1}{2}\left(\sqrt{17}-\sqrt{2}\right)$ ***b***) 8π ***c***) $\frac{16}{3}$

11. $\frac{164}{9}$

12. $\frac{13}{3}(2-\sin 2)$

13. $\frac{17}{16}$

14. $\frac{\pi}{4}$

15. $(0,\ 0,\ 24)$

16. $\left(0,\ 0,\ \frac{63}{10}\right)$

17. $12\pi\ unit^3$

18. $3\ unit^3$

19. $\frac{128}{3}\ unit^3$

20. $\frac{\pi}{3}\ unit^3$

21. ***a***) $\frac{4\pi}{3}$ ***b***) $\frac{16}{3}Q$

22. ***a***) $V=\frac{9\pi}{8},\ W=220.893\ lb$ ***b***) $\frac{5\pi}{24}$