

$$f(x) = \frac{(x+1)^2}{1+x^2}$$

\mathbb{R}

$$\begin{array}{ccc} 1 & 2 & 1 \\ 1 & 0 & 1 \end{array}$$

CN

$$f'(x) = \frac{-2x^2 + 2}{(1+x^2)^2} = 0 \quad -2(x^2-1)(1+x^2)^2$$

$$x^2 = 1 \Rightarrow \underline{\text{CN: } x = \pm 1}$$

$$f''(x) = -2 \frac{2x(1+x^2) - 4x(x^2-1)}{(1+x^2)^3} \quad \begin{array}{c|c|c} -1 & 0 & 1 \\ - & + & - \end{array}$$

$$= -2 \frac{+6x - 2x^3}{(1+x^2)^3}$$

$$= 4 \frac{x(x^2-3)}{(1+x^2)^3} = 0$$

$$\text{Pt of Inf: } x=0, x=\pm\sqrt{3}$$

$$\begin{array}{c|c|c|c} -\sqrt{3} & 0 & \sqrt{3} & \\ \hline - & + & - & + \end{array}$$

$$\begin{array}{c|c} f(x) & \\ \hline -1 & 0 \\ 1 & 2 \end{array}$$

Inc $(-1, 1)$

Dec $(-\infty, -1) (1, \infty)$

LMIN $(-1, 0)$

LMAX $(1, 2)$

Concave up $(-\sqrt{3}, 0) (\sqrt{3}, \infty)$

Concave down $(-\infty, -\sqrt{3}) (0, \sqrt{3})$

Sec 3.3 Optimization

Minimize \rightarrow ☹️
Maximize \rightarrow ☺️ $f'(c)$

Solve

1- Read.

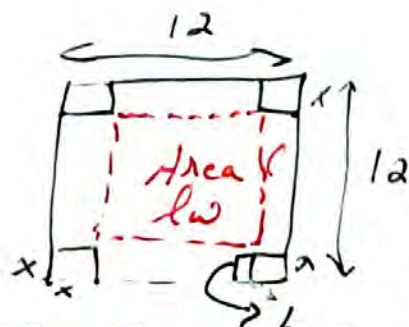
2- Draw (if)

3- read it, Given applied variables (introduction)

4- Write an eqn. (s)

5- Test C.P.

EX



$x?$ C.N.

Goal: Vol Max $h = x$ ($w = 12 - 2x$)

$$V' = l w h$$

$$= (12 - 2x)^2 x \quad (4x^2 - 48x + 144)x$$

$$= 4x^3 - 48x^2 + 144x$$

$$V' = 12x^2 - 96x + 144 = 0$$

$$\Rightarrow x^2 - 8x + 12 = 0$$

$$x = 2, 6 \} \text{ C.N.}$$

$$x = 2 \Rightarrow V = (12 - 4)^2 (2) = 128 \text{ in}^3$$

The Max. vol. is 128 in^3 w/ cutout square is equal to 2 in .

Ex $V' = 1 \text{ l.} = 10^3 \text{ cm}^3$

dim: λ, h

least material, Surf = S_{\min}



Soln $V' = \pi \lambda^2 h = 10^3 \quad (1)$

$S = 2\pi \lambda^2 + 2\pi \lambda h \quad (2)$

circle
Top & Bot.

$(1) \rightarrow h = \frac{10^3}{\pi} \frac{1}{\lambda^2} \quad (3)$

$S = 2\pi \lambda^2 + 2\pi \lambda \frac{10^3}{\pi \lambda^2}$

$S(\lambda) = 2\pi \lambda^2 + \frac{2 \cdot 10^3}{\lambda}$

$S' = 4\pi \lambda - \frac{2 \times 10^3}{\lambda^2} = 0$

$2\pi \lambda = \frac{1000}{\lambda^2}$

$\lambda^3 = \frac{500}{\pi} \Rightarrow \lambda = \sqrt[3]{\frac{500}{\pi}} \quad \frac{10}{\sqrt[3]{20\pi}}$

$(3) \rightarrow h = \frac{10^3}{\pi} \frac{1}{\left(\frac{500}{\pi}\right)^{2/3}}$

$= \frac{10^3}{(500)^{2/3}} \frac{1}{\sqrt[3]{\pi}}$

Ex $\rightarrow ? \text{ Max}$

$$A = 2xy.$$

$$\begin{aligned} A(x) &= 2x \sqrt{4-x^2} \\ &= 2x \underbrace{(4-x^2)}_{v.m}^{\underbrace{\quad}_{u.n}} \frac{1}{2} \end{aligned}$$

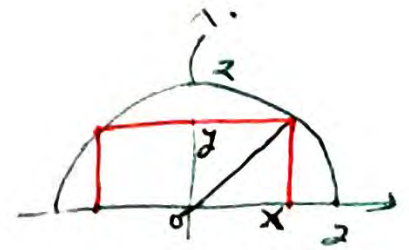
$$f'(x) = 2 \frac{4-x^2 - x^2}{\sqrt{4-x^2}}$$

$$= 4 \frac{2-x^2}{\sqrt{4-x^2}} = 0 \Rightarrow x^2 = 2$$

$$\text{C.N: } x = \pm \sqrt{2}$$

$$\boxed{x = \sqrt{2} \Rightarrow y = \sqrt{2}}$$

$$\begin{aligned} A(\sqrt{2}) &= 2\sqrt{2}\sqrt{2} \\ &= 4 \text{ unit}^2 \end{aligned}$$



$$x^2 + y^2 = 4$$

$$y = \sqrt{4-x^2}$$

Economics.

$P = \text{Profit}$

$R = \text{Revenue}$

$C = \text{Cost.}$

$$\left. \begin{array}{l} P = \text{Profit} \\ R = \text{Revenue} \\ C = \text{Cost.} \end{array} \right\} P = R - C$$

$$\frac{dP}{dx} = \text{Marginal Profit}$$

$$\frac{dR}{dx} = \text{Revenue}$$

$$\frac{dC}{dx} = \text{Cost.}$$

Point of inflection: Break-even point.
diminishing point



EX

$$r = 9x$$

$$C(x) = x^3 - 6x^2 + 15x$$

$x_{\text{mil.}}$

$P_{\text{Max}}?$

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 9x - x^3 + 6x^2 - 15x \\ &= \underline{-x^3 + 6x^2 - 6x} \end{aligned}$$

Break-even, $P'(x) = C'(x)$

$$P'(x) = -3x^2 + 12x - 6$$

$$P''(x) = -6x + 12 = 0 \Rightarrow \underline{x = 2}$$

$$\begin{array}{c|c|c} 0 & 2 & \\ \hline & + & - \end{array}$$

Concave up $(0, 2)$ concave down $(2, \infty)$



$$P'(x) = C'(x)$$

$$9 = 3x^2 - 12x + 15$$

$$3x^2 - 12x + 6 = 0$$

$$x^2 - 4x + 2 = 0$$

$$\left. \begin{array}{l} 3x^2 - 12x + 6 = 0 \\ x^2 - 4x + 2 = 0 \end{array} \right\} x = 2 \pm \sqrt{2}$$

$$\left\{ \begin{array}{l} x = 2 - \sqrt{2} \rightarrow \text{Max loss} \\ x = 2 + \sqrt{2} \rightarrow \text{Max profit} \end{array} \right.$$

$$\left\{ \begin{array}{l} x = 2 - \sqrt{2} \rightarrow \text{Max loss} \\ x = 2 + \sqrt{2} \rightarrow \text{Max profit} \end{array} \right.$$

3.3
#1

$y \geq 0$ $2x + y = 30$ $\exists \frac{xy^2}{2} \text{ Max?}$

① $\Rightarrow y = 30 - 2x$ ②

② $\Rightarrow M = x(30 - 2x)^2$
 $= 4x^3 - 120x^2 + 900x$

$M' = 12x^2 - 240x + 900 = 0$

$x^2 - 20x + 75 = 0$

$\begin{cases} x = 5 & \text{③} \rightarrow y = 20 \\ x = 15 & \rightarrow y = 0 \end{cases}$ #

$x = 5, y = 20$

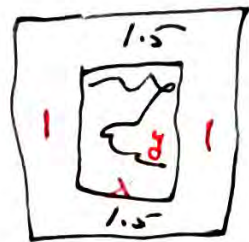
$M = 5(20)^2$
 $= 2000$ } ☺

11.2

$$A_i = 54 \text{ in}^2$$

dim?

$$A_i = xy = 54 \quad (1) \rightarrow y = \frac{54}{x}$$



$$A = (x+2)(y+3)$$

$$= xy + 3x + 2y + 6$$

$$= 54 + 3x + \frac{108}{x} + 6$$

$$A(x) = 3x + \frac{108}{x} + 60$$

$$A' = 3 - \frac{108}{x^2} = 0 \Rightarrow 3 = \frac{108}{x^2}$$

$$x^2 = 36$$

$$x = 6 \Rightarrow y = \frac{54}{6} = 9$$

$$\begin{cases} x+2 = 6+2 = 8 \\ y+3 = 12 \end{cases}$$

dimension of paper: 8×12

#12

$$S(x) = -x^3 + 6x^2 + 288x + 4,000$$

$$4 \leq x \leq 20$$

$$S' = -3x^2 + 12x + 288 = 0$$

$$-x^2 + 6x + 96 = 0$$

$$\left. \begin{array}{l} x = -8 \\ x = 12 \end{array} \right\} \text{ #}$$

#21. Circum. (dist. around) = $P = 2(x+y)$

$$l + G = 108$$

$$L + 4x = 108 \Rightarrow \underline{L = 108 - 4x}$$

a) $V = x^2 L$

$$= x^2 (108 - 4x)$$

$$= 108x^2 - 4x^3$$

$$V' = 216x - 12x^2 = 0 \quad x = 0 \text{ #}$$

$$\underline{Lx = \frac{216}{12} = 18}$$

$$L = 108 - 4(18) = \underline{36 \text{ in}}$$

$$\underline{V = 18^2 (36) \text{ in}^3}$$

b) $L + 2\pi r = 108 \rightarrow L = 108 - 2\pi r$

$$V = \pi r^2 (108 - 2\pi r)$$

$$= 108\pi r^2 - 2\pi^2 r^3$$

$$V' = 216\pi r - 6\pi^2 r^2$$

$$= 6\pi r (36 - \pi r) = 0 \quad \left. \begin{array}{l} r = 0 \text{ #} \\ r = \frac{36}{\pi} \end{array} \right\}$$

$$L = 108 - 72 = 36$$