Section R.2 – Linear Equations and Rational Equations

Definition of a Linear Equation

A linear equation in one variable x is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real number, and $a \neq 0$

Addition and Multiplication Properties of Equalities

If $a = b$, *then* $a + c = b + c$

If $a = b$, *then* $ac = bc$

Example

Solve: $3(2x - 4) = 7 - (x + 5)$

Solution

$6x - 12 = 7 - x - 5$
$6x - 12 + x = 2 - x + x$
$7x - 12 = 2$
$7x - 12 + 12 = 2 + 12$
$7x = 14$
$\frac{7}{7}x = \frac{14}{7}$
$x = 2$

Example

Solve: $\frac{2x+4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$

Solution

$(12)\frac{2x+4}{3} + (12)\frac{1}{2}x = (12)\frac{1}{4}x - (12)\frac{7}{3}$
$4(2x + 4) + 6x = 3x - 28$
$8x + 16 + 6x = 3x - 28$
$14x + 16 = 3x - 28$
$14x - 3x = -28 - 16$
$11x = -44$
$x = -4$

Example

Solve: $\frac{5}{2x} = \frac{17}{18} - \frac{1}{3x}$

Solution

$(18x)\frac{5}{2x} = (18x)\frac{17}{18} - (18x)\frac{1}{3x}$ *Restriction:* $\boldsymbol{x \neq 0}$

$45 = 17\boldsymbol{x} - 6$

$45 + 6 = 17\boldsymbol{x}$

$17\boldsymbol{x} = 51$

$\underline{x = 3}$

Example

Solve: $\frac{x}{x-2} = \frac{2}{x-2} - \frac{2}{3}$

Solution

$3(x-2)\frac{x}{x-2} = 3(x-2)\frac{2}{x-2} - 3(x-2)\frac{2}{3}$ *Restriction*: $\boldsymbol{x \neq 2}$

$3\boldsymbol{x} = 6 - 2(\boldsymbol{x} - 2)$

$3\boldsymbol{x} = 6 - 2\boldsymbol{x} + 4$

$3\boldsymbol{x} + 2\boldsymbol{x} = 10$

$\Rightarrow 5\boldsymbol{x} = 10$

$\Rightarrow \boldsymbol{x} = 2$

No Solution ***or*** $\{\varnothing\}$

Identities, Conditional Equations, and Contradictions

Example

Solve: $-2(x+4)+3x=x-8$

Solution

$-2(x+4)+3x=x-8$

$-2x-8+3x=x-8$

$x-8=x-8$

$0=0$ *True statement*

Solution: All real numbers

Example

Solve: $5x-4=11$

Solution

$5x-4=11$

$5x=15$

$x=3$

This is a conditional equation, and its solution set is {3}

Example

Solve: $3(3x-1)=9x+7$

Solution

$3(3x-1)=9x+7$

$9x-3=9x+7$

$-3=7$ ***False statement***

This is a contradiction equation, and its solution set is empty set {∅} or null

Solving for a Specified Variable

Example

Solve

a) $I = \text{Pr}\,\text{t} \quad for\ t$

$$\frac{I}{\text{Pr}} = \frac{\text{Pr}}{\text{Pr}}\text{t}$$

$$\frac{I}{\text{Pr}} = t$$

b) $A - P = \text{Pr}\,t \quad for\ P$

$$A = Prt + P$$

$$A = P(rt + 1)$$

$$\frac{A}{rt+1} = P \qquad or \quad P = \frac{A}{rt+1}$$

c) $3(2x - 5a) + 4b = 4x - 2 \quad for\ x$

$$6x - 15a + 4b = 4x - 2$$

$$6x - 4x = 15a - 4b - 2$$

$$2x = 15a - 4b - 2$$

$$x = \frac{15a - 4b - 2}{2}$$

Example

Solve the formula $2l + 2w = P$ for $\boldsymbol{w}$

Solution

$$2\boldsymbol{w} = \text{P} - 2l$$

$$\boxed{w = \frac{P-2l}{2}}$$

Example

Solve the formula $P = C + MC$ for C

Solution

$$P = C(1 + M)$$

$$\frac{P}{1+M} = C$$

$$\boxed{C = \frac{P}{1+M}}$$

Exercises *Section* R.2 – Linear Equations and Rational Equations

(1 – 78) Solve

1. $5x-8=72$

2. $14-5x=-41$

3. $2x+6=3x-2$

4. $11x-(6x-5)=40$

5. $9x+11=7x+1$

6. $2x-7=6+x$

7. $5x-2=9x+2$

8. $3(x-2)+7=2(x+5)$

9. $3x+5-5(x+1)=6x+7$

10. $4(-2x+1)=6-(2x-4)$

11. $4(x+7)=2(x+12)+2(x+1)$

12. $6(3x-1)=8-10(10x-14)$

13. $5x-(2x-8)=35$

14. $\frac{1}{14}(3x-2)=\frac{x+10}{10}$

15. $\frac{5}{6}x-2x+\frac{4}{3}=\frac{5}{3}$

16. $\frac{7}{4}+\frac{1}{5}x-\frac{3}{2}=\frac{4}{5}x$

17. $5(x+3)+4x-3=-(2x-4)+2$

18. $2\left[x-(4+2x)+3\right]=2x+3$

19. $2x-\left\{x-\left[3x-(8x+6)\right]\right\}=2x-2$

20. $4(2x+7)=2x+22+3(2x+3)$

21. $4\left[2x-(3-x)+5\right]=-7x-2$

22. $3\left[2x-(4-x)+5\right]=7x-2$

23. $-4(2x-6)+8x=5x+24+x$

24. $-8(3x+4)+6x=4(x-8)+4x$

25. $4(x+7)=2(x+12)+2(x+1)$

26. $-6(2x+1)-3(x-4)=-15x+1$

27. $2(x-1)+3=x-3(x+1)$

28. $3(x-4)-4(x-3)=x+3-(x-2)$

29. $2-(7x+5)=13-3x$

30. $16=3(x-1)-(x-7)$

31. $5x-2(x+1)=x+(3x-5)$

33. $2\left[3x-2(2x-3)\right]=5(x-6)$

34. $.2x-.5=.1x+7$

35. $.01x+3.1=2.03x-2.96$

36. $.08x-.06(x+12)=7.72$

37. $.04(x-12)+.06x=1.52$

38. $.3(x+2)-.5(x+2)=-.2x-.4$

39. $.6(x-5)+.8(x-6)=.2x-1.8$

40. $.5x+\frac{4}{3}x=x+10$

41. $.25x+\frac{2}{3}x=x+2$

42. $\frac{1}{4}(x-2)=\frac{1}{6}(x-5)$

43. $\frac{1}{4}(3x-2)=\frac{1}{5}(x+5)$

44. $\frac{1}{9}(x+2)=\frac{1}{15}(2x+5)$

45. $\frac{1}{2}(4x+8)-16=-\frac{2}{3}(9x-12)$

46. $\frac{3}{4}(24-8x)-16=-\frac{2}{3}(6x-9)$

47. $\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$

48. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

49. $\frac{x-8}{3}+\frac{x-3}{2}=0$

50. $\frac{5}{2x}-\frac{8}{9}=\frac{1}{18}-\frac{1}{3x}$

51. $\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{x^2-16}$

52. $\frac{3x-1}{3}-\frac{2x}{x-1}=x$

53. $\frac{x}{x-2}=\frac{2}{x-2}+2$

54. $\frac{x}{x-7}=\frac{7}{x-7}+8$

55. $\frac{3x}{5}-x=\frac{x}{10}-\frac{5}{2}$

56. $2x-\frac{2x}{7}=\frac{x}{2}+\frac{17}{2}$

57. $\frac{x+3}{6}=\frac{2}{3}+\frac{x-5}{4}$

58. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

32. $7(x+1)=4\left[x-(3-x)\right]$

59. $\frac{x}{4}=2+\frac{x-3}{3}$

60. $5+\frac{x-2}{3}=\frac{x+3}{8}$

61. $\frac{x+1}{3}=5-\frac{x+2}{7}$

62. $\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}$

63. $\frac{3x+2}{x-2}+\frac{1}{x}=\frac{-2}{x^2-2x}$

64. $\frac{-4x}{x-1}+\frac{4}{x+1}=\frac{-8}{x^2-1}$

65. $\frac{4x+3}{x+1}+\frac{2}{x}=\frac{1}{x^2+x}$

66. $\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{x^2+x-6}$

67. $\frac{6}{x+1}-\frac{5}{x+2}=\frac{10}{x^2+3x+2}$

68. $3(x-4)-5(x+2)=3\left[2-(x+24)\right]-2(x-2)$

69. $(2x+3)(6x-1)-9=15x^2-(3x-2)(x-2)$

70. $(3x-1)^2-2x(x-1)=7x^2-5x+2$

71. $(2x+3)(x-1)+(x+1)(x-4)=3x^2$

72. $4x+13-\left\{2x-\left[4(x-3)-5\right]\right\}=2(x-6)$

73. $-2\left\{7-\left[4-2(1-x)+3\right]\right\}=10-\left[4x-2(x-3)\right]$

74. $2(y+2)+(y+3)^2=y(y+5)+2\left(\frac{17}{2}+y\right)$

75. $(y+1)(y-1)=(y+2)(y-3)+4$

76. $45-\left[4-2y-4(y+7)\right]=-4(1+3y)-\left[4-3(y+2)-2(2y-5)\right]$

77. $35-\left[2-3y-4(y+7)\right]=-3(1+3y)+4-3(y+2)-2(2y-5)$

78. $25-\left[2+5y-3(y+2)\right]=-3(2y-5)-\left[5(y-1)-3y+3\right]$

(79 – 100) Solve for the specific variable

79. $V=lwh$, *for* h

80. $A=\frac{1}{2}h(B+b)$, *for* B

81. $A=\frac{1}{2}h(a+b)$, *for* a

82. $S=2\pi rh+2\pi r^2$ *for* h

83. $A=\frac{1}{2}h\left(b_1+b_2\right)$, *for* h

84. $A=\frac{1}{2}h\left(b_1+b_2\right)$, *for* b_2

86. $S=P+\mathrm{Pr}t$ *for* t

87. $S=2lw+2wh+2hl$ *for* h

88. $S=2lw+2wh+2hl$ *for* w

89. $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ *for* R_1

90. $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ *for* R

91. $V=\frac{d_1-d_2}{t}$ *for* d_1

92. $V=\frac{d_1-d_2}{t}$ *for* d_2

85. $A = \frac{1}{2}h\left(b_1 + b_2\right)$, *for* b_1

93. $z = \frac{x-\mu}{s}$ *for* x

94. $z = \frac{x-\mu}{s}$ *for* μ

95. $s = \frac{1}{2}at^2 + vt$ *for* v

96. $s = \frac{1}{2}at^2 + vt$ *for* a

97. $L = a + (n-1)d$ *for* n

98. $L = a + (n-1)d$ *for* d

99. $A = \frac{x_1 + x_2 + x_3}{n}$ *for* x_2

100. $A = \frac{x_1 + x_2 + x_3}{n}$ *for* n