23. $\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array}\right]$ $R_2 - 3R_1$, $R_3 - R_1$

$R_2 - 3R_1$

$$\begin{array}{cccc} 3 & 3 & -1 & 10 \\ -3 & 3 & -15 & 18 \\ \hline 0 & 6 & -16 & 28 \end{array}$$

$R_3 - R_1$

$$\begin{array}{cccc} 1 & 3 & 2 & 5 \\ -1 & 1 & -5 & 6 \\ \hline 0 & 2 & -3 & 11 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 0 & 6 & -16 & 28 \\ 0 & 2 & -3 & 11 \end{array}\right]$$

6.2

Ex $\begin{pmatrix} -4 & 3 \\ 7 & -6 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} -4+6 & 3+(-3) \\ 7+2 & -6+(-4) \end{pmatrix}$

$= \begin{pmatrix} 2 & 0 \\ 9 & -10 \end{pmatrix}$

Ex $\begin{bmatrix} 5 & 4 \\ -3 & 7 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 8 \\ 6 & 0 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ -9 & 7 \\ 5 & -2 \end{bmatrix}$

Ex $\begin{pmatrix} 5 & -6 \\ 8 & 9 \end{pmatrix} + \begin{pmatrix} -4 & 6 \\ 8 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 8 & 6 \end{pmatrix}$

$$\begin{array}{cccc} + & 5 & -6 & 8 & 9 \\ & -4 & 6 & 8 & -3 \end{array}$$

Scalar.

$$kA = k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

$$5\begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 10 & -15 \\ 0 & 20 \end{bmatrix}$$

$$\frac{3}{4}\begin{bmatrix} 20 & 36 \\ 12 & -16 \end{bmatrix} = \begin{bmatrix} 15 & 27 \\ 9 & -12 \end{bmatrix} \qquad \frac{3}{4}20 = 3(5)$$

Ex

$$A = \begin{pmatrix} -4 & 1 \\ 3 & 0 \end{pmatrix} \quad B = \begin{pmatrix} -1 & -2 \\ 8 & 5 \end{pmatrix}$$

a) $-6B = -6\begin{pmatrix} -1 & -2 \\ 8 & 5 \end{pmatrix}$

$$= \begin{pmatrix} 6 & 12 \\ -48 & -30 \end{pmatrix}$$

b) $3A + 2B = 3\begin{pmatrix} -4 & 1 \\ 3 & 0 \end{pmatrix} + 2\begin{pmatrix} -1 & -2 \\ 8 & 5 \end{pmatrix}$

$$= \begin{pmatrix} -12 & 3 \\ 9 & 0 \end{pmatrix} + \begin{pmatrix} -2 & -4 \\ 16 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} -14 & -1 \\ 25 & 10 \end{pmatrix}$$

$A \times B = m \times p$

$m \times n \quad n \times p$

$=$

if it is not AB does not exist $\not\exists$

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{pmatrix} \underset{r_1 c_1}{ae + bg} & \underset{r_1 c_2}{af + bh} \\ \underline{ce + dg} & \underline{cf + dh} \end{pmatrix}$$

$a \cdot e$

$b \cdot g$ $+$

$ae \; bg$

$+ \; a \cdot f$

$b \cdot h$

Ex $A = \underset{2\times 2}{\begin{pmatrix} 1 & -3 \\ 7 & 2 \end{pmatrix}}$ $B = \underset{2\times 4}{\begin{pmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 4 & -1 \end{pmatrix}}$

$$\underset{2\times 2 \quad 2\times 4}{AB} = \begin{pmatrix} 1 & -3 \\ 7 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-9 & 0-3 & -1-12 & 2+3 \\ 7+6 & 2 & -7+14 & 14-2 \end{pmatrix}$$

$$= \begin{pmatrix} -8 & -3 & -13 & 5 \\ 13 & 2 & 7 & 12 \end{pmatrix}$$

$\underset{2\times 4 \quad 2\times 2}{BA} = \not\exists$

Ex $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 6 \\ 1 & 0 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 4 & 6 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4+3 & 6 \\ 8+5 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 6 \\ 13 & 12 \end{pmatrix}$$

$$BA = \begin{pmatrix} 4 & 6 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 4+12 & 12+30 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 42 \\ 1 & 3 \end{pmatrix}$$

$$A = \underset{2\times 3}{\begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}} \qquad B = \underset{3\times 2}{\begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}}$$

$$\underset{2\times 2}{AB} = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+3+2 & 18-5-4 \\ 2-6 & 12+12 \end{bmatrix}$$

$$= \begin{pmatrix} 8 & 9 \\ -4 & 24 \end{pmatrix}$$

Ex $A_{3\times 2}$ $B = 2\times 4$

a) $\underset{3\times 2}{A}\ \underset{2\times 4}{B} = \underset{3\times 4}{\ }$

b) size $AB = 3\times 4$

c) $\underset{2\times 4}{B}\ \underset{\neq 3\times 2}{A}$ can't be calculated.

Ex: $A = \begin{bmatrix} 2 & 0 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$

$$\underset{1\times 3}{A}\ \underset{3\times 1}{B} = \begin{bmatrix} 2 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$$

$$= [2+28]$$

$$= [30]$$

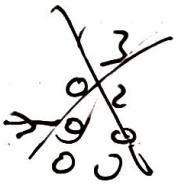
$$\underset{3\times 1}{B}\ \underset{1\times 3}{A} = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}\begin{bmatrix} 2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 4 \\ 6 & 0 & 12 \\ 14 & 0 & 28 \end{bmatrix}$$

Ex $A = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{pmatrix}$

$\underset{2\times 4}{B}\ \underset{2\times 2}{A}$ = can't be calculated.

$$\begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$$



$A + B = B + A$

$A + (B + C) = (A + B) + C$

$A + 0 = A$

Zero matrix $= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $1 \times m$

$AB \neq BA$

#4 $\begin{bmatrix} a+2 & 3b & 4c \\ d & 7f & 8 \end{bmatrix} + \begin{bmatrix} -7 & 2b & 6 \\ -3d & -6 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 25 & 6 \\ -8 & 1 & 6 \end{bmatrix}$

$\begin{bmatrix} a+2-7 & 3b+2b & 4c+6 \\ d-3d & 7f-6 & 8-2 \end{bmatrix} = \begin{bmatrix} 15 & 25 & 6 \\ -8 & 1 & 6 \end{bmatrix}$

$a - 5 = 15 \rightarrow a = 20$

$5b = 25 \Rightarrow b = 5$

$4c + 6 = 6 \Rightarrow c = 0$

$-2d = -8 \Rightarrow d = 4$

$7f - 6 = 1 \Rightarrow 7f = 7$

$f = 1$

$f = \frac{7}{7}$

---

7 $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ $F = \begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix}$

$3F + 2A = 3\begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix} + 2\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

$= \begin{pmatrix} 9 & 9 \\ -3 & -3 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ 8 & 6 \end{pmatrix}$

$= \begin{pmatrix} 11 & 13 \\ 5 & 3 \end{pmatrix}$

$$\begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1-1+4 & 1-2-4 & -4+2 \\ 4-1+3 & 4-2-3 & -4+9 \\ 2-2 & 2+2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -5 & 8 \\ 6 & -1 & 5 \\ 0 & 4 & -6 \end{bmatrix}$$

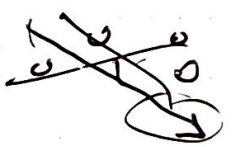
Identity Matrix $\quad a \times 1 = a$

$$AI = IA = A$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$n \times n$ 1 in main diagonal and zero elsewhere

must has to Square Matrix.



main diagonal

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$A \quad I \quad = A.$$

$\frac{a}{a}$ $\quad a \cdot a^{-1} = 1$

$$AA^{-1} = A^{-1}A = I$$

Ex $\quad A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$ B is inverse of A

$$A \cdot B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2-1 & -2+2 \\ 1-1 & -1+2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Finding inverse matrices A^{-1}

$[A|I] \xrightarrow{R/O} [I|A^{-1}]$

ONLY for 2x2 ONLY

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ✗ ✗

$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$= \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$

"if $ad-bc=0 \Rightarrow$ A^{-1} doesn't exist

$A = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$ A^{-1}?

$A^{-1} = \frac{1}{3-2}\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$

3/ $A = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$ A^{-1}?

$A^{-1} = \frac{1}{-4-(-6)}\begin{pmatrix} -2 & 6 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{pmatrix}$

14. $A = \begin{pmatrix} 4 & 6 \\ 2 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{12-12}\begin{pmatrix} & \\ & \end{pmatrix}$

$= \frac{1}{0}$

$= \not{7}$

4) $A = \begin{pmatrix} 10 & -2 \\ -5 & 1 \end{pmatrix}$

$A^{-1} = \frac{1}{10-10}$ $\not{7}$

6) $A = \begin{pmatrix} a & 1 \\ 3 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{3a-3b}\begin{pmatrix} 3 & -b \\ -3 & a \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{3(a-b)} & -\frac{b}{3(a-b)} \\ \frac{-3}{3(a-b)} & \frac{a}{3(a-b)} \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{a-b} & -\frac{b}{3(a-b)} \\ \frac{-1}{a-b} & \frac{a}{3(a-b)} \end{pmatrix}$

9 $A = \begin{pmatrix} 2 & 1 \\ a & a \end{pmatrix}$

$$A^{-1} = \frac{1}{\underbrace{2a-a}_{a}} \begin{pmatrix} a & -1 \\ -a & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\frac{1}{a} \\ -1 & \frac{2}{a} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} \qquad A^{-1}?$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ R_2 + R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{array}{cccccc} -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 & 0 & 0 \\ \hline 0 & 2 & 5 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{cccccc} 1 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & -1 & 0 & 0 \\ \hline 0 & -1 & -2 & -1 & 0 & 1 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right) 2R_3 + R_2$$

$$\begin{array}{cccccc} 0 & -2 & -4 & -2 & 0 & 1 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ \hline 0 & 0 & 1 & -1 & 1 & 1 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right) \begin{matrix} R_1 - 2R_3 \\ R_2 - 5R_3 \end{matrix}$$

$$\begin{array}{cccccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 & -2 & -2 \\ \hline 1 & 0 & 0 & 3 & -2 & -2 \end{array}$$

$$\begin{array}{cccccc} 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & 0 & -5 & 5 & -5 & -5 \\ \hline 0 & 2 & 0 & 6 & -4 & -5 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -2 \\ 0 & 2 & 0 & 6 & -4 & -5 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right) \frac{1}{2} R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -2 \\ 0 & 1 & 0 & 3 & -2 & -\frac{5}{2} \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 3 & -2 & -2 \\ 3 & -2 & -\frac{5}{2} \\ -1 & 1 & 1 \end{pmatrix}$$

Determinant. $\det(A) = |A|$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$A = \begin{pmatrix} -3 & 4 \\ 6 & 8 \end{pmatrix}$ $|A| = ?$ find the determinant

$$|A| = \begin{vmatrix} -3 & 4 \\ 6 & 8 \end{vmatrix}$$
$$= -24 - 24$$
$$= -48$$

$$\begin{vmatrix} 2 & -3 \\ -4 & 1 \end{vmatrix} = 2 - 12$$
$$= -10$$

$$\begin{vmatrix} x & 4x \\ 2x & 8x \end{vmatrix} = x(8x) - (4x)(2x)$$
$$= 8x^2 - 8x^2$$
$$= 0$$

$$\begin{vmatrix} \frac{1}{5} & \frac{1}{6} \\ -6 & -5 \end{vmatrix} = \left(\frac{1}{5}\right)(-5) - \left(\frac{1}{6}\right)(-6)$$
$$= -1 + 1$$
$$= 0$$

$$\begin{vmatrix} x & x^2 \\ 4 & x \end{vmatrix} = x^2 - 4x^2$$
$$= -3x^2$$

$$\begin{vmatrix} -8 & -5 \\ b & a \end{vmatrix} = -8a + 5b$$

$$\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 4 & 6 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 4+3 & 6 \\ 8+5 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 6 \\ 13 & 12 \end{pmatrix}$$

$$\begin{bmatrix} -3 & 4 & 2 \\ 5 & 0 & 4 \end{bmatrix}\begin{bmatrix} -6 & 4 \\ 2 & 3 \\ 3 & -2 \end{bmatrix} = \begin{pmatrix} 18+8+6 & -12+12-4 \\ -30+12 & 20-8 \end{pmatrix}$$

2×3 $\quad$ 3×2

$$= \begin{pmatrix} 32 & -4 \\ -18 & 12 \end{pmatrix}$$

$$\begin{pmatrix} x & 2x+1 & 4 \\ 5 & x-1 & 8 \\ -2 & 3x & 2x+1 \end{pmatrix} + \begin{pmatrix} 2x-1 & -2x-1 & 4x \\ -5 & 6 & x+1 \\ -5 & 2 & -2x \end{pmatrix}$$

$$= \begin{pmatrix} x+2x-1 & 2x+1-2x-1 & 4+4x \\ 5-5 & x-1+6 & 8+x+1 \\ -2-5 & 3x+2 & 2x+1-2x \end{pmatrix}$$

$$= \begin{pmatrix} 3x-1 & 0 & 4+4x \\ 0 & x+5 & x+9 \\ -7 & 3x+2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} -2 & 7 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 7+6 \\ 4 & -14+10 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 13 \\ 4 & -4 \end{pmatrix}$$

$$\begin{vmatrix} 2 & -3 & -2 \\ -1 & -4 & -3 \\ -1 & 0 & 2 \end{vmatrix} \begin{matrix} 2 & -3 \\ -1 & -4 \\ -1 & 0 \end{matrix}$$

Copy 1st 2 Columns

$$= 2(-4)(2) + (-3)(-1)(-1) + (-2)(-1)(0)$$
$$+ 8 + 0 - 6$$
$$= -16 - 9 + 8 - 6$$
$$= -23$$

$$\begin{vmatrix} -8 & 0 & 6 \\ 4 & -6 & 7 \\ -1 & -3 & 5 \end{vmatrix} \begin{matrix} -8 & 0 \\ 4 & -6 \\ -1 & -3 \end{matrix} = 240 - 72 - 168 - 36$$
$$= -36$$

$$\begin{vmatrix} x & 0 & -1 \\ 2 & x & x^2 \\ -3 & x & 1 \end{vmatrix} \begin{matrix} x & 0 \\ 2 & x \\ -3 & x \end{matrix}$$

$$= x^2 - 2x - x^4 - 3x$$
$$= x^2 - 5x - x^4$$

$$A = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \quad A^{-1}??$$

$$A^{-1} = \frac{1}{-2-12} \begin{pmatrix} -1 & -4 \\ -3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-1}{-14} & \frac{-4}{-14} \\ \frac{-3}{-14} & \frac{2}{-14} \end{pmatrix} = \begin{pmatrix} \frac{1}{14} & \frac{2}{7} \\ \frac{3}{14} & -\frac{1}{7} \end{pmatrix}$$