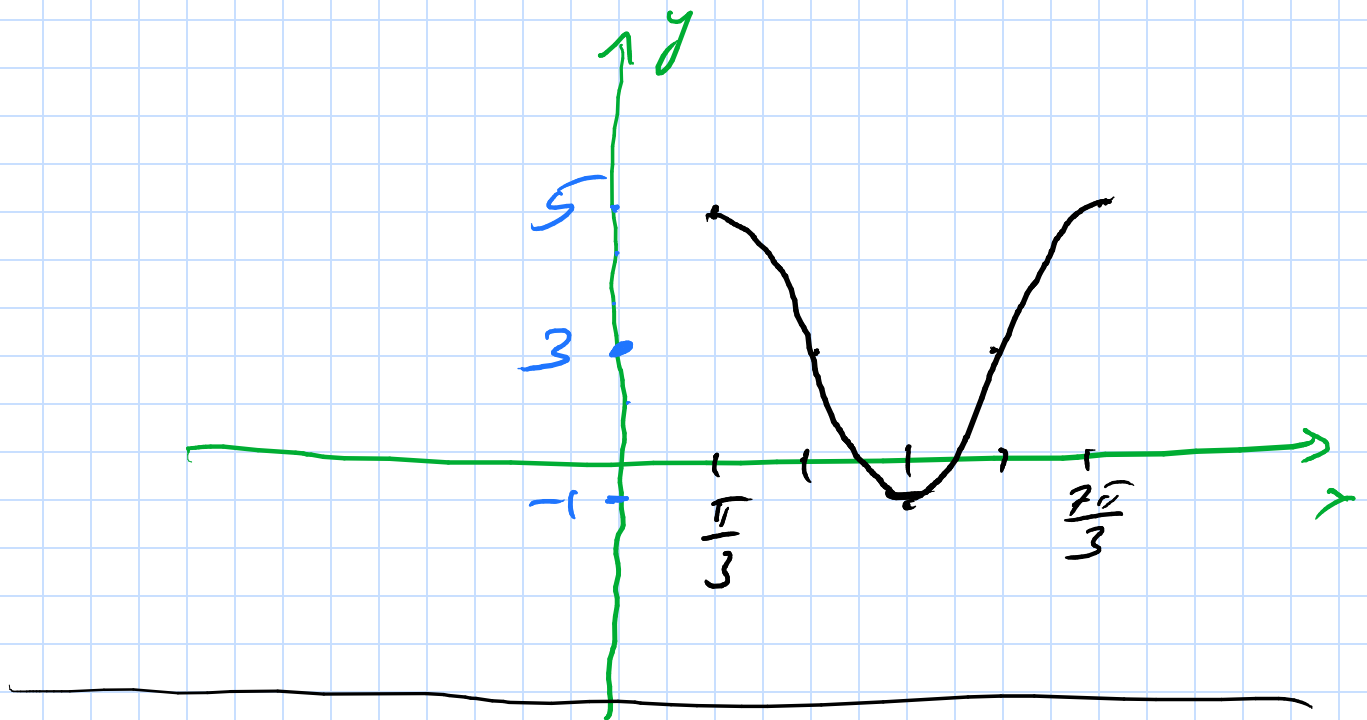


$$y = 3 \cos\left(x - \frac{\pi}{3}\right) + 2 \quad \pm 1$$



$$\begin{aligned} 3, 4 &\rightarrow 5 \\ 5, 12 &\rightarrow 13 \\ 8, 15 &\rightarrow 17 \end{aligned}$$

$$\begin{aligned} 7, 24 &\rightarrow 25 \\ 20, 21 &\rightarrow 29 \end{aligned}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin \theta \csc \theta = 1$$

$$\cos \theta \sec \theta = 1$$

$$\tan \theta \cot \theta = 1$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\begin{cases} \cos^2 \theta = 1 - \sin^2 \theta \\ \sin^2 \theta = 1 - \cos^2 \theta \end{cases}$$

$$\begin{aligned}
 \underline{\text{Ex}} \quad \sec \theta \tan \theta &= \frac{1}{\cos \theta} \frac{\sin \theta}{\cos \theta} \\
 &= \frac{\sin \theta}{\cos^2 \theta} \\
 &= \frac{\sin \theta}{1 - \sin^2 \theta}
 \end{aligned}$$

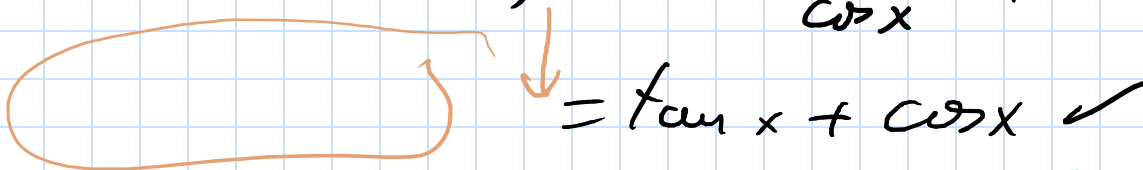
$\cos^2 \theta + \sin^2 \theta = 1$

$$\underline{\text{Ex}} \quad \frac{1}{\sin \theta} + \frac{1}{\cos \theta} = \frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta}$$

$$\begin{aligned}
 \underline{\text{Ex}} \quad \tan \alpha + \cot \alpha &= \frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} \\
 &= \frac{\sin^2 \alpha + \cos^2 \alpha}{\cos \alpha \sin \alpha} \\
 &= \frac{1}{\cos \alpha \sin \alpha}
 \end{aligned}$$

Prove $\tan x + \cot x = \sin x (\sec x + \csc x)$

$$\sin x (\sec x + \csc x) = \sin x \frac{1}{\cos x} + \sin x \frac{\cos x}{\sin x}$$


 $= \tan x + \cot x \quad \checkmark$

$$\begin{aligned}
 \tan x + \cot x &= \frac{\sin x}{\cos x} + \cos x \frac{\sin x}{\sin x} \\
 &= \sin x \left(\frac{1}{\cos x} + \frac{\cos x}{\sin x} \right) \\
 &= \sin x (\sec x + \csc x) \quad \checkmark
 \end{aligned}$$

Ex Prove $\cot \alpha + 1 = \csc \alpha (\cos \alpha + \sin \alpha)$

$$\begin{aligned}\csc \alpha (\cos \alpha + \sin \alpha) &= \frac{1}{\sin \alpha} \cos \alpha + \frac{1}{\sin \alpha} \sin \alpha \\ &= \cot \alpha + 1 \quad \checkmark\end{aligned}$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (\quad + \quad) (\quad - \quad + \quad)$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a - b)(a + b)(a^2 + b^2)$$

$$a^2 + b^2 = (\times) \text{ (C)}$$

$$\frac{\cos^4 t - \sin^4 t}{\cos^2 t} = 1 - \tan^2 t$$

$$\begin{aligned}\frac{\cos^4 t - \sin^4 t}{\cos^2 t} &= \frac{(\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t)}{\cos^2 t} \\ &= \frac{\cos^2 t - \sin^2 t}{\cos^2 t} \\ &= \frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t} \\ &= 1 - \tan^2 t \quad \checkmark\end{aligned}$$

$\left(\frac{\sin t}{\cos t}\right)^2$

Prove

$$1 + \cos \theta = \frac{\sin^2 \theta}{1 - \cos \theta}$$

$$\begin{aligned} \frac{\sin^2 \theta}{1 - \cos \theta} &= \frac{1 - \cos^2 \theta}{1 - \cos \theta} \\ &= \frac{(1 - \cos \theta)(1 + \cos \theta)}{1 - \cos \theta} \\ &= 1 + \cos \theta \quad \checkmark \end{aligned}$$

$$\begin{aligned} 1 + \cos \theta &= (1 + \cos \theta) \frac{1 - \cos \theta}{1 - \cos \theta} \\ &= \frac{1 - \cos^2 \theta}{1 - \cos \theta} \\ &= \frac{\sin^2 \theta}{1 - \cos \theta} \end{aligned}$$

$$1 + \cos \theta \stackrel{?}{=} \frac{\sin^2 \theta}{1 - \cos \theta} \quad ?$$

$$(1 + \cos \theta)(1 - \cos \theta) \stackrel{?}{=} \sin^2 \theta$$

$$1 - \cos^2 \theta \stackrel{?}{=} \sin^2 \theta$$

$$\sin^2 \theta = \sin^2 \theta \quad \checkmark$$

Prove

$$\tan^2 \alpha (1 + \cot^2 \alpha) = \frac{1}{1 - \sin^2 \alpha}$$

$$\begin{aligned}\tan^2 \alpha (1 + \cot^2 \alpha) &= \tan^2 \alpha + \tan^2 \alpha \cot^2 \alpha \\ &= \tan^2 \alpha + 1 \\ &= \sec^2 \alpha \\ &= \frac{1}{\cos^2 \alpha} \\ &= \frac{1}{1 - \sin^2 \alpha} \quad \checkmark\end{aligned}$$

$$\frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} = 2 \csc \alpha$$

$$\begin{aligned}\frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} &= \frac{\sin^2 \alpha + (1 + \cos \alpha)^2}{\sin \alpha (1 + \cos \alpha)} \\ &= \frac{\sin^2 \alpha + 1 + 2 \cos \alpha + \cos^2 \alpha}{\sin \alpha (1 + \cos \alpha)} \\ &= \frac{2 + 2 \cos \alpha}{\sin \alpha (1 + \cos \alpha)} \\ &= \frac{2 (1 + \cos \alpha)}{\sin \alpha (1 + \cos \alpha)} \\ &= \frac{2}{\sin \alpha} \\ &= 2 \csc \alpha \quad \checkmark\end{aligned}$$

$$\frac{1 + \sin x}{\cos x} = \frac{\cos x}{1 - \sin x}$$

$$\begin{aligned} \frac{1 + \sin x}{\cos x} &= \frac{1 + \sin x}{\cos x} \cdot \frac{1 - \sin x}{1 - \sin x} \\ &= \frac{1 - \sin^2 x}{\cos x (1 - \sin x)} \\ &= \frac{\cos^2 x}{\cos x (1 - \sin x)} \\ &= \frac{\cos x}{1 - \sin x} \quad \checkmark \end{aligned}$$

$$\cot^2 \theta + \cos^2 \theta \neq \cot^2 \theta \cos^2 \theta$$

$$\theta = \frac{\pi}{4}$$

$$\cot^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} = 1 + \left(\frac{1}{\sqrt{2}} \right)^2 = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\cot^2 \frac{\pi}{4} \cos^2 \frac{\pi}{4} = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} \neq \frac{3}{2}$$

$$\therefore \cot^2 \theta + \cos^2 \theta \neq \cot^2 \theta \cos^2 \theta$$

$$25/ \quad 7 \csc^2 x - 5 \cot^2 x = 2 \csc^2 x + 5$$

$$\begin{aligned} 7 \csc^2 x - 5 \cot^2 x &= 7 \csc^2 x - 5 (\csc^2 x - 1) \\ &= 7 \csc^2 x - 5 \csc^2 x + 5 \\ &= 2 \csc^2 x + 5 \quad \checkmark \end{aligned}$$

$$33/ \quad (\sec x + \tan x)^2 = \frac{1 + \sin x}{1 - \sin x}$$

$$\begin{aligned} (\sec x + \tan x)^2 &= \sec^2 x + 2 \sec x \tan x + \tan^2 x \\ &= \sec^2 x + 2 \frac{1}{\cos x} \frac{\sin x}{\cos x} + \sec^2 x - 1 \\ &= \frac{2}{\cos^2 x} + \frac{2 \sin x}{\cos^2 x} - 1 \\ &= \frac{2 + 2 \sin x - \cos^2 x}{\cos^2 x} \\ &= \frac{2 + 2 \sin x - (1 - \sin^2 x)}{1 - \sin^2 x} \\ &= \frac{\sin^2 x + 2 \sin x + 1}{(1 - \sin x)(1 + \sin x)} \\ &= \frac{(1 + \sin x)^2}{(1 - \sin x)(1 + \sin x)} \\ &= \frac{1 + \sin x}{1 - \sin x} \quad \checkmark \end{aligned}$$

6.4 $\cot^3 x = \cot x (\csc^2 x - 1)$

$$\cot x (\csc^2 x - 1) = \cot x (\cot^2 x) \\ = \cot^3 x \quad \checkmark$$

Cosine square of $\cos^2 \theta$

" double of $\cos 2\theta$

means

$$\frac{\cos \theta + 1}{\cos(\theta + 1)} \rightarrow \frac{(\cos \theta) + 1}{\cos(\theta + 1)}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

2 cosines $\rightarrow 2\phi$

2 sines $\rightarrow 2\phi$

$$\begin{aligned}
 \cos(75^\circ) &= \cos(45^\circ + 30^\circ) \\
 &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\
 &= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \frac{1}{2} \\
 &= \frac{\sqrt{6} - \sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \cos(x + 2\pi) &= \cos x \cos 2\pi - \sin x \sin 2\pi \\
 &= \cos x \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \cos 3x \cos 2x - \sin 3x \sin 2x \\
 &= \cos(3x + 2x) \\
 &= \cos 5x
 \end{aligned}$$

$$\begin{aligned}
 \sin(90^\circ - A) &= \cos A \\
 \sin(90^\circ - A) &= \sin 90^\circ \cos A - \cos 90^\circ \sin A \\
 &= \cos A \checkmark
 \end{aligned}$$