Lecture One

Section 1.1 - **Linear Equations and Rational Equations**

Definition of a Linear Equation

A linear equation in one variable x is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real number, and $a \neq 0$

Addition and Multiplication Properties of Equalities

If $a = b$, then $a + c = b + c$

If $a = b$, then $ac = bc$

Example

Solve: $3(2x - 4) = 7 - (x + 5)$

Solution

$6x - 12 = 7 - x - 5$

$6x - 12 + x = 2 - x + x$

$7x - 12 = 2$

$7x - 12 + 12 = 2 + 12$

$7x = 14$

$\dfrac{7}{7}x = \dfrac{14}{7}$

$x = 2$

Example

Solve: $\dfrac{2x + 4}{3} + \dfrac{1}{2}x = \dfrac{1}{4}x - \dfrac{7}{3}$

Solution

$(12)\dfrac{2x + 4}{3} + (12)\dfrac{1}{2}x = (12)\dfrac{1}{4}x - (12)\dfrac{7}{3}$

$4(2x + 4) + 6x = 3x - 28$

$8x + 16 + 6x = 3x - 28$

$14x + 16 = 3x - 28$

$14x - 3x = -28 - 16$

$11x = -44$

$\underline{x = -4}$

Example

Solve: $\dfrac{5}{2x} = \dfrac{17}{18} - \dfrac{1}{3x}$

Solution

$(18x)\dfrac{5}{2x} = (18x)\dfrac{17}{18} - (18x)\dfrac{1}{3x}$ *Restriction:* $x \neq 0$

$45 = 17x - 6$

$45 + 6 = 17x$

$17x = 51$

$\underline{x = 3}$

Example

Solve: $\dfrac{x}{x-2} = \dfrac{2}{x-2} - \dfrac{2}{3}$

Solution

$3(x-2)\dfrac{x}{x-2} = 3(x-2)\dfrac{2}{x-2} - 3(x-2)\dfrac{2}{3}$ *Restriction*: $x \neq 2$

$3x = 6 - 2(x-2)$

$3x = 6 - 2x + 4$

$3x + 2x = 10$

$\Rightarrow 5x = 10$

$\Rightarrow x = 2$

*No Solution **or** $\{\varnothing\}$*

Identities, Conditional Equations, and Contradictions

Example

Solve: $-2(x+4)+3x = x-8$

Solution

$$-2(x+4)+3x = x-8$$

$$-2x-8+3x = x-8$$

$$x-8 = x-8$$

$$0 = 0 \quad \textit{True statement}$$

Solution: All real numbers

Example

Solve: $5x-4 = 11$

Solution

$$5x-4 = 11$$

$$5x = 15$$

$$x = 3$$

This is a conditional equation, and its solution set is {3}

Example

Solve: $3(3x-1) = 9x+7$

Solution

$$3(3x-1) = 9x+7$$

$$9x-3 = 9x+7$$

$$-3 = 7 \quad \textit{False statement}$$

This is a contradiction equation, and its solution set is empty set {∅} or null

Solving for a Specified Variable

Example

Solve

a) $I = \Pr t$ *for t*

$$\frac{I}{\Pr} = \frac{\Pr}{\Pr} t$$

$$\frac{I}{\Pr} = t$$

b) $A - P = \Pr t$ *for P*

$$A = Prt + P$$
$$A = P(rt + 1)$$

$$\frac{A}{rt+1} = P \qquad or \quad P = \frac{A}{rt+1}$$

c) $3(2x - 5a) + 4b = 4x - 2$ *for x*

$$6x - 15a + 4b = 4x - 2$$
$$6x - 4x = 15a - 4b - 2$$
$$2x = 15a - 4b - 2$$
$$x = \frac{15a - 4b - 2}{2}$$

Example

Solve the formula $2l + 2w = P$ for **w**

Solution

$$2w = P - 2l$$

$$w = \frac{P - 2l}{2}$$

Example

Solve the formula $P = C + MC$ for C

Solution

$$P = C(1 + M)$$

$$\frac{P}{1+M} = C$$

$$C = \frac{P}{1+M}$$

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Exercises   *Section* 1.1 - Linear Equations and Rational Equations

Solve

1. $5x - 8 = 72$

2. $14 - 5x = -41$

3. $2x + 6 = 3x - 2$

4. $11x - (6x - 5) = 40$

5. $9x + 11 = 7x + 1$

6. $2x - 7 = 6 + x$

7. $5x - 2 = 9x + 2$

8. $3(x - 2) + 7 = 2(x + 5)$

9. $3x + 5 - 5(x + 1) = 6x + 7$

10. $4(-2x + 1) = 6 - (2x - 4)$

11. $4(x + 7) = 2(x + 12) + 2(x + 1)$

12. $6(3x - 1) = 8 - 10(10x - 14)$

13. $5x - (2x - 8) = 35$

14. $\frac{1}{14}(3x - 2) = \frac{x + 10}{10}$

15. $\frac{5}{6}x - 2x + \frac{4}{3} = \frac{5}{3}$

16. $\frac{7}{4} + \frac{1}{5}x - \frac{3}{2} = \frac{4}{5}x$

17. $5(x + 3) + 4x - 3 = -(2x - 4) + 2$

18. $2\left[x - (4 + 2x) + 3\right] = 2x + 3$

19. $2x - \left\{x - \left[3x - (8x + 6)\right]\right\} = 2x - 2$

20. $4(2x + 7) = 2x + 22 + 3(2x + 3)$

21. $4\left[2x - (3 - x) + 5\right] = -7x - 2$

22. $3\left[2x - (4 - x) + 5\right] = 7x - 2$

23. $-4(2x - 6) + 8x = 5x + 24 + x$

24. $-8(3x + 4) + 6x = 4(x - 8) + 4x$

25. $4(x + 7) = 2(x + 12) + 2(x + 1)$

26. $-6(2x + 1) - 3(x - 4) = -15x + 1$

27. $2(x - 1) + 3 = x - 3(x + 1)$

28. $3(x - 4) - 4(x - 3) = x + 3 - (x - 2)$

29. $2 - (7x + 5) = 13 - 3x$

30. $16 = 3(x - 1) - (x - 7)$

31. $5x - 2(x + 1) = x + (3x - 5)$

32. $7(x + 1) = 4\left[x - (3 - x)\right]$

33. $2\left[3x - 2(2x - 3)\right] = 5(x - 6)$

34. $.2x - .5 = .1x + 7$

35. $.01x + 3.1 = 2.03x - 2.96$

36. $.08x - .06(x + 12) = 7.72$

37. $.04(x - 12) + .06x = 1.52$

38. $.3(x + 2) - .5(x + 2) = -.2x - .4$

39. $.6(x - 5) + .8(x - 6) = .2x - 1.8$

40. $.5x + \frac{4}{3}x = x + 10$

41. $.25x + \frac{2}{3}x = x + 2$

42. $\frac{1}{4}(x - 2) = \frac{1}{6}(x - 5)$

43. $\frac{1}{4}(3x - 2) = \frac{1}{5}(x + 5)$

44. $\frac{1}{9}(x + 2) = \frac{1}{15}(2x + 5)$

45. $\frac{1}{2}(4x + 8) - 16 = -\frac{2}{3}(9x - 12)$

46. $\frac{3}{4}(24 - 8x) - 16 = -\frac{2}{3}(6x - 9)$

47. $\frac{x - 3}{4} = \frac{5}{14} - \frac{x + 5}{7}$

48. $\frac{x + 1}{4} = \frac{1}{6} + \frac{2 - x}{3}$

49. $\frac{x - 8}{3} + \frac{x - 3}{2} = 0$

50. $\frac{5}{2x} - \frac{8}{9} = \frac{1}{18} - \frac{1}{3x}$

51. $\frac{1}{x + 4} + \frac{1}{x - 4} = \frac{22}{x^2 - 16}$

52. $\frac{3x - 1}{3} - \frac{2x}{x - 1} = x$

53. $\frac{x}{x - 2} = \frac{2}{x - 2} + 2$

54. $\frac{x}{x - 7} = \frac{7}{x - 7} + 8$

55. $\frac{3x}{5} - x = \frac{x}{10} - \frac{5}{2}$

56. $2x - \frac{2x}{7} = \frac{x}{2} + \frac{17}{2}$

57. $\frac{x + 3}{6} = \frac{2}{3} + \frac{x - 5}{4}$

58. $\frac{x + 1}{4} = \frac{1}{6} + \frac{2 - x}{3}$

59. $\frac{x}{4} = 2 + \frac{x-3}{3}$

60. $5 + \frac{x-2}{3} = \frac{x+3}{8}$

61. $\frac{x+1}{3} = 5 - \frac{x+2}{7}$

62. $\frac{3x}{5} - \frac{x-3}{2} = \frac{x+2}{3}$

63. $\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x^2 - 2x}$

64. $\frac{-4x}{x-1} + \frac{4}{x+1} = \frac{-8}{x^2 - 1}$

65. $\frac{4x+3}{x+1} + \frac{2}{x} = \frac{1}{x^2 + x}$

66. $\frac{6}{x+3} - \frac{5}{x-2} = \frac{-20}{x^2 + x - 6}$

67. $\frac{6}{x+1} - \frac{5}{x+2} = \frac{10}{x^2 + 3x + 2}$

68. $3(x-4) - 5(x+2) = 3[2 - (x+24)] - 2(x-2)$

69. $(2x+3)(6x-1) - 9 = 15x^2 - (3x-2)(x-2)$

70. $(3x-1)^2 - 2x(x-1) = 7x^2 - 5x + 2$

71. $(2x+3)(x-1) + (x+1)(x-4) = 3x^2$

72. $4x + 13 - \{2x - [4(x-3) - 5]\} = 2(x-6)$

73. $-2\{7 - [4 - 2(1-x) + 3]\} = 10 - [4x - 2(x-3)]$

74. $2(y+2) + (y+3)^2 = y(y+5) + 2\left(\frac{17}{2} + y\right)$

75. $(y+1)(y-1) = (y+2)(y-3) + 4$

76. $45 - [4 - 2y - 4(y+7)] = -4(1+3y) - [4 - 3(y+2) - 2(2y-5)]$

77. $35 - [2 - 3y - 4(y+7)] = -3(1+3y) + 4 - 3(y+2) - 2(2y-5)$

78. $25 - [2 + 5y - 3(y+2)] = -3(2y-5) - [5(y-1) - 3y + 3]$

Solve for the specific variable

79. $V = lwh$,   for h

80. $A = \frac{1}{2}h(B+b)$,   for B

81. $A = \frac{1}{2}h(a+b)$,   for a

82. $S = 2\pi rh + 2\pi r^2$ for h

83. $A = \frac{1}{2}h(b_1 + b_2)$,   for h

84. $A = \frac{1}{2}h(b_1 + b_2)$,   for b_2

85. $A = \frac{1}{2}h(b_1 + b_2)$,   for b_1

86. $S = P + \Pr t$ for t

87. $S = 2lw + 2wh + 2hl$ for h

88. $S = 2lw + 2wh + 2hl$ for w

89. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R_1

90. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R

91. $V = \frac{d_1 - d_2}{t}$ for d_1

92. $V = \frac{d_1 - d_2}{t}$ for d_2

93. $z = \dfrac{x - \mu}{s}$ *for* x

94. $z = \dfrac{x - \mu}{s}$ *for* μ

95. $s = \dfrac{1}{2}at^2 + vt$ *for* v

96. $s = \dfrac{1}{2}at^2 + vt$ *for* a

97. $L = a + (n-1)d$ *for* n

98. $L = a + (n-1)d$ *for* d

99. $A = \dfrac{x_1 + x_2 + x_3}{n}$ *for* x_2

100. $A = \dfrac{x_1 + x_2 + x_3}{n}$ *for* n

101. A sewage treatment plant has two inlet pipes to its settling pond. One can fill the pond in 10 *hrs*. the other in 12 *hrs*. If the first pipe is open for 5 *hrs*. and then the second pipe us opened, how long will it take to fill the pond?