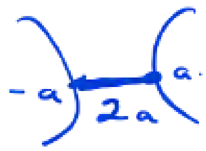
Pre-Cal 9/10

## 5.4 # 27

$$625y^2 - 400x^2 = 250,000$$

Soln

$$25 \times 10^4$$




$$\frac{y^2}{400} - \frac{x^2}{625} = 1$$

$$a^2 = 400$$

$$a = 20$$

closest point: $2a = \underline{40 \text{ yards}}$

---

## Sec 5.5 Infinite Sequences & Summation Notation

$$a_1, a_2, \ldots, a_n, \ldots$$

$n \in \mathbb{N}$ or $\mathbb{Z}^+$ Formula

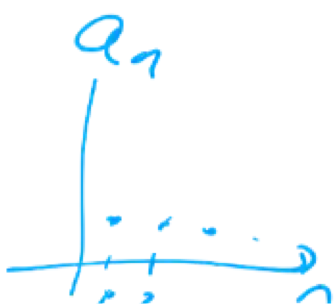
Ex 1st 4 terms, 10th term $\left\{\frac{n}{n+1}\right\}$

Soln

$$n = 1 \rightarrow \frac{1}{1+1} = \frac{1}{2}$$

$$n = 2 \rightarrow \frac{2}{2+1} = \frac{2}{3}$$

$$n = 3 \rightarrow \frac{3}{3+1} = \frac{3}{4}$$



$n=4 \to \frac{4}{4+1} = \frac{4}{5}$

$n=10 \to \frac{10}{10+1} = \frac{10}{11}$

---

$a_n = \frac{n}{n+1} \qquad f(x) = \frac{x}{x+1}$ $\quad$ o(1)

---

Ex $\quad$ 1st 4 terms $\quad$ 10th $\quad \{2+(0.1)^n\}$

$n=1 \Rightarrow 2+(0.1)^1 = 2.1$

$n=2 \Rightarrow 2+(.1)^2 = 2+.01 = 2.01$

$n=3 \Rightarrow 2+(.1)^3 = 2.001$

$n=4 \Rightarrow 2+(.1)^4 = 2.0001$

$n=10 \Rightarrow 2+(.1)^{10} = 2.0000000001$

---

Ex $\quad$ 1st 4 & 10th $\quad \left\{(-1)^{n+1}\frac{n^2}{3n-1}\right\}$

$n=1 \Rightarrow (-1)^2\frac{1^2}{3-1} = \frac{1}{2}$

$n=2 \Rightarrow (-1)^3\frac{4}{6-1} = -\frac{4}{5}$

$n=3 \Rightarrow (-1)^4\frac{9}{9-1} = \frac{9}{8}$

$n=4 \Rightarrow (-1)^5\frac{16}{12-1} = -\frac{16}{11}$

$n=10 \Rightarrow (-1)^{11}\frac{100}{30-1} = -\frac{100}{29}$

---

Ex $\quad \{a_n\}_1^\infty$ 4 3

$n=1 \longrightarrow 4$

$n = 1 \Rightarrow$

$n = 2 \Rightarrow 4$

$n = 3 \Rightarrow 4$

$n = 4 \Rightarrow 4$

$n = 10 \Rightarrow \cancel{4}$

---

#10 $\{c_n\} = \{(-1)^{n+1} n^2\}$ 1st $c_8$

$c_1 = (-1)^2 1^2 = 1$

$c_2 = (-1)^3 (2^2) = -4$

$c_3 = (-1)^4 3^2 = +9$

$c_4 = (-1)^5 4^2 = -16$

$c_8 = (-1)^9 8^6 = -64$

---

Ex 1st 4 : $a_1 = 3$ $a_{n+1} = (n+1) a_n$

$a_1 = 3$

$n=1$ $a_2 = 2 a_1 = 2(3) = 6$

$n=2$ $a_3 = 3 a_2 = 3(6) = 18$

$n=3$ $a_4 = 4 a_3 = 4(18) = 72$

---

#21 $a_1 = \sqrt{2}$ $a_n = \sqrt{2 + a_{n-1}}$

$n=1$ $a_1 = \sqrt{2}$

$n=2$ $a_2 = \sqrt{2 + a_1} = \sqrt{2 + \sqrt{2}}$

$n=3$ $a_3 = \sqrt{2 + a_2} = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$

$n=4$ $a_4 = \sqrt{2 + a_3} = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$

## Summation Notation: $\sum$

$$\underset{\substack{\text{1st value of } n}}{} \sum_{n=1}^{5} \underbrace{(2n+3)}_{\text{Formula}}$$

last value ($\infty$)

1st value of $n$

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \cdots$$

---

Ex

$$\sum_{k=1}^{4} k^2(k-3) = \underbrace{1(1-3)}_{k=1} + \underbrace{2^2(2-3)}_{k=2} + \underbrace{9(0)}_{k=3} + \underbrace{16(1)}_{k=4}$$

$$= -2 - 4 + 16$$

$$= \boxed{10}$$

---

$$\sum_{k=1}^{n} c = nc \qquad \sum_{k=m}^{n} c = (n-m+1)c$$

$$\sum_{k=10}^{20} 5 = 5(20-10+1)$$

$$= \boxed{55}$$

#42 $\sum^{50} 8 = 8(50) \qquad (50-1+1)8$

$$\sum_{k=1} \;=\; 400$$

$$\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_n$$

$$\sum_{k=1}^{n} c a_k = c\left(\sum_{k=1}^{n} a_k\right)$$

$$2^1 + 2^2 + 2^3 + \cdots + 2^{16} = \sum_{n=1}^{16} 2^n$$

#49

$$\sum_{k=1}^{6} (10 - 3k) = \sum_{k=1}^{6} 10 - 3\sum_{k=1}^{6} k$$

$$= 6(10) - 3(1+2+3+4+5+6)$$

$$= 60 - 63$$

$$= \underline{-3}$$

or

$$10-3+10-6+10$$

#52

$$\sum_{k=137}^{428} 2.1 = (428 - 137 + 1)\frac{21}{10}$$

$$= 292\left(\frac{21}{10}\right)$$

$$= \frac{146(21)}{5}$$

146

$$= \frac{3066}{5}$$

$$\begin{array}{r} 146 \\ \hline 292 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ 11 \\ \hline 462 \end{array} \qquad \begin{array}{r} ab \\ 11 \\ \hline a\,(a+b)\,b \end{array} \qquad \begin{array}{r} 76 \\ 11 \\ \hline 836 \end{array}$$

## 5.6 Arithmetic Sequences

Seq.

**Defn** A seq $a_1, a_2, \ldots, a_n, \ldots$ is an arithmetic seq, if $\exists\, d \in \mathbb{R}$ such that

$$a_{k+1} = a_k + d$$

Common difference

$$d = a_{k+1} - a_k$$

$$\boxed{a_n = a_1 + (n-1)d}$$

$1, 4, 7, 10, \ldots, 3n-2, \ldots$

$$a_n = 3n - 2$$

$$a_{k+1} = a_k + d$$
$$a_{k+1} - a_k = d \; ?$$
$$d = 4 - 1 = 3$$

$$a_{k+1} - a_k = [3(k+1) - 2] - [3k - 2]$$
$$= 3k + 3 - 2 - 3k + 2$$
$$= 3 \checkmark$$

---

Ex $\quad 20, 16.5, 13 \qquad a_{15}\,?$

$$d = 16.5 - 20$$
$$\boxed{= -3.5}$$

$$a_n = a_1 + (n-1)d$$

$$a_{15} = a_1 + (15-1)d$$
$$= 20 + 14(-3.5)$$
$$= 20 - 49$$
$$\boxed{= -29}$$

$\frac{42}{7}$

---

Ex $\quad a_4 = 5 \quad ① \qquad a_6\,??$

$\quad a_9 = 20$

$$a_n = a_1 + (n-1)d$$

$$d = \frac{y_2 - y_1}{x_2 - x_1} \; (\text{slope})$$

$$\boxed{d = \frac{20 - 5}{9 - 4} = \frac{15}{5} = 3}$$

$a_4 = a_1 + 3(3) = 5$

$$\boxed{a_1 = 5 - 9 = -4}$$

$a_6 = -4 + (5)(3)$

$= \boxed{11}$

---

$a_4 = a_1 + 3d = 5$

$a_9 = a_1 + 8d = 20$

$5d = 15$

$\boxed{d = 3}$

---

#22 arith. $a_{20}$ : $a_9 = -5$ $a_{15} = 31$

$\boxed{a_n = a_1 + (n-1)d}$

$\boxed{d = \frac{31+5}{15-9} = 6}$

$a_9 = a_1 + 8(6) = -5$

$\boxed{a_1 = -53}$

$a_{20} = -53 + 19(6)$

$= 114 - 53$

$= \boxed{61}$

---

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$= \frac{n}{2}(a_1 + a_n)$

$S_\infty = \infty$

Ex sum all even $2 \to 100$

$n = \frac{100}{2} = 50$

$S_{50} = \frac{50}{2}(2 + 100)$

$= 50 \; \frac{102}{2} \; (51)$

$= 50(51)$

$= 2,550$

Ex $\frac{1}{4} + \frac{2}{9} + \frac{3}{14} + \frac{4}{19} + \frac{5}{24} + \frac{6}{29}$

$= \sum_{n=1}^{6} \frac{n}{5n-1}$

$4, 9, 14, 19, 24, 29$

$d = 5$

$a_n = 4 + (n-1)(5)$

$= 4 + 5n - 5$

$= 5n - 1$

p 318 Geometric Seq.

& hwk #?