Solution Section 4.5 – Exponential and logarithmic Equations

Exercise

Solve: $2^{3x-7} = 32$

Solution

$$2^{3x-7} = 32 = 2^5$$

$3x - 7 = 5$ *add 7 on both sides*

$3x = 12$ *Divide by 3*

$x = 4$

Exercise

Solve $4^{2x-1} = 64$

Solution

$$4^{2x-1} = 4^3$$

$2x\text{-}1 = 3$

$2x = 4$

$x = 2$

Exercise

Solve $3^{1-x} = \frac{1}{27}$

Solution

$$3^{1-x} = \frac{1}{3^3}$$

$$3^{1-x} = 3^{-3}$$

$1 - x = -3$

$-x = -4$

$x = 4$

Exercise

Solve $\left(\frac{1}{3}\right)^{x} = 81$

Solution

$$\left(\frac{1}{3}\right)^{x} = 81$$

$$\left(3^{-1}\right)^{x} = 3^{4}$$

$$3^{-x} = 3^{4}$$

$$-x = 4$$

$$x = -4$$

Exercise

Solve: $5^{x} = 134$

Solution

$$\ln 5^{x} = \ln(134)$$

$$x \ln 5 = \ln(134)$$

$$\boxed{x = \frac{\ln(134)}{\ln 5} = \log_{5} 134}$$

Exercise

Solve: $7^{x} = 12$

Solution

$$7^{x} = 12$$

$\ln 7^{x} = \ln 12$ *Property of logarithm*

$x \ln 7 = \ln 12$ *Power Rule*

$$x = \frac{\ln 12}{\ln 7}$$

$$\boxed{\approx 1.277}$$

Exercise

Solve $\quad 9^x = \frac{1}{\sqrt[3]{3}}$

Solution

$$\left(3^3\right)^x = \frac{1}{3^{1/3}}$$

$$3^{3x} = 3^{-1/3}$$

$$3x = -\frac{1}{3}$$

$$\boxed{x = -\frac{1}{9}}$$

Exercise

Solve $\quad 9e^x = 107$

Solution

$$e^x = \frac{107}{9}$$

$$\ln e^x = \ln\left(\frac{107}{9}\right)$$

$$x \ln e = \ln\left(\frac{107}{9}\right)$$

$$\boxed{x = \ln\left(\frac{107}{9}\right)}$$

Exercise

Solve $\quad 7^{2x+1} = 3^{x+2}$

Solution

$$\ln 7^{2x+1} = \ln 3^{x+2}$$

$$(2x+1)\ln 7 = (x+2)\ln 3$$

$$2x\ln 7 + \ln 7 = x\ln 3 + 2\ln 3$$

$$2x\ln 7 - x\ln 3 = 2\ln 3 - \ln 7$$

$$x(2\ln 7 - \ln 3) = 2\ln 3 - \ln 7$$

$$\boxed{x = \frac{2\ln 3 - \ln 7}{2\ln 7 - \ln 3}}$$

Exercise

Solve: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3)\ln 4 = -x\ln 3$$

$$x\ln 4 + 3\ln 4 = -x\ln 3$$

$$x\ln 4 + x\ln 3 = -3\ln 4$$

$$x(\ln 4 + \ln 3) = -3\ln 4$$

$$x = \frac{-3\ln 4}{(\ln 4 + \ln 3)}$$

$$x \approx -1.6737$$

Exercise

Solve $2^{x+4} = 8^{x-6}$

Solution

$$2^{x+4} = \left(2^3\right)^{x-6}$$

$$2^{x+4} = 2^{3x-18}$$

$$x + 4 = 3x - 18$$

$$x + 4 - 3x - 4 = 3x - 18 - 3x - 4$$

$$-2x = -22$$

$$\boxed{x = 11}$$

Exercise

Solve $8^{x+2} = 4^{x-3}$

Solution

$$\left(2^3\right)^{x+2} = \left(2^2\right)^{x-3}$$

$$2^{3(x+2)} = 2^{2(x-3)}$$

$$3(x+2) = 2(x-3)$$

$$3x + 6 = 2x - 6$$

$$3x - 2x = -6 - 6$$

$$x = -12$$

Exercise

Solve $7^{x} = 12$

Solution

$$\ln 7^{x} = \ln 12 \qquad \textit{Property of logarithm}$$

$$x \ln 7 = \ln 12 \qquad \textit{Power Rule}$$

$$x = \frac{\ln 12}{\ln 7} \approx 1.277$$

Exercise

Solve: $5^{x+4} = 4^{x+5}$

Solution

$$\ln 5^{x+4} = \ln 4^{x+5}$$

$$(x+4)\ln 5 = (x+5)\ln 4$$

$$x\ln 5 + 4\ln 5 = x\ln 4 + 5\ln 4$$

$$x\ln 5 - x\ln 4 = 5\ln 4 - 4\ln 5$$

$$x(\ln 5 - \ln 4) = 5\ln 4 - 4\ln 5$$

$$\boxed{x = \frac{5\ln 4 - 4\ln 5}{\ln 5 - \ln 4}}$$

Exercise

Solve: $5^{x+2} = 4^{1-x}$

Solution

$$\ln 5^{x+2} = \ln 4^{1-x}$$

$$(x+2)\ln 5 = (1-x)\ln 4$$

$$x\ln 5 + 2\ln 5 = \ln 4 - x\ln 4$$

$$x\ln 5 + x\ln 4 = \ln 4 - 2\ln 5$$

$$x(\ln 5 + \ln 4) = \ln 4 - 2\ln 5$$

$$\boxed{x = \frac{\ln 4 - 2\ln 5}{\ln 5 + \ln 4}} \approx -0.612$$

Exercise

Solve: $27 = 3^{5x} 9^{x^2}$

Solution

$$27 = 3^{5x} (3^2)^{x^2}$$

$$3^3 = 3^{5x} 3^{2x^2}$$

$$3^3 = 3^{5x+2x^2}$$

$$3 = 5x + 2x^2$$

$$0 = 5x + 2x^2 - 3$$

$$2x^2 + 5x - 3 = 0$$

$$x = \frac{-(5) \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)} = \frac{-5 \pm \sqrt{49}}{4} = \frac{-5 \pm 7}{4}$$

$$x = \begin{cases} \frac{-5+7}{4} = \frac{1}{2} \\ \frac{-5-7}{4} = -3 \end{cases}$$

Exercise

Solve: $3^{2x-1} = 0.4^{x+2}$

Solution

$$\ln 3^{2x-1} = \ln 0.4^{x+2}$$

$$(2x-1)\ln 3 = (x+2)\ln 0.4 \qquad \textit{Power Property}$$

$$2x\ln 3 - \ln 3 = x\ln 0.4 + 2\ln 0.4 \qquad \textit{Distributive property}$$

$$2x\ln 3 - x\ln 0.4 = 2\ln 0.4 + \ln 3$$

$$x(2\ln 3 - \ln 0.4) = 2\ln 0.4 + \ln 3$$

$$x = \frac{2\ln 0.4 + \ln 3}{2\ln 3 - \ln 0.4} \qquad (2\ln(0.4)+\ln(3)) / (2\ln(3)-\ln(0.4))$$

$$\underline{\approx -.236}$$

Exercise

Solve: $4^{3x-5} = 16$

Solution

$$4^{3x-5} = 4^2$$

$$3x - 5 = 2$$

$$3x = 7$$

$$\boxed{x = \frac{7}{3}}$$

Exercise

Solve: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3)\ln 4 = -x\ln 3$$

$$x\ln 4 + 3\ln 4 = -x\ln 3$$

$$x\ln 4 + x\ln 3 = -3\ln 4$$

$$x(\ln 4 + \ln 3) = -3\ln 4$$

$$\boxed{x = \frac{-3\ln 4}{\ln 4 + \ln 3}} \approx -1.6737$$

Exercise

Solve: $3^{x-1} = 7^{2x+5}$

Solution

$$\ln 3^{x-1} = \ln 7^{2x+5}$$

$$(x-1)\ln 3 = (2x+5)\ln 7$$

$$x\ln 3 - \ln 3 = 2x\ln 7 + 5\ln 7$$

$$x\ln 3 - 2x\ln 7 = \ln 3 + 5\ln 7$$

$$x(\ln 3 - 2\ln 7) = \ln 3 + 5\ln 7$$

$$x = \frac{\ln 3 + 5\ln 7}{\ln 3 - 2\ln 7}$$

$(\ln(3)+5\ln(7))/(\ln(3)-2\ln(7))$

$$\boxed{\approx -3.8766}$$

Exercise

Solve: $4^{x-2} = 2^{3x+3}$

Solution

$$\left(2^2\right)^{x-2} = 2^{3x+3}$$

$$2^{2x-4} = 2^{3x+3}$$

$$2x - 4 = 3x + 3$$

$$2x - 3x = 4 + 3$$

$$-x = 7$$

$$x = -7$$

Exercise

Solve: $2^{3x-7} = 32$

Solution

$$2^{3x-7} = 32 = 2^5$$

$$3x - 7 = 5$$

$$3x = 5+7 = 12$$

$$x = \frac{12}{3} \underline{= 4}$$

Exercise

Solve: $3^{2x-1} = 0.4^{x+2}$

Solution

$$\ln 3^{2x-1} = \ln 0.4^{x+2}$$

$$(2x-1)\ln 3 = (x+2)\ln 0.4$$ *Power Property*

$$2x\ln 3 - \ln 3 = x\ln 0.4 + 2\ln 0.4$$ *Distributive property*

$$2x\ln 3 - x\ln 0.4 = 2\ln 0.4 + \ln 3$$

$$x(2\ln 3 - \ln 0.4) = 2\ln 0.4 + \ln 3$$

$$x = \frac{2\ln 0.4 + \ln 3}{2\ln 3 - \ln 0.4}$$

$(2\ln(0.4)+\ln(3)) / (2\ln(3)-\ln(0.4))$

$$\underline{\approx -.236}$$

Exercise

Solve $e^{2x} - 2e^{x} - 3 = 0$

Solution

$$U^2 - 2U - 3 = 0 \quad \Rightarrow \; U = -1, 3$$

$$\begin{cases} U = e^{x} = -1 \rightarrow Impossible \\ U = e^{x} = 3 \rightarrow ln e^{x} = ln 3 \rightarrow \boxed{x = ln 3} \end{cases}$$

Exercise

Solve: $e^{0.08t} = 2500$

Solution

$$\ln\left(e^{0.08t}\right) = \ln 2500$$

$$0.08t = \ln 2500$$

$$t = \frac{\ln 2500}{0.08} \;\; \underline{\approx 97.8|}$$

Exercise

Solve $e^{x^2} = 200$

Solution

$$\ln e^{x^2} = \ln 200 \qquad \textit{Natural Log both sides}$$

$$x^2 = \ln 200 \qquad \ln e = 1$$

$$x = \pm\sqrt{\ln 200}$$

$$\underline{\approx \pm 2.302|}$$

Exercise

Solve $e^{2x+1} \cdot e^{-4x} = 3e$

Solution

$$e^{2x+1-4x} = 3e$$

$$e^{-2x+1} = 3e$$

$$e^{-2x}e = 3e \qquad \textit{Divide by } e$$

$$e^{-2x} = 3$$

$$\ln e^{-2x} = \ln 3$$

$$-2x = \ln 3$$

$$x = -\frac{1}{2}\ln 3$$

$$\approx -.549$$

Exercise

Solve: $e^{2x} - 8e^{x} + 7 = 0$

Solution

$$Let \;\; u = e^{x} \qquad\qquad \left(e^{x}-1\right)\left(e^{x}-7\right) = 0$$

$$u^2 - 8u + 7 = 0$$

$$\Rightarrow u = e^{x} = 1 \qquad\qquad \Rightarrow u = e^{x} = 7$$

$$ln\, e^{x} = ln 1 \qquad\qquad ln\, e^{x} = ln 7$$

$$\Rightarrow x = 0 \qquad\qquad \Rightarrow x = ln 7$$

Exercise

Solve: $e^{x} + e^{-x} - 6 = 0$

Solution

$$e^{x}e^{x} + e^{x}e^{-x} - e^{x}6 = e^{x}0$$

$$e^{2x} + 1 - 6e^{x} = 0$$

$$e^{2x} - 6e^{x} + 1 = 0$$

$$u^2 - 6u + 1 = 0$$

$$u = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)} = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm 4\sqrt{2}}{2}$$

$$u = 3 \pm 2\sqrt{2}$$

$$e^{x} = 3 \pm 2\sqrt{2} \Rightarrow \ln(e^{x}) = \ln\left(3 \pm 2\sqrt{2}\right)$$

$$\Rightarrow x\ln(e) = \ln\left(3 \pm 2\sqrt{2}\right)$$

$$\Rightarrow \boxed{x = \ln\left(3 \pm 2\sqrt{2}\right)} \qquad \text{or} \pm 1.76$$

Exercise

Solve: $e^{1-3x} \cdot e^{5x} = 2e$

Solution

$e^{1-3x+5x} = 2e$

$e^{1+2x} = 2e$

$e^{1}e^{2x} = 2e$ *Divide by e*

$e^{2x} = 2$ *Natural Log both sides*

$\ln e^{2x} = \ln 2$

$2x = \ln 2$

$\underline{\lfloor x = \frac{1}{2}\ln 2 \approx 0.3456 \rfloor}$

Exercise

Solve $6\,ln(2x) = 30$

Solution

$ln(2x) = \frac{30}{6}$

$ln(2x) = 5$

$2x = e^{5}$

$x = \frac{1}{2}e^{5}$

Exercise

Solve $log_{5}(x-7) = 2$

Solution

$x - 7 = 5^{2}$

$x = 25 + 7$

$\underline{x = 32}$

Exercise

Solve $\log_5 x + \log_5(4x-1) = 1$

Solution

$$\log_5 x(4x-1) = 1$$

$$x(4x-1) = 5^1$$

$$4x^2 - x = 5$$

$$4x^2 - x - 5 = 0$$

$$\to \begin{cases} x = -1 \\ x = \frac{5}{4} \end{cases} \to Check \quad x = \frac{5}{4} \quad only \ \ solution$$

Exercise

Solve: $\log x + \log(x-3) = 1$

Solution

$$\log\left[x(x-3)\right] = 1$$

$$x(x-3) = 10^1 = 10$$

$$x^2 - 3x = 10$$

$$x^2 - 3x - 10 = 0$$

$$(x+2)(x-5) = 0$$

$$\Rightarrow x = -2,\ 5$$

Check: $x = -2 \ \Rightarrow \ \cancel{\log(-2) + \log(x-3) = 1}$

$x = 5 \ \Rightarrow \ \log(5) + \log(5-3) = 1$

Exercise

Solve: $\log x - \log(x+3) = 1$

Solution

$$\log\frac{x}{x+3} = 1$$

$$\frac{x}{x+3} = 10^1 = 10$$

$$x = 10x + 30$$

$$9x = -30$$

$x = -\frac{10}{3}$ ***No Solution***

Exercise

Solve: $\log_3 x = -2$

Solution

$x = 3^{-2}$ *Convert to exponential*

$x = \frac{1}{3^2}$

$\boxed{x = \frac{1}{9}}$

Exercise

Solve: $\log(3x+2) + \log(x-1) = 1$

Solution

$\log(3x+2) + \log(x-1) = 1$ *Product Rule*

$\log\left[(3x+2)(x-1)\right] = 1$ *Convert to exponential form*

$(3x+2)(x-1) = 10^1$

$3x^2 - x - 2 = 10$

$3x^2 - x - 12 = 0$ *Solve for x*

$x = \frac{1-\sqrt{145}}{6} < 0 \qquad x = \frac{1+\sqrt{145}}{6} > 1$

Solution: $\boxed{x = \frac{1+\sqrt{145}}{6}}$

Exercise

Solve: $\log_5(x+2) + \log_5(x-2) = 1$

Solution

$\log_5\left[(x+2)(x-2)\right] = 1$

$(x+2)(x-2) = 5^1$

$x^2 - 4 = 5$

$x^2 = 5 + 4$

$x^2 = 9$

$x = \pm 3$ $\qquad \log_5\left[(-3)+2\right] + \log_5\left[(-3)-2\right] = 1$

$\log_5\left[(3)+2\right] + \log_5\left[(3)-2\right] = 1$

Solution: $\boxed{x = 3}$

Exercise

Solve: $\log x + \log(x-9) = 1$

Solution

$$\log x(x-9) = 1$$

$$x(x-9) = 10^1$$

$$x^2 - 9x - 10 = 0$$

$\Rightarrow x = -1$ (Check; it is not a solution)

$\Rightarrow x = 10$ (only solution)

Exercise

Solve: $\log_2(x+1) + \log_2(x-1) = 3$

Solution

$$\log_2(x+1)(x-1) = 3$$

$$x^2 - 1 = 2^3$$

$$x^2 = 8 + 1 = 9 \Rightarrow x = \pm 3$$

Check: $x = -3 \ \rightarrow \log_2(-3+1) + \log_2(-3-1) = 3 \quad \Rightarrow$ *It is* not a *Solution*

$x = 3 \ \rightarrow \log_2(3+1) + \log_2(3-1) = 3 \quad \Rightarrow$ *Solution*

Exercise

Solve: $\log_8(x+1) - \log_8 x = 2$

Solution

$$\log_8\left(\frac{x+1}{x}\right) = 2$$

$$\frac{x+1}{x} = 8^2 = 64$$

$$x + 1 = 64x$$

$$1 = 63x$$

$$\boxed{x = \frac{1}{63}}$$

Exercise

Solve: $\log(x+6)-\log(x+2)=\log x$

Solution

$\log(x+6)-\log(x+2)=\log x$

$\log\frac{x+6}{x+2}=\log x$ *Quotient Rule*

$\frac{x+6}{x+2}=x$ *Multiply by x + 2*

$x+6=x(x+2)$

$x+6=x^2+2x$

$0=x^2+2x-x-6$

$x^2+x-6=0$ *Solve for x*

$x=-3,\ 2$

Check: $x=-3\rightarrow\log(-3+6)-\log(-3+2)=\log(-3)$ Or **Domain**

$x=2\rightarrow\log(2+6)-\log(2+2)=\log(2)$

Solution: $x=2$

Exercise

Solve: $\ln(x+8)+\ln(x-1)=2\ln x$

Solution

$\ln\left[(x+8)(x-1)\right]=\ln x^2$

$(x+8)(x-1)=x^2$

$x^2-x+8x-8=x^2$

$x^2-x+8x-8-x^2=0$

$7x-8=0$

$7x=8$

$\boxed{x=\frac{8}{7}}$ Check: $\ln(\frac{8}{7}+8)+\ln(\frac{8}{7}-1)=2\ln\frac{8}{7}$

Exercise

Solve: $\ln(4x+6)-\ln(x+5)=\ln x$

Solution

$$\ln\left(\frac{4x+6}{x+5}\right)=\ln x$$

$$\frac{4x+6}{x+5}=x$$

$$4x+6=x(x+5)$$

$$4x+6=x^2+5x$$

$$0=x^2+5x-4x-6$$

$$0=x^2+x-6$$

$$x^2+x-6=0$$

$$(x+3)(x-2)=0$$

$$\Rightarrow x=-3,\ \ 2$$

Check: $x=-3$ *no solution* $\ln(4x+6)-\ln(x+5)=\ln(-3)$

$x=2$ *(only solution)*

Exercise

Solve: $\ln(5+4x)-\ln(x+3)=\ln 3$

Solution

$$\ln\frac{5+4x}{x+3}=\ln 3$$

$$\frac{5+4x}{x+3}=3$$

$$5+4x=3(x+3)$$

$$5+4x=3x+9$$

$$4x-3x=9-5$$

$$x=4$$

Check: $\ln(5+4(4))-\ln((4)+3)=\ln 3$

Solution: $x=4$

Exercise

Solve $ln(x-5)-ln(x+4)=ln(x-1)-ln(x+2)$

Solution

$$ln\frac{x-5}{x+4}=ln\frac{x-1}{x+2}$$

$$\frac{x-5}{x+4}=\frac{x-1}{x+2}$$

$$(x-5)(x+2)=(x-1)(x+4)$$

$$x^2+2x-5x-10=x^2+4x-x-4$$

$$x^2-3x-10=x^2+3x-4$$

$$x^2-3x-10-x^2-3x+4=0$$

$$-6x-6=0$$

$$-6x=6 \Rightarrow x=-1 \;\; \textit{No solution}$$

Exercise

Solve $\ln(x-3)=\ln(7x-23)-\ln(x+1)$

Solution

$$\ln(x-3)=\ln\left(\frac{7x-23}{x+1}\right)$$

$$x-3=\frac{7x-23}{x+1}$$

$$(x-3)(x+1)=7x-23$$

$$x^2-2x-3=7x-23$$

$$x^2-9x+20=0$$

$$\Rightarrow x=4,\ 5$$

Check: $x=4 \;\Rightarrow\; \ln(4-3)=\ln(7(4)-23)-\ln(4+1)$

$x=5 \;\Rightarrow\; \ln(5-3)=\ln(7(5)-23)-\ln(5+1)$

$$\boxed{x=4,\ 5}$$

Exercise

How long, to the nearest tenth of a year, will it take \$1000 to grow to \$3600 at 8% annual interest compounded quarterly?

Solution

Given: $A = \$3600$

$P = \$1000$

$r = 8\% = 0.08$

$n = 4$

$$\Rightarrow A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$3600 = 1000\left(1+\frac{0.08}{4}\right)^{4t}$$

$$3.6 = (1.02)^{4t}$$

$$\ln 3.6 = \ln(1.02)^{4t}$$

$$\ln 3.6 = 4t\ln(1.02)$$

$$\frac{\ln 3.6}{4\ln 1.02} = t$$

$$\boxed{t \approx 16.2\,yr}$$

Exercise

Solve: $27 = 3^{5x}9^{x^2}$

Solution

$$27 = 3^{5x}(3^2)^{x^2}$$

$$3^3 = 3^{5x}3^{2x^2}$$

$$3^3 = 3^{5x+2x^2}$$

$$3 = 5x + 2x^2$$

$$0 = 5x + 2x^2 - 3$$

$$2x^2 + 5x - 3 = 0$$

$$x = \frac{-(5) \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)} = \frac{-5 \pm \sqrt{49}}{4} = \frac{-5 \pm 7}{4} = \begin{cases} \dfrac{-5+7}{4} = \dfrac{1}{2} \\ \dfrac{-5-7}{4} = -3 \end{cases}$$

Exercise

Solve: $ln\sqrt[4]{x} = \sqrt{lnx}$

Solution

$$ln\,x^{1/4} = \sqrt{ln\,x}$$

$$\frac{1}{4}ln\,x = \sqrt{ln\,x}$$

$$\left(\frac{1}{4}ln\,x\right)^2 = \left(\sqrt{ln\,x}\right)^2$$

$$\frac{1}{6}ln^2 x = ln\,x$$

$$ln^2 x = 6\,ln\,x$$

$$ln^2 x - 6\,ln\,x = 0$$

$$ln\,x(ln\,x - 6) = 0$$

$$\begin{cases} \ln x = 0 \to \boxed{x = 1} \\ \ln x - 6 = 0 \Rightarrow \ln x = 6 \to \boxed{x = e^6} \end{cases}$$

Exercise

Solve: $\sqrt{lnx} = ln\sqrt{x}$

Solution

$$\sqrt{ln\,x} = ln\,x^{1/2}$$

$$\sqrt{ln\,x} = \frac{1}{2}ln\,x$$

$$\left(\sqrt{ln\,x}\right)^2 = \left(\frac{1}{2}ln\,x\right)^2$$

$$ln\,x = \frac{1}{4}ln^2 x$$

$$4\,ln\,x = ln^2 x$$

$$ln^2 x - 4\,ln\,x = 0$$

$$ln\,x(ln\,x - 4) = 0$$

$$\begin{cases} \ln x = 0 \to \boxed{x = 1} \\ \ln x - 4 = 0 \Rightarrow \ln x = 4 \to \boxed{x = e^4} \end{cases}$$

Exercise

Solve the equation: $7^{x+6} = 7^{3x-4}$

Solution

$$x + 6 = 3x - 4$$
$$4 + 6 = 3x - x$$
$$10 = 2x$$
$$\boxed{x = 5}$$

Exercise

Solve the equation: $2^{-100x} = (0.5)^{x-4}$

Solution

$$2^{-100x} = \left(\tfrac{1}{2}\right)^{x-4}$$
$$2^{-100x} = \left(2^{-1}\right)^{x-4}$$
$$2^{-100x} = 2^{-x+4}$$
$$-100x = -x + 4$$
$$-100x + x = 4$$
$$-99x = 4$$
$$\boxed{x = -\frac{4}{99}}$$

Exercise

Solve the equation: $4^x \left(\frac{1}{2}\right)^{3-2x} = 8.\left(2^x\right)^2$

Solution

$$\left(2^2\right)^x \left(2^{-1}\right)^{3-2x} = 2^3.2^{2x}$$
$$2^{2x}2^{2x-3} = 2^{3+2x}$$
$$2^{2x+2x-3} = 2^{3+2x}$$
$$2^{4x-3} = 2^{3+2x}$$
$$4x - 3 = 3 + 2x$$
$$4x - 2x = 3 + 3$$
$$2x = 6$$
$$\boxed{x = 3}$$

Exercise

Solve the equation: $5^{3x-6} = 125$

Solution

$$5^{3x-6} = 5^3$$

$$3x - 6 = 3$$

$$3x = 9$$

$$\Rightarrow x = 3$$

Exercise

Solve the equation $e^{x^2} = e^{7x-12}$

Solution

$$e^{x^2} = e^{7x-12}$$

$$x^2 = 7x - 12$$

$$x^2 - 7x + 12 = 0$$

$$\boxed{x = 3,\ 4}$$

Exercise

Solve the equation $f(x) = xe^x + e^x$

Solution

$$xe^x + e^x = 0$$

$$e^x(x+1) = 0$$

$$e^x = 0 \qquad x + 1 = 0$$

$\boxed{x = -1}$ (Only solution)

Exercise

Solve for t using logarithms with base a: $2a^{t/3} = 5$

Solution

$$a^{t/3} = \frac{5}{2}$$

$$\log a^{t/3} = \log \frac{5}{2}$$

$$\frac{t}{3} \log a = \log \frac{5}{2}$$

$$\frac{t}{3} = \frac{\log \frac{5}{2}}{\log a}$$

$$\frac{t}{3} = \log_a \frac{5}{2}$$

$$\boxed{t = 3\log_a \frac{5}{2}}$$

Exercise

Solve for t using logarithms with base a: $K = H - Ca^t$

Solution

$$Ca^t = H - K$$

$$a^t = \frac{H-K}{C}$$

$$\log a^t = \log \frac{H-K}{C}$$

$$t \log a = \log \frac{H-K}{C}$$

$$t = \frac{\log \frac{H-K}{C}}{\log a} = \log_a \frac{H-K}{C}$$

Exercise

Solve the equation: $\log_4 x = \log_4 (8-x)$

Solution

$$x = 8 - x$$

$$x + x = 8$$

$$2x = 8 \to x = 4$$

Check: $\boxed{x = 4}$

Exercise

Solve the equation: $\log_7(x-5)=\log_7(6x)$

Solution

$$x-5=6x$$
$$x-6x=5$$
$$-5x=5$$
$$x=-1$$

Check: $\log_7(-1-5)=\log_7(6(-1))$

No solution (no negative inside the log)

Exercise

Solve the equation: $\ln x^2=\ln(12-x)$

Solution

$$\ln x^2=\ln(12-x)$$
$$x^2=12-x$$
$$x^2+x-12=0 \quad \rightarrow \quad x=-4,\ 3$$

Check: $x=-4 \quad \Rightarrow \quad \ln(-4)^2=\ln(12+4)$

$x=3 \quad \Rightarrow \quad \ln(3)^2=\ln(12-3)$

The solutions are: $\boxed{x=-4,\ 3}$

Exercise

Solve the equation: $e^{x\ln 3}=27$

Solution

$$\ln e^{x\ln 3}=\ln 27$$
$$x\ln 3=\ln 3^3$$
$$x=\frac{3\ln 3}{\ln 3}=3$$

Exercise

Solve the equation: $\log_4 x = \log_4 (8-x)$

Solution

$$x = 8 - x$$

$$x + x = 8$$

$$2x = 8 \rightarrow x = 4$$

Check: $\boxed{x = 4}$

Exercise

Solve the equation: $\log_7 (x-5) = \log_7 (6x)$

Solution

$$x - 5 = 6x$$
$$x - 6x = 5$$
$$-5x = 5$$
$$x = -1$$

Check: $\log_7 (-1-5) = \log_7 (6(-1))$

No solution (no negative inside the log)

Exercise

Solve the equation: $\ln x^2 = \ln(12 - x)$

Solution

$$\ln x^2 = \ln(12 - x)$$

$$x^2 = 12 - x$$

$$x^2 + x - 12 = 0 \quad \rightarrow \quad x = -4,\ 3$$

Check: $x = -4 \quad \Rightarrow \quad \ln(-4)^2 = \ln(12 + 4)$

$x = 3 \quad \Rightarrow \quad \ln(3)^2 = \ln(12 - 3)$

The solutions are: $\boxed{x = -4,\ 3}$

Exercise

Solve the equation: $e^{x\ln 3} = 27$

Solution

$$\ln e^{x\ln 3} = \ln 27$$

$$x\ln 3 = \ln 3^3$$

$$x = \frac{3\ln 3}{\ln 3} \underline{= 3|}$$

Exercise

Solve the equation $\log_6(2x-3) = \log_6 12 - \log_6 3$

Solution

$$\log_6(2x-3) = \log_6 \frac{12}{3}$$

$$\log_6(2x-3) = \log_6 4$$

$$2x-3=4$$

$$2x=7$$

$$\boxed{x = \frac{7}{2}}$$ ***Check***

Exercise

Solve the equation $\ln(-4-x) + \ln 3 = \ln(2-x)$

Solution

$$\ln 3(-4-x) = \ln(2-x)$$

$$-12-3x = 2-x$$

$$-12-2 = 3x-x$$

$$-14 = 2x$$

$$x = -7$$

Check: $\ln(-4-(-7)) + \ln 3 = \ln(2-(-7))$

$$\ln(3) + \ln 3 = \ln(9)$$

$$\ln 3(3) = \ln(9)$$

The solution is $\boxed{x = -7}$

Exercise

Solve the equation $\log_2(x+7)+\log_2 x = 3$

Solution

$$\log_2 x(x+7)=3$$

$$x(x+7)=2^3 \qquad \textit{Convert to Exponential Form}$$

$$x^2+7x=8$$

$$x^2+7x-8=0 \quad \Rightarrow x=1, \ -8$$

Check: $x=1 \Rightarrow \quad \log_2(1+7)+\log_2 1=3 \rightarrow \log_2 8=3$

$x=-8 \Rightarrow \quad \log_2(-8+7)+\cancel{\log_2(-8)}=3$

The solution is $\boxed{x=1}$

Exercise

Solve the equation $\log_3(x+3)+\log_3(x+5)=1$

Solution

$$\log_3(x+3)(x+5)=1$$

$$x^2+3x+5x+15=3^1 \qquad \textit{Convert to Exponential Form}$$

$$x^2+8x+15-3=0$$

$$x^2+8x+12=0$$

$$x=-2, \ -6$$

Check: $x=-2 \Rightarrow \quad \log_3(-2+3)+\log_3(-2+5)=1$

$\log_3(1)+\log_3(3)=1$

$x=-6 \Rightarrow \quad \log_3(-6+3)+\log_3(-6+5)=1$

$\cancel{\log_3(-3)}+\log_3(-1)=1$

The solution is $\boxed{x=-2}$

Exercise

Solve the equation $\ln x = 1 - \ln(x+2)$

Solution

$$\ln x + \ln(x+2) = 1$$

$$\ln x(x+2) = 1$$

$$x^2 + 2x = e^1$$ ***Convert to Exponential Form***

$$x^2 + 2x - e = 0$$

$$x = \frac{-2 \pm \sqrt{4+4e}}{2} = \frac{-2 \pm 2\sqrt{1+e}}{2} = \begin{cases} -1 - \sqrt{1+e} < 0 \\ -1 + \sqrt{1+e} = 0.923 \end{cases}$$

The solution is $\boxed{x = -1 + \sqrt{1+e}}$

Exercise

Solve the equation $\ln x = 1 + \ln(x+1)$

Solution

$$\ln x - \ln(x+1) = 1$$

$$\ln \frac{x}{x+1} = 1$$

$$\frac{x}{x+1} = e^1$$

$$x = (x+1)e$$

$$x = ex + e$$

$$x - ex = e$$

$$x(1-e) = e$$

$$x = \frac{e}{1-e} < 0$$

No solution

Exercise

Solve the equation $\log_3(x-2)=\log_3 27-\log_3(x-4)-5^{\log_5 1}$

Solution

$$\log_3(x-2)+\log_3(x-4)=\log_3 3^3-1$$

$$\log_3(x-2)(x-4)=3-1$$

$$\log_3\left(x^2-6x+8\right)=2$$

$$x^2-6x+8=3^2$$

$$x^2-6x+8=9$$

$$x^2-6x-1=0$$

$$x=3+\sqrt{10} \quad x=3-\sqrt{10}$$

Check: $x=3+\sqrt{10} \quad \Rightarrow \log_3\left(3+\sqrt{10}-2\right)=\log_3 27-\log_3\left(3+\sqrt{10}-4\right)-5^{\log_5 1}$

$x=3+\sqrt{10} \quad \Rightarrow \cancel{\log_3\left(3-\sqrt{10}-2\right)}=\log_3 27-\log_3\left(3-\sqrt{10}-4\right)-5^{\log_5 1}$

The solution is $\boxed{x=3+\sqrt{10}}$

Exercise

Solve the equation $\log_2(x+3)=\log_2(x-3)+\log_3 9+4^{\log_4 3}$

Solution

$$\log_2(x+3)-\log_2(x-3)=2+3$$

$$\log_2\frac{x+3}{x-3}=5$$

$$\frac{x+3}{x-3}=2^5$$

$$x+3=32(x-3)$$

$$x+3=32x-96$$

$$96+3=32x-x$$

$$31x=99$$

$x=\frac{99}{31}>3$ ***Domain***: $x>3$

The solution is: $\boxed{x=\frac{99}{31}}$

Exercise

Find the exact solution (2-decimal place approximation): $3^{x+4} = 2^{1-3x}$

Solution

$\ln 3^{x+4} = \ln 2^{1-3x}$ ***'ln' both sides***

$(x+4)\ln 3 = (1-3x)\ln 2$ ***Power Rule***

$x\ln 3 + 4\ln 3 = \ln 2 - 3x\ln 2$ ***Distribute***

$x\ln 3 + 3x\ln 2 = \ln 2 - 4\ln 3$

$x(\ln 3 + 3\ln 2) = \ln 2 - 4\ln 3$

$$\underline{x = \frac{\ln 2 - 4\ln 3}{\ln 3 + 3\ln 2} \approx -1.16}$$

Exercise

Find the exact solution (2-decimal place approximation): $3^{2-3x} = 4^{2x+1}$

Solution

$\ln 3^{2-3x} = \ln 4^{2x+1}$ ***'ln' both sides***

$(2-3x)\ln 3 = (2x+1)\ln 4$ ***Power Rule***

$2\ln 3 - 3x\ln 3 = 2x\ln 4 + \ln 4$

$-3x\ln 3 - 2x\ln 4 = \ln 4 - 2\ln 3$

$-x(3\ln 3 + 2\ln 4) = \ln 4 - 2\ln 3$

$$x = -\frac{\ln 4 - 2\ln 3}{3\ln 3 + 2\ln 4}$$

$$= -\frac{\ln 4 - \ln 3^2}{\ln 3^3 + \ln 4^2}$$

$$= \frac{\ln 9 - \ln 4}{\ln 27 + \ln 16}$$

$$= \frac{\ln \frac{9}{4}}{\ln 432}$$

$$\underline{\approx 0.13}$$

Exercise

Find the exact solution (2-decimal place approximation): $2^{-x^2} = 5$

Solution

$$\ln 2^{-x^2} = \ln 5$$
$$-x^2 \ln 2 = \ln 5$$
$$x^2 = -\frac{\ln 5}{\ln 2} \quad \Rightarrow \quad \textit{No Solution}$$

Exercise

Find the exact solution (2-decimal place approximation): $2^{-x} = 8$

Solution

$$2^{-x} = 2^3$$
$$-x = 3$$
$$\boxed{x = -3}$$

Exercise

Find the exact solution (2-decimal place approximation): $\log\left(x^2+4\right) - \log(x+2) = 2 + \log(x-2)$

Solution

$$\log\left(x^2+4\right) - \log(x+2) - \log(x-2) = 2$$
$$\log\left(x^2+4\right) - \left[\log(x+2) + \log(x-2)\right] = 2$$
$$\log\left(x^2+4\right) - \log(x+2)(x-2) = 2$$
$$\log\left(\frac{x^2+4}{x^2-4}\right) = 2$$
$$\frac{x^2+4}{x^2-4} = 10^2$$
$$x^2 + 4 = 100x^2 - 400$$
$$400 + 4 = 100x^2 - x^2$$
$$99x^2 = 404$$
$$x^2 = \frac{404}{99}$$
$$x = \pm\sqrt{\frac{404}{99}} \approx \pm 2.02 \qquad \boxed{x = 2.02} \text{ is the only solution}$$

Exercise

Find the exact solution (2-decimal place approximation): $5^x + 125\left(5^{-x}\right) = 30$

Solution

$$5^x 5^x + 125\left(5^{-x}\right)5^x = 30\left(5^x\right)$$

$$5^{2x} + 125 = 30\left(5^x\right)$$

$$5^{2x} - 30\left(5^x\right) + 125 = 0 \qquad \textit{Solve for } 5^x$$

$$5^x = 5 \qquad 5^x = 25 = 5^2$$

$$x = 1 \qquad x = 2$$

$$\boxed{x = 1,\ 2}$$

Exercise

Find the exact solution (2-decimal place approximation): $4^x - 3\left(4^{-x}\right) = 8$

Solution

$$4^x 4^x - 3\left(4^{-x}\right)4^x = 8\left(4^x\right)$$

$$4^{2x} - 3 = 8\left(4^x\right)$$

$$4^{2x} - 8\left(4^x\right) - 3 = 0 \qquad \textit{Solve for } 4^x$$

$$4^x = 4 + \sqrt{19} \qquad 4^x = 4 - \sqrt{19} < 0$$

$$x \ln 4 = \ln\left(4 + \sqrt{19}\right)$$

$$\underline{\left|x = \frac{\ln\left(4+\sqrt{19}\right)}{\ln 4} \approx 1.53\right|}$$

Exercise

Solve the equation without using the calculator: $\log x^2 = (\log x)^2$

Solution

$$2\log x = (\log x)^2$$

$$(\log x)^2 - 2\log x = 0$$

$$\log x(\log x - 2) = 0$$

$$\log x = 0 \qquad\qquad \log x - 2 = 0$$

$$x = 1 \qquad\qquad \log x = 2$$

$$x = 10^2 = 100$$

$$\boxed{x = 1,\ 100}$$

Exercise

Solve the equation without using the calculator: $\log(\log x) = 2$

Solution

$\log x = 10^2$ ***Convert to exponential***

$$\boxed{x = 10^{100}}$$

Exercise

Solve the equation without using the calculator: $\log\sqrt{x^3 - 9} = 2$

Solution

$\sqrt{x^3 - 9} = 10^2$ ***Convert to exponential***

$$\left(\sqrt{x^3 - 9}\right)^2 = (100)^2$$

$$x^3 - 9 = 10000$$

$$x^3 = 10009$$

$$\boxed{x = \sqrt[3]{10,009}}$$

Exercise

Solve the equation without using the calculator: $e^{2x} + 2e^x - 15 = 0$

Solution

$\left(e^x\right)^2 + 2e^x - 15 = 0$ ***Solve for*** e^x

$$e^x = 3 \qquad\qquad e^x \cancel{=} -5 < 0$$

$$\boxed{x = \ln 3}$$

Exercise

Solve the equation: $\log_3 x - \log_9 (x+42) = 0$

Solution

$$\frac{\ln x}{\ln 3} - \frac{\ln(x+42)}{\ln 9} = 0$$

$$\frac{\ln x}{\ln 3} - \frac{\ln(x+42)}{\ln 3^2} = 0$$

$$\frac{\ln x}{\ln 3} - \frac{\ln(x+42)}{2\ln 3} = 0$$

$$\frac{\ln x}{\ln 3} - \frac{1}{2}\frac{\ln(x+42)}{\ln 3} = 0$$

$$\frac{\ln x - \ln(x+42)^{1/2}}{\ln 3} = 0$$

$$\ln x - \ln(x+42)^{1/2} = 0$$

$$\ln x = \ln(x+42)^{1/2}$$

$$x = (x+42)^{1/2}$$

$$(x)^2 = \left((x+42)^{1/2}\right)^2$$

$$x^2 = x + 42$$

$$x^2 - x - 42 = 0 \Rightarrow x = -6, \ 7$$

The solution: $\boxed{x = 7}$

Exercise

Solve the equation $f(x) = x^3\left(4e^{4x}\right) + 3x^2 e^{4x}$

Solution

$$x^3\left(4e^{4x}\right) + 3x^2 e^{4x} = 0$$

$$x^2 e^{4x}(4x+3) = 0$$

$$x^2 = 0 \qquad 4x + 3 = 0$$

$$x = 0, \ 0 \qquad x = -\frac{3}{4}$$

The solutions are: $\boxed{x = 0, \ 0, \ -\frac{3}{4}}$