Hwk 5.2

#1. $$\frac{2s-1}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2}$$

$$2s-1 = A(s-2) + B(s+1)$$

s^1 $A + B = 2 \Rightarrow \underline{B = 1}$

s^0 $-2A + B = +1$

$3A = 3 \rightarrow \underline{A = 1}$

$$\frac{2s-1}{(s+1)(s-2)} = \frac{1}{s+1} + \frac{1}{s-2}$$

#2 $$\frac{2s-2}{(s-4)(s+2)} = \frac{A}{s-4} + \frac{B}{s+2}$$

$s^{2}\ s^{-1}$ $2s-2 = A(s+2) + B(s-4)$

s^1 $A + B = 2$ $\qquad \begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} = -6$

s^0 $2A - 4B = -2$

$$A = \frac{\begin{vmatrix} 2 & 1 \\ -2 & -4 \end{vmatrix}}{-6} = \frac{-6}{-6} = \underline{1}$$

$\underline{B = 1}$

$$\frac{2s-2}{(s-4)(s+2)} = \frac{1}{s-4} + \frac{1}{s+2}$$

#3 $$\frac{s^2+1}{s^3-2s^2-8s} \qquad s(s^2-2s-8)$$

$$\frac{s^2+1}{s^3-2s^2-8s} = \frac{A}{s} + \frac{B}{s-4} + \frac{C}{s+2}$$

$$s^2+1 = A(s^2-2s-8) + Bs(s+2) + Cs(s-4)$$

s^2 $A + B + C = 1$ ①

s^1 $-2A + 2B - 4C = 0$ ②

s^0 $-8A = 1 \Rightarrow \underline{A = -\frac{1}{8}}$

$$\begin{cases} B + C = 1 + \frac{1}{8} = \frac{9}{8} \\ 2B - 4C = -\frac{1}{4} \end{cases}$$

$$\begin{cases} B + C = \frac{9}{8} \\ -B + 2C = +\frac{1}{8} \end{cases}$$

$$3C = \frac{10}{8} = \frac{5}{24} \Rightarrow \boxed{C = \frac{5}{12}}$$

$$B = \frac{9}{8} - \frac{5}{12} = \frac{27-10}{24} = \frac{17}{24}$$

$$\frac{s^2-1}{s^3-2s-8s} = \frac{1}{8s} + \frac{17/24}{s-4} + \frac{5/12}{s+2}$$

$$\begin{cases} B + C = \frac{9}{8} \\ 2B - 4C = -\frac{1}{4} \end{cases} \qquad \begin{cases} 8B + 8C = 9 \\ 8B - 16C = -1 \end{cases}$$

#4 $\frac{1}{x^2+2x} = \frac{A}{x} + \frac{B}{x+2}$

$$1 = A(x+2) + Bx$$

$$\begin{cases} x^1 \quad A + B = 0 \Rightarrow B = -\frac{1}{2} \\ x^0 \quad 2A = 1 \rightarrow A = \frac{1}{2} \end{cases}$$

$$\frac{1}{x^2+2x} = \frac{1}{2}\frac{1}{x} - \frac{1}{2}\frac{1}{x+2}$$

#15 $\frac{2x+1}{x^2-7x+12} = \frac{A}{x-4} + \frac{B}{x-3}$

-1 $\quad 2x+1 = A(x-3) + B(x-4)$

$x^1 \quad {}^{+3}A \; {}^{+3}+B = 2^{+c}$

$x^0 \quad -3A - 4B = 1$

$$\begin{vmatrix} 1 & 1 \\ -3 & -4 \end{vmatrix} = -1$$

$$-B = 7 \Rightarrow B = -7$$

$$B = \frac{\begin{vmatrix} 1 & 2 \\ -3 & 1 \end{vmatrix}}{-1} = -7$$

$$A = 2 + 7 = 9$$

$$\frac{2x+1}{x^2-7x+12} = \frac{9}{x-4} - \frac{7}{x-3}$$

5.6 Arithmetic Seqn

$$a_n = a_1 + (n-1)d$$

Ex $20,\ 16.5,\ 13 \qquad a_{15}\,?$

$d = 16.5 - 20$
$= -3.5$

$a_{15} = 20 + 14(-3.5) \qquad \frac{7}{2}$
$= 20 - 49$
$= -29$

$3.5 = \frac{35}{10} = \frac{7}{2}$

$3.55 = \frac{355}{100}$

Ex $a_4 = 5 \qquad a_9 = 20 \qquad a_6\,?$ $\quad f(4) = 5$, $f(9) = 20$

$a_n = a_1 + (n-1)d$
$a_4 = a_1 + 3d = 5$
$a_9 = a_1 + 8d = 20$
$5d = 15$
$d = 3$

$d = \frac{20-5}{9-4} = \frac{15}{5} = 3$

$a_1 + 3(3) = 5$
$a_1 = -4$

$a_6 = -4 + 5(3)$
$= 11$

#22 a_{20}. $a_9 = -5$ $a_{15} = 31$ HW

$$d = \frac{31+5}{15-9} = \boxed{6}$$

$$a_1 + 8(6) = -5$$

$$\boxed{a_1 = -53}$$

$$a_{20} = -53 + 19(6)$$
$$= -53 + 114$$
$$\boxed{= 61}$$

Formula: $S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$$= \frac{n}{2}(a_1 + a_n)$$

Ex even from $2 \to 100$

$n = 50$

$$S_{50} = \frac{50}{2}(2+100)$$

102

$$= 50(51)$$
$$\boxed{= 2550}$$

$$\frac{1}{4} + \frac{2}{9} + \frac{3}{14} + \frac{4}{19} + \frac{5}{24} + \frac{6}{29} = \sum_{n=1}^{6} \frac{n}{5n-1}$$

$4, 9, 14, 19, \ldots$

$d = 5$

$$a_n = 4 + (n-1)(5)$$
$$= 5n - 1$$

Geometric Series

$$a_{k+1} = a_k r$$

$$a_n = a_1 r^{n-1}$$

Ex $a_1 = 3 \quad r = -\frac{1}{2}$ $\qquad a_n = 3\left(-\frac{1}{2}\right)^{n-1}$

$$a_1 = 3$$

$$a_2 = 3\left(-\frac{1}{2}\right)^1 = -\frac{3}{2}$$

$$a_3 = 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$$

Ex 3^{rd} term $a_3 = 5 \quad a_6 = -40 \quad a_8 ?$

$$r = \left(\frac{y_2}{y_1}\right)^{\frac{1}{n_2 - n_1}}$$

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 r^2 = 5$$

$$a_6 = a_1 r^5 = -40$$

$$\frac{a_6}{a_3} \quad \frac{a_1 r^5}{a_1 r^2} = \frac{-40}{5}$$

$$r^3 = -8 = (-2)^3$$

$$r = -2$$

$$r = \left(-\frac{40}{5}\right)^{1/3}$$

$$= (-8)^{1/3}$$

$$= -2$$

$$r = (-8)^{1/3}$$

$$5 = a_1(-2)^2$$

$$5 = 4a_1$$

$$a_1 = \frac{5}{4}$$

$$a_8 = \frac{5}{4}(-2)^7$$

$$= -5 \cdot 2^5$$

$$= -160$$

$$\frac{(-2)^7}{\cancel{4}} \quad 2(2)$$

#64 a_{10} ; $a_4 = 4$ $a_7 = 12$

$\boxed{r = (3)^{1/3}} = \sqrt[3]{3}$ $a_n = a_1 r^{n-1}$

$a_4 = 4 = a_1 (3^{1/3})^3$

$= 3a_1$

$\boxed{a_1 = \frac{4}{3}}$

$a_{10} = \frac{4}{3} (3^{1/3})^9$ $\frac{4}{3} 3^{9(\frac{1}{3})} = \frac{4}{3} 3^3$

$= \frac{4}{3} 3^3$ $4(9)$

$= 36$

66 a_7 : $a_2 = 3$ $a_3 = -\sqrt{3}$

$\boxed{r = -\frac{\sqrt{3}}{3}}$

$a_2 = a_1 \left(-\frac{\sqrt{3}}{3}\right) = 3$

$a_1 = -\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{= -3\sqrt{3}}$

$a_7 = (-3\sqrt{3}) \left(-\frac{\sqrt{3}}{3}\right)^6$ $\frac{\cancel{3}^2}{3 \cdot 3 \cdot 3}$

$\boxed{= -\frac{\sqrt{3}}{9}}$

$$S_n = a_1 \frac{1-r^n}{1-r} \quad n$$

$$\begin{cases} S = \dfrac{a_1}{1-r} & \text{if } |r| < 1 \\ S = \infty & \text{if } |r| \geq 1 \end{cases}$$

Ex $\displaystyle\sum_{n=1}^{\infty} 3\left(-\frac{2}{3}\right)^{n-1}$

$$|r| = +\frac{2}{3} < 1$$

$$S = \frac{3}{1-\left(\frac{-2}{3}\right)}$$

$$= \frac{3}{1+\frac{2}{3}}$$

$$= \frac{3}{\frac{5}{3}} \qquad 3 \cdot \frac{3}{5}$$

$$= \frac{9}{5}$$

Ex $\displaystyle\sum_{n=1}^{\infty} 3\left(-\frac{3}{2}\right)^{n-1} = \infty \qquad |r| = \frac{3}{2} \geq 1$

$$5.4\overline{27} = 5.4272727\ldots$$
$$= 5.4 + .02727\ldots$$
$$= \frac{54}{10} + .027 + .00027 + \ldots$$

$$r = \frac{.00027}{.027} = .01$$

$$5.4\overline{27} = \frac{54}{10} + \frac{.027}{1-.01}$$
$$= \frac{54}{10} + \frac{.027}{.990}$$
$$= \frac{54}{10} + \frac{27}{990}$$
$$= \frac{54}{10} + \frac{3}{110}$$
$$= \frac{594+3}{110}$$
$$= \frac{597}{}$$

98 $\sum_{k=1}^{20}(3k-5) = \sum_{k=1}^{20} 3k - \sum_{k=1}^{20} 5$

$$= 3\left(\frac{20}{2}(1+20)\right) - 5(20)$$
$$= 30(21) - 100$$
$$= 630 - 100$$
$$= \underline{530}$$

#110 $\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1}$ $\quad |r| = \frac{1}{4} < 1$

$$S = \frac{5}{1-\frac{1}{4}}$$
$$= 5 \cdot \frac{4}{3}$$
$$= \underline{\frac{20}{3}}$$

#111 $\sum_{n=1}^{\infty} 8\left(\frac{1}{3}\right)^{n-1}$ $\quad |r| = \frac{1}{3} < 1$

$$S = \frac{8}{1-\frac{1}{3}}$$
$$= 8\left(\frac{3}{2}\right)$$
$$= \underline{12}$$

#112 $\sum_{k=1}^{\infty} \frac{1}{2}(3)^{k-1} = \infty$ $\quad |r| = 3 \geq 1$

#113 $\sum_{k=1}^{\infty} 6\left(-\frac{2}{3}\right)^{k-1}$ $\quad |r| = \frac{2}{3} < 1$

$$S = \frac{6}{1+\frac{2}{3}}$$
$$= 6 \cdot \frac{3}{5}$$
$$= \underline{\frac{18}{5}}$$

arith.-- $d = \frac{y_2 - y_1}{x_2 - x_1}$ $a_n = a_1 + (n-1)d$

Geom. $r = \left(\frac{y_2}{y_1}\right)^{1/x_2 - x_1}$ $a_n = a_1 r^{n-1}$

$$\begin{cases} S = \frac{a}{1-r} & |r| < 1 \\ S = \infty & |r| \geq 1 \end{cases}$$

Sum of of $n(>0)$ numbers.

P_n: $\frac{n(n+1)}{2}$

Use Mathematical Induction

① P_1 is true

② Assume P_k is true, Prove P_{k+1} also true.

P_n: $\frac{n(n+1)}{2} = 1 + 2 + \cdots + n$

① $1 \stackrel{?}{=} \frac{1(2)}{2}$

$1 = 1$ ✓ Hence, P_1 is true

② Assume: P_k: $1 + 2 + \cdots + k = \frac{k(k+1)}{2}$ is true

P_{k+1}: $1 + 2 + \cdots + k + (k+1) = \frac{(k+1)(k+2)}{2}$? true

$$\underline{1 + 2 + \cdots + k} + (k+1) = \frac{1}{2}k(k+1) + (k+1)$$

$$= (k+1)\left(\frac{k}{2} + 1\right)$$

$$= (k+1)\left(\frac{k+2}{2}\right) ✓$$

P_{k+1} is also true.

∴ By the Mathematical Induction, the proof is completed