Solution   Section 4.4 – Bernoulli Trials & Binomial Distributions

Exercise

If a baseball player has a batting average of 0.350, what is the probability that the player will get the following number of hits in the next four times at bat?

a) Exactly 2 hits

b) At least 2 hits.

Solution

a) ***Given***: $p = .35 \to q = 1 - .35 = .65, \quad n = 4$

$$P(x=2) = C_{4,2}(.35)^2(.65)^2$$

$$\approx 0.311$$

b) $P(x \ge 2) = P(2) + P(3) + P(4)$

$$= C_{4,2}(.35)^2(.65)^2 + C_{4,3}(.35)^3(.65) + C_{4,4}(.35)^4(.65)^0$$

$$= .3105 + .1115 + .015$$

$$\approx 0.437$$

Exercise

If a true-false test with 10 questions is given, what is the probability of scoring

a) Exactly 70% just by guessing?

b) 70% or better just by guessing?

Solution

Given: $p = .5 \to q = 1 - .5 = .5, \quad n = 10$

a) $P(x=7) = C_{10,7}(.5)^7(.5)^3$

$$\approx 0.117$$

b) $P(x \ge 7) = P(7) + P(8) + P(9) + P(10)$

$$= C_{10,7}(.5)^7(.5)^3 + C_{10,8}(.5)^8(.5)^2 + C_{10,9}(.5)^9(.5)^1 + C_{10,10}(.5)^{10}(.5)^0$$

$$\approx 0.172$$

Exercise

If 60% of the electorate supports the mayor, what is the probability that in a random sample of 10 voters, fewer than half support her?

Solution

$$p = P(\text{electorate supports the mayor}) = .6 \rightarrow q = .4, \quad n = 10$$

$$P(x \le 4) = P(4) + P(3) + P(2) + P(1) + P(0)$$

$$= C_{10,4}(.6)^4(.4)^6 + C_{10,3}(.6)^3(.4)^7 + C_{10,2}(.6)^2(.4)^8 + C_{10,1}(.6)^1(.4)^9 + C_{10,0}(.4)^{10}$$

$$\approx 0.166$$

Exercise

Each year a company selects a number of employees for a management training program given by nearby university. On the average, 70% of those sent complete the program. Out of 7 people sent by the company, what is the probability that

a) Exactly 5 complete the program?

b) 5 or more complete the program?

Solution

a) $p = .7 \rightarrow q = 1 - .7 = .3, \ \ n = 7$

$$P(x = 5) = C_{7,5}(.7)^5(.3)^2 = .318$$

$$= .318$$

b) $P(x \ge 5) = P(5) + P(6) + P(7)$

$$= .318 + C_{7,6}(.7)^6(.3) + C_{7,7}(.7)^7(.3)^0$$

$$= .318 + .2471 + .0824$$

$$\approx 0.647$$

Exercise

If the probability of a new employee in a fast-food chain still being with the company at the end of 1 year is 0.6, what is the probability that out of 8 newly hired people?

a) 5 will still be with the company after 1 year?

b) 5 or more will still be with the company after 1 year?

Solution

a) $p = .6 \rightarrow q = .4, \quad n = 8$

$$P(x = 5) = C_{8,5}(.6)^5(.4)^3$$

$$= .279$$

b) $P\left(x \ge 5\right) = P(5) + P(6) + P(7) + P(8)$

$$= C_{8,5}(.6)^5(.4)^3 + C_{8,6}(.6)^6(.4)^2 + C_{8,7}(.6)^7(.4)^1 + C_{8,8}(.6)^8$$

$$\underline{\approx 0.594|}$$

Exercise

A manufacturing process produces, on the average, 6 defective items out of 100. To control quality, each day a sample of 10 completed items is selected at random and inspected. If the sample produces more than 2 defective items, then the whole day's output is inspected and the manufacturing process is reviewed. What is the probability of this happening, assuming that the process is still producing 6% defective items?

Solution

$$p = P(\text{defective}) = .06 \rightarrow q = P(\text{not defective}) = .94 \qquad n = 10$$

$$P(x > 2) = 1 - P(x \le 2) = 1 - \left[P(2) + P(1) + P(0)\right]$$

$$= 1 - \left[C_{10,2}(.06)^2(.94)^8 + C_{10,1}(.06)^1(.94)^9 + C_{10,0}(.06)^0(.94)^{10}\right]$$

$$\approx 1 - (.0988 + .3438 + .5386)$$

$$\underline{\approx 0.188|}$$

A day's output will be inspected with a probability of .0188

Exercise

A manufacturing process produces, on the average, 3% defective items. The company ships 10 items in each box and wishes to guarantee no more than 1 defective item per box. If this guarantee accompanies each box, what is the probability that the box will fail to satisfy the guarantee?

Solution

$$p = .03 \rightarrow q = .97 \quad n = 10$$

$$P(\text{fail to satisfy}) = P(x \ge 2) = 1 - P(x < 2)$$

$$= 1 - P(x < 2)$$

$$= 1 - [P(0) + P(1)]$$

$$= 1 - C_{10,0}(.03)^0(.97)^{10} + C_{10,1}(.03)^1(.97)^9$$

$$\underline{\approx 0.035|}$$

Exercise

A manufacturing process produces, on the average, 5 defective items out of 100. To control quality, each day a random sample of 6 completed items is selected and inspected. If a success on a single trial (inspection of 1 item) is finding the item defective, then the inspection of each of the 6 items in the sample constitutes a binomial experiment, which has a binomial distribution.

a) Write the function defining the distribution
b) Construct a table and histogram for the distribution.
c) Compute the mean and standard deviation.

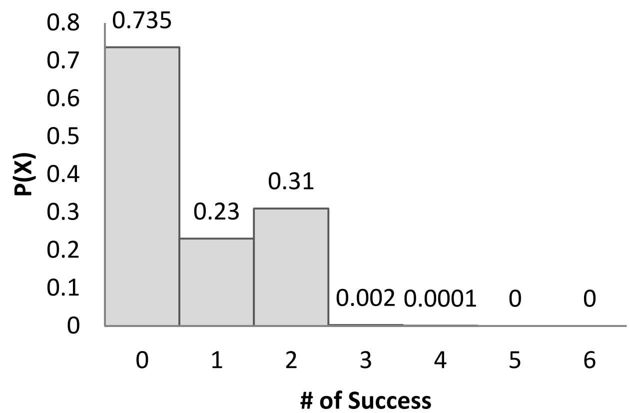
Solution

$$p = \frac{5}{100} = .05 \rightarrow q = .95 \qquad n = 6$$

a) $P(X) = C_{6,x}(.05)^x(.95)^{6-x} \quad , x = 0,1,2,3,4,5,6$

b) Table and histogram for the distribution.

x	*P(x)*
0	.735
1	.23
2	.31
3	.002
4	.0001
5	.000
6	.000



c) $\mu = np$

$= 6 \times .05$

$\underline{= 3}$

$\sigma = \sqrt{npq}$

$= \sqrt{6(.05)(.95)}$

$\underline{= .53}$

Exercise

Each year a company selects 5 employees for a management training program given at a nearly university. On the average, 40% of those sent complete the course in the top 10% of their class. If we consider an employee finishing in the top 10% of the class a success in a binomial experiment, then for the 5 employee entering the program there exists a binomial distribution involving P(x success out of 5).

a) Write the function defining the distribution

b) Construct a table and histogram for the distribution.

c) Compute the mean and standard deviation.

Solution

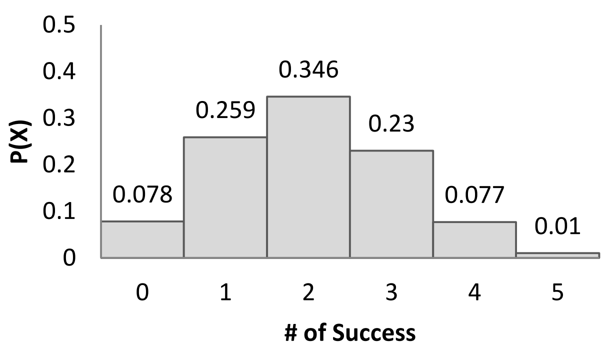
$$p = .4 \to q = .6 \qquad n = 5$$

a) $P(X) = C_{5,x}(.4)^{x}(.6)^{5-x} \quad , \ x = 0, 1, 2, 3, 4, 5$

b) Table

x	$P(x)$
0	0.078
1	0.259
2	0.346
3	0.230
4	0.077
5	0.01

Histogram



c) $\mu = np$

$$= 5 \times .4$$
$$\underline{= 2}$$

$$\sigma = \sqrt{npq}$$
$$= \sqrt{5(.4)(.6)}$$
$$\underline{\approx 1.095}$$

Exercise

A person with tuberculosis is given a chest x-ray. Four tuberculosis x-ray specialists examine each x-ray independently. If each specialist can detect tuberculosis 80% of the time when it is present, what is the probability that at least 1 of the specialists will detect tuberculosis in this person?

Solution

$$p = .8 \rightarrow q = .2 \qquad n = 4$$

$$\begin{aligned} P(x \geq 1) &= 1 - P(x < 1) \\ &= 1 - P(x = 0) \\ &= 1 - C_{4,0}(.8)^4(.2)^4 \approx 0.998 \end{aligned}$$

Exercise

A pharmaceutical laboratory claims that a drug it produces causes serious side effects in 20 people out of 1,000 on the average. To check this claim, a hospital administers the drug to 10 randomly chosen patients and finds that 3 suffer from serious side effects. If the laboratory's claims are correct, what is the probability of the hospital obtaining these results?

Solution

$$p = \frac{20}{1000} = .02 \rightarrow q = .98 \qquad n = 10$$

$$\begin{aligned} P(x = 3) &= C_{10,3}(.02)^3(.98)^7 \\ &\approx 0.0008 \end{aligned}$$

Exercise

The probability that brown-eyed parents, both with the recessive gene for blue, will have a child with brown eye is .75. If such parents have 5 children, what is the probability that they will have

a) All blue-eyed children?

b) Exactly 3 children with brown eyes?

c) At least 3 children with brown eyes?

Solution

$$p = .75 \rightarrow q = .25 \qquad n = 5$$

a) $P(x = 0) = C_{5,0}(.75)^0(.25)^5 \approx 0.00098$

b) $P(x = 3) = C_{5,3}(.75)^3(.25)^2 \approx 0.264$

c) $$\begin{aligned} P(x \geq 3) &= P(3) + P(4) + P(5) \\ &= C_{5,3}(.75)^3(.25)^2 + C_{5,4}(.75)^4(.25)^1 + C_{5,5}(.75)^5(.25)^0 \\ &\approx .897 \end{aligned}$$

Exercise

The probability of gene mutation under a given level of radiation is 3×10^{-5}. What is the probability of the occurrence of at least 1 gene mutation if 10^5 genes are expected to this level of radiation?

Solution

$$p = 3\times10^{-5} \rightarrow q = 1 - 3\times10^{-5}$$

$$P(x \ge 1) = 1 - P(0)$$

$$= 1 - \left(1 - 3\times10^{-5}\right)^{10^5}$$

$$\approx 0.95$$

Exercise

If the probability of a person contracting influenza on exposure is .6 consider the binomial distribution for a family of 6 that has been exposed.

a) Write the function defining the distribution.

b) Compute the mean and standard deviation.

Solution

$$p = 0.6 \rightarrow q = .4, \qquad n = 6$$

a) $P(X) = C_{6,x}(.6)^x(.4)^{6-x}$, $x = 0,1,2,3,4,5,6$

b) $\mu = np = 6(.6) = 3.6$

$$\sigma = \sqrt{npq} = \sqrt{6(.6)(.4)} = 1.2$$

Exercise

The probability that a given drug will produce a serious side effect in a person using the drug is .02. In the binomial distribution for 450 people using the drug, what are the mean and standard deviation?

Solution

$$p = 0.02 \rightarrow q = .98, \qquad n = 450$$

$$\mu = np$$

$$= 450\times.02$$

$$= 9$$

$$\sigma = \sqrt{npq}$$

$$= \sqrt{450\times.02\times.98}$$

$$\approx 2.97$$

Exercise

An opinion poll based on a small sample can be unrepresentative of the population. To see why, let us assume that 40% of the electorate favors a certain candidate. If a random sample of 7 is asked their preference, what is the probability that a majority will favor this candidate?

Solution

$$p = 0.4 \to q = .6, \qquad n = 7$$

$$P(x \ge 4) = P(4) + P(5) + P(6) + P(7)$$

$$P(x \ge 4) = C_{7,4}(.4)^4(.6)^3 + C_{7,5}(.4)^5(.6)^2 + C_{7,6}(.4)^6(.6)^1 + C_{7,7}(.4)^7(.6)^0$$

$$\approx 0.29$$

(Better than one chance out of four)

Exercise

A multiple choice test is given with 5 choices only one is correct, for each of 5 questions. Answering each of the 5 questions by guessing constitutes a binomial experiment with an associated binomial distribution

a) Write the function defining the distribution.

b) Compute the mean and standard deviation.

Solution

$$p = \frac{1}{5} = 0.2 \to q = .8, \qquad n = 5$$

a) $P(X) = C_{5,x}(.2)^x(.8)^{5-x} \quad , x = 0,1,2,3,4,5$

b) $\mu = np = 5(.2) = 1$

$$\sigma = \sqrt{npq}$$

$$= \sqrt{5(.2)(.8)}$$

$$= .894$$

Exercise

Suppose a die is rolled 4 times.

a) Find the probability distribution for the number of times 1 is rolled.

b) What is the expected number of times 1 is rolled

Solution

a) $P(x=0) = C_{4,0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4 \approx 0.482$

$$P(x=1)=C_{4,1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 \underline{\approx 0.386|}$$

$$P(x=2)=C_{4,2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 \underline{\approx 0.116|}$$

$$P(x=3)=C_{4,3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 \underline{\approx 0.0154|}$$

$$P(x=4)=C_{4,4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 \underline{\approx 0.00077|}$$

x	0	1	2	3	4
$P(x)$	0.482	0.386	0.116	0.0154	0.00077

b) $E(x)=0(0.482)+1(0.386)+2(0.116)+3(0.0154)+4(0.00077)$

$\underline{\approx 0.667|}$