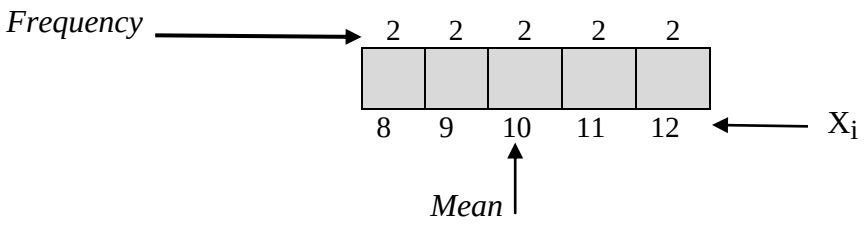
Section 4.3 – Measures of Variation / Dispersion



Range

Range is the difference between the largest and the smallest number in a sample.

Example

Find the range

a) 12, 27, 6, 19, 38, 9, 42, 15
b) 74, 112, 59, 88, 200, 73, 92, 175

Solution

a) ***Range*** = 42 – 6 = 36

b) ***Range*** = 200 – 59 = 141

Deviation from the Mean

Sample Mean $\overline{X} = \frac{\sum x_i}{n}$

The deviation from the mean is the difference between the mean and each number: $\left(x_i - \bar{x}\right)$

Variance

The variance of a sample of n numbers $x_1, x_2, x_3, \ldots, x_n$, with mean $\bar{x}$, is

$$[Variance] = s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1}$$

Standard Deviation

$$[\textit{Standard Deviation}] = s = \sqrt{\frac{\sum x^2 - n\bar{x}^2}{n-1}}$$

TI-8x

Store data: create a list
STAT menu $\rightarrow$ *CALC* $\rightarrow$ ***1: 1-Var Stats***

Example

Find the standard deviation of the numbers: 7, 9, 18, 22, 27, 29, 32, 40.

Solution

$$Mean = \frac{7+9+18+22+27+29+32+40}{8}$$

$$= 23$$

Number, x	Square of the Number, x^2
7	49
9	81
18	324
22	484
27	729
29	841
32	1024
40	1600
	Total: 5132

$$s = \sqrt{\frac{\sum x^2 - n\bar{x}^2}{n-1}}$$

$$= \sqrt{\frac{5132 - 8(23)^2}{8-1}}$$

$$\approx 11.34$$

Standard Deviation for a Grouped Distribution

$$\textit{Standard Deviation: } s = \sqrt{\frac{\sum fx^2 - n\bar{x}^2}{n-1}}$$

Example

Find ***s*** for the grouped

Interval	*Frequency*
0 – 4	3
5 – 9	4
10 – 14	6
15 – 19	8
20 – 24	5
25 – 39	3
30 – 34	1
Total	30

Solution

Interval	x	x^2	f	fx^2
0 – 4	2	4	3	12
5 – 9	7	49	4	196
10 – 14	12	144	6	864
15 – 19	17	289	8	2312
20 – 24	22	484	5	2420
25 – 39	27	729	3	2187
30 – 34	32	1024	1	1024
Total			30	9015

$$s = \sqrt{\frac{\sum fx^2 - n\bar{x}^2}{n-1}}$$

$$= \sqrt{\frac{9015 - 30(15.5)^2}{30-1}}$$

$$\approx 7.89$$

TI-8x

Store data: create 2 lists
STAT menu → *CALC* → ***1: 1-Var Stats L1, L2***

Example

Since 1996, Nathan's Famous Hot Dogs has held an annual hot dog eating contest, in which each contest attempts to consume as many hot dogs with burns as possible in a 12-minute period.

Year	Winner	Hot Dogs Eaten
1997	Hirofumi Nakajima	24.5
1998	Hirofumi Nakajima	19
1999	Steve Keiner	20.25
2000	Kazutoyo Arai	25.125
2001	Takeru Kobayashi	50
2002	Takeru Kobayashi	50.5
2003	Takeru Kobayashi	44.5
2004	Takeru Kobayashi	53.5
2005	Takeru Kobayashi	49
2006	Takeru Kobayashi	53.75
2007	Joey Chestnut	66

In what percent of the contests did the number of hot dogs eaten by the winner fall within one standard deviation of the mean number of hot dogs?

Solution

$$\bar{x} = \frac{24.5+19+20.25+25.125+50+50.5+44.5+53.5+49+53.75+66}{11}$$

$$\approx 41.47$$

$$s \approx 16.21$$

Lower: $\bar{x} - s = 41.47 - 16.21 = 25.26$

Upper: $\bar{x} + s = 41.47 + 16.21 = 57.68$

$\therefore$ 6 out of 11 (node: between the lower and upper) $\rightarrow$ = 55%

Exercises *Section* 4.3 – Measures of Variation / Dispersion

1. Find the range and standard deviation for: $\{3,\ 7,\ 4,\ 12,\ 15,\ 18,\ 19,\ 27,\ 24,\ 11\}$

2. Find the range and standard deviation for: $S = \{1.2, 1.4, 1.7, 1.3, 1.5\}$

3. Find the range and standard deviation for: 72, 61, 57, 83, 52, 66, 85

4. Find the range and standard deviation for: 241, 248, 251, 257, 252, 287

5. Find the range and standard deviation for: 122, 132, 141, 158, 162, 169, 180

6. Find the standard deviation for the following data

Interval	*Frequency*
30 – 39	1
40 – 49	6
50 – 59	13
60 – 69	22
70 – 79	17
80 – 89	13
90 – 99	8

7. Find the standard deviation for the following data

Interval	*Frequency*
0 – 24	4
25 – 49	8
50 – 74	5
75 – 99	10
100 – 124	4
125 – 149	5

8. Forever Power Company analysis conducted tests on the life of its batteries and those of a competitor (Brand X). They found that their batteries have a mean life (in hours) of 26.2, with a standard deviation of 4.1. Their results for a sample of 10 Brand X were as follows: 15, 18, 19, 23, 25, 25, 28, 30, 34, 38.

a) Find the mean and standard deviation for the sample of Brand X batteries.

b) Which batteries have a more uniform life in hours?

c) Which batteries have the highest average life in hours?