6 INTEGRATION

EXERCISE 6-1

2. $\int 10dx = 10x + C$

Check: $\frac{d}{dx}(10x + C) = 10 \cdot 1 + 0 = 10$

4. $\int 14xdx = 7x^2 + C$

Check: $\frac{d}{dx}(7x^2 + C) = 7 \cdot 2x + 0 = 14x$

6. $\int 15x^2 dx = 5x^3 + C$

Check: $\frac{d}{dx}(5x^3 + C) = 5 \cdot 3x^2 + 0 = 15x^2$

8. $\int x^8 dx = \frac{1}{9}x^9 + C$;

Check: $\frac{d}{dx}\left(\frac{1}{9}x^9 + C\right) = \frac{1}{9} \cdot 9x^8 + 0 = x^8$

10. $\int x^{-4} dx = -\frac{1}{3}x^{-3} + C$

Check: $\frac{d}{dx}\left(-\frac{1}{3}x^{-3} + C\right) = -\frac{1}{3}\left(-3x^{-4}\right) + 0 = x^{-4}$

12. $\int 8x^{1/3} dx = 6x^{4/3} + C$

Check: $\frac{d}{dx}\left(6x^{4/3} + C\right) = 6\left(\frac{4}{3}x^{1/3}\right) + 0 = 8x^{1/3}$

14. $\int \frac{7}{z} dz = 7\ln|z| + C$

Check: $\frac{d}{dz}(7\ln|z| + C) = 7\left(\frac{1}{z} + 0\right) = \frac{7}{z}$

16. $\int 5e^u du = 5e^u + C$

Check: $\frac{d}{du}(5e^u + C) = 5e^u + 0 = 5e^u$

18. $\frac{dx}{dt} = 42t^5$

$x = \int 42t^5\, dt = \frac{42}{6}t^6 + C = 7t^6 + C$

20. $\dfrac{dy}{dx} = 3x^2 - 4x^3$

$y = \int (3x^2 - 4x^3)dx \qquad = \int 3x^2\,dx - \int 4x^3\,dx$

$= x^3 - x^4 + C$

22. $\dfrac{dy}{dx} = x - e^x$

$y = \int (x - e^x)dx \qquad = \int x\,dx - \int e^x\,dx$

$= \dfrac{1}{2}x^2 - e^x + C$

24. $\dfrac{du}{dv} = \dfrac{4}{v} + \dfrac{v}{4}$

$du = \int \left(\dfrac{4}{v} + \dfrac{v}{4}\right) dv \qquad = \int \dfrac{4}{v}\,dv + \int \dfrac{v}{4}\,dv$

$= 4\ \ln|v| + \dfrac{1}{4} \cdot \dfrac{1}{2} v^2 + C$

$= 4\ \ln|v| + \dfrac{1}{8} v^2 + C$

26. False, since any antiderivative of $f(x) = \pi$ is of the form $F(x) = \pi x + C$ which is identically 0 only when $C = 0$ and $x = 0$.

28. True, since any antideriative of $k(x) = 0$ is of the form $K(x) = C$ and $K(x) = 0$ is an antiderivative of $k(x) = 0$.

30. False, since any antiderivative of $g(x) = 5e^{\pi}$ is of the form $G(x) = (5e^{\pi})x + C$ which obviously is not equal to $g(x) = 5e^{\pi}$.

32. No, since one graph cannot be obtained from another by a vertical translation.

34. Yes, since one graph can be obtained from another by a vertical translation.

36. $\int x^2(1 + x^3)dx \qquad = \int (x^2 + x^5)dx = \int x^2 dx + \int x^5 dx$

$= \dfrac{1}{3}x^3 + \dfrac{1}{6}x^6 + C$ [using Indefinite Integral Formula]

Check: $\left(\dfrac{1}{3}x^3 + \dfrac{1}{6}x^6 + C\right)' = \dfrac{1}{3} \cdot 3x^2 + \dfrac{1}{6} \cdot 6x^5 + 0 = x^2 + x^5 = x^2(1 + x^3)$

38. $\int \dfrac{dt}{\sqrt[3]{t}} = \int \dfrac{dt}{t^{1/3}} = \int t^{-1/3}\,dt$

$= \dfrac{1}{-\frac{1}{3}+1} t^{(-1/3)+1} + C = \dfrac{3}{2}t^{2/3} + C$

Check: $\left(\dfrac{3}{2}t^{2/3} + C\right)' = \dfrac{3}{2}\left(\dfrac{2}{3}\right)t^{(2/3)-1} + 0 = t^{-1/3} = \dfrac{1}{t^{1/3}} = \dfrac{1}{\sqrt[3]{t}}$

40. $\int \dfrac{6\,dm}{m^2} = 6\int m^{-2}\,dm$

$= 6(-m^{-1}) + C = -\dfrac{6}{m} + C$

Check: $(-6m^{-1} + C)' = (-6)(-1)m^{-2} + 0 = 6m^{-2} = \dfrac{6}{m^2}$

42. $\int \frac{1-y^2}{3y}\,dy = \int \frac{1}{3y}\,dy - \int \frac{y^2}{3y}\,dy$

$= \frac{1}{3}\int \frac{1}{y}\,dy - \frac{1}{3}\int y\,dy$

$= \frac{1}{3}\ln|y| - \frac{1}{3}\cdot\frac{1}{2}y^2 + C$

$= \frac{1}{3}\ln|y| - \frac{1}{6}y^2 + C$

Check: $\left(\frac{1}{3}\ln|y| - \frac{1}{6}y^2 + C\right)' = \frac{1}{3}\cdot\frac{1}{y} - \frac{1}{6}(2y) + 0 = \frac{1}{3y} - \frac{y}{3} = \frac{1-y^2}{3y}$

44. $\int \frac{e^t - t}{2}\,dt = \int \left(\frac{e^t}{2} - \frac{t}{2}\right)dt = \int \frac{e^t}{2}\,dt - \int \frac{t}{2}\,dt$

$= \frac{1}{2}e^t - \frac{t^2}{4} + C$ [using Indefinite Integral Formula]

Check: $\left(\frac{1}{2}e^t - \frac{t^2}{4} + C\right)' = \frac{1}{2}e^t - \frac{2t}{4} + 0 = \frac{e^t}{2} - \frac{t}{2} = \frac{e^t - t}{2}$

46. $\int \left(4x^3 + \frac{2}{x^3}\right)dx = 4\int x^3\,dx + 2\int x^{-3}\,dx$

$= 4\left(\frac{x^4}{4}\right) + 2\left(\frac{1}{-2}x^{-2}\right) + C$

$= x^4 - x^{-2} + C$

Check: $(x^4 - x^{-2} + C)' = 4x^3 - (-2)x^{-3} + 0 = 4x^3 + 2x^{-3} = 4x^3 + \frac{2}{x^3}$

48. $\int \left(\frac{2}{\sqrt[3]{x}} - \sqrt[3]{x^2}\right)dx = \int \left(\frac{2}{x^{1/3}} - x^{2/3}\right)dx$

$= 2\int x^{-1/3}\,dx - \int x^{2/3}\,dx$

$= 2\left(\frac{x^{2/3}}{2/3}\right) - \left(\frac{x^{5/3}}{5/3}\right) + C$

$= 3x^{2/3} - \frac{3}{5}x^{5/3} + C$

Check: $\left(3x^{2/3} - \frac{3}{5}x^{5/3} + C\right)' = 3\left(\frac{2}{3}\right)x^{-1/3} - \frac{3}{5}\left(\frac{5}{3}\right)x^{2/3}$

$= 2x^{-1/3} - x^{2/3} = \frac{2}{x^{1/3}} - \sqrt[3]{x^2} = \frac{2}{\sqrt[3]{x}} - \sqrt[3]{x^2}$

50. $\int \frac{e^x - 3x^2}{2}\,dx = \int \left(\frac{e^x}{2} - \frac{3x^2}{2}\right)dx$

$= \frac{1}{2}\int e^x\,dx - \frac{3}{2}\int x^2\,dx$

$$= \frac{1}{2}e^x - \frac{3}{2}\left(\frac{x^3}{3}\right) + C$$

$$= \frac{1}{2}e^x - \frac{1}{2}x^3 + C$$

Check: $\left(\frac{1}{2}e^x - \frac{1}{2}x^3 + C\right)' = \frac{1}{2}e^x - \frac{1}{2}(3x^2) + 0$

$$= \frac{1}{2}e^x - \frac{3x^2}{2} = \frac{e^x - 3x^2}{2}$$

52. $R'(x) = 600 - 0.6x$

$R(x) = \int (600 - 0.6x)dx = \int 600\,dx - 0.6\int x\,dx$

$$= 600x - 0.6\left(\frac{x^2}{2}\right) + C = 600x - 0.3x^2 + C$$

Given $R(0) = 0$: $0 = 600(0) - 0.3(0)^2 + C$. Hence, $C = 0$ and $R(x) = 600x - 0.3x^2$.

54. $\frac{dR}{dt} = \frac{100}{t^2}$

$R = \int \frac{100}{t^2}\,dt = \int 100t^{-2}\,dt = 100\left(\frac{t^{-1}}{-1}\right) + C = -100t^{-1} + C$

Given $R(1) = 400$: $400 = -100(1)^{-1} + C = -100 + C$. Hence, $C = 500$ and $R = -100t^{-1} + 500 = 500 - \frac{100}{t}$

56. $\frac{dy}{dx} = 3x^{-1} + x^{-2}$

$y = \int (3x^{-1} + x^{-2})dx = 3\int x^{-1}\,dx + \int x^{-2}\,dx$

$$= 3\ln|x| + \left(\frac{x^{-1}}{-1}\right) + C = 3\ln|x| - x^{-1} + C$$

Given $y(1) = 1$: $1 = 3\ln|1| - (1)^{-1} + C$. Hence, $C = 2$ and $y = 3\ln|x| - x^{-1} + 2$.

58. $\frac{dy}{dt} = 5e^t - 4$

$y = \int (5e^t - 4)dt = \int 5e^t\,dt - \int 4\,dt = 5e^t - 4t + C$

Given $y(0) = -1$: $-1 = 5e^0 - 4(0) + C$. Hence, $C = -6$ and $y = 5e^t - 4t - 6$.

60. $\frac{dy}{dx} = 12x^2 - 12x$

$y = \int (12x^2 - 12x)dx \quad = 12\int x^2\,dx - 12\int x\,dx$

$= 12\left(\frac{x^3}{3}\right) - 12\left(\frac{x^2}{2}\right) + C = 4x^3 - 6x^2 + C$

Given $y(1) = 3$: $3 = 4(1)^3 - 6(1)^2 + C$. Hence, $C = 5$ and
$y = 4x^3 - 6x^2 + 5$.

62. $\int \frac{x^{-1} - x^4}{x^2}\,dx \quad = \int \left(\frac{x^{-1}}{x^2} - \frac{x^4}{x^2}\right)dx$

$= \int (x^{-3} - x^2)dx = \int x^{-3}\,dx - \int x^2\,dx$

$= \frac{x^{-2}}{-2} - \frac{x^3}{3} + C = -\frac{1}{2}x^{-2} - \frac{1}{3}x^3 + C$

64. $\int \frac{1 - 3x^4}{x^2}\,dx \quad = \int \left(\frac{1}{x^2} - \frac{3x^4}{x^2}\right)dx$

$= \int (x^{-2} - 3x^2)dx = \int x^{-2}\,dx - 3\int x^2\,dx$

$= \frac{x^{-1}}{-1} - 3\left(\frac{x^3}{3}\right) + C$

$= -x^{-1} - x^3 + C$

66. $\int \frac{1 - xe^x}{x}\,dx \quad = \int \left(\frac{1}{x} - \frac{xe^x}{x}\right)dx$

$= \int (x^{-1} - e^x)dx = \int x^{-1}\,dx - \int e^x\,dx = \ln|x| - e^x + C$

68. $\frac{dR}{dx} = \frac{1 - x^4}{x^3}$

$R = \int \left(\frac{1 - x^4}{x^3}\right)dx \quad = \int \left(\frac{1}{x^3} - \frac{x^4}{x^3}\right)dx$

$= \int (x^{-3} - x)dx = \int x^{-3}\,dx - \int x\,dx$

$= \frac{x^{-2}}{-2} - \frac{x^2}{2} + C$

$= -\frac{1}{2}x^{-2} - \frac{1}{2}x^2 + C$

Given $R(1) = 4$: $4 = -\frac{1}{2}(1)^{-2} - \frac{1}{2}(1)^2 + C$. Hence, $C = 5$ and

$R = -\frac{1}{2}x^{-2} - \frac{1}{2}x^2 + 5$.

70. $\dfrac{dx}{dt} = \dfrac{\sqrt{t^3} - t}{\sqrt{t^3}}$

$$x = \int \left(\frac{\sqrt{t^3} - t}{\sqrt{t^3}}\right) dt = \int \left(1 - \frac{t}{\sqrt{t^3}}\right) dt$$

$$= \int \left(1 - \frac{t}{t^{3/2}}\right) dt$$

$$= \int (1 - t^{-1/2}) dt$$

$$= \int dt - \int t^{-1/2}\, dt$$

$$= t - \frac{t^{1/2}}{1/2} + C = t - 2t^{1/2} + C$$

Given $x(9) = 4$: $4 = 9 - 2\sqrt{9} + C$. Hence, $C = 1$ and $x = t - 2\sqrt{t} + 1$.

72. $p'(x) = \dfrac{10}{x^3}$

$$p(x) = \int \frac{10}{x^3}\, dx = \int 10x^{-3}\, dx = 10\left(\frac{x^{-2}}{-2}\right) + C = -5x^{-2} + C$$

Given $p(1) = 15$: $15 = -5(1)^{-2} + C$. Hence, $C = 20$ and
$p(x) = -5x^{-2} + 20$.

74. $\dfrac{d}{dt}\left(\int \dfrac{\ln t}{t} dt\right) = \dfrac{\ln t}{t} \quad \left(\dfrac{d}{dx}\left(\int f(x)\,dx\right) = f(x)\right)$

76. $\int \dfrac{d}{du}(e^{u^2})du = e^{u^2} + C \quad \left(\int F'(x)dx = F(x) + C\right)$

78. $\dfrac{d}{dx}(e^x + C) = e^x + 0 = e^x$

80. $\dfrac{d}{dx}(\ln|x| + C) = \dfrac{d}{dx}(\ln(-x) + C)$ since $x < 0$

$$= \frac{-1}{-x} + 0 = \frac{1}{x}$$

82. No solution provided.

84. For $f'(t) = 0.004t + 0.062$, we have

$f(t) = 0.002t^2 + 0.062t + C$

In 2007, (47 years after 1960), $f(47) = 6.8$ quadrillion Btu, and therefore, we should have

$6.8 = 0.002(47)^2 + (0.062)(47) + C$

which implies that $C = -0.532$.

Thus

$f(t) = 0.002t^2 + 0.062t - 0.532$

In 2020, $t = 60$, and

$f(60) = 0.002(60)^2 + 0.062(60) - 0.532 = 10.4$ quadrillion Btu.

86. (A) $R'(x) > 0$ for $0 < x < 500$ and $R'(x) < 0$ for $500 < x < 1{,}000$. Therefore, the graph of $R(x)$ is rising from 0 to 500 and falling from 500 to 1,000. $R'(x)$ is decreasing, so $R''(x) < 0$ and hence the graph of $R(x)$ is concave downward on (0, 1,000). It has a local maximum at $x = 500$.

(B) $\dfrac{R'(x) - 100}{0 - 100} = \dfrac{x - 0}{500 - 0}$ or $R'(x) = 100 - 0.2x$

(C) $R(x) = \int R'(x)dx = \int (100 - 0.2x)dx$

$$= \int 100\, dx - \int 0.2\, x\, dx$$

$$= 100x - 0.2\left(\frac{x^2}{2}\right) + C$$

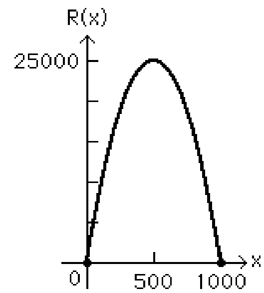
$$= 100x - 0.1x^2 + C$$

Given $R(0) = 0$: $0 = 100(0) - 0.1(0)^2 + C$. Hence, $C = 0$ and $R(x) = 100x - 0.1x^2$.

(D) $p(x) = \dfrac{R(x)}{x} = 100 - 0.1x$

Given $x = 700$: $p(700) = 100 - 0.1(700)$
$= 100 - 70 = 30.$

So the price is \$30 per sports watch when the demand is 700.



88. $S'(t) = 500t^{1/4}$

$S(t) = \int S'(t)dt = \int 500t^{1/4}\, dt = 500\int t^{1/4}\, dt$

$$= 500\left(\frac{1}{1+\frac{1}{4}}\right)t^{1+(1/4)} + C$$

$$= 500\left(\frac{4}{5}\right)t^{5/4} + C$$

$$= 400t^{5/4} + C$$

Given $S(0) = 0$: $0 = 400(0)^{5/4} + C$. Hence, $C = 0$ and $S(t) = 400t^{5/4}$.

We need to solve the following equation for t:

$20{,}000 = 400t^{5/4}$ or $t^{5/4} = 50$ or $t = 50^{4/5} \approx 23$ months

90. $S'(t) = 500t^{1/4} + 300$

$S(t) = \int (500t^{1/4} + 300)dt = 500\int t^{1/4}\, dt + \int 300dt$

$$= 400t^{5/4} + 300t + C$$

Given $S(0) = 0$: This implies that $C = 0$ and hence

$S(t) = 400t^{5/4} + 300t.$

For $S(t) = 20{,}000$, we have

$20{,}000 = 400t^{5/4} + 300t$

Using a graphing utility, we obtain $t \approx 17.83$ months.

92. $L'(x) = 2{,}000x^{-1/3}$

$$L(x) = \int g(x)dx = \int 2{,}000x^{-1/3}\,dx = 2{,}000\int x^{-1/3}\,dx$$
$$= 2{,}000\left(\frac{x^{(-1/3)+1}}{-\frac{1}{3}+1}\right) + C$$
$$= 2{,}000\left(\frac{3}{2}x^{2/3}\right) + C = 3{,}000x^{2/3} + C$$

Given $L(8) = 12{,}000$: $12{,}000 = 3{,}000(8)^{2/3} + C$. Hence,
$C = 0$ and $L(x) = 3{,}000x^{2/3}$.
$L(27) = 3{,}000(27)^{2/3} = 3{,}000(9) = 27{,}000$ labor hours.

94. $\frac{dA}{dt} = -4t^{-3},\ 1 \le t \le 10$

$$A = \int -4t^{-3}\,dt = -4\left(\frac{t^{-2}}{-2}\right) + C = 2t^{-2} + C$$

Given $A(1) = 2$: $2 = 2(1)^{-2} + C$. Hence, $C = 0$ and $A = 2t^{-2}$.
For $t = 10$, $A(10) = 2(10)^{-2} = \frac{2}{100} = 0.02$ square centimeters.

96. $V'(t) = \frac{15}{t},\ 1 \le t \le 5$

$$V(t) = \int \frac{15}{t}\,dt = 15\int t^{-1}\,dt = 15\ln t + C$$

Given: $V(1) = 15$: $15 = 15\ln 1 + C$. Hence, $C = 15$ and
$V(t) = 15\ln t + 15,\ 1 \le t \le 5$.

After 4 hours of study,
$V(4) = 15\ln 4 + 15 \approx 36$ words.

EXERCISE 6-2

2. $\int (6x - 1)^3(6)dx$

Let $u = 6x - 1$, then $du = 6\,dx$ and

$$\int (6x-1)^3(6)dx = \int u^3\,du = \frac{u^4}{4} + C \text{ [using Indefinite Integral Formulas]}$$
$$= \frac{(6x-1)^4}{4} + C$$

Check: $\frac{d}{dx}\left[\frac{(6x-1)^4}{4} + C\right] = \frac{1}{4}(4)(6x-1)^3(6) + 0$
$$= (6x-1)^3(6)$$

4. $\int (x^6 + 1)^4(6x^5)dx$

Let $u = x^6 + 1$, then $du = 6x^5dx$ and

$$\int (x^6 + 1)^4(6x^5)dx = \int u^4\,du \quad = \frac{u^5}{5} + C \text{ [using Indefinite Integral Formulas]}$$

$$= \frac{(x^6+1)^5}{5} + C$$

Check: $\frac{d}{dx}\left[\frac{(x^6+1)^5}{5} + C\right] \quad = \frac{1}{5}(5)(x^6 + 1)^4(6x^5) + 0$

$$= (x^6 + 1)^4(6x^5)$$

6. $\int (4x^2 - 3)^{-6}(8x)dx$

Let $u = 4x^2 - 3$, then $du = 8xdx$ and

$$\int (4x^2 - 3)^{-6}(8x)dx = \int u^{-6}\,du \quad = -\frac{u^{-5}}{5} + C \text{ [using Indefinite Integral Formulas]}$$

$$= -\frac{(4x^2-3)^{-5}}{5} + C$$

Check: $\frac{d}{dx}\left[-\frac{(4x^2-3)^{-5}}{5} + C\right] \quad = \left(-\frac{1}{5}\right)(-5)(4x^2 - 3)^{-6}(8x) + 0$

$$= (4x^2 - 3)^{-6}(8x)$$

8. $\int e^{x^3}(3x^2)dx$

Let $u = x^3$, then $du = 3x^2\,dx$ and

$$\int e^{x^3}(3x^2)dx = \int e^u\,du \quad = e^u + C \text{ [using Indefinite Integral Formulas]}$$

$$= e^{x^3} + C$$

Check: $\frac{d}{dx}(e^{x^3} + C) = e^{x^3}(3x^2) + 0 = e^{x^3}(3x^2)$

10. $\int \frac{1}{5x-7}(5)dx = \int (5x - 7)^{-1}(5)dx$

Let $u = (5x - 7)$, then $du = 5dx$ and

$$\int (5x - 7)^{-1}(5)dx = \int u^{-1}du \quad = \ln|u| + C \text{ [using Indefinite Integral Formulas]}$$

$$= \ln|5x - 7| + C$$

Check: $\frac{d}{dx}(\ln|5x - 7| + C) = \frac{5}{5x-7} + 0 = (5x - 7)^{-1}(5)$

12. $\int (x^2 + 9)^{-1/2}(2x)dx$

Let $u = x^2 + 9$, then $du = 2x\,dx$ and

$$\int (x^2 + 9)^{-1/2}(2x)dx \quad = \int u^{-1/2}du = 2u^{1/2} + C \text{ [using Indefinite Integral Formulas]}$$

$$= 2(x^2 + 9)^{1/2} + C$$

Check: $\frac{d}{dx}[2(x^2 + 9)^{1/2}] = (2)\left(\frac{1}{2}\right)(x^2 + 9)^{-1/2}(2x) + 0 = (x^2 + 9)^{-1/2}(2x)$

14. $\int (x - 3)^{-4}\, dx$

Let $u = x - 3$, then $du = dx$ and

$$\int (x-3)^{-4}\, dx = \int u^{-4} du \quad = \frac{1}{-4+1} u^{-4+1} + C \text{ [using Indefinite Integral Formulas]}$$

$$= -\frac{1}{3} u^{-3} + C$$

$$= -\frac{1}{3}(x-3)^{-3} + C$$

Check: $\frac{d}{dx}\left[-\frac{1}{3}(x-3)^{-3} + C\right] = -\frac{1}{3}(-3)(x-3)^{-4}(1) = (x-3)^{-4}$

16. $\int (5t + 1)^3\, dt$

Let $u = 5t + 1$, then $du = 5\, dt$ and $dt = \frac{1}{5} du$ and

$$\int (5t+1)^3\, dt = \int u^3 \frac{1}{5} du \quad = \frac{1}{5}\int u^3\, du$$

$$= \frac{1}{5}\cdot\frac{u^4}{4} + C \text{ [using Indefinite Integral Formulas]}$$

$$= \frac{1}{20}(5t+1)^4 + C$$

Check: $\frac{d}{dt}\left[\frac{1}{20}(5t+1)^4 + C\right] = \frac{1}{20}(4)(5t+1)^3(5) = (5t+1)^3$

18. $\int (t^3 + 4)^{-2}\, t^2\, dt$

Let $u = t^3 + 4$, then $du = 3t^2 dt$, $t^2 dt = \frac{1}{3} du$ and

$$\int (t^3+4)^{-2} t^2 dt = \int u^{-2}\frac{1}{3} du = \frac{1}{3}\int u^{-2}\, du$$

$$= \frac{1}{3}\cdot\frac{1}{-2+1} u^{-2+1} + C$$

$$= -\frac{1}{3} u^{-1} + C \text{ [using Indefinite Integral Formulas]}$$

$$= -\frac{1}{3}(t^3+4)^{-1} + C$$

Check: $\frac{d}{dt}\left[-\frac{1}{3}(t^3+4)^{-1} + C\right] = -\frac{1}{3}(-1)(t^3+4)^{-2}(3t^2)$

$$= (t^3+4)^{-2}(t^2) = (t^3+4)^{-2} t^2$$

20. $\int e^{-0.01x}\, dx$

Let $u = -0.01x$, then $du = -0.01\, dx$, $dx = -100\, du$ and

$$\int e^{-0.01x}\, dx = \int e^u(-100) du \quad = -100\int e^u\, du$$

$$= -100e^u + C \text{ [using Indefinite Integral Formulas]}$$

$= -100e^{-0.01x} + C$

Check: $\frac{d}{dx}[-100e^{-0.01x} + C] = (-100)e^{-0.01x}(-0.01) = e^{-0.01x}$

22. $\int \frac{x}{1+x^2}\,dx$

Let $u = 1 + x^2$, then $du = 2x\,dx$, $x\,dx = \frac{1}{2}du$ and

$\int \frac{x}{1+x^2}\,dx = \int \frac{1}{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$ [using Indefinite Integral Formulas]

$= \frac{1}{2}\ln(1 + x^2) + C$

Check: $\frac{d}{dx}\left[\frac{1}{2}\ln(1+x^2)+C\right] = \frac{1}{2}\cdot\frac{1}{1+x^2}(2x) = \frac{x}{1+x^2}$

24. $\int \frac{3}{2-t}\,dt$

Let $u = 2 - t$, then $du = -dt$, $dt = -du$ and

$\int \frac{3}{u}(-du) = -3\int \frac{1}{u}\,du = -3\ln|u| + C$ [using Indefinite Integral Formulas]

$= -3\ln|2 - t| + C$

Check: $\frac{d}{dt}[-3\ln|2 - t| + C] = -3\cdot\frac{1}{2-t}(-1) = \frac{3}{2-t}$

26. $\int \frac{t^2}{(t^3-2)^5}\,dt$

Let $u = t^3 - 2$, then $du = 3t^2\,dt$, and

$\int \frac{t^2}{(t^3-2)^5}\,dt = \int (t^3 - 2)^{-5}\,\frac{3}{3}t^2\,dt = \int u^{-5}\,\frac{1}{3}du$

$= \frac{1}{3}\int u^{-5}\,du = \frac{1}{3}\cdot\frac{1}{-5+1}u^{-5+1} + C$

$= -\frac{1}{12}u^{-4} + C$

$= -\frac{1}{12}(t^3 - 2)^{-4} + C$

Check: $\frac{d}{dt}\left[-\frac{1}{12}(t^3-2)^{-4}+C\right] = -\frac{1}{12}(-4)(t^3 - 2)^{-5}(3t^2)$

$= (t^3 - 2)^{-5}t^2 = \frac{t^2}{(t^3-2)^5}$

28. $\int x\sqrt{x-9}\ dx = \int x(x-9)^{1/2}\ dx$

Let $u = (x - 9)$, then $du = dx$ and $x = u + 9$.

$$\begin{aligned}
\int x\sqrt{x-9}\ dx &= \int (u+9)u^{1/2}\ du \\
&= \int (u^{3/2} + 9u^{1/2})du \\
&= \frac{u^{5/2}}{5/2} + \frac{9u^{3/2}}{3/2} + C \\
&= \frac{2}{5}u^{5/2} + 6u^{3/2} + C \\
&= \frac{2}{5}(x-9)^{5/2} + 6(x-9)^{3/2} + C
\end{aligned}$$

Check: $\frac{d}{dx}\left[\frac{2}{5}(x-9)^{5/2} + 6(x-9)^{3/2} + C\right]$

$$\begin{aligned}
&= \frac{2}{5}\left(\frac{5}{2}\right)(x-9)^{3/2}(1) + 6\left(\frac{3}{2}\right)(x-9)^{1/2}(1) \\
&= (x-9)^{3/2} + 9(x-9)^{1/2} \\
&= (x-9)\sqrt{x-9} + 9\sqrt{x-9} \\
&= x\sqrt{x-9} - 9\sqrt{x-9} + 9\sqrt{x-9} \\
&= x\sqrt{x-9}
\end{aligned}$$

30. $\int \frac{x}{\sqrt{x+5}}\ dx = \int x(x+5)^{-1/2}\ dx$

Let $u = x + 5$, then $du = dx$ and $x = u - 5$.

$$\begin{aligned}
\int \frac{x}{\sqrt{x+5}}\ dx &= \int (u-5)u^{-1/2}\ du \\
&= \int (u^{1/2} - 5u^{-1/2})du \\
&= \frac{u^{3/2}}{3/2} - \frac{5u^{1/2}}{1/2} + C \\
&= \frac{2}{3}u^{3/2} - 10u^{1/2} + C \\
&= \frac{2}{3}(x+5)^{3/2} - 10(x+5)^{1/2} + C
\end{aligned}$$

Check: $\frac{d}{dx}\left[\frac{2}{3}(x+5)^{3/2} - 10(x+5)^{1/2} + C\right]$

$$\begin{aligned}
&= \frac{2}{3}\left(\frac{3}{2}\right)(x+5)^{1/2} - 10\left(\frac{1}{2}\right)(x+5)^{-1/2} \\
&= (x+5)^{1/2} - 5(x+5)^{-1/2} \\
&= (x+5)^{1/2} - \frac{5}{(x+5)^{1/2}} = \frac{(x+5)-5}{(x+5)^{1/2}} = \frac{x}{\sqrt{x+5}}
\end{aligned}$$

32. $\int x(x+6)^8\,dx$

Let $u = x + 6$, then $du = dx$ and $x = u - 6$.

$$\int x(x+6)^8\,dx = \int (u-6)u^8\,du$$
$$= \int (u^9 - 6u^8)du$$
$$= \frac{u^{10}}{10} - \frac{6u^9}{9} + C$$
$$= \frac{1}{10}(x+6)^{10} - \frac{2}{3}(x+6)^9 + C$$

Check: $\frac{d}{dx}\left[\frac{1}{10}(x+6)^{10} - \frac{2}{3}(x+6)^9 + C\right]$

$$= \frac{1}{10}(10)(x+6)^9(1) - \frac{2}{3}(9)(x+6)^8(1)$$
$$= (x+6)^9 - 6(x+6)^8$$
$$= (x+6)^8[(x+6) - 6] = (x+6)^8(x)$$
$$= x(x+6)^8$$

34. Let $u = 1 - e^{-x}$, then $du = -e^{-x}(-1)dx = e^{-x}\,dx$.

$$\int e^{-x}(1-e^{-x})^4dx = \int (1-e^{-x})^4 e^{-x}\,dx$$
$$= \int u^4\,du = \frac{u^5}{5} + C$$
$$= \frac{1}{5}(1-e^{-x})^5 + C$$

Check: $\frac{d}{dx}\left[\frac{1}{5}(1-e^{-x})^5 + C\right] = \frac{1}{5}(5)(1-e^{-x})^4(-e^{-x})(-1)$

$$= (1-e^{-x})^4 e^{-x}$$
$$= e^{-x}(1-e^{-x})^4$$

36. Let $u = x^3 - 3x + 7$, then $du = (3x^2 - 3)dx = 3(x^2 - 1)dx$.

$$\int \frac{x^2-1}{x^3-3x+7}dx = \int (x^3 - 3x + 7)^{-1}\,\frac{3}{3}(x^2-1)dx$$
$$= \int u^{-1}\,\frac{1}{3}du = \frac{1}{3}\int u^{-1}\,du$$
$$= \frac{1}{3}\ln|u| + C$$
$$= \frac{1}{3}\ln|x^3 - 3x + 7| + C$$

Check: $\frac{d}{dx}\left[\frac{1}{3}\ln\left|x^3-3x+7\right|+C\right]$
$$= \frac{1}{3}\cdot\frac{1}{x^3-3x+7}(3x^2 - 3)$$
$$= \frac{1}{3}\cdot\frac{3(x^2-1)}{x^3-3x+7} = \frac{x^2-1}{x^3-3x+7}$$

38. Let $u = 4-7x$, then $du = -7dx$.

$$\int -7(4-7x)dx = \int u du$$
$$= \frac{u^2}{2}+C$$
$$= \frac{(4-7x)^2}{2}+C$$
$$= \frac{49}{2}x^2-28x+8+C$$
$$= \frac{49}{2}x^2-28x+C \qquad \text{(8 is incorporated into } C\text{)}$$
$$\int -7(4-7x)dx = \int (49x-28)dx$$
$$= \frac{49x^2}{2}-28x+C$$

40. Let $u = x^3+1$, then $du = 3x^2dx$.

$$\int 3x^2\left(x^3+1\right)dx = \int u du$$
$$= \frac{u^2}{2}+C$$
$$= \frac{\left(x^3+1\right)^2}{2}+C$$
$$\frac{x^6}{2}+x^3+\frac{1}{2}+C$$
$$\frac{x^6}{2}+x^3+C \qquad \left(\frac{1}{2}\text{ is incorporated into } C\right)$$
$$\int 3x^2\left(x^3+1\right)dx = \int\left(3x^5+3x^2\right)dx$$
$$= \frac{3x^6}{6}+\frac{3x^3}{3}+C$$
$$= \frac{x^6}{2}+x^3+C$$

42. Let $u = x^8$, then $du = 8x^7 dx$.

$$\begin{aligned}\int 8x^7 \left(x^8\right)^3 dx &= \int u^3 du \\ &= \frac{u^4}{4} + C \\ &= \frac{\left(x^8\right)^4}{4} + C\end{aligned}$$

$$\begin{aligned}\int 8x^7 \left(x^8\right)^3 dx &= \int 8x^{31} dx \\ &= \frac{8x^{32}}{32} + C \\ &= \frac{\left(x^8\right)^4}{4} + C\end{aligned}$$

44. (A) Differentiate $F(x) = \ln|x^2 + 5| + C$ to see if you get the integrand $f(x) = \frac{x}{x^2+5}$

(B) Wrong: $\frac{d}{dx}[\ln|x^2 + 5| + C] = \frac{2x}{x^2+5} \neq \frac{x}{x^2+5}$

(C) Let $u = x^2 + 5$, then $du = 2x\, dx$.

$$\begin{aligned}\int \frac{x}{x^2+5} dx = \int (x^2+5)^{-1} \frac{2}{2} x\, dx &= \int u^{-1} \frac{1}{2} du \\ &= \frac{1}{2} \int u^{-1} du \\ &= \frac{1}{2} \ln|u| + C \\ &= \frac{1}{2} \ln|x^2 + 5| + C\end{aligned}$$

Check: $\frac{d}{dx}\left[\frac{1}{2}\ln\left|x^2+5\right| + C\right] = \frac{1}{2} \cdot \frac{1}{x^2+5}(2x) = \frac{x}{x^2+5}$

46. (A) Differentiate $F(x) = e^{4x-5} + C$ to see if you get the integrand $f(x) = e^{4x-5}$

(B) Wrong: $\frac{d}{dx}[e^{4x-5} + C] = e^{4x-5}(4) \neq e^{4x-5}$

(C) Let $u = 4x - 5$, then $du = 4\, dx$.

$$\begin{aligned}\int e^{4x-5} dx = \int e^{4x-5} \frac{4}{4} dx &= \int e^u \frac{1}{4} du \\ &= \frac{1}{4} \int e^u du\end{aligned}$$

$$= \frac{1}{4} e^{u} + C$$

$$= \frac{1}{4} e^{4x-5} + C$$

Check: $\frac{d}{dx}\left[\frac{1}{4}e^{4x-5} + C\right] = \frac{1}{4} e^{4x-5}(4) = e^{4x-5}$

48. (A) Differentiate $F(x) = (x^2 - 3)^{-5} + C$ to see if you get the integrand $f(x) = (-10x)(x^2 - 3)^{-4}$

(B) Wrong: $\frac{d}{dx}[(x^2 - 3)^{-5} + C] = (-5)(x^2 - 3)^{-6}(2x) = (-10x)(x^2 - 3)^{-6}$

$$\neq (-10x)(x^2 - 3)^{-4}$$

(C) Let $u = x^2 - 3$, then $du = 2x\,dx$

$$\int (-10x)(x^2 - 3)^{-4}\,dx = -5\int (x^2 - 3)^{-4}(2x)dx = -5\int u^{-4}\,du$$

$$= -5\cdot\frac{1}{-4+1}u^{-4+1} + C = \frac{5}{3}u^{-3} + C$$

$$= \frac{5}{3}(x^2 - 3)^{-3} + C$$

Check: $\frac{d}{dx}\left[\frac{5}{3}(x^2 - 3)^{-3} + C\right] = \frac{5}{3}(-3)(x^2 - 3)^{-4}(2x)$

$$= (-10x)(x^2 - 3)^{-4}$$

50. Let $u = 2x^3 + 1$, then $du = 6x^2\,dx$.

$$\int x^2\sqrt{2x^3+1}\,dx = \int (2x^3 + 1)^{1/2}\,\frac{6}{6}x^2\,dx$$

$$= \int u^{1/2}\,\frac{1}{6}\,du = \frac{1}{6}\int u^{1/2}\,du$$

$$= \frac{1}{6}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{9}u^{3/2} + C$$

$$= \frac{1}{9}(2x^3 + 1)^{3/2} + C$$

Check: $\frac{d}{dx}\left[\frac{1}{9}(2x^3+1)^{3/2} + C\right] = \frac{1}{9}\left(\frac{3}{2}\right)(2x^3 + 1)^{1/2}(6x^2) = x^2\sqrt{2x^3+1}$

52. Let $u = x^2 + 2$, then $du = 2x\,dx$.

$$\int x(x^2+2)^2\,dx = \int (x^2+2)^2\,\frac{2}{2}x\,dx$$
$$= \int u^2\,\frac{1}{2}\,du = \frac{1}{2}\int u^2\,du$$
$$= \frac{1}{2}\cdot\frac{u^3}{3} + C$$
$$= \frac{1}{6}(x^2+2)^3 + C$$

Check: $\frac{d}{dx}\left[\frac{1}{6}(x^2+2)^3 + C\right] = \frac{1}{6}(3)(x^2+2)^2(2x) = x(x^2+2)^2$

54. $$\int (x^2+2)^2\,dx = \int (x^4+4x^2+4)dx$$
$$= \frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x + C$$

Check: $\frac{d}{dx}\left[\frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x + C\right] = \frac{1}{5}(5x^4) + \frac{4}{3}(3x^2) + 4$
$$= x^4 + 4x^2 + 4 = (x^2+2)^2$$

56. Let $u = 4x^3 - 1$, then $du = 12x^2\,dx$.

$$\int \frac{x^2}{\sqrt{4x^3-1}}\,dx = \int \frac{x^2}{(4x^3-1)^{1/2}}\,dx = \int (4x^3-1)^{-1/2}\,\frac{12}{12}x^2\,dx$$
$$= \int u^{-1/2}\,\frac{1}{12}\,du$$
$$= \frac{1}{12}\int u^{-1/2}\,du$$
$$= \frac{1}{12}\cdot\frac{u^{1/2}}{1/2} + C$$
$$= \frac{1}{6}(4x^3-1)^{1/2} + C$$

Check: $\frac{d}{dx}\left[\frac{1}{6}(4x^3-1)^{1/2} + C\right] = \frac{1}{6}\left(\frac{1}{2}\right)(4x^3-1)^{-1/2}(12x^2)$
$$= (4x^3-1)^{-1/2}(x^2)$$
$$= \frac{x^2}{(4x^3-1)^{1/2}} = \frac{x^2}{\sqrt{4x^3-1}}$$

58. Let $u = 1 + e^x$, then $du = e^x\,dx$.

$$\int \frac{e^x}{1+e^x}\,dx = \int (1+e^x)^{-1}e^x\,dx = \int u^{-1}\,du$$
$$= \ln|u| + C$$
$$= \ln(1+e^x) + C$$

Check: $\frac{d}{dx}[\ln(1+e^x) + C] = \frac{1}{1+e^x}(e^x) = \frac{e^x}{1+e^x}$

60. Let $u = \ln x$, then $du = \frac{1}{x}\,dx$.

$$\int \frac{1}{x\ln x}\,dx = \int (\ln x)^{-1}\,\frac{1}{x}\,dx = \int u^{-1}\,du$$
$$= \ln|u| + C$$
$$= \ln|\ln x| + C$$

Check: $\frac{d}{dx}[\ln|\ln x| + C] = \frac{1}{\ln x}\left(\frac{1}{x}\right) = \frac{1}{x\ln x}$

62. $\frac{dm}{dn} = 10n(n^2 - 8)^7$

Let $u = (n^2 - 8)$, then $du = 2n\,dn$.

$$m = \int 10n(n^2 - 8)^7 dn = 10\int (n^2 - 8)^7\,\frac{2}{2}\,n\,dn$$
$$= 10\int u^7\,\frac{1}{2}\,du$$
$$= 5\int u^7\,du$$
$$= 5\cdot\frac{u^8}{8} + C$$
$$= \frac{5}{8}(n^2 - 8)^8 + C$$

64. $\frac{dy}{dx} = \frac{5x^2}{(x^3 - 7)^4}$

Let $u = x^3 - 7$, then $du = 3x^2\,dx$.

$$y = \int \frac{5x^2}{(x^3-7)^4}\,dx = \int (x^3 - 7)^{-4}(5x^2)dx$$
$$= 5\int (x^3 - 7)^{-4}\,\frac{3}{3}x^2\,dx$$
$$= 5\int u^{-4}\,\frac{1}{3}u\,du$$
$$= \frac{5}{3}\int u^{-4}\,du = \frac{5}{3}\cdot\frac{u^{-3}}{-3} + C$$
$$= -\frac{5}{9}u^{-3} + C = -\frac{5}{9}(x^3 - 7)^{-3} + C$$

66. $\frac{dm}{dt} = \frac{\ln(t-5)}{t-5}$

Let $u = \ln(t - 5)$, then $du = \frac{1}{t-5}\,dt$.

$$m = \int \frac{\ln(t-5)}{t-5}\,dt = \int \ln(t-5)\,\frac{1}{t-5}\,dt$$
$$= \int u\,du = \frac{u^2}{2} + C = \frac{1}{2}[\ln(t-5)]^2 + C$$

68. $p'(x) = \frac{300}{(3x+25)^2}$

Let $u = 3x + 25$, then $du = 3\,dx$.

$$p(x) = \int \frac{300}{(3x+25)^2}\,dx = 300\int (3x+25)^{-2}\,dx$$
$$= 300\int (3x+25)^{-2}\,\frac{3}{3}\,dx$$
$$= 300\int u^{-2}\,\frac{1}{3}\,du = 100\int u^{-2}\,du$$
$$= 100\cdot\frac{u^{-1}}{-1} + C = -100u^{-1} + C$$

$$p(x) = -100(3x+25)^{-1} + C = -\frac{100}{3x+25} + C$$

Given: $p(75) = 5.0$:

$$5.0 = -\frac{100}{3(75)+25} + C$$

$5.0 = -\frac{100}{250} + C = -0.4 + C$ or $C = 5.4$ and

$$p(x) = -\frac{100}{3x+25} + 5.4$$

Now, $5.15 = -\frac{100}{3x+25} + 5.4$

$$\frac{100}{3x+25} = 0.25$$
$$0.25(3x+25) = 100$$
$$3x + 25 = 400$$
$$3x = 375$$
$$x = 125$$

Thus, the demand is 125 bottles when the price is \$5.15.

70. $R'(x) = 40 - 0.02x + \dfrac{200}{x+1}$

$R(x) = \int \left(40 - 0.02x + \dfrac{200}{x+1}\right) dx$

$= \int 40\,dx - \int 0.02x\,dx + 200\int \dfrac{1}{x+1}\,dx$

$= 40x - 0.02\left(\dfrac{x^2}{2}\right) + 200\ln(x+1) + C$ $(u = x + 1, du = dx)$

$= 40x - 0.01x^2 + 200\ln(x+1) + C$

Now, $R(0) = 0$. Thus, $C = 0$ and

$R(x) = 40x - 0.01x^2 + 200\ln(x+1)$

$R(1{,}000) = 40(1{,}000) - 0.01(1{,}000)^2 + 200\ln(1{,}000 + 1)$

$= \$31{,}381.75$

72. $S'(t) = 20 - 20e^{-0.05t}$, $0 \le t \le 24$

(A) $S(t) = \int (20 - 20e^{-0.05t})dt$

$= \int 20\,dt - 20\int e^{-0.05t}\,dt$

$= 20t - 20\left(\dfrac{1}{-0.05}\right)e^{-0.05t} + C$

$= 20t + 400e^{-0.05t} + C$

Given: $S(0) = 0$: $0 = 0 + 400 + C$

$C = -400$

Total sales at time t:

$S(t) = 20t + 400e^{-0.05t} - 400$, $0 \le t \le 24$

(B) $S(12) = 20(12) + 400e^{-0.05(12)} - 400 \approx 60$

Total estimated sales for the first twelve months: \$60 million.

(C) On a graphing utility solve

$20t + 400e^{-0.05t} - 400 = 100$

or

$20t + 400e^{-0.05t} = 500$

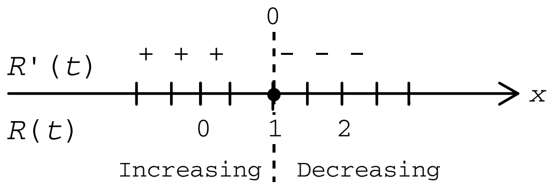
The result is: $t \approx 16.02$ months.

74. (A) $R(t) = \dfrac{120t}{t^2+1} + 3,\ 0 \le t \le 20$

$R'(t) = 120\left(\dfrac{t^2+1-2t^2}{(t^2+1)^2}\right) = \dfrac{120(1-t^2)}{(t^2+1)^2}$

$R'(t) = 0$ when $t = 1$.

Sign chart for $R'(t)$:



Test Numbers

t	$R'(t)$
0	$120(+)$
2	$-\frac{72}{5}(-)$

Thus, the rate of production is greatest at $t = 1$.

(B) $Q(t) = \int R(t)dt = \int \left(\dfrac{120t}{t^2+1} + 3\right)dt$

$= 120\int \dfrac{t}{t^2+1}\,dt + \int 3\,dt$

$= 60\ln(t^2 + 1) + 3t + C$

$Q(0) = 0$: $0 = 0 + 0 + C$ and

$Q(t) = 60\ln(t^2 + 1) + 3t$

$Q(5) = 60\ln(5^2 + 1) + 3(5) = 60\ln(26) + 15$

≈ 210.5 thousand barrels

(C) $Q(t) = 250$ thousands. Now, we need to solve:

$250 = 60\ln(t^2 + 1) + 3t$

Using a graphing utility we obtain $t \approx 6.7$ years.

76. $A'(t) = -0.9e^{-0.1t},\ t \ge 0$

$A(t) = \int -0.9e^{-0.1t}\,dt = -0.9\int e^{-0.1t}\,dt$

$= -0.9\left(\dfrac{1}{-0.1}\right)e^{-0.1t} + C = 9e^{-0.1t} + C$

Given $A(0) = 9$: $9 = 9e^{-0.1(0)} + C$ or $9 = 9 + C$ or $C = 0$

Thus, $A(t) = 9e^{-0.1t}$.

Now, $A(5) = 9e^{-0.1(5)} \approx 5.46$ square centimeters.

78. $\frac{dR}{dt} = \frac{60}{\sqrt{t+9}}$, $t \geq 0$

$R = \int \frac{60}{\sqrt{t+9}}\,dt \quad = 60\int (t+9)^{-1/2}\,dt$

$= 60\left(\frac{(t+9)^{1/2}}{\frac{1}{2}}\right) + C \quad (u = t + 9, du = dt)$

$= 120(t+9)^{1/2} + C$

Given $R(0) = 0$: $0 = 120(0+9)^{1/2} + C$ or $C = -360$, and

$R(t) = 120(t+9)^{1/2} - 360$

Now, $R(16) = 120(16+9)^{1/2} - 360$

$= 120(25)^{1/2} - 360$

$= 120(5) - 360 = 600 - 360 = 240$ feet.

80. $N'(t) = 12e^{-0.06t}$, $0 \leq t \leq 15$

$N(t) = \int 12e^{-0.06t}\,dt \quad = (12)\left(\frac{1}{-0.06}\right)e^{-0.06t} + C$

$= -200e^{-0.06t} + C$

Given $N(0) = 0$: $0 = -200e^{-0.06(0)} + C = -200 + C$ or $C = 200$ and

$N(t) = -200e^{-0.06t} + 200$

or $N(t) = 200(1 - e^{-0.06t})$, $0 \leq t \leq 15$

Now, $N(15) = 200(1 - e^{-0.06(15)}) \approx 118$ words per minute.

EXERCISE 6-3

2. $\frac{dy}{dx} = 3x^{-2}$

$y = \frac{3}{-2+1}x^{-2+1} + C = -3x^{-1} + C$ (General solution)

4. $\frac{dy}{dx} = e^{0.1x}$

$y = \frac{1}{0.1}e^{0.1x} + C = 10e^{0.1x} + C$ (General solution)

6. $\frac{dy}{dx} = 8x^{-1}$

$y = 8\ln|x| + C$ (General solution)

8. $\frac{dy}{dx} = \sqrt{x} = x^{1/2}$

$y = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C$

Given $y(0) = 0$: $0 = \frac{2}{3}(0)^{3/2} + C$ or $C = 0$ and

the particular solution is: $y = \frac{2}{3}x^{3/2}$.

10. $\frac{dy}{dx} = e^{(x-3)}$

$y = e^{(x-3)} + C$

Given $y(3) = -5$: $-5 = e^{(3-3)} + C = 1 + C$ or $C = -6$

and the particular solution is: $y = e^{(x-3)} - 6$.

12. $\frac{dy}{dx} = \frac{1}{4(3-x)}$

$y = -\frac{1}{4}\ln|3 - x| + C$

Given $y(0) = 1$: $1 = -\frac{1}{4}\ln|3 - 0| + C$ or $C = 1 + \frac{1}{4}\ln 3$

and the particular solution is:

$y = -\frac{1}{4}\ln|3 - x| + 1 + \frac{1}{4}\ln 3$

14. Figure (a). When $x = 0$, $\frac{dy}{dx} = -0 = 0$ for any y. When $x = -1$, $\frac{dy}{dx} = -(-1) = 1$ for any y. When $x = 1$, $\frac{dy}{dx} = -1$ for any y.

These facts are consistent with the slope-field in Figure (a); they are not consistent with the slope-field in Figure (b).

16. $\frac{dy}{dx} = -x$

$\int \frac{dy}{dx}dx = \int (-x)dx$

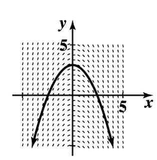
General solution: $y = -\frac{1}{2}x^2 + C$

Given $y(0) = 3$: $-\frac{1}{2}(0)^2 + C = 3$

$C = 3$

Particular solution: $y = -\frac{1}{2}x^2 + 3$

18.



20. $\dfrac{dy}{dt} = -3y$

$$\int \frac{1}{y}\frac{dy}{dt}\,dt = \int -3dt$$

$$\int \frac{1}{y}\,dy = \int -3dt$$

$$\ln |y| = -3t + K \quad [K \text{ an arbitrary constant}]$$

$$|y| = e^{-3t+K} = e^K e^{-3t}$$

$$|y| = Ce^{-3t} \quad [C = e^K]$$

If we assume $y > 0$, we get

General solution: $y = Ce^{-3t}$

22. $\dfrac{dy}{dx} = 0.1y$, $y(0) = -2.5$

$$\int \frac{1}{y}\frac{dy}{dx}\,dx = \int 0.1dx$$

$$\int \frac{1}{y}\,dy = \int 0.1dx$$

$$\ln |y| = 0.1x + K \quad (K \text{ an arbitrary constant})$$

$$|y| = e^{0.1x+K} = e^K e^{0.1x}$$

$$|y| = Ce^{0.1x} \quad (C = e^K)$$

If we assume $y < 0$, we get

General solution: $y = -Ce^{0.1x}$

Given $y(0) = -2.5$: $-2.5 = -Ce^{0.1(0)} = -C$ or $C = 2.5$

and the particular solution is: $y = -2.5e^{0.1x}$

24. $\dfrac{dx}{dt} = 4t$

$x = \dfrac{4t^2}{2} + C = 2t^2 + C$ (General solution)

26. $\dfrac{dx}{dt} = 4x$

$$\int \frac{1}{x}\frac{dx}{dt}\,dt = \int 4\,dt$$

$$\int \frac{1}{x}\,dx = \int 4\,dt$$

$$\ln|x| = 4t + K \quad [K \text{ an arbitrary constant}]$$

$$|x| = e^{4t+K} = e^K e^{4t}$$

$$|x| = Ce^{4t} \quad [C = e^K]$$

If we assume $x > 0$, we get General solution: $x = Ce^{4t}$.

28. Figure (B). When $y = -1$, the slope $\frac{dy}{dx} = -1 + 1 = 0$ for any x.

When $y = 1$, the slope $\frac{dy}{dx} = 1 + 1 = 2$ for any x; and so on. Both are consistent with the slope-field graph in Figure (B).

30. $y = Ce^{x} - 1$

$\frac{dy}{dx} = \frac{d}{dx}[Ce^{x} - 1] = Ce^{x}$

From the original equation,

$Ce^{x} = y + 1$

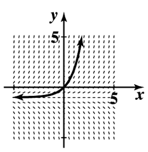
Thus, we have

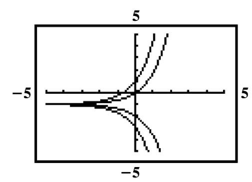
$\frac{dy}{dx} = y + 1$

and $y = Ce^{x} - 1$ is a solution of the differential equation for any number C.

Given $y(0) = 0$: $0 = Ce^{0} - 1 = C - 1$ or $C = 1$

Particular solution: $y = e^{x} - 1$

32. 

34. 

36. Given $y = \sqrt{x^2 + C} = (x^2 + C)^{1/2}$.

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + C)^{1/2} \qquad = \frac{1}{2}(2x)(x^2 + C)^{-1/2}$$

$$= \frac{x}{(x^2 + C)^{1/2}} = \frac{x}{y}$$

So, $y = \sqrt{x^2 + C}$ is a solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$.

Now we should determine the constant C so that the solution curve passes through (-6, 7), i.e.

$7 = \sqrt{(-6)^2 + C}$ or $49 = 36 + C$ or $C = 13$

so the desired particular solution is $y = \sqrt{x^2 + 13}$.

38. Given $y = \frac{C}{x} = Cx^{-1}$,

$$\frac{dy}{dx} = \frac{d}{dx}(Cx^{-1}) = C\frac{d}{dx}(x^{-1}) = C(-1)x^{-2} = -\frac{C}{x^2}$$

$$= -\frac{1}{x} \cdot \frac{C}{x} = -\frac{1}{x} \cdot y = -\frac{y}{x}$$

So, $y = \frac{C}{x}$ is a solution of the differential equation $\frac{dy}{dx} = -\frac{y}{x}$.

Now we should determine the constant c so that the solution curve passes through (2, 5), i.e.

$5 = \frac{c}{2}$ or $c = 10$

so the desired particular solution is $y = \frac{10}{x}$.

40. Given $y = \dfrac{2}{(1+Ce^{-6t})} = 2(1 + Ce^{-6t})^{-1}$

$$\frac{dy}{dt} = \frac{d}{dt}(2(1 + Ce^{-6t})^{-1}) = 2\frac{d}{dt}(1 + Ce^{-6t})^{-1}$$

$$= 2(-1)(-6Ce^{-6t})(1 + Ce^{-6t})^{-2}$$

$$= \frac{12Ce^{-6t}}{(1+Ce^{-6t})^2} \qquad = 3Ce^{-6t}\left(\frac{2}{(1+Ce^{-6t})}\right)^2 = 3Ce^{-6t}y^2$$

Note that from $y = \dfrac{2}{(1+Ce^{-6t})}$ we obtain

$1 + Ce^{-6t} = \dfrac{2}{y}$ or $Ce^{-6t} = \dfrac{2}{y} - 1$

Thus $\dfrac{dy}{dt} = 3Ce^{-6t}y^2 = 3\left(\dfrac{2}{y}-1\right)y^2 = 3(2-y)y.$

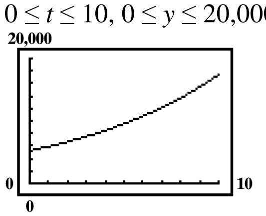
To find the desired particular solution, we have to find the constant C from the following equation:

$$1 = \frac{2}{(1+Ce^{-6(0)})} = \frac{2}{1+C}$$

or $1 + C = 2$ or $C = 1$ and $y = \dfrac{2}{(1+e^{-6t})}$.

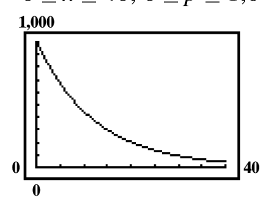
42. $y = 5{,}250e^{0.12t}$

$0 \le t \le 10$, $0 \le y \le 20{,}000$



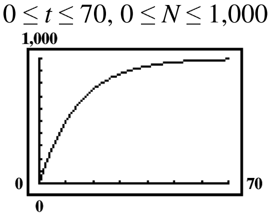
44. $p = 1{,}000e^{-0.08x}$

$0 \le x \le 40$, $0 \le p \le 1{,}000$



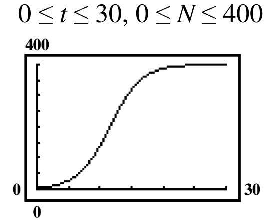
46. $N = 1{,}000(1 - e^{-0.07t})$

$0 \le t \le 70$, $0 \le N \le 1{,}000$



48. $N = \dfrac{400}{1+99e^{-0.4t}}$

$0 \le t \le 30$, $0 \le N \le 400$



50. $y = \dfrac{M}{1+ce^{-kMt}} = \dfrac{M}{2}$. Thus,

$1 + ce^{-kMt} = 2$ or $ce^{-kMt} = 1$ or

$e^{-kMt} = \dfrac{1}{c} = c^{-1}$. Take natural log from both sides.

$-kMt = -\ln c$ or $t = \dfrac{\ln c}{kM}$.

52. r = continuous compound growth rate is not constant, as can be seen from Problem 51.

54. $\frac{dA}{dt} = 0.02A$, $A(0) = 5{,}250$

This is an unlimited growth model. Thus,

$A(t) = 5{,}250e^{0.02t}$

56. $\frac{dA}{dt} = rA$, $A(0) = 5{,}000$

This is an unlimited growth model. Thus,

$A(t) = 5{,}000e^{rt}$

Since $A(5) = 5{,}581.39$, we solve $5{,}000e^{5r} = 5{,}581.39$ for r.

$$e^{5r} = \frac{5{,}581.39}{5{,}000}$$

$$5r = \ln\left(\frac{5{,}581.39}{5{,}000}\right)$$

$$r = \frac{1}{5}\ln\left(\frac{5{,}581.39}{5{,}000}\right) \approx 0.022$$

Thus, $A(t) = 5{,}000e^{0.022t}$.

58. (A) $\frac{dp}{dx} = rp$, $p(0) = 10$

This is an unlimited growth model. Thus,

$p(x) = 10e^{rx}$

Since $p(50) = 12.84$, we have

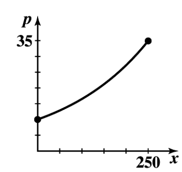
$$\begin{aligned} 12.84 &= 10e^{50r} \\ e^{50r} &= 1.284 \\ 50r &= \ln(1.284) \\ r &= \frac{1}{50}\ln(1.284) \approx 0.005 \end{aligned}$$

Therefore, $p(x) = 10e^{0.005x}$.

(B) $p(100) = 10e^{0.005(100)} = 10e^{0.5}$

$\approx \$16.49$ per unit

(C)



60. $\frac{dN}{dt} = k(L - N)$; $N(0) = 0$

(A) $N(10) = 0.1L$

Approximately 10% of the possible viewers will have been exposed after 10 days.

(B) $\frac{dN}{dt} = k(L - N)$; $N(0) = 0$

This is a limited growth model. Thus,

$N(t) = L(1 - e^{-kt})$

Since $N(10) = 0.1L$, we have

$$\begin{aligned} 0.1L &= L(1 - e^{-10k}) \\ 1 - e^{-10k} &= 0.1 \\ e^{-10k} &= 0.9 \\ -10k &= \ln(0.9) \\ k &= -\frac{1}{10}\ln(0.9) \approx 0.011 \end{aligned}$$

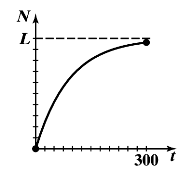
Therefore,

$N(t) = L(1 - e^{-0.011t})$

(C) Solve

$$\begin{aligned} L(1 - e^{-0.011t}) &= 0.5L: \\ 1 - e^{-0.011t} &= 0.5 \\ e^{-0.011t} &= 0.5 \\ -0.011t &= \ln(0.5) \\ t &= -\frac{\ln(0.5)}{0.011} \approx 63 \text{ days} \end{aligned}$$

(D)



62. $\frac{dP}{dt} = -aP,\ P(0) = P_0$

This is an exponential decay model. Thus,

$P(t) = P_0 e^{-at}$

64. (A) $N(0) = \frac{1{,}000}{1+999e^{-0.4(10)}} \approx 52$ people

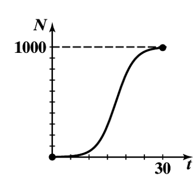
$N(20) = \frac{1{,}000}{1+999e^{-0.4(20)}} \approx 749$ people

(B) Solve $\frac{1{,}000}{2} = \frac{1{,}000}{1+999e^{-0.4t}}$ for t. Thus,

$$\begin{aligned} 1 + 999e^{-0.4t} &= 2 \\ 999e^{-0.4t} &= 1 \\ e^{-0.4t} &= \frac{1}{999} \\ -0.4t &= \ln\left(\frac{1}{999}\right) \\ t &= -\frac{1}{0.4}\ln\left(\frac{1}{999}\right) = \frac{\ln(999)}{0.4} \approx 17 \text{ days} \end{aligned}$$

(C)
$$\begin{aligned} \lim_{t\to\infty} N(t) &= \lim_{t\to\infty} \frac{1{,}000}{1+999e^{-0.4t}} \\ &= \frac{1{,}000}{1} = 1{,}000 \end{aligned}$$

(D)



66. Using the exponential decay model, we have $\frac{dy}{dt} = -ky$, $y(0) = 100$, $k > 0$ where $y = y(t)$ is the amount of DDT present at time t. Therefore,

$$y(t) = 100e^{-kt}$$

Since $y(5) = 75$, we solve:

$$75 = 100e^{-5k}$$

for k to find the continuous compound decay rate:

$$\begin{aligned} 75 &= 100e^{-5k} \\ e^{-5k} &= 0.75 \\ -5k &= \ln(0.75) \\ k &= -\frac{1}{5}\ln(0.75) \approx 0.057536 \end{aligned}$$

68. $N(t) = 100(1 - e^{-0.02t})$

$N'(t) = 100(-e^{-0.02t})(-0.02)$

$\quad = 2e^{-0.02t}$

$N'(10) = 2e^{-0.02(10)} = 2e^{-0.2} \approx 1.64$ words per minute/hour of practice

$N'(40) = 2e^{-0.02(40)} = 2e^{-0.8} \approx 0.9$ words per minute/hour of practice

70. $\frac{dS}{dR} = \frac{k}{R}$

$S = k\int \frac{1}{R}\,dR = k \ln R + C$

Given: $S(R_0) = 0$: $0 = k \ln R_0 + C$ or $C = -k \ln R_0$.

Thus,

$$\begin{aligned} S &= k \ln R - k \ln R_0 \\ &= k(\ln R - \ln R_0) = k \ln \frac{R}{R_0} \end{aligned}$$

72. Solve

$$\begin{aligned} \frac{400}{2} &= \frac{400}{1+399e^{-0.4t}} \text{ for } t. \\ 2 &= 1 + 399e^{-0.4t} \\ 399e^{-0.4t} &= 1 \\ e^{-0.4t} &= \frac{1}{399} \\ -0.4t &= \ln\left(\frac{1}{399}\right) = -\ln(399) \\ t &= \frac{\ln(399)}{0.4} \approx 15 \text{ minutes} \end{aligned}$$

EXERCISE 6-4

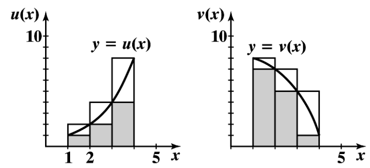
2. A and D

4. None of the rectangles are both left and right rectangles.

6. G and H

8. F and J are neither left or right rectangles.

10.



12. For Figure (C):

$L_3 = u(1)\cdot 1 + u(2)\cdot 1 + u(3)\cdot 1$

$= 1 + 2 + 4 = 7$

$R_3 = u(2)\cdot 1 + u(3)\cdot 1 + u(4)\cdot 1$

$= 2 + 4 + 8 = 14$

For Figure (D):

$L_3 = v(1)\cdot 1 + v(2)\cdot 1 + v(3)\cdot 1$

$= 8 + 7 + 5 = 20$

$R_3 = v(2)\cdot 1 + v(3)\cdot 1 + v(4)\cdot 1$

$= 7 + 5 + 1 = 13$

14. $L_3 \le \int_1^4 u(x)dx \le R_3$, $R_3 \le \int_1^4 v(x)dx \le L_3$; since $u(x)$ is increasing on $[1, 4]$, L_3 underestimates the area and R_3 overestimates the area; since $v(x)$ is decreasing on $[1, 4]$, L_3 overestimates the area and R_3 underestimates the area.

16. For Figure (C):

Error bound for L_3 and R_3:

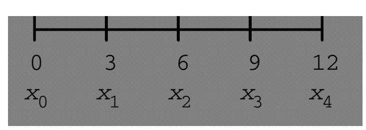
$$\text{Error} \le |u(4) - u(1)|\left(\frac{4-1}{3}\right) = |8 - 1| = 7$$

For Figure (D):

Error bound for L_3 and R_3:

$$\text{Error} \le |v(4) - v(1)|\left(\frac{4-1}{3}\right) = |1 - 8| = 7$$

18. $f(x) = 25 - 3x^2$; $\Delta x = 3$;



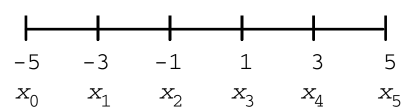
Given $c_i = \frac{x_{i-1} + 2x_i}{3}$

$c_1 = \frac{0 + 2(3)}{3} = 2$, $c_2 = \frac{3 + 2(6)}{3} = 5$, $c_3 = \frac{6 + 2(9)}{3} = 8$, $c_4 = \frac{9 + 2(12)}{3} = 11$

$S_4 = \Delta x\,\{f(c_1) + f(c_2) + f(c_3) + f(c_4)\}$

$= 3\{13 - 50 - 167 - 338\} = -1{,}626$

20. $f(x) = 25 - 3x^2$; $\Delta x = 2$;



Given $c_i = \frac{x_{i-1} + x_i}{2}$

$c_1 = -4$, $c_2 = -2$, $c_3 = 0$, $c_4 = 2$, $c_5 = 4$

$S_5 = \Delta x\,\{f(c_1) + f(c_2) + f(c_3) + f(c_4) + f(c_5)\}$

$= 2\{-23 + 13 + 25 + 13 - 23\} = 10$

22. $f(x) = x^2 - 5x - 6$; $\Delta x = 1$.
Given: $c_1 = 0.2$, $c_2 = 1.5$, $c_3 = 2.8$

$$S_3 = \Delta x\{f(c_1) + f(c_2) + f(c_3)\}$$
$$= 1 \cdot \{-6.96 - 11.25 - 12.16\} = -30.37$$

24. $f(x) = x^2 - 5x - 6$; $\Delta x = 1$.
Given: $c_1 = 2$, $c_2 = 2$, $c_3 = 4$, $c_4 = 4$, $c_5 = 6$, $c_6 = 6$

$$S_6 = \Delta x\{f(c_1) + f(c_2) + f(c_3) + f(c_4) + f(c_5) + f(c_6)\}$$
$$= 1 \cdot \{-12 - 12 - 10 - 10 + 0 + 0\} = -44$$

26. $\int_0^c f(x)dx = \text{Area } C = 5.333$

28. $\int_b^d f(x)dx = -(\text{Area } B) + (\text{Area } C) - (\text{Area } D)$
$= -2.475 + 5.333 - 1.792 = 1.066$

30. $\int_0^d f(x)dx = (\text{Area } C) - (\text{Area } D) = 5.333 - 1.792 = 3.541$

32. $\int_d^a f(x)dx = -\int_a^d f(x)dx = -\{(\text{Area } A) - (\text{Area } B) + (\text{Area } C) - (\text{Area } D)\}$
$= -\{1.408 - 2.475 + 5.333 - 1.792\} = -2.474$

34. $\int_c^a f(x)dx = -\int_a^c f(x)dx = -\{(\text{Area } A) - (\text{Area } B) + (\text{Area } C)\}$
$= -\{1.408 - 2.475 + 5.333\} = -4.266$

36. $\int_c^b f(x)dx = -\int_b^c f(x)dx = -\{-(\text{Area } B) + (\text{Area } C)\}$
$= -\{-2.475 + 5.333\} = -2.858$

38. $\int_1^4 3x^2dx = 3\int_1^4 x^2dx = 3(21) = 63$

40. $\int_1^4 (7x - 2x^2)dx = \int_1^4 7xdx - \int_1^4 2x^2dx$
$= 7\int_1^4 xdx - 2\int_1^4 x^2dx$
$= 7(7.5) - 2(21) = 52.5 - 42 = 10.5$

42. $\int_1^4 (4x^2 - 9x)dx = \int_1^4 4x^2dx - \int_1^4 9xdx$
$= 4\int_1^4 x^2dx - 9\int_1^4 xdx$
$= 4(21) - 9(7.5) = 16.5$

44. $$\int_1^5 -4x^2dx = \int_1^4 -4x^2dx + \int_4^5 -4x^2dx$$
$$= -4\int_1^4 x^2dx - 4\int_4^5 x^2dx$$
$$= -4(21) - 4\left(\frac{61}{3}\right) = -84 - \frac{244}{3} = -\frac{496}{3}$$

46. $$\int_5^5 (10 - 7x + x^2)dx = 0$$

48. $$\int_4^1 x(1 - x)dx = -\int_1^4 x(1 - x)dx$$
$$= -\int_1^4 (x - x^2)dx$$
$$= -\int_1^4 xdx - \int_1^4 -x^2dx$$
$$= -\int_1^4 xdx + \int_1^4 x^2dx$$
$$= -7.5 + 21 = 13.5$$

50. True. The left and right sums will be zero for any n and obviously their limits are zero, which is the value of the integral.

52. True. Take $x_0 = 0, x_1 = 1, x_2 = 2, \ldots, x_{10} = 10$ and $c_1 = \frac{1}{2}, c_2 = \frac{3}{2}, \ldots, c_{10} = \frac{19}{2}$, i.e. $c_1, c_2, \ldots c_{10}$ are the midpoints of the intervals
$(0, 1), (1, 2), \ldots, (9, 10)$ and of course $\Delta x = 1$. Then
$$S_{10} = \Delta x(f(c_1) + f(c_2) + \ldots + f(c_{10}))$$
$$= 1\left(2\left(\frac{1}{2}\right) + 2\left(\frac{3}{2}\right) + \ldots + 2\left(\frac{19}{2}\right)\right)$$
$$= 1 + 3 + 5 + 7 + \ldots + 19 = 100$$

The exact area under the graph of f from $x = 0$ to $x = 10$ is the area of the right triangle with perpendicular sides of lengths 10 and 20 whose total area is $\frac{(10)(20)}{2} = 100$.

54. False. Let $f(x) = -2x$ on $[-10, 0]$. The exact area under the graph of f from $x = -10$ to $x = 0$ is 100 (see problem 52 above). For $n = 10$,
$\Delta x = -1$, we have:
$$L_{10} = \Delta x[f(-10) + f(-9) + \ldots + f(-1)]$$
$$= (-1)[-20 - 18 - 16 - \ldots - 2] = 2(1 + 2 + \ldots + 10) = 110$$
$$R_{10} = \Delta x[f(-9) + f(-8) + \ldots + f(-1) + f(0)]$$
$$= (-1)[-18 - 16 - \ldots - 2] = 2(1 + 2 + \ldots + 9) = 90$$

56. $h(x)$ is an increasing function; $\Delta x = 100$

$$R_{10} = h(100)100 + h(200)100 + h(300)100 + h(400)100 + h(500)100$$
$$+ h(600)100 + h(700)100 + h(800)100 + h(900)100$$
$$+ h(1000)100 = 336{,}100 \text{ ft}^2.$$

Error bound for R_{10}:

$$\text{Error} \le |h(1000) - h(0)|\left(\frac{1000-0}{10}\right) = |500 - 0|(100) = 50{,}000$$

To choose n so that Error ≤ 1000, we have

$$(500)\left(\frac{1000}{n}\right) \le 1000 \text{ or } n \ge 500$$

58. $f(x) = 0.25x^2 - 4$ on $[1, 6]$

$$L_5 = f(1)\Delta x + f(2)\Delta x + f(3)\Delta x + f(4)\Delta x + f(5)\Delta x$$

where $\Delta x = 1$.

Thus,

$$L_5 = [-3.75 - 3 - 1.75 + 0 + 2.25](1) = -6.25$$
$$R_5 = f(2)\Delta x + f(3)\Delta x + f(4)\Delta x + f(5)\Delta x + f(6)\Delta x$$

where $\Delta x = 1$.

Thus,

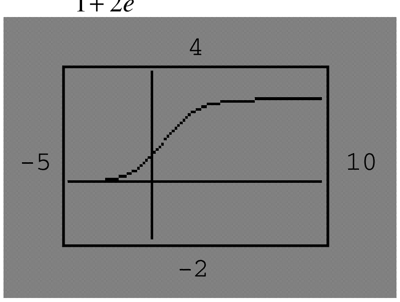
$$R_5 = [-3 - 1.75 + 0 + 2.25 + 5](1) = 2.5$$

Error bound for L_5 and R_5:

$$\text{Error} \le |f(6) - f(1)|\left(\frac{6-1}{5}\right) = |5 - (-3.75)| = 8.75$$

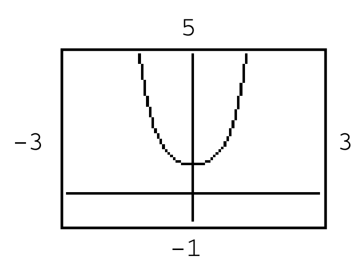
Geometrically, the definite integral over the interval [1, 6] is the sum of the areas between the curve and the x-axis from $x = 1$ to $x = 6$, with the areas below the x-axis counted negatively and those above the x-axis counted positively.

60. $f(x) = \dfrac{3}{1+2e^{-x}}$



Thus, f is increasing on $(-\infty, \infty)$.

62. $f(x) = e^{x^2}$



Thus, f is decreasing on $(-\infty, 0]$ and increasing on $[0, \infty)$.

64. $\int_0^{10} \ln(x^2 + 1)dx$; $f(x) = \ln(x^2 + 1)$

$$|I - L_n| \le |f(10) - f(0)|\left(\frac{10-0}{n}\right) \le 0.5$$

$$|\ln(101) - \ln 1|\frac{10}{n} \le 0.5$$

$$n \ge \frac{10\ln(101)}{0.5} \approx 93$$

66. $\int_1^4 x^x\, dx$; $f(x) = x^x$

$$|I - R_n| = |f(4) - f(1)|\left(\frac{4-1}{n}\right) \le 0.5$$

$$|4^4 - 1^1|\left(\frac{3}{n}\right) \le 0.5$$

$$n \ge \frac{3(255)}{0.5} \approx 1{,}530$$

68. $L_4 = [N(20) + N(40) + N(60) + N(80)]\Delta t$

$= [51 + 68 + 76 + 81](20) = 5{,}520$ units

$R_4 = [N(40) + N(60) + N(80) + N(100)]\Delta t$

$= [68 + 76 + 81 + 84](20) = 6{,}180$ units

Thus, $5{,}520 \le \int_{20}^{100} N(t)dt \le 6{,}180$.

Error bound for L_5 and R_5: Error $\le |84 - 51|(20) = 660$

70. $L_5 = [A'(5) + A'(6) + A'(7) + A'(8) + A'(9)]\Delta t$, $\Delta t = 1$

$= [0.55 + 0.49 + 0.45 + 0.40 + 0.36] = 2.25$

$R_5 = [A'(6) + A'(7) + A'(8) + A'(9) + A'(10)]\Delta t$, $\Delta t = 1$

$= [0.49 + 0.45 + 0.40 + 0.36 + 0.33] = 2.03$

Error bound for L_5 and R_5: Error $\le |0.55 - 0.33|(1) = 0.22$

72. $[0, 6]$, $\Delta x = 2$

$L_3 = [N'(0) + N'(2) + N'(4)]\Delta x$

$= [29 + 26 + 23](2) = 156$

$R_3 = [N'(2) + N'(4) + N'(6)]\Delta x$

$= [26 + 23 + 21](2) = 140$

$R_3 = 140 \le \int_0^6 N'(x)dx \le 156 = L_3$

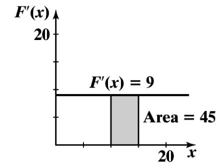
EXERCISE 6-5

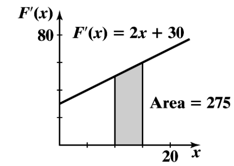
2. $F(x) = 9x + 120$

(A) $F(15) - F(10) = 255 - 210 = 45$

(B) $F'(x) = 9$

Area $= 9 \times 5 = 45$





4. $F(x) = x^2 + 30x + 210$

(A) $F(15) - F(10) = 225 + 450 - 100 - 300 = 275$

(B) $F'(x) = 2x + 30$

Area $= \frac{5}{2}(60 + 50) = 275$

6. $\int_0^8 9x\,dx = \left.\frac{9x^2}{2}\right|_0^8 = \frac{9(8)^2}{2} - \frac{9(0)^2}{2} = \frac{576}{2} = 288$

8. $\int_0^4 x^3\,dx = \left.\frac{x^4}{4}\right|_0^4 = \frac{(4)^4}{4} - \frac{(0)^4}{4} = \frac{256}{4} = 64$

10. $\int_2^5 (2x-1)\,dx = \left.(x^2 - x)\right|_2^5 = \left((5)^2 - (5)\right) - \left((2)^2 - (2)\right) = (20) - (2) = 18$

12. $\int_0^2 4e^x\,dx = \left.4e^x\right|_0^2 = 4e^2 - 4e^0 = 4e^2 - 4 \approx 25.556$

14. $\int_1^5 \frac{2}{x}\,dx = \left.2\ln|x|\right|_1^5 = 2\ln 5 - 2\ln 1 = 2\ln 5 \approx 3.219$

16. $\int_0^8 (0.25x - 1)\,dx = \int_0^8 \left(\frac{1}{4}x - 1\right)dx = \left.\left(\frac{1}{4}\cdot\frac{x^2}{2} - x\right)\right|_0^8 = \left(\frac{(8)^2}{8} - (8)\right) - \left(\frac{(0)^2}{8} - (0)\right) = 8 - 8 = 0$

18. $\int_1^4 (6x-5)\,dx = \left.\left(6\cdot\frac{x^2}{2} - 5x\right)\right|_1^4 = \left(3(4)^2 - 5(4)\right) - \left(3(1)^2 - 5(1)\right) = 28 - (-2) = 30$

20. $\int_4^1 (6x-5)\,dx = \left.\left(6\cdot\frac{x^2}{2} - 5x\right)\right|_4^1 = \left(3(1)^2 - 5(1)\right) - \left(3(4)^2 - 5(4)\right) = -2 - 28 = -30$

22. $\int_6^9 (5 - x^2)\,dx = \left.\left(5x - \frac{x^3}{3}\right)\right|_6^9 = \left(5(9) - \frac{(9)^3}{3}\right) - \left(5(6) - \frac{(6)^3}{3}\right) = (-198) - (-42) = -156$

24. $\int_{-3}^{-3} (x^2 + 4x + 2)^8\,dx = 0$ Note: $\int_a^a f(x)\,dx = 0$

26. $\int_1^2 (5 - 16x^{-3})dx = \left.(5x + 8x^{-2})\right|_1^2 = (5(2) + 8(2)^{-2}) - (5(1) + 8(1)^{-2})$

$= 12 - 13 = -1$

28. $\displaystyle\int_4^{25} \frac{2}{\sqrt{x}}\,dx = \int_4^{25} 2x^{-1/2}\,dx = 4x^{1/2}\Big|_4^{25} = 4(25)^{1/2} - 4(4)^{1/2} = 20 - 8 = 12$

30. $\displaystyle\int_0^1 32(x^2+1)^7 x\,dx$

Let $u = x^2 + 1$, then $du = 2x\,dx$.

$$\int 32(x^2+1)^7 x\,dx = 32\int (x^2+1)^7 \frac{2}{2} x\,dx = 16\int u^7\,du = 2u^8 + C$$
$$= 2(x^2+1)^8 + C$$

Thus,

$$\int_0^1 32(x^2+1)^7 x\,dx = 2(x^2+1)^8\Big|_0^1 = 2(1^2+1)^8 - 2(0^2+1)^8$$
$$= 2^9 - 2 = 512 - 2 = 510$$

32. $\displaystyle\int_2^8 \frac{1}{x+1}\,dx$

Let $u = x + 1$, then $du = dx$.

$$\int \frac{1}{x+1}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|x+1| + C$$

Thus,

$$\int_2^8 \frac{1}{x+1}\,dx = \ln|x+1|\Big|_2^8 = \ln 9 - \ln 3 = \ln \frac{9}{3} = \ln 3 \approx 1.099$$

34. $\displaystyle\int_{-10}^{25} e^{-0.01x}\,dx$

Let $u = -0.01x$, then $du = -0.01dx$.

$$\int e^{-0.01x}\,dx = \int e^{-0.01} \frac{-0.01}{-0.01}\,dx = -100\int e^u\,du = -100e^u + C$$
$$= -100e^{-0.01x} + C$$

$$\int_{-10}^{25} e^{-0.01x}\,dx = -100e^{-0.01x}\Big|_{-10}^{25}$$
$$= -100e^{-0.25} + 100e^{0.1} = 100(e^{0.1} - e^{-0.25}) \approx 32.637$$

36. $\displaystyle\int_e^{e^2} \frac{(\ln t)^2}{t}\,dt$

Let $u = \ln t$, the $du = \frac{1}{t}\,dt$. Thus,

$$\int \frac{(\ln t)^2}{t}\,dt = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln t)^3 + C$$

$$\int_e^{e^2} \frac{(\ln t)^2}{t}\,dt = \frac{1}{3}(\ln t)^3\Big|_e^{e^2} = \frac{1}{3}(\ln e^2)^3 - \frac{1}{3}(\ln e)^3$$
$$= \frac{1}{3}(2\ln e)^3 - \frac{1}{3}(\ln e)^3$$
$$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \approx 2.333$$

38. $\int_0^1 xe^{x^2}dx$

Let $u = x^2$, then $du = 2x\,dx$.

$$\int xe^{x^2}dx = \int e^{x^2}\frac{2}{2}x\,dx = \frac{1}{2}\int e^u\,du$$
$$= \frac{1}{2}e^u + C$$
$$= \frac{1}{2}e^{x^2} + C$$

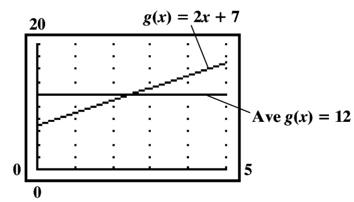
$$\int_0^1 xe^{x^2}dx = \frac{1}{2}e^{x^2}\Big|_0^1$$
$$= \frac{1}{2}e - \frac{1}{2}e^0 = \frac{1}{2}(e - 1) \approx 0.859$$

40. $\int_{-1}^{-1} e^{-x^2}dx = 0$ Note: $\int_a^a f(x)dx = 0$

42. $g(x) = 2x + 7$ on $[0, 5]$

(A) $\text{Ave } g(x) = \frac{1}{5-0}\int_0^5 (2x+7)dx$ (B)

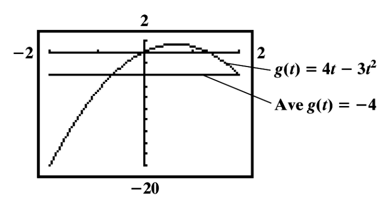
$$= \frac{1}{5}(x^2 + 7x)\Big|_0^5$$
$$= \frac{1}{5}(5^2 + 7(5))$$
$$= \frac{1}{5}(25 + 35) = 12$$



44. $g(t) = 4t - 3t^2$ on $[-2, 2]$

(A) $\text{Ave } g(t) = \frac{1}{2-(-2)}\int_{-2}^2 (4t - 3t^2)dt$ (B)

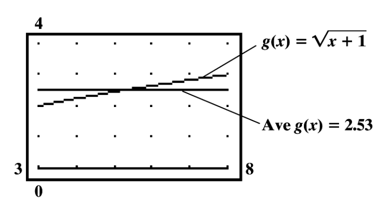
$$= \frac{1}{4}(2t^2 - t^3)\Big|_{-2}^2$$
$$= \frac{1}{4}(8 - 8) - \frac{1}{4}(8 + 8)$$
$$= 0 - 4 = -4$$



46. $g(x) = \sqrt{x+1}$ on $[3, 8]$

(A) $\text{Ave } g(x) = \frac{1}{8-3}\int_3^8 \sqrt{x+1}\,dx$ (B)

$$= \frac{1}{5}\int_3^8 (x+1)^{1/2}\,dx$$
$$= \frac{2}{15}(x+1)^{3/2}\Big|_3^8$$



$= \frac{2}{15}[(9)^{3/2} - (4)^{3/2}]$

$= \frac{2}{15}[27 - 8]$

$= \frac{38}{15} \approx 2.53$

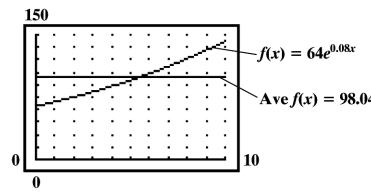
48. $f(x) = 64e^{0.08x}$ on [0, 10]

(A) Ave $f(x) = \frac{1}{10-0}\int_0^{10} 64e^{0.08x}\,dx$ (B)

$= \frac{64}{10}\left(\frac{1}{0.08}e^{0.08x}\right)\Big|_0^{10}$

$= 80(e^{0.8} - 1)$

≈ 98.04



50. $\int_0^1 x\sqrt{3x^2+2}\,dx = \int_0^1 x(3x^2 + 2)^{1/2}\,dx$

Let $u = 3x^2 + 2$, then $du = 6x\,dx$.

$\int x(3x^2 + 2)^{1/2}\,dx = \int (3x^2 + 2)^{1/2}\,\frac{6}{6}x\,dx = \frac{1}{6}\int u^{1/2}\,du$

$= \frac{1}{6}\cdot\frac{u^{3/2}}{3/2} + C$

$= \frac{u^{3/2}}{9} + C$

$= \frac{(3x^2+2)^{3/2}}{9} + C$

$\int_0^1 x\sqrt{3x^2+2}\,dx = \left(\frac{(3x^2+2)^{3/2}}{9}\right)\Big|_0^1$

$= \frac{(3+2)^{3/2}}{9} - \frac{(2)^{3/2}}{9}$

$= \frac{(5^{3/2}-2^{3/2})}{9} = \frac{1}{9}(5^{3/2} - 2^{3/2})$

52. $\int_1^2 \frac{x+1}{2x^2+4x+4}\,dx = \int_1^2 (2x^2 + 4x + 4)^{-1}(x + 1)dx$

Let $u = 2x^2 + 4x + 4$, then $du = (4x + 4)dx = 4(x + 1)dx$.

$\int (2x^2 + 4x + 4)^{-1}(x + 1)dx = \int (2x^2 + 4x + 4)^{-1}\,\frac{4}{4}(x + 1)dx$

$= \int \frac{1}{4}u^{-1}\,du = \frac{1}{4}\ln|u| + C$

$= \frac{1}{4}\ln|2x^2 + 4x + 4| + C$

$$\int_1^2 \frac{x+1}{2x^2+4x+4}\,dx = \left(\frac{1}{4}\ln\left|2x^2+4x+4\right|\right)\Bigg|_1^2$$

$$= \frac{1}{4}\ln(20) - \frac{1}{4}\ln(10) = \frac{1}{4}\ln\left(\frac{20}{10}\right) = \frac{1}{4}\ln 2$$

54. $\int_6^7 \frac{\ln(t-5)}{t-5}\,dt$

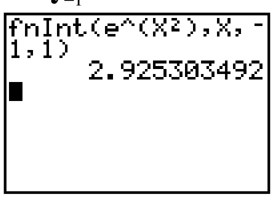
Let $u = \ln(t - 5)$, then $du = \frac{1}{t-5}\,dt$.

$$\int \frac{\ln(t-5)}{t-5}\,dt = \int u\,du = \frac{u^2}{2} + C = \frac{(\ln(t-5))^2}{2} + C$$

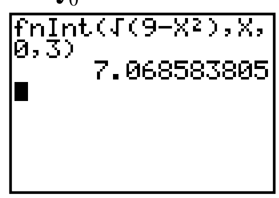
$$\int_6^7 \frac{\ln(t-5)}{t-5}\,dt = \left[\frac{(\ln(t-5))^2}{2}\right]\Bigg|_6^7$$

$$= \frac{(\ln 2)^2}{2} - \frac{(\ln 1)^2}{2} = \frac{(\ln 2)^2}{2} = \frac{1}{2}(\ln 2)^2$$

56. $\int_{-1}^1 e^{x^2}\,dx \approx 2.925$



58. $\int_0^3 \sqrt{9-x^2}\,dx \approx 7.069$



60. No solution provided.

62. $C'(x) = 500 - \frac{x}{3}$ on [0, 600]

The increase in cost from a production level of 0 bikes per month to a production level of 600 bikes per month is given by:

$$\int_0^{600}\left(500-\frac{x}{3}\right)dx = \left(500x-\frac{1}{6}x^2\right)\Bigg|_0^{600}$$

$$= 500(600) - \frac{1}{6}(600)^2$$

$$= 300{,}000 - 60{,}000$$

$$= \$240{,}000$$

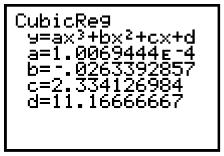
64. Total maintenance costs from the end of the second year to the end of the seventh year:

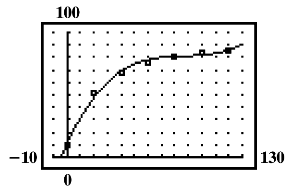
$$M(7) - M(2) = \int_2^7 (90x^2 + 5{,}000)dx = (30x^3 + 5{,}000x)\Bigg|_2^7$$

$$= (30(7)^3 + 5{,}000(7)) - (30(2)^3 + 5{,}000(2))$$

$$= 45{,}290 - 10{,}240 = \$35{,}050$$

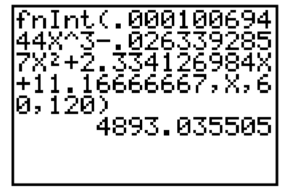
66. (A)





(B) Let $q(t)$ be the quadratic regression model found in part (A). The number of units assembled by a new employee during the second 60 days on the job is given (approximately) by

$$\int_{60}^{120} q(t)dt \approx 4{,}893$$



68. To obtain the useful life, set $C'(t) = R'(t)$ and solve for t.

$$\begin{aligned} 3 &= 15e^{-0.1t} \\ e^{0.1t} &= 5 \\ 0.1t &= \ln 5 \\ t &= 10 \ln 5 \approx 16 \text{ years} \end{aligned}$$

The total profit accumulated during the useful life is:

$$\begin{aligned} P(16) - P(0) &= \int_0^{16} [R'(t) - C'(t)]dt \\ &= \int_0^{16} (15e^{-0.1t} - 3)dt \\ &= \left(-\frac{15}{0.1}e^{-0.1t} - 3t\right)\Bigg|_0^{16} \\ &= -150e^{-1.6} - 48 + 150 \\ &= 102 - 150e^{-1.6} \approx 71.716 \text{ or } \$71{,}716 \end{aligned}$$

70. $C(x) = 20{,}000 + 10x$

(A) Average cost per unit:

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{20{,}000}{x} + 10$$

$$\overline{C}(1{,}000) = \frac{20{,}000}{1{,}000} + 10 = \$30$$

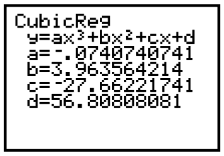
(B) $$\text{Ave } C(x) = \frac{1}{1{,}000}\int_0^{1{,}000} (20{,}000 + 10x)dx$$

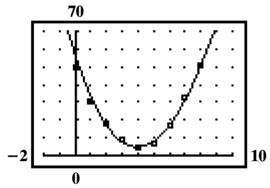
$$= \frac{1}{1{,}000}(20{,}000x + 5x^2)\Bigg|_0^{1{,}000}$$

$$= \$25{,}000$$

(C) $\overline{C}(1{,}000)$ is the average cost per unit at a production level of 1,000 units; Ave $C(x)$ is the average value of the total cost as production increases from 0 units to 1,000 units.

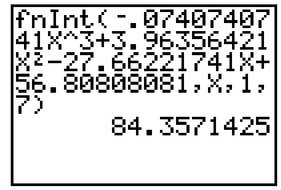
72. (A)





(B) Let $q(x)$ be the cubic regression model found in part (A). The increase in cost in going from a production level of 1 thousand sunglasses per month to 7 thousand sunglasses per month is given (approximately) by

$$\int_1^7 q(x)dx \approx 84.357 \text{ or } \$84{,}357$$



74. Average price:

$$\text{Ave } D(x) = \frac{1}{600-400}\int_{400}^{600} \frac{1{,}000}{x}\,dx$$

$$= \frac{1}{200}(1{,}000 \ln|x|)\Big|_{400}^{600}$$

$$= 5(\ln 600 - \ln 400) = 5 \ln\left(\frac{3}{2}\right) \approx \$2.03$$

76. $g(x) = 2{,}000x^{-1/3}$ and $L'(x) = g(x)$.

The number of labor hours to assemble the 9th through the 27th control units is:

$$L(27) - L(8) = \int_8^{27} g(x)dx = \int_8^{27} 2{,}000x^{-1/3}dx$$

$$= 2{,}000\left(\frac{x^{2/3}}{\frac{2}{3}}\right)\Big|_8^{27}$$

$$= 3{,}000(x^{2/3})\Big|_8^{27}$$

$$= 3{,}000(9 - 4)$$

$$= 15{,}000 \text{ labor hours}$$

78. (A) The inverntory function is obtained by finding the equation of the line joining (0, 1,200) and (4, 0).

Slope: $m = \frac{0-1{,}200}{4-0} = -300$, y intercept: $b = 1{,}200$

Thus, the equation of the line is: $I = -300t + 1{,}200$

(B) The average of I over [0, 4] is given by:

$$\text{Ave } I(t) = \frac{1}{4-0}\int_0^4 I(t)dt = \frac{1}{4}\int_0^4 (-300t + 1{,}200)dt$$

$$= \frac{1}{4}(-150t^2 + 1{,}200t)\Big|_0^4$$

$$= \frac{1}{4}(-150(4)^2 + 1{,}200(4))$$

$$= 600 \text{ units}$$

80. Rate of production: $R(t) = \dfrac{120t}{t^2+1} + 3,\ 0 \le t \le 20$

Total production from year N to year M is given by:

$$P = \int_N^M R(t)dt = \int_N^M \left(\frac{120t}{t^2+1}+3\right)dt$$

$$= 120\int_N^M \frac{t}{t^2+1}dt + \int_N^M 3\ dt$$

$$= 60(\ln(t^2+1))\Big|_N^M + (3t)\Big|_N^M$$

$$= 60(\ln(M^2+1) - \ln(N^2+1)) + 3(M - N)$$

$$= 60 \ln\left(\frac{M^2+1}{N^2+1}\right) + 3(M - N)$$

Thus, for total production during the first 5 years, let $M = 5$ and $N = 0$.

$$P = 60 \ln\left(\frac{26}{1}\right) + 3(5 - 0) = 60 \ln(26) + 15 \approx 210 \text{ thousand barrels}$$

For the total production from the end of the 5th year to the end of the 10th year, let $M = 10$ and $N = 5$.

$$P = 60 \ln\left(\frac{101}{26}\right) + 3(10 - 5) = 60 \ln\left(\frac{101}{26}\right) + 15 \approx 96 \text{ thousand barrels}$$

82. $A'(t) = -0.9e^{-0.1t}$

The change during the first five days is given by:

$$A(5) - A(0) = \int_0^5 -0.9e^{-0.1t}\,dt$$

$$= -0.9\left(\frac{e^{-0.1t}}{-0.1}\right)\Big|_0^5$$

$$= 9(e^{-0.1t})\Big|_0^5$$

$$= 9(e^{-0.5} - 1) \approx -3.54 \text{ square centimeters}$$

The change during the second five days, i.e., from the 5th day to the 10th day, is given by:

$$A(10) - A(5) = \int_5^{10} -0.9e^{-0.1t}\, dt$$

$$= 9(e^{-0.1t})\Big|_5^{10}$$

$$= 9(e^{-1} - e^{-0.5}) \approx -2.15 \text{ square centimeters}$$

84. $C(t) = \dfrac{0.14t}{t^2+1}$

Average concentration during the first hour after injection is given by:

$$\frac{1}{1-0}\int_0^1 \frac{0.14t}{t^2+1}\,dt = 0.07(\ln(t^2 + 1))\Big|_0^1$$

$$= 0.07 \ln 2 \approx 0.0485$$

Average concentration during the first two hours after the injection is given by:

$$\frac{1}{2-0}\int_0^2 \frac{0.14t}{t^2+1}\,dt = \frac{0.07}{2}\ln(t^2 + 1)\Big|_0^2$$

$$= \frac{0.07}{2}\ln 5 = 0.035 \ln 5 \approx 0.056$$

86. The average number of children in the city over the six year time period is given by:

$$\frac{1}{6-0}\int_0^6 N(t)dt = \frac{1}{6}\int_0^6 \left(-\frac{1}{4}t^2 + t + 4\right)dt$$

$$= \frac{1}{6}\left(-\frac{1}{12}t^3 + \frac{t^2}{2} + 4t\right)\Bigg|_0^6$$

$$= \frac{1}{6}\left(-\frac{1}{12}(6)^3 + \frac{(6)^2}{2} + 4(6)\right)$$

$$= -\frac{6^2}{12} + \frac{6}{2} + 4$$

$$= -3 + 3 + 4 = 4 \text{ million}$$