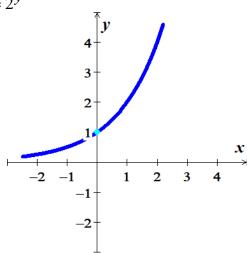
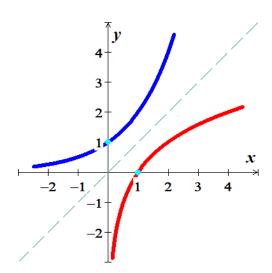
Section 3.3 – Logarithmic Functions

Graph: $x = 2^y$





Find the inverse function of $f(x) = 2^x$

$$y = 2^x$$

$$x = 2^{y}$$

Solve for y?

Logarithmic Function (Definition)

For x > 0 and $b > 0, b \ne 1$

 $y = \log_b x$ is equivalent to $x = b^y$

$$y = \log_b x \Leftrightarrow x = b^{\mathsf{J}}$$

The function $f(x) = \log_b x$ is the logarithmic function with base b.

 $\log_b x : \underline{read} \quad \log \text{ base } b \text{ of } x$

 $\log x$ *means* $\log_{10} x$

Example

Write each equation in its equivalent exponential form:

a)
$$3 = \log_7 x$$
 $\Rightarrow x = 7^3$

$$b) \quad 2 = \log_b 25 \qquad \Rightarrow 25 = b^2$$

Example

Write each equation in its equivalent logarithmic form:

$$a) \quad 2^5 = x \qquad \Rightarrow 5 = \log_2 x$$

$$b) \quad 27 = b^3 \qquad \Rightarrow 3 = \log_b 27$$

Basic Logarithmic Properties

$$log_b b = 1 \rightarrow b = b^1$$

$$\log_b 1 = 0 \longrightarrow 1 = b^0$$

Inverse Properties

$$\log_b b^x = x$$

$$\log_7 7^8 = 8$$

$$b^{\log b} = x$$

$$3^{\log_3 17} = 17$$

Example

Evaluate each expression without using a calculator:

a)
$$\log_5 \frac{1}{125}$$
 b) $\log_3 \sqrt[7]{3}$

b)
$$\log_3 \sqrt[7]{3}$$

Solution

a)
$$\log_5 \frac{1}{125} = \log_5 \frac{1}{5^3}$$

= $\log_5 5^{-3}$

$$= -3$$

b)
$$\log_3 \sqrt[7]{3} = \log_3 3^{1/7}$$

$$=\frac{1}{7}$$

Natural Logarithms

Definition

$$f(x) = \log_{e} x = \ln x$$

The logarithmic function with base e is called natural logarithmic function.

ln x read "el en of x"

$$\log(-1) = doesn't \ exist$$
 $\ln(-1) = doesn't \ exist$

$$\log 0 = doesn't \ exist$$
 $\ln 0 = doesn't \ exist$

$$\log 0.5 \approx -0.3010$$
 $\ln 0.5 \approx -0.6931$

$$\log 1 = 0 \qquad \qquad \ln 1 = 0$$

$$\log 2 \approx 0.3010$$
 $\ln 2 \approx 0.6931$

$$\log 10 = 1 \qquad \qquad \ln e = 1$$

Change-of-Base Logarithmic

$$\log_b M = \frac{\log_a M}{\log_a b} \qquad \qquad \log_b M = \frac{\log M}{\log b} \quad \textit{or} \quad \log_b M = \frac{\ln M}{\ln b}$$

Evaluate

$$\log_{7} 2506 = \frac{\log 2506}{\log 7}$$

$$\approx 4.02$$

Or

$$\log_7 \frac{2506}{\ln 7} \approx 4.02$$
 $\ln(2506) / \ln(7)$

$$\log_5 \frac{17}{\ln 5} \approx 1.7604$$

$$\log_2 \frac{0.1}{\ln 2} \approx -3.3219$$

Domain

The domain of a logarithmic function of the form $f(x) = \log_b x$ is the set of all positive real numbers. (*Inside* the log has to be > 0)

Range: $(-\infty, \infty)$

Example

Find the *domain* of

$$a) \quad f(x) = \log_4(x - 5)$$

$$x-5>0 \implies x>5$$

Domain: $(5, \infty)$

$$b) \quad f(x) = \ln\left(4 - x\right)$$

$$4 - x > 0$$

$$-x > -4$$

Domain: $(-\infty, 4)$

c)
$$h(x) = \ln(x^2)$$

 $x^2 > 0 \Rightarrow$ all real numbers except 0.

Domain: $\{x | x \neq 0\}$

$$or \ (-\infty, \ 0) \cup (0, \ \infty)$$

or
$$\mathbb{R}-\{0\}$$

Graphs of Logarithmic Functions

Example

Graph $g(x) = \log x$

Solution

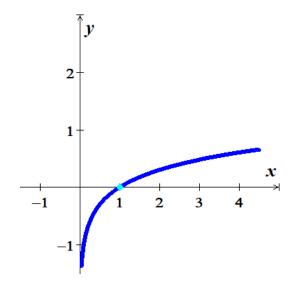
Asymptote: x = 0

(Force inside log to be equal to zero, then solve for x)

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

X	g(x)
<u> </u>	
0.5	3
1	0
2	.3
3	.5



Example

$$f(x) = \log_5 x$$

Solution

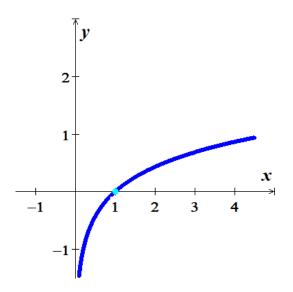
$$f(x) = \frac{\log x}{\log 5}$$

Asymptote: x = 0

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x	f(x)
-0-	
<u>1</u> 5	-1
1	0
5	1



Example

Graph: $f(x) = \log_{1/2} x$

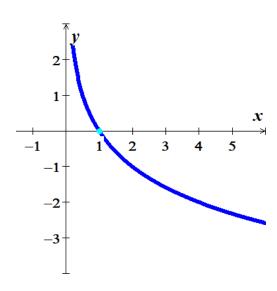
Solution

Asymptote: x = 0

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x	f(x)
-0-	
2	-1
1	0
$\frac{1}{2}$	1



Example

Graph: $f(x) = \log_2(x-1)$

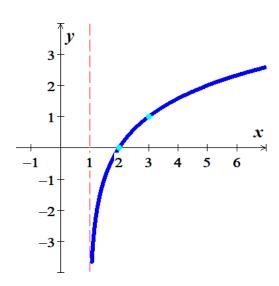
Solution

Asymptote: x = 1

Domain: $(1, \infty)$

Range: $(-\infty, \infty)$

X	f(x)
_ + _ :	
1	
$\frac{1}{3}$	-1
2	0
3	1



Example

$$f(x) = \left| \ln(x - 1) \right|$$

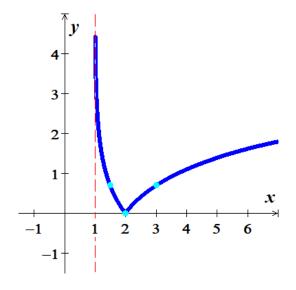
Solution

Asymptote: x = 1

Domain: $(1, \infty)$

Range: $[0, \infty)$

x	f(x)
-1	-
$\frac{3}{2}$	-0.7
2	0
3	0.7



Exercises Section 3.3 – Logarithmic Functions

(1-12) Write the equation in its equivalent logarithmic form

1.
$$2^6 = 64$$

2.
$$5^4 = 625$$

3.
$$5^{-3} = \frac{1}{125}$$
 7. $\sqrt[n]{x} = y$

4.
$$\sqrt[3]{64} = 4$$

5.
$$b^3 = 343$$

6.
$$8^{y} = 300$$

7.
$$\sqrt[n]{x} = y$$

8.
$$\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$$

9.
$$\left(\frac{1}{2}\right)^{-5} = 32$$

10.
$$e^{x-2} = 2y$$

11.
$$e = 3x$$

12.
$$\sqrt[3]{e^{2x}} = y$$

(13-24) Write the equation in its equivalent exponential form

13.
$$\log_5 125 = y$$

17.
$$\log_6 \sqrt{6} = x$$

21.
$$\log_{\sqrt{3}} 81 = 8$$

14.
$$\log_4 16 = x$$

18.
$$\log_3 \frac{1}{\sqrt{3}} = x$$

22.
$$\log_4 \frac{1}{64} = -3$$

15.
$$\log_5 \frac{1}{5} = x$$

19.
$$6 = \log_2 64$$

23.
$$\log_4 26 = y$$

16.
$$\log_2 \frac{1}{8} = x$$

20.
$$2 = \log_{\mathbf{Q}} x$$

24.
$$\ln M = c$$

(25-31) Evaluate the expression without using a calculator

25.
$$\log_4 16$$

27.
$$\log_6 \sqrt{6}$$
 29. $\log_3 \sqrt[7]{3}$

29.
$$\log_2 \sqrt[7]{3}$$

31.
$$\log_{\frac{1}{2}} \sqrt{\frac{1}{2}}$$

26.
$$\log_2 \frac{1}{8}$$

28.
$$\log_3 \frac{1}{\sqrt{3}}$$
 30. $\log_3 \sqrt{9}$

30.
$$\log_3 \sqrt{9}$$

(32-40) Simplify

32.
$$\log_{5} 1$$

35.
$$10^{\log 3}$$

38.
$$\ln e^{x-5}$$

33.
$$\log_{7} 7^2$$

36.
$$e^{2+\ln 3}$$

38.
$$\ln e^{x-5}$$
39. $\log_b b^n$

34.
$$3^{\log_3 8}$$

37.
$$\ln e^{-3}$$

40.
$$\ln e^{x^2 + 3x}$$

(41 - 64) Find the domain of

41.
$$f(x) = \log_5(x+4)$$

42.
$$f(x) = \log_5(x+6)$$

43.
$$f(x) = \log(2 - x)$$

44.
$$f(x) = \log(7 - x)$$

45.
$$f(x) = \ln(x-2)^2$$

46.
$$f(x) = \ln(x-7)^2$$

47.
$$f(x) = \log(x^2 - 4x - 12)$$

$$48. \qquad f(x) = \log\left(\frac{x-2}{x+5}\right)$$

$$49. \qquad f(x) = \log\left(\frac{3-x}{x-2}\right)$$

$$50. \quad f(x) = \ln\left(\frac{x^2}{x-4}\right)$$

$$51. \quad f(x) = \log_3\left(x^3 - x\right)$$

52.
$$f(x) = \log \sqrt{2x-5}$$

53.
$$f(x) = 3\ln(5x-6)$$

$$54. \quad f(x) = \log\left(\frac{x}{x-2}\right)$$

$$55. \quad f(x) = \ln(x^2 + 4)$$

56.
$$f(x) = \ln|4x - 8|$$

$$57. \qquad f(x) = \ln(x^2 - 9)$$

58.
$$f(x) = \ln|5 - x|$$

59.
$$f(x) = \ln(x-4)^2$$

$$60. \qquad f(x) = \ln(x^2 - 4)$$

61.
$$f(x) = \ln(x^2 - 4x + 3)$$

62.
$$f(x) = \ln(2x^2 - 5x + 3)$$

63.
$$f(x) = \log(x^2 + 4x + 3)$$

$$64. \qquad f(x) = \ln\left(x^4 - x^2\right)$$

(65-73) Find the *asymptote*, *domain*, and *range* of the given functions. Then, sketch the graph

65.
$$f(x) = \log_{A} (x-2)$$

68.
$$f(x) = \log(3-x)$$

71.
$$f(x) = \ln(3-x)$$

$$66. \quad f(x) = \log_{A} |x|$$

$$f(x) = \log_4(x-2)$$
 68. $f(x) = \log(3-x)$ 71. $f(x) = \ln(3-x)$
 $f(x) = \log_4|x|$ 69. $f(x) = 2 - \log(x+2)$ 72. $f(x) = 2 + \ln(x+1)$

72.
$$f(x) = 2 + \ln(x+1)$$

67.
$$f(x) = (\log_4 x) - 2$$

70.
$$f(x) = \ln(x-2)$$
 73. $f(x) = 1 - \ln(x-2)$

73.
$$f(x) = 1 - \ln(x - 2)$$

On a study by psychologists Bornstein and Bornstein, it was found that the average walking speed w, 74. in feet per second, of a person living in a city of population P, in *thousands*, is given by the function:

$$w(P) = 0.37 \ln P + 0.05$$

- a) The population is 124,848. Find the average walking speed of people living in Hartford.
- b) The population is 1,236,249. Find the average walking speed of people living in San Antonio.
- *75.* The loudness of sounds is measured in a unit called a decibel. To measure with this unit, we first assign an intensity of I_0 to a very faint sound, called the threshold sound. If a particular sound has intensity I, then the decibel rating of this louder sound is

$$d = 10\log \frac{I}{I_0}$$

Find the exact decibel rating of a sound with intensity $10,000I_0$

76. Students in an accounting class took a final exam and then took equivalent forms of the exam at monthly intervals thereafter. The average score S(t), as a percent, after t months was found to be given by the function

$$S(t) = 78 - 15 \log(t+1); t \ge 0$$

- a) What was the average score when the students initially took the test, t = 0?
- b) What was the average score after 4 months? 24 months?
- 77. A model for advertising response is given by the function

$$N(a) = 1,000 + 200 \ln a, \quad a \ge 1$$

Where N(a) is the number of units sold when a is the amount spent on advertising, in thousands of dollars.

- a) N(1)
- b) N(5)