08-11-2020

Review Exam 4

Exam Thursday 8/13
10:15 { lockdown computer
Team → phone

$\sin A = \frac{12}{13}$ $A \in Q_{II}$,

$\cos B = -\frac{15}{17}$ $B \in Q_{III}$

$\sin A = \frac{12}{13}$ Q_{II} $\cos B = -\frac{15}{17}$ Q_{III}

$\cos A = -\frac{5}{13}$ $\sin B = -\frac{8}{17}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \left(\frac{12}{13}\right)\left(-\frac{15}{17}\right) + \left(\frac{-5}{13}\right)\left(-\frac{8}{17}\right)$$

$$= -\frac{180}{221} + \frac{40}{221}$$

$$= \boxed{-\frac{140}{221}}$$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \left(-\frac{5}{13}\right)\left(-\frac{15}{17}\right) - \left(\frac{12}{13}\right)\left(-\frac{8}{17}\right)$$

$$= \frac{75+96}{221}$$

$$= \frac{171}{221}$$

$$\frac{\quad}{221}$$

c) $\tan(A+B) = -\dfrac{140}{171}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= -\frac{180}{221} - \frac{40}{221}$$

$$= -\frac{220}{221}$$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$= \frac{75-96}{221}$$

$$= -\frac{21}{221}$$

f) $\tan(A-B) = \dfrac{220}{21}$

$\cos\theta = -\dfrac{3}{5}$ $\quad \theta \in Q_{II} \quad 90^\circ \le \theta < 180^\circ$

$\sin\theta = \dfrac{4}{5}$ $\quad - \quad 45^\circ < \dfrac{\theta}{2} < 90^\circ$

$\theta/2 \in Q_{I}$

a) $\sin 2\theta = 2 \sin\theta \cos\theta$

$$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$$

$$= -\frac{24}{25}$$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \frac{9}{25} - \frac{16}{25}$$

$$= -\frac{7}{\quad}$$

$= -\frac{7}{25}$

c) $\boxed{\tan 2\theta = \frac{24}{7}}$

$\theta/2 \in QI$

d) $\sin\frac{\theta}{2} = \sqrt{\frac{1}{2}(1-\cos\theta)}$

$= \sqrt{\frac{1}{2}\left(1+\frac{3}{5}\right)}$

$= \sqrt{\frac{1}{2}\left(\frac{8}{5}\right)}$

$= \boxed{\frac{2}{\sqrt{5}}}$

e) $\cos\frac{\theta}{2} = \sqrt{\frac{1}{2}(1+\cos\theta)}$

$= \sqrt{\frac{1}{2}\left(1-\frac{3}{5}\right)} = \sqrt{\frac{1}{2}\left(\frac{2}{5}\right)}$

$= \boxed{\frac{1}{\sqrt{5}}}$

f) $\tan\frac{\theta}{2} = 2$ $\quad \frac{2}{1}$

Solve $\rightarrow$ $\boxed{3 \text{ out of } 5}$ $[0, 2\pi)$

Solve: $4\cos^2 x + 4\sin x - 5 = 0$

$4(1-\sin^2 x) + 4\sin x - 5 = 0$

$4 - 4\sin^2 x + 4\sin x - 5 = 0$

$-4\sin^2 x + 4\sin x - 1 = 0$

$$\sin x = \frac{-4 \pm \sqrt{16 - 4(6)}}{-8}$$

$$= \left.\begin{array}{l} \frac{-4}{-8} \\ = \frac{1}{2} \end{array}\right\} \#$$

$$\underline{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

$$2 \tan x \csc x + 2 \csc x + \tan x + 1 = 0$$

$$2 \csc x (\tan x + 1) + (\tan x + 1) = 0$$

$$(\tan x + 1)(2 \csc x + 1) = 0$$

$$\tan x = -1 \qquad \csc x = -\frac{1}{2} \ \#$$

$$\underline{x = \frac{3\pi}{4}, \frac{7\pi}{4}} \qquad \sin x = -2$$

$$2 \cos\alpha + \tan\alpha = \sec\alpha \qquad [0, 2\pi)$$

$$\cos\alpha) \ 2 \cos\alpha + \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\cos\alpha} \qquad \cos\alpha \neq 0$$

$$\alpha \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2 \cos^2\alpha + \sin\alpha = 1$$

$$2(1 - \sin^2\alpha) + \sin\alpha = 1$$

$$2 - 2\sin^2\alpha + \sin\alpha - 1 = 0$$

$$-2\sin^2\alpha + \sin\alpha + 1 = 0$$

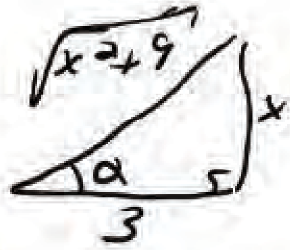
$$\sin\alpha = 1 \qquad \sin\alpha = -\frac{1}{2}$$

$$\alpha = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

6 6

$$\sec\left(\arcsin\underbrace{\frac{x}{\sqrt{x^2+9}}}_{\alpha}\right)$$

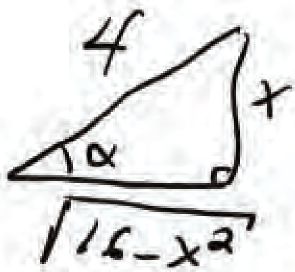
$$\sin\alpha = \frac{x}{\sqrt{x^2+9}}$$



$$\boxed{\sec\alpha = \frac{\sqrt{x^2+9}}{3}}$$

$$\sin\left(\tan^{-1}\underbrace{\frac{x}{\sqrt{16-x^2}}}_{\alpha}\right)$$

$$\tan\alpha = \frac{x}{\sqrt{16-x^2}}$$



$$\boxed{\sin\alpha = \frac{x}{\sqrt{16-x^2}}}$$

$$\left(2, \frac{\pi}{6}\right)$$

$$x = r\cos\theta = 2\cos\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

$$y = r\sin\theta = 2\sin\frac{\pi}{6} = 2\left(\frac{1}{2}\right) = 1$$

$(x, y) = (\sqrt{3}, 1)$

$(2, 2)$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$\theta = \tan^{-1}\frac{2}{2}$

$= \frac{\pi}{4}$

$(r, \theta) = \left(2\sqrt{2}, \frac{\pi}{4}\right)$

$r(\sin\theta + \cos\theta) = 4$

$r\sin\theta + r\cos\theta = 4$

$y + x = 4$

$x - y = 2$

$r\cos\theta - r\sin\theta = 2$

$r(\cos\theta - \sin\theta) = 2$

$r = \dfrac{2}{\cos\theta - \sin\theta}$

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(3) $\left.\begin{matrix} -1 \rightarrow \\ -2 \\ -3 \end{matrix}\right\}$ 2 out 3

$1 + \sec^2 x \sin^2 x = \sec^2 x$

$$1+\sec^2 x\sin^2 x = 1+\frac{1}{\cos^2 x}\sin^2 x$$
$$= 1+\tan^2 x$$
$$= \sec^2 x \checkmark$$

$$\tan^2 x = \sec^2 x - \sin^2 x - \cos^2 x$$

$$\sec^2 x - \sin^2 x - \cos^2 x = \sec^2 x - (\sin^2 x + \cos^2 x)$$
$$= \sec^2 x - 1$$
$$= \tan^2 x \checkmark$$

$$1+\tan^2 x = \sec^2 x$$

$$\sec^4 x - \tan^4 x = \sec^2 x + \tan^2 x$$

$$\sec^4 x - \tan^4 x = (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$$
$$= (\sec^2 x + \tan^2 x)(1)$$
$$= \sec^2 x + \tan^2 x \checkmark$$

$$\frac{\sin(A-B)}{\cos A\cos B} = \tan A - \tan B$$

$$\frac{\sin(A-B)}{\cos A\cos B} = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B}$$
$$= \frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}$$

$$= \tan A - \tan B$$

$$\frac{\cos(x+y)}{\cos(x-y)} = \frac{\cot y - \tan x}{\cot y + \tan x}$$

$$\frac{\cos(x+y)}{\cos(x-y)} = \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y + \sin x \sin y}$$

$$= \frac{\frac{\cos x \cos y}{\cos x \sin y} - \frac{\sin x \sin y}{\cos x \sin y}}{\frac{\cos x \cos y}{\cos x \sin y} + \frac{\sin x \sin y}{\cos x \sin y}}$$

$$= \frac{\cot y - \tan x}{\cot y + \tan x}$$

$$\sec(x+y) = \frac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y}$$

$$\sec(x+y) = \frac{1}{\cos(x+y)} \cdot \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y + \sin x \sin y}$$

$$= \frac{\cos(x-y)}{(\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)}$$

$$= \frac{\cos(x-y)}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}$$

$$= \frac{\cos(x-y)}{\cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y}$$

$$= \frac{\cos(x-y)}{\cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y} \checkmark$$

$$\cos 4x = \cos^4 x - 6 \sin^2 x \cos^2 x + \sin^4 x$$

$$\cos 4x = \cos 2(2x)$$

$$= \cos^2 2x - \sin^2 2x$$

$$= (\cos^2 x - \sin^2 x)^2 - (2 \sin x \cos x)^2$$

$$= \cos^4 x - 2 \sin^2 x \cos^2 x + \sin^4 x - 4 \sin^2 x \cos^2 x$$

$$= \cos^4 x - 6 \sin^2 x \cos^2 x + \sin^4 x \checkmark$$

$$\frac{\cos 2x}{\cos^2 x} = \sec^2 x - 2 \tan^2 x$$

$$\frac{\cos 2x}{\cos^2 x} = \frac{1 - 2\sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - 2 \frac{\sin^2 x}{\cos^2 x}$$

$$= \sec^2 x - 2 \tan^2 x \checkmark$$

$$2 \sin^2 \frac{x}{2} = \frac{\sin^2 x}{1 + \cos x}$$

$$2 \sin^2 \frac{x}{2} = 2 \left(\frac{1 - \cos x}{2} \right)$$

$$= (1 - \cos x) \frac{1+\cos x}{1+\cos x}$$

$$= \frac{1-\cos^2 x}{1+\cos x}$$

$$= \frac{\sin^2 x}{1+\cos x}$$

$$\sec^2\left(\frac{\alpha}{2}\right) = \frac{2\sec\alpha + 2}{\sec\alpha + 2 + \cos\alpha}$$

$$\sec^2\left(\frac{\alpha}{2}\right) = \frac{1}{\cos^2\frac{\alpha}{2}}$$

$$= \frac{1}{\frac{1}{2}(1+\cos\alpha)}$$

$$= \frac{2}{1+\cos\alpha} \cdot \frac{\sec\alpha + 1}{\sec\alpha + 1}$$

$$= \frac{2\sec\alpha + 2}{\sec\alpha + \underbrace{\sec\alpha\cos\alpha}_{=1} + \cos\alpha + 1}$$

$$= \frac{2\sec\alpha + 2}{\sec\alpha + 2 + \cos\alpha}$$

$$\frac{\sin 4t}{4} = \cos^3 t \sin t - \sin^3 t \cos t$$

$$\frac{1}{4}\sin 4t = \frac{1}{4}\sin(2t+2t)$$

$$\ldots = \frac{1}{4} \sin(2t + 2t)$$

$$= \frac{1}{4}\left(\sin 2t \cos 2t + \sin 2t \cos 2t\right)$$

$$= \frac{1}{2} \sin 2t \cos 2t$$

$$= \frac{1}{2}\left(2 \sin t \cos t\right)\left(\cos^2 t - \sin^2 t\right)$$

$$= \sin t \cos^3 t - \sin^3 t \cos t$$

$$\frac{\sin(x+y)}{\cos(x-y)} = \frac{1 + \cot x \tan y}{\cot x + \tan y}$$

$$\frac{\sin(x+y)}{\cos(x-y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y + \sin x \sin y}$$

$$= \frac{\frac{\sin x \cos y}{\sin x \cos y} + \frac{\cos x \sin y}{\sin x \cos y}}{\frac{\cos x \cos y}{\sin x \cos y} + \frac{\sin x \sin y}{\sin x \cos y}}$$

$$= \frac{1 + \cot x \tan x}{\cot x + \tan y}$$