MATH 1314 – College Algebra **EXAM 4** **REVIEW**
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1. Solve the following systems using Gaussian Elimination.

a) $\begin{aligned} 2x-3y&=-8\\ x+3y&=5 \end{aligned}$

b) $\begin{aligned} x+2y-5z&=-1\\ 2x+3y-2z&=2\\ 3x+5y-7z&=4 \end{aligned}$

c) $\begin{aligned} x-y+z&=-4\\ 5x+y-2z&=12\\ 2x-3y+4z&=-15 \end{aligned}$

d) $\begin{aligned} x+y+z&=9\\ 2x-3y+4z&=7\\ x-4y+3z&=-2 \end{aligned}$

e) $\begin{aligned} 2x+y+2z&=4\\ 2x+2y\quad&=5\\ 2x-y+6z&=2 \end{aligned}$

2. Perform the matrix row operation (or operations) and write the new matrix.

a) $\left[\begin{array}{cccc|c} 1&-2&1&3&-2\\ 2&-3&5&-1&0\\ 1&0&3&1&-4\\ -4&3&2&-1&3 \end{array}\right]$ $\begin{aligned} R_2-2R_1\\ R_3-R_1\\ R_4+4R_1 \end{aligned}$

b) $\left[\begin{array}{cccc|c} 1&-2&1&3&-2\\ -3&6&-3&-9&6\\ 2&1&2&3&4\\ 5&3&2&-1&-7 \end{array}\right]$ $\begin{aligned} R_2+3R_1\\ R_3-2R_1\\ R_4-5R_1 \end{aligned}$

3. Find the values that make the equation true.

a) $\begin{bmatrix} 3&5&x\\ -4&y&x-y \end{bmatrix}=\begin{bmatrix} m&5&6\\ n&-1&p \end{bmatrix}$

b) $\begin{pmatrix} a&8&-6\\ 5&4&f \end{pmatrix}+\begin{pmatrix} 6&b&-1\\ d&8&8 \end{pmatrix}=\begin{pmatrix} 8&1&c\\ 3&e&-5 \end{pmatrix}$

4. Use the following definitions for *A*, *B*, and *C*.

$A=\begin{bmatrix} 1&-1&2\\ 2&-3&5 \end{bmatrix}$ $B=\begin{bmatrix} -1&0&5\\ 2&3&4 \end{bmatrix}$ $C=\begin{bmatrix} 1&-1\\ 2&3\\ 4&6 \end{bmatrix}$

a. $-2B$
b. $B-A$
c. $2A-3B$
d. $4A-3C$
e. AB
f. CB
g. BC

5. Find the inverse of the following matrices if they exist.

a) $A=\begin{bmatrix} -3&2\\ -2&1 \end{bmatrix}$ *b*) $B=\begin{bmatrix} 1&-3\\ -2&6 \end{bmatrix}$ *c*) $C=\begin{bmatrix} 2&-4\\ a&b \end{bmatrix}$

6. Write the matrix equation as a system of linear equations.

$\begin{bmatrix} 9&1&1\\ 0&6&6\\ 3&1&9 \end{bmatrix}\begin{bmatrix} x\\ y\\ z \end{bmatrix}=\begin{bmatrix} 1\\ -1\\ 7 \end{bmatrix}$

7. Given the system: $x + 2y = -3$
$2x + 2y = 2$

a) Write this system of equations as the matrix equation $AX = B$.

b) Solve this system using the inverse matrix method.

8. Given the system: $x + 2y + 5z = 2$
$2x + 3y + 8z = 3$
$-x + y + 2z = 3$

a) Write this system of equations as the matrix equation $AX = B$.

b) Solve this system using the inverse matrix method, given $A^{-1} = \begin{bmatrix} 2 & -1 & -1 \\ 12 & -7 & -2 \\ -5 & 3 & 1 \end{bmatrix}$

9. Evaluate the determinant

a) $\begin{vmatrix} 3 & 1 & 2 \\ -2 & 3 & 1 \\ 3 & 4 & -6 \end{vmatrix}$ *b*) $\begin{vmatrix} x & 1 & -1 \\ x^2 & x & x \\ 0 & x & 1 \end{vmatrix}$ *c*) $\begin{vmatrix} 1 & x & x \\ 2 & x^2 & 2x \\ x & 0 & -1 \end{vmatrix}$ *d*) $\begin{vmatrix} a & c \\ -2 & -4 \end{vmatrix}$

10. Solve the system of equations using Cramer's Rule:

a. $x - y + 2z = 0$
$x - 2y + 3z = -1$
$2x - 2y + z = -3$

b. $x - y + z = -4$
$5x + y - 2z = 12$
$2x - 3y + 4z = -15$

11. Give the answer true or false to each of the following statements:

a) Matrix multiplication is commutative

b) If a matrix is square then it has a multiplicative inverse

c) The identity matrix for multiplication is a matrix of ones

d) The identity matrix has one on the main diagonal, and all other elements are zero

12. One group of people purchased 10 hot dogs and 5 soft drinks for $12.50. A second group bought 7 hot dogs and 4 soft drinks for $9.00. What is the cost of a single hot dog and a single soft drink?

13. Fred sold total of 145 Italian sausages and hot dogs from his curbside pushcart and collected $242.05. He sold 45 more hot dogs than sausages. How many of each did he sell?

14. Your instructor bought 4 lab record books and 3 highlighters for $13.93. He bought 3 lab record books and 2 highlighters for $10.25. Find the price of each item.

15. At a garage sale all hardback books sell for one price, all paperback books sell for another price, and all children's books sell for a third price. Mary bought 3 hardbacks, 7 paperbacks and 4 children's books for $25.50. Sarah bought 3 paperbacks and 10 children's books for $12. Jeff bought 9 hardbacks and 11 paperbacks for $52.50. What was the price of each type of book?

16. In a group of 100 adults, 70 say they are most likely to do spring housecleaning in March, April, or May. Of these 70, the number who cleans in April is 14 more than the total number who cleans in March and May. The total number who cleans in April and May is 2 more than three times the number who cleans in March. Write the system equations only (*do not solve*).

17. Casey receives $126 per year in simple interest from three investments. Part is invested at 2%, part at 3%, and part at 4%. There is $500 more invested at 3% than at 2%. The amount invested at 4% is three times the amount invested at 3%. Find the amount invested at each rate. Write the system equations only (*do not solve).*

18. Orange juice, a raisin bagel, and a cup of coffee from Kelly's Koffee Kart cost a total of $4.10. Kelly posts a notice announcing that, effective the following week, the price of orange juice will increase 50% and the price of bagels will increase 20%. After the increase the same purchase will cost a total of $5.10, and orange juice will cost twice as much as coffee. Write the system equations only (*do not solve*);

19. Martin and Eva pool their loose change to buy snacks on their coffee break. One day, they spent $5 on 1 carton of milk, 2 donuts, and 1 cup of coffee. The next day, they spent $5.50 on 3 donuts and 2 cups of coffee. The third day, they bought 1 carton of milk, 1 donut, and 2 cups of coffee and spent $5.25. On the fourth day, they have a total $5.45 left. Is this enough to buy 2 cartons of milk and 2 donuts?

20. Evergreen Landscaping bought 4 tons of topsoil, 3 tons of mulch, and 6 tons of pea gravel for $2825. The next week the firm bought 5 tons of topsoil, 2 tons of mulch, and 5 tons of pea gravel for $2663. Pea gravel costs $17 less per ton than topsoil. Write the system equations only (*do not solve*).

SOLUTIONS

1. *a*) $(-1, 2)$ *b*) No Solution *c*) $(1,\ 3, -2)$

d) $\left(-\frac{7z}{5}+\frac{34}{5}, \frac{2z}{5}+\frac{11}{5}, z\right)$ *e*) $\left(-2z+\frac{3}{2},\ 2z+1,\ z\right)$

2. *a*) $\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ 0 & 1 & 3 & -7 & 4 \\ 0 & 2 & 2 & -2 & -2 \\ 0 & -5 & 6 & 11 & -5 \end{array}\right]$ *b*) $\left[\begin{array}{cccc|c} 1 & -2 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & -3 & 8 \\ 0 & 13 & -3 & -16 & 3 \end{array}\right]$

3. *a)* $x=6,\ m=3,\ y=-1,\ n=-4,\ p=7$

b) $a=2;\ b=-7;\ c=-7;\ d=-2;\ e=12;\ f=-13$

4.

a) $\begin{bmatrix} 2 & 0 & -10 \\ -4 & -6 & -8 \end{bmatrix}$

b) $\begin{bmatrix} -2 & 1 & 3 \\ 0 & 6 & -1 \end{bmatrix}$

c) $\begin{bmatrix} 5 & -2 & -11 \\ -2 & -15 & -2 \end{bmatrix}$

d) Not possible $4A$ is a 2x3 matrix, $-3C$ is a 3x2 matrix.

e) Not Possible A is a 2x3 matrix, B is a 2x3 matrix.

f) $\begin{bmatrix} -3 & -3 & 1 \\ 4 & 9 & 22 \\ 8 & 18 & 44 \end{bmatrix}$

g) $\begin{bmatrix} 19 & 31 \\ 24 & 31 \end{bmatrix}$

5. *a*) $A^{-1}=\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$ *b*) B^{-1} Does not exist *c*) $C^{-1}=\begin{bmatrix} \frac{b}{2b+4a} & \frac{2}{b+2a} \\ -\frac{a}{2b+4a} & \frac{1}{b+2a} \end{bmatrix}$

6.
$$9x+y+z=1$$
$$6y+6z=-1$$
$$3x+y+9z=7$$

7. *a*) $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ where $A=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$, $B=\begin{bmatrix} -3 \\ 2 \end{bmatrix}$

b) $A^{-1}=\begin{bmatrix} -1 & 1 \\ 1 & -1/2 \end{bmatrix}$ $X=A^{-1}B=\begin{bmatrix} 5 \\ -4 \end{bmatrix}$, So solution is $(5, -4)$

8. *a*) $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 8 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$

b) $X = A^{-1}B = \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix}$ So solution is $(-2,-3,2)$

9. *a)* –109 *b*) $-2x^3$ *c*) $-x^4 + 2x^3 - x^2 + 2x$ *d*) $-4a + 2c$

10. *a.* $D = 3 \quad D_x = 0 \quad D_y = 6 \quad D_z = 3 \quad (0,\ 2,\ 1)$

b. $D = 5 \quad D_x = 5 \quad D_y = 15 \quad D_z = -10 \quad (1,\ 3,\ -2)$

11. *a*) False *b*) False *c*) False *d*) True

12. h = price of hot dog, s = price soft drink

$10h + 5s = 12.50$

$7h + 4s = 9.00$

h = \$1.00, s = \$0.50

13. x: sausages y: hot dogs

$\begin{cases} x + y = 145 \\ y = x + 45 \end{cases} \quad \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 145 \\ 45 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} .5 & -.5 \\ .5 & .5 \end{bmatrix}$

50 sausages and 95 hot dogs

14. x: lab record y: highlighters

$\begin{cases} 4x + 3y = 13.93 \\ 3x + 2y = 10.25 \end{cases} \quad \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13.93 \\ 10.25 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$

Lab record: \$2.89 highlighters = \$0.79

15. h = price of one hardback book
p = price of one paperback book
c = price of one children's book

$3h + 7p + 4c = 25.50$

$0h + 3p + 10c = 12$

$9h + 11p + 0c = 52.50$

h = \$4.00,p = \$1.50, c = \$0.75

16. x: March y: April z: May

$$\begin{aligned} x+y+z&=70\\ y&=x+z+14\\ y+z&=3x+2 \end{aligned} \rightarrow \begin{cases} x+y+z=70\\ -x+y-z=14\\ -3x+y+z=2 \end{cases}$$

17. x: 2% y: 3% z: 4%

$$\begin{aligned} .02x+.03y+.04z&=126\\ y&=x+500\\ z&=3y \end{aligned} \rightarrow \begin{cases} .02x+.03y+.04z=126\\ -x+y=500\\ -3y+z=0 \end{cases}$$

18. x: OJ y: Raisin z: Coffee

$$\begin{aligned} x+y+z&=4.1\\ 1.5x+1.2y+z&=5.1\\ 1.5x&=2z \end{aligned} \rightarrow \begin{cases} x+y+z=4.1\\ 1.5x+1.2y+z=5.1\\ 1.5x-2z=0 \end{cases}$$

19. x: milk y: donut z: coffee

$$\begin{aligned} x+2y+z&=5\\ 3y+2z&=5.50\\ x+y+2z&=5.25 \end{aligned}$$

No, they need 5 *cents* more

20. x:topsoil y: mulch z: gravel

$$\begin{aligned} 4x+3y+6z&=2825\\ 5x+2y+5z&=2663\\ z&=x-17 \end{aligned} \rightarrow \begin{cases} 4x+3y+6z=2825\\ 5x+2y+5z=2663\\ x-z=17 \end{cases}$$