Section 2.9 – Applications of the Normal Distribution

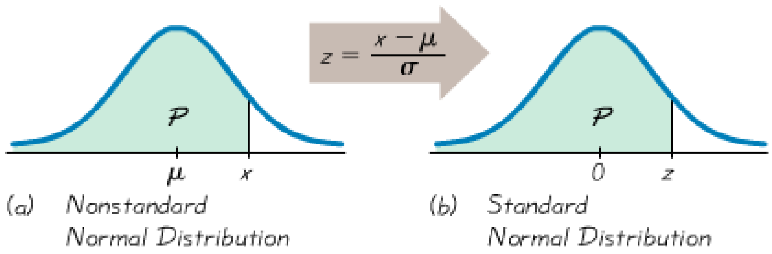
Working with normal distributions that are not standard, that is, the mean is not 0 or the standard deviation is not 1, or both.

The key concept is that we can use a simple conversion that allows us to standardize any normal distribution so that the same methods of the previous section can be used.

Conversion Formula

$$z = \frac{x - \mu}{\sigma}$$

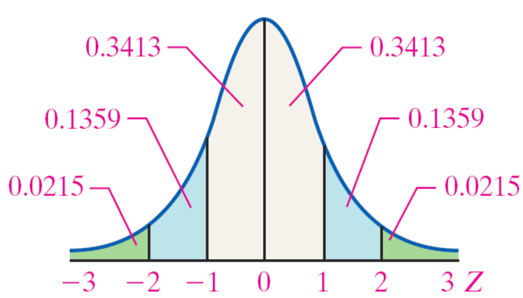
Round z scores to 2 decimal places



Finding Areas with a nonstandard normal distribution

1. Sketch a normal curve, label the mean and the specific x values, then *shade* the region representing the desired probability.
2. For each relevant value x that is a boundary for the shaded region, use the formula to convert that value to the equivalent z-score.
3. Refer the Normal Distribution Table or use the calculator to find the area of the shaded region. This area is the desired probability.

Areas Under the Standard Normal Curve

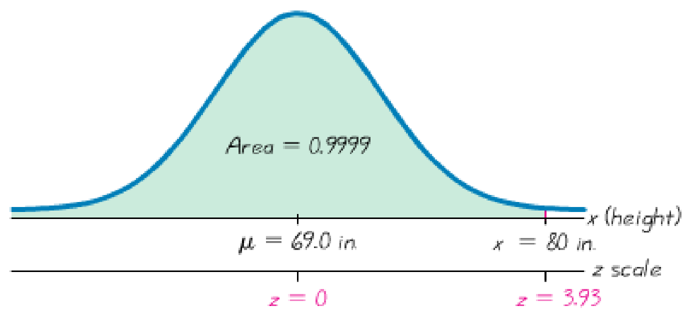


Example

The typical home doorway has a height of 6 ft. 8 in., or 80 in. Because men tend to be taller than women, we will consider only men as we investigate the limitations of that standard doorway height. Given that heights of men are normally distributed with a mean of 69.0 in. and a standard deviation of 2.8 in., find the percentage of men who can fit through the standard doorway without bending or bumping their head. Is that percentage high enough to continue using 80 in. as the standard height? Will a doorway height of 80. Be sufficient in future years?

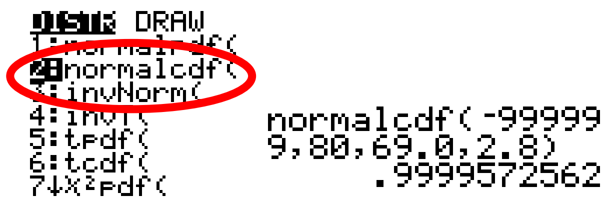
Solution

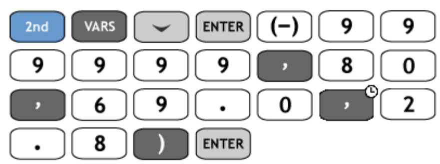
Men have heights that are normally distributed with a mean of 69.0 in. and a standard deviation of 2.8 in. The shaded region represents the men who can fir through a doorway that has a height 80 in.



The z-score: $z = \dfrac{x-\mu}{\sigma} = \dfrac{80-69}{2.8} = 3.93$ $\qquad (80-69)/2.8$

Referring to the table, the z-score values are less than 3.5, therefore; if we use calculator:





TI-83/84 PLUS

- To find the area between two values, press **2nd, VARS, 2** (for normalcdf), then proceed to enter the two values, the mean, and the standard deviation, all separated by commas, as in (left value, right value, mean, standard deviation). *Hint:* If there is no left value, enter the left value as −999999, and if there is no right value, enter the right value as 999999. In Example 1 we want the area to the left of x = 80 in., so use the command **normalcdf (−999999, 80, 69.0, 2.8)** as shown in the accompanying screen display.

$\boxed{normalcdf\left(-999999,\ x,\ \mu,\ \sigma\right)}$

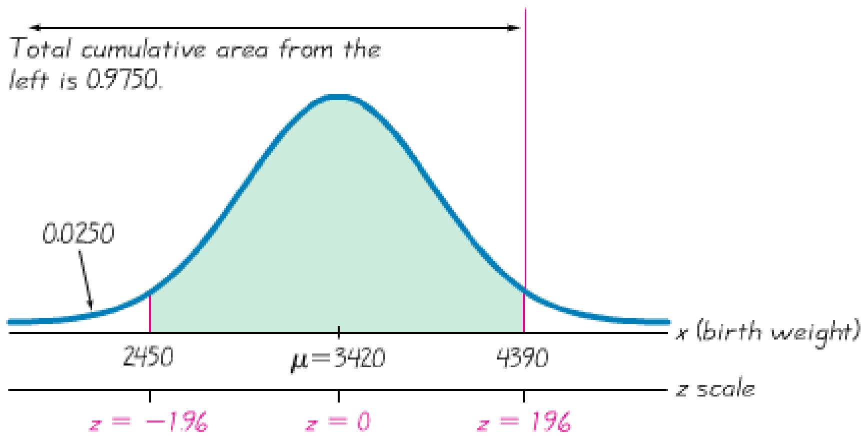
✓ The proportion of men who can fit through the standard doorway height of 80 in. is 0.9999, or 99.99%. Very few men will not be able to fit through the doorway without bending their head. This percentage id high enough to justify the use of 80 in. as the standard doorway height. However, heights of men and women have been increasing gradually but steady over the past decades.

Example

Birth weights in the U.S. are normally distributed with a mean of 3420 g and a standard deviation of 495 g. A hospital requires special treatment for babies that are less than 2450 g (unusual light) or more than 4390 g (unusually heavy). What is the percentage of babies who do not require special treatment because they have birth weights between 2450 g and 4390 g? Under these conditions, do many babies require special treatment?

Solution

Given: $\mu = 3420, \quad \sigma = 495$



The area to the ***left*** of $x = 2450 \Rightarrow \underline{\left|z = \frac{2450-3420}{495} = -1.96\right|}$

$$Area = A_1 \ (<2450) \underline{= 0.0250|}$$

The area to the ***left*** of $x = 4390 \Rightarrow \underline{\left|z = \frac{4390-3420}{495} = 1.96\right|}$

$$Area = A_2 \ (<4390) \underline{= 0.9750|}$$

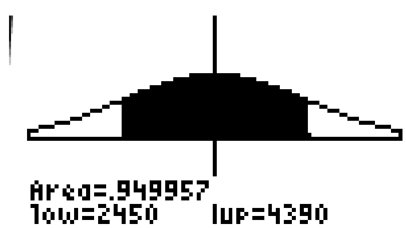
The total shaded area:

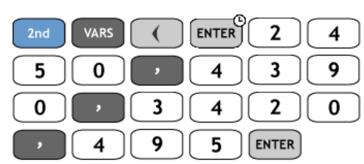
$$Area = A_2 - A_1 = 0.9750 - 0.025 \underline{= 0.950|}$$

[2ND] [VARS]

DISTR DRAW
1:ShadeNorm(

ShadeNorm(2450,4390,3420,495





$\boxed{ShadeNorm(low\ x,\ up\ x,\ \sigma)}$

✓ We conclude that 95.00% of the babies do not require special treatment because they have birth weights between 2450 g and 4390 g. It follows that 5.00 % of the babies do require special treatment.

Helpful *Hints*

1. Don't confuse z scores and areas. z-scores are distances along the horizontal scale, but areas are regions under the normal curve. Table lists z-scores in the left column and across the top row, but areas are found in the body of the table.
2. Choose the correct (right/left) side of the graph.
3. A z-score must be negative whenever it is located in the left half of the normal distribution.
4. Areas (or probabilities) are positive or zero values, but they are never negative.

Procedure for Finding the Value of a Normal Random Variable

***Step* 1**: Draw a normal curve and shade the area corresponding to the proportion, probability, or percentile.

***Step* 2**: Use Table V to find the z-score that corresponds to the shaded area.

***Step* 3**: Obtain the normal value from the formula $x = \mu + z\sigma$.

Example

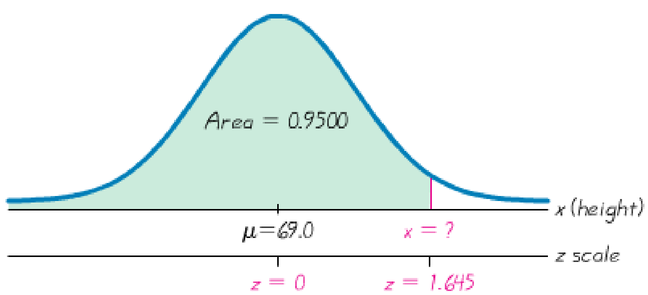
When designing an environment, one common criterion is to use a design that accommodates 95% of the population. How high should doorways be if 95% of men will fit through without bending or bumping their head? That is, find the 95th percentile of heights of men. Heights of men normally distributed with a mean of 69.0 in. and a standard deviation of 2.8 in.

Solution

Given: $\mu = 69.0, \quad \sigma = 2.8$

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449

From the table, we find the areas of 0.9495 and 0.9505. The area 0.95 corresponds to a z-score of 1.645.



$\boxed{x = \mu + z \cdot \sigma = 69.0 + (1.645)(2.8) \; = 73.606}$

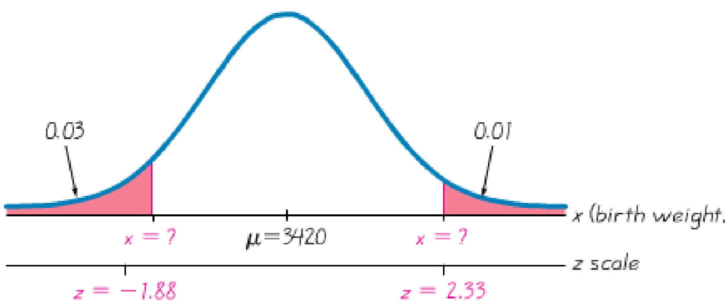
✓ A doorway height of 73.6 in. would allow 95% of men to fit without bending their head.

Example

Hospital wants to redefine the minimum and maximum birth weights that require special treatment because they are unusual low or unusual high. After considering relevant factors, a committee recommends special treatment for birth weights in the lowest 3% and the highest 1%. The committee members soon realize that specific birth weights need to be identified. Help this committee by finding the birth weights that separate the lowest 3% and the highest 1%. Birth weights in the U.S. are normally distributed with a mean of 3420 *g* and a standard deviation of 495 *g*.

Solution

Given: $\mu = 3420, \quad \sigma = 495$



For the leftmost value of x:

The cumulative area from the left is 0.03, from the table: $z = -1.88$

$x = \mu + z \cdot \sigma = 3420.0 + (-1.88)(495) = 2489.4$

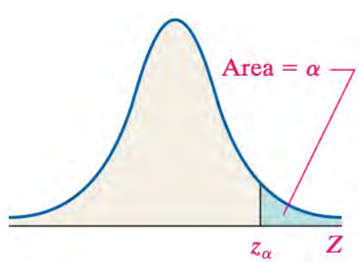
For the rightmost value of x:

The cumulative area from the left is $1 - .01 = .99$, from the table: $z = 2.33$

$x = \mu + z \cdot \sigma = 3420.0 + (2.33)(495) = 4573.35$

✓ The birth weight of 2489 *g* separates the lowest 3% of birth weights, and 4573 *g* separates the lowest 1% of birth weights. The hospital now has well-defined criteria for determining whether a newborn baby should be given special treatment for a birth weight that is unusual lo row high.

The notation z_α (pronounced "*z* sub alpha") is the *z*-score such that the area under the standard normal curve to the right of z_α is α.

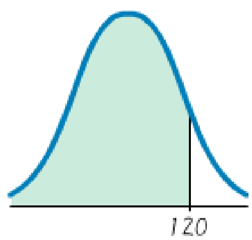


Exercises Section 2.9 – Applications of the Normal Distribution

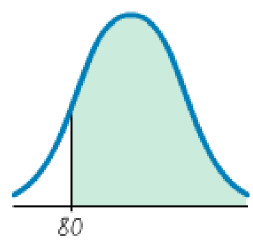
1. The distribution of IQ scores is a nonstandard normal distribution with mean of 100 and standard deviation of 15. What are the values of the mean and standard deviation after all IQ scores have been standardized by converting them to *z*-scores using $z = \dfrac{x-\mu}{\sigma}$?

2. Find the area of the shaded region. The graphs depict IQ scores adults, and those scores are normally distributed with mean of 100 and standard deviation of 15.

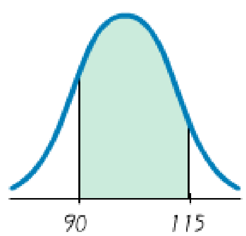
a)



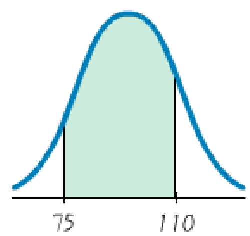
b)



c)

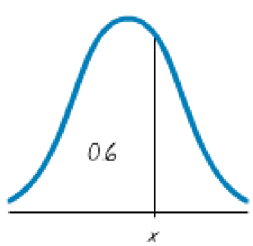


d)

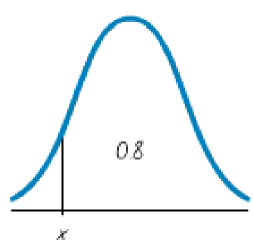


3. Find the Indicated IQ scores. The graphs depict IQ scores adults, and those scores are normally distributed with mean of 100 and standard deviation of 15.

a)



b)



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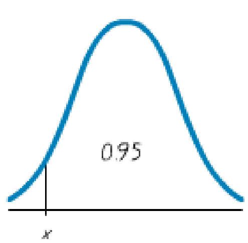

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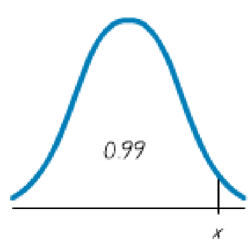

4. Assume that adults have IQ scores that are normally distributed with mean of 100 and standard deviation of 15

a) Find the probability that a randomly selected adult has an IQ that is less than 115.

b) Find the probability that a randomly selected adult has an IQ that is greater than 131.5.

c) Find the probability that a randomly selected adult has an IQ that is between 90 and 110.

d) Find the probability that a randomly selected adult has an IQ that is between 110 and 120.

e) Find P_{30} which is the IQ score separating the bottom 30% from the top 70%.

f) Find the first quartile Q_1 which is the IQ score separating the bottom 25% from the top 75%.

5. The Gulfstream 100 is an executive jet that seats six, and it has a doorway height of 51.6

- ***Men's*** heights are normally distributed with mean 69.0 in. and standard deviation 2.8 in.
- ***Women's*** heights are normally distributed with mean 63.6 in. and standard deviation 2.5 in.

a) What percentage of adult men can fit through the door without bending?

b) What percentage of adult women can fit through the door without bending?

c) Does the door design with a height of 51.6 in. appear to be adequate? Why didn't the engineers design larder door?

d) What doorway height would allow 60% of men to fit without bending?

6. Assume that human body temperatures are normally distributed with a mean of 98.20°F and a standard deviation of 0.62°F.

a) A Hospital uses 100.6°F as the lowest temperature considered to be a fever. What percentage of normal and healthy persons would be considered to have fever? Does this percentage suggest that a cutoff of 100.6°F is appropriate?

b) Physicians want to select a minimum temperature for requiring further medical tests. What should that temperature be, if we want only 5.0% of healthy people to exceed it? (Such a result is a false positive, meaning that the test result is positive, but the subject is not really sick.)

7. The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days.

a) One classical use of the normal distribution is inspired by a letter to "Dear Abby" in which a wife claimed to have given birth 308 days after a brief visit from her husband. Given this information, find the probability of a pregnancy lasting 308 days or longer. What does the result suggest?

b) If we stipulate that a baby is premature if the length of pregnancy is the lowest 4%, find the length that separates premature babies from those who are not premature. Premature babies often require special care, and this result could be helpful to hospital administrators in planning for that care.

8. A statistics professor gives a test and finds that the scores are normally distributed with a mean of 25 and a standard deviation of 5. She plans to curve the scores.

a) If the curves by adding 50 to each grade, what is the new mean? What is the new standard deviation?

b) Is it fair to curve by adding 50 to each grade? Why or why not?

c) If the grades are curved according to the following scheme (instead of adding 50), find the numerical limits for each letter grade.

A: Top 10%

B: Scores above the bottom 70% and below the top 10%.

C: Scores above the bottom 30% and below the top 30%.
D: Scores above the bottom 10% and below the top 70%.
F: Bottom 10%.

d) Which method of curving the grades is fairer: Adding 50 to each grade or using the scheme given in part (*c*)? Explain.