

Solution **Section 4.2 – Matrix operations and Their Applications**

Exercise

Find values for the variables so that the matrices are equal. $\begin{bmatrix} w & x \\ 8 & -12 \end{bmatrix} = \begin{bmatrix} 9 & 17 \\ y & z \end{bmatrix}$

Solution

$$\begin{bmatrix} w & x \\ 8 & -12 \end{bmatrix} = \begin{bmatrix} 9 & 17 \\ y & z \end{bmatrix}$$

$$\Rightarrow \begin{cases} w = 9 & x = 17 \\ y = 8 & z = -12 \end{cases}$$

Exercise

Find values for the variables so that the matrices are equal. $\begin{bmatrix} x & y+3 \\ 2z & 8 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 6 & 8 \end{bmatrix}$

Solution

$$\begin{cases} x = 12 \\ y + 3 = 5 \rightarrow y = 2 \\ 2z = 6 \rightarrow z = 3 \end{cases}$$

Exercise

Find values for the variables so that the matrices are equal. $\begin{bmatrix} 5 & x-4 & 9 \\ 2 & -3 & 8 \\ 6 & 0 & 5 \end{bmatrix} = \begin{bmatrix} y+3 & 2 & 9 \\ z+4 & -3 & 8 \\ 6 & 0 & w \end{bmatrix}$

Solution

$$\begin{bmatrix} 5 = y+3 & x-4 = 2 & 9 = 9 \\ 2 = z+4 & -3 = -3 & 8 = 8 \\ 6 = 6 & 0 = 0 & 5 = w \end{bmatrix}$$

$$\rightarrow \begin{cases} y = 2 & z = -2 \\ x = 6 & w = 5 \end{cases}$$

Exercise

Find values for the variables so that the matrices are equal.

$$\begin{bmatrix} a+2 & 3b & 4c \\ d & 7f & 8 \end{bmatrix} + \begin{bmatrix} -7 & 2b & 6 \\ -3d & -6 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 25 & 6 \\ -8 & 1 & 6 \end{bmatrix}$$

Solution

$$\begin{bmatrix} a-5 & 5b & 4c+6 \\ -2d & 7f-6 & 6 \end{bmatrix} = \begin{bmatrix} 15 & 25 & 6 \\ -8 & 1 & 6 \end{bmatrix}$$

$$\begin{cases} a-5=15 & \rightarrow & a=20 \\ 5b=25 & \rightarrow & b=5 \\ 4c+6=6 & \rightarrow & 4c=0 \rightarrow c=0 \\ -2d=-8 & \rightarrow & d=4 \\ 7f-6=1 & \rightarrow & 7f=7 \rightarrow f=1 \end{cases}$$

Exercise

Find values for the variables so that the matrices are equal.

$$\begin{bmatrix} a+11 & 12z+1 & 5m \\ 11k & 3 & 1 \end{bmatrix} + \begin{bmatrix} 9a & 9z & 4m \\ 12k & 5 & 3 \end{bmatrix} = \begin{bmatrix} 41 & -62 & 72 \\ 92 & 8 & 4 \end{bmatrix}$$

Solution

$$\begin{bmatrix} a+11+9a & 12z+1+9z & 5m+4m \\ 11k+12k & 3+5 & 1+3 \end{bmatrix} = \begin{bmatrix} 41 & -62 & 72 \\ 92 & 8 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 10a+11 & 21z+1 & 9m \\ 23k & 8 & 4 \end{bmatrix} = \begin{bmatrix} 41 & -62 & 72 \\ 92 & 8 & 4 \end{bmatrix}$$

$$10a+11=41 \rightarrow 10a=30$$

$$\underline{a=3}$$

$$21z+1=-62 \rightarrow 21z=-63$$

$$\underline{z=-3}$$

$$9m=72 \rightarrow \underline{m=8}$$

$$23k=92 \rightarrow \underline{k=\frac{92}{23}=4}$$

Exercise

Find values for the variables so that the matrices are equal.

$$\begin{bmatrix} x+2 & 3y+1 & 5z \\ 8w & 2 & 3 \end{bmatrix} + \begin{bmatrix} 3x & 2y & 5z \\ 2w & 5 & -5 \end{bmatrix} = \begin{bmatrix} 10 & -14 & 80 \\ 10 & 7 & -2 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 4x+2 & 5y+1 & 10z \\ 10w & 7 & -2 \end{bmatrix} = \begin{bmatrix} 10 & -14 & 80 \\ 10 & 7 & -2 \end{bmatrix}$$

$$\begin{cases} 4x+2=10 & \rightarrow \underline{x=2} \\ 5y+1=-14 & \rightarrow \underline{y=-3} \\ 10z=80 & \rightarrow \underline{z=8} \\ 10w=10 & \rightarrow \underline{w=1} \end{cases}$$

Exercise

Find values for the variables so that the matrices are equal.

$$\begin{bmatrix} 2x-3 & y-2 & 2z+1 \\ 5 & 2w & 7 \end{bmatrix} + \begin{bmatrix} 3x-3 & y+2 & z-1 \\ -5 & 5w+1 & 3 \end{bmatrix} = \begin{bmatrix} 20 & 8 & 9 \\ 0 & 8 & 10 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 5x-6 & 2y & 3z \\ 0 & 7w+1 & 10 \end{bmatrix} = \begin{bmatrix} 20 & 8 & 9 \\ 0 & 8 & 10 \end{bmatrix}$$

$$\begin{cases} 5x-6=20 & \rightarrow \underline{x=\frac{26}{5}} \\ 2y=8 & \rightarrow \underline{y=4} \\ 3z=9 & \rightarrow \underline{z=3} \\ 7w+1=8 & \rightarrow \underline{w=1} \end{cases}$$

Exercise

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix}$$

Solution

$$\begin{aligned} A - B &= \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 4 & -5 \\ 2 & 1 & 9 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 3A + 2B &= 3 \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix} + 2 \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix} \\
 &= \begin{bmatrix} 9 & 3 & 3 \\ -3 & 6 & 15 \end{bmatrix} + \begin{bmatrix} 4 & -6 & 12 \\ -6 & 2 & -8 \end{bmatrix} \\
 &= \begin{bmatrix} 13 & -3 & 15 \\ -9 & 8 & 7 \end{bmatrix}
 \end{aligned}$$

Exercise

Given $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $F = \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$ Find $3F + 2A$

Solution

$$\begin{aligned}
 3F + 2A &= 3 \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 3(3) & 3(3) \\ 3(-1) & 3(-1) \end{bmatrix} + \begin{bmatrix} 2(1) & 2(2) \\ 2(4) & 2(3) \end{bmatrix} \\
 &= \begin{bmatrix} 9 & 9 \\ -3 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} 9+2 & 9+4 \\ -3+8 & -3+6 \end{bmatrix} \\
 &= \begin{bmatrix} 11 & 13 \\ 5 & 3 \end{bmatrix}
 \end{aligned}$$

Exercise

Evaluate $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 12 \end{bmatrix}$

Solution

$$\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 12 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \\ 20 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} 5 & 8 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 & 1 \\ 4 & 2 & 5 \end{bmatrix}$

Solution

$$\begin{bmatrix} 5 & 8 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 & 1 \\ 4 & 2 & 5 \end{bmatrix}$$

It is **impossible**; 2×2 and 2×3 are not the same size.

Exercise

Evaluate $\begin{bmatrix} -5 & 0 \\ 4 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix}$

Solution

$$\begin{bmatrix} -5 & 0 \\ 4 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -5 + 6 & 0 + (-3) \\ 4 + 2 & \frac{1}{2} + 3 \end{bmatrix} \\ = \begin{bmatrix} 1 & -3 \\ 6 & \frac{7}{2} \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix}$

Solution

$$\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 5 - 4 & -6 + 6 \\ 8 + 8 & 9 - 3 \end{bmatrix} \\ = \begin{bmatrix} 1 & 0 \\ 16 & 6 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} -5 & 6 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 5 & -8 \end{bmatrix}$

Solution

$$\begin{bmatrix} -5 & 6 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 5 & -8 \end{bmatrix} = \begin{bmatrix} -5 - (-3) & 6 - 2 \\ 2 - 5 & 4 - (-8) \end{bmatrix} \\ = \begin{bmatrix} -2 & 4 \\ -3 & 12 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} 8 & 6 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -8 \end{bmatrix}$

Solution

$$\begin{bmatrix} 8 & 6 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -8 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 4 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$

Solution

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(4) + 3(1) & 1(6) + 3(0) \\ 2(4) + 5(1) & 2(6) + 5(0) \end{bmatrix} \\ = \begin{bmatrix} 7 & 6 \\ 13 & 12 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} -3 & 4 & 2 \\ 5 & 0 & 4 \end{bmatrix} \begin{bmatrix} -6 & 4 \\ 2 & 3 \\ 3 & -2 \end{bmatrix}$

Solution

$$\begin{bmatrix} -3 & 4 & 2 \\ 5 & 0 & 4 \end{bmatrix} \begin{bmatrix} -6 & 4 \\ 2 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} -3(-6) + 4(2) + 2(3) & -3(4) + 4(3) + 2(-2) \\ 5(-6) + 0(2) + 4(3) & 5(4) + 0(3) + 4(-2) \end{bmatrix} \\ = \begin{bmatrix} 32 & -4 \\ -18 & 12 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

Solution

$$\begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1(1) - 1(1) + 4(1) & 1(1) - 1(2) + 4(-1) & 1(0) - 1(4) + 4(3) \\ 4(1) - 1(1) + 3(1) & 4(1) - 1(2) + 3(-1) & 4(0) - 1(4) + 3(3) \\ 2(1) + 0(1) - 2(1) & 2(1) + 0(2) - 2(-1) & 2(0) + 0(4) - 2(3) \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -5 & 8 \\ 6 & -1 & 5 \\ 0 & 4 & -6 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}$

Solution

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1(1) + 1(4) + 0(2) & 1(-1) + 1(-1) + 0(0) & 1(4) + 1(3) + 0(-2) \\ 1(1) + 2(4) + 4(2) & 1(-1) + 2(-1) + 4(0) & 1(4) + 2(3) + 4(-2) \\ 1(1) - 1(4) + 3(2) & 1(-1) - 1(-1) + 3(0) & 1(4) - 1(3) + 3(-2) \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -2 & 7 \\ 17 & -3 & 2 \\ 3 & 0 & -5 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} -2 & -3 & -4 \\ 2 & -1 & 0 \\ 4 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix}$

Solution

$$\begin{bmatrix} -2 & -3 & -4 \\ 2 & -1 & 0 \\ 4 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -3 - 12 & -2 - 6 - 8 & -8 + 3 + 8 \\ -1 & 2 - 2 & 8 + 1 \\ -2 + 9 & 4 - 4 + 6 & 16 + 2 - 6 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & -16 & 3 \\ -1 & 0 & 9 \\ 7 & 6 & 12 \end{bmatrix}$$

Exercise

Evaluate $\begin{bmatrix} \sqrt{2} & \sqrt{2} & -\sqrt{18} \\ \sqrt{3} & \sqrt{27} & 0 \end{bmatrix} \begin{bmatrix} 8 & -10 \\ 9 & 12 \\ 0 & 2 \end{bmatrix}$

Solution

$$\begin{bmatrix} \sqrt{2} & \sqrt{2} & -\sqrt{18} \\ \sqrt{3} & \sqrt{27} & 0 \end{bmatrix} \begin{bmatrix} 8 & -10 \\ 9 & 12 \\ 0 & 2 \end{bmatrix} = \begin{pmatrix} 17\sqrt{2} & -4\sqrt{2} \\ 35\sqrt{3} & 26\sqrt{3} \end{pmatrix}$$

Exercise

Evaluate $\begin{bmatrix} x & 2x+1 & 4 \\ 5 & x-1 & 8 \\ -2 & 3x & 2x+1 \end{bmatrix} + \begin{bmatrix} 2x-1 & -2x-1 & 4x \\ -5 & 6 & x+1 \\ -5 & 2 & -2x \end{bmatrix}$

Solution

$$\begin{bmatrix} x & 2x+1 & 4 \\ 5 & x-1 & 8 \\ -2 & 3x & 2x+1 \end{bmatrix} + \begin{bmatrix} 2x-1 & -2x-1 & 4x \\ -5 & 6 & x+1 \\ -5 & 2 & -2x \end{bmatrix} = \begin{bmatrix} 3x-1 & 0 & 4x+4 \\ 0 & x+5 & x+9 \\ -7 & 3x+2 & 1 \end{bmatrix}$$

Exercise

Given $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 7 \\ 0 & 2 \end{bmatrix}$. Find AB and BA .

Solution

$$AB = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} -2 & 7 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 13 \\ 4 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 7 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -16 & 29 \\ -4 & 10 \end{bmatrix}$$

Note: $AB \neq BA$

Exercise

Given $A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned} AB &= \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -6 & 17 \\ 6 & -8 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 14 \\ 1 & -20 \end{pmatrix} \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix}$ $B = \begin{pmatrix} -1 & -1 \\ 0 & 4 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned} AB &= \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & 4 \end{pmatrix} \\ &= \begin{pmatrix} -3 & -11 \\ 4 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} -1 & -1 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -7 & 1 \\ 16 & 4 \end{pmatrix} \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 1 \\ 2 & -3 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned} AB &= \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 2 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 6 \\ 14 & -7 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{pmatrix} 4 & 1 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \\
 &= \begin{pmatrix} 14 & -1 \\ 0 & -11 \end{pmatrix}
 \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned}
 AB &= \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} -4 & 2 \\ 4 & -4 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix} \\
 &= \begin{pmatrix} 4 & -4 \\ 14 & -12 \end{pmatrix}
 \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & -2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned}
 AB &= \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & -4 & 5 \\ 6 & 0 & 3 \\ -3 & -2 & 1 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & -2 \end{pmatrix} \\
 &= \begin{pmatrix} 5 & -13 \\ 3 & -4 \end{pmatrix}
 \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned} AB &= \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 5 & 4 \\ 2 & -3 & 8 \\ -3 & 8 & -5 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & 1 \\ -3 & 2 \end{pmatrix} \\ &= \begin{pmatrix} -14 & 7 \\ -4 & 5 \end{pmatrix} \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ -3 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 0 & -2 \\ -2 & 6 & 2 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned} AB &= \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 3 & 0 & -2 \\ -2 & 6 & 2 \end{pmatrix} \\ &= \begin{pmatrix} -2 & 24 & 4 \\ 2 & -6 & -2 \\ -13 & 12 & 10 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} 3 & 0 & -2 \\ -2 & 6 & 2 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ -3 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 12 & 8 \\ -10 & 10 \end{pmatrix} \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned} AB &= \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 11 & -2 & -6 \\ 0 & -1 & 2 \\ 5 & -3 & -4 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix} \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 6 & -3 & 9 \\ 2 & -3 & 4 \\ 4 & -6 & 3 \end{pmatrix} \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} -1 & 2 & 0 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$. Find AB and BA .

Solution

$$\begin{aligned} AB &= \begin{pmatrix} -1 & 2 & 0 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 8 & -2 \\ 3 & -8 & 4 \\ -2 & 13 & -5 \end{pmatrix} \end{aligned}$$

$$BA = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} -1 & 2 & 0 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 5 & -1 \\ 11 & -5 & 6 \\ -8 & 7 & -4 \end{pmatrix}$$

Exercise

Given $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & -2 & -1 \end{pmatrix}$ $B = \begin{pmatrix} -3 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix}$. Find AB and BA .

Solution

$$AB = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & -2 & -1 \end{pmatrix} \begin{pmatrix} -3 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -7 & 2 \\ -6 & 2 & 2 \\ -8 & -6 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} -3 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & -2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 6 & 1 \\ 7 & 0 & 5 \\ 4 & -4 & -2 \end{pmatrix}$$

Exercise

Given $A = \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}$, Find

a) $A + B$

b) $A - B$

c) $3A$

d) $-2B$

e) $2A + 3B$

f) A^2

g) AB

h) BA

Solution

a) $A + B = \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 5 \\ 3 & -5 \\ 2 & -4 \end{bmatrix}$$

$$\begin{aligned} b) \quad A - B &= \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -7 & 3 \\ 1 & -1 \\ -4 & 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} c) \quad 3A &= 3 \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -9 & 12 \\ 6 & -9 \\ -3 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} d) \quad -2B &= -2 \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -8 & -2 \\ -2 & 4 \\ -6 & 8 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} e) \quad 2A + 3B &= 2 \begin{bmatrix} -3 & 4 \\ 2 & -3 \\ -1 & 0 \end{bmatrix} + 3 \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ 3 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -6 & 8 \\ 4 & -6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 12 & 3 \\ 3 & -6 \\ 9 & -12 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 11 \\ 7 & -12 \\ 7 & -12 \end{bmatrix} \end{aligned}$$

$$f) \quad A^2 = \text{doesn't exist} \quad (\text{not a square matrix})$$

$$g) \quad AB = \cancel{\exists} \quad (2 \times 3 \quad 2 \times 3) \text{ the inner not equal}$$

$$h) \quad BA = \cancel{\exists} \quad (2 \times 3 \quad 2 \times 3) \text{ the inner not equal}$$

Exercise

Given $A = \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix}$, Find

a) $A + B$

c) $3A$

e) $2A + 3B$

g) AB

b) $A - B$

d) $-2B$

f) A^2

h) BA

Solution

$$\begin{aligned} \text{a) } A + B &= \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 6 \\ 5 & 2 \\ -3 & 3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{b) } A - B &= \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -10 \\ 1 & 6 \\ 5 & -3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{c) } 3A &= 3 \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 6 & -6 \\ 9 & 12 \\ 3 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{d) } -2B &= -2 \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -16 \\ -4 & 4 \\ 8 & -6 \end{bmatrix} \end{aligned}$$

$$\text{e) } 2A + 3B = 2 \begin{bmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix} + 3 \begin{bmatrix} -1 & 8 \\ 2 & -2 \\ -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ 6 & 8 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 24 \\ 6 & -6 \\ -12 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 20 \\ 12 & 2 \\ -10 & 9 \end{bmatrix}$$

f) $A^2 = \text{doesn't exist}$ (not a square matrix)

g) $AB = \nexists$ (2×3 2×3) the inner not equal

h) $BA = \nexists$ (2×3 2×3) the inner not equal

Exercise

Given $A = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$, Find

a) $A + B$

c) $3A$

e) $2A + 3B$

g) AB

b) $A - B$

d) $-2B$

f) A^2

h) BA

Solution

a) $A + B = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 1 & -1 \\ 2 & 2 & 1 \\ -1 & 2 & 5 \end{bmatrix}$$

b) $A - B = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 5 & -1 \\ -2 & -4 & 3 \\ -7 & 4 & 1 \end{bmatrix}$$

c) $3A = 3 \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -6 & 9 & -3 \\ 0 & -3 & 6 \\ -12 & 9 & 9 \end{bmatrix}$$

$$\begin{aligned} d) \quad -2B &= -2 \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 4 & 0 \\ -4 & -6 & 2 \\ -6 & 2 & -4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} e) \quad 2A + 3B &= 2 \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} + 3 \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -4 & 6 & -2 \\ 0 & -2 & 4 \\ -8 & 6 & 6 \end{bmatrix} + \begin{bmatrix} 3 & -6 & 0 \\ 6 & 9 & -3 \\ 9 & -3 & 6 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & -2 \\ 6 & 7 & 1 \\ 1 & 3 & 12 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} f) \quad A^2 &= \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 4+4 & -6-3-3 & 2+6-3 \\ -8 & 1+6 & -2+6 \\ 8-12 & -12-3+9 & 4+6+9 \end{bmatrix} \\ &= \begin{bmatrix} 8 & -12 & 5 \\ -8 & 7 & 4 \\ -4 & -6 & 19 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} g) \quad AB &= \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -2+6-3 & 4+9+1 & -3-2 \\ -2+6 & -3-2 & 1+4 \\ -4+6+9 & 8+9-3 & -3+6 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 1 & 14 & -5 \\ 4 & -5 & 5 \\ 11 & 14 & 3 \end{bmatrix}$$

$$\begin{aligned} h) \quad BA &= \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & 2 \\ -4 & 3 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 3+2 & -1-4 \\ -4+4 & 6-3-3 & -2+6-3 \\ -6-8 & 9+1+6 & -3-2+6 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 5 & -5 \\ 0 & 0 & 1 \\ -14 & 16 & 1 \end{bmatrix} \end{aligned}$$

Exercise

Given $A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix}$, Find

a) $A + B$

c) $3A$

e) $2A + 3B$

g) AB

b) $A - B$

d) $-2B$

f) A^2

h) BA

Solution

$$\begin{aligned} a) \quad A + B &= \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 4 & 4 \\ 4 & 0 & 1 \\ 1 & 8 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} b) \quad A - B &= \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & -4 \\ -2 & -6 & 5 \\ 9 & 0 & -5 \end{bmatrix} \end{aligned}$$

$$c) \quad 3A = 3 \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 6 & 0 \\ 3 & -9 & 9 \\ 15 & 12 & -6 \end{bmatrix}$$

$$\begin{aligned} d) \quad -2B &= -2 \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -4 & -8 \\ -6 & -6 & 4 \\ 8 & -8 & -6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} e) \quad 2A + 3B &= 2 \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} + 3 \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 4 & 0 \\ 2 & -6 & 6 \\ 10 & 8 & -4 \end{bmatrix} + \begin{bmatrix} -3 & 6 & 12 \\ 9 & 9 & -6 \\ -12 & 12 & 9 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 10 & 12 \\ 11 & 3 & 0 \\ -2 & 20 & 5 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} f) \quad A^2 &= \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -6 & 6 \\ -3+15 & 2+9+12 & -9-6 \\ 4-10 & 10-12-8 & 12+4 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -6 & 6 \\ 12 & 23 & -15 \\ -6 & -10 & 16 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} g) \quad AB &= \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 6 & -4 \\ -1-9-12 & 2-9+12 & 4+6+9 \\ -5+12+8 & 10+12-8 & 20-8-6 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 6 & 6 & -4 \\ -22 & 5 & 19 \\ 15 & 14 & 6 \end{bmatrix}$$

$$\begin{aligned} h) \quad BA &= \begin{bmatrix} -1 & 2 & 4 \\ 3 & 3 & -2 \\ -4 & 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 1 & -3 & 3 \\ 5 & 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2+10 & -2-6+16 & 6-8 \\ 3-10 & 6-9-8 & 9+4 \\ 4+15 & -8-12+12 & 12-6 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 8 & -2 \\ -7 & -11 & 13 \\ 19 & -8 & 6 \end{bmatrix} \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ $C = \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix}$ $D = \begin{pmatrix} -2 & 3 \\ 2 & -1 \\ 3 & 2 \end{pmatrix}$, Find

a) $4A - 2B$

d) $2A - 3B$

g) A^2

j) CA

b) $3A + C$

e) AB

h) B^3

k) CD

c) $3A + B$

f) BA

i) AC

l) DC

Solution

$$\begin{aligned} a) \quad 4A - 2B &= 4 \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} - 2 \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -4 & 8 \\ -8 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -4 \\ 4 & -2 \end{pmatrix} \\ &= \begin{pmatrix} -6 & 12 \\ -12 & 6 \end{pmatrix} \end{aligned}$$

b) $3A + C = \text{not possible}$

They are not the same order.

$$\begin{aligned} c) \quad 3A + B &= 3 \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -3 & 6 \\ -6 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -2 & 4 \\ -4 & 2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
 d) \quad 2A - 3B &= 2 \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} - 3 \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} -2 & 4 \\ -4 & 2 \end{pmatrix} - \begin{pmatrix} 3 & -6 \\ 6 & -3 \end{pmatrix} \\
 &= \begin{pmatrix} -5 & 10 \\ -10 & 5 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad AB &= \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad BA &= \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 g) \quad A^2 &= \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 h) \quad B^3 &= \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} -3 & 6 \\ -6 & 3 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 i) \quad AC &= \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix} & 2 \times 2 \quad 2 \times 3 \quad \rightarrow 2 \times 3 \\
 &= \begin{pmatrix} -6 & 1 & 0 \\ -9 & -4 & -3 \end{pmatrix}
 \end{aligned}$$

$$j) \quad CB = \text{not defined} \quad 2 \times 3 \quad 2 \times 2$$

C and B are not the same order.

$$\begin{aligned}
 k) \quad CD &= \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 2 & -1 \\ 3 & 2 \end{pmatrix} & 2 \times 3 \quad 3 \times 2 \quad \rightarrow 2 \times 2 \\
 &= \begin{pmatrix} -8+6+6 & 12-3+4 \\ 2+4+3 & -3+2+2 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{pmatrix} 4 & 13 \\ 9 & 1 \end{pmatrix} \\
 l) \quad DC &= \begin{pmatrix} -2 & 3 \\ 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix} \quad 3 \times 2 \quad 2 \times 3 \rightarrow 3 \times 3 \\
 &= \begin{pmatrix} -8-3 & -6+6 & -4+3 \\ 8+1 & 6-2 & 4-1 \\ 12-2 & 9+4 & 6+2 \end{pmatrix} \\
 &= \begin{pmatrix} -11 & 0 & -1 \\ 9 & 4 & 3 \\ 10 & 13 & 8 \end{pmatrix}
 \end{aligned}$$

Exercise

Given $A = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix}$ $D = \begin{pmatrix} 2 & 4 & -2 \\ 0 & 3 & 5 \\ -3 & 1 & 1 \end{pmatrix}$, Find

a) $4A - 2B$

d) $2A - 3B$

g) A^2

j) CB

b) $3A + C$

e) AB

h) B^3

k) CD

c) $3A + B$

f) BA

i) AC

l) DC

Solution

$$\begin{aligned}
 a) \quad 4A - 2B &= 4 \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} - 2 \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} 8 & 16 \\ 12 & -4 \end{pmatrix} - \begin{pmatrix} -2 & 6 \\ 4 & -2 \end{pmatrix} \\
 &= \begin{pmatrix} 10 & 10 \\ 8 & -2 \end{pmatrix}
 \end{aligned}$$

b) $3A + C = \text{✗}$

They are not the same order.

$$\begin{aligned}
 c) \quad 3A + B &= 3 \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} 6 & 12 \\ 9 & -3 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} 5 & 15 \\ 11 & -4 \end{pmatrix}
 \end{aligned}$$

$$d) \quad 2A - 3B = 2 \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} - 3 \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 8 \\ 6 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 9 \\ 6 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -1 \\ 0 & 1 \end{pmatrix}$$

$$e) \quad AB = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 2 \\ -5 & 10 \end{pmatrix}$$

$$f) \quad BA = \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -7 \\ 1 & 9 \end{pmatrix}$$

$$g) \quad A^2 = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 4 \\ 3 & 13 \end{pmatrix}$$

$$h) \quad B^3 = \left(\begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix} \right) \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -6 \\ -4 & 7 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -19 & 27 \\ 18 & -19 \end{pmatrix}$$

$$i) \quad AC = \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix} \quad 2 \times 2 \quad 2 \times 3 \rightarrow 2 \times 3$$

$$= \begin{pmatrix} -6 & 1 & 0 \\ -9 & -4 & -3 \end{pmatrix}$$

$$j) \quad CB = \text{not possible} \quad 2 \times 3 \quad 2 \times 2$$

C and B are not the same order.

$$k) \quad CD = \begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 4 & -2 \\ 0 & 3 & 5 \\ -3 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -12 & 21 & 13 \\ -16 & 5 & 23 \\ 4 & -6 & 0 \end{pmatrix}$$

$$\begin{aligned}
 l) \quad DC &= \begin{pmatrix} 2 & 4 & -2 \\ 0 & 3 & 5 \\ -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ -1 & 0 & -2 \end{pmatrix} \\
 &= \begin{pmatrix} 2-8+2 & 8+12 & 10+16+4 \\ -6-5 & 9 & 12-10 \\ -3-2-1 & -12+3 & -15+4-2 \end{pmatrix} \\
 &= \begin{pmatrix} -4 & 20 & 30 \\ -11 & 9 & 2 \\ -6 & -9 & -12 \end{pmatrix}
 \end{aligned}$$

Exercise

A contractor builds three kinds of houses, models *A*, *B*, and *C*, with a choice of two styles, Spanish and contemporary. Matrix *P* shows the number of each kind of house planned for a new 100-home subdivision. The amounts for each of the exterior materials depend primarily on the style of the house. These amounts are shown in matrix *Q*. (concrete is in cubic yards, lumber in units of 1000 board feet, brick in 1000s, and shingles in units of 100 ft^2 .) Matrix *R* gives the cost in dollars for each kind of material.

- What is the total cost of these materials for each model?
- How much of each of four kinds of material must be ordered?
- What is the total cost for exterior materials?

Solution

$$\begin{array}{cc}
 \text{Spanish} & \text{Contemporary} \\
 \text{Model A} & \begin{bmatrix} 0 & 30 \end{bmatrix} \\
 \text{Model B} & \begin{bmatrix} 10 & 20 \end{bmatrix} \\
 \text{Model C} & \begin{bmatrix} 20 & 20 \end{bmatrix}
 \end{array} = P$$

$$\begin{array}{cc}
 \text{Concrete} & \text{Lumber} & \text{Brick} & \text{Shingles} \\
 \text{Spanish} & \begin{bmatrix} 10 & 2 & 0 & 2 \end{bmatrix} \\
 \text{Contemporary} & \begin{bmatrix} 50 & 1 & 20 & 2 \end{bmatrix}
 \end{array} = Q$$

$$\begin{array}{cc}
 \text{Cost per unit} \\
 \text{Concrete} & \begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix} \\
 \text{Lumber} & \\
 \text{Brick} & \\
 \text{Shingles} &
 \end{array} = R$$

- What is the total cost of these materials for each model?

$$PQ = \begin{bmatrix} 0 & 30 \\ 10 & 20 \\ 20 & 20 \end{bmatrix} \begin{bmatrix} 10 & 2 & 0 & 2 \\ 50 & 1 & 20 & 2 \end{bmatrix}$$

	<i>Concrete</i>	<i>Lumber</i>	<i>Brick</i>	<i>Shingles</i>	
$=$	$\begin{bmatrix} 1500 \\ 100 \\ 1200 \end{bmatrix}$	$\begin{bmatrix} 30 \\ 40 \\ 60 \end{bmatrix}$	$\begin{bmatrix} 600 \\ 400 \\ 400 \end{bmatrix}$	$\begin{bmatrix} 60 \\ 60 \\ 80 \end{bmatrix}$	<i>Model A</i> <i>Model B</i> <i>Model C</i>

$$(PQ)R = \begin{bmatrix} 1500 & 30 & 600 & 60 \\ 100 & 40 & 400 & 60 \\ 1200 & 60 & 400 & 80 \end{bmatrix} \begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix}$$

$$= \begin{bmatrix} 72,900 \\ 54,700 \\ 60,800 \end{bmatrix} \begin{matrix} \textit{Model A} \\ \textit{Model B} \\ \textit{Model C} \end{matrix}$$

The total cost of materials is \$72,900 for model A, \$54,700 for model B, \$60,800 for model C.

b) How much of each of four kinds of material must be ordered

$$\begin{array}{cccc} \begin{bmatrix} 1500 & 30 & 600 & 60 \\ 100 & 40 & 400 & 60 \\ 1200 & 60 & 400 & 80 \end{bmatrix} & & & \\ \hline 3800 & 130 & 1400 & 200 \end{array}$$

$$T = [3800 \quad 130 \quad 1400 \quad 200]$$

3800 yd^3 of concrete, 130,000 board feet of lumber, 1,400,000 bricks, and 20,000 ft^2 of shingles are needed.

c) What is the total cost for exterior materials?

$$TR = [3800 \quad 130 \quad 1400 \quad 200] \begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix}$$

$$= [188,400]$$

The total cost for exterior materials is \$188,400

Exercise

Mitchell Fabricators manufactures three styles of bicycle frames in its two plants. The following table shows the number of each style produced at each plant

	<i>Mountain Bike</i>	<i>Racing Bike</i>	<i>Touring Bike</i>
<i>North Plant</i>	150	120	100
<i>South Plant</i>	180	90	130

- Write a 2×3 matrix A that represents the information in the table
- The manufacturer increased production of each style by 20%. Find a Matrix M that represents the increased production figures.
- Find the matrix $A + M$ and tell what it represents

Solution

$$a) \quad A = \begin{bmatrix} 150 & 120 & 100 \\ 180 & 90 & 130 \end{bmatrix}$$

- b) The 20% production will represent

$$A + 20\%(A)$$

$$\rightarrow A + .2A = 1.2A$$

$$M = (1.2) \begin{bmatrix} 150 & 120 & 100 \\ 180 & 90 & 130 \end{bmatrix}$$
$$= \begin{bmatrix} 180 & 144 & 120 \\ 216 & 108 & 156 \end{bmatrix}$$

$$c) \quad A + M = \begin{bmatrix} 150 & 120 & 100 \\ 180 & 90 & 130 \end{bmatrix} + \begin{bmatrix} 180 & 144 & 120 \\ 216 & 108 & 156 \end{bmatrix}$$
$$= \begin{bmatrix} 330 & 264 & 220 \\ 396 & 198 & 286 \end{bmatrix}$$

The matrix $A + M$ represents the total production of each style at each plant for the time period (2 months)

Exercise

Sal's Shoes and Fred's Footwear both have outlets in California and Arizona. Sal's sells shoes for \$80, sandals for \$40, and boots for \$120. Fred's prices are \$60, \$30, and \$150 for shoes, sandals and boots, respectively. Half of all sales in California stores are shoes, 1/4 are *sandals*, and 1/4 are *boots*. In Arizona the fractions are 1/5 *shoes*, 1/5 are *sandals*, and 3/5 are *boots*.

- Write a 2×3 matrix called P representing prices for the two stores and three types of footwear.
- Write a 2×3 matrix called F representing fraction of each type of footwear sold in each state.
- Only one of the two products PF and FP is meaningful. Determine which one it is, calculate the product, and describe what the entries represent.

Solution

- Write a 2×3 matrix called P representing prices for the two stores and three types of footwear.

$$P = \begin{bmatrix} 80 & 40 & 120 \\ 60 & 30 & 150 \end{bmatrix} \quad \begin{array}{l} \text{Sal's} \\ \text{Fred's} \end{array}$$

- Write a 2×3 matrix called F representing fraction of each type of footwear sold in each state.

$$F = \begin{array}{cc} \text{CA} & \text{AR} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{3}{5} \end{bmatrix} \end{array}$$

$$\begin{aligned} \text{c) } PF &= \begin{bmatrix} 80 & 40 & 120 \\ 60 & 30 & 150 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{3}{5} \end{bmatrix} \\ &= \begin{bmatrix} 80\frac{1}{2} + 40\frac{1}{4} + 120\frac{1}{4} & 80\frac{1}{5} + 40\frac{1}{5} + 120\frac{3}{5} \\ 60\frac{1}{2} + 30\frac{1}{4} + 150\frac{1}{4} & 60\frac{1}{5} + 30\frac{1}{5} + 150\frac{3}{5} \end{bmatrix} \\ &= \begin{bmatrix} 80 & 96 \\ 75 & 108 \end{bmatrix} \end{aligned}$$