

$$\sin A = \frac{a}{c}$$

$$\cos A = \frac{b}{c}$$

$$\tan A = \frac{a}{b}$$

$$\sec A = \frac{c}{a}$$

$$\csc A = \frac{c}{b}$$

$$\cot A = \frac{b}{a}$$

$$\sin B = \frac{b}{c}$$

$$\cos B = \frac{a}{c}$$

$$\tan B = \frac{b}{a}$$

$$\csc B = \frac{c}{b}$$

$$\sec B = \frac{c}{a}$$

$$\cot B = \frac{a}{b}$$

Sine
cosine } hyp + side

tangent 2 sides
 $\frac{\text{opp}}{\text{adj}}$ (No hyp)

Ex.

$$C = 90^\circ \quad A = 40^\circ \quad C = 12$$

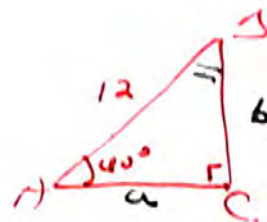
$$B = 90^\circ - 40^\circ = 50^\circ$$

$$\cos 40^\circ = \frac{a}{12}$$

$$a = 12 \cos 40^\circ \approx 9.2$$

$$\sin 40^\circ = \frac{b}{12}$$

$$b = 12 \sin 40^\circ$$

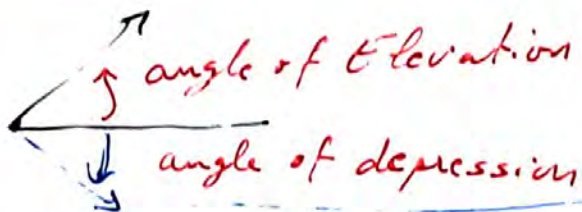
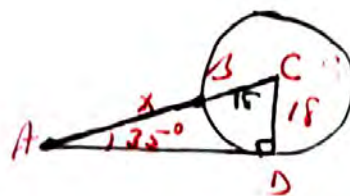


Ex.

$$\sin 35^\circ = \frac{18}{x+18}$$

$$x+18 = \frac{18}{\sin 35^\circ}$$

$$x = \frac{18}{\sin 35^\circ} - 18$$



Ex

$$\sin 52.6^\circ = \frac{h}{213}$$

$$h = 213 \sin 52.6^\circ$$



Ex. $\triangle ABC$ & $\triangle DCB$ $\angle B = h$

$\triangle ABC$:

$$\tan 36.7^\circ = \frac{h}{x}$$

$$\underline{h = x \tan 36.7^\circ} \quad (1)$$

$\triangle DCB$: $\tan 22.2^\circ = \frac{h}{x+50}$

$$\underline{h = (x+50) \tan 22.2^\circ} \quad (2)$$

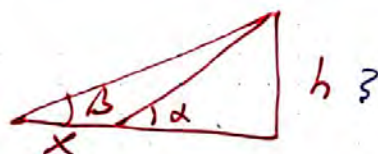
$$(1) = (2) \quad x \tan 36.7^\circ = (x+50) \tan 22.2^\circ$$

$$x(\tan 36.7^\circ - \tan 22.2^\circ) = 50 \tan 22.2^\circ$$

$$x = \frac{50 \tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}$$

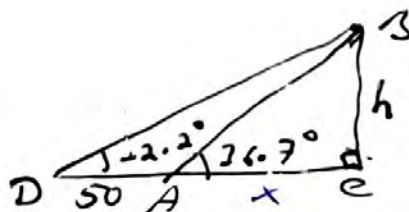
$$\underline{h = \frac{50 \tan 22.2^\circ \cdot \tan 36.7^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}}$$

(-)



$$h = \frac{x \tan \alpha \cdot \tan \beta}{\tan \alpha - \tan \beta}$$

\downarrow
 large small
 angle



Ex 20



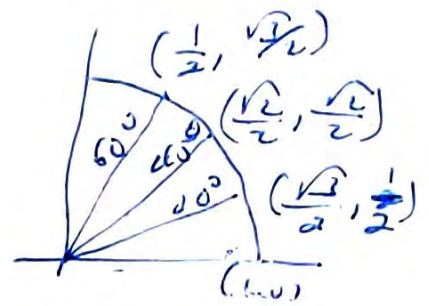
$$h = \frac{100 \tan 30^\circ \tan 60^\circ}{\tan 60^\circ - \tan 30^\circ}$$

$$= \frac{100 \left(\frac{1}{\sqrt{3}}\right) (\sqrt{3})}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$= \frac{100}{\frac{3-1}{\sqrt{3}}}$$

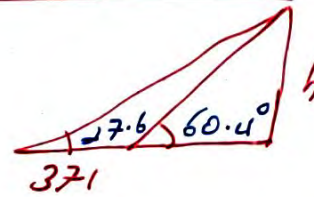
$$= 100 \cdot \frac{\sqrt{3}}{2}$$

$$= 50\sqrt{3}$$



Ex 17

$$h = \frac{371 \tan 60.4^\circ \tan 27.6^\circ}{\tan 60.4^\circ - \tan 27.6^\circ}$$



Ex

$$|AB| = 25$$

$$y \approx 3.5$$

$$\Delta AFC: \tan 13^\circ = \frac{y}{x} \quad (1)$$

$$\Delta FBC: \tan 19^\circ = \frac{y}{25-x} \quad (2)$$

$$(1) \rightarrow y = x \tan 13^\circ \quad (3)$$

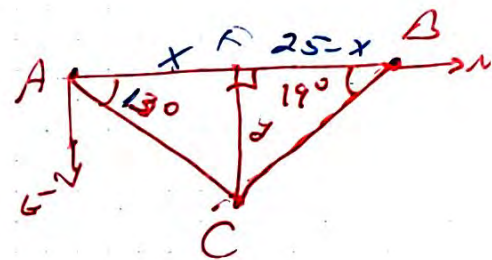
$$(2) \rightarrow y = (25-x) \tan 19^\circ \quad (4)$$

$$x \tan 13^\circ = 25 \tan 19^\circ - x \tan 19^\circ$$

$$x (\tan 13^\circ + \tan 19^\circ) = 25 \tan 19^\circ$$

$$x = \frac{25 \tan 19^\circ}{\tan 13^\circ + \tan 19^\circ}$$

$$y = \frac{25 \tan 19^\circ}{\tan 13^\circ + \tan 19^\circ} \cdot \tan 13^\circ$$



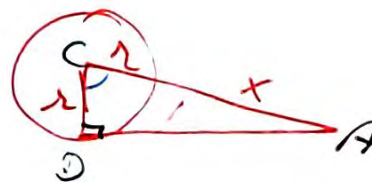
#6 $C = 30^\circ$ $r = 15$ $x??$

$$\cos 30^\circ = \frac{15}{x+15} = \frac{\sqrt{3}}{2}$$

$$30 = \sqrt{3}(x+15)$$

$$x+15 = \frac{30}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} >$$

$$x = 10\sqrt{3} - 15$$



$$\frac{30\sqrt{3}}{3}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

#10 $\tan 36^\circ = \frac{h}{67}$

$$h = 67 \tan 36^\circ$$

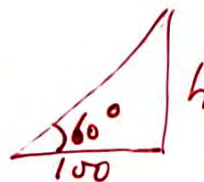


#11

$$\tan 60^\circ = \frac{h}{100}$$

$$h = 100 \tan 60^\circ$$

$$h = 100\sqrt{3}$$



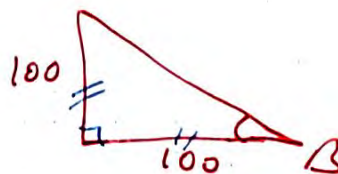
$$\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{3}}{2}$$

#20

$$\tan B = \frac{100}{100} = 1$$

$$B = 45^\circ$$



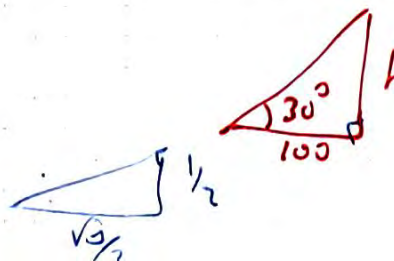
$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

#37

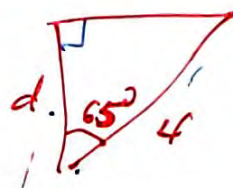
$$\tan 30^\circ = \frac{h}{100}$$

$$h = 100 \cdot \frac{1}{\sqrt{3}}$$

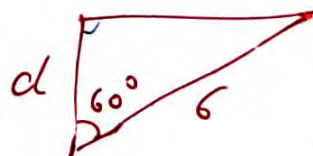
$$= \frac{100}{\sqrt{3}} \text{ or } \frac{100\sqrt{3}}{3}$$



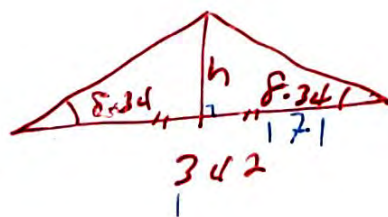
39/ $\cos 65^\circ = \frac{d}{4}$
 $d = 4 \cos 65^\circ$



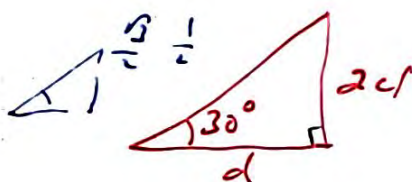
40/ $\cos 60^\circ = \frac{d}{6}$
 $d = 6 \cdot \frac{1}{2}$
 $= 3 \text{ mi}$



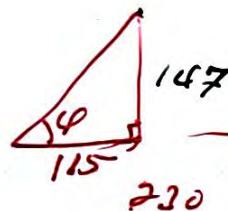
41/ $\tan 8.34^\circ = \frac{h}{171}$
 $h = 171 \tan 8.34^\circ$



40/ $\tan 30^\circ = \frac{2d}{d}$
 $d = \frac{2d}{\frac{1}{\sqrt{3}}}$
 $= 2d\sqrt{3} \text{ in}$

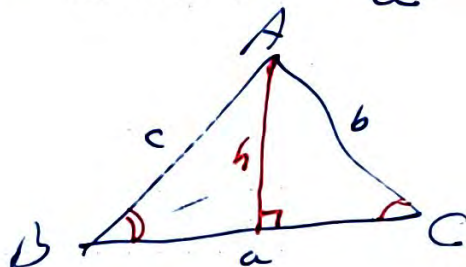


81/ $\tan \varphi = \frac{147}{115}$
 $\varphi = \tan^{-1} \frac{147}{115}$



6.5
(no right triangle)

Law of Sine: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



$$A + B + C = 180^\circ$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\sin B = \frac{h}{c} \quad \sin C = \frac{h}{b}$$

$$h = c \sin B = h = b \sin C$$

$$c \sin B = b \sin C$$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$



Ex

$$A = 30^\circ \quad B = 70^\circ \quad a = 8 \quad c?$$
$$\angle C = 180^\circ - 30^\circ - 70^\circ = 80^\circ$$

$$\frac{c}{\sin C} \Rightarrow \frac{a}{\sin A}$$

$$c = \frac{8 \sin 80^\circ}{\sin 30^\circ}$$

$$= 16 \sin 80^\circ$$

$$\approx 16$$

Ex Given $A = 32^\circ$ $C = 81.8^\circ$ $a = 42.9 \text{ cm}$

$$B = 180^\circ - 32^\circ - 81.8^\circ \\ = 66.2^\circ$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 66.2^\circ} = \frac{42.9}{\sin 32^\circ}$$

$$b = \frac{42.9 \sin 66.2^\circ}{\sin 32^\circ}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{42.9 \sin 81.8^\circ}{\sin 32^\circ}$$

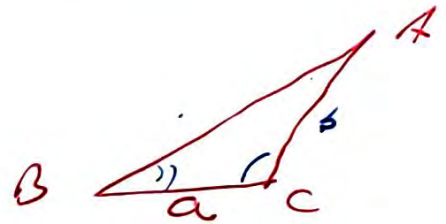
Ex $C = 112.9^\circ$ $A = 31.1^\circ$

$$b = 347.6 \text{ ft}$$

$$B = 180^\circ - 31.1^\circ - 112.9^\circ \\ = 36^\circ$$

$$a = \frac{b \sin A}{\sin B}$$

$$= \frac{347.6 \sin 31.1^\circ}{\sin 36^\circ}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Ex $a = 562$ $B = 5.7^\circ$ $A = 85.3^\circ$

$$\frac{x}{\sin B} = \frac{a}{\sin A}$$

$$x = \frac{562 \sin 5.7^\circ}{\sin 85.3^\circ}$$

$$\textcircled{1} \quad 0 \leq \sin e \leq 1$$

$$\textcircled{2} \quad \sin e (+) \quad Q I \& Q II$$

Ex

$$B? \quad a=2, \quad b=6 \quad A=30^\circ$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{6 \cdot \sin 30^\circ}{2}$$

$$= 3\left(\frac{1}{2}\right)$$

$$= \frac{3}{2} > 1$$

No Triangle.

Ex

$$C=35.4^\circ \quad a=205 \quad c=314 \quad ft$$

$$\sin A = \frac{a \sin C}{c}$$

$$= \frac{205 \sin 35.4^\circ}{314}$$

$$A = \sin^{-1}\left(\frac{205 \sin 35.4^\circ}{314}\right)$$

$$\approx 22.2^\circ$$

$$B \approx 180^\circ - 35.4^\circ - 22.2^\circ$$

$$\approx 122.4^\circ$$

$$A = 180^\circ - 22.2^\circ$$

$$= 157.8^\circ$$

$$B_1 = 180^\circ - 157.8^\circ$$

$$= 35.4^\circ$$

$$= - \quad \#$$

$$b = \frac{c \sin B}{\sin C}$$

$$= \frac{314 \sin 122.4^\circ}{\sin 35.4^\circ}$$

$$\approx 458 ft$$

Ex

$$a = 54$$

$$b = 62$$

$$A = 40^\circ$$

$$\sin B = \frac{b \sin A}{a}$$
$$= \frac{62 \sin 40^\circ}{54}$$

$$B = \sin^{-1} \left(\frac{62 \sin 40^\circ}{54} \right)$$
$$\approx 48^\circ$$

$$C = 180^\circ - 40^\circ - 48^\circ$$
$$= 92^\circ$$

$$c = \frac{54 \sin 92^\circ}{\sin 40^\circ}$$
$$\approx 85 \text{ cm}$$

$$B = 180^\circ - 40^\circ$$
$$= 132^\circ$$

$$C = 180^\circ - 40^\circ - 132^\circ$$
$$= 8^\circ$$

$$c_1 = \frac{a \sin C}{\sin A}$$
$$= \frac{54 \sin 8^\circ}{\sin 40^\circ}$$
$$\approx 12 \text{ cm}$$

(Area) 2 side \rightarrow Angle (3 letters)

$$K = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} ac \sin B$$

Ex 1 $b = 4$ $c = 1$ $A = 120^\circ$

$$\text{Area} = \frac{1}{2} bc \sin A$$
$$= \frac{1}{2} (4)(1) \sin 120^\circ$$

$$= 2 \left(\frac{\sqrt{3}}{2} \right)$$

$$= \sqrt{3} \text{ unit}^2$$

$$\sin 60^\circ$$

180°

$> 90^\circ$

$90^\circ < \quad < 180^\circ$

$180^\circ - 120^\circ = 60^\circ$

46 $a=8$ $b=6$ $C=30^\circ$

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} (8)(6) \sin 30^\circ \\ &= 24 \left(\frac{1}{2}\right) \\ &= 12 \text{ unit}^2 \end{aligned}$$

30°
 150°
 $\frac{1}{2}$

45°
 135°
 $\frac{\sqrt{2}}{2}$

60°
 120°
 $\frac{\sqrt{3}}{2}$

Q I

Q II