Solution Section 3.3 – Logarithmic Functions

Exercise

Write the equation in its equivalent logarithmic form $2^6 = 64$

Solution

$$\boxed{6 = \log_2 64}$$

Exercise

Write the equation in its equivalent logarithmic form $5^4 = 625$

Solution

$$\boxed{4 = \log_5 625}$$

Exercise

Write the equation in its equivalent logarithmic form $5^{-3} = \frac{1}{125}$

Solution

$$\boxed{-3 = \log_5 \frac{1}{125}}$$

Exercise

Write the equation in its equivalent logarithmic form $\sqrt[3]{64} = 4$

Solution

$$64^{1/3} = 4$$

$$\boxed{\log_{64} = \frac{1}{3}}$$

Exercise

Write the equation in its equivalent logarithmic form $b^3 = 343$

Solution

$$\boxed{\log_b 343 = 3}$$

Exercise

Write the equation in its equivalent logarithmic form $8^y = 300$

Solution

$$\boxed{\log_8 300 = y}$$

Exercise

Write the equation in its equivalent logarithmic form: $\sqrt[n]{x} = y$

Solution

$$(x)^{1/n} = y$$

$$\boxed{\log_x(y) = \frac{1}{n}}$$

Exercise

Write the equation in its equivalent logarithmic form: $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$

Solution

$$\boxed{\log_{\frac{2}{3}}\left(\frac{27}{8}\right) = -3}$$

Exercise

Write the equation in its equivalent logarithmic form: $\left(\frac{1}{2}\right)^{-5} = 32$

Solution

$$\boxed{\log_{\frac{1}{2}}(32) = -5}$$

Exercise

Write the equation in its equivalent logarithmic form: $e^{x-2} = 2y$

Solution

$$\boxed{x - 2 = \ln|2y|}$$

Exercise

Write the equation in its equivalent logarithmic form: $e = 3x$

Solution

$\boxed{1 = \ln|3x|}$

Exercise

Write the equation in its equivalent logarithmic form: $\sqrt[3]{e^{2x}} = y$

Solution

$e^{2x/3} = y$

$\boxed{\frac{2x}{3} = \ln|y|}$

Exercise

Write the equation in its equivalent exponential form $\log_5 125 = y$

Solution

$\boxed{5^y = 125}$

Exercise

Write the equation in its equivalent exponential form $\log_4 16 = x$

Solution

$\boxed{16 = 4^x}$

Exercise

Write the equation in its equivalent exponential form $\log_5 \frac{1}{5} = x$

Solution

$\boxed{\frac{1}{5} = 5^x}$

Exercise

Write the equation in its equivalent exponential form $\log_2 \frac{1}{8} = x$

Solution

$$\frac{1}{8} = 2^x$$

Exercise

Write the equation in its equivalent exponential form $\log_6 \sqrt{6} = x$

Solution

$$\sqrt{6} = 6^x$$

Exercise

Write the equation in its equivalent exponential form $\log_3 \frac{1}{\sqrt{3}} = x$

Solution

$$3^{-1/2} = 3^x$$

Exercise

Write the equation in its equivalent exponential form: $6 = \log_2 64$

Solution

$$6 = \log_2 64 \iff 2^6 = 64$$

Exercise

Write the equation in its equivalent exponential form: $2 = \log_9 x$

Solution

$$2 = \log_9 x \iff x = 2^9$$

Exercise

Write the equation in its equivalent exponential form: $\log_{\sqrt{3}} 81 = 8$

Solution

$$\log_{\sqrt{3}} 81 = 8 \iff 81 = \left(\sqrt{3}\right)^8$$

Exercise

Write the equation in its equivalent exponential form: $\log_4 \frac{1}{64} = -3$

Solution

$$\log_4 \frac{1}{64} = -3 \quad \Leftrightarrow \quad \frac{1}{64} = x^{-3}$$

Exercise

Write the equation in its equivalent exponential form: $\log_4 26 = y$

Solution

$$\log_4 26 = y \quad \Leftrightarrow \quad 26 = 4^y$$

Exercise

Write the equation in its equivalent exponential form: $\ln M = c$

Solution

$$\ln M = c \quad \Leftrightarrow \quad M = e^c$$

Exercise

Evaluate the expression without using a calculator: $\log_4 16$

Solution

$$\log_4 16 = \log_4 4^2 \qquad \log_b b^x = x$$

$$= 2$$

Exercise

Evaluate the expression without using a calculator: $\log_2 \frac{1}{8}$

Solution

$$\log_2 \frac{1}{8} = \log_2 \frac{1}{2^3}$$

$$= \log_2 2^{-3} \qquad \log_b b^x = x$$

$$= -3$$

Exercise

Evaluate the expression without using a calculator: $\log_6 \sqrt{6}$

Solution

$$\log_6 \sqrt{6} = \log_6 6^{1/2}$$
$$\boxed{=\frac{1}{2}}$$

Exercise

Evaluate the expression without using a calculator: $\log_3 \frac{1}{\sqrt{3}}$

Solution

$$\log_3 \frac{1}{\sqrt{3}} = \log_3 \frac{1}{3^{1/2}}$$
$$= \log_3 3^{-1/2} \qquad \log_b b^x = x$$
$$\boxed{=-\frac{1}{2}}$$

Exercise

Evaluate the expression without using a calculator: $\log_3 \sqrt[7]{3}$

Solution

$$\log_3 3^{1/7} = x \qquad \textit{Converts to exponential}$$
$$3^{1/7} = 3^x$$
$$x = \frac{1}{7}$$
$$\boxed{\log_3 \sqrt[7]{3} = \frac{1}{7}}$$

Exercise

Evaluate the expression without using a calculator: $\log_3 \sqrt{9}$

Solution

$$\log_3 \sqrt{9} = \log_3 3 \qquad \log_b b^x = x$$
$$\boxed{=1}$$

Exercise

Evaluate the expression without using a calculator: $\log_{\frac{1}{2}} \sqrt{\frac{1}{2}}$

Solution

$$\log_{\frac{1}{2}} \sqrt{\frac{1}{2}} = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{\frac{1}{2}} \qquad \log_b b^x = x$$

$$\boxed{= \frac{1}{2}}$$

Exercise

Simplify $\log_5 1$

Solution

$$\boxed{\log_5 1 = 0}$$

Exercise

Simplify $\log_7 7^2$

Solution

$$\boxed{\log_7 7^2 = 2}$$

Exercise

Simplify $3^{\log_3 8}$

Solution

$$\boxed{3^{\log_3 8} = 8}$$

Exercise

Simplify $10^{\log 3}$

Solution

$$\boxed{10^{\log 3} = 3}$$

Exercise

Simplify $e^{2+\ln 3}$

Solution

$$e^{2+\ln 3} = e^2 e^{\ln 3}$$
$$\boxed{= 3e^2}$$

Exercise

Simplify $\ln e^{-3}$

Solution

$$\boxed{\ln e^{-3} = -3}$$

Exercise

Simplify $\ln e^{x-5}$

Solution

$$\boxed{\ln e^{x-5} = x - 5}$$

Exercise

Simplify $\log_b b^n$

Solution

$$\boxed{\log_b b^n = n}$$

Exercise

Simplify $\ln e^{x^2+3x}$

Solution

$$\boxed{\ln e^{x^2+3x} = x^2 + 3x}$$

Exercise

Find the domain of $f(x) = \log_5 (x+4)$

Solution

Domain: $\underline{x > -4}$

Exercise

Find the domain of $f(x) = \log_5 (x+6)$

Solution

Domain: $\underline{x > -6}$

Exercise

Find the domain of $f(x) = \log(2-x)$

Solution

Domain: $\underline{x < 2}$

Exercise

Find the domain of $f(x) = \log(7-x)$

Solution

Domain: $\underline{x < 7}$

Exercise

Find the domain of $f(x) = \ln(x-2)^2$

Solution

Domain: $\underline{\mathbb{R} - \{2\}}$

$\underline{(-\infty,\ 2) \cup (2,\ \infty)}$

Exercise

Find the domain of $f(x) = \ln(x-7)^2$

Solution

Domain: $\boxed{\mathbb{R}-\{7\}}$

$\boxed{(-\infty,\ 7)\cup(7,\ \infty)}$

Exercise

Find the domain of $f(x)=\log\left(x^2-4x-12\right)$

Solution

$$x^2-4x-12>0$$

$$x=\frac{4\pm\sqrt{16+48}}{2}$$

$$=\begin{cases}\frac{4-8}{2}=-2\\ \frac{4+8}{2}=6\end{cases}$$

Domain: $\boxed{x<-2 \quad x>6}$

$\boxed{(-\infty,\ -2)\cup(6,\ \infty)}$

Exercise

Find the domain of $f(x)=\log\left(\frac{x-2}{x+5}\right)$

Solution

$$\begin{cases}x\neq 2\\ x\neq -5\end{cases}$$

-5	0	2
$+$	$-$	$+$

Domain: $\boxed{x<-5 \quad x>2}$

$\boxed{(-\infty,\ -5)\cup(2,\ \infty)}$

Exercise

Find the domain of $f(x)=\log\left(\frac{3-x}{x-2}\right)$

Solution

$$\begin{cases}x\neq 3\\ x\neq 2\end{cases}$$

0 2		3
$-$	$+$	$-$

Domain: $\boxed{2<x<3}$ $\boxed{(2,\ 3)}$

Exercise

Find the domain of $f(x)=\ln\left(x^2-9\right)$

Solution

$$x^2-9>0$$

Domain: $\underline{x<-3 \quad x>3}$

Exercise

Find the domain of $f(x)=\ln\left(\dfrac{x^2}{x-4}\right)$

Solution

$$\frac{x^2}{x-4}>0$$

$$x^2 \ \to \ \mathbb{R}$$

$$x>4$$

Domain: $\underline{x>4}$

Exercise

Find the domain of $f(x)=\log_3\left(x^3-x\right)$

Solution

$$x^3-x>0$$

$$\underline{x=0,\ 0,\ 1}$$

	0,0	1 2
–	–	+

Domain: $\underline{x>1}$

Exercise

Find the domain of $f(x)=\log\sqrt{2x-5}$

Solution

$$2x-5>0$$

Domain: $\underline{x>\frac{5}{2}}$

Exercise

Find the domain of $\quad f(x) = 3\ln(5x-6)$

Solution

$5x - 6 > 0$

Domain: $\underline{x > \frac{6}{5}}$

Exercise

Find the domain of $\quad f(x) = \log\left(\frac{x}{x-2}\right)$

Solution

$\frac{x}{x-2} > 0$

$\underline{x = 0,\ 2}$

Domain: $\underline{x < 0 \quad x > 2}$

Exercise

Find the domain of $\quad f(x) = \log\left(4 - x^2\right)$

Solution

$4 - x^2 > 0$

$4 - x^2 = 0 \quad \rightarrow \quad x = \pm 2$

Domain: $\underline{-2 < x < 2}$

Exercise

Find the domain of $\quad f(x) = \ln\left(x^2 + 4\right)$

Solution

$x^2 + 4$ always positive.

Domain: $\underline{\mathbb{R}}$

Exercise

Find the domain of $f(x) = \ln|4x-8|$

Solution

$4x - 8 = 0 \quad \to \quad x = 2$

Domain: $\underline{\mathbb{R} - \{2\}}$

Exercise

Find the domain of $f(x) = \ln|5-x|$

Solution

$5 - x = 0 \quad \to \quad x = 5$

Domain: $\underline{\mathbb{R} - \{5\}}$

Exercise

Find the domain of $f(x) = \ln(x-4)^2$

Solution

$x - 4 = 0 \quad \to \quad x = 4$

Domain: $\underline{\mathbb{R} - \{4\}}$

Exercise

Find the domain of $f(x) = \ln\left(x^2 - 4\right)$

Solution

$x^2 - 4 > 0$

$x^2 - 4 = 0 \quad \to \quad x = \pm 2$

Domain: $\underline{x < -2 \quad x > 2}$

Exercise

Find the domain of $f(x) = \ln\left(x^2 - 4x + 3\right)$

Solution

$x^2 - 4x + 3 = 0 \quad \to \quad \underline{x = 1,\ 3}$

$x^2 - 4x + 3 > 0$

Domain: $x < 1 \quad x > 3$

Exercise

Find the domain of $\quad f(x) = \ln\left(2x^2 - 5x + 3\right)$

Solution

$$2x^2 - 5x + 3 = 0 \quad \to \quad x = 1, \ \frac{3}{2}$$

$$2x^2 - 5x + 3 > 0$$

Domain: $x < 1 \quad x > \frac{3}{2}$

Exercise

Find the domain of $\quad f(x) = \log\left(x^2 + 4x + 3\right)$

Solution

$$x^2 + 4x + 3 = 0 \quad \to \quad x = -1, \ -3$$

$$x^2 + 4x + 3 > 0$$

Domain: $x < -3 \quad x > -1$

Exercise

Find the domain of $\quad f(x) = \ln\left(x^4 - x^2\right)$

Solution

$$x^4 - x^2 = 0$$

$$x^2\left(x^2 - 1\right) = 0$$

$$x = 0, \ 0, \ \pm 1$$

$$x^4 - x^2 > 0$$

-1	$0,0$	1	2
$+$	$-$	$-$	$+$

Domain: $x < -1 \quad x > 1$

Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \log_4 (x-2)$

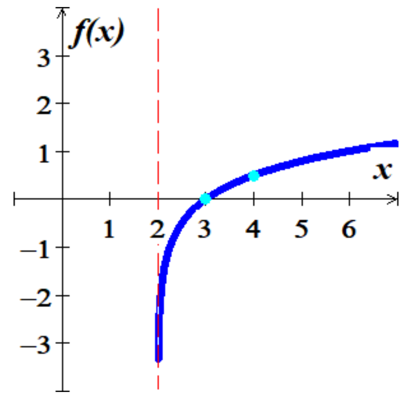
Solution

Asymptote: $x = 2$

Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

x	$f(x)$
2	
3	0
4	.5



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \log_4 |x|$

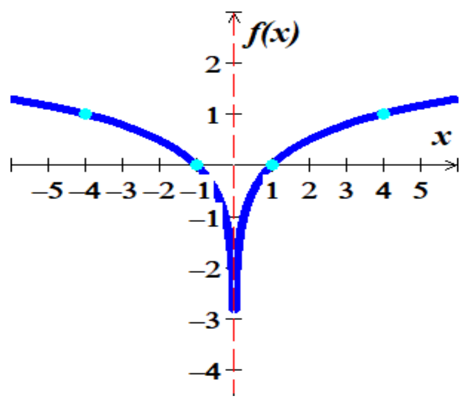
Solution

Asymptote: $x = 0$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, \infty)$

x	$f(x)$
0	
± 1	0
± 4	1



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \left(\log_4 x\right) - 2$

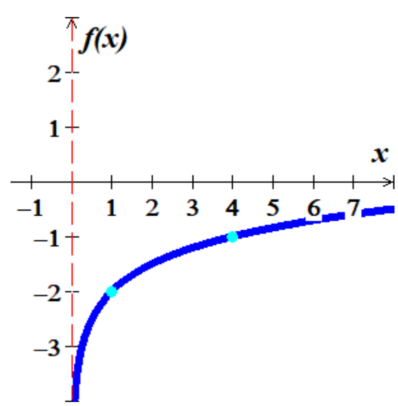
Solution

Asymptote: $x = 0$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x	$f(x)$
0	
1	0
4	-1



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \log(3 - x)$

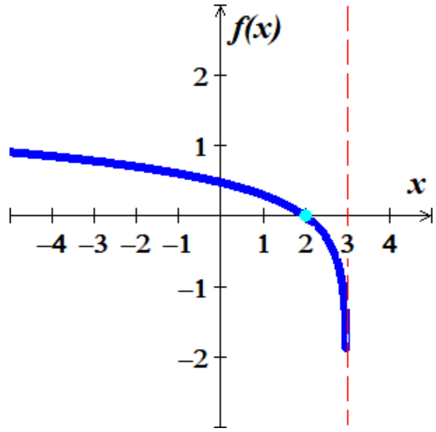
Solution

Asymptote: $x = 3$

Domain: $(-\infty,\ 3)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
3	
2	0



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph
$f(x) = 2 - \log(x + 2)$

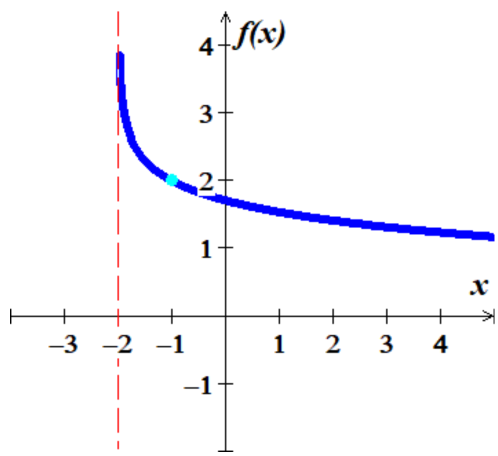
Solution

Asymptote: $x = -2$

Domain: $(-2,\ \infty)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
−2	
−1	2



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \ln(x - 2)$

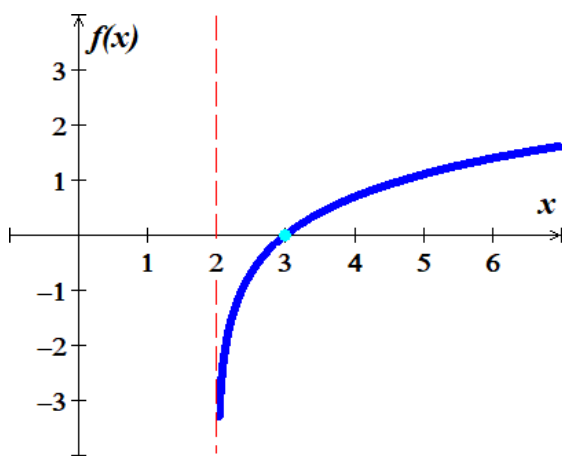
Solution

Asymptote: $x = 2$

Domain: $(2,\ \infty)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
2	
3	0



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph $f(x) = \ln(3 - x)$

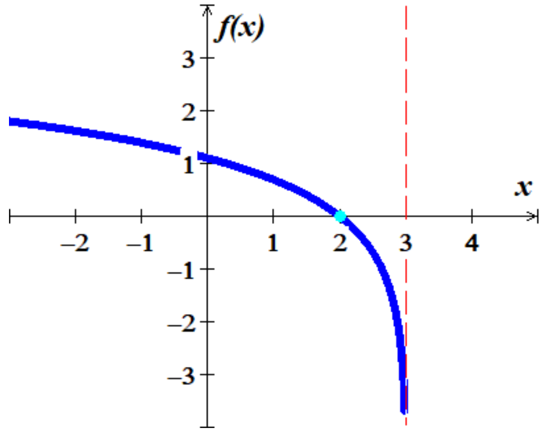
Solution

Asymptote: $x = 3$

Domain: $(-\infty,\ 3)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
3	
2	0



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph

$f(x) = 2 + \ln(x + 1)$

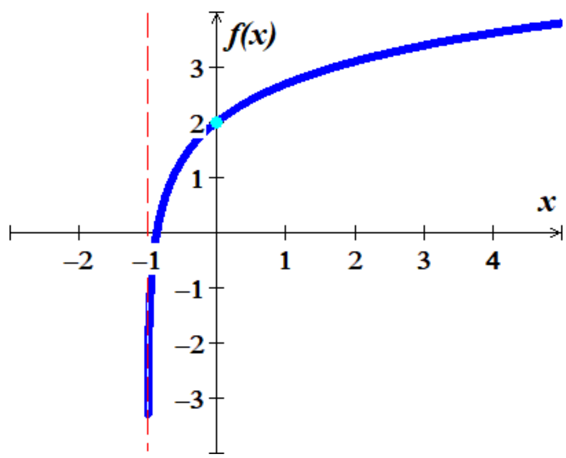
Solution

Asymptote: $x = -1$

Domain: $(-1,\ \infty)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
−1	
0	2



Exercise

Find the ***asymptote***, ***domain***, and ***range*** of the given function. Then, sketch the graph

$f(x) = 1 - \ln(x - 2)$

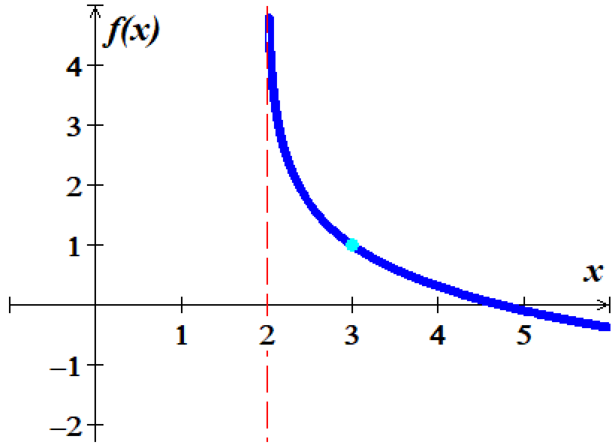
Solution

Asymptote: $x=2$

Domain: $(2,\ \infty)$

Range: $(-\infty,\ \infty)$

x	$f(x)$
2	
3	1



Exercise

On a study by psychologists Bornstein and Bornstein, it was found that the average walking speed w, in feet per second, of a person living in a city of population P, in ***thousands***, is given by the function

$$w(P) = 0.37 \ln P + 0.05$$

a) The population is 124,848. Find the average walking speed of people living in Hartford.

b) The population is 1,236,249. Find the average walking speed of people living in San Antonio.

Solution

124,848 = 124.848 ***thousand***

a) $w(124.848) = 0.37 \ln(124.848) + 0.05$

$$\approx 1.8 \ \ ft/sec$$

b) $w(1,236.249) = 0.37 \ln(1,236.249) + 0.05$

$$\approx 2.7 \ \ ft/sec$$

Exercise

The loudness of sounds is measured in a unit called a decibel. To measure with this unit, we first assign an intensity of I_0 to a very faint sound, called the threshold sound. If a particular sound has intensity I, then the decibel rating of this louder sound is

$$d = 10 \log \frac{I}{I_0}$$

Find the exact decibel rating of a sound with intensity $10,000 I_0$

Solution

$$\begin{aligned} d &= 10 \log \frac{10000 I_0}{I_0} \\ &= 10 \log 10000 \\ &= 40 \ \ db \end{aligned}$$

Exercise

Students in an accounting class took a final exam and then took equivalent forms of the exam at monthly intervals thereafter. The average score *S(t)*, as a percent, after *t* months was found to be given by the function

$$S(t) = 78 - 15 \ \log(t+1); \quad t \geq 0$$

a) What was the average score when the students initially took the test, $t = 0$?

b) What was the average score after 4 *months*? 24 *months*?

Solution

a) $S(0) = 78 - 15\ \log(1)$

$\approx 78\%$

b) After 4 *months*

$S(4) = 78 - 15\ \log(5)$

$\approx 67.5\%$

After 24 *months*

$S(24) = 78 - 15\ \log(25)$

$\approx 57\%$

Exercise

A model for advertising response is given by the function

$$N(a) = 1,000 + 200\ \ln a, \quad a \ge 1$$

Where $N(a)$ is the number of units sold when a is the amount spent on advertising, in *thousands* of *dollars*.

a) $N(1)$

b) $N(5)$

Solution

a) $N(1) = 1,000 + 200\ \ln(1)$

$= 1,000\ \ units$

b) $N(5) = 1,000 + 200\ \ln(5)$

$= 1,322\ \ units$