$\underline{7.4}$

#18 $2\cos^2 t - 9\cos t = 5 \quad [0, 2\pi)$

$$2\cos^2 t - 9\cos t - 5 = 0$$

$$\cos t = \frac{+9 \pm \sqrt{81 + 40}}{4}$$

$$= \begin{cases} \dfrac{9-11}{4} = -\dfrac{1}{2} \\ \dfrac{9+11}{4} = 5 > 1 \not\exists \end{cases}$$

$$\cos t = -\frac{1}{2} \Rightarrow \boxed{t = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

---

22 $2\cos x - 1 = \sec x$

$$= \frac{1}{\cos x} \qquad \boxed{\begin{array}{l}\cos x \neq 0 \\ x \neq \frac{\pi}{2}, \frac{3\pi}{2}\end{array}}$$

$$2\cos^2 x - \cos x = 1$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$\cos x = 1 \qquad \cos x = -\frac{1}{2}$$

$$\boxed{x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}}$$

#23 $4\cos^2 x + 4\sin x - 5 = 0 \qquad \cos^2 x + \sin^2 x = 1$

$$4(1 - \sin^2 x) + 4\sin x - 5 = 0$$

$$-4\sin^2 x + 4\sin x - 1 = 0$$

$$\sin x = \frac{-4 \pm \sqrt{16 - 16}}{-8} = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

5.8

sine of an angle is between -1 & 1

$y = \sin x$ $\quad\quad$ $x = \arcsin y = \sin^{-1} y$

$-1 \le y \le 1$ $\quad\quad$ angle $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

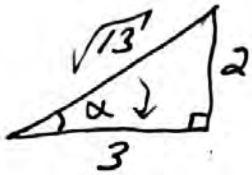
$x = \sin y$

$[-1, 1]$ $\quad\quad$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ex exact $\sec(\underbrace{\arctan \tfrac{2}{3}}_{\alpha})$

$\alpha = \arctan \frac{2}{3}$ $\quad\quad$ $\tan\left(\tan^{-1} \frac{2}{3}\right) = \frac{2}{3}$

$\tan \alpha = \frac{2}{3}$



$\boxed{\sec \alpha = \frac{\sqrt{13}}{3}}$

Ex $\sin\left(\underbrace{\arctan \tfrac{1}{2}}_{\alpha} - \underbrace{\arccos \tfrac{4}{5}}_{\beta}\right)$

$\alpha = \arctan \frac{1}{2} \Rightarrow \tan \alpha = \frac{1}{2}$

$\beta = \arccos \frac{4}{5} \to \cos \beta = \frac{4}{5} \to \sin \beta = \frac{3}{5}$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

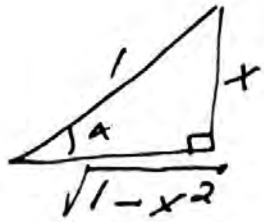
$$= \frac{1}{\sqrt{5}} \frac{4}{5} - \frac{2}{\sqrt{5}} \frac{3}{5}$$

$$= \boxed{\frac{-2}{5\sqrt{5}}} = \boxed{-\frac{2\sqrt{5}}{25}}$$

Ex $\cos(\underbrace{\sin^{-1}x}_{\alpha})$

$\sin^{-1}x = \alpha$

$\sin\alpha = x$

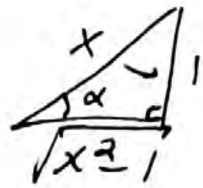


$$\boxed{\cos(\sin^{-1}x) = \sqrt{1-x^2}}$$

#31 $\csc\left(\sin^{-1}\frac{1}{x}\right)$

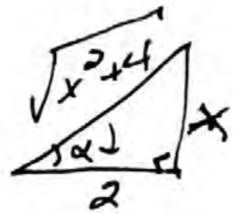
$\alpha = \sin^{-1}\frac{1}{x} \Rightarrow \sin\alpha = \frac{1}{x}$

$\csc\left(\sin^{-1}\frac{1}{x}\right) = x$



33 $\sec\left(\sin^{-1}\frac{x}{\sqrt{x^2+4}}\right)$ $x>0$

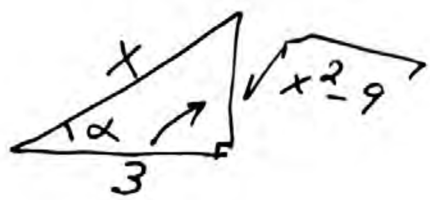
$\alpha = \sin^{-1}\frac{x}{\sqrt{x^2+4}} \rightarrow \sin\alpha = \frac{x}{\sqrt{x^2+4}}$



$\sec\alpha = \frac{\sqrt{x^2+4}}{2}$

34 $\cot(\underbrace{\sin^{-1}\frac{\sqrt{x^2-9}}{x}}_{\alpha})$

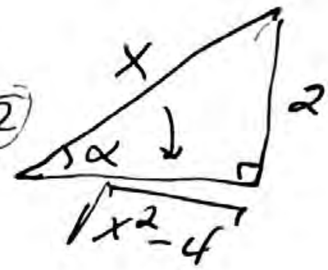
$\sin\alpha = \frac{\sqrt{x^2-9}}{x}$



$$\boxed{\cot\alpha = \frac{3}{\sqrt{x^2-9}}}$$

39/ $\sec\Big(\underbrace{\tan^{-1}\frac{2}{\sqrt{x^2-4}}}_{\alpha}\Big)$

$$\tan\alpha = \frac{2}{\sqrt{x^2-4}} \quad 1 \longrightarrow 2$$

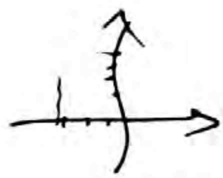


$$2 \longrightarrow 3$$

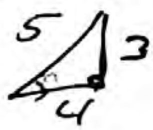
$$\boxed{\sec\alpha = \frac{x}{\sqrt{x^2-4}}}$$

13/ $\cos\Big(\underbrace{\arctan\left(-\frac{3}{4}\right)}_{\alpha} - \underbrace{\arcsin\left(\frac{4}{5}\right)}_{\beta}\Big)$

$$\tan\alpha = -\frac{3}{4}$$



$$\sin\beta = \frac{4}{5} \rightarrow \cos\beta = \frac{3}{5}$$

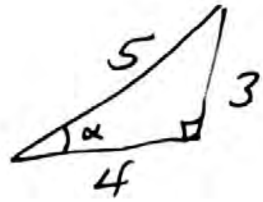


$$\begin{aligned}
\cos(-\alpha-\beta) &= \cos(-(\alpha+\beta)) \\
&= \cos(\alpha+\beta) \\
&= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
&= \frac{4}{5}\,\frac{3}{5} - \frac{3}{5}\,\frac{4}{5} \\
&= 0
\end{aligned}$$

20/ $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$$= \frac{7\pi}{6}$$

26 $\tan(\underbrace{\sin^{-1}\tfrac{3}{5}}_{\alpha})$



$$\sin\alpha = \frac{3}{5}$$

$$\boxed{\tan\alpha = \frac{3}{4}}$$

53 8.1

$$\frac{1-2\sin^2 x}{1+2\sin x\cos x} = \frac{\cos x-\sin x}{\cos x+\sin x}$$

$$\frac{1-2\sin^2 x}{1+2\sin x\cos x} = \frac{\cos^2 x+\sin^2 x-2\sin^2 x}{\cos^2 x+\sin^2 x+2\sin x\cos x}$$

$$= \frac{\cos^2 x-\sin^2 x}{(\cos x+\sin x)^2}$$

$$= \frac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)^2}$$

$$= \frac{\cos x-\sin x}{\cos x+\sin x}$$

64 $\cot^3 x = \cot x(\csc^2 x-1)$ $\cot^2 x+1=\csc^2 x$

$$\cot x(\csc^2 x-1) = \cot x(\cot^2 x)$$

$$= \cot^3 x$$

So $10\csc^2 x - 6\cot^2 x = 4\csc^2 x + 6$

$$10\csc^2 x - 6\cot^2 x = 10\csc^2 x - 6(\csc^2 x - 1)$$
$$= 10\csc^2 x - 6\csc^2 x + 6$$
$$= 4\csc^2 x + 6 \checkmark.$$

8.2: 28, 15, 12

#17 $\frac{\sin(x-y)}{\sin x \cos y} = 1 - \cot x \tan y$

$$\frac{\sin(x-y)}{\sin x \cos y} = \frac{\sin x \cos y}{\sin x \cos y} - \frac{\cos x \sin y}{\sin x \cos y}$$
$$= 1 - \cot x \tan y \checkmark.$$

---

20) $\frac{\sin(x+y)}{\cos(x-y)} = \frac{1 + \cot x \cot y}{\cot x + \tan y}$

$$\frac{\sin(x+y)}{\cos(x-y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y + \sin x \sin y}$$

$$= \frac{\frac{\sin x \cos y}{\sin x \cos y} + \frac{\cos x \sin y}{\sin x \cos y}}{\frac{\cos x \cos y}{\sin x \cos y} + \frac{\sin x \sin y}{\sin x \cos y}}$$

$$= \frac{1 + \cot x \tan y}{\cot x + \tan y} \checkmark$$

27 $\sec(x+y) = \dfrac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y}$

$$\sec(x+y) = \frac{1}{\cos(x+y)} \, \frac{\cos(x-y)}{\cos(x-y)}$$

$$= \frac{\cos(x-y)}{(\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)}$$

$$= \frac{\cos(x-y)}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y}$$

$$= \frac{\cos(x-y)}{\cos^2 x(1-\sin^2 y) - (1-\cos^2 x)\sin^2 y}$$

$$= \frac{\cos(x-y)}{\cos^2 x - \underline{\cos^2 x \sin^2 y} - \sin^2 y + \underline{\cos^2 x \sin^2 y}}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cos^2 x - \sin^2 y}$$

8.3 / 36 $\sin 2\alpha \sin 2\beta = \sin^2(\alpha+\beta) - \sin^2(\alpha-\beta)$

$$\sin^2(\alpha+\beta) - \sin^2(\alpha-\beta) = \underbrace{(\sin(\alpha+\beta) - \sin(\alpha-\beta))}_{①}\underbrace{(\sin(\alpha+\beta) + \sin(\alpha-\beta))}_{②}$$

① $\to \sin\alpha\cos\beta + \cos\alpha\sin\beta - \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\to 2\cos\alpha\sin\beta$.

② $\sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\to 2\sin\alpha\cos\beta$

$$\sin^2(\alpha+\beta) - \sin^2(\alpha-\beta) = (2\cos\alpha\sin\beta)(2\sin\alpha\cos\beta)$$
$$= (2\cos\alpha\sin\alpha)(2\sin\beta\cos\beta)$$
$$= \sin 2\alpha \cos 2\beta$$

46 $\dfrac{1-\cos^2\frac{x}{2}}{1-\sin^2\frac{x}{2}} = \dfrac{1-\cos x}{1+\cos x}$

$$\frac{1-\cos^2\frac{x}{2}}{1-\sin^2\frac{x}{2}} = \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}$$

$$= \tan^2\frac{x}{2}$$

$$= \tan\frac{x}{2}\,\tan\frac{x}{2}$$

$$= \frac{1-\cos x}{\sin x}\,\frac{\sin x}{1+\cos x}$$

$$= \frac{1-\cos x}{1+\cos x}$$

$\rightarrow$

$$= \frac{\frac{1}{2}(1-\cos x)}{\frac{1}{2}(1+\cos x)}$$

$$= \frac{1-\cos x}{1+\cos x} \checkmark$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\frac{1+\cos 2\theta}{2} = \cos^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

32) $\dfrac{\cos 2x}{\sin^2 x} = 2\cot^2 x - \csc^2 x$

$$\frac{\cos 2x}{\sin^2 x} = \frac{2\cos^2 x - 1}{\sin^2 x}$$

$$= 2\,\frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x}$$

$$= 2\cot^2 x - \csc^2 x$$