Section 3.5 – Probability

The type of experiments on which probability studies are based called random experiments: flipping coins, dice.

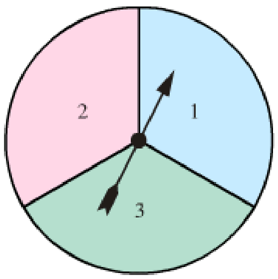
Experiment means a random experiment; an activity with a result.

Each possible result is called an ***outcome*** of the experiment (each trial).

Sample Spaces (*S*)

Sample Space is the set of all possible outcomes for an experiment.

Outcome: Event – subset of S: ***simple or compound***



Example

a) Give the sample space for a the spinner $\quad S=\{1,2,3\}$

b) Give the sample space for an experiment consists of studying the numbers of boys and girls in families with exactly 3 children. Let *b* represent *boy* and *g* represent *girl*.

$$S=\{bbb,\ bbg,\ bgb,\ gbb,\ bgg,\ gbg,\ ggb,\ ggg\}$$

Events

Event is any subset of S including $\varnothing$

$$\begin{cases} \textit{Simple event} & \text{Contains only one element} \\ \textit{Compound event} & \text{Contains more than one element} \end{cases}$$

Example

Give the sample space for an experiment consists of studying the numbers of boys and girls in families with exactly 3 children. Write the event foe the family that has exactly two girls

<u>Solution</u>

$$E=\{bgg, gbg, ggb\}$$

Example

Suppose a coin is flipped until both a head and a tail appear, or until the coin has been flipped four times, whichever comes first.

a) Write the event that the coin flipped exactly three times.

b) Write the event that the coin flipped at least three times.

c) Write the event that the coin flipped at least two times.

<u>*Solution*</u>

a) $S=\{HHH, HHT, HTH, THH, HTT, HTH, TTH, TTT\}$

$E=\{HHT, TTH\}$

b) $E=\{TTH, HHT, HHHH, HHHT, TTTH, TTTT\}$

c) $E=\{HT, TH, TTH, HHT, HHHH, HHHT, TTTH, TTTT\}$

Probability

Let S be a sample space of equally likely outcomes, and let event E be a subset of S. Then the probability that event E occurs is

$$P(E)=\frac{\text{number of elements of } E}{\text{number of elements in } S}=\frac{n(E)}{n(S)}$$

Example

Suppose a single fair die is rolled. Use the sample space $S=\{1,2,3,4,5,6\}$ and give the probability of each event.

a) E: the die shows an even number
b) F: the die show a number less than 10
c) G: the die shows an 8

<u>*Solution*</u>

a) Even number: $E=\{2,4,6\}$

$$P(E)=\frac{n(S)}{n(E)}$$

$$=\frac{3}{6}$$

$$=\frac{1}{2}$$

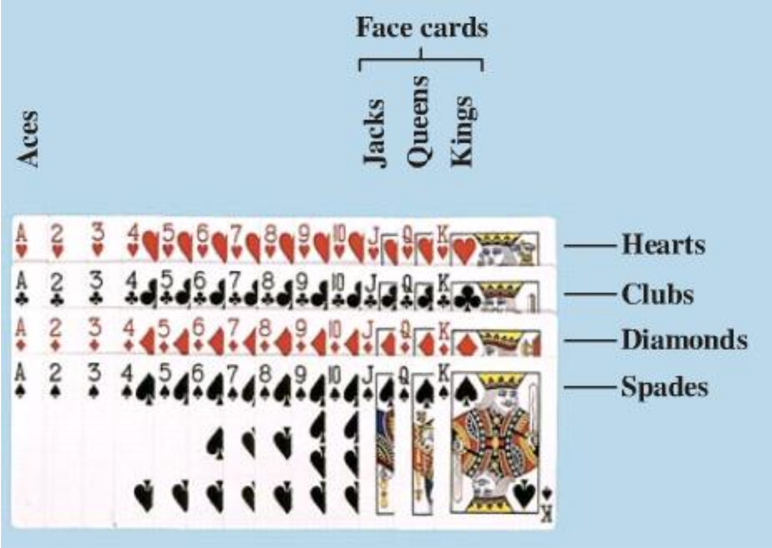
b) Number less than 10

$F=\{1,2,3,4,5,6\}$

$P(F)=\frac{6}{6}=1$

c) Die shows an 8

$\boxed{P(G)=0}$ ***Impossible***



Example

If a single playing card is drawn at random from a standard 52-card deck, find the probability of each event.

a) Drawing an ace
b) Drawing a face card
c) Drawing a spade
d) Drawing a spade or a heart

<u>*Solution*</u>

a) $P(Ace) = \frac{4}{52} = \frac{1}{13}$

b) $P(face\ card) = \frac{12}{52} = \frac{3}{13}$

c) $P(spade) = \frac{13}{52} = \frac{1}{4}$

d) $P(spade\ \text{or}\ heart) = \frac{26}{52} = \frac{1}{2}$

Probability of an Event

$0 \le P(E) \le 1$

$S = \{e_1, \cdots, e_i\}$

Probability of the event e_i : $P(e)$ *or* $\Pr(e)$

$$\begin{cases} 0 \le P \le 1 \\ \sum P_i = 1 \end{cases} \quad \Rightarrow \textit{Acceptable probability assignment}$$

1) If ***E*** is an empty set $\Rightarrow$ P(E) = 0
2) If ***E*** is a simple event $\Rightarrow$ P(E) has already been assigned
3) If ***E*** is a compound event $\Rightarrow$ P(E) is the sum of probability of all sample E
4) If ***E*** is a sample space $\Rightarrow$ P(E) = P(S) = 1

Example

After flipping the nickel and dime 1,000 times, we find that HH turns up 273 times, HT turns up 206 times, TH turns up 312 times, TT turns up 209 times.

Simple Event e_i	*HH*	*HT*	*TH*	*TT*
$P(e_i)$	.273	.206	.312	.209

What are the probabilities of the following events?

a) E_1 = getting at least 1 tail

b) E_2 = getting 2 tails

c) E_3 = getting at least 1 tail or at least 1 head

Solution

a) $P(E_1) = P(HT) + P(TH) + P(TT)$

$= .206 + .312 + .209$

$\underline{= .727}$

b) $P(E_2) = P(TT)$ $\underline{= .209}$

c) $P(E_3) = P(HH) + P(HT) + P(TH) + P(TT)$

$\underline{= 1}$

Set Operations for Events

Let E and F be events for a sample space S.

$E \cup F = \{e \in S \mid e \in E \text{ or } e \in F\}$ Occurs when E ***or*** F ***or both*** occur

$E \cap F = \{e \in S \mid e \in E \text{ and } e \in F\}$ Occurs when both E ***and*** F occur

E' Occurs when E ***does not*** occur

Event

The event E ***or*** $F = E \cup F$

The event E ***and*** $F = E \cap F$

Example

A study of workers earning the minimum wage grouped such workers into various categories, which can be interpreted as events when a worker is selected at random. Consider the following events:

E: worker is under 20;
F: worker is white;
G: worker is female.

a) E'
b) $F \cap G'$
c) $E \cap G$

<u>*Solution*</u>

a) E' is the event that the worker is 20 or over.

b) $F \cap G'$ is the event that the worker is white and not a female

c) $E \cap G$ is the event that the worker is under 20 or is female

Union Rule For Probability

For any events E and F from a sample space S,

$$\boxed{P(E \cup F) = P(E) + P(F) - P(E \cap F)}$$

Example

If a single card is drawn from an ordinary deck of cards, find the probability that it will be a red or a face card

<u>*Solution*</u>

$P(Red \text{ or } Face) = P(R \cup F)$

$$P(R) = \boxed{\frac{26}{52}}$$

$$P(F) = \boxed{\frac{12}{52}}$$

$$P(R \cap F) = \boxed{\frac{6}{52}}$$

$$\begin{aligned} P(R \cup F) &= P(R) + P(F) - P(R \cap F) \\ &= \frac{26}{52} + \frac{12}{52} - \frac{6}{52} \\ &= \frac{32}{52} \\ &= \boxed{\frac{8}{13}} \end{aligned}$$

Example

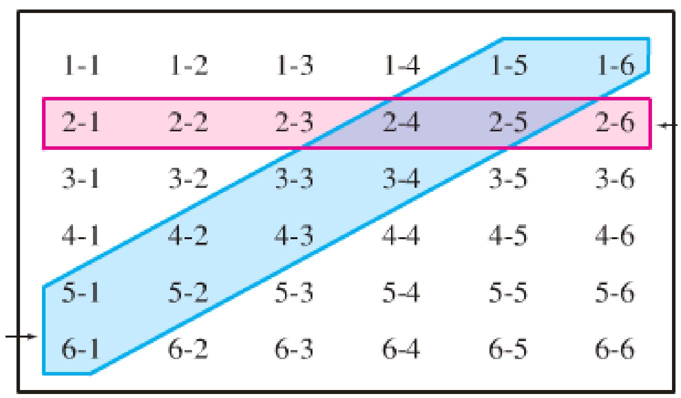
Suppose two fair dice are rolled. Find each probability

a) The first die shows a 2, or the sum of the results is 6 or 7.

b) The sum of the results is 11 or the second die shows a 5.

Solution

a) The first die shows a 2, or the sum of the results is 6 or 7.



$$P\left(2 \text{ or } \sum 6 \text{ or } 7\right) = \frac{15}{36} = \frac{5}{12}$$

b) $P\left(\sum 11 \text{ or second shows } 5\right) = \boxed{\frac{7}{36}}$

Complement Rule

E' is called the complement of E relative to S

$$E \cap E' = \varnothing, \qquad E \cup E' = S \qquad \textit{Mutually Exclusive}$$

$$\mathrm{P}(S) = \mathrm{P}(E \cup E') = \mathrm{P}(E) + \mathrm{P}(E') = 1$$

$$P(E) = 1 - P(E') \quad and \quad P(E') = 1 - P(E)$$

Example

If a fair die is rolled, what is the probability that any number but 5 will come up?

Solution

$$P(E=5) = \frac{1}{6}$$

$$\begin{aligned} P(E' = any\ but\ 5) &= 1 - P(E) \\ &= 1 - \frac{1}{6} \\ &= \frac{5}{6} \end{aligned}$$

Example

Suppose two fair dice are rolled. Find each probability that the sum of the numbers rolled is greater than 3.

Solution

$$P(sum > 3) = 1 - P(sum \le 3)$$

$$\begin{aligned} P(sum \le 3) &= P(sum\ is\ 2) + P(sum\ is\ 3) \\ &= \frac{1}{36} + \frac{2}{36} \\ &= \frac{3}{36} \\ &= \frac{1}{12} \end{aligned}$$

$$\begin{aligned} P(sum > 3) &= 1 - P(sum \le 3) \\ &= 1 - \frac{1}{12} \\ &= \frac{11}{12} \end{aligned}$$

Mutually Exclusive

Events E and F are ***mutually exclusive events*** if $E \cap F = \phi$

Mutually exclusive events are disjoint sets

If ***E*** and F are mutually exclusive (i.e. their intersection is empty), then

$$P(E \cup F) = P(E) + P(F)$$

In looking $P(E)$ always find $P(E')$ first!

Example

Let $E = \{4,5,6\}$ and $G = \{1,2\}$

Then E and G are mutually exclusive events since they have no outcomes in common: $E \cap G = \phi$

Example

A shipment of 40 precision parts, including 8 that are defective, is sent to an assembly plant. The quality control division selects 10 at random for testing and rejects the entire shipment if 1 or more in the sample are found defective. What is the probability that the shipment will be rejected?

Solution

8 ***defective*** $\Rightarrow$ 32 ***non-defective***.

$n(S) = C_{40,10}$

$n(E') = C_{32,10}$

$P(E') = \dfrac{n(E')}{n(S)} \approx 0.08$

$P(E) = 1 - P(E') \approx 0.92$

ODDS

Definition

The ***actual odds against*** event A occurring are the ratio $\frac{P(\bar{A})}{P(A)}$, usually expressed in the form of a:b (or "a to b"), where a and b are integers having no common factors.

The ***actual odds in favor*** of event A occurring are the ratio $\frac{P(A)}{P(\bar{A})}$, which is the reciprocal of the actual odds against the event. If the odds against A are a:b, then the odds in favor of A are b:a.

The ***payoff odds*** against event A occurring are the ratio of the net profit (if you win) to the amount bet.

$$payoff\ \ odds\ against\ event\ A = \frac{net\ profit}{amount\ bet}$$

$$\begin{cases} Odds\ of\ \ E = \dfrac{P(E)}{1-P(E)} = \dfrac{P(E)}{P(E')} & ,\ \ P(E) \neq 1 \ \text{or}\ P(E') \neq 0 \\ Odds\ against\ E = \dfrac{P(E')}{P(E)} & ,\ \ P(E) \neq 0 \end{cases}$$

Note:

Odds are expressed as ratios of whole numbers

- If the odds favoring event ***E*** are ***m*** to ***n***, then

$$P(E) = \frac{m}{m+n} \qquad \textbf{and} \qquad P(E') = \frac{n}{m+n}$$

Example

Suppose the weather forecast says that the probability of rain tomorrow is $\frac{1}{3}$. Find the odds in favor of rain tomorrow.

Solution

$$P(E) = \frac{1}{3} \Rightarrow P(E') = 1 - \frac{1}{3} = \frac{2}{3}$$

$$Odds: \frac{1/3}{2/3} = \frac{1}{2} \quad \Rightarrow Written: \text{"1:2"} \ \ or \ \ \text{"1 to 2"} \qquad Read: 1 \ to\ 2$$

The odds that it will not rain:

$$Odds\ against: \frac{2/3}{1/3} = \frac{2}{1} \qquad \Rightarrow Written: \text{"2:1"} \ \ or \ \ \text{"2 to 1"}$$

Example

The odds that a particular bid will be the low bid are 4 to 5

a) Find the probability that the bid will be the low bid.

$$P(bid\ will\ be\ low) = \frac{4}{4+5} = \frac{4}{9}$$

b) Find the odds against that bid being the low bid.

$$P(bid\ will\ \text{not}\ be\ low) = \frac{5}{4+5} = \frac{5}{9}$$

$$odds\ Against = \frac{P(bid\ will\ \text{not}\ be\ low)}{P(bid\ will\ be\ low)} = \frac{5/9}{4/9} = \frac{5}{4}$$

So, 5:4

Example

If the odds in favor of a particular horse's winning a race are 5 to 7, what is the probability that the horse will win the race?

<u>*Solution*</u>

$$P(winning) = \frac{5}{5+7} = \frac{5}{12}$$

Example

a) What are the odds for rolling a sum of 8 in a single roll of two fair dice?

$$P(E) = \frac{5}{36}$$

$$\Rightarrow\ P(E') = 1 - \frac{5}{36} = \frac{31}{36}$$

$$Odds\ of\ E = \frac{P(E)}{P(E')} = \frac{\frac{5}{36}}{\frac{31}{36}} = \frac{5}{31}$$

$$\Rightarrow read\ "\mathbf{5\ to\ 31}" \to written\ "5{:}31"$$

b) If you bet \$5 that sum of 8 will turn up, what should the house pay (plus returning your \$5 bet) if a sum of 8 does turn up for the game to be fair?

Odds: 5 *to* 31 $\to$ \$31

Empirical Probability

It is not possible to establish exact probabilities for events; instead, useful approximations are often found by drawing on past experience. This approach is called ***Empirical probabilities***.

If an experiment is conducted n times and event E occurs with frequency f(E), then the ratio f(E)/n is called the relative frequency of the occurrence of event E in n trials.

$$P(E)=\frac{frequency\ of\ occurrence\ of\ E}{total\ number\ of\ trials}=\frac{f(E)}{n} \rightarrow \text{Ratio called } \textbf{\textit{relative frequency}}$$

$$P(E)=\frac{number\ of\ elements\ of\ E}{number\ of\ elements\ in\ S}=\frac{n(E)}{n(S)}$$

The larger n is, the better the approximation.

Example

Let consider rolling 2 dice. Find the probabilities of the following events

a) E = Sum of 5 turns up

b) F = a sum that is a prime number greater than 7 turns up

<u>*Solution*</u>

a) $P(E)=\frac{4}{36}\ \underline{=\frac{1}{9}}|$

b) $P(F)=\frac{2}{36}\ \underline{=\frac{1}{18}}|$

$$\mathrm{P}\left(E_1=\sum 7\right)=\frac{n\left(E_1\right)}{n(S)}=\frac{6}{36}\ \underline{=\frac{1}{6}}|$$

$$\mathrm{P}\left(E_2=\sum 11\right)=\frac{n\left(E_2\right)}{n(S)}=\frac{2}{36}\ \underline{=\frac{1}{18}}|$$

Example

The following table lists U.S. advertising volume in millions of dollars by medium in 2004

Medium	Expenditures
Direct mail	52,191
Newspapers	46,614
Broadcast TV	46,264
Cable TV	21,527
Radio	19,581
Yellow pages	14,002
Magazines	12,247
Other	51,340

Find the empirical probability that a dollar of advertising is spent on each medium.

<u>*Solution*</u>

Medium	***Expenditures***	***Probabilities***
Direct mail	53,191	$\frac{53191}{263766}=0.1979$
Newspapers	46,614	$\frac{46614}{263766}=0.1767$
Broadcast TV	46,264	$\frac{46264}{263766}=0.1754$
Cable TV	21,527	$\frac{21527}{263766}=0.0816$
Radio	19,581	$\frac{19581}{263766}=0.0742$
Yellow pages	14,002	$\frac{14002}{263766}=0.0531$
Magazines	12,247	$\frac{12247}{263766}=0.0464$
Other	51,340	$\frac{51340}{263766}=0.1946$
$\sum$	263,766	$0.9999 \approx 1$

Properties of Probability

1. The probability of each outcome is a number between 0 and 1.

$$0 \le P_1 \le 1,\ 0 \le P_2 \le 1,\ \ldots,\ 0 \le P_n \le 1$$

2. The sum of the probabilities of all possible outcomes is 1.

$$p_1 + p_2 + \ldots + P_n = 1$$

Example

In a group of n people, what is the probability that at least 2 people have the same birthday (the same month and day, excluding February 29)? Evaluate *P(E)* for $n = 4$

Solution

Birthday: at least 2 same.

$$S = 365.365 \cdots 365 = 365^n$$

E = at least 2 same birthday

E' no people have the same

$$n(E') = 365.364.363 \cdots \big(365 - (n-1)\big) = 366 - n$$

$$n(E') = \frac{365!}{(365-n)!}$$

$$P(E') = \frac{n(E')}{n(S)}$$

$$= \frac{\frac{365!}{(365-n)!}}{365^n}$$

$$= \frac{365!}{365^n (365-n)!}$$

$$P(E) = 1 - P(E') = 1 - \frac{365!}{365^n (365-n)!}$$

Group of $n = 4$: $P(E) = 1 - \dfrac{365!}{365^4 (365-4)!} \approx 0.016$

Exercises *Section* 3.5 – Probability

1. An experiment consists of recording the boy-girl composition of a two-child family.

a) What is an appropriate sample space if we are interested in the genders of the children in the order of their births? Draw a tree diagram.

b) What is an appropriate sample space if we are interested only in the *number* of the girls in a family?

2. Given $S = \{1, 2, 3, \cdots, 17, 18\}$

a) The outcome is a number divisible by 12

b) The outcome is an even number greater than 15

c) Is divisible by 4

d) Is divisible by 5

3. Consider rolling 2 Dice.

a) What is the event that a sum of 5 turns up

b) What is the event that a sum that is a prime number greater than 7 turns up

4. A single fair die is rolled. Find the probability of each event

a) Getting a 2

b) Getting an odd number

c) Getting a number less than 5

d) Getting a number greater than 2

e) Getting a 3 or a 4

f) Getting any number except 3

5. A card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing the following

a) A 9

b) A black card

c) A black 9

d) A heart

e) The 9 of hearts

f) A face card

g) A 2 or a queen

h) A black 7 or red 8

i) A red card or a 10

j) A spade or a king

6. The student sitting next to you in class concludes that the probability of the ceiling falling down on both of you before class ends is 1/2, because there are two possible outcomes - the ceiling will fall or not fall. What is wrong with this reasoning?

7. A jar contains 3 white, 4 orange, 5 yellow, and 8 black marbles. If a marble is drawn at random, find the probability that it is the following.

a) White

b) Orange

c) Yellow

d) Black

e) Not black

f) Orange or Yellow

8. Let consider rolling 2 dice. Find the probabilities of the following events

a) ***E*** = Sum of 5 turns up

b) ***F*** = a sum that is a prime number greater than 7 turns up

9. The board of regents of a university is made up of 12 men and 16 women. If a committee of 6 chosen at random, what is the probability that it will contain 4 men and 2 women?

10. In drawing 7 cards from a 52-card deck without replacement, what is the probability of getting 7 hearts.

11. A committee of 4 people is to be chosen from a group of 5 men and 6 women. What is the probability that the committee will consist of 2 men and 2 women?

E is the set of all possible ways to select 2 men and 2 women

S is the set of all possible ways to select 4 people from 11

12. A department store receives a shipment of 27 new portable radios. There are 4 defective radios in the shipment. If 6 radios are selected for display, what is the probability that 2 of them are defective?

E is the set of all possible ways to have 2 defective and 4 not defective.

S is the set of all possible ways to select 6 radios from 27.

13. Eight cards are drawn from a standard deck of cards. What is the probability that there are 4 face cards and 4 non-face cards?

E is the set of all possible ways to have 4 faces and 4 non-faces.

S is the set of all possible ways to select 8 cards from 52.

14. Five cards are drawn from a standard deck of cards. What is the probability that there are exactly 3 hearts?

15. There are 11 members on the board of directors for the Coca Cola Company.

a) If they must select a chairperson, first vice chairperson, second vice chairperson, and secretary, how many different slates of candidates are possible?

b) If they must form an ethics subcommittee of 4 members, how many different subcommittees are possible?

16. A poll was conducted preceding an election to determine the relationship between voter persuasion concerning a controversial issue and the area of the city in which the voter lives. Five hundred registered voters were interviewed from three areas of the city. The data are shown below. Compute the probability of having no opinion on the issue or living in the inner city.

Area of city	***Favor***	***Oppose***	***No Opinion***
East	30	40	55
North	25	45	50
Inner	95	65	85

17. When testing for current in a cable with five color-coded wires, the author used a meter to test two wires at a time. How many different tests are required for every possible pairing of two wires?

18. Identity theft often begins by someone discovering your 9-digit social security number. Answer each of the following. Express probabilities as fractions.

a) What is the probability of randomly generating 9 digits and getting your social security number?

b) In the past, many teachers posted grades along with the last 4 digits of your social security number, what is the probability that if they randomly generated the order digits, they would match yours? Is that something to worry about?

19. You become suspicious when a genetics researcher randomly selects groups of 20 newborn babies and seems to consistently get 10 girls and 10 boys. The researchers claims that it is common to get 10 girls and 10 boys in such cases,

a) If 20 newborn babies are randomly selected, how many different gender sequences are possible?

b) How many different ways can 10 girls and 10 boys be arranged in sequence?

c) What is the probability of getting 10 girls and 10 boys when 10 babies are born?

20. Two dice are rolled. Find the probabilities of the following events.

a) The first die is 3 or the sum is 8

b) The second die is 5 or the sum is 10.

21. One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) A 9 or 10

b) A red card or a 3

c) A 9 or a black 10

d) A heart or a black card

e) A face card or a diamond

22. One card is drawn from an ordinary of 52 cards. Find the probabilities of drawing the following cards

a) Less than a 4 (count aces as ones)

b) A diamond or a 7

c) A black card or an ace
d) A heart or a jack
e) A red card or a face card

23. Pam invites relatives to a party: her mother, 2 aunts, 3 uncles, 2 brothers, 1 male cousin, and 4 female cousins. If the chances of any one guest first equally likely, find the probabilities that the first guest to arrive is as follows.
a) A brother or an uncle
b) A brother or a cousin
c) A brother or her mother
d) An uncle or a cousin
e) A male or a cousin
f) A female or a cousin

24. The numbers $\{1,2,3,4, and\ 5\}$ are written on slips of paper, and 2 slips are drawn at random one at a time without replacement. Find the probabilities:
a) The sum of the numbers is 9.
b) The sum of the numbers is 5 or less.
c) The first number is 2 or the sum is 6
d) Both numbers are even.
e) One of the numbers is even or greater than 3.
f) The sum is 5 or the second number is 2.

Use Venn diagrams

25. Suppose $P(E)=0.26, \quad P(F)=0.41, \quad P(E\cap F)=0.16$. Find the following
a) $P(E\cup F)=$
b) $P(E'\cap F)=$
c) $P(E\cap F')=$
d) $P(E'\cup F')=$

26. Suppose $P(E)=0.42, \quad P(F)=0.35, \quad P(E\cap F)=0.59$. Find the following
a) $P(E'\cap F')=$
b) $P(E'\cup F')=$
c) $P(E'\cup F)=$
d) $P(E\cap F')=$

27. A single fair die is rolled. Find the odds in favor of getting the results
a) 3
b) 4, 5, or 6
c) 2, 3, 4, or 5
d) Some number less than 6

28. If in repeated rolls of two fair dice the odds against rolling a 6 before rolling a 7 are 6 to 5, what is the probability of rolling a 6 before rolling 7?

29. From survey involving 1,000 people in the certain city, it was found that 500 people had tried a certain brand of diet cola, 600 had tried a certain brand of regular cola, and 200 had tried both types of cola. If a resident of the city is selected at random, what is the empirical probability that

a) The resident has not tried either cola? What are the empirical odds for this event?

b) The resident has tried the diet or has not tried the regular cola? What are the empirical odds against this event?

30. The odds in favor of a particular horse winning a race are 4:5.

a) Find the probability of the horse winning.

b) Find the odds against the horse winning.

31. Consider the sample space of equally likely events for the rolling of a single fair die.

a) What is the probability of rolling an odd number **and** a prime number?

b) What is the probability of rolling an odd number **or** a prime number?

32. Suppose that 2 fair Dice are rolled

a) What is the probability of that a sum of 2 or 3 turns up?

b) What is the probability of that both dice turn up the same or that a sum greater than 8 turns up?

33. A single card is drawn from an ordinary of 52 cards. Calculate the probabilities of and odds for each event

a) A face card or a club is drawn

b) A king or a heart is drawn

c) A black card or an ace is drawn

d) A heart or a number less than 7 (count an ace as 1) is drawn.

34. What is the probability of getting at least 1 black card in a 7-card hand dealt from a standard 52-card deck?

35. What is the probability that a number selected at random from the first 600 positive integers is (exactly) divisible by 6 or 9?

36. What is the probability that a number selected at random from the first 1,000 positive integers is (exactly) divisible by 6 or 8?

37. From a survey involving 1,000 students at a large university, a market research company found that 750 students owned stereos, 450 owned cars, and 350 owned cars and stereos. If a student at the university is selected at random, what is the (empirical) probability that

a) The student owns either a car or a stereo?

b) The student owns neither a car nor a stereo?

38. In order to test a new car, an automobile manufacturer wants to select 4 employees to test drive the car for 1 year. If 12 management and 8 union employees volunteer to be test drivers and the selection is made at random, what is the probability that at least 1 union employee is selected.

39. A shipment of 60 inexpensive digital watches, including 9 that are defective, is sent to a department store. The receiving department selects 10 at random for testing and rejects the whole shipment if 1 or more in the sample are found defective. What is the probability that the shipment will be rejected?

40. If you bet \$5 on the number 13 in roulette, your probability of winning is $\frac{1}{38}$ and the payoff odds are given by the casino as 35:1.

a) Find the actual odds against the outcome of 13.

b) How much net profit would you make if you win by betting on 13?

c) If the casino was not operating for profit, and the payoff odds were changed to match the actual odds against 13, how much would you win if the outcome were 13?