Homework Sec 2.5

1. Verify that $W = \left\{ \left(x_1,\ x_2,\ x_3,\ 0 \right) : x_1,\ x_2,\ x_3 \in \mathbb{R} \right\}$ is a subspace of $V = \mathbb{R}^4$

2. Verify that W is the set of all 2×2 matrices of the form $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$ is a subspace of $V = M_{2,2}$

3. Verify that W is the set of all vectors in $\mathbb{R}^3$ whose the third component is -1 is a subspace in $\mathbb{R}^3$

4. Verify that W is the set of all 3×3 matrices of the form $\begin{bmatrix} 1 & a & b \\ c & 1 & d \\ e & f & 0 \end{bmatrix}$ is a subspace of $V = M_{3,3}$

5. Verify that W is the set of all positive functions: $f(x) > 0$ is a subspace of $C(-\infty,\ \infty)$

6. Verify that W is the set of all $n \times n$ matrices with integer entries is a subspace of $M_{n,n}$

7. Verify that $W = \left\{ (a,\ a - 3b,\ b) : a,\ b \in \mathbb{R} \right\}$ is a subspace of $V = \mathbb{R}^3$

8. Verify that $W = \left\{ \left(x_1,\ x_2,\ x_1 x_2 \right) : x_1,\ x_2 \in \mathbb{R} \right\}$ is a subspace of $V = \mathbb{R}^3$

9. Write each vector as a linear combination of the vectors in S (if possible)

$S = \left\{ (2,\ -1,\ 3),\ (5,\ 0,\ 4) \right\}$

a) $\vec{z} = (-1,\ -2,\ 2)$ | ***b)*** $\vec{v} = \left(8,\ -\frac{1}{4},\ \frac{27}{4} \right)$ | ***c)*** $\vec{w} = (1,\ -8,\ 12)$ | ***d)*** $\vec{u} = (1,\ 1,\ -1)$

10. Write each vector as a linear combination of the vectors in S (if possible)

$S = \left\{ (2,\ 0,\ 7),\ (2,\ 4,\ 5),\ (2,\ -12,\ 13) \right\}$

a) $\vec{u} = (-1,\ 5,\ -6)$ | ***b)*** $\vec{v} = (-3,\ 15,\ 18)$ | ***c)*** $\vec{w} = \left(\frac{1}{3},\ \frac{4}{3},\ \frac{1}{2} \right)$ | ***d)*** $\vec{z} = (2,\ 20,\ -3)$

11. Determine whether the set $S = \left\{ (2,\ 1),\ (-1,\ 2) \right\}$ spans $\mathbb{R}^2$

12. Determine whether the set $S = \left\{ (-3,\ 5) \right\}$ spans $\mathbb{R}^2$

13. Determine whether the set $S = \left\{ (1,\ 3),\ (-2,\ -6),\ (4,\ 12) \right\}$ spans $\mathbb{R}^2$

14. Determine whether the set $S = \left\{ (4,\ 7,\ 3),\ (-1,\ 2,\ 6),\ (2,\ -3,\ 5) \right\}$ spans $\mathbb{R}^3$