Matrix Factorization

$$\boldsymbol{A} = \boldsymbol{LU} = \begin{pmatrix} \textit{lower triangular } L \\ 1's \textit{ on the diagonal} \end{pmatrix} \begin{pmatrix} \textit{upper triangular } U \\ \textit{pivots on the diagonal} \end{pmatrix}$$

$$\boldsymbol{A} = \boldsymbol{LDU} = \begin{pmatrix} \textit{lower triangular } L \\ 1's \textit{ on the diagonal} \end{pmatrix} \begin{pmatrix} \textit{pivot matrix} \\ D \textit{ is diagonal} \end{pmatrix} \begin{pmatrix} \textit{upper triangular } U \\ 1's \textit{ on the diagonal} \end{pmatrix}$$

$\boldsymbol{PA} = \boldsymbol{LU}$ (Permutation matrix P to avoid zeros in the pivot positions).

$\boldsymbol{EA} = \boldsymbol{R}$ (m by m invertible E) (any A) = rref(A)

$\boldsymbol{A} = \boldsymbol{CC^T}$ = (lower triangular matrix C) (transpose is upper triangular)

$\boldsymbol{A} = \boldsymbol{QR}$ = (orthonormal columns in Q) (upper triangular R)

$\boldsymbol{A} = \boldsymbol{S\Lambda S^{-1}}$ = (eigenvectors in S) (eigenvalues in Λ) (left eigenvectors in S^{-1}).

$= \boldsymbol{PDP^{-1}}$ = (eigenvectors in P) (eigenvalues in D) (left eigenvectors in P^{-1}).

$\boldsymbol{A} = \mathbf{Q}D\mathbf{Q}^{\mathbf{T}}$ = (orthogonal matrix Q) (real eigenvalue matrix D) $\left(Q^T \textit{ is } Q^{-1}\right)$

$\boldsymbol{A} = \boldsymbol{MJM^{-1}}$ = (generalized eigenvectors in M) (**J**ordan blocks in J) $\left(\mathrm{M}^{-1}\right)$

$$\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^{\mathbf{T}} = \begin{pmatrix} \textit{orthogonal} \\ U \textit{ is } m \times m \end{pmatrix} \begin{pmatrix} m \times n \textit{ singular value matrix} \\ \delta_1, \cdots, \delta_r \textit{ on its diagonal} \end{pmatrix} \begin{pmatrix} \textit{orthogonal} \\ V \textit{ is } n \times n \end{pmatrix}$$

$$\boldsymbol{A}^{+} = \boldsymbol{V}\Sigma^{+}\boldsymbol{U}^{\mathbf{T}} = \begin{pmatrix} \textit{orthogonal} \\ n \times n \end{pmatrix} \begin{pmatrix} n \times m \textit{ pseudoinverse of } \Sigma \\ 1/\delta_1, \cdots, 1/\delta_r \textit{ on diagonal} \end{pmatrix} \begin{pmatrix} \textit{orthogonal} \\ m \times m \end{pmatrix}$$

$\boldsymbol{A} = \boldsymbol{QH}$ = (orthogonal matrix Q) (symmetric positive definite matrix H)

$\boldsymbol{A} = \boldsymbol{UDU^{-1}}$ = (unitary U) (eigenvalue matrix D) (U^{-1}which $U^H = \overline{U}^T$).

$\boldsymbol{A} = \boldsymbol{UTU^{-1}}$ = (unitary U) (triangular T with λ 's on diagonal) ($U^{-1} = U^H$).

$$\boldsymbol{F_n} = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} \boldsymbol{F_{n/2}} & \\ & \boldsymbol{F_{n/2}} \end{bmatrix} \begin{bmatrix} \textit{even} - \textit{odd} \\ \textit{permutation} \end{bmatrix}$$ = one step of the **FFT**.