8.3 #1 $\sin A = -\frac{3}{5}$ $A \in$ QIII

$\cos A = -\frac{4}{5}$

a) $\sin 2A = 2 \sin A \cos A$

$= 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= \underline{\frac{24}{25}}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{16}{25} - \frac{9}{25}$

$= \underline{\frac{7}{25}}$

c) $\tan 2A = \underline{\frac{24}{7}}$

$180° < \frac{A}{2} < 270° \rightarrow \frac{A}{2} \in$ QII

d) $\sin \frac{A}{2} = \sqrt{\frac{1}{2}(1 - \cos A)}$

$= \sqrt{\frac{1}{2}\left(1 + \frac{4}{5}\right)}$

$= \sqrt{\frac{9}{10}}$

$= \underline{\frac{3}{\sqrt{10}}}$

e) $\cos\frac{A}{2} = -\sqrt{\frac{1}{2}(1+\cos A)}$ ✓

$= -\sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)}$ ✓

$= -\frac{1}{\sqrt{10}}$ ✓

f) $\tan\frac{A}{2} = -3$ ✓

P5x $(\sin\theta+\cos\theta)^2 = 1+\sin 2\theta$

$(\sin\theta+\cos\theta)^2 = \sin^2\theta + 2\cos\theta\sin\theta + \cos^2\theta$

$= 1 + \sin 2\theta$ ✓

P5x $\sin 2x = \frac{2\cot x}{1+\cot^2 x}$

$\frac{2\cot x}{1+\cot^2 x} = 2\,\frac{\frac{\cos x}{\sin x}}{1+\frac{\cos^2 x}{\sin^2 x}}$

$= 2\,\frac{\cos x}{\sin x}\,\frac{\sin^2 x}{\sin^2 x+\cos^2 x}$ $\quad \sin^2 x+\cos^2 x = 1$

$= 2\cos x \sin x$

$= \sin 2x$ ✓

Ex $\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$

$\cos 4x = \cos(2x + 2x)$

$\cos(2(2x)) = \cos^2 2x - \sin^2 2x$

$= 2\cos^2 2x - 1$

$= 2\left(2\cos^2 x - 1\right)^2 - 1$

$= 2\left(4\cos^4 x - 4\cos^2 x + 1\right) - 1$

$= 8\cos^4 x - 8\cos^2 x + 1$ ✓

Ex $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta} = \dfrac{\sin 2\theta}{1+\cos 2\theta}$

$$\frac{1-\cos 2\theta}{\sin 2\theta} = \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$$

$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$ ✓

Ex $\sin^2\frac{x}{2} = \dfrac{\tan x - \sin x}{2\tan x}$

$$\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)\,\frac{\tan x}{\tan x}$$

$$= \frac{\tan x - \cos x \cdot \frac{\sin x}{\cos x}}{2\tan x}$$

$$= \frac{\tan x - \sin x}{2\tan x} \quad \checkmark$$

#20 $\sin 3x = \sin x(3\cos^2 x - \sin^2 x)$

$$\sin 3x = \sin(2x + x)$$

$$= \sin 2x \cos x + \cos 2x \sin x$$

$$= 2\sin x \cos^2 x + (\cos^2 x - \sin^2 x)\sin x$$

$$= \sin x(2\cos^2 x + \cos^2 x - \sin^2 x)$$

$$= \sin x(3\cos^2 x - \sin^2 x) \quad \checkmark$$

25/ $\dfrac{\cos 2x}{\sin^2 x} = \csc^2 x - 2$

$$\frac{\cos 2x}{\sin^2 x} = \frac{1 - 2\sin^2 x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x} - 2\frac{\sin^2 x}{\sin^2 x}$$

$$= \csc^2 x - 2 \quad \checkmark$$

30/ $\cos 4x = \cos^4 x - 6\sin^2 x \cos^2 x + \sin^4 x$

$$\cos 4x = \cos(2(2x))$$

$$= \cos^2(2x) - \sin^2(2x)$$

$$= (\cos^2 x - \sin^2 x)^2 - (2\sin x \cos x)^2$$

$$= \cos^4 x - 2\sin^2 x \cos^2 x + \sin^4 x - 4\sin^2 x \cos^2 x$$

$$= \cos^4 x - 6\sin^2 x \cos^2 x + \sin^4 x \quad \checkmark$$

(c) $\sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right) = \frac{1}{4}\sin^2 x$

$$\sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right) = \frac{1}{2}(1-\cos x)\left(\frac{1+\cos x}{2}\right)$$

$$= \frac{1}{4}(1-\cos^2 x)$$

$$= \frac{1}{4}\sin^2 x \checkmark$$

$$\sin^2\frac{x}{2}\cos^2\frac{x}{2} = \left(\sin\frac{x}{2}\cos\frac{x}{2}\right)^2$$

$$= \left(\frac{1}{2}\sin x\right)^2$$

$$= \frac{1}{4}\sin^2 x \checkmark$$

(d) $\sec^2\left(\frac{x}{2}\right) = \frac{2\sec x + 2}{\sec x + 2 + \cos x}$

c) if

$$\sec^2\frac{x}{2} = \frac{2\sec x + 2}{\sec x + 2 + \cos x}$$

$$\sec^2\frac{x}{2} = \frac{1}{\cos^2\frac{x}{2}} \qquad \cos^2\frac{x}{2} = \frac{1+\cos x}{2}$$

$$= \frac{2}{1+\cos x}\,\frac{1+\sec x}{1+\sec x}$$

$$= \frac{2 + 2\sec x}{1 + \sec x + \cos x + \underbrace{\cos x \sec x}_{=1}}$$

$$= \frac{2 + 2\sec x}{\sec x + 2 + \cos x} \quad \checkmark$$

8.4 Solving Trig. eqns

Equations

$f(\ ) = a(\ )^2 + b(\ ) + c$

$a + b + c = 0 \rightarrow \begin{cases} ① = 1 \\ ② = \frac{c}{a} \end{cases}$

$a - b + c = 0 \rightarrow \begin{cases} ① = -1 \\ ② = -\frac{c}{a} \end{cases}$

$2\cos^2 x - 3\cos x + 1 = 0$

$\cos x = 1$ $\qquad$ $\cos x = \frac{1}{2}$

$x = 0$ $\qquad$ $\frac{\pi}{3}, \frac{5\pi}{3}$

$2\cos^2 x + 3\cos x + 1 = 0$ $\quad (-3)$

$\cos x = -1$ $\qquad$ $\cos x = -\frac{1}{2}$

$x = \pi$ $\qquad$ $\frac{2\pi}{3}, \frac{4\pi}{3}$

Q I $\quad \theta = \frac{\pi}{n}$

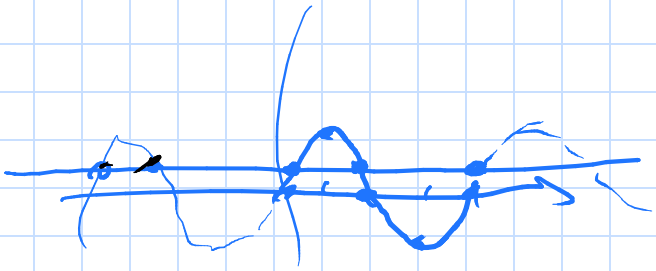
Q II $\quad \frac{(n-1)\pi}{n}$

Q III $\quad \frac{(n+1)\pi}{n}$

Q IV $\quad \frac{(2n-1)\pi}{n}$

Ex $\quad \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$ ✓



a) $[0, 2\pi) \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

b) $\theta \in \mathbb{R}$

$\theta = \frac{\pi}{6} + 2n\pi \qquad \theta = \frac{5\pi}{6} + 2n\pi$

Ex $\quad \sin x \tan x = \sin x$

$\sin x \tan x - \sin x = 0$

$\sin x (\tan x - 1) = 0$ $\quad \tan x - 1 = 0$

$\sin x = 0 \qquad \tan x = 1$

$x = 0, \pi \qquad \frac{\pi}{4}, \frac{5\pi}{4}$

$x = n\pi, \quad \frac{\pi}{4} + n\pi$

Ex $2\sin^2 t - \cos t - 1 = 0$

$2(1-\cos^2 t) - \cos t - 1 = 0$

$2 - 2\cos^2 t - \cos t - 1 = 0$

$-2\cos^2 t - \cos t + 1 = 0$ $\quad -1 \mp (-1)$

$\cos t = -1 \qquad \cos t = +\frac{1}{2}$

$t = \pi \quad , \frac{\pi}{3}, \frac{5\pi}{3} \quad [0, 2\pi)$

$t = \pi + 2n\pi \qquad t = \frac{\pi}{3} + 2n\pi$

$= (2n+1)\pi \qquad t = \frac{5\pi}{3} + 2n\pi$