Solution *Section* 4.1 – Inferences about Two Population Portions

Exercise

A Student surveyed her friends and found that among 20 males, 4 smoke and among 30 female, 6 smoke. Give two reasons why these results should not be used for a hypothesis test of the claim that the proportions of male smokers and female smokers are equal.

Solution

There are two requirements for using the methods of this section, and each of them is violated.

i. The samples should be 2 sample random samples that are independent. These samples are convenience samples, not simple random samples. These samples are likely not independent. Since she surveyed her friends, she may well have males and females that are dating each other (or least that associate with each other) – and people tend to associate with those that have similar behaviors.

ii. The number of successes for each sample should be at least 5, and the number of failures for each sample be at least 5. This is not true for the males, for which $x = 4$.

Using $\hat{p} = \frac{x}{n}$ to estimate p and $\hat{q} = 1 - \frac{x}{n} = \frac{n-x}{n}$ to estimate q.

$$n\hat{p} \ge 5 \qquad\qquad n\hat{q} \ge 5$$

$$n\left(\frac{x}{n}\right) \ge 5 \qquad\qquad n\left(\frac{n-x}{n}\right) \ge 5$$

$$x \ge 5 \qquad\qquad (n-x) \ge 5$$

These inequalities state that the number of successes must be greater than 5, and the number of failures must be greater than 5.

Exercise

In clinical trials of the drug Zocor, some subjects were treated with Zocor and other were given a placebo. The 95% confidence interval estimate of the difference between the proportions of subjects who experienced headaches is $-0.0518 < p_1 - p_2 < 0.0194$. Write a statement interpreting that confidence interval.

Solution

We have 95% confidence that the limits of –0.0518 and 0.01094 contain the true difference between the population proportions of subjects who experience headaches. Repeating the trials many times would result in confidence limits that would include the true difference between the population proportions 95% of the time. Since the interval includes the value 0, there is no significant difference between the two population proportions.

Exercise

Among 8834 malfunctioning pacemakers, in 15.8% the malfunctions were due to batteries. Find the number of successes *x*.

Solution

$$x = (0.158)(8834) \ \underline{\approx 1396}$$

Exercise

Among 129 subjects who took Chantix as an aid to stop smoking, 12.4% experienced nausea. Find the number of successes *x*.

Solution

$$x = (0.124)(129) \ \underline{\approx 16}$$

Exercise

Among 610 adults selected randomly from among the residents of one town, 26.1% said that they have favor stronger gun-control laws. Find the number of successes *x*.

Solution

$$x = (610)(0.261) \ \underline{\approx 159}$$

Exercise

A computer manufacturer randomly selects 2,410 of its computers for quality assurance and finds that 3.13% of these computer are found defective. Find the number of successes *x*.

Solution

$$x = (2,410)(0.0313) \ \underline{\approx 67}$$

Exercise

Assume that you plan to use a significance level of $\alpha = 0.05$ to test the claim that $p_1 = p_2$. Use the given sample sizes and number of successes to find the pooled estimate $\overline{p}$

a) $n_1 = 288, \quad n_2 = 252, \quad x_1 = 75, \quad x_2 = 70$

b) $n_1 = 100, \quad n_2 = 100, \quad \hat{p}_1 = 0.2, \quad \hat{p}_2 = 0.18$

Solution

a) $\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{75}{288} = 0.26 \qquad \hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{70}{252} = 0.278$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{75+70}{288+252} \underline{= 0.269|}$$

b) $x_1 = n_1\hat{p}_1 = (100)(0.2) = 20 \qquad x_2 = n_2\hat{p}_2 = (100)(0.18) = 18$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{20+18}{100+100} \underline{= 0.19|}$$

Exercise

The numbers of online applications from simple random samples of college applications for 2003 and for the current year are given below.

	2003	Current Year
Number of application in sample	36	27
Number of online applications in sample	13	14

Assume that you plan to use a significance level of $\alpha = 0.05$ to test the claim that $p_1 = p_2$. Find

a) The pooled estimate $\bar{p}$

b) The *x* test statistic

c) The critical *z* values

d) The *P*–value

Assume 95% confidence interval

e) The margin of error E

f) The 95% confidence interval.

Solution

$\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{13}{36} = 0.361 \qquad \hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{14}{27} = 0.519$

$\hat{p}_1 - \hat{p}_2 = 0.361 - 0.519 = -0.157$

a) $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{13+14}{36+27} = \dfrac{27}{63} \underline{= 0.429|}$

b)
$$z_{\hat{p}_1 - \hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\overline{pq}}{n_1} + \dfrac{\overline{pq}}{n_2}}}$$

$$= \frac{-0.157 - 0}{\sqrt{\dfrac{(0.429)(0.571)}{36} + \dfrac{(0.429)(0.571)}{27}}}$$

$\underline{= -1.25|}$

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575

c) For $\alpha = 0.05$, the critical values are

$$z = \pm z_{\alpha/2} = \pm z_{0.025} \underline{= \pm 1.96}$$

d) $P - value = 2 \cdot P(z < -1.25)$

z	.00	.01	.02	.03	.04	.05
−1.2	.1151	.1131	.1112	.1093	.1075	.1056

$$= 2(0.1056)$$

$$\underline{= 0.2112}$$

e) $E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

$$= 1.96\sqrt{\frac{(0.361)(0.639)}{36} + \frac{(0.519)(0.481)}{27}}$$

$$\underline{= 0.2452}$$

f) $(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$

$$-0.1574 - 0.2452 < p_1 - p_2 < -0.1574 + 0.2452$$

$$\underline{-0.4026 < p_1 - p_2 < 0.0878}$$

Exercise

Chantix is a drug used as an aid to stop smoking. The numbers of subjects experiencing insomnia for each of two treatment groups in a clinical trial of the drug Chantix are given below:

	Chantix Treatment	Placebo
Number in group	129	805
Number experiencing insomnia	19	13

Assume that you plan to use a significance level of $\alpha = 0.05$ to test the claim that $p_1 = p_2$. Find

a) The pooled estimate $\bar{p}$

b) The *x* test statistic

c) The critical *z* values

d) The *P*–value

Assume 95% confidence interval

e) The margin of error E

f) The 95% confidence interval.

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{19}{129} = 0.147 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{13}{805} = 0.016$$

$$\hat{p}_1 - \hat{p}_2 = 0.147 - 0.016 = 0.131$$

a) $\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{19 + 13}{129 + 805} = \frac{32}{934} \underline{= 0.0343}$

b) $z_{\hat{p}_1-\hat{p}_2} = \dfrac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\dfrac{\overline{pq}}{n_1}+\dfrac{\overline{pq}}{n_2}}}$

$$= \frac{0.131-0}{\sqrt{\dfrac{(0.0343)(0.9657)}{129}+\dfrac{(0.0343)(0.9657)}{805}}}$$

$$\underline{= 7.60|}$$

c) For $\alpha = 0.05$, the critical values are

$\dfrac{\alpha}{2} = \dfrac{1-0.05}{2} = 0.025 \quad \Rightarrow A = 1-0.025 = 0.975$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$z = \pm z_{\alpha/2} = \pm z_{0.025}$

$\underline{= \pm 1.96|}$

d) $P-value = 2\cdot P(z > 7.60)$

$= 2(1-0.9999)$

$\underline{= 0.0002|}$

3.50 and up	.9999

e) $E = z_{\alpha/2}\sqrt{\dfrac{\hat{P}_1\hat{q}_1}{n_1}+\dfrac{\hat{P}_2\hat{q}_2}{n_2}}$

$= 1.96\sqrt{\dfrac{(0.147)(0.853)}{129}+\dfrac{(0.147)(0.853)}{805}}$

$\underline{= 0.0618|}$

f) $(\hat{p}_1-\hat{p}_2)-E < (p_1-p_2) < (\hat{p}_1-\hat{p}_2)+E$

$0.1311-0.0618 < p_1-p_2 < 0.1311+0.0618$

$\underline{0.0694 < p_1-p_2 < 0.1929|}$

Exercise

In a 1993 survey of 560 college students, 171 said that they used illegal drugs during the previous year. In a recent survey of 720 college students, 263 said that they used illegal drugs during the previous year. Use a 0.05 significance level to test the claim that the proportion of college students using illegal drugs in 1993 was less than it is now.

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{171}{560} = 0.305 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{263}{720} = 0.365$$

$$\hat{p}_1 - \hat{p}_2 = 0.305 - 0.365 = -0.60$$

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$= \frac{170 + 263}{560 + 720} \qquad (170 + 263)/(560 + 720)$$

$$\underline{= 0.339}$$

Original Claim: $p_1 - p_2 < 0$

$H_0 : p_1 - p_2 = 0$

$H_1 : p_1 - p_2 < 0$

z score	Area
1.645	0.9500
2.575	0.9950

$\alpha = 0.05$, the critical value

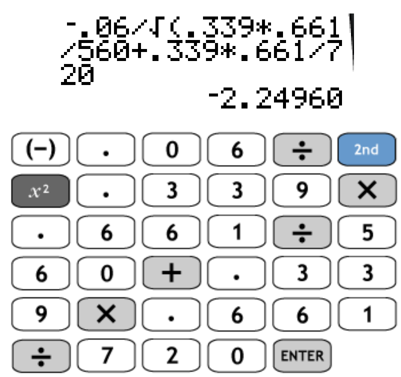
$z = -z_\alpha = -z_{0.05}$

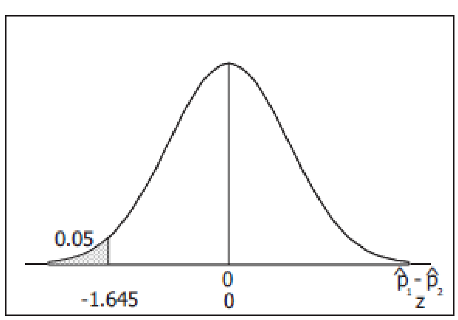
$\underline{= -1.645}$

$$z_{\hat{p}_1 - \hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p}\overline{q}}{n_1} + \frac{\overline{p}\overline{q}}{n_2}}}$$

$$= \frac{-0.060 - 0}{\sqrt{\frac{(0.339)(0.661)}{560} + \frac{(0.339)(0.661)}{720}}}$$

$\underline{= -2.25}$





$P - value = P(z < -2.52)$

$\underline{= 0.0122}$

z	.00	.01	.02	.03	.04	.05
-2.2	.0139	.0136	.0132	.0129	.0125	.0122

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 < 0$. There is sufficient evidence to support the claim that the proportion of college students using illegal drugs in 1993 was less than it is now.

Exercise

In a 1993 survey of 560 college students, 171 said that they used illegal drugs during the previous year. In a recent survey of 720 college students, 263 said that they used illegal drugs during the previous year. Construct the confidence interval corresponding to the hypothesis test conducted with a 0.05 significance level. What conclusion does the confidence interval suggest?

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{171}{560} = 0.305 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{263}{720} = 0.365$$

$$\hat{p}_1 - \hat{p}_2 = 0.305 - 0.365 = -0.60$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sigma_{\hat{p}_1 - \hat{p}_2}$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1\bar{q}_1}{n_1} + \frac{\bar{p}_2\bar{q}_2}{n_2}}$$

$$-0.0599 \pm 1.645\sqrt{\frac{(0.305)(.695)}{560} + \frac{(0.365)(.935)}{720}}$$

$$-0.0599 \pm 0.0480$$

$$-0.0599 - 0.0480 < p_1 - p_2 < -0.0599 + 0.0480$$

$$-0.1079 < p_1 - p_2 < -0.0119$$

Since the confidence interval does not include the value 0, it suggests that the two population proportions are not equal and that the proportion of college students using illegal drugs in 1993 was less than it is now.

Exercise

A simple random sample of front-seat occupants involved in car crashes is obtained. Among 2823 occupants not wearing seat belts, 31 were killed. Among 7765 occupants wearing seat belts, 16 were killed. Construct a 90% confidence interval estimate of the difference between the fatality rates for those not wearing seat belts and those wearing seat belts. What does the result suggest about the effectiveness of seat belts?

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{31}{2823} = 0.01098 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{16}{7765} = 0.00206$$

$$\hat{p}_1 - \hat{p}_2 = 0.01098 - 0.00206 = 0.00892$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sigma_{\hat{p}_1 - \hat{p}_2}$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1\bar{q}_1}{n_1} + \frac{\bar{p}_2\bar{q}_2}{n_2}}$$

$$-0.00892 \pm 1.645\sqrt{\frac{(.01098)(.98902)}{2823} + \frac{(.00206)(.99794)}{7765}}$$

$$-0.00892 \pm 0.00334$$

$$-0.00892 - 0.00334 < p_1 - p_2 < -0.00892 + 0.00334$$

$$0.00558 < p_1 - p_2 < 0.01226$$

Since the confidence interval does not include the value 0, it suggests that the two population proportions are not equal and that seat belts are effective because the proportion of non-users who killed is greater than the proportion of users who are killed.

Exercise

A Pew Research Center poll asked randomly selected subjects if they agreed with the statement that "It is morally wrong fir married people to have an affair" Among the 386 women surveyed, 347 agrees with the statement. Among the 359 men surveyed, 305 agreed with the statement.

a) Use a 0.05 significance level to test the claim that the percentage of women who agree is difference from the percentage of men who agree. Does there appear to be a difference in the way women and men feel about this issue?

b) Construct the confidence interval corresponding to the hypothesis test conducted with a 0.05 significance level. What conclusion does the confidence interval suggest?

Solution

a) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{347}{386} = 0.899 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{305}{359} = 0.850$

$$\hat{p}_1 - \hat{p}_2 = 0.899 - 0.85 = 0.049$$

$$\bar{p} = \frac{437 + 305}{386 + 359} = 0.875$$

Original Claim: $p_1 - p_2 \neq 0$

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 \neq 0$$

$$\alpha = 0.05,\ \frac{\alpha}{2} = \frac{1 - 0.05}{2} = 0.025 \quad \Rightarrow A = 1 - 0.025 = 0.975$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

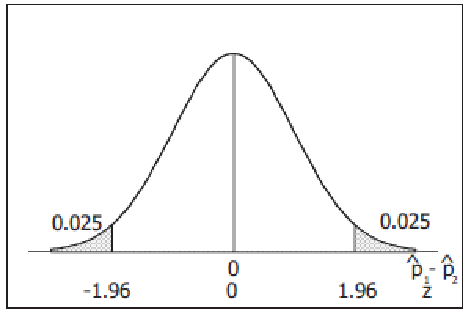
$$z = \pm z_{\alpha/2} = \pm z_{0.025}$$

$$= \pm 1.96$$

$$z_{\hat{p}_1 - \hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{0.049 - 0}{\sqrt{\frac{(.875)(.125)}{386} + \frac{(.875)(.125)}{359}}}$$

$$= 2.04$$



$$P - value = 2 \cdot P(z > 2.04)$$
$$= 2 \cdot (1 - 0.9793)$$
$$= 0.0414$$

z	.00	.01	.02	.03	.04	.05
2.0	.9772	.9778	.9783	.9788	.9793	

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 \neq 0$ (in fact, that $p_1 - p_2 > 0$. There is sufficient evidence to support the claim that the percentage of women who agree is different from the percentage of men who agree. Yes; there does appear to be a difference in the way that women and men feel about the issue.

b) $(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sigma_{\hat{p}_1 - \hat{p}_2}$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1\bar{q}_1}{n_1} + \frac{\bar{p}_2\bar{q}_2}{n_2}}$$

$$0.04938 \pm 1.96\sqrt{\frac{(.899)(.101)}{386} + \frac{(.85)(.15)}{359}}$$

$$0.04938 \pm 0.04766$$

$$0.04938 - 0.04766 < p_1 - p_2 < 0.04938 + 0.04766$$

$$0.00172 < p_1 - p_2 < 0.09704$$

Since the confidence interval does not include the value 0, it suggests that the two population proportions are not equal and the percentage of women who agree is different from the percentage of men who agree. Since the interval includes only positive values, conclude that the percentage of women who agree is greater than the percentage of men who agree.

Exercise

Tax returns include an option of designating \$3 for presidential election campaigns, and it does not cost the taxpayer anything to make that designation. In a simple random sample of 250 tax returns from 1976, 27.6% of the returns designated the \$3 for the campaign. In a simple random sample of 300 recent tax returns, 7.3% of the returns designated the \$3 for the campaign. Use a 0.05 significance level to test the claim that the percentage of returns designating the \$3 for the campaign was greater in 1973 than it is now.

Solution

$$x_1 = (.276)(250) = 69 \qquad x_2 = (.073)(300) = 22$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{69}{250} = 0.276 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{22}{300} = 0.073$$

$$\hat{p}_1 - \hat{p}_2 = 0.276 - 0.073 = 0.203$$

$$\overline{p} = \frac{69+22}{250+300} = 0.165$$

Original Claim: $p_1 - p_2 > 0$

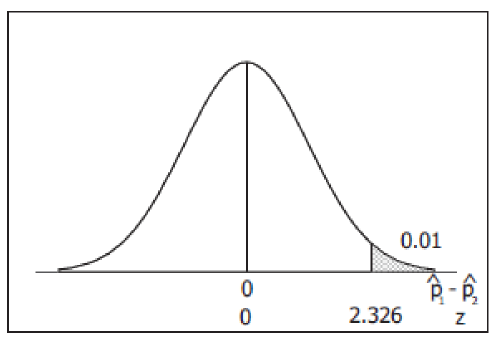
$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 > 0$$

$\alpha = 0.01$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.9893	.9896	.9898	.9901	.9904					

$z = z_\alpha = z_{0.01} = 2.326$



$$z_{\hat{p}_1 - \hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p}\overline{q}}{n_1} + \frac{\overline{p}\overline{q}}{n_2}}}$$

$$= \frac{0.203 - 0}{\sqrt{\frac{(.165)(.835)}{250} + \frac{(.165)(.835)}{300}}}$$

$$= 6.37$$

3.50 and up	.9999

$$P - value = P(z > 6.37)$$

$$= 1 - 0.9999$$

$$= 0.0001$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 > 0$. There is sufficient evidence to support the claim that the percentage of returns designated funds for campaigns was greater on 1976 than it is now.

Exercise

In an experiment, 16% of 734 subjects treated with Viagra experienced headaches. In the same experiment, 4% of 725 subjects given a placebo experienced headaches.

a) Use a 0.01 significance level to test the claim that the proportion of headaches is greater for those treated with Viagra. Do headaches appear to be a concern for those who take Viagra?

b) Construct the confidence interval corresponding to the hypothesis test conducted with a 0.01 significance level. What conclusion does the confidence interval suggest?

Solution

a) $\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{x_1}{734} = 0.16 \qquad \hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{x_2}{725} = 0.04$

$$\hat{p}_1 - \hat{p}_2 = 0.16 - 0.04 = 0.12$$

$$\overline{p} = \frac{(.16)(734) + (.04)(725)}{734 + 725} = 0.100$$

Original Claim: $p_1 - p_2 > 0$

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 > 0$$

$\alpha = 0.01$

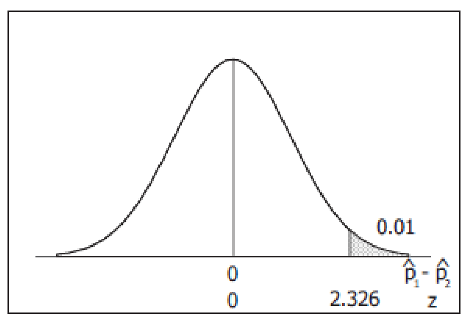
z	.00	.01	.02	.03	.04	.05
2.3	.9893	.9896	.9898	.9901	.9904	

$z = z_\alpha = z_{0.01} = 2.326$

$$z_{\hat{p}_1 - \hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\overline{p}\overline{q}}{n_1} + \dfrac{\overline{p}\overline{q}}{n_2}}}$$

$$= \frac{0.12 - 0}{\sqrt{\dfrac{(0.1)(0.9)}{734} + \dfrac{(0.1)(0.9)}{725}}}$$

$$= 7.63$$



3.50 and up	.9999

$$P - value = P(z > 7.63)$$

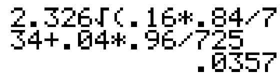
$$= 1 - 0.9999$$

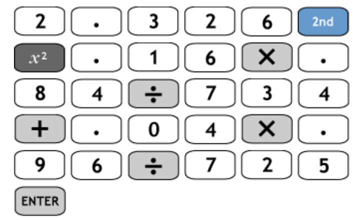
$$= 0.0001$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 > 0$. There is sufficient evidence to support the claim that the proportion of persons experiencing headaches is greater for those treated with Viagra. Yes; headaches do appear to be a concern for those who take Viagra.

b) $\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\dfrac{\bar{p}_1\bar{q}_1}{n_1} + \dfrac{\bar{p}_2\bar{q}_2}{n_2}}$





$$0.12 \pm 2.326\sqrt{\frac{(.16)(.84)}{734} + \frac{(.04)(.96)}{725}}$$

$$0.12 \pm 0.0357$$

$$0.12 - 0.0357 < p_1 - p_2 < 0.12 + 0.0357$$

$$0.0843 < p_1 - p_2 < 0.1557$$

Since the confidence interval does not include the value 0, there is a significant difference the two proportions. Since the confidence interval includes only positive values, the proportion of persons experiencing headaches is greater for those treated with Viagra.

Exercise

Two different simple random samples are drawn from two different populations. The first sample consists of 20 people with 10 having a common attribute. The second sample consists of 2000 people with 1404 of them having the same common attribute. Compare the results from a hypothesis test of $p_1 = p_2$ (with a 0.05 significance level) and a 95% confidence interval estimate of $p_1 - p_2$.

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{10}{20} = 0.5 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{1404}{2000} = 0.702$$

$$\hat{p}_1 - \hat{p}_2 = 0.5 - 0.702 = -0.202$$

$$\bar{p} = \frac{10 + 1404}{20 + 2000} = 0.70$$

Original Claim: $p_1 - p_2 \neq 0$

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 \neq 0$$

$$\alpha = 0.05, \ \frac{\alpha}{2} = \frac{1 - 0.05}{2} = 0.025 \quad \Rightarrow A = 1 - 0.025 = 0.975$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

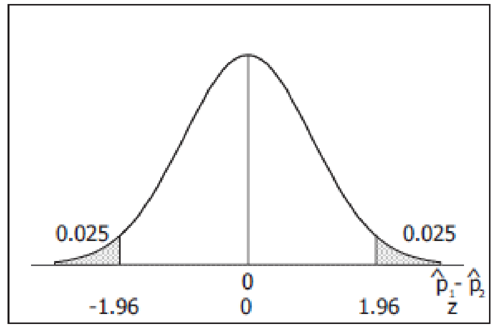
$$z = \pm z_{\alpha/2} = \pm z_{0.025}$$

$$\underline{= \pm 1.96}$$

$$z_{\hat{p}_1-\hat{p}_2} = \frac{\left(\hat{p}_1-\hat{p}_2\right)-\left(p_1-p_2\right)}{\sqrt{\frac{\overline{p}\overline{q}}{n_1}+\frac{\overline{p}\overline{q}}{n_2}}}$$

$$= \frac{-0.202-0}{\sqrt{\frac{(0.7)(0.3)}{20}+\frac{(0.7)(0.3)}{2000}}}$$

$$\underline{=-1.9615}$$



$$P-value = 2 \cdot P\left(z<-1.96\right)$$
$$= 2 \cdot (.025)$$
$$\underline{\approx 0.05}$$

z	.00	.01	.02	.03	.04	.05	.06
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250

Conclusion:

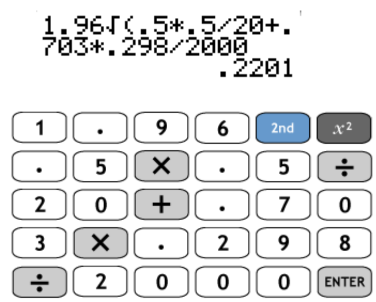
Reject H_0; there is sufficient evidence to conclude that $p_1 - p_2 = 0$ and conclude that $p_1 - p_2 \le 0$ (in fact, that $p_1 - p_2 < 0$).

The confidence interval is:

$$\left(\hat{p}_1-\hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\overline{p}_1\overline{q}_1}{n_1}+\frac{\overline{p}_2\overline{q}_2}{n_2}}$$

$$-0.202 \pm 1.96\sqrt{\frac{(0.5)(0.5)}{20}+\frac{(0.702)(0.298)}{2000}}$$

$$-0.202 \pm 0.220$$
$$-0.202-0.220 < p_1 - p_2 < -0.202+0.220$$
$$-0.422 < p_1 - p_2 < 0.018$$



Since the confidence interval includes the value 0, p_1 *and* p_2 could have the same values and one should not reject the claim that $p_1 - p_2 = 0$.

The test of hypothesis and the confidence interval lead to different conclusions. In this instance, they are not equivalent.

Exercise

A report on the nightly news broadcast stated that 11 out of 142 households with pet dogs were burglarized and 21 out of 217 without pet dogs were burglarized. Find the *z* test statistic for the hypothesis test.

Assume that you plan to use a significance level of $\alpha = 0.05$ to test the claim that $p_1 = p_2$.

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{11}{142} = 0.0775 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{21}{217} = 0.0968$$

$$\hat{p}_1 - \hat{p}_2 = .0775 - .0968 = -0.0193$$

$$\overline{p} = \frac{11+21}{142+217} = 0.089$$

$$z = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\frac{\overline{p}\overline{q}}{n_1} + \frac{\overline{p}\overline{q}}{n_2}}}$$

$$= \frac{-.0193 - 0}{\sqrt{\frac{(.089)(0.911)}{142} + \frac{(.089)(0.911)}{217}}}$$

$$\boxed{= -0.62}$$

Exercise

Assume that the samples are independent and that they have been randomly selected. Construct a 90% confidence interval for the difference between population proportions $p_1 = p_2$

$n_1 = 39, \quad n_2 = 50, \quad x_1 = 13, \quad x_2 = 28$

Solution

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{13}{39} = 0.3333 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{28}{50} = 0.56$$

$$\hat{p}_1 - \hat{p}_2 = 0.33 - 0.56 = -0.23$$

$$\overline{p} = \frac{13+28}{39+50} = 0.461$$

$$A = 0.9 \quad \Rightarrow \; z = 1.645$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\overline{p}_1\overline{q}_1}{n_1} + \frac{\overline{p}_2\overline{q}_2}{n_2}} = -0.23 \pm 1.645\sqrt{\frac{(0.461)(0.539)}{39} + \frac{(0.461)(0.539)}{50}}$$

$$-0.23 \pm 0.175$$

$$-0.23 - 0.175 < p_1 - p_2 < -0.23 + 0.175$$

$$\boxed{-0.405 < p_1 - p_2 < -0.055}$$

Exercise

The sample size needed to estimate the difference between two population proportions ti within a margin of error E with a confidence level of $1 - \alpha$ can be found as follows:

$$E = z_{\alpha/2}\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}.$$

In this expression, replace n_1 and n_2 by n (assuming both samples have the same size) and replace each of p_1, q_1, p_2 *and* q_2 by 0.5 (because their values are not known). Then solve for n.

Use this approach to find the size pf each sample of you want to estimate the difference between the proportions of men and women who plan to vote in the next presidential election. Assume that you want 99% confidence that your error is no more than 0.05.

Solution

$$A = 99\% = 0.99 \quad \Rightarrow \; z = 2.575$$

$$E = z_{\alpha/2}\sqrt{2\frac{p_1 q_1}{n}} \qquad p_1 = q_1 = p_2 = q_2 = 0.5$$

$$0.05 = 2.575\sqrt{2\frac{(0.5)^2}{n}}$$

$$\frac{0.05}{2.575} = \sqrt{\frac{2(0.5)^2}{n}}$$

$$\left(\frac{0.05}{2.575}\right)^2 = \frac{2(0.5)^2}{n}$$

$$n = 2(0.5)^2\left(\frac{2.575}{0.05}\right)^2 \quad \underline{\approx 1,327}$$

Solution ***Section* 4.2 – Inferences about Two Means: Dependent**

Exercise

Listed below are the time intervals (in minutes) before and after eruptions of the Old Faithful geyser. Find the values of $\bar{d}$ and s_d. In general, what does μ_d represent?

Time interval before eruption	98	92	95	87	96
Time interval after eruption	92	95	92	100	90

Solution

The difference values are:

	98	92	95	87	96
	92	95	92	100	90
Difference = d	6	−3	3	−13	6
d^2	36	9	9	169	36

$n=5; \qquad \sum d = 6-3+3-13+6=-1; \quad \sum d^2 = 36+9+9+169+36=259$

$$\bar{d}=\frac{\sum d}{n}=\frac{-1}{5}\ \underline{=-0.2\ \text{min}}$$

$$\begin{aligned} s_d^2 &= \frac{n\sum d^2-\left(\sum d\right)^2}{n(n-1)} \\ &= \frac{5(259)-(-1)^2}{5(4)} \\ &= 64.7 \end{aligned}$$

```
(5*259-(-1)²)/(5*4)
                64.7000
```

$$s_d=\sqrt{64.7}\ \underline{=8.0\ \text{min}}$$

In general, μ_d represents the true mean of the differences from the population of matched pairs (which is mathematically equivalent to the true of the difference between the means of the two populations).

Exercise

Listed below are measured fuel consumption amount (in miles/gal) from a sample of cars.

City fuel consumption	18	22	21	21
Highway fuel consumption	26	31	29	29

Assume that wou want to use a 0.05 significance level to test the claim that the paired sample data come from a population for which the mean differecne is $\mu_d = 0$. Find

a) $\bar{d}$

b) s_d

c) The t test statistic

d) The critical values.

Solution

The difference values are:

	18	22	21	21
	26	31	29	29
Difference = d	-8	-9	-8	-8
d^2	64	81	64	64

$n = 4; \qquad \sum d = -8-9-8-8 = -33; \quad \sum d^2 = 64+81+64+64 = 273$

a) $\bar{d} = \dfrac{\sum d}{n} = \dfrac{-33}{4} \underline{= -8.3\ mpg|}$

b) $s_d^2 = \dfrac{n\sum d^2 - \left(\sum d\right)^2}{n(n-1)} = \dfrac{4(273)-(-33)^2}{4(3)} = 0.25$

$s_d = \sqrt{0.25} \underline{= .5\ mpg|}$

c) $t_{\bar{d}} = \dfrac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{-8.25 - 0}{\frac{0.5}{\sqrt{4}}} = -33.00$

d) With $df = 3 \quad and \quad \alpha = 0.05$, the critical values are $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 3.182$

Exercise

Listed below are predicted high temperatures that were forecast different days.

Predicted high temperatures forecast 3 *days ahead*	79	86	79	83	80
Predicted high temperatures forecast 5 *days ahead*	80	80	79	80	79

Assume that wou want to use a 0.05 significance level to test the claim that the paired sample data come from a population for which the mean differecne is $\mu_d = 0$. Find

a) $\bar{d}$

b) s_d

c) The t test statistic

d) The critical values.

Solution

The difference values are:

	79	83	79	83	80
	80	80	79	80	79
Difference = d	–1	6	0	3	1
d^2	1	36	0	9	1

$n = 4; \qquad \sum d = -1+6+0+3+1 = 9; \quad \sum d^2 = 1+36+0+9+1 = 47$

a) $\bar{d} = \dfrac{\sum d}{n} = \dfrac{9}{5} = 1.8\ {}^\circ F$

b) $s_d^2 = \dfrac{n\sum d^2 - \left(\sum d\right)^2}{n(n-1)} = \dfrac{5(47)-(9)^2}{5(4)} = 7.7$

$s_d = \sqrt{7.7} = 2.8\ {}^\circ F$

c) $t_{\bar{d}} = \dfrac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{1.8-0}{\frac{2.7749}{\sqrt{5}}} = 1.45$

d) With $df = 4 \quad and \quad \alpha = 0.05$, the critical values are $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.776$

Exercise

Listed below are body mass indices (BMI). The BMI of each student was measured in September and April of the freshman year.

a) Use a 0.05 significance level to tet the claim that the mean change in BMI for all students is equal to 0. Does BMI appear to change during freshman year?

b) Construct a 95% confidence interval estimate of the change in BMI during freshman year. Does the confidence interval include 0, and what does that suggest about BMI during freshman year?

April BMI	20.15	19.24	20.77	23.85	21.32
September BMI	20.68	19.48	19.59	24.57	20.96

Solution

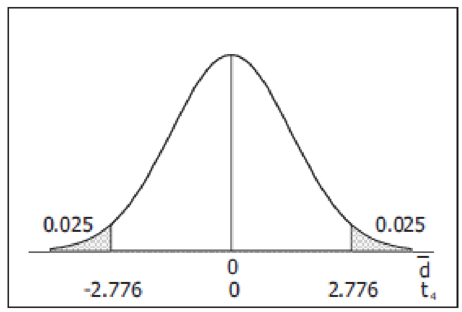
	20.15	19.24	20.77	23.85	21.32
	20.68	19.48	19.59	24.57	20.96
Difference = d	–0.53	–0.24	1.18	–0.72	0.36
d^2	.2809	.0576	1.3924	.5184	.1296

$n = 5; \qquad \sum d = -53 - .24 + 1.18 - .72 + .36 = .05; \quad \sum d^2 = 2.3789$

a) Original claim: $\mu_d = 0$

$$H_0 : \mu_d = 0$$

$$H_1 : \mu_d \neq 0$$



$df = 4 \quad and \quad \alpha = 0.05$

$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.776$

$$s_d = \sqrt{\frac{n\sum d^2 - \left(\sum d\right)^2}{n(n-1)}}$$

$$= \sqrt{\frac{5(2.3789) - (0.05)^2}{5(4)}}$$

$$\underline{= 0.7711}$$

$$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{0.01 - 0}{\frac{0.7711}{\sqrt{5}}} \underline{= 0.029}$$

$P - value = 2 \cdot tcdf\,(0.029,\ 99,\ 4)\ \underline{= 0.9783}$

Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim $\mu_d = 0$. There is not sufficient evidence to reject the claim that the mean change in BMI for all students is equal to 0.

No; BMI does not appear to change during the freshman year.

b) $\bar{d} - E < \mu_d < \bar{d} + E$

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

$$0.01 - 2.776\left(\frac{0.771}{\sqrt{5}}\right) < \mu_d < 0.01 + 2.776\left(\frac{0.771}{\sqrt{5}}\right)$$

$$-0.947 < \mu_d < 0.967$$

Yes; the confidence interval includes 0, which suggests that the mean of the differences could be 0 and that there is no change in BMI during the freshman year

Exercise

Listed below are body temperature (in °F) of subjects measured at 8:00 AM and at 12:00 AM. Construct a 95% confidence interval estimate of the difference between the 8:00 AM temperatures and the 12:00 AM temperatures. Is body temperature basically the same at both times?

8:00 AM	97.0	96.2	97.6	96.4	97.8	99.2
12:00 AM	98.0	98.6	98.8	98.0	98.6	97.6

Solution

	97.0	96.2	97.6	96.4	97.8	99.2
	98.0	98.6	98.8	98.0	98.6	97.6
Difference = d	–1.0	–2.4	–1.2	–1.6	–0.8	1.6
d^2	1	5.76	1.44	2.56	.64	2.56

$n = 6; \qquad \sum d = -5.4; \quad \sum d^2 = 13.96 \quad \bar{d} = \frac{\sum d}{n} = \frac{-5.4}{6} = .9$

$$s_d^2 = \frac{n\sum d^2 - \left(\sum d\right)^2}{n(n-1)} = \frac{6(13.96) - (-5.4)^2}{6(5)} = 1.82$$

$$s_d = \sqrt{1.82} = 1.349$$

$df = 5 \quad and \quad \alpha = 0.05$

$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.571$

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

$$-0.90 - 2.571\left(\frac{1.349}{\sqrt{6}}\right) < \mu_d < -0.90 + 2.571\left(\frac{1.349}{\sqrt{6}}\right)$$

$$-2.32\ ^\circ F < \mu_d < 0.52\ ^\circ F$$

Yes; since the confidence intervals includes 0, body temperature is basically the same at both times.

Exercise

Listed below are systolic blood pressure measuremetns (mm Hg) taking rom the right and left arms of the same woman. Use a 0.05 significance level to test for a difference the measurements from the two arms. What do you conclude?

Right arm	102	101	94	79	79
Left arm	175	169	182	146	144

Solution

	102	101	94	79	79
	175	169	182	146	144
Difference = d	−73	−68	−88	−67	−65
d^2	5329	4624	7744	4489	4225

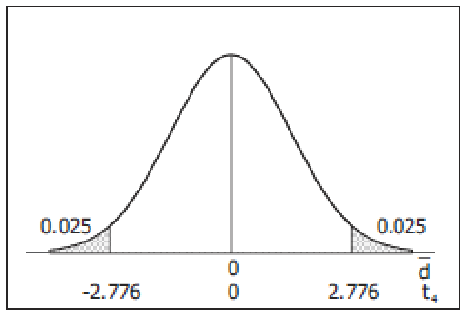
$n = 5; \qquad \sum d = -361; \quad \sum d^2 = 26,411 \quad \bar{d} = \frac{\sum d}{n} = \frac{-361}{5} = -72.2$

$s_d^2 = \frac{n\sum d^2 - (\sum d)^2}{n(n-1)} = \frac{5(26,411) - (-361)^2}{5(4)} = 86.6947$

$s_d = \sqrt{86.6947} \underline{= 9.311}$

$df = 4 \quad and \quad \alpha = 0.05$

$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.776$



$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$

$= \frac{-72.2 - 0}{\frac{9.311}{\sqrt{5}}}$

$\underline{= -17.339}$

$P - value = 2 \cdot tcdf\left(-99, \ -17.338, \ 4\right) \underline{= 6.488E - 5 = 0.00006}$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_d \neq 0$ (in fact, that $\mu_d < 0$). There is sufficient evidence to support the claim that there is a difference in measurements between the two arms. The statistical conclusion is that the right arm. Since the right and left arms should yield the same measurements, the practical conclusion is that a mistake has been made. The most obvious explanation is that diastolic (and not the systolic) values were mistakenly recorded for the right arm. Further investigation is definitely in order.

Exercise

As part of the National Health and Nutrition Examination Survey, the Department of Health and Human Services obtained self-reported heights and measured heights for males ages 12 – 16. All measurement are in inches. Listed below are sample results

Reported height	68	71	63	70	71	60	65	64	54	63	66	72
Measured height	67.9	69.9	64.9	68.3	70.3	60.6	64.5	67.0	55.6	74.2	65.0	70.8

a) Is there sufficient evidence to support the claim that there is a difference between self-reported heights and measured heights of males? Use a 0.05 significance level.

b) Construct a 95% confidence interval estimate of the man difference between reported heights and measured heights. Interpret the resulting confidence interval, and comment on the implications of whether the confidence interval limits contain 0.

Solution

a)

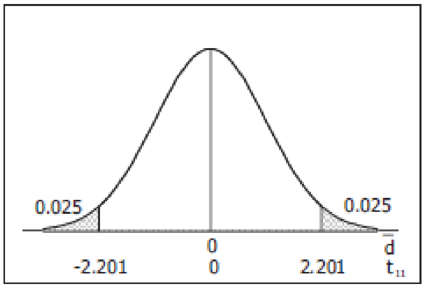
	68	71	63	70	71	60	65	64	54	63	66	72
	67.9	69.9	64.9	68.3	70.3	60.6	64.5	67.0	55.6	74.2	65.0	70.8
Difference = d	0.1	1.1	–1.9	1.7	0.7	–0.6	0.5	–3.0	–1.6	–11.2	1.0	1.2
d^2	.01	1.21	3.61	2.89	.49	.36	.25	9	2.56	125.44	1	1.44

$n=12; \qquad \sum d=-12.0; \quad \sum d^2=148.26 \quad \bar{d}=\frac{\sum d}{n}=\frac{-12}{12}=-1.0$

$$s_d=\sqrt{\frac{n\sum d^2-\left(\sum d\right)^2}{n(n-1)}}$$

$$=\sqrt{\frac{12(148.26)-(-12)^2}{12(11)}}$$

$$\underline{=3.52|}$$



Original claim: $\mu_d \neq 0$ *inches*

$$H_0: \mu_d=0$$

$$H_1: \mu_d \neq 0$$

$df=11 \quad and \quad \alpha=0.05$

$t=\pm t_{\alpha/2}=\pm t_{0.025}=\pm 2.201$

$$t_{\bar{d}}=\frac{\bar{d}-\mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$=\frac{-1.0-0}{\frac{3.52}{\sqrt{12}}}$$

$$\underline{=-0.984|}$$

$P-value = 2\cdot tcdf(-99,\ -0.984,\ 11)$

$\underline{=0.3461}$

Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim $\mu_d \neq 0$. There is not sufficient evidence to support the claim that there is a difference between self-reported heights and measured height of such males.

b) $df = 11 \quad and \quad \alpha = 0.05$

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

$$-1.0 - 2.201\left(\frac{3.52}{\sqrt{12}}\right) < \mu_d < -1.0 + 2.201\left(\frac{3.52}{\sqrt{12}}\right)$$

$$-3.2\ in < \mu_d < 1.2\ in$$

Since the confidence interval contains 0, there is no significant difference between the reported and measured heights.

Exercise

Listed below are combined city – highway fuel consumption ratings (in miles/gal) for different cars measured under both the old rating system and a new rating system introducing in 2008. The new ratings were implemented in response to complaints that the old ratings were too high. Use a 0.01 significance level to test the claim the old ratings are higher than the new ratings.

Old rating	16	18	27	17	33	28	33	18	24	19	18	27	22	18	20	29	19	27	20	21
New rating	15	16	24	15	29	25	29	16	22	17	16	24	20	16	18	26	17	25	18	19

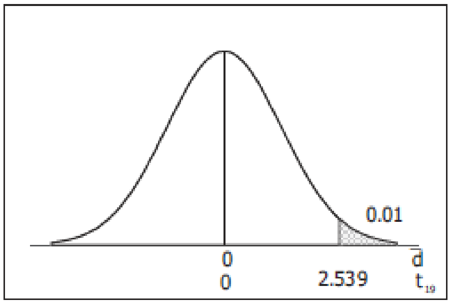
Solution

	16	18	27	17	33	28	33	18	24	19	18	27	22	18	20	29	19	27	20	21
	15	16	24	15	29	25	29	16	22	17	16	24	20	16	18	26	17	25	18	19
Diff = d	1	2	3	2	4	3	4	2	2	2	2	3	2	2	2	3	2	2	2	2
d^2	1	4	9	4	16	9	16	4	4	4	4	9	4	4	4	9	4	4	4	4

$n = 20; \quad \sum d = 47; \quad \sum d^2 = 121 \quad \bar{d} = \frac{\sum d}{n} = \frac{47}{20} = 2.35$

$$s_d = \sqrt{\frac{n\sum d^2 - \left(\sum d\right)^2}{n(n-1)}} = \sqrt{\frac{20(121)-(47)^2}{20(19)}}$$

$\underline{=0.745}$



Original claim: $\mu_d > 0\ mpg$

$H_0:\ \mu_d = 0$

$H_1:\ \mu_d > 0$

$df = 19 \quad and \quad \alpha = 0.01$

$$t = t_{\alpha} = t_{0.01} = 2.539$$

$$t_{\bar{d}} = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$= \frac{2.35 - 0}{\frac{0.745}{\sqrt{20}}}$$

$$\underline{= 14.104}$$

$$P - value = tcdf\,(14.104,\ 99,\ 19)$$

$$\underline{= 9.093E - 12\ \approx 0}$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_d > 0$. There is sufficient evidence to support the claim that the old ratings are higher than the new ratings.

Exercise

Listed below are 2 tables. Construct a 95% confidence interval estimate of the mean of the differences between weights of discarded paper and weights of discarded plastic. Which seems to weigh more: discarded paper or discarded plastic?

Paper

2.41	7.57	9.55	8.82	8.72	6.96	6.83	11.42	16.08	6.38	13.05	11.36	15.09
2.80	6.44	5.86	11.08	12.43	6.05	13.61	6.98	14.33	13.31	3.27	6.67	17.65
12.73	9.83	16.39	6.33	9.19	9.41	9.45	12.32	20.12	7.72	6.16	7.98	9.64
8.08	10.99	13.11	3.26	1.65	10.00	8.96	9.46	5.88	8.26	12.45	10.58	5.87
8.78	11.03	12.29	20.58	12.56	9.92	3.45	9.09	3.69	2.61			

Plastic

0.27	1.41	2.19	2.83	2.19	1.81	0.85	3.05	3.42	2.10	2.93	2.44	2.17
1.41	2.00	0.93	2.97	2.04	0.65	2.13	0.63	1.53	4.69	0.15	1.45	2.68
3.53	1.49	2.31	0.92	0.89	0.80	0.72	2.66	4.37	0.92	1.40	1.45	1.68
1.53	1.44	1.44	1.36	0.38	1.74	2.35	2.30	1.14	2.88	2.13	5.28	1.48
3.36	2.83	2.87	2.96	1.61	1.58	1.15	1.28	0.58	0.74			

Solution

Using Ti-84, store paper into List 1 and plastic in List 2

To create list 3: **[2nd] 1** (L1) **– [2nd] 2** (L2) [***STO→***] **[2nd] 3** (L3)

```
TInterval                TInterval
 Inpt:Data Stats          (6.6106,8.424)
 List:L3                  x̄=7.517258065
 Freq:1                   Sx=3.57036168
 C-Level:.95              n=62
 Calculate
```

$$6.6107\ lbs < \mu_d < 8.424\ lbs$$

Since the confidence interval includes only positive values, there discarded paper appears to weigh more than the discarded plastic.

Exercise

Suppose you wish to test the claim that μ_d, the mean value of the differences d for a population of paired data, is different from 0. Given a sample of $n = 23$ and a significance level of $\alpha = 0.05$, what criterion would be used for rejecting the null hypothesis?

Solution

Given: $n = 23 \Rightarrow df = 23 - 1 = 22$ and $\alpha = 0.05$

Degrees of Freedom	Area in Two Tails 0.01	0.02	0.05	0.10	0.20
22	**2.819**	2.508	**2.074**	1.717	1.321

To reject null hypothesis if test statistic is: $|t| > 2.074$ or $\underline{< -2.074 \quad or \quad > 2.074}$

Exercise

Assume that he paired data came from a population that is normally distributed. Using a 0.05 significance level, find $\bar{d}$, s_d, the t test statistic, and the critical values to test the claim that $\mu_d = 0$

x	14	8	4	14	3	12	4	13
y	15	8	7	13	5	11	6	15

Solution

	14	8	4	14	3	12	4	13	
	15	8	7	13	5	11	6	15	
Difference = d	−1	0	−3	1	−2	1	−2	−2	
$(d-\bar{d})^2$	0	1	4	4	1	4	1	1	16

$$\bar{d} = \frac{-1+0-3+1-2+1-2-2}{8} \underline{= -1}$$

$$s_d = \sqrt{\frac{\sum(d-\bar{d})^2}{n-1}} = \sqrt{\frac{16}{7}} \underline{\approx 1.5119}$$

$$t_{\bar{d}} = \frac{\bar{d}-\mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$= \frac{-1-0}{\frac{1.5119}{\sqrt{8}}}$$

$$\underline{= -1.8708}$$

$df = 7 \quad and \quad \alpha = 0.05$

$t = \pm t_{\alpha/2} = \pm t_{0.025} \underline{= \pm 2.365}$

Exercise

Assume that he paired data came from a population that is normally distributed. Using a 0.05 significance level, find $\bar{d}$, s_d, the t test statistic, and the critical values to test the claim that $\mu_d = 0$

x	12	5	1	20	3	16	12	8
y	7	10	5	15	7	14	10	13

Solution

	12	5	1	20	3	16	12	8
	7	10	5	15	7	14	10	13
Difference = d	5	−5	−4	5	−4	2	2	−5
$(d-\bar{d})^2$	25	25	16	25	16	4	4	25

$$\bar{d} = \frac{5-5-4+5-4+2+2-5}{8} = -0.5$$

$$s_d = \sqrt{\frac{n\sum d^2 - \left(\sum d\right)^2}{n(n-1)}} = \sqrt{\frac{8(140)-(-0.5)^2}{7}} \approx 4.440$$

$$t_{\bar{d}} = \frac{\bar{d}-\mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$= \frac{-0.5-0}{\frac{4.440}{\sqrt{8}}}$$

$$= -0.319$$

$df = 7 \quad and \quad \alpha = 0.05$

$t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.365$

Solution Section 4.3 – Inferences About Two Means: Independent

Exercise

If the pulse rates of men and women shown in the data below

Women:

76	72	88	60	72	68	80	64	68	68	80	76	68	72	96	72	68	72	64	80
64	80	76	76	76	80	104	88	60	76	72	72	88	80	60	72	88	88	124	64

Men:

68	64	88	72	64	72	60	88	76	60	96	72	56	64	60	64	84	76	84	88
72	56	68	64	60	68	60	60	56	84	72	84	88	56	64	56	56	60	64	72

These data are used to construct 95% confidence interval for the difference between the two population means, the result is $-12.2 < \mu_1 - \mu_2 < -1.6$, where pulse rates of men correspond to population 1 and pulse rates of women correspond to population 2. Express the confidence interval with pulse rates of women being population 1 and pulse rates of men being population 2.

Solution

Reversing the designation of which sample is considered group 1 and which sample is considered group 2 changes the sign of the point estimate and the signs of the endpoints of the interval estimate. The confidence interval using the new designation is $1.6 < \mu_1 - \mu_2 < 12.2$

Exercise

Assume that you want to use a 0.01 significance level to test the claim that the mean pulse rate of men is less than the mean pulse rate of women. What confidence level should be used if you want to test that claim using a confidence interval?

Solution

A one-tailed test of hypothesis at the 0.01 level of significance corresponds to a two-sided confidence interval at the 2(0.01) = 0.02 level of significance –i.e., to an interval with a confidence level of 98%

Exercise

To test the effectiveness of Lipitor, cholesterol levels are measured in 250 subjects before and after Lipitor treatments. Determine whether this sample is independent or dependent.

Solution

Dependent, since cholesterol levels and determined by many factors that the Lipitor treatment cannot change. Treatments to lower cholesterol typically reduce everyone’s levels by a certain amount, by persons who were high compared to the others before the treatment, for example, will likely still be high compared to the others after the treatment.

Exercise

On each of 40 different days, you measured the voltage supplied to your home and you also measured the voltage produced by the gasoline-powered generator. One sample consists of the voltages in the house and the second sample consists of the voltages produced by the generator. Determine whether this sample is independent or dependent.

Solution

Independent, since there is no relationship between the voltage supplied to the house by the power company and the voltage generated by a completely separate gasoline-powered generator.

Exercise

In a randomized controlled trial conducted with children suffering from viral croup, 46 children were treated with low humidity while 46 other children were treated with high humidity. Researchers used the Westley Croup Score to assess the results after one hour. The low humidity group had a mean score of 0.98 with standard deviation of 1.22 while the high humidity group had a mean score of 1.09 with standard deviation of 1.11.

a) Use a 0.05 significance level to test the claim that the two groups are from populations with the same mean. What does the result suggest about the common treatment of humidity?
Assume that the two samples are independent simple random samples selected from normally distributed populations.

b) Assume that $\sigma_1 = \sigma_2$, how are the results affected by this additional assumption?

Solution

a) *Original Claim*: $H_0 : \mu_1 - \mu_2 = 0$

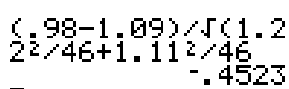
$H_1 : \mu_1 - \mu_2 \neq 0$

$\alpha = 0.05 \quad and \quad df = 45$

Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
45	2.690	2.412	$\boxed{2.014}$	1.679	1.301

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.014$

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$



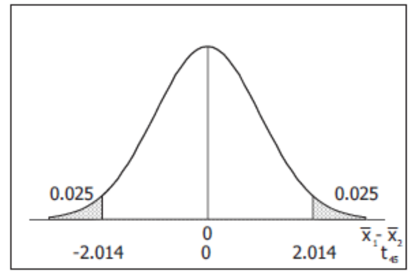
$$= \frac{(.98 - 1.09) - (0)}{\sqrt{\frac{1.22^2}{46} + \frac{1.11^2}{46}}}$$

$$= -0.452$$

$P-value = 2 \cdot tcdf\,(-99, -0.452, 45)$

$\underline{= 0.6534}$

```
2tcdf(-99,-.452,
45
                .65344
```



Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim that $\mu_1 - \mu_2 = 0$. There is not sufficient evidence to reject the claim that the two groups are from populations with the same mean. The results suggest that increasing the humidity does not have a significant effect on the treatment of croup.

b) $df = df_1 + df_2 = 45 + 45 = 90$

$$s_p^2 = \frac{(df_1)s_1^2 + (df_2)s_2^2}{df}$$

$$= \frac{(45)(1.22)^2 + (45)(1.11)^2}{90}$$

$$\underline{= 1.3603}$$

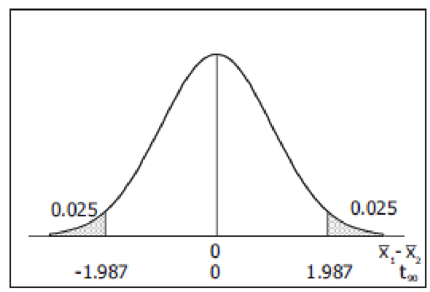
```
(45*1.22²+45*1.1
1²)/90
           1.3603
```

Original Claim: $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 \neq 0$

$\alpha = 0.05 \quad and \quad df = 90$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} \; \underline{= \pm 1.987}$



$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(.98 - 1.09) - (0)}{\sqrt{\frac{1.3603}{46} + \frac{1.3603}{46}}}$$

$\underline{= -0.452}$

$P-value = 2 \cdot tcdf\,(-99,\ -0.452,\ 90)$

$\underline{= 0.6521}$

Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim that $\mu_1 - \mu_2 = 0$. There is not sufficient evidence to reject the claim that the two groups are from populations with the same mean. The results suggest that increasing the humidity does not have a significant effect on the treatment of croup.

When $n_1 - n_2 = 0$, the calculated t statistic does not change at all. The only difference the assumption of equal standard deviations makes in this instance is to change the *df* from 45 to 90 and the *P*-value from 0.6532 to 0.6521. The conclusion is unaffected.

Exercise

The mean tar content of a simple random sample of 25 unfiltered king size cigarettes is 21.1 mg, with a standard deviation of 3.2 mg. The mean tar content of a simple random sample of 25 filtered 100 mm cigarettes is 13.2 mg, with a standard deviation of 3.7 mg.

Assume that the two samples are independent simple random samples selected from normally distributed populations in part a and b.

a) Construct a 90% confidence interval estimate of the difference between the mean tar content of unfiltered king size cigarettes and the mean tar content of filtered 100 mm cigarettes. Does the result suggest that 100 mm filtered cigarettes have less tar than unfiltered king size cigarettes?

b) Use a 0.05 significance level to test the claim that unfiltered king size cigarettes have a mean tar content greater than that of filtered 100 mm cigarettes. What does the result suggest about the effectiveness of cigarette filters?

c) Assume that $\sigma_1 = \sigma_2$, how are the results affected by this additional assumption?

Solution

a) Let the unfiltered cigarettes be group 1.

$\alpha = 0.1 \quad and \quad df = 24$

Critical value: $t = t_{0.05} = 1.711$

Degrees of Freedom	Area in One Tail 0.005	0.01	0.025	0.05	0.10
24	**2.797**	2.492	**2.064**	$\boxed{1.711}$	1.318

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(21.1 - 13.2) - 1.711\sqrt{\frac{3.2^2}{25} + \frac{3.7^2}{25}} < \mu_1 - \mu_2 < (21.1 - 13.2) + 1.711\sqrt{\frac{3.2^2}{25} + \frac{3.7^2}{25}}$$

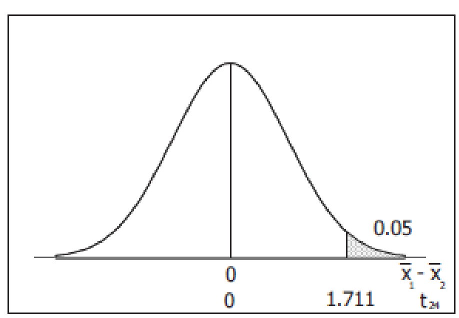
$$6.2 < \mu_1 - \mu_2 < 9.6$$

Yes; since the confidence interval includes only positive values, the results suggest that the filtered cigarettes have less tar than the unfiltered ones.

b) *Original Claim*: $\mu_1 - \mu_2 > 0$

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 > 0$$



$\alpha = 0.1 \quad and \quad df = 24$

Critical value: $t = t_{0.05} = 1.711$

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(21.1 - 13.2) - 0}{\sqrt{\frac{3.2^2}{25} + \frac{3.7^2}{25}}} = 8.075$$

$P - value = tcdf\,(8.075,\ 99,\ 24)$

$= 1.338E - 8$ ≈ 0.00000001

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 > 0$. There is sufficient evidence to support the claim that unfiltered king size cigarettes have a mean tar content greater than that of filtered 100 mm cigarettes. The results suggest that filters are effective in reducing the tar content cigarettes.

c) $df = df_1 + df_2 = 24 + 24 = 48$

$$s_p^2 = \frac{(df_1)s_1^2 + (df_2)s_2^2}{df}$$

$$= \frac{(24)(3.2)^2 + (24)(3.7)^2}{48}$$

$$= 11.965$$

$\alpha = 0.10 \quad and \quad df = 48$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(21.1 - 13.2) - 1.676\sqrt{\frac{11.965}{25} + \frac{11.965}{25}} < \mu_1 - \mu_2 < (21.1 - 13.2) + 1.676\sqrt{\frac{11.965}{25} + \frac{11.965}{25}}$$

$6.3 < \mu_1 - \mu_2 < 9.5$

Yes; since the confidence interval includes only positive values, the results suggest that the filtered cigarettes have less tar than the unfiltered ones.

When $n_1 = n_2$ the value of is unchanged. The only difference the assumption of equal standard deviations makes in this instance is to change the *df* from 24 to 48 and the from 1.711 to 1.676. This makes the interval slightly narrower, but the conclusion is unaffected.

Exercise

The heights are measured for the simple random sample of supermodels Crawford, Bundchen, Pestova, Christenson, Hume, Moss, Campbell, Schiffer, and Taylor. They have a mean of 70.0 in. and a standard deviation of 1.5 in. 40 women who are not supermodels, listed below and they have heights with means of 63.2 in. and a standard deviation of 2.7 in.

64.3	66.4	62.3	62.3	59.6	63.6	59.8	63.3	67.9	61.4	66.7	64.8	63.1	66.7	66.8
64.7	65.1	61.9	64.3	63.4	60.7	63.4	62.6	60.6	63.5	58.6	60.2	67.6	63.4	64.1
62.7	61.3	58.2	63.2	60.5	65.0	61.8	68.0	67.0	57.0					

a) Use a 0.01 significance level to test the claim that the mean height of supermodels is greater than the mean height of women who are not supermodels *b)* Construct a 98% confidence interval level for the difference between the mean height of supermodels and the mean height of women who are not supermodels. What does the result suggest about those two means?

Solution

a) Let the supermodels be group 1. For which $n_1 = 9$

Original Claim: $\mu_1 - \mu_2 > 0$ $(inches)$

$$H_0 : \mu_1 - \mu_2 = 0$$

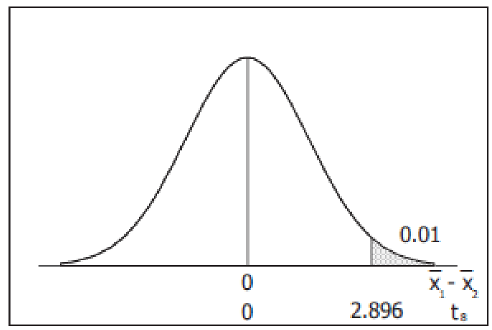
$$H_1 : \mu_1 - \mu_2 > 0$$

$\alpha = 0.01$ *and* $df = 8$

Critical value: $t = t_\alpha = t_{0.01} \underline{= 2.896}$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(70.0 - 63.2) - 0}{\sqrt{\frac{1.5^2}{9} + \frac{2.7^2}{40}}}$$

$\underline{= 10.343}$



$P-value = tcdf\,(10.343,\ 99,\ 8)$

$\underline{= 3.29E-6}$ $\quad \underline{\approx 0.000003}$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 > 0$. There is sufficient evidence to support the claim that the mean height of supermodels is greater than the mean height of women who are not supermodels.

b) Let the supermodels be group 1. For which $n_1 = 9$.

$\alpha = 1 - .98 = 0.2$ *and* $df = 8$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(70 - 63.2) - 2.896\sqrt{\frac{1.5^2}{9} + \frac{2.7^2}{40}} < \mu_1 - \mu_2 < (70 - 63.2) + 2.896\sqrt{\frac{1.5^2}{9} + \frac{2.7^2}{40}}$$

$\underline{4.9 < \mu_1 - \mu_2 < 8.7}$ $\quad (inches)$

Since the confidence interval includes only positive values, the results suggest that the mean height of supermodels is greater than the mean height of women who are not supermodels.

Exercise

Many studies have been conducted to test the effects of marijuana use on mental abilities. In one such study, groups of light and heavy users of marijuana in college were tested for memory recall, with the results given below. Use a 0.01 significance level to test the claim that the population of heavy marijuana users has a lower mean than the light users. Should marijuana use be of concern to college students?

Items sorted correctly by light marijuana users: $n = 64, \quad \bar{x} = 53.3, \quad s = 3.6$

Items sorted correctly by heavy marijuana users: $n = 65, \quad \bar{x} = 51.3, \quad s = 4.5$

Solution

Original Claim: $\mu_1 - \mu_2 > 0$

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 > 0$$

$\alpha = 0.01 \quad and \quad df = 63$

Critical value: $t = t_\alpha = t_{0.01} \underline{= 2.390}$

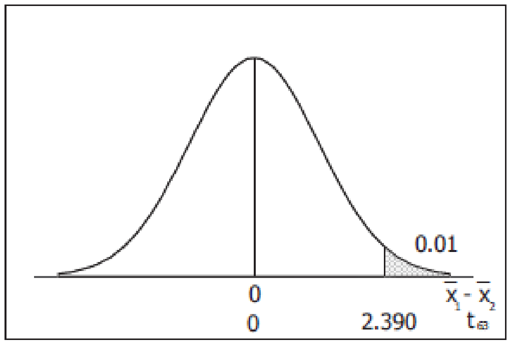
Degrees of Freedom	0.005	0.01	Area in One Tail 0.025	0.05	0.10
60	2.660	2.390	2.000	1.671	1.296

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(53.3 - 51.3) - (0)}{\sqrt{\frac{3.6^2}{64} + \frac{4.5^2}{65}}}$$

$$\underline{= 2.790}$$

```
(53.3-51.3)/√(3.
6²/64+4.5²/65
          2.78954
```



$P - value = tcdf\,(2.790,\ 99,\ 63)$

$$\underline{= 0.0035}$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 > 0$. There is sufficient evidence to support the claim that heavy marijuana users have a lower mean number of recalled items than do light users.

Yes; marijuana use should be of concern to college students – and an even more valuable study might one comparing light users to those who do not use marijuana at all.

Exercise

The trend of thinner Miss America winners has generated charges that the contest encourages unhealthy diet habits among young women. Listed below are body mass indexes (BMI) for Miss America winners from two different time periods. Consider the listed values to be simple random samples selected from larger populations.

a) Use a 0.05 significance level to test the claim that recent winners have a lower mean BMI than winners from the 1920s and 1930s.

b) Construct a 90% Confidence interval for the difference between the mean BMI of recent winners and the mean BMI of winners from the 1920s and 1930s.

BMI (from recent winners):	19.5	20.3	19.6	20.2	17.8	17.9	19.1	18.8	17.6	16.8
BMI (from 1920s and 1930s):	20.4	21.9	22.1	22.3	20.3	18.8	18.9	19.4	18.4	19.1

Solution

Group 1: recent (n = 10)

$$\bar{x}_1 = \frac{\sum x}{n_1} = \frac{187.6}{10} = 18.76; \quad s_1 = 1.186$$

```
1-Var Stats
 x̄=18.76000
 Σx=187.60000
 Σx²=3532.04000
 Sx=1.18622
 σx=1.12534
↓n=10.00000
```

Group 2: 1920,1930 (n = 10)

$$\bar{x}_2 = \frac{\sum x}{n_2} = \frac{201.6}{10} = 20.16; \quad s_2 = 1.479$$

```
1-Var Stats
 x̄=20.16000
 Σx=201.60000
 Σx²=4083.94000
 Sx=1.47889
 σx=1.40300
↓n=10.00000
```

a) *Original Claim*: $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 < 0$

$\alpha = 0.05 \quad and \quad df = 9$

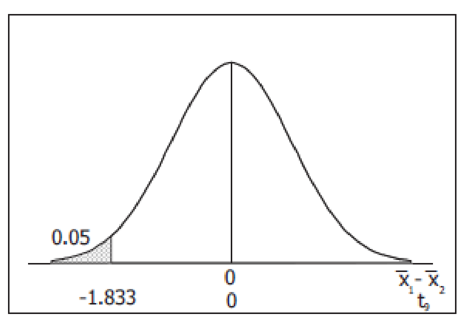
Critical value: $t = -t_\alpha = -t_{0.05} = -1.833$

Degrees of Freedom	0.005	0.01	0.025	0.05	0.10
			Area in One Tail		
9	3.250	2.821	2.262	1.833	1.383

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(18.76 - 20.16) - (0)}{\sqrt{\frac{1.186^2}{10} + \frac{1.479^2}{10}}}$$

$$= -2.335$$



$P - value = tcdf\,(-99, \; -2.335, \; 9) = 0.0222$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 < 0$. There is sufficient evidence to support the claim that recent winners have a lower mean BMI than winners from the 1920s and 1930s.

b) $\alpha = 0.1 \quad and \quad df = 9$.

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(18.76 - 20.16) - 1.833\sqrt{\frac{1.186^2}{10} + \frac{1.479^2}{10}} < \mu_1 - \mu_2 < (18.76 - 20.16) + 1.833\sqrt{\frac{1.186^2}{10} + \frac{1.479^2}{10}}$$

$$\underline{-2.50 < \mu_1 - \mu_2 < -0.30}$$

Exercise

Listed below are amounts of strontium-90 (in millibecquerels or mBq per gram of calcium) in a simple random sample of baby teeth obtained from Pennsylvania residents and New York residents born after 1979.

a) Use a 0.05 significance level to test the claim that the mean amount of strontium-90 from Pennsylvania residents is greater than the mean amount from New York residents.

b) Construct a 90% Confidence interval for the difference between the mean amount of strontium-90 from Pennsylvania residents and the mean amount from New York residents.

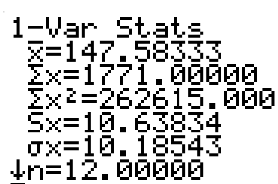
Pennsylvania:	155	142	149	130	151	163	151	142	156	133	138	161
New York:	133	140	142	131	134	129	128	140	140	140	137	143

Solution

Group 1: PA (n = 12)

$$\bar{x}_1 = \frac{\sum x}{n_1} = \frac{1771}{12} = \underline{147.58}$$

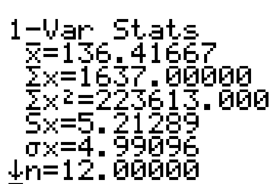
$s_1 = 10.64$



Group 2: NY (n = 12)

$$\bar{x}_2 = \frac{\sum x}{n_2} = \frac{1637}{12} = \underline{136.42}$$

$s_2 = 5.21$



a) *Original Claim*: $H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 > 0$

$\alpha = 0.05 \quad and \quad df = 11$

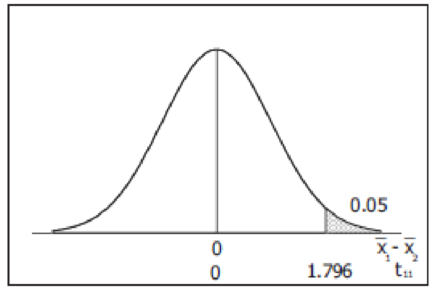
Critical value: $t = t_\alpha = t_{0.05} = \underline{1.796}$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(147.58 - 136.42) - (0)}{\sqrt{\frac{10.64^2}{12} + \frac{5.21^2}{12}}}$$

```
(147.58-136.42)/
√(10.64²/12+5.21
²/12
           3.26319
```

$$\underline{= 3.263}$$




```
tcdf(3.265,99,11
              .0038
```

$$P - value = tcdf\,(3.265,\ 99,\ 11)\ \underline{= 0.0038}$$

Conclusion:

Reject H_0; there is sufficient evidence to conclude that $\mu_1 - \mu_2 > 0$. There is sufficient evidence to support the claim that the mean amount of Strontium-90 from Pennsylvania residents is greater than the mean amount from N.Y. residents.

c) $\alpha = 0.1$ *and* $df = 11$.

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(147.58 - 136.42) - 1.796\sqrt{\frac{10.64^2}{12} + \frac{5.21^2}{12}} < \mu_1 - \mu_2 < (147.58 - 136.42) + 1.796\sqrt{\frac{10.64^2}{12} + \frac{5.21^2}{12}}$$

$$\underline{5.0 < \mu_1 - \mu_2 < 17.3} \qquad (mBq)$$

Exercise

Listed below are the word counts for male and female psychology students.

a) Use a 0.05 significance level to test the claim that male and female psychology students speak the same mean number of words in a day.

b) Construct a 95% Confidence interval estimate of the difference between the mean number of words spoken in a day by male and female psychology students. Do the confidence interval limits include 0, and what does that suggest about the two means?

Male	21143	17791	36571	6724	15430	11552	11748	12169	15581	23858	5269
	12384	11576	17707	15229	18160	22482	18626	1118	5319		

Female	6705	21613	11935	15790	17865	13035	24834	7747	3852	11648	25862
	17183	11010	11156	11351	25693	13383	19992	14926	14128	10345	13516
	12831	9671	17011	28575	23557	13656	8231	10601	8124		

Assume that the two samples are independent simple random samples selected from normally distributed populations. Do not assume that the population standard deviations are equal.

Solution

Group 1: Males ($n = 20$)

$\bar{x}_1 = 15021.9$

$s_1 = 7863.87$

Group 2: Females ($n = 31$)

$\bar{x}_2 = 14704.1$

$s_2 = 6215.35$

a) *Original Claim*: $H_0: \mu_1 - \mu_2 = 0$ ***words/day***

$H_1: \mu_1 - \mu_2 \neq 0$

$\alpha = 0.05$ *and* $df = 19$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.093$

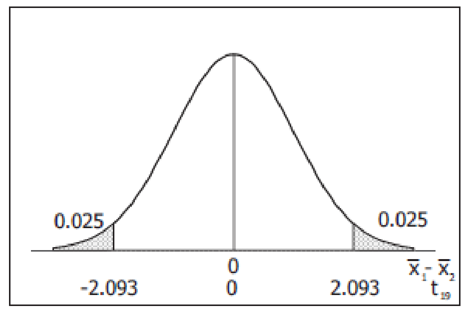
Degrees of Freedom	Area in Two Tails 0.01	0.02	0.05	0.10	0.20
19	2.861	2.539	$\boxed{2.093}$	1.729	1.328

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(15021.9 - 14704.1) - (0)}{\sqrt{\frac{7863.87^2}{20} + \frac{6215.35^2}{31}}}$$

$= 0.153$

```
(15021.9-14704.1
)/√(7863.87²/20+
6215.35²/31
            .1526
```




```
2tcdf(.1526,99,1
9
          .8803
```

$P - value = 2 \cdot tcdf\,(0.1526,\ 99,\ 19) = 0.8803$

Conclusion:

Do not reject H_0; there is not sufficient evidence to reject the claim $\mu_1 - \mu_2 = 0$. There is not sufficient evidence to reject the claim that the male and female students speak the same mean number of words per day.

d) $\alpha = 0.05$ *and* $df = 19$.

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$317.8 - 2.093\sqrt{\frac{7863.87^2}{20} + \frac{6215.35^2}{31}} < \mu_1 - \mu_2 < 317.8 + 2.093\sqrt{\frac{7863.87^2}{20} + \frac{6215.35^2}{31}}$$

$-4041.6 < \mu_1 - \mu_2 < 4677.1$ $(words/day)$

```
317.8-2.093√(786
3.87²/20+6215.35
²/31
       -4041.5578
```

Yes; since the confidence interval includes zero, there does not appear to be significant difference between the mean number of words spoken by the male and female students.

Exercise

Refer to the tables below and test the claim that they contain the same amount of cola, the mean weight of cola cans of regular Coke is the same as the mean weight of cola in cans of Diet Coke. If there is a difference in the mean weights, identify the most likely explanation for that difference.

Coke	0.8192	0.815	0.8163	0.8211	0.8181	0.8247	0.8062	0.8128	0.8172	0.811
	0.8251	0.8264	0.7901	0.8244	0.8073	0.8079	0.8044	0.817	0.8161	0.8194
	0.8189	0.8194	0.8176	0.8284	0.8165	0.8143	0.8229	0.815	0.8152	0.8244
	0.8207	0.8152	0.8126	0.8295	0.8161	0.8192				

Diet	0.7773	0.7758	0.7896	0.7868	0.7844	0.7861	0.7806	0.783	0.7852	0.7879
	0.7881	0.7826	0.7923	0.7852	0.7872	0.7813	0.7885	0.776	0.7822	0.7874
	0.7822	0.7839	0.7802	0.7892	0.7874	0.7907	0.7771	0.787	0.7833	0.7822
	0.7837	0.791	0.7879	0.7923	0.7859	0.7811				

Assume that the two samples are independent simple random samples selected from normally distributed populations. Do not assume that the population standard deviations are equal.

Solution

Group 1: Regular Coke ($n = 36$)

$\bar{x}_1 = 0.8168222$

$s_1 = 0.0075074$

Group 2: Diet Coke ($n = 36$)

$\bar{x}_2 = 0.7847944$

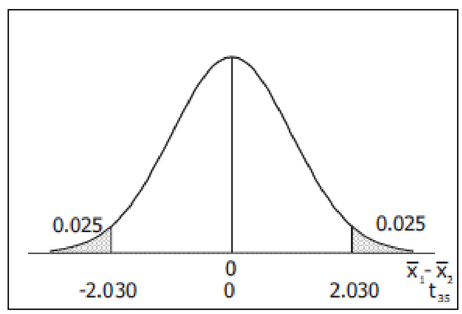
$s_2 = 0.0043909$

Original Claim: $H_0 : \mu_1 - \mu_2 = 0$ ***lbs***

$H_1 : \mu_1 - \mu_2 \neq 0$

$\alpha = 0.05 \quad and \quad df = 35$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.030$



$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(0.8168222 - 0.7847944) - (0)}{\sqrt{\frac{.0075074^2}{36} + \frac{.0043909^2}{36}}}$$

```
(.8168222-.78479
44)/√(.0075074²/
36+.0043909²/36
            22.0953
```

$= 22.095$

$P - value = 2 \cdot tcdf\,(22.095,\ 99,\ 35) = 0.8803$

Conclusion:

Reject H_0; there is sufficient evidence to reject the claim that $\mu_1 - \mu_2 = 0$ and conclude that $\mu_1 - \mu_2 \neq 0$ (in fact, that $\mu_1 - \mu_2 > 0$). There is sufficient evidence to reject the claim that the mean weight of cola in cans of regular Coke is the same as the mean weight of cola in cans of Diet Coke. The regular Coke may weigh more because it contains sugar.

Exercise

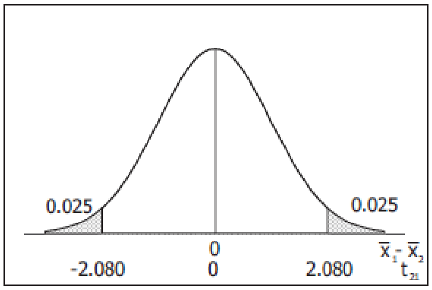
An Experiment was conducted to test the effects of alcohol. Researchers measured the breath alcohol levels for a treatment group of people who drank ethanol and another group given a placebo. The results are given in the accompanying table. Use a 0.05 significance level to test the claim that the two sample groups come from populations with the same mean.

Treatment Group:	$n_1 = 22$	$\bar{x}_1 = 0.049$	$s_1 = 0.015$
Placebo Group:	$n_2 = 22$	$\bar{x}_2 = 0.000$	$s_2 = 0.000$

Solution

Original Claim: $\mu_1 - \mu_2 = 0$ ***lbs***

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_1: \mu_1 - \mu_2 \neq 0$$



$\alpha = 0.05$ *and* $df = 21$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} = \pm 2.080$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(0.049 - 0.0) - (0)}{\sqrt{\frac{.015^2}{22} + \frac{0^2}{22}}}$$

$$= 15.322$$

$$P - value = 2 \cdot tcdf\,(15.322,\ 99,\ 21)$$
$$= 7.14E-13 \approx 0$$

Conclusion:

Reject H_0; there is sufficient evidence to reject the claim that $\mu_1 - \mu_2 = 0$ and conclude that $\mu_1 - \mu_2 \neq 0$ (in fact, that $\mu_1 - \mu_2 > 0$). There is sufficient evidence to reject the claim that the two sample groups come from populations with the same mean.

The fact that there was no variation in the second sample did not affect the calculations or present any special problems. Since there is no variation in x_2, it is really equivalent to the constant value zero – and the test is mathematically equivalent to the one-sample test $H_0: \mu_1 = 0$ for which $t = \dfrac{\bar{x}_1 - 0}{s_{\bar{x}_1}}$

Exercise

A researcher was interested in comparing the GPAs of students at two different colleges. Independent simple populations. Do samples of 8 students from college *A* and 13 students from college *B* yielding the following GPAs.

College *A*	3.7	3.2	3.0	2.5	2.7	3.6	2.8	3.4					
College *B*	3.8	3.2	3.0	3.9	3.8	2.5	3.9	2.8	4.0	3.6	2.6	4.0	3.6

Construct a 95% confidence interval for $\mu_1 - \mu_2$. The difference between the mean GPA of college *A* students and the mean GPA of college *B* students.

$$\left(Note: \ \bar{x}_1 = 3.1125, \ \bar{x}_2 = 3.4385, \ s_1 = 0.4357, \ s_2 = 0.5485 \right)$$

Solution

$\alpha = 0.05 \quad and \quad df = 21$

Critical value: $t = \pm t_{\alpha/2} = \pm t_{0.025} \ \underline{= \pm 2.080|}$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(3.1125 - 3.4385) - 2.08\sqrt{\frac{0.4357^2}{8} + \frac{0.5485^2}{13}} \approx -0.78$$

$$(3.1125 - 3.4385) + 2.08\sqrt{\frac{0.4357^2}{8} + \frac{0.5485^2}{13}} \approx 0.13$$

$$\underline{-0.78 < \mu_1 - \mu_2 < 0.13|}$$

Exercise

Assume that the two samples are independent simple random samples selected from normal distributed populations. Do not assume that the population standard deviations are equal.

A researcher was interested in comparing the heights of women in two different countries. Independent simple random samples of 9 women from country ***A*** and 9 women from ***B*** yielded to the following heights (in inches).

Country A	64.1	66.4	61.7	62.0	67.3	64.9	64.7	68.0	63.6
Country B	65.3	60.2	61.7	65.8	61.0	64.6	60.0	65.4	59.0

Construct a 90% confidence interval for $\mu_1 - \mu_2$ the difference between the mean height of women in country ***A*** and the mean height of women in country ***B***. Round to two decimal places.

$\left(Note: \ \overline{x}_1 = 64.744 \ in, \ \overline{x}_2 = 62.556 \ in, \ s_1 = 2.192 \ in, \ s_2 = 2.697 \ in \right)$

Solution

$\overline{x}_1 = 64.744 \quad \overline{x}_2 = 62.556 \qquad s_1 = 2.192 \quad s_2 = 2.697$

$$A = \frac{s_1^2}{n_1} = 0.53 \quad B = \frac{s_2^2}{n_2} = 0.81$$

$$df = \frac{(A+B)^2}{\frac{A^2}{n_1 - 1} + \frac{B^2}{n_2 - 1}} \approx 15$$

Degrees of Freedom		Area in One Tail			
	0.005	0.01	0.025	0.05	0.10
15	**2.947**	2.602	**2.131**	1.753	1.341

$$\left(\overline{x}_1 - \overline{x}_2\right) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\overline{x}_1 - \overline{x}_2\right) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(64.744 - 62.556) - 1.753\sqrt{\frac{2.192^2}{9} + \frac{2.697^2}{9}} < \mu_1 - \mu_2 < (64.744 - 62.556) + 1.753\sqrt{\frac{2.192^2}{9} + \frac{2.697^2}{9}}$$

$$0.16 < \mu_1 - \mu_2 < 4.22$$

Solution Section 4.4 – Goodness-of-Fit

Exercise

A poll typically involves the selection of random digits to be used for telephone numbers. The New York Times states that "within each (telephone) exchange, random digits were added to form a complete telephone number, thus permitting access to listed and unlisted numbers. "When such digits are randomly generated, what is the distribution of those digits? Given such randomly generated digits, what is a test for "goodness-off-fit"?

Solution

When digits are randomly generated they should form a uniform distribution – i.e., a distribution in which each of the digits is equally likely. The test for goodness-to-fit is a test of the hypothesis that the sample data fit the uniform distribution.

Exercise

When generating random digits, we can test the generated digits for goodness-of-fit with the distribution in which all of the digits are equally likely. What does an exceptionally large value of the χ^2 test statistic suggest about the goodness-of-fit? What does an exceptionally small value of the χ^2 test statistic (such as 0.002) suggest about the goodness-of-fit?

Solution

The calculated χ^2 is a measure of the discrepancy between the hypothesis distribution and the sample data. An exceptionally large value of the χ^2 test statistic suggests a large discrepancy between the hypothesized distribution and the sample data – that there is not goodness-of-fit, and that the observed and expected frequencies are quite different. An exceptionally small of the χ^2 test statistic suggests an extremely good fir – that the observed and expected values are almost identical.

Exercise

You purchased a slot machine, and tested it by playing it 1197 times. There are 10 different categories of outcome, including no win, win jackpot, win with three bells, and so on. When testing the claim the observed outcomes agree with the expected frequencies, the author obtained a test statistic of $\chi^2 = 8.185$. Use a 0.05 significance level to test the claim that the actual outcomes agree with the expected frequencies. Does the slot machine appear to be functioning as expected?
Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Solution

Original claim: the actual outcomes agree with the expected frequencies

H_0 : The actual outcomes agree with the expected frequencies

H_1 : At least one outcome is not as expected

$\alpha = 0.05 \quad and \quad df = 9$

$C.V. \ \chi^2 = \chi^2_\alpha = \chi^2_{0.05} = 16.919$

Calculations:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 8.185$$

P-value = $\chi^2 \ cdf\,(8.185,\ 99,\ 9) = 0.5156$

Conclusion

Do not reject H_0; there is not sufficient evidence to reject the claim that the actual outcomes agree with the expected frequencies. There is no reason to say the slot machine is not functioning as expected.

Exercise

Do "*A*" students tend to sit in a particular part of the classroom? The author recorded the locations of the students who received grades *A*, with these results: 17 sat in the front, 9 sat in the middle, and 5 sat in the back of the classroom. When testing the assumption that the "*A*" students are distributed evenly throughout the room, the author obtained the test statistic of $\chi^2 = 7.226$. If using a 0.05 significance level, is there sufficient evidence to support the claim that the "*A*" students are not evenly distributed throughout the classroom? If so, does that mean you can increase your likelihood of getting an *A* by sitting in the front of the room?

Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Solution

Original claim: "*A*" student are not evenly distributed throughout the classroom

H_0 : "*A*" students are evenly distributed throughout the classroom

H_1 : "*A*" students are not evenly distributed throughout the classroom

$\alpha = 0.05 \quad and \quad df = 2$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597

$C.V. \ \chi^2 = \chi^2_\alpha = \chi^2_{0.05} = 5.991$

Calculations:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 7.226$$

P-value = $\chi^2 \ cdf\,(7.226,\ 99,\ 2) = 0.0270$

Conclusion

Reject H_0; there is sufficient evidence to support the claim "*A*" students are not evenly distributed throughout the classroom.

Exercise

Randomly selected nonfat occupational injuries and illnesses are categorized according to the day of the week that they first occurred, and the results are listed below. Use a 0.05 significance level to test the claim that such injuries and illness occur with equal frequency on the different days of the week Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Day	*Mon*	*Tues*	*Wed*	*Thurs*	*Fri*
Number	23	23	21	21	19

Solution

Original Claim: The injuries and illnesses occur with equal frequencies on the different days.

$H_0: p_M = p_T = p_W = p_{Th} = p_F = \frac{1}{5} = 0.20$

$H_1: at$ least one $\mathrm{p_i} \neq 0$

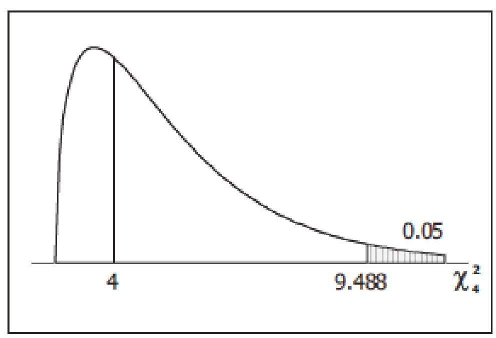
$\alpha = 0.05 \quad and \quad df = 4$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860

$C.V.\ \chi^2 = \chi^2_\alpha = \chi^2_{0.05} = 9.488 \quad E = \frac{1}{5}\sum O = \frac{107}{5} = 21.4$

Calculations:

Day	*O*	*E*	$\frac{(O-E)^2}{E}$
M	23	21.4	0.1196
T	23	21.4	0.1196
W	21	21.4	0.0075
Th	21	21.4	0.075
F	19	21.4	0.2693
	107	107	0.5234



$$\chi^2 = \sum \frac{(O-E)^2}{E} = 0.5234$$

P-value = $\chi^2\ cdf\,(0.523,\ 99,\ 4) = 0.9712$

Conclusion

Do not reject H_0; there is not sufficient evidence to reject the claim that $\mathrm{p_i} \neq 0$ for each day.

There is no sufficient evidence to reject the claim that the injuries and illnesses occur with equal frequencies on the different days of the week.

Exercise

Records of randomly selected births were obtained and categorized according to the day of the week that they occurred. Because babies are unfamiliar with our schedule of weekdays, a reasonable claim is that occur on the different days with equal frequency. Use a 0.01 significance level to test that claim. Can you provide an explanation for the result?
Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Day	*Sun*	*Mon*	*Tues*	*Wed*	*Thurs*	*Fri*	*Sat*
Number of births	77	110	124	122	120	123	97

Solution

Original Claim: births occur on the different days with equal frequency.

$H_0 : p_S = p_M = p_T = p_W = p_{Th} = p_F = p_S = \frac{1}{7}$

$H_1 : \textit{at}$ least one $\mathrm{p_i} \neq \frac{1}{7}$

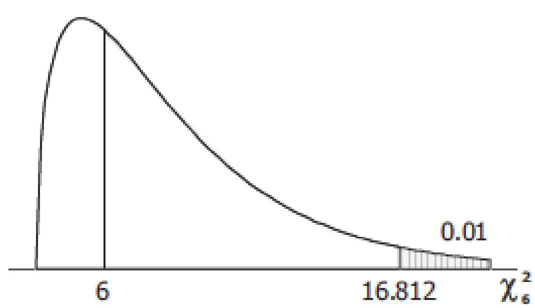
$\alpha = 0.01 \quad \textit{and} \quad df = 6$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548

$C.V. \ \chi^2 = \chi^2_\alpha = \chi^2_{0.05} = 16.812$ $\qquad E = \frac{1}{7}\sum O = \frac{773}{7} = 110.43$

Calculations:

Day	*O*	*E*	$\frac{(O-E)^2}{E}$
S	77	110.43	10.119
M	110	110.43	0.0017
T	124	110.43	1.6679
W	122	110.43	1.2125
Th	120	110.43	0.8296
F	123	110.43	1.4312
S	97	110.43	1.6330
	773	773	16.8952



$$\chi^2 = \sum \frac{(O-E)^2}{E} = 16.8952$$

P-value = $\chi^2 \ cdf\,(16.895,\ 99,\ 6) = 0.0097$

Conclusion

Reject H_0; there is sufficient evidence to support the claim that $\mathrm{p_i} = \frac{1}{7}$ for each day. There is sufficient evidence to reject the claim that births occur on the different days with equal frequency. Births that do not occur naturally (induced, Caesarean sections) are typically not scheduled for Saturday and Sunday, accounting for the smaller than expected numbers of births on those days.

Exercise

The table below lists the frequency of wins for different post positions in the Kentucky Derby horse race. A post position of 1 is closest to the inside rail, so that horse has the shortest distance to run. (Because the number of horses varies from year to year, only the first ten post positions are included.) Use a 0.05 significance level to test the claim that the likelihood of winning is the same for the different post positions. Based on the result, should bettor consider the post position of a horse racing in the Kentucky Derby?

Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Post Position	1	2	3	4	5	6	7	8	9	10
Wins	19	14	11	14	14	7	8	11	5	11

Solution

Original Claim: The likelihood of winning is the same for all post positions.

$$H_0 : p_1 = p_2 = \ldots = p_{10} = \frac{1}{10}$$

$$H_1 : \textit{at} \text{ least one } \mathrm{p_i} \neq \frac{1}{10}$$

$$\alpha = 0.05 \quad \textit{and} \quad df = 9$$

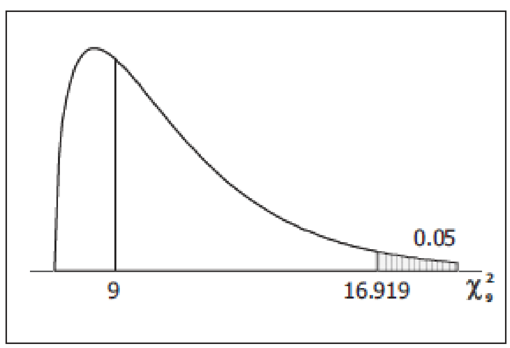
Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589

$$C.V. \ \chi^2 = \chi^2_\alpha = \chi^2_{0.05} = 16.919$$

$$E = \frac{1}{10}\sum O = \frac{114}{10} = 11.4$$

Calculations:

Position	*O*	*E*	$\frac{(O-E)^2}{E}$
1	19	11.4	5.0667
2	14	11.4	0.5930
3	11	11.4	0.0140
4	14	11.4	0.5930
5	14	11.4	0.5930
6	7	11.4	1.6982
7	8	11.4	1.0140
8	11	11.4	0.0140
9	5	11.4	3.5930
10	11	11.4	0.0140
	114	114.0	13.193



$$\chi^2 = \sum \frac{(O-E)^2}{E} = 13.193$$

$$P\text{-value} = \ \chi^2 \ cdf\,(13.193,\ 99,\ 9) = 0.1541$$

Conclusion

Do not reject H_0; there is not sufficient evidence to reject the claim that $p_i = \frac{1}{10}$ for each position. There is no sufficient evidence to reject the claim that the likelihood of winning is the same for all post positions. Based on these results, post position is not a significant consideration when betting on the Kentucky Derby.

Exercise

The table below lists the cases of violent crimes are randomly selected and categorized by month.
Use a 0.01 significance level to test the claim that the rate of violent crime is the same for each month.
Can you explain the result?
Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Number	786	704	835	826	900	868	920	901	856	862	783	797

Solution

Original Claim: The occurrence of violent crime is the same for each month.

$$H_0 : p_{Jan} = p_{Feb} = \ldots = p_{Dec} = \frac{1}{12}$$

$$H_1 : at \text{ least one } p_i \neq \frac{1}{12}$$

$$\alpha = 0.01 \quad and \quad df = 11$$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757

$$C.V.\ \chi^2 = \chi^2_\alpha = \chi^2_{0.01} = 24.725$$

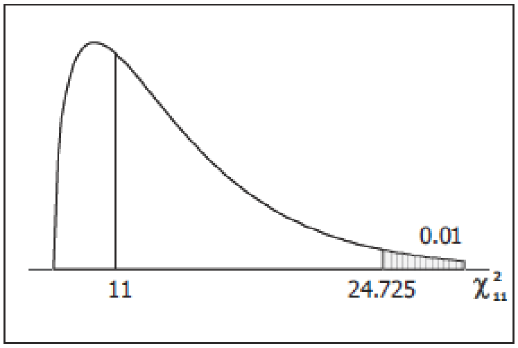
$$E = \frac{1}{12}\sum O$$

$$= \frac{10038}{12}$$

$$= 836.5$$

Calculations:

Month	*O*	*E*	$\frac{(O-E)^2}{E}$
Jan	786	836.5	3.0487
Feb	704	836.5	20.9877
Mar	835	836.5	0.0027
Apr	826	836.5	0.1318
May	900	836.5	4.8204
Jun	868	836.5	1.1862
Jul	920	836.5	8.3350
Aug	901	836.5	4.9734
Sep	856	836.5	0.4546
Oct	862	836.5	0.7773
Nov	783	836.5	3.4217
Dec	797	836.5	1.8652
	10038	10038.0	50.0048



$$\chi^2 = \sum \frac{(O-E)^2}{E} = \underline{50.0048}$$

P-value = $\chi^2\ cdf\,(50.005,\ 99,\ 11) = \underline{0.0000006}$

Conclusion

Reject H_0; there is sufficient evidence to support the claim that $p_i = \frac{1}{12}$ for each month. There is sufficient evidence to reject the claim that the occurrence of violent crime is the same for each month. A major factor involved in this conclusion is the large contribution of the month of February to the calculated χ^2 statistic. The comparison of frequencies for each month is not fair because not all months have the same number of days.

Exercise

The table below lists the results of the Advanced Placement Biology class conducted genetics experiments with fruit flies. Use a 0.05 significance level to test the claim that the observed frequencies agree with the proportions that were expected according to principles of genetics

Conduct the hypothesis test and the test statistic, critical value and/or *P*-value, and state the conclusion.

Characteristic	***Red eye / normal wing***	***Sepia eye / normal wing***	***Red eye / vestigial wing***	***Sepia eye / vestigial wing***
Frequency	59	15	2	4
Expected proportion	$\frac{9}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

Solution

Original Claim: Observed frequencies fit the expected proportions.

$H_0: \ p_1 = \frac{9}{16}, \quad p_2 = \frac{3}{16}, \quad p_3 = \frac{3}{16}, \quad p_4 = \frac{1}{16}$

H_1 : *at least one* p_i is not as claimed

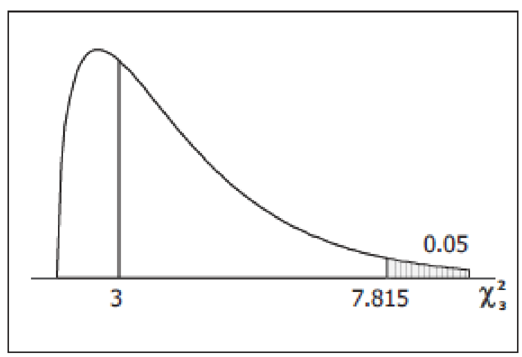
$\alpha = 0.05 \quad and \quad df = 3$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838

$C.V. \ \chi^2 = \chi^2_\alpha = \chi^2_{0.05} = 7.815$

Calculations:

Day	*O*	*E*	$\frac{(O-E)^2}{E}$
1	59	$80 \cdot \frac{9}{16} = 45$	4.3556
2	15	$80 \cdot \frac{3}{16} = 15$	0.00
3	2	$80 \cdot \frac{3}{16} = 15$	11.2667
4	4	$80 \cdot \frac{1}{16} = 5$	0.200
	80	80	15.8222



$$\chi^2 = \sum \frac{(O-E)^2}{E} = \underline{15.8222}$$

P-value = $\chi^2 \ cdf\,(15.822, \ 99, \ 3) = \underline{0.0012}$

Conclusion

Reject H_0 ; there is sufficient evidence to reject the claim that the proportions are as claimed.

There is sufficient evidence to reject the claim that observed frequencies fit the proportions that were expected according to the principles of genetics

Exercise

The table below lists the claims that its M&M plain candies are distributed with the following color percentages: 16% green, 20% orange, 14% yellow, 24% blue, 13% red, and 13% brown. Use a 0.05 significance level to test the claim that the color distribution is as claimed.

Green	Orange	Yellow	Blue	Red	Brown
19	25	8	27	13	8

Solution

Original Claim: The color distribution is as stated

$H_0:\ p_G = .16,\quad p_O = .20,\quad p_Y = .14,\quad p_{Bl} = .24,\quad p_R = .13,\quad p_{BR} = .13$

$H_1: at\ least\ one\ p_i$ is not as claimed

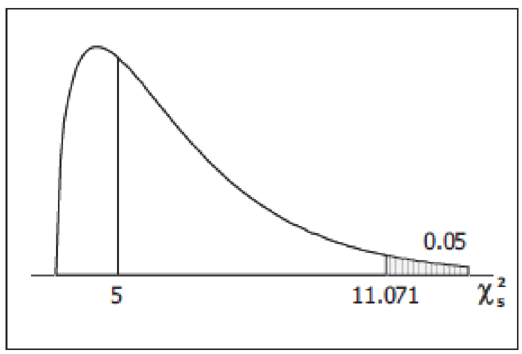
$\alpha = 0.05 \quad and \quad df = 5$

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750

$C.V.\ \chi^2 = \chi^2_\alpha = \chi^2_{0.05} = 11.071$

Calculations:

Day	*O*	*E*	$\frac{(O-E)^2}{E}$
G	19	$100(.16) = 16$	0.5625
O	25	$100(.20) = 20$	1.2500
Y	8	$100(.14) = 14$	2.5714
Bl	27	$100(.24) = 24$	0.3750
R	13	$100(.13) = 13$	0.0
Br	8	$100(.13) = 13$	1.9231
	100	100	6.6820



$$\chi^2 = \sum \frac{(O-E)^2}{E} = 6.682$$

P-value = $\chi^2\ cdf\ (6.682,\ 99,\ 5) = 0.2454$

Conclusion

Do not reject H_0; there is not sufficient evidence to reject the claim that the proportion are as stated. There is no sufficient evidence to reject the claim that the color distribution is as stated.

Solution Section 4.5 – Comparing Three or More Means

Exercise

Fill in the ANOVA table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-Test Statistic
Treatment	565	5		
Error	3560	32		
Total				

Solution

$$MST = \frac{SST}{k-1} = \frac{565}{5} \underline{\underline{=113}}$$

$$MSE = \frac{SSE}{n-k} = \frac{3560}{32} \underline{\underline{=111.25}}$$

$$F_0 = \frac{MST}{MSE} = \frac{113}{111.25} \underline{\underline{=1.016}}$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-Test Statistic
Treatment	565	5	**113**	**1.016**
Error	3560	32	**111.25**	
Total	**4125**	**37**		

Exercise

Fill in the ANOVA table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-Test Statistic
Treatment	490	4		
Error	7267	21		
Total				

Solution

$$MST = \frac{SST}{k-1} = \frac{490}{4} \underline{\underline{=122.5}}$$

$$MSE = \frac{SSE}{n-k} = \frac{7267}{21} \underline{\underline{=346.048}}$$

$$F_0 = \frac{MST}{MSE} = \frac{122.5}{346.048} \underline{\underline{=0.354}}$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-Test Statistic
Treatment	490	4	**122.5**	**0.354**
Error	7267	21	**346.048**	
Total	**7757**	**25**		

Exercise

Determine the ***F***-test statistic based on the given summary statistics

Population	***Sample Size***	***Sample Mean***	***Sample Variance***
1	10	42	35
2	10	41	40
3	10	22	23

Compute $\overline{x}$, the sample mean of the combined data set, by adding up all the observations and dividing by the number pf the observations.

Solution

$$\overline{x} = \frac{\sum n_i x_i}{\sum n_i} = \frac{10(42)+10(41)+10(22)}{10+10+10} \underline{=35|}$$

$$MST = \frac{\sum n_i\left(x_i - \overline{x}\right)^2}{k-1} = \frac{10(42-35)^2 + 10(41-35)^2 + 10(22-35)^2}{3-1} \underline{=1270|}$$

$$MSE = \frac{\sum\left(n_i - 1\right)s_i^2}{n-k} = \frac{(10-1)(35)+(10-1)(40)+(10-1)(23)}{30-3} \underline{=32.667|}$$

$$F = \frac{MST}{MSE} = \frac{1270}{32.667} \underline{=38.88|}$$

Therefore, the value of the ***F***-test statistic is 38.8

Exercise

An engineer wants to know if the mean strengths of three concrete mix designs differ significantly. He randomly selects 9 cylinders that measure 6 inches in diameter and 12 inches in heights in which mixture *A* is poured, 9 cylinders of mixture *B*, and 9 cylinders of mixture *C*. After 28 days, he measures the strength (in pounds per square inch) of the cylinders. The results are presented in the table below.

Mixture A	***Mixture B***	***Mixture C***
3,980	4,070	4,130
4,040	4,340	3,820
3,760	4,620	4,020
3,870	3,730	4,150
3,990	4,870	4,190
4,090	4,120	3,840
3,820	4,640	3,750
3,940	4,180	3,990
4,080	3,850	4,320

a) Write the null and alternative hypotheses

b) Explain why the one-way ANOVA cannot be used to test these hypotheses

Solution

a) $H_0 : \mu_A = \mu_B = \mu_C$

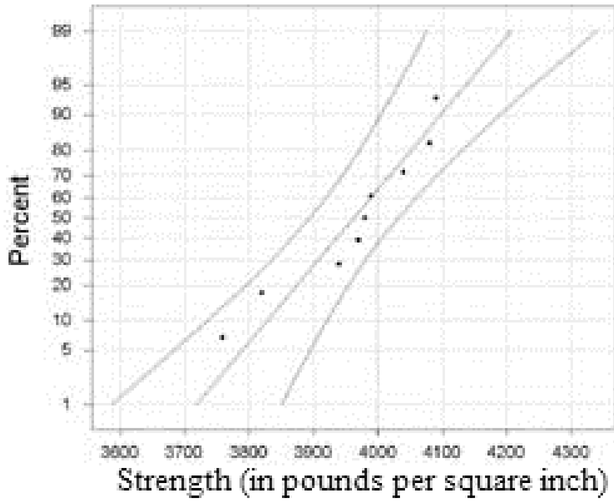
H_1 : at least one of the means is different

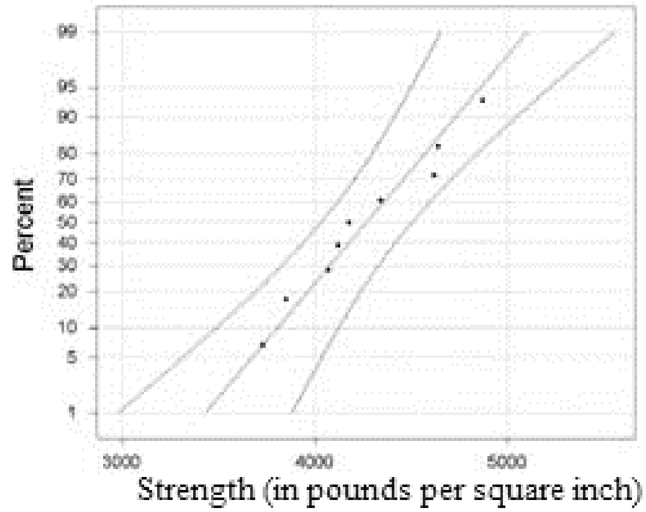
b) The standard deviation for mixture *A* is 115.41

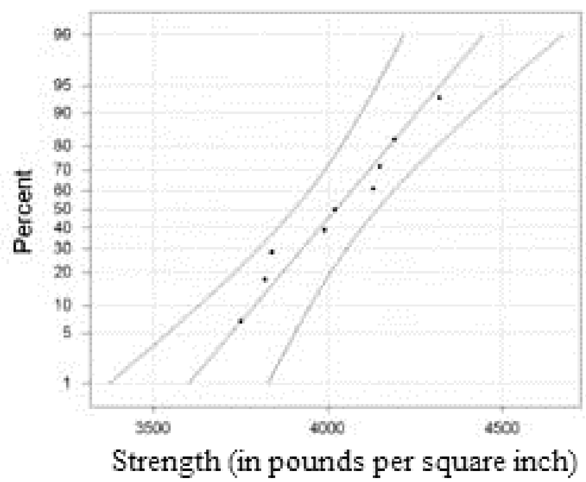
The standard deviation for mixture *B* is 381.26

The standard deviation for mixture *C* is 191.71

The standard deviation for mixture *B* is more than 2 times larger than the standard deviation for mixture *A*.







Exercise

At a community college, the mathematics department has been experimenting with four different delivery mechanisms for content in their Elementary Statistics courses. One method is the traditional lecture (method I), the second is a hybrid format in which half the class time is online and the other half is face-to-face (method II), the third is online (method III), and the fourth is an emporium model from which students obtain their lectures and do their work in a lab with an instructor available for assistance (method IV). To assess the effectiveness of the four methods, students in each approach are given a final exam with the results shown in the accompanying table. Do the data suggest that any method has a different mean score from the others?

Method I	76	81	85	68	88	73	80	65	60	92	83	51	71	63	71	65
Method II	88	52	77	73	64	38	57	63	83	65	78	64	87	92		
Method III	78	60	73	70	62	82	74	80	53	46	84	80	78			
Method IV	89	90	79	62	83	75	54	70	80	94	76	78	81			

a) Write the null and alternative hypotheses

b) State the requirements that must be satisfied to use one-way ANOVA procedure

c) Assuming the requirements stated in part (b) are satisfied, use the following one-way ANOVA table to test the hypothesis of equal means at the $\alpha = 0.05$ level of significance.

d) Interpret the *P*-value.

e) Verify that the residuals are normally distributed

Solution

a) $H_0 : \mu_I = \mu_{II} = \mu_{III} = \mu_{IV}$

H_1 : at least one of the means is different

b) There must be *k* simple samples, one from each of *k* populations.

The populations must be normally distributed.

The populations must have the same variance

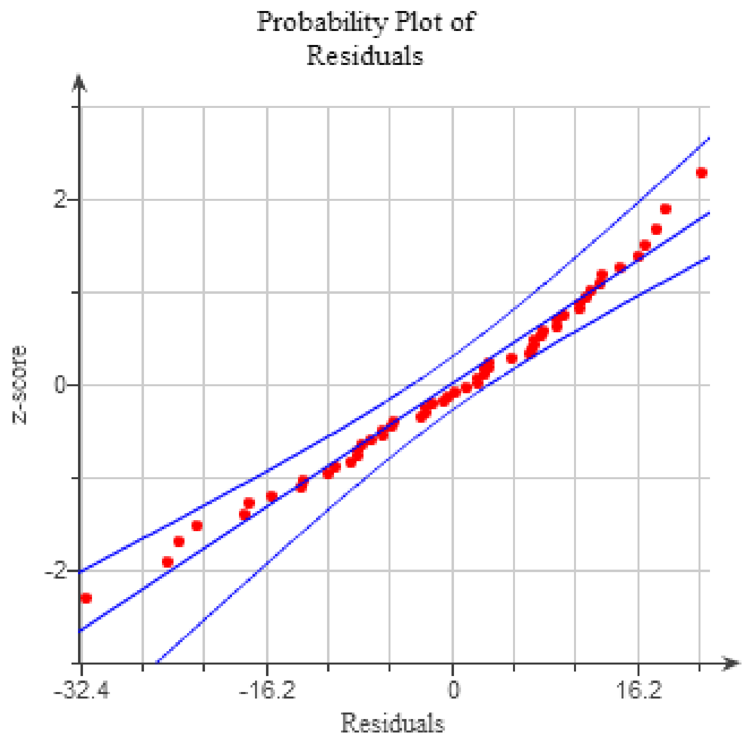
c)

Source of Variation	***Degrees of Freedom***	***Sum of Squares***	***Mean Squares***	***F-Test Statistic***	***P- value***
Treatment	3	481.24	**160.41**	**1.03**	**0.387**
Error	52	8089.54	**155.74**		
Total	**55**	**8579.78**			

Since the *P*-value is greater than α, do not reject H_0, there is insufficient evidence to support H_1

d) Since the *P*-value is greater than or equal to α, conclude that there is insufficient evidence that any one method is more or less effective than the others.

e) $\mu_I = 73.250$; $\mu_{II} = 70.071$; $\mu_{III} = 70.769$; $\mu_{IV} = 77.769$


Probability Plot of
Residuals
z-score
2
0
-2
-32.4
-16.2
0
16.2
Residuals

Solution Section 4.7 – Variation and Prediction Intervals

Exercise

A height of 70 in. is used to find the predicted weight is 180 lb. In your own words, describe a prediction interval in this situation.

Solution

A prediction interval is an interval estimate for a predicted value. In this situation it will be a range of weights centered at the prediction's point estimate of 180 lbs.

Exercise

A height of 70 in. is used to find the predicted weight is 180 lb. What is the major advantage of using a prediction interval instead of the predicted weight of 180 lb.? Why is the terminology of prediction interval used instead of confidence interval?

Solution

By providing a range of values instead of a single point, a prediction interval gives an indication of the accuracy of the prediction. A confidence interval is an internal estimate of a parameter – i.e., of a conceptually fixed, although unknown, value. A prediction interval is an interval estimate of a random variable – i.e., of a value from a distribution of values.

Exercise

Use the value of the linear correlation $r = 0.873$ to find the coefficient of determination and the percentage of the total variation that can be explained by the linear relationship between the 2 variables

$x = tar\ in\ menthol\ cigarettes$

16	13	16	9	14	13	12	14	14	13	13	16	13	13	18
9	19	2	13	14	14	15	16	6	8					

$y = nicotine\ in\ menthol\ cigarettes$

1.1	0.8	1	0.9	0.8	0.8	0.8	0.8	0.9	0.8	0.8	1.2	0.8	0.8	1.3
0.7	1.4	0.2	0.8	1	0.8	0.8	1.2	0.6	0.7					

Solution

The coefficient of determination is $r = (0.873)^2 = 0.762$

The portion of the total variation in y explained by the regression is $r^2 = 0.762 \underline{= 76.2\%}$

Regression Statistics	
Multiple R	0.873034386
R Square	0.762189039
Adjusted R Square	0.751849432
Standard Error	0.120760017
Observations	25

Exercise

Use the value of the linear correlation $r = 0.744$ to find the coefficient of determination and the percentage of the total variation that can be explained by the linear relationship between the 2 variables

$x = movie\ budget$

41	20	116	70	75	52	120	65	6.5	60	125	20	5	150
4.5	7	100	30	225	70	80	40	70	50	74	200	113	68
72	160	68	29	132	40								

$y = movie\ gross$

117	5	103	66	121	116	101	100	55	104	213	34	12	290
47	10	111	100	322	19	117	48	228	47	17	373	380	118
114	120	101	120	234	209								

Solution

The coefficient of determination is $r = (0.744)^2 = 0.554$

The portion of the total variation in y explained by the regression is $r^2 = 0.554 \ \underline{= 55.4\%}$

Exercise

Use the value of the linear correlation $r = -0.865$ to find the coefficient of determination and the percentage of the total variation that can be explained by the linear relationship between the 2 variables

$x = car\ weight, \quad y = city\ fuel\ consumption\ in\ mi/gal$

Solution

The coefficient of determination is $r = (-0.865)^2 = 0.748$

The portion of the total variation in y explained by the regression is $r^2 = 0.748 \ \underline{= 74.8\%}$

Exercise

Use the value of the linear correlation $r = -0.488$ to find the coefficient of determination and the percentage of the total variation that can be explained by the linear relationship between the 2 variables

$x = age\ of\ home, \quad y = home\ selling\ price$

Solution

The coefficient of determination is $r = (-0.488)^2 = 0.238$

The portion of the total variation in y explained by the regression is $r^2 = 0.238 \ \underline{= 23.8\%}$

Exercise

Refer to the display obtained by using the paired data consisting of weights (in *lb*.) of 32 cars and their highway fuel consumption amounts (in *mi/gal*). A car weight of 4000 *lb*. to be used for predicting the highway fuel consumption amount

```
The regression equation is
Highway = 50.5 - 0.00587 Weight

Predictor         Coef     SE Coef       T      P
Constant        50.502       2.860   17.66  0.000
Weight      -0.0058685   0.0007859   -7.47  0.000

S = 2.19498   R-Sq = 65.0%   R-Sq(adj) = 63.9%

Predicted Values for New Observations

New
Obs     Fit   SE Fit         95% CI              95% PI
  1  27.028    0.497  (26.013, 28.042)  (22.431, 31.624)

Values of Predictors for New Observations
New
Obs  Weight
  1    4000
```

a) What percentage of the total variation in highway fuel consumption can be explained by the linear correlation between weight and highway fuel consumption?

b) If a car weighs 4000 *lb*., what is the single value that is the best predicted amount of highway fuel consumption? (Assume that there is a linear correlation between weight and highway fuel consumption.)

Solution

a) $\boxed{R - squared = 65.0\%}$

b) The given point estimate is $\boxed{\hat{y} = 27.028\ \ mpg}$

Exercise

The paired values of the Consumer Price Index (CPI) and the cost of a slice of pizza are shown below

CPI	30.2	48.3	112.3	162.2	191.9	197.8
Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00

a) Find the explained variation

b) Find the unexplained variation

c) Find the total variation

d) Find the coefficient of determination

e) Find the standard error of estimate s_e

f) Find the predicted cost of a slice of pizza for the year 2001, when the CPI was 187.1.

g) Find a 95% prediction interval estimate of the cost of a slice of pizza when the CPI was 187.1

In each case, there is sufficient evidence to support a claim of a linear correlation so that it is reasonable to use the regression equation when making predictions.

Solution

The predicted values:

	Coefficients
Intercept	-0.161601
X Variable	0.0100574

$$\hat{y} = -0.161601 + 0.0100574x$$

x	y	$\hat{y}$	$\bar{y}$	$\hat{y}-\bar{y}$	$(\hat{y}-\bar{y})^2$	$y-\hat{y}$	$(y-\hat{y})^2$	$y-\bar{y}$	$(y-\bar{y})^2$
30.2	0.15	0.142	1.083	–0.940	0.886	0.008	0.000	–0.930	0.871
48.3	0.35	0.324	1.083	–0.760	0.576	0.023	0.001	–0.730	0.538
112.3	1.00	0.968	1.083	–0.120	0.013	0.032	0.001	–0.080	0.007
162.2	1.25	1.470	1.083	0.386	0.149	–0.220	0.048	0.167	0.028
191.9	1.75	1.768	1.083	0.685	0.469	–0.018	0.000	0.667	0.444
197.8	2.00	1.828	1.083	0.744	0.554	0.172	0.030	0.917	0.840
742.7	6.50	6.50	6.50	0.0	2.648	0.0	0.08	0.0	2.728

a) The explained variation is $\sum(\hat{y}-\bar{y})^2 = 2.648$

b) The unexplained variation is $\sum(y-\hat{y})^2 = 0.080$

c) The total variation is $\sum(y-\bar{y})^2 = 2.728$

d) $r^2 = \dfrac{\sum(\hat{y}-\bar{y})^2}{\sum(y-\bar{y})^2} = \dfrac{2.648}{2.728} = 0.971$

e) $s_e^2 = \dfrac{\sum (y-\hat{y})^2}{n-2} = \dfrac{0.08}{4} = 0.02$

$s_e = \sqrt{0.02} \; \underline{= 0.141}$

f) $\hat{y}\big|_{187.1} = -0.161601 + 0.0100574(187.1)$

$= 1.7201$

$\underline{= \$1.72}$

g) Preliminary calculations for $n = 6$

$\bar{x} = \dfrac{\sum x}{n} = \dfrac{742.7}{6} = \underline{123.783}$

$\alpha = 0.05 \quad \text{(2-tails)}$

$t_{\alpha/2} = 2.776; \quad df = 6 - 2 = 4$

x	***y***	x^2
30.2	0.15	912.04
48.3	0.35	2332.89
112.3	1.00	12611.29
162.2	1.25	26308.84
191.9	1.75	36825.61
197.8	2.00	39124.84
742.7	6.50	118115.5

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	**Area in Two Tails**				
	0.01	0.02	0.05	0.10	0.20
4	**4.604**	3.747	**2.776**	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (2.776)(0.141)\sqrt{1 + \frac{1}{6} + \frac{6(187.1 - 123.783)^2}{6(118115.5) - (742.7)^2}}$$

$\underline{\approx 0.4450}$

$\hat{y} - E < y < \hat{y} + E$

$1.7201 - 0.445 < y_{187.1} < 1.7201 + 0.445$

$\underline{\$1.27 < y_{187.1} < \$2.17}$

Exercise

The paired values of the Consumer Price Index (CPI) and the subway fare are shown below

CPI	30.2	48.3	112.3	162.2	191.9	197.8
Subway fare	0.15	0.35	1.00	1.35	1.5	2.00

a) Find the explained variation
b) Find the unexplained variation
c) Find the total variation
d) Find the coefficient of determination
e) Find the standard error of estimate s_e
f) Find the predicted cost of subway fare for the year 2001, when the CPI was 187.1.
g) Find a 95% prediction interval estimate of the cost of subway fare when the CPI was 187.1

In each case, there is sufficient evidence to support a claim of a linear correlation so that it is reasonable to use the regression equation when making predictions.

Solution

The predicted values (from Excel):

	Coefficients
Intercept	-0.124252712
X Variable 1	0.009553677

$$\hat{y} = -0.124253 + 0.00955368x$$

x	y	$\hat{y}$	$\bar{y}$	$\hat{y}-\bar{y}$	$(\hat{y}-\bar{y})^2$	$y-\hat{y}$	$(y-\hat{y})^2$	$y-\bar{y}$	$(y-\bar{y})^2$
30.2	0.15	0.164	1.058	–0.890	0.799	–0.014	0.0	–0.910	0.825
48.3	0.35	0.337	1.058	–0.720	0.520	0.013	0.0	–0.710	0.502
112.3	1.00	0.949	1.058	–0.110	0.012	0.051	0.006	0.292	0.085
162.2	1.35	1.425	1.058	0.367	0.135	–0.075	0.006	0.292	0.085
191.9	1.50	1.709	1.058	0.651	0.423	–0.209	0.044	0.442	0.195
197.8	2.00	1.765	1.58	0.707	0.500	0.235	0.055	0.942	0.887
742.7	6.35	6.350	6.350	0.0	2.930	0.0	0.104	0.0	2.497

a) The explained variation is $\sum(\hat{y}-\bar{y})^2 = \underline{2.390}$

b) The unexplained variation is $\sum(y-\hat{y})^2 = \underline{0.107}$

c) The total variation is $\sum(y-\bar{y})^2 = \underline{2.497}$

d) $$r^2 = \frac{\sum(\hat{y}-\bar{y})^2}{\sum(y-\bar{y})^2} = \frac{2.648}{2.728} \underline{= 0.957}$$

Regression Statistics	
Multiple R	0.978255696
R Square	0.956984207
Adjusted R Square	0.946230258
Standard Error	0.163870391
Observations	6

e) $$s_e^2 = \frac{\sum(y-\hat{y})^2}{n-2} = \frac{0.107}{4} = 0.02675$$

$s_e = \sqrt{0.02675} = 0.164$

f) $\hat{y}\Big|_{187.1} = -0.124253 + 0.00955368(187.1)$

$= \$1.66$

x	y	x^2
30.2	0.15	912.04
48.3	0.35	2332.89
112.3	1.00	12611.29
162.2	1.35	26308.84
191.9	1.50	36825.61
197.8	2.00	39124.84
742.7	6.35	118115.51

g) Preliminary calculations for $n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{742.7}{6} = 123.783$$

$\alpha = 0.05$ (2-tails)

$t_{\alpha/2} = 2.776;\quad df = 6 - 2 = 4$

TABLE A-3 t Distribution: Critical t Values

	Area in One Tail 0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails 0.01	0.02	0.05	0.10	0.20
4	**4.604**	3.747	**2.776**	2.132	1.533

$$E = t_{\alpha/2}\, s_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (2.776)(0.164)\sqrt{1 + \frac{1}{6} + \frac{6(187.1 - 123.783)^2}{6(118115.5) - (742.7)^2}}$$

$$\approx 0.5230$$

$$\hat{y} - E < y < \hat{y} + E$$

$$1.6632 - 0.5230 < y_{187.1} < 1.6632 + 0.5230$$

$$\$1.14 < y_{187.1} < \$2.19$$

Exercise

Find the best predicted temperature for a recent year in which the concentration (in parts per million) of CO_2 and temperature (in °C) for different years

CO_2	314	317	320	326	331	339	346	354	361	369
Temperature	13.9	14.0	13.9	14.1	14.0	14.3	14.1	14.5	14.5	14.4

a) Find the explained variation
b) Find the unexplained variation
c) Find the total variation
d) Find the coefficient of determination
e) Find the standard error of estimate s_e
f) Find the predicted temperature (in °C) when CO_2 concentration is 370.9 parts per million.
g) Find a 99% prediction interval estimate temperature (in °C) when CO_2 concentration is 370.9 parts per million

In each case, there is sufficient evidence to support a claim of a linear correlation so that it is reasonable to use the regression equation when making predictions.

Solution

The predicted values (from Excel):

	Coefficients
Intercept	10.48308065
X Variable 1	0.010917736

$\hat{y} = 10.4831 + 0.0109177x$

x	**y**	$\hat{y}$	$\bar{y}$	$\hat{y}-\bar{y}$	$(\hat{y}-\bar{y})^2$	$y-\hat{y}$	$(y-\hat{y})^2$	$y-\bar{y}$	$(y-\bar{y})^2$
314	13.9	13.911	14.17	–0.259	0.067	–0.011	0.0	–0.27	0.073
317	14	13.944	14.17	–0.266	0.051	0.056	0.003	–0.17	0.029
320	13.9	13.977	14.17	–0.193	0.037	–0.077	0.006	–0.27	0.073
326	14.1	14.042	14.17	–0.128	0.016	0.058	0.003	–0.07	0.005
331	14	14.097	14.17	–0.073	0.005	–0.097	0.009	–0.17	0.029
339	14.3	14.184	14.17	0.014	0.0	0.116	0.013	0.13	0.017
346	14.1	14.261	14.17	0.091	0.008	–0.161	0.026	–0.07	0.005
354	14.5	14.348	14.17	0.178	0.032	0.152	0.023	0.33	0.109
361	14.5	14.424	14.17	0.254	0.065	0.076	0.006	0.33	0.109
369	14.4	14.512	14.17	0.342	0.117	–0.112	0.012	0.23	0.053
3377	141.7	141.7	141.70	0.0	0.399	0.0	0.102	0.0	0.501

a) The explained variation is $\sum(\hat{y}-\bar{y})^2 = 0.399$

b) The unexplained variation is $\sum(y-\hat{y})^2 = 0.102$

c) The total variation is $\sum(y-\bar{y})^2 = 0.501$

Regression Statistics	
Multiple R	0.891976355
R Square	0.795621818
Adjusted R Square	0.770074545
Standard Error	0.113133477
Observations	10

d) $$r^2 = \frac{\sum(\hat{y}-\bar{y})^2}{\sum(y-\bar{y})^2} = \frac{0.399}{0.501} = 0.796$$

e) $$s_e^2 = \frac{\sum(y-\hat{y})^2}{n-2} = \frac{0.102}{8} = 0.01275$$

$$s_e = \sqrt{0.01275} = 0.113$$

f) $$\hat{y}\big|_{370.9} = 10.4831 + 0.0109177(370.9)$$

$$= 14.53\ °C$$

g) Preliminary calculations for $n = 8$

$$\bar{x} = \frac{\sum x}{n} = \frac{3377}{10} = 337.7$$

$$\alpha = 0.01 \quad and \quad df = n-2 = 8 \quad \text{(2-tails)}$$

$$t_{\alpha/2} = t_{0.005} = 3.355$$

x	y	x^2
314	13.9	985696
317	14	100489
320	13.9	102400
326	14.1	106276
331	14	109561
339	14.3	114921
346	14.1	119716
354	14.5	125316
361	14.5	130321
369	14.4	136161
3377	141.7	1143757

TABLE A-3 t Distribution: Critical t Values

			Area in One Tail		
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom			Area in Two Tails		
	0.01	0.02	0.05	0.10	0.20
8	$\boxed{3.355}$	2.896	2.306	1.860	1.397

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0-\bar{x})^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (3.355)(0.113)\sqrt{1 + \frac{1}{10} + \frac{10(370.9-337.7)^2}{10(1143757) - (3377)^2}}$$

$$\approx 0.4533$$

$$\hat{y} - E < y < \hat{y} + E$$

$$14.5324 - 0.4533 < y_{370.9} < 14.5324 + 0.4533$$

$$14.08\ °C < y_{370.9} < 14.99\ °C$$

Exercise

Find a prediction interval data listed below.

Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00
Subway Fare	0.15	0.35	1.00	1.35	1.50	2.00

Using: *Cost of a slice of pizza:* \$2.10; 99% *confidence*

Solution

The predicted values (from Excel):

	Coefficients
Intercept	0.03456017
X Variable 1	0.94502138

$$\hat{y} = 0.034560 + 0.945021x$$

$$\hat{y}\big|_{2.1} = 0.034560 + 0.945021(2.1)$$

$$= 2.019$$

$\alpha = 0.01$ *and* $df = n - 2 = 4$

$t_{\alpha/2} = t_{0.005} = 4.604$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	**Area in Two Tails**				
	0.01	0.02	0.05	0.10	0.20
4	4.604	3.747	2.776	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (4.604)(0.122987)\sqrt{1 + \frac{1}{6} + \frac{6(2.1 - 1.083333)^2}{6(9.77) - (6.5)^2}}$$

$$\approx 0.704$$

x	x^2
0.15	0.0225
0.35	0.1225
1	1
1.25	1.5625
1.75	3.0625
2	4
6.5	9.77

$$\hat{y} - E < y < \hat{y} + E$$

$$2.019 - 0.704 < y_{2.1} < 2.019 + 0.704$$

$$\$1.32 < y_{2.1} < \$2.72$$

Exercise

Find a prediction interval data listed below.

Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00
Subway Fare	0.15	0.35	1.00	1.35	1.50	2.00

Using: *Cost of a slice of pizza:* \$2.10; 90% *confidence*

Solution

The predicted values (from Excel):

	Coefficients
Intercept	0.03456017
X Variable 1	0.94502138

$$\hat{y} = 0.034560 + 0.945021x$$

$$\hat{y}\Big|_{2.1} = 0.034560 + 0.945021(2.1)$$

$$= 2.019$$

Regression Statistics	
Multiple R	0.98781094
R Square	0.97577045
Adjusted R Square	0.96971306
Standard Error	0.122987
Observations	6

$$\alpha = 0.1 \quad and \quad df = n-2 = 4$$

$$t_{\alpha/2} = t_{0.05} = 2.132$$

TABLE A-3 *t* Distribution: Critical *t* Values

Degrees of Freedom					
Area in One Tail	0.005	0.01	0.025	0.05	0.10
Area in Two Tails	0.01	0.02	0.05	0.10	0.20
4	**4.604**	3.747	**2.776**	2.132	1.533

$$E = t_{\alpha/2}s_e\sqrt{1+\frac{1}{n}+\frac{n\left(x_0-\bar{x}\right)^2}{n\left(\sum x^2\right)-\left(\sum x\right)^2}}$$

$$=(2.132)(0.122987)\sqrt{1+\frac{1}{6}+\frac{6(2.10-1.083333)^2}{6(9.77)-(6.5)^2}}$$

$$\approx 0.326$$

x	x^2
0.15	0.0225
0.35	0.1225
1	1
1.25	1.5625
1.75	3.0625
2	4
6.5	9.77

$$\hat{y}-E < y < \hat{y}+E$$

$$2.019 - 0.326 < y_{2.1} < 2.019 + 0.326$$

$$\$1.69 < y_{2.1} < \$2.34$$

Exercise

Find a prediction interval data listed below.

Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00
Subway Fare	0.15	0.35	1.00	1.35	1.50	2.00

Using: *Cost of a slice of pizza:* \$0.50; 95% *confidence*

Solution

The predicted values (from Excel):

	Coefficients
Intercept	0.03456017
X Variable 1	0.94502138

$\hat{y} = 0.034560 + 0.945021x$

$\hat{y}\big|_{0.50} = 0.034560 + 0.945021(0.5)$

$= 0.507$

Regression Statistics	
Multiple R	0.98781094
R Square	0.97577045
Adjusted R Square	0.96971306
Standard Error	0.122987
Observations	6

$\alpha = 0.05 \quad and \quad df = n - 2 = 4$

$t_{\alpha/2} = t_{0.025} = 2.776$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	**Area in Two Tails**				
	0.01	0.02	0.05	0.10	0.20
4	4.604	3.747	2.776	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (2.776)(0.122987)\sqrt{1 + \frac{1}{6} + \frac{6(0.5 - 1.083333)^2}{6(9.77) - (6.5)^2}}$$

$$\approx 0.388$$

$\hat{y} - E < y < \hat{y} + E$

$0.507 - 0.388 < y_{0.5} < 0.507 + 0.388$

$\$0.12 < y_{0.5} < \$0.89$

Exercise

Find a prediction interval data listed below.

Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00
Subway Fare	0.15	0.35	1.00	1.35	1.50	2.00

Using: *Cost of a slice of pizza:* \$0.75; 99% *confidence*

Solution

The predicted values (from Excel):

	Coefficients
Intercept	0.03456017
X Variable 1	0.94502138

$$\hat{y} = 0.034560 + 0.945021x$$

$$\hat{y}\big|_{0.75} = 0.034560 + 0.945021(0.75)$$

$$= 0.743$$

$$\alpha = 0.01 \quad and \quad df = n - 2 = 4$$

$$t_{\alpha/2} = t_{0.005} = 4.604$$

TABLE A-3	*t* Distribution: Critical *t* Values				
	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
4	$\boxed{4.604}$	3.747	2.776	2.132	1.533

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x^2\right) - \left(\sum x\right)^2}}$$

$$= (4.604)(0.122987)\sqrt{1 + \frac{1}{6} + \frac{6(0.75 - 1.083333)^2}{6(9.77) - (6.5)^2}}$$

$$\approx 0.622$$

$$\hat{y} - E < y < \hat{y} + E$$

$$0.743 - 0.622 < y_{0.75} < 0.743 + 0.622$$

$$\$0.12 < y_{0.75} < \$1.37$$