Hwk 5.6

1/ $\displaystyle\sum_{k=0}^{19} \frac{k-3}{4} = \frac{1}{4}\left(\sum_{k=0}^{19} k - \sum_{k=0}^{19} 3\right)$

$= \frac{1}{4}\left(\frac{1}{2}\, 19(19+1) - 3\,(19-0+1)\right)$

$= \frac{1}{4}(190 - 60)$

$= \frac{130}{4}$

$= \frac{65}{2}$

#2/ $\displaystyle\sum_{k=2}^{50} (2{,}000 - 3k) = \sum_{k=2}^{50} 2{,}000 - 3\sum_{k=2}^{50} k$

$= 2{,}000\,(50-2+1) - 3\left(\frac{1}{2}50(51) - 1\right)$

$= 98{,}000 - 3{,}822$

$= 94{,}178$

$.\overline{78} = ?$

$.\overline{78} = .787878\ldots$

$= .78 + .0078 + \ldots$

$= 78(.01 + .0001 + \ldots)$ $\quad r = \frac{.0001}{.01} = .01 < 1$

$= 78\left(\frac{.01}{1-.01}\right)$

$= 78\left(\frac{.01}{.99}\right)$

$= 78\left(\frac{1}{99}\right)$

$= \frac{26}{33}$

5.7

#1 $4+8+12+\dots+4n=2n(n+1)$

For $n=1 \Rightarrow 4 \stackrel{?}{=} 2(2)$

$4=4$ ✓ P_1 is true

Assume P_k: $4+8+\dots+4k=2k(k+1)$ is true

is P_{k+1}: $4+\dots+4k+4(k+1)=2(k+1)(k+2)$?

$$4+\dots+4k+4(k+1)=2k(k+1)+4(k+1)$$
$$=(k+1)(2k+4)$$
$$=2(k+1)(k+2) \checkmark$$

P_{k+1} is also true

$\therefore$ By the mathematical Induction, the proof is completed

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#2 $1+5+9+\dots+(4n-3)=n(2n-1)$

2

$n=1 \Rightarrow 1 = 1(2-1)$

$1=1$ ✓ P_1 is true!

1 Let P_k is true: $1+5+\dots+(4k-3)=k(2k-1)$

1 is P_{k+1}: $1+\dots+(4k-3)+[4(k+1)-3]=(k+1)[2(k+1)-1]$

1 $1+\dots+(4k-3)+(4k+1)=(k+1)(2k+1)$?

$$1+\dots+(4k-3)+(4k+1)=k(2k-1)+(4k+1)$$
$$=2k^2-k+4k+1$$
1 $$=2k^2+3k+1$$
$$=(k+1)(2k+1) \checkmark$$

1 P_{k+1} is also true.

1 $\therefore$ By the mathematical induction, the proof is completed.

$$\frac{3x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$3x = A(x-1) + B(x+2)$$

$$x^1 \quad A + B = 3 \rightarrow \boxed{A = 3-1 = 2}$$

$$x^0 \quad -A + 2B = 0$$

$$3B = 3 \Rightarrow \boxed{B = 1}$$

$$\frac{3x}{(x+2)(x-1)} = \boxed{\frac{2}{x+2} + \frac{1}{x-1}}$$

$$\frac{2x+1}{x^2-7x+12} = \frac{A}{x-3} + \frac{B}{x-4}$$

$$2x+1 = A(x-4) + B(x-3)$$

$$x^1: \quad A + B = 2 \qquad\qquad D = \begin{vmatrix} 1 & 1 \\ -4 & -3 \end{vmatrix} = 1$$

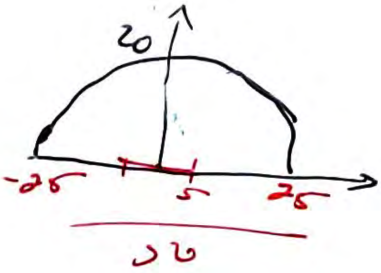
$$x^0 \quad -4A - 3B = 1$$

$$A = \frac{1}{1}\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} = -7$$

$$B = 2 + 7 = 9$$

$$\frac{2x+1}{x^2-7x+12} = \frac{-7}{x-3} + \frac{9}{x-4}$$

37 T: h:14 w=10



$$\frac{x^2}{25^2}+\frac{y^2}{20^2}=1$$

$$\frac{y^2}{20^2}=1-\frac{5^2}{25^2}$$

$$y^2=\frac{20^2}{25^2}\left(\frac{625-25}{}\right)$$

$$y=\frac{20}{25}\sqrt{600}$$

$$=\frac{40}{5}\sqrt{6}$$

$$14 \ ? \ \frac{40}{5}\sqrt{6}$$

$$70 \quad 40\sqrt{6}$$

$$\frac{7}{4}<\sqrt{6}$$

<2 $\quad$ >2

2nd $\quad$ 120

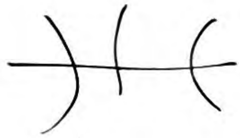
$$14^2=\left(\frac{4}{5}\right)^2 600$$

$$\frac{16}{25}\,600$$

$$14^2 \quad 16(24)$$

$$14\times 14 < 16\times 24$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$



$$\left\{(-1)^{n+1}\frac{n+7}{2n}\right\}$$

1st term & a_8

$n=1 \rightarrow (-1)^2\frac{8}{2}=4$

$n=2 \rightarrow (-1)^3\frac{9}{4}=-\frac{9}{4}$

$n=3 \rightarrow (-1)^4\frac{10}{6}=\frac{5}{3}$

$n=4 \rightarrow (-1)^5\frac{11}{8}=-\frac{11}{8}$

$n=8 \rightarrow (-1)^9\frac{15}{16}=-\frac{15}{16}$

Arithm

a_{20} : $a_9 = -5 \quad a_{15} = 31$

$d = \frac{31+5}{15-9} = 6$

$a_9 = a_1 + 8(6) = -5$

$\underline{a_1 = -53}$

$a_{20} = -53 + 19(6)$

$= -53 + 114$

$= \underline{61}$

Geometric a_8 : $a_2 = 3 \quad a_4 = 6$

$r = \left(\frac{6}{3}\right)^{1/4-2} = 2^{1/2} = \sqrt{2}$

$a_2 = a_1 (\sqrt{2})^1 = 3$

$a_1 = \frac{3}{\sqrt{2}}$

$\frac{1}{2} \quad 7 \quad (3)$

$a_8 = \frac{3}{\sqrt{2}} (\sqrt{2})^7$

$= \frac{3}{\sqrt{2}} 2^3 \sqrt{2}$

$= \underline{24}$

$4 + 11 + 18 + 25 + 32 = \sum_{n=1}^{5} (7n - 3)$

$d = 7, \ a_1 = 4$

$a_n = 4 + (n-1)(7)$

$= 4 + 7n - 7$

$4 + 11 + 18 + \dots + 466 = \sum_{n=1}^{66} (7n - 3)$

$\frac{466 - 4}{d} = \frac{462}{7}$

$$\sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^{n-1} = \infty \qquad |r| = \frac{5}{3} > 1$$

$$\sum_{n=1}^{\infty} 2\left(\frac{3}{5}\right)^{n-1} = \frac{2}{1-\frac{3}{5}} \qquad r = \left|\frac{3}{5}\right| < 1$$

$$= 2 \cdot \frac{5}{2}$$

$$= 5$$

$$\sum_{k=1}^{20} 5 = 5 \times 20$$

$$= 100$$

$$\sum_{k=11}^{55} 4 = 4\,(55 - 11 + 1)$$

$$= 4\,(45)$$

$$= 180$$

$$\sum_{k=2}^{6} (3k - 5) = (6-5) + (9-5) + (12-5) + (15-5) + (18-5)$$

$$= 1 + 4 + 7 + 10 + 13$$

$$= 35$$

2- Partial.

2- 1st form --

1- Arth. a_i $d = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad a_n = a_1 + (n-1)d$

1- Geom $a_?$ $r = \left(\frac{y_2}{y_1}\right)^{1/x_2 - x_1}$ $\quad a_n = a_1 r^{n-1}$

1- Prove

Appl 1- Ellipse

1- Hyperbola

$\Sigma \Rightarrow 5$

$+ - - = \Sigma\ ?$

$$\frac{3x+2}{x^2-5x+4} = \frac{A}{x-1} + \frac{B}{x-4}$$

$$3x+2 = A(x-4) + B(x-1)$$

x^1 $A + B = 3 \rightarrow \boxed{B = 3 + \frac{5}{3} = \frac{14}{3}}$

x^0 $-4A - B = 2$

$-3A = 5$

$\boxed{A = -\frac{5}{3}}$

$$\frac{3x+2}{x^2-5x+4} = -\frac{5}{3}\frac{1}{x-1} + \frac{14}{3}\frac{1}{x-4}$$

$$\boxed{\frac{-5/3}{x-1} + \frac{14/3}{x-4}}$$

$n = 1$ ✓ P_1 is true.

let P_k : $\underline{\quad\quad}^{k}$ = true

is P_{k+1} : $\underbrace{\underline{\quad\quad} + (k+1)}_{1} = k \to k+1$

1 =

P_{k+1} is also true

∴

$$3+6+9+\cdots+3n=\frac{3n(n+1)}{2}$$

For $n=1 \Rightarrow 3=\frac{3(1)(2)}{2}$

$3=3$ ✓ P_1 is true.

P_k is true: $3+6+\cdots+3k=\frac{3k(k+1)}{2}$

is P_{k+1} : $3+\cdots+3k+3(k+1)=\frac{3}{2}(k+1)(k+2)$

$$3+\cdots+3k+3(k+1)=\frac{3}{2}k(k+1)+3(k+1)$$
$$=3(k+1)\left(\frac{1}{2}k+1\right)$$
$$=3(k+1)\left(\frac{k+2}{2}\right) ✓$$

P_{k+1} is also true.

$\therefore$ By the mathematical induction, the proof is completed

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Geom $\quad a_{10}: a_4=4 \quad a_7=12$

$r=\left(\frac{12}{4}\right)^{\frac{1}{7-4}}=3^{1/3}$ $\qquad r=\left(\frac{a_7}{a_4}\right)^{1/(7-4)}$

$a_4=a_1(3^{1/3})^3=4$ $\qquad a_n=a_0 r^{n-1}$

$3a_1=4$

$\boxed{a_1=\frac{4}{3}}$

$a_{10}=\frac{4}{3}(3^{1/3})^9$ $\qquad \frac{4}{3}3^3 \; 3^{-1}$

$=4(3^2)$

$\boxed{=36}$

a_{12} : $a_8 = 4 \quad a_{18} = -96$

$$d = \frac{y_2 - y_1}{x_2 - x_1}$$

$$d = \frac{-96-4}{18-8} = \frac{-100}{10} = -10$$

$$a_n = a_1 + (n-1)d$$

$$a_8 = a_1 + 7(-10) = 4$$

$$a_1 - 70 = 4$$

$$\underline{a_1 = 74}$$

$$a_{12} = 74 + 11(-10)$$

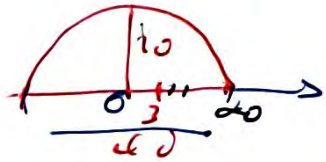
$$= 74 - 110$$

$$\underline{= -36}$$

$$\begin{array}{r} 110 \\ -\ 74 \\ \hline 36 \end{array}$$

T: $h = 9$, $w = 10$

$x = 5$



$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

$$\frac{y^2}{10^2} = 1 - \frac{5^2}{20^2}$$

$$y^2 = 10^2\left(\frac{400-25}{20^2}\right)$$

$$= \frac{10^2}{20^2}(375)$$

$$= \frac{1}{4}\, 375$$

$$\left(\frac{10}{20}\right)^2 = \left(\frac{1}{2}\right)^2$$

$81 \ ? \ \frac{375}{4}$ 4×

$324 < 375$ will be cleared.