5.1 Antiderivative

$$f(x) = x^3 \Rightarrow f'(x) = 3x^2$$

C

$$f'(x) = (C)' = 0$$

$$F'(x) = f(x)$$

differential

$$\int \underbrace{f(x)}_{\text{integrand}}\, dx = \underbrace{F(x) + C}_{\text{Antiderivative}}$$

Integral sign

$$\int k\,dx = kx + C$$

$$\int k f(x)\,dx = k \int f(x)\,dx$$

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \underline{n \neq -1}$$

Ex
$$\int \frac{dx}{x^2} = \int x^{-2}\,dx$$

$$= \frac{x^{-2+1}}{-2+1} + C$$

$$= -x^{-1} + C$$

$$\boxed{= -\frac{1}{x} + C}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\int \frac{1}{x^2}\,dx = \frac{-1}{x}$$

Ex $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx$

$\frac{a}{b}+1$

$\frac{a+b}{b}$

$= \frac{3}{4}x^{4/3} + C$

$= \frac{3}{4}\, x\, \sqrt[3]{x} + C$

Ex $\int (4x^3 - 5x + 2)\,dx = x^4 - \frac{5}{2}x^2 + 2x + C$

Ex $\int (x^2 - 2x + 5)\,dx = \frac{1}{3}x^3 - x^2 + 5x + C$

Ex $\int \sin x\,dx = -\cos x + C$

Ex $\int \cos 3x\,dx = \frac{1}{3}\sin 3x + C$

$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C.$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad n \neq -1$$

$$n = -1 \Rightarrow \int x^{-1} dx = \int \frac{dx}{x}$$

$$= \ln|x| + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$$

Ex $\int e^{-10x} dx = -\frac{1}{10} e^{-10x} + C$

Ex $\int \frac{5}{x} dx = 5 \ln|x| + C$

Ex $\int \frac{4}{\sqrt{9 - x^2}} dx = 4 \sin^{-1}\left(\frac{x}{3}\right) + C$

$$\int \frac{dx}{16x^2+1} = \int \frac{dx}{16\left(x^2+\frac{1}{16}\right)}$$

$$= \frac{1}{16} \, 4 \tan^{-1} \frac{x}{\frac{1}{4}} + C$$

$$= \frac{1}{4} \tan^{-1}(4x) + C$$

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1 $\int v^2 \, dv = \frac{1}{3} v^3 + C$

2/ $\int x^{1/2} \, dx = \frac{2}{3} x^{3/2} + C$

3/ $\int 4 y^{-3} \, dy = -2 y^{-2} + C$

4/ $\int (x^3 - 4x + 2) \, dx = \frac{1}{4} x^4 - 2x^2 + 2x + C$

* $\int (ax+b) \, dy = (ax+b)y + C$

6/ $\int (x^2-1)^2 \, dx = \int (x^4 - 2x^2 + 1) \, dx$

$= \frac{1}{5} x^5 - \frac{2}{3} x^3 + x + C$

14. $\int \frac{x^2+1}{\sqrt{x}}\,dx = \int (x^{3/2}+x^{-1/2})\,dx$

$\frac{x^2}{x^{1/2}} + \frac{1}{x^{1/2}}$

$= \frac{2}{5}x^{5/2} + 2x^{1/2} + C$

$= \frac{2}{5}x^2\sqrt{x} + 2\sqrt{x} + C$

15. $\int (\sqrt{x} + \sqrt[3]{x})\,dx = \int (x^{1/2} + x^{1/3})\,dx$

$= \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

18. $\int (-2\cos t)\,dt = -2\sin t + C$

19. $\int 7\sin\frac{\theta}{3}\,d\theta = -21\cos\frac{\theta}{3} + C$

21. $\int (4\sec x\tan x - 2\sec^2 x)\,dx$

$= 4\sec x - 2\tan x + C$

22. $\int (2\cos 2x - 3\sin 3x)\,dx$

$= \sin 2x + \cos 3x + C$

#23 $\int (1+\tan^2\theta)\,d\theta = \int \sec^2\theta\,d\theta$

$= \tan\theta + C$

$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

#24 $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\,d\theta = \int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}\,d\theta$

$= \int \frac{\frac{1}{\sin\theta}}{\frac{1-\sin^2\theta}{\sin\theta}}\,d\theta$

$= \int \frac{1}{\cos^2\theta}\,d\theta$

$= \int \sec^2\theta\,d\theta$

$= \tan\theta + C$

#25 $\int (2e^x - 3e^{-2x})dx = 2e^x + \frac{3}{2}e^{-2x} + C$

#32 $\int \left(\frac{1}{x^2} - \frac{2}{x^{5/2}}\right) dx = -\frac{1}{x} - 2\int x^{-5/2} dx$

$= -\frac{1}{x} + \frac{4}{3}x^{-3/2} + C$

#36 $\int \sec 2x \tan 2x \, dx = \frac{1}{2}\sec 2x + C$

#38 $\int \frac{12}{x} dx = 12 \ln|x| + C$

#41 $\int \frac{1+\tan\theta}{\sec\theta} d\theta = \int \frac{1 + \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} d\theta$

$\int \frac{\frac{\cos\theta + \sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$ $= \int (\cos\theta + \sin\theta) d\theta$

$= \sin\theta - \cos\theta + C$

#42 $\int (\sqrt[4]{x^3} + \sqrt{x^5}) dx = \int (x^{3/4} + x^{5/2}) dx$

$= \frac{4}{7}x^{7/4} + \frac{2}{7}x^{7/2} + C$

$= \frac{4}{7}x\sqrt[4]{x^3} + \frac{2}{7}x^3\sqrt{x} + C$

46 $\int \left(5x^{-4/3} + 3x^{-2/3} + 2x^{-1/3}\right) dx$

$= -15x^{-1/3} + 9x^{1/3} + 3x^{2/3} + C$

54 $\int \sec\theta(\sec\theta + \tan\theta)\, d\theta = \int (\sec^2\theta + \sec\theta\tan\theta)\, d\theta$

$= \tan\theta + \sec\theta + C$

56 $\int (\cos^4\theta - \sin^4\theta)\, d\theta = \int (\cos^2\theta - \sin^2\theta)\overbrace{(\cos^2\theta + \sin^2\theta)}^{=1}\, d\theta$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$\sin^2\theta = \frac{1 - \sin 2\theta}{2}$

$= \int (\cos^2\theta - \sin^2\theta)\, d\theta$

$= \int \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta - \frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta$

$= \int \cos 2\theta\, d\theta$

$= \frac{1}{2}\sin 2\theta + C$

59 $\int (\cos 2x \cos 4x - \sin 2x \sin 4x)\, dx$

$= \int \cos(2x + 4x)\, dx$

$= \int \cos 6x\, dx$

$= \frac{1}{6}\sin 6x + C$

6/ $\int \cos 2x \sin 2x \, dx = \frac{1}{2}\int \sin 4x \, dx$

$= -\frac{1}{8}\cos 4x + C$

7/ $\int (4x - \frac{3}{x} - \csc^2 x) dx = 2x^2 - 3\ln|x| + \cot x + C$

7/ $\int (e^{4x} - \frac{3}{x} + 2\csc x \cot x) dx = \frac{1}{4}e^{4x} - 3\ln|x| - 2\csc x + C$

74/ $\int (a^2 - b^2) e^{(a-b)x} dx = \frac{a^2 - b^2}{a-b} e^{(a-b)x} + C$

$= (a+b) e^{(a-b)x} + C$

$V(r) = cr^2(r_0 - r)$

$= c r_0 r^2 - cr^3$

$V'(r) = 2 c r_0 r - 3cr^2 = 0$

$2r_0 - 3r = 0$

$r = \frac{2}{3} r_0$