$$\int_{-2}^{2}(3x^4-2x+1)\,dx = \frac{3}{5}x^5 - x^2 + x \Big|_{-2}^{2}$$

$$= \frac{96}{5} - 4 + 2 - \left(-\frac{96}{5} - 4 - 2\right)$$

$$= \frac{192}{5} + 4$$

$$= \frac{212}{5}$$

$$\int_{-2}^{2}(3x^4-\underline{2x}+1)\,dx = 2\left(\frac{3}{5}x^5 + x\right)\Big|_{0}^{2}$$

$$= 2\left(\frac{96}{5} + 2\right)$$

$$= 2\left(\frac{106}{5}\right)$$

$$= \frac{212}{5}$$

H.2 $\int_0^1 (4x^{21} - 2x^{16} + 1)\,dx = \frac{2}{11}x^{22} - \frac{2}{17}x^{17} + x \Big|_0^1$

$$= \frac{2}{11} - \frac{2}{17} + 1$$

$$= \frac{34 - 22 + 187}{187}$$

$$= \frac{199}{187}$$

H 3 $f(x) = 16 - x^2 \qquad [-4, 4]$

$$16 - x^2 = 0 \Rightarrow x = \pm 4$$

$$A = \int_{-4}^{4}(16-x^2)\,dx \longrightarrow 2\int_0^4 (16-x^2)\,dx$$

$$= 2\left(16x - \frac{1}{3}x^3\right)\Big|_0^4$$

$$= 2\left(64 - \frac{64}{3}\right) \longrightarrow 2(64)\left(1-\frac{1}{3}\right)$$

$$= 128\left(\frac{2}{3}\right)$$

$$= \frac{256}{3} \text{ unit}^2$$

#4 $f(x) = x^3 - x = 0 \quad [-1, 0]$

$x = 0. \qquad x^2 - 1 = 0 \qquad x(x^2-1) = 0$

$x = 0, \cancel{1}, -1$

$Area = \int_{-1}^{0} (x^3 - x)\,dx$

$= \frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_{-1}^{0}$

$= -\left(\frac{1}{4} - \frac{1}{2}\right)$

$= \frac{1}{4} \text{ unit}^2$

#5 $f(x) = x^2 - x = 0 \qquad [0, 3]$

$x = 0, 1$

$A = -\int_0^1 (x^2 - x)\,dx + \int_1^3 (x^2 - x)\,dx$

$= -\left(\frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_0^1 + \left(\frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_1^3\right.\right.$

$= -\left(\frac{1}{3} - \frac{1}{2}\right) + 9 - \frac{9}{2} - \frac{1}{3} + \frac{1}{2}$

$= \cancel{2}\frac{1}{\cancel{6}_3} + \frac{9}{2}$

$= \frac{29}{6} \text{ unit}^2$

\# 6 $f(x) = x^4 - x^2$ $[-1, 1]$

$$x^2(x^2 - 1) = 0$$

$$x = \boxed{0, 0}, \pm 1$$

$\frac{1}{16} - \frac{1}{4}$

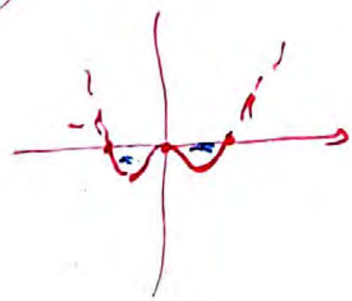
$$Area = -\int_{-1}^{0} (x^4 - x^2)\,dx + \int_{0}^{1} (x^4 - x^2)\,dx$$

$$= \left| \int_{-1}^{1} (x^4 - x^2)\,dx \right|$$

$$= 2 \left| \left(\frac{1}{5}x^5 - \frac{1}{3}x^3 \right|_0^1 \right|$$

$$= 2 \left| \left(\frac{1}{5} - \frac{1}{3} \right) \right|$$

$$= \frac{4}{5} \text{ unit}^2$$



Sec 4.6

#12 $\int r^4 \left(7 - \frac{r^5}{10}\right)^3 dr$ $\qquad d\left(7 - \frac{r^5}{10}\right) = -\frac{1}{2} r^4 dr$

$$= -2 \int \left(7 - \frac{r^5}{10}\right)^3 d\left(7 - \frac{r^5}{10}\right)$$

$$= -\frac{1}{2}\left(7 - \frac{r^5}{10}\right)^4 + C$$

#13 $\int x^{1/2} \sin(x^{3/2}+1)\, dx$ $\qquad d(x^{3/2}+1) = \frac{3}{2} x^{1/2} dx$

$$= \frac{2}{3} \int \sin(x^{3/2}+1)\, d(x^{3/2}+1)$$

$$= -\frac{2}{3} \cos(x^{3/2}+1) + C$$

#14 $\int \csc\left(\frac{v-\pi}{2}\right) \cot\left(\frac{v-\pi}{2}\right) dv$ $\qquad d\left(\frac{v-\pi}{2}\right) = \frac{1}{2} dv$

$$= 2 \int \csc\left(\frac{v-\pi}{2}\right) \cot\left(\frac{v-\pi}{2}\right) d\left(\frac{v-\pi}{2}\right)$$

$$= -2 \csc\left(\frac{v-\pi}{2}\right) + C$$

#15 $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt$ $\qquad d\cos(2t+1) = -2\sin(2t+1)dt$

$$= -\frac{1}{2} \int \frac{d\cos(2t+1)}{\cos^2(2t+1)} \qquad \int \frac{du}{u^2} = \frac{-1}{u}$$

$$= \frac{1}{2} \frac{1}{\cos(2t+1)} + C$$

#16. $\int \frac{\sec z \tan z}{\sqrt{\sec z}} dz$ $\qquad d(\sec z) = \sec z \tan z\, dz$

$$= \int (\sec z)^{-1/2} d(\sec z)$$

$$= \frac{1}{2}\sqrt{\sec z} + C$$

#17 $\int \frac{1}{\sqrt{t}} \cos(\sqrt{t}+3)\,dt$ $\qquad d(\sqrt{t}+3) = \frac{1}{2\sqrt{t}}\,dt$

$$= 2\int \cos(\sqrt{t}+3)\, d(\sqrt{t}+3)$$

$$= \boxed{2\sin(\sqrt{t}+3) + C}$$

#18 $\int \frac{1}{\theta^2} \sin\frac{1}{\theta} \cos\frac{1}{\theta}\,d\theta$ $\qquad d\left(\cos\frac{1}{\theta}\right) = +\frac{1}{\theta^2}\sin\frac{1}{\theta}\,d\theta$

$$= \int \cos\frac{1}{\theta}\, d\left(\cos\frac{1}{\theta}\right)$$

$$= \boxed{\frac{1}{2}\cos^2\frac{1}{\theta} + C}$$

* $= \frac{1}{2}\int \frac{1}{\theta^2}\sin\frac{2}{\theta}\,d\theta$ $\qquad d\left(\frac{2}{\theta}\right) = -\frac{2}{\theta^2}\,d\theta$

$$= -\frac{1}{4}\int -\sin\left(\frac{2}{\theta}\right) d\left(\frac{2}{\theta}\right)$$

$$= \frac{1}{4}\cos\left(\frac{2}{\theta}\right) + C.$$

$$\frac{1}{4}\left(2\cos^2\frac{1}{\theta} - 1\right)$$

$$\frac{1}{2}\cos^2\frac{1}{\theta} \underbrace{- \frac{1}{4} + C}_{C_1}$$

#20 $\int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx = \int \frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx$

$$d(1-x^{-2}) = 2x^{-3}dx = \frac{2}{x^3}dx$$

$$= \frac{1}{2}\int \left(1-\frac{1}{x^2}\right)^{1/2} d\left(1-\frac{1}{x^2}\right)$$

$$= \boxed{\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} + C}$$

#21 $\int x^3 \sqrt{x^2+1}\, dx$

$u = x^2+1 \to x^2 = u-1$

$\frac{du}{2} = \frac{2x\,dx}{2}$ $\to \frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int x^2 u^{1/2}\, du$$

$$= \frac{1}{2}\int (u-1)u^{1/2}\, du$$

$$= \frac{1}{2}\int (u^{3/2} - u^{1/2})\, du$$

$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$$

#22 $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\cos^3\sqrt{\theta}}\, d\theta$

$d(\cos\sqrt{\theta}) = \frac{-1}{2\sqrt{\theta}}\sin\sqrt{\theta}\, d\theta$

$$= \int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\sqrt{\cos^3\sqrt{\theta}}}\, d\theta$$

$$= -2\int (\cos\sqrt{\theta})^{-3/2}\, d(\cos\sqrt{\theta})$$

$$= 4\frac{1}{\sqrt{\cos\sqrt{\theta}}} + C$$

$$-2\,\frac{(\cos\sqrt{\theta})^{-1/2}}{-1/2}$$

#30 $\displaystyle\int \frac{(2r-1)\cos\sqrt{3(2r-1)^2+6}}{\sqrt{3(2r-1)^2+6}}\,dr$

$$d\left(\sqrt{3(2r-1)^2+6}\right) = \frac{\overset{6}{\cancel{12}}(2r-1)}{\cancel{2}\sqrt{3(2r-1)^2+6}}\,dr$$

$$= \frac{1}{6}\int \cos\left(\sqrt{3(2r-1)^2+6}\right) d\left(\sqrt{3(2r-1)^2+6}\right)$$

$$= \frac{1}{6}\sin\left(\sqrt{3(2r-1)^6+6}\right) + C$$

#51 $\displaystyle\int \frac{dx}{x(\ln x)^2}$ $\qquad d(\ln x) = \frac{1}{x}\,dx$

$$= \int \frac{d(\ln x)}{(\ln x)^2}$$

$$= -\frac{1}{\ln x} + C$$

#53 $\displaystyle\int \frac{3x}{x^2+4}\,dx = \frac{3}{2}\int \frac{d(x^2+4)}{x^2+4}$ $\qquad d(x^2+4) = 2x\,dx$

$$= \frac{3}{2}\ln(x^2+4) + C$$

#55 $\displaystyle\int \frac{\sec x}{\sqrt{\ln(\sec x+\tan x)}}\,dx$

$$d(\ln(\sec x+\tan x)) = \frac{\sec x\tan x + \sec^2 x}{\sec x+\tan x}\,dx$$

$$= \frac{\sec x(\tan x+\sec x)}{\sec x+\tan x}\,dx$$

$$= \sec x\,dx$$

$$= \int (\ln(\sec x+\tan x))^{-1/2}\, d(\ln(\sec x+\tan x))$$

$$= 2\sqrt{\ln(\sec x+\tan x)} + C$$

#57 $\int 4xe^{x^2}dx = 2\int e^{x^2}d(x^2)$ $\quad d(x^2) = 2x\,dx$

$= 2e^{x^2} + C$

#64 $\int \frac{2}{e^{-x}+1}dx = \int \frac{2}{e^{-x}+1}\frac{e^x}{e^x}dx$

$= \int \frac{2e^x}{1+e^x}dx$ $\quad d(1+e^x) = e^x dx$

$= 2\int \frac{d(1+e^x)}{1+e^x}$

$= 2\ln(1+e^x) + C$

#146 $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}dx$ $\quad d(x^4+9) = 4x^3dx$

$= \frac{1}{4}\int_0^1 (x^4+9)^{-1/2}d(x^4+9)$

$= \frac{1}{2}\sqrt{x^4+9}\Big|_0^1$

$= \frac{1}{2}(\sqrt{10} - 3)$

#147 $\int_0^{\pi/6} (1-\cos 3t)\sin 3t\,dt$ $\quad d(1-\cos 3t) = 3\sin 3t\,dt$

$= \frac{1}{3}\int_0^{\pi/6} (1-\cos 3t)\,d(1-\cos 3t)$

$= \frac{1}{6}(1-\cos 3t)^2\Big|_0^{\pi/6}$

$= \frac{1}{6}(1-0)$

$= \frac{1}{6}$

$\sin 3t - \cos 3t \sin 3t$

$\int (\sin 3t - \frac{1}{2} \sin 6t)\, dt$

$-\frac{1}{3} \cos 3t + \frac{1}{12} \cos 6t$

#148 $\displaystyle\int_{-\pi/2}^{\pi/2} \left(2 + \tan \frac{t}{2}\right) \sec^2 \frac{t}{2}\, dt$

$d\left(2 + \tan \frac{t}{2}\right) = \frac{1}{2} \sec^2 \frac{t}{2}\, dt$

$= 2 \displaystyle\int_{-\pi/2}^{\pi/2} \left(2 + \tan \frac{t}{2}\right) d\left(2 + \tan \frac{t}{2}\right)$

$= \left(2 + \tan \frac{t}{2}\right)^2 \Big|_{-\pi/2}^{\pi/2}$

$= 9 - 1$

$= \boxed{8}$

#149 $\displaystyle\int_{-\pi}^{\pi} \frac{\cos z}{\sqrt{4 + 3 \sin z}}\, dz$ $\quad d(4 + 3\sin z) = 3 \cos z\, dz$

$= \frac{1}{3} \displaystyle\int_{-\pi}^{\pi} (4 + 3 \sin z)^{-1/2}\, d(4 + 3 \sin z)$

$= \frac{2}{3} \sqrt{4 + 3 \sin z} \Big|_{-\pi}^{\pi}$

$= \frac{2}{3} (2 - 2)$

$= \boxed{0}$

152/ $\displaystyle\int_1^4 \frac{dy}{2\sqrt{y}\,(1+\sqrt{y})^2}$ $\qquad d(1+\sqrt{y}) = \frac{1}{2\sqrt{y}}\,dy$

$$= \int_1^4 \frac{d(1+\sqrt{y})}{(1+\sqrt{y})^2}$$

$$= -\frac{1}{1+\sqrt{y}}\Big|_{+1}^{4}$$

$$= -\left(\frac{1}{3}-\frac{1}{2}\right)$$

$$= \frac{1}{6}$$

155/ $\displaystyle\int_0^{\pi/2} e^{\sin x}\cos x\,dx = \int_0^{\pi/2} e^{\sin x}\,d(\sin x)$ $\qquad d(\sin x) = \cos x\,dx$

$$= e^{\sin x}\Big|_0^{\pi/2}$$

$$= e - 1$$

#160/ $\displaystyle\int_0^{\pi/2} \tan\frac{x}{2}\,dx = \int_0^{\pi/2} \tan\left(\frac{x}{2}\right) d\left(\frac{x}{2}\right)$ $\qquad d\left(\frac{x}{2}\right) = \frac{1}{2}\,dx$

$$= \ln\left|\sec\frac{x}{2}\right|\Big|_0^{\pi/2} \qquad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$= \ln\sqrt{2} - \ln 1$$

$$= \frac{1}{2}\ln 2$$

#164 $\int_0^{\sqrt{\ln \pi}} 2x e^{x^2} \cos(e^{x^2})\,dx$

$$d(e^{x^2}) = 2x e^{x^2} dx$$

$$= \int_0^{\sqrt{\ln \pi}} \cos(e^{x^2})\, d(e^{x^2})$$

$$= \sin e^{x^2} \Big|_0^{\sqrt{\ln \pi}} \qquad e^{\ln x} = x$$

$$= \sin e^{\ln \pi} - \sin e^0$$

$$= \sin \pi - \sin 1$$

$$= \boxed{-\sin 1}$$