Homework

Find the determinant of the matrix

1. $\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}$

2. $\begin{vmatrix} 5 & 3 \\ -6 & 3 \end{vmatrix}$

3. $\begin{vmatrix} -7 & 6 \\ \frac{1}{2} & 3 \end{vmatrix}$

4. $\begin{vmatrix} 0 & 8 \\ 0 & 4 \end{vmatrix}$

5. $\begin{vmatrix} \lambda-3 & 2 \\ 4 & \lambda-1 \end{vmatrix}$

6. $\begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix}$

7. $\begin{vmatrix} x & \ln x \\ 1 & \dfrac{1}{x} \end{vmatrix}$

8. $\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$

9. $\begin{vmatrix} \sin\theta & 1 \\ 1 & \sin\theta \end{vmatrix}$

10. $\begin{vmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 4 & 3 \end{vmatrix}$

11. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{vmatrix}$

12. $\begin{vmatrix} 1 & 0 & 0 \\ k & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

13. $\begin{vmatrix} 2 & 4 & 6 \\ 0 & 3 & 1 \\ 0 & 0 & -5 \end{vmatrix}$

14. $\begin{vmatrix} x & y & -1 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

15. $\begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$

16. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

17. $\begin{vmatrix} 5 & 3 & 0 & 6 \\ 4 & 6 & 4 & 12 \\ 0 & 2 & -3 & 4 \\ 0 & 1 & -2 & 2 \end{vmatrix}$

Find the values of λ for which the determinant is zero.

18. $\begin{vmatrix} \lambda+2 & 2 \\ 1 & \lambda \end{vmatrix}$

19. $\begin{vmatrix} \lambda & 2 & 0 \\ 0 & \lambda+1 & 2 \\ 0 & 1 & \lambda \end{vmatrix}$

Find **a)** $|A|$ **b)** $|B|$ **c)** AB **d)** $|AB|$. Then verify that $|A|\,|B|=|AB|$

20. $A = \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$

21. $A = \begin{pmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Use the fact that $|cA| = c^n |A|$ to evaluate the determinant of the $n\times n$ matrix

22. $A = \begin{pmatrix} 5 & 15 \\ 10 & -20 \end{pmatrix}$

23. $A = \begin{pmatrix} -3 & 6 & 9 \\ 6 & 9 & 12 \\ 9 & 12 & 15 \end{pmatrix}$

Find **a)** $|A|$ **b)** $|B|$ **c)** $A+B$ **d)** $|A+B|$. Then verify that $|A|+|B| \neq |A+B|$

24. $A = \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix}$

Use a determinant to decide whether the matrix is singular or nonsingular.

25. $\begin{pmatrix} 5 & 4 \\ 10 & 8 \end{pmatrix}$

26. $\begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 2 \\ \frac{2}{3} & -\frac{1}{3} & 0 \\ 1 & 1 & 1 \end{pmatrix}$

Verify that $|A^{-1}| = \dfrac{1}{|A|}$

27. $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

28. $A = \begin{pmatrix} 2 & -2 & 3 \\ 1 & -1 & 2 \\ 3 & 0 & 3 \end{pmatrix}$

29. Let A and B be $n \times n$ matrices such that $AB = I$. Prove that $|A| \neq 0$ and $|B| \neq 0$

30. Let A and B be $n \times n$ matrices such that AB is singular. Prove that A or B is singular.

31. Find two 2×2 matrices such that $|A| + |B| = |A+B|$

32. Verify the equation $\begin{vmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{vmatrix}$

33. Let A be an $n \times n$ matrix in which the entries of each row sum to zero. Find $|A|$

Find the inverse matrix using adjoint of the matrix

34. $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 6 \\ 0 & -4 & -12 \end{pmatrix}$

35. $A = \begin{pmatrix} -3 & -5 & -7 \\ 2 & 4 & 3 \\ 0 & 1 & -1 \end{pmatrix}$

Use Cramer's Rule

36. $\begin{cases} x_1 + 2x_2 = 5 \\ -x_1 + x_2 = 1 \end{cases}$

37. $\begin{cases} 3x + 4y = -2 \\ 5x + 3y = 4 \end{cases}$

38. $\begin{cases} 20x + 8y = 11 \\ 12x - 24y = 21 \end{cases}$

39. $\begin{cases} 4x - y - z = 1 \\ 2x + 2y + 3z = 10 \\ 5x - 2y - 2z = -1 \end{cases}$

40. $\begin{cases} 3x + 4y + 4z = 11 \\ 4x - 4y + 6z = 11 \\ 6x - 6y = 3 \end{cases}$