7/ $S = \{(-4,5), (0,0)\}$

$\begin{vmatrix} -4 & 0 \\ 5 & 0 \end{vmatrix} = 0 \rightarrow$ Linearly dependent & S doesn't span R^2

11/ $S = \{(1,2), (1,0), (0,1)\}$

$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} R_2 - 2R_1 \rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \rightarrow c_1 = -c_2 \rightarrow 2c_2 = c_3$

Linearly dependent

15/ $S = \{(1,3,0), (4,1,2), (-2,5,-2)\}$

$\begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 5 \\ 0 & 2 & -2 \end{vmatrix} = 0$ Linearly dependent & S doesn't span R^3

17/ $S = \{(7,0,3), (8,-4,1)\}$

$\begin{pmatrix} 7 & 8 \\ 0 & -4 \\ 3 & 1 \end{pmatrix} 7R_3 - 3R_1 \rightarrow \begin{pmatrix} 7 & 8 \\ 0 & -4 \\ 0 & -17 \end{pmatrix} \rightarrow c_1 = 0 \rightarrow c_2 = 0$

L.I.

S doesn't span R^3 (2 pts make line)

is not basis

23/ $S = \{1, 2x, -4+x^2, 5x\}$

$\begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{pmatrix} \frac{1}{2}R_2 \rightarrow \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & 0 \end{pmatrix} \rightarrow c_1 = 0 \rightarrow c_2 = -\frac{5}{2}c_4 \rightarrow c_3 = 0$

S is linearly dependent.

31/ $S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \rightarrow c_1 = c_2 = 0.$ S doesn't span $M_{2\times 2}$

39/ $S = \{(4,-3), (5,2)\}$ for R^2

$\begin{vmatrix} 4 & 5 \\ -3 & 2 \end{vmatrix} = 23 \neq 0$ basis.

$\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix} \begin{matrix} \\ 4R_2 + 3R_1 \end{matrix} \rightarrow \begin{pmatrix} 4 & 5 \\ 0 & 23 \end{pmatrix} 23R_1 - 5R_2$

$\begin{pmatrix} 92 & 0 \\ 0 & 23 \end{pmatrix} \begin{matrix} \frac{1}{92}R_1 \\ \frac{1}{23}R_2 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

S is a basis

41/ $S = \{(1,5,3), (0,1,2), (0,0,6)\}$ R^3

$\begin{vmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 3 & 2 & 6 \end{vmatrix} = 6 \neq 0$ S is a basis

43/ $S = \{(0,3,-2), (4,0,3), (-8,15,-16)\}$ for R^3

$\begin{vmatrix} 0 & 4 & -8 \\ 3 & 0 & 15 \\ -2 & 3 & -16 \end{vmatrix} = 0$ S is not a basis

47/ $S = \{1 - 2t^2 + t^3,\ -4 + t^2,\ 2t + t^3,\ 5t\}$

$$\begin{vmatrix} 1 & -4 & 0 & 0 \\ 0 & 0 & 2 & 5 \\ -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 2 & 5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} + 4 \begin{vmatrix} 0 & 2 & 5 \\ -2 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix}$$

$= 5 - 40$

$= -35 \neq 0$. ✓

(or)

$$\begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & 5 & 0 & 2 \\ -2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{matrix} \\ \frac{1}{5}R_2 \\ R_3 + 2R_1 \\ R_4 - R_1 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & 1 & 0 & 2/5 \\ 0 & 0 & -7 & 0 \\ 0 & 0 & 4 & 1 \end{pmatrix} -\frac{1}{7}R_3$$

$$\begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & 1 & 0 & 2/5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 1 \end{pmatrix} \begin{matrix} R_1 + 4R_3 \\ \\ \\ R_4 - 4R_3 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2/5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} R_2 - \frac{5}{2}R_4$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

S is a basis

51/ $S = \left\{\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}\right\}$

$$\begin{vmatrix} 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 3 & 1 & 2 & 0 \end{vmatrix} = 2 \begin{vmatrix} 4 & 1 & 1 \\ 0 & 3 & 2 \\ 1 & 2 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 1 \\ 0 & 3 & 2 \\ 3 & 2 & 0 \end{vmatrix}$$

$= -34 + 3$

$= -31 \neq 0$

S is a basis.