Exercise

Find the limit as $x \to \infty$ and as $x \to -\infty$ of $h(x) = \dfrac{-5 + \frac{7}{x}}{3 - \frac{1}{x^2}}$

Solution

$$\lim_{x\to\infty} \frac{-5+\frac{7}{x}}{3-\frac{1}{x^2}} = \boxed{-\frac{5}{3}}$$

$$\lim_{x\to-\infty} \frac{-5+\frac{7}{x}}{3-\frac{1}{x^2}} = \boxed{-\frac{5}{3}}$$

Exercise

Find the limit as $x \to \infty$ and as $x \to -\infty$ of $f(x) = \dfrac{2x+3}{5x+7}$

Solution

$$\lim_{x\to\infty} \frac{2x+3}{5x+7} = \lim_{x\to\infty} \frac{2+\frac{3}{x}}{5+\frac{7}{x}}$$

$$= \boxed{\frac{2}{5}}$$

$$\lim_{x\to-\infty} \frac{2x+3}{5x+7} = \lim_{x\to-\infty} \frac{2+\frac{3}{x}}{5+\frac{7}{x}}$$

$$= \boxed{\frac{2}{5}}$$

Exercise

Find the limit as $x \to \infty$ and as $x \to -\infty$ of $f(x) = \dfrac{2x^3+7}{x^3-x^2+x+7}$

Solution

$$\lim_{x\to\infty} \frac{2x^3+7}{x^3-x^2+x+7} = \lim_{x\to\infty} \frac{2+\frac{7}{x^3}}{1-\frac{1}{x}+\frac{1}{x^2}+\frac{7}{x^3}}$$

$= 2$

$$\lim_{x\to-\infty} \frac{2x^3+7}{x^3-x^2+x+7} = \lim_{x\to-\infty} \frac{2+\frac{7}{x^3}}{1-\frac{1}{x}+\frac{1}{x^2}+\frac{7}{x^3}}$$

$= 2$

Exercise

Find the limit as $x\to\infty$ and as $x\to-\infty$ of $f(x)=\frac{x+1}{x^2+3}$

Solution

$$\lim_{x\to\infty} \frac{x+1}{x^2+3} = \lim_{x\to\infty} \frac{\frac{x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{3}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{1}{x^2}}{1+\frac{3}{x^2}}$$

$= 0$

$$\lim_{x\to-\infty} \frac{x+1}{x^2+3} = \lim_{x\to-\infty} \frac{\frac{1}{x}+\frac{1}{x^2}}{1+\frac{3}{x^2}}$$

$= 0$

Exercise

Find the limit as $x\to\infty$ and as $x\to-\infty$ of $f(x)=\frac{7x^3}{x^3-3x^2+6x}$

Solution

$$\lim_{x\to\infty} \frac{7x^3}{x^3-3x^2+6x} = \lim_{x\to\infty} \frac{7}{1-\frac{3}{x}+\frac{6}{x^2}}$$

$= 7$

$$\lim_{x\to-\infty} \frac{7x^3}{x^3-3x^2+6x} = \lim_{x\to-\infty} \frac{7}{1-\frac{3}{x}+\frac{6}{x^2}}$$

$= 7$

Exercise

Find the limit as $x \to \infty$ and as $x \to -\infty$ of $f(x) = \dfrac{9x^4 + x}{2x^4 + 5x^2 - x + 6}$

Solution

$$\lim_{x\to\infty} \frac{9x^4 + x}{2x^4 + 5x^2 - x + 6} = \lim_{x\to\infty} \frac{\frac{9x^4}{x^4} + \frac{x}{x^4}}{\frac{2x^4}{x^4} + \frac{5x^2}{x^4} - \frac{x}{x^4} + \frac{6}{x^4}}$$

$$= \lim_{x\to\infty} \frac{9 + \frac{1}{x^3}}{2 + \frac{5}{x^2} - \frac{1}{x^3} + \frac{6}{x^4}}$$

$$= \frac{9}{2}$$

$$\lim_{x\to-\infty} \frac{9x^4 + x}{2x^4 + 5x^2 - x + 6} = \lim_{x\to-\infty} \frac{9 + \frac{1}{x^3}}{2 + \frac{5}{x^2} - \frac{1}{x^3} + \frac{6}{x^4}}$$

$$= \frac{9}{2}$$

Exercise

Find the limit as $x \to \infty$ and as $x \to -\infty$ of $f(x) = \dfrac{-2x^3 - 2x + 3}{3x^3 + 3x^2 - 5x}$

Solution

$$\lim_{x\to\infty} \frac{-2x^3 - 2x + 3}{3x^3 + 3x^2 - 5x} = \lim_{x\to\infty} \frac{-2 - \frac{2}{x^2} + \frac{3}{x^3}}{3 + \frac{3}{x} - \frac{5}{x^2}}$$

$$= -\frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{-2x^3 - 2x + 3}{3x^3 + 3x^2 - 5x} = \lim_{x\to-\infty} \frac{-2 - \frac{2}{x^2} + \frac{3}{x^3}}{3 + \frac{3}{x} - \frac{5}{x^2}}$$

$$= -\frac{2}{3}$$

Exercise

Find $\lim_{x\to\infty} x^{12}$

Solution

$$\lim_{x\to\infty} x^{12} = \infty \rfloor$$

Exercise

Find $\lim_{x\to-\infty} 3x^{9}$

Solution

$$\lim_{x\to-\infty} 3x^{9} = -\infty \rfloor$$

Exercise

Find $\lim_{x\to-\infty} x^{-8}$

Solution

$$\lim_{x\to-\infty} x^{-8} = \frac{1}{(-\infty)^{8}}$$

$$= 0 \rfloor$$

Exercise

Find $\lim_{x\to-\infty} x^{-9}$

Solution

$$\lim_{x\to-\infty} x^{-9} = \frac{1}{(-\infty)^{9}}$$

$$= 0 \rfloor$$

Exercise

Find $\lim_{x\to-\infty} 2x^{-6}$

Solution

$$\lim_{x\to-\infty} 2x^{-6} = \frac{2}{\infty}$$

$$= 0 \rfloor$$

Exercise

Find $\lim_{x\to\infty}\left(3x^{12}-9x^{7}\right)$

Solution

$$\lim_{x\to\infty}\left(3x^{12}-9x^{7}\right)=\infty$$

Exercise

Find $\lim_{x\to-\infty}\left(3x^{7}+x^{2}\right)$

Solution

$$\lim_{x\to-\infty}\left(3x^{7}+x^{2}\right)=\lim_{x\to-\infty}x^{2}\left(3x^{5}+1\right)$$
$$=-\infty$$

Exercise

Find $\lim_{x\to-\infty}\left(-2x^{16}+2\right)$

Solution

$$\lim_{x\to-\infty}\left(-2x^{16}+2\right)=-\infty$$

Exercise

Find $\lim_{x\to-\infty}\left(2x^{-6}+4x^{5}\right)$

Solution

$$\lim_{x\to-\infty}\left(2x^{-6}+4x^{5}\right)=\lim_{x\to-\infty}x^{-6}\left(2+4x^{11}\right) \qquad +\infty\left(-\infty\right)$$
$$=-\infty$$

Exercise

Find $\lim_{x\to-\infty}\frac{\cos x}{3x}$

Solution

$$-\frac{1}{3x} \le \frac{\cos x}{3x} \le \frac{1}{3x}, \quad -1 \le \cos x \le 1$$

$$\lim_{x\to-\infty} \frac{\cos x}{3x} = 0$$ *By the Sandwich Theorem*

Exercise

Find $\lim_{x\to\infty} \frac{x+\sin x}{2x+7-5\sin x}$

Solution

$$\lim_{x\to\infty} \frac{x+\sin x}{2x+7-5\sin x} = \lim_{x\to\infty} \frac{1+\frac{\sin x}{x}}{2+\frac{7}{x}-\frac{5\sin x}{x}}$$

$$= \frac{1+0}{2+0-0}$$

$$= \frac{1}{2}$$

Exercise

Find $\lim_{x\to\infty} \sqrt{\frac{8x^2-3}{2x^2+x}}$

Solution

$$\lim_{x\to\infty} \sqrt{\frac{8x^2-3}{2x^2+x}} = \lim_{x\to\infty} \sqrt{\frac{8-\frac{3}{x^2}}{2+\frac{1}{x}}}$$

$$= \sqrt{\frac{8}{2}}$$

$$= 2$$

Exercise

Find $\lim_{x\to-\infty} \left(\frac{x^2+x-1}{8x^2-3}\right)^{1/3}$

Solution

$$\lim_{x\to-\infty} \left(\frac{x^2+x-1}{8x^2-3}\right)^{1/3} = \lim_{x\to-\infty} \left(\frac{1+\frac{1}{x}-\frac{1}{x^2}}{8-\frac{3}{x^2}}\right)^{1/3}$$

$$= \left(\frac{1}{8}\right)^{1/3}$$

$$= \frac{1}{2}$$

Exercise

Find $\lim_{x\to\infty} \dfrac{2\sqrt{x} + x^{-1}}{3x - 7}$

Solution

$$\lim_{x\to\infty} \frac{2\sqrt{x} + x^{-1}}{3x - 7} = \lim_{x\to\infty} \frac{\frac{2\sqrt{x}}{x} + \frac{x^{-1}}{x}}{3 - \frac{7}{x}}$$

$$= \lim_{x\to\infty} \frac{\frac{2}{x^{1/2}} + \frac{1}{x^2}}{3 - \frac{7}{x}}$$

$$= 0$$

Exercise

Find $\lim_{x\to\infty} \dfrac{x^{-1} + x^{-4}}{x^{-2} + x^{-3}}$

Solution

$$\lim_{x\to\infty} \frac{x^{-1} + x^{-4}}{x^{-2} + x^{-3}} = \lim_{x\to\infty} \frac{\frac{x^{-1}}{x^{-2}} + \frac{x^{-4}}{x^{-2}}}{\frac{x^{-2}}{x^{-2}} + \frac{x^{-3}}{x^{-2}}}$$

$$= \lim_{x\to\infty} \frac{x + \frac{1}{x^2}}{1 + \frac{1}{x}}$$

$$= \infty$$

Exercise

Find $\lim_{x\to-\infty} \dfrac{4 - 3x^3}{\sqrt{x^6 + 9}}$

Solution

$$\lim_{x\to-\infty}\frac{4-3x^3}{\sqrt{x^6+9}}=\lim_{x\to-\infty}\frac{\frac{4-3x^3}{\sqrt{x^6}}}{\frac{\sqrt{x^6+9}}{\sqrt{x^6}}}$$

$$=\lim_{x\to-\infty}\frac{\frac{4-3x^3}{x^3}}{\sqrt{\frac{x^6+9}{x^6}}}$$

$$=\lim_{x\to-\infty}\frac{\frac{4}{x3}-3}{\sqrt{1+\frac{9}{x^6}}}$$

$$=\frac{-3}{\sqrt{1}}$$

$$=-3$$

Exercise

Find $\lim_{x\to-\infty}\left(\sqrt{x^2+3}+x\right)$

Solution

$$\lim_{x\to-\infty}\left(\sqrt{x^2+3}+x\right)=\lim_{x\to-\infty}\left(\sqrt{x^2+3}+x\right)\frac{\sqrt{x^2+3}-x}{\sqrt{x^2+3}-x}$$

$$=\lim_{x\to-\infty}\frac{x^2+3-x^2}{\sqrt{x^2+3}-x}$$

$$=\lim_{x\to-\infty}\frac{3}{\sqrt{x^2+3}-x}$$

$$=\lim_{x\to-\infty}\frac{\frac{3}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{3}{x^2}}-\frac{x}{x}}$$

$$=\lim_{x\to-\infty}\frac{\frac{3}{x}}{\sqrt{1+\frac{3}{x^2}}+1}$$

$$=\frac{0}{\sqrt{1}+1}$$

$$=0$$

Exercise

Find $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-2x}\right)$

Solution

$$\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-2x}\right)=\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-2x}\right)\frac{\sqrt{x^2+3x}+\sqrt{x^2-2x}}{\sqrt{x^2+3x}+\sqrt{x^2-2x}}$$

$$=\lim_{x\to\infty}\frac{\left(x^2+3x\right)-\left(x^2-2x\right)}{\sqrt{x^2+3x}+\sqrt{x^2-2x}}$$

$$=\lim_{x\to\infty}\frac{x^2+3x-x^2+2x}{\sqrt{x^2+3x}+\sqrt{x^2-2x}}$$

$$=\lim_{x\to\infty}\frac{5x}{\sqrt{x^2+3x}+\sqrt{x^2-2x}}$$

$$=\lim_{x\to\infty}\frac{\frac{5x}{\sqrt{x^2}}}{\sqrt{\frac{x^2}{x^2}+\frac{3x}{x^2}}+\sqrt{\frac{x^2}{x^2}-\frac{2x}{x^2}}}$$

$$=\lim_{x\to\infty}\frac{5}{\sqrt{1+\frac{3}{x}}+\sqrt{1-\frac{2}{x}}}$$

$$=\frac{5}{\sqrt{1}+\sqrt{1}}$$

$$=\frac{5}{2}$$

Exercise

Find $\lim_{x\to\infty}\frac{2x-3}{4x+10}$

Solution

$$\lim_{x\to\infty}\frac{2x-3}{4x+10}=\frac{1}{2}$$

Exercise

Find $\lim_{x\to\infty}\frac{x^4-1}{x^5+2}$

Solution

$$\lim_{x\to\infty}\frac{x^4-1}{x^5+2} = 0 \rfloor$$

Exercise

Find $\lim_{x\to-\infty}\left(-3x^3+5\right)$

Solution

$$\lim_{x\to-\infty}\left(-3x^3+5\right) = \infty \rfloor$$

Exercise

Find $\lim_{x\to\infty}\left(e^{-2x}+\frac{2}{x}\right)$

Solution

$$\lim_{x\to\infty}\left(e^{-2x}+\frac{2}{x}\right) = e^{-\infty}+0$$

$$= 0 \rfloor$$

Exercise

Find $\lim_{x\to\infty}\frac{1}{\ln x+1}$

Solution

$$\lim_{x\to\infty}\frac{1}{\ln x+1} = \frac{1}{\infty}$$

$$= 0 \rfloor$$

Exercise

Find $\lim_{x\to\infty}\left(3+\frac{10}{x^2}\right)$

Solution

$$\lim_{x\to\infty}\left(3+\frac{10}{x^2}\right) = 3+0$$

$$= 3 \rfloor$$

Exercise

Find $\lim_{x\to\infty}\left(5+\frac{1}{x}+\frac{10}{x^2}\right)$

Solution

$$\lim_{x\to\infty}\left(5+\frac{1}{x}+\frac{10}{x^2}\right)=5+0+0$$

$$=5 \rfloor$$

Exercise

Find $\lim_{x\to\infty}\frac{4x^2+2x+3}{x^2}$

Solution

$$\lim_{x\to\infty}\frac{4x^2+2x+3}{x^2}=\lim_{x\to\infty}\frac{4x^2}{x^2}$$

$$=4 \rfloor$$

Exercise

Find $\lim_{x\to\infty}\left(5+\frac{100}{x}+\frac{\sin^4 x^3}{x^2}\right)$

Solution

$$-1\le\sin\theta\le 1$$

$$0\le\sin^4\theta\le 1$$

$$0\le\frac{\sin^4\theta}{x^2}\le\frac{1}{x^2}\ \to 0$$

$$\lim_{x\to\infty}\left(5+\frac{100}{x}+\frac{\sin^4 x^3}{x^2}\right)=5 \rfloor$$

Exercise

Find $\lim_{\theta\to\infty}\frac{\cos\theta}{\theta^2}$

Solution

$$-1\le\cos\theta\le 1$$

$$-\frac{1}{\theta^2}\le\frac{\cos\theta}{\theta^2}\le\frac{1}{\theta^2}\ \to 0$$

$$\lim_{\theta\to\infty} \frac{\cos\theta}{\theta^2} = 0$$

Exercise

Find $\lim_{\theta\to\infty} \frac{\cos\theta^5}{\sqrt{\theta}}$

Solution

$$-1 \le \cos\theta^5 \le 1$$

$$-\frac{1}{\sqrt{\theta}} \le \frac{\cos\theta^5}{\sqrt{\theta}} \le \frac{1}{\sqrt{\theta}} \to 0$$

$$\lim_{\theta\to\infty} \frac{\cos\theta^5}{\sqrt{\theta}} = 0$$

Exercise

Find $\lim_{x\to\infty} \frac{4x}{20x+1}$

Solution

$$\lim_{x\to\infty} \frac{4x}{20x+1} = \frac{4}{20}$$

$$= \frac{1}{5}$$

Exercise

Find $\lim_{x\to-\infty} \frac{4x}{20x+1}$

Solution

$$\lim_{x\to-\infty} \frac{4x}{20x+1} = \lim_{x\to-\infty} \frac{4x}{20x}$$

$$= \frac{1}{5}$$

Exercise

Find $\lim_{x\to\infty} \frac{3x^2-7}{x^2+5x}$

Solution

$$\lim_{x\to\infty} \frac{3x^2-7}{x^2+5x} = 3 \rfloor$$

Exercise

Find $\lim_{x\to-\infty} \frac{3x^2-7}{x^2+5x}$

Solution

$$\lim_{x\to-\infty} \frac{3x^2-7}{x^2+5x} = \lim_{x\to-\infty} \frac{3x^2}{x^2}$$
$$= 3 \rfloor$$

Exercise

Find $\lim_{x\to\infty} \frac{6x^2-9x+8}{3x^2+2}$

Solution

$$\lim_{x\to\infty} \frac{6x^2-9x+8}{3x^2+2} = \lim_{x\to\infty} \frac{6x^2}{3x^2}$$
$$= \frac{6}{3}$$
$$= 2 \rfloor$$

Exercise

Find $\lim_{x\to-\infty} \frac{6x^2-9x+8}{3x^2+2}$

Solution

$$\lim_{x\to-\infty} \frac{6x^2-9x+8}{3x^2+2} = \lim_{x\to-\infty} \frac{6x^2}{3x^2}$$
$$= \frac{6}{3}$$
$$= 2 \rfloor$$

Exercise

Find $\lim_{x\to\infty} \dfrac{4x^2-7}{8x^2+5x+2}$

Solution

$$\lim_{x\to\infty} \frac{4x^2-7}{8x^2+5x+2} = \lim_{x\to\infty} \frac{4x^2}{8x^2}$$

$$= \frac{4}{8}$$

$$= \boxed{\frac{1}{2}}$$

Exercise

Find $\lim_{x\to-\infty} \dfrac{4x^2-7}{8x^2+5x+2}$

Solution

$$\lim_{x\to-\infty} \frac{4x^2-7}{8x^2+5x+2} = \lim_{x\to-\infty} \frac{4x^2}{8x^2}$$

$$= \frac{4}{8}$$

$$= \boxed{\frac{1}{2}}$$

Exercise

Find $\lim_{x\to\infty} \dfrac{\sqrt{16x^4+64x^2}+x^2}{2x^2-4}$

Solution

$$\lim_{x\to\infty} \frac{\sqrt{16x^4+64x^2}+x^2}{2x^2-4} = \lim_{x\to\infty} \frac{\sqrt{16x^4}+x^2}{2x^2}$$

$$= \lim_{x\to\infty} \frac{4x^2+x^2}{2x^2}$$

$$= \lim_{x\to\infty} \frac{5x^2}{2x^2}$$

$$= \boxed{\frac{5}{2}}$$

Exercise

Find $\lim_{x\to-\infty} \dfrac{\sqrt{16x^4+64x^2}+x^2}{2x^2-4}$

Solution

$$\lim_{x\to-\infty} \frac{\sqrt{16x^4+64x^2}+x^2}{2x^2-4} = \lim_{x\to-\infty} \frac{\sqrt{16x^4}+x^2}{2x^2}$$

$$= \lim_{x\to-\infty} \frac{4x^2+x^2}{2x^2}$$

$$= \lim_{x\to-\infty} \frac{5x^2}{2x^2}$$

$$= \frac{5}{2}$$

Exercise

Find $\lim_{x\to\infty} \dfrac{3x^4+3x^3-36x^2}{x^4-25x^2+144}$

Solution

$$\lim_{x\to\infty} \frac{3x^4+3x^3-36x^2}{x^4-25x^2+144} = \lim_{x\to\infty} \frac{3x^4}{x^4}$$

$$= 3$$

Exercise

Find $\lim_{x\to-\infty} \dfrac{3x^4+3x^3-36x^2}{x^4-25x^2+144}$

Solution

$$\lim_{x\to-\infty} \frac{3x^4+3x^3-36x^2}{x^4-25x^2+144} = \lim_{x\to-\infty} \frac{3x^4}{x^4}$$

$$= 3$$

Exercise

Find $\lim_{x\to\infty} 16x^2\left(4x^2-\sqrt{16x^4+1}\right)$

Solution

$$\lim_{x\to\infty} 16x^2\left(4x^2-\sqrt{16x^4+1}\right)=\infty-\infty$$

$$=\lim_{x\to\infty} 16x^2\left(4x^2-\sqrt{16x^4+1}\right)\cdot\frac{4x^2+\sqrt{16x^4+1}}{4x^2+\sqrt{16x^4+1}}$$

$$=\lim_{x\to\infty} 16x^2\,\frac{16x^4-16x^4-1}{4x^2+\sqrt{16x^4+1}}$$

$$=\lim_{x\to\infty} 16x^2\,\frac{-1}{4x^2+\sqrt{16x^4+1}}$$

$$=\lim_{x\to\infty} \frac{-16x^2}{4x^2+4x^2}$$

$$=\lim_{x\to\infty} \frac{-16x^2}{8x^2}$$

$$=-2$$

Exercise

Find $\lim_{x\to-\infty} 16x^2\left(4x^2-\sqrt{16x^4+1}\right)$

Solution

$$\lim_{x\to-\infty} 16x^2\left(4x^2-\sqrt{16x^4+1}\right)=\infty-\infty$$

$$=\lim_{x\to-\infty} 16x^2\left(4x^2-\sqrt{16x^4+1}\right)\cdot\frac{4x^2+\sqrt{16x^4+1}}{4x^2+\sqrt{16x^4+1}}$$

$$=\lim_{x\to-\infty} 16x^2\,\frac{16x^4-16x^4-1}{4x^2+\sqrt{16x^4+1}}$$

$$=\lim_{x\to-\infty} 16x^2\,\frac{-1}{4x^2+\sqrt{16x^4+1}}$$

$$=\lim_{x\to-\infty} \frac{-16x^2}{4x^2+4x^2}$$

$$=\lim_{x\to-\infty} \frac{-16x^2}{8x^2}$$

$$=-2$$

Exercise

Find $\lim_{x\to\infty} \frac{x-1}{x^{2/3}-1}$

Solution

$$\lim_{x\to\infty} \frac{x-1}{x^{2/3}-1} = \lim_{x\to\infty} \frac{x}{x^{2/3}}$$

$$= \lim_{x\to\infty} x^{1/3}$$

$$= \infty$$

Exercise

Find $\lim_{x\to-\infty} \frac{x-1}{x^{2/3}-1}$

Solution

$$\lim_{x\to-\infty} \frac{x-1}{x^{2/3}-1} = \lim_{x\to-\infty} \frac{x}{x^{2/3}}$$

$$= \lim_{x\to-\infty} x^{1/3}$$

$$= -\infty$$

Exercise

Find $\lim_{x\to\infty} \frac{\sqrt{x^2+2x+6}-3}{x-1}$

Solution

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2x+6}-3}{x-1} = \lim_{x\to\infty} \frac{\sqrt{x^2}}{x}$$

$$= \lim_{x\to\infty} \frac{x}{x}$$

$$= 1$$

Exercise

Find $\lim_{x\to\infty} \frac{\left|1-x^2\right|}{x(x+1)}$

Solution

$$\lim_{x\to\infty}\frac{\left|1-x^2\right|}{x(x+1)}=\lim_{x\to\infty}\frac{x^2-1}{x^2+1}$$

$$=\lim_{x\to\infty}\frac{x^2}{x^2}$$

$$=1$$

Exercise

Find $\lim_{x\to\infty}\left(\sqrt{|x|}-\sqrt{|x-1|}\right)$

Solution

$$\lim_{x\to\infty}\left(\sqrt{|x|}-\sqrt{|x-1|}\right)=\infty-\infty \qquad x\to\infty \Rightarrow |x|=x \ \& \ |x-1|=x-1$$

$$=\lim_{x\to\infty}\left(\sqrt{x}-\sqrt{x-1}\right)\cdot\frac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}$$

$$=\lim_{x\to\infty}\frac{x-x+1}{\sqrt{x}+\sqrt{x-1}}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{x}+\sqrt{x-1}}$$

$$=\frac{1}{\infty}$$

$$=0$$

Exercise

Find $\lim_{x\to\infty}\frac{\tan^{-1}x}{x}$

Solution

$$-\frac{\pi}{2}\le\tan^{-1}x\le\frac{\pi}{2}$$

$$-\frac{\pi}{2x}\le\frac{\tan^{-1}x}{x}\le\frac{\pi}{2x} \quad\to 0$$

$$\lim_{x\to\infty}\frac{\tan^{-1}x}{x}=0$$

Exercise

Find $\lim_{x\to\infty} \frac{\cos x}{e^{3x}}$

Solution

$$-1 \le \cos x \le 1$$

$$-\frac{1}{e^{3x}} \le \frac{\cos x}{e^{3x}} \le \frac{1}{e^{3x}} \quad \to \ 0$$

$$\lim_{x\to\infty} \frac{\cos x}{e^{3x}} = 0$$

Exercise

Find $\lim_{x\to 0} \frac{2e^{x}+10e^{-x}}{e^{x}+e^{-x}}$

Solution

$$\lim_{x\to 0} \frac{2e^{x}+10e^{-x}}{e^{x}+e^{-x}} = \frac{2+10}{1+1}$$

$$= 6$$

Exercise

Find $\lim_{x\to\infty} \frac{2e^{x}+10e^{-x}}{e^{x}+e^{-x}}$

Solution

$$\lim_{x\to\infty} \frac{2e^{x}+10e^{-x}}{e^{x}+e^{-x}} = \lim_{x\to\infty} \frac{2e^{x}}{e^{x}} \qquad \lim_{x\to\infty} e^{-x} = 0$$

$$= 2$$

Exercise

Find $\lim_{x\to-\infty} \frac{2e^{x}+10e^{-x}}{e^{x}+e^{-x}}$

Solution

$$\lim_{x\to\infty} \frac{2e^{x}+10e^{-x}}{e^{x}+e^{-x}} = \lim_{x\to\infty} \frac{10e^{-x}}{e^{-x}} \qquad \lim_{x\to-\infty} e^{x} = 0$$

$$= 10$$

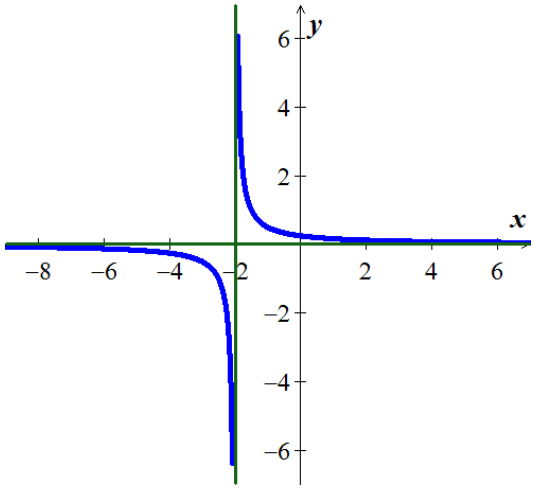
Exercise

Graph the rational function $y = \dfrac{1}{2x+4}$. Include the equations of the asymptotes.

Solution

VA: $2x = 4 = 0 \;\Rightarrow\; \underline{x = -2}$

HA: $\underline{y = 0}$



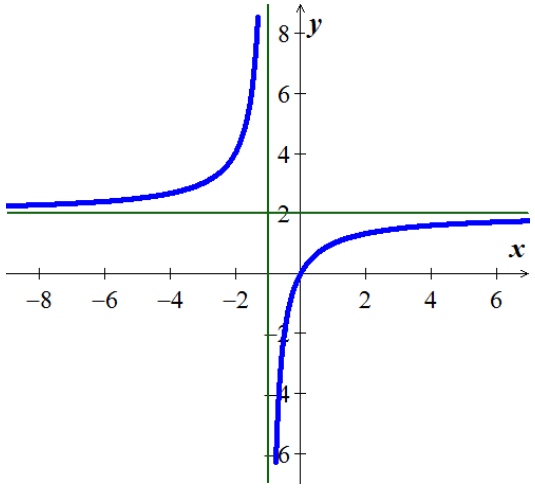
Exercise

Graph the rational function $y = \dfrac{2x}{x+1}$. Include the equations of the asymptotes.

Solution

VA: $\underline{x = -1}$

HA: $\underline{y = 2}$



Exercise

Graph the rational function $y = \frac{x^2}{x-1}$. Include the equations of the asymptotes.

Solution

$$x-1\overline{)\,x^2}$$

$$\underline{x^2 - x}$$

$$x$$

$$\underline{x - 1}$$

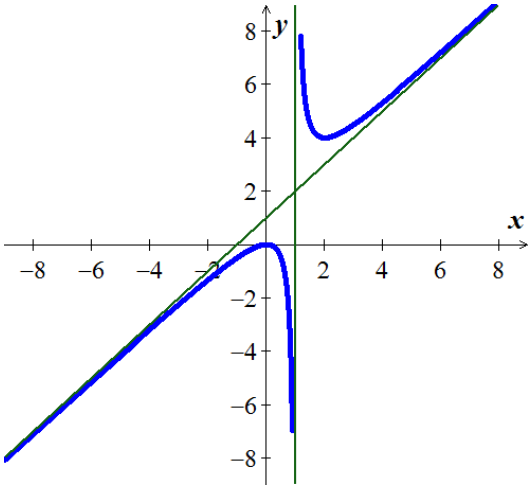
$$1$$

$$y = \frac{x^2}{x-1}$$

$$= x + 1 + \frac{1}{x-1}$$

VA: $x = 1$

Oblique Asymptote: $y = x + 1$



Exercise

Graph the rational function $y = \frac{x^3+1}{x^2}$. Include the equations of the asymptotes.

Solution

$$x^2\overline{)\,x^3+1}$$

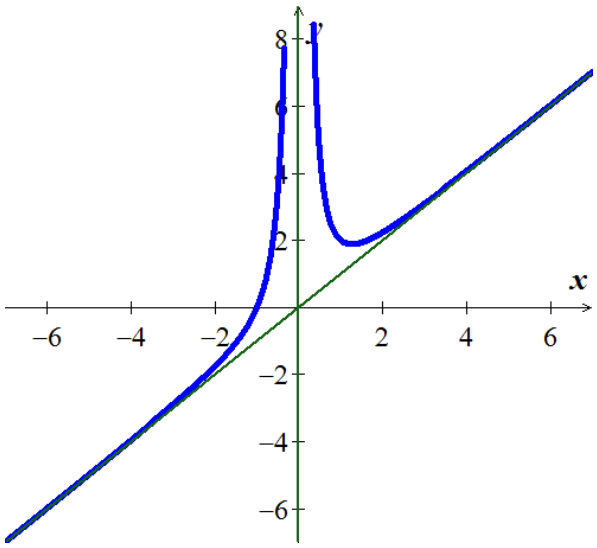
$$\underline{x^3}$$

$$1$$

$$y = \frac{x^3+1}{x^2} = x + \frac{1}{x^2}$$

VA: $\underline{x = 0}$

Oblique Asymptote: $\underline{y = x}$



Exercise

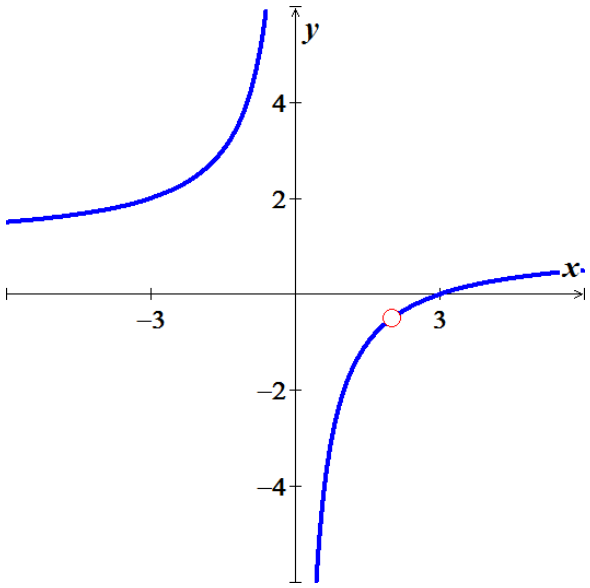
Let $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 2x}$

a) Analyze $\lim\limits_{x \to 0^-} f(x)$, $\lim\limits_{x \to 0^+} f(x)$, $\lim\limits_{x \to 2^-} f(x)$, and $\lim\limits_{x \to 2^+} f(x)$

b) Does the graph of f have any vertical asymptotes? Explain?

Solution

$$f(x) = \frac{x^2 - 5x + 6}{x^2 - 2x}$$
$$= \frac{(x-2)(x-3)}{x(x-2)}$$
$$= \frac{x-3}{x}$$



a) $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \dfrac{x-3}{x}$

$$= \frac{-3}{0^-}$$
$$= \infty$$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \dfrac{x-3}{x}$

$$= \frac{-3}{0^+}$$

$$= -\infty$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x-3}{x}$$

$$= \frac{2-3}{2}$$

$$= -\frac{1}{2}$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x-3}{x}$$

$$= \frac{2-3}{2}$$

$$= -\frac{1}{2}$$

b) ***VA*** : $x = 0$ ***Hole*** : $x = 2 \to f(2) = -\frac{1}{2}$

HA : $y = 1$ ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y = \frac{3x}{1-x}$

Solution

VA : $x = 1$, ***Hole*** : n/a, ***HA*** : $y = -3$, ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y = \frac{x^2}{x^2+9}$

Solution

VA : n/a; ***Hole*** : n/a; ***HA*** : $y = 1$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y = \frac{x-2}{x^2-4x+3}$

Solution

VA : $x = 1,\ 3$; ***Hole*** : n/a; ***HA*** : $y = 0$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y=\dfrac{5x-1}{1-3x}$

Solution

VA : $x=\dfrac{1}{3}$; ***Hole*** : n/a; ***HA*** : $y=-\dfrac{5}{3}$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y=\dfrac{3}{x-5}$

Solution

VA : $x=5$, ***Hole*** : n/a, ***HA*** : $y=0$, ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y=\dfrac{x^3-1}{x^2+1}$

Solution

$$x^2+1\overline{)\,x^3-1}\qquad x$$
$$x^3+x$$
$$\boxed{-x-1}$$

$$y=\frac{x^3-1}{x^2+1}$$
$$=x+\frac{-x-1}{x^2+1}$$
$$=x-\frac{x+1}{x^2+1}$$

VA : n/a, ***Hole*** : n/a, ***HA*** : n/a, ***OA*** : $y=x$

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y=\dfrac{3x^2-27}{(x+3)(2x+1)}$

Solution

VA : $x=-3,\ -\dfrac{1}{2}$; ***Hole*** : n/a; ***HA*** : $y=\dfrac{3}{2}$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y = \dfrac{x^3 + 3x^2 - 2}{x^2 - 4}$

Solution

$$x^2 - 4 \overline{)\, x^3 + 3x^2 - 2} \quad x + 3$$
$$\underline{x^3 \quad - 4x}$$
$$3x^2 + 4x - 2$$

$$y = \frac{x^3 + 3x^2 - 2}{x^2 - 4}$$
$$= x + 3 + \frac{4x + 10}{x^2 - 4}$$

VA : $x = \pm 2$, ***Hole*** : n/a, ***HA*** : n/a, ***OA*** : $y = x + 3$

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y = \dfrac{x - 3}{x^2 - 9}$

Solution

VA : $x = -3$; ***Hole*** : $x = 3$; ***HA*** : $y = 0$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $y = \dfrac{6}{\sqrt{x^2 - 4x}}$

Solution

VA : $x = 0,\ 4$; ***Hole*** : n/a; ***HA*** : $y = 0$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x) = \dfrac{4x^3 + 1}{1 - x^3}$

Solution

VA : $x = 1$; ***Hole*** : n/a; ***HA*** : $y = -4$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=\dfrac{x+1}{\sqrt{9x^2+x}}$

Solution

VA : $x=0,\ -\frac{1}{9}$; ***Hole*** : n/a; ***HA*** : $y=\frac{1}{3}$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=1-e^{-2x}$

Solution

VA : n/a; ***Hole*** : n/a; ***HA*** : $y=1$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=\dfrac{1}{\ln x^2}$

Solution

VA : $x=0$; ***Hole*** : n/a; ***HA*** : $y=0$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=\dfrac{1}{\tan^{-1}x}$

Solution

VA : $x=0$; ***Hole*** : n/a; ***HA*** : $y=\dfrac{1}{\frac{\pi}{2}}=\dfrac{2}{\pi}$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=\dfrac{2x^2+6}{2x^2+3x-2}$

Solution

VA : $x=-2,\ \frac{1}{2}$; ***Hole*** : n/a; ***HA*** : $y=1$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=\dfrac{3x^2+2x-1}{4x+1}$

Solution

$$\begin{array}{r} \frac{3}{4}x+\frac{5}{16} \\ 4x+1\overline{)\,3x^2+2x-1} \\ \underline{3x^2+\frac{3}{4}x} \\ \frac{5}{4}x-1 \end{array}$$

VA : $x=-\dfrac{1}{4}$; ***Hole*** : n/a; ***HA*** : n/a; ***OA*** : $y=\dfrac{3}{4}x+\dfrac{5}{16}$

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=\dfrac{9x^2+4}{(2x-1)^2}$

Solution

VA : $x=\dfrac{1}{2}$; ***Hole*** : n/a; ***HA*** : $y=\dfrac{9}{4}$; ***OA*** : n/a

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=\dfrac{1+x-2x^2-x^3}{x^2+1}$

Solution

$$\begin{array}{r} -x-2 \\ x^2+1\overline{)\,-x^3-2x^2+x+1} \\ \underline{-x^3\qquad -x} \\ -2x^2+2x \end{array}$$

VA : n/a; ***Hole*** : n/a; ***HA*** : n/a; ***OA*** : $y=-x-2$

Exercise

Find the vertical, horizontal, hole, and oblique asymptotes (if any) of $f(x)=\dfrac{x(x+2)^3}{3x^2-4x}$

Solution

$$f(x)=\frac{x\left(x^3+6x^2+12x+8\right)}{x(3x-4)}$$

$$=\frac{x^3+6x^2+12x+8}{3x-4}$$

$$\begin{array}{r|l} & \frac{1}{3}x^2+\frac{22}{9}x+\frac{196}{27} \\ \hline 3x-4 & x^3+6x^2+12x+8 \\ & \underline{x^3-\frac{4}{3}x^2} \\ & \frac{22}{3}x^2+12x \\ & \underline{\frac{22}{3}x^2-\frac{88}{9}x} \\ & \frac{196}{9}x \end{array}$$

VA : $x=\frac{4}{3}$; ***Hole*** : $(0,\ -2)$; ***HA*** : n/a; ***OA*** : $y=\frac{1}{3}x^2+\frac{22}{9}x+\frac{196}{27}$

Exercise

Find the limit $\lim\limits_{x\to 0}\ \dfrac{x^2-4x+4}{x^3+5x^2-14x}$

Solution

$$\lim_{x\to 0}\ \frac{x^2-4x+4}{x^3+5x^2-14x}=\frac{4}{0}$$

$$=\infty$$

Exercise

Find the limit $\lim\limits_{x\to 2}\ \dfrac{x^2-4x+4}{x^3+5x^2-14x}$

Solution

$$\lim_{x\to 2}\ \frac{x^2-4x+4}{x^3+5x^2-14x}=\frac{4-8+4}{8+20-28}=\frac{0}{0}$$

$$=\lim_{x\to 2}\ \frac{(x-2)(x-2)}{x(x-2)(x+7)}$$

$$=\lim_{x\to 2}\ \frac{x-2}{x(x+7)}$$

$$=\frac{0}{18}$$

$$=0$$

Exercise

Find the limit $\lim_{x\to a} \dfrac{x^2-a^2}{x^4-a^4}$

Solution

$$\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4} = \frac{a^2-a^2}{a^4-a^4} = \frac{0}{0}$$

$$= \lim_{x\to a} \frac{x^2-a^2}{\left(x^2-a^2\right)\left(x^2+a^2\right)}$$

$$= \lim_{x\to a} \frac{1}{x^2+a^2}$$

$$= \frac{1}{a^2+a^2}$$

$$= \underline{\boxed{\frac{1}{2a^2}}}$$

Exercise

Find the limit $\lim_{x\to 0} \dfrac{(x+h)^2-x^2}{h}$

Solution

$$\lim_{x\to 0} \frac{(x+h)^2-x^2}{h} = \frac{h^2}{h}$$

$$\underline{\boxed{= h}}$$

Exercise

Find the limit $\lim_{h\to 0} \dfrac{(x+h)^2-x^2}{h}$

Solution

$$\lim_{h\to 0} \frac{(x+h)^2-x^2}{h} = \frac{x^2-x^2}{0} = \frac{0}{0}$$

$$= \lim_{h\to 0} \frac{x^2+2hx+h^2-x^2}{h}$$

$$= \lim_{h\to 0} \frac{2hx+h^2}{h}$$

$$= \lim_{h\to 0} (2x+h)$$
$$= 2x$$

Exercise

Find the limit $\lim_{x\to 1} \dfrac{1-\sqrt{x}}{1-x}$

Solution

$$\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = \frac{1-1}{1-1} = \frac{0}{0}$$

$$= \lim_{x\to 1} \frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})}$$

$$= \lim_{x\to 1} \frac{1}{1+\sqrt{x}}$$

$$= \frac{1}{2}$$

$$\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = \lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}}$$

$$= \lim_{x\to 1} \frac{1-x}{(1-x)(1+\sqrt{x})}$$

$$= \lim_{x\to 1} \frac{1}{1+\sqrt{x}}$$

$$= \frac{1}{2}$$

Exercise

Find the limit $\lim_{x\to 0} \dfrac{\frac{1}{2+x} - \frac{1}{2}}{x}$

Solution

$$\lim_{x\to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = \frac{\frac{1}{2} - \frac{1}{2}}{0} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{1}{x}\left(\frac{2-2-x}{2(2+x)}\right)$$

$$= \frac{1}{2} \lim_{x\to 0} \frac{1}{x}\left(\frac{-x}{2+x}\right)$$

$$= -\frac{1}{2}\lim_{x\to 0}\ \frac{1}{2+x}$$

$$= -\frac{1}{2}\left(\frac{1}{2}\right)$$

$$\underline{= -\frac{1}{4}}$$

Exercise

Find the limit $\lim\limits_{x\to 1}\ \dfrac{x^{1/3}-1}{\sqrt{x}-1}$

Solution

$$\lim_{x\to 1}\ \frac{x^{1/3}-1}{\sqrt{x}-1} = \frac{1-1}{1-1} = \frac{0}{0}$$

$$= \lim_{x\to 1}\ \frac{x^{1/3}-1}{\sqrt{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$= \lim_{x\to 1}\ \frac{\left(x^{1/3}-1\right)\left(\sqrt{x}+1\right)}{x-1}$$

$$= \lim_{x\to 1}\ \frac{\left(x^{1/3}-1\right)\left(\sqrt{x}+1\right)}{\left(x^{1/3}\right)^3-1^3} \qquad a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$$

$$= \lim_{x\to 1}\ \frac{\left(x^{1/3}-1\right)\left(\sqrt{x}+1\right)}{\left(x^{1/3}-1\right)\left(x^{2/3}+x^{1/3}+1\right)}$$

$$= \lim_{x\to 1}\ \frac{\sqrt{x}+1}{x^{2/3}+x^{1/3}+1}$$

$$\underline{= \frac{2}{3}}$$

Exercise

Find the limit $\lim\limits_{x\to 64}\ \dfrac{x^{2/3}-16}{\sqrt{x}-8}$

Solution

$$\lim_{x\to 64}\ \frac{x^{2/3}-16}{\sqrt{x}-8} = \frac{\left(4^3\right)^{2/3}-16}{8-8}$$

$$= \frac{16-16}{0} = \frac{0}{0}$$

$$= \lim_{x\to 64} \frac{\left(x^{1/3}\right)^2 - 16}{\sqrt{x}-8} \cdot \frac{\sqrt{x}+8}{\sqrt{x}+8}$$

$$= \lim_{x\to 64} \frac{\left(x^{1/3}-4\right)\left(x^{1/3}+4\right)\left(\sqrt{x}+8\right)}{x-64}$$

$$= \lim_{x\to 64} \frac{\left(x^{1/3}-4\right)\left(x^{1/3}+4\right)\left(\sqrt{x}+8\right)}{\left(x^{1/3}\right)^3 - 4^3} \qquad a^3-b^3 = (a-b)\left(a^2+ab+b^2\right)$$

$$= \lim_{x\to 64} \frac{\left(x^{1/3}-4\right)\left(x^{1/3}+4\right)\left(\sqrt{x}+8\right)}{\left(x^{1/3}-4\right)\left(x^{2/3}+4x^{1/3}+16\right)}$$

$$= \lim_{x\to 64} \frac{\left(x^{1/3}+4\right)\left(\sqrt{x}+8\right)}{x^{2/3}+4x^{1/3}+16}$$

$$= \frac{(4+4)(8+8)}{16+16+16}$$

$$= \boxed{\frac{8}{3}}$$

Exercise

Find the limit $\lim_{x\to 0} \frac{\tan(2x)}{\tan(\pi x)}$

Solution

$$\lim_{x\to 0} \frac{\tan(2x)}{\tan(\pi x)} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\sin 2x}{\cos 2x} \cdot \frac{\cos(\pi x)}{\sin(\pi x)}$$

$$= \lim_{x\to 0} \frac{\cos(\pi x)}{\cos 2x} \cdot \frac{\sin 2x}{2x} \cdot \frac{2x}{\pi x} \cdot \frac{\pi x}{\sin(\pi x)}$$

$$= \frac{2}{\pi} \frac{\cos 0}{\cos 0} \cdot \lim_{2x\to 0} \frac{\sin 2x}{2x} \lim_{\pi x\to 0} \frac{1}{\frac{\sin \pi x}{\pi x}}$$

$$= \boxed{\frac{2}{\pi}}$$

Exercise

Find the limit $\lim_{x\to\pi^-} \csc x$

Solution

$$\lim_{x\to\pi^-} \csc x = \frac{1}{\sin \pi^-}$$
$$= \frac{1}{0^-}$$
$$= -\infty$$

Exercise

Find the limit $\lim_{x\to\pi} \sin\left(\frac{x}{2} + \sin x\right)$

Solution

$$\lim_{x\to\pi} \sin\left(\frac{x}{2} + \sin x\right) = \sin\left(\frac{\pi}{2} + \sin \pi\right)$$
$$= \sin \frac{\pi}{2}$$
$$= 1$$

Exercise

Find the limit $\lim_{x\to\pi} \cos^2\left(x - \tan x\right)$

Solution

$$\lim_{x\to\pi} \cos^2\left(x - \tan x\right) = \cos^2\left(\pi - \tan \pi\right)$$
$$= \cos^2\left(\pi\right)$$
$$= \left(-1\right)^2$$
$$= 1$$

Exercise

Find the limit $\lim_{x\to 0} \frac{8x}{3\sin x - x}$

Solution

$$\lim_{x\to 0}\frac{8x}{3\sin x - x} = \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{8\frac{x}{x}}{3\frac{\sin x}{x} - \frac{x}{x}}$$

$$= \frac{8}{3\lim\limits_{x\to 0}\frac{\sin x}{x} - 1} \qquad \lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$= \frac{8}{3-1}$$

$$= 4$$

Exercise

Find the limit $\lim\limits_{x\to 0}\dfrac{\cos 2x - 1}{\sin x}$

Solution

$$\lim_{x\to 0}\frac{\cos 2x - 1}{\sin x} = \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{1 - 2\sin^2 x - 1}{\sin x}$$

$$= \lim_{x\to 0}\frac{-2\sin^2 x}{\sin x}$$

$$= -2\lim_{x\to 0}\sin x$$

$$= 0$$

Exercise

Find the limit $\lim\limits_{x\to -\infty}\dfrac{4 - 3x^3}{\sqrt{x^6 + 9}}$

Solution

$$\lim_{x\to -\infty}\frac{4 - 3x^3}{\sqrt{x^6 + 9}} = \lim_{x\to -\infty}\frac{3x^3}{\sqrt{x^6}}$$

$$= \lim_{x\to -\infty}\frac{3x^3}{x^3}$$

$$= 3$$

Exercise

Find the limit $\lim_{x\to-\infty} \dfrac{x^2-4x+8}{3x^3}$

Solution

$$\lim_{x\to-\infty} \frac{x^2-4x+8}{3x^3} = \lim_{x\to-\infty} \frac{x^2}{3x^3}$$

$$= \lim_{x\to-\infty} \frac{1}{3x}$$

$$= 0$$

Exercise

Find the limit $\lim_{x\to-\infty} \dfrac{2x^2+3}{5x^2+7}$

Solution

$$\lim_{x\to-\infty} \frac{2x^2+3}{5x^2+7} = \frac{2}{5}$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{x^4+x^3}{12x^3+128}$

Solution

$$\lim_{x\to\infty} \frac{x^4+x^3}{12x^3+128} = \infty$$

Exercise

Find the limit $\lim_{x\to-\infty} \dfrac{2+\sqrt{x}}{2-\sqrt{x}}$

Solution

Since $x \to -\infty$ and inside the square root can't be negative

$$\lim_{x\to-\infty} \frac{2+\sqrt{x}}{2-\sqrt{x}} = \nexists$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{2+\sqrt{x}}{2-\sqrt{x}}$

Solution

$$\lim_{x\to\infty} \frac{2+\sqrt{x}}{2-\sqrt{x}} = \lim_{x\to\infty} \frac{\sqrt{x}}{-\sqrt{x}}$$

$$= -1$$

Exercise

Find the limit $\lim_{x\to-\infty} \dfrac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}}$

Solution

$$\lim_{x\to-\infty} \frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}} = \lim_{x\to-\infty} \frac{\sqrt[3]{x}}{\sqrt[3]{x}}$$

$$= 1$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{\frac{1}{x}+\frac{1}{x^4}}{\frac{1}{x^2}-\frac{1}{x^3}}$

Solution

$$\lim_{x\to\infty} \frac{\frac{1}{x}+\frac{1}{x^4}}{\frac{1}{x^2}-\frac{1}{x^3}} = \frac{0}{0}$$

$$= \lim_{x\to\infty} \frac{\frac{x^3+1}{x^4}}{\frac{x-1}{x^3}}$$

$$= \lim_{x\to\infty} \frac{x^3+1}{x-1}\cdot\frac{x^3}{x^4}$$

$$= \lim_{x\to\infty} \frac{x^3+1}{x(x-1)}$$

$$= \lim_{x\to\infty} \frac{x^3}{x^2}$$

$$= \infty$$

Exercise

Find the limit $\lim\limits_{x\to\infty} \dfrac{2x^{5/3} - x^{1/3} + 7}{x^{8/5} + 3x + \sqrt{x}}$

Solution

$$\lim_{x\to\infty} \frac{2x^{5/3} - x^{1/3} + 7}{x^{8/5} + 3x + \sqrt{x}} = \lim_{x\to\infty} \frac{2x^{5/3}}{x^{8/5}}$$

$$= \lim_{x\to\infty} 2x^{\left(\frac{5}{3}-\frac{8}{5}\right)}$$

$$= \lim_{x\to\infty} 2x^{\frac{1}{15}}$$

$$= \infty$$

Exercise

Find the limit $\lim\limits_{x\to 2^+} \ln(x-2)$

Solution

$$\lim_{x\to 2^+} \ln(x-2) = \ln\left(0^+\right)$$

$$= -\infty$$

Exercise

Find the limit $\lim\limits_{x\to 1} x^2 \ln\left(2-\sqrt{x}\right)$

Solution

$$\lim_{x\to 1} x^2 \ln\left(2-\sqrt{x}\right) = \ln(2-1)$$

$$= \ln 1$$

$$= 0$$

Exercise

Find the limit $\lim\limits_{\theta\to 0^+} \sqrt{\theta}\, e^{\cos\frac{\pi}{\theta}}$

Solution

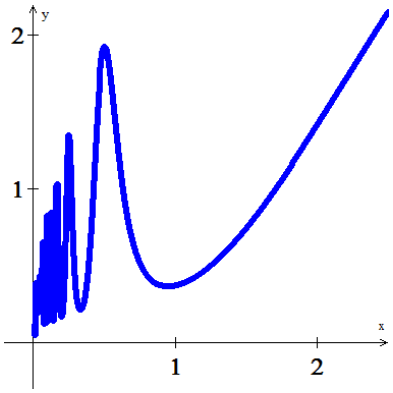
$$\lim_{\theta\to 0^+} \sqrt{\theta}\, e^{\cos\frac{\pi}{\theta}} = 0\cdot e^{\cos\infty}$$

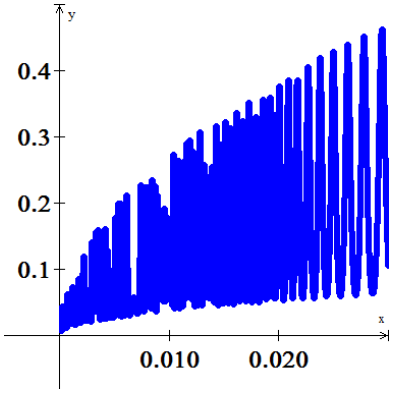
$$-1 \le \cos\frac{\pi}{\theta} \le 1$$

$$e^{-1} \le e^{\cos\frac{\pi}{\theta}} \le e$$

$$0\cdot\frac{1}{e} \le 0\cdot e^{\cos\frac{\pi}{\theta}} \le 0\cdot e$$

$$\boxed{\lim_{\theta\to 0^+} \sqrt{\theta}\, e^{\cos\frac{\pi}{\theta}} = 0}$$





Exercise

Find the limit $\lim_{x\to\infty} \frac{2x-3}{5x+6}$

Solution

$$\lim_{x\to\infty} \frac{2x-3}{5x+6} = \lim_{x\to\infty} \frac{2x}{5x}$$

$$\boxed{= \frac{2}{5}}$$

Exercise

Find the limit $\lim_{x\to\infty} \frac{2x^2-3}{5x^2+6}$

Solution

$$\lim_{x\to\infty} \frac{2x^2-3}{5x^2+6} = \lim_{x\to\infty} \frac{2x^2}{5x^2}$$

$$\boxed{= \frac{2}{5}}$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{2x-3}{5x^3+6}$

Solution

$$\lim_{x\to\infty} \frac{2x-3}{5x^3+6} = \lim_{x\to\infty} \frac{2x}{5x^3}$$

$$= 0$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{1}{5x^2-3x+6}$

Solution

$$\lim_{x\to\infty} \frac{1}{5x^2-3x+6} = \lim_{x\to\infty} \frac{1}{5x^2}$$

$$= 0$$

Exercise

Find the limit $\lim_{\theta\to 0} \dfrac{\theta \cot 4\theta}{\sin^2\theta \cot^2 2\theta}$

Solution

$$\lim_{\theta\to 0} \frac{\theta \cot 4\theta}{\sin^2\theta \cot^2 2\theta} = \frac{0}{0}$$

$$= \lim_{\theta\to 0} \frac{1}{\frac{\sin\theta}{\theta}} \cdot \frac{1}{\sin\theta} \cdot \frac{\cos 4\theta}{\sin 4\theta} \cdot \frac{\sin^2 2\theta}{\cos^2 2\theta}$$

$$= \lim_{\theta\to 0} \frac{1}{\frac{\sin\theta}{\theta}} \lim_{\theta\to 0} \frac{\cos 4\theta}{\cos^2 2\theta} \lim_{\theta\to 0} \frac{1}{\sin\theta} \cdot \frac{\sin 2\theta \sin 2\theta}{2\sin 2\theta \cos 2\theta}$$

$$= (1)(1) \lim_{\theta\to 0} \frac{1}{\sin\theta} \cdot \frac{2\sin\theta\cos\theta}{2\cos 2\theta}$$

$$= \lim_{\theta\to 0} \frac{\cos\theta}{\cos 2\theta}$$

$$= 1$$

Exercise

Find the limit $\lim\limits_{x\to 0^+} \dfrac{\sqrt{x^2+4x+5}-\sqrt{5}}{x}$

Solution

$$\lim_{x\to 0^+} \frac{\sqrt{x^2+4x+5}-\sqrt{5}}{x} = \frac{\sqrt{5}-\sqrt{5}}{0} = \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x^2+4x+5}-\sqrt{5}}{x} \cdot \frac{\sqrt{x^2+4x+5}+\sqrt{5}}{\sqrt{x^2+4x+5}+\sqrt{5}}$$

$$= \lim_{x\to 0^+} \frac{x^2+4x+5-5}{x\left(\sqrt{x^2+4x+5}+\sqrt{5}\right)}$$

$$= \lim_{x\to 0^+} \frac{x(x+4)}{x\left(\sqrt{x^2+4x+5}+\sqrt{5}\right)}$$

$$= \lim_{x\to 0^+} \frac{x+4}{\sqrt{x^2+4x+5}+\sqrt{5}}$$

$$= \frac{4}{\sqrt{5}+\sqrt{5}}$$

$$= \frac{4}{2\sqrt{5}}$$

$$= \underline{\frac{2}{\sqrt{5}}}$$

Exercise

Find the limit $\lim\limits_{x\to 2} \dfrac{x^4-16}{x-2}$

Solution

$$\lim_{x\to 2} \frac{x^4-16}{x-2} = \frac{16-16}{2-2} = \frac{0}{0}$$

$$= \lim_{x\to 2} \frac{(x-2)(x+2)\left(x^2+4\right)}{x-2} \qquad a^4-b^4 = (a-b)(a+b)\left(a^2+b^2\right)$$

$$= \lim_{x\to 2} (x+2)\left(x^2+4\right)$$

$$= (4)(8)$$

$$= \underline{32}$$

Exercise

Find the limit $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$

Solution

$$\lim_{x\to 2}\frac{x^3-8}{x-2}=\frac{0}{0}$$

$$=\lim_{x\to 2}\frac{(x-2)\left(x^2+2x+4\right)}{x-2} \qquad a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$$

$$=\lim_{x\to 2}\left(x^2+2x+4\right)$$

$$=4+4+4$$

$$=12$$

Exercise

Find the limit $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x}-5x+3}{2x+x^{2/3}-4}$

Solution

$$\lim_{x\to-\infty}\frac{\sqrt[3]{x}-5x+3}{2x+x^{2/3}-4}=\lim_{x\to-\infty}\frac{-5x}{2x}$$

$$=-\frac{5}{2}$$

Exercise

Find the limit $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+1}}{x+1}$

Solution

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{x+1}=\lim_{x\to-\infty}\frac{\sqrt{x^2}}{x}$$

$$=\lim_{x\to-\infty}\frac{|x|}{x}$$

$$=-1$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{\sqrt{x^2+1}}{x+1}$

Solution

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1} = \lim_{x\to\infty} \frac{\sqrt{x^2}}{x}$$

$$= \lim_{x\to\infty} \frac{|x|}{x}$$

$$= 1$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{x-3}{\sqrt{4x^2+25}}$

Solution

$$\lim_{x\to\infty} \frac{x-3}{\sqrt{4x^2+25}} = \lim_{x\to\infty} \frac{x}{2|x|}$$

$$= \frac{1}{2}$$

Exercise

Find the limit $\lim_{x\to-\infty} \dfrac{4-3x^3}{\sqrt{x^6+9}}$

Solution

$$\lim_{x\to-\infty} \frac{4-3x^3}{\sqrt{x^6+9}} = \lim_{x\to-\infty} \frac{3x^3}{x^3}$$

$$= 3$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{x^4-x}{15x^3+4}$

Solution

$$\lim_{x\to\infty}\frac{x^4-x}{15x^3+4}=\infty$$

Exercise

Find the limit $\lim_{x\to\infty}\frac{x+\sin x+2\sqrt{x}}{x+\sin x}$

Solution

$$-1\le\sin x\le 1$$

$$\lim_{x\to\infty}\frac{x+\sin x+2\sqrt{x}}{x+\sin x}=\lim_{x\to\infty}\frac{x}{x}$$

$$=1$$

Exercise

Find the limit $\lim_{x\to\infty}\frac{x^{2/3}-x^{-1}}{x^{2/3}+\cos^2 x}$

Solution

$$-1\le\cos x\le 1$$

$$0\le\cos^2 x\le 1$$

$$\lim_{x\to\infty}\frac{x^{2/3}-\frac{1}{x}}{x^{2/3}+\cos^2 x}=\lim_{x\to\infty}\frac{x^{2/3}}{x^{2/3}}$$

$$=1$$

Exercise

Find the limit $\lim_{x\to\infty}\frac{\sin 2x}{x}$

Solution

$$-1\le\sin 2x\le 1$$

$$-\lim_{x\to\infty}\frac{1}{x}\le\lim_{x\to\infty}\frac{\sin 2x}{x}\le\lim_{x\to\infty}\frac{1}{x}$$

$$0\le\lim_{x\to\infty}\frac{\sin 2x}{x}\le 0$$

$$\lim_{x\to\infty}\frac{\sin 2x}{x}=0$$

Exercise

Find the limit $\lim\limits_{x\to 0} \dfrac{\sin 5x}{3x}$

Solution

$$\lim_{x\to 0} \frac{\sin 5x}{3x} = \lim_{5x\to 0} \frac{5}{3}.\frac{\sin 5x}{5x}$$

$$= \underline{\left| \frac{5}{3} \right.}$$

Exercise

Find the limit $\lim\limits_{x\to -\infty} \dfrac{\cos x}{2x}$

Solution

$$-1 \le \cos x \le 1$$

$$-\lim_{x\to\infty} \frac{1}{2x} \le \lim_{x\to\infty} \frac{\cos x}{2x} \le \lim_{x\to\infty} \frac{1}{2x}$$

$$0 \le \lim_{x\to\infty} \frac{\cos x}{2x} \le 0$$

$$\underline{\left| \lim_{x\to\infty} \frac{\cos x}{2x} = 0 \right.}$$

Exercise

Find the limit $\lim\limits_{x\to -\infty} \left(\dfrac{x^2+x-1}{8x^2-3} \right)^{1/3}$

Solution

$$\lim_{x\to -\infty} \left(\frac{x^2+x-1}{8x^2-3} \right)^{1/3} = \lim_{x\to -\infty} \left(\frac{x^2}{8x^2} \right)^{1/3}$$

$$= \left(\frac{1}{2^3} \right)^{1/3}$$

$$\underline{\left| = \frac{1}{2} \right.}$$

Exercise

Find the limit $\lim\limits_{x\to -1} \dfrac{\sqrt{x^2+8}-3}{x+1}$

Solution

$$\lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1} = \frac{3-3}{-1+1} = \frac{0}{0}$$

$$= \lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1} \cdot \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}$$

$$= \lim_{x\to -1} \frac{x^2+8-9}{(x+1)\left(\sqrt{x^2+8}+3\right)}$$

$$= \lim_{x\to -1} \frac{x^2-1}{(x+1)\left(\sqrt{x^2+8}+3\right)}$$

$$= \lim_{x\to -1} \frac{(x-1)(x+1)}{(x+1)\left(\sqrt{x^2+8}+3\right)}$$

$$= \lim_{x\to -1} \frac{x+1}{\sqrt{x^2+8}+3}$$

$$= \frac{0}{6}$$

$$= 0$$

Exercise

Find the limit $\lim\limits_{x\to -\infty} \left(\dfrac{1-x^3}{x^2+7x}\right)^5$

Solution

$$\lim_{x\to -\infty} \left(\frac{1-x^3}{x^2+7x}\right)^5 = \lim_{x\to -\infty} \left(\frac{-x^3}{x^2}\right)^5$$

$$= \lim_{x\to -\infty} \left(-x^5\right)$$

$$= \infty$$

Exercise

Find the limit $\lim_{x\to\infty} \sqrt{\dfrac{x^2-5x}{x^3+x-2}}$

Solution

$$\lim_{x\to\infty} \sqrt{\frac{x^2-5x}{x^3+x-2}} = \lim_{x\to\infty} \sqrt{\frac{x^2}{x^3}}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{x}}$$

$$= 0$$

Exercise

Find the limit $\lim_{x\to\infty} \dfrac{2\sqrt{x}+x^{-1}}{3x-7}$

Solution

$$\lim_{x\to\infty} \frac{2\sqrt{x}+x^{-1}}{3x-7} = \lim_{x\to\infty} \frac{2\sqrt{x}}{3x}$$

$$= \lim_{x\to\infty} \frac{2}{3\sqrt{x}}$$

$$= 0$$

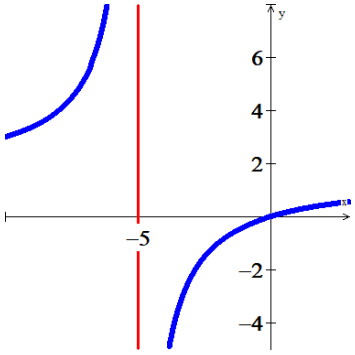
Exercise

Find the limit $\lim_{x\to -5^-} \dfrac{3x}{2x+10}$

Solution

$$\lim_{x\to -5^-} \frac{3x}{2x+10} = \frac{-15}{0^-}$$

$$= \infty$$



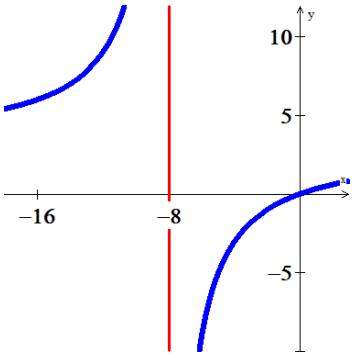
Exercise

Find the limit $\lim_{x\to -8^+} \dfrac{3x}{x+8}$

Solution

$$\lim_{x\to -8^+} \frac{3x}{x+8} = \frac{-24}{0^+}$$

$$= -\infty$$



Exercise

Find the limit $\lim_{x\to 0} \dfrac{-1}{x^2(x+1)}$

Solution

$$\lim_{x\to 0} \frac{-1}{x^2(x+1)} = -\frac{1}{0}$$

$$= -\infty$$

Exercise

Find the limit $\lim_{x\to 7} \dfrac{4}{(x-7)^2}$

Solution

$$\lim_{x\to 7} \frac{4}{(x-7)^2} = \frac{4}{0}$$

$$= \infty$$

Exercise

Find the limit $\lim_{x\to 0} \dfrac{1}{x^{2/3}}$

Solution

$$\lim_{x\to 0} \frac{1}{x^{2/3}} = \infty$$

Exercise

Find the limit $\lim_{x\to -\infty} \left(x+\sqrt{x^2-4x+2}\right)$

Solution

$$\lim_{x\to -\infty} \left(x+\sqrt{x^2-4x+2}\right) = -\infty + \infty$$

$$= \lim_{x\to -\infty} \left(x+\sqrt{x^2-4x+2}\right)\cdot \frac{x-\sqrt{x^2-4x+2}}{x-\sqrt{x^2-4x+2}}$$

$$= \lim_{x\to -\infty} \frac{x^2-x^2+4x-2}{x-\sqrt{x^2-4x+2}}\ .$$

$$= \lim_{x \to -\infty} \frac{4x - 2}{x - \sqrt{x^2 - 4x + 2}}$$

$$= \lim_{x \to -\infty} \frac{4x - 2}{x - |x|} \qquad x \to -\infty \quad (x < 0) \to \quad |x| = -x$$

$$= \lim_{x \to -\infty} \frac{4x}{x + x}$$

$$= \lim_{x \to -\infty} \frac{4x}{2x}$$

$= 2$