1.1 Limits

$g = 32.2 \text{ ft/sec}^2 = 9.8 \text{ m/sec}^2$

Position / displacement $s(t) = -\frac{1}{2} g t^2 + v_0 t + s_0$

$= -16t^2 + v_0 t + s_0$ ← initial displacement

↑ initial velocity

Average Rate $= \frac{\Delta s}{\Delta t}$ $\quad \Delta s = s_2 - s_1$

Ex $s(t) = 16t^2$

a) average speed during 1st 2 sec $\quad t = 0 \to 2$

average speed $= \frac{\Delta s}{\Delta t}$

$= \frac{s(2) - s(0)}{2 - 0}$

$= \frac{64 - 0}{2}$

$= 32 \text{ ft/sec}$

b) $t = 1 \to 2$ sec

Average speed $= \frac{s(2) - s(1)}{2 - 1}$

$= \frac{64 - 16}{1}$

$= 48 \text{ ft/sec}$

Ex $y(t) = 16t^2 \quad [t_0, t_0+h]$

soln

$$\frac{\Delta y}{\Delta t} = \frac{16(t_0+h)^2 - 16t_0^2}{t_0+h-t_0}$$

$$= \frac{16}{h}\left(t_0^2 + 2ht_0 + h^2 - t_0^2\right)$$

$$= \frac{16}{h}\left(h^2 + 2ht_0\right)$$

$$= 16\left(h + 2t_0\right)$$

$$= 32t_0 + 16h$$

$t_0 = 1 \Rightarrow \quad \frac{\Delta y}{\Delta t} = 32 + 16h \Big|_{h=0}$

$= 32$ ft/sec

$\lim_{h \to 0} \frac{\Delta y}{\Delta t} = 32$

Limits

$$\lim_{x \to x_0} f(x) = L$$

limit of $f(x)$ as x approaches x_0 is L

$x \to a^-$ left less than a

$x \to a^+$ Right greater than a

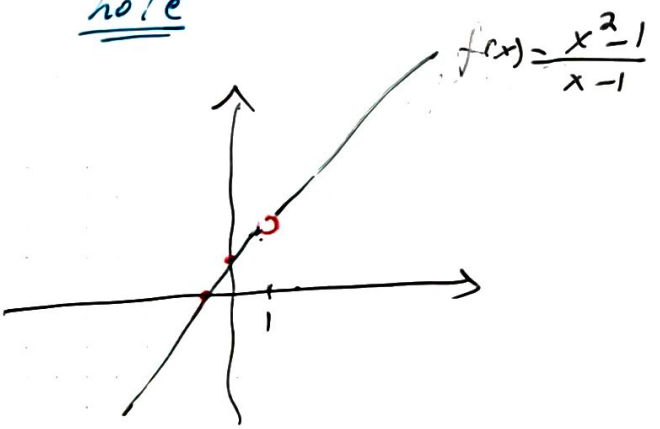
Ex $f(x) = \dfrac{x^2-1}{x-1}$ near $x=1$ behavior.

$f(x=1) = \dfrac{0}{0}$

$f(x) = \dfrac{(x-1)(x+1)}{x-1}$ hole

$= x+1$

$f(1) = 2.$

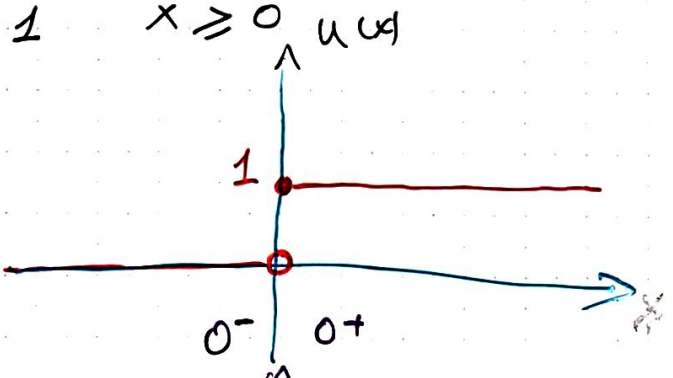


1^-

1^+

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2.$$

$$u(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$



Ex $\lim_{x \to c} (x^3 + 4x^2 - 3) = c^3 + 4c^2 - 3$

Ex $\lim_{x \to -2} \sqrt{4x^2 - 3} = \sqrt{16 - 3}$

$= \sqrt{13}$

$\lim_{x \to a} (\text{constant}) = \text{constant}.$

Ex $\lim_{x \to -1} \frac{x^3 + 4x^2 - 3}{x^2 + 5} = \frac{-1 + 4 - 3}{6}$

$= \frac{0}{6}$

$= 0$

Ex $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x} = \frac{1 + 1 - 2}{1 - 1} = \frac{0}{0}$

$= \lim_{x \to 1} \frac{(x-1)(x+2)}{(x-1)x}$

$= \lim_{x \to 1} \frac{x+2}{x}$

$= \frac{3}{1}$

$= 3$

Ex $\lim_{x\to 0} \dfrac{\sqrt{x^2+100}-10}{x^2} = \dfrac{10-10}{0} = \dfrac{0}{0}$

$$= \lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2} \cdot \frac{\sqrt{x^2+100}+10}{\sqrt{x^2+100}+10}$$

$$= \lim_{x\to 0} \frac{x^2+100-100}{x^2\left(\sqrt{x^2+100}+10\right)}$$

$$= \lim_{x\to 0} \frac{x^2}{x^2\left(\sqrt{x^2+100}+10\right)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+100}+10}$$

$$= \frac{1}{20}$$

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