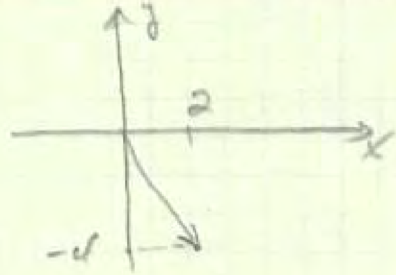
3) $\vec{u} = (2, -4)$



7/ $\vec{u} = (1, 3) \quad \vec{v}\,(2, -2)$

$\vec{u} + \vec{v} = (1, 3) + (2, -2)$
$= (1+2,\ 3-2)$
$= (3, 1)$

9/ $\vec{u} = (2, -3) \quad \vec{v} = (-3, -1)$

$\vec{u} + \vec{v} = (2, -3) + (-3, -1)$
$= (-1, -4)$

#11/ $\vec{u} = (-2, 3) \quad \vec{w} = (-3, -2)$

$\vec{v} = \frac{3}{2}\vec{u}$
$= \frac{3}{2}(-2, 3)$
$= (-3, \frac{9}{2})$

13/ $\vec{u} = (-2, 3) \quad \vec{w} = (-3, -2)$

$\vec{v} = \vec{u} + 2\vec{w}$
$= (-2, 3) + 2(-3, -2)$
$= (-2, 3) + (-6, -4)$
$= (-8, -1)$

15/ $\vec{u} = (-2, 3) \quad \vec{w} = (-3, -2)$

$\vec{v} = \frac{1}{2}(3\vec{u} + \vec{w})$
$= \frac{1}{2}(3(-2, 3) + (-3, -2))$
$= \frac{1}{2}(-9, 7)$
$= (-\frac{9}{2}, \frac{7}{2})$

19/ $\vec{u} = (1,2,3)$ $\vec{v} = (2,2,-1)$

$\vec{u} - \vec{v} = (1,2,3) - (2,2,-1)$
$= (-1, 0, 4)$

$\vec{v} - \vec{u} = (2,2,-1) - (1,2,3)$
$= (1, 0, -4)$

21/ $\vec{u} = (1,2,3)$ $\vec{v} = (2,2,-1)$ $\vec{w} = (4,0,-4)$

$2\vec{u} + 4\vec{v} - \vec{w} = 2(1,2,3) + 4(2,2,-1) - (4,0,-4)$
$= (6, 12, 6)$

23/ $\vec{u} = (1,2,3)$ $\vec{w} = (4,0,-4)$

$\vec{z}$?

$3\vec{u} - 4\vec{z} = \vec{w}$

$4\vec{z} = 3\vec{u} - \vec{w}$

$\vec{z} = \frac{1}{4}\left(3(1,2,3) - (4,0,-4)\right)$

$= \frac{1}{4}(-1, 6, 13)$

$= \left(-\frac{1}{4}, \frac{3}{2}, \frac{13}{4}\right)$

#29 $\vec{u} = (4,0,-3,5)$ $\vec{v} = (0,2,5,4)$

a) $\vec{u} - \vec{v} = (4,0,-3,5) - (0,2,5,4)$
$= (4, -2, -8, 1)$

b) $2(\vec{u} + 3\vec{v}) = 2\left[(4,0,-3,5) + 3(0,2,5,4)\right]$
$= 2(4, 6, 12, 17)$
$= (8, 12, 24, 34)$

c) $2\vec{v} - \vec{u} = 2(0,2,5,4) - (4,0,-3,5)$
$= (-4, 4, 13, 3)$

33) $\vec{u} = (1, 2, -3, 1)$ $\vec{v} = (0, 2, -1, -2)$
$\vec{w} = (2, -2, 1, 3)$

a) $\vec{u} + 2\vec{v} = (1, 2, -3, 1) + 2(0, 2, -1, -2)$
$= (1, 6, -5, -3)$

b) $\vec{w} - 3\vec{u} = (2, -2, 1, 3) - 3(1, 2, -3, 1)$
$= (-1, -8, 10, 0)$

c) $4\vec{v} + \frac{1}{2}\vec{u} - \vec{w} = 4(0, 2, -1, -2) + \frac{1}{2}(1, 2, -3, 1) - (2, -2, 1, 3)$
$= \left(-\frac{3}{2}, 11, -\frac{13}{2}, -\frac{21}{2}\right)$

35) $\vec{u} = (1, -1, 0, 1)$ $\vec{v} = (0, 2, 3, -1)$ $\vec{w}$?
$3\vec{w} = \vec{u} - 2\vec{v}$
$\vec{w} = \frac{1}{3}\left[(1, -1, 0, 1) - 2(0, 2, 3, -1)\right]$
$= \frac{1}{3}(1, -5, -6, 3)$
$= \left(\frac{1}{3}, \frac{-5}{3}, -2, 1\right)$

41) $\vec{u} = (1, 2)$ $\vec{w} = (1, -1)$
$\vec{v} = (2, 1)$
$\vec{v} = a\vec{u} + b\vec{w}$
$(2, 1) = (a, 2a) + (b, -b)$
$= (a + b, 2a - b)$

$\begin{cases} a + b = 2 \rightarrow b = 1 \\ 2a - b = 1 \end{cases}$
$3a = 3 \Rightarrow a = 1$

$\vec{v} = \vec{u} + \vec{v}$

47/ $\vec{v} = (10, 1, 4)$ $\vec{u}_1 = (2, 3, 5)$ $u_2 = (1, 2, 4)$

$\vec{u}_3 = (-2, 2, 3)$

$\vec{v} = x_1 \vec{u}_1 + x_2 \vec{u}_2 + x_3 \vec{u}_3$

$(10, 1, 4) = (2x_1 + x_2 - 2x_3, 3x_1 + 2x_2 + 2x_3, 5x_1 + 4x_2 + 3x_3)$

$$\begin{cases} 2x_1 + x_2 - 2x_3 = 10 \\ 3x_1 + 2x_2 + 2x_3 = 1 \\ 5x_1 + 4x_2 + 3x_3 = 4 \end{cases}$$

$$\Delta = \begin{vmatrix} 2 & 1 & -2 \\ 3 & 2 & 2 \\ 5 & 4 & 3 \end{vmatrix} = -7 \qquad \Delta_1 = \begin{vmatrix} 10 & 1 & -2 \\ 1 & 2 & 2 \\ 4 & 4 & 3 \end{vmatrix} = -7$$

$$\Delta_2 = \begin{vmatrix} 2 & 10 & -2 \\ 3 & 1 & 2 \\ 5 & 4 & 3 \end{vmatrix} = -14 \qquad \Delta_3 = \begin{vmatrix} 2 & 1 & 10 \\ 3 & 2 & 1 \\ 5 & 4 & 4 \end{vmatrix} = 21$$

$x_1 = \frac{-7}{-7} = 1,$ $x_2 = \frac{-14}{-7} = 2$ $x_3 = \frac{21}{-7} = -3$

$\Rightarrow \vec{v} = \vec{u}_1 + 2\vec{u}_2 - 3\vec{u}_3$

51/ $$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{array}\right] \begin{array}{l} \\ R_2 - 7R_1 \\ R_3 - 4R_1 \end{array} \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -6 & -12 \\ 0 & -3 & -6 \end{array}\right] 2R_3 - R_2$$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -6 & -12 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} \rightarrow a + 2b = 3 \quad (1) \\ \rightarrow -6b = -12 \Rightarrow b = 2 \\ \\ \end{array}$$

$(1) \rightarrow a = -1$

$$\begin{pmatrix} 3 \\ 9 \\ 6 \end{pmatrix} = (-1) \begin{pmatrix} 1 \\ 7 \\ 4 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ 8 \\ 5 \end{pmatrix}$$

1/ R^4: $(0, 0, 0, 0)$

3/ $M_{4,3}$: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5/ P_3: $0 + 0x + 0x^2 + 0x^3$

7/ R^3: (v_1, v_2, v_3)

$-(v_1, v_2, v_3) = (-v_1, -v_2, -v_3)$

15/ $p_1(x) = x^3 + x^2 - x$

1) $p_2(x) = -x^3 - x^2 + 2x$

$(p_1 + p_2)(x) = x^3 + x^2 - x - x^3 - x^2 + 2x$

$= x$ is not 3^{rd} deg polyn.

$\therefore$ This set is not a vector space

#21 $\{(x, y) : x \geq 0, y \in \mathbb{R}\}$

Axiom 6: $cu \in V$.

Assume $c = -1 \Rightarrow -1(2, y) = (-2, y)$

$\Rightarrow -2 \ngeq 0$

Axiom 6 failed

$\therefore$ This set is not a vector space

25 set $\begin{bmatrix} a & b \\ c & 0 \end{bmatrix} \Rightarrow M_{2\times2}$

let $M_1 = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} d & e \\ f & 0 \end{pmatrix}$ $M_3 = \begin{pmatrix} g & h \\ i & 0 \end{pmatrix}$

Axiom 1: $M_1 + M_2 = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + \begin{pmatrix} d & e \\ f & 0 \end{pmatrix}$

$= \begin{pmatrix} a+d & b+e \\ c+f & 0 \end{pmatrix} \in M_{2\times2}$

Axiom 2: $M_1 - M_2 = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} - \begin{pmatrix} d & e \\ f & 0 \end{pmatrix}$

$= \begin{pmatrix} a-d & b-e \\ c-f & 0 \end{pmatrix} \in M_{2\times2}$

Axiom 3: $(M_1 + M_2) + M_3 = \left[\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + \begin{pmatrix} d & e \\ f & 0 \end{pmatrix}\right] + \begin{pmatrix} g & h \\ i & 0 \end{pmatrix}$

$= \begin{pmatrix} a+d & b+e \\ c+f & 0 \end{pmatrix} + \begin{pmatrix} g & h \\ i & 0 \end{pmatrix}$

$= \begin{pmatrix} (a+d)+g & (b+e)+h \\ (c+f)+i & 0 \end{pmatrix}$

$= \begin{pmatrix} a+(d+g) & b+(e+h) \\ c+(f+i) & 0 \end{pmatrix}$

$= \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + \begin{pmatrix} d+g & e+h \\ f+i & 0 \end{pmatrix}$

$= M_1 + \left[\begin{pmatrix} d & e \\ f & 0 \end{pmatrix} + \begin{pmatrix} g & h \\ i & 0 \end{pmatrix}\right]$

$= M_1 + (M_2 + M_3)$

Axiom 4: $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$M_1 + 0 = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = M_1$

Axiom 5: $M_1 + (-M_1) = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + \begin{pmatrix} -a & -b \\ -c & 0 \end{pmatrix}$

$= \begin{pmatrix} a-a & b-b \\ c-c & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in M_{2\times2}$

25 cont

Ax. 6':
$$kM_1 = k\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = \begin{pmatrix} ak & bk \\ ck & 0 \end{pmatrix} \in M_{2\times 2} \stackrel{!}{=} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & 0 \end{pmatrix}$$

Axiom 7:
$$k(M_1+M_2) = k\left[\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + \begin{pmatrix} d & e \\ f & 0 \end{pmatrix}\right]$$
$$= k\begin{pmatrix} a+d & b+e \\ c+f & 0 \end{pmatrix}$$
$$= \begin{pmatrix} ak+dk & bk+ek \\ ck+fk & 0 \end{pmatrix}$$
$$= \begin{pmatrix} ak & bk \\ ck & 0 \end{pmatrix} + \begin{pmatrix} dk & ek \\ fk & 0 \end{pmatrix}$$
$$= k\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + k\begin{pmatrix} d & e \\ f & 0 \end{pmatrix}$$
$$= kM_1 + kM_2 \checkmark$$

Axiom 8:
$$(k_1+k_2)M_1 = (k_1+k_2)\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$$
$$= \begin{pmatrix} a(k_1+k_2) & b(k_1+k_2) \\ c(k_1+k_2) & 0 \end{pmatrix}$$
$$= \begin{pmatrix} ak_1+ak_2 & bk_1+bk_2 \\ ck_1+ck_2 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} ak_1 & bk_1 \\ ck_1 & 0 \end{pmatrix} + \begin{pmatrix} ak_2 & bk_2 \\ ck_2 & 0 \end{pmatrix}$$
$$= k_1\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + k_2\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$$
$$= k_1M_1 + k_2M_2 \checkmark$$

25 cont

Axiom 9: $(k_1 k_2)M_1 = (k_1 k_2)\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$

$$= \begin{pmatrix} ak_1k_2 & bk_1k_2 \\ ck_1k_2 & 0 \end{pmatrix}$$

$$= k_1\begin{pmatrix} ak_2 & bk_2 \\ ck_2 & 0 \end{pmatrix}$$

$$= k_1\left(k_2\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}\right)$$

$$= k_1(k_2 M_1) \checkmark$$

Axiom 10: $1M_1 = 1\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$

$$= \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$$

$$= M_1 \checkmark$$

$\therefore$ set is a vector space

39/ $V = \{(x, 2x) : x \in \mathbb{R}\}$

Axiom 1: $u = (x_1, 2x_1)$ $v = (x_2, 2x_2)$ $w(x_3, 2x_3)$

$$u + v = (x_1, 2x_1) + (x_2, 2x_2)$$
$$= (x_1 + x_2, 2(x_1 + x_2)) \quad \text{let } x = x_1 + x_2$$
$$= (x, 2x) \in V.$$

Axiom 2: $u + v = (x_1, 2x_1) + (x_2, 2x_2)$

$$= (x_1 + x_2, 2x_1 + 2x_2)$$
$$= (x_2 + x_1, 2x_2 + 2x_1)$$
$$= (x_2, 2x_2) + (x_1, 2x_1)$$
$$= v + u \checkmark$$

Axiom 4: $u + 0 = (x_1, 2x_1) + (0, 0)$

$$= (x_1 + 0, 2x_1 + 0)$$
$$= (x_1, 2x_1)$$
$$= u \checkmark$$

Axiom 3: $(u + v) + w = ((x_1, 2x_1) + (x_2, 2x_2)) + (x_3, 2x_3)$

$$= (x_1 + x_2, 2x_1 + 2x_2) + (x_3, 2x_3)$$
$$= ((x_1 + x_2) + x_3, (2x_1 + 2x_2) + 2x_3)$$
$$= (x_1 + (x_2 + x_3), 2x_1 + (2x_2 + 2x_3))$$
$$= (x_1, 2x_1) + (x_2 + x_3, 2x_2 + 2x_3)$$
$$= u + ((x_2, 2x_2) + (x_3, 2x_3))$$
$$= u + (v + w) \checkmark.$$

Axiom 5: $u + (-u) = (x_1, 2x_1) + (-(x_1, 2x_1))$

$$= (x_1, 2x_1) + (-x_1, -2x_1)$$
$$= (x_1 - x_1, 2x_1 - 2x_1)$$
$$= (0, 0)$$
$$= 0 \checkmark$$

<u>39 cont</u>

Axiom 6: $cu = c(x_1, 2x_1)$
$= (cx_1, 2cx_1)$ let $x = cx_1$
$= (x_1, 2x_1) \in V$.

Axiom 7: $c(u+v) = c\big((x_1, 2x_1) + (x_2, 2x_2)\big)$
$= c(x_1 + x_2, 2x_1 + 2x_2)$
$= \big(c(x_1 + x_2), c(2x_1 + 2x_2)\big)$
$= (cx_1 + cx_2, 2cx_1 + 2cx_2)$
$= (cx_1, 2cx_1) + (cx_2, 2cx_2)$
$= c(x_1, 2x_1) + c(x_2, 2x_2)$
$= cu + cv$ ✓

Axiom 8: $(c+d)u = (c+d)(x_1, 2x_1)$
$= \big((c+d)x_1, 2(c+d)x_1\big)$
$= (cx_1 + dx_1, 2cx_1 + 2dx_1)$
$= (cx_1, 2cx_1) + (dx_1, 2dx_1)$
$= c(x_1, 2x_1) + d(x_1, 2x_1)$
$= cu + du$ ✓

Axiom 9: $(cd)u = (cd)(x_1, 2x_1)$
$= \big((cd)x_1, (cd)(2x_1)\big)$
$= \big(c(dx_1), c(2dx_1)\big)$
$= c(dx_1, 2dx_1)$
$= c[d(x_1, 2x_1)]$
$= c(du)$ ✓

Axiom 10: $1u = 1(x_1, 2x_1)$
$= (1x_1, 1(2x_1))$
$= (x_1, 2x_1)$
$= u$ ✓

$\therefore$ The set is a vector space