Solution   *Section* 1.4 – Quadratic Functions

Exercise

For the function $f(x) = x^2 + 6x + 3$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{6}{2(1)} = -3$

$y = f(-3) = (-3)^2 + 6(-3) + 3 = -6$ ***Vertex point*** $(-3, -6)$

b) Line of *symmetry*: $x = -3$

c) *Minimum* point, value $(-3, -6)$

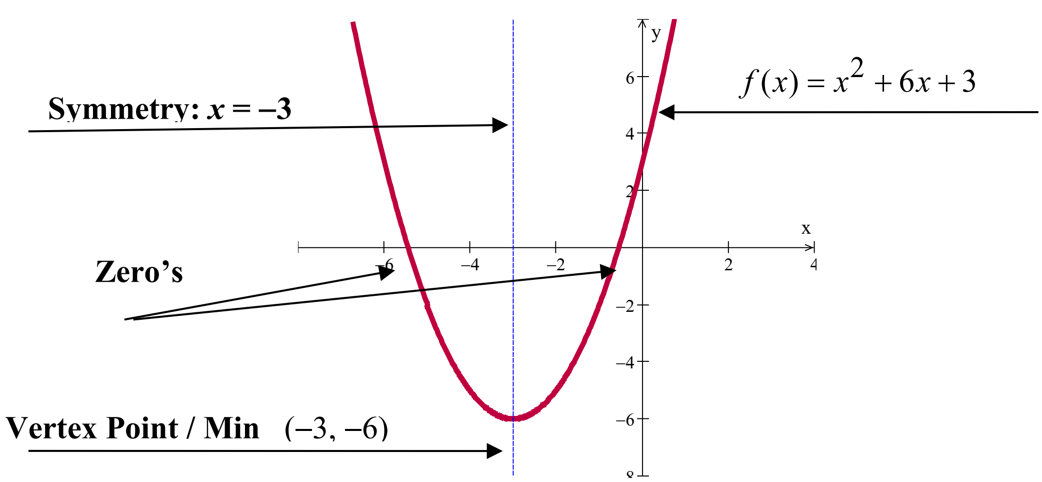
d) $x = \frac{-6 \pm \sqrt{36-12}}{2} = \frac{-6 \pm \sqrt{24}}{2} = \frac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$

$$x = \begin{cases} -3 + \sqrt{6} = -0.5 \\ -3 - \sqrt{6} = -5.45 \end{cases}$$

e) y-intercept $y = 3$

f) *Range*: $[-6, \infty)$ *Domain*: $(-\infty, \infty)$

g)



h) *Decreasing*: $(-\infty, -3)$ *Increasing*: $(-3, \infty)$

Exercise

For the function $f(x) = x^2 + 6x + 5$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{6}{2}$ $\qquad\qquad x = -\frac{b}{2a}$

$= -3$

$y = f(-3) = (-3)^2 + 6(-3) + 5$

$= -4$

Vertex point: $(-3, -4)$

b) Axis of *symmetry*: $x = -3$

c) *Minimum* point @ $(-3, -4)$

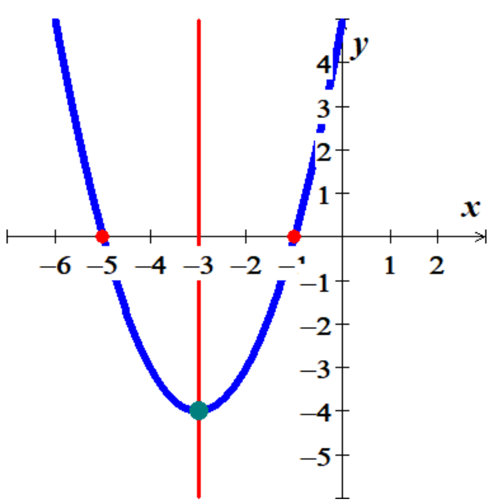
d) $x^2 + 6x + 5 = 0$

$x = -5, \; -1$

e) $x = 0 \;\; \rightarrow \;\; y = 5$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $[-4, \infty)$

g)



h) *Increasing:* $(-3, \infty)$ $\qquad$ Decreasing: $(-\infty, -3)$

Exercise

For the function $f(x) = -x^2 - 6x - 5$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{-6}{-2}$ $\qquad x = -\frac{b}{2a}$

$\quad = -3$

$y = f(-3) = -9 + 18 - 5$

$\quad = 4$

Vertex point: $(-3,\ 4)$

b) Axis of *symmetry*: $x = -3$

c) *Maximum* point @ $(-3,\ 4)$

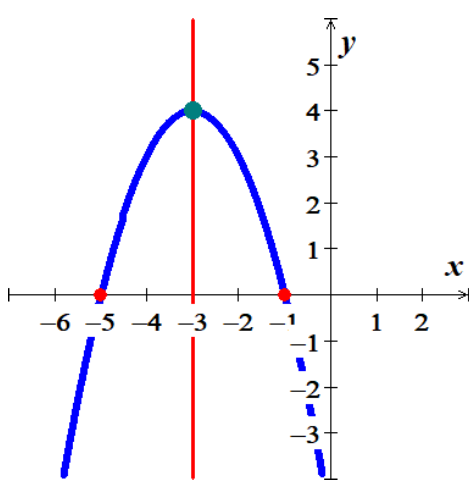
d) $-\left(x^2 + 6x + 5\right) = 0$

$x = -5,\ -1$

e) $x = 0 \quad \rightarrow \quad y = -5$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $(-\infty,\ 4]$

g)



h) *Increasing*: $(-\infty,\ -3)$ $\qquad$ *Decreasing*: $(-3,\ \infty)$

Exercise

For the function $f(x) = x^2 - 4x + 2$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the *y*-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{-4}{2}$ $\qquad x = -\frac{b}{2a}$

$\underline{= 2|}$

$f(2) = 4 - 8 + 2$

$\underline{= -2|}$

Vertex point: $\underline{(2,\ -2)|}$

b) Axis of *symmetry*: $\underline{x = 2|}$

c) *Minimum* point @ $\underline{(2,\ -2)|}$

d) $x^2 - 4x + 2 = 0$

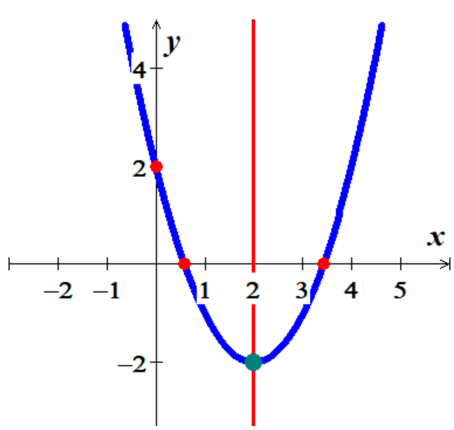
$x = \frac{4 \pm \sqrt{8}}{2}$

$\underline{x = 2 \pm \sqrt{2}|}$

e) $x = 0 \quad \rightarrow \quad \underline{y = 2|}$

f) *Domain*: $\underline{\mathbb{R}|}$ *Range*: $\underline{[-2,\ \infty)|}$

g)



h) *Increasing:* $\underline{(2,\ \infty)|}$ *Decreasing*: $\underline{(-\infty,\ 2)|}$

Exercise

For the function $f(x) = -2x^2 + 16x - 26$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{16}{-4}$ $\qquad x = -\frac{b}{2a}$

$= 4$

$f(4) = -32 + 64 - 26$

$= 6$

Vertex point: $(4,\ 6)$

b) Axis of *symmetry*: $x = 4$

c) *Maximum* point @ $(4,\ 6)$

d) $-2x^2 + 16x - 26 = 0$

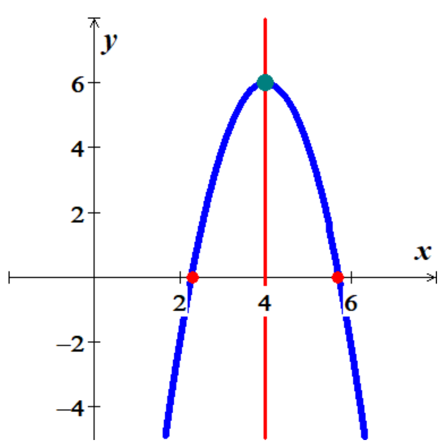
$x = \frac{-16 \pm \sqrt{128}}{-4}$

$x = 4 \pm 2\sqrt{2}$

e) $x = 0 \quad \rightarrow \quad y = -26$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $(-\infty,\ 6]$

g)



h) *Increasing:* $(-\infty,\ 4)$ $\qquad$ *Decreasing*: $(4,\ \infty)$

Exercise

For the function $f(x)=x^2+4x+1$

a) Find the vertex point

b) Find the line of symmetry

c) State whether there is a *maximum* or *minimum* value *and* find that value

d) Find the zeros of $f(x)$

e) Find the y-intercept

f) Find the *range* and the *domain* of the function.

g) Graph the function

h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x=-\frac{4}{2}$ $\qquad x=-\frac{b}{2a}$

$\quad =-2$

$f(-2)=4-8+1$

$\quad =-3$

Vertex point: $(-2,\ -3)$

b) Axis of *symmetry*: $x=-2$

c) *Minimum* point @ $(-2,\ -3)$

d) $x^2+4x+1=0$

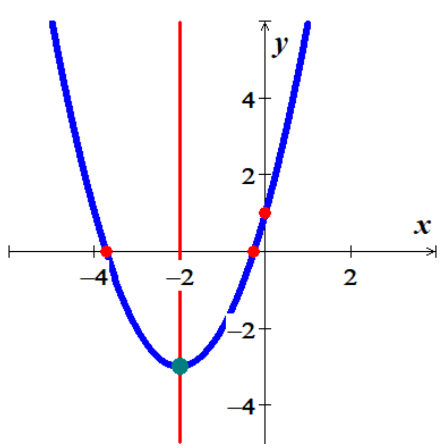
$x=\frac{-4\pm\sqrt{12}}{2}$

$x=-2\pm\sqrt{3}$

e) $x=0 \quad\to\quad y=1$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $[-3,\ \infty)$

g)



h) *Increasing:* $(-2,\ \infty)$ $\qquad$ *Decreasing*: $(-\infty,\ -2)$

Exercise

For the function $f(x) = x^2 - 8x + 5$

a) Find the vertex point

b) Find the line of symmetry

c) State whether there is a *maximum* or *minimum* value *and* find that value

d) Find the zeros of $f(x)$

e) Find the *y*-intercept

f) Find the *range* and the *domain* of the function.

g) Graph the function

h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{-8}{2}$ $\qquad x = -\frac{b}{2a}$

$= 4$

$f(4) = 16 - 32 + 5$

$= -11$

Vertex point: $(4, \ -11)$

b) Axis of *symmetry*: $x = 4$

c) *Minimum* point @ $(4, \ -11)$

d) $x^2 - 8x + 5 = 0$

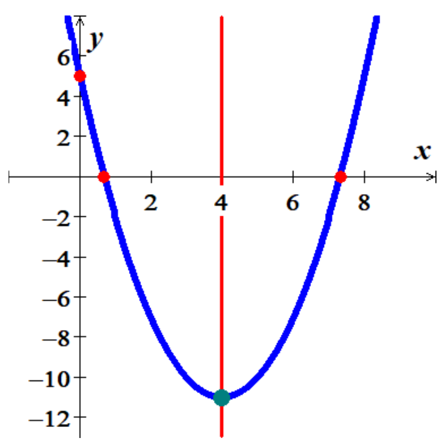
$x = \frac{8 \pm \sqrt{44}}{2}$

$x = 4 \pm \sqrt{11}$

e) $x = 0 \ \rightarrow \ y = 5$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $[-11, \ \infty)$

g)



h) *Increasing:* $(4, \ \infty)$ $\qquad$ *Decreasing*: $(-\infty, \ 4)$

Exercise

For the function $f(x) = x^2 + 6x - 1$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{6}{2}$ $\qquad\qquad$ $x = -\frac{b}{2a}$

$\quad = -3$

$f(-3) = 9 - 18 - 1$

$\qquad = -10$

Vertex point: $(-3, \ -10)$

b) Axis of *symmetry*: $x = -3$

c) *Minimum* point @ $(-3, \ -10)$

d) $x^2 + 6x - 1 = 0$

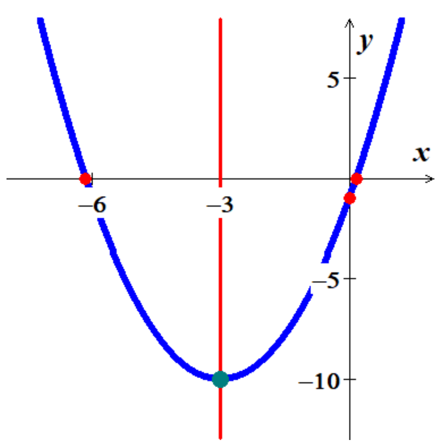
$x = \dfrac{-6 \pm \sqrt{40}}{2}$

$x = -3 \pm \sqrt{10}$

e) $x = 0 \ \rightarrow \ y = -1$

f) *Domain*: $\mathbb{R}$ $\qquad\qquad$ *Range*: $[-10, \ \infty)$

g)



h) *Increasing:* $(-3, \ \infty)$ $\qquad\qquad$ *Decreasing*: $(-\infty, \ -3)$

Exercise

For the function $f(x) = x^2 + 6x + 3$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the *y*-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{6}{2}$ $\qquad x = -\frac{b}{2a}$

$\underline{= -3}$

$f(-3) = 9 - 18 + 3$

$\underline{= -6}$

Vertex point: $\underline{(-3,\ -6)}$

b) Axis of *symmetry*: $\underline{x = -3}$

c) *Minimum* point @ $\underline{(-3,\ -6)}$

d) $x^2 + 6x + 3 = 0$

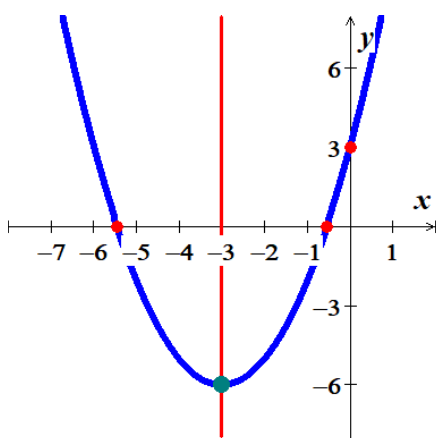
$x = \frac{-6 \pm \sqrt{24}}{2}$

$\underline{x = -3 \pm \sqrt{6}}$

e) $x = 0 \quad \rightarrow \quad \underline{y = 3}$

f) *Domain*: $\underline{\mathbb{R}}$ $\qquad$ *Range*: $\underline{[-6,\ \infty)}$

g)



h) *Increasing:* $\underline{(-3,\ \infty)}$ $\qquad$ *Decreasing*: $\underline{(-\infty,\ -3)}$

Exercise

For the function $f(x) = x^2 - 10x + 3$

a) Find the vertex point

b) Find the line of symmetry

c) State whether there is a *maximum* or *minimum* value *and* find that value

d) Find the zeros of $f(x)$

e) Find the y-intercept

f) Find the *range* and the *domain* of the function.

g) Graph the function

h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{-10}{2}$ $\qquad$ $x = -\frac{b}{2a}$

$\underline{= 5}|$

$f(5) = 25 - 50 + 3$

$\underline{= -22}|$

Vertex point: $\underline{(5, \ -22)}|$

b) Axis of *symmetry*: $\underline{x = 5}|$

c) *Minimum* point @ $\underline{(5, \ -22)}|$

d) $x^2 - 10x + 3 = 0$

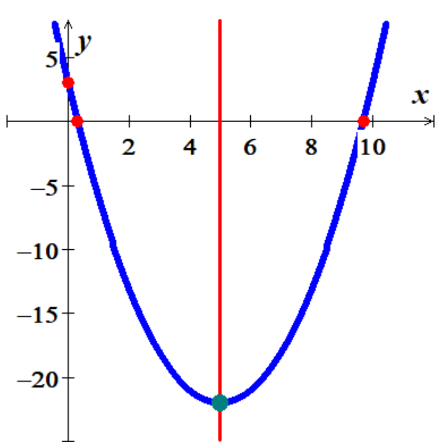
$x = \frac{10 \pm \sqrt{88}}{2}$

$\underline{x = 5 \pm \sqrt{22}}|$

e) $x = 0 \quad \rightarrow \quad \underline{y = 3}|$

f) *Domain*: $\underline{\mathbb{R}}|$ $\qquad$ *Range*: $\underline{[-22, \ \infty)}|$

g)



h) *Increasing:* $\underline{(5, \ \infty)}|$ $\qquad$ *Decreasing*: $\underline{(-\infty, \ 5)}|$

Exercise

For the function $f(x) = x^2 - 3x + 4$

a) Find the vertex point

b) Find the line of symmetry

c) State whether there is a *maximum* or *minimum* value *and* find that value

d) Find the zeros of $f(x)$

e) Find the y-intercept

f) Find the *range* and the *domain* of the function.

g) Graph the function

h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $\boxed{x = \frac{3}{2}}$ $\qquad$ $x = -\frac{b}{2a}$

$$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} + 4$$

$$= \boxed{\frac{7}{4}}$$

Vertex point: $\boxed{\left(\frac{3}{2}, \frac{7}{4}\right)}$

b) Axis of *symmetry*: $\boxed{x = \frac{3}{2}}$

c) *Minimum* point @ $\boxed{\left(\frac{3}{2}, \frac{7}{4}\right)}$

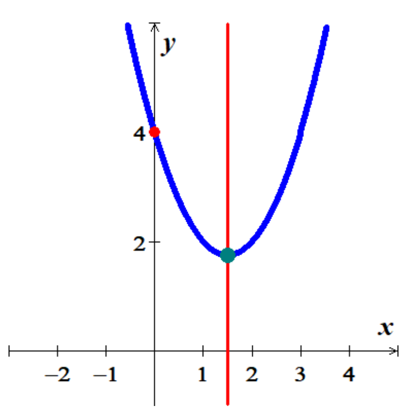
d) $x^2 - 3x + 4 = 0$

$$x = \frac{3 \pm \sqrt{-7}}{2} \qquad \mathbb{C}$$

e) $x = 0 \rightarrow \boxed{y = 4}$

f) *Domain*: $\boxed{\mathbb{R}}$ $\qquad$ *Range*: $\boxed{\left[\frac{7}{4}, \infty\right)}$

g)



h) *Increasing:* $\boxed{\left(\frac{3}{2}, \infty\right)}$ $\qquad$ *Decreasing*: $\boxed{\left(-\infty, \frac{3}{2}\right)}$

Exercise

For the function $f(x) = x^2 - 3x - 4$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the *y*-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $\boxed{x = \frac{3}{2}}$ $\qquad x = -\frac{b}{2a}$

$$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} - 4$$
$$= \boxed{-\frac{25}{4}}$$

Vertex point: $\boxed{\left(\frac{3}{2},\ -\frac{25}{4}\right)}$

b) Axis of *symmetry*: $\boxed{x = \frac{3}{2}}$

c) *Minimum* point @ $\boxed{\left(\frac{3}{2},\ -\frac{25}{4}\right)}$

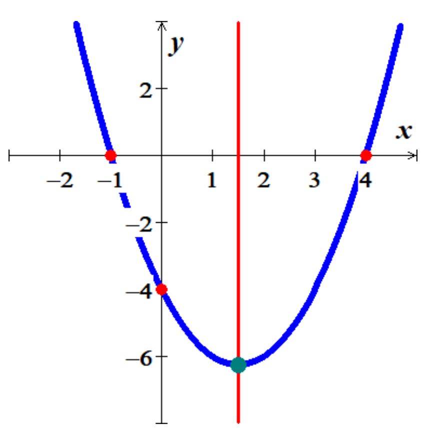
d) $x^2 - 3x - 4 = 0$

$\boxed{x = -1,\ 4}$

e) $x = 0 \quad \rightarrow \quad \boxed{y = -4}$

f) *Domain*: $\boxed{\mathbb{R}}$ $\qquad$ *Range*: $\boxed{\left[-\frac{25}{4},\ \infty\right)}$

g)



h) *Increasing:* $\boxed{\left(\frac{3}{2},\ \infty\right)}$ $\qquad$ *Decreasing*: $\boxed{\left(-\infty,\ \frac{3}{2}\right)}$

Exercise

For the function $f(x) = x^2 - 4x - 5$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $\underline{x = 2}$ $\qquad x = -\frac{b}{2a}$

$f(2) = 4 - 8 - 5$

$\quad = -9$

Vertex point: $\boxed{(2, \ -9)}$

b) Axis of *symmetry*: $\boxed{x = 2}$

c) *Minimum* point @ $\boxed{(2, \ -9)}$

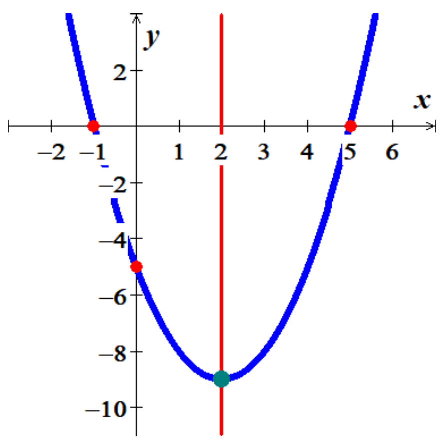
d) $x^2 - 4x - 5 = 0$

$\boxed{x = -1, \ 5}$

e) $x = 0 \ \rightarrow \ \boxed{y = -5}$

f) *Domain*: $\boxed{\mathbb{R}}$ $\qquad$ *Range*: $\boxed{[-9, \ \infty)}$

g)



h) *Increasing*: $\boxed{(2, \ \infty)}$ $\qquad$ *Decreasing*: $\boxed{(-\infty, \ 2)}$

Exercise

For the function $f(x) = 2x^2 - 3x + 1$

a) Find the vertex point

b) Find the line of symmetry

c) State whether there is a *maximum* or *minimum* value *and* find that value

d) Find the zeros of $f(x)$

e) Find the y-intercept

f) Find the *range* and the *domain* of the function.

g) Graph the function

h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = \frac{3}{4}$ $\qquad\qquad x = -\frac{b}{2a}$

$$f\left(\frac{3}{4}\right) = \frac{9}{8} - \frac{9}{4} + 1$$

$$= -\frac{1}{8}$$

Vertex point: $\left(\frac{3}{4},\ -\frac{1}{8}\right)$

b) Axis of *symmetry*: $x = \frac{3}{4}$

c) *Minimum* point @ $\left(\frac{3}{4},\ -\frac{1}{8}\right)$

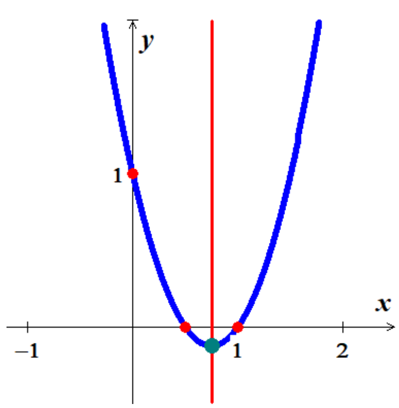
d) $2x^2 - 3x + 1 = 0$

$x = 1,\ \frac{1}{2}$

e) $x = 0 \quad \rightarrow \quad y = 1$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $\left[-\frac{1}{8},\ \infty\right)$

g)



h) *Increasing:* $\left(\frac{3}{4},\ \infty\right)$ $\qquad$ *Decreasing*: $\left(-\infty,\ \frac{3}{4}\right)$

Exercise

For the function $f(x) = -x^2 - 3x + 4$

a) Find the vertex point

b) Find the line of symmetry

c) State whether there is a *maximum* or *minimum* value *and* find that value

d) Find the zeros of $f(x)$

e) Find the y-intercept

f) Find the *range* and the *domain* of the function.

g) Graph the function

h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{3}{2}$ $\qquad x = -\frac{b}{2a}$

$$f\left(-\frac{3}{2}\right) = -\frac{9}{4} + \frac{9}{2} + 4$$

$$= \frac{7}{2}$$

Vertex point: $\left(-\frac{3}{2},\ \frac{7}{2}\right)$

b) Axis of *symmetry*: $x = -\frac{3}{2}$

c) *Maximum* point @ $\left(-\frac{3}{2},\ \frac{7}{2}\right)$

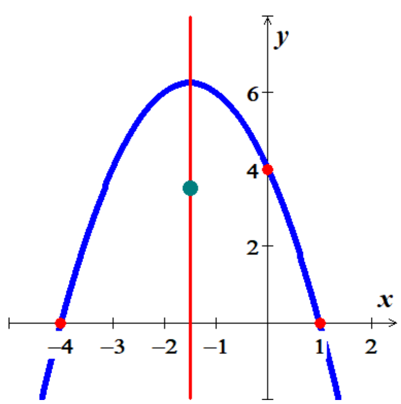
d) $-x^2 - 3x + 4 = 0$

$x = 1,\ -4$

e) $x = 0 \ \rightarrow\ y = 4$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $\left(-\infty,\ \frac{7}{2}\right]$

g)



h) *Increasing:* $\left(-\infty,\ -\frac{3}{2}\right)$ $\qquad$ *Decreasing*: $\left(-\frac{3}{2},\ \infty\right)$

Exercise

For the function $f(x) = -2x^2 + 3x - 1$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the *y*-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $\boxed{x = \frac{3}{4}}$ $\qquad x = -\frac{b}{2a}$

$f\left(\frac{3}{4}\right) = -\frac{9}{8} + \frac{9}{4} - 1$

$= \boxed{\frac{1}{8}}$

Vertex point: $\boxed{\left(\frac{3}{4}, \frac{1}{8}\right)}$

b) Axis of *symmetry*: $\boxed{x = \frac{3}{4}}$

c) *Maximum* point @ $\boxed{\left(\frac{3}{4}, \frac{1}{8}\right)}$

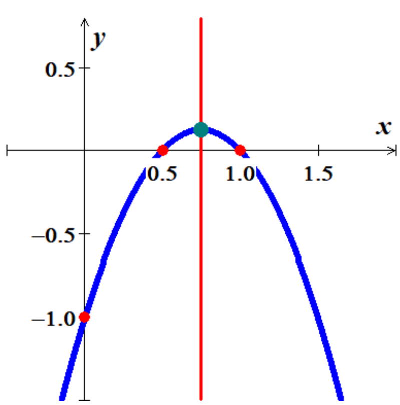
d) $-2x^2 + 3x - 1 = 0$

$\boxed{x = 1, \ \frac{1}{2}}$

e) $x = 0 \quad \rightarrow \quad \boxed{y = -1}$

f) *Domain*: $\boxed{\mathbb{R}}$ $\qquad$ *Range*: $\boxed{\left(-\infty, \frac{1}{8}\right]}$

g)



h) *Increasing:* $\boxed{\left(-\infty, \frac{3}{4}\right)}$ $\qquad$ *Decreasing*: $\boxed{\left(\frac{3}{4}, \infty\right)}$

Exercise

For the function $f(x) = -2x^2 - 3x - 1$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = -\frac{3}{4}$ $\qquad\qquad x = -\frac{b}{2a}$

$$f\left(-\frac{3}{4}\right) = -\frac{9}{8} + \frac{9}{4} - 1$$

$$= \frac{1}{8}$$

Vertex point: $\left(-\frac{3}{4}, \frac{1}{8}\right)$

b) Axis of *symmetry*: $x = -\frac{3}{4}$

c) *Maximum* point @ $\left(-\frac{3}{4}, \frac{1}{8}\right)$

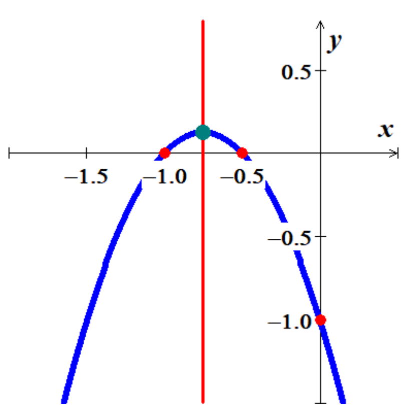
d) $-2x^2 - 3x - 1 = 0$

$x = -1, \ -\frac{1}{2}$

e) $x = 0 \quad \rightarrow \quad y = -1$

f) *Domain*: $\mathbb{R}$ $\qquad\qquad$ *Range*: $\left(-\infty, \frac{1}{8}\right]$

g)



h) *Increasing:* $\left(-\infty, -\frac{3}{4}\right)$ $\qquad$ *Decreasing*: $\left(-\frac{3}{4}, \infty\right)$

Exercise

For the function $f(x) = -x^2 - 4x + 5$

a) Find the vertex point

b) Find the line of symmetry

c) State whether there is a *maximum* or *minimum* value *and* find that value

d) Find the zeros of $f(x)$

e) Find the y-intercept

f) Find the *range* and the *domain* of the function.

g) Graph the function

h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $\underline{x = -2}$ $\qquad x = -\frac{b}{2a}$

$f(-2) = -4 + 8 + 5$

$\underline{= 9}$

Vertex point: $\underline{(-2,\ 9)}$

b) Axis of *symmetry*: $\underline{x = -2}$

c) *Maximum* point @ $\underline{(-2,\ 9)}$

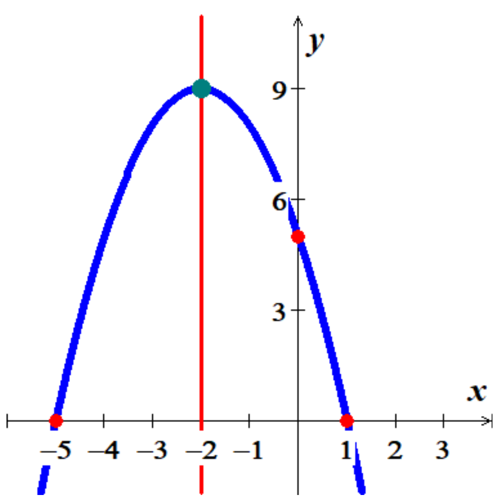
d) $-x^2 - 4x + 5 = 0$

$\underline{x = 1,\ \ -5}$

e) $x = 0 \quad \rightarrow \quad \underline{y = 5}$

f) *Domain*: $\underline{\mathbb{R}}$ $\qquad$ *Range*: $\underline{(-\infty,\ 9]}$

g)



h) *Increasing:* $\underline{(-\infty,\ -2)}$ $\qquad$ *Decreasing*: $\underline{(-2,\ \infty)}$

Exercise

For the function $f(x)=-x^2+4x+2$

a) Find the vertex point

b) Find the line of symmetry

c) State whether there is a *maximum* or *minimum* value *and* find that value

d) Find the zeros of $f(x)$

e) Find the y-intercept

f) Find the *range* and the *domain* of the function.

g) Graph the function

h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $\underline{x=2}$ $\qquad x=-\frac{b}{2a}$

$f(2)=-4+8+2$

$\underline{=6}$

Vertex point: $\underline{(2,\ 6)}$

b) Axis of *symmetry*: $\underline{x=2}$

c) *Maximum* point @ $\underline{(2,\ 6)}$

d) $-x^2+4x+2=0$

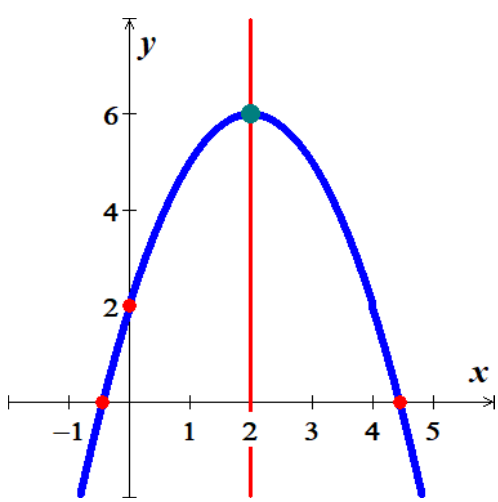
$x=\dfrac{-4\pm\sqrt{16+8}}{-2}$

$\underline{x=2\pm\sqrt{6}}$

e) $x=0 \quad\to\quad \underline{y=2}$

f) *Domain*: $\underline{\mathbb{R}}$ $\qquad$ *Range*: $\underline{(-\infty,\ 6]}$

g)



h) *Increasing:* $\underline{(-\infty,\ 2)}$ $\qquad$ *Decreasing*: $\underline{(2,\ \infty)}$

Exercise

For the function $f(x) = -3x^2 + 3x + 7$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $x = \frac{1}{2}$ $\qquad x = -\frac{b}{2a}$

$f\left(\frac{1}{2}\right) = -\frac{3}{4} + \frac{3}{2} + 7 \quad = \frac{31}{4}$

Vertex point: $\left(\frac{1}{2}, \frac{31}{4}\right)$

b) Axis of *symmetry*: $x = \frac{1}{2}$

c) *Maximum* point @ $\left(\frac{1}{2}, \frac{31}{4}\right)$

d) $-3x^2 + 3x + 7 = 0$

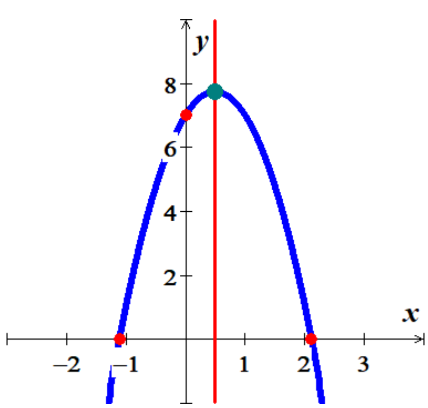
$x = \frac{-3 \pm \sqrt{93}}{-6}$

$x = \frac{3 \pm \sqrt{93}}{6}$

e) $x = 0 \quad \rightarrow \quad y = 7$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $\left(-\infty, \frac{31}{4}\right]$

g)



h) *Increasing:* $\left(-\infty, \frac{1}{2}\right)$ $\qquad$ *Decreasing*: $\left(\frac{1}{2}, \infty\right)$

Exercise

For the function $f(x) = -x^2 + 2x - 2$

a) Find the vertex point
b) Find the line of symmetry
c) State whether there is a *maximum* or *minimum* value *and* find that value
d) Find the zeros of $f(x)$
e) Find the y-intercept
f) Find the *range* and the *domain* of the function.
g) Graph the function
h) On what intervals is the function *increasing*? *decreasing*?

Solution

a) $\underline{x = 1}$ $\qquad x = -\dfrac{b}{2a}$

$$f(1) = -1 + 2 - 2$$
$$= -1$$

Vertex point: $(1, \ -1)$

b) Axis of *symmetry*: $x = 1$

c) *Maximum* point @ $(1, \ -1)$

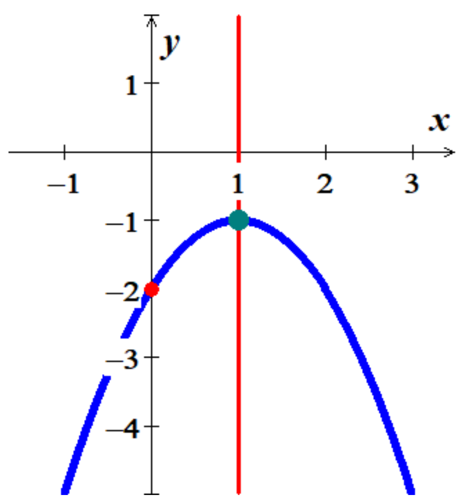
d) $-x^2 + 2x - 2 = 0$

$$x = \frac{-2 \pm \sqrt{-4}}{-2} \quad \mathbb{C}$$

e) $x = 0 \ \rightarrow \ y = -2$

f) *Domain*: $\mathbb{R}$ $\qquad$ *Range*: $(-\infty, \ -1]$

g)



h) *Increasing*: $(-\infty, \ 1)$ $\qquad$ *Decreasing*: $(1, \ \infty)$

Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $20x = y^2$

Solution

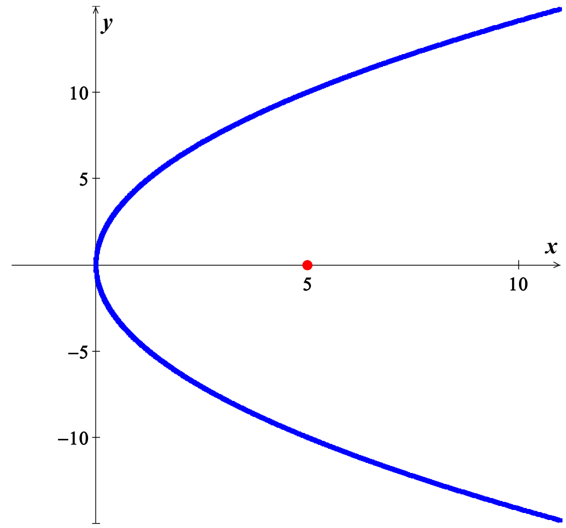
$20x = y^2 \qquad 4px = y^2$

$4p = 20 \quad \Rightarrow \quad \boxed{p = 5}$

Vertex: $(0, 0)$

Focus $(5,\ 0)$

Directrix: $x = -5$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $2y^2 = -3x$

Solution

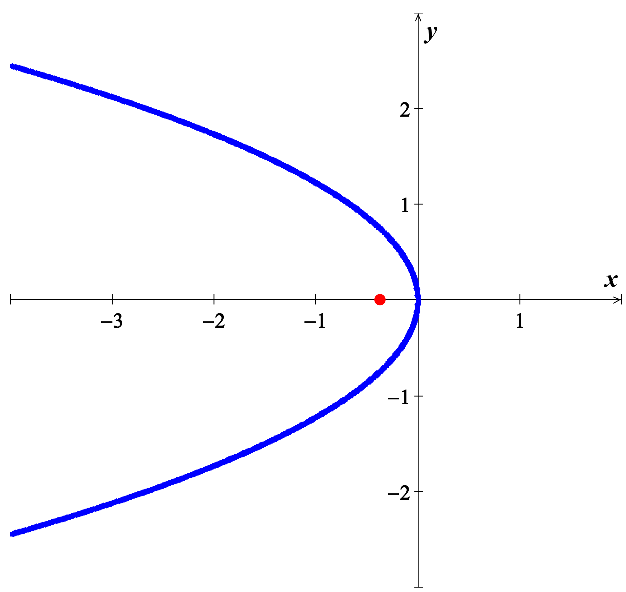
$y^2 = -\frac{3}{2}x = 4px$

$4p = -\frac{3}{2} \quad \Rightarrow \quad \boxed{p = -\frac{3}{8}}$

Vertex: $(0, 0)$

Focus: $\left(-\frac{3}{8},\ 0\right)$

Directrix: $x = \frac{3}{8}$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $(x+2)^2 = -8(y-1)$

Solution

$$(x+2)^2 = 4p(y-1)$$

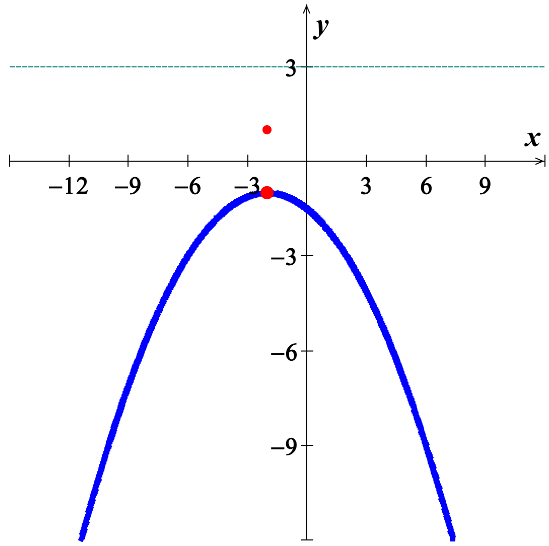
$$4p = -8 \quad \Rightarrow \quad \boxed{p = -2}$$

Vertex: $(-2,\ 1)$

Focus: $(-2,\ 1-2) = (-2,\ -1)$

Directrix: $y = 1+2$

$$\underline{= 3}$$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $(x-3)^2 = \frac{1}{2}(y+1)$

Solution

$$(x-3)^2 = 4p(y+1)$$

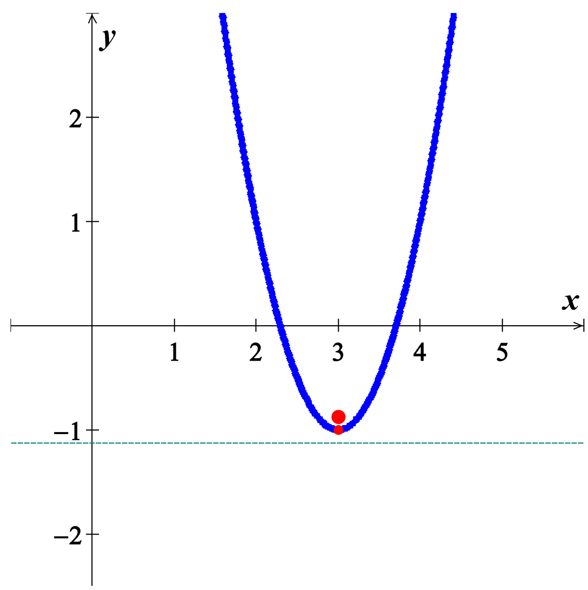
$$4p = \frac{1}{2} \quad \Rightarrow \quad \boxed{p = \frac{1}{8}}$$

Vertex: $(3,\ -1)$

Focus: $\left(3,\ -1+\frac{1}{8}\right) = \left(3,\ -\frac{7}{8}\right)$

Directrix: $y = -1-\frac{1}{8}$

$$\underline{= -\frac{9}{8}}$$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $(y+1)^2 = -12(x+2)$

Solution

$$(y+1)^2 = 4p(x+2)$$

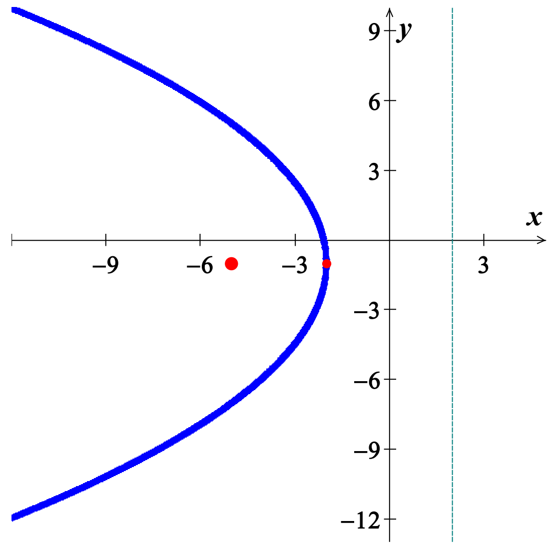
$$4p = -12 \quad \Rightarrow \quad \boxed{p = -3}$$

Vertex: $(-2, \ -1)$

Focus: $(-2-3, \ -1) \ \underline{= (-5, \ -1)}$

Directrix: $x = -1 + 3$

$\underline{= 2}$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $y = x^2 - 4x + 2$

Solution

$$y = ax^2 + bx + c \quad \Rightarrow \quad a = 1$$

$$p = \frac{1}{4a} = \frac{1}{4}$$

$$\underline{p = \frac{1}{4}}$$

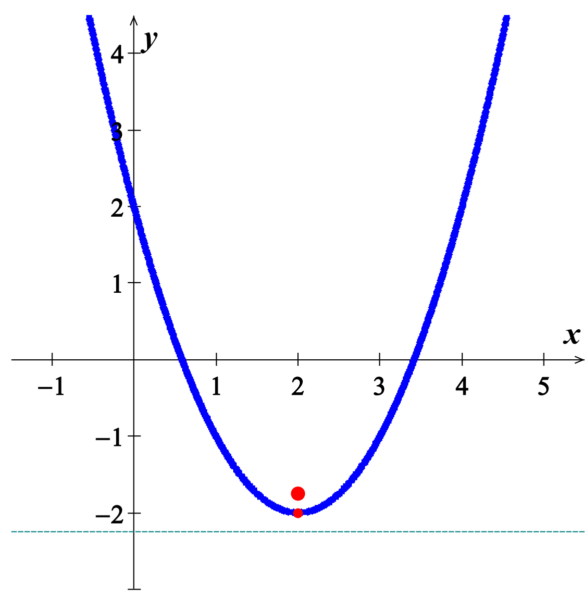
Vertex: $\begin{cases} h = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2 \\ k = 2^2 - 4(2) + 2 = -2 \end{cases}$

$\underline{V = (2, \ -2)}$

Focus: $\left(2, \ -2 + \frac{1}{4}\right) = \left(2, \ -\frac{7}{4}\right)$

Directrix: $y = -2 - \frac{1}{4}$

$\underline{= -\frac{9}{4}}$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $y^2+14y+4x+45=0$

Solution

$$y^2+14y=-4x-45$$

$$y^2+14y+(7)^2=-4x-45+(7)^2$$

$$(y+7)^2=-4x+4$$

$$(y+7)^2=-4(x-1)$$

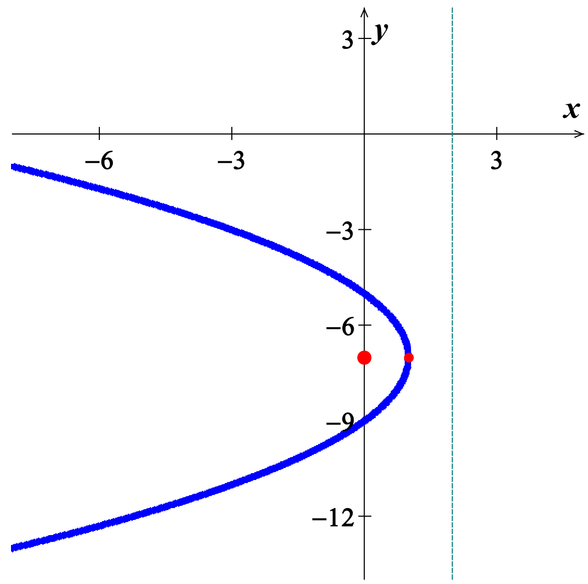
$$4p=-4 \quad \Rightarrow \quad \boxed{p=-1}$$

Vertex: $(1,\ -7)$

Focus: $(1-1,\ -7)$ $=(0,\ -7)$

Directrix: $x=1+1$

$=2$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $x^2+20y=10$

Solution

$$x^2=-20y+10$$

$$x^2=-20\left(y-\frac{1}{2}\right)$$

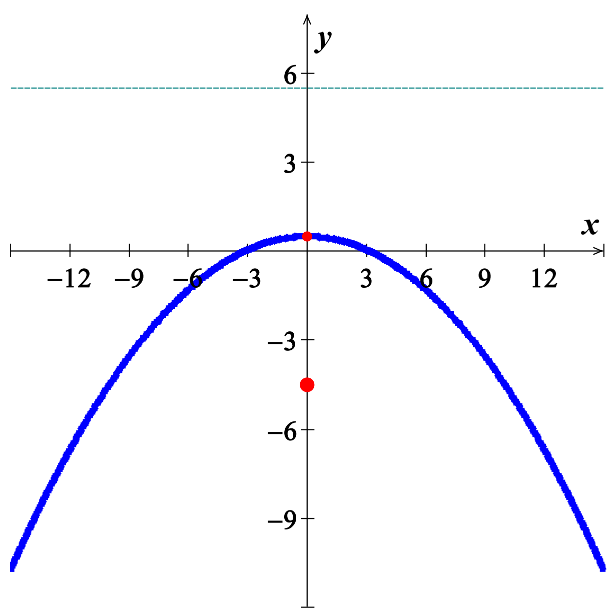
$$4p=-20 \quad \Rightarrow \quad \boxed{p=-5}$$

Vertex: $\left(0,\ \frac{1}{2}\right)$

Focus: $\left(0,\ \frac{1}{2}-5\right)$ $=\left(0,\ -\frac{9}{2}\right)$

Directrix: $y=\frac{1}{2}+5$

$=\frac{11}{2}$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $x^2 = 16y$

Solution

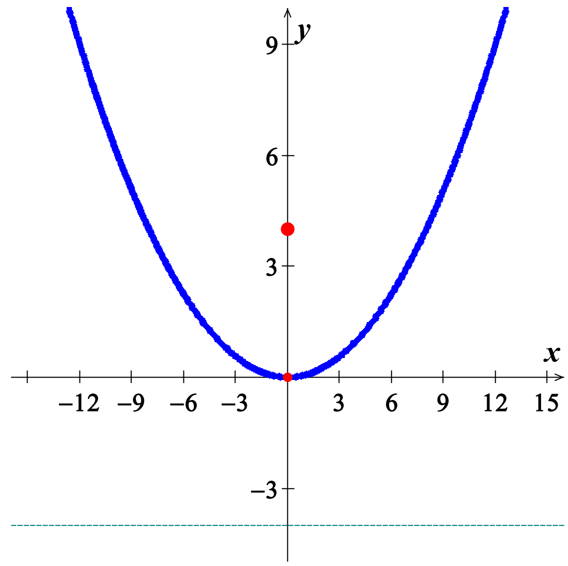
$x^2 = 16y = 4py$

$4p = 16 \quad \Rightarrow \quad \boxed{p = 4}$

Vertex: $\underline{(0,\ 0)|}$

Focus: $\underline{(0,\ 4)|}$

Directrix: $\underline{y = -4|}$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $x^2 = -\frac{1}{2}y$

Solution

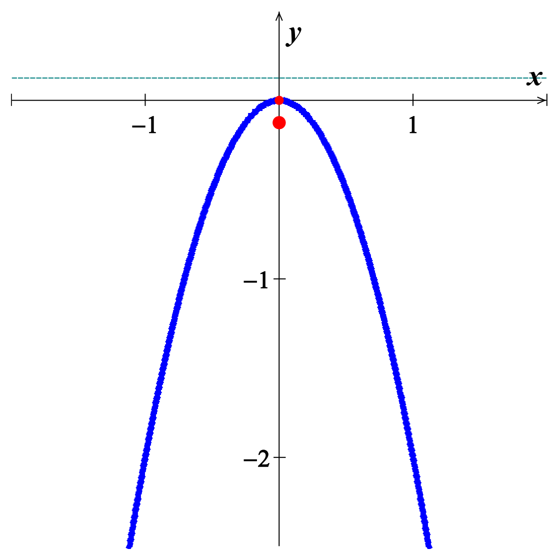
$x^2 = -\frac{1}{2}y = 4py$

$4p = -\frac{1}{2} \quad \Rightarrow \quad \boxed{p = -\frac{1}{8}}$

Vertex: $\underline{(0,\ 0)|}$

Focus: $\underline{\left(0,\ -\frac{1}{8}\right)|}$

Directrix: $\underline{y = \frac{1}{8}|}$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $(y+1)^2 = -4(x-2)$

Solution

$$(y+1)^2 = 4p(x-2)$$

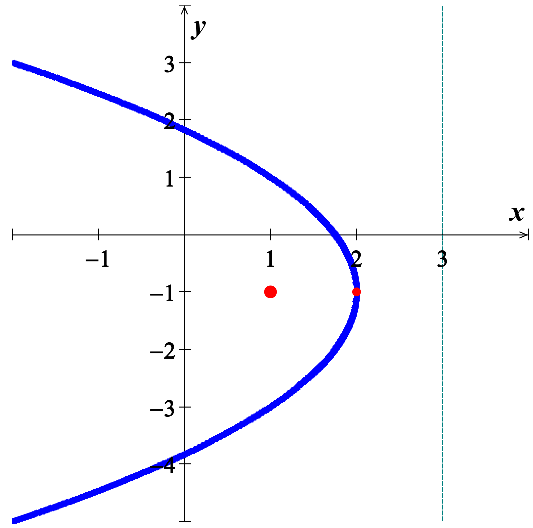
$4p = -4 \quad \Rightarrow \quad \boxed{p = -1}$

Vertex: $(2, \ -1)$

Focus: $(2-1, \ -1) = (1, \ -1)$

Directrix: $x = 2+1$

$= 3$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $x^2 + 6x - 4y + 1 = 0$

Solution

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 4y - 1 + (3)^2$$

$$(x+3)^2 = 4y + 8$$

$$(x+3)^2 = 4(y+2)$$

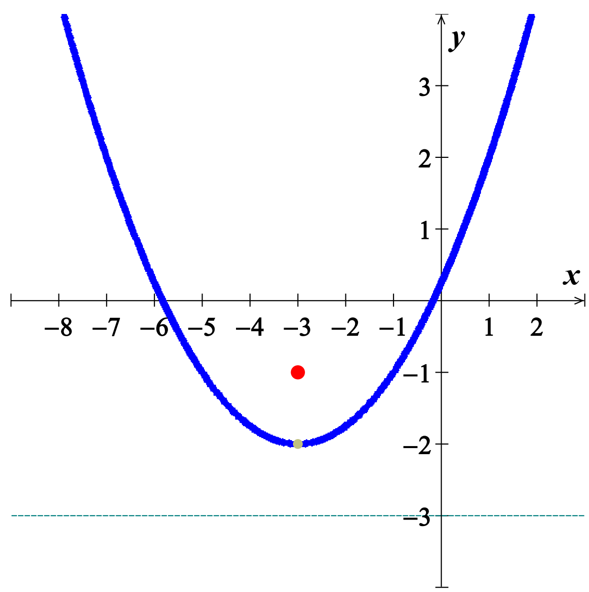
$4p = 4 \quad \Rightarrow \quad \boxed{p = 1}$

Vertex: $(-3, \ -2)$

Focus: $(-3, \ -2+1) = (-3, \ -1)$

Directrix: $y = -2-1$

$= -3$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $y^2 + 2y - x = 0$

Solution

$$y^2 + 2y = x$$

$$y^2 + 2y + \left(\frac{2}{2}\right)^2 = x + (1)^2$$

$$(y+1)^2 = (x+1)$$

$$4p = 1 \quad \Rightarrow \quad \boxed{p = \frac{1}{4}}$$

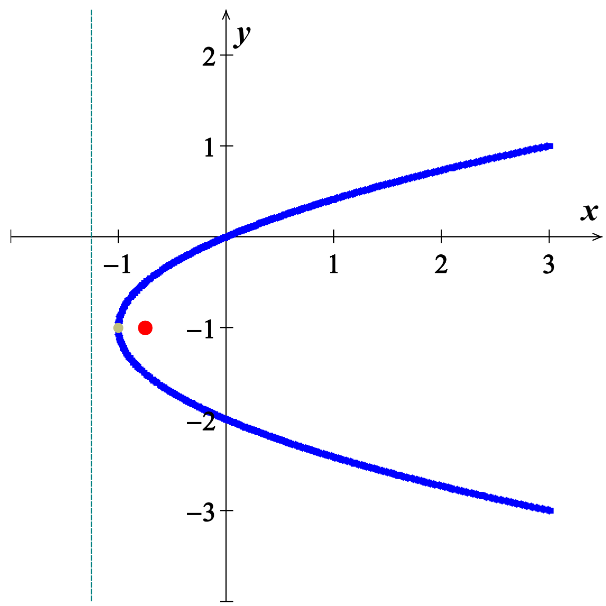
Vertex: $V = (-1,\ -1)$

Focus: $F = \left(-1 + \frac{1}{4},\ -1\right)$

$$= \left(-\frac{3}{4},\ -1\right)$$

Directrix: $x = -1 - \frac{1}{4}$

$$= -\frac{5}{4}$$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $y^2 - 4y + 4x + 4 = 0$

Solution

$$y^2 - 4y = -4x - 4$$

$$y^2 - 4y + \left(\frac{-4}{2}\right)^2 = -4x - 4 + (-2)^2$$

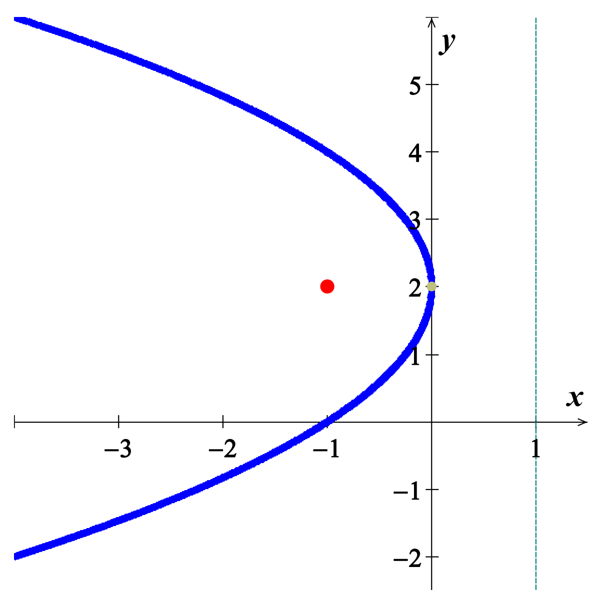
$$(y-2)^2 = -4x$$

$$4p = -4 \quad \Rightarrow \quad \boxed{p = -1}$$

Vertex: $V = (0,\ 2)$

Focus: $F = (-1,\ 2)$

Directrix: $x = 1$



Exercise

Find the ***vertex***, ***focus***, and ***directrix*** of the parabola. Sketch its graph. $x^2 - 4x - 4y = 4$

Solution

$$x^2 - 4x = 4y + 4$$

$$x^2 - 4x + \left(\frac{-4}{2}\right)^2 = 4y + 4 + (-2)^2$$

$$(x-2)^2 = 4(y+2)$$

$$4p = 4 \quad \Rightarrow \quad \boxed{p = 1}$$

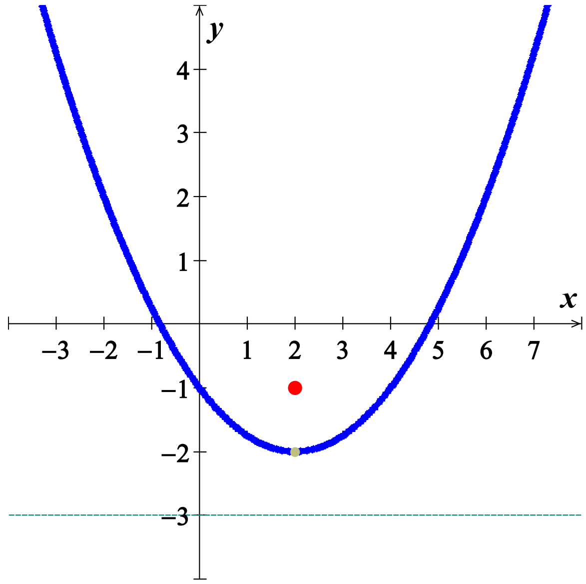
Vertex: $V = (2,\ -2)$

Focus: $F = (2,\ -2+1)$

$= (2,\ -1)$

Directrix: $y = -2 - 1$

$= -3$



Exercise

Find an equation of the parabola that satisfies the given conditions *Focus* : $F(2,0)$ *directrix* : $x = -2$

Solution

$$x = -2 = -p \quad \rightarrow \quad p = 2$$

$$y^2 = 4px$$

$$y^2 = 8x$$

Exercise

Find an equation of the parabola that satisfies the given conditions *Focus* : $F(0,-40)$ *directrix* : $y = 4$

Solution

$$y = 4 = -p \quad \rightarrow \quad p = -4$$

$$x^2 = 4py$$

$$x^2 = -16y$$

Exercise

Find an equation of the parabola that satisfies the given conditions $Focus: F(-3,-2)$ $\quad directrix: y=1$

Solution

$$y=1=k-p \quad \rightarrow \quad k-p=1$$

$$\begin{cases} \boxed{h=-3} \\ k+p=-2 \end{cases} \rightarrow \begin{cases} k+p=-2 \\ k-p=1 \end{cases}$$

$$\Rightarrow 2k=-1 \rightarrow k=-\frac{1}{2}$$

$$k-p=1 \quad \rightarrow \quad p=k-1$$

$$p=-\frac{1}{2}-1$$

$$=-\frac{3}{2}$$

Vertex: $V=\left(-3,\ -\frac{1}{2}\right)$

$$(x+3)^2=4\left(-\frac{3}{2}\right)\left(y+\frac{1}{2}\right)$$

$$(x+3)^2=-6\left(y+\frac{1}{2}\right)$$

Exercise

Find an equation of the parabola that satisfies the given conditions $Vertex: V(3,-5)$ $\quad directrix: x=2$

Solution

$$Vertex: V(3,-5) \quad \begin{cases} h=3 \\ k=-5 \end{cases}$$

$$directrix: x=2=h-p$$

$$p=h-2$$

$$=3-2$$

$$=1$$

$$(y-k)^2=4p(x-h)$$

$$(y+5)^2=4(x-3)$$

Exercise

Find an equation of the parabola that satisfies the given conditions $Vertex: V(-2,3) \quad directrix: y=5$

Solution

$$Vertex: V(-2,\ 3) \quad \begin{cases} h=-2 \\ k=3 \end{cases}$$

$$directrix:\ \ y=5=k-p$$

$$\begin{aligned} p &= k-5 \\ &= 3-5 \\ &= -2 \end{aligned}$$

$$(x-h)^2 = 4p(y-k)$$

$$\boxed{(x+2)^2 = -8(y-3)}$$

Exercise

Find an equation of the parabola that satisfies the given conditions $Vertex: V(-1,0) \quad focus: F(-4,0)$

Solution

$$Vertex: V(-1,\ 0) \quad \begin{cases} h=-1 \\ k=0 \end{cases}$$

$$focus:\ F(-4,0) \quad \begin{cases} h+p=-4 \\ k=0 \end{cases}$$

$$\begin{aligned} p &= -4-h \\ &= -4+1 \\ &= -3 \end{aligned}$$

$$(y-k)^2 = 4p(x-h)$$

$$\boxed{y^2 = -12(x+1)}$$

Exercise

Find an equation of the parabola that satisfies the given conditions $Vertex: V(1,-2) \quad focus: F(1,0)$

Solution

$$Vertex: V(1,\ -2) \quad \begin{cases} h=1 \\ k=-2 \end{cases}$$

$focus: F(1,\ 0)\ \begin{cases} h=1 \\ k+p=0 \end{cases} \Rightarrow \underline{p=-k=2}$

$(x-h)^2 = 4p(y-k)$

$\underline{(x-1)^2 = 8(y+2)}$

Exercise

Find an equation of the parabola that satisfies the given conditions $Vertex: V(0,\ 1)\quad focus: F(0,\ 2)$

Solution

$Vertex: V(0,\ 1)\ \begin{cases} h=0 \\ k=1 \end{cases}$

$focus: F(0,\ 2)\ \begin{cases} h=0 \\ k+p=2 \end{cases} \Rightarrow \underline{p=2-1=1}$

$(x-h)^2 = 4p(y-k)$

$\underline{x^2 = 4(y-1)}$

Exercise

Find an equation of the parabola that satisfies the given conditions $Vertex: V(3,\ 2)\quad focus: F(-1,\ 2)$

Solution

$Vertex: V(3,\ 2)\ \begin{cases} h=3 \\ k=2 \end{cases}$

$focus: F(-1,2)\ \begin{cases} h+p=-1 \Rightarrow \underline{p=-1-3=-4} \\ k=2 \end{cases}$

$(y-k)^2 = 4p(x-h)$

$\underline{(y-2)^2 = -16(x-3)}$