Exam 2 - Review

+, -, unit vector

$$proj_{\vec{a}} \vec{u} = \frac{\vec{u} \cdot \vec{a}}{\|\vec{a}\|^2} \vec{a}$$

* cross product (x)

Vol?

L.I or dependent

* subspace $\vec{a}, \vec{b}, \vec{c}$ $\cdot$

* r, dim, F, P, - —

Prove ~~6~~ 5 out 8

$\vec{u} = (-3, 1, 0)$ $\vec{v} = (1, 3, 4)$

$$2\vec{u} - 3\vec{v} = 2(-3, 1, 0) - 3(1, 3, 4)$$

$$= (-9, -7, -12)$$

$$\text{unit vector of } \vec{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$= \frac{(1, 3, 4)}{\sqrt{1+9+16}}$$

$$= \left(\frac{1}{\sqrt{26}}, \frac{3}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 1 & 0 \\ 1 & 3 & 4 \end{vmatrix}$$

$$= 4\vec{i} + 12\vec{j} - 10\vec{k}$$

$\vec{u} = (1, 2, 3) \qquad \vec{a} = (0, -1, 2)$

$$\text{proj}_{\vec{a}}\,\vec{u} = \frac{\vec{u} \cdot \vec{a}}{\|\vec{a}\|^2}\,\vec{a}$$

$$= \frac{(1,2,3) \cdot (0,-1,2)}{1+4}\,(0, -1, 2)$$

$$= \frac{4}{5}(0, -1, 2)$$

$$= \left(0, -\frac{4}{5}, \frac{8}{5}\right)$$

$$\text{Volume} = \underbrace{\left\| \det \right\|}_{\text{absolute value}}$$

$(1, 1, 2) \qquad (1, 0, 1) \qquad (2, 1, 3)$

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 0 \quad \text{Linearly dependent}$$

$\neq 0$ - L.I

$S = \{ (a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 + 2a_2 - 3a_3 = 0 \}$

a) closed under addition?

let $\begin{cases} \vec{v}_1 = (a_1, a_2, a_3) & : a_1 + 2a_2 - 3a_3 = 0 \\ \vec{v}_2 = (b_1, b_2, b_3) & : b_1 + 2b_2 - 3b_3 = 0 \end{cases}$

a) $\vec{v}_1 + \vec{v}_2 = (a_1, a_2, a_3) + (b_1, b_2, b_3)$

$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

$a_1 + b_1 + 2(a_2 + b_2) - 3(a_3 + b_3)$

$= a_1 + 2a_2 - 3a_3 + b_1 + 2b_2 - 3b_3$

$= 0 + 0$

$= 0$ ✓

∴ it's closed under addition

b) let $k \in \mathbb{R}$

$k\vec{v}_1 = k(a_1, a_2, a_3)$

$= (ka_1, ka_2, ka_3)$

$ka_1 + 2ka_2 - 3ka_3 = k(a_1 + 2a_2 - 3a_3)$

$= k(0)$

$= 0$ ✓

it's closed under scalar multiplication

c) Since it's closed under addition & scalar multiplication, S is a subspace $\mathbb{R}^3$

$$A = \begin{array}{c} \begin{pmatrix} 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \end{array} \qquad b = A \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

~~a) 3~~

~~b) = 2~~

$\varnothing$

a) $\operatorname{rank}(A) = 3$

b) $\dim(A) = 2$

c) pivots var: x_1, x_3, x_5

d) Free var: x_2, x_4

e) $$\begin{cases} x_1 = -2x_2 + x_4 \\ x_3 = -3x_4 \\ x_5 = 0 \end{cases}$$

$x_2 = 1 \quad x_4 = 0 \qquad s_1 = (-2, 1, 0, 0, 0)$

$x_2 = 0 \quad x_4 = 1 \qquad s_2 = (1, 0, -3, 1, 0)$

f) $$N(A) = \begin{pmatrix} -2 & 1 \\ 1 & 0 \\ 0 & -3 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

g) $$x_p = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad b = A x_p$$

h) $$x = x_p + x_N$$
$$= \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{u} \circ \vec{u} = \|\vec{u}\|^2 \geq 0$$

let $\vec{u} = (u_1, u_2, ---, u_n)$

$$\vec{u} \bullet \vec{u} = (u_1, u_2, ---, u_n) \bullet (u_1, u_2, ---, u_n)$$

$$= u_1 u_1 + u_2 u_2 + --- + u_n \cdot u_n$$

$$= u_1^2 + u_2^2 + --- + u_n^2$$

$$\|\vec{u}\|^2 = \left(\sqrt{u_1^2 + u_2^2 + --- + u_n^2}\right)^2$$

$$= u_1^2 + u_2^2 + --- + u_n^2$$

$$\therefore \vec{u} \bullet \vec{u} = \|\vec{u}\|^2 \checkmark$$

$$\vec{u} \circ (\vec{v} + \vec{w}) = \vec{u} \circ \vec{v} + \vec{u} \circ \vec{w}$$

let $\vec{u} = (u_1, u_2, ---, u_n)$
$\vec{v} = (v_1, v_2, ---, v_n)$
$\vec{w} = (w_1, w_2, ---, w_n)$

$$\vec{u} \circ (\vec{v} + \vec{w}) = (u_1, ---, u_n) \bullet \left[(v_1, ---, v_n) + (w_1, ---, w_n)\right]$$

$$= (u_1, ---, u_n) \circ (v_1 + w_1, v_2 + w_2, ---, v_n + w_n)$$

$$= u_1(v_1 + w_1) + u_2(v_2 + w_2) + --- + u_n(v_n + w_n)$$

$$= u_1 v_1 + u_1 w_1 + u_2 v_2 + u_2 w_2 + --- + u_n v_n + u_n w_n$$

$$= (u_1 v_1 + --- + u_n v_n) + (u_1 w_1 + --- + u_n w_n)$$

$$= (u_1, ---, u_n) \bullet (v_1, ---, v_n) + (u_1, ---, u_n) \bullet (w_1, ---, w_n)$$

$$= \vec{u} \bullet \vec{v} + \vec{u} \bullet \vec{w} \checkmark$$

$f_1 = \sin x \qquad f_2 = \cos x \qquad f_3(x) = x\cos x$

$$W = \begin{vmatrix} \sin x & \cos x & x\cos x \\ \cos x & -\sin x & \cos x - x\sin x \\ -\sin x & -\cos x & -2\sin x - x\cos x \end{vmatrix}$$

$$= +2\sin^3 x + \cancel{x\sin^2 x\cos x} - \cancel{\sin x\cos^2 x}$$
$$+ \cancel{x\sin^2 x\cos x} - \cancel{x\cos^3 x} - \cancel{x\sin^2 x\cos x}$$
$$+ \cancel{\sin x\cos^2 x} - \cancel{x\sin^2 x\cos x} + 2\sin x\cos^2 x$$
$$+ \cancel{x\cos^3 x}$$

$$= 2\sin^3 x + 2\sin x\cos^2 x$$
$$= 2\sin x(\sin^2 x + \cos^2 x)$$
$$= 2\sin x \neq 0$$

L.I

$$f_1 = e^x \qquad f_2 = xe^x \qquad f_3 = x^2e^x$$

$$W = \begin{vmatrix} e^x & xe^x & x^2e^x \\ e^x & (x+1)e^x & (2x+x^2)e^x \\ e^x & (x+2)e^x & (2+x^2+4x)\,e^x \end{vmatrix}$$

$$= \left(e^x\, e^x\, e^x\right)\begin{vmatrix} 1 & x & x^2 \\ 1 & x+1 & 2x+x^2 \\ 1 & x+2 & x^2+4x+2 \end{vmatrix}$$

$$= e^{3x}\left(\begin{array}{cccc} x^3 & x^2 & x^1 & x^0 \\ \cancel{2} & \cancel{4} & \cancel{2} & 2 \\ \cancel{1} & \cancel{+2} & \cancel{4} & \\ \cancel{-1} & \cancel{2} & \cancel{+4} & \\ \cancel{-1} & \cancel{-1} & \cancel{-2} & \\ \cancel{-1} & \cancel{-2} & & \\ & \cancel{-2} & & \\ & \cancel{-4} & & \end{array} \right)$$

$$\boxed{= 2e^{3x} \neq 0}$$