$$\sin(36°) = 2\sin(18°)\cos(18°)$$

$$\cos(36°) = 2\cos^2(18°) - 1$$

$$\cos(9°) = \frac{\sqrt{1+\cos 18°}}{\sqrt{2}}$$

$$\sin(9°) = \frac{\sqrt{1-\cos 18°}}{\sqrt{2}}$$

$$\sin 45° = \sin(36° + 9°)$$

$$= \sin(36°)\cos(9°) + \cos(36°)\sin(9°)$$

$$\frac{1}{\sqrt{2}} = 2\sin(18°)\cos(18°)\frac{\sqrt{1+\cos(18°)}}{\sqrt{2}} + \left(2\cos^2(18°) - 1\right)\frac{\sqrt{1-\cos(18°)}}{\sqrt{2}}$$

$$1 = 2\sin(18°)\cos(18°)\sqrt{1+\cos(18°)} + \left(2\cos^2(18°) - 1\right)\sqrt{1-\cos(18°)}$$

$$\textit{Let:}\quad \cos(18°) = x \quad \Rightarrow \sin(18°) = \sqrt{1-\cos^2(18°)} = \sqrt{1-x^2}$$

$$1 = 2\sqrt{1-x^2}\,(x)\left(\sqrt{1+x}\right) + \left(2x^2 - 1\right)\sqrt{1-x}$$

$$1 = 2x\sqrt{1-x^2}\left(\sqrt{1+x}\right) + \left(2x^2 - 1\right)\sqrt{1-x}$$

$$1 = 2x\sqrt{(1-x)(1+x)}\left(\sqrt{1+x}\right) + \left(2x^2 - 1\right)\sqrt{1-x}$$

$$1 = 2x\sqrt{(1-x)(1+x)^2} + \left(2x^2 - 1\right)\sqrt{1-x}$$

$$1 = 2x(1+x)\sqrt{(1-x)} + \left(2x^2 - 1\right)\sqrt{1-x}$$

$$1 = (2x + 2x^2)\sqrt{(1-x)} + \left(2x^2 - 1\right)\sqrt{1-x}$$

$$1 = \left(2x + 2x^2 + 2x^2 - 1\right)\sqrt{1-x}$$

$$1 = \left(4x^2 + 2x - 1\right)\sqrt{1-x}$$

$$(1)^2 = \left(\left(4x^2 + 2x - 1\right)\sqrt{1-x}\right)^2$$

$$1 = \left(4x^2 + 2x - 1\right)^2(1-x)$$

$$1 = \left(16x^4 + 8x^3 - 4x^2 + 8x^3 + 4x^2 - 2x - 4x^2 - 2x + 1\right)(1-x)$$

$$1 = \left(16x^4 + 16x^3 - 4x^2 - 4x + 1\right)(1-x)$$

$$1 = 16x^4 + 16x^3 - 4x^2 - 4x + 1 - 16x^5 - 16x^4 + 4x^3 + 4x^2 - x$$

$$16x^5 - 20x^3 + 5x = 0$$

$$x(16x^4 - 20x^2 + 5) = 0$$

$$16x^4 - 20x^2 + 5 = 0$$

$$x^2 = \frac{-(-20) \pm \sqrt{20^2 - 4(16)(5)}}{2(16)}$$

$$= \frac{20 \pm \sqrt{80}}{32}$$

$$= \frac{20 \pm 4\sqrt{5}}{32}$$

$$= \frac{5 \pm \sqrt{5}}{8}$$

$$x = \begin{cases} \pm\dfrac{\sqrt{5+\sqrt{5}}}{\sqrt{8}} \\ \pm\dfrac{\sqrt{5-\sqrt{5}}}{\sqrt{8}} \end{cases} \qquad x = \begin{cases} \dfrac{\sqrt{5+\sqrt{5}}}{2\sqrt{2}} \approx 0.951 \\ \dfrac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}} \approx 0.588 \end{cases} = \cos(18°)$$

$$\cos(18°) = \sqrt{\frac{5+\sqrt{5}}{8}} = \sin(90° - 18°) = \sin(72°)$$

$$\cos^2(72°) = 1 - \left(\sqrt{\frac{5+\sqrt{5}}{8}}\right)^2$$

$$= 1 - \frac{5+\sqrt{5}}{8}$$

$$= \frac{3-\sqrt{5}}{8}$$

$$= \frac{2}{2}\,\frac{3-\sqrt{5}}{8} = \frac{6-2\sqrt{5}}{16} = \frac{1+5-2\sqrt{5}}{16} = \frac{\left(\sqrt{5}-1\right)^2}{16}$$

$$\cos(72°) = \frac{\sqrt{5}-1}{4}$$

$$\cos(72°) = \frac{a/2}{b} = \frac{a}{2b} = \frac{\sqrt{5}-1}{4}$$

$$\frac{a}{b} = \frac{\sqrt{5}-1}{2}$$