Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \frac{3x}{1-x}$

Solution

$1 - x = 0 \Rightarrow x = 1$

$y = \frac{3x}{-x} = \frac{3}{-1} = -3$

VA	$x = 1$
HA	$y = -3$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \frac{x^2}{x^2 + 9}$

Solution

VA: *n / a*
HA: $y = 1$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \frac{x-2}{x^2 - 4x + 3}$

Solution

$x^2 - 4x + 3 = 0 \Rightarrow x = 1,\ 3$

$y = \frac{x}{x^2} \to 0$

VA: $x = 1,\ x = 3$ ***HA***: $y = 0$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \frac{3}{x-5}$

Solution

VA: $x = 5$ ***HA***: $y = 0$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{x^3 - 1}{x^2 + 1}$

Solution

VA: *none* ***/HA***: *none*

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{3x^2 - 27}{(x+3)(2x+1)}$

Solution

$$x + 3 = 0 \rightarrow x = -3$$
$$2x + 1 = 0 \rightarrow x = -\frac{1}{2}$$
$$y = \frac{3x^2}{(x)(2x)} = \frac{3x^2}{2x^2} = \frac{3}{2}$$

VA: $x = -3,\ -\frac{1}{2}$ ***HA***: $y = \frac{3}{2}$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{x^3 + 3x^2 - 2}{x^2 - 4}$

Solution

VA: $x = \pm 2$ ***HA***: *n / a*

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{x - 3}{x^2 - 9}$

Solution

$$x^2 - 9 = 0 \rightarrow \boxed{x = \pm 3}$$
$$y = \frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3}$$

VA: $x = 3$ ***HA***: $y = 0$ ***Hole***: $x = 3 \rightarrow y = \frac{1}{6}$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{6}{\sqrt{x^2 - 4x}}$

Solution

$x^2 - 4x = 0 \Rightarrow x(x-4) = 0 \rightarrow \boxed{x = 0, 4}$

VA: $x = 0,\ x = 4$ ***HA***: $y = 0$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $y = \dfrac{5x - 1}{1 - 3x}$

Solution

VA: $x = \frac{1}{3}$ ***HA***: $y = -\frac{5}{3}$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $f(x) = \dfrac{2x - 11}{x^2 + 2x - 8}$

Solution

VA: $x = 2,\ x = -4$ ***HA***: $y = 0$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $f(x) = \dfrac{x^2 - 4x}{x^3 - x}$

Solution

$f(x) = \dfrac{x(x-4)}{x(x^2 - 1)} = \dfrac{x - 4}{x^2 - 1}$

VA: $x = -1,\ x = 1$ ***HA***: $y = 0$ ***Hole***: $x = 0 \rightarrow y = 4$

Exercise

Find the vertical and horizontal asymptotes (if any) of: $f(x) = \dfrac{x - 2}{x^3 - 5x}$

Solution

VA: $x = 0,\ x = \pm\sqrt{5}$ ***HA***: $y = 0$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{4x}{x^2+10x}$

Solution

$x^2+10x=0 \;\rightarrow\; x=0,\; -10$ ***Domain***: $(-\infty,\; -10)\cup(-10,\; 0)\cup(0,\; \infty)$

$f(x)=\dfrac{4x}{x(x+10)}=\dfrac{4}{x+10}$

VA: $x=-10$ ***HA***: $y=0$

Hole: $x=0 \;\rightarrow\; y=\dfrac{4}{10}\Rightarrow$ *hole* $\left(0,\; \dfrac{2}{5}\right)$

Oblique asymptote: *n / a*

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{3-x}{(x-4)(x+6)}$

Solution

Domain: $(-\infty,\; -6)\cup(-6,\; 4)\cup(4,\; \infty)$

VA: $x=-6$ *and* $x=4$ ***HA***: $y=0$

Hole: *n / a* ***Oblique asymptote***: *n / a*

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3}{2x^3-x^2-3x}$

Solution

$2x^3-x^2-3x=x\left(2x^2-x-3\right)=0 \;\rightarrow\; x=0,\; -1,\; \dfrac{3}{2}$

Domain: $(-\infty,\; -1)\cup(-1,\; 0)\cup\left(0,\; \dfrac{3}{2}\right)\cup\left(\dfrac{3}{2},\; \infty\right)$

$f(x)=\dfrac{x^3}{2x^3-x^2-3x}=\dfrac{x^3}{x\left(2x^2-x-3\right)}=\dfrac{x^2}{2x^2-x-3}$

VA: $x=-1$ *and* $x=\dfrac{3}{2}$ ***HA***: $y=\dfrac{1}{2}$

Hole: $x=0 \;\rightarrow\; y=0\Rightarrow$ *hole* $(0,\; 0)$

Oblique asymptote: *n / a*

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{3x^2+5}{4x^2-3}$

Solution

$4x^2-3=0 \quad \rightarrow \quad x=\pm\dfrac{\sqrt{3}}{2}$ ***Domain***: $\left(-\infty,\ -\dfrac{\sqrt{3}}{2}\right)\cup\left(-\dfrac{\sqrt{3}}{2},\ \dfrac{\sqrt{3}}{2}\right)\cup\left(\dfrac{\sqrt{3}}{2},\ \infty\right)$

VA: $x=-\dfrac{\sqrt{3}}{2}$ *and* $x=\dfrac{\sqrt{3}}{2}$ ***HA***: $y=\dfrac{3}{4}$

Hole: n/a ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x+6}{x^3+2x^2}$

Solution

$x^3+2x^2=x^2(x+2)=0 \quad \rightarrow \quad x=0,-2$ ***Domain***: $(-\infty,\ -2)\cup(-2,\ 0)\cup(0,\ \infty)$

VA: $x=0$ *and* $x=2$ ***HA***: $y=0$

Hole: n/a ***Oblique asymptote***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2+4x-1}{x+3}$

Solution

$x+3=0 \quad \rightarrow \quad x=-3$ ***Domain***: $(-\infty,\ -3)\cup(-3,\ \infty)$

$$\begin{array}{r|l} & x+1 \\ \hline x+3 & x^2+4x-1 \\ & -x^2-3x \\ \hline & x-1 \\ & -x-3 \\ \hline & -4 \end{array}$$

$f(x)=\dfrac{x^2+4x-1}{x+3}=x+1-\dfrac{4}{x+3}$

VA: $x=-3$ ***HA***: n/a

Hole: n/a ***Oblique asymptote***: $y=x+1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2-6x}{x-5}$

Solution

$x-5=0 \;\rightarrow\; x=5$ ***Domain***: $(-\infty,\ 5)\cup(5,\ \infty)$

$$\begin{array}{r|l} x-5 & x^2-6x \\ & x-1 \\ \hline & -x^2+5x \\ & -x \\ \hline & x-5 \\ & -5 \end{array}$$

VA: $x=5$

HA: N/A

Hole: N/A

Oblique asymptote: $y=x-1$

$$f(x)=\frac{x^2-6x}{x-5}=x-1-\frac{5}{x-5}$$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3-x^2+x-4}{x^2+2x-1}$

Solution

$x^2+2x-1=0 \;\rightarrow\; x=-1\pm\sqrt{2}$

Domain: $\left(-\infty,\ -1-\sqrt{2}\right)\cup\left(-1-\sqrt{2},\ -1+\sqrt{2}\right)\cup\left(-1+\sqrt{2},\ \infty\right)$

$$\begin{array}{r|l} x^2+2x-1 & x^3-x^2+x-4 \\ & x-3 \\ \hline & -x^3-2x^2+x \\ & -3x^2+2x-4 \\ \hline & 3x^2+6x-3 \\ & 8x-7 \end{array}$$

VA: $x=-1\pm\sqrt{2}$

HA: N/A

Hole: N/A

Oblique asymptote: $y=x-3$

$$f(x)=\frac{x^3-x^2+x-4}{x^2+2x-1}=x-3+\frac{8x-7}{x^2+2x-1}$$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{-3x}{x+2}$

Solution

VA: $x=-2$ ***HA***: $y=-3$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x+1}{x^2+2x-3}$

Solution

VA: $x=-3,\ 1$ ***HA***: $y=0$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{2x^2-2x-4}{x^2+x-12}$

Solution

VA: $x=-4,\ 3$ ***HA***: $y=2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{-2x^2+10x-12}{x^2+x}$

Solution

VA: $x=-1,\ 0$ ***HA***: $y=-2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2-x-6}{x+1}$

Solution

$$x+1\overline{)\,x^2-x-6}\quad x-2$$

$$\begin{array}{r} x^2+x \\ -2x-6 \\ -2x-2 \\ \hline \boxed{-4} \end{array}$$

$$f(x)=\frac{x^2-x-6}{x+1}=x-2-\frac{4}{x+1}$$

The oblique asymptote is: $y=x-2$

The vertical asymptote is: $x=-1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3+1}{x-2}$

Solution

$$
\begin{array}{r}
x^2+2x+4 \\
x-2\overline{)\,x^3-1} \\
\underline{x^3-2x^2} \\
2x^2 \\
\underline{2x^2-4x} \\
4x-1 \\
\underline{4x-8} \\
7
\end{array}
$$

The oblique asymptote is:

$y = x^2 + 2x + 4$

$f(x) = x^2 + 2x + 4 + \frac{7}{x-2}$

The vertical asymptote is: $x = 2$

Exercise

Determine all asymptotes of the function $f(x) = \frac{2x^2 + x - 6}{x^2 + 3x + 2}$

Solution

$$f(x) = \frac{(2x-3)(x+2)}{(x+1)(x+2)}$$

$$= \frac{2x-3}{x+1}$$

VA	$x = -1$
HA	$y = 2$
Hole	$x = -2$

Exercise

Determine all asymptotes of the function $f(x) = \frac{x-1}{1-x^2}$

Solution

$$f(x) = \frac{x-1}{(1-x)(1+x)}$$

$$f(x) = -\frac{1}{1+x}$$

VA	$x = -1$
HA	$y = 0$
Hole	$x = 1$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2+x-2}{x+2}$

Solution

$$f(x)=\frac{(x+2)(x-1)}{x+2}$$
$$=x-1$$

VA	*na*
HA	*na*
Hole	$x=-2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^3-2x^2-4x+8}{x-2}$

Solution

$$f(x)=\frac{\left(x^2-4\right)(x-2)}{x-2}=x^2-4$$

Hole	$x=2$

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{2x^2-3x-1}{x-2}$

Solution

$$\begin{array}{r} 2x+1 \\ x-2\overline{)\,2x^2-3x-1} \\ \underline{-2x^2+4x} \\ x-1 \\ \underline{-x+2} \\ 1 \end{array}$$

$$f(x)=\frac{2x^2-3x-1}{x-2}=(2x+1)+\frac{1}{x-2}$$

The ***oblique asymptote*** is the line $y=2x+1$

VA: $x=2$ ***HA***: $y=1$ ***Hole***: n/a

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{2x+3}{3x^2+7x-6}$

Solution

$3x^2+7x-6=0 \quad \Rightarrow \quad x=-3,\ \dfrac{2}{3}$

VA: $x=-3$ *and* $x=\dfrac{2}{3}$ ***HA***: $y=0$

Hole: *n / a* ***OA:*** *n / a*

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{x^2-1}{x^2+x-6}$

Solution

$x^2+x-6=0 \quad \Rightarrow \quad x=-3,\ 2$

$1=\dfrac{x^2-1}{x^2+x-6} \Rightarrow x^2+x-6=x^2-1$

$\boxed{x=5}$

VA: $x=-3$ *and* $x=2$ ***HA***: $y=1$

Hole: *n / a* ***OA:*** *n / a*

Exercise

Determine all asymptotes of the function $f(x)=\dfrac{-2x^2-x+15}{x^2-x-12}$

Solution

$x^2-x-12=0 \quad \Rightarrow \quad x=-3,\ 4$

Domain: $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

$f(x)=\dfrac{(-2x+5)(x+3)}{(x-4)(x+3)}=\dfrac{-2x+5}{x-4}$

VA: $x=4$ ***HA***: $y=-2$

Hole: $x=-3 \quad \rightarrow \quad y=-\dfrac{11}{7}$ ***OA:*** *n / a*

hole $\left(-3,\ -\dfrac{11}{7}\right)$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=4 \\ \textit{horizontal asymptote}: \ y=-1 \\ x-\textit{intercept}: \ 3 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\dfrac{}{x-4}$

Horizontal Asymptote: $f(x)=\dfrac{-x+a}{x-4}$

x-intercept: $f(x=3)=\dfrac{-3+a}{3-4}$

$$0=-3+a$$

$$a=3$$

$$f(x)=\frac{-x+3}{x-4}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=-3, x=1 \\ \textit{horizontal asymptote}: \ y=0 \\ x-\textit{intercept}: \ -1, \ f(0)=-2 \\ \textit{hole at} \ \ x=2 \end{cases}$$

Solution

Vertical Asymptote: $f(x)=\dfrac{}{(x+3)(x-1)}$

Horizontal Asymptote: $f(x)=\dfrac{ax+b}{(x+3)(x-1)}$

x-intercept: $f(x=-1)=\dfrac{a(-1)+b}{(-1+3)(-1-1)}=\dfrac{-a+b}{-4}=0$

$$-a+b=0 \ \Rightarrow \ a=b$$

$$f(x=0)=\frac{a(0)+b}{(0+3)(0-1)}=\frac{b}{-3}=-2$$

$$\boxed{b=6=a}$$

$$f(x)=\frac{6x+6}{(x+3)(x-1)}$$

Hole at $x=2$: $f(x)=\dfrac{6x+6}{(x+3)(x-1)}\dfrac{x-2}{x-2}$

$$= \frac{6(x+1)(x-2)}{(x^2+2x-3)(x-2)}$$

$$= \boxed{\frac{6(x^2-x-2)}{x^3-7x+6}}$$

Exercise

Find an equation of a rational function f that satisfies the given conditions

$$\begin{cases} \textit{vertical asymptote}: \ x=-4, x=5 \\ \textit{horizontal asymptote}: \ y=\frac{3}{2} \\ x-\textit{intercept}: \ -2 \end{cases}$$

Solution

Vertical Asymptote: $f(x) = \frac{}{(x+4)(x-5)}$

Horizontal Asymptote: $f(x) = \frac{3}{2}\frac{(x+a)(x+b)}{(x+4)(x-5)}$

x-intercept: $f(x=-2) = \frac{3}{2}\frac{(-2+a)(-2+b)}{}$

$$0 = (-2+a)(-2+b)$$

$$\boxed{a=b=2}$$

$$f(x) = \frac{3}{2}\frac{(x-2)^2}{x^2-x-20}$$

$$= \boxed{\frac{3x^2-12x+12}{2x^2-2x-40}}$$