#1 a) $f(t) = \sqrt{t-4}$

$$f'(t) = \frac{1}{2\sqrt{t-4}}$$

b) $f(x) = \frac{1}{x+2}$ $\quad f'(x) = \frac{-1}{(x+2)^2}$

c) $f(x) = 3x^4 - 3x^3 + 6x^2 - x + 5$

$$f'(x) = 12x^3 - 9x^2 + 12x - 1$$

d) $y = \frac{2}{\sqrt[3]{x^2}} = 2x^{-2/3}$

$$y' = \frac{-4}{3x^{5/3}}$$

e) $g(t) = \frac{t^2-1}{t+4}$

$$g'(t) = \frac{2t^2+8t - t^2+1}{(t+4)^2}$$

$$= \frac{t^2+8t+1}{(t+4)^2}$$

f) $y = (x^5 - 3x)\left(\frac{1}{x^2}\right) = x^3 - \frac{3}{x}$

$$y' = 3x^2 + \frac{3}{x^2}$$

g) $f(x) = (x+3)\left(1 - \frac{2}{x-3}\right)$

$$f'(x) = 1 - \frac{2}{x-3} + \frac{2}{(x-3)^2}(x+3)$$

$$= \frac{x^2-6x+9-2x+6+2x+6}{(x-3)^2}$$

$$= \frac{x^2-6x+21}{(x-3)^2}$$

h) $R(s) = \frac{s^3-2s^2+3}{\sqrt{s-2}}$

$$R'(s) = \frac{(3s^2-4s)(s-2) - \frac{1}{2}(s^3-2s^2+3)}{(s-2)^2}$$

$$= \frac{6s^3-20s^2+16s-s^3+2s^2-3}{2(s-2)^2}$$

$$= \frac{5s^3-18s^2+16s-3}{2(s-2)^2}$$

#1 cont

i) $y = \frac{x+3}{x-4}(x+5) = \frac{x^2+8x+15}{x-4}$ $\left(\frac{u}{v}\right)'$

$y' = \frac{(2x+8)(x-4) - x^2 - 8x - 15}{(x-4)^2}$

$= \frac{x^2 - 8x - 47}{(x-4)^2}$

j) $f(x) = \sqrt{x^2 - 3x + 5}$

$f'(x) = \frac{2x-3}{2\sqrt{x^2-3x+5}}$

k) $h(t) = (2t^2 - 3t + 4)^3 \sqrt{t^2 - 3}$ $(u^n v^m)' = u^{n-1} v^{m-1} (n u' v + m u v')$

$h'(t) = \frac{(2t^2-3t+4)^2}{\sqrt{t^2-3}} \left(3(4t-3)(t^2-3) + t(2t^2-3t+4) \right)$

$= \frac{(2t^2-3t+4)^2 (14t^3 - 12t^2 - 32t + 27)}{\sqrt{t^2-3}}$

l) $y = \sqrt{x}\,(x+2)^2$ $(u^n v^m)'$

$y' = \frac{x+2}{\sqrt{x}} \left(\frac{1}{2}x + 1 + 2x \right)$

$= \frac{(x+2)(5x+2)}{2\sqrt{x}}$

m) $Q(w) = \frac{w+1}{\sqrt{2w+3}}$ $(u^n v^m)' =$

$Q'(w) = \frac{1}{(2w+3)^{3/2}} (2w + 3 - w - 1)$

$= \frac{w+2}{(2w+3)^{3/2}}$

n) $y = x^7 + \sqrt{7}\,x - \frac{1}{7+1}$

$y' = 7x^6 + \sqrt{7}$

#1 cont

o) $f(t) = \frac{\sqrt{t}}{1+\sqrt{t}}$ $\qquad (u^n v^m)' = u^{n-1} v^{m-1}(nu'v + muv')$

$$f'(t) = \frac{1}{\sqrt{t}\,(1+\sqrt{t})^2}\left(\frac{1}{2} + \frac{1}{2}\sqrt{t} - \frac{1}{2\sqrt{t}}\,t\right)$$

$$= \frac{1}{2\sqrt{t}\,(1+\sqrt{t})^2}$$

p) $f(x) = \left(\frac{2\sqrt{x}}{1+2\sqrt{x}}\right)^2 = \frac{4x}{(1+2\sqrt{x})^2}$ $\qquad (u^n v^m)'$

$$f'(x) = \frac{4}{(1+2\sqrt{x})^3}\left(1+2\sqrt{x} - 2\frac{1}{\sqrt{x}}\,x\right)$$

$$= \frac{4}{(1+2\sqrt{x})^3}$$

q) $y = \sqrt{\frac{x^2+x}{x^2}} = \left(1+\frac{1}{x}\right)^{1/2}$

$$y' = \frac{1}{2}\left(-\frac{1}{x^2}\right)\left(1+\frac{1}{x}\right)^{-1/2}$$

$$= \frac{-1}{2x^2}\left(\frac{x}{1+x}\right)^{1/2}$$

r) $y = (2x+1)\sqrt{2x+1}$

$$= (2x+1)^{3/2}$$

$$y' = 3(2x+1)^{1/2} = \boxed{3\sqrt{2x+1}}$$

#2

a) $y = 2\tan^2 x - \sec^2 x$

$y' = 4\tan x \sec^2 x - 2\sec^2 x \tan x$
$= 2\tan x \sec^2 x$

b) $y = \dfrac{1}{\sin^2 x} - \dfrac{2}{\sin x}$

$y' = \dfrac{-2\cos x}{\sin^3 x} + \dfrac{2\cos x}{\sin x}$
$= 2\cot x - 2\cot x \csc^2 x$
$= 2\cot x\,(1 - \csc^2 x)$

c) $y = (\sec x + \tan x)^5$

$y' = 5(\sec x + \tan x)^4 (\sec x \tan x + \sec^2 x)$
$= \boxed{5\sec x\,(\sec x + \tan x)^5}$

d) $r = \sqrt{2\theta \sin\theta}$

$r' = \dfrac{2(\sin\theta + \theta\cos\theta)}{2\sqrt{2\theta\sin\theta}} = \dfrac{\sin\theta + \theta\cos\theta}{\sqrt{2\theta\cdot\sin\theta}}$

e) $r = \sin\cdot(\theta + \sqrt{\theta+1})$

$r' = \left(1 + \dfrac{1}{2\sqrt{\theta+1}}\right)\cos\left(\theta + \sqrt{\theta+1}\right)$

f) $y = 2\sqrt{x}\sin\sqrt{x}$

$(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$

$y' = \dfrac{\sin\sqrt{x}}{\sqrt{x}} + \cos\sqrt{x}$

g) $y = x^2 \sin^2(2x^2)$

$y' = 2x\sin^2(2x^2) + 8x^3 \sin(2x^2)\cos(2x^2)$

#2 cont

b) $r = \left(\frac{\sin\theta}{\cos\theta - 1}\right)^2 = \frac{\sin^2\theta}{(\cos\theta-1)^2}$

$r' = \frac{\sin\theta}{(\cos\theta-1)^3}\left(2\cos\theta(\cos-1) + 2\sin^2\theta\right)$

$= \frac{\sin\theta}{(\cos\theta-1)^3}(2 - 2\cos\theta)$

$= \boxed{\frac{-2\sin\theta}{(\cos\theta-1)^2}}$

c) $y = (3+\cos^3 3x)^{-1/3}$

$y' = -\frac{1}{3}\left(9\cos^2 3x(-\sin 3x)\right)(3+\cos^3 3x)^{-4/3}$

$= \frac{3\sin 3x\cos^2 3x}{(3+\cos^3 3x)^{4/3}}$

#3

a) $f(x) = 3x^4 - 3x^3 + 6x^2 - x + 5$

$f^{(4)}(x) = 3(4!) = \boxed{72}$

b) $f(x) = 6x^5 - 3x^4 - 2x + e$

$f^{(5)}(x) = 6(5!) = \boxed{720}$

c) $y = \frac{x^2+7}{x} = x + \frac{7}{x}$

$y' = 1 - \frac{7}{x^2}$

$y'' = \frac{14}{x^3}$

$y''' = \frac{-42}{x^4}$

#4

a) $y = \sqrt{2}\, e^{\sqrt{2}x}$

$y' = 2e^{\sqrt{2}x}$

b) $y = \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$

$y' = \frac{1}{4}e^{4x} + xe^{4x} - \frac{1}{4}e^{4x}$

$= xe^{4x}$

c) $y = \ln(\sec^2\theta)$

$y' = \frac{2\sec^2\theta\tan\theta}{\sec^2\theta} = 2\tan\theta$

d) $y = \log_5(3x-7)$

$y' = \frac{3}{(3x-7)\ln 5}$

e) $y = (x+2)^{x+2}$

$\ln y = (x+2)\ln(x+2)$

$\frac{y'}{y} = \ln(x+2) + 1$

$y' = (x+2)^{x+2}(\ln(x+2)+1)$

f) $y = \sin^{-1}\left(\frac{1}{\sqrt{x}}\right)$

$y' = \frac{-\frac{1}{2x\sqrt{x}}}{\sqrt{1-\frac{1}{x}}}$

$= \frac{-\sqrt{x}}{2x\sqrt{x}\sqrt{x-1}}$

$= \frac{-1}{2x\sqrt{x-1}}$

#4 cont

g) $y = z\cos^{-1}z - \sqrt{1-z^2}$

$$y' = \cos^{-1}z + z\frac{1}{\sqrt{1-z^2}} - \frac{z}{\sqrt{1-z^2}}$$

$$= \cos^{-1}z$$

h) $y = t\tan^{-1}t - \frac{1}{2}\ln t$

$$y' = \tan^{-1}t + \frac{t}{1+t^2} - \frac{1}{2t}$$

i) $y = \sqrt[10]{\dfrac{3x+4}{2x-4}}$

$$= \left(\frac{3x+4}{2x-4}\right)^{1/10}$$

$$y' = \frac{1}{10}\left(\frac{3x+4}{2x-4}\right)^{-9/10}\left(\frac{6x-8-6x-8}{(2x-4)^2}\right)$$

$$= \frac{1}{10}\left(\frac{2x-4}{3x+4}\right)^{9/10}\frac{-16}{(2x-4)^2}$$

$$= -\frac{8}{5}\frac{1}{(2x-4)^2}\left(\frac{2x-4}{3x+4}\right)^{9/10}$$

$$\ln y = \frac{1}{10}\left(\ln(3x+4) - \ln(2x-4)\right)$$

$$\frac{y'}{y} = \frac{1}{10}\left(\frac{3}{3x+4} - \frac{2}{2x-4}\right)$$

$$y' = \frac{1}{10}\sqrt[10]{\frac{3x+4}{2x-4}}\left(\frac{3}{3x+4} - \frac{2}{2x-4}\right)$$

j) $y = \left(\dfrac{(t+1)(t-1)}{(t-2)(t+3)}\right)^5$

$$\ln y = 5\left(\ln(t+1) + \ln(t-1) - \ln(t-2) - \ln(t+3)\right)$$

$$\frac{y'}{y} = 5\left(\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t+3}\right)$$

$$y' = 5\left(\frac{(t+1)(t-1)}{(t-2)(t+3)}\right)^5\left(\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t+3}\right)$$

#4 cont

k) $y = (\sin\theta)^{\sqrt{\theta}}$

$\ln y = \sqrt{\theta}\,\ln \sin\theta$

$\frac{y'}{y} = \frac{1}{2\sqrt{\theta}} \ln \sin\theta + \sqrt{\theta}\,\frac{\cos\theta}{\sin\theta}$

$y' = \left(\frac{\ln \sin\theta}{2\sqrt{\theta}} + \sqrt{\theta}\cot\theta\right)(\sin\theta)^{\sqrt{\theta}}$

#5

a) $xy + 2x + 3y = 1$

$y + xy' + 2 + 3y' = 0$

$y'(x+3) = -y - 2$

$\boxed{\frac{dy}{dx} = -\frac{y+2}{x+3}}$

b) $x^3 + 4xy - 3y^{4/3} = 2x$

$3x^2 + 4y + 4xy' - 4y^{1/3}y' = 2$

$y'(4x - 4y^{1/3}) = 2 - 3x^2 - 4y$

$\boxed{y' = \frac{2 - 3x^2 - 4y}{4x - 4y^{1/3}}}$

c) $x^2y^2 = 1$

$2xy^2 + 2x^2yy' = 0$

$\boxed{y' = -\frac{y}{x}}$

d) $y^2 = \sqrt{\frac{1+x}{1-x}}$

$2\ln y = \frac{1}{2}(\ln(1+x) - \ln(1-x))$

$\frac{y'}{y} = \frac{1}{4}\left(\frac{1}{1+x} - \frac{1}{1-x}\right) \Rightarrow \boxed{y' = \frac{-1}{2}\left(\frac{1+x}{1-x}\right)^{1/4}\left(\frac{x}{1-x^2}\right)}$

#6 $x^3+y^3=1$

$3x^2+3y^2y'=0$

$x^2+y^2y'=0 \to y'=-\frac{x^2}{y^2}$

$2x+2y(y')^2+y^2y''=0$

$y^2y''=-2-2y\left(-\frac{x^2}{y^2}\right)^2$

$=-2+\frac{2x^4}{y^3}$

$y''=-\frac{2}{y^2}+\frac{2x^4}{y^5}$

$$\boxed{=\frac{2x^4-2y^3}{y^5}}$$

#7 $y=x^2+C$; tangent to $y=x$

$y=x \to m=y'=1.$

$y'=2x=1 \to x=\frac{1}{2}=y$

$\frac{1}{2}=\frac{1}{4}+C \to \boxed{C=\frac{1}{4}}$

#8 $x^2+2y^2=9$ @ $(1,2)$

$2x+4yy'=0 \to y'=\frac{-x}{2y}\Big|_{(1,2)}=-\frac{1}{4}=m$

tangent line: $y=-\frac{1}{4}(x-1)+2$

$=-\frac{1}{x}x+\frac{9}{4}$

normal line: $m=4$

$y=4(x-1)+2$

$=4x-2$

#9 $\frac{dV}{dt} = 8 \text{ cm}^3/\text{sec}$ $V = \frac{4}{3}\pi r^3$ $\frac{dr}{dt}$ @ $r = 10\text{cm}$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2} \left.\frac{dV}{dt}\right|_{r=10}$$

$$= \frac{1}{400\pi} 8$$

$$= \boxed{\frac{1}{50\pi} \text{ cm/sec}}$$

#10 $x = -0.01t^4 + 0.3t^3 + 0.4t^2 + 12t$

$$v = x' = -.04t^3 + .9t^2 + .8t + 12$$

$$v(20) = -.04(20)^3 + .9(20)^2 + .8(20) + 12$$

$$= \boxed{68 \text{ ft/sec}}$$

#11 $f(t) = 10.72(.9t+10)^{.3}$; $0 \le t \le 20$

$$f'(t) = 2.8944(.9t+10)^{-.7}$$

2010 $\rightarrow t = 10$

$$f'(10) = \frac{2.8944}{19^{.7}} \approx \boxed{.3685}$$

#12 $S = 2\pi r^2 + 2\pi r h$

a) $\frac{dS}{dt} = 4\pi r \frac{dr}{dt} + 2\pi h \frac{dr}{dt}$

$$= 2\pi(2r+h)\frac{dr}{dt}$$

b) $\frac{dS}{dt} = 2\pi r \frac{dh}{dt}$

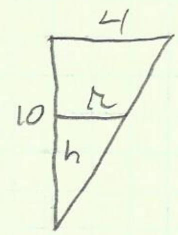
c) $\frac{dS}{dt} = 2\pi(2r+h)\frac{dr}{dt} + 2\pi r \frac{dh}{dt}$.

#13 $y = x^{3/2}$ $\quad \left.\frac{dx}{dt}\right|_{x=3} = ?$ $\quad \frac{dy}{dt} = 11$ units/sec

$$\frac{dy}{dt} = \frac{3}{2} x^{1/2} \frac{dx}{dt}$$

$$11 = \frac{3}{2}\sqrt{3}\,\frac{dx}{dt} \Rightarrow \boxed{\frac{dx}{dt} = \frac{22}{3\sqrt{3}} = \frac{22\sqrt{3}}{9} \text{ units/sec}}$$

#14 $\frac{dV}{dt} = 5\ ft^3/min$



a) $\frac{h}{r} = \frac{10}{4} \Rightarrow 2h = 5r$

b) $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\left(\frac{2h}{5}\right)^2 h$$

$$= \frac{4}{75}\pi h^3$$

$$\frac{dV}{dt} = \frac{4}{25}\pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{25}{4\pi h^2}\frac{dV}{dt} \quad @\ h = 6$$

$$= \frac{25}{144\pi}(-5)$$

$$= \boxed{\frac{-125}{144\pi}} \text{ ft/min}$$

#15 $s = r\theta$ $\quad r = 1.2$ ft $\quad \frac{ds}{dt} = 6$ ft/sec

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$6 = 1.2\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \boxed{5 \text{ rad/sec}}$$