$$\int \frac{6}{x^2-1}\,dx$$

$$\frac{6}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$6 = (A+B)x + A - B$$

$$\begin{cases} A+B=0 \\ A-B=6 \end{cases} \Rightarrow A=3,\ B=-3$$

$$\int \frac{6}{x^2-1}\,dx = 3\int \frac{dx}{x-1} - 3\int \frac{dx}{x+1}$$

$$= 3\ln|x-1| - 3\ln|x+1| + C$$

$$= \ln\left|\frac{x-1}{x+1}\right|^3 + C$$

$$\int \frac{21x^2}{x^3-x^2-12x}\,dx$$

$$\frac{21x}{x^2-x-12} = \frac{A}{x+3} + \frac{B}{x-4}$$

$$21x = (A+B)x - 4A + 3B$$

$$\begin{cases} A+B=21 \\ -4A+3B=0 \end{cases} \Rightarrow B=12\ \ A=9$$

$$\int \frac{21x^2}{x^3-x^2-12x}\,dx = 9\int \frac{dx}{x+3} + 12\int \frac{dx}{x-4}$$

$$= 9\ln|x+3| + 12\ln|x-4| + C$$

$$\int \frac{10x}{x^2-2x-24}\,dx$$

$$\frac{10x}{x^2-2x-24} = \frac{A}{x+4} + \frac{B}{x-6}$$

$$10x = (A+B)x - 6A + 4B$$

$$\begin{cases} A+B=10 \\ -6A+4B=0 \end{cases} \quad B=6 \Rightarrow A=4$$

$$\int \frac{10x}{x^2-2x-24}\,dx = 4\int \frac{dx}{x+4} + 6\int \frac{dx}{x-6}$$

$$= 4\ln|x+4| + 6\ln|x-6| + C$$

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$$\int \frac{x+1}{x^3+3x^2-18x}\,dx$$

$$\frac{x+1}{x^3+3x^2-18x} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+6}$$

$$x+1 = (A+B+C)x^2 + (3A+6B-3C)x - 18A$$

$$\begin{cases} -18A = 1 \Rightarrow \boxed{A = -\tfrac{1}{18}} \\ 3A+6B-3C = 1 \\ A+B+C = 0 \end{cases} \quad \begin{cases} 6B-3C = \tfrac{7}{6} \\ B+C = \tfrac{1}{18} \end{cases}$$

$$9B = \frac{4}{3} \Rightarrow \boxed{B = \frac{4}{27}} \quad \boxed{C = \frac{-5}{54}}$$

$$\int \frac{x+1}{x^3+3x^2-18x}\,dx = -\frac{1}{18}\int \frac{dx}{x} + \frac{4}{27}\int \frac{dx}{x-3} - \frac{5}{54}\int \frac{dx}{x+6}$$

$$= \boxed{\frac{-1}{18}\ln|x| + \frac{4}{27}\ln|x-3| - \frac{5}{54}\ln|x+6| + K}$$

$$\int \frac{x^2+12x-4}{x^3-4x}\,dx$$

$$\frac{x^2+12x-4}{x^3-4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$x^2+12x-4 = (A+B+C)x^2 + (2B-2C)x - 4A$$

$$\begin{cases} -4A = -4 \Rightarrow \boxed{A=1} \\ 2B-2C = 12 \\ A+B+C = 1 \end{cases} \rightarrow \begin{cases} B-C = 6 \\ B+C = 0 \end{cases} \quad \boxed{\begin{matrix} B=3 \\ C=-3 \end{matrix}}$$

$$\int \frac{x^2+12x-4}{x^3-4x}\,dx = \int \frac{dx}{x} + 3\int \frac{dx}{x-2} - 3\int \frac{dx}{x+2}$$

$$= \boxed{\ln|x| + 3\ln|x-2| - 3\ln|x+2| + K}$$

$$= \boxed{\ln\left|\frac{x(x-2)^3}{(x+2)^3}\right| + K}$$

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$$\int \frac{6x^2}{x^4-5x^2+4}\,dx$$

$(x^2-1)(x^2-4)$

$$\frac{6x^2}{x^4-5x^2+4} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2} + \frac{D}{x+2}$$

$$6x^2 = A(x+1)(x^2-4) + B(x-1)(x^2-4) + C(x+2)(x^2-1) + D(x-2)(x^2-1)$$

$$= x^3(A + B + C + D$$
$$x^2(A - B + 2C - 2D$$
$$x^1(-4A - 4B - C - D$$
$$x^0(-4A + 4B - 2C + 2D$$

$$\left|\begin{array}{l} A + B + C + D = 0 \\ A - B + 2C - 2D = 6 \\ +4A + 4B + C + D = 0 \\ -4A + 4B - 2C + 2D = 0 \end{array}\right. \rightarrow \begin{array}{ll} A = -1 & B = 1 \\ C = 2 & D = -2 \end{array}$$

$$\int \frac{6x^2}{x^4-5x^2+4}\,dx = -\int\frac{dx}{x-1} + \int\frac{dx}{x+1} + 2\int\frac{dx}{x-2} - 2\int\frac{dx}{x+2}$$

$$= -\ln|x-1| + \ln|x+1| + 2\ln|x-2| - 2\ln|x+2| + K$$

$$= \ln\left|\frac{(x-2)^2(x+1)}{(x+2)^2(x-1)}\right| + K$$

$$\int \frac{4x-2}{x^3-x}\,dx$$

$$\frac{4x-2}{x^3-x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$4x-2 = Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx$$

$$\begin{array}{lll} A + B + C = 0 & \left.\begin{array}{l} B + C = -2 \\ B - C = 4 \end{array}\right\} & \begin{array}{l} B = 1 \\ C = -3 \end{array} \\ B - C = 4 & & \\ -A = -2 \Rightarrow A = 2 & & \end{array}$$

$$\int \frac{4x-2}{x^3-x}\,dx = 2\int\frac{dx}{x} + \int\frac{dx}{x-1} - 3\int\frac{dx}{x+1}$$

$$= 2\ln|x| + \ln|x-1| - 3\ln|x+1| + K$$

$$= \ln\left|\frac{x^2(x-1)}{(x+1)^3}\right| + K$$

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$$\int_{-1}^{2} \frac{5x}{x^2-x-6}\,dx$$

$$\frac{5x}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$5x = (A+B)x + 2A - 3B$$

$$\begin{cases} A+B=5 \\ 2A-3B=0 \end{cases} \Rightarrow A=3 \quad B=2$$

$$\int_{-1}^{2} \frac{5x}{x^2-x-6}\,dx = 3\int_{-1}^{2}\frac{dx}{x-3} + 2\int_{-1}^{2}\frac{dx}{x+2}$$

$$= 3\ln|x-3| + 2\ln|x+2| \Big|_{-1}^{2}$$

$$= 2\ln 4 - 3\ln 4$$

$$= \ln 16 - \ln 64$$

$$= \ln \frac{16}{64}$$

$$= \ln \frac{1}{4}$$

$$= -\ln 4$$

$$\int_{0}^{5} \frac{2}{x^2-4x-32}\,dx$$

$$\frac{2}{x^2-4x-32} = \frac{A}{x-8} + \frac{B}{x+4}$$

$$2 = (A+B)x + 4A - 8B$$

$$\begin{cases} A+B=0 \\ 4A-8B=2 \end{cases} \rightarrow A=\frac{1}{6},\ B=-\frac{1}{6}$$

$$\int_{0}^{5} \frac{2}{x^2-4x-32}\,dx = \frac{1}{6}\int_{0}^{5}\frac{dx}{x-8} - \frac{1}{6}\int_{0}^{5}\frac{dx}{x+4}$$

$$= \frac{1}{6}\ln|x-8| - \frac{1}{6}\ln|x+4| \Big|_{0}^{5}$$

$$= \frac{1}{6}\left[\ln 3 - \ln 9 - \ln 8 + \ln 4\right]$$

$$= \frac{1}{6}\left(\ln\frac{1}{3} - \ln 2\right)$$

$$= \frac{1}{6}\ln\frac{1}{6}$$

$$= -\frac{\ln 6}{6}$$

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$$\int \frac{81}{x^3-9x^2}\,dx$$

$$\frac{81}{x^3-9x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-9}$$

$$81 = Ax^2 - 9Ax + Bx - 9B + Cx^2$$

$$\begin{cases} A + C = 0 & C = 1 \\ -9A + B = 0 & \rightarrow A = -1 \\ -9B = 81 & \rightarrow B = -9 \end{cases}$$

$$\int \frac{81}{x^3-9x^2}\,dx = -\int \frac{dx}{x} - 9\int \frac{dx}{x^2} + \int \frac{dx}{x-9}$$

$$= -\ln|x| + \frac{9}{x} + \ln|x-9| + K$$

$$= \frac{9}{x} + \ln\left|\frac{x-9}{x}\right| + K$$

$$\int \frac{16x^2}{(x-6)(x+2)^2}\,dx$$

$$\frac{16x^2}{(x-6)(x+2)^2} = \frac{A}{x-6} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$16x^2 = Ax^2 + 4Ax + 4A + Bx^2 - 4Bx - 12B + Cx - 6C$$

$$\begin{cases} A + B = 16 & A = 9 & C = -8 \\ 4A - 4B + C = 0 & B = 7 \\ 4A - 12B - 6C = 0 \end{cases}$$

$$\int \frac{16x^2}{(x-6)(x+2)^2}\,dx = 9\int \frac{dx}{x-6} + 7\int \frac{dx}{x+2} - 8\int \frac{dx}{(x+2)^2}$$

$$= 9\ln|x-6| + 7\ln|x+2| - 8\frac{1}{x+2} + K$$

$$= \ln\left|(x-6)^9(x+2)^7\right| - \frac{8}{x+2} + K$$

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$$\int \frac{8(x^2+4)}{x(x^2+8)}\,dx =$$

$$\frac{8x^2+32}{x(x^2+8)} = \frac{A}{x} + \frac{Bx+C}{x^2+8}$$

$$8x^2+32 = Ax^2+8A+Bx^2+Cx$$

$$\begin{cases} A+B=8 \rightarrow B=4 \\ C=0 \\ 8A=32 \rightarrow A=4 \end{cases}$$

$$\int \frac{8(x^2+4)}{x(x^2+8)}\,dx = 4\int\frac{dx}{x} + \int\frac{4x}{x^2+8}\,dx$$

$$= 4\ln|x| + 2\int\frac{d(x^2+8)}{x^2+8}$$

$$= \boxed{4\ln|x| + 2\ln(x^2+8) + K}$$

$$\int \frac{x^2+x+2}{(x+1)(x^2+1)}\,dx \qquad \frac{x^2+x+2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$x^2+x+2 = Ax^2+A+Bx^2+Bx+Cx+C$$

$$\begin{cases} A+B=1 \\ B+C=1 \\ A+C=2 \end{cases} \rightarrow \quad A=1,\ B=0,\ C=1$$

$$\int \frac{x^2+x+2}{(x+1)(x^2+1)}\,dx = \int\frac{dx}{x+1} + \int\frac{dx}{x^2+1}$$

$$= \boxed{\ln|x+1| + \tan^{-1}x + K}$$

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$$\int \frac{2}{x(x^2+1)^2}\,dx \qquad \frac{2}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$2 = Ax^4 + 2Ax^2 + A + (Bx+C)(x^3+x) + Dx^2 + Ex$$

$$\begin{array}{lll} x^4 & A+B=0 & \rightarrow \underline{B=-2} \\ x^3 & \underline{C=0} & \\ x^2 & 2A+B+D=0 & \rightarrow D=-2 \\ x & C+E=0 & \rightarrow E=0 \\ & \underline{A=2} & \end{array}$$

$$\int \frac{2\,dx}{x(x^2+1)^2} = 2\int\frac{dx}{x} - 2\int\frac{x\,dx}{x^2+1} - 2\int\frac{x\,dx}{(x^2+1)^2}$$

$$= 2\ln|x| - \int\frac{d(x^2+1)}{x^2+1} - \int\frac{d(x^2+1)}{(x^2+1)^2}$$

$$= \underline{2\ln|x| - \ln(x^2+1) + \frac{1}{x^2+1} + K}$$

$$\int \frac{dx}{(x+1)(x^2+2x+2)^2} \qquad \frac{1}{(x+1)(x^2+2x+2)^2} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2x+2} + \frac{Dx+E}{(x^2+2x+2)^2}$$

$$1 = A(x^2+2x+2)^2 + (Bx+C)(x+1)(x^2+2x+2) + (Dx+E)(x+1)$$

$$= Ax^4 + 4Ax^3 + 8Ax^2 + 8Ax + 4A + Bx^4 + 2Bx^3 + 4Bx^2$$
$$+ Bx^3 + Cx^3 + 2Bx + 2Cx + 3Cx^2 + 2Cx + 2C$$
$$+ Dx^2 + Dx + Ex + E$$

$$\begin{array}{ll} x^4 \; A+B=0 & A=1,\; B=-1 \\ 4A+3B+C=0 & C=-1,\; D=-1 \\ 8A+4B+3C+D=0 & E=-1 \\ 8A+2B+4C+D+E=0 & \\ 4A+2C+E=1 & \end{array}$$

$$\int \frac{dx}{(x+1)(x^2+2x+2)^2} = \int\frac{dx}{x+1} - \int\frac{(x+1)\,dx}{x^2+2x+2} - \int\frac{(x+1)\,dx}{(x^2+2x+2)^2}$$

$$= \ln|x+1| - \frac{1}{2}\int\frac{d(x^2+2x+2)}{x^2+2x+2} - \frac{1}{2}\int\frac{d(x^2+2x+2)}{(x^2+2x+2)^2}$$

$$= \underline{\ln|x+1| - \frac{1}{2}\ln|x^2+2x+2| + \frac{1}{2}\frac{1}{(2x^2+2x+2)^2} + K}$$