Solution *Section* 3.5 – Language of Hypothesis Testing

Exercise

Bottles of Bayer aspirin are labeled with a statement that the tablets each contain 325 mg of aspirin. A quality control manager claims that a large sample of data can be used to support the claim that the mean amount of aspirin in the tablets is equal to 325 mg, as the label indicates. Can a hypothesis test be used to support that claim? Why or Why not?

Solution

No. Since the claim that the mean is equal to a specific value must be the null hypothesis, the only possible conclusions are to reject that claim or to fail to reject that claim, Hypothesis testing cannot be used to support a claim that a parameter is equal to a particular value.

Exercise

In the preliminary results from couples using the Gender Choice method of gender selection to increase the likelihood of having a baby girl, 20 couples used the Gender Choice method with the result that 8 of them had baby girls and 12 had baby boys. Given that the sample proportion of girls is $\frac{8}{20}$ *or* 0.4, can the sample data support the claim that the proportion of girls is greater than 0.5? Can any sample proportion less than 0.5 be used to support a claim that the population proportion is greater than 0.5?

Solution

No. Sample data that is not consistent with a claim can't be used to support that claim. In particular, no sample proportion less than 0.5 can ever be used to support a claim that the population proportion is greater than 0.5.

Exercise

Express the null hypothesis H_0 and alternative hypothesis H_1 in symbolic form. Be sure to use the correct symbol $(\mu,\ p,\ \sigma)$ for indicated parameter

a) The mean annual income of employees who took a statistics course is greater than \$60,000.
b) The proportion of people aged 18 to 25 who currently use illicit drugs is equal to 0.20 (or 20%).
c) The standard deviation of human body temperatures is equal to 0.62°F.
d) The majority of college students have credit cards.
e) The proportion of homes with fire extinguishers is 0.80.
f) The mean weight of plastic discarded by households in one week is less than 1 kg.

Solution

a) Original claim: $\mu > \$60,000$

$H_0: \mu = \$60,000 \qquad H_1: \mu > \$60,000$

b) Original claim: $p = 0.20$

$H_0 : p = 0.20 \qquad H_1 : p \neq 0.20$

c) Original claim: $p = 0.20$

$H_0 : \sigma = 0.62°F \qquad H_1 : p \neq 0.62°F$

d) Original claim: $p > 0.5$

$H_0 : p = 0.5 \qquad H_1 : p > 0.5$

e) Original claim: $p = 0.80$

$H_0 : p = 0.80 \qquad H_1 : p \neq 0.80$

f) Original claim: $\mu < 1\ kg$

$H_0 : \mu = 1\ kg \qquad H_1 : \mu < 1\ kg$

Exercise

Assume that the normal distribution applies and find the critical *z* values.

a) Two-tailed test: $\alpha = 0.01$.

b) Right-tailed test: $\alpha = 0.02$.

c) Left-tailed test: $\alpha = 0.10$.

d) $\alpha = 0.05;\quad H_1$ *is* $p \neq 0.4$

e) $\alpha = 0.01;\quad H_1$ *is* $p > 0.5$

f) $\alpha = 0.005;\quad H_1$ *is* $p < 0.8$

g) $\alpha = 0.05$ for two-tailed test

h) $\alpha = 0.05$ for left-tailed test

i) $\alpha = 0.08;\quad H_1$ *is* $\mu \neq 3.25$

Solution

a) Two-tailed test; place $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$ in each tail.

z score	Area
1.645	0.9500
2.575	0.9950

$A = 1 - \frac{\alpha}{2} = 0.995$

Critical value: $\pm z_{\alpha/2} = \pm z_{0.005}\ \underline{= \pm 2.575}$

b) Right-tailed test; place $\alpha = 0.02$ in the upper tail. $\Rightarrow A = 1 - \alpha = 0.98$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

Critical value: $z_{\alpha/2} = z_{0.02}\ \underline{= 2.05}$

c) Left-tailed test; place $\alpha = 0.10$ in the lower tail. $\Rightarrow A = \alpha = 0.1$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985

Critical value: $z_{\alpha} = z_{0.1}\ \underline{= -1.28}$

d) Two-tailed test; place $\frac{\alpha}{2}=\frac{0.05}{2}=0.025$ in each tail. $\rightarrow A=1-\frac{\alpha}{2}=0.975$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

Critical value: $\pm z_{\alpha/2}=\pm z_{0.025}\ \underline{=\pm 1.96|}$

e) Right-tailed test; place $\alpha=0.01$ in the upper tail. $A=1-\alpha=0.99$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916

Critical value: $z_{\alpha}=z_{0.02}\ \underline{=2.33|}$

f) Left-tailed test; place $\alpha=0.005$ in the lower tail. $\Rightarrow A=1-\alpha=0.995$

Critical value: $-z_{\alpha}=-z_{0.005}\ \underline{=-2.575|}$

z score	Area
1.645	0.9500
2.575	0.9950

g) Two-tailed test; place $\frac{\alpha}{2}=\frac{0.05}{2}=0.025$ in each tail.

$A=1-\frac{\alpha}{2}=0.975$

Critical value: $\pm z_{\alpha/2}=\pm z_{0.025}\ \underline{=\pm 1.96|}$

h) Left-tailed test; $\alpha=0.05\ \Rightarrow A=1-\alpha=0.95$

Critical value: $z_{\alpha}=z_{0.05}\ \underline{=-1.645|}$

i) Two-tailed test; place $\frac{\alpha}{2}=\frac{0.08}{2}=0.04$ in each tail. $A=\alpha=0.04$

Critical value: $\pm z_{\alpha/2}=\pm z_{0.04}\ \underline{=\pm 1.75|}$

Exercise

The claim is that the proportion of peas with yellow pods is equal to 0.25 (or 25%). The sample statistics from one of Mendel's experiments include 580 peas with 152 of them having yellow pods. Find the value of the test statistic *z* using $z=\frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$

Solution

$$\hat{p}=\frac{x}{n}=\frac{152}{580}=0.262$$

$$z=\frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}=\frac{0.262-0.250}{\sqrt{\frac{(0.25)(.75)}{580}}}\ \underline{=0.67|}$$

Exercise

The claim is that less than $\frac{1}{2}$ of adults in U.S. have carbon monoxide detectors. A KRC Research survey of 1005 adults resulted in 462 who have carbon monoxide detectors. Find the value of the test statistic z using $z=\dfrac{\hat{p}-p}{\sqrt{\dfrac{pq}{n}}}$

Solution

$$\hat{p}=\frac{x}{n}=\frac{462}{1005}=0.460$$

$$z=\frac{\hat{p}-p}{\sqrt{\dfrac{pq}{n}}}=\frac{0.46-0.5}{\sqrt{\dfrac{(0.5)(.5)}{1005}}}\;\underline{=-2.56}$$

Exercise

The claim is that more than 25% of adults prefer Italian food as their favorite ethnic food. A Harris Interactive survey of 1122 adults resulted in 314 who say that Italian food is their favorite ethnic food. Find the value of the test statistic z using $z=\dfrac{\hat{p}-p}{\sqrt{\dfrac{pq}{n}}}$

Solution

$$\hat{p}=\frac{x}{n}=\frac{314}{1122}=0.28$$

$$z=\frac{\hat{p}-p}{\sqrt{\dfrac{pq}{n}}}=\frac{0.28-0.25}{\sqrt{\dfrac{(0.25)(.75)}{1122}}}\;\underline{=2.31}$$

Exercise

Find P-value by using a 0.05 significance level and state the conclusion about the null hypothesis. (Reject the null hypothesis or fail to reject the null hypothesis)

a) The test statistic in a left-tailed test is $z = -1.25$

b) The test statistic in a right-tailed test is $z = 2.50$

c) The test statistic in a two-tailed test is $z = 1.75$

d) With $H_1: p \neq 0.707$, the test statistic is $z = -2.75$

e) With $H_1: p > \frac{1}{4}$, the test statistic is $z = 2.30$

f) With $H_1: p < 0.777$, the test statistic is $z = -2.95$

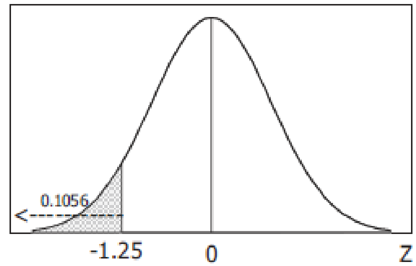
Solution

a) $P\text{-value} = P(z < -1.25)$

$= 0.1056$

z	.00	.01	.02	.03	.04	.05
−1.2	.1151	.1131	.1112	.1093	.1075	.1056

Since 0.1056 > 0.05, fail to reject H_0



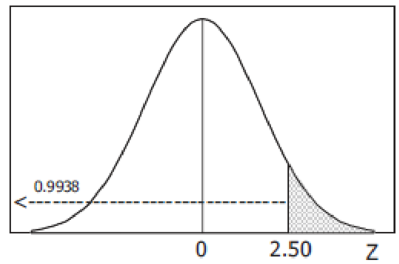
b) $P\text{-value} = P(z > 2.5)$

$= 1 - 0.9938$

$\underline{= 0.0062}$

z	.00	.01
2.5	.9938	

Since 0.0062 < 0.05, reject H_0

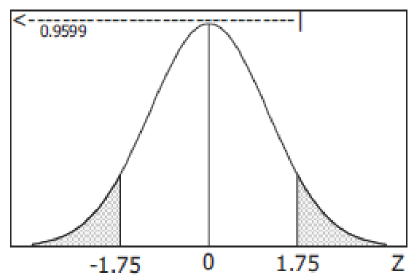


c) $P\text{-value} = 2 \cdot P(z > 1.75)$

$= 2(1 - 0.9599)$

$\underline{= 0.0802}$

Since 0.0802 > 0.05, fail to reject H_0

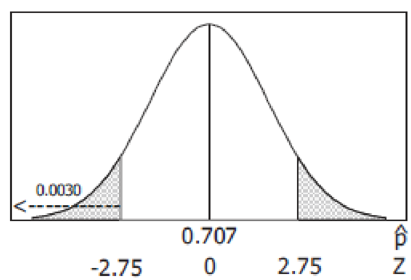


d) $P\text{-value} = 2 \cdot P(z < -2.75)$

$= 2(0.003)$

$\underline{= 0.006}$

Since 0.006 > 0.05, reject H_0

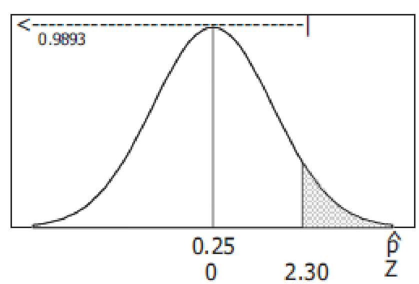


e) $P\text{-value} = P(z > 2.3)$

$= 1 - 0.9893$

$\underline{= 0.0107}$

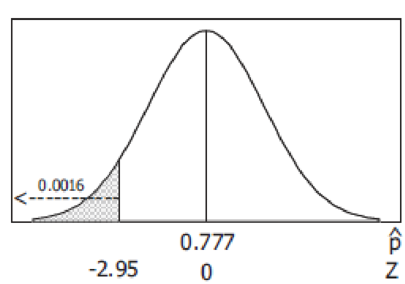
Since 0.0107 < 0.05, reject H_0



f) $P\text{-value} = P(z < -2.95)$

$\underline{= 0.0016}$

Since 0.0016 < 0.05, reject H_0



Exercise

The percentage of nonsmokers exposed to secondhand smoke is equal to 41%. Identify the type I error and type II error.

Solution

Original claim: $p = 0.41$

$H_0 : p = 0.41$

Type I error: rejecting H_0 when H_0 is actually true rejecting the claim that the percentage of non-smokers exposed to secondhand smoke is 41% when that percentage actually is 41%

Type II error: failing to reject H_0 when H_1 is actually true failing to reject the claim that the percentage is actually different from 41%

Exercise

The percentage of Americans who believe that life exists only on earth is equal to 20%. Identify the type I error and type II error.

Solution

Original claim: $p = 0.20$ $\qquad H_0 : p = 0.20$

Type I error: rejecting H_0 when H_0 is actually true rejecting the claim that the percentage of Americans who believe that life exists only on earth is 20% when that percentage actually is 20%

Type II error: failing to reject H_0 when H_1 is actually true failing to reject the claim that the percentage of Americans who believe that life exists only on earth is 20% when that percentage is actually different from 20%

Exercise

The percentage of college students who consume alcohol is greater than 70%. Identify the type I error and type II error.

Solution

Original claim: $p > 0.70$ $\qquad H_0 : p = 0.70$

Type I error: rejecting H_0 when H_0 is actually true rejecting the claim that the percentage of college students who use alcohol is 70% when that percentage actually is 70%.

Type II error: failing to reject H_0 when H_1 is actually true failing to reject the claim that the percentage of college students who use alcohol is 70% when that percentage actually is actually greater than 70%

Exercise

An entomologist writes an article in a scientific journal which claims that fewer than 13 in 10,000 male fireflies are unable to produce light due to a genetic mutation. Use the parameter p, the true proportion of fireflies unable to produce light. Express the null hypothesis and the alternative hypothesis in symbolic form. $(\mu,\ p,\ \sigma)$

<u>Solution</u>

$$p = \frac{13}{10,000} = 0.0013$$

Since the claims are fewer than it will be "<"

$$H_0 : p = 0.0013$$

$$H_1 : p < 0.0013$$