

$$\#1 \quad \frac{2s-1}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2}$$

$$2s-1 = A(s-2) + B(s+1)$$

$$s^1: A + B = 2 \rightarrow [B = 2 - 1 = 1]$$

$$s^0: -2A + B = -1$$

$$-3A = -3$$

$$A = 1$$

$$A = \frac{\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{3}{3} = 1$$

$$\frac{2s-1}{(s+1)(s-2)} = \frac{1}{s+1} + \frac{1}{s-2}$$


---

$$\#2 \quad \frac{2s-2}{(s-4)(s+2)} = \frac{A}{s-4} + \frac{B}{s+2}$$

$$2s-2 = As+2A + Bs-4B$$

$$\begin{cases} s^1: A + B = 2 \\ s^0: 2A - 4B = -2 \end{cases}$$

$$B = 1, A = 1$$

$$\frac{2s-2}{(s-4)(s+2)} = \frac{1}{s-4} + \frac{1}{s+2}$$


---

$$\#3 \quad \frac{7s^2 + 3s + 16}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4}$$

$$7s^2 + 3s + 16 = A(s^2+4) + (s+1)(Bs+C)$$

$$s^2 \quad A + B = 7 \rightarrow [A = 7 - B = 4]$$

$$s^1 \quad B + C = 3 \rightarrow [C = 3 - B = 0]$$

$$s^0 \quad 4A + C = 16$$

$$28 - 4B + 3 - B = 16$$

$$-5B = -15 \Rightarrow B = 3$$

$$\frac{7s^2 + 3s + 16}{(s+1)(s^2+4)} = \frac{4}{s+1} + \frac{3s}{s^2+4}$$

$$\#4 \quad \frac{1}{(s+2)^2(s^2+9)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+9}$$

$$\textcircled{1} = A(s+2)(s^2+9) + B(s^2+9) + (Cs+D)(s^2+4s+4)$$

$$s^3 \quad A + C = 0 \Rightarrow A = -C$$

$$s^2 \quad 2A + B + 4C + D = 0 \quad \textcircled{1}$$

$$s^1 \quad 9A + 4C + 4D = 0 \quad \textcircled{2}$$

$$s^0 \quad 18A + 9B + 4D = 1 \quad \textcircled{3}$$

$$\left\{ \begin{array}{l} \textcircled{1} \rightarrow B + 2C + D = 0 \\ \textcircled{2} \rightarrow -5C + 4D = 0 \\ 9B - 18C + 4D = 1 \end{array} \right.$$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 0 & -5 & 4 & 0 \\ 9 & -18 & 4 & 9 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 0 & -5 \\ 9 & -18 \end{vmatrix} \begin{matrix} s^2 \\ s^2 \\ s^2 \end{matrix}$$

$$= 169$$

$$\Delta_B = \begin{vmatrix} 0 & 2 & 1 \\ 0 & -5 & 4 \\ 1 & -18 & 4 \end{vmatrix} = 13 \quad \Delta_C = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 4 \\ 9 & 1 & 4 \end{vmatrix} = -4$$

$$B = \frac{13}{169} = \frac{1}{13} \quad C = \frac{-4}{169}$$

$$D = \frac{5}{4} C = -\frac{5}{169}$$

$$A = \frac{4}{169}$$

$$\frac{1}{(s+2)^2(s^2+9)} = \frac{\frac{4}{169}}{s+2} + \frac{\frac{1}{13}}{(s+2)^2} + \frac{\frac{-4}{169}s - \frac{5}{169}}{s^2+9}$$

$$\frac{s}{(s+2)^2(s^2+9)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+9}$$

$$A + C = 0 \rightarrow A = -C$$

$$\begin{cases} 2A + B + 4C + D = 0 \\ 9A + 4C + 4D = 1 \\ 18A + 9B + 4D = 0 \end{cases}$$

$$\begin{cases} B + 2C + D = 0 \\ -5C + 4D = 1 \\ 9B - 18C + 4D = 0 \end{cases}$$

$$\Delta = 169$$

$$\Delta_B = \begin{vmatrix} 0 & 2 & 1 \\ 1 & -5 & 4 \\ 0 & -18 & 4 \end{vmatrix} = -26$$

$$B = -\frac{26}{169} = -\frac{2}{13}$$

$$\Delta_C = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 9 & 0 & 4 \end{vmatrix} = -5$$

$$C = \frac{-5}{169}$$

$$\Delta_D = \begin{vmatrix} 1 & 2 & 0 \\ 0 & -5 & 1 \\ 9 & -18 & 0 \end{vmatrix} = 36$$

$$D = \frac{36}{169}$$

$$\frac{s}{(s+2)^2(s^2+9)} = \frac{5/169}{s+2} + \frac{-2/13}{(s+2)^2} + \frac{\frac{-5}{169}s + \frac{36}{169}}{s^2+9}$$



$$\text{#6} \quad \frac{1}{(s+1)^2(s^2-4)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s-2} + \frac{D}{s+2}$$

$$1 = A(s+1)(s^2-4) + B(s^2-4) + C(s+2)(s^2+2s+1) + D(s-2)(s^2+2s+1)$$

$$\left. \begin{array}{l} s^3 \quad A + C + D = 0 \\ s^2 \quad A + B + 4C = 0 \\ s^1 \quad -4A + 5C - 3D = 0 \\ s^0 \quad -4A - 4B + 2C - 2D = 1 \end{array} \right\} \begin{array}{l} -B - 3C + D = 0 \\ 4B + 3C - D = -1 \\ 3B = -1 \end{array}$$

$$\boxed{B = -\frac{1}{3}}$$

$$\left\{ \begin{array}{l} A + C + D = 0 \\ A + 4C = \frac{1}{3} \rightarrow 3A + 12C = 1 \\ -4A + 5C - 3D = 0 \\ -4A + 2C - 2D = 1 - \frac{4}{3} = -\frac{1}{3} \end{array} \right.$$

$$\left\{ \begin{array}{l} 12A - 6C + 6D = 1 \\ A + C + D = 0 \rightarrow D = -A - C \\ 3A + 12C = 1 \quad \textcircled{3} \\ -4A + 5C - 3D = 0 \rightarrow -4A + 5C + 3A + 3C = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} 3A + 12C = 1 \\ -A + 8C = 0 \end{array} \right. \quad \left[ A = \frac{-8}{36} = -\frac{2}{9} \right]$$

$$\hookrightarrow 8C = A = -\frac{2}{9}$$

$$\boxed{C = -\frac{1}{36}}$$

$$\boxed{D = \frac{2}{3} + \frac{1}{36} = \frac{25}{36}}$$

$$\frac{1}{(s+1)^2(s^2-4)} = \frac{-2/9}{s+1} - \frac{1/3}{(s+1)^2} - \frac{1/36}{s-2} + \frac{25/36}{s+2}$$

#9  $\frac{1}{x^2+2x} = \frac{A}{x} + \frac{B}{x+2}$   
 $x(x+2)$

$$1 = A(x+2) + Bx$$

$$\left. \begin{array}{l} x^1 \quad A + B = 0 \Rightarrow B = -\frac{1}{2} \\ x^0 \quad 2A = 1 \Rightarrow A = \frac{1}{2} \end{array} \right\}$$

$$\frac{1}{x^2+2x} = \frac{\frac{1}{2}}{x} - \frac{\frac{1}{2}}{x+2}$$

#10  $\frac{2x+1}{x^2-7x+12} = \frac{A}{x-3} + \frac{B}{x-4}$

$$2x+1 = A(x-4) + B(x-3)$$

$$x^1 \quad A + B = 2 \quad A = -7$$

$$x^0 \quad -4A - 3B = 1 \quad B = 9$$

$$\frac{2x+1}{x^2-7x+12} = \frac{-7}{x-3} + \frac{9}{x-4}$$