y', $\underbrace{f', f'(x)}_{\frac{d}{dx}f(x)}$ $\qquad \frac{d}{dx}f \quad dy \quad D_x y$

$\frac{dy}{dx}$

$D_x y$

$$\frac{d}{dx}$$

$f(x) = c \Rightarrow f'(x) = 0$

Proof $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{c - c}{h}$

$= 0$

Ex $f(x) = 9 \Rightarrow f'(x) = 0$

Ex $h(t) = \pi$
$h'(t) = 0$

$f(x) = x^n \Rightarrow$
$f'(x) = n x^{n-1}$

$f'(x) = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$

$= \lim_{h\to 0} \frac{x^n + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \cdots + h^n - x^n}{h}$

$= \lim_{h\to 0} \left(n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} h + \cdots + h^{n-1} \right)$

$= n x^{n-1}$

Ex find derivative x^3

$$\boxed{(x^3)' = 3x^2}$$

Ex $y = x^{2/3}$

$y' = \frac{2}{3} x^{-1/3}$

Ex $\left(\frac{1}{x^4}\right)' = (x^{-4})'$

$= -4x^{-5}$

$= \boxed{-\frac{4}{x^5}}$

Ex $y = x^{\sqrt{2}}$

$$\boxed{y' = \sqrt{2}\, x^{\sqrt{2}-1}}$$

Ex $y = \sqrt{x^{2+\pi}}$

$= (x^{2+\pi})^{1/2}$

$= x^{1+\pi/2}$

$y' = (1+\frac{\pi}{2}) x^{1+\frac{\pi}{2}-1}$

$= (1+\frac{\pi}{2}) x^{\pi/2}$

$= \boxed{(1+\frac{\pi}{2})\sqrt{x^{\pi}}}$

$y = \frac{1}{x}$
$= x^{-1}$
$\boxed{y' = -x^{-2} = -\frac{1}{x^2}}$

$y = \sqrt{x} = x^{1/2}$
$y' = \frac{1}{2} x^{-1/2}$
$= \frac{1}{2\sqrt{x}}$

$(cx^n)' = ncx^{n-1}$

$y = 8x^4 \quad \frac{dy}{dx}?$

$\frac{dy}{dx} = \underbrace{4(8)\,x^{\cancel{3}4-1}}$
$\underline{= 32x^3}$

$y = -\frac{3}{4} x^{12} \qquad 12\left(\frac{3}{4}\right)$

$y' = -9x^{11}$

$y = x^3 + \frac{4}{3}x^2 - 5x + 1$

$\underline{y' = 3x^2 + \frac{8}{3}x - 5}$

$$y = x^{5/2} + x^3 + \frac{1}{2}x^2 + 4$$

$$y' = \frac{5}{2}x^{3/2} + 3x^2 + x$$

$$y = x^4 - 2x^2 + 2$$

$$\boxed{y' = 4x^3 - 4x}$$

#7 $y = 3x^4 - 6x^3 + \frac{1}{8}x^2 + 5$

$$y' = 12x^3 - 18x^2 + \frac{1}{4}x$$

#8 $p(t) = 2t^4 + 6\sqrt{t} + \frac{5}{t}$

$$p'(t) = 8t^3 + \frac{3}{\sqrt{t}} - \frac{5}{t^2}$$

$$p : 2t^4 + 6t^{1/2} + 5t^{-1}$$

$$p' = 8t^3 + 3t^{-1/2} - 5t^{-2}$$

#10 $y = \frac{x^3 - 4x}{\sqrt{x}}$

$$= \frac{x^3}{x^{1/2}} - 4\frac{x}{x^{1/2}}$$

$$= x^{5/2} - 4x^{1/2}$$

$$y' = \frac{5}{2}x^{3/2} - 2x^{-1/2}$$

$$= \frac{5}{2}x\sqrt{x} - \frac{2}{\sqrt{x}}$$

$$y$$

$$\frac{dy}{dx}$$

$$y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

$$y = x^3 - 3x^2 + 2$$

$$y^{(4)} \quad y^{\overline{IV}}$$

$$y' = 3x^2 - 6x$$

$$y'' = 6x - 6$$

$$y''' = 6$$

$$y^{(4)} = 0$$

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

$$f^{(n)}(x) = n!\, a_n$$

$$y = x^3 - 3x^2 + 2$$

$$n! = 1 \cdot 2 \cdots n$$

$$0! = 1$$

$$y^{(3)} = 3! = 6$$

$$y^{(4)} = 0$$

#33 $f(x) = 3x^4 - 6x^3 + \frac{x^2}{8} + 5$

$2 \times 3 \times 4$

$$f^{(4)}(x) = 3(4!)$$

$$= 3(24)$$

$$= 72$$

34 $f^{(5)}(x) = 0$

2.3 Product.

$$(uv)' = u'v + uv'$$

Ex $f(x) = (\underbrace{2x+3}_{u})(\underbrace{3x^2}_{v})$

$u = 2x+3 \qquad v = 3x^2$

$u' = 2 \qquad v' = 6x$

$$f'(x) = \overset{u'}{2}(\overset{v}{3x^2}) + \overset{v'}{6x}(\overset{u}{2x+3})$$

$$= 6x^2 + 12x^2 + 18x$$

$$= \underline{18x^2 + 18x}$$

use $(uv)' = u'v + v'u$

$$f'(x) = 2(3x^2) + 6x(2x+3)$$

$$= 6x^2 + 12x^2 + 18x$$

$$= \underline{18x^2 + 18x}$$

$$f(x) = (2x+3)(3x^2)$$

$$= 6x^3 + 9x^2$$

$$f'(x) = \underline{18x^2 + 18x}$$

$$y = (\underbrace{3x^2+1}_{u})(\underbrace{x^3+3}_{v}) \qquad (uv)' = u'v + v'u$$

$$y' = \overbrace{6x}^{u'}(\overbrace{x^3+3}^{v}) + \overbrace{3x^2}^{v'}(\overbrace{3x^2+1}^{u})$$

$$= 6x^4 + 18x + 9x^4 + 3x^2$$

$$= 15x^4 + 3x^2 + 18x$$

or

$$y = 3x^5 + 9x^2 + x^3 + 3$$

$$y' = 15x^4 + 18x + 3x^2$$

$$y = (\underbrace{3x^3+2x+5}_{u})(\underbrace{x^2-2x+4}_{v})$$

$$y' = (\overbrace{9x^2+2}^{u'})(\overbrace{x^2-2x+4}^{v}) + (\overbrace{2x-2}^{v'})(\overbrace{3x^3+2x+5}^{u})$$

x^4	x^3	x^2	x^1	x^0
9	-18	36	-4	8
	-6	2	10	-10
6		4	-4	

$$y' = 15x^4 - 24x^3 + 42x^2 + 2x - 2$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$6+4$

$$f(x) = \frac{2x-1}{4x+3} \Longrightarrow f'(x) = \frac{10}{(4x+3)^2}$$

$u = 2x-1$ $\quad$ $v = 4x+3$

$u' = 2$ $\quad$ $v' = 4$

$$f'(x) = \frac{8x+6-(8x-4)}{(4x+3)^2}$$

$$= \frac{8x+6-\cancel{8x}+4}{(4x+3)^2}$$

$$= \frac{10}{(4x+3)^2}$$

$$y = \frac{x-4}{5x-2}$$

$\begin{vmatrix} 1 & -4 \\ 5 & -2 \end{vmatrix}$ $\quad -2+20$

$$y' = \frac{18}{(5x-2)^2}$$

$$\left(\frac{ax^2+bx+c}{dx^2+ex+f}\right)' = \frac{(ae-bd)x^2+2(af-dc)x+(bf-ce)}{(dx^2+ex+f)^2}$$

$$y = \frac{x^2 + x - 1}{x - 1}$$

$$\begin{vmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$2\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix}$$

$$y' = \frac{x^2 - 2x}{(x-1)^2}$$

#52 $$y = \frac{x^2 - 4x + 1}{5x^2 - 2x - 1}$$

$$\begin{matrix} 1 & -4 & 1 \\ 5 & -2 & -1 \end{matrix}$$

$$y' = \frac{18x^2 - 12x + 6x}{(5x^2 - 2x - 1)^2}$$

$$\begin{vmatrix} 1 & -4 \\ 5 & -2 \end{vmatrix} x^2$$

$$2\begin{vmatrix} 1 & 1 \\ 5 & -1 \end{vmatrix} +$$

$$\begin{vmatrix} -4 & 1 \\ -2 & -1 \end{vmatrix}$$

#32 $$y = \frac{3x + 4}{2x + 1}$$

$3 - 8$

$$y' = \frac{-5}{(2x+1)^2}$$

---

36 $$\left(\frac{3x}{3x-2}\right)' = \frac{-6}{(3x-2)^2}$$

$$\begin{matrix} 3 & 0 \\ 3 & -2 \end{matrix}$$

---

40 $$y = \frac{x^2 - 4}{5x^2 - 2}$$

$$2\begin{vmatrix} 1 & -4 \\ 5 & -2 \end{vmatrix}$$

$$\boxed{y' = \frac{-36x}{(5x^2 - 2)^2}}$$

41

$$y = \frac{3x^2-4}{2x^2-1} \qquad 2\begin{vmatrix} 3 & -4 \\ 2 & -1 \end{vmatrix}$$

$$y' = \frac{-10x}{(2x^2-1)^2}$$

42

$$y = \frac{3x^2+4}{2x^2+1} \longrightarrow 2$$

$$y' = \frac{-10x}{(2x^2+1)^2}$$

54

$$y = \frac{2x^4-3}{2x^4+1} \qquad 4\begin{vmatrix} 2 & -3 \\ 2 & 1 \end{vmatrix}$$

$$y' = \frac{32x^3}{(2x^4+1)^2}$$

#20

$$f(x) = \frac{x^9 + x^8 + 4x^5 - 7x}{x^4 - 3x^2 + 2x + 1}$$

$u = x^9 + x^8 + 4x^5 - 7x$ $\quad v = x^4 - 3x^2 + 2x + 1$

$u' = 9x^8 + 8x^7 + 20x^4 - 7$ $\quad v' = -4x^3 + 6x + 2$

x^{12}	x^{11}	x^{10}	x^9	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x^1	x^0
9	8	-27	18	9	8	-60	40	20		21	-14	-7
-4	-4	6	-24	16		24	-8	-7		-42	14	
			-2	20				28				
			6	-2								
				-16								

$$f'(x) = \frac{5x^{12} + 4x^{11} - 21x^{10} - 2x^9 + 27x^8 + 8x^7 - 36x^6 + 32x^5 + 41x^4 - 2x^2 - 7}{(x^4 - 3x^2 + 2x + 1)^2}$$

$f(x) = (\sqrt{x} + 3)(x^3 - 5x) = (x^{1/2} + 3)(x^3 - 5x)$

$= x^{7/2} - 5x^{3/2} + 3x^3 - 15x$

$$f'(x) = \frac{7}{2}x^{5/2} - \frac{15}{2}x^{1/2} + 9x^2 - 15$$

$$\boxed{f^{(n)}(x) = n!\,a_n}$$