Review

$\cos A = -\frac{15}{17}$ $\quad A \in Q\,\text{II}$ $\qquad \cos B = \frac{12}{13}$ $\quad B \in Q\,\text{I}$

$\sin A = \frac{8}{17}$ $\qquad \sin B = \frac{5}{13}$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \frac{8}{17}\,\frac{12}{13} + \left(-\frac{15}{17}\right)\left(\frac{5}{13}\right)$$

$$= \frac{96 - 75}{221}$$

$$= \boxed{\frac{21}{221}}$$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \frac{-15}{17}\,\frac{12}{13} - \frac{8}{17}\,\frac{5}{13}$$

$$= \frac{-180 - 40}{221}$$

$$= \boxed{\frac{-220}{221}}$$

c) $\tan(A+B) = \boxed{-\frac{21}{220}}$

d) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$= \frac{96+75}{221}$$

$$= \boxed{\frac{171}{221}}$$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$= \frac{-180+40}{221}$$

$$= \boxed{-\frac{140}{221}}$$

f) $$\tan(A-B) = \boxed{-\frac{171}{140}}$$

$\cos A = -\frac{4}{5}$ $A \in Q\,II$ $\frac{90^\circ}{2} < \frac{A}{2} < \frac{180^\circ}{2}$

$\sin A = \frac{3}{5}$ $\frac{A}{2} \in Q\,I$

a) $\sin 2A = 2\sin A\cos A$

$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= \boxed{-\frac{24}{25}}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{16}{25} - \frac{9}{25}$

$= \boxed{\frac{7}{25}}$

c) $\tan 2A = \boxed{-\frac{24}{7}}$

d) $\sin\frac{A}{2} = \sqrt{\frac{1}{2}(1-\cos A)}$

$= \sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)}$

$= \boxed{\frac{3}{\sqrt{10}}}$

e) $\cos\frac{A}{2} = \sqrt{\frac{1}{2}(1+\cos A)}$

$= \sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)}$

$= \boxed{\frac{1}{\sqrt{10}}}$

f) $\tan\frac{A}{2} = \boxed{3}$

$4\sin^2 x + 4\cos x - 5 = 0 \qquad [0, 2\pi)$

$4(1-\cos^2 x) + 4\cos x - 5 = 0$

$-4\cos^2 x + 4\cos x - 1 = 0$

$\cos x = \dfrac{-4 \pm 0}{-8} = \dfrac{1}{2}$

$\boxed{x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}}$

$\tan^2 x \sin x = \sin x$

$\tan^2 x \sin x - \sin x = 0$

$\sin x(\tan^2 x - 1) = 0$

$\sin x = 0 \qquad \tan x = \pm 1$

$\boxed{x = 0, \pi, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}}$

$2\tan x \csc x + 2\csc x + \tan x + 1 = 0$

$2\csc x(\tan x + 1) + (\tan x + 1) = 0$

$(\tan x + 1)(2\csc x + 1) = 0$

$\tan x = -1 \qquad \csc x = -\dfrac{1}{2}$

$\sin x = -2$ #

$\boxed{x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}}$

1 $\sin\theta - \sqrt{3}\cos\theta = 1$

$$\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta = \frac{1}{2}$$

$$\sin\theta\cos\frac{\pi}{3} - \cos\theta\sin\frac{\pi}{3} = \frac{1}{2}$$

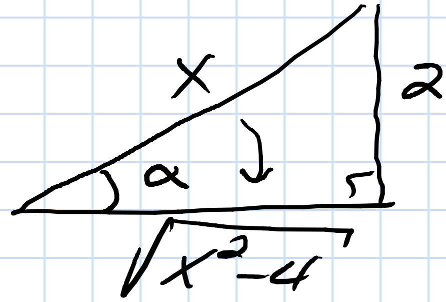
$$\sin\left(\theta - \frac{\pi}{3}\right) = \frac{1}{2}\ ?$$

$$\theta - \frac{\pi}{3} = \frac{\pi}{6} \qquad\qquad \theta - \frac{\pi}{3} = \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{6} + \frac{\pi}{3} \qquad\qquad \theta = \frac{5\pi}{6} + \frac{\pi}{3}$$

$$= \frac{\pi}{2} \qquad\qquad = \frac{7\pi}{6}$$

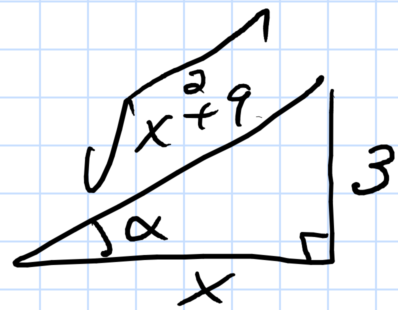
$$\sec\left(\underbrace{\tan^{-1}\frac{2}{\sqrt{x^2-4}}}_{\alpha}\right)$$



$$\tan\alpha = \frac{2}{\sqrt{x^2-4}}$$

$$\boxed{\sec\alpha = \frac{x}{\sqrt{x^2-4}}}$$

$$\sin\left(\underbrace{\cos^{-1}\frac{x}{\sqrt{x^2+9}}}_{\alpha}\right)$$



$$\cos\alpha = \frac{x}{\sqrt{x^2+9}}$$

$$\boxed{\sin\alpha = \frac{3}{\sqrt{x^2+9}}}$$

$(3, 270^\circ)$ $\quad (x, y)?$

$x = r\cos\theta$
$= 3\cos 270^\circ$
$= \underline{0}$

$y = r\sin\theta$
$= 3\sin 270^\circ$
$= \underline{-3}$

$$\boxed{(x, y) = (0, -3)}$$

$(-1, \sqrt{3})$ $\quad (r, \theta)?$

$r = \sqrt{x^2 + y^2}$
$= \sqrt{1 + 3}$
$= \underline{2}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$ $\quad \sqrt{3}/2$
$= \tan^{-1}\frac{\sqrt{3}}{1}$
$= 60^\circ$

$(r, \theta) = (2, 120^\circ)$ $\quad$ II

$r = 8\sin\theta - 2\cos\theta$

$r^2 = 8r\sin\theta - 2r\cos\theta$

$x^2 + y^2 = 8y - 2x$

$x^2 + 2x + (1)^2 + y^2 - 8y + (-4)^2 = 1 + 16$

$(x+1)^2 + (y-4)^2 = 17$

center $(-1, 4)$ $\quad$ radius $\sqrt{17}$

$x^2+y^2 = 4x$

$r^2\cos^2\theta + r^2\sin^2\theta = 4r\cos\theta \quad (r\neq 0)$

$r(\cos^2\theta+\sin^2\theta) = 4\cos\theta$

$\boxed{r = 4\cos\theta}$

$\sqrt{3} - i$

$x = \sqrt{3}$

$y = -1$

$r = \sqrt{x^2+y^2}$

$= \sqrt{3+1}$

$= \boxed{2}$

$\theta = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right| \quad \frac{\pi}{6}$

$\boxed{\sqrt{3} - i = 2\,\mathrm{cis}\frac{11\pi}{6}}$

$3\,\mathrm{cis}\,210^\circ = 3(\cos 210^\circ + i\sin 210^\circ)$

$= -3\frac{\sqrt{3}}{2} - i\,3\,\frac{1}{2}$

$= \boxed{-\frac{3\sqrt{3}}{2} - \frac{3}{2}i}$

$$\frac{15 \operatorname{cis}(210^\circ)}{3 \operatorname{cis}(150^\circ)} = 5 \operatorname{cis}(210^\circ - 150^\circ)$$
$$= 5 \operatorname{cis} 60^\circ$$
$$= 5 \cos 60^\circ + i\, 5 \sin 60^\circ$$
$$= \frac{5}{2} + \frac{5}{2} i$$

$$(15 \operatorname{cis} 210^\circ)(5 \operatorname{cis} 150^\circ) = 75 \operatorname{cis} 360^\circ$$
$$= 75(\cos 360^\circ + \cancel{i \sin 360^\circ})$$
$$= 75$$