1/ $\int x^5 e^{4x} dx$

$$= \left(\frac{1}{4}x^5 - \frac{5}{16}x^4 + \frac{5}{16}x^3 - \frac{15}{64}x^2 + \frac{15}{128}x - \frac{15}{512}\right)e^{4x}$$

| | $\int e^{4x}$ |
|---|---|
| $+\ x^5$ | $\frac{1}{4}e^{4x}$ |
| $-\ 5x^4$ | $\frac{1}{2^4}e^{4x}$ |
| $+\ 20x^3$ | $\frac{1}{2^6}e^{4x}$ |
| $-\ 60x^2$ | $\frac{1}{2^8}e^{4x}$ |
| $+\ 120x$ | $\frac{1}{2^{10}}e^{4x}$ |
| $-\ 120$ | $\frac{1}{2^{12}}e^{4x}$ |

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2/ $\int \cos 2x\, e^{3x} dx$

| | $\int \cos 2x$ |
|---|---|
| $+\ e^{3x}$ | $\frac{1}{2}\sin 2x$ |
| $-\ 3e^{3x}$ | $-\frac{1}{4}\cos 2x$ |
| $+\ 9e^{3x}$ | |

$$\int \cos 2x\, e^{3x} dx = \left(\frac{1}{2}\sin 2x + \frac{3}{4}\cos 2x\right)e^{3x} - \frac{9}{4}\int e^{3x}\cos 2x\, dx$$

$$\frac{13}{4}\int \cos 2x\, e^{3x} dx = \frac{1}{4}\left(2\sin 2x + 3\cos 2x\right)$$

$$\int \cos 2x\, e^{3x} dx = \boxed{\frac{1}{13}\left(2\sin 2x + 3\cos 2x\right) + C}$$

3/ $\int x^6 \cos 4x \, dx$

| | $\int \cos 4x$ |
|---|---|
| $+ \; x^6$ | $\frac{1}{4} \sin 4x$ |
| $- \; 6x^5$ | $-\frac{1}{2^4} \cos 4x$ |
| $+ \; 30x^4$ | $-\frac{1}{2^6} \sin 4x$ |
| $- \; 120x^3$ | $\frac{1}{2^8} \cos 4x$ |
| $+ \; 260x^2$ | $\frac{1}{2^{10}} \sin 4x$ |
| $- \; 720x$ | $\frac{-1}{2^{12}} \cos 4x$ |
| $+ \; 720$ | $\frac{-1}{2^{14}} \sin 4x$ |

$$\int x^6 \cos 4x \, dx = \left(\frac{x^6}{4} - \frac{30}{2^6} x^4 + \frac{260}{2^{10}} x^2 - \frac{720}{2^{14}}\right) \sin 4x$$

$$+ \left(\frac{6}{2^4} x^5 - \frac{120}{2^8} x^3 + \frac{720}{2^{12}} x\right) \cos 4x$$

$$= \left(\frac{x^6}{4} - \frac{15}{32} x^4 + \frac{45}{128} x^2 - \frac{45}{1{,}024}\right) \sin 4x$$

$$+ \left(\frac{3}{8} x^5 - \frac{15}{32} x^3 + \frac{45}{256} x\right) \cos 4x + C$$

4/ $\int \cos(\ln x)\,dx$

$u = \cos(\ln x)$ | $v = \int dx$

$du = -\frac{1}{x}\sin(\ln x)\,dx$ | $= x$

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + \underline{\int \sin(\ln x)\,dx}$$

$u = \sin(\ln x)$ $\quad v = x$

$du = \frac{1}{x}\cos(\ln x)\,dx$

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\,dx$$

$$2\int \cos(\ln x)\,dx = x\left(\cos(\ln(x)) + \sin(\ln x)\right)$$

$$\int \cos(\ln x)\,dx = \boxed{\frac{1}{2}x\left(\cos(\ln x) + \sin(\ln x)\right) + C}$$

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5/ $\int_0^{\pi} e^{\cos t}\sin 2t\,dt = 2\int_0^{\pi} e^{\cos t}\sin t\cos t\,dt$

$$= -2\int_0^{\pi} e^{\cos t}\cos t\,d(\cos t)$$

$$= -2\int_0^{\pi} e^{u}\,u\,du$$

| | $\int e^u$ |
|---|---|
| $+u$ | $e^u$ |
| $-1$ | $e^u$ |

$$= -2(\cos t - 1)e^{\cos t}\Big|_0^{\pi}$$

$$= -2\left(-2e^{-1} - 0\right)$$

$$= \boxed{\frac{4}{e}}$$