Hwk 5.6

1) $$\sum_{k=0}^{19} \frac{k-3}{4} = \frac{1}{4}\left(\sum_{k=0}^{19} k - \sum_{k=0}^{19} 3\right)$$

$$= \frac{1}{4}\left(\frac{1}{2}\,19(19+1) - 3(19-0+1)\right)$$

$$= \frac{1}{4}(190 - 60)$$

$$= \frac{130}{4}$$

$$= \frac{65}{2}$$

#2) $$\sum_{k=2}^{50}(2,000 - 3k) = \sum_{k=2}^{50} 2,000 - 3\sum_{k=2}^{50} k$$

$$= 2,000(50-2+1) - 3\left(\frac{1}{2}50(51) - 1\right)$$

$$= 98,000 - 3,822$$

$$= 94,178$$

. $\overline{78}$ =?

$$.\overline{78} = .787878 \cdots$$

$$= .78 + .0078 + \cdots$$

$$= 78(.01 + .0001 + \cdots) \qquad r = \frac{.0001}{.01} = .01 < 1$$

$$= 78\left(\frac{.01}{1-.01}\right)$$

$$= 78\left(\frac{.01}{.99}\right)$$

$$= 78\left(\frac{1}{99}\right)$$

$$= \frac{26}{33}$$

 # 1 $4 + 8 + 12 + \cdots + 4n = 2n(n+1)$

For $n = 1 \implies 4 \overset{?}{=} 2(2)$

$4 = 4 \checkmark$ P_1 is true

Assume P_k : $4 + 8 + \cdots + 4k = 2k(k+1)$ is true

is P_{k+1} : $4 + \cdots + 4k + 4(k+1) = 2(k+1)(k+2)$?

$4 + \cdots + 4k + 4(k+1) = 2k(k+1) + 4(k+1)$
$$= (k+1)(2k+4)$$
$$= 2(k+1)(k+2) \checkmark$$

P_{k+1} is also true

∴ By the mathematical Induction, the proof is completed

#2 $1 + 5 + 9 + \cdots + (4n-3) = n(2n-1)$

$n = 1 \implies 1 = 1(2-1)$
$1 = 1 \checkmark$ P_1 is true !

Let P_k is true : $1 + 5 + \cdots + (4k-3) = k(2k-1)$

is P_{k+1} : $1 + \cdots + (4k-3) + [4(k+1)-3] = (k+1)[2(k+1)-1]$

$1 + \cdots + (4k-3) + (4k+1) = (k+1)(2k+1)$?

$1 + \cdots + (4k-3) + (4k+1) = k(2k-1) + (4k+1)$
$$= 2k^2 - k + 4k + 1$$
$$= 2k^2 + 3k + 1$$
$$= (k+1)(2k+1) \checkmark$$

P_{k+1} is also true.

∴ By the mathematical induction, the proof is completed.

$$\frac{3x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$3x = A(x-1) + B(x+2)$$

$x^1 \quad A + B = 3 \rightarrow \boxed{A = 3-1 = 2}$

$x^0 \quad -A + 2B = 0$

$$3B = 3 \Rightarrow \boxed{B = 1}$$

$$\frac{3x}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$$

$$\frac{2x+1}{x^2 - 7x + 12} = \frac{A}{x-3} + \frac{B}{x-4}$$

$$2x+1 = A(x-4) + B(x-3)$$

$x^1 : \quad A + B = 2$

$x^0 \quad -4A - 3B = 1$

$D = \begin{vmatrix} 1 & 1 \\ -4 & -3 \end{vmatrix} = 1$

$A = \frac{1}{1} \begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} = -7$

$B = 2 + 7 = 9$

$$\frac{2x+1}{x^2 - 7x + 12} = \frac{-7}{x-3} + \frac{9}{x-4}$$

37. T: h:14 w=10

$$\frac{x^2}{25^2} + \frac{y^2}{20^2} = 1$$

$$\frac{y^2}{20^2} = 1 - \frac{5^2}{25^2}$$

$$y^2 = \frac{20^2}{25^2}\left(\frac{625-25}{}\right)$$
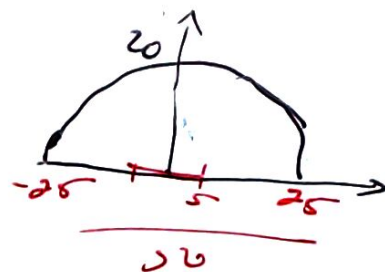
$$y = \frac{20}{25}\sqrt{600}$$

$$= \frac{40}{5}\sqrt{6}$$

$$14 \ ? \ \frac{40}{5}\sqrt{6}$$

$$70 \qquad 40\sqrt{6}$$

$$\frac{7}{4} < \sqrt{6}$$

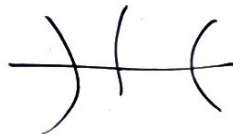$$\boxed{<2} \qquad >2$$

2nd   120

$$14^2 = \left(\frac{4}{5}\right)^2 600$$

$$\frac{16}{25} \, 600$$

$$14^2 \quad 16(24)$$

$$14 \times 14 < 16 \times 24$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\left\{(-1)^{n+1}\frac{n+7}{2n}\right\} \qquad {}^{st}\text{el term} + a_8$$

$$n=1 \rightarrow (-1)^2\frac{8}{2} = 4$$

$$n=2 \rightarrow (-1)^3\frac{9}{4} = -\frac{9}{4}$$

$$n=3 \rightarrow (-1)^4\frac{10}{6} = \frac{5}{3}$$

$$n=4 \rightarrow (-1)^5\frac{11}{8} = -\frac{11}{8}$$

$$n=8 \rightarrow (-1)^9\frac{15}{16} = -\frac{15}{16}$$

Arthur

a_{20} : $a_9 = -5$ $a_{15} = 31$

$$d = \frac{31+5}{15-9} = 6$$

$$a_9 = a_1 + 8(6) = -5$$
$$\underline{a_1 = -53}$$

$$a_{20} = -53 + 19(6)$$
$$= -53 + 114$$
$$= \underline{61}$$

Geometric a_8 : $a_2 = 3$ $a_4 = 6$

$$r = \left(\frac{6}{3}\right)^{\frac{1}{4-2}} = 2^{\frac{1}{2}} = \sqrt{2}$$

$$a_2 = a_1 (\sqrt{2})^1 = 3$$
$$a_1 = \frac{3}{\sqrt{2}}$$
$$a_8 = \frac{3}{\sqrt{2}} (\sqrt{2})^7$$
$$= \frac{3}{\sqrt{2}} 2^3 \sqrt{2}$$
$$= \underline{24}$$

$\frac{1}{2}$ $\underset{(3)}{}$ 7

$$4 + 11 + 18 + 25 + 32 = \sum_{n=1}^{5} (7n - 3)$$

$$d = 7, \quad a_1 = 4$$
$$a_n = 4 + (n-1)(7)$$
$$= 4 + 7n - 7$$

$$4 + 11 + 18 + \ldots + 466 = \sum_{n=1}^{66} (7n - 3)$$

$$\frac{466 - 4}{d} = \frac{462}{7}$$

$$\sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^{n-1} = \infty \qquad |r| = \frac{5}{3} > 1$$

$$\sum_{n=1}^{\infty} 2\left(\frac{3}{5}\right)^{n-1} = \frac{2}{1 - \frac{3}{5}} \qquad r = \left|\frac{3}{5}\right| < 1$$

$$= 2 \cdot \frac{5}{2}$$

$$= \underline{5}$$

$$\sum_{k=1}^{20} 5 = 5 \times 20$$

$$= \underline{100}$$

$$\sum_{k=11}^{55} 4 = 4(55 - 11 + 1)$$

$$= 4(45)$$

$$= \underline{180}$$

$$\sum_{k=2}^{6} (3k - 5) = (6-5) + (9-5) + (12-5) + (15-5) + (18-5)$$

$$= 1 + 4 + 7 + 10 + 13$$

$$= \underline{35}$$

2- Partial.

2- 1st form --

1- Arth. a_1 $d = \frac{y_2 - y_1}{x_2 - x_1}$ $a_n = a_1 + (n-1)d$

1- Geom a_2 $r = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2} - x_1}$ $a_n = a_1 r^{n-1}$

1- Prove

App $\left\{ \begin{array}{l} \text{1- Ellipse} \\ \text{1- Hyperbola} \end{array} \right.$

$\sum \Rightarrow \boxed{5}$

$+ - - = \sum ?$

$$\frac{3x+2}{x^2-5x+4} = \frac{A}{x-1} + \frac{B}{x-4}$$

$$3x+2 = A(x-4) + B(x-1)$$

$x^1 \qquad A + B = 3 \rightarrow \boxed{B = 3 + \frac{5}{3} = \frac{14}{3}}$

$x^0 \qquad \underline{-4A - B = 2}$

$$-3A = 5$$

$$\boxed{A = -\frac{5}{3}}$$

$$\frac{3x+2}{x^2-5x+4} = -\frac{5}{3}\frac{1}{x-1} + \frac{14}{3}\frac{1}{x-4}$$

$$\boxed{\frac{-5/3}{x-1} + \frac{14/3}{x-4}}$$

$n = 1 \qquad\qquad \checkmark \quad P_1 \text{ is true.}$

Let P_k : $\underbrace{\qquad}_{} k = $ true

is P_{k+1} : $\underbrace{\qquad + (k+1)}_{①} = k \rightarrow k+1)$

$① = $

P_{k+1} is also true

$$3+6+9+\cdots+3n = \frac{3n(n+1)}{2}$$

For $n=1 \Rightarrow \quad 3 = \frac{3(1)(2)}{2}$

$\qquad\qquad 3 = 3 \checkmark \quad$ P_1 is true.

P_k is true: $\quad 3+6+\cdots+3k = \frac{3k(k+1)}{2}$

is P_{k+1}: $\quad 3+\cdots+3k+3(k+1) = \frac{3}{2}(k+1)(k+2)$

$3+\cdots+3k+3(k+1) = \frac{3}{2}k(k+1)+3(k+1)$

$\qquad\qquad\qquad\qquad = 3(k+1)\left(\frac{1}{2}k+1\right)$

$\qquad\qquad\qquad\qquad = 3(k+1)\left(\frac{k+2}{2}\right) \checkmark$

$\qquad\qquad P_{k+1}$ is also true.

$\therefore$ By the mathematical induction, the proof is completed

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Geom $\qquad a_{10} : \quad a_4 = 4 \qquad a_7 = 12$

$r = \left(\frac{12}{4}\right)^{\frac{1}{7-4}} = 3^{1/3} \qquad\qquad r = \left(\frac{a_7}{a_4}\right)^{\frac{1}{7-4}}$

$a_4 = a_1\left(3^{1/3}\right)^3 = 4 \qquad\qquad a_n = a_0 r^{n-1}$

$\qquad 3a_1 = 4$

$\qquad a_1 = \frac{4}{3}$

$a_{10} = \frac{4}{3}\left(3^{1/3}\right)^9 \qquad \frac{4}{3}3^3 \, 3^{-1}$

$\qquad = 4(3^2)$

$\qquad = 36$

$a_{12} : a_8 = 4 \quad a_{18} = -96$

$d = \dfrac{-96-4}{18-8} = \dfrac{-100}{10} = -10$

$\qquad\qquad d = \dfrac{y_2-y_1}{x_2-x_1}$

$\qquad\qquad a_n = a_1 + (n-1)d$

$a_8 = a_1 + 7(-10) = 4$

$a_1 - 70 = 4$

$\underline{a_1 = 74}$

$a_{12} = 74 + 11(-10)$

$\qquad = 74 - 110$

$\qquad \underline{= -36}$

$\begin{array}{r} 1\;1\!\!\!/0 \\ 74 \\ \hline 6 \end{array}$

$T: \quad h = 4, \quad w = 10$

$\qquad x = 5$

$\dfrac{x^2}{20^2} + \dfrac{y^2}{10^2} = 1$

$\dfrac{y^2}{10^2} = 1 - \dfrac{5^2}{20^2}$

$y^2 = 10^2 \left(\dfrac{400 - 25}{20^2} \right)$

$\qquad = \dfrac{10^2}{20^2}(375) \qquad\longrightarrow \quad \left(\dfrac{10}{20}\right)^2 = \left(\dfrac{1}{2}\right)^2$

$\qquad = \dfrac{1}{4} \, 375$

$81 \quad ? \quad \dfrac{375}{4} \quad yp.$

$324 < 375$ \qquad will be cleared.