Review :-)

det → ②

② A^{-1} A^n $n \times n$

1- det

→ Coding 2x2 Given A,
A^{-1} crypto
page

⑤

A^{-1} 3x3

show

Prove (sort F)

$\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 5$

$|A| = ??$

( ) det(A) =

$$\begin{vmatrix} \lambda-3 & 2 \\ 4 & \lambda-1 \end{vmatrix} = \lambda^2 - 4\lambda - 5$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{vmatrix} = k$$

(diagonal)
product of main diagonal

$$\begin{vmatrix} 2 & 3 & 5 & 2 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 5 \end{vmatrix} = 40$$

why

upper triangular
(product of main diagonal)

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any specific matrix $\neq$ 2x2

1, 5 # 16

$$(A+B)(A-B) \neq A^2 - B^2$$

$$(A+B)(A-B) = A^2 - AB + BA - B^2$$

since product not commutative

$$AB \neq BA$$

$$\underbrace{2\times 3 \;\; 3\times 2}_{2\times 2} \qquad \underbrace{3\times 2 \;\; 2\times 3}_{3\times 3}$$

$AB \neq BA$ (not necessarily)

$$\therefore (A+B)(A-B) \neq A^2 - B^2$$

1.5 #7 $(A+B)(A+B) \neq A^2+2AB+B^2$

$(A+B)(A+B) = AA + AB + BA + BB$

$= A^2 + AB + BA + B^2$

since AB is not necessarily commutative

$AB \neq BA$

$\therefore (A+B)(A+B) \neq A^2+2AB+B^2$

#18 A $m \times n$ $\Rightarrow$ AA^T & A^TA symm?

$AA^T = (AA^T)^T$?

$(AA^T)^T = (A^T)^T A^T$

$= AA^T$ ✓

$(A^T)^T = A$

$(AB)^T = B^T A^T$

$AA^T = (A^T)^T A^T$

$= (A A^T)^T$ ✓

matrix is symm : matrix $= (\text{mat})^T$

A is invertible : $AA^{-1} = A^{-1}A = I$

#9 $A_{n\times n}$ sym $\Rightarrow A = A^T$

$B_{n\times n}$ sym $\Rightarrow B = B^T$

a) $A = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$ $\quad B = \begin{pmatrix} 0 & 1 \\ 1 & 3 \end{pmatrix}$

$AB = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 2 & 7 \\ 0 & 2 \end{pmatrix}$ it's not sym.

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$ $\quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{pmatrix}$

$AB = \begin{pmatrix} a_{11} b_{11} \cdots & a_{11} b_{12} + a_{12} b_{22} \\ a_{12} b_{11} + a_{22} b_{12} & \end{pmatrix}$

$a_{11} b_{12} + a_{12} b_{22} \neq a_{12} b_{11} + a_{22} b_{12}$

b) AB is symmetric iff $AB = BA$

$AB = BA \Rightarrow AB$ is sym.

$AB \stackrel{?}{=} (AB)^T$

$(AB)^T = B^T A^T$

$= BA$

$= AB$

$\left\{ \begin{array}{l} B = B^T \\ A = A^T \end{array} \right.$

AB is sym $\Rightarrow$ $AB = BA$

$A = A^T$, $B = B^T$, $AB = (AB)^T$

$$AB = (AB)^T$$
$$= B^T A^T$$
$$= BA \checkmark$$

Coding.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$$

-1

$$A^{-1} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} \qquad A^{-1} = -1 \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}$$

$[11 \ \ 21][64 \ \ 112]$

$$[11 \ \ 21] \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} = [8 \ \ 1]$$

$$[64 \ \ 112] \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} = [18 \ \ 16]$$

0

8 1 18 16

H A P ~~P~~ Y

$[\quad]\left(\quad\right) =$

1.d #10

$A^2 = A \Rightarrow I - 2A = (I-2A)^{-1}$

$(I-2A)(I-2A)^{-1} = I$

$I - 2A$ means invertible $AB = I$

$$(I-2A)(I-2A) = I^2 - 2IA - 2AI + 4A^2$$
$$= I - 2A - 2A + 4A^2$$
$$= I - 4A + 4A^2$$
$$= I - 4A + 4A \quad (A = A^2)$$
$$= I \checkmark$$

$(I-2A) = (I-2A)^{-1}$

#11 A sym $\Rightarrow A^{-1}$ is symmetric

$A = A^T$

nonsingular: A^{-1} exist $\Rightarrow AA^{-1} = A^{-1}A = I$

$A^{-1} = (A^{-1})^T$??

$A^{-1} = (A)^{-1} \quad A = A^T$
$$= (A^T)^{-1}$$
$$= (A^{-1})^T \checkmark$$

$$(A^{-1})^T = (A^T)^{-1}$$
$$= A^{-1}$$

$\therefore A^{-1}$ is symmetric

\# ~~12~~ $\left.\begin{array}{l} A = A^T,\ B = B^T,\ C = C^T \quad n\times n \\ ABC = I \end{array}\right.$

A, B inve