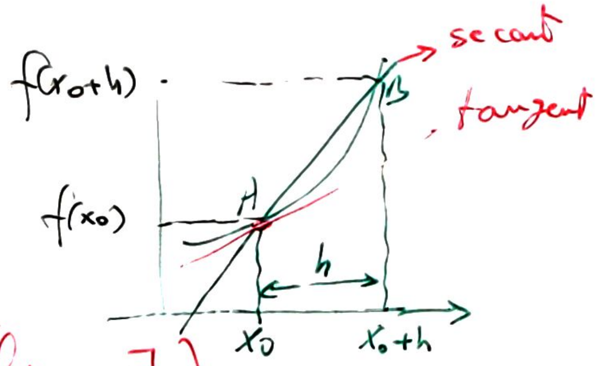
$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+h) - f(x_0)}{x_0 + h - x_0}$$



$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} \quad (\lim \exists)$$

$$\text{slope} = \frac{\Delta y}{\Delta x} = \text{change of Rate}$$

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$f(x) \longrightarrow$ derivative

$$f'(x),\ f',\ \frac{d}{dx}f(x),\ \frac{df}{dx},\ \frac{dy}{dx},\ y'$$

$$\longrightarrow D_x y,\ \dot{y}$$

$f(x) = \sqrt{x} \Rightarrow f'$ use limit to find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{0}{0}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$f(x) = \frac{x}{x-1} \qquad f'(x)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{x+h}{x+h-1} - \frac{x}{x-1}\right] \qquad = \frac{1}{0}\left(\frac{x}{x-1} - \frac{x}{x-1}\right)$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{\cancel{x^2} - \cancel{x} + \cancel{hx} - h - \cancel{x^2} - \cancel{xh} + \cancel{x}}{(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{-h}{(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{-1}{(x-1)(x+h-1)}$$

$$= \frac{-1}{(x-1)^2}$$

#43 $\lim_{x \to -0.5^-} \sqrt{\frac{x+2}{x+1}} = \sqrt{\frac{-.5+2}{-.5+1}}$

$$= \sqrt{\frac{1.5}{.5}}$$

$$= \sqrt{3}$$

#47 $\lim_{x \to -2^+} (x+3) \frac{|x+2|}{x+2} = \frac{0}{0}$

$$= \lim_{x \to -2^+} \frac{|x+2|}{(x+2)} = 1$$

$$|x+2| \xrightarrow[x \to -2^+]{} +(x+2)$$

#56

$$\lim_{\theta \to 0} \frac{\theta \cot 4\theta}{\sin^2\theta \cot^2 2\theta} = \frac{0}{0}$$

$$= \lim_{\theta \to 0} \frac{1}{\underbrace{\frac{\sin\theta}{\theta}}_{\to 1}} \cdot \frac{1}{\sin\theta} \frac{\overbrace{\cos 4\theta}^{\to 1}}{\sin 4\theta} \cdot \frac{\sin^2 2\theta}{\underbrace{\cos^2 2\theta}_{\to 1}}$$

$$= \lim_{\theta \to 0} \frac{\sin^2 2\theta}{\sin\theta \; 2\sin 2\theta \underbrace{\cos 2\theta}_{\to 1}}$$

$$= \frac{1}{2} \lim_{\theta \to 0} \frac{\sin 2\theta}{\sin\theta}$$

$$= \frac{1}{2} \lim_{\theta \to 0} \frac{2 \cancel{\sin\theta} \cos\theta}{\cancel{\sin\theta}}$$

$$\boxed{= 1}$$

#73 $$\lim_{x \to a} \frac{x^n - a^n}{x - a} = \frac{0}{0}$$

$$= \lim_{x \to a} \frac{(x-a)}{x-a}$$

$$\begin{array}{r|l} & x^{n-1} + a x^{n-2} \\ x-a & x^n - a^n \\ & x^n - a x^{n-1} \\ \hline & a x^{n-1} \\ & a x^{n-1} - a^2 x^{n-2} \end{array} \quad - a^n$$

$$= \lim_{x \to a} \frac{(x-a)(x^{n-1} + a x^{n-2} + a^2 x^{n-3} + \cdots + a^{n-1})}{x-a}$$

$$= \underbrace{a^{n-1} + a a^{n-2} + \overset{a^2 a^{n-3}}{\cdots} + a^{n-1}}$$

$$= a^{n-1} + a^{n-1} + a^{n-1} \cdots + a^{n-1}$$

$$\boxed{= n a^{n-1}}$$

#63 $\lim_{x \to 3} \frac{1}{x-3}\left(\frac{1}{\sqrt{x+1}} - \frac{1}{2}\right) = \frac{1}{0}\left(\frac{1}{2}-\frac{1}{2}\right)$

$$= \frac{0}{0}$$

$$= \lim_{x \to 3} \frac{1}{x-3} \frac{2-\sqrt{x+1}}{2\sqrt{x+1}} \cdot \frac{2+\sqrt{x+1}}{2+\sqrt{x+1}}$$

$$= \lim_{x \to 3} \frac{1}{x-3} \frac{4-x-1}{2\sqrt{x+1}\,(2+\sqrt{x+1})}$$

$$= \frac{1}{2} \lim_{x \to 3} \frac{\cancel{3-x}}{\cancel{(x-3)}\sqrt{x+1}\,(2+\sqrt{x+1})}$$

$$= \frac{1}{2} \frac{-1}{2(4)}$$

$$= -\frac{1}{16}$$

87/ $\lim_{x \to \frac{3\pi}{2}} \frac{\sin^2 x + 6\sin x + 5}{\sin^2 x - 1} = \frac{1-6+5}{0} = \frac{0}{0}$

$$= \lim_{x \to \frac{3\pi}{2}} \frac{(\sin x + 1)(\sin x + 5)}{(\sin x - 1)(\sin x + 1)}$$

$$= \lim_{x \to \frac{3\pi}{2}} \frac{\sin x + 5}{\sin x - 1}$$

$$= \frac{4}{-2}$$

$$= -2$$

#49 $\lim_{x \to 0^-} \dfrac{2e^x + 5e^{3x}}{e^{2x} - e^{3x}} = \dfrac{2+5}{1-1} = \dfrac{7}{0} = \infty$

#43 $\lim_{x \to \frac{\pi}{2}^-} \dfrac{1+\sin x}{\cos x} = \dfrac{2}{0} = \infty$

$x \to \frac{\pi}{2}^+ \qquad -\infty$

$f(x) = \dfrac{x^2 - 7x + 12}{x - a}$

a) a? $\lim_{x \to a^+} \dfrac{x^2 - 7x + 12}{x - a} = \dfrac{a^2 - 7a + 12}{0}$

$= \lim_{x \to a^+} \dfrac{(x-3)(x-4)}{x-a}$

$\boxed{a = 3, 4}$

b) $\lim_{x \to a^+} \dfrac{(x-3)(x-4)}{x-a} = \infty = \dfrac{}{0}$

$a \neq 3, 4$, $x < 3$, $x > 4$

c) $\lim_{x \to a^+} \dfrac{(x-3)(x-4)}{x-a} = -\infty$

$a \neq 3, 4$ $\boxed{3 < x < 4}$

$$\lim_{x\to 64}\frac{x^{2/3}-16}{\sqrt{x}-8} = \frac{(4^3)^{2/3}-16}{8-8} = \frac{0}{0}$$

$\frac{2}{3}+\frac{1}{2}$

$\overline{\quad 6}$

$$= \lim_{x\to 64}\frac{x^{2/3}-16}{\sqrt{x}-8}\cdot\frac{\sqrt{x}+8}{\sqrt{x}+8}$$

$$= \lim_{x\to 64}\frac{x^{7/6}+8x^{2/3}-16\sqrt{x}-128}{x-64}$$

$$\lim_{x\to 64}\frac{(x^{1/3})^2-16}{\sqrt{x}-8} = \lim_{x\to 64}\frac{(x^{1/3}-4)(x^{1/3}+4)}{\sqrt{x}-8}$$

$$\begin{array}{r|l}
 & x^{1/6}+8x^{-2/3}-\cdots \\ \hline
x^{1/2}-8 & x^{2/3}-16 \\
 & x^{2/3}-8x^{1/6} \\ \hline
 & 8x^{1/6}-16
\end{array}$$

$$\lim_{x\to 64}\frac{x^{2/3}-16}{\sqrt{x}-8} = \frac{\sqrt{x}+8}{\sqrt{x}+8}$$

$$= \lim_{x\to 64}\frac{(x^{1/3}-4)(x^{1/3}+4)(\sqrt{x}+8)}{x-64}$$

$$= \lim_{x\to 64}\frac{(x^{1/3}-4)(x^{1/3}+4)(\sqrt{x}+8)}{(x^{1/3})^3-(4)^3}$$

$$= \lim_{x\to 64}\frac{\cancel{(x^{1/3}-4)}(x^{1/3}+4)(\sqrt{x}+8)}{\cancel{(x^{1/3}-4)}(x^{2/3}+4x^{1/3}+16)}$$

$$= \lim_{x\to 64}\frac{(x^{1/3}+4)(\sqrt{x}+8)}{x^{2/3}+4x^{1/3}+16}$$

$$= \frac{8(16)}{16+16+16} = 8/3$$