Solution   *Section* 1.5 – Exponential and Logarithmic Equations

Exercise

Express the following in terms of sums and differences of logarithms: $\log_3(ab)$

Solution

$$\log_3(ab) = \boxed{\log_3 a + \log_3 b}$$

Exercise

Express the following in terms of sums and differences of logarithms: $\log_7(7x)$

Solution

$$\log_7(7x) = \log_7 7 + \log_7 x$$

$$= \boxed{1 + \log_7 x}$$

Exercise

Express the following in terms of sums and differences of logarithms: $\log \frac{x}{1000}$

Solution

$$\log \frac{x}{1000} = \log x - \log 10^3$$

$$= \boxed{\log x - 3}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_5\left(\frac{125}{y}\right)$

Solution

$$\log_5\left(\frac{125}{y}\right) = \log_5 5^3 - \log_5 y$$

$$= \boxed{3 - \log_5 y}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b x^7$

Solution

$$\boxed{\log_b x^7 = 7\log_b x}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\sqrt[7]{x}$

Solution

$$\ln\sqrt[7]{x} = \ln x^{1/7}$$

$$\boxed{= \frac{1}{7}\ln x}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \frac{x^2 y}{z^4}$

Solution

$$\log_a \frac{x^2 y}{z^4} = \log_a x^2 y - \log_a z^4 \qquad \textit{Quotient Rule}$$

$$= \log_a x^2 + \log_a y - \log_a z^4 \qquad \textit{Product Rule}$$

$$\boxed{= 2\log_a x + \log_a y - 4\log_a z} \qquad \textit{Power Rule}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \frac{x^2 y}{b^3}$

Solution

$$\log_b\left(\frac{x^2 y}{b^3}\right) = \log_b x^2 y - \log_b b^3$$

$$= \log_b x^2 + \log_b y - \log_b b^3$$

$$= 2\log_b x + \log_b y - 3\log_b b$$

$$\boxed{= 2\log_b x + \log_b y - 3}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b\left(\frac{x^3y}{z^2}\right)$

Solution

$$\log_b\left(\frac{x^3y}{z^2}\right) = \log_b\left(x^3y\right) - \log_b z^2$$

$$= \log_b x^3 + \log_b y - \log_b z^2$$

$$= 3\log_b x + \log_b y - 2\log_b z$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$

Solution

$$\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right) = \log_b\left(\sqrt[3]{x}y^4\right) - \log_b\left(z^5\right)$$

$$= \log_b\left(x^{1/3}\right) + \log_b\left(y^4\right) - \log_b\left(z^5\right)$$

Exercise

Express the following in terms of sums and differences of logarithms $\log\left(\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right)$

Solution

$$\log\left(\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right) = \log\left(100x^3\sqrt[3]{5-x}\right) - \log\left(3(x+7)^2\right)$$

$$= \log 10^2 + \log x^3 + \log(5-x)^{1/3} - \left[\log 3 + \log\left((x+7)^2\right)\right]$$

$$= 2\log 10 + 3\log x + \tfrac{1}{3}\log(5-x) - \log 3 - 2\log(x+7)$$

$$= 2 + 3\log x + \tfrac{1}{3}\log(5-x) - \log 3 - 2\log(x+7)$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}}$

Solution

$$\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}} = \log_a \left(\frac{m^8 n^{12}}{a^3 b^5}\right)^{1/4}$$ *Power Rule*

$$= \frac{1}{4}\log_a \left(\frac{m^8 n^{12}}{a^3 b^5}\right)$$ *Quotient Rule*

$$= \frac{1}{4}\left[\log_a m^8 n^{12} - \log_a a^3 b^5\right]$$ *Product Rule*

$$= \frac{1}{4}\left[\log_a m^8 + \log_a n^{12} - \left(\log_a a^3 + \log_a b^5\right)\right]$$ *Power Rule*

$$= \frac{1}{4}\left[8\log_a m + 12\log_a n - 3 - 5\log_a b\right]$$

$$\underline{= 2\log_a m + 3\log_a n - \frac{3}{4} - \frac{5}{4}\log_a b}$$

Exercise

Use the properties of logarithms to rewrite: $\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}}$

Solution

$$\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}} = \log_p \left(\frac{m^5 n^4}{t^2}\right)^{1/3}$$ *Power Rule*

$$= \frac{1}{3}\log_p \left(\frac{m^5 n^4}{t^2}\right)$$ *Quotient Rule*

$$= \frac{1}{3}\left(\log_p m^5 n^4 - \log_p t^2\right)$$ *Product Rule*

$$= \frac{1}{3}\left(\log_p m^5 + \log_p n^4 - \log_p t^2\right)$$ *Power Rule*

$$= \frac{1}{3}\left(5\log_p m + 4\log_p n - 2\log_p t\right)$$

$$\underline{= \frac{5}{3}\log_p m + \frac{4}{3}\log_p n - \frac{2}{3}\log_p t}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \sqrt[n]{\frac{x^3 y^5}{z^m}}$

Solution

$$\log_b \sqrt[n]{\frac{x^3 y^5}{z^m}} = \log_b \left(\frac{x^3 y^5}{z^m}\right)^{1/n}$$

$$=\frac{1}{n}\log_b\left(\frac{x^3y^5}{z^m}\right)$$ *Power Rule*

$$=\frac{1}{n}\left(\log_b x^3y^5-\log_b z^m\right)$$ *Quotient Rule*

$$=\frac{1}{n}\left(\log_b x^3+\log_b y^5-\log_b z^m\right)$$ *Product Rule*

$$=\frac{1}{n}\left(3\log_b x+5\log_b y-m\log_b z\right)$$ *Power Rule*

$$=\frac{3}{n}\log_b x+\frac{5}{n}\log_b y-\frac{m}{n}\log_b z$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \sqrt[3]{\frac{a^2 b}{c^5}}$

Solution

$$\log_a \sqrt[3]{\frac{a^2 b}{c^5}}=\log_a\left(\frac{a^2 b}{c^5}\right)^{1/3}$$ ***Convert the radical to power***

$$=\frac{1}{3}\log_a\left(\frac{a^2 b}{c^5}\right)$$ ***Power Rule***

$$=\frac{1}{3}\left[\log_a a^2 b-\log_a c^5\right]$$ ***Quotient Rule***

$$=\frac{1}{3}\left[\log_a a^2+\log_a b-\log_a c^5\right]$$ ***Product Rule***

$$=\frac{1}{3}\left[2\log_a a+\log_a b-5\log_a c\right]$$ ***Power Rule***

$$=\frac{2}{3}\log_a a+\frac{1}{3}\log_a b-\frac{5}{3}\log_a c$$

$$=\frac{2}{3}+\frac{1}{3}\log_a b-\frac{5}{3}\log_a c$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b\left(x^4\sqrt[3]{y}\right)$

Solution

$$\log_b\left(x^4\sqrt[3]{y}\right)=\log_b\left(x^4\right)+\log_b\left(\sqrt[3]{y}\right)$$

$$=\log_b\left(x^4\right)+\log_b\left(y^{1/3}\right)$$

$$= 4\log_b x + \frac{1}{3}\log_b y$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_5\left(\dfrac{\sqrt{x}}{25y^3}\right)$

Solution

$$\log_5\left(\frac{\sqrt{x}}{25y^3}\right) = \log_5\left(x^{1/2}\right) - \log_5\left(25y^3\right)$$

$$= \log_5\left(x^{1/2}\right) - \left[\log_5\left(5^2\right) + \log_5\left(y^3\right)\right]$$

$$= \log_5\left(x^{1/2}\right) - \log_5\left(5^2\right) - \log_5\left(y^3\right)$$

$$= \frac{1}{2}\log_5(x) - 2\log_5(5) - 3\log_5(y)$$

$$= \frac{1}{2}\log_5(x) - 2 - 3\log_5(y)$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \dfrac{x^3 w}{y^2 z^4}$

Solution

$$\log_a \frac{x^3 w}{y^2 z^4} = \log_a x^3 w - \log_a y^2 z^4$$ ***Quotient rule***

$$= \log_a x^3 + \log_a w - \left(\log_a y^2 + \log_a z^4\right)$$ ***Product rule***

$$= \log_a x^3 + \log_a w - \log_a y^2 - \log_a z^4$$ ***Distribute minus***

$$= 3\log_a x + \log_a w - 2\log_a y - 4\log_a z$$ ***Power rule***

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \dfrac{\sqrt{y}}{x^4\sqrt[3]{z}}$

Solution

$$\log_a \frac{\sqrt{y}}{x^4\sqrt[3]{z}} = \log_a y^{1/2} - \log_a x^4 z^{1/3}$$ ***Quotient rule***

$$= \log_a y^{1/2} - \left(\log_a x^4 + \log_a z^{1/3}\right)$$ ***Product rule***

$$= \log_a y^{1/2} - \log_a x^4 - \log_a z^{1/3}$$ ***Distribute minus***

$$\boxed{= \frac{1}{2}\log_a y - 4\log_a x - \frac{1}{3}\log_a z}$$ ***Power rule***

Exercise

Express the following in terms of sums and differences of logarithms $\ln \sqrt[4]{\dfrac{x^7}{y^5 z}}$

Solution

$$\ln \sqrt[4]{\frac{x^7}{y^5 z}} = \ln\left(\frac{x^7}{y^5 z}\right)^{1/4}$$

$$= \frac{1}{4}\ln\left(\frac{x^7}{y^5 z}\right)$$ ***Power rule***

$$= \frac{1}{4}\left(\ln x^7 - \ln y^5 z\right)$$ ***Quotient rule***

$$= \frac{1}{4}\left(\ln x^7 - \left(\ln y^5 + \ln z\right)\right)$$ ***Product rule***

$$= \frac{1}{4}\left(\ln x^7 - \ln y^5 - \ln z\right)$$

$$= \frac{1}{4}\left(7\ln x - 5\ln y - \ln z\right)$$ ***Power rule***

$$\boxed{= \frac{7}{4}\ln x - \frac{5}{4}\ln y - \ln z}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln x\sqrt[3]{\dfrac{y^4}{z^5}}$

Solution

$$\ln x\sqrt[3]{\frac{y^4}{z^5}} = \ln x + \ln\left(\frac{y^4}{z^5}\right)^{1/3}$$ ***Product rule***

$$= \ln x + \ln\left(\frac{y^{4/3}}{z^{5/3}}\right)$$

$$= \ln x + \ln y^{4/3} - \ln z^{5/3}$$ ***Quotient rule***

$$\boxed{= \ln x + \frac{4}{3}\ln y - \frac{5}{3}\ln z}$$ ***Power rule***

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \sqrt[5]{\dfrac{m^4 n^5}{x^2 a b^{10}}}$

Solution

$$\begin{aligned}
\log_b \sqrt[5]{\frac{m^4 n^5}{x^2 a b^{10}}} &= \log_b \left(\frac{m^4 n^5}{x^2 a b^{10}}\right)^{1/5} \\
&= \frac{1}{5}\log_b \left(\frac{m^4 n^5}{x^2 a b^{10}}\right) \\
&= \frac{1}{5}\left(\log_b \left(m^4 n^5\right) - \log_b \left(x^2 a b^{10}\right)\right) \\
&= \frac{1}{5}\left(\left(\log_b \left(m^4\right) + \log_b \left(n^5\right)\right) - \left(\log_b \left(x^2\right) + \log_b (a) + \log_b \left(b^{10}\right)\right)\right) \\
&= \frac{1}{5}\left(4\log_b m + 5\log_b n - 2\log_b x - \log_b a - 10\right) \\
&= \frac{4}{5}\log_b m + \log_b n - \frac{2}{5}\log_b x - \frac{1}{5}\log_b (a) - 2
\end{aligned}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \dfrac{a^5 b^{10}}{c^2 \sqrt[4]{d^3}}$

Solution

$$\begin{aligned}
\log_b \frac{a^5 b^{10}}{c^2 \sqrt[4]{d^3}} &= \log_b \left(a^5 b^{10}\right) - \log_b \left(c^2\, d^{3/4}\right) \\
&= \log_b \left(a^5\right) + \log_b \left(b^{10}\right) - \left(\log_b \left(c^2\right) + \log_b \left(d^{3/4}\right)\right) \\
&= 5\log_b a + 10 - 2\log_b c - \frac{3}{4}\log_b d
\end{aligned}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\left(x^2\sqrt{x^2+1}\right)$

Solution

$$\begin{aligned}
\ln\left(x^2\sqrt{x^2+1}\right) &= \ln x^2 + \ln\left(x^2+1\right)^{1/2} \\
&= 2\ln x + \frac{1}{2}\ln\left(x^2+1\right)
\end{aligned}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln \dfrac{x^2}{x^2+1}$

Solution

$$\ln \frac{x^2}{x^2+1} = \ln x^2 - \ln\left(x^2+1\right)$$

$$\boxed{= 2\ln x - \ln\left(x^2+1\right)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\left(\dfrac{x^2(x+1)^3}{(x+3)^{1/2}}\right)$

Solution

$$\ln\left(\frac{x^2(x+1)^3}{(x+3)^{1/2}}\right) = \ln\left(x^2(x+1)^3\right) - \ln(x+3)^{1/2}$$

$$= \ln x^2 + \ln(x+1)^3 - \frac{1}{2}\ln(x+3)$$

$$\boxed{= 2\ln x + 3\ln(x+1) - \frac{1}{2}\ln(x+3)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\sqrt{\dfrac{(x+1)^5}{(x+2)^{20}}}$

Solution

$$\ln\sqrt{\frac{(x+1)^5}{(x+2)^{20}}} = \ln\left(\frac{(x+1)^5}{(x+2)^{20}}\right)^{1/2}$$

$$= \frac{1}{2}\ln\left(\frac{(x+1)^5}{(x+2)^{20}}\right)$$

$$= \frac{1}{2}\left(\ln(x+1)^5 - \ln(x+2)^{20}\right)$$

$$= \frac{1}{2}\left(5\ln(x+1) - 20\ln(x+2)\right)$$

$$\boxed{= \frac{5}{2}\ln(x+1) - 10\ln(x+2)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\quad \ln \frac{\left(x^2+1\right)^5}{\sqrt{1-x}}$

Solution

$$\ln \frac{\left(x^2+1\right)^5}{\sqrt{1-x}} = \ln\left(x^2+1\right)^5 - \ln(1-x)^{1/2}$$

$$= 5\ln\left(x^2+1\right) - \frac{1}{2}\ln(1-x)$$

Exercise

Express the following in terms of sums and differences of logarithms $\quad \ln\left(\sqrt[3]{\frac{x(x+1)(x-2)}{\left(x^2+1\right)(2x+3)}}\right)$

Solution

$$\ln\left(\sqrt[3]{\frac{x(x+1)(x-2)}{\left(x^2+1\right)(2x+3)}}\right) = \ln\left(\frac{x(x+1)(x-2)}{\left(x^2+1\right)(2x+3)}\right)^{1/3}$$

$$= \frac{1}{3}\ln\left(\frac{x(x+1)(x-2)}{\left(x^2+1\right)(2x+3)}\right)$$

$$= \frac{1}{3}\left(\ln\left(x(x+1)(x-2)\right) - \ln\left(\left(x^2+1\right)(2x+3)\right)\right)$$

$$= \frac{1}{3}\left(\ln x + \ln(x+1) + \ln(x-2) - \left(\ln\left(x^2+1\right) + \ln(2x+3)\right)\right)$$

$$= \frac{1}{3}\left(\ln x + \ln(x+1) + \ln(x-2) - \ln\left(x^2+1\right) - \ln(2x+3)\right)$$

$$= \frac{1}{3}\ln x + \frac{1}{3}\ln(x+1) + \frac{1}{3}\ln(x-2) - \frac{1}{3}\ln\left(x^2+1\right) - \frac{1}{3}\ln(2x+3)$$

Exercise

Express the following in terms of sums and differences of logarithms $\quad \ln\left(\sqrt{\frac{1}{x(x+1)}}\right)$

Solution

$$\ln\left(\sqrt{\frac{1}{x(x+1)}}\right) = \ln\left(\frac{1}{x(x+1)}\right)^{1/2}$$

$$= \frac{1}{2}\left(\ln 1 - \ln\left(x(x+1)\right)\right)$$

$$= -\frac{1}{2}\left(\ln x + \ln(x+1)\right)$$

$$\boxed{= -\frac{1}{2}\ln x - \frac{1}{2}\ln(x+1)}$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\left(\sqrt{\left(x^2+1\right)(x-1)^2}\right)$

Solution

$$\ln\left(\sqrt{\left(x^2+1\right)(x-1)^2}\right) = \ln\left(\left(x^2+1\right)(x-1)^2\right)^{1/2}$$

$$= \frac{1}{2}\ln\left(\left(x^2+1\right)(x-1)^2\right)$$

$$= \frac{1}{2}\left(\ln\left(x^2+1\right) + \ln(x-1)^2\right)$$

$$= \frac{1}{2}\left(\ln\left(x^2+1\right) + 2\ln(x-1)\right)$$

$$\boxed{= \frac{1}{2}\ln\left(x^2+1\right) + \ln(x-1)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\log(x+5) + 2\log x$

Solution

$$\log(x+5) + 2\log x = \log(x+5) + \log x^2$$

$$\boxed{= \log\left(x^2(x+5)\right)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $3\log_b x - \frac{1}{3}\log_b y + 4\log_b z$

Solution

$$3\log_b x - \frac{1}{3}\log_b y + 4\log_b z = \log_b x^3 + \log_b z^4 - \log_b y^{1/3}$$

$$= \log_b\left(x^3 z^4\right) - \log_b \sqrt[3]{y}$$

$$\boxed{= \log_b\left(\frac{x^3 z^4}{\sqrt[3]{y}}\right)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\frac{1}{2}\log_b(x+5)-5\log_b y$

Solution

$$\frac{1}{2}\log_b(x+5)-5\log_b y=\log_b(x+5)^{1/2}-\log_b y^5$$

$$=\boxed{\log_b\left(\frac{\sqrt{x+5}}{y^5}\right)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\ln\left(x^2-y^2\right)-\ln(x-y)$

Solution

$$\ln\left(x^2-y^2\right)-\ln(x-y)=\ln\frac{x^2-y^2}{x-y}$$

$$=\ln\frac{(x-y)(x+y)}{x-y}$$

$$=\boxed{\ln(x+y)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\ln(xz)-\ln\left(x\sqrt{y}\right)+2\ln\frac{y}{z}$

Solution

$$\ln(xz)-\ln\left(x\sqrt{y}\right)+2\ln\frac{y}{z}=\ln(xz)+\ln\left(\frac{y}{z}\right)^2-\ln\left(x\sqrt{y}\right)$$

$$=\ln\left(\frac{xzy^2}{z^2}\right)-\ln\left(x\sqrt{y}\right)$$

$$=\ln\left(\frac{xy^2}{z}\frac{1}{x\sqrt{y}}\right)$$

$$=\boxed{\ln\left(\frac{y^{3/2}}{z}\right)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\log\left(x^2 y\right) - \log z$

Solution

$$\boxed{\log\left(x^2 y\right) - \log z = \log\left(\frac{x^2 y}{z}\right)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary: $\log\left(z^2\sqrt{y}\right) - \log z^{1/2}$

Solution

$$\begin{aligned}\log\left(z^2\sqrt{y}\right) - \log z^{1/2} &= \log\left(\frac{z^2\sqrt{y}}{z^{1/2}}\right)\\ &= \log\left(z^{3/2}\sqrt{y}\right)\\ &= \boxed{\log\left(\sqrt{z^3 y}\right)}\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$2\log_a x + \frac{1}{3}\log_a(x-2) - 5\log_a(2x+3)$

Solution

$$\begin{aligned}2\log_a x + \tfrac{1}{3}\log_a(x-2) - 5\log_a(2x+3) &= \log_a x^2 + \log_a(x-2)^{1/3} - \log_a(2x+3)^5\\ &= \log_a x^2(x-2)^{1/3} - \log_a(2x+3)^5\\ &= \boxed{\log_a \frac{x^2(x-2)^{1/3}}{(2x+3)^5}}\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$5\log_a x - \frac{1}{2}\log_a(3x-4) - 3\log_a(5x+1)$

Solution

$$5\log_a x - \tfrac{1}{2}\log_a (3x-4) - 3\log_a (5x+1) = \log_a x^5 - \log_a (3x-4)^{1/2} - \log_a (5x+1)^3$$

$$= \log_a x^5 - \left[\log_a (3x-4)^{1/2} + \log_a (5x+1)^3\right]$$

$$= \log_a x^5 - \left[\log_a (3x-4)^{1/2}(5x+1)^3\right]$$

$$= \log_a \frac{x^5}{(3x-4)^{1/2}(5x+1)^3}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\log\left(x^3y^2\right) - 2\log\left(x\sqrt[3]{y}\right) - 3\log\left(\frac{x}{y}\right)$$

Solution

$$\log\left(x^3y^2\right) - 2\log\left(x\sqrt[3]{y}\right) - 3\log\left(\frac{x}{y}\right) = \log\left(x^3y^2\right) - \log\left(xy^{1/3}\right)^2 - \log\left(xy^{-1}\right)^3$$

$$= \log\left(x^3y^2\right) - \left[\log\left(x^2y^{2/3}\right) + \log\left(x^3y^{-3}\right)\right]$$

$$= \log\left(x^3y^2\right) - \log\left(x^2y^{2/3}x^3y^{-3}\right)$$

$$= \log\left(x^3y^2\right) - \log\left(x^5y^{-7/3}\right)$$

$$= \log\left(\frac{x^3y^2}{x^5y^{-7/3}}\right)$$

$$= \log\left(\frac{y^2y^{7/3}}{x^2}\right)$$

$$= \log\left(\frac{y^{13/3}}{x^2}\right)$$

$$= \log\left(\frac{\sqrt[3]{y^{13}}}{x^2}\right)$$

$$= \log\left(\frac{y^4\sqrt[3]{y}}{x^2}\right)$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\ln y^3 + \frac{1}{3}\ln\left(x^3y^6\right) - 5\ln y$

Solution

$$\begin{aligned}
\ln y^3 + \tfrac{1}{3}\ln\left(x^3y^6\right) - 5\ln y &= \ln y^3 + \ln\left(x^3y^6\right)^{1/3} - \ln y^5 \\
&= \ln y^3 + \ln\left(x^{3/3}y^{6/3}\right) - \ln y^5 \\
&= \ln y^3 + \ln\left(xy^2\right) - \ln y^5 \\
&= \ln\left(y^3xy^2\right) - \ln y^5 \\
&= \ln\left(\frac{y^5x}{y^5}\right) \\
&= \ln x
\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$2\ln x - 4\ln\left(\frac{1}{y}\right) - 3\ln(xy)$

Solution

$$\begin{aligned}
2\ln x - 4\ln\left(\frac{1}{y}\right) - 3\ln(xy) &= \ln x^2 - \ln\left(\frac{1}{y}\right)^4 - \ln(xy)^3 \\
&= \ln x^2 - \left[\ln\left(y^{-4}\right) + \ln\left(x^3y^3\right)\right] \\
&= \ln x^2 - \ln\left(y^{-4}x^3y^3\right) \\
&= \ln x^2 - \ln\left(y^{-1}x^3\right) \\
&= \ln\frac{x^2}{y^{-1}x^3} \\
&= \ln\frac{y}{x}
\end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$4\ln x + 7\ln y - 3\ln z$

Solution

$$\begin{aligned} 4\ln x + 7\ln y - 3\ln z &= \ln x^4 + \ln y^7 - \ln z^3 \\ &= \ln\left(x^4 y^7\right) - \ln z^3 \\ &= \ln\left(\frac{x^4 y^7}{z^3}\right) \end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{1}{3}\left[5\ln(x+6) - \ln x - \ln(x^2 - 25)\right]$

Solution

$$\begin{aligned} \frac{1}{3}\left[5\ln(x+6) - \ln x - \ln(x^2 - 25)\right] &= \frac{1}{3}\left[5\ln(x+6) - \left(\ln x + \ln(x^2 - 25)\right)\right] \\ &= \frac{1}{3}\left[\ln(x+6)^5 - \ln x(x^2 - 25)\right] \\ &= \frac{1}{3}\left[\ln\frac{(x+6)^5}{x(x^2-25)}\right] \\ &= \ln\left(\frac{(x+6)^5}{x(x^2-25)}\right)^{1/3} \end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{2}{3}\left[\ln\left(x^2 - 4\right) - \ln\left(x+2\right)\right] + \ln(x+y)$

Solution

$$\begin{aligned} \frac{2}{3}\left[\ln\left(x^2 - 4\right) - \ln\left(x+2\right)\right] + \ln(x+y) &= \frac{2}{3}\left[\ln\frac{x^2-4}{x+2}\right] + \ln(x+y) \\ &= \frac{2}{3}\left[\ln\frac{(x+2)(x-2)}{x+2}\right] + \ln(x+y) \\ &= \frac{2}{3}\ln(x-2) + \ln(x+y) \end{aligned}$$

$$= \ln(x-2)^{2/3} + \ln(x+y)$$

$$= \ln(x-2)^{2/3}(x+y)$$

$$\boxed{= \ln(x+y)\ \sqrt[3]{(x-2)^2}}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{1}{2}\log_b m + \frac{3}{2}\log_b 2n - \log_b m^2 n$

Solution

$$\frac{1}{2}\log_b m + \frac{3}{2}\log_b 2n - \log_b m^2 n = \log_b m^{1/2} + \log_b (2n)^{3/2} - \log_b m^2 n$$

$$= \log_b \left(m^{1/2}(2n)^{3/2}\right) - \log_b m^2 n$$

$$= \log_b \frac{m^{1/2}2^{3/2}n^{3/2}}{m^2 n}$$

$$= \log_b \frac{2^{3/2}n^{1/2}}{m^{3/2}}$$

$$= \log_b \left(\frac{2^3 n}{m^3}\right)^{1/2}$$

$$\boxed{= \log_b \sqrt{\frac{8n}{m^3}}}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{1}{2}\log_y p^3 q^4 - \frac{2}{3}\log_y p^4 q^3$

Solution

$$\frac{1}{2}\log_y p^3 q^4 - \frac{2}{3}\log_y p^4 q^3 = \log_y \left(p^3 q^4\right)^{1/2} - \log_y \left(p^4 q^3\right)^{2/3}$$

$$= \log_y \frac{\left(p^3 q^4\right)^{1/2}}{\left(p^4 q^3\right)^{2/3}}$$

$$= \log_y \frac{\left(p^3\right)^{1/2}\left(q^4\right)^{1/2}}{\left(p^4\right)^{2/3}\left(q^3\right)^{2/3}}$$

$$= \log_y \frac{p^{3/2}q^2}{p^{8/3}q^2}$$

$$= \log_y \frac{p^{3/2}}{p^{8/3}}$$

$$= \log_y \frac{1}{p^{8/3-3/2}}$$

$$\boxed{= \log_y \frac{1}{p^{7/6}}}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x$

Solution

$$\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x = 4\log_a y - \frac{5}{2}\log_a x$$

$$= \log_a y^4 - \log_a x^{5/2}$$

$$\boxed{= \log_a \frac{y^4}{\sqrt{x^5}}}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{2}{3}\left[\ln\left(x^2-9\right) - \ln(x+3)\right] + \ln(x+y)$

Solution

$$\frac{2}{3}\left[\ln\left(x^2-9\right) - \ln(x+3)\right] + \ln(x+y) = \frac{2}{3}\ln\frac{x^2-9}{x+3} + \ln(x+y)$$

$$= \frac{2}{3}\ln\frac{(x+3)(x-3)}{x+3} + \ln(x+y)$$

$$= \frac{2}{3}\ln(x-3) + \ln(x+y)$$

$$= \ln(x-3)^{2/3} + \ln(x+y)$$

$$= \ln\left((x-3)^{2/3}(x+y)\right)$$

$$\boxed{= \ln\left((x+y)\sqrt[3]{(x-3)^2}\right)}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\frac{1}{4}\log_b x - 2\log_b 5 - 10\log_b y$

Solution

$$
\begin{aligned}
\frac{1}{4}\log_b x - 2\log_b 5 - 10\log_b y &= \log_b x^{1/4} - \log_b 5^2 - \log_b y^{10} \\
&= \log_b x^{1/4} - \left[\log_b 5^2 + \log_b y^{10}\right] \\
&= \log_b x^{1/4} - \log_b \left(5^2 y^{10}\right) \\
&= \log_b \frac{\sqrt[4]{x}}{25y^{10}}
\end{aligned}
$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$2\ln(x+4) - \ln x - \ln\left(x^2-3\right)$

Solution

$$
\begin{aligned}
2\ln(x+4) - \ln x - \ln\left(x^2-3\right) &= \ln(x+4)^2 - \left(\ln x + \ln\left(x^2-3\right)\right) \\
&= \ln(x+4)^2 - \ln\left(x\left(x^2-3\right)\right) \\
&= \ln\frac{(x+4)^2}{x\left(x^2-3\right)}
\end{aligned}
$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$\ln x + \ln(y+3) + \ln(y+2) - \ln\left(y^2+5y+6\right)$

Solution

$$
\begin{aligned}
\ln x + \ln(y+3) + \ln(y+2) - \ln\left(y^2+5y+6\right) &= \ln\left(x(y+3)(y+2)\right) - \ln\left((y+3)(y+2)\right) \\
&= \ln\left(\frac{x(y+3)(y+2)}{(y+3)(y+2)}\right) \\
&= \ln x
\end{aligned}
$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\ln x + \ln(x+4) + \ln(x+1) - \ln\left(x^2 + 5x + 4\right)$$

Solution

$$\begin{aligned} \ln x + \ln(x+4) + \ln(x+1) - \ln\left(x^2 + 5x + 4\right) &= \ln\left(x(x+4)(x+1)\right) - \ln\left((x+4)(x+1)\right) \\ &= \ln\left(\frac{x(x+4)(x+1)}{(x+4)(x+1)}\right) \\ &= \ln x \end{aligned}$$

Exercise

Write the expression as a single logarithm and simplify if necessary:

$$\ln\left(x^2 - 25\right) - 2\ln(x+5) + \ln(x-5)$$

Solution

$$\begin{aligned} \ln\left(x^2 - 25\right) - 2\ln(x+5) + \ln(x-5) &= \ln\left(x^2 - 25\right) + \ln(x-5) - \ln(x+5)^2 \\ &= \ln\frac{(x-5)(x+5)(x-5)}{(x+5)^2} \\ &= \ln\left(\frac{(x-5)^2}{x+5}\right) \end{aligned}$$

Exercise

Solve the equation: $2^x = 128$

Solution

$2^x = 2^7$

$x = 7$

Exercise

Solve the equation: $3^x = 243$

Solution

$3^x = 3^5$

$\boxed{x=5}$

Exercise

Solve the equation: $5^x = 70$

Solution

$\boxed{x = \log_5 70}$

Exercise

Solve the equation: $6^x = 50$

Solution

$\boxed{x = \log_6 50}$

Exercise

Solve the equation: $5^x = 134$

Solution

$\boxed{x = \log_5 134}$

Exercise

Solve the equation: $7^x = 12$

Solution

$$\boxed{x = \log_7 12}$$

Exercise

Solve the equation: $9^x = \dfrac{1}{\sqrt[3]{3}}$

Solution

$$\left(3^2\right)^x = \frac{1}{3^{1/3}}$$

$$3^{2x} = 3^{-1/3}$$

$$2x = -\frac{1}{3}$$

$$\boxed{x = -\frac{1}{6}}$$

Exercise

Solve the equation: $49^x = \dfrac{1}{343}$

Solution

$$\left(7^2\right)^x = \frac{1}{7^3}$$

$$7^{2x} = 7^{-3}$$

$$2x = -3$$

$$\boxed{x = -\frac{3}{2}}$$

Exercise

Solve the equation: $2^{5x+3} = \dfrac{1}{16}$

Solution

$$2^{5x+3} = 2^{-4}$$

$$5x + 3 = -4$$

$$5x = -7$$

$$\boxed{x = -\frac{7}{5}}$$

Exercise

Solve the equation: $\left(\frac{2}{5}\right)^x = \frac{8}{125}$

Solution

$$\left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^3$$

$$\underline{x = 3}$$

Exercise

Solve the equation: $2^{3x-7} = 32$

Solution

$$\begin{aligned} 2^{3x-7} &= 32 \\ &= 2^5 \end{aligned}$$

$3x - 7 = 5$ *add 7 on both sides*

$3x = 12$ *Divide by 3*

$$\underline{x = 4}$$

Exercise

Solve the equation: $4^{2x-1} = 64$

Solution

$$4^{2x-1} = 4^3$$

$$2x - 1 = 3$$

$$2x = 4$$

$$\underline{x = 2}$$

Exercise

Solve the equation: $3^{1-x} = \frac{1}{27}$

Solution

$$3^{1-x} = \frac{1}{3^3}$$

$$3^{1-x} = 3^{-3}$$

$$1 - x = -3$$

$\underline{x = 4}$

Exercise

Solve the equation: $2^{-x^2} = 5$

Solution

$\ln 2^{-x^2} = \ln 5$

$-x^2 \ln 2 = \ln 5$

$x^2 = -\dfrac{\ln 5}{\ln 2} \quad \Rightarrow \quad$ *No Solution*

Exercise

Solve the equation: $2^{-x} = 8$

Solution

$2^{-x} = 2^3$

$-x = 3$

$\underline{x = -3}$

Exercise

Solve the equation: $\left(\dfrac{1}{3}\right)^x = 81$

Solution

$\left(\dfrac{1}{3}\right)^x = 81$

$\left(3^{-1}\right)^x = 3^4$

$3^{-x} = 3^4$

$-x = 4$

$\underline{x = -4}$

Exercise

Solve the equation: $3^{-x} = 120$

Solution

$-x = \log_3 120$ *Convert to Log*

$x = -\log_3 120$

$$\boxed{= \log_3 \frac{1}{120}}$$

Exercise

Solve the equation: $27 = 3^{5x}\ 9^{x^2}$

Solution

$$3^3 = 3^{5x}\left(3^2\right)^{x^2}$$

$$= 3^{5x}\ 3^{2x^2}$$

$$= 3^{5x+2x^2}$$

$$2x^2 + 5x = 3$$

$$2x^2 + 5x - 3 = 0$$

$$x = \frac{-5 \pm \sqrt{25+24}}{6}$$

$$x = \begin{cases} \frac{-5-7}{6} & \boxed{= -2} \\ \frac{-5+7}{6} & \boxed{= \frac{1}{3}} \end{cases}$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3)\ln 4 = -x\ln 3$$

$$x\ln 4 + 3\ln 4 = -x\ln 3$$

$$x\ln 4 + x\ln 3 = -3\ln 4$$

$$x(\ln 4 + \ln 3) = -3\ln 4$$

$$\boxed{x = \frac{-3\ln 4}{(\ln 4 + \ln 3)}}$$

Exercise

Solve the equation: $2^{x+4} = 8^{x-6}$

Solution

$$2^{x+4} = \left(2^3\right)^{x-6}$$

$$2^{x+4} = 2^{3x-18}$$

$x+4=3x-18$

$2x=22$

$\boxed{x=11}$

Exercise

Solve the equation: $8^{x+2}=4^{x-3}$

Solution

$\left(2^3\right)^{x+2}=\left(2^2\right)^{x-3}$

$2^{3(x+2)}=2^{2(x-3)}$

$3(x+2)=2(x-3)$

$3x+6=2x-6$

$3x-2x=-6-6$

$\boxed{x=-12}$

Exercise

Solve the equation: $7^x=12$

Solution

$\boxed{x=\log_7 12}$ ***Convert to Log***

Exercise

Solve the equation: $5^{x+4}=4^{x+5}$

Solution

$\ln 5^{x+4}=\ln 4^{x+5}$

$(x+4)\ln 5=(x+5)\ln 4$

$x\ln 5+4\ln 5=x\ln 4+5\ln 4$

$(\ln 5-\ln 4)x=5\ln 4-4\ln 5$

$\boxed{x=\dfrac{5\ln 4-4\ln 5}{\ln 5-\ln 4}}$

Exercise

Solve the equation: $5^{x+2}=4^{1-x}$

Solution

$$\ln 5^{x+2} = \ln 4^{1-x}$$

$$(x+2)\ln 5 = (1-x)\ln 4$$

$$x\ln 5 + 2\ln 5 = \ln 4 - x\ln 4$$

$$(\ln 5 + \ln 4)x = \ln 4 - 2\ln 5$$

$$\boxed{x = \frac{\ln 4 - 2\ln 5}{\ln 5 + \ln 4}}$$

Exercise

Solve the equation: $3^{2x-1} = 0.4^{x+2}$

Solution

$$\ln 3^{2x-1} = \ln\left(0.4^{x+2}\right)$$

$$(2x-1)\ln 3 = (x+2)\ln \frac{4}{10}$$

$$2x\ln 3 - \ln 3 = x\ln \frac{2}{5} + 2\ln \frac{2}{5}$$

$$\left(2\ln 3 - \ln \frac{2}{5}\right)x = \ln 3 + 2\ln \frac{2}{5}$$

$$\boxed{x = \frac{\ln 3 + 2\ln 0.4}{2\ln 3 - \ln 0.4}}$$

Exercise

Solve the equation: $4^{3x-5} = 16$

Solution

$$4^{3x-5} = 4^2$$

$$3x - 5 = 2$$

$$3x = 7$$

$$\boxed{x = \frac{7}{3}}$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3)\ln 4 = -x\ln 3$$

$$x\ln 4 + 3\ln 4 = -x\ln 3$$

$$(\ln 4 + \ln 3)\,x = -3\ln 4$$

$$\boxed{x = -\frac{3\ln 4}{\ln 4 + \ln 3}}$$

Exercise

Solve the equation: $7^{2x+1} = 3^{x+2}$

Solution

$$\ln 7^{2x+1} = \ln 3^{x+2}$$

$$(2x+1)\ln 7 = (x+2)\ln 3$$

$$2x\ln 7 + \ln 7 = x\ln 3 + 2\ln 3$$

$$2x\ln 7 - x\ln 3 = 2\ln 3 - \ln 7$$

$$x(2\ln 7 - \ln 3) = 2\ln 3 - \ln 7$$

$$\boxed{x = \frac{2\ln 3 - \ln 7}{2\ln 7 - \ln 3}}$$

Exercise

Solve the equation: $3^{x-1} = 7^{2x+5}$

Solution

$$\ln 3^{x-1} = \ln 7^{2x+5}$$

$$(x-1)\ln 3 = (2x+5)\ln 7$$

$$x\ln 3 - \ln 3 = 2x\ln 7 + 5\ln 7$$

$$x\ln 3 - 2x\ln 7 = \ln 3 + 5\ln 7$$

$$x(\ln 3 - 2\ln 7) = \ln 3 + 5\ln 7$$

$$\boxed{x = \frac{\ln 3 + 5\ln 7}{\ln 3 - 2\ln 7}}$$

Exercise

Solve the equation: $4^{x-2} = 2^{3x+3}$

Solution

$$\left(2^2\right)^{x-2} = 2^{3x+3}$$

$$2^{2x-4} = 2^{3x+3}$$

$$2x - 4 = 3x + 3$$

$$2x - 3x = 4 + 3$$

$$-x = 7$$

$$\underline{x = -7}$$

Exercise

Solve the equation: $3^{5x-8} = 9^{x+2}$

Solution

$$3^{5x-8} = \left(3^2\right)^{x+2}$$

$$3^{5x-8} = 3^{2x+4}$$

$$5x - 8 = 2x + 4$$

$$5x - 2x = 8 + 4$$

$$3x = 12$$

$$\underline{x = 4}$$

Exercise

Solve the equation: $3^{x+4} = 2^{1-3x}$

Solution

$\ln 3^{x+4} = \ln 2^{1-3x}$ ***'ln' both sides***

$(x+4)\ln 3 = (1-3x)\ln 2$ ***Power Rule***

$x\ln 3 + 4\ln 3 = \ln 2 - 3x\ln 2$ ***Distribute***

$$x\ln 3 + 3x\ln 2 = \ln 2 - 4\ln 3$$

$$x(\ln 3 + 3\ln 2) = \ln 2 - 4\ln 3$$

$$\underline{x = \frac{\ln 2 - 4\ln 3}{\ln 3 + 3\ln 2}}$$

Exercise

Solve the equation: $3^{2-3x} = 4^{2x+1}$

Solution

$\ln 3^{2-3x} = \ln 4^{2x+1}$ *'ln' both sides*

$(2-3x)\ln 3 = (2x+1)\ln 4$ *Power Rule*

$2\ln 3 - 3x\ln 3 = 2x\ln 4 + \ln 4$

$-3x\ln 3 - 2x\ln 4 = \ln 4 - 2\ln 3$

$-x(3\ln 3 + 2\ln 4) = \ln 4 - 2\ln 3$

$$x = -\frac{\ln 4 - 2\ln 3}{3\ln 3 + 2\ln 4}$$

$$= -\frac{\ln 4 - \ln 3^2}{\ln 3^3 + \ln 4^2}$$

$$= \frac{\ln 9 - \ln 4}{\ln 27 + \ln 16}$$

$$= \frac{\ln \frac{9}{4}}{\ln 432}$$

$$\boxed{= \log_{432} \frac{9}{4}}$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

Solution

$\ln 4^{x+3} = \ln 3^{-x}$

$(x+3)\ln 4 = -x\ln 3$

$x\ln 4 + 3\ln 4 = -x\ln 3$

$x\ln 4 + x\ln 3 = -3\ln 4$

$x(\ln 4 + \ln 3) = -3\ln 4$

$$\boxed{x = \frac{-3\ln 4}{(\ln 4 + \ln 3)}}$$

Exercise

Solve the equation: $7^{x+6} = 7^{3x-4}$

Solution

$x + 6 = 3x - 4$

$4 + 6 = 3x - x$

$10 = 2x$

$\boxed{x = 5}$

Exercise

Solve the equation: $2^{-100x} = (0.5)^{x-4}$

Solution

$$2^{-100x} = \left(\frac{1}{2}\right)^{x-4}$$

$$2^{-100x} = \left(2^{-1}\right)^{x-4}$$

$$2^{-100x} = 2^{-x+4}$$

$$-100x = -x + 4$$

$$-100x + x = 4$$

$$-99x = 4$$

$$x = -\frac{4}{99}$$

Exercise

Solve the equation: $4^x \left(\frac{1}{2}\right)^{3-2x} = 8.\left(2^x\right)^2$

Solution

$$\left(2^2\right)^x \left(2^{-1}\right)^{3-2x} = 2^3.2^{2x}$$

$$2^{2x}2^{2x-3} = 2^{3+2x}$$

$$2^{2x+2x-3} = 2^{3+2x}$$

$$2^{4x-3} = 2^{3+2x}$$

$$4x - 3 = 3 + 2x$$

$$4x - 2x = 3 + 3$$

$$2x = 6$$

$$x = 3$$

Exercise

$$5^x + 125\left(5^{-x}\right) = 30$$

Solution

$$5^x 5^x + 125\left(5^{-x}\right)5^x = 30\left(5^x\right)$$

$$5^{2x} + 125 = 30\left(5^x\right)$$

$$5^{2x} - 30\left(5^x\right) + 125 = 0 \qquad \textit{Solve for } 5^x$$

$$5^x = 5 \qquad 5^x = 25 = 5^2$$

$$x = 1 \qquad x = 2$$

$$\boxed{x = 1,\ 2}$$

Exercise

$$4^x - 3\left(4^{-x}\right) = 8$$

Solution

$$4^x 4^x - 3\left(4^{-x}\right)4^x = 8\left(4^x\right)$$

$$4^{2x} - 3 = 8\left(4^x\right)$$

$$4^{2x} - 8\left(4^x\right) - 3 = 0 \qquad \textit{Solve for } 4^x$$

$$4^x = 4 + \sqrt{19} \qquad 4^x = 4 - \sqrt{19} < 0$$

$$x \ln 4 = \ln\left(4 + \sqrt{19}\right)$$

$$\boxed{x = \frac{\ln\left(4+\sqrt{19}\right)}{\ln 4}}$$

Exercise

Solve the equation: $5^{3x-6} = 125$

Solution

$$5^{3x-6} = 5^3$$

$$3x - 6 = 3$$

$$3x = 9$$

$$\boxed{x = 3}$$

Exercise

Solve the equation: $e^x = 15$

Solution

$$\boxed{x = \ln 5} \qquad \textit{Convert to Log}$$

Exercise

Solve the equation: $e^{x+1} = 20$

Solution

$$x+1 = \ln 20 \qquad \textit{Convert to Log}$$

$$\boxed{x = -1+\ln 20}$$

Exercise

Solve the equation: $9e^{x} = 107$

Solution

$$e^{x} = \frac{107}{9}$$

$$\ln e^{x} = \ln\left(\frac{107}{9}\right)$$

$$x\ln e = \ln\left(\frac{107}{9}\right)$$

$$\boxed{x = \ln\left(\frac{107}{9}\right)}$$

Exercise

Solve the equation: $e^{x\ln 3} = 27$

Solution

$$x\ln 3 = \ln 27 \qquad \textit{Convert to Log}$$

$$x\ln 3 = \ln 3^{3}$$

$$x = \frac{3\ln 3}{\ln 3}$$

$$\boxed{= 3}$$

Exercise

Solve the equation: $e^{x^{2}} = e^{7x-12}$

Solution

$$e^{x^{2}} = e^{7x-12}$$

$$x^{2} = 7x-12$$

$$x^{2} - 7x + 12 = 0$$

$$\boxed{x = 3,\ 4}$$

Exercise

Solve the equation: $f(x) = xe^{x} + e^{x}$

Solution

$$xe^{x} + e^{x} = 0$$

$$e^{x}(x+1) = 0$$

$$e^{x} \neq 0 \qquad x+1=0$$

$$\underline{x=-1} \ \ (Only\ solution)$$

Exercise

Solve the equation $f(x) = x^{3}\left(4e^{4x}\right) + 3x^{2}e^{4x}$

Solution

$$x^{3}\left(4e^{4x}\right) + 3x^{2}e^{4x} = 0$$

$$x^{2}e^{4x}(4x+3) = 0$$

$$x^{2} = 0 \qquad 4x+3=0$$

$$x = 0,\ 0 \qquad x = -\frac{3}{4}$$

The solutions are: $\underline{x = 0,\ 0,\ -\frac{3}{4}}$

Exercise

Solve the equation: $e^{2x} - 2e^{x} - 3 = 0$

Solution

$$\left(e^{x}\right)^{2} - 2e^{x} - 3 = 0$$

$$\begin{cases} e^{x} = -1 \ \times & \rightarrow \ \textit{Impossible} \\ e^{x} = 3 & \rightarrow \ \underline{x = \ln 3} \end{cases}$$

Exercise

Solve the equation: $e^{0.08t} = 2500$

Solution

$$\ln\left(e^{0.08t}\right) = \ln 2500$$

$$0.08t = \ln(50)^2$$

$$t = \frac{200\ln 50}{8}$$

$$\underline{= 25\ln 50\,|}$$

Exercise

Solve the equation: $e^{x^2} = 200$

Solution

$\ln e^{x^2} = \ln 200$ *Natural Log both sides*

$x^2 = \ln 200$ $\ln e = 1$

$$\underline{x = \pm\sqrt{\ln 200}\,|}$$

Exercise

Solve the equation: $e^{2x+1} \cdot e^{-4x} = 3e$

Solution

$$e^{2x+1-4x} = 3e$$

$$e^{-2x+1} = 3e$$

$e^{-2x}e = 3e$ *Divide by e*

$$e^{-2x} = 3$$

$$\ln e^{-2x} = \ln 3$$

$$-2x = \ln 3$$

$$\underline{x = -\frac{1}{2}\ln 3\,|}$$

Exercise

Solve the equation: $e^{2x} - 8e^x + 7 = 0$

Solution

$\left(e^x\right)^2 - 8e^x + 7 = 0$ $a+b+c=0 \rightarrow x = 1, \frac{c}{a}$

$$\begin{cases} e^x = 1 & \rightarrow \underline{x = 0\,|} \\ e^x = 7 & \rightarrow \underline{x = \ln 7\,|} \end{cases}$$

Exercise

Solve the equation without using the calculator: $e^{2x} + 2e^x - 15 = 0$

Solution

$$\left(e^x\right)^2 + 2e^x - 15 = 0 \qquad \textit{Solve for } e^x$$

$$e^x = 3 \qquad e^x \cancel{=} -5 < 0$$

$$\boxed{x = \ln 3}$$

Exercise

Solve the equation: $e^x + e^{-x} - 6 = 0$

Solution

$$e^x e^x + e^x e^{-x} - e^x 6 = e^x 0$$

$$e^{2x} + 1 - 6e^x = 0$$

$$\left(e^x\right)^2 - 6e^x + 1 = 0$$

$$e^x = \frac{6 \pm \sqrt{36 - 4}}{2}$$

$$= \frac{6 \pm 4\sqrt{2}}{2}$$

$$e^x = 3 \pm 2\sqrt{2}$$

$$\boxed{x = \ln\left(3 \pm 2\sqrt{2}\right)}$$

Exercise

Solve the equation: $e^{1-3x} \cdot e^{5x} = 2e$

Solution

$$e^{1-3x+5x} = 2e$$

$$e^{1+2x} = 2e$$

$$e^1 e^{2x} = 2e \qquad \textit{Divide by e}$$

$$e^{2x} = 2 \qquad \textit{Natural Log both sides}$$

$$\ln e^{2x} = \ln 2$$

$$2x = \ln 2$$

$$\boxed{x = \frac{1}{2}\ln 2}$$

Exercise

Solve the equation: $6\ln(2x) = 30$

Solution

$$\ln(2x) = \frac{30}{6}$$

$$\ln(2x) = 5$$

$$2x = e^5$$

$$x = \frac{1}{2}e^5$$

Exercise

Solve the equation: $\log_5(x-7) = 2$

Solution

$$x - 7 = 5^2$$

$$x = 25 + 7$$

$$x = 32$$

Exercise

Solve the equation: $\log_4(5+x) = 3$

Solution

$$5 + x = 4^3$$

$$x = 64 - 5$$

$$= 59$$

Check: $\log_4(5+59)$

Exercise

Solve the equation: $\log(4x-18) = 1$

Solution

$$4x - 18 = 10$$

$$4x = 28$$

$$x = 7$$

Exercise

Solve the equation: $\log\left(x^2+19\right)=2$

Solution

$x^2+19=10^2$

$x^2=81$

$\underline{x=\pm 9}\quad\quad (\pm 9)^2+19>0$

Exercise

Solve the equation: $\ln\left(x^2-12\right)=\ln x$

Solution

$\ln\left(x^2-12\right)=\ln x$

$x^2-12=x$

$x^2-x-12=0$

$\underline{x=-3,\ 4}$

Check: $x=-3 \quad \ln(9-12)=\ln(-3)$ ✗

$x=4 \quad \ln(16-12)=\ln(4)$

$\therefore$ ***Solution***: $\underline{x=4}$

Exercise

Solve the equation: $\log\left(2x^2+3x\right)=\log(10x+30)$

Solution

$\log\left(2x^2+3x\right)=\log(10x+30)$

$2x^2+3x=10x+30$

$2x^2-7x-30=0$

$$x=\frac{7\pm\sqrt{49+240}}{4}$$

$$=\begin{cases}\frac{7-17}{4}=-\frac{5}{2}\\ \frac{7+17}{4}=6\end{cases}$$

Check: $x=-\frac{5}{2}$ $\log\left(\frac{25}{2}-\frac{15}{2}\right)=\log(-25+30)$

$x=4$ $\log(32+12)=\log(40+30)$

$\therefore$ ***Solution***: $\boxed{x=-\frac{5}{2},\ 4}$

Exercise

Solve the equation: $\log_5(2x+3)=\log_5 11+\log_5 3$

Solution

$$\log_5(2x+3)=\log_5(11\times 3)$$

$$2x+3=33$$

$$2x=30$$

$\boxed{x=15}$ ***Check:*** $\log_5(30+3)$

Exercise

Solve the equation: $\log_3 x-\log_9(x+42)=0$

Solution

$$\frac{\log x}{\log 3}-\frac{\log(x+42)}{\log 9}=0$$

$$\frac{\log x}{\log 3}-\frac{\log(x+42)}{\log 3^2}=0$$

$$\frac{\log x}{\log 3}-\frac{1}{2}\frac{\log(x+42)}{\log 3}=0$$

$$\log x-\frac{1}{2}\log(x+42)=0$$

$$2\log x=\log(x+42)$$

$$\log x^2=\log(x+42)$$

$$x^2=x+42$$

$$x^2-x-42=0$$

$\boxed{x=-6,\ 7}$

Check: $x=-6$ $\log_3(-6)-\log_9(-6+42)$ ✗

$x=7$ $\log_3 7-\log_9(7+42)=0$

$\therefore$ ***Solution***: $\boxed{x=7}$

Exercise

Solve the equation: $\log_5 x + \log_5 (4x-1) = 1$

Solution

$\log_5 x(4x-1) = 1$

$4x^2 - x = 5$

$4x^2 - x - 5 = 0 \qquad a-b+c=0 \rightarrow x=-1, \; -\frac{c}{a}$

$x = -\frac{5}{2}, \; 4$

Check: $x = -\frac{5}{2} \quad \log_5\left(-\frac{5}{2}\right) + \log_5(10-1)$ ✗

$x = 4 \quad \log_5(4) + \log_5(15)$

∴ **Solution**: $x = 4$

Exercise

Solve the equation: $\log x - \log(x+3) = 1$

Solution

$\log \frac{x}{x+3} = 1$

$\frac{x}{x+3} = 10$

$x = 10x + 30$

$9x = -30$

$x = -\frac{10}{3}$

Check: $x = -\frac{10}{3} \quad \log\left(-\frac{10}{3}\right) - \log(x+3)$ ✗

∴ *No Solution*

Exercise

Solve the equation: $\log x + \log(x-9) = 1$

Solution

$\log x(x-9) = 1$

$x^2 - 9x = 10$

$x^2 - 9x - 10 = 0 \qquad a-b+c=0 \rightarrow x=-1, \; -\frac{c}{a}$

$x = -1, \; 10$

Check: $x=-1 \quad \log(-1)+\log(x-9)$ ╳

$x=10 \quad \log(10)+\log(10-9)$

$\therefore$ ***Solution***: $\boxed{x=10}$

Exercise

Solve the equation: $\log_2(x+1)+\log_2(x-1)=3$

Solution

$\log_2(x+1)(x-1)=3$

$x^2-1=2^3$

$x^2=9$

$\boxed{x=\pm 3}$

Check: $x=-3 \quad \log_2(-2)+\log_2(x-1)$ ╳

$x=3 \quad \log_2(4)+\log_2(2)$

$\therefore$ ***Solution***: $\boxed{x=3}$

Exercise

Solve the equation: $\log_8(x+1)-\log_8 x=2$

Solution

$\log_8 \frac{x+1}{x}=2$

$\frac{x+1}{x}=8^2$

$x+1=64x$

$63x=1$

$\boxed{x=\frac{1}{63}}$

Check: $x=\frac{1}{63} \quad \log_8\left(\frac{1}{63}+1\right)-\log_8\frac{1}{63}$

$\therefore$ ***Solution***: $\boxed{x=\frac{1}{63}}$

Exercise

Solve the equation: $\ln(x+8)+\ln(x-1)=2\ln x$

Solution

$$\ln(x+8)(x-1)=\ln x^2$$

$$x^2+7x-8=x^2$$

$$7x-8=0$$

$$\underline{x=\frac{8}{7}}$$

Check: $x=\frac{8}{7}$ $\ln\left(\frac{8}{7}+8\right)+\ln\left(\frac{8}{7}-1\right)=2\ln\frac{8}{7}$

$\therefore$ ***Solution***: $\underline{x=\frac{8}{7}}$

Exercise

Solve the equation: $\ln(4x+6)-\ln(x+5)=\ln x$

Solution

$$\ln\frac{4x+6}{x+5}=\ln x$$

$$\frac{4x+6}{x+5}=x$$

$$4x+6=x^2+5x$$

$$x^2+x-6=0$$

$$\underline{x=-3,\ 2}$$

Check: $x=-3$ $\ln(-6)-\ln(x+5)=\ln x$ ✗

$x=2$ $\ln(14)-\ln(7)=\ln 2$

$\therefore$ ***Solution***: $\underline{x=2}$

Exercise

Solve the equation: $\ln(5+4x)-\ln(x+3)=\ln 3$

Solution

$$\ln\frac{5+4x}{x+3}=\ln 3$$

$$\frac{5+4x}{x+3}=3$$

$$5+4x=3x+9$$

$$\underline{x=4}$$

Check: $x = 4 \quad \ln(21) - \ln(7) = \ln 3$

$\therefore$ ***Solution***: $\boxed{x = 4}$

Exercise

Solve the equation: $\ln \sqrt[4]{x} = \sqrt{\ln x}$

Solution

Domain: $\boxed{x \ge 1}$

$$\ln x^{1/4} = \sqrt{\ln x}$$

$$\frac{1}{4}\ln x = \sqrt{\ln x}$$

$$\left(\frac{1}{4}\ln x\right)^2 = \left(\sqrt{\ln x}\right)^2$$

$$\frac{1}{6}\ln^2 x = \ln x$$

$$\ln^2 x = 6\ln x$$

$$\ln^2 x - 6\ln x = 0$$

$$(\ln x)(\ln x - 6) = 0$$

$$\begin{cases} \ln x = 0 \ \rightarrow \ \boxed{x = 1} \\ \ln x = 6 \ \rightarrow \ \boxed{x = e^6} \end{cases}$$

$\therefore$ ***Solution***: $\boxed{x = 1, \ e^6}$

Exercise

Solve the equation: $\sqrt{\ln x} = \ln \sqrt{x}$

Solution

Domain: $\boxed{x \ge 1}$

$$\sqrt{\ln x} = \ln x^{1/2}$$

$$\sqrt{\ln x} = \frac{1}{2}\ln x$$

$$\left(\sqrt{\ln x}\right)^2 = \left(\frac{1}{2}\ln x\right)^2$$

$$\ln x = \frac{1}{4}\ln^2 x$$

$$4\ln x = \ln^2 x$$

$$\ln^2 x - 4\ln x = 0$$

$$\ln x(\ln x - 4) = 0$$

$$\begin{cases} \ln x = 0 \ \rightarrow \ \boxed{x = 1} \\ \ln x = 4 \ \rightarrow \ \boxed{x = e^4} \end{cases}$$

$\therefore$ ***Solution***: $\boxed{x = 1,\ e^4}$

Exercise

Solve the equation: $\quad \log x^2 = (\log x)^2$

Solution

Domain: $\boxed{x \ge 1}$

$$2\log x = (\log x)^2$$

$$(\log x)^2 - 2\log x = 0$$

$$\log x(\log x - 2) = 0$$

$$\begin{cases} \log x = 0 \ \rightarrow \ \boxed{x = 1} \\ \log x = 2 \ \rightarrow \ \boxed{x = 100} \end{cases}$$

$\therefore$ ***Solution***: $\boxed{x = 1,\ 100}$

Exercise

Solve the equation: $\quad \log x^3 = (\log x)^2$

Solution

Domain: $\boxed{x \ge 1}$

$$3\log x = (\log x)^2$$

$$(\log x)^2 - 3\log x = 0$$

$$\log x(\log x - 3) = 0$$

$$\begin{cases} \log x = 0 \ \rightarrow \ \boxed{x = 1} \\ \log x = 3 \ \rightarrow \ \boxed{x = 10^3} \end{cases}$$ ***Convert to exponential***

$\therefore$ ***Solution***: $\boxed{x = 1,\ 10^3}$

Exercise

Solve the equation: $\quad \log(\log x) = 1$

Solution

$\log x = 10$ *Convert to exponential*

$\therefore$ ***Solution***: $\boxed{x = 10^{10}}$

Exercise

Solve the equation: $\log(\log x) = 2$

Solution

$\log x = 10^2$ *Convert to exponential*

$\therefore$ ***Solution***: $\boxed{x = 10^{100}}$

Exercise

Solve the equation: $\ln(\ln x) = 2$

Solution

$\ln x = e^2$ *Convert to exponential*

$\therefore$ ***Solution***: $\boxed{x = e^{e^2}}$

Exercise

Solve the equation: $\ln\left(e^{x^2}\right) = 64$

Solution

$e^{x^2} = e^{64}$ *Convert to exponential*

$x^2 = 64$

$\therefore$ ***Solution***: $\boxed{x = \pm 8}$

Exercise

Solve the equation: $e^{\ln(x-1)} = 4$

Solution

$x - 1 = 4$

$\therefore$ ***Solution***: $\boxed{x = 5}$

Exercise

Solve the equation: $10^{\log(2x+5)} = 9$

Solution

$2x + 5 = 9$

$2x = 4$

$\therefore$ ***Solution***: $\boxed{x = 2}$

Exercise

Solve the equation: $\log\sqrt{x^3 - 9} = 2$

Solution

$\sqrt{x^3 - 9} = 10^2$

$x^3 - 9 = 10^4$

$x^3 = 10,009$

$\therefore$ ***Solution***: $\boxed{x = \sqrt[3]{10,009}}$

Exercise

Solve the equation: $\log\sqrt{x^3 - 17} = \frac{1}{2}$

Solution

$\log\left(x^3 - 17\right)^{1/2} = \frac{1}{2}$

$\frac{1}{2}\log\left(x^3 - 17\right) = \frac{1}{2}$

$\log\left(x^3 - 17\right) = 1$

$x^3 - 17 = 10$

$x^3 = 27$

$\boxed{x = 3}$

Check: $x = 3 \quad \log\sqrt{27 - 17}$

$\therefore$ ***Solution***: $\boxed{x = 3}$

Exercise

Solve the equation: $\log_4 x = \log_4 (8-x)$

Solution

$x = 8 - x$

$x + x = 8$

$2x = 8$

$\underline{x = 4}$

Check: $x = 4 \quad \log_4 4 = \log_4 (8-4)$

$\therefore$ ***Solution***: $\underline{x = 4}$

Exercise

Solve the equation: $\log_7 (x-5) = \log_7 (6x)$

Solution

$x - 5 = 6x$

$x - 6x = 5$

$-5x = 5$

$\underline{x = -1}$

Check: $x = -1 \quad \log_7 (-6) = \log_7 (6x)$ ╳

$\therefore$ ***No Solution***

Exercise

Solve the equation: $\ln x^2 = \ln (12 - x)$

Solution

$\ln x^2 = \ln (12 - x)$

$x^2 = 12 - x$

$x^2 + x - 12 = 0$

$\underline{x = -4,\ 3}$

Check: $x = -4 \quad \ln (16) = \ln (16)$

$x = 3 \quad \ln (9) = \ln (12 - 3)$

$\therefore$ ***Solution***: $\underline{x = -4,\ 3}$

Exercise

Solve the equation $\log_2(x+7)+\log_2 x=3$

Solution

$$\log_2 x(x+7)=3$$

$$x(x+7)=2^3$$ ***Convert to Exponential Form***

$$x^2+7x=8$$

$$x^2+7x-8=0$$

$$\underline{x=1,\ \ -8|}$$

Check: $x=-8 \quad \log_2(x+7)+\log_2(-8)$ ✗

$x=1 \quad \log_2(1+7)+\log_2 1$

$\therefore$ ***Solution***: $\underline{x=1|}$

Exercise

Solve the equation $\ln x=1-\ln(x+2)$

Solution

$$\ln x+\ln(x+2)=1$$

$$\ln x(x+2)=1$$

$$x^2+2x=e$$

$$x^2+2x-e=0$$

$$x=\frac{-2\pm\sqrt{4+4e}}{2}$$

$$=\frac{-2\pm2\sqrt{1+e}}{2}$$

$$=\begin{cases}-1-\sqrt{1+e}<0\\ -1+\sqrt{1+e}>0\end{cases}$$

$\therefore$ ***Solution***: $\underline{x=-1+\sqrt{1+e}\ |}$

Exercise

Solve the equation $\ln x=1+\ln(x+1)$

Solution

$$\ln x-\ln(x+1)=1$$

$$\ln \frac{x}{x+1} = 1$$

$$\frac{x}{x+1} = e^1$$

$$x = (x+1)e$$

$$x = ex + e$$

$$x - ex = e$$

$$x(1-e) = e$$

$$x = \frac{e}{1-e} < 0$$

$\therefore$ ***No solution***

Exercise

Solve the equation $\log_6 (2x-3) = \log_6 12 - \log_6 3$

Solution

$$\log_6 (2x-3) = \log_6 \frac{12}{3}$$

$$\log_6 (2x-3) = \log_6 4$$

$$2x - 3 = 4$$

$$2x = 7$$

$$\boxed{x = \frac{7}{2}}$$

Check: $\quad x = \frac{7}{2} \quad \log_6 (7-3) = \log_6 12 - \log_6 3$

$\therefore$ ***Solution***: $\boxed{x = \frac{7}{2}}$

Exercise

Solve the equation: $\quad \log(3x+2) + \log(x-1) = 1$

Solution

Domain: $x > 1$

$$\log(3x+2)(x-1) = 1$$ ***Convert to exponential form***

$$3x^2 - x - 2 = 10$$

$$3x^2 - x - 12 = 0$$ ***Solve for x***

$$x = \frac{1 \pm \sqrt{1+144}}{6}$$

$$= \begin{cases} \frac{1-\sqrt{145}}{6} < 0 \\ \frac{1+\sqrt{145}}{6} > 1 \end{cases}$$

$\therefore$ ***Solution***: $\underline{x = \frac{1+\sqrt{145}}{6}}$

Exercise

Solve the equation: $\log_5 (x+2) + \log_5 (x-2) = 1$

Solution

$\log_5 (x+2)(x-2) = 1$

$(x+2)(x-2) = 5^1$

$x^2 - 4 = 5$

$x^2 = 5 + 4$

$x^2 = 9$

$\underline{x = \pm 3}$

Check: $x = -3 \quad \log_5 (-1) + \log_5 (x-2)$ ╳

$x = 3 \quad \log_5 (3+2) + \log_5 (3-2)$

$\therefore$ ***Solution***: $\underline{x = 3}$

Exercise

Solve the equation: $\log_2 x + \log_2 (x-4) = 2$

Solution

Domain: $x > 4$

$\log_2 x(x-4) = 2$

$x^2 - 4x = 2^2$

$x^2 - 4x - 4 = 0$

$$x = \frac{4 \pm \sqrt{32}}{2}$$

$$= \begin{cases} 2 - 2\sqrt{2} < 4 \text{ ╳} \\ 2 + 2\sqrt{2} > 4 \end{cases}$$

$\therefore$ ***Solution***: $\underline{x = 2 + 2\sqrt{2}}$

Exercise

Solve the equation: $\log_3 x + \log_3 (x+6) = 3$

Solution

Domain: $x > 0$

$\log_3 x(x+6) = 3$

$x^2 + 6x = 3^3$

$x^2 + 6x - 27 = 0$

$$x = \frac{-6 \pm \sqrt{36+108}}{2}$$

$$= \begin{cases} \frac{-6-12}{2} = -9 < 0 \times \\ \frac{-6+12}{2} = 3 > 0 \end{cases}$$

∴ ***Solution***: $\underline{x = 3}$

Exercise

Solve the equation: $\log_3 (x+3) + \log_3 (x+5) = 1$

Solution

Domain: $x > -3$

$\log_3 (x+3)(x+5) = 1$

$x^2 + 8x + 15 = 3$

$x^2 + 8x + 12 = 0$

$$x = \frac{-8 \pm \sqrt{64-48}}{2}$$

$$= \begin{cases} \frac{-8-4}{2} = -6 < -3 \times \\ \frac{-8+4}{2} = -2 > -3 \end{cases}$$

∴ ***Solution***: $\underline{x = -2}$

Exercise

Solve the equation: $\ln x = \frac{1}{2} \ln\left(2x + \frac{5}{2}\right) + \frac{1}{2} \ln 2$

Solution

Domain: $x > 0$

$$2\ln x = \ln\left(2x+\frac{5}{2}\right)+\ln 2$$

$$\ln x^2 = \ln 2\left(2x+\frac{5}{2}\right)$$

$$x^2 = 4x+5$$

$$x^2-4x-5=0 \qquad\qquad a-b+c=0 \;\rightarrow\; x=-1,\; -\frac{c}{a}$$

$$x=-1,\; 5$$

$\therefore$ ***Solution***: $x=5$

Exercise

Solve the equation $\ln(-4-x)+\ln 3=\ln(2-x)$

Solution

Domain: $x<5$

$$\ln 3(-4-x)=\ln(2-x)$$
$$-12-3x=2-x$$
$$-12-2=3x-x$$
$$-14=2x$$

$\therefore$ ***Solution***: $x=-7$

Exercise

Solve the equation: $\log_4 x+\log_4(x-2)=\log_4(15)$

Solution

Domain: $x>2$

$$\log_4 x(x-2)=\log_4(15)$$

$$x^2-2x=15$$

$$x^2-2x-15=0$$

$$x=\frac{2\pm\sqrt{4+60}}{2}$$

$$=\begin{cases}\frac{2-8}{2}=-4<2 \times\\ \frac{2+8}{2}=5>2\end{cases}$$

$\therefore$ ***Solution***: $x=5$

Exercise

Solve the equation: $\ln(x-5)-\ln(x+4)=\ln(x-1)-\ln(x+2)$

Solution

Domain: $x>5$

$$\ln\frac{x-5}{x+4}=\ln\frac{x-1}{x+2}$$

$$\frac{x-5}{x+4}=\frac{x-1}{x+2}$$

$$(x-5)(x+2)=(x-1)(x+4)$$

$$x^2+2x-5x-10=x^2+4x-x-4$$

$$x^2-3x-10=x^2+3x-4$$

$$x^2-3x-10-x^2-3x+4=0$$

$$-6x-6=0$$

$$x=-1$$

$\therefore$ ***No solution***

Exercise

Solve the equation: $\log\left(x^2+4\right)-\log\left(x+2\right)=2+\log\left(x-2\right)$

Solution

Domain: $x>-2$

$$\log\left(x^2+4\right)-\log\left(x+2\right)-\log\left(x-2\right)=2$$

$$\log\left(x^2+4\right)-\left[\log\left(x+2\right)+\log\left(x-2\right)\right]=2$$

$$\log\left(x^2+4\right)-\log\left(x+2\right)\left(x-2\right)=2$$

$$\log\left(\frac{x^2+4}{x^2-4}\right)=2$$

$$\frac{x^2+4}{x^2-4}=10^2$$

$$x^2+4=100x^2-400$$

$$400+4=100x^2-x^2$$

$$99x^2=404$$

$$x^2=\frac{404}{99}$$

$\therefore$ ***Solution***: $x=\frac{2\sqrt{101}}{3\sqrt{11}}$ is the only solution

Exercise

Solve the equation $\log_3(x-2) = \log_3 27 - \log_3(x-4) - 5^{\log_5 1}$

Solution

Domain: $x > 4$

$$\log_3(x-2) + \log_3(x-4) = \log_3 3^3 - 1$$

$$\log_3(x-2)(x-4) = 3 - 1$$

$$\log_3\left(x^2 - 6x + 8\right) = 2$$

$$x^2 - 6x + 8 = 3^2$$

$$x^2 - 6x + 8 = 9$$

$$x^2 - 6x - 1 = 0$$

$\rightarrow$ $x = 3 \pm \sqrt{10}$

Check: $x = 3 + \sqrt{10} > 4$

$x = 3 - \sqrt{10} < 4$ ✗

$\therefore$ ***Solution***: $x = 3 + \sqrt{10}$

Exercise

Solve the equation $\log_2(x+3) = \log_2(x-3) + \log_3 9 + 4^{\log_4 3}$

Solution

Domain: $x > 3$

$$\log_2(x+3) - \log_2(x-3) = 2 + 3$$

$$\log_2 \frac{x+3}{x-3} = 5$$

$$\frac{x+3}{x-3} = 2^5$$

$$x + 3 = 32(x-3)$$

$$x + 3 = 32x - 96$$

$$96 + 3 = 32x - x$$

$$31x = 99$$

$$x = \frac{99}{31} > 3$$

$\therefore$ ***Solution***: $x = \frac{99}{31}$

Exercise

Solve the equation $\quad \dfrac{10^x - 10^{-x}}{2} = 20$

Solution

$$\frac{10^x - 10^{-x}}{2} = 20$$

$$10^x - 10^{-x} = 40$$

$$10^x \times \quad 10^x - 40 - 10^{-x} = 0$$

$$\left(10^x\right)^2 - 40\left(10^x\right) - 1 = 0$$

$$10^x = \frac{40 \pm \sqrt{1604}}{2}$$

$$= \frac{40 \pm 2\sqrt{401}}{2}$$

$$= \begin{cases} 20 - \sqrt{401} < 0 \times \\ 20 + \sqrt{401} > 0 \end{cases}$$

$$10^x = 20 + \sqrt{401}$$

$$\underline{x = \log\left(20 + \sqrt{401}\right)\,|}$$

Exercise

Solve the equation $\quad \dfrac{10^x + 10^{-x}}{2} = 8$

Solution

$$10^x - 10^{-x} = 16$$

$$10^x \times \quad 10^x - 40 - 10^{-x} = 0$$

$$\left(10^x\right)^2 - 16\left(10^x\right) - 1 = 0$$

$$10^x = \frac{16 \pm \sqrt{260}}{2}$$

$$= \frac{16 \pm 2\sqrt{65}}{2}$$

$$= \begin{cases} 16 - \sqrt{65} < 0 \times \\ 16 + \sqrt{65} > 0 \end{cases}$$

$$10^x = 16 + \sqrt{65}$$

$$\underline{x = \log\left(16 + \sqrt{65}\right)\,|}$$

Exercise

Solve the equation $\dfrac{10^x + 10^{-x}}{10^x - 10^{-x}} = 5$

Solution

$$10^x + 10^{-x} = 5\left(10^x\right) - 5\left(10^{-x}\right)$$

$$10^x \times \quad 4\left(10^x\right) = 6\left(10^{-x}\right)$$

$$\left(10^x\right)^2 = \frac{3}{2}$$

$$10^x = \pm\sqrt{\frac{3}{2}}$$

$$10^x = \sqrt{\frac{3}{2}} \qquad\qquad 10^x = -\sqrt{\frac{3}{2}} \times$$

$$x = \log\left(\frac{3}{2}\right)^{1/2}$$

$$= \frac{1}{2}\log\frac{3}{2}$$

Exercise

Solve the equation $\dfrac{10^x + 10^{-x}}{10^x - 10^{-x}} = 2$

Solution

$$10^x + 10^{-x} = 2\left(10^x\right) - 2\left(10^{-x}\right)$$

$$10^x \times \quad \left(10^x\right) = 3\left(10^{-x}\right)$$

$$\left(10^x\right)^2 = 3$$

$$10^x = \pm\sqrt{3}$$

$$10^x = \sqrt{3} \qquad\qquad 10^x = -\sqrt{3} \times$$

$\therefore$ ***Solution***: $x = \log\sqrt{3}$

Exercise

Solve the equation $\dfrac{e^x + e^{-x}}{2} = 15$

Solution

$$e^x + e^{-x} = 30$$

$$e^x \times \quad e^x - 30 + e^{-x} = 0$$

$$\left(e^x\right)^2 - 30e^x + 1 = 0$$

$$e^x = \frac{30 \pm \sqrt{896}}{2}$$

$$= \frac{30 \pm 8\sqrt{14}}{2}$$

$$= 15 \pm 4\sqrt{14}$$

$\therefore$ ***Solution***: $\quad x = \ln\left(15 \pm 4\sqrt{14}\right)$

Exercise

Solve the equation $\qquad \dfrac{e^x - e^{-x}}{2} = 15$

Solution

$$e^x - e^{-x} = 30$$

$$e^x \times \quad e^x - 30 - e^{-x} = 0$$

$$\left(e^x\right)^2 - 30e^x - 1 = 0$$

$$e^x = \frac{30 \pm \sqrt{904}}{2}$$

$$= \frac{30 \pm 2\sqrt{226}}{2}$$

$$15 - \sqrt{226} < 0$$

$$e^x = 15 + \sqrt{226}$$

$\therefore$ ***Solution***: $\quad x = \ln\left(15 + \sqrt{226}\right)$

Exercise

Solve the equation $\qquad \dfrac{1}{e^x - e^{-x}} = 4$

Solution

$$4e^x - 4e^{-x} = 1$$

$$e^x \times \quad 4e^x - 1 - 4e^{-x} = 0$$

$$4\left(e^x\right)^2 - e^x - 4 = 0$$

$$e^x = \frac{1 \pm \sqrt{65}}{2}$$

$\therefore$ ***Solution***: $\boxed{x = \ln\left(\frac{1 \pm \sqrt{65}}{2}\right)}$

Exercise

Solve the equation $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$

Solution

$$e^x + e^{-x} = 3e^x - 3e^{-x}$$

$$-2e^x = -4e^{-x}$$

$$e^x \times \quad e^x = 2e^{-x}$$

$$\left(e^x\right)^2 = 2$$

Since, e^x can't be negative, then

$$\boxed{e^x = \sqrt{2}}$$

$\therefore$ ***Solution***: $\boxed{x = \ln\sqrt{2}}$

Exercise

Solve the equation $\dfrac{e^x - e^{-x}}{e^x + e^{-x}} = 6$

Solution

$$e^x - e^{-x} = 6e^x + 6e^{-x}$$

$$-5e^x = 7e^{-x}$$

$$e^x \times \quad -5e^x = 7e^{-x}$$

$$\left(e^x\right)^2 = -\frac{7}{5} \times$$

$\therefore$ ***No Solution***

Exercise

Use common logarithms to solve for $\boldsymbol{x}$ in terms of $\boldsymbol{y}$: $y=\dfrac{10^x+10^{-x}}{2}$

Solution

$$2y=10^x+10^{-x}$$

$$10^x\left(10^x\right)+10^{-x}\left(10^x\right)-2y\left(10^x\right)=0$$

$$\left(10^x\right)^2-2y\left(10^x\right)+1=0$$

Using the quadratic formula:

$$10^x=\frac{2y\pm\sqrt{(2y)^2-4(1)(1)}}{2(1)}$$

$$=\frac{2y\pm\sqrt{4y^2-4}}{2}$$

$$=\frac{2y\pm2\sqrt{y^2-1}}{2}$$

$$=y\pm\sqrt{y^2-1}$$

$y-\sqrt{y^2-1}>0\Rightarrow y>\sqrt{y^2-1}\Rightarrow y^2>y^2-1$ (***True for any y > 1***)

$y^2-1\geq0\ \Rightarrow\ \cancel{y\leq-1}\ \ or\ \ y\geq1$

$10^x=y-\sqrt{y^2-1}$ $\qquad$ $10^x=y+\sqrt{y^2-1}$

$\boxed{x=\log\left(y-\sqrt{y^2-1}\right)}$ $\qquad$ $\boxed{x=\log\left(y+\sqrt{y^2-1}\right)}$

Exercise

Use common logarithms to solve for $\boldsymbol{x}$ in terms of $\boldsymbol{y}$: $y=\dfrac{10^x-10^{-x}}{10^x+10^{-x}}$

Solution

$$y\left(10^x+10^{-x}\right)=10^x-10^{-x}$$

$$y10^x+y10^{-x}=10^x-10^{-x}$$

$$y10^x-10^x=-10^{-x}-y10^{-x}$$

$$10^x(y-1)=-10^{-x}(1+y)$$

$$10^x10^x(y-1)=-10^x10^{-x}(1+y)$$

$$\left(10^x\right)^2(y-1)=-(1+y)$$

$$\left(10^x\right)^2 = -\frac{y+1}{y-1}$$

$$\left(10^x\right)^2 = \frac{y+1}{1-y}$$

$$10^x = \left(\frac{y+1}{1-y}\right)^{1/2}$$

$$\boxed{x = \log\left(\frac{y+1}{1-y}\right)^{1/2}}$$

Exercise

Use natural logarithms to solve for ***x*** in terms of ***y***: $y = \frac{e^x - e^{-x}}{2}$

Solution

$$2y = e^x - e^{-x}$$

$$2ye^x = e^x e^x - e^{-x}e^x$$

$$2ye^x = \left(e^x\right)^2 - 1$$

$$\left(e^x\right)^2 - 2ye^x - 1 = 0$$

$$e^x = \frac{2y \pm \sqrt{4y^2+4}}{2}$$

$$= \frac{2y \pm 2\sqrt{y^2+1}}{2}$$

$$\boxed{= y \pm \sqrt{y^2+1}}$$

$e^x = y - \sqrt{y^2+1} < 0$ (***not a solution***)

$$e^x = y + \sqrt{y^2+1}$$

$$\boxed{x = \ln\left(y + \sqrt{y^2+1}\right)}$$

Exercise

Use natural logarithms to solve for ***x*** in terms of ***y***: $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Solution

$$ye^x + ye^{-x} = e^x - e^{-x}$$

$$ye^{-x} + e^{-x} = e^x - ye^x$$

$$(y+1)e^{-x} = (1-y)e^x$$

$$(y+1)e^{-x}e^x = (1-y)e^x e^x$$

$$y+1 = (1-y)\left(e^x\right)^2$$

$$\left(e^x\right)^2 = \frac{y+1}{1-y}$$

$$e^x = \pm\sqrt{\frac{y+1}{1-y}}$$

$$e^x = -\sqrt{\frac{y+1}{1-y}} < 0 \quad (\textbf{\textit{not a solution}})$$

$$e^x = \sqrt{\frac{y+1}{1-y}}$$

$$\boxed{x = \ln\sqrt{\frac{y+1}{1-y}}}$$

Exercise

Solve for t using logarithms with base a: $2a^{t/3} = 5$

Solution

$$a^{t/3} = \frac{5}{2}$$

$$\log a^{t/3} = \log\frac{5}{2}$$

$$\frac{t}{3}\log a = \log\frac{5}{2}$$

$$\frac{t}{3} = \frac{\log\frac{5}{2}}{\log a}$$

$$\frac{t}{3} = \log_a \frac{5}{2}$$

$$\boxed{t = 3\log_a \frac{5}{2}}$$

Exercise

Solve for t using logarithms with base a: $K = H - Ca^t$

Solution

$$Ca^t = H - K$$

$$a^t = \frac{H - K}{C}$$

$$\log a^t = \log \frac{H - K}{C}$$

$$t \log a = \log \frac{H - K}{C}$$

$$t = \frac{\log \frac{H-K}{C}}{\log a}$$

$$= \log_a \frac{H-K}{C}$$