ODE.

1st Order.

y' $\frac{dy}{dt}$ $D_t y$

$y' = y - t$ form $y' + p_{(t)} y = f(t)$

$y' + 4y = e^{-3t}$

second order: y'' $\ddot{y}$

$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$ partial derivative

$\frac{dy}{dt} = y' = f(t, y) = 0$

$y'' = f(t, y, y') = 0$

$y^{(n)} = f(t, y, y', \cdots, y^{(n-1)})$

Ex $y(t) = Ce^{-t^2}$ is a solution $y' = -2ty$

$y = Ce^{-t^2}$

$y' = -2t\, Ce^{-t^2}$

$= -2ty$ ✓

Ex $y(t) = \cos t$ is a soln. $y' = 1 + y^2$

$y = \cos t$

$y' = -\sin t = 1 + y^2$

$-\sin t = 1 + \cos^2 t$

not a soln.

1.2 Separable Eqns

$y' = y(x) = \frac{dy}{dx}$

$\int g(x)\,dx = \int dy$

$\frac{dy}{dx} = y + \sin x$ not separable.

Ex $y' = t y^2$

$\frac{dy}{dt} = t y^2$

$\int \frac{dy}{y^2} = \int t\,dt$

$-\frac{1}{y} = \frac{1}{2}t^2 + C$

$= \frac{t^2 + 2C}{2}$

$y(t) = -\frac{2}{t^2 + 2C}$

Newton's Law of Cooling

$$\frac{dT}{dt} = -k(T-A)$$

$$\int_{T_0}^{T} \frac{dT}{T-A} = -\int_0^t k\,dt \qquad d(T-A) = dT$$

$$\ln|T-A|\Big|_{T_0}^{T} = -kt\Big|_0^t$$

$$\ln|T-A| - \ln|T_0-A| = -kt$$

$$\ln\frac{|T-A|}{|T_0-A|} = -kt$$

$$\left|\frac{T-A}{T_0-A}\right| = e^{-kt}$$

$$\frac{T-A}{T_0-A} = e^{-kt}$$

$$T-A = (T_0-A)e^{-kt}$$

$$\boxed{T = A + (T_0-A)e^{-kt}}$$

$A = 70^\circ \quad T_0 = 40^\circ$

$T(t=10) = 50^\circ$

$$50^\circ = 70^\circ + (40^\circ - 70^\circ)e^{-10k}$$

$$+20 = +30\,e^{-10k}$$

$$e^{-10k} = \frac{2}{3} \Rightarrow -10k = \ln\left(\frac{2}{3}\right)$$

$$k = \frac{-1}{10}\ln\left(\frac{2}{3}\right) \quad \text{or} \quad \frac{1}{10}\ln\left(\frac{3}{2}\right)$$

≈ 0.0405

$$T(30) = 70 - 30e^{-(.0405)(30)}$$

$\approx 63.6^\circ F$