Lecture One

Section 1.1 - Linear Equations and Rational Equations

Definition of a Linear Equation

A linear equation in one variable x is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real number, and $a \neq 0$

Addition and Multiplication Properties of Equalities

If $a = b$, *then* $a + c = b + c$

If $a = b$, *then* $ac = bc$

Example

Solve: $3(2x - 4) = 7 - (x + 5)$

Solution

$$6x - 12 = 7 - x - 5$$
$$6x - 12 + x = 2 - x + x$$
$$7x - 12 = 2$$
$$7x - 12 + 12 = 2 + 12$$
$$7x = 14$$
$$\frac{7}{7}x = \frac{14}{7}$$
$$\underline{x = 2}$$

Example

Solve: $\frac{2x+4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$

Solution

$$(12)\frac{2x+4}{3} + (12)\frac{1}{2}x = (12)\frac{1}{4}x - (12)\frac{7}{3}$$
$$4(2x + 4) + 6x = 3x - 28$$
$$8x + 16 + 6x = 3x - 28$$
$$14x + 16 = 3x - 28$$

$14x - 3x = -28 - 16$

$11x = -44$

$\underline{x = -4}$

Example

Solve: $\frac{5}{2x} = \frac{17}{18} - \frac{1}{3x}$

Solution

$(18x)\frac{5}{2x} = (18x)\frac{17}{18} - (18x)\frac{1}{3x}$ *Restriction:* $\boldsymbol{x \neq 0}$

$45 = 17\boldsymbol{x} - 6$

$45 + 6 = 17\boldsymbol{x}$

$17\boldsymbol{x} = 51$

$\underline{x = 3}$

Example

Solve: $\frac{x}{x-2} = \frac{2}{x-2} - \frac{2}{3}$

Solution

$3(x-2)\frac{x}{x-2} = 3(x-2)\frac{2}{x-2} - 3(x-2)\frac{2}{3}$ *Restriction*: $\boldsymbol{x \neq 2}$

$3\boldsymbol{x} = 6 - 2(\boldsymbol{x} - 2)$

$3\boldsymbol{x} = 6 - 2\boldsymbol{x} + 4$

$3\boldsymbol{x} + 2\boldsymbol{x} = 10$

$\Rightarrow 5\boldsymbol{x} = 10$

$\Rightarrow \boldsymbol{x} = 2$

No Solution ***or*** $\{\varnothing\}$

Identities, Conditional Equations, and Contradictions

Example

Solve: $-2(x+4)+3x=x-8$

Solution

$-2(x+4)+3x=x-8$

$-2x-8+3x=x-8$

$x-8=x-8$

$0=0$ *True statement*

Solution: All real numbers

Example

Solve: $5x-4=11$

Solution

$5x-4=11$

$5x=15$

$x=3$

This is a conditional equation, and its solution set is {3}

Example

Solve: $3(3x-1)=9x+7$

Solution

$3(3x-1)=9x+7$

$9x-3=9x+7$

$-3=7$ ***False statement***

This is a contradiction equation, and its solution set is empty set {∅} or null

Solving for a Specified Variable

Example

Solve

a) $I = \text{Pr}\,\text{t} \quad for\ t$

$$\frac{I}{\text{Pr}} = \frac{\text{Pr}}{\text{Pr}}\,\text{t}$$

$$\frac{I}{\text{Pr}} = t$$

b) $A - P = \text{Pr}\,t \quad for\ P$

$$A = Prt + P$$

$$A = P(rt + 1)$$

$$\frac{A}{rt+1} = P \qquad or \quad P = \frac{A}{rt+1}$$

c) $3(2x - 5a) + 4b = 4x - 2 \quad for\ x$

$$6x - 15a + 4b = 4x - 2$$

$$6x - 4x = 15a - 4b - 2$$

$$2x = 15a - 4b - 2$$

$$x = \frac{15a - 4b - 2}{2}$$

Example

Solve the formula $2l + 2w = P$ for $\boldsymbol{w}$

Solution

$$2\boldsymbol{w} = \text{P} - 2l$$

$$\boxed{w = \frac{P - 2l}{2}}$$

Example

Solve the formula $P = C + MC$ for C

Solution

$$P = C(1 + M)$$

$$\frac{P}{1+M} = C$$

$$\boxed{C = \frac{P}{1+M}}$$

Exercises *Section* 1.1 - Linear Equations and Rational Equations

Solve

1. $5x-8=72$

2. $14-5x=-41$

3. $2x+6=3x-2$

4. $11x-(6x-5)=40$

5. $9x+11=7x+1$

6. $2x-7=6+x$

7. $5x-2=9x+2$

8. $3(x-2)+7=2(x+5)$

9. $3x+5-5(x+1)=6x+7$

10. $4(-2x+1)=6-(2x-4)$

11. $4(x+7)=2(x+12)+2(x+1)$

12. $6(3x-1)=8-10(10x-14)$

13. $5x-(2x-8)=35$

14. $\frac{1}{14}(3x-2)=\frac{x+10}{10}$

15. $\frac{5}{6}x-2x+\frac{4}{3}=\frac{5}{3}$

16. $\frac{7}{4}+\frac{1}{5}x-\frac{3}{2}=\frac{4}{5}x$

17. $5(x+3)+4x-3=-(2x-4)+2$

18. $2\left[x-(4+2x)+3\right]=2x+3$

19. $2x-\left\{x-\left[3x-(8x+6)\right]\right\}=2x-2$

20. $4(2x+7)=2x+22+3(2x+3)$

21. $4\left[2x-(3-x)+5\right]=-7x-2$

22. $3\left[2x-(4-x)+5\right]=7x-2$

23. $-4(2x-6)+8x=5x+24+x$

24. $-8(3x+4)+6x=4(x-8)+4x$

25. $4(x+7)=2(x+12)+2(x+1)$

26. $-6(2x+1)-3(x-4)=-15x+1$

27. $2(x-1)+3=x-3(x+1)$

28. $3(x-4)-4(x-3)=x+3-(x-2)$

29. $2-(7x+5)=13-3x$

30. $16=3(x-1)-(x-7)$

31. $5x-2(x+1)=x+(3x-5)$

32. $7(x+1)=4\left[x-(3-x)\right]$

33. $2\left[3x-2(2x-3)\right]=5(x-6)$

34. $.2x-.5=.1x+7$

35. $.01x+3.1=2.03x-2.96$

36. $.08x-.06(x+12)=7.72$

37. $.04(x-12)+.06x=1.52$

38. $.3(x+2)-.5(x+2)=-.2x-.4$

39. $.6(x-5)+.8(x-6)=.2x-1.8$

40. $.5x+\frac{4}{3}x=x+10$

41. $.25x+\frac{2}{3}x=x+2$

42. $\frac{1}{4}(x-2)=\frac{1}{6}(x-5)$

43. $\frac{1}{4}(3x-2)=\frac{1}{5}(x+5)$

44. $\frac{1}{9}(x+2)=\frac{1}{15}(2x+5)$

45. $\frac{1}{2}(4x+8)-16=-\frac{2}{3}(9x-12)$

46. $\frac{3}{4}(24-8x)-16=-\frac{2}{3}(6x-9)$

47. $\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$

48. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

49. $\frac{x-8}{3}+\frac{x-3}{2}=0$

50. $\frac{5}{2x}-\frac{8}{9}=\frac{1}{18}-\frac{1}{3x}$

51. $\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{x^2-16}$

52. $\frac{3x-1}{3}-\frac{2x}{x-1}=x$

53. $\frac{x}{x-2}=\frac{2}{x-2}+2$

54. $\frac{x}{x-7}=\frac{7}{x-7}+8$

55. $\frac{3x}{5}-x=\frac{x}{10}-\frac{5}{2}$

56. $2x-\frac{2x}{7}=\frac{x}{2}+\frac{17}{2}$

57. $\frac{x+3}{6}=\frac{2}{3}+\frac{x-5}{4}$

58. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

59. $\frac{x}{4} = 2 + \frac{x-3}{3}$

60. $5 + \frac{x-2}{3} = \frac{x+3}{8}$

61. $\frac{x+1}{3} = 5 - \frac{x+2}{7}$

62. $\frac{3x}{5} - \frac{x-3}{2} = \frac{x+2}{3}$

63. $\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x^2-2x}$

64. $\frac{-4x}{x-1} + \frac{4}{x+1} = \frac{-8}{x^2-1}$

65. $\frac{4x+3}{x+1} + \frac{2}{x} = \frac{1}{x^2+x}$

66. $\frac{6}{x+3} - \frac{5}{x-2} = \frac{-20}{x^2+x-6}$

67. $\frac{6}{x+1} - \frac{5}{x+2} = \frac{10}{x^2+3x+2}$

68. $3(x-4) - 5(x+2) = 3[2-(x+24)] - 2(x-2)$

69. $(2x+3)(6x-1) - 9 = 15x^2 - (3x-2)(x-2)$

70. $(3x-1)^2 - 2x(x-1) = 7x^2 - 5x + 2$

71. $(2x+3)(x-1) + (x+1)(x-4) = 3x^2$

72. $4x + 13 - \{2x - [4(x-3) - 5]\} = 2(x-6)$

73. $-2\{7 - [4 - 2(1-x) + 3]\} = 10 - [4x - 2(x-3)]$

74. $2(y+2) + (y+3)^2 = y(y+5) + 2\left(\frac{17}{2} + y\right)$

75. $(y+1)(y-1) = (y+2)(y-3) + 4$

76. $45 - [4 - 2y - 4(y+7)] = -4(1+3y) - [4 - 3(y+2) - 2(2y-5)]$

77. $35 - [2 - 3y - 4(y+7)] = -3(1+3y) + 4 - 3(y+2) - 2(2y-5)$

78. $25 - [2 + 5y - 3(y+2)] = -3(2y-5) - [5(y-1) - 3y + 3]$

Solve for the specific variable

79. $V = lwh$, *for* h

80. $A = \frac{1}{2}h(B+b)$, *for* B

81. $A = \frac{1}{2}h(a+b)$, *for* a

82. $S = 2\pi rh + 2\pi r^2$ *for* h

83. $A = \frac{1}{2}h(b_1 + b_2)$, *for* h

84. $A = \frac{1}{2}h(b_1 + b_2)$, *for* b_2

85. $A = \frac{1}{2}h(b_1 + b_2)$, *for* b_1

86. $S = P + \Pr t$ *for* t

87. $S = 2lw + 2wh + 2hl$ *for* h

88. $S = 2lw + 2wh + 2hl$ *for* w

89. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ *for* R_1

90. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ *for* R

91. $V = \frac{d_1 - d_2}{t}$ *for* d_1

92. $V = \frac{d_1 - d_2}{t}$ *for* d_2

93. $z = \dfrac{x-\mu}{s}$ *for* x

94. $z = \dfrac{x-\mu}{s}$ *for* μ

95. $s = \frac{1}{2}at^2 + vt$ *for* v

96. $s = \frac{1}{2}at^2 + vt$ *for* a

97. $L = a + (n-1)d$ *for* n

98. $L = a + (n-1)d$ *for* d

99. $A = \dfrac{x_1 + x_2 + x_3}{n}$ *for* x_2

100. $A = \dfrac{x_1 + x_2 + x_3}{n}$ *for* n

101. A sewage treatment plant has two inlet pipes to its settling pond. One can fill the pond in 10 *hrs*. the other in 12 *hrs*. If the first pipe is open for 5 *hrs*. and then the second pipe us opened, how long will it take to fill the pond?

Section 1.2 - Applications and Model

Solving an Applied Problem

1. ***Read*** the problem carefully until you understand what is given and what is to be found
2. ***Assign a variable*** to represent the unknown value.
3. ***Write an equation*** using the variable expression(s).
4. ***Solve*** the equation.
5. ***State the answer*** to the problem. Does it seem reasonable?
6. ***Check*** the answer.

Example

According to the US Department of Education (2007 data), there is a gap between teaching salaries for men and women at private colleges and universities. The average salary for men exceeds the average salary for women by $14,037. Combined their average salaries are $130,015. Determine the average teaching salaries at private colleges for women and for men.

Solution

The average salary for men exceeds the average salary for women by $14,037

$m = w + 14037$ **(1)**

Combined their average salaries are $130,015

$m + w = 130015$ **(2)**

$w + 14037 + w = 130015$ *Substitute* ***m*** *with equation* **(1)**

$2w + 14037 = 130015$

$2w = 130015 - 14037$

$2w = 115978$

$w = \frac{115978}{2} = \$57,989.00$

$m = 57989 + 14037$

$= \$\ 72,026.00$

Average salary for women $\$57,989.00$ and men $\$72,026.00$

Example

You are choosing between two long-distance telephone plans.
Plan A has a monthly fee of $15 with a charge of $0.08 per minute for all long distance calls.
Plan B has a monthly fee of $3 with a charge of $0.12 per minute for all long distance calls.
For how many minutes of long-distance calls will the costs for the two plans be the same?

Solution

Plan A has a monthly fee of $15 with a charge of $0.08 per minute for all long distance calls.

$\boldsymbol{A} = 15 + .08x$

Plan B has a monthly fee of $3 with a charge of $0.12 per minute for all long distance calls.

$\boldsymbol{B} = 3 + .12x$

Costs for the two plans be the same

$\boldsymbol{A = B}$

$15 + .08\boldsymbol{x} = 3 + .12\boldsymbol{x}$

$.08\boldsymbol{x} - 0.12\boldsymbol{x} = 3 - 15$

$-0.04\boldsymbol{x} = -12$

$$x = \frac{-12}{-0.04}$$

$= 300$ *minutes*

Example

You inherit $5000 with the stipulation that for the first year the money had to be invested in two funds paying 9% and 11% annual interest. How much did you invest at each rate if the total interest earned for the year was $487?

Solution

$.09\boldsymbol{x} + .11(5000 - \boldsymbol{x}) = 487$
$.09x + 550 - .11x = 487$
$-.02x = 487 - 550$
$-.02x = -63$

$$x = \frac{63}{.02}$$

$$= \frac{6300}{2}$$

$\$3{,}150.00$ **9%**

for **11%**: $5000 - 3150 = \$1{,}850.00$

Amount	***Rate***	***Year***	***I = Prt***
$\boldsymbol{x}$	.09	1	$.09x$
$5000 - \boldsymbol{x}$	.11	1	$.11(5000 - \boldsymbol{x})$
$5000			$487

Example

After a 30% price reduction, you purchase a new computer for $840. What was the computer's price before the reduction?

Solution

$x - .3x = 840$ ***Or 70 % = $840***

$.7\,x = 840$

$x = \dfrac{840}{0.7}$

$\boxed{x = \$1,200.00}$

Example

The length of a rectangular basketball court is 44 *feet* more than the width. If the perimeter of the basketball court is 288 *feet*. What are the dimensions?

Solution

Length of a rectangular basketball court is 44 feet more than the width

$l = w + 44$ **(1)**

$P = 288 = 2l + 2w$ ***Divide by 2 both sides***

$144 = l + w$

$l + w = 144$ **(2)**

From **(1)** $\rightarrow$ **(2)**: $w + 44 + w = 144$

$2w = 100$

$\boxed{w = 50\ ft}$

$\rightarrow l = 50 + 44$

$\boxed{= 94\ ft}$

Motion Problems

$$d = rt \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

d: *D*istance

r: *R*ate, speed, or velocity

t: *T*ime

Example

Maria and Eduardo are traveling to a business conference. The trip takes 2 *hr*. for Maria and 2.5 *hr*. for Eduardo, since he lives 40 *mi* farther away. Eduardo travels 5 *mph* faster than Maria. Find their average rates.

Solution

	r	***t***	***d***
Maria	x	2	$2x$
Eduardo	$x + 5$	2.5	$2.5(x + 5)$

He lives 40 *mi* farther away

$$2.5(x+5) = 2x + 40$$
$$2.5x + 12.5 = 2x + 40$$
$$2.5x - 2x = 40 - 12.5$$
$$0.5x = 27.5$$
$$x = \frac{27.5}{0.5}$$
$$= 55$$

Maria's rate is 55 *mph*

Eduardo's rate is 55 + 5 = 60 *mph*

Exercises *Section* 1.2 - Applications and Model

1. When a number is decreased by 30% of itself, the result is 28. What is the number?

2. When 80% a number is added to the number, the result is 252. What is the number?

3. If the length of each side of a square is increased by 3 *cm*, the perimeter of the new square is 40 *cm* more than twice of each side of the original square. Find the dimensions of the origin square.

4. The length of a rectangular label is 2.5 *cm* less than twice the width. The perimeter is 40.6 *cm*. Find the width.

5. An Automobile repair shop charged a customer $448, listing $63 for parts and the remainder for labor. If the cost of labor is $35 per hour, how many hours of labor did it take to repair the car?

6. In the morning, Margaret drove to a business appointment at 50 *mph*. Her average speed on the return trip in the afternoon was 40 *mph*. The return trip took $\frac{1}{4}$ *hr*. longer because of heavy traffic. How far did she travel to the appointment?

7. Marie borrowed $5240 for new furniture. She will pay it off in 11 months at an annual simple interest rate of 4.5%. How much interest will she pay?

8. One of the most effective ways of removing contaminants such as carbon monoxide and nitrogen dioxide from the air while cooking is to use a vented range hood. If a range hood removes contaminants at a rate of F liters of air per second, then the percent P of contaminants that are also removed from the surrounding air can be modeled by the linear equation

$$P = 1.06F + 7.18$$

Where $10 \le F \le 75$. What flow F must a range hood have to remove 50% of the contaminants from the air?

9. Americans spent about $511 billion dining out in 2006. This was a 5.1% increase over the amount spent in 2005. How much was spent dining out in 2005?

10. For households with at least one credit card, the average U.S. credit-card debt per household was $9312 in 2004. This was $6346 more than the average credit-card debt in 1990. What was the average credit-card debt per household in 1990?

11. Morgan's Seeds has a rectangular test plot with a perimeter of 322 *m*. The length is 25 *m* more than the width. Find the dimensions of the plot?

12. Together, a dog owner and a cat owner spend an average of $376 annually for veterinary-related expenses. A dog owner spends $150 more per year than a cat owner. Find the average annual veterinary-related expenses of a dog owner and of a cat owner.

13. America West Airlines fleet includes Boeing, each with a cruising speed of 500 *mph*, and Bombardier Dash each with a cruising speed of 302 *mph*. Suppose that a Dash takes off and travels at its cruising speed. One hour later, a Boeing takes off and follows the same route, traveling at its cruising speed. How long will it take the Boeing to overtake the Dash?

14. Two airline jets, jet ***A*** with a cruising speed of 517 *mph*, and jet ***B*** with a cruising speed of 290 *mph*. Suppose that jet ***B*** takes off and travels at it cruising speed. One hour later, jet ***A*** takes off and follows the same route, traveling at its cruising speed. How long will it take the jet ***A*** to overtake the jet ***B***?

15. Two airline jets, jet ***A*** with a cruising speed of 900 *km/h*, and jet ***B*** with a cruising speed of 180 *km/h*. Suppose that jet ***B*** takes off and travels at it cruising speed. Two hours later, jet ***A*** takes off and follows the same route, traveling at its cruising speed. How long will it take the jet ***A*** to overtake the jet ***B***?

16. A central Railway freight train leaves a station and travels due north at a speed of 60 *mph*. One hour later, An Amtrak passenger train leaves the same station and travels due north on a parallel track at a speed of 80 *mph*. How long will it take the passenger train to overtake the freight train?

17. An airplane that travels 450 *mph* in still air encounters a 30-*mph* headwind. How long will it take the plane to travel 1050 *mi* into the wind?

18. An airplane that travels 375 *mph* in still air is flying with a 25-*mph* tailwind. How long will it take the plane to travel 700 *mi* with the wind?

19. A kayak moves at a rate of 12 *mph* in still water. If the river's current flows at a rate of 4 *mph*, how long does it take the boat to travel 36 *miles* upstream?

20. A kayak travels at a rate of 14 *km/h* in still water. If the river's current flows at a rate of 2 *km/h*, how long does it take the boat to travel 20 *km* downstream?

21. Ron's two student loans total $28,000. One loan is at 5% simple interest and the other is at 3% simple interest. After 1 *year*, Ron owes $1,040 in interest. What is the amount of each loan?

22. You borrowed money at 5% simple interest rate to pay your tuition. At the end of 1 *year*, you owed a total of $1,365 in principal and interest. How much did you borrow?

23. You make an investment at 4% simple interest rate. At the end of 1 *year*, the total value of the investment is $1,560 in principal and interest. How much did you invest originally?

24. You invested a total of $5,000, part at 3% simple interest and a part at 4% simple interest. At the end of 1 *year* the investments had earned $176 interest. How much was invested at each rate?

25. You worked 48 *hr*. one week and earned a $1066 paycheck. You earn time and a half (1.5 times your regular hourly wage) for the number of hours you work in excess of 40. What is your regular hourly wage?

26. In 2010, 40.9% of federal tax returns had zero or negative tax liability. This amount is 15.7% more than the percentage of filers who had zero or negative tax liability in 2000. Find the percentage of tax filers in 2000 who had zero or negative tax liability.

27. The average annual salary of a restaurant manager is 24.8% less than the average annual salary of an office manager. The average annual salary of a restaurant manager is $48,533. Find the average annual salary of an office manager.

28. Jared's two student loans total $12,000. One loan is at 5% simple interest and the other is at 8% simple interest. After 1 *yr*. Jared owes $750 in interest. What is the amount of each loan?

29. Cody wishes to sell a piece of property for $240,000. He wants the money to be paid off in two ways – a short-term note at 6% interest and a long-term note at 5%. Find the amount of each note if the total annual interest paid is $13,000.

30. You inherit $5000 with the stipulation that for the first year the money had to be invested in two funds paying 9% and 11% annual interest. How much did you invest at each rate if the total interest earned for the year was $487?

31. An artist has sold a painting for $410,000. He needs some of the money in 6 months and the rest in 1 yr. He can get a treasury bond for 6 months at 4.65% and for one year at 4.91%. His broker tells him the two investments will earn a total of $14,961. How much should be invested at each rate to obtain that amount of interest?

32. The number of steps needed to burn off a Cheeseburger exceeds the number needed to burn off a 12-*ounce* Soda by 4140. The number needed to burn off a Doughnut exceeds the number needed to burn off 12-*ounce* soda by 2300. If you chow down a cheeseburger, doughnut, and 12-*ounce* soda, a 16790 step walk is needed to burn off the calories (and perhaps alleviate the guilt). Determine the number of steps it takes to burn off a cheeseburger, a doughnut, and a 12-*ounce* soda.

33. Although organic milk accounts for only 12% of the market, consumption is increasing. In 2004, Americans purchased 40.7 million gallons of organic milk, increasing at a rate of 5.6 million gallons per year. If this trend continues, when will Americans purchase 79.9 million gallons of organic milk?

34. How many gallons of a 5% acid solution must be fixed with 5 *gal* of a 10% solution to obtain 7% solution?

35. In 1969, 88% of the women considered this objective essential or very important. Since then, this percentage has decreased by approximately 1.1 each year. If this trend continues, by which year will only 33% of female freshmen consider "developing a meaningful philosophy of life" essential or very important?

36. Charlotte is a chemist. She needs a 20% solution of alcohol. She has a 15% solution on hand, as well as a 30% solution. How many liters of the 15% solution should she add to 3 L of the 30% solution to obtain her 20% solution?

37. You are choosing between two texting plans. Plan ***A*** has a monthly fee of $20 with a charge of $0.05 per text. Plan ***B*** has monthly fee of $5 with charge of $0.10 per text.
Both plans include photo and video texts. For how many text messages will the costs for the two plans be the same?

38. A computer store is having a sale on digital cameras. After a 40% price reduction, your purchase a digital camera for $276. What was the camera's price before the reduction?

39. In a triangle, the measure of the first angle is twice the measure of the second angle. The measure of the third angle is $8°$ less than the measure of the second angle. What is the measure of each angle?

40. In a triangle, the measure of the first angle is three times the measure of the second angle. The measure of the third angle is $35°$ less than the measure of the second angle. What is the measure of each angle?

41. In a triangle, the measures of the three angles are consecutive integers. What is the measure of each angle?

42. In a triangle, the measures of the three angles are consecutive ***even*** integers. What is the measure of each angle?

43. In a triangle, the measure of the first angle is *five* times the measure of the second angle. The measure of the third angle is $2°$ less than the measure of the second angle. What is the measure of each angle?

44. In a triangle, the measure of the first angle is *twice* as large as the second angle. The measure of the third angle is $20°$ more than the measure of the second angle. What is the measure of each angle?

45. In 2000, 31% of U.S. adults viewed a college education as essential for success. For the period 2000 through 2010, the percentage viewing a college education as essential for success increased on average by approximately 2.4 each year. If this trend continues, by which year will be 67% of all American adults view college education as essential for success?

46. Each day, the number of births in the world exceeds twice the number of deaths by 61 *thousand*.

a) If the population increase in a single day is 214 *thousand*, determine the number of births and deaths per day.

b) If the population increase in a single day is 214 *thousand*, by how many millions of people does the worldwide population increase each year?

c) Based on your answer to part (*b*), approximately how many years does it take for the population of the world to increase by an amount greater than the entire U.S. population (308 million)?

47. You are choosing between two health clubs. Club ***A*** offers membership for a fee of $40 plus a monthly fee of $25. Club ***B*** offers membership for a fee of $15 plus a monthly fee of $30. After how many months will the total cost at each health club be the same? What will be the total cost for each club?

48. Video Store ***A*** charges $9 to rent a video game for one week. Although only members can rent from the store, membership is free. Video Store ***B*** charges $4 to rent a video game for one week. Only members can rent from the store and membership is $50 per year. After how many video-game rentals will the total amount spent at each store be the same? What will be the total amount spent at each store?

49. The bus fare in a city is $1.25. people who use the bus have the option of purchasing a monthly discount pass for $15.00. with the discount pass, the fare is reduced to $0.75. Determine the number of times in a month the bus must be used so that the total monthly cost without the discount pass is the same as the total monthly cost with the discount pass.

50. A discount pass for a bridge costs $30 per month. The toll for the bridge is normally $5.00, but it is reduced to $3.50 for people who have purchased the discount pass. Determine the number of times in a month the bridge must be crossed so that the total monthly cost without the discount pass is the same as the total monthly cost with the discount pass.

51. After a 30% reduction, you purchase a dictionary for $30.80. What was the dictionary's price before the reduction?

52. After a 20% reduction, you purchase a television for $336. What was the television's price before the reduction?

53. Including 8% sales tax, an inn charges $162 per night. Find the inn's nightly cost before the tax is added.

54. Including 5% sales tax, an inn charges $252 per night. Find the inn's nightly cost before the tax is added.

55. The selling price of a refrigerator is $584. If the markup is 25% of the dealer's cost, what is the dealer's cost of the refrigerator?

56. The selling price of a scientific calculator is $15. If the markup is 25% of the dealer's cost, what is the dealer's cost of the calculator?

57. A job pays an annual salary of $33,150, which includes a holiday bonus of $750. If paychecks are issued twice a month, what is the gross amount for each paycheck?

58. The price of a coat is reduced by 40%. Where the coat still does not sell, it is reduced by 40% of the reduced price. If the price of the dress after both reductions is $72. What was the original price?

59. For an international call, a telephone company charges $0.43 for the first minute, $0.32 for each additional minute, and a $2.10 service charge. If the cost of a call is $5.73, how long did the person talk?

60. Metro taxi charges $2.50 pickup fee and $2 per mile traveled. The cab fare from the airport to his hotel is $32.50. how many miles did you travel in the cab?

61. The children at Tiny Tots Day Care plant a rectangular vegetable garden with a perimeter of 39 *m*. The length is twice the width. Find the dimensions of the garden.

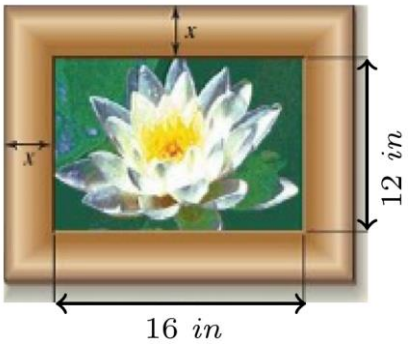
62. A rectangular field is twice as long as it is wide. If the perimeter of the field is 300 *yards*, what are its dimensions?

63. A rectangular swimming pool is three times as long as it is wide. If the perimeter of the pool is 320 *feet*, what are its dimensions?

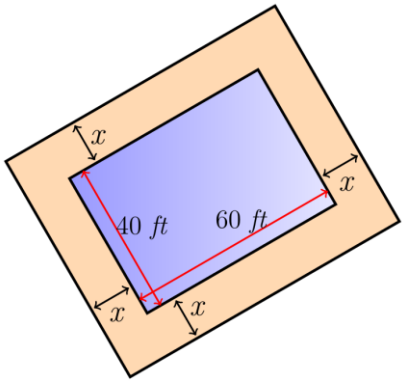
64. The length of the rectangular tennis court is 6 *feet* longer than twice the width. If the court's perimeter is 228 *feet*, what are its dimensions?

65. The length of the rectangular pool is 6 *meters* less than twice the width. If the pool's perimeter is 126 *meters*, what are its dimensions?

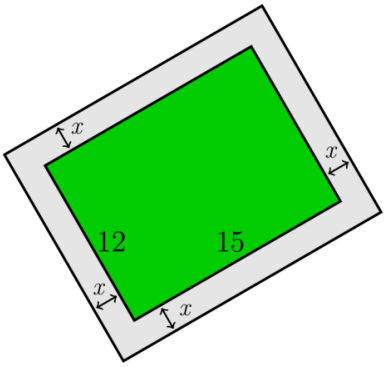
66. The rectangular painting measures 12 *inches* by 16 *inches* and contains a frame of uniform width around the four edges. The perimeter of the rectangle formed by the painting and its frame is 72 *inches*. Determine the width of the frame.



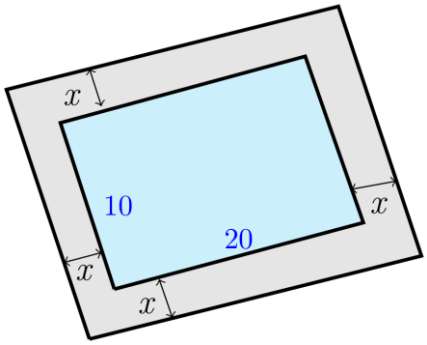
67. The rectangular swimming pool measures 40 *feet* by 60 *feet* and contains a path of uniform width around the four edges. The perimeter of the rectangle formed by the pool and the surrounding path is 248 *feet*. Determine the width of the path.



68. You paved your vegetable garden measuring 15 *meters* by 12 *meters* with stones. A path of uniform width is to surround the garden. If the perimeter of the garden and path combined is 70 *meters*, find the width of the path.



69. A pool measuring 10 *m* by 20 *m* is surrounded by a path of uniform width. If the perimeter of the pool and the path combined is 92 *m*, what is the width of the path?



70. A thief steals a number of rare plants from a nursery. On the way out, the thief meets three security guards, one after another. To each security guard, the thief is forced to give one-half the plants that he still has, plus 2 more. Finally, the thief leaves the nursery with 1 lone palm. How many plants were originally stolen?

Section 1.3 - Quadratic Equations

Basic Complex Number

$i^2 = -1 \qquad \Rightarrow i = \sqrt{-1} \qquad \Rightarrow \sqrt{-1} = i$

The number i is called the ***imaginary unit***.

Example

$\sqrt{-8} = i2\sqrt{2} \qquad 2i\sqrt{2}$

$\sqrt{-7}\sqrt{-7} = i\sqrt{7}\ i\sqrt{7}$

$= i^2\left(\sqrt{7}\right)^2$

$= -7$

Complex number is written in a form: $z = a + ib$

a is the real part

b is the imaginary part

Conjugate of a complex number $a + bi$ is $a - bi$

A ***quadratic equation*** in x is an equation that can be written in the general form:

$ax^2 + bx + c = 0$ where a, b, and c are real numbers,

$4x^2 - 3x + 2 = 0 \qquad a = 4 \quad b = -3 \quad c = 2$

Solving Quadratic Equations by Factoring

The Zero-Product Principle

If $AB = 0$ then $A = 0$ or $B = 0$.

Example

Solve $6x^2 + 7x - 3 = 0$

Solution

$(3x - 1)(2x + 3) = 0$

$3x - 1 = 0 \qquad 2x + 3 = 0$

$x = \frac{1}{3} \qquad x = -\frac{3}{2}$

The Square Root Property

If u is an algebraic expression and d is a nonzero real number, then $u^2 = d$ has exactly two solutions:

$$\text{If } \; u^2 = d, \text{ then } \; u = \sqrt{d} \; \text{ or } \; u = -\sqrt{d}$$

Equivalently,

$$\text{If } \; u^2 = d \; \Rightarrow \; u = \pm\sqrt{d} \; .$$

Example

Solve $3x^2 - 21 = 0$

Solution

$3x^2 = 21$

$x^2 = 7$

$\Rightarrow \; \underline{x = \pm\sqrt{7}}$

Example

Solve $5x^2 + 45 = 0$

Solution

$5x^2 = -45$

$x^2 = -9$

$x = \pm\sqrt{-9}$

$\Rightarrow \; \underline{x = \pm 3i}$

Example

Solve $(x + 5)^2 = 11$

Solution

$x + 5 = \pm\sqrt{11}$

$\Rightarrow \; \underline{x = -5 \pm \sqrt{11}}$

Completing the Square

If $x^2 + bx$ is a binomial, then by ***adding*** $\left(\frac{b}{2}\right)^2$ which is the square of half the coefficient of *x*. a perfect square trinomial will result. That is.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 \qquad x^2 + bx + \left(\frac{1}{2}b\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example

Solve: $x^2 + 4x - 1 = 0$

Solution

$$x^2 + 4x = 1$$

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 1 + \left(\frac{4}{2}\right)^2$$

$$x^2 + 4x + (2)^2 = 1 + 4$$

$$(x + 2)^2 = 5$$

$$x + 2 = \pm\sqrt{5}$$

$$\underline{x = -2 \pm \sqrt{5}}$$

Quadratic Formula

(Using Completing the Square)

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{1}{2}\frac{b}{a}\right)^2 = -\frac{c}{a} + \left(\frac{1}{2}\frac{b}{a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$*** \quad ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{cases} b^2 - 4ac > 0 \rightarrow 2 \text{ Real numbers} \\ b^2 - 4ac < 0 \rightarrow 2 \text{ Complex numbers} \\ b^2 - 4ac = 0 \rightarrow \text{One solution (repeated)} \end{cases}$$

Example

Solve: $2x^2 + 2x - 1 = 0$

Solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \Rightarrow a = 2 \quad b = 2 \quad c = -1$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4 + 8}}{4}$$

$$= -\frac{2}{4} \pm \frac{\sqrt{12}}{4}$$

$$= -\frac{1}{2} \pm \frac{2\sqrt{3}}{4}$$

$$\boxed{= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}}$$

$$= \frac{-2 \pm \sqrt{12}}{4}$$

$$= \frac{-2 \pm 2\sqrt{3}}{4}$$

$$= \frac{2\left(-1 \pm \sqrt{3}\right)}{4}$$

$$\boxed{= \frac{-1 \pm \sqrt{3}}{2}}$$

Example

Solve $x^2 - 4x = -2$

Solution

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} \qquad \Rightarrow a = 1 \quad b = -4 \quad c = 2 \; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$$= \frac{2\left(2 \pm \sqrt{2}\right)}{2}$$

$$\boxed{= 2 \pm \sqrt{2}}$$

Example

Solve: $x^2 - 2x + 2 = 0$

Solution

$\Rightarrow a = 1 \quad b = -2 \quad c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2}{2} \pm \frac{\sqrt{-4}}{2}$$

$$= 1 \pm \frac{2i}{2}$$

$$= 1 \pm i$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

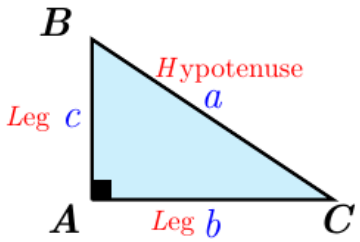
$$= \frac{2 \pm 2i}{2}$$

$$= \frac{2(1 \pm i)}{2}$$

$$= 1 \pm i$$

Pythagorean Theorem

The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If the legs have lengths a and b, and the hypotenuse has length c, then:



$$a^2 = b^2 + c^2$$

Example

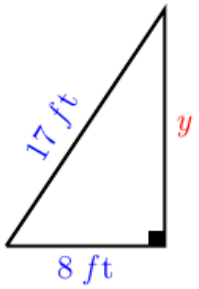
A ladder that is 17 *feet* long is 8 *feet* from the base of a wall. How far up the wall does the ladder reach?

Solution

$$8^2 + y^2 = 17^2$$

$$y^2 = 17^2 - 8^2$$

$$y = \sqrt{17^2 - 8^2} = 15 \text{ ft}$$



Height of a Projected Object *(Position Function)*

An object that is falling or vertically projected into the air has its height above the ground, $s(t)$, in feet, given by

$$s(t) = -16t^2 + v_0 t + s_0$$

v_0 is the original velocity (initial velocity) of the object, in feet per second

t is the time that the object is in motion, in second

s_0 is the original height (initial height) of the object, in feet

Example

If a projectile is shot vertically upward from the ground with an initial velocity of 100 $ft/\sec$, neglecting air resistance, its height s (in *feet*) above the ground t seconds after projection is given by

$$s = -16t^2 + 100t$$

a) After how many seconds will it be 50 *feet*. above the ground?

b) How long will it take for the projectile to return to the ground?

Solution

a) *After how many seconds will it be 50 feet above the ground?*

$$50 = -16t^2 + 100t$$

$$16t^2 - 100t + 50 = 0$$

$$8t^2 - 50t + 25 = 0$$

$$t = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(8)(25)}}{2(8)}$$

$$= \frac{50 \pm \sqrt{1700}}{16}$$

$$t = \frac{50 - 10\sqrt{17}}{16} \qquad t = \frac{50 + 10\sqrt{17}}{16}$$

$$= \frac{25 - 5\sqrt{17}}{8} \approx 0.55 \qquad = \frac{25 + 5\sqrt{17}}{8} \approx 5.70$$

b) *How long will it take for the projectile to return to the ground?*

$$0 = -16t^2 + 100t$$

$$0 = -4t(4t - 25)$$

$$-4t = 0 \qquad 4t - 25 = 0$$

$$t = 0 \qquad t = \frac{25}{4} = 6.25$$

Exercises *Section* 1.3 - Quadratic Equations

Solve

1. $x^2 = -25$

2. $x^2 = 49$

3. $9x^2 = 100$

4. $4x^2 + 25 = 0$

5. $5x^2 + 35 = 0$

6. $5x^2 - 45 = 0$

7. $(x-4)^2 = 12$

8. $(x+3)^2 = -16$

9. $(x-2)^2 = -20$

10. $(4x+1)^2 = 20$

11. $x^2 - 6x = -7$

12. $-6x^2 = 3x + 2$

13. $3x^2 + 2x = 7$

14. $3x^2 + 6 = 10x$

15. $5x^2 + 2 = x$

16. $5x^2 = 2x - 3$

17. $x^2 + 8x + 15 = 0$

18. $x^2 + 5x + 2 = 0$

19. $x^2 + x - 12 = 0$

20. $x^2 - 2x - 15 = 0$

21. $x^2 - 4x - 45 = 0$

22. $x^2 - 6x - 10 = 0$

23. $2x^2 + 3x - 4 = 0$

24. $x^2 - x + 8 = 0$

25. $2x^2 - 13x = 1$

26. $r^2 + 3r - 3 = 0$

27. $x^3 + 8 = 0$

28. $4x^2 - 12x + 9 = 0$

29. $9x^2 - 30x + 25 = 0$

30. $x^2 - 14x + 49 = 0$

31. $x^2 - 8x + 16 = 0$

32. $x^2 + 6x + 13 = 0$

33. $2x^2 - 2x + 13 = 0$

34. $x^2 + 2x + 29 = 0$

35. $4x^2 + 4x + 13 = 0$

36. $x^2 - 2x + 26 = 0$

37. $9x^2 - 4x + 20 = 0$

38. $x^2 + 6x + 21 = 0$

39. $9x^2 - 12x - 49 = 0$

40. $x(x-3) = 18$

41. $x(x-4) - 21 = 0$

42. $(x-1)(x+4) = 14$

43. $(x-3)(x+8) = -30$

44. $x(x+8) = 16(x-1)$

45. $x(x+9) = 4(2x+5)$

46. $(x+1)^2 = 2(x+3)$

47. $(x+1)^2 - 5(x+2) = 3x + 7$

48. $x(8x+1) = 3x^2 - 2x + 2$

Solve

49. $x^2 + 6x - 7 = 0$

50. $x^2 - 6x - 7 = 0$

51. $3x^2 + 4x - 7 = 0$

52. $3x^2 - 4x - 7 = 0$

53. $3x^2 - x - 2 = 0$

54. $3x^2 + x - 2 = 0$

55. $2x^2 + 3x - 5 = 0$

56. $2x^2 - 3x - 5 = 0$

57. $x^2 - 3x - 4 = 0$

58. $x^2 + 3x - 4 = 0$

59. $x^2 + 2x + 1 = 0$

60. $4x^2 - x - 5 = 0$

61. Solve for the specified variable $A = \frac{\pi d^2}{4}$, *for* d

62. Solve for the specified variable $rt^2 - st - k = 0 \ \ (r \neq 0)$, *for* t

63. A rectangular park is 6 *miles* long and 2 *miles* wide. How long is a pedestrian route that runs diagonally across the park?

64. What is the width of a 25-*inch* television set whose height is 15 *inches*?

65. The length of a rectangular sign is 3 *feet* longer than the width. If the sign's area is 54 square *feet*, find its length and width.

66. A rectangular parking lot has a length that is 3 *yards* greater than the width. The area of the parking lot is 180 square *yards*, find the length and the width.

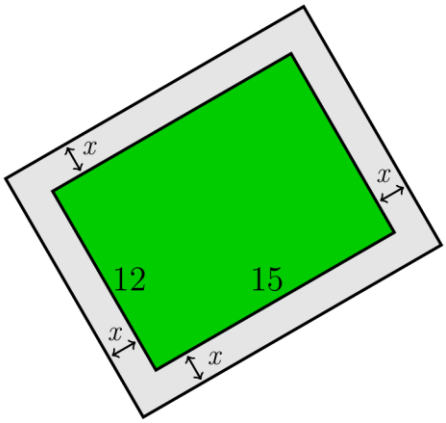
67. Each side of a square is lengthened by 3 *inches*. The area of this new, larger square is 64 square *inches*. Find the length of a side of the original square.

68. Each side of a square is lengthened by 2 *inches*. The area of this new, larger square is 36 square *inches*. Find the length of a side of the original square.

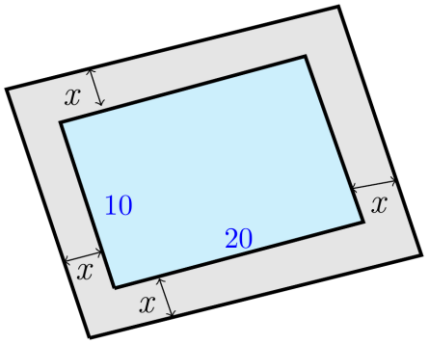
69. One number is 5 greater than another. The product of the numbers is 36. Find the numbers.

70. One number is 6 less than another. The product of the numbers is 72. Find the numbers.

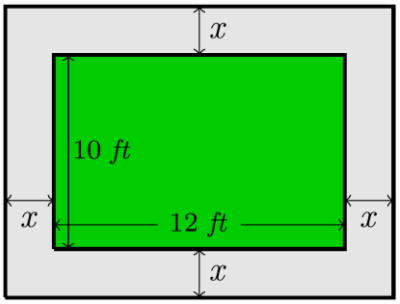
71. A vacant rectangular lot is being turned into a community vegetable garden measuring 15 *meters* by 12 *meters*. A path of uniform width is to surround the garden. If the area of the garden and path combined is 378 *square meters*, find the width of the path.



72. A pool measuring 10 *m* by 20 *m* is surrounded by a path of uniform width. If the area of the pool and the path combined is 600 m^2, what is the width of the path?



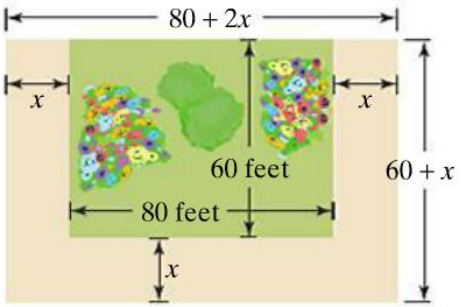
73. You put in flower bed measuring 10 *feet* by 12 *feet*. You plan to surround the bed with uniform border of low-growing plants.



a) Write a polynomial that describes the area of the uniform border that surrounds your flowers.

b) The low growing plants surrounding the flower bed require 1 square *foot* each when mature. If you have 168 of these plants, how wide a strip around the flower bed should you prepare for the border?

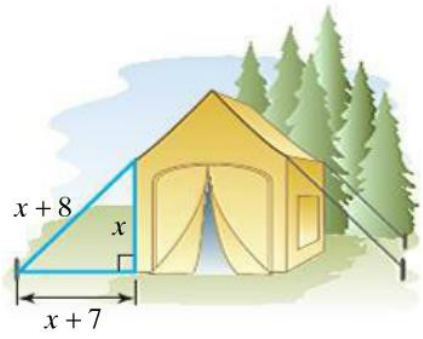
74. A rectangular garden measures 80 *feet* by 60 *feet*. A large path of uniform width is to be added along both shorter sides and one longer side of the garden. The landscape designer doing the work wants to double the garden's area with the addition of this path. How wide should the path be?



75. The length of a rectangular poster is 1 *foot* more than the width, and a diagonal of the poster is 5 *feet*. Find the length and the width.

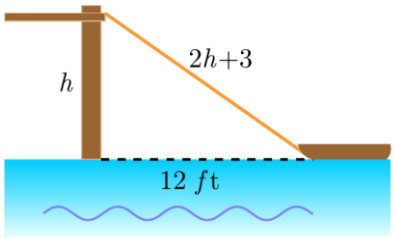
76. One leg of a right triangle is 7 *cm* less than the length of the other leg. The length of the hypotenuse is 13 *cm*. find the lengths of the legs.

77. A tent with wires attached to help stabilize it, as shown below. The length of each wire is 8 *feet* greater than the distance from the ground to where it is attached to the tent.

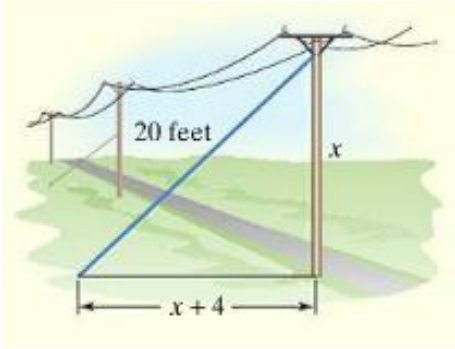


The distance from the base of the tent to where the wire is anchored exceeds this height by 7 feet, Find the length of each wire used to stabilize the tent.

78. A boat is being pulled into a dock with a rope attached to the boat at water level. Where the boat is 12 *feet*. from the dock, the length of the rope from the boat to the dock is 3 *feet*. longer than twice the height of the dock above the water. Find the height of the dock.

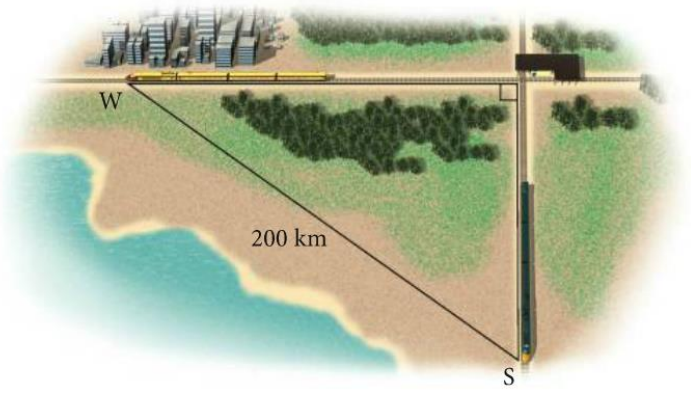


79. A piece of wire measuring 20 *feet* is attached to a telephone pole as a guy wire. The distance along the ground from the bottom of the pole to the end of the wire is 4 *feet* greater than the height where the wire is attached to the pole. How far up the pole does the guy wire reach?



80. Logan and Cassidy leave a campsite, Logan biking due north and Cassidy biking due east. Logan bikes 7 *km/h* slower than Cassidy. After 4 *hrs*, they are 68 *km* apart. Find the speed of each bicyclist.

81. Two trains leave a station at the same time. One train travels due west, and the other travels due south. The train traveling west travels 20 *km/hr* faster than the train traveling south. After 2 *hr*., the trains are 200 *km* apart. Find the speed of each train.



82. Towers are 1482 *feet.* tall. How long would it take an object dropped from the top to reach the ground? Given $\boldsymbol{s} = 16\boldsymbol{t}^2$

83. The formula $P = 0.01A^2 + .05A + 107$ models a woman's normal Point systolic blood pressure, *P*, an age *A*. Use this formula to find the age, to the nearest year, of a woman whose normal systolic blood pressure is 115 *mm* Hg.

84. A rectangular piece of metal is 10 *in*. longer than it is wide. Squares with sides 2 *in*. long are cut from the four corners, and the flaps folded upward to form an open box. If the volume of the box is 832 in^3, what were the original dimensions of the piece of metal?

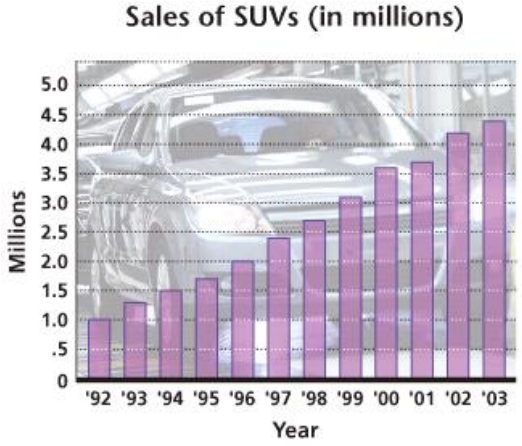
85. An astronaut on the moon throws a baseball upward. The astronaut is 6 *ft*., 6 *in*., tall, and the initial velocity of the ball is 30 *ft/sec* . The height *s* of the ball in feet is given by the equation

$$s = -2.7t^2 + 30t + 6.5$$

Where ***t*** is the number of seconds after the ball was thrown.

a) After how many seconds is the ball 12 *ft*. above the moon's surface?
b) How many seconds will it take for the ball to return to the surface?

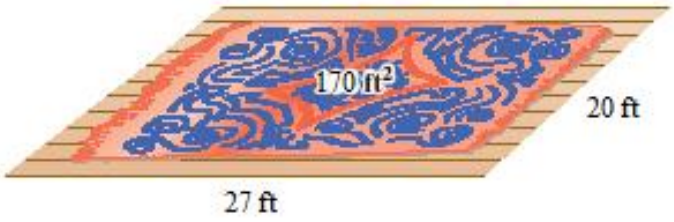
86. The bar graph shows of SUVs (sport utility vehicles in the US, in *millions*. The quadratic equation $S = .00579x^2 + .2579x + .9703$ models sales of SUVs from 1992 to 2003, where *S* represents sales in *millions*, and $x = 0$ represents 1992, $x = 1$ represents 1993 and so on.



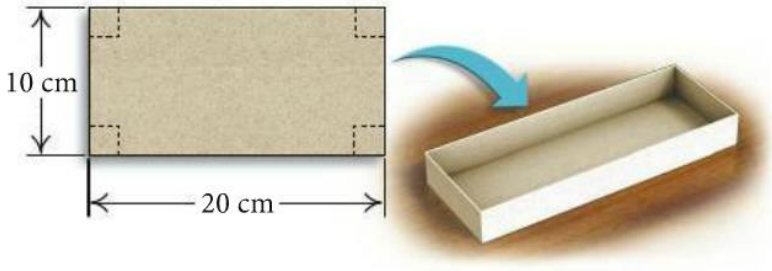
a) Use the model to determine sales in 2002 and 2003. Compare the results to the actual figures of 4.2 million and 4.4 *million* from the graph.
b) According to the model, in what year do sales reach 3.5 million? Is the result accurate?

87. Erik finds a piece of property in the shape of a right triangle. He finds that the longer leg is 20 *m* longer than twice the length of the shorter leg. The hypotenuse is 10 *m* longer than the length of the longer leg. Find the lengths of the sides of the triangular lot.

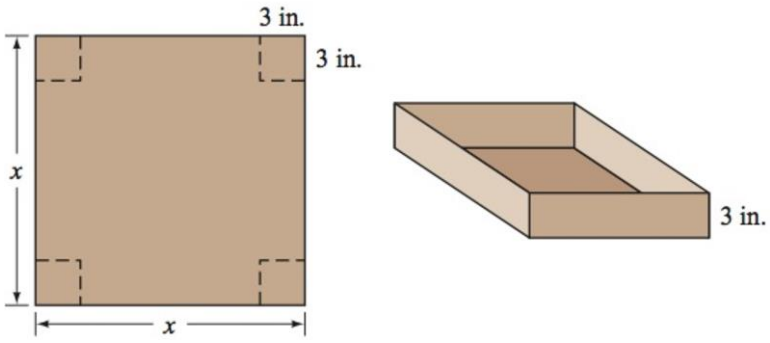
88. Cynthia wants to buy a rug for a room that is 20 *feet.* wide and 27 *feet*. long. She wants to leave a uniform strip of floor around the rug. She can afford to buy 170 square *feet* of carpeting. What dimension should the rug have?



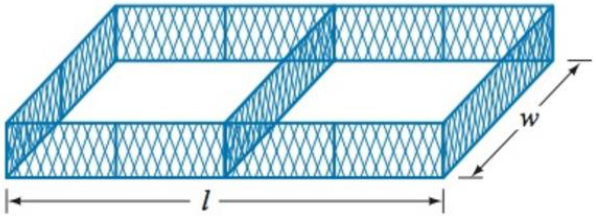
89. An open box is made from a 10-*cm* by 20-*cm* piece of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2. What is the length of the sides of the squares?



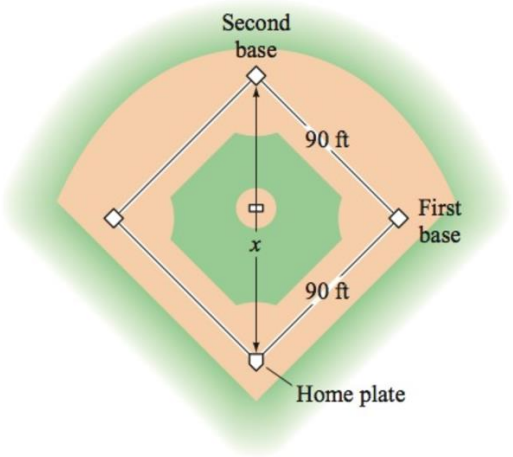
90. A square piece of cardboard is formed into a box by cutting out 3-*inch* squares from each of the corners and folding up the sides. If the volume of the box needs to be 126.75 cubic *inches*, what size square piece of cardboard is needed?



91. You want to use 132 *feet* of chain-link fencing to enclose a rectangular region and subdivide the region into two smaller rectangular regions. If the total enclosed area is 576 *square feet*, find the dimensions of the enclosed region.

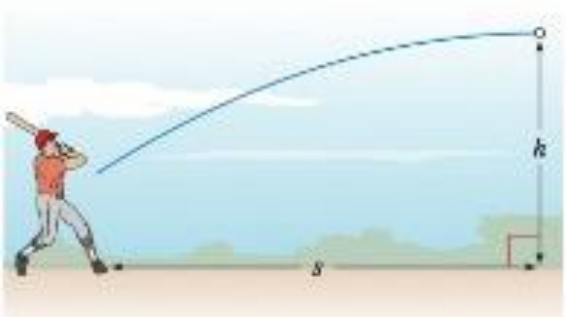


92. How far is it from home plate to second base on a baseball diamond?



93. Two equations can be used to track the position of a baseball t seconds after it is hit.

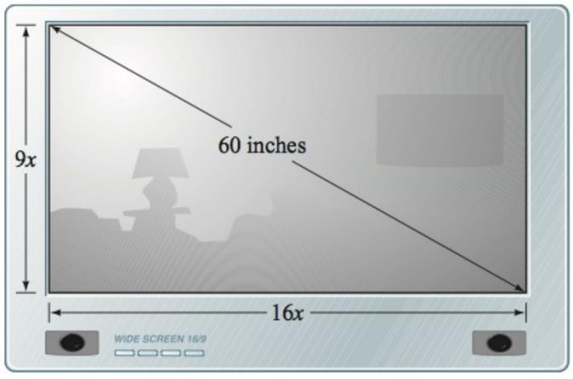
For instance, suppose $h = -16t^2 + 50t + 4.5$ gives the height, in *feet*, of a baseball t seconds after it is hit and $s = 103.9t$ gives the horizontal distance, in *feet*, of the ball from home plate t seconds after it is hit.



Use these equations to determine whether this particular baseball will clear a 10-*foot* fence positioned 360 *feet* from home plate.

94. A ball is thrown downward with an initial velocity of 5 *feet* per *second* from the Golden Gate Bridge, which is 220 *feet* above the water. How long will it take for the ball to hit the water?

95. A television screen measures 60 *inches* diagonally, and its aspect ratio is 16 to 9. This means that the ratio of the width of the screen to the height of the screen is 16 to 9.
Find the width and height of the screen.



96. A company makes rectangular solid candy bars that measures 5 *inches* by 2 *inches* by 0.5 *inch*. Due to difficult financial times, the company has decided to keep the price of the candy bar fixed and reduce the volume of the bar by 20%. What should the dimensions be for the new candy bar if the company keeps the height at 0.5 *inch* and makes length of the candy bar 3 *inches* longer than the width?

97. A company makes rectangular solid candy bars that measures 5 *inches* by 2 *inches* by 0.5 *inch*. Due to difficult financial times, the company has decided to keep the price of the candy bar fixed and reduce the volume of the bar by 20%. What should the dimensions be for the new candy bar if the company keeps the height at 0.5 *inch* and makes length of the candy bar 2.5 times as long as its width?

Section 1.4 - Other Types of Equations

The numbers of solutions to a polynomial with ***n*** degree, where ***n*** is Natural Number, are ***n*** solutions.

Solving a Polynomial Equation by factoring

Example

Solve: $4x^4 = 12x^2$

Solution

$4x^4 - 12x^2 = 0$

$4x^2(x^2 - 3) = 0$

$4x^2 = 0$ $\qquad$ $x^2 - 3 = 0$

$x^2 = 0$ $\qquad$ $x^2 = 3$

$\rightarrow \underline{x = 0, 0}$ $\qquad$ $\underline{x = \pm\sqrt{3}}$

Example

Solve: $2x^3 + 3x^2 = 8x + 12$

Solution

$2x^3 + 3x^2 - 8x - 12 = 0$

$x^2(2x + 3) - 4(2x + 3) = 0$

$(2x + 3)(x^2 - 4) = 0$

$2x + 3 = 0$ $\qquad$ $x^2 - 4 = 0$

$2x = -3$ $\qquad$ $x^2 = 4$

$\underline{x = -\frac{3}{2}}$ $\qquad$ $\underline{x = \pm\sqrt{4} \quad = \pm 2}$

Equations that Are Quadratic in Form

$$ax^2+bx+c=0$$
$$a(x)^2+b(x)^1+c=0$$
$$a(u)^2+b(u)^1+c=0$$
$$a\left(x^n\right)^2+b\left(x^n\right)^1+c=0$$
$$au^2+bu+c=0$$

Example

Solve: $x^4-5x^2+6=0$

Solution

$$\left(x^2\right)^2-5\left(x^2\right)+6=0$$
$$(U)^2-5(U)+6=0$$
$$U^2-5U+6=0$$

Solve for U

$$\Rightarrow U=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(6)}}{2(1)}$$
$$=\frac{5\pm\sqrt{25-24}}{2}$$
$$=\frac{5\pm\sqrt{1}}{2}$$
$$\rightarrow\begin{cases}U=\frac{5-1}{2}=2\\U=\frac{5+1}{2}=3\end{cases}$$

$$x^2=U\qquad\rightarrow\begin{cases}x^2=2\rightarrow \underline{x=\pm\sqrt{2}}\\x^2=3\rightarrow \underline{x=\pm\sqrt{3}}\end{cases}$$

or $\left(x^2-2\right)\left(x^2-3\right)=0$

$x^2-2=0$ $\quad x^2-3=0$

$x^2=2$ $\quad x^2=3$

$x=\pm\sqrt{2}$ $\quad x=\pm\sqrt{3}$

Example

Solve: $(x+1)^{2/3}-(x+1)^{1/3}-2=0$

Solution

$u=(x+1)^{1/3}$

$u^2-u-2=0$

$(u-2)(u+1)=0$

$u-2=0 \qquad u+1=0$

$u=2 \qquad u=-1$

$u=(x+1)^{1/3}=2 \qquad u=(x+1)^{1/3}=-1$

$x+1=2^3 \qquad x+1=(-1)^3$

$x+1=8 \qquad x+1=-1$

$\underline{x=7}$ $\qquad$ $\underline{x=-2}$

$\left((x+1)^{1/3}-2\right)\left((x+1)^{1/3}+1\right)=0$

Example

Solve: $3x^{2/3}-11x^{1/3}-4=0$

Solution

$3\left(x^{1/3}\right)^2-11\left(x^{1/3}\right)-4=0$

$3(U)^2-11(U)-4=0$

$3U^2-11U-4=0$

Solve for U

$\Rightarrow U=\dfrac{-(-11)\pm\sqrt{11^2-4(3)(-4)}}{2(3)}$

$=\dfrac{11\pm13}{6}$

$x^{1/3}=U \qquad x^{1/3}=\dfrac{11-13}{6} \qquad x^{1/3}=\dfrac{11+13}{6}$

$=-\dfrac{1}{3} \qquad =4$

$\rightarrow x=\left(-\dfrac{1}{3}\right)^3 \qquad x=4^3$

$\underline{=-\dfrac{1}{27}}$ $\qquad$ $\underline{=64}$

Or factor

$\left(3x^{1/3}+1\right)\left(x^{1/3}-4\right)=0$

$3x^{1/3}+1=0 \qquad x^{1/3}-4=0$

Solving a *Radical* Equation

Power Property

If P and Q are algebraic expressions, then every solution of the equation $P = Q$ is also a solution of the equation $P^n = Q^n$; for any positive integer n.

Example

Solve $x - \sqrt{15 - 2x} = 0$

Solution

$x = \sqrt{15 - 2x}$

$x^2 = \left(\sqrt{15 - 2x}\right)^2$

$x^2 = 15 - 2x$

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

$x - 3 = 0$ $\qquad$ $x + 5 = 0$

$x = 3$ $\qquad$ $x = -5$

Check

$x = 3$	$x = -5$
$3 - \sqrt{15 - 2(3)} = 0$	$-5 - \sqrt{15 - 2(-5)} = 0$
$3 - \sqrt{9} = 0$	$-5 - \sqrt{25} = 0$
$3 - 3 = 0$ (*true*)	$-5 - 5 \neq 0$ (*false*)

$\boxed{x = 3}$ is the only solution

Solving Radical Equations of the Form $x^{\frac{m}{n}} = k$

Assume that *m* and *n* are positive integers

If *m* is ***even***: $x^{\frac{m}{n}} = k \quad \Rightarrow \left(x^{\frac{m}{n}} \right)^{\frac{n}{m}} = k^{\frac{n}{m}} \quad \Rightarrow x = \pm k^{\frac{n}{m}}$

If *m* is ***odd***: $x^{\frac{m}{n}} = k \quad \Rightarrow \left(x^{\frac{m}{n}} \right)^{\frac{n}{m}} = k^{\frac{n}{m}} \quad \Rightarrow x = k^{\frac{n}{m}}$

Example

Solve: $5x^{3/2} - 25 = 0$

Solution

$$5x^{3/2} = 25$$
$$x^{3/2} = \frac{25}{5} = 5$$
$$x = 5^{\frac{2}{3}}$$
$$= \sqrt[3]{5^2}$$
$$= \sqrt[3]{25}$$

Example

Solve: $x^{2/3} - 8 = -4$

Solution

$$x^{2/3} = 4$$
$$x = \pm(4)^{3/2}$$
$$= \pm\left(2^2\right)^{3/2}$$
$$= \pm 2^3$$
$$= \pm 8$$

Solving an Absolute Value Equation

If c is a positive real number and X represents any algebraic expression, then $|X| = c$ is equivalent to $X = c$ or $X = -c$

$$|X| = c \quad \rightarrow \quad X = c \text{ or } X = -c$$

Properties of Absolute Value

1. For b > 0, $|a| = b$ if and only if (*iff*) $a = b$ *or* $a = -b$
2. $|a| = |b|$ *iff* $a = b$ *or* $a = -b$

For any positive number b:

3. $|a| < b$ *iff* $-b < a < b$
4. $|a| < b$ *iff* $a < -b$ *or* $a > b$

Example

Solve: $|2x - 1| = 5$

Solution

$2x - 1 = 5$ $\qquad$ $2x - 1 = -5$

$2\boldsymbol{x} = 6$ $\qquad$ $2\boldsymbol{x} = -4$

$\boldsymbol{x} = 3$ $\qquad$ $\boldsymbol{x} = -2$

Solutions: $x = -2,\ 3$

Example

Solve: $4|1 - 2x| - 20 = 0$

Solution

$4|1 - 2x| = 20$

$|1 - 2x| = 5$

$1 - 2x = 5$ $\qquad$ $1 - 2x = -5$

$-2x = 4$ $\qquad$ $-2x = -6$

$x = -2$ $\qquad$ $x = 3$

Solutions: $x = -2,\ 3$

Exercise *Section* 1.4- Other Types of Equations

Solve

1. $3x^3+2x^2=12x+8$

2. $x^3+x^2-4x-4=0$

3. $x^3+x^2+4x+4=0$

4. $x^3+4x^2-25x-100=0$

5. $x^3-2x^2-x+2=0$

6. $x^3-x^2-25x+25=0$

7. $x^3-x^2=16x-16$

8. $x^3+x^2+25x+25=0$

9. $x^3+2x^2=16x+32$

10. $2x^3+3x^2-6x-9=0$

11. $2x^3+x^2-8x-4=0$

12. $2x^3+16x^2+30x=0$

13. $3x^3-9x^2-30x=0$

14. $x^4+3x^2=10$

15. $5x^4=40x$

16. $9x^4-9x^2+2=0$

17. $x^4+720=89x^2$

18. $12x^4-11x^2+2=0$

19. $2x^4-7x^2+5=0$

20. $x^4-5x^2+4=0$

21. $x^4+3x^2=10$

22. $3x^4-48x^2=0$

23. $5x^4-20x^2=0$

24. $x^4-4x^3-4x^2=0$

25. $x^4-6x^3+9x^2=0$

26. $x^4-4x^3+3x^2=0$

27. $x^4-4x^2+3=0$

28. $x^4+4x^2+3=0$

29. $x^4+6x^2-7=0$

30. $x^4-6x^2-7=0$

31. $3x^4+4x^2-7=0$

32. $3x^4-4x^2-7=0$

33. $3x^4-x^2-2=0$

34. $3x^4+x^2-2=0$

---

35. $x-3\sqrt{x}-4=0$

36. $\left(5x^2-6\right)^{1/4}=x$

37. $\left(x^2+24x\right)^{1/4}=3$

38. $x^{5/2}=32$

39. $\sqrt[3]{2x+11}=3$

40. $\sqrt[3]{6x-3}=3$

41. $\sqrt[3]{2x-6}=4$

42. $\sqrt[3]{4x-3}-5=0$

43. $(3x-1)^{1/3}+4=0$

44. $(2x+3)^{1/3}+4=6$

45. $(3x-6)^{1/3}+5=8$

46. $(3x+1)^{1/4}+7=9$

47. $(2x+3)^{1/4}+7=10$

48. $\sqrt[3]{4x^2-4x+1}-\sqrt[3]{x}=0$

---

49. $\sqrt{2x+3}=5$

50. $\sqrt{x-3}+6=5$

51. $\sqrt{3x-2}=4$

52. $\sqrt{5x-4}=9$

53. $\sqrt{5x-1}=8$

54. $\sqrt{3x-2}-5=0$

55. $\sqrt{2x+5}+11=6$

56. $\sqrt{3x+7}+10=4$

57. $x=\sqrt{7x+8}$

58. $x=\sqrt{6x+7}$

59. $\sqrt{5x+1}=x+1$

60. $x=\sqrt{2x-2}+1$

61. $x-2\sqrt{x-3}=3$

62. $x+\sqrt{26-11x}=4$

63. $x-\sqrt{2x+3}=0$

64. $\sqrt{x+3}+3=x$

65. $x-\sqrt{x+11}=1$

66. $\sqrt{x-7}=7-\sqrt{x}$

67. $\sqrt{x-8}=\sqrt{x}-2$

68. $\sqrt{2x-5}=\sqrt{x+4}$

69. $\sqrt{6x+2}=\sqrt{5x+3}$

70. $\sqrt{3x+1}-\sqrt{x+4}=1$

71. $\sqrt{x+2}+\sqrt{x-1}=3$

72. $\sqrt{x-4}+\sqrt{x+4}=4$

73. $\sqrt{2x-3}-\sqrt{x-2}=1$

74. $\sqrt{x+2}+\sqrt{3x+7}=1$

75. $2\sqrt{4x+1}-9=x-5$

76. $\sqrt{2x-3}+\sqrt{x-2}=1$

77. $\sqrt{2x+3}=1+\sqrt{x+1}$

78. $\sqrt{x+5}-\sqrt{x-3}=2$

79. $|x| = -9$

80. $|x| = 9$

81. $|x-2| = 7$

82. $|x-2| = 0$

83. $|2x-3| = 6$

84. $|2x-1| = 11$

85. $7|5x| + 2 = 16$

86. $4\left|1 - \frac{3}{4}x\right| + 7 = 10$

87. $|x+7| + 6 = 2$

88. $|5-3x| = 12$

89. $|4x+2| = 5$

90. $3|x+5| = 12$

91. $2|x-6| = 8$

92. $3|2x-1| = 21$

93. $2|3x-2| = 14$

94. $|3x-1| + 2 = 16$

95. $|6x-2| + 4 = 32$

96. $7|5x| + 2 = 16$

97. $|4x+1| + 10 = 4$

98. $|4x+1| + 4 = 10$

99. $|3x-2| + 8 = 1$

100. $|3x-2| + 1 = 8$

101. $\left|\frac{6x+1}{x-1}\right| = 3$

102. $|x+1| = |1-3x|$

103. $|3x-1| = |x+5|$

104. $|5x-8| = |3x+2|$

105. $|4x-9| = |2x+1|$

106. $|2x-4| = |x-1|$

107. $|3x-4| = |3x+4|$

108. $|3x-5| = |3x+5|$

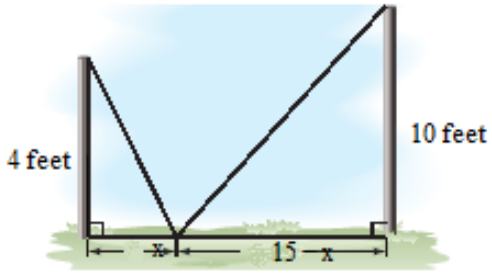
109. $|x-3| = |5-x|$

110. $|x-3| = |6-x|$

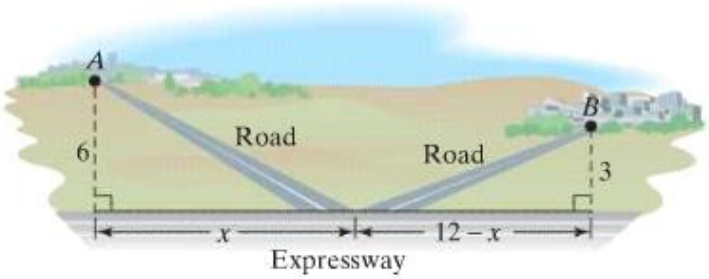
111. $\left|\frac{2}{3}x - 2\right| = \left|\frac{1}{3}x + 3\right|$

112. $\left|\frac{1}{2}x - 2\right| = \left|x - \frac{1}{2}\right|$

113. Two vertical poles of lengths 4 *feet* and 10 *feet* stand 15 *feet* apart. A cable reaches from the top of one pole to some point on the ground between the poles and then to the top of the other pole. Where should this point be located to use 24 *feet* of cable?



114. Towns ***A*** and ***B*** are located 6 *miles* and 3 *miles*, respectively, from a major expressway. The point on the expressway closet to town ***A*** is 12 *miles* from the point on the expressway closet to town ***B***. Two new roads are to be built from ***A*** to the expressway and then to ***B***.



a) Express the combined lengths of the new road in terms of x.

b) If the combined lengths of the new roads is 15 *miles*, what distance does x represent?

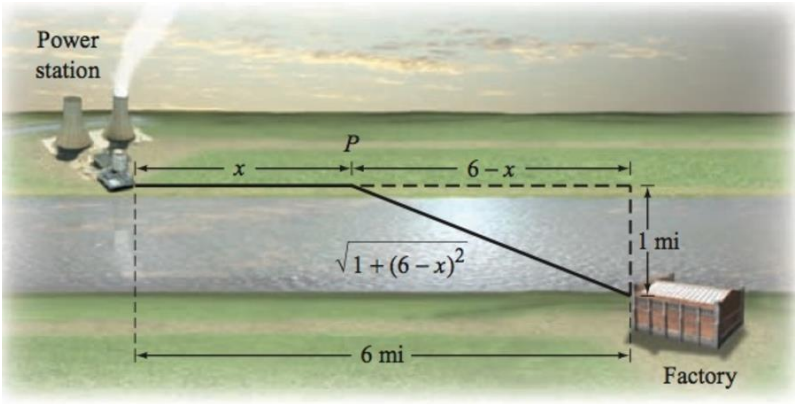
115. A solid silver sphere has a diameter of 8 *millimeters*, and a second silver has a diameter of 12 *millimeters*. The spheres are melted down and recast to form a single cube. What is the length s of each edge of the cube?

116. The period T of the pendulum is the time it takes the pendulum to complete one swing from left to right and back. For a pendulum near the surface of Earth

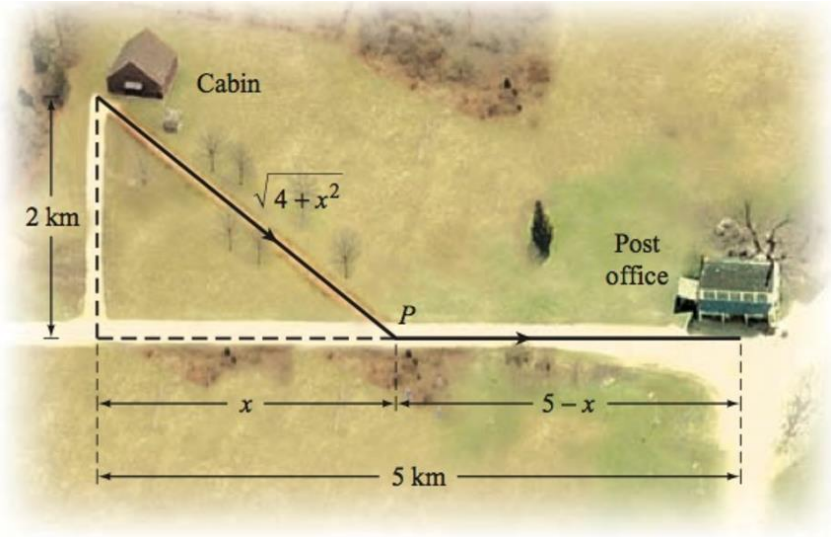
$$T = 2\pi\sqrt{\frac{L}{32}}$$

Where T is measured in *seconds* and L is the length of the pendulum in feet. Find the length of a pendulum that has a period of 4 *seconds*.

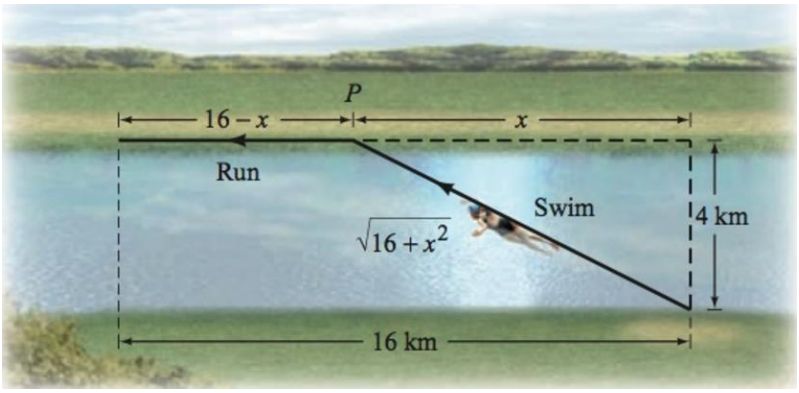
117. A power station is on one side of a river that is 1 *mile* wide, and a factory is 6 *miles* down-stream on the other side of the river, the cost is \$0.125 *million* per *mile* to run power lines over land and \$0.2 *million* per *mile* to run power lines under water. How far over the land should the power line be run if the total cost of the project is to be \$1 *million*?



118. A cabin is located in a meadow at the end of a straight driveway 2 *km* long. A post office is located 5 *km* from the driveway along a straight road. A woman walks 2 *km/hr* through the meadow to point ***P*** and then 5 *km/hr* along the road to the post office. If it takes the woman 2.25 *hours* to reach the post office, what is the distance x of point ***P*** from the end of the driveway?



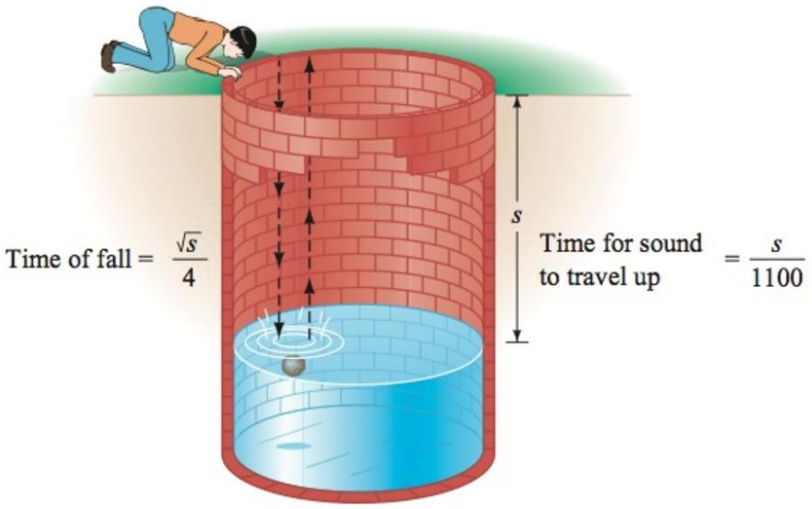
119. To prepare for a triathlon, a person swims across a river to point ***P*** and then runs along a path.



The person swims at 7 *km/hr* and runs at 22 *km/hr* . For what distance *x* is the total time for swimming and running 2 *hours*?

120. The depth *s* from the opening of a well to the water below can be determined by measuring the total time between the instant you drop a stone and the moment you heat it hit the water. The time, in *seconds*, it takes the stone to hit the water is given by $\frac{\sqrt{s}}{4}$, where s is measured in *feet*. The time, also in seconds, required for the sound of the impact to travel up to your ears is given by $\frac{s}{1{,}100}$. Thus, the total time *T*, in *seconds*, between the instant you drop the stone and the moment you hear its impact is

$$T = \frac{\sqrt{s}}{4} + \frac{s}{1{,}100}$$



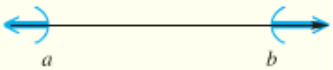
a) One of the world's deepest water wells is 7,320 *feet* deep. Find the time between the instant you drop a stone and the time you hear it hit the water if the surface of the water if the surface of the water is 7,100 *feet* below the opening of the well.

b) Find the depth from the opening of a well to the water level if the time between the instant you drop a stone and the moment you heat its impact is 3 *seconds*.

121. On a ship, the distance *d* that you can see to the horizon is given by $d = \sqrt{1.5h}$, where h is the height of your eye measured in *feet* above the sea level and *d* is measured in *miles*. How high is the eye level of a navigator who can see 14 *miles* to the horizon?

Section 1.5 – Inequalities

Notation

Type of Interval	Set	Interval Notation	Graph
Open interval	$\{x \mid x > a\}$	(a, ∞)	
	$\{x \mid a < x < b\}$	(a, b)	
	$\{x \mid x < b\}$	$(-\infty, b)$	
Other intervals	$\{x \mid x \geq a\}$	$[a, \infty)$	
	$\{x \mid a < x \leq b\}$	$(a, b]$	
	$\{x \mid a \leq x < b\}$	$[a, b)$	
	$\{x \mid x \leq b\}$	$(-\infty, b]$	
Closed interval	$\{x \mid a \leq x \leq b\}$	$[a, b]$	
Disjoint interval	$\{x \mid x < a \text{ or } x > b\}$	$(-\infty, a) \cup (b, \infty)$	
All real numbers	$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	

Properties of inequality

1. ***If a < b, then a + c < b + c***
2. ***If a < b and if c > 0, then ac < bc***
3. ***If a < b and if c < 0, then ac > bc***

Example

Solve $3x + 1 > 7x - 15$

Solution

$3x - 7x > -1 - 15$

$-4x > -16$ *Divide by – 4 both sides*

$x < 4$ or $(-\infty, 4)$ or $\{x \mid x < 4\}$

Example

Solve $\frac{x-4}{2} \geq \frac{x-2}{3} + \frac{5}{6}$ *LCD*: 2, 3, 6

Solution

$(6)\frac{x-4}{2} \geq (6)\frac{x-2}{3} + (6)\frac{5}{6}$

$3(x-4) \geq 2(x-2) + 5$

$3x - 12 \geq 2x - 4 + 5$

$3x - 12 \geq 2x + 1$

$3x - 2x \geq 12 + 1$

$\underline{x \geq 13}$

Example

a) $3(x + 1) > 3x + 2$

$3x + 3 > 3x + 2$

$3x - 3x > -3 + 1$

$0 > -1$ (*True statement*)

Sol.: $\mathbb{R}$ ***or*** $\{x|$ *All Real numbers*$\}$ ***or*** $(-\infty, \infty)$

b) $x + 1 \leq x - 1$

$x - x \leq -1 - 1$

$0 \leq -2$

Sol.: $\varnothing$

Example

Solve $-2 < 5 + 3x < 20$ Give the solution set in interval notation and graph it.

Solution

$-2 - 5 < 5 + 3x - 5 < 20 - 5$

$-7 < 3x < 15$

$-\frac{7}{3} < \frac{3}{3}x < \frac{15}{3}$

$-\frac{7}{3} < x < 5$

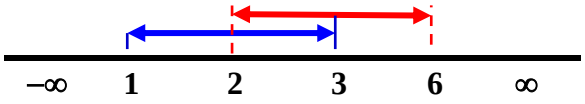
Solution: $\left(-\frac{7}{3}, 5\right)$

Intersections of Interval $\cap$

To find the intersection, take the portion of the number line that the two graphs have in ***common***

Example

$[1, 3] \cap (2, 6) = (2, 3]$

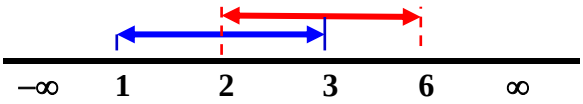


Unions of Interval $\cup$

To find the union, take the portion of the number line representing the total ***collection*** of numbers in the two graphs.

Example

$[1, 3] \cup (2, 6) = [1, 6)$



Solving an *Absolute Value* Inequality:

If X is an algebraic expression and c is a positive number,

1. The solutions of $|X| < c$ are the numbers that satisfy $-c < X < c$.
2. The solutions of $|X| > c$ are the numbers that satisfy $X < -c$ or $X > c$.

Example

Solve: $-3|5x - 2| + 20 \geq -19$

Solution

$-3|5x - 2| \geq -39$

$-|5x - 2| \geq -13$

$|5x - 2| \leq 13$

$-13 \leq 5x - 2 \leq 13$

$-11 \leq 5x \leq 15$

$\boxed{-\frac{11}{5} \leq x \leq 3}$ or $\boxed{\left[-\frac{11}{5}, 3\right]}$

Example

Solve: $18 < |6 - 3x|$

Solution

$|6-3x|>18$

$6-3x<-18$	$6-3x>18$
$-3x<-18-6$	$-3x>18-6$
$-3x<-24$	$-3x>12$
$\frac{-3}{-3}x>-\frac{24}{-3}$	$\frac{-3}{-3}x<\frac{12}{-3}$
$x>8$	$x<-4$

Solution: $(-\infty,-4)\cup(8,\infty)$

Special Cases

Example

Solve the inequality $|2-5x|\geq -4$

Solution

$|2-5x|\geq -4$

It is always ***true***

$\therefore$ The solution set is: $\mathbb{R}$ All real numbers $(-\infty,\ \infty)$

Example

Solve the inequality $|4x-7|<-3$

Solution

$|4x-7|<-3$

Any absolute value can't be less than any negative number.

$\therefore$ No solution or $\varnothing$

Example

Solve the inequality $|5x+15|=0$

Solution

$|5x+15|=0$

$5x+15=0$

$5x=-15$

$\therefore$ Solution: $x=-3$

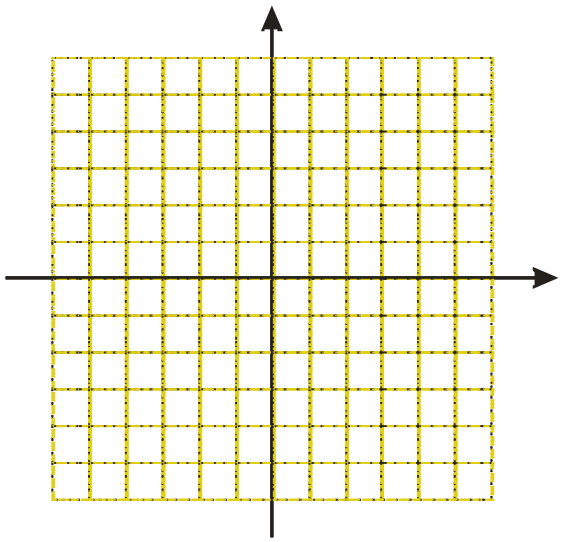
Definition of a Polynomial Inequality

A polynomial inequality is any inequality that can be put into one of the forms

$$f(x) < 0 \qquad f(x) > 0 \qquad f(x) \le 0 \qquad f(x) \ge 0$$

Where f is a polynomial function.

$f(x) = x^2 - 5x + 4 \quad (x = 1, 4)$



Procedure* for Solving Polynomial Inequalities** ***Example

1. Express the inequality in the form $f(x)$? 0	$x^2 - x < 12$ $x^2 - x - 12 < 0$
2. Solve $f(x) = 0$	$x^2 - x - 12 = 0$ $x = -3,\ 4$
3. Locate the boundary 4. Choose one test value	$-3 \quad 0 \quad 4$ $+ \quad - \quad +$
5. Write the solution set	$(-3,\ 4)$

✓ $ax^2 + bx + c \ge 0 \rightarrow$ *if* $a > 0 \Rightarrow x \le x_1,\ x \ge x_2$

✓ $ax^2 + bx + c \le 0 \rightarrow$ *if* $a > 0 \Rightarrow x_1 \le x \le x_2$

Example

Solve $2x^2+5x-12\geq 0$

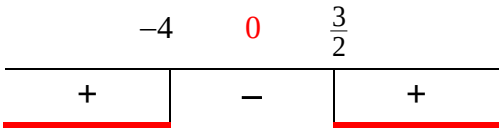
Solution

$2x^2+5x-12=0$

$(2x-3)(x+4)=0$

$x=-4,\ \frac{3}{2}$

Solution: $x\leq -4 \quad x\geq \frac{3}{2}$



$(-\infty,\ -4]\ \cup\ \left[\frac{3}{2},\ \infty\right)$

Example

Solve: $x^3+3x^2\leq x+3$

Solution

$x^3+3x^2-x-3=0$

$x^2(x+3)-(x+3)=0$

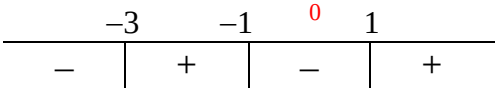
$(x+3)(x^2-1)=0$

$x+3=0$ $\qquad$ $x^2-1=0$

$x=-3$ $\qquad$ $x^2=1$

$x=-3$ $\qquad$ $x=\pm 1$

Solution: $(-\infty,\ -3]\cup[-1,\ 1]$



Rational Inequality

Example

Solve: $\frac{2x}{x+1} \geq 1$

Solution

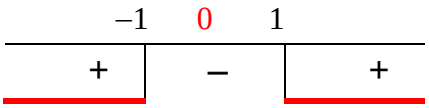
$\frac{2x}{x+1} = 1 \quad \rightarrow \; Cond.: x+1 \neq 0 \Rightarrow \; \underline{x \neq -1}$

$(x+1)\frac{2x}{x+1} - 1(x+1) = 0$

$2x - x - 1 = 0$

$x - 1 = 0$

$x = 1$



Solution: $\underline{x \leq -1 \quad x \geq 1}$ $\underline{(-\infty, -1) \cup [1, \infty)}$

Example

Solve $\frac{5}{x+4} \geq 1$

Solution

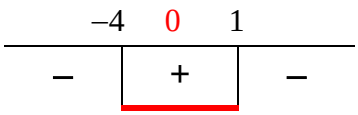
$\frac{5}{x+4} - 1 = 0$ *Exception:* $x+4 \neq 0 \Rightarrow x \neq -4$

$(x+4)\frac{5}{x+4} - 1(x+4) = 0$

$5 - x - 4 = 0$

$1 - x = 0$

$\underline{x = 1}$



Solution: $\underline{-4 < x \leq 1}$ $\underline{(-4, 1]}$

Example

Solve $\frac{2x-1}{3x+4} < 5$

Solution

$\frac{2x-1}{3x+4} - 5 = 0$ *Restriction:* $3x+4 \neq 0 \Rightarrow \underline{x \neq -\frac{4}{3}}$

$(3x+4)\frac{2x-1}{3x+4} - 5(3x+4) = 0$

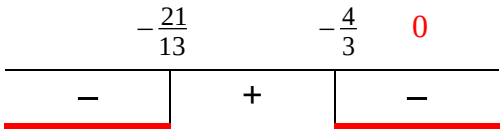
$2x - 1 - 15x - 20 = 0$

$-13x - 21 = 0$

$x = -\frac{21}{13}$

Solution: $x < -\frac{21}{13} \quad x > -\frac{4}{3}$

$\left(-\infty, -\frac{21}{13}\right) \cup \left(-\frac{4}{3}, \infty\right)$



Exercises *Section* 1.5 – Inequalities

Find:

1. $(-3,0)\cap[-1,2]$

2. $(-3,0)\cup[-1,2]$

3. $(-4,0)\cap[-2,1]$

4. $(-4,0)\cup[-2,1]$

5. $(-\infty,5)\cap[1,8)$

6. $(-\infty,5)\cup[1,8)$

Solve the inequality equation

7. $-3x+5>-7$

8. $2-3x\le 5$

9. $4-3x\le 7+2x$

10. $5x+11<26$

11. $3x-8\ge 13$

12. $-9x\ge 36$

13. $-4x\le 64$

14. $8x-11\le 3x-13$

15. $18x+45\le 12x-8$

16. $4(x+1)+2\ge 3x+6$

17. $8x+3>3(2x+1)+x+5$

18. $2x-11<-3(x+2)$

19. $-4(x+2)>3x+20$

20. $1-(x+3)\ge 4-2x$

21. $5(3-x)\le 3x-1$

22. $\frac{x}{4}-\frac{1}{2}\le\frac{x}{2}+1$

23. $\frac{3x}{10}+1\ge\frac{1}{5}-\frac{x}{10}$

24. $6x-(2x+3)\ge 4x-5$

25. $\frac{2x-5}{-8}\le 1-x$

26. $1-\frac{x}{2}>4$

27. $7-\frac{4}{5}x<\frac{3}{5}$

28. $\frac{x-4}{6}\ge\frac{x-2}{9}+\frac{5}{18}$

29. $\frac{4x-3}{6}+2\ge\frac{2x-1}{12}$

30. $4(3x-2)-3x<3(1+3x)-7$

31. $3(x-8)-2(10-x)<5(x-1)$

32. $8(x+1)\le 7(x+5)+x$

33. $4(x-1)\ge 3(x-2)+x$

34. $7(x+4)-13>12+13(3+x)$

35. $-2[7x-(2x-3)]<-2(x+1)$

36. $6-\frac{2}{3}(3x-12)\le\frac{2}{5}(10x+50)$

37. $\frac{2}{7}(7-21x)-4<10-\frac{3}{11}(11x-11)$

38. $3[3(x+5)+8x+7]+5[3(x-6)-2(3x-5)]<2(4x+3)$

39. $5[3(2-3x)-2(5-x)]-6[5(x-2)-2(4x-3)]<3x+19$

40. $0\le 3x-1\le 10$

41. $0\le 1-3x\le 10$

42. $0\le 2x+6\le 54$

43. $-3\le\frac{2}{3}x-5\le -1$

44. $-6\le 6x+3\le 21$

45. $1\le 2x+3\le 11$

Solve the inequality equation

46. $|x| < 2$

47. $|x| \geq 2$

48. $|x-2| < 1$

49. $|x-1| < 4$

50. $|x+2| \geq 1$

51. $|x+1| \geq 4$

52. $|3x+5| < 17$

53. $|5x-2| < 13$

54. $|5x-2| \geq 13$

55. $|2(x-1)+4| \leq 8$

56. $|3(x-1)+2| \leq 20$

57. $\left|\frac{2x+6}{3}\right| > 2$

58. $\left|\frac{3x-3}{4}\right| < 6$

59. $\left|\frac{2x+2}{4}\right| \geq 2$

60. $\left|\frac{3x-3}{9}\right| \leq 1$

61. $\left|3-\frac{2x}{3}\right| > 5$

62. $\left|3-\frac{3x}{4}\right| < 9$

63. $|x-2| < -1$

64. $|x+2| < -3$

65. $|x+6| > -10$

66. $|x+2| > -8$

67. $|x+2|+9 \leq 16$

68. $|x-2|+4 \geq 5$

69. $2|2x-3|+10 > 12$

70. $3|2x-1|+2 < 8$

71. $-4|1-x| < -16$

72. $-2|5-x| < -6$

73. $3 \leq |2x-1|$

74. $9 \leq |4x+7|$

75. $12 < \left|-2x+\frac{6}{7}\right|+\frac{3}{7}$

76. $4+\left|3-\frac{x}{3}\right| \geq 9$

77. $|x-2| < 5$

78. $|2x+1| < 7$

79. $|5x+2|-2 < 3$

80. $|2-7x|-1 > 4$

81. $|3x-4| < 2$

82. $|2x+5| \geq 3$

83. $|12-9x| \geq -12$

84. $|6-3x| < -11$

85. $|7+2x| < 0$

Solve the inequality equation

86. $x^2-7x+10 > 0$

87. $2x^2-9x \leq 18$

88. $x^2-5x+4 > 0$

89. $x^2+x-2 > 0$

90. $x^2-4x+12 < 0$

91. $x^2+7x > 0$

92. $x^2-49 < 0$

93. $x^2-5x \geq 0$

94. $x^2-16 \leq 0$

95. $x^2+7x+10 < 0$

96. $x^2-3x \geq 28$

97. $x^2+5x+6 < 0$

98. $x^2 < -x+30$

99. $x^3-3x^2-9x+27 < 0$

100. $x^3-x > 0$

101. $x^3+3x^2 \leq x+3$

102. $x^3+x^2 \geq 48x$

103. $x^3-x^2-16x+16 < 0$

104. $x^3+x^2-9x-9 > 0$

105. $x^3+3x^2-4x-12 \geq 0$

106. $x^4-20x^2+64 \leq 0$

107. $x^4-10x^2+9 \geq 0$

Solve the inequality equation

108. $\frac{x+4}{x-1} < 0$

109. $\frac{x-2}{x+3} > 0$

110. $\frac{x-5}{x+8} \geq 3$

111. $\frac{x-4}{x+6} \leq 1$

112. $\frac{x}{2x+7} \geq 4$

113. $\frac{x}{3x-5} \leq -5$

114. $\frac{x+2}{x-5} \leq 2$

115. $\frac{3x+1}{x-2} \geq 4$

116. $\frac{x}{x-3} > 0$

117. $\frac{x-3}{x+2} \geq 0$

118. $\frac{x-2}{x+2} \leq 2$

119. $\frac{x+2}{x-2} \geq 2$

120. $\frac{x+2}{3+2x} \leq 5$

121. $\frac{x+6}{x-14} \geq 1$

122. $\frac{x-3}{x+4} \geq \frac{x+2}{x-5}$

123. $\frac{x-4}{x+3} - \frac{x+2}{x-1} \leq 0$

124. $\frac{2x-1}{x+3} \geq \frac{x+1}{3x+1}$

125. $\frac{(x+1)(x-4)}{x-2} < 0$

126. $\frac{x(x-4)}{x+5} > 0$

127. $\frac{6x^2-11x-10}{x} > 0$

128. $\frac{3x^2-2x-8}{x-1} \geq 0$

129. $\frac{x^2-6x+9}{x-5} \leq 0$

130. $\frac{x^2+10x+25}{x+1} \ll 0$

131. A car can be rented from Basic Rental for $260 per week with no extra charge for mileage. Continental charges $80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

132. If a projectile is launched from ground level with an initial velocity of 96 *ft.* per *sec*, its height in feet ***t*** seconds after launching is ***s*** feet, where

$$s = -16t^2 + 96t$$

When will the projectile be greater than 80 *ft*. above the ground?

133. A projectile is fired straight up from ground level. After ***t*** seconds, its height above the ground is ***s*** *ft*., where

$$s = -16t^2 + 220t$$

For what time period is the projectile at least 624 *ft*. above the ground?

134. Your test scores of 70 and 81 in your math class. To receive a *C* grade, you must obtain an average greater than or equal to 72 but less than 82. What range of test scores on the one remaining test will enable you to get a *C* for the course.

135. A truck can be rented from Basic Rental for $50 a day plus $0.20 per *mile*. Continental charges $20 per day plus $0.50 per *mile* to rent the same truck. How many miles must be driven in a day to make the rental cost for Basic Rental a better deal than Constiental's?

136. You are choosing between two telephone plans. Plan ***A*** has a monthly fee of $15 with a charge of $0.08 per *minute* for all calls. Plan ***B*** has a monthly fee of $3 with a charge of $0.12 per *minute* for all calls. How many calling minutes in a month make plan ***A*** the better deal?

137. A City commission has proposed two tax bills. The first bill requires that a homeowner pay $1,800 plus 3% of the assesses home value in taxes. The second bill requires taxes of $200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal for the homeowner?

138. A local bank charges $8 per month plus $0.05 per check. The credit union charges $2 per month $0.08 per check. How many checks should be written each month to make the credit union a better deal?

139. A company manufactures and sells blank audiocassette tapes. The weekly fixed cost is $10,000 and it costs $0.40 to produce each tape. The selling price is $2.00 per tape. How many tapes must be produced and sold each week for the company to have a profit?

140. A company manufactures and sells stationery. The weekly fixed cost is $3,000 and it costs $3.00 to produce each package of stationery. The selling price is $5.50 per package. How many packages of stationery must be produced and sold each week for the company to have a profit?

141. An elevator at a construction site has a maximum capacity of 3,000 *pounds*. If the elevator operator weighs 200 *pounds* and each cement bag weighs 70 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

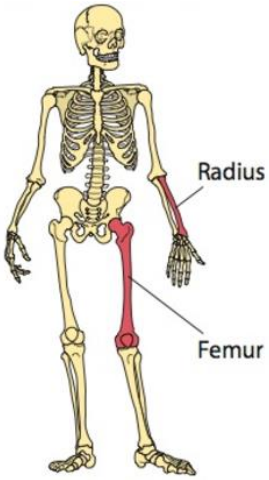
142. An elevator at a construction site has a maximum capacity of 2,500 *pounds*. If the elevator operator weighs 160 *pounds* and each cement bag weighs 60 *pounds*, how many bags of cement can be safely lifted on the elevator in one trip?

143. You can rent a car for the day from Company ***A*** for $29.00 plus $0.12 a *mile*. Company ***B*** charges $22.00 plus $0.21 a *mile*. Find the number of miles m per day for which it is cheaper to rent from Company ***A***.

144. UPS will only ship packages for which the length is less than or equal to 108 *inches* and the length plus the girth is less than or equal to 130 *inches*. The length of a package is defined as the length of the longest side. The girth is defined as twice the width plus twice the height of the package. If a box has a length of 34 *inches* and a width of 22 *inches*, determine the possible range of heights h for this package if you wish to ship it by UPS.

145. The sum of three consecutive odd integers is between 63 and 81. Find all possible sets of integers that satisfy these conditions.

146. Forensic specialists can estimate the height of a deceased person from the lengths of the person's bones. For instance, an inequality that relates the height h, in *cm*, of an adult female and the length f, in *cm*, of her femur is $\left|h-\left(2.47f+54.10\right)\right|\le 3.72$. Use the inequalities to estimate the possible range of heights for an adult female whose measures 32.24 *cm*.



147. An inequality that is used to calculate the height h of an adult male from the length r of his radius is

$$\left|h-\left(3.32r+85.43\right)\right| \le 4.57$$

Where h and r are both in *cm*. Use this inequality to estimate the possible range of heights for an adult male whose radius measures 26.36 *cm*.