$$\lim_{(x,y)\to(0,1)} \frac{x - xy + 3}{x^2y + 5xy - y^3} = \frac{0 - 0 + 3}{0 + 0 - 1} = \boxed{-3}$$

$$\lim_{(x,y)\to(3,-4)} \sqrt{x^2 + y^2} = \sqrt{9 + 16}$$
$$= \boxed{5}$$

19/ $$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}} = \frac{0}{0}$$

$$= \lim_{(x,y)\to(0,0)} \frac{x(x-y)}{\sqrt{x} - \sqrt{y}} \cdot \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}}$$

$$= \lim_{(x,y)\to(0,0)} \frac{x(x-y)(\sqrt{x} + \sqrt{y})}{x - y}$$

$$= \lim_{(x,y)\to(0,0)} x(\sqrt{x} + \sqrt{y})$$

$$= \boxed{0}$$

$$\frac{x(x-y)}{\sqrt{x} - \sqrt{y}} = \frac{x(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})}{\sqrt{x} - \sqrt{y}}$$
$$= x(\sqrt{x} + \sqrt{y})$$

---

#1/ $$\lim_{(x,y)\to(0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2} = \boxed{\frac{5}{2}}$$

4/ $$\lim_{(x,y)\to(0,0)} \cos\frac{x^2 + y^3}{x + y + 1} = \cos 0 = \boxed{1}$$

15/ $$\lim_{P\to(2,-3,6)} \ln\sqrt{x^2 + y^2 + z^2} = \ln\sqrt{4 + 9 + 36}$$
$$= \boxed{\ln 7}$$

10) $\lim_{\substack{(x,y)\to(4,3) \\ x\neq y+1}} \dfrac{\sqrt{x}-\sqrt{y+1}}{x-y-1} = \dfrac{2-2}{4-3-1} = \dfrac{0}{0}$

$= \lim_{\substack{(x,y)\to(4,3) \\ x\neq y+1}} \dfrac{\sqrt{x}-\sqrt{y+1}}{x-y-1} \cdot \dfrac{\sqrt{x}+\sqrt{y+1}}{\sqrt{x}+\sqrt{y+1}}$

$= \lim_{\substack{(x,y)\to(4,3) \\ x\neq y+1}} \dfrac{\cancel{x-y-1}}{(\cancel{x-y-1})(\sqrt{x}+\sqrt{y+1})}$

$x-y-1\neq 0$

$= \lim_{\substack{(x,y)\to(4,3) \\ x\neq y+1}} \dfrac{1}{\sqrt{x}+\sqrt{y+1}}$

$= \dfrac{1}{2+2}$

$= \boxed{\dfrac{1}{4}}$

---

#20 $\lim_{(x,y)\to(0,0)} \dfrac{\sin xy}{x^2+y^2} = \dfrac{0}{0}$

$\lim_{\substack{(x,y)\to(0,0) \\ y=x}} \dfrac{\sin xy}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \dfrac{\sin x^2}{2x^2} = \dfrac{0}{0}$

$= \lim_{x^2\to 0} \dfrac{\sin x^2}{2x^2}$

$= \dfrac{1}{2}$

$\lim_{(x,y)\to(0,0)} \dfrac{2x\cos x^2}{\cancel{4}x}$

$= \lim_{(x,y)\to(0,0)} \dfrac{1}{2}\cos x^2$

$= \dfrac{1}{2}$

$x^2+y^2=0$

$x=\pm y$

$\lim_{y=-x}$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=-x}} \frac{\sin xy}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{\sin(-x^2)}{x^2+x^2}$$

$$= \lim_{x^2\to 0} \frac{-\sin x^2}{2x^2}$$

$$= -\frac{1}{2}$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin xy}{x^2+y^2} = \nexists$$