Solution *Section* 1.8 – Measures of Central Tendency

Exercise

In what sense are the mean, median, mode and midrange measures of "center"?

Solution

The mean, median, mode, and midrange are measures of "center" in the sense that they each attempt to determine (by various criteria – i.e., by using different approaches) what might be designated as a typical or representative value.

Exercise

A headline in USA Today stated that "Average family income drops 2.3%." What is the role of the term average in statistics? Should another term be used in place of average?

Solution

The term *average* is not used in statistics because it is imprecisely used by the general public as a synonym for *typical* – as in, the average American had blue eyes. When referring to the result obtained by dividing a sum by the number of values contributing to that sum, the term ***mean*** should be used.

Exercise

In an editorial, the Poughkeepsie Journal printed this statement: "The median price – the price exactly in between the highest and lowest -- …" Does that statement correctly describes the median? Why or why not?

Solution

No. The price exactly in between the highest and the lowest would be the mean of the highest and lowest values – which is the midrange, and not the median.

Exercise

A simple random sample of pages from Merriam-Webster's Collegiate Dictionary, 11th edition, was obtained. Listed below are the numbers of words defined on those pages. Given that this dictionary has 1459 pages with defined words, estimate the total number of defined words in the dictionary.

51 63 36 43 34 62 73 39 53 79

Find the

a) mean *b*) median *c*) mode *d*) midrange

e) Is that estimate likely to be an accurate estimate of the number of words in the English language?

Solution

34 36 39 43 51 53 62 63 73 79

a) *Mean*: $\bar{x}=\frac{\sum x}{n}=\frac{51+63+36+43+34+62+73+39+53+79}{10}=\frac{533}{10}\underline{\ =53.3\ words|}$

b) *Median*: $\tilde{x}=\frac{51+53}{2}\underline{\ =52.0\ words|}$

c) *Mode*: None

d) *Midrange*: $\frac{34+79}{2}\underline{\ =56.5\ words|}$

e) Using the mean of 53.3 words per page, a reasonable estimate for the total number of words in the dictionary is $(53.3)(1459)=77,765$. Since the sample is a simple random sample, it should be representative of the population and the estimate of 77,675 is a valid estimate for the number of words in the dictionary – but not for the total number of words in the English language, since the dictionary does not claim to contain every word.

Exercise

The National Highway Traffic Administration conducted crash tests of child booster seats for cars. Listed below are results from those teats, with the measurements given in *hic* (standard head injury condition units.

774 249 1210 546 431 612

Find the

a) mean *b*) median *c*) mode *d*) midrange

e) According to the safety requirement, the *hic* measurement should be less than 1000 *hic*. Do the results suggest that all of the child booster seats meet the specified requirement?

Solution

431 546 612 649 774 1210

a) *Mean*: $\bar{x}=\frac{\sum x}{n}=\frac{431+546+612+649+774+1210}{6}=\frac{4222}{6}$

$\underline{=703.7\ hic|}$

b) *Median*: $\tilde{x}=\frac{612+649}{2}\underline{\ =630.5\ hic|}$

c) *Mode*: None

d) *Midrange*: $\frac{431+1210}{2}\underline{\ =820.5\ hic|}$

e) No. Even though all four measures of center fall within with accepted guidelines, all the individual values do not. Since one result exceeds the guidelines, it is clear that all child booster seats do not meet the requirement.

Exercise

The insurance Institution for Highway Safety conducted tests with crashes of new cars traveling at 6 mi/h. The total cost of the damages was found for a simple random sample of the tested cars and listed below

$7448 $4911 $9051 $6374 $4277

Find the

a) mean *b*) median *c*) mode *d*) midrange

e) Do the different measures of center differ very much?

Solution

4277 4911 6374 7448 9051

a) *Mean*: $\bar{x} = \frac{\sum x}{n} = \frac{4277+4911+6374+7448+9051}{5} \underline{\boxed{= \$6412.2}}$

b) *Median*: $\boxed{= \$6,374}$

c) *Mode*: None

d) *Midrange*: $\frac{4277+9051}{2} \underline{\boxed{= \$6,664.00}}$

e) No. Even though the sample values cover a fairly wide range, the measures of center do not differ very much.

Exercise

Listed below are the durations (in hours) of a simple random sample of all flights (as of this writing) of NASA's Space Transport System (space shuttle).

73 95 235 192 165 262 191 376 259 235 381 331 221 244 0

Find the

a) mean *b*) median *c*) mode *d*) midrange

e) How might that duration time be explained?

Solution

0 73 95 165 191 192 221 235 235 244 259 262 331 376 381

a) *Mean*: $\bar{x} = \frac{\sum x}{n}$

$$= \frac{0+73+95+165+191+192+221+235+235+244+259+262+331+376+381}{15}$$

$$\boxed{= 217.3\ hrs}$$

b) *Median*: $\boxed{= 235\ hrs}$

c) *Mode*: 235 ***hrs***.

d) *Midrange*: $\frac{0+381}{2} \underline{\boxed{= 190.5\ \ hrs}}$

e) The duration of time of 0 appears to be very unusual. It likely represents Challenger disaster of Jan 1986, when the mission ended in an explosion shortly after takeoff.

Exercise

Listed below are the playing times (in seconds) of songs that were popular at the time of this writing.

448 242 231 246 246 293 280 227 213 262 239 213 258 255 257 244

Find the

a) mean *b*) median *c*) mode *d*) midrange

e) Is there on time that is very different from the others?

Solution

213 213 227 231 239 242 244 246 246 255 257 258 262 280 293 448

a) *Mean*: $\overline{x} = \dfrac{\sum x}{n}$

$$= \frac{213+213+227+231+239+242+244+246+246+255+257+258+262+280+293+448}{16}$$

$$\underline{= 259.6 \text{ sec}}$$

b) *Median*: $= \dfrac{246+246}{2} \underline{= 246 \text{ sec}}$

c) *Mode*: 213, 246 ***sec***.

d) *Midrange*: $\dfrac{213+448}{2} \underline{= 330.5 \text{ sec}}$

e) Yes the time of 448 seconds appears to be very different from the others.

Exercise

Listed below are numbers of satellites in orbit from different countries.

158 17 15 17 7 3 5 1 8 3 4 2 4 1 2 3 1 1 1 1 1 1 1 1

Find the

a) mean *b*) median *c*) mode *d*) midrange

e) Does on country have an exceptional number of satellites?

f) Can you guess which country has the most satellites?

Solution

1 1 1 1 1 1 1 1 1 1 2 2 3 3 3 4 4 5 7 8 15 17 18 158

a) *Mean*: $\overline{x} = \dfrac{\sum x}{n}$

$$= \frac{1+1+1+1+1+1+1+1+1+1+2+2+3+3+3+4+4+5+7+8+15+17+18+158}{24}$$

$$\underline{= 10.8 \textit{ satellites}}$$

b) *Median*: $= \dfrac{2+3}{2} \underline{= 2.5 \textit{ satellites}}$

c) *Mode*: 1 *satellite*.

d) *Midrange*: $\dfrac{1+158}{2} \underline{= 79.5 \textit{ satellites}}$

e) Yes the country with 158 satellites has an exceptional number of them.

f) That country is most likely the United States.

Exercise

Listed below are costs (in dollars) of roundtrip flights from JFK airport in NY City to San Francisco. (All flights involve one stop and a two-week stay.) The airlines are US Air, Continental, Delta, United, American, Alaska, and Northwest.

30 Days in Advance	244	260	264	264	278	318	280
1 Day in Advance	456	614	567	943	628	1088	536

a) Find the mean and median for each then compare the two sets of results.

b) Does it make much difference if the tickets are purchased 30 days in advance or 1 day in advance?

Solution

30 Days in Advance	244	260	264	264	278	280	318
1 Day in Advance	456	536	567	614	628	943	1088

a) $Mean: \ \bar{x}\Big|_{30\ days} = \frac{\sum x}{n} = \frac{244+260+264+278+280+318}{7} \underline{=\$272}|$

$Median: \ \underline{=\$264}|$

$Mean: \ \bar{x}\Big|_{1\ day} = \frac{456+536+567+614+628+943+1088}{7} \underline{=\$690.3}|$

$Median: \ \underline{=\$614}|$

b) The tickets purchased 30 days in advance are considerably less expensive – they appear to cost less than half of the tickets purchased one day in advance.

Exercise

The trend of thinner Miss America winners has generated charges that the contest encourages unhealthy diet habits among young women. Listed below are body mass indexes (BMI) for Miss America winners from two different periods.

BMI (1920 – 1930)	20.4	21.9	22.1	22.3	20.3	18.8	18.9	19.4	18.4	19.1
BMI – (from recent winners)	19.5	20.3	19.6	20.2	17.8	17.9	19.1	18.8	17.6	16.8

Find the mean and median for each then compare the two sets of results.

Solution

BMI (1920 – 1930)	18.4	18.8	18.9	19.1	19.4	20.3	20.4	21.9	22.1	22.3
BMI – (*from recent winners*)	16.8	17.6	17.8	17.9	18.8	19.1	19.5	19.6	20.2	20.3

a) *Mean*: $\bar{x}\Big|_{past} = \dfrac{18.4+18.8+18.9+19.1+19.4+20.3+20.4+21.9+22.1+22.3}{10} \underline{= 20.16|}$

Median: $= \dfrac{19.4+20.3}{2} \underline{= 19.85|}$

Mean: $\bar{x}\Big|_{present} = \dfrac{16.8+17.6+17.8+17.9+18.8+19.1+19.5+19.6+20.2+20.3}{10} \underline{= 18.76|}$

Median: $= \dfrac{18.8+19.1}{2} \underline{= 18.95|}$

b) The recent winners appear to have lower measures of BMI than do the former winners.

Exercise

Find the mean of the data summarized in the given frequency distribution.

a)

Tar (mg) in Nonfiltered Cigarettes	***Frequency***
10 – 13	1
14 – 17	0
18 – 21	15
22 – 25	7
26 – 29	2

b)

Pulse Rates of Females	***Frequency***
60 – 69	12
70 – 79	14
80 – 89	11
90 – 99	1
100 – 109	1
110 – 119	0
120 – 129	1

Solution

a) $\bar{x} = \dfrac{\sum(f \cdot x)}{\sum f} = \dfrac{523.5}{25} \underline{= 20.9\ mg|}$

x	**f**	**f · x**
11.5	1	11.5
15.5	0	0
19.5	15	292.5
23.5	7	164.5
27.5	2	55.0
	25	523.5

b)

x	**f**	**f · x**
64.5	12	774.0
74.5	14	1043.0
84.5	11	929.5
94.5	1	94.5
104.5	1	104.5
114.5	0	0.0
124.5	1	124.5
	40	3070.0

$\bar{x} = \dfrac{\sum(f \cdot x)}{\sum f} = \dfrac{3070.0}{40} \underline{= 76.8\ \ beats\ per\ minute|}$

Exercise

A student of the author earned grades of B, C, B, A, and D. Those courses has these corresponding numbers credit hours: 3, 3, 4, 4, and 1. The grading system assigns quality points to letter grades as follows: A = 4; B = 3; C = 2; D = 1; F =0. Compute the grade point average (GPA) and round the result with two decimal places. If the Dean's list requires a GPA 3.00 or greater, did this student make the Dean's list?

Solution

The *x* values are the numerical values of the letter grades, and the corresponding weights are the numbers of credit hours.

x	w	$w \cdot x$
3	3	9
2	3	6
3	4	12
4	4	16
1	1	1
	15	44

$$\overline{x} = \frac{\sum(f \cdot x)}{\sum f} = \frac{44}{15} \underline{= 2.93|}$$

No; since 2.93 is below 3.00, the student did not make the Dean's list.

Exercise

A student of the author earned grades of 92, 83, 77, 84, and 82 on her five regular tests. She earned grades of 88 on the final exam and 95 on her class projects. Her combined homework grade was 77. The five regular tests count for 60% of the final grade, the final exam counts for 10%, the project counts for 15%, and homework counts for 15%. What is her weighted mean grade? What letter grade did she earn? (A, B, C, D, or F)

Solution

The *x* values are the individual grades, and the corresponding weights are the percent of the final grade.
Since the 5 regular tests count for 60% of the final grade, each regular test counts for:

$\frac{60\%}{5} = 12\% = 0.12$ of the final grade.

$$\overline{x} = \frac{\sum(w \cdot x)}{\sum w} = \frac{84.76}{1.00} \underline{= 84.76|}$$

Her weighted mean grade is 84.8. Within 90-80-70-60 grading system for *A*-*B*-*C*-*D*, that corresponds to *B*.

x	w	$w \cdot x$
92	0.12	11.04
83	0.12	9.96
77	0.12	9.24
84	0.12	10.08
82	0.12	9.84
88	0.10	8.80
95	0.15	14.25
77	0.15	11.55
	1.0	84.76

Exercise

You are taking a class in which your grade is determined from five sources: 50% from your test mean, 15% from your midterm, 20% from your final exam, 10% from your computer lab work, and 5% from your homework. Your scores are 86 (test mean), 96 (midterm), 82 (final exam), 98 (computer lab), and 100 (homework). What is the weighted mean of your scores? If the minimum average for an A is 90, did you get an A?

Solution

Source	***x***	***w***	$\boldsymbol{w \cdot x}$
Test mean	86	0.50	73.0
Midterm	96	0.15	14.4
Final Exam	82	0.20	16.4
Computer lab	98	0.10	9.8
Homework	100	0.05	5.0
		1	88.6

$$\overline{x} = \frac{\sum(w \cdot x)}{\sum w} = \frac{88.6}{1.00} \ \underline{= 88.6|}$$

Your weighted mean for the course is 88.6. So you did not get an *A*.

Exercise

During a quality assurance check, the actual coffee contents (in ounces) of six jars of instant coffee were recorded as 6.03, 5.59, 6.40, 6.00, 5.99, and 6.02.

a) Find the mean and the median of the coffee content.

b) The third value was incorrectly measured and is actually 6.04. Find the mean and median of the coffee content again.

c) Which measure of central tendency, the mean or the median, was affected more by the data entry error?

Solution

a) $\overline{x} = \frac{\sum x}{n} = \frac{6.03 + 5.59 + 6.40 + 6.00 + 5.99 + 6.02}{6} = \frac{36.03}{6} \ \underline{= 6.005|}$

$5.59 \quad 5.99 \quad \underbrace{6.0 \quad 6.02}_{median} \quad 6.03 \quad 6.4$

$median = \frac{6 + 6.02}{2} \ \underline{= 6.01|}$

b) $\overline{x} = \frac{\sum x}{n} = \frac{6.03 + 5.59 + 6.04 + 6.00 + 5.99 + 6.02}{6} = \frac{35.67}{6} \ \underline{= 5.945|}$

$5.59 \quad 5.99 \quad \underbrace{6.0 \quad 6.02}_{median} \quad 6.03 \quad 6.4$

$median = \frac{6 + 6.02}{2} \ \underline{= 6.01|}$

c) The mean was affected more

Exercise

The table below shows the U.S. exports (in billions of dollars) to 19 countries for a recent year.

U.S. Exports (in billions of dollars)		
Canada: 261.1	Mexico: 151.2	Germany: 54.5
Taiwan: 24.9	Netherlands: 39.7	China: 69.7
Australia: 22.2	Malaysia: 12.9	Switzerland: 22.0
Saudi Arabia: 12.5	United Kingdom: 53.6	Japan: 65.1
South Korea: 34.7	Singapore: 27.9	France: 28.8
Brazil: 32.3	Belgium: 28.9	Italy: 15.5
Thailand: 9.1		

a) Find the mean and the median.

b) Find the mean and median without the U.S. exports to Canada. Which measure of central tendency, the mean or the median, was affected more by the elimination of the Canadian exports?

c) The U.S. Exports to India were \$17.7 billion. Find the mean and median with the Indian exports added to the original data set. Which measure of central tendency was affected more by adding the Indian exports?

Solution

a) $\bar{x} = \frac{\sum x}{n} = \frac{966.6}{19} \approx 50.87$

9.1 12.5 12.9 15.5 22.0 22.2 24.9 27.9 28.8 $\boxed{28.9}$

32.3 34.7 39.7 53.6 54.5 65.1 69.7 151.2 261.1

$median = 28.9$

b) $\bar{x} = \frac{\sum x}{n} = \frac{705.5}{18} \approx 39.19$

9.1 12.5 12.9 15.5 22.0 22.2 24.9 27.9 $\underbrace{28.8 \quad 28.9}_{median}$

32.3 34.7 39.7 53.6 54.5 65.1 69.7 151.2

$median = \frac{28.8 + 28.9}{2} = 28.85$

The mean was affected more

c) $\bar{x} = \frac{\sum x}{n} = \frac{984.3}{20} \approx 49.22$

9.1 12.5 12.9 15.5 *17.7* 22.0 22.2 24.9 27.9 $\underbrace{28.8 \quad 28.9}_{median}$

32.3 34.7 39.7 53.6 54.5 65.1 69.7 151.2

$median = \frac{28.8 + 28.9}{2} = 28.85$

The mean was affected more