Lecture Two – Trigonometric

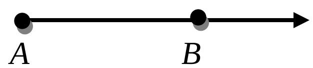
Section 2.1 – Angles

Basic Terminology

Two distinct points determine line *AB*.

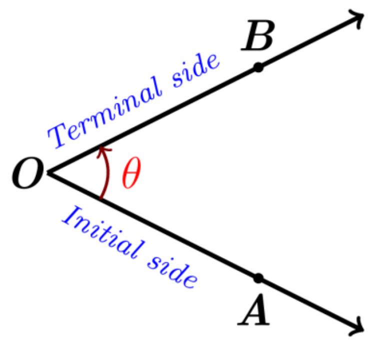
Line segment AB: portion of the line between *A* and *B*.

Ray AB: portion of the line *AB* starts at *A* and continues through *B*, and past *B*.



Angles in General

An angle is formed by 2 rays with the same end point.

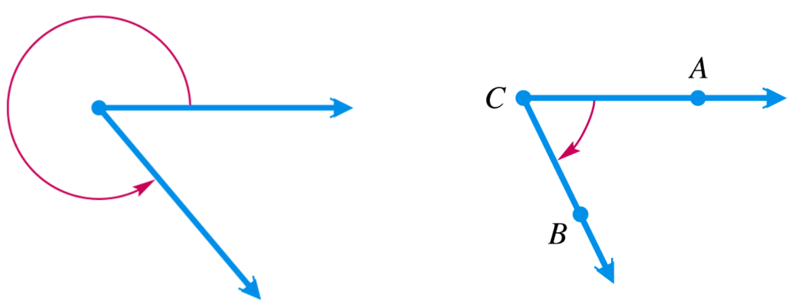


The two rays are the sides of the angle, angle $\theta = AOB$

O is the common endpoint and it is called ***vertex*** of the angle**.**

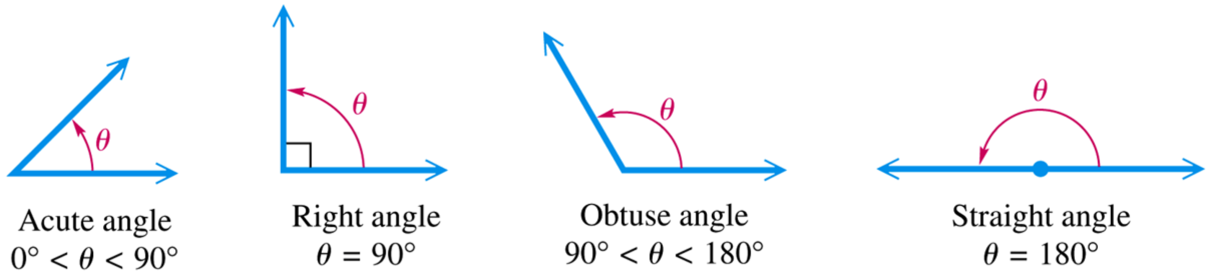
An angle is in a Counterclockwise (***CCW***) direction: positive angle.

An angle is in a Clockwise (***CW***) direction: negative angle.



Positive angle Negative angle

Type of Angles: *Degree*



Complementary angles: $\alpha + \beta = 90°$

Supplementary angles: $\alpha + \beta = 180°$

Example

Give the complement and the supplement of each angle: 40° 110°

Solution

a. 40° Complement: $90° - 40° = 50°$ Supplement: $180° - 40° = 140°$

b. 110° Complement: $90° - 110° = -20°$ Supplement: $180° - 110° = 70°$

Degrees, Minutes, Seconds

$1°$: 1 *degree*
$1'$: 1 *minute*
$1''$: 1 *second*

1 ***full Rotation*** or ***Revolution*** **= 360°** $1° = 60' = 3600''$ $1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ}$

Example

Change 27.25° to degrees and minutes

Solution

$$\begin{aligned} 27.25° &= 27° + .25° \\ &= 27° + .25(\mathbf{60'}) \\ &= 27° + 15' \\ &= 27°\ 15' \end{aligned}$$

Example

Add $48°\ 49'$ and $72°\ 26'$

Solution

$$\begin{array}{r} 48^\circ \quad 49' \\ +\ 72^\circ \quad 26' \\ \hline 120^\circ \quad 75' \end{array}$$

$$\begin{aligned} 120^\circ\, 75' &= 120^\circ\ \ 60'+15' \\ &= 121^\circ\ \ 15' \end{aligned}$$

Example

Subtract $24^\circ\, 14'$ and 90°

Solution

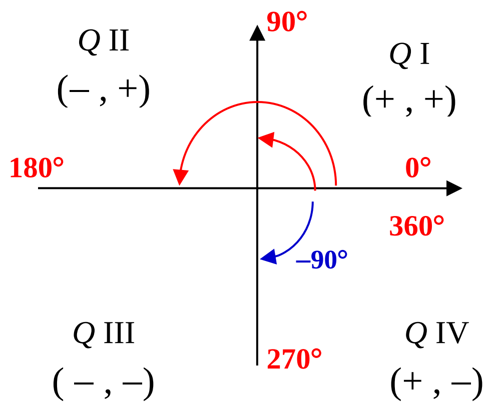
$$\begin{array}{r} 90^\circ \qquad \\ -\,24^\circ \quad 14' \\ \hline \end{array} \quad = \quad \begin{array}{r} 89^\circ \quad 60' \\ -\,24^\circ \quad 14' \\ \hline 65^\circ \quad 46' \end{array}$$

Angles in Standard Position

An angle is said to be in standard position if its initial side is along the positive x-axis and its vertex is at the origin. If angle θ is in standard position and the terminal side of θ lies in quadrant I, then we say θ lies in QI

$$\theta \in QI$$

If the terminal side of an angle in standard position lies along one of the axes (x-axis or y-axis), such as angles with measures 90°, 180°, 270°, then that called a ***quadrantal*** *angle*.
Two angles in standard position with the same terminal side are called ***coterminal*** *angles*.



Example

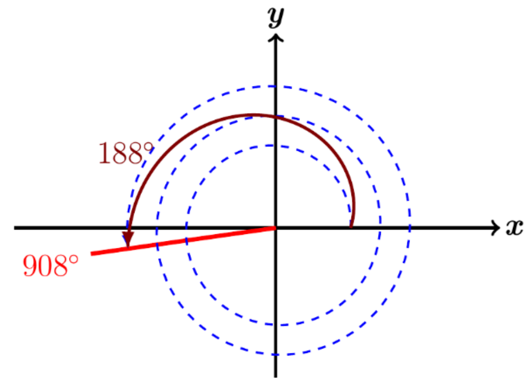
Find all angles that are coterminal with 120°.

Solution

$120^\circ + 360^\circ k$

Example

Find the angle of least possible positive measure coterminal with an angle of 908°.

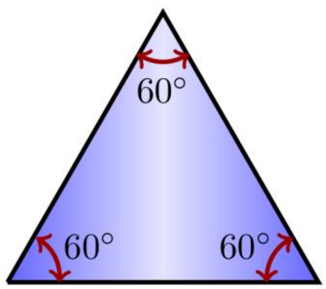
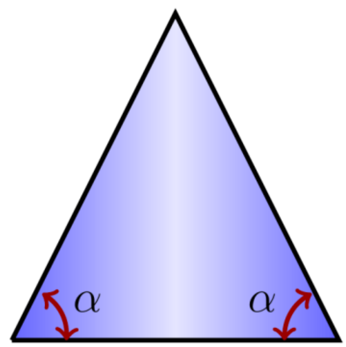
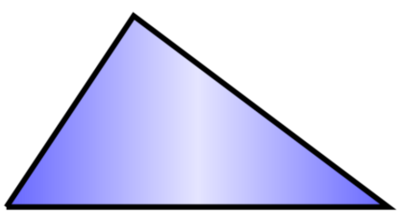
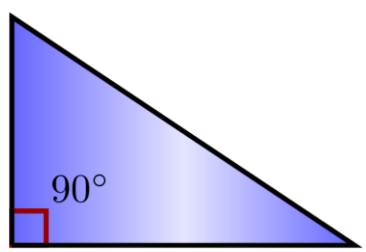
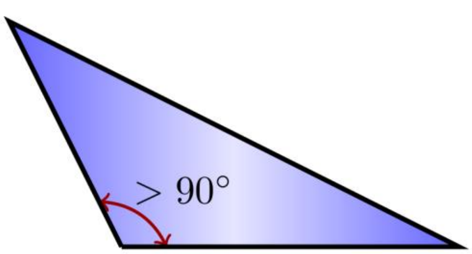
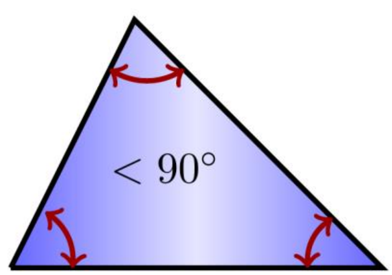


Solution

$908° - 2.360° = 188°$

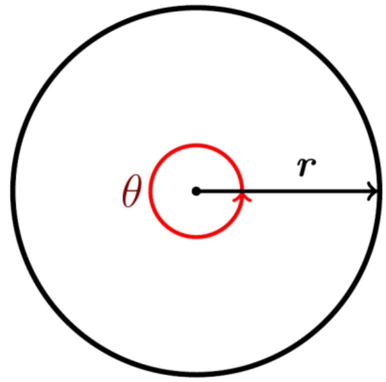
An angle of 908° is coterminal with an angle of 188°

Triangles

Equilateral – All angles always equal to 60°& all sides are equals 	***Isosceles***: 2 sides and angles are equals 
Scalene: No equal sides or angles 	***Right***: Has a right angle 90°. 
Obtuse: Has an angle more than 90°. 	***Acute***: All angles are less than 90°. 

Radians

Degrees - Radians



θ measures one full rotation $\quad \theta = 2\pi \quad$ *The measure of θ in radians is 2π*

$$\boxed{1 = 1\ rad}$$

$1° = 1$ degree

If no unit of angle measure is specified, then the angle is to be measured in radians.

$$Full\ Rotation:\ 360° = 2\pi\ rad$$

$$180° = \pi\ rad$$

Converting from Degrees to Radians

$$\frac{180°}{180} = \frac{\pi}{180}\ rad \qquad \Rightarrow 1° = \frac{\pi}{180}\ rad$$

Multiply a degree measure by $\frac{\pi}{180}\ rad$ and simplify to convert to radians.

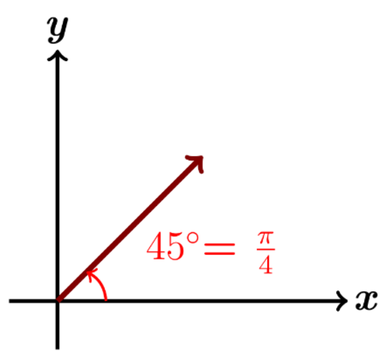
Example

Convert 45° to radians

Solution

$$45° = 45\left(\frac{\pi}{180}\right) rad$$

$$= \frac{\pi}{4}\ rad$$



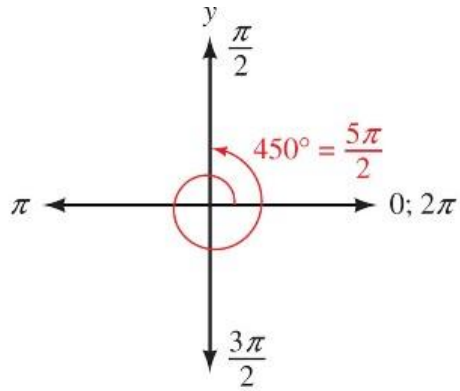
Example

Convert -450° to radians

Solution

$$-450^\circ = -450\left(\frac{\pi}{180}\right)rad$$

$$= -\frac{5\pi}{2}\ rad$$



Example

Convert 249.8° to radians

Solution

$$249.8^\circ = \frac{2498}{10}\left(\frac{\pi}{180}\right)rad$$

$$= \frac{1,249\pi}{900}\ rad$$

$$\approx 4.360\ rad$$

Converting from Radians to Degrees

Multiply a radian measure by $\frac{180^\circ}{\pi}$ radian and simplify to convert to degrees.

$$\frac{180^\circ}{\pi} = \frac{\pi}{\pi}\ rad \qquad \boxed{\left(\frac{180}{\pi}\right)^\circ = 1\ rad}$$

Example

Convert 1 to degrees

Solution

$$1\ rad = 1\left(\frac{180}{\pi}\right)^\circ = 1\left(\frac{180}{3.14}\right)^\circ = 57.3^\circ$$

Example

Convert $\frac{4\pi}{3}$ to degrees

Solution

$$\frac{4\pi}{3} = \frac{4\pi}{3}\left(\frac{180}{\pi}\right)^\circ = 240^\circ$$

Example

Convert –4.5 to degrees

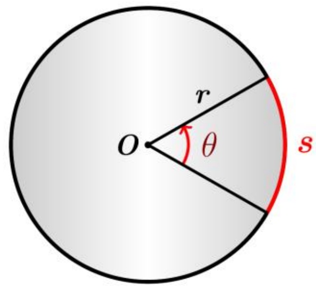
Solution

$$-4.5 = -4.5\left(\frac{180}{\pi}\right)^\circ \approx -257.8^\circ$$

Arc Length

Definition

If a central angle θ, in a circle of a radius r, cuts off an arc of length s, then the measure of θ, in radians is:

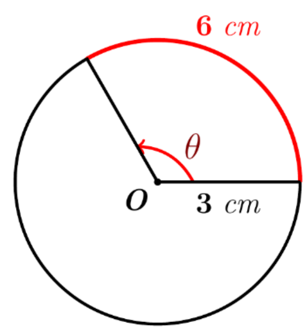


$\theta r = \frac{s}{r} r$ $$s = r\theta \qquad (\theta \text{ in radians})$$

Note : *When applying the formula, the value of* ***must*** *be in* ***radian***

Example

A central angle θ in a circle of radius 3 *cm* cuts off an arc of length 6 *cm*.
What is the radian measure of θ.



Solution

$$\theta = \frac{s}{r} = \frac{6\ cm}{3\ cm}$$

$$\boxed{= 2\ rad}$$

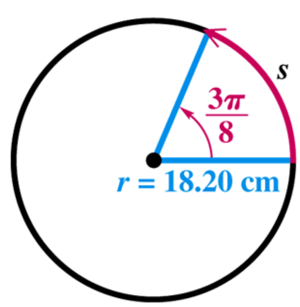
Example

A circle has radius 18.20 *cm*. Find the length of the arc intercepted by a central angle with measure $\frac{3\pi}{8}$ radians.

Solution

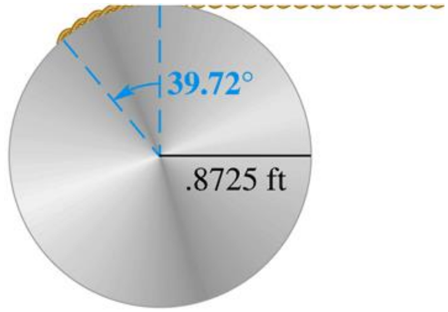
Given: $\theta = \frac{3\pi}{8}\ rad, \quad r = 18.20\ cm$

$$s = r\theta$$
$$= \frac{182}{10}\left(\frac{3\pi}{8}\right)$$
$$\boxed{= \frac{273\pi}{40}\ cm}$$
$$\boxed{\approx 21.44\ cm}$$



Example

A rope is being wound around a drum with radius 0.8725 *feet*. How much rope will be wound around the drum if the drum is rotated through an angle of 39.72°?

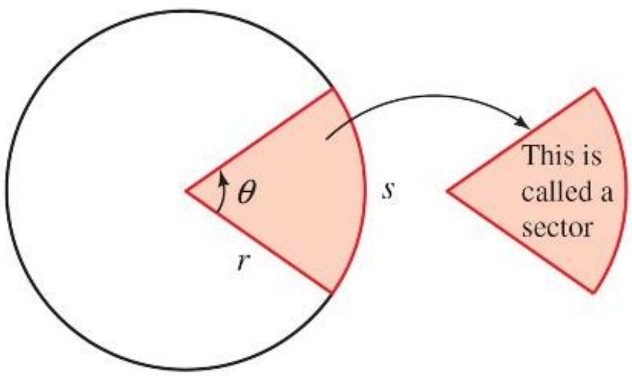


Solution

$$
\begin{aligned}
s &= r\theta \\
&= 0.8725\left(39.72^\circ \frac{\pi}{180^\circ}\right) \\
&= 8725\left(3972 \frac{\pi}{180}\right)10^{-6} \\
&= \frac{1,732,785\pi}{9}10^{-7} \quad feet \\
&\approx 0.6049\ feet
\end{aligned}
$$

Area of a Sector

A sector of a circle is a portion of the interior of a circle intercepted by a central angle.



$$
\begin{matrix} \textit{Area of sector} & \rightarrow \\ \textit{Area of circle} & \rightarrow \end{matrix} \quad \frac{A}{\pi r^2} = \frac{\theta}{2\pi} \quad \begin{matrix} \leftarrow & \textit{Central angle } \theta \\ \leftarrow & \textit{One full rotation} \end{matrix}
$$

$$\frac{A}{\pi r^2}\pi r^2 = \frac{\theta}{2\pi}\pi r^2$$

$$A = \frac{1}{2}r^2\theta$$

Definition

If θ (in *radians*) is a central angle in a circle with radius r, then the area of the sector formed by an angle θ is given by

$$A = \frac{1}{2}r^2\theta \qquad (\boldsymbol{\theta} \ \textbf{\textit{in radians}})$$

Example

Find the area of the sector formed by a central angle of 1.4 *radians* in a circle of radius 2.1 *meters*.

Solution

Given: $r = 2.1\ m,\ \ \theta = 1.4$

$$A = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}\left(\frac{21}{10}\right)^2\left(\frac{14}{10}\right)$$

$$= \frac{3,087}{1,000}\ m^2$$

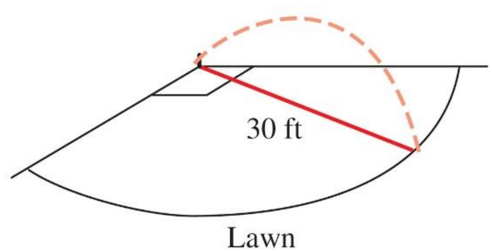
$$= 0.3087\ m^2$$

Example

A lawn sprinkler located at the corner of a yard is set to rotate 90° and project water out 30.0 *feet*. To three significant digits, what area of lawn is watered by the sprinkler?

Solution

Given: $\theta = 90° = \frac{\pi}{2};\ \ r = 30\ ft$



$$A = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}(30)^2\frac{\pi}{2}$$

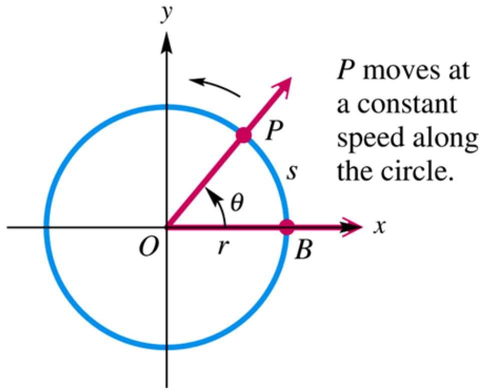
$$= 225\pi\ \ ft^2$$

$$\approx 707\ ft^2$$

Linear Speed

Definition

If P is a point on a circle of radius r, and P moves a distance s on the circumference of the circle in an amount of time t, then the ***linear velocity***, v, of P is given by the formula



$$speed = \frac{distance}{time}$$

$$v = \frac{s}{t}$$

Example

A point on a circle travels 5 *cm* in 2 *sec*. Find the linear velocity of the point.

Solution

Given: $s = 5\ cm;\quad t = 2\ sec$

$$v = \frac{s}{t} = \frac{5\ cm}{2\ sec}$$

$$= 2.5\ cm/sec$$

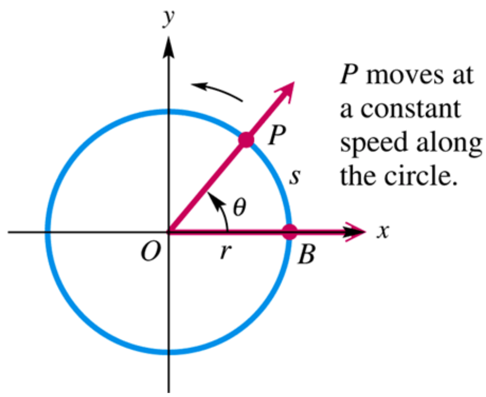
The most intuitive measure of the rate at which the rider is traveling around the wheel is what we call ***linear velocity***.

Another way to specify how fast the rider is traveling around the wheel is with what we call ***angular velocity***.

Angular Speed

Definition

If P is a point moving with uniform circular motion on a circle of radius r, and the line from the center of the circle through P sweeps out a central angle θ in an amount of time t, then the *angular velocity*, ω (omega), of P is given by the formula



$$\omega = \frac{\theta}{t} \qquad \textit{where } \theta \textit{ is measured in radians}$$

Example

A point on a circle rotates through $\frac{3\pi}{4}$ *radians* in 3 *sec*. Give the angular velocity of the point.

Solution

Given: $\theta = \frac{3\pi}{4}\,rad;\ t = 3\ sec$

$$\omega = \frac{\frac{3\pi}{4}\ rad}{3\ sec}$$

$$\boxed{= \frac{\pi}{4}\ rad/sec}$$

Example

A bicycle wheel with a radius of 13.0 *in*. turns with an angular velocity of 3 *radians per seconds*. Find the distance traveled by a point on the bicycle tire in 1 *minute*.

Solution

Given: $r = 13.0\ in.;\ \omega = 3\ rad/sec;\ t = 1\ min = 60\ sec.$

$$\omega = \frac{\theta}{t} \Rightarrow \omega t = \theta$$

$$s = r\theta \Rightarrow \theta = \frac{s}{r}$$

$$\omega t = \frac{s}{r}$$

$$s = \omega\, tr$$
$$= 3 \times 60 \times 13$$
$$\boxed{= 2{,}340\ inches}$$

$$or\ \ \frac{2{,}340}{12} = 195\, ft$$

Relationship between the Two Velocities

If $\quad s = r\theta$

$$\frac{s}{t} = \frac{r\theta}{t}$$

$$\frac{s}{t} = r\frac{\theta}{t}$$

$$v = r\omega$$

$$v = r\frac{\theta}{t}$$

Linear and Angular Velocity

If a point is moving with uniform circular motion on a circle of radius r, then the linear velocity v and angular velocity ω of the point are related by the formula

$$v = r\omega$$

Example

A phonograph record is turning at 45 revolutions per minute (*rpm*). If the distance from the center of the record to a point on the edge of the record is 3 *inches*, find the angular velocity and the linear velocity of the point in *feet* per *minute*.

Solution

$$\omega = 45\ rpm$$

$$= 45\ \frac{rev}{min} \qquad\qquad 1\ revolution = 2\pi rad$$

$$= 45\ \frac{rev}{min}\frac{2\pi\ rad}{1\ rev}$$

$$\boxed{= 90\pi\ rad\ /\ min}$$

$$v = r\omega$$

$$= (3\ in.)\left(90\pi\frac{rad}{min}\right)$$

$$= 270\pi\frac{in}{min}$$

$$\boxed{\approx 848\ \ in\ /\ min}$$

$$v = 848\ \frac{in}{min}\frac{1\,ft}{12in}$$

$$= \frac{212}{3}\ ft/\mathrm{min}$$

$$v \approx 70.7\ ft/\mathrm{min}$$

Exercises *Section* 2.1– Angles

1. Indicate the angle if it is an acute or obtuse. Then give the complement and the supplement of each angle.

a) $10°$ *b)* $52°$ *c)* $90°$ *d)* $120°$ *e)* $150°$

2. Change to decimal degrees.

a) $10°\ 45'$ *c)* $274°\ 18'\ 59''$ *e)* $98°\ 22'\ 45''$ *g)* $1°\ 2'\ 3''$

b) $34°\ 51'\ 35''$ *d)* $74°\ 8'\ 14''$ *f)* $9°\ 9'\ 9''$ *h)* $73°\ 40'\ 40''$

3. Convert to degrees, minutes, and seconds.

a) $89.9004°$ *c)* $122.6853°$ *e)* $44.01°$ *g)* $29.411°$

b) $34.817°$ *d)* $178.5994°$ *f)* $19.99°$ *h)* $18.255°$

4. Perform each calculation

a) $51°\ 29' + 32°\ 46'$ *b)* $90° - 73°12'$ *c)* $90° - 36°\ 18'\ 47''$ *d)* $75°\ 15' + 83°\ 32'$

5. Find the angle of least possible positive measure coterminal with an angle of

a) $-75°$ *b)* $-800°$ *c)* $270°$

6. A vertical rise of the Forest Double chair lift 1,170 *feet* and the length of the chair lift as 5,570 *feet*. To the nearest foot, find the horizontal distance covered by a person riding this lift.

7. A tire is rotating 600 times per *minute*. Through how many degrees does a point of the edge of the tire move in $\frac{1}{2}$ second?

8. A windmill makes 90 revolutions per minute. How many revolutions does it make per second?

9. Convert to radians

a) $256°\ 20'$ *b)* $-78.4°$ *c)* $330°$ *d)* $-60°$ *e)* $-225°$

10. Convert to degrees

a) $\frac{11\pi}{6}$ *c)* $\frac{\pi}{6}$ *e)* $\frac{\pi}{3}$ *g)* -4π

b) $-\frac{5\pi}{3}$ *d)* 2.4 *f)* $-\frac{5\pi}{12}$ *h)* $\frac{7\pi}{13}$

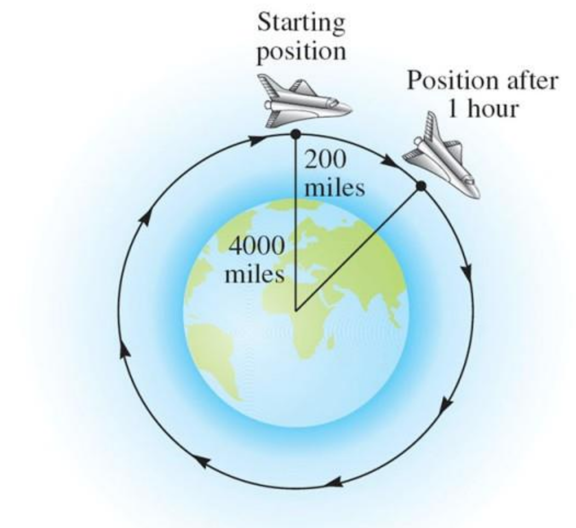
11. The minute hand of a clock is 1.2 *cm* long. How far does the tip of the minute hand travel in 40 *minutes*?

12. Find the radian measure if angle θ, if θ is a central angle in a circle of radius $r = 4$ *inches*, and θ cuts off an arc of length $s = 12\pi$ *inches*.

13. Give the length of the arc cut off by a central angle of 2 *radians* in a circle of radius 4.3 *inches.*

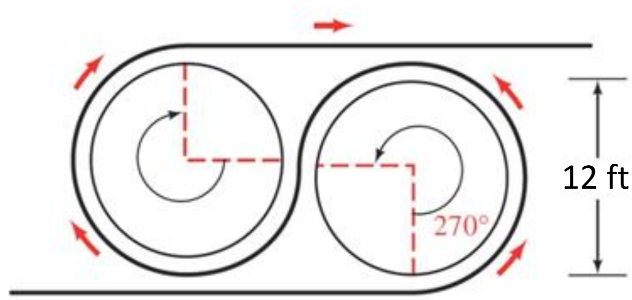
14. Reno, Nevada is due north of Los Angeles. The latitude of Reno is 40°, while that of Los Angeles is 34° N. The radius of Earth is about 4000 *mi*. Find the north-south distance between the two cities.

15. A space shuttle 200 *miles* above the earth is orbiting the earth once every 6 *hours*. How long, in hours, does it take the space shuttle to travel 8,400 *miles*? (Assume the radius of the earth is 4,000 *miles*.) Give both the exact value and an approximate value for your answer.



16. The pendulum on a grandfather clock swings from side to side once every second. If the length of the pendulum is 4 *feet* and the angle through which it swings is 20°. Find the total distance traveled in 1 *minute* by the tip of the pendulum on the grandfather clock.

17. The first cable railway to make use of the figure-eight drive system was a Sutter Street Railway. Each drive sheave was 12 *feet* in diameter. Find the length of cable riding on one of the drive sheaves.

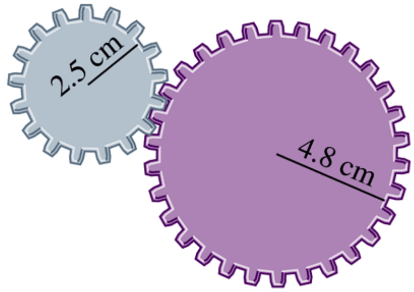


18. The diameter of a model of George Ferris's Ferris wheel is 250 *feet*, and θ is the central angle formed as a rider travels from his or her initial position P_0 to position P_1. Find the distance traveled by the rider if $\theta = 45°$ and if $\theta = 105°$.

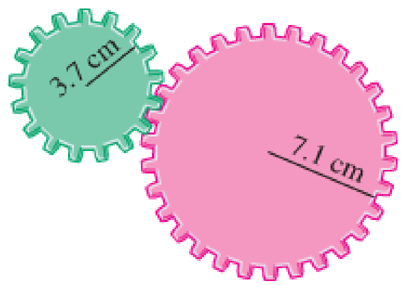
19. The rotation of the smaller wheel causes the larger wheel to rotate. Through how many degrees will the larger wheel rotate if the smaller one rotates through 60.0°?

20. Find the number of regular (statute) miles in 1 *nautical mile* to the nearest hundredth of a mile. (Use 4,000 miles for the radius of the earth).

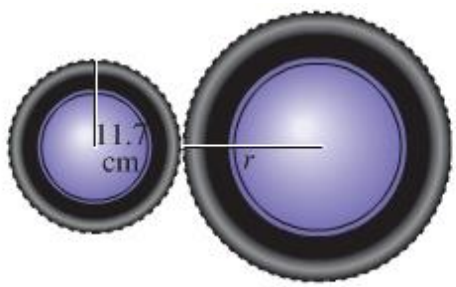
21. Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of 225°, through how many degrees will the larger gear rotate?



22. Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of 300°, through how many degrees will the larger rotate?



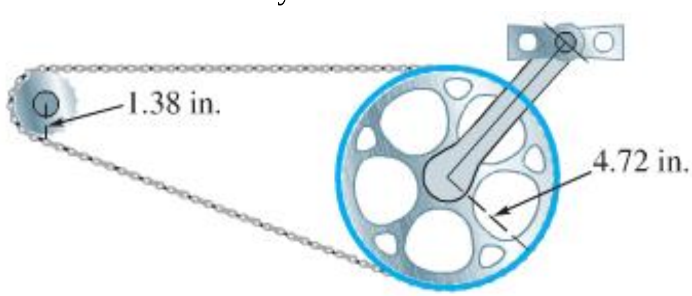
23. Find the radius of the larger wheel if the smaller wheel rotates 80° when the larger wheel rotates 50°.



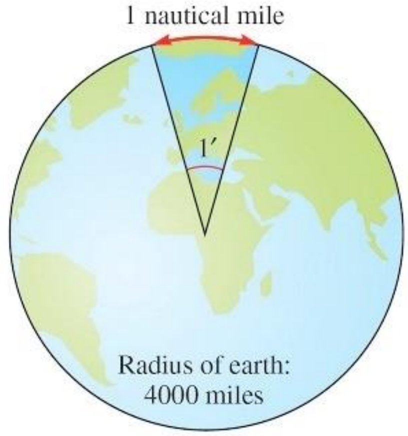
24. Los Angeles and New York City are approximately 2,500 *miles* apart on the surface of the earth. Assuming that the radius of the earth is 4,000 *miles*, find the radian measure of the central angle with its vertex at the center of the earth that has Los Angeles on one side and New York City in the other side.

25. If two ships are 20 *nautical miles* apart on the ocean, how many statute miles apart are they?

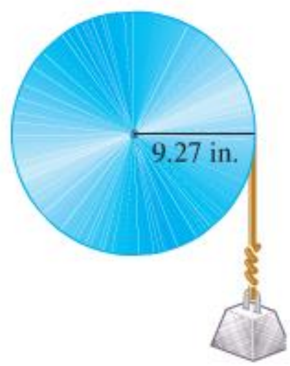
26. The figure shows the chain drive of a bicycle. How far will the bicycle move if the pedals are rotated through 180°? Assume the radius of the bicycle wheel is 13.6 *in*.



27. If a central angle with its vertex at the center of the earth has a measure of 1′, then the arc on the surface of the earth that is cut off by this angle (knows as the great circle distance) has a measure of 1 nautical mile.

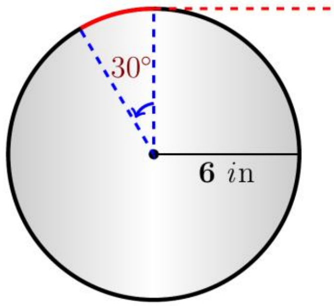


28. How many inches will the weight rise if the pulley is rotated through an angle of 71° 50′? Through what angle, to the nearest minute, must the pulley be rotated to raise the weight 6 *in*?

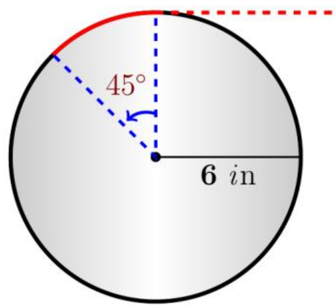


29. Find the radius of the pulley if a rotation of 51.6° raises the weight 11.4 *cm*.

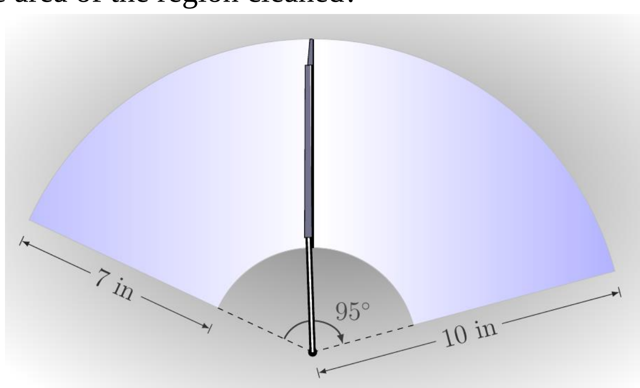
30. A rope is being wound around a drum with radius 6 *inches*. How much rope will be wound around the drum if the drum is rotated through an angle of 30°?



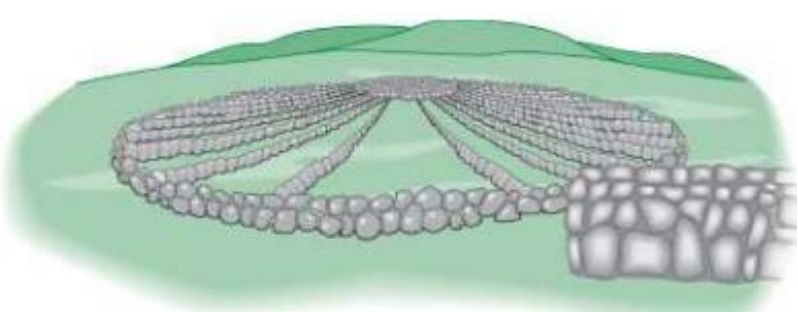
31. A rope is being wound around a drum with radius 6 *inches*. How much rope will be wound around the drum if the drum is rotated through an angle of 45°?



32. The total arm and blade of a single windshield wiper was 10 *in*. long and rotated back and forth through an angle of 95°. The shaded region in the figure is the portion of the windshield cleaned by the 7-*in*. wiper blade. What is the area of the region cleaned?



33. The circular of a Medicine Wheel is 2500 *yrs* old. There are 27 aboriginal spokes in the wheel, all equally spaced.

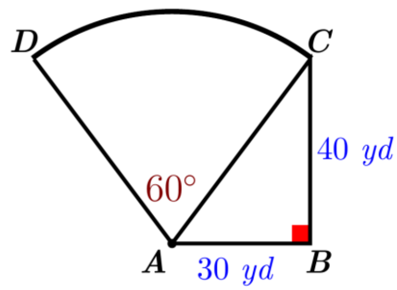


a) Find the measure of each central angle in degrees and in radians.

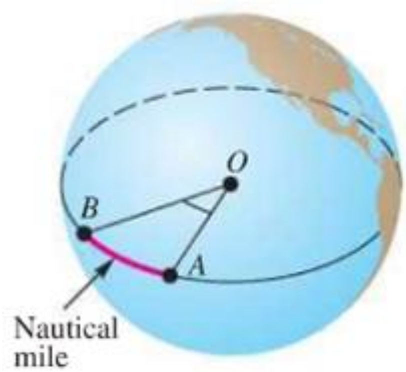
b) The radius measure of each of the wheel is 76.0 *feet*, find the circumference.

c) Find the length of each arc intercepted by consecutive pairs of spokes.
d) Find the area of each sector formed by consecutive spokes,

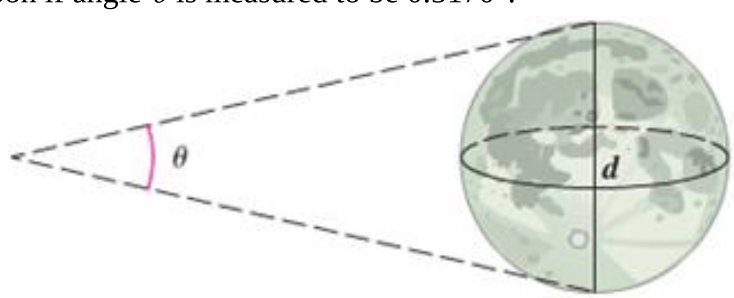
34. A frequent problem in surveying city lots and rural lands adjacent to curves of highways and railways is that of finding the area when one or more of the boundary lines is the arc of the circle. Find the area of the lot.



35. Nautical miles are used by ships and airplanes. They are different from statue miles, which equal 5280 *feet*. A nautical mile is defined to be the arc length along the equator intercepted by a central angle AOB of 1 *min*. If the equatorial radius is 3963 *mi*, use the arc length formula to approximate the number of statute miles in 1 *nautical mile*.



36. The distance to the moon is approximately 238,900 *mi*. Use the arc length formula to estimate the diameter d of the moon if angle θ is measured to be 0.5170°.



37. The minute hand of a clock is 1.2 *cm* long. To two significant digits, how far does the tip of the minute hand move in 20 *minutes*?

38. If the sector formed by a central angle of 15° has an area of $\frac{\pi}{3}\,cm^2$, find the radius of a circle.

39. Suppose that P is on a circle with radius 10 *cm*, and ray OP is rotating with angular speed $\frac{\pi}{18}$ $rad/\sec$.

a) Find the angle generated by P in 6 *seconds*

b) Find the distance traveled by P along the circle in 6 seconds.

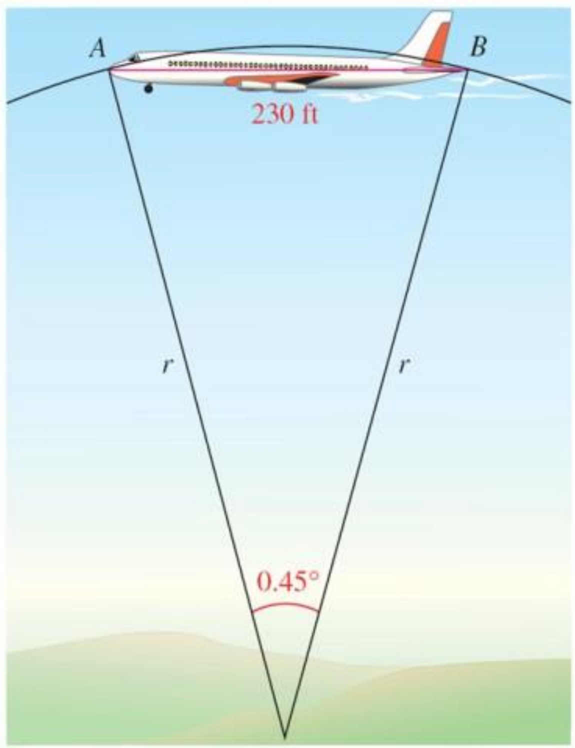
c) Find the linear speed of P in *cm* per *sec*.

40. A belt runs a pulley of radius 6 *cm* at $80\ rev/\min$.

a) Find the angular speed of the pulley in *radians* per *sec*.

b) Find the linear speed of the belt in *cm* per *sec*.

41. A person standing on the earth notices that a 747 jet flying overhead subtends an angle 0.45°. If the length of the jet is 230 *feet.*, find its altitude to the nearest thousand feet.



42. Find the linear velocity of a point moving with uniform circular motion, if s = 12 *cm* and t = 2 *sec*.

43. Find the distance s covered by a point moving with linear velocity v = 55 *mi/hr* and t = 0.5 *hr*.

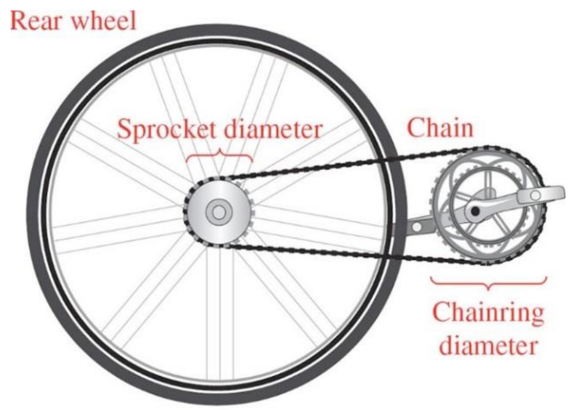
44. Point ***P*** sweeps out central angle $\theta = 12\pi$ as it rotates on a circle of radius r with t = 5π sec. Find the angular velocity of point ***P***.

45. Find the angular velocity, in *radians* per *minute*, associated with given 7.2 *rpm*.

46. Suppose that point P is on a circle with radius 60 *cm*, and ray OP is rotating with angular speed $\frac{\pi}{12}$ radian per sec.

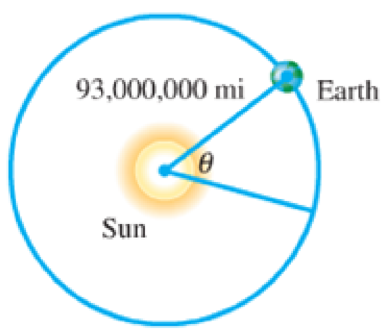
a) Find the angle generated by P in 8 *sec*.
b) Find the distance traveled by P along the circle in 8 *sec*.
c) Find the linear speed of P.

47. When Lance Armstrong blazed up Mount Ventoux in the 2002 tour, he was equipped with a 150-millimeter-diameter chainring and a 95-*millimeter*-diameter sprocket. Lance is known for maintaining a very high cadence, or pedal rate. If he was pedaling at a rate of 90 revolutions per minute, find his speed in kilometers per hour. (1 *km* = 1,000,000 *mm* or 10^6 *mm*)



48. Tires of a bicycle have radius 13 *in*. and are turning at the rate of 215 revolutions per min. How fast is the bicycle traveling in miles per hour? (*Hint*: 1 *mi* = 5280 *ft*.)

49. Earth travels about the sun in an orbit that is almost circular. Assume that the orbit is a circle with radius 93,000,000 *mi*. Its angular and linear speeds are used in designing solar-power facilities.



a) Assume that a year is 365 *days*, and find the angle formed by Earth's movement in one day.
b) Give the angular speed in *radians* per *hour*.
c) Find the linear speed of Earth in miles per hour.

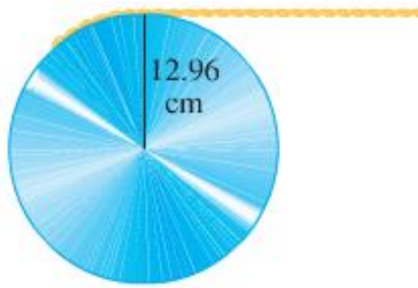
50. Earth revolves on its axis once every 24 *hr*. Assuming that earth's radius is 6400 *km*, find the following.

a) Angular speed of Earth in radians per day and radians per hour.

b) Linear speed at the North Pole or South Pole

c) Linear speed ar a city on the equator

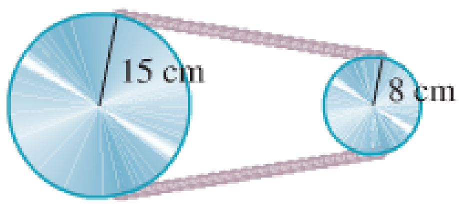
51. The pulley has a radius of 12.96 *cm*. Suppose it takes 18 *sec* for 56 *cm* of belt to go around the pulley.



a) Find the linear speed of the belt in *cm* per *sec*.

b) Find the angular speed of the pulley in *rad* per *sec*.

52. The two pulleys have radii of 15 *cm* and 8 *cm*, respectively. The larger pulley rotates 25 times in 36 *sec*. Find the angular speed of each pulley in *rad* per *sec*.



53. A thread is being pulled off a spool at the rate of 59.4 *cm* per *sec*. Find the radius of the spool if it makes 152 revolutions per min.

54. A railroad track is laid along the arc of a circle of radius 1800 *feet*. The circular part of the track subtends a central angle of 40°. How long (in seconds) will it take a point on the front of a train traveling 30 *mph* to go around this portion of the track?

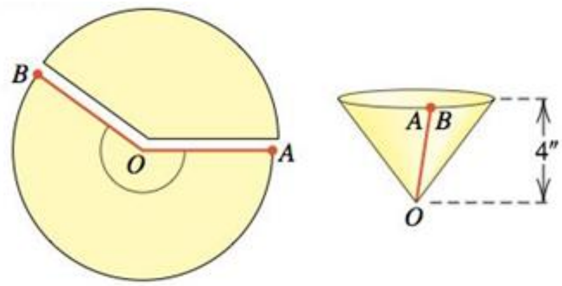
55. A 90-horsepower outboard motor at full throttle will rotate it propeller at exactly 5000 revolutions per min. Find the angular speed of the propeller in radians per second.

56. The shoulder joint can rotate at 25 *rad/min*. If a golfer's arm is straight and the distance from the shoulder to the club head is 5.00 *feet*., find the linear speed of the club head from the shoulder rotation

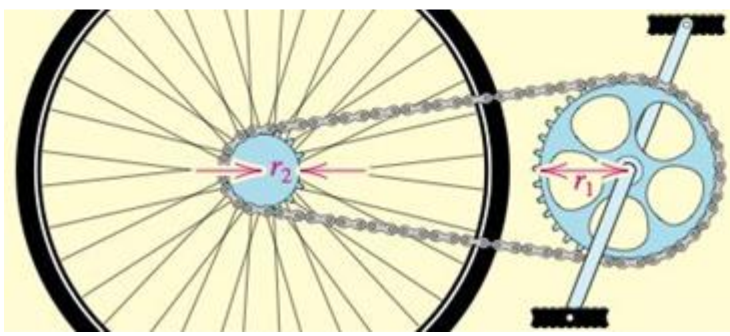
57. A vendor sells two sizes of pizza by the slice. The small slice is $\frac{1}{6}$ of a circular 18–*inch*–diameter pizza, and it sells for \$2.00. The large slice is $\frac{1}{8}$ of a circular 26–*inch*–diameter pizza, and it sells for \$3.00. Which slice provides more pizza per dollar?

58. A cone–shaped tent is made from a circular piece of canvas 24 *feet* in diameter by removing a sector with central angle $100°$ and connecting the ends. What is the surface area of the tent?

59. A conical paper cup is constructed by removing a sector from a circle of radius 5 *inches* and attaching edge *OA* to *OB*. Find angle *AOB* so that the cap has a depth of 4 *inches*.



60. The sprocket assembly for a bicycle is show in the figure. If the sprocket of radius r_1 rotates through an angle of θ_1 *radians*, find the corresponding angle of rotation for the sprocket of radius r_2



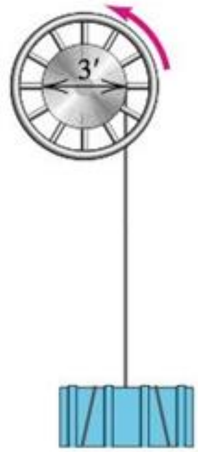
61. A simple model of the core of a tornado is a right circular cylinder that rotates about its axis. If a tornado has a core diameter of 200 *feet* and maximum wind speed of 180 *mi/hr*. (or 264 *ft/sec*) at the perimeter of the core, approximate the number of revolutions the core makes each minute.

62. Earth rotates about its axis once every 23 *hours*, 56 *minutes*, and 4 *seconds*. Approximate the number of radians Earth rotates in one *second*

63. A typical tire for a compact car is 22 *inches* in diameter. If the car is traveling at a speed of 60 *mi/hr*., find the number of revolutions the tire makes per minute.

64. A pendulum in a grandfather clock is 4 *feet* long and swings back and forth along a 6–*inch* arc. Approximate the angle (in *degrees*) through which the pendulum passes during one swing.

65. A large winch of diameter 3 *feet* is used to hoist cargo.



a) Find the distance the cargo is lifted if the winch rotates through an angle measure $\frac{7\pi}{4}$.

b) Find the angle (in *radians*) through which the winch must rotate in order to lift the cargo d *feet*.