Math 2415 – Calculus III ***Exam 2*** **Review**

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1. Find partial derivatives of the function with respect to each variables

a) $g(r,\theta)=r\cos\theta+r\sin\theta$

b) $f(x,y)=\frac{1}{2}\ln\left(x^2+y^2\right)+\tan^{-1}\frac{y}{x}$

c) $h(x,y,z)=\sin(2\pi x+y-3z)$

d) $f(r,l,T,w)=\frac{1}{2rl}\sqrt{\frac{T}{\pi w}}$

2. Find second-order partial derivatives of the functions

a) $g(x,y)=y+\frac{x}{y}$ *b)* $g(x,y)=e^x+y\sin x$ *c)* $f(x,y)=y^2-3xy+\cos y+7e^y$

3. Find $\frac{dw}{dt}$ at $t=0$ if $w=\sin(xy+\pi)$, $x=e^t$, *and* $y=\ln(t+1)$

4. Find $\frac{dw}{dt}$ at $t=1$ if $w=xe^y+y\sin z-\cos z$, $x=2\sqrt{t}$, $y=t-1+\ln t$ *and* $z=\pi t$

5. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ when $r=\pi$ and $s=0$ if $w=\sin(2x-y)$, $x=r+\sin s$, $y=rs$

6. Find the value of the derivative of $f(x,y,z)=xy+yz+xz$ with respect to *t* on the curve $x=\cos t$, $y=\sin t$, $z=\cos 2t$ at $t=1$

7. Define *y* as a differentiable function of *x* for $2xy+e^{x+y}-2=0$, find the values of $\frac{dy}{dx}$ at point $P(0,\ \ln 2)$

8. Find the direction in which f increases and decreases most rapidly at P_0 and find the derivative of f in each direction. Also, find the derivative of f at P_0 in the direction of the vector $\boldsymbol{v}$.

a) $f(x,y)=\cos x\cos y$, $P_0\left(\frac{\pi}{4},\frac{\pi}{4}\right)$, $\boldsymbol{v}=3\boldsymbol{i}+4\boldsymbol{j}$

b) $f(x,y)=x^2e^{-2y}$, $P_0(1,\ 0)$, $\boldsymbol{v}=\boldsymbol{i}+\boldsymbol{j}$

c) $f(x,y,z)=\ln(2x+3y+6z)$, $P_0(-1,-1,1)$, $\boldsymbol{v}=2\boldsymbol{i}+3\boldsymbol{j}+6\boldsymbol{k}$

d) $f(x,y,z)=x^2+3xy-z^2+2y+z+4$, $P_0(0,0,0)$, $\boldsymbol{v}=\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}$

9. Find an equation for the plane tangent to the level surface $f(x,y,z)=x^2-y-5z$ at the point $P_0(2,\ -1,\ 1)$. Also, find parametric equations for the line is normal to the surface at P_0.

10. Find an equation for the plane tangent to the surface $z=f(x,y)=\frac{1}{x^2+y^2}$ at the point $\left(1,\ 1,\ \frac{1}{2}\right)$.

11. What is the largest value that the directional derivative of $f(x,y,z)=xyz$ can have at the point $(1,\ 1,\ 1)$?

12. You plan to calculate the volume inside a stretch of pipeline that is about 36 in. in diameter and 1 mile long. With which measurement should you be more careful, the length or the diameter? Why?

13. Find all the local maxima, local minima, and saddle points of the function

a) $f(x,y) = x^2 - xy + y^2 + 2x + 2y - 4$

b) $f(x,y) = x^3 + y^3 - 3xy + 15$

c) $f(x,y) = x^4 - 8x^2 + 3y^2 - 6y$

14. Find the extreme values of $f(x,y) = x^3 + y^2$ on the circle $x^2 + y^2 = 1$

15. Find the extreme values of $f(x,y) = x^2 + y^2 - 3x - xy$ on the circle $x^2 + y^2 \le 9$

16. A closed rectangular box is to have volume $V\ cm^3$. The cost of the material used in the box is a $a\ cents/cm^2$ for top and bottom, $b\ cents/cm^2$ for front and back, and $c\ cents/cm^2$ for the remaining sides. What dimensions minimize the total cost of materials?

17. Find the extreme values of $f(x,y,z) = x(y+z)$ on the curve of intersection of the right circular cylinder $x^2 + y^2 = 1$ and the hyperbolic cylinder $xz = 1$.

18. Find the point closest to the origin on the curve of intersection of the plane $x + y + z = 1$ and the cone $z^2 = 2x^2 + 2y^2$

Solution

1. ***a***) $\dfrac{\partial g}{\partial r}=\cos\theta+\sin\theta \quad \dfrac{\partial g}{\partial \theta}=-r\sin\theta+r\cos\theta$ ***b***) $\dfrac{\partial f}{\partial x}=\dfrac{x-y}{x^2+y^2} \quad \dfrac{\partial f}{\partial y}=\dfrac{x+y}{x^2+y^2}$

c) $h_x=2\pi\cos(2\pi x+y-3z), \quad h_y=\cos(2\pi x+y-3z), \quad h_z=-3\cos(2\pi x+y-3z)$

d) $f_r=-\dfrac{1}{2r^2 l}\sqrt{\dfrac{T}{\pi w}} \quad f_l=-\dfrac{1}{2rl^2}\sqrt{\dfrac{T}{\pi w}} \quad f_T=\dfrac{1}{4rl}\sqrt{\dfrac{1}{\pi wT}} \quad f_w=-\dfrac{1}{4\pi rlw}\sqrt{\dfrac{T}{\pi w}}$

2. ***a***) $g_{xx}=0, \quad g_{yy}=\dfrac{2x}{y^3}, \quad g_{xy}=g_{yx}=-\dfrac{1}{y^2}$

b) $g_x=e^x+y\cos x, \quad g_y=\sin x, \quad g_{xx}=e^x-y\sin x, \quad g_{yy}=0, \quad g_{xy}=g_{yx}=\cos x$

c) $f_{xx}=0, \quad f_{yy}=y-\cos y+7e^y, \quad f_{xy}=f_{yx}=-3$

3. $\left.\dfrac{\partial w}{\partial t}\right|_{t=0}=-1$

4. $\dfrac{\partial w}{\partial t}=5$

5. $\dfrac{\partial w}{\partial r}=2, \quad \dfrac{\partial w}{\partial s}=2-\pi$

6. $\left.\dfrac{df}{dt}\right|_{t=1}=-(\sin 1+\cos 2)\cdot(\sin 1)+(\cos 1+\cos 2)\cdot\cos 1-2(\cos 1+\sin 1)\cdot(\sin 2)$

7. $\left.\dfrac{dy}{dx}\right|_{(0,\ln 2)}=-\ln 2-1$

8. ***a***) *increases* $\boldsymbol{u}=-\dfrac{1}{\sqrt{2}}\boldsymbol{i}-\dfrac{1}{\sqrt{2}}\boldsymbol{j}$ *decreases* $-\boldsymbol{u}=\dfrac{1}{\sqrt{2}}\boldsymbol{i}+\dfrac{1}{\sqrt{2}}\boldsymbol{j}$ $\left(D_{\boldsymbol{u}}f\right)_{P_0}=-\dfrac{7}{10}$

b) *increases* $\boldsymbol{u}=\dfrac{1}{\sqrt{2}}\boldsymbol{i}-\dfrac{1}{\sqrt{2}}\boldsymbol{j},$ *decreases* $-\boldsymbol{u}=-\dfrac{1}{\sqrt{2}}\boldsymbol{i}+\dfrac{1}{\sqrt{2}}\boldsymbol{j},$ $\left(D_{\boldsymbol{u}}f\right)_{P_0}=0$

c) *increases* $\boldsymbol{u}=\frac{2}{7}\boldsymbol{i}+\frac{3}{7}\boldsymbol{j}+\frac{6}{7}\boldsymbol{k},$ *decreases* $-\boldsymbol{u}=-\frac{2}{7}\boldsymbol{i}-\frac{3}{7}\boldsymbol{j}-\frac{6}{7}\boldsymbol{k},$ $\left(D_{\boldsymbol{u}}f\right)_{P_0}=7$

d) *increases* $\boldsymbol{u}=\dfrac{2}{\sqrt{5}}\boldsymbol{i}+\dfrac{1}{\sqrt{5}}\boldsymbol{k},$ *decreases* $-\boldsymbol{u}=-\dfrac{2}{\sqrt{5}}\boldsymbol{i}-\dfrac{1}{\sqrt{5}}\boldsymbol{k},$ $\left(D_{\boldsymbol{u}}f\right)_{P_0}=\sqrt{3}$

9. Tangent plane: $4x-y-5z=4$ Normal line: $x=2+4t, \quad y=-1-t, \quad z=1-5t$

10. Tangent plane: $2y-z-2=0$

11. $\sqrt{3}$

12. $dV=15{,}840\pi dr+2.25\pi dh$, the diameter has a greater effect on dV

13. ***a***) *Local Min*: $f(-2,-2)=-8$

b) $Local\ Min: f(1,1)=14 \quad saddle\ point: f(0,0)=15$

c) $Local\ Min: f(2,1)=f(-2,1)=14 \quad saddle\ point: f(0,1)=-3$

14. $Local\ Max: 1 @ (0,\pm1),(1,0) \quad Local\ Min: -1 @ (-1,0)$

15. $Abs.\ Max: 9+\frac{27\sqrt{3}}{4}\ @\ \left(-\frac{3\sqrt{3}}{2},\frac{3}{2}\right) \quad Abs.\ Min: -3 @ (2,1)$

16. $width = x = \left(\frac{c^2V}{ab}\right)^{1/3} \quad depth = y = \left(\frac{b^2V}{ac}\right)^{1/3} \quad height = z = \left(\frac{a^2V}{bc}\right)^{1/3}$

17. $Abs.\ Max: \frac{3}{2} @ \left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\sqrt{2}\right)\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},-\sqrt{2}\right)$

$Abs.\ Min: \frac{1}{2} @ \left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},-\sqrt{2}\right)\left(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},\sqrt{2}\right)$

18. $\left(\frac{1}{4},\frac{1}{4},\frac{1}{2}\right)$