Critical Points CP (x, y)

$f'(c) = 0$

- $f'(x) = 0$, then, solve for x (critical #)
C.N

Ex. abs. (absolute) extreme

$f(x) = x^2$ $[-2, 1]$

Soln. $f'(x) = 2x$

$\underline{x = 0}$ C.N

x	$f(x)$	
-2	4	→ abs. Max @ $(-2, 4)$
0	0	→ abs. Min @ $(0, 0)$
1	1	

Ex. $g(t) = 8t - t^4$ $[-2, 1]$ abs. Ext.

Soln. $g'(t) = 8 - 4t^3 = 0$ C.N

$t^3 = 2 \Rightarrow \boxed{t = \sqrt[3]{2} > 1}$

t	$g(t)$	
-2	-32	→ abs. Min $(-2, -32)$
1	7	→ abs. Max $(1, 7)$

Ex $f(x) = x^{2/3}$ $[-2, 3]$ Extrem.

Soln $f'(x) = \frac{2}{3} x^{-1/3}$ $\frac{2}{3x^{1/3}} \neq 0$

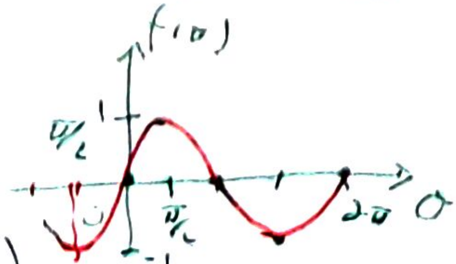
$\boxed{\text{C.N: } x = 0}$

x	$f(x)$	
-2	$\sqrt[3]{4}$	
0	0	$\to$ Abs Min $(0, 0)$
3	$\sqrt[3]{9}$	$\to$ abs Max $(3, \sqrt[3]{9})$

#5 $f(\theta) = \sin\theta$ $-\frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$, QII

$f'(\theta) = \cos\theta = 0$

$\theta = \pm\frac{\pi}{2}, \cancel{\frac{3\pi}{2}}$



θ	$f(\theta)$	
$-\frac{\pi}{2}$	-1	$\to$ abs Min $(-\frac{\pi}{2}, -1)$
$\frac{\pi}{2}$	1	$\to$ abs. Max $(\frac{\pi}{2}, 1)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	

3.2 Increasing Decreasing

(Incr) Decr.

Ex $f(x) = x^2 - 12x - 5$ Inc? Dec?

$f'(x) = 2x - 12 = 0$

CN: $\underline{x = 6}$

$f' \to ?$

$$\begin{array}{c|c} & 6 \\ \hline - & + \end{array} \leftarrow f'(7)$$

5

Incr: $(6, \infty)$ Decr: $(-\infty, 6)$

$f(x) = x^3 - 12x - 5$ deg 3 | $3 - 1 = 2$ extr.
| $3 \to$ I & D.

$f'(x) = 3x^2 - 12 = 0$

$x^2 = 4 \to$ CN: $\underline{x = \pm 2}$

$$\begin{array}{c|c|c} -2 & 0 & 2 \\ \hline + & - & + \end{array}$$

Testing use f' sign

→ , and ∠

Incr: $(-\infty, -2)$, $(2, \infty)$

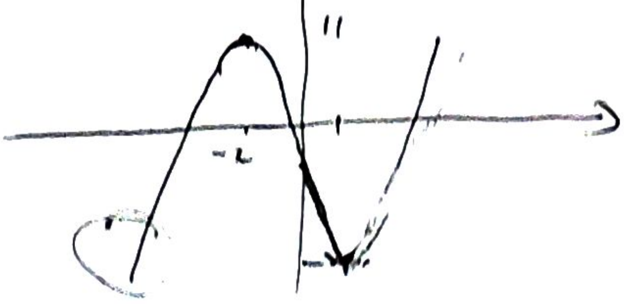
Dec: $(-2, 2)$

RMax

x	f(x)
-2	11
2	-21

→ LMAX $(-2, 11)$

LMIN $(2, -21)$



→ CN, Abs. Ext, Ext, (Inc, Dec)

<u>Ex</u> $f(x) = x^{1/3}(x-4)$

$= x^{4/3} - 4x^{1/3}$

$f'(x) = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3}$ → $x^{-2/3} = \frac{1}{x^{2/3}}$ $\boxed{x \neq 0}$

$= \frac{4}{3}\left(x^{1/3} - x^{-2/3}\right) = 0$

$x^{1/3} = x^{-2/3}$ CN $\begin{cases} x = 0 \\ x = 1 \end{cases}$

0		1		8
−	|	−	|	+

$8 \to 2 - \frac{1}{4}$

Inc: $(1, \infty)$ Dec: $(-\infty, 1)$

LMIN. $(1, -3)$ → Min. Point

Concavity $\begin{cases} \text{Concave Upward (up) } + \\ \text{" downward (down) } - \end{cases}$

→ Point of Inflection

Pt. Infl.

To find Pt. Infl

$f''(x) = 0 \to$

Ex $f(x) = x^4 - 8x^3 + 18x^2$

$f'(x) = 4x^3 - 24x^2 + 36x$

$f''(x) = 12x^2 - 48x + 36 = 0$

pt. of Infl: $x = 1, 3$

0	1	3
+	−	+

Concave up: $(-\infty, 1)$ $(3, \infty)$

" down: $(1, 3)$

Ex $y = 3 + \sin x$ $[0, 2\pi]$ Concavities??

$y' = \cos x$

$y'' = -\sin x = 0 \Rightarrow$ pt. Inf: $x = 0, \pi, 2\pi$

0 $\pi/2$	π	2π
−	+	

concave up: $(\pi, 2\pi)$

" down: $(0, \pi)$

Ex $\quad s(t) = 2t^3 - 14t^2 + 22t - 5 \qquad t \geq 0$

$s' = 6t^2 - 28t + 22 = 0$

CN: $t = 1, \frac{11}{3}$

$s''(t) = 12t - 28 = 0 \qquad$ pt. Infl. $t = \frac{7}{3}$

t	$s(t)$
1	5
$\frac{11}{3}$	-14

$$\begin{array}{c|c|c} 0 \quad 1 & & \frac{11}{3} \\ \hline + & - & + \end{array}$$

$$\begin{array}{c|c} 0 \quad \frac{7}{3} & \\ \hline - & + \end{array}$$

$t \geq 0$

L MIN $\left(\frac{11}{3}, -14\right)$ $\quad$ L MAX: $(1, 5)$

Inc: $(0, 1)\ \left(\frac{11}{3}, \infty\right)$

Dec: $\left(1, \frac{11}{3}\right)$

Concave up: $\left(\frac{7}{3}, \infty\right)$

" down: $\left(0, \frac{7}{3}\right)$

2

- CN
- abs. Ext.
- Ext. $\longrightarrow$ Critical Point (CP)
- Inc Dec
- Pt of Infl. ?
- Concavity

$f(x) = x^4 - 4x^3 + 10$

$f'(x) = 4x^3 - 12x^2$

$= 4x^2(x-3) = 0$

$\underline{CN: x = 0, 0, 3}$

$f''(x) = 12x^2 - 24x$

$= 12x(x-2) = 0$ Pt. Infl: $x = 0, 2$

	0	1	3	
$-$		$-$		$+$

	0	1	2	
$+$		$-$		$+$

LMIN: $(3, -17)$

Inc: $(3, \infty)$

Dec: $(-\infty, 3)$

Concave up: $(-\infty, 0)$ $(2, \infty)$

" down: $(0, 2)$

Maximum / Maximize

or

Minimize.

$f' = 0$

C.N