$$\int 4y^{-3}\,dy = -2y^{-2} + C$$

$$\int \pi xy\,dt = \pi xyt + C$$

$$\int \frac{x^4 - 2\sqrt{x} + 2}{x^2}\,dx = \int \left(x^2 - 2x^{-3/2} + \frac{2}{x^2}\right)dx$$

$$= \frac{1}{3}x^3 + \frac{4}{\sqrt{x}} - \frac{2}{x} + C$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int \sec 2x \tan 2x\,d(2x) \quad \{ d(2x) = 2dx$$

$$= \frac{1}{2}\sec 2x + C$$

$$\int (2\sin\theta - 5e^{\theta})\,d\theta = -2\cos\theta - 5e^{\theta} + C$$

$$\int \left(\frac{3}{x} + \sec^2 x\right)dx = 3\ln|x| + \tan x + C$$

$$\int (a^2 - b^2)\,e^{(a-b)x} = \frac{a^2 - b^2}{a-b}e^{(a-b)x} + C$$

$$= \frac{(a-b)(a+b)}{a-b}e^{(a-b)x} + C$$

$$= (a+b)\,e^{(a-b)x} + C$$

$$\int_0^2 x(x-2)\,dx = \int_0^2 (x^2-2x)\,dx$$
$$= \frac{1}{3}x^3 - x^2 \Big|_0^2$$
$$= \frac{8}{3} - 4$$
$$= -\frac{4}{3}$$

$$\int_0^{\pi/3} 4\sec u \tan u\,du = 4\sec u \Big|_0^{\pi/3}$$
$$= 4(2-1)$$
$$= 4$$

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right)dt = 4\tan t - \frac{\pi}{t} \Big|_{-\pi/3}^{-\pi/4}$$
$$= -4 + 4 - (-4\sqrt{3} + 3)$$
$$= 4\sqrt{3} - 3$$

$$\int_{-2}^{-1} \left(3e^{3x} + \frac{2}{x}\right)dx = e^{3x} + 2\ln|x| \Big|_{-2}^{-1}$$
$$= e^{-3} + 2\ln 1 - (e^{-6} + 2\ln 2)$$
$$= e^{-3} - e^{-6} - 2\ln 2$$

$2x^2 - 4x + 2 = 0$ $\quad 0 \le x \le 2$ A?

$x = 1, 1$

$$A = 2\int_0^2 (x^2 - 2x + 1)\,dx$$
$$= 2\left(\frac{1}{3}x^3 - x^2 + x\right)\Big|_0^2$$
$$= 2\left(\frac{8}{3} - 4 + 2\right)$$
$$= \frac{4}{3} \text{ unit}^2$$

(6) $f(x) = x^2 + 4x + 3 = 0$ $\quad -3 \le x \le 0$

$x = -1, -3$

$$A = -\int_{-3}^{-1} (x^2+4x+3)\,dx + \int_{-1}^{0} (x^2+4x+3)\,dx$$

$$= -\left(\tfrac{1}{3}x^3 + 2x^2 + 3x\right|_{-3}^{-1} + \left(\tfrac{1}{3}x^3 + 2x^2 + 3x\right|_{-1}^{0}$$

$$= -\left(-\tfrac{1}{3} + 2 - 3 - (-9 + 18 - 9)\right) - \left(-\tfrac{1}{3} + 2 - 3\right)$$

$$= \tfrac{4}{3} + \tfrac{4}{3}$$

$$= \tfrac{8}{3} \text{ unit}^2$$

A x-axis $\quad y = x^4 - 16 = 0$

$x^4 = 2^4 \quad x = \pm 2$

$$A = -\int_{-2}^{2} (x^4 - 16)\,dx$$

$$= -2\left(\tfrac{1}{5}x^5 - 16x\right|_{0}^{2}$$

$$= -2\left(\tfrac{32}{5} - 32\right)$$

$$= -64\left(\tfrac{1}{5} - 1\right)$$

$$= \tfrac{256}{5} \text{ unit}^2$$

$\int_{-a}^{a} \text{even} = 2\int_0^a \text{even}$

if $f(x)$ odd then $\int_{-a}^{a} \text{odd} = 0$.

$$\int_0^1 (2t+3)^3\,dt = \frac{1}{2}\int_0^1 (2t+3)^2\,d(2t+3) \qquad d(2t+3) = 2dt$$

$$= \frac{1}{6}(2t+3)^3\Big|_0^1$$

$$= \frac{1}{6}(125 - 27)$$

$$= \frac{49}{3}$$

$$\int_{-\pi}^{\pi} \frac{\cos z}{\sqrt{4+3\sin z}}\,dz \qquad d(4+3\sin z) = 3\,\underline{\cos z\,dz}$$

$$= \frac{1}{3}\int_{-\pi}^{\pi} (4+3\sin z)^{-1/2}\,dz$$

$$= \frac{2}{3}(4+3\sin z)^{1/2}\Big|_{-\pi}^{\pi}$$

$$= \frac{2}{3}(2-2)$$

$$= 0$$

$$\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\,dt \qquad d(t^5+2t) = (5t^4+2)dt$$

$$= \int_0^1 (t^5+2t)^{1/2}\,d(t^5+2t)$$

$$= \frac{2}{3}(t^5+2t)^{3/2}\Big|_0^1$$

$$= \frac{2}{3}(3^{3/2} - 0)$$

$$= 2\sqrt{3}$$

$$\int_0^{\pi/2} e^{\sin x} \cos x\,dx \qquad d(\sin x) = \cos x\,dx$$

$$= \int_0^{\pi/2} e^{\sin x}\, d(\sin x)$$

$$= e^{\sin x} \Big|_0^{\pi/2}$$

$$\underline{= e - 1}$$

$$\int_0^{\pi/2} \tan\frac{x}{2}\,dx = 2\int_0^{\pi/2} \tan\frac{x}{2}\, d\left(\frac{x}{2}\right) \qquad d\left(\frac{x}{2}\right) = \frac{1}{2}\,dx$$

$$= 2\ln\left|\sec\frac{x}{2}\right| \Big|_0^{\pi/2} \qquad -\ln|\cos x|$$

$$= 2\left(\ln\sqrt{2} - \ln 1\right) \qquad \ln 2^{1/2} = \frac{1}{2}\ln 2$$

$$= 2\left(\frac{1}{2}\ln 2\right)$$

$$= \ln 2$$

$$\int_1^{e^x} \frac{1}{t}\,dt = \ln|t| \Big|_1^{e^x}$$

$$= \ln e^x - \ln 1$$

$$\underline{= x}$$

$$\int_0^{\sqrt{\ln \pi}} 2x e^{x^2} \cos(e^{x^2})\,dx \qquad d(e^{x^2}) = 2x e^{x^2}\,dx$$

$$= \int_0^{\sqrt{\ln \pi}} \cos(e^{x^2})\, d(e^{x^2})$$

$$= \sin(e^{x^2}) \Big|_0^{\sqrt{\ln \pi}} \qquad e^{\ln x} = x$$

$$= \sin e^{\ln \pi} - \sin e^0$$

$$= -\sin 1$$

$$\int_0^{\ln 2} \frac{e^x}{1+e^{2x}}\,dx = \int_0^{\ln 2} \frac{e^x}{1+(e^x)^2}\,dx \qquad d(e^x) = e^x dx$$

$$= \int_0^{\ln 2} \frac{d(e^x)}{1+(e^x)^2}$$

$$= \tan^{-1} e^x \Big|_0^{\ln 2}$$

$$= \tan^{-1} 2 - \tan^{-1} 1$$

$$= \boxed{\tan^{-1} 2 - \frac{\pi}{4}}$$

$$\int_{-2}^{2} e^{4x+8}\,dx = \frac{1}{4}\int_{-2}^{2} e^{4x+8}\,d(4x+8) \qquad d(4x+8) = 4dx$$

$$= \frac{1}{4} e^{4x+8} \Big|_{-2}^{2}$$

$$= \boxed{\frac{1}{4}\left(e^{16} - 1\right)}$$

$$\int_0^{\ln 4} \frac{2e^x}{3+2e^x}\,dx = \int_0^{\ln 4} \frac{d(3+2e^x)}{3+2e^x} \qquad d(3+2e^x) = 2e^x dx$$

$$= \ln(3+2e^x) \Big|_0^{\ln 4}$$

$$= \ln 11 - \ln 5 = \ln \frac{11}{5}$$

$$\int_{-1}^{1} (x-1)(x^2-2x)^7 dx \qquad d(x^2-2x) = (2x-2)dx = 2(x-1)dx$$

$$= \frac{1}{2}\int_{-1}^{1} (x^2-2x)^7 \, d(x^2-2x)$$

$$= \frac{1}{16}(x^2-2x)^8 \Big|_{-1}^{1}$$

$$= \frac{1}{16}(1-3^8)$$

$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^2 x} dx = \int_{\pi/4}^{\pi/2} \frac{d(\sin x)}{\sin^2 x} \qquad d(\sin x) = \cos x \, dx$$

$$= -\frac{1}{\sin x}\Big|_{\pi/4}^{\pi/2} \qquad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{2}}{2}$$

$$= -(1-\sqrt{2})$$

$$= \sqrt{2} - 1$$

$$\int_{-1}^{2} x^2 e^{x^3+1} dx = \frac{1}{3}\int_{-1}^{2} e^{x^3+1} d(x^3+1) \qquad d(x^3+1) = 3x^2 dx$$

$$= \frac{1}{3} e^{x^3+1}\Big|_{-1}^{2}$$

$$= \frac{1}{3}(e^9 - 1)$$

$$\int_{1}^{e^2} \frac{\ln x}{x} dx = \int_{1}^{e^2} (\ln x)\, d(\ln x) \qquad d(\ln x) = \frac{1}{x}dx$$

$$= \frac{1}{2}(\ln x)^2 \Big|_{1}^{e^2}$$

$$= \frac{1}{2}(4-0)$$

$$= 2$$

$$\int_0^{\pi/4} \frac{\sin\theta}{\cos^3\theta}\,d\theta = -\int_0^{\pi/4} \cos^{-3}\theta\, d(\cos x) \qquad d(\cos\theta) = -\sin\theta\, d\theta$$

$$= \frac{1}{2}\cos^{-2}\theta \Big|_0^{\pi/4}$$

$$= \frac{1}{2}\,\frac{1}{\cos^2\theta}\Big|_0^{\pi/4}$$

$$= \frac{1}{2}(2-1)$$

$$= \frac{1}{2}$$

$$\int_0^{\pi/4} e^{\sin^2 x}\sin 2x\,dx \qquad d(\sin^2 x) = \overbrace{2\sin x\cos x}\,dx = \sin 2x\,dx$$

$$= \int_0^{\pi/4} e^{\sin^2 x}\, d(\sin^2 x)$$

$$= e^{\sin^2 x}\Big|_0^{\pi/4}$$

$$= e^{1/2} - 1 \qquad (\sqrt{e} - 1)$$

$$\int_{-1}^{2} x^2 e^{x^3+1}\,dx = \frac{1}{3}\int_{-1}^{2} e^{x^3+1}\,d(x^3+1) \qquad d(x^3+1) = 3x^2\,dx$$

$$= \frac{1}{3}\, e^{x^3+1}\Big|_{-1}^{2}$$

$$= \frac{1}{3}\big($$