Solution Section 2.5 – Polynomial Functions

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^3 + 7x^2 - x + 9$

Solution

Leading term: $5x^3$ with 3^{rd} degree (n is ***odd***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 11x^3 - 6x^2 + x + 3$

Solution

Leading term: $11x^3$ with 3^{rd} degree (n is ***odd***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -11x^3 - 6x^2 + x + 3$

Solution

Leading term: $-11x^3$ with 3^{rd} degree (n is ***odd***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to \infty \qquad f(x)$ rises left

$x \to \infty \quad \Rightarrow \quad f(x) \to -\infty \qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 2x^3 + 3x^2 - 23x - 42$

Solution

Leading term: $2x^3$ with 3^{rd} degree (n is ***odd***)

$x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \qquad f(x)$ falls left

$x \to \infty \quad \Rightarrow \quad f(x) \to \infty \qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=5x^4+7x^2-x+9$

Solution

Leading term: $5x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=11x^4-6x^2+x+3$

Solution

Leading term: $11x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-5x^4+7x^2-x+9$

Solution

Leading term: $-5x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-11x^4-6x^2+x+3$

Solution

Leading term: $-11x^4$ with 4^{rd} degree (n is ***even***)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 5x^5 - 16x^2 - 20x + 64$

Solution

Leading term: $5x^5$ with 5^{th} degree (n is ***odd***)

$x \to -\infty \ \Rightarrow \ f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \ \Rightarrow \ f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -5x^5 - 16x^2 - 20x + 64$

Solution

Leading term: $-5x^5$ with 5^{th} degree (n is ***odd***)

$x \to -\infty \ \Rightarrow \ f(x) \to \infty$ $\qquad f(x)$ rises left

$x \to \infty \ \Rightarrow \ f(x) \to -\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -3x^6 - 16x^3 + 64$

Solution

Leading term: $-3x^6$ with 6^{th} degree (n is ***even***)

$x \to -\infty \ \Rightarrow \ f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \ \Rightarrow \ f(x) \to -\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 3x^6 - 16x^3 + 4$

Solution

Leading term: $3x^6$ with 6^{th} degree (n is ***even***)

$x \to -\infty \ \Rightarrow \ f(x) \to \infty$ $\qquad f(x)$ rises left

$x \to \infty \ \Rightarrow \ f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - x - 1; \quad between \ \ 1 \ and \ \ 2$

Solution

$$f(1) = (1)^3 - (1) - 1$$

$$= -1$$

$$f(2) = (2)^3 - (2) - 1$$

$$= 5$$

Since $f(1) \ and \ f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - 4x^2 + 2; \quad between \ \ 0 \ and \ 1$

Solution

$$f(0) = (0)^3 - 4(0)^2 + 2$$

$$= 2$$

$$f(1) = (1)^3 - 4(1)^2 + 2$$

$$= -1$$

Since $f(0) \ and \ f(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = 2x^4 - 4x^2 + 1; \quad between \ \ -1 \ and \ \ 0$

Solution

$$f(-1) = 2(-1)^4 - 4(-1)^2 + 1$$

$$= -1$$

$$f(0) = 2(0)^4 - 4(0)^2 + 1$$

$$= 1$$

Since $f(0)$ *and* $f(-1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between -1 and 0.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^4 + 6x^3 - 18x^2$; *between* 2 *and* 3

Solution

$$f(2) = (2)^4 + 6(2)^3 - 18(2)^2$$
$$\underline{= -8}$$

$$f(3) = (3)^4 + 6(3)^3 - 18(3)^2$$
$$\underline{= 81}$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 + x^2 - 2x + 1$; *between* -3 *and* -2

Solution

$$f(-3) = (-3)^3 + (-3)^2 - 2(-3) + 1$$
$$\underline{= -11}$$

$$f(-2) = (-2)^3 + (-2)^2 - 2(-2) + 1$$
$$\underline{= 1}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between -2 and -3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^5 - x^3 - 1$; *between* 1 *and* 2

Solution

$$f(1)=(1)^5-(1)^3-1$$
$$\underline{=-1}$$
$$f(2)=(2)^5-(2)^3-1$$
$$\underline{=23}$$

Since $f(1)$ *and* $f(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-10x+9;$ *between* -3 *and* -2

Solution

$$f(-3)=3(-3)^3-10(-3)+9$$
$$\underline{=-42}$$
$$f(-2)=3(-2)^3-10(-2)+9$$
$$\underline{=5}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between –3 and –2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-8x^2+x+2;$ *between* 2 *and* 3

Solution

$$f(2)=3(2)^3-8(2)^2+(2)+2$$
$$\underline{=-4}$$
$$f(3)=3(3)^3-8(3)^2+(3)+2$$
$$\underline{=14}$$

Since $f(2)$ *and* $f(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-8x^2+x+2;\quad between\ \ 1\ and\ \ 2$

Solution

$$f(1)=3(1)^3-8(1)^2+(1)+2$$

$$\underline{=-2\ }|$$

$$f(2)=3(2)^3-8(2)^2+(2)+2$$

$$\underline{=-4\ }|$$

Since $f(1)$ *and* $f(2)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^5+2x^4-6x^3+2x-3;\quad between\ \ 0\ and\ \ 1$

Solution

$$f(0)=(0)^5+2(0)^4-6(0)^3+2(0)-3$$

$$\underline{=-3\ }|$$

$$f(1)=(1)^5+2(1)^4-6(1)^3+2(1)-3$$

$$\underline{=-4\ }|$$

Since $f(0)$ *and* $f(1)$ have same signs.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=2x^3+3x^2-23x-42,\quad a=3,\quad b=4$

Solution

$$P(3)=54+27-69-42$$

$$\underline{=-30\ }|$$

$$P(4)=128+48-92-42$$

$$\underline{=90\ }|$$

Since $P(3)$ *and* $P(4)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and 4.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 4x^3 - x^2 - 6x + 1, \quad a = 0, \quad b = 1$

Solution

$P(0) \underline{= 1}$

$P(1) = 4 - 1 - 6 + 1$

$\underline{= -2}$

Since $P(0)$ *and* $P(1)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 3x^3 + 7x^2 + 3x + 7, \quad a = -3, \quad b = -2$

Solution

$P(-3) = -81 + 63 - 9 + 7$

$\underline{= -20}$

$P(-2) = -24 + 28 - 6 + 7$

$\underline{= 5}$

Since $P(-3)$ *and* $P(-2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between –3 and –2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 2x^3 - 21x^2 - 2x + 25, \quad a = 1, \quad b = 2$

Solution

$P(1) = 2 - 21 - 2 + 25$

$\underline{= 4}$

$$P(2) = 16 - 84 - 4 + 25$$

$$\boxed{= -47}$$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 4x^4 + 7x^3 - 11x^2 + 7x - 15, \quad a = 1, \quad b = \frac{3}{2}$

Solution

$$P(1) = 4 + 7 - 11 + 7 - 15$$

$$\boxed{= -8}$$

$$P\left(\frac{3}{2}\right) = 81 + \frac{189}{8} - \frac{99}{4} + \frac{21}{2} - 15$$

$$= 66 + \frac{189 - 198 + 84}{8}$$

$$= 66 + \frac{75}{8}$$

$$\boxed{= \frac{603}{8}}$$

Since $P(1)$ *and* $P\left(\frac{3}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and $\frac{3}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = 5x^3 - 16x^2 - 20x + 64, \quad a = 3, \quad b = \frac{7}{2}$

Solution

$$P(3) = 135 - 144 - 60 + 64$$

$$\boxed{= -5}$$

$$P\left(\frac{7}{2}\right) = \frac{1715}{8} - 196 - 70 + 64$$

$$= \frac{1715}{8} - 202$$

$$\boxed{= \frac{99}{8}}$$

Since $P(3)$ *and* $P\left(\frac{7}{2}\right)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 3 and $\frac{7}{2}$.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^4-x^2-x-4, \quad a=1, \quad b=2$

Solution

$P(1)=1-1-1-4$

$\quad\quad = -5$

$P(2)=16-4-2-4$

$\quad\quad = 6$

Since $P(1)$ *and* $P(2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^3-x-8, \quad a=2, \quad b=3$

Solution

$P(2)=8-2-8$

$\quad\quad = -2$

$P(3)=27-3-8$

$\quad\quad = 16$

Since $P(2)$ *and* $P(3)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x)=x^3-x-8, \quad a=0, \quad b=1$

Solution

$P(0)=-8$

$$P(1) = 1 - 1 - 8$$

$$\underline{= -8}$$

Since $P(0)$ *and* $P(1)$ have same sign.

Therefore, ***cannot be determined***.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $P(x) = x^3 - x - 8, \quad a = 2.1, \quad b = 2.2$

Solution

$$\begin{aligned} P(2.1) &= P\left(\frac{21}{10}\right) \\ &= \frac{9261}{1000} - \frac{21}{10} - 8 \\ &= \frac{9261 - 2100 - 8000}{1000} \\ &= -\frac{839}{1,000} \end{aligned}$$

$$\begin{aligned} P(2.2) &= P\left(\frac{2.2}{10}\right) \\ &= \frac{10,648}{1000} - \frac{22}{10} - 8 \\ &= \frac{10,648 - 2,200 - 8,000}{1000} \\ &= \frac{448}{1,000} \\ &= 0.448 \end{aligned}$$

Since $P(2.1)$ *and* $P(2.2)$ have opposite signs.

Therefore, the polynomial ***has a real zero*** between 2.1 and 2.2.