Solution *Section R.2* – Integration

Exercise

Find each indefinite integral. $\displaystyle\int \frac{x+2}{\sqrt{x}}\,dx$

Solution

$$\begin{aligned}
\int \frac{x+2}{\sqrt{x}}dx &= \int \left[\frac{x}{x^{1/2}} + \frac{2}{x^{1/2}}\right]dx \\
&= \int \frac{x}{x^{1/2}}dx + \int \frac{2}{x^{1/2}}dx \\
&= \int x^{1/2}dx + 2\int x^{-1/2}dx \\
&= \frac{x^{3/2}}{3/2} + 2\frac{x^{1/2}}{1/2} + C \\
&= \frac{2}{3}x^{3/2} + 4x^{1/2} + C
\end{aligned}$$

Exercise

Find each indefinite integral $\displaystyle\int 4y^{-3}dy$

Solution

$$\begin{aligned}
\int 4y^{-3}dy &= 4\frac{y^{-2}}{-2} + C \\
&= -\frac{2}{y^2} + C
\end{aligned}$$

Exercise

Find each indefinite integral $\displaystyle\int (x^3 - 4x + 2)dx$

Solution

$$\int \left(x^3 - 4x + 2\right)dx = \frac{1}{4}x^4 - 2x^2 + 2x + C$$

Exercise

Find each indefinite integral $\int \left(\sqrt[4]{x^3}+1\right)dx$

Solution

$$\int \left(x^{3/4}+1\right)dx = \underline{\tfrac{4}{7}x^{7/4}+x+C}$$

Exercise

Find each indefinite integral $\int \sqrt{x}(x+1)dx$

Solution

$$\int x^{1/2}(x+1)dx = \int \left(x^{3/2}+x^{1/2}\right)dx$$

$$= \tfrac{2}{5}x^{5/2}+\tfrac{2}{3}x^{3/2}+C$$

Exercise

Find each indefinite integral $\int (1+3t)t^2 dt$

Solution

$$\int \left(t^2+3t^3\right)dt = \tfrac{1}{3}t^3+\tfrac{3}{4}t^4+C$$

Exercise

Find each indefinite integral $\int \frac{x^2-5}{x^2}dx$

Solution

$$\int \frac{x^2-5}{x^2}dx = \int \left(1-\frac{5}{x^2}\right)dx$$

$$= \int \left(1-5x^{-2}\right)dx$$

$$= x+5x^{-1}+C$$

$$= x+\frac{5}{x}+C$$

Exercise

Find each indefinite integral $\int(-40x+250)\,dx$

Solution

$$\int(-40x+250)\,dx = \underline{-20x^2+250x+C}$$

Exercise

Find each indefinite integral $\int\left(7-3x-3x^2\right)(2x+1)\,dx$

Solution

$$\begin{aligned}\int\left(7-3x-3x^2\right)(2x+1)\,dx &= \int\left(14x+7-6x^2-3x-6x^3-3x^2\right)dx\\ &= \int\left(-6x^3-9x^2+11x+7\right)dx\\ &= \underline{-\frac{3}{2}x^4-3x^3+\frac{11}{2}x^2+7x+C}\end{aligned}$$

Exercise

Find the integral $\int(1+\cos 3\theta)\,d\theta$

Solution

$$\int(1+\cos 3\theta)\,d\theta \underline{=\theta+\frac{1}{3}\sin 3\theta+C}$$

Exercise

Find the integral $\int 2\sec^2\theta\,d\theta$

Solution

$$\int 2\sec^2\theta\,d\theta \underline{=2\tan\theta+C}$$

Exercise

Find the integral $\int \sec 2x\tan 2x\,dx$

Solution

$$\int \sec 2x \tan 2x \, dx = \boxed{\frac{1}{2}\sec 2x + C}$$

Exercise

Find the integral $\int 2e^{2x} dx$

Solution

$$\int 2e^{2x} dx = \boxed{e^{2x} + C}$$

Exercise

Find the integral $\int \frac{12}{x} dx$

Solution

$$\int \frac{12}{x} dx = \boxed{12\ln|x| + C}$$

Exercise

Find the integral $\int \frac{dx}{\sqrt{1-x^2}}$

Solution

$$\int \frac{dx}{\sqrt{1-x^2}} = \boxed{\sin^{-1} x + C}$$

Exercise

Find the integral $\int \frac{dx}{x^2+1}$

Solution

$$\int \frac{dx}{x^2+1} = \boxed{\tan^{-1} x + C}$$

Exercise

Find the integral $\displaystyle\int \frac{1+\tan\theta}{\sec\theta}\,d\theta$

Solution

$$\int \frac{1+\tan\theta}{\sec\theta}\,d\theta = \int \left(\frac{1}{\sec\theta}+\frac{\tan\theta}{\sec\theta}\right)d\theta$$

$$= \int (\cos\theta+\sin\theta)\,d\theta$$

$$\underline{= \sin\theta - \cos\theta + C|}$$

Exercise

Find the general solution of the differential equation $y' = 2t+3$

Solution

$$dy = (2t+3)\,dt$$

$$\int dy = \int (2t+3)\,dt$$

$$\boxed{y = t^2 + 3t + C}$$

Exercise

Find the general solution of the differential equation $y' = 3t^2 + 2t + 3$

Solution

$$\int dy = \int \left(3t^2 + 2t + 3\right)dt$$

$$\boxed{y = t^3 + t^2 + 3t + C}$$

Exercise

Find the general solution of the differential equation $y' = \sin 2t + 2\cos 3t$

Solution

$$\int dy = \int (\sin 2t + 2\cos 3t)\,dt$$

$$\boxed{y(t) = -\tfrac{1}{2}\cos 2t + \tfrac{2}{3}\sin 3t + C}$$

Exercise

Find the general solution of the differential equation: $y' = x^3(3x^4+1)^2$

Solution

$$\int x^3\left(3x^4+1\right)^2 dx$$

$u = 3x^4+1 \Rightarrow du = 12x^3 dx$

$\Rightarrow \frac{1}{12}du = x^3 dx$

$$\int x^3(3x^4+1)^2 dx = \int \frac{1}{12}u^2 du$$

$$= \frac{1}{12}\frac{(3x^4+1)^3}{3}+C$$

$$= \frac{1}{36}(3x^4+1)^3+C$$

$$\boxed{y = \frac{1}{36}\left(3x^4+1\right)^3+C}$$

Exercise

Find the general solution of the differential equation: $y' = 5x\sqrt{x^2-1}$

Solution

$$\int 5x\left(x^2-1\right)^{1/2} dx = \frac{5}{2}\int\left(x^2-1\right)^{1/2} d\left(x^2-1\right) \qquad d\left(x^2-1\right) = 2xdx$$

$$= \frac{5}{3}(x^2-1)^{3/2}+C$$

Exercise

Find the general solution of the differential equation: $y' = x\sqrt{x^2+4}$

Solution

$$\int \sqrt{x^2+4}\; xdx = \frac{1}{2}\int\left(x^2+4\right)^{1/2} d\left(x^2+4\right)$$

$$= \frac{1}{3}(x^2+4)^{3/2}+C$$

Exercise

Evaluate the integrals $\int_{-2}^{2}\left(x^3-2x+3\right)dx$

Solution

$$\int_{-2}^{2}\left(x^3-2x+3\right)dx=\left[\frac{x^4}{4}-x^2+3x\right]_{-2}^{2}$$

$$=\left(\frac{(2)^4}{4}-(2)^2+3(2)\right)-\left(\frac{(-2)^4}{4}-(-2)^2+3(-2)\right)$$

$$=12$$

Exercise

Evaluate the integrals $\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx$

Solution

$$\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx=\left[\frac{x^3}{3}+\frac{2}{3}x^{3/2}\right]_{0}^{1}$$

$$=\left(\frac{(1)^3}{3}+\frac{2}{3}(1)^{3/2}\right)-0$$

$$=1$$

Exercise

Evaluate the integrals $\int_{0}^{\pi/3}4\sec u\tan u\ du$

Solution

$$\int_{0}^{\pi/3}4\sec u\tan u\ du=4\sec u\Big|_{0}^{\pi/3}$$

$$=4\left(\sec\frac{\pi}{3}-\sec 0\right)$$

$$=4(2-1)$$

$$=4$$

Exercise

Evaluate the integrals $\displaystyle\int_{\pi/4}^{3\pi/4} \csc\theta\cot\theta\, d\theta$

Solution

$$\int_{\pi/4}^{3\pi/4} \csc\theta\cot\theta\, d\theta = -\csc\theta\Big|_{\pi/4}^{3\pi/4}$$

$$= -\left(\csc\frac{3\pi}{4} - \csc\frac{\pi}{4}\right)$$

$$= -\left(\sqrt{2} - \sqrt{2}\right)$$

$$= 0$$

Exercise

Evaluate the integrals $\displaystyle\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt$

Solution

$$\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt = \int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \pi t^{-2}\right) dt$$

$$= \left[4\tan t - \pi t^{-1}\right]_{-\pi/3}^{-\pi/4}$$

$$= \left(4\tan\left(-\frac{\pi}{4}\right) - \pi\left(-\frac{4}{\pi}\right)\right) - \left(4\tan\left(-\frac{\pi}{3}\right) - \pi\left(-\frac{3}{\pi}\right)\right)$$

$$= \left(4(-1) + 4\right) - \left(4\left(-\sqrt{3}\right) + 3\right)$$

$$= -\left(-4\sqrt{3} + 3\right)$$

$$= 4\sqrt{3} - 3$$

Exercise

Evaluate the integrals $\displaystyle\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} dy$

Solution

$$\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} dy = \int_{-3}^{-1} \left(\frac{y^5}{y^3} - \frac{2y}{y^3}\right) dy$$

$$= \int_{-3}^{-1} \left(y^2 - 2y^{-2}\right) dy$$

$$= \left[\frac{1}{3}y^3 + 2y^{-1}\right]_{-3}^{-1}$$

$$= \left(\frac{1}{3}(-1)^3 + \frac{2}{-1}\right) - \left(\frac{1}{3}(-3)^3 + \frac{2}{-3}\right)$$

$$= \frac{22}{3}$$

Exercise

Evaluate the integrals $\displaystyle\int_1^8 \frac{\left(x^{1/3}+1\right)\left(2-x^{2/3}\right)}{x^{1/3}}dx$

Solution

$$\int_1^8 \frac{\left(x^{1/3}+1\right)\left(2-x^{2/3}\right)}{x^{1/3}}dx = \int_1^8 \frac{2x^{1/3}-x+2-x^{2/3}}{x^{1/3}}dx$$

$$= \int_1^8 \left(2-x^{2/3}+2x^{-1/3}-x^{1/3}\right)dx$$

$$= \left[2x - \frac{3}{5}x^{5/3} + 3x^{2/3} - \frac{3}{4}x^{4/3}\right]_1^8$$

$$= \left(2(8) - \frac{3}{5}(8)^{5/3} + 3(8)^{2/3} - \frac{3}{4}(8)^{4/3}\right) - \left(2(1) - \frac{3}{5}(1)^{5/3} + 3(1)^{2/3} - \frac{3}{4}(1)^{4/3}\right)$$

$$= \left(-\frac{16}{5}\right) - \left(\frac{73}{20}\right)$$

$$= -\frac{137}{20}$$

Exercise

Evaluate: $\displaystyle\int_0^1 (2t+3)^3 dt$

Solution

$$\int_0^1 (2t+3)^3 dt = \frac{1}{2}\int_0^1 (2t+3)^3 d(2t+3) \qquad d(2t+3) = 2dt$$

$$= \frac{1}{8}(2t+3)^4 \Big|_0^1$$

$$= \frac{1}{8}\left[5^4 - 3^4\right]$$

$$= 68$$

Exercise

Evaluate the integral $\int_{-1}^{1} r\sqrt{1-r^2}\ dr$

Solution

$$\int_{-1}^{1} r\sqrt{1-r^2}\ dr = -\frac{1}{2}\int_{-1}^{1}\left(1-r^2\right)^{1/2}\ d\left(1-r^2\right)$$

$$= -\frac{1}{3}\left[\left(1-r^2\right)^{3/2}\right]_{-1}^{1}$$

$$= -\frac{1}{3}[0-0]$$

$$\underline{= 0}$$