Solution *Section* 2.1 – Functions and Graphs

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(1,\ 2),\ (3,\ 4),\ (5,\ 6),\ (5,\ 8)\}$

Solution

Not a function

Domain: {1, 3, 5}

Range: {2, 4, 6, 8}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(1,\ 2),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5)\}$

Solution

It is a Function

Domain: {1, 3, 6, 8}

Range: {2, 4, 5}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(9,\ -5),\ (9,\ 5),\ (2,\ 4)\}$

Solution

It is ***not*** a function

Domain = {2, 9}

Range = {–5, 5, 4}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$\{(-2,\ 5),\ (5,\ 7),\ (0,\ 1),\ (4,\ -2)\}$

Solution

It is a function

Domain = {–2, 0, 4, 5}

Range = {–2, 1, 5, 7}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(-5,\ 3),\ (0,\ 3),\ (6,\ 3)\}$$

Solution

It is a function

Domain = {–5, 0, 6}

Range = {3}

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(1,\ 2),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5),\ (1,\ 5)\}$$

Solution

It is ***not*** a function

Domain = $\{1,\ 3,\ 6,\ 8\}$

Range = $\{2,\ 4,\ 5\}$

Exercise

Determine whether each relation is a function and *find the domain and the range*.

$$\{(-1,\ 3),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5),\ (1,\ 5)\}$$

Solution

It is a function

Domain = $\{-1,\ 1,\ 3,\ 6,\ 8\}$

Range = $\{3,\ 4,\ 5\}$

Exercise

Find the domain and the range of the relation:

$$\{(5,\ 12.8),\ (10,\ 16.2),\ (15,\ 18.9),\ (20,\ 20.7),\ (25,\ 21.81)\}$$

Solution

Domain: {5, 10, 15, 20, 25}

Range: {12.8, 16.2, 18.9, 20.7, 21.81}

Exercise

Let $f(x) = -3x + 4$, find $f(0)$

Solution

$$f(0) = -3(0) + 4$$

$$\underline{= 4}$$

Exercise

Let $g(x) = -x^2 + 4x - 1$, find $g(-x)$

Solution

$$g(-x) = -(-x)^2 + 4(-x) - 1$$

$$\underline{= -x^2 - 4x - 1}$$

Exercise

Let $f(x) = -3x + 4$, find $f(a+4)$

Solution

$$f(a+4) = -3(a+4) + 4$$

$$= -3a - 12 + 4$$

$$\underline{= -3a - 8}$$

Exercise

Given: $f(x) = 2|x| + 3x$, find $f(2-h)$.

Solution

$$f(2-h) = 2|2-h| + 3(2-h)$$

$$\underline{= 2|2-h| + 6 - 3h}$$

Exercise

Given: $g(x) = \dfrac{x-4}{x+3}$, find $g(x+h)$

Solution

$$g(x+h) \underline{= \frac{x+h-4}{x+h+3}}$$

Exercise

Given: $g(x)=\frac{x}{\sqrt{1-x^2}}$, find $g(0)$ *and* $g(-1)$

Solution

$$g(0)=\frac{0}{\sqrt{1-0^2}}$$

$$=0$$

$$g(-1)=\frac{-1}{\sqrt{1-(-1)^2}}$$

$$=\frac{-1}{0} \quad \textit{undefined}$$

Exercise

Given that $g(x)=2x^2+2x+3$. Find $g(p+3)$

Solution

$$g(p+3)=2(p+3)^2+2(p+3)+3 \qquad (a+b)^2=a^2+2ab+b^2$$

$$=2\left(p^2+2(p)(3)+3^2\right)+2p+6+3$$

$$=2\left(p^2+6p+9\right)+2p+9$$

$$=2p^2+12p+18+2p+9$$

$$=2p^2+14p+27$$

Exercise

If $f(x)=x^2-2x+7$, evaluate each of the following: $f(-5)$, $f(x+4)$, $f(-x)$

Solution

$$f(-5)=(-5)^2-2(-5)+7$$

$$= 25 + 10 + 7$$

$$= 42$$

$$f(x+4)=(x+4)^2-2(x+4)+7 \qquad (a+b)^2=a^2+2ab+b^2$$

$$=x^2+2(4)x+4^2-2x-8+7$$

$$=x^2+8x+16-2x-1$$

$$=x^2+6x+15$$

$$=x^2+2x+7$$

Exercise

Find $g(0),\ g(-4),\ g(7),\ and\ g\left(\frac{3}{2}\right)$ *for* $g(x)=\dfrac{x}{\sqrt{16-x^2}}$

Solution

$$g(0)=\frac{0}{\sqrt{16-0^2}}$$

$$=\frac{0}{\sqrt{16}}$$

$$\boxed{=0}$$

$$g(-4)=\frac{-4}{\sqrt{16-(-4)^2}}$$

$$=\frac{-4}{\sqrt{16-16}}$$

$$\boxed{=\frac{-4}{0}\quad \nexists}$$

$$g(7)=\frac{7}{\sqrt{16-7^2}}$$

$$=\frac{7}{\sqrt{16-49}}$$

$$\boxed{=\frac{7}{\sqrt{-33}}\quad \textit{doesn't exist in real number}}$$

$$g\left(\frac{3}{2}\right)=\frac{\frac{3}{2}}{\sqrt{16-\left(\frac{3}{2}\right)^2}}$$

$$=\frac{\frac{3}{2}}{\sqrt{16-\frac{9}{4}}}$$

$$=\frac{\frac{3}{2}}{\sqrt{\frac{4(16)-9}{4}}}$$

$$=\frac{\frac{3}{2}}{\frac{\sqrt{55}}{2}}$$

$$=\frac{3}{\sqrt{55}}$$

$$\boxed{=\frac{3\sqrt{55}}{55}}$$

Exercise

$f(x) = 3x - 4$

a) $f(0)$ *b)* $f\left(\frac{5}{3}\right)$ *c)* $f(-2a)$ *d)* $f(x+h)$

Solution

a) $f(0) \underline{= -4}$

b) $f\left(\frac{5}{3}\right) = 3\frac{5}{3} - 4$

$= 5 - 1$

$\underline{= 1}$

c) $f(-2a) = 3(-2a) - 4$

$\underline{= -6a - 4}$

d) $f(x+h) = 3(x+h) - 4$

$\underline{= 3x + 3h - 4}$

Exercise

$f(x) = 3x^2 + 3x - 1$

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(2)$ *d)* $f(h)$

Solution

a) $f(0) \underline{= -1}$

b) $f(x+h) = 3(x+h)^2 + 3(x+h) - 1$

$= 3\left(x^2 + 2hx + h^2\right) + 3x + 3h - 1$

$\underline{= 3x^2 + 6hx + 3h^2 + 3x + 3h - 1}$

c) $f(2) = 12 + 6 - 1$

$\underline{= 17}$

d) $\underline{f(h) = 3h^2 + 3h - 1}$

Exercise

$f(x) = 2x^2 - 4$

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(2)$ *d)* $f(2) - f(-3)$

Solution

a) $f(0) \underline{= -4}$

b)
$$f(x+h) = 2(x+h)^2 - 4$$
$$= 2\left(x^2 + 2hx + h^2\right) - 4$$
$$\underline{= 2x^2 + 4hx + 2h^2 - 4}$$

c)
$$f(2) = 8 - 4$$
$$\underline{= 4}$$

d)
$$f(2) - f(-3) = 8 - 4 - (18 - 4)$$
$$= 4 - 14$$
$$\underline{= -10}$$

Exercise

$f(x) = 3x^2 + 4x - 2$

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(3)$ *d)* $f(-5)$

Solution

a) $f(0) \underline{= -2}$

b)
$$f(x+h) = 3(x+h)^2 + 4(x+h) - 2$$
$$= 3\left(x^2 + 2hx + h^2\right) + 4x + 4h - 2$$
$$\underline{= 3x^2 + 6hx + 3h^2 + 4x + 4h - 2}$$

c)
$$f(3) = 27 + 12 - 2$$
$$\underline{= 37}$$

d)
$$f(-5) = 75 - 20 - 2$$
$$\underline{= 53}$$

Exercise

$f(x) = -x^3 - x^2 - x + 10$

a) $f(0)$ *b)* $f(-1)$ *c)* $f(2)$ *d)* $f(1) - f(-2)$

Solution

a) $f(0)\ \underline{=10}$

b) $f(-1) = 1 - 1 + 1 + 10$

$\underline{=11}$

c) $f(2) = -8 - 4 - 2 + 10$

$\underline{=-4}$

d) $f(1) - f(-2) = -1 - 1 - 1 + 10 - (8 - 4 + 2 + 10)$

$= 7 - 16$

$\underline{=-9}$

Exercise

For $\frac{1}{10}x^{10} - \frac{1}{2}x^6 + \frac{2}{3}x^3 - 10x$, determine

a) $f(2) - f(-2)$ *b)* $f(1) - f(-1)$ *c)* $f(2) - f(0)$

Solution

a) $f(2) - f(-2) = \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2}{3}2^3 - 20 - \left(\frac{2^{10}}{10} - \frac{2^6}{2} - \frac{2}{3}2^3 + 20\right)$

$= \frac{2^{10}}{10} - \frac{2^6}{2} + \frac{2^4}{3} - 20 - \frac{2^{10}}{10} + \frac{2^6}{2} + \frac{2^4}{3} - 20$

$= \frac{2^5}{3} - 40$

$= \frac{32}{3} - 40$

$\underline{= -\frac{88}{3}}$

b) $f(1) - f(-1) = \frac{1}{10} - \frac{1}{2} + \frac{2}{3} - 10 - \left(\frac{1}{10} - \frac{1}{2} - \frac{2}{3} + 10\right)$

$= \frac{1}{10} - \frac{1}{2} + \frac{2}{3} - 10 - \frac{1}{10} + \frac{1}{2} + \frac{2}{3} - 10$

$= \frac{4}{3} - 20$

$\underline{= -\frac{56}{3}}$

c) $f(2)-f(0)=\frac{2^{10}}{10}-\frac{2^6}{2}+\frac{2}{3}2^3-20-(0)$

$$=\frac{2^9}{5}-2^5+\frac{2^4}{3}-5\left(2^2\right)$$

$$=2^2\left(\frac{128}{5}-8+\frac{4}{3}-5\right)$$

$$=4\left(\frac{384+20-195}{15}\right)$$

$$=4\left(\frac{209}{15}\right)$$

$$=\frac{836}{15}$$

Exercise

For $f(x)=3x^4+x^2-4$, determine

a) $f(2)-f(-2)$ *b)* $f(1)-f(-1)$ *c)* $f(2)-f(0)$

Solution

a) $f(2)-f(-2)=3(16)+4-4-(3(16)+4-4)$

$$=48+4-4-48-4+4$$

$$=0$$

b) $f(1)-f(-1)=3+1-4-(3+1-4)$

$$=0$$

c) $f(2)-f(0)=3(16)+4-4-(0)$

$$=48$$

Exercise

For $f(x)=-\frac{2}{3}x^3+4x$, determine

a) $f(2)-f(-2)$ *b)* $f(1)-f(-1)$ *c)* $f(2)-f(0)$

Solution

a) $f(2)-f(-2)=-\frac{2}{3}\left(2^3\right)+8-\left(-\frac{2}{3}(-2)^3-8\right)$

$$=-\frac{16}{3}+8-\frac{16}{3}+8$$

$$=2\left(-\frac{16}{3}+8\right)$$

$$=16\left(-\frac{1}{3}+1\right)$$

$$=16\left(\frac{2}{3}\right)$$

$$=\frac{32}{3}$$

b) $f(1)-f(-1)=-\frac{2}{3}+4-\left(\frac{2}{3}-4\right)$

$$=2\left(-\frac{2}{3}+4\right)$$

$$=\frac{20}{3}$$

c) $f(2)-f(0)=-\frac{16}{3}+8-(0)$

$$=\frac{8}{3}$$

Exercise

$f(x)=\frac{2x-3}{x-4}$

a) $f(0)$ *b)* $f(3)$ *c)* $f(x+h)$ *d)* $f(-4)$

Solution

a) $f(0)=\frac{3}{4}$

b) $f(3)=\frac{6-3}{3-4}$

$$=-3$$

c) $f(x+h)=\frac{2(x+h)-3}{x+h-4}$

$$=\frac{2x+2h-3}{x+h-4}$$

d) $f(-4)=\frac{-8-3}{-4-4}$

$$=\frac{11}{8}$$

Exercise

$$f(x)=\frac{3x-1}{x-5}$$

a) $f(0)$ *b)* $f(3)$ *c)* $f(x+h)$ *d)* $f(-5)$

Solution

a) $f(0)=\frac{1}{5}$

b) $f(3)=\frac{9-1}{3-5}$

$=-4$

c) $f(x+h)=\frac{3(x+h)-1}{x+h-5}$

$=\frac{3x+3h-1}{x+h-5}$

d) $f(-5)=\frac{-12-1}{-4-5}$

$=\frac{13}{9}$

Exercise

$$f(x)=\begin{cases} 2+x & \text{if } x<-4 \\ -x & \text{if } -4\le x\le 2 \\ 3x & \text{if } x>2 \end{cases}$$

Find: $f(-5),\ f(-1),\ f(0),\ \text{and } f(3)$

Solution

a) $f(-5)=2-5=-3$

b) $f(-1)=-(-1)=1$

c) $f(0)=-0=0$

d) $f(3)=3(3)=9$

Exercise

$$f(x)=\begin{cases} -2x & \text{if } x<-3 \\ 3x-1 & \text{if } -3\le x\le 2 \\ -4x & \text{if } x>2 \end{cases}$$

Find: $f(-5),\ f(-1),\ f(0),\ \text{and } f(3)$

Solution

a) $f(-5)=-2(-5)=10$

b) $f(-1) = 3(-1) - 1 = -4$

c) $f(0) = 3(0) - 1 = -1$

d) $f(3) = -4(3) = -12$

Exercise

$$f(x) = \begin{cases} x^3 + 3 & \text{if } \ -2 \le x \le 0 \\ x + 3 & \text{if } \ 0 < x < 1 \\ 4 + x - x^2 & \text{if } \ 1 \le x \le 3 \end{cases} \quad \text{Find: } f(-5),\ f(-1),\ f(0),\ and\ f(3)$$

Solution

a) $f(-5) = \ doesn't\ exist$

b) $f(-1) = (-1)^3 + 3$

$\underline{= 2}$

c) $f(0) = (0)^3 + 3$

$\underline{= 3}$

d) $f(3) = 4 + (3) - (3)^2$

$\underline{= -2}$

Exercise

$$h(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } \ x \ne 3 \\ 6 & \text{if } \ x = 3 \end{cases} \qquad \text{Find: } h(5),\ h(0),\ and\ h(3)$$

Solution

a) $h(5) = \dfrac{5^2 - 9}{5 - 3}$

$\underline{= 8}$

b) $h(0) = \dfrac{0^2 - 9}{0 - 3}$

$\underline{= 3}$

c) $\underline{h(3) = 6}$

Exercise

$$f(x)=\begin{cases} 3x+5 & if \quad x<0 \\ 4x+7 & if \quad x\ge 0 \end{cases} \qquad \text{Find}$$

a) $f(0)$ *b)* $f(-2)$ *c)* $f(1)$ *d)* $f(3)+f(-3)$ *e)* Graph $f(x)$

Solution

a) $f(0)=4(0)+7$

$\qquad = 7$

b) $f(-2)=3(-2)+5$

$\qquad = -1$

c) $f(1)=4(1)+7$

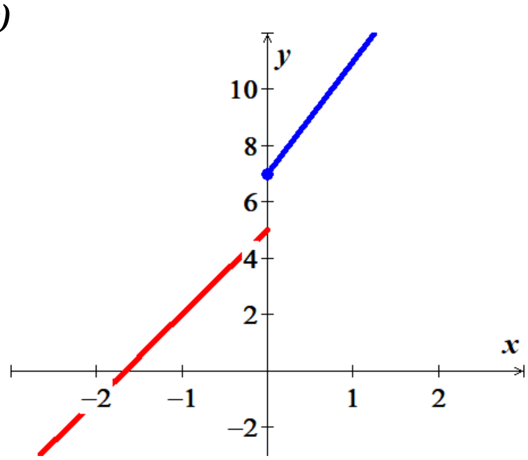
$\qquad = 11$

d) $f(3)+f(-3)=4(3)+7+3(-3)+5$

$\qquad =12+7-9+5$

$\qquad = 15$

e)



Exercise

$$f(x)=\begin{cases} 6x-1 & if \quad x<0 \\ 7x+3 & if \quad x\ge 0 \end{cases} \qquad \text{Find}$$

a) $f(0)$ *b)* $f(-1)$ *c)* $f(4)$ *d)* $f(2)+f(-2)$ *e)* Graph $f(x)$

Solution

a) $f(0)=7(0)+3$

$\qquad = 3$

b) $f(-2)=6(-1)-1$

$\boxed{= -7}$

c) $f(4) = 7(4) + 3$

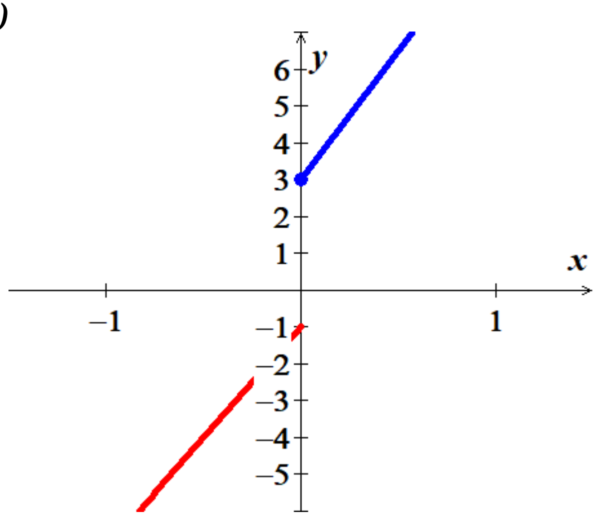
$\boxed{= 31}$

d) $f(2) + f(-2) = 7(2) + 3 + 6(-2) - 1$

$= 14 + 3 - 12 - 1$

$\boxed{= 4}$

e)



Exercise

$$f(x) = \begin{cases} 2x+1 & if \quad x \le 1 \\ 3x-2 & if \quad x > 1 \end{cases}$$ Find

a) $f(0)$ *b)* $f(2)$ *c)* $f(-2)$ *d)* $f(1) + f(-1)$ *e)* Graph $f(x)$

Solution

a) $f(0) = 2(0) + 1$

$\boxed{= 1}$

b) $f(2) = 3(2) - 2$

$\boxed{= 4}$

c) $f(-2) = 2(-2) + 1$

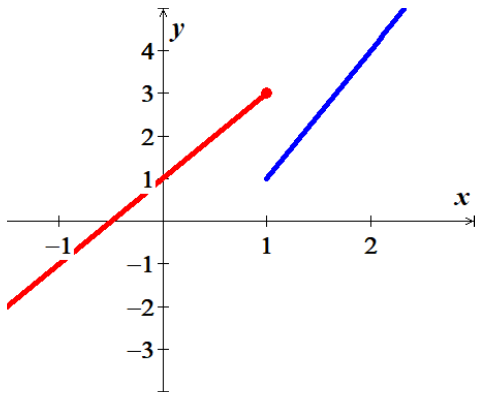
$\boxed{= -3}$

d) $f(1) + f(-1) = 2(1) + 1 + 2(-1) + 1$

$= 2 + 1 - 2 + 1$

$\boxed{= 2}$

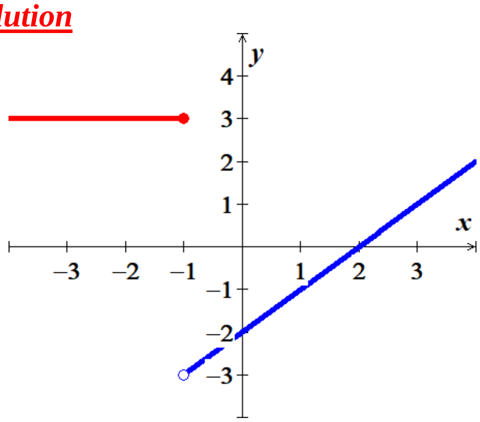
e)



Exercise

Graph the piecewise function defined by $f(x)=\begin{cases} 3 & if \quad x\le -1 \\ x-2 & if \quad x>-1 \end{cases}$

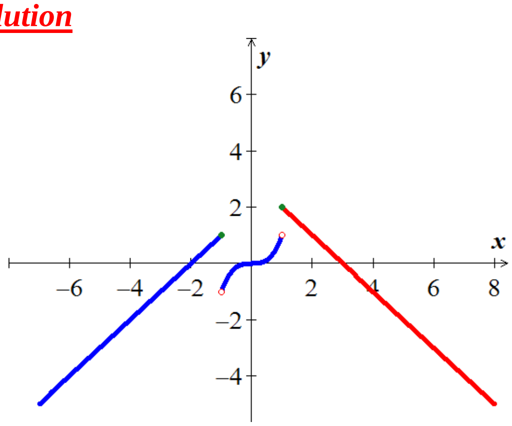
Solution



Exercise

Sketch the graph $f(x)=\begin{cases} x+2 & if & x\le -1 \\ x^3 & if & -1<x<1 \\ -x+3 & if & x\ge 1 \end{cases}$

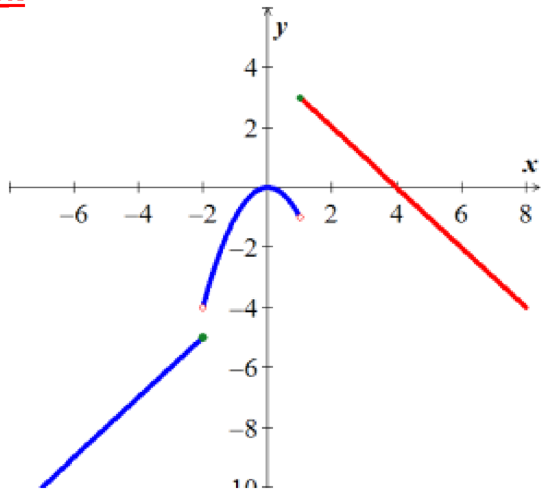
Solution



Exercise

Sketch the graph $f(x)=\begin{cases} x-3 & if \quad x \le -2 \\ -x^2 & if \quad -2 < x < 1 \\ -x+4 & if \quad x \ge 1 \end{cases}$

Solution



Exercise

Determine any ***relative maximum*** or ***minimum*** of the function, determine the intervals on which the function ***increasing*** or ***decreasing***, and then find the ***domain*** and the ***range***.

$f(x)=x^2-2x+3$

Solution

Relative Maximum: *None*

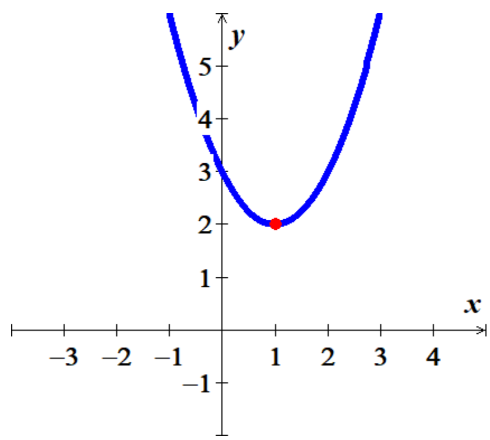
Minimum Point: $(1,\ 2)$

Increasing: $(1,\ \infty)$

Decreasing: $(-\infty,\ 1)$

Domain: $\mathbb{R}$

Range: $[2,\ \infty)$



Exercise

Determine any ***relative maximum*** or ***minimum*** of the function, determine the intervals on which the function ***increasing*** or ***decreasing***, and then find the ***domain*** and the ***range***.

$$f(x) = -x^2 - 2x + 3$$

Solution

Maximum Point: $(-1,\ 4)$

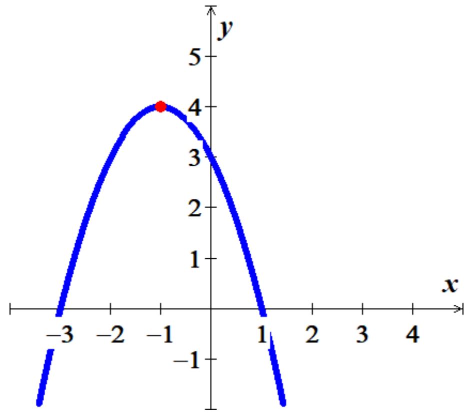
Relative Minimum: *None*

Increasing: $(-\infty,\ -1)$

Decreasing: $(-1,\ \infty)$

Domain: $\mathbb{R}$

Range: $(-\infty,\ 4]$



Exercise

Determine any ***relative maximum*** or ***minimum*** of the function, determine the intervals on which the function ***increasing*** or ***decreasing***, and then find the ***domain*** and the ***range***.

$$f(x) = -x^3 + 3x^2$$

Solution

Relative Maximum: $(2,\ 4)$

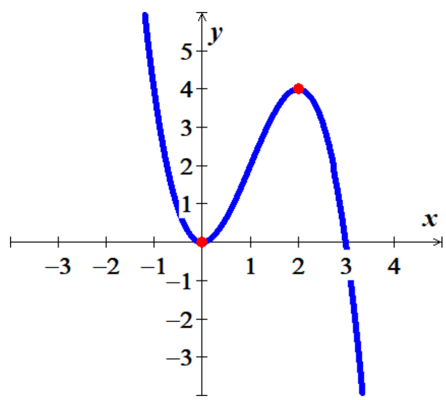
Relative Minimum: $(0,\ 0)$

Increasing: $(0,\ 2)$

Decreasing: $(-\infty,\ 0)\quad (2,\ \infty)$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$



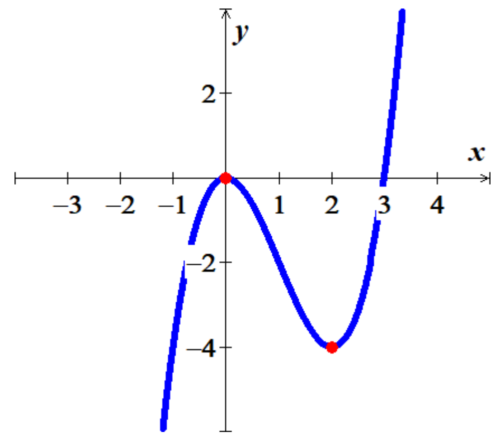
Exercise

Determine any ***relative maximum*** or ***minimum*** of the function, determine the intervals on which the function ***increasing*** or ***decreasing***, and then find the ***domain*** and the ***range***.

$$f(x) = x^3 - 3x^2$$

Solution

Relative Maximum:	$(0,\ 0)$
Relative Minimum:	$(2,\ -4)$
Increasing:	$(-\infty,\ 0)\ (2,\ \infty)$
Decreasing:	$(0,\ 2)$
Domain:	$\mathbb{R}$
Range:	$\mathbb{R}$



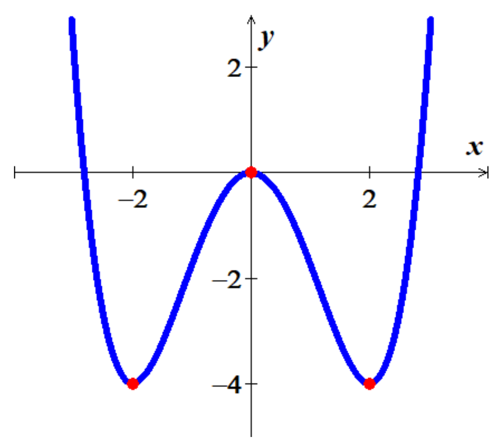
Exercise

Determine any ***relative maximum*** or ***minimum*** of the function, determine the intervals on which the function ***increasing*** or ***decreasing***, and then find the ***domain*** and the ***range***.

$$f(x) = \frac{1}{4}x^4 - 2x^2$$

Solution

Relative Maximum:	$(0,\ 0)$
Minimum Points:	$(-2,\ -4)\ \&\ (2,\ -4)$
Increasing:	$(-2,\ 0)\cup(2,\ \infty)$
Decreasing:	$(-\infty,\ -2)\cup(0,\ 2)$
Domain:	$\mathbb{R}$
Range:	$[-4,\ \infty)$



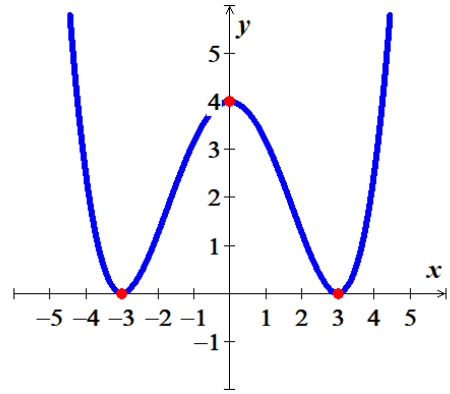
Exercise

Determine any ***relative maximum*** or ***minimum*** of the function, determine the intervals on which the function ***increasing*** or ***decreasing***, and then find the ***domain*** and the ***range***.

$$f(x) = \frac{4}{81}x^4 - \frac{8}{9}x^2 + 4$$

Solution

Relative Maximum:	$(0,\ 4)$
Minimum Points:	$(-3,\ 0)$ & $(3,\ 0)$
Increasing:	$(-3,\ 0)\cup(3,\ \infty)$
Decreasing:	$(-\infty,\ -3)\cup(0,\ 3)$
Domain:	$\mathbb{R}$
Range:	$[0,\ \infty)$



Exercise

The elevation H, in *meters*, above sea level at which the boiling point of water is in t degrees Celsius is given by the function

$$H(t)=1000(100-t)+580(100-t)^2$$

At what elevation is the boiling point 99.5°.

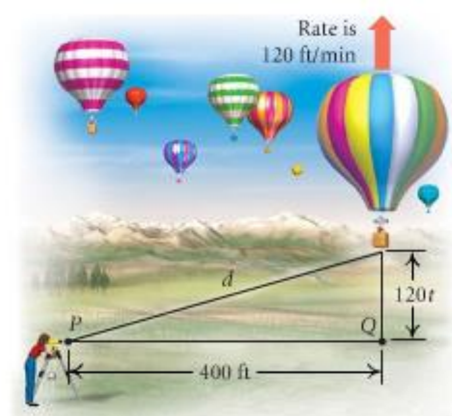
Solution

$$H(99.5)=1000(100-99.5)+580(100-99.5)^2$$
$$\boxed{=645\ m}$$

Exercise

A hot-air balloon rises straight up from the ground at a rate of 120 ft./min. The balloon is tracked from a rangefinder on the ground at point P, which is 400 ft. from the release point Q of the balloon. Let $\boldsymbol{d}$ = the distance from the balloon to the rangefinder and $\boldsymbol{t}$ – the time, in minutes, since the balloon was released. Express $\boldsymbol{d}$ as a function of $\boldsymbol{t}$.

Solution

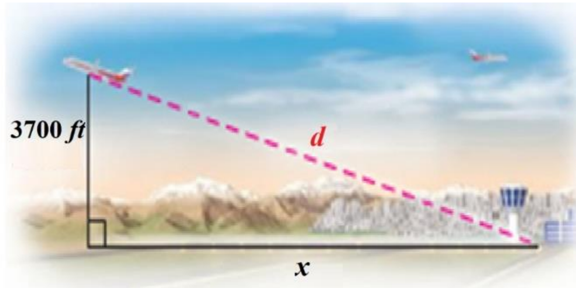


$$d^2=(120t)^2+400^2$$
$$d=\sqrt{14400t^2+160000}$$
$$d=\sqrt{1600\left(9t^2+100\right)}$$
$$\boxed{d(t)=40\sqrt{9t^2+100}}$$

Exercise

An airplane is flying at an altitude of 3700 *feet*. The slanted distance directly to the airport is d *feet*. Express the horizontal distance x as a function of d.

Solution



$$d^2 = (3,700)^2 + x^2$$

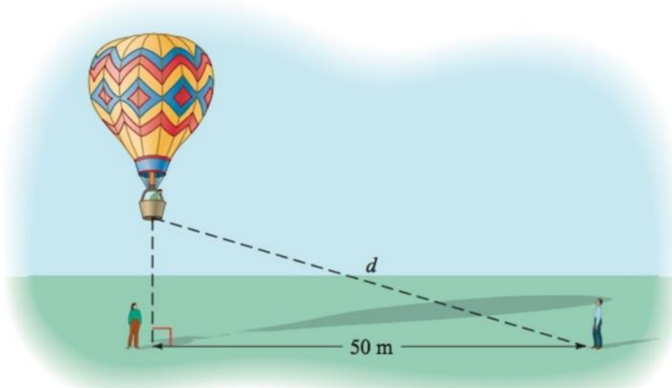
$$h^2 = d^2 - (3700)^2$$

$$\boxed{h(t) = \sqrt{d^2 - (3,700)^2}}$$

Exercise

For the first minute of flight, a hot air balloon rises vertically at a rate of 3 *m/sec*. If t is the time in *seconds* that the balloon has been airborne, write the distance d between the balloon and a point on the ground 50 *meters* from the point to lift off as a function of t.

Solution



$$\boxed{h = 3t} \qquad v = \frac{h}{t}$$

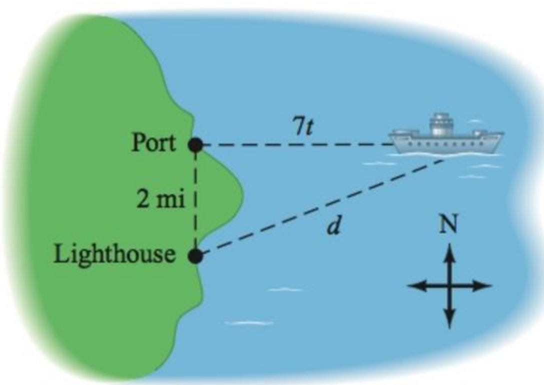
$$d^2 = h^2 + 50^2$$

$$\boxed{d(t) = \sqrt{9t^2 + 2,500}}$$

Exercise

A light house is 2 *miles* south of a port. A ship leaves port and sails east at a rate of 7 *miles* per *hour*. Express the distance d between the ship and the lighthouse as a function of time, given that the ship has been sailing for t hours.

Solution

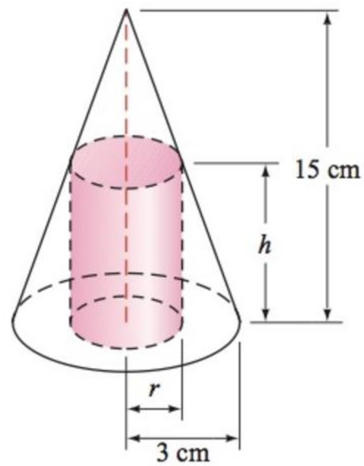


$$d^2 = 4^2 + (7t)^2$$

$$\boxed{d(t) = \sqrt{16 + 49t^2}}$$

Exercise

A cone has an altitude of 15 *cm* and a radius of 3 *cm*. A right circular cylinder of radius *r* and height *h* is inscribed in the cone. Use similar triangles to write *h* as a function of *r*.



Solution

$$\frac{15-h}{15}=\frac{r}{3}$$

$$15-h=5r$$

$$\boxed{h(r)=15-5r}$$

Exercise

Water is flowing into a conical drinking cup with an altitude of 4 *inches* am a radius of 2 *inches*.

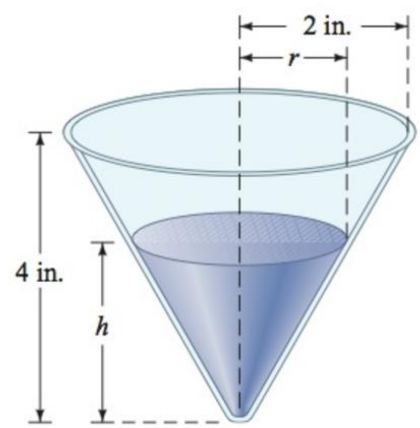
a) Write the radius *r* of the surface of the water as a function of its depth *h*.

b) Write the volume *V* of the water as a function of its depth *h*.

Solution

a) $\frac{h}{4}=\frac{r}{2}$

$$\boxed{r(h)=\frac{1}{2}h}$$



b) $Area=\pi r^2$

$$V=\frac{1}{3}\pi r^2 h$$

$$=\frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$$

$$\boxed{=\frac{1}{12}\pi h^3}$$

Exercise

A water tank has the shape of a right circular cone with height 16 *feet* and radius 8 *feet*. Water is running into the tank so that the radius *r* (in *feet*) of the surface of the water is given by $r=1.5t$, where *t* is the time (in *minutes*) that the water has been running.

a) The area *A* of the surface of the water is $A=\pi r^2$. Find $A(t)$ and use it to determine the area of the surface of the water when $t=2$ *minutes*.

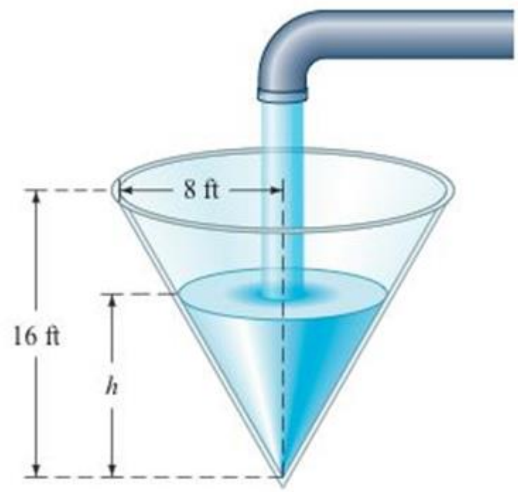
b) The volume *V* of the water is given by $V=\frac{1}{3}\pi r^2 h$. Find $V(t)$ and use it to determine the volume of the water when $t=3$ *minutes*

Solution

c) $Area = \pi r^2$

$$A(t) = \pi\left(\frac{3}{2}t\right)^2$$

$$\boxed{= \frac{9\pi}{4}t^2}$$



d) $\frac{h}{16} = \frac{r}{8}$

$$\boxed{h = 2r}$$

$$V(t) = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2 (2r)$$

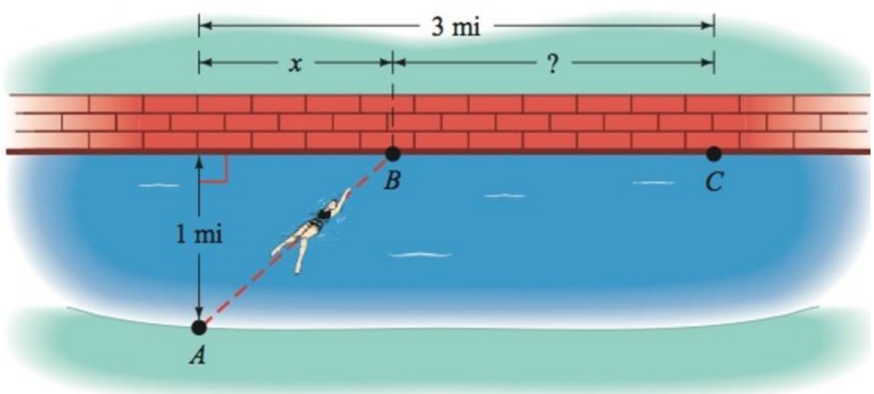
$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi\left(\frac{3}{2}t\right)^3$$

$$\boxed{= \frac{9}{4}\pi t^3}$$

Exercise

An athlete swims from point ***A*** to point ***B*** at a rate of 2 *miles* per *hour* and runs from point ***B*** to point ***C*** at a rate of 8 *miles* per *hour*. Use the dimensions in the figure to write the time t required to reach point ***C*** as a function of x.



Solution

$Swimming\ distance = \sqrt{x^2+1}$

$$t_{swim} = \frac{\sqrt{x^2+1}}{2} \qquad t = \frac{d}{v}$$

$Running\ distance = 3 - x$

$$t_{run} = \frac{3-x}{8} \qquad\qquad t = \frac{d}{v}$$

$$\boxed{t_{total} = \frac{\sqrt{x^2+1}}{2} + \frac{3-x}{8}}$$

Exercise

A device used in golf to estimate the distance d, in *yards*, to a hole measures the size $\boldsymbol{s}$, in *inches*, that the 7-*foot* pin appears to be in a viewfinder. Express the distance d as a function of $\boldsymbol{s}$.

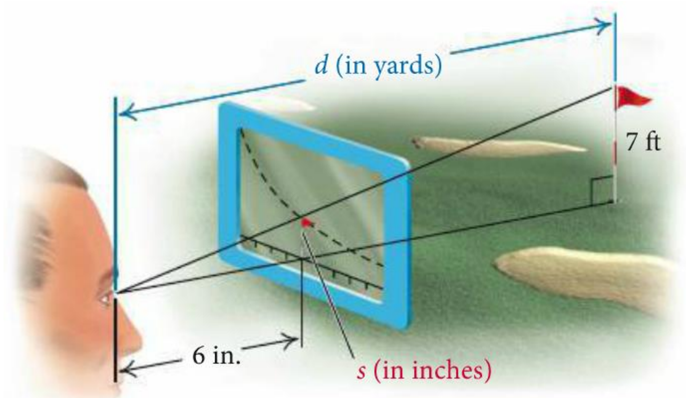
Solution

$$\frac{d}{6} = \frac{7}{s}\frac{ft}{in}$$

$$d = \frac{7}{s}\frac{ft}{in}6in$$

$$d = \frac{42}{s}\,ft\,\frac{1yd}{3\,ft}$$

$$\boxed{d(s) = \frac{14}{s}}$$



Exercise

A *rhombus* is inscribed in a rectangle that is w *meters* wide with a perimeter of 40 m. Each vertex of the rhombus is a midpoint of a side of the rectangle. Express the area of the *rhombus* as a function of the rectangle's width.

Solution

The area of the rhombus = $\frac{1}{2}$ area of the rectangle, since each vertex of the rhombus is a midpoint of a side of the rectangle.

$$Perimter: \quad 2l + 2w = 40 \qquad \textit{Divide both sides by } 2$$

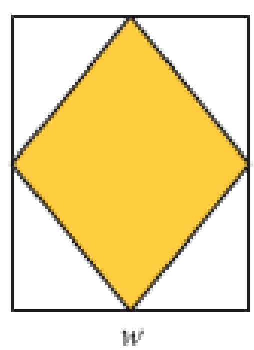
$$l + w = 20$$

$$l = 20 - w$$

$$Area\ of\ the\ rectangle = lw = (20-w)w$$

$$Area\ of\ the\ rhombus = \frac{1}{2}\left(20w - w^2\right)$$

$$\boxed{= -\frac{1}{2}w^2 + 10w}$$



Exercise

The surface area S of a right circular cylinder is given by the formula $S = 2\pi rh + 2\pi r^2$. if the height is twice the radius, find each of the following.

a) A function $S(r)$ for the surface area as a function of r.

b) A function $S(h)$ for the surface area as a function of h.

Solution

Given: $h = 2r$

a) $S = 2\pi rh + 2\pi r^2$

$$S(r) = 2\pi r(2r) + 2\pi r^2$$

$$= 4\pi r^2 + 2\pi r^2$$

$$\boxed{= 6\pi r^2}$$

b) $r = \frac{1}{2}h$

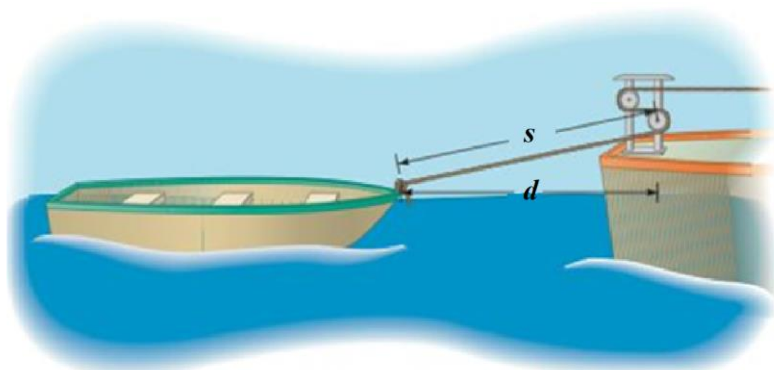
$$S(h) = 2\pi\left(\frac{1}{2}h\right)h + 2\pi\left(\frac{1}{2}h\right)^2$$

$$= \pi h^2 + \frac{1}{2}\pi h^2$$

$$\boxed{= \frac{3}{2}\pi h^2}$$

Exercise

A boat is towed by a rope that runs through a pulley that is 4 *feet* above the point where the rope is tied to the boat. The length (in *feet*) of the rope from the boat to the pulley is given by $s = 48 - t$, where t is the time in *seconds* that the boat has been in tow. The horizontal distance from the pulley to the boat is d.



a) Find $d(t)$

b) Evaluate $s(35)$ and $d(35)$

Solution

a) $s^2 = d^2 + 4^2$

$$d^2 = (48-t)^2 - 16$$

$$d(t) = \sqrt{2,304 - 96t + t^2 - 16}$$

$$\boxed{= \sqrt{t^2 - 96t + 2,288}}$$

b) $s(35) = 48 - 35$

$$\boxed{= 13 \ \ feet}$$

$$d(35) = \sqrt{(48-35)^2 - 16}$$

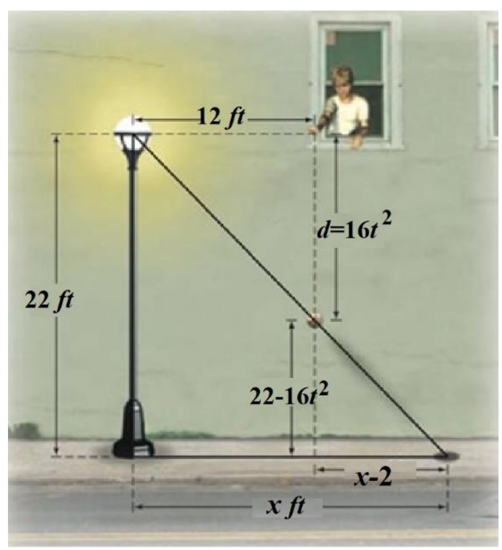
$$= \sqrt{13^2 - 16}$$

$$\boxed{= \sqrt{153} \ \ feet}$$

Exercise

The light from a lamppost casts a shadow from a ball that was dropped from a height of 22 *feet* above the ground. The distance *d*, in *feet*, the ball has dropped *t seconds* after it is released is given by $d(t) = 16t^2$. Find the distance *x*, in *feet*, of the shadow from the base of the lamppost as a function of time *t*.

Solution



$$\frac{22-16t^2}{22} = \frac{x-12}{x}$$

$$\left(22-16t^2\right)x = 22(x-12)$$

$$\left(22-16t^2\right)x = 22x - 264$$

$$\left(22-16t^2 - 22\right)x = -264$$

$$-16t^2 x = -264$$

$$\boxed{x(t) = \frac{33}{2t^2}}$$

Exercise

A right circular cylinder of height *h* and a radius *r* is inscribed in a right circular cone with a height of 10 *feet* and a base with radius 6 *feet*.

a) Express the height *h* of the cylinder as a function of *r*.

b) Express the volume *V* of the cylinder as a function of *r*.

c) Express the volume *V* of the cylinder as a function of *h*.

Solution

a) $\dfrac{h}{10} = \dfrac{6-r}{6}$

$\boxed{h(r) = \dfrac{5}{3}(6-r)}$

b) $V = \pi r^2 h$

$V(r) = \dfrac{5}{3}\pi r^2 (6-r)$

$\boxed{= \dfrac{5}{3}\pi\left(6r^2 - r^3\right)}$

c) $\dfrac{3}{5}h = 6 - r$

$r = 6 - \dfrac{3}{5}h$

$V = \pi r^2 h$

$V(h) = \pi\left(\dfrac{30-3h}{5}\right)^2 h$

$\boxed{= \dfrac{1}{25}\pi h(30-3h)^2}$