- abs extrem 2
- Incr/decr 2 } C.N
- Extreme 2
- Concave 2 } profile $\underline{\underline{x}}$
- Hôpital (4)
- 2 applications

Exam 3 – Review

#1-a) $f(x) = \sin 2x + 3 \quad [-\pi, \pi]$

$f'(x) = 2\cos 2x = 0$

$2x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$

$\boxed{CN: \ x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}}$

x	$f(x)$
$-\pi$	3
$-\frac{3\pi}{4}$	4
$-\pi/4$	2
$\pi/4$	4
$3\pi/4$	2
π	3

abs Min : $(-\frac{\pi}{4}, 2)$ $(\frac{3\pi}{4}, 2)$

abs Max : $(-\frac{3\pi}{4}, 4)$ $(\frac{\pi}{4}, 4)$

1-d $f(x) = 2x\ln x + 10 \quad (0, 4)$

$f'(x) = 2(\ln x + 1) = 0$

$\ln x = -1$

$\boxed{CN: \ x = \frac{1}{e}}$

$\ln e = 1 \quad \ln\frac{1}{e} = -\ln e$

$f(\frac{1}{e}) = -\frac{2}{e} + 10$

abs Min: $\boxed{(\frac{1}{e}, 10 - \frac{2}{e})}$

2. $f(x) = \frac{x^2+8}{x+1}$ $[-5, 5]$

$f'(x) = \frac{2x(x+1) - x^2 - 8}{(x+1)^2}$

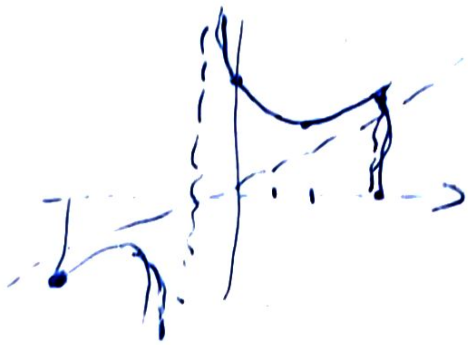
$= \frac{x^2+2x-8}{(x+1)^2}$

CN: $x = -1, 2, -4$

x	$f(x)$
-5	$-33/4$
-4	-8
-1	—
2	4
5	$\frac{33}{6}$

$-33/4$ → abs Min $(-5, -\frac{33}{4})$

$\frac{33}{6}$ → abs Max (



Since asymptote: $x = -1 \in [-5, 5]$
No abs. extreme.

2 - a $f(x) = x^3 - 3x + 2$ Inc & dec

$f'(x) = 3x^2 - 3 = 0$

$\boxed{CN: \underline{x = \pm 1}}$

$$\begin{array}{c|c|c} & -1 \qquad 1 & \\ + & - & + \end{array}$$

Inc: $(-\infty, -1)$ & $(1, \infty)$

Dec: $(-1, 1)$

g) $f(x) = x(x-1)e^{-x}$

$= (x^2 - x)e^{-x}$

$f'(x) = (2x-1)e^{-x} - (x^2 - x)e^{-x}$

$= e^{-x}(2x - 1 - x^2 + x)$

$= (-x^2 + 3x - 1)e^{-x} = 0$

$x = \frac{-3 \pm \sqrt{5}}{-2}$

$\boxed{CN: x = \frac{3 \mp \sqrt{5}}{2}}$

$$\begin{array}{c|c|c} 0 & \frac{3-\sqrt{5}}{2} \qquad \frac{3+\sqrt{5}}{2} & \\ - & + & - \end{array}$$

Inc: $\left(\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right)$

Dec: $\left(-\infty, \frac{3-\sqrt{5}}{2}\right)$ & $\left(\frac{3+\sqrt{5}}{2}, \infty\right)$

3-a $f(x) = x^3 - 3x^2 + 3$

$f'(x) = 3x^2 - 6x$

$3x(x-2) = 0$

CN: $x = 0, 2$

$f(0) = 3$

$f(2) = -1$

RMAX $(0,3)$ RMIN $(2,-1)$

3-c) $f(x) = x\sqrt{3-x}$ $x \leq 3$

$= x(3-x)^{1/2}$ $(u^n v^m)' = nu'v + muv'$

$f'(x) = \frac{1}{\sqrt{3-x}}\left(3 - x - \frac{1}{2}x\right)$

$= \frac{3 - \frac{3}{2}x}{\sqrt{3-x}} = 0$

$3 = \frac{3}{2}x \Rightarrow x = 2$: CN

$f(2) = 2$

MAX $(2,2)$

3	0
0	$\sqrt{3}$

5-b $y = -x^3 + 6x^2 - 9x + 3$

$y' = -3x^2 + 12x - 9$

$y'' = -6x + 12 = 0$

Point of infl: $x = 2$

$0 \quad 2$

$+ \mid -$

concave up: $(-\infty, 2)$

" down: $(2, \infty)$

$f(x) = 2x^3 - 3x^2 - 36x + 12$

$f'(x) = 6x^2 - 6x - 36$

$f''(x) = 12x - 6 = 0$

$x = \frac{1}{2}$: point of inflection

$0 \quad 1/2$

$- \mid +$

concave up: $(\frac{1}{2}, \infty)$

" down: $(-\infty, \frac{1}{2})$

4 - a $\lim_{x \to 1} \frac{x^a - 1}{x^b - 1} = \frac{1-1}{1-1} = \frac{0}{0}$

$$= \lim_{x \to 1} \frac{a x^{a-1}}{b x^{b-1}}$$

$$= \frac{a}{b}$$

b) $\lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{\cos x}{1}$$

$$= 1$$

c) $\lim_{x \to 0} \frac{2^{-\sin x} - 1}{e^x - 1} = \frac{1-1}{1-1} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{(-\cos x) 2^{-\sin x} \ln 2}{e^x}$$

$$= -\ln 2$$

d) $\lim_{x \to 4} \frac{\sin^2(\pi x)}{e^{x-4} + 3 - x} = \frac{0}{1+3-4} = \frac{0}{0}$

$$= \lim_{x \to 4} \frac{2\pi \sin(\pi x) \cos(\pi x)}{e^{x-4} - 1}$$

$$= \frac{0}{0}$$

$$= \pi \lim_{x \to 4} \frac{\sin(2\pi x)}{e^{x-4} - 1}$$

$$= \pi \lim_{x \to 4} \frac{2\pi \cos(2\pi x)}{e^{x-4}}$$

$$= 2\pi^2$$

7) $\lim_{x\to\infty}\left(\frac{e^x+1}{e^x-1}\right)^{\ln x} = 1^\infty$

$$\ln\left(\left(\frac{e^x+1}{e^x-1}\right)^{\ln x}\right) = \ln x \cdot \ln\left(\frac{e^x+1}{e^x-1}\right)$$

$$= \frac{\ln(e^x+1) - \ln(e^x-1)}{\frac{1}{\ln x}}$$

$$\lim_{x\to\infty} \ln\left(\left(\frac{e^x+1}{e^x-1}\right)^{\ln x}\right) = \lim_{x\to\infty} \frac{\frac{e^x}{e^x+1} - \frac{e^x}{e^x-1}}{-\frac{1/x}{(\ln x)^2}} \quad 3 \quad \frac{1}{x(\ln x)^2}$$

$$= -\lim_{x\to\infty} x(\ln x)^2 \cdot \frac{-2}{e^{2x}-1} \cdot e^x$$

$$= 2\lim_{x\to\infty} \frac{x(\ln x)^2 e^x}{e^{2x}-1} = \frac{\infty}{\infty}$$

$$= 2\lim_{x\to\infty} \frac{(\ln x)^2 + 2x(\ln x)\frac{1}{x} + x(\ln x)^2 e^x}{2e^{2x}}$$

$$= 2\lim_{x\to\infty} \frac{(\ln x)^2 + 2\ln x + x(\ln x)^2}{2e^x}$$

$$= 2\lim_{x\to\infty} \frac{2\frac{\ln x}{x} + \frac{2}{x} + (\ln x)^2 + 2\ln x}{2e^x}$$

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = 1^\infty$$

$$\ln\left(1+\frac{a}{x}\right)^x = x\ln\left(\frac{x+a}{x}\right)$$

$$= \frac{\ln\left(\frac{x+a}{x}\right)}{1/x}$$

$$\lim_{x\to\infty}\ln\left(1+\frac{a}{x}\right)^x = \lim_{x\to\infty}\frac{\ln\left(1+\frac{a}{x}\right)}{1/x} = \frac{0}{0}$$

$$= \lim_{x\to\infty}\frac{\left\{\frac{-a/x^2}{1+a/x}\right\}}{-\frac{1}{x^2}}$$

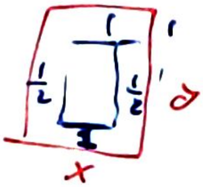
$$= \lim_{x\to\infty}\overbrace{\frac{a}{x^2}\cdot\frac{x}{x+a}}\,x^2$$

$$= \lim_{x\to\infty} a\cdot\frac{x}{x+a}$$

$$= a$$

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a$$

$A = 98\ m^2 = xy \to y = \frac{98}{x}$



$A_1 = (x-1)(y-2)$

$= x\left(\frac{98}{x}\right) - 2x - \frac{98}{x} + 2$

$A_1(x) = 100 - 2x - \frac{98}{x}$

$A_1' = -2 + \frac{98}{x^2} = 0$

$\frac{98}{x^2} = 2$

$x^2 = 49 \to x = 7$

$y = \frac{98}{7} = 14$

dimension: 7 by 14

Rev 15 $\quad V = (42-2x)^2 x \qquad (uv)'$

$V' = (42-2x)(2x(-2) + 42 - 2x)$

$= (42-2x)(-6x + 42) = 0$

CN: $x = \cancel{21}, 7$

Value of x is 7