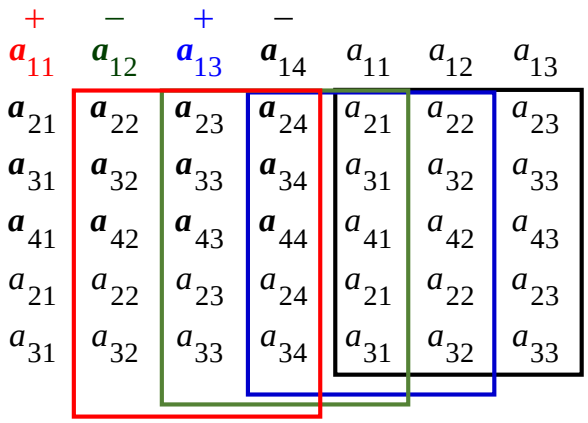
Finding the determinant of a matrix (4×4) different than using co-factor?

This method will explain the step by step on how to use the diagonal (3×3) method and co-factor to determine the matrix (4×4) determinant.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

1- Copy the 2^{nd} & 3^{rd} rows bellow 4^{th} row respectively.

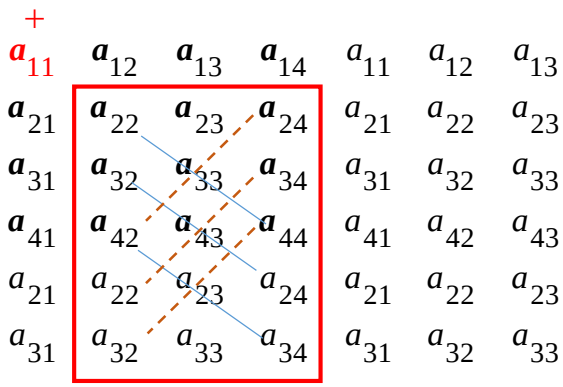
2- Copy the 1^{st}, 2^{nd} and 3^{rd} column next to the 4^{th} column respectively as is shown below



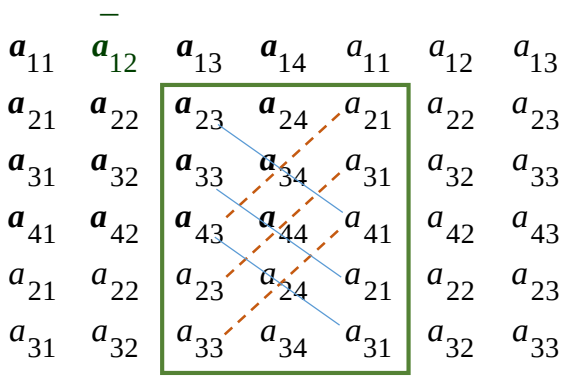
Determinant =

$$\begin{aligned}
&a_{11}\left(a_{22}a_{33}a_{44}+a_{24}a_{32}a_{43}+a_{23}a_{34}a_{42}-a_{24}a_{33}a_{42}-a_{22}a_{34}a_{43}-a_{23}a_{32}a_{44}\right)\\
&-a_{12}\left(a_{23}a_{34}a_{41}+a_{21}a_{33}a_{44}+a_{24}a_{31}a_{43}-a_{21}a_{34}a_{43}-a_{23}a_{31}a_{44}-a_{24}a_{33}a_{41}\right)\\
&+a_{13}\left(a_{24}a_{31}a_{42}+a_{22}a_{34}a_{41}+a_{21}a_{32}a_{44}-a_{22}a_{31}a_{44}-a_{24}a_{32}a_{41}-a_{21}a_{34}a_{42}\right)\\
&-a_{14}\left(a_{21}a_{32}a_{43}+a_{22}a_{33}a_{41}+a_{23}a_{31}a_{42}-a_{21}a_{33}a_{42}-a_{22}a_{31}a_{43}-a_{23}a_{32}a_{41}\right)
\end{aligned}$$

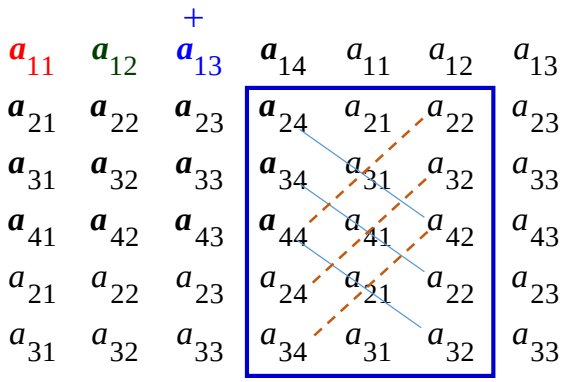
The steps to determine the determinant are:



$$a_{11}\left(a_{22}a_{33}a_{44}+a_{24}a_{32}a_{43}+a_{23}a_{34}a_{42}-a_{24}a_{33}a_{42}-a_{22}a_{34}a_{43}-a_{23}a_{32}a_{44}\right)$$



$$-a_{12}\left(a_{23}a_{34}a_{41}+a_{21}a_{33}a_{44}+a_{24}a_{31}a_{43}-a_{21}a_{34}a_{43}-a_{23}a_{31}a_{44}-a_{24}a_{33}a_{41}\right)$$



$$+a_{13}\left(a_{24}a_{31}a_{42}+a_{22}a_{34}a_{41}+a_{21}a_{32}a_{44}-a_{22}a_{31}a_{44}-a_{24}a_{32}a_{41}-a_{21}a_{34}a_{42}\right)$$

$$
\begin{array}{cccc|ccc}
\boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \boldsymbol{a_{13}} & \boldsymbol{a_{14}} & a_{11} & a_{12} & a_{13} \\
\boldsymbol{a_{21}} & \boldsymbol{a_{22}} & \boldsymbol{a_{23}} & \boldsymbol{a_{24}} & a_{21} & a_{22} & a_{23} \\
\boldsymbol{a_{31}} & \boldsymbol{a_{32}} & \boldsymbol{a_{33}} & \boldsymbol{a_{34}} & a_{31} & a_{32} & a_{33} \\
\boldsymbol{a_{41}} & \boldsymbol{a_{42}} & \boldsymbol{a_{43}} & \boldsymbol{a_{44}} & a_{41} & a_{42} & a_{43} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{31} & a_{32} & a_{33}
\end{array}
$$

$$
-a_{14}\left(a_{21}a_{32}a_{43}+a_{22}a_{33}a_{41}+a_{23}a_{31}a_{42}-a_{23}a_{32}a_{41}-a_{21}a_{33}a_{42}-a_{22}a_{31}a_{43}\right)
$$