# Solution Section 1.10 – Measures of Position, Outliers, and Boxplots

## *Exercise*

When Reese Witherspoon won an Oscar as Best Actress for the movie *Walk the Line*, her age was converted to a $z$-score of $-0.61$ when included among the ages of all other Oscar-winning Best Actress at the time of this writing. Was her age above the mean or below the mean? How many standard deviations away from the mean is her age?

**Solution**

For a $z$ score of $-0.61$, the negative sign indicates that her age is below the mean and the numerical portion indicates that her age is 0.61 standard deviations away from the mean.

## *Exercise*

Hoffman was 38 years of age when he won a Best Actor Oscar for his role in Capote. The Oscar-winning Best Actors have a mean age of 43.8 years and a standard deviation of 8.9 years.

*a)* What is the difference between Hoffman's age and the mean age?

*b)* How many standard deviations is that (the difference found in part (a))?

*c)* Convert Hoffman's age to a $z$-score.

*d)* If we consider "usual" ages to be those that convert to $z$-scores between $-2$ and 2, is Hoffman's age usual or unusual?

**Solution**

***a)***
$$\begin{aligned} |x-\bar{x}| &= |38-43.8| \\ &= |-5.8| \\ &= 5.8 \ \ years \end{aligned}$$

***b)*** $\dfrac{5.8}{8.9} = 0.65$

***c)***
$$\begin{aligned} z &= \frac{x-\bar{x}}{s} \\ &= \frac{38-43.8}{8.9} \qquad (38-43.8)/8.9 \\ &= -0.65 \end{aligned}$$

***d)*** Since $-2.00 < -0.65 < 2.00$, Hoffman's age is not considered unusual in this context.

### Exercise

Eruptions of the Old Faithful geyser have duration times with a mean of 245.0 sec and a standard deviation of 36.4 sec. One eruption had a duration time of 110 sec.

*a)* What is the difference between a duration time of 110 sec and the mea?

*b)* How many standard deviations is that (the difference found in part (a))?

*c)* Convert duration time of 110 sec to a *z*-score.

*d)* If we consider "usual" ages to be those that convert to *z*-scores between –2 and 2, is a duration time of 110 sec usual or unusual?

***Solution***

***a)*** $|x-\overline{x}| = |110-245.0| = |-135.0| \ \underline{= 135.0 \ \ sec|}$

***b)*** $\dfrac{135.0}{36.4} = 3.71$

***c)*** $z - \dfrac{x-\overline{x}}{s} = \dfrac{110-245}{36.4} \ \underline{= -3.71|}$ $\qquad (110-245)/36.4$

***d)*** Since $-3.71 < -2.00$, a duration time of 110 seconds is considered unusual in this context.

### Exercise

Human body temperatures have a mean of 98.20°F and a standard deviation of 0.62°F. Convert each given temperature to a *z*-score and determine whether it is usual and unusual.

*a*) 101.00°F $\qquad$ *b*) 96.90°F $\qquad$ *c*) 96.98°F

***Solution***

***a)*** $z - \dfrac{x-\overline{x}}{s} = \dfrac{101.0-98.20}{0.62} \ \underline{= 4.52|}$; since $4.52 > 2.00$ it is unusual. $\qquad (101-98.2)/.62$

***b)*** $z - \dfrac{x-\overline{x}}{s} = \dfrac{96.9-98.20}{0.62} \ \underline{= -2.10|}$; since $-2.10 < 2.00$ it is unusual.

***c)*** $z - \dfrac{x-\overline{x}}{s} = \dfrac{96.98-98.20}{0.62} \ \underline{= -1.97|}$; since $-2.00 < -1.97 < 2.00$ it is usual.

### Exercise

Scores on SAT test have a mean of 1518 and a standard deviation of 325. Scores on the ACT test have a mean of 21.1 and standard deviation of 4.8. Which is relatively better: a score of 1840 on the SAT test or a score of 26.0 on the ACT test? Why?

***Solution***

SAT: $z - \dfrac{x-\overline{x}}{s} = \dfrac{1840-1518}{325} \ \underline{= 0.99|}$ $\qquad (1840-1518)/325$

ACT: $z - \dfrac{x-\overline{x}}{s} = \dfrac{26.0-21.1}{4.8} \ \underline{= 1.02|}$ $\qquad (26-21.1)/4.8$

Since 1.02 > 0.99, the ACT score of 26.0 is the relatively better score.

## Exercise

Scores on SAT test have a mean of 1518 and a standard deviation of 325. Scores on the ACT test have a mean of 21.1 and standard deviation of 4.8. Which is relatively better: a score of 1190 on the SAT test or a score of 16.0 on the ACT test? Why?

**Solution**

SAT: $z - \frac{x-\overline{x}}{s} = \frac{1190-1518}{325} \underline{=-1.01|}$ $\quad (1190-1518)/325$

ACT: $z - \frac{x-\overline{x}}{s} = \frac{16.0-21.1}{4.8} \underline{=-1.06|}$ $\quad (16-21.1)/4.8$

Since $-1.01 > -1.06$, the SAT score of 119 is the relatively better score.

## Exercise

Use the given sorted values, which are the numbers of points scored in the Super Bowl for a recent period of 24 years. Find the percentile corresponding to the given number of points

36 37 37 39 39 41 43 44 44 47 50 53 54 55 56 56 57 59 61 61 65 69 69 75

*a)* 47   *b)* 65   *c)* 54   *d)* 41

**Solution**

Let $b$ to the number of scores below $x$; $n$ is the total number of scores.

The percentile score of $x$ is $\frac{b}{n}\cdot 100$

***a)*** The percentile score of 47 is $\frac{9}{24}\cdot 100 \underline{=38|}$   36 37 37 39 39 41 43 44 44 $\rightarrow b=9$

***b)*** The percentile score of 65 is $\frac{20}{24}\cdot 100 \underline{=83|}$

***c)*** The percentile score of 54 is $\frac{12}{24}\cdot 100 \underline{=50|}$

***d)*** The percentile score of 41 is $\frac{5}{24}\cdot 100 \underline{=21|}$

## Exercise

For the given data, find the indicated percentile or quartile

36 37 37 39 39 41 43 44 44 47 50 53 54 55 56 56 57 59 61 61 65 69 69 75

*a)* $P_{20}$   *c)* $P_{50}$   *e)* $P_{25}$   *g)* $Q_1$

*b)* $P_{80}$   *d)* $P_{75}$   *f)* $P_{95}$   *h)* $Q_3$

**Solution**

$N = 24$

***a)*** $L = \frac{20}{100} \cdot 24 = 4.8 \approx 5$. Since the 5th score is 39, then $\underline{P_{20} = 39}|$

***b)*** $L = \frac{80}{100} \cdot 24 = 19.2 \approx 20$. Since the 20th score is 61, then $\underline{P_{80} = 61}|$

***c)*** $L = \frac{50}{100} \cdot 24 = 12$ (it is a whole number).

The mean of the 12th and 13th score, then $P_{50} = \frac{53+54}{2} \underline{= 53.5}|$

***d)*** $L = \frac{75}{100} \cdot 24 = 18$ (it is a whole number).

The mean of the 18th and 19th score, then $P_{75} = \frac{59+61}{2} \underline{= 60}|$

***e)*** $L = \frac{25}{100} \cdot 24 = 6$ (it is a whole number).

The mean of the 6th and 7th score, then $P_{25} = \frac{41+43}{2} \underline{= 42}|$

***f)*** $L = \frac{95}{100} \cdot 24 = 22.8 \approx 23$. Since the 23th score is 69, then $P_{95} \underline{= 69}|$

***g)*** $Q_1 = P_{25}$, $L = \frac{25}{100} \cdot 24 = 6$ (it is a whole number).

The mean of the 6th and 7th score, then $Q_1 = \frac{41+43}{2} \underline{= 42}|$

***h)*** $Q_3 = P_{75}$, $L = \frac{75}{100} \cdot 24 = 18$ (it is a whole number).

The mean of the 18th and 19th score, then $Q_3 = \frac{59+61}{2} \underline{= 60}|$

## *Exercise*

The number of hours of television watched per day by a sample of 28 people

2 4 1 5 7 2 5 4 4 2 3 6 4 3 5 2 0 3 5 9 4 5 2 1 3 6 7 2

*a)* Find the data set's first, second, and third quartiles.
*b)* Draw a box-and-whisker plot that represents the data set.
*c)* About 75% of the people watched no more than now many hours of television per day?
*d)* What percent of the people watched more than 4 hours of television per day?
*e)* If you randomly selected one person from the sample, what is the likelihood that the person watched less than 2 hours of television per day? Write your answer as a percent.

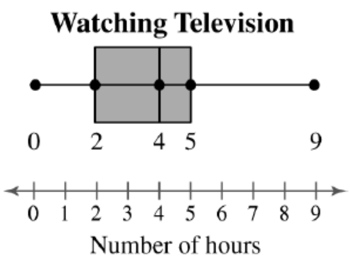
***Solution***

$N = 24 \rightarrow \quad \frac{1}{4}(28) = 7 \quad \frac{1}{2}(28) = 14 \quad \frac{3}{4}(28) = 21$

0 1 1 2 2 2 **2** 2 2 3 3 3 3 **4** 4 4 4 4 5 5 5 **5** 5 6 6 7 7 9

***a)*** $Q_1 = 2; \quad Q_2 = 4; \quad Q_3 = 5$

***b)***




**c)** 75% $\to Q_3$ $\boxed{5}$

**d)** The percentage of the people watched more than 4 hours of television per day is 50%

**e)** Less than 2 hours of television per day is 25%

## *Exercise*

The hourly earnings (in dollars) of a sample of 25 railroad equipment manufacturers

15.6 18.75 14.6 15.8 14.35 13.9 17.5 17.55 13.0 14.2 19.05 15.35 15.2
19.45 15.95 16.5 16.3 15.25 15.05 19.1 15.2 16.22 17.75 18.4 15.25

*a)* Find the data set's first, second, and third quartiles.
*b)* Draw a box-and-whisker plot that represents the data set.
*c)* About 75% of the manufacturers made less than $15.80 per hour?
*d)* What percent of the manufacturers made more than $15.80 per hour?
*e)* If you randomly selected one manufacturer from the sample, what is the likelihood that the manufacturer made less than $15.80 per hour? Write your answer as a percent.

***Solution***

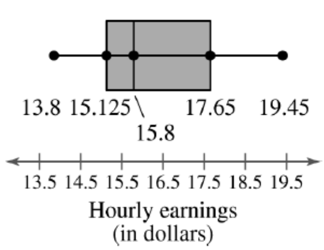
13.0 13.9 14.2 14.35 14.6 **15.05** **15.2** 15.2 15.25 15.25 15.35 15.6 **15.8**
15.95 16.22 16.3 16.5 17.5 **17.55** **17.75** 18.4 18.75 19.05 19.1 19.45

**a)** $Q_1 = P_{25}$, $L = \frac{25}{100} \cdot 25 = 6.25$, then $Q_1 = \frac{15.05 + 15.2}{2}$ $\boxed{= 15.125}$

$Q_2 = P_{50}$, $L = \frac{50}{100} \cdot 25 = 12.5 \approx 13$, then $Q_2$ $\boxed{= 15.8}$

$Q_3 = P_{75}$, $L = \frac{75}{100} \cdot 25 = 18.75$, then $Q_3 = \frac{17.55 + 17.75}{2}$ $\boxed{= 17.65}$

**b)** Railroad equipment manufacturers




**c)** 75% $\to Q_3$ $\boxed{17.65}$

**d)** The percentage of the manufacturers made more than $15.80 per hour is 50%

**e)** The manufacturer made less than $15.80 per hour is 50%

### *Exercise*

A certain brand of automobile tire has a mean life span of 35,000 miles, with a standard deviation of 2250 miles. (Assume the life spans of the tires have a bell-shaped distribution)

*a)* The life spans of three randomly selected tires are 34,000 miles, 37,000 miles, and 30,000 miles. Find the *z*-score that corresponds to each life span. According to the *z*-scores, would the life spans of any of these tires be considered unusual?

*b)* The life spans of three randomly selected tires are 30,500 miles, 37,250 miles, and 35,000 miles. Using the Empirical Rule, find the percentile that corresponds to each life span.

***Solution***

***a)*** $x = 34,000 \quad z - \frac{x-\overline{x}}{s} = \frac{34,000-35,000}{2,250} \approx -0.44$

$x = 37,000 \quad z - \frac{x-\overline{x}}{s} = \frac{37,000-35,000}{2,250} \approx 0.89$

$x = 30,000 \quad z - \frac{x-\overline{x}}{s} = \frac{30,000-35,000}{2,250} \approx -2.22$

The tire with a life span of 30,000 miles has an unusual short life span.

***b)*** $x = 30,500 \quad z - \frac{x-\overline{x}}{s} = \frac{30,500-35,000}{2,250} = -2 \quad \Rightarrow 2.5^{\text{th}}$ percentile

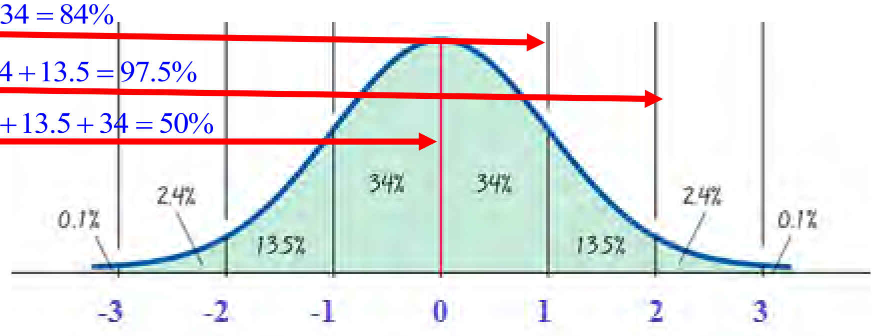
$x = 37,250 \quad z - \frac{x-\overline{x}}{s} = \frac{37,250-35,000}{2,250} = 1 \quad \Rightarrow 84^{\text{th}}$ percentile

$x = 35,000 \quad z - \frac{x-\overline{x}}{s} = \frac{35,000-35,000}{2,250} = 0 \quad \Rightarrow 50^{\text{th}}$ percentile

$1 \rightarrow 50+34 = 84\%$

$2 \rightarrow 50+34+13.5 = 97.5\%$

$0 \rightarrow 0.1+2.4+13.5+34 = 50\%$



### *Exercise*

The life spans of species of fruit fly have a bell shaped distribution, with mean of 33 days and a standard deviation of 4 days.

*a)* The life spans of three randomly selected fruit flies are 34 days, 30 days, and 42 days. Find the *z*-score that corresponds to each life span and determine if any of these life spans are unusual.

*b)* The life spans of three randomly selected fruit flies are 29 days, 41 days, and 25 days. Using the Empirical Rule, find the percentile that corresponds to each life span.

**Solution**

***a)*** $x=34 \quad z-\frac{x-\mu}{\delta}=\frac{34-33}{4}\ \underline{\approx 0.25}|$

$x=30 \quad z-\frac{x-\mu}{\delta}=\frac{30-33}{4}\ \underline{\approx -0.75}|$

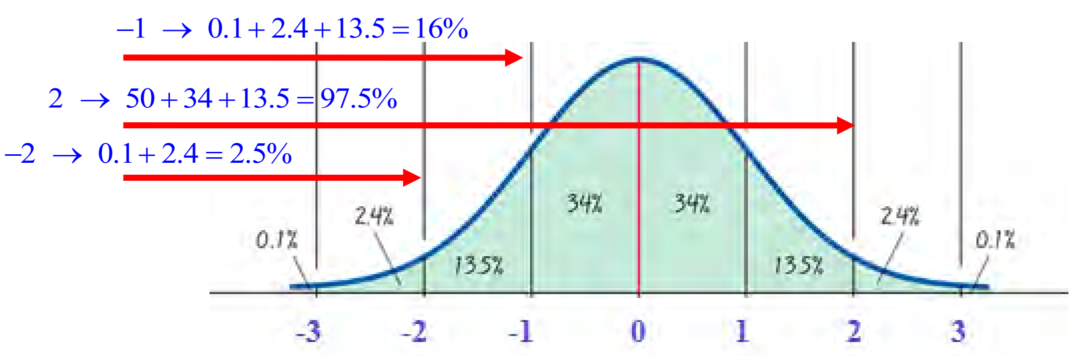
$x=42 \quad z-\frac{x-\mu}{\delta}=\frac{42-33}{4}\ \underline{\approx 2.25}|$

The fruit fly with a life span of 42 days has an unusual short life span.

***b)*** $x=29 \quad z-\frac{x-\mu}{\delta}=\frac{29-33}{4}\ \underline{\approx -1}| \qquad \Rightarrow 16^{\text{th}}$ percentile

$x=41 \quad z-\frac{x-\mu}{\delta}=\frac{41-33}{4}\ \underline{\approx 2}| \qquad \Rightarrow 97.5^{\text{th}}$ percentile

$x=25 \quad z-\frac{x-\mu}{\delta}=\frac{25-33}{4}\ \underline{\approx -2}| \qquad \Rightarrow 2.5^{\text{th}}$ percentile




### *Exercise*

Find the $Q_1$ and $Q_3$ for the given data: 49 52 52 52 74 67 55 55

**Solution**

49 52 52 52 55 55 67 74

$Q_1\ \underline{=52}|$

$Q_3=\frac{55+67}{2}\ \underline{=61}|$

### *Exercise*

Find the $Q_1$ and $Q_3$ for the given weights (in pounds) of 30 newborn babies listed below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5.5 | 5.7 | 5.8 | 6.0 | 6.1 | 6.1 | 6.3 | 6.4 | 6.5 | 6.6 |
| 6.7 | 6.7 | 6.7 | 6.9 | 7.0 | 7.0 | 7.0 | 7.1 | 7.2 | 7.2 |
| 7.4 | 7.5 | 7.7 | 7.7 | 7.8 | 8.0 | 8.1 | 8.1 | 8.3 | 8.7 |

***Solution***

$$Q_1 \underline{= 6.4\ \ lb|}$$

$$Q_3 = \left(L_{23}\right) \underline{= 7.7\ lb|}$$

### *Exercise*

Find the percentile for the data value:

113 125 117 111 119 121 111 109 116 113 117 127 109 113 115 110

Data value: 119

***Solution***

Since there is 12 number less than 119

$$percentile = \frac{12 \times 100}{16} \underline{= 75|}$$

### *Exercise*

The test scores of 40 students are listed below:

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 30 | 35 | 43 | 44 | 47 | 48 | 54 | 55 | 56 | 57 | 59 | 62 | 63 | 65 | 66 | 68 | 69 | 69 | 71 | 72 |
| 72 | 73 | 74 | 76 | 77 | 77 | 78 | 79 | 80 | 81 | 81 | 82 | 83 | 85 | 89 | 92 | 93 | 94 | 97 | 98 |

Find $P_{56}$

***Solution***

$$L = \frac{56 \times 40}{100} = 22.4 \approx 23$$

$$P_{56} \underline{= 74|}$$