8.1 $\cos\theta = \frac{1}{\sec\theta}$ $\sin\theta = \frac{1}{\csc\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sec\theta\tan\theta = \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos^2\theta}$

$\frac{1}{\sin\theta} + \frac{1}{\cos\theta} = \frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}$

$\tan\alpha + \cot\alpha = \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}$

$= \frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha\sin\alpha}$

$= \frac{1}{\cos\alpha\sin\alpha}$

Prove: $\tan x + \cos x = \sin x(\sec x + \cot x)$

$\sin x(\sec x + \cot x) = \sin x\left(\frac{1}{\cos x} + \frac{\cos x}{\sin x}\right)$

$= \frac{\sin x}{\cos x} + \cos x$

$= \tan x + \cos x$ ✓

Prove $\cot\alpha + 1 = \csc\alpha(\cos\alpha + \sin\alpha)$

$$\csc\alpha(\cos\alpha+\sin\alpha) = \frac{1}{\sin\alpha}(\cos\alpha+\sin\alpha)$$
$$= \frac{\cos\alpha}{\sin\alpha} + 1$$
$$= \cot\alpha + 1 \checkmark$$

Prove $\dfrac{\cos^4 t - \sin^4 t}{\cos^2 t} = 1 - \tan^2 t$ $\quad \cos^4 t = (\cos^2 t)^2$

$$\frac{\cos^4 t - \sin^4 t}{\cos^2 t} = \frac{(\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t)}{\cos^2 t}$$
$$= \frac{\cos^2 t - \sin^2 t}{\cos^2 t}$$
$$= \frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t}$$
$$= 1 - \tan^2 t \checkmark$$

$1 + \cos\theta = \dfrac{\sin^2\theta}{1-\cos\theta}$? $\quad \cos^2\theta + \sin^2\theta = 1$

$$\frac{\sin^2\theta}{1-\cos\theta} = \frac{1-\cos^2\theta}{1-\cos\theta}$$
$$= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$
$$= 1 + \cos\theta \checkmark$$

$$1 + \cos\theta = (1+\cos\theta)\frac{1-\cos\theta}{1-\cos\theta}$$
$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$
$$= \frac{\sin^2\theta}{1-\cos\theta} \checkmark$$

$$(1+\cos\theta)(1-\cos\theta) \overset{?}{=} \sin^2\theta$$
$$1 - \cos^2\theta \overset{?}{=} \sin^2\theta$$
$$\sin^2\theta = \sin^2\theta \checkmark$$

$$\tan^2\alpha\,(1+\cot^2\alpha) \stackrel{?}{=} \frac{1}{1-\sin^2\alpha}$$

$$\tan^2\alpha\,(1+\cot^2\alpha) = \tan^2\alpha + \overbrace{\tan^2\alpha\cot^2\alpha}^{=1}$$

$$= \tan^2\alpha + 1$$

$$= \sec^2\alpha$$

$$= \frac{1}{\cos^2\alpha} \qquad \cos^2\alpha + \sin^2\alpha = 1$$

$$= \frac{1}{1-\sin^2\alpha}$$ ✓

$$\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} \stackrel{?}{=} 2\csc\alpha$$

$$\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + (1+\cos\alpha)^2}{\sin\alpha\,(1+\cos\alpha)}$$

$$= \frac{\sin^2\alpha + 1 + 2\cos\alpha + \cos^2\alpha}{\sin\alpha\,(1+\cos\alpha)}$$

$$= \frac{2+2\cos\alpha}{\sin\alpha\,(1+\cos\alpha)}$$

$$= \frac{2\,(1+\cos\alpha)}{\sin\alpha\,(1+\cos\alpha)}$$

$$= \frac{2}{\sin\alpha}$$

$$= 2\csc\alpha$$ ✓

$$\frac{1+\sin t}{\cos t} = \frac{\cos t}{1-\sin t}$$

$$\frac{1+\sin t}{\cos t} = \frac{1+\sin t}{\cos t} \cdot \frac{1-\sin t}{1-\sin t}$$

$$= \frac{1-\sin^2 t}{\cos t(1-\sin t)} \qquad \cos^2 t + \sin^2 t = 1$$

$$= \frac{\cos^2 t}{\cos t(1-\sin t)}$$

$$= \frac{\cos t}{1-\sin t} \quad \checkmark$$

$$\cot^2\theta + \cos^2\theta \neq \cot^2\theta \cos^2\theta$$

$\theta = \frac{\pi}{4} \rightarrow \cot\frac{\pi}{4} = 1 \quad , \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \frac{2}{4}$

$$\cot^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} \stackrel{?}{=} \cot^2\theta \cos^2\theta$$

$$1 + \frac{1}{2} \stackrel{?}{=} 1 \cdot \frac{1}{2}$$

$$\frac{3}{2} \neq \frac{1}{2} \quad \checkmark$$

#21 $$\frac{\cot^2\theta + 3\cot\theta - 4}{\cot\theta + 4} = \cot\theta - 1$$

$$\frac{\cot^2\theta + 3\cot\theta - 4}{\cot\theta + 4} = \frac{(\cot\theta + 4)(\cot\theta - 1)}{(\cot\theta + 4)}$$

$$= \cot\theta - 1 \quad \checkmark$$

#24 $$\sin\alpha(\tan\alpha\cos\alpha - \cot\alpha\cos\alpha) = 1 - 2\cos^2\alpha$$

$$\sin\alpha(\tan\alpha\cos\alpha - \cot\alpha\cos\alpha) = \sin\alpha\frac{\sin\alpha}{\cos\alpha}\cos\alpha - \sin\alpha\frac{\cos\alpha}{\sin\alpha}\cos\alpha$$

$$= \sin^2\alpha - \cos^2\alpha$$

$$= 1 - \cos^2\alpha - \cos^2\alpha$$

$$= 1 - 2\cos^2\alpha \quad \checkmark$$