#15 $\displaystyle\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$ $\quad d(\cos(2t+1)) = -2\sin(2t+1)\,dt$

$$= -\frac{1}{2}\int \frac{d(\cos(2t+1))}{\cos^2(2t+1)}$$

$$= \frac{1}{2}\,\frac{1}{\cos(2t+1)} + C$$

#16 $\displaystyle\int \frac{\sec z \tan z}{\sqrt{\sec z}}\,dz$ $\quad d(\sec z) = \sec z \tan z\,dz$

$$= \int (\sec z)^{-1/2}\, d(\sec z)$$

$$= \frac{1}{2}\sqrt{\sec z} + C$$

#17 $\displaystyle\int \frac{1}{\sqrt{t}}\cos(\sqrt{t}+3)\,dt$ $\quad d(\sqrt{t}+3) = \frac{1}{2\sqrt{t}}\,dt$

$$= 2\int \cos(\sqrt{t}+3)\, d(\sqrt{t}+3)$$

$$= 2\sin(\sqrt{t}+3) + C$$

18 $\displaystyle\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\,d\theta = \frac{1}{2}\int \frac{1}{\theta^2}\sin\frac{2}{\theta}\,d\theta$

$$d\left(\frac{2}{\theta}\right) = -\frac{2}{\theta^2}\,d\theta$$

$$= \frac{1}{2}\left(-\frac{1}{2}\right)\int \sin\left(\frac{2}{\theta}\right) d\left(\frac{2}{\theta}\right)$$

$$= +\frac{1}{4}\cos\frac{2}{\theta} + C$$

7- $\int t^3(1+t^4)^3\,dt$ $\qquad d(1+t^4) = 4t^3\,dt$

$$= \frac{1}{4}\int (1+t^4)^3\, d(1+t^4)$$

$$= \frac{1}{16}(1+t^4)^4 + C$$

20/ $\int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx = \int \frac{1}{x^3}\left(1-\frac{1}{x^2}\right)^{1/2} dx$

$$d\left(1-\frac{1}{x^2}\right) = \frac{2}{x^3}\,dx$$

$$= \int \left(1-\frac{1}{x^2}\right)^{1/2} d\left(1-\frac{1}{x^2}\right)$$

$$= \frac{2}{3}\left(1-\frac{1}{x^2}\right)^{3/2} + C$$

91/ $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\cos^3\sqrt{\theta}}\,d\theta$ $\qquad d(\cos\sqrt{\theta}) = -\frac{1}{2\sqrt{\theta}}\sin\sqrt{\theta}\,d\theta$

$$= -2\int \cos^{-3/2}(\sqrt{\theta})\, d(\cos\sqrt{\theta})$$

$$= 4\cos^{-1/2}(\sqrt{\theta}) + C$$

$$= \frac{4}{\sqrt{\cos\sqrt{\theta}}} + C$$

23/ $\int 2x\sqrt{x^2-2}\,dx = \int (x^2-2)^{1/2} d(x^2-2) \quad \Big\{ d(x^2-2) = 2x\,dx$

$$= \frac{2}{3}(x^2-2)^{3/2} + C$$

24/ $\int \frac{x\,dx}{(x^2-4)^3} = \frac{1}{2}\int (x^2-4)^{-3} d(x^2-4) \quad \Big\} d(x^2-4) = 2x\,dx$

$$= -\frac{1}{4}(x^2-4)^{-2} + C$$

25/ $\int x^3(3x^4+1)^2 dx \qquad d(3x^4+1) = 12x^3 dx$

$$= \frac{1}{12}\int (3x^4+1)^2 d(3x^4+1)$$

$$= \frac{1}{36}(3x^4+1)^3 + C$$

26/ $\int 2(3x^4+1)^2 dx = 2\int (9x^8+6x^4+1)\,dx$

$$= 2.x^9 + \frac{12}{5}x^5 + 2x + C$$

30/ $\int \frac{(2r-1)\cos\sqrt{3(2r-1)^2+6}}{\sqrt{3(2r-1)^2+6}}\,dr \qquad (u^n)' = n u' u^{n-1}$

$$d\left(\sqrt{3(2r-1)^2+6}\right) = \frac{\overset{=6}{12}(2r-1)\,dr}{2\sqrt{3(2r-1)^2+6}}$$

$$= \frac{1}{6}\int \cos\sqrt{3(2r-1)^2+6}\; d\left(\sqrt{3(2r-1)^2+6}\right)$$

$$= \frac{1}{6}\sin\sqrt{3(2r-1)^2+6} + C$$

32 $\displaystyle\int \frac{t+2t^2}{\sqrt{t}}\,dt = \int (t^{1/2}+2t^{3/2})\,dt$

$$= \frac{2}{3}t^{3/2} + \frac{4}{5}t^{5/2} + C$$

33/ $\displaystyle\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2}\,dt \qquad d\left(1+\frac{1}{t}\right) = -\frac{1}{t^2}\,dt$

$$= -\int \left(1+\frac{1}{t}\right)^3 d\left(1+\frac{1}{t}\right)$$

$$= -\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + C$$

#36 $\displaystyle\int \frac{dx}{\sqrt{x}+\sqrt{x+1}}$

$u = \sqrt{x+1}$

$u^2 = x+1 \Rightarrow x = u^2-1$

$2u\,du = dx$

$$\int \frac{dx}{\sqrt{x}+\sqrt{x+1}} = \int \frac{2u\,du}{(u^2-1)^{1/2}+u} \quad \frac{1}{u}$$

$$= 2\int \frac{du}{\frac{(u^2-1)^{1/2}}{u}+1}$$

$$= 2\int \frac{du}{1+\left(\frac{u^2-1}{u^2}\right)^{1/2}}$$

$$= 2\int \frac{du}{1+\left(1-\frac{1}{u^2}\right)^{1/2}}$$

$d(\sqrt{x}+\sqrt{x+1}) = \left(\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x+1}}\right)dx$

$= 2\left(\frac{\sqrt{x}+\sqrt{x+1}}{\sqrt{x}\sqrt{x+1}}\right)dx$

36. $\int \frac{dx}{\sqrt{x}+\sqrt{x+1}} \cdot \frac{\sqrt{x}-\sqrt{x+1}}{\sqrt{x}-\sqrt{x+1}} = \int \frac{(x)^{1/2}-(x+1)^{1/2}}{x^2-x^2-1}dx$

$= -\int x^{1/2}dx + \int (x+1)^{1/2} d(x+1)$

$d(x+1) = dx$

$= -\frac{2}{3}x^{3/2} + \frac{2}{3}(x+1)^{3/2} + C$

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49. $\int \frac{dx}{6x-5} = \frac{1}{6}\int \frac{d(6x-5)}{6x-5}$ $\quad d(6x-5) = 6\,dx$

$= \frac{1}{2} \ln|6x-5| + C$

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139. $\int_{-1}^{1} r\sqrt{1-r^2}\,dr$ $\quad d(1-r^2) = -2r\,dr$

$= -\frac{1}{2}\int_{-1}^{1}(1-r^2)^{1/2}\,d(1-r^2)$

$= -\frac{1}{3}(1-r^2)^{3/2}\Big|_{-1}^{1}$

$= -\frac{1}{3}(0-0)$

$= 0$

40) $$\int_0^{\pi/4} \tan x \sec^2 x\,dx \qquad d(\tan x) = \sec^2 x\,dx$$

$$= \int_0^{\pi/4} \tan x \, d(\tan x)$$

$$= \frac{1}{2}(\tan x)^2 \Big|_0^{\pi/4}$$

$$= \frac{1}{2}(1 - 0)$$

$$= \boxed{\frac{1}{2}}$$

41) $$\int_{2\pi}^{3\pi} 3\cos^2 x \sin x\,dx \qquad d(\cos x) = -\sin x\,dx$$

$$= -3\int_{2\pi}^{3\pi} \cos^2 x \, d(\cos x)$$

$$= - \cos^3 x \Big|_{2\pi}^{3\pi}$$

$$= -(-1-1)$$

$$= \boxed{2}$$

44) $$\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\,dv \qquad d(1+v^{3/2}) = \frac{3}{2}v^{1/2}\,dv$$

$$= \frac{20}{3}\int_0^1 \frac{d(1+v^{3/2})}{(1+v^{3/2})^2}$$

$$= -\frac{20}{3} \frac{1}{1+v^{3/2}} \Big|_0^1$$

$$= -\frac{20}{3}\left(\frac{1}{2} - 1\right)$$

$$= \boxed{\frac{10}{3}}$$

53/ $\int_0^1 (4y-y^2+4y^3+1)^{-2/3}(12y^2-2y+4)\,dy$

$$d(4y-y^2+4y^3) = (4-2y+12y^2)\,dy$$

$$= \int_0^1 (4y-y^2+4y^3+1)^{-2/3}\,d(4y-y^2+4y^3+1)$$

$$= 3\,(4y-y^2+4y^3+1)^{1/3}\Big|_0^1$$

$$= 3\left(\underline{(4-1+4+1)}^{1/3} - 1\right)$$

$$= 3\left(8^{1/3} - 1\right)$$

$$= 3\left((2^3)^{1/3} - 1\right)$$

$$= 3(2-1)$$

$$= \underline{3}$$

155/ $\int_0^{\pi/2} e^{\sin x}\cos x\,dx =$ $d(\sin x) = \cos x\,dx$

$$= \int_0^{\pi/2} e^{\sin x}\,d(\sin x)$$

$$= e^{\sin x}\Big|_0^{\pi/2}$$

$$= e^1 - e^0$$

$$= \underline{e-1}$$

57 $\displaystyle\int_0^{\pi/3} \frac{4\sin\theta}{1-4\cos\theta}\,d\theta$ $\quad d(1-4\cos\theta) = 4\sin\theta\,d\theta$

$$= \int_0^{\pi/3} \frac{d(1-4\cos\theta)}{1-4\cos\theta}$$

$$= \ln|1-4\cos\theta|\Big|_0^{\pi/3}$$

$$= \ln|1-2| - \ln|1-4| \leftarrow$$

$$= \ln 1 - \ln 3$$

$$= -\ln 3$$

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58 $\displaystyle\int_1^2 \frac{2\ln x}{x}\,dx = 2\int_1^2 \ln x\, d(\ln x)$ $\Big\{ d(\ln x) = \frac{1}{x}\,dx$

$$= (\ln x)^2\Big|_1^2$$

$$= (\ln 2)^2$$

59 $\displaystyle\int_2^{16} \frac{dx}{2x\sqrt{\ln x}} = \frac{1}{2}\int_2^{16} (\ln x)^{-1/2} d(\ln x)$ $\quad d(\ln x) = \frac{dx}{x}$

$$= (\ln x)^{1/2}\Big|_2^{16}$$

$\ln a^x = x\ln a$

$$= \sqrt{\ln 2^4} - \sqrt{\ln 2}$$

$$= 2\sqrt{\ln 2} - \sqrt{\ln 2}$$

$$= \sqrt{\ln 2}$$

160 $\int_0^{\pi/2} \tan\frac{x}{2}\,dx = 2\int_0^{\pi/2} \tan\frac{x}{2}\, d\left(\frac{x}{2}\right)$ $\quad d\left(\frac{x}{2}\right) = \frac{1}{2}dx$

$= 2\ \ln\left|\sec\frac{x}{2}\right|\Big|_0^{\pi/2}$

$= 2\left(\ln\sqrt{2} - \underline{\ln 1}\right)$

$= 2\ \ln 2^{1/2}$

$= \underline{\ln 2}$

$\tan x \quad \frac{\sin}{\cos} \quad -\frac{d\cos}{\cos} \quad -\ln\cos$

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162 $\int_{-\ln 2}^{0} e^{-x}\,dx = -e^{-x}\Big|_{-\ln 2}^{0}$

$= -\left(1 - e^{\ln 2}\right)$

$= -(1-2)$

$= 1$

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163 $\int_{\pi/4}^{\pi/2} \left(1+e^{\cot\theta}\right)\csc^2\theta\, d\theta$ $\quad d(\cot\theta) = -\csc^2\theta\, d\theta$

$= -\int_{\pi/4}^{\pi/2} \left(1+e^{\cot\theta}\right) d(\cot\theta)$

$= -\left(\cot\theta + e^{\cot\theta}\right)\Big|_{\pi/4}^{\pi/2}$

$= -\left[0 + 1 - (1 - e)\right]$

$= \underline{-e}$

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$$\int_0^{\sqrt{\ln \pi}} 2x e^{x^2} \cos(e^{x^2})\,dx$$

$$d(e^{x^2}) = 2x e^{x^2} dx$$

$$= \int_0^{\sqrt{\ln \pi}} \cos(e^{x^2})\, d(e^{x^2})$$

$$= \sin e^{x^2} \Big|_0^{\sqrt{\ln \pi}}$$

$$= \sin e^{\ln \pi} - \sin e^{0}$$

$$= \sin \pi - \sin 1$$

$$= 0 - \sin 1$$

$$\boxed{= -\sin 1}$$

167

$$\int_1^e x^{(\ln 2)-1}\,dx = \frac{1}{\ln 2} x^{(\ln 2)\cancel{-1+1}} \Big|_1^e$$

$$= \frac{1}{\ln 2}\left(e^{\ln 2} - 1\right)$$

$$\boxed{= \frac{1}{\ln 2}}$$

$$\int y\,dx = yx + c$$

$$\boxed{\int \pi\,dx = \pi x + c}$$

170. $\int_1^{e^x} \frac{1}{t}\,dt = \ln t \Big|_1^{e^x}$

$= \ln e^x - \ln 1$

$\boxed{= x}$

174. $\int_1^{e^{\pi/4}} \frac{4}{t(1+\ln^2 t)}\,dt$

$x = \ln t$

$dx = \frac{dt}{t}$

$\int_1^{e^{\pi/4}} \frac{4}{1+x^2}\,dx = 4\tan^{-1} x \Big|_1^{e^{\pi/4}}$

$= 4\tan^{-1}(\ln t) \Big|_1^{e^{\pi/4}}$

$\boxed{= 4 \cdot \tan^{-1}\frac{\pi}{4}}$

202. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^2 x}\,dx$ $\qquad d(\sin x) = \cos x\,dx$

$= \int_{\pi/4}^{\pi/2} \frac{d(\sin x)}{\sin^2 x}$

$= -\frac{1}{\sin x}\Big|_{\pi/4}^{\pi/2}$

$= -(1-\sqrt{2})$

$\boxed{= \sqrt{2} - 1}$

203 $\int_{-1}^{1} (x-1)(x^2-2x)^7 dx$ $\qquad d(x^2-2x)=(2x-2)dx$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 2(x-1)dx$

$$= \frac{1}{2}\int_{-1}^{1}(x^2-2x)^7 \, d(x^2-2x)$$

$$= \frac{1}{16}(x^2-2x)^8 \Big|_{-1}^{1}$$

$$= \frac{1}{16}(1-3^8)$$

217 $\int_{0}^{3} \frac{x^2+1}{\sqrt{x^3+3x+4}} dx$ $\qquad d(x^3+3x+4)=(3x^2+3)dx$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 3(x^2+1)dx$

$$= \frac{1}{3}\int_{0}^{3}(x^3+3x+4)^{-1/2} \, d(x^3+3x+4)$$

$$= \frac{2}{3}\sqrt{x^3+3x+4}\Big|_{0}^{3}$$

$$= \frac{2}{3}(2\sqrt{10}-2)$$

$$= \frac{4}{3}(\sqrt{10}-1)$$

222 $\int_{-1}^{2} x^2 e^{x^3+1} dx$ $\qquad d(x^3+1)=3x^2dx$

$$= \frac{1}{3}\int_{-1}^{2} e^{x^3+1} \, d(x^3+1)$$

$$= \frac{1}{3} e^{x^3+1}\Big|_{-1}^{2}$$

$$= \frac{1}{3}(e^9-1)$$