### *Exercise*

Find the period, show the asymptotes, and sketch the graph of $y = \tan\left(x - \frac{\pi}{4}\right)$

***Solution***

***Amplitude***: $n / a$

***Period***: $P = \pi$

***Phase Shift***: $\varphi = \frac{\pi}{4}$

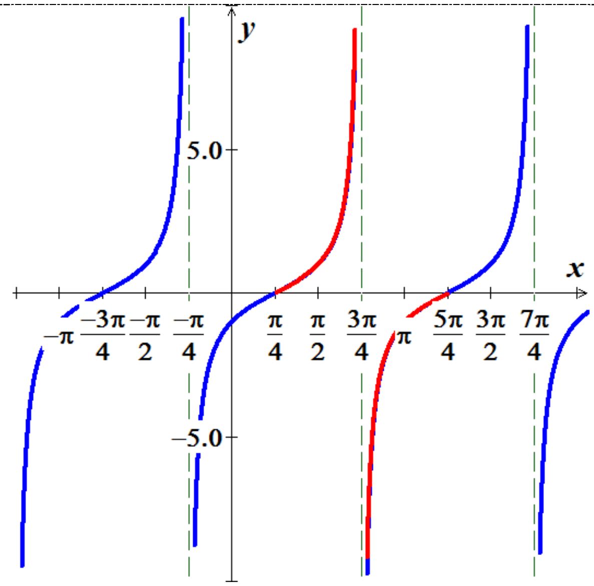
***VT:*** $y = 0$

***Asymptotes***:

$x = -\frac{\pi}{4} + n\pi$

$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$

| | $x$ | $y$ |
|---|---|---|
| $0 + \frac{\pi}{4}$ | $\frac{\pi}{4}$ | 0 |
| $\frac{\pi}{4} + \frac{\pi}{4}$ | $\frac{\pi}{2}$ | 1 |
| $\frac{\pi}{2} + \frac{\pi}{4}$ | $\frac{3\pi}{4}$ | $\infty$ |
| $\frac{3\pi}{4} + \frac{\pi}{4}$ | $\pi$ | $-1$ |
| $\pi + \frac{\pi}{4}$ | $\frac{5\pi}{4}$ | 0 |




### *Exercise*

Find the period, show the asymptotes, and sketch the graph of $y = 2\tan\left(2x + \frac{\pi}{2}\right)$

***Solution***

***Amplitude***: $n / a$

***Period***: $P = \frac{\pi}{2}$

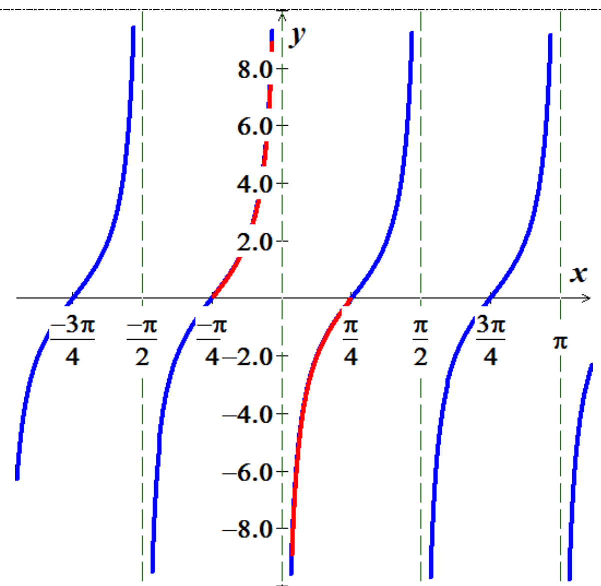
***Phase Shift***: $\varphi = -\frac{\pi}{4}$

***VT:*** $y = 0$

***Asymptotes***: $x = n\frac{\pi}{2}$

$x = \ldots, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \ldots$

| | $x$ | $y$ |
|---|---|---|
| $0 - \frac{\pi}{4}$ | $-\frac{\pi}{4}$ | 0 |
| $\frac{\pi}{8} - \frac{\pi}{4}$ | $-\frac{\pi}{8}$ | 2 |
| $\frac{\pi}{4} - \frac{\pi}{4}$ | 0 | $\infty$ |
| $\frac{3\pi}{8} - \frac{\pi}{4}$ | $\frac{\pi}{8}$ | $-2$ |
| $\frac{\pi}{2} - \frac{\pi}{4}$ | $\frac{\pi}{4}$ | 0 |



## *Exercise*

Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{4}\tan\left(\frac{1}{2}x + \frac{\pi}{3}\right)$

***Solution***

***Amplitude***: $n/a$

***Period***: $P = 2\pi$

***Phase Shift***: $\varphi = -\frac{2\pi}{3}$

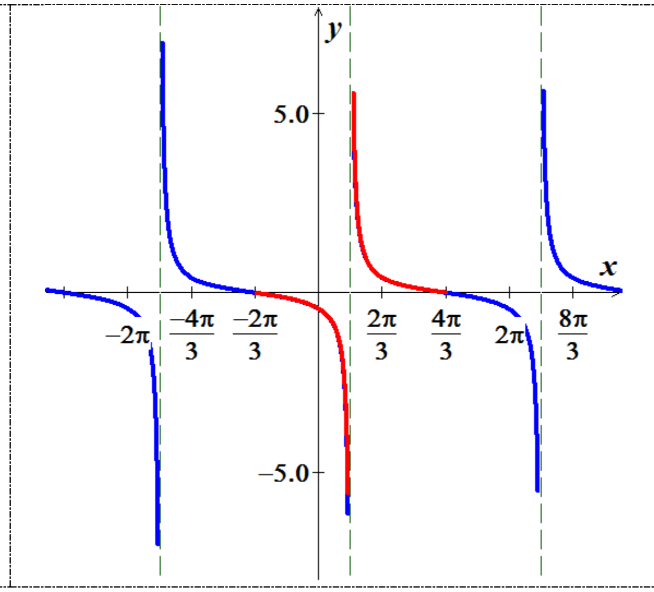
***VT:*** $y = 0$

***Asymptotes***:

$$x = -\frac{5\pi}{3} + 2n\pi$$

$$x = \ldots, -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3}, \ldots$$

| | $x$ | $y$ |
|---|---|---|
| $0 - \frac{2\pi}{3}$ | $-\frac{2\pi}{3}$ | 0 |
| $\frac{\pi}{2} - \frac{2\pi}{3}$ | $-\frac{\pi}{6}$ | $-\frac{1}{4}$ |
| $\pi - \frac{2\pi}{3}$ | $\frac{\pi}{3}$ | $\infty$ |
| $\frac{3\pi}{2} - \frac{2\pi}{3}$ | $\frac{5\pi}{6}$ | $\frac{1}{4}$ |
| $2\pi - \frac{2\pi}{3}$ | $\frac{4\pi}{3}$ | 0 |




## *Exercise*

Find the period, show the asymptotes, and sketch the graph of $y = \cot\left(x + \frac{\pi}{4}\right)$

***Solution***

***Amplitude***: $n/a$

***Period***: $P = \pi$

***Phase Shift***: $\varphi = -\frac{\pi}{4}$

***VT:*** $y = 0$

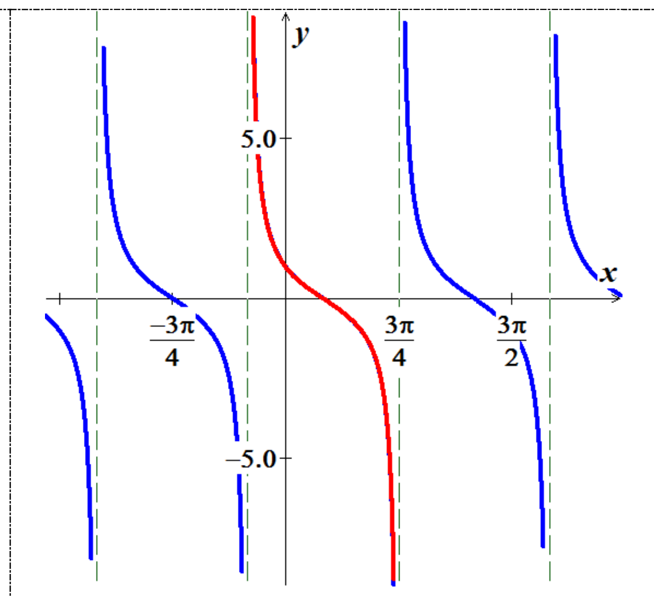
***Asymptotes***:

$$x = 2n\pi + \pi - \frac{\pi}{4}$$

$$x = 2n\pi + \frac{3\pi}{4}$$

$$x = \ldots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$$

| | $x$ | $y$ |
|---|---|---|
| $0 - \frac{\pi}{4}$ | $-\frac{\pi}{4}$ | $\infty$ |
| $\frac{\pi}{4} - \frac{\pi}{4}$ | 0 | 1 |
| $\frac{\pi}{2} - \frac{\pi}{4}$ | $\frac{\pi}{4}$ | 0 |
| $\frac{3\pi}{4} - \frac{\pi}{4}$ | $\frac{\pi}{2}$ | $-1$ |
| $\pi - \frac{\pi}{4}$ | $\frac{3\pi}{4}$ | $\infty$ |



## Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\cot\left(2x + \frac{\pi}{2}\right)$

***Solution***

***Amplitude***: $n/a$

***Period***: $P = \frac{\pi}{2}$

***Phase Shift***: $\varphi = -\frac{\pi}{4}$

***VT:*** $y = 0$

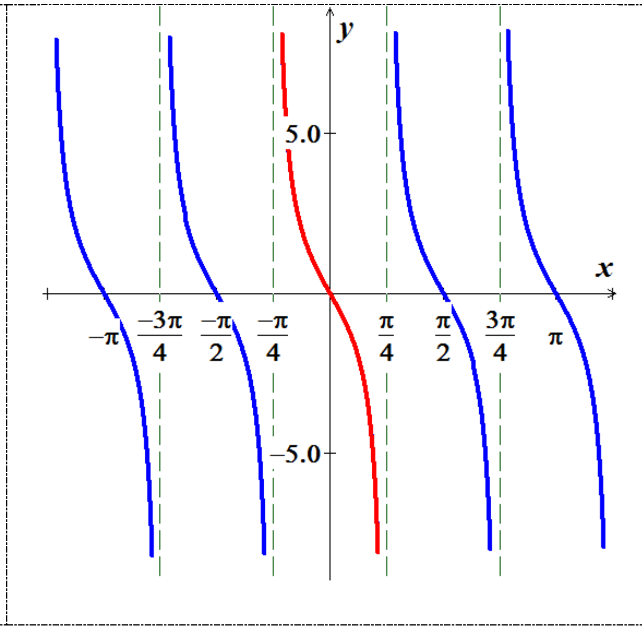
***Asymptotes***:

$$2x + \frac{\pi}{2} = (2n+1)\pi$$

$$2x = 2n\pi + \frac{\pi}{2}$$

$$x = \dots, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \dots$$

| | ***x*** | ***y*** |
|---|---|---|
| $0 - \frac{\pi}{4}$ | $-\frac{\pi}{4}$ | $\infty$ |
| $\frac{\pi}{8} - \frac{\pi}{4}$ | $-\frac{\pi}{8}$ | 2 |
| $\frac{\pi}{4} - \frac{\pi}{4}$ | 0 | 0 |
| $\frac{3\pi}{8} - \frac{\pi}{4}$ | $\frac{\pi}{8}$ | $-2$ |
| $\frac{\pi}{2} - \frac{\pi}{4}$ | $\frac{\pi}{4}$ | $\infty$ |




## Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{2}\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

***Solution***

***Amplitude***: $n/a$

***Period***: $P = 2\pi$

***Phase Shift***: $\varphi = -\frac{\pi}{2}$

***VT:*** $y = 0$

***Asymptotes***:

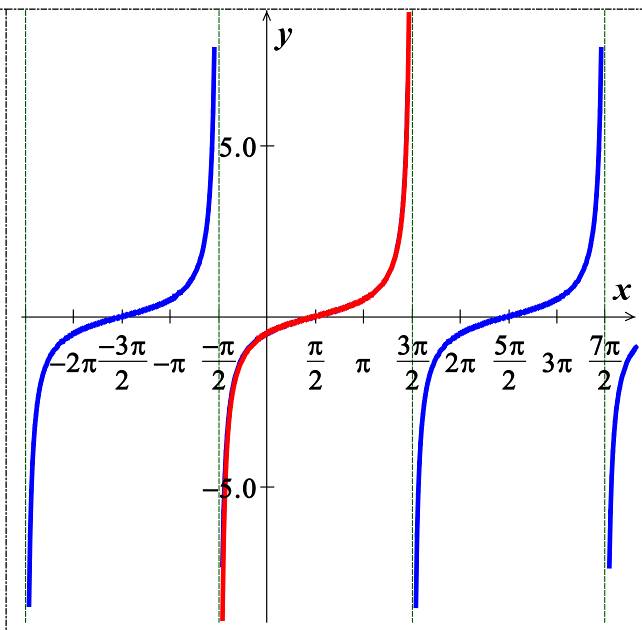
$$\frac{1}{2}x + \frac{\pi}{4} = 2n\pi + \pi$$

$$\frac{1}{2}x = 2n\pi + \frac{3\pi}{4}$$

$$x = 4n\pi + \frac{3\pi}{2}$$

$$x = \dots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{11\pi}{2}, \dots$$

| | ***x*** | ***y*** |
|---|---|---|
| $0 - \frac{\pi}{2}$ | $-\frac{\pi}{2}$ | $\infty$ |
| $\frac{\pi}{2} - \frac{\pi}{2}$ | 0 | $-\frac{1}{2}$ |
| $\pi - \frac{\pi}{2}$ | $\frac{\pi}{2}$ | 0 |
| $\frac{3\pi}{2} - \frac{\pi}{2}$ | $\pi$ | $\frac{1}{2}$ |
| $2\pi - \frac{\pi}{2}$ | $\frac{3\pi}{2}$ | $\infty$ |



### *Exercise*

Graph over a 1-period interval $y = 1 - 2\cot 2\left(x + \frac{\pi}{2}\right)$

***Solution***

***Amplitude***: $n/a$

***Period***: $P = \frac{\pi}{2}$

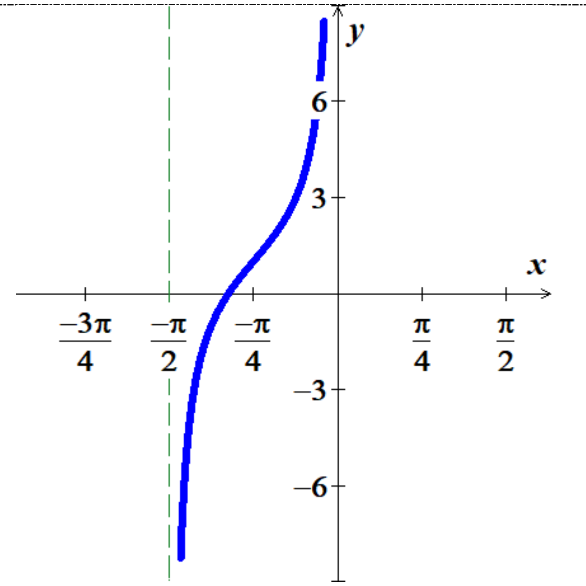
***Phase Shift***: $\varphi = -\frac{\pi}{2}$

***VT:*** $y = 1$

***Asymptotes***:

$-\frac{\pi}{2} + n\pi$

| | $x$ | $y$ |
|---|---|---|
| $0 - \frac{\pi}{2}$ | $-\frac{\pi}{2}$ | $\infty$ |
| $\frac{\pi}{8} - \frac{\pi}{2}$ | $-\frac{3\pi}{8}$ | $-1$ |
| $\frac{\pi}{4} - \frac{\pi}{2}$ | $-\frac{\pi}{4}$ | $1$ |
| $\frac{3\pi}{8} - \frac{\pi}{2}$ | $-\frac{\pi}{8}$ | $3$ |
| $\frac{\pi}{2} - \frac{\pi}{2}$ | $0$ | $\infty$ |




### *Exercise*

Graph over a 1-period interval $y = \frac{2}{3}\tan\left(\frac{3}{4}x - \pi\right) - 2$

***Solution***

***Amplitude***: $n/a$

***Period***: $P = \frac{4\pi}{3}$

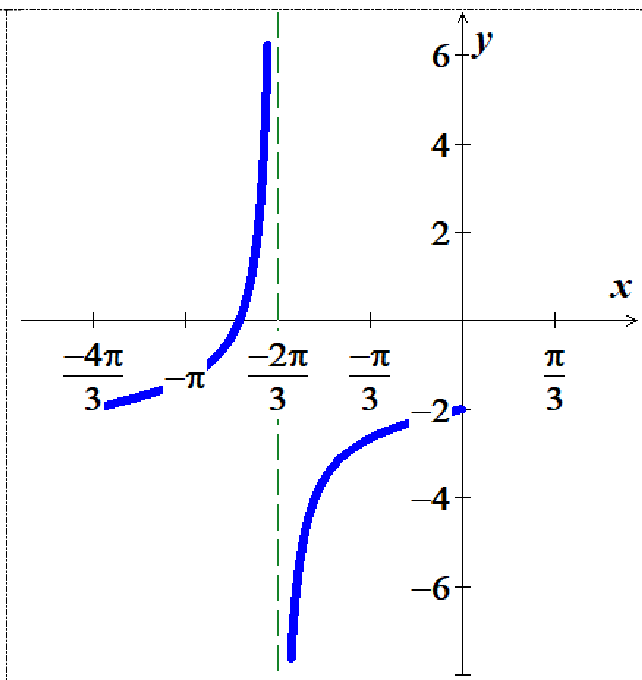
***Phase Shift***: $\varphi = -\frac{4\pi}{3}$

***VT:*** $y = -2$

***Asymptotes***:

$x = \frac{3\pi}{4} + n\pi$

| | $x$ | $y$ |
|---|---|---|
| $0 - \frac{4\pi}{3}$ | $-\frac{4\pi}{3}$ | $-2$ |
| $\frac{\pi}{3} - \frac{4\pi}{3}$ | $-\pi$ | $-\frac{4}{3}$ |
| $\frac{2\pi}{3} - \frac{4\pi}{3}$ | $-\frac{2\pi}{3}$ | $\infty$ |
| $\pi - \frac{4\pi}{3}$ | $-\frac{\pi}{3}$ | $-\frac{8}{3}$ |
| $\frac{4\pi}{3} - \frac{4\pi}{3}$ | $0$ | $-2$ |



### *Exercise*

Graph one complete cycle $y = 3 + 2\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)$

***Solution***

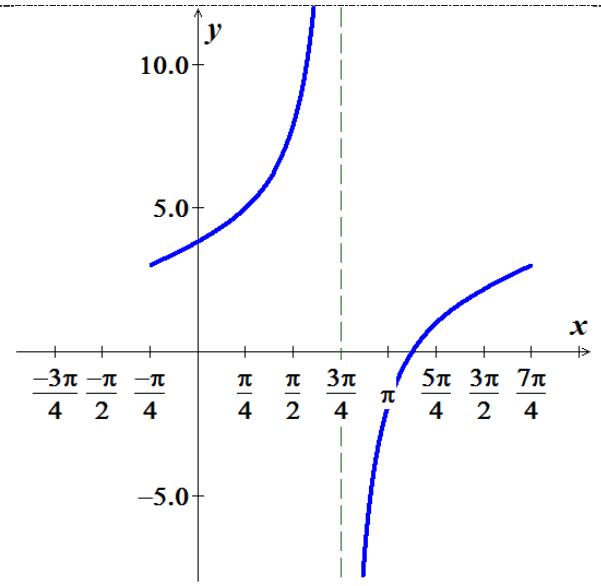
***Amplitude***: $n/a$

***Period***: $P = 2\pi$

***Phase Shift***: $\varphi = -\frac{\pi}{4}$

***VT:*** $y = 3$

| | $x$ | $y$ |
|---|---|---|
| $0 - \frac{\pi}{4}$ | $-\frac{\pi}{4}$ | 3 |
| $\frac{\pi}{2} - \frac{\pi}{4}$ | $\frac{\pi}{4}$ | 5 |
| $\pi - \frac{\pi}{4}$ | $\frac{3\pi}{4}$ | $\infty$ |
| $\frac{3\pi}{2} - \frac{\pi}{4}$ | $\frac{5\pi}{4}$ | 1 |
| $2\pi - \frac{\pi}{4}$ | $\frac{7\pi}{4}$ | 3 |




### *Exercise*

Graph one complete cycles $y = -2 - \cot\left(x - \frac{\pi}{4}\right)$

***Solution***

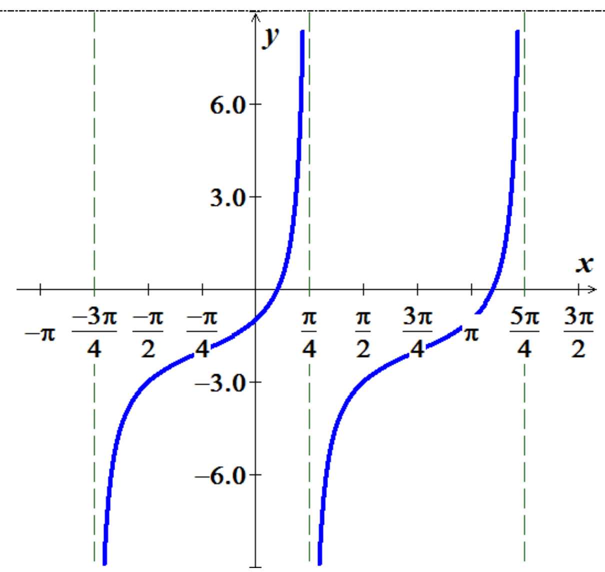
***Amplitude***: $n/a$

***Period***: $P = \pi$

***Phase Shift***: $\varphi = \frac{\pi}{4}$

***VT:*** $y = -2$

| | $x$ | $y$ |
|---|---|---|
| $0 + \frac{\pi}{4}$ | $\frac{\pi}{4}$ | $\infty$ |
| $\frac{\pi}{4} + \frac{\pi}{4}$ | $\frac{\pi}{2}$ | $-3$ |
| $\frac{\pi}{2} + \frac{\pi}{4}$ | $\frac{3\pi}{4}$ | $-2$ |
| $\frac{3\pi}{4} + \frac{\pi}{4}$ | $\pi$ | $-1$ |
| $\pi + \frac{\pi}{4}$ | $\frac{5\pi}{4}$ | $\infty$ |



### ***Exercise***

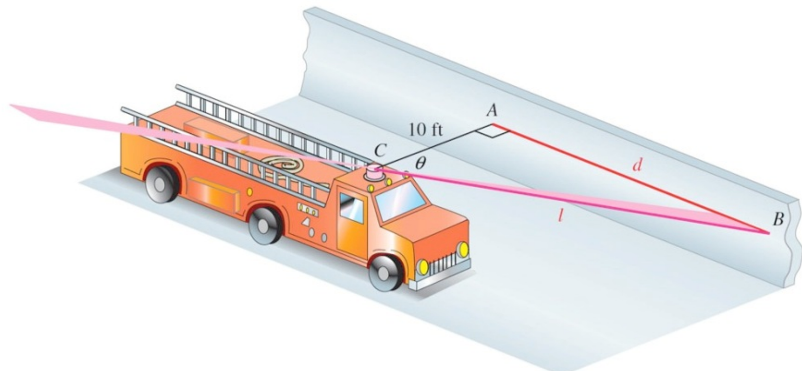
A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top is 10 *feet* from the wall and rotates through one complete revolution every 2 *seconds*. Graph the function that gives the length $d$ in terms of time $t$ from $t = 0$ to $t = 2$.

***Solution***

$$\omega = \frac{\theta}{t} = \frac{2\pi}{2} = \pi \ rad / sec$$

$$\tan\theta = \frac{d}{10} \quad \rightarrow \ d = 10\tan\theta$$

$$d(t) = 10\tan \pi t$$

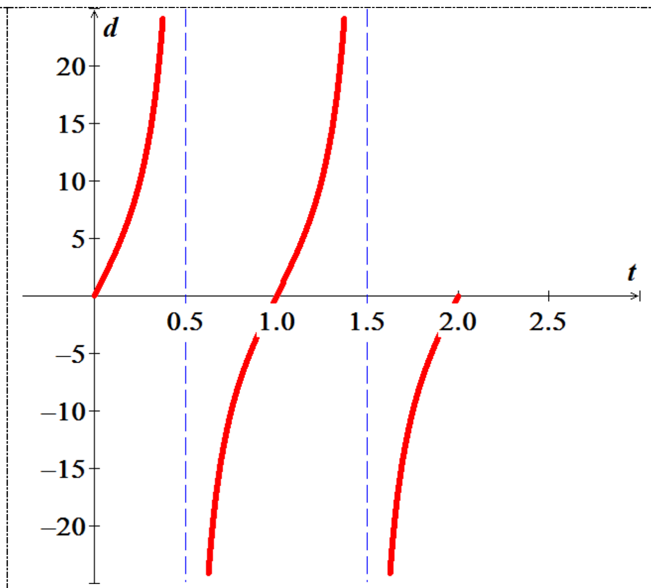



***Period*** $= \frac{\pi}{\pi} = 1$

One cycle: $0 \le \pi t \le \pi$

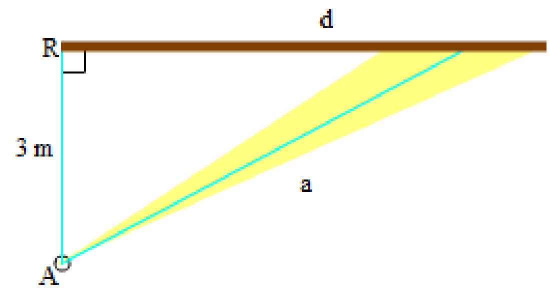
$0 \le t \le 1$

| $t$ | $d = 10\tan \pi t$ |
|---|---|
| 0 | 0 |
| $\frac{1}{4}$ | 10 |
| $\frac{1}{2}$ | $\infty$ |
| $\frac{3}{4}$ | $-10$ |
| 1 | 0 |



### *Exercise*

A rotating beacon is located 3 *m* south of point *R* on an east-west wall. *d*, the length of the light display along the wall from *R*, is given by $d = 3\tan 2\pi t$, where *t* is time measured in seconds since the beacon started rotating. (When *t* = 0, the beacon is aimed at point *R*. When the beacon is aimed to the right of *R*, the value of *d* is positive; *d* is negative if the beacon is aimed to the left of *R*.) Find a for *t* = 0.8




***Solution***

$$d = 3\tan\left(2\pi(0.8)\right)$$

$$\approx -9.23\ m$$

### *Exercise*

Let a person whose eyes are $h_1$ feet from the ground stand ***d*** feet from an object $h_1$ feet tall, where $h_2 > h_1$ feet. Let θ be the angle of elevation to the top of the object.

*a)* Show that $d = \left(h_2 - h_1\right)\cot\theta$

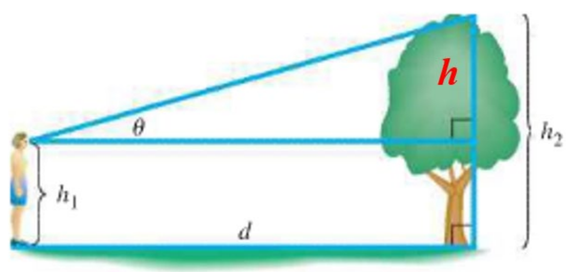
*b)* Let $h_2 = 55$ and $h_1 = 5$. Graph ***d*** for the interval $0 < \theta \le \frac{\pi}{2}$

***Solution***

***a)*** $h = h_2 - h_1$

$\cot\theta = \frac{d}{h}$

$d = \left(h_2 - h_1\right)\cot\theta$




***b)*** $d = (55 - 5)\cot\theta$

$d = 50\cot\theta \quad 0 < \theta \le \frac{\pi}{2}$