Solution   *Section* 3.2 – Polynomial Functions

Exercise

$f(x)$ Determine the end behavior of the graph of the polynomial function $f(x)=5x^3+7x^2-x+9$

Solution

Leading term: $5x^3$ with 3rd degree (n is odd)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises right

$11x^3$

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=11x^3-6x^2+x+3$

Solution

Leading term: with 3rd degree (n is odd)

$x\to-\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\quad$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=-11x^3-6x^2+x+3$

Solution

Leading term: $-11x^3$ with 3rd degree (n is odd)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to-\infty$ $\quad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x)=5x^4+7x^2-x+9$

Solution

Leading term: $5x^4$ with 4rd degree (n is even)

$x\to-\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises left

$x\to\infty \;\Rightarrow\; f(x)\to\infty$ $\quad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = 11x^4 - 6x^2 + x + 3$

Solution

Leading term: $11x^4$ with 4^{rd} degree (*n* is even)

$x \to -\infty \;\Rightarrow\; f(x) \to \infty$ $\qquad f(x)$ rises left

$x \to \infty \;\Rightarrow\; f(x) \to \infty$ $\qquad f(x)$ rises right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -5x^4 + 7x^2 - x + 9$

Solution

Leading term: $-5x^4$ with 4^{rd} degree (*n* is even)

$x \to -\infty \;\Rightarrow\; f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \;\Rightarrow\; f(x) \to -\infty$ $\qquad f(x)$ falls right

Exercise

Determine the end behavior of the graph of the polynomial function $f(x) = -11x^4 - 6x^2 + x + 3$

Solution

Leading term: $-11x^4$ with 4^{rd} degree (*n* is even)

$x \to -\infty \;\Rightarrow\; f(x) \to -\infty$ $\qquad f(x)$ falls left

$x \to \infty \;\Rightarrow\; f(x) \to -\infty$ $\qquad f(x)$ falls right

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x) = x^3 - x - 1;$ *between* 1 *and* 2

Solution

$$f(1) = (1)^3 - (1) - 1 \underline{= -1|}$$

$$f(2) = (2)^3 - (2) - 1 \underline{= 5|}$$

Since $f(1)$ *and* $f(2)$ have opposite signs; therefore, the polynomial has a real zero between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^3-4x^2+2;\quad between\ \ 0\ and\ 1$

Solution

$$f(0)=(0)^3-4(0)^2+2 \underline{=2|}$$

$$f(1)=(1)^3-4(1)^2+2 \underline{=-1|}$$

Since $f(0)\ and\ f(1)$ have opposite signs; therefore, the polynomial has a real zero between 0 and 1.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=2x^4-4x^2+1;\quad between\ \ -1\ and\ 0$

Solution

$$f(-1)=2(-1)^4-4(-1)^2+1 \underline{=-1|}$$

$$f(0)=2(0)^4-4(0)^2+1 \underline{=1|}$$

Since $f(0)\ and\ f(-1)$ have opposite signs; therefore, the polynomial has a real zero between -1 and 0.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^4+6x^3-18x^2;\quad between\ \ 2\ and\ 3$

Solution

$$f(2)=(2)^4+6(2)^3-18(2)^2 \ \underline{=-8|}$$

$$f(3)=(3)^4+6(3)^3-18(3)^2 \ \underline{=81|}$$

Since $f(2)\ and\ f(3)$ have opposite signs; therefore, the polynomial has a real zero between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^3+x^2-2x+1;\quad between\ \ -3\ and\ -2$

Solution

$$f(-3)=(-3)^3+(-3)^2-2(-3)+1 \underline{=-11}$$

$$f(-2)=(-2)^3+(-2)^2-2(-2)+1 \underline{=1}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs; therefore, the polynomial has a real zero between –2 and –3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^5-x^3-1;$ *between* 1 *and* 2

Solution

$$f(1)=(1)^5-(1)^3-1 \underline{=-1}$$

$$f(2)=(2)^5-(2)^3-1 \underline{=23}$$

Since $f(1)$ *and* $f(2)$ have opposite signs; therefore, the polynomial has a real zero between 1 and 2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-10x+9;$ *between* -3 *and* -2

Solution

$$f(-3)=3(-3)^3-10(-3)+9 \underline{=-42}$$

$$f(-2)=3(-2)^3-10(-2)+9 \underline{=5}$$

Since $f(-3)$ *and* $f(-2)$ have opposite signs; therefore, the polynomial has a real zero between –3 and –2.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-8x^2+x+2;$ *between* 2 *and* 3

Solution

$$f(2)=3(2)^3-8(2)^2+(2)+2 \underline{=-4}$$

$$f(3)=3(3)^3-8(3)^2+(3)+2 \underline{=14}$$

Since $f(2)$ *and* $f(3)$ have opposite signs; therefore, the polynomial has a real zero between 2 and 3.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=3x^3-8x^2+x+2;$ *between* 1 *and* 2

Solution

$$f(1)=3(1)^3-8(1)^2+(1)+2 \underline{=-2}$$

$$f(2)=3(2)^3-8(2)^2+(2)+2 \underline{=-4}$$

Since $f(1)$ *and* $f(2)$ have same signs; therefore, cannot be determined.

Exercise

Use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers. $f(x)=x^5+2x^4-6x^3+2x-3;$ *between* 0 *and* 1

Solution

$$f(0)=(0)^5+2(0)^4-6(0)^3+2(0)-3 \underline{=-3}$$

$$f(1)=(1)^5+2(1)^4-6(1)^3+2(1)-3 \underline{=-4}$$

Since $f(0)$ *and* $f(1)$ have same signs; therefore, cannot be determined.