

Solution **Section 3.7 – Implicit Differentiation and related Rates**

Exercise

Find dy/dx for the equation $y^2 + x^2 - 2y - 4x = 4$

Solution

$$\frac{d}{dx}[y^2 + x^2 - 2y - 4x] = \frac{d}{dx}[4]$$

$$\frac{d}{dx}\left[y^2\right] + \frac{d}{dx}\left[x^2\right] - \frac{d}{dx}[2y] - \frac{d}{dx}[4x] = \frac{d}{dx}[4]$$

$$2y \frac{dy}{dx} + 2x - 2 \frac{dy}{dx} - 4 = 0$$

$$2(y-1) \frac{dy}{dx} = 4 - 2x$$

$$(y-1) \frac{dy}{dx} = 2 - x$$

$$\frac{dy}{dx} = \frac{2-x}{y-1}$$

Exercise

Find dy/dx : $x^2y^2 - 2x = 3$

Solution

$$2xy^2 + 2x^2yy' - 2 = 0$$

$$2x^2yy' = 2 - 2xy^2$$

$$y' = \frac{2(1-xy^2)}{2x^2y}$$

$$\frac{dy}{dx} = \frac{1-xy^2}{x^2y}$$

Exercise

Find dy/dx : $x^2 - xy + y^2 = 4$ and evaluate the derivative at the given point $(0, -2)$

Solution

$$2x - (y + xy') + 2yy' = 0$$

$$-y - xy' + 2yy' = -2x$$

$$(2y - x)y' = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

$$\begin{aligned} @ (0, -2) \rightarrow \frac{dy}{dx} &= \frac{-2 - 2(0)}{2(-2) - (0)} \\ &= \frac{-2}{-4} \\ &= \frac{1}{2} \end{aligned}$$

Exercise

Find the rate of change of x with respect to p . $p = \sqrt{\frac{200-x}{2x}}$, $0 < x \leq 200$

Solution

$$p^2 = \frac{200-x}{2x}$$

$$2xp^2 = 200 - x$$

$$2p^2 \frac{dx}{dp} + 4xp = -\frac{dx}{dp}$$

$$2p^2 \frac{dx}{dp} + \frac{dx}{dp} = -4xp$$

$$\frac{dx}{dp} (2p^2 + 1) = -4xp$$

$$\frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}$$

Exercise

Implicit Differentiation: Find $\frac{dy}{dx}$, $e^{xy} + x^2 - y^2 = 10$

Solution

$$\frac{d}{dx}(e^{xy}) + \frac{d}{dx}(x^2) - \frac{d}{dx}(y^2) = \frac{d}{dx}(10)$$

$$e^{xy} \frac{d}{dx}(xy) + 2x - 2yy' = 0$$

$$e^{xy}(xy' + y) + 2x - 2yy' = 0$$

$$e^{xy}xy' + e^{xy}y + 2x - 2yy' = 0$$

$$y'(xe^{xy} - 2y) = -2x - e^{xy}y$$

$$y' = \frac{-2x - e^{xy}y}{xe^{xy} - 2y}$$

Exercise

Find the slope of the tangent line to the circle $x^2 - 9y^2 = 16$ at the point (5, 1)

Solution

$$2x - 18y \frac{dy}{dx} = 0$$

$$-18y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{-18y} = \frac{x}{9y}$$

$$@ (5, 1) \rightarrow \frac{dy}{dx} = \frac{5}{9(1)} = \frac{5}{9}$$

Exercise

The demand function for a product is given by $P = \frac{2}{0.001x^2 + x + 1}$. Find dx / dp implicitly.

Solution

$$0.001x^2 + x + 1 = \frac{2}{p}$$

$$\frac{d}{dp} \left[0.001x^2 + x + 1 \right] = \frac{d}{dp} \left[2p^{-1} \right]$$

$$0.002x \frac{dx}{dp} + \frac{dx}{dp} = -2p^{-2}$$

$$(0.002x + 1) \frac{dx}{dp} = -\frac{2}{p^2}$$

$$\frac{dx}{dp} = -\frac{2}{(0.002x + 1)p^2}$$