1/ $\{(2,-4),(2,1)\}$

a) orthogonal?

$(2,-4)\cdot(2,1) = 4-4=0$

it's orthogonal

b) orthonormal? $\sqrt{4+16} \neq 1$

it's <u>not</u> orthonormal

c) Since it's orthogonal $\Rightarrow$ the set is a basis for R^2

$\left(\begin{vmatrix} 2 & 2 \\ -4 & 1 \end{vmatrix} = 10 \neq 0\right)$.

5/ $\{(4,-1,1),(-1,0,4),(-4,-17,-1)\}$

a) $(4,-1,1)\cdot(-1,0,4) = -4+4=0$

$(4,-1,1)\cdot(-4,-17,-1) = -16+17-1=0$

$(-1,0,4)\cdot(-4,-17,-1) = 4-4=0$

The set is orthogonal

b) $\sqrt{16+1+1} = \sqrt{18} \neq 0$

The set is <u>not</u> orthonormal

c) Since it's orthogonal $\Rightarrow$ the set is a basis in R^3

$\det \neq 0$.

11) $\left\{\left(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}\right), \left(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right), \left(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)\right\}$

a) $\left(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}\right) \cdot \left(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right) = 0$

$\left(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}\right) \cdot \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = -\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} = 0$

$\left(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right) \cdot \left(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right) = \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4} = 0$

The set is orthogonal

b) $\sqrt{\frac{2}{4} + \frac{2}{4}} = 1$ –

$\sqrt{\frac{2}{4} + \frac{2}{4}} = 1$

$\sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = 1$

The set is an orthonormal.

c) set is in R^4 & we only have 3 vectors (4x3)

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{1}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{1}{2} \end{bmatrix} \xrightarrow{\times 2} \begin{bmatrix} \sqrt{2} & 0 & 1 \\ 0 & \sqrt{2} & 1 \\ 0 & \sqrt{2} & -1 \\ \sqrt{2} & 0 & 1 \end{bmatrix} \begin{matrix} \\ \\ R_3 - R_2 \\ R_4 - R_1 \end{matrix}$$

$$\begin{bmatrix} \sqrt{2} & 0 & 1 \\ 0 & \sqrt{2} & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

15) $\{(\sqrt{3}, \sqrt{3}, \sqrt{3}), (-\sqrt{2}, 0, \sqrt{2})\}$

a) $(\sqrt{3}, \sqrt{3}, \sqrt{3}) \cdot (-\sqrt{2}, 0, \sqrt{2}) = -\sqrt{6} + \sqrt{6} = 0$

set is orthogonal

b) $\dfrac{(\sqrt{3}, \sqrt{3}, \sqrt{3})}{\|\sqrt{3}, \sqrt{3}, \sqrt{3}\|} = \dfrac{(\sqrt{3}, \sqrt{3}, \sqrt{3})}{\sqrt{3+3+3}} = \frac{1}{3}(\sqrt{3}, \sqrt{3}, \sqrt{3})$

$\vec{u}_1 = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

$\|(-\sqrt{2}, 0, \sqrt{2})\| = \sqrt{2+2} = 2$

$\vec{u}_2 = \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$

25) $A = \{(3,4), (1,0)\}$

$\vec{v}_1 = (3,4) \quad \vec{v}_2 = (1,0)$

$\vec{w}_1 = \vec{v}_1 = (3,4)$

$\vec{u}_1 = \dfrac{\vec{w}_1}{\|\vec{w}_1\|} = \left(\frac{3}{5}, \frac{4}{5}\right)$

$\vec{w}_2 = \vec{v}_2 - \dfrac{\vec{v}_2 \cdot \vec{w}_1}{\|\vec{w}_1\|^2} \vec{w}_1$

$= (1,0) - \dfrac{(1,0) \cdot (3,4)}{5}(3,4)$

$= (1,0) - \frac{3}{25}(3,4)$

$= (1,0) - \left(\frac{9}{25}, \frac{12}{25}\right)$

$= \left(\frac{16}{25}, -\frac{12}{25}\right)$

$\|\vec{w}_2\| = \sqrt{\frac{256 + 144}{252}}$

$= \frac{20}{25}$

$\vec{u}_2 = \dfrac{\vec{w}_2}{\|\vec{w}_2\|} = \frac{25}{20}\left(\frac{16}{25}, -\frac{12}{25}\right)$

$= \left(\frac{4}{5}, -\frac{3}{5}\right)$

29/ $B = \{(2,1,-2), (1,2,2), (2,-2,1)\}$
$\vec{v}_1 \quad \vec{v}_2 \quad \vec{v}_3$

$\vec{w}_1 = \vec{v}_1 = (2,1,-2)$ $\quad \vec{u}_1 = \frac{(2,1,-2)}{\sqrt{4+1+4}} = \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$

$\vec{w}_2 = \vec{v}_2 - \frac{\vec{v}_2 \cdot \vec{w}_1}{\|\vec{w}_1\|^2} \vec{w}_1$

$= (1,2,2) - \frac{(1,2,2)\cdot(2,1,-2)}{9}(2,1,-2)$

$= (1,2,2) - 0$

$= (1,2,2)$

$\vec{u}_2 = \frac{(1,2,2)}{3} = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$\vec{w}_3 = \vec{v}_3 - \frac{\langle \vec{v}_3, \vec{w}_1 \rangle}{\|\vec{w}_1\|^2} \vec{w}_1 - \frac{\langle \vec{v}_3, \vec{w}_2 \rangle}{\|\vec{w}_2\|^2} \vec{w}_2$

$= (2,-2,1) - \frac{(2,-2,1)\cdot(2,1,-2)}{3}(2,1,-2)$

$- \frac{(2,-2,1)\cdot(1,2,2)}{3}(1,2,2)$

$= (2,-2,1) - 0 - 0$

$= (2,-2,1)$

$\vec{u}_3 = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$

$\left\{\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right), \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)\right\}$

39/ $B = \{(1,2,-1,0), (2,2,0,1), (1,1,-1,0)\}$ — v_1, v_2, v_3

$\vec{w}_1 = \vec{v}_1 = (1,2,-1,0)$

$\|\vec{w}_1\| = \sqrt{1+4+1} = \sqrt{6}$

$\vec{u}_1 = \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, 0\right)$

$\vec{w}_2 = \vec{v}_2 - \frac{\langle \vec{v}_2, \vec{w}_1 \rangle}{\|\vec{w}_1\|^2} \vec{w}_1$

$= (2,2,0,1) - \frac{(2,2,0,1)\cdot(1,2,-1,0)}{6}(1,2,-1,0)$

$= (2,2,0,1) - (1,2,-1,0)$

$= (1,0,1,1)$

$\|\vec{w}_2\| = \sqrt{1+1+1} = \sqrt{3}$

$\vec{u}_2 = \left(\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$\vec{w}_3 = \vec{v}_3 - \frac{\langle \vec{v}_3, \vec{w}_1 \rangle}{\|\vec{w}_1\|^2} \vec{w}_1 - \frac{\langle \vec{v}_3, \vec{w}_2 \rangle}{\|\vec{w}_2\|^2} \vec{w}_2$

$= (1,1,-1,0) - \frac{(1,1,-1,0)\cdot(1,2,-1,0)}{6}(1,2,-1,0)$

$- \frac{(1,1,-1,0)\cdot(1,0,1,1)}{3}(1,0,1,1)$

$= (1,1,-1,0) - \frac{2}{3}(1,2,-1,0) - 0$

$= \left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0\right)$

$\|\vec{w}_3\| = \sqrt{\frac{1}{9} + \frac{1}{9}\ \frac{1}{9}} = \frac{\sqrt{3}}{3}$

$\vec{u}_3 = \frac{3}{\sqrt{3}}\left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0\right)$

$= \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, 0\right)$