Solution   *Section* 3.1 – Increasing and Decreasing Functions

Exercise

Find the critical numbers and decide on which the function $f(x) = x - 4\ln(3x-9)$ is increasing or decreasing.

Solution

$$3x - 9 > 0 \Rightarrow \boxed{x > 3}$$

$$\begin{aligned} f'(x) &= 1 - 4\frac{3}{3x-9} \\ &= 1 - \frac{12}{3x-9} \\ &= \frac{3x-9-12}{3x-9} \\ &= \frac{3x-21}{3x-9} = 0 \end{aligned}$$

$$3x - 21 = 0$$
$$3x = 21$$
$$\boxed{x = 7}$$

3	7	∞
	$f'(4) < 0$	$f'(8) > 0$
	Decreasing	*Increasing*

$CN: x = 3,\ 7$

Increasing: $(7, \infty)$

Decreasing: $(3, 7)$

Exercise

Find the open intervals on which the function $f(x) = x^3 - 12x$ is increasing or decreasing

Solution

$$f'(x) = 3x^2 - 12 = 0$$
$$\Rightarrow 3x^2 = 12$$
$$x^2 = 4$$
$$\Rightarrow x = \pm 2 \quad \text{(Critical Numbers - } CN\text{)}$$

-∞	-2	2	∞
$f'(-3) > 0$	$f'(1) < 0$	$f'(3) > 0$	
Increasing	*Decreasing*	*Increasing*	

Increasing: (-∞, -2) and (2, ∞)

Decreasing: (-2, 2)

Exercise

Find the open intervals on which the function $f(x) = x^{2/3}$ is increasing or decreasing

Solution

$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$= \frac{2}{3x^{1/3}} = 0$$

$\Rightarrow$ *Undefined* $x = 0$ (CN)

-∞	**0**	∞
$f'(-1) < 0$	$f'(1) > 0$	
Decreasing	*Increasing*	

Decreasing: (-∞, 0)

Increasing: (0, ∞)

Exercise

Find the critical numbers and the open intervals on which the function is increasing or decreasing.

$f(x) = x\sqrt{x+1}$

Solution

$$f'(x) = \sqrt{x+1} + \frac{1}{2}x(x+1)^{-1/2}$$

$$= \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$$

$$= \frac{2x+2+x}{2\sqrt{x+1}}$$

$$= \frac{3x+2}{2\sqrt{x+1}}$$

Critical numbers are $x = -\frac{2}{3}$ and $x = -1$, but the domain is $[-1, \infty)$.

Interval(s)	$(-1, -2/3)$	$(-2/3, \infty)$
Sign of f'	$f'(-0.9) < 0$	$f'(0) > 0$
Conclusion for f	*decreasing*	*increasing*

The function is decreasing on $\left(-1,\ -\frac{2}{3}\right)$

The function is increasing on $\left(-\frac{2}{3},\ \infty\right)$

Exercise

Find the critical numbers and the open intervals on which the function is increasing or decreasing.

$$f(x)=\frac{x}{x^2+4}$$

Solution

$$f'(x)=\frac{(1)\left(x^2+4\right)-x(2x)}{\left(x^2+4\right)^2}$$

$$=\frac{x^2+4-2x^2}{\left(x^2+4\right)^2}$$

$$=\frac{-x^2+4}{\left(x^2+4\right)^2} \qquad -x^2+4=0 \Rightarrow x^2=4 \rightarrow x=\pm 2$$

Critical numbers are $x=\pm 2$.

Interval(s)	$(-\infty,-2)$	$(-2,\ 2)$	$(2,\ \infty)$
Sign of f'	$f'(-2)<0$	$f'(0)>0$	$f'(0)<0$
Conclusion for f	*decreasing*	*increasing*	*decreasing*

Decreasing: $(-\infty,-2)\cup(2,\infty)$.
Increasing: $(-2,\ 2)$.

Exercise

Find the critical numbers and the open intervals on which the function is increasing or decreasing.

$$f(x)=\frac{x}{x^2+1}$$

Solution

$$f'(x)=-\frac{(x-1)(x+1)}{(x^2+1)^2}$$

Critical numbers are x = 1, and x=-1.

Interval(s)	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
Sign of f'	$f'(-2)<0$	$f'(0)>0$	$f'(0)<0$
Conclusion for f	*decreasing*	*increasing*	*decreasing*

Decreasing: $(-\infty,-1)\cup(1,\infty)$.
Increasing: $(-1,\ 1)$.

Exercise

Find the critical numbers and the open intervals on which the function is increasing or decreasing.

$$f(x) = x\sqrt{x+1}$$

Solution

$$f'(x) = \frac{3x+2}{2\sqrt{x+1}}$$

Critical numbers are $x = -\frac{2}{3}$ *and* $x = -1$, but the domain is $[-1, \infty)$.

Interval(s)	$(-1, -2/3)$	$(-2/3, \infty)$
Sign of f'	$f'(-0.9) < 0$	$f'(0) > 0$
Conclusion for f	decreasing	increasing

The function is decreasing on $(-1, -2/3)$

The function is increasing on $(-2/3, \infty)$

Exercise

A county realty group estimates that the number of housing starts per year over the next three years will be

$$H(r) = \frac{300}{1+0.03r^2}$$

Where *r* is the mortgage rate (in percent).

a) Where is $H(r)$ increasing?

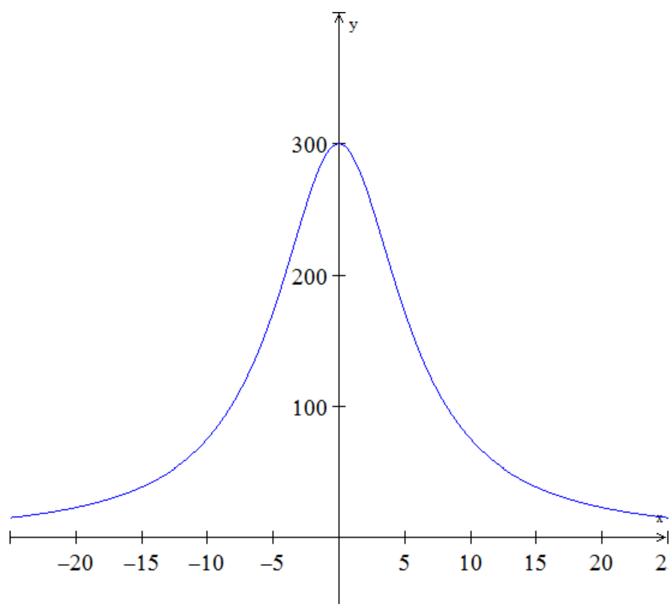
b) Where is $H(r)$ decreasing?

Solution

$$H'(r) = \frac{-300(0.06r)}{\left(1+0.03r^2\right)^2}$$

$$H'(r) = \frac{-18r}{\left(1+0.03r^2\right)^2}$$

$$-18r = 0 \Rightarrow \boxed{r=0} \quad (CN)$$



a) $H(r)$ is increasing on the interval $(-\infty,\ 0)$

b) $H(r)$ is decreasing on the interval $(0,\ \infty)$

Exercise

Suppose the total cost $C(x)$ to manufacture a quantity x of insecticide (in hundreds of liters) is given by $C(x) = x^3 - 27x^2 + 240x + 750$. Where is $C(x)$ decreasing?

Solution

$$C'(x) = 3x^2 - 54x + 240 = 0$$

$$\Rightarrow x = 8,\ 10$$

0 8	10	
$C'(1) = 189 > 0$	$C' < 0$	$C' > 0$
Increasing	***Decreasing***	***Increasing***

$C(x)$ is decreasing $(8,\ 10)$

Exercise

A manufacturer sells telephones with cost function $C(x) = 6.14x - 0.0002x^2,\quad 0 \le x \le 950$ and revenue function $R(x) = 9.2x - 0.002x^2,\quad 0 \le x \le 950$. Determine the interval(s) on which the profit function is increasing.

Solution

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 9.2x - 0.002x^2 - \left(6.14x - 0.0002x^2\right) \\ &= 9.2x - 0.002x^2 - 6.14x + 0.0002x^2 \\ &= -0.0018x^2 + 3.06x \end{aligned}$$

$$P'(x) = -0.0036x + 3.06 = 0$$

$$-0.0036x = -3.06$$

$$x = \frac{-3.06}{-0.0036} = 850$$

The profit function is increasing on the interval $(850,\ 950]$

Exercise

The cost of a computer system increases with increased processor speeds. The cost C of a system as a function of processor speed is estimated as $C(x)=14x^2-4x+1200$, where x is the processor speed in MHz. Determine the intervals where the cost function $C(x)$ is decreasing.

Solution

$$C'(x)=28x-4=\mathbf{0}$$

$$\Rightarrow x=\frac{4}{28}=\frac{1}{7}$$

	$\mathbf{\frac{1}{7}}$	
$C'(0)=-4<0$		$C'>0$
Decreasing		***Increasing***

The cost function $C(x)$ is decreasing $\left(0,\ \frac{1}{7}\right)$

Exercise

The percent of concentration of a drug in the bloodstream t hours after the drug is administered is given by $K(t)=\dfrac{t}{t^2+36}$. On what time interval is the concentration of the drug increasing?

Solution

$$f=t \qquad f'=1$$

$$g=t^2+36 \quad g'=2t$$

$$K'(t)=\frac{1\left(t^2+36\right)-2t(t)}{\left(t^2+36\right)^2} \qquad\qquad K=\frac{f}{g}\Rightarrow K'=\frac{f'g+g'f}{g^2}$$

$$=\frac{t^2+36-2t^2}{\left(t^2+36\right)^2}$$

$$=\frac{36-t^2}{\left(t^2+36\right)^2}$$

$$K'(t)=0$$

$$\frac{36-t^2}{\left(t^2+36\right)^2}=0\Rightarrow 36-t^2=0$$

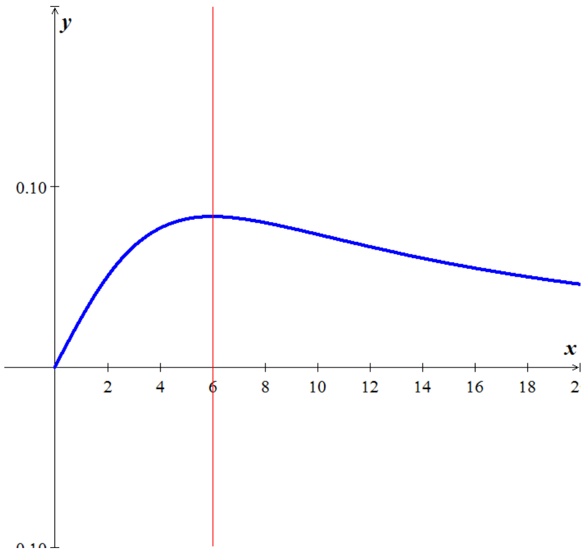
$$t^2=36$$

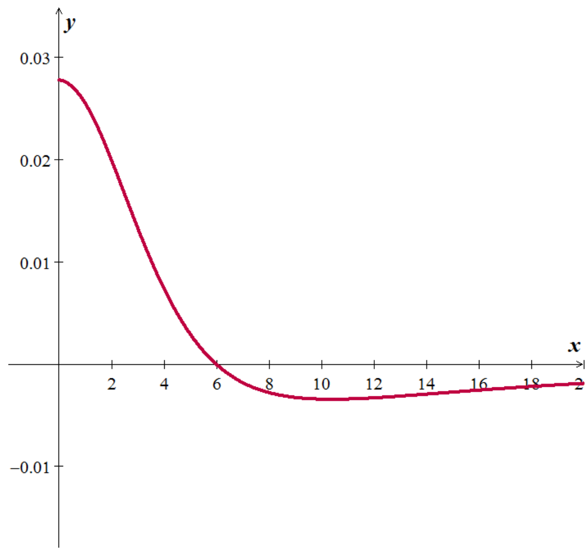
$t = \pm\sqrt{36} = \pm 6$

$\Rightarrow \boxed{t = 6}$

0	6
$K'(1) = \frac{35}{37^2} > 0$	$K'(7) < 0$
Increasing	***Decreasing***

The concentration of the drug is increasing over $\boxed{(0,\ 6)}$





Exercise

A probability function is defined by $f(x) = \frac{1}{\sqrt{6\pi}} e^{-x^2/8}$. Give the intervals where the function is increasing and decreasing.

Solution

$$f'(x) = \frac{1}{\sqrt{6\pi}} e^{-x^2/8} \overbrace{\left(-\frac{2x}{8}\right)}^{\left(e^{-x^2/8}\right)'}$$

$$= -\frac{x}{4\sqrt{6\pi}} e^{-x^2/8}$$

$$f'(x) = 0 \quad \Rightarrow x = 0$$

0

$f'(-1) = -\frac{-1}{4\sqrt{6\pi}} e^{-(-1)^2/8} > 0$	$f'(1) = -\frac{1}{4\sqrt{6\pi}} e^{-(1)^2/8} < 0$
Increasing	***Decreasing***

The function is increasing on $(-\infty,\ 0)$ and decreasing on $(0,\ \infty)$