Exercise

Solve $3x^3 + 2x^2 = 12x + 8$

Solution

$3x^3 + 2x^2 - (12x + 8) = 0$

$x^2(3x + 2) - 4(3x + 2) = 0$

$(3x + 2)(x^2 - 4) = 0$

$3x + 2 = 0$	$x^2 - 4 = 0$
$3x = -2$	$x^2 = 4$
$x = -\frac{2}{3}$	$x = \pm 2$

∴ Solutions: $x = -\frac{2}{3},\ \pm 2$

Exercise

Solve: $x^3 + x^2 - 4x - 4 = 0$

Solution

$x^2(x + 1) - 4(x + 1) = 0$

$(x + 1)(x^2 - 4) = 0$

$x + 1 = 0$	$x^2 - 4 = 0$
$x = -1$	$x^2 = 4$
	$x = \pm 2$

∴ Solutions: $x = -1,\ \pm 2$

Exercise

Solve: $x^3 + x^2 + 4x + 4 = 0$

Solution

$x^2(x + 1) + 4(x + 1) = 0$

$$(x+1)\left(x^2+4\right)=0$$

$x+1=0$ | $x^2+4=0$

$\underline{x=-1}$ | $x^2=-4$

$\underline{x=\pm 2i}$

∴ Solutions: $\underline{x=-1,\ \pm 2i}$

Exercise

Solve: $x^3+4x^2-25x-100=0$

Solution

$$x^2(x+4)-25(x+4)=0$$

$$(x+4)\left(x^2-25\right)=0$$

$x+4=0$ | $x^2-25=0$

$\underline{x=-4}$ | $x^2=25$

$\underline{x=\pm 5}$

∴ Solutions: $\underline{x=-4,\ \pm 5}$

Exercise

Solve: $x^3-2x^2-x+2=0$

Solution

$$x^2(x-2)-(x-2)=0$$

$$(x-2)\left(x^2-1\right)=0$$

$x-2=0$ | $x^2-1=0$

$\underline{x=2}$ | $x^2=1$

$\underline{x=\pm 1}$

∴ Solutions: $\underline{x=2,\ \pm 1}$

Exercise

Solve: $x^3 - x^2 - 25x + 25 = 0$

Solution

$$x^2(x-1) - 25(x-1) = 0$$

$$(x-1)\left(x^2 - 25\right) = 0$$

$x - 1 = 0$	$x^2 - 25 = 0$
$\underline{x = 1}$	$x^2 = 25$
	$\underline{x = \pm 5}$

$\therefore$ Solutions: $\underline{x = 1,\ \pm 5}$

Exercise

Solve: $x^3 - x^2 = 16x - 16$

Solution

$$x^3 - x^2 - 16x + 16 = 0$$

$$x^2(x-1) - 16(x-1) = 0$$

$$(x-1)\left(x^2 - 16\right) = 0$$

$x - 1 = 0$	$x^2 - 16 = 0$
$\underline{x = 1}$	$x^2 = 16$
	$\underline{x = \pm 4}$

$\therefore$ Solutions: $\underline{x = 1,\ \pm 4}$

Exercise

Solve: $x^3 + x^2 + 25x + 25 = 0$

Solution

$$x^2(x+1) + 25(x+1) = 0$$

$$(x+1)\left(x^2 + 25\right) = 0$$

$x + 1 = 0$	$x^2 + 25 = 0$
$\underline{x = -1}$	$x^2 = -25$
	$\underline{x = \pm 5i}$

$\therefore$ Solutions: $\underline{x = -1,\ \pm 5i}$

Exercise

Solve: $x^3 + 2x^2 = 16x + 32$

Solution

$$x^3 + 2x^2 - 16x - 32 = 0$$
$$x^2(x+2) - 16(x+2) = 0$$
$$(x+2)(x^2 - 16) = 0$$

$x + 2 = 0$	$x^2 - 16 = 0$
$\underline{x = -2}$	$x^2 = 16$
	$\underline{x = \pm 4}$

$\therefore$ Solutions: $\underline{x = -2,\ \pm 4}$

Exercise

Solve: $2x^3 + 3x^2 - 6x - 9 = 0$

Solution

$$x^2(2x+3) - 3(2x+3) = 0$$
$$(2x+3)(x^2 - 3) = 0$$

$2x + 3 = 0$	$x^2 - 3 = 0$
$\underline{x = -\frac{3}{2}}$	$x^2 = 3$
	$\underline{x = \pm\sqrt{3}}$

$\therefore$ Solutions: $\underline{x = -\frac{3}{2},\ \pm\sqrt{3}}$

Exercise

Solve: $2x^3 + x^2 - 8x - 4 = 0$

Solution

$$x^2(2x+1) - 4(2x+1) = 0$$
$$(2x+1)(x^2 - 4) = 0$$

$2x+1=0$

$x=-\frac{1}{2}$

$x^2-4=0$

$x^2=4$

$x=\pm 2$

∴ Solutions: $x=-\frac{1}{2},\ \pm 2$

Exercise

Solve: $2x^3+16x^2+30x=0$

Solution

$2x\left(x^2+8x+15\right)=0$

$2x(x+3)(x+5)=0$

∴ Solutions: $x=0,\ -3,\ -5$

Exercise

Solve: $3x^3-9x^2-30x=0$

Solution

$3x\left(x^2-3x-10\right)=0$

$3x(x+2)(x-5)=0$

∴ Solutions: $x=0,\ -2,\ 5$

Exercise

Solve $x^4+3x^2=10$

Solution

$x^4+3x^2-10=0$

$\left(x^2+5\right)\left(x^2-2\right)=0$

$x^2+5=0$

$x^2=-5$

$x=\pm i\sqrt{5}$

$x^2-2=0$

$x^2=2$

$x=\pm\sqrt{2}$

∴ Solutions: $x=\pm i\sqrt{5},\ \pm\sqrt{2}$

Exercise

Solve: $5x^4 = 40x$

Solution

$$5x^4 - 40x = 0$$
$$5x\left(x^3 - 8\right) = 0$$
$$5x\left(x^3 - 2^3\right) = 0$$
$$5x(x-2)\left(x^2 + 2x + 2^2\right) = 0$$
$$5x(x-2)\left(x^2 + 2x + 4\right) = 0$$

$x = 0$ | $x = 2$ |

$$x^2 + 2x + 4 = 0$$
$$x = \frac{-2 \pm \sqrt{4-16}}{2}$$
$$= \frac{-2 \pm \sqrt{-12}}{2}$$
$$= \frac{-2 \pm 2i\sqrt{3}}{2}$$
$$= -1 \pm i\sqrt{3}$$

∴ Solutions: $x = 0,\ 2,\ -1 \pm i\sqrt{3}$

Exercise

Solve $9x^4 - 9x^2 + 2 = 0$

Solution

$$x^2 = \frac{9 \pm \sqrt{81-72}}{18}$$
$$= \frac{9 \pm \sqrt{9}}{18}$$
$$= \frac{9 \pm 3}{18}$$
$$= \begin{cases} \frac{9-3}{18} = \frac{6}{18} = \frac{1}{3} \\ \frac{9+3}{18} = \frac{12}{18} = \frac{2}{3} \end{cases}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{cases} x^2 = \frac{1}{3} \quad \Rightarrow \quad \underline{\left| x = \pm \frac{1}{\sqrt{3}} \right|} \\ x^2 = \frac{2}{3} \quad \Rightarrow \quad \underline{\left| x = \pm \frac{\sqrt{2}}{\sqrt{3}} \right|} \end{cases}$$

$$\begin{cases} x = \pm \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3} \\ x = \pm \frac{\sqrt{2}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{6}}{3} \end{cases}$$

$\therefore$ Solutions: $\underline{\left| x = \pm \frac{\sqrt{3}}{3}, \ \pm \frac{\sqrt{6}}{3} \right|}$

Exercise

Solve: $x^4 + 720 = 89x^2$

Solution

$$x^4 - 89x^2 + 720 = 0$$

$$x^2 = \frac{89 \pm \sqrt{7,921 - 2,880}}{2}$$

$$= \frac{89 \pm \sqrt{5041}}{2}$$

$$= \frac{89 \pm 71}{2}$$

$$= \begin{cases} \frac{89 - 71}{2} = 9 \\ \frac{89 + 71}{2} = 80 \end{cases}$$

$x^2 = 9 \quad \Rightarrow \quad \underline{\left| x = \pm 3 \right|}$

$x^2 = 80 \quad \Rightarrow x = \pm\sqrt{80}$

$$= \pm\sqrt{(16)(5)}$$

$$\underline{= \pm 4\sqrt{5}\,\Big|}$$

$\therefore$ Solutions: $\underline{\left| x = \pm 3, \ \pm 4\sqrt{5} \right|}$

Exercise

Solve $12x^4 - 11x^2 + 2 = 0$

Solution

$$x^2 = \frac{11 \pm \sqrt{121 - 96}}{24}$$

$$= \frac{11 \pm \sqrt{25}}{24}$$

$$= \frac{11 \pm 5}{24}$$

$$= \begin{cases} \frac{11-5}{24} = \frac{1}{4} \\ \frac{11+5}{24} = \frac{2}{3} \end{cases}$$

$$x^2 = \frac{1}{4} \qquad\qquad x^2 = \frac{2}{3}$$

$$\underline{x = \pm\frac{1}{2}} \qquad\qquad \underline{x = \pm\frac{\sqrt{6}}{3}}$$

$\therefore$ Solutions: $\left\{\pm\frac{\sqrt{6}}{3},\ \pm\frac{1}{2}\right\}$

$$\left(3x^2 - 2\right)\left(4x^2 - 1\right) = 0$$

Exercise

Solve $2x^4 - 7x^2 + 5 = 0$

Solution

$$\left(2x^2 - 5\right)\left(x^2 - 1\right) = 0$$

$$2x^2 - 5 = 0 \qquad\qquad x^2 - 1 = 0$$

$$x^2 = \frac{5}{2} \qquad\qquad x^2 = 1$$

$$x = \pm\frac{\sqrt{5}}{\sqrt{2}} \qquad\qquad x = \pm 1$$

$\therefore$ Solutions: $\left\{\pm\frac{\sqrt{10}}{2},\ \pm 1\right\}$

Exercise

Solve $x^4 - 5x^2 + 4 = 0$

Solution

$1 - 5 + 4 = 0 \qquad a + b + c = 0$

$x^2 = 1,\ 4$ $\qquad\qquad x_1 = 1, \quad x_2 = \frac{c}{a}$

$x^2 = 1$	$x^2 = 4$
$x = \pm 1$	$x = \pm 2$

∴ Solutions: $x = \pm 1,\ \pm 2$

Exercise

Solve $x^4 + 3x^2 = 10$

Solution

$x^4 + 3x^2 - 10 = 0$

$\left(x^2 + 5\right)\left(x^2 - 2\right) = 0$

$x^2 + 5 = 0$	$x^2 - 2 = 0$
$x^2 = -5$	$x^2 = 2$
$x = \pm\sqrt{-5}$	$x = \pm\sqrt{2}$
$= \pm i\sqrt{5}$	

∴ Solutions: $x = \pm i\sqrt{5},\ \pm\sqrt{2}$

Exercise

Solve: $3x^4 - 48x^2 = 0$

Solution

$3x^2\left(x^2 - 16\right) = 0$

$x^2 = 0$	$x^2 - 16 = 0$
$x = 0,\ 0$	$x^2 = 16$
	$x = \pm 4$

∴ Solutions: $x = 0,\ 0,\ \pm 4$

Exercise

Solve: $5x^4 - 20x^2 = 0$

Solution

$$5x^2\left(x^2 - 4\right) = 0$$

$x^2 = 0$ | $x^2 - 4 = 0$

$x = 0,\ 0$ | $x^2 = 4$

$x = \pm 2$

∴ Solutions: $x = 0,\ 0,\ \pm 2$

Exercise

Solve: $x^4 - 4x^3 - 4x^2 = 0$

Solution

$$x^2\left(x^2 - 4x - 4\right) = 0$$

$x^2 = 0$ | $x^2 - 4x - 4 = 0$

$x = 0,\ 0$ | $x = \dfrac{4 \pm \sqrt{16 + 16}}{2}$

$= \dfrac{4 \pm 4\sqrt{2}}{2}$

$x = 2 \pm 2\sqrt{2}$

∴ Solutions: $x = 0,\ 0,\ 2 \pm 2\sqrt{2}$

Exercise

Solve: $x^4 - 6x^3 + 9x^2 = 0$

Solution

$$x^2\left(x^2 - 6x + 9\right) = 0$$

$x^2 = 0$ | $x^2 - 6x + 9 = 0$

$x = 0,\ 0$ | $(x - 3)^2 = 0$

$x = 3,\ 3$

∴ Solutions: $x = 0,\ 0,\ 3,\ 3$

Exercise

Solve: $x^4 - 4x^3 + 3x^2 = 0$

Solution

$$x^2\left(x^2 - 4x + 3\right) = 0$$

$x^2 = 0$	$x^2 - 4x + 3 = 0$
$\underline{x = 0,\ 0}$	$\underline{x = 1,\ 3}$

∴ Solutions: $\underline{x = 0,\ 0,\ 1,\ 3}$

Exercise

Solve: $x^4 - 4x^2 + 3 = 0$

Solution

$1 - 4 + 3 = 0 \qquad a + b + c = 0$

$\underline{x^2 = 1,\ 3} \qquad x_1 = 1, \quad x_2 = \frac{c}{a}$

$x^2 = 1$	$x^2 = 3$
$\underline{x = \pm 1}$	$\underline{x = \pm\sqrt{3}}$

∴ Solutions: $\underline{x = \pm 1,\ \pm\sqrt{3}}$

Exercise

Solve: $x^4 + 4x^2 + 3 = 0$

Solution

$1 - 4 + 3 = 0 \qquad a - b + c = 0$

$\underline{x^2 = -1,\ -3} \qquad x_1 = -1, \quad x_2 = -\frac{c}{a}$

$x^2 = -1$	$x^2 = -3$
$\underline{x = \pm i}$	$\underline{x = \pm i\sqrt{3}}$

∴ Solutions: $\underline{x = \pm i,\ \pm i\sqrt{3}}$

Exercise

Solve: $x^4 + 6x^2 - 7 = 0$

Solution

$1 + 6 - 7 = 0 \qquad a + b + c = 0$

$\underline{x^2 = 1,\ -7}$ $\qquad x_1 = 1, \quad x_2 = \frac{c}{a}$

$x^2 = 1$ | $x^2 = -7$

$\underline{x = \pm 1}$ | $\underline{x = \pm i\sqrt{7}}$

∴ Solutions: $\underline{x = \pm 1,\ \pm i\sqrt{7}}$

Exercise

Solve: $x^4 - 6x^2 - 7 = 0$

Solution

$1 - (-6) - 7 = 0 \qquad a - b + c = 0$

$\underline{x^2 = -1,\ 7}$ $\qquad x_1 = -1, \quad x_2 = -\frac{c}{a}$

$x^2 = -1$ | $x^2 = 7$

$\underline{x = \pm i}$ | $\underline{x = \pm\sqrt{7}}$

∴ Solutions: $\underline{x = \pm i,\ \pm\sqrt{7}}$

Exercise

Solve: $3x^4 + 4x^2 - 7 = 0$

Solution

$3 + 4 - 7 = 0 \qquad a + b + c = 0$

$\underline{x^2 = 1,\ -\frac{7}{3}}$ $\qquad x_1 = 1, \quad x_2 = \frac{c}{a}$

$x^2 = 1$ | $x^2 = -\frac{7}{3}$

$\underline{x = \pm 1}$ | $x = \pm i\sqrt{\frac{7}{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$\underline{= \pm i\frac{\sqrt{21}}{3}}$

$\therefore$ Solutions: $\underline{x = \pm 1,\ \pm i\frac{\sqrt{21}}{3}}$

Exercise

Solve: $3x^4 - 4x^2 - 7 = 0$

Solution

$3 - (-4) - 7 = 0 \qquad a - b + c = 0$

$\underline{x^2 = -1,\ \frac{7}{3}} \qquad x_1 = -1, \quad x_2 = -\frac{c}{a}$

$x^2 = -1$

$\underline{x = \pm i}$

$x^2 = \frac{7}{3}$

$x = \pm\sqrt{\frac{7}{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$= \pm\frac{\sqrt{21}}{3}$

$\therefore$ Solutions: $\underline{x = \pm i,\ \pm\frac{\sqrt{21}}{3}}$

Exercise

Solve: $3x^4 - x^2 - 2 = 0$

Solution

$3 - 1 - 2 = 0 \qquad a + b + c = 0$

$\underline{x^2 = 1,\ -\frac{2}{3}} \qquad x_1 = 1, \quad x_2 = \frac{c}{a}$

$x^2 = 1$

$\underline{x = \pm 1}$

$x^2 = -\frac{2}{3}$

$x = \pm i\sqrt{\frac{2}{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$= \pm i\frac{\sqrt{6}}{3}$

$\therefore$ Solutions: $\underline{x = \pm 1,\ \pm i\frac{\sqrt{6}}{3}}$

Exercise

Solve: $3x^4 + x^2 - 2 = 0$

Solution

$3-1-2=0 \qquad a-b+c=0$

$x^2=-1,\ \frac{2}{3}$ $\qquad x_1=-1, \quad x_2=-\frac{c}{a}$

$x^2=-1$

$x=\pm i$

$x^2=\frac{2}{3}$

$x=\pm\sqrt{\frac{2}{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$=\pm\frac{\sqrt{6}}{3}$

∴ Solutions: $x=\pm i,\ \pm\frac{\sqrt{6}}{3}$

Exercise

Solve $x-3\sqrt{x}-4=0$

Solution

$(\sqrt{x}-4)(\sqrt{x}+1)=0$

$\sqrt{x}-4=0 \qquad \sqrt{x}+1=0$

$\sqrt{x}=4 \qquad \sqrt{x}=-1 \qquad$ *Impossible*

$x=16$

∴ Solution: $x=16$

Exercise

Solve $\left(5x^2-6\right)^{1/4}=x$

Solution

$\left[\left(5x^2-6\right)^{1/4}\right]^4=x^4$

$5x^2-6=x^4$

$x^4-5x^2+6=0$

$(x^2-3)(x^2-2)=0$

$x^2=3 \qquad x^2=2$

$x=\pm\sqrt{3} \qquad x=\pm\sqrt{2}$

∴ Solutions: $x = \pm\sqrt{3},\ \pm\sqrt{2}$

Exercise

Solve $\left(x^2 + 24x\right)^{1/4} = 3$

Solution

$$\left[\left(x^2 + 24x\right)^{1/4}\right]^4 = 3^4$$

$$x^2 + 24x = 81$$

$$x^2 + 24x - 81 = 0$$

$$(x + 27)(x - 3) = 0$$

$x + 27 = 0$ $\qquad$ $x - 3 = 0$

$x = -27$ $\qquad$ $x = 3$

∴ Solutions: $x = -27,\ 3$

Exercise

Solve: $x^{5/2} = 32$

Solution

$x = 32^{2/5}$ $\qquad$ ***Reciprocal***

$= \left(2^5\right)^{2/5}$

$= 4$

Exercise

Solve: $\sqrt[3]{2x + 11} = 3$

Solution

$$2x + 11 = 3^3$$

$$2x = 27 - 11$$

$$x = \frac{16}{2}$$

$= 8$

Exercise

Solve: $\sqrt[3]{6x-3}=3$

Solution

$$6x-3=3^3$$
$$6x=27+3$$
$$x=\frac{30}{6}$$
$$=5$$

Exercise

Solve: $\sqrt[3]{2x-6}=4$

Solution

$$2x-6=4^3$$
$$2x=64+6$$
$$x=\frac{70}{2}$$
$$=35$$

Exercise

Solve: $\sqrt[3]{4x-3}-5=0$

Solution

$$\sqrt[3]{4x-3}=5$$
$$4x-3=5^3$$
$$4x=125+3$$
$$x=\frac{128}{4}$$
$$=32$$

Exercise

Solve: $(3x-1)^{1/3}+4=0$

Solution

$$(3x-1)^{1/3}=-4$$

$$3x-1=(-4)^3$$
$$3x=1-64$$
$$x=-\frac{63}{3}$$
$$=-21$$

Exercise

Solve: $(2x+3)^{1/3}+4=6$

Solution

$$(2x+3)^{1/3}=2$$
$$2x+3=2^3$$
$$2x=8-3$$
$$x=\frac{5}{2}$$

Exercise

Solve: $(3x-6)^{1/3}+5=8$

Solution

$$(3x-6)^{1/3}=3$$
$$3x-6=3^3$$
$$3x=27+6$$
$$x=\frac{33}{3}$$
$$=11$$

Exercise

Solve: $(3x+1)^{1/4}+7=9$

Solution

$$(3x+1)^{1/4}=2$$
$$3x+1=2^4$$
$$3x=16-1$$

$$x = \frac{15}{3}$$

$$= 5$$

∴ Solution set is: $\{5\}$

Exercise

Solve: $(2x+3)^{1/4} + 7 = 10$

Solution

$$(2x+3)^{1/4} = 3$$

$$2x + 3 = 3^4$$

$$2x = 81 - 3$$

$$x = \frac{78}{2}$$

$$= 39$$

∴ Solution set is: $\{39\}$

Exercise

Solve: $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$

Solution

$$\left(\sqrt[3]{4x^2 - 4x + 1}\right)^3 = \left(\sqrt[3]{x}\right)^3$$

$$4x^2 - 4x + 1 = x$$

$$4x^2 - 5x + 1 = 0$$

$$(4x - 1)(x - 1) = 0$$

$4x - 1 = 0$	$x - 1 = 0$
$x = \frac{1}{4}$	$x = 1$

Check

$x = \frac{1}{4}$	$x = 1$
$\sqrt[3]{4\left(\frac{1}{4}\right)^2 - 4\frac{1}{4} + 1} - \sqrt[3]{\frac{1}{4}} = 0$	$\sqrt[3]{4(1)^2 - 4(1) + 1} - \sqrt[3]{1} = 0$
$\sqrt[3]{\frac{1}{4}} - \sqrt[3]{\frac{1}{4}} = 0$	$\sqrt[3]{4 - 4 + 1} - \sqrt[3]{1} = 0$

$0 = 0 \quad (true)$ $\qquad\qquad$ $0 = 0 \quad (true)$

$\therefore$ Solution set is: $\underline{\left\{-1, \ \frac{1}{4}\right\}}$

Exercise

Solve: $\sqrt{2x+3} = 5$

Solution

$2x + 3 = 5^2 \qquad \sqrt[n]{u} = a \ \rightarrow \ u = a^n$

$2x = 25 - 3$

$x = \frac{22}{2}$

$= 11$

Check: $\sqrt{2(11)+3} \overset{?}{=} 5$

$\sqrt{25} = 5 \quad \checkmark$

$\therefore$ Solution set is: $\underline{\{11\}}$

Exercise

Solve: $\sqrt{x-3} + 6 = 5$

Solution

$\sqrt{x-3} = -1 \ \times$

$\therefore$ ***No*** solution.

Exercise

Solve: $\sqrt{3x-2} = 4$

Solution

$3x - 2 = 4^2 \qquad \sqrt[n]{u} = a \ \rightarrow \ u = a^n$

$3x = 16 + 2$

$x = \frac{18}{3}$

$= 6$

Check: $\sqrt{3(6)-2} \overset{?}{=} 4$

$\sqrt{16} = 4 \quad \checkmark$

$\therefore$ Solution set is: $\underline{\{6\}}$

Exercise

Solve: $\sqrt{5x-4}=9$

Solution

$5x-4=9^2$ $\qquad \sqrt[n]{u}=a \rightarrow u=a^n$

$5x=81+4$

$x=\frac{85}{5}$

$\underline{=17}$

Check: $\sqrt{5(17)-4}\overset{?}{=}9$

$\sqrt{81}=9$ ✓

$\therefore$ Solution set is: $\underline{\{17\}}$

Exercise

Solve: $\sqrt{5x-1}=8$

Solution

$5x-1=8^2$ $\qquad \sqrt[n]{u}=a \rightarrow u=a^n$

$5x=64+1$

$x=\frac{65}{5}$

$\underline{=13}$

Check: $\sqrt{5(13)-1}\overset{?}{=}8$

$\sqrt{64}=8$ ✓

$\therefore$ Solution set is: $\underline{\{13\}}$

Exercise

Solve: $\sqrt{3x-2}-5=0$

Solution

$\sqrt{3x-2}=5$

$3x-2=5^2$ $\qquad \sqrt[n]{u}=a \rightarrow u=a^n$

$3x=25+2$

$$x = \frac{27}{3}$$

$$= 9 \quad \boxed{}$$

Check: $\sqrt{3(9)-2} - 5 \overset{?}{=} 0$

$5 - 5 = 0$ ✓

∴ Solution set is: $\{9\}$ $\boxed{}$

Exercise

Solve: $\sqrt{2x+5} + 11 = 6$

Solution

$\sqrt{2x+5} = -5$ ✗

∴ ***No*** solution.

Exercise

Solve: $\sqrt{3x+7} + 10 = 4$

Solution

$\sqrt{3x+7} = -6$ ✗

∴ ***No*** solution.

Exercise

Solve: $x = \sqrt{7x+8}$

Solution

$x^2 = 7x + 8$

$x^2 - 7x - 8 = 0$

$1 - (-7) - 8 = 0 \qquad a - b + c = 0$

$x = -1,\ 8$ $\boxed{}$ $\qquad x_1 = -1, \quad x_2 = -\frac{c}{a}$

Check:

$x = -1$

$-1 \neq \sqrt{7(-1)+8}$

$x = 8$

$8 \overset{?}{=} \sqrt{7(8)+8}$

$8 = \sqrt{64}$ ✓

∴ Solution set is: $\underline{\{8\}}$

Exercise

Solve: $x = \sqrt{6x+7}$

Solution

$x^2 = 6x + 7$

$x^2 - 6x - 7 = 0$

$1 - (-6) - 7 = 0 \qquad a - b + c = 0$

$\underline{x = -1,\ 7}$ $\qquad\qquad x_1 = -1, \quad x_2 = -\frac{c}{a}$

Check:

$x = -1$	$x = 7$
$-1 \neq \sqrt{\ }$	$7 \overset{?}{=} \sqrt{6(7)+7}$
	$7 = \sqrt{49}$ ✓

∴ Solution is: $\underline{x = 7}$

Exercise

Solve: $\sqrt{5x+1} = x + 1$

Solution

$5x + 1 = (x+1)^2$

$5x + 1 = x^2 + 2x + 1$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\underline{x = 0,\ 3}$

Check:

$x = 0$	$x = 3$
$\sqrt{1} = 1$ ✓	$\sqrt{5(3)+1} \overset{?}{=} 3 + 1$
	$\sqrt{16} = 4$ ✓

∴ Solutions are: $\underline{x = 0,\ 3}$

Exercise

Solve: $x = \sqrt{2x-2} + 1$

Solution

$$x-1=\sqrt{2x-2}$$

$$(x-1)^2=2x-2$$

$$x^2-2x+1=2x-2$$

$$x^2-4x+3=0$$

$1-4+3=0$ $\qquad a+b+c=0$

$\underline{x=1,\ 3}$ $\qquad x_1=1,\quad x_2=\frac{c}{a}$

Check:

$x=1$

$1=1$ ✓

$x=3$

$3\overset{?}{=}\sqrt{4}+1$

$3=3$ ✓

∴ Solutions are: $\underline{x=1,\ 3}$

Exercise

Solve: $\quad x-2\sqrt{x-3}=3$

Solution

$$x-3=2\sqrt{x-3}$$

$$(x-3)^2=\left(2\sqrt{x-3}\right)^2$$

$$x^2-6x+9=4(x-3)$$

$$x^2-6x+9=4x-12$$

$$x^2-10x+21=0$$

$\underline{x=3,\ 7}$

Check:

$x=3$

$3-2(0)\overset{?}{=}3$

$3=3$ ✓

$x=7$

$7-2\sqrt{4}\overset{?}{=}3$

$7-4=3$ ✓

∴ Solution set is: $\underline{\{3,\ 7\}}$

Exercise

Solve: $\quad x+\sqrt{26-11x}=4$

Solution

$$\sqrt{26-11x}=4-x$$

$$26-11x=(4-x)^2$$

$$26-11x=16-8x+x^2$$

$$x^2+3x-10=0$$

$$(x-2)(x+5)=0$$

$$\underline{x=2,\ -5}$$

Check:

$x=2$	$x=-5$
$2+\sqrt{26-22}\overset{?}{=}4$	$-5+\sqrt{26+55}\overset{?}{=}4$
$2+2=4$ ✓	$-5+9=4$ ✓

∴ Solutions are: $\underline{x=2,\ -5}$

Exercise

Solve $x-\sqrt{2x+3}=0$

Solution

$$x=\sqrt{2x+3}$$

$$(x)^2=\left(\sqrt{2x+3}\right)^2$$

$$x^2=2x+3$$

$$x^2-2x-3=0$$

$$(x+1)(x-3)=0$$

$$x+1=0 \qquad x-3=0$$

$$x=-1 \qquad x=3$$

Check

$x=-1$	$x=3$
$(-1)-\sqrt{2(-1)+3}=0$	$(3)-\sqrt{2(3)+3}=0$
$-1-\sqrt{1}=0$	$3-\sqrt{9}=0$
False	*True*

∴ Solution set is: $\underline{\{3\}}$

Exercise

Solve: $\sqrt{x+3}+3=x$

Solution

$$\sqrt{x+3} = x-3$$

$$\left(\sqrt{x+3}\right)^2 = (x-3)^2$$

$$x+3 = x^2 - 6x + 9$$

$$x^2 - 7x + 6 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{25}}{2}$$

$$= \frac{7 \pm 5}{2} = \begin{cases} \frac{7+5}{2} = \frac{12}{2} = 6 \\ \frac{7-5}{2} = \frac{2}{2} = 1 \end{cases} \qquad \Rightarrow x = 1, 6$$

Check:

$x = 1 \quad \Rightarrow \sqrt{1+3} + 3 = 1 \Rightarrow 5 = 1$ (*Not a solution*)

$x = 6 \quad \Rightarrow \sqrt{6+3} + 3 = 6 \Rightarrow 6 = 6 \rightarrow x = 6$ is the only solution

Exercise

Solve $x - \sqrt{x+11} = 1$

Solution

$$-\sqrt{x+11} = 1 - x$$ ***Square both side***

$$\left(-\sqrt{x+11}\right)^2 = (1-x)^2 \qquad (a-b)^2 = a^2 - 2ab + b^2$$

$$x + 11 = 1 - 2x + x^2$$

$$0 = x^2 - 2x + 1 - x - 11$$

$$0 = x^2 - 3x - 10$$

$$x^2 - 3x - 10 = 0$$ ***Solve for*** x

$$x = 5, -2$$

Check:

$x = 5 \Rightarrow 5 - \sqrt{5+11} = 1 \Rightarrow 5 - \sqrt{16} = 1 \Rightarrow 5 - 4 = 1 \Rightarrow 1 = 1$

$x = -2 \Rightarrow -2 - \sqrt{-2+11} = 1 \Rightarrow -2 - \sqrt{9} = 1 \Rightarrow -2 - 3 = 1 \Rightarrow -5 = 1$ (*False*)

∴ Solution set is: $\{5\}$

Exercise

Solve: $\sqrt{x-7} = 7 - \sqrt{x}$

Solution

$$\left(\sqrt{x-7}\right)^2 = \left(7-\sqrt{x}\right)^2$$

$$x - 7 = 49 - 14\sqrt{x} + x$$

$$14\sqrt{x} = 56$$

$$\sqrt{x} = \frac{56}{14}$$

$$= 4$$

$$\underline{x = 16}$$

Check:

$$x = 16$$

$$\sqrt{16-7} \overset{?}{=} 7 - 4$$

$$3 = 3 \quad \checkmark$$

∴ Solution set is: $\underline{\{16\}}$

Exercise

Solve: $\sqrt{x-8} = \sqrt{x} - 2$

Solution

$$\left(\sqrt{x-8}\right)^2 = \left(\sqrt{x}-2\right)^2$$

$$x - 8 = x - 4\sqrt{x} + 4$$

$$4\sqrt{x} = 12$$

$$\sqrt{x} = 3$$

$$\underline{x = 9}$$

Check:

$$x = 9$$

$$\sqrt{9-8} \overset{?}{=} \sqrt{9} - 2$$

$$1 = 3 - 2 \quad \checkmark$$

∴ Solution set is: $\underline{\{9\}}$

Exercise

Solve: $\sqrt{2x-5}=\sqrt{x+4}$

Solution

$$\left(\sqrt{2x-5}\right)^2=\left(\sqrt{x+4}\right)^2$$

$$2x-5=x+4$$

$$\underline{x=9}$$

Check:

$x=9$

$$\sqrt{18-5}\overset{?}{=}\sqrt{9+4}$$

$$\sqrt{13}=\sqrt{13}\quad\checkmark$$

∴ Solution set is: $\underline{\{9\}}$

Exercise

Solve: $\sqrt{6x+2}=\sqrt{5x+3}$

Solution

$$\left(\sqrt{6x+2}\right)^2=\left(\sqrt{5x+3}\right)^2$$

$$6x+2=5x+3$$

$$\underline{x=1}$$

Check:

$x=1$

$$\sqrt{6+2}\overset{?}{=}\sqrt{5+3}$$

$$\sqrt{8}=\sqrt{8}\quad\checkmark$$

∴ Solution set is: $\underline{\{1\}}$

Exercise

Solve: $\sqrt{3x+1}-\sqrt{x+4}=1$

Solution

$$\sqrt{3x+1}=1+\sqrt{x+4}$$

$$\left(\sqrt{3x+1}\right)^2=\left(1+\sqrt{x+4}\right)^2$$

$$3x+1=1+2\sqrt{x+4}+x+4$$
$$2x-4=2\sqrt{x+4}$$
$$x-2=\sqrt{x+4}$$
$$(x-2)^2=x+4$$
$$x^2-4x+4=x+4$$
$$x^2-5x=0$$
$$x(x-5)=0$$
$$\underline{x=0,\ 5}$$

Check:

$x=0$	$x=5$
$1-\sqrt{4}\overset{?}{=}1$	$\sqrt{15+1}-\sqrt{5+4}\overset{?}{=}1$
$-1\neq 1$ ✗	$4-3=1$ ✓

∴ Solution is: $\underline{x=5}$

Exercise

Solve: $\sqrt{x+2}+\sqrt{x-1}=3$

Solution

$$\sqrt{x+2}=3-\sqrt{x-1}$$
$$x+2=\left(3-\sqrt{x-1}\right)^2$$
$$x+2=9-6\sqrt{x-1}+x-1$$
$$6\sqrt{x-1}=6$$
$$\sqrt{x-1}=1$$
$$x-1=1^2$$
$$\underline{x=2}$$

Check:

$x=2$

$\sqrt{4}+1\overset{?}{=}3$

$2+1=3$ ✓

∴ Solution is: $\underline{x=2}$

Exercise

Solve: $\sqrt{x-4}+\sqrt{x+4}=4$

Solution

$$\sqrt{x-4}=4-\sqrt{x+4}$$

$$x-4=\left(4-\sqrt{x+4}\right)^2$$

$$x-4=16-8\sqrt{x+4}+x+4$$

$$8\sqrt{x+4}=24$$

$$\sqrt{x+4}=3$$

$$x+4=9$$

$$\underline{x=5}$$

Check:

$x=5$

$\sqrt{1}+\sqrt{9}\overset{?}{=}4$

$1+3=4$ ✓

∴ Solution is: $\underline{x=5}$

Exercise

Solve: $\sqrt{2x-3}-\sqrt{x-2}=1$

Solution

$$\sqrt{2x-3}=1+\sqrt{x-2}$$

$$2x-3=\left(1+\sqrt{x-2}\right)^2$$

$$2x-3=1+2\sqrt{x-2}+x-2$$

$$x-2=2\sqrt{x-2}$$

$$(x-2)^2=\left(2\sqrt{x-2}\right)^2$$

$$x^2-4x+4=4(x-2)$$

$$x^2-4x+4=4x-8$$

$$x^2-8x+12=0$$

$$\underline{x=2,\ 6}$$

Check:

$x=2$ | $x=6$

$\sqrt{4-3}-\sqrt{2-2}\overset{?}{=}1$ | $\sqrt{12-3}-\sqrt{6-2}\overset{?}{=}1$

$1=1$ ✓ | $3-2=1$ ✓

∴ Solution is: $\underline{x=2,\ 6}$

Exercise

Solve: $\sqrt{x+2}+\sqrt{3x+7}=1$

Solution

$$\sqrt{x+2}=1-\sqrt{3x+7}$$
$$x+2=\left(1-\sqrt{3x+7}\right)^2$$
$$x+2=1-2\sqrt{3x+7}+3x+7$$
$$2\sqrt{3x+7}=2x+6$$
$$\sqrt{3x+7}=x+3$$
$$3x+7=(x+3)^2$$
$$3x+7=x^2+6x+9$$
$$x^2+3x+2=0$$
$$\underline{x=-1,\ -2}$$

Check:

$x=-1$ | $x=-2$

$\sqrt{-1+2}\overset{?}{=}1-\sqrt{-3+7}$ | $\sqrt{-2+2}\overset{?}{=}1-\sqrt{-6+7}$

$1\neq 1-2$ ✗ | $0=1-1$ ✓

∴ Solution is: $\underline{x=-2}$

Exercise

Solve: $2\sqrt{4x+1}-9=x-5$

Solution

$$2\sqrt{4x+1}=x+4$$
$$\left(2\sqrt{4x+1}\right)^2=(x+4)^2$$
$$4(4x+1)=x^2+8x+16$$
$$16x+4=x^2+8x+16$$

$x^2 - 8x + 12 = 0$

$\underline{x = 2,\ 6}$

Check:

$x = 2$	$x = 6$
$2\sqrt{8+1} - 9 \overset{?}{=} 2 - 5$	$2\sqrt{24+1} - 9 \overset{?}{=} 6 - 5$
$6 - 9 = -3$ ✓	$10 - 9 = 1$ ✓

∴ Solution is: $\underline{x = 2,\ 6}$

Exercise

Solve: $x\sqrt{x-3} + 4 = x + 1$

Solution

$x\sqrt{x-3} = x - 3$

$\left(x\sqrt{x-3}\right)^2 = (x-3)^2$

$x^2(x-3) - (x-3)^2 = 0$

$(x-3)\left(x^2 - x + 3\right) = 0$

$x^2 - x + 3 = 0 \quad \rightarrow \quad x = \frac{1 \pm \sqrt{1-12}}{2} \in \mathbb{C}$

$\underline{x = 3}$ *Only* result.

Check:

$x = 3$

$3\sqrt{3-3} + 4 \overset{?}{=} 3 + 1$

$4 = 4$ ✓

∴ Solution is: $\underline{x = 3}$

Exercise

Solve: $\sqrt{2x-3} + \sqrt{x-2} = 1$

Solution

$\sqrt{2x-3} = 1 - \sqrt{x-2}$

$\left(\sqrt{2x-3}\right)^2 = \left(1 - \sqrt{x-2}\right)^2$

$$2x-3=1-2\sqrt{x-2}+\left(\sqrt{x-2}\right)^2$$

$$2x-3-1=-2\sqrt{x-2}+x-2$$

$$2x-4-x+2=-2\sqrt{x-2}$$

$$x-2=-2\sqrt{x-2}$$

$$(x-2)^2=\left(-2\sqrt{x-2}\right)^2$$

$$x^2-4x+4=4(x-2)$$

$$x^2-4x+4=4x-8$$

$$x^2-4x+4-4x+8=0$$

$$x^2-8x+12=0$$

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(12)}}{2(1)}=\frac{8\pm\sqrt{64-48}}{2}=\frac{8\pm\sqrt{16}}{2}=\frac{8\pm4}{2}$$

$$x=2,\ 6$$

Check

$$x=2\Rightarrow\sqrt{2(2)-3}+\sqrt{2-2}=1\Rightarrow1+0=1$$

$$x=6\Rightarrow\sqrt{2(6)-3}+\sqrt{6-2}=1\Rightarrow3+2=1\Rightarrow5\neq1$$

∴ Solution is: $x=2$

Exercise

Solve: $\sqrt{x+5}-\sqrt{x-3}=2$

Solution

$$\sqrt{x+5}=2+\sqrt{x-3}$$

$$\left(\sqrt{x+5}\right)^2=\left(2+\sqrt{x-3}\right)^2$$

$$x+5=4+4\sqrt{x-3}+\left(\sqrt{x-3}\right)^2$$

$$x+5=4+4\sqrt{x-3}+x-3$$

$$x-x+5-4-3=4\sqrt{x-3}$$

$$4=4\sqrt{x-3}$$

$$1=\sqrt{x-3}$$

$$1=x-3$$

$$\Rightarrow x=4$$

Check: $\sqrt{4+5}-\sqrt{4-3}=2$

$3 - 1 = 2$ (True statement)

∴ Solution is: $\underline{x = 4}$

Exercise

Solve: $\sqrt{2x+3} = 1 + \sqrt{x+1}$

Solution

$$\left(\sqrt{2x+3}\right)^2 = \left(1+\sqrt{x+1}\right)^2 \qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$2x+3 = 1 + 2\sqrt{x+1} + x + 1$$

$$2x+3 = 2\sqrt{x+1} + x + 2$$

$$x+1 = 2\sqrt{x+1}$$

$$\left(x+1\right)^2 = \left(2\sqrt{x+1}\right)^2$$

$$x^2 + 2x + 1 = 4(x+1)$$

$$x^2 + 2x + 1 = 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x-3=0 \qquad x+1=0$$

$$x=3 \qquad x=-1$$

Check

$x = 3$

$\sqrt{2(3)+3} = 1 + \sqrt{(3)+1}$

$\sqrt{9} = 1 + \sqrt{4}$

$3 = 3 \quad (true)$

$x = -1$

$\sqrt{2(-1)+3} = 1 + \sqrt{(-1)+1}$

$\sqrt{1} = 1 + \sqrt{0}$

$1 = 1 \quad (true)$

∴ Solution: $x = -1,\ 3$

Exercise

Solve: $\sqrt{x+5} - \sqrt{x-3} = 2$

Solution

$$\sqrt{x+5} = 2 + \sqrt{x-3}$$

$$\left(\sqrt{x+5}\right)^2 = \left(2+\sqrt{x-3}\right)^2$$

$$x+5 = 4 + 4\sqrt{x-3} + \left(\sqrt{x-3}\right)^2$$

$$x+5 = 4 + 4\sqrt{x-3} + x - 3$$

$$x - x + 5 - 4 - 3 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$1 = \sqrt{x-3}$

$1 = x - 3$

$\Rightarrow x = 4$

Check: $\sqrt{4+5} - \sqrt{4-3} = 2$

$3 - 1 = 2$ (True statement)

$\therefore$ Solution is: $\underline{x = 4}$

Exercise

Solve: $|x| = -9$

Solution

$|x| = -9$ Not True

$\therefore$ ***No*** Solution

Exercise

Solve: $|x| = 9$

Solution

$\therefore$ Solutions: $\underline{x = \pm 9}$

Exercise

Solve: $|x - 2| = 7$

Solution

$x - 2 = 7$ $\quad$ $x - 2 = -7$

$\underline{x = 9}$ $\quad$ $\underline{x = -5}$

$\therefore$ Solutions: $\underline{x = -5,\ 9}$

Exercise

Solve: $|x - 2| = 0$

Solution

$x - 2 = 0$

$\therefore$ Solution: $\underline{x = 2}$

Exercise

Solve: $|2x-3|=6$

Solution

$2x-3=6$ $\quad$ $2x-3=-6$

$2x=9$ $\quad$ $2x=-3$

$x=\frac{9}{2}$ $\quad$ $x=-\frac{3}{2}$

$\therefore$ Solutions: $x=-\frac{3}{2},\ \frac{9}{2}$

Exercise

Solve: $|2x-1|=11$

Solution

$2x-1=11$ $\quad$ $2x-1=-11$

$2x=12$ $\quad$ $2x=-10$

$x=6$ $\quad$ $x=-5$

$\therefore$ Solutions: $x=-5,\ 6$

Exercise

Solve $7|5x|+2=16$

Solution

$7|5x|=16-2$

$7|5x|=14$

$|5x|=\frac{14}{7}$

$|5x|=2$

$5x=2$ $\quad$ $5x=-2$

$x=\frac{2}{5}$ $\quad$ $x=-\frac{2}{5}$

$\therefore$ Solution: $x=\pm\frac{2}{5}$

Exercise

Solve $4\left|1-\frac{3}{4}x\right|+7=10$

Solution

$4\left|1-\frac{3}{4}x\right|=10-7$ $\qquad$ $|4-3x|=10-7$ $\qquad$ ***Distribute 4***

$4\left|1-\frac{3}{4}x\right|=3$ $\qquad$ $|4-3x|=3$

$\left|1-\frac{3}{4}x\right|=\frac{3}{4}$

$1-\frac{3}{4}x=\frac{3}{4}$	$1-\frac{3}{4}x=-\frac{3}{4}$	$4-3x=-3$	$4-3x=3$
$-\frac{3}{4}x=\frac{3}{4}-1$	$-\frac{3}{4}x=-\frac{3}{4}-1$	$-3x=-7$	$-3x=-1$
$-\frac{3}{4}x=-\frac{1}{4}$	$-\frac{3}{4}x=-\frac{7}{4}$	$x=\frac{7}{3}$	$x=\frac{1}{3}$
$x=-\frac{1}{4}\left(-\frac{4}{3}\right)$	$x=-\frac{7}{4}\left(-\frac{4}{3}\right)$		
$x=\frac{1}{3}$	$x=\frac{7}{3}$		

$\therefore$ Solutions: $\underline{x=\frac{1}{3},\ \frac{7}{3}}$

Exercise

Solve $|x+7|+6=2$

Solution

$|x+7|=2-6$

$|x+7|=-4$

$\Rightarrow$ No solution or $\varnothing$, since the absolute value can't be equal to a negative.

Exercise

Solve equation: $|5-3x|=12$

Solution

$5-3x=12$	$5-3x=-12$
$5-3x-5=12-5$	$5-3x-5=-12-5$
$-3x=7$	$-3x=-17$
$x=-\frac{7}{3}$	$x=\frac{17}{3}$

∴ Solutions: $x = \frac{17}{3},\ -\frac{7}{3}$

Exercise

Solve equation: $|4x + 2| = 5$

Solution

$4x + 2 = -5$ $\qquad$ $4x + 2 = 5$

$4x = -7$ $\qquad$ $4x = 3$

$x = -\frac{7}{4}$ $\qquad$ $x = \frac{3}{4}$

∴ Solutions: $x = -\frac{7}{4},\ \frac{3}{4}$

Exercise

Solve: $3|x + 5| = 12$

Solution

$|x + 5| = 4$

$x + 5 = 4$ $\qquad$ $x + 5 = -4$

$x = -1$ $\qquad$ $x = -9$

∴ Solutions: $x = -9,\ -1$

Exercise

Solve: $2|x - 6| = 8$

Solution

$x - 6 = 4$

$x - 6 = 4$ $\qquad$ $x - 6 = -4$

$x = 10$ $\qquad$ $x = 2$

∴ Solutions: $x = 2,\ 10$

Exercise

Solve: $3|2x - 1| = 21$

Solution

$|2x - 1| = 7$

$2x-1=7$ $\quad$ $2x-1=-7$

$2x=8$ $\quad$ $2x=-6$

$\underline{x=4}$ $\quad$ $\underline{x=-3}$

$\therefore$ Solutions: $\underline{x=-3,\ 4}$

Exercise

Solve: $2|3x-2|=14$

Solution

$|3x-2|=7$

$3x-2=7$ $\quad$ $3x-2=-7$

$3x=9$ $\quad$ $3x=-5$

$\underline{x=3}$ $\quad$ $\underline{x=-\frac{5}{3}}$

$\therefore$ Solutions: $\underline{x=-\frac{5}{3},\ 3}$

Exercise

Solve: $|3x-1|+2=16$

Solution

$|3x-1|=14$

$3x-1=14$ $\quad$ $3x-1=-14$

$3x=15$ $\quad$ $3x=-13$

$\underline{x=5}$ $\quad$ $\underline{x=-\frac{13}{3}}$

$\therefore$ Solutions: $\underline{x=-\frac{13}{3},\ 5}$

Exercise

Solve: $|6x-2|+4=32$

Solution

$|6x-2|=28$

$6x-2=28$ $\quad$ $6x-2=-28$

$6x=30$ $\quad$ $6x=-26$

$x = 5$ $\qquad$ $x = -\frac{13}{3}$

$\therefore$ Solutions: $x = -\frac{13}{3},\ 5$

Exercise

Solve: $\quad 7|5x| + 2 = 16$

Solution

$7|5x| = 14$

$|5x| = 2$

$5x = 2 \qquad 5x = -2$

$x = \frac{2}{5} \qquad x = -\frac{2}{5}$

$\therefore$ Solutions: $x = -\frac{2}{5},\ \frac{2}{5}$

Exercise

Solve: $\quad |4x + 1| + 10 = 4$

Solution

$|4x + 1| = -6$

$\therefore$ ***No*** Solution

Exercise

Solve: $\quad |4x + 1| + 4 = 10$

Solution

$|4x + 1| = 6$

$4x + 1 = 6 \qquad 4x + 1 = -6$

$4x = 5 \qquad 4x = -7$

$x = \frac{5}{4} \qquad x = -\frac{7}{4}$

$\therefore$ Solutions: $x = -\frac{7}{4},\ \frac{5}{4}$

Exercise

Solve: $|3x-2|+8=1$

Solution

$|3x-2|=-7$

$\therefore$ ***No*** Solution

Exercise

Solve: $|3x-2|+1=8$

Solution

$|3x-2|=7$

$4x+1=6$ $\qquad$ $4x+1=-6$

$4x=5$ $\qquad$ $4x=-7$

$\underline{x=\frac{5}{4}}$ $\qquad$ $\underline{x=-\frac{7}{4}}$

$\therefore$ Solutions: $\underline{x=-\frac{7}{4},\ \frac{5}{4}}$

Exercise

Solve equation: $\left|\frac{6x+1}{x-1}\right|=3$

Solution

$\frac{6x+1}{x-1}=-3$ $\qquad$ $\frac{6x+1}{x-1}=3$

$(x-1)\frac{6x+1}{x-1}=-3(x-1)$ $\qquad$ $6x+1=3(x-1)$

$6x+1=-3x+3$ $\qquad$ $6x+1=3x-3$

$6x+1+3x=-3x+3+3x$ $\qquad$ $6x+1-3x=3x-3-3x$

$9x+1=3$ $\qquad$ $3x+1=-3$

$9x=2$ $\qquad$ $3x=-4$

$x=\frac{2}{9}$ $\qquad$ $x=-\frac{4}{3}$

$\therefore$ Solutions: $\underline{x=-\frac{4}{3},\ \frac{2}{9}}$

Exercise

Solve equation: $|x+1| = |1-3x|$

Solution

$x+1 = -(1-3x)$
$x+1 = -1+3x$
$x-3x = -1-1$
$-2x = -2$
$x = 1$

$x+1 = 1-3x$
$x+3x = 1-1$
$4x = 0$
$x = 0$

∴ Solutions: $\underline{x = 0,\ 1}$

Exercise

Solve: $|3x-1| = |x+5|$

Solution

$3x-1 = x+5$
$2x = 6$
$\underline{x = 3}$

$3x-1 = -(x+5)$
$3x-1 = -x-5$
$4x = -4$
$\underline{x = -1}$

∴ Solutions: $\underline{x = -1,\ 3}$

Exercise

Solve: $|5x-8| = |3x+2|$

Solution

$5x-8 = 3x+2$
$2x = 10$
$\underline{x = 5}$

$5x-8 = -(3x+2)$
$5x-8 = -3x-2$
$8x = 6$
$\underline{x = \frac{3}{4}}$

∴ Solutions: $\underline{x = \frac{3}{4},\ 5}$

Exercise

Solve: $|4x-9|=|2x+1|$

Solution

$4x-9=2x+1$
$2x=10$
$\underline{x=5}$

$4x-9=-(2x+1)$
$5x-8=-3x-2$
$8x=6$
$\underline{x=\frac{3}{4}}$

$\therefore$ Solutions: $\underline{x=\frac{3}{4},\ 5}$

Exercise

Solve: $|2x-4|=|x-1|$

Solution

$2x-4=x-1$
$\underline{x=3}$

$2x-4=-x+1$
$3x=-5$
$\underline{x=-\frac{5}{3}}$

$\therefore$ Solutions: $\underline{x=-\frac{5}{3},\ 3}$

Exercise

Solve: $|3x-4|=|3x+4|$

Solution

$3x-4=3x+4$
$-4\neq 4$

$3x-4=-3x+4$
$6x=8$
$\underline{x=\frac{4}{3}}$

$\therefore$ Solution: $\underline{x=\frac{4}{3}}$

Exercise

Solve: $|3x-5|=|3x+5|$

Solution

$3x-5=3x+5$ $\qquad$ $3x-5=-3x+5$

$-5\neq 5$ $\qquad$ $6x=10$

$\underline{x=\frac{5}{3}}$

$\therefore$ Solution: $\underline{x=\frac{5}{3}}$

Exercise

Solve: $|x-3|=|5-x|$

Solution

$x-3=5-x$ $\qquad$ $x-3=-5+x$

$2x=8$ $\qquad$ $-3\neq -5$

$\underline{x=4}$

$\therefore$ Solutions: $\underline{x=4}$

Exercise

Solve: $|x-3|=|6-x|$

Solution

$x-3=6-x$ $\qquad$ $x-3=-6+x$

$2x=9$ $\qquad$ $-3\neq -6$

$\underline{x=\frac{9}{2}}$

$\therefore$ Solutions: $\underline{x=\frac{9}{2}}$

Exercise

Solve: $\left|\frac{2}{3}x-2\right|=\left|\frac{1}{3}x+3\right|$

Solution

$\frac{2}{3}x-2=\frac{1}{3}x+3$ $\qquad$ $\frac{2}{3}x-2=-\frac{1}{3}x-3$

$\frac{2}{3}x-\frac{1}{3}x=3+2$ $\frac{2}{3}x+\frac{1}{3}x=-3+2$

$\frac{1}{3}x=5$ $\underline{x=-1}$

$\underline{x=15}$

∴ Solutions: $\underline{x=-1,\ 15}$

Exercise

Solve: $\left|\frac{1}{2}x-2\right|=\left|x-\frac{1}{2}\right|$

Solution

$\frac{1}{2}x-2=x-\frac{1}{2}$ $\frac{1}{2}x-2=-x+\frac{1}{2}$

$\frac{1}{2}x-x=2-\frac{1}{2}$ $\frac{1}{2}x+x=2+\frac{1}{2}$

$\frac{1}{2}x=\frac{3}{2}$ $\frac{3}{2}x=\frac{5}{2}$

$\underline{x=3}$ $\underline{x=\frac{5}{3}}$

∴ Solutions: $\underline{x=3,\ \frac{5}{3}}$