

Section 2.3 – Solving Right Triangle Trigonometry

Example

In the right triangle ABC , $A = 40^\circ$ and $c = 12$ cm. Find a , b , and B .

Solution

$$\sin 40^\circ = \frac{a}{c} = \frac{a}{12}$$

$$a = 12 \sin 40^\circ$$

$$\approx 7.7 \text{ cm}$$

$$\cos 40^\circ = \frac{b}{c} = \frac{b}{12}$$

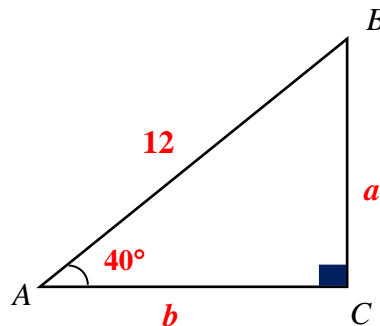
$$b = 12 \cos 40^\circ$$

$$\approx 9.2 \text{ cm}$$

$$B = 90^\circ - A$$

$$= 90^\circ - 40^\circ$$

$$\approx 50^\circ$$



Example

A circle has its center at C and a radius of 18 inches. If triangle ADC is a right triangle and $A = 35^\circ$. Find x , the distance from A to B .

Solution

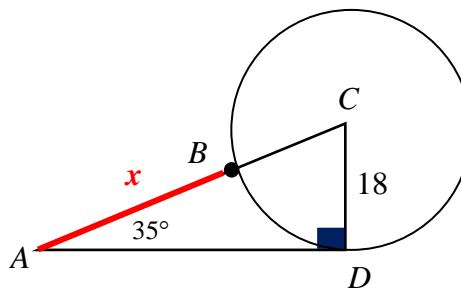
$$\sin 35^\circ = \frac{18}{x+18}$$

$$(x+18)\sin 35^\circ = 18$$

$$x+18 = \frac{18}{\sin 35^\circ}$$

$$x = \frac{18}{\sin 35^\circ} - 18$$

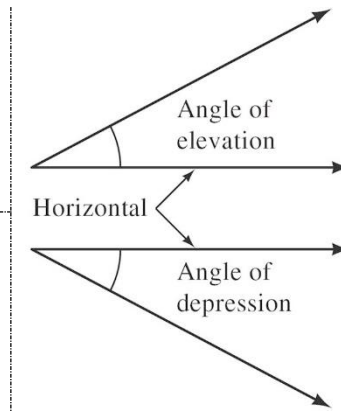
$$\approx 13 \text{ in}$$



Definitions

An angle measured from the horizontal up is called an **angle of elevation**.

An angle measured from the horizontal down is called an **angle of depression**.



Example

The two equal sides of an isosceles triangle are each 24 cm. If each of the two equal angles measures 52° , find the length of the base and the altitude.

Solution

$$\sin 52^\circ = \frac{x}{24}$$

$$x = 24 \sin 52^\circ$$

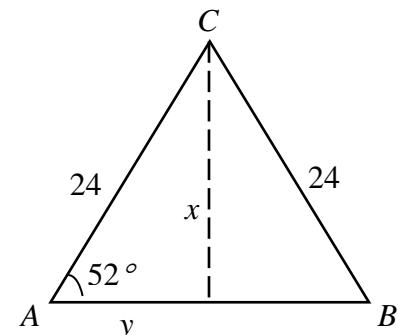
$$x \approx 19 \text{ cm}$$

$$\cos 52^\circ = \frac{y}{24}$$

$$y = 24 \cos 52^\circ$$

$$y \approx 15 \text{ cm}$$

$$\Rightarrow AB = 2y \approx 30 \text{ cm}$$



Example

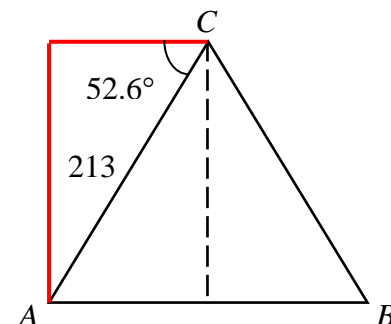
A man climbs 213 meters up the side of a pyramid. Find that the angle of depression to his starting point is 52.6° . How high off of the ground is he?

Solution

$$\sin 52.6^\circ = \frac{h}{213}$$

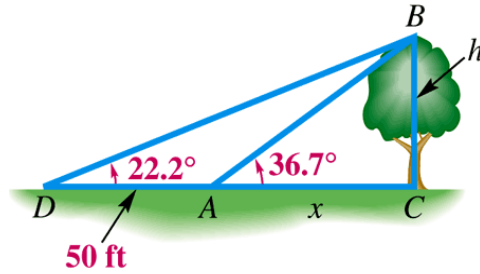
$$h = 213 \sin 52.6^\circ$$

$$h \approx 169 \text{ m}$$



Example

From a given point on the ground, the angle of elevation to the top of a tree is 36.7° . From a second point, 50 feet back, the angle of elevation to the top of the tree is 22.2° . Find the height of the tree to the nearest foot.



Solution

Triangle **DCB**

$$\Rightarrow \tan 22.2^\circ = \frac{h}{50 + x}$$

$$h = (50 + x) \tan 22.2^\circ$$

Triangle **ACB**

$$\Rightarrow \tan 36.7^\circ = \frac{h}{x}$$

$$h = x \tan 36.7^\circ$$

$$x \tan 36.7^\circ = (50 + x) \tan 22.2^\circ$$

$$x \tan 36.7^\circ = 50 \tan 22.2^\circ + x \tan 22.2^\circ$$

$$x \tan 36.7^\circ - x \tan 22.2^\circ = 50 \tan 22.2^\circ$$

$$x(\tan 36.7^\circ - \tan 22.2^\circ) = 50 \tan 22.2^\circ$$

$$x = \frac{50 \tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}$$

$$h = x \tan 36.7^\circ$$

$$= \left(\frac{50 \tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ} \right) \tan 36.7^\circ$$

$$\approx 45 \text{ ft}$$

The tree is about 45 feet tall.

OR

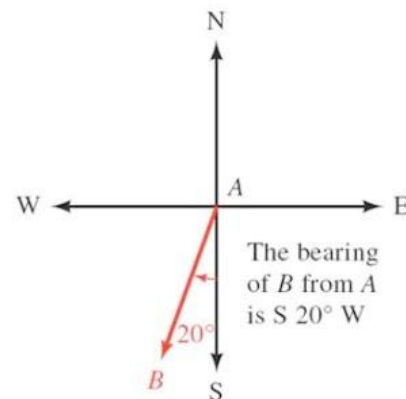
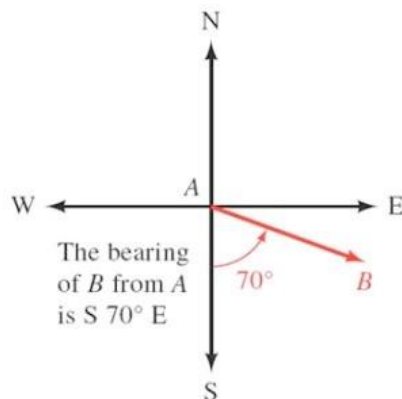
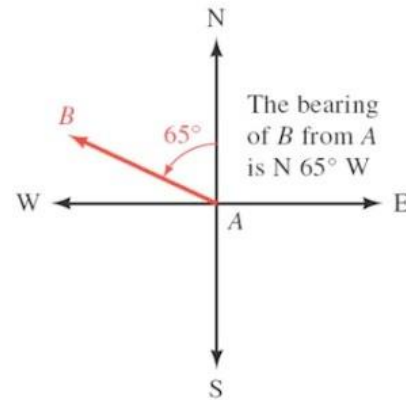
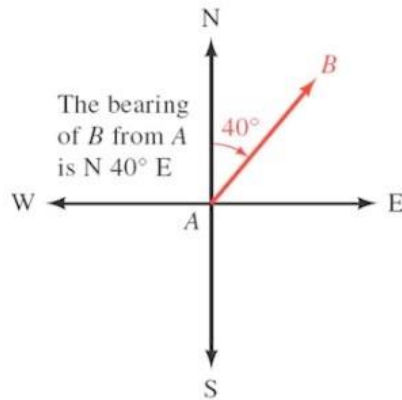
$$h = \frac{x \tan \alpha \tan \beta}{\tan \beta - \tan \alpha} = \frac{50 \tan 22.2^\circ \tan 36.7^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}$$

Bearing

Definition

The *bearing of a line ℓ* is the acute angle formed by the *north-south* line and the line ℓ .

The notation used to designate the bearing of a line begins with *N* (for **north**) or *S* (for **south**), followed by the number of degrees in the angle, and ends with *E* (for **east**) or *W* (for **west**).



Example

A boat travels on a course of bearing N $52^{\circ} 40'$ E for distance of 238 *miles*. How many miles north and how many miles east have the boat traveled?

Solution

$$52^{\circ} 40' = 52^{\circ} + 40' \frac{1^{\circ}}{60'}$$

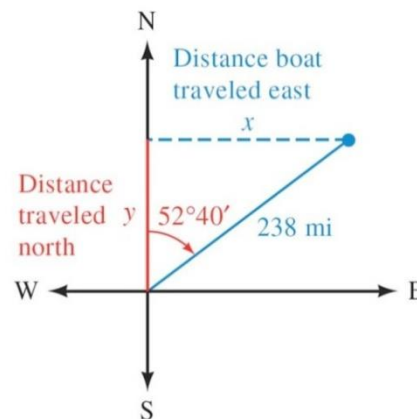
$$\approx 52.6667^{\circ}$$

$$\sin 52.6667^{\circ} = \frac{x}{238}$$

$$x = 238 \sin 52.6667^{\circ} \approx 189 \text{ mi}$$

$$\cos 52.6667^{\circ} = \frac{y}{238}$$

$$y = 238 \cos 52.6667^{\circ} \approx 144 \text{ mi}$$



Example

A helicopter is hovering over the desert when it develops mechanical problems and is forced to land. After landing, the pilot radios his position to a pair of radar station located 25 *miles* apart along a straight road running north and south. The bearing of the helicopter from one station is N 13° E, and from the other it is S 19° E. After doing a few trigonometric calculations, one of the stations instructs the pilot to walk due west for 3.5 *miles* to reach the road. Is this information correct?

Solution

In triangle AFC

$$\tan 13^\circ = \frac{y}{x}$$
$$y = x \tan 13^\circ$$

In triangle BFC

$$\tan 19^\circ = \frac{y}{25 - x}$$
$$y = (25 - x) \tan 19^\circ$$

$$y = y$$

$$(25 - x) \tan 19^\circ = x \tan 13^\circ$$

$$25 \tan 19^\circ - x \tan 19^\circ = x \tan 13^\circ$$

$$25 \tan 19^\circ = x \tan 13^\circ + x \tan 19^\circ$$

$$25 \tan 19^\circ = x(\tan 13^\circ + \tan 19^\circ)$$

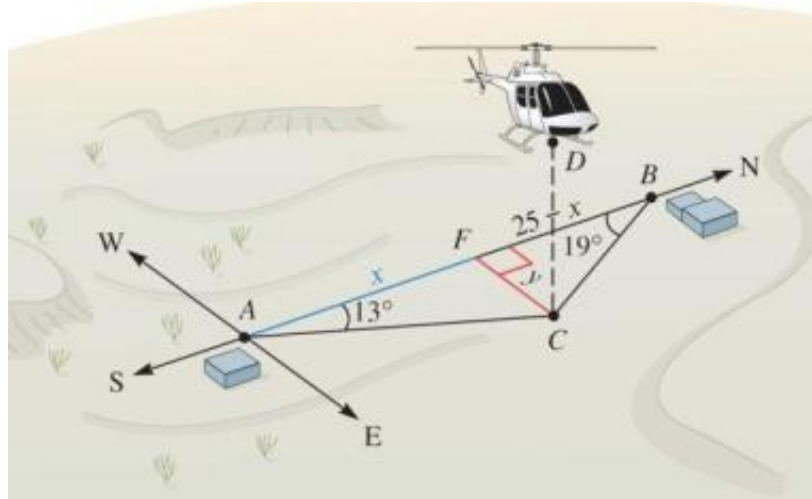
$$\frac{25 \tan 19^\circ}{\tan 13^\circ + \tan 19^\circ} = x$$

$$x = 14.966$$

$$y = x \tan 13^\circ$$

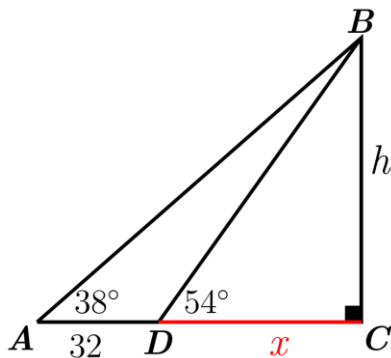
$$= 14.966 \tan 13^\circ$$

$$\approx 3.5 \text{ mi}$$

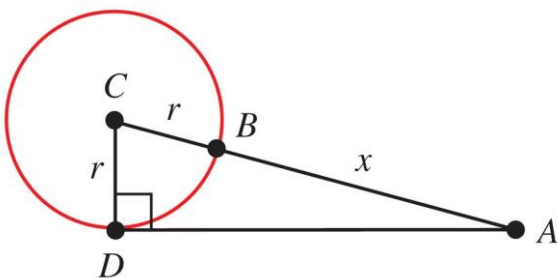


Exercises Section 2.3 – Solving Right Triangle Trigonometry

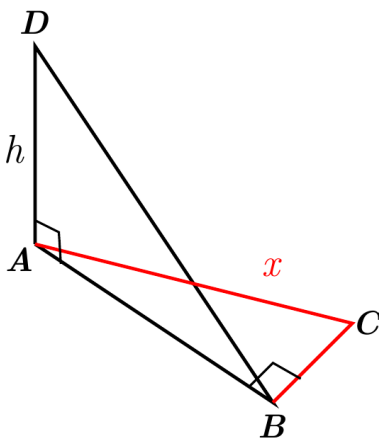
1. In the right triangle ABC , $a = 29.43$ and $c = 53.58$. Find the remaining side and angles.
2. In the right triangle ABC , $a = 2.73$ and $b = 3.41$. Find the remaining side and angles.
3. The distance from A to D is 32 feet. Use the information in figure to solve x , the distance between D and C .



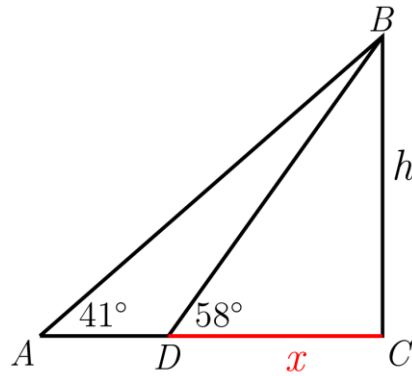
Find x .



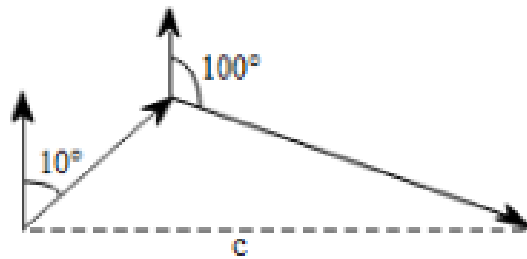
4. If $C = 26^\circ$ and $r = 19$
5. If $C = 30^\circ$ and $r = 15$
6. If $\angle ABD = 53^\circ$, $C = 48^\circ$, and $BC = 42$, find x and then find h .



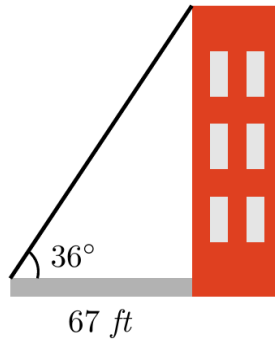
7. If $A = 41^\circ$, $\angle BDC = 58^\circ$, and $AB = 28$, find h , then x .



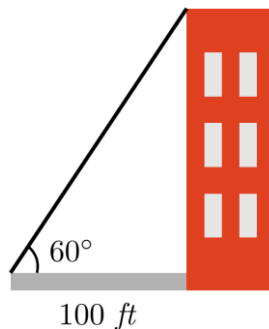
8. A plane flies 1.7 hours at 120 mph on a bearing of 10° . It then turns and flies 9.6 hours at the same speed on a bearing of 100° . How far is the plane from its starting point?



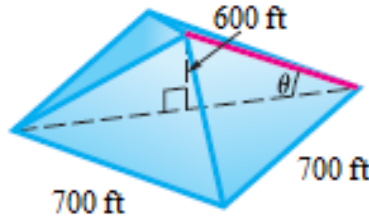
9. The shadow of a vertical tower is 67.0 feet long when the angle of elevation of the sun is 36.0° . Find the height of the tower.



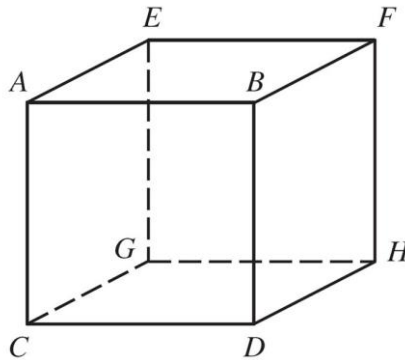
10. The shadow of a vertical tower is 100 feet long when the angle of elevation of the sun is 60° . Find the height of the tower.



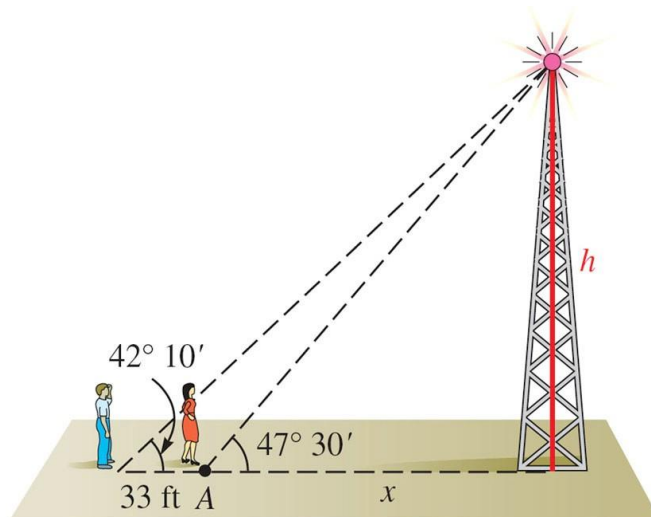
11. The base of a pyramid is square with sides 700 *feet* long, and the height of the pyramid is 600 *feet*. Find the angle of elevation of the edge indicated in the figure to two significant digits. (Hint: The base of the triangle in the figure is half the diagonal of the square base of the pyramid.)



12. If a 73-*foot* flagpole casts a shadow 51 *feet* long, what is the angle of elevation of the sun (to the nearest tenth of a degree)?
13. If a 75-*foot* flagpole casts a shadow 43 *feet* long, to the nearest 10 *minutes* what is the angle of elevation of the sun from the tip of the shadow?
14. Suppose each edge of the cube is 3.00 *inches* long. Find the measure of the angle formed by diagonals DE and DG. Round your answer to the nearest tenth of a degree.

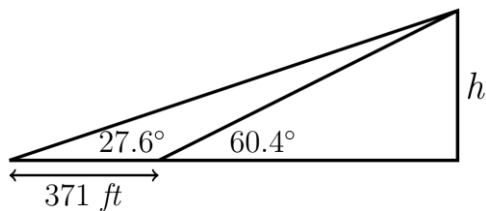


15. A person standing at point A notices that the angle of elevation to the top of the antenna is $47^\circ 30'$. A second person standing 33.0 *feet* farther from the antenna than the person at A finds the angle of elevation to the top of the antenna to be $42^\circ 10'$. How far is the person at A from the base of the antenna?

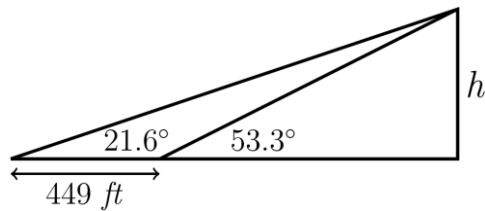


Find h as indicated in the figure.

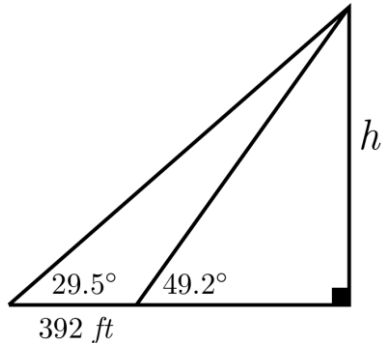
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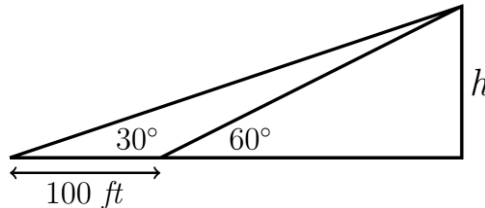
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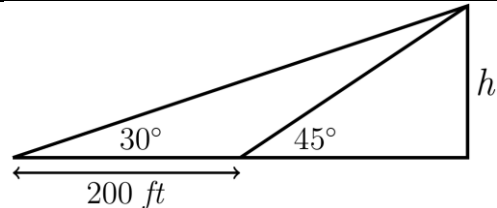
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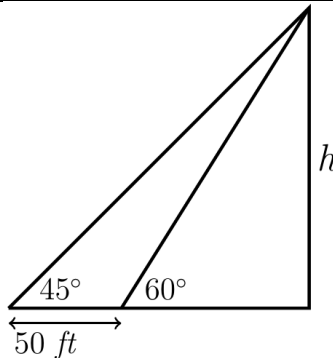
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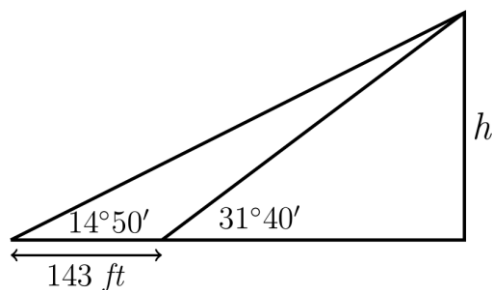
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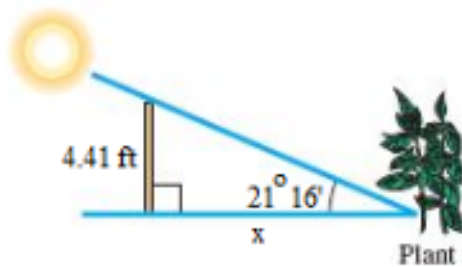
21.



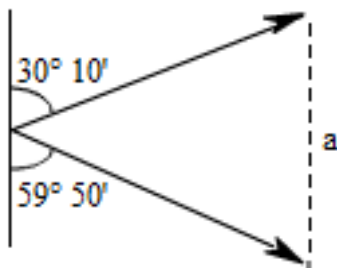
22. The angle of elevation from a point on the ground to the top of a pyramid is $31^\circ 40'$. The angle of elevation from a point 143 feet farther back to the top of the pyramid is $14^\circ 50'$. Find the height of the pyramid.



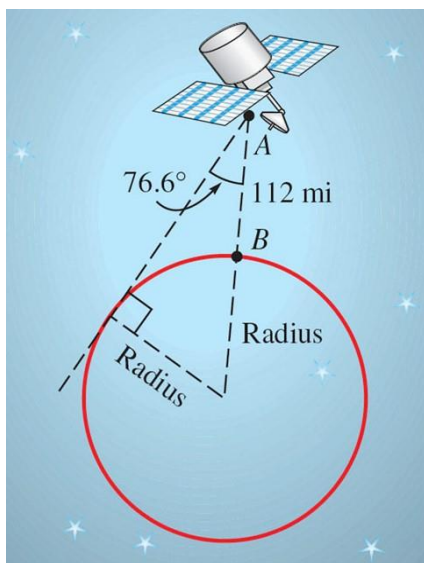
23. In one area, the lowest angle of elevation of the sun in winter is $21^\circ 16'$. Find the minimum distance, x , that a plant needing full sun can be placed from a fence 4.41 feet high.



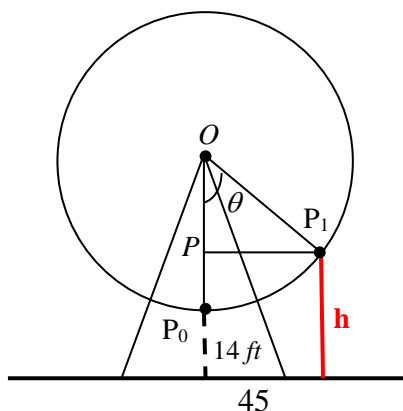
24. A ship leaves its port and sails on a bearing of $N 30^\circ 10' E$, at speed 29.4 mph . Another ship leaves the same port at the same time and sails on a bearing of $S 59^\circ 50' E$, at speed 17.1 mph . Find the distance between the two ships after 2 hrs .



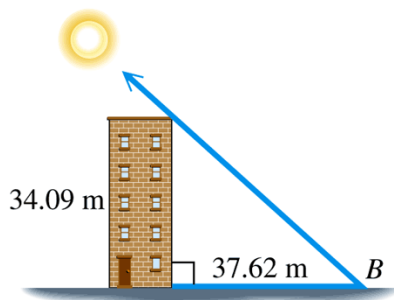
25. Radar stations A and B are on the east-west line, 3.7 km apart. Station A detects a plane at C , on a bearing of 61° . Station B simultaneously detects the same plane, on a bearing of 331° . Find the distance from A to C .
26. Suppose the figure below is exaggerated diagram of a plane flying above the earth. If the plane is 4.55 miles above the earth and the radius of the earth is $3,960 \text{ miles}$, how far is it from the plane to the horizon? What is the measure of angle A ?



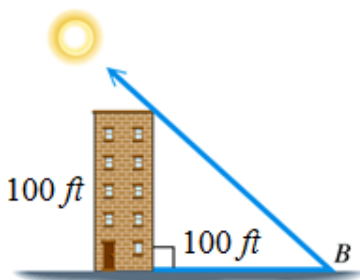
27. The Ferry wheel has a 250 feet diameter and 14 feet above the ground. If θ is the central angle formed as a rider moves from position P_0 to position P_1 , find the rider's height above the ground h when θ is 45° .



28. The length of the shadow of a building 34.09 *m* tall is 37.62 *m*. Find the angle of the elevation of the sun.



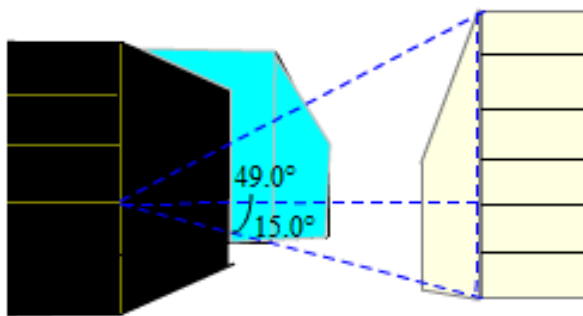
29. The length of the shadow of a building 100 *feet* tall is 100 *feet*. Find the angle of the elevation of the sun.



30. San Luis Obispo, California is 12 *miles* due north of Grover Beach. If Arroyo Grande is 4.6 *miles* due east of Grover Beach, what is the bearing of San Luis Obispo from Arroyo Grande?

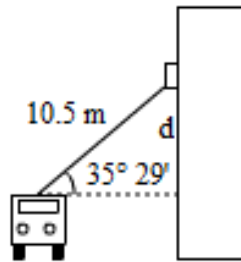
31. The bearing from *A* to *C* is S 52° E. The bearing from *A* to *B* is N 84° E. The bearing from *B* to *C* is S 38° W. A plane flying at 250 *mph* takes 2.4 hours to go from *A* to *B*. Find the distance from *A* to *C*.

32. From a window 31.0 *feet*. above the street, the angle of elevation to the top of the building across the street is 49.0° and the angle of depression to the base of this building is 15.0° . Find the height of the building across the street.

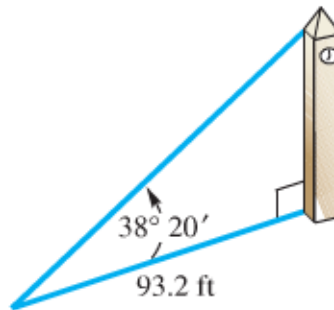


33. A man wondering in the desert walks 2.3 *miles* in the direction S 31° W. He then turns 90° and walks 3.5 *miles* in the direction N 59° W. At that time, how far is he from his starting point, and what is his bearing from his starting point?

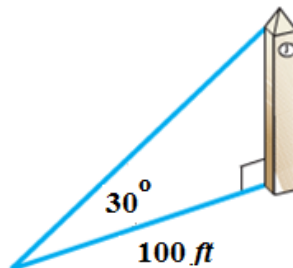
34. A 10.5-m fire truck ladder is leaning against a wall. Find the distance d the ladder goes up the wall (above the fire truck) if the ladder makes an angle of $35^\circ 29'$ with the horizontal.



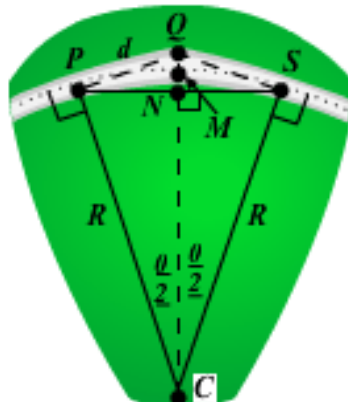
35. The angle of elevation from a point 93.2 feet from the base of a tower to the top of the tower is $38^\circ 20'$. Find the height of the tower.



36. The angle of elevation from a point 100 feet from the base of a tower to the top of the tower is 30° . Find the height of the tower.



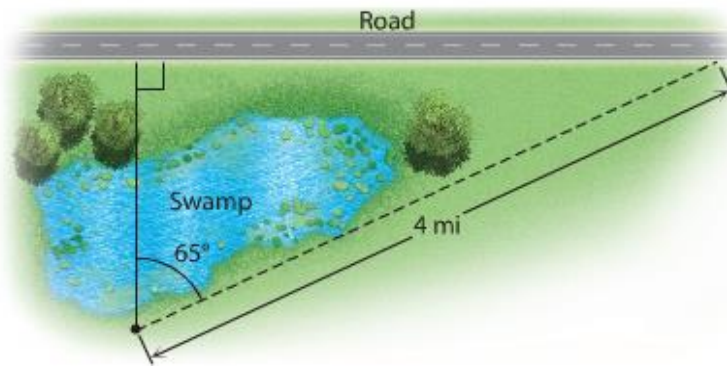
37. A basic curve connecting two straight sections of road is often circular. In the figure, the points P and S mark the beginning and end of the curve. Let Q be the point of intersection where the two straight sections of highway leading into the curve would meet if extended. The radius of the curve is R , and the central angle denotes how many degrees the curve turns.



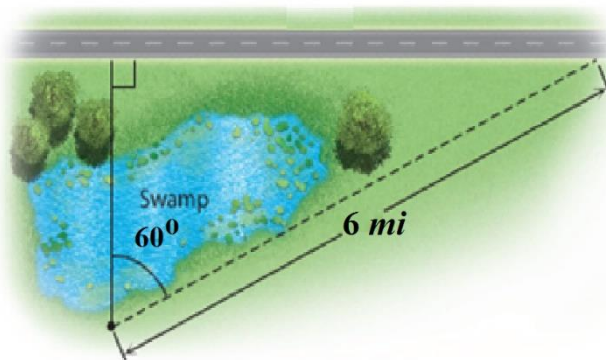
a) If $R = 965 \text{ ft.}$ and $\theta = 37^\circ$, find the distance d between P and Q .

b) Find an expression in terms of R and θ for the distance between points M and N .

38. Jane was hiking directly toward a long straight road when she encountered a swamp. She turned 65° to the right and hiked 4 mi in that direction to reach the road. How far was she from the road when she encountered the swamp?

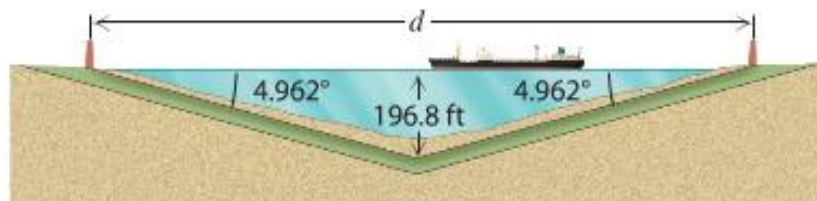


39. You were hiking directly toward a long straight road when you encountered a swamp. you turned 60° to the right and hiked 6 mi in that direction to reach the road. How far were you from the road when you encountered the swamp?

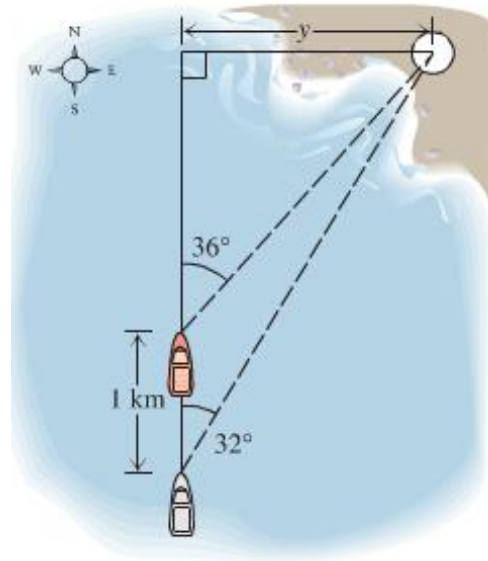


40. From a highway overpass, 14.3 m above the road, the angle of depression of an oncoming car is measured at 18.3° . How far is the car from a point on the highway directly below the observer?

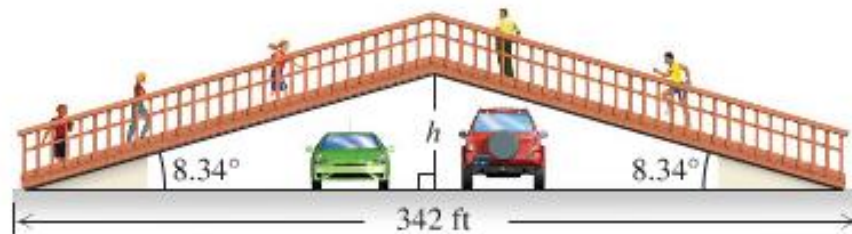
41. A tunnel under a river is 196.8 feet. below the surface at its lowest point. If the angle of depression of the tunnel is 4.962° , then how far apart on the surface are the entrances to the tunnel? How long is the tunnel?



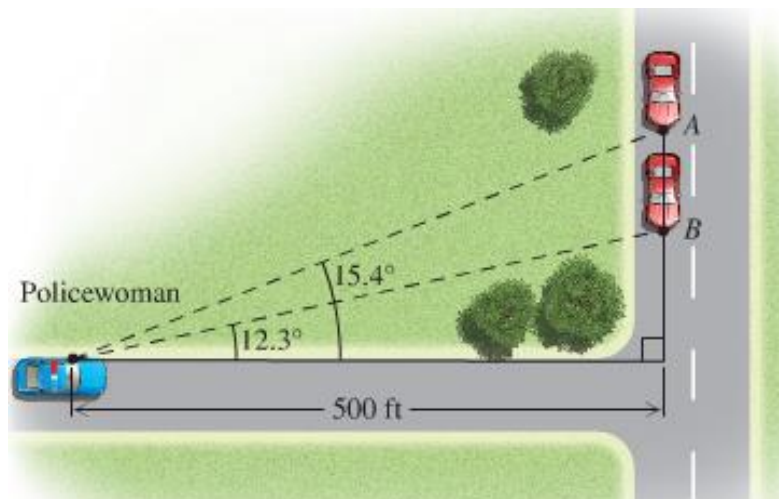
42. A boat sailing north sights a lighthouse to the east at an angle of 32° from the north. After the boat travels one more *kilometer*, the angle of the lighthouse from the north is 36° . If the boat continues to sail north, then how close will the boat come to the lighthouse?



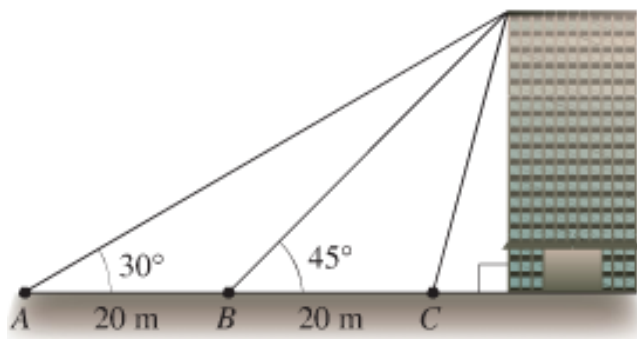
43. The angle of elevation of a pedestrian crosswalk over a busy highway is 8.34° , as shown in the drawing. If the distance between the ends of the crosswalk measured on the ground is 342 *feet*., then what is the height h of the crosswalk at the center?



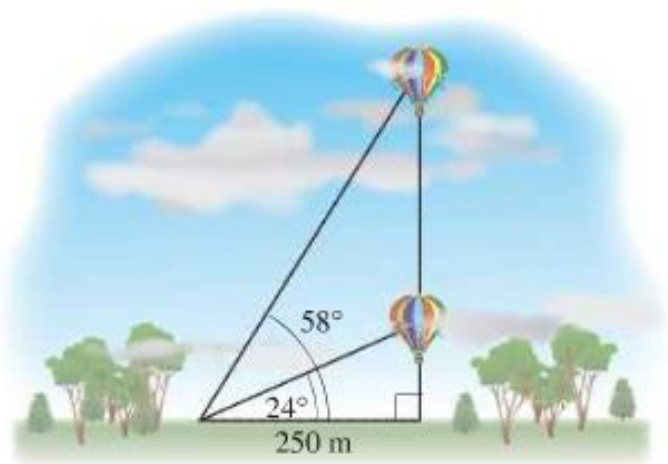
44. A policewoman has positioned herself 500 *feet*. from the intersection of two roads. She has carefully measured the angles of the lines of sight to points A and B. If a car passes from A to B is 1.75 *sec* and the speed limit is 55 *mph*, is the car speeding? (Hint: Find the distance from B to A and use $R = D/T$)



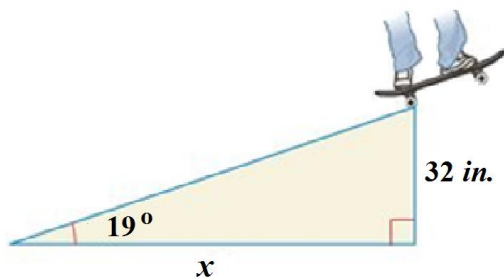
36. From point A the angle of elevation to the top of the building is 30° . From point B , 20 *meters* closer to the building, the angle of elevation is 45° . Find the angle of elevation of the building from point C , which is another 20 *meters* closer to the building.



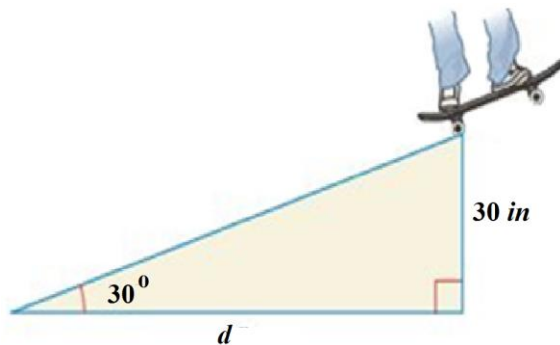
37. A hot air balloon is rising upward from the earth at a constant rate. An observer 250 *m* away spots the balloon at an angle of elevation of 24° . Two minutes later the angle of elevation of the balloon is 58° . At what rate is the balloon ascending?



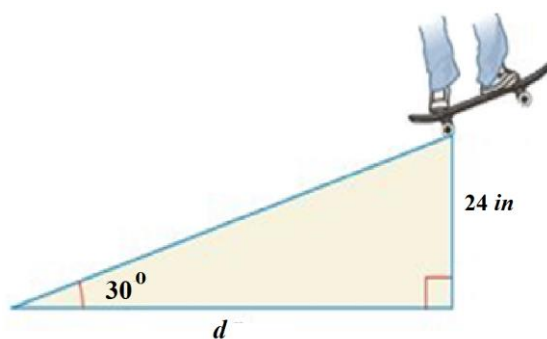
38. A skateboarder wishes to build a jump ramp that is inclined at a 19° angle and that has a maximum height of 32.0 *inches*. Find the horizontal width x of the ramp.



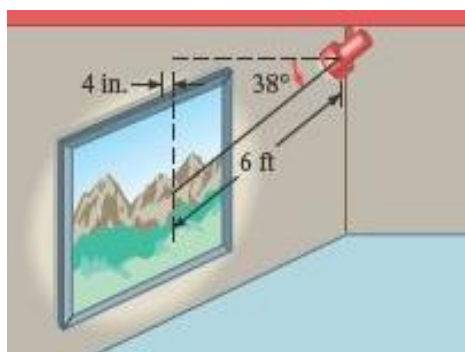
39. A skateboarder wishes to build a jump ramp that is inclined at a 30° angle and that has a maximum height of 30 *inches*. Find the horizontal width d of the ramp.



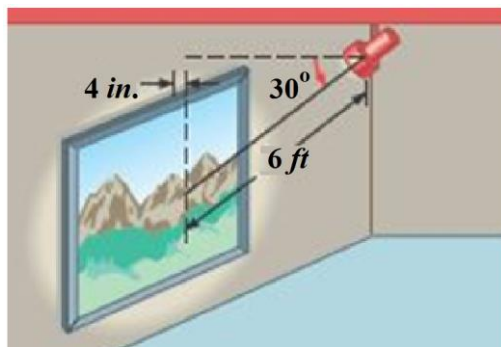
40. A skateboarder wishes to build a jump ramp that is inclined at a 30° angle and that has a maximum height of 24 *inches*. Find the horizontal width d of the ramp.



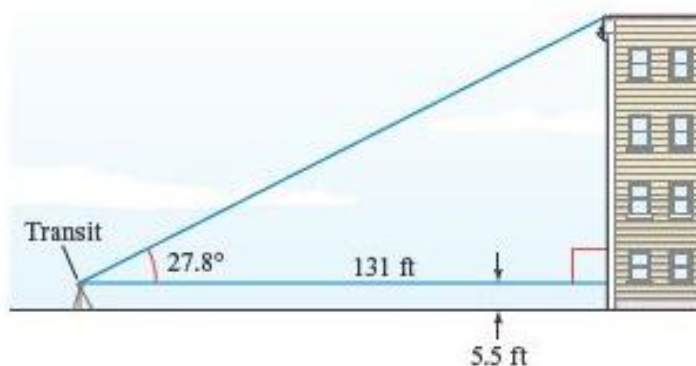
41. For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 *feet* from the piece of art and that the angle of depression of the light be 38° . How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 *inches* from the wall.



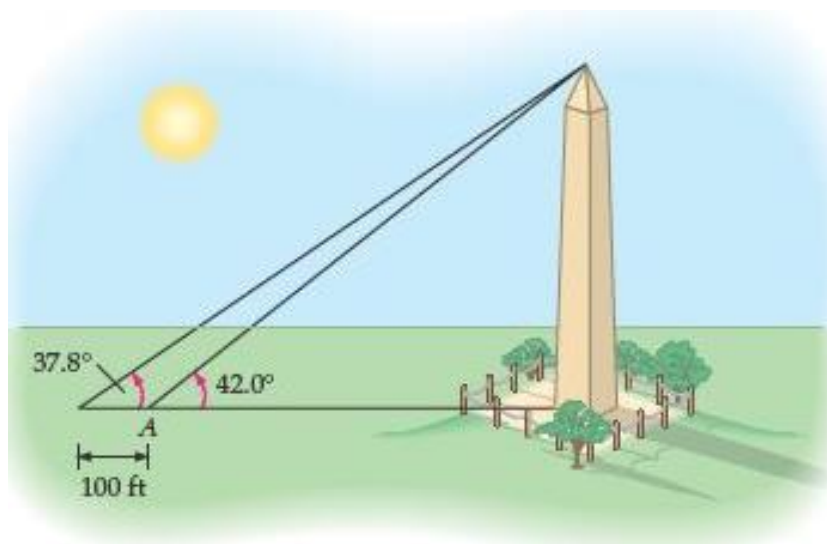
42. For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 *feet* from the piece of art and that the angle of depression of the light be 30° . How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 *inches* from the wall.



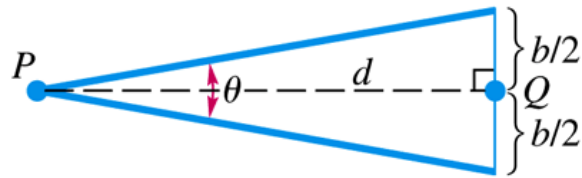
43. A surveyor determines that the angle of elevation from a transit to the top of a building is 27.8° . The transit is positioned 5.5 feet above ground level and 131 feet from the building. Find the height of the building to the nearest tenth of a foot.



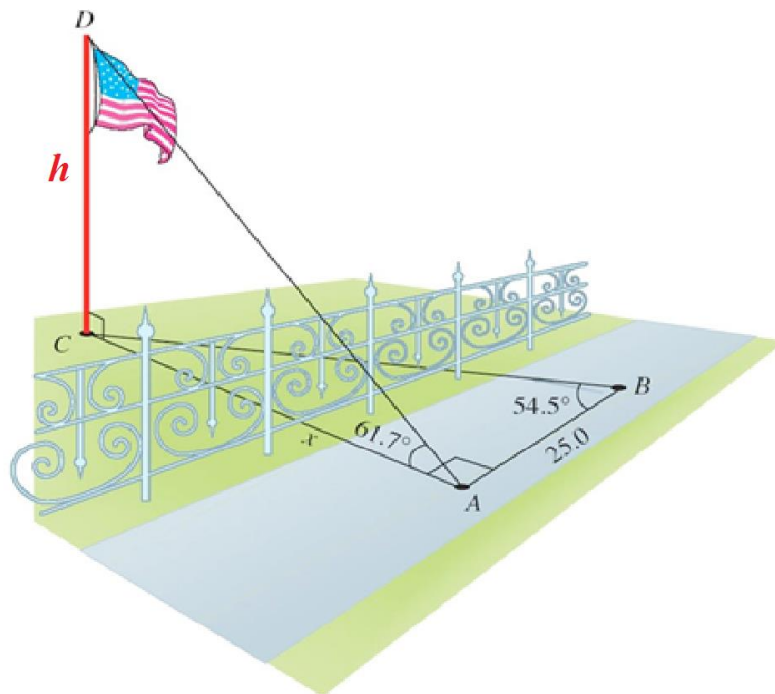
44. From a point A on a line from the base of the Washington Monument, the angle of elevation to the top of the monument is 42.0° . From a point 100 feet away from A and on the same line, the angle to the top is 37.8° . Find the height, to the nearest foot, of the Monument.



45. A method that surveyors use to determine a small distance d between two points P and Q is called the **subtense bar method**. The subtense bar with length b is centered at Q and situated perpendicular to the line of sight between P and Q . Angle θ is measured, then the distance d can be determined.



- a) Find d with $\theta = 1^\circ 23' 12''$ and $b = 2.000 \text{ cm}$
- b) Angle θ usually cannot be measured more accurately than to the nearest $1''$. How much change would there be in the value of d if θ were measured $1''$ larger?
46. A diagram that shows how Diane estimates the height of a flagpole. She can't measure the distance between herself and the flagpole directly because there is a fence in the way. So, she stands at point A facing the pole and finds the angle of elevation from point A to the top of the pole to be 61.7° . Then she turns 90° and walks 25.0 feet to point B , where she measures the angle between her path and a line from B to the base of the pole. She finds that angle is 54.5° . Use this information to find the height of the pole.

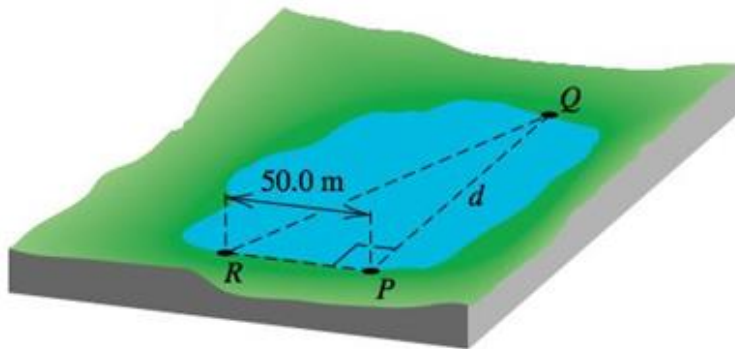


47. From a point 15 feet above level ground, a surveyor measures the angle of depression of an object on the ground at 68° . Approximate the distance from the object to the point on the ground directly beneath the surveyor.
48. A pilot, flying at an altitude of $5,000 \text{ feet}$ wishes to approach the numbers on a runway at an angle of 10° . Approximate, to the nearest 100 feet , the distance from the airplane to the numbers at the beginning of the descent.

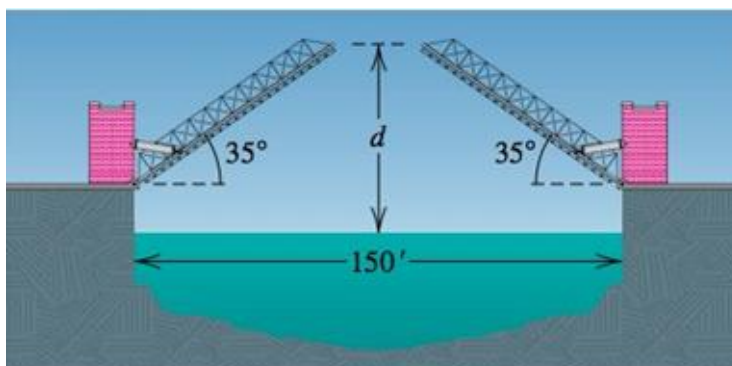
49. A person flying a kite holds the string 4 feet above ground level. The string of the kite is taut and makes an angle of 60° with the horizontal. Approximate the height of the kite above level ground if 500 feet of string is paved out.



50. To find the distance d between two points P and Q on opposite shores of a lake, a surveyor locates a point R that is 50.0 meters from P such that RP is perpendicular to PQ . Next, using a transit, the surveyor measures angle PRQ as $72^\circ 40'$. Find d .

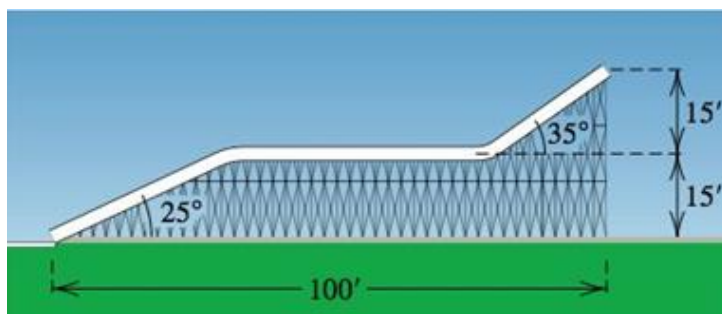


51. A drawbridge is 150 feet long when stretched across a river. The two sections of the bridge can be rotated upward through an angle of 35° .

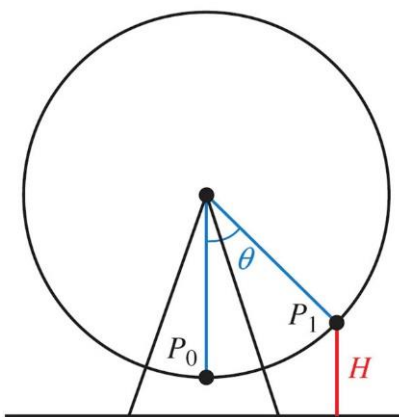


- a) If the water level is 15 feet below the closed bridge, find the distance d between the end of a section and the water level when the bridge is fully open.
- b) Approximately how far apart are the ends of the two sections when the bridge is fully opened?

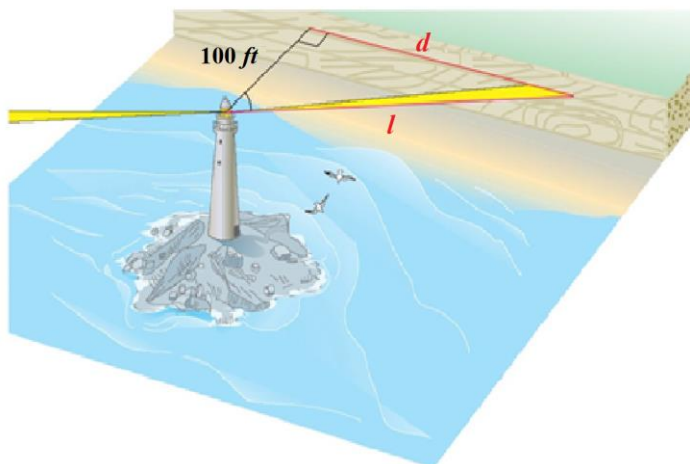
52. Find the total length of a design for a water slide to the nearest foot.



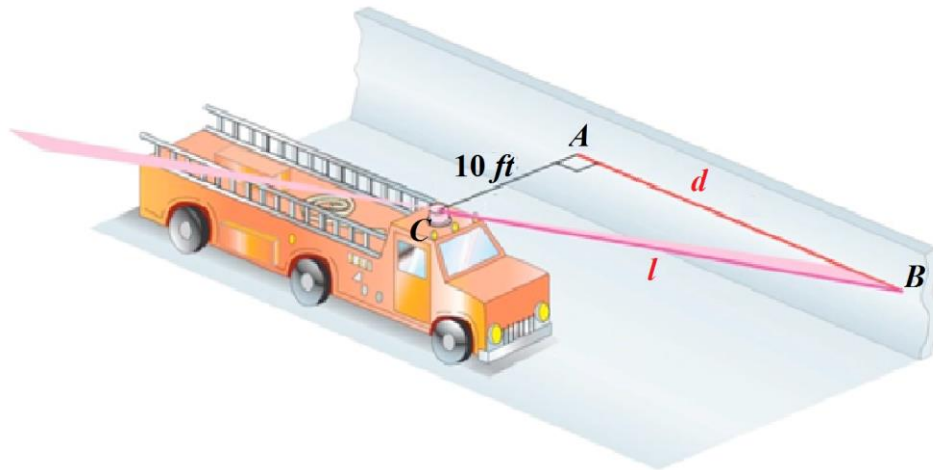
53. A Ferris wheel has radius 50.0 *feet*. A person takes a seat and then the wheel turns $\frac{2\pi}{3}$ *rad*.
- How far is the person above the ground?
 - If it takes 30 *sec* for the wheel to turn $\frac{2\pi}{3}$ *rad*, what is the angular speed of the wheel?
54. The diameter of the Ferris wheel is 250 *feet*, the distance from the ground to the bottom of the wheel is 14 *feet*, and one complete revolution takes 20 *minutes*, find



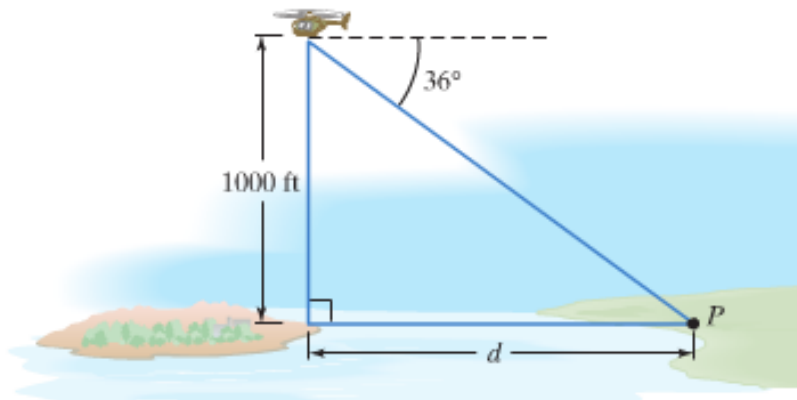
- The linear velocity, in miles per hour, of a person riding on the wheel.
 - The height of the rider in terms of the time t , where t is measured in minutes.
55. Find an equation that expresses l in terms of time t . Find l when t is 0.5 *sec*, 1.0 *sec*, and 1.5 *sec*. (assume the light goes through one rotation every 4 *seconds*.)



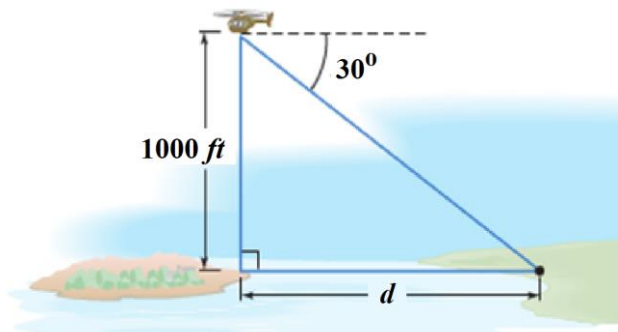
56. A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top of the truck is 10 feet from the wall and rotates through a complete revolution every 2 seconds. Find the equations that give the lengths d and ℓ in terms of time.



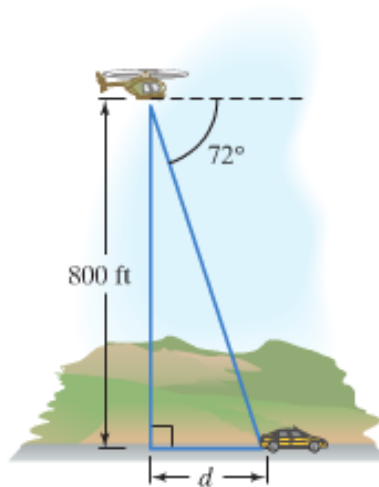
57. A helicopter hovers 1,000 feet above a small island. The angle of depression from the helicopter to point P on the coast is 36° . How far off the coast is the island?



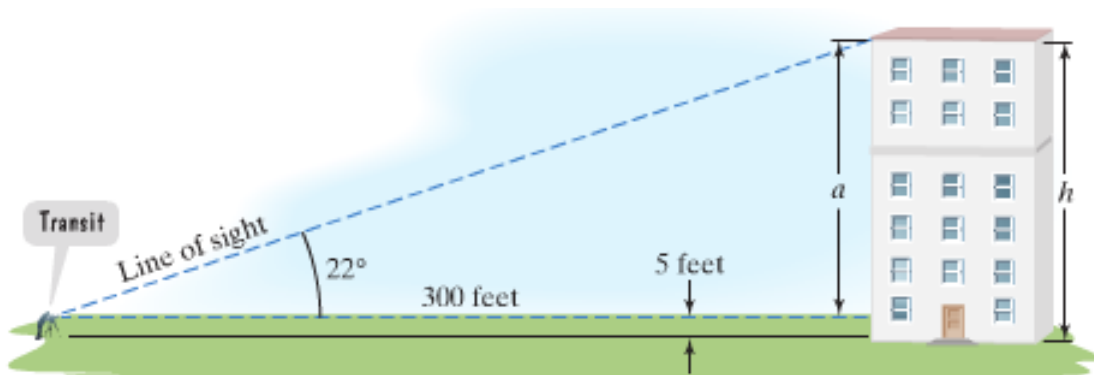
58. A helicopter hovers 1,000 feet above a small island. The angle of depression from the helicopter to point P on the coast is 30° . How far off the coast is the island?



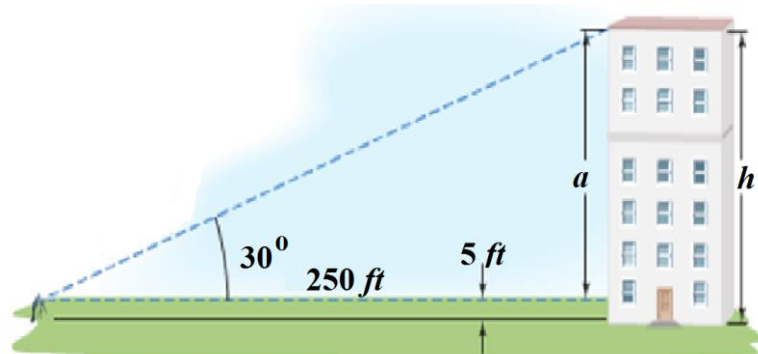
59. A police helicopter is flying at 800 *feet*. A stolen car is sighted at an angle of depression of 72° . Find the distance of the stolen car from a point directly below the helicopter.



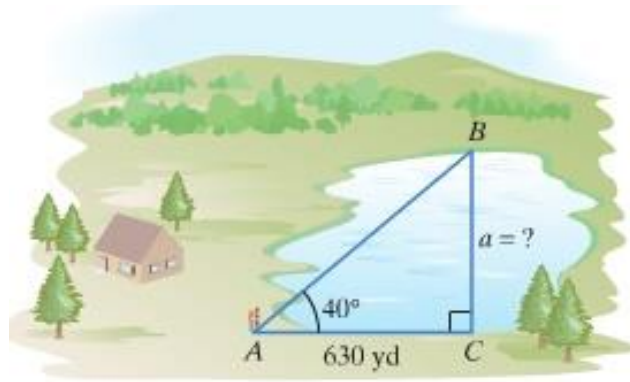
60. Sighting the top of a building a surveyor measured the angle of elevation to be 22° . The transit is 5 *feet* above the ground and 300 *feet* from the building. Find the building's height.



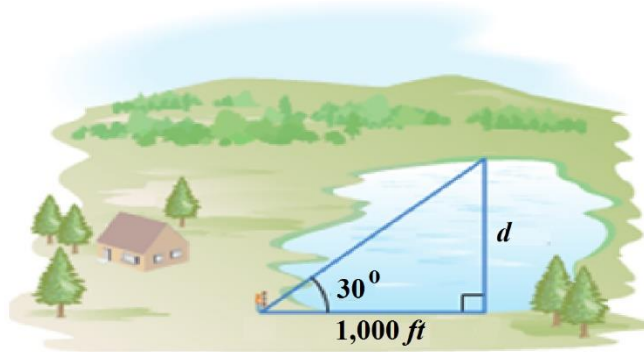
61. Sighting the top of a building a surveyor measured the angle of elevation to be 30° . The transit is 5 *feet* above the ground and 250 *feet* from the building. Find the building's height.



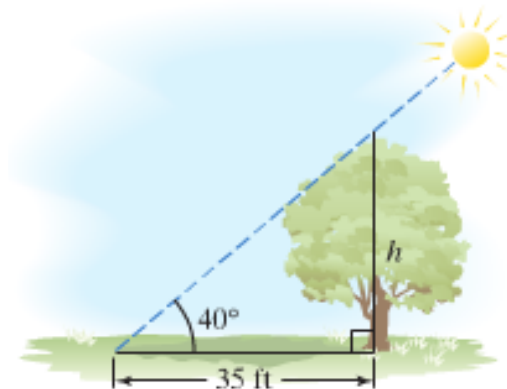
62. Determine how far it is across the lake.



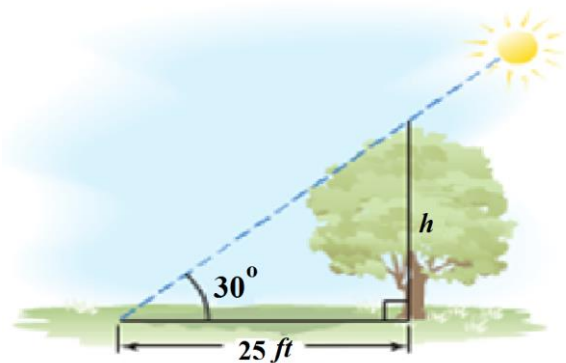
63. Determine how far it is across the lake.



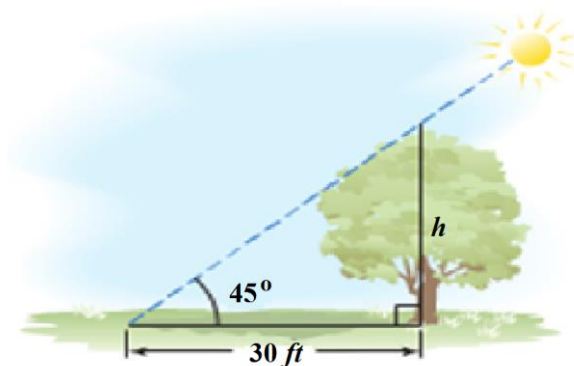
64. At a certain time of day, the angle of elevation of the sun is 40° . Find the height of a tree whose shadow is 35 feet long.



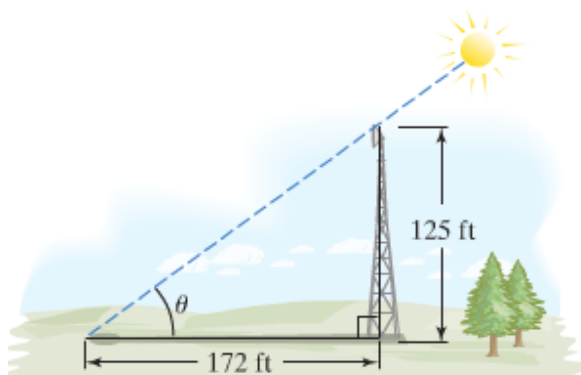
65. At a certain time of day, the angle of elevation of the sun is 30° . Find the height of a tree whose shadow is 25 feet long.



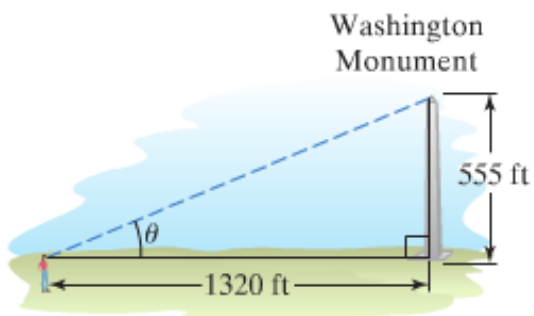
66. At a certain time of day, the angle of elevation of the sun is 45° . Find the height of a tree whose shadow is 30 *feet* long.



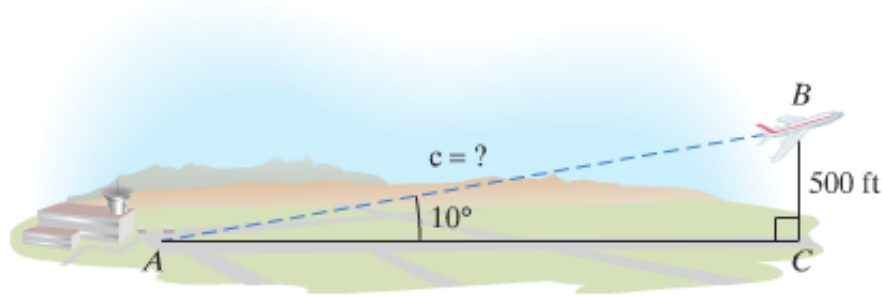
67. A tower that is 125 *feet* casts a shadow 172 *feet* long. Find the angle of elevation of the sun.



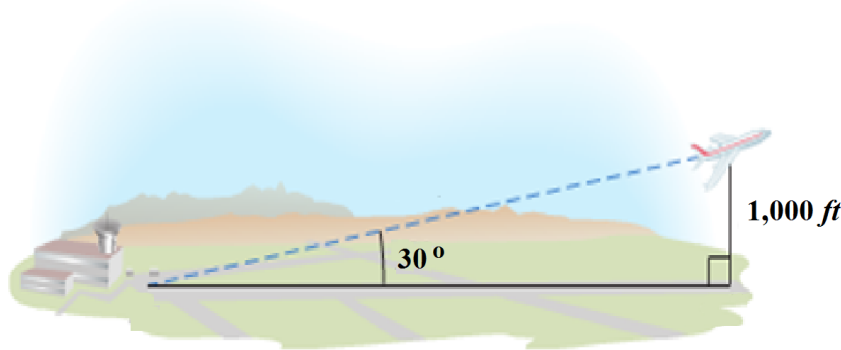
68. The Washington Monument is 555 *feet* high. If you are standing one quarter of a mile, or 1,320 *feet*, from the base of the monument and looking to the top, find the angle of elevation.



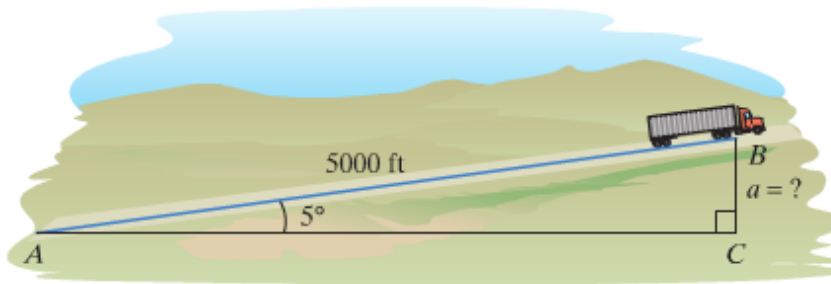
69. A plane rises from take-off and flies at an angle of 10° with the horizontal runway. When it has gained 500 *feet*, find the distance the plane has flown.



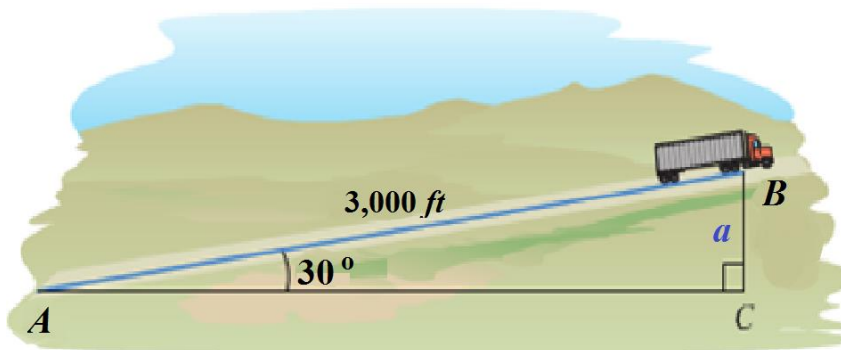
70. A plane rises from take-off and flies at an angle of 30° with the horizontal runway. When it has gained 1,000 *feet*, find the distance the plane has flown.



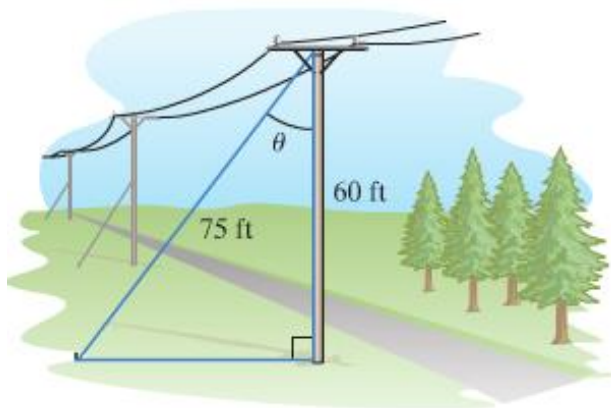
71. A road is inclined at an angle of 5° . After driving 5,000 *feet* along this road, find the driver's increase in altitude.



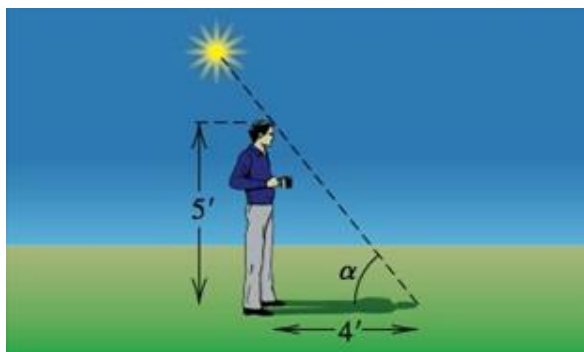
72. A road is inclined at an angle of 30° . After driving 3,000 *feet* along this road, find the driver's increase in altitude.



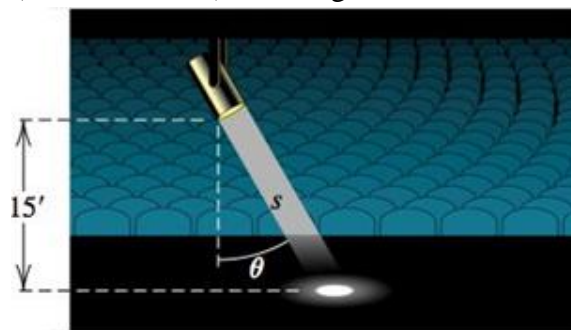
73. A telephone pole is 60 *feet* tall. A guy wire 75 *feet* long is attached from the ground to the top of the pole. Find the angle between the wire and the pole.



74. Approximate the angle of elevation α of the sun if a person 5.0 *feet* tall casts a shadow 4.0 *feet* long on level ground.



75. A spotlight with intensity 5000 candles is located 15 *feet* above a stage. If the spotlight is rotated through an angle θ , the illuminance E (in foot-candles) in the lighted area of the stage is given by

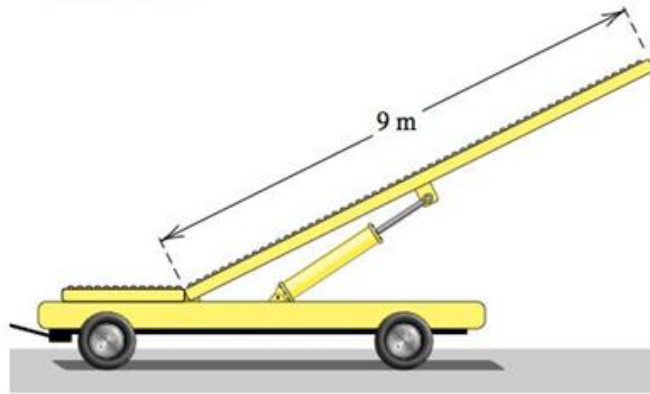


$$E = \frac{5,000 \cos \theta}{s^2}$$

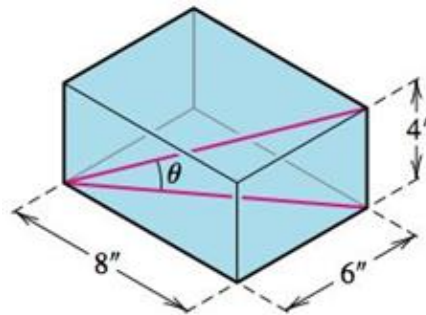
Where s is the distance (in *feet*) that the light must travel.

- Find the illuminance if the spotlight is rotated through an angle of 30° .
- The maximum illuminance occurs when $\theta = 0^\circ$. For what value of θ is the illuminance one-half the maximum value.

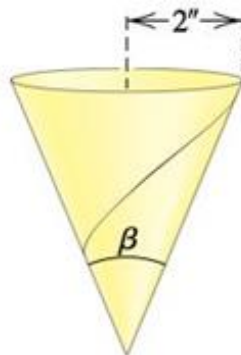
76. A conveyor belt 9 meters long can be hydraulically rotated up to an angle of 40° to unload cargo from airplanes.



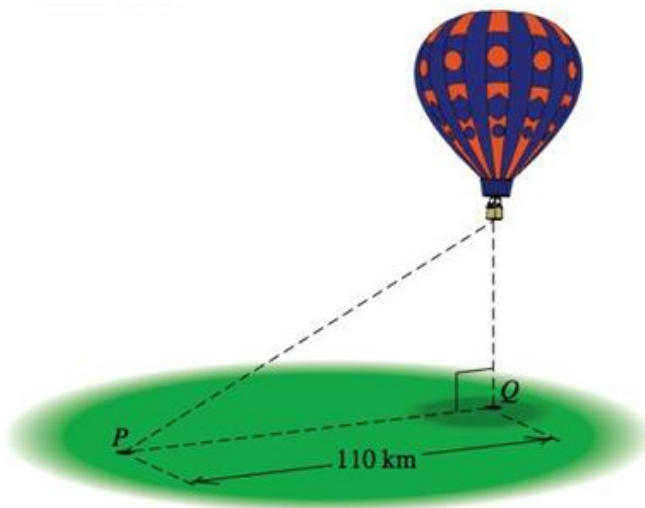
- a) Find, to the nearest degree, the angle through which the conveyor belt should be rotated up to reach a door that is 4 meters above the platform supporting the belt.
- b) Approximate the maximum height above the platform that the belt can reach.
77. A rectangular box has dimensions $8'' \times 6'' \times 4''$. Approximate, to the nearest tenth of a degree, the angle θ formed by a diagonal of the base and the diagonal of the box.



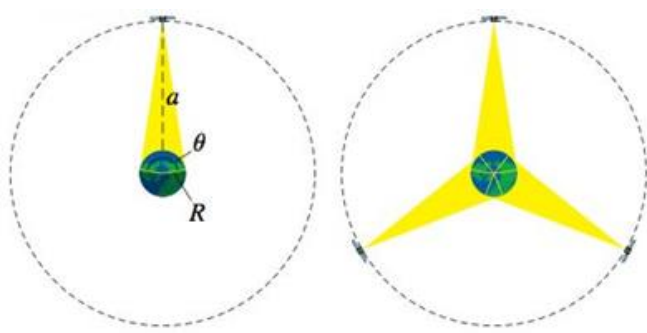
78. A conical paper cup has a radius of 2 inches, approximate, to the nearest degree, the angle β so that the cone will have a volume of 20 in^3 .



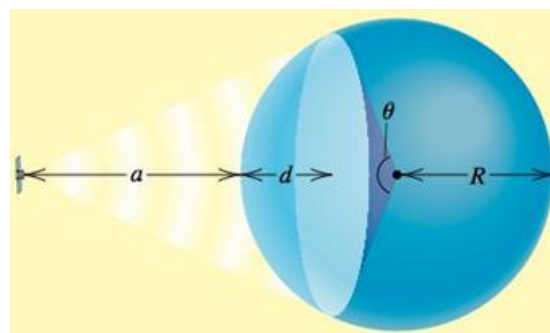
79. As a hot-air balloon rises vertically, its angle of elevation from a point P on level ground 100 km from the point Q directly underneath the balloon changes from $19^\circ 20'$ to $31^\circ 50'$. Approximately how far does the balloon rise during this period?



80. Shown in the left part of the figure is a communications satellite with an equatorial orbit—that is, a nearly circular orbit in the plane determined by Earth's equator. If the satellite circles Earth at an altitude of $a = 22,300 \text{ mi}$, its speed is the same as the rotational speed of Earth; to an observer on the equator, the satellite appears to be stationary—that is, its orbit is synchronous.



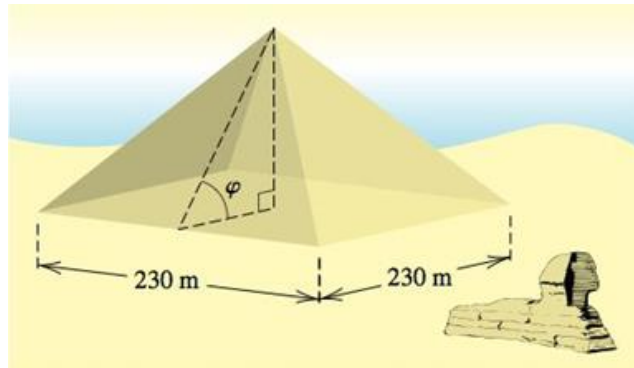
a



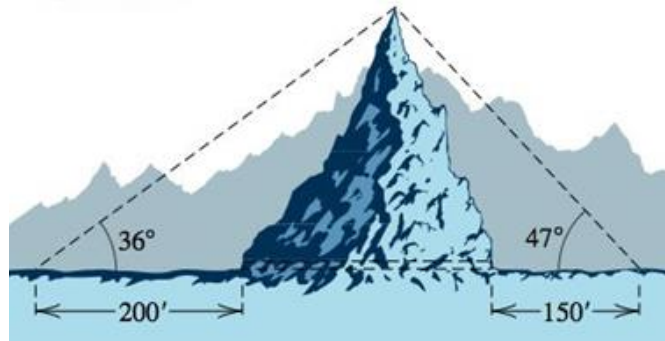
b

- Using $R = 4,000 \text{ mi}$ for the radius of Earth, determine the percentage of the equator that is within signal range of such a satellite.
- As shown in the right part of the figure (*a*), three satellites are equally spaced in equatorial synchronous orbits. Use the value of θ obtained in part (*a*) to explain why all points on the equator are within signal range of at least one of the three satellites.
- The figure (*b*) shows the area served by a communication satellite circling a planet of radius R at an altitude a . The portion of the planet's surface within range of the satellite is a spherical cap of depth d and surface area $A = 2\pi R d$. Express d in terms of R and θ .
- Estimate the percentage of the planet's surface that is within signal range of a single satellite in equatorial synchronous orbit.

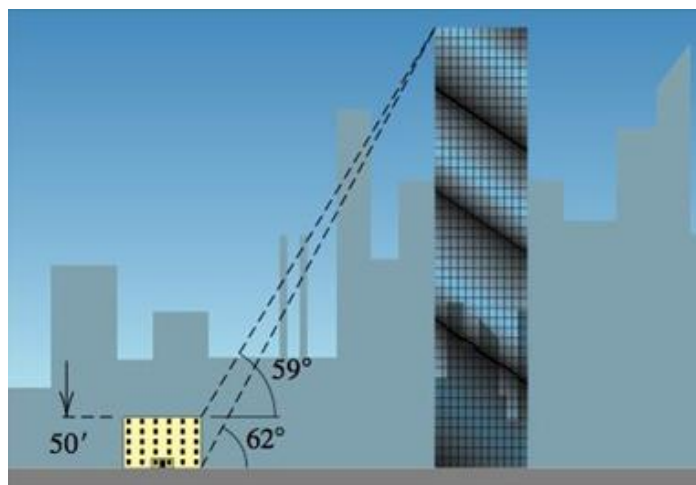
81. The great Pyramid of Egypt is 147 *meters* high, with a square base of side 230 *meters*. Approximate, to the nearest degree, the angle φ formed when an observer stands at the midpoint of one the sides and views the apex of the pyramid.



82. A tunnel for a new highway is to be cut through a mountain that is 260 *feet* high. At a distance of 200 *feet* from the base of the mountain, the angle of elevation is 36° . From a distance of 150 *feet* on the other side, the angle of elevation is 47° . Approximate the length of the tunnel to the nearest foot.

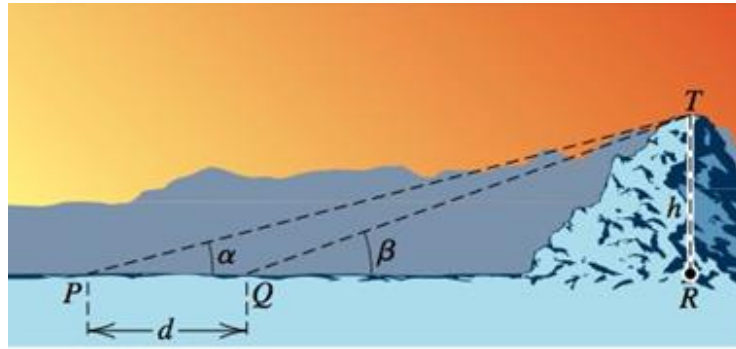


83. When a certain skyscraper is viewed from the top of a building 50 *feet* tall, the angle of elevation is 59° . When viewed from the street next to the shorter building, the angle of elevation is 62° .

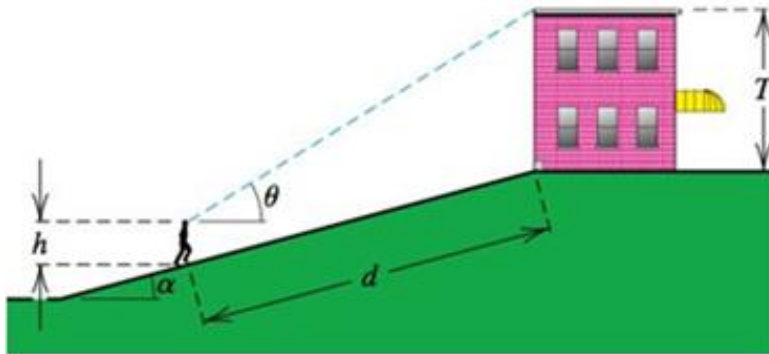


- Approximately how far apart are the two structures?
- Approximate the height of the skyscraper to the nearest tenth of a *foot*.

84. When a mountaintop is viewed from the point P , the angle of elevation is α . From a point Q , which is d miles closer to the mountain, the angle of elevation increases to β .



- a) Show that the height h of the mountain is given by: $h = \frac{d}{\cot \alpha - \cot \beta}$.
- b) If $d = 2\text{mi}$, $\alpha = 15^\circ$, and $\beta = 20^\circ$, approximate the height of the mountain.
85. An observer of height h stands on an incline at a distance d from the base of a building of height T . The angle of elevation from the observer to the top of the building is θ , and the incline makes an angle of α with the horizontal.



- a) Express T in terms of h , d , α , and θ .
- b) If $d = 50\text{ft}$, $h = 6\text{ft}$, $\alpha = 15^\circ$, and $\theta = 31.4^\circ$, estimate the height of the building.