

see 1.3, 1.4

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1/ $y = x^3$, $y = 8$, $x = 0$ about y -axis.

$$V = \pi \int_0^8 (y^{\frac{1}{3}})^2 dy$$

$$= \frac{3\pi}{5} y^{\frac{5}{3}} \Big|_0^8$$

$$= \frac{3\pi}{5} (2^5 - 0)$$

$$= \frac{96\pi}{5} \text{ unit}^3$$

$$8 = 2^3$$

2/ $y = x^2$, $y = x$ about x -axis

$$y = x^2 = x \rightarrow x = 0, 1$$

$$V = \pi \int_0^1 (x^2 - x^4) dx$$

$$= \pi \left(\frac{1}{3} x^3 - \frac{1}{5} x^5 \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{5} \right)$$

$$= \frac{2\pi}{15} \text{ unit}^3$$

3/ $y = x^2$ $y = x$ line: $y = 2$.

$$y = x^2 = x \rightarrow x = 0, 1.$$

$$V = \pi \int_0^1 ((2-x^2)^2 - (2-x)^2) dx$$

$$= \pi \int_0^1 (4 - 4x^2 + x^4 - 4 + 4x - x^2) dx$$

$$= \int_0^1 (x^4 - 5x^2 + 4x) dx$$

$$= \pi \left(\frac{1}{5} x^5 - \frac{5}{3} x^3 + 2x^2 \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{5} - \frac{5}{3} + 2 \right)$$

$$= \frac{8\pi}{15} \text{ unit}^3$$

4/ $y = \sqrt{x}$ $0 < y < 1$ x -axis
 $y^2 = x$

$$V = 2\pi \int_0^1 y(1-y^2) dy$$

$$= 2\pi \int_0^1 (y - y^3) dy$$

$$= 2\pi \left(\frac{1}{2} y^2 - \frac{1}{4} y^4 \right) \Big|_0^1$$

$$= 2\pi \left(\frac{1}{2} - \frac{1}{4} \right)$$

$$= \frac{\pi}{2} \text{ unit}^3$$

5/ $y = x - x^2$, $y = 0$ about $x = 2$.
 $x=0, 1$

$$V = 2\pi \int_0^1 (2-x)(x-x^2) dx$$

$$= 2\pi \int_0^1 (2x - 3x^2 + x^3) dx$$

$$= 2\pi \left(x^2 - x^3 + \frac{1}{4}x^4 \right) \Big|_0^1$$

$$= 2\pi \left(1 - 1 + \frac{1}{4} \right)$$

$$= \frac{\pi}{2} \text{ unit}^3$$

6/ $y = -x^2 + 6x - 8 = 0$ $y = 0$ $\approx y$ -axis
 $x = 2, 4$

$$V = 2\pi \int_2^4 x(-x^2 + 6x - 8) dx \quad \text{shell-method}$$

$$= 2\pi \int_2^4 (-x^3 + 6x^2 - 8x) dx$$

$$= 2\pi \left(-\frac{1}{4}x^4 + 2x^3 - 4x^2 \right) \Big|_2^4$$

$$= 2\pi (-64 + 128 - 64 - (-4 + 16 - 16))$$

$$= 8\pi \text{ unit}^3$$

$$7/ \quad y = -x^2 + 6x - 8 = 0 \quad y=0 \quad -x \text{ axis}$$

$$x = 2, 4$$

$$\begin{aligned}
 V &= \pi \int_2^4 (-x^2 + 6x - 8)^2 dx \\
 &= \pi \int_2^4 (x^4 - 12x^3 + 16x^2 + 36x^2 + 64 - 96x) dx \\
 &= \pi \int_2^4 (x^4 - 12x^3 + 52x^2 - 96x + 64) dx \\
 &= \pi \left(\frac{1}{5}x^5 - 3x^4 + \frac{52}{3}x^3 - 48x^2 + 64x \right) \Big|_2^4 \\
 &= \pi \left(\frac{1024}{5} - 768 + \frac{3,328}{3} - 768 + 256 \right) \\
 &\quad - \frac{32}{5} + 48 - \frac{416}{3} + 192 - 128 \\
 &= \pi \left(\frac{992}{5} + \frac{2,912}{3} - 1168 \right) \\
 &= \frac{16\pi}{15} \text{ unit}^3
 \end{aligned}$$

$$8/ \quad x = (y-3)^2 \quad x=4 \quad \sim y=1$$

$$x = (y-3)^2 = 4 \rightarrow y-3 = \pm 2 \quad \left. \begin{array}{l} y=1, 5 \end{array} \right\}$$

$$\begin{aligned}
 V &= 2\pi \int_1^5 (y-1)(4 - (y-3)^2) dy \\
 &= 2\pi \int_1^5 (y-1)(-5 - y^2 + 6y) dy \\
 &= 2\pi \int_1^5 (-y^3 - 11y + 7y^2 + 5) dy \\
 &= 2\pi \left(-\frac{1}{4}y^4 - \frac{11}{2}y^2 + \frac{7}{3}y^3 + 5y \right) \Big|_1^5 \\
 &= 2\pi \left(-\frac{625}{4} - \frac{275}{2} + \frac{875}{3} + 25 + \frac{1}{4} + \frac{11}{2} - \frac{7}{3} - 5 \right) \\
 &= 2\pi \left(-288 + 20 + \frac{868}{3} \right) \rightarrow \frac{64}{3} \\
 &= \frac{128\pi}{3} \text{ unit}^3
 \end{aligned}$$