A group of 5 boys and 3 girls is to be photographed.

- How many ways can they be arranged in one row?
  $8! = 40320$ ways

- How many ways can they be arranged with the girls in the front row and the boys in the back?
  Front row = $3! = 6$ Back row = $5! = 120$
  They can be arranged in: $6.120 = 720$ ways.

- What if a boy is to sit in the end chairs? (All the chairs in one row)
  There exists 5 choices for seating a boy in the left seat, then 4 choices right end.
  Left (8-2 =) 6 people may be seated with no restriction:
  $5.(6!\,(4)) = 14400$ ways b _ _ _ _ _ _ b
  5 6 5 4 3 2 1 4

- How many ways can this be done if boys sit side by side & girls side by side?
  Boys-Girls or Girls-Boys $\Rightarrow$ Total numbers = $(5!\,3!) + (3!\,5!) = 1440$

- In how many ways can be seated on a bench if only 4 seats are available?
  Number of arrangements of 8 people taken 4 at a time = $P(8,4) = 8.7.6.5 = 1680$

- How many ways can be arranged if the girls occupy the even places? (All the chairs in one row)
  The boys can be seated in $P(5,5) = 5! = 120$ ways, and the girls in $P(3,3) = 3! = 6$ ways.
  Number of arrangements = $(120)(6) = 720$

- In how many ways 2 particular people must not sit next to each other?
  Consider 2 particular people as 1 person. Then there are 7 people altogether and they can be arranged in $6! = 720$ ways. But the 2 people can be arranged in $2!$ ways. Therefore, the number of ways to arranging 8 people with 2 particular people together is = $6!\,2! = 1440$.
  The total number of ways in which 8 people can be seated so that the 2 particular people do not sit together

  = (# of 1 seated anywhere) - (# 2 particular people seated together)
  = $7! - 1440 = 3600$ ways.

- How many possibilities for 2 boys and 3 girls.
  $C(5,2)\,C(3,2) = 10\,.(3) = 30$ possibilities.

- To select 3 how many ways for any mixture of boys and girls $C(8,3) = \frac{8.7.6}{3.2.1} = 56$ ways

- How many to select 3 with majority boys

  All boys and no girl: $C(5,3).C(3,0) = 10.(1) = 10$
  2 boys and 1 girl: $C(5,2).C(3,1) = 10.(3) = 30$
  $\Rightarrow$ the # of selecting majority boys = $10 + 30 = 40$ ways.

12 Boys & 10 Girls. 7 are chosen.

- What is the probability that at least 2 Girls are chosen?

  $$1 - [\Pr(\text{no Girls}) + \Pr(1\text{ G})] = 1 - \frac{C(12,7)+C(12,6)C(10,1)}{C(22,7)} \approx .94$$

- What is the probability no boys are chosen? $\Pr(\text{no boys}) = \Pr(\text{all Girls}) = \dfrac{C(10,7)}{C(22,7)} = \dfrac{5}{7106}$

- What is the probability that 1st three are boys? $\Pr = \dfrac{12.11.10}{22.21.20} = \dfrac{1}{7}$

- What is the probability that more boys than girls are chosen?

  $$\Pr(B > G) = \frac{C(12,4).C(10,3)+C(12,5).C(10,2)+C(12,6).C(10,1)+C(12,7)}{C(22,7)} \approx .616$$

Poker hand consists of 5 cards selected from a deck of 52 cards.

- How many different poker hands are there?
  C(52,5) = 2598960 hands   ({1,2,3,4,5} = {5,3,4,2,1})

- How many different poker hands consist entirely of aces and kings?
  Number of aces + kings = 8; C(8,5) = 56 hands.

- How many different poker hands consist entirely of clubs?   # of clubs = 13, C(13,5) = 1287 hands.

- How many consist of 3 aces and 2 kings?   C(4,3).C(4,2) = 4.6 = 24

- How many different poker hands consist entirely of red cards?
  # red cards = 26; C(26,5) = 65780 hands.

- How many combinations have cards from exactly 2 suits?
  a)Consider one from the 1^{st} suit, then there are C(4,1) = 4, and left 4 for the other suit then there are C(3,1) = 3. Therefore there are 4.C(13,1) . 3C(13,4) = 111540 ways.
  b) Consider 2 from the 1^{st} suit, then there are C(4,1) = 4, and left 3 for the other suit then there are C(3,1) = 3. Therefore there are 4.C(13,2) . 3C(13,3) = 267696 ways
  c)Total = 111540 + 267696 = 379236 ways

- How many ways all the cards from the same suit?
  Select a suit, there are C(4,1) = 4 ways to do this. For each selection of a suit there are C(13,5) = 1287.
  Final = 4. C(13,5) = 5148 ways.

- How many ways 3 from one suit and 2 from another?
  Select a suit, there are C(4,1) = 4 ways to do this. The other suit is C(3,1) = 3 (since 3 suits left to choose from). First 3 from 1 suit there are 4.C(13,3) = 286 ways, and 2 from another 3.C(13,2) = 78.
  Total = 4.C(13,3) . 3C(13,2) = 22308 ways.

- How many ways 2 aces, 2 cards of another denomination, and 1 card of a 3^{rd} denomination.
  - For 2 aces = C(4,2) = 6
  - 2 cards of another denomination are C(4,2) = 6 ways, there are 12 ways for the 2^{nd} denomination.
    Therefore, there are 12.(6) = 72 ways
  - 3^{rd} denomination the are 11 ways, 1 card $\Rightarrow$ 11.C(4,1) = 44
    The outcomes: 6.(72).(44) = 19008 hands.

- How many hands are in 2 cards of 1 denomination, 2 cards of another different denomination, and 1 card of a 3^{rd} denomination.

  Select 2 cards of 1 denomination = C(13,2) = 78 ways.
  Select 2 of one denomination, there are C(4,2) = 6
  Then select 2 of the other = C(4,2) = 6
  Select the 3^{rd} denomination, there are 11.C(4,1) = 44
  # of poker hands = 78.6.6.44 = 123552 hands.

Let the event A: card is a spade, and B is the face card.

- ***What is the probability of A?*** Pr (A={card is a spade}) = $\frac{\#\text{spades}}{\#\text{cards}} = \frac{13}{52} = \frac{1}{4}$

- ***What is the probability of B?*** Pr (B={face card}) = $\frac{\#\text{faces}}{\#\text{cards}} = \frac{12}{52} = \frac{3}{13}$

- ***What is the probability of A $\cap$ B?*** Pr (A $\cap$ B) = $\frac{\#\text{spade face cards}}{\#\text{cards}} = \frac{3}{52}$

* 13 cards are dealt from a deck of 52 cards

a) ***What is the probability that the ace of spades is one of the 13 cards***?

Pr (ace of spades) = $\frac{13}{52}$ = 1/4

b) ***Suppose 1 of the 13 cards is chosen at random and not found to be the ace spades. What is the probability that none of the 13 cards is the ace of spades***?

$$\text{Pr(none| 1 is not)} = \frac{\text{Pr(none)x Pr(random 1 is not found |none)}}{\text{Pr(none)xPr(1 is not found |none) + Pr(1 is)xPr(Ace is not| ace is)}} = \frac{\frac{3}{4}\text{x}1}{\frac{3}{4}\text{x}1+(\frac{1}{4}\text{x}\frac{12}{13})} = \frac{13}{17}$$

c) Repeat previous experiment 10 times (replacing the card), and the ace of spades is not seen. What is the probability that the ace of spades actually is one of the 13 cards?

$$\text{Pr(aces is one| 10 is not)} = \frac{\text{Pr(1 } is) x \text{ Pr(10 are not |is)}}{\text{Pr(1 is)xPr(10 are not|is) + Pr(0 is)xPr(10 are not| 0 is)}}$$

$$= \frac{\frac{1}{4}x\left(\frac{12}{13}\right)^{10}}{\frac{1}{4}x\left(\frac{12}{13}\right)^{10} + (\frac{3}{4}x1)} \approx .13$$

COIN

- 3 coins to be tossed.
  - ***What is the probability at least one head appears***?

    $\Pr(\{1,2,3\}) = \Pr(1) + \Pr(2) + \Pr(3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$

  - ***What is the probability all heads or all tails***?

    $\Pr(\text{all heads or all tails}) = \Pr(0) + \Pr(3) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

- 1 coin, ***what is the probability that heads is twice as likely to appear as tails***?

  Let $\Pr(T) = p \Rightarrow \Pr(H) = 2p \Rightarrow p + 2p = 1 \Rightarrow p = 1/3$ $\quad\Pr(T) = 1/3 \Rightarrow \Pr(H) = 2/3$

- 1 coin is tossed 10 times?

  a) ***How many different outcomes are possible***? $2^{10} = 1024$ outcomes.

  b) ***How many different outcomes have exactly 4 heads***? $C(10,4) = 210$ outcomes

  c) ***How many different outcomes at the most 2 heads?*** $C(10,0) + C(10,1) + C(10,2) = 56$

  d) ***How many different outcomes at least 3 heads?***

  All outcomes – (at most 2 H) = 1024 – 56 = 968

  e) ***What is the probability of obtaining exactly 4 heads?***

  $$P(4H) = \frac{C(10,4)}{2^{10}} = \frac{210}{1024} \approx .205$$

- ***Tosses 3 times, what is the conditional probability that the outcome is HHH given that at least 2H occurs?***

  $\Pr(\{HHH\}|\text{ at least 2H}) = \frac{\#\{HHH\}\cap\{\text{least 2H}\}}{\#\{\text{least 2H}\}} = \frac{1}{4}$

  at least 2 H = {THH,HTH,HHT,HHH}

DICE

- Toss 2 dice come up 7 or 11 (you win \$7), for any (loose \$2).

Determine the player's mathematical expectation?

36 possible pairs with 2 dice

8 possible for 7 or 11 $\quad Pr(7 \text{ or } 11) = \frac{8}{36}$ and $Pr(\text{not } (7 \text{ or} 11)) = 1 - \frac{8}{36} = \frac{28}{36}$

$E = 7 x \frac{8}{36} + (-2)\frac{28}{36} = 0$

Events E= 1st die is a 3, F = 2nd die is 6. Pr(E) = ? Pr(F) = ? E and F are independent?

$Pr(E) = Pr(F) = 1/6$, $E \cap F = \{(3,6)\}$ $\quad \Rightarrow Pr(E|F) = 1/6 = Pr(E)$

$\Rightarrow Pr(F|E) = 1/6 = Pr(F)$ $\quad \therefore$ E and F are independent

What is the probability that the 2 dice show the same number?

$E = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$ $\quad Pr(E) = \frac{6}{36} = \frac{1}{6}$

What is the probability that the number add up to 8?

$Pr(=8) = \frac{5}{36}$ $\quad E = \{(2,6),(6,2),(3,5),(5,3),(4,4)\}$

What is the probability that the sum is less than 5?

$Pr(2) + Pr(3) + Pr(4) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{1}{6}$

- Toss 1 die

What is the probability that an odd number will appear?

$E = \{1,3,5\} \Rightarrow Pr(E) = 3/6 = ½$

What is the probability that the result is odd and greater than 4?

$F = \{5,6\}$ $\quad Pr(F) = \frac{2}{6} = \frac{1}{3}$ $\quad \Rightarrow Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$