Section 3.2 – Exponential Functions

Definition

The exponential function f with base $\boldsymbol{b}$ is defined by

$$\underbrace{f(x)=b^{x}}_{\text{Base}} \qquad \text{or} \qquad y=b^{x}$$

where $b > 0$, $b \neq 1$ and $\boldsymbol{x}$ is any real number.

$$f(x)=2^{x} \qquad f(x)=\left(\tfrac{1}{2}\right)^{2x+1} \qquad f(x)=3^{-x} \qquad \cancel{f(x)=(-2)^{x}}$$

Example

Given: $f(x)=13.49\,(0.967)^{x}-1$, find $f(60)$

Solution

$$f(\mathbf{60})=13.49\,(0.967)^{\mathbf{60}}-1$$
$$=0.8014$$

Example

If $f(x)=2^{x}$, find each of the following. $f(-1)$, $f(3)$, $f\left(\frac{5}{2}\right)$

Solution

a) $f(-1)=2^{-1}$

$$=\frac{1}{2}$$

b) $f(3)=2^{3}$

$$=8$$

c) $f\left(\frac{5}{2}\right)=2^{\frac{5}{2}}$

$$=4\sqrt{2}$$
$$=5.6569$$

Graphing Exponential

1. Define the Horizontal Asymptote $f(x) = b^{x} \pm d$

$y = 0 \pm \text{d}$

The exponential function always equals to 0

$x \to \infty \;\; or \;\; x \to -\infty \Rightarrow f(x) \to 0$

2. Define/Make a table

(Force your exponential to = 0, then solve for x)

x	***f(x)***
$x-2$	
$x-1$	
x	
$x+1$	
$x+2$	

x	*f(x)*
–2	1/9
–1	1/3
0	1
1	3
2	9

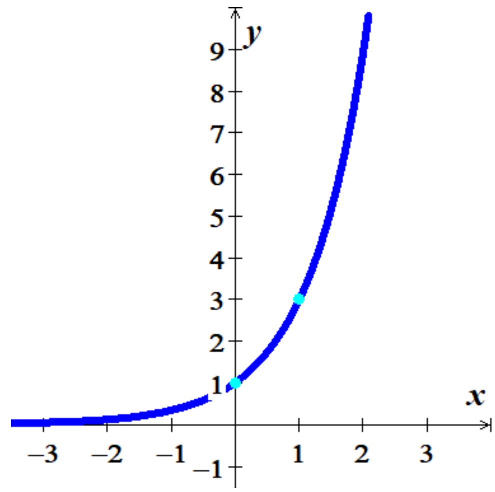
Domain: $(-\infty,\ \infty)$

Range: (d, ∞)

Example

$f(x) = 3^{x}$

Asymptote: $y = 0$



Example

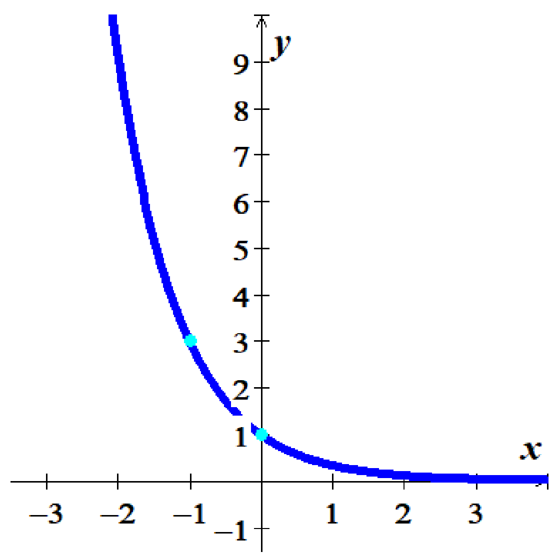
$$f(x) = \left(\tfrac{1}{3}\right)^{x} = \left(3^{-1}\right)^{x} = 3^{-x}$$

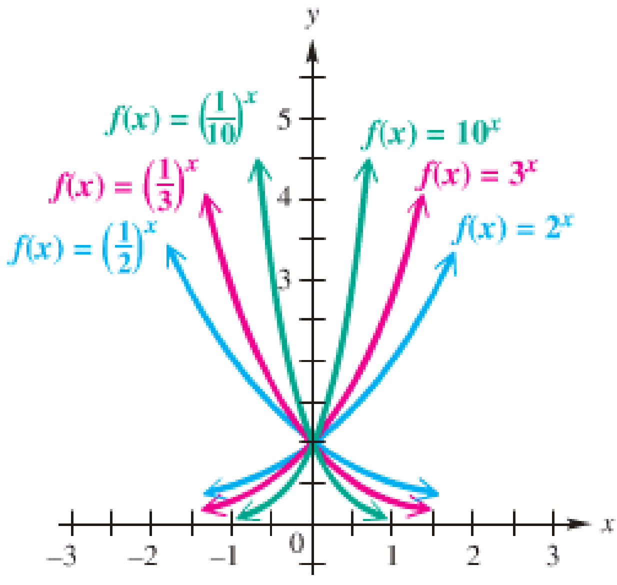
Reflected across y-axis

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$





$f(x)=a^x$

Example

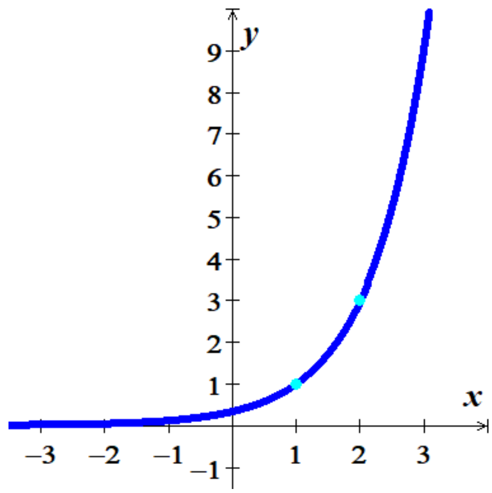
$f(x)=3^{x-1}$

Shift right 1 unit

Asymptote: $y = 0$

Domain: $(-\infty,\infty)$

Range: $(0,\infty)$



Example

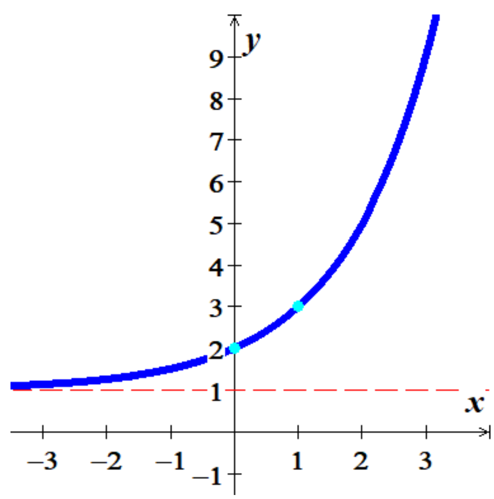
$f(x)=2^x+1$

Shift up 1 unit

Asymptote: $y = 1$

Domain: $(-\infty,\ \infty)$

Range: $(1,\ \infty)$



Example

$f(x) = 5 - 2^{-x}$

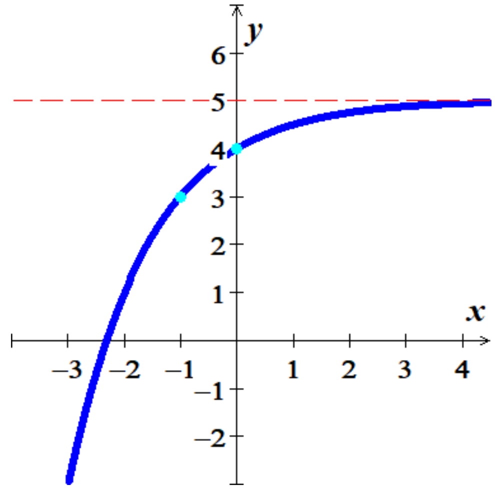
Shifted up 5 units

Reflected across x-axis and y-axis

Asymptote: $y = 5$

Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ 5)$



Example

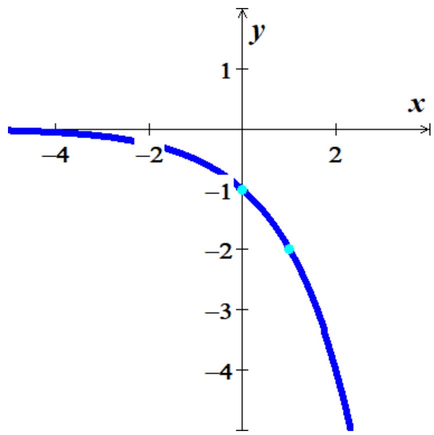
Give the *asymptote*, *domain* and *range*.

a) $f(x) = -2^{x}$

Asymptote: $y = 0$

Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ 0)$

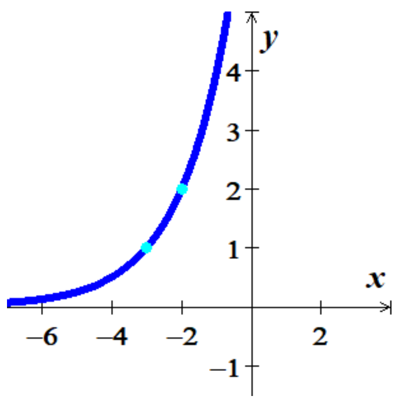


b) $f(x) = 2^{x+3}$

Asymptote: $y = 0$

Domain: $(-\infty,\ \infty)$

Range: $(0, \infty)$

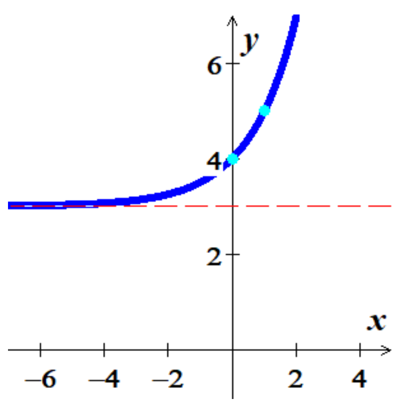


c) $f(x) = 2^{x} + 3$

Asymptote: $y = 3$

Domain: $(-\infty, \infty)$

Range: $(3, \infty)$



Natural Base e

The irrational number e is called natural base

$f(x) = e^x$ is called natural exponential function

$e^0 = 1$ $\quad$ $e \approx 2.7183$ $\quad$ $e^2 \approx 7.3891$ $\quad$ $e^{-1} \approx 0.3679$

Example

The exponential function $f(x) = 1066e^{0.042x}$ models the gray wolf population of the Western Great Lakes, $f(x)$, in billions, x *years* after 1978. Project the gray population in the recovery area in 2012.

Solution

$x = 2012 - 1978 = 34$

$$f(x = 34) = 1066e^{0.042(34)}$$
$$= 4445.6$$
$$\approx 4446$$

Example

Graph $\quad f(x) = e^x$

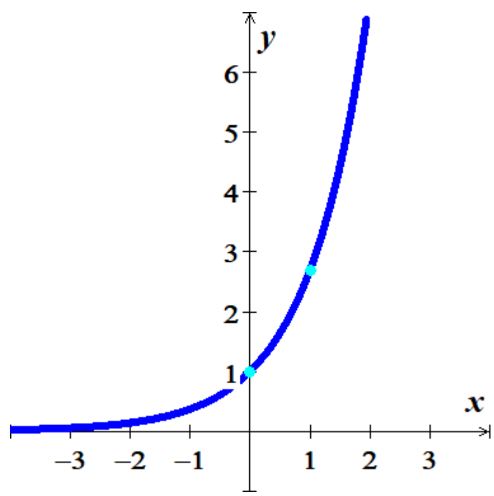
Solution

Asymptote: $y = 0$

x	*f(x)*
–1	.4
0	1
1	2.7

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$



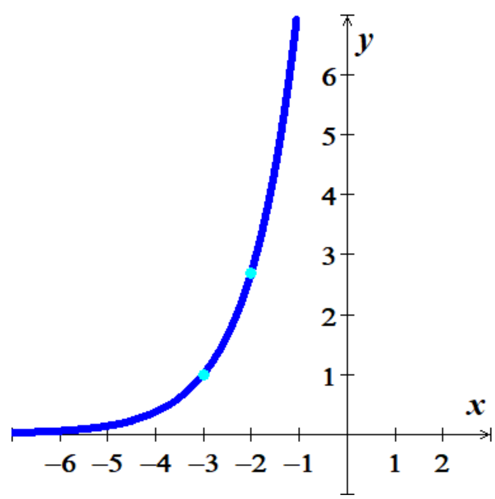
Example

$f(x) = e^{x+3}$

Solution

Shifted left 3 units

Asymptote: $y = 0$



Exercises Section 3.2 – Exponential Functions

(**1 – 8**) Evaluate to four decimal places using a calculator

1. $2^{3.4}$

2. $5^{\sqrt{3}}$

3. $6^{-1.2}$

4. $e^{-0.75}$

5. $e^{2.3}$

6. $e^{-0.95}$

7. $\pi^{\sqrt{\pi}}$

8. $e^{\sqrt{2}}$

(**9 – 20**) Find the ***asymptote***, ***domain***, and ***range*** of the given functions. Then, sketch the graph

9. $f(x) = 2^x + 3$

10. $f(x) = 2^{3-x}$

11. $f(x) = \left(\frac{2}{5}\right)^{-x}$

12. $f(x) = -\left(\frac{1}{2}\right)^x + 4$

13. $f(x) = 4^x$

14. $f(x) = 2 - 4^x$

15. $f(x) = -3 + 4^{x-1}$

16. $f(x) = 1 + \left(\frac{1}{4}\right)^{x+1}$

17. $f(x) = e^{x-2}$

18. $f(x) = 3 - e^{x-2}$

19. $f(x) = e^{x+4}$

20. $f(x) = 2 + e^{x-1}$

21. The exponential function $f(x) = 1066e^{0.042x}$ models the gray wolf population of the Western Great Lakes, $f(x)$, in *billions*, *x years* after 1978. Project the gray population in the recovery area in 2012.

22. The function $f(x) = 6.4e^{0.0123x}$ describes world population, $f(x)$, in billions, x years after 2004 subject to a growth rate of 1.23% *annually*. Use the function to predict world population in 2050.

23. A cup of coffee is heated to $160°F$ and placed in a room that maintains a temperature of $70°F$. The temperature *T* of the coffee, in *degree Fahrenheit*, after *t* minutes is given by

$$T(t) = 70 + 90e^{-0.0485t}$$

a) Find the temperature of the coffee 20 *minutes* after it is placed in the room

b) Determine when the temperature of the coffee will reach $90°F$

24. A cup of coffee is heated to $180°F$ and placed in a room that maintains a temperature of $65°F$. The temperature *T* of the coffee, in *degree Fahrenheit*, after *t* minutes is given by

$$T(t) = 65 + 115e^{-0.042t}$$

a) Find the temperature of the coffee 10 *minutes* after it is placed in the room

b) Determine when the temperature of the coffee will reach $100°F$

25. The percent $I(x)$ of the original intensity of light striking the surface of a lake that is available *x feet* below the surface of the lake is given by the equation

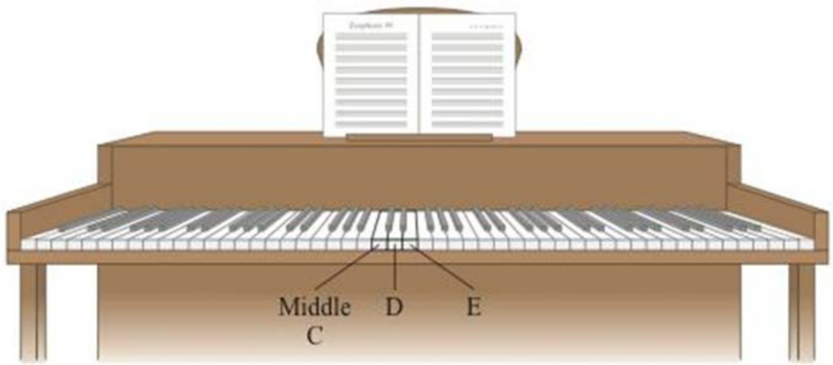
$$I(x) = 100e^{-.95x}$$

a) What percentage of the light is available 2 *feet* below the surface of the lake?

b) At what depth is the intensity of the light one-half the intensity at the surface?

26. Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

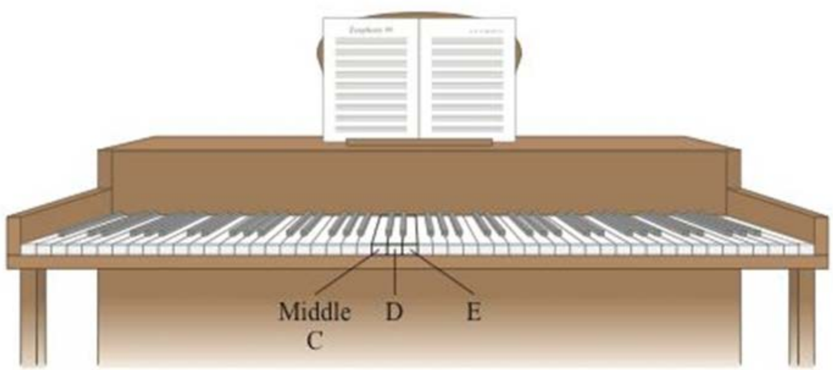
$$f(n) = (2.75)\ 2^{\frac{n-1}{12}}$$



a) Determine the frequency of middle *C*, key number 40 on an 88–*key* piano.
b) Is the difference in frequency between middle *C* (key number 40) and *D* (key number 42) the same as the difference in frequency between *D* (key number 42) and *E* (key number 44)?

27. Starting on the left side of a standard 88–*key* piano, the frequency, in *vibrations* per *second*, of the *n*th note is given by

$$f(n) = (27.5)\ 2^{\frac{n-1}{12}}$$



a) Determine the frequency of middle *C*, key number 40 on an 88–*key* piano.

b) Is the difference in frequency between middle *C* (key number 40) and *D* (key number 42) the same as the difference in frequency between *D* (key number 42) and *E* (key number 44)?