8.3 #4 $\cos A = \frac{5}{13}$ $A \in QI$

$$\sin A = \frac{12}{13}$$

a) $\sin 2A = 2 \sin A \cos A$

$= 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)$

$= \boxed{\frac{120}{169}}$

b) $\cos 2A = \cos^2 A - \sin^2 A$

$= \frac{25}{169} - \frac{144}{169}$

$= \boxed{-\frac{119}{169}}$

c) $\tan 2A = \boxed{-\frac{120}{119}}$

$A \in QI \qquad \frac{A}{2} \in QI \qquad 0^\circ < \frac{A}{2} < \frac{90}{2}^\circ$

d) $\sin \frac{A}{2} = \sqrt{\frac{1}{2}(1 - \cos A)}$

$= \sqrt{\frac{1}{2}\left(1 - \frac{5}{13}\right)}$

$= \frac{2\sqrt{2}}{\sqrt{26}}$

e) $\cos\frac{A}{2} = \sqrt{\frac{1}{2}(1+\cos A)}$

$= \sqrt{\frac{1}{2}\left(1+\frac{5}{13}\right)}$

$= \frac{3\sqrt{2}}{\sqrt{26}}$

f) $\tan\frac{A}{2} = \frac{2}{3}$

Ex $(\sin\theta+\cos\theta)^2 = 1+\sin 2\theta$

$(\sin\theta+\cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= 1+\sin 2\theta$ ✓

Ex $\sin 2x = \frac{2\cot x}{1+\cot^2 x}$

$\frac{2\cot x}{1+\cot^2 x} = 2\frac{\frac{\cos x}{\sin x}}{1+\frac{\cos^2 x}{\sin^2 x}}$

$= 2\frac{\cos x}{\sin x}\cdot\frac{\sin^2 x}{\sin^2 x+\cos^2 x}$ ($\sin^2 x+\cos^2 x = 1$)

$= 2\cos x\sin x$

$= \sin 2x$ ✓

Ex $\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$

$$\begin{aligned}\cos 4x &= \cos(2(2x)) \\ &= 2\cos^2(2x) - 1 \\ &= 2(2\cos^2 x - 1)^2 - 1 \\ &= 2(4\cos^4 x - 4\cos^2 x + 1) - 1 \\ &= 8\cos^4 x - 8\cos^2 x + 1 \checkmark\end{aligned}$$

Ex $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta}$

$$\begin{aligned}\frac{1-\cos 2\theta}{\sin 2\theta} &= \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta} \\ &= \frac{2\sin^2\theta}{2\sin\theta\cos\theta} \\ &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta \checkmark\end{aligned}$$

Ex $\sin^2\frac{x}{2} = \dfrac{\tan x - \sin x}{2\tan x}$

$$\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)\frac{\tan x}{\tan x}$$

$$= \frac{\tan x - \cos x\left(\frac{\sin x}{\cos x}\right)}{2\tan x}$$

$$= \frac{\tan x - \sin x}{2\tan x}$$ ✓

#27 $\dfrac{\cos 2x}{\cos^2 x} = \sec^2 x - 2\tan^2 x$

$$\frac{\cos 2x}{\cos^2 x} = \frac{1-2\sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - 2\frac{\sin^2 x}{\cos^2 x}$$

$$= \sec^2 x - 2\tan^2 x$$ ✓

#28 $\sin 4x = 4\sin x\cos x(2\cos^2 x - 1)$

$$\sin 4x = \sin 2(2x)$$

$$= 2\sin 2x\cos 2x$$

$$= 4\sin x\cos x(2\cos^2 x - 1)$$ ✓

#32 $\cos 4x = \cos^4 x - 6\sin^2 x \cos^2 x + \sin^4 x$

$\cos 4x = \cos 2(2x)$

$= \cos^2 2x - \sin^2 2x$

$= (\cos^2 x - \sin^2 x)^2 - (2\sin x \cos x)^2$

$= \cos^4 x - 2\sin^2 x \cos^2 x + \sin^4 x$

$\quad - 4\sin^2 x \cos^2 x$

$= \cos^4 x - 6\sin^2 x \cos^2 x + \sin^4 x$ ✓

#33 $\tan^2 \frac{x}{2} = \frac{\sec x + \cos x - 2}{\sec x - \cos x}$

$\tan^2 \frac{x}{2} = \left(\frac{1 - \cos x}{\sin x}\right)^2$

$= \frac{1 - 2\cos x + \cos^2 x}{\sin^2 x}$

$= \frac{1 - 2\cos x + \cos^2 x}{1 - \cos^2 x}$

$= \frac{\frac{1}{\cos x} - 2\frac{\cos x}{\cos x} + \frac{\cos^2 x}{\cos x}}{\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}}$

$= \frac{\sec x - 2 + \cos x}{\sec x - \cos x}$ ✓

$$a(\ )^2 + b(\ ) + c = 0$$

$a + b + c = 0$ $\quad x = 1, \frac{c}{a}$

$a - b + c = 0$ $\quad x = -1, -\frac{c}{a}$

Ex $\quad 2\cos^2 x + 3\cos x + 1 = 0$

$\cos x = -1$ $\qquad \cos x = -\frac{1}{2}$

$x = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

Ex $\quad -2\sin^2 x + 3\sin x - 1 = 0$

$\sin x = 1$ $\qquad \sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

QI $\quad \frac{\pi}{n}$

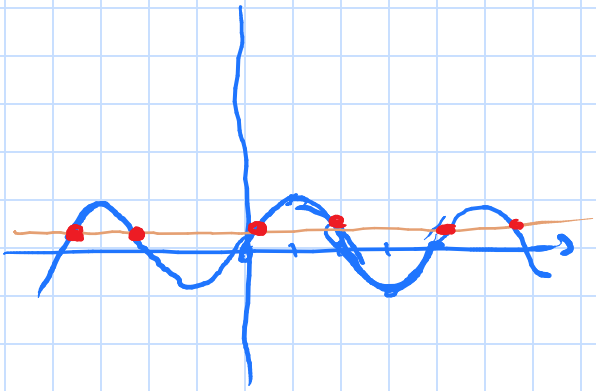
QII $\quad \frac{(n-1)\pi}{n}$

QIII $\quad \frac{(n+1)\pi}{n}$

QIV $\quad \frac{(2n-1)\pi}{n}$

Ex $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $[0, 2\pi)$



$x \in \mathbb{R}$

$$\begin{cases} x = \frac{\pi}{6} + 2n\pi \\ x = \frac{5\pi}{6} + 2n\pi \end{cases}$$

Ex $\sin x \tan x = \sin x$ $[0, 2\pi)$

$\sin x \tan x - \sin x = 0$

$\sin x (\tan x - 1) = 0$

$\sin x = 0$ $\quad$ $\tan x = 1$

$x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$

$x = n\pi, \frac{\pi}{4} + n\pi$

Ex $2\sin^2 t - \cos t - 1 = 0$ $[0, 2\pi]$

$2(1-\cos^2 t) - \cos t - 1 = 0$

$2 - 2\cos^2 t - \cos t - 1 = 0$

$-2\cos^2 t - \cos t + 1 = 0$

$\cos t = -1 \qquad \cos t = \frac{1}{2}$

$t = \pi, \ \frac{\pi}{3}, \ \frac{5\pi}{3}$

$n\pi, \ \frac{\pi}{3} + 2n\pi, \ \frac{5\pi}{3} + 2n\pi$

Ex $4\sin^2 x \tan x - \tan x = 0$ $[0, 2\pi)$

$\tan x (4\sin^2 x - 1) = 0$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\tan x = 0 \qquad \sin x = \pm \frac{1}{2}$

$x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Ex $\csc^4 2u - 4 = 0 \quad [0, 2\pi)$

$(\csc^2 2u - 2)(\csc^2 2u + 2) = 0$

$\csc 2u = \pm\sqrt{2}$

$\sin 2u = \pm \frac{1}{\sqrt{2}}$

$2u = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$u = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

$\frac{13\pi}{8}, \frac{15\pi}{8}$ —

Ex $5\sin\theta\tan\theta - 10\tan\theta + 3\sin\theta - 6 = 0$

$5\tan\theta(\sin\theta - 2) + 3(\sin\theta - 2) = 0$

$(\sin\theta - 2)(5\tan\theta + 3) = 0$

$\sin\theta = 2$ ##

>1

$\tan\theta = -\frac{3}{5}$

$\hat{\theta} = \tan^{-1}\left(\frac{3}{5}\right)$ } $\frac{\pi}{16}$

$\theta = \pi - \hat{\theta}$

$\theta = 2\pi - \hat{\theta}$