

Inverse Trig.

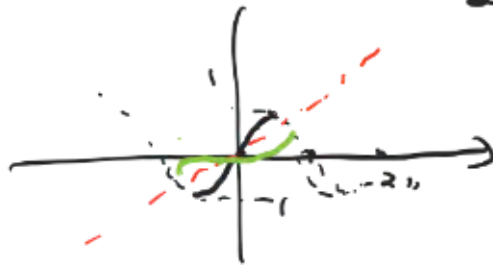
$$y = \sin x \Rightarrow \overset{\text{angle}}{\overbrace{x = \sin^{-1} y}^{\text{angle}}}$$

$$-1 \leq y \leq 1$$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$x = \arcsin y$$

$$\begin{aligned} -1 &\leq y \leq 1 \\ -\frac{\pi}{2} &\leq x \leq \frac{\pi}{2} \end{aligned}$$



$$\sin(\sin^{-1} \frac{1}{2}) = \frac{1}{2}$$

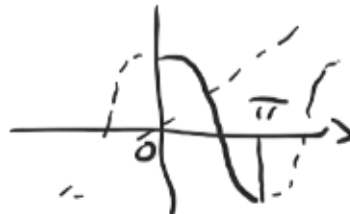
$$\sin(\sin^{-1} \frac{1}{2}) = \sin(\frac{\pi}{6})$$

$$= \frac{1}{2} \quad \checkmark$$

$$y = \cos^{-1} x \quad \text{iff} \quad x = \cos y$$

$$-1 \leq x \leq 1$$

$$0 \leq y \leq \pi$$



$$\cos^{-1} x = \arccos x$$

$$\cos(\cos^{-1} x) = x$$

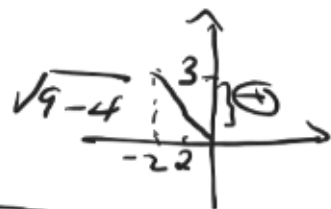
Arccos

$$\sin(\underbrace{\arccos(-\frac{2}{3})}_{\alpha})$$

$$\alpha = \arccos(-\frac{2}{3})$$

$$\cos \alpha = -\frac{2}{3} \quad \frac{\pi}{2} < \alpha < \pi$$

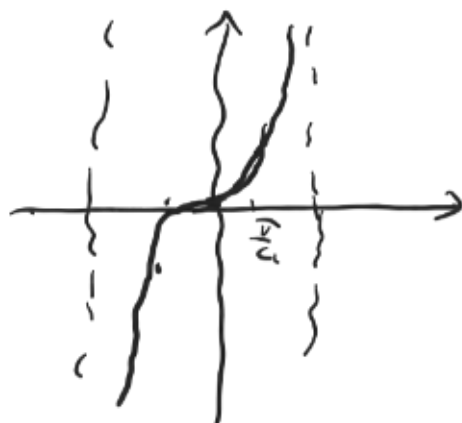
$$\sin \alpha = \frac{\sqrt{5}}{3}$$



Inverse Tangent

$$x = \tan y \Rightarrow y = \tan^{-1} x$$

$$y = \arctan x$$



$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

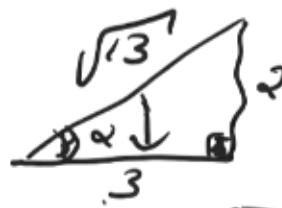
$$\tan^{-1}(\tan \frac{\pi}{4}) = \frac{\pi}{4}$$

$$\sec(\underbrace{\arctan \frac{2}{3}}_{\alpha})$$

$$\alpha = \arctan \frac{2}{3}$$

$$\tan \alpha = \frac{2}{3}$$

$$\sec \alpha = \frac{\sqrt{13}}{3}$$



$$r^2 = x^2 + y^2 \quad \sqrt{9+4}$$

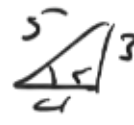
Ex $\sin(\underbrace{\arctan \frac{1}{2}}_{\alpha} - \underbrace{\arccos \frac{4}{5}}_{\beta})$

$$\tan \alpha = \frac{1}{2}$$



$$\cos \beta = \frac{4}{5}$$

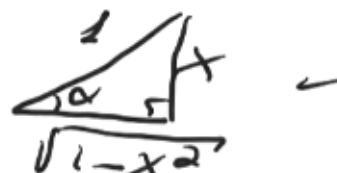
$$\sin \beta = \frac{3}{5}$$



$$\begin{aligned}
 \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\
 &= \frac{1}{\sqrt{5}} \cdot \frac{4}{5} - \frac{2}{\sqrt{5}} \cdot \frac{3}{5} \\
 &= \frac{4-6}{5\sqrt{5}} \\
 &= -\frac{2}{5\sqrt{5}} \quad (2) \quad -\frac{2\sqrt{5}}{25}
 \end{aligned}$$

Ex $\cos(\sin^{-1} x) = ?$

$$\begin{aligned}
 \alpha &= \sin^{-1} x \\
 \sin \alpha &= x \quad \left(\frac{x}{1}\right) \\
 \cos \alpha &= \sqrt{1-x^2}
 \end{aligned}$$



#34 $\cot(\sin^{-1} \frac{\sqrt{x^2-9}}{x})$

$$\sin \alpha = \frac{\sqrt{x^2-9}}{x}$$

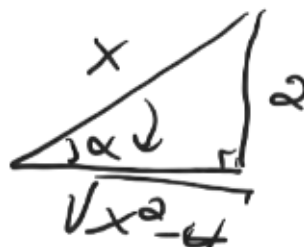
$$\cot \alpha = \frac{3}{x}$$



#39 $\sec(\tan^{-1} \frac{2}{\sqrt{x^2-4}})$

$$\tan \alpha = \frac{2}{\sqrt{x^2-4}}$$

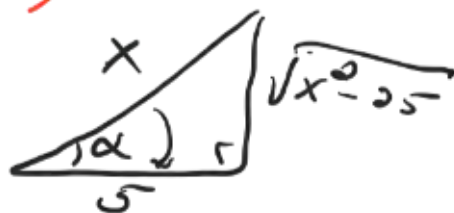
$$\sec \alpha = \frac{x}{\sqrt{x^2-4}}$$



do $\sec(\sin^{-1} \frac{\sqrt{x^2-25}}{x})$

$$\sin \alpha = \frac{\sqrt{x^2-25}}{x}$$

$$\sec \alpha = \frac{x}{5}$$



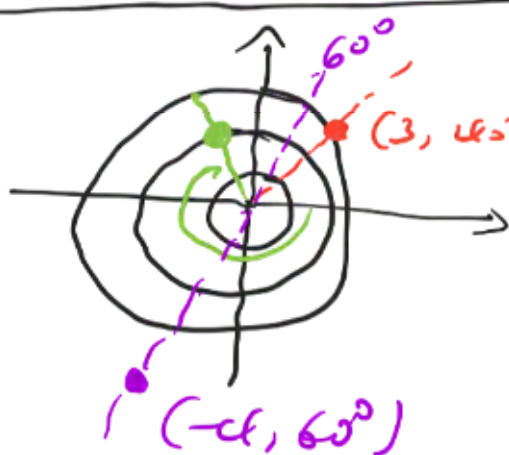
8.6 Polar Coordinates.

(r, θ) $\xrightarrow{\text{Relation}}$ (x, y)
 $\xrightarrow{\text{Rectangular}}$



$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1} \frac{y}{x} \end{cases}$$



$(3, 45^\circ)$
 $(2, -240^\circ)$
 $(-4, 60^\circ)$

Ex $(r, \theta) = (4, \frac{7\pi}{6})$ $(x, y)?$

$$\begin{aligned} x &= r \cos \theta \\ &= 4 \cos \frac{7\pi}{6} \\ &= -2\sqrt{3} \end{aligned}$$

$$\begin{aligned} y &= r \sin \theta \\ &= 4 \sin \frac{7\pi}{6} \\ &= -2 \end{aligned}$$

$$= 4 \left(-\frac{\sqrt{3}}{2} \right) = -2\sqrt{3}$$

$$= 4 \left(-\frac{1}{2} \right) = -2$$

$$(x, y) = (-2\sqrt{3}, -2)$$

$$(x, y) = (-1, \sqrt{3}) \quad (r, \theta)?$$

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{1 + 3}$$

$$= 2$$

$$\hat{\theta} = \tan^{-1} \frac{\sqrt{3}}{1} = \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

$$(r, \theta) = (2, \frac{2\pi}{3}) \quad 120^\circ$$



$x =$
 $y =$



line $ax + by = c$

$$a r \cos \theta + b r \sin \theta = c$$

$$r (a \cos \theta + b \sin \theta) = c$$

$$r(a \cos \theta + b \sin \theta) = c$$

$$r = \frac{c}{a \cos \theta + b \sin \theta}$$

$$x^2 - y^2 = 16$$

$$(r \cos \theta)^2 - (r \sin \theta)^2 = 16$$

$$r^2 \cos^2 \theta - r^2 \sin^2 \theta = 16$$

$$r^2 (\cos^2 \theta - \sin^2 \theta) = 16$$

$$r^2 = \frac{16}{\cos 2\theta}$$

$$r = a \sin \theta \quad (a \neq 0)$$

$$r^2 = a r \sin \theta$$

$$x^2 + y^2 = a y$$

Graph

$$0, 1, \frac{1}{2}$$

$$\begin{cases} \cos 60^\circ = \frac{1}{2} \\ \sin 30^\circ = \frac{1}{2} \end{cases}$$

$$r = 2 + 2 \cos \theta \quad (\text{cardioid})$$

θ	r
0	4
60°	3
90°	2
120°	1
$\frac{\pi}{2}$	0



$$\begin{aligned} & a(1 + \cos \theta) \\ & a(1 - \cos \theta) \rightarrow \\ & a(1 + \sin \theta) \\ & a(1 - \sin \theta) \end{aligned}$$



#1 $(4, 30^\circ)$

$$\begin{aligned} x &= r \cos \theta \\ &= 4 \cos 30^\circ \\ &= 4 \left(\frac{\sqrt{3}}{2} \right) \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} y &= r \sin \theta \\ &= 4 \sin 30^\circ \\ &= 4 \left(\frac{1}{2} \right) \\ &= 2 \end{aligned}$$

$$(x, y) = (2\sqrt{3}, 2)$$

#2 $(-\sqrt{2}, \frac{3\pi}{4})$

$$\begin{aligned} x &= r \cos \theta \\ &= -\sqrt{2} \cos \frac{3\pi}{4} \\ &= -\sqrt{2} \left(-\frac{\sqrt{2}}{2} \right) \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= r \sin \theta \\ &= -\sqrt{2} \sin \frac{3\pi}{4} \\ &= -\sqrt{2} \left(\frac{\sqrt{2}}{2} \right) \\ &= -1 \end{aligned}$$

$$(x, y) = (1, -1)$$

$(x, y) = (-1, \sqrt{3})$

$$\theta \in QII$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1 + 3} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} \frac{\sqrt{3}}{-1} \\ &= \frac{2\pi}{3} \\ \theta &= 201^\circ \end{aligned}$$

-2)

3)

$$(r, \theta) = (2, \frac{2\pi}{3})$$

$$\#13) (-3, -3) = (x, y)$$

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{9 + 9}$$

$$= 3\sqrt{2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

$$\theta = \frac{5\pi}{4}$$

$$(r, \theta) = (3\sqrt{2}, \frac{5\pi}{4})$$

$$\#23 \quad r = 6 \cos \theta$$

$$r^2 = 6r \cos \theta$$

$$\underline{x^2 + y^2 = 6x}$$

$$\#29 \quad r^2 (4 \sin^2 \theta - 9 \cos^2 \theta) = 36$$

$$4r^2 \sin^2 \theta - 9r^2 \cos^2 \theta = 36$$

$$4y^2 - 9x^2 = 36$$

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

$$\#34 \quad r (\sin \theta + r \cos^2 \theta) = 1$$

$$r \sin \theta + r^2 \cos^2 \theta = 1$$

$$y + x^2 = 1$$

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$$y^- = 6x$$

$$(r \sin \theta)^2 = 6 r \cos \theta$$

$$r^2 \sin^2 \theta = 6 r \cos \theta \quad (r \neq 0)$$

$$r \sin^2 \theta = 6 \cos \theta$$

$$r = \frac{6 \cos \theta}{\sin^2 \theta}$$

$$= 6 \cot \theta$$

37 $(x+2)^2 + (y-3)^2 = 13$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 13$$

$$x^2 + y^2 + 4x - 6y = 0$$

$$r^2 + 4r \cos \theta - 6r \sin \theta = 0$$

$$(r \neq 0)$$

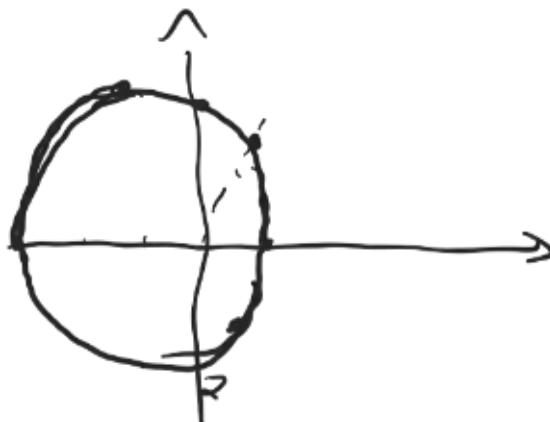
$$r + 4 \cos \theta - 6 \sin \theta = 0$$

$$r = 6 \sin \theta - 4 \cos \theta = \underline{\underline{f(\theta)}}$$

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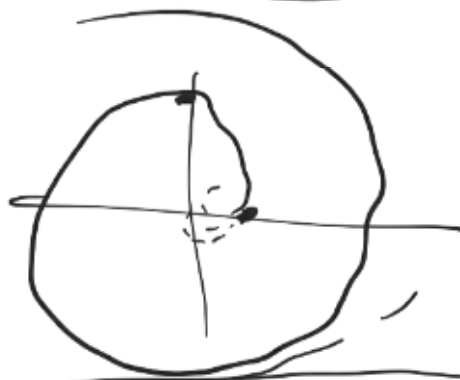
$$r = 2 - \cos \theta$$

θ	r
0	1
$\frac{\pi}{2}$	$\frac{3}{2}$
π	2
$\frac{3\pi}{2}$	$\frac{5}{2}$
2π	1



$$\frac{10}{3} \mid 2$$

$$h = e^{2\sigma}$$



8.7 Trig Form.

①

$$\sqrt{-1} = i$$

$$-1 = i^2$$

Real part

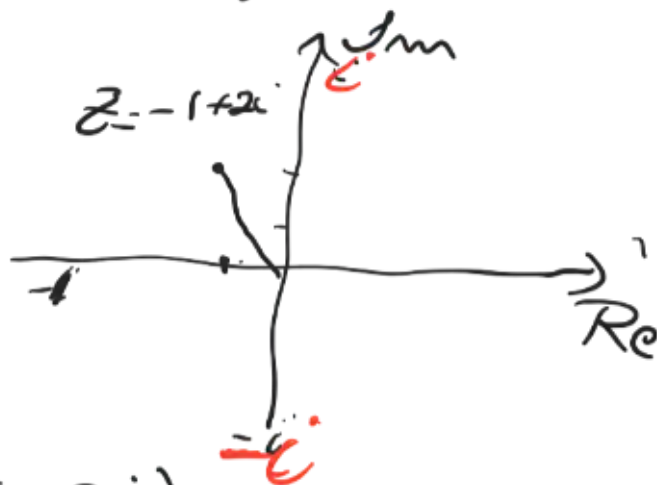
$$-i = i^3 \quad i^2$$

$$1 = i^4$$

$$z = \underbrace{a}_{\text{Re}} + i \underbrace{b}_{\text{Im}} \quad \text{imaginary part}$$

$$a, b \in \mathbb{R}$$

$$z = -1 + 2i$$



$$(6 - 2i) + (-4 - 3i)$$

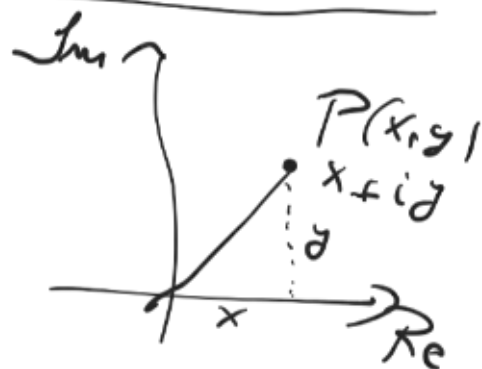
$$= 6 - 4 + i(-2 - 3)$$

$$= 2 - 5i$$



$$r = \sqrt{x^2 + y^2}$$

modulus.



θ : argument

$$\theta = \tan^{-1} \frac{y}{x}$$

$$z = x + yi$$

$$= r \cos \theta + i(r \sin \theta)$$

$$= r (\cos \theta + i \sin \theta)$$

$$= r \operatorname{cis} \theta$$

} Trig Form

$$z = -1 + i$$

Trig Form

$$r = \sqrt{(-1)^2 + 1^2}$$

$$= \sqrt{2}$$

$$\theta = \tan^{-1} \frac{1}{-1} = \frac{3\pi}{4}$$

$$\theta = \frac{3\pi}{4}$$

$$z = \sqrt{2} \operatorname{cis} \frac{3\pi}{4}$$

$$\left(\sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \right)$$

$$z = 6 \operatorname{cis} 60^\circ$$

$$= 6 \cos 60^\circ + i 6 \sin 60^\circ$$

$$\begin{aligned}
 &= 6 \cos 60^\circ + i 6 \sin 60^\circ \\
 &= 6 \left(\frac{1}{2} \right) + i 6 \left(\frac{\sqrt{3}}{2} \right) \\
 &= 3 + 3i\sqrt{3}
 \end{aligned}$$

$$\begin{array}{ccc}
 |r| ? & z = 5i & z = 7 \\
 & r = 5 & r = 7
 \end{array}$$

$$\begin{aligned}
 3 + 4i &\rightarrow r = 5 \\
 \sqrt{9 + 16} &= \sqrt{25}
 \end{aligned}$$

$$\cos z \leftarrow z \in \mathbb{C}$$

$$(r_1 \operatorname{cis} \theta_1)(r_2 \operatorname{cis} \theta_2) = r_1 r_2 \operatorname{cis} (\theta_1 + \theta_2)$$

$$\begin{aligned}
 \underline{\text{Ex}} \quad 3(\operatorname{cis} 45^\circ)(2 \operatorname{cis} 135^\circ) &= 6 \operatorname{cis} (45^\circ + 135^\circ) \\
 &= 6 \operatorname{cis} 180^\circ \\
 &= 6 \cos 180^\circ + i 6 \sin 180^\circ \\
 &= \underline{-6}
 \end{aligned}$$

$$a^2 + b^2 = (a + ib)(a - ib)$$

$$\begin{aligned}
 &= a^2 - (ib)^2 \\
 &= a^2 - i^2 b^2 \\
 &= a^2 - (-1)b^2 \\
 &= a^2 + b^2
 \end{aligned}$$

$$a^2 + b^2$$

$$a+b = (\sqrt{a}-i\sqrt{b})(\sqrt{a}+i\sqrt{b})$$

$$a-b = (\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$$

$$\frac{r_1 \operatorname{cis} \theta_1}{r_2 \operatorname{cis} \theta_2} = \frac{r_1}{r_2} \operatorname{cis} (\theta_1 - \theta_2)$$

$$\begin{aligned} \frac{10 \operatorname{cis} (-60^\circ)}{5 \operatorname{cis} (150^\circ)} &= \frac{10}{5} \operatorname{cis} (-60^\circ - 150^\circ) \\ &= 2 \operatorname{cis} (-210^\circ) \\ &= 2 \cos(-210^\circ) + i 2 \sin(-210^\circ) \\ &= 2\left(-\frac{\sqrt{3}}{2}\right) + i 2\left(+\frac{1}{2}\right) \\ &= -\sqrt{3} + i \end{aligned} \quad \begin{matrix} 210^\circ \\ (-) \end{matrix} \equiv \text{III}$$

De Moivre's Theorem

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} n\theta$$

$$(1 + i\sqrt{3})^8 \quad \theta \in \mathbb{Q}\pi$$

$$r = \sqrt{1+3} = 2$$

$$\theta = \tan^{-1} \frac{\sqrt{3}}{1} = 60^\circ$$

$$(1 + i\sqrt{3})^8 = (2 \operatorname{cis} 60^\circ)^8$$

$$= 2^8 \operatorname{cis} (480^\circ)$$

$$= 256 \operatorname{cis} (120^\circ)$$

$$= 256 (\cos 120^\circ + i \sin 120^\circ)$$

$$= 256 \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)$$

$$= -128 + i 128\sqrt{3}$$

n^{th} root $(r \operatorname{cis} \theta)^{1/n} = \sqrt[n]{r} \operatorname{cis} \left(\frac{\theta}{n} \right)$

$$\alpha = \frac{\theta}{n} + \frac{2\pi k}{n}$$

