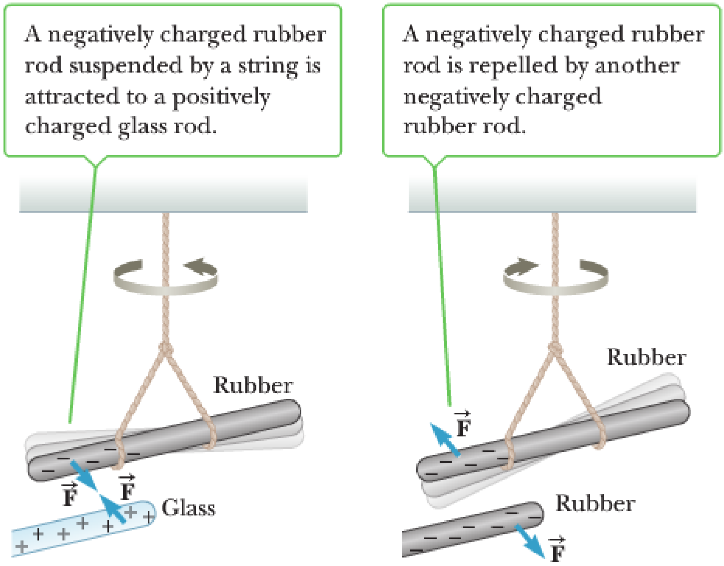
2.1 – *Electric Fields*

Electric Force

Experiment shows that when rubber (or plastic) and fur are rubbed together, they develop the property of attracting each other. This kind of force that arises after objects are rubbed together is called ***electrical force***. The change that occurred during the rubbing process that is responsible for this force is called ***charge***.

When a glass that has been rubbed with silk brought near the rubber rod, the rubber rod is attracted towards the glass rod.



If two charges rubber rods (or 2 charged glass rods) are brought near each other, the force between them is repulsive.

This shows that there are two kinds of charge and that opposite charges attract and similar charges repel mathematically, these two different kind of charges are identified as positive and negative. The SI unit of measurement for charge is the ***Coulomb***, or "***C***" (as abbreviation).

We can say that like charges repel one another and unlike charges attract one another.
Objects usually contain equal amounts of positive and negative charge; electrical forces between objects arise when those objects have net negative or positive charges.

According to the Rutherford model of the atom, an atom consists of Q nucleus with protons and neutrons with electron are revolving around.

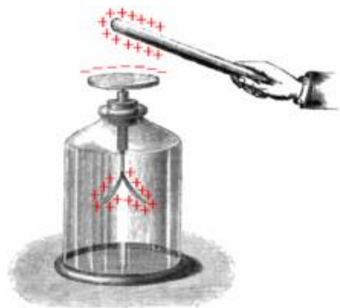
Neutrons are neutral (do not have electric charge)
Electron has charge of $-e$.
Proton has an equal and opposite charge of $+e$.

The value of $e = 1.60219 \times 10^{-19}$ C .

Particle	***Charge (C)***	***Mass (Kg)***
Electron	-1.60×10^{-19}	9.11×10^{-31}
Proton	$+1.60 \times 10^{-19}$	1.67×10^{-27}
Neutron	0	1.67×10^{-27}

According to the current understanding of charges, when rubber and fur are rubbed together. Electrons are transferred from one to the other. The one that last electrons becomes positively charges because it is losing negatively charges electrons, and the one that gained electrons becomes negatively charges because it is gaining negative charges.

Charges are measured by a device called ***electroscope***. An electroscope consists of a jar with a pair of gold leaves as shown below



When the gold leaves are brought into contact with a charges objects, the two leaves acquire the same charges and repel each other forming a deflection angle between them.

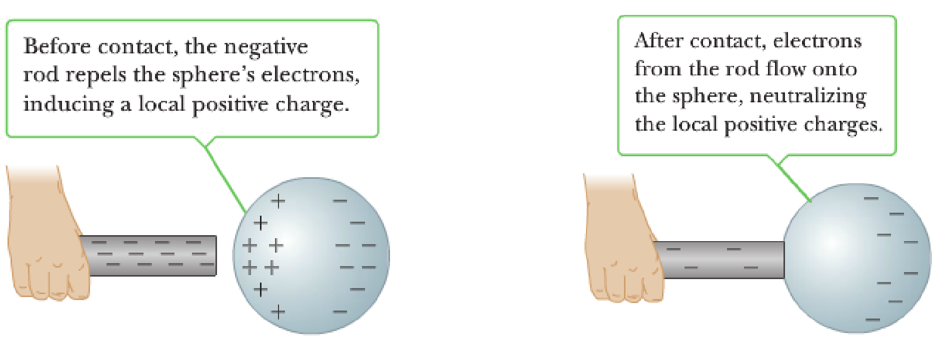
This deflection angle is proportional to the amount of charge; that is $\frac{Q}{\theta} = constant$ where Q is charge and θ is the deflection end. The constant can be determined from single pair of charge & deflection angle. Thus the charge can be obtained by measuring the deflection angle.

There are 2 ways by which an object can be charged, they are called ***conduction*** and ***induction***.

Conduction: is a process by which the neutral object is brought in contact with the neutral object transferring charge of the same sign to the neutral object.

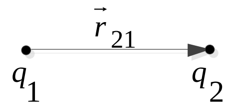
Induction: is a process by which a charges object is brought closer to the grounded neutral object and then the neutral object is disconnected from the ground. In this process the neutral object acquires a charge opposite in sign to that of the charging object.

Substances are classified into two based on whether they have free (valence) electrons or not, objects with free electrons are called ***conductors*** and substances without free electrons are called ***insulators***.
Conductors are good conductors of heat and electrons, shiny and ductile.
Insulators are bad conductors of heat and electricity, dull and brittle.

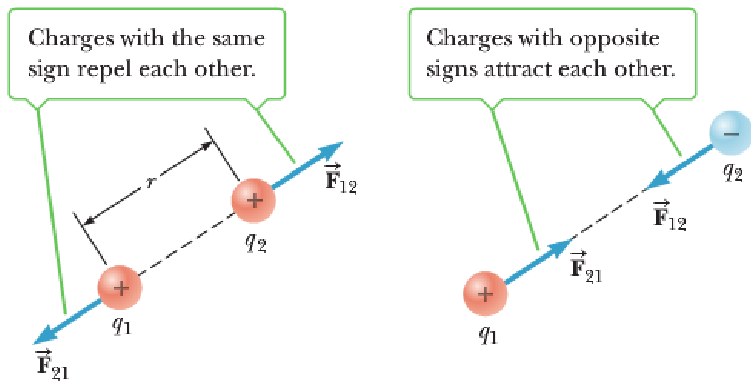


Coulomb's Law

Coulomb's law states that any two charges objects exert electrical force on each other which is directly proportional to the product of the charges and inversely proportional to the square of the distance separating them.



The direction of the force exerted by charge 1 (q_1) on charge 2 (q_2) is in the direction of the position vector of q_2 with respect to q_1 ($\vec{r}_{21}$ – position vector whose tail is at charge q_1 & whose head is on q_2).



If the charges are of opposite sign and repulsive if the charges have the same sign.
Coulomb's law can be written mathematically as

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{21}^2}\hat{e}_{21}$$

Where $\vec{F}_{21}$ is electric force exerted on q_2 by q_1.

$\hat{e}_{21} = \dfrac{\vec{r}_{21}}{r_{21}}$ is a unit vector in the direction of the position vector of q_2 with respect to q_1

Coulomb's law can be rewritten as

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{21}^2}\hat{e}_{21} = k\frac{q_1 q_2}{r_{21}^2}\frac{\vec{r}_{21}}{r_{21}} \boxed{= k\frac{q_1 q_2}{r_{21}^3}\vec{r}_{21}}$$

The magnitude of the electric force *F* between charges q_1 and q_2 separated by a distance *r* is given by

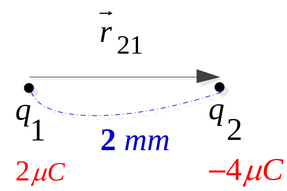
$$F = k\frac{|q_1||q_2|}{r^2}$$

k: is a universal constant called Coulomb's constant whose value is given by

$$k = 8.9875\times10^9\ N\cdot m^2/C^2 \approx 9\times10^9\ N\cdot m^2/C^2$$

Example

Consider the charges shown



a) Determine the electric force exerted by q_1 on q_2

$$q_1 = 2\mu C = 2\times10^{-6}\ C$$

$$q_2 = -4\mu C = -4\times10^{-6}\ C$$

$$r_{21} = 2mm = 2\times10^{-3}\ m$$

$$\vec{r}_{21} = r_{21}\cos\theta_{21}\hat{i} + r_{21}\sin\theta_{21}\hat{j} \qquad \left(\theta_{21} = 0^\circ\right)$$

$$= 2\times10^{-3}\cos(0)\hat{i} + 2\times10^{-3}\sin(0)\hat{j}$$

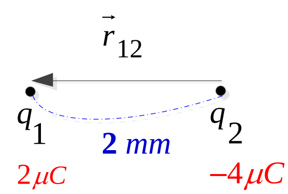
$$= 2\times10^{-3}\hat{i}$$

$$\vec{F}_{21} = k\frac{q_1q_2}{r_{21}^3}\vec{r}_{21}$$

$$= \frac{\left(9\times10^9\right)\left(2\times10^{-6}\right)\left(-4\times10^{-6}\right)}{\left(2\times10^{-3}\right)^3}\left(2\times10^{-3}\right)\hat{i}$$

$$\underline{= -18\times10^3\ \hat{i}\quad N}$$

b) Determine the electric force exerted by charge q_2 on charge q_1



$$\vec{r}_{12} = -\vec{r}_{21} = -2\times10^{-3}\hat{i}$$

$$\vec{F}_{12} = -\vec{F}_{21} \underline{= 18\times10^3\ \hat{i}\quad N}$$

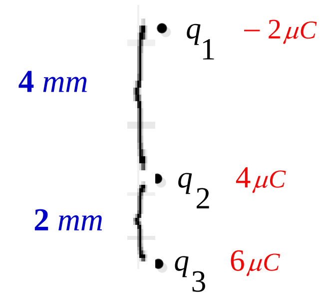
$\vec{F}_{12}$ *and* $\vec{F}_{21}$ are action reaction forces. They have the same magnitude but opposite direction.

The superposition principle for electric forces

If a charge is in the vicinity of a number of charges, the net force acting on the charge is the vector sum of all of the individual forces due to the individual charges.
If charge q_0 is in the vicinity of charges q_1, q_2, q_3, $\ldots$, then the net force acting on charge q_0 is given by

$$\vec{F}_0^{net} = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \ldots = \sum_{i=1} \vec{F}_{0i}$$

Example



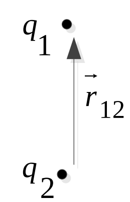
Consider the charges shown

Calculate the net force exerted on charge q_1 by charges q_2 and q_3

Solution

$$q_1 = -2\mu C = -2\times10^{-6}\ C$$
$$q_2 = 4\mu C = 4\times10^{-6}\ C$$
$$q_3 = 6\mu C = 6\times10^{-6}\ C$$

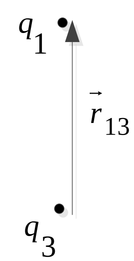
$$\vec{F}_1^{net} = \vec{F}_{12} + \vec{F}_{13}$$



$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^3}\vec{r}_{12} = k\frac{q_1 q_2}{r_{12}^3}\left(r_{12}\cos\theta_{12}\,\hat{i} + r_{12}\sin\theta_{12}\,\hat{j}\right) \qquad \left(\theta_{12} = 90°\right)$$

$$= \frac{\left(9\times10^{9}\right)\left(-2\times10^{-6}\right)\left(4\times10^{-6}\right)}{\left(4\times10^{-3}\right)^3}\left(4\times10^{-3}(\cos 90°)\,\hat{i} + 4\times10^{-3}(\sin 90°)\,\hat{j}\right)$$

$$\boxed{= -4.5\times10^{-3}\,\hat{j}\quad N}$$



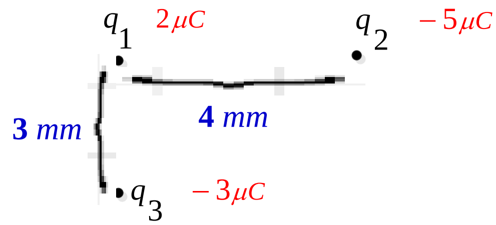
$$\vec{F}_{13} = k\frac{q_1 q_3}{r_{13}^3}\left(r_{13}\cos\theta_{13}\,\hat{i} + r_{13}\sin\theta_{13}\,\hat{j}\right) \qquad \left(\theta_{13} = 90°\right)$$

$$= \frac{\left(9\times10^{9}\right)\left(-2\times10^{-6}\right)\left(6\times10^{-6}\right)}{\left(6\times10^{-3}\right)^3}\left(6\times10^{-3}(\cos 90°)\,\hat{i} + 6\times10^{-3}(\sin 90°)\,\hat{j}\right)$$

$$\boxed{= -3\times10^{-3}\,\hat{j}\quad N}$$

$$\vec{F}_1^{net} = \vec{F}_{12} + \vec{F}_{13} = -4.5\times10^{-3}\,\hat{j} - 3\times10^{-3}\,\hat{j}\ \boxed{= -7.5\times10^{-3} N\ \hat{j}}$$

Example



Consider the charges shown

Calculate the net force exerted on charge q_2 by charges q_1 and q_3

Solution

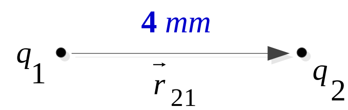
$$q_1 = 2\mu C = 2\times10^{-6}\ C$$

$$q_2 = -5\mu C = -5\times10^{-6}\ C$$

$$q_3 = -3\mu C = -3\times10^{-6}\ C$$

$$r_{21} = 4mm = 4\times10^{-3}\ m$$

$$r_{13} = 3mm = 3\times10^{-3}\ m$$



$$\vec{F}_2^{net} = \vec{F}_{21} + \vec{F}_{23}$$

$$\vec{r}_{21} = r_{21}\cos\theta_{21}\hat{i} + r_{21}\sin\theta_{21}\hat{j} \qquad (\theta_{21} = 0°)$$

$$= 4\times10^{-3}\cos(0)\hat{i} + 4\times10^{-3}\sin(0)\hat{j}$$

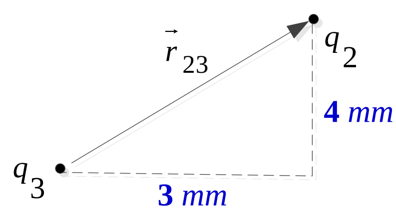
$$= 4\times10^{-3}\hat{i}$$

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{21}^3}\vec{r}_{21}$$

$$= \frac{(9\times10^9)(2\times10^{-6})(-5\times10^{-6})}{(4\times10^{-3})^3}(4\times10^{-3})\hat{i}$$

$$\boxed{= -5.6\times10^3\ N\ \hat{i}}$$

$$r_{23} = \sqrt{(4\times10^{-3})^2 + (3\times10^{-3})^2} = 5\times10^{-3}$$



$$\theta_{23} = \tan^{-1}\left(\frac{4\times10^{-3}}{3\times10^{-3}}\right) = 53°$$

$$\vec{r}_{23} = r_{23}\cos\theta_{23}\hat{i} + r_{23}\sin\theta_{23}\hat{j}$$

$$= 5\times10^{-3}\cos(53°)\hat{i} + 5\times10^{-3}\sin(53°)\hat{j}$$

$$= 3\times10^{-3}\hat{i} + 4\times10^{-3}\hat{j}$$

$$\vec{F}_{23} = k\frac{q_1 q_2}{r_{23}^3}\vec{r}_{23}$$

$$= \frac{\left(9\times10^{9}\right)\left(-5\times10^{-6}\right)\left(-3\times10^{-6}\right)}{\left(5\times10^{-3}\right)^{3}}\left(3\times10^{-3}\hat{i}+4\times10^{-3}\hat{j}\right)$$

$$= 1.08\times10^{6}\left(3\times10^{-3}\hat{i}+4\times10^{-3}\hat{j}\right)$$

$$\underline{= 3.24\times10^{3}\,N\,\hat{i}+4.32\times10^{3}\,N\,\hat{j}}$$

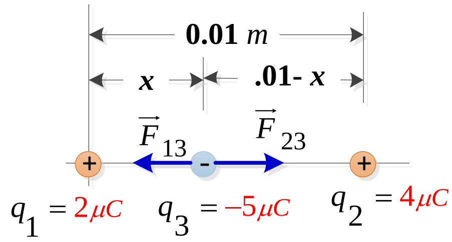
$$\vec{F}_{2}^{net} = \vec{F}_{21}+\vec{F}_{23}$$

$$= -5.6\times10^{3}\ \hat{i}+3.24\times10^{3}\ \hat{i}+4.32\times10^{3}\ j$$

$$\underline{= -2.36\times10^{3}\ \hat{i}+4.32\times10^{3}\ j \quad (N)}$$

Example

A $2\mu C$ charge and a $4\mu C$ charge are separated by a distance of 10 *mm* as shown.



Where a third charge should $q_3 = -5\mu C$ be placed if the net force exerted on it due to q_1 and q_2 is to be zero.

Solution

Let x be the distance between q_1 and q_3

Then the distance between q_3 and q_2 is $(.01 - x)$, the value of x is such that

$$\vec{F}_3^{net} = \vec{F}_{31} + \vec{F}_{32} = 0$$

$$\vec{F}_{31} = k\frac{q_1 q_3}{r_{13}^3}\vec{r}_{13}$$

$$= \frac{\left(9\times 10^9\right)\left(2\times 10^{-6}\right)\left(-5\times 10^{-6}\right)}{x^3}\left(x\,\hat{i}\right)$$

$$= -\frac{90\times 10^{-3}}{x^2}\;\hat{i}$$

$$\vec{F}_{32} = k\frac{q_2 q_3}{r_{32}^3}\vec{r}_{32}$$

$$= \frac{\left(9\times 10^9\right)\left(4\times 10^{-6}\right)\left(-5\times 10^{-6}\right)}{(.01-x)^3}(.01-x)\left(-\hat{i}\right)$$

$$= \frac{180\times 10^{-3}}{(.01-x)^2}\;\hat{i}$$

$$\vec{F}_3^{net} = \vec{F}_{31} + \vec{F}_{32} = 0$$

$$0 = -\frac{90\times 10^{-3}}{x^2}\;\hat{i} + \frac{180\times 10^{-3}}{(.01-x)^2}\;\hat{i}$$

$$\frac{9}{x^2} = \frac{18}{(.01-x)^2} \quad \Rightarrow \quad 9\left(.01^2 - .02x + x^2\right) = 18x^2$$

$$0 = -.0001 + .02x - x^2 + 2x^2$$

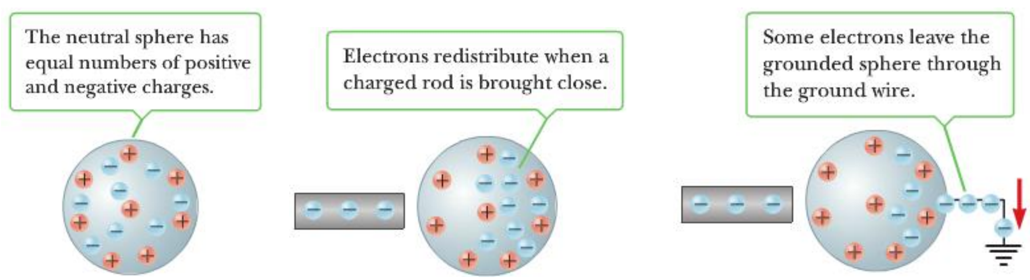
$$x^2 + .02x - .0001 = 0$$

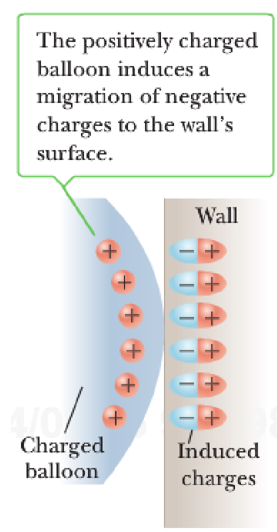
$$x = \frac{-.02 \pm \sqrt{(.02)^2 - 4(-.0001)}}{2}$$

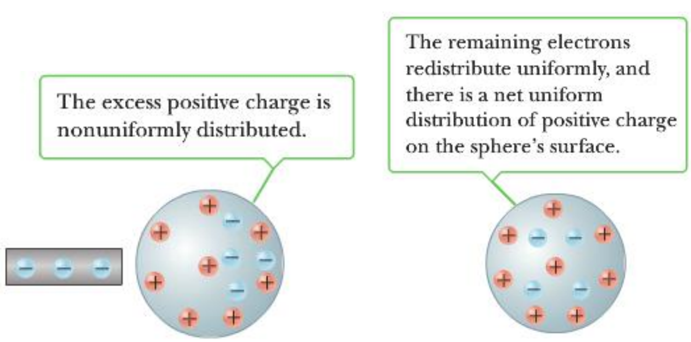
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-.02 \pm \sqrt{.0008}}{2}$$

$$= .004 \;\; m \qquad \cancel{=> 0.024}$$







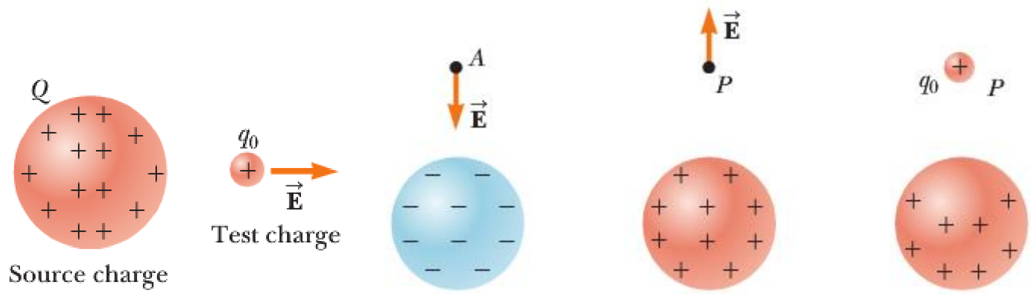
Electric Fields

Field Theory: Instead of saying q_1, exerts electrical force on charge q_2, in field theory, we say q_1 sets up electric field throughout space and this field exerts force or charge q_2.

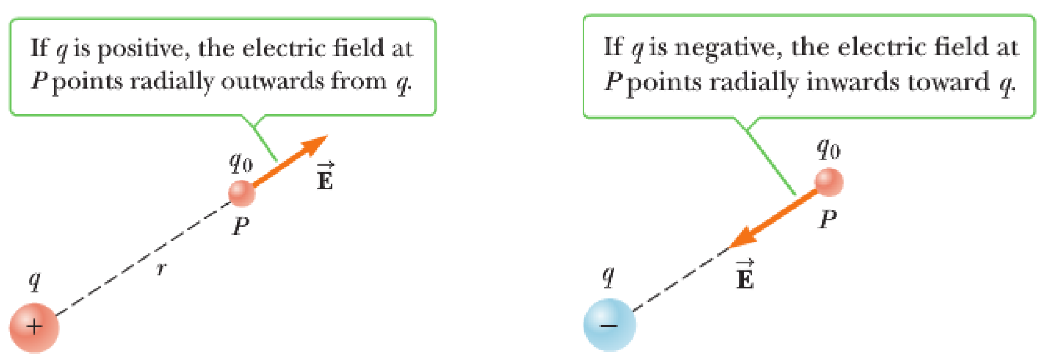
Electric Field: at a given point *P* is defined to be the electric force exerted on per unit charge on a charge placed at the given point.

$$\vec{E} = \frac{\vec{F}}{q}$$

$\vec{E}$: Electric field produces by a charge Q
$\vec{F}$: Electric force exerted on a charge q placed at point *P*.



Customarily, this equation is written as $\vec{F} = q\vec{E}$. If *q* is a positive charge then the electric force and electric field at the given point have the same direction. And if the charge is negative have opposite directions. The unit of measurement for electric field is *N/C*.



The equation relating the magnitude of $\vec{F}$ & $\vec{E}$ is obtained by taking the magnitude of both sides of the equation $\vec{F} = q\vec{E}$

$$F = |q|E$$

Example

The electric field at a certain point P is 2 N/C North. Determine the force exerted on q.

a) 4μC charge placed at point P

Given: $q = 4\mu C$ $\qquad \vec{E} = 2 \ \dfrac{N}{C} \qquad north \Rightarrow \theta = 90°$

Find: $\vec{F} = ?$

$$\vec{E} = E\cos(\theta)\hat{i} + E\sin(\theta)\hat{j}$$

$$= 2\frac{N}{C}\cos(90°)\hat{i} + 2\frac{N}{C}\sin(90°)\hat{j}$$

$$= 2\frac{N}{C}\cos(90°)\hat{i} + 2\frac{N}{C}\sin(90°)\hat{j}$$

$$\boxed{= 2\hat{j} \ \frac{N}{C}}$$

$$\vec{F} = q\vec{E}$$

$$= (4\mu C)\left(2\frac{N}{C}\right)\hat{j}$$

$$\boxed{= 8\mu N \ \hat{j}}$$

b) $-3\mu C$ charge placed at point P

$$\vec{F} = q\vec{E}$$

$$= (-3\mu C)\left(2\frac{N}{C}\right)\hat{j}$$

$$\boxed{= -6\mu N \ \hat{j}}$$

Example

Determine the electric field at point P if $q = -6nC$ charge placed at point experience a force of $12\mu N$ 37° North of East.

Solution

Given: $q = -6nC = -6\times 10^{-9} C$

$\vec{F} = 12\mu N = 12\times 10^{-6} N$

$\theta = 37°$

$$\vec{F} = F\cos(\theta)\hat{i} + F\sin(\theta)\hat{j}$$

$$= \left(12\times 10^{-6}\right)\cos(37°)\hat{i} + \left(12\times 10^{-6}\right)\sin(37°)\hat{j}$$

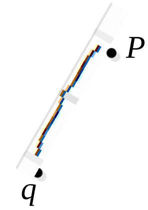
$$= 9.6\times 10^{-6} \ \hat{i} + 7.2\times 10^{-6}\hat{j}$$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{9.6\times10^{-6}\ \hat{i} + 7.2\times10^{-6}\ \hat{j}}{-6\times10^{-9}}$$

$$= -1.6\times10^{3}\ \hat{i} - 1.2\times10^{3}\ \hat{j}$$

Electric field due to a point charge

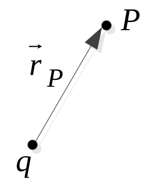
Consider a point P at a distance r from a charge q as shown.



Now suppose a small positive test charge q' is placed at point P. Then the electrical force exerted on q' by q is given by

$$\vec{F}_{q'} = k\frac{qq'}{r_p^3}\vec{r}_p$$

Where $\vec{r}_p$ is the position vector of point P with respect to charge q.



The electric field at point p is

$$\vec{E}_p = \frac{\vec{F}_{q'}}{q'} = \frac{kqq'}{q'r_p^2}\vec{r}_p = \frac{kq}{r^3}\vec{r}_P$$

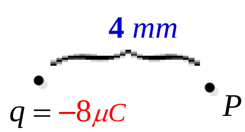
$\vec{E}_P$: Electric field at point P whose position vector with respect to the charge q is $\vec{r}_P$

r_p : Distance between q and P.

If the change q is positive $\vec{E}_P$ and $\vec{r}_p$ will have the same direction which means the direction of the electrical field is directed radially outward from the charge. If q is negative the direction of the field is opposite to that of $\vec{r}_p$; which means the direction of the electric field is towards the charge.

Example

Determine the electric field at point P due to the point charge $q = -8\mu C$.



Solution

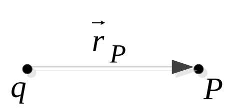
$$\vec{r}_p = r_p\cos(\theta_p)\hat{i} + r_p\sin(\theta_p)\hat{j}$$

$$= 4\times10^{-3}\cos(0°)\hat{i} + 4\times10^{-3}\sin(0°)\hat{j}$$

$$\boxed{= 4\times10^{-3}\hat{i}}$$

$$\vec{E}_p = \frac{kq}{r^3}\vec{r}_P$$

$$= \frac{(9\times10^9)(-8\times10^{-6})}{(4\times10^{-3})^3}(4\times10^{-3}\hat{i})$$



$$\boxed{= -4.5\times10^{-9}\ N/C\ \hat{i}}$$

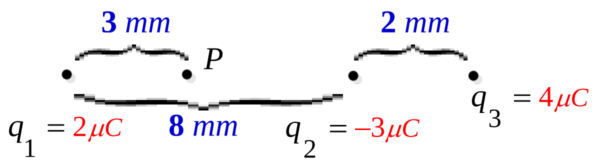
Superposition Principle

If there are a number of charges in the vicinity of a point *P*, then the net electric field at point *P* is equal to the vector sum of all the electric fields due to all the individual charges. If charges $q_1, q_2, \cdots$ are in the vicinity of point *P*, then

$$\vec{E}_p^{net} = \overrightarrow{E_1} + \overrightarrow{E_2} + \overrightarrow{E_3} + \ldots$$

Example

Consider the charges shown

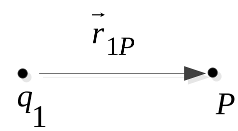


Determine the electric field at point *P* due to charges q_1, q_2 & q_3.

Solution

$$\vec{E}_p = \vec{E}_{1p} + \vec{E}_{2p} + \vec{E}_{3p}$$

$\vec{E}_{1p}$: $\vec{E}_{1p} = \dfrac{kq_1}{r_{1P}^3}\vec{r}_{1P}$



Given: $q_1 = 2\mu C = 2\times 10^{-6} C \qquad r_{1P} = 3\times 10^{-3} m$

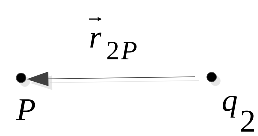
$$\vec{E}_{1p} = \frac{\left(9\times 10^{9}\right)\left(2\times 10^{-6}\right)}{\left(3\times 10^{-3}\right)^3}\left(3\times 10^{-3}\hat{\imath}\right)$$

$$\boxed{= 2\times 10^{9}\ N/C\ \hat{\imath}}$$

$\vec{E}_{2p}$: $\vec{E}_{2p} = \dfrac{kq_2}{r_{2P}^3}\vec{r}_{2P}$

Given: $q_2 = -3\mu C = -3\times 10^{-6} C; \quad \theta = 180°; \quad r_{2P} = (8-3)\times 10^{-3} m = 5\times 10^{-3} m$

$$\vec{E}_{2p} = \frac{\left(9\times 10^{9}\right)\left(-3\times 10^{-6}\right)}{\left(5\times 10^{-3}\right)^3}\left(-5\times 10^{-3}\hat{\imath}\right)$$

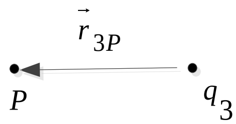


$$\boxed{= 1.08\times 10^{9}\ N/C\ \hat{\imath}}$$

$\vec{E}_{3p}$: $\vec{E}_{3p} = \dfrac{kq_3}{r_{3P}^3}\vec{r}_{3P}$

Given: $q_3 = 4\times10^{-6}C; \quad \theta = 180°; \quad r_{3P} = (8-3+2)\times10^{-3}m = 7\times10^{-3}m$

$$\vec{E}_{3p} = \frac{\left(9\times10^{9}\right)\left(4\times10^{-6}\right)}{\left(7\times10^{-3}\right)^3}\left(-7\times10^{-3}\hat{i}\right)$$



$$\boxed{= -0.735\times10^{9}\ N/C\ \hat{i}}$$

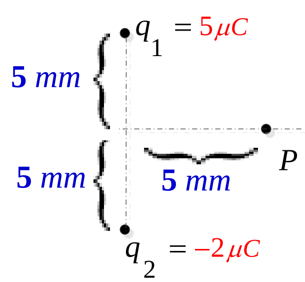
$$\vec{E}_P^{net} = \vec{E}_{1p} + \vec{E}_{2p} + \vec{E}_{3p}$$

$$= 2\times10^{9}\ \hat{i} + 1.08\times10^{9}\ \hat{i} - 0.735\times10^{9}\hat{i}$$

$$\boxed{= 2.345\times10^{9}\ \left(\frac{N}{C}\right)\ \hat{i}}$$

Example

Consider the two charges shown determine the electric filed at point P due to the charges q_1 & q_2 .



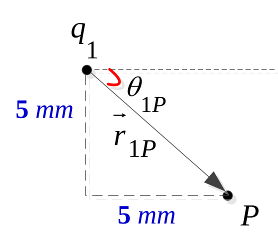
<u>Solution</u>

$$\vec{E}_p = \vec{E}_{1p} + \vec{E}_{2p}$$

$\vec{E}_{1p}$: $\vec{E}_{1p} = \dfrac{kq_1}{r_{1P}^3}\ \vec{r}_{1P}$

Given: $q_1 = 5\times10^{-6}C \qquad r_{1P} = \sqrt{\left(5\times10^{-3}\right)^2 + \left(5\times10^{-3}\right)^2} = 7.07\times10^{-3}m \approx 7\times10^{-3}m$

$$\theta_{1P} = -\tan^{-1}\left(\frac{5}{5}\right) = -45°$$



$$\vec{r}_{1P} = r_{1P}\cos\left(\theta_{1P}\right)\hat{i} + r_{1P}\sin\left(\theta_{1P}\right)\hat{j}$$

$$= 7\times10^{-3}\cos(-45°)\hat{i} + 7\times10^{-3}\sin(-45°)\hat{j}$$

$$= 4.9\times10^{-3}\hat{i} - 4.9\times10^{-3}\hat{j}$$

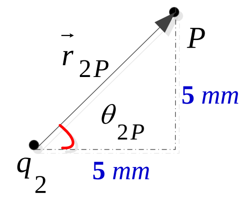
$$\vec{E}_{1p} = \frac{\left(9\times10^{9}\right)\left(5\times10^{-6}\right)}{\left(7\times10^{-3}\right)^3}\left(4.9\times10^{-3}\hat{i} - 4.9\times10^{-3}\hat{j}\right)$$

$$\boxed{= 0.6429\times10^{9}\hat{i} - 0.6429\times10^{9}\hat{j}}$$

$\vec{E}_{2p}$: $\vec{E}_{2p} = \dfrac{kq_2}{r_{2P}^3}\,\vec{r}_{2P}$

Given: $q_2 = -2\times10^{-6}C$ $\qquad r_{2P} = \sqrt{\left(5\times10^{-3}\right)^2 + \left(5\times10^{-3}\right)^2} \approx 7\times10^{-3}m$

$\theta_{2P} = \tan^{-1}\left(\frac{5}{5}\right) = 45°$



$$\vec{r}_{2P} = r_{2P}\cos\left(\theta_{2P}\right)\hat{i} + r_{1P}\sin\left(\theta_{2P}\right)\hat{j}$$

$$= 7\times10^{-3}\cos(45°)\hat{i} + 7\times10^{-3}\sin(45°)\hat{j}$$

$$= 4.9\times10^{-3}\hat{i} + 4.9\times10^{-3}\hat{j}$$

$$\vec{E}_{2p} = \frac{\left(9\times10^9\right)\left(-2\times10^{-6}\right)}{\left(7\times10^{-3}\right)^3}\left(4.9\times10^{-3}\hat{i} + 4.9\times10^{-3}\hat{j}\right)$$

$$\boxed{= 0.257\times10^9\hat{i} + 0.257\times10^9\hat{j}}$$

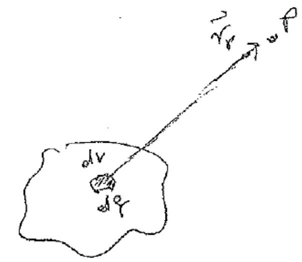
$$\vec{E}_P^{net} = \vec{E}_{1p} + \vec{E}_{2p}$$

$$== 0.6429\times10^9\hat{i} - 0.6429\times10^9\hat{j} + 0.257\times10^9\hat{i} + 0.257\times10^9\hat{j}$$

$$\boxed{\approx 0.9\times10^9\hat{i} - 0.39\times10^9\hat{j}\ \left(\frac{N}{C}\right)}$$

Electric Field due to a constant continuous distribution of charges

Suppose the charge contained in a small volume element dV is dq_0 then the electric field due to dq at point P is given by



$$d\overrightarrow{E_p} = k\frac{dq}{r_p^2}\overrightarrow{r_p}$$

The total electric field at point p is obtained by adding (*integrating*) the electric fields due to all dq in the distribution.

$$\overrightarrow{E_p} = \int k\frac{dq}{r_p^3}\overrightarrow{r_p}$$

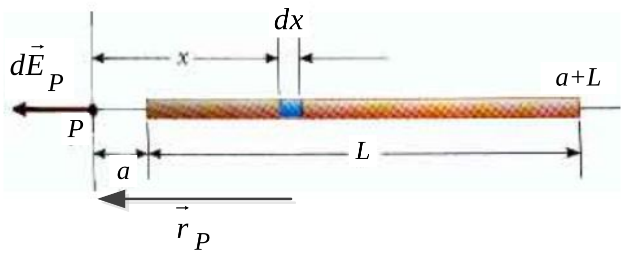
This integral over charge element dq can be conserved into integral over volume by defining a charge P as

$$\rho = \frac{dq}{dv} \Rightarrow dq = \rho dv$$

$$\boxed{\overrightarrow{E_p} = k\int_V \frac{\overrightarrow{r_p}\ \rho}{r_\rho^3}dV}$$

Example

The rod shown has a length L and a uniform linear charge density λ. The total charge is Q obtains an expression for the electric field at point P due to this charge distribution.



Consider a small length rod containing charge dq .

<u>Solution</u>

The charge density λ is given by $\rho = \frac{dq}{dx} = \frac{Q}{L}$ (*because the charge density is uniform*)

$$\Rightarrow\ dq = \frac{Q}{L}dx$$

The electric field due to charge dq in dx at point P is given by

$$d\overrightarrow{E_p} = k\frac{dq}{r_p^2}\overrightarrow{r_p} \qquad \left(\overrightarrow{r_p} = -x\hat{i}\right)$$ (*negative because it is directed towards west*)

$$\overrightarrow{E_p} = k\int_a^{a+L} \frac{dq}{x^3}\left(-x\hat{i}\right)$$

$$= \left(-\hat{i}\right)k\int_a^{a+L} \frac{1}{x^2}\frac{Q}{L}dx \qquad \left(since\ \ dq = \frac{Q}{L}dx\right)$$

$$= \left(-\hat{i}\right)k\frac{Q}{L}\left[-\frac{1}{x}\right]_a^{a+L}$$

$$= \left(\frac{kQ}{L}\hat{i}\right)\left(\frac{1}{a+L} - \frac{1}{a}\right)$$

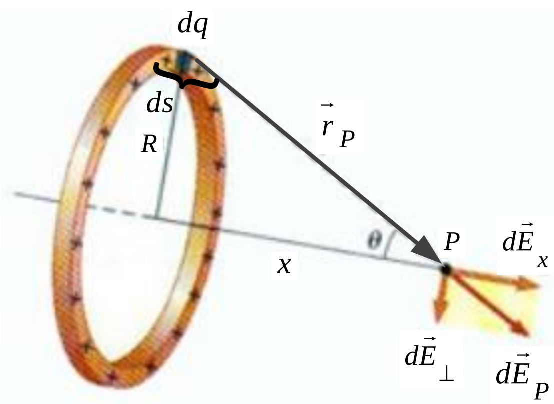
$$= \frac{kQ}{L}\cdot\frac{-L}{a\left(a+L\right)}\hat{i}$$

$$\boxed{= -\frac{kQ}{a\left(a+L\right)}\hat{i}}$$

The magnitude of the field is $\boxed{E_P = \frac{kQ}{a\left(a+L\right)}}$

Example

A ring of radius R has a uniform linear charge density λ. The total charge is Q.



Find an expression for the electric field at point P.

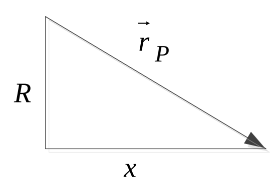
Solution

Consider the electric field $d\overrightarrow{E_p}$ due to a small charge element ds. Then the charge density λ is given by

$$\lambda = \frac{dq}{ds} = \frac{Q}{2\pi R} \ \Rightarrow\ dq = \frac{Q}{2\pi R}ds \qquad \left(\text{Since the charge density is uniform } \lambda = \frac{Q}{2\pi R}\right)$$

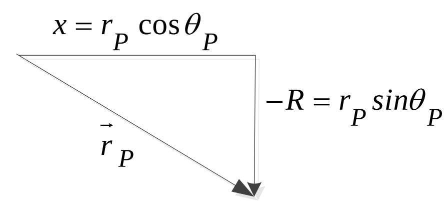
Therefore the electric field at point P due to dq is given by

$$d\overrightarrow{E_p} = k\frac{dq}{r_p^2}\overrightarrow{r_p} \qquad \left(r_p = \sqrt{R^2 + x^2}\right)$$



$$\vec{r}_P = r_P \cos\left(\theta_P\right)\hat{i} + r_P \sin\left(\theta_P\right)\hat{j}$$

$$= x\hat{i} - R\hat{j}$$



$$d\overrightarrow{E_p} = \frac{kdq}{r_p^2}\overrightarrow{r_p} = k\left(\frac{Q}{2\pi R}ds\right)\frac{1}{\left(x^2 + R^2\right)^{3/2}}\left(x\hat{i} - R\hat{j}\right)$$

From symmetry, upon integration all the $y\left(\hat{j}\right)$ components will cancel each other. That is for every dE_y from one point, there will be $-dE_y$ contribution from another point. Therefore we need to consider only $x\left(\hat{i}\right)$ components

$$E_p = E_{px} = \int_0^{2\pi R} \frac{kQ}{2\pi R}\frac{x}{\left(x^2 + R^2\right)^{3/2}}ds$$

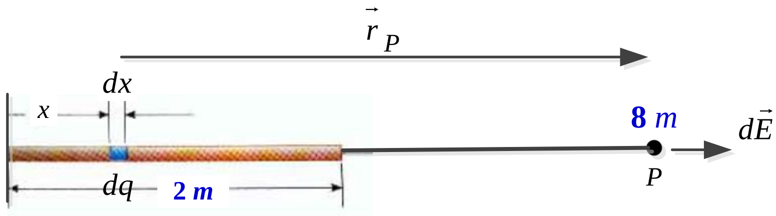
$$= \frac{kQ}{2\pi R}\frac{x}{\left(x^2 + R^2\right)^{3/2}}\int_0^{2\pi R} ds \qquad but \int_0^{2\pi R} ds = 2\pi R$$

$$= \frac{kQ}{2\pi R}\frac{x}{\left(x^2 + R^2\right)^{3/2}}\left(2\pi R\right)$$

$$= \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

Example

Consider a rod that extends from $x = 0$ to $x = 2m$ on the x-axis. The charge density in the rod varies on x according to the equation $\lambda(x) = 2\sqrt{8-x}$. Calculate the electric field due to this charged rod at a point located at $x = 8m$



Solution

Let the electric field due to charge dg located at x away from the origin be $d\vec{E}$. Then

$$\overrightarrow{r_P} = (8-x)\hat{i}$$

$$r_P = (8-x)$$

$$\frac{dq}{dx} = \lambda(x) = 2\sqrt{8-x} \quad \Rightarrow \quad dq\lambda(x) = 2\sqrt{8-x}$$

$$d\overrightarrow{E_P} = \frac{kdq}{r_P^2}\overrightarrow{r_P} = k\frac{2\sqrt{8-x}}{(8-x)^2}dx\ \hat{i}$$

$$E = \int_{x=0}^{x=2} k\frac{2\sqrt{8-x}}{(8-x)^2}dx\ \hat{i} \qquad\qquad d(8-x) = -dx \quad \Rightarrow \quad dx = -d(8-x)$$

$$= -2k\ \hat{i}\int_0^2 (8-x)^{-3/2}d(8-x)$$

$$= -2k\ \hat{i}\,\frac{2}{5}\left[(8-x)^{-1/2}\right]_0^2$$

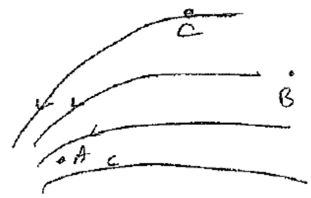
$$= -\frac{4}{5}\left(9\times10^9\right)\left(6^{-1/2} - 8^{-1/2}\right)\ \hat{i}$$

$$\boxed{= 0.4\times10^9\ \left(\frac{N}{C}\right)\ \hat{i}}$$

Electric Field Lines

Electric field lines are lines used to represent electric field graphically. These lines should represent both magnitude and the direction of the field at any point. To represent magnitude they are drawn in such a way that the number of lines crossing a unit perpendicular area (*density of line*) is directly proportional to the magnitude of the field at any point. To represent direction, these lines are drawn in such a way that the line of action of the field is tangent to the curves at any point. To distinguish between the two possible directions of the tangent line, arrows are included in the lines. Electric field lines originate in a positive charge and sink in a negatives charge.

Example



Consider the electric field lines shown.

a) Where is the field stronger at point *A* or *B*

At point *A* because the lines are denser at point *A*

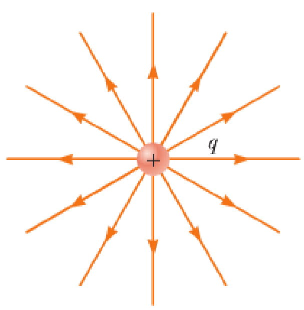
b) Determine the direction of the field at point *C* approximately.

The tangent line at point *C* is approximately horizontal.

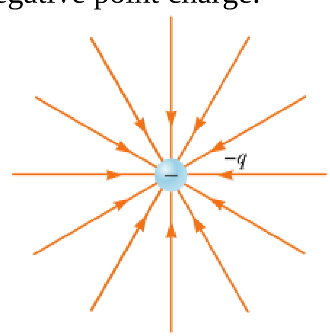
Therefore the direction is west as indicated by the arrows.

Electric Field Lines Due to Point Charges

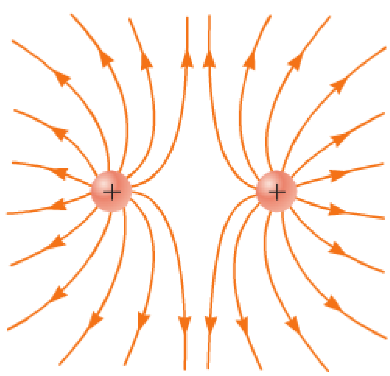
a) A positive point charge



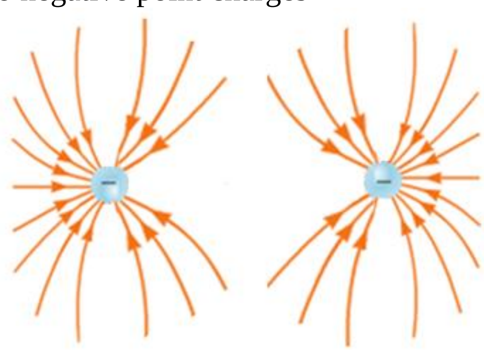
b) A negative point charge.



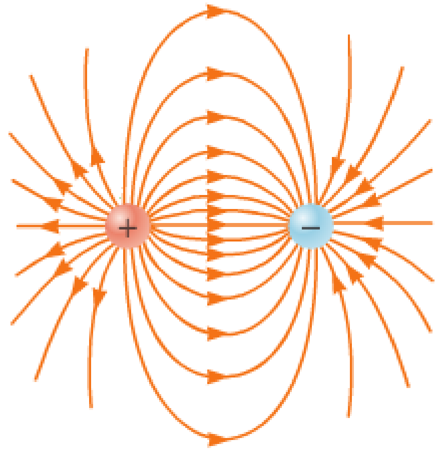
c) Two positive point charges



d) Two negative point charges



e) A positive and a negative charge.



<u>Note</u>: Electric field lines do not cross each other. Because if they do that would mean two tangent lines at the intersection point and hence two directions for a field at the intersection point which is not possible.