Solution Section 1.3 – Infinite Limits

Exercise

Find $\lim\limits_{x\to 5} \dfrac{x-7}{x(x-5)^2}$

Solution

$$\lim\limits_{x\to 5} \frac{x-7}{x(x-5)^2} = \frac{-2}{0}$$

$$\underline{=\infty|}$$

Exercise

Find $\lim\limits_{x\to -5^+} \dfrac{x-5}{x+5}$

Solution

$$\lim\limits_{x\to -5^+} \frac{x-5}{x+5} = \frac{-10}{0^+}$$

$$\underline{=-\infty|}$$

Exercise

Find $\lim\limits_{x\to 3^-} \dfrac{x-4}{x^2-3x}$

Solution

$$\lim\limits_{x\to 3^-} \frac{x-4}{x^2-3x} = \frac{-1}{0^-}$$

$$\underline{=\infty|}$$

Exercise

Find $\lim\limits_{x\to 0^+} \dfrac{1}{3x}$

Solution

$$\lim\limits_{x\to 0^+} \frac{1}{3x} = \frac{1}{0^+}$$

$$\underline{=\infty|}$$

Exercise

Find $\lim\limits_{x\to -5^-} \dfrac{3x}{2x+10}$

Solution

$$\lim_{x\to -5^-} \frac{3x}{2x+10} = \lim_{x\to -5^-} \frac{3}{2+\frac{10}{x}}$$

$$\underline{=\infty}$$

Exercise

Find $\lim\limits_{x\to 0} \dfrac{1}{x^{2/3}}$

Solution

$$\lim_{x\to 0} \frac{1}{x^{2/3}} = \lim_{x\to 0} \frac{1}{\left(x^{1/3}\right)^2}$$

$$\underline{=\infty}$$

Exercise

Find $\lim\limits_{x\to 0^-} \dfrac{1}{3x^{1/3}}$

Solution

$$\lim_{x\to 0^-} \frac{1}{3x^{1/3}} = \frac{1}{0^-}$$

$$\underline{=-\infty}$$

Exercise

Find $\lim\limits_{x\to\left(-\frac{\pi}{2}\right)^+} \sec x$

Solution

$$\lim_{x\to\left(-\frac{\pi}{2}\right)^+} \sec x \underline{=\infty}$$

Exercise

Find $\lim\limits_{\theta\to 0^-} (1+\csc\theta)$

Solution

$$\lim_{\theta\to 0^-}(1+\csc\theta)=\lim_{\theta\to 0^-}\left(1+\frac{1}{\sin\theta}\right)$$

$$\boxed{=-\infty}$$

Exercise

Find $\quad \lim_{\theta\to 0^+}\csc\theta$

Solution

$$\lim_{\theta\to 0^+}\csc\theta=\lim_{\theta\to 0^+}\frac{1}{\sin\theta} \qquad \text{As } \theta\to 0^+ \ \sin\theta>0$$

$$\boxed{=+\infty}$$

Exercise

Find $\quad \lim_{x\to 0+}(-10\cot x)$

Solution

$$\lim_{x\to 0^+}(-10\cot x)=-10\lim_{x\to 0^+}\frac{\cos\theta}{\sin\theta}=-10\left(\frac{1}{0}\right) \qquad \text{As } x\to 0^+ \ \cos\theta>0; \ \sin\theta>0$$

$$\boxed{=-\infty}$$

Exercise

Find $\quad \lim_{\theta\to\frac{\pi}{2}^+}\frac{1}{3}\tan\theta$

Solution

$$\lim_{\theta\to\frac{\pi}{2}^+}\frac{1}{3}\tan\theta=\frac{1}{3}\lim_{\theta\to\frac{\pi}{2}^+}\frac{\sin\theta}{\cos\theta}=\frac{1}{3}\left(-\frac{1}{0}\right) \qquad \text{As } \theta\to\frac{\pi}{2}^+ \ \cos\theta<0; \ \sin\theta>0$$

$$\boxed{=-\infty}$$

Exercise

Find $\quad \lim_{x\to 2^+}\frac{1}{x-2}$

Solution

$$\lim_{x\to 2^+}\frac{1}{x-2}=\frac{1}{2^+-2}=\frac{1}{0^+}$$

$$\boxed{=\infty}$$

Exercise

Find $\lim\limits_{x\to 2^-} \frac{1}{x-2}$

Solution

$$\lim_{x\to 2^-} \frac{1}{x-2} = \frac{1}{2^- - 2} = \frac{1}{0^-}$$

$$\boxed{= -\infty}$$

Exercise

Find $\lim\limits_{x\to 2} \frac{1}{x-2}$

Solution

$$\lim_{x\to 2} \frac{1}{x-2} = \frac{1}{0}$$

$$\boxed{= \infty}$$

Exercise

Find $\lim\limits_{x\to 3^+} \frac{2}{(x-3)^3}$

Solution

$$\lim_{x\to 3^+} \frac{2}{(x-3)^3} = \frac{2}{0^+}$$

$$\boxed{= \infty}$$

Exercise

Find $\lim\limits_{x\to 3^-} \frac{2}{(x-3)^3}$

Solution

$$\lim_{x\to 3^-} \frac{2}{(x-3)^3} = \frac{2}{0^-}$$

$$\boxed{= -\infty}$$

Exercise

Find $\lim\limits_{x\to 3} \frac{2}{(x-3)^3}$

Solution

$$\lim_{x\to 3} \frac{2}{(x-3)^3} = \frac{2}{0}$$

$$= \infty$$

Exercise

Find $\displaystyle \lim_{x\to 4^+} \frac{x-5}{(x-4)^2}$

Solution

$$\lim_{x\to 4^+} \frac{x-5}{(x-4)^2} = \frac{-1}{0}$$

$$= -\infty$$

Exercise

Find $\displaystyle \lim_{x\to 4^-} \frac{x-5}{(x-4)^2}$

Solution

$$\lim_{x\to 4^-} \frac{x-5}{(x-4)^2} = \frac{-1}{0}$$

$$= -\infty$$

Exercise

Find $\displaystyle \lim_{x\to 4} \frac{x-5}{(x-4)^2}$

Solution

$$\lim_{x\to 4^-} \frac{x-5}{(x-4)^2} = \frac{-1}{0}$$

$$= -\infty$$

Exercise

Find $\displaystyle \lim_{x\to 1^+} \frac{x-2}{(x-1)^3}$

Solution

$$\lim_{x\to 1^+} \frac{x-2}{(x-1)^3} = \frac{-1}{0^+}$$

$$= -\infty$$

Exercise

Find $\displaystyle\lim_{x\to 1^-} \frac{x-2}{(x-1)^3}$

Solution

$$\lim_{x\to 1^-} \frac{x-2}{(x-1)^3} = \frac{-1}{0^-}$$

$$= \infty$$

Exercise

Find $\displaystyle\lim_{x\to 1} \frac{x-2}{(x-1)^3}$

Solution

$$\lim_{x\to 1} \frac{x-2}{(x-1)^3} = \frac{-1}{0^+}$$

$$= \nexists$$

Exercise

Find $\displaystyle\lim_{x\to 3^+} \frac{(x-1)(x-2)}{x-3}$

Solution

$$\lim_{x\to 3^+} \frac{(x-1)(x-2)}{x-3} = \frac{2}{0}$$

$$= \infty$$

Exercise

Find $\displaystyle\lim_{x\to 3^-} \frac{(x-1)(x-2)}{x-3}$

Solution

$$\lim_{x\to 3^-} \frac{(x-1)(x-2)}{x-3} = \frac{2}{0^-}$$

$$= -\infty$$

Exercise

Find $\displaystyle\lim_{x\to 3}\frac{(x-1)(x-2)}{x-3}$

Solution

$$\lim_{x\to 3^-}\frac{(x-1)(x-2)}{x-3}=\frac{2}{0^-}$$

$$\underline{=\nexists}$$

$$\lim_{x\to 3^-}\frac{(x-1)(x-2)}{x-3}=-\infty \qquad \lim_{x\to 3^+}\frac{(x-1)(x-2)}{x-3}=\infty$$

Exercise

Find $\displaystyle\lim_{x\to -2^+}\frac{x-4}{x(x+2)}$

Solution

$$\lim_{x\to -2^+}\frac{x-4}{x(x+2)}=\frac{-6}{-0^+}$$

$$\underline{=\infty}$$

Exercise

Find $\displaystyle\lim_{x\to -2^-}\frac{x-4}{x(x+2)}$

Solution

$$\lim_{x\to -2^-}\frac{x-4}{x(x+2)}=\frac{-6}{0^+}$$

$$\underline{=-\infty}$$

Exercise

Find $\displaystyle\lim_{x\to -2}\frac{x-4}{x(x+2)}$

Solution

$$\lim_{x\to -2}\frac{x-4}{x(x+2)}\underline{=\nexists}$$

$$\lim_{x\to -2^+}\frac{x-4}{x(x+2)}=\infty \qquad \lim_{x\to -2^-}\frac{x-4}{x(x+2)}=-\infty$$

Exercise

Find $\displaystyle\lim_{x\to 2^+}\frac{x^2-4x+3}{(x-2)^2}$

Solution

$$\lim_{x\to 2^+} \frac{x^2-4x+3}{(x-2)^2} = \frac{-1}{0^+}$$

$$= -\infty$$

Exercise

Find $\lim_{x\to 2^-} \frac{x^2-4x+3}{(x-2)^2}$

Solution

$$\lim_{x\to 2^-} \frac{x^2-4x+3}{(x-2)^2} = \frac{-1}{0^+}$$

$$= -\infty$$

Exercise

Find $\lim_{x\to 2} \frac{x^2-4x+3}{(x-2)^2}$

Solution

$$\lim_{x\to 2} \frac{x^2-4x+3}{(x-2)^2} = \frac{-1}{0}$$

$$= -\infty$$

Exercise

Find $\lim_{x\to -2^+} \frac{x^3-5x^2+6x}{x^4-4x^2}$

Solution

$$\lim_{x\to -2^+} \frac{x^3-5x^2+6x}{x^4-4x^2} = \lim_{x\to -2^+} \frac{x(x-2)(x-3)}{x^2(x-2)(x+2)}$$

$$= \lim_{x\to -2^+} \frac{x-3}{x(x+2)} \qquad \frac{-}{-(+)}$$

$$= \infty$$

Exercise

Find $\lim_{x\to -2^-} \frac{x^3-5x^2+6x}{x^4-4x^2}$

Solution

$$\lim_{x\to -2^-} \frac{x^3-5x^2+6x}{x^4-4x^2} = \lim_{x\to -2^-} \frac{x(x-2)(x-3)}{x^2(x-2)(x+2)}$$

$$= \lim_{x \to -2^-} \frac{x-3}{x(x+2)} \quad \frac{-}{-(-)}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim_{x \to -2} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2}$

Solution

$$\lim_{x \to -2} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2} = \frac{-8 - 20 - 12}{16 - 16}$$

$$= \frac{-40}{0}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim_{u \to 0^+} \frac{u-1}{\sin u}$

Solution

$$\lim_{u \to 0^+} \frac{u-1}{\sin u} = \frac{-1}{0^+}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim_{x \to 0^-} \frac{2}{\tan x}$

Solution

$$\lim_{x \to 0^-} \frac{2}{\tan x} = \frac{2}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim_{x \to 1^+} \frac{x^2 - 5x + 6}{x-1}$

Solution

$$\lim_{x \to 1^+} \frac{x^2 - 5x + 6}{x-1} = \frac{2}{0^+}$$

$$\underline{= \infty}$$

Exercise

Find $\lim\limits_{x\to 2\pi^-} \csc x$

Solution

$$\lim_{x\to 2\pi^-} \csc x = \frac{1}{\sin\left(2\pi^-\right)} = \frac{1}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim\limits_{x\to 0^+} e^{\sqrt{x}}$

Solution

$$\lim_{x\to 0^+} e^{\sqrt{x}} \underline{=1}$$

Exercise

Find $\lim\limits_{x\to \frac{\pi}{2}^-} \frac{1+\sin x}{\cos x}$

Solution

$$\lim_{x\to \frac{\pi}{2}^-} \frac{1+\sin x}{\cos x} = \frac{2}{0^+}$$

$$\underline{= \infty}$$

Exercise

Find $\lim\limits_{x\to \frac{\pi}{2}^+} \frac{1+\sin x}{\cos x}$

Solution

$$\lim_{x\to \frac{\pi}{2}^+} \frac{1+\sin x}{\cos x} = \frac{2}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\lim\limits_{x\to 0^-} \frac{e^x}{1+e^x}$

Solution

$$\lim_{x\to 0^-} \frac{e^x}{1-e^x} = \frac{1}{0^+}$$

$$\underline{= \infty}$$

Exercise

Find $\quad \lim_{x\to 0^+} \frac{e^x}{1-e^x}$

Solution

$$\lim_{x\to 0^+} \frac{e^x}{1-e^x} = \frac{1}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\quad \lim_{x\to 1^-} \frac{x}{\ln x}$

Solution

$$\lim_{x\to 1^-} \frac{x}{\ln x} = \frac{1}{0^-}$$

$$\underline{= -\infty}$$

Exercise

Find $\quad \lim_{x\to 0^+} \frac{x}{\ln x}$

Solution

$$\lim_{x\to 0^+} \frac{x}{\ln x} = \frac{0}{-\infty}$$

$$\underline{= 0}$$

Exercise

Find $\quad \lim_{x\to 0^-} \frac{2e^x+5e^{3x}}{e^{2x}-e^{3x}}$

Solution

$$\lim_{x\to 0^-} \frac{2e^x+5e^{3x}}{e^{2x}-e^{3x}} = \lim_{x\to 0^-} \frac{2e^x+5e^{3x}}{e^{2x}\left(1-e^x\right)}$$

$$= \frac{7}{0}$$

$$\underline{= \infty}$$

Exercise

Find $\displaystyle\lim_{x\to 0^+} \frac{2e^x+5e^{3x}}{e^{2x}-e^{3x}}$

Solution

$$\lim_{x\to 0^+} \frac{2e^x+5e^{3x}}{e^{2x}-e^{3x}} = \lim_{x\to 0^+} \frac{2e^x+5e^{3x}}{e^{2x}\left(1-e^x\right)}$$

$$= \frac{7}{0^-}$$

$$\boxed{= -\infty}$$

Exercise

Find $\displaystyle\lim_{x\to 1^-} \frac{\ln x}{\sin^{-1} x}$

Solution

$$\lim_{x\to 1^-} \frac{\ln x}{\sin^{-1} x} = \frac{\ln 1}{\sin^{-1} 1}$$

$$= \frac{0}{\frac{\pi}{2}}$$

$$\boxed{= 0}$$

Exercise

Let $f(x) = \dfrac{x^2-7x+12}{x-a}$

a) For what values of a, if any, does $\displaystyle\lim_{x\to a^+} f(x)$ equal a finite number?

b) For what values of a, if any, does $\displaystyle\lim_{x\to a^+} f(x) = \infty$?

c) For what values of a, if any, does $\displaystyle\lim_{x\to a^+} f(x) = -\infty$?

Solution

$$f(x) = \frac{x^2-7x+12}{x-a} = \frac{(x-3)(x-4)}{x-a}$$

a) If $a=3$, then $\displaystyle\lim_{x\to 3} \frac{(x-3)(x-4)}{x-3} = \lim_{x\to 3} (x-4) \boxed{= -1}$

If $a=4$, then $\displaystyle\lim_{x\to 4} \frac{(x-3)(x-4)}{x-4} = \lim_{x\to 4} (x-1) \boxed{= 1}$

b) $\lim\limits_{x\to a^+} f(x) = \infty$ for any number other than 3 or 4.

As $x \to a^+$, then $(x-a)$ is always positive.

$(x-3)(x-4) > 0 \quad \Rightarrow \quad (-\infty,\ 3) \cup (4,\ \infty)$

c) $\lim\limits_{x\to a^+} f(x) = -\infty$ for any number other than 3 or 4.

As $x \to a^+$, then $(x-a)$ is always positive, and $(3,\ 4)$

Exercise

Analyze $\lim\limits_{x\to 1^+} \sqrt{\dfrac{x-1}{x-3}}$ and $\lim\limits_{x\to 1^-} \sqrt{\dfrac{x-1}{x-3}}$

Solution

$$\lim_{x\to 1^+} \sqrt{\frac{x-1}{x-3}} = \sqrt{\frac{0^+}{-2}} \qquad \nexists$$

$$\lim_{x\to 1^-} \sqrt{\frac{x-1}{x-3}} = \sqrt{\frac{0^-}{-2}} \ \underline{= 0}$$