Solution *Section* 1.3 – Polynomial Functions & Graphs

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=2x^4-x^3+7x-12; \quad p(x)=x^2-3$

Solution

$$\begin{array}{rl}
 & 2x^2 - x + 6 \\
x^2-3 & \overline{)\,2x^4 - x^3 + 0x^2 + 7x - 12} \\
 & \underline{2x^4 \qquad\; - 6x^2} \\
 & \; - x^3 + 6x^2 + 7x \\
 & \; \underline{-x^3 \qquad\quad + 3x} \\
 & \; 6x^2 + 4x - 12 \\
 & \; \underline{6x^2 \qquad - 18} \\
 & \; 4x + 6
\end{array}$$

$$\underline{Q(x)=2x^2-x+6; \quad R(x)=4x+6}\,\big|$$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=3x^3+2x-4; \quad p(x)=2x^2+1$

Solution

$$\begin{array}{rl}
 & \; \frac{3}{2}x \\
2x^2+1 & \overline{)\,3x^3 + 0x^2 + 2x - 4} \\
 & \underline{3x^3 \qquad\quad + \frac{3}{2}x \qquad} \\
 & \; \frac{1}{2}x - 4
\end{array}$$

$$\underline{Q(x)=\frac{3}{2}x; \quad R(x)=\frac{1}{2}x-4}\,\big|$$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x)=7x+2; \quad p(x)=2x^2-x-4$

Solution

$$P(x)=\frac{7x+2}{2x^2-x-4}$$

$$\underline{Q(x)=0; \quad R(x)=7x+2}\,\big|$$

Exercise

Find the quotient and remainder if $f(x)$ is divided by $p(x)$: $f(x) = 9x + 4; \quad p(x) = 2x - 5$

Solution

$$\begin{array}{rl} & \frac{9}{2} \\ 2x-5 & \overline{\smash{)}\,9x+4} \\ & \underline{\;9x-\frac{45}{2}\;} \\ & \qquad -\frac{37}{2} \end{array}$$

$$\boxed{Q(x) = \frac{9}{2}; \quad R(x) = -\frac{37}{2}}$$

Exercise

Use the remainder theorem to find $f(c)$: $f(x) = x^4 - 6x^2 + 4x - 8; \quad c = -3$

Solution

$$f(-3) = (-3)^4 - 6(-3)^2 + 4(-3) - 8$$

$$\boxed{= 7}$$

Exercise

Use the remainder theorem to find $f(c)$: $f(x) = x^4 + 3x^2 - 12; \quad c = -2$

Solution

$$f(-2) = (-2)^4 + 3(-2)^2 - 12$$

$$\boxed{= 16}$$

Exercise

Use the factor theorem to show that $x - c$ is a factor of $f(x)$: $f(x) = x^3 + x^2 - 2x + 12; \quad c = -3$

Solution

$$f(-3) = (-3)^3 + (-3)^2 - 2(-3) + 12$$

$$\boxed{= 0}$$

From the factor theorem; $x + 3$ is a factor of $f(x)$.

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second:

$2x^3 - 3x^2 + 4x - 5; \quad x - 2$

Solution

$$\begin{array}{r|rrrr} 2 & 2 & -3 & 4 & -5 \\ & & 4 & 2 & 12 \\ \hline & 2 & 1 & 6 & \boxed{7} \end{array}$$

$Q(x) = 2x^2 + x + 6 \qquad R(x) = 7$

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second:

$5x^3 - 6x^2 + 15; \quad x - 4$

Solution

$$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 15 \\ & & 20 & 56 & 224 \\ \hline & 5 & 14 & 56 & \boxed{239} \end{array}$$

$Q(x) = 5x^2 + 14x + 56 \qquad R(x) = 239$

Exercise

Use the synthetic division to find the quotient and remainder if the first polynomial is divided by the second:

$9x^3 - 6x^2 + 3x - 4; \quad x - \frac{1}{3}$

Solution

$$\begin{array}{r|rrrr} \frac{1}{3} & 9 & -6 & 3 & -4 \\ & & 3 & -1 & \frac{2}{3} \\ \hline & 9 & -3 & 2 & \boxed{-\frac{10}{3}} \end{array}$$

$Q(x) = 9x^2 - 3x + 2 \qquad R(x) = -\frac{10}{3}$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=2x^3+3x^2-4x+4; \quad c=3$

Solution

$$\begin{array}{r|rrrr} 3 & 2 & 3 & -4 & 4 \\ & & 6 & 27 & 69 \\ \hline & 2 & 9 & 23 & \boxed{73} \end{array}$$

$f(3)=73$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=8x^5-3x^2+7; \quad c=\frac{1}{2}$

Solution

$$\begin{array}{r|rrrrrr} \frac{1}{2} & 8 & 0 & 0 & -3 & 0 & 7 \\ & & 4 & 2 & 1 & -1 & -\frac{1}{2} \\ \hline & 8 & 4 & 2 & -2 & -1 & \boxed{\frac{13}{2}} \end{array}$$

$f\left(\frac{1}{2}\right)=\frac{13}{2}$

Exercise

Use the synthetic division to find $f(c)$: $f(x)=x^3-3x^2-8; \quad c=1+\sqrt{2}$

Solution

$$\begin{array}{r|cccc} 1+\sqrt{2} & 3 & -3 & 0 & -8 \\ & & 3+3\sqrt{2} & 6+3\sqrt{2} & 12+9\sqrt{2} \\ \hline & 3 & 3\sqrt{2} & 6+3\sqrt{2} & \boxed{4+9\sqrt{2}} \end{array}$$

$f\left(1+\sqrt{2}\right)=4+9\sqrt{2}$

Exercise

Use the synthetic division to show that c is a zero of $f(x)$: $f(x)=3x^4+8x^3-2x^2-10x+4; \quad c=-2$

Solution

$$\begin{array}{r|rrrrr} -2 & 3 & 8 & -2 & -10 & 4 \\ & & -6 & -4 & 12 & -4 \\ \hline & 3 & 2 & -6 & 2 & \boxed{0} \end{array}$$

$$\boxed{f(-2)=0}$$

Exercise

Use the synthetic division to show that c is a zero of $f(x)$: $f(x)=27x^4-9x^3+3x^2+6x+1;\quad c=-\frac{1}{3}$

Solution

$$\begin{array}{r|rrrrr} -\frac{1}{3} & 27 & -9 & 3 & 6 & 1 \\ & & -9 & 6 & -3 & -1 \\ \hline & 27 & -18 & 9 & 3 & \boxed{0} \end{array}$$

$$\boxed{f\left(-\tfrac{1}{3}\right)=0}$$

Exercise

Find all values of k such that $f(x)$ is divisible by the given linear polynomial:

$f(x)=kx^3+x^2+k^2x+3k^2+11;\quad x+2$

Solution

$$\begin{array}{r|cccc} -2 & k & 1 & k^2 & 3k^2+11 \\ & & -2k & 4k-2 & -2k^2-8k+4 \\ \hline & k & 1-2k & k^2+4k-2 & k^2-8k+15 \end{array}$$

$$k^2-8k+15=0 \Rightarrow \boxed{k=3,\ 5}$$

Exercise

Find all values of k such that $f(x)$ is divisible by the given linear polynomial:

$f(x)=x^3+k^3x^2++2kx-2k^4;\quad x-1.6$

Solution

$$\begin{array}{r|cccc} 1.6 & 1 & k^3 & 2k & -2k^4 \\ & & 1.6 & 1.6k^3+2.56 & 2.56k^3+3.2k+4.096 \\ \hline & 1 & k^3+1.6 & 1.6k^3+2k+2.56 & -2k^4+2.56k^3+3.2k+4.096 \end{array}$$

$$-2k^4 + 2.56k^3 + 3.2k + 4.096 = 0$$

Using the calculator, the result will show that the solutions are: $\underline{x = -0.75,\ \ 1.96}|$ $\quad 0.032 \pm 1.18i$

Exercise

Find all values of k such that $f(x)$ is divisible by the given linear polynomial:

$f(x) = k^2x^3 - 4kx + 3; \quad x-1$

Solution

$$\begin{array}{r|llll} 1 & k^2 & 0 & -4k & 3 \\ & & k^2 & k^2-4k & k^2-4k \\ \hline & k^2 & k^2 & k^2-4k & k^2-4k+3 \end{array}$$

$k^2 - 4k + 3 = 0 \quad \Rightarrow \quad \underline{k = 1,\ 3}|$

Exercise

Find all solutions of the equation: $\quad x^3 - x^2 - 10x - 8 = 0$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & 0 \end{array} \rightarrow \quad x^2 - 2x - 8 = 0$$

The solutions are: $\underline{x = -1,\ -2,\ 4}|$

Exercise

Find all solutions of the equation: $\quad x^3 + x^2 - 14x - 24 = 0$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & 0 \end{array} \rightarrow \quad x^2 - x - 12 = 0$$

The solutions are: $\underline{x = -2,\ -3,\ 4}|$

Exercise

Find all solutions of the equation: $2x^3-3x^2-17x+30=0$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{30}{2}\right\}$

$$=\pm\left\{1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30,\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{15}{2}\right\}$$

$$\begin{array}{c|cccc} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & \boxed{0} \end{array} \rightarrow\ 2x^2+x-15=0$$

The solutions are: $x=2,\ -3,\ \frac{5}{2}$

Exercise

Find all solutions of the equation: $12x^3+8x^2-3x-2=0$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{2}{12}\right\}=\pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ \frac{1}{6},\ \frac{1}{12}\right\}$

$$\begin{array}{c|cccc} \frac{1}{2} & 12 & 8 & -3 & -2 \\ & & 6 & 7 & 2 \\ \hline & 12 & 14 & 4 & \boxed{0} \end{array} \rightarrow\ 12x^2+14x+4=0$$

$$6x^2+7x+2=0$$

$$x=\frac{-7\pm\sqrt{49-48}}{12}$$

$$=\begin{cases} \frac{-7-1}{12}=-\frac{2}{3} \\ \frac{-7+1}{12}=-\frac{1}{2} \end{cases}$$

The solutions are: $x=\frac{1}{2},\ -\frac{1}{2},\ -\frac{2}{3}$

Exercise

Find all solutions of the equation: $x^4+3x^3-30x^2-6x+56=0$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{56}{1}\right\}=\pm\{1,\ 2,\ 4,\ 7,\ 8,\ 14,\ 28,\ 56\}$

$$\begin{array}{r|rrrrr} 4 & 1 & 3 & -3 & -6 & 56\\ & & 4 & 28 & -8 & -56\\ \hline -7 & 1 & 7 & -2 & -14 & 0\\ & & -7 & 0 & 14 & \\ \hline & 1 & 0 & -2 & & \end{array}$$

$$\to \quad x^3+7x^2-2x-14=0 \quad \Rightarrow \quad \frac{c}{d}=\pm\left\{\frac{14}{1}\right\}=\pm\{1,\ 2,\ 7,\ 14\}$$

$$\to \quad x^2-2=0 \quad \Rightarrow \quad x=\pm\sqrt{2}$$

The solutions are: $x=4,\ \ -7,\ \ \pm\sqrt{2}$

Exercise

Find all solutions of the equation: $3x^5-10x^4-6x^3+24x^2+11x-6=0$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{6}{3}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrrrr} -1 & 3 & -10 & -6 & 24 & 11 & -6\\ & & -3 & 13 & -7 & -17 & 6\\ \hline -1 & 3 & -13 & 7 & 17 & -6 & 0\\ & & -3 & 16 & -23 & 6 & \\ \hline 2 & 3 & -16 & 23 & -6 & 0 & \\ & & 6 & 20 & 6 & & \\ \hline & 3 & -10 & 3 & 0 & & \end{array}$$

$$x^4-13x^3+7x^2+17x-6=0 \to \pm\left\{\frac{6}{3}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^3-16x^2+26x-6=0 \to \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^2-10x+3=0$$

$$3x^2-10x+3=0$$

$$x=\frac{10\pm\sqrt{100-36}}{6}$$

$$=\begin{cases} \frac{10-8}{6}=\frac{1}{3} \\ \frac{10+8}{6}=3 \end{cases}$$

The solutions are: $x=-1,\ \ -1,\ \ \frac{1}{3},\ 2,\ 3$

Exercise

Find all solutions of the equation: $6x^5+19x^4+x^3-6x^2=0$

Solution

$$x^2\left(6x^3+19x^2+x-6\right)=0 \quad \to \quad \boxed{x=0,\ \ 0}$$

$$6x^3+19x^2+x-6=0$$

possibilities for $\frac{c}{d}: \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrr} -3 & 6 & 19 & 1 & -6 \\ & & -18 & -3 & 6 \\ \hline & 6 & 1 & -2 & \boxed{0} \end{array} \quad 6x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1+48}}{12}$$

$$= \begin{cases} \frac{-1-7}{12} = -\frac{2}{3} \\ \frac{-1+7}{12} = \frac{1}{2} \end{cases}$$

The solutions are: $x = 0,\ 0,\ -\frac{2}{3},\ -3,\ \frac{1}{2}$

Exercise

Find all solutions of the equation: $x^4 - x^3 - 9x^2 + 3x + 18 = 0$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{18}{1}\right\} = \pm\{1,\ 2,\ 3,\ 6,\ 9,\ 18\}$

$$\begin{array}{r|rrrrr} -2 & 1 & -1 & -9 & 3 & 18 \\ & & -2 & 6 & 6 & -18 \\ \hline 3 & 1 & -3 & -3 & 9 & 0 \\ & & 3 & 0 & -9 & \\ \hline & 1 & 0 & -3 & 0 & \end{array} \rightarrow x^3 - 3x^2 - 3x + 9 = 0 \rightarrow \pm\left\{\frac{9}{1}\right\} = \pm\{1,\ 3,\ 9\}$$

$$\rightarrow x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$$

The solutions are: $x = -2,\ 3,\ \pm\sqrt{3}$

Exercise

Find all solutions of the equation: $2x^4 - 9x^3 + 9x^2 + x - 3 = 0$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & -9 & 9 & 1 & -3 \\
 & & 2 & -7 & 2 & 3 \\ \hline
1 & 2 & -7 & 2 & 3 & 0 \\
 & & 2 & -5 & -3 & \\ \hline
 & 2 & -5 & -3 & 0 &
\end{array}$$

$$\rightarrow\ 2x^3 - 7x^2 + 2x + 3 = 0 \ \rightarrow\ \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\rightarrow\ 2x^2 - 5x - 3 = 0$$

$$x = \frac{5 \pm \sqrt{25+24}}{4}$$

$$= \begin{cases} \dfrac{5-7}{4} = -\dfrac{1}{2} \\ \dfrac{5+7}{4} = 3 \end{cases}$$

The solutions are: $\underline{x = 1,\ 1,\ -\frac{1}{2},\ 3}$

Exercise

Find all solutions of the equation: $8x^3 + 18x^2 + 45x + 27 = 0$

Solution

possibilities : $\pm\left\{\frac{27}{8}\right\} = \pm\left\{\frac{1,\ 3,\ 9,\ 27}{1,\ 2,\ 4,\ 8}\right\}$

$$= \pm\left\{1,\ 3,\ 9,\ 27,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{3}{2},\ \frac{3}{4},\ \frac{3}{8},\ \frac{9}{2},\ \frac{9}{4},\ \frac{9}{8},\ \frac{27}{2},\ \frac{27}{4},\ \frac{27}{8}\right\}$$

$$\begin{array}{r|rrrr}
-\frac{3}{4} & 8 & 18 & 45 & 27 \\
 & & -6 & -9 & -27 \\ \hline
 & 8 & 12 & 36 & \boxed{0}
\end{array} \ \rightarrow\ 8x^2 + 12x + 36 = 0$$

$$2x^2 + 3x + 9 = 0$$

$$x = \frac{-3 \pm \sqrt{9-72}}{4}$$

$$= \frac{-3 \pm \sqrt{-63}}{4}$$

$$= \frac{-3 \pm 3i\sqrt{7}}{4}$$

The solutions are: $\underline{x = -\frac{3}{4},\ -\frac{3}{4} \pm i\frac{3\sqrt{7}}{4}}$

Exercise

Find all solutions of the equation: $3x^3 - x^2 + 11x - 20 = 0$

Solution

$$\textit{possibilities}: \ \pm\left\{\frac{20}{3}\right\} = \pm\left\{\frac{1,\ 2,\ 4,\ 5,\ 10,\ 20}{1,\ 3}\right\}$$

$$= \pm\left\{1,\ 2,\ 4,\ 5,\ 10,\ 20,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{5}{3},\ \frac{10}{3},\ \frac{20}{3}\right\}$$

A result will show that one solution is: $\boxed{x = \frac{4}{3}}$

$$\begin{array}{r|rrrr} \frac{4}{3} & 3 & -1 & 11 & -20 \\ & & 4 & 4 & 20 \\ \hline & 3 & 3 & 15 & \boxed{0} \end{array} \quad \to \ 3x^2 + 3x + 15 = 0$$

$$x^2 + x + 5 = 0$$

$$x = \frac{-1 \pm \sqrt{1-20}}{2}$$

The solutions are: $\boxed{x = \frac{4}{3},\ \ -\frac{1}{2} \pm i\frac{\sqrt{19}}{2}}$

Exercise

Find all solutions of the equation: $6x^4 + 5x^3 - 17x^2 - 6x = 0$

Solution

$$x\left(6x^3 + 5x^2 - 17x - 6\right) = 0 \quad \to \quad \boxed{x = 0}$$

$$\textit{possibilities}: \ \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrr} -2 & 6 & 5 & -17 & -6 \\ & & -12 & 14 & 6 \\ \hline & 6 & -7 & -3 & \boxed{0} \end{array} \quad \to \ 6x^2 - 7x - 3 = 0$$

$$x = \frac{7 \pm \sqrt{49+72}}{12}$$

$$= \left\{\begin{array}{l} \frac{7-11}{12} = -\frac{1}{3} \\ \frac{7+11}{12} = \frac{3}{2} \end{array}\right.$$

The solutions are: $\boxed{x = 0,\ \ -2,\ \ -\frac{1}{3},\ \frac{3}{2}}$

Exercise

If $f(x) = 3x^3 - kx^2 + x - 5k$, find a number k such that the graph of f contains the point $(-1, 4)$.

Solution

$$f(-1) = 3(-1)^3 - k(-1)^2 + (-1) - 5k$$
$$4 = -3 - k - 1 - 5k$$
$$4 = -4 - 6k$$

Add 4 on both side

$$8 = -6k$$
$$k = -\frac{8}{6}$$
$$= -\frac{4}{3}$$

Exercise

If $f(x) = kx^3 + x^2 - kx + 2$, find a number k such that the graph of f contains the point $(2, 12)$.

Solution

$$f(2) = k(2)^3 + (2)^2 - k(2) + 2$$
$$12 = 8k + 4 - 2k + 2$$
$$12 = 6k + 6$$
$$6k = 6$$
$$k = 1$$

Exercise

If one zero of $f(x) = x^3 - 2x^2 - 16x + 16k$ is 2, find two other zeros.

Solution

$$f(x) = x^2(x-2) - 16(x-k) \qquad k = 2$$
$$= (x-2)\left(x^2 - 16\right)$$
$$= (x-2)(x-4)(x+4)$$

The other zeros are: 4, –4

Exercise

If one zero of $f(x) = x^3 - 3x^2 - kx + 12$ is -2, find two other zeros.

Solution

$$f(x)=x^2(x-3)-k\left(x-\frac{12}{k}\right) \qquad \frac{12}{k} \textit{ has to be equal to } 3. \Rightarrow k=4$$

$$f(x)=x^2(x-3)-4(x-3)$$

$$=(x-3)\left(x^2-4\right)$$

$$=(x-3)(x-2)(x+2)$$

The zeros of $f(x)$ are: 3, –2, 2

Exercise

Find a polynomial $f(x)$ of degree 3 that has the zeros –1, 2, 3; and satisfies the given condition: $f(-2)=80$

Solution

$$f(x)=k(x+1)(x-2)(x-3)$$

$$=k\left(x^2-x-2\right)(x-3)$$

$$=k\left(x^3-3x^2-x^2+3x-2x+6\right)$$

$$=k\left(x^3-4x^2+x+6\right)$$

$$f(-2)=k\left((-2)^3-4(-2)^2+(-2)+6\right)$$

$$80=k(-20)$$

$$k=\frac{80}{-20}=-4$$

$$f(x)=-4\left(x^3-4x^2+x+6\right)$$

$$f(x)=-4x^3+16x^2-4x-24$$

Exercise

Find a polynomial $f(x)$ of degree 3 that has the zeros $-2i$, $2i$, 3; and satisfies the given condition: $f(1)=20$

Solution

$$f(x)=k(x+2i)(x-2i)(x-3)$$

$$=k\left(x^2+4\right)(x-3)$$

$$=k\left(x^3-3x^2+4x-12\right)$$

$$f(1)=k\left((1)^3-3(1)^2+4(1)-12\right)$$

$$20=k(-10)$$

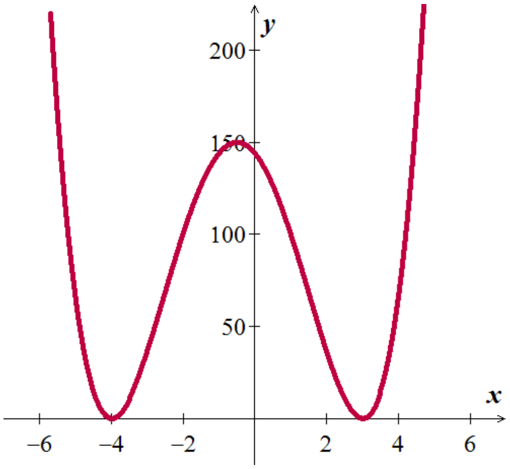
$k = -2$

$f(x) = -2\left(x^3 - 3x^2 + 4x - 12\right)$

$\underline{f(x) = -2x^3 + 6x^2 - 8x + 24}|$

Exercise

Find a polynomial $f(x)$ of degree 4 with leading coefficient 1 such that both -4 and 3 are zeros of multiplicity 2, and sketch the graph of f.



Solution

$$f(x) = a\left(x - x_1\right)\left(x - x_2\right)\left(x - x_3\right)\left(x - x_4\right)$$

$a = 1 \quad x_1 = x_2 = -4 \quad x_3 = x_4 = 3$

$$\begin{aligned} f(x) &= (x+4)(x+4)(x-3)(x-3) \\ &= \left(x^2 + 8x + 16\right)\left(x^2 - 6x + 9\right) \\ &= x^4 - 6x^3 + 9x^2 + 8x^3 - 48x^2 + 72x + 16x^2 - 96x + 144 \\ &= x^4 + 2x^3 - 23x^2 - 24x + 144 \end{aligned}$$

Exercise

Find the zeros of $f(x) = x^2(3x+2)(2x-5)^3$, and state the multiplicity of each zero.

Solution

$f(x) = x^2(3x+2)(2x-5)^3 = 0$

The zeros are: $x = 0 \quad (multiplicity\ of\ \ 2)$

$x = -\frac{2}{3}$

$x = \frac{5}{2} \quad (multiplicity\ of\ \ 3)$

Exercise

Find the zeros of $f(x) = 4x^5 + 12x^4 + 9x^3$, and state the multiplicity of each zero.

Solution

$$\begin{aligned} f(x) &= x^3\left(4x^2 + 12x + 9\right) = 0 \\ &= x^3(2x+3)^2 = 0 \end{aligned}$$

The zeros are: $x=0$ $(multiplicity\ of\ 3)$

$x=-\frac{3}{2}$ $(multiplicity\ of\ 2)$

Exercise

Find the zeros of $f(x)=\left(x^2+x-12\right)^3\left(x^2-9\right)^2$, and state the multiplicity of each zero.

Solution

$$f(x)=\left(x^2+x-12\right)^3\left(x^2-9\right)^2=0$$

$$x^2+x-12=0 \qquad x^2-9=0$$

$$x=-4,\ 3 \qquad x=\pm 3$$

The zeros are: $x=-4$ $(multiplicity\ of\ 3)$

$x=-3$ $(multiplicity\ of\ 2)$

$x=3$ $(multiplicity\ of\ 5)$

Exercise

Find the zeros of $f(x)=\left(6x^2+7x-5\right)^4\left(4x^2-1\right)^2$, and state the multiplicity of each zero.

Solution

$$f(x)=\left(6x^2+7x-5\right)^4\left(4x^2-1\right)^2=0$$

$$6x^2+7x-5=0 \qquad 4x^2-1=0\rightarrow x^2=\frac{1}{4}$$

$$x=-\frac{5}{3},\ \frac{1}{2} \qquad x=\pm\frac{1}{2}$$

The zeros are: $x=-\frac{5}{3}$ $(multiplicity\ of\ 4)$

$x=-\frac{1}{2}$ $(multiplicity\ of\ 2)$

$x=\frac{1}{2}$ $(multiplicity\ of\ 6)$

Exercise

Find the zeros of $f(x)=x^4+7x^2-144$, and state the multiplicity of each zero.

Solution

$$f(x)=x^4+7x^2-144$$

$$=\left(x^2-9\right)\left(x^2+16\right)=0$$

$$x^2 - 9 = 0 \qquad\qquad x^2 + 16 = 0$$

$$x = \pm 3 \qquad\qquad x^2 = -16 \ \ (\mathbb{C})$$

The zeros are: $x = \pm 3$

Exercise

Find the zeros of $f(x) = x^4 + 21x^2 - 100$, and state the multiplicity of each zero.

Solution

$$f(x) = x^4 + 21x^2 - 100$$

$$= \left(x^2 - 4\right)\left(x^2 + 25\right) = 0$$

$$x^2 - 4 = 0 \qquad\qquad x^2 + 25 = 0$$

$$x = \pm 2 \qquad\qquad x^2 = -25 \ \ (\mathbb{C})$$

The zeros are: $x = \pm 2$

Exercise

Let $f(x) = x^4 - 4x^2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$f(x) = x^2\left(x^2 - 4\right)$$

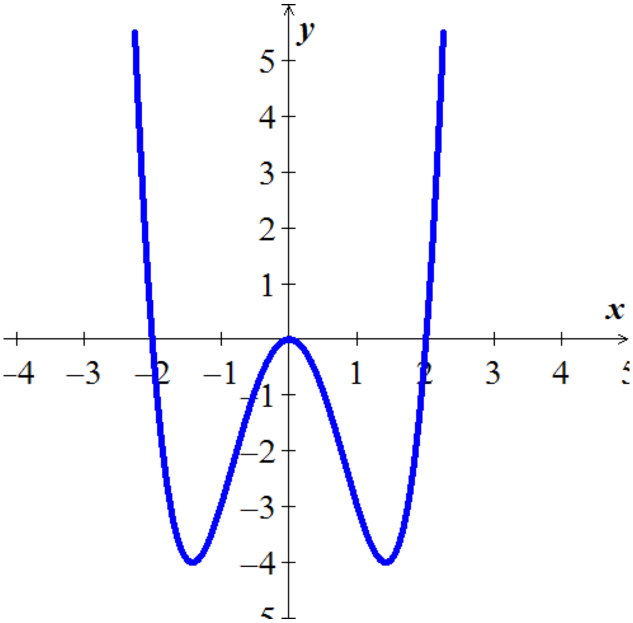
$$= x^2(x-2)(x+2)$$

The zeros are: 0, 0, 2, −2.

$-\infty$ $\quad -2$	**0,0**	2 $\quad \infty$
+	−	+

$f(x) < 0 \quad (-2,\ 0) \cup (0,\ 2)$

$f(x) > 0 \quad (-\infty,\ -2) \cup (2,\ \infty)$



Exercise

Let $f(x) = x^4 + 3x^3 - 4x^2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

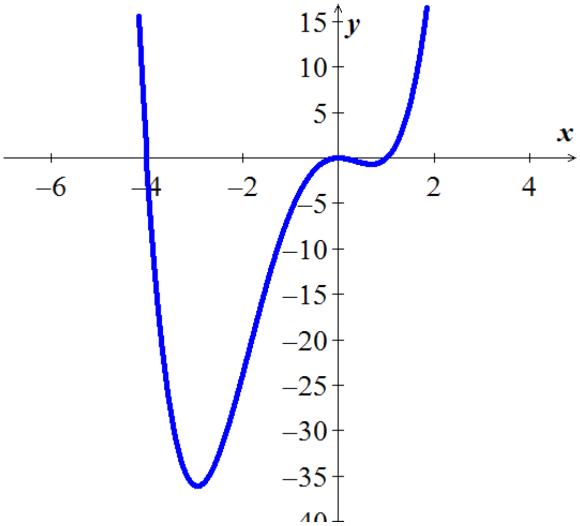
$$f(x) = x^2\left(x^2 + 3x - 4\right)$$

The zeros are: 0, 0, 1, −4.

$-\infty$	-4	**$0,0$**	1	∞
+		−		+

$f(x) > 0$ $\boxed{(-\infty,\ -4) \cup (1,\ \infty)}$

$f(x) < 0$ $\boxed{(-4,\ 0) \cup (0,\ 1)}$



Exercise

Let $f(x) = x^3 + 2x^2 - 4x - 8$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

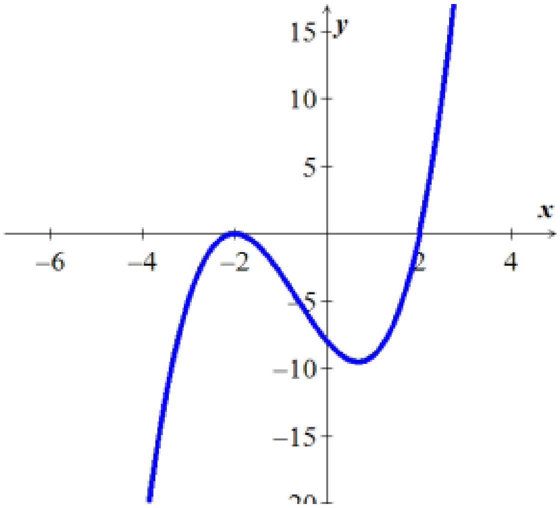
$$\begin{aligned} f(x) &= x^2(x+2) - 4(x+2) \\ &= (x+2)\left(x^2 - 4\right) \\ &= (x+2)(x+2)(x-2) = 0 \end{aligned}$$

The zeros are: 2, −2, −2

$-\infty$	-2	0	2	∞
−		−		+

$f(x) > 0$ $\boxed{(2,\ \infty)}$

$f(x) < 0$ $\boxed{(-\infty,\ -2) \cup (-2,\ 2)}$



Exercise

Let $f(x) = x^3 - 3x^2 - 9x + 27$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

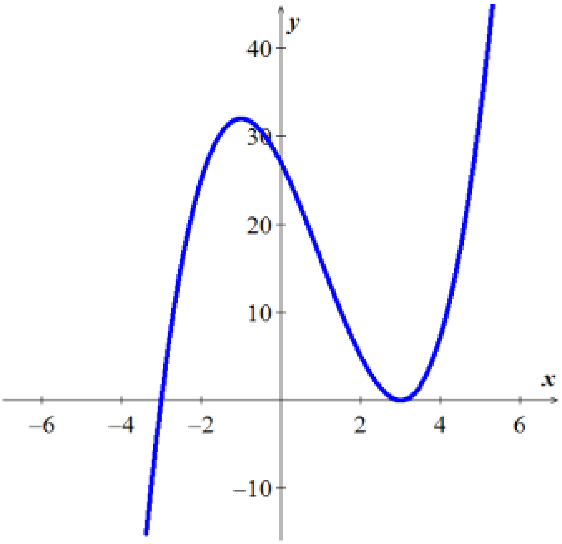
Solution

$$
\begin{aligned}
f(x) &= x^2(x-3) - 9(x-3) \\
&= (x-3)\left(x^2-9\right) \\
&= (x-3)(x-3)(x+3)
\end{aligned}
$$

The zeros are: −3, 3 (multiplicity)

$-\infty \qquad -3 \qquad 0 \qquad 3 \qquad \infty$

−	+	+



$f(x) > 0 \quad (-3,\ 3)\cup(3,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -3)$

Exercise

Let $f(x) = -x^4 + 12x^2 - 27$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

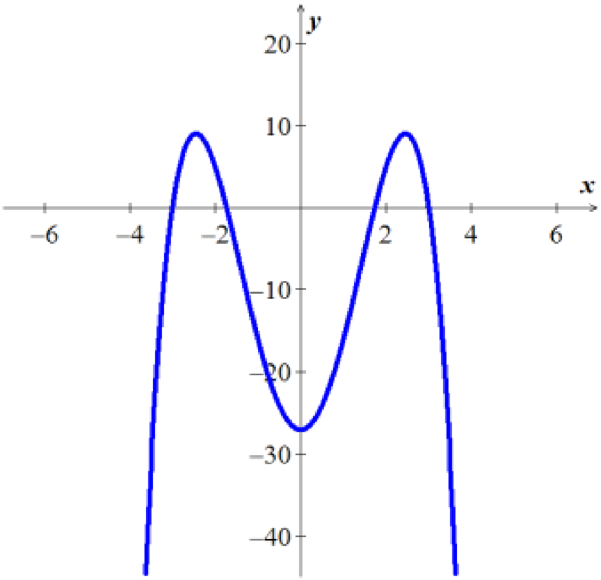
$$
x^2 = \frac{-12 \pm \sqrt{36}}{-2}
$$

$$
= \begin{cases} \dfrac{-12-6}{-2} = 9 \\[2ex] \dfrac{-12+6}{-2} = 3 \end{cases}
$$

$$
\to \begin{cases} x^2 = 9 \\ x^2 = 3 \end{cases} \quad \Rightarrow \quad \begin{matrix} x = \pm 3 \\ x = \pm\sqrt{3} \end{matrix}
$$

$-3 \qquad -\sqrt{3} \qquad \sqrt{3} \qquad 3$

−	+	−	+	−



$f(x) > 0 \quad \left(-3,\ -\sqrt{3}\right)\cup\left(\sqrt{3},\ 3\right)$

$f(x) < 0 \quad (-\infty,\ -3)\cup\left(-\sqrt{3},\ \sqrt{3}\right)\cup(3,\ \infty)$

Exercise

Let $f(x)=x^2(x+2)(x-1)^2(x-2)$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

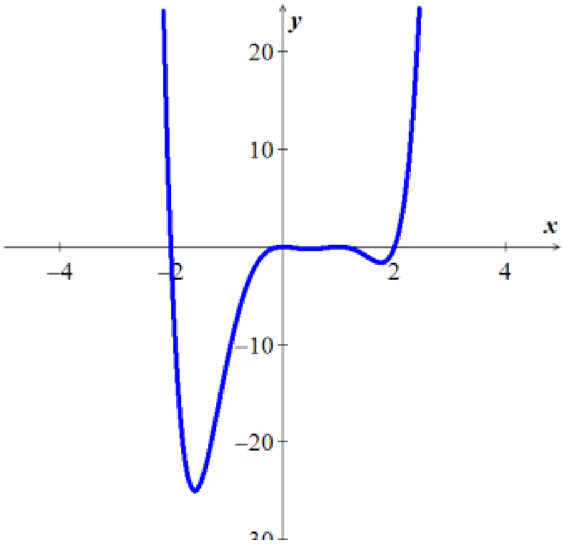
The zeros are: −2, 2, 0, 0, 1, 1

$-2 \qquad \mathbf{0,0} \quad \mathbf{1,1} \qquad 2$

+	−	+

$f(x)>0 \quad \boxed{(-\infty,\ -2)\cup(2,\ \infty)}$

$f(x)<0 \quad \boxed{(-2,\ 0)\cup(0,\ 1)\cup(1,\ 2)}$



Exercise

Let $f(x)=2x^3+11x^2-7x-6$. Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of f.

Solution

$$possibilities: \pm\left\{\frac{6}{2}\right\}=\pm\left\{\frac{1,\ 2,\ 3,\ 6}{1,\ 2}\right\}$$

$$=\pm\left\{1,\ 2,\ 3,\ 5,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\begin{array}{r|rrrr} 1 & 2 & 11 & -7 & -6 \\ & & 2 & 13 & 6 \\ \hline & 2 & 13 & 6 & \boxed{0} \end{array} \quad \rightarrow \ 2x^2+13x+6=0$$

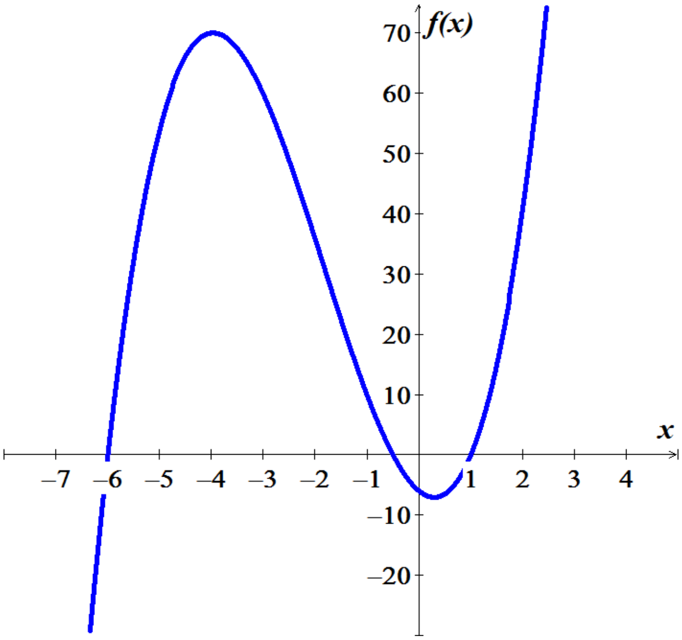
The zeros are: $\boxed{x=1,\ -\frac{1}{2},\ -6}$

$-6 \qquad -\frac{1}{2} \qquad 1$

−	+	−	+

$f(x)>0 \quad \boxed{\left(-6,\ -\frac{1}{2}\right)\cup(1,\ \infty)}$

$f(x)<0 \quad \boxed{(-\infty,\ -6)\cup\left(-\frac{1}{2},\ 1\right)}$



Exercise

Let $f(x) = x^3 + 2x^2 - 5x - 6$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\textit{possibilities}\ : \pm\left\{\frac{6}{1}\right\} = \pm\{1,\ 2,\ 3,\ 6\}$$

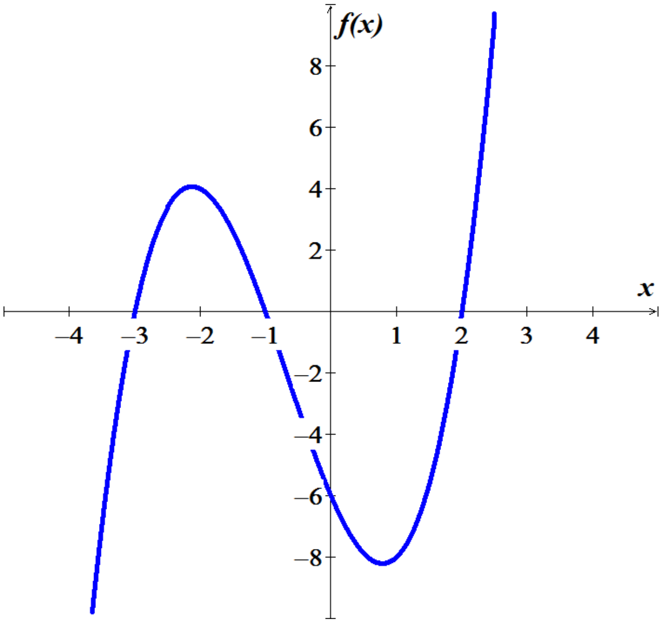
$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array} \quad \to\ x^2 + x - 6 = 0$$

The zeros are: $x = -1,\ -3,\ 2$

$$\begin{array}{cccccccc} & & -3 & & -1 & & 2 & \\ & - & | & + & | & - & | & + \end{array}$$

$f(x) > 0$ $\quad (-3,\ -1) \cup (2,\ \infty)$

$f(x) < 0$ $\quad (-\infty,\ -3) \cup (-1,\ 2)$



Exercise

Let $f(x) = x^3 + 8x^2 + 11x - 20$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\textit{possibilities}\ : \pm\left\{\frac{20}{1}\right\} = \pm\{1,\ 2,\ 4,\ 5,\ 20,\ 20\}$$

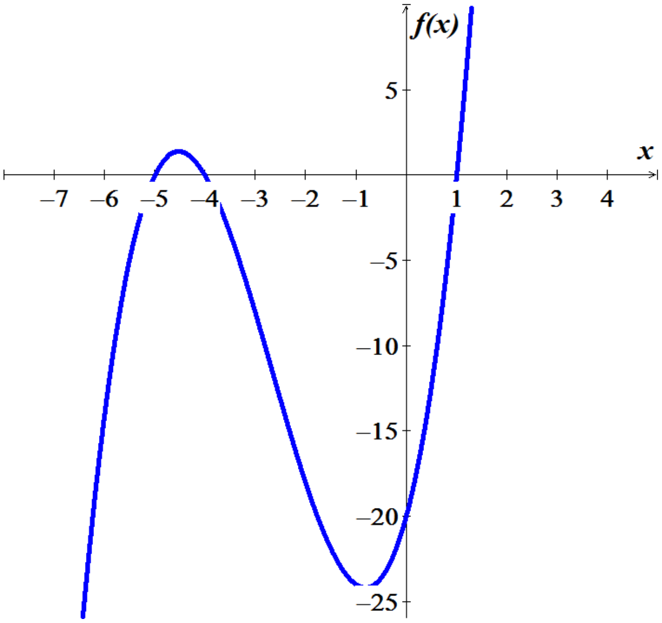
$$\begin{array}{r|rrrr} 1 & 1 & 8 & 11 & -20 \\ & & 1 & 9 & 20 \\ \hline & 1 & 9 & 20 & \boxed{0} \end{array} \quad \to\ x^2 + 9x + 20 = 0$$

The zeros are: $x = -5,\ -4,\ 1$

$$\begin{array}{cccccccc} & & -5 & & -4 & & 1 & \\ & - & | & + & | & - & | & + \end{array}$$

$f(x) > 0$ $\quad (-5,\ -1) \cup (1,\ \infty)$

$f(x) < 0$ $\quad (-\infty,\ -5) \cup (-4,\ 1)$



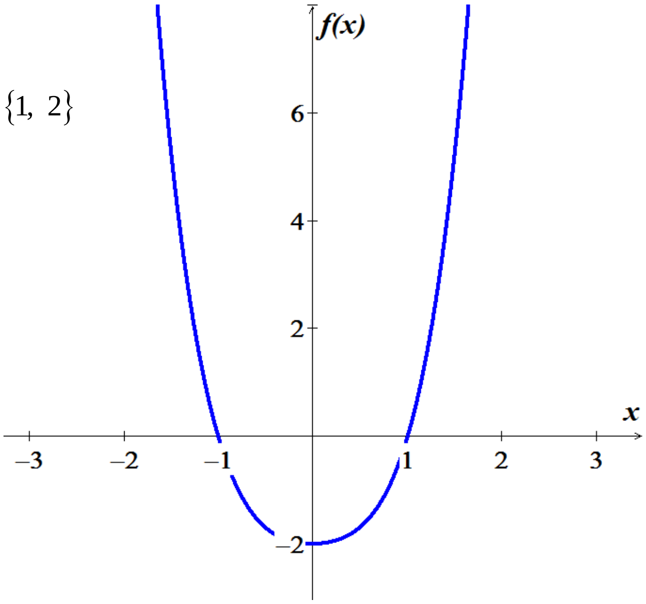
Exercise

Let $f(x) = x^4 + x^2 - 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

possibilities : $\pm\{1,\ 2\}$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & 1 & 0 & -2 \\ & & 1 & 1 & 2 & 1 \\ \hline -1 & 1 & 1 & 2 & 2 & 0 \\ & & -1 & 0 & -2 & \\ \hline & 1 & 0 & 2 & 0 & \end{array} \rightarrow x^3 + x^2 + 2x + 1 = 0 \rightarrow \pm\{1,\ 2\}$$



$\rightarrow x^2 + 2 = 0 \quad \Rightarrow \quad x = \pm i\sqrt{2}$

The zeros are: $x = \pm 1$

$$\begin{array}{|c|c|c|} \multicolumn{1}{c}{} & \multicolumn{1}{c}{-1} & \multicolumn{1}{c}{1} \\ \hline + & - & + \end{array}$$

$f(x) > 0 \quad (-\infty,\ -1) \cup (1,\ \infty)$

$f(x) < 0 \quad (-1,\ 1)$

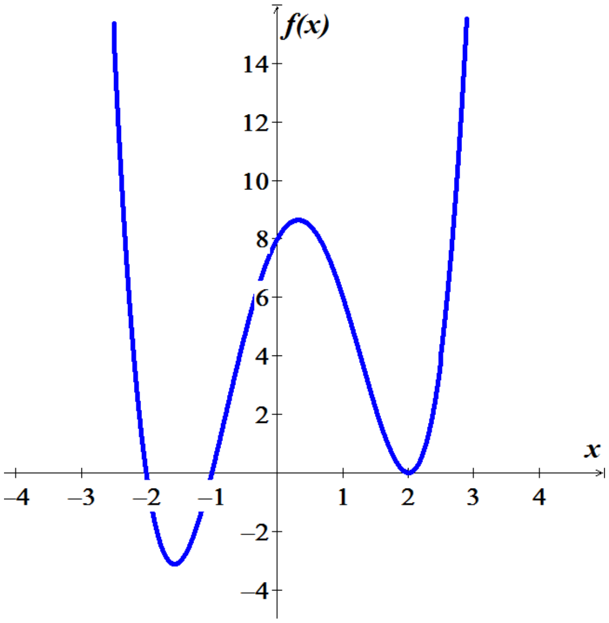
Exercise

Let $f(x) = x^4 - x^3 - 6x^2 + 4x + 8$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

possibilities : $\pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -6 & 4 & 8 \\ & & -1 & 2 & 4 & -8 \\ \hline -2 & 1 & -2 & -4 & 8 & 0 \\ & & -2 & 8 & -8 & \\ \hline & 1 & -4 & 4 & 0 & \end{array} \rightarrow x^3 - 2x^2 - 4x + 8 = 0 \rightarrow \pm\{1,\ 2,\ 4,\ 8\}$$



$\rightarrow x^2 - 4x + 4 = 0 \quad \Rightarrow \quad x = 2,\ 2$

The zeros are: $x = -2,\ -1,\ 2,\ 2$

$$\begin{array}{|c|c|c||c|} \multicolumn{1}{c}{} & \multicolumn{1}{c}{-2} & \multicolumn{1}{c}{-1} & \multicolumn{1}{c}{2} \\ \hline + & - & + & + \end{array}$$

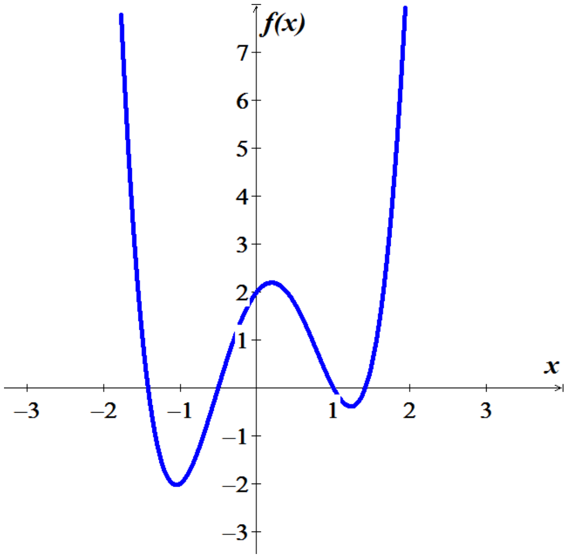
$f(x) > 0 \quad \boxed{(-\infty,\ -1)\cup(1,\ \infty)}$

$f(x) < 0 \quad \boxed{(-1,\ 1)}$

Exercise

Let $f(x) = 2x^4 - x^3 - 5x^2 + 2x + 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution



$$possibilities: \pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$$

$$\begin{array}{c|ccccc}
1 & 2 & -1 & -5 & 2 & 2 \\
 & & 2 & 1 & -4 & -2 \\ \hline
-\frac{1}{2} & 2 & 1 & -4 & -2 & 0 \\
 & & -1 & 0 & 2 & \\ \hline
 & 2 & 0 & -4 & 0 &
\end{array}
\quad \begin{array}{l} \to 2x^3+x^2-4x-2=0 \\ \to \pm\left\{1,\ 2,\ \frac{1}{2}\right\} \end{array}$$

$$\to 2x^2 - 4 = 0 \Rightarrow \boxed{x = \pm\sqrt{2}}$$

The zeros are: $\boxed{x = -\frac{1}{2},\ 1,\ -\sqrt{2},\ \sqrt{2}}$

	$-\sqrt{2}$		$-\frac{1}{2}$		1		$\sqrt{2}$	
+		−		+		−		+

$f(x) > 0 \quad \boxed{(-\infty,\ -\sqrt{2})\cup\left(-\frac{1}{2},\ 1\right)\cup\left(\sqrt{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-\sqrt{2},\ -\frac{1}{2}\right)\cup\left(1,\ \sqrt{2}\right)}$

Exercise

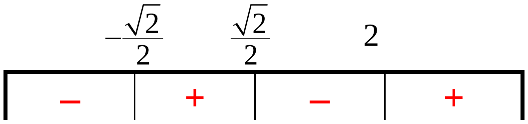
Let $f(x) = 4x^5 - 8x^4 - x + 2$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$$\begin{aligned} f(x) &= 4x^4(x-2) - (x-2) \\ &= (x-2)\left(4x^4 - 1\right) = 0 \end{aligned}$$

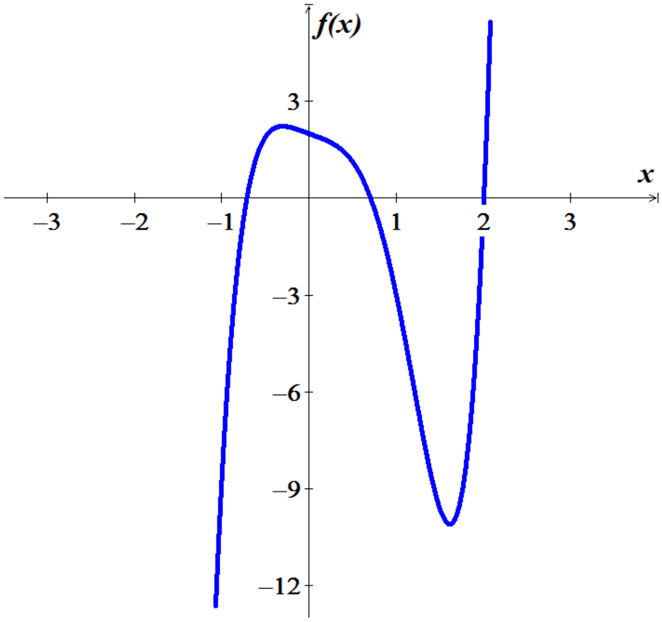
$$4x^4 - 1 = 0 \quad \Rightarrow \quad \begin{cases} x^2 = -\frac{1}{2} & \mathbb{C} \\ x^2 = \frac{1}{2} & x = \pm\frac{\sqrt{2}}{2} \end{cases}$$

The zeros are: $\boxed{x = 2,\ -\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}}$



$f(x) > 0 \quad \boxed{\left(-\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right) \cup (2,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-\infty,\ -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2},\ 2\right)}$



Exercise

Let $f(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of f.

Solution

$possibilities: \ \pm\left\{\frac{36}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 9,\ 12,\ 18,\ 36\}$

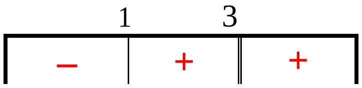
$$\begin{array}{r|rrrrrr}
1 & 1 & -7 & 19 & -37 & 60 & -36 \\
 & & 1 & -6 & 13 & -24 & 36 \\
\hline
3 & 1 & -6 & 13 & -24 & 36 & 0 \\
 & & 3 & -9 & 12 & -36 & \\
\hline
3 & 1 & -3 & 4 & -12 & 0 & \\
 & & 3 & 0 & 12 & & \\
\hline
 & 1 & 0 & 4 & 0 & &
\end{array}$$

$x^4 - 6x^3 + 13x^2 - 24x + 36 = 0 \ \rightarrow \ \pm\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$x^3 - 3x^2 + 4x - 12 = 0 \ \rightarrow \ \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

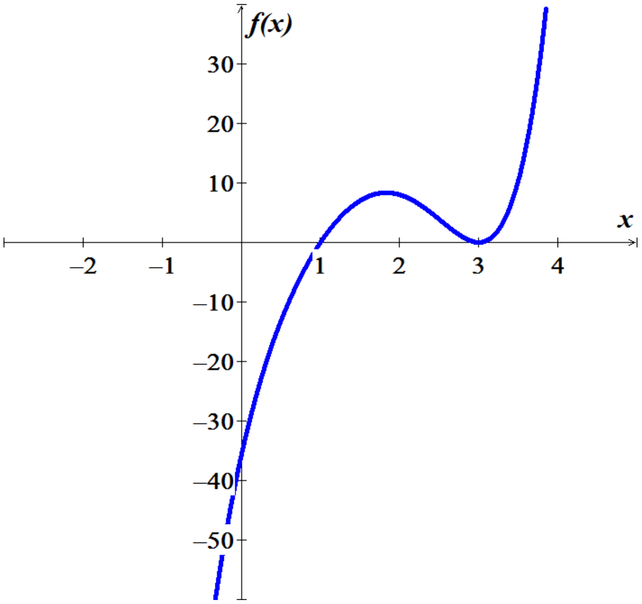
$x^2 + 4 = 0 \ \Rightarrow \ x = \pm 2i$

The zeros are: $\boxed{x = 1,\ 3,\ 3}$



$f(x) > 0 \quad \boxed{(1,\ 3) \cup (3,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ 1)}$



Exercise

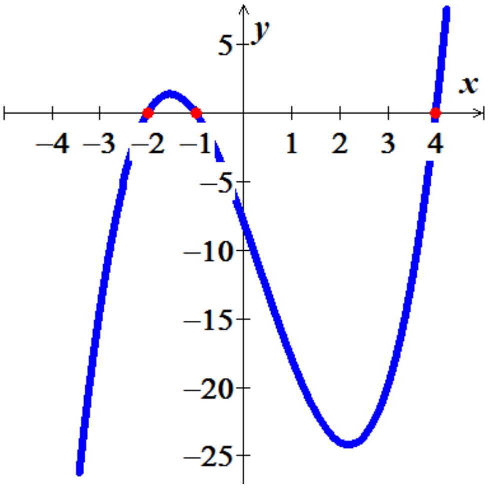
Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - x^2 - 10x - 8$$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & 0 \end{array} \quad \to \quad x^2 - 2x - 8 = 0$$



$\boxed{x = -1,\ -2,\ 4}$

	−2		−1		4	
−		+		−		+

$f(x) > 0$ $\quad \boxed{(-2,\ -1) \cup (4,\ \infty)}$

$f(x) < 0$ $\quad \boxed{(-\infty,\ -2) \cup (-1,\ 4)}$

Exercise

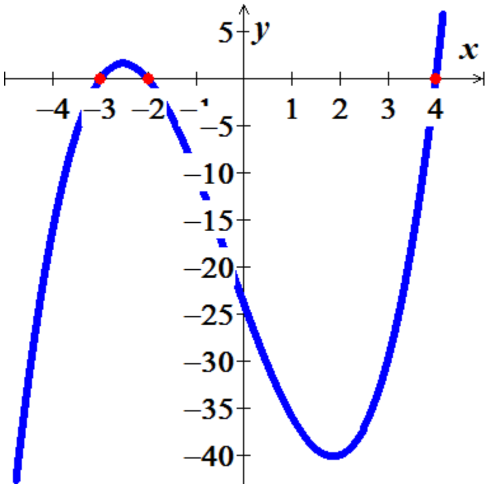
Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + x^2 - 14x - 24$$

Solution

possibilities for $\frac{c}{d}: \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & 0 \end{array} \quad \to \quad x^2 - x - 12 = 0$$



$\boxed{x = -2,\ -3,\ 4}$

	−3		−2		4	
−		+		−		+

$f(x) > 0$ $\quad \boxed{(-3,\ -2) \cup (4,\ \infty)}$

$f(x) < 0$ $\quad \boxed{(-\infty,\ -3) \cup (-2,\ 4)}$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 - 3x^2 - 17x + 30$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{30}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30,\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{15}{2}\right\}$

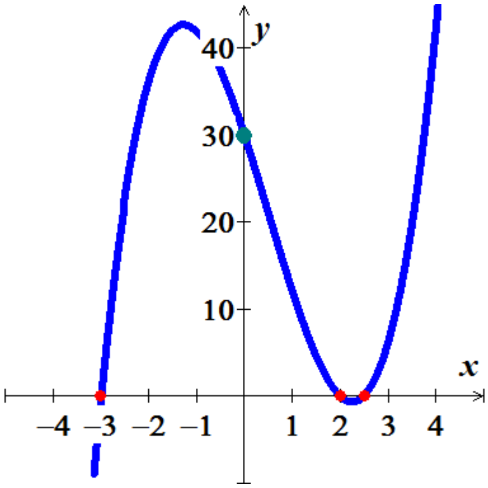
$$\begin{array}{c|cccc} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & \boxed{0} \end{array} \rightarrow \quad 2x^2 + x - 15 = 0$$

$x = 2,\ -3,\ \frac{5}{2}$

$$\begin{array}{ccccccc} & -3 & & 2 & & \frac{5}{2} & \\ - & | & + & | & - & | & + \end{array}$$

$f(x) > 0 \quad (-3,\ 2) \cup \left(\frac{5}{2},\ \infty\right)$

$f(x) < 0 \quad (-\infty,\ -3) \cup \left(2,\ \frac{5}{2}\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 12x^3 + 8x^2 - 3x - 2$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{12}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ \frac{1}{6},\ \frac{1}{12}\right\}$

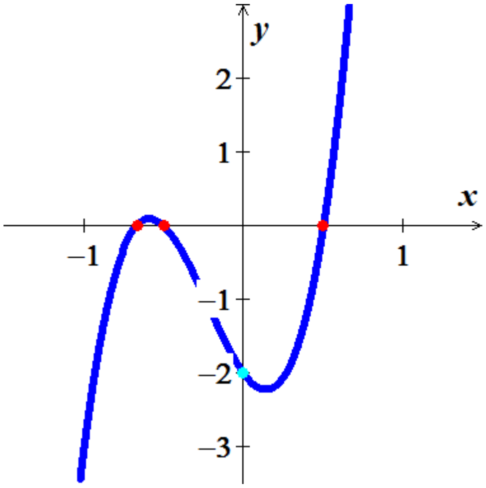
$$\begin{array}{c|cccc} \frac{1}{2} & 12 & 8 & -3 & -2 \\ & & 6 & 7 & 2 \\ \hline & 12 & 14 & 4 & \boxed{0} \end{array} \rightarrow \quad 12x^2 + 14x + 4 = 0$$

$x = \frac{1}{2},\ -\frac{1}{2},\ -\frac{2}{3}$

$$\begin{array}{ccccccc} & -\frac{2}{3} & & -\frac{1}{2} & & \frac{1}{2} & \\ - & | & + & | & - & | & + \end{array}$$

$f(x) > 0 \quad \left(-\frac{2}{3},\ -\frac{1}{2}\right) \cup \left(\frac{1}{2},\ \infty\right)$

$f(x) < 0 \quad \left(-\infty,\ -\frac{2}{3}\right) \cup \left(-\frac{1}{2},\ \frac{1}{2}\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + x^2 - 6x - 8$$

Solution

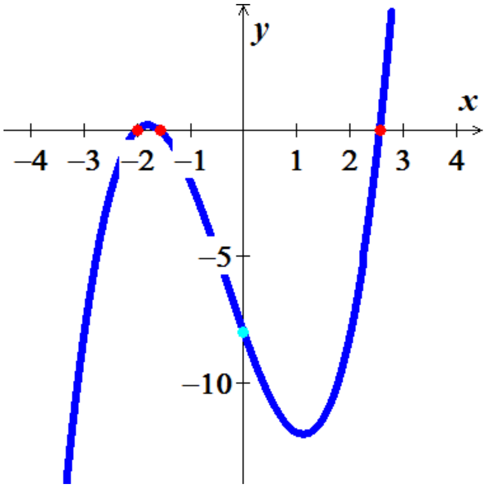
possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1,\ 2,\ 4,\ 8\}$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -6 & -8 \\ & & -2 & 2 & 8 \\ \hline & 1 & -1 & -4 & \boxed{0} \end{array} \to \quad x^2 - x - 4 = 0$$

$$x = \frac{1 \pm \sqrt{1+16}}{2}$$

$$x = -2,\ \frac{1 \pm \sqrt{17}}{2}$$

$$\begin{array}{c|c|c|c|c} & -2 & \frac{1-\sqrt{17}}{2} & \frac{1+\sqrt{17}}{2} & \\ \hline - & + & - & + & \end{array}$$



$$f(x) > 0 \quad \left(-2,\ \frac{1-\sqrt{17}}{2}\right) \cup \left(\frac{1+\sqrt{17}}{2},\ \infty\right)$$

$$f(x) < 0 \quad (-\infty,\ -2) \cup \left(\frac{1-\sqrt{17}}{2},\ \frac{1+\sqrt{17}}{2}\right)$$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 19x - 30$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{30}{1}\right\} = \pm\{1,\ 2,\ 3,\ 5,\ 6,\ 15,\ 30\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & \boxed{0} \end{array} \to \quad x^2 - 2x - 15$$

$$x = \frac{2 \pm \sqrt{4+60}}{2}$$

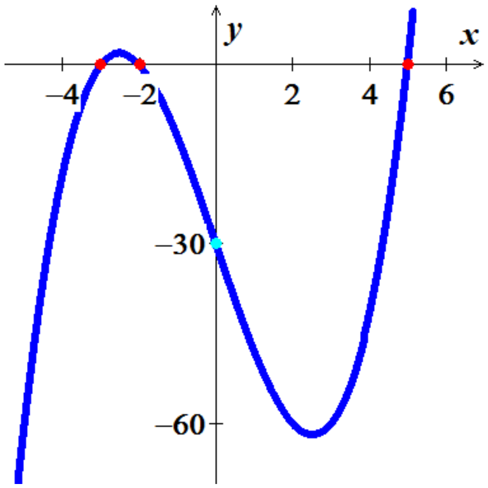
$$=\begin{cases}\dfrac{2-8}{2}=-3\\[2mm] \dfrac{2+8}{2}=5\end{cases}$$

$$\boxed{x=-2,\ -3,\ 5}$$

	−3		−2		5	
−		+		−		+

$f(x)>0 \quad \boxed{(-3,\ -2)\cup(5,\ \infty)}$

$f(x)<0 \quad \boxed{(-\infty,\ -3)\cup(-2,\ 5)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=2x^3+x^2-25x+12$$

Solution

$$possibilities\ for\ \frac{c}{d}:\pm\left\{\frac{12}{2}\right\}=\pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 12,\ \frac{1}{2},\ \frac{3}{2}\right\}$$

$$\begin{array}{r|rrrr} 3 & 2 & 1 & -25 & 12\\ & & 6 & 21 & -12\\ \hline & 2 & 7 & -4 & \boxed{0}\end{array} \quad \to \quad 2x^2+7x-4$$

$$x=\frac{-7\pm\sqrt{49+32}}{4}$$

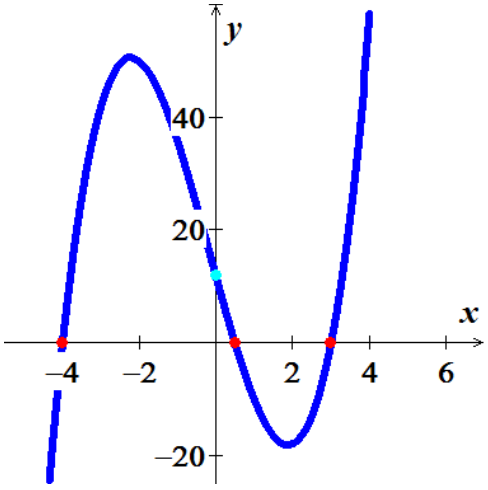
$$=\begin{cases}\dfrac{-7-9}{4}=-4\\[2mm] \dfrac{-7+9}{4}=\dfrac{1}{2}\end{cases}$$

$$\boxed{x=-4,\ \frac{1}{2},\ 3}$$

	−4		$\frac{1}{2}$		3	
−		+		−		+

$f(x)>0 \quad \boxed{\left(-4,\ \frac{1}{2}\right)\cup(3,\ \infty)}$

$f(x)<0 \quad \boxed{(-\infty,\ -1)\cup\left(\frac{1}{2},\ 3\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^3 + 11x^2 - 6x - 8$$

Solution

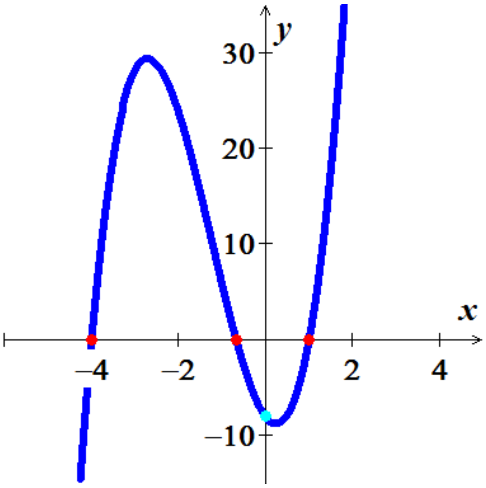
possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ 8,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{8}{3}\right\}$

$$\begin{array}{r|rrrr} 1 & 3 & 11 & -6 & -8 \\ & & 3 & 14 & 8 \\ \hline & 3 & 14 & 8 & \boxed{0} \end{array} \quad \rightarrow \quad 3x^2 + 14x + 8$$

$$x = \frac{-14 \pm \sqrt{196 - 96}}{6}$$

$$= \begin{cases} \frac{-14 - 10}{6} = -4 \\ \frac{-14 + 10}{6} = -\frac{2}{3} \end{cases}$$

$$x = -4,\ -\frac{2}{3},\ 1$$



	-4		$-\frac{2}{3}$		1	
–		+		–		+

$f(x) > 0 \quad \left(-4,\ -\frac{2}{3}\right) \cup (1,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -4) \cup \left(-\frac{2}{3},\ 1\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^3 + 9x^2 - 2x - 9$$

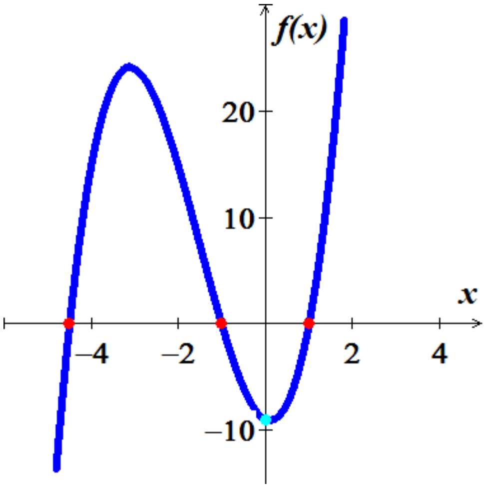
Solution

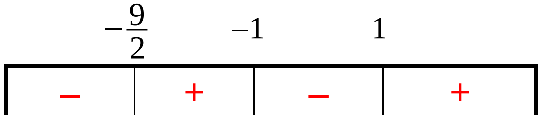
possibilities for $\frac{c}{d} : \pm\left\{\frac{9}{2}\right\} = \pm\left\{1,\ 3,\ 9,\ \frac{1}{2},\ \frac{3}{2},\ \frac{9}{2}\right\}$

$$\begin{array}{r|rrrr} 1 & 2 & 9 & -2 & -9 \\ & & 2 & 11 & 9 \\ \hline & 2 & 11 & 9 & \boxed{0} \end{array} \quad \rightarrow \quad 2x^2 + 11x + 9$$

$x = -1, \ -\frac{9}{2}$ $\qquad a - b + c = 0 \ \rightarrow \ x = -1, \ -\frac{c}{a}$

$x = -\frac{9}{2}, \ -1, \ 1$





$f(x) > 0 \quad \left(-\frac{9}{2}, \ -1\right) \cup (1, \ \infty)$

$f(x) < 0 \quad \left(-\infty, \ -\frac{9}{2}\right) \cup (-1, \ 1)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 + 3x^2 - 6x - 8$$

Solution

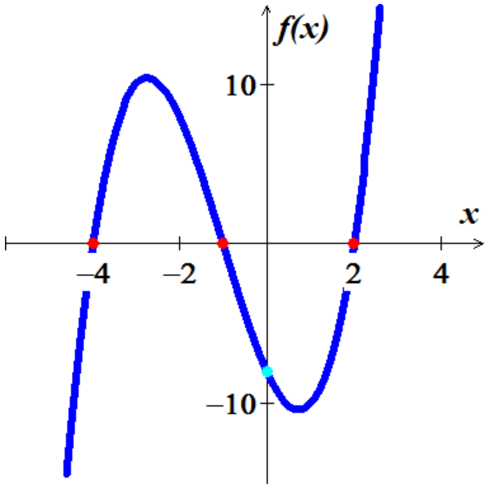
possibilities for $\frac{c}{d} : \pm\left\{\frac{8}{1}\right\} = \pm\{1, \ 2, \ 4, \ 8\}$

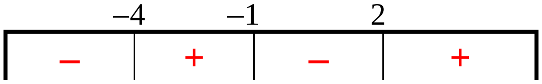
$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ & & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & \boxed{0} \end{array} \rightarrow \quad x^2 + 2x - 8 = 0$$

$$x = \frac{-2 \pm \sqrt{4+32}}{2}$$

$$= \begin{cases} \frac{-2-6}{2} = -4 \\ \frac{-2+6}{2} = 2 \end{cases}$$

$x = -4, \ -1, \ 2$





$f(x) > 0 \quad (-4, \ -1) \cup (2, \ \infty)$

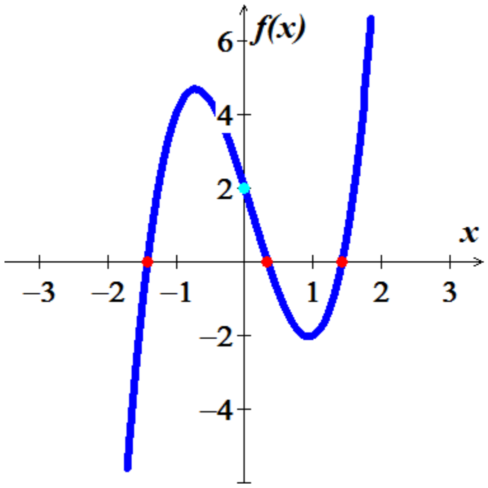
$f(x) < 0 \quad (-\infty, \ -4) \cup (-1, \ 2)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^3 - x^2 - 6x + 2$$

Solution



possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{3}\right\} = \pm\left\{1,\ 2,\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -6 & 2 \\ & & 1 & 0 & -2 \\ \hline & 3 & 0 & -6 & \boxed{0} \end{array} \quad \rightarrow \quad 3x^2 - 6 = 0$$

$x = \frac{1}{3},\ \pm\sqrt{2}$

	$-\sqrt{2}$		$\frac{1}{3}$		$\sqrt{2}$	
–		+		–		+

$f(x) > 0 \quad \left(-\sqrt{2},\ \frac{1}{3}\right) \cup \left(\sqrt{2},\ \infty\right)$

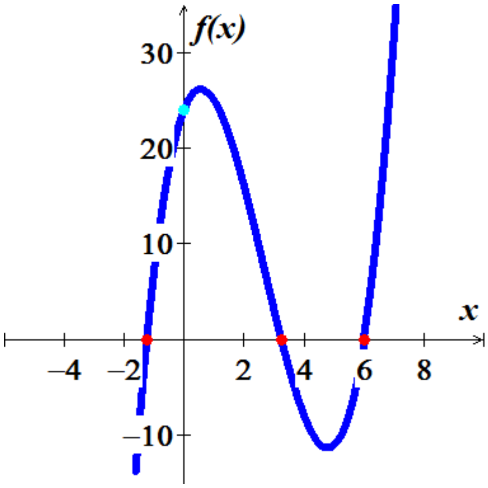
$f(x) < 0 \quad \left(-\infty,\ -\sqrt{2}\right) \cup \left(\frac{1}{3},\ \sqrt{2}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 8x^2 + 8x + 24$$

Solution



possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24\}$

$$\begin{array}{r|rrrr} 6 & 1 & -8 & 8 & 24 \\ & & 6 & -12 & -24 \\ \hline & 1 & -2 & -4 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - 2x - 4 = 0$$

$x = \dfrac{2 \pm \sqrt{4+16}}{2}$

$x = 6,\ 1 \pm \sqrt{5}$

	$1-\sqrt{5}$		$1+\sqrt{5}$		6	
–		+		–		+

$$f(x) > 0 \quad \left(1-\sqrt{5},\ 1+\sqrt{5}\right) \cup (6,\ \infty)$$

$$f(x) < 0 \quad \left(-\infty,\ 1-\sqrt{5}\right) \cup \left(1+\sqrt{5},\ 6\right)$$

Exercise

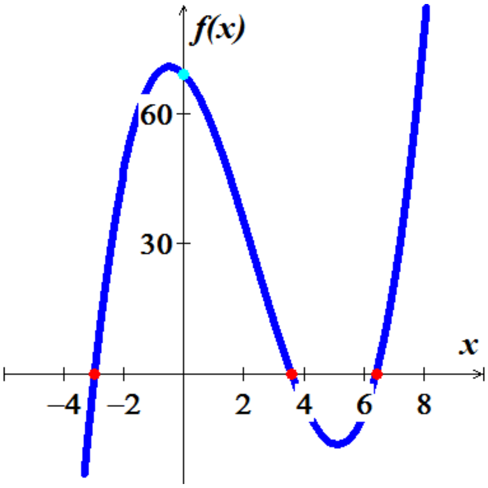
Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 7x^2 - 7x + 69$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{69}{1}\right\} = \pm\{1,\ 3,\ 23,\ 69\}$

$$\begin{array}{r|rrrr} -3 & 1 & -7 & -7 & 69 \\ & & -3 & 30 & -69 \\ \hline & 1 & -10 & 23 & \boxed{0} \end{array} \quad \to \quad x^2 - 10x + 23 = 0$$



$$x = \frac{10 \pm \sqrt{100-92}}{2}$$

$$= \frac{10 \pm 2\sqrt{2}}{2}$$

$$x = -3,\ 5 \pm \sqrt{2}$$

$$\begin{array}{c|c|c|c} & -3 \quad & 5-\sqrt{2} \quad & 5+\sqrt{2} \\ - & + & - & + \end{array}$$

$$f(x) > 0 \quad \left(-3,\ 5-\sqrt{2}\right) \cup \left(5+\sqrt{2},\ \infty\right)$$

$$f(x) < 0 \quad (-\infty,\ -3) \cup \left(5-\sqrt{2},\ 5+\sqrt{2}\right)$$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 3x - 2$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -3 & -2 \\ & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 - x - 2 = 0$$

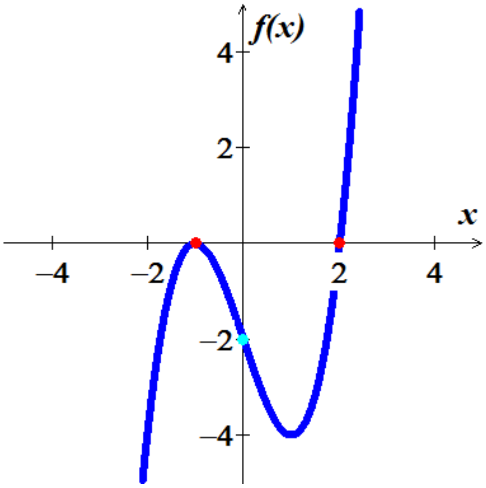
$x = -1,\ 2$ $\qquad\qquad$ $a - b + c = 0 \quad \rightarrow \quad x = -1,\ -\frac{c}{a}$

$x = -1,\ -1,\ 2$

$$\begin{array}{c|c|c|c} & -1 & 2 & \\ - & \| & - & + \end{array}$$

$f(x) > 0 \quad (2,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -1) \cup (-1,\ 2)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 2x + 1$$

Solution

possibilities for $\frac{c}{d} : \pm\{1\}$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -2 & 1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & -1 & \boxed{0} \end{array} \quad \rightarrow \quad x^2 + x - 1 = 0$$

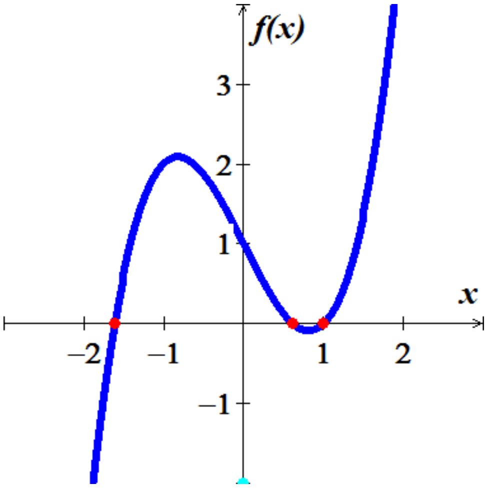
$x = \frac{-1 \pm \sqrt{5}}{2}$

$x = 1,\ \frac{-1 \pm \sqrt{5}}{2}$

$$\begin{array}{c|c|c|c} & \frac{-1-\sqrt{5}}{2} & \frac{-1+\sqrt{5}}{2} & 1 \\ - & + & - & + \end{array}$$

$f(x) > 0 \quad \left(\frac{-1-\sqrt{5}}{2},\ \frac{-1+\sqrt{5}}{2}\right) \cup (1,\ \infty)$

$f(x) < 0 \quad \left(-\infty,\ \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2},\ 1\right)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3-2x^2-11x+12$$

Solution

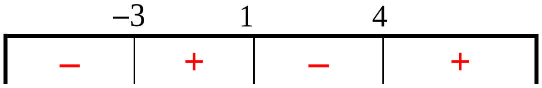
possibilities for $\frac{c}{d}:\pm\left\{\frac{12}{1}\right\}=\pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -11 & 12 \\ & & 1 & -1 & 12 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array} \quad \to \quad x^2-x-12=0$$

$$x=\frac{1\pm\sqrt{1+48}}{2}$$

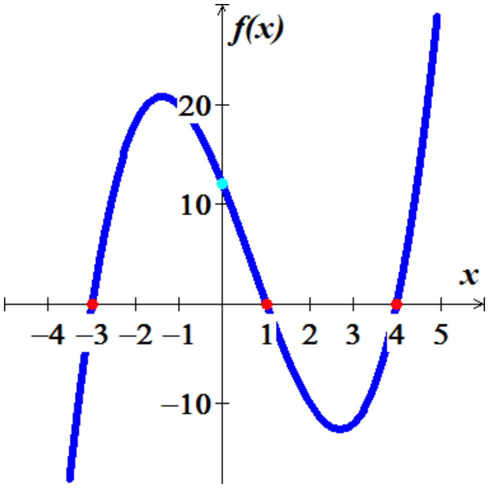
$$=\begin{cases} \frac{1-7}{2}=-3 \\ \frac{1+7}{2}=4 \end{cases}$$

$$\underline{x=-3,\ 1,\ 4 \,|}$$



$f(x)>0 \quad \underline{(-3,\ 1)\cup(4,\ \infty)\,|}$

$f(x)<0 \quad \underline{(-\infty,\ -3)\cup(1,\ 4)\,|}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3-2x^2-7x-4$$

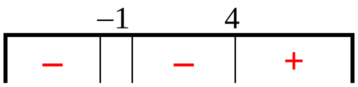
Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{4}{1}\right\}=\pm\{1,\ 2,\ 4\}$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -7 & -4 \\ & & -1 & 3 & 4 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array} \quad \to \quad x^2-3x-4=0$$

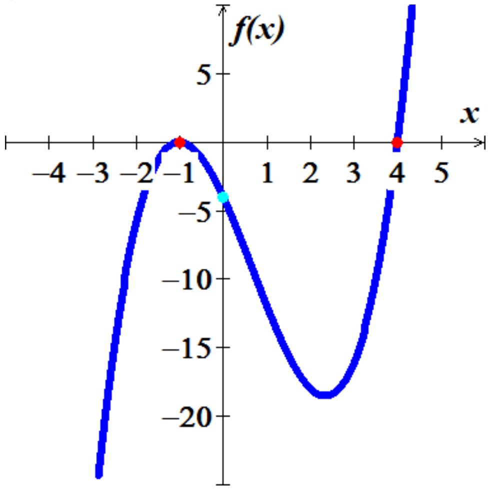
$\underline{x=-1,\ 4\,|}$ $\qquad a-b+c=0 \quad \to \quad x=-1,\ -\frac{c}{a}$

$\underline{x=-1,\ -1,\ 4\,|}$



$f(x) > 0 \quad (4,\ \infty)$

$f(x) < 0 \quad (-\infty,\ -1) \cup (-1,\ 4)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^3 - 10x - 12$$

Solution

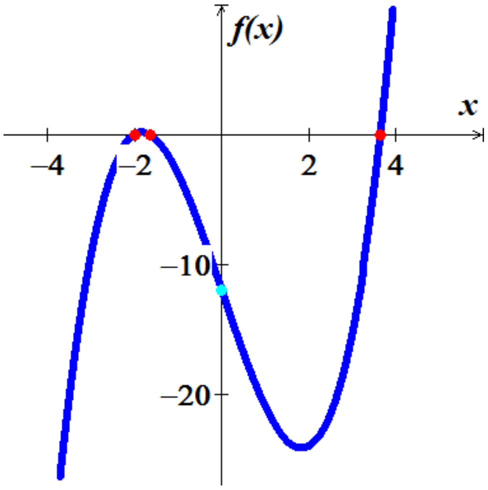
possibilities for $\frac{c}{d} : \pm\left\{\frac{12}{1}\right\} = \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

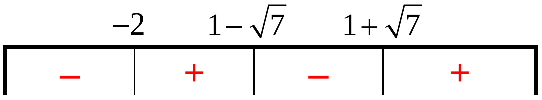
$$\begin{array}{r|rrrr} -2 & 1 & 0 & -10 & -12 \\ & & -2 & 4 & 12 \\ \hline & 1 & -2 & -6 & \boxed{0} \end{array} \quad \to \quad x^2 - 2x - 6 = 0$$

$$x = \frac{2 \pm \sqrt{4+24}}{2}$$

$$= \frac{2 \pm 2\sqrt{7}}{2}$$

$$x = -2,\ 1 \pm \sqrt{7}$$





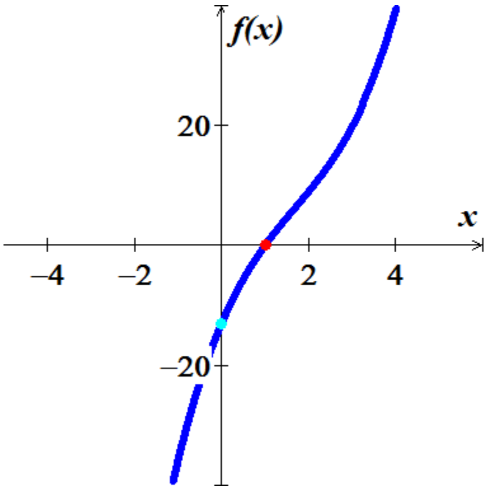
$f(x) > 0 \quad \left(-2,\ 1-\sqrt{7}\right) \cup \left(1+\sqrt{7},\ \infty\right)$

$f(x) < 0 \quad (-\infty,\ -2) \cup \left(1-\sqrt{7},\ 1+\sqrt{7}\right)$

Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^3-5x^2+17x-13$$



Solution

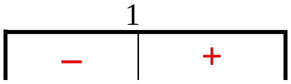
$$\textit{possibilities for } \frac{c}{d}:\pm\left\{\frac{13}{1}\right\}=\pm\{1,\ 13\}$$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 17 & -13 \\ & & 1 & -4 & 13 \\ \hline & 1 & -4 & 13 & \boxed{0} \end{array} \quad \to \quad x^2-4x+13=0$$

$$x=\frac{4\pm\sqrt{16-52}}{2}$$

$$=\frac{4\pm 6i}{2}$$

$$x=1,\ 2\pm 3i$$



$f(x)>0$ $\quad(1,\ \infty)$

$f(x)<0$ $\quad(-\infty,\ 1)$

Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=6x^3+25x^2-24x+5$$

Solution

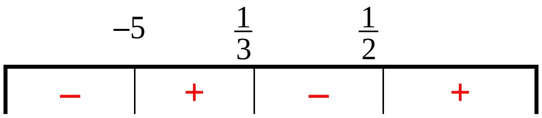
$$\textit{possibilities for } \frac{c}{d}:\pm\left\{\frac{5}{6}\right\}=\pm\left\{1,\ 5,\ \frac{1}{2},\ \frac{5}{2},\ \frac{1}{3},\ \frac{5}{3},\ \frac{1}{6},\ \frac{5}{6}\right\}$$

$$\begin{array}{r|rrrr} -5 & 6 & 25 & -24 & 5 \\ & & -30 & 25 & -5 \\ \hline & 6 & -5 & 1 & \boxed{0} \end{array} \quad \to \quad 6x^2-5x+1=0$$

$$x=\frac{5\pm\sqrt{25-24}}{12}$$

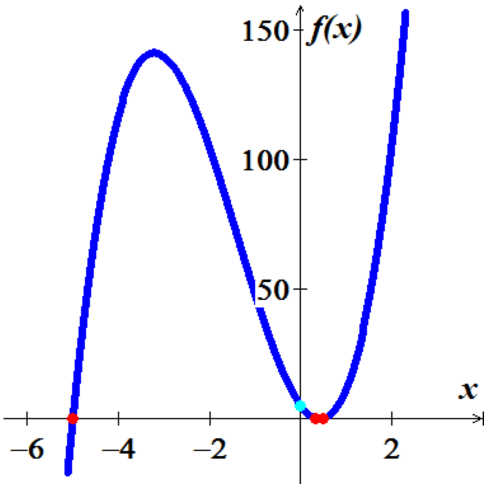
$$=\begin{cases} \frac{5-1}{12}=\frac{1}{3} \\ \frac{5+1}{12}=\frac{1}{2} \end{cases}$$

$x = -5,\ \frac{1}{3},\ \frac{1}{2}$



$f(x) > 0 \quad \left(-5,\ \frac{1}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)$

$f(x) < 0 \quad (-\infty,\ -5) \cup \left(\frac{1}{3},\ \frac{1}{2}\right)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 8x^3 + 18x^2 + 45x + 27$$

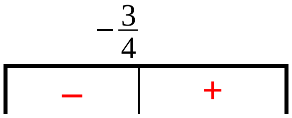
Solution

$possibilities: \quad \pm\left\{\frac{27}{8}\right\} = \pm\left\{\frac{1,\ 3,\ 9,\ 27}{1,\ 2,\ 4,\ 8}\right\}$

$= \pm\left\{1,\ 3,\ 9,\ 27,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{3}{2},\ \frac{3}{4},\ \frac{3}{8},\ \frac{9}{2},\ \frac{9}{4},\ \frac{9}{8},\ \frac{27}{2},\ \frac{27}{4},\ \frac{27}{8}\right\}$

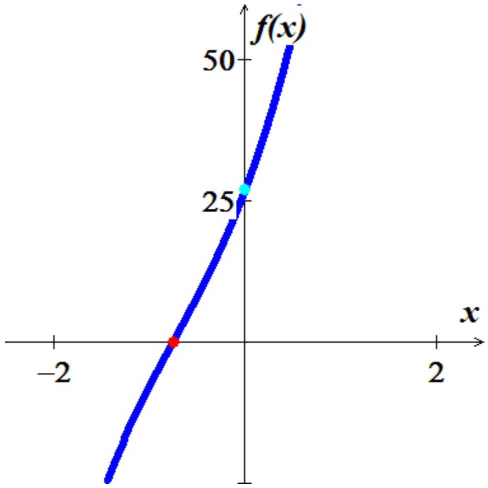
$$\begin{array}{r|rrrr} -\frac{3}{4} & 8 & 18 & 45 & 27 \\ & & -6 & -9 & -27 \\ \hline & 8 & 12 & 36 & \boxed{0} \end{array} \quad \to \ 8x^2 + 12x + 36 = 0$$

$x = -\frac{3}{4},\ \ -\frac{3}{4} \pm i\frac{3\sqrt{7}}{4}$



$f(x) > 0 \quad \left(-\frac{3}{4},\ \infty\right)$

$f(x) < 0 \quad \left(-\infty,\ -\frac{3}{4}\right)$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=3x^3-x^2+11x-20$$

Solution

$$\textit{possibilities}:\ \pm\left\{\frac{20}{3}\right\}=\pm\left\{\frac{1,\ 2,\ 4,\ 5,\ 10,\ 20}{1,\ 3}\right\}$$

$$=\pm\left\{1,\ 2,\ 4,\ 5,\ 10,\ 20,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{5}{3},\ \frac{10}{3},\ \frac{20}{3}\right\}$$

$$\begin{array}{r|rrrr}
\frac{4}{3} & 3 & -1 & 11 & -20 \\
 & & 4 & 4 & 20 \\
\hline
 & 3 & 3 & 15 & \boxed{0}
\end{array}\quad\rightarrow\ 3x^2+3x+15=0$$

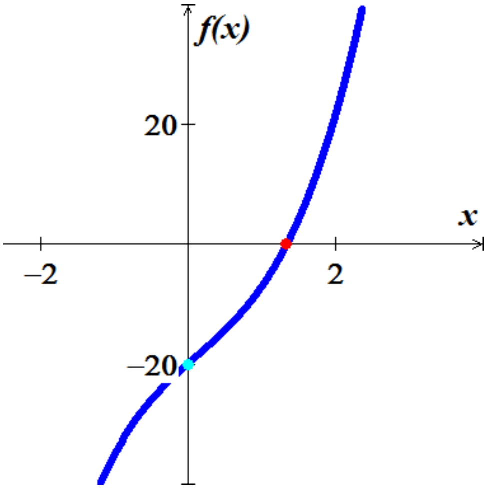
$$x=\frac{-3\pm\sqrt{9-180}}{6}$$

$$\boxed{x=\frac{4}{3},\ -\frac{1}{2}\pm i\frac{\sqrt{19}}{2}}$$

$$\begin{array}{c|c}
\multicolumn{2}{c}{\frac{4}{3}} \\
- & +
\end{array}$$

$f(x)>0$ $\boxed{\left(\frac{4}{3},\ \infty\right)}$

$f(x)<0$ $\boxed{\left(-\infty,\ \frac{4}{3}\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=x^4-x^3-9x^2+3x+18$$

Solution

$$\textit{possibilities for }\frac{c}{d}:\pm\left\{\frac{18}{1}\right\}=\pm\{1,\ 2,\ 3,\ 6,\ 9,\ 18\}$$

$$\begin{array}{r|rrrrr}
-2 & 1 & -1 & -9 & 3 & 18 \\
 & & -2 & 6 & 6 & -18 \\
\hline
3 & 1 & -3 & -3 & 9 & 0 \\
 & & 3 & 0 & -9 & \\
\hline
 & 1 & 0 & -3 & 0 &
\end{array}$$

$\rightarrow x^3-3x^2-3x+9=0\ \rightarrow\ \pm\left\{\frac{9}{1}\right\}=\pm\{1,\ 3,\ 9\}$

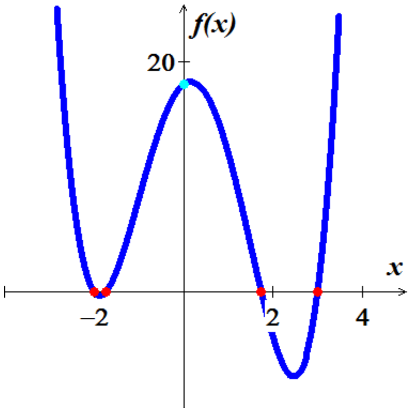
$\rightarrow x^2-3=0\Rightarrow \boxed{x=\pm\sqrt{3}}$

$$\boxed{x=-2,\ 3,\ \pm\sqrt{3}}$$

+	-2	–	$-\sqrt{3}$	+	$\sqrt{3}$	–	3	+

$f(x) > 0$ $\quad (-\infty,\ -2)\cup(-\sqrt{3},\ \sqrt{3})\cup(3,\ \infty)$

$f(x) < 0$ $\quad (-2,\ -\sqrt{3})\cup(\sqrt{3},\ 3)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 9x^3 + 9x^2 + x - 3$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrrr} 1 & 2 & -9 & 9 & 1 & -3 \\ & & 2 & -7 & 2 & 3 \\ \hline 1 & 2 & -7 & 2 & 3 & 0 \\ & & 2 & -5 & -3 & \\ \hline & 2 & -5 & -3 & 0 & \end{array}$$

$\rightarrow\ 2x^3 - 7x^2 + 2x + 3 = 0\ \rightarrow\ \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$\rightarrow\ 2x^2 - 5x - 3 = 0$

$$x = \frac{5 \pm \sqrt{25+24}}{4}$$

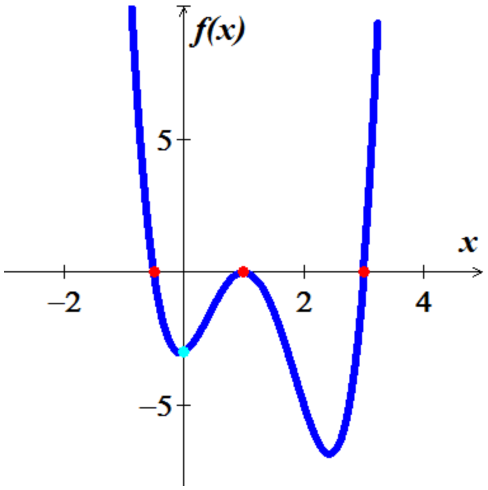
$$= \begin{cases} \frac{5-7}{4} = -\frac{1}{2} \\ \frac{5+7}{4} = 3 \end{cases}$$

$x = 1,\ 1,\ -\frac{1}{2},\ 3$

+	$-\frac{1}{2}$	–	1	–	3	+

$f(x) > 0$ $\quad \left(-\infty,\ -\frac{1}{2}\right)\cup(3,\ \infty)$

$f(x) < 0$ $\quad \left(-\frac{1}{2},\ 1\right)\cup(1,\ 3)$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 + 5x^3 - 17x^2 - 6x$$

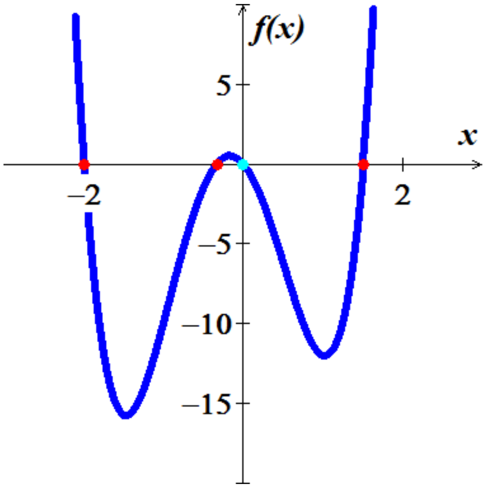
Solution

$$x\left(6x^3 + 5x^2 - 17x - 6\right) = 0 \quad \rightarrow \quad x = 0$$

$$\textit{possibilities}: \ \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrr} -2 & 6 & 5 & -17 & -6 \\ & & -12 & 14 & 6 \\ \hline & 6 & -7 & -3 & \boxed{0} \end{array} \quad \rightarrow \ 6x^2 - 7x - 3 = 0$$

$$x = 0,\ -2,\ -\frac{1}{3},\ \frac{3}{2}$$



	-2		$-\frac{1}{3}$		0		$\frac{3}{2}$	
+		−		+		−		+

$f(x) > 0$ $\quad (-\infty,\ -2) \cup \left(-\frac{1}{3},\ 0\right) \cup \left(\frac{3}{2},\ \infty\right)$

$f(x) < 0$ $\quad \left(-2,\ -\frac{1}{3}\right) \cup \left(0,\ \frac{3}{2}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 2x^2 - 16x - 15$$

Solution

$$\textit{possibilities}: \ \pm\left\{\frac{15}{1}\right\} = \pm\{1,\ 3,\ 5,\ 15\}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -2 & -16 & -15 \\ & & -1 & 1 & 1 & 15 \\ \hline 3 & 1 & -1 & -1 & -15 & 0 \\ & & 3 & 6 & 15 & \\ \hline & 1 & 2 & 5 & 0 & \end{array} \quad \begin{array}{l} \rightarrow \ x^3 - x^2 - x - 15 = 0 \ \rightarrow \ \pm\left\{\frac{15}{1}\right\} = \pm\{1,\ 3,\ 5,\ 15\} \\ \\ \rightarrow \ x^2 + 2x + 5 = 0 \end{array}$$

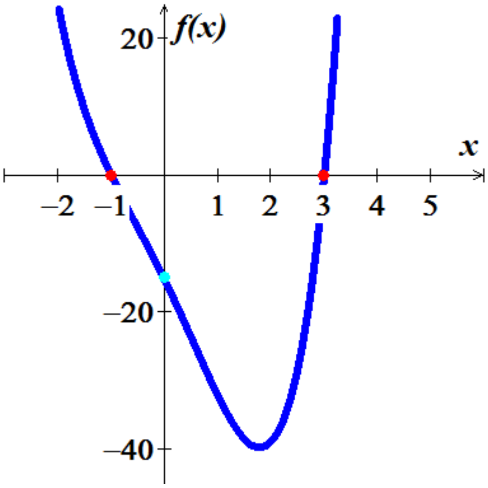
$$\begin{aligned} x &= \frac{-2 \pm \sqrt{-16}}{2} \\ &= -1 \pm 2i \end{aligned}$$

$\boxed{x = -1,\ 3,\ \ -1 \pm 2i}$

	−1		3	
+		−		+

$f(x) > 0 \quad \boxed{(-\infty,\ -1) \cup (3,\ \infty)}$

$f(x) < 0 \quad \boxed{(-1,\ 3)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 2x^3 - 5x^2 + 8x + 4$$

Solution

possibilities : $\pm\left\{\frac{4}{1}\right\} = \pm\{1,\ 2,\ 4\}$

$$\begin{array}{r|rrrrr} 2 & 1 & -2 & -5 & 8 & 4 \\ & & 2 & 0 & -10 & -4 \\ \hline -2 & 1 & 0 & -5 & -2 & 0 \\ & & -2 & 4 & 2 & \\ \hline & 1 & -2 & -1 & 0 & \end{array}$$

$\rightarrow x^3 - 5x - 2 = 0 \rightarrow \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$\rightarrow x^2 - 2x - 1 = 0$

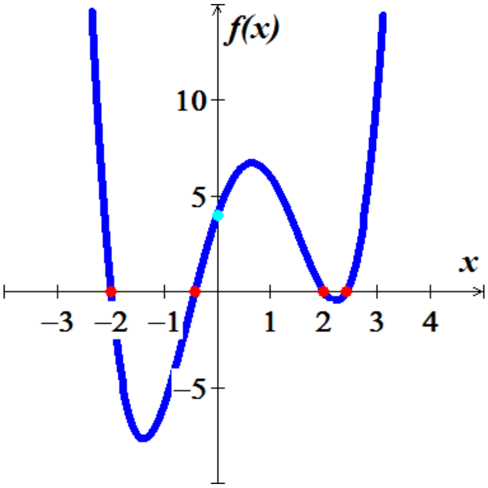
$$x = \frac{2 \pm \sqrt{8}}{2}$$

$\boxed{x = -2,\ 2,\ 1 \pm \sqrt{2}}$

	−2		$1-\sqrt{2}$		2		$1+\sqrt{2}$	
+		−		+		−		+

$f(x) > 0 \quad \boxed{(-\infty,\ -2) \cup \left(1-\sqrt{2},\ 2\right) \cup \left(1+\sqrt{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-2,\ 1-\sqrt{2}\right) \cup \left(2,\ 1+\sqrt{2}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 17x^3 + 4x^2 + 35x - 24$$

Solution

possibilities : $\pm\left\{\frac{24}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & -17 & 4 & 35 & -24 \\
 & & 2 & -15 & -11 & 24 \\
\hline
1 & 2 & -15 & -11 & 24 & 0 \\
 & & 2 & -13 & 24 & \\
\hline
 & 2 & -13 & -24 & 0 &
\end{array}$$

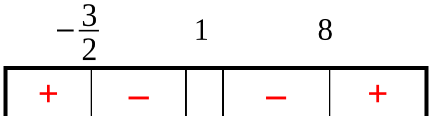
$\rightarrow\ 2x^3 - 15x^2 - 11x + 24 = 0 \ \rightarrow\ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$\rightarrow\ 2x^2 - 13x - 24 = 0$

$$x = \frac{13 \pm \sqrt{169 + 192}}{4}$$

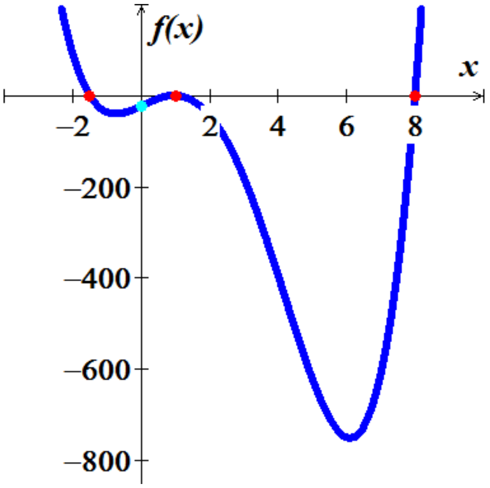
$$= \begin{cases} \frac{13-19}{4} = -\frac{3}{2} \\ \frac{13+19}{4} = 8 \end{cases}$$

$\boxed{x = -\frac{3}{2},\ 1,\ 1,\ 8}$



$f(x) > 0 \quad \boxed{\left(-\infty,\ -\frac{3}{2}\right) \cup (8,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-\frac{3}{2},\ 1\right) \cup (1,\ 8)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 + x^3 - 3x^2 - 5x - 2$$

Solution

$$possibilities: \ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$$

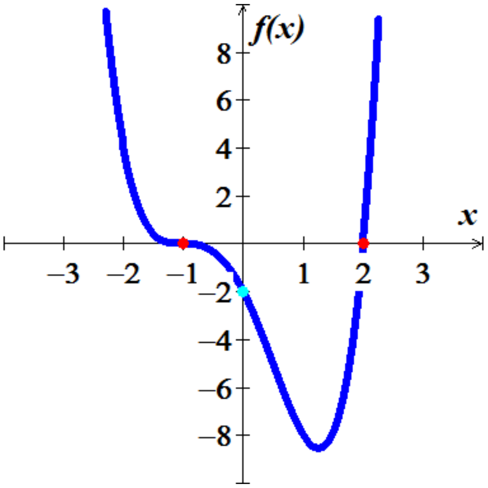
$$\begin{array}{r|rrrrr} -1 & 1 & 1 & -3 & -5 & -2 \\ & & -1 & 0 & 3 & 2 \\ \hline -1 & 1 & 0 & -3 & -2 & 0 \\ & & -1 & 1 & 2 & \\ \hline & 1 & -1 & -2 & 0 & \end{array} \quad \to \ x^3 - 3x - 2 = 0 \ \to \ \pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$$

$$\to \ x^2 - x - 2 = 0$$

$$x = \frac{1 \pm \sqrt{1+8}}{2}$$

$$= \begin{cases} \frac{1-3}{2} = -1 \\ \frac{1+3}{2} = 2 \end{cases}$$

$$x = -1,\ -1,\ -1,\ 2$$



$-1 \qquad 2$

+	–	+

$f(x) > 0 \quad (-\infty,\ -1) \cup (2,\ \infty)$

$f(x) < 0 \quad (-2,\ 2)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 - 17x^3 - 11x^2 + 42x$$

Solution

$$x\left(6x^3 - 17x^2 - 11x + 42\right) = 0$$

$$x = 0 \quad 6x^3 - 17x^2 - 11x + 42 = 0$$

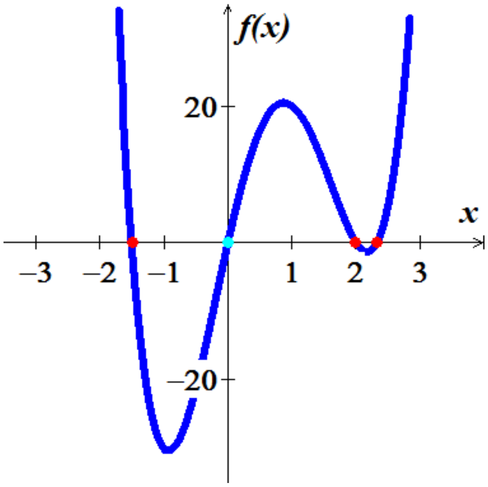
$$possibilities: \ \pm\left\{\frac{42}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ 7,\ 14,\ 21,\ 42,\ \frac{1}{2},\ \frac{3}{2},\ \frac{7}{2},\ \frac{21}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{7}{3},\ \frac{14}{3},\ \frac{1}{6},\ \frac{7}{6},\ \frac{21}{6}\right\}$$

$$\begin{array}{r|rrrr} 2 & 6 & -17 & -11 & 42 \\ & & 12 & -10 & -42 \\ \hline & 6 & -5 & -21 & \boxed{0} \end{array} \quad \to \ 6x^2 - 5x - 21 = 0$$

$$x = \frac{5 \pm \sqrt{25+504}}{12}$$

$$= \begin{cases} \frac{5-23}{12} = -\frac{3}{2} \\ \frac{5+23}{12} = \frac{7}{3} \end{cases}$$

$$\underline{x = -\frac{3}{2},\ 0,\ 2,\ \frac{7}{3}\ \Big|}$$



	$-\frac{3}{2}$		0		2		$\frac{7}{3}$	
+		–		+		–		+

$f(x) > 0 \quad \left(-\infty,\ -\frac{3}{2}\right) \cup (0,\ 2) \cup \left(\frac{7}{3},\ \infty\right)$

$f(x) < 0 \quad \left(-\frac{3}{2},\ 0\right) \cup \left(2,\ \frac{7}{3}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 - 5x^2 - 2x$$

Solution

$x\left(x^3 - 5x - 2\right) = 0$

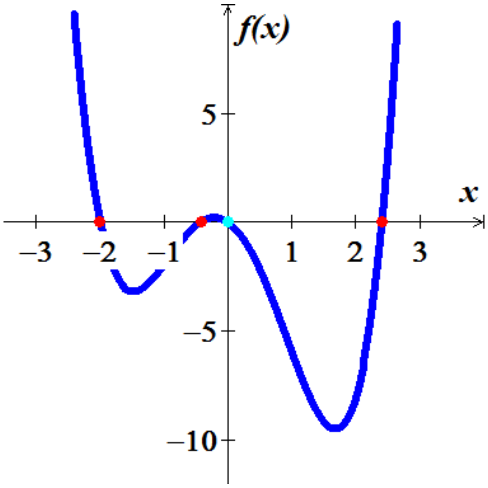
$x = 0 \quad x^3 - 5x - 2 = 0$

possibilities : $\pm\left\{\frac{2}{1}\right\} = \pm\{1,\ 2\}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -5 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & \boxed{0} \end{array} \rightarrow x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$\underline{x = -2,\ 0,\ 1 \pm \sqrt{2}\ \Big|}$$



	−2		$1-\sqrt{2}$		2		$1+\sqrt{2}$	
+		–		+		–		+

$f(x) > 0 \quad (-\infty,\ -2) \cup \left(1-\sqrt{2},\ 2\right) \cup \left(1+\sqrt{2},\ \infty\right)$

$f(x) < 0 \quad \left(-2,\ 1-\sqrt{2}\right) \cup \left(2,\ 1+\sqrt{2}\right)$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^4 - 4x^3 - 11x^2 + 16x - 4$$

Solution

$$\text{possibilities}: \ \pm\left\{\frac{4}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3}\right\}$$

$$\begin{array}{r|rrrrr} 1 & 3 & -4 & -11 & 16 & -4 \\ & & 3 & -1 & -12 & 4 \\ \hline 2 & 3 & -1 & -12 & 4 & 0 \\ & & 6 & 10 & -4 & \\ \hline & 3 & 5 & -2 & 0 & \end{array}$$

$$\rightarrow 3x^3 - x^2 - 12x + 4 = 0 \ \rightarrow \ \pm\left\{\frac{4}{3}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3}\right\}$$

$$\rightarrow 3x^2 + 5x - 2 = 0$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{6}$$

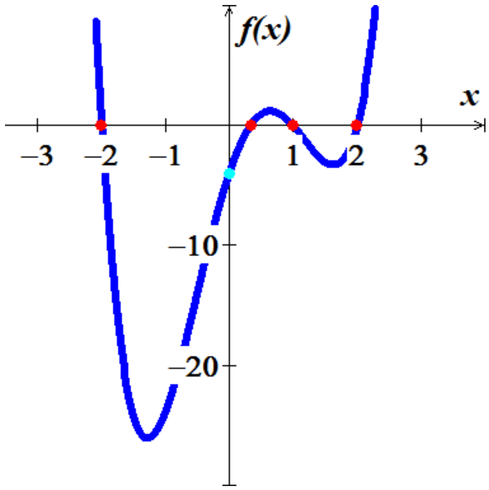
$$= \begin{cases} \dfrac{-5-7}{6} = -2 \\ \dfrac{-5+7}{6} = \dfrac{1}{3} \end{cases}$$

$$\boxed{x = -2,\ \frac{1}{3},\ 1,\ 2}$$

	−2		$\frac{1}{3}$		1		2	
+		−		+		−		+

$f(x) > 0$ $\quad \boxed{(-\infty,\ -2) \cup \left(\frac{1}{3},\ 1\right) \cup (2,\ \infty)}$

$f(x) < 0$ $\quad \boxed{\left(-2,\ \frac{1}{3}\right) \cup (1,\ 2)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^4 + 23x^3 + 19x^2 - 8x - 4$$

Solution

$$\text{possibilities}: \ \pm\left\{\frac{4}{6}\right\} = \pm\left\{1,\ 2,\ 4,\ \frac{1}{6},\ \frac{2}{3},\ \frac{2}{3}\right\}$$

$$\begin{array}{r|rrrrr} -2 & 6 & 23 & 19 & -8 & -4 \\ & & -12 & -22 & 6 & 4 \\ \hline -2 & 6 & 11 & -3 & -2 & 0 \\ & & -12 & 2 & 2 & \\ \hline & 6 & -1 & -1 & 0 & \end{array}$$

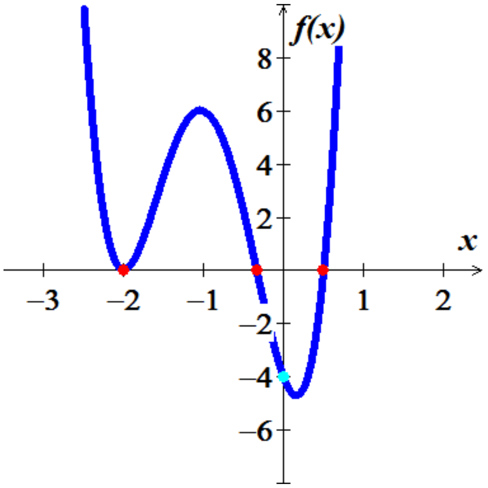
$\rightarrow 6x^3+11x^2-3x-2=0 \rightarrow \pm\left\{\frac{2}{6}\right\}=\pm\left\{1,\ 2,\ \frac{1}{6},\ \frac{1}{3}\right\}$

$\rightarrow 6x^2-x-1=0$

$$x=\frac{1\pm\sqrt{25}}{12}$$

$$=\begin{cases}\frac{1-5}{12}=-\frac{1}{3}\\ \frac{1+5}{12}=\frac{1}{2}\end{cases}$$

$$x=-2,\ -2,\ -\frac{1}{3},\ \frac{1}{2}$$



$$\begin{array}{ccccccc} & -2 & & -\frac{1}{3} & & \frac{1}{2} & \\ \hline + & & + & & - & & + \end{array}$$

$f(x)>0$ $\quad(-\infty,\ -2)\cup\left(-2,\ -\frac{1}{3}\right)\cup\left(\frac{1}{2},\ \infty\right)$

$f(x)<0$ $\quad\left(-\frac{1}{3},\ \frac{1}{2}\right)$

Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=4x^4-12x^3+3x^2+12x-7$$

Solution

possibilities : $\pm\left\{\frac{7}{4}\right\}=\pm\left\{1,\ 7,\ \frac{1}{2},\ \frac{7}{2},\ \frac{1}{4},\ \frac{7}{4}\right\}$

$$\begin{array}{r|rrrrr} 1 & 4 & -12 & 3 & 12 & -7 \\ & & 4 & -8 & -5 & 7 \\ \hline -1 & 4 & -8 & -5 & 7 & 0 \\ & & -4 & 12 & -7 & \\ \hline & 4 & -12 & 7 & 0 & \end{array}$$

$\rightarrow 4x^3-8x^2-5x+7=0 \rightarrow \pm\left\{\frac{7}{4}\right\}=\pm\left\{1,\ 7,\ \frac{1}{2},\ \frac{7}{2},\ \frac{1}{4},\ \frac{7}{4}\right\}$

$\rightarrow 4x^2-12x+7=0$

$$x=\frac{12\pm\sqrt{144-112}}{8}$$

$$=\frac{12\pm4\sqrt{2}}{8}$$

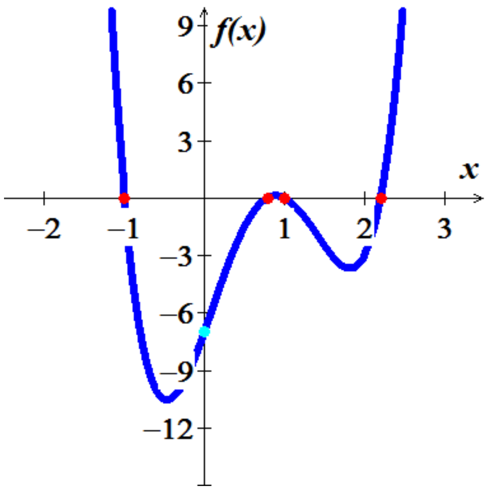
$$\boxed{x = -1,\ 1,\ \frac{3 \pm \sqrt{2}}{2}}$$

$-1 \quad \frac{3-\sqrt{2}}{2} \quad 1 \quad \frac{3+\sqrt{2}}{2}$

+	−	+	−	+

$f(x) > 0 \quad \boxed{(-\infty,\ -1) \cup \left(\frac{3-\sqrt{2}}{2},\ 1\right) \cup \left(\frac{3+\sqrt{2}}{2},\ \infty\right)}$

$f(x) < 0 \quad \boxed{\left(-1,\ \frac{3-\sqrt{2}}{2}\right) \cup \left(1,\ \frac{3+\sqrt{2}}{2}\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 9x^3 - 2x^2 + 27x - 12$$

Solution

possibilities : $\pm\left\{\frac{12}{2}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 12,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$$\begin{array}{r|rrrrr} 4 & 2 & -9 & -2 & 27 & -12 \\ & & 8 & -4 & -24 & 12 \\ \hline \frac{1}{2} & 2 & -1 & -6 & 3 & 0 \\ & & 1 & 0 & -3 & \\ \hline & 2 & 0 & -6 & 0 & \end{array}$$

$\to 2x^3 - x^2 - 6x + 3 = 0 \ \to\ \pm\left\{\frac{3}{2}\right\} = \pm\left\{1,\ 3,\ \frac{1}{2},\ \frac{3}{2}\right\}$

$\to 2x^2 - 6 = 0$

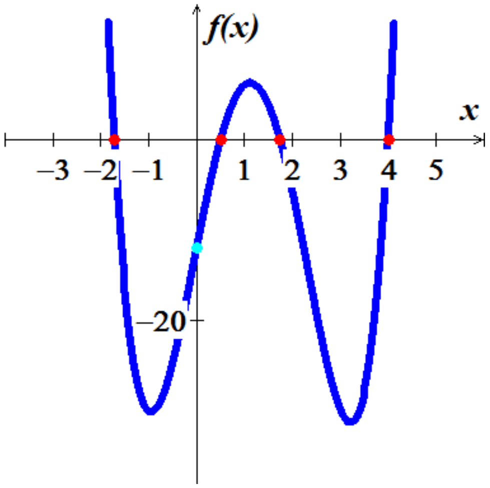
$$\boxed{x = \frac{1}{2},\ 4,\ \pm\sqrt{3}}$$

$-\sqrt{3} \quad \frac{1}{2} \quad \sqrt{3} \quad 4$

+	−	+	−	+

$f(x) > 0 \quad \boxed{(-\infty,\ -\sqrt{3}) \cup \left(\frac{1}{2},\ \sqrt{3}\right) \cup (4,\ \infty)}$

$f(x) < 0 \quad \boxed{\left(-\sqrt{3},\ \frac{1}{2}\right) \cup (\sqrt{3},\ 4)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 - 19x^3 + 51x^2 - 31x + 5$$

Solution

$$\text{possibilities}: \ \pm\left\{\frac{5}{2}\right\} = \pm\left\{1,\ 5,\ \frac{1}{2},\ \frac{5}{2}\right\}$$

$$\begin{array}{r|rrrrr} 5 & 2 & -19 & 51 & -31 & 5 \\ & & 10 & -45 & 30 & -5 \\ \hline \frac{1}{2} & 2 & -9 & 6 & -1 & 0 \\ & & 1 & -4 & 1 & \\ \hline & 2 & -8 & 2 & 0 & \end{array}$$

$$\rightarrow 2x^3 - 9x^2 + 6x - 1 = 0 \ \rightarrow \ \pm\left\{\frac{1}{2}\right\}$$

$$\rightarrow 2x^2 - 8x + 2 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 16}}{4}$$

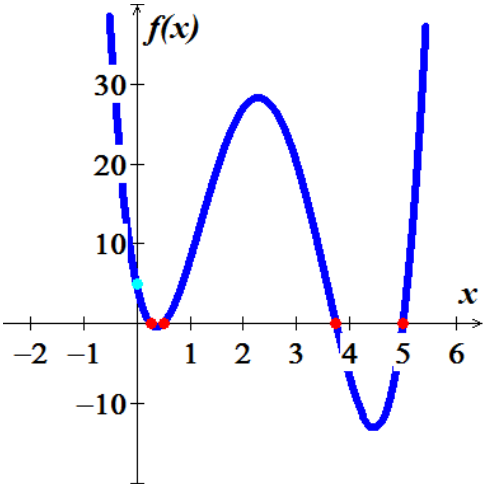
$$= \frac{8 \pm 4\sqrt{3}}{4}$$

$$\boxed{x = \frac{1}{2},\ 5,\ 2 \pm \sqrt{3}}$$

$$\begin{array}{ccccccccc} & 2-\sqrt{3} & & \frac{1}{2} & & 2+\sqrt{3} & & 5 & \\ + & & - & & + & & - & & + \end{array}$$

$$f(x) > 0 \quad \boxed{\left(-\infty,\ 2-\sqrt{3}\right) \cup \left(\frac{1}{2},\ 2+\sqrt{3}\right) \cup (5,\ \infty)}$$

$$f(x) < 0 \quad \boxed{\left(2-\sqrt{3},\ \frac{1}{2}\right) \cup \left(2+\sqrt{3},\ 5\right)}$$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 4x^4 - 35x^3 + 71x^2 - 4x - 6$$

Solution

$$\text{possibilities}: \ \pm\left\{\frac{6}{4}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{4},\ \frac{3}{4}\right\}$$

$$\begin{array}{r|rrrrr}
3 & 4 & -35 & 71 & -4 & -6\\
 & & 12 & -69 & 6 & 6\\ \hline
-\frac{1}{4} & 4 & -23 & 2 & 2 & 0\\
 & & -1 & 6 & -2 & \\ \hline
 & 4 & -24 & 8 & 0 &
\end{array}$$

$$\to 4x^3 - 23x^2 + 2x + 2 = 0 \to \pm\left\{\frac{2}{4}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2},\ \frac{1}{4}\right\}$$

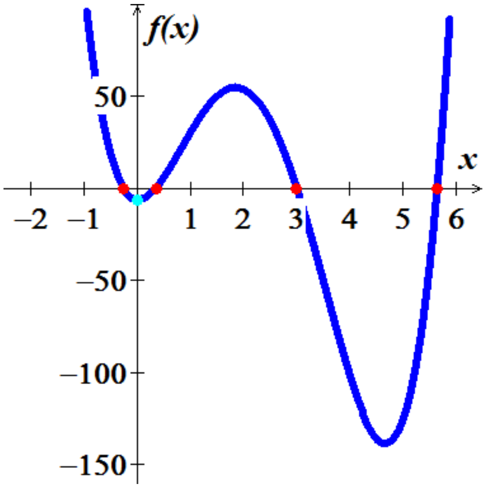
$$\to 4x^2 - 24x + 8 = 0$$

$$x^2 - 6x + 2 = 0$$

$$x = \frac{6 \pm \sqrt{36-8}}{2}$$

$$= \frac{6 \pm 2\sqrt{7}}{4}$$

$$\boxed{x = -\frac{1}{4},\ 3,\ 3 \pm \sqrt{7}}$$



$$\begin{array}{ccccc}
 & -\frac{1}{4} & 3-\sqrt{7} & 3 & 3+\sqrt{7} \\
+ & - & + & - & +
\end{array}$$

$$f(x) > 0 \quad \boxed{\left(-\infty,\ -\frac{1}{4}\right) \cup \left(3-\sqrt{7},\ 3\right) \cup \left(3+\sqrt{7},\ \infty\right)}$$

$$f(x) < 0 \quad \boxed{\left(-\frac{1}{4},\ 3-\sqrt{7}\right) \cup \left(3,\ 3+\sqrt{7}\right)}$$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$$

Solution

possibilities : $\pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$

$$\begin{array}{r|rrrrr}
1 & 2 & 3 & -4 & -3 & 2\\
 & & 2 & 5 & 1 & -2\\ \hline
-1 & 2 & 5 & 1 & -2 & 0\\
 & & -2 & -3 & 2 & \\ \hline
 & 2 & 3 & -2 & 0 &
\end{array}$$

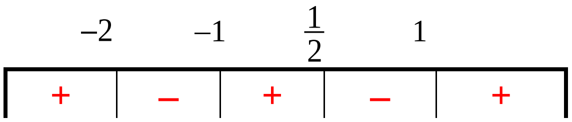
$$\to 2x^3 - 23x^2 + 2x - 2 = 0 \to \pm\left\{\frac{2}{2}\right\} = \pm\left\{1,\ 2,\ \frac{1}{2}\right\}$$

$$\to 2x^2 + 3x - 2 = 0$$

$$x = \frac{-3 \pm \sqrt{9+16}}{4}$$

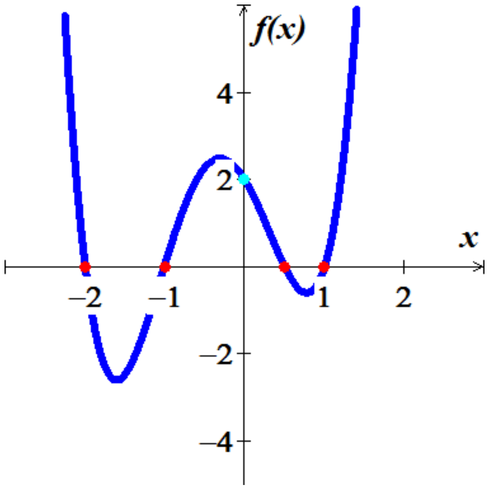
$$= \begin{cases} \frac{-3-5}{4} = -2 \\ \frac{-3+5}{4} = \frac{1}{2} \end{cases}$$

$\boxed{x = -2, \ -1, \ \frac{1}{2}, \ 1}$



$f(x) > 0 \quad \boxed{(-\infty, \ -2) \cup \left(-1, \ \frac{1}{2}\right) \cup (1, \ \infty)}$

$f(x) < 0 \quad \boxed{(-2, \ -1) \cup \left(\frac{1}{2}, \ 1\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^4 + 3x^3 - 30x^2 - 6x + 56$$

Solution

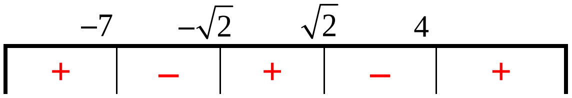
possibilities for $\frac{c}{d} : \pm\left\{\frac{56}{1}\right\} = \pm\{1, \ 2, \ 4, \ 7, \ 8, \ 14, \ 28, \ 56\}$

$$\begin{array}{r|rrrrr}
4 & 1 & 3 & -3 & -6 & 56 \\
 & & 4 & 28 & -8 & -56 \\
\hline
-7 & 1 & 7 & -2 & -14 & 0 \\
 & & -7 & 0 & 14 & \\
\hline
 & 1 & 0 & -2 & &
\end{array}$$

$\rightarrow \ x^3 + 7x^2 - 2x - 14 = 0 \ \Rightarrow \ \frac{c}{d} = \pm\left\{\frac{14}{1}\right\} = \pm\{1, \ 2, \ 7, \ 14\}$

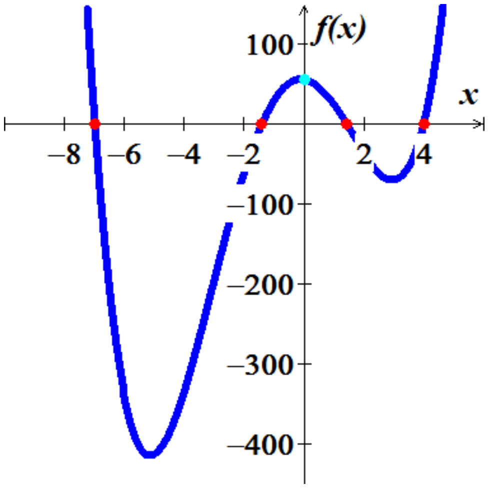
$\rightarrow \ x^2 - 2 = 0 \Rightarrow \boxed{x = \pm\sqrt{2}}$

$\boxed{x = 4, \ -7, \ \pm\sqrt{2}}$



$f(x) > 0 \quad \boxed{(-\infty, \ -7) \cup \left(-\sqrt{2}, \ \sqrt{2}\right) \cup (4, \ \infty)}$

$f(x) < 0 \quad \boxed{\left(-7, \ -\sqrt{2}\right) \cup \left(\sqrt{2}, \ 4\right)}$



Exercise

Find all values of x such that $f(x)>0$ and all x such that $f(x)<0$, and then sketch the graph of $f(x)$

$$f(x)=3x^5-10x^4-6x^3+24x^2+11x-6$$

Solution

possibilities for $\frac{c}{d}:\pm\left\{\frac{6}{3}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrrrr}
-1 & 3 & -10 & -6 & 24 & 11 & -6 \\
 & & -3 & 13 & -7 & -17 & 6 \\
\hline
-1 & 3 & -13 & 7 & 17 & -6 & 0 \\
 & & -3 & 16 & -23 & 6 & \\
\hline
2 & 3 & -16 & 23 & -6 & 0 & \\
 & & 6 & 20 & 6 & & \\
\hline
 & 3 & -10 & 3 & 0 & &
\end{array}$$

$$x^4-13x^3+7x^2+17x-6=0\to\pm\left\{\frac{6}{3}\right\}=\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^3-16x^2+26x-6=0\to\pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{3},\ \frac{2}{3}\right\}$$

$$3x^2-10x+3=0$$

$$x=\frac{10\pm\sqrt{100-36}}{6}$$

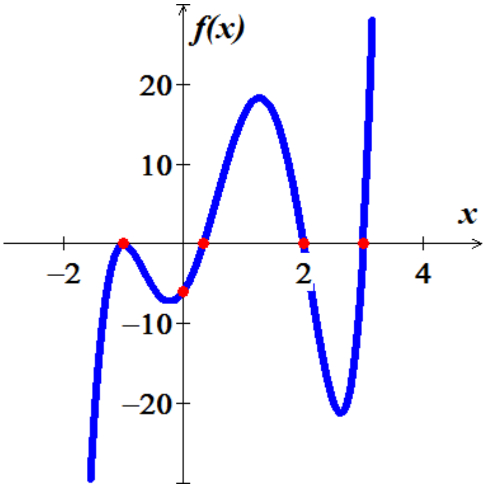
$$=\begin{cases}\frac{10-8}{6}=\frac{1}{3}\\ \frac{10+8}{4}=3\end{cases}$$

$$\boxed{x=-1,\ -1,\ \frac{1}{3},\ 2,\ 3}$$

$$\begin{array}{ccccccccc}
 & -1 & & \frac{1}{3} & & 2 & & 3 & \\
\hline
- & | & - & | & + & | & - & | & +
\end{array}$$

$f(x)>0$ $\boxed{\left(\frac{1}{3},\ 2\right)\cup(3,\ \infty)}$

$f(x)<0$ $\boxed{(-\infty,\ -1)\cup\left(-1,\ \frac{1}{3}\right)\cup(2,\ 3)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 6x^5 + 19x^4 + x^3 - 6x^2$$

Solution

$$x^2\left(6x^3 + 19x^2 + x - 6\right) = 0 \quad \rightarrow \quad \underline{x = 0,\ 0|}$$

$$6x^3 + 19x^2 + x - 6 = 0$$

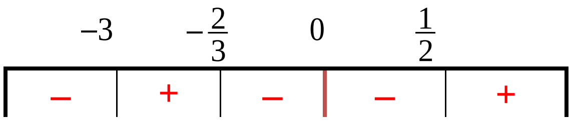
possibilities for $\frac{c}{d} : \pm\left\{\frac{6}{6}\right\} = \pm\left\{1,\ 2,\ 3,\ 6,\ \frac{1}{2},\ \frac{3}{2},\ \frac{1}{3},\ \frac{2}{3}\right\}$

$$\begin{array}{r|rrrr} -3 & 6 & 19 & 1 & -6 \\ & & -18 & -3 & 6 \\ \hline & 6 & 1 & -2 & \boxed{0} \end{array} \qquad 6x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1+48}}{12}$$

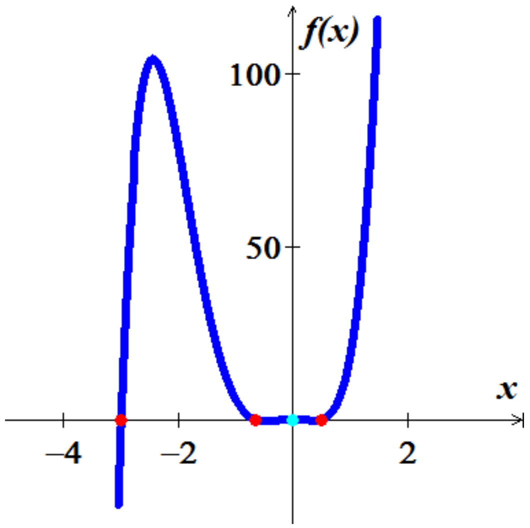
$$= \begin{cases} \frac{-1-7}{12} = -\frac{2}{3} \\ \frac{-1+7}{12} = \frac{1}{2} \end{cases}$$

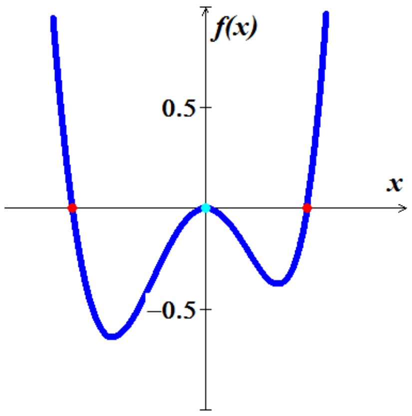
$$\underline{x = 0,\ 0,\ -\frac{2}{3},\ -3,\ \frac{1}{2}\ |}$$



$f(x) > 0 \quad \underline{\left(-3,\ -\frac{2}{3}\right) \cup \left(\frac{1}{2},\ \infty\right)|}$

$f(x) < 0 \quad \underline{\left(-\infty,\ -3\right) \cup \left(-\frac{2}{3},\ 0\right) \cup \left(0,\ \frac{1}{2}\right)|}$





Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Solution

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1 = (x+1)^5 = 0$$

possibilities for $\frac{c}{d} : \pm\{1\}$

$$\begin{array}{r|rrrrrr}
-1 & 1 & 5 & 10 & 10 & 5 & 1 \\
 & & -1 & -4 & -6 & -4 & -1 \\
\hline
-1 & 1 & 4 & 6 & 4 & 1 & 0 \\
 & & -1 & -3 & -3 & -1 & \\
\hline
-1 & 1 & 3 & 3 & 1 & 0 & \\
 & & -1 & -2 & -1 & & \\
\hline
 & 1 & 2 & 1 & 0 & &
\end{array}$$

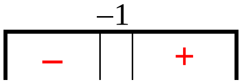
$\rightarrow\ x^4 + 4x^3 + 6x^2 + 4x + 1 = 0 \rightarrow \pm\{1\}$

$\rightarrow\ x^3 + 3x^2 + 3x + 1 = 0 \rightarrow \pm\{1\}$

$\rightarrow\ x^2 + 2x + 1 = 0$

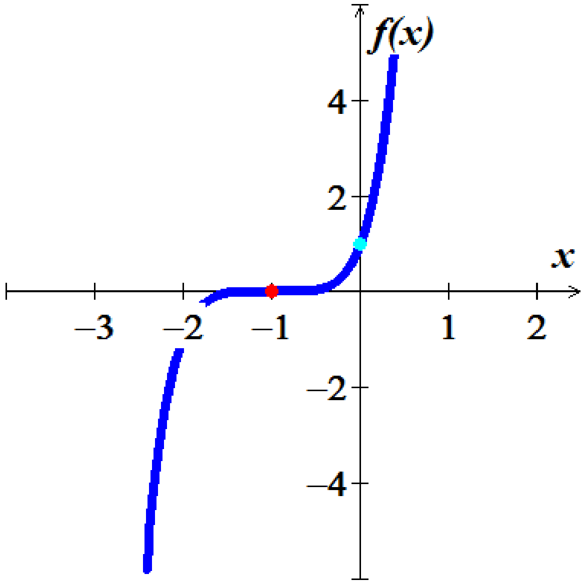
$$x^2 + 2x + 1 = (x+1)^2$$

$\underline{x = -1}$ $\left(\textit{multiplicity}\text{ of } 5\right)$



$f(x) > 0$ $\underline{(-1,\ \infty)}$

$f(x) < 0$ $\underline{(-\infty,\ -1)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^5 - x^4 - 7x^3 + 7x^2 + 12x - 12$$

Solution

possibilities for $\frac{c}{d} : \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$$\begin{array}{r|rrrrrr}
1 & 1 & -1 & -7 & 7 & 12 & -12 \\
 & & 1 & 0 & -7 & 0 & 12 \\
\hline
2 & 1 & 0 & -7 & 0 & 12 & 0 \\
 & & 2 & 4 & -6 & -12 & \\
\hline
-2 & 1 & 2 & -3 & -6 & 0 & \\
 & & -2 & 0 & 6 & & \\
\hline
 & 1 & 0 & -3 & 0 & &
\end{array}$$

$\rightarrow x^4 - 7x^2 - 12 = 0 \rightarrow \pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

$\rightarrow x^3 + 2x^2 - 3x - 6 = 0 \rightarrow \pm\{1,\ 2,\ 3,\ 6\}$

$\rightarrow x^2 - 3 = 0$

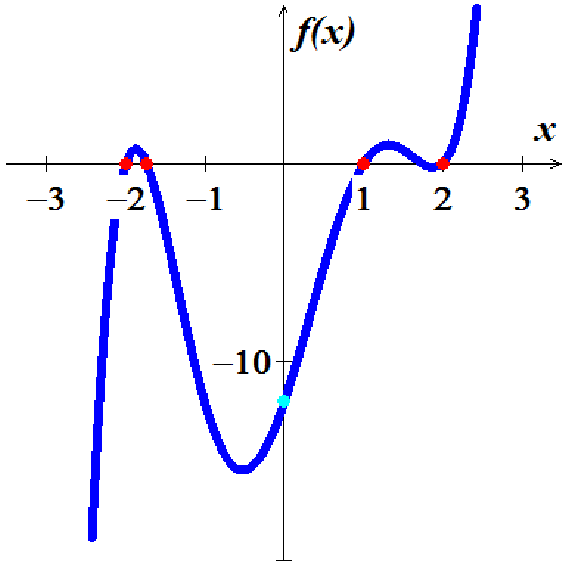
$x^2 = 3$

$\boxed{x = -2,\ 1,\ 2,\ \pm\sqrt{3}}$

$$\begin{array}{cccccccccccc}
 & -2 & & -\sqrt{3} & & 1 & & \sqrt{3} & & 2 & \\
- & | & + & | & - & | & + & | & - & | & +
\end{array}$$

$f(x) > 0 \quad \boxed{\left(-2,\ -\sqrt{3}\right) \cup \left(1,\ \sqrt{3}\right) \cup (2,\ \infty)}$

$f(x) < 0 \quad \boxed{(-\infty,\ -2) \cup \left(-\sqrt{3},\ 1\right) \cup \left(\sqrt{3},\ 2\right)}$



Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = x^5 - 2x^3 - 8x$$

Solution

$$x\left(x^4 - 2x^2 - 8\right) = 0$$

$$\underline{x = 0}\,|$$

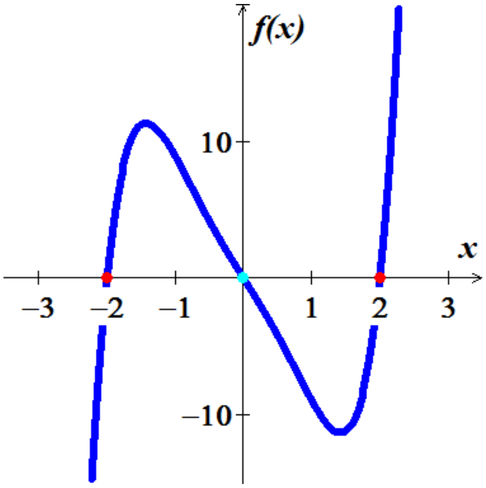
$$x^4 - 2x^2 - 8 = 0.$$

$$x^2 = \frac{2 \pm \sqrt{4+32}}{2}$$

$$= \begin{cases} \frac{2-6}{2} = -2 \\ \frac{2+6}{2} = 4 \end{cases}$$

$$\begin{cases} x^2 = -2 & \rightarrow \quad x = \pm i\sqrt{2} \\ x^2 = 4 & \rightarrow \quad x = \pm 2 \end{cases}$$

$$\underline{x = 0, \ \pm 2, \ \pm i\sqrt{2}}\,|$$



−2 0 2

−	+	−	+

$f(x) > 0$ $\quad \underline{(-2,\ 0) \cup (2,\ \infty)}\,|$

$f(x) < 0$ $\quad \underline{(-\infty,\ -2) \cup (0,\ 2)}\,|$

Exercise

Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and then sketch the graph of $f(x)$

$$f(x) = 3x^6 - 10x^5 - 29x^4 + 34x^3 + 50x^2 - 24x - 24$$

Solution

possibilities for $\frac{c}{d} : \pm\left\{\frac{24}{3}\right\} = \pm\left\{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{8}{3}\right\}$

1	3	−10	−29	34	50	−24	−24	
		3	−7	−36	−2	48	24	
−1	3	−7	−36	−2	48	24	0	→ $3x^5-7x^4-36x^3-2x^2+48x+24=0$
		−3	10	26	−24	−24		
−2	3	−10	−26	24	24	0		→ $3x^4-10x^3-26x^2+24x+24=0$
		−6	32	−12	−24			
$-\frac{2}{3}$	3	−16	6	12	0			→ $3x^3-16x^2+12x-12=0$
		−2	12	−12				
	3	−18	18	0				→ $3x^2-18x+18=0$

$$x^2-6x+6=0$$

$$x=\frac{6\pm\sqrt{36-24}}{2}$$

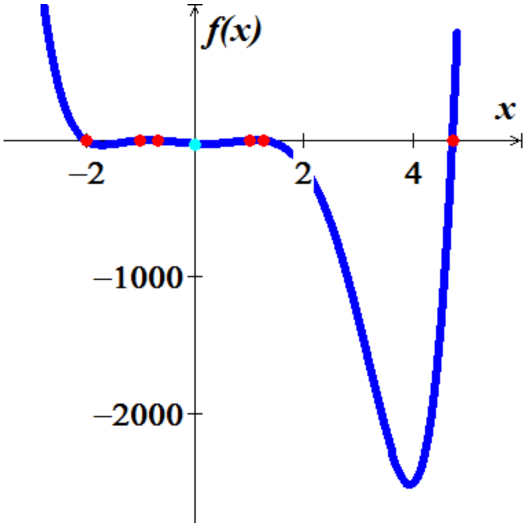
$$=\frac{6\pm 2\sqrt{3}}{2}$$

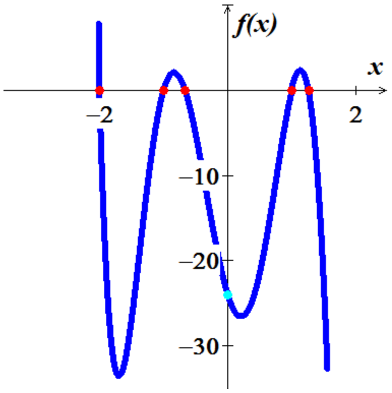
$$\boxed{x=-2,\ -1,\ 1,\ -\frac{2}{3},\ 3\pm\sqrt{3}}$$

	−2		−1		$-\frac{2}{3}$		1		$3-\sqrt{3}$		$3+\sqrt{3}$	
+		−		+		−		+		−		+

$f(x)>0$ $\quad \boxed{(-\infty,\ -2)\cup\left(-1,\ -\frac{2}{3}\right)\cup\left(1,\ 3-\sqrt{3}\right)\cup\left(3+\sqrt{3},\ \infty\right)}$

$f(x)<0$ $\quad \boxed{(-2,\ -1)\cup\left(-\frac{2}{3},\ 1\right)\cup\left(3-\sqrt{3},\ 3+\sqrt{3}\right)}$



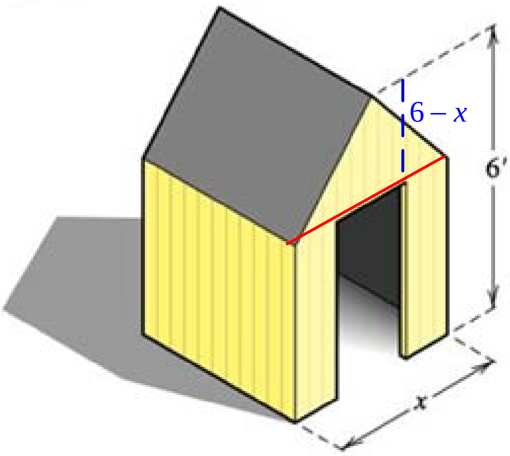


Exercise

A storage shelter is to be constructed in the shape of a cube with a triangular prism forming the roof. The length x of a side of the cube is yet to be determined.

a) If the total height of the structure is 6 *feet*, show that its volume V is given by $V = x^3 + \frac{1}{2}x^2(6-x)$

b) Determine x so that the volume is $80\ ft^3$



Solution

a) $V = V_{cube} + V_{triangle}$

$$= x^3 + \frac{1}{2}x(x)(6-x)$$

$$= \frac{1}{2}x^2(2x+6-x)$$

$$= \frac{1}{2}x^2(x+6)$$

b) $V = \frac{1}{2}x^2(x+6) = 80$

$$x^3 + 6x^2 - 160 = 0$$

possibilities : $\pm\left\{\frac{160}{1}\right\} = \pm\{1,\ 2,\ 4,\ 5,\ 8,\ 10,\ 16,\ 20,\ 32,\ 40,\ 80,\ 160\}$

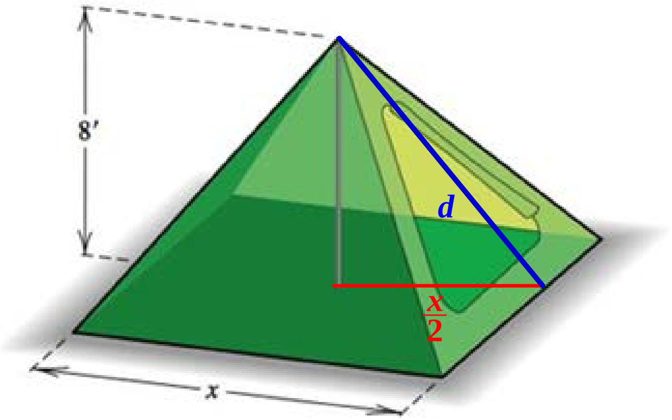
$$\begin{array}{r|rrrr} 4 & 1 & 6 & 0 & -160 \\ & & 4 & 40 & 160 \\ \hline & 1 & 10 & 40 & \boxed{0} \end{array} \quad \to \quad x^2 + 10x + 40 = 0 \Rightarrow x = -5 \pm i\sqrt{15}$$

The solution is: $x = 4$

Exercise

A canvas camping tent is to be constructed in the shape of a pyramid with a square base. An 8–foot pole will form the center support. Find the length x of a side of the base so that the total amount of canvas needed for the sides and bottom is $384\ ft^2$.

Solution



$$d = \sqrt{64 + \frac{x^2}{4}} = \frac{1}{2}\sqrt{x^2 + 256}$$

$$A_{bottom} = x^2$$

$$A_{1-side} = \frac{1}{2}xd$$

$$= \frac{1}{4}x\sqrt{x^2 + 256}$$

$$A_{total} = A_{bottom} + 4A_{1-side}$$

$$= x^2 + x\sqrt{x^2 + 256} = 384$$

$$x\sqrt{x^2 + 256} = 384 - x^2$$

$$\left(x\sqrt{x^2 + 256}\right)^2 = \left(384 - x^2\right)^2$$

$$x^2\left(x^2 + 256\right) = 147,456 - 768x^2 + x^4$$

$$-1,024x^2 + 147,456 = 0$$

$$x = \pm\sqrt{\frac{147,456}{1,024}}$$

$$\underline{= 12\ \ ft|}$$

Exercise

Glasses can be stacked to form a triangular pyramid. The total number of glasses in one of these pyramids is given by

$$T(k) = \frac{1}{6}\left(k^3 + 3k^2 + 2k\right)$$

Where k is the number of levels in the pyramid. If 220 glasses are used to form a triangle pyramid, how many levels are in the pyramid?

Solution

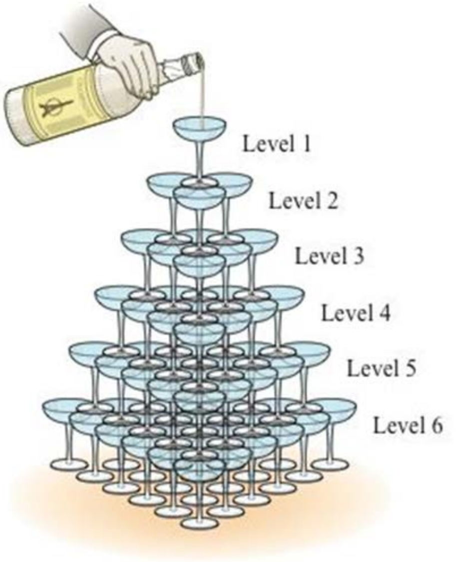
$$\frac{1}{6}\left(k^3 + 3k^2 + 2k\right) = 220$$

$$k^3 + 3k^2 + 2k - 1,320 = 0$$

$$\begin{array}{r|rrrr} 10 & 1 & 3 & 2 & -1320 \\ & & 10 & 130 & 1320 \\ \hline & 1 & 13 & 132 & 0 \end{array} \quad \rightarrow \quad k^2 + 13k + 132 = 0$$

$$k = \frac{-13 \pm \sqrt{-359}}{2} \quad \mathbb{C}$$

The are **10** levels in the pyramid.

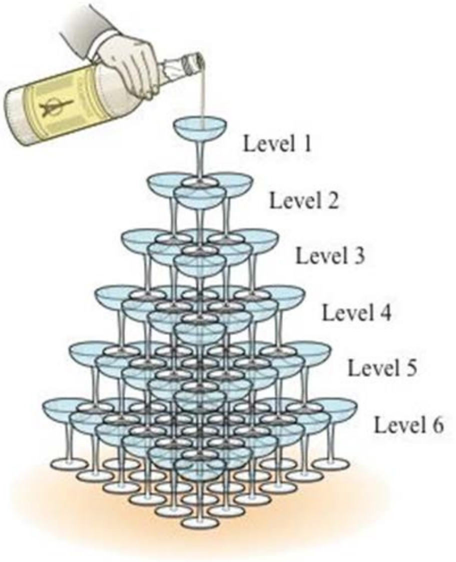


Exercise

Glasses can be stacked to form a triangular pyramid. The total number of glasses in one of these pyramids is given by

$$T(k) = \frac{1}{6}\left(2k^3 + 3k^2 + k\right)$$

Where k is the number of levels in the pyramid. If 140 glasses are used to form a triangle pyramid, how many levels are in the pyramid?



Solution

$$\frac{1}{6}\left(2k^3+3k^2+k\right)=150$$

$$2k^3+3k^2+k-840=0$$

$$\begin{array}{r|rrrr} 7 & 2 & 3 & 1 & -840 \\ & & 14 & 119 & 840 \\ \hline & 2 & 17 & 120 & 0 \end{array} \quad \to \quad 2k^2+17k+120=0$$

$$k=\frac{-17\pm\sqrt{-671}}{4} \qquad \mathbb{C}$$

The are 7 levels in the pyramid.

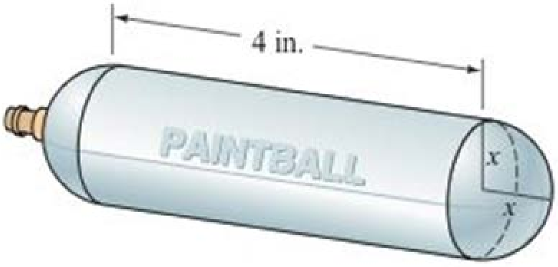
Exercise

A carbon dioxide cartridge for a paintball rifle has the shape of a right circular cylinder with a hemisphere at each end. The cylinder is 4 *inches* long, and the volume of the cartridge is $2\pi \ \ in^3$.

The common interior radius of the cylinder and the hemispheres is denoted by x. Estimate the length of the radius x.

Solution

Volume of the Cartridge = 2 × (Volume of Hemisphere) + Volume of Cylinder



$$Volume\ of\ \ Sphere=\frac{4}{3}\pi x^3$$

$$Volume\ of\ \ Cylinder=4\pi x^2$$

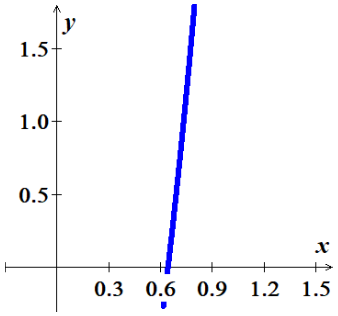
$$Volume\ of\ \ Cartridge=\frac{4}{3}\pi x^3+4\pi x^2$$

$$2\pi=\frac{4}{3}\pi x^3+4\pi x^2$$

$$2x^3+6x^2=3$$

$$2x^3+6x^2-3=0$$

Using Graph:



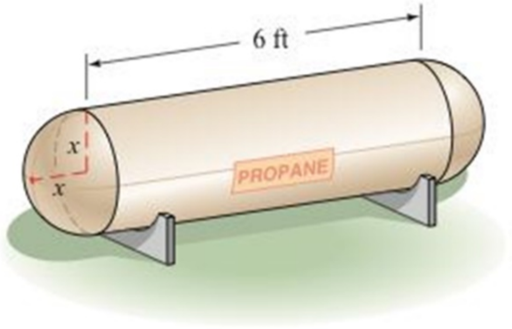
$x\approx 0.64\ \ in.$

Exercise

A propane tank has the shape of a circular cylinder with a hemisphere at each end. The cylinder is 6 *feet* long and the volume of the tank is $9\pi \ ft^3$. Find the length of the radius *x*.

Solution

Volume of the Cartridge = 2 × (Volume of Hemisphere) + Volume of Cylinder



$$Volume\ of\ Sphere = \frac{4}{3}\pi x^3$$

$$Volume\ of\ Cylinder = 6\pi x^2$$

$$Volume\ of\ Cartridge = \frac{4}{3}\pi x^3 + 6\pi x^2$$

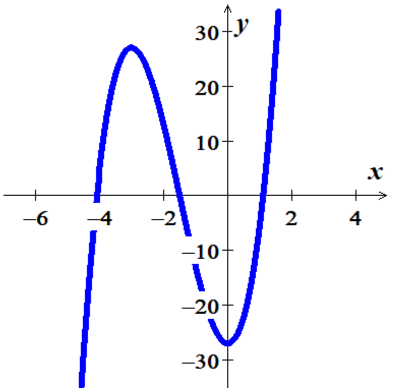
$$9\pi = \frac{4}{3}\pi x^3 + 6\pi x^2$$

$$27 = 4x^3 + 18x^2$$

$$4x^3 + 18x^2 - 27 = 0$$

$$possibilities\ for\ \frac{c}{d} : \pm\left\{\frac{27}{4}\right\} = \pm\left\{1,\ 3,\ 9,\ 27,\ \frac{1}{2},\ \frac{3}{2},\ \frac{9}{2},\ \frac{27}{2},\ \frac{1}{4},\ \frac{3}{4},\ \frac{9}{4},\ \frac{27}{4}\right\}$$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 4 & 18 & 0 & -27 \\ & & -6 & -18 & 27 \\ \hline & 4 & 12 & -18 & 0 \end{array} \quad \to \quad 4x^2 + 12x - 18 = 0$$



$$2x^2 + 6x - 9 = 0$$

$$x = \frac{-6 \pm \sqrt{36+72}}{4}$$

$$= \frac{-6 \pm 6\sqrt{3}}{4}$$

$$= \frac{-3 \pm 3\sqrt{3}}{2}$$

$$x = -\frac{3}{2},\ \frac{-3-3\sqrt{3}}{2},\ \frac{-3+3\sqrt{3}}{2}$$

∴ the length of the radius *x* is $\frac{-3+3\sqrt{3}}{2} \approx 1.1 \ foot$

Exercise

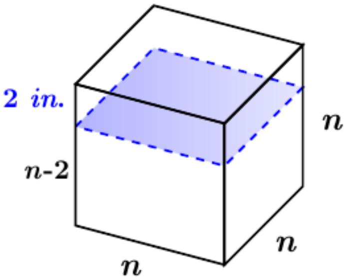
A cube measures *n* inches on each edge. If a slice 2 *inches* thick is cut from one face of the cube, the resulting solid has a volume of $567 \ in^3$. Find *n*.

Solution

$$Volume = n^2(n-2)$$

$$n^3-2n^2=567$$

$$n^3-2n^2-567=0$$



$$\text{possibilities for } \frac{c}{d} := \pm\{1,\ 3,\ 7,\ 9,\ 21,\ 27,\ 63,\ 81,\ 189,\ 567\}$$

$$\begin{array}{r|rrrr} 9 & 1 & -2 & 0 & -567 \\ & & 9 & 63 & 567 \\ \hline & 1 & 7 & 63 & 0 \end{array} \quad \rightarrow \quad n^2+7n+63=0$$

$$n=\frac{-7\pm\sqrt{49-252}}{2}$$

$$=\frac{-7\pm i\sqrt{203}}{2} \quad ✗$$

$$\therefore \quad \underline{n=9}\,\rvert$$

Exercise

A cube measures n inches on each edge. If a slice 1 *inch* thick is cut from one face of the cube and then a slice 3 inches thick is cut from another face of the cube, the resulting solid has a volume of 1560 in^3. Find the dimensions of the original cube.

Solution

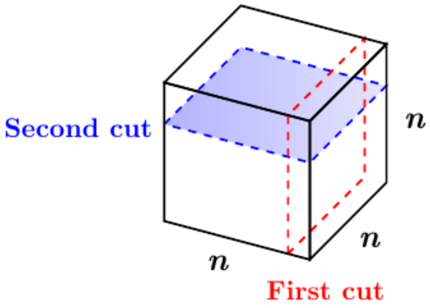
$$Volume=n(n-1)(n-3)$$

$$n^3-4n^2+3n=1560$$

$$n^3-4n^2+3n-1560=0$$

$$\text{possibilities for } \frac{c}{d} := \pm\left\{\begin{matrix}1,\ 2,\ 4,\ 5,\ 6,\ 8,\ 10,\ 12,\ 13,\ 15,\ 20,\ 24,\ 26,\ 30,\ 39, \\ 40,\ 52,\ 60,\ 65,\ 78,\ 104,\ 120,\ 130,\ 156,\ 195,\ 260,\ 312,\ 390,\ 780,\ 1560\end{matrix}\right\}$$

$$\begin{array}{r|rrrr} 13 & 1 & -4 & 3 & -1560 \\ & & 13 & 117 & 1560 \\ \hline & 1 & 9 & 120 & 0 \end{array} \quad \rightarrow \quad n^2+9n+120=0$$



$$n=\frac{-9\pm\sqrt{81-480}}{2}$$

$$=\frac{-9\pm i\sqrt{399}}{2} \quad ✗$$

$$\therefore \quad \underline{n=13}\,\rvert$$

Exercise

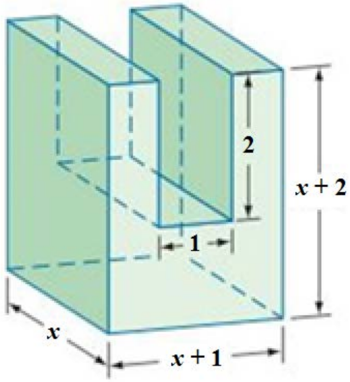
For what value of x will the volume of the following solid be $112 \ \ in^3$

Solution

$$\text{Volume of the bottom portion } = x^2(x+1)$$

$$\text{Volume of one side portion } = 2x\left(\frac{1}{2}x\right)$$

$$= x^2$$



$$\text{Total Volume } = x^2(x+1) + 2x^2$$

$$112 = x^3 + 3x^2$$

$$x^3 + 3x^2 - 112 = 0$$

possibilities for $\frac{c}{d} := \pm\{1,\ 2,\ 4,\ 8,\ 14,\ 28,\ 56,\ 112\}$

$$\begin{array}{r|rrrr} 4 & 1 & 3 & 0 & -112 \\ & & 4 & 28 & 112 \\ \hline & 1 & 7 & 28 & 0 \end{array} \quad \rightarrow \quad x^2 + 7x + 28 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 112}}{2}$$

$$= \frac{-7 \pm 3i\sqrt{7}}{2} \quad ✗$$

$$\therefore \quad \underline{x = 4}|$$

Exercise

For what value of x will the volume of the following solid be $208 \ \ in^3$

Solution

$$\text{Volume of the bottom portion } = (2x+1)(x+5)(x+2-3)$$

$$= \left(2x^2 + 11x + 5\right)(x-1)$$

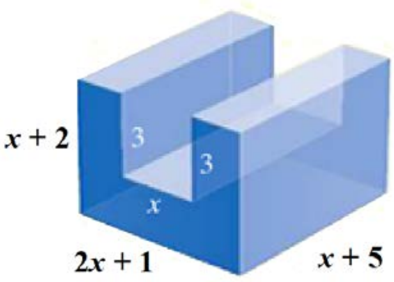
$$= 2x^3 + 11x^2 + 5x - 2x^2 - 11x - 5$$

$$= 2x^3 + 9x^2 - 6x - 5$$

$$\text{Volume of one side portion } = (3)\frac{1}{2}(2x+1-x)(x+5)$$

$$= \frac{3}{2}(x+1)(x+5)$$

$$= \frac{3}{2}\left(x^2 + 6x + 5\right)$$



$$\text{Total Volume } = 2x^3 + 9x^2 - 6x - 5 + 2\left(\frac{3}{2}\right)\left(x^2 + 6x + 5\right)$$

$$208 = 2x^3 + 9x^2 - 6x - 5 + 3x^2 + 18x + 15$$

$$2x^3 + 12x^2 + 12x - 198 = 0$$

$$x^3 + 6x^2 + 6x - 99 = 0$$

possibilities for $\frac{c}{d} := \pm\{1,\ 3,\ 9,\ 11,\ 33,\ 99\}$

$$\begin{array}{r|rrrr} 3 & 1 & 6 & 6 & -99 \\ & & 3 & 27 & 99 \\ \hline & 1 & 9 & 33 & 0 \end{array} \quad \rightarrow \quad x^2 + 9x + 33 = 0$$

$$x = \frac{-9 \pm \sqrt{81 - 132}}{2}$$

$$= \frac{-9 \pm i\sqrt{51}}{2} \quad \times$$

$$\therefore \quad \underline{x = 3}$$

Exercise

The length of rectangular box is 1 *inch* more than twice the height of the box, and the width is 3 *inches* more than the height. If the volume of the box is 126 in^3, find the dimensions of the box.

Solution

$$Volume = x(2x + 1)(x + 3)$$

$$2x^3 + 7x^2 + 3x = 126$$

$$2x^3 + 7x^2 + 3x - 126 = 0$$

possibilities for $\frac{c}{d} := \pm\left\{\frac{126}{2}\right\}$

$$= \pm\left\{1,\ 2,\ 3,\ 6,\ 9,\ 14,\ 21,\ 42,\ 63,\ 126,\ \frac{1}{2},\ \frac{3}{2},\ \frac{9}{2},\ \frac{21}{2},\ \frac{63}{2}\right\}$$

$$\begin{array}{r|rrrr} 3 & 2 & 7 & 3 & -126 \\ & & 6 & 39 & 126 \\ \hline & 2 & 13 & 42 & 0 \end{array} \quad \rightarrow \quad 2x^2 + 13x + 42 = 0$$

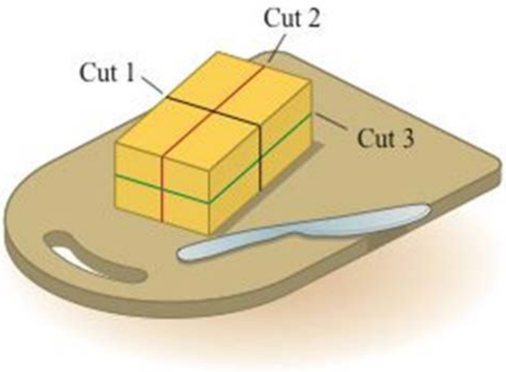
$$x = \frac{-13 \pm \sqrt{169 - 336}}{4}$$

$$= \frac{-13 \pm i\sqrt{167}}{4} \quad \times$$

$$\therefore \quad \underline{x = 3}$$

Exercise

One straight cut through a thick piece of cheese produces two pieces. Two straight cuts can produce a maximum of four pieces. Two straight cuts can produce a maximum of four pieces. Three straight cuts can produce a maximum of eight pieces.



You might be inclined to think that every additional cut doubles number of pieces. However, for four straight cuts, you get a maximum of 15 pieces. The maximum number of pieces *P* that can be produced by *n* straight cuts is given by

$$P(n)=\frac{n^3+5n+6}{6}$$

a) Determine number of pieces that can be produces by five straight cuts.

b) What is the fewest number of straight cuts that are needed to produce 64 pieces?

Solution

a) $P(5)=\frac{5^3+25+6}{6}$

$$\underline{=26\,}\big|$$

b) $\frac{n^3+5n+6}{6}=64$

$$n^3+5n+6=384$$

$$n^3+5n-378=0$$

$$possibilities\ for\ \frac{c}{d}:=\pm\{378\}$$

$$=\pm\{1,\ 2,\ 3,\ 6,\ 7,\ 9,\ 14,\ 18,\ 21,\ 27,\ 42,\ 54,\ 63,\ 126,\ 189,\ 378\}$$

$$\begin{array}{r|rrrr} 7 & 1 & 0 & 5 & -378 \\ & & 7 & 49 & 378 \\ \hline & 1 & 7 & 54 & 0 \end{array} \quad \rightarrow \quad n^2+7n+54=0$$

$$n=\frac{-7\pm\sqrt{49-216}}{2}$$

$$=\frac{-7\pm i\sqrt{167}}{2} \quad \times$$

$$\therefore\ \underline{n=7\,}\big|$$

Exercise

The number of ways one can select three cards from a group of *n* cards (the order of the selection matters), where $n \ge 3$, is given by $P(n) = n^3 - 3n^2 + 2n$. For a certain card trick, a magician has determined that there are exactly 504 *ways* to choose three cards from a given group. How many cards are in the group?

Solution

$$P(n) = n^3 - 3n^2 + 2n = 504$$

$$n^3 - 3n^2 + 2n - 504 = 0$$

possibilities for $\frac{c}{d} := \pm\{504\}$

$$= \pm\left\{\begin{matrix} 1,\ 2,\ 3,\ 4,\ 6,\ 7,\ 8,\ 9,\ 12,\ 14,\ 18,\ 21, \\ 24,\ 28,\ 36,\ 42,\ 56,\ 63,\ 72,\ 84,\ 126,\ 168,\ 252,\ 504 \end{matrix}\right\}$$

$$\begin{array}{r|rrrr} 9 & 1 & -3 & 2 & -504 \\ & & 9 & 54 & 504 \\ \hline & 1 & 6 & 56 & 0 \end{array} \rightarrow \quad n^2 + 6n + 56 = 0$$

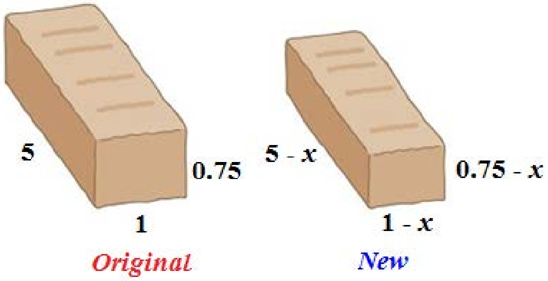
$$n = \frac{-6 \pm \sqrt{36 - 224}}{2}$$

$$= -3 \pm i\sqrt{47} \quad \times$$

$$\therefore \quad \underline{n = 9}\,\rvert$$

Exercise

A nutrition bar in the shape of a rectangular solid measure 0.75 *in*. by 1 *in*. by 5 *inches*.



To reduce costs, the manufacturer has decided to decrease each of the dimensions of the nutrition bar by *x inches*, what value of *x* will produce a new bar with a volume that is $0.75\ in^3$ less than the present bar's volume.

Solution

$$V_{original} = (5)(1)\left(\frac{3}{4}\right)$$

$$= \frac{15}{4}$$

$V_{new} = (5-x)(1-x)\left(\frac{3}{4}-x\right) \qquad \left(x < \frac{3}{4}\right)$

$\left(5-6x+x^2\right)\left(\frac{3-4x}{4}\right) = \frac{15}{4} - \frac{3}{4}$

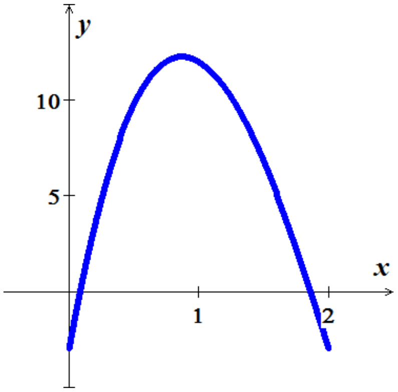
$15 - 20x - 18x + 24x^2 + 3x^2 - 4x^3 = 4(3)$

$4x^3 - 27x^2 + 38x - 3 = 0$

From graph table:

```
0.08200  -0.06334
0.08400   0.00386
```

$\underline{x \approx 0.083 \ \ in.}|$



Exercise

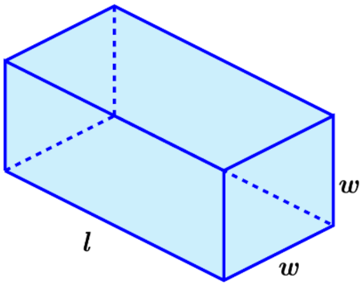
A rectangular box is square on two ends and has length plus girth of 81 *inches*. (Girth: distance *around* the box). Determine the possible lengths $l \ (l > w)$ of the box if its volume is $4900 \ \ in^3$.

Solution

$81 = l + 4w$
$l = 81 - 4w$

$V = lw^2$

$\quad = (81-4w)w^2$



$-4w^3 + 81w^2 = 4900$

$4w^3 - 81w^2 + 4900 = 0$

$possibilities \ for \ \frac{c}{d} := \pm\left\{\frac{4900}{4}\right\} = \pm\left\{\begin{array}{l}1, \ 2, \ 4, \ 7, \ 10, \ 14, \ 20, \ 28, \ 49, \\ \ 100, \ 175, \ 245, \ 350, \ 490, \ 700, \ 1225, \ 2450, \ 4900, \cdots\end{array}\right\}$

$$\begin{array}{r|rrrr} 14 & 4 & -81 & 0 & 4900 \\ & & 56 & -350 & -4900 \\ \hline & 4 & -25 & -350 & 0 \end{array} \rightarrow \ 4w^2 - 25w - 350 = 0$$

$w = \frac{25 \pm \sqrt{625+5600}}{8}$

$\quad = \frac{25 \pm 5\sqrt{249}}{8}$

$\quad = \begin{cases} \frac{25 - 5\sqrt{249}}{8} < 0 \\ \frac{25 + 5\sqrt{249}}{8} \approx 13 \end{cases}$

$l = 81 - 4(14) = 25$

$l = 81 - 4(13) = 29$

∴ the possible lengths l are around **25** *in*. ***or*** **29** *in*.