u point

$\vec{u}$ vector

Ex

$\vec{u} = \langle -1, 3, 1 \rangle$

$\vec{v} = \langle 4, 7, 0 \rangle$

a) $2\vec{u} + 3\vec{v} = 2\langle -1, 3, 1\rangle + 3\langle 4, 7, 0\rangle$

$= \langle -2, 6, 2\rangle + \langle 12, 21, 0\rangle$

$= \boxed{\langle 10, 27, 2\rangle}$

b) $\vec{u} - \vec{v} = \langle -1, 3, 1\rangle - \langle 4, 7, 0\rangle$

$= \boxed{\langle -5, -4, 1\rangle}$

c) $\left|\frac{1}{2}\vec{u}\right| = \frac{1}{2}|\vec{u}|$ $\qquad |k\vec{u}| = |k|\|\vec{u}\|$

$= \frac{1}{2}\sqrt{1+9+1}$

$= \boxed{\frac{\sqrt{11}}{2}}$

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$

$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + w$

$\vec{u} + \vec{0} = \vec{u}$

$\vec{u} + (-\vec{u}) = \vec{0}$

$0\vec{u} = \vec{0}$

$1\vec{u} = \vec{u}$

$a(b\vec{u}) = (ab)\vec{u}$

$(a+b)\vec{u} = a\vec{u} + b\vec{u}$

$a\vec{u} + a\vec{v} = a(\vec{u} + \vec{v})$

Def^n

$\vec{v} = \langle v_1 , v_2 \rangle$

$\vec{v} = \langle v_1 , v_2 , v_3 \rangle$

$|\vec{v}|_2 = \sqrt{v_1^2 + v_2^2}$

$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

$\vec{v} = \vec{PQ}$ where $P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$

$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + \underbrace{(y_2 - y_1)^2}_{[y_1 - y_2]^2} + (z_2 - z_1)^2}$

Ex

$P(-3, 4, 1)$

$Q(-5, 2, 2)$

$|\vec{PQ}| = \sqrt{(-3+5)^2 + (4-2)^2 + (1-2)^2}$

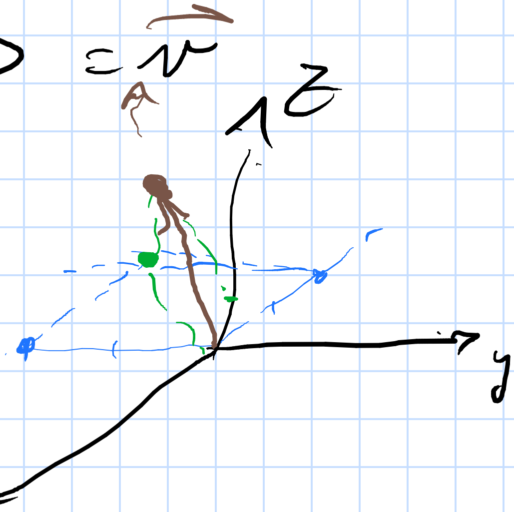
$= \sqrt{4 + 4 + 1}$

$= 3$ unit

$\vec{PQ} = \langle -2, -2, 1 \rangle = \vec{v}$

$\vec{QP} = \langle 2, 2, -1 \rangle$

2nd — 1st



$\hat{i}$, $\hat{j}$, $\hat{k}$

$$\left.\begin{aligned} \hat{i} &= \langle 1, 0, 0 \rangle \\ \hat{j} &= \langle 0, 1, 0 \rangle \\ \hat{k} &= \langle 0, 0, 1 \rangle \end{aligned}\right\} \text{ \underline{unit vectors}}$$

$$\vec{N} = \langle N_1, N_2, N_3 \rangle$$
$$= N_1 \hat{i} + N_2 \hat{j} + N_3 \hat{k}$$

Ex $P_1 (1, 0, 1)$ $P_2 (3, 2, 0)$

$$\text{unit vector} = \frac{\text{vector}}{|\text{vector}|} \quad \text{of } P_1 P_2$$

$$\overrightarrow{P_1 P_2} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{u} = \frac{\overrightarrow{P_1 P_2}}{|\overrightarrow{P_1 P_2}|} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{4+4+1}}$$

$$= \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$

Ex $\vec{v} = 3\hat{i} - 4\hat{j}$

Speed : magnitude $= |\vec{v}|$

$|\vec{v}| = \sqrt{9+16}$
$= \boxed{5}$

unit vector : direction $= \frac{\vec{v}}{|\vec{v}|}$

$\vec{u} = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$

$\vec{v} = 3\hat{i} - 4\hat{j}$
$= \underbrace{5}_{\text{speed}} \underbrace{\left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)}_{\text{direction}}$

direction $= \frac{\vec{v}}{|\vec{v}|}$ unit vector

$\vec{v} = |\vec{v}| \frac{\vec{v}}{|\vec{v}|}$: magnitude & direction

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Ex $\vec{v} = 2\hat{i} + 2\hat{j} - \hat{k}$

$|\vec{v}| = \sqrt{4+4+1}$
$= \boxed{3}$

$\vec{F} = |\vec{v}| \frac{\vec{v}}{|\vec{v}|} = 3\left(\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}\right)$

$= 3 \langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \rangle$

Midpoint

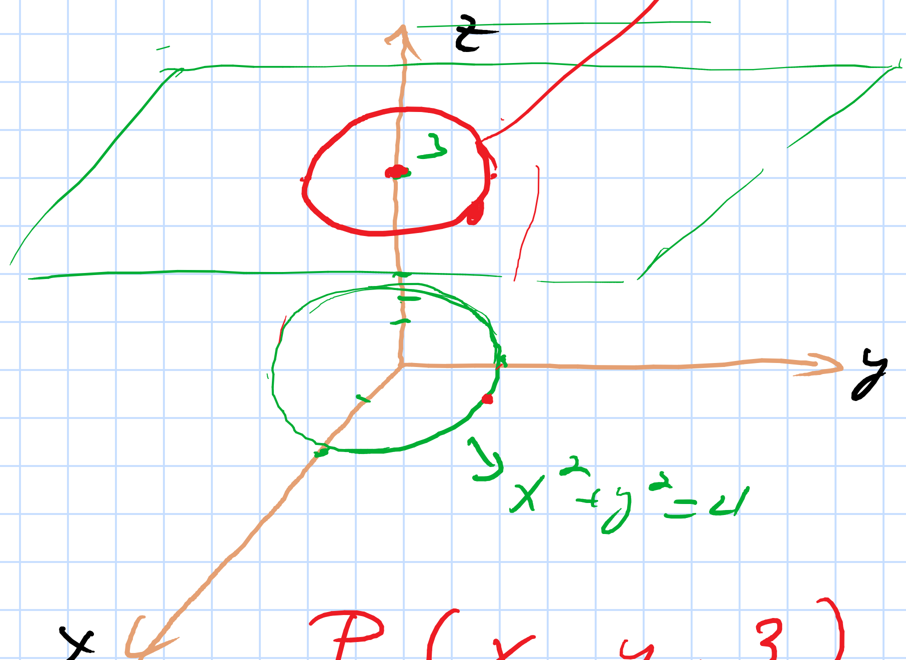
$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

Ex $P_1(3,-2,0)$ $P_2(7,4,4)$

$M=(5,1,2)$

(point)

Ex $P(x,y,z)$ $\begin{cases} x^2+y^2=4 \\ z=3 \end{cases}$



$P(x,y,3)$

$$|\overrightarrow{P_1P_2}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

2-dim: circle: $(x-x_0)^2+(y-y_0)^2=r^2$

3-dim: sphere

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=a^2$$

2 sphere $r < R$ $\boxed{R^2}$

$$x^2+y^2+z^2+3x-4z+1=0$$

$$\underbrace{x^2+3x+\left(\frac{3}{2}\right)^2}+y^2+z^2-4z+(-2)^2=-1+\frac{9}{4}+4$$

$$\left(x+\frac{3}{2}\right)^2+y^2+(z-2)^2=\frac{21}{4}$$

center: $\left(-\frac{3}{2}, 0, 2\right)$

radius: $\frac{\sqrt{21}}{2}$