

Solution

Section 3.5 – Exponential and logarithmic Equations

Exercise

Solve the equation: $2^x = 128$

Solution

$$2^x = 2^7$$

$$\underline{x = 7}$$

Exercise

Solve the equation: $3^x = 243$

Solution

$$3^x = 3^5$$

$$\underline{x = 5}$$

Exercise

Solve the equation: $5^x = 70$

Solution

$$\underline{x = \log_5 70}$$

Exercise

Solve the equation: $6^x = 50$

Solution

$$\underline{x = \log_6 50}$$

Exercise

Solve the equation: $5^x = 134$

Solution

$$\underline{x = \log_5 134}$$

Exercise

Solve the equation: $7^x = 12$

Solution

$$\underline{x = \log_7 12}$$

Exercise

Solve the equation: $9^x = \frac{1}{\sqrt[3]{3}}$

Solution

$$(3^2)^x = \frac{1}{3^{1/3}}$$

$$3^{2x} = 3^{-1/3}$$

$$2x = -\frac{1}{3}$$

$$\underline{x = -\frac{1}{6}}$$

Exercise

Solve the equation: $49^x = \frac{1}{343}$

Solution

$$(7^2)^x = \frac{1}{7^3}$$

$$7^{2x} = 7^{-3}$$

$$2x = -3$$

$$\underline{x = -\frac{3}{2}}$$

Exercise

Solve the equation: $2^{5x+3} = \frac{1}{16}$

Solution

$$2^{5x+3} = 2^{-4}$$

$$5x + 3 = -4$$

$$5x = -7$$

$$\underline{x = -\frac{7}{5}}$$

Exercise

Solve the equation: $\left(\frac{2}{5}\right)^x = \frac{8}{125}$

Solution

$$\left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^3$$

$$\underline{x = 3}$$

Exercise

Solve the equation: $2^{3x-7} = 32$

Solution

$$\begin{aligned} 2^{3x-7} &= 32 \\ &= 2^5 \end{aligned}$$

$$3x - 7 = 5$$

add 7 on both sides

$$3x = 12$$

Divide by 3

$$\underline{x = 4}$$

Exercise

Solve the equation: $4^{2x-1} = 64$

Solution

$$4^{2x-1} = 4^3$$

$$2x - 1 = 3$$

$$2x = 4$$

$$\underline{x = 2}$$

Exercise

Solve the equation: $3^{1-x} = \frac{1}{27}$

Solution

$$3^{1-x} = \frac{1}{3^3}$$

$$3^{1-x} = 3^{-3}$$

$$1 - x = -3$$

$$\underline{x = 4}$$

Exercise

Solve the equation: $2^{-x^2} = 5$

Solution

$$\ln 2^{-x^2} = \ln 5$$

$$-x^2 \ln 2 = \ln 5$$

$$x^2 = -\frac{\ln 5}{\ln 2} \Rightarrow \text{No Solution}$$

Exercise

Solve the equation: $2^{-x} = 8$

Solution

$$2^{-x} = 2^3$$

$$-x = 3$$

$$\underline{x = -3}$$

Exercise

Solve the equation: $\left(\frac{1}{3}\right)^x = 81$

Solution

$$\left(\frac{1}{3}\right)^x = 81$$

$$\left(3^{-1}\right)^x = 3^4$$

$$3^{-x} = 3^4$$

$$-x = 4$$

$$\underline{x = -4}$$

Exercise

Solve the equation: $3^{-x} = 120$

Solution

$$-x = \log_3 120$$

Convert to Log

$$x = -\log_3 120$$

$$\underline{= \log_3 \frac{1}{120}} \quad |$$

Exercise

Solve the equation: $27 = 3^{5x} 9^{x^2}$

Solution

$$\begin{aligned} 3^3 &= 3^{5x} (3^2)^{x^2} \\ &= 3^{5x} 3^{2x^2} \\ &= 3^{5x+2x^2} \end{aligned}$$

$$2x^2 + 5x = 3$$

$$2x^2 + 5x - 3 = 0$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$x = \left\{ \begin{array}{l} \frac{-5-7}{6} = -2 \\ \frac{-5+7}{6} = \frac{1}{3} \end{array} \right. |$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3) \ln 4 = -x \ln 3$$

$$x \ln 4 + 3 \ln 4 = -x \ln 3$$

$$x \ln 4 + x \ln 3 = -3 \ln 4$$

$$x(\ln 4 + \ln 3) = -3 \ln 4$$

$$\underline{x = \frac{-3 \ln 4}{(\ln 4 + \ln 3)}} \quad |$$

Exercise

Solve the equation: $2^{x+4} = 8^{x-6}$

Solution

$$2^{x+4} = (2^3)^{x-6}$$

$$2^{x+4} = 2^{3x-18}$$

$$x + 4 = 3x - 18$$

$$2x = 22$$

$$\underline{x = 11}$$

Exercise

Solve the equation: $8^{x+2} = 4^{x-3}$

Solution

$$(2^3)^{x+2} = (2^2)^{x-3}$$

$$2^{3(x+2)} = 2^{2(x-3)}$$

$$3(x+2) = 2(x-3)$$

$$3x + 6 = 2x - 6$$

$$3x - 2x = -6 - 6$$

$$\underline{x = -12}$$

Exercise

Solve the equation: $7^x = 12$

Solution

$$\underline{x = \log_7 12}$$

Convert to Log

Exercise

Solve the equation: $5^{x+4} = 4^{x+5}$

Solution

$$\ln 5^{x+4} = \ln 4^{x+5}$$

$$(x+4) \ln 5 = (x+5) \ln 4$$

$$x \ln 5 + 4 \ln 5 = x \ln 4 + 5 \ln 4$$

$$(\ln 5 - \ln 4)x = 5 \ln 4 - 4 \ln 5$$

$$\underline{x = \frac{5 \ln 4 - 4 \ln 5}{\ln 5 - \ln 4}}$$

Exercise

Solve the equation: $5^{x+2} = 4^{1-x}$

Solution

$$\ln 5^{x+2} = \ln 4^{1-x}$$

$$(x+2)\ln 5 = (1-x)\ln 4$$

$$x\ln 5 + 2\ln 5 = \ln 4 - x\ln 4$$

$$(\ln 5 + \ln 4)x = \ln 4 - 2\ln 5$$

$$x = \frac{\ln 4 - 2\ln 5}{\ln 5 + \ln 4}$$

Exercise

Solve the equation: $3^{2x-1} = 0.4^{x+2}$

Solution

$$\ln 3^{2x-1} = \ln(0.4^{x+2})$$

$$(2x-1)\ln 3 = (x+2)\ln \frac{4}{10}$$

$$2x\ln 3 - \ln 3 = x\ln \frac{2}{5} + 2\ln \frac{2}{5}$$

$$(2\ln 3 - \ln \frac{2}{5})x = \ln 3 + 2\ln \frac{2}{5}$$

$$x = \frac{\ln 3 + 2\ln 0.4}{2\ln 3 - \ln 0.4}$$

Exercise

Solve the equation: $4^{3x-5} = 16$

Solution

$$4^{3x-5} = 4^2$$

$$3x-5 = 2$$

$$3x = 7$$

$$x = \frac{7}{3}$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3)\ln 4 = -x\ln 3$$

$$x\ln 4 + 3\ln 4 = -x\ln 3$$

$$(\ln 4 + \ln 3)x = -3\ln 4$$

$$x = -\frac{3\ln 4}{\ln 4 + \ln 3}$$

Exercise

Solve the equation: $7^{2x+1} = 3^{x+2}$

Solution

$$\ln 7^{2x+1} = \ln 3^{x+2}$$

$$(2x+1)\ln 7 = (x+2)\ln 3$$

$$2x\ln 7 + \ln 7 = x\ln 3 + 2\ln 3$$

$$2x\ln 7 - x\ln 3 = 2\ln 3 - \ln 7$$

$$x(2\ln 7 - \ln 3) = 2\ln 3 - \ln 7$$

$$x = \frac{2\ln 3 - \ln 7}{2\ln 7 - \ln 3}$$

Exercise

Solve the equation: $3^{x-1} = 7^{2x+5}$

Solution

$$\ln 3^{x-1} = \ln 7^{2x+5}$$

$$(x-1)\ln 3 = (2x+5)\ln 7$$

$$x\ln 3 - \ln 3 = 2x\ln 7 + 5\ln 7$$

$$x\ln 3 - 2x\ln 7 = \ln 3 + 5\ln 7$$

$$x(\ln 3 - 2\ln 7) = \ln 3 + 5\ln 7$$

$$x = \frac{\ln 3 + 5\ln 7}{\ln 3 - 2\ln 7}$$

Exercise

Solve the equation: $4^{x-2} = 2^{3x+3}$

Solution

$$(2^2)^{x-2} = 2^{3x+3}$$

$$2^{2x-4} = 2^{3x+3}$$

$$2x - 4 = 3x + 3$$

$$2x - 3x = 4 + 3$$

$$-x = 7$$

$$\underline{x = -7}$$

Exercise

Solve the equation: $3^{5x-8} = 9^{x+2}$

Solution

$$3^{5x-8} = (3^2)^{x+2}$$

$$3^{5x-8} = 3^{2x+4}$$

$$5x - 8 = 2x + 4$$

$$5x - 2x = 8 + 4$$

$$3x = 12$$

$$\underline{x = 4}$$

Exercise

Solve the equation: $3^{x+4} = 2^{1-3x}$

Solution

$$\ln 3^{x+4} = \ln 2^{1-3x}$$

$$(x+4) \ln 3 = (1-3x) \ln 2$$

$$x \ln 3 + 4 \ln 3 = \ln 2 - 3x \ln 2$$

$$x \ln 3 + 3x \ln 2 = \ln 2 - 4 \ln 3$$

$$x(\ln 3 + 3 \ln 2) = \ln 2 - 4 \ln 3$$

$$\underline{x = \frac{\ln 2 - 4 \ln 3}{\ln 3 + 3 \ln 2}}$$

'ln' both sides

Power Rule

Distribute

Exercise

Solve the equation: $3^{2-3x} = 4^{2x+1}$

Solution

$$\ln 3^{2-3x} = \ln 4^{2x+1}$$

'ln' both sides

$$(2-3x)\ln 3 = (2x+1)\ln 4$$

Power Rule

$$2\ln 3 - 3x\ln 3 = 2x\ln 4 + \ln 4$$

$$-3x\ln 3 - 2x\ln 4 = \ln 4 - 2\ln 3$$

$$-x(3\ln 3 + 2\ln 4) = \ln 4 - 2\ln 3$$

$$x = -\frac{\ln 4 - 2\ln 3}{3\ln 3 + 2\ln 4}$$

$$= -\frac{\ln 4 - \ln 3^2}{\ln 3^3 + \ln 4^2}$$

$$= \frac{\ln 9 - \ln 4}{\ln 27 + \ln 16}$$

$$= \frac{\ln \frac{9}{4}}{\ln 432}$$

$$= \log_{432} \frac{9}{4}$$

Exercise

Solve the equation: $4^{x+3} = 3^{-x}$

Solution

$$\ln 4^{x+3} = \ln 3^{-x}$$

$$(x+3)\ln 4 = -x\ln 3$$

$$x\ln 4 + 3\ln 4 = -x\ln 3$$

$$x\ln 4 + x\ln 3 = -3\ln 4$$

$$x(\ln 4 + \ln 3) = -3\ln 4$$

$$x = \frac{-3\ln 4}{(\ln 4 + \ln 3)}$$

Exercise

Solve the equation: $7^{x+6} = 7^{3x-4}$

Solution

$$x + 6 = 3x - 4$$

$$4 + 6 = 3x - x$$

$$10 = 2x$$

$$\underline{x = 5}$$

Exercise

Solve the equation: $2^{-100x} = (0.5)^{x-4}$

Solution

$$2^{-100x} = \left(\frac{1}{2}\right)^{x-4}$$

$$2^{-100x} = \left(2^{-1}\right)^{x-4}$$

$$2^{-100x} = 2^{-x+4}$$

$$-100x = -x + 4$$

$$-100x + x = 4$$

$$-99x = 4$$

$$\underline{x = -\frac{4}{99}}$$

Exercise

Solve the equation: $4^x \left(\frac{1}{2}\right)^{3-2x} = 8 \cdot (2^x)^2$

Solution

$$\left(2^2\right)^x \left(2^{-1}\right)^{3-2x} = 2^3 \cdot 2^{2x}$$

$$2^{2x} 2^{2x-3} = 2^{3+2x}$$

$$2^{2x+2x-3} = 2^{3+2x}$$

$$2^{4x-3} = 2^{3+2x}$$

$$4x - 3 = 3 + 2x$$

$$4x - 2x = 3 + 3$$

$$2x = 6$$

$$\underline{x = 3}$$

Exercise

Solve the equation: $5^x + 125(5^{-x}) = 30$

Solution

$$5^x \cdot 5^x + 125(5^{-x}) \cdot 5^x = 30(5^x)$$

$$5^{2x} + 125 = 30(5^x)$$

$$5^{2x} - 30(5^x) + 125 = 0 \quad \text{Solve for } 5^x$$

$$5^x = 5$$

$$x = 1$$

$$5^x = 25 = 5^2$$

$$x = 2$$

$$\underline{x = 1, 2}$$

Exercise

Solve the equation: $4^x - 3(4^{-x}) = 8$

Solution

$$4^x \cdot 4^x - 3(4^{-x}) \cdot 4^x = 8(4^x)$$

$$4^{2x} - 3 = 8(4^x)$$

$$4^{2x} - 8(4^x) - 3 = 0 \quad \text{Solve for } 4^x$$

$$4^x = 4 + \sqrt{19}$$

$$4^x = 4 - \sqrt{19} < 0$$

$$x \ln 4 = \ln(4 + \sqrt{19})$$

$$\underline{x = \frac{\ln(4 + \sqrt{19})}{\ln 4}}$$

Exercise

Solve the equation: $5^{3x-6} = 125$

Solution

$$5^{3x-6} = 5^3$$

$$3x - 6 = 3$$

$$3x = 9$$

$$\underline{x = 3}$$

Exercise

Solve the equation: $e^x = 15$

Solution

$$x = \ln 5 \quad |$$

Convert to Log

Exercise

Solve the equation: $e^{x+1} = 20$

Solution

$$x + 1 = \ln 20$$

Convert to Log

$$x = -1 + \ln 20 \quad |$$

Exercise

Solve the equation: $9e^x = 107$

Solution

$$e^x = \frac{107}{9}$$

$$\ln e^x = \ln \left(\frac{107}{9} \right)$$

$$x \ln e = \ln \left(\frac{107}{9} \right)$$

$$x = \ln \left(\frac{107}{9} \right) \quad |$$

Exercise

Solve the equation: $e^{x \ln 3} = 27$

Solution

$$x \ln 3 = \ln 27$$

Convert to Log

$$x \ln 3 = \ln 3^3$$

$$x = \frac{3 \ln 3}{\ln 3}$$

$$= 3 \quad |$$

Exercise

Solve the equation: $e^{x^2} = e^{7x-12}$

Solution

$$e^{x^2} = e^{7x-12}$$

$$x^2 = 7x - 12$$

$$x^2 - 7x + 12 = 0$$

$$\underline{x = 3, 4}$$

Exercise

Solve the equation: $f(x) = xe^x + e^x$

Solution

$$xe^x + e^x = 0$$

$$e^x(x+1) = 0$$

$$e^x \neq 0 \quad x+1 = 0$$

$$\underline{x = -1} \quad (\text{Only solution})$$

Exercise

Solve the equation $f(x) = x^3(4e^{4x}) + 3x^2e^{4x}$

Solution

$$x^3(4e^{4x}) + 3x^2e^{4x} = 0$$

$$x^2e^{4x}(4x+3) = 0$$

$$x^2 = 0 \quad 4x+3 = 0$$

$$x = 0, 0 \quad x = -\frac{3}{4}$$

$$\text{The solutions are: } \underline{x = 0, 0, -\frac{3}{4}}$$

Exercise

Solve the equation: $e^{2x} - 2e^x - 3 = 0$

Solution

$$(e^x)^2 - 2e^x - 3 = 0$$

$$\begin{cases} e^x = -1 \quad \times \rightarrow \text{Impossible} \\ e^x = 3 \rightarrow \underline{x = \ln 3} \end{cases}$$

Exercise

Solve the equation: $e^{0.08t} = 2500$

Solution

$$\ln(e^{0.08t}) = \ln 2500$$

$$0.08t = \ln(50)^2$$

$$t = \frac{200 \ln 50}{8} \\ = \underline{25 \ln 50}$$

Exercise

Solve the equation: $e^{x^2} = 200$

Solution

$$\ln e^{x^2} = \ln 200$$

Natural Log both sides

$$x^2 = \ln 200$$

$$\ln e = 1$$

$$x = \pm \sqrt{\ln 200}$$

Exercise

Solve the equation: $e^{2x+1} \cdot e^{-4x} = 3e$

Solution

$$e^{2x+1-4x} = 3e$$

$$e^{-2x+1} = 3e$$

$$e^{-2x}e = 3e$$

Divide by e

$$e^{-2x} = 3$$

$$\ln e^{-2x} = \ln 3$$

$$-2x = \ln 3$$

$$x = -\frac{1}{2} \ln 3$$

Exercise

Solve the equation: $e^{2x} - 8e^x + 7 = 0$

Solution

$$(e^x)^2 - 8e^x + 7 = 0 \quad a + b + c = 0 \rightarrow x = 1, \frac{c}{a}$$

$$\begin{cases} e^x = 1 \rightarrow \underline{x = 0} \\ e^x = 7 \rightarrow \underline{x = \ln 7} \end{cases}$$

Exercise

Solve the equation without using the calculator: $e^{2x} + 2e^x - 15 = 0$

Solution

$$(e^x)^2 + 2e^x - 15 = 0 \quad \text{Solve for } e^x$$

$$e^x = 3 \quad e^x \not= -5 < 0$$
$$\underline{x = \ln 3}$$

Exercise

Solve the equation: $e^x + e^{-x} - 6 = 0$

Solution

$$e^x e^x + e^x e^{-x} - e^x 6 = e^x 0$$

$$e^{2x} + 1 - 6e^x = 0$$

$$(e^x)^2 - 6e^x + 1 = 0$$

$$e^x = \frac{6 \pm \sqrt{36 - 4}}{2}$$

$$= \frac{6 \pm 4\sqrt{2}}{2}$$

$$e^x = 3 \pm 2\sqrt{2}$$

$$\underline{x = \ln(3 \pm 2\sqrt{2})}$$

Exercise

Solve the equation: $e^{1-3x} \cdot e^{5x} = 2e$

Solution

$$e^{1-3x+5x} = 2e$$

$$e^{1+2x} = 2e$$

$$e^1 e^{2x} = 2e$$

Divide by e

$$e^{2x} = 2$$

Natural Log both sides

$$\ln e^{2x} = \ln 2$$

$$2x = \ln 2$$

$$x = \frac{1}{2} \ln 2$$

Exercise

Solve the equation: $6 \ln(2x) = 30$

Solution

$$\ln(2x) = \frac{30}{6}$$

$$\ln(2x) = 5$$

$$2x = e^5$$

$$x = \frac{1}{2} e^5$$

Exercise

Solve the equation: $\log_5(x-7) = 2$

Solution

$$x-7 = 5^2$$

$$x = 25 + 7$$

$$x = 32$$

Exercise

Solve the equation: $\log_4(5+x) = 3$

Solution

$$5 + x = 4^3$$

$$x = 64 - 5$$

$$= \underline{59}$$

$$\text{Check: } \log_4 (5 + 59)$$

Exercise

Solve the equation: $\log(4x - 18) = 1$

Solution

$$4x - 18 = 10$$

$$4x = 28$$

$$x = \underline{7}$$

Exercise

Solve the equation: $\log(x^2 + 19) = 2$

Solution

$$x^2 + 19 = 10^2$$

$$x^2 = 81$$

$$x = \underline{\pm 9}$$

$$(\pm 9)^2 + 19 > 0$$

Exercise

Solve the equation: $\ln(x^2 - 12) = \ln x$

Solution

$$\ln(x^2 - 12) = \ln x$$

$$x^2 - 12 = x$$

$$x^2 - x - 12 = 0$$

$$x = \underline{-3, 4}$$

$$\text{Check: } x = -3 \quad \ln(9 - 12) = \ln(-3) \quad \times$$

$$x = 4 \quad \ln(16 - 12) = \ln(4)$$

$$\therefore \text{Solution: } x = \underline{4}$$

Exercise

Solve the equation: $\log(2x^2 + 3x) = \log(10x + 30)$

Solution

$$\log(2x^2 + 3x) = \log(10x + 30)$$

$$2x^2 + 3x = 10x + 30$$

$$2x^2 - 7x - 30 = 0$$

$$x = \frac{7 \pm \sqrt{49 + 240}}{4}$$

$$= \begin{cases} \frac{7-17}{4} = -\frac{5}{2} \\ \frac{7+17}{4} = 6 \end{cases}$$

Check: $x = -\frac{5}{2} \quad \log\left(\frac{25}{2} - \frac{15}{2}\right) = \log(-25 + 30)$

$$x = 4 \quad \log(32 + 12) = \log(40 + 30)$$

\therefore **Solution:** $x = -\frac{5}{2}, 4$

Exercise

Solve the equation: $\log_5(2x + 3) = \log_5 11 + \log_5 3$

Solution

$$\log_5(2x + 3) = \log_5(11 \times 3)$$

$$2x + 3 = 33$$

$$2x = 30$$

$x = 15$ | **Check:** $\log_5(30 + 3)$

Exercise

Solve the equation: $\log_3 x - \log_9(x + 42) = 0$

Solution

$$\frac{\log x}{\log 3} - \frac{\log(x + 42)}{\log 9} = 0$$

$$\frac{\log x}{\log 3} - \frac{\log(x + 42)}{\log 3^2} = 0$$

$$\frac{\log x}{\log 3} - \frac{1}{2} \frac{\log(x+42)}{\log 3} = 0$$

$$\log x - \frac{1}{2} \log(x+42) = 0$$

$$2 \log x = \log(x+42)$$

$$\log x^2 = \log(x+42)$$

$$x^2 = x + 42$$

$$x^2 - x - 42 = 0$$

$$x = -6, 7$$

Check: $x = -6 \quad \log_3(-6) - \log_9(-6+42) \quad \times$

$$x = 7 \quad \log_3 7 - \log_9(7+42) = 0$$

∴ Solution: $x = 7$

Exercise

Solve the equation: $\log_5 x + \log_5(4x-1) = 1$

Solution

$$\log_5 x(4x-1) = 1$$

$$4x^2 - x = 5$$

$$4x^2 - x - 5 = 0 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x = -\frac{5}{4}, 1$$

Check: $x = -\frac{5}{4} \quad \log_5\left(-\frac{5}{4}\right) + \log_5(10-1) \quad \times$

$$x = 1 \quad \log_5(4) + \log_5(15)$$

∴ Solution: $x = 1$

Exercise

Solve the equation: $\log x - \log(x+3) = 1$

Solution

$$\log \frac{x}{x+3} = 1$$

$$\frac{x}{x+3} = 10$$

$$x = 10x + 30$$

$$9x = -30$$

$$x = -\frac{10}{3} \quad |$$

$$\text{Check: } x = -\frac{10}{3} \quad \log\left(-\frac{10}{3}\right) - \log(x+3) \quad \times$$

\therefore *No Solution*

Exercise

Solve the equation: $\log x + \log(x-9) = 1$

Solution

$$\log x(x-9) = 1$$

$$x^2 - 9x = 10$$

$$x^2 - 9x - 10 = 0 \quad a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x = -1, 10 \quad |$$

$$\text{Check: } x = -1 \quad \log(-1) + \log(x-9) \quad \times$$

$$x = 10 \quad \log(10) + \log(10-9)$$

\therefore *Solution:* $x = 10$ |

Exercise

Solve the equation: $\log_2(x+1) + \log_2(x-1) = 3$

Solution

$$\log_2(x+1)(x-1) = 3$$

$$x^2 - 1 = 2^3$$

$$x^2 = 9$$

$$x = \pm 3 \quad |$$

$$\text{Check: } x = -3 \quad \log_2(-2) + \log_2(x-1) \quad \times$$

$$x = 3 \quad \log_2(4) + \log_2(2)$$

\therefore *Solution:* $x = 3$ |

Exercise

Solve the equation: $\log_8 (x+1) - \log_8 x = 2$

Solution

$$\log_8 \frac{x+1}{x} = 2$$

$$\frac{x+1}{x} = 8^2$$

$$x+1 = 64x$$

$$63x = 1$$

$$x = \frac{1}{63} \quad \Bigg|$$

$$\text{Check: } x = \frac{1}{63} \quad \log_8 \left(\frac{1}{63} + 1 \right) - \log_8 \frac{1}{63}$$

$$\therefore \text{Solution: } x = \frac{1}{63} \quad \Bigg|$$

Exercise

Solve the equation: $\ln(x+8) + \ln(x-1) = 2 \ln x$

Solution

$$\ln(x+8)(x-1) = \ln x^2$$

$$x^2 + 7x - 8 = x^2$$

$$7x - 8 = 0$$

$$x = \frac{8}{7} \quad \Bigg|$$

$$\text{Check: } x = \frac{8}{7} \quad \ln\left(\frac{8}{7} + 8\right) + \ln\left(\frac{8}{7} - 1\right) = 2 \ln \frac{8}{7}$$

$$\therefore \text{Solution: } x = \frac{8}{7} \quad \Bigg|$$

Exercise

Solve the equation: $\ln(4x+6) - \ln(x+5) = \ln x$

Solution

$$\ln \frac{4x+6}{x+5} = \ln x$$

$$\frac{4x+6}{x+5} = x$$

$$4x+6 = x^2 + 5x$$

$$x^2 + x - 6 = 0$$

$$\underline{x = -3, 2}$$

$$\text{Check: } x = -3 \quad \ln(-6) - \ln(x+5) = \ln x \quad \times$$

$$x = 2 \quad \ln(14) - \ln(7) = \ln 2$$

$$\therefore \text{Solution: } \underline{x = 2}$$

Exercise

Solve the equation: $\ln(5+4x) - \ln(x+3) = \ln 3$

Solution

$$\ln \frac{5+4x}{x+3} = \ln 3$$

$$\frac{5+4x}{x+3} = 3$$

$$5+4x = 3x+9$$

$$\underline{x = 4}$$

$$\text{Check: } x = 4 \quad \ln(21) - \ln(7) = \ln 3$$

$$\therefore \text{Solution: } \underline{x = 4}$$

Exercise

Solve the equation: $\ln \sqrt[4]{x} = \sqrt{\ln x}$

Solution

$$\text{Domain: } \underline{x \geq 1}$$

$$\ln x^{1/4} = \sqrt{\ln x}$$

$$\frac{1}{4} \ln x = \sqrt{\ln x}$$

$$\left(\frac{1}{4} \ln x\right)^2 = (\sqrt{\ln x})^2$$

$$\frac{1}{6} \ln^2 x = \ln x$$

$$\ln^2 x = 6 \ln x$$

$$\ln^2 x - 6 \ln x = 0$$

$$(\ln x)(\ln x - 6) = 0$$

$$\begin{cases} \ln x = 0 \rightarrow \underline{x = 1} \\ \ln x = 6 \rightarrow \underline{x = e^6} \end{cases}$$

$$\therefore \text{Solution: } \underline{x = 1, e^6}$$

Exercise

Solve the equation: $\sqrt{\ln x} = \ln \sqrt{x}$

Solution

Domain: $\underline{x \geq 1}$

$$\sqrt{\ln x} = \ln x^{1/2}$$

$$\sqrt{\ln x} = \frac{1}{2} \ln x$$

$$\left(\sqrt{\ln x}\right)^2 = \left(\frac{1}{2} \ln x\right)^2$$

$$\ln x = \frac{1}{4} \ln^2 x$$

$$4 \ln x = \ln^2 x$$

$$\ln^2 x - 4 \ln x = 0$$

$$\ln x(\ln x - 4) = 0$$

$$\begin{cases} \ln x = 0 \rightarrow \underline{x = 1} \\ \ln x = 4 \rightarrow \underline{x = e^4} \end{cases}$$

\therefore **Solution:** $\underline{x = 1, e^4}$

Exercise

Solve the equation: $\log x^2 = (\log x)^2$

Solution

Domain: $\underline{x \geq 1}$

$$2 \log x = (\log x)^2$$

$$(\log x)^2 - 2 \log x = 0$$

$$\log x(\log x - 2) = 0$$

$$\begin{cases} \log x = 0 \rightarrow \underline{x = 1} \\ \log x = 2 \rightarrow \underline{x = 100} \end{cases}$$

\therefore **Solution:** $\underline{x = 1, 100}$

Exercise

Solve the equation: $\log x^3 = (\log x)^2$

Solution

Domain: $x \geq 1$

$$3 \log x = (\log x)^2$$

$$(\log x)^2 - 3 \log x = 0$$

$$\log x (\log x - 3) = 0$$

$$\begin{cases} \log x = 0 \rightarrow x = 1 \\ \log x = 3 \rightarrow x = 10^3 \end{cases}$$

Convert to exponential

$$\therefore \text{Solution: } x = 1, 10^3$$

Exercise

Solve the equation: $\log(\log x) = 1$

Solution

$$\log x = 10$$

Convert to exponential

$$\therefore \text{Solution: } x = 10^{10}$$

Exercise

Solve the equation: $\log(\log x) = 2$

Solution

$$\log x = 10^2$$

Convert to exponential

$$\therefore \text{Solution: } x = 10^{100}$$

Exercise

Solve the equation: $\ln(\ln x) = 2$

Solution

$$\ln x = e^2$$

Convert to exponential

$$\therefore \text{Solution: } x = e^{e^2}$$

Exercise

Solve the equation: $\ln\left(e^{x^2}\right) = 64$

Solution

$$e^{x^2} = e^{64}$$

Convert to exponential

$$x^2 = 64$$

$$\therefore \text{Solution: } \underline{x = \pm 8}$$

Exercise

Solve the equation: $e^{\ln(x-1)} = 4$

Solution

$$x - 1 = 4$$

$$\therefore \text{Solution: } \underline{x = 5}$$

Exercise

Solve the equation: $10^{\log(2x+5)} = 9$

Solution

$$2x + 5 = 9$$

$$2x = 4$$

$$\therefore \text{Solution: } \underline{x = 2}$$

Exercise

Solve the equation: $\log\sqrt{x^3 - 9} = 2$

Solution

$$\sqrt{x^3 - 9} = 10^2$$

$$x^3 - 9 = 10^4$$

$$x^3 = 10,009$$

$$\therefore \text{Solution: } \underline{x = \sqrt[3]{10,009}}$$

Exercise

Solve the equation: $\log \sqrt{x^3 - 17} = \frac{1}{2}$

Solution

$$\log(x^3 - 17)^{1/2} = \frac{1}{2}$$

$$\frac{1}{2} \log(x^3 - 17) = \frac{1}{2}$$

$$\log(x^3 - 17) = 1$$

$$x^3 - 17 = 10$$

$$x^3 = 27$$

$$\underline{x = 3}$$

Check: $x = 3 \quad \log \sqrt{27 - 17}$

\therefore **Solution:** $\underline{x = 3}$

Exercise

Solve the equation: $\log_4 x = \log_4 (8 - x)$

Solution

$$x = 8 - x$$

$$x + x = 8$$

$$2x = 8$$

$$\underline{x = 4}$$

Check: $x = 4 \quad \log_4 4 = \log_4 (8 - 4)$

\therefore **Solution:** $\underline{x = 4}$

Exercise

Solve the equation: $\log_7 (x - 5) = \log_7 (6x)$

Solution

$$x - 5 = 6x$$

$$x - 6x = 5$$

$$-5x = 5$$

$$\underline{x = -1}$$

Check: $x = -1 \quad \log_7 (-6) = \log_7 (6x) \quad \times$

\therefore **No Solution**

Exercise

Solve the equation: $\ln x^2 = \ln(12 - x)$

Solution

$$\ln x^2 = \ln(12 - x)$$

$$x^2 = 12 - x$$

$$x^2 + x - 12 = 0$$

$$x = -4, 3$$

Check: $x = -4 \quad \ln(16) = \ln(16)$

$x = 3 \quad \ln(9) = \ln(12 - 3)$

\therefore **Solution:** $x = -4, 3$

Exercise

Solve the equation $\log_2(x + 7) + \log_2 x = 3$

Solution

$$\log_2 x(x + 7) = 3$$

$$x(x + 7) = 2^3$$

Convert to Exponential Form

$$x^2 + 7x = 8$$

$$x^2 + 7x - 8 = 0$$

$$x = 1, -8$$

Check: $x = -8 \quad \log_2(x + 7) + \log_2(-8) \quad \times$

$x = 1 \quad \log_2(1 + 7) + \log_2 1$

\therefore **Solution:** $x = 1$

Exercise

Solve the equation $\ln x = 1 - \ln(x + 2)$

Solution

$$\ln x + \ln(x + 2) = 1$$

$$\ln x(x + 2) = 1$$

$$x^2 + 2x = e$$

$$x^2 + 2x - e = 0$$

$$\begin{aligned}
 x &= \frac{-2 \pm \sqrt{4+4e}}{2} \\
 &= \frac{-2 \pm 2\sqrt{1+e}}{2} \\
 &= \begin{cases} -1 - \sqrt{1+e} < 0 \\ -1 + \sqrt{1+e} > 0 \end{cases}
 \end{aligned}$$

$$\therefore \text{Solution: } \underline{x = -1 + \sqrt{1+e}}$$

Exercise

Solve the equation $\ln x = 1 + \ln(x+1)$

Solution

$$\ln x - \ln(x+1) = 1$$

$$\ln \frac{x}{x+1} = 1$$

$$\frac{x}{x+1} = e^1$$

$$x = (x+1)e$$

$$x = ex + e$$

$$x - ex = e$$

$$x(1-e) = e$$

$$x = \frac{e}{1-e} < 0$$

\therefore **No solution**

Exercise

Solve the equation $\log_6(2x-3) = \log_6 12 - \log_6 3$

Solution

$$\log_6(2x-3) = \log_6 \frac{12}{3}$$

$$\log_6(2x-3) = \log_6 4$$

$$2x-3 = 4$$

$$2x = 7$$

$$x = \frac{7}{2}$$

$$\text{Check: } x = \frac{7}{2} \quad \log_6(7-3) = \log_6 12 - \log_6 3$$

$$\therefore \text{Solution: } \underline{x = \frac{7}{2}}$$

Exercise

Solve the equation: $\log(3x+2) + \log(x-1) = 1$

Solution

Domain: $x > 1$

$$\log(3x+2)(x-1) = 1$$

Convert to exponential form

$$3x^2 - x - 2 = 10$$

$$3x^2 - x - 12 = 0$$

Solve for x

$$x = \frac{1 \pm \sqrt{1+144}}{6}$$

$$= \begin{cases} \frac{1 - \sqrt{145}}{6} < 0 \\ \frac{1 + \sqrt{145}}{6} > 1 \end{cases}$$

$$\therefore \text{Solution: } x = \frac{1 + \sqrt{145}}{6}$$

Exercise

Solve the equation: $\log_5(x+2) + \log_5(x-2) = 1$

Solution

$$\log_5(x+2)(x-2) = 1$$

$$(x+2)(x-2) = 5^1$$

$$x^2 - 4 = 5$$

$$x^2 = 5 + 4$$

$$x^2 = 9$$

$$x = \pm 3$$

Check: $x = -3$ $\log_5(-1) + \log_5(x-2)$ ✗

$$x = 3 \quad \log_5(3+2) + \log_5(3-2)$$

$$\therefore \text{Solution: } x = 3$$

Exercise

Solve the equation: $\log_2 x + \log_2(x-4) = 2$

Solution

Domain: $x > 4$

$$\log_2 x(x-4) = 2$$

$$x^2 - 4x = 2^2$$

$$x^2 - 4x - 4 = 0$$

$$x = \frac{4 \pm \sqrt{32}}{2}$$

$$= \begin{cases} 2 - 2\sqrt{2} < 4 \times \\ 2 + 2\sqrt{2} > 4 \end{cases}$$

\therefore **Solution:** $x = 2 + 2\sqrt{2}$ |

Exercise

Solve the equation: $\log_3 x + \log_3 (x+6) = 3$

Solution

Domain: $x > 0$

$$\log_3 x(x+6) = 3$$

$$x^2 + 6x = 3^3$$

$$x^2 + 6x - 27 = 0$$

$$x = \frac{-6 \pm \sqrt{36 + 108}}{2}$$

$$= \begin{cases} \frac{-6-12}{2} = -9 < 0 \times \\ \frac{-6+12}{2} = 3 > 0 \end{cases}$$

\therefore **Solution:** $x = 3$ |

Exercise

Solve the equation: $\log_3 (x+3) + \log_3 (x+5) = 1$

Solution

Domain: $x > -3$

$$\log_3 (x+3)(x+5) = 1$$

$$x^2 + 8x + 15 = 3$$

$$x^2 + 8x + 12 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 48}}{2}$$

$$= \begin{cases} \frac{-8-4}{2} = -6 < -3 \text{ X} \\ \frac{-8+4}{2} = -2 > -3 \end{cases}$$

\therefore **Solution:** $x = -2$ |

Exercise

Solve the equation: $\ln x = \frac{1}{2} \ln \left(2x + \frac{5}{2} \right) + \frac{1}{2} \ln 2$

Solution

Domain: $x > 0$

$$2 \ln x = \ln \left(2x + \frac{5}{2} \right) + \ln 2$$

$$\ln x^2 = \ln 2 \left(2x + \frac{5}{2} \right)$$

$$x^2 = 4x + 5$$

$$x^2 - 4x - 5 = 0$$

$$a - b + c = 0 \rightarrow x = -1, -\frac{c}{a}$$

$$x = -1, 5 \text{ |}$$

\therefore **Solution:** $x = 5$ |

Exercise

Solve the equation $\ln(-4 - x) + \ln 3 = \ln(2 - x)$

Solution

Domain: $x < 5$

$$\ln 3(-4 - x) = \ln(2 - x)$$

$$-12 - 3x = 2 - x$$

$$-12 - 2 = 3x - x$$

$$-14 = 2x$$

\therefore **Solution:** $x = -7$ |

Exercise

Solve the equation: $\log_4 x + \log_4 (x - 2) = \log_4 (15)$

Solution

Domain: $x > 2$

$$\log_4 x(x-2) = \log_4 (15)$$

$$x^2 - 2x = 15$$

$$x^2 - 2x - 15 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 60}}{2}$$

$$= \begin{cases} \frac{2-8}{2} = -4 < 2 \times \\ \frac{2+8}{2} = 5 > 2 \end{cases}$$

\therefore **Solution:** $x = 5$

Exercise

Solve the equation: $\ln(x-5) - \ln(x+4) = \ln(x-1) - \ln(x+2)$

Solution

Domain: $x > 5$

$$\ln \frac{x-5}{x+4} = \ln \frac{x-1}{x+2}$$

$$\frac{x-5}{x+4} = \frac{x-1}{x+2}$$

$$(x-5)(x+2) = (x-1)(x+4)$$

$$x^2 + 2x - 5x - 10 = x^2 + 4x - x - 4$$

$$x^2 - 3x - 10 = x^2 + 3x - 4$$

$$x^2 - 3x - 10 - x^2 - 3x + 4 = 0$$

$$-6x - 6 = 0$$

$$x = -1$$

\therefore **No solution**

Exercise

Solve the equation: $\log(x^2 + 4) - \log(x+2) = 2 + \log(x-2)$

Solution

Domain: $x > -2$

$$\log(x^2 + 4) - \log(x+2) - \log(x-2) = 2$$

$$\log(x^2 + 4) - [\log(x+2) + \log(x-2)] = 2$$

$$\log(x^2 + 4) - \log(x+2)(x-2) = 2$$

$$\log \left(\frac{x^2 + 4}{x^2 - 4} \right) = 2$$

$$\frac{x^2 + 4}{x^2 - 4} = 10^2$$

$$x^2 + 4 = 100x^2 - 400$$

$$400 + 4 = 100x^2 - x^2$$

$$99x^2 = 404$$

$$x^2 = \frac{404}{99}$$

$$\therefore \textbf{Solution: } \underline{x = \frac{2\sqrt{101}}{3\sqrt{11}}} \text{ is the only solution}$$

Exercise

Solve the equation $\log_3 (x - 2) = \log_3 27 - \log_3 (x - 4) - 5^{\log_5 1}$

Solution

Domain: $x > 4$

$$\log_3 (x - 2) + \log_3 (x - 4) = \log_3 3^3 - 1$$

$$\log_3 (x - 2)(x - 4) = 3 - 1$$

$$\log_3 (x^2 - 6x + 8) = 2$$

$$x^2 - 6x + 8 = 3^2$$

$$x^2 - 6x + 8 = 9$$

$$x^2 - 6x - 1 = 0$$

$$\rightarrow \underline{x = 3 \pm \sqrt{10}}$$

$$\textbf{Check: } x = 3 + \sqrt{10} > 4$$

$$x = 3 - \sqrt{10} < 4 \quad \times$$

$$\therefore \textbf{Solution: } \underline{x = 3 + \sqrt{10}}$$

Exercise

Solve the equation $\log_2 (x + 3) = \log_2 (x - 3) + \log_3 9 + 4^{\log_4 3}$

Solution

Domain: $x > 3$

$$\log_2 (x+3) - \log_2 (x-3) = 2+3$$

$$\log_2 \frac{x+3}{x-3} = 5$$

$$\frac{x+3}{x-3} = 2^5$$

$$x+3 = 32(x-3)$$

$$x+3 = 32x-96$$

$$96+3 = 32x-x$$

$$31x = 99$$

$$x = \frac{99}{31} > 3$$

$$\therefore \text{Solution: } x = \frac{99}{31}$$

Exercise

Solve for t using logarithms with base a : $2a^{t/3} = 5$

Solution

$$a^{t/3} = \frac{5}{2}$$

$$\log a^{t/3} = \log \frac{5}{2}$$

$$\frac{t}{3} \log a = \log \frac{5}{2}$$

$$\frac{t}{3} = \frac{\log \frac{5}{2}}{\log a}$$

$$\frac{t}{3} = \log_a \frac{5}{2}$$

$$t = 3 \log_a \frac{5}{2}$$

Exercise

Solve for t using logarithms with base a : $K = H - Ca^t$

Solution

$$Ca^t = H - K$$

$$a^t = \frac{H-K}{C}$$

$$\log a^t = \log \frac{H-K}{C}$$

$$t \log a = \log \frac{H-K}{C}$$

$$t = \frac{\log \frac{H-K}{C}}{\log a}$$

$$= \log_a \frac{H-K}{C}$$