$3, 4 \to 5$

$5, 12 \to 13$

$8, 15 \to 17$

8.2

$$\begin{cases} \cos(A+B) = \cos A \cos B - \sin A \sin B \\ \cos(A-B) = \cos A \cos B + \sin A \sin B \end{cases}$$

cosine , coscos , sinsin ☹

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin A \cos A \quad \boxed{\sin B \cos B}$

$\cos(75°) = \cos(30°+45°)$

$= \cos 30° \cos 45° - \sin 30° \sin 45°$

$= \dfrac{\sqrt{3}}{2} \dfrac{\sqrt{2}}{2} - \dfrac{1}{2} \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{6} - \sqrt{2}}{4}$

$\cos(90° - A) = \cos 90° \cdot \cos A + \sin 90° \sin A$

$= \sin A \checkmark$

$\sin \dfrac{\pi}{12} = \sin\left(\dfrac{\pi}{3} - \dfrac{\pi}{4}\right)$

$= \sin \dfrac{\pi}{3} \cos \dfrac{\pi}{4} - \cos \dfrac{\pi}{3} \sin \dfrac{\pi}{4}$

$= \dfrac{\sqrt{3}}{2} \dfrac{\sqrt{2}}{2} - \dfrac{1}{2} \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{6} - \sqrt{2}}{4}$

$\dfrac{\pi}{3} - \dfrac{\pi}{4} = \dfrac{\textcircled{1}\pi}{12} \quad \dfrac{\overset{15°}{100}}{12}$

$\dfrac{\pi}{4} - \dfrac{\pi}{6} \quad 60 - 45°$

$\tan(A+B) = \boxed{\dfrac{\sin(A+B)}{\cos(A+B)}} = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

EX $\sin A = \frac{3}{5}$ $A \in QI$ $\cos B = -\frac{5}{13}$ $B \in QIII$

   $3, 4 \to 5$

   $\cos A = \frac{4}{5}$ $\sin B = -\frac{12}{13}$ $5, 12 \to 13$

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{3}{5}\left(-\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right)$

$= \frac{-15 - 48}{65}$

$= -\frac{63}{65}$

b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{4}{5}\left(-\frac{5}{13}\right) - \frac{3}{5}\left(-\frac{12}{13}\right)$

$= \frac{-20 + 36}{65}$

$= \frac{16}{65}$

c) $\tan(A+B) = -\frac{63}{16}$

d) $\sin(A-B) = \sin A \cos B - \cos B \sin A$

$= \frac{-15 + 48}{65}$

$= \frac{33}{65}$

e) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{4}{5}\left(-\frac{5}{13}\right) + \frac{3}{5}\left(-\frac{12}{13}\right)$ ☺

$= \frac{-20 - 36}{65}$

$= -\frac{56}{65}$

f) $\tan(A-B) = -\frac{33}{56}$

Ex Prove: $\dfrac{\cos(x-y)}{\sin x \sin y} = \cot x \cot y + 1$

$$\frac{\cos(x-y)}{\sin x \sin y} = \frac{\cos x \cos y + \sin x \sin y}{\sin x \sin y}$$

$$= \frac{\cos x \cos y}{\sin x \sin y} + \frac{\sin x \sin y}{\sin x \sin y}$$

$$= \cot x \cot y + 1 \checkmark$$

.2, 17, 18, 19.

Prove $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

$$\cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \sin y \cos x}$$

$$= \frac{\dfrac{\cos x \cos y}{\sin x \sin y} - \dfrac{\sin x \sin y}{\sin x \sin y}}{\dfrac{\sin x \cos y}{\sin x \sin y} + \dfrac{\cos x \sin y}{\sin x \sin y}}$$

$$= \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

Prove: $\sec(x-y) = \dfrac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \sin^2 y}$

$$\sec(x-y) = \frac{1}{\cos(x-y)} \cdot \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{(\cos x \cos y + \sin x \sin y)(\cos x \cos y - \sin x \sin y)}$$

$$= \frac{\cos(x+y)}{\cos^2 x \cos^2 y - \sin^2 x \sin^2 y} \qquad \cos^2 x + \sin^2 x = 1$$

$$= \frac{\cos(x+y)}{\cos^2 x(1-\sin^2 y) - (1-\cos^2 x)\sin^2 y}$$

$$= \frac{\cos(x+y)}{\cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos^2 x - \sin^2 y} \qquad \checkmark$$

$\cos(2A)$ double angle

$(A+A)$

$\cos^2 A = (\cos A)^2$ square.

$\cos A^2 = \cos(A^2)$

Cube $\cos^3 A$

$3A \qquad \cos 3A \qquad \begin{array}{l}(A+A+A)\\(3\times A)\end{array}$