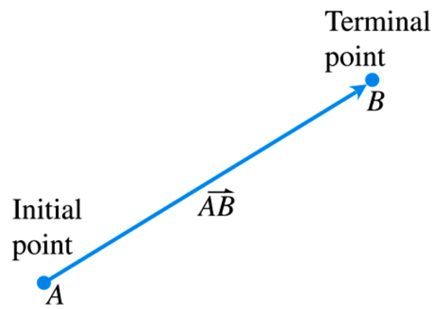
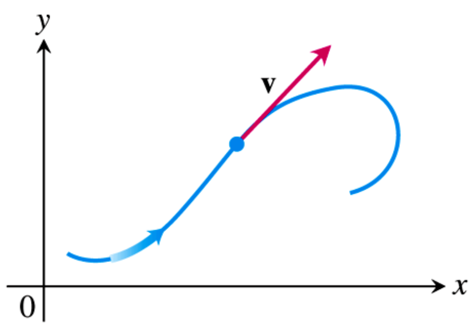
Lecture One – Vectors and Vector-Values Functions

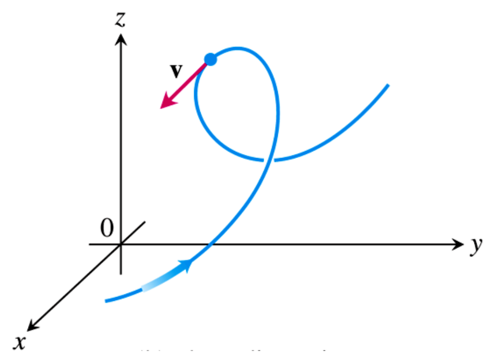
Section 1.1 – Vectors

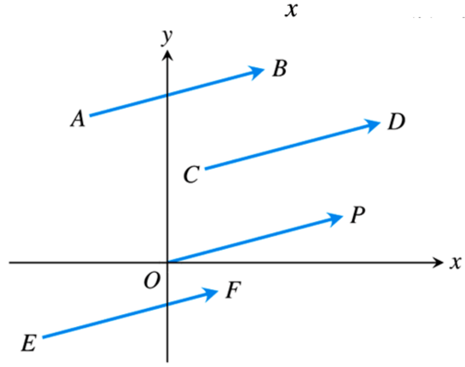


Component Form

A quantity such as force, or velocity is called a vector and is represented by a directed line segment.

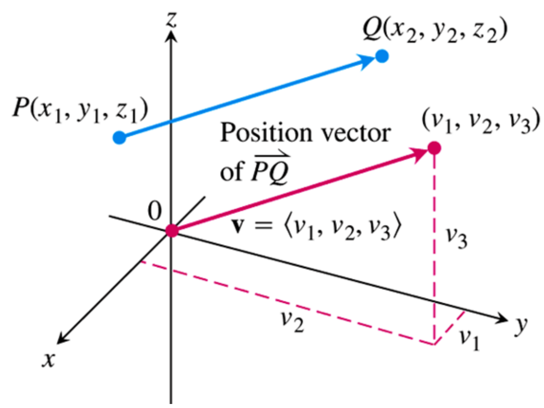






Definition

The vector represented by the directed line segment $\overrightarrow{PQ}$ has initial point P and terminal point Q and its length is denoted by $\left|\overrightarrow{PQ}\right|$



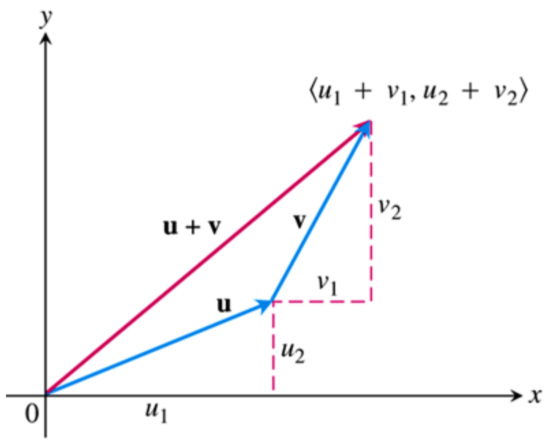
Vector Algebra Operations

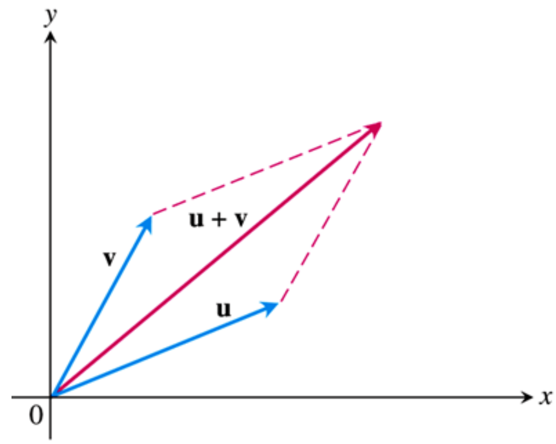
Definitions

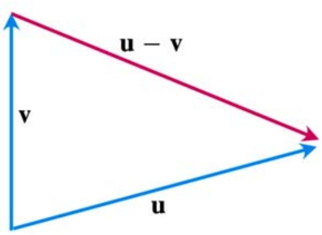
Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ be vectors with $\boldsymbol{k}$ a scalar

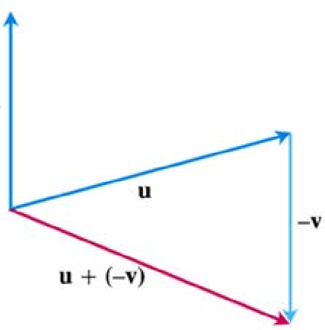
Addition: $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$

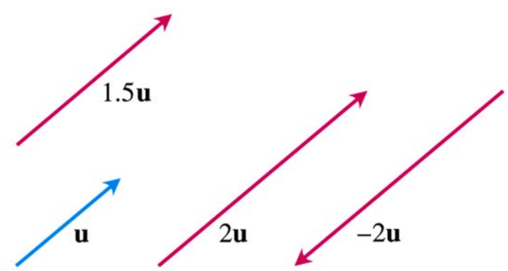
Scalar multiplication: $k\vec{u} = \langle ku_1, ku_2, ku_3 \rangle$











Example

Let $\boldsymbol{u} = \langle -1,\ 3,\ 1 \rangle$ and $\boldsymbol{v} = \langle 4,\ 7,\ 0 \rangle$. Find the components of

a) $2\boldsymbol{u} + 3\boldsymbol{v}$ *b)* $\boldsymbol{u} - \boldsymbol{v}$ *c)* $\left| \frac{1}{2}\boldsymbol{u} \right|$

Solution

a) $2\boldsymbol{u} + 3\boldsymbol{v} = 2\langle -1,\ 3,\ 1 \rangle + 3\langle 4,\ 7,\ 0 \rangle$

$= \langle -2,\ 6,\ 2 \rangle + \langle 12,\ 21,\ 0 \rangle$

$= \langle 10,\ 27,\ 2 \rangle$

b) $\boldsymbol{u} - \boldsymbol{v} = \langle -1,\ 3,\ 1 \rangle - \langle 4,\ 7,\ 0 \rangle$

$= \langle -5,\ -4,\ 1 \rangle$

c) $\left|\frac{1}{2}\boldsymbol{u}\right| = \left|\left\langle -\frac{1}{2},\ \frac{3}{2},\ \frac{1}{2}\right\rangle\right|$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{9}{4} + \frac{1}{4}}$$

$$= \sqrt{\frac{11}{4}}$$

$$= \frac{\sqrt{11}}{2}$$

Proporties of Vector Operations

Let ***u, v, w*** be vectors and a, b be scalars

1. $\boldsymbol{u}+\boldsymbol{v}=\boldsymbol{v}+\boldsymbol{u}$
2. $(\boldsymbol{u}+\boldsymbol{v})+\boldsymbol{w}=\boldsymbol{u}+(\boldsymbol{v}+\boldsymbol{w})$
3. $\boldsymbol{u}+\boldsymbol{0}=\boldsymbol{u}$
4. $\boldsymbol{u}+(-\boldsymbol{u})=\boldsymbol{0}$
5. $0\boldsymbol{u}=\boldsymbol{0}$
6. $1\boldsymbol{u}=\boldsymbol{u}$
7. $a(b\boldsymbol{u})=(ab)\boldsymbol{u}$
8. $(a+b)\boldsymbol{u}=a\boldsymbol{u}+b\boldsymbol{u}$
9. $a(\boldsymbol{u}+\boldsymbol{v})=a\boldsymbol{u}+a\boldsymbol{v}$

Definition

If $\boldsymbol{v}$ is a **two-dimensional** vector in the plane equal to the vector with initial point at the origin and terminal point $(v_1,\ v_2)$, then the ***component form*** of $\boldsymbol{v}$ is

$$\boldsymbol{v}=\langle v_1,\ v_2\rangle$$

If $\boldsymbol{v}$ is a **three-dimensional** vector in the plane equal to the vector with initial point at the origin and terminal point $(v_1,\ v_2,\ v_3)$, then the ***component form*** of $\boldsymbol{v}$ is

$$\boldsymbol{v}=\langle v_1,\ v_2,\ v_3\rangle$$

The magnitude or length of the vector $\boldsymbol{v}=\overrightarrow{PQ}$ is the nonegative number

$$|\boldsymbol{v}|=\sqrt{v_1^2+v_2^2+v_3^2}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

The only vector with length 0 is the ***zero vector*** $\boldsymbol{0}=\langle 0,\ 0,\ 0\rangle$

Example

Find the component form and the length of the vector with initial point $P(-3,\ 4,\ 1)$ and terminal point $Q(-5,\ 2,\ 2)$

Solution

The component form of $\overrightarrow{PQ}$ is

$$\overrightarrow{PQ} = \langle -5-(-3),\ 2-4,\ 2-1\rangle$$

$$\underline{= \langle -2,\ -2,\ 1\rangle}$$

The length is

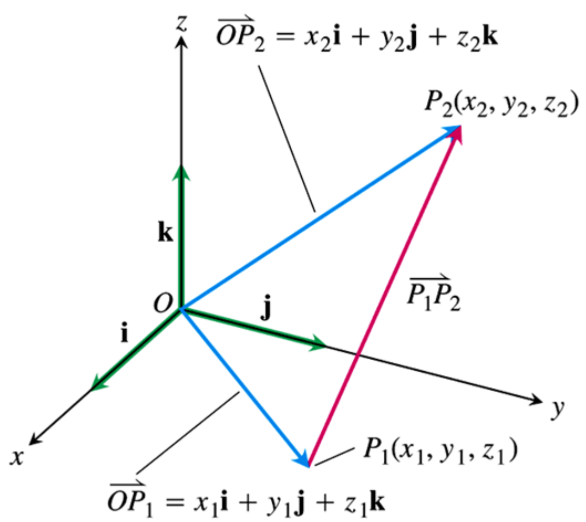
$$\left|\overrightarrow{PQ}\right| = \sqrt{(-2)^2+(-2)^2+1^2}$$

$$\underline{=3}$$

Unit Vectors

A vector ***v*** of length 1 is called a ***unit vector***. The ***standard unit vectors*** are

$$\hat{i} = \langle 1,\ 0,\ 0\rangle, \quad \hat{j} = \langle 0,\ 1,\ 0\rangle, \quad \textit{and} \quad \hat{k} = \langle 0,\ 0,\ 1\rangle$$



Any vector $\vec{v} = \langle v_1,\ v_2,\ v_3\rangle$ can be written as a linear combination of the standard unit vectors as follows:

$$\begin{aligned}\vec{v} &= \langle v_1,\ v_2,\ v_3\rangle \\ &= \langle v_1,\ 0,\ 0\rangle + \langle 0,\ v_2,\ 0\rangle + \langle 0,\ 0,\ v_3\rangle \\ &= v_1\langle 1,\ 0,\ 0\rangle + v_2\langle 0,\ 1,\ 0\rangle + v_3\langle 0,\ 0,\ 1\rangle \\ &= v_1\hat{i} + v_2\hat{j} + v_3\hat{k}\end{aligned}$$

Example

Find a unit vector $\vec{u}$ in the direction of the vector from $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

Solution

$$\overrightarrow{P_1P_2} = (3-1)\hat{i} + (2-0)\hat{j} + (0-1)\hat{k}$$

$$= 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\left|\overrightarrow{P_1P_2}\right| = \sqrt{2^2 + 2^2 + (-1)^2}$$

$$= \sqrt{9}$$

$$= 3$$

$$\vec{u} = \frac{\overrightarrow{P_1P_2}}{\left|\overrightarrow{P_1P_2}\right|}$$

$$= \frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$$

$$= \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$

Example

If $\vec{v} = 3\hat{i} - 4\hat{j}$ is a velocity vector, express $\vec{v}$ as a product of its speed times a unit vector in the direction of motion.

Solution

Speed is the magnitude (length) of $\vec{v}$:

$$|\vec{v}| = \sqrt{3^2 + (-4)^2}$$

$$= 5$$

The unit vector has the same direction as $\vec{v}$:

$$\frac{\vec{v}}{|\vec{v}|} = \frac{3\hat{i} - 4\hat{j}}{5}$$

$$= \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$$

$$\vec{v} = 3\hat{i} - 4\hat{j} = \underset{\substack{\textbf{Length} \\ \textbf{(speed)}}}{5}\underbrace{\left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)}_{\substack{Direction \\ of\ motion}}$$

***Note*:**

If $\vec{v} \neq 0$, then

1. $\dfrac{\vec{v}}{|\vec{v}|}$ is a unit vector in the direction of $\vec{v}$;

2. The equation $\vec{v} = |\vec{v}|\dfrac{\vec{v}}{|\vec{v}|}$ expresses $\vec{v}$ as its length times its direction.

Example

A force of 6 *Newton* is applied in the direction of the vector $\vec{v} = 2\hat{i} + 2\hat{j} - \hat{k}$. Express the force $\vec{F}$ as a product of its magnitude and direction.

Solution

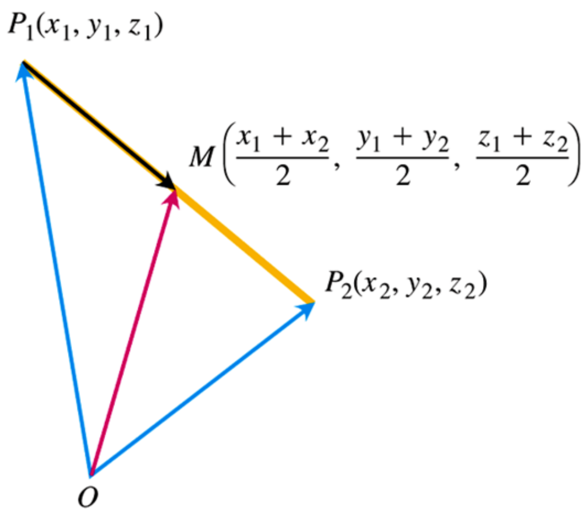
$$|v| = \sqrt{2^2 + 2^2 + (-1)^2} = 3$$

$$\begin{aligned} \boldsymbol{F} &= |\boldsymbol{v}|\frac{\boldsymbol{v}}{|\boldsymbol{v}|} \\ &= 3\frac{2\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}}{3} \\ &= 3\left(\frac{2}{3}\hat{\boldsymbol{i}} + \frac{2}{3}\hat{\boldsymbol{j}} - \frac{1}{3}\hat{\boldsymbol{k}}\right) \end{aligned}$$

Midpoint of a Line Segment

The midpoint M of the line segment joining points $P_1\left(x_1, y_1, z_1\right)$ and $P_2\left(x_2, y_2, z_2\right)$ is the point

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$



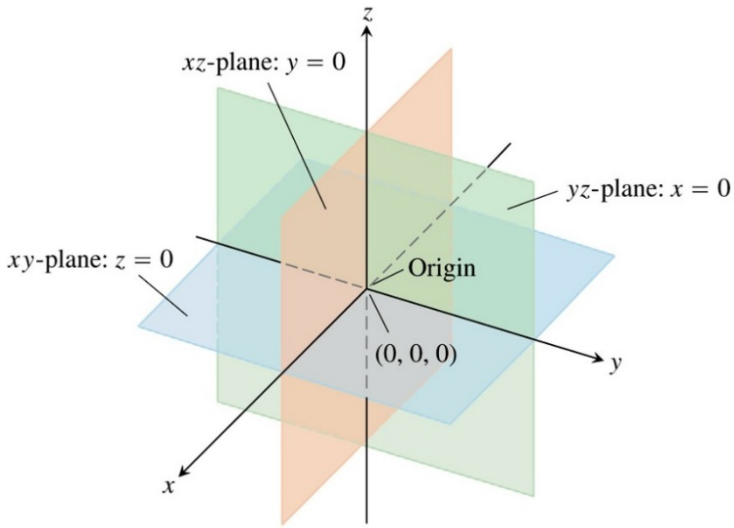
Example

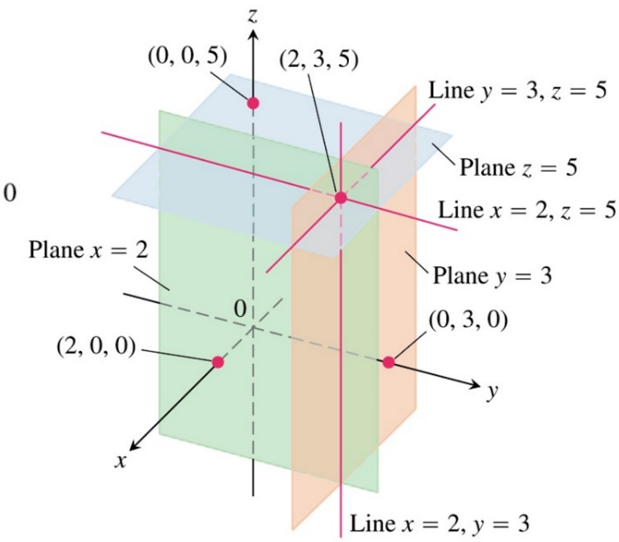
Find the midpoint of the segment $P_1(3, -2, 0)$ and $P_2(7, 4, 4)$

Solution

$$M = \left(\frac{3+7}{2}, \frac{-2+4}{2}, \frac{0+4}{2}\right)$$

$$= (5, 1, 2)$$

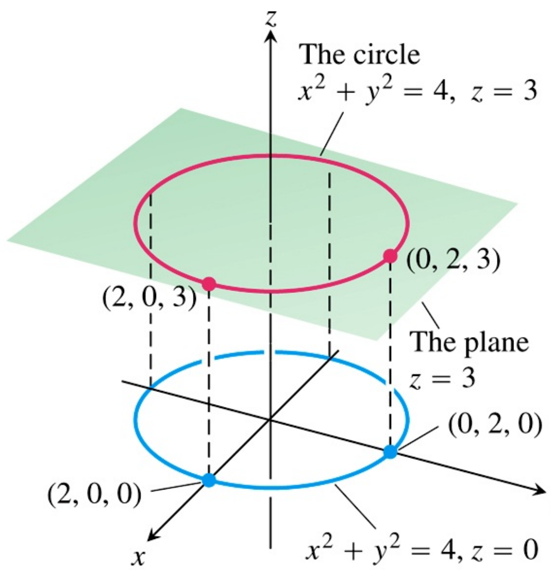




Example

What points $P(x, y, z)$ satisfy the equations $x^2 + y^2 = 4$ *and* $z = 3$

Solution



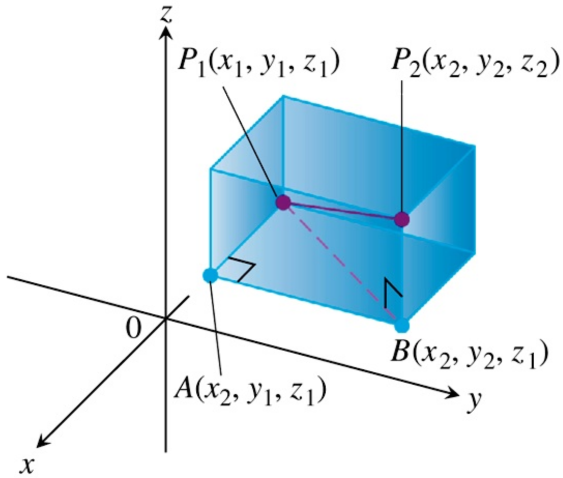
The point lie in the horizontal plane $z = 3$ and the circle $x^2 + y^2 = 4$.

The solution is the set of points: "the circle $x^2 + y^2 = 4$ in the plane $z = 3$"

Distance in Space

The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



Proof

$$|P_1A| = |x_2 - x_1|$$
$$|AB| = |y_2 - y_1|$$
$$|BP_2| = |z_2 - z_1|$$

From the right triangles P_1AB and P_1BP_2:

$$|P_1B|^2 = |P_1A|^2 + |AB|^2$$
$$|P_1P_2|^2 = |P_1B|^2 + |BP_2|^2$$

$$\begin{aligned}|P_1P_2|^2 &= |P_1B|^2 + |BP_2|^2 \\ &= |P_1A|^2 + |AB|^2 + |BP_2|^2 \\ &= |x_2 - x_1|^2 + |y_2 - y_1|^2 + |z_2 - z_1|^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \quad \checkmark\end{aligned}$$

Example

Find the distance between $P_1(2,\ 1,\ 5)$ and $P_2(-2,\ 3,\ 0)$

Solution

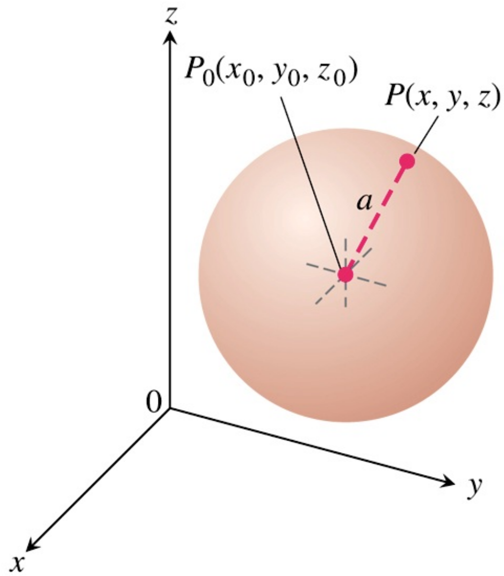
$$\left|P_1P_2\right| = \sqrt{(-2-2)^2 + (3-1)^2 + (0-5)^2}$$

$$= \sqrt{16+4+25}$$

$$\underline{= \sqrt{45} \ \ \textit{or} \ \ \approx 6.708}$$

The Standard Equation for the Sphere of Radius a and Center $\left(x_0, y_0, z_0\right)$

$$\left(x-x_0\right)^2+\left(y-y_0\right)^2+\left(z-z_0\right)^2=a^2$$



Example

Find the center and radius of the sphere $\quad x^2+y^2+z^2+3x-4z+1=0$

Solution

$$\left(x^2+3x\right)+y^2+\left(z^2-4z\right)=-1$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+y^2+\left(z^2-4z+\left(\frac{-4}{2}\right)^2\right)=-1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+y^2+(z-2)^2=-1+\frac{9}{4}+4$$

$$\left(x+\frac{3}{2}\right)^2+y^2+(z-2)^2=\frac{21}{4}$$

Therefore; the center is $\left(-\frac{3}{2},\ 0,\ 2\right)$ and the radius is $\frac{\sqrt{21}}{2}$

Applications

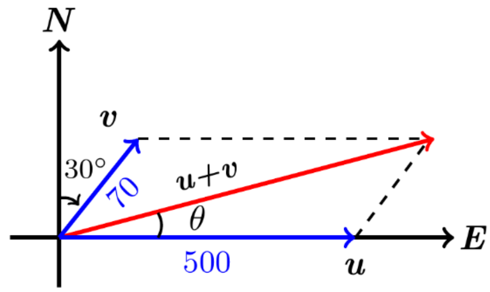
Example

A jet airliner, flying due east at 500 *mph* in still air, encounters a 70-*mph* tailwind blowing in the direction 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?

Solution

$\vec{u}$ = the velocity of the airplane
$\vec{v}$ = the velocity of the tailwind

Given: $|\vec{u}| = 500 \quad |\vec{v}| = 70$



$$\vec{u} = \langle 500,\ 0\rangle$$

$$\vec{v} = \langle 70\cos 60°,\ 70\sin 60°\rangle$$

$$= \langle 35,\ 35\sqrt{3}\rangle$$

$$\vec{u} + \vec{v} = \langle 535,\ 35\sqrt{3}\rangle \qquad = 535\hat{i} + 35\sqrt{3}\,\hat{j}$$

$$|\vec{u} + \vec{v}| = \sqrt{535^2 + \left(35\sqrt{3}\right)^2}$$

$$\approx 538.4$$

$$\theta = \tan^{-1}\frac{35\sqrt{3}}{535}$$

$$\approx 6.5°$$

The ground speed of the airplane is about 538.4 *mph*, and its direction is about 6.5° north of east.

Example

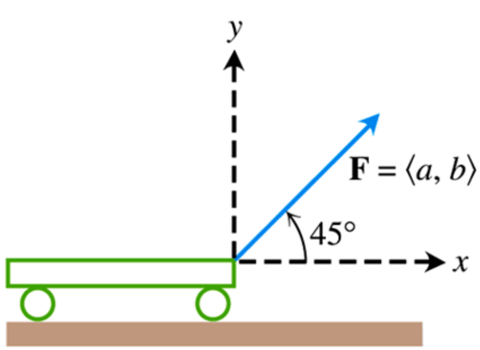
A small cart is being pulled along a 20-*lb* smooth horizontal floor with a force $\vec{F}$ making a 45° angle to the floor. What is the effective force moving the cart forward?

Solution

$$a = |\vec{F}|\cos 45°$$

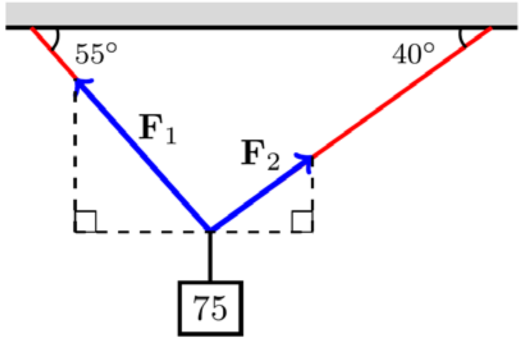
$$= (20)\left(\frac{\sqrt{2}}{2}\right)$$

$$\equiv 14.14\ \ lb$$



Example

A 75-N weight is suspended by two wires.

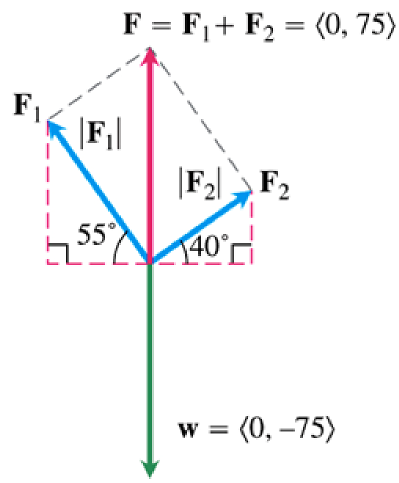


Find the forces $\vec{F}_1$ and $\vec{F}_2$ acting both wires

Solution

$$\vec{F}_1 = \left\langle -\left|\vec{F}_1\right|\cos 55°,\ \left|\vec{F}_1\right|\sin 55° \right\rangle$$

$$\vec{F}_2 = \left\langle \left|\vec{F}_2\right|\cos 40°,\ \left|\vec{F}_2\right|\sin 40° \right\rangle$$



$$\vec{F}_1 + \vec{F}_2 = \langle 0,\ 75 \rangle$$

$$-\left|\vec{F}_1\right|\cos 55° + \left|\vec{F}_2\right|\cos 40° = 0$$

$$\Rightarrow \quad \left|\vec{F}_2\right| = \left|\vec{F}_1\right|\frac{\cos 55°}{\cos 40°}$$

$$\left|\vec{F}_1\right|\sin 55° + \left|\vec{F}_2\right|\sin 40° = 75$$

$$\left|\vec{F}_1\right|\sin 55° + \left|\vec{F}_1\right|\frac{\cos 55°}{\cos 40°}\sin 40° = 75$$

$$\left|\vec{F}_1\right|\left(\sin 55° + \cos 55° \tan 40°\right) = 75$$

$$\left|\vec{F}_1\right| = \frac{75}{\sin 55° + \cos 55° \tan 40°}$$

$$\approx 57.67\ N$$

$$\left|\vec{F}_2\right| = 57.67\frac{\cos 55°}{\cos 40°}$$

$$\approx 43.18\ N$$

The force vectors are then:

$$\vec{F}_1 = \left\langle -\left|\vec{F}_1\right|\cos 55°,\ \left|\vec{F}_1\right|\sin 55° \right\rangle$$

$$= \left\langle -57.67\cos 55°,\ 57.67\sin 55° \right\rangle$$

$$= \langle -33.08,\ 47.24 \rangle$$

$$\vec{F}_2 = \left\langle \left|\vec{F}_2\right| \cos 40°,\ \left|\vec{F}_2\right| \sin 40° \right\rangle$$

$$= \langle 43.18 \cos 40°,\ 43.18 \sin 40° \rangle$$

$$= \langle 33.08,\ 27.76 \rangle$$

Exercises *Section* 1.1 – Vectors

Give a geometric description of the set of points in space whose coordinates satisfy the given pairs of equations

1. $x^2+z^2=4, \quad y=0$

2. $x^2+y^2=4, \quad z=-2$

3. $x^2+y^2+z^2=1, \quad x=0$

4. $x^2+(y-1)^2+z^2=4, \quad y=0$

5. $x^2+y^2+z^2=4, \quad y=x$

Find the distance between points P_1 *and* P_2

6. $P_1(1,\ 1,\ 1), \quad P_2(3,\ 3,\ 0)$

7. $P_1(-1,\ 1,\ 5), \quad P_2(2,\ 5,\ 0)$

8. $P_1(1,\ 4,\ 5), \quad P_2(4,\ -2,\ 7)$

9. $P_1(3,\ 4,\ 5), \quad P_2(2,\ 3,\ 4)$

Find the center and radii of the spheres

10. $x^2+y^2+z^2+4x-4z=0$

11. $x^2+y^2+z^2-6y+8z=0$

12. $2x^2+2y^2+2z^2+x+y+z=9$

13. Find a formula for the distance from the point $P(x,\ y,\ z)$ to x-axis

14. Find a formula for the distance from the point $P(x,\ y,\ z)$ to xz-plane.

15. Let $\boldsymbol{u}=\langle -3,\ 4\rangle$ *and* $\boldsymbol{v}=\langle 2,\ -5\rangle$. Find the component form and the magnitude if the vector

a) $3\boldsymbol{u}-4\boldsymbol{v}$ ***b)*** $-2\boldsymbol{u}$ ***c)*** $\boldsymbol{u}+\boldsymbol{v}$

16. Let $\boldsymbol{u}=\langle 3,\ -2\rangle$ *and* $\boldsymbol{v}=\langle -2,\ 5\rangle$. Find the component form and the magnitude if the vector

a) $3\boldsymbol{u}$

b) $\boldsymbol{u}-\boldsymbol{v}$

c) $2\boldsymbol{u}-3\boldsymbol{v}$

d) $-2\boldsymbol{u}+5\boldsymbol{v}$

e) $-\frac{5}{13}\boldsymbol{u}+\frac{12}{13}\boldsymbol{v}$

17. Find scalars a, b, and c such that $\langle 2,\ 2,\ 2\rangle = a\langle 1,\ 1,\ 0\rangle + b\langle 0,\ 1,\ 1\rangle + c\langle 1,\ 0,\ 1\rangle$

18. Find the component form of the vector: The sum of $\overrightarrow{AB}$ and $\overrightarrow{CD}$ where

$A=(1,-1), \quad B=(2,0), \quad C=(-1,3), \quad and \quad D=(-2,2)$

Find the component form of the vector:

19. The unit vector that makes an angle $\theta=\frac{2\pi}{3}$ with the positive x-axis

20. The unit vector obtained by rotating the vector $\langle 0,\ 1\rangle$ 120° counterclockwise about the origin.

21. The unit vector obtained by rotating the vector $\langle 1,\ 0\rangle$ $135°$ counterclockwise about the origin.

22. The unit vector that makes an angle $\theta = \frac{\pi}{6}$ with the positive x-axis

23. The vector 5 units long in the direction opposite to the direction of $\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$

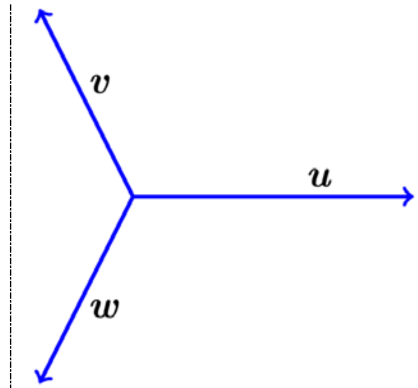
24. Express the velocity vector $\vec{v} = \left(e^t \cos t - e^t \sin t\right)i + \left(e^t \cos t + e^t \sin t\right)\hat{j}$ *when* $t = \ln 2$ in terms of its length and direction.

25. Sketch the indicated vector

a) $\boldsymbol{u} - \mathbf{v}$

b) $2\boldsymbol{u} - \mathbf{v}$

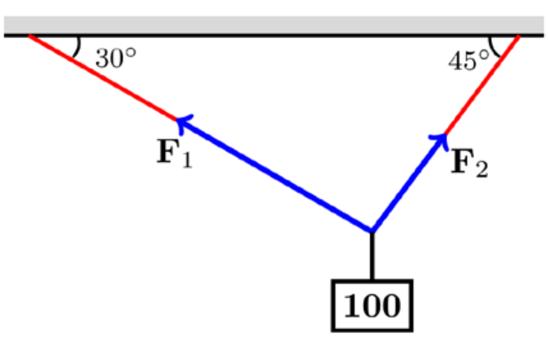
c) $\boldsymbol{u} - \mathbf{v} + \boldsymbol{w}$



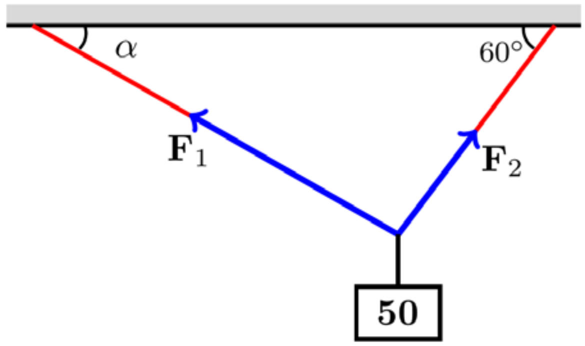
26. An Airplane is flying in the direction 25° west of north at 800 *km/h*. Find the component form of the velocity of the airplane, assuming that the positive x-axis represents due east and the positive y-axis represents due north.

27. A jet airliner, flying due east at 500 *mph* in still air, encounters a 70-*mph* tailwind blowing in the direction 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What speed and direction should the jetliner have in order for the resultant vector to be 500 *mph* due east?

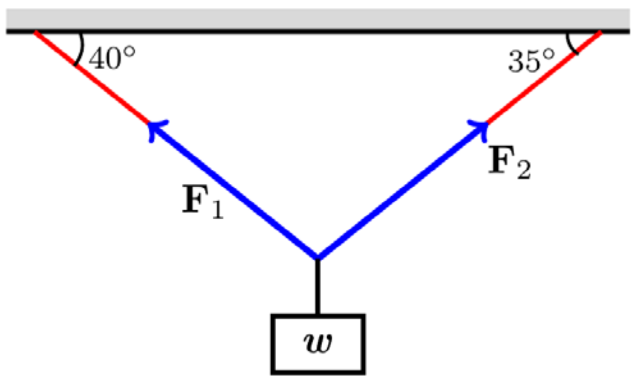
28. Consider a 100-N weight suspended by two wires. Find the magnitudes and components of the force vectors $\overrightarrow{F_1}$ *and* $\overrightarrow{F_2}$



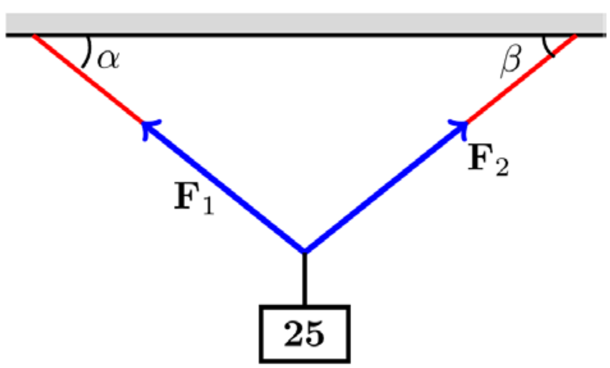
29. Consider a 50-N weight suspended by two wires, If the magnitude of vector $\overrightarrow{F_1} = 35\ N$, find the angle α and the magnitude of vector $\overrightarrow{F_2}$



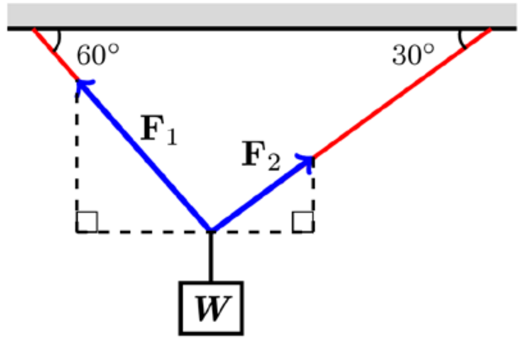
30. Consider a ***w***-N weight suspended by two wires, If the magnitude of vector $\left|\overrightarrow{F_2}\right| = 100\ N$, find ***w*** and the magnitude of vector $\overrightarrow{F_1}$



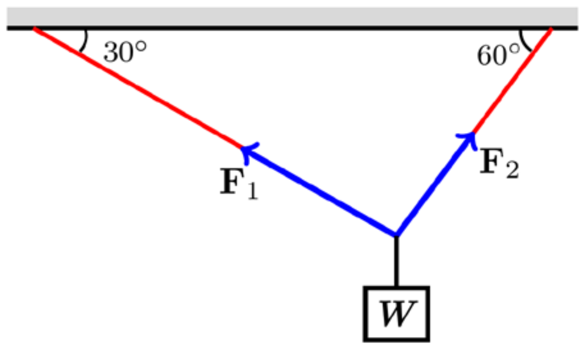
31. Consider a 25-N weight suspended by two wires, If the magnitude of vector $\overrightarrow{F_1}$ *and* $\overrightarrow{F_2}$ are both 75 N, then angles α and β are equal. Find α.



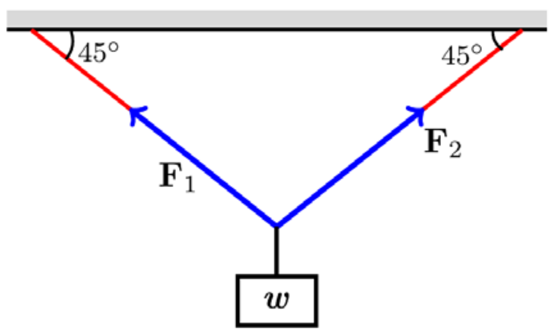
32. Consider a $W = 100\ N$ weight suspended by two wires. Find the magnitudes and components of the force vectors $\overrightarrow{F_1}$ *and* $\overrightarrow{F_2}$



33. Consider a $W = 50\ N$ weight suspended by two wires. Find the magnitudes and components of the force vectors $\overrightarrow{F_1}$ *and* $\overrightarrow{F_2}$



34. Consider a $W = 100\ N$ weight suspended by two wires. Find the magnitudes and components of the force vectors $\overrightarrow{F_1}$ *and* $\overrightarrow{F_2}$

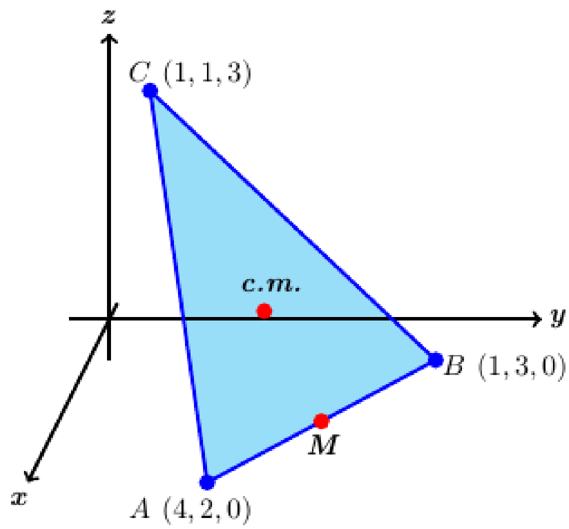


35. A bird flies from its nest 5 *km* in the direction 60° north east, where it stops to rest on a tree. It then flies 10 *km* in the direction due southeast and lands atop a telephone pole. Place an *xy*-coordinate system so that the origin is the bird's nest, the *x*-axis points east, and the *y*-axis points north.

a) At what point is the tree located?

b) At what point is the telephone pole?

36. Suppose that *A*, *B*, and *C* are the corner points of the thin triangular plate of constant density.



a) Find the vector from *C* to the midpoint *M* of side *AB*.

b) Find the vector from *C* to the point that lies two-thirds of the way from *C* to *M* on the median *CM*.

c) Find the coordinates of the point in which the medians of $\triangle ABC$ intersect (this point is the plate's center of mass).

37. Show that a unit vector in the plane can be expressed as $\vec{u} = (\cos\theta)\hat{i} + (\sin\theta)\hat{j}$, obtained by rotating $\hat{i}$ through an angle θ in the counterclockwise direction. Explain why this form gives ***every*** *unit vector* in the plane.

38. Assume the positive *x-axis* points east and the positive *y-axis* points north.

a) An airliner flies northeast at a constant altitude at 550 *mi/hr* in calm air. Find *a* and *b* such that it velocity may be expressed in the form $\vec{v} = a\hat{i} + b\hat{j}$

b) An airliner flies northeast at a constant altitude at 550 *mi/hr* relative to the air in a southerly crosswind $\vec{w} = \langle 0,\ 40\rangle$. Find the velocity of the airliner relative to the ground.

39. Let $\overrightarrow{PQ}$ extended from $P(2,\ 0,\ 6)$ to $Q(2,\ -8,\ 5)$

a) Find the position vector equal to $\overrightarrow{PQ}$.

b) Find the midpoint *M* of the line segment PQ. Then find the magnitude of $\overrightarrow{PM}$

c) Find a vector of length 8 with direction opposite that of $\overrightarrow{PQ}$

40. An object at the origin is acted on by the forces $\overrightarrow{F_1} = -10\hat{i} + 20\hat{k}$, $\overrightarrow{F_2} = 40\hat{j} + 10\hat{k}$, and $\overrightarrow{F_3} = -50\hat{i} + 20\hat{j}$. Find the magnitude of the combined force and use a sketch to illustrate the direction of the combined force.

41. A remote sensing probe falls vertically with a terminal of 60 *m/s* when it encounters a horizontal crosswind blowing north at 4 *m/s* and an updraft blowing vertically at 10 *m/s*. find the magnitude and direction of the resulting velocity relative to the ground.

42. A small plane is flying north in calm air at 250 *mi/hr* when it is hit by a horizontal crosswind blowing northeast at 40 *mi/hr* and a 25 *mi/hr* downdraft. Find the resulting velocity and speed of the plane.