Lecture 4

4.1 Relation (R)

$(a, b) \in R \qquad a R b$

$(a, b) \in R \qquad a \not R b$

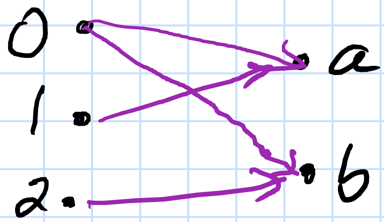
Ex

$A = \overbrace{\{0, 1, 2\}}^{3} \qquad B = \overbrace{\{a, b\}}^{2}$

$R: \{(0,a), (0,b)\ (1,a), (2,b)\}$

$0 R a \qquad 1 \not R b$

$2 \not R a$

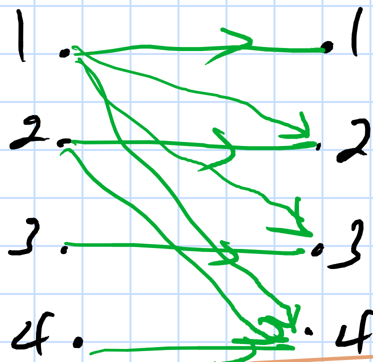


R	a	b
0	×	×
1	×	
2		×

Ex $A = \{1, 2, 3, 4\}$

$R = \{(a,b) \mid a \text{ divides } b\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2)$
$(2,4), (3,3), (4,4)\}$



$R_1 \to a \le b$ $\quad R_2 \to a > b$ $\quad R_3 \quad a = \pm b$

$R_4 \Rightarrow a = b$ $\quad R_5 \quad a = b+1$ $\quad R_6 \quad a+b \le 3$

$(1,1)$ $\quad R_1, R_3, R_4, R_6$

$(1,2)$ $\quad R_1, R_6$

$(2,1)$ $\quad R_2, R_5, R_6$

$(1,-1)$ $\quad R_2, R_3, R_6$

$(2,2)$ $\quad R_1, R_3, R_4$

Ex of n elments ?

$A \times A$ $\quad$ n^2 elements for A^2

$n \quad n$

2^{n^2}

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① Reflexive

<u>Defn</u> R on a set A is called reflexive if $(a,a) \in R \quad \forall a \in A$.

(graphically a⟲

Ex $R_1 = \{ (1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4) \}$

R_1 is not reflexive because $(3,3) \notin R_1$

R_3 is reflexive: $(1,1), (2,2), (3,3), (4,4)$

② Symmetric

Defn A relation R is called symmetric if $(b,a) \in R \rightarrow (a,b) \in R$

$$\forall a \forall b \left((a,b) \in R \rightarrow (b,a) \in R \right)$$

Ex "divides"

$1 \mid 2 \qquad 2 \nmid 1 \qquad$ antisymmetric

Ex $R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$

R_1 is antisymmetric (not symmetric)

$(4,3) \notin R$

$R_2 = \{(1,1), (1,2), (2,1)\}$

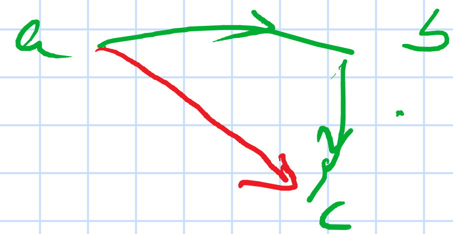
R_2 is symmetric

to be symmetric

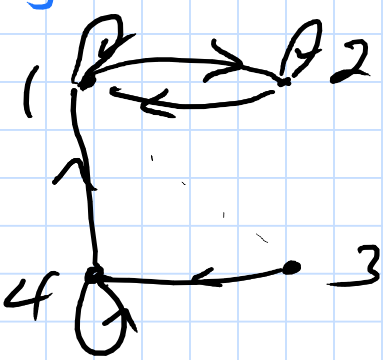
3 Transitive

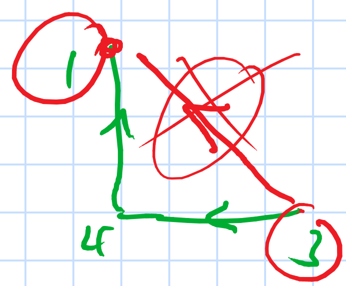
Defn $(a,b) \in R$ & $(b,c) \in R$

then $(a,c) \in R$



$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1),$
$(4,4)\}$





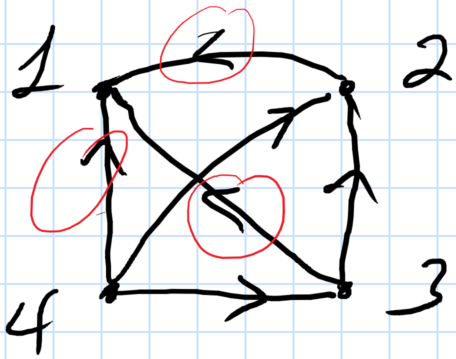
not reflexive $3 \not R\, 3$

" symmetric $1 \not R\, 4$ $\quad (1,4) \in R$

not Transitive!

$3 \not R\, 1$

Ex $R_4 = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)\}$



R is not reflexive $a \not R a$

R is not symmetric $3 \not R 4$

R is transitive.

$R_1 = \{(a,b) \mid a \leq b\}$

$a \leq a \Rightarrow R_1$ is reflexive

$a \leq b \Rightarrow b \not\leq a$ is antisymmetric

$1 \leq 2 \Rightarrow 2 \not\leq 1$

$a \leq b,\ b \leq c \Rightarrow a \leq c$

R_1 is transitive

$R_5 = \{(a,b) \mid a = b+1\}$

$a \neq a+1 \Rightarrow a \not R a$

R_5 is not reflexive

$aRb \rightarrow a = b+1 \Rightarrow bRa \rightarrow b = a+1$ #

$\hookrightarrow b = a-1 \neq a+1$

R_5 is not symmetric

$aRb \rightarrow a = b+1$

$bRc \rightarrow b = c+1$

$aRc \rightarrow a = c+1$?

$$a = b+1$$
$$= (c+1)+1$$
$$= c+2$$

$\Rightarrow 1 \neq 2$ R_5 is not transitive.

$R_1 = \{(1,1), (2,2), (3,3)\}$

$R_2 = \{(1,1), (1,2), (1,3), (1,4)\}$

$R_1 \cup R_2 = \{(1,1), (1,2), (1,2), (1,4), (2,2), (3,3)\}$

$R_1 \cap R_2 = \{(1,1)\}$

$R_1 - R_2 = \{(2,2), (3,3)\}$

$R_2 - R_1 = \{(1,2), (1,3), (1,4)\}$

$$R_1 \oplus R_2 = (R_1 \cup R_2) - (R_1 \cap R_2)$$

$(a,b) \in R$ $\quad$ $(b,c) \in S$

$S \circ R$

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$

$R \circ S = \{(1,0), (1,1), (2,1), (2,0), (3,1), (2,2)\}$

R	S	
$(1,1)$	$(1,0)$	$(1,0)$
$(1,4)$	$(4,1)$	$(1,1)$
$(2,3)$	$(3,1)$	$(2,1)$
$(3,1)$	$(1,0)$	$(3,0)$
$(3,4)$	$(4,1)$	$(3,1)$
$(2,3)$	$(3,2)$	$(2,2)$

$$R = R^1$$
$$R^{n+1} = R^n \circ R$$

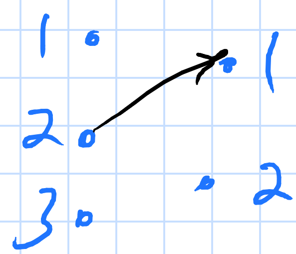
4.2

$R = \{(2,1), (3,1), (3,2)\}$

$$\boxed{m_{ij} = \begin{cases} 1 & (a_i, b_j) \in R \\ 0 & (a_i, b_j) \notin R \end{cases}}$$

$A = \{1, 2, 3\} \quad B = \{1, 2\}$

$$M = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$



Ex

$A = \{a_1, a_2, a_3\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

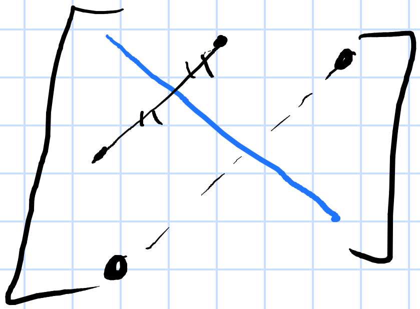
$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_5)\}$

Reflexive $(n \times n)$

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

main diagonal has to be 1 always (all)



$$a_{ij} = a_{ji}$$

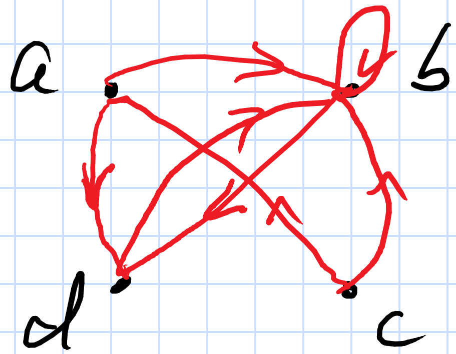
$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

M_R is reflexive, all entries in the main diagonal has 1.

M_R is symmetric

$(a,b), (a,d), (b,b), (b,d), (c,a), (c,b)$
(d,b)

directed graph



It's not reflexive $a \not R a$

not symmetric $a R b$ but $b \not R a$

$c R b$, $b R d \Rightarrow c \not R d$

it'not transitive.

$\Delta = \{(a,a) \mid a \in A\} = \{(a,a), (c,c), (d,d)\}$

reflexive closure

$R \cup \Delta = \{(a,a), (b,b), (c,c), (d,d) \ldots\}$

$a < b$ - Δ: $a = a$

$a \leq b$

Path