Solution Section 1.2 – Dot Products

Exercise

Find for $\vec{v} = 2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}, \quad \vec{u} = -2\hat{i} + 4\hat{j} - \sqrt{5}\hat{k}$

a) $\vec{v} \bullet \vec{u}, \ |\vec{v}|, \ |\vec{u}|$

b) The cosine of the angle between $\vec{v}$ and $\vec{u}$

c) The scalar component of $\vec{u}$ in the direction of $\vec{v}$

d) The vector $proj_{\vec{v}}\vec{u}$

Solution

a)

$$\begin{aligned}\vec{v} \bullet \vec{u} &= \left(2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}\right)\cdot\left(-2\hat{i} + 4\hat{j} - \sqrt{5}\hat{k}\right)\\ &= -4 - 16 - 5\\ &= -25\end{aligned}$$

$$\begin{aligned}|\vec{v}| &= \sqrt{2^2 + (-4)^2 + \left(\sqrt{5}\right)^2}\\ &= \sqrt{4 + 16 + 5}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

$$\begin{aligned}|\vec{u}| &= \sqrt{(-2)^2 + 4^2 + \left(-\sqrt{5}\right)^2}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

b)

$$\begin{aligned}\cos\theta &= \frac{-25}{(5)(5)}\\ &= -1\end{aligned}$$

$$\cos\theta = \frac{\vec{u} \bullet \vec{v}}{|\vec{u}| \ |\vec{v}|}$$

c)

$$\begin{aligned}|\vec{u}|\cos\theta &= (5)(-1)\\ &= -5\end{aligned}$$

d)

$$\begin{aligned}proj_{\vec{v}}\vec{u} &= \left(\frac{\vec{u} \bullet \vec{v}}{|\vec{v}|^2}\right)\vec{v}\\ &= \left(\frac{-25}{5^2}\right)\left(2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}\right)\\ &= -\left(2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}\right)\\ &= -2\hat{i} + 4\hat{j} - \sqrt{5}\hat{k}\end{aligned}$$

Exercise

Find for $\vec{v} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}, \quad \vec{u} = 5\hat{i} + 12\hat{j}$

a) $\vec{v} \bullet \vec{u}$, $|\vec{v}|$, $|\vec{u}|$

b) The cosine of the angle between $\vec{v}$ and $\vec{u}$

c) The scalar component of $\vec{u}$ in the direction of $\vec{v}$

d) The vector $proj_{\vec{v}}\vec{u}$

Solution

a) $\vec{v} \bullet \vec{u} = \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}\right) \bullet \left(5\hat{i} + 12\hat{j}\right)$

$$= 3$$

$$|\vec{v}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{16}{25}}$$

$$= \sqrt{\frac{25}{25}}$$

$$= 1$$

$$|\vec{u}| = \sqrt{5^2 + 12^2}$$

$$= 13$$

b) $\cos\theta = \dfrac{\vec{u} \bullet \vec{v}}{|\vec{u}|\ |\vec{v}|}$

$$= \frac{3}{(1)(13)}$$

$$= \frac{3}{13}$$

c) $|\vec{u}|\cos\theta = (13)\left(\frac{3}{13}\right) = 3$

d) $proj_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u} \bullet \vec{v}}{|\vec{v}|^2}\right)\vec{v}$

$$= \left(\frac{3}{1^2}\right)\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}\right)$$

$$= \frac{9}{5}\hat{i} + \frac{12}{5}\hat{k}$$

Exercise

Find for $\vec{v} = 2\hat{i} + 10\hat{j} - 11\hat{k}, \quad \vec{u} = 2\hat{i} + 2\hat{j} + \hat{k}$

a) $\vec{v} \bullet \vec{u},\ |\vec{v}|,\ |\vec{u}|$

b) The cosine of the angle between $\vec{v}$ and $\vec{u}$

c) The scalar component of $\vec{u}$ in the direction of $\vec{v}$

d) The vector $proj_{\vec{v}}\vec{u}$

Solution

a)
$$\begin{aligned}\vec{v} \bullet \vec{u} &= \left(2\hat{i} + 10\hat{j} - 11\hat{k}\right) \bullet \left(2\hat{i} + 2\hat{j} + \hat{k}\right) \\ &= 4 + 20 - 11 \\ &= 13\end{aligned}$$

$$\begin{aligned}|\vec{v}| &= \sqrt{2^2 + 10^2 + (-11)^2} \\ &= \sqrt{4 + 100 + 121} \\ &= \sqrt{225} \\ &= 15\end{aligned}$$

$$\begin{aligned}|\vec{u}| &= \sqrt{2^2 + 2^2 + 1^2} \\ &= 3\end{aligned}$$

b)
$$\begin{aligned}\cos\theta &= \frac{\vec{u} \bullet \vec{v}}{|\vec{u}|\ |\vec{v}|} \\ &= \frac{13}{(3)(15)} \\ &= \frac{13}{45}\end{aligned}$$

c)
$$\begin{aligned}|\vec{u}|\ \cos\theta &= (3)\left(\frac{13}{45}\right) \\ &= \frac{13}{15}\end{aligned}$$

d)
$$\begin{aligned}proj_{\vec{v}}\vec{u} &= \left(\frac{\vec{u} \bullet \vec{v}}{|\vec{v}|^2}\right)\vec{v} \\ &= \left(\frac{13}{15^2}\right)\left(2\hat{i} + 10\hat{j} - 11\hat{k}\right) \\ &= \frac{13}{225}\left(2\hat{i} + 10\hat{j} - 11\hat{k}\right)\end{aligned}$$

Exercise

Find for $\vec{v} = -\hat{i} + \hat{j}$, $\vec{u} = 2\hat{i} + \sqrt{17}\,\hat{j}$

a) $\vec{v} \bullet \vec{u},\ |\vec{v}|,\ |\vec{u}|$

b) The cosine of the angle between $\vec{v}$ and $\vec{u}$

c) The scalar component of $\vec{u}$ in the direction of $\vec{v}$

d) The vector $proj_{\vec{v}}\vec{u}$

Solution

a)
$$\vec{v} \bullet \vec{u} = \left(5\hat{i} + \hat{j}\right) \bullet \left(2\hat{i} + \sqrt{17}\,\hat{j}\right)$$
$$= 10 + \sqrt{17}$$

$$|\vec{v}| = \sqrt{25+1}$$
$$= \sqrt{26}$$

$$|\vec{u}| = \sqrt{4+17}$$
$$= \sqrt{21}$$

b)
$$\cos\theta = \frac{\vec{u} \bullet \vec{v}}{|\vec{u}|\ |\vec{v}|}$$
$$= \frac{10+\sqrt{17}}{\sqrt{21}\sqrt{26}}$$
$$= \frac{10+\sqrt{17}}{\sqrt{546}}$$

c)
$$|\vec{u}|\cos\theta = \left(\sqrt{21}\right)\left(\frac{10+\sqrt{17}}{\sqrt{546}}\right)$$
$$= \frac{10+\sqrt{17}}{\sqrt{26}}$$

d)
$$proj_{\vec{v}}\vec{u} = \left(\frac{\vec{u} \bullet \vec{v}}{|\vec{v}|^2}\right)\vec{v}$$
$$= \left(\frac{10+\sqrt{17}}{26}\right)\left(5\hat{i} + \hat{j}\right)$$

Exercise

Find for $\vec{v}=\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}} \right\rangle,\quad \vec{u}=\left\langle \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{3}} \right\rangle$

a) $\vec{v}\bullet\vec{u},\ |\vec{v}|,\ |\vec{u}|$

b) The cosine of the angle between $\vec{v}$ and $\vec{u}$

c) The scalar component of $\vec{u}$ in the direction of $\vec{v}$

d) The vector $proj_{\vec{v}}\vec{u}$

Solution

a) $\vec{v}\bullet\vec{u}=\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}} \right\rangle\bullet\left\langle \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{3}} \right\rangle$

$$=\frac{1}{2}-\frac{1}{3}$$

$$=\frac{1}{6}$$

$$|\vec{v}|=\sqrt{\frac{1}{2}+\frac{1}{3}}$$

$$=\frac{\sqrt{5}}{\sqrt{6}}\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{\sqrt{30}}{6}$$

$$|\vec{u}|=\sqrt{\frac{1}{2}+\frac{1}{3}}$$

$$=\frac{\sqrt{5}}{\sqrt{6}}\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{\sqrt{30}}{6}$$

b) $\cos\theta=\frac{\vec{u}\bullet\vec{v}}{|\vec{u}|\ |\vec{v}|}$

$$=\frac{\frac{1}{6}}{\frac{\sqrt{30}}{6}\frac{\sqrt{30}}{6}}$$

$$=\frac{1}{6}\left(\frac{36}{30}\right)$$

$$=\frac{1}{5}$$

c) $|\vec{u}|\cos\theta=\left(\frac{\sqrt{30}}{6}\right)\left(\frac{1}{5}\right)$

$$= \frac{\sqrt{30}}{30}$$

$$\boxed{= \frac{1}{\sqrt{30}}}$$

d) $proj_{\vec{v}}\,\vec{u} = \left(\frac{\vec{u} \bullet \vec{v}}{|\vec{v}|^2}\right)\vec{v}$

$$= \frac{1}{6}\left(\frac{36}{30}\right)\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}} \right\rangle$$

$$\boxed{= \frac{1}{5}\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}} \right\rangle}$$

Exercise

Find the angles between the vectors $\vec{u} = 2\hat{i} + \hat{j}, \quad \vec{v} = \hat{i} + 2\hat{j} - \hat{k}$

Solution

$$\theta = \cos^{-1}\left(\frac{2+2+0}{\sqrt{4+1}\sqrt{1+4+1}}\right) \qquad \cos\theta = \frac{\vec{u} \bullet \vec{v}}{|\vec{u}|\ |\vec{v}|}$$

$$= \cos^{-1}\left(\frac{4}{\sqrt{5}\sqrt{6}}\right)$$

$$= \cos^{-1}\left(\frac{4}{\sqrt{30}}\right)$$

$$\boxed{\approx 0.84\ \ rad}$$

Exercise

Find the angles between the vectors $\vec{u} = \sqrt{3}\hat{i} - 7\hat{j}, \quad \vec{v} = \sqrt{3}\hat{i} + \hat{j} + \hat{k}$

Solution

$$\theta = \cos^{-1}\left(\frac{3-7+0}{\sqrt{3+49}\sqrt{3+1+1}}\right) \qquad \cos\theta = \frac{\vec{u} \bullet \vec{v}}{|\vec{u}|\ |\vec{v}|}$$

$$= \cos^{-1}\left(\frac{-4}{\sqrt{52}\sqrt{5}}\right)$$

$$= \cos^{-1}\left(-\frac{4}{\sqrt{260}}\right)$$

$$\boxed{\approx 1.82\ \ rad}$$

Exercise

Find the angles between the vectors $\vec{u} = \hat{i} + \sqrt{2}\hat{j} - \sqrt{2}\hat{k}, \quad \vec{v} = -\hat{i} + \hat{j} + \hat{k}$

Solution

$$\theta = \cos^{-1}\left(\frac{-1+\sqrt{2}-\sqrt{2}}{\sqrt{1+2+2}\sqrt{1+1+1}}\right) \qquad \cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\ |\vec{v}|}$$

$$= \cos^{-1}\left(\frac{-1}{\sqrt{5}\sqrt{3}}\right)$$

$$= \cos^{-1}\left(-\frac{1}{\sqrt{15}}\right)$$

$$\boxed{\approx 1.83\ \ rad}$$

Exercise

Consider $\vec{u} = -3\hat{j} + 4\hat{k}$, $\vec{v} = -4\hat{i} + \hat{j} + 5\hat{k}$

a) Find the angle between $\vec{u}$ and $\vec{v}$.

b) Compute $proj_{\vec{v}}\vec{u}$ and $scal_{\vec{v}}\vec{u}$

c) Compute $proj_{\vec{u}}\vec{v}$ and $scal_{\vec{u}}\vec{v}$

Solution

a) $$\theta = \cos^{-1}\frac{\left(-3\hat{j}+4\hat{k}\right)\cdot\left(-4\hat{i}+\hat{j}+5\hat{k}\right)}{\sqrt{9+16}\ \sqrt{16+1+25}} \qquad \cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\ |\vec{v}|}$$

$$= \cos^{-1}\frac{-3+20}{\sqrt{25}\ \sqrt{42}}$$

$$\boxed{= \cos^{-1}\frac{17}{5\sqrt{42}}}$$

$$\boxed{\approx 1.02\ \ rad}$$

b) $$proj_{\vec{v}}\vec{u} = \frac{17}{42}\left(-4\hat{i}+\hat{j}+5\hat{k}\right) \qquad proj_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v}$$

$$\boxed{= \frac{17}{42}\langle -4,\ \ 1,\ \ 5\rangle}$$

$$\boxed{scal_{\vec{v}}\vec{u} = \frac{17}{\sqrt{42}}} \qquad scal_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$$

c) $$proj_{\vec{u}}\vec{v} = \frac{17}{25}\left(-3\hat{j}+4\hat{k}\right) \qquad proj_{\vec{u}}\vec{v} = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|^2}\vec{u}$$

$$\boxed{= \frac{17}{25}\langle 0,\ \ -3,\ \ 4\rangle}$$

$$\boxed{scal_{\vec{u}}\,\vec{v} = \frac{17}{5}}$$

$$scal_{\vec{v}}\,\vec{u} = \frac{\vec{u}\bullet\vec{v}}{|\vec{v}|}$$

Exercise

Consider $\vec{u} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{v} = 3\hat{i} + 6\hat{j} + 6\hat{k}$

a) Find the angle between $\vec{u}$ and $\vec{v}$.

b) Compute $proj_{\vec{v}}\vec{u}$ and $scal_{\vec{v}}\vec{u}$

c) Compute $proj_{\vec{u}}\vec{v}$ and $scal_{\vec{u}}\vec{v}$

Solution

a)
$$\theta = \cos^{-1}\frac{\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)\bullet\left(3\hat{i} + 6\hat{j} + 6\hat{k}\right)}{\sqrt{1+4+4}\ \sqrt{9+36+36}}$$
$$= \cos^{-1}\frac{-3+12+12}{3\ (9)}$$
$$= \cos^{-1}\frac{21}{27}$$
$$\boxed{= \cos^{-1}\frac{7}{9}}$$
$$\boxed{\approx 0.68\ \ rad}$$

$$\cos\theta = \frac{\vec{u}\bullet\vec{v}}{|\vec{u}|\ |\vec{v}|}$$

b)
$$proj_{\vec{v}}\,\vec{u} = \frac{21}{81}\langle 3,\ 6,\ 6\rangle$$
$$\boxed{= \frac{7}{9}\langle 1,\ 2,\ 2\rangle}$$

$$proj_{\vec{v}}\,\vec{u} = \frac{\vec{u}\bullet\vec{v}}{|\vec{v}|^2}\vec{v}$$

$$scal_{\vec{v}}\,\vec{u} = \frac{21}{9}$$
$$\boxed{= \frac{7}{3}}$$

$$scal_{\vec{v}}\,\vec{u} = \frac{\vec{u}\bullet\vec{v}}{|\vec{v}|}$$

c)
$$proj_{\vec{u}}\,\vec{v} = \frac{21}{9}\langle -1,\ 2,\ 2\rangle$$
$$\boxed{= \frac{7}{3}\langle -1,\ 2,\ 2\rangle}$$

$$proj_{\vec{u}}\,\vec{v} = \frac{\vec{u}\bullet\vec{v}}{|\vec{u}|^2}\vec{u}$$

$$scal_{\vec{u}}\,\vec{v} = \frac{21}{3}$$
$$\boxed{= 7}$$

$$scal_{\vec{v}}\,\vec{u} = \frac{\vec{u}\bullet\vec{v}}{|\vec{v}|}$$

Exercise

The direction angles α, β, and γ of a vector $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ are defined as follows:

is the angle between $\vec{v}$ and the positive x-axis $(0 \le \alpha \le \pi)$

is the angle between $\vec{v}$ and the positive y-axis $(0 \le \beta \le \pi)$

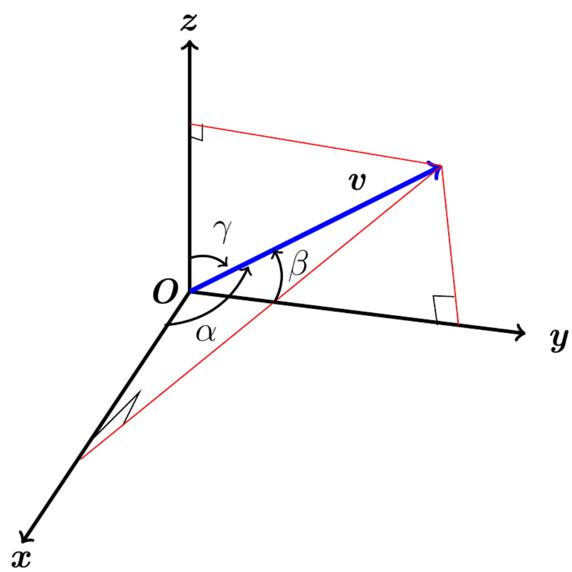
is the angle between $\vec{v}$ and the positive z-axis $(0 \le \gamma \le \pi)$

a) Show that $\cos\alpha = \frac{a}{|\vec{v}|}$, $\cos\beta = \frac{b}{|\vec{v}|}$, $\cos\gamma = \frac{c}{|\vec{v}|}$, and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. These cosines are called the direction cosines of $\vec{v}$.

b) Show that if $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ is a unit vector, then a, b, and c are the direction cosines of $\vec{v}$.

Solution

a)
$$\cos\alpha = \frac{\hat{i}\bullet\vec{v}}{|\hat{i}||\vec{v}|}$$
$$= \frac{\hat{i}\bullet\left(a\hat{i} + b\hat{j} + c\hat{k}\right)}{|\vec{v}|}$$
$$\boxed{= \frac{a}{|\vec{v}|}}$$



$$\cos\beta = \frac{\hat{j}\bullet\vec{v}}{|\hat{j}||\vec{v}|}$$
$$= \frac{\hat{j}\bullet\left(a\hat{i} + b\hat{j} + c\hat{k}\right)}{|\vec{v}|}$$
$$\boxed{= \frac{b}{|\vec{v}|}}$$

$$\cos\gamma = \frac{\hat{k}\bullet\vec{v}}{|\hat{k}||\vec{v}|}$$
$$= \frac{\hat{k}\bullet\left(a\hat{i} + b\hat{j} + c\hat{k}\right)}{|\vec{v}|}$$
$$\boxed{= \frac{c}{|\vec{v}|}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\frac{a}{|\vec{v}|}\right)^2 + \left(\frac{b}{|\vec{v}|}\right)^2 + \left(\frac{c}{|\vec{v}|}\right)^2$$
$$= \frac{a^2}{|\vec{v}|^2} + \frac{b^2}{|\vec{v}|^2} + \frac{c^2}{|\vec{v}|^2}$$

$$= \frac{a^2+b^2+c^2}{|\vec{v}|^2}$$

$$= \frac{a^2+b^2+c^2}{a^2+b^2+c^2}$$

$$\underline{=1}$$

b) If $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ is a unit vector $\Rightarrow |\vec{v}| = 1$

$\cos\alpha = \frac{a}{|\vec{v}|} = a, \quad \cos\beta = \frac{b}{|\vec{v}|} = b, \quad \cos\gamma = \frac{c}{|\vec{v}|} = c$ are the direction cosines of $\vec{v}$.

Exercise

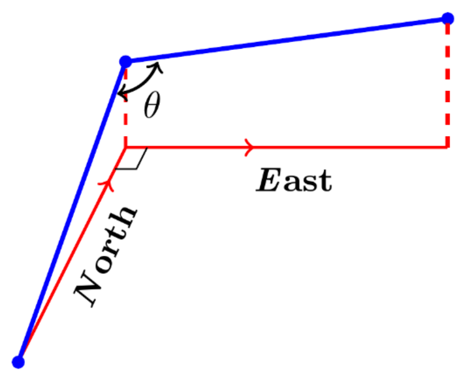
A water main is to be constructed with 20% grade in the north direction and a 10% grade in the east direction. Determine the angle θ required in the water main for the turn from north to east.

Solution

20% grade in the north direction

$z\hat{k} = 20\% x\hat{i} = .2x\hat{i}$

$\to If \ \ x = 10 \quad z = 2$



Let $\vec{u} = 10\hat{i} + 2\hat{k}$ be parallel to the pipe in the north direction.

$\vec{v} = 10\hat{j} + \hat{k}$ be parallel to the pipe in the east direction.

$$\theta = \cos^{-1}\frac{0+0+2}{\sqrt{100+4}\sqrt{100+1}} \qquad \theta = \cos^{-1}\frac{\vec{u}\bullet\vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \cos^{-1}\frac{2}{\sqrt{104}\sqrt{101}}$$

$$\underline{\approx 88.88^\circ}$$

Exercise

A gun with muzzle velocity of 1200 *ft/sec* is fired at an angle of 8° above the horizontal. Find the horizontal and vertical components of the velocity.

Solution

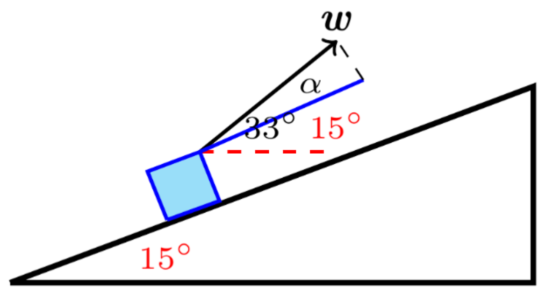
Horizontal component: $1200\cos 8^\circ \ \underline{\approx 1188 \ \ ft/s}$

Vertical component: $1200\sin 8^\circ \ \underline{\approx 167 \ \ ft/s}$

Exercise

Suppose that a box is being towed up an inclined plane. Find the force ***w*** needed to make the component of the force parallel to the indicated plane equal to 2.5 *lb*.

Solution



$$2.5 = |w|\cos\alpha$$

$$|\vec{w}| = \frac{2.5}{\cos(33° - 15°)}$$

$$= \frac{2.5}{\cos 18°}$$

$$\vec{w} = \frac{2.5}{\cos 18°}\langle \cos 33°,\ \sin 33° \rangle$$

$$= \langle 2.205,\ 1.432 \rangle$$

Exercise

Find the work done by a force $\vec{F} = 5\hat{i}$ (magnitude 5 *N*) in moving an object along the line from the origin to the point (1, 1) (distance in meters)

Solution

$$P(1,\ 1) \quad \Rightarrow \quad \overrightarrow{OP} = \hat{i} + \hat{j}$$

$$W = F \bullet \overrightarrow{OP}$$

$$= 5\hat{i} \bullet (\hat{i} + \hat{j})$$

$$= 5\ \ J$$

Exercise

How much work does it take to slide a crate 20 *m* along a loading dock by pulling on it with a 200 *N* force at an angle of 30° from the horizontal?

Solution

$$W = |F|\left|\overrightarrow{PQ}\right|\cos\theta$$

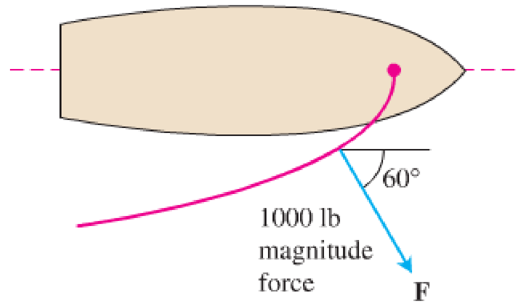
$$= (200)(20)\cos 30°$$

$$= 3464.10\ \ J$$

Exercise

The wind passing over a boat's sail exerted a 1000-*lb* magnitude force *F*. How much work did the wind perform in moving the boat forward 1 *mi*? Answer in foot-pounds.

Solution



$$W = |F|\left|\overrightarrow{PQ}\right|\cos\theta$$

$$= (1000N)\left(1\ mi\,\frac{5280\,ft}{1\ mi}\right)\cos 60°$$

$$\boxed{= 2,640,000\quad ft\cdot lb}$$

Exercise

Use a dot product to find an equation of the line in the *xy*-plane passing through the point (x_0, y_0) perpendicular to the vector $\langle a,\ b\rangle$.

Solution

$$\langle x - x_0,\ y - y_0\rangle \bullet \langle a,\ b\rangle = 0$$

$$\boxed{a(x - x_0) + b(y - y_0) = 0}$$

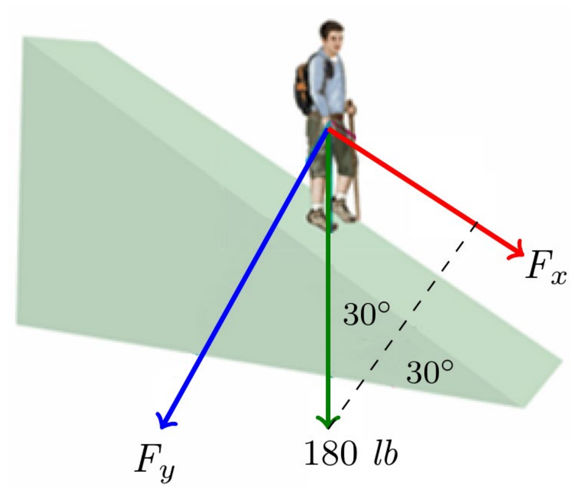
Exercise

A 180-*lb* man stands on a hillside that makes an angle of 30° with the horizontal, producing a force of $W = \langle 0,\ -180\rangle$ *lbs*.

a) Find the component of his weight in the downward direction perpendicular to the hillside and in the downward parallel to the hillside.

b) How much work is done when the man moves 10 *ft* up the hillside?

Solution



a) $|F_\perp| = |F_y| = 180\cos 30°$

$$= 180\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{= 90\sqrt{3}\ \ lb}$$

$$|F_{/\!/}| = |F_x| = 180\sin 30°$$

$$= 180\left(\frac{1}{2}\right)$$

$$\boxed{= 90\ \ lb}$$

b) $Work = d \cdot F_x$

$$= 10(90)$$

$$\boxed{= 900 \;\; ft\text{-}lbs}$$