Math 2312 – Pre-Calculus

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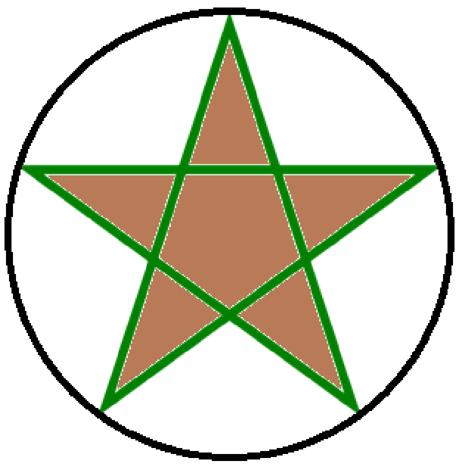
***Assignment* 3**

***Directions*: Show your work whenever possible: a correct answer is worth 0 point without any supporting work.**

1. In any triangle *ABC*, prove that:

$$a = b\cos C \ + \ c \ \cos B$$
$$b = c\cos A \ + \ a \ \cos C$$
$$c = a\cos B \ + \ b \ \cos A$$

2. Find the area of the shaded star that is inscribed in a circle with a radius 1.



3. Evaluate:

$$\sin 1° + \sin 2° + \sin 3° + \cdots\cdots + \sin 357° + \sin 358° + \sin 359°$$
$$\sin^2 1° + \sin^2 2° + \sin^2 3° + \cdots\cdots + \sin^2 357° + \sin^2 358° + \sin^2 359°$$

4. Find the solution(s) for: $\cos 2x + \cos 4x = \cos x$

Solution

1. $b\cos C + c\cos B = b\dfrac{a^2+b^2-c^2}{2ab} + c\dfrac{a^2+c^2-b^2}{2ac}$

$$= \frac{a^2+b^2-c^2}{2a} + \frac{a^2+c^2-b^2}{2a}$$

$$= \frac{a^2+b^2-c^2+a^2+c^2-b^2}{2a}$$

$$= \frac{2a^2}{2a}$$

$$\underline{= a|}$$

$$c\cos A + a\cos C = c\frac{b^2+c^2-a^2}{2bc} + a\frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{b^2+c^2-a^2}{2b} + \frac{a^2+b^2-c^2}{2b}$$

$$= \frac{b^2+c^2-a^2+a^2+b^2-c^2}{2b}$$

$$= \frac{2b^2}{2b}$$

$$\underline{= b|}$$

$$a\cos B + b\cos A = a\frac{b^2+c^2-a^2}{2ac} + b\frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{b^2+c^2-a^2}{2c} + \frac{b^2+c^2-a^2}{2c}$$

$$= \frac{b^2+c^2-a^2+b^2+c^2-a^2}{2c}$$

$$= \frac{2c^2}{2c}$$

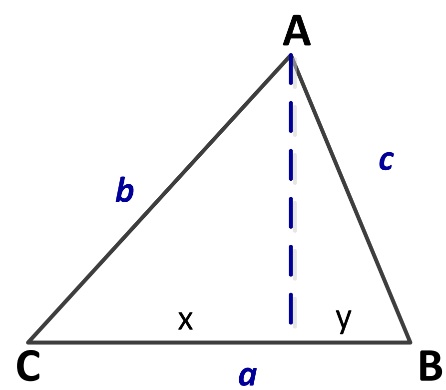
$$\underline{= c|}$$

$\cos C = \dfrac{x}{b} \to x = b\cos C$

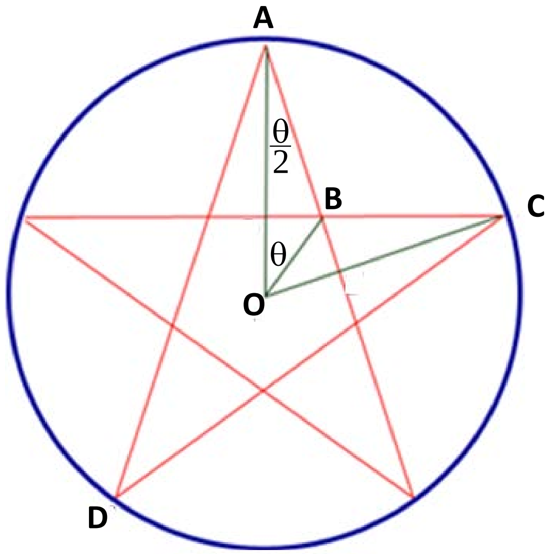
$\cos B = \dfrac{y}{c} \to y = c\cos B$

$a = x + y$

$a = b\cos C + c\cos B$



2.



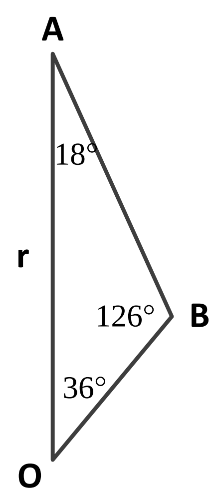
Given: $OA = OC = r = 1$

The star divides the circle into 5 equal sections. $\Rightarrow \angle AOC = \frac{360°}{5} = 72°$.

The segment OB cut the angle $\angle AOC$ into half $\Rightarrow \theta = \frac{72°}{2} = 36°$

By definition: $\angle CDA = \frac{1}{2}\angle AOC = \frac{1}{2}\theta = \angle OAB \Rightarrow \boxed{\angle OAB = \frac{36°}{2} = 18°}$

$$\begin{aligned}\angle OBA &= 180° - \angle BOA - \angle OAB \\ &= 180° - 36° - 18° \\ &= 126°\end{aligned}$$



Consider the triangle AOB:

Using the Law of sine:

$$\frac{AB}{\sin 36°} = \frac{r}{\sin 126°} \Rightarrow AB = \frac{1.\sin 36°}{\sin 126°} = 0.727$$

$$\frac{OB}{\sin 18°} = \frac{r}{\sin 126°} \Rightarrow OB = \frac{1.\sin 18°}{\sin 126°} = 0.382$$

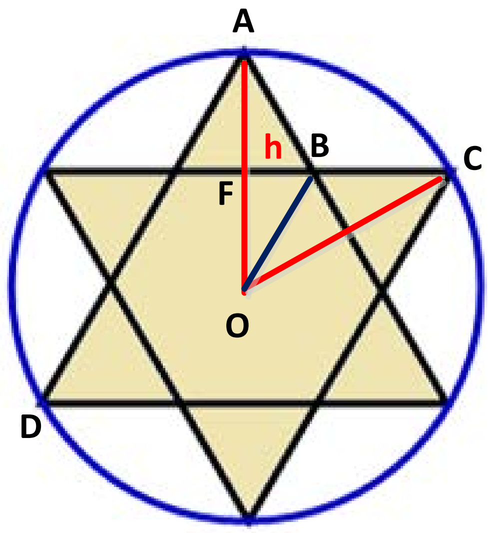
Use *Heron's formula* to find the area of the triangle:

$$s = \frac{1}{2}(1 + 0.727 + 0.382) = 1.055$$

$$\begin{aligned}A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{1.055(1.055-1)(1.055-.727)(1.055-.382)} \\ &= .113\end{aligned}$$

The triangle AOB is equal to OBC and so on.

The total area of the star $= 10(.113) = 1.13$



The star divides the circle into 6 equal sections. $\Rightarrow \angle AOC = \frac{360°}{6} = 60°$.

$\Rightarrow \angle AOB = \frac{60°}{2} = 30° \qquad \Rightarrow \boxed{\angle OAB = \frac{60°}{2} = 30°}$

$$\begin{aligned}\angle OBA &= 180° - \angle BOA - \angle OAB \\ &= 180° - 30° - 30° \\ &= 120°\end{aligned}$$

Consider the triangle AOB:

Using the Law of sine:

$$\frac{AB}{\sin 60°} = \frac{r}{\sin 120°} \Rightarrow AB = \frac{\sin 30°}{\sin 120°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = OB$$

OR

Consider the right triangle AFB with

$$F = 90°, \quad \angle FAB = 30° \quad \rightarrow \tan 30° = \frac{h}{AF} \Rightarrow \boxed{h = AF \tan 30° = \frac{1}{2}\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{6}}$$

The area of the triangle AOB = 2 times Area of AFB

$$A_1 = \frac{1}{2}h|AF| = \frac{1}{2}\frac{\sqrt{3}}{6}\cdot 1 = \frac{\sqrt{3}}{12}$$

There are 12 equal triangles that cover the star:

$$\boxed{A = 12A_1 = 12\frac{\sqrt{3}}{12} = \sqrt{3}}$$

3. Evaluate:

$$y = -y$$

$$\sin 1^\circ = -\sin 359^\circ \quad \rightarrow \quad \sin 1^\circ + \sin 359^\circ = 0$$

$$\sin 2^\circ = -\sin 358^\circ \quad \rightarrow \quad \sin 2^\circ + \sin 358^\circ = 0$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\sin 90^\circ = -\sin 270^\circ \quad \rightarrow \quad \sin 90^\circ + \sin 270^\circ = 0$$

$$\vdots \qquad \vdots$$

$$\sin 180^\circ = 0$$

$$\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots\cdots + \sin 357^\circ + \sin 358^\circ + \sin 359^\circ = 0$$

The first quadrant:

$$\sin^2 1^\circ + \cos^2 1^\circ = 1 \quad \Rightarrow \sin^2 1^\circ + \sin^2 (90^\circ - 1^\circ) = 1 \quad \Rightarrow \boxed{\sin^2 1^\circ + \sin^2 (89^\circ) = 1}$$

$$\sin^2 2^\circ + \cos^2 2^\circ = 1 \quad \Rightarrow \sin^2 2^\circ + \sin^2 (90^\circ - 2^\circ) = 1 \quad \Rightarrow \boxed{\sin^2 2^\circ + \sin^2 (88^\circ) = 1}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\boxed{\sin^2 44^\circ + \sin^2 (46^\circ) = 1}$$

$$\sin^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 \qquad \sin^2 45^\circ = \frac{1}{2}$$

$$\sin^2 1^\circ + \cdots\cdots + \sin^2 44^\circ + \sin^2 45^\circ + \sin^2 46^\circ + \cdots\cdots + \sin^2 89^\circ = 44 + \frac{1}{2} = 44.5$$

There are four quadrant and $\sin^2 90^\circ = 1 \quad \sin^2 180^\circ = 0 \quad \sin^2 270^\circ = 1$

$$\sin^2 1^\circ + \sin^2 2^\circ + \cdots\cdots + \sin^2 358^\circ + \sin^2 359^\circ = 4(44.5) + 1 + 0 + 1 = 180$$

1. Find one solution for: $\cos 2x + \cos 4x = \cos x$

$$2\cos\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right) = \cos x$$

$$2\cos(3x)\cos(-x) = \cos x$$

$$2\cos(3x)\cos(x) - \cos x = 0$$

$$\cos(x)(2\cos(3x) - 1) = 0$$

$$\cos(x) = 0 \qquad 2\cos(3x) - 1 = 0$$

$$\cos(3x) = \frac{1}{2}$$

$$3x = 60° \qquad 3x = 300°$$

$$x = 90°, 270° \qquad x = 20°, 100°$$

$$x = 20°, 90°, 100°, 270°$$

$$x = \frac{\pi}{9}, \frac{\pi}{2}, \frac{5\pi}{9}, \frac{3\pi}{2}$$