Solution Section 2.9 – Derivatives of Inverse Trigonometric Functions

Exercise

Find the value of $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

Solution

$$\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) = \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}$$

Exercise

Find the value of $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

Solution

$$\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \cot\left(-\frac{\pi}{3}\right)$$

$$= -\frac{1}{\sqrt{3}}$$

Exercise

Find the limit: $\lim\limits_{x\to -1^+} \cos^{-1} x$

Solution

$$\lim_{x\to -1^+} \cos^{-1} x = \cos^{-1}(-1)$$

$$= \pi$$

Exercise

Find the limit: $\lim\limits_{x\to -\infty} \tan^{-1} x$

Solution

$$\lim_{x\to -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

Exercise

Find the limit: $\lim\limits_{x\to\infty} \csc^{-1} x$

Solution

$$\lim_{x\to\infty} \csc^{-1} x = \lim_{x\to\infty} \sin^{-1}\left(\frac{1}{x}\right)$$

$$= \sin^{-1}\left(\frac{1}{\infty}\right)$$

$$\underline{= 0}$$

Exercise

Find the derivative $y = \cos^{-1}\left(\frac{1}{x}\right)$

Solution

$$y = \cos^{-1}\left(\frac{1}{x}\right)$$

$$= \sec^{-1}(x)$$

$$y' = \frac{1}{|x| \cdot \sqrt{x^2 - 1}}$$

Exercise

Find the derivative $y = \sin^{-1}\sqrt{2}t$

Solution

$$y' = \frac{\sqrt{2}}{\sqrt{1 - \left(\sqrt{2}t\right)^2}}$$

$$= \frac{\sqrt{2}}{\sqrt{1 - 2t^2}}$$

Exercise

Find the derivative $y = \sec^{-1}(5s)$

Solution

$$y' = \frac{5s}{|5s|\sqrt{(5s)^2 - 1}}$$

$$= \boxed{\frac{s}{|s|\sqrt{25s^2 - 1}}}$$

Exercise

Find the derivative $y = \cot^{-1}\sqrt{t-1}$

Solution

$$y' = -\frac{\frac{1}{2}(t-1)^{-1/2}}{1 + \left[(t-1)^{1/2}\right]^2}$$

$$= -\frac{1}{2(t-1)^{1/2}(1+t-1)}$$

$$= \boxed{-\frac{1}{2t\sqrt{t-1}}}$$

Exercise

Find the derivative $y = \ln\left(\tan^{-1} x\right)$

Solution

$$y' = \frac{\frac{1}{1+x^2}}{\tan^{-1} x}$$

$$= \boxed{\frac{1}{\left(1+x^2\right)\tan^{-1} x}}$$

Exercise

Find the derivative $y = \tan^{-1}(\ln x)$

Solution

$$y' = \frac{\frac{1}{x}}{1 + (\ln x)^2} \qquad \left(\tan^{-1} u\right)' = \frac{u'}{1+u^2}$$

$$= \boxed{\frac{1}{x\left[1 + (\ln x)^2\right]}}$$

Exercise

Find the derivative $y = \csc^{-1}\left(e^t\right)$

Solution

$$y' = -\frac{e^t}{\left|e^t\right|\sqrt{\left(e^t\right)^2 - 1}}$$

$$\boxed{= -\frac{1}{\sqrt{e^{2t} - 1}}}$$

Exercise

Find the derivative $y = x\sqrt{1-x^2} + \cos^{-1} x$

Solution

$$y' = \sqrt{1-x^2} + x\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x) - \frac{1}{\sqrt{1-x^2}}$$

$$= \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{1-x^2-x^2-1}{\sqrt{1-x^2}}$$

$$\boxed{= \frac{-2x^2}{\sqrt{1-x^2}}}$$

Exercise

Find the derivative $y = \ln\left(x^2+4\right) - x\tan^{-1}\left(\frac{x}{2}\right)$

Solution

$$y' = \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - x\frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}$$

$$= \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{x}{2}\cdot\frac{1}{1+\frac{x^2}{4}}$$

$$= \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{x}{2}\cdot\frac{4}{4+x^2}$$

$$= \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{2x}{4+x^2}$$

$$= -\tan^{-1}\left(\frac{x}{2}\right)$$

Exercise

Find the derivative $f(x) = \sin^{-1}\frac{1}{x}$

Solution

$$f'(x) = -\frac{1}{x^2}\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$$

$$= \frac{-1}{|x|\sqrt{x^2-1}}$$

Exercise

Find the derivative $\frac{d}{dx}\left(x\sec^{-1}x\right)\Big|_{x=\frac{2}{\sqrt{3}}}$

Solution

$$\frac{d}{dx}\left(x\sec^{-1}x\right) = \sec^{-1}x + \frac{x}{x\sqrt{x^2-1}}\Bigg|_{x=\frac{2}{\sqrt{3}}}$$

$$= \sec^{-1}\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{\frac{4}{3}-1}}$$

$$= \frac{\pi}{6} + \sqrt{3}$$

Exercise

Find the derivative $\frac{d}{dx}\left(\tan^{-1}e^{-x}\right)\Big|_{x=0}$

Solution

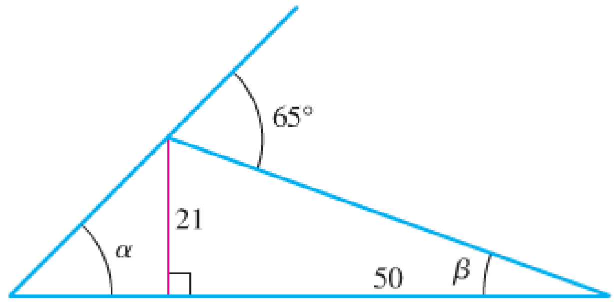
$$\frac{d}{dx}\left(\tan^{-1}e^{-x}\right) = \frac{-e^{-x}}{1+e^{-2x}}\Bigg|_{x=0}$$

$$= -\frac{1}{2}$$

Exercise

Find the angle α

Solution



$$65° + (90° - \beta) + (90° - \alpha) = 180°$$

$$65° + 180° - \beta - \alpha = 180°$$

$$\beta + \alpha = 65°$$

$$\underline{\alpha = 65° - \beta}$$

$$\tan \beta = \frac{21}{50}$$

$$\beta = \tan^{-1}\left(\frac{21}{50}\right)$$

$$\approx 22.78°$$

$$\alpha \approx 65° - 22.78°$$

$$\approx 42.22°$$