Section 4.3 – Multiplicative Inverses of Matrices

Identity Matrix

The $n \; x \; n$ identity matrix with 1's on the main diagonal and 0's elsewhere and is denoted by

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2x2} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3} \qquad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{I} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

The Multiplicative Identity Matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Then $AI = IA = A$

Example

$$A = \begin{bmatrix} 4 & -7 \\ -3 & 2 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solution

$$AI = \begin{bmatrix} 4 & -7 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4(1) - 7(0) & 4(0) - 7(1) \\ -3(1) + 2(0) & -3(0) + 2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -7 \\ -3 & 2 \end{bmatrix} = A$$

$$= A$$

Multiplicative inverse of a matrix

Multiplicative inverse of a matrix $\underset{n\times n}{A}$ and $\underset{n\times n}{A^{-1}}$ if exists, then:

$$A\cdot A^{-1}=A^{-1}\cdot A=I$$

Example

Show that B is Multiplicative inverse of A.

$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $\qquad$ $B = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

Solution

$$\boldsymbol{A.B} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2(1)-1(1) & 2(-1)+1(2) \\ 1(1)+1(-1) & 1(-1)+1(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{=I}\,\rfloor$$

$$BA = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{=I}\,\rfloor$$

$\therefore$ B is multiplicative inverse of a matrix A: $B = A^{-1}$

Finding Inverse matrix

To find inverse matrix using Gauss-Jordan method:

$$\left[A \middle| I\right] \rightarrow \left[I \middle| A^{-1}\right] \qquad \textit{where } A^{-1} \textit{ read as "A inverse"}$$

For 2 by 2 matrices (*only*)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{d}{ad-bc} & \dfrac{-b}{ad-bc} \\ \dfrac{-c}{ad-bc} & \dfrac{a}{ad-bc} \end{bmatrix}$$

If $ad - bc = 0$, then A^{-1} doesn't exist

Example

$$A = \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix} \Rightarrow A^{-1} = ?$$

Solution

$$A^{-1} = \frac{1}{(-1)(4) - (-2)(3)}\begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

Example

$$A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \Rightarrow A^{-1} = ?$$

Solution

$$A^{-1} = \frac{1}{(3)(1)-(-2)(-1)} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

To find inverse matrix using Gauss-Jordan method:

$$\left[A|I\right] \rightarrow \left[I|A^{-1}\right] \qquad \textit{where } A^{-1} \textit{ read as "A inverse"}$$

Example

$$A=\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \qquad \text{Find } A^{-1}$$

Solution

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2+R_1 \\ R_3-R_1 \end{array}$$

$$\begin{array}{rrrrrr} -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 & 0 & 0 \\ \hline 0 & 2 & 5 & 1 & 1 & 0 \end{array} \qquad \begin{array}{rrrrrr} 1 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & -1 & 0 & 0 \\ \hline 0 & -1 & -2 & -1 & 0 & 1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ 2R_3+R_2 \end{array}$$

$$\begin{array}{rrrrrr} 0 & -2 & -4 & -2 & 0 & 2 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ \hline 0 & 0 & 1 & -1 & 1 & 2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right] \begin{array}{l} R_1-2R_3 \\ R_2-5R_3 \\ \end{array}$$

$$\begin{array}{rrrrrr} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 & -2 & -4 \\ \hline 0 & 0 & 0 & 3 & -2 & -4 \end{array} \qquad \begin{array}{rrrrrr} 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & 0 & -5 & 5 & -5 & -10 \\ \hline 0 & 2 & 0 & 6 & -4 & -10 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -4 \\ 0 & 2 & 0 & 6 & -4 & -10 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right] \begin{array}{l} \\ \frac{1}{2}R_2 \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -4 \\ 0 & 1 & 0 & 3 & -2 & -5 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right]$$

$$\Rightarrow \qquad A^{-1}=\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Example

$$A=\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \qquad \text{Find } A^{-1}$$

Solution

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2+R_1 \\ R_3-R_1 \end{array} \qquad \begin{array}{cccccc} -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 & 0 & 0 \\ \hline 0 & 2 & 5 & 1 & 1 & 0 \end{array} \qquad \begin{array}{cccccc} 1 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & -1 & 0 & 0 \\ \hline 0 & -1 & -2 & -1 & 0 & 1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right] \tfrac{1}{2}R_2 \qquad \begin{array}{cccccc} 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right] R_3+R_2 \qquad \begin{array}{cccccc} 0 & -1 & -2 & -1 & 0 & 1 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ \hline 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 \end{array}\right] 2R_3 \qquad \begin{array}{cccccc} 0 & 0 & 1 & -1 & 1 & 2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right] \begin{array}{l} R_1-2R_3 \\ R_2-\frac{5}{2}R_3 \\ \end{array} \qquad \begin{array}{cccccc} 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{5}{2} & \frac{5}{2} & -\frac{5}{2} & -5 \\ \hline 0 & 1 & 0 & 3 & -2 & -5 \end{array} \qquad \begin{array}{cccccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 & -2 & -4 \\ \hline 1 & 0 & 0 & 3 & -2 & -4 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -4 \\ 0 & 1 & 0 & 3 & -2 & -5 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right]$$

$$\Rightarrow \qquad A^{-1}=\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Solving a System Using A^{-1}

To solve the matrix equation $AX = B$.

- $\boldsymbol{X}$: matrix of the variables
- $\boldsymbol{A}$: Coefficient matrix
- $\boldsymbol{B}$: Constant matrix

$AX = B$	
$A^{-1}(AX) = A^{-1}B$	*Multiply both side by* A^{-1}
$(A^{-1}A)X = A^{-1}B$	*Associate property*
$IX = A^{-1}B$	*Multiplicative inverse property*
$X = A^{-1}B$	*Identity property*

Example

Solve the system using A^{-1}

$$\begin{cases} x \quad\;\; + 2z = 6 \\ -x + 2y + 3z = -5 \\ x - y \quad\;\; = 6 \end{cases}$$

Given $A^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Solution

$$\underbrace{\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} \underset{=}{=} \underbrace{\begin{bmatrix} 6 \\ -5 \\ 6 \end{bmatrix}}_{B}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} . \begin{bmatrix} 6 \\ -5 \\ 6 \end{bmatrix} = \begin{bmatrix} 3(6)-2(-5)-4(6) \\ 3(6)-2(-5)-5(6) \\ -1(6)+1(-5)+2(6) \end{bmatrix} = \begin{bmatrix} 18+10-24 \\ 18+10-30 \\ -6-5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

Solution: $\{(4, -2, 1)\}$

Example

Use the inverse of the coefficient matrix to solve the linear system

$$\begin{aligned} 2x - 3y &= 4 \\ x + 5y &= 2 \end{aligned}$$

Solution

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \end{bmatrix} \qquad B = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{5}{13} & \frac{3}{13} \\ -\frac{1}{13} & \frac{2}{13} \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{5}{13} & \frac{3}{13} \\ -\frac{1}{13} & \frac{2}{13} \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The solution of the system is $(2, 0)$

Exercise Section 4.3 – Multiplicative Inverses of Matrices

Show that B is Multiplicative inverse of A

1. $A=\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}$ $B=\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

2. $A=\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ & $B=\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$

Find the inverse, if exists, of

3. $A=\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

4. $A=\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

5. $A=\begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix}$

6. $A=\begin{bmatrix} a & b \\ 3 & 3 \end{bmatrix}$

7. $A=\begin{bmatrix} -2 & a \\ 4 & a \end{bmatrix}$

8. $A=\begin{bmatrix} 4 & 4 \\ b & a \end{bmatrix}$

9. $A=\begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$

10. $A=\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$

11. $A=\begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$

12. $A=\begin{pmatrix} -1 & 3 \\ 2 & -1 \end{pmatrix}$

13. $A=\begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}$

14. $A=\begin{pmatrix} 4 & 6 \\ 2 & 3 \end{pmatrix}$

15. $A=\begin{pmatrix} -6 & 9 \\ 2 & -3 \end{pmatrix}$

16. $A=\begin{pmatrix} -2 & 7 \\ 0 & 2 \end{pmatrix}$

17. $A=\begin{pmatrix} 4 & -16 \\ 1 & -4 \end{pmatrix}$

18. $A=\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

19. $A=\begin{pmatrix} 2 & 1 \\ a & a \end{pmatrix}$

20. $A=\begin{pmatrix} b & 3 \\ b & 2 \end{pmatrix}$

21. $A=\begin{pmatrix} 1 & a \\ 3 & a \end{pmatrix}$

22. $A=\begin{pmatrix} a & 2 \\ 2 & a \end{pmatrix}$

23. $A=\begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}$

24. $A=\begin{pmatrix} -3 & \frac{1}{2} \\ 6 & -1 \end{pmatrix}$

25. $A=\begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{bmatrix}$

26. $A=\begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & 3 \\ 2 & 4 & 3 \end{bmatrix}$

27. $A=\begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

28. $A=\begin{bmatrix} -2 & 5 & 3 \\ 4 & -1 & 3 \\ 7 & -2 & 5 \end{bmatrix}$

29. $A=\begin{pmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{pmatrix}$

30. $A=\begin{pmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{pmatrix}$

31. $A=\begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix}$

32. $A=\begin{pmatrix} 1 & 1 & 1 \\ 3 & 2 & -1 \\ 3 & 1 & 2 \end{pmatrix}$

33. $A=\begin{pmatrix} 3 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$

34. $A=\begin{pmatrix} -3 & 1 & -1 \\ 1 & -4 & -7 \\ 1 & 2 & 5 \end{pmatrix}$

35. $A=\begin{pmatrix} 1 & 1 & -3 \\ 2 & -4 & 1 \\ -5 & 7 & 1 \end{pmatrix}$

36. $A=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 5 & 3 \\ 2 & 4 & 3 \end{pmatrix}$

37. $A=\begin{bmatrix} -2 & -3 & 4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 4 & -6 & 1 \\ -2 & -2 & 5 & 1 \end{bmatrix}$

38. $A=\begin{bmatrix} 1 & -14 & 7 & 38 \\ -1 & 2 & 1 & -2 \\ 1 & 2 & -1 & -6 \\ 1 & -2 & 3 & 6 \end{bmatrix}$

39. $A=\begin{bmatrix} 10 & 20 & -30 & 15 \\ 3 & -7 & 14 & -8 \\ -7 & -2 & -1 & 2 \\ 4 & 4 & -3 & 1 \end{bmatrix}$

State the conditions under which A^{-1} exists. Then find a formula for A^{-1}

40. $A=[x]$

41. $A=\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$

42. $A=\begin{bmatrix} 0 & 0 & x \\ 0 & y & 0 \\ z & 0 & 0 \end{bmatrix}$

43. $A=\begin{bmatrix} x & 1 & 1 & 1 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & w \end{bmatrix}$

44. Solve the system using A^{-1} $\begin{cases} x + 2z = 6 \\ -x + 2y + 3z = -5 \\ x - y = 6 \end{cases}$ Given $A^{-1}=\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

45. Solve the system using A^{-1}

$$\begin{cases} x + 2y + 5z = 2 \\ 2x + 3y + 8z = 3 \\ -x + y + 2z = 3 \end{cases}$$

a) Write the linear system as a matrix equation in the form *AX*= *B*

b) Solve the system using the inverse that is given for the coefficient matrix

$$\textit{the inverse of } \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 8 \\ -1 & 1 & 2 \end{bmatrix} \textit{ is } \begin{bmatrix} 2 & -1 & -1 \\ 12 & -7 & -2 \\ -5 & 3 & 1 \end{bmatrix}$$

46. Solve the system using A^{-1}

$$\begin{cases} x - y + z = 8 \\ \quad 2y - z = -7 \\ 2x + 3y \quad = 1 \end{cases}$$

a) Write the linear system as a matrix equation in the form $AX = B$

b) Solve the system using the inverse that is given for the coefficient matrix

$$\textit{the inverse is } \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

(47– 75) Use the ***inverse*** of the coefficient matrix to solve the linear system

47. $\begin{cases} 3x + 2y = -4 \\ 2x - y = -5 \end{cases}$

48. $\begin{cases} 2x + 5y = 7 \\ 5x - 2y = -3 \end{cases}$

49. $\begin{cases} 4x - 7y = -16 \\ 2x + 5y = 9 \end{cases}$

50. $\begin{cases} 3x + 2y = 4 \\ 2x + y = 1 \end{cases}$

51. $\begin{cases} 3x + 4y = 2 \\ 2x + 5y = -1 \end{cases}$

52. $\begin{cases} 5x - 2y = 4 \\ -10x + 4y = 7 \end{cases}$

53. $\begin{cases} x - 4y = -8 \\ 5x - 20y = -40 \end{cases}$

54. $\begin{cases} 2x + y = 3 \\ x - y = 3 \end{cases}$

55. $\begin{cases} 2x + 10y = -14 \\ 7x - 2y = -16 \end{cases}$

56. $\begin{cases} 4x - 3y = 24 \\ -3x + 9y = -1 \end{cases}$

57. $\begin{cases} 4x + 2y = 12 \\ 3x - 2y = 16 \end{cases}$

58. $\begin{cases} x + 2y = -1 \\ 4x - 2y = 6 \end{cases}$

59. $\begin{cases} x - 2y = 5 \\ -10x + 2y = 4 \end{cases}$

60. $\begin{cases} 12x + 15y = -27 \\ 30x - 15y = -15 \end{cases}$

61. $\begin{cases} 4x - 4y = -12 \\ 4x + 4y = -20 \end{cases}$

62. $\begin{cases} -2x + 3y = 4 \\ -3x + 4y = 5 \end{cases}$

63. $\begin{cases} x - 2y = 6 \\ 4x + 3y = 2 \end{cases}$

64. $\begin{cases} 2x - 3y = 7 \\ 4x + y = -7 \end{cases}$

65. $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$

66. $\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \\ 3x - y + z = 9 \end{cases}$

67. $\begin{cases} 3y - z = -1 \\ x + 5y - z = -4 \\ -3x + 6y + 2z = 11 \end{cases}$

68. $\begin{cases} x + 3y + 4z = 14 \\ 2x - 3y + 2z = 10 \\ 3x - y + z = 9 \end{cases}$

69. $\begin{cases} x + 4y - z = 20 \\ 3x + 2y + z = 8 \\ 2x - 3y + 2z = -16 \end{cases}$

70. $\begin{cases} 2y - z = 7 \\ x + 2y + z = 17 \\ 2x - 3y + 2z = -1 \end{cases}$

71. $\begin{cases} -2x + 6y + 7z = 3 \\ -4x + 5y + 3z = 7 \\ -6x + 3y + 5z = -4 \end{cases}$

72. $\begin{cases} 2x - y + z = 1 \\ 3x - 3y + 4z = 5 \\ 4x - 2y + 3z = 4 \end{cases}$

73. $\begin{cases} x - 2y - z = 2 \\ 2x - y + z = 4 \\ -x + y + z = 4 \end{cases}$