#1 $\dfrac{2s-1}{(s+1)(s-2)} = \dfrac{A}{s+1} + \dfrac{B}{s-2}$

$$2s-1 = A(s-2) + B(s+1)$$

$s^1 \quad A + B = 2 \rightarrow \boxed{B = 2 - 1 = 1}$

$s^0 \quad \underline{-2A + B = -1}$

$\qquad -3A = -3$

$\qquad \underline{A = 1}$

$A = \dfrac{\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \dfrac{3}{3} = 1$

$$\boxed{\dfrac{2s-1}{(s+1)(s-2)} = \dfrac{1}{s+1} + \dfrac{1}{s-2}}$$

#2 $\dfrac{2s-2}{(s-4)(s+2)} = \dfrac{A}{s-4} + \dfrac{B}{s+2}$

$$2s-2 = As + 2A + Bs - 4B$$

$\begin{cases} s^1 \quad A + B = 2 \\ s^0 \quad 2A - 4B = -2 \end{cases}$ $\qquad B = 1, \ A = 1$ ✓

$$\dfrac{2s-2}{(s-4)(s+2)} = \dfrac{1}{s-4} + \dfrac{1}{s+2}$$

#3 $\dfrac{7s^2 + 3s + 16}{(s+1)(s^2+4)} = \dfrac{7}{s+1} + \dfrac{Bs+C}{s^2+4}$

$7s^2 + 3s + 16 = A(s^2+4) + (s+1)(Bs+C)$

$s^2 \qquad A + B = 7 \qquad \Rightarrow \quad \boxed{A = 7 - B = 4}$

$s^1 \qquad B + C = 3 \qquad \rightarrow \boxed{C = 3 - B = 0}$

$s^0 \qquad 4A + C = 16$

$$28 - 4B + 3 - B = 16$$

$$-5B = -15 \iff \boxed{B = 3}$$

$$\dfrac{7s^2 + 3s + 16}{(s+1)(s^2+4)} = \dfrac{4}{s+1} + \dfrac{3s}{s^2+4}$$

---

#4 $\dfrac{1}{(s+2)^2(s^2+9)} = \dfrac{A}{s+2} + \dfrac{B}{(s+2)^2} + \dfrac{Cs+D}{s^2+9}$

① $= A(s+2)(s^2+9) + B(s^2+9) + (Cs+D)(s^2+4s+4)$

$s^3 \qquad A + C \qquad\qquad = 0 \quad \Rightarrow \quad 7 = -C$

$s^2 \qquad 2A + B + 4C + D = 0 \quad ①$

$s^1 \qquad 9A + 4C + 4D = 0 \quad ②$

$s^0 \qquad 18A + 9B + 4D = 1 \quad ③$

$\begin{cases} ① \rightarrow \qquad B + 2C + D = 0 \\ ② \rightarrow \qquad\quad -5C + 4D = 0 \end{cases}$

$$9B - 18C + 4D = 1$$

$$\Delta D = \begin{vmatrix} 1 & 2 & 1 \\ 0 & -5 & 4 \\ 9 & -18 & 4 \end{vmatrix} \begin{matrix} 1 & 2 \\ 0 & -5 \\ 9 & -18 \end{matrix} \quad \Rightarrow \qquad \begin{matrix} 5 & 2 \\ 7 & 2 \\ 4 & 5 \\ 7 & 2 \end{matrix}$$

$$= 169$$

$$D_B = \begin{vmatrix} 0 & 2 & 1 \\ 0 & -5 & 4 \\ 1 & -18 & 4 \end{vmatrix} = 13 \qquad A_C = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 4 \\ 9 & 1 & 4 \end{vmatrix} = -4$$

$$B = \frac{13}{169} = \frac{1}{13} \qquad\qquad C = \frac{-4}{169}$$

$$D = \frac{5}{4} C = -\frac{5}{169}$$

$$A = \frac{-4}{169}$$

$$\frac{1}{(s+2)^2(s^2+9)} = \frac{4/169}{s+2} + \frac{1/13}{(s+2)^2} + \frac{-\frac{4}{169}s - \frac{5}{169}}{s^2+9}$$

$$\frac{s}{(s+2)^2(s^2+9)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+9}$$

$$A + C = 0 \qquad \rightarrow \qquad A = -C$$

$$\begin{cases} 2A + B + 4C + D = 0 \\ 9A + 4C + 4D = 1 \\ 18A + 9B + 4D = 0 \end{cases}$$

$$\begin{cases} B + 2C + D = 0 \\ -5C + 4D = 1 \qquad\qquad \Delta = 169 \\ 9B - 18C + 4D = 0 \end{cases}$$

$$\Delta_B = \begin{vmatrix} 0 & 2 & 1 \\ 1 & -5 & 4 \\ 0 & -18 & 4 \end{vmatrix} = -26 \qquad B = -\frac{2}{13} \qquad -\frac{26}{169}$$

$$\Delta_C = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 9 & 0 & 4 \end{vmatrix} = -5 \qquad C = \frac{-5}{169}$$

$$\Delta_D = \begin{vmatrix} 1 & 2 & 0 \\ 0 & -5 & 1 \\ 9 & -18 & 0 \end{vmatrix} = 36 \qquad D = \frac{36}{169}$$

$$\frac{s}{(s+2)^2(s^2+9)} = \frac{5/169}{s+2} + \frac{-2/13}{(s+2)^2} + \frac{-\frac{5}{169}s + \frac{36}{169}}{s^2+9}$$

#6 
$$\frac{1}{(s+1)^2(s^2-4)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s-2} + \frac{D}{s+2}$$

$$1 = A(s+1)(s^2-4) + B(s^2-4)$$
$$+ C(s+2)(s^2+2s+1) + D(s-2)(s^2+2s+1)$$

$$
\begin{array}{ll}
s^3 & A + C + D = 0 \\
s^2 & A + B + 4C = 0 \\
s^1 & -4A + 5C - 3D = 0 \\
s^0 & -4A - 4B + 2C - 2D = 1
\end{array}
\left\}
\begin{array}{l}
-B - 3C + D = 0 \\
\\
4B + 3C - D = -1 \\
\hline
3B = -1
\end{array}
\right.
$$

$$\boxed{B = -\tfrac{1}{3}}$$

$$
\begin{cases}
A + C + D = 0 \\
A + 4C = \tfrac{1}{3} \rightarrow 3A + 12C = 1 \\
-4A + 5C - 3D = 0 \\
-4A + 2C - 2D = 1 - \tfrac{4}{3} = -\tfrac{1}{3}
\end{cases}
$$

$$
\begin{cases}
12A - 6C + 6D = 1 \\
A + C + D = 0 \rightarrow D = -A - C \\
3A + 12C = 1 \quad ③ \\
-4A + 5C - 3D = 0 \Rightarrow -4A + 5C + 3A + 3C = 0
\end{cases}
$$

$$
\begin{cases}
3A + 12C = 1 \\
-A + 8C = 0
\end{cases}
\quad
\boxed{A = \frac{-8}{36} = \frac{-2}{9}}
$$
$$\hookrightarrow 8C = A = -\tfrac{2}{9}$$

$$\boxed{C = -\tfrac{1}{36}}$$

$$\boxed{D = \tfrac{2}{9} + \tfrac{1}{36} = \tfrac{25}{36}}$$

$$\frac{1}{(s+1)^2(s^2-4)} = \frac{-2/9}{s+1} - \frac{1/3}{(s+1)^2} - \frac{1/36}{s-2} + \frac{25/36}{s+2}$$

**#9** $\dfrac{1}{x^2+2x} = \dfrac{A}{x} + \dfrac{B}{x+2}$

$x(x+2)$

$1 = A(x+2) + Bx$

$x^1 \quad A+B = 0 \quad \Rightarrow B = -\frac{1}{2}$

$x^0 \quad 2A = 1 \quad \Rightarrow A = \frac{1}{2}$

$\dfrac{1}{x^2+2x} = \dfrac{\frac{1}{2}}{x} - \dfrac{\frac{1}{2}}{x+2}$

---

**#10** $\dfrac{2x+1}{x^2-7x+12} = \dfrac{A}{x-3} + \dfrac{B}{x-4}$

$2x+1 = A(x-4) + B(x-3)$

$x^1 \quad A+B = 2 \qquad A = -7$

$x^0 \quad -4A -3B = 1 \qquad B = 9$

$\dfrac{2x+1}{x^2-7x+12} = \dfrac{-7}{x-3} + \dfrac{9}{x-4}$