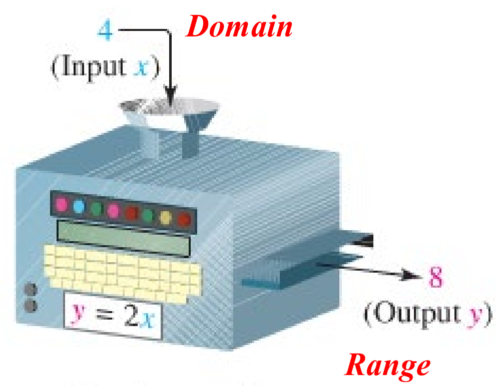
Section 1.2 – Functions

Relations

A ***relation*** is any set of ordered pairs. The set of all first components of ordered pairs is called the domain of the relation and the set of second components is called the range of the relation.

Definition of a Function



Function machine

A ***function*** is a relation between two variables such that to matches each element of a first set (called ***domain***) to an element of a second set (called ***range***) in such way that no element in the first set is assigned to two different elements in the second set.

The ***domain*** of the function is the set of all values of the independent variable for which the function is defined.

The ***range*** of the function is the set of all values taken on by the dependent variable.

Example

Determine whether each relation is a function and *find the domain and the range*.

a) $F = \{(1,\ 2),\ (-2,\ 4),\ (3,\ -1)\}$

Function: Yes

Domain: $\{-2,\ 1,\ 3\}$

Range: $\{-1,\ 2,\ 4\}$

b) $G = \{(1,\ 1),\ (1,\ 2),\ (1,\ 3),\ (2,\ 3)\}$

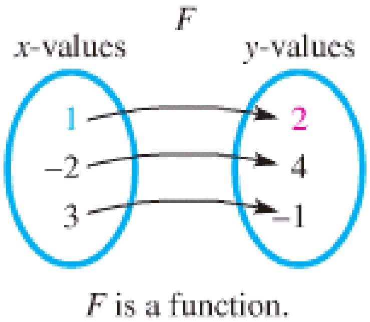
Function: No
Domain: {1, 2}
Range: {1, 2, 3}

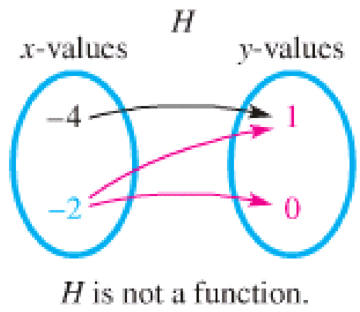
c) $H = \{(-4,\ 1),\ (-2,\ 1),\ (-2,\ 0)\}$

Function: No

Domain: $\{-4,\ -2\}$

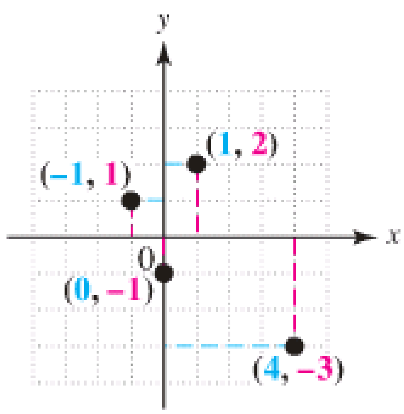
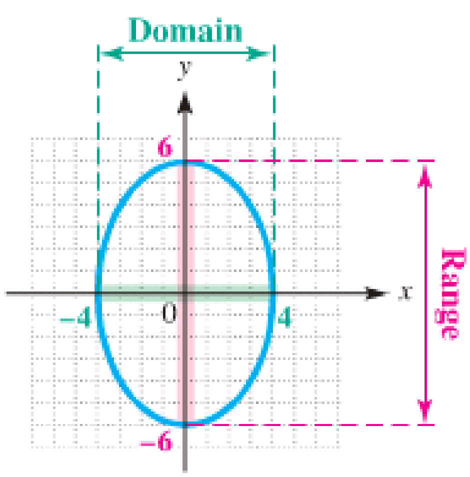
Range: {0, 1}





Example

Give the domain and range of each relation

	***Domain***: {-1, 0, 1, 4} ***Range***: {-3, -1, 1, 2}
	***Domain***: $[-4,\ 4]$ ***Range***: $[-6,\ 6]$

Functions as Equations $y = -0.016x^2 + 0.93x + 8.5$

x: independent
y: depend on x

Notation for Functions

$\boldsymbol{f(x)}$ read "f of x" or "f at x" represents the value of the function at the number x.

Example

Let $f(x) = -x^2 + 5x - 3$

a) $f(2)$

$$f(x) = -x^2 + 5x - 3$$
$$f(\text{—}) = -(\text{—})^2 + 5(\text{—}) - 3$$
$$f(2) = -(2)^2 + 5(2) - 3$$
$$\underline{= 3}$$

b) $f(q)$

$$f(q) = -(q)^2 + 5(q) - 3$$
$$\underline{= -q^2 + 5q - 3}$$

Example

If $f(x) = x^2 - 2x + 7$, evaluate each of the following:

a) $f(-5)$
b) $f(x+4)$

Solution

a) $f(-5) = ?$

$$f(\text{—}) = (\text{—})^2 - 2(\text{—}) + 7$$
$$f(-5) = (-5)^2 - 2(-5) + 7$$
$$= 25 + 10 + 7$$
$$\underline{= 42}$$

b) $f(x+4)=?$

$$f(\text{—})=(\text{—})^2-2(\text{—})+7$$

$$f(x+4)=(x+4)^2-2(x+4)+7 \qquad (a+b)^2=a^2+2ab+b^2$$

$$=x^2+2(4)x+4^2-2x-8+7$$

$$=x^2+8x+16-2x-1$$

$$\underline{=x^2+6x+15}$$

Example

Let $g(x)=2x+3$, find $g(a+1)$

Solution

$$g(x)=2x+3$$

$$g(a+1)=2(a+1)+3$$

$$=2a+2+3$$

$$\underline{=2a+5}$$

Example

Given: $f(x)=2x^2-x+3$, find the following.

a) $f(0)$

b) $f(-7)$

c) $f(5a)$

Solution

a) $f(x=0)=2(0)^2-(0)+3$

$$\underline{=3}$$

b) $f(-7)=2(-7)^2-(-7)+3$

$$\underline{=108}$$

c) $f(5a)=2(5a)^2-(5a)+3$

$$\underline{=50a^2-5a+3}$$

Exercises *Section* 1.2 – Functions

(1 – 7) Determine whether each relation is a function and *find the domain and the range*.

1. $\{(1,\ 2),\ (3,\ 4),\ (5,\ 6),\ (5,\ 8)\}$

2. $\{(1,\ 2),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5)\}$

3. $\{(9,\ -5),\ (9,\ 5),\ (2,\ 4)\}$

4. $\{(-2,\ 5),\ (5,\ 7),\ (0,\ 1),\ (4,\ -2)\}$

5. $\{(-5,\ 3),\ (0,\ 3),\ (6,\ 3)\}$

6. $\{(1,\ 2),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5),\ (1,\ 5)\}$

7. $\{(-1,\ 3),\ (3,\ 4),\ (6,\ 5),\ (8,\ 5),\ (1,\ 5)\}$

8. Let $f(x)=-3x+4$, find $f(0)$, $f(-1)$, $f(h)$, and $f(a-1)$

9. Let $g(x)=-x^2+4x-1$, find $g(-x)$, $g(2)$, and $g(-2)$

10. Let $f(x)=-3x+4$, find $f(a+4)$

11. Given: $f(x)=2|x|+3x$, find $f(2-h)$.

12. Given: $g(x)=\frac{x-4}{x+3}$, find $g(x+h)$

13. Given: $g(x)=\frac{x}{\sqrt{1-x^2}}$, find $g(0)$ *and* $g(-1)$

14. Given that $g(x)=2x^2+2x+3$. Find $g(p+3)$

15. If $f(x)=x^2-2x+7$, evaluate each of the following: $f(-5), f(x+4), f(-x)$

16. Find $g(0),\ g(-4),\ g(7),\ \textit{and}\ g\left(\frac{3}{2}\right)$ *for* $g(x)=\frac{x}{\sqrt{16-x^2}}$

17. For $f(x)=3x-4$, determine

a) $f(0)$ *b)* $f\left(\frac{5}{3}\right)$ *c)* $f(-2a)$ *d)* $f(x+h)$

18. For $f(x)=3x^2+3x-1$, determine

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(2)$ *d)* $f(h)$

19. For $f(x)=2x^2-4$, determine

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(2)$ *d)* $f(2)-f(-3)$

20. For $f(x) = 3x^2 + 4x - 2$, determine

a) $f(0)$ *b)* $f(x+h)$ *c)* $f(3)$ *d)* $f(-5)$

21. For $f(x) = -x^3 - x^2 - x + 10$, determine

a) $f(0)$ *b)* $f(-1)$ *c)* $f(2)$ *d)* $f(1) - f(-2)$

22. For $\frac{1}{10}x^{10} - \frac{1}{2}x^6 + \frac{2}{3}x^3 - 10x$, determine

a) $f(2) - f(-2)$ *b)* $f(1) - f(-1)$ *c)* $f(2) - f(0)$

23. For $f(x) = 3x^4 + x^2 - 4$, determine

a) $f(2) - f(-2)$ *b)* $f(1) - f(-1)$ *c)* $f(2) - f(0)$

24. For $f(x) = -\frac{2}{3}x^3 + 4x$, determine

a) $f(2) - f(-2)$ *b)* $f(1) - f(-1)$ *c)* $f(2) - f(0)$

25. For $f(x) = \frac{2x-3}{x-4}$, determine

a) $f(0)$ *b)* $f(3)$ *c)* $f(x+h)$ *d)* $f(-4)$

26. For $f(x) = \frac{3x-1}{x-5}$, determine

a) $f(0)$ *b)* $f(3)$ *c)* $f(x+h)$ *d)* $f(-5)$