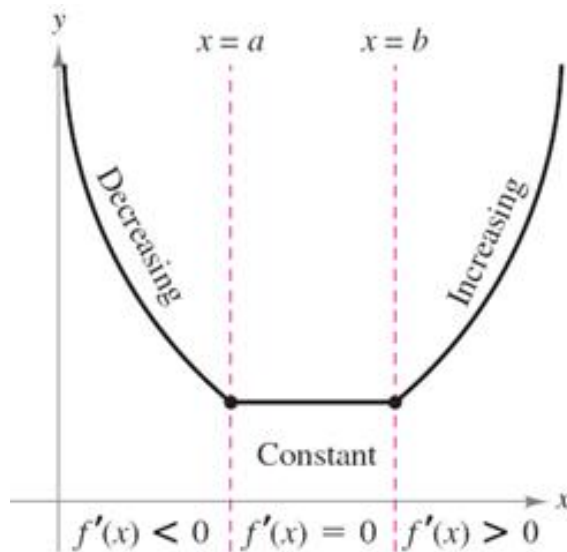


Lecture Three - Graphs and the Derivative

Section 3.1 – Increasing and Decreasing Functions



Test for Increasing and Decreasing Functions

Let f be differentiable on the interval (a, b)

1. If $f'(x) > 0$ for all x in (a, b) , then f is increasing on (a, b)
2. If $f'(x) < 0$ for all x in (a, b) , then f is decreasing on (a, b)
3. If $f'(x) = 0$ for all x in (a, b) , then f is constant on (a, b)

Critical Numbers (CN)

The critical numbers for a function f are those numbers c in the domain of f for which $f'(c) = 0$ or $f'(c)$ doesn't exist. A critical point is a point whose x -coordinate is the critical number c , and whose y -coordinate is $f(c)$

$$f(x) = x^2$$

$$\Rightarrow f'(x) = 2x = 0$$

$\rightarrow x = 0$ is a critical point.

If $f'(x) = 0$ undefined

Example

Find the open intervals on which the function $f(x) = x^3 + 3x^2 - 9x + 4$ is increasing or decreasing

Solution

$$f'(x) = 3x^2 + 6x - 9$$

$$3x^2 + 6x - 9 = 0 \Rightarrow \boxed{x = -3, 1} \text{ (CN)}$$

$-\infty$	-3	1	∞
$f'(-4) > 0$	$f'(0) < 0$	$f'(2) > 0$	
<i>Increasing</i>	<i>Decreasing</i>	<i>Increasing</i>	

Increasing: $(-\infty, -3)$ and $(1, \infty)$

Decreasing: $(-3, 1)$

Example

Find the critical numbers and decide on which the function $f(x) = (x-1)^{2/3}$ is increasing or decreasing

Solution

$$f'(x) = \frac{2}{3}(x-1)^{-1/3}$$

$$= \frac{2}{3(x-1)^{1/3}} = 0$$

$$f'(x) \neq 0$$

$$x-1=0 \Rightarrow \boxed{x=1} \text{ is the only critical number}$$

$-\infty$	1	∞
$f'(0) < 0$	$f'(2) > 0$	
<i>Decreasing</i>	<i>Increasing</i>	

Decreasing: $(-\infty, 1)$

Increasing: $(1, \infty)$

Exercise **Section 3.1 – Increasing and Decreasing Functions**

Find the critical numbers and the open intervals on which the function is increasing or decreasing.

1. $f(x) = x - 4\ln(3x - 9)$
2. $f(x) = \frac{x}{x^2 + 4}$
3. $f(x) = \frac{x}{x^2 + 1}$
4. $f(x) = x\sqrt{x+1}$
5. $f(x) = x^3 - 12x$
6. $f(x) = x^{2/3}$
7. $f(x) = 2.4 + 5.2x - 1.1x^2$
8. A county realty group estimates that the number of housing starts per year over the next three years will be
$$H(r) = \frac{300}{1 + 0.03r^2}$$
Where r is the mortgage rate (in percent).
 - a) Where is $H(r)$ increasing?
 - b) Where is $H(r)$ decreasing?
9. Suppose the total cost $C(x)$ to manufacture a quantity x of insecticide (in hundreds of liters) is given by $C(x) = x^3 - 27x^2 + 240x + 750$. Where is $C(x)$ decreasing?
10. A manufacturer sells telephones with cost function $C(x) = 6.14x - 0.0002x^2$, $0 \leq x \leq 950$ and revenue function $R(x) = 9.2x - 0.002x^2$, $0 \leq x \leq 950$. Determine the interval(s) on which the profit function is increasing.
11. The cost of a computer system increases with increased processor speeds. The cost C of a system as a function of processor speed is estimated as $C(x) = 14x^2 - 4x + 1200$, where x is the processor speed in MHz. Determine the intervals where the cost function $C(x)$ is decreasing.
12. The percent of concentration of a drug in the bloodstream t hours after the drug is administered is given by $K(t) = \frac{t}{t^2 + 36}$. On what time interval is the concentration of the drug increasing?

- 13.** A probability function is defined by $f(x) = \frac{1}{\sqrt{6\pi}} e^{-x^2/8}$. Give the intervals where the function is increasing and decreasing.