5.2 Partial Fraction

(1) $G(x) = (x-x_1)(x-x_2) \cdots (x-x_n)$

$$\frac{P}{Q} = \frac{A_1}{x-x_1} + \frac{A_2}{x-x_2} + \cdots + \frac{A_n}{x-x_n}$$

Ex $\dfrac{x}{x^2-5x+6}$ $\qquad x^2-5x+6 = (x-2)(x-3)$

$$\frac{x}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}, \qquad \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$$

$$x = A(x-3) + B(x-2) \quad \text{—(1)}$$

$x^1 \quad A + B = 1$

$x^0 \quad -3A - 2B = 0$

$$D = \begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} = -2 + 3 = 1$$

$$D_A = \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} = -2 \qquad A = \frac{D_A}{D} = -2$$

$$D_B = \begin{vmatrix} 1 & 1 \\ -3 & 0 \end{vmatrix} = +3 \qquad B = 3$$

$$\boxed{\frac{x}{x^2-5x+6} = \frac{-2}{x-2} + \frac{3}{x-3}}$$

$3A + 3B = 3$

$-3A - 2B = 0$

$\boxed{B = 3} \qquad A = 1 - 3 = -2$

$Q: (x-a)^n$

$$\frac{P}{Q} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_n}{(x-a)^n}$$

Ex $\quad \dfrac{x+2}{x^3-2x^2+x} \qquad x(x^2-2x+1) = x(x-1)^2$

$$\frac{x+2}{x^3-2x^2+x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$x+2 = A(x^2-2x+1) + Bx(x-1) + Cx$$

$x^2 \quad A + B = 0 \rightarrow \boxed{B = -2}$

$x^1 \quad -2A - B + C = 1 \rightarrow C = 1 + 2(2) + (-2) = \boxed{3}$

$x^0 \quad \boxed{A = 2}$

$$\boxed{\frac{x+2}{x^3-2x^2+x} = \frac{2}{x} - \frac{2}{x-1} + \frac{3}{(x-1)^2}}$$

Ex $\dfrac{x^3-8}{x^2(x-1)^3} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2} + \dfrac{E}{(x-1)^3}$

$$x^3-8 = Ax(x^3-3x^2+3x-1) + B(x^3-3x^2+3x-1) + Cx^2(x^2-2x+1) + Dx^2(x-1) + Ex^2$$

x^4 $\quad A + C = 0$ (2)

x^3 $\quad -3A + B - 2C + D = 1$ (3)

x^2 $\quad 3A - 3B + C - D + E = 0$ (4)

x^1 $\quad -A + 3B = 0$ (1)

x^0 $\quad -B = -8 \Rightarrow B = 8$

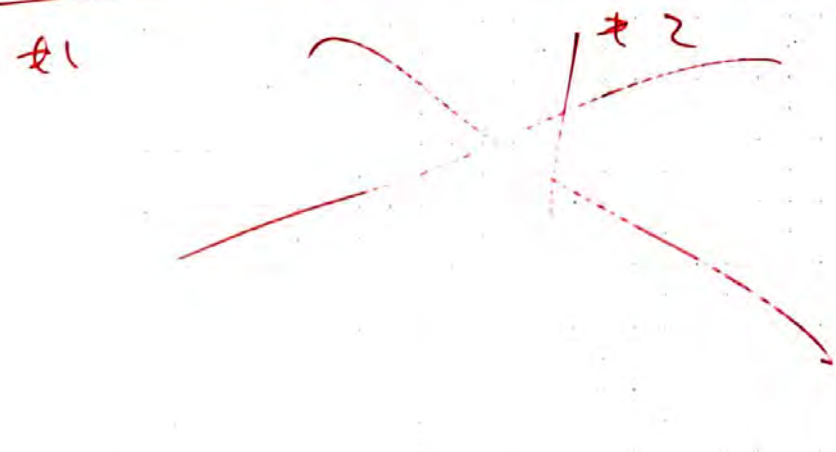
(1) $A = 3B = 24$

(2) $C = -24$

(3) $D = 1 + 3(24) - 8 + 2(-24)$
$= 17$

(4) $E = -3(24) + 3(8) + 24 + 17$
$= -7$

$$\frac{x^3-8}{x^2(x-1)^3} = \frac{24}{x} + \frac{8}{x^2} - \frac{24}{x-1} + \frac{17}{(x-1)^2} - \frac{7}{(x-1)^3}$$



$$\frac{Ax+B}{ax^2+bx+c}$$ a can't be factorable

Ex. $\dfrac{3x-5}{x^3-1}$ $\quad a^3-b^3=(a-b)(a^2+ab+b^2)$

$$\frac{3x-5}{x^3-1}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$$

$$3x-5=A(x^2+x+1)+(Bx+C)(x-1)$$

x^2 $\quad A+B=0 \Rightarrow \boxed{B=-A=\frac{2}{3}}$

x^1 $\quad A-B+C=3$ ①

x^0 $\quad A-C=-5 \Rightarrow C=A+5$

$=\frac{-2}{3}+5$

$=\boxed{\frac{13}{3}}$

① $\quad A+A+A+5=3$

$3A=-2$

$\boxed{A=\frac{-2}{3}}$

$$\frac{3x-5}{x^3-1}=\frac{-2/3}{x-1}+\frac{\frac{2}{3}x+\frac{13}{3}}{x^2+x+1}$$

$$=-\frac{2}{3}\frac{1}{x-1}+\frac{1}{3}\frac{2x+13}{x^2+x+1}$$

Ex :
$$\frac{x^3+x^2}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2}$$

$$x^3+x^2 = (Ax+B)(x^2+4) + Cx+D$$

x^3 $\quad A = 1$

x^2 $\quad B = 1$

x^1 $\quad 4A + C = 0 \rightarrow C = -4$

x^0 $\quad 4B + D = 0 \rightarrow D = -4$

$$\frac{x^3+x^2}{(x^2+4)^2} = \frac{x+1}{x^2+4} + \frac{-4x-4}{(x^2+4)^2}$$

$1x = x$ // $\Rightarrow$ implay / $\equiv$ Equal $\rightarrow$

5.2-1
$$\frac{4}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$4 = A(x-1) + Bx$$

x^1 $\quad A + B = 0 \Rightarrow B = 4$

x^0 $\quad -A = 4 \rightarrow A = -4$

$$\frac{4}{x(x-1)} = \frac{-4}{x} + \frac{4}{x-1}$$

#18 $\dfrac{4}{2x^2-5x+3} = \dfrac{A}{x-1} + \dfrac{B}{2x-3}$

$4 = A(2x-3) + B(x-1)$

$x^1 \quad 2A + B = 0$

$x^0 \quad -3A - B = 4$

$-A = 4 \Rightarrow \boxed{A = -4}$

$\boxed{B = -2(-4) = 8}$

$$\boxed{\frac{4}{2x^2-5x+3} = \frac{-4}{x-1} + \frac{8}{2x-3}}$$

Sec. 5.3 Ellipse

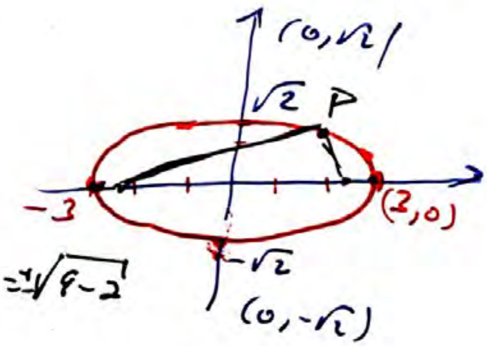
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

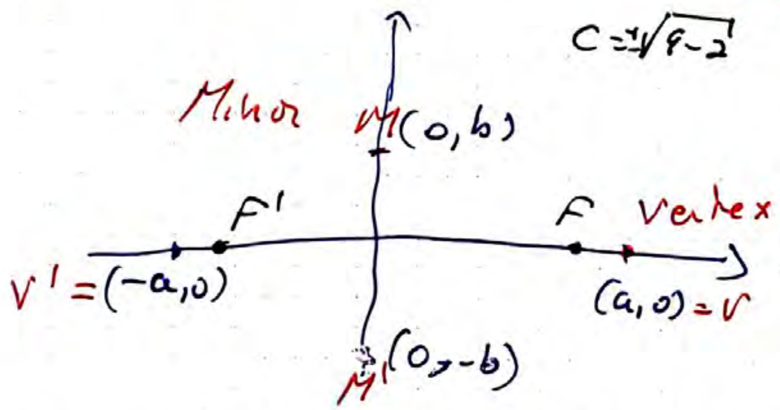
if $a = b \Rightarrow x^2 + y^2 = a^2$ circle

Ex $\dfrac{2x^2}{18} + \dfrac{9y^2}{18} = \dfrac{18}{18}$

$$\frac{x^2}{9} + \frac{y^2}{2} = 1$$

$c = \pm\sqrt{9-2}$





$c^2 = a^2 - b^2$ foci (focus)

Ex $\frac{9x^2}{25} + \frac{4y^2}{25} = \frac{25}{25}$

$$\frac{x^2}{\frac{25}{9}} + \frac{y^2}{\frac{25}{4}} = 1$$

$9 > 4$

$\frac{1}{9} < \frac{1}{4}$

$a^2 = \frac{25}{4} \to a = \pm\frac{5}{2}$

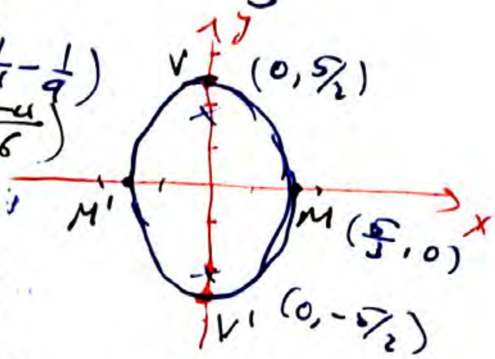
$b^2 = \frac{25}{9} \to b = \pm\frac{5}{3}$

$c^2 = 25\left(\frac{1}{4} - \frac{1}{9}\right)$

$= 25\left(\frac{9-4}{36}\right)$

$= \frac{125}{36}$

$c = \pm\frac{5\sqrt{5}}{6}$



Foci: $\left(0, \pm\frac{5\sqrt{5}}{6}\right)$

Ex

$w_T = 10$ ft.

$h_T = 9$

$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

$$\frac{y^2}{10^2} = 1 - \frac{x^2}{20^2}$$

$$y^2 = 10^2\left(\frac{20^2 - 5^2}{20^2}\right)$$

$$= \frac{10^2}{20^2}(400 - 25)$$

$\left(\frac{10}{20}\right)^2$

$$= \left(\frac{1}{2}\right)^2 (375)$$

$$= \frac{375}{4}$$

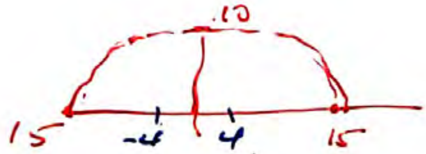
$4\overline{)375}$ 93, 15

$81 \ ?\ 93$

$81 < \frac{375}{4}$

Truck will clear.

36



$w=8$, $h=7$

$$\frac{x^2}{15^2}+\frac{y^2}{10^2}=1$$

$$\frac{y^2}{10^2}=1-\frac{x^2}{15^2}$$

$$=\frac{15^2-x^2}{15^2}$$

$$y^2=\frac{10^2}{15^2}\left(15^2-4^2\right)$$

$$=\left(\frac{10}{15}\right)^2(209)$$

$$49 \ ? \ \left(\frac{2}{3}\right)^2(209)$$

$$\frac{4}{9}(209)$$

$$441 < 836$$

clear

$$\begin{array}{r} 15 \\ \times\,15 \\ \hline 75 \\ 15 \\ \hline 225 \\ -\,16 \\ \hline \end{array}$$

$\frac{10}{15}=$