Solution Section 4.1 – Law of Sines

Exercise

In triangle *ABC*, $B=110°$, $C=40°$ and $b=18\,in$. Find the length of side *c*.

Solution

$$A=180°-(B+C)$$
$$=180°-110°-40°$$
$$=30°$$

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$
$$\frac{a}{\sin 30°}=\frac{18}{\sin 110°}$$
$$a=\frac{18\sin 30°}{\sin 110°}$$
$$\approx 9.6\ in$$

$$\frac{c}{\sin C}=\frac{b}{\sin B}$$
$$\frac{c}{\sin 40°}=\frac{18}{\sin 110°}$$
$$c=\frac{18\sin 40°}{\sin 110°}$$
$$\approx 12.3\ in$$

Exercise

In triangle *ABC*, $A=110.4°$, $C=21.8°$ and $c=246\,in$. Find all the missing parts.

Solution

$$B=180°-A-C$$
$$=180°-110.4°-21.8°$$
$$=47.8°$$

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$
$$\frac{a}{\sin 110.4°}=\frac{246}{\sin 21.8°}$$
$$a=\frac{246\sin 110.4°}{\sin 21.8°}$$
$$\approx 621\ in$$

$$\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$\frac{b}{47.8}=\frac{246}{\sin 21.8°}$$
$$b=\frac{246\sin 47.8°}{\sin 21.8°}$$
$$\approx 491\ in$$

Exercise

Find the missing parts of triangle *ABC* if $B = 34°$, $C = 82°$, and $a = 5.6\,cm$.

Solution

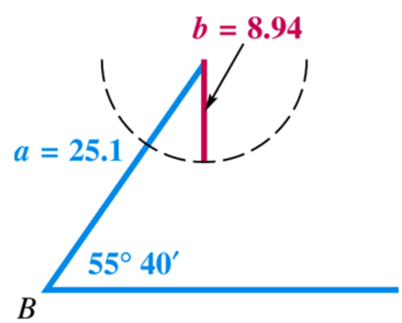
$$A = 180° - (B + C)$$
$$= 180° - (34° + 82°)$$
$$= 180° - 116°$$
$$= 64°$$

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $b = \frac{a \sin B}{\sin A}$ $= \frac{5.6 \sin 34°}{\sin 64°}$ $= 3.5\,cm$	$\frac{c}{\sin C} = \frac{a}{\sin A}$ $c = \frac{a \sin C}{\sin A}$ $= \frac{5.6 \sin 82°}{\sin 64°}$ $= 6.2\,cm$

Exercise

Solve triangle *ABC* if $B = 55°40'$, $b = 8.94$ m, and $a = 25.1$ m.

Solution



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{25.1} = \frac{\sin\left(55° + \frac{40°}{60}\right)}{8.94}$$

$$\sin A = \frac{25.1 \sin(55.667°)}{8.94} \approx 2.3184 > 1$$

Since $\sin A > 1$ is impossible, no such triangle exists.

Exercise

Solve triangle *ABC* if $A = 55.3°$, $a = 22.8$ *ft*, and $b = 24.9$ ft

Solution

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

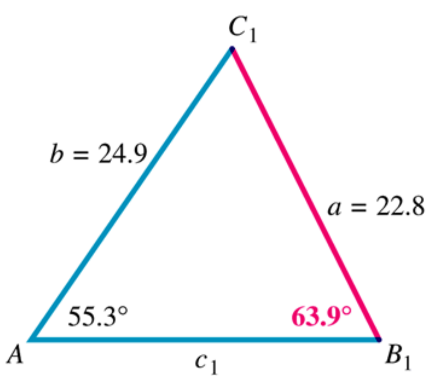
$$\sin B = \frac{24.9 \sin 55.3°}{22.8} \approx 0.89787$$

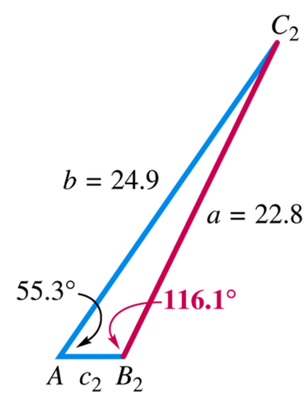
$$B = \sin^{-1}(0.89787)$$

$$B = 63.9° \qquad \textit{and} \qquad B = 180° - 63.9° = 116.1°$$

$$C = 180° - A - B$$

$C = 180° - 55.3° - 63.9°$	$C = 180° - 55.3° - 116.1°$
$C = 60.8°$	$C = 8.6°$
$c = \dfrac{a \sin C}{\sin A}$	$c = \dfrac{a \sin C}{\sin A}$
$= \dfrac{22.8 \sin 60.8°}{\sin 55.3°}$	$= \dfrac{22.8 \sin 8.6°}{\sin 55.3°}$
$= 24.2$ *ft*	$= 4.15$ *ft*

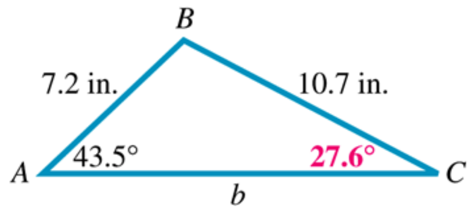




Exercise

Solve triangle *ABC* given $A = 43.5°$, $a = 10.7$ in., and $c = 7.2$ in

Solution



$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = \frac{7.2 \sin 43.5°}{10.7} \approx 0.4632$$

$$C = \sin^{-1}(0.4632)$$

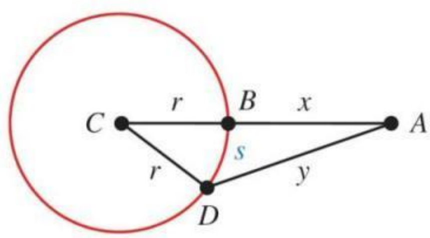
$$C = 27.6° \quad \text{and} \quad C = 180° - 27.6° = 152.4°$$

$$B = 180° - A - C$$

$B = 180° - 43.5° - 27.6°$ $B = 108.9°$ $b = \frac{a \sin B}{\sin A}$ $= \frac{10.7 \sin 108.9°}{\sin 43.5°}$ $= 14.7\ in$	$B = 180° - 43.5° - 152.4°$ $B = -15.9°$ Is not possible

Exercise

If $A = 26°, s = 22, and\ \ r = 19$ find x



Solution

$$C = \theta = \frac{s}{r}\ rad = \frac{22}{19}\frac{180°}{\pi} \approx 66°$$

$$\underline{D =}\ 180 - A - C = 180° - 26° - 66°\ \underline{= 88°}$$

$$\frac{r + x}{\sin D} = \frac{r}{\sin A}$$

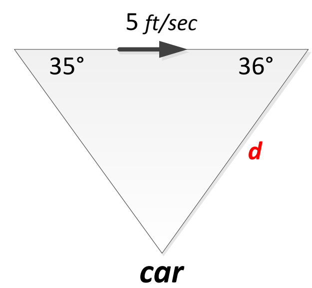
$$19 + x = \frac{19 \sin 88°}{\sin 26°}$$

$$\underline{x =}\ \frac{19 \sin 88°}{\sin 26°} - 19\ \underline{\approx 24}$$

Exercise

A man flying in a hot-air balloon in a straight line at a constant rate of 5 feet per second, while keeping it at a constant altitude. As he approaches the parking lot of a market, he notices that the angle of depression from his balloon to a friend's car in the parking lot is 35°. A minute and a half later, after flying directly over this friend's car, he looks back to see his friend getting into the car and observes the angle of depression to be 36°. At that time, what is the distance between him and his friend?

Solution



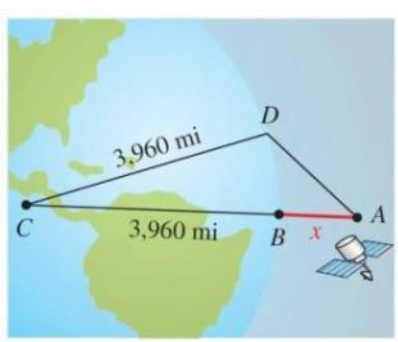
$$\angle car = 180° - 35° - 36° = 109°$$

$$\frac{d}{\sin 35°} = \frac{450}{\sin 109°}$$

$$\boxed{d = \frac{450 \sin 35°}{\sin 109°} \approx 273 \; ft}$$

Exercise

A satellite is circling above the earth. When the satellite is directly above point B, angle A is 75.4°. If the distance between points B and D on the circumference of the earth is 910 miles and the radius of the earth is 3,960 miles, how far above the earth is the satellite?



Solution

$$\theta = \frac{s}{r}$$

$C = arc\ length\ BD$ divides by $radius$

$$C = \frac{910}{3960} rad$$

$$= \frac{910}{3960} \frac{180°}{\pi}$$

$$= 13.2°$$

$$D = 180° - (A + C)$$

$$= 180° - (75.4° + 13.2°)$$

$= 91.4°$

$$\frac{CA}{\sin D} = \frac{3960}{\sin A}$$

$$\frac{x + 3960}{\sin 91.4°} = \frac{3960}{\sin 75.4°}$$

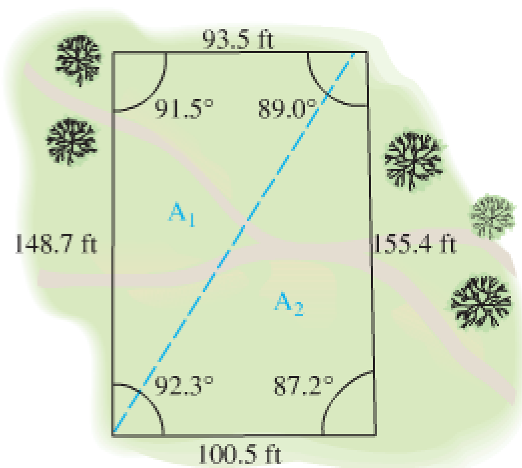
$$x + 3960 = \frac{3960 \sin 91.4°}{\sin 75.4°}$$

$$x = \frac{3960 \sin 91.4°}{\sin 75.4°} - 3960$$

$$x = 130\ mi$$

Exercise

The dimensions of a land are given in the figure. Find the area of the property in square feet.



Solution

$$A_1 = \frac{1}{2}(148.7)(93.5)\sin 91.5° \approx 6949.3\ ft^2$$

$$A_2 = \frac{1}{2}(100.5)(155.4)\sin 87.2° \approx 7799.5\ ft^2$$

The total area = $A_1 + A_2 = 6949.3 + 7799.5 \,\boxed{= 14,748.8\ ft^2}$

Exercise

A pilot left Fairbanks in a light plane and flew 100 miles toward Fort in still air on a course with bearing of 18°. She then flew due east (bearing 90°) for some time drop supplies to a snowbound family. After the drop, her course to return to Fairbanks had bearing of 225°. What was her maximum distance from Fairbanks?

Solution

From the triangle ABC:

$\angle ABC = 90° + 18° = 108°$

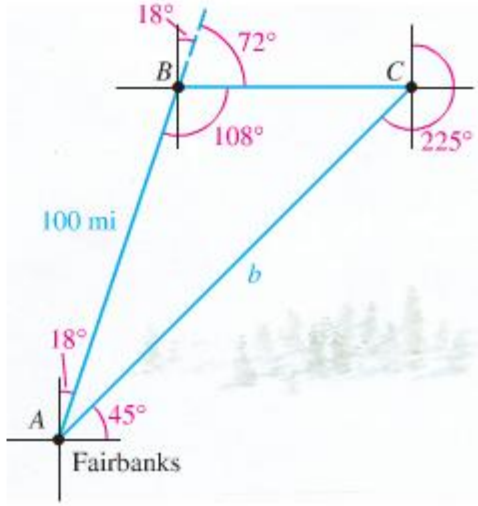
$\angle ACB = 360° - 225° - 90° = 45°$

$\angle BAC = 90° - 18° - 45° = 27°$

The length AC is the maximum distance from Fairbanks:

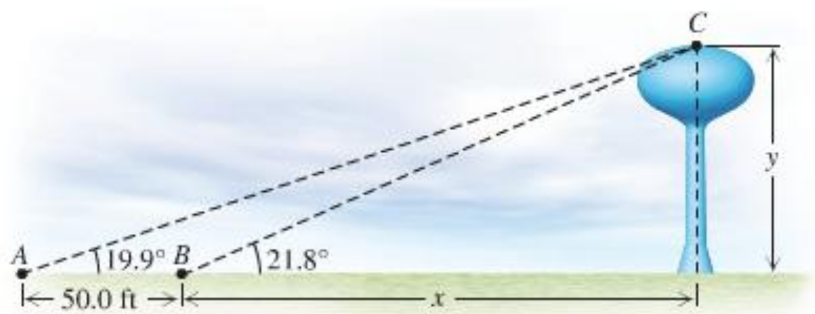
$$\frac{b}{\sin 108°} = \frac{100}{\sin 45°}$$

$$b = \frac{100 \sin 108°}{\sin 45°} \approx 134.5 \text{ miles}$$



Exercise

The angle of elevation of the top of a water tower from point A on the ground is 19.9°. From point B, 50.0 feet closer to the tower, the angle of elevation is 21.8°. What is the height of the tower?



Solution

$\angle ABC = 180° - 21.8° = 158.2°$

$\angle ACB = 180° - 19.9° - 158.2° = 1.9°$

Apply the law of sines in triangle ABC:

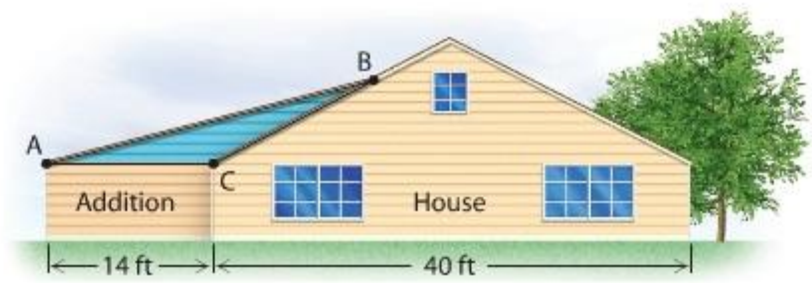
$$\frac{BC}{\sin 19.9°} = \frac{50}{\sin 1.9°} \Rightarrow BC = \frac{50 \sin 19.9°}{\sin 1.9°} \approx 513.3$$

Using the right triangle: $\sin 21.8° = \dfrac{y}{BC}$

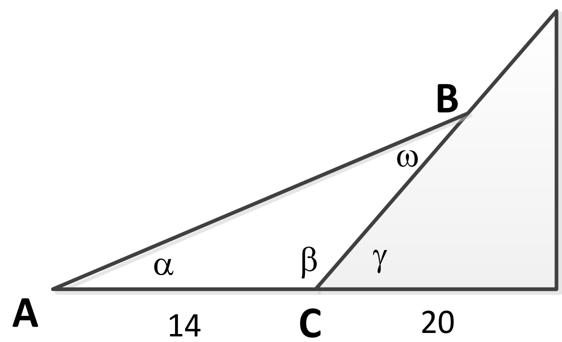
$$y = 513.3 \sin 21.8° \approx 191 \text{ ft}$$

Exercise

A 40-ft wide house has a roof with a 6-12 pitch (the roof rises 6 ft for a run of 12 ft). The owner plans a 14-ft wide addition that will have a 3-12 pitch to its roof. Find the lengths of $\overline{AB}$ and $\overline{BC}$.



Solution



$$\tan\gamma = \frac{6}{12} \Rightarrow \gamma = \tan^{-1}\left(\frac{6}{12}\right) = 26.565°$$

$$\tan\alpha = \frac{3}{12} \Rightarrow \alpha = \tan^{-1}\left(\frac{3}{12}\right) = 14.036°$$

$$\beta = 180° - \gamma = 180° - 26.565° = 153.435°$$

$$\omega = 180° - 14.036° - 153.435° = 12.529°$$

$$\frac{AB}{\sin 153.435°} = \frac{14}{\sin 12.529°}$$

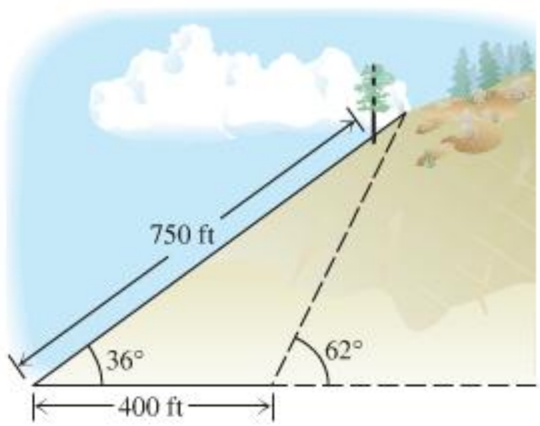
$$\Rightarrow \underline{|AB} = \frac{14\sin 153.435°}{\sin 12.529°} \underline{\approx 28.9\ ft|}$$

$$\frac{BC}{\sin 14.036°} = \frac{14}{\sin 12.529°}$$

$$\Rightarrow \underline{|BC} = \frac{14\sin 14.036°}{\sin 12.529°} \underline{\approx 15.7\ ft|}$$

Exercise

A hill has an angle of inclination of 36°. A study completed by a state's highway commission showed that the placement of a highway requires that 400 ft of the hill, measured horizontally, be removed. The engineers plan to leave a slope alongside the highway with an angle of inclination of 62°. Located 750 ft up the hill measured from the base is a tree containing the nest of an endangered hawk. Will this tree be removed in the excavation?



Solution

$\angle ACB = 180^\circ - 62^\circ = 118^\circ$

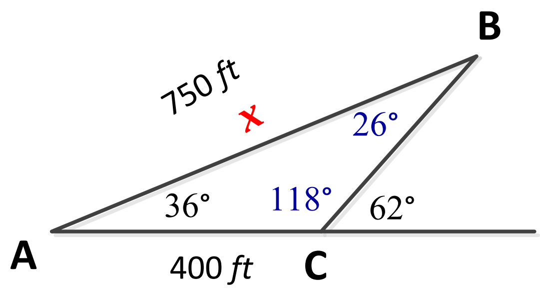
$\angle ABC = 180^\circ - 118^\circ - 36^\circ = 26^\circ$

Using the law of sines:

$$\frac{x}{\sin 118^\circ} = \frac{400}{\sin 26^\circ}$$

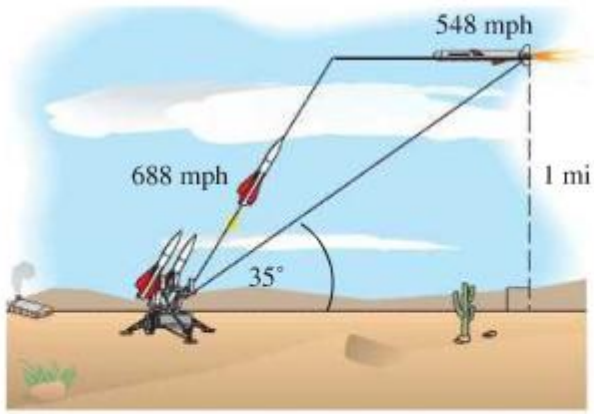
$$x = \frac{400 \sin 118^\circ}{\sin 26^\circ} \approx 805.7\ ft$$

Yes, the tree will have to be excavated.



Exercise

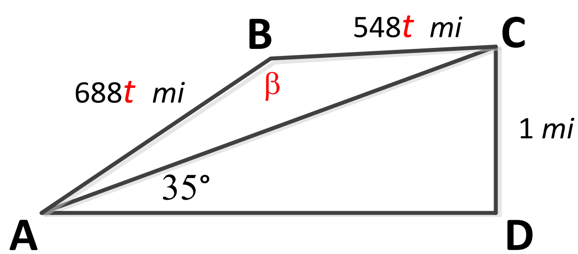
A cruise missile is traveling straight across the desert at 548 mph at an altitude of 1 mile. A gunner spots the missile coming in his direction and fires a projectile at the missile when the angle of elevation of the missile is 35°. If the speed of the projectile is 688 mph, then for what angle of elevation of the gun will the projectile hit the missile?



Solution

$\angle ACB = 35°$

$\angle BAC = 180° - 35° - \beta$



After ***t*** seconds;

The cruise missile distance: $548 \dfrac{t}{3600}$ *miles*

The Projectile distance: $688 \dfrac{t}{3600}$ *miles*

Using the law of sines:

$$\frac{\frac{548t}{3600}}{\sin(145° - \beta)} = \frac{\frac{688t}{3600}}{\sin 35°}$$

$$\frac{548t}{3600} \sin 35° = \frac{688t}{3600} \sin(145° - \beta)$$

$$548 \sin 35° = 688 \sin(145° - \beta)$$

$$\sin(145° - \beta) = \frac{548}{688} \sin 35°$$

$$145° - \beta = \sin^{-1}\left(\frac{548}{688} \sin 35°\right)$$

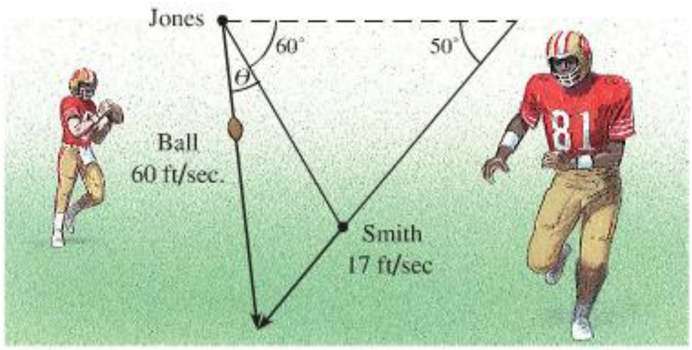
$$\boxed{\beta = 145° - \sin^{-1}\left(\frac{548}{688} \sin 35°\right) \approx 117.8°}$$

$$\Rightarrow \angle BAC = 180° - 35° - 117.8° = 27.2°$$

The angle of elevation of the projectile must be $(= 35° + 27.2°)$ 62.2°

Exercise

When the ball is snapped, Smith starts running at a 50° angle to the line of scrimmage. At the moment when Smith is at a 60° angle from Jones, Smith is running at 17 ft/sec and Jones passes the ball at 60 ft/sec to Smith. However, to complete the pass, Jones must lead Smith by the angle θ. Find θ (find θ in his head. Note that θ can be found without knowing any distances.)



Solution

$$\angle ABD = 180° - 60° - 50° = 70°$$

$$\angle ABC = 180° - 70° = 110°$$

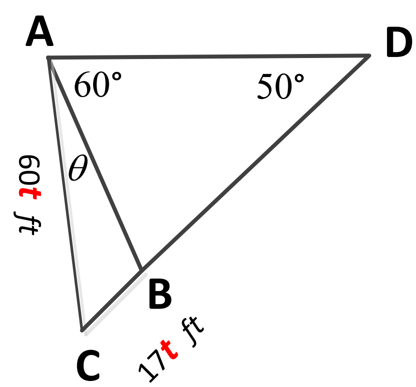
Using the law of sines:

$$\frac{17t}{\sin\theta} = \frac{60t}{\sin 110°}$$

$$\frac{17}{\sin\theta} = \frac{60}{\sin 110°}$$

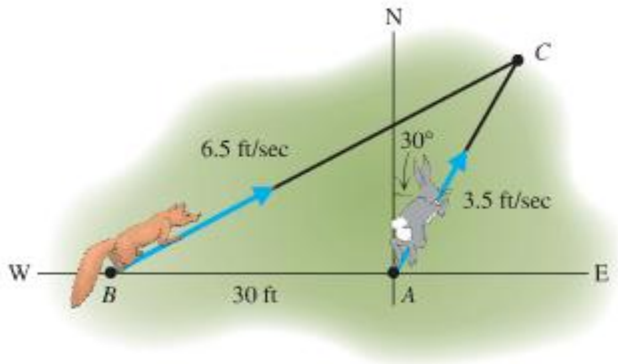
$$\sin\theta = \frac{17\sin 110°}{60}$$

$$\theta = \sin^{-1}\left(\frac{17\sin 110°}{60}\right) = 15.4°$$



Exercise

A rabbit starts running from point A in a straight line in the direction 30° from the north at 3.5 *ft/sec*. At the same time a fox starts running in a straight line from a position 30 *ft* to the west of the rabbit 6.5 *ft/sec*. The fox chooses his path so that he will catch the rabbit at point C. In how many seconds will the fox catch the rabbit?



Solution

$$\angle BAC = 90° + 30° = 120°$$

$$\frac{6.5t}{\sin 120°} = \frac{3.5t}{\sin B}$$

$$\frac{6.5}{\sin 120°} = \frac{3.5}{\sin B}$$

$$\sin B = \frac{3.5 \sin 120°}{6.5}$$

$$B = \sin^{-1}\left(\frac{3.5 \sin 120°}{6.5}\right) \approx 28°$$

$$C = 180° - 120° - 28° = 32°$$

$$\frac{3.5t}{\sin 28} = \frac{30}{\sin 32°}$$

$$\boxed{t = \frac{30 \sin 28}{3.5 \sin 32°} \approx 7.6 \text{ sec}}$$

It will take 7.6 sec. to catch the rabbit.