$y = 3 + \sin x \qquad x \in [0, 2\pi]$

concavity:

$y' = \cos x$

$y'' = -\sin x = 0$

$x = 0, \pi, 2\pi$ } point of inflection

pt. infl.

0	$\frac{\pi}{2}$	π	2π
	$-$		$+$

concave up: $(\pi, 2\pi)$

" down: $(0, \pi)$

5. [illegible]

$s'(t) = v(t)$

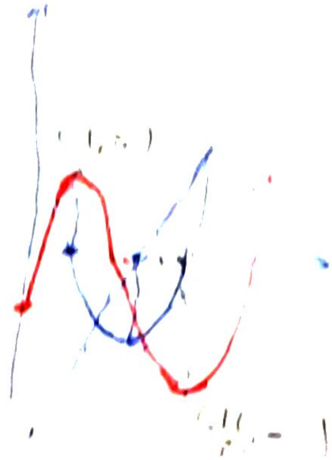
$= 6t^2 - 28t + 22 = 0$

C.N.: $t = 1, \frac{11}{3}$

t	$s'(t)$
1	[illegible]
$\frac{11}{3}$	$-$

[illegible] $(1, \infty)$

[illegible] $\frac{11}{3}$ [illegible]



	0	1	2	11/3	5
s' sign	$+$		$-$		$+$

Inc.: $(0, 1)$ $(\frac{11}{3}, \infty)$

Dec.: $(1, \frac{11}{3})$

$s''(t) = a(t)$

$= 12t - 28 = 0$

point of infl.: $\boxed{t = \frac{7}{3}}$

0	1	7/3	
	$-$		$+$

concave down: $(0, \frac{7}{3})$

" up: $(\frac{7}{3}, \infty)$

Ex

$f(x) = x^4 - 4x^3 + 10$

$f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

CN: $x = 0, 0, 3$

0		3	
−	−		+

Incr: $(3, \infty)$

Decr: $(-\infty, 0)\ (0, 3)$

$f(3) = -17$

L MIN: $(3, -17)$

concavity?

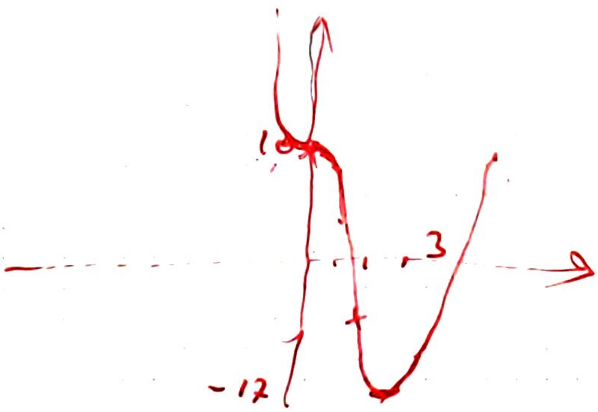
$f''(x) = 12x^2 - 24x = 0 \quad 12x(x-2) = 0$

Point of infl? $x = 0, 2$

0	1	2	
+		−	+

concave up: $(-\infty, 0)\ (2, \infty)$

" down: $(0, 2)$



$f(x) = \frac{(x+1)^2}{1+x^2}$ $\quad (x+1)^2 \, (1+x^2)^{-1}$

$(u^n v^m)' = u^{n-1} v^{m-1} (n u' v + m u v')$

$f'(x) = \frac{x+1}{(1+x^2)^2} \left(2(1+x^2) - 2x(x+1)\right)$

$= \frac{(x+1)(2+2x^2-2x^2-2x)}{(1+x^2)^2}$

$= \frac{(x+1)(2-2x)}{(1+x^2)^2} = 0$

CV: $X = -1, 1$

	-1		0		1	
	$-$		$+$		$-$	

Incr: $(-1, 1)$

Decr: $(-\infty, -1)\ (1, \infty)$

x	$f(x)$
-1	0
1	2

RMIN $(-1, 0)$

RMAX $(1, 2)$

$f'(x) = 2\frac{1-x^2}{(1+x^2)^2}$ $\quad (1-x^2)^1 (1+x^2)^{-2}$

$f''(x) = \frac{2}{(1+x^2)^3} \left(-2x(1+x^2) - 2(2x)(1-x^2)\right)$

$= \frac{2(-2x - 2x^3 - 4x + 4x^3)}{(1+x^2)^3}$

$= \frac{2(2x^3 - 6x)}{(1+x^2)^2}$

$= \frac{4x(x^2-3)}{(1+x^2)^2} = 0$

$x = 0, \pm\sqrt{3}$ point of infl.

$-\infty$	$-\sqrt{3}$	0	$\sqrt{3}$
$-$	$+$	$-$	$+$

Concave up: $(-\sqrt{3}, 0)\ (\sqrt{3}, \infty)$

" down: $(-\infty, -\sqrt{3})\ (0, \sqrt{3})$

3.3 Optimization

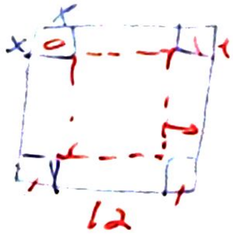
$\left\{\begin{array}{l}\text{Maximize}\\ \text{Minimize}\end{array}\right\}$ 1st deriv.

$\left\{\begin{array}{l}\text{point of infl.}\\ \text{diminishing}\end{array}\right\}$ 2nd deriv $= 0$

$y = ax^2 + bx + c$

$y' = 2ax + b = 0 \quad x = -\dfrac{b}{2a}$

Ex. 12×12



largest x?

$V = (12-2x)^2(x)$

$= x(144 - 48x + 4x^2)$

$= 4x^3 - 48x^2 + 144x$

$\dfrac{dV}{dx} = 12x^2 - 96x + 144 = 0$

$x^2 - 8x + 12 = 0$

CN: $x = 2, 6$

$V(2) = 2(64) = 128$

$V(6) = 6(0)$ ∴

$x = 2$ w/ large Volume of 128 unit^3

Ex $V = 1L = 10^3 \, cm^3 = \pi r^2 h$ (1)

Material (surface)

Surface $= 2\pi r^2 + 2\pi r h = A$ (2)

(1) $\rightarrow$ $h = \frac{10^3}{\pi r^2}$ (3)

(2) $A(r) = 2\pi r^2 + 2\pi r \cdot \frac{10^3}{\pi r^2}$

$= 2\pi r^2 + 2\frac{10^3}{r}$

$\frac{dA}{dr} = 4\pi r - \frac{2 \times 10^3}{r^2} = 0$

$2\pi r = \frac{1000}{r^2}$

$r^3 = \frac{500}{\pi}$

$\boxed{r = \sqrt[3]{\frac{500}{\pi}}} = \sqrt[3]{\frac{10^3}{2\pi}} = \frac{10}{(2\pi)^{1/3}}$

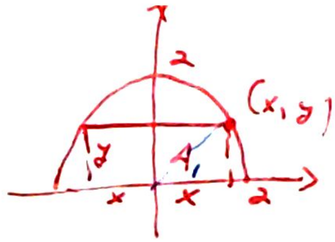
(3) $\rightarrow$ $h = \frac{10^3}{\pi \frac{100}{(2\pi)^{2/3}}}$

$= \frac{10 \times 2^{2/3} \pi^{2/3}}{\pi}$

$= \frac{10 (2^{2/3})}{\pi^{1/3}}$

$x^2 + y^2 = 4$

A_{max}? dim



$y = \sqrt{4 - x^2}$

$A_1 = xy$

$A(x) = x(4 - x^2)^{1/2}$

$\frac{dA_1}{dx} = \frac{1}{\sqrt{4-x^2}} \left(4 - x^2 + \frac{1}{2}(-2x)x\right)$

$= \frac{4 - 2x^2}{\sqrt{4-x^2}} = 0 \Rightarrow x^2 = 2$

$x = \pm\sqrt{2}$

CN: $\left.\begin{array}{l} x = \sqrt{2} \\ y = \sqrt{2} \end{array}\right]$

$l = 2\sqrt{2}, \quad w = \sqrt{2}$

$A_{rect} = lw$

$= 2\sqrt{2}(\sqrt{2})$

$= 4 \text{ unit}^2$