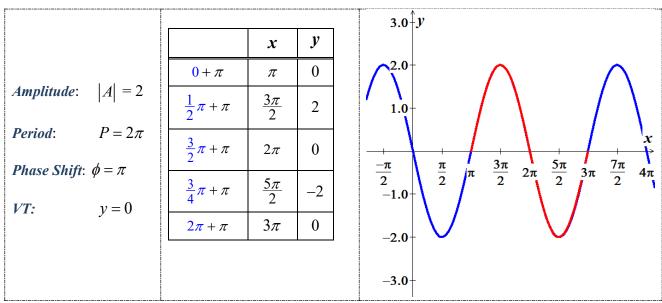
Solution

Section 7.1 – Graphing Sine & Cosine

Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 2\sin(x - \pi)$

Solution



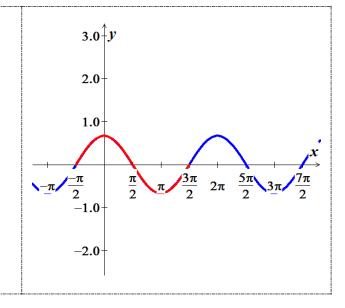
Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = \frac{2}{3}\sin\left(x + \frac{\pi}{2}\right)$

Solution

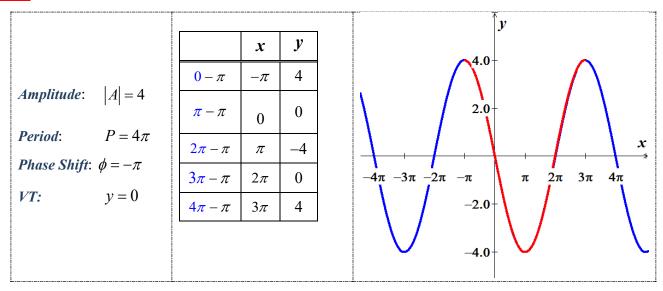
Amplitude: $|A| = \frac{2}{3}$ Period: $P = 2\pi$ Phase Shift: $\phi = -\frac{\pi}{2}$ VT: y = 0

	X	y
$0-\frac{\pi}{2}$	$-\frac{\pi}{2}$	0
$\frac{1}{2}\pi - \frac{\pi}{2}$	0	$\frac{2}{3}$
$\frac{3}{2}\pi - \frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{3}{4}\pi - \frac{\pi}{2}$	π	$-\frac{2}{3}$
$2\pi - \frac{\pi}{2}$	$\frac{3\pi}{2}$	0



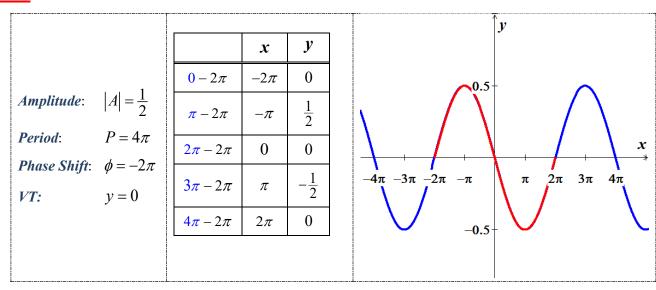
Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 4\cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

Solution



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = \frac{1}{2}\sin(\frac{1}{2}x + \pi)$



Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 3\cos\left[\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right]$

Solution

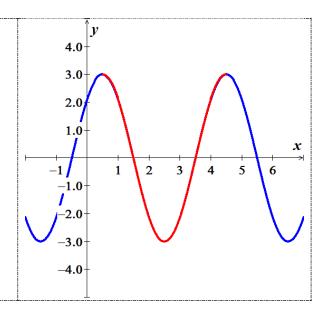
Amplitude: |A| = 3

Period: $P = \frac{2\pi}{\frac{\pi}{2}} = 4$

Phase Shift: $\varphi = \frac{1}{2}$

VT: y = 0

	X	y
$0 + \frac{1}{2}$	$\frac{1}{2}$	3
$1 + \frac{1}{2}$	$\frac{3}{2}$	0
$2 + \frac{1}{2}$	<u>5</u> 2	-3
$3 + \frac{1}{2}$	7/2	0
$4 + \frac{1}{2}$	9 2	3



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = -\cos \pi \left(x - \frac{1}{3}\right)$

Solution

Amplitude: |A| = 1Period: P = 2

Phase Shift: $\phi = \frac{1}{3}$

VT: y=0

 $\begin{array}{c|ccccc} & x & y \\ \hline 0 + \frac{1}{3} & \frac{1}{3} & -1 \\ \hline \frac{1}{2} + \frac{1}{3} & \frac{5}{6} & 0 \\ \hline 1 + \frac{1}{3} & \frac{4}{3} & 1 \\ \hline \frac{3}{2} + \frac{1}{3} & \frac{11}{6} & 0 \\ \hline 2 + \frac{1}{3} & \frac{7}{3} & -1 \\ \hline \end{array}$

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 2 - \sin\left(3x - \frac{\pi}{5}\right)$

Solution

		x	y	$\int y$
Amplitude: $ A = 1$	$0+\frac{\pi}{15}$	$\frac{\pi}{15}$	2	3.0+
Period : $P = \frac{2\pi}{3}$	$\frac{\pi}{6} + \frac{\pi}{15}$	$\frac{7\pi}{30}$	1	2.0+
Phase Shift: $\phi = \frac{\pi}{15}$	$\frac{\pi}{3} + \frac{\pi}{15}$	$\frac{6\pi}{15}$	2	
VT: $y=2$	$\frac{\pi}{2} + \frac{\pi}{15}$	$\frac{17\pi}{30}$	3	1.0
	$\frac{2\pi}{3} + \frac{\pi}{15}$	$\frac{11\pi}{15}$	2	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
				<u> </u>

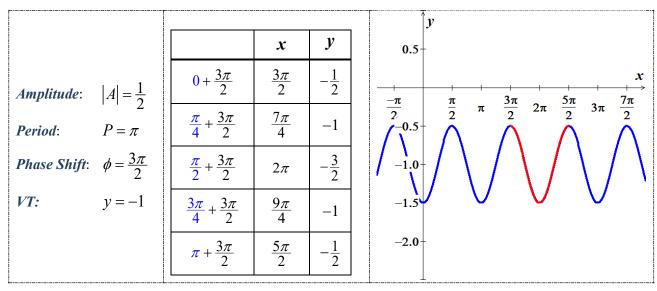
Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = -\frac{2}{3}\sin\left(3x - \frac{\pi}{2}\right)$

Amplitude: $ A = \frac{2}{3}$ Period: $P = \frac{2\pi}{3}$	$0 + \frac{\pi}{6}$ $\frac{\pi}{6} + \frac{\pi}{6}$	$\frac{x}{6}$ $\frac{\pi}{3}$	y 0 $-\frac{2}{3}$	1.0- 1.0-
Period: $P = \frac{2\pi}{3}$ Phase Shift: $\phi = \frac{\pi}{6}$	$\frac{\pi}{3} + \frac{\pi}{6}$	$\frac{\pi}{2}$	0	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
VT: $y=0$		$\frac{2\pi}{3}$ $\frac{5\pi}{6}$	$\frac{2}{3}$	-1.0-

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = -1 + \frac{1}{2}\cos(2x - 3\pi)$

Solution



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = 2 - \frac{1}{3}\cos\left(\pi x + \frac{3\pi}{2}\right)$

					↑v
			x	y	3.0
Amplitude: $ A = \frac{1}{3}$		$0 - \frac{3}{2}$	$-\frac{3}{2}$	<u>5</u>	2.5
Period : $P = \frac{2\pi}{\pi} = 2$		$\frac{1}{2} - \frac{3}{2}$	-1	2	1.5
Phase Shift: $\phi = -\frac{3\pi}{2} = -$	3 2	$1 - \frac{3}{2}$	$-\frac{1}{2}$	7/3	1.0-
VT: $y=2$		$\frac{3}{2} - \frac{3}{2}$	0	2	0.5- x
		$2 - \frac{3}{2}$	1/2	<u>5</u> 3	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = \frac{5}{2} - 3\cos\left(\pi x - \frac{\pi}{4}\right)$

Solution

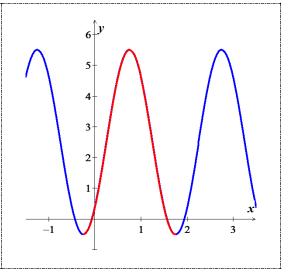
Amplitude: |A| = 3

Period: $P = \frac{2\pi}{\pi} = 2$

Phase Shift: $\phi = -\frac{-\frac{\pi}{4}}{\pi} = \frac{1}{4}$

VT: $y = \frac{5}{2}$

	x	у
$0 - \frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$
$\frac{1}{2} - \frac{1}{4}$	<u>1</u>	<u>5</u> 2
$1 - \frac{1}{4}$	<u>3</u>	<u>11</u> 2
$\frac{3}{2} - \frac{1}{4}$	<u>5</u>	<u>5</u> 2
$2 - \frac{1}{4}$	7/4	$-\frac{1}{2}$



Exercise

Find the amplitude, the period, the phase shift, and the vertical translation and sketch the graph of the equation $y = \frac{2}{3} - \frac{4}{3}\cos(3x - \pi)$

Solution

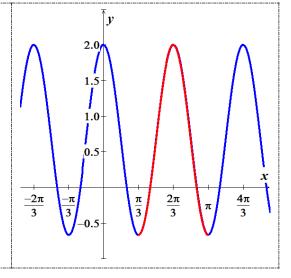
Amplitude:	$ A = \frac{4}{3}$

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\phi = \frac{\pi}{3}$

VT: $y = \frac{2}{3}$

	x	y
$0+\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{2}{3} - \frac{4}{3} = -\frac{2}{3}$
$\frac{\pi}{6} + \frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}$
$\frac{\pi}{3} + \frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{2}{3} + \frac{4}{3} = 2$
$\frac{\pi}{2} + \frac{\pi}{3}$	$\frac{5\pi}{6}$	<u>2</u> 3
$\frac{2\pi}{3} + \frac{\pi}{3}$	π	$-\frac{2}{3}$

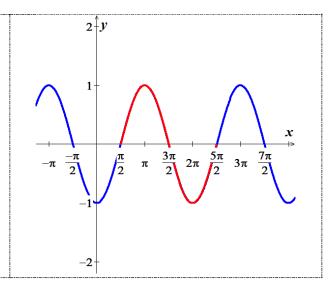


Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = \sin\left(x - \frac{\pi}{2}\right)$

Solution

Amplitude: |A| = 1Period: $P = 2\pi$ Phase Shift: $\phi = \frac{\pi}{2}$ VT: y = 0

	x	y
$0+\frac{\pi}{2}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2} + \frac{\pi}{2}$	π	1
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$\frac{3\pi}{2} + \frac{\pi}{2}$	2π	-1
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	0



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 2\sin\left(x - \frac{\pi}{3}\right)$

Solution

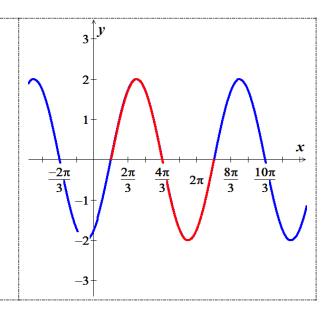
Amplitude: |A| = 2

Period: $P = 2\pi$

Phase Shift: $\phi = \frac{\pi}{3}$

VT: y=0

	X	y
$0+\frac{\pi}{3}$	$\frac{\pi}{3}$	0
$\frac{\pi}{2} + \frac{\pi}{3}$	$\frac{5\pi}{6}$	2
$\pi + \frac{\pi}{3}$	$\frac{4\pi}{3}$	0
$\frac{3\pi}{2} + \frac{\pi}{3}$	$\frac{11\pi}{6}$	-2
$2\pi + \frac{\pi}{3}$	$\frac{7\pi}{3}$	0



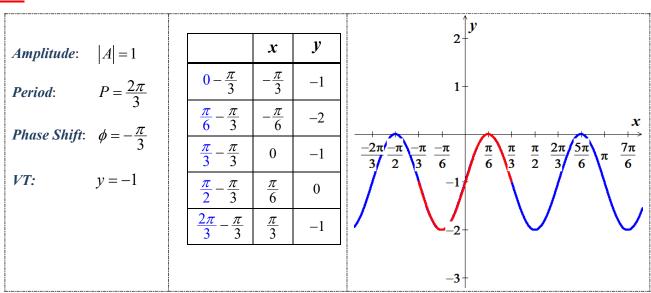
Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 4\cos\left(x - \frac{\pi}{4}\right)$

Solution

		6 † <i>y</i>
Amerikandan d	x y	
Amplitude: $ A = 4$	$0+\frac{\pi}{4}$ $\frac{\pi}{4}$ 4	↑ ↑ ↑ ↑
Period : $P = 2\pi$		24
Phase Shift: $\phi = \frac{\pi}{4}$		\
0	$\pi + \frac{\pi}{4}$ $\frac{5\pi}{4}$ -4	$\frac{-3\pi}{2} - \pi \frac{-\pi}{2} \qquad \frac{\pi}{2} \pi \frac{3\pi}{2} 2\pi \frac{5\pi}{2} 3\pi \frac{7\pi}{2}$
VT: $y=0$	$\left \frac{3\pi}{2} + \frac{\pi}{4} \right \left \frac{7\pi}{4} \right 0$	$\left \begin{array}{c cccccccccccccccccccccccccccccccccc$
	$2\pi + \frac{\pi}{4} \qquad \frac{9\pi}{4} \qquad 4$	\bigcup_{-4} \bigcup

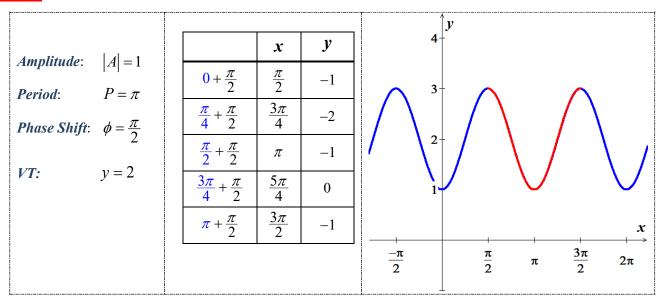
Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = -\sin(3x + \pi) - 1$



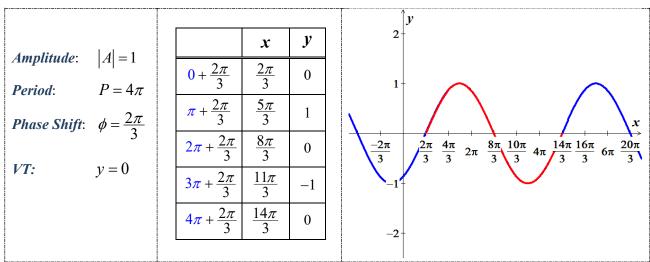
Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = \cos(2x - \pi) + 2$

Solution



Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = \sin\left(\frac{1}{2}x - \frac{\pi}{3}\right)$



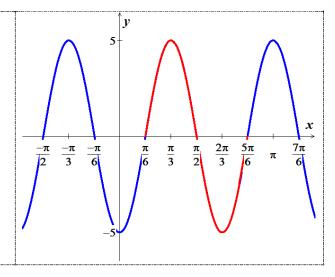
Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 5\sin\left(3x - \frac{\pi}{2}\right)$

Solution

Amplitude:	A = 5
Period:	$P = \frac{2\pi}{3}$
Phase Shift:	$\varphi = \frac{\pi}{6}$

2 100000 2	 4	6
VT:	<i>y</i> =	0

	x	y
$0+\frac{\pi}{6}$	$\frac{\pi}{6}$	0
$\frac{\pi}{6} + \frac{\pi}{6}$	$\frac{\pi}{3}$	5
$\frac{\pi}{3} + \frac{\pi}{6}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2} + \frac{\pi}{6}$	$\frac{2\pi}{3}$	-5
$\frac{2\pi}{3} + \frac{\pi}{6}$	$\frac{5\pi}{6}$	0

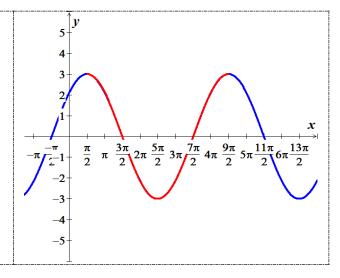


Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 3\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

	i	
Amplituda.	4 2	
Amplitude:	' '	0+
Period:	$P = 4\pi$	π
Phase Shift:	$\varphi = \frac{\pi}{2}$	70 1
I //F	0	2π +
VT:	y = 0	3π +
		4 .

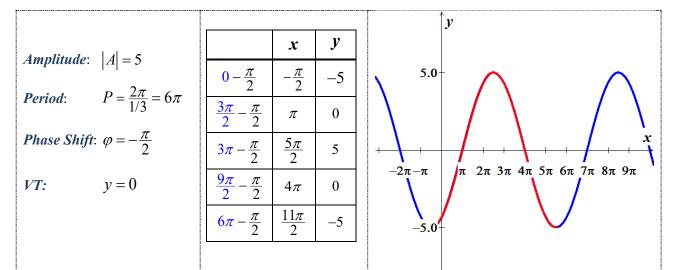
	x	y
$0+\frac{\pi}{2}$	$\frac{\pi}{2}$	3
$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	-3
$3\pi + \frac{\pi}{2}$	$\frac{7\pi}{2}$	0
$4\pi + \frac{\pi}{2}$	$\frac{9\pi}{2}$	3



Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$$y = -5\cos\left(\frac{1}{3}x + \frac{\pi}{6}\right)$$

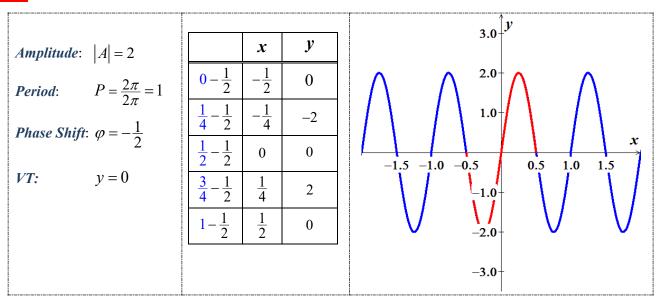
Solution



Exercise

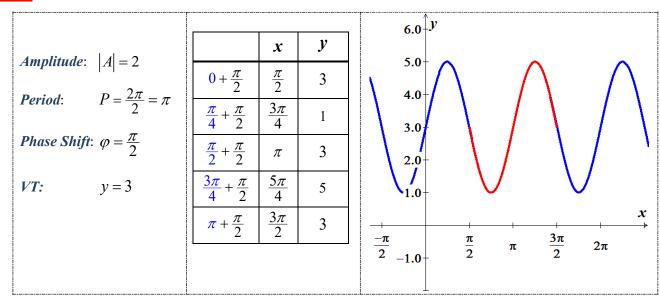
Find the amplitude, the period, and the phase shift and sketch the graph of the equation

$$y = -2\sin(2\pi x + \pi)$$



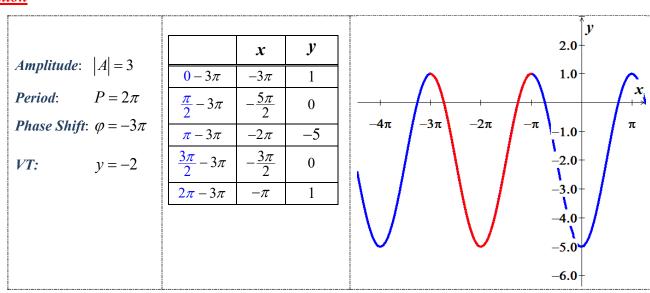
Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = -2\sin(2x - \pi) + 3$

Solution



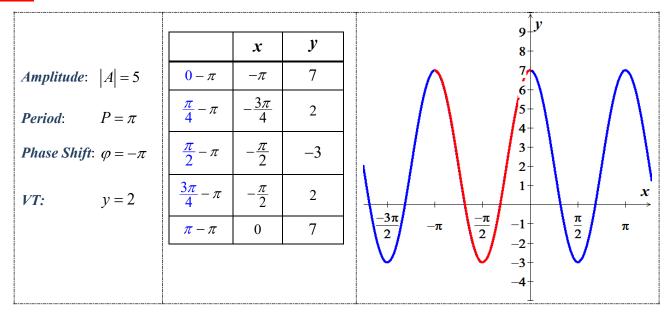
Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 3\cos(x + 3\pi) - 2$



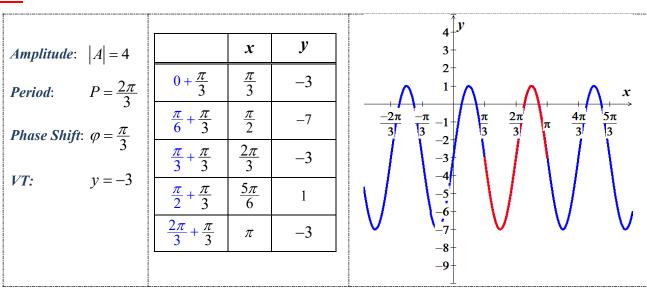
Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = 5\cos(2x + 2\pi) + 2$

Solution



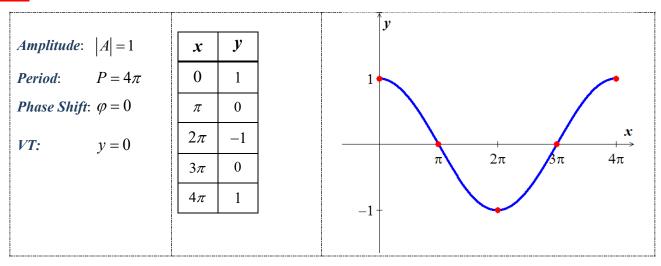
Exercise

Find the amplitude, the period, and the phase shift and sketch the graph of the equation $y = -4\sin(3x - \pi) - 3$



Find the amplitude, the period, any vertical translation, and any phase shift. Then graph a one complete cycle of $y = \cos \frac{1}{2}x$

Solution



Exercise

Find the amplitude, the period, any vertical translation, and any phase shift. Then graph

$$y = 2\sin(-\pi x) \quad for \quad -3 \le x \le 3$$

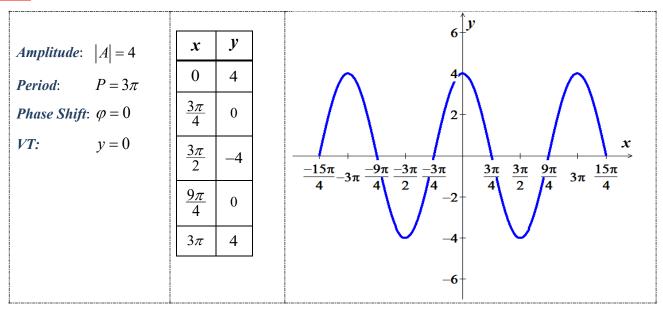
$$y = 2\sin(-\pi x) = -2\sin(\pi x)$$

	T		1 v
	x	y	3+
Amplitude: A = 2	0	0	
Period: $P = 2$ Phase Shift: $\varphi = 0$	1/2	-2	
VT: $y=0$	1	0	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
	$\frac{3}{2}$	2	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
	2		V ₋₂ V V
			-3-
	<u> </u>		<u> </u>

Find the amplitude, the period, any vertical translation, and any phase shift. Then graph

$$y = 4\cos\left(-\frac{2}{3}x\right)$$
 for $-\frac{15\pi}{4} \le x \le \frac{15\pi}{4}$

Solution



Exercise

Graph one complete cycle $y = \cos\left(x - \frac{\pi}{6}\right)$

Amplitude: $ A = 1$		x	у	
Period : $P = 2\pi$	$\frac{\pi}{6}$ + 0	$\frac{\pi}{6}$	1	1+
	$\frac{\pi}{6} + \frac{1}{2}\pi$	$\frac{2\pi}{3}$	0	
VT: $y=0$	$\frac{\pi}{6} + \pi$	$\frac{7\pi}{6}$	-1	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$
	$\frac{\pi}{6} + \frac{3}{2}\pi$	$\frac{5\pi}{3}$	0	-1-
	$\frac{\pi}{6} + 2\pi$	$\frac{13\pi}{6}$	1	

Graph one complete cycle $y = \frac{2}{3} - \frac{4}{3}\cos(3x - \pi)$

Solution

Amplitude:	$ A = \frac{4}{3}$
------------	---------------------

Period: $P = \frac{2\pi}{3}$

Phase Shift: $\varphi = \frac{\pi}{3}$

VT: $y = \frac{2}{3}$

	x	у
$0+\frac{\pi}{3}$	$\frac{\pi}{3}$	$-\frac{2}{3}$
$\frac{\pi}{6} + \frac{\pi}{3}$	$\frac{\pi}{2}$	<u>2</u> 3
$\frac{\pi}{3} + \frac{\pi}{3}$	$\frac{2\pi}{3}$	2
$\frac{\pi}{2} + \frac{\pi}{3}$	$\frac{5\pi}{6}$	<u>2</u> 3
$\frac{2\pi}{3} + \frac{\pi}{3}$	π	$-\frac{2}{3}$

2.7 y		
2.0 -	\wedge	
1.3 -	/ \	
0.7	<i>+</i> +	
		<i>x</i>
-0.7 +	$\pi/3$ $2\pi/3$ π $4\pi/3$	

Exercise

Graph one complete cycle $y = -3 + \sin\left(\pi x + \frac{\pi}{2}\right)$

Solution

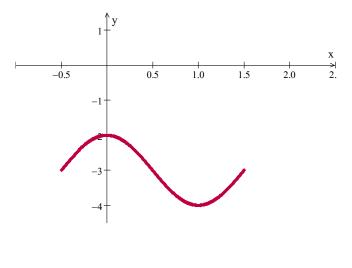
Amplitude:
$$|A| = 1$$

Period: P = 2

Phase Shift: $\varphi = -\frac{1}{2}$

VT: y = -3

	x	у
$0 - \frac{1}{2}$	$-\frac{1}{2}$	-3
$\frac{1}{2} - \frac{1}{2}$	0	-2
$1 - \frac{1}{2}$	1/2	-3
$\frac{3}{2} - \frac{1}{2}$	1	-4
$2-\frac{1}{2}$	$\frac{3}{2}$	-3



Graph $y = -1 + 2\sin(4x + \pi)$ over two periods.

Solution

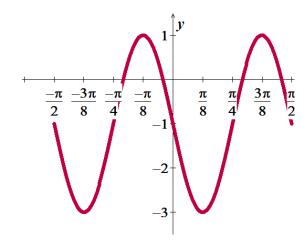
Amplitude:
$$|A| = 2$$

Period:
$$P = \frac{\pi}{2}$$

Phase Shift:
$$\varphi = -\frac{\pi}{4}$$

VT:
$$y = -1$$

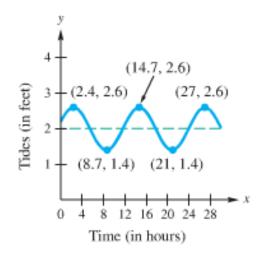
	x	у
$0-\frac{\pi}{4}$	$-\frac{\pi}{4}$	-1
$\frac{\pi}{8} - \frac{\pi}{4}$	$-\frac{\pi}{8}$	1
$\frac{\pi}{4} - \frac{\pi}{4}$	0	-1
$\frac{3\pi}{8} - \frac{\pi}{4}$	$\frac{\pi}{8}$	-3
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	-1



Exercise

The figure shows a function f that models the tides in feet at Clearwater Beach, x hours after midnight starting on Aug. 26,

- a) Find the time between high tides.
- b) What is the difference in water levels between high tide and low tide?
- c) The tides can be modeled by $f(x) = 0.6\cos[0.511x 2.4] + 2$. Estimate the tides when x = 10.



17

Solution

a) Time between high tides = 14.7 - 2.4

$$= 12.3 hrs$$

b) Difference in water levels between high tide and low tide = 2.6 - 1.4

$$=1.2 ft$$

c)
$$f(x=10) = 0.6 \cos [0.511(10) - 2.4]_{rad} + 2$$

 $\approx 1.45 \ ft$

The maximum afternoon temperature in a given city might be modeled by $t = 60 - 30\cos\frac{\pi x}{6}$

Where t represents the maximum afternoon temperature in month x, with x = 0 representing January, x = 1 representing February, and so on.. Find the maximum afternoon temperature to the nearest degree for each month.

a) Jan.

b) Apr.

c) May.

d) Jun.

e) Oct.

Solution

a) Jan.
$$t = 60 - 30\cos\frac{\pi(0)}{6} = 30^{\circ}$$

b) Apr.
$$t = 60 - 30\cos\frac{\pi(4)}{6} = 75^{\circ}$$

c) May.
$$t = 60 - 30\cos\frac{\pi(5)}{6} = 86^{\circ}$$

d) Jun.
$$t = 60 - 30\cos\frac{\pi(6)}{6} = 90^{\circ}$$

e) Oct.
$$t = 60 - 30\cos\frac{\pi(10)}{6} = 45^{\circ}$$

Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph

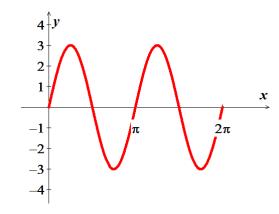
Solution

$$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$$

Amplitude = 3

Phase shift: $\phi = 0$

 $y = 3\sin 2x \qquad 0 \le x \le 2\pi$



Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph

Solution

$$P = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$$

$$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$$

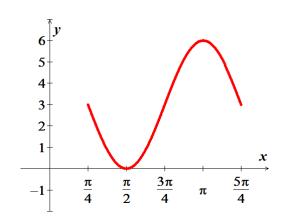
Amplitude = 3

$$A = -3$$

Phase shift: $\phi = \frac{\pi}{4} = -\frac{C}{B}$

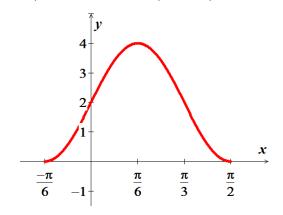
$$C = -\frac{\pi}{2}$$

$$y = -3\sin\left(2x - \frac{\pi}{2}\right) \qquad \frac{\pi}{4} \le x \le \frac{5\pi}{4} \qquad \frac{\pi}{4} \le x \le \frac{5\pi}{4}$$



Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



$$P = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$$

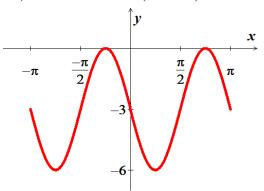
$$B = \frac{2\pi}{P} = \frac{2\pi}{\frac{2\pi}{3}} = 3$$

$$\phi = -\frac{\pi}{6} = -\frac{C}{B} \Rightarrow C = \frac{\pi B}{6} = \frac{\pi}{2}$$

$$Amplitude = 2 \Rightarrow \boxed{A = -2}$$

$$y = 2 - 2\cos\left(3x + \frac{\pi}{2}\right) \qquad -\frac{\pi}{6} \le x \le \frac{\pi}{2}$$

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



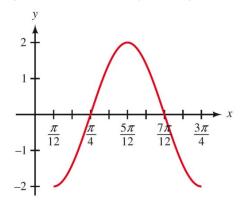
Solution

$P = \pi$	$B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$	D=-3
$\phi = 0$	$Amplitude = 3 \boxed{A = -3}$	

$$y = -3 - 3\sin 2x \quad -\pi \le x \le \pi$$

Exercise

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



$$P = \frac{3\pi}{4} - \frac{\pi}{12} = \frac{2\pi}{3}$$

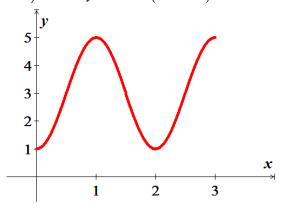
$$B = \frac{2\pi}{P} = \frac{2\pi}{\frac{2\pi}{3}} = 3$$

$$D = 0$$

$$\phi = \frac{\pi}{12} \Rightarrow C = -B\phi = -3\frac{\pi}{12} = -\frac{\pi}{4}$$
Amplitude = 2 $\boxed{A = -2}$

$$y = -2\cos\left(3x - \frac{\pi}{4}\right) \quad \frac{\pi}{12} \le x \le \frac{3\pi}{4}$$

Find an equation $y = A\sin(Bx + C) + D$ or $y = A\cos(Bx + C) + D$ to match the graph



Solution

 $Amplitude = 2 \qquad \boxed{A = -2}$

$$P = 2$$

$$B = \frac{2\pi}{P}$$

$$=\frac{2\pi}{2}$$

$$=\pi$$

$$\phi = 0$$

$$D = 3$$

$$y = 3 - 2\cos(\pi x) \quad 0 \le x \le 3$$

Exercise

The diameter of the Ferris wheel is 250 *feet*, the distance from the ground to the bottom of the wheel is 14 *feet*. We found the height of a rider on that Ferris wheel was given by the function:

$$H = 139 - 125\cos\left(\frac{\pi}{10}t\right)$$

Where *t* is the number of minutes from the beginning of a ride. Graph a complete cycle of this function.

Solution

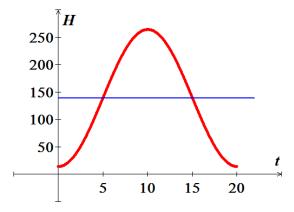
Amplitude: A = 125

Period: $P = \frac{2\pi}{\frac{\pi}{10}} = 20$

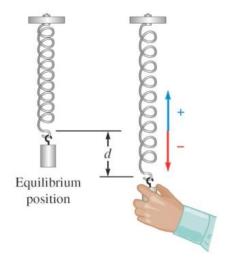
Phase Shift: $\phi = 0$

VT: H = 139

t	$H = 139 - 125\cos\left(\frac{\pi}{10}t\right)$
0	139-125=14
5	139
10	139+125=264
15	139
20	14



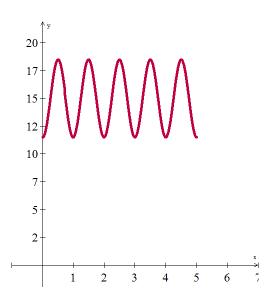
A mass attached to a spring oscillates upward and downward. The length L of the spring after t seconds is given by the function $L = 15 - 3.5\cos(2\pi t)$, where L is measured in cm.



- a) Sketch the graph of this function for $0 \le t \le 5$
- b) What is the length the spring when it is at equilibrium?
- c) What is the length the spring when it is shortest?
- d) What is the length the spring when it is longest?

Solution

a)



- **b)** The length the spring when it is at equilibrium L = 15 cm
- c) L = 15 3.5= 11.5 cm
- d) L = 15 + 3.5= $18.5 \ cm$

Based on years of weather data, the expected low temperature T (in °F) in Fairbanks, Alaska, can be approximated by

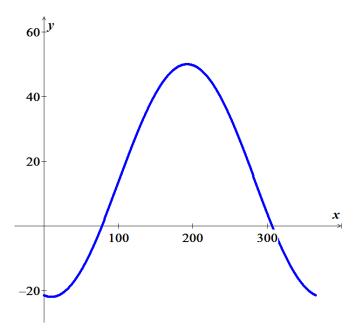
$$T = 36\sin\left(\frac{2\pi}{365}(t - 101)\right) + 14$$

- a) Sketch the graph T for $0 \le t \le 365$
- b) Predict when the coldest day of the year will occur.

Solution

a)

Amplitude: $ A = 36$		
	x	у
Period : $P = 2\pi \frac{365}{2\pi} = 365$	101	14
Phase Shift: $\phi = 101$	$\frac{365}{4} + 101 = \frac{769}{4}$	50
VT: $y = 14$	$\frac{365}{2} + 101 = \frac{567}{2}$	14
	$\frac{1095}{4} + 101 = \frac{1,499}{4}$	-22
	365 + 101 = 466	14



b) From the table the coldest temperature is -22 °F at $t = \frac{1499}{4} = 374.75 > 365$

$$t = 374.75 - 365$$

= 9.75 days |

Exercise

To simulate the response of a structure to an earthquake, an engineer must choose a shape for the initial displacement of the beams in the building. When the beam has length L feet and the maximum displacement is a feet, the equation

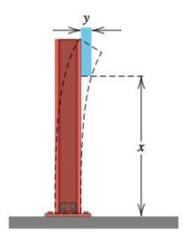
$$y = a - a\cos\frac{\pi}{2L}x$$

Has been used by engineers to estimate the displacement y. if a = 1 and L = 10, sketch the graph of the equation for $0 \le x \le 10$.

Given:
$$a = 1 \& L = 10$$

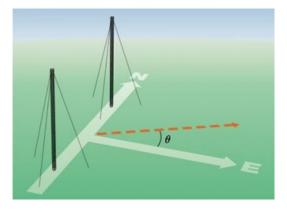
$$y = a - a\cos\frac{\pi}{2L}x$$

$$=1-\cos\left(\frac{\pi}{20}x\right)$$



		∫ <i>y</i>
	x y	
Amplitude: $ A = 1$	0 0	2.0-
	10 1	
Period : $P = \frac{2\pi}{\frac{\pi}{20}} = 40$	20 2	
20	30 1	1.0+
Phase Shift : $\phi = 0$	40 0	
VT: $y=1$	$0 \le x \le 10$	10 20 30 40

Radio stations often have more than one broadcasting tower because federal guidelines do not usually permit a radio station to broadcast its signal in all directions with equal power. Since radio waves cam travel over long distances, it is important to control their directional patterns so that radio stations do not interfere with one another. Suppose that a radio station has two broadcasting towers located along a north-south line.



If the radio station is broadcasting at a wavelength λ and the distance between the two radio towers is equal to $\frac{1}{2}\lambda$, then the intensity *I* of the signal in the direction θ is given by

$$I = \frac{1}{2}I_0 \left[1 + \cos(\pi \sin \theta) \right]$$

where I_0 is the maximum intensity.

a) Approximate I in terms of I_0 for each θ .

$$i. \quad \theta = 0$$

$$i. \qquad \theta = 0 \qquad \qquad ii. \qquad \theta = \frac{\pi}{3}$$

iii.
$$\theta = \frac{\pi}{7}$$

b) Determine the direction in which I has maximum or minimum values.

c) Graph I on the interval $[0, 2\pi)$. Graphically approximate θ to three decimal places, when I is equal to $\frac{1}{3}I_0$. (Hint: let $I_0=1$)

Solution

a)

θ	$I = \frac{1}{2}I_0 \left[1 + \cos(\pi \sin \theta) \right]$
0	$I = \frac{1}{2}I_0 \left[1 + \cos(\pi \sin 0) \right] = \frac{1}{2}I_0 \left[1 + \cos(0) \right] = \frac{1}{2}I_0 \left(2 \right) = \frac{I_0}{2}$
$\frac{\pi}{3}$	$I = \frac{1}{2}I_0 \left[1 + \cos\left(\pi \sin\frac{\pi}{3}\right) \right] = \frac{1}{2}I_0 \left[1 + \cos\left(\frac{\sqrt{3}}{2}\pi\right) \right] \approx 0.044I_0$
$\frac{\pi}{7}$	$I = \frac{1}{2}I_0 \left[1 + \cos\left(\pi \sin\frac{\pi}{7}\right) \right] \approx I_0$

- **b)** I to have a maximum when cos(0) = 1
 - ∴ *I* has *maximum* at $\theta = 0$

I to have a *minimum* when $-1 = \cos(\pi)$

$$\cos(\pi\sin\theta) = \cos(\pi)$$

$$\sin \theta = 1 \rightarrow \theta = \frac{\pi}{2}$$

I has *minimum* at $\theta = \frac{\pi}{2}$



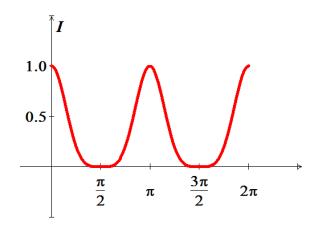
$$I = \frac{1}{2} + \frac{1}{2}\cos(\pi\sin\theta)$$

Amplitude: $|A| = \frac{1}{2}$

Period: $P = \frac{2\pi}{\pi} = 2$

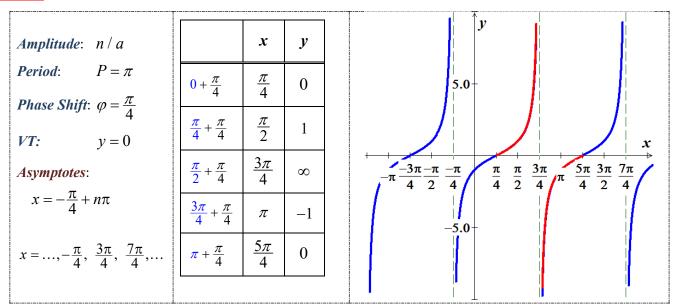
Phase Shift: $\varphi = 0$

VT: $y = \frac{1}{2}$



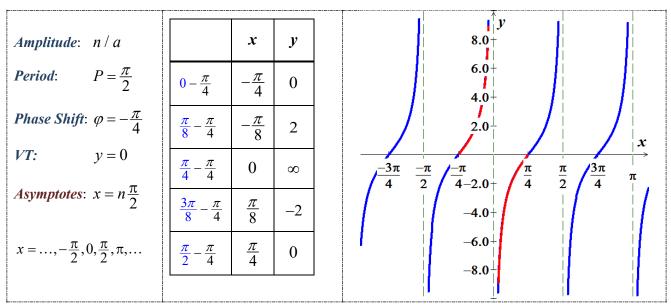
Find the period, show the asymptotes, and sketch the graph of $y = \tan\left(x - \frac{\pi}{4}\right)$

Solution



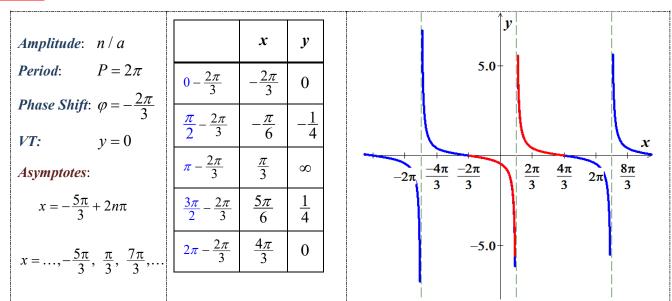
Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2 \tan \left(2x + \frac{\pi}{2}\right)$



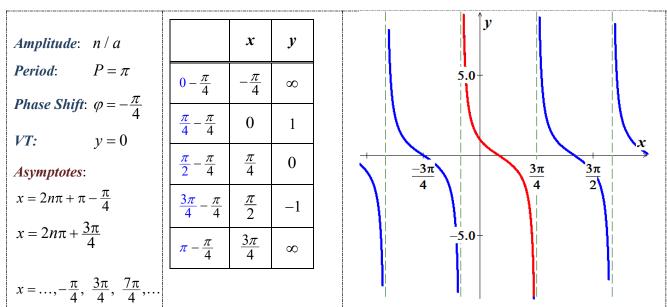
Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{4}\tan\left(\frac{1}{2}x + \frac{\pi}{3}\right)$

Solution



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \cot\left(x + \frac{\pi}{4}\right)$



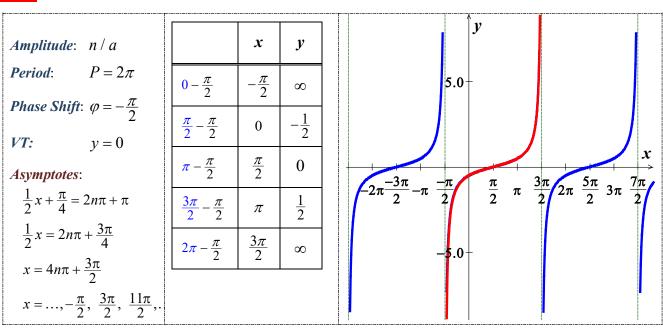
Find the period, show the asymptotes, and sketch the graph of $y = 2\cot\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a \boldsymbol{x} y **Period**: $P = \frac{\pi}{2}$ *Phase Shift*: $\varphi = -\frac{\pi}{4}$ 2 \boldsymbol{x} VT: y = 00 0 $\frac{3\pi}{4}$ Asymptotes: -2 $2x + \frac{\pi}{2} = (2n+1)\pi$ $2x = 2n\pi + \frac{\pi}{2}$ ∞ $x = \dots, -\frac{\pi}{4}, \ \frac{3\pi}{4}, \ \frac{7\pi}{4},$

Exercise

Find the period, show the asymptotes, and sketch the graph of $y = -\frac{1}{2}\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

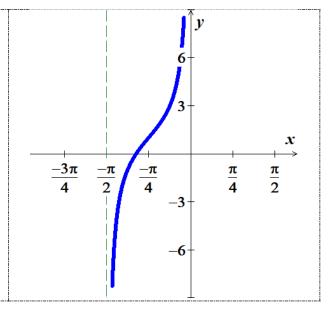


Graph over a 1-period interval $y = 1 - 2 \cot 2\left(x + \frac{\pi}{2}\right)$

Solution

Amplitude: n / aPeriod: $P = \frac{\pi}{2}$ Phase Shift: $\varphi = -\frac{\pi}{2}$ VT: y = 1Asymptotes: $-\frac{\pi}{2} + n\pi$

	x	у
$0-\frac{\pi}{2}$	$-\frac{\pi}{2}$	8
$\frac{\pi}{8} - \frac{\pi}{2}$	$-\frac{3\pi}{8}$	-1
$\frac{\pi}{4} - \frac{\pi}{2}$	$-\frac{\pi}{4}$	1
$\frac{3\pi}{8} - \frac{\pi}{2}$	$-\frac{\pi}{8}$	3
$\frac{\pi}{2} - \frac{\pi}{2}$	0	8

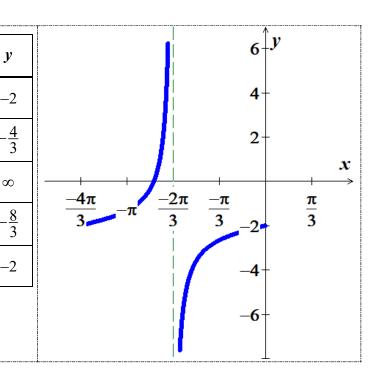


Exercise

Graph over a 1-period interval $y = \frac{2}{3} \tan \left(\frac{3}{4} x - \pi \right) - 2$

Solution

Amplitude: n / aPeriod: $P = \frac{4\pi}{3}$ Phase Shift: $\varphi = -\frac{4\pi}{3}$ VT: y = -2Asymptotes: $x = \frac{3\pi}{4} + n\pi$ $\frac{2\pi}{3} - \frac{4\pi}{3} - \frac{2\pi}{3}$ $\frac{2\pi}{3} - \frac{4\pi}{3} - \frac{2\pi}{3}$ $\frac{2\pi}{3} - \frac{4\pi}{3} - \frac{2\pi}{3}$ $\frac{4\pi}{3} - \frac{4\pi}{3} = 0$



Graph one complete cycle $y = 3 + 2 \tan \left(\frac{x}{2} + \frac{\pi}{8} \right)$

Solution

Amplitude: n/a **Period**: $P = 2\pi$

Phase Shift: $\varphi = -\frac{\pi}{4}$

VT: y = 3

	x	У
$0-\frac{\pi}{4}$	$-\frac{\pi}{4}$	3
$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	5
$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	8
$\frac{3\pi}{2} - \frac{\pi}{4}$	$\frac{5\pi}{4}$	1
$2\pi - \frac{\pi}{4}$	$\frac{7\pi}{4}$	3

10.0 [†]	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \frac{\pi}{4} \frac{\pi}{2} \frac{3\pi}{4} \pi \frac{5\pi}{4} \frac{3\pi}{2} \frac{7\pi}{4} $
4 2 4 -5.0+	4 2 4

Exercise

Graph one complete cycles $y = -2 - \cot\left(x - \frac{\pi}{4}\right)$

Solution

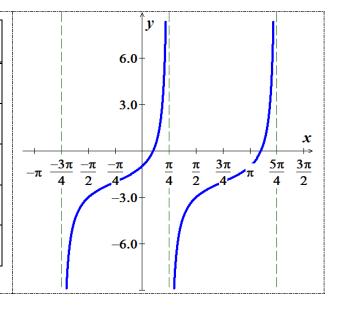
Amplitude: n/a

 $P = \pi$ Period:

Phase Shift: $\varphi = \frac{\pi}{4}$

y = -2VT:

	x	у
$0+\frac{\pi}{4}$	$\frac{\pi}{4}$	8
$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	-3
$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	-2
$\frac{3\pi}{4} + \frac{\pi}{4}$	π	-1
$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	8



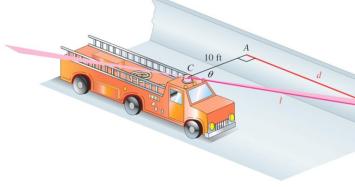
A fire truck parked on the shoulder of a freeway next to a long block wall. The red light on the top is 10 *feet* from the wall and rotates through one complete revolution every 2 *seconds*. Graph the function that gives the length d in terms of time t from t = 0 to t = 2.

Solution

$$\omega = \frac{\theta}{t} = \frac{2\pi}{2} = \pi \ rad \ / \ sec$$

$$\tan \theta = \frac{d}{10} \longrightarrow d = 10 \tan \theta$$

$$d(t) = 10 \tan \pi t$$



Period =
$$\frac{\pi}{\pi} = 1$$

One cycle: $0 \le \pi t \le \pi$ $0 \le t \le 1$

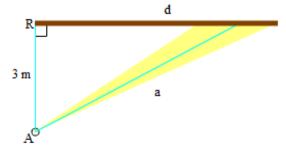
t	$d = 10 \tan \pi t$	↑ d 20+
0	0	15-
1/4	10	10-
$\frac{1}{2}$	∞	5 t
<u>3</u>	-10	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
1	0	-10-
L		-15-
		-20-

A rotating beacon is located 3 m south of point R on an east-west wall. d, the length of the light display along the wall from R, is given by $d = 3\tan 2\pi t$, where t is time measured in seconds since the beacon started rotating. (When t = 0, the beacon is aimed at point R. When the beacon is aimed to the right of R, the value of d is positive; d is negative if the beacon is aimed to the left of R.) Find a for t = 0.8

Solution

$$d = 3\tan(2\pi(0.8))$$

$$\approx -9.23 \ m$$



Exercise

Let a person whose eyes are h_1 feet from the ground stand d feet from an object h_1 feet tall, where $h_2 > h_1$ feet. Let θ be the angle of elevation to the top of the object.

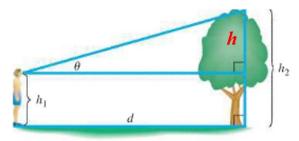
- a) Show that $d = (h_2 h_1)\cot\theta$
- b) Let $h_2 = 55$ and $h_1 = 5$. Graph **d** for the interval $0 < \theta \le \frac{\pi}{2}$

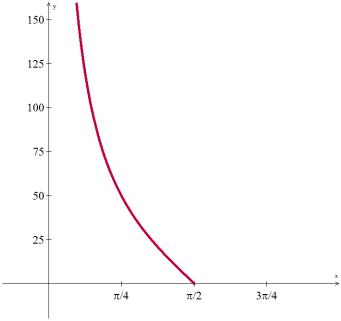
a)
$$h = h_2 - h_1$$

 $\cot \theta = \frac{d}{h}$
 $d = (h_2 - h_1)\cot \theta$

b)
$$d = (55-5)\cot\theta$$

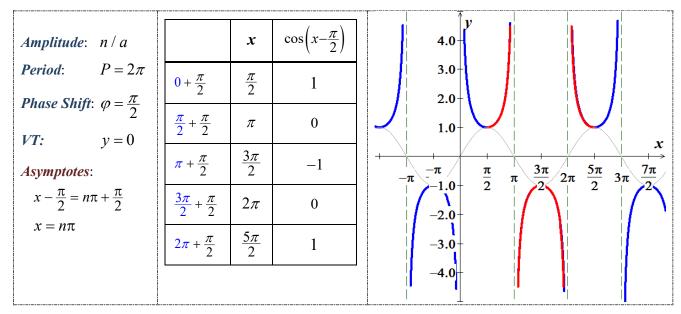
 $d = 50\cot\theta \quad 0 < \theta \le \frac{\pi}{2}$





Find the period, show the asymptotes, and sketch the graph of $y = \sec\left(x - \frac{\pi}{2}\right)$

Solution



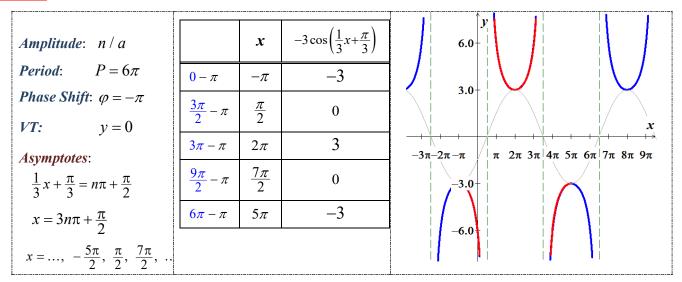
Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 2\sec\left(2x - \frac{\pi}{2}\right)$

Amplitude: n/a		x	$2\cos\left(2x-\frac{\pi}{2}\right)$	4.0+
Period: $P = \pi$	$0+\frac{\pi}{4}$	$\frac{\pi}{4}$	2	2.0+
Phase Shift: $\varphi = \frac{\pi}{4}$ VT: $y = 0$	$\frac{\pi}{4} + \frac{\pi}{4}$	$\frac{\pi}{2}$	0	2.0
Asymptotes:	$\frac{\pi}{2} + \frac{\pi}{4}$	$\frac{3\pi}{4}$	-2	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$2x - \frac{\pi}{2} = n\pi + \frac{\pi}{2}$	$\frac{3\pi}{4} + \frac{\pi}{4}$	2π	0	
$x = \left(\frac{n+1}{2}\right)\pi$	$\pi + \frac{\pi}{4}$	$\frac{5\pi}{4}$	2	-4.0+

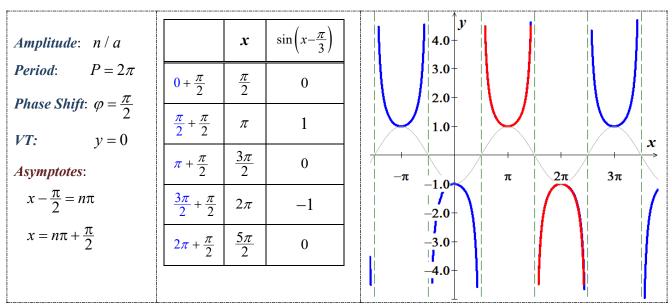
Find the period, show the asymptotes, and sketch the graph of $y = -3\sec\left(\frac{1}{3}x + \frac{\pi}{3}\right)$

Solution



Exercise

Find the period, show the asymptotes, and sketch the graph of $y = \csc\left(x - \frac{\pi}{2}\right)$



Find the period, show the asymptotes, and sketch the graph of $y = 2\csc\left(2x + \frac{\pi}{2}\right)$

Solution

Amplitude: n/a		x	$2\sin\left(2x+\frac{\pi}{2}\right)$	4.0+
Period: $P = \pi$	$0-\frac{\pi}{4}$	$-\frac{\pi}{4}$	0	
Phase Shift: $\varphi = -\frac{\pi}{4}$ VT: $y = 0$	$\frac{\pi}{4} - \frac{\pi}{4}$	0	2	
Asymptotes:	$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{\pi}{4}$	0	$-\pi$ $\frac{-\pi}{2}$ $\frac{\pi}{2}$ π $\frac{3\pi}{2}$
$x = -\frac{\pi}{4} + n\frac{\pi}{2}$	$\frac{3\pi}{4} - \frac{\pi}{4}$	$\frac{\pi}{2}$	-2	-2.0
	$\pi - \frac{\pi}{4}$	$\frac{3\pi}{4}$	0	-4.0
	1			

Exercise

Find the period, show the asymptotes, and sketch the graph of $y = 4\csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

Amplitude: n/a		x	$4\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$	12.0
Period: $P = 4\pi$	$0+\frac{\pi}{2}$	$\frac{\pi}{2}$	0	
Phase Shift: $\varphi = \frac{\pi}{2}$ VT: $y = 0$	$\pi + \frac{\pi}{2}$	$\frac{3\pi}{2}$	4	4.0
Asymptotes:	$2\pi + \frac{\pi}{2}$	$\frac{5\pi}{2}$	0	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$x = \frac{\pi}{2} + 2\pi n$	$3\pi + \frac{\pi}{2}$	$\frac{7\pi}{2}$	-4	
	$4\pi + \frac{\pi}{2}$	$\frac{9\pi}{2}$	0	
				-12.0+

Graph over a one-period interval $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$

Solution

Amplitude:	n/a
Period:	$P = 2\pi$
Phase Shift	$\varphi = \frac{3\pi}{4}$

VT: y=1

Asymptotes:

$$x = \frac{3\pi}{4} + 2\pi n$$

	x	$4\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$
$0+\frac{3\pi}{4}$	$\frac{3\pi}{4}$	1
$\frac{\pi}{2} + \frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{1}{2}$
$\pi + \frac{3\pi}{4}$	$\frac{7\pi}{4}$	1
$\frac{3\pi}{2} + \frac{3\pi}{4}$	$\frac{9\pi}{4}$	$-\frac{1}{2}$
$2\pi + \frac{3\pi}{4}$	$\frac{11\pi}{4}$	1

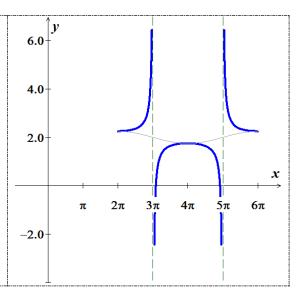
$\mathbf{\hat{y}}$			
4.0			
3.0		\ /	
2.0-			
1.0	< /		
			\mathbf{x}
	/ \	 	
3:	π 3π	9π	2_
$-1.0 - \frac{4}{1}$	$\frac{\pi}{2}$ $\frac{3\pi}{2}$	$\frac{9\pi}{4}$	3π
-2.0			
-3.0+			
1	l l		

Exercise

Graph over a one-period interval $y = 2 + \frac{1}{4}\sec(\frac{1}{2}x - \pi)$

		x
Amplitude: n/a Period: $P = 4\pi$	$0+2\pi$	2π
Phase Shift: $\varphi = 2\pi$	$\pi + 2\pi$	3π
VT: $y = 2x$	$2\pi + 2\pi$	4π
y - 2	$3\pi + 2\pi$	5π
	$4\pi + 2\pi$	6π

	x	$2 + \frac{1}{4}\cos\left(\frac{1}{2}x - \pi\right)$
$0+2\pi$	2π	9/4
$\pi + 2\pi$	3π	2
$2\pi + 2\pi$	4π	$\frac{7}{4}$
$3\pi + 2\pi$	5π	2
$4\pi + 2\pi$	6π	94



Graph
$$y = \frac{1}{3}\sec 2x$$
 for $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$

Solution

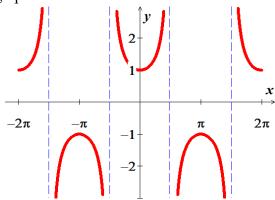
	x	$\frac{1}{3}\cos 2x$	3.0
Amplitude: n / a Period: $P = \pi$	0	$\frac{1}{3}$	2.0
Phase Shift: $\varphi = 0$	$\frac{\pi}{4}$	0	1.0
VT: $y=0$	$\frac{\pi}{2}$	$-\frac{1}{3}$	$\frac{-3\pi}{2}$ $-\pi$ $\frac{\pi}{2}$ π $\frac{3\pi}{2}$
	$\frac{3\pi}{4}$	0	
	π	$\frac{1}{3}$	-2.0+
			-3.0

Exercise

Graph one complete cycle $y = -1 - 3\csc\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$

				, , , , , , , , , , , , , , , , , , ,
		x	$-1-3\sin\left(\frac{\pi x}{2}+\frac{3\pi}{4}\right)$	6.0
Amplitude: n/a	0 3	3	1	4.0
Period : $P = 4$	$\frac{0-\frac{3}{2}}{}$	$-\overline{2}$	—I 	2.0
Phase Shift: $\varphi = -\frac{3}{2}$	$1 - \frac{3}{2}$	$-\frac{1}{2}$	-4	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
VT: $y = -1$	$2 - \frac{3}{2}$	$\frac{1}{2}$	-1	-2.0+
	$3 - \frac{3}{2}$	$\frac{3}{2}$	2	-6.0
	$4 - \frac{3}{2}$	<u>5</u>	-1	-8.0

Find an equation to match the graph



Solution

$$P = 2\pi$$

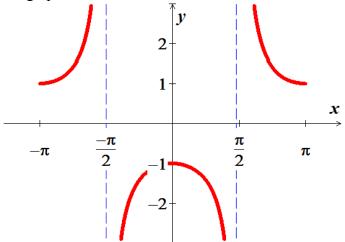
$$\phi = 0$$

$$A = \frac{1+1}{2} = 1$$

$$y = \sec x \quad -2\pi \le x \le 2\pi$$

Exercise

Find an equation to match the graph



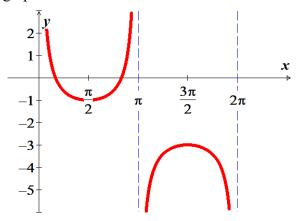
$$B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$$

$$\phi = 0 \rightarrow C = 0$$

$$A = \frac{1+1}{2} = 1$$

$$y = -\sec(x) - \pi \le x \le \pi$$

Find an equation to match the graph



Solution

$$B = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$$

$$\phi = 0 \rightarrow C = 0$$

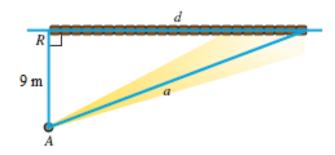
$$A = \frac{-3 - 1}{2} = -2$$

$$y = -2 + \csc(x) \quad -2\pi \le x \le 2\pi$$

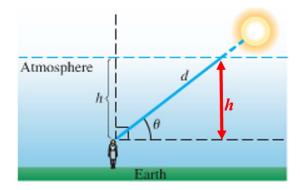
Exercise

A rotating beacon is located at point A next to a long wall. The beacon is 9 m from the wall. The distance a is given by $a = 9|\sec 2\pi t|$, where t is time measured in seconds since the beacon started rotating. (When t = 0, the beacon is aimed at point R.) Find a for t = 0.45

$$a = 9 \left| \sec(2\pi (0.45)) \right|$$
$$= \frac{9}{\left| \cos(2\pi (0.45)) \right|}$$
$$\approx 9.5 \ m$$



The shortest path for the sun's rays through Earth's atmosphere occurs when the sun is directly overhead. Disregarding the curvature of Earth, as the sun moves lower on the horizon, the distance that sunlight passes through the atmosphere increases by a factor of $\csc\theta$, where θ is the angle of elevation of the sun. This increased distance reduces both the intensity of the sun and the amount of ultraviolet light that reached Earth's surface.



- a) Verify that $d = h \csc \theta$
- b) Determine θ when d = 2h
- c) The atmosphere filters out the ultraviolet light that causes skin to burn, Compare the difference between sunbathing when $\theta = \frac{\pi}{2}$ and when $\theta = \frac{\pi}{3}$. Which measure gives less ultraviolet light?

Solution

a)
$$\sin \theta = \frac{h}{d}$$

$$= \frac{1}{\csc \theta}$$

$$d = h \csc \theta$$

(cross-multiplication)

b)
$$\sin \theta = \frac{h}{d}$$

$$= \frac{h}{2h}$$

$$= \frac{1}{2}$$

$$\theta = \sin^{-1} \frac{1}{2}$$

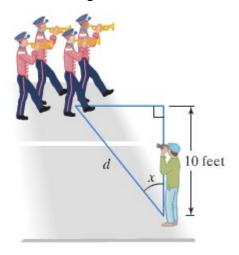
$$= \frac{\pi}{6}$$

c)
$$\begin{cases} \csc\frac{\pi}{2} = 1\\ \csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3} \approx 1.15 \end{cases}$$

When the distance to the sun is lager $\left(\theta = \frac{\pi}{3}\right)$, there is less ultraviolet light reaching the earth's surface. In this case, sunlight passes through 15% more atmosphere.

Exercise

Your friend is marching with a band and has asked you to film him. You have set yourself up 10 *feet* from the street where your friend will be passing from left to right. If *d* represents your distance, in feet, from your friend and *x* is the radian measure of the angle.



- a) Express d in terms of a trigonometric function of x.
- b) Graph the function for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

a)
$$\cos x = \frac{10}{d}$$

$$d = 10 \sec x$$

