

Find the following

$$\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \sin B = \frac{12}{13}, \cos B = \frac{5}{13}$$

$$\sin A = \frac{3}{5}, \cos A = \frac{4}{5}$$

$$\sin B = \frac{12}{13}, \cos B = \frac{5}{13}$$

$$\begin{aligned} \text{a) } \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{3}{5} \left(\frac{5}{13} \right) + \left(\frac{4}{5} \right) \left(\frac{12}{13} \right) \\ &= \frac{15}{65} + \frac{48}{65} \\ &= \frac{63}{65} \end{aligned}$$

$$\begin{aligned} \text{b) } \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \left(\frac{4}{5} \right) \left(\frac{5}{13} \right) - \left(\frac{3}{5} \right) \left(\frac{12}{13} \right) \\ &= \frac{20}{65} - \frac{36}{65} \\ &= -\frac{16}{65} \end{aligned}$$

$$\text{c) } \tan(A+B) = -\frac{16}{63}$$

$$\begin{aligned} \text{d) } \sin(A-B) &= \sin A \cos B - \cos A \sin B \\ &= \frac{15}{65} - \frac{48}{65} \\ &= -\frac{33}{65} \end{aligned}$$

$$\begin{aligned} \text{e) } \cos(A-B) &= \cos A \cos B + \sin A \sin B \\ &= \frac{20}{65} + \frac{36}{65} \\ &= \frac{56}{65} \end{aligned}$$

$$\text{f) } \tan(A-B) = \frac{33}{56}$$

8.3. $\sin A = -\frac{3}{5}$ $A \in \text{QIV}$

$$\cos A = -\frac{4}{5}$$

$$\frac{180^\circ}{2} < \frac{A}{2} < \frac{270^\circ}{2}$$

$$\frac{A}{2} \in \text{QIII}$$

$$\begin{aligned} a) \sin 2A &= 2 \sin A \cos A \\ &= 2 \left(-\frac{3}{5}\right) \left(-\frac{4}{5}\right) \\ &= \frac{24}{25} \end{aligned}$$

$$\begin{aligned} b) \cos 2A &= \cos^2 A - \sin^2 A \\ &= \frac{16}{25} - \frac{9}{25} \\ &= \frac{7}{25} \end{aligned}$$

$$c) \tan 2A = \frac{24}{7}$$

$$d) \sin \frac{A}{2} = \sqrt{\frac{1}{2}(1 - \cos A)} \quad \frac{A}{2} \in \text{QIV}$$

$$= \sqrt{\frac{1}{2} \left(1 + \frac{4}{5}\right)}$$

$$= \sqrt{\frac{9}{10}}$$

$$= \frac{3}{\sqrt{10}}$$

$$e) \cos \frac{A}{2} = -\sqrt{\frac{1}{2}(1 + \cos A)}$$

$$= -\sqrt{\frac{1}{2} \left(1 - \frac{4}{5}\right)}$$

$$= -\frac{1}{\sqrt{10}}$$

$$f) \tan \frac{A}{2} = -3$$

Prob #34

$$\cos 2\theta + 3\sin\theta - 2 = 0$$

$$1 - 2\sin^2\theta + 3\sin\theta - 2 = 0$$

$$-2\sin^2\theta + 3\sin\theta - 1 = 0$$

$$\sin\theta = 1$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

14 $\sin\theta \tan\theta = \sin\theta$

$$(\sin\theta \tan\theta - \sin\theta = 0$$

$$\sin\theta (\tan\theta - 1) = 0$$

$$\sin\theta = 0$$

$$\tan\theta = 1$$

$$\begin{matrix} \tan\theta = 1 \Rightarrow \\ \tan\theta = 1 \end{matrix}$$

$$\theta = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$$

15 $2\tan x \cos x + 2\cos x + \tan x + 1 = 0$

$$2\cos x (\tan x + 1) + (\tan x + 1) = 0$$

$$(\tan x + 1) (2\cos x + 1) = 0$$

$$\tan x = -1$$

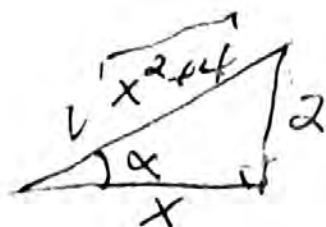
$$\cos x = -\frac{1}{2} = \frac{1}{\sin x} \Rightarrow \sin x$$

$$\sin x = -2 \quad \#$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

7.5 $\sin(\cos^{-1} \frac{x}{\sqrt{x^2+4}})$

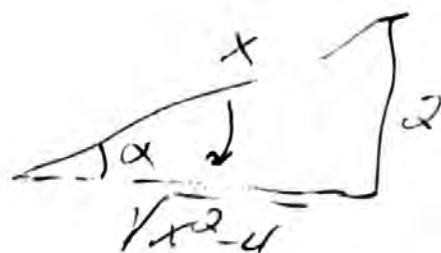
$$\cos \alpha = \frac{x}{\sqrt{x^2+4}}$$



$$\sin \alpha = \frac{2}{\sqrt{x^2+4}}$$

39 $\sec(\tan^{-1} \frac{2}{\sqrt{x^2-4}})$

$$\tan \alpha = \frac{2}{\sqrt{x^2-4}}$$



$$\sec \alpha = \frac{x}{\sqrt{x^2-4}}$$

8.6 $(4\sqrt{3}, -\frac{\pi}{6})$

$$\begin{aligned} x &= r \cos \theta \\ &= 4\sqrt{3} \cos(-\frac{\pi}{6}) \\ &= 4\sqrt{3} (\frac{\sqrt{3}}{2}) \\ &= 6 \end{aligned}$$

$$\begin{aligned} y &= r \sin \theta \\ &= 4\sqrt{3} \sin(-\frac{\pi}{6}) \\ &= -4\sqrt{3} (\frac{1}{2}) \\ &= -2\sqrt{3} \end{aligned}$$

$$(x, y) = (6, -2\sqrt{3})$$

16) $(-1, -\sqrt{3})$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1 + 3} \\ &= \underline{2} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{\sqrt{3}}{1} \right) \quad \frac{\sqrt{3}/2}{1/2} \\ &= \frac{\pi}{3} \\ &\in [0, 2\pi] \quad \theta = \underline{\frac{4\pi}{3}} \end{aligned}$$

$$\underline{(r, \theta) = (2, \frac{4\pi}{3})}$$

32) $r = 8 \sin \theta - 2 \cos \theta$

$$\frac{r}{r^2} = 8 \sin \theta - 2 \cos \theta$$

$$\underline{x^2 + y^2 = 8y - 2x}$$

43) $x + y = 4$

$$r \cos \theta + r \sin \theta = 4$$

$$r (\cos \theta + \sin \theta) = 4$$

$$\underline{r = \frac{4}{\cos \theta + \sin \theta}}$$

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$$\cos^4 x - \sin^4 x = \cos 2x$$

$$\begin{aligned}\cos^4 x - \sin^4 x &= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \\ &= \cos 2x \quad (1) \\ &= \cos 2x \quad \checkmark\end{aligned}$$

51 $\frac{1 - \cos^2 x}{1 + \cos x} = \frac{\sec x - 1}{\sec x}$

$$\begin{aligned}\frac{\sec x - 1}{\sec x} &= \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x}} \\ &= \frac{1 - \cos x}{\cos x} \cdot \frac{\cos x}{1} \\ &= 1 - \cos x \cdot \frac{1 + \cos x}{1 + \cos x} \\ &= \frac{1 - \cos^2 x}{1 + \cos x} \quad \checkmark\end{aligned}$$

63 $\csc x - \sin x = \cos x \cot x$

$$\begin{aligned}\csc x - \sin x &= \frac{1}{\sin x} - \sin x \\ &= \frac{1 - \sin^2 x}{\sin x} \\ &= \frac{\cos^2 x}{\sin x} \\ &= \cos x \cdot \frac{\cos x}{\sin x} \\ &= \cos x \cot x \quad \checkmark\end{aligned}$$

8.2. $\sec(A+B) = \frac{\cos(A-B)}{\cos^2 A - \sin^2 B}$

$$\begin{aligned}
 \sec(A+B) &= \frac{1}{\cos(A+B)} \cdot \frac{\cos(A-B)}{\cos(A-B)} \\
 &= \frac{\cos(A-B)}{(\cos A \cos B - \sin A \sin B)(\cos A \cos B + \sin A \sin B)} \\
 &= \frac{\cos(A-B)}{\cos^2 A \cos^2 B - \sin^2 A \sin^2 B} \\
 &= \frac{\cos(A-B)}{\cos^2 A (1 - \sin^2 B) - (1 - \cos^2 A) \sin^2 B} \\
 &= \frac{\cos(A-B)}{\cos^2 A - \cos^2 A \sin^2 B - \sin^2 B + \cos^2 A \sin^2 B} \\
 &= \frac{\cos(A-B)}{\cos^2 A - \sin^2 B} \quad \checkmark
 \end{aligned}$$

16. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\cot y + \cot x}{\cot y - \cot x}$

$$\begin{aligned}
 \frac{\sin(x+y)}{\sin(x-y)} &= \frac{\frac{\sin x \cos y}{\sin x \sin y} + \frac{\cos x \sin y}{\sin x \sin y}}{\frac{\sin x \cos y}{\sin x \sin y} - \frac{\cos x \sin y}{\sin x \sin y}} \\
 &= \frac{\cot y + \cot x}{\cot y - \cot x} \quad \checkmark
 \end{aligned}$$

$$11.17 \quad \frac{\sin(x-y)}{\sin x \cos y} = 1 - \cot x \tan y$$

$$\begin{aligned} \frac{\sin(x-y)}{\sin x \cos y} &= \frac{\sin x \cos y - \cos x \sin y}{\sin x \cos y} \\ &= \frac{\sin x \cos y}{\sin x \cos y} - \frac{\cos x \sin y}{\sin x \cos y} \\ &= 1 - \cot x \tan y \end{aligned}$$

$$5.3 \quad (27) \quad \frac{\cos 2x}{\cos^2 x} = \sec^2 x - 2 \tan^2 x$$

$$\begin{aligned} \frac{\cos 2x}{\cos^2 x} &= \frac{1 - 2 \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} - 2 \frac{\sin^2 x}{\cos^2 x} \\ &= \sec^2 x - 2 \tan^2 x \end{aligned}$$

$$4.2 \quad 2 \sin^2 \frac{x}{2} = \frac{\sin^2 x}{1 + \cos x}$$

$$\begin{aligned} 2 \sin^2 \frac{x}{2} &= 2 \frac{1 - \cos x}{2} \\ &= (1 - \cos x) \frac{1 + \cos x}{1 + \cos x} \\ &= \frac{1 - \cos^2 x}{1 + \cos x} \\ &= \frac{\sin^2 x}{1 + \cos x} \end{aligned}$$

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

$$\text{Ex. 10. } \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} = \frac{\sin^2 x}{4}$$

$$\begin{aligned} \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} &= \left(\sin \frac{x}{2} \cos \frac{x}{2} \right)^2 \\ &= \left(\frac{1}{2} \sin x \right)^2 \quad \left(\sin 2\left(\frac{x}{2}\right) \right) \\ &= \frac{1}{4} \sin^2 x \checkmark \end{aligned}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\begin{aligned} \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} &= \frac{1 - \cos x}{2} \cdot \frac{1 + \cos x}{2} \\ &= \frac{1}{4} (1 - \cos^2 x) \\ &= \frac{1}{4} \sin^2 x \checkmark \end{aligned}$$

Ex. 4

$$12/ \quad 1 - \sin x = \sqrt{3} \cos x \quad)^2 = ()^2$$

$$\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x = \frac{1}{2}$$

$$\sin \frac{\pi}{6} \sin x + \cos \frac{\pi}{6} \cos x = \cos \frac{\pi}{3} ; \quad \frac{5\pi}{3}$$

$$\cos \left(x - \frac{\pi}{6} \right) = \cos \frac{\pi}{3} \quad , \quad \frac{5\pi}{3}$$

$$x - \frac{\pi}{6} = \frac{\pi}{3}$$

$$x - \frac{\pi}{6} = \frac{5\pi}{3}$$

$$x = \frac{\pi}{3} + \frac{\pi}{6}$$

$$x = \frac{5\pi}{3} + \frac{\pi}{6}$$

$$= \frac{11\pi}{6}$$

$$= \frac{11\pi}{6}$$

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$$\tan x + \sqrt{3} = \sec x$$

$$\frac{\sin x}{\cos x} + \sqrt{3} = \frac{1}{\cos x}$$

$$\cos x \neq 0$$
$$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x + \sqrt{3} \cos x = 1$$

$$x = \frac{\pi}{2}, \frac{11\pi}{6}$$
