Exercise

Find the derivative of $y = -10x + 3\cos x$

Solution

$$y' = -10 - 3\sin x$$

Exercise

Find the derivative of $y = \csc x - 4\sqrt{x} + 7$

Solution

$$y' = -\csc x \cot x - 4\left(\frac{1}{2}x^{-1/2}\right)$$

$$= -\csc x \cot x - \frac{2}{\sqrt{x}}$$

Exercise

Find the derivative of $y = x^2 \cos x$

Solution

$$y = 2x\cos x + x^2(-\sin x) \qquad (uv)' = u'v + v'u$$

$$= 2x\cos x - x^2 \sin x$$

Exercise

Find the derivative of $y = \csc x \ \cot x$

Solution

$$y' = (-\csc x \cot x)\cot x + \csc x\left(-\csc^2 x\right) \qquad (uv)' = u'v + uv'$$

$$= -\csc x \cot^2 x - \csc^3 x$$

$$= -\csc x\left(\cot^2 x + \csc^2 x\right)$$

Exercise

Find the derivative of $y=(\sin x+\cos x)\sec x$

Solution

$u=\sin x+\cos x \qquad v=\sec x$

$u'=\cos x-\sin x \quad v'=\sec x\tan x$

$$y'=(\cos x-\sin x)\sec x+(\sin x+\cos x)(\sec x\tan x)$$

$$=\sec x\left[\cos x-\sin x+(\sin x+\cos x)\frac{\sin x}{\cos x}\right]$$

$$=\sec x\left[\cos x-\sin x+\frac{\sin^2 x}{\cos x}+\sin x\right]$$

$$=\sec x\left[\cos x+\frac{\sin^2 x}{\cos x}\right]$$

$$=\sec x\left[\frac{\cos^2 x+\sin^2 x}{\cos x}\right]$$

$$=\sec x\left(\frac{1}{\cos x}\right)$$

$$=\sec x\sec x$$

$$=\sec^2 x$$

$$y=(\sin x+\cos x)\frac{1}{\cos x}$$

$$=\tan x+1$$

$$y'=\sec^2 x$$

Exercise

Find the derivative of $y=(\sec x+\tan x)(\sec x-\tan x)$

Solution

$$y=(\sec x+\tan x)(\sec x-\tan x)$$

$$=\sec^2 x-\tan^2 x$$

$$=1+\tan^2 x-\tan^2 x$$

$$=1$$

$$y'=0$$

$$y'=(\sec x+\tan x)'(\sec x-\tan x)+(\sec x+\tan x)(\sec x-\tan x)'$$

$$=(\sec x\tan x+\sec^2 x)(\sec x-\tan x)$$

$$+(\sec x+\tan x)(\sec x\tan x-\sec^2 x)$$

$$=\sec^2 x\tan x-\sec x\tan^2 x+\sec^3 x-\sec^2 x\tan x$$

$$+\sec^2 x\tan x-\sec^3 x+\sec x\tan^2 x-\sec^2 x\tan x$$

$$=0$$

Exercise

Find the derivative of $y = \frac{\cos x}{x} + \frac{x}{\cos x}$

Solution

$$y = \frac{\cos^2 x + x^2}{x\cos x}$$

$$u = \cos^2 x + x^2 \qquad v = x\cos x$$

$$u' = 2\cos x(-\sin x) + 2x \qquad v' = \cos x - x\sin x$$

$$y' = \frac{(-2\cos x\sin x + 2x)x\cos x - (\cos x - x\sin x)(\cos^2 x + x^2)}{(x\cos x)^2} \qquad \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$= \frac{-2x\sin x\cos^2 x + 2x^2\cos x - \cos^3 x - x^2\cos x + x\sin x\cos^2 x + x^3\sin x}{(x\cos x)^2}$$

$$= \underline{\frac{-x\sin x\cos^2 x - x^2\cos x - \cos^3 x + x^3\sin x}{(x\cos x)^2}}$$

Exercise

Find the derivative of $y = x^2\cos x - 2x\sin x - 2\cos x$

Solution

$$y' = 2x\cos x - x^2\sin x - 2(\sin x + x\cos x) - 2(-\sin x)$$

$$= 2x\cos x - x^2\sin x - 2\sin x - 2x\cos x + 2\sin x$$

$$= \underline{-x^2\sin x}$$

Exercise

Find the derivative of $y = (2 - x)\tan^2 x$

Solution

$$y' = -\tan^2 x + (2 - x)(2\tan x\sec^2 x)$$

$$= \tan x(-\tan x + 2(2 - x)\sec^2 x)$$

$$= \underline{\tan x(2(2 - x)\sec^2 x - \tan x)}$$

Exercise

Find the derivative of $y = t^2 - \sec t + 1$

Solution

$$y' = 2t - \sec t \tan t$$

Exercise

Find the derivative of $y = \dfrac{1+\csc t}{1-\csc t}$

Solution

$$y' = \frac{(-\csc x \cot x)(1-\csc t) - (1+\csc t)(\csc x \cot x)}{(1-\csc t)^2} \qquad u = 1+\csc t \quad v = 1-\csc t$$
$$u' = -\csc x \cot x \quad v' = \csc x \cot x$$
$$= \frac{-\csc x \cot x + \csc^2 x \cot x - \csc x \cot x - \csc^2 x \cot x}{(1-\csc t)^2}$$
$$= -\frac{2\csc x \cot x}{(1-\csc t)^2}$$

Exercise

Find the derivative of $r = \theta \sin\theta + \cos\theta$

Solution

$$r' = \sin\theta + \theta\cos\theta - \sin\theta$$
$$= \theta\cos\theta$$

Exercise

Find the derivative of $p = \dfrac{\sin q + \cos q}{\cos q}$

Solution

$$p' = \frac{(\cos q - \sin q)\cos q - (-\sin q)(\sin q + \cos q)}{\cos^2 q} \qquad u = \sin q + \cos q \quad v = \cos q$$
$$u' = \cos q - \sin q \quad v' = -\sin q$$
$$= \frac{\cos^2 q - \sin q \cos q + \sin^2 q + \sin q \cos q}{\cos^2 q}$$
$$= \frac{\cos^2 q + \sin^2 q}{\cos^2 q}$$
$$= \frac{1}{\cos^2 q}$$
$$= \sec^2 q$$

Exercise

Find the derivative of $p = \dfrac{3q + \tan q}{q \sec q}$

Solution

$$u = 3q + \tan q \qquad v = q \sec q$$

$$u' = 3 + \sec^2 q \qquad v' = \sec q + q \sec q \tan q$$

$$p' = \frac{\left(3 + \sec^2 q\right)\left(q \sec q\right) - \left(3q + \tan q\right)\left(\sec q + q \sec q \tan q\right)}{\left(q \sec q\right)^2} \qquad \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$= \frac{3q \sec q + q \sec^3 q - 3q \sec q - 3q^2 \sec q \tan q - \tan q \sec q - q \sec q \tan^2 q}{q^2 \sec^2 q}$$

$$= \underline{\left| \frac{q \sec^3 q - 3q^2 \sec q \tan q - \tan q \sec q - q \sec q \tan^2 q}{q^2 \sec^2 q} \right|}$$

Exercise

Find the derivative of $f(x) = \dfrac{\sin x + 2x}{x}$

Solution

$$f'(x) = \frac{x \cos x + 2x - \sin x - 2x}{x^2}$$

$$= \underline{\left| \frac{x \cos x - \sin x}{x^2} \right|}$$

Exercise

Find the derivative of $f(x) = \dfrac{\sin x}{x^2}$

Solution

$$f'(x) = \frac{x^2 \cos x - 2x \sin x}{x^4}$$

$$= \underline{\left| \frac{x \cos x - 2 \sin x}{x^3} \right|}$$

Exercise

Find the derivative of $f(x) = x^3 \cos x$

Solution

$$\underline{\left| f'(x) = 3x^2 \cos x - x^3 \sin x \right|}$$

Exercise

Find the derivative of $f(x) = \frac{1}{x} - 12\sec x$

Solution

$$f'(x) = -\frac{1}{x^2} - 12\sec x \tan x$$

Exercise

Find the derivative of $f(\theta) = 5\theta\sec\theta + \theta\tan\theta$

Solution

$$f'(\theta) = 5\sec\theta + 5\theta\sec\theta\tan\theta + \tan\theta + \theta\sec^2\theta$$

Exercise

Find the derivative of $y = \sec\pi x$

Solution

$$y' = \pi\sec\pi x\tan\pi x$$

Exercise

Find the derivative of $y = \cos 5x$

Solution

$$y' = -5\sin 5x$$

Exercise

Find the derivative of $y = \cos(4 - 3x)$

Solution

$$y' = 3\sin(4 - 3x)$$

Exercise

Find the derivative of $f(x) = \sin(4 - 3x)$

Solution

$$f'(x) = -3\cos(4 - 3x)$$

Exercise

Find the derivative of $f(\theta)=\dfrac{\sin a\theta}{\cos b\theta}$

Solution

$$f'(\theta)=\frac{a\cos a\theta\cos b\theta+b\sin a\theta\sin b\theta}{\cos^2 b\theta}$$

$u=\sin a\theta \qquad v=\cos b\theta$

$u'=a\cos a\theta \qquad v'=-b\sin b\theta$

Exercise

Find the derivative of $f(\theta)=\sin 2\theta-\cos 2\theta$

Solution

$$f'(\theta)=2\cos 2\theta+2\sin 2\theta$$

Exercise

Find the derivative of $f(\theta)=\tan\theta-\cot\theta$

Solution

$$f'(\theta)=\sec^2\theta+\csc^2\theta$$

Exercise

Find the derivative of $\dfrac{d}{dx}\left(5x^2\sin x\right)$

Solution

$$\frac{d}{dx}\left(5x^2\sin x\right)=10x\sin x+5x^2\cos x$$

Exercise

Find the derivative of $\dfrac{d}{dx}\left(2x(\sin x)\sqrt{3x-1}\right)$

Solution

$$\frac{d}{dx}\left(2x(\sin x)\sqrt{3x-1}\right)=\underbrace{2(\sin x)\sqrt{3x-1}}_{u'}+2x\underbrace{(\cos x)}_{v'}\sqrt{3x-1}+2x(\sin x)\underbrace{\frac{1}{2}(3)(3x-1)^{-1/2}}_{w'}$$

$$=2(\sin x)\sqrt{3x-1}+2x(\cos x)\sqrt{3x-1}+\frac{3x\sin x}{\sqrt{3x-1}}$$

Exercise

Find $y^{(4)}$ *if* $y = 9\cos x$

Solution

$$\boxed{y' = -9\sin x} \qquad \boxed{y'' = -9\cos x} \qquad \boxed{y''' = 9\sin x} \qquad \boxed{y^{(4)} = 9\cos x}$$

Exercise

Find $\dfrac{d^{999}}{dx^{999}}(\cos x)$

Solution

$$\boxed{y' = -\sin x} \qquad \boxed{y'' = -\cos x} \qquad \boxed{y''' = \sin x} \qquad \boxed{y^{(4)} = \cos x}$$

$$999 = 249 \times 4 + 3$$

$$\Rightarrow \quad \frac{d^{999}}{dx^{999}}(\cos x) = \frac{d^3}{dx^3}(\cos x) \ \boxed{= \sin x}$$

Exercise

Find y', y'', y''' $\qquad y = \sin\sqrt{x}$

Solution

$$\boxed{y' = \frac{1}{2\sqrt{x}}\cos\sqrt{x}}$$

$$\boxed{y'' = -\frac{1}{4x^{3/2}}\cos\sqrt{x} - \frac{1}{4x}\sin\sqrt{x}}$$

$$y''' = \frac{3}{8x^{5/2}}\cos\sqrt{x} + \frac{1}{8x^2}\sin\sqrt{x} + \frac{1}{4x^2}\sin\sqrt{x} - \frac{1}{8x^{3/2}}\cos\sqrt{x}$$

$$\boxed{= \frac{3}{8x^2}\sin\sqrt{x} + \frac{3-x}{8x^{5/2}}\cos\sqrt{x}}$$

Exercise

Find $\lim\limits_{x \to -\frac{\pi}{6}} \sqrt{1 + \cos(\pi \csc x)}$

Solution

$$\lim_{x \to -\frac{\pi}{6}} \sqrt{1 + \cos(\pi \csc x)} = \sqrt{1 + \cos\left(\pi \csc\left(-\frac{\pi}{6}\right)\right)}$$

$$= \sqrt{1+\cos\left(\pi(-2)\right)}$$

$$= \sqrt{1+\cos(-2\pi)}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

Exercise

Assume that a particle's position on the *x*-axis is given by $x = 3\cos t + 4\sin t; \quad ft$

a) Find the particle's position when $t = 0, \quad t = \frac{\pi}{2}, \quad and \quad t = \pi$

b) Find the particle's velocity when $t = 0, \quad t = \frac{\pi}{2}, \quad and \quad t = \pi$

Solution

a) $t = 0 \quad \Rightarrow \quad x = 3\cos 0 + 4\sin 0 = 3\ ft$

$t = \frac{\pi}{2} \quad \Rightarrow \quad x = 3\cos\frac{\pi}{2} + 4\sin\frac{\pi}{2} = 0 + 4 = 4\ ft$

$t = \pi \quad \Rightarrow \quad x = 3\cos\pi + 4\sin\pi = 3(-1) + 0 = -3\ ft$

b) $v = x' = -3\sin t + 4\cos t$

$t = 0 \quad \Rightarrow \quad x = -3\sin 0 + 4\cos 0 = 4\ ft/\sec$

$t = \frac{\pi}{2} \quad \Rightarrow \quad x = -3\sin\frac{\pi}{2} + 4\cos\frac{\pi}{2} = -3 + 0 = -3\ ft/\sec$

$t = \pi \quad \Rightarrow \quad x = -3\sin\pi + 4\cos\pi = 0 - 4 = -4\ ft/\sec$

Exercise

A weight is attached to a spring and reaches its equilibrium position $(x = 0)$. It is then set in motion resulting in a displacement of $x = 10\cos t$

Where *x* is measured in centimeters and *t* is measured in seconds.

a) Find the spring's displacement when $t = 0, \quad t = \frac{\pi}{3}, \quad and \quad t = \frac{3\pi}{4}$

b) Find the spring's velocity when $t = 0, \quad t = \frac{\pi}{3}, \quad and \quad t = \frac{3\pi}{4}$

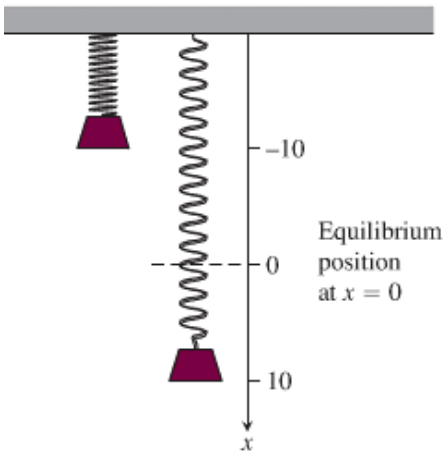
Solution

a) $t = 0 \quad \Rightarrow \quad x = 10\cos 0 = 10\ cm$

$t = \frac{\pi}{3} \quad \Rightarrow \quad x = 10\cos\frac{\pi}{3} = 10\left(\frac{1}{2}\right) = 5\ cm$

$$t=\frac{3\pi}{4} \quad \Rightarrow \quad x=10\cos\frac{3\pi}{4}=10\frac{\sqrt{2}}{2}$$

$$=5\sqrt{2}\ cm$$



b) $v=x'=-10\sin t$

$$t=0 \quad \Rightarrow \quad x=-10\sin 0 \ =0\ cm/\sec$$

$$t=\frac{\pi}{3} \quad \Rightarrow \quad x=-10\sin\frac{\pi}{3}=10\left(\frac{\sqrt{3}}{2}\right) \ =5\sqrt{3}\ cm/\sec$$

$$t=\frac{3\pi}{4} \quad \Rightarrow \quad x=-10\sin\frac{3\pi}{4}=-10\frac{\sqrt{2}}{2} \ =-5\sqrt{2}\ cm/\sec$$