Solution Section R.1 – Derivative

Exercise

Find the derivative of $f(t) = -3t^2 + 2t - 4$

Solution

$$f'(t) = \boxed{-6t + 2}$$

Exercise

Find the derivative of $g(x) = 4\sqrt[3]{x} + 2$

Solution

$$g(x) = 4x^{1/3} + 2$$

$$g'(x) = \frac{4}{3}x^{-2/3}$$

$$= \frac{4}{3x^{2/3}}$$

$$= \boxed{\frac{4}{3\sqrt[3]{x^2}}}$$

Exercise

Find the derivative of $f(x) = x\left(x^2 + 1\right)$

Solution

$$f(x) = x^3 + x$$

$$f'(x) = \boxed{3x^2 + 1}$$

Exercise

Find the derivative of $f(x) = \frac{2x^2 - 3x + 1}{x}$

Solution

$$f(x) = \frac{2x^2}{x} - \frac{3x}{x} + \frac{1}{x}$$

$$= 2x - 3 + \frac{1}{x}$$

$$f'(x) = \boxed{2 - \frac{1}{x^2}} \qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

Exercise

Find the derivative of $f(x)=\dfrac{4x^3-3x^2+2x+5}{x^2}$

Solution

$$f(x)=4x-3+\frac{2}{x}+5x^{-2} \qquad\qquad \left(\frac{1}{x}\right)'=-\frac{1}{x^2}$$

$$f'(x)=4-\frac{2}{x^2}-10x^{-3}$$

$$=4-\frac{2}{x^2}-\frac{10}{x^3}$$

Exercise

Find the derivative of $f(x)=\dfrac{-6x^3+3x^2-2x+1}{x}$

Solution

$$f(x)=-6x^2+3x-2+\frac{1}{x} \qquad\qquad \left(\frac{1}{x}\right)'=-\frac{1}{x^2}$$

$$f'(x)=-12x+3-\frac{1}{x^2}$$

Exercise

Find the derivative of $f(x)=x\left(1-\dfrac{2}{x+1}\right)$

Solution

$$f(x)=x-\frac{2x}{x+1}$$

$$\left(\frac{2x}{x+1}\right)' \Rightarrow \qquad \begin{array}{ll} f=2x & f'=2 \\ g=x+1 & g'=1 \end{array}$$

$$f'(x)=1-\frac{2(x+1)-2x}{(x+1)^2}$$

$$=1-\frac{2x+2-2x}{(x+1)^2}$$

$$=1-\frac{2}{(x+1)^2}$$

Exercise

Find the derivative of $g(s)=\dfrac{s^2-2s+5}{\sqrt{s}}$

Solution

$$g(s)=\frac{s^2}{s^{1/2}}-2\frac{s}{s^{1/2}}+\frac{5}{s^{1/2}}$$

$$=s^{3/2}-2s^{1/2}+5s^{-1/2}$$

$$g'(s)=\tfrac{3}{2}s^{1/2}-2\tfrac{1}{2}s^{-1/2}+5\left(-\tfrac{1}{2}\right)s^{-3/2}$$

$$=\tfrac{3}{2}s^{1/2}-s^{-1/2}-\tfrac{5}{2}s^{-3/2}$$

$$=\frac{3}{2}\sqrt{s}-\frac{1}{\sqrt{s}}-\frac{5}{2s^{3/2}}$$

$$=\frac{3}{2}\sqrt{s}-\frac{1}{\sqrt{s}}-\frac{5}{2s\sqrt{s}}$$

Exercise

Find the derivative of $f(x)=\dfrac{x+1}{\sqrt{x}}$

Solution

$$f(x)=\frac{x}{x^{1/2}}+\frac{1}{x^{1/2}}$$

$$=x^{1/2}+x^{-1/2}$$

$$f'(x)=\tfrac{1}{2}x^{-1/2}-\tfrac{1}{2}x^{-3/2}$$

$$=\frac{1}{2x^{1/2}}-\frac{1}{2x^{3/2}}$$

Exercise

Find the derivative to the following functions $y=3x\left(2x^2+5x\right)$

Solution

$$y=6x^3+15x^2$$

$$\Rightarrow y'=18x^2+30x$$

Exercise

Find the derivative to the following functions $y = 3\left(2x^2 + 5x\right)$

Solution

$$y = 6x^2 + 15x$$

$$\Rightarrow \boxed{y' = 12x + 15}$$

Exercise

Find the derivative to the following functions $y = \dfrac{x^2 + 4x}{5}$

Solution

$$y = \frac{1}{5}\left[x^2 + 4x\right]$$

$$\boxed{y' = \frac{1}{5}(2x + 4)}$$

Exercise

Find the derivative to the following functions $y = \dfrac{3x^4}{5}$

Solution

$$y = \frac{3}{5}x^4$$

$$\boxed{y' = \frac{12}{5}x^3}$$

Exercise

Find the derivative to the following functions $y = \dfrac{x^2 - 4}{2x + 5}$

Solution

$$y' = \frac{(2x+5)(2x) - (x^2 - 4)(2)}{(2x+5)^2}$$

$$= \frac{4x^2 + 10x - 2x^2 + 8}{(2x+5)^2}$$

$$\boxed{= \frac{2x^2 + 10x + 8}{(2x+5)^2}}$$

Exercise

Find the derivative to the following functions $y = \dfrac{(1+x)(2x-1)}{x-1}$

Solution

$$y = \frac{(1+x)(2x-1)}{x-1}$$

$$= \frac{2x-1+2x^2-x}{x-1}$$

$$= \frac{2x^2+x-1}{x-1}$$

$$y' = \frac{(x-1)(4x+1)-(2x^2+x-1)(1)}{(x-1)^2}$$

$$= \frac{4x^2+x-4x-1-2x^2-x+1}{(x-1)^2}$$

$$= \frac{2x^2-4x}{(x-1)^2}$$

Exercise

Find the derivative to the following functions $y = \dfrac{4}{2x+1}$

Solution

$$y = 4(2x+1)^{-1}$$

$$y' = -4(2x+1)^{-2}(2)$$

$$= -8(2x+1)^{-2}$$

$$= -\frac{8}{(2x+1)^2}$$

Exercise

Find the derivative to the following functions $y = \dfrac{2}{(x-1)^3} = 2(x-1)^{-3}$

Solution

$$y = 2(x-1)^{-3}$$

$$y' = -\frac{6}{(x-1)^4}$$

Exercise

Find the derivative to the following functions $y=\sqrt[3]{(x+4)^2}$

Solution

$$y=(x+4)^{2/3}$$

$$y'=\frac{2}{3}(x+4)^{-1/3}$$

$$=\frac{2}{3}\frac{1}{(x+4)^{1/3}}$$

$$=\frac{2}{3\sqrt[3]{x+4}}$$

Exercise

Find the derivative of $f(x)=\sqrt{2t^2+5t+2}$

Solution

$$f(t)=\left(2t^2+5t+2\right)^{1/2}$$

$$U=2t^2+5t+2 \;\rightarrow\; U'=4t+5$$

$$f'(t)=\frac{1}{2}(4t+5)\left(2t^2+5t+2\right)^{-1/2}$$

$$\left(U^n\right)'=nU'U^{n-1}$$

$$=\frac{1}{2}\frac{4t+5}{\sqrt{2t^2+5t+2}}$$

Exercise

Find the derivative of $f(x)=\dfrac{1}{\left(x^2-3x\right)^2}$

Solution

$$f(x)=\left(x^2-3x\right)^{-2}$$

$$f'(x)=-2(2x-3)\left(x^2-3x\right)^{-3}$$

$$=-\frac{2(2x-3)}{\left(x^2-3x\right)^3}$$

Exercise

Find the derivative of $y = t^2\sqrt{t-2}$

Solution

$$y' = 2t\sqrt{t-2} + t^2\frac{1}{2}(t-2)^{-1/2}$$

$f = t^2 \qquad f' = 2t$

$g = (t-2)^{1/2} \qquad g' = \frac{1}{2}(t-2)^{-1/2}$

$$= \left[2t(t-2)^{1/2} + t^2\frac{1}{2}(t-2)^{-1/2}\right]\frac{2(t-2)^{1/2}}{2(t-2)^{1/2}}$$

$$= \frac{4t(t-2)+t^2}{2(t-2)^{1/2}}$$

$$= \frac{4t^2-8t+t^2}{2\sqrt{t-2}}$$

$$= \frac{5t^2-4t}{2\sqrt{t-2}}$$

Exercise

Find the derivative of $y = \left(\frac{6-5x}{x^2-1}\right)^2$

Solution

$f = 6-5x \qquad g = x^2-1$

$f' = -5 \qquad g' = 2x$

$$y' = 2\frac{-5\left(x^2-1\right)-2x(6-5x)}{\left(x^2-1\right)^2}\left(\frac{6-5x}{x^2-1}\right)$$

$\left(U^n\right)' = nU'U^{n-1}$

$$= 2\frac{-5x^2+5-12x+10x^2}{\left(x^2-1\right)^3}(6-5x)$$

$$= \frac{2\left(5x^2-12x+5\right)(6-5x)}{\left(x^2-1\right)^3}$$

Exercise

Find the derivative to the following functions $y = x^2\sqrt{x^2+1}$

Solution

$$y = x^2\left(x^2+1\right)^{1/2}$$

$$y' = x^2\left[\frac{1}{2}(x^2+1)^{-1/2}(2x)\right] + (x^2+1)^{1/2}[2x]$$

$$= x^3(x^2+1)^{-1/2} + 2x(x^2+1)^{1/2}$$

$$= \frac{(x^2+1)^{1/2}}{(x^2+1)^{1/2}}\left[x^3(x^2+1)^{-1/2} + 2x(x^2+1)^{1/2}\right]$$

$$= \frac{x^3(x^2+1)^{-1/2}(x^2+1)^{1/2} + 2x(x^2+1)^{1/2}(x^2+1)^{1/2}}{(x^2+1)^{1/2}}$$

$$= \frac{x^3+2x(x^2+1)}{(x^2+1)^{1/2}}$$

$$= \frac{x^3+2x^3+2x}{\sqrt{x^2+1}}$$

$$= \frac{3x^3+2x}{\sqrt{x^2+1}}$$

$$= \frac{x(3x^2+2)}{\sqrt{x^2+1}}$$

Exercise

Find the derivative to the following functions $y = \left(\frac{x+1}{x-5}\right)^2$

Solution

$$y' = 2\left(\frac{x+1}{x-5}\right)\frac{d}{dx}\left[\frac{x+1}{x-5}\right]$$

$$= 2\left(\frac{x+1}{x-5}\right)\left[\frac{(1)(x-5)-(1)(x+1)}{(x-5)^2}\right]$$

$$= 2\left(\frac{x+1}{x-5}\right)\left(\frac{x-5-x-1}{(x-5)^2}\right)$$

$$= 2\left(\frac{x+1}{x-5}\right)\left(\frac{-6}{(x-5)^2}\right)$$

$$= -\frac{12(x+1)}{(x-5)^3}$$

Exercise

Find the derivative to the following functions $y = x^2 \sin x$

Solution

$$y' = \boxed{2x\sin x + x^2 \cos x}$$

$u = x^2 \quad v = \sin x$

$u' = 2x \quad v' = \cos x$

Exercise

Find the derivative to the following functions $y = \dfrac{\sin x}{x}$

Solution

$$y' = \boxed{\frac{x\cos x - \sin x}{x^2}}$$

$u = \sin x \quad v = x$

$u' = \cos x \quad v' = 1$

Exercise

Find the derivative to the following functions $y = \dfrac{\cot x}{1 + \cot x}$

Solution

$$\begin{aligned} y' &= \frac{-\csc^2 x(1+\cot x) + \csc^2 x \cot x}{(1+\cot x)^2} \\ &= \frac{-\csc^2 x - \csc^2 x \cot x + \csc^2 x \cot x}{(1+\cot x)^2} \\ &= \boxed{\frac{-\csc^2 x}{(1+\cot x)^2}} \end{aligned}$$

$u = \cot x \quad v = 1 + \cot x$

$u' = -\csc^2 x \quad v' = -\csc^2 x$

Exercise

Find the derivative to the following functions $y = x^2 \sin x + 2x\cos x - 2\sin x$

Solution

$$\begin{aligned} y' &= 2x\sin x + x^2 \cos x + 2\cos x - 2x\sin x - 2\cos x \\ &= \boxed{x^2 \cos x} \end{aligned}$$

Exercise

Find the derivative to the following functions $y = x^3 \sin x \cos x$

Solution

$$y' = \left(x^3\right)' \sin x \cos x + x^3 (\sin x)' \cos x + x^3 \sin x (\cos x)'$$

$$= 3x^2 \sin x \cos x + x^3 \cos^2 x - x^3 \sin^2 x$$

Exercise

Find the derivative to the following functions $y = \dfrac{4}{\cos x} + \dfrac{1}{\tan x}$

Solution

$$y' = \frac{-4\sin x}{\cos^2 x} - \frac{\sec^2 x}{\tan^2 x} \qquad \left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$= -4\frac{\sin x}{\cos x}\frac{1}{\cos x} - \frac{1}{\cos^2 x}\frac{\cos^2 x}{\sin^2 x}$$

$$= -4\tan x \sec x - \csc^2 x$$

Exercise

Find the derivative of $f(x) = \dfrac{\left(x^2 - 6x\right)^5}{\left(3x^2 + 5x - 2\right)^4}$

Solution

$$f(x) = \left(x^2 - 6x\right)^5 \left(3x^2 + 5x - 2\right)^{-4} \qquad \left(U^m V^n\right)' = U^{m-1} V^{n-1} \left(mU'V + nUV'\right)$$

$$f'(x) = \left(x^2 - 6x\right)^4 \left(3x^2 + 5x - 2\right)^{-5} \left[5(2x-6)\left(3x^2 + 5x - 2\right) - 4\left(x^2 - 6x\right)(6x+5)\right]$$

$$= \left(x^2 - 6x\right)^4 \left(3x^2 + 5x - 2\right)^{-5} \left[(10x - 30)\left(3x^2 + 5x - 2\right) - 4\left(6x^3 - 31x^2 - 30x\right)\right]$$

$$= \left(x^2 - 6x\right)^4 \left(3x^2 + 5x - 2\right)^{-5}$$

$$\left[\begin{array}{ll} x^3 & 30 - 24 \\ x^2 & 50 - 90 + 124 \\ x & -20 - 150 + 120 \\ x^0 & 60 \end{array}\right]$$

$$= \frac{\left(x^2 - 6x\right)^4 \left(6x^3 + 84x^2 - 50x + 60\right)}{\left(3x^2 + 5x - 2\right)^5}$$

Exercise

Find the derivative of $y = \ln\sqrt{x+5}$

Solution

$$y = \ln\left(x+5\right)^{1/2} = \tfrac{1}{2}\ln\left(x+5\right)$$

$$\boxed{y' = \frac{1}{2\left(x+5\right)}}$$

Exercise

Find the Derivatives of $y = \left(3x+7\right)\ln\left(2x-1\right)$

Solution

$$f = 3x+7 \qquad f' = 3$$

$$g = \ln\left(2x-1\right) \qquad g' = \frac{2}{2x-1}$$

$$\boxed{y' = 3x\ln\left(2x-1\right) + \frac{2\left(3x+7\right)}{2x-1}}$$

Exercise

Find the Derivatives of $f(x) = \ln\sqrt[3]{x+1}$

Solution

$$f(x) = \ln(x+1)^{1/3}$$

$$= \tfrac{1}{3}\ln(x+1)$$

$$u = x+1 \Rightarrow \frac{du}{dx} = 1$$

$$f'(x) = \frac{1}{3}\frac{1}{x+1}$$

$$\boxed{= \frac{1}{3(x+1)}}$$

Exercise

Find the Derivatives of $f(x) = \ln\left[x^2\sqrt{x^2+1}\right]$

Solution

$$f(x) = \ln(x^2) + \ln\sqrt{x^2+1}$$ ***Product Property***

$$= \ln(x^2) + \ln(x^2+1)^{1/2}$$

$$= 2\ln x + \tfrac{1}{2}\ln\left(x^2+1\right)$$ ***Power Property***

$$f'(x) = 2\frac{1}{x} + \frac{1}{2}\frac{2x}{x^2+1}$$ ***Differentiate***

$$\underline{= \frac{2}{x} + \frac{x}{x^2+1}}$$

Exercise

Find the Derivatives of $y = \ln\dfrac{x^2}{x^2+1}$

Solution

$$y = \ln x^2 - \ln x^2 + 1$$

$$y' = \frac{2x}{x^2} - \frac{2x}{x^2+1} \quad \underline{= \frac{2}{x} - \frac{2x}{x^2+1}}$$

Exercise

Find the derivative of $f(x) = e^{-2x^3}$

Solution

$$f'(x) = e^{-2x^3}\left(-6x^2\right) \quad \underline{= -\frac{6x^2}{e^{2x^3}}}$$

Exercise

Find the derivative of $f(x) = 4e^{x^2}$

Solution

$$f'(x) = 4e^{x^2}(2x) \qquad \underline{= 8xe^{x^2}}$$

Exercise

Find the derivative of $f(x) = x^2e^x$

Solution

$$f'(x) = e^x\frac{d}{dx}[x^2] + x^2\frac{d}{dx}[e^x]$$

$$= e^x(2x) + x^2e^x$$

$$\underline{= xe^x(2+x)}$$

Exercise

Find the derivative $\quad f(x) = 2x^3e^x$

Solution

$$f'(x) = 6x^2e^x + 2x^3e^x$$

$$u = 2x^3 \quad v = e^x$$

$$u' = 6x^2 \quad v' = e^x$$

$$\boxed{= 2x^2e^x(3+x)}$$

Exercise

Find the derivative $\quad f(x) = \dfrac{3e^x}{1+e^x}$

Solution

$$f'(x) = \frac{3e^x\left(1+e^x\right) - 3e^xe^x}{\left(1+e^x\right)^2}$$

$$u = 3e^x \quad v = 1+e^x$$

$$u' = 3e^x \quad v' = e^x$$

$$= \frac{3e^x + 3e^{2x} - 3e^{2x}}{\left(1+e^x\right)^2}$$

$$\boxed{= \frac{3e^x}{\left(1+e^x\right)^2}}$$

Exercise

Find the derivative $\quad f(x) = 5e^x + 3x + 1$

Solution

$$f'(x) \ \boxed{= 5e^x + 3}$$

Exercise

Find the derivative of $f(x) = \frac{e^x + e^{-x}}{2}$

Solution

$$f(x) = \tfrac{1}{2}(e^x + e^{-x})$$

$$f'(x) \ \boxed{= \tfrac{1}{2}(e^x - e^{-x})}$$

Exercise

Find the derivative of $f(x) = \frac{e^x}{x^2}$

Solution

$$f'(x) = \frac{x^2e^x - 2xe^x}{x^4}$$

$$= \frac{xe^x(x-2)}{x^4}$$

$$\boxed{= \frac{e^x(x-2)}{x^3}}$$

Exercise

Find the derivative of $f(x) = x^2e^x - e^x$

Solution

$$f'(x) = e^x \frac{d}{dx}[x^2] + x^2 \frac{d}{dx}[e^x] - \frac{d}{dx}[e^x]$$

$$= e^x(2x) + x^2e^x - e^x$$

$$\boxed{= e^x(x^2 + 2x - 1)}$$

Exercise

Find the derivative of $f(x) = (1+2x)e^{4x}$

Solution

$$f'(x) = (2)e^{4x} + (1+2x)(4e^{4x})$$

$$= 2e^{4x} + (1+2x)(4e^{4x})$$

$$= 2e^{4x}(1 + 2(1+2x))$$

$$= 2e^{4x}(1 + 2 + 4x)$$

$$\boxed{= 2e^{4x}(3 + 4x)}$$

Exercise

Find the derivative of $y = x^2e^{5x}$

Solution

$$y' = x^2\left(5e^{5x}\right) + 2x\left(e^{5x}\right)$$

$$= xe^{5x}(5x+2)$$

Exercise

Find the derivative of $y = x^2 e^{-2x}$

Solution

$$y' = 2xe^{-2x} - 2x^3 e^{-2x}$$

$$= 2xe^{-2x}\left(1 - x^2\right)$$

Exercise

Find the derivative $\quad f(x) = \dfrac{e^x}{x^2+1}$

Solution

$$f'(x) = \frac{e^x\left(x^2+1\right) - 2xe^x}{\left(x^2+1\right)^2} \qquad u = e^x \quad v = x^2+1 \qquad u' = e^x \quad v' = 2x$$

$$= \frac{\left(x^2+1-2x\right)e^x}{\left(x^2+1\right)^2}$$

Exercise

Find the derivative $\quad f(x) = \dfrac{1-e^x}{1+e^x}$

Solution

$$f'(x) = \frac{-e^x\left(1+e^x\right) - e^x\left(1-e^x\right)}{\left(1+e^x\right)^2} \qquad u = 1-e^x \quad v = 1+e^x \qquad u' = -e^x \quad v' = e^x$$

$$= \frac{-e^x - e^{2x} - e^x + e^{2x}}{\left(1+e^x\right)^2}$$

$$= -\frac{2e^x}{\left(1+e^x\right)^2}$$

Exercise

Find the Derivatives of $y = \dfrac{\ln x}{e^{2x}}$

Solution

$$y' = \frac{e^{2x}(1/x) - \ln x(2e^{2x})}{e^{4x}}$$

$$= \frac{e^{2x} - 2x\ln x(e^{2x})}{e^{4x}}$$

$$= \boxed{\frac{e^{2x}(1 - 2x\ln x)}{e^{4x}}}$$

Exercise

Find the Derivatives of $f(x) = e^{2x}\ln\left(xe^x + 1\right)$

Solution

$$f'(x) = 2e^{2x}\ln\left(xe^x + 1\right) + e^{2x}\frac{e^x + xe^x}{xe^x + 1}$$

$$= \boxed{e^{2x}\left[2\ln\left(xe^x + 1\right) + \frac{e^x(1+x)}{xe^x + 1}\right]}$$

$f = e^{2x}$ $\quad U = 2x \to U' = 2$ $\quad f' = 2e^{2x}$

$g = \ln\left(xe^x + 1\right)$ $\quad U = xe^x + 1 \to U' = e^x + xe^x$ $\quad g' = \frac{e^x + xe^x}{xe^x + 1}$

Exercise

Find the Derivatives of $f(x) = \dfrac{xe^x}{\ln\left(x^2 + 1\right)}$

Solution

$$f'(x) = \frac{e^x(1+x)\ln\left(x^2+1\right) - \frac{2x}{x^2+1}xe^x}{\left[\ln\left(x^2+1\right)\right]^2}$$

$$= \frac{e^x\left[(1+x)\ln\left(x^2+1\right) - \frac{2x^2}{x^2+1}\right]}{\left[\ln\left(x^2+1\right)\right]^2}$$

$$= \frac{e^x\left[\frac{\left(x^2+1\right)(1+x)\ln\left(x^2+1\right) - 2x^2}{x^2+1}\right]}{\left[\ln\left(x^2+1\right)\right]^2}$$

$u = xe^x$ $\quad u' = e^x + xe^x$

$v = \ln\left(x^2 + 1\right)$ $\quad v' = \frac{2x}{x^2+1}$

$$= \boxed{\frac{e^x\left[\left(x^2+1\right)(1+x)\ln\left(x^2+1\right)-2x^2\right]}{\left(x^2+1\right)\left[\ln\left(x^2+1\right)\right]^2}}$$

Exercise

Find the derivative $y=\cos^{-1}\left(\frac{1}{x}\right)$

Solution

$$y=\cos^{-1}\left(\frac{1}{x}\right)=\sec^{-1}(x)$$

$$y'=\boxed{\frac{1}{|x|\cdot\sqrt{x^2-1}}}$$

Exercise

Find the derivative $y=\sin^{-1}\sqrt{2}t$

Solution

$$y'=\frac{\sqrt{2}}{\sqrt{1-\left(\sqrt{2}t\right)^2}}$$

$$=\boxed{\frac{\sqrt{2}}{\sqrt{1-2t^2}}}$$

Exercise

Find the derivative $y=\sec^{-1}(5s)$

Solution

$$y'=\frac{5s}{|5s|\sqrt{(5s)^2-1}}$$

$$=\boxed{\frac{s}{|s|\sqrt{25s^2-1}}}$$

Exercise

Find the derivative $y = \cot^{-1}\sqrt{t-1}$

Solution

$$y' = -\frac{\frac{1}{2}(t-1)^{-1/2}}{1+\left[(t-1)^{1/2}\right]^2}$$

$$= -\frac{1}{2(t-1)^{1/2}(1+t-1)}$$

$$\boxed{= -\frac{1}{2t\sqrt{t-1}}}$$

Exercise

Find the derivative $y = \ln\left(\tan^{-1} x\right)$

Solution

$$y' = \frac{\frac{1}{1+x^2}}{\tan^{-1} x} = \boxed{\frac{1}{\left(1+x^2\right)\tan^{-1} x}}$$

Exercise

Find the derivative $y = \tan^{-1}(\ln x)$

Solution

$$y' = \frac{\frac{1}{x}}{1+(\ln x)^2}$$

$$\left(\tan^{-1} u\right)' = \frac{u'}{1+u^2}$$

$$= \boxed{\frac{1}{x\left[1+(\ln x)^2\right]}}$$

Solution *Section R.2* – Integration

Exercise

Find each indefinite integral. $\int \frac{x+2}{\sqrt{x}}\,dx$

Solution

$$\int \frac{x+2}{\sqrt{x}}dx = \int \left[\frac{x}{x^{1/2}} + \frac{2}{x^{1/2}}\right]dx$$

$$= \int \frac{x}{x^{1/2}}dx + \int \frac{2}{x^{1/2}}dx$$

$$= \int x^{1/2}dx + 2\int x^{-1/2}dx$$

$$= \frac{x^{3/2}}{3/2} + 2\frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{3}x^{3/2} + 4x^{1/2} + C$$

Exercise

Find each indefinite integral $\int 4y^{-3}dy$

Solution

$$\int 4y^{-3}dy = 4\frac{y^{-2}}{-2} + C$$

$$= -\frac{2}{y^2} + C$$

Exercise

Find each indefinite integral $\int (x^3 - 4x + 2)dx$

Solution

$$\int \left(x^3 - 4x + 2\right)dx = \frac{1}{4}x^4 - 2x^2 + 2x + C$$

Exercise

Find each indefinite integral $\int \left(\sqrt[4]{x^3}+1\right)dx$

Solution

$$\int \left(x^{3/4}+1\right)dx = \underline{\frac{4}{7}x^{7/4}+x+C}$$

Exercise

Find each indefinite integral $\int \sqrt{x}(x+1)dx$

Solution

$$\int x^{1/2}(x+1)dx = \int \left(x^{3/2}+x^{1/2}\right)dx$$

$$\underline{=\frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}+C}$$

Exercise

Find each indefinite integral $\int (1+3t)t^2dt$

Solution

$$\int \left(t^2+3t^3\right)dt = \underline{\frac{1}{3}t^3+\frac{3}{4}t^4+C}$$

Exercise

Find each indefinite integral $\int \frac{x^2-5}{x^2}dx$

Solution

$$\int \frac{x^2-5}{x^2}dx = \int \left(1-\frac{5}{x^2}\right)dx$$

$$= \int \left(1-5x^{-2}\right)dx$$

$$= x+5x^{-1}+C$$

$$\underline{= x+\frac{5}{x}+C}$$

Exercise

Find each indefinite integral $\int(-40x+250)\,dx$

Solution

$$\int(-40x+250)\,dx = \underline{-20x^2+250x+C}$$

Exercise

Find each indefinite integral $\int\left(7-3x-3x^2\right)(2x+1)\,dx$

Solution

$$\begin{aligned}\int\left(7-3x-3x^2\right)(2x+1)\,dx &= \int\left(14x+7-6x^2-3x-6x^3-3x^2\right)dx\\ &= \int\left(-6x^3-9x^2+11x+7\right)dx\\ &= \underline{-\frac{3}{2}x^4-3x^3+\frac{11}{2}x^2+7x+C}\end{aligned}$$

Exercise

Find the integral $\int(1+\cos 3\theta)\,d\theta$

Solution

$$\int(1+\cos 3\theta)\,d\theta \underline{=\theta+\frac{1}{3}\sin 3\theta+C}$$

Exercise

Find the integral $\int 2\sec^2\theta\,d\theta$

Solution

$$\int 2\sec^2\theta\,d\theta \underline{=2\tan\theta+C}$$

Exercise

Find the integral $\int \sec 2x\tan 2x\,dx$

Solution

$$\int \sec 2x \tan 2x \, dx = \frac{1}{2}\sec 2x + C$$

Exercise

Find the integral $\int 2e^{2x}dx$

Solution

$$\int 2e^{2x}dx = e^{2x} + C$$

Exercise

Find the integral $\int \frac{12}{x}dx$

Solution

$$\int \frac{12}{x}dx = 12\ln|x| + C$$

Exercise

Find the integral $\int \frac{dx}{\sqrt{1-x^2}}$

Solution

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$$

Exercise

Find the integral $\int \frac{dx}{x^2+1}$

Solution

$$\int \frac{dx}{x^2+1} = \tan^{-1}x + C$$

Exercise

Find the integral $\displaystyle\int \frac{1+\tan\theta}{\sec\theta}\,d\theta$

Solution

$$\int \frac{1+\tan\theta}{\sec\theta}\,d\theta = \int \left(\frac{1}{\sec\theta}+\frac{\tan\theta}{\sec\theta}\right)d\theta$$

$$= \int (\cos\theta+\sin\theta)\,d\theta$$

$$\underline{= \sin\theta-\cos\theta+C}$$

Exercise

Find the general solution of the differential equation $y' = 2t+3$

Solution

$$dy = (2t+3)\,dt$$

$$\int dy = \int (2t+3)\,dt$$

$$\boxed{y = t^2+3t+C}$$

Exercise

Find the general solution of the differential equation $y' = 3t^2+2t+3$

Solution

$$\int dy = \int \left(3t^2+2t+3\right)dt$$

$$\boxed{y = t^3+t^2+3t+C}$$

Exercise

Find the general solution of the differential equation $y' = \sin 2t + 2\cos 3t$

Solution

$$\int dy = \int (\sin 2t+2\cos 3t)\,dt$$

$$\boxed{y(t) = -\frac{1}{2}\cos 2t+\frac{2}{3}\sin 3t+C}$$

Exercise

Find the general solution of the differential equation: $y' = x^3(3x^4+1)^2$

Solution

$$\int x^3\left(3x^4+1\right)^2 dx$$

$u = 3x^4+1 \Rightarrow du = 12x^3 dx$

$\Rightarrow \frac{1}{12}du = x^3 dx$

$$\int x^3(3x^4+1)^2 dx = \int \frac{1}{12}u^2 du$$

$$= \frac{1}{12}\frac{(3x^4+1)^3}{3} + C$$

$$= \frac{1}{36}(3x^4+1)^3 + C$$

$$\boxed{y = \frac{1}{36}\left(3x^4+1\right)^3 + C}$$

Exercise

Find the general solution of the differential equation: $y' = 5x\sqrt{x^2-1}$

Solution

$$\int 5x\left(x^2-1\right)^{1/2} dx = \frac{5}{2}\int\left(x^2-1\right)^{1/2} d\left(x^2-1\right) \qquad d\left(x^2-1\right) = 2xdx$$

$$= \frac{5}{3}(x^2-1)^{3/2} + C$$

Exercise

Find the general solution of the differential equation: $y' = x\sqrt{x^2+4}$

Solution

$$\int \sqrt{x^2+4}\ xdx = \frac{1}{2}\int\left(x^2+4\right)^{1/2} d\left(x^2+4\right)$$

$$= \frac{1}{3}(x^2+4)^{3/2} + C$$

Exercise

Evaluate the integrals $\displaystyle\int_{-2}^{2}\left(x^3-2x+3\right)dx$

Solution

$$\int_{-2}^{2}\left(x^3-2x+3\right)dx=\left[\frac{x^4}{4}-x^2+3x\right]_{-2}^{2}$$

$$=\left(\frac{(2)^4}{4}-(2)^2+3(2)\right)-\left(\frac{(-2)^4}{4}-(-2)^2+3(-2)\right)$$

$$=12$$

Exercise

Evaluate the integrals $\displaystyle\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx$

Solution

$$\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx=\left[\frac{x^3}{3}+\frac{2}{3}x^{3/2}\right]_{0}^{1}$$

$$=\left(\frac{(1)^3}{3}+\frac{2}{3}(1)^{3/2}\right)-0$$

$$=1$$

Exercise

Evaluate the integrals $\displaystyle\int_{0}^{\pi/3}4\sec u\tan u\ du$

Solution

$$\int_{0}^{\pi/3}4\sec u\tan u\ du=4\sec u\Big|_{0}^{\pi/3}$$

$$=4\left(\sec\frac{\pi}{3}-\sec 0\right)$$

$$=4(2-1)$$

$$=4$$

Exercise

Evaluate the integrals $\displaystyle\int_{\pi/4}^{3\pi/4} \csc\theta\cot\theta\, d\theta$

Solution

$$\int_{\pi/4}^{3\pi/4} \csc\theta\cot\theta\, d\theta = -\csc\theta\Big|_{\pi/4}^{3\pi/4}$$

$$= -\left(\csc\frac{3\pi}{4} - \csc\frac{\pi}{4}\right)$$

$$= -\left(\sqrt{2} - \sqrt{2}\right)$$

$$\underline{= 0}$$

Exercise

Evaluate the integrals $\displaystyle\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt$

Solution

$$\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt = \int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \pi t^{-2}\right) dt$$

$$= \left[4\tan t - \pi t^{-1}\right]_{-\pi/3}^{-\pi/4}$$

$$= \left(4\tan\left(-\frac{\pi}{4}\right) - \pi\left(-\frac{4}{\pi}\right)\right) - \left(4\tan\left(-\frac{\pi}{3}\right) - \pi\left(-\frac{3}{\pi}\right)\right)$$

$$= \left(4(-1) + 4\right) - \left(4\left(-\sqrt{3}\right) + 3\right)$$

$$= -\left(-4\sqrt{3} + 3\right)$$

$$\underline{= 4\sqrt{3} - 3}$$

Exercise

Evaluate the integrals $\displaystyle\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} dy$

Solution

$$\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} dy = \int_{-3}^{-1} \left(\frac{y^5}{y^3} - \frac{2y}{y^3}\right) dy$$

$$= \int_{-3}^{-1} \left(y^2 - 2y^{-2}\right) dy$$

$$=\left[\frac{1}{3}y^3+2y^{-1}\right]_{-3}^{-1}$$

$$=\left(\frac{1}{3}(-1)^3+\frac{2}{-1}\right)-\left(\frac{1}{3}(-3)^3+\frac{2}{-3}\right)$$

$$\underline{=\frac{22}{3}}$$

Exercise

Evaluate the integrals $\displaystyle\int_1^8 \frac{\left(x^{1/3}+1\right)\left(2-x^{2/3}\right)}{x^{1/3}}dx$

Solution

$$\int_1^8 \frac{\left(x^{1/3}+1\right)\left(2-x^{2/3}\right)}{x^{1/3}}dx=\int_1^8 \frac{2x^{1/3}-x+2-x^{2/3}}{x^{1/3}}dx$$

$$=\int_1^8 \left(2-x^{2/3}+2x^{-1/3}-x^{1/3}\right)dx$$

$$=\left[2x-\frac{3}{5}x^{5/3}+3x^{2/3}-\frac{3}{4}x^{4/3}\right]_1^8$$

$$=\left(2(8)-\frac{3}{5}(8)^{5/3}+3(8)^{2/3}-\frac{3}{4}(8)^{4/3}\right)-\left(2(1)-\frac{3}{5}(1)^{5/3}+3(1)^{2/3}-\frac{3}{4}(1)^{4/3}\right)$$

$$=\left(-\frac{16}{5}\right)-\left(\frac{73}{20}\right)$$

$$\underline{=-\frac{137}{20}}$$

Exercise

Evaluate: $\displaystyle\int_0^1 (2t+3)^3\,dt$

Solution

$$\int_0^1 (2t+3)^3\,dt=\frac{1}{2}\int_0^1 (2t+3)^3\,d(2t+3) \qquad d(2t+3)=2dt$$

$$=\frac{1}{8}(2t+3)^4\Big|_0^1$$

$$=\frac{1}{8}\left[5^4-3^4\right]$$

$$\underline{=68}$$

Exercise

Evaluate the integral $\displaystyle\int_{-1}^{1} r\sqrt{1-r^2}\, dr$

Solution

$$\int_{-1}^{1} r\sqrt{1-r^2}\, dr = -\frac{1}{2}\int_{-1}^{1} \left(1-r^2\right)^{1/2}\, d\left(1-r^2\right)$$

$$= -\frac{1}{3}\left[\left(1-r^2\right)^{3/2}\right]_{-1}^{1}$$

$$= -\frac{1}{3}\left[0-0\right]$$

$$\underline{= 0}$$