Solution Section 2.3 – Present Value of an Annuity Amortization

Exercise

How much should you deposit in an account paying 8% compounded quarterly in order to receive quarterly payments of $1,000 for the next 4 years?

Solution

Given: $PMT = 1,000 \quad r = 8\% = .08, \quad m = 4, \quad t = 4$

$$i = \frac{r}{m} = \frac{.08}{4} = .02 \quad n = 4(4) = 16$$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$= 1000\,\frac{1-(1+.02)^{-16}}{.02} \qquad 1000(1-1.02 \wedge (-)16)/.02$$

$$\approx \$13,577.71$$

Exercise

You have negotiated a price of $25,200 for a new truck. Now you must choose between 0% financing for 48 months or a $3,000 rebate. If you choose the rebate, you can obtain a loan for the balance at 4.5% compounded monthly for 48 months . Which option should you choose?

Solution

0% ***financing***: ***Given***: $P = 25,200 \quad r = 0\% = 0, \quad t = 48\ mth$

$$PMT_1 = \frac{25,200}{48} = \$525$$

Rebate: ***Given***: $P = 25,200 \quad Rebate = \$3,000 \quad r = 4.5\% = .045, \quad m = 12 \quad n = t = 48\ mth$

$$PV = 25,200 - 3,000 = \$22,200$$

$$i = \frac{.045}{12} = .00375$$

$$PMT_2 = PV\,\frac{i}{1-(1+i)^{-n}}$$

$$= 22,200\,\frac{.00375}{1-1.00375^{-48}} \qquad 22200(.00375/(1-1.00375 \wedge (-)48))$$

$$= \$506.24$$

$\Rightarrow$ Rebate is better and you save 525 – 506.24 = $ 18.76 *per month*

Or 18.76 * 48 = $ 900.48 (over the loan)

Exercise

Suppose you have selected a new car to purchase for $19,500. If the car can be financed over a period of 4 years at an annual rate of 6.9% compounded monthly, how much will your monthly payments be? Construct an amortization table for the first 3 months.

Solution

$$PMT = 19500\left(\frac{0.069/12}{1-(1+0.069/12)^{-48}}\right) = 466.05 \qquad 19500(0.069/12)/(1-(1+0.069/12)\wedge(-)48$$

$$I_1 = 19500(0.069/12) = 112.13$$

Pmt #	Pmt Amount	Interest	Reduction	Unpaid Bal.
0	-------------	----	---------	19500
1	466.05	112.13	353.93	19146.08
2	466.05	110.09	355.96	18790.12
3	466.05	108.04	358.01	18432.11

Exercise

Suppose your parents decide to give you $10,000 to be put in a college trust fund that will be paid in equally quarterly installments over a 5 year period. If you deposit the money into an account paying 1.5% per quarter, how much are the quarterly payments (Assume the account will have a zero balance at the end of period.)

Solution

Given: $PV = 10,000 \quad r = 1.5\% = .015, \quad m = 4, \quad t = 5$

$i = r = .015 \quad n = 4(5) = 20$

$$\text{PMT} = 10,000\left(\frac{0.015}{1-(1+0.015)^{-20}}\right) = \$582.46 \qquad 10000(0.015)/(1-(1+0.015)\wedge(-)20)$$

Exercise

You finally found your dream home. It sells for $120,000 and can be purchased by paying 10% down and financing the balance at an annual rate of 9.6% compounded monthly.

a) How much are your payments if you pay monthly for 30 years?
b) Determine how much would be paid in interest .
c) Determine the payoff after 100 payments have been made.
d) Change the rate to 8.4% and the time to 15 years and calculate the payment.
e) Determine how much would be paid in interest and compare with the previous interest.

Solution

$$\boldsymbol{a})\ PMT = 108000\left(\frac{0.096/12}{1-(1+0.096/12)^{-360}}\right) = \$916.01$$

$$\boldsymbol{b})\ I_1 = 360(916.01) - 108000 = \$221763.60$$

$$\boldsymbol{c})\ PV = 916.01\left(\frac{1-(1+0.096/12)^{-260}}{0.096/12}\right) = \$100077.71$$

$$\boldsymbol{d})\ PMT = 108000\left(\frac{0.084/12}{1-(1+0.084/12)^{-180}}\right) = \$1057.20$$

$$\boldsymbol{e})\ I_2 = 180(1057.20) - 108000 = \$82296$$

$$\mathrm{I}_1 - \mathrm{I}_2 = 221763.60 - 82296 = 139467.60$$

Exercise

Sharon has found the perfect car for her family (anew mini-van) at a price of $24,500. She will receive a $3500 credit toward the purchase by trading in her old Gremlin, and will finance the balance at an annual rate of 4.8% compounded monthly.

a) How much are her payments if she pays monthly for 5 years?
b) How long would it take for her to pay off the car paying an extra $100 per mo., beginning with the first month?

Solution

$$\boldsymbol{a})\ PMT = 21000\left(\frac{0.048/12}{1-(1+0.048/12)^{-60}}\right) = \$394.37$$

$$\boldsymbol{b})\ 21000 = 494.37\left(\frac{1-(1+0.048/12)^{n}}{0.048/12}\right)$$

Divide both sides by 494.37

$$42.4783 = \frac{1-(1+0.004)^{-n}}{0.004}$$ ***multiply by 0.004***

$$0.1699 = 1 - 1.004^{-n}$$

$$1.004^{-n} = 1 - 0.1699$$

$$1.004^{-n} = 0.8301$$ ***"ln" both sides***

$$-n \ln 1.004 = \ln 0.8301$$

$$n = -\ln 0.8301 / \ln 1.004 = 46.65 \quad n = -\frac{\ln 0.8301}{\ln 1.004}$$

$$\boxed{n = 47 mo.}$$

Money is compounded monthly; it can't be compounded at 46.65 months. Bump to 47mo.

Exercise

Marie has determined that she will need \$5000 per month in retirement over a 30-year period. She has forecasted that her money will earn 7.2% compounded monthly. Marie will spend 25-years working toward this goal investing monthly at an annual rate of 7.2%. How much should Marie's monthly payments be during her working years in order to satisfy her retirement needs? *This is a 2-part problem: 1st calculate the PV for retirement. Then use that value as FV for working years.*

Solution

$$PV = 5000\left(\frac{1-(1+0.072/12)^{-360}}{0.072/12}\right) = 736606.78$$

Exercise

American General offers a 10-year ordinary annuity with a guaranteed rate of 6.65% compounded annually. How much should you pay for one of these annuities if you want to receive payments of \$5,000 annually over the 10-year period?

Solution

Given: $PMT = 5{,}000 \quad r = 6.65\% = .0665, \quad m = 1, \quad t = 10$

$$i = \frac{r}{m} = .0665 \quad n = 1(10) = 10$$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$= 5{,}000\left(\frac{1-(1+0.0665)^{-10}}{.0665}\right) \qquad = 5000\left(1-(1.0665)\,^\wedge(-)10\right)/.0665$$

$$\underline{= \$35{,}693.18}$$

Exercise

American General offers a 7-year ordinary annuity with a guaranteed rate of 6.35% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $10,000 annually over the 7-year period?

Solution

Given: $PMT = 10,000 \quad r = 6.35\% = .0635, \quad m = 1, \quad t = 7$

$$i = \frac{r}{m} = .0635 \quad n = 7$$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$= 10,000\left(\frac{1-(1+0.0635)^{-7}}{.0635}\right) \qquad = 10000\left(1-(1.0635)\wedge(-)7\right)/.0635$$

$$\underline{= \$55,135.98}$$

Exercise

You want to purchase an automobile for $27,300. The dealer offers you 0% financing for 60 months or a $5,000 rebate. You can obtain 6.3% financial for 60 months at the local bank. Which option should you choose? Explain.

Solution

0% ***financing***: ***Given***: $P = 27,300 \quad r = 0\% = 0, \quad t = 60\ mo$

$$PMT_1 = \frac{27,300}{60} \underline{= \$455.00}$$

Rebate: ***Given***: $Rebate = \$5,000 \quad r = 6.3\% = .063, \quad n = 60$

$$PV = 27,300 - 5,000 = \$22,300. \quad i = \frac{.063}{12}$$

$$PMT_2 = PV\,\frac{i}{1-(1+i)^{-n}}$$

$$= 22,300\,\frac{\frac{.063}{12}}{1-\left(1+\frac{.063}{12}\right)^{-60}} \qquad 22300(.00375/(1-(1+.063/12)\wedge(-)60))$$

$$= \underline{\$434.24}$$

$\Rightarrow$ Rebate is better and you save $\$455 - \$434.24 = \$20.76$ *per month*

Or $60(20.76) = \$1,245.60$ (over the life of the loan)

Exercise

You want to purchase an automobile for $28,500. The dealer offers you 0% financing for 60 months or a $6,000 rebate. You can obtain 6.2% financial for 60 months at the local bank. Which option should you choose? Explain.

Solution

0% ***financing***: ***Given***: $P = 28,500 \quad r = 0\% = 0, \quad t = 60\ mo$

$$\boxed{PMT_1 = \frac{28,500}{60} = \$475.00}$$

Rebate: ***Given***: $Rebate = \$6,000 \quad r = 6.2\% = .062, \quad n = 60$

$$PV = 28,500 - 6,000 = \$22,500. \quad i = \frac{.062}{12}$$

$$PMT_2 = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 22,500 \frac{\frac{.062}{12}}{1-\left(1+\frac{.062}{12}\right)^{-60}}$$

$$= \$437.08$$

$\Rightarrow$ Rebate is better and you save $\$475 - \$437.08 = \$37.92$ *per month*

Or $60(37.92) = \$2,275.20$ (over the life of the loan)

Exercise

Construct the amortization schedule for a $5,000 debt that is to be amortized in eight equal quarterly payments at 2.8% interest per quarter on the unpaid balance.

Solution

Given: $PV = 5,000 \quad i = r = 2.8\% = .028, \quad n = 8$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 5,000 \frac{.028}{1-(1+.028)^{-8}} \qquad 5000(.028 / (1-1.028 \wedge (-)8))$$

$$= \$706.29$$

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0		$0		$5,000.00
1	$706.29	$.028(5000) = \$140.00$	$706.29 - 140.00 = \$566.29$	$5000 - 566.29 = \$4,433.71$
2	$706.29	$.028(4433.71) = \$124.14$	$706.29 - 124.14 = \$582.15$	$4433.71 - 582.15 = \$3,851.56$
3	$706.29	$.028(3851.56) = \$107.84$	$706.29 - 107.84 = \$598.45$	$3851.56 - 598.45 = \$3,253.11$
4	$706.29	$.028(3253.11) = \$91.09$	$706.29 - 91.09 = \$615.20$	$3253.11 - 615.20 = \$2,637.91$
5	$706.29	$.028(2637.91) = \$73.86$	$706.29 - 73.86 = \$632.43$	$2637.91 - 632.43 = \$2,005.48$
6	$706.29	$.028(2005.48) = \$56.15$	$706.29 - 56.15 = \$650.14$	$2005.48 - 650.14 = \$1,355.34$
7	$706.29	$.028(1355.34) = \$37.95$	$706.29 - 37.95 = \$668.34$	$1355.34 - 668.34 = \$687.00$
8	$706.29	$.028(687) = \$19.24$	$706.29 - 19.24 = \$687.00$	$0.00
Total	**$5,650.27**	**$650.27**	**$5,000.00**	

Exercise

Construct the amortization schedule for a $10,000 debt that is to be amortized in six equal quarterly payments at 2.6% interest per quarter on the unpaid balance.

Solution

Given: $PV = 10,000 \quad i = r = 2.6\% = .026, \quad n = 6$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 10,000 \frac{.026}{1-(1+.026)^{-6}}$$

$10000(.026 / (1-1.026 \wedge (-) 6))$

$$= \underline{\$1,821.58}$$

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0		$0		$10,000.00
1	$1,821.58	$.026(10000) = \$260.00$	$1821.58 - 260.00 = \$1,561.58$	$10000 - 1561.58 = \$8,438.42$
2	$1,821.58	$.026(8438.42) = \$219.40$	$1821.58 - 219.40 = \$1,602.18$	$8438.42 - 1602.18 = \$6,836.24$
3	$1,821.58	$.026(6836.24) = \$177.74$	$1821.58 - 177.74 = \$1,643.84$	$6836.24 - 1643.84 = \$5,192.40$
4	$1,821.58	$.026(5192.40) = \$135.00$	$1821.58 - 135.00 = \$1,686.58$	$5192.40 - 1686.58 = \$3,505.82$
5	$1,821.58	$.026(3505.82) = \$91.15$	$1821.58 - 91.15 = \$1,730.43$	$3505.82 - 1730.43 = \$1,775.39$
6	$1,821.58	$.026(1775.39) = \$46.16$	$1821.58 - 46.16 = \$1,775.39$	$0.00
Total	**$10,929.45**	**$929.45**	**$10,000.00**	

Exercise

A loan of $37,948 with interest at 6.5% compounded annually, to be paid with equal annual payments over 10 years

Solution

Given: $PV = 37,948. \quad m=1, \quad i = r = 6.5\% = .065, \quad n = 10$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 37,948 \frac{.065}{1-(1+.065)^{-10}}$$

37948(.065 / (1 – 1.065 ^ (–)10))

$$= \boxed{\$5,278.74}$$

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0				$37,948.00
1	$5,278.74	$2,466.62	$2,812.12	$35,135.88
2	$5,278.74	$2,283.83	$2,994.91	$32,140.97
3	$5,278.74	$2,089.16	$3,189.58	$28,951.40
4	$5,278.74	$1,881.84	$3,396.90	$25,554.50
5	$5,278.74	$1,661.04	$3,617.70	$21,936.80
6	$5,278.74	$1,425.89	$3,852.85	$18,083.95
7	$5,278.74	$1,175.46	$4,103.28	$13,980.67
8	$5,278.74	$908.74	$4,370.00	$9,610.67
9	$5,278.74	$624.69	$4,654.05	$4,956.62
10	$5,278.74	$322.18	$4,956.62	$0.00
Total	**$52,787.40**	**$14,839.40**	**$37,948.00**	

Exercise

A loan of $4,836 with interest at 7.25% compounded semi-annually, to be repaid in 5 years in equal semi-annual payments.

Solution

Given: $PV = 4,836. \quad m = 2, \quad r = 7.25\% = .0725, \quad t = 5$

$$i = \frac{r}{m} = \frac{.0725}{2} = .03625 \quad n = 2(5) = 10$$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 4,836 \frac{.03625}{1-(1+.03625)^{-10}}$$

4836(.03625 / (1 – 1.03625 ^ (–)10))

$$= \underline{\$585.16}$$

Pmt #	***Payment***	***Interest***	***Reduction***	***Unpaid Balance***
0				$4,836.00
1	$585.16	$175.31	$409.85	$4,426.15
2	$585.16	$160.45	$424.71	$4,001.43
3	$585.16	$145.05	$440.11	$3,561.32
4	$585.16	$129.10	$456.06	$3,105.26
5	$585.16	$112.57	$472.59	$2,632.67
6	$585.16	$95.43	$489.73	$2,142.94
7	$585.16	$77.68	$507.48	$1,635.46
8	$585.16	$59.29	$525.87	$1,108.59
9	$585.16	$40.22	$544.94	$564.65
10	$585.16	$20.47	$564.65	$0.00
Total	**$5,851.60**	**$1,015.60**	**$4,836.00**	