3.1 Inverse

relation $\{(x, y)\}$

inverse relation $\{(y, x)\}$

Ex $G = \{(4, 2), (3, -1), (-2, 0)\}$

inverse $G = \{(2, 4), (-1, 3), (0, -2)\}$

$$\left.\begin{array}{l} x \to y \\ y \to x \end{array}\right\} \quad \underbrace{y = x}_{\text{symmetric}}$$

Function has an inverse

One-to-One fctns
1-1 fctns

$$\left\{\begin{array}{l} f(a) = f(b) \Rightarrow a = b \\ a \neq b \Rightarrow f(a) \neq f(b) \end{array}\right.$$

$f(x) = x^2$

$-1 \neq 1$

$$\left.\begin{array}{l} f(-1) = 1 \\ f(1) = 1 \end{array}\right\} \Rightarrow f(-1) = f(1)$$

$\therefore$ Inverse fctn doesn't exist.
it's not 1-1 fctn

Ex $f(x) = 2x - 3$

$$f(a) = f(b)$$
$$2a - 3 = 2b - 3$$
$$2a = 2b$$
$$a = b \checkmark$$

$f(x)$ is 1-1.

$$a^2 = b^2 \Rightarrow a = \pm b \begin{cases} a = b \\ a = -b \end{cases} \#$$

Definition of Inverse fctn

$$\begin{tikzcd} x \arrow[r, "f"] \arrow[rr, bend right, "g(f(x))"'] & y = f(x) \arrow[r, "g"] & g(y) \end{tikzcd}$$

$$\begin{tikzcd} \text{Domain } f \;\; \text{Range } f^{-1} \;\; x \arrow[r, bend left, "f"] & y \;\; \text{Range } f \;\; \text{domain } y \arrow[l, bend left, "g = f^{-1}"] \end{tikzcd}$$

1   1

input $\longrightarrow$ output

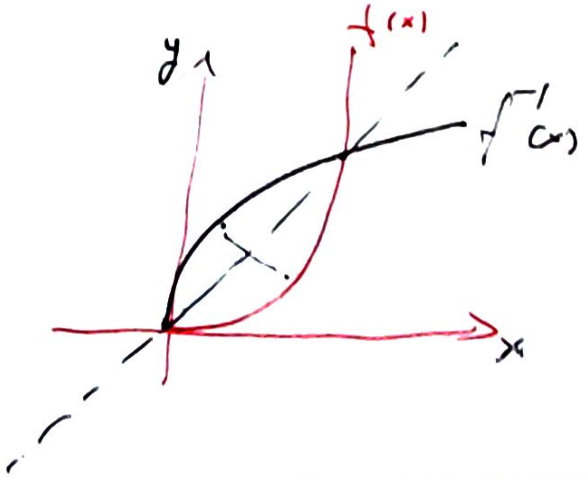
$$g(f(x)) = x$$

f^{-1}: f inverse

$$\text{Domain } f(x) = \text{Range } f^{-1}(x) = \mathbb{R} - \{\ \}$$
$$\text{"} \quad f^{-1}(x) = \quad \text{"} \quad f(x)$$
$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

$f(x) = x^2$ is not 1-1.

$f(x) = x^2 \quad x \geq 0 \rightarrow$ restriction



$y = x$

Ex $\quad f(x) = x^3 - 1 \qquad g(x) = \sqrt[3]{x+1}$

g is inverse of f.

$f(g(x)) = f(\sqrt[3]{x+1}) \qquad (\sqrt[3]{x+1})^3$

$= x + 1 - 1$

$= x$ ✓

$g(f(x)) = g(x^3 - 1)$

$= \sqrt[3]{x^3 - 1 + 1}$

$= \sqrt[3]{x^3}$

$= x$

Finding inverse fctn

Ex

$f(x) = 2x+7$

Replace f(x) w/ y $\quad y = 2x+7$

sub x + y (interchange $\quad x = 2y+7$

solve for y $\quad x-7 = 2y$

replace y w/ $f^{-1}(x)$ $\quad \boxed{y = \frac{x-7}{2} = f^{-1}(x)}$

$f(x) = \frac{5x-3}{2x+1}$ $\qquad f^{-1}(x) = \frac{-x-3}{2x-5}$

$y = \frac{5x-3}{2x+1}$

$x = \frac{5y-3}{2y+1} \rightarrow x(2y+1) = 5y-3$

$2xy + x = 5y-3$

$2xy - 5y = -x-3$

$(2x-5)y = -x-3$

$\underline{y = \frac{-x-3}{2x-5} = f^{-1}(x)}$

$f(x) = \frac{ax+b}{cx+d} \Rightarrow f^{-1}(x) = \frac{-dx+b}{cx-a}$

$f(x) = \frac{2x+7}{1} \Rightarrow f^{-1}(x) = \frac{-x+7}{-2} = \frac{x-7}{2}$

#1 $f(x) = \frac{x+1}{x-1}$ $f^{-1}(x)$? d. R. f f^{-1}

a) $y = \frac{x+1}{x-1}$

$x = \frac{y+1}{y-1}$

$x(y-1) = y+1$

$xy - x = y+1$

$xy - y = x+1$

$(x-1)y = x+1$

$y = \frac{x+1}{x-1} = f^{-1}(x)$

b) domain $f(x)$ = Range $f^{-1}(x)$: $\mathbb{R} - \{1\}$

" $f^{-1}(x)$ = Range $f(x)$: $\mathbb{R} - \{1\}$

* $f(x) = \frac{ax+b}{cx+d}$ → domain.

Range

56. $f(x) = \dfrac{3x+2}{2x-5}$

a) $y = \dfrac{3x+2}{2x-5}$

$x = \dfrac{3y+2}{2y-5}$

$x(2y-5) = 3y+2$

$2xy - 5x = 3y + 2$

$2xy - 3y = 5x + 2$

$(2x-3)y = 5x+2$

$\boxed{y = \dfrac{5x+2}{2x-3} = f^{-1}(x)}$

b) Domain of $f(x)$ = Range $f^{-1}(x)$: $\mathbb{R} - \{\frac{5}{2}\}$

" $f^{-1}(x)$ = " $f(x)$: $\mathbb{R} - \{\frac{3}{2}\}$