Review

$$A = \begin{pmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & 7 & -1 \\ 0 & 1-\lambda & 0 \\ 0 & 15 & -2-\lambda \end{vmatrix}$$

$$= (-1-\lambda)(1-\lambda)(-2-\lambda) = 0$$

$$\boxed{\lambda_{1,2,3} = -1, 1, -2}$$

For $\lambda_1 = -1 \Rightarrow (A - \lambda_1 I) V_1 = 0$

$$\begin{pmatrix} 0 & 7 & -1 \\ 0 & 2 & 0 \\ 0 & 15 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} \rightarrow (1) \\ \rightarrow 2y_1 = 0 \\ \boxed{y_1 = 0} \end{matrix}$$

$x_1 = 1$

$-z_1 = -7y_1 = \boxed{0}$

$$\boxed{V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$$

For $\lambda_2 = 1 \Rightarrow (A - \lambda_2 I) V_2 = 0$

$$\begin{pmatrix} -2 & 7 & -1 \\ 0 & 0 & 0 \\ 0 & 15 & -3 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(3) $15 y_2 = 3 z_2 \rightarrow \boxed{5 y_2 = z_2}$

(1) $-2x_2 = z_2 - 7y_2 \Rightarrow \boxed{x_2 = y_2}$

$$V_2 = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$$

For $\lambda_3 = -2 \Rightarrow (A - \lambda_3 I) V_3 = 0$

$$\begin{pmatrix} 1 & 7 & -1 \\ 0 & 3 & 0 \\ 0 & 15 & 0 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \to \underline{y_3 = 0}$$

$$x_3 - z_3 = 0 \Rightarrow x_3 = z_3$$

$$V_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$ax + by + cz = 0$$

$$\begin{pmatrix} -b \\ a \\ 0 \end{pmatrix} \quad \begin{pmatrix} -c \\ 0 \\ a \end{pmatrix} \quad \begin{pmatrix} 0 \\ -c \\ b \end{pmatrix}$$

$$ax = -by \quad ax = -cz \quad by = -cz$$

$$\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = A$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)^2 = 0$$

Eigenvalues : $\boxed{\lambda_{1,2} = 2}$

since it mult — A is not diagonalizable.

<u>$P^{-1}AP$:</u>

$$A = \begin{pmatrix} -14 & 12 \\ -20 & 17 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -14-\lambda & 12 \\ -20 & 17-\lambda \end{vmatrix}$$

$$= \lambda^2 - 3\lambda + 2 = 0$$

$\boxed{\lambda_{1,2} = 1, 2}$

For $\lambda_1 = 1 \Rightarrow (A - \lambda_1 I) V_1 = 0$

$$\begin{pmatrix} -15 & 12 \\ -20 & 16 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{array}{l} 15x_1 = 12y_1 \\ 5x_1 = 4y_1 \end{array}$$

$\boxed{V_1 = \begin{pmatrix} 4 \\ 5 \end{pmatrix}}$

For $\lambda_2 = 2 \Rightarrow (A - \lambda_2 I) V_2 = 0$

$$\begin{pmatrix} -16 & 12 \\ -20 & 15 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow 4x_2 = 3y_2$$

$$V_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$P = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix} \Rightarrow P^{-1} = \begin{pmatrix} 4 & -3 \\ -5 & 4 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 4 & -3 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} -14 & 12 \\ -20 & 17 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -3 \\ -10 & 8 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \checkmark$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}$$

$$|A| = 1 \qquad |B| = -2$$

since $|A| \neq |B|$

$\therefore$ A & B are not similar

$$\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix} \qquad \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$|\,| = 3 \qquad |\,| = 3$$

$$\begin{pmatrix} 3-\lambda & 1 \\ 0 & 3-\lambda \end{pmatrix} \qquad \begin{vmatrix} 3-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 = 0$$

$$(3-\lambda)^2 = 0 \qquad \lambda_{1,2} = 3$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ line $\qquad V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ plane

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \cancel{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

$$y_2 = 1 \qquad \cancel{0 \neq 1}$$

$$V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow > 0$$

$a \neq 0$

$$(a-d)^2 + 4bc > 0$$

(3

$bc = 0$

$b = 0$ $\quad c \neq 0$ $\rightarrow$ $\overline{c}$ $\begin{vmatrix} a-\lambda & 0 \\ c & d-\lambda \end{vmatrix}$

$c = 0$ $\quad b \neq 0$ $\rightarrow$ $\overline{b}$ $\begin{vmatrix} 0 & b \\ & \end{vmatrix}$

$b = c = 0$ $\rightarrow$ $\begin{vmatrix} & 0 \\ 0 & \end{vmatrix}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$