$$\int \sin^3 x \cos^5 x\,dx = \int \sin^2 x \cos^5 x \sin x\,dx$$
$$= -\int (1-\cos^2 x)\cos^5 x\, d(\cos x)$$
$$= \int (\cos^7 x - \cos^5 x)\, d(\cos x)$$
$$= \frac{1}{8}\cos^8 x - \frac{1}{6}\cos^6 x + C$$

$$\int \sin^2 \theta \cos^5 \theta\,d\theta = \int \sin^2\theta \cos^4\theta \cos\theta\,d\theta$$
$$= \int \sin^2\theta\,(1-\sin^2\theta)^2\,d(\sin\theta)$$
$$= \int \sin^2\theta\,(1-2\sin^2\theta+\sin^4\theta)\,d(\sin\theta)$$
$$= \int (\sin^2\theta - 2\sin^4\theta + \sin^6\theta)\,d(\sin\theta)$$
$$= \frac{1}{3}\sin^3\theta - \frac{2}{5}\sin^5\theta + \frac{1}{7}\sin^7\theta + C$$

$$\int \sin^5 x \cos^{-2} x\,dx = \int \sin^4 x \cos^{-2} x \sin x\,dx$$
$$= -\int (1-\cos^2 x)^2 \cos^{-2} x\, d(\cos x)$$
$$= -\int (1-2\cos^2 x + \cos^4 x)\cos^{-2} x\, d(\cos x)$$
$$= -\int (\cos^{-2} x - 2 + \cos^2 x)\, d(\cos x)$$
$$= \cos^{-1} x + 2\cos x - \frac{1}{3}\cos^3 x + C$$

$$\int \sin^{-3/2} \cos^3 x\, dx = \int \sin^{-3/2} \cos^2 x \cos x\, dx$$

$$= \int \sin^{-3/2} (1 - \sin^2 x)\, d(\sin x)$$

$$= \int (\sin^{-3/2} - \sin^{1/2} x)\, d(\sin x)$$

$$= -2 \sin^{-1/2} - \frac{2}{3} \sin^{3/2} x + C$$

$$\int \sin^3 x \cos^{3/2} x\, dx = \int \sin^2 x \cos^{3/2} x \sin x\, dx$$

$$= -\int (1 - \cos^2 x) \cos^{3/2} x\, d(\cos x)$$

$$= \int (\cos^{7/2} x - \cos^{3/2} x)\, d(\cos x)$$

$$= \frac{2}{9} \cos^{9/2} x - \frac{2}{5} \cos^{5/2} x + C$$

$$\int 6 \sec^4 x\, dx = 6 \int \sec^2 x (\tan^2 x + 1)\, dx$$

$$= 6 \int (\tan^2 x + 1)\, d(\tan x)$$

$$= 6 \left(\frac{1}{3} \tan^3 x + \tan x \right) + C$$

$$= 2 \tan^3 x + 6 \tan x + C$$

$$\int \cot^4 x\, dx = \int \cot^2 x (\csc^2 x - 1)\, dx$$

$$= \int (\cot^2 x \csc^2 x - \cot^2 x)\, dx$$

$$= \int \cot^2 x \csc^2 x\, dx - \int (\csc^2 x - 1)\, dx$$

$$= -\int \cot^2 x\, d(\cot x) - \cot x + x$$

$$= -\frac{1}{3} \cot^3 x - \cot x + x + C$$

$$\int 20\tan^6 x\,dx = 20\int \tan^4 x\,(\sec^2 x - 1)\,dx$$
$$= 20\int(\tan^4 x \sec^2 x - \tan^4 x)\,dx$$
$$= 20\left[\int \tan^4 x\,d(\tan x) - \int \tan^2 x\,(\sec^2 x - 1)\,dx\right]$$
$$= 4\tan^5 x - 20\int(\tan^2 x\sec^2 x - \tan^2 x)\,dx$$
$$= 4\tan^5 x - 20\int \tan^2 x\,d(\tan x) + 20\int \tan^2 x\,dx$$
$$= 4\tan^5 x - \frac{20}{3}\tan^3 x + 20\int(\sec^2 x - 1)\,dx$$
$$= 4\tan^5 x - \frac{20}{3}\tan^3 x + 20\tan x - 20x + C$$

$$\int \cot^5 3x\,dx = \int \cot^3 3x\,(\csc^2 3x - 1)\,dx$$
$$= \int(\cot^3 3x\csc^2 3x - \cot^3 3x)\,dx$$
$$= -\frac{1}{3}\int \cot^3 3x\,d(\cot 3x) - \int \cot 3x\,(\csc^2 3x - 1)\,dx$$
$$= -\frac{1}{12}\cot^4 3x - \int \cot 3x\csc^2 3x\,dx + \int \cot 3x\,dx$$
$$= -\frac{1}{12}\cot^4 3x + \frac{1}{3}\int \cot 3x\,d(\cot 3x) + \int \frac{\cos 3x}{\sin 3x}\,dx$$
$$= -\frac{1}{12}\cot^4 3x + \frac{1}{6}\cot^2 3x + \frac{1}{3}\int \frac{d\sin 3x}{\sin 3x}$$
$$= -\frac{1}{12}\cot^4 3x + \frac{1}{6}\cot^2 3x + \frac{1}{3}\ln|\sin 3x| + C$$

$$\int \sqrt{\tan x}\,\sec^4 x\,dx = \int \sqrt{\tan x}\,(\tan^2 x + 1)\sec^2 x\,dx$$
$$= \int (\tan x)^{1/2}(\tan^2 x + 1)\,d(\tan x)$$
$$= \int\left[(\tan x)^{5/2} + (\tan x)^{1/2}\right]d(\tan x)$$
$$= \frac{2}{7}\tan^{7/2} x + \frac{2}{3}\tan^{3/2} x + C$$

$$\int \tan^3 4x\,dx = \int \tan 4x\,(\sec^2 4x - 1)\,dx$$

$$= \int \tan 4x \sec^2 4x\,dx - \int \tan 4x\,dx$$

$$= \frac{1}{4}\int \tan 4x\; d(\tan 4x) - \int \frac{\sin 4x}{\cos 4x}\,dx$$

$$= \frac{1}{8}\tan^2 4x + \frac{1}{4}\int \frac{d(\cos 4x)}{\cos 4x}$$

$$= \underline{\frac{1}{8}\tan^2 4x + \frac{1}{4}\ln|\cos 4x| + C}$$

$$\int \frac{\sec^2 x}{\tan^5 x}\,dx = \int \tan^{-5} x\; d(\tan x)$$

$$= -\frac{1}{4}\tan^{-4} x + C$$

$$= \underline{\frac{-1}{4\tan^4 x} + C}$$

$$\int \sec^{-2} x\,\tan^3 x\,dx = \int \sec^{-2} x\,(\sec^2 x - 1)\tan x\,dx$$

$$= \int (1 - \sec^{-2} x)\tan x\,dx$$

$$= \int \tan x\,dx - \int \sec^{-2} x\tan x\,dx$$

$$= \int \frac{\sin x}{\cos x}\,dx - \int \cos^2 x\,\frac{\sin x}{\cos x}\,dx$$

$$= -\int \frac{d(\cos x)}{\cos x} + \int \cos x\; d(\cos x)$$

$$= \underline{-\ln|\cos x| + \frac{1}{2}\cos^2 x + C}$$

$$\int \frac{\csc^4 x}{\cot^2 x}\,dx = \int \frac{\csc^2 x\,(\cot^2 x + 1)}{\cot^2 x}\,dx$$

$d\cot x = -\csc^2 x\,dx$

$$= -\int \left(1 + \frac{1}{\cot^2 x}\right) d(\cot x)$$

$$= -\left(\cot x - \frac{1}{\cot x}\right) + C$$

$$= \boxed{-\cot x + \tan x + C}$$

$$\int \csc^{10} x \cot x\,dx = -\int \csc^9 x\,d(\csc x)$$

$$= \boxed{-\frac{1}{10}\csc^{10} x + C}$$

$$\int \frac{\sec^4(\ln x)}{x}\,dx = \int \sec^4(\ln x)\,d(\ln x)$$

$$= \int \sec^2(\ln x)\,(1 + \tan^2 \ln x)\,d(\ln x)$$

$$= \int \sec^2(\ln x)\,d(\ln x) + \int \sec^2(\ln x)\tan^2(\ln x)\,d(\ln x)$$

$$= \tan(\ln x) + \int \tan^2(\ln x)\,d(\sec^2 \ln x)$$

$$= \boxed{\tan(\ln x) + \frac{1}{3}\tan^3 \ln x) + C}$$

$$\int_0^{\sqrt{\pi/2}} x \sin^3(x^2)\,dx = \frac{1}{2}\int_0^{\sqrt{\pi/2}} \sin^3(x^2)\,d(x^2)$$

$$= \frac{1}{2}\int_0^{\sqrt{\pi/2}} \sin^2(x^2)\sin(x^2)\,d(x^2)$$

$$= \frac{-1}{2}\int_0^{\sqrt{\pi/2}} (1 - \cos^2(x^2))\,d(\cos x^2)$$

$$= -\frac{1}{2}\left[\cos x^2 - \frac{1}{3}\cos^3 x^2\right|_0^{\sqrt{\pi/2}}$$

$$= -\frac{1}{2}\left(\cos\frac{\pi}{2} - \frac{1}{3}\cos^3\frac{\pi}{2} - 1 + \frac{1}{3}\right)$$

$$= -\frac{1}{2}\left(-\frac{2}{3}\right)$$

$$= \boxed{\frac{1}{3}}$$

$$\int_{\pi/6}^{\pi/3} \cot^3\theta\, d\theta = \int_{\pi/6}^{\pi/3} \cot\theta(\csc^2\theta - 1)\, d\theta$$

$$= \int_{\pi/6}^{\pi/3} \cot\theta \csc^2\theta d\theta - \int_{\pi/6}^{\pi/3} \frac{\cos\theta}{\sin\theta}\, d\theta$$

$$= -\int_{\pi/6}^{\pi/3} \cot\theta\, d(\cot\theta) - \int_{\pi/6}^{\pi/3} \frac{d\sin\theta}{\sin\theta}$$

$$= -\frac{1}{2}\cot^2\theta - \ln|\sin\theta| \Big|_{\pi/6}^{\pi/3}$$

$$= -\frac{1}{2}\frac{1}{3} - \ln\frac{\sqrt{3}}{2} + \frac{1}{2}3 + \ln\frac{1}{2}$$

$$= \frac{4}{3} + \ln\frac{1}{\sqrt{3}}$$

$$= \boxed{\frac{4}{3} - \ln\sqrt{3}}$$

$$\int \tan^5\theta \sec^4\theta\, d\theta = \int \tan^5\theta\,(1+\tan^2\theta)\sec^2\theta\, d\theta$$

$$= \int(\tan^5\theta + \tan^7\theta)\, d(\tan\theta)$$

$$= \boxed{\frac{1}{6}\tan^6\theta + \frac{1}{8}\tan^8\theta + C}$$

$$\int_{\pi/6}^{\pi/2} \frac{dy}{\sin y} = \int_{\pi/6}^{\pi/2} \csc y\, \frac{\csc y + \cot y}{\csc y + \cot y}\, dy$$

$$= \int_{\pi/6}^{\pi/2} \frac{\csc^2 y + \csc y \cot y}{\csc y + \cot y}\, dy$$

$d\csc y = -\csc^2 y\, dy$

$d\cot y = -\csc y \cot y\, dy$

$$= -\int_{\pi/6}^{\pi/2} \frac{d(\csc y + \cot y)}{\csc y + \cot y}$$

$$= -\ln|\csc y + \cot y| \Big|_{\pi/6}^{\pi/2}$$

$$= -\ln 1 + \ln|2+\sqrt{3}|$$

$$= \boxed{\ln(2+\sqrt{3})}$$

$$\int_{-\pi/3}^{\pi/3} \sqrt{\sec^2\theta - 1}\, d\theta = 2\int_0^{\pi/3} \tan\theta \, d\theta$$

$$= -2 \ln|\cos\theta| \Big|_0^{\pi/3}$$

$$= -2\left(\ln\tfrac{1}{2} - \ln 1\right)$$

$$= \boxed{2\ln 2}$$

$$\int_0^{\pi} (1-\cos 2x)^{3/2}\, dx = \int_0^{\pi} (2\sin^2 x)^{3/2}\, dx$$

$$= \int_0^{\pi} 2\sqrt{2}\, \sin^3 x \, dx$$

$$= 2\sqrt{2} \int_0^{\pi} \sin^2 x \sin x \, dx$$

$$= -2\sqrt{2} \int_0^{\pi} (1-\cos^2 x)\, d(\cos x)$$

$$= -2\sqrt{2}\left[\cos x - \tfrac{1}{3}\cos^3 x \right|_0^{\pi}$$

$$= -2\sqrt{2}\left[-1 + \tfrac{1}{3} - 1 + \tfrac{1}{3}\right]$$

$$= -2\sqrt{2}\left(\tfrac{-4}{3}\right)$$

$$= \boxed{\frac{8\sqrt{2}}{3}}$$

$$\int e^x \sec(e^x+1)\, dx = \int \sec(e^x+1)\, d(e^x+1)$$

$$= \boxed{\ln\left|\sec(e^x+1) + \tan(e^x+1)\right| + C}$$
