

# Lecture Two – Functions

## Section 2.1 – Functions and Graphs

### Increasing and Decreasing Functions

- ✚ A function *ris*es from left to right (*x*-coordinate), the function  $f$  is said to be **increasing** on an open interval  $I(a, b)$  (*x*-coordinate)

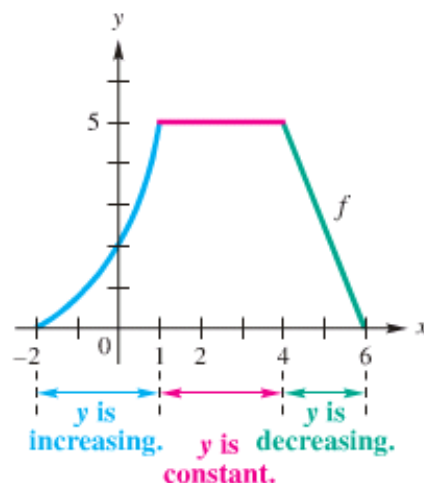
$$a < b \Rightarrow f(a) < f(b)$$

- ✚ A function  $f$  is said to be **decreasing** on an open interval  $I$

$$a < b \Rightarrow f(a) > f(b)$$

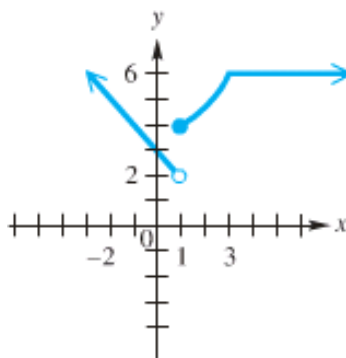
- ✚ A function  $f$  is said to be **constant** on an open interval  $I$

$$a < b \Rightarrow f(a) = f(b)$$



### Example

Determine the intervals over which the function is increasing, decreasing, or constant



Increasing:  $[1, 3]$

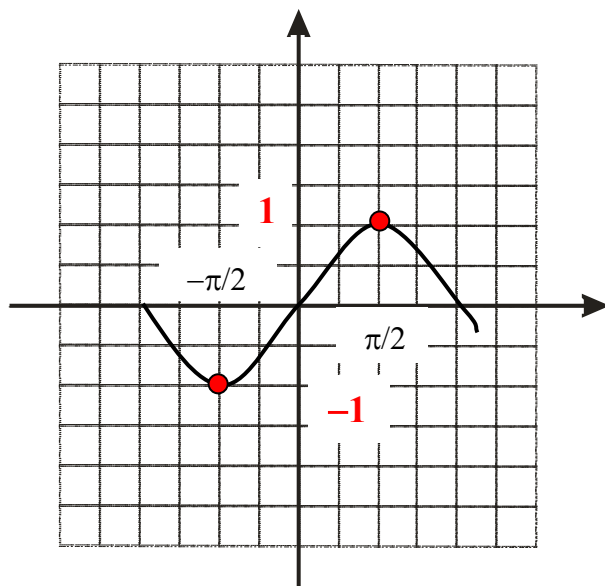
Decreasing:  $(-\infty, 1)$

Constant:  $[3, \infty)$

## Relative *Maxima* (um) and *Minima* (um)

$f(a)$  is a relative maximum if there exists an open interval  $I$  about  $a$  such that  $f(a) > f(x)$ , for all  $x$  in  $I$ .

$f(a)$  is a relative minimum if there exists an open interval  $I$  about  $a$  such that  $f(a) < f(x)$ , for all  $x$  in  $I$ .

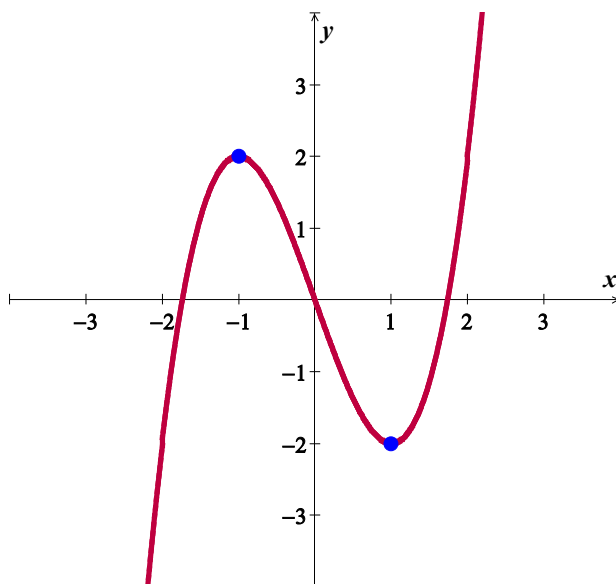


The relative minimum value of the function is  $-1$  @  $x = -\pi/2$

The relative maximum value of the function is  $1$  @  $x = \pi/2$

## Example

State the intervals on which the given function  $f(x) = x^3 - 3x$  is increasing, decreasing, or constant, and determine the extreme values



**Increasing**  $(-\infty, -1) \cup (1, \infty)$

**Decreasing**  $(-1, 1)$

**RMIN**  $(1, -2)$

**RMAX**  $(-1, 2)$

## Piecewise-Defined Functions

Function are sometimes described by more than one expression, we call such functions *piecewise-defined functions*.

### Example

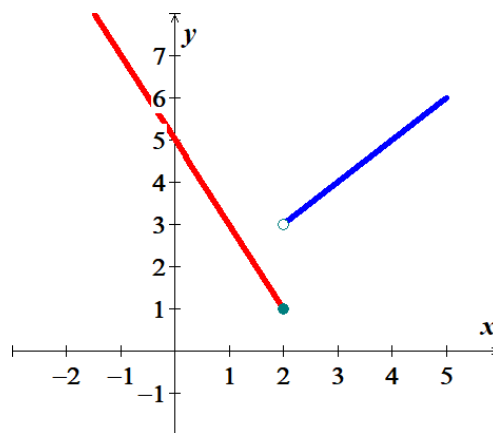
Graph function

$$f(x) = \begin{cases} -2x+5 & \text{if } x \leq 2 \\ x+1 & \text{if } x > 2 \end{cases}$$

Find:  $f(2) = -2(\textcolor{red}{2}) + 5 = \textcolor{blue}{1}$

$$f(0) = -2(\textcolor{red}{0}) + 5 = \textcolor{blue}{5}$$

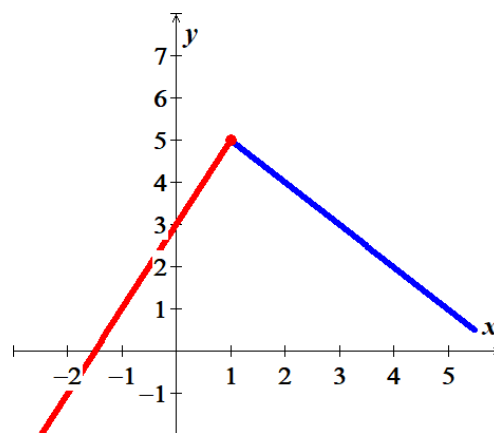
$$f(4) = \textcolor{red}{4} + 1 = \textcolor{blue}{5}$$



### Example

Graph function

$$f(x) = \begin{cases} 2x+3 & \text{if } x \leq 1 \\ -x+6 & \text{if } x > 1 \end{cases}$$



### Example

$$C(t) = \begin{cases} 20 & \text{if } 0 \leq t \leq 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$$

Find  $C(40)$ ,  $C(80)$ , and  $C(60)$

### Solution

a)  $C(40) = 20$

b)  $C(80) = 20 + 0.40(80 - 60) = 28$

c)  $C(60) = 20$

## Exercise Section 2.1 – Functions and Graphs

1.  $f(x) = \begin{cases} 2+x & \text{if } x < -4 \\ -x & \text{if } -4 \leq x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$  **Find:**  $f(-5)$ ,  $f(-1)$ ,  $f(0)$ , and  $f(3)$

2.  $f(x) = \begin{cases} -2x & \text{if } x < -3 \\ 3x-1 & \text{if } -3 \leq x \leq 2 \\ -4x & \text{if } x > 2 \end{cases}$  **Find:**  $f(-5)$ ,  $f(-1)$ ,  $f(0)$ , and  $f(3)$

3.  $f(x) = \begin{cases} x^3+3 & \text{if } -2 \leq x \leq 0 \\ x+3 & \text{if } 0 < x < 1 \\ 4+x-x^2 & \text{if } 1 \leq x \leq 3 \end{cases}$  **Find:**  $f(-5)$ ,  $f(-1)$ ,  $f(0)$ , and  $f(3)$

4.  $h(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$  **Find:**  $h(5)$ ,  $h(0)$ , and  $h(3)$

5.  $f(x) = \begin{cases} 3x+5 & \text{if } x < 0 \\ 4x+7 & \text{if } x \geq 0 \end{cases}$  **Find**

a)  $f(0)$       b)  $f(-2)$       c)  $f(1)$       d)  $f(3)+f(-3)$       e) Graph  $f(x)$

6.  $f(x) = \begin{cases} 6x-1 & \text{if } x < 0 \\ 7x+3 & \text{if } x \geq 0 \end{cases}$  **Find**

a)  $f(0)$       b)  $f(-1)$       c)  $f(4)$       d)  $f(2)+f(-2)$       e) Graph  $f(x)$

7.  $f(x) = \begin{cases} 2x+1 & \text{if } x \leq 1 \\ 3x-2 & \text{if } x > 1 \end{cases}$  **Find**

a)  $f(0)$       b)  $f(2)$       c)  $f(-2)$       d)  $f(1)+f(-1)$       e) Graph  $f(x)$

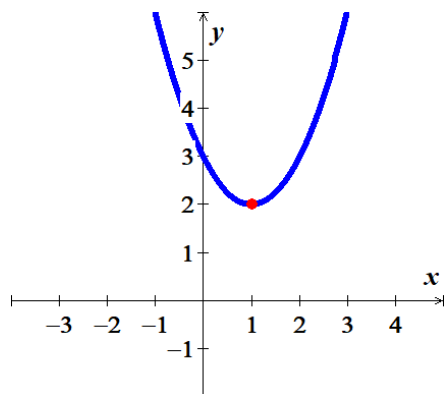
8. Graph the piecewise function defined by  $f(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ x-2 & \text{if } x > -1 \end{cases}$

9. Sketch the graph  $f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x^3 & \text{if } -1 < x < 1 \\ -x+3 & \text{if } x \geq 1 \end{cases}$

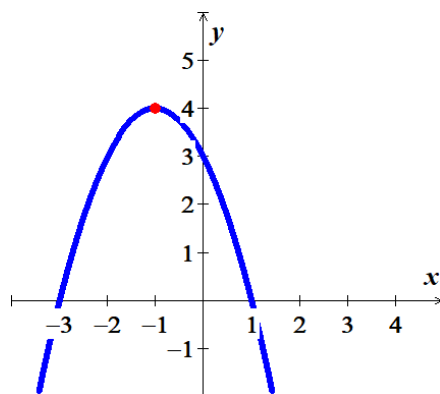
10. Sketch the graph  $f(x) = \begin{cases} x-3 & \text{if } x \leq -2 \\ -x^2 & \text{if } -2 < x < 1 \\ -x+4 & \text{if } x \geq 1 \end{cases}$

(37 – 42) Determine any **relative maximum** or **minimum** of the function, determine the intervals on which the function **increasing** or **decreasing**, and then find the **domain** and the **range**.

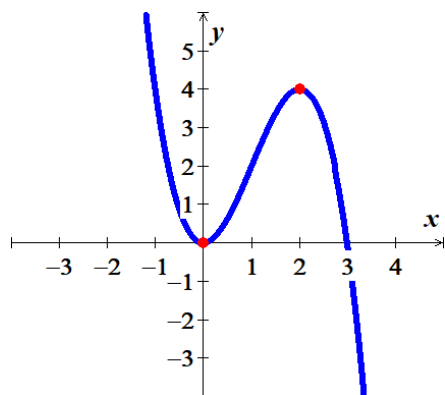
11.  $f(x) = x^2 - 2x + 3$



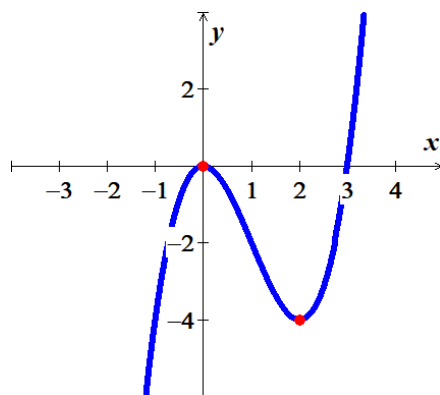
12.  $f(x) = -x^2 - 2x + 3$



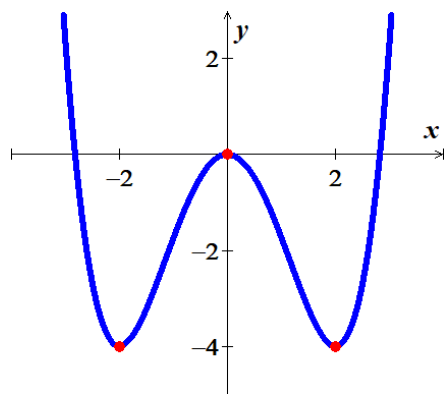
13.  $f(x) = -x^3 + 3x^2$



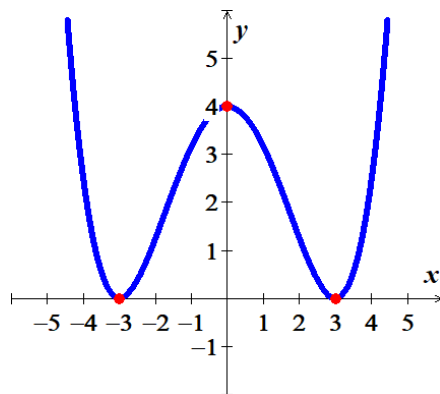
14.  $f(x) = x^3 - 3x^2$



15.  $f(x) = \frac{1}{4}x^4 - 2x^2$



16.  $f(x) = \frac{4}{81}x^4 - \frac{8}{9}x^2 + 4$

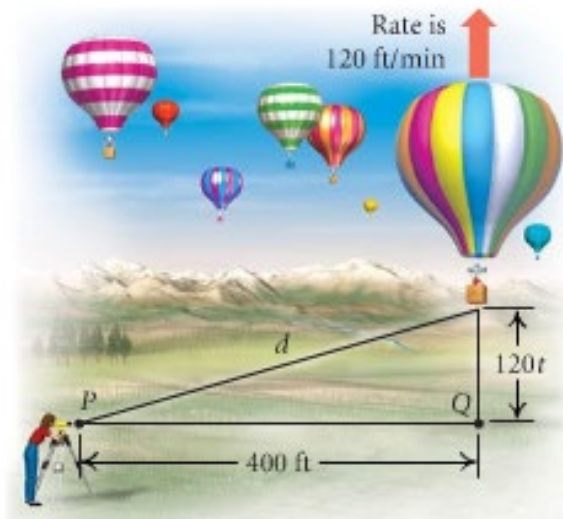


17. The elevation  $H$ , in *meters*, above sea level at which the boiling point of water is in  $t$  *degrees Celsius* is given by the function

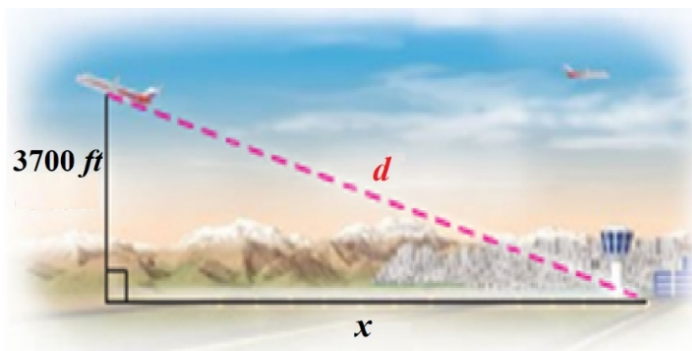
$$H(t) = 1000(100 - t) + 580(100 - t)^2$$

At what elevation is the boiling point  $99.5^\circ$ .

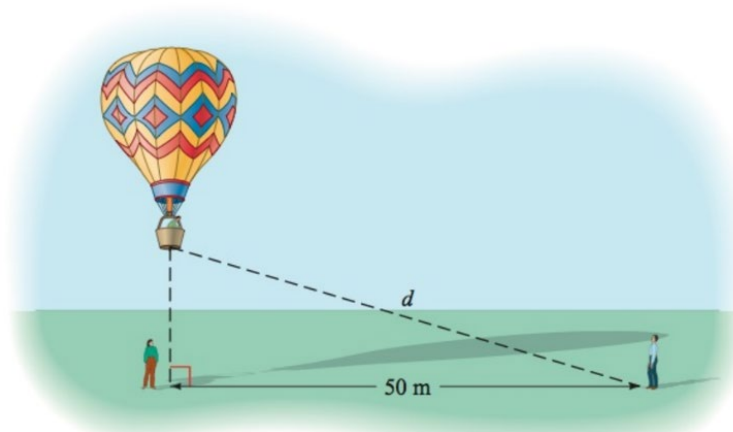
18. A hot-air balloon rises straight up from the ground at a rate of  $120 \text{ ft./min.}$  The balloon is tracked from a rangefinder on the ground at point  $P$ , which is  $400 \text{ feet.}$  from the release point  $Q$  of the balloon. Let  $d$  be the distance from the balloon to the rangefinder and  $t$  – the time, in *minutes*, since the balloon was released. Express  $d$  as a function of  $t$ .



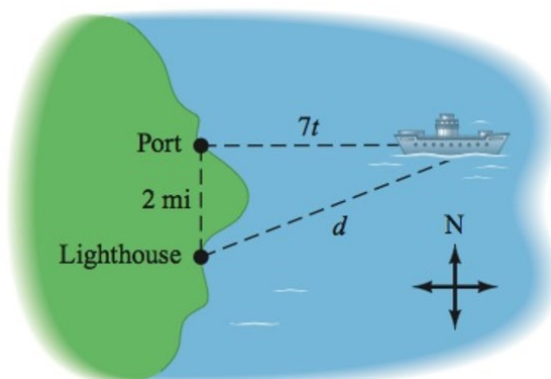
19. An airplane is flying at an altitude of  $3700 \text{ feet.}$  The slanted distance directly to the airport is  $d \text{ feet.}$  Express the horizontal distance  $x$  as a function of  $d$ .



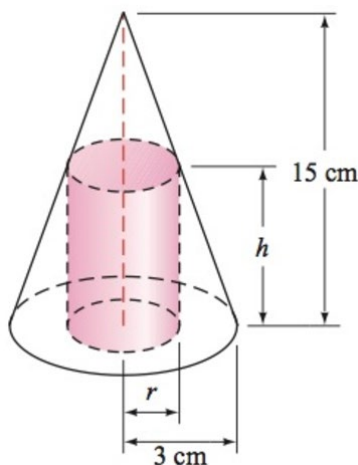
20. For the first minute of flight, a hot air balloon rises vertically at a rate of  $3 \text{ m/sec}$ . If  $t$  is the time in *seconds* that the balloon has been airborne, write the distance  $d$  between the balloon and a point on the ground  $50 \text{ meters}$  from the point to lift off as a function of  $t$ .



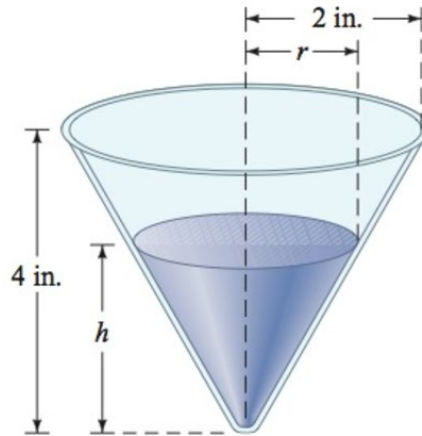
21. A light house is  $2 \text{ miles}$  south of a port. A ship leaves port and sails east at a rate of  $7 \text{ miles per hour}$ . Express the distance  $d$  between the ship and the lighthouse as a function of time, given that the ship has been sailing for  $t \text{ hours}$ .



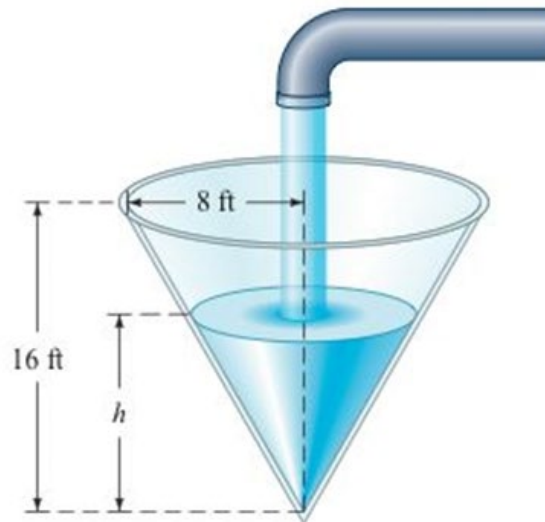
22. A cone has an altitude of  $15 \text{ cm}$  and a radius of  $3 \text{ cm}$ . A right circular cylinder of radius  $r$  and height  $h$  is inscribed in the cone. Use similar triangles to write  $h$  as a function of  $r$ .



23. Water is flowing into a conical drinking cup with an altitude of 4 inches and a radius of 2 inches.

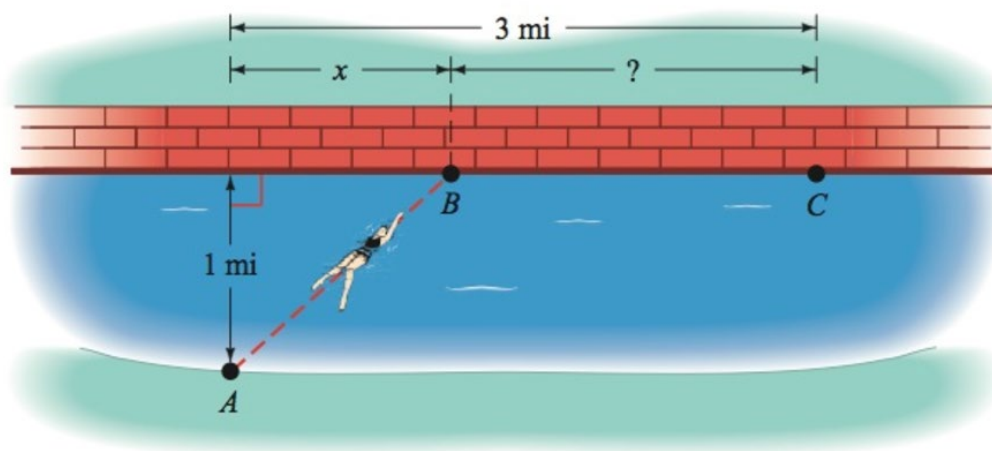


- Write the radius  $r$  of the surface of the water as a function of its depth  $h$ .
  - Write the volume  $V$  of the water as a function of its depth  $h$ .
24. A water tank has the shape of a right circular cone with height 16 feet and radius 8 feet. Water is running into the tank so that the radius  $r$  (in feet) of the surface of the water is given by  $r = 1.5t$ , where  $t$  is the time (in minutes) that the water has been running.

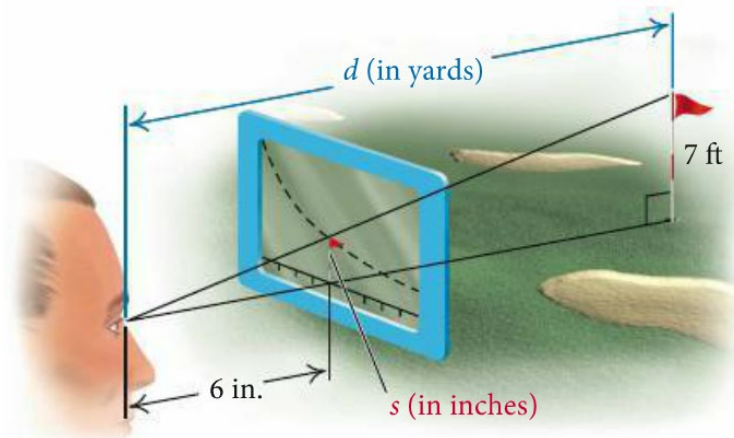


- The area  $A$  of the surface of the water is  $A = \pi r^2$ . Find  $A(t)$  and use it to determine the area of the surface of the water when  $t = 2$  minutes.
  - The volume  $V$  of the water is given by  $V = \frac{1}{3}\pi r^2 h$ . Find  $V(t)$  and use it to determine the volume of the water when  $t = 3$  minutes.
25. An athlete swims from point  $A$  to point  $B$  at a rate of 2 miles per hour and runs from point  $B$  to point  $C$  at a rate of 8 miles per hour. Use the dimensions in the figure to write the time  $t$  required to reach point  $C$  as a function of  $x$ .

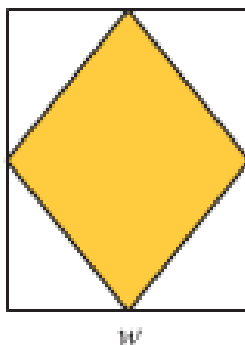




26. A device used in golf to estimate the distance  $d$ , in *yards*, to a hole measures the size  $s$ , in *inches*, that the 7-foot pin appears to be in a viewfinder. Express the distance  $d$  as a function of  $s$ .



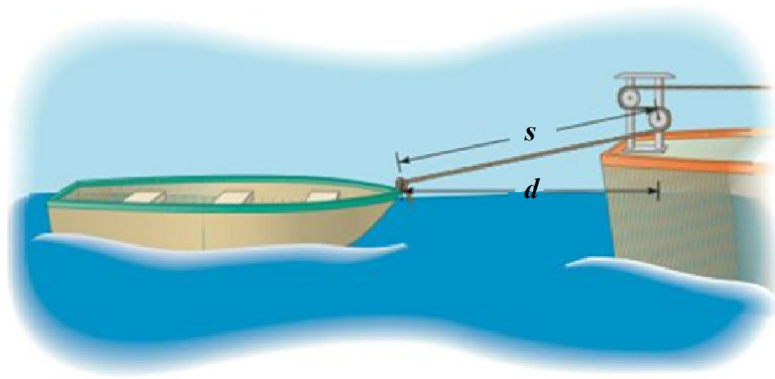
27. A rhombus is inscribed in a rectangle that is  $w$  meters wide with a perimeter of 40  $m$ . Each vertex of the rhombus is a midpoint of a side of the rectangle. Express the area of the rhombus as a function of the rectangle's width.



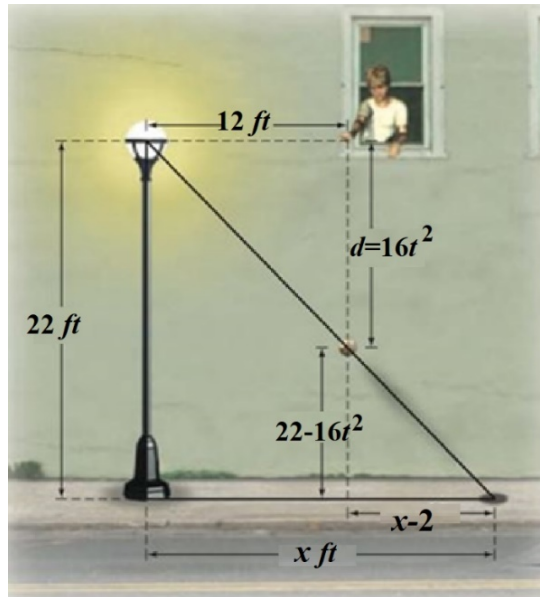
28. The surface area  $S$  of a right circular cylinder is given by the formula  $S = 2\pi rh + 2\pi r^2$ . If the height is twice the radius, find each of the following.



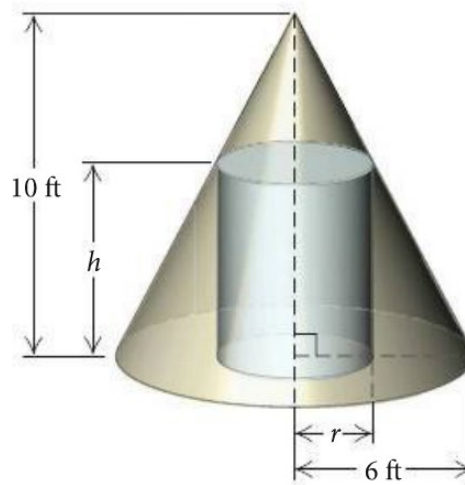
- a) A function  $S(r)$  for the surface area as a function of  $r$ .  
b) A function  $S(h)$  for the surface area as a function of  $h$ .
29. A boat is towed by a rope that runs through a pulley that is 4 feet above the point where the rope is tied to the boat. The length (in feet) of the rope from the boat to the pulley is given by  $s = 48 - t$ , where  $t$  is the time in seconds that the boat has been in tow. The horizontal distance from the pulley to the boat is  $d$ .



- a) Find  $d(t)$   
b) Evaluate  $s(35)$  and  $d(35)$
30. The light from a lamppost casts a shadow from a ball that was dropped from a height of 22 feet above the ground. The distance  $d$ , in feet, the ball has dropped  $t$  seconds after it is released is given by  $d(t) = 16t^2$ . Find the distance  $x$ , in feet, of the shadow from the base of the lamppost as a function of time  $t$ .



31. \*A right circular cylinder of height  $h$  and a radius  $r$  is inscribed in a right circular cone with a height of 10 feet and a base with radius 6 feet.



- Express the height  $h$  of the cylinder as a function of  $r$ .
- Express the volume  $V$  of the cylinder as a function of  $r$ .
- Express the volume  $V$  of the cylinder as a function of  $h$ .