Solution Section 2.1 – Simple and Compound Interest

Exercise

If you want to earn an annual rate of 10% on your investments, how much should you pay for a note that will be worth $5,000 in 6 month?

Solution

$$A = P(1+rt)$$

$$5000 = P\left(1+.1(\tfrac{6}{12})\right)$$

$$5000 = P\left(1+\tfrac{.1}{2}\right)$$

$$P = \frac{5000}{\left(1+\frac{.1}{2}\right)} = \$4761.90$$

5000 / (1 + .1 / 2)

Exercise

a) How much should you deposit initially in an account paying 10% compounded semiannually in order to have $1,000,000 in 30 years?

b) Compounded monthly?

c) Compounded daily?

Solution

$$A = P\left(1+\frac{r}{m}\right)^{mt} \Rightarrow P = \frac{A}{\left(1+\frac{r}{m}\right)^{mt}} = or = A\left(1+\frac{r}{m}\right)^{-mt}$$

$$\textbf{a)}\ P = \frac{1000000}{(1+0.10/2)^{60}} = \$53,535.52$$

$$or\ \ 1000000(1+0.10/2)^{-60} = \$53,535.52$$

$$\textbf{b)}\ P = 1000000(1+0.10/12)^{-360} = \$50,409.83$$

$$\textbf{c)}\ P = 1000000(1+0.10/365)^{-10950} = \$49,807.53$$

Exercise

You have \$7,000 toward the purchase of a \$10,000 automobile. How long will it take the \$7000 to grow to the \$10,000 if it is invested at 9% compounded quarterly? (Round up to the next highest quarter if not exact.)

Solution

$$10,000 = 7,000(1+0.09/4)^{n}$$

$$10/7 = 1.0225^{n}$$

$$\rightarrow \ln(10/7) = \ln 1.0225^{n}$$

Log Power Property: $\ln a^{x} = x\ln a$

$$\rightarrow \ln(10/7) = n\ln 1.0225$$

$$n = \frac{\ln(10/7)}{\ln 1.0225} = 16.03 \ \ or \ \ \underline{17\ quarters}$$

Exercise

How long, to the nearest tenth of a year, will it take \$1000 to grow to \$3600 at 8% annual interest compounded quarterly?

Solution

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$3600 = 1000\left(1+\frac{0.08}{4}\right)^{4t}$$

$$3.6 = (1.02)^{4t}$$

$$\ln 3.6 = \ln(1.02)^{4t}$$

$$\ln 3.6 = 4t\ln(1.02)$$

$$\frac{\ln 3.6}{4\ln 1.02} = t$$

$$\Rightarrow \boxed{t \approx 16.2\,yr}$$

Exercise

Jennifer invested \$4,000 in her savings account for 4 years. When she withdrew it, she had \$4,350.52. Interest was compounded continuously. What was the interest rate on the account? Round to the nearest tenth of a percent.

Solution

$$A = Pe^{rt}$$

$$4350.52 = 4000e^{r4}$$

$$\frac{4350.52}{4000} = e^{4r}$$

$$\ln\left(\frac{4350.52}{4000}\right) = \ln e^{4r}$$

Inverse Property: $\ln e^{x} = x$

$$\ln\left(\frac{4350.52}{4000}\right) = 4r$$

$$\to r = \frac{1}{4}\ln\left(\frac{4350.52}{4000}\right) \approx .021$$

$$\Rightarrow \boxed{r = 2.1\%}$$

Exercise

An actuary for a pension fund need to have \$14.6 million grow to \$22 million in 6 years. What interest rate compounded annually does he need for this investment to growth as specified. Round your answer to the nearest hundredth of a percent.

Solution

$$22 = 14.6\left(1 + \frac{r}{1}\right)^{(1)(6)}$$

$$22 = 14.6(1 + r)^{6}$$

$$\frac{22}{14.6} = (1 + r)^{6}$$

$$\left(\frac{22}{14.6}\right)^{1/6} = 1 + r$$

$$r = \left(\frac{22}{14.6}\right)^{1/6} - 1$$

$$\approx .0707$$

$$\to \boxed{r \approx 7.07\%}$$

Exercise

Which is the better investment: 9% compounded monthly or 9.1% compounded quarterly?

Solution

For 9%: $APY = r_e = \left(1+\frac{0.09}{12}\right)^{12} - 1 = 9.38\%$

For 9.1%: $r_e = \left(1+\frac{0.091}{4}\right)^{4} - 1 = 9.42\%$

9.1% is better

Exercise

Sun Kang borrowed $5200 from his friend to pay for remodeling work on his house. He repaid the loan 10 months later with simple interest at 7%. His friend then invested the proceeds in a 5-year CD paying 6.3% compounded quarterly. How much will his friend have at the end of the 5 years?

Solution

For 7%: $A_1 = P(1+rt)$

$$A_1 = 5200\left(1+0.07\tfrac{10}{12}\right) \underline{= \$5503.33}$$

For 6.3%: $A_2 = 5503.33\left(1+\frac{0.063}{4}\right)^{20} \underline{= \$7,522.50}$

Exercise

The consumption of electricity has increased historically at 6% per year. If it continues to increase at this rate indefinitely, find the number of years before the electric utilities will need to double their generating capacity. {Round up to the next highest year}

Solution

$$2P = P\left(1+\frac{0.06}{1}\right)^n$$

$$\Rightarrow 2 = 1.06^n$$ *ln both sides*

$$\ln 2 = \ln 1.06^n$$

$$\ln 2 = n\ln 1.06$$

$$\underline{n} = \frac{\ln 2}{\ln 1.06} \underline{= 12 \ years}$$

Exercise

In the New Testament, Jesus commends a widow who contributed 2 mites (roughly ¼ cent) to the temple treasury. Suppose the temple invested those mites at 4% compounded quarterly. How much would the money be worth 2000 years later?

Solution

Given: $P = \frac{1}{4}¢ = .25¢\,\frac{\$1}{100¢} = \$0.0025$

$r = 0.04 \quad m = 4 \quad t = 2000$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$A = 0.0025\left(1+\frac{0.04}{4}\right)^{4(2000)}$$

0.0025(1+0.04/4)^(4*2000)

$$\boxed{= \$9.3\,x\,10^{31}}$$

Exercise

If \$1,000 is invested in an account that earns 9.75% compounded annually for 6 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

Solution

Given: $P = 1,000 \quad r = .0975 \quad m = 1 \quad t = 6$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

1st year: $t = 1 \Rightarrow \quad A_1 = 1000\left(1+\frac{.0975}{1}\right)^{1(1)} = \$1,097.50$

$\mathit{Interest} = \$1,097.50 - \$1,000 = \$97.50$

2nd year: $t = 2 \Rightarrow \quad A_2 = 1000(1+.0975)^2 = \$1,204.51$

$\mathit{Interest} = \$1,204.51 - \$1,097.50 = \$107.01$

3rd year: $t = 3 \Rightarrow \quad A_3 = 1000(1+.0975)^3 = \$1,321.95$

4th year: $t = 4 \Rightarrow \quad A_4 = 1000(1+.0975)^4 = \$1,450.84$

5th year: $t = 5 \Rightarrow \quad A_5 = 1000(1+.0975)^5 = \$1,592.29$

6th year: $t = 6 \Rightarrow \quad A_6 = 1000(1+.0975)^6 = \$1,747.54$

Period	***Amount***	***Interest***
0	\$1,000.00	
1	\$1,097.50	\$97.50
2	\$1,204.51	\$107.01
3	\$1,321.95	\$117.44
4	\$1,450.84	\$128.89
5	\$1,582.29	\$141.46
6	\$1,747.54	\$155.25

Exercise

If \$2,000 is invested in an account that earns 8.25% compounded annually for 5 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

Solution

Given: $P = 2,000 \quad r = 8.25\% = .08.25 \quad m = 1 \quad t = 5$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

1^{st} year: $t = 1 \Rightarrow \quad A_1 = 1000\left(1 + \frac{.0825}{1}\right)^{1(1)} = \$2,165.00$

$\quad$ *Interest* $= \$2,165.00 - \$2,000 = \$165.00$

2^{nd} year: $t = 2 \Rightarrow \quad A_2 = 2000(1 + .0825)^2 = \$2,343.61$

$\quad$ *Interest* $= \$2,343.61 - \$2,165.00 = \$178.61$

3^{rd} year: $t = 3 \Rightarrow \quad A_3 = 2000(1 + .0825)^3 = \$2,536.96$

4^{th} year: $t = 4 \Rightarrow \quad A_4 = 2000(1 + .0825)^4 = \$2,746.26$

5^{th} year: $t = 5 \Rightarrow \quad A_5 = 2000(1 + .0825)^5 = \$2,972.83$

Period	***Amount***	***Interest***
0	\$2,000.00	
1	\$2,165.00	\$165.00
2	\$2,343.61	\$178.61
3	\$2,536.96	\$193.35
4	\$2,746.26	\$209.30
5	\$2,972.83	\$226.57

Exercise

If an investment company pays 6% compounded semiannually, how much you should deposit now to have \$10,000

a) 5 years from now?

b) 10 years from now?

Solution

Given: $A = 10,000 \quad r = .06 \quad m = 2$

a) $t = 5 \quad A = P\left(1 + \frac{r}{m}\right)^{mt}$

$$10,000 = P\left(1 + \frac{.06}{2}\right)^{2(5)}$$

$$10,000 = P(1.03)^{10}$$

$$\boxed{P = \frac{10,000}{(1.03)^{10}} = \$7,440.94}$$

b) $t = 10$

$$10,000 = P(1.03)^{2(10)}$$

$$\underline{|P} = \frac{10,000}{(1.03)^{20}} \ \underline{= \$5,536.76|}$$

Exercise

If an investment company pays 8% compounded quarterly, how much you should deposit now to have $6,000

a) 3 years from now?

b) 6 years from now?

Solution

Given: $A = 6,000 \quad r = 8\% = .05 \quad m = 4$

a) $t = 3 \quad A = P\left(1 + \frac{r}{m}\right)^{mt}$

$$6,000 = P\left(1 + \frac{.08}{4}\right)^{4(3)}$$

$$6,000 = P(1.02)^{12}$$

$$\underline{|P} = \frac{6,000}{(1.02)^{12}} \ \underline{= \$4,730.96|}$$

6000 / 1.02 ^ 12

b) $t = 6$

$$6,000 = P\left(1 + \frac{.08}{4}\right)^{4(6)}$$

$$6,000 = P(1.02)^{24}$$

$$\underline{|P} = \frac{6,000}{(1.02)^{24}} \ \underline{= \$3,730.33|}$$

Exercise

What is the annual percentage yield (*APY*) for money invested at:

a) 4.5% compounded monthly?

b) 5.8% compounded quarterly?

Solution

a) ***Given***: $r = 0.045 \quad m = 12$

$$APY = \left(1+\frac{.045}{12}\right)^{12} - 1 \approx 0.04594 \qquad (1+.045/12)\wedge 12-1$$

$\boxed{APY:\ 4.594\%}$

b) ***Given***: $r = 0.058 \quad m = 4$

$$APY = \left(1+\frac{.058}{4}\right)^{4} - 1 \approx 0.05927 \qquad (1+.058/4)\wedge 4-1$$

$\boxed{APY:\ 5.927\%}$

Exercise

48.

What is the annual percentage yield (APY) for money invested at

a) 6.2% compounded semiannually?

b) 7.1% compounded monthly?

Solution

a) ***Given***: $r = 6.2\% = 0.062 \quad m = 2$

$$APY = \left(1+\frac{.062}{2}\right)^{2} - 1 \approx 0.06296 \qquad (1+.062/2)\wedge 2-1$$

$\boxed{APY:\ 6.296\%}$

b) ***Given***: $r = 7.1\% = 0.071 \quad m = 12$

$$APY = \left(1+\frac{.071}{12}\right)^{12} - 1 \approx 0.07336 \qquad (1+.071/12)\wedge 12-1$$

$\boxed{APY:\ 7.336\%}$

Exercise

A newborn child receives a $20,000 gift toward a college education from her grandparents. How much will the $20,000 be worth in 17 years if it is invested at 7% compounded quarterly?

Solution

Given: $P = 20,000 \quad r = 7\% = .07 \quad m = 4 \quad t = 17$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$= 20,000\left(1 + \frac{.07}{4}\right)^{4(17)} \qquad 20000(1+.07/4)\text{^}(4*17)$$

$$= \$65,068.44$$

Exercise

A person with $14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy more expensive car. How much will be available for the purchase of a car at the end of 3 years?

Solution

Given: $P = 14,000 \quad r = 6.5\% = .065 \quad m = 2 \quad t = 3$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$= 14,000\left(1 + \frac{.065}{2}\right)^{2(3)} \qquad 14000(1+.065/2)\text{^}(2*3)$$

$$= \$16,961.66$$

Exercise

You borrowed $7200 from a bank to buy a car. You repaid the bank after 9 months at an annual interest rate of 6.2%. Find the total amount you repaid. How much of this amount is interest?

Solution

Given: $P = 7,200 \quad r = 6.2\% = .062 \quad t = \frac{9}{12}$

$$A = P(1 + rt)$$

$$= 7200\left[1 + .062\left(\frac{9}{12}\right)\right] \qquad 7200(1+.07(9/12))$$

$$= \$7,534.80$$

Exercise

An account for a corporation forgot to pay the firm's income tax of \$321,812.85 on time. The government changed a penalty based on an annual interest rate of 13.4% for the 29 days the money was late. Find the total amount (tax and penalty) that was paid. (Use 365 days a year.)

Solution

Given: $P = 321,812.85 \quad r = 13.4\% = .13.4 \quad t = \dfrac{29}{365}$

$$A = P(1 + rt)$$

$$= 321,812.85\left[1 + .134\left(\frac{29}{365}\right)\right] \qquad\qquad 7200(1 + .07(9/12))$$

$$\underline{= \$325,239.05}$$

Exercise

A bond with a face value of \$10,000 in 10 years can be purchased now for \$5,988.02. What is the simple interest rate?

Solution

The interest earned is: \$10,000 – \$5,988.02 = \$4011.98

Given: $P = 5988.02 \quad I = 4011.98 \quad t = 10$

$$I = \Pr t$$

$$4011.98 = 5988.02\, \boldsymbol{r}\ (10)$$

$$\boldsymbol{r} = \frac{4011.98}{5988.02(10)} \approx 0.067 \qquad\qquad 4011.98/(5988.02*10)$$

The interest rate was about $\underline{6.7\%}$

Exercise

A stock that sold for \$22 at the beginning of the year was selling for \$24 at the end of the year. If the stock paid a dividend of \$0.50 per share, what is the simple interest rate on an investment in this stock?

Solution

The interest earned is: $(\$24 - \$22) + \$0.50 = \$2.50$

Given: $P = 22 \quad I = 2.50 \quad t = 1$

$$I = \Pr t$$

$$2.50 = 22\, \boldsymbol{r}\ (1)$$

$$\boldsymbol{r} = \frac{2.5}{22} \approx 0.11364$$

The interest rate was about $\underline{11.36\%}$

Exercise

The Frank Russell Company is an investment fund that tracks the average performance of various groups of stocks. On average, a \$10,000 investment in midcap growth funds over a recent 10-year period would have grown to \$63,000. What annual nominal rate would produce the same growth if

a) Annually

b) Continuously

Solution

a) Annually: $m = 1$

$$63,000 = 10,000\left(1+\frac{r}{1}\right)^{1(10)}$$

$$\frac{63000}{10000} = (1+r)^{10}$$

$$6.3 = (1+r)^{10}$$

$$\sqrt[10]{6.3} = 1+r$$

$$r = \sqrt[10]{6.3} - 1 \approx 0.20208 \quad or \quad \underline{20.208\%}$$

b) Continuously : $A = Pe^{rt}$

$$63,000 = 10,000e^{r(10)}$$

$$6.3 = e^{10r}$$

$$ln 6.3 = ln e^{10r} \qquad\qquad \ln e^x = x$$

$$ln 6.3 = 10r$$

$$r = \frac{ln 6.3}{10 ln e} \approx 0.18405 \quad or \quad \underline{18.405\%}$$

Solution Section 2.2 – Future Value of an Annuity

Exercise

Recently, Guaranty Income Life offered an annuity that pays 6.65% compounded monthly. If $500 is deposited into this annuity every month, how much is in the account after 10 years? How much of this is interest?

Solution

Given: $PMT = 500 \quad r = 6.65\% = .0665 \quad m = 12 \quad t = 10$

$$i = \frac{r}{m} = \frac{.0665}{12} \quad n = mt = 12(10) = 120$$

$$\begin{aligned} FV &= PMT\,\frac{(1+i)^n - 1}{i} \\ &= 500\,\frac{\left(1+\frac{.0665}{12}\right)^{120} - 1}{\frac{.0665}{12}} \\ &= \$84,895.10 \end{aligned}$$

Total deposits: $500(120) = \$60,000.00$

$$\begin{aligned} Interest &= FV - Deposits \\ &= 84,895.40 - 60,000 \\ &= \underline{\$24,895.40} \end{aligned}$$

Exercise

Recently, USG Annuity Life offered an annuity that pays 4.25% compounded monthly. If $1,000 is deposited into this annuity every month, how much is in the account after 15 years? How much of this is interest?

Solution

Given: $PMT = 1,000 \quad r = 4.25\% = .0425 \quad m = 12 \quad t = 15$

$$i = \frac{r}{m} = \frac{.0425}{12} \quad n = mt = 12(15) = 180$$

$$\begin{aligned} FV &= PMT\,\frac{(1+i)^n - 1}{i} \\ &= 1000\,\frac{\left(1+\frac{.0425}{12}\right)^{180} - 1}{\frac{.0425}{12}} \\ &= \$251,185.76 \end{aligned}$$

Total deposits: $1,000(180) = \$180,000.00$

$$\begin{aligned} Interest &= FV - Deposits \\ &= 251,185.76 - 180,000 \\ &= \underline{\$71,185.76} \end{aligned}$$

Exercise

In order to accumulate enough money for a down payment on a house, a couple deposits \$300 per month into an account paying 6% compounded monthly. If payments are made at the end of each period, how much money will be in the account in 5 years?

Solution

Given: $PMT = 300 \quad r = 6\% = .06 \quad m = 12 \quad t = 5$

$$i = \frac{r}{m} = \frac{.06}{12} = 0.005 \qquad n = mt = 12(5) = 60$$

$$\begin{aligned} FV &= PMT\,\frac{(1+i)^n - 1}{i} \\ &= 300\,\frac{(1+.005)^{60} - 1}{.005} \\ &= \underline{\$20,931.01} \end{aligned}$$

Exercise

A self-employed person has a Keogh retirement plan. (This type of plan is free of taxes until money is withdrawn.) If deposits of \$7,500 are made each year into an account paying 8% compounded annually, how much will be in the account after 20 years?

Solution

Given: $PMT = 7,500 \quad r = 8\% = .08 \quad m = 1 \quad t = 20$

$$i = \frac{r}{m} = \frac{.08}{1} = 0.08 \qquad n = mt = 1(20) = 20$$

$$\begin{aligned} FV &= PMT\,\frac{(1+i)^n - 1}{i} \\ &= 7,500\,\frac{(1+.08)^{20} - 1}{.08} \\ &= \underline{\$343,214.73} \end{aligned}$$

Exercise

Sun America recently offered an annuity that pays 6.35% compounded monthly. What equal monthly deposit should be made into this annuity in order to have \$200,000 in 15 years?

Solution

Given: $FV = 200,000 \quad r = 6.35\% = .0635, \quad m = 12, \quad t = 15$

$$i = \frac{r}{m} = \frac{.0635}{12} \qquad n = mt = 12(15) = 180$$

$$PMT = FV\frac{i}{(1+i)^n - 1}$$

$$= 200,000\frac{\frac{.0635}{12}}{\left(1+\frac{.0635}{12}\right)^{180} - 1} \qquad 200000(.0635/12)/((1+.0635/12)\wedge 180-1)$$

$$\underline{= \$667.43|} \quad per\ month$$

Exercise

Recently, The Hartford offered an annuity that pays 5.5% compounded monthly. What equal monthly deposit should be made into this annuity in order to have \$100,000 in 10 years?

Solution

Given: $FV = 100,000 \quad r = 5.5\% = .055, \quad m = 12, \quad t = 10$

$$i = \frac{r}{m} = \frac{.055}{12} \qquad n = mt = 12(10) = 120$$

$$PMT = FV\frac{i}{(1+i)^n - 1}$$

$$= 100,000\frac{\frac{.055}{12}}{\left(1+\frac{.055}{12}\right)^{120} - 1} \qquad 100000(.055/12)/((1+.055/12)\wedge 120-1)$$

$$\underline{= \$626.93|} \quad per\ month$$

Exercise

Compu-bank, an online banking service, offered a money market account with an APY of 4.86%.

a) If interest is compounded monthly, what is the equivalent annual nominal rate?

b) If you wish to have \$10,000 in the account after 4 years, what equal deposit should you make each month?

Solution

Given: $APY = 4.86\% = .0486$

$$APY = \left(1+\frac{r}{m}\right)^m - 1$$

a) $m = 12$

$$.0486 = \left(1+\frac{r}{12}\right)^{12} - 1$$

Add1 on both sides

$$1.0486 = \left(1+\frac{r}{12}\right)^{12}$$

$$(1.0486)^{1/12} = 1+\frac{r}{12}$$

$$\frac{r}{12} = (1.0486)^{1/12} - 1$$

$$r = 12\left[(1.0486)^{1/12} - 1\right]$$

$12\left(1.0486\ \wedge\left(1/12\right)-1\right)$

$$\approx 0.0475$$

The equivalent annual nominal rate $\boxed{r = 4.75\%}$

b) ***Given***: $FV = \$10,000 \quad r = .0475, \quad m = 12, \quad t = 4$

$$i = \frac{r}{m} = \frac{.0475}{12} \qquad n = mt = 12(4) = 48$$

$$PMT = FV\frac{i}{(1+i)^n - 1}$$

$$= 10,000\frac{\frac{.0475}{12}}{\left(1+\frac{.0475}{12}\right)^{48} - 1}$$

$10000\left(.0475/12\right)/\left(\left(1+.0475/12\right)\wedge 60-1\right)$

$\underline{= \$189.58|}$ *per month*

Exercise

American Express's online banking division offered a money market account with an APY of 5.65%.

a) If interest is compounded monthly, what is the equivalent annual nominal rate?

b) If you wish to have $1,000,000 in the account after 8 years, what equal deposit should you make each month?

Solution

Given: $APY = 5.65\% = .0565$

$$APY = \left(1+\frac{r}{m}\right)^m - 1$$

a) $m = 12$

$$.0565 = \left(1+\frac{r}{12}\right)^{12} - 1$$

Add1 on both sides

$$1.0565 = \left(1+\frac{r}{12}\right)^{12}$$

$$(1.0565)^{1/12} = 1+\frac{r}{12}$$

$$\frac{r}{12} = (1.0565)^{1/12} - 1$$

$$r = 12\left[(1.0565)^{1/12} - 1\right]$$

$12\left(1.0565 \wedge (1/12) - 1\right)$

$$\approx 0.0551$$

The equivalent annual nominal rate $\boxed{r = 5.51\%}$

b) ***Given***: $FV = \$1,000,000 \quad r = .0551, \quad m = 12, \quad t = 8$

$$i = \frac{r}{m} = \frac{.0551}{12} \quad n = mt = 12(8) = 96$$

$$PMT = FV\frac{i}{(1+i)^n - 1}$$

$$= 1,000,000\frac{\frac{.0551}{12}}{\left(1+\frac{.0551}{12}\right)^{96} - 1}$$

$1000000(.0551/12)/\left((1+.051/12) \wedge 96 - 1\right)$

$$\boxed{= \$8,312.47} \quad per\ month$$

Exercise

Find the future value of an annuity due if payments of \$500 are made at the beginning of each quarter for 7 years, in an account paying 6% compounded quarterly.

Solution

Given: $PMT = 500 \quad r = 6\% = .06 \quad m = 4 \quad t = 7$

$$i = \frac{r}{m} = \frac{.06}{4} = 0.015 \qquad \underline{n = mt + 1 = 4(7) + 1 = 29}$$

Since you put money at the beginning of each month, we need to add the first payment.

$$FV = PMT\,\frac{(1+i)^n - 1}{i}$$

$$= 500\,\frac{(1+.015)^{29} - 1}{.015} \qquad 500\left((1+.015)\,\wedge\,29 - 1\right)/.015$$

$$\underline{= \$17,499.35}$$

Exercise

A 45 year-old man puts \$2500 in a retirement account at the end of each quarter until he reaches the age of 60, then makes no further deposits. If the account pays 6% interest compounded quarterly, how much will be in the account when the man retires at age 65?

Solution

For the 15 years $(60 - 45 = 15)$:

$PMT = 2,500 \quad r = 6\% = .06 \quad m = 4 \quad t = 15$

$$i = \frac{r}{m} = \frac{.06}{4} = 0.015 \qquad n = mt + 1 = 4(15) = 60$$

$$FV = PMT\,\frac{(1+i)^n - 1}{i}$$

$$= 2,500\,\frac{(1+.015)^{60} - 1}{.015} \qquad 2500\left((1+.015)\,\wedge\,60 - 1\right)/.015$$

$$\underline{= \$240,536.63}$$

For the remaining 5 years, the FV amount is the present amount (P) at 6% compounded quarterly.

$$A = P(1+i)^n$$

$$= 240,536.63(1+.015)^{4(5)} \qquad 240536.63(1+.015)\,\wedge\,(5*4)$$

$$\underline{= \$323,967.96}$$

Exercise

A father opened a savings account for his daughter on the day she was born, depositing \$1000. Each year on her birthday he deposits another \$1000, making the last deposit on her 21^{st} birthday. If the account pays 5.25% interest compounded annually, how much is in the account at the end of the day on his daughter's 21^{st} birthday? How much interest has been earned?

Solution

Given: $PMT = 1,000 \quad r = 5.25\% = .0525 \quad m = 1 \quad t = 21$

$$i = \frac{r}{m} = \frac{.0525}{1} = 0.0525 \quad \underline{|n} = mt + 1 = 1(21) + 1 \underline{= 22|}$$

Since you put money at the beginning of each year, we need to add the first payment.

$$FV = PMT\,\frac{(1+i)^n - 1}{i}$$

$$= 1,000\,\frac{(1+.0525)^{22} - 1}{.0525} \qquad 1000\left(\left(1+.0525\right)^\wedge 22 - 1\right)/.0525$$

$$\underline{= \$39,664.40|}$$

The Total contribution: $1000\left(22\right) = \$22,000.00$

The interest earned: $39,664.40 - 22,000 \ \underline{= \$17,664.40|}$

Exercise

You deposits \$10,000 at the beginning of each year for 12 years in an account paying 5% compounded annually. Then you put the total amount on deposit in another account paying 6% compounded semi-annually for another 9 years. Find the final amount on deposit after the entire 21-year period.

Solution

Given: $PMT = 10,000 \quad r = 5.\% = .05 \quad m = 1 \quad t = 12$

$$i = \frac{r}{m} = \frac{.05}{1} = 0.05 \quad \underline{|n} = mt + 1 = 12 + 1 \underline{= 13|}$$

$$FV_{12} = PMT\,\frac{(1+i)^n - 1}{i}$$

$$= 10,000\,\frac{(1+.05)^{13} - 1}{.05} \qquad 10000\left(\left(1+.05\right)^\wedge 13 - 1\right)/.05$$

$$\underline{= \$177,129.83|}$$

Since the last deposit did mature yet when roll over, then:

$$P = 177,129.83 - 10,000 = \$167,129.83$$

$$i = \frac{r}{m} = \frac{.06}{2} = 0.03 \quad \underline{|n} = 9(2) \underline{= 18|}$$

$A = P(1+i)^n$

$= 167,129.83(1+.03)^{18}$ $\qquad$ $167129.83(1.03)\wedge 18$

$\underline{= \$284,527.35}$

Exercise

You need $10,000 in 8 years.

a) What amount should be deposit at the end of each quarter at 8% compounded quarterly so that he will have his $10,000?

b) Find your quarterly deposit if the money is deposited at 6% compounded quarterly.

Solution

a) ***Given***: $FV = 10,000 \quad r = 8\% = .08, \quad m = 4, \quad t = 8$

$$i = \frac{r}{m} = \frac{.08}{4} = .02 \quad n = mt = 4(8) = 32$$

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

$$= 10,000 \frac{.02}{(1+.02)^{32} - 1} \qquad 10000(.02)/(1.02 \wedge 32 - 1)$$

$\underline{= \$226.11}$ *each quarter*

b) ***Given***: $FV = 10,000 \quad r = 6\% = .06, \quad m = 4, \quad t = 8$

$$i = \frac{r}{m} = \frac{.06}{4} = .015 \quad n = 4(8) = 32$$

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

$$= 10,000 \frac{.015}{(1+.015)^{32} - 1} \qquad 10000(.015)/(1.015 \wedge 32 - 1)$$

$\underline{= \$245.77}$ *each quarter*

Exercise

You want to have a $20,000 down payment when you buy a car in 6 years. How much money must you deposit at the end of each quarter in an account paying 3.2% compounded quarterly so that you will have the down payment you desire?

Solution

Given: $FV = 20{,}000 \quad r = 3.2\% = .032, \quad m = 4, \quad t = 6$

$$i = \frac{r}{m} = \frac{.032}{4} = .008 \quad n = 4(6) = 24$$

$$PMT = FV\frac{i}{(1+i)^n - 1}$$

$$= 20{,}000\frac{.008}{(1+.008)^{24} - 1}$$

$100000(.055/12)/((1+.055/12)\wedge 120-1)$

$$\underline{= \$759.21|} \quad \textit{quarterly}$$

Exercise

You sell a land and then you will be paid a lump sum of $60,000 in 7 years. Until then, the buyer pays 8% simple interest quarterly.

a) Find the amount of each quarterly interest payment on the $60,000

b) The buyer sets up a sinking fund so that enough money will be present to pay off the $60,000. The buyer will make semiannual payments into the sinking fund; the account pays 6% compounded semiannually. Find the amount of each payment into the fund.

Solution

Given: $P = 60{,}000 \quad r = 8\% = .08, \quad m = 4, \quad t = 7$

a) $I = Prt$

$$= 60{,}000(.08)\left(\frac{1}{4}\right)$$

$$= \$1{,}200.00$$

b) **Given**: $FV = 60{,}000 \quad r = 6\% = .06, \quad m = 2, \quad t = 7$

$$i = \frac{r}{m} = \frac{.06}{2} = .03 \quad n = 2(7) = 14$$

$$PMT = FV\frac{i}{(1+i)^n - 1}$$

$$= 60{,}000\frac{.03}{(1+.03)^{14} - 1}$$

$60000(.03)/((1.03)\wedge 14-1)$

$$\underline{= \$3511.58|}$$

$i = \frac{.06}{2} = .03$

$(Balance)i = .03(Balance)$

Pmt #	**Deposit Amount**	$I = .03 * Balance$	**Interest Earned**		**Balance**
1	$3,511.58	----	$0		$3,511.58
2	$3,511.58	$.03 * 3,511.58$	$105.35	$2(3,511.58) + 105.35$	$7,128.51
3	$3,511.58	$.03 * 7128.51$	$213.86	$7128.51 + 3,511.58 + 213.86$	$10,853.95
4	$3,511.58	$.03 * 10853.95$	$325.62	$10853.95 + 3511.58 + 325.62$	$14,691.15
5	$3,511.58	$.03 * 14691.15$	$440.73	$14691.15 + 3511.58 + 440.73$	$18,643.46
6	$3,511.58	$.03 * 18643.46$	$559.30	$18643.46 + 3511.58 + 559.30$	$22,714.34
7	$3,511.58	$.03 * 22714.34$	$681.43	$22714.34 + 3511.58 + 681.43$	$26,907.35
8	$3,511.58	$.03 * 26907.35$	$807.22	$26907.35 + 3511.58 + 807.22$	$31,226.15
9	$3,511.58	$.03 * 31226.15$	$936.78	$31226.15 + 3511.58 + 936.78$	$35,674.51
10	$3,511.58	$.03 * 35674.51$	$1,070.24	$35674.51 + 3511.58 + 1070.24$	$40,256.33
11	$3,511.58	$.03 * 40256.33$	$1,207.69	$40256.33 + 3511.58 + 1207.69$	$44,975.60
12	$3,511.58	$.03 * 44975.60$	$1,349.57	$44975.60 + 3511.58 + 1349.57$	$49,843.13
13	$3,511.58	$.03 * 49.843.13$	$1,495.09	$49843.13 + 3511.58 + 1495.09$	$54,843.13
14	$3,511.58	$.03 * 54843.13$	$1,645.29	$54843.13 + 3511.58 + 1645.29$	$60,000.00

Solution Section 2.3 – Present Value of an Annuity Amortization

Exercise

How much should you deposit in an account paying 8% compounded quarterly in order to receive quarterly payments of $1,000 for the next 4 years?

Solution

Given: $PMT = 1,000 \quad r = 8\% = .08, \quad m = 4, \quad t = 4$

$$i = \frac{r}{m} = \frac{.08}{4} = .02 \quad n = 4(4) = 16$$

$$PV = PMT\frac{1-(1+i)^{-n}}{i}$$

$$= 1000\frac{1-(1+.02)^{-16}}{.02} \qquad 1000(1-1.02\,^\wedge(-)16)\,/\,.02$$

$$\approx \$13,577.71$$

Exercise

You have negotiated a price of $25,200 for a new truck. Now you must choose between 0% financing for 48 months or a $3,000 rebate. If you choose the rebate, you can obtain a loan for the balance at 4.5% compounded monthly for 48 months . Which option should you choose?

Solution

0% ***financing***: **Given**: $P = 25,200 \quad r = 0\% = 0, \quad t = 48\ mth$

$$PMT_1 = \frac{25,200}{48} = \$525$$

Rebate: **Given**: $P = 25,200 \quad Rebate = \$3,000 \quad r = 4.5\% = .045, \quad m = 12 \quad n = t = 48\ mth$

$$PV = 25,200 - 3,000 = \$22,200$$

$$i = \frac{.045}{12} = .00375$$

$$PMT_2 = PV\frac{i}{1-(1+i)^{-n}}$$

$$= 22,200\frac{.00375}{1-1.00375^{-48}} \qquad 22200(.00375\,/\,(1-1.00375\,^\wedge(-)\,48))$$

$$= \$506.24$$

$\Rightarrow$ Rebate is better and you save 525 – 506.24 = $ 18.76 *per month*

Or 18.76 * 48 = $ 900.48 (over the loan)

Exercise

Suppose you have selected a new car to purchase for $19,500. If the car can be financed over a period of 4 years at an annual rate of 6.9% compounded monthly, how much will your monthly payments be? Construct an amortization table for the first 3 months.

Solution

$$PMT = 19500\left(\frac{0.069/12}{1-(1+0.069/12)^{-48}}\right) = 466.05$$

$19500(0.069/12)/(1-(1+0.069/12)\wedge(-)48$

$$I_1 = 19500(0.069/12) = 112.13$$

Pmt #	*Pmt Amount*	*Interest*	*Reduction*	*Unpaid Bal.*
0	-------------	----	---------	19500
1	466.05	112.13	353.93	19146.08
2	466.05	110.09	355.96	18790.12
3	466.05	108.04	358.01	18432.11

Exercise

Suppose your parents decide to give you $10,000 to be put in a college trust fund that will be paid in equally quarterly installments over a 5 year period. If you deposit the money into an account paying 1.5% per quarter, how much are the quarterly payments (Assume the account will have a zero balance at the end of period.)

Solution

Given: $PV = 10,000 \quad r = 1.5\% = .015, \quad m = 4, \quad t = 5$

$$i = r = .015 \quad n = 4(5) = 20$$

$$\text{PMT} = 10,000\left(\frac{0.015}{1-(1+0.015)^{-20}}\right) = \$582.46$$

$10000(0.015)/(1-(1+0.015)\wedge(-)20)$

Exercise

You finally found your dream home. It sells for $120,000 and can be purchased by paying 10% down and financing the balance at an annual rate of 9.6% compounded monthly.

a) How much are your payments if you pay monthly for 30 years?
b) Determine how much would be paid in interest .
c) Determine the payoff after 100 payments have been made.
d) Change the rate to 8.4% and the time to 15 years and calculate the payment.
e) Determine how much would be paid in interest and compare with the previous interest.

Solution

$$\boldsymbol{a})\ PMT = 108000\left(\frac{0.096/12}{1-(1+0.096/12)^{-360}}\right) = \$916.01$$

$$\boldsymbol{b})\ I_1 = 360(916.01) - 108000 = \$221763.60$$

$$\boldsymbol{c})\ PV = 916.01\left(\frac{1-(1+0.096/12)^{-260}}{0.096/12}\right) = \$100077.71$$

$$\boldsymbol{d})\ PMT = 108000\left(\frac{0.084/12}{1-(1+0.084/12)^{-180}}\right) = \$1057.20$$

$$\boldsymbol{e})\ I_2 = 180(1057.20) - 108000 = \$82296$$

$$\mathrm{I}_1 - \mathrm{I}_2 = 221763.60 - 82296 = 139467.60$$

Exercise

Sharon has found the perfect car for her family (anew mini-van) at a price of $24,500. She will receive a $3500 credit toward the purchase by trading in her old Gremlin, and will finance the balance at an annual rate of 4.8% compounded monthly.

a) How much are her payments if she pays monthly for 5 years?
b) How long would it take for her to pay off the car paying an extra $100 per mo., beginning with the first month?

Solution

$$\boldsymbol{a})\ PMT = 21000\left(\frac{0.048/12}{1-(1+0.048/12)^{-60}}\right) = \$394.37$$

$$\boldsymbol{b})\ 21000 = 494.37\left(\frac{1-(1+0.048/12)^{n}}{0.048/12}\right)$$

Divide both sides by 494.37

$$42.4783 = \frac{1-(1+0.004)^{-n}}{0.004}$$ ***multiply by 0.004***

$$0.1699 = 1 - 1.004^{-n}$$

$$1.004^{-n} = 1 - 0.1699$$

$$1.004^{-n} = 0.8301$$ ***"ln" both sides***

$$-n\ln 1.004 = \ln 0.8301$$

$$n = -\ln 0.8301/\ln 1.004 = 46.65 \quad n = -\frac{\ln 0.8301}{\ln 1.004}$$

$$\boxed{n = 47mo.}$$

Money is compounded monthly; it can't be compounded at 46.65 months. Bump to 47mo.

Exercise

Marie has determined that she will need $5000 per month in retirement over a 30-year period. She has forecasted that her money will earn 7.2% compounded monthly. Marie will spend 25-years working toward this goal investing monthly at an annual rate of 7.2%. How much should Marie's monthly payments be during her working years in order to satisfy her retirement needs? *This is a 2-part problem: 1st calculate the PV for retirement. Then use that value as FV for working years.*

Solution

$$PV = 5000\left(\frac{1-(1+0.072/12)^{-360}}{0.072/12}\right) = 736606.78$$

Exercise

American General offers a 10-year ordinary annuity with a guaranteed rate of 6.65% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $5,000 annually over the 10-year period?

Solution

Given: $PMT = 5,000 \quad r = 6.65\% = .0665, \quad m = 1, \quad t = 10$

$$i = \frac{r}{m} = .0665 \quad n = 1(10) = 10$$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$= 5,000\left(\frac{1-(1+0.0665)^{-10}}{.0665}\right) \qquad = 5000\left(1-(1.0665)\wedge(-)10\right)/.0665$$

$$\underline{= \$35,693.18}$$

Exercise

American General offers a 7-year ordinary annuity with a guaranteed rate of 6.35% compounded annually. How much should you pay for one of these annuities if you want to receive payments of \$10,000 annually over the 7-year period?

Solution

Given: $PMT = 10,000 \quad r = 6.35\% = .0635, \quad m = 1, \quad t = 7$

$$i = \frac{r}{m} = .0635 \quad n = 7$$

$$PV = PMT\frac{1-(1+i)^{-n}}{i}$$

$$= 10,000\left(\frac{1-(1+0.0635)^{-7}}{.0635}\right) \qquad = 10000\left(1-(1.0635)\wedge(-)7\right)/.0635$$

$$\boxed{= \$55,135.98}$$

Exercise

You want to purchase an automobile for \$27,300. The dealer offers you 0% financing for 60 months or a \$5,000 rebate. You can obtain 6.3% financial for 60 months at the local bank. Which option should you choose? Explain.

Solution

0% ***financing***: ***Given***: $P = 27,300 \quad r = 0\% = 0, \quad t = 60\ mo$

$$\boxed{PMT_1} = \frac{27,300}{60}\ \boxed{= \$455.00}$$

Rebate: ***Given***: $Rebate = \$5,000 \quad r = 6.3\% = .063, \quad n = 60$

$$PV = 27,300 - 5,000 = \$22,300. \quad i = \frac{.063}{12}$$

$$PMT_2 = PV\frac{i}{1-(1+i)^{-n}}$$

$$= 22,300\frac{\frac{.063}{12}}{1-\left(1+\frac{.063}{12}\right)^{-60}} \qquad 22300(.00375/(1-(1+.063/12)\wedge(-)60))$$

$$= \boxed{\$434.24}$$

$\Rightarrow$ Rebate is better and you save $\$455 - \$434.24 = \$20.76$ *per month*

Or $60(20.76) = \$1,245.60$ (over the life of the loan)

Exercise

You want to purchase an automobile for \$28,500. The dealer offers you 0% financing for 60 months or a \$6,000 rebate. You can obtain 6.2% financial for 60 months at the local bank. Which option should you choose? Explain.

Solution

0% ***financing***: ***Given***: $P = 28,500 \quad r = 0\% = 0, \quad t = 60\ mo$

$$\boxed{PMT_1} = \frac{28,500}{60} = \$475.00$$

Rebate: ***Given***: $Rebate = \$6,000 \quad r = 6.2\% = .062, \quad n = 60$

$$PV = 28,500 - 6,000 = \$22,500. \quad i = \frac{.062}{12}$$

$$\begin{aligned} PMT_2 &= PV \frac{i}{1-(1+i)^{-n}} \\ &= 22,500 \frac{\frac{.062}{12}}{1-\left(1+\frac{.062}{12}\right)^{-60}} \\ &= \$437.08 \end{aligned}$$

$\Rightarrow$ Rebate is better and you save $\$475 - \$437.08 = \$37.92$ *per month*

Or $60(37.92) = \$2,275.20$ (over the life of the loan)

Exercise

Construct the amortization schedule for a \$5,000 debt that is to be amortized in eight equal quarterly payments at 2.8% interest per quarter on the unpaid balance.

Solution

Given: $PV = 5,000 \quad i = r = 2.8\% = .028, \quad n = 8$

$$\begin{aligned} PMT &= PV \frac{i}{1-(1+i)^{-n}} \\ &= 5,000 \frac{.028}{1-(1+.028)^{-8}} \\ &= \$706.29 \end{aligned}$$

5000(.028 / (1 − 1.028 ^ (−) 8))

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0		$0		$5,000.00
1	$706.29	$.028(5000) = \$140.00$	$706.29 - 140.00 = \$566.29$	$5000 - 566.29 = \$4,433.71$
2	$706.29	$.028(4433.71) = \$124.14$	$706.29 - 124.14 = \$582.15$	$4433.71 - 582.15 = \$3,851.56$
3	$706.29	$.028(3851.56) = \$107.84$	$706.29 - 107.84 = \$598.45$	$3851.56 - 598.45 = \$3,253.11$
4	$706.29	$.028(3253.11) = \$91.09$	$706.29 - 91.09 = \$615.20$	$3253.11 - 615.20 = \$2,637.91$
5	$706.29	$.028(2637.91) = \$73.86$	$706.29 - 73.86 = \$632.43$	$2637.91 - 632.43 = \$2,005.48$
6	$706.29	$.028(2005.48) = \$56.15$	$706.29 - 56.15 = \$650.14$	$2005.48 - 650.14 = \$1,355.34$
7	$706.29	$.028(1355.34) = \$37.95$	$706.29 - 37.95 = \$668.34$	$1355.34 - 668.34 = \$687.00$
8	$706.29	$.028(687) = \$19.24$	$706.29 - 19.24 = \$687.00$	$0.00
Total	**$5,650.27**	**$650.27**	**$5,000.00**	

Exercise

Construct the amortization schedule for a $10,000 debt that is to be amortized in six equal quarterly payments at 2.6% interest per quarter on the unpaid balance.

Solution

Given: $PV = 10,000 \quad i = r = 2.6\% = .026, \quad n = 6$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 10,000 \frac{.026}{1-(1+.026)^{-6}}$$

$10000(.026 / (1 - 1.026 \wedge (-) 6))$

$$= \underline{\$1,821.58}$$

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0		$0		$10,000.00
1	$1,821.58	$.026(10000) = \$260.00$	$1821.58 - 260.00 = \$1,561.58$	$10000 - 1561.58 = \$8,438.42$
2	$1,821.58	$.026(8438.42) = \$219.40$	$1821.58 - 219.40 = \$1,602.18$	$8438.42 - 1602.18 = \$6,836.24$
3	$1,821.58	$.026(6836.24) = \$177.74$	$1821.58 - 177.74 = \$1,643.84$	$6836.24 - 1643.84 = \$5,192.40$
4	$1,821.58	$.026(5192.40) = \$135.00$	$1821.58 - 135.00 = \$1,686.58$	$5192.40 - 1686.58 = \$3,505.82$
5	$1,821.58	$.026(3505.82) = \$91.15$	$1821.58 - 91.15 = \$1,730.43$	$3505.82 - 1730.43 = \$1,775.39$
6	$1,821.58	$.026(1775.39) = \$46.16$	$1821.58 - 46.16 = \$1,775.39$	$0.00
Total	**$10,929.45**	**$929.45**	**$10,000.00**	

Exercise

A loan of $37,948 with interest at 6.5% compounded annually, to be paid with equal annual payments over 10 years

Solution

Given: $PV = 37,948. \quad m = 1, \quad i = r = 6.5\% = .065, \quad n = 10$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 37,948 \frac{.065}{1-(1+.065)^{-10}}$$

37948(.065 / (1 – 1.065 ^ (–)10))

$$= \boxed{\$5,278.74}$$

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0				$37,948.00
1	$5,278.74	$2,466.62	$2,812.12	$35,135.88
2	$5,278.74	$2,283.83	$2,994.91	$32,140.97
3	$5,278.74	$2,089.16	$3,189.58	$28,951.40
4	$5,278.74	$1,881.84	$3,396.90	$25,554.50
5	$5,278.74	$1,661.04	$3,617.70	$21,936.80
6	$5,278.74	$1,425.89	$3,852.85	$18,083.95
7	$5,278.74	$1,175.46	$4,103.28	$13,980.67
8	$5,278.74	$908.74	$4,370.00	$9,610.67
9	$5,278.74	$624.69	$4,654.05	$4,956.62
10	$5,278.74	$322.18	$4,956.62	$0.00
Total	**$52,787.40**	**$14,839.40**	**$37,948.00**	

Exercise

A loan of $4,836 with interest at 7.25% compounded semi-annually, to be repaid in 5 years in equal semi-annual payments.

Solution

Given: $PV = 4,836. \quad m = 2, \quad r = 7.25\% = .0725, \quad t = 5$

$$i = \frac{r}{m} = \frac{.0725}{2} = .03625 \qquad n = 2(5) = 10$$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 4,836 \frac{.03625}{1-(1+.03625)^{-10}}$$

4836(.03625 / (1 − 1.03625 ^ (−)10))

$$= \underline{\$585.16}$$

Pmt #	*Payment*	*Interest*	*Reduction*	*Unpaid Balance*
0				$4,836.00
1	$585.16	$175.31	$409.85	$4,426.15
2	$585.16	$160.45	$424.71	$4,001.43
3	$585.16	$145.05	$440.11	$3,561.32
4	$585.16	$129.10	$456.06	$3,105.26
5	$585.16	$112.57	$472.59	$2,632.67
6	$585.16	$95.43	$489.73	$2,142.94
7	$585.16	$77.68	$507.48	$1,635.46
8	$585.16	$59.29	$525.87	$1,108.59
9	$585.16	$40.22	$544.94	$564.65
10	$585.16	$20.47	$564.65	$0.00
Total	**$5,851.60**	**$1,015.60**	**$4,836.00**	