

$$\begin{cases} x_1 - x_2 = 0 & \textcircled{1} \\ 3x_1 - 2x_2 = -1 & \textcircled{2} \end{cases}$$

$$\textcircled{1} \rightarrow x_1 = x_2$$

$$\textcircled{2} \quad 3x_1 - 2x_1 = -1$$

$$\underline{x_1 = -1 = x_2}$$

$$\begin{cases} 3u + v = 240 \\ -3u + v = 240 & \textcircled{1} \end{cases}$$

$$\begin{cases} 3u + v = 240 \\ -3u - 9v = -720 \end{cases}$$

$$\underline{-8v = -480}$$

$$\underline{v = 60}$$

$$\textcircled{1} \quad u = 240 - 3(60)$$

$$\underline{= 60}$$

$$\begin{cases} 3x_1 - 2x_2 + 4x_3 = 1 & \textcircled{1} \\ x_1 + x_2 - 2x_3 = 3 & \textcircled{2} \\ 2x_1 - 3x_2 + 6x_3 = 8 & \textcircled{3} \end{cases}$$

$$\begin{cases} 3x_1 - 2x_2 + 4x_3 = 1 \\ 3\textcircled{2} - \textcircled{1} \quad x_2 - 2x_3 = 8 & \textcircled{4} \\ 3\textcircled{3} - 2\textcircled{1} \quad -5x_2 + 10x_3 = 22 & \textcircled{5} \end{cases}$$

$$\begin{cases} 3x_1 - 2x_2 + 4x_3 = 1 \\ x_2 - 2x_3 = 8 \\ \textcircled{5} + 5\textcircled{4} \quad 0 = 42 \Rightarrow \text{(impossible)} \end{cases}$$

No solution.

$$4/ \begin{bmatrix} 1 & 2 & -4 \\ 3 & -4 & 6 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{size: } \begin{matrix} 3 \times 3 \\ R \quad C \end{matrix}$$

$$5/ \begin{bmatrix} 2 & -1 & -1 & 1 \\ -6 & 2 & 0 & 1 \end{bmatrix} \quad \text{size } \begin{matrix} 2 \times 4 \\ R, C \end{matrix}$$

$$6/ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{matrix} x_1 = 0 \\ x_2 = 2 \end{matrix}$$

$$8/ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Reduced}$$

$$9/ \begin{cases} x_1 - 3x_3 = -2 \\ 3x_1 + x_2 - 2x_3 = 5 \\ 2x_1 + 2x_2 + x_3 = 4 \end{cases}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{array} \right] \begin{matrix} \\ R_2 - 3R_1 \\ R_3 - 2R_1 \end{matrix}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{array} \right] \quad R_3 - 2R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{array} \right] \begin{matrix} \rightarrow x_1 - 3x_3 = -2 \quad (2) \\ \rightarrow x_2 + 7x_3 = 11 \quad (1) \\ \rightarrow -7x_3 = -14 \\ \quad \underline{x_3 = 2} \end{matrix}$$

$$(1) \quad \underline{x_2 = 11 - 14 = -3}$$

$$(2) \quad \underline{x_1 = -2 + 6 = 4}$$

$$\underline{\therefore (4, -3, 2)}$$



$$10 \rightarrow \begin{cases} 2x_1 + 3x_3 = 3 \\ 4x_1 - 3x_2 + 7x_3 = 5 \\ 8x_1 - 9x_2 + 15x_3 = 10 \end{cases}$$

$$\left[ \begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 4 & -3 & 7 & 5 \\ 8 & -9 & 15 & 10 \end{array} \right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 4R_1 \end{array}$$

$$\left[ \begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & -9 & 3 & -2 \end{array} \right] R_3 - 3R_2$$

$$\left[ \begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array} \right] \rightarrow 0 = 1 \text{ (imp)}$$

$\therefore$  No solution

$$11 \rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \rightarrow x_1 = 0 \\ \rightarrow x_2 + x_3 = 0 \\ x_2 = -x_3 \end{array}$$

$$\text{let } x_3 = t \Rightarrow x_2 = -t$$

$$\therefore (0, -t, t)$$

$$12 \rightarrow A = \left[ \begin{array}{cc|c} 1 & k & 2 \\ -3 & 4 & 1 \end{array} \right] \text{ Augmented matrix}$$

a) 2 eqns w 2 variables.

$$b) \left[ \begin{array}{cc|c} 1 & k & 2 \\ -3 & 4 & 1 \end{array} \right] R_2 + 3R_1$$

$$\left[ \begin{array}{cc|c} 1 & k & 2 \\ 0 & 4+3k & 7 \end{array} \right]$$

For the system to be consistent:  $4+3k \neq 0$   
 $\Rightarrow k \neq -\frac{4}{3} \quad k \in \mathbb{R} - \left\{ -\frac{4}{3} \right\}$

$\rightarrow$

11 cont

System is homogeneous

$$a) \left[ \begin{array}{ccc|c} 1 & k & 2 & 0 \\ -3 & 4 & 1 & 0 \end{array} \right] \quad \text{2 eqns w/ 3 variables}$$

$$b) R_2 + 3R_1$$

$$\left[ \begin{array}{ccc|c} 1 & k & 2 & 0 \\ 0 & 4+3k & 7 & 0 \end{array} \right] \rightarrow \begin{array}{l} x_1 + kx_2 + 2x_3 = 0 \quad (2) \\ (4+3k)x_2 + 7x_3 = 0 \quad (1) \end{array}$$

$$k \in \mathbb{R}$$



12/ (2, 5) (3, 2) (4, 5)

a)  $P(x) = a_0 + a_1x + a_2x^2$

$$P(2) = \begin{cases} a_0 + 2a_1 + 4a_2 = 5 \\ a_0 + 3a_1 + 9a_2 = 2 \\ a_0 + 4a_1 + 16a_2 = 5 \end{cases}$$

$$P(3) =$$

$$P(4) =$$

$$\begin{bmatrix} 1 & 2 & 4 & | & 5 \\ 1 & 3 & 9 & | & 2 \\ 1 & 4 & 16 & | & 5 \end{bmatrix} \begin{array}{l} \\ R_2 - R_1 \\ R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 4 & | & 5 \\ 0 & 1 & 5 & | & -3 \\ 0 & 2 & 12 & | & 0 \end{bmatrix} \begin{array}{l} \\ R_3 - 2R_2 \\ a_0 + 2a_1 + 4a_2 = 5 \text{ (2)} \end{array}$$

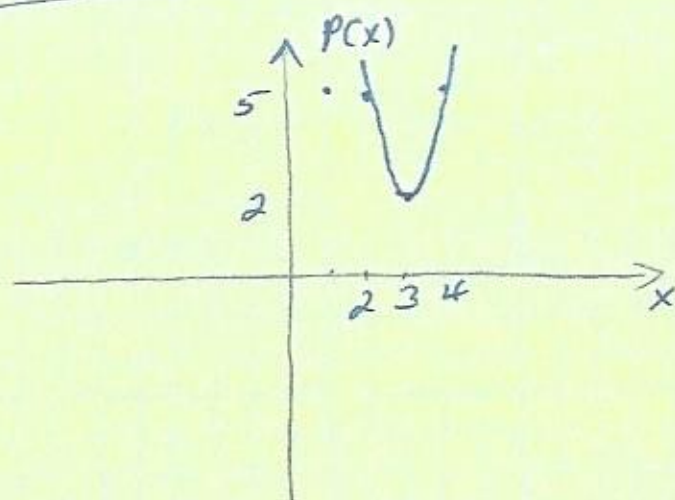
$$\begin{bmatrix} 1 & 2 & 4 & | & 5 \\ 0 & 1 & 5 & | & -3 \\ 0 & 0 & 2 & | & 6 \end{bmatrix} \begin{array}{l} a_1 + 5a_2 = -3 \text{ (1)} \\ 2a_2 = 6 \Rightarrow a_2 = 3 \end{array}$$

$$\textcircled{1} \rightarrow a_1 = -3 - 15 = -18$$

$$\textcircled{2} \rightarrow a_0 = 5 + 36 - 12 = 29$$

$$P(x) = 29 - 18x + 3x^2$$

b)



14)  $(-1, 3) (0, 0) (1, 1) (4, 58)$

a)  $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$p(-1) \quad a_0 - a_1 + a_2 - a_3 = 3 \quad (1)$

$p(0) \quad \underline{a_0 = 0}$

$p(1) \quad a_0 + a_1 + a_2 + a_3 = 1 \quad (2)$

$p(4) \quad a_0 + 4a_1 + 16a_2 + 64a_3 = 58 \quad (3)$

$(1) \rightarrow -a_1 + a_2 - a_3 = 3 \quad +$

$(2) \rightarrow \underline{a_1 + a_2 + a_3 = 1}$   
 $2a_2 = 4 \Rightarrow \underline{a_2 = 2}$

$(2) \Rightarrow a_1 + a_3 = -1$

$(3) \rightarrow 4a_1 + 64a_3 = 58 - 32 = 26$

$-2x \times \begin{cases} a_1 + a_3 = -1 \\ 2a_1 + 32a_3 = 13 \end{cases}$

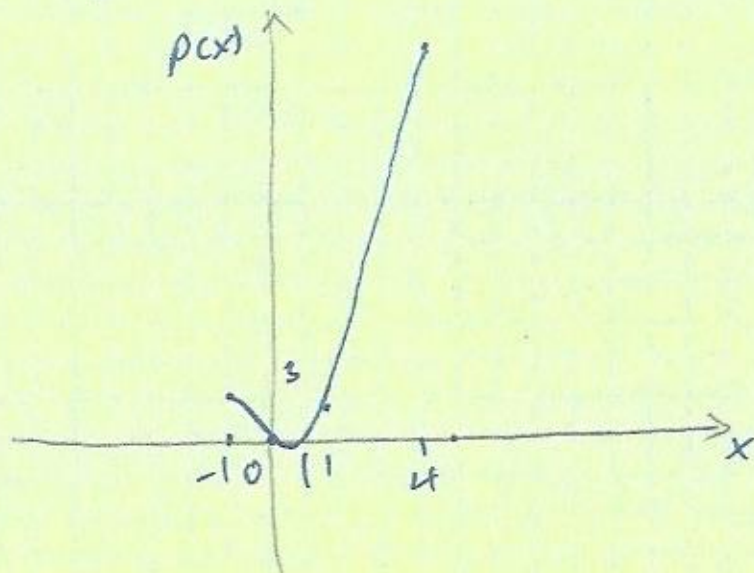
$\begin{cases} -2a_1 - 2a_3 = 2 \\ 2a_1 + 32a_3 = 13 \end{cases}$

$\underline{30a_3 = 15} \Rightarrow \underline{a_3 = \frac{1}{2}}$

$\underline{a_1 = -1 - \frac{1}{2} = -\frac{3}{2}}$

$\therefore p(x) = -\frac{3}{2}x + 2x^2 + \frac{1}{2}x^3$

b)





15 let assume  $x=0$  @ year 1990  
 $\Rightarrow (0, 249)$

$$2000 \rightarrow (10, 282)$$

$$2010 \rightarrow (20, 309)$$

$$P(x) = a_0 + a_1x + a_2x^2$$

$$P(0): \underline{a_0 = 249}$$

$$P(10): \begin{cases} a_0 + 10a_1 + 100a_2 = 282 \end{cases}$$

$$P(20): \begin{cases} a_0 + 20a_1 + 400a_2 = 309 \end{cases}$$

$$\times (-2) \begin{cases} 10a_1 + 100a_2 = 33 \quad (1) \\ 20a_1 + 400a_2 = 60 \end{cases}$$

$$\begin{cases} -20a_1 - 200a_2 = -66 \\ 20a_1 + 400a_2 = 60 \end{cases}$$

$$200a_2 = -6 \Rightarrow \underline{a_2 = -\frac{3}{100}}$$

$$\begin{aligned} (1) \rightarrow 10a_1 &= 33 + 3 \\ \underline{a_1 = \frac{36}{10} = \frac{18}{5}} \end{aligned}$$

$$P(x) = 249 + \frac{18}{5}x - \frac{3}{100}x^2$$

$$\text{year: } 2020 \rightarrow x=30$$

$$P(30) = 249 + 18(6) - 27 = \underline{330 \text{ mill}} \quad \text{in } 2020$$

$$\text{year } 2030 \rightarrow x=40$$

$$\begin{aligned} P(40) &= 249 + 18(8) - 3(16) \\ &= \underline{345 \text{ million}} \quad \text{in } 2030 \end{aligned}$$

16/21

$$\begin{cases} x_3 = x_1 + 100 \\ x_1 = 200 + x_2 \\ x_4 = 100 + x_2 \\ x_3 = 200 + x_4 \end{cases}$$

$$\begin{cases} -x_1 + x_3 = 100 \\ x_1 - x_2 = 200 \\ -x_2 + x_4 = 100 \\ x_3 - x_4 = 200 \end{cases}$$

$$\left[ \begin{array}{cccc|c} -1 & 0 & 1 & 0 & 100 \\ 1 & -1 & 0 & 0 & 200 \\ 0 & -1 & 0 & 1 & 100 \\ 0 & 0 & 1 & -1 & 200 \end{array} \right] \begin{array}{l} \\ R_2 + R_1 \\ \\ -R_1 \end{array}$$

$$\left[ \begin{array}{cccc|c} +1 & 0 & -1 & 0 & 100 \\ 0 & +1 & -1 & 0 & -300 \\ 0 & -1 & 0 & 1 & 100 \\ 0 & 0 & 1 & -1 & 200 \end{array} \right] R_3 + R_2$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & -300 \\ 0 & 0 & -1 & 1 & -200 \\ 0 & 0 & 1 & -1 & 200 \end{array} \right] R_4 + R_3$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & -300 \\ 0 & 0 & -1 & 1 & -200 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} x_1 - x_3 = -100 \\ x_2 - x_3 = -300 \\ -x_3 + x_4 = -200 \end{array} \text{ or}$$

$$\text{Let } x_4 = t$$

$$x_3 = t + 200$$

$$x_2 = t + 200 - 300 = t - 100$$

$$x_1 = -100 + t + 200$$

$$\begin{cases} x_1 = t + 100 \\ x_2 = t - 100 \\ x_3 = t + 200 \\ x_4 = t \end{cases}$$

$$b) \quad t = 0 \Rightarrow \begin{cases} x_1 = 100 \\ x_2 = -100 \\ x_3 = 200 \\ x_4 = 0 \end{cases}$$



76. cont.

$$c) \quad x_4 = 100 = t.$$

$$\begin{cases} x_1 = 200 \\ x_2 = 0 \\ x_3 = 300 \\ x_4 = 100 \end{cases}$$

$$d) \quad x_1 = 2x_2$$

$$t + 100 = 2(t - 100)$$

$$t + 100 = 2t - 200$$

$$\underline{t = 300}$$

$$\begin{cases} x_1 = 400 \\ x_2 = 200 \\ x_3 = 500 \\ x_4 = 300 \end{cases}$$