

5.1 Cramer's Rule.

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Determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ X

D or Δ
 $\det(A)$

Ex $\begin{cases} 5x + 7y = -1 \\ 6x + 8y = 1 \end{cases}$

$$D = \begin{vmatrix} 5 & 7 \\ 6 & 8 \end{vmatrix} = 40 - 42$$

$$= -2$$

$$D_x = \begin{vmatrix} -1 & 7 \\ 1 & 8 \end{vmatrix} = -8 - 7 = -15$$

$$D_y = \begin{vmatrix} 5 & -1 \\ 6 & 1 \end{vmatrix} = 5 + 6 = 11$$

$$x = \frac{D_x}{D} = \frac{-15}{-2} = \frac{15}{2}$$

$$y = -\frac{11}{2} \quad \text{Solution } \left(\frac{15}{2}, -\frac{11}{2} \right)$$

Diagonal method (3x3)

$$3! = 6$$

$$2! = 2 \times 1$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Diagram illustrating the diagonal method for a 3x3 matrix. The matrix is shown with its elements a_{ij} . Blue arrows indicate the downward diagonal elements a_{11}, a_{22}, a_{33} and the upward diagonal elements a_{31}, a_{22}, a_{13} . Red arrows indicate the elements a_{11}, a_{21}, a_{31} and a_{12}, a_{22}, a_{32} . A green arrow points from a_{13} to a_{32} .

E_x

$$x - 3y + 7z = 13$$

$$x + y + z = 1$$

$$x - 2y + 3z = 4$$

$$D = \begin{vmatrix} 1 & -3 & 7 & 1 & -3 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & -2 & 3 & 1 & -2 \end{vmatrix}$$

$$= 3 - 3 - 14 - 7 + 2 + 9$$

$$= -10$$

$$D_x = \begin{vmatrix} 13 & -3 & 7 & 13 & -3 \\ 1 & 1 & 1 & 1 & 1 \\ 4 & -2 & 3 & 4 & -2 \end{vmatrix}$$

$$= 39 - 12 - 14 - 28 + 26 + 9$$

$$= 20$$

$$D_y = \begin{vmatrix} 1 & 13 & 7 & 1 & 13 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 4 & 3 & 1 & 4 \end{vmatrix}$$

$$= 3 + 13 + 28 - 7 - 4 - 39$$

$$= -6$$

$$D_2 = \begin{vmatrix} 1 & -3 & 13 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix} \begin{vmatrix} 1 & -3 \\ 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 4 - 3 - 26 - 13 + 2 + 12$$

$$= \underline{-24}$$

$$\left(-2, -\frac{6}{10}, -\frac{24}{10} \right)$$

$$\therefore \left(-2, \frac{3}{5}, \frac{12}{5} \right)$$

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2. Partial Fraction Decomposition

$$\frac{P(x)}{(x-a_1) \cdots (x-a_n)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \cdots + \frac{A_n}{x-a_n}$$

Ex: $\frac{x}{x^2 - 5x + 6} = \frac{A}{x-2} + \frac{B}{x-3}$

$$\frac{x}{x^2 - 5x + 6} = \frac{A(x-3) + B(x-2)}{(x-2) + (x-3)}$$

$$x = A(x-3) + B(x-2)$$

$$x^1: A + B = 1 \rightarrow \begin{cases} A = 1 - 3 \\ = -2 \end{cases}$$

$$x^0: -3A - 2B = 0$$

$$\underline{B = 3}$$

$$\frac{x}{x^2 - 5x + 6} = \frac{-2}{x-2} + \frac{3}{x-3}$$

$$\begin{aligned} 3A + 3B &= 3, \Leftrightarrow \textcircled{\sim} \textcircled{\times 3} \\ -3A - 2B &= 0 \\ \hline 0 \quad B &= 3 \end{aligned}$$

$$\begin{aligned} A + B &= 1 \\ -3A - 2B &= 0 \end{aligned} \quad A = \frac{-2}{1} = \textcircled{-2}$$

$$\left(A = \frac{\begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix}} = \frac{-2}{1} = \underline{-2} \right)$$

$$\left[B = \frac{\begin{vmatrix} 1 & 1 \\ -3 & 0 \end{vmatrix}}{1} = \underline{3} \right]$$

$$\overline{(x-a)^n} = \overline{x-a} + \overline{(x-a)^2} + \dots + \overline{(x-a)^n}$$

$$\boxed{Ex} \quad \frac{x-2}{x^3-2x^2+x}$$

$$x(x^2-2x+1) = x(x-1)(x-1)$$

$$\frac{x-2}{x^3-2x^2+x} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$\boxed{x-2} = A(x^2-2x+1) + Bx(x-1) + Cx$$

$$x^2 \quad A + B = 0 \Rightarrow \underline{B=2}$$

$$x^1 \quad -2A - B + C = 1 \quad (1)$$

$$x^0 \quad \underline{A = -2}$$

$$(1) \rightarrow C = 1 + 2(-2) + 2 = -1$$

$$\frac{x-2}{x^3-2x^2+x} = \frac{-2}{x} + \frac{2}{x-1} - \frac{1}{(x-1)^2}$$

$$\frac{x^3-8}{x^2(x-1)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}$$

$$x^3-8 = A x (x-1)^3 + B (x-1)^3 + C x^2 (x-1)^2 + D x^2 (x-1) + E x^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$x^3 - 8 = A x (x^3 - 3x^2 + 3x - 1) \\ + B (x^3 - 3x^2 + 3x - 1) \\ + C x^2 (x^2 - 2x + 1) \\ + D (x^3 - x^2) + E x^2$$

$$x^4 \quad A + C = 0 \rightarrow \underline{C = -24} \quad (1)$$

$$x^3 \quad -3A + B + D - 2C = 1 \quad (2)$$

$$x^2 \quad 3A - 3B - D + E + C = 0 \quad (3)$$

$$x^1 \quad -A + 3B = 0 \rightarrow \underline{A = 3/8 = 24} \quad (4)$$

$$x^0 \quad -B = -8 \Rightarrow \underline{B = 8}$$

$$(1) \rightarrow D = 1 + 3(24) - 8 + 2(-24) \\ = 17$$

$$(2) \quad E = -3(24) + 3(8) + 17 + 24 \\ = -7$$

$$\frac{x^3 - 8}{x^2(x-1)^3} = \frac{24}{x} + \frac{8}{x^2} - \frac{24}{x-1} + \frac{17}{(x-1)^2} - \frac{7}{(x-1)^3}$$

$$\frac{Ax+B}{ax^2+bx+c} \leftarrow \text{not factorable}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\frac{3x-5}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$3x-5 = A(x^2+x+1) + (Bx+C)(x-1)$$

$$x^2 \quad A+B=0 \rightarrow B=-A=\frac{2}{3}$$

$$x^1 \quad A-B+C=3 \quad (1)$$

$$x^0 \quad A-C=-5 \rightarrow C=A+5 = 5 - \frac{2}{3} = \frac{13}{3}$$

$$(1) \quad A+A+A+5=3 \Rightarrow 3A=-2 \Rightarrow A=-\frac{2}{3}$$

$$\frac{3x-5}{x^3-1} = \frac{-2/3}{x-1} + \frac{2/3x + 13/3}{x^2+x+1}$$

$$= -\frac{2}{3} \frac{1}{x-1} + \frac{1}{3} \frac{2x+13}{x^2+x+1}$$