2.9 $y = \sin^{-1} u$

$y = \sin^{-1} x$
$x = \sin y$
$\sin^2(y)$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$= \frac{1}{\cos(\sin^{-1} x)}$$

$$= \frac{1}{\sqrt{1 - \sin^2 \sin^{-1}(x)}}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

Ex $f^{-1}(x) = \ln x = y \Rightarrow x = e^y$

$f(x) = e^x$

$f'(x) = e^x$

$$\frac{d}{dx}(f^{-1}(x)) = \frac{d}{dx}(\ln x)$$

$$= \frac{1}{f'(\ln x)}$$

$$= \frac{1}{e^{\ln x}}$$

$$= \frac{1}{x}$$

Ex $\frac{d}{dx}\left(\sin^{-1} x^2\right) = \frac{2x}{\sqrt{1-x^4}}$

Ex $\left(\sec^{-1} 5x^4\right)' = \frac{20x^3}{5x^4\sqrt{25x^8-1}}$

$= \frac{4}{x\sqrt{25x^8-1}}$

2.10 Related Rates

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$ V'

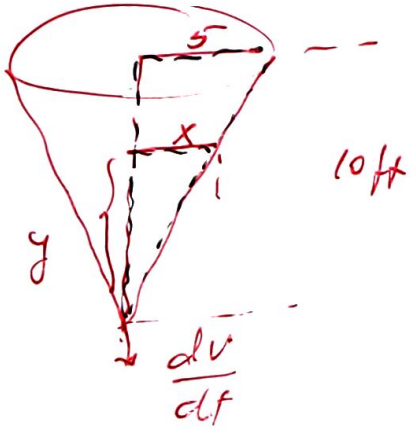
$\frac{dV}{dt} = V'\frac{dr}{dt}$

$= 4\pi r^2 \frac{dr}{dt}$

5x

$\frac{dV}{dt} = 9\ ft^3/min$

$V = \frac{1}{3}\pi r^2 h$



$\frac{dy}{dt}$? $y = 6$ ft

$V = \frac{1}{3}\pi x^2 y$

$\frac{x}{y} = \frac{5}{10} \Rightarrow x = \frac{1}{2}y$

$V = \frac{\pi}{3}\left(\frac{1}{4}y^2\right)y$ $\qquad \frac{1}{12}\pi y^2$

$V = \frac{\pi}{12}y^3$

$\frac{dV}{dt} = \frac{\pi}{4}y^2\frac{dy}{dt}$

$9 = \frac{\pi}{4}6^2\frac{dy}{dt}$ $\qquad \frac{4\times 9}{36} =$

$\frac{dy}{dt} = \frac{1}{\pi}$ ft/min

$V = \frac{1}{3}\pi r^2 y$

$V = \frac{\pi}{3}\frac{r^2}{h^2}y^3$

$\frac{dV}{dt} = \pi\frac{r^2}{h^2}y^2\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{h^2}{\pi r^2 y^2}\frac{dV}{dt}$

$\frac{x}{y} = \frac{r}{h}$

7x

Given $\theta = \frac{\pi}{4}$

$\frac{d\theta}{dt} = 0.14$ rad/min

Find: $\frac{dy}{dt}$

$\tan\theta = \frac{y}{500}$

$y = 500 \tan\theta$.

$\frac{dy}{dt} = 500 \sec^2\theta \frac{d\theta}{dt}$

$= 500\,(2)\,(\frac{14}{100}$

$= 140$ ft/min

$\frac{1}{\cos^2\frac{\pi}{4}} = \frac{1}{(\frac{1}{\sqrt{2}})^2}$

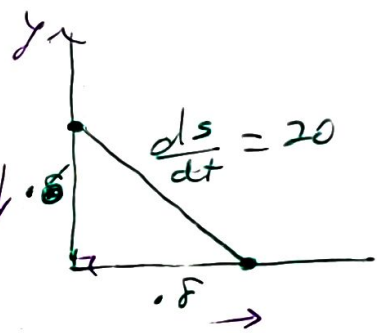
$= \frac{1}{\frac{1}{2}}$

$= 2$

---

x z

7x Given $(.8, .6)$



$\frac{dy}{dt} = -60$ $\frac{ds}{dt} = 20$

$\frac{dx}{dt} = ?$

$\Rightarrow s^2 = x^2 + y^2$

$= \frac{36}{100} + \frac{64}{100} = 1 \Rightarrow s = 1$

$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$.8\frac{dx}{dt} = (20) - (.6)(-60)$

$= 20 + 36$

$\frac{dx}{dt} = \frac{56}{.8} = \frac{560}{8} = 70$ mph

Ex

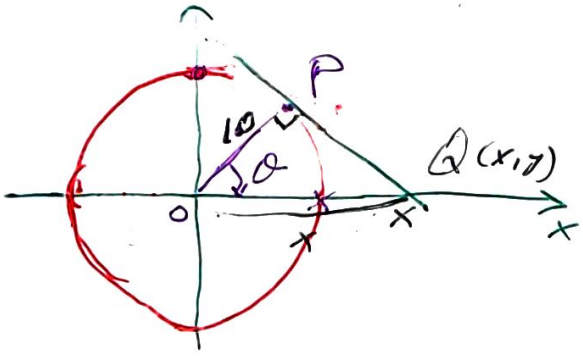
Given $r = 10$

$P_e = (0, 10)$

$P_f = (10, 0)$

$t = 30 \text{ sec} \rightarrow \theta: \frac{\pi}{2} \rightarrow 0$

$x = 20 \text{ ft} \rightarrow \frac{dx}{dt}$

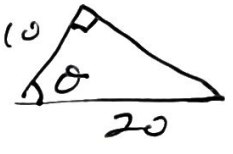


$\frac{d\theta}{dt} ? = \frac{-\pi/2}{30 \text{ sec}} = -\frac{\pi}{60} \cdot \frac{60 \text{ sec}}{1 \text{ min}}$

$\frac{y_1 - y_1}{x_2 - x_1}$

$= -\pi$ rad/min

$\cos\theta = \frac{10}{x}$



$x = \frac{10}{\cos\theta} = 10 \sec\theta$

$\frac{dx}{dt} = 10 \sec\theta \tan\theta \frac{d\theta}{dt}$ | $x = 20 \rightarrow \theta = \frac{\pi}{3}$

$\cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}$

$\frac{\sqrt{3}}{1}$

$= 10 \cdot (2) \cdot (\sqrt{3}) (-\pi)$

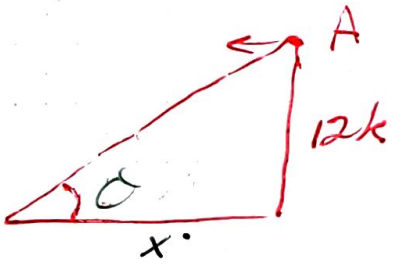
$= -20\pi\sqrt{3}$ ft/min

Ex

Given: $\theta = 30°$

$\frac{dx}{dt}$?

$\frac{d\theta}{dt} = \frac{2}{3}$ deg/sec



$\tan\theta = \frac{12k}{x} \Rightarrow x = 12k \frac{1}{\tan\theta}$

$= \frac{12000}{5280} \cot\theta.$

$\frac{dx}{dt} = -\frac{12000}{5280} \csc^2\theta \cdot \frac{d\theta}{dt}$

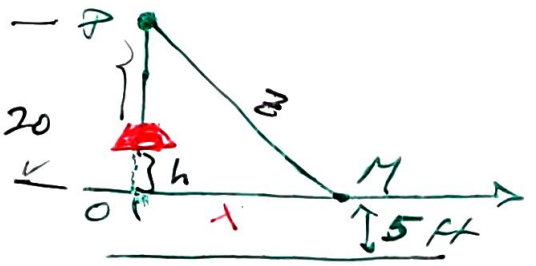
$= -\frac{300}{132} \csc^2 30° \quad 1 \text{ deg} \frac{\pi}{180°} \frac{2}{3} \frac{1}{1 \text{ sec}} \frac{3600 \text{ sec}}{1 \text{ hr}}$

$= -\frac{75}{33}(2)^2 \frac{(2)(12000)}{189}$

$= \frac{12000}{99} \text{ mi/hr}$

$= \frac{4000}{11}$

Ex



Given
$$\frac{dx}{dt} = 6$$
$$45 = 20 - h + z$$
$$z = h + 25$$

$\frac{dh}{dt}$? $x = 21$

$\triangle OPM \Rightarrow x^2 + 20^2 = z^2$

$$x^2 + 20^2 = (h+25)^2$$

$$2x\frac{dx}{dt} = 2(h+25)\frac{dh}{dt}$$

$$21^2 + 20^2 = (h+25)^2$$

$$\sqrt{841} = h + 25 = 29$$

~~$h = 4$~~

$$29\frac{dh}{dt} = 21.(6)$$

$$\frac{dh}{dt} = \frac{126}{29} \text{ ft/sec}$$

$\sqrt{841}$ | 29
441 x 2
841