$\overrightarrow{P_0P} = t\vec{v}$

$P_0\ (x_0, y_0, z_0)$

$P\ (x, y, z)$

$\vec{v} = \langle v_1, v_2, v_3 \rangle$

$$(x-x_0)\hat{i} + (y-y_0)\hat{j} + (z-z_0)\hat{k} = t(v_1\hat{i} + v_2\hat{j} + v_3\hat{k})$$

Path $\vec{r}(t)$

$$r(t) = \vec{r}_0 + \overrightarrow{P_0P}$$
$$= \vec{r}_0 + t\vec{v}$$

$\overrightarrow{P_0P} \parallel \vec{v}$

$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r}_0(t) = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$ (P_0)

Eqn of line

$$\begin{cases} x = x_0 + tv_1 \\ y = y_0 + tv_2 \\ z = z_0 + tv_3 \end{cases}$$

x, y, z: path $\quad x_0, y_0, z_0$: P_0 $\quad v_1, v_2, v_3$: $\vec{v}$

Ex eqn line? $(-2, 0, 4) \parallel \vec{v} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

$$\begin{cases} x = -2 + 2t \\ y = 4t \\ z = 4 - 2t \end{cases}$$

Ex line? $P\ (-3, 2, -3)$ $Q\ (1, -1, 4)$

Soln $\vec{v} = \overrightarrow{PQ}$

$= 4\hat{i} - 3\hat{j} + 7\hat{k}$

$$\begin{cases} x = -3 + 4t \\ y = 2 - 3t \\ z = -3 + 7t \end{cases}$$

$$\vec{r}(t) = r_0 + t\vec{v}$$

$$= r_0 + t\,|\vec{v}|\,\frac{\vec{v}}{|\vec{v}|}$$

$\frac{\vec{v}}{|\vec{v}|}$: unit vector, direction

$|\vec{v}|$: magnitude, speed

Ex $\underset{(0,0,0)}{O} \rightarrow (1,1,1)$ @ $|\vec{v}| = 60$ ft/sec

$r(t=10)$?

$$\vec{v} = \hat{i} + \hat{j} + \hat{k}$$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$$

$$r(t) = 0 + t(60)\left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right) \qquad \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

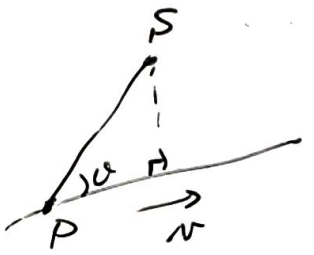
$$= 20t\sqrt{3}\,(\hat{i}+\hat{j}+\hat{k})$$

$$\vec{r}(10) = 200\sqrt{3}\,(\hat{i}+\hat{j}+\hat{k})$$

$$|\vec{r}(10)| = 200\sqrt{3}\,(\sqrt{3})$$

$$= 600 \text{ ft}$$

Distance $S \to$ line $(P \parallel \vec{v})$



$$d = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|}$$

Ex d? $S(1,1,5) \to$ line $\begin{cases} x = 1+t \\ y = 3-t \\ z = 2t \end{cases}$

soln

$P(1,3,0)$

$\vec{v} = \langle 1, -1, 2 \rangle$

$|\vec{v}| = \sqrt{6}$

$\vec{PS} = \langle 0, -2, 5 \rangle$

$$\vec{PS} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix} \begin{matrix} 0 & -2 \\ 1 & -1 \end{matrix}$$

$$= \hat{i} + 5\hat{j} + 2\hat{k}$$

$$d = \frac{\sqrt{1+25+4}}{\sqrt{6}} \qquad \sqrt{\frac{30}{6}}$$

$$= \sqrt{5} \text{ unit}$$

Plane in Space

$\vec{n} = A\hat{i} + B\hat{j} + C\hat{k}$

$P(x, y, z)$ $P_0(x_0, y_0, z_0)$

$\vec{n} \perp \vec{P_0P} \Rightarrow \vec{n} \cdot \vec{P_0P} = 0$

$\vec{n}$

$(A\hat{i} + B\hat{j} + C\hat{k}) \cdot [(x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k}] = 0$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$Ax + By + Cz = Ax_0 + By_0 + Cz_0$

$$Ax + By + Cz = D$$

Ex Eqn of plane? $P_0(-3, 0, 7)$ & $\vec{n} = 5\hat{i} + 2\hat{j} - \hat{k}$

soln $5x + 2y - z = -22$ ($-15 - 7$)

Ex $A(0, 0, 1)$ $B(2, 0, 0)$ $C(0, 3, 0)$ eqn plane ??

soln

$\vec{AB} = \langle 2, 0, -1 \rangle$

$\vec{AC} = \langle 0, 3, -1 \rangle$

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} \begin{matrix} 2 & 0 \\ 0 & 3 \end{matrix}$$

$= 3\hat{i} + 2\hat{j} + 6\hat{k}$ ← (A)

$3(0) + 2(0) + 6(1)$

eqn of a plane:

$$3x + 2y + 6z = 6$$

Lines of Intersection

Ex $\vec{n} \parallel$ $\begin{cases} 3x - 6y - 2z = 15 & (1) \\ 2x + y - 2z = 5 & (2) \end{cases}$

$\vec{n_1} = 3\hat{i} - 6\hat{j} - 2\hat{k}$

$\vec{n_2} = 2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} \begin{matrix} 3 & -6 \\ 2 & 1 \end{matrix}$$

$$= 14\hat{i} + 2\hat{j} + 15\hat{k}$$

Ex Point? $\begin{cases} x = \frac{8}{3} + 2t \\ y = -2t \\ z = 1 + t \end{cases}$ (1) $3x + 2y + 6z = 6$

$3\left(\frac{8}{3} + 2t\right) + 2(-2t) + 6(1 + t) = 6$

$8 + 6t - 4t + 6 + 6t = 6$

$8t = -8$

$t = -1$

$x = \frac{8}{3} - 2 = \frac{2}{3}$, $y = +2$, $z = 1 - 1 = 0$

Point $\left(\frac{2}{3}, 2, 0\right)$

Distance from a Pt to a plane

$$d = \left| \vec{PS} \cdot \frac{\vec{n}}{|\vec{n}|} \right|$$

Ex d? from $S(1,1,3) \rightarrow 3x+2y+6z=6$

Soln: $\vec{n} = 3\hat{i}+2\hat{j}+6\hat{k}$ $\quad (0,0,1)$ $(0,3,0)$

$P = (2, 0, 0)$

$\vec{PS} = -\hat{i}+\hat{j}+3\hat{k}$

$$d = \left| (-\hat{i}+\hat{j}+3\hat{k}) \cdot (3\hat{i}+2\hat{j}+6\hat{k}) \right| \left(\frac{1}{\sqrt{9+4+36}} \right)$$

$$= \frac{1}{7}(-3+2+18)$$

$$= \frac{17}{7} \text{ unit}$$

Ex $\vec{v}$, $|\vec{v}|$, $\vec{a}(t)$

$\vec{r}(t) = 2\cos t\,\hat{i} + 2\sin t\,\hat{j} + 5\cos^2 t\,\hat{k}$

$t = \frac{7\pi}{4}$

soln $\vec{v}(t) = -2\sin t\,\hat{i} + 2\cos t\,\hat{j} - 10\cos t\sin t\,\hat{k}$

$= -2\sin t\,\hat{i} + 2\cos t\,\hat{j} - 5\sin 2t\,\hat{k}$

$\vec{a}(t) = -2\cos t\,\hat{i} - 2\sin t\,\hat{j} - 10\cos 2t\,\hat{k}$

$|\vec{v}| = \sqrt{4\sin^2 t + 4\cos^2 t + 25\sin^2 2t}$

$= \sqrt{4(\sin^2 t + \cos^2 t) + 25\sin^2 2t}$

$= \sqrt{4 + 25\sin^2 2t}$

$\vec{v}\Big|_{t=\frac{7\pi}{4}} = -2\sin\frac{7\pi}{4}\,\hat{i} + 2\cos\frac{7\pi}{4}\,\hat{j} - 5\sin\frac{7\pi}{2}\,\hat{k}$

(QIV; $-\sqrt{2}/2$; $\sqrt{2}/2$; -1)

$= \sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j} + 5\,\hat{k}$

$|\vec{v}| = \sqrt{2+2+25}$

$= \sqrt{29}$

$a(t)\Big|_{t=\frac{7\pi}{4}} = -\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}$

#10 $\vec{r}(t) = 2\ln(t+1)\,\hat{i} + t^2\hat{j} + \frac{1}{2}t^2\hat{k}$ $t=1$

soln

$$\vec{v}(t) = \frac{2}{t+1}\hat{i} + 2t\hat{j} + t\hat{k}\Big|_{t=1}$$

$$= \hat{i} + 2\hat{j} + \hat{k}$$

$$|\vec{v}| = \sqrt{1+4+1} = \sqrt{6}$$

$$\vec{a}(t) = -\frac{2}{(t+1)^2}\hat{i} + 2\hat{j} + \hat{k}\Big|_{t=1}$$

$$= -\frac{1}{2}\hat{i} + 2\hat{j} + \hat{k}$$

Ex $\int (\cos t\,\hat{i} + \hat{j} - 2t\hat{k})\,dt$

$$= (\sin t + c_1)\hat{i} + (t + c_2)\hat{j} + (-t^2 + c_3)\hat{k}$$

$$= \sin t\,\hat{i} + t\hat{j} - t^2\hat{k} + c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$= \sin t\,\hat{i} + t\hat{j} - t^2\hat{k} + \vec{C}$$

$$\vec{C} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$