#1 $y = x^2$, $y = \sqrt{x}$ $\quad 0 \le x \le 1$

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}\left(\sqrt{x} - x^2\right)^2 = \frac{\pi}{4}\left(x - \underbrace{2\sqrt{x}\,x^2}_{=5/2} + x^4\right)$$

$$V = \int_0^1 A(x)\,dx = \frac{\pi}{4}\int_0^1 \left(x - 2x^2 x^{1/2} + x^4\right)dx$$

$$= \frac{\pi}{4}\left[\frac{1}{2}x^2 - 2\,\frac{2}{7}x^{7/2} + \frac{1}{5}x^5\right]_0^1$$

$$= \frac{\pi}{4}\left[\frac{1}{2} - \frac{4}{7} + \frac{1}{5}\right] = \boxed{\frac{9\pi}{280}}$$

#2 $y = x$; $y = 2\sqrt{x}$

$$y = \underbrace{x = 2\sqrt{x}} \Rightarrow x^2 = 4x$$

$$x^2 - 4x = 0 \to x(x-4) = 0$$

$$\Rightarrow x = \underset{a}{0},\ x = \underset{b}{4}$$

Area (equilateral Δ) $= \frac{1}{2}(\text{side})^2(\sin \pi/3)$

$$= \frac{1}{2}\left(2\sqrt{x} - x\right)^2\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{4}\left(4x - 4x\sqrt{x} + x^2\right)$$

$$V = \int_0^4 A(x)\,dx = \int_0^4 \frac{\sqrt{3}}{4}\left(4x - 4x^{3/2} + x^2\right)dx$$

$$= \frac{\sqrt{3}}{4}\left[2x^2 - 4\,\frac{2}{5}x^{5/2} + \frac{1}{3}x^3\right]_0^4$$

$$= \frac{\sqrt{3}}{4}\left[2(4)^2 - \frac{8}{5}4^{5/2} + \frac{1}{3}4^3\right]$$

$$= \frac{8\sqrt{3}}{15}$$

#3 $x^{1/2} + y^{1/2} = \sqrt{6} \Rightarrow 0 \le x \le 6$

$$A(x) = (edge)^2 = \left((\sqrt{6} - \sqrt{x})^2 - 0\right)^2$$
$$= (\sqrt{6} - \sqrt{x})^4$$
$$= 36 - 24\sqrt{6}\sqrt{x} + 36x - 4\sqrt{6}\, x^{3/2} + x^2$$

$$V = \int_0^6 (36 - 24\sqrt{6}\, x^{1/2} + 36x - 4\sqrt{6}\, x^{3/2} + x^2)\, dx$$
$$= 36x - 24\sqrt{6}\, \tfrac{2}{3} x^{3/2} + 18x^2 + 4\sqrt{6}\, \tfrac{2}{5} x^{5/2} + \tfrac{1}{3} x^3 \Big|_0^6$$
$$= 36(6) - 16\sqrt{6}(6)^{3/2} + 18(6)^2 - \tfrac{8}{5}\sqrt{6}\,(6)^{5/2} + \tfrac{1}{3}(6)^3$$
$$= \boxed{\tfrac{72}{5}}$$

#4 $y = \frac{4}{x^3}$ $\quad \underset{(a)}{x = 1}$ $\quad y = \frac{1}{2}$ $\quad \left\{ \frac{4}{x^3} = \frac{1}{2} \Rightarrow x^3 = 8 \right.$ $\boxed{x = 2} = b$

a) x-axis $\rightarrow$ washer method

$$R(x) = \frac{4}{x^3}, \quad r(x) = \frac{1}{2}$$

$$V = \pi \int_1^2 [R^2(x) - r^2(x)]\, dx = \pi \int_1^2 \left[\left(\frac{4}{x^3}\right)^2 - \left(\frac{1}{2}\right)^2\right] dx$$
$$= \pi \int_1^2 \left(16x^{-6} - \frac{1}{4}\right) dx$$
$$= \pi \left[16\frac{x^{-5}}{-5} - \frac{1}{4}x\right]_1^2 = \cancel{\pi\left[\left(-\frac{16}{5}2^{-5} - \frac{1}{2}\right)\right.}$$
$$= \pi\left[-\frac{16}{5}2^{-5} - \frac{1}{4}(2) - \left(-\frac{16}{5} - \frac{1}{4}\right)\right]$$
$$= \frac{57\pi}{20}$$

4 cont. b) shell method

$$V = 2\pi \int_1^2 x\left(\frac{4}{x^3} - \frac{1}{2}\right)dx = 2\pi \int_1^2 \left(4x^{-2} - \frac{1}{2}x\right)dx$$

$$= 2\pi \left[\frac{4x^{-3}}{-3} - \frac{1}{4}x^2\right]_1^2$$

$$= 2\pi \left(-\frac{4}{3}2^{-3} - \frac{1}{4}2^2\right) - \left(-\frac{4}{3} - \frac{1}{4}\right)\Big]$$

$$= 2\pi\left(\frac{5}{4}\right)$$

$$= \frac{5\pi}{2}$$

c) shell method : $x = 2 \Rightarrow$ shell radius $= 2 - x$

$$V = 2\pi \int_1^2 (2-x)\left(\frac{4}{x^3} - \frac{1}{2}\right)dx$$

$$= 2\pi \int_1^2 \left(\frac{8}{x^3} - 1 - 4x^{-2} + \frac{1}{2}x\right)dx$$

$$= 2\pi \left[8\frac{x^{-2}}{-2} - x - 4\frac{x^{-3}}{-3} + \frac{1}{4}x^2\right]_1^2$$

$$= 2\pi \Big[-4(2)^{-2} - 2 + \frac{4}{3}2^{-3} + \frac{1}{4}2^2 - \left(-4 - 1 + \frac{4}{3} + \frac{1}{4}\right)$$

$$= \frac{3\pi}{2}$$

d) washer method. $y = 4$

$R(x) = 4 - \frac{1}{2} = \frac{8-1}{2} = 7/2$

$r(x) = 4 - 4/x^3$

$$V = \pi \int_1^2 \left[\left(\frac{7}{2}\right)^2 - \left(4 - \frac{4}{x^2}\right)^2\right]dx$$

$$= \pi \int_1^2 \left[\frac{49}{4} - \left(16 - 32x^{-2} + 16x^{-4}\right)\right]dx$$

$$= \pi \int_1^2 \left(\frac{49}{4} - 16 + 32x^{-2} - 16x^{-4}\right)dx$$

4-d cont.

$$V = \pi \int_1^2 \left(-\frac{15}{4} + 32x^{-2} - 16x^{-4}\right) dx$$

$$= \pi \left[-\frac{15}{4}x + 32\frac{x^{-1}}{-1} - 16\frac{x^{-3}}{-3}\right]_1^2$$

$$= \pi \left[-\frac{15}{4}x - 32x^{-1} + \frac{16}{3}x^{-3}\right|_1^2$$

$$= \pi \left[-\frac{15}{4}2 - \frac{32}{2} + \frac{16}{3}2^{-3} - \left(-\frac{15}{4} - 32 + \frac{16}{3}\right)\right]$$

$$= \frac{103}{20}\pi$$

#5 $y = \sin x \quad x = 0, \pi \ \& \ y = 2$ about $y = 2$

$$V = \pi \int_0^\pi (2 - \sin x)^2 dx = \pi \int_0^\pi (4 - 4\sin x + \sin^2 x) dx$$

$$= \pi \left[4x + 4\cos x + \frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^\pi$$

$$= \pi \left[4\pi + 4\cos\pi + \frac{\pi}{2} - \frac{1}{4}\underbrace{\sin 2\pi}_{0} - (4)\right]$$

$$= \pi \left[\frac{9}{2}\pi - 4 - 4\right)$$

$$= \pi \left(\frac{9}{2}\pi - 8\right)$$

$$= \frac{\pi}{2}(9\pi - 16)$$

#6 $\frac{4x^2}{121} + \frac{y^2}{12} = 1 \Rightarrow \frac{y^2}{12} = 1 - \frac{4x^2}{121}$

$$y^2 = 12\left(1 - \frac{4x^2}{121}\right) \Rightarrow y = \sqrt{12\left(1 - \frac{4x^2}{121}\right)}$$

$$V = \pi \int_{-11/2}^{11/2} R^2(x)\,dx = \pi \int_{-11/2}^{11/2} \left(\sqrt{12\left(1 - \frac{4x^2}{121}\right)}\right)^2 dx$$

$$= \pi \int_{-11/2}^{11/2} 12\left(1 - \frac{4}{121}x^2\right) dx = 12\pi \left[x - \frac{4}{121}\frac{x^3}{3}\right]_{-11/2}^{11/2}$$

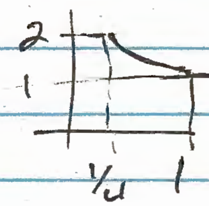
$$= 24\pi \left(x - \frac{4}{363}x^3\right)_0^{11/2}$$

$$= 24\pi \left(\frac{11}{2} - \frac{4}{363}\left(\frac{11}{2}\right)^3\right) = 88\pi \approx \boxed{276.\ in^3}$$

#7 a) Shell Method. $y = \frac{1}{\sqrt{x}} \Rightarrow \sqrt{x} = \frac{1}{y} \Rightarrow x = \frac{1}{y^2}$

$$V = \pi \int_1^2 [R^2(y) - r^2(y)]\,dy = \pi \int_1^2 \left(\frac{1}{y^4} - \frac{1}{16}\right) dy$$

$$= \pi \left[\frac{y^{-3}}{-3} - \frac{1}{16}y\right]_1^2 = \pi$$



$$= \pi \left[-\frac{1}{3}2^{-3} - \frac{1}{16}(2) - \left(-\frac{1}{3} - \frac{1}{16}\right)\right]$$

$$= \boxed{\frac{11\pi}{48}}$$

b) Washer Method. $V = 2\pi \int_{1/4}^{1} x\left(\frac{1}{\sqrt{x}} - 1\right) dx$

$$V = 2\pi \int_{1/4}^{1} (x^{1/2} - x)\,dx = 2\pi \left[\frac{2}{3}x^{3/2} - \frac{1}{2}x^2\right]_{1/4}^{1}$$

$$= 2\pi \left[\frac{2}{3} - \frac{1}{2} - \left(\frac{2}{3}\left(\frac{1}{4}\right)^{3/2} - \frac{1}{2}\left(\frac{1}{4}\right)^2\right)\right]$$

$$= \boxed{\frac{11\pi}{48}}$$

#8 $y = x^{1/2} - \frac{1}{3}x^{3/2} \qquad 1 \le x \le 4$

$$y' = \frac{1}{2}x^{-1/2} - \frac{1}{3}\cdot\frac{3}{2}x^{1/2} = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{1/2}$$

$$(y')^2 = \frac{1}{4}\left(x^{-1/2} - x^{1/2}\right)^2 = \frac{1}{4}\left(x^{-1} - 2 + x\right)$$

$$L = \int_1^4 \sqrt{1 + \frac{1}{4}\left(\frac{1}{x} - 2 + x\right)}\,dx$$

$$= \int_1^4 \sqrt{1 + \frac{1}{4}\frac{1}{x} - \frac{1}{2} + \frac{1}{4}x}\,dx$$

$$= \int_1^4 \sqrt{\frac{1}{2} + \frac{1}{4}\frac{1}{x} + \frac{1}{4}x}\,dx = \int_1^4 \sqrt{\frac{1}{4}\left(\frac{1}{x} + 2 + x\right)}\,dx$$

$$\cancel{= \frac{1}{2}\int_1^4 \sqrt{\left(\frac{1}{x^{1/2}} + x\right)^2}\,dx - \frac{1}{2}\int_1^4 \left(\frac{1}{x^{1/2}} + x^{1/2}\right)dx}$$

$$= \frac{1}{2}\int_1^4 \sqrt{\left(x^{-1/2} + x^{1/2}\right)^2}\,dx = \frac{1}{2}\int_1^4 \left(x^{-1/2} + x^{1/2}\right)dx$$

$$= \frac{1}{2}\left[2x^{1/2} + \frac{2}{3}x^{3/2}\right]_1^4$$

$$= \frac{1}{2}\left[2\sqrt{4} + \frac{2}{3}4^{3/2} - \left(2 + \frac{2}{3}\right)\right]$$

$$= \boxed{\frac{10}{3}}$$

#9 $x = y^{2/3} \quad 1 \le y \le 8$

$x' = \frac{2}{3} y^{-1/3} \Rightarrow (x')^2 = \frac{4}{9} y^{-2/3}$

$L = \int_1^8 \sqrt{1 + \frac{4}{9} y^{-2/3}}\, dy = \int_1^8 \sqrt{1 + \frac{4}{9y^{2/3}}}\, dy$

$= \int_1^8 \sqrt{\frac{9y^{2/3} + 4}{9y^{2/3}}}\, dy = \frac{1}{3} \int_1^8 \sqrt{9y^{2/3} + 4}\,(y^{-1/3})\, dy$

$(\sqrt{y^{2/3}} = y^{1/3})$

$u = 9y^{2/3} + 4$

$du = 9 \cdot \frac{2}{3} y^{-1/3} dy$

$du = 6y^{-1/3} dy$

$\frac{1}{6} du = y^{-1/3} dy$

$L = \frac{1}{3} \int_1^8 (9y^{2/3}+4)^{1/2} \frac{1}{6} d(9y^{2/3}+4)$

$= \frac{1}{18} \cdot \frac{2}{3} (9y^{2/3}+4)^{3/2} \Big|_1^8$

$= \frac{1}{27} \left[(9 \underbrace{(8)^{2/3}}_{4} + 4)^{3/2} - (9+4)^{3/2} \right]$

$= \frac{1}{27} \left[40^{3/2} - 13^{3/2} \right]$

$\boxed{\approx 7.634}$

\# 10 Area of the surface $y = \frac{1}{3}x^3$ $\quad 0 \le x \le 1 \quad$ x-axis

$$y' = x^2 \Rightarrow (y')^2 = x^4$$

$$S = 2\pi\int_0^1 y\sqrt{1+(y')^2}\,dx = 2\pi\int_0^1 \frac{1}{3}x^3\sqrt{1+x^4}\,dx$$

$$= \frac{2}{3}\pi\int_0^1 x^3(1+x^4)^{1/2}\,dx \qquad d(1+x^4) = 4x^3\,dx$$

$$\frac{1}{4}\,d(1+x^4) = x^3\,dx$$

$$= \frac{2}{3}\pi\int_0^1 (1+x^4)^{1/2}\,\frac{1}{4}\,d(1+x^4)$$

$$= \frac{\pi}{6}\int_0^1 (1+x^4)^{1/2}\,d(1+x^4)$$

$$= \frac{\pi}{6}\,\frac{2}{3}(1+x^4)^{3/2}\Big|_0^1$$

$$= \frac{\pi}{9}\left[2^{3/2} - 1^{3/2}\right]$$

$$= \frac{\pi}{9}\left(2\sqrt{2} - 1\right)$$

$$\#11 \qquad x = \sqrt{4y - y^2} \qquad 1 \le y \le 2 \qquad y\text{-axis}$$

$$x' = \frac{1}{2}(4y - y^2)^{-1/2}\underbrace{(4 - 2y)}_{2(\quad)} = (2-y)(4y-y^2)^{-1/2}$$

$$(x')^2 = (2-y)^2(4y-y^2)^{-1} = \frac{4 - 4y + y^2}{4y - y^2}$$

$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{4 - 4y + y^2}{4y - y^2} = \frac{4y - y^2 + 4 - 4y + y^2}{4y - y^2}$$

$$= \frac{4}{4y - y^2}$$

$$S = 2\pi \int_1^2 x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= 2\pi \int_1^2 \cancel{(4y - y^2)^{1/2}} \frac{2}{\cancel{\sqrt{4y - y^2}}}\, dy$$

$$= 4\pi \int_1^2 dy = 4\pi\, y\Big|_1^2 = 4\pi(2-1)$$

$$= \boxed{4\pi}$$

#12 $y = 2\sqrt{x} \Rightarrow y' = 2\frac{1}{2}x^{-1/2} = x^{-1/2} = \frac{1}{\sqrt{x}}$

$(y')^2 = \frac{1}{x} \Rightarrow ds = \sqrt{(y')^2+1}\,dx$

$= \sqrt{\frac{1}{x}+1}\,dx$

$$A = \int_0^3 y\,ds$$

$$= \int_0^3 2\sqrt{x}\sqrt{\frac{1+x}{x}}\,dx$$

$d(1+x) = dx$

$$= 2\int_0^3 \sqrt{1+x}\,dx = 2\int_0^3 (1+x)^{1/2}\,d(x+1)$$

$$= 2\,\frac{2}{3}(1+x)^{3/2}\Big|_0^3$$

$$= \frac{4}{3}\left[4^{3/2} - 1^{3/2}\right]$$

$$= \boxed{\frac{28}{3}}$$

#13 Equipment alone: $F_1 = 100N$.

$$W_1 = \int_0^{40} F_1\,dx = \int_0^{40} 100\,dx = 100x\Big|_0^{40} = 4000\,J.$$

Rope: $F_2 = kx = .8(40-x)$

$$W_2 = \int_0^{40} F_2\,dx = \int_0^{40} .8(40-x)\,dx$$

$$= .8\left(40x - \frac{1}{2}x^2\right)\Big|_0^{40} = .8\left[40^2 - \frac{1}{2}40^2 - 0\right]$$

$$= 640\,J.$$

$$\boxed{\text{Total work} = 4000 + 640 = 4640\,J}$$

#14

$800 \to 400$ gal.

$8\frac{lb}{gal} \cdot 800$ gal $\to 8\frac{lb}{gal} \cdot 400$ gal.

6400 lb $\to 3200$ lb.

$$F(x) = 6400\left(\frac{2(4750)-x}{2(4750)}\right) = 6400\left(1-\frac{x}{9500}\right) \text{ lb}$$

$$W = \int_0^{4750} 6400\left(1-\frac{x}{9500}\right)dx = 6400\left[x - \frac{x^2}{19000}\right]_0^{4750}$$

$$= 6400\left[4750 - \frac{4750^2}{4(4750)}\right] = 6400\left(4750 - \frac{4750}{3}\right)$$

$$= 22,800,000. \text{ ft. lb}$$

#15

$F = kx \Rightarrow 200 = k(0.8) \Rightarrow k = \frac{200}{.8} = 250$ N/m

$300 = 250x \Rightarrow \boxed{x = \frac{300}{250} = 1.2\text{m}}$

$$W = \int_0^{1.2} F(x)\,dx = \int_0^{1.2} 250x\,dx = 125x^2\Big|_0^{1.2} = 125(1.2)^2$$

$\boxed{= 180 \text{ J}}$

#16

a) $y = \sqrt{2}e^{\sqrt{2}x} \to y' = 2e^{\sqrt{2}x}$

b) $y = \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} \Rightarrow y' = \frac{1}{4}e^{4x} + \frac{1}{4}x(4e^{4x}) - \frac{4}{16}e^{4x}$

$y' = \frac{1}{4}e^{4x} + xe^{4x} - \frac{1}{4}e^{4x}$

$= xe^{4x}$

#16 cont c) $y = \ln(\sec^2\theta) \Rightarrow y' = \dfrac{(\sec^2\theta)'}{\sec^2\theta} = \dfrac{2\sec\theta(\sec\theta)'}{\sec^2\theta}$

$$= \frac{2\sec\theta\sec\theta\tan\theta}{\sec^2\theta}$$

$$= \underline{2\tan\theta}$$

d) $y = \log_5(3x-7) = \dfrac{\ln(3x-7)}{\ln 5}$

$$y' = \frac{1}{\ln 5}\,\frac{(3x-7)'}{3x-7} = \frac{3}{(\ln 5)(3x-7)}$$

e) $y = (x+2)^{x+2}$

$\ln y = \ln(x+2)^{x+2} = (x+2)\ln(x+2)$

$(\ln y)' = [\underbrace{(x+2)}_{u}\underbrace{\ln(x+2)}_{v}]'$ $\quad u'v + v'u$

$$\frac{y'}{y} = \ln(x+2) + (x+2)\frac{(x+2)'}{x+2}$$

$$= \ln(x+2) + 1$$

$$y' = y[\ln(x+2)+1]$$

$$= (x+2)^{x+2}[\ln(x+2)+1]$$

f) $y = \sin^{-1}\left(\dfrac{1}{\sqrt{x}}\right) \quad x > 1$

$$y' = \frac{(x^{-1/2})'}{\sqrt{1-\left(\frac{1}{\sqrt{x}}\right)^2}} = \frac{-\frac{1}{2}x^{-3/2}}{\sqrt{1-\frac{1}{x}}} = -\frac{1}{2}\,\frac{1}{x^{3/2}\sqrt{\frac{x-1}{x}}}$$

$$= -\frac{1}{2x^{3/2}\frac{\sqrt{x-1}}{\sqrt{x}}} = \boxed{-\frac{1}{2x\sqrt{x-1}}}$$

16 cont

g) $y = z\cos^{-1}z - \sqrt{1-z^2}$

$$y' = \cos^{-1}z + z\left(\frac{-1}{\sqrt{1-z^2}}\right) - \frac{1}{2}(1-z^2)^{-1/2}(-2z)$$

$$= \cos^{-1}z - \frac{z}{\sqrt{1-z^2}} + \frac{z}{1-z^2}$$

$$= \cos^{-1}z$$

h) $y = t\tan^{-1}t - \frac{1}{2}\ \ln t$

$$y' = \tan^{-1}t + t\left(\frac{1}{1+t^2}\right) - \frac{1}{2}\left(\frac{1}{t}\right)$$

$$= \tan^{-1}t + \frac{t}{1+t^2} - \frac{1}{2t}$$

i) $y = \sqrt[10]{\frac{3x+4}{2x-4}}$

$$\ln y = \ln\left(\frac{3x+4}{2x-4}\right)^{1/10} = \frac{1}{10}\ln\frac{3x+4}{2x-4}$$

$$= \frac{1}{10}\left[\ln(3x+4) - \ln(2x-4)\right]$$

derivative:

$$\frac{y'}{y} = \frac{1}{10}\left[\frac{3}{3x+4} - \frac{2}{2x-4}\right]$$

$$y' = y\,\frac{1}{10}\left(\frac{3}{3x+4} - \frac{2}{2x-4}\right)$$

$$= \frac{1}{10}\sqrt[10]{\frac{3x+4}{2x-4}}\left(\frac{3}{3x+4} - \frac{2}{2x-4}\right)$$

16 cont d) $y = \left(\frac{(t+1)(t-1)}{(t-2)(t+3)} \right)^5 \quad t > 2.$

$$\ln y = 5 \ln \frac{(t+1)(t-1)}{(t-2)(t+3)}$$

$$= 5 \left[\ln(t+1) + \ln(t-1) - \ln(t-2) - \ln(t+3) \right]$$

$$\frac{y'}{y} = 5 \left(\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t+3} \right)$$

$$y' = 5y \left(\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t+3} \right)$$

$$y' = 5 \left(\frac{(t+1)(t-1)}{(t-2)(t+3)} \right)^5 \left(\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t+3} \right)$$

k) $y = (\sin\theta)^{\sqrt{\theta}}$

$$\ln y = \sqrt{\theta} \ln \sin\theta$$

$$\frac{y'}{y} = \frac{1}{2} \theta^{-1/2} \ln \sin\theta + \sqrt{\theta} \frac{\cos\theta}{\sin\theta}$$

$$y' = y \left(\frac{1}{2\sqrt{\theta}} \ln \sin\theta + \sqrt{\theta} \cot\theta \right)$$

$$= (\sin\theta)^{\sqrt{\theta}} \left(\frac{1}{2\sqrt{\theta}} \ln \sin\theta + \sqrt{\theta} \cot\theta \right)$$

#17 a) $\int e^t \cos(3e^t-2)\,dt$.

$d(3e^t-2) = 3e^t\,dt$

$\frac{1}{3}d(3e^t-2) = e^t\,dt$.

$$\int e^t \cos(3e^t-2)\,dt = \frac{1}{3}\int \cos(3e^t-2)\,d(3e^t-2)$$

$$= \frac{1}{3}\sin(3e^t-2) + C.$$

b) $\int e^y \csc(e^y+1)\cot(e^y+1)\,dy$.

$u = e^y+1$

$du = e^y\,dy$

$$\int e^y \csc(e^y+1)\cot(e^y+1)\,dy = \int \csc u \cot u\,du$$

$$= -\csc u + C$$

$$= -\csc(e^y+1) + C.$$

c) $\int (\csc^2 x)\,e^{\cot x}\,dx$

$u = \cot x \rightarrow du = -\csc^2 x\,dx$

$$= \int e^u(-du) = -e^u + C = -e^{\cot x} + C$$

d) $\int_1^e \frac{\sqrt{\ln x}}{x}\,dx$

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$$= \int_1^e u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_1^e$$

$$= \frac{2}{3}(\ln x)^{3/2}\Big|_1^e$$

$$= \frac{2}{3}\left[(\ln e)^{3/2} - (\ln 1)^{3/2}\right] = \frac{2}{3}(1-0)$$

$$= \boxed{\frac{2}{3}}$$

#17 cont.

e) $\displaystyle\int_{-\pi/2}^{\pi/6} \frac{\cos t}{1-\sin t}\,dt$ $\qquad u = 1-\sin t \to du = -\cos t\,dt$

$\qquad -du = \cos t\,dt$

$$= \int_{-\pi/2}^{\pi/6} \frac{-du}{u} = -\ln|u|\Big|_{-\pi/2}^{\pi/6}$$

$$= -\ln|1-\sin t|\Big|_{-\pi/2}^{\pi/6} = -\left[\ln\left|1-\sin\tfrac{\pi}{6}\right| - \ln\left|1-\sin(-\tfrac{\pi}{2})\right|\right]$$

$$= -\left(\ln\left|\tfrac{1}{2}\right| - \ln|2|\right)$$

$$= -\left(-\ln 2 - \ln 2\right)$$

$$= 2\ln 2 = \ln 2^2$$

$$\boxed{= \ln 4}$$

f) $\displaystyle\int 2^{\tan x}\sec^2 x\,dx$ $\qquad u = \tan x \to du = \sec^2 x\,dx$

$$= \int 2^u\,du = \frac{1}{\ln 2}2^u + C = \frac{1}{\ln 2}2^{\tan x} + C.$$

g) $\displaystyle\int_1^8 \left(\frac{2}{3x} - \frac{8}{x^2}\right)dx = \int\left(\frac{2}{3}\,\frac{1}{x} - 8x^{-2}\right)dx$

$$= \frac{2}{3}\ln|x| - 8\,\frac{x^{-1}}{-1}\Big|_1^8$$

$$= \frac{2}{3}\left[\ln|x| + \frac{8}{x}\right]_1^8$$

$$= \frac{2}{3}\left[\ln 8 + \frac{8}{8} - \left(\ln 1 + \frac{8}{1}\right)\right]$$

$$= \frac{2}{3}\left(\ln 8 + 1 - (0+8)\right)$$

$$= \frac{2}{3}\left(\ln 8 + 1 - 8\right)$$

$$= \frac{2}{3}(\ln 8 - 7) = \frac{2}{3}\ln 8 \cancel{-6}\,{-7} = \ln 8^{2/3} \cancel{-6} - 7$$

$$\boxed{\ln 4 - 7}$$

\# 17. cont.

h) $\displaystyle\int_0^{\ln 9} e^{\theta}(e^{\theta}-1)^{1/2}\,d\theta \qquad u = e^{\theta}-1 \Rightarrow du\ e^{\theta} d\theta$

$$= \int_0^{\ln 9} u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_0^{\ln 9}$$

$$= \frac{2}{3}(e^{\theta}-1)^{3/2}\Big|_0^{\ln 9}$$

$$= \frac{2}{3}\left[(e^{\ln 9}-1)^{3/2} - \overbrace{(e^{0}-1)}^{=0}\right]$$

$$= \frac{2}{3}(9-1)^{3/2} = \frac{2}{3}(8^{3/2}) =$$

$$= \frac{2}{3}\sqrt{(2^3)^3} = \frac{2}{3}\sqrt{2^9} = \frac{2}{3}2^4\sqrt{2}$$

$$= \boxed{\frac{32\sqrt{2}}{3}}$$

i) $\displaystyle\int_{-1/5}^{1/5} \frac{6\,dx}{\sqrt{4-25x^2}} \qquad \begin{cases} a^2 = 4 \Rightarrow a = 2 \\ u^2 = 25x^2 \Rightarrow u = 5x \end{cases}$

$$du = 5\,dx$$
$$\tfrac{1}{5}du = dx$$

$$= \int_{-1/5}^{1/5} \frac{6}{\sqrt{a^2-u^2}}\,\frac{1}{5}du$$

$$= \frac{6}{5}\left[\sin^{-1}\left(\frac{u}{2}\right)\right]_{-1/5}^{1/5} = \frac{6}{5}\left[\sin^{-1}\left(\frac{5x}{2}\right)\right]_{-1/5}^{1/5}$$

$$= \frac{6}{5}\left(\sin^{-1}\left(\frac{5}{2}\cdot\frac{1}{5}\right) - \sin^{-1}\left(\frac{5}{2}\cdot\frac{-1}{5}\right)\right]$$

$$= \frac{6}{5}\left(\sin^{-1}\frac{1}{2} + \sin^{-1}\left(-\frac{1}{2}\right)\right)$$

$$= \frac{6}{5}\left(\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)\right) = \frac{6}{5}\left(\frac{\pi}{3}\right)$$

$$= \boxed{\frac{2\pi}{5}}$$

17 cont.

j) $\int \frac{24\,dy}{y\sqrt{y^2-16}} = 24\left(\frac{1}{4}\sec^{-1}\left|\frac{y}{4}\right|\right)+C$ $\qquad a^2=16$, $a=4$

$= 6\sec^{-1}\left|\frac{y}{4}\right| + C$

k) $\int \frac{dx}{\sqrt{-x^2+4x-1}}$

$= \int \frac{du}{\sqrt{a^2-u^2}}$

$= \sin^{-1}\left(\frac{x-2}{\sqrt{3}}\right)+C$

$-x^2+4x-1 = -(x^2-4x)-1$
$= -(x^2-4x)-1+4-4$
$= -(x^2-4x+4)-1+4$
$= -(x-2)^2+3$
$= 3-(x-2)^2$

$a^2=3 \Rightarrow a=\sqrt{3}$
$u^2=(x-2)^2 \Rightarrow u=(x-2)$
$du = dx$

l) $\int_{-1}^{1} \frac{3\,dv}{4v^2+4v+4}$

$= \int_{-1}^{1} \frac{3\,dv}{4(v^2+v+1)}$

$= \frac{3}{4}\int_{-1}^{1} \frac{dv}{\frac{3}{4}+(v+\frac{1}{2})^2}$

$= \frac{3}{4}\int_{-1}^{1} \frac{du}{a^2+u^2}$

$= \frac{3}{4}\,\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\frac{2u}{\sqrt{3}}\Big|_{-1}^{1}$

$= \frac{3}{2\sqrt{3}}\tan^{-1}\frac{2}{\sqrt{3}}(v+\frac{1}{2})\Big|_{-1}^{1} = \frac{\sqrt{3}}{2}\left[\tan^{-1}\left(\frac{2}{\sqrt{3}}\frac{3}{2}\right)-\tan^{-1}\frac{2}{\sqrt{3}}\left(-\frac{1}{2}\right)\right]$

$= \frac{\sqrt{3}}{2}\left(\tan^{-1}\frac{3}{\sqrt{3}}-\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)\right) = \frac{\sqrt{3}}{2}\left(\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right) = \boxed{\frac{\sqrt{3}}{4}\pi}$

~~$4v^2+4v+4$~~
$v^2+v+1 = (v^2+v)+\frac{1}{4}-\frac{1}{4}+1$
$= (v^2+v+\frac{1}{4})+\frac{3}{4}$
$= (v+\frac{1}{2})^2+\frac{3}{4}$

$a = \frac{3}{4} \Rightarrow a = \frac{\sqrt{3}}{2}$
$u^2 = (v+\frac{1}{2})^2$
$u = v+\frac{1}{2}$
$du = dv$

#17 cont m) $\displaystyle\int \frac{dt}{(3t+1)\sqrt{9t^2+6t}}$

$9t^2+6t = 9t^2+6t+1-1$
$= (3t+1)^2 - 1$

$u = 3t+1$
$du = 3\,dt \Rightarrow \frac{1}{3}du = dt$

$$= \int \frac{(1/3)\,du}{u\sqrt{u^2-1}}$$

$$= \frac{1}{3}\sec^{-1}|u| + C$$

$$= \frac{1}{3}\sec^{-1}|3t+1| + C.$$

#18 a) $\displaystyle\lim_{x\to 1} \frac{x^a-1}{x^b-1} = \frac{1^a-1}{1^b-1} = \frac{1-1}{1-1} = \frac{0}{0}$

$$= \lim_{x\to 1} \frac{a x^{a-1}}{b x^{b-1}} = \boxed{\frac{a}{b}}$$

b) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = \cos 0 = \boxed{1}$

c) $\displaystyle\lim_{x\to 0} \frac{\sin mx}{\sin nx} = \lim_{x\to 0} \frac{m\cos mx}{n\cos nx} = \frac{m\cos 0}{n\cos 0} = \boxed{\frac{m}{n}}$

d) $\displaystyle\lim_{x\to\infty} \left(\frac{x^3}{x^2-1} - \frac{x^3}{x^2+1}\right) = \infty - \infty$

$$= \lim \frac{x^3(x^2+1) - x^3(x^2-1)}{(x^2-1)(x^2+1)} = \lim \frac{x^5+x^3-x^5+x^3}{x^4-1}$$

$$= \lim_{x\to\infty} \frac{2x^3}{x^4} = \lim_{x\to\infty} \frac{2}{x} = \boxed{0}$$

#18 cont.

g) $$\lim_{x\to 0}\frac{2^{-\sin x}-1}{e^x-1} = \lim_{x\to 0}\frac{2^{-\sin x}(\ln 2)(-\cos x)}{e^x} = -\ln 2$$

f) $$\lim_{x\to 4}\frac{\sin^2 \pi x}{e^{x-4}+3-x} = \lim_{x\to 4}\frac{2(\sin \pi x)(\pi \cos \pi x)}{e^{x-4}-1}$$

$$= \lim_{x\to 4}\frac{\pi \sin 2\pi x}{e^{x-4}-1} = \frac{0}{0}$$

$$= \lim \frac{\pi(2\pi)\cos 2\pi x}{e^{x-4}} = \boxed{2\pi^2}$$

g) $$\lim \left(\frac{e^x+1}{e^x-1}\right)^{\ln x}$$

$$f(x) = \left(\frac{e^x+1}{e^x-1}\right)^{\ln x} \Rightarrow \ln f(x) = \ln x \ln\left(\frac{e^x+1}{e^x-1}\right)$$

$$\left\{ \frac{e^x+1}{e^x+1}\cdot\frac{e^{-x/2}}{e^{-x/2}} = \frac{e^{x/2}-e^{-x/2}}{e^{x/2}+e^{-x/2}} = \coth\left(\frac{x}{2}\right) \right\}$$

$$\lim_{x\to\infty} \ln x \ln\left(\frac{e^x+1}{e^x-1}\right) = \lim_{x\to\infty} \ln x \ln \coth(x/2)$$

$$= \lim_{x\to\infty}\frac{\ln\coth(x/2)}{\frac{1}{\ln x}} = \frac{0}{0}$$

$$= \lim\left(\frac{\frac{\operatorname{csch}^2(\frac{x}{2})}{\coth(x/2)}(-1/2)}{-\frac{1}{(\ln x)^2}\frac{1}{x}}\right) = \lim \frac{x(\ln x)^2}{2\sinh\frac{x}{2}\cosh\frac{x}{2}}$$

$$= \lim\frac{x(\ln x)^2}{\sinh x} = \lim_{x\to\infty}\frac{(\ln x)^2 + 2x\ln(x)\left(\frac{1}{x}\right)}{\cancel{\sinh x}\cosh x}$$

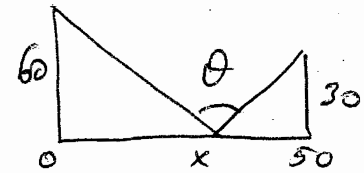
$$= \lim_{x\to\infty}\frac{(\ln x)^2+2\ln x}{\cosh x} = \lim_{x\to\infty}\left(\frac{2\left(\frac{1}{x}\right)+2(\ln x)\frac{1}{x}}{\sinh x}\right)$$

$$= \lim_{x\to\infty}\frac{2+2\ln x}{x\sinh x} = \lim_{x\to\infty} 2\,\frac{1/x}{x\cosh x+\sinh x}$$

$$= \lim_{x\to\infty}\frac{2}{x^2\cosh x + x\sinh x} = 0 \Rightarrow$$

$$\ln f(x) = 0 \Rightarrow \boxed{f(x) = 1}$$

#19



$$\theta = \pi - \cot^{-1}\left(\frac{x}{60}\right) - \cot^{-1}\left(\frac{50-x}{30}\right)$$

$$0 < x < 50$$

$$\theta' = \frac{1/60}{1+\left(\frac{x}{60}\right)^2} + \frac{-1/30}{1+\left(\frac{50-x}{30}\right)^2} = \frac{1}{60}\,\frac{1}{1+\frac{x^2}{60^2}} - \frac{1}{30}\,\frac{1}{1+\frac{(50-x)^2}{30^2}}$$

$$= \frac{1}{60}\,\frac{1}{\frac{60^2+x^2}{60^2}} - \frac{1}{30}\,\frac{1}{\frac{30^2+(50-x)^2}{30^2}}$$

$$= 60\,\frac{1}{60^2+x^2} - 30\,\frac{1}{30^2+(50-x)^2}$$

$$= 30\left(\frac{2}{60^2+x^2} - \frac{1}{30^2+(50-x)^2}\right)$$

$$= 30\left[\frac{2(30^2+50^2-100x+x^2) - 60^2 - x^2}{(60^2+x^2)\,(30^2+(50-x)^2)}\right] = 0$$

$$2(900+2500-100x+x^2) - 3600 - x^2 = 0$$

$$1800 + 5000 - 200x + 2x^2 - 3600 - x^2 = 0$$

$$x^2 - 200x + 3200 = 0$$

$$\Rightarrow x = 100 \pm 20\sqrt{17} \Rightarrow \begin{cases} x = 80.46 \underline{\underline{> 50}} \\ x = 17.54 \end{cases}$$

to maximize $\theta \Rightarrow \underline{x = 17.54\ \text{m}}$

#20

$$v = x^2 \ln\left(\tfrac{1}{x}\right) = x^2\left(\overset{=0}{\ln 1} - \ln x\right) = -x^2 \ln x$$

$$v' = -2x\ln x - x^2\left(\tfrac{1}{x}\right)$$
$$= -2x\ln x - x$$
$$= -x\,(2\ln x + 1) = 0$$

$$\begin{cases} x = 0 \\ 2\ln x + 1 = 0 \Rightarrow 2\ln x = -1 \Rightarrow \ln x = -\tfrac{1}{2} \end{cases}$$

$$\ln x = -\tfrac{1}{2} \Rightarrow x = e^{-1/2}$$

x	0		$e^{-1/2}$	
v'		>0	Max	<0

RMAX @ $x = e^{-1/2}$

$$x = \frac{r}{h} \Rightarrow \boxed{h = \frac{r}{x} = \frac{1}{e^{-1/2}} = e^{1/2} = \sqrt{e} \approx 1.65\text{ cm}}$$