Solution   *Section* 3.2 – Estimating a Population Proportion

Exercise

Find the critical value $z_{\alpha/2}$ that corresponds to a 99% confidence level.

Solution

For 99% confidence, $\alpha = 1 - 0.99 = 0.01 \quad \Rightarrow \quad \frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

For the upper 0.005: $A = 0.995 \quad \Rightarrow \quad z = 2.575$

$z_{\alpha/2} = z_{0.005} = 2.575$

z score	Area
1.645	0.9500
2.575	0.9950

Exercise

Find the critical value $z_{\alpha/2}$ that corresponds to a 99.5% confidence level.

Solution

For 95% confidence, $\alpha = 1 - 0.995 = 0.005 \quad \Rightarrow \quad \frac{\alpha}{2} = \frac{0.005}{2} = 0.0025$

For the upper 0.0025: $A = 0.9975 \quad \Rightarrow \quad z = 2.81$

$z_{\alpha/2} = z_{0.0025} = 2.81$

z	.00	.01	.02
2.8	.9974	.9975	.9976

Exercise

Find the critical value $z_{\alpha/2}$ that corresponds to a 98% confidence level.

Solution

For 95% confidence, $\alpha = 1 - 0.98 = 0.02 \quad \Rightarrow \quad \frac{\alpha}{2} = \frac{0.02}{2} = 0.01$

For the upper 0.01: $A = 0.99 \quad \Rightarrow \quad z = 2.33$

$z_{\alpha/2} = z_{0.01} = 2.33$

Exercise

Find $z_{\alpha/2}$ for $\alpha = 0.10$.

Solution

For $\alpha = 0.1 \quad \Rightarrow \quad \frac{\alpha}{2} = \frac{0.1}{2} = 0.05$

For the upper 0.05: $A = 0.95 \quad \Rightarrow \quad z = 1.645$

$z_{\alpha/2} = z_{0.05} = 1.645$

z score	Area
1.645	0.9500
2.575	0.9950

Exercise

Find $z_{\alpha/2}$ for $\alpha = 0.02$.

Solution

For $\alpha = 0.02 \quad \Rightarrow \quad \frac{\alpha}{2} = \frac{0.02}{2} = 0.01$

For the upper 0.01: $A = 0.99 \quad \Rightarrow \quad z = 2.33$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916

$z_{\alpha/2} = z_{0.01} = 2.33$

Exercise

Express the confidence interval $0.200 < p < 0.500$ in the form $\hat{p} \pm E$

Solution

Let L = the lower confidence limit $\qquad U$ = the upper confidence limit

$\hat{p} = \frac{L+U}{2} = \frac{0.2+0.5}{2} = 0.35$

$E = \frac{U-L}{2} = \frac{0.5-0.2}{2} = 0.15$

$\therefore$ The interval can be expressed as 0.35 ± 0.15

Exercise

Express the confidence interval $0.42 < p < 0.54$ in the form $\hat{p} \pm E$

Solution

$\hat{p} = \frac{L+U}{2} = \frac{0.42+0.54}{2} = 0.48$

$E = \frac{U-L}{2} = \frac{0.54-0.42}{2} = 0.06$

$\therefore$ The interval can be expressed as 0.48 ± 0.06

Exercise

Express the confidence interval 0.222 ± 0.044 in the form $\hat{p} - E < p < \hat{p} + E$

Solution

Given: $\quad \hat{p} = 0.222 \quad$ *and* $\quad E = 0.044$

$L = \hat{p} - E = 0.222 - 0.044 = 0.178 \qquad\qquad L = \hat{p} + E = 0.222 + 0.044 = 0.266$

$\therefore$ The interval can be expressed as $0.178 < p < 0.266$

Exercise

Find the point estimate $\hat{p}$ and the margin of error E of $(0.320,\ 0.420)$

Solution

$$\hat{p} = \frac{L+U}{2} = \frac{0.32+0.42}{2} \underline{=0.370|}$$

$$E = \frac{U-L}{2} = \frac{0.42-0.32}{2} \underline{=0.050|}$$

Exercise

Find the margin of error E of $0.542 < p < 0.576$

Solution

$$E = \frac{U-L}{2} = \frac{0.576-0.542}{2} \underline{=0.017|}$$

Exercise

Find the point estimate $\hat{p}$ of $0.824 < p < 0.868$

Solution

$$\hat{p} = \frac{L+U}{2} = \frac{0.824+0.868}{2} \underline{=0.846|}$$

Exercise

Find the point estimate $\hat{p}$ and the margin of error E of $0.772 < p < 0.776$

Solution

$$\hat{p} = \frac{L+U}{2} = \frac{0.772+0.776}{2} \underline{=0.774|}$$

$$E = \frac{U-L}{2} = \frac{0.776-0.772}{2} \underline{=0.002|}$$

Exercise

Find the point estimate $\hat{p}$ and the margin of error E of $0.433 < p < 0.527$

Solution

$$\hat{p} = \frac{L+U}{2} = \frac{0.433+0.527}{2} \underline{=0.480|}$$

$$E = \frac{U-L}{2} = \frac{0.527-0.433}{2} \underline{=0.047|}$$

Exercise

Assume that a sample is used to estimate a population proportion p. Find the margin of error E that corresponds to the given $n=1000, \quad x=400, \quad 95\%$ *confidence*

Solution

$$\frac{\alpha}{2}=\frac{1-0.95}{2}=0.025 \quad \Rightarrow \quad A=1-0.025=0.975$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$$A=0.95 \quad \Rightarrow \quad z_{\alpha/2}=z_{0.025}=1.96$$

$$\hat{p}=\frac{x}{n}=\frac{400}{1000}\ \underline{=0.40} \qquad \hat{q}=1-\hat{p}=1-0.4\ \underline{=0.6}$$

$$E=z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=1.96\sqrt{\frac{(0.4)(0.6)}{1000}}\ \underline{=0.0304}$$

Exercise

Assume that a sample is used to estimate a population proportion p. Find the margin of error E that corresponds to the given $n=500, \quad x=220, \quad 99\%$ *confidence*

Solution

$$\frac{\alpha}{2}=\frac{1-0.99}{2}=0.005 \quad \Rightarrow \quad A=1-0.005=0.995$$

$$A=0.995 \quad \Rightarrow \quad z_{\alpha/2}=z_{0.005}=2.575$$

z score	Area
1.645	0.9500
2.575	0.9950

$$\hat{p}=\frac{x}{n}=\frac{220}{500}\ \underline{=0.44} \qquad \hat{q}=1-\hat{p}=1-0.44\ \underline{=0.56}$$

$$E=z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=2.575\sqrt{\frac{(0.44)(0.56)}{500}}\ \underline{=0.0572}$$

Exercise

Assume that a sample is used to estimate a population proportion p. Find the margin of error E that corresponds to the given $n=390, \quad x=130, \quad 90\%$ *confidence*

Solution

z score	Area
1.645	0.9500
2.575	0.9950

$$\frac{\alpha}{2}=\frac{1-0.90}{2}=0.05 \quad \Rightarrow \quad A=1-0.05=0.95$$

$$A=0.95 \quad \Rightarrow \quad z_{\alpha/2}=z_{0.05}=1.645$$

$$\hat{p}=\frac{x}{n}=\frac{130}{390}\ \underline{=0.33} \qquad \hat{q}=1-0.33\ \underline{=0.67}$$

$$E=z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=1.645\sqrt{\frac{(0.33)(0.67)}{390}}\ \underline{=0.0392}$$

Exercise

Assume that a sample is used to estimate a population proportion p. Find the margin of error E that corresponds to the given 98% confidence; the sample size is 1230, of which 40% are successes.

Solution

$$\frac{\alpha}{2}=\frac{1-0.98}{2}=0.01 \quad \Rightarrow \quad A=1-0.01=0.99$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916

$$A=0.99 \quad \Rightarrow \quad z_{\alpha/2}=z_{0.01}=2.33$$

$$\hat{p}=\frac{x}{n}=\frac{492}{1230}\ \underline{=0.4}$$

$$\hat{q}=1-\hat{p}=1-0.4\ \underline{=0.6}$$

$$E=z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=2.33\sqrt{\frac{(0.4)(0.6)}{1230}}\ \underline{=0.0325}$$

Exercise

Construct the confidence interval estimate of the population proportion p that corresponds to the given $n=200, \quad x=40, \quad 95\%$ *confidence*

Solution

$$\frac{\alpha}{2}=\frac{1-0.95}{2}=0.025 \quad \Rightarrow A=1-0.025=0.975 \quad \to z_{\alpha/2}=z_{0.025}=1.96$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$$\hat{p}=\frac{x}{n}=\frac{40}{200}\ \underline{=0.2}$$

$$\hat{q}=1-\hat{p}=1-0.2\ \underline{=0.8}$$

$$p\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.20\pm 1.96\sqrt{\frac{(0.2)(0.8)}{200}}$$

$$0.20\pm 0.0554$$

$$0.20-0.0554<p<0.20+0.0554$$

$$\underline{0.145<p<0.255}$$

Exercise

Construct the confidence interval estimate of the population proportion p that corresponds to the given $n = 1236, \quad x = 109, \quad 99\%\ \textit{confidence}$

Solution

$$\frac{\alpha}{2} = \frac{1-0.99}{2} = 0.005 \quad \Rightarrow A = 1 - 0.005 = 0.995 \quad \rightarrow z_{\alpha/2} = z_{0.005} = 2.575$$

z score	Area
1.645	0.9500
2.575	0.9950

$$\hat{p} = \frac{x}{n} = \frac{109}{1236} \underline{= 0.0882}$$

$$\hat{q} = 1 - \hat{p} = 1 - 0.0882 \underline{= 0.9118}$$

$$p \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.0882 \pm 2.575\sqrt{\frac{(0.0882)(0.9118)}{1236}}$$

$$0.0882 \pm 0.0207$$

$$0.0882 - 0.0207 < p < 0.0882 + 0.0207$$

$$\underline{0.0674 < p < 0.109}$$

Exercise

Construct the confidence interval estimate of the population proportion p that corresponds to the given $n = 5200, \quad x = 4821, \quad 99\%\ \textit{confidence}$

Solution

$$\frac{\alpha}{2} = \frac{1-0.99}{2} = 0.005 \quad \Rightarrow A = 1 - 0.005 = 0.995 \quad \rightarrow z_{\alpha/2} = z_{0.005} = 2.575$$

z score	Area
1.645	0.9500
2.575	0.9950

$$\hat{p} = \frac{x}{n} = \frac{4821}{5200} \underline{= 0.9271}$$

$$\hat{q} = 1 - \hat{p} = 1 - 0.9271 \underline{= 0.0729}$$

$$p \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.9271 \pm 2.575\sqrt{\frac{(0.9271)(0.0729)}{5200}}$$

$$0.9271 \pm 0.0093$$

$$0.9271 - 0.00937 < p < 0.9271 + 0.0093$$

$$\underline{0.918 < p < 0.936}$$

Exercise

Find the minimum sample size requires to estimate a population proportion or percentage:
Margin of error: 0.045; confidence level: 95%: $\hat{p}$ *and* $\hat{q}$ *unknown*

Solution

$$\frac{\alpha}{2}=\frac{1-0.95}{2}=0.025 \quad \Rightarrow A=1-0.025=0.975 \quad \to z_{\alpha/2}=z_{0.025}=1.96$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$E=0.045;\quad \hat{p}$ *unknown* $\quad$ *use* $\hat{p}=0.5$

$$n=\frac{\left(z_{\alpha/2}\right)^2\hat{p}\hat{q}}{E^2}$$

$$=\frac{(1.96)^2(0.5)(.5)}{(.045)^2}$$

$$\approx 475$$

Exercise

Find the minimum sample size requires to estimate a population proportion or percentage:
Margin of error: 2% points; confidence level: 99%: from prior study, $\hat{p}$ is estimate by the decimal equivalent of 14%

Solution

$$\frac{\alpha}{2}=\frac{1-0.99}{2}=0.005 \quad \Rightarrow \quad A=1-0.005=0.995 \quad \to z_{\alpha/2}=z_{0.005}=2.575$$

$E=0.02;\quad \hat{p}\approx 0.14$

z score	Area
1.645	0.9500
2.575	0.9950

$$n=\frac{\left(z_{\alpha/2}\right)^2\hat{p}\hat{q}}{E^2}$$

$$=\frac{(2.575)^2(0.14)(.86)}{(.02)^2}$$

$$\approx 1996$$

Exercise

The Genetics and IVF Institute conducted a clinical trial of the XSORT method designed to increase the probability of conceiving a girl. As of this writing, 574 babies were born to parents using the XSORT method, and 525 of them were girls.

a) What is the best point estimate of the population proportion of girls born to parents using the XSORT method?

b) Use the sample data to construct a 95% confidence interval estimate of the percentage of girls born to parents using the XSORT method.

c) Based on the results, does the XSORT method appear to be effective? Why or why not?

Solution

Let x = the number of girls born using the method

a) $\hat{p} = \frac{x}{n} = \frac{525}{574} \approx 0.9146$

b) $\frac{\alpha}{2} = \frac{1-0.95}{2} = 0.025 \quad \Rightarrow A = 0.975 \quad \to z_{\alpha/2} = z_{0.025} = 1.96$

$$p \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.9146 \pm 1.96\sqrt{\frac{(0.9146)(.0854)}{5200}}$$

$$0.9146 \pm 0.0229$$

$$0.9146 - 0.0229 < p < 0.9146 + 0.0229$$

$$0.892 < p < 0.937$$

c) Yes. Since 0.5 is not within the confidence interval, and below the interval, we can be 95% certain that the method is effective.

Exercise

An important issue facing Americans is the large number of medical malpractice lawsuits and the expenses that they generate. In a study of 1228 randomly selected medical malpractice lawsuits, it is found that 856 of them were later dropped or dismissed.

a) What is the best point estimate of the proportion of medical malpractice lawsuits that are dropped or dismissed?

b) Construct a 99% confidence interval estimate of the proportion of medical malpractice lawsuits that are dropped or dismissed.

c) Does it appear that the majority of such suits are dropped or dismissed?

Solution

Let x = the number of suits dropped or dismissed

a) $\hat{p} = \frac{x}{n} = \frac{856}{1228} = 0.6971$

b) $\frac{\alpha}{2} = \frac{1-0.99}{2} = 0.005 \quad \Rightarrow A = 0.995 \quad \to z_{\alpha/2} = z_{0.005} = 2.575$

z score	Area
1.645	0.9500
2.575	0.9950

$$p \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.6971 \pm 2.575\sqrt{\frac{(0.6971)(0.3029)}{1228}}$$

$$0.6971 \pm 0.0338$$

$$0.6971 - 0.0338 < p < 0.6971 + 0.0338$$

$$0.663 < p < 0.731$$

c) Yes. Since 0.5 is not within the confidence interval, and below the interval, we can be 99% certain that more than half the suits are dropped or dismissed.

Exercise

A study of 420,095 Danish cell phone users found that 135 of them developed cancer was found to be 0.0340% for those not using cell phones.

a) Use the sample data to construct a 95% confidence interval estimate of the percentage of cell phone users who develop cancer of the brain or nervous system.

b) Do cell phone users appear to have a rate of cancer of the brain or nervous system that is different from the rate of such cancer among those not using cells phones? Why or why not?

Solution

Let x = the number that develop those types of cancer.

a) $\hat{p} = \frac{x}{n} = \frac{135}{420{,}095}$ 0.0003214

b) $\frac{\alpha}{2} = \frac{1-0.95}{2} = 0.025 \quad \Rightarrow A = 0.975 \quad \to z_{\alpha/2} = z_{0.025} = 1.96$

$$p \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.0003214 \pm 1.96\sqrt{\frac{(0.0003214)(0.9996786)}{420095}}$$

$$0.0003214 \pm 0.0000542$$

$$0.0003214 - 0.0000542 < p < 0.0003214 + 0.0000542$$

$$0.0267\% < p < 0.0376\%$$

c) No. Since 0.034% is within the confidence interval, it is a reasonable possibility for the true population value. The results do not provide evidence that cell phone users have a different cancer rate than the general population.

Exercise

In an Account survey of 150 senior executives, 47% said that the most common job interview mistake is to have little or no knowledge of the company. Construct a 99% confidence interval estimate of the proportion of all senior executives who have that same opinion. Is it possible that exactly half of all senior executives believe that the most common job interview mistake is to have little or no knowledge of the company? Why or why not?

Solution

Let x = the number who display little or no knowledge of the company.

$$\frac{\alpha}{2} = \frac{1-0.99}{2} = 0.005 \quad \Rightarrow A = 0.995 \quad \to z_{\alpha/2} = z_{0.005} = 2.575$$

$$\hat{p} = \frac{x}{n} = \frac{x}{150} = 0.47 \qquad \Rightarrow x \approx 71$$

$$p \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.47 \pm 2.575\sqrt{\frac{(.47)(.53)}{150}}$$

$$0.47 \pm 0.1049$$

$$0.47 - 0.1049 < p < 0.47 + 0.1049$$

$$0.365 < p < 0.575$$

Yes. Since 0.50 is within the confidence interval, it is a likely value for the true population proportion.