Section 3.6 – Conditional Probability, Independent Events

Conditional Probabilities

The probability of the occurrence of an event *A*, given the occurrence of another event B is called a ***conditional probability***.

"***Changes due the occurrence of another event***"

Example

Age > 21 $\Rightarrow$ the probability of having cancer would be too high

Class of 21 students $\Rightarrow$ passing and > 90 (*conditional*)

Conditional Probability $$P(A \mid B) = \frac{n(A \cap B)}{n(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{P(A \cap B)}{P(B)}; \quad P(B) \neq 0$$

Example

A pointer is spun once, the probability assigned to the pointer landing on a given integer (1 to 6) as given in the table

e_i	1	2	3	4	5	6
$P(e_i)$	.1	.2	.1	.1	.3	.2

a) What is the probability of the pointer landing on a number greater than 4?

$$\begin{aligned} P(>4) &= P(E) \\ &= P(5) + P(6) \\ &= .3 + .2 \\ &= .5 \end{aligned}$$

b) What is the probability of the pointer landing on a number greater than 4 given that it landed on an even number?

E: > 4

F: even number

$$\Rightarrow P(F) = .2 + .1 + .2 = .5$$

$$\begin{aligned} P(E \mid F) &= \frac{P(E \cap F)}{P(F)} = \frac{.2}{.5} = .4 \\ &= \frac{.2}{.5} = .4 \\ &= .4 \end{aligned}$$

***Intersection* of Events:** Product Rule

$$P(A|B)=\frac{P(A\cap B)}{P(B)} \qquad \Rightarrow P(A\cap B)=P(A|B)P(B)$$

$$P(A)\neq 0 \;\; \& \;\; P(B)\neq 0$$

$$P(B|A)=\frac{P(A\cap B)}{P(A)} \qquad \Rightarrow P(A\cap B)=P(B|A)P(A)$$

Product Rule: $P(A\cap B)=P(A|B)P(B)=P(B|A)P(A)$

Example

If 40% 0f the department store's customers are male and 80% of the male customers of the department store have charge accounts, what is the probability that a customer selected at random is a male and has a charge account?

Solution

M: Male customer

C: Customers with a charge account

$P(M)=0.4$

$P\left(C|M\right)=0.8$

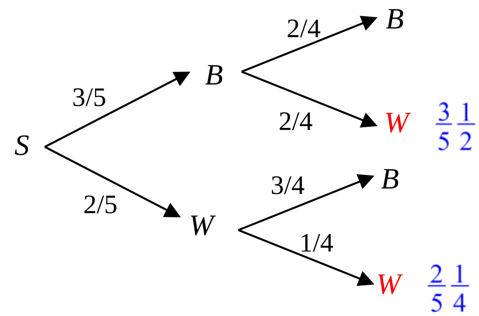
$$\begin{aligned} P(M\cap C) &= P(M)P(C|M) \\ &= (.4)(.8) \\ &= .32 \end{aligned}$$

Probability Tree

Example

Two balls are drawn in succession, without replacement, from a box containing 3 blue and 2 white balls. What is the probability of drawing a white ball on the second draw?

<u>*Solution*</u>



$$P(2^{nd}\ \ White) = P(B \cap W) + P(W \cap W)$$

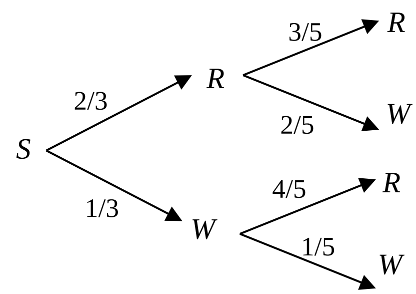
$$= \frac{1}{10} + \frac{3}{10}$$

$$= \frac{2}{5}$$

Example

Two balls are drawn in succession, without replacement, from a box containing 4 red and 2 white balls. What is the probability of drawing a red ball on the second draw?

<u>*Solution*</u>



$$P\left(2^{nd}\ \ Red\right) = \frac{2}{3}\frac{3}{5} + \frac{4}{5}\frac{1}{3}$$

$$= .67$$

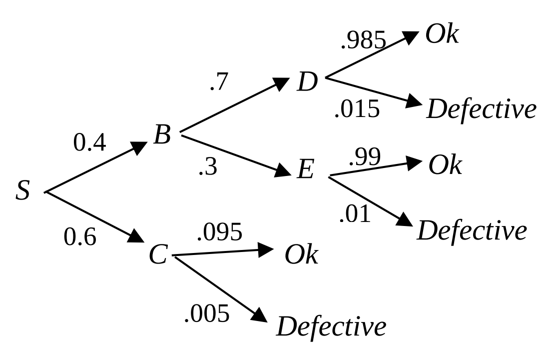
Example

A large computer company A subcontracts the manufacturing of it circuit boards to two companies, 40% to company B and 60% to company C. Company B in turn subcontracts 70% of the orders it receives from company A to company D and the remaining 30% to company E, both subsidiaries of company B. When the boards are completed by companies D, E, and C, they are shipped to company A to be used in various computer models. It has been found that 1.5%, 1%, and .5% of the boards from D, E, and C respectively, prove defective during the 90-day warranty period after a computer is first sold. What is the probability that a given board in a computer will be defective during the 90-day warranty period?

<u>*Solution*</u>

$$P\left(defective\right) = .4\left(.7\right)\left(.015\right) + .4\left(.3\right)\left(01\right) + .6\left(.005\right)$$

$$= 0.0084$$



Independent Events

A & B are independent if and only if $P(A \cap B) = P(A)P(B)$

$$P(A \mid B) = P(A)$$

$$P(B \mid A) = P(B)$$

Otherwise, A & B are said to be dependent

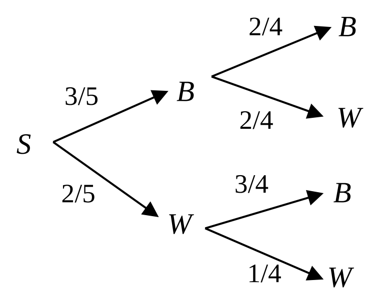
With ***or*** Without Replacement

Example

Two balls are drawn in succession, without replacement, from a box containing 3 blue and 2 white balls. What is the probability of drawing a white ball on the second draw?

Solution

Without



$$P(w) = \frac{2}{5}\frac{1}{4} + \frac{3}{5}\frac{2}{4} = 0.4$$

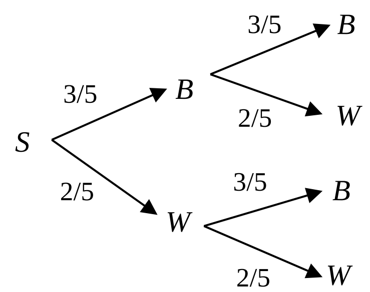
$$P\left(w \mid w_1\right) = \frac{P\left(w \cap w_1\right)}{P(w) + P\left(w_1\right)}$$

$$= \frac{\frac{2}{5}\frac{1}{4}}{\frac{2}{5}\frac{1}{4} + \frac{3}{5}\frac{2}{4}}$$

$$\underline{= 0.25|}$$

$$\Rightarrow P\left(w \mid w_1\right) \neq P(w) \rightarrow \textit{Dependent}$$

With



$$P(w) = \frac{2}{5}\frac{2}{5} + \frac{3}{5}\frac{2}{5} = 0.4$$

$$P(w \mid w_1) = \frac{\frac{2}{5}\frac{2}{5}}{\frac{2}{5}\frac{2}{5} + \frac{3}{5}\frac{2}{5}} = 0.4$$

$$\underline{= 0.4|}$$

$$\Rightarrow P\left(w \mid w_1\right) = P(w) \rightarrow \textit{Independent}$$

Example

A single card is drawn from a standard 52-card deck. Test the following events for independence:

a) E: Red card F: divisible by 5

b) G: Kings H: Queen

<u>*Solution*</u>

a) $P(E \cap F) = \frac{4}{52} = \frac{1}{13}$

$P(E) = \frac{26}{52} \quad P(F) = \frac{8}{52}$

$P(E)P(F) = \frac{26}{52}\frac{8}{52} = \frac{1}{13}$ ***Independent***

b) $P(G \cap H) = 0$

$P(G) = \frac{4}{52} \quad P(H) = \frac{4}{52}$

$P(G)P(H) = \frac{4}{52}\frac{4}{52} = \frac{1}{169}$ ***Dependent***

Independent Set of Events

$$\Rightarrow P\left(E_1 \cap E_2 \cap \cdots\right) = P\left(E_1\right) \bullet P\left(E_2\right) \bullet \cdots$$

Exercises *Section* 3.6 – Conditional Probability, Independent Events

1. In building the space shuttle, NASA contracts for certain guidance components to be supplied by three different companies: 41% by company A, 25% by company B, and 34% by company C. It has been found that 1%, 1.75%, and 2% of the components from companies A, B, and C, respectively, are defective. If one of these guidance components is selected at random, what is the probability that it is defective?

2. Suppose the probability of A is $P(A)=\frac{1}{4}$ and the probability of B is $P(B)=\frac{2}{3}$. What would the probability of A intersect B need to be for A and B to be independent events?

3. In 2 throws of a fair die, what is the probability that you will get at least 5 on each throw? At least 5 on the first or second throw?

4. 2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that the second ball was red, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

5. 2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that at least 1 ball was red, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

6. 2 balls are drawn in succession out a box containing 2 red and 5 white balls. Find the probability that both balls were the same color, given that the first ball was

a) Replaced before the second draw

b) Not replaced before the second draw

7. An automobile manufacturer produces 37% of its cars at plant A. If 5% of the cars manufactured at plant A have defective emission control devices, what is the probability that one of this manufacturer's cars was manufactured at plant A and has a defective emission control device?

8. To transfer into a particular department, a company requires an employee to pass a screening test. A maximum of 3 attempts are allowed at 6-month intervals between trials. From past records it is found that 40% pass on the first trial; of those that fail the first trial and take the test a second time, 60% pass; and of those that fail on the second trial and take the test a third time, 20% pass. For an employee wishing to transfer:

a) What is the probability of passing the test on the first or second try?

b) What is the probability of failing on the first 2 trials and passing on the third?

c) What is the probability of failing on all 3 attempts?

9. A survey of the residents of a precinct in a large city revealed that 55% of the residents where members of the Democratic party and that 60% of the Democratic party members voted in the last election. What is the probability that a person selected at random from the residents of this precinct is a member of the Democratic party and voted in the last election?