4.6 – Mirrors and Lenses

An image of a point formed by a mirror is the point at which light rays from the point converge or seem to converge after reflection. An image of a point formed by a lens (a piece of glass with spherical surfaces) is the point at which light rays from the point converge or seem to converge after refraction. There are two kinds of images: real and virtual. A ***real image*** is an image where actual light rays converge. A real image can be captured in a screen. An example is an image formed by a cinema projector. A ***virtual image*** is an image where actual light rays do not converge but seem to converge. A virtual image cannot be captured in a screen. An example is an image formed by a flat mirror.

Mirrors

Flat Mirrors

The following diagram shows image formation by a flat mirror.

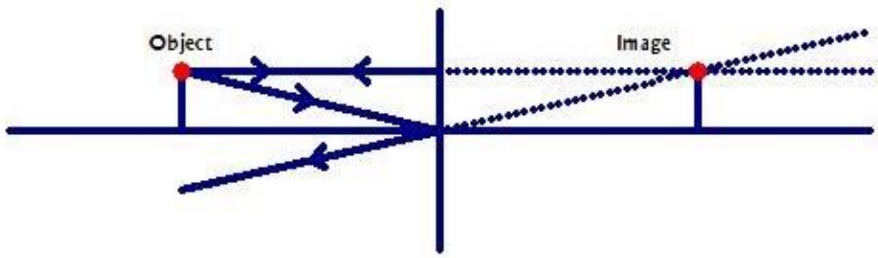


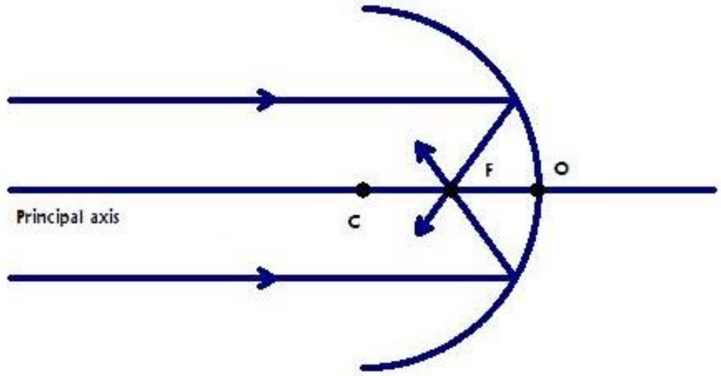
Image formed by a flat mirror

The image formed by a flat mirror has the following properties.

1. It is a virtual image.
2. It is located behind the mirror.
3. It has the same size as the object. Its perpendicular distance from the mirror is equal to the perpendicular distance of the object from the mirror.
4. It is erect (not inverted) in a direction parallel to the mirror.
5. It is laterally inverted. In other words the image is inverted in a direction perpendicular to the mirror. For example the image of an arrow pointing towards the mirror is an arrow pointing towards the arrow itself.

Concave Mirror

A concave mirror is a spherical mirror with the reflecting surface being the inner surface. The following diagram shows a concave mirror.



Concave mirror

The center of the spherical surface is called the ***center of curvature*** (point C in the diagram) of the mirror. The mid-point of the mirror is called the ***center of the mirror*** (point O in the diagram). The line joining the center of curvature and the center of the mirror is called the ***principal axis*** of the mirror. The point at which light rays parallel to the principal axis converge after reflection is called the focus (point F in the diagram) of the mirror. The focus of a concave mirror is real because actual light rays meet at the point. The focus is located midway between the center of curvature and the center of the mirror. The distance between the focus and the center of the mirror is called the ***focal length*** (f) of the mirror.

Only two light rays originating from a point are needed to construct its image. The image is is the point at which these light rays converge or seem to converge after reflection. There are 3 special light rays that can be used when constructing an image.

1. A light ray parallel to the principal axis is reflected through the focus.
2. A light ray through the focus is reflected parallel to the principal axis.
3. A light ray through the center of curvature returns in its own path.

The following diagram shows the construction of the image formed by a concave mirror when the object is placed beyond the center of curvature.

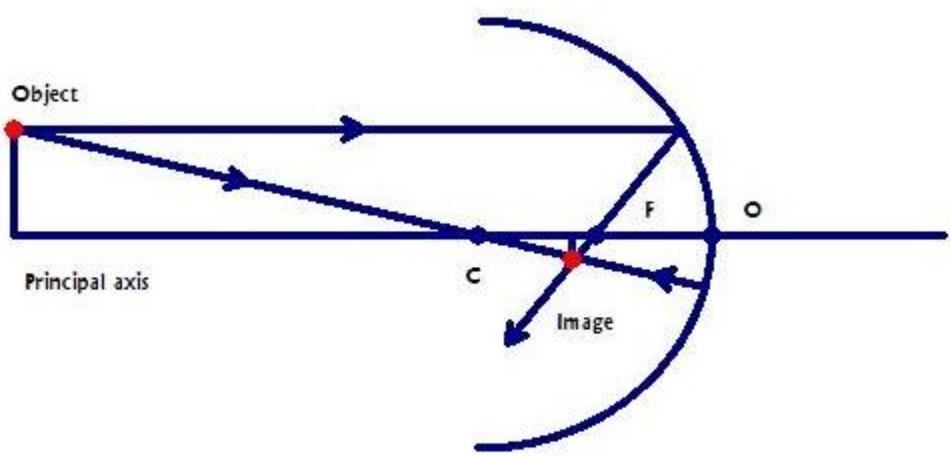


Image formed by a concave mirror when the object is placed beyond the center of curvature

The image formed by a concave mirror when the object is located beyond the center of curvature has the following properties.

1. The image is real.
2. The image is inverted.
3. The image is diminished.
4. The image is located between the focus and the center of curvature on the same side as the object.

The following diagram shows the construction of the image formed by a concave mirror when the object is located between the center of curvature and the focus.

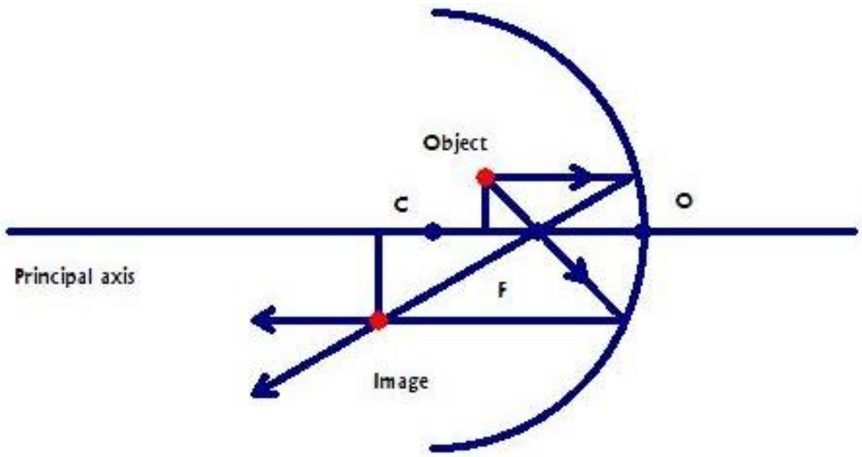


Image formed by a concave mirror when the object between the center of curvature and the focal point

An image formed by a concave mirror when the object is placed between the center of curvature and the focus has the following properties.

1. The image is real.
2. The image is inverted.
3. The image is enlarged.
4. The image is located beyond the center of curvature on the same side as the object.

The following diagram shows the image construction of the image formed by a concave mirror when the object is located between the focus and the center of the mirror.

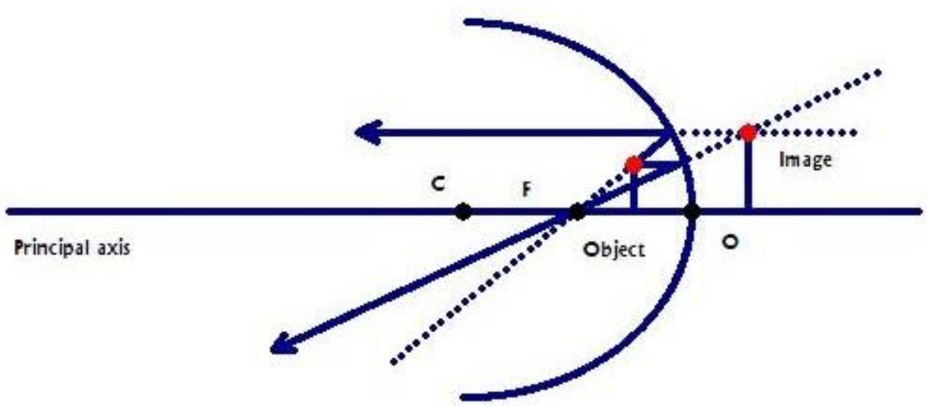


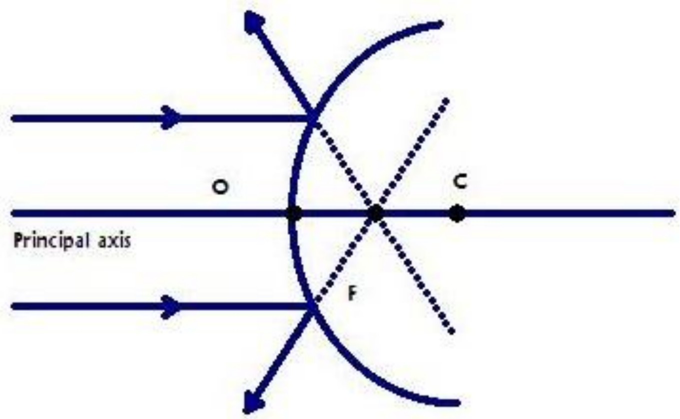
Image formed by a concave mirror when the object between the center and the focal point of mirror

The image formed by a concave mirror when the object is placed between the focus and the center of the mirror has the following properties.

1. The image is virtual.
2. The image is erect.
3. The image is enlarged.
4. The image is located behind the mirror.

Convex Mirror

A ***convex mirror*** is a spherical mirror with the outside surface being the reflecting surface. The following diagram shows a convex mirror.



Convex mirror

The center of the spherical surface is called the ***center of curvature*** (point *C* on the diagram) of the mirror. The mid-point of the mirror is called the ***center of the mirror*** (point *O* on the diagram). The line joining the center of curvature and the center of the mirror is called the ***principal axis*** of the mirror. The point from which light rays parallel to the principal axis seem to come from after reflection is called the ***focus*** (point *F* on the diagram) of the mirror. The focus of a convex mirror is virtual because actual light rays do not meet at the focus. The focus is located midway between the center of curvature and the the center of the mirror. The distance between the focus and the center of the mirror is called the ***focal length*** (*f*) of the mirror.

There are 3 special light rays that can be used in constructing images formed by a convex mirror.

1. A light ray parallel to the principal axis seems to come from the focus after reflection.
2. A light ray directed towards the focus is reflected back parallel to the principal axis.
3. A light ray directed towards the center of curvature is reflected in its own path.

The following diagram shows the image construction of the image formed by a convex mirror.

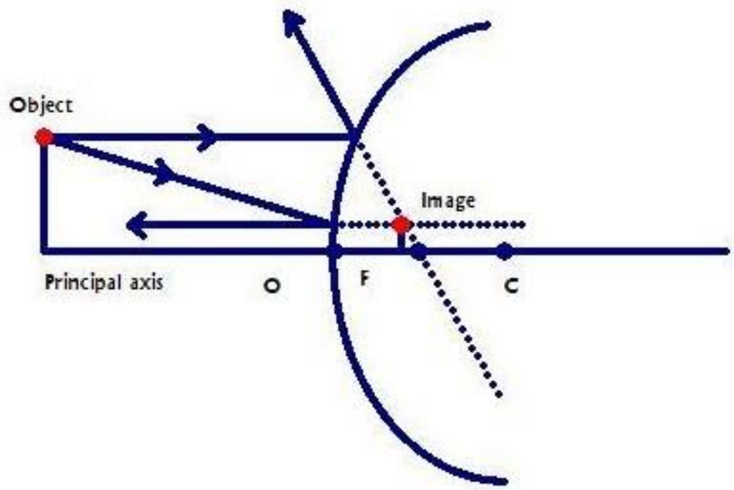


Image formed by a Convex mirror

The image formed by a convex mirror has the following properties.

1. The image is virtual.
2. The image is erect.
3. The image is diminished.
4. The image is located behind the mirror.

The Mirror Equation

The mirror equation is an equation that relates the distance of the object from the center of the mirror, the distance of the image from the center of the mirror and the focal length (distance between focus and the center of the mirror). The distance between the object and the center of the mirror is called ***object distance*** (p). It is taken to be positive if the object is real and negative if the object is virtual. A virtual object is possible when more than one mirrors are involved. The distance between the image and the center of the mirror is called the ***image distance*** (q). The image distance is taken to be positive if the image is real and negative if the image is virtual. The focal length (f) is taken to be positive if the focus is real and negative if the focus is virtual. Thus, the focal length of a concave mirror is positive since its focus is real and that of a convex mirror is negative because its focus is virtual. The focal length of a mirror is half the radius of curvature: $|f| = \frac{R}{2}$ where R is the radius of curvature of the mirror. The following equation is the mirror equation.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

The ***magnification*** (M) of a mirror is defined to be the ratio between the size of the image $\left(h_i\right)$ and the size of the object $\left(h_0\right)$. The size of the object (image) is taken to be positive if the object (image) is erect and negative if the object (image) is inverted.

$$M = \frac{h_i}{h_0}$$

It can also be shown that the magnification is equal to the negative of the ratio between image distance and object distance.

$$M = -\frac{q}{p}$$

Example

An object of height 0.02*m* is placed 0.4*m* in front of a concave mirror whose radius of curvature is 0.1*m*.

a) Determine its focal length
b) Calculate the distance of the image from the mirror
c) Is the image real or virtual?
d) Calculate the magnification
e) Calculate the size of the image
f) Is the image erect or inverted?

Solution

Given: $h_0 = 0.02m; \quad p = 0.4m; \quad R = 0.1m$

a) The focal length of a concave mirror is positive.

$$|f| = \frac{R}{2} = \frac{0.1}{2} = 0.05\ m$$

$$f \ \underline{= 0.05\ m|}$$

b) $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \quad \Rightarrow \quad \frac{1}{q} = \frac{1}{f} - \frac{1}{p}$

$$\frac{1}{q} = \frac{1}{0.05} - \frac{1}{0.4} = 17.5$$

$$q = \frac{1}{17.5} \ \underline{= 0.06\ \ m|}$$

c) The image is real because the image distance is positive.

d) $M = -\frac{q}{p} = -\frac{0.06}{0.4} \ \underline{= -0.15|}$

e) $M = \frac{h_i}{h_0} \quad \Rightarrow \quad h_i = Mh_0$

$$h_i = (-0.15)(0.02) \ \underline{= -0.003\ m|}$$

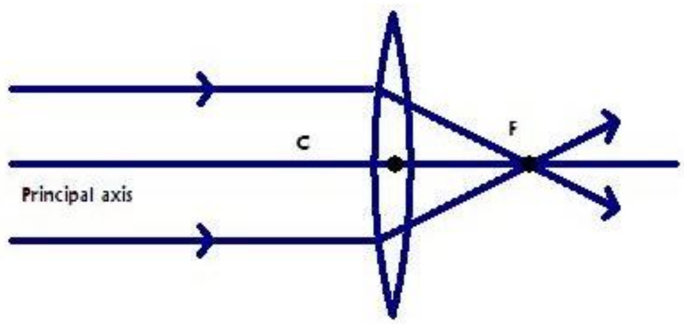
f) The image is inverted because h_i is negative

Lenses

A ***lens*** is a piece of glass with spherical surfaces. There are two types of lenses. They are convex (converging) lens and concave (diverging) lens.

Convex Lens

The following diagram shows a convex lens.



Convex Lens

The mid-point of the lens (point *C* in the diagram) is called the ***center of the lens***. The line joining the centers of curvature of both surfaces and the center of the lens is called the ***principal axis*** of the lens. The point at which light rays parallel to the principal axis converge after refraction is called the ***focus*** (point *F* on the diagram) of the lens. The focus of a convex lens is real because actual light rays meet at the point. The distance between the focus and the center of the lens is called the ***focal length*** of the lens.

There are three special light rays that can be used to construct images formed by a convex lens.

1. A light ray parallel to the principal axis passes through the focal point after refraction.
2. A light ray through the focal point is refracted parallel to the principal axis.
3. A light ray through the center of the lens passes undeflected.

The following diagram shows the image construction of the image formed by a convex lens when the object is placed beyond twice the focal length.

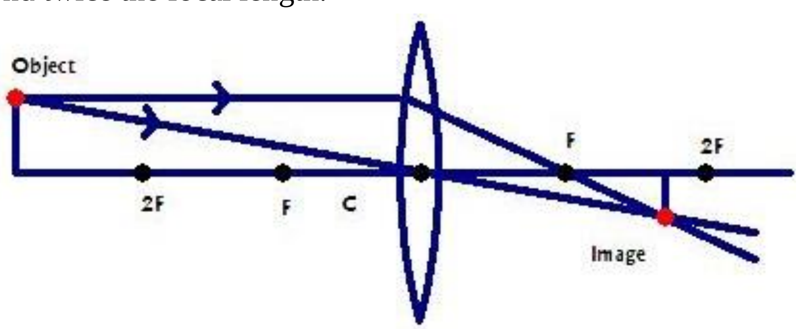


Image formed by a convex lens when the object is placed beyond twice the focal length

The image formed by a convex lens when the object is placed beyond twice the focal length has the following properties.

1. The image is real.
2. The image is inverted.

3. The image is diminished.
4. The image is located at a distance greater than the focal length but smaller than twice the focal length on the other side of the lens.

The following diagram shows the image construction of the image formed by a convex lens when the object is located at a distance greater than the focal length but less than twice the focal length.

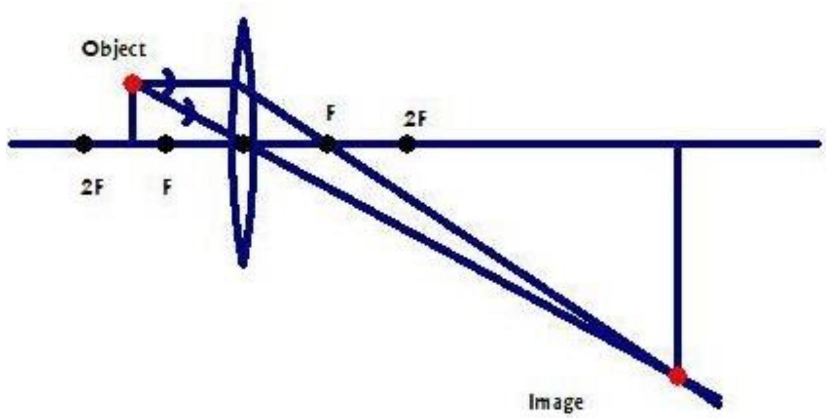


Image formed by a convex lens when the object is located between F and 2F

The image formed by a convex lens when the object is placed at a distance greater than the focal length but less than twice the focal length has the following properties.
1. The image is real.
2. The image is inverted.
3. The image is enlarged.
4. The image is located beyond twice the focal length on the other side of the lens.

The following diagram shows the image construction of the image formed by a convex lens when the object is placed between the focus and the center of the lens.

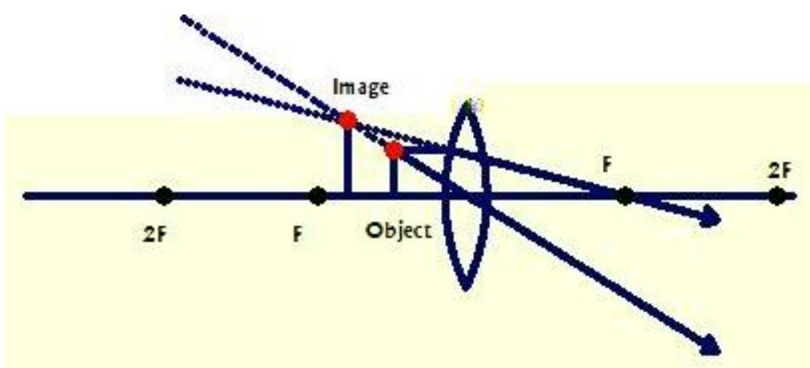
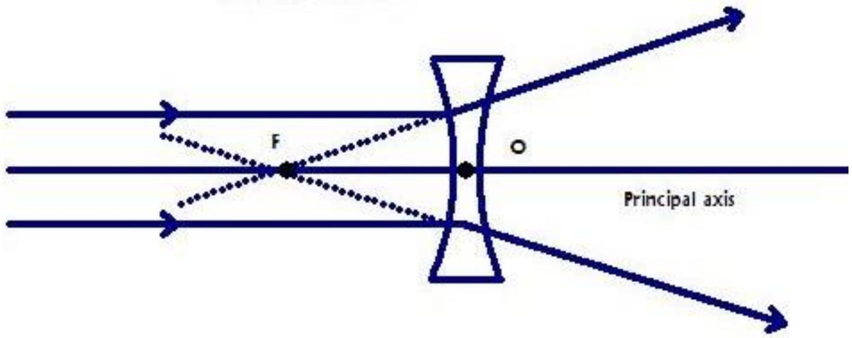


Image formed by a convex lens when the object is placed between the lens and the focal point

The image formed by a convex lens when the object is located between the focus and the center of the lens has the following properties.
1. The image is virtual.
2. The image is erect.
3. The image is enlarged.
4. The image is formed on the same side as the object.

Concave Lens

The following diagram shows a concave lens.



Concave lens

The mid-point of the lens (point *O* in the diagram) is called the ***center of the lens***. The line joining the centers of curvature of the surfaces of the lens and the center of the lens is called the ***principal axis*** of the lens. The point from which light rays parallel to the principal axis seem to come from after refraction is called the ***focus*** of the lens. The focus of a concave lens is virtual because actual light rays do not meet at the focus. The distance between the focus and the center of the lens is called the ***focal length*** of the lens.

There are three special light rays used to construct images formed by a concave lens.

1. A light ray parallel to the principal axis seems to come from the focal point after refraction.
2. A light ray directed towards the focal point is refracted parallel to the principal axis.
3. A light ray directed towards the center of the lens passes undeflected.

The following diagram shows the image construction of the image formed by a concave lens.

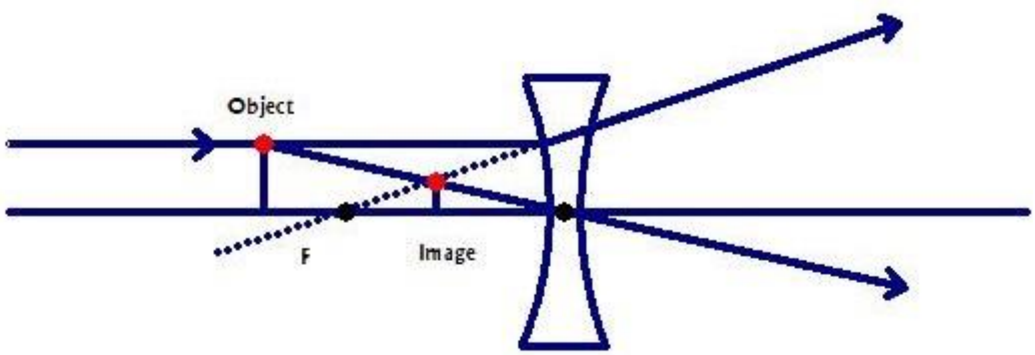


Image formed by a concave lens

The image formed by a concave lens has the following properties.

1. The image is virtual.
2. The image is erect.
3. The image is diminished.
4. The image is located on the same side as the object.

The Lens Equation

The lens equation is an equation that relates the object distance, image distance and the focal length. The ***object distance*** (p) is the distance between the object and the center of the lens. It is taken to be positive if the object is real and negative if the object is virtual. The ***image distance*** (q) is the distance between the image and the center of the lens. It is taken to be positive if the image is real and negative if the image is virtual. The focal length (f) is the distance between the focal point and the center of the lens. The focal length is taken to be positive if the focus is real and negative if the focus is virtual. This means the focal length of a convex lens is positive (because its focus is real and that of a concave lens is negative (because its focus is virtual). The following equation is the so called lens equation.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

The magnification of a lens is defined to be the ratio between the size of the image $\left(h_i\right)$ and the size of the object $\left(h_0\right)$. The size of the object (image) is taken to be positive if the object (image) is erect and negative if the object (image) is inverted.

$$M = \frac{h_i}{h_0}$$

The magnification is also equal to the negative of the ratio between the image distance and object distance.

$$M = -\frac{q}{p}$$

Example

An object of height 0.03 *m* is placed 0.3 *m* in front of a concave lens whose focal length is 0.06 *m*.

a) Calculate the distance of the image from the center of the lens
b) Is the image real or virtual?
c) Calculate its magnification
d) Calculate the height of the image and determine if the image is erect or inverted

Solution

Given: $h_0 = 0.03m; \quad p = 0.3m; \quad f = 0.06m$

a) The focal length is negative because the lens is concave

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{-0.06} - \frac{1}{0.3} = -20$$

$$q = \frac{1}{-20} \underline{= -0.05 \ m}$$

b) It is virtual because the image distance is negative

c) $M = -\frac{q}{p} = -\frac{-0.05}{0,3} \underline{= 0.17}$

d) $h_i = Mh_0 = \left(0.17\right)\left(0.03\right) \underline{= 0.0051 \ m}$ The image is erect because h_i is positive.

Lens Makers Equation

If the radius of curvature of the surface of a lens upon which the light rays are incident is R_1 and the radius of curvature of the other surface is R_2, then the focal length of the lens is given by

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Where *n* is the refractive index of the lens. A radius of curvature of the surface of a lens is taken to be positive if the direction from the surface towards the center of curvature of the surface is the same as the direction of the incident light rays and negative if opposite to the direction of the incident light rays.

Example

Both surfaces of a convex lens have a radius of curvature of 0.05 *m*. The refractive index of the glass is 1.5. Calculate the focal length of the lens.

Solution

The radius of curvature of the surface upon which the light rays are incident (R_1) is positive because the direction from the surface towards its center of curvature is the same as the direction of the incident light rays. The radius of curvature of the other surface is negative because the direction from the surface to its center of curvature is opposite to the direction of the incident light rays.

Given: $n=1.5$; $R_1=0.05m$; $R_2=0.05m$ *Find*: $f=?$

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1.5-1)\left(\frac{1}{0.05}-\frac{1}{-0.05}\right)$$

$$=\frac{1}{0.05}\ m^{-1}$$

$$\boxed{f=0.05\ m}$$

Example

Both surfaces of a concave lens have a radius of curvature of 0.04 m. The refractive index of the glass is 1.5. Calculate the focal length of the lens.

Solution

The radius of curvature of the surface upon which the light rays are incident (R_1) is negative because the direction from the surface towards its center of curvature is opposite to the direction of the incident light rays. The radius of curvature of the other surface is positive because the direction from the surface to its center of curvature is the same as the direction of the incident light rays.

Given: $n = 1.5$; $R_1 = -0.04m$; $R_2 = 0.04m$ *Find*: $f = ?$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5-1)\left(\frac{1}{-0.04} - \frac{1}{0.04}\right)$$

$$= -\frac{1}{0.04}\ m^{-1}$$

$$\underline{f = -0.04\ m|}$$