Exercise

Express as a sum of logarithms: $\log_3(ab)$

Solution

$$\log_3(ab) = \log_3 a + \log_3 b$$

Exercise

Express as a sum of logarithms: $\log_7(7x)$

Solution

$$\log_7(7x) = \log_7 7 + \log_7 x$$
$$= 1 + \log_7 x$$

Exercise

Express the following in terms of sums and differences of logarithms $log\frac{x}{1000}$

Solution

$$log\frac{x}{1000} = log\, x - log 1000$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_5\left(\frac{125}{y}\right)$

Solution

$$\log_5\left(\frac{125}{y}\right) = \log_5 125 - \log_5 y$$

Exercise

Express the following in terms of sums and differences of logarithms $log_b x^7$

Solution

$$log_b x^7 = 7 log_b x$$

Exercise

Express the following in terms of sums and differences of logarithms $\ln\sqrt[7]{x}$

Solution

$$\ln\sqrt[7]{x} = \ln x^{1/7}$$

$$= \frac{1}{7}\ln x$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \frac{x^2 y}{z^4}$

Solution

$$\log_a \frac{x^2 y}{z^4} = \log_a x^2 y - \log_a z^4 \qquad \textit{Quotient Rule}$$

$$= \log_a x^2 + \log_a y - \log_a z^4 \qquad \textit{Product Rule}$$

$$= 2\log_a x + \log_a y - 4\log_a z \qquad \textit{Power Rule}$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \frac{x^2 y}{b^3}$

Solution

$$\log_b \frac{x^2 y}{b^3} = \log_b x^2 y - \log_b b^3$$

$$= \log_b x^2 + \log_b y - \log_b b^3$$

$$= 2\log_b x + \log_b y - 3\log_b b$$

$$= 2\log_b x + \log_b y - 3$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b\left(\frac{x^3y}{z^2}\right)$

Solution

$$\log_b\left(\frac{x^3y}{z^2}\right)=\log_b\left(x^3y\right)-\log_b z^2$$
$$=\log_b x^3+\log_b y-\log_b z^2$$
$$=3\log_b x+\log_b y-2\log_b z$$

Exercise

Express the following in terms of sums and differences of logarithms $log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$

Solution

$$\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)=\log_b\left(\sqrt[3]{x}y^4\right)-\log_b\left(z^5\right)$$
$$=log_b\left(x^{1/3}\right)+log_b\left(y^4\right)-log_b\left(z^5\right)$$
$$=\frac{1}{3}log_b(x)+4log_b(y)-5log_b(z)$$

Exercise

Express the following in terms of sums and differences of logarithms $\log\left(\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right)$

Solution

$$\log\left(\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right)=\log\left(100x^3\sqrt[3]{5-x}\right)-\log\left(3(x+7)^2\right)$$
$$=\log 10^2+\log x^3+\log(5-x)^{1/3}-\left[\log 3+\log\left((x+7)^2\right)\right]$$
$$=2\log 10+3\log x+\tfrac{1}{3}\log(5-x)-\log 3-2\log(x+7)$$
$$=2+3\log x+\tfrac{1}{3}\log(5-x)-\log 3-2\log(x+7)$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}}$

Solution

$$\log_a \sqrt[4]{\frac{m^8 n^{12}}{a^3 b^5}} = \log_a \left(\frac{m^8 n^{12}}{a^3 b^5}\right)^{1/4} \quad \textit{Power Rule}$$

$$= \frac{1}{4}\log_a \left(\frac{m^8 n^{12}}{a^3 b^5}\right) \quad \textit{Quotient Rule}$$

$$= \frac{1}{4}\left[\log_a m^8 n^{12} - \log_a a^3 b^5\right] \quad \textit{Product Rule}$$

$$= \frac{1}{4}\left[\log_a m^8 + \log_a n^{12} - \left(\log_a a^3 + \log_a b^5\right)\right] \quad \textit{Power Rule}$$

$$= \frac{1}{4}\left[8\log_a m + 12\log_a n - 3 - 5\log_a b\right]$$

$$= 2\log_a m + 3\log_a n - \frac{3}{4} - \frac{5}{4}\log_a b$$

Exercise

Use the properties of logarithms to rewrite: $\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}}$

Solution

$$\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}} = \log_p \left(\frac{m^5 n^4}{t^2}\right)^{1/3} \quad \textit{Power Rule}$$

$$= \frac{1}{3}\log_p \left(\frac{m^5 n^4}{t^2}\right) \quad \textit{Quotient Rule}$$

$$= \frac{1}{3}\left(\log_p m^5 n^4 - \log_p t^2\right) \quad \textit{Product Rule}$$

$$= \frac{1}{3}\left(\log_p m^5 + \log_p n^4 - \log_p t^2\right) \quad \textit{Power Rule}$$

$$= \frac{1}{3}\left(5\log_p m + 4\log_p n - 2\log_p t\right)$$

$$= \frac{5}{3}\log_p m + \frac{4}{3}\log_p n - \frac{2}{3}\log_p t$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b \sqrt[n]{\frac{x^3y^5}{z^m}}$

Solution

$$\log_b \sqrt[n]{\frac{x^3y^5}{z^m}} = \log_b \left(\frac{x^3y^5}{z^m}\right)^{1/n}$$

$$= \frac{1}{n}\log_b \left(\frac{x^3y^5}{z^m}\right) \qquad \textit{Power Rule}$$

$$= \frac{1}{n}\left(\log_b x^3y^5 - \log_b z^m\right) \qquad \textit{Quotient Rule}$$

$$= \frac{1}{n}\left(\log_b x^3 + \log_b y^5 - \log_b z^m\right) \qquad \textit{Product Rule}$$

$$= \frac{1}{n}\left(3\log_b x + 5\log_b y - m\log_b z\right) \qquad \textit{Power Rule}$$

$$= \frac{3}{n}\log_b x + \frac{5}{n}\log_b y - \frac{m}{n}\log_b z$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_a \sqrt[3]{\frac{a^2\,b}{c^5}}$

Solution

$$\log_a \sqrt[3]{\frac{a^2\,b}{c^5}} = \log_a \left(\frac{a^2\,b}{c^5}\right)^{1/3} \qquad \textbf{\textit{Convert the radical to power}}$$

$$= \frac{1}{3}\log_a \left(\frac{a^2\,b}{c^5}\right) \qquad \textbf{\textit{Power Rule}}$$

$$= \frac{1}{3}\left[\log_a a^2\,b - \log_a c^5\right] \qquad \textbf{\textit{Quotient Rule}}$$

$$= \frac{1}{3}\left[\log_a a^2 + \log_a b - \log_a c^5\right] \qquad \textbf{\textit{Product Rule}}$$

$$= \frac{1}{3}\left[2\log_a a + \log_a b - 5\log_a c\right] \qquad \textbf{\textit{Power Rule}}$$

$$= \frac{2}{3}\log_a a + \frac{1}{3}\log_a b - \frac{5}{3}\log_a c$$

$$= \frac{2}{3} + \frac{1}{3}\log_a b - \frac{5}{3}\log_a c$$

Exercise

Express the following in terms of sums and differences of logarithms $\log_b\left(x^4\sqrt[3]{y}\right)$

Solution

$$\log_b\left(x^4\sqrt[3]{y}\right) = \log_b\left(x^4\right)+\log_b\left(\sqrt[3]{y}\right)$$
$$= \log_b\left(x^4\right) + \log_b\left(y^{1/3}\right)$$
$$= 4\log_b(x) + \frac{1}{3}\log_b(y)$$

Exercise

Express the following in terms of sums and differences of logarithms $log_5\left(\frac{\sqrt{x}}{25y^3}\right)$

Solution

$$log_5\left(\frac{\sqrt{x}}{25y^3}\right) = log_5\left(x^{1/2}\right) - log_5\left(25y^3\right)$$
$$= log_5\left(x^{1/2}\right) - \left[log_5\left(5^2\right)+log_5\left(y^3\right)\right]$$
$$= log_5\left(x^{1/2}\right) - log_5\left(5^2\right) - log_5\left(y^3\right)$$
$$= \frac{1}{2}log_5\, x - 2 - 3log_5\, y$$

Exercise

Express $\log_a\frac{x^3w}{y^2z^4}$ in terms of logarithms of x, y, z, and w.

Solution

$$\log_a\frac{x^3w}{y^2z^4} = \log_a x^3w - \log_a y^2z^4 \qquad \textit{Quotient rule}$$
$$= \log_a x^3 + \log_a w - \left(\log_a y^2 + \log_a z^4\right) \qquad \textit{Product rule}$$
$$= \log_a x^3 + \log_a w - \log_a y^2 - \log_a z^4 \qquad \textit{Distribute minus}$$
$$= 3\log_a x + \log_a w - 2\log_a y - 4\log_a z \qquad \textit{Power rule}$$

Exercise

Express $\log_a \dfrac{\sqrt{y}}{x^4\sqrt[3]{z}}$ in terms of logarithms of x, y, and z.

Solution

$$\log_a \frac{\sqrt{y}}{x^4\sqrt[3]{z}} = \log_a y^{1/2} - \log_a x^4 z^{1/3} \qquad \textit{Quotient rule}$$

$$= \log_a y^{1/2} - \left(\log_a x^4 + \log_a z^{1/3}\right) \qquad \textit{Product rule}$$

$$= \log_a y^{1/2} - \log_a x^4 - \log_a z^{1/3} \qquad \textit{Distribute minus}$$

$$= \frac{1}{2}\log_a y - 4\log_a x - \frac{1}{3}\log_a z \qquad \textit{Power rule}$$

Exercise

Express $\ln \sqrt[4]{\dfrac{x^7}{y^5 z}}$ in terms of logarithms of x, y, and z.

Solution

$$\ln \sqrt[4]{\frac{x^7}{y^5 z}} = \ln\left(\frac{x^7}{y^5 z}\right)^{1/4}$$

$$= \frac{1}{4}\ln\left(\frac{x^7}{y^5 z}\right) \qquad \textit{Power rule}$$

$$= \frac{1}{4}\left(\ln x^7 - \ln y^5 z\right) \qquad \textit{Quotient rule}$$

$$= \frac{1}{4}\left(\ln x^7 - \left(\ln y^5 + \ln z\right)\right) \qquad \textit{Product rule}$$

$$= \frac{1}{4}\left(\ln x^7 - \ln y^5 - \ln z\right)$$

$$= \frac{1}{4}\left(7\ln x - 5\ln y - \ln z\right) \qquad \textit{Power rule}$$

$$= \frac{7}{4}\ln x - \frac{5}{4}\ln y - \ln z$$

Exercise

Express $\ln x\sqrt[3]{\frac{y^4}{z^5}}$ in terms of logarithms of x, y, and z.

Solution

$$\ln x\sqrt[3]{\frac{y^4}{z^5}} = \ln x + \ln\left(\frac{y^4}{z^5}\right)^{1/3} \qquad \textit{Product rule}$$

$$= \ln x + \ln\left(\frac{y^{4/3}}{z^{5/3}}\right)$$

$$= \ln x + \ln y^{4/3} - \ln z^{5/3} \qquad \textit{Quotient rule}$$

$$= \ln x + \frac{4}{3}\ln y - \frac{5}{3}\ln z \qquad \textit{Power rule}$$

Exercise

Express as one logarithm: $2\log_a x + \frac{1}{3}\log_a(x-2) - 5\log_a(2x+3)$

Solution

$$2\log_a x + \frac{1}{3}\log_a(x-2) - 5\log_a(2x+3) = \log_a x^2 + \log_a(x-2)^{1/3} - \log_a(2x+3)^5$$

$$= \log_a x^2(x-2)^{1/3} - \log_a(2x+3)^5$$

$$= \log_a \frac{x^2(x-2)^{1/3}}{(2x+3)^5}$$

Exercise

Express as one logarithm: $5\log_a x - \frac{1}{2}\log_a(3x-4) - 3\log_a(5x+1)$

Solution

$$5\log_a x - \frac{1}{2}\log_a(3x-4) - 3\log_a(5x+1) = \log_a x^5 - \log_a(3x-4)^{1/2} - \log_a(5x+1)^3$$

$$= \log_a x^5 - \left[\log_a(3x-4)^{1/2} + \log_a(5x+1)^3\right]$$

$$= \log_a x^5 - \left[\log_a(3x-4)^{1/2}(5x+1)^3\right]$$

$$= \log_a \frac{x^5}{(3x-4)^{1/2}(5x+1)^3}$$

Exercise

Express as one logarithm: $\log\left(x^3y^2\right)-2\log\left(x\sqrt[3]{y}\right)-3\log\left(\frac{x}{y}\right)$

Solution

$$\begin{aligned}
\log\left(x^3y^2\right)-2\log\left(x\sqrt[3]{y}\right)-3\log\left(\frac{x}{y}\right) &= \log\left(x^3y^2\right)-\log\left(xy^{1/3}\right)^2-\log\left(xy^{-1}\right)^3 \\
&= \log\left(x^3y^2\right)-\left[\log\left(x^2y^{2/3}\right)+\log\left(x^3y^{-3}\right)\right] \\
&= \log\left(x^3y^2\right)-\log\left(x^2y^{2/3}x^3y^{-3}\right) \\
&= \log\left(x^3y^2\right)-\log\left(x^5y^{-7/3}\right) \\
&= \log\left(\frac{x^3y^2}{x^5y^{-7/3}}\right) \\
&= \log\left(\frac{y^2y^{7/3}}{x^2}\right) \\
&= \log\left(\frac{y^{13/3}}{x^2}\right) \\
&= \log\left(\frac{\sqrt[3]{y^{13}}}{x^2}\right) \\
&= \log\left(\frac{y^4\sqrt[3]{y}}{x^2}\right)
\end{aligned}$$

Exercise

Express as one logarithm: $\ln y^3+\frac{1}{3}\ln\left(x^3y^6\right)-5\ln y$

Solution

$$\begin{aligned}
\ln y^3+\tfrac{1}{3}\ln\left(x^3y^6\right)-5\ln y &= \ln y^3+\ln\left(x^3y^6\right)^{1/3}-\ln y^5 \\
&= \ln y^3+\ln\left(x^{3/3}y^{6/3}\right)-\ln y^5 \\
&= \ln y^3+\ln\left(xy^2\right)-\ln y^5 \\
&= \ln\left(y^3xy^2\right)-\ln y^5 \\
&= \ln\left(\frac{y^5x}{y^5}\right) \\
&= \ln x
\end{aligned}$$

Exercise

Express as one logarithm: $2\ln x - 4\ln\left(\frac{1}{y}\right) - 3\ln(xy)$

Solution

$$\begin{aligned}
2\ln x - 4\ln\left(\frac{1}{y}\right) - 3\ln(xy) &= \ln x^2 - \ln\left(\frac{1}{y}\right)^4 - \ln(xy)^3 \\
&= \ln x^2 - \left[\ln\left(y^{-4}\right) + \ln\left(x^3y^3\right)\right] \\
&= \ln x^2 - \ln\left(y^{-4}x^3y^3\right) \\
&= \ln x^2 - \ln\left(y^{-1}x^3\right) \\
&= \ln\frac{x^2}{y^{-1}x^3} \\
&= \underline{\ln\frac{y}{x}}
\end{aligned}$$

Exercise

Write as a single logarithmic $4\ln x + 7\ln y - 3\ln z$

Solution

$$\begin{aligned}
4\ln x + 7\ln y - 3\ln z &= \ln x^4 + \ln y^7 - \ln z^3 \\
&= \ln\left(x^4y^7\right) - \ln z^3 \\
&= \underline{\ln\left(\frac{x^4y^7}{z^3}\right)}
\end{aligned}$$

Exercise

Write as a single logarithmic $\frac{1}{3}\left[5\ln(x+6) - \ln x - \ln(x^2-25)\right]$

Solution

$$\begin{aligned}
\frac{1}{3}\left[5\ln(x+6) - \ln x - \ln(x^2-25)\right] &= \frac{1}{3}\left[5\ln(x+6) - \left(\ln x + \ln(x^2-25)\right)\right] \\
&= \frac{1}{3}\left[\ln(x+6)^5 - \ln x(x^2-25)\right] \\
&= \frac{1}{3}\left[\ln\frac{(x+6)^5}{x(x^2-25)}\right] \\
&= \underline{\ln\left(\frac{(x+6)^5}{x(x^2-25)}\right)^{1/3}}
\end{aligned}$$

Exercise

Write as a single logarithmic $\frac{2}{3}\left[\ln\left(x^2-4\right)-\ln\left(x+2\right)\right]+\ln(x+y)$

Solution

$$\frac{2}{3}\left[\ln\left(x^2-4\right)-\ln\left(x+2\right)\right]+\ln(x+y)=\frac{2}{3}\left[\ln\frac{x^2-4}{x+2}\right]+\ln(x+y)$$

$$=\frac{2}{3}\left[\ln\frac{(x+2)(x-2)}{x+2}\right]+\ln(x+y)$$

$$=\frac{2}{3}\ln(x-2)+\ln(x+y)$$

$$=\ln(x-2)^{2/3}+\ln(x+y)$$

$$=\ln(x-2)^{2/3}(x+y)$$

$$=\ln(x+y)\sqrt[3]{(x-2)^2}$$

Exercise

Write as a single logarithmic $\frac{1}{2}\log_b m+\frac{3}{2}\log_b 2n-\log_b m^2n$

Solution

$$\frac{1}{2}\log_b m+\frac{3}{2}\log_b 2n-\log_b m^2n=\log_b m^{1/2}+\log_b\left(2n\right)^{3/2}-\log_b m^2n$$

$$=\log_b\left(m^{1/2}\left(2n\right)^{3/2}\right)-\log_b m^2n$$

$$=\log_b\frac{m^{1/2}2^{3/2}n^{3/2}}{m^2n}$$

$$=\log_b\frac{2^{3/2}n^{1/2}}{m^{3/2}}$$

$$=\log_b\left(\frac{2^3n}{m^3}\right)^{1/2}$$

$$=\log_b\sqrt{\frac{8n}{m^3}}$$

Exercise

Write the expression as a single logarithm. $\frac{1}{2}\log_y p^3q^4 - \frac{2}{3}\log_y p^4q^3$

Solution

$$\frac{1}{2}\log_y p^3q^4 - \frac{2}{3}\log_y p^4q^3 = \log_y \left(p^3q^4\right)^{1/2} - \log_y \left(p^4q^3\right)^{2/3}$$

$$= \log_y \frac{\left(p^3q^4\right)^{1/2}}{\left(p^4q^3\right)^{2/3}}$$

$$= \log_y \frac{\left(p^3\right)^{1/2}\left(q^4\right)^{1/2}}{\left(p^4\right)^{2/3}\left(q^3\right)^{2/3}}$$

$$= \log_y \frac{p^{3/2}q^2}{p^{8/3}q^2}$$

$$= \log_y \frac{p^{3/2}}{p^{8/3}}$$

$$= \log_y \frac{1}{p^{8/3-3/2}}$$

$$\boxed{= \log_y \frac{1}{p^{7/6}}}$$

Exercise

Write the expression as a single logarithm. $\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x$

Solution

$$\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x = 4\log_a y - \frac{5}{2}\log_a x$$

$$= \log_a y^4 - \log_a x^{5/2}$$

$$\boxed{= \log_a \frac{y^4}{\sqrt{x^5}}}$$

Exercise

Write the expression as a single logarithm. $\frac{2}{3}\left[\ln\left(x^2-9\right)-\ln\left(x+3\right)\right]+\ln\left(x+y\right)$

Solution

$$\frac{2}{3}\left[\ln\left(x^2-9\right)-\ln\left(x+3\right)\right]+\ln\left(x+y\right)=\frac{2}{3}\ln\frac{x^2-9}{x+3}+\ln\left(x+y\right)$$

$$=\frac{2}{3}\ln\frac{(x+3)(x-3)}{x+3}+\ln\left(x+y\right)$$

$$=\frac{2}{3}\ln(x-3)+\ln(x+y)$$

$$=\ln\left(x-3\right)^{2/3}+\ln(x+y)$$

$$=\ln\left((x-3)^{2/3}(x+y)\right)$$

$$=\ln\left((x+y)\sqrt[3]{(x-3)^2}\right)$$

Exercise

Write the expression as a single logarithm. $\frac{1}{4}\log_b x-2\log_b 5-10\log_b y$

Solution

$$\frac{1}{4}\log_b x-2\log_b 5-10\log_b y=\log_b x^{1/4}-\log_b 5^2-\log_b y^{10}$$

$$=\log_b x^{1/4}-\left[\log_b 5^2+\log_b y^{10}\right]$$

$$=\log_b x^{1/4}-\left[\log_b\left(5^2y^{10}\right)\right]$$

$$=\log_b\frac{\sqrt[4]{x}}{5^2y^{10}}$$

Exercise

Assume that $\log_{10}2=.3010$. Find each logarithm $\log_{10}4$, $\log_{10}5$

Solution

a) $\log_{10}4=\log_{10}2^2$

$$=2\log_{10}2$$

$$=2(.301)$$

$$=.6020$$

b) $\log_{10} 5 = \log_{10} \frac{10}{2}$

$= \log_{10} 10 - \log_{10} 2$

$= 1 - .03010$

$= .6990$

Exercise

Given that: $\log_a 2 \approx 0.301, \log_a 7 \approx 0.845$, and $\log_a 11 \approx 1.041$ find each of the following:

$\log_a \frac{2}{11}$, $\log_a 14$, $\log_a 98$, $\log_a \frac{1}{7}$, $\log_a 9$

Solution

$\log_a \frac{2}{11} = \log_a 2 - \log_a 11$

$= 0.301 - 1.041 \approx -0.74$

$\log_a 14 = \log_a 2(7) = \log_a 2 + \log_a 7 = 0.301 + 0.845 \approx 1.146$

$\log_a 98 = \log_a 2(7^2) = \log_a 2 + \log_a 7^2 = \log_a 2 + 2\log_a 7 = 0.301 + 2(0.845) \approx 1.991$

$\log_a \frac{1}{7} = \log_a 1 - \log_a 7 \approx 0 - 0.845 = -0.845$

$\log_a 9$ Can't be found from the given information