***Lecture* Two**

***Section* 2.1 – Definition of the Derivative**

**Derivative**

The derivative of the function at  is defined as



A function is differentiable @ *x* if its derivative exists at *x*.

The process of finding derivatives is called ***differentiation***.



**Differentiability ⇒ Continuity**

If a function *f* is differentiable @ *x* = *c* ⇒ *f* is continuous @ *x* = *c*

***Example***

Find the derivative of 

*Solution*















***Example***

Find the derivative of 

*Solution*















***Exercises Section* 2.1 – Definition of the Derivative**

1. Find the derivative of *y* with the respect to *t* for the function 
2. Find the equation of the tangent line to  that is parallel to 2*x* + *y* = 0
3. Use the definition of limits to find the derivative: 
4. Use the definition of limits to find the derivative: 

***Section* 2.2 – Techniques for Finding Derivatives**

**Notations for the Derivative**

The derivative of  may be written in any of the following ways:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1st *derivative* |  |  |  |  |  |

***Constant Rule***

 *c* is constant

***Proof:***

Let *f*(*x*) = *c*





 So, 

***Example***

Find the derivative

1. 



1. 



1. 



***Power Rule***

 ***n*** is any real number

***Proof***

Let 









***Example***

Find the derivative

1. 





1. 







1. 





1. 



1. 





***Constant Times a Function***

If is a differentiable function of *x*, and *c* is a real number, then

If  

***Example***

1. If , find 





1. If , find 





1. If 



1. If , find 







1. 







***Sum or Difference Rule***

The derivative of the sum or difference of two differentiable functions is the sum or difference of their derivatives.

***Example***

Find the derivative of each function

1. 



1. 







1. 











1.  





***Example***

Find the slope of the graph of  at the point (2, -5)

*Solution*



Slope = 





***Example***

Researchers have determined that the daily energy requirements of female beagles who are at least 1 year old change with respect to age according to the function



where is the daily energy requirements  for a dog that is *t* years old.

1. Find 





1. Determine the rate of change of the daily energy requirements of a 2-year old female beagle





The daily energy requirements of a 2-year old female beagle are decreasing at the rate

***Example***

From 1998 through 2005, the revenue per share *R* (in dollars) for McDonald’s Corporation can be modeled by



Where t represents the year, with *t* = 8 corresponding to 1998. At what rate was McDonald’s revenue per share changing in 2003?

*Solution*

2003 ⇒*t* = 13



→



***Marginal Analysis***

Profit = *P* Revenue = *R* Cost = *C* *P = R - C*

The derivatives of these quantities are called ***Marginal***

Marginal Profit

Marginal Revenue

Marginal Cost

***Marginal Cost***

***Example***

Suppose that the total cost in hundreds of dollars to produce *x* thousand barrels of a beverage is given by



Find the marginal cost for the following values of *x*.

1. 

*Solution*







After 5 thousand barrels, the cost will be 140 (hundred dollars) or $14,000.00

1. 

*Solution*





After 30 thousand barrels, the cost will be $34,000.00

***Demand Functions***

The numbers of unit *q* that are willing to purchase at a given price per unit *p*



**Total Revenue *R***

Related to the price per unit and the quantity demanded (or sold): 

***Example***

The demand function for a certain product is given by 

Find the marginal revenue when  units and *p* is in dollars.

*Solution*

















When  units; the marginal revenue is $1.20 per unit.

***Example***

Find the revenue function and marginal revenue for a demand function of 

*Solution*

Revenue = quantity \* price

Revenue:





Marginal: 

***Marginal Profit***

***Example***

Suppose that the function for the product  is given by

 where 

Find the marginal profit from the production of the following numbers of units.

1. 
2. 

*Solution*

1. 

The profit is given by: 













The marginal profit is $0.55 per unit

1. 



The marginal profit is  per unit, which will reduce the profit.

***Exercises Section* 2.2 – Techniques for Finding Derivatives**

Find the derivative of

1. 
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31. 
32. 
33. Find the slope of the graph of  at the point (2, −5)
34. Find an equation of the tangent line to the graph of  at the point (2, 0)
35. Find the slope of the graph of when *x* = −1, 0, and 1.
36. The height *h* (in feet) of a free-falling object at time (in seconds) is given by . Find the average velocity of the object over each interval.
37. [0, 1]
38. [1, 2]
39. Give the position function of a diver who jumps from a board 12 feet high with initial velocity 16 feet per second. Then find the diver’s velocity function.
40. An analyst has found that a company’s costs and revenues in dollars for one product are given by

Respectively, where *x* is the number of items produced.

1. Find the marginal cost function
2. Find the marginal revenue function
3. Using the fact that profit is the difference between revenue and costs, find the marginal profit function.
4. What value of *x* makes the marginal profit is 0.
5. Find the profit when the marginal profit is 0.
6. A business sells 2000 units per month at a price $10 each. If monthly sales increases 200 units for each $0.10 reduction in price.
7. From 1998 through 2005, the revenue per share R (in dollars) for McDonald’s Corporation can be modeled by

******

Where *t* represents the year, with *t* = 8 corresponding to 1998. At what rate was McDonald’s revenue per share changing in 2003?

1. The cost ***C*** (in dollars) of producing *x* units of a product is given by 

*a*) Find the additional cost when the production increases from 9 to 10 units.

*b*) Find the marginal cost when

*c*) Compare the results of parts (*a*) and (*b*)

1. The revenue ***R*** (in dollars) of renting *x* apartments can be modeled by 

*a*) Find the additional revenue when the number of rentals is increased from 14 to 15

*b*) Find the marginal revenue when

*c*) Compare the results of parts (*a*) and (*b*)

1. The profit ***P*** (in dollars) of selling *x* units of calculus textbooks is given by



*a*) Find the additional profit when the sales increase from 150 to 151 units.

*b*) Find the marginal profit when

*c*) Compare the results of parts (*a*) and (*b*)

1. From 1998 through 2005, the revenue per share *R* (in dollars) for McDonald’s Corporation can be modeled by



Where *t* represents the year, with *t* = 8 corresponding to 1998. At what rate was McDonald’s revenue per share changing in 2003?

1. The profit derived from selling *x* units, is given by , find the marginal profit for a production level of 100 units. Compare this with the actual gain in profit by increasing production from 100 to 101 units.
2. The Cost of producing *x* hamburgers is  and the revenue function is given by



Compare the marginal profit when 10,000 units are produced with the actual increase in profit from 10,000 units to 10,001 units

1. An object moves along the *y*-axis (marked in feet) so that its position at time *x* (in seconds) is



1. Find the instantaneous velocity function *v*.
2. Find the velocity at *x* = 2 and *x* = 5 seconds
3. Find the time(s) when the velocity is 0.
4. A company’s total sales (in millions of dollars) *t* months from now are given by



1. Find .
2. Find  and  (to two decimal places). Write a brief verbal interpretation of these results.
3. Find  and  (to two decimal places). Write a brief verbal interpretation of these results.
4. A company’s total sales (in millions of dollars) *t* months from now are given by



1. Find .
2. Find  and  (to two decimal places). Write a brief verbal interpretation of these results.
3. Find  and  (to two decimal places). Write a brief verbal interpretation of these results.
4. A marine manufacturer will sell  power boats after spending $*x* thousand on advertising, as given by



1. Find .
2. Find  and  (to two decimal places). Write a brief verbal interpretation of these results.
3. A company manufactures and sells *x* transistor radios per week. If the weekly cost and revenue equations are



Then find the approximate changes in revenue and profit if production is increased from 2,000 to 2,010 per week.

1. A company manufactures fuel tanks for cars. The total weekly cost (in dollars) of producing *x* tanks given by



1. Find the marginal cost function.
2. Find the marginal cost at a production level of 500 tanks per week.
3. Interpret the result of part b.
4. Find the exact cost of producing the 501st item.
5. A company’s market research department recommends the manufacture and marketing of a new headphone set for MP3 players. After suitable test marketing, the research department presents the following *price-demand* equation:



Where *x* is the number of headphones that retailers are likely to buy at $*p* per set.

The financial department provides the cost function



Where $7,000 is the estimate of fixed costs (tooling and overhead) and $2 is the estimate of variable costs per headphone set (materials, labor, marketing, transportation, storage, etc.).

1. Find the domain of the function defined by the price demand function.
2. Find and interpret the marginal cost function .
3. Find the revenue function as a function of *x* and find its domain.
4. Find the marginal revenue at *x* = 2,000, 5,000, and 7,000. Interpret these results.
5. Graph the cost function and the revenue function in the same coordinate system, Find the intersection points of these two graphs and interpret the results.
6. Find the profit function and its domain and sketch the graph of the function.
7. Find the marginal profit at *x* = 1,000, 4,000, and 6,000. Interpret these results.
8. A small machine shop manufactures drill bits used in the petroleum industry. The manager estimates that the total daily cost (in dollars) of producing *x* bits is



1. Find  and 
2. Find  and . Interpret these quantities.
3. Use the results in part (***b***) to estimate the average cost per bit at a production level of 11 bits per day.
4. The total profit (in dollars) from the sale of *x* calendars is



1. Find the exact profit from the sale of the 41st calendar.
2. Use the marginal profit to approximate the profit from the sale of the 41st calendar.
3. The total profit (in dollars) from the sale of *x* cameras is



Evaluate the marginal profit at the given values of *x*, and interpret the results.

1. .
2. .
3. The total profit (in dollars) from the sale of *x* gas grills is



1. Find the average profit per grill if 40 grills are produced.
2. Find the marginal average profit at a production level of 40 grills and interpret the results.
3. Use the results from parts (*a*) and (*b*) to estimate the average profit per grill if 41 grills are produced.
4. The price *p* (in dollars) and the demand *x* for a particular steam iron are related by the equation



1. Express the price *p* in terms of the demand *x*, and find the domain of this function.
2. Find the revenue  from the sale of x steam irons. What is the domain of R?
3. Find the marginal revenue at a production level of 400 steam irons and interpret the results.
4. Find the marginal revenue at a production level of 650 steam irons and interpret the results.
5. The price-demand equation and the cost function for the production of TVs are given respectively, by



Where *x* is the number of TVs that can be sold at a price of $*p* per TV and  is the total cost (in dollars) of producing *x* TVs.

1. Express the price *p* as a function of the demand *x*, and find the domain of this function.
2. Find the marginal cost.
3. Find the revenue function and state its domain.
4. Find the marginal revenue.
5. Find  and  and interpret these quantities.
6. Graph the cost function and the revenue function on the same coordinate system for . Find the break−even points and indicate regions of loss and profit.
7. Find the profit function in terms of *x*.
8. Find the marginal profit.
9. Find  and  and interpret these quantities
10. The total cost and the total revenue (in dollars) for the production and sale of *x* hair dryers are given, respectively, by



1. Find the value of *x* where the graph of  has a horizontal tangent line.
2. Find the profit function .
3. Find the value of *x* where the graph of  has a horizontal tangent line.
4. Graph , , and  on the same coordinate system for . Find the break−even points. Find the *x* intercept of the graph of .

***Section* 2.3 – Derivatives of Products and Quotients**

***Product Rule***

The derivative of the product of two differentiable functions is equal to the first function times the derivative of the second plus the second function times the derivative of the first,





***Example***

Find the derivative of 

*Solution*











***Example***

Find the derivative of 

*Solution*







***Quotient Rule***



***Example***

Find  if 

*Solution*







***Example***

Find the derivative of 

*Solution*







***Example***

Find an equation of the tangent line to the graph of when *x* = 0

*Solution*















***Combining the product and Quotient Rules***

***Example***

Find the derivative of 

*Solution*





























**Average Cost Function**

To Study the effects of production levels on cost, economist use the average cost function , which is defined as



Where  is the total cost function and *x* the number of units produced.

***Marginal Average Cost Function*** 

***Example***

Suppose the cost in dollars of manufacturing *x* hundred small motors is given by

1. Find the average cost per hundred motors







1. Find the marginal average cost





1. Average cost is generally minimized when the marginal average cost is zero. Find the level of production that minimizes average cost





**16,000 *motors*** will minimize average cost.

***Exercises Section* 2.3 – Derivatives of Products and Quotients**

Find the derivative

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 
17. 
18. 
19. 
20. 
21. 
22. Find an equation of the tangent line to the graph of when 
23. A company that manufactures bicycles has determined that a new employee can assemble  bicycles per day after *d* days of on-the-job training, where



1. Find the rate of change function for the number of bicycles assembled with respect to time.
2. Find and interpret  and 
3. A small business invests $25,000.00 in a new product. In addition, the product will cost $0.75 per unit to produce. Find the cost function and the average cost function. What is the limit of the average cost function as production increase?
4. A communications company has installed a new cable TV system in a city. The total number *N* (in thousands) of subscribers *t* months after the installation of the system is given by



1. Find 
2. Find  and . Write a brief interpretation of these results.
3. Use the results from part (***b***) to estimate the total number of subscribers after 17 months.
4. One hour after a dose of x milligrams of a particular drug is administered to a person, the change in body temperature , in degrees Fahrenheit, is given approximately by



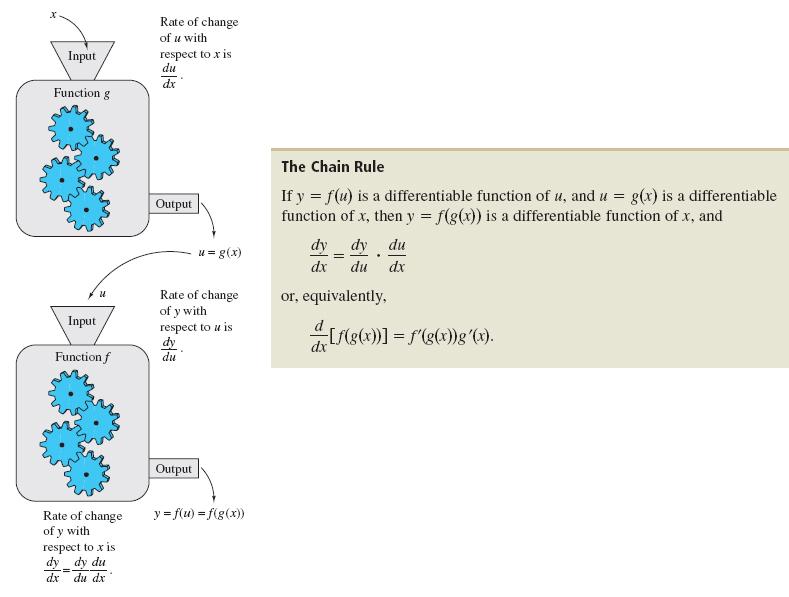
The rate  at which  changes with respect to the size of the dosage *x* is called the sensitivity of the body to the dosage.

1. Find 
2. Find , , and 
3. According to economic theory, the supply *x* of a quantity in a free market increases as the price *p* increases. Suppose that the number *x* of DVD players a retail chain is willing to sell per week at a price of $*p* is given by



1. Find 
2. Find the supply and the instantaneous rate of change of supply with respect to price is $40. Write a brief interpretation of these results.
3. Use the results from part (***b***) to estimate the supply if the price is increased to $41.

***Section* 2.4 – The Chain Rule**









***The General Power Rule***







***Example***

Find  if 

*Solution*











***Example***

Find 

*Solution*







***Example***

Find the derivative 

*Solution*







***Example***

Find the derivative 

*Solution*









***Example***

Find the tangent line to the graph of  when *x* = 4.

*Solution*



















***Exercises Section* 2.4 – The Chain Rule**

Find the derivative of

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 
17. 
18. 
19. 
20. 
21. Suppose a demand function is given by



Where *q* is the demand for a product and *p* is the price per item in dollars. Find the rate of change in the demand for the product per unit change in price (i.e. find *dq/dp*)

1. Find the tangent line to the graph of  when *x* = 4.
2. The revenue realized by a small city from the collection of fines from parking tickets is given by



where n is the number of work-hours each day that can be devoted to parking patrol. At the outbreak of a flu epidemic, 30 work-hours are used daily in parking patrol, but during the epidemic that number is decreasing at the rate of 6 work-hours per day. How fast is revenue from parking fines decreasing at the outbreak of the epidemic?

1. To test an individual’s use of a certain mineral, a researcher injects small amount form of that material into the person’s bloodstream. The mineral remaining in the bloodstream is measured each day for several days. Suppose the amount of the mineral remaining in the bloodstream (in milligrams per cubic centimeter) *t* days after the injection is approximated by . Find the rate of change of the mineral level with respect to time for 4 days.
2. The total cost (in hundreds of dollars) of producing *x* cameras per week is



1. Find 
2. Find  and . Interpret the results

***Section* 2.5 – Higher Order Derivatives**

**Higher Derivatives**





|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ***Notation for Higher-Order Derivatives*** | | | | | | |
| 1. | **1*st derivative*** |  |  |  |  |  |
| 2. | **2*nd derivative*** |  |  |  |  |  |
| 3. | **3*rd derivative*** |  |  |  |  |  |
| 4. | **4*th derivative*** |  |  |  |  |  |
| 5. | ***nth derivative*** |  |  |  |  |  |

***Example***

Find the first four derivatives of 

*Solution*









***Example***

Find the value of  for 

*Solution*









***Example***

Find the fourth derivative of 

*Solution*











***Acceleration***

 ***P***osition function

 ***V***elocity function

 ***A***cceleration function

***Example***

A ball is thrown upward from the top of an 80-foot cliff with an initial velocity of 64 feet per second. Give the position function. Then find the velocity and acceleration functions.

*Solution*







***Exercises Section* 2.5 – Higher Order Derivatives**

1. Find the second derivative: 
2. Find the third derivative: 
3. Find  the given value: 
4. Find the 4th derivative of 
5. Find the second derivative of 
6. Find  for , then find  and 
7. Find  for , then find  and 
8. The position function on Earth, where *s* is measured in meters, *t* is measured in seconds, *v*0 is the initial velocity in meters per second, and *h*0 is the initial height in meters, is



If the initial velocity is 2.2 and the initial height is 3.6, what is the acceleration due to gravity on Earth in meters per second per second?

***Section* 2.6 – Exponential & Logarithmic Functions**

***Exponential***

***Definition***

The exponential function *f* with base *b* is defined by



Where *b* > 0, *b* ≠ 1 and *x* is any real number.

*Example:*

**Exponential Equations**

 for any b > 0, ≠ 1

***Example***

Solve 

*Solution*









***Example***

Solve 

*Solution*











**Natural Base *e***

The irrational number *e* is called natural base

is called natural exponential function

***Example***

Biologists studying salmon have found that the oxygen consumption of yearling salmon (in appropriate units) increases exponentially with the speed of swimming according to the function defined by



where *x* is the speed in feet per second. Find the following

1. The oxygen consumption when the fish are still





1. The oxygen consumption at a speed of 2 *ft* per second





***Logarithmic* Function (*Definition)***

For  and 

 is equivalent to 



**Base**

The function  is the logarithmic function with base *b*.

: *read* log base *b* of *x* 

|  |  |
| --- | --- |
| ***Logarithmic Form*** | ***Exponential Form*** |
|  |  |
|  |  |
|  |  |
|  |  |

***Natural Logarithms***

***Definition***



The logarithmic function with base ***e*** is called natural logarithmic function.

 *read* "el en of *x*"

log(-1) = *doesn’t exist* ln(-1) = *doesn’t exist*

log0 = *doesn’t exist* ln0 = *doesn’t exist*

log1 = 0 ln1 = 0

log10 = 1 ln*e* = 1

***Domain***

The domain of a logarithmic function of the form is the set of all positive real numbers.

(Inside the log has to be > 0)

Find the domain of 

 *Domain:*

**Properties of Logarithmic Functions**

***Product Rule***



***Power Rule***



***Quotient Rule***



***Example***

Use the properties of logarithms to rewrite 

*Solution*

 *Quotient Rule*

 *Product Rule*



 *Power Rule*

***Changing Logarithmic Bases***

***Example***

Find: 

*Solution*



**Property of *Logarithmic***

* 

1. Isolate the exponential expression
2. Take the natural logarithm on both sides of the equation
3. Simplify using one of the following properties: 
4. Solve for the variable

***Example***

Solve 

*Solution*













**Solving Logarithmic Equations**

1. Express the equation in the form 
2. Use the definition of a logarithm to rewrite the equation in exponential form:



1. Solve for the variable
2. Check proposed solution in the original equation. Include only the set for M > 0

***Example***

Solve 

*Solution*

 *Write in exponential form*









 *Check*: 

* For any M > 0, N > 0, b > 0, ≠ 1



Check proposed solution in the original equation. Include only the set inside the log for > 0

***Example***

Solve: 

*Solution*

 *Quotient Rule*



 *Multiply by x* + 2







 *Solve for x*



*Check*:  *OrDomain*



Solution: 





***Exercises Section* 2.6 – Exponential & Logarithmic Functions**

Solve

1. 
2. 
3. 
4. 
5. 

Solve

1. 
2. 
3. 

Use the properties of logarithms to rewrite

1. 
2. 
3. 
4. 
5. ****

***Section* 2.7 – Derivatives of Exponential and Logarithmic Functions**

**Derivative of the Natural Exponential Function**

**Derivative of**

***Example***

Find the derivative of each function

1. 





1. 



1. 







1. 





***Example***

Find the derivative of 

*Solution*













***Example***

The demand function for the product is modeled by where *p* is the price per unit in dollars and *x* is the number of units. What price will yield maximum revenue?

*Solution*



















***Example***

A company sells 990 units of a new product in the first year and 3213 units in the fourth year. They expect that sales can be approximated by a logistic function, leveling off at around 100,000 in the long run given by the formula



1. Find ***k*** and rewrite the function

*Solution*







 *Subtract 3213 from both sides*

 *Divide both sides by 321300*

 *ln both sides*









1. Find the rate of change of sales after 4 years

*Solution*





***Derivatives of Logarithmic***

***Derivative* of **

** **

***Derivative of ln***

** **

***Other Bases***

***Example***

Find the derivative of each function

1. 







1. 



1.  ****



***Example***

Find the derivative of function 

*Solution*

 ****



***Example***

Find the derivative of function

*Solution*





***Example***

Find the derivative of function 

*Solution*





***Example***

Find the derivative of function 

*Solution*







***Example***

Find the derivative of function 

*Solution*









***Exercises Section* 2.7 – Derivatives of Exponential and Logarithmic Functions**

Find the derivative:

1. 
2. 
3. 
4. 
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11. 
12. 
13. 
14. ******
15. ******
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21. ******
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48. 
49. 
50. 
51. 
52. 
53. 
54. 
55. 
56. Find the second derivative of ******
57. Find the equation of the tangent line to  at the point (0, 1)
58. Find the equation of the tangent line to  at the point (1, *e*)
59. Find the equation of the tangent lines to  at the points (0, 4)
60. Find the equation of the tangent line to ****** at 
61. The percentage of people of any particular age group that will die in a given year may be approximated by the formula



where *t* is the age of the person in years

1. Find 
2. Find 
3. Assume the cost of a gallon of milk is $2.90. With continuous compounding, find the time it would take the cost to be 5 times as much (to the nearest tenth of a year), at an annual inflation rate of 6 %.
4. The sales in thousands of a new type of product are given by , where *t* represents time in years. Find the rate of change of sales at the time when 
5. A company's total cost, in millions of dollars, is given by  where *t* = time in years. Find the marginal cost when .
6. A company's total cost, in millions of dollars, is given by  where *t* = time in years. Find the marginal cost when .
7. The demand function for a certain book is given by the function . Find the marginal demand 
8. Suppose that the amount in grams of a radioactive substance present at time *t* (in years) is given by . Find the rate of change of the quantity present at the time when .
9. Researchers have found that the maximum number of successful trials that a laboratory rat can complete in a week is given by



where *t* is the number of weeks the rat has been trained. Find the rate of change 

1. When a telephone wire is hung between two poles, the wire forms a U-shape curve called a Catenary. For instance, the function   models the shape of the telephone wire strung between two poles that are 60 ft. apart (*x* & *y* are measured in ft.). Show that the lowest point on the wire is midway between two poles. How much does the wire sag between the two poles?
2. Find  for , then find  and 
3. Suppose the average test score *p* and was modeled by , where *t* is the time in months. How would the rate at which the average test score changed after 1 year?
4. Suppose that the population of a certain collection of rare ants is given by



Where *t* represents the time in days. Find the rates of change of the population on the second day and on the eighth day.

1. Suppose that the demand function for *x* units of a certain item is  where *P* is the price per unit, in dollars. Find the marginal revenue.
2. The population of coyotes in the northwestern portion of Alabama is given by the formula , where *t* represents the time in years since 2000 (the year 2000 corresponds to  Find the rate of change of the coyote population in 2013 .
3. Students in a math class took a final exam. They took equivalent forms of the exam in monthly intervals thereafter. The average score , in percent, after t months was found to be given by



Find .

1. Suppose that the population of a town is given by  where *t* is the time in years after 1980 and *P* is the population of the town in thousands. Find .
2. The following formula accurately models the relationship between the size of a certain type of tumor and the amount of time that it has been growing:



where *t* is in months and is measured in cubic centimeters. Calculate the rate of change of tumor volume at 80 months.

1. A yeast culture at room temperature (68° F) is placed in a refrigerator set at a constant temperature of 38° F. After *t* hours, the temperature *T* of the culture is given approximately by



What is the rate of change of temperature of the culture at the end of 1 hour? At the end of 4 hours?

1. A mathematical model for the average age of a group of people learning to type is given by



Where  is the number of words per minute typed after *t* hours of instruction and practice (2 hours per day, 5 days per week). What is the rate of learning after 10 hours of instruction and practice? After 100 hours?