***Section* 1.4 – Limits at Infinity**

|  |  |
| --- | --- |
| ***Notation*** | ***Terminology*** |
|  | increases without bound (can be made as large positive as desired) |
|  | decreases without bound (can be made as large negative as desired) |

**Horizontal Asymptote (*HA*)**

The line  is a ***horizontal asymptote*** for the graph of a function  if



Let  be a rational function. (***Proof*** !)

1. If the degree of numerator is less than of denominator (*n* < *m*) ⇒ *y* = 0

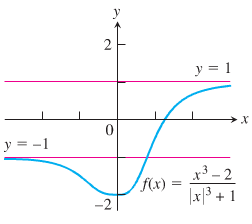


1. If the degree of numerator is equal of denominator (*n* = *m*) ⇒



1. If the degree of numerator is greater than of denominator (*n* > *m*) ⇒ No horizontal asymptote



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***Example***

Find the horizontal asymptotes of the graph of 

***Solution***





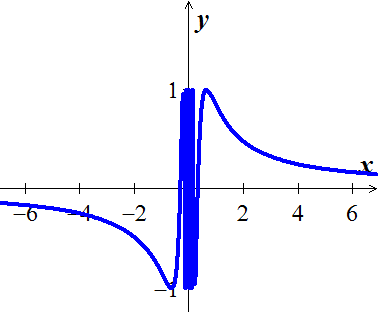








The ***HA*** are  and .

***Example***

Find 

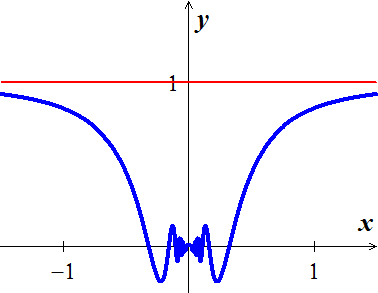
***Solution***

Let 





***Example***

Find 

***Solution***

Let 





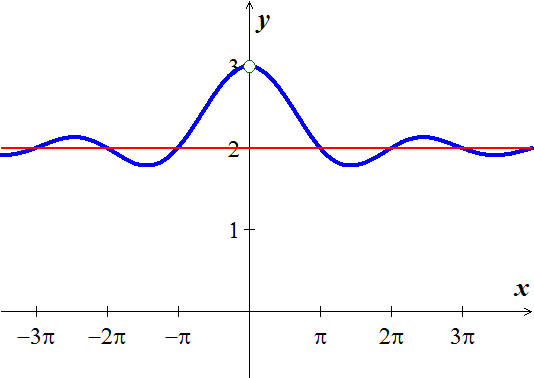




***Example***

Find the horizontal asymptote of 

***Solution***

****Since 









***HA*:** 

***Example***

Find 

***Solution***





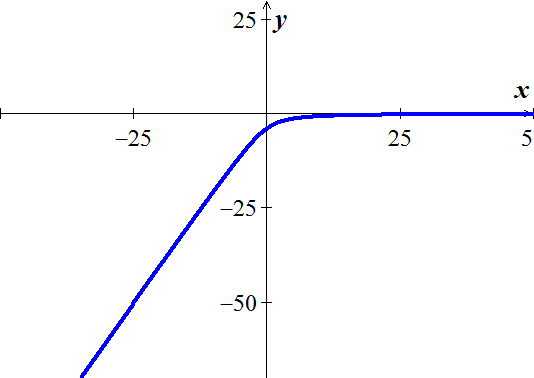








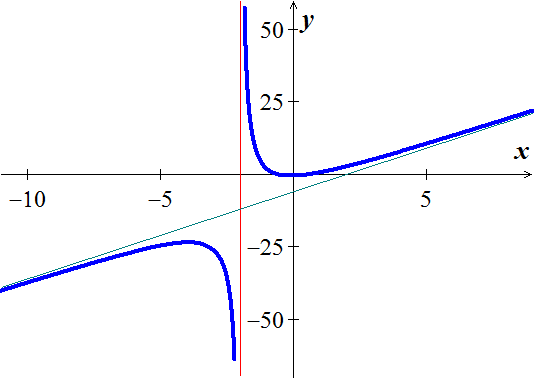




***Slant* or *Oblique* Asymptotes**

When the degree of the numerator is one greater than the degree of the numerator, the graph has a ***slant*** or ***oblique*** asymptote and it is a line . To find the slant asymptote, divide the fraction using long division. The quotient (not remainder) is the slant asymptote.

***Example***

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The ***oblique*** ***asymptote*** is the line ***y* = 3*x* − 6**

***Example***

Find the horizontal and vertical asymptotes of the curve 

***Solution***

***HA***: 

***VA***: 

***Example***

Find the horizontal and vertical asymptotes of the curve 

***Solution***

***HA***: 

***VA***: 



**Infinite Limits**

The limit has a value of infinity or minus infinity, such a function . It is convenient to describe the behavior of  by saying that  approaches ∞ as .

***Definition***

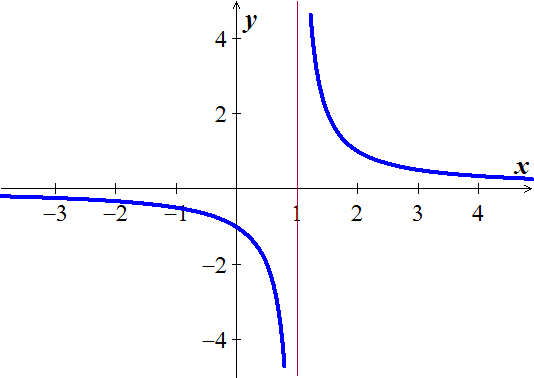
We say 

That  doesn’t exist because  becomes arbitrary large and positive as .

We say 

That  doesn’t exist because  becomes arbitrary large and negative as .

***Example***

Find 

***Solution***

As 





***Examples***

* 







* 





* 



* 



* 



***Exercises*** ***Section* 1.4 – Limits at Infinity**

(**1 – 8**) Find the limit as  and as  of

|  |  |  |
| --- | --- | --- |
|  |  |  |

(**8 – 60**) Evaluate the limits

|  |  |
| --- | --- |
|  |  |

|  |  |
| --- | --- |
|  |  |

(**61 – 64**) Graph the rational function and include the equations of the asymptotes

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

1. Let 
2. Analyze ,, , and 
3. Does the graph of *f* have any vertical asymptotes? Explain?

(**66 – 85**) Find the vertical, horizontal, hole, and oblique asymptotes (if any) of

|  |  |  |
| --- | --- | --- |
|  |  |  |

(**85 – 142**) Find the limits

|  |  |  |
| --- | --- | --- |
|  |  |  |

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| --- | --- | --- |
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