***Section* 4.3 – Definite Integral**

***Definition***

Let  be a function defined on a closed interval [*a, b*]. We say that a number *J* is the ***definite integral* of *f* over [*a, b*]** and that *J* is the limit of the Riemann sums  if the following condition is satisfied:

Given any number ε > 0 there is a corresponding number δ > 0 such that for every partition

 of [a, b] with  and any choice of  in , we have



***Leibniz*** introduced a notation for the definite integral that captures its construction as a limit of Riemann sums.



**Integral of *f* from *a* to *b*.**





***Theorem* – Integrability of Continuous Functions**

If a function *f* is continuous over the interval [*a, b*], or if *f* has at most finitely many jump discontinuities there, then the definite integral  exists and *f* in integrable over [*a, b*]

**Properties of Definite Integrals**

 

***Theorem***

When *f* and *g* are integrable over the interval [*a, b*], the definite integral satisfies the rules:

*Order of Integration*: 

*Zero Width Interval*: 

*Constant Multiple*: 

*Sum and Difference*: 

*Additivity*: 

*Max-Min* Inequality: If *f* has ***maximum*** value *max* *f* and ***minimum*** value *min* *f* on [*a, b*], then



Domination: 



|  |  |
| --- | --- |
|  |  |
| ***Zero Width Interval***: | ***Constant Multiple***: (*k* = 2) |
|  |  |
| ***Sum***: (*areas add*) | Additive for definite integrals: |
|  |  |
| Max-Min Inequality: | Domination |

***Example***

Suppose that . Find:

1. 
2. 

***Solution***

1. 
2. 





***Example***

Show that the value of  is less than or equal to 

***Solution***

: is the lower bound

: is the upper bound

The maximum value of  on [0, 1] is 

So, 

**Area Under the Graph of a Nonnegative Function**

***Definition***

If  is nonnegative and integrable over a closed interval [*a, b*], then the area under the curve  over [*a, b*] is the integral of *f* from *a* to *b*,



***Example***

Compute  and find the area *A* under *y* = *x* over the interval [0, *b*], *b* > 0.

***Solution***

To Compute the definite integral, we consider the partition *P* subdivides the interval [0, *b*] into *n* subintervals of equal width .

















  











