**Math 2415 – Calculus III** ***Exam* 4 *Review***

*Professor*: Fred Khoury

1. Let *C* be the circle of radius 2 centered at the origin with counterclockwise orientation
2. Give the unit outward vector at any point  on *C*.
3. Find the normal component of the vector field  at any point on *C*.
4. Find the normal component of the vector field  at any point on *C*.
5. Evaluate the line integral  *C* is the upper half of a circle 
6. Evaluate the line integral  *C* is the path 
7. Integrate  over the circle 
8. Integrate  over the involute curve 
9. Find the work required to move an object on the given curve  on the path 
10. Find the work done by  over the plane curve  from the point  to the point  by using the parametrization of the curve to evaluate the work integral
11. Find the circulation and the outward flux of the vector field  for the curve 
12. Find the flow of the field 
13. Once around the ellipse *C* in which the plane  intersects the cylinder, clockwise as viewed from the positive *y*-axis.
14. Along the curved boundary of the helicoid  from  to 
15. Evaluate the line integral  for the following vector field ***F*** and curves *C* in two ways
16. By parameterizing C.
17. By using the Fundamental Theorem for the integrals, if possible.
18. ; 
19. ; *C*: is the square with vertices  with *counterclockwise* orientation.
20. Prove that the radial field  where  and *p* is a real number, is conservative on  with the origin removed. For what value of *p* is ***F*** conservative on  (including the origin)?
21. Evaluate  *C* is the circle 
22. Find the area of the elliptical region cut from the plane  by the cylinder 
23. Find the area of the cap cut from the paraboloid  by the plane 
24. Use Green’s Theorem to find the counterclockwise circulation and outward flux for the field and curves  
25. Use Green’s Theorem to find the counterclockwise circulation and outward flux for the field and curves 



1. Show that  for any closed curve C to which Green’s Theorem applies.
2. Use either form of Green’s Theorem to evaluate the line integral ; *C* is the square with vertices  with *counterclockwise* orientation
3. Use either form of Green’s Theorem to evaluate the line integral ; *C* is the circle of radius 4 centered at the origin with *clockwise* orientation.
4. Find the area of the region bounded by the hypocycloid  for , using a line integral
5. Compute the divergence and curl of the following vector fields. State whether the field is *source*-*free* or *irrotational*.
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9. Prove that  and use the result to prove that 
10. Find the surface area of the helicoid 
11. Use a surface integral to find the area of the hemisphere  for  (excluding the base)
12. Use a surface integral to find the area of the surface  above the origin 
13. Find the flux of  across the sphere of radius a centered at the origin, where . Assume the normal vectors to the surface point outward.
14. Evaluate the surface integrals ; *S* is the entire surface including the base of the hemisphere , for 
15. Evaluate the line integral  using the Stoke’s Theorem ; *C* is the boundary of the plane  in the first octant and has counterclockwise orientation.
16. Use Stoke’s Theorem to evaluate the surface integral ; , where *S* is the hyperboloid . Assume that ***n*** is the *outward normal*.
17. Use the Divergence Theorem to compute the outward flux of the vector field ; *S* is the surface of the cube cut from the first octant by the planes .
18. Use the Divergence Theorem to compute the outward flux of the vector field ; *S* is the cylinder 
19. Compute the outward flux of the field  across the surface *S* that is the boundary of the prism bounded by the planes 
20. Consider the surface *S* consisting of the quarter-sphere , for , and the half disk in the *yz*-plane , for . The boundary of *S* in the *xy-*plane is *C*, which consists of the semicircle , for , and the line segment  on the *y-*axis, with a counterclockwise orientation. Let 
21. Describe the direction in which the normal vectors point on *S*.
22. Evaluate 
23. Evaluate  and check for segment with part (*b*).
24. Let *S* be the hemisphere , for , and let *T* be the paraboloid , for , where *a* > 0. Assume the surfaces have outward normal vectors.
25. Verify that *S* and *T* have the same base  and the same high point .
26. Which surface has the greater area?
27. Show that the flux of the radial field  across *S* is .
28. Show that the flux of the radial field  across *T* is .

***Solution***

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