***Solution*** ***Section* 4.2 – Line Integrals**

***Exercise***

Evaluate  where *C* is the straight-line segment  from (0, 1, 0) to .

***Solution***















***Exercise***

Evaluate  where *C* is the straight-line segment  from  to .

***Solution***















***Exercise***

Evaluate  along the curve 

***Solution***

















***Exercise***

Find the integral of  over the straight line segment from  to 

***Solution***

















***Exercise***

Find the integral of  over the curve 

***Solution***















***Exercise***

Evaluate  where *C* is

1. The straight-line segment , from (0, 0) to (4, 2).
2. The parabolic curve , from (0, 0) to (2, 4).

***Solution***

1.  











1.  













***Exercise***

Evaluate  where *C* is

1. The straight-line segment , from (0, 0) to (1, 4).
2.   is the line segment (0, 0) to (1, 0) and  is the line segment (1, 0) to (1, 2).

***Solution***

1.  













1. 



















***Exercise***

Find the line integral of  along the curve 

***Solution***







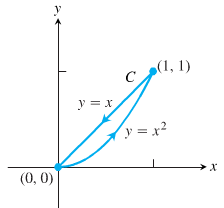








***Exercise***

Evaluate  where *C* is

***Solution***













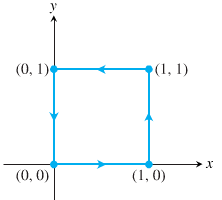








***Exercise***

Evaluate  where *C* is

***Solution***





















***Exercise***

Find the line integral of  over the curve 

***Solution***













***Exercise***

Find the line integral of  over the curve  in the first quadrant from (0, 2) to 

*********Solution***



















***Solution*** ***Section* 4.3 – Conservative Vector Fields**

***Exercise***

Find the gradient field of the function 

***Solution***











***Exercise***

Find the gradient field of the function 

***Solution***











***Exercise***

Find the gradient field of the function 

***Solution***



***Exercise***

Find the line integral of  where 

***Solution***





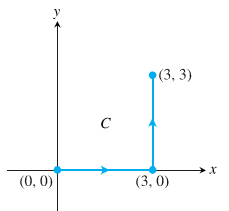










***Exercise***

Find the line integral of  where *C* is

***Solution***

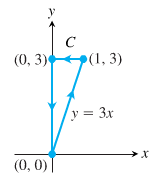








***Exercise***

Find the line integral of  where *C* is

***Solution***



















***Exercise***

Find the work done by the force field  over the curve .

***Solution***























***Exercise***

Find the work done by the force field  over the curve .

***Solution***



























***Exercise***

Find the work done by the force field  over the curve .

***Solution***













|  |  |  |
| --- | --- | --- |
|  |  |  |
| **+** |  |  |
| **−** |  |  |











***Exercise***

Evaluate  for the vector field  along the curve  from  to 

***Solution***

















***Exercise***

Find the circulation and flux of the fields  around and across each of the following curves.

1. The circle 
2. The ellipse 

***Solution***

1. 





























|  |  |
| --- | --- |
|  |  |

1. 

































|  |  |
| --- | --- |
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***Exercise***

Find the circulation and flux of the fields  across the circle 

***Solution***















|  |  |
| --- | --- |
|  |  |































***Exercise***

Find a field  in the *xy*-plane with the property that at each point , ***F*** points toward the origin and  is

1. The distance from (*x, y*) to the origin
2. Inversely proportional to the distance from (*x, y*) to the origin. (The field is undefined at (0, 0).)

***Solution***

1. The slope of the line through the origin and a point  is: 

The vector parallel to the line is given by: 

Pointing away from the origin:  is the unit vector pointing toward the origin.





1. 



***Exercise***

A fluid’s velocity field is . Find the flow along the curve 

***Solution***









***Exercise***

A fluid’s velocity field is . Find the flow along the curve 

***Solution***

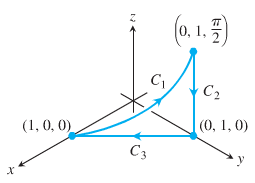








***Exercise***

Find the circulation of  around the closed path consisting of the following three curves traversed in the direction of increasing *t*.







***Solution***













|  |  |  |
| --- | --- | --- |
|  |  |  |
| **+** |  |  |
| **−** |  |  |











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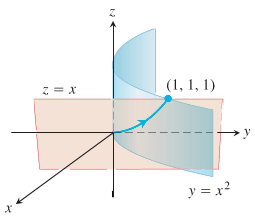






***Exercise***

The field  is the velocity field of a flow in space. Find the flow from  to  along the curve of intersection of the cylinder  and the plane . (***Hint***: Use  as the parameter.)

***Solution***

Let 















***Solution*** ***Section* 4.4 – Green's Theorem**

***Exercise***

Use Green’s theorem to find the counterclockwise circulation and outward flux for the field  and curve *C* is the square bounded by 

***Solution***















***Exercise***

Use Green’s theorem to find the counterclockwise circulation and outward flux for the field  and curve *C* is the square bounded by 

***Solution***



|  |  |
| --- | --- |
|  |  |

***Exercise***

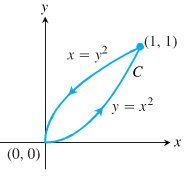
Use Green’s theorem to find the counterclockwise circulation and outward flux for the field  and curve *C* is the triangle bounded by 

***Solution***



|  |  |
| --- | --- |
|  |  |

***Exercise***

Use Green’s theorem to find the counterclockwise circulation and outward flux for the field  and curve *C*

***Solution***



























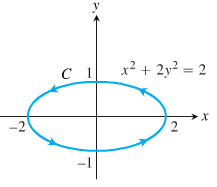






***Exercise***

Use Green’s theorem to find the counterclockwise circulation and outward flux for the field  and curve *C*

***Solution***

























***Exercise***

Use Green’s theorem to find the counterclockwise circulation and outward flux for the field  and curve *C* is the right-hand loop of the lemniscate 

***Solution***



|  |  |
| --- | --- |
|  |  |

***Exercise***

Find the outward flux for the field  across the cardioid 

***Solution***





















***Exercise***

Find the work done by  in moving a particle once counterclockwise around the curve *C*: The boundary of the triangular region in the first quadrant enclosed by the *x*-axis, the line  and the curve 

***Solution***















***Exercise***

Apply Green’s Theorem to evaluate the integral  *C*: The triangle bounded by 

***Solution***



















***Exercise***

Apply Green’s Theorem to evaluate the integral  *C*: The boundary of 

***Solution***













***Exercise***

Evaluate  counterclockwise around the triangle with vertices (0, 0),  and 

***Solution***

Along : 







Along : 







Along : 



















***Solution Section* 4.5 – Divergence and Curl**

***Exercise***

Find the divergence of the following vector fields 

***Solution***





***Exercise***

Find the divergence of the following vector fields 

***Solution***



***Exercise***

Find the divergence of the following vector fields 

***Solution***





***Exercise***

Find the divergence of the following vector fields 

***Solution***



***Exercise***

Find the divergence of the following vector fields 

***Solution***



***Exercise***

Find the divergence of the following vector fields 

***Solution***





***Exercise***

Calculate the divergence of the radial fields. 

Express the result in terms of the position vector ***r*** and its length .

***Solution***







***Exercise***

Calculate the divergence of the radial fields. 

Express the result in terms of the position vector ***r*** and its length .

***Solution***











***Exercise***

Calculate the divergence of the radial fields. 

Express the result in terms of the position vector ***r*** and its length .

***Solution***



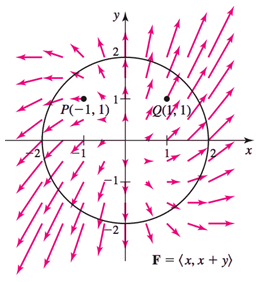




***Exercise***

Consider the following vector fields , the circle *C*, and two points *P* and *Q*.

1. Without computing the divergence, does the graph suggest that the divergence is positive or negative at *P* and *Q*?
2. Compute the divergence and confirm your conjecture in part (*a*).
3. On what part of *C* is the flux outward? Inward?
4. Is the net outward flux across *C* positive or negative?vector

***Solution***

1. At both *P* and *Q*, the arrows going away from the point are larger in both number and magnitude than those going in, so we would expect the divergence to be positive at both points.
2. 

It is positive everywhere.

1. The arrows all point roughly away from the origin, so we the flux is outward everywhere.
2. The net flux across *C* should be positive.

***Exercise***

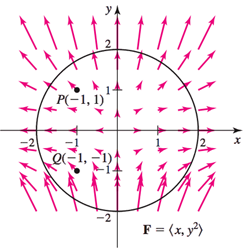
Consider the following vector fields , the circle *C*, and two points *P* and *Q*.

1. Without computing the divergence, does the graph suggest that the divergence is positive or negative at *P* and *Q*?
2. Compute the divergence and confirm your conjecture in part (*a*).
3. On what part of *C* is the flux outward? Inward?
4. Is the net outward flux across *C* positive or negative?

***Solution***

1. At *P*, the divergence should be positive.

At *Q*, the larger arrows point in towards *Q*, so the divergence should be negative.



1. 

At  At 

1. The flux is outward above the line ; below this line, the flux is inward across *C*.
2. The size of the narrows pointing outward at the top of the circle seems to roughly equal those pointing inward at the bottom, so the remaining outward-pointing arrows result in a net positive flux across *C*.

***Exercise***

Consider the vector fields , where 

1. Compute the curl field and verify that it has the same direction as the axis of rotation
2. Compute the magnitude of the curl of the field

***Solution***

1. 







The curl is the same direction as the axis of rotation.

1. The magnitude of the curl is 

***Exercise***

Consider the vector fields , where 

1. Compute the curl field and verify that it has the same direction as the axis of rotation
2. Compute the magnitude of the curl of the field

***Solution***

1. 







The curl is the same direction as the axis of rotation.

1. The magnitude of the curl is 

***Exercise***

Consider the vector fields , where 

1. Compute the curl field and verify that it has the same direction as the axis of rotation
2. Compute the magnitude of the curl of the field

***Solution***

1. 







The curl is the same direction as the axis of rotation.

1. The magnitude of the curl is 

***Exercise***

Consider the vector fields , where 

1. Compute the curl field and verify that it has the same direction as the axis of rotation
2. Compute the magnitude of the curl of the field

***Solution***

1. 









The curl is the same direction as the axis of rotation.

1. The magnitude of the curl is 

***Exercise***

Compute the curl of the vector fields 

***Solution***





***Exercise***

Compute the curl of the vector fields 

***Solution***





***Exercise***

Compute the curl of the vector fields 

***Solution***





***Exercise***

Compute the curl of the vector fields 

***Solution***





***Exercise***

Compute the curl of the vector fields 

***Solution***











***Exercise***

Compute the curl of the vector fields 

***Solution***





***Exercise***

Show that the general rotation field , where ***a*** is a nonzero constant vector and , has zero divergence.

***Solution***

Let 











***Exercise***

Let ,  and consider the rotation field . Use the right-hand rule for cross product to find the direction of ***F*** at the points , , , and 

***Solution***

; ***F*** points in the positive *x-*direction

; ***F*** points in the negative *z-*direction

; ***F*** points in the negative *x-*direction

; ***F*** points in the positive *z-*direction

***Exercise***

Find the exact points on the circle  at which the field  switches from pointing inward to outward on the circle, or vice versa.

***Solution***

The field switches from inward-pointing to outward-pointing at points where it is tangent to the circle , where it is orthogonal to the normal to the circle.

The normal to the circle at  is a multiple of , so we want to find  so that







The solutions are: 

***Exercise***

Suppose a solid object in  has a temperature distribution given by . The heat flow vector field in the object is , where the conductivity *k* > 0 is a property of the material. Note that the heat flow vector points in the direction opposite to that of the gradient, which is the direction of greatest temperature decrease. The divergence of the heat flow vector is  (the Laplacian of *T*). Compute the heat flow vector field and its divergence for the following temperature distribution.

|  |  |
| --- | --- |
|  |  |

***Solution***

1. 























1. 









1. 













***Solution*** ***Section* 4.6 – Surfaces and Area**

***Exercise***

Find a parametrization of the surface: The paraboloid 

***Solution***



Then:  

***Exercise***

Find a parametrization of the surface: The portion of the cone  between the planes  and 

***Solution***



Then:  

***Exercise***

Find a parametrization of the surface cut from the sphere  by the plane 

***Solution***











Then:  

***Exercise***

Use a parametrization to express the area of the surface as a double integral. Then evaluate the integral of the portion of the plane  inside the cylinder 

***Solution***





Then:  













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***Exercise***

Use a parametrization to express the area of the surface as a double integral. Then evaluate the integral of the portion of the cone  between the planes  and 

***Solution***



Then:  













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***Exercise***

Use a parametrization to express the area of the surface as a double integral. Then evaluate the integral of the portion of the cylinder  between the planes  and 

***Solution***







Then: 













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***Exercise***

Use a parametrization to express the area of the surface as a double integral. Then evaluate the integral of the portion of the cap cut from the paraboloid  between the planes  and 

***Solution***



Then:  

















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***Exercise***

Find the area of the surface cut from the bottom of the paraboloid  by the plane .

***Solution***















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***Exercise***

Find the area of the portion of the surface that lies above the triangle bounded by the lines , and  in the *xy*-plane.

***Solution***

























***Exercise***

Find the area of the cap cut from the sphere  by the cone .

***Solution***



























***Exercise***

Find the area of the ellipse cut from the plane  (*c* a constant) by the cylinder .

***Solution***





















***Exercise***

Find the area of the surface cut from the nose of the paraboloid  by *yz*-plane.

***Solution***



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***Exercise***

Find the area of the surface in the first octant cut from the cylinder  by the planes  and 

***Solution***









****







***Solution*** ***Section* 4.7 – Stokes’ Theorem**

***Exercise***

Verify that the line integral and the surface integral of Stokes’ Theorem are equal for the following vector fields, surfaces *S*, and closed curves *C*. Assume that *C* has counterclockwise orientation and *S* has a consistent orientation.

; *S* is the upper half of the sphere  and *C* is the circle  in the *xy-*plane

***Solution***



























***Or***

Using the standard parametrization of the sphere













***Exercise***

Verify that the line integral and the surface integral of Stokes’ Theorem are equal for the following vector fields, surfaces *S*, and closed curves *C*. Assume that *C* has counterclockwise orientation and *S* has a consistent orientation.

; *S* is the upper half of the sphere  and *C* is the circle  in the *xy-*plane

***Solution***





































***Exercise***

Verify that the line integral and the surface integral of Stokes’ Theorem are equal for the following vector fields, surfaces *S*, and closed curves *C*. Assume that *C* has counterclockwise orientation and *S* has a consistent orientation.

; *S* is the paraboloid  for  and *C* is the circle  in the *xy-*plane

***Solution***











Surface integral: 

***Exercise***

Verify that the line integral and the surface integral of Stokes’ Theorem are equal for the following vector fields, surfaces *S*, and closed curves *C*. Assume that *C* has counterclockwise orientation and *S* has a consistent orientation.

; *S* is the cap of the sphere  above the plane  and *C* is the boundary of *S*.

***Solution***

 is the intersection of the sphere with the plane .













































***Exercise***

Verify that the line integral and the surface integral of Stokes’ Theorem are equal for the following vector fields, surfaces *S*, and closed curves *C*. Assume that *C* has counterclockwise orientation and *S* has a consistent orientation.

; *S* is the cap of the sphere  above the plane  and *C* is the boundary of *S*.

***Solution***

 is the intersection of the sphere with the plane .





































***Exercise***

Evaluate the line integral  by evaluating the surface integral in Stokes’ Theorem with an appropriate choice of *S*. Assume that *C* has a counterclockwise orientation,

; *C* is the circle  in the plane .

***Solution***















***Exercise***

Evaluate the line integral  by evaluating the surface integral in Stokes’ Theorem with an appropriate choice of *S*. Assume that *C* has a counterclockwise orientation,

; *C* is the ellipse  in the plane .

***Solution***







 Because 





***Exercise***

Evaluate the line integral  by evaluating the surface integral in Stokes’ Theorem with an appropriate choice of *S*. Assume that *C* has a counterclockwise orientation,

; *C* is the boundary of the plane  in the plane first octant.

***Solution***























***Exercise***

Evaluate the line integral  by evaluating the surface integral in Stokes’ Theorem with an appropriate choice of *S*. Assume that *C* has a counterclockwise orientation,

; *C* is the circle  for  .

***Solution***



*S* is the disk 





















***Exercise***

Evaluate the line integral  by evaluating the surface integral in Stokes’ Theorem with an appropriate choice of *S*. Assume that *C* has a counterclockwise orientation,

; *C* is the boundary of the plane  in the first octant.

***Solution***









***Exercise***

Evaluate the line integral in Stokes’ Theorem to evaluate the surface integral . Assume that ***n*** points in an upward direction,

; *S* is the upper half of the ellipsoid 

***Solution***





Let 



















***Exercise***

Evaluate the line integral in Stokes’ Theorem to evaluate the surface integral . Assume that ***n*** points in an upward direction,

; *S* is the cap of the sphere 

***Solution***

The boundary of the surface is the intersection of the plane  and 

At 























***Exercise***

Evaluate the line integral in Stokes’ Theorem to evaluate the surface integral . Assume that ***n*** points in an upward direction,

; *S* is the tilted disk enclosed 

***Solution***























*S* is the disk 











***Exercise***

Use Stokes’ Theorem and a surface integral to find the circulation on *C* of the vector field  as a function of . For what value of  is the circulation a maximum?

***Solution***





















The maximum value of the circulation when  which is 

***Exercise***

A circle *C* in the plane  has a radius of 4 and center (2, 3, 3). Evaluate  for  where *C* has a counterclockwise orientation when viewed from above. Does the circulation depend on the radius of the circle? Does it depend on the location of the center of the circle?

***Solution***





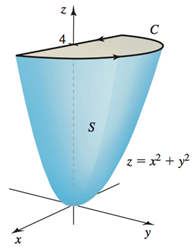








***Exercise***

Begin with the paraboloid , for , and slice it with the plane . Let *S* be the surface that remains for  (including the planar surface in the *xz-*plane). Let *C* be the semicircle and line segment that bound the cap of *S* in the plane  with counterclockwise orientation. Let 

1. Describe the direction of the vectors normal to the surface that are consistent with the orientation of *C*.
2. Evaluate 
3. Evaluate  and check for argument with part (*b*).

***Solution***

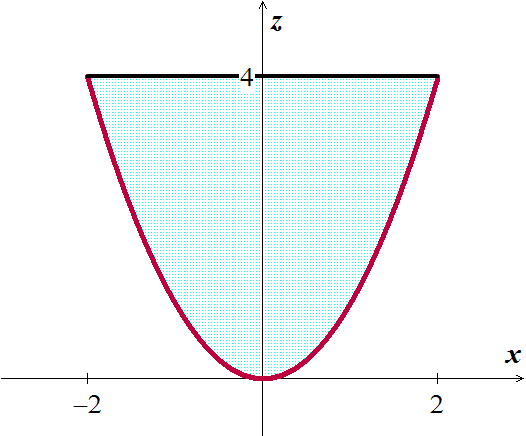
1. The normal vector point toward the *z-*axis on the curved surface of *S* and in the direction  on the flat surface of *S*.
2. 

The planar surface in the *xz-*plane, then let  be the surface parameterized by .

Where, since ,  and 





















Let  be the surface of the half of the paraboloid for , parametrized as



























1. 









































***Exercise***

The French Physicist André-Marie Ampère discovered that an electrical current *I* in a wire produces a magnetic field *B*. A special case of Ampère’s Law relates the current to the magnetic field through the equation , where *C* is any closed curve through which the wire passes and *μ* is a physical constant. Assume that the current *I* is given in terms of the current density ***J*** as , where *S* is an oriented surface with *C* as a boundary. Use Stokes’ Theorem to show that an equivalent form of Ampère’s Law is .

***Solution***





Thus 

For all surfaces *S* bounded by any given closed curve *C*.

It is clear that given the freedom to choose *C* and *S*, that it follows that the integrand is identically zero, i.e. that for any surface *S*, .

From this, it is easy to see that we must have , since we are free to make normal vector point in any direction at any given point by choosing *S* appropriately.

***Exercise***

Let S be the paraboloid , for , where *a* > 0 is a real number. Let . For what value(s) of *a* (if any) does  have its maximum value?

***Solution***



















∴ The integral is independent of *a*.

***Exercise***

The goal is to evaluate , where  and *S* ids the surface of the upper half of the ellipsoid 

1. Evaluate a surface integral over a more convenient surface to find the value of *A*.
2. Evaluate *A* using a line integral.

***Solution***

1. The boundary of this surface is the circle  at 





At 







1. With the parameterization of the boundary circle and , we have







***Solution*** ***Section* 4.8 – Divergence Theorem**

***Exercise***

Evaluate both integrals of the Divergence Theorem for the following vector field and region. Check for agreement. 

***Solution***













































***Exercise***

Evaluate both integrals of the Divergence Theorem for the following vector field and region. Check for agreement. 

***Solution***









Since the surface has a form of cube, therefore we have 6 surfaces











***Exercise***

Evaluate both integrals of the Divergence Theorem for the following vector field and region. Check for agreement. 

***Solution***









































***Exercise***

Evaluate both integrals of the Divergence Theorem for the following vector field and region. Check for agreement. 

***Solution***



























***Exercise***

Find the net outward flux of the field  across the sphere of radius 1 centered at the origin.

***Solution***



So by the Divergence Theorem, the net outward flux is zero since the volume integral of  is zero.

***Exercise***

Find the net outward flux of the field  across any smooth closed surface , where *a, b*, and *c* are constants.

***Solution***



So by the Divergence Theorem, the net outward flux is zero since the volume integral of  is zero.

***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *S*.

; *S* is the sphere 

***Solution***



The sphere has a radius , therefore the volume of the sphere is 







***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *S*.

; *S* is surface of the cube cut from the first octant by the planes *x* = 1, *y* = 1, and 

***Solution***













***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *S*.

; *S* is boundary of the tetrahedron in the first octant formed by the plane 

***Solution***



So by the Divergence Theorem, the net outward flux is 4 times the volume of the tetrahedron.

Volume of the tetrahedron = 







***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *S*.

; *S* is the sphere 

***Solution***



So by the Divergence Theorem, the net outward flux is

















***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *S*.

; *S* is the surface of the paraboloid , for , plus its base in the *xy-*plane

***Solution***



So by the Divergence Theorem, the net outward flux is











***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *S*.

; *S* is the surface of the cone , for , plus its top surface in the plane 

***Solution***



Volume of a cone 

So by the Divergence Theorem, the net outward flux is



***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *D*.

; *D* is the region between the spheres of radius 2 and 4 centered at origin.

***Solution***



Volume between 2 spheres 



***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *D*.

; *D* is the region between the spheres of radius 1 and 2 centered at origin.

***Solution***































***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *D*.

; *D* is the region between the spheres of radius 1 and 2 centered at origin.

***Solution***

























***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *D*.

;  is the region between two cubes

***Solution***



Therefore by the Divergence Theorem, the net outward flux is ***zero***.

***Exercise***

Use the Divergence Theorem to compute the net outward flux of the following field across the given surface *D*.

; *D* is the region between the cylinders  and  for 

***Solution***



|  |  |
| --- | --- |
| Volume of the sphere            ***Or*** | Volume of the sphere            ***Or*** |



Therefore, the net outward flux is 

***Exercise***

Compute the outward flux of the following vector field across the given surface; *S* is the boundary of the ellipsoid 

***Solution***







Therefore by the Divergence Theorem, the net outward flux is ***zero***.

***Exercise***

Compute the outward flux of the following vector field across the given surface ; *S* is the boundary of the ellipsoid 

***Solution***



Therefore by the Divergence Theorem, the net outward flux is ***zero***.

***Exercise***

Compute the outward flux of the following vector field across the given surface; *S* is the boundary of the region bounded by the planes , , , , and 

***Solution***

















***Exercise***

The electric field due to a point charge *Q* is , where  and  is a constant

1. Show that the flux of the field across a sphere of radius *a* centered at the origin is 
2. Let *S* be the boundary of the origin between two spheres centered of radius *a* and *b* with . Use the Divergence Theorem to show that the net outward flux across *S* is zero.
3. Suppose there is a distribution of charge within a region *D*. Let  be the charge density (charge per unit volume). Interpret the statement that



1. Assuming ***E*** satisfies the conditions of the Divergence Theorem, conclude from part (*c*) that 
2. Because the electric force is conservative, it has a potential function . From part (*d*) conclude that 

***Solution***

1. 



***Area*** of sphere 



1. The net outward flux across *S* is the difference of the fluxes across the inner and outer sphere; by part (*a*), these are equal (independent of the radius), so the net flux across *S* is zero.
2. The left-hand side is the flux across the boundary of *D*, while the right-hand side is the sum of the charge densities at each point of *D*.





The statement says that the flux across the boundary, up to multiplication by a constant, is the sum of the charge densities in the region.

1. 

This holds for all regions *D*.

Therefore; that implies that 

1. 

***Exercise***

***Fourier’s Law*** of heat transfer (or heat conduction) states that the heat flow vector ***F*** at a point is proportional to the negative gradient of the temperature that is, , which means that heat energy flows from hot regions to cold region. The constant  is called the *conductivity*, which has metric units of . A temperature function for a region *D* is given. Find the net outward heat flux  across the boundary *S* of *D*. In some cases it may be easier to use the Divergence Theorem and evaluate a triple integral. Assume .



***Solution***









Therefore, the heat flux is ***zero***.

***Exercise***

***Fourier’s Law*** of heat transfer (or heat conduction) states that the heat flow vector ***F*** at a point is proportional to the negative gradient of the temperature that is, , which means that heat energy flows from hot regions to cold region. The constant  is called the *conductivity*, which has metric units of . A temperature function for a region *D* is given.

Find the net outward heat flux  across the boundary *S* of *D*. In some cases it may be easier to use the Divergence Theorem and evaluate a triple integral. Assume .



***Solution***







Therefore, the heat flux is ***−*6** times the volume of the region (or ***−*6**).

***Exercise***

***Fourier’s Law*** of heat transfer (or heat conduction) states that the heat flow vector ***F*** at a point is proportional to the negative gradient of the temperature that is, , which means that heat energy flows from hot regions to cold region. The constant  is called the *conductivity*, which has metric units of . A temperature function for a region *D* is given.

Find the net outward heat flux  across the boundary *S* of *D*. In some cases it may be easier to use the Divergence Theorem and evaluate a triple integral. Assume .



***Solution***







Therefore, the heat flux is







***Exercise***

***Fourier’s Law*** of heat transfer (or heat conduction) states that the heat flow vector ***F*** at a point is proportional to the negative gradient of the temperature that is, , which means that heat energy flows from hot regions to cold region. The constant  is called the *conductivity*, which has metric units of . A temperature function for a region *D* is given.

Find the net outward heat flux  across the boundary *S* of *D*. In some cases it may be easier to use the Divergence Theorem and evaluate a triple integral. Assume .

 *D* is the sphere of radius *a* centered at the origin.

***Solution***



















