***Section* 5.5 – Infinite Sequences and Summation Notation**

An arbitrary ***infinite sequence*** may be denoted as follows:



An infinite sequence is a function whose domain is the set of positive integers.

***Example***

Find the first four terms and the tenth term of the sequence: 

***Solution***





***Example***

Find the first four terms and the tenth term of the sequence: 

***Solution***





***Example***

Find the first four terms and the tenth term of the sequence: 

***Solution***





***Example***

Find the first four terms and the tenth term of the sequence: 

***Solution***





***Example***

Find the first four terms of the recursively defined infinite sequence 

***Solution***









***Summation Notation***

To find the sum of many terms of an infinite sequence, it is easy to express using summation notation.



***Example***

Find the sum: 

***Solution***







**Theorem on the Sum of a *Constant***



***Proof***:



***Example***

Find the sum: 

***Solution***





***Theorem on Sums***

If  and  are infinite sequences, then for every positive integer ***n***,







***Proof***







***Example***

Express the sum using summation notation 

***Solution***



***Exercises*** ***Section* 5.5 – Infinite Sequences and Summation Notation**

(**1 – 13**) Find the first four terms and the eight term of the sequence:

|  |  |  |
| --- | --- | --- |
|  |  |  |

1. Graph the sequence 
2. Find the first four terms of the sequence of partial sums for the given sequence. 

(**16 – 27**) Find the first five terms of the recursively defined infinite sequence

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| --- | --- |
|  |  |

(**28 – 37**) Express each sum using summation notation

|  |  |
| --- | --- |
|  |  |

(**38 – 52**) Find the sum

|  |  |  |
| --- | --- | --- |
|  |  |  |

(**53 – 56**) Write out each sum

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| --- | --- | --- |
|  |  |  |

1. Fred has a balance of $3,000 on his card which charges 1% interest per month on any unpaid balance. Fred can afford to pay $100 toward the balance each month. His balance each month after making a $100 payment is given by the recursively defined sequence



Determine Fred’s balance after making the first payment. That is, determine 

1. A pond currently has 2,000 trout in it. A fish hatchery decides to add an additional 20 trout each month. Is it also known that the trout population is grwoing at a rate of 3% per month. The size of the population after *n* months is given bu the recursively defined sequence



How many trout are in the pond after 2 months? That is, what is ?

1. Fred bought a car by taking out a loan for $18,500 at 0.5% interest per month. Fred’s normal monthly payment is $434.47 per month, but he decides that he can afford to pay $100 extra toward the balance each month. His balance each month is given by the recursively defined sequence



Determine Fred’s balance after making the first payment. That is, determine 

1. The Environmental Protection Agency (EPA) determines that Maple Lake has 250 *tons* of pollutant as a result of industrial waste and that 10% of the pollutant present is neuttralized by solar oxidation every year. The EPA imposes new pollution control laws that result in 15 *tons* of new pollutant entering the lake each year. The amount of pollutant in the lake after *n* years is given by the recursively defined sequence



Determine the amount of pollutant in the lake after 2 years? That is, what is ?

1. Let 

Define the *n*th term of a sequence

1. Show that  and 
2. Show that 
3. Draw the conclusion that  is a Fibonacci sequence
4. Find the first ten terms of the sequence from part (*c*)