***Section* 5.7 – Mathematical Induction**

If *n* is a positive integer and we let  denote the mathematical statement , we obtained the following ***infinite sequence*** of statements:









***Principle* of Mathematical Induction**

If with each positive integer *n* there is associated a statement  then all the statements  are true, provided the following two conditions are satisfied.

1.  is true.
2. Whenever *k* is a positive integer such that  is true, then  is also true.

***Steps in Applying the Principle of Mathematical Induction***

1. Show that  is true.
2. Assume that  is true, and then prove that  is true.

***Example***

Use the mathematical induction to prove that for every positive integer *n*, the sum of the first *n* positive integers is:



***Solution***

1. For *n* = 1 ⇒ 

 ***√***

Hence  is true.

1. Assume that  is true.

Thus, the induction hypothesis is: 

For *k* + 1: 



 ***Induction hypothesis***



 ***Factor out* k + 1**

 ***√*** ***Change form of* k + 2**

This shows that  is also true.

**∴** By the mathematical induction, the proof is completed

***Example***

Prove that for every positive integer *n*,



***Solution***

1. For ***n* = 1** ⇒ 



 ***√*** hence  is true.

1. 

For *k* + 1:















**√**

This shows that  is also true.

**∴** By the mathematical induction, the proof is completed

***Example***

Prove that 2 is a factor of  for every positive integer *n*,

***Solution***

1. For ***n* = 1** ⇒ 



 **√**

Thus, 2 is a factor of  for *n* = 1; hence  is true.

1. 2 is a factor of  

is 2 a factor of ?









 **√**

Thus, 2 is a factor of the last expression; hence  is also true.

**∴** By the mathematical induction, the proof is completed

***Steps in Applying the Extended Principle of Mathematical Induction***

1. Show that  is true.
2. Assume that  is true with , and then prove that  is true.

***Example***

Let *a* be a nonzero real number such that *a* > −1. Prove that  for every integer *n* ≥ 2.

***Solution***

For ***n* = 1** ⇒  ⇒  is false.

***Step* 1**. For ***n* = 2** ⇒ 

 **√**

⇒  is true.

***Step* 2**. Assume that  is true 

We need to prove that  is true, that is 















 ***√***

Thus,  is also true.

**∴** By the mathematical induction, the proof is completed

***Exercises Section* 5.7 – Mathematical Induction**

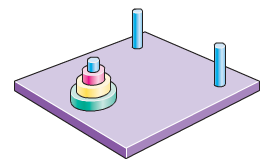
1. Find all positive integers *n* for which the given statement is not true

1. Prove that the statement is true for every positive integer *n*. 
2. Prove that the statement is true for every positive integer *n*. 
3. Prove that the statement is true for every positive integer *n*. 

(**5 – 35**) Prove that the statement is true by the mathematical induction

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 
17. 
18. 
19. 
20. 
21. 
22. For every positive integer *n*. 
23. For every positive integer *n*. 3 is a factor of 
24. For every positive integer *n*. 4 is a factor of 
25.  (*a* and *m* are constant)
26. 
27. If , then 
28. If , then 
29. 
30. 
31. 
32. A pile of *n* rings, each smaller than the one below it, is on a peg on board. Two other pegs are attached to the board. In the game called the Tower of Hanoi puzzle, all the rings must moved one at a time, to a different peg with no ring ever placed on top of a smaller ring.



Find the least number of moves that would be required.

Prove your result by mathematical induction.