***Section* 5.7 – Mathematical Induction**

If *n* is a positive integer and we let  denote the mathematical statement , we obtained the following ***infinite sequence*** of statements:









***Principle* of Mathematical Induction**

If with each positive integer *n* there is associated a statement  then all the statements  are true, provided the following two conditions are satisfied.

1.  is true.
2. Whenever *k* is a positive integer such that  is true, then  is also true.

***Steps in Applying the Principle of Mathematical Induction***

1. Show that  is true.
2. Assume that  is true, and then prove that  is true.

***Example***

Use the mathematical induction to prove that for every positive integer *n*, the sum of the first *n* positive integers is:



***Solution***

1. For *n* = 1 ⇒ 

 ***√***

Hence  is true.

1. Assume that  is true.

Thus, the induction hypothesis is: 

For *k* + 1: 



 ***Induction hypothesis***



 ***Factor out* k + 1**

 ***√*** ***Change form of* k + 2**

This shows that  is also true.

**∴** By the mathematical induction, the proof is completed

***Example***

Prove that for every positive integer *n*,



***Solution***

1. For ***n* = 1** ⇒ 



 ***√*** hence  is true.

1. 

For *k* + 1:















**√**

This shows that  is also true.

**∴** By the mathematical induction, the proof is completed

***Example***

Prove that 2 is a factor of  for every positive integer *n*,

***Solution***

1. For ***n* = 1** ⇒ 



 **√**

Thus, 2 is a factor of  for *n* = 1; hence  is true.

1. 2 is a factor of  

is 2 a factor of ?









 **√**

Thus, 2 is a factor of the last expression; hence  is also true.

**∴** By the mathematical induction, the proof is completed

***Steps in Applying the Extended Principle of Mathematical Induction***

1. Show that  is true.
2. Assume that  is true with , and then prove that  is true.

***Example***

Let *a* be a nonzero real number such that *a* > −1. Prove that  for every integer *n* ≥ 2.

***Solution***

For ***n* = 1** ⇒  ⇒  is false.

***Step* 1**. For ***n* = 2** ⇒ 

 **√**

⇒  is true.

***Step* 2**. Assume that  is true 

We need to prove that  is true, that is 















 **√**

Thus,  is also true.

**∴** By the mathematical induction, the proof is completed

***Exercises Section* 5.7 – Mathematical Induction**

1. Find all positive integers *n* for which the given statement is not true

   

1. Prove that the statement is true for every positive integer *n*. 
2. Prove that the statement is true for every positive integer *n*. 
3. Prove that the statement is true for every positive integer *n*. 
4. Prove that the statement is true: 
5. Prove that the statement is true: 
6. Prove that the statement is true: 
7. Prove that the statement is true: 
8. Prove that the statement is true: 
9. Prove that the statement is true: 
10. Prove that the statement is true: 
11. Prove that the statement is true: 
12. Prove that the statement is true: 
13. Prove that the statement is true: 
14. Prove that the statement is true: 
15. Prove that the statement is true: 
16. Prove that the statement is true: 
17. Prove that the statement is true: 
18. Prove that the statement is true: 
19. Prove that the statement is true for every positive integer *n*. 
20. Prove that the statement is true for every positive integer *n*. 3 is a factor of 
21. Prove that the statement is true for every positive integer *n*. 4 is a factor of 
22. Prove that the statement by mathematical induction:  (*a* and *m* are constant)
23. Prove that the statement by mathematical induction: 
24. Prove that the statement by mathematical induction: If , then 
25. Prove that the statement by mathematical induction: If , then 
26. Prove that the statement by mathematical induction: 
27. Prove that the statement by mathematical induction: 
28. Prove that the statement by mathematical induction: 
29. A pile of *n* rings, each smaller than the one below it, is on a peg on board. Two other pegs are attached to the board. In the game called the Tower of Hanoi puzzle, all the rings must moved one at a time, to a different peg with no ring ever placed on top of a smaller ring. Find the least number of moves that would be required. Prove your result by mathematical induction.

