***Solution Section* 5.7 – Mathematical Induction**

***Exercise***

Find all positive integers *n* for which the given statement is not true

***Solution***

1. 





The statement is true for all 

The statement is not true for 

1. 



The statement is true for all 

The statement is not true for 

1. 









The statement is true for all 

The statement is not true for 

1. 







The statement is true for all 

The statement is not true for 

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

1. For ***n* = 1** ⇒ ; hence  is true.
2. Assume  is true





 ***Factor* (k + 1)**



 ***√*** Hence  is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

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Prove that the statement is true: 

***Solution***

1. For ***n* = 1** ⇒  ***√*** ;  is true.
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1. For ***n* = 1** ⇒  ***√*** ;  is true.
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Prove that the statement is true: 

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 is also true.

∴ By the mathematical induction, the proof is completed.

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Prove that the statement is true: 

***Solution***

1. For ***n* = 1** ⇒  ***√*** ;  is true.
2.  is true













 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

1. For ***n* = 1** ⇒  ***√*** ;  is true.
2.  is true **→** Is 







 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true: 

***Solution***

1. For ***n* = 1** ⇒ 



 ***√*** ;  is true.

1.  is true









 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true: 

***Solution***

1. For ***n* = 1** ⇒  ***√*** ;  is true.
2.  is true











 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true: 

***Solution***

1. For ***n* = 1** ⇒  ***√*** ;  is true.
2.  is true











 ***√***

 is also true.

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Prove that the statement is true: 

***Solution***

1. For ***n* = 1** ⇒  ***√***;  is true.
2.  is true









 ***√***

 is also true.

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Prove that the statement is true: 

***Solution***

1. For ***n* = 1** ⇒  ***√***;  is true.
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1. For ***n* = 1** ⇒  ***√*** ;  is true.
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 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true: 

***Solution***

1. For ***n* = 1** ⇒  ***√*** ;  is true.
2.  is true











 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement by mathematical induction:  (*a* and *m* are constant)

***Solution***

* For ***n* = 1** ⇒  ***√*** ;  is true.
*  is true









 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

***Step* 1**. For ***n* = 1** ⇒  ***√*** ⇒  is true.

***Step* 2**. Assume that  is true 

We need to prove that  is true, that is 





 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true for every positive integer *n*. 3 is a factor of 

***Solution***

* For ***n* = 1** ⇒  ***√*** ⇒  is true.
* Assume that  is true 3 is a factor of 

We need to prove that  is true, that is 







 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true for every positive integer *n*. 4 is a factor of 

***Solution***

* For ***n* = 1** ⇒  ***√*** ⇒  is true.
* Assume that  is true 4 is a factor of 

We need to prove that  is true, that is 







By the induction hypothesis, 4 is a factor of  and 4 is a factor of 4, so 4 is a factor of the  term. ***√***

Thus,  is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement by mathematical induction: 

***Solution***

* For ***n* = 3** ⇒ ***√*** ⇒  is true.
* Assume that  is true: ;

we need to prove that :  is true







***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement by mathematical induction: If , then 

***Solution***

* For ***n* = 1** ⇒ ***√*** since ****** ⇒  is true.
* Assume that  is true: ;

We need to prove that :  is true



 ***√***

 is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement by mathematical induction: If , then 

***Solution***

* For ***n* = 4** ⇒ ***√***  ⇒  is true.
* Assume that  is true: ;

We need to prove that :  is true





 ***√***

Thus,  is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement by mathematical induction: 

***Solution***

* For ***n* = 2** ⇒ ***√***  ⇒  is true.
* Assume that  is true: 

We need to prove that :  is true







 ***√***

Thus,  is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement by mathematical induction: 

***Solution***

* For ***n* = 5**  ⇒ ***√***  ⇒  is true.
* Assume that  is true: 

We need to prove that :  is true



 ***√***

Thus,  is also true

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement by mathematical induction: 

***Solution***

* For ***n* = 5**  ⇒ ***√***  ⇒  is true.
* Assume that  is true: 

We need to prove that :  is true



 ***√***

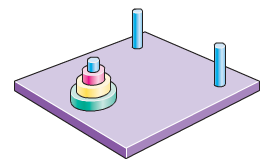
Thus,  is also true

∴ By the mathematical induction, the proof is completed.

***Exercise***

A pile of *n* rings, each smaller than the one below it, is on a peg on board. Two other pegs are attached to the board. In the game called the Tower of Hanoi puzzle, all the rings must moved, one at a time, to a different peg with no ring ever placed on top of a smaller ring. Find the least number of moves that would be required. Prove your result by mathematical induction.

***Solution***

With 1 ring, 1 move is required.

With 2 rings, 3 moves are required 

With 3 rings, 7 moves are required 

With *n* rings,  moves are required

* For ***n* = 1**  ⇒ ***√***  ⇒  is true.
* Assume that  is true: 







 ***√***

∴ By the mathematical induction, the proof is completed.