***Solution Section* 5.7 – Mathematical Induction**

***Exercise***

Find all positive integers *n* for which the given statement is not true

***Solution***

1. 





The statement is true for all 

The statement is not true for 

1. 



The statement is true for all 

The statement is not true for 

1. 









The statement is true for all 

The statement is not true for 

1. 







The statement is true for all 

The statement is not true for 

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

1. For ***n* = 1** ⇒ ; hence  is true.
2. Assume  is true





 ***Factor* (k + 1)**



 ***√*** Hence  is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

1. For ***n* = 1** ⇒ ; hence  is true.
2. Assume  is true









***√*** Hence  is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

1. For ***n* = 1** ⇒ ; hence  is true.
2. Assume  is true















 ***√***  is also true.

∴ By the mathematical induction, the proof is completed.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

For ***n* = 1**



Hence  is true.

 is true













 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

For ***n* = 1**

***√***

Hence  is true.

 is true

















 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

For ***n* = 1**

***√***

Hence  is true.

 is true













 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true













 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true









 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence, is true.

 is true









 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true













 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true









 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**





 ***√***

Hence,  is true.

 is true









 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true











 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true









 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true









 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true









 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true







 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true: 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

 is true











 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true by mathematical induction:



***Solution***

For ***n* = 1**



 ***√***

Hence,  is true.

For ***k***: 





 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true by mathematical induction:



***Solution***

For ***n* = 1**





 ***√***

Hence,  is true.

For ***k***: 

Is : 















 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true by mathematical induction: 

***Solution***

For ***n* = 1**



 ***√***

Hence,  is true.

For ***k***: 

Is : 











 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true by mathematical induction:



***Solution***

For ***n* = 1**

***√***

Hence,  is true.

For ***k***: 

Is : 











 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true by mathematical induction: 

***Solution***

For ***n* = 1**





 *Square both sides*

 ***√***

Hence,  is true.

For ***k***: 



Is : 







 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true by mathematical induction: 

***Solution***

For ***n* = 2**



 ***√***

Hence,  is true.

For ***k***: 

Is : 











 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true by mathematical induction:  (*a* and *m* are constant)

***Solution***

For ***n* = 1**



 ***√***

Hence,  is true.

 is true



****

****

****

** *√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true for every positive integer *n*. 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

Assume that  is true 

We need to prove that  is true, that is 





 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true for every positive integer *n*. 3 is a factor of 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

Assume that  is true 3 is a factor of 

We need to prove that  is true, that is 





 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement is true for every positive integer *n*. 4 is a factor of 

***Solution***

For ***n* = 1**

 ***√***

Hence,  is true.

Assume that  is true 4 is a factor of 

We need to prove that  is true, that is 







By the induction hypothesis, 4 is a factor of  and 4 is a factor of 4, so 4 is a factor of the  term. ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement by mathematical induction: 

***Solution***

For ***n* = 3**



 ***√***

Hence,  is true.

Assume that  is true: 

We need to prove that :  is true







***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement by mathematical induction: If , then 

***Solution***

For ***n* = 1**



 ***√***

since ****** ⇒  is true.

Assume that  is true: 

We need to prove that :  is true



 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement by mathematical induction: If , then 

***Solution***

For ***n* = 4**



***√***

Hence,  is true.

Assume that  is true: 

We need to prove that :  is true





 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement by mathematical induction: 

***Solution***

For ***n* = 2**



***√***

Hence,  is true.

Assume that  is true: ;

We need to prove that :  is true







 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement by mathematical induction: 

***Solution***

For ***n* = 5**



 ***√***

Hence,  is true.

Assume that  is true: 

We need to prove that :  is true









 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

Prove that the statement by mathematical induction: 

***Solution***

For ***n* = 5**



 ***√***

Hence,  is true.

Assume that  is true: 

We need to prove that :  is true











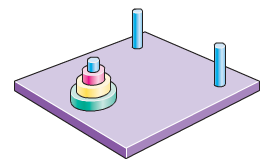


 ***√***

Hence  is true.

∴ By the mathematical induction, the given statement is true.

***Exercise***

A pile of *n* rings, each smaller than the one below it, is on a peg on board. Two other pegs are attached to the board. In the game called the Tower of Hanoi puzzle, all the rings must be moved, one at a time, to a different peg with no ring ever placed on top of a smaller ring.

Find the least number of moves that would be required.

Prove your result by mathematical induction.

***Solution***

With 1 ring, 1 move is required.

With 2 rings, 3 moves are required 

With 3 rings, 7 moves are required 

With *n* rings,  moves are required

For ***n* = 1**

***√***

Hence,  is true.

Assume that  is true: 







 ***√***

∴ By the mathematical induction, the given statement is true.