***Chapter Two***

***Solution Section 2.1* – Differential Equations & Solutions**

***Exercise***

Show that  is a solution of the 1st order equation  

***Solution***

***Exercise***

Show that  is a solution of the 1st order equation 

***Solution***









***Exercise***

A general solution may fail to produce all solutions of a differential equation. In exercise 6, show that  is a solution of the differential equation, but no value of C in the given general solution will produce this solution.

***Solution***





***Exercise***

Use the given general solution to find a solution of the differential equation having the given initial condition.

***Solution***

















The interval of existence is 

***Solution Section 2.2* – Solutions to Separable Equations**

***Exercise***

Find the general solution of the differential equation 

*Solution*















Where 

***Exercise***

Find the general solution of the differential equation 

*Solution*

















***Solution Section 2.3* – Models of Motions**

***Exercise***

A stone is released from rest and dropped into a deep well. Eight seconds later, the sound of the stone splashing into the water at the bottom of the well returns to the ear of the person who released the stone. How long does it take the stone to drop to the bottom of the well? How deep is the well? Ignore air resistance. The speed of sound is 340 *m/s*.

*Solution*





















***Exercise***

A rocket is fired vertically and ascends with constant acceleration  for 1.0 *min*. At that point, the rocket motor shuts off and the rocket continues upward under the influence of gravity. Find the maximum altitude acquired by the rocket and the total time elapsed from the take-off until the rocket returns to the earth. *Ignore air resistance*.

*Solution*



















The velocity will reduced: 



The altitude: 





Back to the ground: 



Total time: 

***Exercise***

A ball having mass  falls from rest under the influence of gravity in a medium that provides a resistance that is proportional to its velocity. For a velocity of the force due to the resistance.

*Solution*

The resistance force: 





The terminal velocity: 





***Solution Section* 2.4– Linear Equations**

***Exercise***

Find the general solution of 

*Solution*















***Exercise***

Find the general solution of 

*Solution*















***Exercise***

Find the general solution of 

*Solution*























***Exercise***

Find the general solution of  

*Solution*































***Exercise***

Solve  

*Solution*



















*Integration by part*























***Exercise***

Find the general solution of the given differential equation. Then find the particular solution satisfying the given initial condition of



*Solution*























***Solution Section* 2.5– Mixing Problems**

***Exercise***

Consider two tanks, label tank A and tank B for reference. Tank A contains 100 *gal* of solution in which is dissolved 20 *lb* of salt. Tank B contains 200 *gal* of solution which is dissolved 40 *lb* of salt. Pure water flows into the tank A at rate of 5 *gal/s*. There is a drain at the bottom of tank A. The solution leaves tank A via the drain at a rate of 5 *gal/s* and flows immediately into tank B at the same rate. A drain at the bottom of tank B allows the solution to leave tank B at a rate of 2.5 *gal/s*. What is the salt content in tank B at the precise moment that tank B contains 250 *gal* of solution?

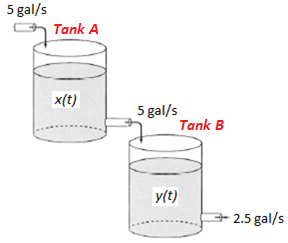
*Solution*

Tank ***A*** contains 100 *gal* of solution in which is dissolved 20 *lb* of salt

*Volume*: 

Concentration at time *t*: 

Rate out = Volume Rate *x* Concentration























Tank ***B*** contains 200 *gal* of solution which is dissolved 40 *lb* of salt.

*Volume*: 

































































***Exercise***

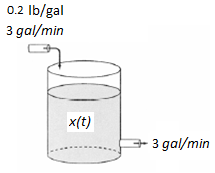
A tank contains 100 *gal* of pure water. At time zero, a sugar-water solution containing 0.2 *lb* of sugar per gal enters the tank at a rate of 3 *gal/min*. Simultaneously, a drain is opened at the bottom of the tank allowing the sugar solution to leave the tank at 3 *gal/min*. Assume that the solution in the tank is kept perfectly mixed at all times.

*Solution*

1. What will be the sugar content in the tank after 20 minutes?





















1. How long will it take the sugar content in the tank to reach 15 lb?













1. What will be the eventual sugar content in the tank?

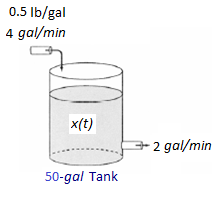




***Exercise***

A 50-gal tank initially contains 20 *gal* of pure water. Salt-water solution containing 0.5 *lb* of salt for each gallon of water begins entering the tank at a rate of 4 *gal/min*. Simultaneously, a drain is opened at the bottom of the tank, allowing the salt-water solution to leave the tank at a rate of 2 *gal/min*. What is the salt content (*lb*) in the tank at the precise moment that the tank is full of salt-water solution?

*Solution*



































Full tank: 









***Solution Section* 2.6 - Existence and Uniqueness of Solutions**

***Exercise***

Which of the initial value problems are guaranteed a unique solution. 

*Solution*

→ *f* is continuous

 is also continuous on the whole plane.

Hence the hypotheses are satisfied and guarantees a unique solution.

***Exercise***

Which of the initial value problems are guaranteed a unique solution. 

*Solution*





Initial condition: 

Both  are not continuous in the rectangle containing 

Hence the hypotheses are not satisfied.

***Exercise***

Show that and  are both solutions of the initial value problem , where . Explain why this fact doesn't contradict Theorem

*Solution*



whichis not continuous at 

***Exercise***

Use a numerical solver to sketch the solution of the given initial value problem



1. Where does your solver experience difficulty? why? Use the image of your solution to estimate the interval of existence.



1. Find an explicit solution; then use your formula to determine the interval of existence. How does it compare with the approximation found in part (*i*).

***Solution Section* 2.7 - Autonomous Equations and Stability**

***Exercise***

The graph of the right-hand side  is shown. Identify the equilibrium points and sketch the equilibrium solutions in the *ty*-plane. Classify each equilibrium point as either unstable or asymptotically stable.

*Solution*



The equilibrium points are:  and both are unstable

***Exercise***

The graph of the right-hand side  is shown. Identify the equilibrium points and sketch the equilibrium solutions in the *ty*-plane. Classify each equilibrium point as either unstable or asymptotically stable.

*Solution*



The equilibrium points are: 

 are asymptotically stable

 are unstable

***Exercise***



*Solution*

***i***) *Sketch a graph of *



***ii***) *Use the graph of f to develop a phase line for the autonomous equation. Classify each equilibrium point as either unstable or asymptotically stable.*

-1 4

***iii***) *Sketch the equilibrium solutions in the ty-plane into regions. Sketch at least one solution trajectory in each of these regions.*

 is asymptotically stable and  is unstable

***Exercise***

Determine the stability of the equilibrium solutions 

*Solution*





The equilibrium points 



  is unstable

  is asymptotically stable

***Solution Section* 2.8 - Modeling Population Growth**

***Exercise***

A biologist starts with 100 cells in a culture. After 24 *hrs,* he counts 300. Assuming a Malthusian model, what the reproductive rate? What will be the number of cells of the end of 5 days?

*Solution*



 *24 hrs = 1 day P = 300*













***Exercise***

Consider a lake that is stocked with walleye pike and that the population of pike is governed by the logistic equation



where time is measured in days and *P* in thousands of fish. Suppose that fishing is started in this lake and that 100 fish are removed each day.

*Solution*

1. Modify the logistic model to account for the fishing.

The modified logistic model







1. Find and classify the equilibrium points for your model.

 *Multiply 100 each term*



 *Solve for P*



 Asymptotically stable

 Unstable

1. Use qualitative analysis to completely discuss the fate of the fish population with this model. In particular, if the initial fish population is 1000, what happens to the fish as time passes? what will happen to an initial population having 2000 fish?

For the 1000 (= 1) population, the population decreases until it dies out (doomed);

For the 2000 (= 2) population, the population tend towards the equilibrium 



