***Chapter* 7– Series Solution to Differential Equations**

***Section* 7.1 – Power Series**

**Definition**

A ***power series*** *about the point*  is a series of the form



The series is said to converge at  if the sequence of partial sums





Converges as . The sum of the series at the point  is defined to be the limit at the partial sums,



***Example***

Show that the geometric series  converges for  and that



Show that the series diverges for .

*Solution*

The partial sums  can be evaluates as follows.









If , then 

If , then  diverges and therefore the  diverges

If , then 

**Interval of convergence**

***Theorem***

For any power series  there is an ***R***, either a nonnegative number or ∞, such that the series converges if  and diverges if 

**The ratio Test**

***Theorem***

Suppose the terms of the series  have the property that



exists. If  the series converges, while if  the series diverges

***Example***

Find the radius of convergence for the series. 

*Solution*











By the ratio test, the series converges if , so the radius of convergence is 



**Algebraic Operations on Series**

The ***sum*** and ***difference*** of two series

 



 

***Differentiating Power Series***

**Theorem**

The function 

Can be differentiating the series by terms









***Identity* Theorem**

Suppose that the series  has a positive radius of convergence. Then



The coefficients of a power series are determined by the values of the sum .

***Integrating Power Series***

**Theorem**

Suppose the power series  converges for 



***Section* 7.2 – Series Solutions near Ordinary Points**

***Example of a First-Order Equation***

Find the series solution for the differential equation



*Solution*

We look for a solution of the form: 









 













By the identity theorem: 

 

 

 









***Example***

Find the general series solution to the equation



Find the particular solution with  and 

*Solution*























 

 

 

 

The general solution can be written as:



For the given initial  and , the solution is:



***Exercises*** ***Section* 7.2 – Series Solutions near Ordinary Points**

Find a power series solution.

1. 
2. 
3. 
4. 
5. 

Solve the initial value problems

1. 
2. 

***Section* 7.3 – Legendre’s Equation**