***Solution*** ***Section* 4.2 – Series Solutions near Ordinary Points**

***Exercise***

Find a power series solution. 

***Solution***

The equation  is separable with solution













With 













 ***√***

***Exercise***

Find a power series solution. 

***Solution***









With 























 ***√***

***Exercise***

Find a power series solution. 

***Solution***















































 ***√***

***Exercise***

Find a power series solution. 

***Solution***































If we set , then























 ***√***

***Exercise***

Find a power series solution. 

***Solution***



























 ***√***

***Exercise***

Find a power series solution. 

***Solution***

The equation  has a characteristic equation 

∴ The general solution: 

With 











































Which are identical.

***Exercise***

Find a power series solution. 

***Solution***

The equation  has a characteristic equation 

∴ The general solution: 

With 























 ***√***

***Exercise***

Find a power series solution. 

***Solution***



























***Exercise***

Find a power series solution. 

***Solution***



















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***Exercise***

Find a power series solution. 

***Solution***

















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***Exercise***

Find a power series solution. 

***Solution***

















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***Exercise***

Find a power series solution. 

***Solution***

















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***Exercise***

Find a power series solution. 

***Solution***



















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***Exercise***

Find a power series solution. 

***Solution***



















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***Exercise***

Find a power series solution. 

***Solution***

























***Exercise***

Find a power series solution. 

***Solution***



























***Exercise***

Find a power series solution. 

***Solution***





























***Exercise***

Find a power series solution. 

***Solution***

















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***Exercise***

Find a power series solution. 

***Solution***

























***Exercise***

Find a power series solution. 

***Solution***























***Exercise***

Find a power series solution. 

***Solution***























***Exercise***

Find the series solution to the initial value problem 

***Solution***





















***Exercise***

Find the series solution to the initial value problem 

***Solution***





















***Exercise***

Find the series solution to the initial value problem 

***Solution***





















***Exercise***

Find the series solution to the initial value problem 

***Solution***



















***Exercise***

Find the series solution to the initial value problem 

***Solution***















***Given***: 

******











***Exercise***

Find the series solution to the initial value problem



***Solution***

















***Given***: 

******





***Exercise***

Find the series solution to the initial value problem 

***Solution***















***Given***: 







***Exercise***

Find the series solution to the initial value problem 

***Solution***















***Given***: 







***Solution Section* 4.3 – Legendre’s Equation**

***Exercise***

Establish the recursion formula using the following two steps

1. Differentiate both sides of equation

 with respect to *t* to show that



1. Equate the coefficients of  in this equation to show that

 ***and*** 

***Solution***

1. Let: 

Differentiate both sides with respect to *t*: 



 ***Multiply both sides by: ***



1. 







Thus,



Therefore;





That implies:









If *n* = 1 then:   ***√***

***Exercise***

Show that 

***Solution***

Given the formula:



By letting , then the formula can be:



Replacing *n* with 2*n*, then























With 



***Exercise***

Show that 

***Hint***: Use Legendre’s equation 

***Solution***

Because  is a solution of Legendre’s equation, then



Let , then





Let , then





However, 



***Exercise***

The differential equation  is called ***Airy’s equation***, and its solutions are called ***Airy functions***. Find the series for the solutions  and  where  and , while  and . What is the radius of convergence for these two series?

***Solution***

Let   





 ***Shifting the index to get a common power of x***









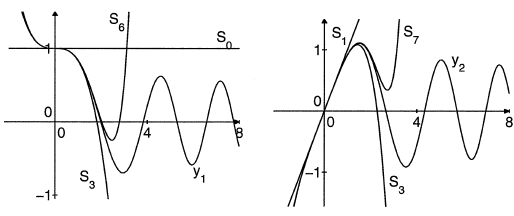
  









***Solution Section* 4.4 – Solution about Singular Points**

***Exercise***

Find the Frobenius series solutions of 

***Solution***

 ***Divide each term by*** 

Therefore,  is a regular singular point, and that 

 are polynomials.

The Frobenius series will converge for all . The indicial equation is



So the roots are .

The two possible Frobenius series solutions are then of the forms

















For 

 ***√***

For 

 Therefore 









***Exercise***

Find the general solution to the equation 

***Solution***

























***Exercise***

Find a Frobenius solution of Bessel’s equation of order zero 

***Solution***



Therefore,  is a regular singular point, and that  and .

The indicial equation is: 

There is only one Frobenius series solution: 

























The choice  gives us the Bessel function of order zero of the first kind.

