***Solution Section* 2.1 – Simple and Compound Interest**

***Exercise***

If you want to earn an annual rate of 10% on your investments, how much should you pay for a note that will be worth $5,000 in 6 month?

***Solution***







 **5000 / (1 + .1 / 2)**

***Exercise***

1. How much should you deposit initially in an account paying 10% compounded semiannually in order to have $1,000,000 in 30 years?
2. Compounded monthly?
3. Compounded daily?

***Solution***



***a***) 



***b***) 

***c***) 

***Exercise***

You have $7,000 toward the purchase of a $10,000 automobile. How long will it take the $7000 to grow to the $10,000 if it is invested at 9% compounded quarterly? (Round up to the next highest quarter if not exact.)

***Solution***





 ***Log Power Property: lnax= xlna***





***Exercise***

How long, to the nearest tenth of a year, will it take $1000 to grow to $3600 at 8% annual interest compounded quarterly?

***Solution***















***Exercise***

Jennifer invested $4,000 in her savings account for 4 years. When she withdrew it, she had $4,350.52. Interest was compounded continuously. What was the interest rate on the account? Round to the nearest tenth of a percent.

***Solution***







 ***Inverse Property:*** 







***Exercise***

An actuary for a pension fund need to have $14.6 million grow to $22 million in 6 years. What interest rate compounded annually does he need for this investment to growth as specified. Round your answer to the nearest hundredth of a percent.

***Solution***















***Exercise***

Which is the better investment: 9% compounded monthly or 9.1% compounded quarterly?

***Solution***

For 9%: 

For 9.1%: 

**9.1% is better**

***Exercise***

Sun Kang borrowed $5200 from his friend to pay for remodeling work on his house. He repaid the loan 10 months later with simple interest at 7%. His friend then invested the proceeds in a 5-year CD paying 6.3% compounded quarterly. How much will his friend have at the end of the 5 years?

***Solution***

For 7%: 



For 6.3%: 

***Exercise***

The consumption of electricity has increased historically at 6% per year. If it continues to increase at this rate indefinitely, find the number of years before the electric utilities will need to double their generating capacity. {Round up to the next highest year}

***Solution***



 ***ln both sides***







***Exercise***

In the New Testament, Jesus commends a widow who contributed 2 mites (roughly ¼ cent) to the temple treasury. Suppose the temple invested those mites at 4% compounded quarterly. How much would the money be worth 2000 years later?

***Solution***

***Given***: 





 



***Exercise***

If $1,000 is invested in an account that earns 9.75% compounded annually for 6 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

***Solution***

***Given***: 



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1st year:    2nd year:    3rd year:  4th year:  5th year:  6th year: | |  |  |  | | --- | --- | --- | | ***Period*** | ***Amount*** | ***Interest*** | | **0** | $1,000.00 |  | | **1** | $1,097.50 | $97.50 | | **2** | $1,204.51 | $107.01 | | **3** | $1,321.95 | $117.44 | | **4** | $1,450.84 | $128.89 | | **5** | $1,582.29 | $141.46 | | **6** | $1,747.54 | $155.25 | |

***Exercise***

If $2,000 is invested in an account that earns 8.25% compounded annually for 5 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

***Solution***

***Given***: 



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1st year:    2nd year:    3rd year:  4th year:  5th year: | |  |  |  | | --- | --- | --- | | ***Period*** | ***Amount*** | ***Interest*** | | **0** | $2,000.00 |  | | **1** | $2,165.00 | $165.00 | | **2** | $2,343.61 | $178.61 | | **3** | $2,536.96 | $193.35 | | **4** | $2,746.26 | $209.30 | | **5** | $2,972.83 | $226.57 | |

***Exercise***

If an investment company pays 6% compounded semiannually, how much you should deposit now to have $10,000

1. 5 years from now?
2. 10 years from now?

***Solution***

***Given***: 

1. 







1. 





***Exercise***

If an investment company pays 8% compounded quarterly, how much you should deposit now to have $6,000

1. 3 years from now?
2. 6 years from now?

***Solution***

***Given***: 

1. 





 

1. 







***Exercise***

What is the annual percentage yield (*APY*) for money invested at:

1. 4.5% compounded monthly?
2. 5.8% compounded quarterly?

***Solution***

1. ***Given***: 

 



1. ***Given***: 

 



***Exercise***

48.

What is the annual percentage yield (APY) for money invested at

1. 6.2% compounded semiannually?
2. 7.1% compounded monthly?

***Solution***

1. ***Given***: 

 



1. ***Given***: 

 



***Exercise***

A newborn child receives a $20,000 gift toward a college education from her grandparents. How much will the $20,000 be worth in 17 years if it is invested at 7% compounded quarterly?

***Solution***

***Given***: 



 



***Exercise***

A person with $14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy more expensive car. How much will be available for the purchase of a car at the end of 3 years?

***Solution***

***Given***: 



 



***Exercise***

You borrowed $7200 from a bank to buy a car. You repaid the bank after 9 months at an annual interest rate of 6.2%. Find the total amount you repaid. How much of this amount is interest?

***Solution***

***Given***: 



 



***Exercise***

An account for a corporation forgot to pay the firm’s income tax of $321,812.85 on time. The government changed a penalty based on an annual interest rate of 13.4% for the 29 days the money was late. Find the total amount (tax and penalty) that was paid. (Use 365 days a year.)

***Solution***

***Given***: 



 



***Exercise***

A bond with a face value of $10,000 in 10 years can be purchased now for $5,988.02. What is the simple interest rate?

***Solution***

The interest earned is: $10,000 − $5,988.02 = $4011.98

***Given***: 





 

The interest rate was about 

***Exercise***

A stock that sold for $22 at the beginning of the year was selling for $24 at the end of the year. If the stock paid a dividend of $0.50 per share, what is the simple interest rate on an investment in this stock?

***Solution***

The interest earned is: 

***Given***: 







The interest rate was about 

***Exercise***

The Frank Russell Company is an investment fund that tracks the average performance of various groups of stocks. On average, a $10,000 investment in midcap growth funds over a recent 10-year period would have grown to $63,000. What annual nominal rate would produce the same growth if

1. Annually
2. Continuously

***Solution***

1. Annually: *m* = 1











1. Continuously : 





 





***Solution Section* 2.2 – Future Value of an Annuity**

***Exercise***

Recently, Guaranty Income Life offered an annuity that pays 6.65% compounded monthly. If $500 is deposited into this annuity every month, how much is in the account after 10 years? How much of this is interest?

***Solution***

***Given***: 









Total deposits: 







***Exercise***

Recently, USG Annuity Life offered an annuity that pays 4.25% compounded monthly. If $1,000 is deposited into this annuity every month, how much is in the account after 15 years? How much of this is interest?

***Solution***

***Given***: 









Total deposits: 







***Exercise***

In order to accumulate enough money for a down payment on a house, a couple deposits $300 per month into an account paying 6% compounded monthly. If payments are made at the end of each period, how much money will be in the account in 5 years?

***Solution***

***Given***: 









***Exercise***

A self-employed person has a Keogh retirement plan. (This type of plan is free of taxes until money is withdrawn.) If deposits of $7,500 are made each year into an account paying 8% compounded annually, how much will be in the account after 20 years?

***Solution***

***Given***: 









***Exercise***

Sun America recently offered an annuity that pays 6.35% compounded monthly. What equal monthly deposit should be made into this annuity in order to have $200,000 in 15 years?

***Solution***

***Given***: 





 



***Exercise***

Recently, The Hartford offered an annuity that pays 5.5% compounded monthly. What equal monthly deposit should be made into this annuity in order to have $100,000 in 10 years?

***Solution***

***Given***: 





 



***Exercise***

Compu-bank, an online banking service, offered a money market account with an APY of 4.86%.

1. If interest is compounded monthly, what is the equivalent annual nominal rate?
2. If you wish to have $10,000 in the account after 4 years, what equal deposit should you make each month?

***Solution***

***Given***: 



1. 

 ***Add*1 *on both sides***







 



The equivalent annual nominal rate 

1. ***Given***: 





 



***Exercise***

American Express’s online banking division offered a money market account with an APY of 5.65%.

1. If interest is compounded monthly, what is the equivalent annual nominal rate?
2. If you wish to have $1,000,000 in the account after 8 years, what equal deposit should you make each month?

***Solution***

***Given***: 



1. 

 ***Add*1 *on both sides***







 



The equivalent annual nominal rate 

1. ***Given***: 





 



***Exercise***

Find the future value of an annuity due if payments of $500 are made at the beginning of each quarter for 7 years, in an account paying 6% compounded quarterly.

***Solution***

***Given***: 



***Since you put money at the beginning of each month, we need to add the first payment.***



 



***Exercise***

A 45 year-old man puts $2500 in a retirement account at the end of each quarter until he reaches the age of 60, then makes no further deposits. If the account pays 6% interest compounded quarterly, how much will be in the account when the man retires at age 65?

***Solution***

For the 15 years :







 



For the remaining 5 years, the *FV* amount is the present amount  at 6% compounded quarterly.



 



***Exercise***

A father opened a savings account for his daughter on the day she was born, depositing $1000. Each year on her birthday he deposits another $1000, making the last deposit on her 21st birthday. If the account pays 5.25% interest compounded annually, how much is in the account at the end of the day on his daughter’s 21st birthday? How much interest has been earned?

***Solution***

***Given***: 



***Since you put money at the beginning of each year, we need to add the first payment.***



 



The Total contribution: 

The interest earned: 

***Exercise***

You deposits $10,000 at the beginning of each year for 12 years in an account paying 5% compounded annually. Then you put the total amount on deposit in another account paying 6% compounded semi-annually for another 9 years. Find the final amount on deposit after the entire 21-year period.

***Solution***

***Given***: 





 



Since the last deposit did mature yet when roll over, then:







 



***Exercise***

You need $10,000 in 8 years.

1. What amount should be deposit at the end of each quarter at 8% compounded quarterly so that he will have his $10,000?
2. Find your quarterly deposit if the money is deposited at 6% compounded quarterly.

***Solution***

1. ***Given***: 





 



1. ***Given***: 





 



***Exercise***

You want to have a $20,000 down payment when you buy a car in 6 years. How much money must you deposit at the end of each quarter in an account paying 3.2% compounded quarterly so that you will have the down payment you desire?

***Solution***

***Given***: 





 



***Exercise***

You sell a land and then you will be paid a lump sum of $60,000 in 7 years. Until then, the buyer pays 8% simple interest quarterly.

1. Find the amount of each quarterly interest payment on the $60,000
2. The buyer sets up a sinking fund so that enough money will be present to pay off the $60,000. The buyer will make semiannual payments into the sinking fund; the account pays 6% compounded semiannually. Find the amount of each payment into the fund.

***Solution***

***Given***: 

1. 





1. ***Given***: 





 



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***Pmt #*** | ***Deposit***  ***Amount*** |  | ***Interest***  ***Earned*** |  | ***Balance*** |
| 1 | $3,511.58 | ---- | $0 |  | $3,511.58 |
| 2 | $3,511.58 |  | $105.35 |  | $7,128.51 |
| 3 | $3,511.58 |  | $213.86 |  | $10,853.95 |
| 4 | $3,511.58 |  | $325.62 |  | $14,691.15 |
| 5 | $3,511.58 |  | $440.73 |  | $18,643.46 |
| 6 | $3,511.58 |  | $559.30 |  | $22,714.34 |
| 7 | $3,511.58 |  | $681.43 |  | $26,907.35 |
| 8 | $3,511.58 |  | $807.22 |  | $31,226.15 |
| 9 | $3,511.58 |  | $936.78 |  | $35,674.51 |
| 10 | $3,511.58 |  | $1,070.24 |  | $40,256.33 |
| 11 | $3,511.58 |  | $1,207.69 |  | $44,975.60 |
| 12 | $3,511.58 |  | $1,349.57 |  | $49,843.13 |
| 13 | $3,511.58 |  | $1,495.09 |  | $54,843.13 |
| 14 | $3,511.58 |  | $1,645.29 |  | $60,000.00 |





***Solution Section* 2.3 – Present Value of an Annuity Amortization**

***Exercise***

How much should you deposit in an account paying 8% compounded quarterly in order to receive quarterly payments of $1,000 for the next 4 years?

***Solution***

***Given***: 





 



***Exercise***

You have negotiated a price of $25,200 for a new truck. Now you must choose between 0% financing for 48 months or a $3,000 rebate. If you choose the rebate, you can obtain a loan for the balance at 4.5% compounded monthly for 48 months . Which option should you choose?

***Solution***

0%***financing****:*  ***Given***: 



***Rebate***: ***Given***: 







 



⇒ Rebate is better and you save 525 – 506.24 = $ 18.76 *per month*

Or 18.76 \* 48 = $ 900.48 (over the loan)

***Exercise***

Suppose you have selected a new car to purchase for $19,500. If the car can be financed over a period of 4 years at an annual rate of 6.9% compounded monthly, how much will your monthly payments be? Construct an amortization table for the first 3 months.

***Solution***

* *



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Pmt #*** | ***Pmt Amount*** | ***Interest*** | ***Reduction*** | ***Unpaid Bal.*** |
| 0 | ------------- | ---- | --------- | 19500 |
| 1 | 466.05 | 112.13 | 353.93 | 19146.08 |
| 2 | 466.05 | 110.09 | 355.96 | 18790.12 |
| 3 | 466.05 | 108.04 | 358.01 | 18432.11 |

***Exercise***

Suppose your parents decide to give you $10,000 to be put in a college trust fund that will be paid in equally quarterly installments over a 5 year period. If you deposit the money into an account paying 1.5% per quarter, how much are the quarterly payments (Assume the account will have a zero balance at the end of period.)

***Solution***

***Given***: 



* *

***Exercise***

You finally found your dream home. It sells for $120,000 and can be purchased by paying 10% down and financing the balance at an annual rate of 9.6% compounded monthly.

1. How much are your payments if you pay monthly for 30 years?
2. Determine how much would be paid in interest .
3. Determine the payoff after 100 payments have been made.
4. Change the rate to 8.4% and the time to 15 years and calculate the payment.
5. Determine how much would be paid in interest and compare with the previous interest.

***Solution***

***a***) **

***b***) 

***c***) **

***d***) **

***e)*** 



***Exercise***

Sharon has found the perfect car for her family (anew mini-van) at a price of $24,500. She will receive a $3500 credit toward the purchase by trading in her old Gremlin, and will finance the balance at an annual rate of 4.8% compounded monthly.

1. How much are her payments if she pays monthly for 5 years?
2. How long would it take for her to pay off the car paying an extra $100 per mo., beginning with the first month?

***Solution***

***a***) ******

***b***) ****** ***Divide both sides by*** **494.37**

****** ***multiply by* 0.004**





 ***“ln” both sides***







*Money is compounded monthly; it can’t be compounded at 46.65 months. Bump to 47mo.*

***Exercise***

Marie has determined that she will need $5000 per month in retirement over a 30-year period. She has forecasted that her money will earn 7.2% compounded monthly. Marie will spend 25-years working toward this goal investing monthly at an annual rate of 7.2%. How much should Marie’s monthly payments be during her working years in order to satisfy her retirement needs? *This is a 2-part problem: 1st calculate the PV for retirement. Then use that value as FV for working years.*

***Solution***



***Exercise***

American General offers a 10-year ordinary annuity with a guaranteed rate of 6.65% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $5,000 annually over the 10-year period?

***Solution***

***Given***: 





 



***Exercise***

American General offers a 7-year ordinary annuity with a guaranteed rate of 6.35% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $10,000 annually over the 7-year period?

***Solution***

***Given***: 





 



***Exercise***

You want to purchase an automobile for $27,300. The dealer offers you 0% financing for 60 months or a $5,000 rebate. You can obtain 6.3% financial for 60 months at the local bank. Which option should you choose? Explain.

***Solution***

0%***financing****:*  ***Given***: 



***Rebate***: ***Given***: 





 



⇒ Rebate is better and you save  *per month*

Or  (over the life of the loan)

***Exercise***

You want to purchase an automobile for $28,500. The dealer offers you 0% financing for 60 months or a $6,000 rebate. You can obtain 6.2% financial for 60 months at the local bank. Which option should you choose? Explain.

***Solution***

0%***financing****:*  ***Given***: 



***Rebate***: ***Given***: 









⇒ Rebate is better and you save  *per month*

Or  (over the life of the loan)

***Exercise***

Construct the amortization schedule for a $5,000 debt that is to be amortized in eight equal quarterly payments at 2.8% interest per quarter on the unpaid balance.

***Solution***

***Given***: 



 



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Pmt***  ***#*** | ***Payment*** | ***Interest*** | ***Reduction*** | ***Unpaid Balance*** |
| 0 |  | $0 |  | $5,000.00 |
| 1 | $706.29 |  |  |  |
| 2 | $706.29 |  |  |  |
| 3 | $706.29 |  |  |  |
| 4 | $706.29 |  |  |  |
| 5 | $706.29 |  |  |  |
| 6 | $706.29 |  |  |  |
| 7 | $706.29 |  |  |  |
| 8 | $706.29 |  |  | $0.00 |
| ***Total*** | **$5,650.27** | **$650.27** | **$5,000.00** |  |

***Exercise***

Construct the amortization schedule for a $10,000 debt that is to be amortized in six equal quarterly payments at 2.6% interest per quarter on the unpaid balance.

***Solution***

***Given***: 



 



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Pmt***  ***#*** | ***Payment*** | ***Interest*** | ***Reduction*** | ***Unpaid Balance*** |
| 0 |  | $0 |  | $10,000.00 |
| 1 | $1,821.58 |  |  |  |
| 2 | $1,821.58 |  |  |  |
| 3 | $1,821.58 |  |  |  |
| 4 | $1,821.58 |  |  |  |
| 5 | $1,821.58 |  |  |  |
| 6 | $1,821.58 |  |  | $0.00 |
| ***Total*** | **$10,929.45** | **$929.45** | **$10,000.00** |  |

***Exercise***

A loan of $37,948 with interest at 6.5% compounded annually, to be paid with equal annual payments over 10 years

***Solution***

***Given***: 



 



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Pmt #*** | ***Payment*** | ***Interest*** | ***Reduction*** | ***Unpaid Balance*** |
| 0 |  |  |  | $37,948.00 |
| 1 | $5,278.74 | $2,466.62 | $2,812.12 | $35,135.88 |
| 2 | $5,278.74 | $2,283.83 | $2,994.91 | $32,140.97 |
| 3 | $5,278.74 | $2,089.16 | $3,189.58 | $28,951.40 |
| 4 | $5,278.74 | $1,881.84 | $3,396.90 | $25,554.50 |
| 5 | $5,278.74 | $1,661.04 | $3,617.70 | $21,936.80 |
| 6 | $5,278.74 | $1,425.89 | $3,852.85 | $18,083.95 |
| 7 | $5,278.74 | $1,175.46 | $4,103.28 | $13,980.67 |
| 8 | $5,278.74 | $908.74 | $4,370.00 | $9,610.67 |
| 9 | $5,278.74 | $624.69 | $4,654.05 | $4,956.62 |
| 10 | $5,278.74 | $322.18 | $4,956.62 | $0.00 |
| ***Total*** | **$52,787.40** | **$14,839.40** | **$37,948.00** |  |

***Exercise***

A loan of $4,836 with interest at 7.25% compounded semi-annually, to be repaid in 5 years in equal semi-annual payments.

***Solution***

***Given***: 

******



 



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Pmt #*** | ***Payment*** | ***Interest*** | ***Reduction*** | ***Unpaid Balance*** |
| 0 |  |  |  | $4,836.00 |
| 1 | $585.16 | $175.31 | $409.85 | $4,426.15 |
| 2 | $585.16 | $160.45 | $424.71 | $4,001.43 |
| 3 | $585.16 | $145.05 | $440.11 | $3,561.32 |
| 4 | $585.16 | $129.10 | $456.06 | $3,105.26 |
| 5 | $585.16 | $112.57 | $472.59 | $2,632.67 |
| 6 | $585.16 | $95.43 | $489.73 | $2,142.94 |
| 7 | $585.16 | $77.68 | $507.48 | $1,635.46 |
| 8 | $585.16 | $59.29 | $525.87 | $1,108.59 |
| 9 | $585.16 | $40.22 | $544.94 | $564.65 |
| 10 | $585.16 | $20.47 | $564.65 | $0.00 |
| ***Total*** | **$5,851.60** | **$1,015.60** | **$4,836.00** |  |