***A group of 5 boys and 3 girls is to be photographed.***

* How many ways can they be arranged in one row?

8! = 40320 ways

* How many ways can they be arranged with the girls in the front row and the boys in the back?

Front row = 3! = 6 Back row = 5! = 120

They can be arranged in: 6.120 = 720 ways.

* What if a boy is to sit in the end chairs? (All the chairs in one row)

There exists 5 choices for seating a boy in the left seat, then 4 choices right end.

Left (8-2 =) 6 people may be seated with no restriction:

5.(6! (4)) = 14400 ways b \_ \_ \_ \_ \_ \_ b

5 6 5 4 3 2 1 4

* How many ways can this be done if boys sit side by side & girls side by side?

Boys-Girls or Girls-Boys ⇒ Total numbers = (5! 3!) + (3! 5!) = 1440

* In how many ways can be seated on a bench if only 4 seats are available?

Number of arrangements of 8 people taken 4 at a time = P(8,4) = 8.7.6.5 = 1680

* How many ways can be arranged if the girls occupy the even places? (All the chairs in one row)

The boys can be seated in P(5,5) = 5! = 120 ways, and the girls in P(3,3) = 3! = 6 ways.

Number of arrangements = (120)(6) = 720

* In how many ways 2 particular people must not sit next to each other?

Consider 2 particular people as 1 person. Then there are 7 people altogether and they can be arranged in 6! = 720 ways. But the 2 people can be arranged in 2! ways. Therefore, the number of ways to arranging 8 people with 2 particular people together is = 6! 2! = 1440.

The total number of ways in which 8 people can be seated so that the 2 particular people do not sit together

= (# of 1 seated anywhere) - (# 2 particular people seated together)

= 7! – 1440 = 3600 ways.

* How many possibilities for 2 boys and 3 girls.

C(5,2) C(3,2) = 10 .(3) = 30 possibilities.

* To select 3 how many ways for any mixture of boys and girls C(8,3) =  = 56 ways
* How many to select 3 with majority boys

All boys and no girl: C(5,3).C(3,0) = 10.(1) = 10

2 boys and 1 girl: C(5,2).C(3,1) = 10.(3) = 30

⇒ the # of selecting majority boys = 10 + 30 = 40 ways.

**12 Boys & 10 Girls. 7 are chosen.**

* What is the probability that at least 2 Girls are chosen?

1 – [Pr(no Girls) + Pr(1 G)] = 1 -  ≈ .94

* What is the probability no boys are chosen?Pr(no boys) = Pr (all Girls) = 
* What is the probability that 1st three are boys? Pr = 
* What is the probability that more boys than girls are chosen?

Pr(B > G) =  ≈ .616

***Poker hand consists of 5 cards selected from a deck of 52 cards.***

* How many different poker hands are there?

C(52,5) = 2598960 hands ({1,2,3,4,5} = {5,3,4,2,1})

* How many different poker hands consist entirely of aces and kings?

Number of aces + kings = 8; C(8,5) = 56 hands.

* How many different poker hands consist entirely of clubs? # of clubs = 13, C(13,5) = 1287 hands.
* How many consist of 3 aces and 2 kings? C(4,3).C(4,2) = 4.6 = 24
* How many different poker hands consist entirely of red cards?

# red cards = 26; C(26,5) = 65780 hands.

* How many combinations have cards from exactly 2 suits?

1. Consider one from the 1st suit, then there are C(4,1) = 4, and left 4 for the other suit then there are C(3,1) = 3. Therefore there are 4.C(13,1) . 3C(13,4) = 111540 ways.
2. Consider 2 from the 1st suit, then there are C(4,1) = 4, and left 3 for the other suit then there are C(3,1) = 3. Therefore there are 4.C(13,2) . 3C(13,3) = 267696 ways
3. Total = 111540 + 267696 = 379236 ways

* How many ways all the cards from the same suit?

Select a suit, there are C(4,1) = 4 ways to do this. For each selection of a suit there are C(13,5) = 1287.

Final = 4. C(13,5) = 5148 ways.

* How many ways 3 from one suit and 2 from another?

Select a suit, there are C(4,1) = 4 ways to do this. The other suit is C(3,1) = 3 (since 3 suits left to choose from). First 3 from 1 suit there are 4.C(13,3) = 286 ways, and 2 from another 3.C(13,2) = 78.

Total = 4.C(13,3) . 3C(13,2) = 22308 ways.

* How many ways 2 aces, 2 cards of another denomination, and 1 card of a 3rd denomination.

- For 2 aces = C(4,2) = 6

- 2 cards of another denomination are C(4,2) = 6 ways, there are 12 ways for the 2nd denomination.

Therefore, there are 12.(6) = 72 ways

- 3rd denomination the are 11 ways, 1 card ⇒ 11.C(4,1) = 44

The outcomes: 6.(72).(44) = 19008 hands.

* How many hands are in 2 cards of 1 denomination, 2 cards of another different denomination, and 1 card of a 3rd denomination.

Select 2 cards of 1 denomination = C(13,2) = 78 ways.

Select 2 of one denomination, there are C(4,2) = 6

Then select 2 of the other = C(4,2) = 6

Select the 3rd denomination, there are 11.C(4,1) = 44

# of poker hands = 78.6.6.44 = 123552 hands.

Let the event A: card is a spade, and B is the face card.

- ***What is the probability of A?*** Pr (A={card is a spade}) = 

- ***What is the probability of B?*** Pr (B={face card}) = 

- ***What is the probability of A ∩ B?*** Pr (A ∩ B) = 

\* 13 cards are dealt from a deck of 52 cards

1. ***What is the probability that the ace of spades is one of the 13 cards***?

Pr (ace of spades) = = 1/4

1. ***Suppose 1 of the 13 cards is chosen at random and not found to be the ace spades. What is the probability that none of the 13 cards is the ace of spades***?

Pr(none| 1 is not) = 

1. ***Repeat previous experiment 10 times (replacing the card), and the ace of spades is not seen.***

***What is the probability that the ace of spades actually is one of the 13 cards?***

Pr(aces is one| 10 is not) = 



# *COIN*

- 3 coins to be tossed.

- ***What is the probability at least one head appears***?

Pr({1,2,3}) = Pr(1) + Pr(2) + Pr(3) = 

- ***What is the probability all heads or all tails***?

Pr(all heads or all tails) = Pr(0) + Pr(3) = 

- 1 coin, ***what is the probability that heads is twice as likely to appear as tails***?

Let Pr(T) = p ⇒ Pr(H) = 2p ⇒ p + 2p = 1 ⇒ p = 1/3 Pr(T) = 1/3 ⇒ Pr(H) = 2/3

- 1 coin is tossed 10 times?

1. ***How many different outcomes are possible***? 210 = 1024 outcomes.
2. ***How many different outcomes have exactly 4 heads***? C(10,4) = 210 outcomes
3. ***How many different outcomes at the most 2 heads?*** C(10,0) + C(10,1) + C(10,2) = 56
4. ***How many different outcomes at least 3 heads?***

All outcomes – (at most 2 H) = 1024 – 56 = 968

1. ***What is the probability of obtaining exactly 4 heads?***



***- Tosses 3 times, what is the conditional probability that the outcome is HHH given that at least 2H occurs?***

Pr({HHH}| at least 2H) = 

at least 2 H = {THH,HTH,HHT,HHH}

# *DICE*

**- Toss 2 dice** come up 7 or 11 (you win $7), for any (loose $2).

Determine the player’s mathematical expectation?

36 possible pairs with 2 dice

8 possible for 7 or 11 Pr(7 or 11) =  and Pr (not (7 or11)) = 1- = 

E = 7x + (-2)  = 0

Events E= 1st die is a 3, F = 2nd die is 6. Pr(E) = ? Pr(F) = ? E and F are independent?

Pr(E) = Pr(F) = 1/6 , E ∩ F = {(3,6)} ⇒ Pr(E|F) = 1/6 = Pr(E)

⇒ Pr(F|E) = 1/6 = Pr(F) ∴ E and F are independent

What is the probability that the 2 dice show the same number?

E = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)} Pr(E) = 

What is the probability that the number add up to 8?

Pr(=8) =  E ={(2,6),(6,2),(3,5),(5,3),(4,4)}

What is the probability that the sum is less than 5?

Pr(2) + Pr(3) + Pr(4) = 

**- Toss 1 die**

What is the probability that an odd number will appear?

E = {1,3,5} ⇒ Pr(E) = 3/6 = ½

What is the probability that the result is odd and greater than 4?

F = {5,6} Pr(F) =  ⇒ Pr(E ∪ F) = Pr(E) + Pr(F) – Pr(E ∩ F) = 