***Exercise* 1**

*Ox* and *Oy* are bisector to 2 adjacent acute angles,  and  where the difference is  and . *Oz* is the bisector of the angle . Determine the angle 

***Solution***



























***Exercise* 2**

*Ox* and *Oy* are bisector to 2 adjacent acute angles,  and  where the difference is . *Oz* is the bisector of the angle . Determine the angle 

***Solution***

*Ox* is the bisector  

*OB* is the bisector  

*OM* is the bisector  

*Oz* is the bisector  

*Oy* is the bisector  







































***Exercise* 3**

Four consecutive half-lines (segments): *OA*, *OB*, *OC*, and *OD* formed angles such as

 and 

Calculate the angles to demonstrate that the bisectors of  and  are in a straight line.

***Solution***













Let:

*Ox* is the bisector 

*Oy* is the bisector 









Therefore; the bisectors of  and  are in a straight line

***Exercise* 4**

The segments *OA* and *OB* formed with *OX* the angles *𝛼* and *𝛽*.

1. Demonstrate that the bisector *OC* of the angle  made with *OX* an angle .
2. Examine the cases where
   1. 
   2. 

***Solution***

***Given***:







1. 



1. ***i.*** If , then



Let:  that implies *OC* is the bisector of 

Since *OC* is the bisector of , then





***ii.*** If , then



Let:  that implies *OC* is the bisector of 

Since *OC* is the bisector of , then





***Exercise* 5**

A point *O* takes on an infinite right  be conducted the same side half-lines *OA* and *OB*, as well as the bisectors of angles , , and .

Calculate the angles of the figure such that the bisector of the angle  is perpendicular to  and the bisectors of the extreme angles formed an angle of .

***Solution***

***Given***: 



*OC* is the bisector 



*Oz* is the bisector 



 is the bisector 















***Exercise* 6**

Four consecutive half-lines *OA*, *OB*, *OC*, and *OD* formed four adjacent consecutive angles which are between them like 1, 2, 3, 4.

Calculate the angles and the adjacent consecutive angles formed by their bisectors.

***Solution***















































***Exercise* 7**

A point *P* is on the base *BC* of an isosceles triangle *ABC*. The two points *M* and *N* are the middle points of the segments *PB* and *PC*, respectively, which lead the perpendicular to the base *BC*; these perpendiculars meet *AB* in *E*, *AC* in *F*.

Demonstrate that the angle *EPF* is equal to *A*.

***Solution***



*M* is the middle of the segment *BP* and *EM*  to *BP*, therefore



*N* is the middle of the segment *CP* and *FN*  to *CP*, therefore









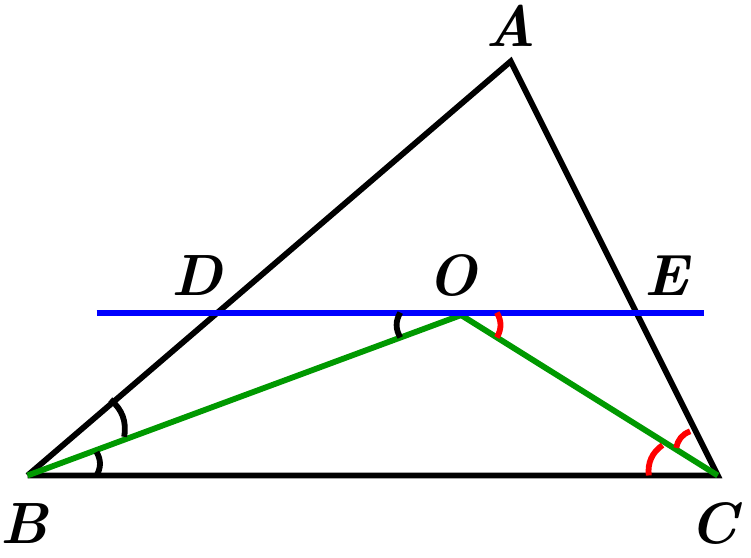
 *√*

***Exercise* 8**

Given the triangle *ABC* and the bisectors *BO* and *CO* of the angles of the base, where the point *O* is the intersection of the 2 bisectors. A line *DOE* passes through the point *O* parallel to base *BC*.

Prove that 

***Solution***

*CO* is the bisector of  





Similar; *BO* is the bisector of  









***Exercise* 9**

A right triangle *ABC* at *A* with a height *AH*. We drop perpendiculars *HE* and *HD* from *H* to sides *AB* and *AC* respectively.

1. Prove that 
2. Prove that *AM* is perpendicular to *DE*, where *M* is the middle point of *BC*.
3. Prove that *MN* (*N* is the middle point of *AB*) and the segment *Bx* (parallel to *DE*) are intersect on *AH*.
4. Prove that *AM* and *HD* are intersect on *Bx*.

***Solution***

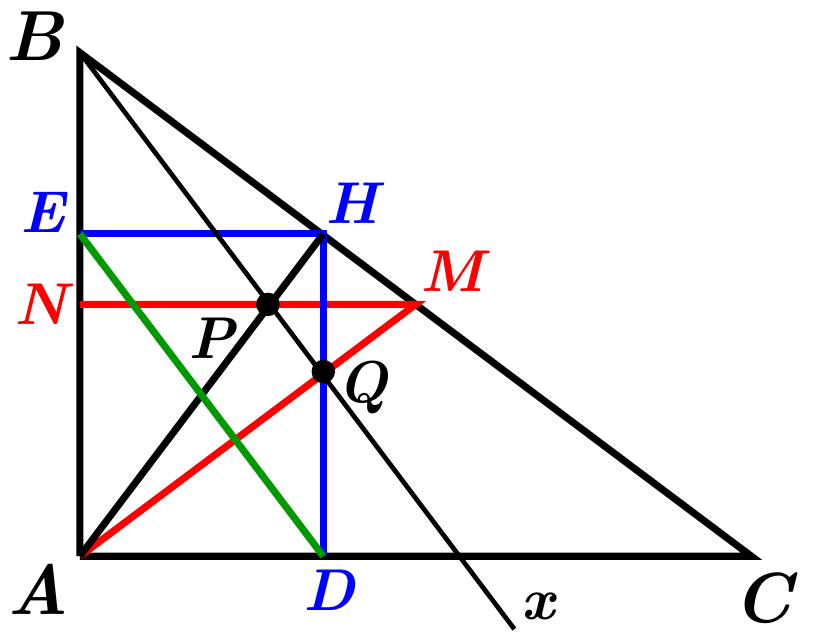
1. The triangles *AEH* and *ADH* are right triangles and angle *A* is right angle.

Then *AEHD* is a rectangle.

Therefore, 

1. A middle point of a hypotenuse of a right triangle is the center of the circle of that triangle.

Therefore, 

That implies to: 

From the rectangle *ADHE*: 











Therefore, *AM* is perpendicular to *DE*.

1. *N* is the middle point of *AB* 

*Bx* parallel to *DE* 

Let point *P* the intersection of B*x* and *AH*. Since, then the triangle *BPA* is an isosceles. *PN* is the perpendicular to *AB* as well *MN*. Which gives us that points *M*, *P*, *N* are on the same line.

Therefore, segment *MN* and *AH* intersect at point *P*.

1. Let Point *Q* be the intersection of *AM* and *Bx*.



Then, the triangles *BHA* and *BQA* are equivalent, therefore with hypotenuse *AB*.

, line *HQ* has to be perpendicular to *AC*.

*AM* and *HD* are intersect on *Bx* at *Q*.

***Exercise* 10**

Given an isosceles triangle *ABC* with a peak at *A*. Extend base *BC* the length , then extend *AB* of a length , at the end draw a line *EHF*, *H* is the middle point of *BC* and *F* is located on *AD*.

1. Prove that 
2. Prove that 
3. Prove that 
4. Calculate the value of the angles  and  where .

***Solution***

1. Triangle *ABC* is isosceles, then 

Since, , then 











1.  *H* the middle point of *BC*







 ***√***

1. 

















 ***√***

1. 











Triangle *AFH* is isosceles then,











***Exercise* 11**

Demonstrate that the heights of a triangle share the angles of triangle that equal to each other.

***Solution***



Consider the 2 right triangles *APB* and *ANC*, which they have the same angle *A*.

Therefore, .

Similar, consider the 2 right triangles *BPC* and *AMC*, which they have the same angle *C*.

Therefore, .

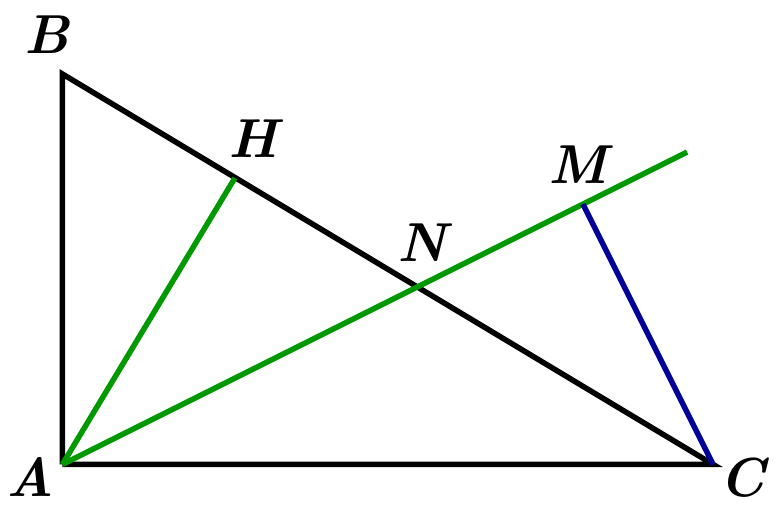
Similar, consider the 2 right triangles *BNC* and *AMB*, which they have the same angle *B*.

Therefore, .

***Exercise* 12**

A right triangle *ABC* at *A* where , drop a perpendicular *AH* from *A* to the hypotenuse *BC* where . From *C* drops a perpendicular *CM* at *AN*. Demonstrate that *BC* is the bisector of the angle.

***Solution***



Consider the 2 right triangles *ABC* and *ABH* with a common angle *B*, then



*Given*: , then 





Consider the 2 right triangles *AHN* and *CMC*, where 

Therefore, 

Since 

Then 

Therefore, *BC* is the bisector of the angle 

***Exercise* 13**

On the sides of an angle that it takes the length *OA* and *OB*, so that  (is given) and construct a parallelogram *OABC*. What is the place of the summit *C* of parallelogram?

***Solution***

Let segment *OE* extension of segment *OA* such that 

Let segment *OF* extension of segment *OB* such that 

Then, the triangle *OEF* is an isosceles.











Therefore, the point *C*, *E*, and *F* are aligned.

***Exercise* 14**

***Solution***