***Solution Section* 1.2 – Gaussian Elimination**

***Exercise***

When elimination is applied to the matrix 

1. What are the first and second pivots?
2. What is the multiplier in the first step ( times row 1 is subtracted from row 2)?
3. What entry in the 2, 2 position (instead of 9) would force an exchange of rows 2 and 3?
4. What is the multiplier , subtracting 0 times row 1 from row 3?

***Solution***

1. The first pivot is 3 and when 2 times row 1 is subtracted from row 2, the second pivot is revealed as 7.



1. The multiplier in the first step is .
2. If we reduce the entry 9 to 2, that drop of 7 in the position would force a row exchange.



1. The multiplier  is already zero because  and no needs row elimination.

***Exercise***

Use elimination to reach upper triangular matrices ***U***. Solve by back substitution or explain why this impossible. What are the pivots (never zero)? Exchange equations when necessary. The only difference is the  in equation (3).

 

***Solution***

For the *first* system:





The solutions are:  and the pivots are 1, -2, -2.

For the *second* system:





The three planes don’t meet. But if we change ‘3’ in the last equation to ‘-5’



 There are unique infinite many solutions!

The three planes now meet along a whole line.

***Exercise***

For which numbers *a* does the elimination break down (1) permanently (2) temporarily



Solve for *x* and *y* after fixing the second breakdown by a row change.

***Solution***

The matrix form is: 

If , the elimination brakes down temporarily.



The system is in upper triangular form and entry row 2 column 2 is not equal to zero, therefore the system has a solution.

If ,











If ,

, the system will fail and has no solution.

If ;

, the system has a unique solution.

***Exercise***

Find the pivots and the solution for these four equations:



***Solution***











The pivots are diagonal entries and the solution is: 

***Exercise***

Look for a matrix that has row sums 4 and 8, and column sums 2 and *s*.



The four equations are solvable only if *s* = \_\_\_\_. Then find two different matrices that have the correct row and column sums.

***Solution***







***Exercise***

Three planes can fail to have an intersection point, even if no planes are parallel. The system is singular if row 3 of *A* is a \_\_\_\_\_\_\_ of the first two rows. Find a third equation that can’t be solved together with  and 

***Solution***

The system is singular if row 3 of *A* is a ***linear combination*** of the first two rows.

There are many possible of a third equation that can’t be solved together with  and .



***Exercise***

Use the Gauss-Jordan method to solve the system 

***Solution***

 

 

 

 

  



S***olution***: ****

***Exercise***

Use the Gauss-Jordan method to solve the system ****

***Solution***

 1 - 2 -

 

 0 1 8 3

 

 0 0 1 -

 



***Solution***: ****

***Exercise***

Use the Gauss-Jordan method to solve the system  ****

***Solution***

 

 

 

 

 

 



***Solution***: ****

***Exercise***

Use the Gauss-Jordan method to solve the system  ****

***Solution***



 



 



 



***Solution***: ****

***Exercise***

Use the Gauss-Jordan method to solve the system 

***Solution***

 



 



let *z* be the variable

From Row 1 ⇒ *y* + 2*z* = 



From Row 1 









***Solution***: 

***Exercise***

Use the Gauss-Jordan method to solve the system 

***Solution***

 1  1 2

  

  



From Row 3: 0 = 0 is a true statement. Let ***z*** be the variable.

From Row 2: *y* − 2*z* = 1



From Row 1: *x* + 2*z* = 



***∴ Solution***: 

***Exercise***

Use the Gauss-Jordan method to solve the system 

***Solution***

  



  



  



***∴ Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

 



 





***∴ Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

















***∴ Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 

















**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

 

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 

















**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 















**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 

















**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  









**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

 

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***



  

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 















**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

 

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***





Since all three equations are the same.

**∴ *Solution***: is the plane 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

 















**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***

  

  

 























**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***



  

  

 



















**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***





 *Interchange* 

















***∴ Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***







***∴ Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***











***∴ Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***









***∴ Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***







**∴ *Solution***: 

***Exercise***

Solve the linear system by Gauss-Jordan elimination.



***Solution***













**∴ *Solution***: 

***Exercise***

Solve the linear system by Gauss-Jordan elimination.



***Solution***













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***











**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***













**∴ *Solution***: 

***Exercise***

Use augmented elimination to solve linear system 

***Solution***











 

The general solution of the system: 

**∴ *Solution***: 

***Exercise***

Add 3 times the second row to the first of 

***Solution***









***Exercise***

For what value(s) of *k*, if any, does the system  have

1. A unique solution?
2. Infinitely many solutions?
3. No solution?

***Solution***









1. Unique solution if 
2. Infinitely solution if 
3. No solution if 

***Exercise***

Choose a coefficient *b* that makes the system singular.



Then choose a right-hand side *g* that makes it solvable.

Find 2 solutions in that singular case.

***Solution***



So, the system is singular if



& 



If 

***Exercise***

This system us not linear, in some sense,



Does the system have a solution?

***Solution***







The system has *no* *solution* since  cannot be equal 2. 