***Solution Section* 1.3 – Matrices and Matrix operations**

***Exercise***

For the matrices:  and , when does 

***Solution***















***Exercise***

Find values for the variables so that the matrices are equal. 

***Solution***





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***Solution***





***Exercise***

Find values for the variables so that the matrices are equal.



***Solution***





***Exercise***

Find a combination  that gives the zero vector:



Those vectors are independent or dependent?

The vectors lie in a \_\_\_\_\_\_.

The matrix W with those columns is not invertible.

***Solution***

; Therefore those vectors are dependent

The vectors lie in a plane.

***Exercise***

The very last words say that the 5 by 5 centered difference matrix is not invertible, Write down the 5 equations . Find a combination of left sides that gives zero. What combination of must be zero?

***Solution***

The 5 by 5 centered difference matrix is



The five equations  are:



Observe that the sum of the first





***Exercise***

A direct graph starts with *n* nodes. There are possible edges, each edge leaves one of the *n* nodes and enters one of the *n* nodes (possibly itself). The *n* by *n* adjacency matrix has  when edge leaves node *i* and enter node *j*; if no edge then . Here are directed graphs and their adjacency matrices:

 

 

The *i*, *j* entry of is .

Why does that sum count the two-step paths from *i* to any node to *j*?

The *i*, *j* entry of  counts *k*-steps paths:



List all 3-step paths between each pair of nodes and compare with . When  has ***no zeros***, that number *k* is the diameter of the graph – the number of edges needed to connect the most pair of nodes. What is the diameter of the second graph?

***Solution***

The number  will be “**1**” if there is an edge from node *i* to *k* and an edge from *k* to *j*.

This is a 2-step path. The number  will be “**0**” if either of those edge (from node *i* to *k* and from *k* to *j*) is missing.

The sum of  is the number of 2-step paths leaving *i* and entering *j*.

Matrix multiplication is right for this count.

The 3-step paths are counted by ; we look at paths to node 2:



The  contain Fibonacci numbers 0, 1, 1, 2, 3, 5, 8, 13, ….

Fibonacci’s rule  show up in 



There are ***13 six-step*** paths from node one to node 1.

***Exercise***

*A* is 3 by 5, B is 5 by 3, *C* is 5 by 1, and *D* is 3 by 1. All entries are 1. Which of these matrix operations are allowed, and what are the results?

1. 
2. 
3. 
4. 
5. 
6. 
7. 

***Solution***

   

1. 



1. 



1. 





1. 
2. 
3. 
4. 

Matrices *B* and *C* are not the same size.

***Exercise***

What rows or columns or matrices do you multiply to find.

1. The third column of *AB*?
2. The second column of *AB*?
3. The first row of *AB*?
4. The second row of *AB*?
5. The entry in row 3, column 4 of *AB*?
6. The entry in row 2, column 3 of *AB*?

***Solution***

1. *A* (column 3 of *B*)
2. *A* (column 2 of *B*)
3. (Row 1 of *A*) *B*
4. (Row 2 of *A*) *B*
5. (Row 3 of *A*) (Column 4 of *B*)
6. (Row 2 of *A*) (Column 3 of *B*)

***Exercise***

Add *AB* to *AC* and compare with :



***Solution***

















***Exercise***

True or False

1. If  is defined then *A* is necessarily square.
2. If  and  are defined then *A* and *B* are square.
3. If  and  are defined then and  are square.
4. If , then 

***Solution***

1. True
2. False, if *A* has an order *m* by *n* and *B* *n* by *m*: 
3. True; 
4. False, if *B* is the matrix of all zeros.

***Exercise***

*a*) Find a nonzero matrix *A* such that 

*b*) Find a matrix that has  but 

***Solution***

1. A nonzero matrix *A* such that 



1. A matrix that has  but 













***Exercise***

Suppose you solve  for three special right sides *b*:



If the three solutions  are the columns of a matrix *X*, what is *A* times *X*?

***Solution***



Therefore, 

***Exercise***

Show that  is different from , when



Write down the correct rule for 

***Solution***



































***Exercise***

Find the product of the 2 matrices by rows or by columns: 

***Solution***

By rows: 



By columns: 



***Exercise***

Find the product of the 2 matrices by rows or by columns: 

***Solution***

By rows: 



By columns: 



***Exercise***

Find the product of the 2 matrices by rows or by columns: 

***Solution***

By rows: 





By columns: 



***Exercise***

Find the product of the 2 matrices by rows or by columns: 

***Solution***

By rows: 



By columns: 



***Exercise***

Given   Find 

***Solution***













***Exercise***

Given. Find *AB* and *BA*.

***Solution***







***Exercise***

Given  . Find *AB* and *BA*.

***Solution***









***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Given   Find 

***Solution***













***Exercise***

Given   Find 

***Solution***

***Undefined***





***Exercise***

Given   Find 

***Solution***







***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Given . Find *AB* and *BA*.

***Solution***









***Exercise***

Consider the matrices

    

Compute the following (where possible):

***a***)  ***b***)  ***c***)  ***d***)  ***e***)  ***f***) 

***Solution***

1. 



1. 



1. 



1. 



1. Since *B* and *C* are not the same size



1. 





***Exercise***

Consider the matrices

   

Compute the following (where possible):

***a***)  ***b***)  ***c***)  ***d***)  ***e***)  ***f***) 

***g***)  ***h***)  ***i***)  ***j***)  ***k***) 

***Solution***

1. Since *A* and *B* are not the same size, then



1. 



1. 

, since the inner are not equal.

1. 





1. 





1. 

, since the inner are not equal.

1. 

, since the inner are not equal.

1. 

, since the inner are not equal.

1. , since *A* is not square matrix.
2. 



1. 



***Exercise***

Let , show that 

***Solution***







 ***√***

***Exercise***

Let , show that 

***Solution***



 ***√***

Let assume  is true

We need to also prove that it is true for 







 ***√***

***Exercise***

Let . Prove that  if 

***Solution***

Using the principle of mathematical induction.

For  ***√***  is true

Assume that  is true, 

We need to prove that :



 is also *true*.







***√***  is also true

∴ By mathematical induction, the proof of  is completed.

***Exercise***

Let . Prove that  if 

***Solution***

Using the principle of mathematical induction.

For  ***√***  is true

Assume that  is true, 

We need to prove that :

 is also *true*.









***√***  is also true

∴ By mathematical induction, the proof of  is completed.

***Exercise***

The following system of recurrence relations holds for all 



Solve the system for  and  in terms of  and 

***Solution***















 





Since, when  that implies  (from previous prove).





∴ 

***Exercise***

If , prove that 

***Solution***













 ***√***

***Exercise***

If , use the fact  and mathematical induction, to prove that



***Solution***













Using mathematical induction model

For  

 ***√*** is true for 

Assume is true for  

We need to prove that is also true for  





 











 ***√*** is also true for 

By mathematical induction, the proof that  is completed.

***Exercise***

A sequence of numbers  satisfies the recurrence relation  for , where *a* and *b* are constants. Prove that



Where  and hence express  in terms of .

If  and , use the previous question to find a formula for  in terms  and 

***Solution***





















 

















