***Section* 1.6 – Determinants and Properties**

The determinant is a number that contains information about matrix. It is used to find formulas for inverse matrices, pivots, and solutions.

 has inverse 

Determinant of the matrix  is written  and is define as



The determinant is zero when the matrix has no inverse.

**Properties of the Determinants**

By using these property rules, we can compute the determinant of any square matrix.

1. ***Determinant of the n by n identity matrix is* 1.**



1. ***Determinant changes sign when 2 rows are exchanged.***



1. ***Determinant is a linear function of each row separately***.

Multiply row 1 by any number *t*: 

Add row 1 of *A* to row 1 of *A* ′: 

* ***For 2 by 2 determinants, if you expand to a rectangle, the determinants equal areas.***
* ***For n-dimensional, the determinants equal volumes.***

1. ***If 2 rows of A are equal, then***.



1. ***Subtracting a multiple of one row from another row leaves***  ***unchanged.***



1. ***A matrix with a row of zeros has*** .



1. ***If A is triangular then***  ***= product of diagonal entries.***



1. ***If A is singular then*** **.**
2. ***If A is invertible then*** **.**
3. ***The determinant of AB***  ***is times*** ***:*** 
4. ***The transpose*** ***has the same determinant as A:***  

* 

***Big Formula* for Determinants (Diagonal)**

***Determinant Using Diagonal Method***



 

 

*Determinant***: *D* = (1) + (2)**



***Example***

Evaluate: 

***Solution***







**Determinant by *Cofactors***



***Minor***

For a square matrix, the minor . Of an element  is the ***determinant*** of the matrix formed by deleting the *ith* row and the *j*th column of *A*.

***Example***

Let  Find 

***Solution***







***Theorem***

The determinant is the dot product of any row ***i*** of ***A*** with its cofactors:

Cofactor Formula: 





***Example***

Find the determinant of the matrix:



***Solution***





= −8(−30 – (−21)) – 0 + 6(−12 − 6)

= −8(−9) + 6(−18)



* By the property of determinants, If ***A*** is triangular then  = product of diagonal entries.

***Example***





***Theorem***

Let ***A*** be any *n* by *n* matrix.

1. If ***A*′** is the matrix that results when a single row of ***A*** is multiplied by a constant ***k***, then .
2. If ***A*′** is the matrix that results when two rows of ***A*** are interchanged, then 
3. If ***A*′** is the matrix that results when a multiple of one row of ***A*** is added to another row then 

***Example***

Evaluate 

***Solution***

 ***Interchanged* 1s*t and* 2*nd row***

 ***A common factor of* 3 *from the first row* (*no need*)**











***Exercises Section* 1.6 – Determinants and Properties**

1. Verify that  when: 
2. For which value(s) of ***k*** does *A* fail to be invertible? 
3. Without directly evaluating, show that 
4. If the entries in every row of *A* add to zero, solve ***Ax*** = 0 to prove. If those entries add to one, show that . Does this mean ?
5. Does  in general?
6. True or false if ***A*** and ***B*** are square *n* x *n* matrices?
7. True or false if ***A*** is *m* x *n* and B is *n* x *m* with ?
8. True or false, with a reason if true or a counterexample if false:
9. The determinant of  is .
10. The determinant of ABC is .
11. The determinant of 4*A* is 
12. The determinant of *AB – BA* is zero. (try an example)
13. If *A* is not invertible then *AB* is not invertible.
14. The determinant of *A – B* equals to .
15. Use row operations to show the 3 by 3 “Vandermonde determinant” is



1. The inverse of a 2 by 2 matrix seems to have determinant = 1:



What is wrong with this calculation? What is the correct 

1. A *Hessenberg* matrix is a triangular matrix with one extra diagonal. Use cofactors of row 1 to show that the 4 by 4 determinant satisfies Fibonacci’s rule . The same rule will continue for all sizes . Which Fibonacci number is ?



(**10 − 44**) Evaluate

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | |  | |
|  | |  | |  | |

1. Find all the values of λ for which 



1. Prove that if a square matrix ***A*** has a column of zeros, then 
2. With 2 by 2 blocks in 4 by 4 matrices, you cannot always use block determinants:



1. Why is the first statement true? Somehow *B* doesn’t enter.
2. Show by example that equality fails (as shown) when *C* enters.
3. Show by example that the answer  is also wrong.
4. Show that the value of the following determinant is independent of *θ*.



1. Show that the matrices 

commute if and only if 

1. Show that  for every  matrix *A*.
2. What is the maximum number of zeros that a  matrix can have without a zero determinant? Explain your reasoning.
3. Evaluate *,* , and . Then verify that 



1. Show that  is not invertible for any values of *α, β, γ*
2. The determinant of a  matrix  is .

Assuming no rows swaps are required, perform elimination on *A* and show explicitly that  is the product of the pivots.

1. If *A* is a  matrix and let . What is ?
2. Explain without computations why the following determinant is equal to zero



1. Let *A* be  real matrix.
2. Show that if  and *n* is odd, then .
3. Show that if , then *n* must be even.
4. Does part (*b*) remain true for complex matrices?
5. Let *A* and *C* be  and  matrices, respectively.
6. Show that 
7. Evaluate
8. 
9. 
10. 
11. 
12. Find a formula for 
13. Let  and let



1. Show that, if  ,



1. Show that, if  ,



Where  means  with factor  missing.

1. Use part (*b*) to evaluate



1. Let *A, B, C, D* 
2. Show that when *A* is invertible: 
3. Show that when : 
4. Can *B* and *C* on the right-hand side of the identity be switched?
5. Does part (*b*) remain true if the condition is dropped?