***Section* 2.2 – Norm, Dot product, and distance in **

***Norm* of a Vector**

The ***length*** (or ***norm***) of a vector  is the square root of 



 **2-*dimension***

 **3-*dimension***

***Definition***

If  is a vector in , then the norm of  (also called the length of ****** or the magnitude of ) is denoted by , and is defined by the formula



***Example***

Find the length of the vector 

***Solution***





***Theorem***

If  is a vector in , and if ***k*** is any scalar, then:

1. 
2. 
3. 

**Unit Vectors**

***Definition***

A ***unit vector*** is a vector whose length equals to one. Then 

Divide any nonzero vector  by its length. Then  is a unit vector in the same direction as .

***Example***

Find the unit vector  that has the same direction as  = (2, 2, −1)

***Solution***

















 ***√***

***Example of unit vectors***



In 



In general, these formulas can be defined as ***standard unit vector*** in 







***Example***



***Distance*** **in** 

In  

In  

***Definition***

If  and  are points in , then we denote the distance between u and v by  and define it to be





***Dot Product***

If  and  are nonzero vectors in  or , and if *θ* is he angle between ****** and , then the ***dot product*** (also called the ***Euclidean inner product***) of  and  is denoted by  and is defined as



***Cosine Formula***

If  and  are nonzero vectors that implies



***Example***

Find the dot product of the vectors  and  and have an angle of 45°.

***Solution***

















***Component Form of the Dot Product***

The ***dot product*** or ***inner product*** of  and  is the number



***Example***

Find the dot product of  and 

***Solution***





* ***For dot products, zero means that the 2 vectors are perpendicular*** (= 90°).

***Example***

Put a weight of 4 at the point  and weight of 2 at the point . The *x*-axis will balance on the center point .

***Solution***

The weight balance is  (*dot product*).

In 3-dimensionals the dot product:



***Theorem***

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 

**Right Angles**

The dot product is  when  is perpendicular to 

***Proof***

Perpendicular vectors: 

Let 





 







If  and *U* are unit vectors, then 

Certainly,







***Schwarz Inequality***

If  and  are any vectors 

***Proof***

The dot product of  and  is  and both lengths are .

Then, the Schwarz inequality says that: 







This proves the Schwarz inequality:





***Theorem* − Parallelogram Equation for Vectors**

If  and  are vectors in , then



***Proof***









***Theorem***

If  and  are vectors in  with the Euclidean Inner product, then



***Exercises Section* 2.2 – Norm, Dot product, and distance in *Rn***

1. If  and , what are the smallest and largest possible values of  and ?
2. If  and , what are the smallest and largest possible values of  and ?
3. Given that  and . Similarly, and . The angle  is . Substitute into the trigonometry formula  for  to find 
4. Can three vectors in the *xy* plane have ,  and ?
5. Find the norm of, a unit vector that has the same direction as , and a unit vector that is oppositely directed.
6. = (4, −3)
7. = (1, −1, 2)
8. = (−2, 3, 3, −1)
9. Evaluate the given expression with ****** = (2, −2, 3),  = (1, −3, 4), and  = (3, 6, −4)

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1. Let ***v*** = (1, 1, 2, −3, 1). Find all scalars *k* such that 
2. Find 
3. = (3, 1, 4), = (2, 2, −4)
4. = (1, 1, 4, 6), = (2, −2, 3, −2)
5. = (2, −1, 1, 0, −2), = (1, 2, 2, 2, 1)
6. Find the Euclidean distance between and ******, then find the angle between them
7. = (3, 3, 3),  = (1, 0, 4)
8. = (1, 2, −3, 0),  = (5, 1, 2, −2)
9. = (0, 1, 1, 1, 2),  = (2, 1, 0, −1, 3)
10. Find a unit vector that has the same direction as the given vector

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| --- | --- | --- |
| 1. (−4, −3) |  | 1. (1, 2, 3, 4, 5) |

1. Find a unit vector that is oppositely to the given vector

|  |  |  |
| --- | --- | --- |
| 1. (−12, −5) | 1. (3, −3, 3) |  |

1. Verify that the Cauchy-Schwarz inequality holds
2.  = (−3, 1, 0),  = (2, −1, 3)
3.  = (0, 2, 2, 1),  = (1, 1, 1, 1)
4.  = (1, 3, 5, 2, 0, 1),  = (0, 2, 4, 1, 3, 5)
5. Find  and then the angle  *θ* between  and ****** 
6. Find the norm: ,  for 
7. Find all numbers *r* such that: 
8. Find the distance between and 
9. Given  = (1, −5, 4), ****** = (3, 3, 3)
10. Find 
11. Find the cosine of the angle *θ* between  and ******.
12. Let . Find 
13. Let . Find 
14. Let . Find 
15. Let  and  . Calculate the following:

|  |  |  |  |  |  |
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1. Let  and  . Calculate the following:

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1. Let  and  . Calculate the following:

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1. Let ,  and  . Calculate the following:

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1. Let ,  and  . Calculate the following:

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1. Let ,  and  . Calculate the following:

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1. Suppose , , and  are vectors in  such that , , and . If possible, calculate the following values:

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1. You are in an airplane flying from Chicago to Boston for a job interview. The compass in the cockpit of the plane shows that your plane is pointed due East, and the airspeed indicator on the plane shows that the plane is traveling through the air at 400 *mph*. there is a crosswind that affects your plane however, and the crosswind is blowing due South at 40 *mph*.

Given the crosswind you wonder; relative to the ground, in what direction are you really flying and how fast are you really traveling?

1. A jet airliner, flying due east at 500 *mph* in still air, encounters a 70-*mph* tailwind blowing in the direction 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What speed and direction should the jetliner have in order for the resultant vector to be 500 *mph* due east?
2. A jet airliner, flying due east at 500 *mph* in still air, encounters a 70-*mph* tailwind blowing in the direction 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?
3. A bird flies from its nest 5 *km* in the direction 60° north east, where it stops to rest on a tree. It then flies 10 *km* in the direction due southeast and lands atop a telephone pole. Place an *xy*-coordinate system so that the origin is the bird’s nest, the *x*-axis points east, and the *y*-axis points north.
4. At what point is the tree located?
5. At what point is the telephone pole?
6. Prove 
7. Prove, for any vectors and  in  and any scalars *c* and *d*, 
8. Prove 
9. Prove Minkowski theorem: 