***Solution Section* 2.3 – Orthogonality**

***Exercise***

Determine whether  and  are orthogonal

1. 
2. 
3. 
4. 

***Solution***

1. 





∴  and  are not orthogonal

1. 



∴  and  are orthogonal

1. 



∴  and  are orthogonal

1. 



∴  and  are orthogonal

***Exercise***

Determine whether the vectors form an orthogonal set

1. 
2. 
3. 
4. 
5. 
6. 
7. 

***Solution***

1. 



∴ Vectors don’t form an orthogonal set

1. 



∴ Vectors don’t form an orthogonal set

1. 



∴ These vectors are not orthogonal

1. ******



∴ These vectors are orthogonal

1. 











∴ Vectors form an orthogonal set

1. 



∴ Vectors don’t form an orthogonal set

1. 











∴ Vectors form an orthogonal set

***Exercise***

Find a unit vector that is orthogonal to both  = (1, 0, 1) and  = (0, 1, 1)

***Solution***

Let  be the unit vector that is orthogonal to both  and .















The orthogonal vector to both  and  is , therefore the unit vector is







The possible vectors are: 

***Exercise***

*a*) Show that  = (*a, b*) and  = (−*b, a*) are orthogonal vectors.

*b*) Use the result to find two vectors that are orthogonal to  = (2, −3).

*c*) Find two unit vectors that are orthogonal to (−3, 4)

***Solution***

1. 





 and  are orthogonal vectors.

1. (2, 3) and (−2, 3).
2. 







***Exercise***

Find the vector component of  along  and the vector component of  orthogonal to

|  |  |
| --- | --- |
|  |  |

***Solution***

1. 











1.  









1.  









1.  









1.  









1.  









***Exercise***

Project the vector  onto the line through , check that is perpendicular to :

1. 
2. 
3. 

***Solution***

1.  

















 is perpendicular to 

1.  















 is perpendicular to 

1.  

















 is perpendicular to 

***Exercise***

Find the projection matrix  onto the line through 

***Solution***









***Exercise***

Draw the projection of  onto  and also compute it from 



***Solution***











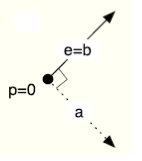




***Exercise***

Draw the projection of  onto  and also compute it from 



***Solution***















***Exercise***

Show that if  is orthogonal to both and , then  is orthogonal to for all scalars and .

***Solution***



 ***If***  ***is orthogonal to ***& 







***Exercise***

1. Project the vector  = (3, 4, 4) onto the line through  = (2, 2, 1) and then onto the plane that also contains .
2. Check that the first error vector  is perpendicular to  , and the second error vector  is also perpendicular to .

***Solution***

1. 







The plane contains the vectors  and  is the column space of ***A***.

















1. The error vector:













Therefore,  is perpendicular to 







The error vector:













Therefore,  is perpendicular to 

***Exercise***

Compute the projection matrices  onto the lines through  and . Multiply those projection matrices and explain why their product  is what it is. Project  onto the lines , , and also onto .Add up the three projections .

***Solution***

For 













For 



















This because are perpendicular.

For 











































The reason is that  is perpendicular to .

Hence, when you compute the three projections of a vector and add them up you get back to the vector you start with.

***Exercise***

If  show that . When *P* projects onto the column space of *A*, *I – P* projects onto the \_\_\_\_.

***Solution***







  ***By given definition***









When *P* projects onto the column space of *A*, then *I – P* projects onto the left nullspace.

Because ; if *P* is in the column space of *A*, then  is a vector perpendicular to *C*(*A*).

***Exercise***

What linear combination of  and  is closest to ?

***Solution***





So, this  is actually in the span of the two given vectors.

***Exercise***

Show that  is orthogonal to  if and only if 

***Solution***

Suppose that  is orthogonal to . Then











So .

Therefore, .

Suppose . Then













So, we can see that  is orthogonal to 

We conclude that  is orthogonal to  if and only if , as desired.

***Exercise***

Given 

1. Find 
2. Find  and then the angle *θ* between  and .

***Solution***

1. 









1. 







***Exercise***

Given: 

1. Compute the projection  of  on 
2. Find  and show that  is perpendicular to .

***Solution***

1. 













1. 









 is perpendicular to  .

***Exercise***

1. Show that  = (*a, b*) and  = (*−b, a*) are orthogonal vectors
2. Use the result in part (*a*) to find two vectors that are orthogonal to  = (2, −3)
3. Find two unit vectors that are orthogonal to (−3, 4)

***Solution***

1. 



The 2 vectors are orthogonal vectors.

1.  = (2, −3)

 = (−3, −2) and  = (3, 2)

1. (−3, 4)







***Exercise***

Show that *A*(3, 0, 2), .*B*(4, 3, 0), and *C*(8, 1, −1) are vertices of a right triangle. At which vertex is the right angle?

***Solution***













The right triangle at point *B*

***Exercise***

Establish the identity: 

***Solution***

Let 























Therefore;  is true.

**2nd *method*:**











***Exercise***

Find the Euclidean inner product : 

***Solution***





***Exercise***

Find the Euclidean distance between  and : 

***Solution***













***Exercise***

Find for 

1. 
2. The cosine of the angle between  and 
3. The scalar component of  in the direction of 
4. The vector 

***Solution***

1. 



















1. 





1. 



1. 







***Exercise***

Find for 

1. 
2. The cosine of the angle between  and 
3. The scalar component of  in the direction of 
4. The vector 

***Solution***

1. 















1. 





1. 



1. 





***Exercise***

Find for 

1. 
2. The cosine of the angle between  and 
3. The scalar component of  in the direction of 
4. The vector 

***Solution***

1. 

















1. 





1. 



1. 





***Exercise***

Find for 

1. 
2. The cosine of the angle between  and 
3. The scalar component of  in the direction of 
4. The vector 

***Solution***

1. 











1. 





1. 



1. 



***Exercise***

Find for 

1. 
2. The cosine of the angle between  and 
3. The scalar component of  in the direction of 
4. The vector 

***Solution***

1. 

















1. 







1. 





1. 





***Exercise***

Suppose Ted weighs 180 *lb*. and he is sitting on an inclined plane that drops 3 *units* for every 4 horizontal units. The gravitational force vector is .

1. Find the force pushing Ted down the slope.
2. Find the force acting to hold Ted against the slope

***Solution***

A vector parallel to the slope of the inclined plane is .

1. The vector of the force acting to push Ted down the slope is









The magnitude of the force pushing Ted down the slope is







1. The vector of the force acting to hold Ted against the slope is













***Exercise***

Prove that is two vectors  and  in  are orthogonal to nonzero vector  in , then  and  are scalar multiples of each other.

***Solution***

Since  is orthogonal to  

 is orthogonal to  



There exist  such that 





Therefore,  and  are scalar multiples of each other