***Lecture Three***

***Section* 3.1 – Inner Products**

***Definition***

An ***inner product*** on a real vector space *V* is a function that associates a real number  with each pair of vectors in *V* in such a way that the following axioms are satisfies for all vectors , and  in *V* and all scalars *k*.

1.  ***Symmetry axiom***
2.  ***Additivity axiom***
3.  ***Homogeneity axiom***
4.  and  iff  ***Positivity axiom***

A real vector space with an inner product is called a ***real inner product space***.



This is called the ***Euclidean inner product*** (or the ***standard*** ***inner product***)

***Definition***

If *V* is a real inner product space, then the norm (or length) of a vector  in *V* is denoted by  and is defined by



And the ***distance*** between two vectors is denoted by  and is defined by



A vector of norm 1 is called a ***unit vector***.

***Theorem***

If ***u*** and ***v*** are vectors in a real inner product space *V*, and if *k* is a scalar, then:

1.  with equality *iff* 
2. 
3. 
4.  with equality *iff*  

Although the Euclidean inner product is the most important inner product on , there are various applications in which is desirable to modify it by weighing each term differently. More precisely, if

 are positive real numbers, which we will call weighs, and if  and are vectors in , then it can be shown that the formula



Defines an inner product on  that we call the ***weighted Euclidean inner product*** with weights 

***Example***

Let  and  be vectors in , verify that the weighted Euclidean inner product  satisfies the four inner product axioms.

***Solution***

*Axiom* 1: 





*Axiom* 2: 









*Axiom* 3: 

**



*Axiom* 4: 

**



***Exercises Section* 3.1 – Inner Products**

1. Let  be the Euclidean inner product on , and let , , , and . Compute the following.
2. 
3. 
4. 
5. 
6. 
7. 
8. Let  be the Euclidean inner product on , and let , , and . Compute the following for the weighted Euclidean inner product  .
9. 
10. 
11. 
12. 
13. 
14. 
15. Let  be the Euclidean inner product on , and let , , , and . Verify the following.

|  |  |
| --- | --- |
|  |  |

1. Let  be the Euclidean inner product on , and let , , , and . Verify the following for the weighted Euclidean inner product

|  |  |
| --- | --- |
|  |  |

1. Let  and . Show that the following are inner product on  by verifying that the inner product axioms hold. 
2. Show that the following identity holds for the vectors in any inner product space



1. Show that the following identity holds for the vectors in any inner product space



1. Prove that 